Investigating student understanding of a heat engine: a case study of a Stirling engine

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Abstract
We report on the study of student difficulties regarding a heat engine in the context of a Stirling cycle by the method of measurement. An in-class test about a Stirling engine with a regenerator was taken by three classes, and the students were asked to perform one of the most basic activities—calculate the efficiency of the heat engine. Our data indicate that quite a few students have not developed a robust conceptual understanding of basic engineering knowledge of the heat engine. Notably, the error ratio of the class given a simple tutorial of engineering knowledge is smaller than those of the other two classes by more than 20%. In addition, both the written answers and post-test interviews show that most of the students cannot associate Carnot’s theorem with a Stirling cycle. Our results suggest that both scientific and engineering knowledge are important and should be included in instructional approaches, especially in the thermodynamics course taught in the countries and regions with a tradition of not paying much attention to experimental education or engineering training.

Keywords: heat engine, Stirling cycle, efficiency, regenerator, conceptual understanding

1. Introduction
Thermodynamics is taught across several science and engineering disciplines due to its relevance in several different domains. Understanding what students know about thermodynamical concepts is one important aspect of providing the pedagogical content knowledge needed to teach thermodynamics well, not only in physics, but also in applied disciplines. In thermodynamics, instructions usually emphasise ideal and simplified models, such as ideal gas and Carnot engine. These models are used to demonstrate the fundamental principles and laws of thermodynamics.
The related central topic in thermodynamics is the second law of thermodynamics. It is the key element of the introductory curriculum for undergraduate students in a wide variety of science and engineering fields. Some studies have focused on the student ideas and learning difficulties related to certain aspects of the second law of thermodynamics [1–3]. In the conventional contexts, one of the major applications of the second law of thermodynamics is the heat engine, which transforms heat partly into work or mechanical energy. Several works on various approaches are central to teaching and learning the subtle and complex nature of heat engines and their applications [4–9], but little research on student understanding of heat engines has been reported [3, 10].

A heat engine operates in a cycle. Figure 1 shows the pressure vs volume ($P$–$V$) diagrams of four classical cycles: (a) Carnot cycle (two adiabatic and two isothermal processes), (b) Stirling cycle (two isochoric and two isothermal processes), (c) Otto cycle (two adiabatic and two isochoric processes) and (d) Diesel cycle (one isochoric, one isobaric and two adiabatic processes). Each cycle has four distinct processes. Among them, both the Carnot cycle [4] and Stirling cycle [5–7] are used in external combustion engines, while an Otto cycle [8] and Diesel cycle [9] are used in internal combustion engines. Actually, besides the four cycles, there are other cycles describing the operation of certain heat engines, such as a Bartyon cycle, which consists of two isobaric and two adiabatic processes [11]. All engines absorb heat from a reservoir at a relatively high temperature, perform some mechanical work, and reject some heat at a lower temperature. To accomplish this, a heat engine needs three things: a high temperature thermal reservoir, a low temperature thermal reservoir and a working substance. Interestingly, all the thermodynamics textbooks describe a Carnot cycle in great detail, but many of them just introduce the other three cycles briefly [12] and some of them even do not cover them at all. Coincidentally, or perhaps correspondingly, a number of studies have shed light on student understanding of basic concepts of a Carnot cycle [3, 10, 13–15], but very few studies have focused on the topics related to the Stirling cycle, Otto cycle or Diesel cycle [16–19]. In fact, the Stirling engine was one of the earliest heat engines put to practical use and is still used in various facilities such as submarines and concentrated solar power systems [6, 20]. Other basic cycles such as the Otto cycle and Diesel cycle were proposed later in the 19th century and have been used in the engines of various vehicles such as automobiles and many other types of machinery since then. In this sense, evaluating student understanding of basic knowledge of different kinds of heat engines, for instance, the Carnot engine and Stirling engine, is important and necessary for guiding instruction and curriculum development. It is worth noting that both the Carnot cycle and Stirling cycle are reversible cycles working between two heat reservoirs, which means Carnot’s theorem is applicable to both of them.

Among the various educational evaluation techniques, post-testing is the most widely adopted method in physics education research (PER) [21–23]. PER focuses on understanding how students learn physics at all levels and developing strategies to help students learn physics more effectively [24, 25]. Based on these facts, we designed an in-class test about the Stirling cycle at the end of the course and then performed this study without being taught about a Stirling cycle before. The nature and prevalence of the errors made by the students can motivate us to probe more deeply into their understanding of heat engines.
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The objectives of this study are as follows. First of all, according to Carnot’s theorem, every reversible heat engine between a pair of heat reservoirs, such as a Carnot engine and Stirling engine, is equally efficient, regardless of the working substance or the operation details. In fact, this conclusion has been demonstrated very clearly in the proving procedure of Carnot’s theorem in most of the textbooks. This means that, a student can use Carnot’s theorem to directly get the efficiency of a Stirling engine, no matter what complication a Stirling cycle may add with respect to the Carnot cycle. However, it is possible that some students would take it for granted that the heat engine here only refers to a Carnot engine if other types of heat engines are just introduced briefly or even not taught. In this sense, our study can estimate to what extent the students understand Carnot’s theorem. Secondly, it is noted that in the last decade or so, a growing number of researchers have shown interest in student understanding of thermodynamics beyond the theoretical level, leading to upper-division engineering instruction on heat engines [26–28]. Interestingly, Stirling engine with a regenerator in this study is such an exquisite design that the more the students work on it, the more the students will be stimulated to be curious in thermodynamics and be interested in integrating theory with practice.

This paper is organised as follows: in the next section, we first introduce the Stirling cycle. In section 3, we describe the overall the research context including the student population and the methodology used. Then we give an overview of student performance and discuss the student difficulties identified as well as implications for instruction in section 4. Finally, a summary is given in section 5.

2. Stirling cycle

A Stirling cycle is similar to a Carnot cycle, except that the two adiabatic processes are replaced by two isochoric processes. In practice, a Stirling heat engine includes a cylinder containing two opposed pistons and a regenerator between the pistons. The regenerator serves as a thermodynamic sponge that alternatively absorbs and releases heat. The cylinder is divided by the regenerator into two parts: the volume between the regenerator and a piston maintained at high temperature $T_{\text{max}}$ is called the expansion space, while the other, maintained at low temperature $T_{\text{min}}$ the compression space, as illustrated in figure 2(b), which shows the piston arrangement at the terminal points of the Stirling cycle. The efficiency of a heat engine is defined as

$$\eta = \frac{Q_1 - Q_2}{Q_1},$$

with $Q_1$ and $Q_2$ being, respectively, the quantities of heat transferred from the hot and cold reservoirs during one cycle, as shown in figure 1. It should be pointed out that the heat transferred per cycle in the engine should not be contained.

The question on a Stirling cycle, shown in figure 2, was developed to assess student understanding of the efficiency calculation of heat engine. The working material is assumed to be an ideal diatomic gas for the calculation of the cycle efficiency. It should be emphasised that during the cycle, the heat transferred in the isochoric processes $2 \rightarrow 3$ and $4 \rightarrow 1$ is contained in the regenerator. In this sense, the regenerator adds an additional layer of complexity. After having a clear picture of the function of regenerator, the students can do the calculation by the definition of heat efficiency equation (1). Furthermore, for Stirling cycle, the heat supply and heat rejection at constant temperature satisfies the requirement of the second law of thermodynamics for maximum thermal efficiency, so that the efficiency of the Stirling cycle is the same as the Carnot cycle,

$$\eta = \frac{T_{\text{max}} - T_{\text{min}}}{T_{\text{max}}}.$$
similar elusive entities in the teaching and learning processes. The quantitative methods can be categorised in different ways. In [29], the quantitative methods that are commonly used for empirical PER studies are classified into three genres: measurement [30], controlled exploration of relations [31, 32] and data mining [33, 34]. Each genre has its own ontological assumptions, epistemological commitments and methodological implications. Measurement is one important genre of quantitative methods, which is to generate numerical values for a certain construct of interest. For example, the investigators can assign grades or create scores to reflect student understanding of a specific physics concept. In this work, we adopt the method of measurement to measure students’ conceptual understanding of heat engine.

This investigation was carried out with three classes of physics undergraduates enrolled in the Thermodynamics and Statistical Mechanics course at Sichuan University (SCU) in 2019, which is a renowned four-year public research university in China. This course is typically taken during the autumn semester of the junior year and usually lasts 17 weeks, which meets for four 45 min periods each week. The students were randomly assigned to different classes and given course credit for participating in in-class tests during the semester and a final exam at the end. The sizes of the classes varied from 14 to 48 students. The three classes were taught by different instructors, but they used the same Chinese textbook, Thermodynamics and Statistical Physics written by Zhi-Cheng Wang. Generally speaking, the content of this textbook is very similar to Parts I and II of the English textbook, Thermodynamics and Statistical Mechanics, edited by Greiner et al [35]. The lectures, homework assignments and in-class tests were presented in traditional formats. Furthermore, the same practice was also required for the three classes to enhance the understanding of Carnot’s theorem by calculating the efficiencies of other heat engines, such as Brayton cycle and Otto cycle, while the Stirling cycle was not mentioned in class. The only nontraditional aspect was the addition of voluntary tutorial given by one of the authors (Xiang). The tutorial is a general introduction of the past, the present and the future of engineering. For instance, one early example of engineering is the water control and irrigation dam called Dujiangyan Dam in Chengdu, China, which was designed by Li Bing about 2200 years ago. Starting in the 19th century, great engineering advances were made in all areas, such as the inventions of the internal-combustion engine, typewriter and telephone. In the 20th century, even greater engineering advances were made, such as the invention of the refrigerator, car, computer,
Internet and cell phone. Importantly, the engineering knowledge related to Thermodynamics was emphasised, such as how the refrigerator is scientifically and mechanically related to the Carnot cycle and how the engineering improvement of internal combustion engines have deeply changed our society and our daily life. Finally, the future of engineering was envisioned. In the future, we need more engineering advances to develop efficient and renewable forms of energy and help to improve our health with safer environments and better medical treatments. To achieve these goals, we need not only the advance of science and technology, but also the opportunity for individuals to learn, play and work in creative and interesting ways. All these contents and related engineering knowledge were introduced briefly. This tutorial is new and unusual to the participants, especially when one recognises that traditional college courses of physics in China do not attach much importance to experimental education or engineering training. It is noted that neither the structure of the Stirling engine, nor the calculation of the efficiency of the Stirling cycle was covered in the tutorial. Therefore, the preliminary analysis showed no significant differences in answering the questions of what a Stirling cycle is and how it works between tutorial participants and nonparticipants.

It should be emphasised that there is only one question in the in-class test, which has been presented in figure 2. Our work focuses on the student results for this particular question on the efficiency of Stirling cycle. There were 101 students taking the in-class test. They were about 21 years old. These students completed the test individually in quiet rooms and were cut off after 30 min. Any answer that satisfies a few key criteria was considered as a correct one. These criteria include correct calculations of heat rejection in the isothermal process $1 \rightarrow 2$, heat supply in the isothermal process $3 \rightarrow 4$ and efficiency $\eta$ of the cycle, or clear expressions of using Carnot’s Theorem to obtain correct efficiency for the Stirling cycle. Therefore, the Stirling cycle not only gives us an opportunity to estimate student understanding of general knowledge about the heat, work and efficiency involved in a heat engine, but also provides novelty to the students which can be used to explore their understanding of Carnot’s theorem. The students have learned that all reversible heat engines between two heat reservoirs have the same efficiency, regardless of other details. Unfortunately, our study shows that most of the students can only associate Carnot’s theorem with a Carnot cycle, but not with a Stirling cycle.

In addition to the collection and analysis of the written answers, post-test interviews were also carried out with the students. About 40% of them (40 students) were randomly selected to take the interviews. Those students come from three classes and gave the correct answer to the in-class test or not. Each interview lasted an average of about 10 min. During the interviews, the students were required to talk aloud, and the interviewers provided minimal interventions only for reminding the students to keep talking or asking them to clarify explanations not understood by the interviewers. Obviously, after having a clear picture of the student errors from the written answers, the researchers can flexibly conduct the interviews. Besides being constrained to cover all the questions about a Stirling cycle, the interviews also included the most commonly discussed or typical questions that are related to the heat engines, such as the second law of thermodynamics and Carnot’s theorem, since we are particularly interested in the connections students do or do not make between Carnot’s theorem, reversibility and Stirling cycle as dictated by the second law of thermodynamics. Some typical interviews will be presented later. In this way, the interviews can provide us with more valuable information for student understanding on heat engines from a different perspective.

4. Results and analysis

The data from the in-class test on Stirling cycle were collected and analysed at SCU in 2019 to investigate student performance. Two independent researchers assigned the grades and tallied instances of specific errors. The responses produced by the students were graded using a three-level scale, where ‘A’ is entirely correct, ‘B’ is mostly correct with minor miscalculation, and ‘C’ has multiple errors, especially misunderstanding of the function of regenerator. Figure 3 establishes
an overview of the student performance. Each pie corresponds to one class. Class 1, 2 and 3 were instructed by three instructors, Wu, Zhu and Xiang, respectively. The three grades ‘A’, ‘B’ and ‘C’ are represented by different colours in figure 3. Firstly, the blue parts present the percentages of students who gave a correct response with a correct explanation for the Stirling engine question in figure 2. Surprisingly, 46% of the students in Class 3 got ‘A’, while only 23% in Class 1 and 14% in Class 2 were entirely correct. Secondly, more than half of students in Class 1 and 2 got ‘C’ labelled by red. It should be noticed that the C-level student number in Class 3 is almost 20% lower than those in the other two classes. The results reveal that quite a few students would have few intuitive ideas about a Stirling engine even after all the relevant instruction. Generally speaking, the student performance is not satisfying. Therefore, it is absolutely necessary to figure out what leads to this bad performance. In the following, we will discuss the test results in terms of qualitative and quantitative description.

### 4.1. Qualitative description of student difficulties

To better understand student thinking behind their written answers, interviews were conducted, which involved an undergraduate student and an interviewer each time. It was found that the most common error made by the students was the misunderstanding of the function of a regenerator. As a result, the heat transferred in the two isochoric processes \(2 \rightarrow 3\) and \(4 \rightarrow 1\) was included when calculating the efficiency of a Stirling cycle. Some typical interviews are listed as follows.

**Interview 1**

I: Can you please tell me why you consider the heat supply in the process \(2 \rightarrow 3\) and heat rejection in the process \(4 \rightarrow 1\) when calculating the efficiency of Stirling cycle?

S1: That’s just how I learned. The heat supply and heat rejection of all processes should be included according to the formula of heat engine’s efficiency.

I: You mean the heat transferred in the engine should also be considered?

S1: Yes.

I: How about the regenerator? You did not consider it during the calculation.

S1: Um… I am not sure. Maybe it is designed for improving the efficiency.

**Interview 2**

I: Can you please explain why you only include the heat supply and the heat rejection in the two isothermal processes \(1 \rightarrow 2\) and \(3 \rightarrow 4\) when calculating the efficiency?

S2: Because in this way, I can get the efficiency of \((T_1 - T_2)/T_1\).

I: You mean Stirling engine has the same efficiency as Carnot engine?

S2: Yes.
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I: Okay, why?
S2: Um… I am not sure.
I: Let me ask in a slightly different way. Where do the two isochoric processes \(2 \rightarrow 3\) and \(4 \rightarrow 1\) happen?
S2: In the engine?
I: So you think the heat transferred for the whole cycle is independent of the regenerator?
S2: Yes. Actually, I think it is just for the practical engineering requirement.

Both S1 and S2 (and indeed most of the interviewed students) did clearly know that the efficiency of heat engine is defined as equation (1). This appeared to be extremely stable in that students never questioned this, even when confronted with other issues. In other words, the heat transferred per cycle in the engine should not be contained. But most students misunderstood the physical meaning of \(Q_1\) and \(Q_2\). S1 was firmer in the belief that all heat transferred during the whole cycle should be considered. Actually, S1 did not understand the physical meaning of efficiency of heat engine. S2 was trying to speak that all reversible engines have the same efficiency, but was unable to properly verbalise it. Even though Carnot’s theorem is fundamental in the Thermodynamics course, S2 was not familiar with it. Therefore, S2 also may not understand that only the heat transferred between the heat engine and hot (cold) reservoir rather than the total heat transferred during the whole cycle should be considered when calculating the efficiency of heat engine.

These interviews were fairly representative of the students as a whole. Most of the students were not able to describe how the regenerator works, indicating a lack of understanding of the relevant engineering concepts of heat engine. As indicated by the interviews, the students in general had a more varied mastery of the function of regenerator. In some cases, the students only treated the regenerator as a required engineering component without knowing its function. This appears to stem from the students’ lack of engineering knowledge of heat engines. In the Thermodynamics course, students majoring in physics usually only learn the theoretical knowledge of heat engines. Relevant engineering aspects are not introduced in the classes, unless the instructor places emphasis on them. This assertion is supported by students’ preformation in the test. Only three students gave a correct description of the regenerator, who were from the same class. Their instructor (Xiang) did give them some general engineering introduction of heat engine.

4.2. Quantitative description of student difficulties

The answers given by the students were analysed case by case. The amount of incorrect answers indicate that after standard instruction many students still have serious difficulties in understanding heat engines. Conceptual, reasoning and mathematical difficulties are intertwined and cannot be completely separated. The nature and prevalence of the errors motivate us to probe more deeply into their understanding of heat engines. Our analysis of the results will guide the design of curriculum to address these specific difficulties. In the following analysis, the errors made by students are mainly divided into four categories: (1) miscalculation, (2) misusing formulas, (3) misunderstanding cycle process, and (4) misunderstanding regenerator. More detailed explanation for the errors will be given later.

To get a clear picture of the student performance, we present the percentages of occurrence of the four errors for the three classes lectured by Wu (Class 1), Zhu (Class 2) and Xiang (Class 3) in figure 4. The number of students taking the test on the Stirling cycle in the three classes were 39, 14 and 48, respectively. We can see that less than 20% of students in the three classes made the two errors of miscalculation and misusing formulas shown in figure 4(a). It indicates that most of the students adequately understood the definition of the efficiency of heat engine and knew how to do the calculation with the formulas in the textbooks, since the three instructors illustrated the heat engine carefully. Especially, for the error of miscalculation, no student made this error in Class 2 and Class 3. The percentages of the other two errors of misunderstanding the cycle process and misunderstanding regenerator are given in figure 4(b). The percentages of misunderstanding the cycle process for the three classes are not high, varying between 15.4% (Class 1)
and 2.1% (Class 3). It means that most of the students have comprehended basic processes of a heat engine. But the situation is totally different for the error of misunderstanding a regenerator. For each class, misunderstanding the regenerator is the most common error (above 50%). The result demonstrates that most of the students do not know the function of regenerator, which is probably related to the fact the students had never been taught about Stirling cycle before and learned the basic knowledge of the regenerator for the first time from the in-class test problem. This is due to the pedagogical techniques we used during the teaching. It should be pointed out that for Class 1 and 2, more than 70% of students misunderstood the function of a regenerator, which is much larger (>10%) than that of Class 3. Similarly, the students in Class 1 and class 2 who misunderstood cycle process were more (>5%) than those in Class 3. We will further discuss this later.

Since miscalculation and misusing formulas were largely related to the student ability of understanding thermodynamics theory and performing mathematical calculation, they were classified as science-related errors, shown by the empty black squares in figure 4(a). And since misunderstanding cycle process and misunderstanding regenerator were basically related to the students’ ability to understand the mechanical processes in a Stirling cycle, they were classified as engineering-related errors, given by the full black squares in figure 4(b). In fact, we found that many students who misunderstood the cycle process also misunderstood the regenerator, which makes sense since both the errors were caused by the poor comprehensive ability of engineering aspects of Stirling engine. The results of the quantitative statistics of the errors are listed in table 1. It is remarkable that more students got the correct answer in Class 3 (above 40%) than the other two classes (~20%). This could be because the instructor Xiang gave the students an engineering tutorial of the heat engine.

The good news is that the science-related error ratios in the three classes were low, which was 10.2%, 14.3% and 8.3% for Class 1, 2 and 3, respectively. The roughly 10% error ratio in all the classes shows that the importance of the basic thermodynamics theories was recognised both in the teaching and learning aspects very well and at approximately same the level. This is actually a tradition in the teaching and learning process of most of the physics courses in China.

The bad news is that the engineering-related error ratios were extremely high, which were above 50% for all the classes. Detailed analysis
show that among the engineering-related errors, the error of misunderstanding regenerator dominates. For instance, in Class 3, 52.1% participants made engineering-related errors, 50% participants made the error of misunderstanding regenerator, and 2.1% participant made the error of misunderstanding cycle process, respectively. The participants who misunderstood regenerator tended to count in the heat absorbed and/or released by the regenerator and obtained the wrong answer for the efficiency. The participant who misunderstood cycle process thought the heat engine did nonzero work during the isovolumic process. Similar results were found in Class 1 and 2.

Although the engineering-related errors were high for the three classes, there was a big difference between Class 3 and the other two classes: the engineering ratio of Class 3 was smaller by approximately 20% than those of the other two. As we mentioned before, the only difference between the lectures of the three classes was that Class 3 was given a tutorial by instructor Xiang, which did not cover anything related to a Stirling cycle but did introduce general knowledge such as mechanical counterparts of refrigerator corresponding to the reverse Carnot cycle and the meaning of the engineering improvement of heat engines to our society. The tutorial was very preliminary, but it could help some of the students realise that engineering is as important as science in terms of applications of the thermodynamics and really inspire their interest in the engineering aspects of the heat engines. To some extent, the difference between not giving any tutorials and even just giving a preliminary tutorial is like the different between ‘0’ and ‘1’ in computer science. Our results demonstrated how the preliminary tutorial emphasising engineering aspects of basic knowledges can influence the teaching results.

Another interesting finding is that we can actually use the quantitative results to investigate how the students understand Carnot’s theorem. Very few students, for instance, two students in Class 3 and one student in Class 1, made errors in the solution process, but got the right answer for the efficiency of the Stirling engine. The written answer sheets or the interviews showed that they used Carnot’s theorem to obtain the answer directly after they got stuck in analysing the cycle processes and performing efficiency calculations. But their way of usage of Carnot’s theorem was kind of passive instead of active, i.e. the students did not realise to use Carnot’s theorem until they failed in other ways. In other words, most of the students only correlate Carnot’s theorem with Carnot cycle and not with other reversible cycles such as Stirling cycle. The very poor performance on Carnot’s theorem was somewhat unexpected, as explicit instruction and practice was involved in all classes. Our speculation is that this is because for a long time traditional Thermodynamics textbooks only emphasise the Carnot
cycle and neglect other important cycles. In this sense, it is necessary to emphasise the importance of other cycles besides Carnot cycle. Our results suggest that the Stirling cycle could be an important supplement to Carnot cycle.

5. Summary
This paper describes an in-depth investigation of student understanding of heat engine in the context of Stirling engine. Our findings indicate that the majority of students in the three classes studied could comprehend the scientific theory of a heat engine reasonably well and make the correct mathematical calculations. However, above 50% students for all the classes did not understand the basic engineering knowledge of Stirling cycle, especially the function of the regenerator. Importantly, the engineering-related error ratio of the class that was given a tutorial of engineering knowledge of heat engine was smaller by about 20% than those of the other two. The results suggest that it is useful to include both scientific and engineering aspects of basic knowledge in instructional approaches, especially in the course of Thermodynamics taught in the countries and regions with a tradition of not paying much attention to experimental teaching or engineering training. In addition, findings from this investigation also suggest that it is necessary to attach more importance to other cycles besides the Carnot cycle in the Thermodynamics textbooks to improve student understanding of Carnot’s theorem and related knowledge of heat engines.

Data availability statement
The data that support the findings of this study are available upon reasonable request from the authors.

Acknowledgments
The authors thank the support from New Century Education Reform Project of Sichuan University (Grant No. SCU8147) and College Teaching Research Project of Thermodynamics in China (Grant No. JZW-21-RX-04).

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Received 13 August 2021, in final form 30 September 2021
Accepted for publication 28 October 2021
https://doi.org/10.1088/1361-6552/ac342b

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