How to take fractional-order derivative experimentally?

D D Stupin\textsuperscript{1,}, A I Lihachev\textsuperscript{1,2,}, and A V Nashchekin\textsuperscript{1,2}

\textsuperscript{1}St. Petersburg Academic University, Khlopina 8/3, 194021 St. Petersburg, Russia
\textsuperscript{2}Ioffe Institute, Russian Academy of sciences, 194021 St. Petersburg, Russia

E-mail: Stu87@ya.ru, Stupin@spbau.ru

Abstract. In this study we demonstrate a simple and compact implementation of the fractional-order differentiator. At the heart of the proposed approach lies the "universal power law" of the capacitance dispersion of the interface between nanoporous TiN micro-electrode and NaCl 0.9% solution. Using of this phenomenon we take experimentally 0.68- and 0.77-order derivatives of various functions.

1. Introduction
The great interest in nowadays material science is connected with unusual properties of the micro- and nanoscale structures. Such properties as quantum-size effects \cite{1, 2}, inverse population in heterojunctions \cite{1}, negative differential resistance \cite{3}, negative refraction \cite{4}, photons band gap in photonic crystals \cite{5, 6} and many others are hot topics in modern electronics and photonics. In this paper we use unique behavior of the electrical signal on the interface between the nanoporous micro-electrode and electrolyte for creating analogue device, which realize fractional-order derivation (FOD) \cite{7, 8, 9} – a progressive tool for nowadays materials modeling, signal processing, automatic control, and bioelectrical measurements \cite{10}.

2. Working principle
The interface between nanoporous metal electrode and electrolyte in general could be considered as constant phase element (CPE) \cite{11, 12, 13, 14}, which admittance $Y$ is described by "universal power law" of capacitance dispersion \cite{12, 15}. Namely, the current response $J$ through CPE depend on excitation voltage (EV) $V$ as

$$J(t) \doteq J(\omega) = YV(\omega) = Q(i\omega)^\alpha V(\omega) \doteq Q\frac{d^\alpha}{dt^\alpha}V(t), \quad (1)$$

where $\omega$ is frequency, $\alpha$ – nonideality parameter, $Q$ is multiplier with $\Omega^{-1}\text{Hz}^{-\alpha}$ dimension, $\doteq$ means Fourier transformation. Thus the current through CPE element is proportional to FOD of the excitation voltage. This phenomenon is used in this study for analogue fractional differentiator creating.

3. Materials and methods
As electrochemical cell we have used multielectrode array MEA 200/30 ITO (Multichannel Systems, Germany), which was filled with 1 ml 0.9% NaCl solution (Biolot, Russia).
Figure 1. Micro-electrodes characterization and differentiator’s properties. (a) Electrodes micro-photography; (b) electrodes morphology; (c) electrodes elemental composition obtained by X-ray microanalysis; (d) differentiator’s electronic scheme; (e) and (f) – admittance spectra for different EV amplitudes.
Optical photograph Fig. 1(a) of the MEA 200/30 ITO electrodes was made on IMN100 microscope (Vistec Semiconductors Systems GmbH, Germany). Electrodes surface characterization and X-ray microanalysis were performed using scanning electron microscope JSM 7001F (JEOL, Japan) [Fig. 1(b,c)].

The proposed differentiator scheme is based on operational amplifier AD8606 (Analog Devices, USA) connected into current-to-voltage converter circuit [Fig. 1(d)]. The output voltage of this scheme is equal to \(-JR\), where \(R = 100 \text{ k}\Omega\) is feedback resistor, \(J\) is current through electrode/electrolyte interface with admittance \(Y\). Current \(J\) shown in last section is proportional to fractional-order derivative of EV (Eq. 1) that’s why scheme presented on Fig. 1(d) realizes fraction-derivation.

For producing EV was used AKIP-3413/3 generator (AKIP, Russia), which has build-in waveforms library. The EV and output voltage of the differentiator were recorded by analogue-to-digital converter (ADC) L-Card E20-10 (L-Card, Russia, 4 channels). Hardware setup and the MATLAB code nelm which were described in ref. [16] were used for admittance measurement and for it’s complex non-linear least square fitting [17] by Eq. 1. The measurement frequency range was 2 Hz ÷ 1 kHz.

4. Results and discussion
The data presented on Fig. 1(a-c) indicate that electrodes of multi-electrode array have developed surface and mainly consist of TiN, oxygen and indium. Thus they are a classical systems where "universal power law" of the capacitance dispersion should be observed [11, 14] that confirmed by presented on Fig. 1(e,f) admittance measurement results.

Obtained by proposed differentiator fraction-order derivatives are shown on Fig. 2. These results indicate FOD qualitatively corresponds to Heaviside’s FOD definition [7]. It should be noticed, that FOD of the step function is not identically equal to zero and FOD of the exponent is not equal to exponent. The FOD of the parabolic function is asymmetric, because the excitation voltage is equal to zero at negative time. It is worth to emphasize that FOD of the cardiac-shape function does not dramatically different from the original cardiac shape function.
Figure 2. Experimentally obtained fractional orders derivatives (red lines, $J$) for various functions (black lines, $V$). The order 0.68 is for 15-mV EV, order 0.77 is for 27.5-mV EV.
5. Conclusion
Here we have proposed a simple implementation of the fractional-order differentiator and experimentally confirmed by using it unusual properties of the FOD, namely non-zero value of the FOD of constant, non-exponential shape of the FOD of the exponent, and non-linear shape of FOD of parabolic function. Developed device is low cost and could be assembled in micro-scale that is important for portable applications, e.g. biosensors. The FOD obtained by proposed differentiator is found to be in perfect qualitatively agreement with Heavisides FOD definition. Finally, the observed little distortion of the cardiac-shape function after fraction-derivation opens a direct way of the current-domain measurements usage in cardiology, that could be simply realized in comparison with voltage-domain.

We believe, that results of our study will be useful in the area of the modern audio signal processing, as sample for fractional calculus, in the bioelectrical experiments and biosensorics.

Acknowledgments
The authors acknowledges SPbOPEN Organizing Committee for holding wonderful conference, and Sergei V. Koniakhin, Nikolay A. Verlov, Anton S. Bukatin, and Michael V. Dubina for multifaceted assistance and support. SEM characterization were performed using equipment owned by the Federal Joint Research Center "Material science and characterization in advanced technology" with financial support by Ministry of Education and Science of the Russian Federation (id RFMEFI62117X0018). This study was funded by Ministry of Education and Science of Russian Federation, governmental order 16.9790.2017/BCh.

References
[1] Alferov Z I 2001 Rev. Mod. Phys. 73(3) 767–782
[2] Ivchenko E L and Pikus G 2012 Superlattices and other heterostructures: symmetry and optical phenomena vol 110 (Springer Science & Business Media)
[3] Léonard F m c and Tersoff J 2000 Phys. Rev. Lett. 85(22) 4767–4770
[4] Veselago V and Narimanov E 2006 Nature materials 5 759
[5] Yablonovitch E, Gmitter T and Leung K 1991 Physical review letters 67 2295
[6] Joannopoulos J D, Johnson S G, Winn J N and Meade R D 2011 Photonic crystals: molding the flow of light (Princeton university press)
[7] Kenneth S Miller B R 1993 An introduction to the fractional calculus and fractional differential equations 1st ed (Wiley)
[8] Charef A 2006 IEE Proceedings-Control Theory and Applications 153 714–720
[9] Sheng H, Sun H, Coopmans C, Chen Y and Bohannan G 2011 The European Physical Journal Special Topics 193 93–104
[10] Krishna B 2011 Signal Processing 91 386–426
[11] Kerner Z and Pajkossy T 2000 Electrochimica Acta 46 207–211
[12] De Levie R 1965 Electrochimica Acta 10 113–130
[13] Singh M B and Kant R 2013 Journal of Electroanalytical Chemistry 704 197–207
[14] Pajkossy T and Nyikos L 1990 Physical Review B 42 709
[15] Martin M and Lasia A 2011 Electrochimica Acta 56 8058–8068
[16] Stupin D D, Koniakhin S V, Verlov N A and Dubina M V 2017 Phys. Rev. Applied 7(5) 054024
[17] Macdonald J R and Garber J 1977 Journal of the Electrochemical Society 124 1022–1030