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Protocol for generation of high-dimensional entanglement from an array of non-interacting photon emitters

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Abstract

Encoding high-dimensional quantum information into single photons can provide a variety of benefits for quantum technologies, such as improved noise resilience. However, the efficient generation of on-demand, high-dimensional entanglement was thought to be out of reach for current and near-future photonic quantum technologies. We present a protocol for the near-deterministic generation of $N$-photon, $d$-dimensional photonic Greenberger–Horne–Zeilinger (GHZ) states using an array of $d$ non-interacting single-photon emitters. We analyse the impact on performance of common sources of error for quantum emitters, such as photon spectral distinguishability and temporal mismatch, and find they are readily correctable with time-resolved detection to yield high fidelity GHZ states of multiple qudits. When applied to a quantum key distribution scenario, our protocol exhibits improved loss tolerance and key rates when increasing the dimensionality beyond binary encodings.

Encoding quantum information into single photons is a promising route to realising a variety of quantum technologies. Most quantum information processing focuses on two-level systems—qubits—but for single photons it is possible to encode and process systems of arbitrary dimension—qudits. This is exploited for high-dimensional quantum communication [1–4], where moving beyond a binary encoding offers improved bandwidth and noise robustness [5, 6]. Furthermore, qudits may enable fault-tolerant quantum computation with significantly lower error thresholds [7].

Universal single-qudit gates for photons are possible using efficient interferometer decompositions [8, 9] and have been demonstrated with high fidelity [10]. However, generating entanglement between photonic qudits is a challenge for current technologies. Recent work on qudit teleportation introduced probabilistic, linear optical Bell state measurements [11, 12], but ancillary resources increase and success probabilities decrease as the qudit dimension increases. Some solutions to this involve the creation of highly entangled resource states, such as GHZ states, which can be fused to create states which are useful for quantum computation [13, 14] and communication [15]. Computation with four-dimensional entangled states has been considered [16], and even qudit Bell pairs (two photon GHZ states) have been shown to offer advantages in quantum communication over qudit states [17]. All-optical schemes for heralded qudit GHZ state generation were recently shown to be possible [18], but their low success probabilities, which decrease exponentially with qudit dimension and number of photons, require large overheads to make the entanglement generation near-deterministic. Experimental progress has been made in creating postselected high-dimensional, multiphoton entanglement [19], including three-photon, three-dimensional GHZ states [20]. However these postselected schemes are limited in the types of entanglement they can generate [21],
and because the entanglement is generated upon detection of the photons, they cannot be used for scalable quantum computation.

In this work, we propose a scheme for the near-deterministic generation of entangled states of photonic qudits. To produce an $N$-photon, $d$-dimensional GHZ state, we require $d$ quantum emitters, each comprising a spin capable of state-dependent photon emission. Importantly, our scheme does not require any spin–spin gates. It takes inspiration from the photon machine gun protocol [22] that uses the coherence of a spin-1/2 system with a light–matter interface to deterministically emit entangled photons. That protocol has been demonstrated in quantum dot systems [23, 24] and there is progress towards a realisation of a spin-1/2 system with a light–matter interface to deterministically emit entangled photons. That interfaces that can lead to deterioration of operation or state fidelity [26]. We investigate how spin dephasing, photon distinguishability, loss, and threshold detection affect the generated photonic qudit entanglement, and show that the requirement of indistinguishable photons can be lifted using time-resolved dephasing, photon distinguishability, loss, and threshold detection affect the generated photonic qudit entanglement, bringing the feasibility of this scheme towards near-term implementation.

Figure 1 shows a platform-agnostic schematic of the idealised setup for generating photonic qudit GHZ states. Figure 1(d) outlines the required qubit control sequence for each of the three stages of the state preparation—I. W state preparation in the emitter array, II. Sequential photon emission, III. Disentangling qubit measurements. We consider the matter system to have three states in a $\Lambda$-configuration with two stable long-lived ground states, only one of which is coupled to a higher level via a radiative transition (see figure 1(a)). In the following, the dark and bright ground states correspond to the logical matter qubit states, denoted by $|0\rangle$ and $|1\rangle$ respectively, and the excited state is $|e\rangle$. This level structure enables photon emission conditioned on the state of the spin qubit; application of a $\pi$-pulse and subsequent radiative decay enacts the transformations $|0\rangle|\Omega\rangle \rightarrow |0\rangle|\Omega\rangle$ and $|1\rangle|\Omega\rangle \rightarrow \hat{a}|1\rangle|\Omega\rangle = |1\rangle|1\rangle$, where $|\Omega\rangle$ denotes vacuum of the electromagnetic field and $\hat{a}$ is a photon creation operator. This is an effective CNOT gate between the spin qubit and a photonic qubit in the single rail encoding, provided the photonic qubit is initialised in a $\sqrt{d}$-dimensional qudit, of $d$-dimensional qudits. (d) Steps of the protocol with simplified control pulse sequence on each emitter, with qubit state control in blue, excitation $\pi$-pulses in green, and qubit measurements in purple.

![Figure 1](image-url)

**Figure 1.** (a) Level structure for the matter qubit, depicted as an emitter coupled to a cavity. $d$ of these are initialised in state $|p\rangle$. (b) The emitters are $\pi$-pulsed on one transition and emission is routed to a discrete Fourier transform (DFT) interferometer. Detection of one photon (red circle) heralds and projects the emitter array into an entangled $W$ state. This step is repeated until success, at which point (c) emission is switched away from the DFT interferometer. Sequential $\pi$-pulsing of the emitters then results in emission of a train of $N$ photons, each entangled with one another and the emitters. Each photon (or qudit) is shown here as an ellipse distributed over $d$ waveguides. Emitters are disentangled via $X$-basis measurements, yielding an $N$ photon qudit GHZ state, of $d$-dimensional qudits. (d) Steps of the protocol with simplified control pulse sequence on each emitter, with qubit state control in blue, excitation $\pi$-pulses in green, and qubit measurements in purple.
\(|s_j\rangle\) is the state in which the jth qubit is \(|1\rangle\) and all others are \(|0\rangle\). Pulses are repeated until a successful detection event. Note that any detection event will collapse the state of the emitter array, so detection of more than one photon requires re-initialisation of the emitters and is a protocol failure.

Further \(\pi\)-pulsing of the emitter array, prepared in \(|W_d\rangle\), results in emission of a single photon distributed over all \(d\) spatial modes (figure 1(c)). Labelling the state of a single photon in mode \(j\) as \(|j\rangle\), this can be interpreted as the superposition over all the basis states of a \(d\)-dimensional qudit, entangled with the array of matter qubits, \(d^{-1/2}\sum_{j=0}^{d-1}|s_j\rangle|j\rangle\). Repeating this process \(N\) times gives a sequence of \(N\) time-bin encoded photonic qudits entangled with the matter qubits. After disentangling the emitters by measurement in the \(X\)-basis, the photons are left in a qudit GHZ state

\[
|\Psi_{\text{GHZ}}\rangle_d^N = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} (-1)^{m_j} |j\rangle^{\otimes N}. \tag{3}
\]

\(m_j \in \{0, 1\}\) is the measurement outcome for the \(j\)th emitter. Any phases incurred due to the particular mode that heralded the \(W\) state, and the emitter measurement pattern, can be corrected either by applying phases to individual modes of the output, or by applying rotation gates to the spins. Here and in the following, we have assumed that the detection was at the 0th detector. The time-bin encoding of the protocol may be deterministically converted to any other mode encoding, e.g. to a spatial mode encoding using fast switching and delay lines.

**Distinguishable photons**—a notable difference between this protocol and other machine gun-inspired protocols is the use of multiple emitters, placing additional challenges on the simultaneous control of multiple systems. In practice, non-identical emitters and spectral distinguishability of the emitted photons can degrade the which-path erasure in the DFT, introducing errors that would propagate to the resultant GHZ state. However, interference between different colour photons can be recovered using detectors that resolve inside the photon wavepacket [27, 28]. In our scheme, this technique, combined with corrective single-qubit rotations, mitigates spectral distinguishability and enables entanglement generation between quantum emitters. Denoting the spectrum of the photon from the \(j\)th emitter as \(\Phi_j(\omega)\), the creation operators for mode \(j\) can be re-written \(\hat{a}_j^\dagger \rightarrow \int d\omega \Phi_j(\omega) \hat{a}_j^\dagger(\omega)\), and the analysis from the ideal case can be repeated. Detection of a single photon at precisely time \(t_0\) in mode \(j\) transforms the state via \(\hat{\rho} \rightarrow \hat{E}_j^+(t_0)\hat{\rho}\hat{E}_j^-(t_0)/P_j(t_0)\), where \(P_j\) is a probability density function, and the electric field operators are Fourier transforms of the spectral operators, \(\hat{E}_j^+(t) = \text{FT}\left[\hat{a}_j^\dagger(\omega)\right]\). Imperfect detector resolution can be treated by convolving this with the detector response function \(R(t_0, t)\), giving the resultant state of the matter qubits as an incoherent sum over different detection times. After detection of a single photon in the top rail, and no other detection events, the array of qubits is described by the density matrix

\[
\hat{\rho}(t_0) = \frac{1}{P(t_0)} \int dt R(t-t_0) \sum_{ij} \zeta_j(t) \zeta_j^\dagger(t)|s_i\rangle\langle s_j|. \tag{4}
\]

The temporal mode functions \(\zeta_j(t)\) are inverse Fourier transforms of the spectra \(\Phi_j(\omega)\). The fidelity to the \(W\) state \(F_W = \langle W_d|\hat{\rho}|W_d\rangle\) is then used to evaluate the effect of distinguishable emitters. The consequences for the resultant GHZ state are discussed later.

We consider a Gaussian detector response function and Lorentzian photon lineshapes, given respectively by

\[
R(t) = \frac{1}{\sigma \sqrt{2\pi}} e^{-t^2/2\sigma^2},
\]

\[
\Phi_j(\omega) = \sqrt{\frac{\Gamma_j}{\pi}} \frac{1}{\Gamma_j - i(\omega - \omega_j)} . \tag{5}
\]

The parameters \(\sigma, \omega_j\) and \(\Gamma_j\) have been introduced as the characteristic width of the detector response, characterised by it is jitter, and the central emission frequencies and linewidths of the \(j\)th emitter. Substituting these functions into equation (4) gives the state of the qubit array. We present some descriptive examples here, and results for the general case in the supplementary information (https://stacks.iop.org/NJP/24/013032/mmedia) [29].

We first consider the case of equal linewidths and different central emission frequencies. A phase factor oscillating at the frequency difference of each pair of emitters, \(\Delta_j = \omega_j - \omega_{j+1}\), appears in the relevant term of the density matrix. For a narrow \(R(t_0, t)\) (small \(\sigma\)), this phase is well defined, and can be corrected by single qubit rotations on the emitters or phases on the output rails of the eventual GHZ state, so that time-filtering is not required. Phase correction does not fully lift the dependence on detection time, so a useful metric to consider is the time averaged fidelity of the corrected state, which is found to have the
Figure 2. (a) Heralded W state fidelity as a function of detector resolution for \( d = 3 \) emitters with frequencies \( \omega_0, \omega_0 \pm \Delta \), averaged over detection time. Dashed lines show W state fidelity without time resolved detection. Here all emitters have equal linewidths \( \Gamma_j = 1 \text{ GHz} \). (b) Mismatched linewidths: red curves show probability density for detection of a photon with linewidth \([0.6, 0.8, 1, 1.2, 1.4] \Gamma_0\), as a function of time (in units of \(1/\Gamma_0\)), and the corresponding fidelity in black. The maximum fidelity achieved here for detection at \( t \sim 0.5 \) is 99.9%, with a time-averaged fidelity of 98%, obtained using the analytic expression given in the supplementary information [29], and verified with numerical integration.

The simple form \( F_W = \frac{1}{d^2} \sum_{j,k} e^{-\frac{1}{\sigma^2} \Delta^2_{jk}} \). As seen in figure 2(a), high-fidelity path erasure between distinguishable emitters can be achieved when the detector can resolve their frequency difference. A larger \( \sigma \) results in a detector that samples a mixture of phases. The phase correction is then imperfect, resulting in a loss of coherence and effective dephasing of the resultant W state, with a dephasing parameter that is a monotonic function of the detector resolution. 3 ps resolution has recently been demonstrated for threshold detectors [30], corresponding to a correctable frequency splitting of \( \sim 50 \text{ GHz} \), and 16 ps for photon number-resolving detectors (PNRDs) [31]. If the detector resolution and emitter beatnotes are long compared to the characteristic photon lifetime, it is not necessary or useful to correct for the additional phase factor. The uncorrected time-averaged fidelity is \( \frac{1}{d^2} \sum_{j,k} \frac{2t}{\Gamma_0} \frac{\Delta^2_{jk}}{} \).

We also consider the effect of variation in photon linewidths, for example as a result of different cavity leakage rates. Linewidth variation due to differing decay constants leads in general to a ‘tilted’ state with imbalanced amplitudes [32]. For example, in the \( d = 2 \) case a heralding event at time \( t \) yields the state \( \lambda_1(t) |10\rangle + \lambda_2(t) |01\rangle \), with a critical time \( t_c \) for which \( \lambda_1(t_c) = \lambda_2(t_c) \), and we can achieve perfect path erasure [32]. For higher dimensions the behaviour is analogous, and while generally there will no longer be a critical time that recovers perfect interference, high fidelities are achievable. Figure 2(b) shows that up to 99.9% fidelity can be recovered for a five-dimensional W state with linewidths varying by \( \pm 40\% \), with a time-average value of 98%. The highest fidelities are achieved by post-selecting on optimal arrival times \( t_0 \sim 0.5/\Gamma_0 \) s in figure 2(b).

Different photon arrival times may be similarly described in this framework by substituting \( \zeta_j(t) \rightarrow \zeta_j(t - \delta t) \) in equation (4). For like emitters, time delays induce both tilting of the resultant W state, as particular emitters are more or less likely to have emitted the detected photon depending on detection time, as well as a relative phase that oscillates at the carrier frequency. It is therefore of critical importance to ensure path length stabilisation in the interferometer as well as highly synchronised pumping of the emitters. Integrated photonics, enabling monolithic integration of emitters and circuits [33], is an ideal platform to maintain high path length stability when increasing the dimensionality.

Loss—photon loss due to, for example, finite collection efficiency, propagation in waveguides and detector inefficiency is an important source of error in this protocol. We assume equal losses on each mode of the DFT interferometer, with overall photon capture probability per photon denoted \( \eta \). Loss in the first step of the protocol (figure 1(b)) can lead to false heralding events, whereby multiple photons could be
emitted and all but one lost prior to detection. This gives the mixed state:

$$\rho = \alpha_0 |W_d\rangle\langle W_d| + \sum_{q} \alpha_q |1_q\rangle \otimes |W\rangle\langle W|_q^f.$$  \hspace{1cm} (6)

The sum runs over all partitions $\mathcal{Q}$ of the mode indices $[d]$ into two sets $\{q, q'\}$, where $q$ represents the modes where photons have been lost. This corresponds to summing over all possible emission configurations. The partition $q = \emptyset, q' = [d]$ has been explicitly separated, as this represents the target W state, where no photons have been lost. The coefficients $\{\alpha_q\}$ are determined by binomial statistics.

The fidelity is used to determine the quality of W state preparation in the presence of loss. Analytic expressions for $F_W$ and success probability $P_W$ are derived in the supplementary information [29], and we find that the fidelity of the heralded state can be made arbitrarily close to unity at the cost of vanishing success probability, by reducing $p$, the weighting of the bright state in the initial superposition. In this manner, losses are converted into time overheads, with a 'repeat-until-success' approach. In suitable parameter regimes, the impact of losses can be mitigated using a 'many-successes' scheme. Emitters are $\pi$-pulsed again after a successful generation attempt, without re-initialisation. If there are multiple successive single click events, the erroneous part of the state can be suppressed [29]. Assuming the emitter array has been prepared in an ideal W state, robustness against loss is naturally achieved for protocols which permit postselection. Because the qudit is encoded by a single photon distributed over $d$ waveguides, loss will take the state out of the qudit subspace. This is a heralded error, corresponding to qudit erasure, and its probability is independent of the qudit dimension.

The entanglement structure of the GHZ state causes this to propagate to a non-local error on the final state, which becomes a significant error channel on the qubit array. For high $d$, success rates approach those achievable using PNRDs, $W$ are derived in the supplementary information [29], and we find that the fidelity of the W state by a factor $(1 - \gamma_T)^2$, quadratically worse than the dephasing on a single emitter:

$$|W_d\rangle\langle W_d| \to (1 - \gamma)^2 |W_d\rangle\langle W_d| + \frac{1}{d} \sum_{j=0}^{d-1} |j\rangle\langle j|.$$  \hspace{1cm} (7)

The entanglement structure of the GHZ state causes this to propagate to a non-local error on the final state, to a degree dictated by the total run-time of the protocol. Comparing the magnitude of this error to another possible imperfection—spin relaxation errors, which cause a linear degradation of the state fidelity (as discussed in the supplementary information [29])—we see that dephasing is likely to be the most significant error channel on the qubit array.

Threshold detectors can operate at much faster rates than PNRDs [30], but photon bunching can no longer be differentiated from the target single photon outcomes. Similarly to loss, this degrades $F_W$ by introducing additional mixedness to the state in the form of higher photon number terms. Expressions for $F_W$ and $P_W$ with threshold detectors can be determined by calculating the size of multi-photon detection events, derived in the supplementary information [29]. In order to achieve the same fidelity as PNRDs, $p$ must be reduced and this incurs a significant penalty in the success probability. However, as $d$ is increased, the impact of threshold detectors is lessened, due to the reduced likelihood of photon bunching in the interferometer. For high $d$, success rates approach those achievable using PNRDs. We also consider time-resolved threshold detection with different colour photons in the supplementary information [29], and find that distinguishable emitters reduce photon bunching in the DFT, lessening the detrimental effects of threshold detection—in fact, the W state fidelity achievable with distinguishable photons and time-resolved threshold detection can exceed that achievable by threshold detectors in the indistinguishable case.

To calculate the ultimate GHZ state fidelity, we consider each source of error separately, and calculate the resultant fidelity by taking the product $F_{\text{GHZ}} = F_{\text{dist}} \cdot F_{\text{loss}} \cdot F_{\text{dephase}}$. The dephasing-type errors arising from distinguishability and pure-dephasing do not affect the photon emission probability, but reduce its coherence. It can therefore be verified that the degradation of the GHZ-state fidelity is equal to the degradation of W state fidelity prior to the $X$-measurement. The pure-dephasing parameter is determined
Figure 3. (a) Fidelity of a three qutrit GHZ state generated using emitters with frequencies \([f_0, f_0 \pm 10 \text{ GHz}]\) in this protocol. Solid lines show the fidelity achievable with state of the art 3 ps detector resolution, calculated analytically, and dashed lines with no time resolution. Dots are simulated results. Dephasing occurs at a rate 0.0088 per round of emission. The W state is assumed to have been heralded after \(1/P_W\) entanglement generation attempts. (b) Success probability of the protocol \(P_{\text{GHZ}}\) (solid lines) and expected number of repetitions until non-vacuum herald \(N_W\) (dashed). Larger \(p\) reduces protocol run-time, but increases the probability of multiple-photon detection events in the repeat-until-success scheme, a failure event which requires re-initialisation. 

by considering the mean runtime of the protocol, using analytically calculated success probabilities. Multiplying the dephasing-type errors in this way will lower bound the resultant fidelity, giving good accuracy when one source of dephasing is small. To determine the impact of loss on the final GHZ state we calculate the size of the \(k\)-excitation terms (when \(k\) photons are emitted) in the noisy W state (equation (6)), and find the probability they give rise to single photon events in the second step of the protocol (figure 1(c)). The fidelity can be calculated by finding the overlap of each \(k\)-excitation term with the target GHZ state after \(n\) noisy emission events. This approach is verified with numerical simulation. The overall protocol success probability \(P_{\text{GHZ}}\) is determined by the probability of multiple detection events during W-state generation, which constitute an intolerable error requiring re-initialisation of the emitters. \(P_{\text{GHZ}}\) is the probability that the first non-vacuum detection is a valid heralding event, \(P_{\text{GHZ}} = P_W/(1 - P_{\text{vac}})\). As shown in figure 3(b), this approaches unity for small \(p\), but the number of W-state generation attempts required in the repeat-until-success step (and protocol run time) becomes exponentially large as \(p \to 0\). In figure 3(a), we plot the expected state fidelity when using this protocol to generate a three-qutrit GHZ state from distinguishable emitters with and without time resolving detection, using realistic emitter parameters—recent experiments with quantum dot sources [34] have shown end-to-end efficiencies of 53% with 76 MHz repetition rate and \(>1.5\) \(\mu\)s dephasing time, a corresponding dephasing rate of \(\gamma < 0.0088\) per photon. We observe that 80% fidelity is achievable in this regime, with success probability of \(>95\%\), which can be boosted to \(F_{\text{GHZ}} > 90\%\) with high probability for modest improvements to capture efficiency and dephasing time. Recent developments in detector timing resolution [30, 31] and efficiency [35, 36] raise the prospect of simultaneously fast and efficient detectors in the near future.

In the supplementary information [29], we calculate secure key rates that could be achieved using these figures of merit for qudit Bell state generation. We follow the methods in [6, 17], and find that five such emitters could be used to distribute \(d = 5\) qudit Bell pairs with secure bit rates of 1.3 Mbps (using the optimal value of \(p = 0.056\)). Higher \(d\) results in recoverable key rates where losses are prohibitive in the lower-dimensional case.

Discussion. We have shown in this work that highly entangled photonic qudit states can be generated with rate overheads that scale promisingly with \(d\), even between non-identical sources. Practically, significant work must be done to simultaneously control and synchronise multiple emitters, but through the use of time resolving detection we have shown that the effects of distinguishability from non-identical
emitters can be mitigated without time-filtering, significantly relaxing this challenging experimental requirement. Looking ahead, one could consider entanglement distillation protocols with a broker-client scheme, as in [37] to boost success probabilities from other herald patterns.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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