Hadronization in heavy ion collisions: Recombination and fragmentation of partons

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We argue that the emission of hadrons with transverse momentum up to about 5 GeV/c in central relativistic heavy ion collisions is dominated by recombination, rather than fragmentation of partons. This mechanism provides a natural explanation for the observed constant baryon-to-meson ratio of about one and the apparent lack of a nuclear suppression of the baryon yield in this momentum range. Fragmentation becomes dominant at higher transverse momentum, but the transition point is delayed by the energy loss of fast partons in dense matter.

Data from the Relativistic Heavy Ion Collider (RHIC) have shown a strong nuclear suppression of the pion yield at transverse momenta larger than 2 GeV/c in central Au + Au collisions, compared with $p+p$ interactions. The emission of protons and antiprotons does not appear to be similarly suppressed, and the $p/\pi^+$ ratio reaches or even exceeds unity for transverse momenta above 2 GeV/c [2,3,4,5]. These results lack a consistent explanation in the standard picture of hadron production at high transverse momentum, which assumes that hadrons are created by the fragmentation of energetic partons. Whereas the observed suppression of the pion yield is attributed to the energy loss of partons during their propagation through the hot and dense matter created in the nuclear collision – a phenomenon commonly referred to as jet quenching [6] – the absence of a similar effect in the proton spectrum is puzzling [7].

We propose that hadron production at momenta of a few GeV/c in an environment with a high density of partons occurs by recombination, rather than fragmentation, of partons. Below we show that recombination always dominates over fragmentation for an exponentially falling parton spectrum, but that fragmentation wins out eventually, when the spectrum takes the form of a power law. We also show that recombination can explain some of the surprising features of the RHIC data, as first suggested by Voloshin [8].

In the fragmentation picture [2], the single parton spectrum is convoluted with the probability for a parton $i$ to hadronize into a hadron $h$, which carries a fraction $z < 1$ of the momentum of the parent parton. It has been argued that the fragmentation functions $D_{i\rightarrow h}(z)$ can be altered by the environment [10]. The dominant modification mechanism is the energy loss of the propagating parton in the surrounding medium, which leads, in first approximation, to a rescaling of the variable $z$. This would affect all produced hadrons in the same way, in contradiction with the observations at RHIC.

Another mechanism of hadron production is quark recombination. In the recombination picture, three quarks or a quark/antiquark pair in a densely populated phase space can form a baryon or meson respectively. The amplitude for this process is determined by the hadron wave function. This mechanism has recently been identified as the source of unnatural isospin ratios in the production of $D$-mesons in the fragmentation region in $\pi^-+\Lambda$ interactions at Fermilab [11]. Hadron production in heavy ion collisions by recombination of quarks has been considered before [12], primarily at small transverse momentum. Quark recombination has recently been invoked to explain some aspects of the RHIC data, such as the flavor pattern of elliptic flow [13], and in the context of a scaling model [14].

In the formalism of perturbative QCD, recombination is a more exclusive process, and falls off faster than fragmentation with increasing transverse momentum. On the other hand, for a meson of given momentum $P$ fragmentation starts out with a parton with much higher momentum $p = P/z$, ($z < 1$), whereas recombination requires a quark/antiquark pair where each parton carries only about $P/2$ in average. However, the spectrum of high-momentum partons is steeply falling and further reduced by the energy loss in dense matter. Our main result is that the momentum range, in which recombination processes can successfully compete with fragmentation, may extend up to 5 GeV/c in the favorable environment of central heavy ion collisions at RHIC. We will show below that recombination is effective whenever the phase-space distribution of a system of partons has thermal character.

Let us consider an expanding system of quarks and antiquarks. We assume that the recombination of these partons into hadrons occurs on a space-like hypersurface $\Sigma_l$. The RHIC experiments indicate that the freeze-out is very rapid. The measured two-particle correlation functions are consistent with an extremely short emission time in the local rest frame, suggesting a sudden transition after which individual hadrons interact only rarely [15]. In our treatment, we assume that the dense parton matter is devoid of dynamical thermal gluons and predominantly composed of quarks and antiquarks at the...
moment of hadronization. We neglect a possible effective quark mass, because we are here interested in hadron production at high transverse momentum.

Denoting the density matrix of the parton system on \( \Sigma_t \) as \( \hat{\rho}_t \), we can express the number of mesons at freeze-out as \( E dN_M / d^3 P = (2\pi)^{-3} \langle M; P | \hat{\rho}_t | M; P \rangle \). Here \( | M; P \rangle \) is a meson state with momentum \( P \). We now introduce the single-particle Wigner functions for quarks and antiquarks, \( w_a(r; p) \) and \( \bar{w}_b(r; p) \), respectively, and neglect multi-particle correlations in the density matrix. Further introducing the time-like, future oriented unit vector \( u^\mu(r) \) orthogonal to the freeze-out hypersurface at \( r \in \Sigma_t \), we obtain the following expression for the meson emission spectrum:

\[
E \frac{dN_M}{d^3 P} = \int_{\Sigma_t} d\sigma \frac{P \cdot u(r)}{(2\pi)^3} \sum_{a, b} \int dz \left| \psi_{ab}^M(z) \right|^2 w_a(r; zP^+) \bar{w}_b(r; (1 - z)P^+). \tag{1}
\]

Because we are considering the emission of a meson with high momentum, it is most convenient to use a distribution amplitude in terms of light-cone coordinates in a frame where \( P^+ \gg P^- \) and the transverse momentum of the meson vanishes. \( z \) and \( 1 - z \) are the momentum fractions carried by the quark and antiquark, respectively, and \( a, b \) denote the internal quantum numbers of the quarks (spin, flavor, color). For baryons, a similar calculation yields:

\[
E \frac{dN_B}{d^3 P} = \int_{\Sigma_t} d\sigma \frac{P \cdot u(r)}{(2\pi)^3} \sum_{a, b, c} \int dz_1 \int_0^{1 - z_1} dz_2 \left| \psi_{abc}^B(z_1, z_2) \right|^2 w_a(r; z_1 P^+) w_b(r; z_2 P^+) w_c(r; (1 - z_1 - z_2)P^+). \tag{2}
\]

The overall normalization is fixed by the number of quarks on the hypersurface \( \Sigma_t \):

\[
N_a = \int_{\Sigma_t} d\sigma \frac{d^3 p}{(2\pi)^3 p_0} p \cdot u(r) w_a(r; p). \tag{3}
\]

It is important to note that our results are not really sensitive to the model used for the recombination process. A complete dynamical description in terms of QCD is hard to achieve. However, for the observables that we discuss below, it is essential to use two common features: the probability for the emission of a meson (baryon) is proportional to the single parton distribution squared (cubed), and the parton momenta sum up to the hadron momentum. At this level of sophistication a description with the light-cone wave functions replaced by three-dimensional, spatial quark wave functions leads to identical results.

To make further progress, we need to specify the parton Wigner functions. We first consider the case that these are described by the exponential tail of a thermal distribution \( w_a(r; p) = \exp(-(p \cdot u - \mu)/T) \) with local temperature \( T(r) \) and chemical potential \( \mu(r) \), independent of the internal quantum numbers. The constraint that the momentum fractions of all partons in the hadron wave function must add up to the total momentum \( P \) of the hadron then insures that the product of all Wigner functions entering into the hadron yield is solely dependent on the hadron momentum:

\[
w_a(r; zP^+) \bar{w}_b(r; (1 - z)P^+) = \exp(-(P \cdot u - \mu_B)/T),
\]

\[
w_a(r; z_1 P^+) w_b(r; z_2 P^+) w_c(r; (1 - z_1 - z_2)P^+) = \exp(-(P \cdot u - 3\mu)/T), \tag{4}
\]

where \( \mu_B = 3\mu \). One can then perform the integrations over the momentum fractions in (1) and (2) and obtains the result that the baryon-to-meson ratio is independent of the momentum \( P \) and simply given by the ratio of the number of quark degrees of freedom contributing to the emission of the specific hadrons: \( dN_B/dN_M = \sum_{a, b} e^{\mu_B/T} / \sum_{a, b} \).

The summations over color, flavor, and helicity give rise to degeneracy factors for mesons and baryons, \( C_M \) and \( C_B \), respectively. In order to derive an upper limit to the nucleon-to-pion ratio, we make the simplifying assumption that all decuplet baryon states \( \Delta \) contribute to the nucleon yield through decay, but neglect the contributions to the high-\( P_T \) pion spectrum from the decay of unstable hadrons. If we require the partons to form color singlets at recombination, we obtain \( C_{\pi^+} = 1 \) and \( C_p = 20/(2 \times 3!) = 5/3 \), yielding the result

\[
dN_p/dN_{\pi^+} = e^{\mu_B/T} C_p / C_{\pi^+} = \frac{5}{3} e^{\mu_B/T}. \tag{5}
\]

Decay of unstable hadron states is likely to reduce this value, but will not change the prediction that the \( p/\pi^+ \) ratio is constant as a function of particle momentum and significantly larger than the value expected from
quark fragmentation. We note that this result is in good agreement with the RHIC data, which show $dN_\pi/dN_{\pi^+}$ reaching a plateau around unity in the range $2 \text{ GeV}/c \leq P_T \leq 4 \text{ GeV}/c$ [18]. For the $\Lambda$-hyperon yield, we need to include the channels $\Lambda, \Sigma^0$ and $\Sigma^{*0}$, leading to $C_\Lambda = 4/3$. This would predict a plateau at 4/3 in the $\Lambda/K^0$ yield, which will be diluted by kaons from the decay of $K^*$. The $p/\pi$ ratio is also predicted to be independent of momentum and equal to $\exp(-2\mu_B/T)$, again in rather good agreement with the RHIC data [3].

The predictions are radically different, when one considers a power law spectrum as it is characteristic in perturbative QCD at large transverse momentum:

$$w_a^{\text{pert}}(r;p) = A_a \left(1 + \frac{P_T}{B}\right)^{-\alpha}.$$  \hspace{1cm} (6)

One then finds that mesons always dominate over baryons at large momentum, $dN_B/dN_M = (27/4\pi)\alpha B A_{\Lambda}^0/C_M A_{\pi}$, and that eventually parton fragmentation wins out over quark recombination. For pions, the local ratio of the two contributions is $dN_{\text{frag}}/dN_{\text{rec}} \propto P^\alpha$. On the other hand, for an exponential quark spectrum, fragmentation is always suppressed with respect to recombination.

This result constitutes the main insight gained from our considerations: Hadron emission from a thermal parton ensemble is always dominated by parton recombination; only when the thermal distribution gives way to a perturbative power law at high momentum, does fragmentation become the leading hadronization mechanism. The threshold between the two domains depends on the size of the emitting system and the hadron species.

That the meson spectrum from recombination is determined, on the average, by $w(P/2)$, whereas the baryon spectrum depends on $w(P/3)$, implies that those kinematic properties of the hadron spectrum, which are due to collective flow of partons, extend to higher values of the transverse momentum for baryons than for mesons [13]. This effect is clearly visible in the RHIC data [18, 19], which exhibit a linear rise of the elliptic flow velocity $v_2$ with $P_T$, which continues further in $P_T$ for protons and hyperons than for pions and kaons.

Realistic calculations require the specification of the freeze-out hypersurface $\Sigma_f$ and the parton spectrum. Here we assume boost invariance of $\Sigma_f$ according to $\tau = \sqrt{t_f^2 - z_f^2} = \text{const}$. For the thermal part of the quark spectrum we use an axially and longitudinally expanding thermal source:

$$w_a^{\text{th}}(\eta; y, P_T) = A_{\text{th}} e^{-\sqrt{m^2 + y^2}/2\Delta^2},$$  \hspace{1cm} (7)

characterized by an effective temperature $T \approx 350 \text{ MeV}$, which includes the blue shift caused by the radial expansion, and a rapidity width $\Delta \approx 2$. Here $\eta$ and $y$ are the rapidities in space-time and momentum. The form of this spectrum agrees with the results of the parton cascade VNI/BMS [20], which yields a parton distribution exhibiting an exponential shape at low transverse momentum and a power law shape for high transverse momentum. For the power law tail of the parton spectrum we choose the results given by a lowest-order perturbative QCD calculation [21], shifted by $\Delta p_T = -\sqrt{\lambda p_T}$ with $\lambda = 1 \text{ GeV}$ to account for the energy loss of fast partons, and standard fragmentation functions [22]. The normalization of the thermal part of the spectrum is adjusted to fit the measured inclusive spectrum of charged hadrons from PHENIX [23], as shown in the upper frame of Fig. 1. The contributions from recombination and fragmentation are shown separately to exhibit the location of the rapid crossover between these two mechanisms at about $5 \text{ GeV}/c$. We note that in the parton spectrum $w^{\text{th}} + w^{\text{pert}}$ itself, the crossover between the thermal and the perturbative part occurs already at about $3 \text{ GeV}/c$, consistent with parton cascade predictions [21]. The recombination mechanism shifts this point to higher values of $p_T$ in the hadron spectrum.

The lower frame of Fig. 1 shows our prediction for the $p/\pi^+$ ratio. The rapid drop of its value in the range $4–5 \text{ GeV}/c$ is an unambiguous prediction of our model. Experiments at RHIC have not yet been able to probe this $P_T$ range, because the identification of protons has not been feasible beyond $4 \text{ GeV}/c$. The identification of hyperons is possible to higher $P_T$, and indications of a rapid drop in the $\Lambda/K^0$ ratio have been found [4].

Figure 2 shows the scaled ratio of particle yields in
Au + Au and p + p collisions, called $R_{AA}$. (We use fits to data for the $p + p$ yields and our predictions for Au + Au.) The energy loss of fast partons leads to a nuclear suppression in the fragmentation region ($P_T > 5$ GeV/c). For low $P_T$ this suppression is counteracted by the recombination mechanism, which is absent in $p + p$ reactions. Recombination is more important for protons than for pions, resulting in much less nuclear suppression for protons.

RHIC data exhibit a strong increase of the anisotropic flow parameter $v_2$ for mesons and baryons at small $P_T$, which finally saturates. This happens earlier for mesons than for baryons. It has been argued that the flow anisotropy originates in the partonic phase. In the recombination region mesons at transverse momentum $P_T$ reflect the properties of partons with an average transverse momentum $P_T/2$, while baryons reflect those of partons with $P_T/3$. It follows that $v_2$ saturates later for baryons than for mesons. The transition to the fragmentation region would provide such a mechanism, but it occurs at too high momentum. The observed saturation of $v_2$ must, therefore, be due to some other mechanism or require a more realistic description of the space-time evolution of the system.

In summary, we propose a two component behaviour of hadronic observables in heavy ion collisions at RHIC. These components include fragmentation of high-$p_T$ partons and recombination from a thermal parton distribution. The competition between recombination and fragmentation of partons can explain several of the surprising features of the published data. In particular, the proton excess at intermediate $P_T$, the different nuclear suppression observed in pion and proton spectra, and the different saturation thresholds in the elliptic flow, are easily explained. We predict that all baryon spectra will exhibit a rapid transition around 5 GeV/c to a region dominated by parton fragmentation. Finally, our scenario requires the assumption of a thermalized partonic phase characterized by an exponential momentum spectrum. Such a phase may be appropriately called a quark-gluon plasma.

Note added: We draw attention to the closely related recent work of Greco et al., who also propose that parton recombination can explain the large baryon/meson ratio observed at RHIC.

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{ratio_2.png}
\caption{Ratio $R_{AA}$ of hadron yields in central Au+Au to $p+p$ collisions, scaled by the number of binary nucleon-nucleon collisions. Data for pions taken from PHENIX.}
\end{figure}

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