Learning Choice Functions via Pareto-Embeddings

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Abstract

We consider the problem of learning to choose from a given set of objects, where each object is represented by a feature vector. Traditional approaches in choice modelling are mainly based on learning a latent, real-valued utility function, thereby inducing a linear order on choice alternatives. While this approach is suitable for discrete (top-1) choices, it is not straightforward how to use it for subset choices. Instead of mapping choice alternatives to the real number line, we propose to embed them into a higher-dimensional utility space, in which we identify choice sets with Pareto-optimal points. To this end, we propose a learning algorithm that minimizes a differentiable loss function suitable for this task. We demonstrate the feasibility of learning a Pareto-embedding on a suite of benchmark datasets.

Keywords Choice function · Pareto-embedding · Generalized utility

1 Introduction

The quest for understanding and modeling human decision making has a long history in various scientific disciplines, including economics and psychology [4]. Starting with the seminal work by Arrow [1], choice functions have been analyzed as a key concept of a formal theory of choice. In simple terms, a decision maker is confronted with a (possibly varying) set of alternatives and the choices made are observed. The ultimate goal is to explain and predict the choice behavior.

In machine learning, the task of “learning to choose” is part of the broader field of preference learning, which attracted increased attention in recent years [5]. The task for a learner is to observe choices from multiple sets of objects, and to produce a function which maps from candidate sets to choice sets. An important special case is the setting in which the decision maker only chooses one object from each given set, which is known as discrete choice. A popular strategy to tackle the learning problem is to posit that the choice probabilities depend on an underlying real-valued utility function of the decision maker. Under this assumption, learning can be accomplished by identifying the parameters of such a function. The more general problem of predicting choices in the form of subsets of objects has been considered only very recently [2, 11]. Extending the approach based on utility functions toward this setting turns out to be non-trivial. Either one faces combinatorial problems calculating the probabilities for many subsets [2], or has to resort to thresholding techniques [11].

We propose to solve this problem by embedding the objects in a higher-dimensional utility space, in which subset choices are naturally identified by Pareto-optimal points (Section 3). To learn a suitable embedding function, we devise a differentiable loss function tailored to this task. We then utilize the loss function as part of a deep learning pipeline to investigate the feasibility of learning such a Pareto-embedding (Section 4).
We proceed from a reference set of objects (choice alternatives) \( X \subset \mathbb{R}^d \), which, for ease of exposition, is assumed to be finite. Each \( x \in X \) is represented as a vector of real-valued features \( (x_1, \ldots, x_d) \). We call a finite subset of objects \( Q \subseteq X \) a choice task and allow the size \(|Q| \in \mathbb{N}\) to vary across tasks. For each choice task \( Q = \{x_1, \ldots, x_m\} \), we assume that a preference is expressed in terms of a choice set \( C \subseteq Q \). A useful representation of a choice set is in terms of a binary vector \( c \in \{0, 1\}^m \), where \( c_i = 1 \) if \( x_i \in C \) and \( c_i = 0 \) if \( x_i \not\in C \).

One of the first approaches to explaining choices was to assume that a decision maker can assign a (latent) utility to each of the choice alternatives. Formally, we represent such a utility function as a function \( X \rightarrow \mathbb{R} \) from the space of objects to the real numbers. Based on these utilities, a rational decision maker will always pick the alternative with the highest utility, i.e., the top-1 object. To explain variability in choices, noise can be added to the utilities, which results in what is called a random utility model [7, 8, 12].

At first glance, it may appear that this approach can easily be generalized to modeling subset choices: Instead of only selecting the top-1 object, one could consider to select the top-\( k \) objects, where \( 1 \leq k \leq |Q| \). One major drawback of this approach is that the subset size is predetermined to be \( k \), so it is not possible to produce subsets of varying size. Another possibility is to specify a threshold for the utilities, and to include all objects with a utility higher than the threshold in the choice set [11]. While this allows for the prediction of subsets of arbitrary size, the decision of whether to include an object in the choice set is now completely independent of all the other objects.

As we shall see in the next section, there is a natural way to define subset choices, if we embed the objects in a higher-dimensional utility space.

### 2 Modeling Choice

The basic idea of a Pareto-embedding is illustrated in Fig. 1. On the left side, we depict the original set of objects in the object space \( X \). The function \( \varphi: X \rightarrow Z \) maps each point into a new embedding space \( Z \subset \mathbb{R}^p \). This space can be thought of as a higher-dimensional utility space, i.e., each dimension corresponds to the utility on a certain aspect or criterion. As we can see, the choice set \( C \) forms what is called a Pareto-set in this new space, i.e., the set of points that are not dominated by any other point.

More formally, let \( Q = \{x_1, \ldots, x_m\} \) be the original set of objects and \( Z = \varphi(Q) = \{z_1, \ldots, z_m\} = \{\varphi(x_1), \ldots, \varphi(x_m)\} \) the corresponding points in the embedding space. A point \( z_i \) in the embedding space is dominated by another point \( z_j \) if \( z_{i,k} \leq z_{j,k} \) for all \( k \in [d'] \) and \( z_{i,k} < z_{j,k} \) for at least one such \( k \). Then, a point \( z_i \) is called Pareto-optimal (with respect to \( Z \)), if it is not dominated by any other point \( z_j \in Z \), \( 1 \leq j \neq i \leq m \). We denote by \( P_\varphi(Q) \) the subset of points that are Pareto-optimal in \( Q \) under the mapping \( \varphi \), i.e., the points \( x_i \in Q \) such that \( \varphi(x_i) \) not dominated by any point in \( \{\varphi(x_1), \ldots, \varphi(x_m)\} \).

It is interesting to note that the traditional one-dimensional utility always imposes a total order relation on the available objects, whereas the Pareto-embedding generalizes this to a partial order. Therefore, richer preference structures with multiple layers of incomparability can be modeled.

Given a set of observed choices \( D = \{(Q_n, C_n)\}_{n=1}^N \) as training data, where \( Q_n \subseteq X \) is a choice task and...
With these two terms, we can ensure that if the loss is zero, we have a valid Pareto-embedding of the points. Furthermore, we add two more terms that are useful. To preserve as much of the original structure present in the object space $X$, we use multidimensional scaling (MDS) [9]. It ensures that objects close to each other in the object space $X$ will also be close in the embedding space $Z$. In addition, all the losses so far are shift-invariant in the embedding space. To make the solution identifiable, we regularize the mapped points towards 0 using an $L_2$ loss. We define the complete Pareto-embedding loss as a convex combination

$$L(Q, Z, c) = \alpha_1 L_{PO}(Z, c) + \alpha_2 L_{DOM}(Z, c) + \alpha_3 L_{MDS}(Q, Z) + \alpha_4 \sum_{i=1}^{\left|Z\right|} \|z_i\|_2$$

with weights $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \geq 0$ such that $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1$. These weights can be treated as hyperparameters of the learning algorithm. Given a space $\Phi$ of embedding functions, this algorithm seeks to find a minimizer

$$\varphi^* \in \arg\min_{\varphi \in \Phi} \sum_{n=1}^{N} L(Q_n, P_\varphi(Q_n), c_n)$$

of the overall loss on the training data $\mathcal{D}$.
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4 Empirical Evaluation

As for the space of embedding functions, any model class amenable to training by gradient descent can in principle be used. Here, as a proof of concept, we use a simple fully connected multi-layer perceptron as a learner. The architecture is depicted in Fig. 3. We take each object $x_i$ for $1 \leq i \leq |Q|$ of the task $Q$ and pass it through the (deep) multi-layer perceptron. Rectified linear units are used here as the nonlinearities. Batch normalization is applied after each layer to speed up and stabilize training. In the final layer, we pass the output of the multi-layer perceptron through a linear layer with $d'$ outputs. After the same network (using weight sharing) was applied to all objects in $Q$, we end up with the transformed set $Z$. To obtain the final prediction, we take the set $Z$ and compute the corresponding Pareto-set. The network can be trained using standard backpropagation of the loss.

To ascertain the feasibility of learning a Pareto-embedding from data, we evaluate our approach on a suite of benchmark problems from the field of multi-criteria optimization. We use the well-known DTLZ test suite by Deb et al. and the ZDT test suite by Zitzler et al., containing datasets of varying difficulty. Adding a simple two-dimensional two parabola (TP) dataset, we end up with 5 benchmark problems in total. We generate 40 960 object sets of size with features each for every problem. Exceptions are the TP dataset with only 2 features and the ZDT5 dataset, which has 35 binary features by definition. For the DTLZ problems, we set the dimensionality of the underlying objective space to 5.

We evaluate our approach on every problem by repetitions of a Monte Carlo cross validation with a 90/10% split into training and test data. The remaining instances are split into 1/9 validation instances and 8/9 training instances. We use the validation set to jointly optimize the hyperparameters of the learner, which are (a) the loss weights $\alpha_1, \alpha_2, \alpha_3, \alpha_4$, (b) the maximum learning rate of the cyclical learning rate scheduler, and (c) the number of hidden units and layers, using 60 iterations of Bayesian optimization. The neural network was trained for epochs. The number of embedding dimensions $d'$ we set to 2, since this allows us to move from a total order (only one utility dimension) to a partial order.

Finally, we need a suitable metric to compare the ground truth subsets to the predicted ones. Since the shape of the Pareto-sets has an impact on how many points end up in the chosen subset, we have varying levels of positives across the datasets. Therefore, we choose a metric that is unbiased with respect to the prevalence of positives and well-suited for problems with class imbalance, called the $A$-mean, the arithmetic mean of the true positive and true negative rate.

The results are shown in Fig. 4. For five of the problems, the embedding approach is able to achieve an average $A$-mean of over 90%, indicating that for these problems we often identify the choice set correctly. This is important, as it shows that the loss function is able to steer the model parameters towards a valid Pareto-embedding. For comparison, a random selection in which each object is included in the choice set with a fixed probability (independently of the others) achieves an average $A$-mean of 50%. Thus, it is apparent that our learner is performing better than random guessing on all datasets. We also did an ablation experiment, where we removed the MDS term from the loss function and repeated the complete training procedure (including optimization of all the other hyperparameters). This resulted in a significant decrease in performance, showing that the MDS term is not only useful to preserve distances, but adds a helpful inductive bias.

Figure 3: Architecture of our approach. Each object is passed through a (deep) multi-layer perceptron followed by a linear output layer to produce the embedding. The Pareto-optimal points are selected to obtain a prediction $C$. 

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We proposed a novel way to tackle the problem of learning choice functions. Viewing it as an embedding problem and transforming the given objects into a utility space of more than one dimension, subset choice are naturally identified by the criterion of Pareto-optimality. To learn an embedding from a given set of observed choices as training data, we developed a suitable loss function that penalizes violations of the Pareto condition. Encouraged by the promising first results on benchmark problems, we are now looking forward to a more extensive empirical evaluation and applications to real-world choice problems.

5 Conclusion and Outlook

We proposed a novel way to tackle the problem of learning choice functions. Viewing it as an embedding problem and transforming the given objects into a utility space of more than one dimension, subset choice are naturally identified by the criterion of Pareto-optimality. To learn an embedding from a given set of observed choices as training data, we developed a suitable loss function that penalizes violations of the Pareto condition. Encouraged by the promising first results on benchmark problems, we are now looking forward to a more extensive empirical evaluation and applications to real-world choice problems.

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