A mathematical optimization model for secondary settler

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Abstract. The study of secondary settler modelling, which aims to establish the main model (one-dimensional-1D model), which is involved in some fundamental processes of the hydrodynamic behaviour of this liquid / solid separation unit and to engender variations of the sludge blanket height as a function of the operating parameters and maintaining of the municipal wastewater treatment plant of Setif. The objective of this research is focused on solid/liquid separation in the secondary settler by attempting a mathematical model that allows us to evaluate the sedimentation velocity as a function of the sludge settleability parameters.

1. Introduction
The activated sludge treatment process is the most used in Algeria and in the world to treat urban wastewater. The wastewater treatment plant of Setif city is a good example of this type of process. It should be noted that it was dimensioned for a capacity of 330,000 population equivalent to accommodate 66,000 m³/d in dry weather. The activated sludge processes have a very high purification performance and reliability, especially for organic pollution. This type of treatment has several limitations, such as control the pollution degradation phenomena in the biological reactor and the separation problems in the secondary settler, which are strongly interconnected. The optimal operation of the wastewater treatment plant depends on all the parameters of the different processes. The activated sludge process is based on a highly active bacteriological culture mixed with wastewater, which results in the aerobic degradation of the organic matter in the aeration tank. The biomass formed in this system, which constitutes the solid phase, is eliminated by settling in the secondary settler. Biological treatment by activated sludge is generally based on a microbial culture maintained in suspension in an aerobic condition in the aeration tank fed by the wastewater to be treated [1]. The general organization of an activated sludge treatment plant is shown as follows (Figure 1). The final stage of treatment involves the simultaneous separation of sludge from the treated water by settling. Liu (2003) [2] assumes that settling is the last step in the treatment chain before the treated water is released into the natural environment. This settling is provided by the secondary settler. The degradation of organic matter carried out in the aeration tank, where microorganisms degrade organic matter by oxidation and the total suspended solids retained by bio-flocs [3,4]. It also includes the aeration system ensure oxygen, necessary for the growth of microorganisms, usually using high powerful turbines. The mixing equipment ensure that flocs are not settling and facilitate the homogeneous oxygenation of the aeration tank.
In this process, several types of hydraulic or biological dysfunctions may appear. Most often is the overflow of the sludge blanket to the receiving environment, due to disturbance of the hydraulic flow and excessive development of filamentous bacteria in the biological reactor. These disturbances generate difficulties settling sludge in the secondary settler. We specify that the excessive growth of filamentous bacteria, characterized by increasing the sludge volume index (> 200 mL / g), causes foaming by the presence of gas bubbles and thus leads to poor sludge settling and starting towards the receiving environment. On the other hand, modelling is one of the tools used to improve the knowledge processes and the best management of wastewater treatment plants (WWTP). It is essential to the improvement of the control strategies for wastewater treatment plants by optimizing the parameters of the different processes.

For example, the increase of the suspended solids and the flux causes an important affecting the purification efficiency of the secondary settler because the residence time decreases, and the upward velocity of the suspended solids increases.

The modeling as an optimum operation of wastewater treatment plants via activated sludge is widely used for design, optimization and management of installations.

Many authors [5-12] have been particularly interested in two main objectives of dynamic modeling of wastewater treatment plants: the first is to optimize the operation of wastewater treatment plants (WWTP) and the second is to size installations and structures. The modelling of wastewater treatment plants is used to meet different objectives. It based on the operating parameters of the WWTP, such as residence time, aeration time, organic load, age of sludge. Modelling of WWTPs, in particular activated sludge plants, is a widely used tool to optimize the operation of the equipment. According to Gernaey et al. (2004) [13], modelling can be used for modifying the operating parameters of wastewater treatment plants to ensure economic treatment. It is also used to understand the performance of the aeration basin and the secondary settler.

The main objective of this work is the development of a mathematical model to simulate the variations of the height of the sludge blanket as a function of concentrations. An integrating the various operating parameters of the Setif wastewater treatment plant. This work focuses mainly on the hydrodynamic model of the one-dimensional secondary settler, based on the theory of the flow of solid particles and the continuity equation.

2. Average loads received by the treatment plant

Samples should be taken with greater accuracy using instantaneous sampling to provide qualitative information. We were particularly interested in the different elements that each have a specific role in the biological purification operations.

The average concentrations, the maximum recorded and the ratios characterizing the urban wastewater of the city of Setif are presented in Table 1.
Table 1. Parameters and ratios characterizing of the effluents coming in WWTP

| Parameters | pH  | BOD₅ | COD  | TSS  | NTK | N-NH₄⁺ | N-NO₂⁻ | N-NO₃⁻ | Pt    |
|------------|-----|-------|------|------|------|---------|---------|---------|-------|
| Units      | --  | mg/L  | mg/L | mg/L | mg/L | mg/L    | mg/L    | mg/L    | mg/L  |
| Average    | 7.58 | 380   | 1020 | 475  | 71.3 | 58.4    | 5.8     | 7.48    | 5.5   |
| Maximum registered | 8.30 | 450   | 1070 | 665  | 84.5 | 65.7    | 7.2     | 8.40    | 7.1   |
| Minimum registered | 6.65 | 270   | 720  | 315  | 35.8 | 24.2    | 2.8     | 3.1     | 2.4   |

Ratios

| Ratios               | COD/BOD₅ | TSS/BOD₅ | NTK/COD | COD/NTK | COD/Pt | N-NH₄⁺/NTK |
|----------------------|-----------|-----------|---------|---------|--------|-------------|
| Average              | 2.68      | 1.25      | 0.07    | 14.31   | 185.45 | 0.82        |
| Maximum registered   | 3.96      | 2.46      | 0.12    | 29.89   | 445.83 | 1.84        |
| Minimum registered   | 1.60      | 0.70      | 0.03    | 8.52    | 101.41 | 0.29        |

The results shown in Table 1 allowed us to conclude that the average concentrations of the pollution parameters are by those of the literature. In the general case, the main objective of wastewater treatment plants is to ensure the treatment of suspended solids (TSS), organic load (COD), ammonium (NH₄⁺), nitrate (NO₃⁻) and phosphorus. For the ratios, we observe the same average values as in the literature. The values of different forms of nitrogen and even for total phosphorus sometimes exceed the standards required by Executive Decrees No. 06-141 of April 19, 2006, and No. 10-23 of January 12, 2010.

3. Dynamic modelling of the secondary settler

The main design criteria for a solids separation unit from the liquid phase are mass loading, hydraulic inflow, peak flow rate, minimum depth, and sludge blanket [14]. Other criteria are defined in the literature, such as Daigger (1995) [15], Schuler and Hoon Jang (2007) [16], Zhang et al. (2006) [17], for conventional activated sludge for large plants designed to operate under optimal solid/liquid separation conditions, among these criteria, mixed liquor concentration, sludge volume index and recirculation ratio.

Our initial model is therefore a one-dimensional model based on solid particle flow theory and including dispersion.

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3.1. Numerical resolution

The hydraulic flow (Feed flux, $Q_f$) is divided into two flows: upward bulk flux (ascending clarification zone, $Q_s$) and and downward bulk flux (descending thickening zone, $Q_r$). We have added a graphic design (Figure 2) that will be useful for more detail.

Generally, three modes of sedimentation are considered: clarification, thickening, and compression. Because the last two zones are of the same nature, we assume only as a single zone. For this reason, the term thickening has been reserved for this manuscript.
3.2. Numerical resolution

Figure 2 shows the solid particle fluxes, sedimentation \( (F_s) \), hydraulic \( (F_h) \), and dispersive \( (F_D) \), in a layer of thickness \( \Delta Z \).

The balance equation (Figure 2) is as follows:

\[
S \left[ F_s \right]_{z} + S \left[ \frac{dF_h}{dz} \right]_{z} + S \left[ \frac{dF_D}{dz} \right]_{z} = S \left[ F_s \right]_{z+dz} + S \left[ \frac{dF_h}{dz} \right]_{z+dz} + S \left[ \frac{dF_D}{dz} \right]_{z+dz} + V \frac{dX}{dt} + V(-r_X)
\]

(1)

Where \( S \) is the section (m\(^2\)), \( V \) is the volume of settler, \( X \) is the total suspended solids (g/L), \( (-r_X) \) is the rate of reaction, (g/L.min). After differentiation, Equation 1 can be represented as

\[
dF_s + dF_h + dF_D + \frac{dX}{dt} + (-r_X)dz = 0
\]

(2)

Dividing by \( dz \), Equation 2 takes the following form:

\[
\frac{dF_s}{dz} + \frac{dF_h}{dz} + \frac{dF_D}{dz} + \frac{dX}{dt} + (-r_X) = 0
\]

(3)

After the replacement of the expressions of the fluxes, so
\[
\frac{d}{dz}(XV_s) + \frac{d}{dz}(V_h X) + \frac{d}{dz} \left( -D_s \frac{dX}{dz} \right) + \frac{dX}{dt} + \left( -r_x \right) = 0
\]  
(4)

Where \( V_s = f(X) \) is the settling velocity (m/h), \( V_h = Q/S \) is the hydraulic velocity (m/h), \( Q_f, Q_e \) and \( Q_r \) are the volumetric flow rates (\( f : \) feed, \( e : \) exit, and \( r : \) recycled) (L/h), \( D_s \) is the dispersion coefficient of solids in the thickening zone (m²/h), \( z \) is the height of the secondary settler (m). The material balance in the thickening zone is as shown below:

\[
\frac{d}{dz}(XV_s) + V_h \frac{dX}{dz} - D_s \frac{d^2X}{dz^2} + \frac{dX}{dt} + \left( -r_x \right) = 0
\]  
(5)

Applying the same reasoning in the clarification zone, i.e. considering a layer and performing the mass balance with a relation to the biomass, we obtain the following equation (6):

\[
-\frac{d}{dz}(XV_s) + V_h \frac{dX}{dz} - D_s \frac{d^2X}{dz^2} + \frac{dX}{dt} + \left( -r_x \right) = 0
\]  
(6)

\( D_s \) is the dispersion coefficient in the clarification zone (m²/h).

Equations (5) to (6) represent the dynamic model of the secondary settler, considering the axial dispersion of the N layers constituting the height (Z) of the settler as a function of the biomass concentration (X).

In steady-state, these non-linear differential equations are transformed into algebraic equations. It is solved by the iterative method, fixing the concentration at the surface of the settler and calculating the different concentrations of the layers constituting the two zones of clarification and thickening, which have been divided into five layers each.

4. Results and discussion

The sedimentation parameters must be affected by the pollution parameters and we have summarized the characteristics of the wastewater treated during the study period (Table 2). The pollutants are eliminated in the presence of microorganisms (organic matter) and by the growth of the bacterial culture (transformation metabolism of bacterial flocs), and by oxidation to \( \text{CO}_2 \) and \( \text{H}_2\text{O} \) which produces the energy necessary for the operation and production of new cellular materials. These parameters are strongly affected by filamentous bacteria and are manifested in two forms: foaming and filamentous expansion. The ability of sludge to be decanted is typically estimated by settling tests. The sludge index (Molhman index, MI) is influenced by the nature of the organic load. Several authors, such as Lee (1999) [10] and Hamilton et al. (1992) [22], include the dispersion term in their models and use a constant dispersion coefficient. In our case, we have also considered a constant dispersion coefficient for the settling process.

4.1.1. Sedimentation velocity and sludge index

An experimental example of relating sludge volume indice (SVI) to sedimentation velocity is summarized in Table 2. This table shows the physical and chemical characteristics of the sludge to be decanted (mixed liquor entering the secondary settler). They are microorganisms which are not the same from one station to another. It is very important to identify these characteristics to control the settling process.
Table 2. Examples of experimental data for the sludge settleability.

| Settability (mL/L) | TSS (g/L) | SVI (mL/g) | V_s (m/h) |
|-------------------|-----------|------------|-----------|
| 325               | 2.80      | 116        | 2.05      |
| 296               | 2.10      | 141        | 2.60      |
| 231               | 1.60      | 144        | 4.55      |
| 184               | 1.10      | 167        | 6.50      |
| 154               | 0.90      | 171        | 7.60      |

These low concentration data were obtained from the aeration tank of the wastewater treatment plant of the city of Setif. Several experiments were performed. During this period, the ability of sludge to settle and its qualities remained relatively homogeneous during these experiments. Note that the sludge indices (SVI) varied between 116 and 171 mL/g, with an average close to 140 mL/g.

Sub-models were developed, allowing us to estimate the settling velocity, by exploiting a large number of experimental results taken from the aeration tank of the Setif city wastewater treatment plant.

As a general practice, exponential functions are used to model the populations. According to the experimental results obtained, the variation of \( V_0 \) as a function of SVI is an increasing exponential function (same for population growth). In the literature, we find three types of models describing the variation in sedimentation velocity. The most widely used current models are usually basing on Vesilind [26] exponential model (\( V_{s1} = k_1 e^{-nX} \)) and Yoshioka [27] power model (\( V_{s2} = k_2X^{-n_2} \)). Other authors use the Cho et al. [28] function. This function is simply the expression of Vesilind divided by the biomass concentration (\( V_{s3} = k_3e^{-nX}/X \)).

Where \( k_i \) and \( n_i \) are the characteristic biomass parameters, \( X \) is the sludge concentration (total suspended solids), and \( V_{si} \) is the settling velocities. There are many different types of expressions for settling velocity, as a function of sludge concentration that can be found in literature.

The proposed model for the average stabilized values of \( n=140 \text{ mL/g} \) and \( V_o=2.6 \text{ m/h} \) for WWTP of Setif. The initial conditions in these model’s calibration are shown in Table 2. Note that our model is based on the Vesilind function.

This concentration represents the initial concentration (\( C_i \)), i.e. concentration of the mixed liquor in the aeration tank entering the secondary settler. It varies during the day and depending on the peak flow rate (low and medium load). This concentration is subjected at a settling in the secondary settler. It is concentrated at the bottom of this tank, and it is almost zero at the surface. For example, the concentration feeding the secondary settler is 5g/L. This concentration is almost zero at the top layer, and it can attend at 12 g/L at the bottom of the settler.
In order to validate the developed model, a comparison between the experimental results of WWTP-Setif and the predictions of the main decanter model is presented. The experimental results of WWTP were obtained in a settler with a cross section of 1661 m², a diameter of 46 m, a depth of 3 m, and an operating flow rate of 650 m³/h. Figure 3 shows the variation of the sludge concentration as a function of the depth in secondary settler. We see from this figure that the model predictions are encouraging.

5. Conclusion
The model predictions developed is satisfactory, by comparison with the experimental results obtained in this work. This model is expressed as a function of the sludge settleability indices (SVI), whose range of variation is from 116 to 171 mL/g, obtained by performing more than 96 experiments. A one-dimensional dynamic model of the secondary clarifier has been developed, considering both clarification and thickening zones. In this model, we integrated the sedimentation velocity as a function of the sludge settleability index (SVI). The simulation of sludge concentrations and sludge blanket height also fits well with our experimental results. This simulation would be very complementary to the scheme adopted for the management and control of hydraulic surcharges.

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