Electric and magnetic microfields inside and outside space-limited configurations of ions and ionic currents

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Abstract.

The problem of electric and magnetic microfields inside finite spherical systems of stochastically moving ions and outside them is studied. The first possible field of applications is high temperature ion clusters created by laser fields [1]. Other possible applications are nearly spherical liquid systems at room-temperature containing electrolytes. Looking for biological applications we may also think about a cell which is a complicated electrolytic system or even a brain which is a still more complicated system of electrolytic currents. The essential model assumption is the random character of charges motion. We assume in our basic model that we have a finite nearly spherical system of randomly moving charges. Even taking into account that this is at best a caricature of any real system, it might be of interest as a limiting case, which admits a full theoretical treatment.

For symmetry reasons, a random configuration of moving charges cannot generate a macroscopic magnetic field, but there will be microscopic fluctuating magnetic fields. Distributions for electric and magnetic microfields inside and outside such space-limited systems are calculated. Spherical systems of randomly distributed moving charges are investigated. Starting from earlier results for infinitely large systems, which lead to Holtsmark-type distributions, we show that the fluctuations in finite charge distributions are larger (in comparison to infinite systems of the same charge density).

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1. Introduction

Investigations of random microfields inside and outside space-limited bodies have clear physical applications. The first example is the electrical microfields inside spherical clusters of randomly distributed ions [1]. This is a generalization of the classical theory of electrical field fluctuations in infinite one-component systems of random charges [2, 3, 4]. In agreement with the general principles of statistical physics, we have shown that the microfields in small systems of charged particles are comparably stronger than in infinite systems.

Electric microfields outside the randomly charged objects are interesting also, especially electric microfields outside small biological objects. These microfields are good diagnostic instrument.
Magnetic field fluctuations can be considered both inside and outside random moved charged bodies too. There is the generalization of the earlier work devoted to the magnetic field distributions in infinite systems [7]: it uses a recently developed theory of field fluctuations in finite systems [5, 6] which has been applied already to the case of electrical fields. Some estimations for living electrolitic systems will be demonstrated and discussed.

2. The distribution of the electric microfield
Let us start with a short review of our knowledge on the electrical microfield in finite clusters of ions following [1]. We model the clusters as spherical systems of randomly distributed charged particles. Let us consider a system of \( N \) identical charged particles, randomly located in a spherical volume. Let us assume that the particles have the charge \( q \), then the electric field created by a charge at the distance \( r \) is \( q r / r^3 \), here \( r \) is the radius-vector. The \( N \) charged particles create the total field \( E(0) \) at the center of the cluster:

\[
E(0) = -\sum_i q_{eff} r_i / r_i^3
\]

Here, the sum is taken over all particles from \( i = 1 \) to \( i = N \). The notation for the charges of particles \( q_{eff} = q / \epsilon \) means that permeability coefficient \( \epsilon \) is included into the charge. Below, we keep the notation \( q \) for the charge, taking into account to return to \( \epsilon \) if it is not equals unit.

Assume that the probability to find the first particle in the infinitesimal volume \( d r_1 \) around the point \( r_1 \), the second particle in the volume \( d r_2 \) around the point \( r_2 \), etc. up to the \( N \)-th particle is given by the function \( P_N(r_1, .. r_N) \). Then the field distribution function can be represented as:

\[
W(E) = \int \delta(E - \sum_i q r_i / r_i^3) P_N(r_1, .. r_N) dr_1 ... dr_N
\]

Using the representation

\[
\delta(E - E(0)) = \left( \frac{1}{2\pi} \right)^3 \int_{-\infty}^{\infty} dK \exp(iK(E - E(0))),
\]

the basic formula for the calculation of the the function \( W(E) \) is the following:

\[
W(E) = \left( \frac{1}{2\pi} \right)^3 \int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} dK P_N(r_1, .. r_N) dr_1 ... dr_N \exp(iK(E - \sum_i q r_i / r_i^3))
\]

The integral in eq. (3) is solvable if the probability function \( P_N \) can be factorized:

\[
P_N(r_1, .. r_N) = \Pi_{i=1}^N P_1(r_i)
\]

Then the expression (3) can be rewritten as

\[
W(E) = \left( \frac{1}{2\pi} \right)^3 \int_{-\infty}^{\infty} dK \exp(iKE) \Pi_{i=1}^N I_i(K)
\]

where

\[
I_i(K) = \int_V \exp(-i q r_i / r_i^3) P_1(r_i) dr_i
\]
We demonstrate now how this integral can be calculated for uniform probability functions \(^1\):

\[
P_1(r_i) = \frac{1}{V}
\]

where \(V = 4\pi b^3/3\) is the volume of the charged sphere, \(b\) is its radius. In fact, it is a very rough approximation: even factorized probability function \(P_1(r_i)\) has to depend on \(r_i\). Nevertheless, this approximation is analyzed easily and will be very informative. It can be clearly improved numerically. Here and follow we suppose that investigated bodies have a near-spherical shape. Since all integrals \(I_i\) are identical, we obtain:

\[
I(K) = \frac{2}{5} \cos(p_1) + \frac{3\sin(p_1)}{5p_1} - \frac{4\sqrt{2\pi p_1}^{3/2}}{5} \left( \frac{1}{2} - C(\sqrt{\frac{p_1}{2\pi}}) \right) \quad (4)
\]

where \(p_1 = Kq/b^2\) and \(C(z)\) is the Fresnel cosine. The function \(I(K)\) depends only on the absolute value \(K\) of the vector \(K\), this is due to the spherical symmetry. We can perform the limit to very large volumes \(b \to \infty\) in order to get the electric microfield in infinite charged media (plasmas, electrolytes). This transition is done in two steps: first we expand \(I(K)\) into a Taylor series with respect to the (now small) values \(p\) for the function (4):

\[
I(p_1 \to \infty) \simeq 1 - \frac{4\sqrt{2\pi} p_1^{3/2}}{15} \quad (5)
\]

This approximation is valid if the value of \(p_1\) are very small, in fact the argument should be much smaller than 0.1 (see inset Fig.1). We note that the deviations should be less than about \(10^{-3}\) since the distribution function of an electric microfield for a large volume is obtained by carrying out the transition

\[
(1 - a)^N \to \exp(-Na).
\]

This requires that the error in the value \(Na\) has to be small (less than \(1/e\)). For charged media with large volumes this condition is in fact satisfied. Indeed, \(p_1 \sim N^{-2/3}\), and the condition \(p_1 \ll 0.1\) is fulfilled already for \(N \sim 10^6\), i.e for so-called large clusters (see [8, 9]). Let us demonstrate this fact. First of all, the general function \(W(E)\) can be expressed through \(E\) as:

\[
W(E) = \frac{2E}{\pi} \int_0^A dKK \sin(KE)(1 - (KE_1)^{3/2}/N)^N
\]

where \(E_1\) is the characteristic microfield for the case of strict boundary spheres \(E_1 = 2\pi(4/15)^{2/3} qn^{2/3}/V\). The value \(A\) in this expression equals \(p_1 N^{2/3}\) and has to be very large in order to transit to the regular distribution function (3). Indeed, if \(N \sim 10^6\), the value \(A\) achieves 100. This number is enough to make a transition to the limit \(A \to \infty\), and one finds this way again the well-known Holtsmark result:

\[
W_1(\beta_1 = \frac{E}{E_1}) = \frac{2\beta_1}{\pi} \int_{0}^{\infty} x \sin(\beta_1 x) \exp(-x^{3/2})dx
\]

The field strength \(E_1\) is the characteristic unit of the electrical field in the Holtsmark distribution, \(x = KE_1\).

\(^1\) The choice of this form of the function \(P_1\) is due to the fact that all objects below estimated for microfields have strict boundaries
Figure 1. The exact function (4) and approximate fit of an integral $I$. The dashed curve represents the charged medium sphere (exact $I$), the solid curve represents the fit $I = 1 - \frac{b_1}{2}$. The inset: the exact and approximate functions of an integral $I (4)$, (5). The dashed curve represents the charged medium sphere (exact $I$), the solid curve represents the approximation $I_1 = 1 - \frac{4\sqrt{2\pi}p_1^{3/2}}{15}$.

Thus, the normal Holtsmark distribution for the electric microfield fluctuations in the center of charged medium sphere arises if the number of ions $N$ inside the sphere is essentially larger than a million. At the same time, many experiments with high-temperature ion clusters [10, 11] were performed for much smaller clusters: $N \approx 10^3 - 10^4$. This means that the condition for obtaining a proper Holtsmark distribution corresponding to $A \approx p_1 N^{2/3} \sim 3 - 7$ is not satisfied here. In order to include finite size corrections we need an approximation for $p_1 N^{2/3}$ beyond 100, therefore we have to extend the approximation of the integral $I(K)$ up to $p_1 \sim 1 - 5$ at least. The Fig.1 demonstrates the possibility of a linear approximation for this region, since the integral $I(K)$ are well-fitted by the linear function $(1 - p_1/2)$ up to the value $p \approx 2$.

This gives for the upper border of the integral $A \sim 300$, now we may extend the integration to infinity. Of course, due to these approximations the further results for the distribution functions can be considered as an estimation only. Nevertheless, this linear approximation can be used for microfield calculations in clusters since the errors are rather small. A more systematic study of the new field distribution we have to leave to future work.

Using the above linear fits $I(K) \approx 1-p_1/2$, we find the final form of the microfield distribution
functions of strict boundary spheres of charged medium:

\[ W_2 = \frac{4E_2^2E_2}{\pi(E_2^2 + E_2^2)^2} \]  

(7)

Here the characteristic field of the distribution is \( E_2 \sim CN^{1/3}E_1 \) where \( C \) is a numerical constant \( C \approx 0.5 \). The microfield values, corresponding to the distribution given by eq.(7) are larger than those for the normal Holtsmark distribution due to the factor \( N^{1/3} \). Above that, the asymptotics of the distribution (7) has a stronger "tail" of order \( E^{-2} \) in comparison with the Holtsmark distribution which has the tail \( E^{-5/2} \).

What is the physical meaning of the new field distributions? Due to the final size effects, the distribution of the ions in comparably small spheres of charged medium sometimes generate very large fields. In big systems, this effect is averaged out due to the compensation of fields in opposite directions. Therefore we have to expect that in some of the charged medium spheres very large electrical microfields are generated.

There are two interesting explamles of electric microfields in such spherical charged media. The first medium is the above cluster contained deuterium ions. Deuterium nuclei may collide and give the nuclear fusion reaction. This effect may be enhanced provided by a cluster electric microfield \[12\]. For typical high-temperature ion clusters with radius \( R \approx 0.25 \cdot 10^{-6} cm \approx \lambda^{-1} \), \( N \approx 3200 \) \[11\], \( q = e \), \( \epsilon = 1 \) (hydrogen plasma), \( n = n_0 = 5 \cdot 10^{22} cm^{-3} \) (the initial density of ions inside expanded cluster), the characteristic field strengths are \( E_2 \sim 1.6 \cdot 10^7 CGS \). This means, that magnitudes of stochastic electric microfield provided by the "tail" of the distribution (7) can be very large and comparable with the effects found for astrophysical objects \[12\]. The estimated yield of fusion reaction enhanced by this electric microfield is about the experimentally registrated one \[10, 11\].

The electric microfields in live tissues and microobjects is the second example. The live body contains 80 percent of water (brain and blood about 95 percent), therefore it can be considered as the water solution, i.e. electrolyte. The concentration of \( OH^- \) and \( H^+ \) ions in the neutral water with \( pH = 7 \) is about \( 10^{15} cm^{-3} \). The characteristic size of bodies "basic" tissues (brain, for example) \( b \) is \( (1 - 10) cm \). Therefore, the total number of charged particles inside the "basic" tissue is very large \( N \gg 10^6 \), and the first case of microfield distribution in infinite media has to be realized. Considering the probability function \( P_N = V^{-N} \) as the first approximation (this question is still open, see \[13\]), one finds the Holtsmark distribution of an electric microfield inside the large "basic" tissue. The value of permeability coefficient in live tissues is about 10 – 20 for long-wavelength limit of electromagnetic field. Therefore, in this case \( E_1 \sim 10^6 CGS \).

All basic tissues contain small objects: mitohondriues, organells, eritrocites in blood, etc. Their characteristic sizes are about \( (1 - 10) \mu m \). Above that, the concentration of ions inside such objects is one-two order larger than in basic tissue. It means that the total number of charged particles inside such separated objects is about \( 10^3 - 10^5 \), and the microfield distribution corresponds to the second case of finite systems (7). Now the characteristic microfield \( E_2 \) is one-two order larger than in the case of basic tissue: \( E_2 \sim (10^4 - 10^5) CGS \).

Electric microfields outside random charged body can be calculated also. Re-writing the expression (1)

\[ E(R) = -\Sigma_i \frac{q}{s_i^3} s_i \]  

(8)

where \( s_i = R - r_i \), the origin of coordinates is in the center of this sphere contained random charges (ions), \( R \) is the observation point, \( r_i \) is the coordinate of the \( i \)-th ion inside the sphere. Supposing \( R \gg r_i \), one find

\[ \frac{s_i}{s_i^3} \simeq \frac{R - r_i}{R^3} (1 + 3\frac{(Rr_i)}{R^2}) \]
Than the expression (8) can be re-written as

$$
E(R) \simeq -\Sigma_i \frac{q}{R^3} \mathbf{R} + \Sigma_i \frac{q}{R^3} \mathbf{r}_i - 3\Sigma_i \frac{q(\mathbf{R}\mathbf{r}_i)}{R^5} \mathbf{R} =
$$

$$
= E_0 + \Delta E
$$

(9)

Introducing (9) instead of (8) into the function $W(E)$,

$$
W(E) = \left( \frac{1}{2\pi} \right)^3 \int_{-\infty}^{\infty} d\mathbf{K} \exp(i\mathbf{K}(E - E_0)) \Pi_{i=1}^{N} I'_i(K)
$$

where

$$
I'_i(K) = \int_{\Omega} \exp(-i \frac{qK\mathbf{r}_i}{R^3} + i \frac{3q(\mathbf{R}\mathbf{r}_i)}{R^5} \mathbf{K}\mathbf{R}) P_1(r_i) d\mathbf{r}_i
$$

The coulombic term $E_0$ gives the shift in the distribution $W(E)$, i.e. the observable mean field if the total charge of the investigated body is non-zero. Making all calculation with the new value $I'_i(K)$, one finds the fluctuation part of distribution function of microfield at the observation point $\mathbf{R}$ located outside the volume of randomly charged particles.

The most interesting case now is $N \gg 10^6$. It means that the value $I'_i(K)$ can be evaluated easily by expansion into Taylor series over $K$ using homogeniously distributed charges $P_1(r_i) = 1/V$. The exact value of $I'_i(K)$

$$
I'_i(K) = \frac{4\pi b^3}{V(2qKb)^3} (\sin(\frac{2qKb}{R^3}) - \frac{2qKb}{R^3} \cos(\frac{2qKb}{R^3}))
$$

where $b$ is the radius of the investigated sphere. The simple Taylor expansion gives

$$
I'_i(K) = 1 - \frac{2K^2 q^2 b^2}{5R^6}
$$

(10)

and

$$
W(\beta) = \frac{\exp(- (\beta - \beta_0)^2)}{2\sqrt{\pi}}
$$

(11)

where $\beta = E/E'_2$, $\beta_0 = E_0/E'_2$, the characteristic fluctuation field

$$
E'_2 = \sqrt{\frac{2N}{5} \frac{qb}{R^3}}
$$

(12)

One need to remind once more that the condition of the expression fulfillment is $R \gg b$.

For typical microobject size $b = 1 \mu m$, $R = 2b$ newertheless, $n = 10^{16}cm^{-3}$, $N \approx 4 \cdot 10^4$, $q = e$ (hydrogen ion), the characteristic outside electric microfield strengths are $E'_2 \sim (5 - 10) CGS$. This means, that magnitudes of stochastic electric microfield can be very large.
3. Distributions of the magnetic microfield

As shown in [7], the magnetic field distribution has, under appropriate assumptions, a shape which is very similar to the electrical field distribution. If a charge $q$ is located at $\mathbf{r}_i$ and has the velocity $\mathbf{v}_i$ the (nonrelativistic) value of the magnetic field is

$$H_i = \frac{q}{cr_i^3} \mathbf{r}_i \mathbf{v}_i$$

$c$ is light velocity. We model now a configuration of moving ions as a spherical system of randomly distributed moving charged particles. Let us consider a system of $N$ identical charged particles which are randomly moving. Then the magnetic field created at the origin is:

$$H(0) = -\sum_i \frac{q}{cr_i^3} \mathbf{r}_i \mathbf{v}_i$$

Here, the sum is to be taken over all particles from $i = 1$ to $i = N$. Assume that the probability to find the first particle in the infinitesimal volume $d\mathbf{r}_1 d\mathbf{v}_1$ of the phase space the second particle in the volume $d\mathbf{r}_2 d\mathbf{v}_2$ etc. up to the $N$–th particle is given by the function

$$P_N(\mathbf{r}_1, \ldots, \mathbf{r}_N; \mathbf{v}_1, \ldots, \mathbf{v}_N) d\mathbf{r}_1 d\mathbf{v}_1 \ldots d\mathbf{r}_N d\mathbf{v}_N.$$ (15)

Then the field distribution function can be represented as:

$$W(H) = \int \delta(H - \sum_i \frac{q}{cr_i^3} \mathbf{r}_i \mathbf{v}_i) P_N(\mathbf{r}_1, \ldots, \mathbf{r}_N; \mathbf{v}_1, \ldots, \mathbf{v}_N) d\mathbf{r}_1 d\mathbf{r}_N d\mathbf{v}_1 \ldots d\mathbf{v}_N.$$ (16)

Using the representation

$$\delta(H - H(0)) = \left(\frac{1}{2\pi}\right)^3 \int_{-\infty}^{\infty} d\mathbf{K} \exp(i\mathbf{K}(H - H(0))),$$

we find the following basic formula for the calculation of the the function $W(H)$ is the following:

$$W(H) = \left(\frac{1}{2\pi}\right)^3 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} d\mathbf{K} \exp(i\mathbf{K}(H - \sum_i \frac{q}{cr_i^3} \mathbf{r}_i \mathbf{v}_i)))$$

$$P_N(\mathbf{r}_1, \ldots, \mathbf{r}_N; \mathbf{v}_1, \ldots, \mathbf{v}_N) d\mathbf{r}_1 d\mathbf{r}_N d\mathbf{v}_1 \ldots d\mathbf{v}_N.$$ (17)

The integral in eq. (17) over the volume of the phase space $\Omega$ can be evaluated if the probability function $P_N$ can be factorized (see above for $P_1(\mathbf{r}_i)$):

$$P_N(\mathbf{r}_1, \ldots, \mathbf{r}_N; \mathbf{v}_1, \ldots, \mathbf{v}_N) = \Pi_{i=1}^N P_1(\mathbf{r}_i; \mathbf{v}_i)$$

Then the expression (17) can be rewritten as

$$W(H) = \left(\frac{1}{2\pi}\right)^3 \int_{-\infty}^{\infty} d\mathbf{K} \exp(i\mathbf{K}H) \Pi_{i=1}^N I_i(K)$$

where

$$I_i(K) = \int_{\Omega} \exp(-i \frac{q\mathbf{K}[\mathbf{r}_i \mathbf{v}_i]}{cr_i^3}) P_1(\mathbf{r}_i; \mathbf{v}_i) d\mathbf{r}_i d\mathbf{v}_i$$
We demonstrate now how this integral can be calculated for Gaussian velocity and uniform space probability functions assuming:

$$P_1(\mathbf{r}_i; \mathbf{v}_i) = \frac{1}{V} \left(\frac{\Lambda}{\sqrt{\pi}}\right)^3 \exp(-\Lambda^2 v_i^2)$$  \tag{18}$$

Here $\Lambda$ the corresponding value for the velocity space ($1/\Lambda$ is reciprocal characteristic velocity).

Since all integrals $I_i$ are identical for the $P_1$ functions, the integrals can be evaluated. Denoting $p_2 = Kq/2c\Lambda b^2$, we obtain:

$$I(K) = \frac{1}{2} \int_0^\pi \exp(-p_2^2 \sin^2(\theta)) \sin(\theta) d\theta - \frac{p_2^{3/2}}{2} \int_0^\pi \sin^{5/2}(\theta) \int_0^\infty \exp(-x) x^{3/4} dx d\theta$$  \tag{19}$$

The function $I(K)$ depends only on the absolute value $K$ of the vector $\mathbf{K}$, this is due to the spherical symmetry.

Now we can perform the limit to very large volumes $b \rightarrow \infty$ or $p_2 \rightarrow 0$ in order to get the magnetic microfield in infinite plasmas. This transition is done in two steps like it was done for the electric microfield in the previous section. First we expand into a Taylor series with respect to the (now small) values $p_2$ for the function (19):

$$I(p_2 \rightarrow 0) \simeq 1 - p_2^{3/2} \frac{3\pi^{3/2}\sqrt{2}}{8}$$  \tag{20}$$

The validity of this approximation is the same like in the previous section for an electric microfield $p_1$: the value of $p_2$ are very small, in fact the argument should be much smaller than 0.1 (see inset Fig.2). The deviations should be less than about $10^{-3}$ since the distribution function of a magnetic microfield for a large volume is obtained by carrying out the transition

$$(1 - a)^N \rightarrow \exp(-Na).$$

This requires that the error in the value $Na$ has to be small (less than $1/e$). For plasmas with large volumes this condition is in fact satisfied. Indeed, $p_2 \sim N^{-2/3}$, and the condition $p_2 \ll 0.1$ is fulfilled already for $N \sim 10^6$ as in the previous section. Finally, one finds the Holtsmark’s distribution for the magnetic microfield $H_1 = \pi^{5/3}q\Lambda n^{2/3}/2^{3/4}c$ [7]. The value $A$ in this expression equals $pN^{2/3}$ and has to be very large once more in order to transit to the regular distribution function (16). For basic tissues, $pH = 7$ and $n$ is about $10^{15} \text{cm}^{-3}$. The velocity $1/\Lambda$ has the thermal character and reach maximal value for ions $H^+$. Under room temperatures, the factor $1/\Lambda c \sim 0.8 \cdot 10^{-5}$, and $H_1 \simeq 1.1 \cdot 10^{-4} \text{CGS}$. This value is small enough.

Thus, the normal Holtsmark distribution for the magnetic microfield fluctuations in the center of charged medium sphere arises if the number of charges $N$ inside the sphere is essentially larger than a million. Total number of ions in above small objects in live tissues: mitohondriis, organells, eritrocites in blood, etc is about $10^3-10^5$. It means that the condition for a Holtsmark distribution corresponding to $A \simeq p_2N^{2/3} \sim 3 - 7$ is not satisfied once more. In order to obtain an improved distribution we need an approximation for $p_2N^{2/3}$ beyond 100, therefore we have to extend the approximation of the integral $I$ up to $p \sim 1$ at least. The Fig.2 demonstrates the possibility of a linear approximation for this region, since the integral $I(K)$ are well-fitted by the linear function $(1 - 5p_2/6)$ up to the value $p_2 \simeq 1$. 
Figure 2. The exact function (19) and approximate fit of an integral $I$. The dashed curve represents the charged medium sphere (exact $I$), the solid curve represents the fit $I = 1 - \frac{5p_2}{6}$. The inset: the exact and approximate functions of an integral $I$ (19), (20). The dashed curve represents the charged medium sphere (exact $I$), the solid curve represents the approximation $1 - p_2^{3/2} \frac{3\pi^{3/2} \sqrt{2}}{8}$.

This gives for the upper border of the integral $A \sim 100$, now we may extend the integration to infinity.

Using the above linear fits $I(K) \simeq 1 - \frac{5p_2}{6}$, we find the final form of the magnetic microfield distribution function inside small microobjects:

$$W_3 = \frac{4H^2_H_2}{\pi(H^2 + H_2^2)^2}$$  \hspace{1cm} (21)

Here the characteristic magnetic field of the distribution is the value $H_2 = (5/12)(4\pi/3)^{2/3} qn^{2/3} N^{1/3} \simeq 0.4N^{1/3} H_1$. The microfield values, corresponding to the new distributions given by eq.(21) are slightly larger than those for the normal Holtsmark distribution due to the factor $N^{1/3}$. Above that, the asymptotics of the distributions (21) have a stronger "tail" of order $H^{-2}$ in comparison with the Holtsmark distribution which has the tail $H^{-5/2}$.

In principle, the above formula (21) solves the problem of the magnetic microfield distribution inside the volume. Considering live object, no experimental observations of such fields can be performed. In those experiments [13], the fields outside live object were measured. Therefore, the question "how to calculate the distribution function of the (magnetic) microfield outside a live (spherical, in the first approximation) object" has to be generated.
It is clear way to calculate this distribution at the distance larger than the characteristic size of an investigated live object. Re-writing the expression (14)

\[ H(R) = -\sum_i q \frac{c s_i}{s_i} [s_i v_i] \] (22)

where \( s_i = R + r_i \), the origin of coordinates is in the center of this sphere contained moving ions (ionic currents), \( R \) is the observation point, \( r_i \) is the coordinate of the \( i \)-th ion inside the sphere. Supposing \( R \gg r_i \), one find

\[ s_i \approx R - r_i R^3 (1 + 3 (R r_i / R^2)) \]

Than the expression (22) can be re-written as

\[ H(R) \approx -\sum_i q c R^3 [v_i] + 3 \sum_i q c (R r_i / R^2) [v_i] = H_0 + \Delta H \] (23)

Introducing (23) instead of (14) into the function \( W(H) \),

\[ W(H) = \left( \frac{1}{2\pi} \right)^3 \int_{-\infty}^{\infty} dK \exp(i K (H - H_0)) \prod_{i=1}^N I_i'(K) \]

where

\[ I_i'(K) = \int_{\Omega} \exp(-i q K [r_i v_i] / c R^3) + i [K [R v_i]] P_1(r_i; v_i) dr_i dv_i \]

The "regular" term \( H_0 \) gives the shift in the distribution \( W(H) \), i.e. the observable mean field. Making all calculation with the new value \( I_i'(K) \), one finds the fluctuation part of distribution function of microfield at the observation point \( R \) located outside the volume of randomly moved ions (random ionic currents).

The most interesting case now is \( N \gg 10^6 \). It means that the value \( I_i'(K) \) can be evaluated easily by expansion into Taylor series over \( K \) using \( P_1(r_i; v_i) \) from (19). The exact value of \( I_i'(K) \)

\[ I_i'(K) = 2 V \left( \frac{\sqrt{\pi}}{K q} \right)^3 \int_0^{bKq} x \Phi(x) dx \]

where \( b \) is the radius of the investigated sphere, \( \Phi(x) \) is the integral of errors. The simple Taylor expansion gives

\[ I_i'(K) = 1 - \frac{K^2 q^2 b^2}{5 R^3 c^2 \Lambda^2} \] (24)

and

\[ W(\beta) = \frac{\exp(-(\beta - \beta_0)^2)}{2 \sqrt{\pi}} \] (25)
where $\beta = H/H_0^2$, $\beta_0 = H_0/H_0^2$, the characteristic fluctuation field

$$
H_0^2 = \sqrt{\frac{N}{5}} \frac{2qb}{cR^3\Lambda}
$$

(26)

One need to remind once more that the condition of the expression fulfillment is $R \gg b$.

For typical microobject size $R = 1\mu m$, $n = 10^{16} cm^{-3}$, $N \simeq 4 \cdot 10^4$, $q = e$ (hydrogen ion), $1/\Lambda c \sim 0.8 \cdot 10^{-5}$, the characteristic magnetic microfield strengths are $H_2 \sim 1.4 \cdot 10^{-3} CGS$. This means, that magnitudes of stochastic electric microfield provided by the "tail" of the distribution (7) can be very large and produce effects comparable with effects provided by the earth magnetic field.

Note that the magnetic microfield outside live biological object has to be larger than the microfield of the same non-live object. The last object has a thermal motion of free charge in biological electrolites only. There are several addition mechanisms of magnetic microfield gnretation in live objects: protons membrane transport, nerve pulses, etc. Thus the magnetic microfield could be considered as the measure of alive existence of biological bodies.

The ratio of characteristic field of microfield distributions inside inside large object $N \gg 10^6$ and outside large object is about $1 : \frac{b^3}{R^3N^{1/6}}$ for the same $n$ and $N$. For a brain $R = 10 cm$, $n = 10^{15} cm^{-3}$, $q = e$ (hydrogen ion), $1/\Lambda c \sim 0.8 \cdot 10^{-5}$, the characteristic measurable magnetic microfield strength outside a head $R \simeq 2b$ is $H_2 \simeq 1.2 \cdot 10^{-8} CGS$. The experimentally observed values [13, 14] were about $3 \cdot 10^{-8} - 10^{-7} CGS$. The calculated microfield corresponds to the charcteristic Holtsmark fluctuation field inside the brain which is about $3 - 4$ orders larger.

The characteristic field inside cell microobjects can be larger than the above "mean" fluctuation characteristic field (see above).

4. Discussion and conclusions

Again, what is the physical reason for the enchancement of field fluctuations in small clusters? In big systems, the magnitude of electrical field fluctuations is limited due to the compensation of fields with opposite directions. In small system of randomly located charged particles, sometimes very large non-compensated fields are generated. For these reasons, we find a distribution function deviating from the Holtsmark distribution in the region of high fields.

As it was shown in [1] for deuterium ion clusters, an acceleration of the rate of nuclear fusion by $2 - 3$ orders could be due to the action of the above high electric microfields inside the cluster ions.

The above estimations for electric microfields inside live electrolitic structures should be compared with electric fields arizing by some non-random biological process. From this point of view, the Holtsmark microfield in large systems (like brain) could be considered as the "basic chaos". The characteristic microfield value is now about $1 CGS$. The microfield inside small live electrolitic systems: mithohondries, organells, eritrocites in blood, etc., is one-two order larger than in basic tissue. The microfield distribution is correspond to the second case of finite systems (7) with the comparably large tail of strong fluctuations. Now the characteristic microfield is one-two order larger than in the case of basic tissue: $E_2 \sim 10^4 - 10^5 CGS$. This microfield should be compared with, for example, the electric field arizing under cell membrane conductivity. It is known that the electric potential diffrence between two surfaces of membrane is about $100 mV$. The standard membrane width $10 nm$ gives the value of membrane electric field $\sim 10^5 CGS$. This value is one-two order larger than the respective characteristic microfield values. In case of large fluctuation, there is some probability that the electric microfield inside a cell membrane will be compared with the "working" field. For example, the distribution (7) gives that the probability to find total number of microfields $100$ times larger than the characteristic one is about one
percent. If the space correlation length of electric microfield is compared with the membrane depth, the electric microfield can correct the biological process of membrane conductivity.

It looks that the "basic chaos" of measured magnetic microfield outside large live objects (like a brain) gives very small magnitudes \( H^2 \sim 10^{-8} \text{CGS} \) and is too small to make any actions. At the same time, these values outside large objects correspond to Holtsmark magnetic microfields inside large biological systems like brain, etc., are about several orders larger: \( H^2 \sim 10^{-4-5} \text{CGS} \). Large fluctuations of magnetic microfield (with values about the geomagnetic field) are much more probable inside the object comparing with the fluctuations outside them.

The characteristic magnetic microfield inside small object \( H_1 \) is about one-two orders larger than the above value, and the distribution function provides larger number of strong fluctuations than the Holtsmark distribution provides. The "working" magnetic field arizing due to the ionic current through the membrane is about the \( J/cr_m \) where \( J \) is the total current through the membrane, \( r_m \) is the membrane radius. Since \( J \sim nqv_i r_m^2 \) where \( v_i \) is the thermal ionic velocity, one finds the value of "working" magnetic field is about \( 5 \cdot 10^{-2} \). It means that the probability to find the comparable microfield is high enough, and the magnetic microfield may correct the process of "working" charge transfers inside live cells. The comparably stronger action of magnetic microfields with respect to the action of electric ones is explained by the denomination of electric microfield by the factor \( \epsilon \sim 10 \) in live electrolites (note that the magnetic permeability coefficient \( \mu = 1 \) for all interesting media).

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2 Estimating the space correlation length of such microfields by the Debye length in corresponding electrilites, one finds the correlation length \( \sim 10 \text{nm} \) i.e. of the order of the membrane width