STABILITY ANALYSIS OF STAGNATION POINT FLOW IN NANOFLUID OVER STRETCHING/SHRINKING SHEET WITH SLIP EFFECT USING BUONGIORNO’S MODEL

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ABSTRACT. The study on stagnation boundary layer flow in nanofluid over stretching/shrinking sheet with the effect of slip at the boundary was considered by applying the Buongiorno’s model. The partial differential equations of the governing equations were transformed into ordinary differential equations by using appropriate similarity transformation in order to obtain the similarity equations. The equations then were substituted into bvp4c code in Matlab software to get the numerical results. The results of skin friction coefficient, heat transfer coefficient as well as mass transfer coefficient on the governing parameters such as slip parameter, Brownian motion parameter, and thermophoresis parameter are shown graphically. The presence of slip parameter is significantly affected the skin friction, heat and mass transfer coefficient. The smallest number of Brownian motion is sufficient to increase the heat transfer coefficient while largest number of thermophoresis parameter is required to increase mass transfer coefficient. The stability analysis results expressed that the first solution is stable and physically realizable whereas the second solution is not.

1. Introduction. Nanofluid term was first introduced by Stephen U.S. Choi in 1995 which describes the fluid that can enhance the heat transfer rate [3]. One method or model in solving nanofluids problem on boundary layer flow and heat transfer was proposed by Buongiorno [2] in 2006. He claimed that only two main mechanisms out of seven are very important to be studied using the proposed model which was Brownian motion and thermophoresis effect. After that, many of the researchers started to study the stagnation boundary layer flow and heat transfer in nanofluid past a stretching/shrinking sheet because of the numerous application in industrial processes such as in polymer extrusion, drawing of plastics films and wires.

2010 Mathematics Subject Classification. 76D10.
Key words and phrases. Stability analysis, nanofluid, stretching/shrinking, slip effect, bvp4c, Buongiorno model.

The first author is supported by Putra grant of Universiti Putra Malaysia.
The reviewing process of the paper is handled by Gafurjan Ibragimov, Siti Hasana Sapid and Siti Nur Iqmal Ibrahim.

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glass fiber, paper production, blood flow in capillary system as well as blowing of hot air balloon. The quality of final product is depending on the stretching/shrinking rate and also heat transfer rate on stretching/shrinking sheet. The study on the flow and heat transfer can be found in Dzulkifli et al. [4] who considered the Soret and Dufour effects in boundary layer flow depending on the time. Jusoh and Nazar [9] and Hamid et al. [5] have done the work by considering the convective boundary condition as well as non-alignment stagnation boundary layer flow in nanofluid past a stretching/shrinking sheet. Furthermore, Mansur et al. [11, 13] investigates the stagnation boundary layer flow in nanofluid and also in the presence of magnetic effect using Buongiorno’s model.

In some cases or phenomena, the slip effect cannot be ignored because of the relation to the shear rate at the surface. For example, the study on MHD boundary layer flow in a cylindrical case, radiation effect and also thermal convective boundary condition with the effect of slip at the boundary can be found in the paper by Mukhopadhyay [18], Khan et al. [10] and also Mansur and Ishak [12]. Zaimi and Ishak [24] considered the slip flow over a stretching vertical plate. Other than that, Mohamed et al. [17] investigate the slip flow over a stretching sheet in a viscoelastic nanofluid.

The dual or more solutions obtained lead the researchers to perform the stability solutions in order to verify which solution is stable and physically reliable. The first pioneer who studied the stability analysis in their work was Merkin [16]. Later, Weidman et al. [23], Harris et al. [6] and Merill et al. [15] used the implemented method to do the stability solutions. Ishak [8] and Najib et al. [19] have performed the stability analysis on the boundary layer flow in shrinking sheet and cylindrical case, respectively. The investigation of stability solution on boundary layer flow and heat transfer in nanofluid pass a moving plate have been done by Bachok et al. [1] and also Najib et al. [20]. Furthermore, Sharma and Ishak [22] and Rosca and Pop [21] studied the stability analysis in the flow and heat transfer in the presence of second order slip in micropolar fluid and viscous fluid, respectively. The main purpose of this paper is to study the effects of Brownian motion as well as thermophoresis parameter in nanofluid near the stagnation boundary layer flow in the presence of slip effect when the plate stretches/shrinks. The results and graphs obtained will be further discussed in this paper.

2. Mathematical problem. Consider the flow of incompressible nanofluid in the region $y > 0$ driven by a stretching/shrinking surface located at $y = 0$ with a fixed stagnation point at $x = 0$ in the presence of slip effect at the wall. It is assumed that the free stream and stretching/shrinking velocity are assumed in the linear form $U_\infty = cx$ and $U_w = bx$, respectively where $c$ and $b$ are constant with $c > 0$. Note that $b > 0$ and $b < 0$ correspond to stretching and shrinking sheet, respectively. Under these conditions the boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + \nu \frac{\partial^2 u}{\partial y^2},$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \delta \left( D_B \frac{\partial C}{\partial y} + D_T \frac{T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right),$$
\[
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2},
\]
along with the initial and boundary conditions

\[
t < 0 : u = 0, \quad v = 0, \quad T = T_\infty, \quad C = C_\infty, \quad \text{for any} \quad x, y
\]

\[
t \leq 0 : u = U_w + A \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad v = 0, \quad T = T_w, \quad C = C_w, \quad \text{at} \quad y = 0,
\]

\[
u \to U_\infty, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty,
\]

where \( u \) is velocity component along \( x \)-axes, \( v \) is the velocity components along \( y \)-axes, \( T \) is the nanofluid temperature. Navier in 1823 who introduced the slip model which assumed that the velocity of the fluid on the surface of the solid body is proportional to the shear rate on the surface \( \partial u/\partial y \), which is written as \( u = A(\partial u/\partial y) \), see Mehmood and Ali [14]. The no-slip condition is applied for the case \( A = 0 \). The slip condition occurs on the surface of the fluid-solid if the finite slip length \( A \) is taken into account [14].

3. Steady-state Solution (\( \partial/\partial t = 0 \)). Introduce the suitable similarity transformation

\[
\psi = \sqrt{\nu x f(\eta)}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad \eta = \sqrt{\frac{c}{\nu}} y,
\]

where \( \eta \) is the similarity variable and \( \psi \) is the stream function defined as \( u = \partial \psi/\partial y \) and \( v = -\partial \psi/\partial x \), which automatically satisfied Eq. (1). Substitute the similarity transformation (6) into Eqs. (2) to (4) we obtain the ordinary (similarity) differential equations as below.

\[
f''' + ff'' - f'^2 + 1 = 0,
\]

\[
\frac{1}{Pr} \theta''' + f \theta' + Nb \theta' \phi' + Nt \theta'^2 = 0,
\]

\[
\phi'' + Le f \phi' + \frac{Nt}{Nb} \phi'' = 0,
\]

subject to the boundary conditions (5) which becomes boundary condition

\[
f(0) = 0, \quad f'(0) = \varepsilon + \sigma f''(0), \quad \theta(0) = 1, \quad \phi(0) = 1,
\]

\[
f'(\infty) \to 1, \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0.
\]

In the above equations, primes denote the differentiation with respect to \( \eta \). Here \( Pr \) corresponds to Prandt number, \( Le \) refers to Lewis number, \( \sigma \) is the velocity slip parameter and \( \varepsilon \) is the velocity ratio parameter which are defined as

\[
Pr = \frac{\nu}{\alpha}, \quad Le = \frac{\nu}{D_B}, \quad Nb = \frac{\delta D_B}{\nu} (C_w - C_\infty),
\]

\[
Nt = \frac{\delta D_T}{\nu T_\infty} (T_w - T_\infty), \quad \sigma = A \sqrt{\frac{c}{\nu}}, \quad \varepsilon = \frac{b}{c},
\]

where \( \varepsilon > 0 \) for stretching and \( \varepsilon < 0 \) for shrinking.

The physical quantities of interest are the local skin friction coefficient \( C_f \), local Nusselt number \( Nu_x \) and local Sherwood number \( Sh_x \) which are defined as

\[
C_f = \frac{\tau_w}{\rho U_\infty^2}, \quad Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, \quad Sh_x = \frac{x q_m}{D_B (C_w - C_\infty)},
\]
where $\tau_w$ is the skin friction or the shear stresses on the stretching/shrinking sheet, $q_w$ is the heat flux from the surface of the plate and $q_m$ is the mass flux from the surface of the plate, which are given by

$$\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0}, \quad q_m = -D_B \left( \frac{\partial C}{\partial y} \right)_{y=0}. \quad (13)$$

Using Eq. (6) in Eqs. (12) and (13), we obtain

$$Re_x^{1/2} C_f = f''(0), \quad Re_x^{1/2} Nu_x = -\theta'(0), \quad Re_x^{-1/2} Sh_x = -\phi'(0), \quad \text{(14)}$$

where $Re_x = cx^2/\nu$ refers to the local Reynolds number.

4. Stability Analysis. Weidman et al. [23] and Rosca and Pop [21] have stated that the lower branch solutions (second solutions) are not stable (not realizable physically), while the upper branch solutions (first solutions) are stable (physically realizable). So, consider Eqs. (2) – (4) in unsteady form in order to test the characteristics of the solutions mentioned above. Thus, introduce the new non-dimensional time variable $\tau$ which related to an initial value problem. It is corresponding to prove that which solution obtained is in practice (physically realizable). Then, by adding $\tau$ into Eq. (6) which becomes

$$\psi = \sqrt{c^2} x f(\eta, \tau), \quad \theta(\eta, \tau) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta, \tau) = \frac{C - C_\infty}{C_w - C_\infty}, \quad \eta = \sqrt{\frac{c}{\nu}} y, \quad \tau = ct,$$

Hence rewrite Eqs. (2) – (4) which can be expressed as follow

$$\frac{\partial^3 f}{\partial \eta^3} + f \frac{\partial^2 f}{\partial \eta^2} - \frac{\partial f}{\partial \eta} \frac{\partial f}{\partial \eta} + 1 - \frac{\partial^2 f}{\partial \eta \partial \tau} = 0, \quad \text{(16)}$$

$$1 \frac{\partial^2 \theta}{Pr \partial \eta^2} + f \frac{\partial \theta}{\partial \eta} + Nb \frac{\partial \phi}{\partial \eta} \frac{\partial \phi}{\partial \eta} + Nt \left( \frac{\partial \theta}{\partial \eta} \right)^2 - \frac{\partial \theta}{\partial \tau} = 0, \quad \text{(17)}$$

$$\frac{\partial^2 \phi}{\partial \eta^2} + Le f \frac{\partial \phi}{\partial \eta} + Nt \frac{\partial^2 \theta}{Nb \partial \eta^2} - Le \frac{\partial \phi}{\partial \tau} = 0, \quad \text{(18)}$$

correspond to the boundary conditions

$$f(0, \tau) = 0, \quad \frac{\partial f}{\partial \eta}(0, \tau) = \varepsilon + \sigma \frac{\partial^2 f}{\partial \eta^2}, \quad \theta(0, \tau) = 1, \quad \phi(0, \tau) = 1,$$

$$\frac{\partial f}{\partial \eta}(\infty, \tau) \rightarrow 1, \quad \theta(\infty, \tau) \rightarrow 0, \quad \phi(\infty, \tau) \rightarrow 0. \quad \text{(19)}$$

In order to get the stability of the solution $f(\eta) = f_0(\eta)$, $\theta(\eta) = \theta_0(\eta)$ and $\phi(\eta) = \phi_0(\eta)$ fulfill the boundary-value problem (16) – (19), we write

$$f(\eta, \tau) = f_0(\eta) + e^{-\gamma \tau} F(\eta),$$
$$\theta(\eta, \tau) = \theta_0(\eta) + e^{-\gamma \tau} G(\eta),$$
$$\phi(\eta, \tau) = \phi_0(\eta) + e^{-\gamma \tau} H(\eta), \quad \text{(20)}$$

where unknown eigenvalue parameter denoted as $\gamma$, and $F(\eta), G(\eta)$ and $H(\eta)$ are the small relative to $f_0(\eta), \theta_0(\eta)$ and $\phi_0(\eta)$. Solutions of the eigenvalue problem (16) – (19) give an infinite set of eigenvalues $\gamma_1 < \gamma_2 < \gamma_3 \cdots$; if $\gamma_1$ is negative, there is an initial growth of disturbances and the flow is unstable but when $\gamma_1$ is
positive, there is an initial decay and the flow is stable. Introduce Eq. (20) into Eqs. (16) – (19), we obtain the following equations as below.

$$F''''_0 + f_0 F'''_0 + f_0'' F'_0 - 2 f_0' F''_0 + \gamma F'_0 = 0,$$

(21)

$$\frac{1}{Pr} G''''_0 + f_0 G'''_0 + F_0 G''_0 + \frac{Nb}{Nt} \phi_0'' H'_0 + \frac{Nb}{Nt} \phi_0' G'_0 + 2 Nt \phi_0' G'_0 + \gamma G'_0 = 0,$$

(22)

$$H''''_0 + Le f_0 H'''_0 + Le \phi_0' F'_0 + \frac{Nt}{Nb} G''''_0 + Le \gamma H'_0 = 0,$$

(23)

correspond to the boundary conditions

$$F_0(0) = 0, \quad F'_0(0) = \sigma F''''_0(0), \quad G_0(0) = 0, \quad H_0(0) = 0,$$

$$F'_0(\infty) \to 0, \quad G_0(\infty) \to 0, \quad H_0(\infty) \to 0.$$

(24)

It is worth to mention that for particular values of $\varepsilon$ the stability of the corresponding steady flow solutions $f_0(\eta)$, $\theta_0(\eta)$ and $\phi_0(\eta)$ is determined by the smallest eigenvalue $\gamma$. The range of probable eigenvalues can be found by relaxing a boundary condition either on $F'_0(\eta)$, $G_0(\eta)$ and $H_0(\eta)$, see Harris et al. [6]. In this paper, we relax the condition on $F'_0(\infty) \to 0$ and for fix eigenvalue $\gamma$ we complete the set of Eqs. (21) – (23) subject to boundary condition (24) together with the new condition $F''''_0 = 1$.

5. Results and Discussion. The stagnation point flow of nanofluid passing through stretching/shrinking sheet in the presence of slip effect at the boundary using Buongiorno’s model is considered. The numerical results are obtained by substituting Eqs. (7) – (9) along with boundary condition (10) in bvp4c code in Matlab software, where the relative tolerance is set at $10^{-10}$. The results of skin friction coefficient, heat transfer as well as mass transfer coefficient together with velocity, temperature and concentration profiles are physically plotted in Figs. 1 – 4. From our observation, unique solution exists when $\varepsilon > -1$, dual solutions occur in between $\varepsilon_c < \varepsilon \leq -1$ and no solution is found for $\varepsilon < \varepsilon_c$. For some values of slip parameter, $\sigma$ the skin friction coefficient is decreasing as slip parameter is increasing, meanwhile heat transfer coefficient and mass transfer coefficient are increasing as slip parameter is increasing, see Fig. 1. The effects of Brownian motion and thermophoresis parameter on the heat transfer rate at the surface are shown in Figs. 2 – 3. The heat transfer coefficient is increasing with stretching/shrinking parameter $\varepsilon$. Despite that the heat transfer rate on the surface is decreases as $Nb$ and $Nt$ are increasing. This is because once $Nb$ and $Nt$ are increasing the thermal boundary layer thickness will increase. As the thermal boundary layer thickness grows thicker, the temperature gradient at the surface is expected to grow smaller (see Ibrahim et al. [7]). Besides that, the mass transfer rate on the surface is decreasing as $Nb$ is increasing. However, for increasing $Nt$ the mass transfer rate is increasing. Note that the negative value of mass transfer coefficient implies that the surface is losing the nanoparticles to the surrounding area (see Mansur et al. [13]). From our observation, the values of heat and mass transfer rate are converging approximately 0 as $\varepsilon = \varepsilon_c$ and the values of the second solution is seen to approach 0 as $\varepsilon$ approaches $-1$.

Figure 4 shows the velocity, temperature and concentration profiles for different values of . These three profiles obviously fulfill the far field boundary condition (10) asymptotically, at once contribute on the validity of the results obtained and dual solution. From all three profiles, it is clearly seen that the boundary layer thickness
for the first solution is increasing as is increasing whereas for second solution the boundary layer thickness is decreasing as $\varepsilon$ is increasing.

Finally, since we obtained dual solutions in this problem then we introduced the stability solution to identify which solution is stable and physically realizable. Hence, substitute the eigenvalue problems (21) – (23) together with new boundary condition (24) into the third and forth code of bvp4c solver to obtain the smallest eigenvalue $\gamma$. The smallest eigenvalue $\gamma$ is approaching 0 ($\gamma \to 0$) as $\varepsilon$ is approaching $\varepsilon_c$. From Table 1 we can see that all values of the first solution in positive value while all values of the second solution is in negative value. Therefore, the first solution is said to be a stable solution and the physical properties can be realized physically due to slightly disturbance in the flow system that does not interrupt the boundary layer separation whereas the second solution is an unstable solution which express as an early growth of disturbance that interrupt the boundary layer separation. So, the first solution is stable and physically realizable but the second solution is not.

![Figure 1](image_url)

**Figure 1.** Skin friction coefficient $f''(0)$, heat transfer coefficient $-\theta'(0)$ and mass transfer coefficient $-\phi'(0)$ vs $\varepsilon$ for different $\sigma$.  

6. **Conclusions.** This paper considers the effects of slip parameter, Brownian motion as well as thermophoresis parameter along a stagnation boundary layer flow immersed in nanofluid passing through stretching/shrinking surface. The results obtained indicate that the increment of slip parameter decreases the skin friction coefficient but increases the heat and mass transfer coefficient. In addition, the smallest value of Brownian motion parameter is sufficient to increase the heat transfer coefficient. Meanwhile, the largest value of thermophoresis parameter is required to increase the mass transfer coefficient. Lastly, the results of stability solutions expressed that the first solution is a stable solution and physically realizable whereas second solution is not a stable solution and the physical properties cannot be realized physically.
Figure 2. Heat transfer coefficient $-\theta'(0)$ and mass transfer coefficient $-\phi'(0)$ vs $\varepsilon$ for different $Nb$

Figure 3. Heat transfer coefficient $-\theta'(0)$ and mass transfer coefficient $-\phi'(0)$ vs $\varepsilon$ for different $Nt$

Figure 4. Velocity profile $f'(\eta)$, temperature profile $\theta(\eta)$ and concentration profile $\phi(\eta)$ for different $\varepsilon$
Table 1. Smallest eigenvalues $\gamma$ for selected values of $\varepsilon$ with different $\sigma$

| $\sigma$ | $\varepsilon$ | First solution | Second solution |
|----------|----------------|----------------|-----------------|
| 0        | -1.246         | 0.0622         | -0.0614         |
|          | -1.24          | 0.2121         | -0.2036         |
|          | -1.2           | 0.5780         | -0.5172         |
| 0.2      | -1.388         | 0.0802         | -0.0791         |
|          | -1.38          | 0.2390         | -0.2300         |
|          | -1.3           | 0.7707         | -0.6783         |
| 0.4      | -1.582         | 0.0719         | -0.0712         |
|          | -1.58          | 0.1273         | -0.1251         |
|          | -1.5           | 0.6936         | -0.6301         |

Acknowledgments. A great appreciation to Putra Grant of Universiti Putra Malaysia (Project code: GP-IPS/2016/9513000) for the financial support received.

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Received January 2018; 1st revision March 2018; final revision September 2018.

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