Minimal $E_6$ Supersymmetric Standard Model

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Abstract

We propose a Minimal $E_6$ Supersymmetric Standard Model (ME$_6$SSM) which allows Planck scale unification, provides a solution to the $\mu$ problem and predicts a new $Z'$. Above the conventional GUT scale $M_{GUT} \sim 10^{16}$ GeV the gauge group corresponds to a left-right symmetric Supersymmetric Pati-Salam model, together with an additional $U(1)_\psi$ gauge group arising from an $E_6$ gauge group broken near the Planck scale. Below $M_{GUT}$ the ME$_6$SSM contains three reducible 27 representations of the Standard Model gauge group together with an additional $U(1)_X$ gauge group, consisting of a novel and non-trivial linear combination of $U(1)_\psi$ and two Pati-Salam generators, which is broken at the TeV scale by the same singlet which also generates the effective $\mu$ term, resulting in a new low energy $Z'$ gauge boson. We discuss the phenomenology of the new $Z'$ gauge boson in some detail.

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1 Introduction

The question of unification of all the forces of Nature is one of the most bold and intriguing in all of physics. It would appear to be somewhat esoteric or premature but for the possibility that that discoveries at the CERN LHC may provide an unprecedented opportunity to shed light on this question. For example, it is well known that, without new physics, the electroweak and strong gauge couplings extracted from LEP data and extrapolated to high energies using the renormalisation group (RG) evolution do not meet within the Standard Model (SM), so unification appears to require some new physics. One example of new physics that can lead to unification is TeV scale supersymmetry (SUSY). For example, in the minimal supersymmetric standard model (MSSM) [1], the gauge couplings converge to a common value at a high energy scale $M_{\text{GUT}} \sim 10^{16}$ GeV, allowing some supersymmetric Grand Unified Theory (GUT) to emerge above this scale. Thus evidence for the MSSM at the LHC would provide support for unification at $M_{\text{GUT}}$.

However, despite its obvious attractions, the standard paradigm of SUSY GUTs based on the MSSM faces some serious shortcomings. On the one hand, the failure to discover superpartners or the Higgs boson by the LEP and the Tevatron indicates that the scale of SUSY breaking must be higher than previously thought, leading to fine-tuning at the per cent level. On the other hand experimental limits on proton decay and the requirement of Higgs doublet-triplet splitting provides some theoretical challenges at the high scale [2–4]. Related to the doublet-triplet splitting problem is the origin of $\mu$, the SUSY Higgs and Higgsino mass parameter, which from phenomenology must be of order the SUSY breaking scale, but which a priori is independent of the SUSY breaking scale. Apart from these problems, unification of gauge couplings near $M_{\text{GUT}} \sim 10^{16}$ GeV leaves open the question of a full unification of all the forces with gravity, although this may be achieved in the framework of string unification, including high energy threshold effects [5].

The challenges facing SUSY GUTs based on the MSSM motivate alternative approaches that successfully overcome these problems. Recently an $E_6$ Supersymmetric Standard Model ($E_6\text{SSM}$) has been proposed [6, 7], in which the low energy particle content consists of three irreducible $\mathbf{27}$ representations of the gauge group $E_6$ plus, in addition, a pair of non-Higgs doublets $H', \overline{H'}$ arising from incomplete $\mathbf{27}', \overline{\mathbf{27}}$ representations. In the $E_6\text{SSM}$ gauge coupling unification works very well, even better than in the MSSM [8]. Moreover, the $E_6\text{SSM}$ also solves the $\mu$ problem via a singlet coupling to two Higgs doublets. In the $E_6\text{SSM}$ a special role is played by a low energy gauged $U(1)_{N}$ symmetry, which consists of a particular linear combination of Abelian generators contained in the $SU(5)$ breaking chain of $E_6$. The $U(1)_{N}$ results from GUT scale Higgs which develop vacuum expectation values (VEVs) in their right-handed neutrino components. As a consequence the right-handed neutrino carries zero charge under $U(1)_{N}$, thereby allowing a conventional see-saw mechanism via heavy right-handed Majorana masses. The $U(1)_{N}$ is anomaly-free since the low energy theory contains complete $\mathbf{27}'s$ (minus the heavy chargeless right-handed neutrinos) and $H', \overline{H'}$, which have opposite $U(1)_{N}$ charges. The $U(1)_{N}$ also serves to forbid the $\mu$ term, but allow the singlet cou-
pling, and is broken by the singlet VEV which generates the effective $\mu$ term. Without the gauged $U(1)_X$ symmetry there would be a Peccei-Quinn global symmetry of the theory resulting in an unwanted Goldstone boson. In the next-to-MSSM (NMSSM) this global symmetry is broken explicitly by a cubic singlet term, however this only reduces the symmetry to $Z_3$ and so the singlet VEV then leads to dangerous cosmological domain walls. In the $E_6$SSM, with a gauged $U(1)_X$ symmetry, the cubic singlet term is forbidden and the resulting would-be Goldstone boson of the theory is eaten by the Higgs mechanism to produce a massive $Z'$ gauge boson.

However, although the $E_6$SSM solves the $\mu$ problem, the presence of the non-Higgs doublets $H', \overline{H'}$ cannot be regarded as completely satisfactory since it introduces a new $\mu'$ problem and in this case a singlet coupling generating the mass $\mu'$ for $H', \overline{H'}$ is not readily achieved [6, 7]. Similarly, although the $E_6$SSM solves the usual doublet-triplet splitting problem, since the usual Higgs doublets are contained along with colour triplets in complete low energy 27 representations, the presence of the low energy $H', \overline{H'}$ introduces a new doublet-triplet splitting problem because their triplet partners are assumed to be very heavy. However, since the only purpose of including the non-Higgs states $H', \overline{H'}$ is to help achieve gauge coupling unification at $M_{\text{GUT}} \sim 10^{16}$ GeV, it is possible to consider not introducing $H', \overline{H'}$ at all, thereby allowing a solution to the $\mu$ problem and the doublet-triplet splitting problem, without re-introducing new ones. An immediate objection to removing the non-Higgs states $H', \overline{H'}$ of the $E_6$SSM is that the gauge couplings will no longer converge at $M_{\text{GUT}} \sim 10^{16}$ GeV, so at first sight this possibility looks unpromising. In a recent paper we showed how this objection could be overcome by embedding the theory into a left-right symmetric Pati-Salam theory at $M_{\text{GUT}} \sim 10^{16}$ GeV, leading to a unification of all forces with gravity close to the Planck scale [9]. However, the analysis did not include the effects of an additional low energy $U(1)'$ gauge group that would be required for a consistent resolution of the $\mu$ problem.

The purpose of the present paper is to propose a Minimal $E_6$ Supersymmetric Standard Model (ME$_6$SSM) based on three low energy reducible 27 representations of the Standard Model gauge group which allows Planck scale unification and provides a solution to the $\mu$ problem and doublet-triplet splitting problem, without re-introducing either of these problems. Above the conventional GUT scale the ME$_6$SSM is embedded into a left-right symmetric Supersymmetric Pati-Salam model with an additional $U(1)'_\psi$ gauge group arising from an $E_6$ gauge group broken near the Planck scale. In our previous analysis [9] we assumed for simplicity that the $U(1)'_\psi$ gauge group was broken at the Planck scale. Here we assume that $U(1)'_\psi$ remains unbroken down to $M_{\text{GUT}}$ and that below $M_{\text{GUT}}$ an additional $U(1)_X$ gauge group, consisting of a novel and non-trivial linear combination of $U(1)'_\psi$ and two Pati-Salam generators, survives down to low energies. Eventually $U(1)_X$ is broken at the TeV scale by the same singlet that also generates the effective $\mu$ term, resulting in a new low energy $Z'$ gauge boson. We discuss the phenomenology of the new $Z'$ gauge boson in some detail. We compare the $Z'$ of the ME$_6$SSM (produced via the Pati-Salam breaking chain of $E_6$, where $E_6$ is broken at the Planck scale) to the $Z'$ of the $E_6$SSM (from the $SU(5)$ breaking chain of $E_6$, where $E_6$ is broken at the GUT scale) and discuss how they can be distinguished by their different couplings, which enable the two models to be resolved experimentally. In the case of the
ME$_6$SSM the $Z'$ gauge boson properties can be said to provide a window on Planck scale physics. In both cases the right-handed neutrinos carry zero charge under the extra low energy $U(1)'$ gauge groups, allowing a conventional see-saw mechanism.

The layout of the rest of this paper is as follows. In the section 2 we consider the pattern of symmetry breaking assumed in this paper. In section 3 we consider the two loop RG evolution of gauge couplings in this model from low energies, through the Pati-Salam breaking scale at $M_{GUT} \sim 10^{16}$ GeV (assuming various Pati-Salam breaking Higgs sectors) and show that the Pati-Salam gauge couplings converge close to the Planck scale $M_p \sim 10^{19}$ GeV. In section 4 we discuss the phenomenology of the new $Z'$ of the ME$_6$SSM and compare it to that of the E$_6$SSM. In section 5 we shall construct an explicit supersymmetric model of the kind we are considering. Finally we conclude the paper in section 5.

2 Pattern of Symmetry Breaking

The two step pattern of gauge group symmetry breaking that we analyse in this paper is:

$$E_6 \overset{M_p}{\longrightarrow} G_{4221} \otimes D_{LR} \overset{M_{GUT}}{\longrightarrow} G_{3211}$$

(1)

where the gauge groups are defined by:

$$G_{4221} \equiv SU(4) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_\psi,$$
$$G_{3211} \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$$

(2)

and we have assumed that the first stage of symmetry breaking happens close to the Planck scale and the second stage happens close to the conventional GUT scale. The first stage of symmetry breaking is based on the maximal $E_6$ subgroup $SO(10) \otimes U(1)_\psi$ and the maximal $SO(10)$ subgroup $G_{422} \otimes D_{LR}$ corresponding to a Pati-Salam symmetry with $D_{LR}$ being a discrete left-right symmetry.$^3$ The pattern of symmetry breaking assumed in this paper is different from that commonly assumed in the literature based on the maximal $SO(10)$ subgroup $SU(5) \otimes U(1)_\chi$ [6,7,11]. In particular, the Pati-Salam subgroup does not contain the Abelian gauge group factor $U(1)_\chi$. The only Abelian gauge group factor involved in this pattern of symmetry breaking is $U(1)_\psi$, with a low energy $U(1)_X$ emerging along with $U(1)_Y$ below the Pati-Salam symmetry breaking scale.

The first stage of symmetry breaking close to $M_p$ will not be considered explicitly in this paper. We only remark that the Planck scale theory may or may not be based on a higher dimensional string theory. Whatever the quantum gravity theory is, it will involve some high energy threshold effects, which will depend on the details of the high

$^3$Under $D_{LR}$ the matter multiplets transform as $q_L \rightarrow q'_L$, and the gauge groups $SU(2)_L$ and $SU(2)_R$ become interchanged [10].
energy theory and which we do not consider in our analysis. Under \( E_6 \to G_{4221} \) the fundamental \( E_6 \) representation \( 27 \) decomposes as:

\[
27 \to (4, 2, 1)_{\frac{1}{2}} + (\overline{4}, 1, \overline{2})_{\frac{1}{2}} + (1, 2, 2)_{-1} + (6, 1, 1)_{-1} + (1, 1, 1)_2
\]

(3)

where the subscripts are related to the \( U(1)_\psi \) symmetry’s charge assignments \([12, 13]\). One family of the left-handed quarks and leptons can come from the \( (4, 2, 1)_{\frac{1}{2}} \); one family of the charge-conjugated quarks and leptons, including a charge-conjugated neutrino \( \nu^c \), can come from the \( (\overline{4}, 1, \overline{2})_{\frac{1}{2}} \); the MSSM Higgs bosons can come from the \( (1, 2, 2)_{-1} \); two colour-triplet weak-singlet particles can come from the \( (6, 1, 1)_{-1} \); and the \( (1, 1, 1)_2 \) is a MSSM singlet.

The second stage of symmetry breaking close to \( M_{GUT} \) is within the realm of conventional quantum field theory and requires some sort of Higgs sector, in addition to the assumed matter content of three \( 27 \) representations of the gauge group \( E_6 \). In order to break the symmetry \( G_{4221} \) to \( G_{3211} \) at \( M_{GUT} \), the minimal Higgs sector required are the \( G_{4221} \) representations \( \overline{H_R} = (\overline{4}, 1, \overline{2})_{\frac{1}{2}} \) and \( H_R = (4, 1, 2)_{-\frac{1}{2}} \). When these particles obtain VEVs in the right-handed neutrino directions \( \langle \overline{H_R} \rangle = \langle \nu^c_R \rangle \) and \( \langle H_R \rangle = \langle \nu^H_R \rangle \) they break the \( SU(4) \otimes SU(2)_R \otimes U(1)_\psi \) symmetry to \( SU(3)_c \otimes U(1)_Y \otimes U(1)_X \). Six of the off-diagonal \( SU(4) \) and two of the off-diagonal \( SU(2)_R \) fields receive masses related to the VEV of the Higgs bosons. The gauge bosons associated with the diagonal \( SU(4) \) generator \( T^+_4 \), the diagonal \( SU(2)_R \) generator \( T^3_R \) and the \( U(1)_\psi \) generator \( T_\psi \), are rotated by the Higgs bosons to create one heavy gauge boson and two massless gauge bosons associated with \( U(1)_Y \) and \( U(1)_X \). The part of the symmetry breaking \( G_{4221} \) to \( G_{3211} \) involving the diagonal generators is then:

\[
U(1)T^+_4 \otimes U(1)T^3_R \otimes U(1)_\psi \to U(1)_Y \otimes U(1)_X,
\]

(4)

where the charges of the “right-handed neutrino” component of the Higgs which gets the VEV are:

\[
\nu^H_R = \left( \begin{array}{c}
-1/2 \sqrt{3/2} \\
1/2 \\
-1/2 \sqrt{1/6}
\end{array} \right)
\]

(5)

under the corresponding correctly normalized diagonal generators\(^4\)

\[
T^+_4 = \sqrt{3/2} \text{diag}(1, 1/6, 1/6, -1/2), \quad T^3_R = \frac{1}{2} \text{diag}(1, -1), \quad T_\psi/\sqrt{6}.
\]

(6)

\(^4\)We show in the Appendix that the symmetry breaking \( G_{4221} \) to \( G_{3211} \) also requires an MSSM singlet \( S \), from a \( 27 \) multiplet of \( E_6 \), to get a low-energy VEV. The VEV of this MSSM singlet is also used to solve the \( \mu \) problem.

\(^5\)Note that we choose to normalize the \( E_6 \) generators \( G^a \) by \( Tr(G^a G^b) = 3\delta^{ab} \). It then follows that the Pati-Salam and standard model operators are conventionally normalized by \( Tr(T^a T^b) = \frac{1}{2} \delta^{ab} \). The correctly normalized \( E_6 \) generator corresponding to \( U(1)_\psi \) is \( T_\psi/\sqrt{6} \) where \( T_\psi \) corresponds to the charges in Eq\(^3\).
In the Appendix we discuss the symmetry breaking in Eq.4 in detail. To simplify the discussion we observe that
\[ T_{15}^1 = \sqrt{3} \frac{(B-L)}{2} \]
where \( B \) and \( L \) are the baryon and lepton number assignments of each Standard Model particle. The Higgs charges can then be written as
\[ \nu_R^H = \left( -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right) \]
under the corresponding generators \( T_{B-L}^1 = \frac{B-L}{2}, T_R^3 \) and \( T_\psi \). Then it is clear to see why the hypercharge generator \( Y \) is preserved by the Higgs \( H_R \) and \( \overline{H}_R \) since
\[ Y = T_R^3 + \frac{(B-L)}{2} \]
takes a zero value for the right-handed neutrino and anti-neutrino Higgs components which develop VEVs. However this is not the only Abelian generator that is preserved by this Higgs sector. The Higgs \( H_R \) and \( \overline{H}_R \) VEVs also preserve the combinations of generators \( T_\psi - T_{B-L} \) and \( T_\psi + T_R^3 \).

As shown in the Appendix, precisely one additional Abelian generator orthogonal to \( U(1)_Y \) is preserved, namely:
\[ X = (T_\psi + T_R^3) - c_{12}^2 Y \]
where \( c_{12} = \cos \theta_{12} \) and the mixing angle is given by
\[ \tan \theta_{12} = \frac{g_{2R}}{g_{B-L}}, \quad g_{B-L} = \sqrt{\frac{3}{2}} g_4, \]
where the \( E_6 \) normalized Pati-Salam coupling constants \( g_{2R} \) and \( g_4 \) are evaluated at the \( G_{4221} \) symmetry breaking scale \( M_{GUT} \). Note that this Abelian generator \( X \) depends on the values that the Pati-Salam coupling constants take at a particular energy scale. It is easy to prove that it is a general rule that, if three massless gauge fields are mixed, then at least two of the resulting mass eigenstate fields must have a charge that depends on the value of the original gauge coupling constants. See the Appendix for more discussion on this unusual aspect of \( X \).

The “GUT” (in this case \( E_6 \)) normalized generator is
\[ T_X = \frac{1}{N_X} X \]
where the normalization constant \( N_X \) is given by:
\[ N_X^2 \equiv 7 - 2c_{12}^2 + \frac{5}{3} c_{12}^4 \]
where \( g \) is a constant and \( G \) is the symmetry breaking scale, therefore \( X \) represents the normalized hypercharge generator \( T_\psi \). From Eq.8 one finds the following relation between the hypercharge gauge coupling constant \( g_Y \) and the \( SU(4) \) and \( SU(2)_R \) gauge coupling constants \( g_4 \) and \( g_{2R} \) respectively:

\[
\frac{1}{\alpha_Y} = \frac{1}{\alpha_{2R}} + \frac{1}{\frac{3}{2} \alpha_4} \tag{15}
\]

From Eq.9, the corresponding gauge coupling constant \( g_X^0 \) may be expressed in terms of the \( SU(4) \), \( SU(2)_R \) and \( U(1)_\psi \) gauge coupling constants \( g_4, g_{2R} \) and \( g_\psi \) as:

\[
\frac{1}{\alpha_X^0} = \frac{1}{\alpha_\psi} + \frac{1}{\frac{2}{3} \alpha_4 + \alpha_{2R}} \tag{13}
\]

where \( \alpha_X^0 = \frac{(g_Y)^2}{4\pi} \), \( \alpha_{2R} = \frac{g_{2R}^2}{4\pi} \), \( \alpha_4 = \frac{g_4^2}{4\pi} \); and \( \alpha_\psi = \frac{g_\psi^2}{4\pi} \). In terms of the \( E_6 \) normalized generator \( T_X = X/N_X \), the normalized gauge coupling constant \( g_X = g_X^0 N_X \) so that \( \alpha_X = \alpha_X^0 N_X^2 \) giving:

\[
\frac{N_X^2}{\alpha_X} = \frac{6}{\alpha_\psi} + \frac{2}{3 \alpha_4 + 2 \alpha_{2R}}. \tag{14}
\]

The boundary condition in Eq.14 applies at the symmetry breaking scale \( M_{\text{GUT}} \).

From Eq.8 one finds the following relation between the hypercharge gauge coupling constant \( g_Y \) and the \( SU(4) \) and \( SU(2)_R \) gauge coupling constants \( g_4 \) and \( g_{2R} \) respectively:

\[
\frac{1}{\alpha_Y} = \frac{1}{\alpha_{2R}} + \frac{1}{\frac{3}{2} \alpha_4} \tag{15}
\]

where \( \alpha_Y = \frac{g_Y^2}{4\pi} \), \( \alpha_{2R} = \frac{g_{2R}^2}{4\pi} \) and \( \alpha_4 = \frac{g_4^2}{4\pi} \). In terms of the “GUT” (in this case \( E_6 \)) normalized hypercharge generator \( T_Y = \sqrt{3} Y \), the coupling constant is \( g_1 \equiv \sqrt{\frac{3}{2}} g_Y \):

\[
\frac{5}{\alpha_1} = \frac{3}{\alpha_{2R}} + \frac{2}{\alpha_4} \tag{16}
\]

where \( \alpha_1 = \frac{g_1^2}{4\pi} \). Eq.15 is a boundary condition for the gauge couplings at the Pati-Salam symmetry breaking scale, in this case \( M_{\text{GUT}} \). Due to the left-right symmetry \( D_{LR} \), at the \( G_{4221} \) symmetry breaking scale we have the additional boundary condition \( \alpha_{2L} = \alpha_{2R} \).

In Table 1 we list the values that the generators \( Y, T_{B-L}, T^3_R, T_\psi \) and \( T_\psi + T^3_R \) (and therefore \( X \)) take for the \( G_{3211} \) representations of the \( 27 \) multiplet. Note that both

| \( T_{B-L} \) | \( T^3_R \) | \( T_\psi \) | \( Y \) | \( T_\psi + T^3_R \) |
|---|---|---|---|---|
| \( \frac{1}{6} \) | \( 0 \) | \( \frac{1}{6} \) | \( 0 \) | \( 0 \) |
| \( \frac{1}{2} \) | \( 0 \) | \( \frac{1}{2} \) | \( 0 \) | \( 0 \) |
| \( 0 \) | \( \frac{1}{3} \) | \( \frac{1}{3} \) | \( 1 \) | \( -1 \) |
| \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) |
which is then broken to the standard model gauge group and a right symmetric Higgs sector capable of breaking the extended Higgs sector including their left-right symmetric partners. A minimal left-inning the 27 according to our assumptions there are three complete corresponding to the pattern of symmetry breaking discussed in the previous section.

In this section we perform a SUSY two-loop RG analysis of the gauge coupling constants, from those usually considered in the literature based on linear combinations of the subgroups $U(1)_\psi$ and $U(1)_X$ since, in the ME$_6$SSM, $U(1)_X$ is necessarily broken at $M_p$. In particular the $Z'$ of the ME$_6$SSM based on $U(1)_X$ and that of the E$_6$SSM based on $U(1)_X$ will have different physical properties, which we will explore later.

3 Two-Loop RGEs Analysis

In this section we perform a SUSY two-loop RG analysis of the gauge coupling constants, corresponding to the pattern of symmetry breaking discussed in the previous section. According to our assumptions there are three complete 27 SUSY representations of the gauge group $E_6$ in the spectrum which survive down to low energies, but, unlike the original E$_6$SSM, there are no additional $H'$, $\overline{H}$ states and so the gauge couplings are not expected to converge at $M_{GUT}$. We therefore envisage the pattern of symmetry breaking shown in Eq.(1) Above the $G_{4221}$ symmetry breaking scale $M_{GUT}$ we assume, in addition to the three 27 representations, some $G_{4221}$ symmetry breaking Higgs sector.

Although a Higgs sector consisting of $H_R$ and $\overline{H}_R$ is perfectly adequate for breaking the $G_{4221}$ symmetry, it does not satisfy $D_{LR}$. We must therefore also consider an extended Higgs sector including their left-right symmetric partners. A minimal left-right symmetric Higgs sector capable of breaking the $G_{4221}$ symmetry consists of the $SO(10) \otimes U(1)_\psi$ Higgs states $(\overline{16}_H)_1$ and $(\overline{16}_H)_{-\frac{1}{2}}$, where $(\overline{16}_H)_1 = (4, 2, 1)_{\frac{1}{2}} + (4, 1, 2)_{\frac{1}{2}}$ and $(\overline{16}_H)_{-\frac{1}{2}} = (4, 2, 1)_{-\frac{1}{2}} + (4, 1, 2)_{-\frac{1}{2}}$, where the components which get VEVs are $\overline{H}_R = (4, 1, 2)_{\frac{1}{2}}$ and $H_R = (4, 1, 2)_{-\frac{1}{2}}$. If complete $E_6$ multiplets are demanded in the entire theory below $M_p$, then the Pati-Salam breaking Higgs sector at $M_{GUT}$ may be assumed to be $27_H$ and $\overline{27}_H$. Therefore, in our analysis we shall consider two possible Higgs sectors which contribute to the SUSY beta functions in the region between $M_{GUT}$ and $M_p$, namely either $(\overline{16}_H)_{\frac{1}{2}} + (\overline{16}_H)_{-\frac{1}{2}}$ or $27_H + \overline{27}_H$, where it is understood that only the $G_{4221}$ gauge group exists in this region, and these Higgs representations must be decomposed under the $G_{4221}$ gauge group.

We therefore investigate the running of the gauge coupling constants at two-loops for an $E_6$ theory that contains three complete 27 multiplets at low energies and either a $(\overline{16}_H)_{\frac{1}{2}} + (\overline{16}_H)_{-\frac{1}{2}}$ or a $27_H + \overline{27}_H$ above the $G_{4221}$ symmetry breaking scale. The $E_6$ symmetry is assumed to be broken to a left-right symmetric $G_{4221}$ gauge symmetry which is then broken to the standard model gauge group and a $U(1)_X$ group as described.
Figure 1: This figure illustrates the two-loop RGEs running of the gauge coupling constants of two different $E_6$ GUTs with an intermediate left-right symmetric $G_{4221}$ symmetry. Both models contain three $27$ supermultiplets of $E_6$ at low energies which contain all the MSSM states as well as new (non-MSSM) states. For the $E_6$ model in the left-panel we assume effective MSSM and non-MSSM thresholds of $250$ GeV and $1.5$ TeV respectively. For the $E_6$ model in the right-panel we assume slightly larger effective MSSM and non-MSSM thresholds of $350$ GeV and $2.1$ TeV respectively. Above the $G_{4221}$ symmetry breaking scale we include the effects of $SO(10) \otimes U(1)_Y$ supermultiplets $(16_H)^+ + (\overline{16}_H)^-$ or $E_6$ supermultiplets $27_H + \overline{27}_H$ for the left-panel and right-panel respectively. From the two-loop RGE analysis we find that the left-right symmetric $G_{4221}$ symmetry is broken at $10^{16.45(3)}$ GeV or $10^{16.40(3)}$ GeV and that gauge coupling unification occurs at $10^{19.05(8)}$ GeV or $10^{19.10(10)}$ GeV for the left-panel and right-panel respectively. For both panels we calculate that $c_{12}^2 \equiv \cos \theta_{12}$ is equal to $0.71$ to two significant figures where $\tan \theta_{12} \equiv g_{2R}/g_{2L}$ and the gauge coupling constants are evaluated at the $G_{4221}$ symmetry breaking scale. The $U(1)_X$ charge is given by $X/N_X$ where $X = (T_R^H + T_L^\psi) - c_{12}^2 Y$ and $N_X^2 = 7 - 2c_{12}^2 + \frac{5}{3}c_{12}^4$.

In section 2. In the previous section we discussed the relation in Eq.(16) between the hypercharge and Pati-Salam gauge coupling constants at the $G_{4221}$ symmetry breaking scale. This can be turned into a boundary condition involving purely Standard Model gauge couplings constants at the $G_{4221}$ breaking scale, since $SU(3)_c$ comes from $SU(4)_c$ so $\alpha_3 = \alpha_4$ at this scale, and, as remarked, the $D_{LR}$ symmetry requires that $\alpha_{2R} = \alpha_{2L}$ at the $G_{4221}$ symmetry breaking scale. Therefore Eq.(16) can be re-expressed as:

\[
\frac{5}{\alpha_1} = \frac{3}{\alpha_{2L}} + \frac{2}{\alpha_3}
\]  

(17)

Having specified the low energy matter content and thresholds, Eq.(17) allows a unique determination of the Pati-Salam breaking scale, by running up the gauge couplings until the condition in Eq.(17) is satisfied. In practice we determine the Pati-Salam symmetry breaking scale to be close to the conventional GUT energy scale, and this justifies our use of the notation $M_{GUT}$ to denote the Pati-Salam breaking scale. Above the scale $M_{GUT}$ we run up the two Pati-Salam gauge couplings, namely $\alpha_4$ and $\alpha_{2L} = \alpha_{2R}$, including, in addition to the three SUSY $27$ matter representations, also a Pati-Salam SUSY Higgs breaking sector consisting of either $(16_H)^+ + (\overline{16}_H)^-$ or $27_H + \overline{27}_H$. The values of the gauge coupling constants meet at a high energy scale close to the Planck scale $M_p = 1.2 \times 10^{19}$ GeV, which suggests a Planck scale unification of all forces with
Table 2: This table lists the values that the charges $Y$, $X$ and $N$ take for the all the $G_{3211}$ representations of the $E_6$ 27 multiplet. $Y$ is hypercharge, $X$ is the charge of $U(1)_X$ for the model presented in sections 3 and 4 and $N$ is the charge associated with the $U(1)_N$ group in the $E_6$SSM. These charges are normalized by the $E_6$ normalization constants $N_X^Y = \frac{2}{3}$, $N_X^X = \frac{6}{27}$ and $N_X^N = 40$ so that the $E_6$ normalized charges of the $U(1)_Y$, $U(1)_X$ and $U(1)_N$ groups are given by $T_Y \equiv Y/N_Y$, $T_X \equiv X/N_X$ and $T_N \equiv N/N_N$ respectively, which are also given in the table. $N_X$ and $X$ have been calculated for the case when the mixing angle $\tan \theta_{12} = g_{2R}/g_{B-L}$ is given by $\cos^2 \theta_{12} \equiv \frac{e_{12}^2}{4} = 5/7$ which, to two significant figures, agrees with the two-loop RGEs analysis presented for the two $E_6$ theories in section 3.

The results are shown in Figure 1. For the $E_6$ theory that contains the $(16_H)_{\frac{2}{3}} + (\overline{16}_H)_{-\frac{1}{3}}$ particles, we take a low energy effective threshold of 250 GeV for the MSSM states and therefore an effective threshold of $(6 \times 250) = 1.5$ TeV for the rest of states of the three complete 27 multiplets as assumed in [9], which follows the analysis of effective MSSM thresholds from [8]. For the $E_6$ theory that contains the $27_H + \overline{27}_H$ particles, the MSSM threshold must be increased to 350 GeV (and hence the 1.5 TeV threshold is increased to 2.1 TeV) to ensure unification for the gauge coupling constants of the $G_{4221}$ symmetry. We run the gauge couplings up from low energies to high energies, using as input the SM gauge coupling constants measured on the Z-pole at LEP, which are as follows [14]: $\alpha_1(M_Z) = 0.016947(6)$, $\alpha_2(M_Z) = 0.033813(27)$ and $\alpha_3(M_Z) = 0.1187(20)$. The general two-loop beta functions used to run the gauge couplings can be found in [15]. Using a two-loop renormalization group analysis, we calculate that the $G_{4221}$ symmetry is broken at $10^{16.45(3)}$ GeV or $10^{16.40(3)}$ GeV and that gauge coupling unification occurs at $10^{18.95(8)}$ GeV or $10^{19.10(10)}$ GeV for the $E_6$ theories that contain $(16_H)_{\frac{2}{3}} + (\overline{16}_H)_{-\frac{1}{3}}$ or $27_H + \overline{27}_H$ respectively. In [9] the same two-loop RGE analysis was carried out for the $E_6$ model but without the inclusion of the $U(1)_\psi$ and $U(1)_X$ groups. In terms of a logarithmic scale, the Pati-Salam symmetry breaking scale and unification scale have not been significantly changed from [9] by the inclusion of $U(1)_X$ and $U(1)_\psi$; and Planck scale unification and GUT scale Pati-Salam symmetry breaking is still a possibility.

The value of the gauge coupling constant at the unification scales $10^{18.95(8)}$ GeV or $10^{19.10(10)}$ GeV is $\alpha_P = 0.183(10)$ or $\alpha_P = 0.432(121)$ for the $(16_H)_{\frac{2}{3}} + (\overline{16}_H)_{-\frac{1}{3}}$ or
The values of the unified gauge coupling at the Planck scale are much larger than the conventional values of $\alpha_{GUT}$ and indeed are larger even than $\alpha_3(M_Z)$, however they are still in the perturbative regime. Of course there are expected to be large threshold corrections coming from Planck scale physics which are not included in our analysis. Indeed, we would expect that QFT breaks down as we approach the Planck scale, so that our RG analysis ceases to be valid as we approach the Planck scale, as remarked in the Introduction. The precise energy scale $E_{\text{new}}$ at which quantum field theories of gravity are expected to break down and new physics takes over is discussed in [16] based on estimates of the scale of violation of (tree-level) unitarity. An upper bound for this new physics energy scale is given by

$$E_{\text{new}}^2 = 20\left[G\left(\frac{2}{3}N_s + N_f + 4N_V\right)\right]^{-1}$$

where $N_s$, $N_f$ and $N_V$ are the number of scalars, fermions and vectors respectively that gravity couples to. Assuming three low-energy $27$ multiplets, $E_{\text{new}}$ would be equal to $10^{18.6}$ GeV which sets an upper bound for the scale at which our quantum field theory analysis (and with any corrections from effective quantum gravity theories included) can no longer be trusted. We have shown that the gauge coupling constants are predicted to be very close to one another at this scale and that, if extrapolated, unify just below $M_p$. We have naively extrapolated the RGEs up to $M_p$, even though new physics associated with quantum gravity must enter an order of magnitude below this. The fact that the two PS couplings are very close to each other at $E_{\text{new}}$, and are on a convergent trajectory must be regarded, at best, as a suggestive hint of a unification of the gauge fields with gravity in this approach.

From the previous section, the charge of the $U(1)_X$ group $T_X$ depends on the values that the $g_4$ and $g_{2R}$ coupling constants take at the $G_{4221}$ symmetry breaking scale, which is written into the cosine $c_{12}$ of the mixing angle $\tan \theta_{12} \equiv g_{2R}/g_{B-L}$. The value that $g_4$ and $g_{2R}$ take at this scale is automatically set by the unification of the $G_{4221}$ gauge coupling constants. We calculate that, with either $(16_H)_{\frac{3}{2}} + (\overline{16}_H)_{-\frac{3}{2}}$ or $27_H + \overline{27}_H$ included above the $G_{4221}$ symmetry breaking scale, $c_{12}^2$ is equal 0.71 to two significant figures. However, for convenience we take the physical value of $c_{12}^2$ to be equal to $\frac{5}{7}$ ($\sim 0.71$) so that $T_X$ can be written in terms of fractions. Using this value of $c_{12}^2$ in equation Eq.28, we can calculate $T_X$ for all the standard model particles of the three low-energy $27$ multiplets. The values that $T_X$, $T_Y$ and $T_N$ take for the particles of the $27$ multiplets are given in Table 2, where $T_N$ is the generator associated with the $U(1)_N$ group in the $E_6$SSM.

4 Phenomenology of the new $Z'$ in the ME6SSM

In this section we investigate certain phenological implications of the $Z'$ gauge boson in the ME6SSM. We compare the results to those calculated for the $Z'$ in the $E_6$SSM to see if a possible distinction could be made between the two models in future experiments. We start by reviewing the covariant derivatives for the $E_6$SSM and ME6SSM symmetries below the GUT scale and compare the different $U(1)'$ groups from the two models. The mixing between the $Z'$ of the ME6SSM and the Standard Model $Z$ gauge boson is then calculated and shown to be negligible as in the $E_6$SSM. We then calculate the axial and
vector couplings of the $Z'$ to the low energy particle spectrum and show that the charged lepton vector couplings do differ in the $E_6$SSM and ME$_6$SSM, which could potentially lead to a distinction between the two models in future experiments.

4.1 The $Z'$ of the $E_6$SSM

In the $E_6$SSM the $E_6$ symmetry is not broken through a Pati-Salam intermediate symmetry but instead breaks to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_N$ via a $E_6 \rightarrow SO(10) \otimes U(1)_\psi \rightarrow SU(5) \otimes U(1)_X \otimes U(1)_\psi$ symmetry breaking chain. The covariant derivative for the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_N$ symmetry can be written as:

$$D_\mu = \partial_\mu + ig_3 T^a_{3c} A^a_{3c\mu} + ig_2 T^s_L A^s_{L\mu} + ig_1 T_Y B_{Y\mu} + ig_N T_N B_{N\mu}$$  \hspace{1cm} (18)$$

where $n = 1 \ldots 8$ and $s = 1 \ldots 3$; $A^a_{3c\mu}$, $A^s_{L\mu}$, $B_{Y\mu}$ and $B_{N\mu}$ are the $SU(3)_c$, $SU(2)_L$, $U(1)_Y$ and $U(1)_N$ quantum fields respectively; $g_3$, $g_2$, $g_1$ and $g_N$ denote the universal gauge coupling constants of the respective fields and $T^a_{3c}$, $T^s_L$, $T_Y$ and $T_N$ represent their generators. At low energies the $U(1)_N$ gauge group will be broken, giving rise to a massive $Z'$ gauge boson associated with the $E_6$SSM.

The $g_N$ gauge coupling constant is equal to $g_1$ to an excellent approximation [6], independent of the energy scale of interest. This is to be compared to the universal gauge coupling constant $g_X$ of the group $U(1)_X$ in the models presented in this section, which is always less than $g_1$.

Similar to $T_Y$ and $T_X$, we can split $T_N$ into an $E_6$ normalization constant $N_N$ and a non-normalized charge $\tilde{N}$ so that $T_N \equiv \tilde{N}/N_N$ where the conventional choice is $N^2_N \equiv 40$ and $N \equiv N^2_N \equiv 20\chi + \frac{9}{2}T_\psi$ where $\chi \equiv 2\sqrt{10}T_X$ [6].

4.2 The $Z'$ of the ME$_6$SSM

The covariant derivative of the $G_{4221}$ symmetry is discussed in the Appendix and is given by Eq(21) as:

$$D_\mu = \partial_\mu + ig_4 T^m_4 A^m_{4\mu} + ig_2 T^r_R A^r_{R\mu} + ig_Y T^r_4 A^r_{Y\mu} + \frac{1}{\sqrt{6}} ig_\psi T_\psi A_{\psi\mu}$$

where $m = 1 \ldots 15$ and $r, s = 1 \ldots 3$; $A^m_{4\mu}$, $A^r_{R\mu}$ and $A_{\psi\mu}$ are the $SU(4)$, $SU(2)_R$ and $U(1)_\psi$ quantum fields respectively; $g_4$, $g_2$ and $g_\psi$ denote the universal gauge coupling constants of the respective fields; and $T^m_4$, $T^r_R$ and $T_\psi$ represent their generators.

The covariant derivative of the $G_{3211}$ symmetry is derived in the Appendix and is given by Eq(32) as:

$$D_\mu = \partial_\mu + ig_3 T^a_{3c} A^a_{3c\mu} + ig_2 T^s_L A^s_{L\mu} + ig_Y T_Y B_{Y\mu} + ig_X T_X B_{X\mu}$$  \hspace{1cm} (19)$$
where \( n = 1 \ldots 8 \) and \( s = 1 \ldots 3 \); and \( B_{X\mu} \) and \( T_X \) are the gauge field of the \( U(1)_X \) group and its \((E_6 \text{ normalized})\) charge respectively. At low energies the \( U(1)_X \) gauge group will be broken, giving rise to a massive \( Z' \) gauge boson associated with the ME\(_6\)SSM.

As is clear from Table 2, for \( c_{12}^2 = \frac{5}{7} \), the \( T_X \) and \( T_N \) charges are different for all of the \( G_{3211} \) representations of the \( 27 \) multiplets. However, in the limit \( c_{12}^2 = \frac{3}{5} \), corresponding to \( g_{2R} = g_4 \) at the \( G_{4221} \) symmetry breaking scale, then \( T_X \) and \( T_N \) are identical. This can be seen if one sets \( g_{2R} = g_4 = \sqrt{\frac{5}{3}} g_{B-L} \) in Eq.9 and Eq.12, in which case \( T_X \) is given by:

\[
T_X = \frac{1}{4} \left[ \frac{4}{2\sqrt{10}} (T_R^3 - \frac{3}{2} T_{B-L}) + \sqrt{15} (T_\psi / \sqrt{6}) \right] \\
\equiv \frac{1}{4} \left[ T_\chi + \sqrt{15} (T_\psi / \sqrt{6}) \right] \\
\equiv T_\chi \cos \theta + (T_\psi / \sqrt{6}) \sin \theta \\
\equiv T_N \quad \text{(see [14])}
\]

where \( \theta = \arctan \sqrt{15} \) and \( T_\chi \) is the \( E_6 \) normalized charge for the \( U(1)_X \) group, which is defined by \( SO(10) \rightarrow SU(5) \otimes U(1)_X \) [13]. In the ME\(_6\)SSM the \( U(1)_N \) group is defined as the linear combination of the two groups \( U(1)_X \) and \( U(1)_\psi \) of which the right-handed neutrino is a singlet of the symmetry [6]. This linear combination is \( U(1)_X \cos \theta + U(1)_\psi \sin \theta \), where \( \theta = \arctan \sqrt{15} \) [6], which is the same linear combination of \( U(1)_X \) and \( U(1)_\psi \) that \( U(1)_X \) becomes if \( g_R = g_4 \) as shown above. Thus if \( g_R = g_4 \) at the \( G_{4221} \) symmetry breaking scale, then the covariant derivative for the \( E_6\)SSM, Eq.18 and the covariant derivative for \( G_{3211} \), Eq.19 become equivalent because of the reasons stated above. However, in the \( E_6 \) theories that we have proposed, \( c_{12}^2 \sim \frac{5}{7} \) not \( \frac{3}{5} \) so that, in general, one expects \( g_R \neq g_4 \) at the \( G_{4221} \) symmetry breaking scale in realistic models.

It is clearly of interest to try to distinguish the \( Z' \) of the \( E_6\)SSM from that of the ME\(_6\)SSM, since the former one is associated with GUT scale unification, while the latter is associated with Planck scale unification. In the remainder of this section we discuss the phenomenology of the new \( Z' \) of the ME\(_6\)SSM, comparing it to that of the \( E_6\)SSM. In principle different \( Z' \) gauge bosons can be distinguished at the LHC by measuring the leptonic forward-backward charge asymmetries as discussed in [17], providing the mass of the \( Z' \) is not much larger than about 2 TeV.

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\(^6\)Although \( T_X \) and \( T_N \) are identical for \( c_{12}^2 = 3/5 \), \( X \) and \( N \) and hence \( N_X \) and \( N_N \) are not. However, we could have defined \( X \) and \( N_X \) differently so that they agree with \( N \) and \( N_N \) when \( c_{12}^2 = 3/5 \).

\(^7\)When \( g_{2L} = g_{2R} = g_4 \) the Pati-Salam generators can be thought of as \( SO(10) \) generators, on the same footing as the \( SU(5) \) and \( U(1)_X \) generators when their gauge couplings are equal, as in the \( E_6\)SSM. In this limit the above argument shows that there is no distinction between \( U(1)_N \) and \( U(1)_X \).
4.3 Mixing between $Z$ and the $Z'$ of the ME$_6$SSM

In this section we investigate the mixing between the $Z$ gauge boson and the $Z'$ gauge boson of $U(1)_X$ which is generated once the MSSM Higgs doublets get vacuum expectation values and break the electroweak symmetry. When the MSSM singlet particle $S$ from the low-energy 27 multiplets of the ME$_6$SSM gets a VEV, the $U(1)_X$ group will be broken and a heavy $Z'$ gauge boson will be produced. Then, when $h_u$ and $h_d$ get VEVs, the $SU(2)_L \otimes U(1)_Y$ symmetry will be broken to $U(1)_{em}$ and a heavy neutral $Z$ gauge boson, which is the following mixture of the $SU(2)_L$ and $U(1)_Y$ fields: \\
$Z_\mu = W_\mu^3 \cos \theta_W - A_Y \sin \theta_W$ where $\theta_W$ is the Electroweak (EW) symmetry breaking angle. Since $h_u$ and $h_d$ transform under $U(1)_X$, they couple to $Z'$ and so mix the $Z'$ and $Z$ gauge bosons when they get VEVs. After $S$, $h_u$ and $h_d$ get VEVs the mass squared mixing matrix for the $Z$ and $Z'$ gauge bosons is given by [18, 21]:

$$\delta M^2 = \left( \begin{array}{cc} M_Z^2 & \delta M^2 \\ \delta M^2 & M_{Z'}^2 \end{array} \right)$$

where:

$$M_Z^2 = (g_{2L}^2 + g_Y^2)(Y^h)^2 v_h^2$$

$$M_{Z'}^2 = g_X^2 v_h^2 (T_X^{h_1})^2 \cos^2 \beta + (T_X^{h_2})^2 \sin^2 \beta + g_X^2 (T_X^S)^2 s^2$$

$$\delta M^2 = \sqrt{g_{2L}^2 + g_Y^2} g_X (Y^h (T_X^{h_1} \cos^2 \beta - T_X^{h_2} \sin^2 \beta) v_h^2$$

where $Y^h$ is the magnitude of the $h_u$ and $h_d$ Higgs bosons’ hypercharge; $T_X^{h_1}$, $T_X^{h_2}$ and $T_X^S$ are the values that the $E_6$ normalized $U(1)_X$ charge, $T_X$, takes for the $h_1$, $h_2$ and $S$ states respectively; $g_{2L}$ and $g_Y$ are the $SU(2)_L$ and (non-GUT normalized) hypercharge gauge coupling constants evaluated at the EW symmetry breaking scale; $g_X$ is the $U(1)_X$ gauge coupling constant evaluated at the $U(1)_{em}$ symmetry breaking scale; $s$ is the VEV of the MSSM singlet $S$; $v_h = \sqrt{v_u^2 + v_d^2}$ and $\tan \beta = \frac{v_d}{v_u}$ where $v_u$ and $v_d$ are the vacuum expectation values for the $h_u$ and $h_d$ MSSM Higgs bosons respectively.

The mass eigenstates generated by this mass mixing matrix are:

$$Z_1 = Z \cos \theta_{ZZ'} + Z' \sin \theta_{ZZ'}$$

$$Z_2 = -Z \sin \theta_{ZZ'} + Z' \cos \theta_{ZZ'}$$

with masses $M_{Z_1, Z_2} = \frac{1}{2} [M_Z^2 + M_{Z'}^2 \mp \sqrt{(M_Z^2 - M_{Z'}^2)^2 + 4 \delta M^4}]$ respectively. The mixing angle $\theta_{ZZ'}$ is given by:

$$\tan(2\theta_{ZZ'}) = \frac{2\delta M^2}{M_{Z'}^2 - M_Z^2}$$

---

The non-GUT normalized hypercharge coupling constant $g_Y$ is identified as $g_Y \equiv \sqrt{\frac{4}{3}} g_1$. 

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13
In terms of this mixing angle the covariant derivative for the mass eigenstate gauge bosons $Z_1$ and $Z_2$ is:

$$
D_\mu = \partial_\mu + i \frac{1}{\sqrt{g_Y^2 + g_{2L}^2}} Z_\mu \left( g_{2L}^2 T^3_L - g_Y^2 Y \right) - g_X T_X \sin \theta_{ZZ'} \right) Z_{1\mu} \\
+ i \left( g_X T_X \cos \theta_{ZZ'} + \frac{\sin \theta_{ZZ'} \left( g_{2L}^2 \right) - g_Y^2 Y \right) \right) Z_{2\mu}
$$

where $g_Y$ and $g_{2L}$ are evaluated at the EW symmetry breaking scale and $g_X$ is evaluated at the scale at which $S$ gets a VEV to break the $U(1)_X$ symmetry. Phenomenology constrains the mixing angle $\theta_{ZZ'}$ to be typically less than $2 - 3 \times 10^{-3}$ [19] and the mass of the extra neutral gauge boson to be heavier than $500 - 600$ GeV [20]. We calculate that, if the $S$ particle gets a VEV at $1.5$ TeV in the ME$_6$SSM, then $\theta_{ZZ'} = 3 \times 10^{-3}$ and $M_{Z'} = 544$ GeV so that phenomenologically acceptable values are therefore produced for $s > 1.5$ TeV. This vacuum expectation value is consistent with the RGEs analysis in section 3 and the scale of electroweak symmetry breaking.

Since the mixing angle $\theta_{ZZ'}$ is very small in the ME$_6$SSM, we approximate the two mass eigenstate gauge bosons to be just $Z$ and $Z'$, which are the neutral gauge bosons of the broken $SU(2)_L \otimes U(1)_Y$ and $U(1)_X$ symmetries respectively. The above covariant derivative is then simplified to:

$$
D_\mu = \partial_\mu + i \frac{1}{\sqrt{g_Y^2 + g_{2L}^2}} Z_\mu \left( g_{2L}^2 T^3_L - g_Y^2 Y \right) + ig_X Z' T_X.
$$

### 4.4 Axial and Vector Couplings for $Z'$ in the ME$_6$SSM

If we ignore the mixing between the $Z$ and $Z'$ gauge bosons, then the most general Lagrangian for the $U(1)_X$ group is [21, 22]:

$$
\mathcal{L}_X = \frac{1}{2} M_{Z'} Z'^\mu Z_\mu - \frac{g_X}{2} \sum_i \bar{\psi}_i \gamma^\mu (f^i_V - f^i_A \gamma^5) \psi_i Z_\mu - \frac{1}{4} F'^{\mu\nu} F'^{\rho\sigma} - \frac{\sin \chi}{2} F'^{\mu\nu} F^{\mu\nu}
$$

where $F'^{\mu\nu}$ and $F^{\mu\nu}$ are the field strength tensors for $U(1)_X$ and $U(1)_Y$ respectively; $\psi_i$ are the chiral fermions and $f^i_V$ and $f^i_A$ are their vector and axial charges which are given by $f^i_V \equiv \frac{1}{N_X} (X^i_L + X^i_R)$ and $f^i_A \equiv \frac{1}{N_X} (X^i_L - X^i_R)$ where $X_L$ and $X_R$ are the $X$ charges for the left-handed and right-handed particles respectively.

The $-\frac{\sin \chi}{2} F'^{\mu\nu} F^{\mu\nu}$ term in the above Lagrangian represents the kinetic term mixing for the two Abelian symmetries $U(1)_Y$ and $U(1)_X$. In general, the kinetic term mixing of two Abelian gauge groups is non-zero because the field strength tensor is gauge-invariant for an Abelian theory. However, if both Abelian groups come from a simple gauge group, such as $E_6$, then $\sin \chi$ is equal to zero at the tree-level, although non-zero elements could arise at higher orders if the trace of the $U(1)$ charges is not equal to zero for the states.
lighter than the energy scale of interest [22]. The trace of the $U(1)_Y$ and $U(1)_X$ charges is given by:

$$\text{Tr} \left( T_Y T_X \right) = \sum_{i=\text{chiral fields}} \left( T^i_Y T^i_X \right).$$

This trace is only non-zero if incomplete GUT multiplets are present in the low energy particle spectrum. There are no low-energy incomplete $E_6$ multiplets in the model we presented in section 4 and so $\sin \chi = 0$ at the tree-level and at higher orders in this particular case. There is therefore no kinetic term mixing between the $U(1)_Y$ and $U(1)_X$ groups in the ME$_6$SSM. In the E$_6$SSM two additional EW doublets from incomplete E$_6$ multiplets are kept light so that, in this case, $\sin \chi$ is non-zero [6].

The second term in the $U(1)_X$ Lagrangian $\mathcal{L}_X$ represents the interaction between the $Z'$ gauge boson and the fermions. In table 3 we list the vector and axial $U(1)_X$ charges for the $G_{3211}$ representations of the complete $27$ low-energy $E_6$ multiplets in a general $E_6$ theory and the ME$_6$SSM, which has $c_{12}^2 = 5/7$. We also list the vector and axial $U(1)_X$ charges of the E$_6$SSM for the low-energy $27$ multiplets for a comparison. The differences between the values of the vector and axial couplings of the two $Z'$ gauge bosons of the $U(1)_X$ and $U(1)_N$ groups are due to the difference in value between the $E_6$ normalized $T_X$ and $T_N$ charges and the fact that the kinetic term mixing between the $U(1)_Y$ and the $U(1)'$ groups is non-zero in the E$_6$SSM but zero in the ME$_6$SSM. The largest difference between the vector and axial couplings of $U(1)_X$ and $U(1)_N$ exists for the charged leptons where the vector coupling for $U(1)_X$ is a factor of two larger than for $U(1)_N$.

|       | $u$         | $d$         | $e$         | $\nu$      | $D$         | $h$         | $S$         |
|-------|-------------|-------------|-------------|------------|-------------|-------------|-------------|
| $f_V/N_X$ | $\frac{1}{2} - \frac{c_{12}^2}{2} + \frac{d_{12}^2}{2}$ | $\frac{1}{2} + \frac{c_{12}^2}{2} - \frac{d_{12}^2}{2}$ | $\frac{1}{2} + \frac{d_{12}^2}{2} - \frac{c_{12}^2}{2}$ | $\frac{1}{2} + \frac{d_{12}^2}{2}$ | $\frac{1}{2} - \frac{d_{12}^2}{2}$ | $-1 + \frac{d_{12}^2}{2}$ | $2$         |
| $f_A/N_X$  | $\frac{1}{2} + \frac{c_{12}^2}{2}$ | $\frac{1}{2} - \frac{c_{12}^2}{2}$ | $\frac{1}{2} - \frac{c_{12}^2}{2}$ | $\frac{1}{2} + \frac{c_{12}^2}{2}$ | $\frac{1}{2} + \frac{c_{12}^2}{2}$ | $-2$ | $-2$ | $2$         |
| $f_V$  | $-0.0376$ | $-0.1503$ | $0.2255$ | $0.3382$ | $0.1879$ | $-0.1127$ | $0.7892$ |
| $f_A$  | $0.3382$ | $0.4510$ | $0.4510$ | $0.3382$ | $-0.7892$ | $-0.7892$ | $0.7892$ |
| $f_V^0$ | $0.0278$ | $-0.1637$ | $0.1081$ | $0.2996$ | $0.1359$ | $-0.1915$ | $0.7906$ |
| $f_A^0$ | $0.2996$ | $0.4910$ | $0.4910$ | $0.2996$ | $-0.7906$ | $-0.7906$ | $0.7906$ |

Table 3: In this table we list the axial $f_A$ and vector $f_V$ $U(1)_X$ charge assignments for the $G_{3211}$ representations of the complete $27$ $E_6$ multiplet in the ME$_6$SSM. The assignments for a general ME$_6$SSM model and for the model presented in sections 3 and 5, which has $c_{12}^2 = 5/7$, are both given. The $E_6$ normalization factor $N_X$ is given by $N_X^2 = 7 - 2c_{12}^2 + \frac{5}{3}c_{12}^4$ for a general model and is equal to $6\frac{62}{47}$ when $c_{12}^2 = 5/7$. We have also included the axial and vector $U(1)_N$ charge assignments $f_V^0$ and $f_A^0$ in the E$_6$SSM so that a comparison can be made to the corresponding ME$_6$SSM quantities $f_V$ and $f_A$. 
Table 4: This table lists all the charge assignments for the $G_{4221}$ representations of the $\text{ME}_6\text{SSM}$, where $i = 1 \ldots 3$ is a family index and $\alpha = 1, 2$. The $U(1)_R$ is an R-symmetry and $Z_2^H$ distinguishes the third family Higgs which get VEVs. These symmetries obey the $G_{4221}$ symmetry but not the $E_6$ symmetry since the latter is assumed to be broken by quantum gravity effects. The superpotential terms that are allowed by these symmetries are given in Table 5. The $h_3$ supermultiplet contains the MSSM Higgs bosons and $S_3$ is the MSSM singlet that generates an effective $\mu$-term $\lambda_S S_3 h_3 h_3$. The $H_L$ and $\overline{H}_L$ Higgs bosons are required to satisfy the left-right discrete operator $D_{LR}$ that is defined by $E_6 \rightarrow G_{4221} \otimes D_{LR}$. $\Sigma$ and $M$ are $E_6$ singlets that are assumed to get VEVs at $10^{7−11}$ GeV and $10^{16.4}$ GeV respectively.

5 A Realistic Model

In this section we construct a realistic $\text{ME}_6\text{SSM}$, focussing on the model building issues. The $\text{ME}_6\text{SSM}$ has a more ‘minimal’ particle content than the $E_6\text{SSM}$ since it only contains three complete $27$ multiplets at low energies whereas the $E_6\text{SSM}$ contains two additional EW doublets which can be considered as states of incomplete $27$ and $\overline{27}$ $E_6$ multiplets. From the RGEs analysis, unification of the $G_{4221}$ gauge coupling constants occurs near the Planck scale where an $E_6$ symmetry should in principle exist. However, given the expected strength of quantum gravity at this scale, it is likely that any such $E_6$ symmetry is for all practical purposes broken by gravitational effects. Therefore, the model that we propose in this section is chosen to not respect an $E_6$ symmetry but instead obey the $G_{4221}$ symmetry that exists between the conventional GUT and Planck scales where quantum gravity effects will not be so significant.

Under $E_6 \rightarrow SO(10) \otimes U(1)_\psi \rightarrow G_{4221}$, the fundamental $E_6$ representation breaks into the following: $27 \rightarrow 16_{2} + 10_{-1} + 1_{2} \rightarrow F + F^c + h + D + S$ where $F \equiv (4, 2, 1)_{\frac{1}{6}}$ contains one family of the left-handed quarks and leptons, $F^c \equiv (\overline{1}, 1, 2)_{\frac{1}{6}}$ can contain one family of the charge-conjugated quarks and leptons, which includes a charge-conjugated neutrino, $h \equiv (1, 2, 2)_{-1}$ contains the MSSM Higgs doublets $h_u$ and $h_d$, while $D \equiv (6, 1, 1)_{-1}$ contains two colour-triplet weak-singlet particles, and $S \equiv (1, 1, 1)_{2}$ is a MSSM singlet. Including three families contained in these $27_i$ reps, then, without further constraints on the theory, the allowed couplings are contained in the tensor products $[6, 12]$:

$$27_i 27_j 27_k \rightarrow F_i F_j h_k + F_i F_j D_k + F_i F^c_j D_k + S_i h_j h_k + S_i D_j D_k$$  \hspace{1cm} (20)$$

where $i, j, k = 1 \ldots 3$ are family indices. However not all these terms are desirable since the presence of extra Higgs doublets can give rise to flavour changing neutral currents.
(FCNCs) and the presence of light colour triplets can induce proton decay. Therefore extra symmetries are required to control the couplings, a suitable choice being the R-symmetry and the discrete $Z_2^H$ symmetry displayed in Table 4, which reduces the allowed couplings to those shown in Table 5, where we also display the lowest order non-renormalizable terms. We now discuss the physics of the allowed and suppressed terms.

5.1 Suppressed Flavour Changing Neutral Currents

We take the $F_iF_j^c h_3$ superpotential terms to contain the MSSM Yukawa couplings since we assume that the third generation $h_3$ gives the MSSM Higgs doublets. The other $h_\alpha$ states are taken to not get VEVs and will cause FCNCs unless the superpotential term $F_iF_j^c h_\alpha$, where $\alpha = 1, 2$, is forbidden or highly suppressed by some new symmetry [6]. Here we forbid these terms using a $Z_2^H$ discrete symmetry that respects the $G_{4221}$ symmetry but not the Planck-scale $E_6$ symmetry since the latter is assumed to be broken by quantum gravity. Under this $Z_2^H$ symmetry the 'matter particles' $F_i$ and $F_j^c$ and 'non-Higgs' particles $h_\alpha$ are taken to have $Z_2^H = -1$ and the MSSM Higgs doublets from $h_3$ are assumed to have $Z_2^H = +1$. The FCNC inducing terms $F_iF_j^c h_\alpha$ are therefore forbidden by the $Z_2^H$ symmetry and the MSSM superpotential generating terms $F_iF_j^c h_3$ are allowed.

However, we show later that, although the $F_iF_j^c h_\alpha$ terms are forbidden at the renormalizable level by $Z_2^H$, they are still generated from non-renormalizable terms, which are heavily suppressed so that the induced FCNCs are not significant. The $Z_2^H$ symmetry used here forbids the FCNCs in the same way that the $Z_2^H$ symmetry of the $E_6$SSM forbids the FCNCs from the $h_\alpha$ 'non-Higgs' particles in that model [6].

5.2 The $\mu$-Term and Exotic Mass Terms

As with $h_i$, we assume that only the third generation of the $S_i$ states gets a vacuum expectation value so that the $S_3 h_3 h_3$ term, from the $G_{4221}$ superpotential term $S_i h_j h_k$, will generate an effective MSSM $\mu$-term. For this term to be allowed by the $Z_2^H$ symmetry, the $S_3$ particles must have $Z_2^H = +1$.

This $S_3$ particle is also used to give mass to the 'non-Higgs' particles $h_\alpha$ and colour-triplet particles $D_i$ via the terms $S_3 h_\alpha h_\beta$ and $S_3 D_i D_j$ respectively where $\beta = 1, 2$. For general $U(1)'$ models, the $S_3 D_i D_j$ superpotential term has been shown to induce a VEV for the singlet $S_3$ so that it can generate an effective $\mu$-term [24,25]. The Yukawa coupling constant for the $S_3 D_i D_j$ term will, in general, contribute to the renormalization group evolution of the soft singlet mass $m_S^2$ causing it to run negative in the scalar potential. The VEV of $S_3$ then carries information about soft supersymmetry breaking from the parameter $m_S^2$. Therefore, the effective $\mu$ parameter is now correlated in some way to the SUSY breaking mechanism and its observed correlation with the soft Higgs mass terms.

\footnote{Forbidding or highly suppressing these terms could explain why only $h_3$ gets a VEV since then the other $h_i$ states won’t directly couple to the top Yukawa coupling.}
| Allowed couplings                                      | Physics |
|-------------------------------------------------------|---------|
| $F_i F_j^c D_k$                                       | MSSM superpotential |
| $S_3 h_3 h_3$                                         | Effective MSSM $\mu$-term |
| $S_3 h_i h_2$                                         | $h_\alpha$ mass |
| $S_3 D_i D_j$                                         | $D_i$ mass |
| $S_3 h_3 h_3$                                         | $S_\alpha$ mass |
| $\frac{1}{M_p} \Sigma (F_i F_j^c D_k + F_i^c F_j^c D_k)$ | Allows $D_i$ and proton decay |
| $\frac{1}{M_p} \Sigma S_3 F_i F_j^c h_\alpha$         | Heavily suppressed FCNCs |
| $\frac{1}{M_p} \Sigma S_3 S_i D_i D_j$                | Harmless |
| $\frac{1}{M_p} \Sigma S_3 h_i h_2 h_3$               | Harmless |
| $\frac{1}{M_p} \Sigma S_3 D_i D_j$                    | Harmless |
| $\frac{1}{M_p} F_i F_j H_R H_R$                       | $\nu^c$ mass |
| $\frac{1}{M_p} F_i F_j H_L H_L$                       | Harmless |
| $M (H_R^2 + H_L^2)$                                   | $16_H + 16_H$ mass |

Table 5: This table lists the $G_{4211}$ superpotential terms that are obtained from all the renormalizable and first-order non-renormalizable $E_6$ tensor products of $27_i, (16_H)_\pm + (16_H)_- \pm$ (from a $27 + \overline{27}$), $M$ and $\Sigma$ that are allowed by the $Z_2^H$ and $U(1)_R$ symmetries of the $E_6$ SSM, as discussed in section 5 and table 4. The indices $i, j, k = 1 \ldots 3$ and $\alpha, \beta, \gamma = 1, 2$ are family indices.

can be understood [24, 26]. That is, the $\mu$ problem of the MSSM should not exist in this model. We show below that the $S_\alpha D_i D_j$ and $S_\alpha h_\beta h_3$ (where $\gamma = 1, 2$) superpotential terms are forbidden at tree-level so that $S_\alpha$ should not acquire VEVs. These $S_\alpha$ particles will instead get mass from the $S_\alpha h_\beta h_3$ superpotential terms where $S_\alpha$ has $Z_2^H = -1$.

5.3 Exotic Decay and Suppressed Proton Decay

The remaining $G_{4221}$ superpotential terms to be discussed from Eq[20] are $F_i F_j D_k$ and $F_i^c F_j^c D_k$. These will cause rapid proton decay in this model unless they are highly suppressed or forbidden by some symmetry [4, 6]. The Standard Model representations of these superpotential terms are often found to some degree in other GUTs and the rapid proton decay problems are often solved using some doublet-triplet splitting mechanism that gives large (above the GUT scale) mass to the analogue of the $D_i$ (triplet) particles, but EW mass to the Higgs doublets. However, in our model we do not give a large mass to the $D_i$ particles because gauge anomalies would then exist, due to the $U(1)_X$ group, and Planck scale unification would be lost. Also, as discussed above, the $D_i$ particles can be used to help induce a VEV for the $S_3$ particle, around the EW scale, if they contribute to the low energy theory. We must therefore highly suppress the $F_i F_j D_k$ and $F_i^c F_j^c D_k$ superpotential terms using a small Yukawa coupling constant rather than using the general GUT method of creating large $D_i$ masses.

Note that these superpotential terms must be suppressed rather than forbidden since the $D_i$ particles would become stable, strongly interacting particles with TeV
scale masses. Such particles cannot exist in nature and in fact could potentially cause problems for nucleosynthesis even if they are unstable with a lifetime greater than just 0.1s [27]. Therefore, the $F_i F_j D_k$ and $F_i^c F_j^c D_k$ terms should not be suppressed by too small a Yukawa coupling constant for the lifetime of $D_i$ to exceed 0.1s, or too large a Yukawa coupling constant for the proton’s lifetime to be smaller than the present experimental limits.

To overcome these problems we use the same method that is used in the model presented in [9]. That is, we forbid the $F_i F_j D_k$ and $F_i^c F_j^c D_k$ superpotential terms at the tree-level but generate them from the non-renormalizable terms $\Sigma F_i F_j D_k$ and $\Sigma F_i^c F_j^c D_k$, where $\Sigma$ is an $E_6$ singlet which is assumed to get a VEV at some high energy scale, by taking both both $\Sigma$ and $D_i$ to have $Z_2^H = -1$. These non-renormalizable superpotential terms are expected to survive from the Planck scale and so will likely be suppressed by a factor of $1/M_p$. We can therefore control the degree of suppression of the $F_i F_j D_k$ and $F_i^c F_j^c D_k$ terms by choosing the energy scale at which $\Sigma$ gets a VEV. Below we estimate the energy scales at which $\Sigma$ can get a VEV so that the proton’s lifetime is above the experimental limits and the $D_i$ particles have a lifetime less than 0.1s.

The superpotential terms $\lambda_{ijk} F_i F_j D_k$ and $\lambda_{ijk} F_i^c F_j^c D_k$ cause proton decay through the decay channels $p \rightarrow K^+ \pi$ via $d = 5$ operators (through the $S_3 D_i D_j$ term which is responsible for the triplet mass $m_D$) and $p \rightarrow \pi^0 e^+$ via $d = 6$ operators with matrix elements proportional to $\lambda^2/m_D m_{SUSY}$ and $\lambda^2/m_D^2$ respectively [28,29], where $m_{SUSY}$ is the mass scale for the Standard Model’s superpartners. The present experimental limits on the proton’s lifetime for the $p \rightarrow K^+ \pi$ and $p \rightarrow \pi^0 e^+$ decay channels are $1.6 \times 10^{33}$ years and $5.0 \times 10^{33}$ years respectively [14]. The mass $m_D$ of the $D$ particles required to suppress the $p \rightarrow K^+ \pi$ and $p \rightarrow \pi^0 e^+$ matrix elements enough for proton decay to not have been observed is given in various papers (see for example [3,30]) where no fine tuning of the Yukawa coupling $\lambda$ is used. These calculations assume that a doublet-triplet splitting mechanism can be implemented to give large GUT scale masses to the triplet $D$ particles but EW scale masses to the Standard Model Higgs doublets. Here we instead assume that the mass of $D$ is not very different from the EW scale (e.g. $m_D = 1.5$ TeV) and that the Yukawa coupling $\lambda$ is very small compared to the Yukawa couplings of the Standard Model. We make a rough order of magnitude estimate for the value of the Yukawa coupling $\lambda$ required for unobservable proton decay through the $d = 5$ and $d = 6$ channels, for $m_D = 1.5$ TeV, by scaling the results obtained from [3,30] and using the fact that the matrix elements for $p \rightarrow K^+ \pi$ and $p \rightarrow \pi^0 e^+$ are proportional to $\lambda^2/m_D m_{SUSY}$ and $\lambda^2/m_D^2$ respectively. In the case of triplets that are much heavier than the doublets, it is the $d = 5$ channel that sets the higher limit on the mass of the triplets and this is usually higher than the GUT scale [3,28]. For $m_D = 1.5$ TeV, however, the $d = 5$ and $d = 6$ decay channels have similar decay rates since $M_{SUSY}$ is expected to be close to the TeV scale and the decay rates depend on the square of the matrix elements. In what follows we shall choose to scale the results based on the $d = 6$ operator, since these turn out to give slightly stronger limits on $\lambda$.

According to [3] the triplets $D$ must have mass larger than $10^{10-11}$ GeV for the lifetime of the proton to be greater than $5.0 \times 10^{33}$ years for the non-SUSY $d = 6$ channel $p \rightarrow K^+ \pi$ and for no fine-tuning of the Yukawa coupling constant of the $F_i F_j D_k$
and $F_i^c F_j^c D_k$ superpotential terms. Assuming that the Yukawa coupling used in [3] is of order unity, since it is not specified, we use the square of the matrix element of $p \rightarrow K^+ \nu$ to estimate that the Yukawa coupling $\lambda$ must be less than the following when the $D$ triplets have mass equal to $m_D = 1.5$ TeV:

$$\lambda^4 \lesssim \frac{1}{5 \times 10^{33} \text{yrs}} \times \left( \frac{1.5 \text{ TeV}}{10^{11-12} \text{ GeV}} \right)^4 \Rightarrow \lambda \lesssim 10^{-8}.$$  

As mentioned above, we forbid the $F_i F_j D_k$ and $F_i^c F_j^c D_k$ superpotential terms by the $Z_2^H$ symmetry but effectively generate them from the non-renormalizable terms $\frac{1}{M_p} \Sigma F_i F_j D_k$ and $\frac{1}{M_p} \Sigma F_i^c F_j^c D_k$ when $\Sigma$ gets a VEV. To generate an effective Yukawa coupling smaller than $10^{-8}$ to avoid experimentally observable proton decay, $\Sigma$ must get a VEV less than $10^{11}$ GeV.

The effective superpotential terms $F_i F_j D_k$ and $F_i^c F_j^c D_k$, generated from $\frac{1}{M_p} \Sigma F_i F_j D_k$ and $\frac{1}{M_p} \Sigma F_i^c F_j^c D_k$, are the only source for the $D_i$ particles to decay. Assuming that $m_{\tilde{t}} < m_D$, where $m_{\tilde{t}}$ is the mass of the heaviest stop, the $D^c$ Standard Model representation of the $G_{1221}$ $D$ particle will predominantly decay through the channel $D^c \rightarrow \tilde{t} + b$ [6]. Using the standard 2-body decay kinematic formula [14] we estimate that the decay rate for $D^c \rightarrow \tilde{t} + b$, under the assumption that $m_b \ll m_{\tilde{t}}$, is:

$$d\Gamma \sim \frac{1}{32\pi^2} |\mathcal{M}|^2 \frac{m_D^2 - m_{\tilde{t}}^2}{2m_D^3} d\Omega$$

At tree-level, a rough order of magnitude estimate of the matrix $\mathcal{M}$ for the $D^c \rightarrow \tilde{t} + b$ decay channel gives:

$$|\mathcal{M}|^2 \sim 2(m_D^2 - m_{\tilde{t}}^2)\lambda^2$$

Taking the mass of the stop to be around the TeV scale, we estimate that the $F_i F_j D_k$ and $F_i^c F_j^c D_k$ operators must be multiplied by an effective Yukawa coupling $\lambda$ that is greater than $10^{-13}$ for the $D_i$ particles to have a lifetime less than 0.1s.

If the stop is assumed to be heavier than $D^c$ (e.g. $m_{\tilde{t}} = 2$ TeV, $m_D = 1.5$ TeV) then the predominant decay channel will most likely be $D^c \rightarrow b + c + \chi_0$ where $\chi_0$ is a neutralino [6]. Under the assumption that $m_c \ll m_b \ll m_{\chi_0}$, a tree-level order of magnitude estimate for the decay rate of this channel can be shown to require an effective Yukawa coupling $\lambda$ about an order of magnitude larger than when the stop is lighter than $D$. We therefore require that $\lambda \gtrsim 10^{-12}$ for the $D_i$ particles to have a lifetime less than 0.1s.

The superpotential terms $\lambda_{ijk} F_i F_j D_k$ and $\lambda_{ijk} F_i^c F_j^c D_k$ are effectively generated from the Planck-suppressed operators $\frac{1}{M_p} \Sigma F_i F_j D_k$ and $\frac{1}{M_p} \Sigma F_i^c F_j^c D_k$ and so the Yukawa coupling $\lambda$ is given by $< \Sigma > / M_p$ where $< \Sigma >$ is the VEV of the $\Sigma$ particle. To avoid cosmological difficulties from the $\Sigma$ particles, $\Sigma$ must therefore get a VEV greater than about $10^5$ GeV. Therefore, to avoid experimentally observable proton decay and cosmological issues with the $D_i$ particles, we require that the $E_6$ singlet $\Sigma$ should get a VEV between $10^7$–$10^{11}$ GeV.
5.4 R-Symmetry and R-Parity

To ensure that the LSP is stable in this model, so that it is a candidate for dark matter, we derive R-parity from the $U(1)_R$ symmetry [23], which commutes with the $G_{4221}$ symmetry but not the $E_6$ symmetry because the latter may not be respected by low energy symmetries as it is assumed to be broken by quantum gravity effects. To allow the $G_{4221}$ superpotential terms, which respect the $Z_2^H$ discrete symmetry, and to derive a generalization of the MSSM R-parity, the $G_{4221}$ supermultiplets of the three $27 E_6$ have the following $U(1)_R$ R-charge assignments: $F_i$ and $F^c_i$ have $R = +1$; $h_3$, $h_\alpha$, $D_i$ and $\Sigma$ have $R = 0$; and $S_3$ and $S_\alpha$ have $R = +2$ (see table 4). The $16_H$ state also has $R = +2$ so that when it gets a VEV the $U(1)_R$ is broken to a $Z_2$ discrete symmetry, which we call $Z_2^R$. Under this $Z_2^R$ symmetry the scalar components of $F_i$, $F^c_i$ and the fermionic components of $h_3$ (the MSSM sparticles) all have $Z_2^R = -1$ while the fermionic components of $F_i$ and $F^c_i$ and the scalar components of $h_3$ (the MSSM particles) all have $Z_2^R = +1$. The $Z_2^R$ symmetry is therefore equivalent to the R-parity of the MSSM for the $F_i$, $F^c_i$ and $h_3$ supermultiplets. The $h_\alpha$, $D_i$, $S_i$ and $\Sigma$ supermultiplets are not in the MSSM. All the scalar components of these ‘new’ supermultiplets can be shown to have $Z_2^R = +1$ while all the fermionic components have $Z_2^R = -1$. Therefore $F_i$ and $F^c_i$ are the only supermultiplets in the theory which have $Z_2^R = +1$ for their fermionic components and $Z_2^R = -1$ for their scalar components. This $Z_2^R$ symmetry stops the ‘non-MSSM’ particles from allowing the MSSM LSP to decay as well as operating as the R-parity of the MSSM. The introduction of the $Z_2^R$ symmetry therefore ensures a stable dark matter candidate, the MSSM LSP.

Note that the $Z_2^H$ symmetry in table 4 is equivalent to an MSSM matter-parity. Therefore if it was left unbroken then it would also prevent the MSSM LSP from decaying. However, as discussed in section 5.3, the $Z_2^H$ symmetry is broken by the $E_6$ singlet $\Sigma$ at around $10^{7-11}$ GeV generating the effective operators $F_i F_j D_k$, $F^c_i F^c_j D_k$ and $F_i F^c_j h_\alpha$ that disrespect $Z_2^H$, and enabling the MSSM LSP to decay. Hence the $Z_2^R$ symmetry must be introduced in addition to the $Z_2^H$ symmetry so that the MSSM LSP is stable.

5.5 $16_H + \overline{16}_H$ Mass

In addition to the three $27 E_6$ multiplets, which are present at low energies, the Higgs states from $(16_H)_{\frac{1}{2}} + (\overline{16}_H)_{-\frac{1}{2}}$ or $27_H + \overline{27}_H$ must be given a mass at the conventional GUT scale so that the $G_{4221}$ symmetry can be broken and the gauge coupling constants can unify at the Planck scale. Here we only consider a model that contains the $(16_H)_{\frac{1}{2}} + (\overline{16}_H)_{-\frac{1}{2}}$ above the GUT scale and briefly mention what changes should be made to such a model if $27_H + \overline{27}_H$ are used to break the $G_{4221}$ symmetry instead.

To give the required GUT scale masses to the $(16_H)_{\frac{1}{2}} + (\overline{16}_H)_{-\frac{1}{2}}$ multiplets, we introduce an $E_6$ singlet $M$ that is assumed to get a VEV at this particular energy scale and which couples to the $G_{4221}$ representations of the $(16_H)_{\frac{1}{2}}$ and $(\overline{16}_H)_{-\frac{1}{2}}$ multiplets via the superpotential term $M(16_H)_{\frac{1}{2}}(\overline{16}_H)_{-\frac{1}{2}}$. We take the $(16_H)_{\frac{1}{2}}$ supermultiplet to have an R-charge of +2 so that certain phenomenologically problematic operators are forbidden.
Then, to allow the $M(16_H)^{1/2}(\overline{16}_H)^{-1/2}$ term, the $M$ and $(\overline{16}_H)^{-1/2}$ supermultiplets are both given an R-charge of $0$. If $27_H + \overline{27}_H$ is included between the GUT and Planck scales, rather than $(16_H)^{1/2} + (\overline{16}_H)^{-1/2}$, then another $Z_2$ must be applied to the $27_H$ and $\overline{27}_H$ multiplets to forbid certain phenomenologically problematic superpotential terms.

5.6 Neutrino Mass

These R-charge assignments forbid phenomenologically-problematic terms and allow the charge-conjugated neutrinos, from $F_c^i$, to obtain a large Majorana mass $\mathcal{O}(M_{GUT}^3/M_p)$ from a $\frac{1}{M_p}F_i^cF_j^c(\overline{16}_H)^{-1/2}(\overline{16}_H)^{-1/2} \equiv \frac{1}{M_p}F_i^cF_j^cH_RH_L$ superpotential term. This term will create a conventional see-saw mechanism for the left-handed neutrinos when $h_3$ gets a VEV in the superpotential term $F_i^cF_j^ch_3$. This and $\frac{1}{M_p}F_i^c\overline{H}_L\overline{H}_L$, which is phenomenologically harmless, are the only superpotential terms that contain interactions between the three $27$ $E_6$ multiplets and the $(16_H)^{1/2} + (\overline{16}_H)^{-1/2}$ and $M$ multiplets.

6 Summary and Conclusion

We have proposed a Minimal $E_6$ Supersymmetric Standard Model (ME$_6$SSM) based on three low energy reducible $27$ representations of the Standard Model gauge group which has many attractive features compared to the MSSM. In particular it provides a solution to the $\mu$ problem and doublet-triplet splitting problem, without re-introducing either of these problems. Above the conventional GUT scale the ME$_6$SSM is embedded into a left-right symmetric Supersymmetric Pati-Salam model, which allows complete gauge unification at the Planck scale, subject to gravitational uncertainties. Although we have not studied it here, it is also clear that fine-tuning in such models will be significantly reduced compared to the MSSM due to the enhanced Higgs mass.

At low energies there is an additional $U(1)_X$ gauge group, consisting of a novel and non-trivial linear combination of one Abelian and two non-Abelian Pati-Salam generators. The $U(1)_X$ is broken at the TeV scale by the same singlet that also generates the effective $\mu$ term, resulting in a new low energy $Z'$ gauge boson. We compared the $Z'$ of the ME$_6$SSM (produced via the Pati-Salam breaking chain of $E_6$, where $E_6$ is broken at the Planck scale) to the $Z'$ of the $E_6$SSM (from the $SU(5)$ breaking chain of $E_6$, where $E_6$ is broken at the GUT scale) and discussed how they can be distinguished by their different couplings. The possible discovery of such $Z'$ gauge bosons is straightforward at the LHC and the different couplings should enable the two models to be resolved experimentally. In particular, the most significant difference between the vector and axial couplings of the $Z'$ of the $E_6$SSM and ME$_6$SSM is in the vector coupling of the charges leptons, which is twice as large in the ME$_6$SSM as in the $E_6$SSM.

We emphasise that the presence of additional threshold corrections at the Planck scale will not change the Pati-Salam breaking scale or the values of the Standard Model

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Note that the bilinear term $(16_H)^{1/2}(\overline{16}_H)^{-1/2}$ is also allowed by the symmetries of the model. We assume that the dimensional coupling constant for this term is less than or equal to $M_{GUT}$. 

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gauge couplings at this scale to one loop order. However, since these quantities are determined by running up the couplings from low energies, there will be some sensitivity to TeV scale threshold corrections. Since the vector and axial vector couplings of the $Z'$ are determined from the values of the Standard Model gauge couplings at the Pati-Salam breaking scale, there will therefore be little sensitivity to Planck scale threshold corrections on the determined vector and axial vector couplings of the $Z'$.

We have introduced an R-symmetry and discrete $Z_2^H$ symmetry that addresses the potential major phenomenological problems such as flavour changing neutral currents and proton decay, which would otherwise be introduced to the theory by colour triplet fermions and extra non-Higgs doublets from the three copies of the 27 multiplet. In the ME$_6$SSM, right-handed Majorana masses of the correct order of magnitude naturally arise from the Higgs mechanism that breaks the intermediate Pati-Salam and $U(1)_\psi$ symmetry to the standard model and $U(1)_X$ gauge group, leading to a conventional see-saw mechanism. It should be possible to embed the model presented here into a realistic flavour model describing all quark and lepton masses, leading to predictions for the exotic colour triplet non-Higgs fermion masses, which will be the subject of a future study.

In conclusion, the ME$_6$SSM has clear advantages over both the MSSM and NMSSM, and even the E$_6$SSM, which make it a serious candidate SUSY Standard Model. It also has a certain elegance in the way that the low energy theory contains only complete reducible 27 representations that also allow for anomaly cancellation of the gauged $U(1)_X$, which we find quite appealing. We have shown that the potentially dangerous couplings of the exotic particles can readily be tamed by simple symmetries, leading to exciting predictions at the LHC of exotic colour triplet fermions and a new $Z'$ with distinctive couplings. The discovery and study of such new particles could provide a glimpse into the physics of unification at the Planck scale.

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**Appendix: Symmetry Breaking $G_{4221}$ to $G_{3211}$**

Since the $U(1)_X$ group does not appear to have been considered in the literature, we illustrate in detail how the $H_R$ and $\bar{H}_R$ Higgs bosons can generate it and also break the $G_{4221} = SU(4) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_\psi$ symmetry to the $G_{3211} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ symmetry.

We start with the covariant derivative of the $G_{4221}$ symmetry, which can be written
as:

\[ D_\mu = \partial_\mu + ig_4 T^m_4 A^m_4 + ig_2 L T^s_L A^s_L + ig_2 R T^r_R A^r_R + \frac{1}{\sqrt{6}} ig_\psi T_\psi A_\psi \]  

(21)

where \( m = 1 \ldots 15 \) and \( r, s = 1 \ldots 3 \); \( A^m_4, A^s_L, A^r_R \) and \( A_\psi \) are the \( SU(4)_c, SU(2)_L, SU(2)_R \) and \( U(1)_\psi \) quantum fields respectively; \( g_4, g_2 L, g_2 R \) and \( g_\psi \) denote the universal gauge coupling constants of the respective fields and \( T^m_4, T^s_L, T^r_R \) and \( T_\psi \) represent their generators. All of the \( T^m_4, T^r_R, T^s_L \) and \( T_\psi \) generators are derived from components of the \( E_6 \) generators \( G^a \), which we choose to normalize, for the fundamental representation 27, by:

\[ \text{Tr}(G^a G^b) = 3 \delta^{ab} \]  

(22)

where \( a, b = 1 \ldots 78 \).

Then, with this normalization, the Pati-Salam generators \( T^m_4, T^r_R \) and \( T^s_L \) are normalized for the fundamental representations of \( SU(4)_c, SU(2)_R \) and \( SU(2)_L \) respectively, by:

\[ \text{Tr}(T^m_4 T^m_4) = \frac{1}{2} \delta^{mn}, \]

\[ \text{Tr}(T^r_R T^s_L) = \text{Tr}(T^r_R T^r_L) = \frac{1}{2} \delta^{rs} \]

where \( m, n = 1 \ldots 15 \).

The \( U(1)_\psi \) charge \( \frac{1}{\sqrt{6}} T_\psi \) is a diagonal \( E_6 \) generator, which we choose to be the 78th generator \( G^{78} = \frac{1}{\sqrt{6}} T_\psi \), and is therefore normalized by Eq.22 to give:

\[ \frac{1}{6} \sum_{27} T^2_\psi = 3 \]  

(23)

where the sum is over all the \( G_{4221} \) representations that make up the fundamental 27 multiplet of \( E_6 \).

To break \( G_{4221} \) to \( SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X \) we use the Higgs bosons \( H_R \) and \( H_R \) that transform as \( (4, 1, 2, -\frac{1}{2}) \) and \( (\overline{3}, 1, \overline{2}, \frac{1}{2}) \) under \( G_{4221} \) respectively. These are the smallest \( G_{4221} \) multiplets that can be used to break the Pati-Salam symmetry directly to the standard model gauge group. When \( H_R \) and \( H_R \) develop VEVs in the \( \nu_R \) and \( \nu^c \) components respectively, they will break \( SU(4)_c \rightarrow SU(3)_c \) [31] and mix the field

---

These normalizations are necessary for the standard model generators \( T_{SM} \) of \( SU(3)_c \) and \( SU(2)_L \) to be normalized in the conventional way: \( \text{Tr}(T^d_c T^e_c) = \frac{1}{2} \delta^{de} \) and \( \text{Tr}(T^d_L T^e_L) = \frac{1}{2} \delta^{de} \) for the fundamental representations, where \( T^d_c \) and \( T^e_L \) are the generators for the \( SU(3)_c \) and \( SU(2)_L \) groups respectively and \( d, e = 1 \ldots 8 \).
associated with the remaining $SU(4)_c$ diagonal generator, $A^4_{15}$, with the field associated with the diagonal generator of $SU(2)_R$, $A^3_R$, and the $U(1)_{\psi}$ field $A_{\psi}$. The rest of the $SU(4)_c$ and $SU(2)_R$ fields are given square mass proportional to $v^2$, the sum of the square of the $H_R$ and $\overline{H}_R$ VEVs.

The diagonal generators for the $A^4_{15}$ and $A^3_R$ fields are $T^4_{15}$ and $T^3_R$. For the fundamental representations of $SU(4)$ and $SU(2)_R$ respectively [12]:

$$T^4_{15} = \sqrt{\frac{3}{2}} \text{diag}(0, 0, -\frac{1}{2}, -\frac{1}{2}), \quad T^3_R = \text{diag}(\frac{1}{2}, -\frac{1}{2}).$$

The part of the symmetry breaking $G_{4221}$ to $G_{3211}$ involving the diagonal generators $T^4_{15}$, $T^3_R$ and $T_{\psi}$ is then equivalent to:

$$U(1)_{T^4_{15}} \otimes U(1)_{T^3_R} \otimes U(1)_{\psi} \rightarrow U(1)_Y \otimes U(1)_X.$$ 

In the rest of this Appendix we explain this particular symmetry breaking in detail. Using the $G_{4221}$ covariant derivative, Eq. [21] the covariant derivative for the $U(1)_{T^4_{15}} \otimes U(1)_{T^3_R} \otimes U(1)_{\psi}$ symmetry is:

$$D_\mu = \partial_\mu + ig_4 T^4_{15} A^4_{15} + ig_2 R T^3_R A^3_R + \frac{1}{\sqrt{6}}ig_{\psi} T_{\psi} A_{\psi}$$

$$\equiv \partial_\mu + ig_{B-L} T_{B-L} A^4_{15} + ig_2 R T^3_R A^3_R + ig_{N_{\psi}} T_{\psi} A_{\psi}$$

where $g_{B-L} \equiv \sqrt{\frac{3}{2}} g_4$, $g_{N_{\psi}} \equiv \frac{1}{\sqrt{6}} g_{\psi}$, $T_{B-L} \equiv \sqrt{\frac{3}{2}} T^4_{15} = \frac{(B-L)}{2}$ and $B$ and $L$ are baryon and lepton number respectively.

In terms of the diagonal generators $T_{B-L}$, $T^3_R$ and $T_{\psi}$, the $\nu_R$ component of $H_R$ and the $\nu^c$ component of $\overline{H}_R$ transform under $U(1)_{T^4_{15}} \otimes U(1)_{T^3_R} \otimes U(1)_{\psi}$ as:

$$\nu^H_R = \left( -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right), \quad \nu^c_{\overline{H}} = \left( \frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right).$$

Therefore, once $H_R$ and $\overline{H}_R$ get their VEVs, the square of the covariant derivative for the $A^4_{15}$, $A^3_R$ and $A_{\psi}$ fields becomes:

$$\left| D_\mu \nu^H_R \right|^2 = \frac{1}{4} v^2 \left( -g_{B-L} A^4_{15} + g_2 R A^3_R + g_{N_{\psi}} A_{\psi} \right)^2$$

where $g_{B-L}$, $g_2 R$ and $g_{N_{\psi}}$ are the $g_{B-L}$, $g_2 R$ and $g_{N_{\psi}}$ gauge coupling constants evaluated at the $G_{4221}$ symmetry breaking scale. The above squared covariant derivative can be written in matrix form as:

$$\frac{1}{4} v^2 \begin{pmatrix} A^3_R & A^4_{15} & A_{\psi} \end{pmatrix} \begin{pmatrix} g_2 R & -g_2 R & -g_2 R & g_{B-L} & -g_2 R & g_{N_{\psi}} \\ -g_2 R & g_{B-L} & g_{B-L} & g_{B-L} & g_{N_{\psi}} & g_{N_{\psi}} \\ -g_2 R & g_{N_{\psi}} & g_{B-L} & g_{N_{\psi}} & g_{N_{\psi}} & g_{N_{\psi}} \end{pmatrix} \begin{pmatrix} A^3_R \\ A^4_{15} \\ A_{\psi} \end{pmatrix} = \begin{pmatrix} A^3_R \\ A^4_{15} \\ A_{\psi} \end{pmatrix}.$$
Diagonalizing this matrix equation determines the mass eigenstate fields generated by the mixing of the $G_{4221}$ fields $A_R^3$, $A_{415}^1$ and $A_\psi$. The $3 \times 3$ square mass mixing matrix has two zero eigenvalues and one non-zero eigenvalue so that two massless gauge bosons and one massive gauge boson appear to have been created by the mixing. The massive gauge boson $B_H$ is the following mixture of $G_{4221}$ fields:

$$B_H = \frac{1}{b} \left( -g_{2R}A_R^3 + g_{B-L}A_{415}^1 + g_{N\psi}A_\psi \right)$$

where $b^2 \equiv g_{2R}^2 + g_{B-L}^2 + g_{N\psi}^2$.

This massive field is an unique mass eigenstate field. However, the degeneracy in the zero-eigenvalue eigenvectors of the square mass mixing matrix implies that all orthogonal combinations of any chosen two massless eigenstate fields also describe two massless eigenstate fields. All the orthogonal combinations of two massless eigenstate fields are physically distinct and so the symmetry breaking mechanism does not generate two unique massless eigenstate fields. We therefore require something in addition to this symmetry breaking mechanism that lifts the degeneracy of the zero-eigenvalue eigenvectors and selects two unique massless gauge fields.

We show below that when we include the low-energy VEV of the $S$ particle from the third generation of the $27$ multiplets, the degeneracy in the zero-eigenvalue eigenvectors of the square mass mixing matrix implies that all orthogonal combinations of any chosen two massless eigenstate fields also describe two massless eigenstate fields. All the orthogonal combinations of two massless eigenstate fields are physically distinct and so the symmetry breaking mechanism does not generate two unique massless eigenstate fields. We therefore require something in addition to this symmetry breaking mechanism that lifts the degeneracy of the zero-eigenvalue eigenvectors and selects two unique massless gauge fields.

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The $S$ particle only couples to $A_\psi$ and so its VEV, $s$, therefore introduces a perturbation proportional to $s^2/v^2$ to the 33 component of the $3 \times 3$ square mass mixing matrix in Eq.26. From Section 3, $v$ is determined to be of the order $10^{16}$ GeV and we require that $s \sim 10^3$ GeV for EW symmetry breaking.

Diagonalizing the $3 \times 3$ square mass mixing matrix with this extremely small perturbation in the 33 component determines the mass eigenstate fields to be the massless
hypercharge gauge field $B_Y$, and an extremely small mass gauge field and large mass gauge field that can be taken to be the $B_X$ and $B_H$ gauge fields, respectively, in the excellent approximation that $s^2/v^2 = 0$.\(^{12}\)

It is easy to see why the hypercharge gauge field of the Standard Model is the exact massless gauge field of this symmetry breaking. The hypercharge field is the only massless gauge field generated by the $H_R$ and $\bar{H}_R$ VEVs that does not contain the $A$ field and therefore the only massless gauge field that $S$ does not couple to. If the $A$ field is removed from the $G_{4221}$ symmetry then the mixing of the remaining $G_{4221}$ diagonal generators becomes equivalent to $U(1)_{T_{4}^3} \otimes U(1)_{T_{4}^R} \rightarrow U(1)_Y$ when $H_R$ and $\bar{H}_R$ get VEVs [9, 31].

The mass eigenstate fields $B_Y$, $B_X$ and $B_H$ can be written in terms of the $G_{4221}$ fields $A^3_R$, $A^4_1$ and $A_\psi$ in the following matrix form:

\[
\begin{pmatrix}
B_Y \\
B_X \\
B_H
\end{pmatrix} =
\begin{pmatrix}
g_{B-L}/a & g_{2R}/a & 0 \\
g_{2R}/b & g_{N_\psi}/ab & (g_{2R}^2 + g_{B-L}^2)/ab \\
-g_{2R}/b & g_{B-L}/b & g_{N_\psi}/b
\end{pmatrix}
\begin{pmatrix}
A^3_R \\
A^4_1 \\
A_\psi
\end{pmatrix}.
\]

We choose to parameterize this orthogonal $3 \times 3$ matrix in terms of rotation and reflection matrices in the following way:

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
= \begin{pmatrix}
c_{12} & s_{12} & 0 \\
c_{23}s_{12} & -c_{23}c_{12} & s_{23} \\
-s_{23}s_{12} & s_{23}c_{12} & c_{23}
\end{pmatrix}
\]

where $c_{12} = g_{B-L}/a$, $s_{12} = g_{2R}/a$, $c_{23} = g_{N_\psi}/b$ and $s_{23} = a/b$. The mixing angles $\theta_{12}$ and $\theta_{23}$ are therefore given by $\tan \theta_{12} = g_{2R}/g_{B-L}$ and $\tan \theta_{23} = a/g_{N_\psi}$.

Taking the transpose of Eq.27 the $G_{4221}$ fields $A^3_R$, $A^4_1$ and $A_\psi$ can be written in terms of the mass eigenstate fields $B_Y$, $B_X$ and $B_H$ as:

\[
\begin{pmatrix}
A^3_R \\
A^4_1 \\
A_\psi
\end{pmatrix} =
\begin{pmatrix}
c_{12} & s_{12}c_{23} & -s_{12}s_{23} \\
s_{12} & -c_{12}c_{23} & c_{12}s_{23} \\
0 & s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
B_Y \\
B_X \\
B_H
\end{pmatrix}.
\]

Putting this matrix equation into the covariant derivative for the $U(1)_{T_{4}^3} \otimes U(1)_{T_{4}^R} \otimes U(1)_Y$ symmetry, Eq.24 determines the covariant derivative for the massless gauge fields $B_Y$ and $B_X$ to be:

\[
D_\mu = \partial_\mu + i g_Y Y B_\mu + i g_X X B_\mu
\]

where:

\[
Y = T^3_R + T_{B-L} = T^3_R + (B - L)/2
\]

\(^{12}\)We have ignored the VEV of the Standard Model Higgs boson in this symmetry breaking.
is the Standard Model hypercharge [31],

\[ X = (T_\psi + T_R^3) - c_{12}^2 Y \] (28)

is the non-normalized charge of the \( B_X \) gauge field. \( g_Y \) and \( g_X^0 \) are the non-normalized universal gauge coupling constants of the \( B_Y \) and \( B_X \) fields respectively and, at the \( G_{4221} \) symmetry breaking scale, are given by Eq.29 and Eq.30:

\[ g_Y = \frac{g_{2R} g_{B-L}}{a} \] (29)

\[ g_X^0 = \frac{a}{b} g_{N\psi}. \] (30)

Eq.29 and Eq.30 can be written in terms of \( \alpha_Y = \frac{g_Y}{4\pi} \) and \( \alpha_X^0 = \frac{(g_X^0)^2}{4\pi} \), see Eq.15 and Eq.13 in Section 3.

The charges \( X \) and \( Y \) are not \( E_6 \) normalized. We write the normalized respective charges as \( T_X \) and \( T_Y \) where:

\[ T_X = X/N_X, \quad T_Y = Y/N_Y \]

and the normalization constants \( N_X \) and \( N_Y \) are given by:

\[ N_X^2 = 7 - 2c_{12}^2 + \frac{5}{3}c_{12}^4, \quad N_Y^2 = \frac{3}{5} \]

Note that the Abelian generator \( T_Y \) is just the conventional GUT normalized hypercharge. \( T_X \) and \( T_Y \) have been \( E_6 \) normalized using Eq.22 which is equivalent to:

\[ \sum_{27}^{27} T_Y^2 = \sum_{27}^{27} T_X^2 = 3 \]

where the sum is over all the \( G_{3211} \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X \) representations of the fundamental \( 27 \) \( E_6 \) multiplet and \( U(1)_X \) is the unitary group of the \( B_X \) field [14].

In terms of the \( E_6 \) normalized charges \( T_X \) and \( T_Y \), the covariant derivative for the \( B_X \) and \( B_Y \) gauge fields becomes:

\[ D_\mu = \partial_\mu + ig_Y T_Y B_{Y\mu} + ig_X T_X B_{X\mu} \] (31)

---

\textsuperscript{13}Note that Eq.29 is the relation that \( g_Y \) must satisfy if the Pati-Salam symmetry, without the \( U(1)_\psi \), was broken to the standard model gauge group using a Higgs boson that transforms as \( (4, 1, 2) \) and gets a VEV in the \( \nu_\mu \) direction [9].

\textsuperscript{14}We could have defined \( X \) and \( N_X \) differently as long as \( T_X \) is the same. Here we have chosen to define \( X \) so that it can be written in terms of hypercharge \( Y \).
where $g_1$ and $g_X$ are the normalized universal gauge coupling constants of the $B_Y$ and $B_X$ fields respectively. At the $G_{4221}$ symmetry breaking scale, the normalized gauge coupling constants $g_1$ and $g_X$ are the following combinations of $G_{4221}$ gauge coupling constants:

$$g_1 = N_Y \frac{g_{2R}}{a} g_{B-L}, \quad g_X = N_X \frac{a}{b} g_{N\psi}.$$ 

From Eq.28, the charge $T_X$ of the $U(1)_X$ group depends on the Pati-Salam gauge coupling constants $g_{2R}$ and $g_{B-L}$ evaluated at the $G_{4221}$ symmetry breaking scale. Therefore, under the excellent approximation that $s^2/v^2 = 0$, a massless gauge boson exists that couples to particles with a charge that depends on the values that certain coupling constants take at some high energy scale. Although this may be unusual, it does not appear to pose any problems. Indeed, like any other quantum charge, $T_X$ is a dimensionless constant that is independent of the energy scale at which the interaction between the particle and the $A_X$ field occurs and, although the numbers that $X$ takes may not be able to be arranged into fractions like $Y$, they are still discrete and sum to zero for a complete $E_6$ representation. However, unlike conventional $U(1)$ charges, $T_X$ is obviously very model dependent since different $E_6$ models with an intermediate Pati-Salam symmetry will, in general, contain different values of the gauge coupling constants $g_{2R}$ and $g_4$ evaluated at the $G_{4221}$ symmetry breaking scale. It is easy to prove that it is a general rule that, if three massless gauge fields are mixed, then at least two of the resulting mass eigenstate fields must have a charge that depends on the value of the original gauge coupling constants. Therefore this gauge coupling dependence is not peculiar to the Higgs symmetry breaking mechanism discussed in this Appendix, but to any symmetry breaking mechanism involving three fields.

In this Appendix we have illustrated how the $G_{4221} \equiv SU(4) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_Y \otimes U(1)_X$ symmetry can be broken to the symmetry $G_{3211} \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ when the $G_{4221}$ multiplets $H_R$, $\overline{H}_R$ and $S$ get vacuum expectation values. Using the covariant derivatives for the $G_{4221}$ symmetry, Eq.21 and the $U(1)_Y \otimes U(1)_X$ symmetry, Eq.31, the covariant derivative for the $G_{3211}$ symmetry is given by:

$$D_\mu = \partial_\mu + ig_3 T_{3c}^n A^n_{3c\mu} + ig_{2L} T_L^s A^s_{L\mu} + ig_1 T_Y B_{Y\mu} + ig_X T_X B_{X\mu} \quad (32)$$

where $A^n_{3c}$ and $T_{3c}$ are the $SU(3)_c$ fields and generators derived from the $SU(4)$ symmetry respectively (with $n = 1 \ldots 8$) and $g_{3c}$ is the universal gauge coupling constant of $A^n_{3c}$.

We consider this $G_{3211}$ symmetry as an effective high energy symmetry under the assumption that the low-energy VEVs of the MSSM singlet $S$ and MSSM Higgs bosons can be neglected at higher energy scales.

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