Entanglement and violation of the discrete symmetries in oscillating systems

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Abstract. We analyse how the self-gravitational interaction of a system consisted of identical mixed particles affects the probability oscillations of any single element of the system. Such a self-gravitational interaction induces a violation of the $CPT$-symmetry due to the breaking of the $T$-symmetry and the simultaneous preservation of the $CP$-symmetry. The violation of the $T$-symmetry could be traced back to the establishment of a non-vanishing entanglement among the different particles. Being a pure many-body effect, it scales with the system’s size and, hence, could have played a key role in situations characterized by extremely high density as the first stages of the Universe or in the core of very dense systems. Next experiments based on Rydberg atoms confined in microtraps and optically manipulated could simulate the discussed effect.

1. Introduction
Oscillatory phenomena play a crucial role in physics and occur in a plethora of systems at different scales including atoms (Rabi oscillations) and elementary particles. Latter are characterized by particle mixing and oscillations which occur when the physical (“flavor”) fields are superpositions of free fields with definite different masses. Some typical examples are axion–photon [1, 2, 3], $\eta-\eta'$ [4], neutral meson [5, 6], and neutrino flavor oscillations [7, 8]. In recent years, many studies have been devoted to probing the very foundations of quantum theory and quantum gravity using these phenomena [9, 10, 11, 12, 13, 14, 15, 16]. In particular, for mixed neutral particles, the frequency of flavor oscillations is highly sensitive even to a weak perturbation because of small differences of masses of the free fields. Hence, they provide a suitable system to analyse the effects of gravity which could produce measurable deviations from vacuum oscillation frequencies [12]. On the other hand, gravity could induce the violation
of the $\mathcal{CP}$- and $\mathcal{CPT}$-symmetries in particle mixing [17, 18, 19, 20, 21, 22, 13] if the particle is considered as an open system interacting with the environment and gravity as a source of decoherence in flavor oscillations [23, 24, 25].

A different $\mathcal{CPT}$ violation caused by the $T$-violation and the preserved $\mathcal{CP}$-symmetry can be found for a closed system of self-interacting particles [14]. Indeed, in a system of mixed neutral particles evolving under the self-gravity, a non-zero difference of masses the free fields induces a non-vanishing entanglement among the elements of the system. In turn, the rising entanglement causes the $\mathcal{CPT}$ violation, which appears to be proportional to the number of elements of the system and its density and, hence, is a many-body effect. Therefore, it could play an important role in very dense astrophysical objects and have affected the early stages of the Universe [26].

In what follows, we analyse the emergence of the entanglement and the resulting $\mathcal{CPT}$ violation in a system of self-interacting neutral mixed particles.

2. Oscillations of interacting neutral particles

Let us consider the mixing of two flavor fields named $n_A$ and $n_B$. The corresponding mixing relations are given by

$$|n_A\rangle = \cos(\theta) |m_1\rangle + e^{i\phi} \sin(\theta) |m_2\rangle,$$

$$|n_B\rangle = -e^{-i\phi} \sin(\theta) |m_1\rangle + \cos(\theta) |m_2\rangle,$$

with $\theta$ being the mixing angle, $\phi$ being the Majorana phase, and $|m_i\rangle$ being the states with definite masses $m_i$. If the particle is considered as a closed system, i.e., propagating in vacuum with its energy $E \gg m_i$, the Hamiltonian of mixed fields can be written as

$$\hat{H}^{(1)} = \omega_0 \hat{\sigma}_z,$$

$$\omega_0 = \frac{1}{4E} (m_2^2 - m_1^2).$$

where $\hbar = c = 1$ is assumed, $\hat{\sigma}_z = |m_1\rangle \langle m_1| - |m_2\rangle \langle m_2|$ is the Pauli $Z$-operator, and the state-independent terms proportional to the identity operator are neglected. If the system is initially in one of the flavor states $|n_A\rangle$, there is a non-vanishing probability for $t > 0$ to observe a flavor transition given by the Pontecorvo formula [7],

$$P_{n_A \to n_B}(t) = \sin^2(2\theta) \sin^2(\omega_0 t),$$

which is invariant under exchange of flavors.

Let us now consider a system of two mixed particles interacting gravitationally. For the sake of simplicity, we assume

(i) the validity of the equivalence principle between inertial and gravitational masses,
(ii) the validity of the Newtonian potential,
(iii) the invariance of the relative distances among the fields during the time evolution.

Within these assumptions, the Hamiltonian of the system becomes

$$\hat{H}^{(2)} = \omega (\hat{\sigma}_i^z + \hat{\sigma}_j^z) + \Omega \hat{\sigma}_i^z \cdot \hat{\sigma}_j^z,$$

$$\omega = \omega_0 + g(m_i^2 - m_j^2),$$

$$\Omega = g(m_i - m_j)^2,$$
Notice that, if the difference of masses is non-zero ($m_1 \neq m_2$), the interaction changes the value of $\omega$ and induces a new term with amplitude $\Omega$. The latter involves operators which act simultaneously on both the fields. Hence, it is non-local and induces an evolution which can increase (or decrease) the value of the entanglement presented in the system [27]. Indeed, let us assume that the initially the particles are not entangled, i.e., initial state of the system is separable, $|\psi(0)\rangle = |n_\eta\rangle_1 |n_\chi\rangle_2$, where $\eta$ and $\chi$ could assume all possible combinations of $A$ and $B$. Evolving under the action of the Hamiltonian (6) the state $|\psi(t)\rangle$ at $t > 0$ holds, in general, a non-vanishing entanglement between the two particles if $\Omega \neq 0$. This fact indicates the quantum nature of gravity because quantum correlations and entanglement can be created only through a quantum channel [28].

The presented entanglement can be quantified by relating it to the purity of the projection of $|\psi(t)\rangle$ on any of the two particles, which would be, generally speaking, a mixed state. Indeed, the purity can be defined as

$$P(\tilde{\rho}_i(t)) = \text{Tr}(\tilde{\rho}_i^2(t)),$$

where $\tilde{\rho}_i(t)$ is the projection over the $i$-th particle of the state $|\psi(t)\rangle$ of the system [29]. In turn, it can be related to the 2–Renyi entropy $S_2 = -\ln(P(\tilde{\rho}_i(t)))$, which quantifies the entanglement between the $i$-th particle and the rest of the system [30, 31, 32]. Calculating the purity explicitly we obtain

$$P(\tilde{\rho}_i(t)) = 1 - \frac{1}{2} \sin^4(2\theta) \sin^2(2\Omega t),$$

independently on the initial state. Notice that, in the presence of flavor mixing ($\theta \neq \pi k \frac{2}{T}$, $k \in \mathbb{Z}$) and for any time $t \neq \pi k \frac{2}{T}$ (with $k \in \mathbb{Z}$), we have $P(\tilde{\rho}_i(t)) < 1$. This means that the single-particle state is not pure and, hence, highlights entanglement in $|\psi(t)\rangle$.

Now we consider two copies of the system which differ by the initial states of the particles. In the first copy, $|\psi(0)\rangle = |n_A\rangle |n_A\rangle$, whereas, in the second one, $|\psi(0)\rangle = |n_B\rangle |n_B\rangle$. If we observe one of the particles at $t > 0$ in both copies, the probabilities of a flavor transition are given by $P_{n_A \rightarrow n_B} = \text{Tr}(\rho_A(t) |n_B\rangle \langle n_B|)$ and $P_{n_B \rightarrow n_A} = \text{Tr}(\rho_B(t) |n_A\rangle \langle n_A|)$. Indeed, we obtain

$$P_{n_A \rightarrow n_B} = \frac{1}{2} \sin^2(2\theta)[1 - \cos(2\omega t) \cos(2\Omega t) \pm \cos(2\theta) \sin(2\omega t) \sin(2\Omega t)].$$

Unlike the probabilities (5), these probabilities are not invariant under exchange of flavors. Hence, the $T$-symmetry is violated,

$$\Delta_T \equiv P_{n_A \rightarrow n_B} - P_{n_B \rightarrow n_A} = \sin^2(2\theta) \cos(2\theta) \sin(2\omega t) \sin(2\Omega t),$$

if $t \neq \pi k \frac{2}{T}$ and $\theta = \pi k \frac{1}{T}$, $k \in \mathbb{Z}$. On the other hand, probabilities (12) do not depend on the Majorana phase $\phi$, and the $CPT$-symmetry is not violated. Hence, the $T$-violation (13) also implies the violation of the $CPT$-symmetry. Hence, we conclude that the emergence of entanglement between the two particles breaks the $CPT$-symmetry.

3. Many-body effect

Unfortunately, the observed $CPT$ violation is tiny. However, it is possible to show that it increases with the number of constituents of the system and, hence, is a many-body effect. Let us consider a system of a large number $N$ of particles. The Hamiltonian (6) can be generalized
straightforwardly,
\[
\hat{H}^{(N)} = \sum_i \omega_i \hat{\sigma}_i^z + \frac{1}{2} \sum_{i,j} \Omega_{i,j} \hat{\sigma}_i^z \cdot \hat{\sigma}_j^z \quad (14)
\]
\[
\omega_i = \omega_0 + \sum_j g_{i,j}(m_1^2 - m_2^2), \quad (15)
\]
\[
\Omega_{i,j} = g_{i,j}(m_1 - m_2)^2, \quad (16)
\]
where \(g_{i,j} = \frac{G}{D_{i,j}}, \) \(d_{i,j}\) is the relative distance between the \(i\)-th and the \(j\)-th fields, and \(\Omega_{i,i} = 0\) by definition. At first, we assume that the first \(M\) particles are initially created in the state \(|n_A\rangle\) and the rest is in the state \(|n_B\rangle\), so that the initial state is
\[
|\psi^{(N)}(0)\rangle = \bigotimes_{\alpha=1}^{M} |n_A\rangle_{\alpha} \bigotimes_{\beta=M+1}^{N} |n_B\rangle_{\beta}. \quad (17)
\]

Generally speaking, evolving under the action of the Hamiltonian (14) the state of the \(i\)-th particle at \(t > 0\) is no more pure,
\[
\hat{\rho}_k(t) = \frac{1}{2} \left( \begin{array}{cc} 1 + \zeta_k \cos(2\theta) & \zeta_k e^{-i\phi} \sin(2\theta) a_k^*(t) \\ \zeta_k e^{i\phi} \sin(2\theta) a_k(t) & 1 - \zeta_k \cos(2\theta) \end{array} \right), \quad (18)
\]
where \(\zeta_k\) depends on the flavor of the \(k\)-th particle and is equal to 1 for \(k \leq M\) and \(-1\) for \(k > M\). The function \(a_k(t)\) is given by
\[
a_k(t) = e^{2i\omega_k t} \prod_{j=1}^{N} (\cos(2\Omega_{k,j} t) + i\zeta_k \cos(2\theta) \sin(2\Omega_{k,j} t)), \quad (19)
\]
Notice that \(|a_k(t)|^2 < 1\) at \(t > 0\) since the amplitudes \(\Omega_{k,j}\) are not invariant under the change of fields. Hence, the state (18) is not pure at any \(t > 0\) with respect to the purity
\[
\mathcal{P}(\rho_k(t)) = 1 - \sin^2(2\theta) \left( 1 - |a_k(t)|^2 \right). \quad (20)
\]
Hence, in contrast to the case of a system of two interacting particles [cf. Eq. (11)], any single particle is entangled with the rest of the system at \(t > 0\).

As done for the two-particle system, we consider two copies of the \(N\)-particle system which differ by the initial states of the particles. In the first copy, we assume \(M = N\), whereas, in the second one, \(M = 0\) is taken. In order to obtain the flavor transition probabilities, we average them over all elements of the system since the single-particle states \(\hat{\rho}_k(t)\) are site-dependent. Indeed, we obtain
\[
P_{n_A,n_B} = \frac{1}{2} \sin^2(2\theta) \left( 1 - \frac{1}{N} \sum_{k=1}^{N} \text{Re}(a_k^{(A/B)}(t)) \right), \quad (21)
\]
As in the two-particle case, these probabilities do not depend on the Majorana phase, and the \(CP\)-symmetry is preserved. On the other hand, \(a_k^{(A)}(t) \neq a_k^{(B)}(t)\), hence, the transition probabilities (21) are not invariant under exchange of flavors, and \(\Delta_{\mathcal{T}} \neq 0\) implying the violation of the \(CPT\)-symmetry. If we assume \(\Omega_{k,j} t \ll 1\), we obtain
\[
a_k(t) \approx e^{2i\omega_k t} \left( 1 \pm 2i \cos(2\theta) \sum_{j=1}^{N} \Omega_{k,j} t \right), \quad (22)
\]
where the upper sign refers to the copy of the system initially consisted only of $n_A$-particles, whereas the lower sign refers to the copy initially consisted only of $n_B$-particles. In turn, the resulting violation of the $T$-symmetry is equal to

$$
\Delta_T = \sin^2(2\theta) \cos(2\theta) \frac{2t}{N} \sum_{k,j=1}^{N} \sin(2\omega_k t) \Omega_{k,j},
$$

and the $CPT$-symmetry is broken. Finally, expressing (23) in terms of average values with respect to the set of the relative distances, we obtain

$$
\Delta_T = \sin^2(2\theta) \cos(2\theta) 2NtF,
$$

where $f_k = \frac{\sin(2\omega_k t)}{N} \sum_j \Omega_{k,j}$ and $F = \frac{1}{N} \sum_{k=1}^{N} f_k$. Hence, the found $T$ violation and, in turn, the $CPT$ violation are proportional to the number of particles of the system. Similar $CPT$ violation can be obtained for all configurations in which the difference between the numbers of the $n_A$-particles and the $n_B$-particles initially presented in the system is of the same order of magnitude of $N$. On the other hand, if the $n_A$-particles and the $n_B$-particles are equally represented in the system at $t = 0$, it is possible to show that $\Delta_T \sim \sqrt{N}$.

4. Conclusions

The gravity in a system of self-interacting mixed particles breaks the $CPT$-symmetry. This effect is related to the emergence of a non-vanishing entanglement among the elements of the system, which, in turn, is induced by a non-zero difference of mass of the free fields. Interestingly, the mechanism of the found $CPT$ violation is associated with the violation of the $T$-symmetry with preserved $CP$-symmetry. Hence, it differs from the $CPT$-symmetry violation induced by dissipative dynamics [20, 21, 22]. Furthermore, the found $CPT$ violation is proportional to the number of elements of the system and its density. Therefore, this effect could play a crucial role in galactic objects and in the first stage of the Universe where the densities and the number of particles are very high.

The presented mechanism of the violation of the discrete symmetries is not only limited to gravitational interaction. In fact, it has two requirements,

(i) the presence of neutral particles whose flavor states are superpositions of the eigenstates of a free field Hamiltonian,

(ii) the presence of interaction depending on the eigenstates of the free Hamiltonian.

Within these requirements, interaction among the particles generates entanglement and, hence, induces a $CPT$-symmetry violation similar to the analysed one. Indeed, table-top experiments, based on Rydberg atoms confined in microtraps and optically manipulated [33], can simulate the mixing and the mutual interaction. In this system, the two internal states, i.e., the ground state and the excited Rydberg level can represent the mass eigenstates, whereas two particular orthonormal superpositions can simulate the flavor states, and the dipole-dipole interaction can play the role of the gravity [34, 35, 36]. Thus, next experiments on atomic physics could allow to test the fundamental laws and symmetries of nature.

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