Investigation the effect discovery based learning on students metacognition in solving rainbow 2-connection numbers

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Abstract. This study aims to investigate the success of the application of discovery based learning model to improve students’ metacognition in solving the problem of mathematical modeling rainbow 2-connection numbers. The research model used is a mixed method that is a combination of quantitative and qualitative models. The quantitative model used a t-test to test the student learning outcomes while questionnaires and interviews are used to achieve students' metacognition skills. The results showed that the learning model of discovery based learning can improve students' metacognitive skills in solving mathematical modeling problem of rainbow 2-connection number, as indicated by the difference of t-test between experimental class and control class pre-test score of two groups [t(48) = 3.308, p < 0.005] not significant. For the post-test class experimental was 65, 10 (SD = 11,875) and for control class was 53, 85 (SD = 8,274). There is a difference between the two significant mean post-test [t(48) = −11.25, p < 0.005]. The results of questionnaires and interviews also showed that the DBL model can improve students' metacognition skills, especially in conducting activities to collect, compare, categorize, analyze, integrate, organize materials and make conclusions.

1. Introduction
Education is the determinant for the development of people's lives. The main problem in the world of education in Indonesia is the lack of student learning activities at school and at home in the context of mathematics, learning activities not only on aspects of cognitive or mathematical understanding, but aspects of attitude in mathematics. We now carry a massive storehouse of information and knowledge and face unprecedented challenge, brought by the converging impact of globalization, increasing impact of knowledge as a principal drive of growth and the ICT revolution. Student activity can not only be seen from the physical aspect, but also the mental and intellectual activity. In the context of educational learning, students are given the authority to compile, and implement the learning program and conduct evaluation on the program independently, so that students not only have skills but also a good way or model in problem Discovery Based Learning. One of the models that was applied was the Discovery Based Learning. Learning discovery learning model is a learning theory that is defined as a form of learning process that occurs if students are not served with lessons in the final form, but are expected to organize themselves. "Discovery Learning is an inquiry-based approach in which the students are given a question of answer, a problem of solving, or a set of observations to explain, and then work in a largely self-directed manner to complete their assigned task and draw appropriate inferences from the outcomes, discovering the desired factual and conceptual knowledge in the process"[2].
Discovery Based Learning has the same principle of the inquiry (inquiry). There is no principal difference in these two terms; the Discovery Learning emphasizes the discovery of previously unknown concepts or principles. The difference with discovery is that on the discovery of problems faced by students such a problem engineered by the teacher, while the inquiry itself is not the result of engineering, so the student must muster all thoughts and his skills for finding the findings in the problem through the research process [5].

This model postulates that in the learning process there are differences in ability. To support the learning process the environment needs to facilitate students' curiosity during the exploration stage. This environment is called the Discovery Learning Environment, an environment in which students can explore, discover new unknowns or notions that are similar to those already known. Such an environment aims to make students in the learning process work better and be more creative. To facilitate a good and creative learning process should be based on the manipulation of instructional materials in accordance with the level of cognitive development of students [5]. The manipulation of instructional materials aims to facilitate students' ability to think (represent what is understood) according to their level of development.

In the application, the researchers act as mentors by providing opportunities for students to learn actively, as the opinion of teachers should be able to guide and direct student learning activities in accordance with the objectives [3]. Conditions like this aim to change teaching-learning activities from teacher oriented into student oriented. From Discovery Based Learning model allows to know the metacognition of each student. The purpose of grouping based on student metacognition is to refer to one's knowledge of processes and products about the person himself. Metacognition is a form of the ability to look at oneself, so that what they do can be controlled optimally. This ability can understand the knowledge and experience as well as the ability to make decisions in choosing strategies to meet the shortcomings of learning [1].

Classifying metacognition activities in solving mathematical problems consists of planning, monitoring (monitoring), and reflection [9]. In this study, discovery based learning model to improve students' metacognition in solving problems mathematical modeling of rainbow 2-connections number. Knowledge of metacognition is knowledge of cognition in general equal's awareness and knowledge of self-cognition. Metacognition knowledge is an indicator of how well a person uses models and strategies to control and improve his learning and knowledge. Therefore it can be said that metacognition knowledge is an awareness of what is known and what is unknown [1]. NCREL presents three basic elements of metacognition specifically in dealing with tasks, namely: developing action plans, organizing / monitoring plans and evaluating plans.
### Table 1. Metakognition Indicators

| No | Aspect     | Indicators                                                                                                                                 |
|----|------------|---------------------------------------------------------------------------------------------------------------------------------------------|
| 1  | Planning   | Determine what is known from the problem given  
|    |            | Presenting problems in their own language or other forms  
|    |            | Think about the relationship between what is known and what is being asked  
|    |            | Plan to implement the chosen strategy / method  |
| 2  | Monitoring | Monitor what is known in a given problem  
|    |            | Monitor the right concept or strategy to solve the given problem  
|    |            | Monitor the corrective steps in the correct path or not  
|    |            | Re-check the accuracy of the answers obtained in accordance with what was asked  |
| 3  | Evaluating | Decide which answers obtained are those that are in accordance with the original intent  
|    |            | Decide on the right strategy / way  
|    |            | Decides that each step in the answer given is correct  
|    |            | Decide that a strategy is used effectively to solve a given problem  |

In the study, the control group received traditional mathematical instruction, while the experimental group received a mathematical instruction using a discovery based learning model. Researchers used pre-test and post-test designs and compared the mean scores. The students of the experimental group were taught using the discovery based learning model while the control group was taught using the lecture model. The findings reveal the discovery-based learning model has a significant effect on mathematical achievement. In their experimental study, comparing the effects of discovery-based learning models with traditional teaching models of mathematics achievement of students at the secondary level and they found that learning together with students outperformed traditional teaching students on their performance in mathematics. Therefore, this study aims to identify the effects of discovery based learning model on students achieving mathematics. The objectives of this research are: (1) To examine the effect of discovery based learning model in improving students metacognition skill in solving mathematical modeling problem of rainbow 2-connection number, (2) To identify the process of the student metacognition when we apply a discovery based learning. In this paper we will investigate the effect when students solve rainbow 2-connection number problems. What do we mean by rainbow 2-connection number, we will give the following definition.

**Definition 1:** Let $G = (V, E)$ be a simple, nontrivial, finite, connected and undirected graph. Let $c$ be a coloring $c : E(G) \rightarrow \{1, 2, \cdots, s\}, s \in N$. A path of edge colored graph is said to be a rainbow path if no two edges on the path have the same color. An edge colored graph $G$ is said to be a rainbow connected graph if there exists a rainbow $u - v$ path for every two vertices $u$ and $v$ of $G$. The rainbow connection number of a graph $G$, denoted by $rc(G)$, is the smallest number of $k$ colors required to edge color the graph such that the graph is rainbow connected. Furthermore, for an 2-connected graph $G$, the rainbow 2-connection number $rc_2(G)$ of $G$ is defined to be the minimum number of colors required to color the edges of $G$ such that every two distinct vertices of $G$ are connected by at least 2 internally disjoint rainbow paths [13].
2. Methodology
The research was conducted in two parts, namely the quantitative research model and the qualitative research model or mix methods. This research uses mix methods for maximum research results [11]. Quantitative research model used is quasi-experimental design while the qualitative model used in this research is descriptive qualitative model, because this research is studying and analyzing student ability in solving problems that arise through student metacognition. The instruments used in this study are the test of student learning outcomes, student worksheets, test research activities and interviews. The population in this study is all PPG students who take combinatorial and trigonometric courses. The number of students is 48 people. The respondents in this study were students of PPG class A and B. A total of 48 students, 24 class A students for experimental group and 24 class B students for control group. The duration of this study is one month with 4 times face to face or meeting. The first day of the experimental class is the pre-test and division of the Student Worksheet 1, the second day of the Student 2 Worksheet, the third day of the Student Worksheet 3 and the fourth day is post-test. For the control class on the first day is pre-test and then followed by conventional learning with the initial material according to the student worksheet 1 which is the rainbow connection number, the second day is the rainbow 2-connection material with sub-edges and vertices, the third day of the rainbow 2-connection number material with the sub-matter of the formulation of the function formula. On the fourth day of the post-test.

The steps in this research are divided into three stages: preparation stage, implementation stage, and final stage of the research. The preparatory phase includes the formation of a research group consisting of several researchers / lecturers with expertise in the field to be studied, developing the MFI. Implementation phase, covering the implementation of learning process based on metacognition and giving Research Activity Test (TAR). This stage includes (1) providing basic information about the material being studied, (2) showing the results of lecturer's research in the study group or the research group related to the material being discussed; (3) assigning the assignment of the student worker (b) the research process, (c) the way of analysis, (d) the formulation of conclusions, and (e) the values emerging from the research results (4) with the student lecturers to draw conclusions. In this stage students are more involved in learning. Lecturers play more role as facilitator. If possible during the discussion, if there are problems that require literature, lecturers can show it through visual media so that the problems faced by students can be answered The final stage, which is processing, data analysis and drawing conclusions.
To determine the effect of discovery based learning model in comparison to conventional teaching, this study used quasi-experimental equivalent group control with pre-test and post-test. In this study, pre-tests were performed on experimental class students and control classes and the results illustrate that both classes are equivalent. Learning discovery based learning model consists of: (1) stimulation (i.e. stimulation) i.e. students are faced with something that raises question marks to arise desire to investigate themselves (2) problem statement (identification problem) include identifying and analyzing the problems they face (3) data collection (collecting data) includes collecting (collection) of relevant information, reading literature, observing objects, interviews with speakers, self-test and so on (4) data processing (data processing) i.e. conceptualization and generalization (5) generalization (drawing conclusions) is based on verification results then formulated the principles underlying generalization.

Students in the experimental class are taught using treatment measures discovery based learning while students in the control class are taught the contents of the same student worksheet using conventional teaching models during the study.

**Figure 1.** Research Design

For data analysis, researchers focused on student learning outcomes, diversity of answers or new ideas and student responses, and the relationship between discovery based learning model learning to improve students’ metacognition in solving mathematical modeling problems of 2-connection numbers of rainbow.

### 3. Tasks

In this study, researchers give the following tasks related to student numbers combinatorial problem rainbow 2-connection on a graph G. AG is defined as the pair set \((V(G),E(G))\) with \(V(G) = \{v_1, v_2, v_3, \ldots, v_n\}\) is non-empty finite set and its elements are called points (Vertex), while \(E(G) = \{e_1, e_2, e_3, \ldots, e_n\}\) is the set side (Edge) which connect two or more points [13]. Meanwhile, from any point \(v_i\) to any point \(v_j\) is \((v_i, v_j)\). Rainbow Connection is side coloring on graph G where every two distinct points have at least one path with color along different paths as the colors of the rainbow. The rainbow 2-connection number is an extension of the regular rainbow connection where every two different points in G are connected with at least two rainbow paths. Develop a rainbow connection that was previously rainbow 1-connected that only has one path \(u - v\) to be rainbow 2-connected. If there
are at least 2 parts in the rainbow path separate (internaly disjoin rainbow path) that connects every two points different in $G$, then graph $G$ is said to be rainbow 2-connected which is notated with $rc_2(G)$ [16]. As the following illustration is given a rainbow 2-connection illustration on the Fan graph, see Figure 1. In the research students are given the task of searching for 2-rainbow connections from any special graph.

![Figure 2. Example 2-rainbow connection of any fan graph](image)

Since the purpose of this research is investigations the effect discovery based learning on students metacognition in solving rainbow 2-connection numbers we assumed some students would have experienced more problem in rainbow 2-connection we felt it desisase to have a deeper understanding of students. Hence some cases were selected for understanding the problem they important, Qualitativ approach was insed to collect information.

4. Data Collection and Data Analysis
Students in both experiment and control classes complete pre-test and post-test student mathematics achievement and attitudes toward mathematics. Quantitative data is processed using SPSS statistical software. This quantitative data is treated as the interval scale in the study. The qualitative data, the researchers conducted interviews with students who are categorized as low, medium and high metacognition students. Descriptive and inferential statistics are used to analyze quantitative data while Analysis is used to analyze qualitative data. Data from students such as frequency, mean and standard deviation are summarized using descriptive statistics to test hypotheses, inferential statistics i.e. independent-sample t-test Because students in the control class are different from the experimental class students so the data becomes independent, therefore the t- sample is done to know the effect of discovery based learning model to improve students' metacognition of experimental class and control class. The independent sample t-test was used to compare the mean values of the two groups to find out the difference significant at 0.05.

5. Results And Discussion
The results of the study are described in relation to the research objectives described above. To test the effect of the discovery based learning model of learning to improve student metacognition, the Independent Sample T-Test was used to analyze the pre-test and the mean post-test grades of students in the experimental and control classes. The test of Independent Sample T-Test is determined by examining the data normality. Students' learning outcomes are pre-test and post-test for 48 students. The second test for normality divided by standard slope error must be between two negative and positive two. As illustrated in Table 1, the mean pre-test score for the experimental class is 65.10 (Std Deviation = 11.875) and the control class is 53.85 (Std Deviation = 8,274).
Table 2. Descriptive Statistics Pre-Test

|          | N  | Mean | Std. Deviation | Minimum | Maximum |
|----------|----|------|----------------|---------|---------|
| Experim  | 24 | 59.38| 11.064         | 40      | 80      |
| ents     |    |      |                |         |         |
| Control  | 24 | 48.29| 7.849          | 40      | 75      |
|          |    |      |                |         |         |

Table 3. One-Sample Kolmogorov-Smirnov Test

|          | experiments | Pre-control |
|----------|-------------|-------------|
| N        | 24          | 24          |
| Normal   | Mean        | Mean        |
| Parameters | Std. Deviation | Std. Deviation |
| Most Extreme | Absolute | 189         | 155       |
| Differences | Positive | 135         | 155       |
|            | Negative   | 189         | 145       |
| Test Statistic | 189     | 142^c       |
| Asym. Sig. (2-tailed) | 026^c   | 142^c       |

a. Test distribution is Normal.
b. Calculated from data.
c. Lilliefors Significance Correction.

Table 4. Descriptive Statistics Post-Test

|          | N  | Mean | Std. Deviation | Minimum | Maximum |
|----------|----|------|----------------|---------|---------|
| experiments | 24 | 65.10| 11.875         | 40      | 90      |
| Controls  | 24 | 53.85| 8.274          | 45      | 83      |

Table 5. One-Sample Kolmogorov-Smirnov Test

|          | experiments | Post-control |
|----------|-------------|--------------|
| N        | 24          | 24           |
| Normal   | Mean        | Mean         |
| Parameters | Std. Deviation | Std. Deviation |
| Most Extreme | Absolute | 163         | 148       |
| Differences | Positive | 107         | 148       |
|            | Negative   | 163         | 142       |
| Test Statistic | 163     | 148       |

Table 2 shows the results kolmogorov-Smirnov test known that the value of Sig. on Asymp. (2-tailed) is 0.098 for the experimental class and 0.185 for the control class. sig value on the experimental class and control class are > 0.05 (level of significant). So it can be concluded that the
results of data values of student learning outcomes in the class of experiments and control classes are normally distributed so that it can be tested Independent Sample T-Test.

| Table 6. Group Statistics |
|--------------------------|
| class | N | Mean | Std. Deviation | Std. Error Mean |
| V experimental | 2 | 65.10 | 11.875 | 2.424 |
| V controls | 4 | 53.85 | 8.274 | 1.689 |

| Table 7. Independent Samples Test |
|----------------------------------|
| Levene's Test for Equality of Variances | t-test for Equality of Means |
| F | Sig. | T | df | Sig. | (2-tailed) | Mean Difference | Std. Error Difference | 95% Confidence Interval of the Difference |
| Equal variances assumed | 3.187 | .081 | 3.808 | 46 | .000 | 11,250 | 2,954 | 5,303 | 17,197 |
| Equal variances not assumed | 3.808 | 41,073 | .000 | 11,250 | 2,954 | 5,284 | 17,216 |

Based on Table 4 above, it can be seen that the value of Sig. on Levene's test for equality of variances of 0.081 > 0.05 so that the results of students' creative thinking learning is said to be homogeneous then decision making using lane equal variances assumed. In the line of assumed equal variances obtained value Sig. (2-tailed) of 0.000 < 0.05). Because the significance value is less than 0.05, meaning H₀ is rejected, Hₐ is accepted. If it is consulted with the above decision-making guidance then the learning result of learning discovery based learning model to improve student metacognition in experiment class is better than control class. From the above quantitative analysis it can be concluded that the learning model of discovery based learning to improve students' metacognition on the experimental class is better than the control class.

Based on the results of research and discussion, obtained some conclusions that is the application of learning discovery based learning model to improve students metacognition in solving the problem of mathematical modeling rainbow 2-connection number has met the valid, practical and effective criteria. After doing kolmogorov smirnov test, can be seen that the value of Sig. on Levene's test for equality of variances of 0.081 > 0.05 so that the results of student creative thinking learning is said to be homogeneous then decision making using lane equal variances assumed. On the lane equal variances assumed obtained Sig (2-tailed) value of 0.000. (< 0.05). Because the significance value is less than 0.05, it means that H₀ is rejected, Hₐ is accepted, so the learning result of the learning discovery based learning model in the experimental class is better than the control class.
The first step uses a qualitative method.

**Figure 3. Analysis of Pre-test result**

In the result of pre-test value in the control class, there were no students who have high metacognition category, 87.5% students with medium metacognition category and 12.5% students with low metacognition category. In the experimental class there were 16.7% of students with high metacognition category, 50% of students with medium metacognition category and 33.3% students with low metacognition category.

**Figure 4. Analysis of Post-test result**

The value of post-test in the control class there are 15% of students with high metacognition to planning indicator, 15% to monitoring and 30% to evaluation, 20% of students with medium metacognition to indicator planning, 25% to monitoring and 26% to evaluation. 20% students with low metacognition to planning indicator, 10% to monitoring indicator and 34% to evaluation indicator. In experiment class there are 22% of students with high metacognition to planning indicator, 33% to monitoring and 45% to evaluation. 30% of students with medium metacognition to indicator planning,
30% to monitoring and 40% to evaluation. 40% students with low metacognition to planning indicator, 20% to monitoring indicator and 40% to evaluation indicator.

![Figure 6. concept map subject](image)

![Figure 7. LKM subject 1](image)

From the data described in the steps of the learning discovery based learning model, the students’ assessment process in Problem solving begins with action steps: Stimulation, Problem Statement, Data Processing, Generalization (drawing conclusions) and then students internalize the action into prose by discovering, predicting patterns and finding functional formulas of special graphs found. The structure of the study presented a phase portrait in the following figure.
From the image above there is a way of students thinking who is described as a portrait phase above, where there are steps ways of thinking. In the portrait phase above there are two circles of black circles and white circles. Black circle on the portrait phase is the activity or way of thinking in the model of discovery based learning conducted by students while the white circle on the portrait phase above is the covering or elaboration of the way of thinking in the model of discovery based learning conducted by students or translation of the black circle. There can be some subjects or students who do the work unlike the above example and the result of the students' metacognition on LKM 1, the MFI 2 researcher goes into rainbow 2-connection number material, so that the student already has the concept of material in the previous MFI. Here is an example of the results of the steps of 5 subjects or students:

| Step | Activity                                    | Step | Activity                                    |
|------|---------------------------------------------|------|---------------------------------------------|
| 1    | Make observation                            | 7    | Complete pattern 1 on the provided graph    |
| 2    | Understand some graphs provided             | 8    | Understand pattern 1 on the provided graph  |
| 2a   | Understand definition                       | 9    | Looking for patterns of staining new from the graph that the banks have provided |
| 2b   | Understanding characteristic                | 9a   | As the number rainbow 2-connection          |
| 3    | Completing parts graph empty                | 9b   | Not according to the number rainbow 2-connection |
| 4    | Viewing the cardinality of some graph provided | 10  | Creating 1 special graph                    |
| 5    | Understanding the cardinality of some graphs provided | 11  | Determining cardinality, the number of points and the number of sides |
| 5a   | Understanding point                         | 11a  | Determining cardinality                     |
| 5b   | Understanding side                          | 11b  | Determining the number of dots              |
| 5c   | Understanding points and sides              | 11c  | Specifies the number of sides               |
| 6    | determine the number of points and the number of sides | 12  | Giving labeling color number rainbow 2-connection, expanding patterns and determine the formula for functions |
| 6a   | Specifies the number of point               | 12a  | Provides labeling color number rainbow 2-connection |
| 6b   | Specifies the number of sides               | 12b  | Gives a pattern                            |
| 6c   | Specifies the number of dots and sides      | 12c  | Specifies the function formula              |
Subject 1 (High Metacognition)
For the concept map on the subject 1 as follows:

![Concept Map](image)

The first step on the subject 1 is to observe and then understand some graphs that have provided authors, in the process of understanding subject 1 only understand the notion of a rainbow connection number alone is not the characteristic of the rainbow 2-connection number. The student then completes the activity of completing the blank portion of the graph and proceeds to the understanding of the cardinality of the graph provided in which subject 1 has an understanding of the point and side along with determining the number of points and the number of sides on the graph. The next step of subject 1 completes the pattern of staining I on the graph that has been provided and does not do step 8 but straight to step 9 that is making a special graph, subject 1 has an understanding of the making of special graph with the pattern of rainbow 2-connection number and subject 1 only continue to define the cardinality of the graph, labeling it so that it forms a pattern, expands the pattern and determines the function formula.
After the researcher analyzed the results of the pre-test, post-test and analysis of the LKM, the results of the subject 1 pre-test (E23) in the experimental class were obtained in the high category, which had a value of 80. At the results of post-test and analysis of subject 1 (E23) is categorized as high with a value of 90.

The excerpt from the interview above can be seen that the subject 1 metacognition aspect (E23) is very good because it can plan, monitor and evaluate the answers or results of their thoughts. After the interview, the researcher conducted a test. The researcher gives the test directly to the subject by giving a note, where the note contains a flow of understanding about the 2-rainbow connection number.

To strengthen the results, the researchers conducted interviews with the subject 2. In the interview the researcher provides a phase portrait media that will show how to think the subject 2 how to do the troubleshooting steps directly. After observing the results the researcher menguwanatan results by conducting the interview with subject 1 (E23), the following excerpts of wawanca with the E23:

Researcher : What did you do after knowing the problem?
E23 : Understanding available graphs, cardinality and graph coloring patterns, complete each available column.

Researcher : Is it finished?
E23 : No, there is still a next step which is to expose the available graph coloring pattern and then create a special graph.

Researcher : How do you find the special graph?
E23 : I made it by trial and error until I found the graph with labeling and coloring that corresponds to the 2-rainbow connection number.

Researcher : Can you develop a special graph that you find in a larger form with the coloring pattern that you find?
E23 : Of course, because I've tried it and when I expose my special graph the formula for the number of points, sides and formulas for coloring functions remains the same and according to what I found.

Researcher : Have you checked the coloring of the side that you specified to a minimum?
E23 : Yes, I have.

Figure 9. phase potrait subject 1
Subject 2 (Medium Metacognition)

In subject 2 did not take observation action but understanding some of the graphs that have been provided, also understanding the meaning and characteristics of rainbow connection numbers and subject 2 is able to complete the blank graph. The next step to observe some graphs that have been provided, to understand the cardinality of graphs in the form of dots and sides and determine the number of points and sides on the graph that has been provided. Subject 2 complete and understand the pattern of coloring then look for new coloring pattern on the graph that has been provided. Next in step 10 the subject 2 creates 1 specific graph, determines the cardinality, and gives the labeling. Subject 2 in step 12 does not exploit the coloring pattern but continues to determine the function formula.

After the researcher analyzed the results of the pre-test, post-test and analysis of the LKM, the results of the subject 2 pre-test (E13) in the experimental class were obtained in the medium category which had a value of 60. At the results of post-test and analysis of subject 2 (E13 ) is categorized as moderate with a value of 70 so that subject 2 metacognition does not increase.
Subject 2 (E13) is quite good because procedural knowledge and conditional knowledge are good, subject 2 (E13) can state, demonstrate and know well the results of their work. Subject 2 (E13) can also plan and monitor and evaluate the results of the work but at the stage of evaluating the answer the subject 2 has problems with determining the function formula. The results of the work belong to subject 2 itself and it is evident that these findings can be developed even though subject 2 says it cannot find the function formula. After observing the results, the researcher understood the results by conducting an interview with subject 2 (E13), following the excerpts of the interview with the E13:

Researcher : What do you think about the test questions?
E13 : Medium, sometimes there is something easy and sometimes it's difficult.
Researcher : Are you interested in finding coloring patterns on graphs?
E13 : Yes, although sometimes finding a new pattern is not easy because it is often affected by the previous pattern.
Researcher : Did you find new patterns that are more than the patterns you make?
E13 : Don't know yet, because making a pattern is difficult and long and thirsty to try first.
Researcher : Can you make a formula for its function?
E13 : No, I have tried but can't find it yet.
Researcher : Have you experienced difficulties in the last test?
E13 : Yes, I can't find generalization formulas and function formulas.

To strengthen the results, the researchers conducted interviews with the subject 2. In the interview the researcher provides a phase portrait media phase that will show how to think the subject 2 how to do the troubleshooting steps directly.

![Phase Portrait Subject 2](image12)

**Figure 12.** Phase portrait subject 2

![Concept Map Subject 3](image13)

**Figure 13.** Concept map subject 3

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The graph is $n = 1$, in that graph each connected point contains of the rainbow connection.

The red mark on the Graph is $n = 2$, in the graph for example the circled point if it is connected does not contain of the rainbow connection.

**Figure 14.** LKM subject 3
Subject 3 (Low Metacognition)

Subject 3 observation then understands some graphs that have been provided and able to understand the meaning and characteristics of rainbow connection numbers. The next step of subject 5 does not do step 4 but proceeds to understand the cardinality but subject 3 can only understand the cardinality at the point as well as determine the number of points. Subject 3 is able to complete the pattern of staining I but on the exposure of the pattern of coloring subject 3 has difficulty so that the work is wrong and the next step look for new coloring pattern from the graph that has been provided. Creating a custom graph, determining cardinality, the number of dots and sides and providing labeling can be done but on the exploitation of the pattern and determining the formula of the subject function 5 is still incorrect.

After the researchers analyzed the results of the pre-test, post-test and LKM analysis, the results of the subject 3 (E5) pre-test results in the experimental class were obtained in the low category, which had a value of 40. In the results of post-test and analysis of subject 3 (E5 ) is categorized as low with a value of 45 so that subject 3 metacognition does not increase. Subject 3 (E5) is not good because of lack of procedural knowledge and conditional knowledge, subject 3 (E5) can express, demonstrate and know the results of his work. Subject 3 (E5) can also plan and monitor and evaluate the results of the work but subject 3 has problems with not understanding about the 2-rainbow connection number.

After observing the results, the researcher menguwatan results by conducting interviews with subject 3 (E5), the following excerpts of the interview with E5:

Researcher : How do you explain your graph?
E5 : The graph I made is a dental graph.

Researcher : How do you explain your pattern?
E5 : I make a coloring pattern that runs, so that every time the side graph expands on the edge of the run.

Researcher : What is your next assignment after developing the graph?
E5 : Cardinality

Researcher : After determining the cardinality what should you do next?
E5 : Checking each edge of the neighboring coloring should not be the same color

Researcher : Can the pattern you find be developed?
E5 : Don’t try yet.

Researcher : Do you experience difficulties in the problem?
E5 : Yes, there is.

For an intelligent result, the researcher conducted an interview with subject 3. At the time of the interview, the researcher provided a combined phase media phase portrait that will reveal the way of thinking of subject 3 Doing the steps directly.

![Figure 15. phase portrait subject 3](image-url)
From the results of learning test described above there are students having low, medium and high metacognition. Student metacognition result in LKM 1, in LKM 1 researcher gives basic material in the form of say rainbow connection is not rainbow 2-connection number this is because to embed the concept or initial material first before moving to material rainbow 2-connection number.

To find out student perception about applying discovery based learning model learning about mathematical modeling problem 2nd number of rainbow connection, researcher conduct interview / give questionnaire to one student in experimental class. Here is one of the results of an open questionnaire with students: Explain what are the applications of the 2nd number of rainbow connections? Answer: Distribution of UN question-sharing routes, electoral boxes. When you give color and labeling to the graph, describe what strategies do you use to get the coloring pattern on the graph? Answer: Perform staining with the minimal amount of color possible, giving color to the graph so as to form a certain pattern. Is the coloring pattern that you find quite beautiful / complicated? Link whether the more complicated will be the more beautiful coloring pattern? Answer: No, I think the coloring pattern does not affect the beauty, but will affect the complexity of determining or finding the formula of the graph function. Will each pattern always be generalizable to the formula? If you do not find generalizations what you do? Answer: Every pattern may not be generalizable formula, if when generalized formula used is not the same then must change the pattern of coloring. Is it after you find a certain pattern, you imagine another pattern that is better but the level of complexity is higher? Whichever you prefer, give the reason of the choice! Answer: No, because with a more complicated pattern it will affect the generalization of the formula. When you find the pattern then after you have checked carefully, is it always possible to find generalizations of the formula? Answer: Yes, generalizations of the formula can be found when the coloring is patterned. Do you always give labeling and coloring to the graph until it really looks the regularity of the dye pattern and there is no doubt to exploit it? Answer: Yes, because when the pattern is already regular coloring will make it easier to determine the coloring pattern when expanded.

6. Discussion
This research was conducted to examine students’ metacognition in solving mathematical modeling problem of rainbow 2-connection number. The findings of this study indicate that the discovery-based learning model has a significant effect on student learning outcomes. Students in the experimental class showed a significant improvement in achievement compared to the control class. The result of the study shows that the increase of the students score on the post-test average score for the experimental class because of the influence of the learning model used and the treatment while using the learning model. Learning outcomes with discovery based learning model is better than conventional learning this is in accordance with the research results [2] and supported by [10] stated that the learning model of discovery based learning can improve student learning outcomes who have a high learning interest. In the classroom grouping of experiments and student control classes are formed randomly, i.e. each class consists of high-ability, medium-skilled and low-ability students so that it is more effective in comparison with traditional teaching in teaching and learning combinatorics of rainbows 2-connection numbers. The data generated from the interviews illustrates that positive student comments on discovery based learning model. Comments obtained from interview sessions show that the implementation of discovery based learning model encourages students to have more ideas to reinforce their self-concept and explain or inform their friends to help solve problems. Students are always active in finding new ideas as well as new graphs that support combinatorial of rainbow 2-connections number. The result is also expressed by [10] stated that the implementation of learning discovery based learning model to improve the learning outcomes goes well, the students look more enthusiastic and interested to follow the learning and can improve student activeness shown through the work of Student Worksheet. Discuss the findings of quantitative approach and than state how qualitative and shared our understanding of discovery based leaning.
7. Conclusion
The implementation of discovery based learning model to improve students’ metacognition in solving mathematical modeling problem 2-connection number of rainbow has given a significant effect on student metacognition. These findings reveal that students’ metacognition for the experimental group is better due to the significant effect of the discovery based learning model. Students in discovery based learning models seem to prefer and be active in learning so as to improve cognitive skills and processes as well as students thinking and working on their own initiative. From the results of the research above illustrates that the learning model of discovery based learning accepted both by students and learning model of discovery based learning is effective in improving student learning performance in the classroom.

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