Each instant of time a new Universe

Yakir Aharonov\textsuperscript{1,2}, Sandu Popescu\textsuperscript{3}, and Jeff Tollaksen\textsuperscript{2}

\textsuperscript{1} School of Physics and Astronomy, Tel Aviv University, Tel Aviv, Israel
\textsuperscript{2} Chapman University, Schmid College of Sciences, Orange, CA 92866, USA and
\textsuperscript{3} H.H.Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol BS8 1TL, U.K.

(Dated:)

We present an alternative view of quantum evolution in which each moment of time is viewed as a new "universe" and time evolution is given by correlations between them.

INTRODUCTION

Since the dawn of civilization, mankind tried to understand the meaning of the inexorable flow of time. One of the philosophical ideas, whose origins can be traced back to Heraclit from Efes, states that the universe gets recreated again and again, at every instant of time. “You never bathe twice in the same river” said Heraclit. Every instant new water, every instant a new universe. This idea, of course, is different from the usual one in which we view the universe as unique, and the objects which inhabit it as just changing their state in time.

Going from philosophy to physics, the way physics is presently formulated is based on the idea of unique universe and evolving objects. But is it possible to reformulate physics to incorporate the idea of a new universe for each instant? As far as classical physics is concerned, the reformulation is rather trivial. We find however that quantum mechanically things are more complicated. The standard formalism of quantum mechanics appears not to allow such a reformulation. It turns out nevertheless that the reformulation is possible if we use the two-states formalism\cite{1,2}.

TOY MODELS: THE DIFFICULTY

In preparation for discussing the time evolution from this alternative point of view, let us start by asking a much simpler question. Consider first classical mechanics. Suppose a particle evolves such that its trajectory is $x(t)$ (fig.1). Consider now a set of $N$ time moments, $t_1, t_2, \ldots, t_N$. Is it possible to prepare instead of this single particle a set of $N$ particles such that if we perform at some given time $\tau$ measurements on these $N$ particles we’ll get the same information as we would have obtained by making measurements at $t_1, t_2, \ldots, t_N$ on the original single particle considered before?

The solution is quite simple: One has to prepare the $N$ particles (fig. 2) such that

\begin{equation}
\begin{align*}
x_1(\tau) &= x(t_1) \\
x_2(\tau) &= x(t_2) \\
& \vdots \\
x_N(\tau) &= x(t_N).
\end{align*}
\end{equation}

When $N$ increases and the time intervals $t_{i+1} - t_i$ decrease, we could say that the $N$ particles lay down, at a single moment of time (at $\tau$) the entire history of the original particle. One particle at $N$ times is thus equivalent to $N$ particles at one time.

Can we do the same for a quantum mechanical particle?

Consider the simple case of a spin 1/2 particle, prepared in some initial state $|\psi\rangle$ and having the hamiltonian $H = 0$. In this case the time evolution of the particle is trivial,
First of all, the state (3) contains too much information. Indeed, suppose somebody prepared the original particle in state $|\psi\rangle$, and gave it to us without telling us what the state is. Then, as we have a single particle (i.e. a single spin 1/2 particle), the task can be accomplished (fig. 3) by preparing the $N$ particles each in the same state $|\psi\rangle$, that is

$$|\Psi\rangle_1|\Psi\rangle_2...|\Psi\rangle_N. \quad (3)$$

But this mapping is not appropriate for many reasons. First of all, the state (3) contains too much information. Indeed, suppose somebody prepared the original particle in state $|\psi\rangle$ and gave it to us without telling us what the state is. Then, as we have a single particle (i.e. a single copy of an unknown state $|\psi\rangle$), there is no way of learning what its state is. However, in (3) we have $N$ particles, all in the same state - by making different measurements on the different copies and looking at the statistics of the results we can learn the state (better and better as $N$ becomes larger).

Second, the time evolution (2) contains subtle correlations, which usually are not noticed, and which do not appear in the state (3). Suppose, for example, that the state $|\psi\rangle = |\sigma_z = 1\rangle$, i.e. the spin is polarized “up” along the $z$ axis. It is generally considered that since the particle is at every moment in a definite state of the $z$ spin component, the $z$ spin component is the only thing we know with certainty about the particle - no other spin component commutes with $\sigma_z$, hence it is not well-defined. However, there are multi-time variables whose values are known with certainty; given the evolution (2). For example, although the $x$ spin component is not well defined when the spin is in the $|\sigma_z = 1\rangle$ state, we know that it is constant in time, since the Hamiltonian is zero. Thus, in particular, the two-time observable

$$\sigma_x(t_2) - \sigma_x(t_1) = 0. \quad (4)$$

As described in [3, 2], this observable can be measured in the following way. Following von Neuman’s measuring formalism, consider a measuring device whose pointer position is denoted by $q$ and its canonical conjugate momentum $p$ and let the interaction between the spin and the measuring device be described by the interaction Hamiltonian

$$H_{\text{int}} = -\delta(t-t_1)p\sigma_x + \delta(t-t_2)p\sigma_x. \quad (5)$$

We also assume that the Hamiltonian of the measuring device at all other times is zero (i.e. the pointer doesn’t move by itself). Due to the strong coupling between the spin and the measuring device during the measurement (the delta function coupling) we can neglect during the measurement any other interaction affecting the spin. From the Heisenberg equations of motion we obtain

$$\frac{dq}{dt} = i[q, H_{\text{int}}] = (\delta(t-t_2) - \delta(t-t_1))\sigma_x(t). \quad (6)$$

The equation can be integrated easily, by noting that $\sigma_x$ doesn’t change in the two (infinitesimally) short interaction times $(t_1 - \epsilon$ to $t_1 + \epsilon$ and $t_2 - \epsilon$ to $t_2 + \epsilon$) when the measuring interaction takes place since it commutes with the interaction Hamiltonian. Hence, integrating (6) we obtain

$$q(t_2 + \epsilon) - q(t_1 - \epsilon) = \sigma_x(t_2) - \sigma_x(t_1), \quad (7)$$

in other words, the difference between the final and initial positions of the pointer indicates the value of the two-time observable $\sigma_x(t_2) - \sigma_x(t_1)$. Crucially, this is a measurement which tells only the value of $\sigma_x(t_2) - \sigma_x(t_1)$ but not the value of $\sigma_x(t_1)$ or $\sigma_x(t_2)$ separately. Now, when the Hamiltonian acting on the spin is zero, $\sigma_x(t_1) = \sigma_x(t_2)$ and therefore the measuring device will indicate

$$q(t_2 + \epsilon) - q(t_1 - \epsilon) = \sigma_x(t_2) - \sigma_x(t_1) = 0. \quad (8)$$

It is also important to note that, given that the spin Hamiltonian is zero, after the measurement is finished, i.e. after $t_2$, the spin is brought back in the initial state, regardless of what this state is. Indeed, suppose that the initial state of the spin is $|\psi\rangle$. By decomposing $|\psi\rangle$ in the $x$-basis, in the Schrodinger representation the evolution of the spin and measuring device is given by

$$|\psi\rangle|q = 0\rangle = (\alpha|+\rangle + \beta|\downarrow\rangle)|q = 0\rangle \rightarrow \alpha|+\rangle|q = 1\rangle + \beta|\downarrow\rangle|q = 1\rangle \quad (9)$$

where the two arrows describe the evolution of the spin and measuring device at $t_1$ and $t_2$ respectively. (Note that the first interaction shifts the pointer with the value $-\sigma_x$ while the second shifts the pointer with $+\sigma_x$.)

Coming back to the $N$ spins in the state (3), there is no correlation whatsoever in between the $x$ components of, say, particles 1 and 2 which were intended to describe the original particle at times $t_1$ and $t_2$. More over, it is not
only the $x$ spin component for which the original particle presents such time correlations, but all spin components. That is, \[ (\hat{\sigma}^x_1(t_1) = \hat{\sigma}^x_2(t_2) = \ldots = \hat{\sigma}^x_N(t_N) \] for any direction $\hat{n}$. (Following the above procedure, we would then obtain $\hat{\sigma}_i(t_i) - \hat{\sigma}_j(t_j) = 0$ for any $i$ and $j$.) Obviously, the $\hat{n}$ spin components of the $N$ spins (except for the $z$ components) are not correlated in this way.

We reached thus the conclusion that the $N$ particles in the state $|3\rangle$ do not describe faithfully the behavior of the original particle at $t_1...t_N$. The question is now whether there exists any state of $N$ particles that could be such a faithful representation? It is easy to see that the answer is “no”. Indeed, there is no state of $N$ spins such that

\[ \hat{\sigma}^1_1 = \hat{\sigma}^2_2 = \ldots = \hat{\sigma}^N_N \] for every direction $\hat{n}$. At best, one may find a two particle state - the singlet state - for which the spins are anti-correlated instead of correlated i.e.

\[ \hat{\sigma}^1_1 = -\hat{\sigma}^2_2 \] for every $\hat{n}$. And even anti-correlation cannot be extended to more than two particles. It appears thus that we reached a dead end.

It is tempting to think that the reason we arrived at this dead end is that we didn’t take into account that quantum mechanically measurements modify the state of the measured system. We could say that the state of the original particle is constant in time, \[ |2\rangle \], and thus is modeled by the $N$ spins in state \[ |3\rangle \] only if no measurements are performed. If measurements are performed, the time evolution of the original particle is no longer given by \[ |2\rangle \], so we shouldn’t expect to model it by \[ |3\rangle \]. But this is actually not the true reason for our difficulties. Indeed, even if we don’t actually measure $\hat{\sigma}_x(t_2) - \hat{\sigma}_x(t_1)$ but merely compute it for the evolution \[ |2\rangle \], we see that it has not the same value as if we compute it for the state \[ |3\rangle \]. So problems arise already at this stage - the state \[ |3\rangle \] simply doesn’t contain the correlations which the free evolution of the original particle prescribes.

**TOY MODELS: THE SOLUTION**

We now arrived at a crucial point. Although a state of $N$ spin 1/2 particles with complete correlations among all their spin components as required by \[ |11\rangle \] doesn’t exist in the usual sense, there exist pre- and post-selected states \[ |2\rangle \] with this property. As we show now, $N$ spin 1/2 particles in a suitably prepared pre- and post-selected state can, at one time, $\tau$, mimic $N$ moments of time in the evolution of a single spin 1/2 particle.

![Figure 4: $N + 1$ "spins" and $N$ ancillas. At time $\tau - \epsilon$ spin $S_0$ is prepared in state $\psi$ while ancilla $A_k$ is maximally entangled with spin $S_k$ (the maximal entanglement is represented by the continuous line connecting the ancilla with the spin). At time $\tau + \epsilon$ a "Bell" state (BSM) is performed on spin $S_{k-1}$ and ancilla $A_k$. The experiment is deemed successful if and only if each of the BSMs yields the outcome corresponding to the maximally entangled state $|\Phi\rangle_{S_{k-1},A_k} = 1/\sqrt{2} \sum_{i=0}^1 |i\rangle_{S_{k-1}} |i\rangle_{A_k}$. In case of a successful experiment, if measurements were performed on spins $S_0...S_{N-1}$ at time $\tau$, their results simulate measurements performed at $N$ moments of time on a single spin prepared in state $\psi$ and evolving with a Hamiltonian equal to zero.](attachment:image.png)

The procedure we will use has three steps. At time $\tau - \epsilon$ we prepare an initial state. At time $\tau$ we perform the measurements that are supposed to simulate the evolution of the original single spin. At time $\tau + \epsilon$ we perform an additional measurement; only if this measurement is successful, we deem our simulation procedure to have succeeded. Specifically, consider $2N - 1$ spin 1/2 particles. $N$ of them will be used as "spins", and we denote them by $S_0,S_1...S_N$. They will simulate $N$ time moments of the evolution of our original spin. For example, a measurement at time $\tau$ on the spin $S_k$ should simulate the measurement on the original spin at time $t = t_k$. Furthermore a two time measurement on the original spin, say at $t_k$ and $t_l$ will be simulated by a measurement on spins $S_k$ and $S_l$ and so on. The other $N - 1$ spins are ancillas, which we denote by $A_1,A_2...A_N$. They are used for helping in our procedure, however, no measurements will be performed on them at $\tau$. We arrange the "spins" and ancillas as illustrated in fig4.

At $\tau - \epsilon$ we prepare the particles in the initial state

\[ |\psi\rangle_{S_0} |\Phi\rangle_{A_1,S_1} \ldots |\Phi\rangle_{A_N,S_N} \] (13)

where $|\Phi\rangle$ is a maximally entangled state of the ancilla $A_k$ and the associated spin, $S_k$, namely $|\Phi\rangle_{A_k,S_k} = 1/\sqrt{2} \sum_{i=0}^1 |i\rangle_{A_k} |i\rangle_{S_k}$ and where $|i\rangle$, $i = 0,1$ represent some arbitrary base vectors. At a later time, $\tau + \epsilon$ we subject the pairs of spins composed by spin $S_{k-1}$ and ancilla $A_k$ to a measurement of an operator that has the maximally entangled state $|\Phi\rangle_{S_{k-1},A_k} = 1/\sqrt{2} \sum_{i=0}^1 |i\rangle_{S_{k-1}} |i\rangle_{A_k}$ as one of its nondegenerated eigenstates (for example the "Bell" operator). Now, as it is easy to directly verify, in the case when all these measurements performed at $\tau + \epsilon$ yield the outcome corresponding to this state, then measurements performed at $\tau$ on the "spins" reproduce the same statistics as measurements on the original single spin.
in our case the pre-and post-selected state of the spins is where \( \tau \).

The above procedure may seem rather convoluted. However, it has a very simple interpretation in the language of pre- and post selected states \([2, 3]\). The procedure simply prepared a particular pre- and post-selected state of the \( N \) spins \( S_1 \ldots S_N \). The defining characteristic of this state is that the post-selected state of one particle is completely correlated to the pre-selected state of the next particle as illustrated in fig. 3. Technically this is possible because post-selected states propagate backwards in time and behave as complex conjugates of pre-selected states. This accounts for the correlations that cannot be created when we consider a pre-selected only state (i.e. a state prepared at \( \tau = \epsilon \)).

The idea of pre- and post-selected states and the above way of preparing them was discussed in detail in [2]. Using the notation \( \Phi_{k+1,k}^{\tau_{+},\tau_{-}} \) for the maximally entangled two-time state

\[
\Phi_{k+1,k}^{\tau_{+},\tau_{-}} = \sum_i |i\rangle_{S_{k+1}}^{\tau_{-}} \phi_k^{\tau_{+}} (i) \tag{14}
\]

where \( \tau_{\pm} = \tau \pm \epsilon \), from the results in \([2]\) it follows that in our case the pre- and post-selected state of the spins is

\[
\Phi_{N,N-i}^{\tau_{+},\tau_{-}} \ldots \Phi_{2,1}^{\tau_{-},\tau_{+}} \Phi_{1,0}^{\tau_{+}} |\Psi\rangle = |\Phi\rangle, \tag{15}
\]

Note that this pre-and post-selected state refers solely to the spins; the ancillas were there only to help prepare this state.

The pre- and post-selected state \([15]\) explicitly shows the two main characteristics of the time evolution of the original spin. On one hand, at the initial time \( t_0 \) the original spin was prepared in the state \( |\Psi\rangle \). To this corresponds the fact that in \([15]\) the pre-selected state of \( S_0 \) is the same as that of the original spin. On the other hand, we also know \([10]\) that the spin components along any direction, although undefined, are constant in time, that is, they are fully correlated. These correlations are realised in \([15]\) via the complete correlations between the post-selected state of one spin and the pre-selected state of the next \([14]\). (Indeed, it is easy to verify that the maximally entangled state \([14]\) is invariant under a simultaneous change of basis for \( S_{k+1} \) and \( S_k \).

**EVERY MOMENT OF TIME A NEW UNIVERSE**

Up to this point we only dealt with a toy model. We now come to the main question, namely how to interpret the time evolution of a quantum particle from the point of view of the philosophical idea of “each moment of time a new universe”. As far as classical physics is concerned, we could formalize this idea by associating a separate configuration-space to each moment of time. A moving particle would then correspond to one particle in each space, having their positions appropriately correlated. In effect, this would mean associating a different configuration space to each of the \( N \) particles in fig 1b and “stacking” them on top of the other along the time axis, see fig?. Naively, one would expect that quantum mechanically this would correspond to associating to each moment of time a separate Hilbert space. The total Hilbert space would be therefore \( \mathcal{H} = \mathcal{H}_0 \otimes \mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_N \). The problem however, as we saw before, is that no state in such a Hilbert space can account for the desired correlations. The solution follows from the above described mapping between the time evolution of the spin 1/2 particle and \( N \) spin 1/2 particles: we have to associate two Hilbert spaces to each moment of time and “stack” them on top of each other along the time axis (fig. 6a).

We consider time as being discrete, made out of finite time intervals, stacked one on top of the other like little bricks. To each moment of time we associate a Hilbert space of bra vectors at the time boundary towards the past and a Hilbert space of ket vectors at
the time boundary towards the future. The total Hilbert space is therefore of the form $\mathcal{H}_N \otimes \cdots \otimes \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_0$. The state of a quantum system during each instant of time is thus determined by two wave-functions. One of them is fixed at the past time boundary of the time interval and is “evolving” towards the future, the other one is fixed at the future boundary of the time interval and it is “evolving” towards the past. We shall call these two states “pre-selected” and “post-selected”.

Consider now the example we presented at the beginning of this paper: At time $t_0$ the spin was prepared in state $|\Psi\rangle$ and then the evolution is trivial, i.e. the Hamiltonian is zero. In our new formalism the preparation at time $t_0$ corresponds having at time $t_0$ the forward evolving state $|\Psi\rangle_{t_0} \in H_0$. At all other moments of time we know that all spin components are completely correlated. We describe this by taking the wavefunctions at the common boundary of two subsequent moments (i.e. the pre-selected state of the earlier moment and the post-selected state of the later moment) to be maximally entangled and completely correlated: $\sum_i |i\rangle_{t_1} |i\rangle_{t_0}$ and so all at all times. Here the vectors $|i\rangle_{t_1}$ and $|i\rangle_{t_0}$ form arbitrary orthonormal bases in $\mathcal{H}_1$ and $\mathcal{H}_0$ respectively and represent the same physical state (such as $|\sigma_z = 1\rangle$ and $|\sigma_z = 1\rangle$). Since all time moments are correlated this way, the time evolution now looks like a chain of connected time intervals. Hence the total state is

$$\sum_k |k\rangle_{t_k} \cdots |k\rangle_{t_1} \cdots |i\rangle_{t_0} |i\rangle_{t_0} \langle |\Psi\rangle_{t_0}. \tag{16}$$

The example above can be easily generalized to arbitrary quantum evolutions which consist of unitary evolutions and measurement induced collapses (fig 6b). When the hamiltonian is non-zero subsequent moments of time continue to be maximally entangled, but the correlation is now “skewed”. That is, the correlation between two moments of time is now of the form

$$\sum_i |u_i\rangle_{t_2} |i\rangle \tag{17}$$

where

$$|u_i\rangle_{t_2} = U_{2,1} |i\rangle_{t_2}. \tag{18}$$

Here $U_{2,1}$ represents a unitary transformation acting on $\mathcal{H}_2$, the pre-selected Hilbert space at time $t_2$, numerically equal to $U(t_2, t_1)$, the unitary that describes the evolution of the particle from $t_1$ to $t_2$ in the standard quantum description.

Note that, similarly to $\sum_i |i\rangle_{t_1} |i\rangle_{t_0}$ the state $|\Psi\rangle_{t_0}$ is also a maximally entangled state between the two moments of time, and also leads to full correlations. The only difference is that the correlations are now not between an arbitrary vector $t_1 (\xi)$ and the corresponding vector $|\xi\rangle_{t_2}$ but between $t_1 (\xi)$ and $U_{2,1} |\xi\rangle_{t_2}$.

On the other hand when a measurement occurs it completely disturbs all the observables that do not commute with the measured observable - their values before the measurement are no longer correlated with their values after the measurement. At the same time a collapse means the introduction of a new boundary condition for times following the collapse as well as a post-selection for times previous to the collapse. Suppose an instantaneous ideal von Neuman measurement took place at time $t_k$ and let $|\phi\rangle$ be the eigenstate corresponding to the observed eigenvalue. In our alternative formalism this collapse is implemented simply by $|\phi\rangle_{t_{k+1}} |\phi\rangle_{t_k}$.

We have now reached our desired alternative description of time evolution. To summarize:

- Each moment of time is indeed one "Universe" but it has associated to it not one but two Hilbert spaces, one corresponding to the "past" boundary of this time moment and one to its "future" boundary.
- Time evolution is represented by correlations between subsequent moments of time, more precisely between the "future" boundary of the earlier time moment and the "past" boundary of the later time moment.
- Unitary time evolution is implemented by maximal entanglement between subsequent moments of time.
- A measurement induced collapse destroys the entanglement and effectively decouples the entire time evolution up to that moment by what happens later; technically, the state before the moment of collapse is in a direct product with the state after it.
- A partial collapse, such as one due to an incomplete measurement will result inentanglement but less than maximal.

**Probabilities**

In the above section we described the time evolution of a quantum system in the "each moment of time a new universe" paradigm. To complete our description there is one more item to address: how to calculate probabilities for the different events. The probability for a particular history to happen can be computed from the "history" state in a straightforward way, with one subtlety. As it stands now, the history is open ended, both towards the remote past and the remote future; without putting boundary conditions at those two end, nothing can be said about the overall
porobability of the history. The standard experimental questions however remove the need for these "cosmological" implications by completely separating a piece of time from its past and future by making complete measurements. The most well-known case is preparing at initial time $t_0$ the system in some state $|\Psi\rangle$ and then performing a measurement at the final time $t_f$ and asking what is the probability to find the system in state $|\phi\rangle$. Looking at our description above, we see that indeed these two measurements cut out a piece of time, i.e. the state starting at $t_1$ and ending at $t_2$ is completely disentangled from the rest. For example, if the Hamiltonian is zero, this piece is:

$$t_f \langle \phi | \sum_k |k\rangle_{t_N} \cdot \cdots \cdot <k| \sum_i |j\rangle_{t_2} \cdot \langle j| \sum_i |i\rangle_{t_1} \cdot \langle i| \cdot |\Psi\rangle_{t_0}. \tag{19}$$

To find the probability of obtaining for $|\phi\rangle$ when the system was prepared in $|\Psi\rangle$ all we do is the following: for each moment of time we contract the vectors belonging to the past and future boundary conditions, i.e. we take the scalar product between the bra and ket vectors corresponding to the same time $t$. The result is the scalar product $\langle \phi | \Psi \rangle$; the absolute value of this, $|\langle \phi | \Psi \rangle|^2$ is the probability we are loong for. It is also immediate to show that in case the Hamiltonian is non-zero, our recepy leads to $|\langle \phi | U(t_f, t_0) \cdot |\Psi\rangle|^2$, the expected result.

**DISCUSSION**

We would like to emphasize what is arguably the most important property of this alternative view of time evolution, namely that there is a complete harmony in between the actions to which the system is subjected and their representation: If a measurement and its associated collapse occur at time $t$ it is there and only there that this information is present - the state $\Phi$ prepared by the collapse appears only at this time. When there are more measurements, each measurement and the state associated to its outcome appear at the time of measurement and only there. At other times, when no measurement is performed all we know is that the time correlations are preserved, and this is what our formalism shows.

Our description is in stark contrast with the usual one in which once we prepare a system in a state $|\psi\rangle$ and we leave it undisturbed than at every subsequent moment of time the state continues to be $|\psi\rangle$. As far as we are concerned however, the state $|\psi\rangle$ doesn’t characterize directly any other moment of time except when it was prepared; it does influence the physics at these other moments, but it does so only indirectly, via a chain of time correlations. What does directly characterize a time when no measurement is performed is that it is an unbroken link in a chain of correlations, nothing more than this; what propagates along the chain is a completely independent issue.

It is very interesting to ponder more carefully on the difference between a measurement and a unitary evolution from our point of view. What we see is a certain complementarity between kinematics and dynamics. When a measurement is performed we know the state at each of the two subsequent moments of time when the measurement took place:

$$|\phi\rangle_{t_k+1} \cdot t_k \cdot |\phi\rangle. \tag{20}$$

On the other hand, when a unitary evolution takes place, the state at each of the two subsequent moments of time is completely uncertain, the state at one moment being entangled with the state at the next moment

$$\sum_i |\psi_i\rangle_{t_k+1} \cdot t_k \cdot |\psi_i\rangle. \tag{21}$$

Furthermore, we note that every measurement is effectively an uncertain time evolution. This is a fact that, as far as we know, it is very rarely mentioned in discussions about quantum measurements. Yet, it is quite obvious. Indeed, as is well known all the observables that do not commute with the measured one are randomized up to some extent, hence their Heisenberg equations of motion must show an uncertain evolution. In its turn, this due to the fact that during the measurement the hamiltonian of the system is uncertain. Indeed, in the standard von Neumann measurement formalism (as used above in [5]) in order to measure an observable $A$ and register its value in the indication $q$ of a pointer we use an interaction Hamiltonian of the form

$$H_{int} = \delta(t) A p \tag{22}$$

where $p$ is the canonical momentum conjugate to the position $q$ of the pointer. Since the initial state of the pointer is well defined, say $|q = 0\rangle$, the momentum of the pointer, $p$ has a large uncertainty $\Delta p = \infty$. In its turn, since $p$ enters the interaction Hamiltonian, it means that as far as the system is concerned, its Hamiltonian is uncertain during the measurement.

The above observations, although not very commonly known, are nevertheless rather straightforward. What our new formalism shows however, is something more subtle: although a measurement is equivalent to an uncertain evolution, the collapse on a particular eigenstate of the measured observable is equivalent to a well-defined superposition of different time evolutions[5]. Indeed, take for example a measurement of the $\sigma_x$, the $x$ component of the spin of a spin 1/2 particle performed at $t_k$. Suppose we found $\sigma_x = 1$. According to our formalism the quantum state at the time of measurement is

$$|\uparrow_x\rangle_{t_k+1} \cdot t_k \cdot |\uparrow_x\rangle. \tag{23}$$

This can be viewed as the superposition of two unitary time evolutions.
| \uparrow x \rangle_{t_{k+1},t_k} \langle \uparrow x | = \\
\frac{1}{\sqrt{2}} \left( | \uparrow x \rangle_{t_{k+1},t_k} \langle \uparrow x | + | \downarrow x \rangle_{t_{k+1},t_k} \langle \downarrow x | \right) + \\
\frac{1}{\sqrt{2}} \left( | \uparrow x \rangle_{t_{k+1},t_k} \langle \downarrow x | - | \downarrow x \rangle_{t_{k+1},t_k} \langle \uparrow x | \right). \\
(24)

Hence our picture suggests a new kind of complementarity between having information about the state of a system versus having information about the dynamics: if one does not know the state, then our picture describes the dynamics as complete correlation. If one obtains information about the state, then the multi-time correlations between conjugate operators are made uncertain, i.e. one loses information about the dynamics in that interval of time. And as for a proper conjugacy relationship, there is a continuous graduation between the extremes. To see this complementarity, consider a partial measurement of \( \sigma_x \) in which the measuring device gives the correct answer (i.e. \( \sigma_x = \pm 1 \)) with probability \( |\alpha|^2 \) and the wrong answer with probability \( |\beta|^2 \) and does this in a way which minimizes the disturbance to the state. This is obtained when the measuring device interacts with the spin via the unitary evolution

\[
| \uparrow x \rangle |0\rangle_M \rightarrow | \uparrow x \rangle (|\alpha|_M + |\beta| - 1)_M \\
| \downarrow x \rangle |0\rangle_M \rightarrow | \downarrow x \rangle (|\alpha| - 1)_M + |\beta|_M) \\
(25)
\]

where \( |0\rangle_M, |1\rangle_M \) and \( | -1\rangle_M \) are different states of the measuring device. Obtaining the value +1 corresponds in our picture to partially destroying the complete correlation between the moments when the measurement occurred and leading to only non-maximal correlations:

\[
\alpha | \uparrow x \rangle_{t_1} \langle \uparrow x | + \beta | \downarrow x \rangle_{t_1} \langle \downarrow x | \\
(26)
\]

We see that in the case \( \alpha = \beta \), we have complete correlation, thus modeling the dynamics. As \( \alpha \rightarrow 1 \) and \( \beta \rightarrow 0 \) we obtain more and more knowledge about the state, while the entanglement, i.e. the dynamics, becomes more and more uncertain.

**MEASUREMENTS ON EPR STATES**

It is very interesting to analyze using our point of view the time evolution of two spin 1/2 particles in a singlet state. The evolution is illustrated in fig 7. At \( t_0 \) the two particles \( A \) and \( B \) are prepared in the singlet state

\[
|S\rangle_{AB,t_0} = \frac{1}{\sqrt{2}} | \uparrow x \rangle_A | \downarrow x \rangle_B + \frac{1}{\sqrt{2}} | \downarrow x \rangle_A | \uparrow x \rangle_B. \\
(27)
\]

![FIG. 7: Two entangled spin 1/2 particles. Entanglement characterizes solely time \( t_0 \) where entanglement is produced. All other times are characterized by trivial time evolution, i.e. maximal entanglement between subsequent moments of time; there is however no entanglement between the particles associated to these times. Alice’s measurement disentangles the time moments of her particle but have no effect on Bob’s particle.](image)

Then each particle evolves separately. That is, the time moments describing particle \( A \) are maximally entangled with each other and the time moments describing the evolution of \( B \) are maximally entangled with each other. There is however no entanglement between particles \( A \) and \( B \) at any other subsequent moment. This situation continues until we disturb the particles.

Suppose now that at time \( T \) Alice performs a measurement on particle \( A \). For example, suppose she measures \( \sigma_x^A \) and finds the value +1. According to our point of view, for particle \( A \) the entanglement between the moments of time \( T \) and \( T + \epsilon \) is broken and in its place we have a direct product state \( | \uparrow x \rangle_A | \downarrow x \rangle_B. \) On the other hand, nothing happens to particle \( B \) - its time moments continue to be maximally entangled.

This conclusion seems to contradict the standard quantum mechanical description. Indeed, in the usual description the state is \( |S\rangle_{AB} \) from time \( t_0 \) until time \( T \) and at time \( T \) the state collapses into the direct product state \( | \uparrow x \rangle_A | \downarrow x \rangle_B. \) In particular, the collapse is symmetric with respect to who produces it: The time evolution is the same whether Alice measured \( \sigma_x^A \) and found +1 or Bob measured \( \sigma_x^B \) and found −1. In our description however, if Alice performs the measurement, the time evolution of her particle is affected and nothing happens to Bob’s, while the opposite would be true if Bob were to perform the measurement. One can check directly however that all observational consequences, i.e. the probabilities for all measurements, are the same in both descriptions. Our point of view however has two main advantages.

First of all, it is relativistically covariant at the level of states. Of course, both views are relativistically covariant at the level of observed results. The standard way however is non-covariant as far as the state description...
is concerned. Indeed, the collapse occurs both at Alice and at Bob at time $T$, i.e. simultaneously in the reference frame in which we chose to work. Had we chosen a different reference frame, the moment at which the collapse occurs for Bob’s particle could have been different. On the other hand, in our description, nothing happens to Bob’s particle when Alice performs a measurement, so no covariance problems arise.

We would like to emphasize however that the relativistic covariance at the level of wave-functions does not necessarily require to consider each moment of time a new universe; it is already present in a simpler version of time evolution, with a “single universe” but with two wave-functions, one propagating forward and the other backward in time [4].

A second interesting feature of our description is that it makes it clear that the evolution of Alice’s particle is different from Bob’s particle, while in the standard description they looked the same. Indeed, in the standard description they appear symmetric - they are in a singlet until time $T$ and then they collapse together on a direct-product state. In our description is clear that for particle $A$ the time moments before and after $T$ are not maximally entangled while for particle $B$ they are. This difference could be checked if in addition to the measurement at time $T$ Alice also measures a two-time variable, say $\sigma_A^z(t_1) - \sigma_A^z(t_2)$ for $t_0 < t_1 < T < t_2$. Since the spin components along the $z$ direction before and after $T$ are not correlated, Alice could obtain $+2$, $0$ or $-2$. On the other hand, if Bob were to measure $\sigma_B^z(t_1) - \sigma_B^z(t_2)$ he would obtain with certainty the value $0$.

**Extensions**

Many more interesting situations are possible. An amusing one is illustrated in fig8. Here every moment of time is fully correlated with the second next. If effect this particle has a “double life” - the even time moments describe a particle whose time evolution is $|\psi(t)| = |\psi_1\rangle$ and the odd moments describe a particle whose time evolution is $|\psi(t)| = |\psi_2\rangle$. As long as we do not take any action to connect them, such as a two-time measurement involving an odd and an even moment, the two lives of this particle do not interact with each other. It is interesting to speculate if such things exist in nature, and what their meaning would be.

**Acknowledgements** SP acknowledges support from the European Research Council ERC Advanced Grant NLST, EU grant Q-Essence and the Templeton Foundation.

![Fig. 8: Two independent ”lives” lived in parallel by the same particle.](image)

[1] Y. Aharonov, P. G. Bergmann and J. Lebowitz, Phys. Rev 134, B1410 (1964).
[2] Y. Aharonov, S. Popescu, J. Tollaksen and L. Vaidman, Phys. Rev. A 79, 052110 (2009).
[3] Y. Aharonov and D. Z Albert, Phys. Rev. D 29, 223, (1982).
[4] Y. Aharonov and D. Albert, Phys. Rev. D 29, 228, (1982); Y. Aharonov, D. Albert and S. D’Amato, Phys. Rev. D 32, 1976 (1985).
[5] In a completely different context, superpositions of time evolutions were discussed in [6] and experimentally verified in [7].
[6] Y Aharonov, J Anandan, S Popescu and L Vaidman, Phys.Rev.Lett 64, 2965 (1990).
[7] Dieter Suter, M. Ernst and R. R. Ernst, Molecular Physics 78:1, 95-102 (1998).
[8] In the standard language this outcome corresponds to the Krauss operator $\alpha|\uparrow_x\rangle\langle\uparrow_x| + \beta|\downarrow_x\rangle\langle\downarrow_x|$.