On the physical nature of accretion disc viscosity

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Abstract

We use well-established observational evidence to draw conclusions about the fundamental nature of the viscosity in accretion discs. To do this, we first summarise the observational evidence for the value of the dimensionless accretion disc viscosity parameter $\alpha$, defined by Shakura & Sunyaev (1973, 1976). We find that, for fully ionized discs, the value of $\alpha$ is readily amenable to reliable estimation and that the observations are consistent with the hypothesis that $\alpha \sim 0.2 - 0.3$. In contrast in discs that are not fully ionized, estimates of the value of $\alpha$ are generally less direct and the values obtained are generally $< 0.01$ and often $\ll 0.01$. We conclude that this gives us crucial information about the nature of viscosity in accretion discs. First, in fully ionized discs the strength of the turbulence is always limited by being at most trans-sonic. This implies that it is necessary that credible models of the turbulence reflect this fact. Second, the smaller values of $\alpha$ found for less ionized, and therefore less strongly conducting, discs imply that magnetism plays a dominant role. This provides important observational support for the concept of magneto-rotational instability (MRI) driven hydromagnetic turbulence.

Keywords: accretion, accretion discs, galaxies: nuclei, magnetohydrodynamics (MHD), black hole physics, stars: pre-main sequence

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1. Introduction

Accretion discs are ubiquitous in the Universe and form on all scales from planetary to stellar to galactic (e.g. Pringle, 1981; Frank et al., 2002). In a thin accretion disc, material orbits a central object of mass $M$ at radius $R$ with Keplerian angular velocity, $\Omega = \sqrt{GM/R^3}$. Some kind of viscous mechanism in the accretion disc drives angular momentum transport and thus allows mass to spiral inwards through the disc (e.g. Pringle & Rees, 1972; Lynden-Bell & Pringle, 1974; Pringle, 1981). During the inspiral, gravitational energy is converted into the kinetic energy of rotation and thermal energy that is radiated from the disc.

It has long been known that ordinary molecular viscosity is far too small to allow accretion to occur on astronomically interesting timescales. Peek (1942) and von Weizsäcker (1943) argued that the likely mechanism for the angular momentum transfer was hydrodynamical turbulence. In two papers, Shakura & Sunyaev (1973, 1976) took this idea a step further and suggested specific physical reasoning behind a means of parametrizing the strength of disc turbulence in terms of a dimensionless parameter $\alpha$.

In Section 2 we outline the derivation of the $\alpha$-parameter, with particular emphasis on the physics behind it, and the limitations that this might impose on its magnitude. We then consider what current observations tell us about the value of the effective disc viscosity in discs that are fully ionised (Section 3) and in discs that are not (Section 4). In Section 5 we discuss the physical implications for the nature of accretion disc viscosity that can be drawn from the observations, and finally our conclusions.

2. The derivation of $\alpha$

In their first paper, Shakura & Sunyaev (1973) introduced a means of parametrizing the effective viscosity in accretion discs by means of a dimensionless parameter $\alpha$. They argued that angular momentum transport, which occurs through the $(R, \phi)$ stress $\tau_{R\phi}$, is most likely caused by [hydrodynamic] turbulence and by magnetism. Here $R$ is the radius and $\phi$ the azimuthal angle in cylindrical coordinates. They noted that the existence of such turbulence is not definite, but that magnetism is always present. They gave the effective viscosity in a turbulent flow, with largest eddy sizes $L$ and largest eddy velocities $u_t$ as $\eta_h = \rho u_t L$, where $\rho$ is the local fluid density. In this paper they took the size of the largest eddies to be $L = H$, where $H$ is the vertical scale-height of the disc and therefore found that for fluid
turbulence one would attain
\[ w_{R\phi} \sim \eta R d\Omega / dR \sim -\eta R u_{\phi} / R \sim -\rho c_s^2 u_t / c_s. \]  
(1)

Here \( u_{\phi} = R\Omega \), and they have used the vertical pressure support equation to deduce \( H \sim R c_s / u_{\phi} \) (e.g. Pringle 1981).

With regard to the \((R, \phi)\) magnetic (or Maxwell) stress they argued that because of plasma instabilities\(^1\) and reconnection\(^2\), the magnetic energy density is unlikely to exceed the thermal energy density, and thus that \( B^2 / 4\pi < \rho c_s^2 \), where \( B \) is a measure of the magnetic field strength. Thus, in this paper, they wrote the defining equation for \( \alpha \) as
\[ w_{R\phi} = \rho c_s^2 \left\{ \frac{u_t}{c_s} + \frac{B^2}{4\pi \rho c_s^2} \right\}. \]  
(2)

In their later paper, Shakura & Sunyaev (1976), refined these arguments. They added the assumption that the largest turbulent eddy sizes, \( L \), might not be the disc scale-height, \( H \), but noted that it is to be expected that \( L \lesssim H \). They also worked in terms of the vertically integrated stress
\[ W_{R\phi} = 2 \int_0^H w_{R\phi} \, dz, \]  
(3)
so that \( \alpha \) is now a vertically averaged quantity defined by
\[ W_{R\phi} = \alpha \Sigma c_s^2, \]  
(4)
where \( \Sigma \) is the disc surface density. With these refinements, they wrote
\[ w_{R\phi} = \rho c_s^2 \left\{ \frac{u_t}{c_s} \frac{L}{H} + \frac{B^2}{4\pi \rho c_s^2} \right\}, \]  
(5)
or, equivalently, the kinematic viscosity \( \nu \) can be written as
\[ \nu = \alpha c_s H, \]  
(6)
where
\[ \alpha = \frac{u_t}{c_s} \frac{L}{H} + \frac{B^2}{4\pi \rho c_s^2}. \]  
(7)

\(^1\)By this they presumably meant mainly magnetic buoyancy, see for example Parker (1979).

\(^2\)In other words, turbulent magnetic diffusivity, in order to prevent the shear causing the magnetic field strength to grow without limit.
They noted, without comment, that it is normally assumed that \( u_t < c_s \) and \( L < H \).

We note here that while the assumption that \( L < H \) is fairly self-evident (unless the turbulence is strongly anisotropic) the demand that \( u_t < c_s \), with the corollary that \( \alpha < 1 \), is less clear cut; there is no reason in principle why the turbulence cannot be supersonic. The argument for subsonic turbulence, given in Shakura & Sunyaev (1973), is simply that if a situation arose in which \( \alpha > 1 \) this would imply that the turbulence in the disc is supersonic, which in turn would lead to strongly enhanced dissipation (presumably through shocks) and rapid disc heating, causing the turbulent velocities to drop rapidly to subsonic values. This is, however, an incomplete argument. The local mass flow, \( \dot{M} \), in the disc is given by

\[
\frac{1}{2} \dot{M} \Omega R = 2\pi \frac{\partial}{\partial R}(W R\phi R^2)
\]

(Shakura & Sunyaev, 1976) and the heating rate (per unit area for, say, the top half of the disc) is given by

\[
Q^+ = -\frac{1}{2} W R\phi R \frac{d\Omega}{dR}.
\]

Both these quantities depend linearly on the magnitude of \( W R\phi \) and hence linearly on the value of \( \alpha \). Thus while it is true that a large \( \alpha \) leads to a large amount of energy dissipation, it also leads to a large accretion rate which can provide the necessary energy to be dissipated. We return to this in Section 5.

3. Fully ionized discs

In this section we summarise determinations of the viscosity parameter \( \alpha \) from observations of accretion discs that are thought to be fully ionized. In general, because the evolution of an accretion disc takes place on the viscous timescale

\[
t_\nu = \frac{R^2}{\nu},
\]

the most reliable measurements for \( \alpha \) come from modeling the time-dependence of evolving discs.

3.1. Dwarf nova outbursts

Cataclysmic variables are binary systems in which a secondary star fills its Roche lobe and transfers mass that is accreted on to a primary white
dwarf through an accretion disc. Dwarf novae are catastrophic variables that undergo outbursts on a timescale of days to months (Warner, 2003). The normal outbursts are thought to be a result of the thermal–viscous instability in the accretion disc around the white dwarf. This occurs due to changes in ionisation state of hydrogen below some critical accretion rate that depends upon the orbital period of the binary (e.g. Cannizzo, 1993; Lasota, 2001). The disc cycles between a hot, high-viscosity state, the outburst phase, where hydrogen is fully ionised and a faint, low-viscosity state, the quiescent state, where hydrogen is mostly neutral.

The thermal–viscous instability is well described by the “S-curve” diagram that shows the steady state disc solutions for the accretion rate (or temperature) through the disc as a function of the surface density at a fixed disc radius (Bath & Pringle, 1982; Faulkner et al., 1983; Meyer & Meyer-Hofmeister, 1983, 1984). Around the temperature at which hydrogen is ionised, the solutions have an “S” shape. As one radius in the disc reaches the critical temperature required for hydrogen to be ionised, its temperature jumps up to the hot state. The heating front propagates through the disc with a snowplough effect (see also Martin & Lubow, 2013). During this outburst phase, the disc evolves on the viscous timescale given in equation (10). In a similar way, a cooling front propagates through the disc, shutting off the high accretion rate. The decay timescale of the outburst allows for a measurement of $\alpha$ in the hot state from modeling the outburst light curve. The disc size is known from the properties of the system and the disc temperature is obtained from the spectra. All models point to relatively large values of $\alpha$, and the most recent models imply that $\alpha \approx 0.1 - 0.3$ (e.g. Bath & Pringle, 1981; Pringle et al., 1986; Smak, 1993, 1999; Buat-Ménard et al., 2001; Cannizzo, 2001a,b; Schreiber et al., 2003, 2004; Balman & Revnivtsev, 2012; Kotko & Lasota, 2012; Coleman et al., 2016).

3.2. X-ray Binary outbursts

Soft X-ray transients (SXTs) are semi-detached binaries with an accreting black hole that also display outbursts. The thermal–viscous disc instability model can be successfully applied to SXTs when X-ray heating is included (van Paradijs, 1996; King & Ritter, 1998). Dubus et al. (2001) modeled SXT light curves and found $\alpha \approx 0.2 - 0.4$. More recently, Tetarenko et al. (2018) analyzed X-ray light curves of twenty-one black hole X-ray binary outbursts and found $\alpha \approx 0.2 - 1$. However, they found a lack of correlation between their estimates of the $\alpha$ parameter and the accretion state, suggesting that outflows may remove significant amounts of mass. Malanchev & Shakura (2015) modeled the light curve of A0620–00 1975 and
found $\alpha \approx 0.5 - 0.6$. Lipunova & Malanchev (2017) modeled the accretion disc in 4U 1543-37 during the 2002 outburst and compared with the accretion rate that is observed from spectral modeling of data from the RXTE observatory. They found that the value for $\alpha$ in X-ray binary outbursts depends upon the self–irradiation, but all models suggest that $\alpha \gtrsim 0.1$. In summary, calculations of $\alpha$ in X-ray binary outbursts are consistent with a relatively large value.

3.3. Be Star decretion discs

Be stars are hot, rapidly rotating, massive stars that are of B spectral type but their spectrum has at some point shown Balmer lines in emission (e.g. Slettebak, 1982; Porter, 1996; Porter & Rivinius, 2003; Rivinius et al., 2013). Be stars eject a circumstellar decretion disc (Pringle, 1991), or an outward flowing disc, that are well described by an $\alpha$ disc model (Lee et al., 1991; Hamuschik, 1996; Porter, 1999; Sigut & Jones, 2007; Jones et al., 2008; Martin et al., 2011). The disc goes through phases of active formation and dissipation (e.g. Bjorkman et al., 2002; Haubois et al., 2012). A value for $\alpha$ may be calculated for this evolving disc.

The first measurement of the viscosity parameter was performed by Carciofi et al. (2012) who examined the Be star 28 CMA and measured the rate of decline of the V-band excess. They found a viscosity parameter of $\alpha = 1.0 \pm 0.2$ during the dissipation phase for the disc. However, it was later determined that the history of the disc has to be taken into account when fitting the dissipation of the light curve and this was revised to $\alpha = 0.2$ (Ghoreyshi & Carciofi, 2017). More recently, Rimulo et al. (2018) used a sample of 54 Be stars and found $\alpha$ values of a few tenths. On average the viscosity parameter is larger during the build–up phase for the disc, $\alpha \approx 0.6$ and lower during the dissipation phase, $\alpha \approx 0.26$. Ghoreyshi et al. (2018) examined $\omega$ CMa with V–band photometry and found that $\alpha$ ranges from 0.1 to 1.0 over the cycles. While more work is required to determine if this trend depends upon the model assumptions, the values are consistent with a relatively high $\alpha$.

4. Partially ionized discs

For discs that are not fully ionized, the estimates of $\alpha$ obtained are typically an order of magnitude, and sometimes many orders of magnitude, smaller than those found for fully ionized discs. Measuring values for $\alpha$ that are small is much more difficult, and therefore much less direct, since the evolutionary timescale is much longer and we typically cannot rely on the
time-dependence of the disc. Here, we discuss well defined observations that suggest much smaller values for $\alpha$.

4.1. The quiescent state for dwarf novae and X-ray binaries

The quiescent state of dwarf novae is defined by the fact that the disc is cool, and therefore only partially ionised. In the quiescent state, $\alpha_{\text{cold}}$ may be estimated through the disc modeling of the outburst. If $\alpha$ were to have the same value during the outburst and in the quiescent state, then it was quickly realized that the observed duration and brightness of the outburst cannot be reproduced. The outburst is too small. However, a smaller value for $\alpha$ in the quiescent state leads to large enough outbursts. For dwarf novae, the two different values for $\alpha$ must be different by a factor of greater than about 10. For typical parameters, $\alpha_{\text{cold}} \approx 0.01$ (Lasota, 2001). Similarly, for X-ray binaries in the cold quiescent state, the value for $\alpha$ is about an order of magnitude smaller than in the outburst state, around 0.02–0.04.

4.2. Dwarf nova superhump decay

Some dwarf novae also show superhump outbursts that are brighter and longer than the normal outbursts but occur less frequently (Warner, 2003). The disc becomes eccentric when it is larger than the location of the 3:1 mean motion resonance with the binary orbital period (Lubow, 1991a,b). The eccentric disc precesses in a prograde direction. Some systems, for example V503 Cyg, precess in a retrograde direction and these are called negative superhumps. In this case the disc is tilted and is precessing due to the tides in a retrograde direction (Wood & Burke, 2007). During the precession, the location of the accretion hot spot where the stream hits the disc varies in distance from the white dwarf, and in brightness. Since the tilt of the disc is a crucial part of this model, the alignment timescale for the disc to the binary orbital plane cannot be too short. In order to keep the disc misaligned, King et al. (2013) argued that the viscosity parameter in the quiescent phase must be small and estimated $\alpha \lesssim 10^{-4}$ (King et al., 2013).

4.3. FU Orionis outbursts

The young stellar object FU Orionis has been observed in outburst. During this time, the majority of the disc is thought to be hot enough to be thermally ionised. With a decay timescale of around 100 yr, Zhu et al. (2007) find $\alpha \approx 0.02 – 0.2$. This phase lasts only a few tens of years and we expect the outbursts to recur on a timescale of around $10^5$ yr. The disc spends most of its time in the quiescent phase.
4.4. Protostellar discs

There is debate on the value of $\alpha$ in observed protoplanetary discs as the results are model dependent. The outer parts of protostellar discs, where most of the mass resides, are, for most of their lives, too cool to be fully ionised (e.g. Gammie, 1996; Gammie & Menou, 1998). Only the inner parts of the disc ($R \lesssim 0.1$ au) are thermally ionised and farther out only the surface layers may be ionised by external sources such as cosmic rays or X-rays from the central star (Glassgold et al., 2004).

A simple estimate of the value of $\alpha$ in these discs comes from comparing estimated disc masses ($M_d$) with estimated central accretion rates ($\dot{M}_c$) and from these deducing an accretion timescale $\nu \sim M_d/\dot{M}_c$. Modeling the outer disc properties (in particular $c_s$ or $H$) then gives an estimate of $\alpha$. Hartmann et al. (1998, see also Hartmann 2000) estimate that $\alpha \sim 0.01$ on distance scales of $10^{-1}$ to $100$ au.

More recently, Andrews et al. (2009) observed protoplanetary discs in Ophiuchus and fitted the continuum visibilities and broadband spectral energy distributions to a parametric disc model. They found $\alpha \sim 0.0005 - 0.08$ for radius $R = 10$ au. Hueso & Guillot (2005) found similarly small values of $0.001 < \alpha < 0.1$ for DM Tau and $4 \times 10^{-4} < \alpha < 0.01$ for GM Aur. More recently, Rafikov (2014) used resolved disc observations by ALMA (Ansdell et al., 2016; Alcalá et al., 2014, 2017) and used a self-similar disc solution to calculate $0.0001 < \alpha < 0.04$. Ansdell et al. (2018) measured the gas disc sizes and refined this calculation and found $0.0003 < \alpha < 0.09$.

In addition, the value of $\alpha$ determines the timescale of the evolution of the disc and how quickly the disc spreads outwards. The viscous timescale is given in equation (10). Numerical models find that if $\alpha = 0.1$ then the outer disc of T Tauri stars expands too quickly to be compatible with observations of disc sizes (Hartmann et al., 1998). The disc radius reaches $> 1000$ AU in a time of 1 Myr. While some discs have been observed to be this large (e.g. Schaefer & Fegley, 2000), typically the discs are a few hundred au (e.g. Dutrey et al., 1996; Vicente & Alves, 2003; Hughes et al., 2008; Andrews et al., 2010; Ansdell et al., 2018).

Recently, Hartmann & Bae (2018) have suggested that viscous protoplanetary disc models with $\alpha \gtrsim 10^{-4}$ can explain observed T Tauri mass accretion rates and lifetimes provided that mass surface densities are sufficiently large.

4.4.1. Direct turbulence measurements

Measuring the turbulence in a disc directly is complicated because the turbulent motions are hidden by the Keplerian and thermal motions (e.g.
Flaherty et al., 2018). Heavier molecules have small thermal motions, so observing them yields a direct measure of the turbulent velocity. Recently observations have measured the turbulent velocity of the gas in the disc, $u_t$. Comparing this to a value for $\alpha$ is complex, but, roughly, we can estimate

$$\alpha = \left( \frac{u_t}{c_s} \right)^2$$  \hspace{1cm} (11)

(Cuzzi et al., 2001; Simon et al., 2013, 2015). Complexities in the distribution of CO abundance affect the measurements leading to underestimates for the turbulent disc speeds (Yu et al., 2017a,b).

Teague et al. (2016) found $u_t \sim 0.2 - 0.4 c_s$ for TW Hya by fitting high resolution spectra. Observations of DM Tau (Dartois et al., 2003), MWC 480 and LkCa 15 (e.g. Piétu et al. 2007) have also found higher values for the turbulence in the range $u_t \lesssim 0.3 - 0.5 c_s$ (Hughes et al., 2011; Guilloteau et al., 2012). These values may be consistent with a much higher value for $\alpha$. Hughes et al. (2011) suggest that these high values for the turbulent velocity imply an $\alpha \sim 0.01$ by assuming that the linewidth drops by a factor of a few between the upper layers (that the observations probe) and the disc midplane. Justification for this comes from observations of FU Orionis (Hartmann et al., 2004) and global MHD simulations (Fromang & Nelson, 2006; Flock et al., 2015, 2017).

More recently, Flaherty et al. (2018) used a parametric disc model that self-consistently calculates the density and temperature of the disc. These parameters are used in a ray–tracing radiative transfer code to find visibilities that are compared to the data. They found that the turbulent broadening in TW Hya gives an upper limit of $\alpha < 0.007$ in the region $2 - 3$ pressure scale heights above the midplane. Similarly, they measured the turbulence in HD 163296 to be small at $\alpha < 0.0025$ (Flaherty et al., 2015, 2017).

5. Discussion and Conclusions

We have summarised estimates found in the literature of the values of the viscosity parameter $\alpha$. We find, in agreement with earlier work by King et al. (2007), that for fully ionized discs reliable estimates can be made and in all cases it is found that the values obtained are consistent with $\alpha \approx 0.2 - 0.3$. This has an important physical implication. Namely, that whatever the origin of the turbulent behaviour within the disc that gives rise to the observed effective viscosity, whether it is purely hydrodynamic, or (as is generally believed) magneto-hydrodynamic, the mechanism that produces
it is able to drive the fluid motions only up to, or close to, the sound speed. The fact that $\alpha$ is always found to be close to this limit (for these discs) implies that whatever instability might give rise to the driving mechanism in this case is able to grow until the motions become trans-sonic. Thus, in agreement with the original conjecture of [Shakura & Sunyaev (1973)], the driving mechanism for the turbulence is limited once the motions become trans-sonic. We have noted that such a limitation does not come about because of energy arguments. Rather, it must be the result of the fact that once the motions approach the sound speed, the nature of the turbulence changes in a fundamental fashion\footnote{For example, a disc powered by supersonic, magnetic turbulence would be strongly clumped in the manner described by [Pustilnik & Shvartsman, 1974, and reference therein]; see also [Begelman & Pringle, 2007].}. Returning to the ideas of [Shakura & Sunyaev (1973, 1976)], described briefly in Section 2, it is evident, from equations (2) and (7), that the change in the nature of the turbulence might occur for one, or both, of two physical reasons. First, in the case of hydrodynamic turbulence, as the turbulence becomes trans-sonic, $u_t \rightarrow c_s$, shocks begin to dominate the dissipative process. Second, once the Alfvén speed $v_A$ approaches the sound speed, $v_A = \sqrt{B^2/8\pi\rho} \rightarrow c_s$, the timescale for the Parker instability (leading to loss of magnetic flux from the disc) becomes comparable with the shearing timescale (growth timescale for magnetic flux) $\sim \Omega$ (cf. Tout & Pringle, 1992).

The corollary of this basic finding is that any numerical simulations of disc turbulence (for fully ionized discs) which do not find that the strength of the turbulence grows until limited by the sound speed (and which therefore do not find the large values of $\alpha$ implied by the observational data) must be missing some fundamental physics. Some of the problems inherent in such simulations were discussed by [King et al, 2007].

For discs that are partially (or barely) ionized, estimates of $\alpha$ are generally less reliable. Nevertheless, a consistent picture seems to emerge that in such discs the values of $\alpha$ are smaller than those found for fully ionized discs by at least an order of magnitude and often by several orders of magnitude. This too has an important physical implication. The point here is that the main difference between a fully ionized and a partially ionized disc lies not in its hydrodynamic, but rather in its magnetic properties. As a disc becomes less ionized, its electrical conductivity decreases and therefore its ability to interact with magnetic fields decreases. This, we would argue, provides strong support for the concept that the main driving mechanism for
the turbulence in viscous accretion discs is magnetic. The obvious candidate for such driving stems from the magneto-rotational instability (MRI), whose importance was stressed by Balbus & Hawley (1991). As was remarked by Gammie & Menou (1998), in the case of quiescent discs in dwarf novae, the driving from such an instability is much weaker, if not non-existent, once the ionization fraction drops.

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