On the Motion of the Local Group and its Substructures

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1 INTRODUCTION

The peculiar motion of the Local Group (LG) seems to become one of the essential problems of observational cosmology, since it can provide at least crucial constraints on properties of the local region of the Universe. In more optimistic evaluation, it can be the cornerstone for the linear theory of gravitational instabilities and, hence, for many aspects of the Big Bang cosmology, in general.

The important question here is, obviously, the convergence of various dipoles, e.g. of optical galaxies and clusters of galaxies, IRAS galaxies, X-ray clusters, X-ray galaxies, etc., with the dipole of the Cosmic Microwave Background radiation, if the latter is caused by the Doppler effect. The reliable determination of the LG CMB dipole itself is the central point here.

Though there is no absolute convergence, all the dipoles, in general, either are aligned in the direction of the defined CMB dipole or differ within limits, which presumably are affected by the choice of particular samples. For example, it is noticed, that the convergence level is improved, if deeper samples of extragalactic objects are involved, i.e. the matter distribution on more larger cosmic volumes is taken into account (Branchini\&Plionis, 1996).

While the result by Lauer\&Postman (1994) on the quite differently oriented dipole obtained from the clusters of galaxies with velocities up to 15 000 km s\(^{-1}\), seems to challenge the situation, the subsequent analysis by Branchini\&Plionis (1996) involving even deeper survey of Abell/ACO clusters - up to 20 000 km s\(^{-1}\), showed alignment within 10\(^{\circ}\) with respect to the CMB dipole, if the Virgocentric flow is also taken into account.

The main common concern in these results based on the analysis of samples of various types of extragalactic objects, is in which degree the given population really traces the large-scale mass distribution. Though the general alignment of the dipoles of a given population with the CMB one, can imply the positive answer to this question, the reliably obtained divergence level of any of dipoles can be not less informative. Therefore, any alternative means of estimation of the motion of the LG should be of particular interest.

In the present study we consider the possibility to estimate the peculiar velocity of the LG using not the extragalactic information, but its internal dynamical properties combined with data on CMB dipole. Namely, we analyse the substructure of the LG, and perform a procedure of 3D reconstruction of the velocity of the whole system and its main subgroups. Thus, we continue to use the approach of 3D reconstruction of the tangential velocities of galaxy systems, developed in Gurzadyan\&Rauzy (1997), with the difference that in the present study the line-of-sight velocities have no components representing the Hubble flow.

The first step is performed by the S-tree technique (see Gurzadyan\&Kocharyan, 1994) enabling, in particular, the analysis of systems such as groups or clusters of galaxies. For the second step, we use the data on the distribution of...
the line-of-sight velocities of the galaxies which are members of the LG and its subsystems, as revealed by the first step. Thus, we obtain the peculiar motions of the Milky Way and M31 subsystems with respect to each other. By the third step, from the bulk motions of the mentioned both subsystems we obtain the velocity vector of the Local Group in CMB frame. These results are discussed in terms of the convergence of the various luminous dipoles with the one observed in the Cosmic Microwave Background Radiation.

2 THE S-TREE TECHNIQUE

The S-tree technique is developed for the investigation of properties of many-dimensional nonlinear systems and essentially uses the concepts of the theory of dynamical systems. Referring for details to (Gurzadyan et al. 1991, 1994; Gurzadyan & Kocharyan 1994 Bekarian & Melkonian 1997), here we outline its key points only. Its idea is based on the property of structural stability well known in theory of dynamical systems enabling to study the robust properties of the systems with limited amount of information. The gravitating systems were known to be exponentially unstable and hence being among systems with strong statistical properties. The advantage of the method in the context of galaxy clusters is in the self-consistent use of both kinematical and positional information on the clusters, as well as of the data on individual observable properties of galaxies - the magnitudes.

This approach is introducing the concept of the degree of boundness between the members of the given N-body system, i.e. the definition of a nonnegative function \( P \) called the boundness function which describes the degree of interaction of a given subset \( Y \) of the initial set \( A \) with another subset \( X \setminus Y \) according to some criterion. Particularly one can define a function

\[
P: S(A) \rightarrow R_+: (X, Y) \rightarrow P_X(Y),
\]

where \( S(A) \) is the set of all subsets of the initial set, when one of the sets contains the other one. The procedure of such splitting of \( A \) can be measured by a non-negative real number \( \rho \) using the boundness function, so that the problem is reduced to finding out a function \( \Sigma(\rho) \) denoting the set of all possible \( \rho \)-subsystems \( \{A_1, \ldots, A_d\} \) of \( (A, P) \).

Attributing for the given \( \rho \) the matrix \( D \) to another matrix \( \Gamma \) in a following way:

\[
\begin{align*}
\Gamma_{ab} &= 0 \quad \text{if} \quad D_{ab} < \rho, \quad D_{ba} < \rho, \\
\Gamma_{ab} &= 1 \quad \text{if} \quad D_{ab} \geq \rho, \quad D_{ba} \geq \rho;
\end{align*}
\]

(1)
the problem of the search of a \( \rho \)-bound cluster is reduced to that of the connected parts of the graph \( \Gamma(\rho) \) - a S-tree diagram.

For the matrix \( D_{ab} \) a representation via the two-dimensional curvature \( K \) of the phase space of the system can be used:

\[
D_{ab} = \max \left\{ -K^\mu_{v \nu} \right\},
\]

where \( \mu = (a, i), \nu = (b, j) \). The two-dimensional curvature is represented by the Riemann tensor \( R \) via the expression:

\[
K^\mu_{v \nu} = R^\mu_{\nu \rho \sigma} u^\rho u^\sigma,
\]

where \( u^\nu \) is the velocity of the geodesics; the explicit expression of two-dimensional curvature for N-body gravitating system is derived in (Gurzadyan & Savvidy, 1986) and has rather complicated form to be represented here. The advantage of the use of geometric characteristics such as Riemann, Ricci tensors, is well known in theory in dynamical systems, and has been used in astrophysical problems as well.

As a result, the degree of boundness between the members and subgroups of the given N-body system can be obtained, thus revealing the physically interacting system (cluster) and its hierarchical substructure.

The computer code based on the S-tree method has been used for the study of the substructure of the Local Group of galaxies (Gurzadyan et al. (1993)), of the core of Virgo (Petrosian et al. 1997) and Abell clusters (Gurzadyan & Mazure, 1997). In these studies the information on the masses of galaxies has been used via the mass-to-luminosity ratio \( M = \text{const} L^* \), \( n = 1 \), though other relations – \( (n = 0.1/2) \), have been checked as well, and the robust character of the results on the subgrouping has been revealed; note, that these assumptions take into account also the existence of the dark matter associated with the galaxies.

3 BULK FLOW RECONSTRUCTION

The reconstruction of the 3D velocity distribution function from its observed line-of-sight velocity distribution is an interesting inverse problem. It was analytically solved by Ambartsumian (1936) for stellar systems without any \( a \) priori assumption on the form of the sight function. The only assumption made was the independence of the distribution function on the spatial regions (directions), or equivalently, that the 3D velocity distribution \( \phi(v_1, v_2, v_3) \) \( dv_1 dv_2 dv_3 \) of such systems was invariant under spatial translations. It means, that the theoretical probability density \( dP_{ab} \) of the system reads as follows

\[
dP_{ab} = \phi(v_1, v_2, v_3) \, dv_1 \, dv_2 \, dv_3 \, \rho(r, l, b) \, r^2 \, \cos b \, dr \, dl \, db
\]

where \( \phi(v_1, v_2, v_3) \) is the 3D velocity distribution function (in galactic cartesian coordinates) and \( \rho(r, l, b) \) is the 3D spatial distribution function (in galactic coordinates). Under this assumption, Ambartsumian had proven the theoretical possibility of reconstructing of \( \phi(v_1, v_2, v_3) \) from the observed radial velocity distribution function \( f(v_r, l, b) \)

\[
dP_{obs} = f(v_r, l, b) \, \cos b \, dl \, db \, dv_r
\]

(3)

However, computer experiments show that the direct application of the Ambartsumian’s formula is hardly possible for that purpose. It is natural, since the derivation of a smooth function based on discrete information on relatively small number of particles (say, less than 1000) is a nonlinear problem. The number of points should exceed essentially the number of galaxies of real clusters in order to apply this formula with success. This fact is a consequence of the principal difference between the N-body problems in stellar dynamics and dynamics of clusters of galaxies.

Nevertheless, some quantities of interest such as the first order moment of this distribution, i.e. the mean 3D velocity of the system, can be obtained. Additional hypotheses on the distribution function \( \phi(v_1, v_2, v_3) \) are then necessary.
Hereafter, we assume that $\phi(v_1, v_2, v_3)$ can be written as follows:

$$\phi(v_1, v_2, v_3) \, dv_1 \, dv_2 \, dv_3 = \prod_{i=1}^{3} \phi_i(v_i) \, dv_i \quad (4)$$

where the distribution functions $\phi_i$ are centered on $\bar{v}_i$ and of variances $\sigma_i (\phi_i(v_i) = \phi_i(v_i, \sigma_i))$. The statistics of the mean 3D velocity, i.e. the bulk flow velocity, are derived in the appendix A. The use of the maximum likelihood technique forces us to entirely specify the functions $\phi_i$. We choose the Gaussian representation ($\bar{v}_1 = \bar{v}_2 = \bar{v}_3 = g$ with $g$ Gaussian) and assume the isotropic velocity dispersions ($\sigma_1 = \sigma_2 = \sigma_3 = \sigma_v$). The velocity field of the system is, thus, split into a mean 3D velocity $\mathbf{v}_B = (v_x, v_y, v_z)$ = $(\bar{v}_1, \bar{v}_2, \bar{v}_3)$ (i.e. a bulk flow) plus a 3D random component, isotropic Maxwellian, centered on $\mathbf{0}$ and of velocity dispersion $\sigma_v$.

The fact that gravitating N-body systems do possess strong statistical properties peculiar to Kolmogorov systems (Gurzadyan & Savvidy, 1986; see also Gurzadyan & Pfenniger, 1994), can essentially influence the substructure of the LG. Moreover, it is shown in appendix A. The use of the maximum likelihood technique forces us to entirely specify the functions $\phi_i$. We choose the Gaussian representation ($\bar{v}_1 = \bar{v}_2 = \bar{v}_3 = g$ with $g$ Gaussian) and assume the isotropic velocity dispersions ($\sigma_1 = \sigma_2 = \sigma_3 = \sigma_v$). The velocity field of the system is, thus, split into a mean 3D velocity $\mathbf{v}_B = (v_x, v_y, v_z)$ = $(\bar{v}_1, \bar{v}_2, \bar{v}_3)$ (i.e. a bulk flow) plus a 3D random component, isotropic Maxwellian, centered on $\mathbf{0}$ and of velocity dispersion $\sigma_v$.

The derived bulk flow statistics is a robust estimator. It does not depend on the distances of galaxies under consideration. This is a positive feature, since there is still disagreement between various authors in the distance estimates of the members of LG. Moreover, it is shown in appendix A that the $\bar{v}_x$, $\bar{v}_y$ and $\bar{v}_z$ estimators do not depend on the projected angular distribution function $\eta(\ell, b)$ of the sample, even if their accuracies do.

4 APPLICATION TO THE LOCAL GROUP

4.1 The Local Group subsystems

In the S-tree study of the substructure of the LG (Gurzadyan et al. (1993)) a sample of 39 galaxies has been used, compiled from various studies. The membership of galaxies in two main subgroups - dominated by the Milky Way and of M31, has been revealed, in general, confirming the conventional views. Physical connections between some individual galaxies have been also indicated, not reported before (e.g. of NGC 6822 and IC 1613). Some galaxies appeared to have no actual influence on the dynamics of LG and vice versa, and therefore, were considered not to be its members. The LG as a physical system of the influenced galaxies only, was shown to extend on less than 2 Mpc. These galaxies and their membership are listed Table 1. The recent studies (see Karachentsev (1995) and references therein), though record some differences in the data on the galaxies, in general, no drastic reliable change is observed. Existence of galaxies obscured by the Galactic disk, like newly discovered Dwingeloo 1,2 ones (Burton et al. (1996)), also cannot be ruled out, though it seems unlikely that, at least the latter ones can essentially influence the substructure of the LG (Lynden-Bell, 1996).

Table 1. Local Group data used for this analysis: the equatorial coordinates, radial velocity $v_r$ (WITH RESPECT TO THE MILKY WAY REST FRAME) and their distance $r$, are listed for 32 objects within 2 Mpc. M31 and Milky Way subsystems were deduced by using the S-tree method (Gurzadyan et al. (1993). Some typographical mistakes occurred in Table 1 of the mentioned paper are corrected in the present Table).

| Object | $\alpha$ (1950) | $\delta$ (1950) | $v_r$ (km s$^{-1}$) | $r$ (Mpc) |
|--------|----------------|----------------|--------------------|-----------|
| Milky Way | 00 00.00 | 00 00.0 | 0 | 0.0 |
| SMC | 00 51.00 | -73 06.0 | -21±5 | 0.063 |
| Sculptor S | 00 57.60 | -33 58.0 | 74±? | 0.085 |
| IC1613 | 01 02.22 | 01 51.0 | -153±3 | 0.64 |
| Fornax D | 02 37.84 | -34 44.4 | -51±? | 0.19 |
| LMC | 05 24.00 | -69 48.0 | 130±? | 0.052 |
| Carina | 06 40.40 | -50 55.0 | 14±? | 0.093 |
| Leo I | 10 05.77 | 12 33.2 | 177±? | 0.22 |
| Leo II | 11 10.83 | 22 26.1 | 15±? | 0.22 |
| UM | 15 08.20 | 67 18.0 | -88±? | 0.065 |
| Draco | 17 19.40 | 57 58.0 | -95±? | 0.075 |
| N6822 | 19 42.12 | -14 55.7 | 55±5 | 0.62 |

| Milky Way subsystem : 12 galaxies |
|-------------------------------|
| N147 | 00 30.46 | 48 13.8 | 36±50 | 0.7 |
| N185 | 00 36.19 | 48 06.7 | 67±22 | 0.72 |
| N205 | 00 37.64 | 41 24.9 | 49±11 | 0.64 |
| M31 | 00 40.00 | 40 59.7 | -110±1 | 0.67 |
| M32 | 00 39.97 | 40 35.5 | 86±6 | 0.66 |
| LGS-3 | 01 01.20 | 21 37.0 | -100±8 | 0.72 |
| M33 | 01 31.05 | 30 23.9 | -39±0.5 | 0.82 |

| M31 subsystem : 7 galaxies |
|----------------------------|
| N147 | 00 30.46 | 48 13.8 | 36±50 | 0.7 |
| N185 | 00 36.19 | 48 06.7 | 67±22 | 0.72 |
| N205 | 00 37.64 | 41 24.9 | 49±11 | 0.64 |
| M31 | 00 40.00 | 40 59.7 | -110±1 | 0.67 |
| M32 | 00 39.97 | 40 35.5 | 86±6 | 0.66 |
| LGS-3 | 01 01.20 | 21 37.0 | -100±8 | 0.72 |
| M33 | 01 31.05 | 30 23.9 | -39±0.5 | 0.82 |

| Others : 13 galaxies (with $r \leq 2$ Mpc) |
|------------------------------------------|
| IC10 | 00 17.69 | 59 00.9 | -144±2 | 1.25 |
| N55 | 00 12.40 | -39 28.0 | 97±3 | 1.4 |
| IC342 | 03 41.95 | 67 56.4 | 221±? | 1.84 |
| Leo A | 09 56.53 | 30 59.2 | -17±10 | 1.59 |
| N3109 | 10 00.80 | -25 54.8 | 193±? | 1.6 |
| Sex A | 10 08.57 | -04 27.7 | 154±5 | 1.3 |
| DDO155 | 12 56.20 | 14 29.2 | 183±5 | 1.0 |
| Sagittarius | 19 27.01 | -17 47.0 | 10.9±1.4 | 1.11 |
| DDO210 | 20 44.13 | -13 02.0 | -13±10 | 1.5 |
| IC5152 | 21 59.00 | -51 32.0 | 78±15 | 1.51 |
| Hog | 23 23.80 | -32 40.0 | 75±5 | 1.3 |
| Pegasus | 23 26.05 | 14 28.3 | -14±2 | 1.0 |
| W-L-M | 23 59.40 | -15 44.6 | -56±8 | 1.0 |

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The choice of membership criteria is always crucial while studying the substructures of clusters of galaxies. In Karachentsev (1995), only closed Keplerian orbits are considered as criteria of a membership, while strictly speaking, precise Keplerian closed orbits are never typical in few-body problem, as one has in the case of LG. As it is mentioned above, the S-tree technique enables to reveal the degree of influence, whatever the exact shape of the orbits is.

Our aim here is to show the principal possibility of the reconstruction of 3D vector of the LG based on the substructure analysis, having in mind, that the latter can be easily revised, if some observational input data on galaxies should be essentially modified with high enough confidence level.

4.2 Bulk flow estimate

The MW and M31 subsystems contain respectively 12 and 7 galaxies. Such a small number of objects forbids indeed the use of the Ambarsumian’s formulae in order to reconstruct the 3D velocity distribution functions of these two systems. Nevertheless, in this section we show that it is possible to evaluate their 3D mean velocity, i.e. their bulk flow.

For each subsystem, as it is assumed Eq. (2), the velocity distribution function $\phi(v_1,v_2,v_3)$ can be split into a bulk flow $\mathbf{v}_B = (v_x,v_y,v_z)$ plus a 3D random component, isotropic Maxwellian, centered on $\mathbf{0}$ and of velocity dispersion $\sigma_v$ (see section 3 Eq.(4)). According to the bulk flow statistics derived in appendix A, Tables 2 and 3 give the values of the bulk flow estimates for the MW and M31 subsystems, respectively. These velocities are expressed in km s$^{-1}$ and with respect to rest frame of the Milky Way (MWR frame).

For each system, the values of the velocity dispersion $\sigma_v$ and of the accuracy of the bulk flow estimate are calculated by using numerical simulations. Large number of samples have been simulated according to the following characteristics:

- The angular position of each object of the simulated system is identical with the observed one.

- The 3D velocity of each object is the estimated bulk flow (for example, $\mathbf{v}_B = (104, -116, 45)$ in km s$^{-1}$ for the MW subsystem) plus a 3D random component, Maxwellian and isotropic, of velocity dispersion $\sigma_v$.

- The simulated radial velocity of each object is the line-of-sight projection of its 3D velocity, plus a white noise accounting for measurement errors on $v_r$ (see table 1). When this information is missing, a value of 10 km s$^{-1}$ is adopted.

Since the quantity $\sigma_v$ is unknown, we adopt the value of the biased estimator $\hat{\sigma}_v$ of $\sigma_v$ as a fiducial starting point (see appendix A). After few iterations, the correct value of the velocity dispersion $\sigma_v$ is reached when the $\hat{\sigma}_v$ estimate of the simulated samples corresponds to the observed one. Errors bars on the $v_x, v_y, v_z$ and $\sigma_v$ estimates are afterwards calculated by using a large number of simulated samples.

The effect of the Hubble expansion have been also investigated. The radial velocities listed in table 1 are indeed not corrected for the Hubble flow. We have applied this

| Object       | $v_r$ | $v_r - \mathbf{\hat{v}} \cdot \mathbf{v}_B$ |
|--------------|-------|-----------------------------------------|
| SMC          | -21   | -99                                     |
| Sculptor S   | 74    | 107                                     |
| IC1613       | -153  | -37                                     |
| Fornax D     | -51   | -23                                     |
| LMC          | 130   | 45                                      |
| Carina       | 14    | 55                                      |
| Leo I        | 177   | 76                                      |
| Leo II       | 15    | -25                                     |
| UM           | -88   | -21                                     |
| Draco        | -95   | -31                                     |
| N6822        | 55    | 31                                      |

| Object       | $v_r$ | $v_r - \mathbf{\hat{v}} \cdot \mathbf{v}_B$ |
|--------------|-------|-----------------------------------------|
| N147         | 36    | -16                                     |
| N185         | 67    | 15                                      |
| N205         | 49    | 36                                      |
| M31          | -110  | -120                                    |
| M32          | 86    | 78                                      |
| LGS-3        | -100  | -2                                      |
| M33          | -39   | 0                                       |

Table 2. Estimates of the bulk flow and of the velocity dispersion for the Milky Way subsystem. Residuals are listed for each object. Standard deviations are computed by using numerical simulations.

Table 3. Estimates of the bulk flow and of the velocity dispersion for the M31 subsystem. Residuals are listed for each object. Standard deviations are computed by using numerical simulations.
estimates are given for the M31 subsystem : $v_B = 137$ km s$^{-1}$ pointing toward $l = 308$ and $b = 16$, with a velocity dispersion of $71 \pm 16$ km s$^{-1}$.

We have also performed our analysis on the 32 Local Group galaxies situated nearer than 2 Mpc, ignoring the MW and M31 dynamical substructures. Bulk flow and velocity dispersion estimates are shown in Table 4. Note, however, that these estimates have no much sense since the presence of the MW and M31 subsystems rules out our main assumption, i.e. the velocity distribution of the 32 Local Group galaxies is not invariant under spatial translations. The values of the biased velocity dispersion estimator $\sigma_\star^B$ for the MW and M31 subsamples are more interesting. They are significantly greater than the $\sigma_\star^B$ estimates of tables 2 and 3. This fact indicates the existence of kinematic substructures in the Local Group, and so, strengthens the present analysis.

5 CONCLUSION AND DISCUSSION

The motions of the two subsystems of the Local Group have been estimated. The M31 and MW dynamical substructures, containing respectively 7 and 12 galaxies, have been identified via the S-tree technique, which takes into account in a self-consistent way the degree of influence of one object (or a set of objects) on another, whatever their or-
bits can be. In the rest frame of the Milky Way (MWR frame) the bulk flow statistic derived above gives 
\[ \mathbf{v}_B = (104 \pm 64, -116 \pm 42, 45 \pm 33) \text{ in km s}^{-1} \] 
and in cartesian galactic coordinates (or \( v_B = 162 \text{ km s}^{-1} \) pointing toward \( l = 312 \) and \( b = 16 \)) for the motion of the MW subsystem, and 
\[ \mathbf{v}_B = (-117 \pm 541, 85 \pm 267, 309 \pm 254) \text{ or } v_B = 341 \text{ km s}^{-1} \] 
toward \( l = 144 \) and \( b = 65 \) for the motion of the subsystem dynamically associated with M31. While these estimates have been derived from the radial velocity and angular position of galaxies, the information on the distance has not been used (the point has its importance since the main ambiguity in the data concerns the galaxy distances). Note, that the independence on the distances concerns only the velocity reconstruction procedure, while the information on distances is used for S-tree analysis; however, the statistical results of the latter concerning the subgroups properties are robust relative the error-boxes of data, unless some data will be modified drastically. Note the following two points. On one hand, the relative velocity of M31 subsystem with respect to the MW subsystem when projected on the line joining MW to M31 is \(-165 \pm 66\), confirming the conventional views. On the other hand, the 3D inner motions inside the Local Group, if significant, are found to be surprisingly large (M31 subsystem has a relative 3D velocity of amplitude 399 km s\(^{-1}\) with respect to the MW subsystem). It is interesting that the conclusion on the existence of transverse velocity of MW relative to M31 has been concluded previously by Peebles (1994) using its least action method (Peebles 1989). The point is discussed below in term of the convergence of the various luminous dipoles with the one observed in the Cosmic Microwave Background Radiation.

The CMB temperature dipole, if interpreted as the signature of our motion with respect to the rest frame of this radiation (CMB frame), gives for the Milky Way a peculiar velocity \( V_{\text{MW}} \rightarrow \text{CMB} \) of 552 km s\(^{-1}\) pointing toward the galactic coordinates \( l = 266 \) and \( b = 29 \) (see Smoot et al. (1991, 1992), Kogut et al. (1993)). This MW motion in the CMB frame is traditionally split into two components

\[ V_{\text{MW}} \rightarrow \text{CMB} = V_{\text{MW}} \rightarrow \text{LG} + V_{\text{LG}} \rightarrow \text{CMB} \quad (5) \]

where \( V_{\text{MW}} \rightarrow \text{LG} \) is the velocity of the Milky Way relative to Local Group rest frame, which originates from the internal non-linear dynamics governing the Local Group and \( V_{\text{LG}} \rightarrow \text{CMB} \) is the peculiar motion of the Local Group as a whole in the CMB frame, created by large scale mass fluctuations present in the Universe.

The latter can be inferred in some way from the various luminous dipoles found in the literature (X-ray galaxies dipole by Miyaji & Boldt (1990); \( l = 313 \) and \( b = 38 \); Optical galaxies within 8000 km s\(^{-1}\) dipole by Hudson (1993); \( l = 242 \) and \( b = 49 \) or \( l = 231 \) and \( b = 40 \); IRAS dipole, the least for shell (142.8-157.3 Mpc) by Plionis, Coles and Catelan (1993); \( l = 260.7 \) and \( b = 39.1 \); Abell/AC0 clusters within 20 000 km s\(^{-1}\) by Branchini & Plionis (1996); \( l = 265 \) and \( b = 16 \) if corrected from Virgo-centric flow). Though there is no absolute convergence, it is noticed that convergence level is improved if deeper samples of extragalactic objects are involved. This fact indicates at least that X-ray, optical and IR observed objects have approximately the same spatial distribution.

Conversion of luminous dipole in terms of the motion \( V_{\text{LG}} \rightarrow \text{CMB} \) of the Local Group in the CMB frame assumes the following hypotheses:

- H1) The luminous objects trace the large scale mass density field.
- H2) The linear approximation holds (in particular, peculiar velocity remains parallel to acceleration throughout the evolution of large scale structures).
- H3) The sample of objects under consideration is at rest in the CMB frame.

In what extent assumptions H1 and H2 are satisfied is yet an open question, while results obtained from peculiar velocity analysis seem to challenge assumption H3 (Bulk flow of the shell of galaxies (within 3500-6500 km s\(^{-1}\)) by Rubin et al. (1976); \( v_B = 950 \text{ km s}^{-1} \) toward \( l = 308 \) and \( b = 25 \) in the CMB frame; Bulk flow of clusters of galaxies (up to 15 000 km s\(^{-1}\)) by Lauher & Postman (1994); \( v_B = 700 \text{ km s}^{-1} \) toward \( l = 340 \) and \( b = 50 \) in the CMB frame, recently revisited by Graham (1996); \( v_B = 738 \text{ km s}^{-1} \) toward \( l = 330 \) and \( b = 45 \). On the other hand, the peculiar motion of the Milky Way in the Local Group rest frame \( V_{\text{MW}} \rightarrow \text{LG} \) can be obtained from the analysis of the dynamics of the Local Group and its substructures. Subtracting it to the observed \( V_{\text{MW}} \rightarrow \text{CMB} \) thus gives a local estimate of \( V_{\text{LG}} \rightarrow \text{CMB} \), which can be compared with its values extracted from dipoles analysis, as mentioned above.

Herein, a rough kinematical model is adopted, assigning to the Local Group the mean motion of its main substructures (i.e. MW and M31 subsystems, equally weighted). It gives \( V_{\text{MW}} \rightarrow \text{LG} = (7 \pm 303, 15 \pm 155, -177 \pm 144) \) or 178 km s\(^{-1}\) toward \( l = 65 \) and \( b = -85 \). Finally, our local estimate of the LG peculiar velocity in the CMB frame yields \( V_{\text{LG}} \rightarrow \text{CMB} = (-41 \pm 303, -497 \pm 155, 445 \pm 144) \) or 668 km s\(^{-1}\) in amplitude pointing toward \( l = 265 \) and \( b = 42 \). This result can be directly compared with the well-known estimation of Yahil et al. (1977), based on the Solar system motion relative to the LG centroid: \( V_{\text{LG}} \rightarrow \text{CMB} = 622 \text{ km s}^{-1} \) toward \( l = 277 \) and \( b = 30 \) or \( 66, -535, 311 \) in galactic cartesian coordinates.

Thus we have obtained the LG motion in CMB frame in an alternative way. Therefore it is remarkable that this result is in good agreement with the result of Yahil et al. (1977) within 1 \( \sigma \) level. The existence of some discrepancy between these two values, if significant, can be interpreted as follows. The LG centroid had been defined by specific choice of the main and satellite populations and with further search of the best-fit solution for the Solar system motion. In the present analysis we have found statistically significant indication of the bulk flow of the two main subsystems of LG, which can influence the definition of its centroid, and hence, the final result.

The understanding of the cause of each discrepancy is the main problem to be solved. The LG substructure’s 3D dynamics, as discussed above, could be essential for that problem. The fact of the existence of the bulk flow of the substructures should be crucial also while studying the past and future evolution of the Local Group.

Moreover, bulk flows can be common properties of subgroups of clusters of galaxies (Gurzadyan&Mazure, 1997) – galaxy associations, thus reflecting the role of merging and other basic trends in the formation mechanisms of the clusters.
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APPENDIX A: THE BULK FLOW STATISTIC

In this appendix, we use the maximum likelihood technique in order to derive the statistics of the 3 components of the bulk flow. Quantities, such as the variance of these estimators or the velocity dispersion estimate, are obtained by using numerical simulations.

We define the statistical model as follows. It is assumed, that the data sample consists of N independent objects, which follow the theoretical probability density dPth of Eq. (2):

\[
dP_{th} = \phi(v_1, v_2, v_3) \ dv_1 \ dv_2 \ dv_3 \times \rho(r, l, b) \ \cos \theta \ dr \ dl \ db
\]

where \( \phi(v_1, v_2, v_3) \) is the 3D velocity distribution function (in galactic cartesian coordinates) and \( \rho(r, l, b) \) is the 3D spatial distribution function (in galactic coordinates). Hereafter, we assume that the 3D velocity distribution function of the sample adopts the following form

\[
\phi(v_1, v_2, v_3) = g(v_1; v_2, \sigma_v) \times g(v_2; v_3, \sigma_v) \times g(v_3; v_2, \sigma_v)
\]

where \( g(x; x_0, \sigma) \) is a Gaussian of mean \( x_0 \) and of dispersion \( \sigma_x \). The velocity field of the sample is thus split into a mean 3D velocity \( \mathbf{v}_B = (v_x, v_y, v_z) \) (i.e. a bulk flow) plus a 3D random component, isotropic Maxwellian, centered on \( \mathbf{0} \) and of velocity dispersion \( \sigma_v \).

For a given object, the observables are the galactic longitude \( l \) and latitude \( b \) and the radial velocity \( v_r \) given in the galactocentric frame (MWR frame):

\[
v_r = v_1 \cos l \cos b + v_2 \sin l \cos b + v_3 \sin b
\]

where \( \mathbf{v} = (v_1, v_2, v_3) \) is the line-of-sight direction and \( \mathbf{v} = (v_1, v_2, v_3) \) the 3D velocity of the galaxy in the MWR frame. By successively integrating over the distance \( r \) and over 2 components of the 3D velocity (say, of \( v_1 \) and \( v_2 \)), we express the observed probability density \( dP_{obs} \) in terms of the observables:

\[
dP_{obs} = g(v_r; v_x \cos l \cos b + v_y \sin l \cos b + v_z \sin b, \sigma_v) \ dv_r
\]

\[
\times \eta(l, b) \ \cos b \ dl \ db
\]

where \( \eta(l, b) \) is the projected angular distribution function of the objects under consideration (i.e. \( \eta(l, b) = \int \rho(r, l, b) \ r^2 \ dr \)). Note, that the information on the distances \( r \) of the galaxies is not used. For a data sample of \( N \) galaxies with measured \( \{h_b, b^k, v_k^b\}_{k=1}^{N} \), the efficient part of the natural logarithm of the likelihood function \( \mathcal{L} = \mathcal{L}(v_x, v_y, v_z, \sigma_v) \) reads thus:

\[
\mathcal{L} = -\ln \sigma_v - \frac{1}{N} \sum_{k=1}^{N} \left[ (v_k^b - v_x h_k^b - v_y \hat{v}_k^b - v_z \hat{r}_k^b)^2 \right] \frac{1}{2 \sigma_v^2}
\]

where \( h_k^b = \cos b^k \cos b^b \), \( \hat{r}_k^b = \sin b^k \cos b^k \) and \( \hat{r}_k^b = \sin b^k \) (see Eq. (3)). Maximizing \( \mathcal{L} \) with respect to \( v_x, v_y \) and \( v_z \) gives the following set of equations:

\[
\langle \hat{r}_1 (v_r - v_x \hat{r}_1 - v_y \hat{r}_2 - v_z \hat{r}_3) \rangle = 0
\]

\[
\langle \hat{r}_2 (v_r - v_x \hat{r}_1 - v_y \hat{r}_2 - v_z \hat{r}_3) \rangle = 0
\]

\[
\langle \hat{r}_3 (v_r - v_x \hat{r}_1 - v_y \hat{r}_2 - v_z \hat{r}_3) \rangle = 0
\]

where \( \langle . \rangle \) denotes the average on the sample (for example \( \langle v_r \hat{r}_2 \rangle = 1/N \sum_{k=1}^{N} v_k^b h_k^b \hat{r}_k^b = 1/N \sum_{k=1}^{N} v_k^b \sin b^k \)). This set of linear equations can be rewritten:

\[
M \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} \langle v_r \hat{r}_1 \rangle \\ \langle v_r \hat{r}_2 \rangle \\ \langle v_r \hat{r}_3 \rangle \end{bmatrix}
\]

where \( M \) is the 3×3 symmetric matrix:

\[
M = \begin{bmatrix} \langle \hat{r}_1 \hat{r}_1 \rangle & \langle \hat{r}_1 \hat{r}_2 \rangle & \langle \hat{r}_1 \hat{r}_3 \rangle \\ \langle \hat{r}_2 \hat{r}_1 \rangle & \langle \hat{r}_2 \hat{r}_2 \rangle & \langle \hat{r}_2 \hat{r}_3 \rangle \\ \langle \hat{r}_3 \hat{r}_1 \rangle & \langle \hat{r}_3 \hat{r}_2 \rangle & \langle \hat{r}_3 \hat{r}_3 \rangle \end{bmatrix}
\]

Finally, the unbiased statistics \( \mathbf{v}_B = (\hat{v}_x, \hat{v}_y, \hat{v}_z) \) of the 3D bulk flow \( \mathbf{v}_B = (v_x, v_y, v_z) \) is obtained by inverting the 3×3 matrix \( M \):

\[
\begin{bmatrix} \hat{v}_x \\ \hat{v}_y \\ \hat{v}_z \end{bmatrix} = M^{-1} \begin{bmatrix} \langle v_r \hat{r}_1 \rangle \\ \langle v_r \hat{r}_2 \rangle \\ \langle v_r \hat{r}_3 \rangle \end{bmatrix}
\]
This bulk flow statistic is robust. It does not depend on the distance of galaxies nor on the projected angular distribution function \( \eta(l, b) \) of the sample. However, the accuracy of the \( \tilde{v}_x, \tilde{v}_y \) and \( \tilde{v}_z \) estimators depends on \( \eta(l, b) \) and on the velocity dispersion \( \sigma_v \) of the dynamical system. We thus, compute the variance of these estimators by using numerical simulations which are supposed to mimic the real data.

The derivation of the velocity dispersion estimator \( \sigma_v \) is not straightforward. Maximizing the efficient part of the likelihood function \( L \) of Eq. (3) with respect to \( \sigma_v \), we have the following equation

\[
\sigma_v^2 = \langle (v_r - \tilde{v}_x \hat{r}_1 - \tilde{v}_y \hat{r}_2 - \tilde{v}_z \hat{r}_3)^2 \rangle \quad (8)
\]

Unfortunately, the velocity dispersion estimator \( \sigma_v^* \) :

\[
\sigma_v^* = \langle (v_r - \tilde{v}_x \hat{r}_1 - \tilde{v}_y \hat{r}_2 - \tilde{v}_z \hat{r}_3)^2 \rangle \quad (9)
\]

obtained while replacing the bulk flow \( v_B \) by its estimate \( \tilde{v}_B \), is biased. The reasons of this bias are twofold. On one hand, the variance of the \( \tilde{v}_x, \tilde{v}_y \) and \( \tilde{v}_z \) estimates leads to enhance the value of \( \sigma_v^* \) and thus, to overestimate the velocity dispersion \( \sigma_v \). On the other hand, because of the finite size of the sample, the random variables \( \tilde{v}_x, \tilde{v}_y, \tilde{v}_z \) and the radial velocity \( v_r \) of each objects are correlated. This feature contributes to underestimate \( \sigma_v \) when using the \( \sigma_v^* \) statistic. As a matter of fact, the unbiased estimator of the velocity dispersion depends on the accuracy of the bulk flow estimate and thus on the velocity dispersion itself. In this paper, we have applied a computational iterative process on numerical simulations which furnishes an unbiased value for \( \sigma_v \).

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