Design of Pose Controller of Underwater Supercavitating Vehicles Based on Variable Structure

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Abstract. On the basis of dynamic modeling for underwater supercavitating vehicles, the paper designed controllers adopting variable structure control strategy whose robustness can suppress unknown disturbance caused by imprecision existed in dynamic modeling. Theory analysis and simulation show that the controller designed can make pose angle track course angle of trajectory steadily, realize decoupling controlling between three pose moving subsystems and have good robustness for outside interference and model’s error, which assure that supercavitating vehicles have good maneuverability.

1. Introduction

Since the advent of supercavitation drag reduction technology, Good drag reduction effect has been used by many military powers in the world to develop underwater hypervelocity weapons. In order to realize the effective control of the supercavitation vehicle, it is necessary to establish a dynamic model that can describe the dynamic characteristics of the supercavitation vehicle and be beneficial to the effective control. A lot of research work has been carried out in the field of supercavitation calculation by scholars both at home and abroad. The early use of the potential flow theory to model has been widely used in the design of supercavitation torpedo [1]. In the past decade, the method of modeling the cavitation by solving the N-S equation, such as the interface tracking method and the homogeneous equilibrium flow theory, began to prevail in [2]. In recent years, many scholars at home and abroad have proposed different dynamic modeling methods for supercavitating vehicles. For example, Dzielski and other proposed a four state two degree of freedom model in a vertical plane [3], and Zhang Xuewei and other developed a method for calculating the morphology of the supercavitation of unsteady ventilation according to the principle of Logvinovich independent expansion [4]. Although many scholars at home and abroad have studied in the field of supercavitation modeling, the cavitation shape of supercavitating is influenced by many uncertain factors, such as the memory effect of the cavitation, the uncertainty of the skidding force and the jump, which makes the dynamic modeling of the supercavitation vehicle extremely difficult, leading to the stability control of the body has always been a difficult problem to solve. On the basis of constructing the dynamic model of the vehicle, this paper describes the part which is difficult to accurately model, and uses variable structure control strategy to design the controller and suppress the random disturbance by the strong robustness of the controller.

2. Dynamic Modeling of Navigation Body

According to different operational requirements, multiple UAVs are often required to perform operational tasks in clusters. How to ensure that multiple UAVs fly steadily in accordance with the set
formation without collision is very important for UAVs formation control. The flying formations often used by UAVs are guided by following formations and left and right diamond formations. In this paper, the left and right diamond formation is taken as the research object, and the state equation of the control object is established based on the relative motion relationship between the main engine and the bureaucrat, and the controller is designed on this basis.

After the cavitation is produced by a supercavitation vehicle, most of the navigational body are covered with bubbles (except for the head cavitation and tail). The buoyancy of the navigation body is lost. If no effective control is taken, the navigation experience will sink rapidly. Through the design of cavitation deflector and tail rudder, lift force can be generated to counteract the loss buoyancy, so as to maintain the balance of the supercavitating vehicle. In this paper, the dynamic analysis of the vehicle is first carried out, then the dynamic equation of the rotation of the vehicle is established, and the variable structure control strategy is used to design the controller, and the attitude angle of the vehicle is controlled steadily.

![Figure 1. A schematic diagram of the force of a cavitation device](image)

2.1. Cavitation Force

As shown in Fig.1, the cavitation device can be rotated around a fixed axis parallel to the Oy axis in the longitudinal plane. The deflection angle of the cavitation device is expressed as $\delta$ which is positive when the cavitation is rotated forward against the clockwise Oy axis, and the opposite direction is negative. For the cavitation coordinate system $O_c - x_c y_c z_c$, the origin of the coordinate system is in the geometric center of the cavitation device, and the force analysis of the cavitation is shown in Fig.1. The transformation matrix of the body coordinate system to the cavitation coordinate system is $T_c$. According to the coordinate transformation method $T_c$ can be obtained as follows:

$$
\begin{bmatrix}
  x_c \\
  y_c \\
  z_c 
\end{bmatrix} = T_c \begin{bmatrix}
  x \\
  y \\
  z 
\end{bmatrix} = \begin{bmatrix}
  \cos \delta & 0 & -\sin \delta \\
  0 & 1 & 0 \\
  \sin \delta & 0 & \cos \delta 
\end{bmatrix} \begin{bmatrix}
  x \\
  y \\
  z 
\end{bmatrix}
$$

(1)

The hydrodynamic force $F_c$ acting on the cavitation device can provide lift and resistance for the underwater vehicle. The lift coefficient $c_{l_c}$ and the drag coefficient $c_{d_c}$ are the function of the angle of attack. According to references [5, 6], the hydrodynamic coefficients acting on disc-cavitation have the following empirical formula:

$$
c_{l_c} = 0.82(1 + \sigma) \cos \alpha \sin \alpha
$$

(2)

$$
c_{d_c} = 0.82(1 + \sigma) \cos^2 \alpha
$$

(3)

In which $\sigma$ is the number of cavitation, and $\alpha$ is the angle of attack of the cavitation device.

Based on the above empirical formula, the expression of lift $F_l$ and drag $F_d$ acting on the cavitation device can be obtained as follows:

$$
F_l = \frac{1}{2} \rho V_c^2 S_c c_{l_c} = \frac{0.82}{2} \rho V_c^2 S_c (1 + \sigma) \cos \alpha \sin \alpha
$$

(4)
\[
F_d = \frac{1}{2} \rho V_e^2 S_c c l_d = \frac{0.82}{2} \rho V_e^2 S_c (1 + \sigma) \cos^2 \alpha
\]  

(5)

Among them, \( \rho \) is the fluid density, \( V_e \) is the speed of the vehicle, \( S_c \) is the cross-sectional area of the cavitation device and \( \alpha \) is the attack angle of the cavitation vehicle.

The force of the cavitation apparatus in the speed coordinate system of the cavitation is obtained, and the force of the cavitation device under the body coordinate system of the navigational body can be obtained by using the coordinate transformation matrix:

\[
\begin{align*}
F_{x,c} &= -(\cos \delta \cos \alpha \cos \beta + \sin \delta \sin \alpha \cos \beta) F_x + (\sin \delta \cos \alpha \cos \beta - \cos \delta \sin \alpha \sin \beta) F_y = -\frac{0.82}{2} \rho V_e^2 S_c (1 + \sigma) \cos \alpha \cos \beta \cos \delta \\
F_{y,c} &= -(\cos \delta \cos \alpha \sin \beta + \sin \delta \sin \alpha \sin \beta) F_x + (\sin \delta \cos \alpha \sin \beta - \cos \delta \sin \alpha \cos \beta) F_y = -\frac{0.82}{2} \rho V_e^2 S_c (1 + \sigma) \cos \alpha \sin \beta \cos \delta \\
F_{z,c} &= (\cos \delta \sin \alpha - \sin \delta \cos \alpha) F_x - (\sin \delta \cos \alpha + \cos \delta \cos \alpha) F_y = -\frac{0.82}{2} \rho V_e^2 S_c (1 + \sigma) \cos \alpha \sin \delta
\end{align*}
\]  

(6)

\[
\begin{align*}
M_{c,x} &= 0 \\
M_{c,y} &= -x_{cg} F_{c,z} = -\frac{0.82}{2} x_{cg} \rho V_e^2 S_c (1 + \sigma) \cos \alpha \cos \alpha \cos \delta \\
M_{c,z} &= x_{cg} F_{c,z} = -\frac{0.82}{2} x_{cg} \rho V_e^2 S_c (1 + \sigma) \cos \alpha \sin \beta \cos \delta
\end{align*}
\]  

(7)

In which, \( F_{x,c} \), \( F_{y,c} \), \( F_{z,c} \) are three components of the force of the cavitation device in the body coordinate system.

When the cavitation device does not have a deflection angle, the coordinate of the pressure center in the body coordinate system is \( (x_{cg}, 0, 0) \) and neglects the small change of coordinates caused by the deflection of the cavitation device, then the rotational torque of the hydrodynamic force relative to the center of gravity of the navigational body can be obtained as followed:

\[
\begin{align*}
M_{c,x1} &= 0 \\
M_{c,y1} &= -x_{cg} F_{c,z1} = -\frac{0.82}{2} x_{cg} \rho V_e^2 S_c (1 + \sigma) \cos \alpha \cos \beta \cos \delta \\
M_{c,z1} &= x_{cg} F_{c,z1} = -\frac{0.82}{2} x_{cg} \rho V_e^2 S_c (1 + \sigma) \cos \alpha \sin \beta \cos \delta
\end{align*}
\]  

(8)

In which, \( M_{c,x1}, M_{c,y1}, M_{c,z1} \) are the moments of the cavitation relative to the center of gravity of the vehicle in the body coordinate system.

2.2. Skidding Force

The sliding force is the most complex in all kinds of fluid forces acting on a supercavitating vehicle. When the navigation speed, the pitching angle and the turning rate of the supercavitation vehicle change, the tail of the vehicle will penetrate the cavitation wall and interact with the fluid and the cavitation wall to form the sliding force. Due to the complexity of the cavitation, the sliding force is nonlinear and the slope of the force curve is discontinuous. Taking into account the complexity of the modeling of the sliding force, this paper uses random functions to simulate it. The sliding force is treated as the external interference, and suppressed by the robustness of the control system.

2.3. Dynamic Modeling of Supercavitation Vehicle

The dynamic model of the supercavitation vehicle is constructed by the dynamic model of the main components of the navigation body. The dynamic equation of the supercavitition vehicle rotation is considered as follows, considering the loss of the buoyancy of the navigation body.

\[
\begin{bmatrix}
J_{n} & \omega_{n} \\
\omega_{n} & 0 \\
0 & \omega_{n}
\end{bmatrix} + \begin{bmatrix}
0 & -\omega_{n} & \omega_{n} \\
-\omega_{n} & 0 & -\omega_{n} \\
-\omega_{n} & \omega_{n} & 0
\end{bmatrix} \begin{bmatrix}
J_{n} & \omega_{n} \\
\omega_{n} & 0 \\
0 & \omega_{n}
\end{bmatrix} = \begin{bmatrix}
M_{n} \\
M_{n} + T_{z_{p}} \\
M_{n} - T_{h}
\end{bmatrix}
\]

(12)

The rotational kinematics equation of the vehicle:
\[
\begin{bmatrix}
\dot{\psi} \\
\dot{\theta} \\
\dot{\phi}
\end{bmatrix} =
\begin{bmatrix}
0 & \sec \theta \cos \phi & -\cos \theta \sin \phi \\
0 & \sin \phi & \cos \phi \\
1 & -\tan \theta \cos \phi & \tan \theta \sin \phi
\end{bmatrix}
\begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix}
\]

(13)

In formula (13), \(\psi\) is the yaw angle of the navigation body, \(\theta\) is the elevation angle of the navigation body, \(\phi\) is the roll angle of the navigation body, \(\omega_x\) is the roll angular velocity, \(\omega_y\) is the yaw angular velocity and \(\omega_z\) is the elevation angle velocity, \(J_x\) is the moment of inertia about the longitudinal axis of a navigation body, \(J_y\) is the moment of inertia around the center of gravity, \(J_z\) is the moment of inertia around the center of gravity, \(M_{\psi}\) is the differential moment, \(M_{\theta}\) is the pitching moment, \(M_{\phi}\) is the elevation moment, \(T\) is the thrust of the navigation body.

The main part of the differential moment \((M_{\phi})\) and yaw moment \((M_{\psi})\) is generated by the rudder, the main part of the pitching moment \((M_{\theta})\) is produced by tail rudder and the cavitation device. Suppose that \(M_{\phi} = M_{\psi} + M_{\delta}\), in which \(M_{\psi}\) is the pitching moment produced by the tail rudder and \(M_{\delta}\) is the pitching moment produced by the cavitation device. It is known from the analysis of the front force as following.

\[
M_{\delta} = M_{c,2,1} = \frac{0.82}{2} x_c \rho V^2 S_c (1 + \alpha) \cos \alpha \cos \delta
\]

(14)

Considering that the main function of the cavitation deflector is to produce lift to offset the loss of buoyancy to maintain the balance of the vehicle in the direction of gravity, the design of the deflection angle of the cavitation device can be designed in the longitudinal motion, such as the fixed depth control system, and the pitching force produced by the cavitation device in the design of the attitude control system in this paper are treated as interference.

The equation of state of the supercavitating vehicle’s attitude angle can be derived by the equation (14) and the equation (12) if we plug \(M_{\phi} = M_{\psi} + M_{\delta}\) into the equation (12).

3. Design of Vehicle Controller

On the basis of the dynamic equations of the navigation body, the state equations of the pitch yaw and roller systems of the navigation body are derived respectively. The variable structure control strategy is used to design the controller, and the stability of the controller is proved by the Lyapunov stability theory [7, 8].

3.1. Pitching Subsystem

It can be obtained as follows:

\[
\dot{\theta} = \dot{\phi} \cos \phi \omega_{x} - \dot{\phi} \sin \phi \omega_{x} + \omega_{x} \sin \phi + \omega_{x} \cos \phi
\]

(15)

From equation (16), we can obtain the following equation:

\[
\dot{\omega}_x = \frac{1}{J_1} (M_{\psi} + M_{\delta} - Th + J_x \omega_x \omega_{x} - J_x \omega_x \omega_{x})
\]

(16)

From equation (15) and equation (16), we can get as follows:

\[
\ddot{\phi} = \frac{\cos \phi}{J_1} (M_{\psi} + M_{\delta}) + \dot{\phi} \cos \phi \omega_{x} - \dot{\phi} \sin \phi \omega_{x} + \omega_{x} \sin \phi + \frac{1}{J_1} (-Th + J_x \omega_x \omega_{x} - J_x \omega_x \omega_{x}) \cos \phi
\]

(17)

Supposing \(x_{1z} = \dot{\phi} \cdot x_{2z} = \dot{\phi}\), then
\[
\begin{bmatrix}
 x_{t1} \\
 x_{t2}
\end{bmatrix} = \begin{bmatrix}
 0 & 1 \\
 0 & 0
\end{bmatrix} \begin{bmatrix}
 x_{t1} \\
 x_{t2}
\end{bmatrix} + \begin{bmatrix}
 0 \\
 \cos \varphi / J_z
\end{bmatrix} M_{cw} + \begin{bmatrix}
 0 \\
 f_z
\end{bmatrix}
\]

in which
\[ f_z = \varphi \cos \omega_1 - \varphi \sin \omega_1 \sin \varphi + \frac{1}{J_z} (M_{z} - Th + J_z \omega_1 \omega_1 - J_z \omega_1 \omega_1) \cos \varphi \]

Based on the variable structure control theory [9], for the pitch subsystem state equation (18), the sliding mode surface is selected as follows.

\[ s_z = c_z (x_{tc} - \theta_0) + x_{t2} = c_z (\theta - \theta_0) + \theta \]

Among them, \( c_z > 0 \) is constant and \( \theta_0 \) is the elevation angle required for the underwater vehicle trajectory. Obviously, the system is asymptotically stable on the sliding surface \( s_z = 0 \).

When differentiate equation (19), we can get:

\[ s_z = c_z \dot{\theta} - \dot{\theta} + c_z \dot{\theta} + \frac{\cos \varphi}{J_z} M_{cw} + f_z \]

Because the underwater vehicle is a non-rolling vehicle and the rolling angle \( \varphi = \pi / 2 \), we can get \( \cos \varphi > 0 \). Because the pitching moment is mainly caused by a horizontal rudder, its size is limited. So we can suppose \( |M_{cw}| \leq K_z \), then design controller as follows.

\[ M_{cw} = \begin{cases}
 -K_z & s_z > 0 \\
 K_z & s_z < 0
\end{cases} \]

By the arrival condition of the sliding mode we can get it as follows.

\[ s_z = \begin{cases}
 c_z \dot{\theta} - \frac{\cos \varphi}{J_z} K_z + f_z < 0 & s_z > 0 \\
 c_z \dot{\theta} + \frac{\cos \varphi}{J_z} K_z + f_z > 0 & s_z < 0
\end{cases} \]

In order to prove that the pitch subsystem can follow the pitch angle of the command stably under the function of the controller \( M_{cw} = \begin{cases}
 K_z & s_z > 0 \\
 -K_z & s_z < 0
\end{cases} \), take the Lyapunov function \( V_z = \frac{1}{2} s_z^2 \). Then \( V_z = s_z \dot{s}_z \). In order to prove equation (20), the inverse method is adopted, supposing equation (20) is false. Because \( K_z > 0 \) in \( M_{cw} = \begin{cases}
 -K_z & s_z > 0 \\
 K_z & s_z < 0
\end{cases} \), in engineering, the pitch angle speed of the underwater vehicle \( \dot{\theta} \) is limited, and the other two channels’ coupling for the pitching channel considered as interference \( f_z \) is limited, the absolute value of the upper and lower limits of the controller \( M_{cw} \) can be large enough to always satisfy the equation (20). So there is \( V_z = s_z \dot{s}_z < 0 \), and \( V_z > 0 \), so \( V_z \to 0 \), that is \( s_z \to 0 \). According equation (19), \( s_z = c_z (x_{tc} - \theta_0) + x_{t2} = c_z (\theta - \theta_0) + \theta \). Because \( c_z > 0 \), \( \theta \) converges steadily to \( \theta_0 \). It can be seen that the pitching subsystem can make the system enter the sliding mode state under the action of the controller \( M_{cw} = \begin{cases}
 -K_z & s_z > 0 \\
 K_z & s_z < 0
\end{cases} \), and the elevation angle is uniformly converged to the command pitch angle \( \theta_0 \) in the sliding mode state that is \( s_z \to 0 \).

Because the controller \( M_{cw} \) is a switching function, it is difficult to realize the high speed switching of the rudder in engineering. Because the rudder has the response time, it cannot be switched instantaneously. Even if the response time is very short, the high speed switching can be approximated, and the chattering is not allowed in the engineering. Therefore, the method of continuous switching is used to eliminate chattering [10, 11].
Supposing $M_{\psi} = -K_c \frac{s_1}{|J| + \delta}$, where $\delta$ is a minimum normal constant. Similarly, by using the Lyapunov stability theorem, it is proved that the controller $M_{\psi} = -K_c \frac{s_1}{|J| + \delta}$ can control the pitch angle in a very small neighborhood near the instruction pitch angle.

### 3.2. Yaw Subsystem

Similar to the pitching subsystem, the state equation of the yaw subsystem can be obtained as follows.

$$
\begin{bmatrix}
  x_{1y} \\
  x_{2y}
\end{bmatrix} =
\begin{bmatrix}
  0 & 1 \\
  0 & 0
\end{bmatrix}
\begin{bmatrix}
  x_{1y} \\
  x_{2y}
\end{bmatrix} +
\begin{bmatrix}
  0 \\
  \frac{\sec \theta \cdot \cos \phi}{J_n}
\end{bmatrix}
M_{\psi} +
\begin{bmatrix}
  1 \\
  1
\end{bmatrix}
f_y.
$$

In equation (21), $x_{1y} = \psi, x_{2y} = \psi, M_{\psi}$ is the controller in the yaw subsystem, that is the yaw moment produced by the straight rudder, and $f_y$ is the coupling effect of the pitching and rolling on the yaw, which is considered as the interference.

$$
f_y = \frac{J - J_n \cdot \omega_n \cdot \omega_n \cdot \sec \theta \cdot \cos \phi}{J_n} \cdot \frac{\sec \theta \cdot \cos \phi}{J_n} (T \cdot z_e) + \frac{\vartheta \cdot \sin \theta \cdot \cos \phi \cdot \omega_n}{\cos \theta} + \vartheta \cdot \sin \theta \cdot \sin \phi \cdot \omega_n - \cos \theta \cdot \sin \phi \cdot \omega_n - \vartheta \cdot \cos \phi \cdot \cos \theta \cdot \omega_n.
$$

The variable structure control strategy is used to design the controller $M_{\psi}$ and the sliding surface is selected as follows.

$$
s_y = c_y (x_{1y} - \psi_0) + x_{2y} = c_y (\psi - \psi_0) + \psi
$$

Among them, $c_y$ is the normal constant and $\psi_0$ is the command yaw angle of the trajectory.

Because $|\psi| < \frac{\pi}{2}, |\phi| < \frac{\pi}{2}$, so $\sec \theta \cdot \cos \phi / J_n > 0$.

Obviously, the system is asymptotically stable on the sliding surface that is $s_y = 0$. It can be obtained as follows.

$$
s_y = c_y \psi + \psi = c_y \psi + \frac{\sec \theta \cdot \cos \phi}{J_n} \cdot M_{\psi} + f_y.
$$

Because the yaw moment $M_{\psi}$ is mainly produced by the direct rudder, its size is limited supposing $|M_{\psi}| < K_c$. Supposing $M_{\psi} = \begin{bmatrix} -K_s & s_1 \end{bmatrix}$, similar to the pitch subsystem controller, it can be proved that the yaw subsystem can track the instruction yaw angle steadily under the action of the controller $M_{\psi} = \begin{bmatrix} -K_s & s_1 \end{bmatrix}$. Similar to the pitching subsystem, let $M_{\psi} = \begin{bmatrix} -K_s & s_1 \end{bmatrix}$.

### 3.3. Roller Subsystem

Using the same derivation method as the pitch subsystem, the state equation of the roller subsystem can be obtained as follows.

$$
\begin{bmatrix}
  x_{1r} \\
  x_{2r}
\end{bmatrix} =
\begin{bmatrix}
  0 & 1 \\
  0 & 0
\end{bmatrix}
\begin{bmatrix}
  x_{1r} \\
  x_{2r}
\end{bmatrix} +
\begin{bmatrix}
  0 \\
  \frac{1}{J_n}
\end{bmatrix}
M_{\psi} +
\begin{bmatrix}
  0 \\
  1
\end{bmatrix}
f_r.
$$
Where \( x_{1x} = \theta, x_{2x} = \phi, M_{x_\phi} \) is the controller in the roller system, that is the torque produced by the differential rudder, and \( f_x \) is the coupling effect of the pitching and yaw subsystem on the roller system, which is considered as interference.

\[
f_x = \frac{J_z - J_x}{J_y} \omega_y \omega_y + \frac{\theta}{\cos^2 \theta} (\sin \theta \omega_x - \cos \theta \omega_x) + \tan \theta (\cos \theta \omega_x - \sin \theta \omega_x) + \cos \theta \omega_x + \sin \theta \omega_x.
\]

We can select sliding surface as follows.

\[
s_x = c_x x_{1x} + x_{2x} = c_x \theta + \phi
\]

(24)

Where \( c_x \) is the normal constant, and it is obvious that the system is asymptotically stable on the sliding surface which is \( s_x = 0 \). It can be obtained as follows.

\[
s_x = c_x \theta + \phi = c_x \theta + \frac{1}{J_{x_\phi}} M_{x_\phi} + f_x
\]

(25)

Because the controller \( M_{x_\phi} \) is generated by the differential rudder angle, its size is limited supposing \( |M_{x_\phi}| < K_{x_\phi} \), and the controller is designed as follows.

\[
M_{x_\phi} = \begin{cases} 
-K_{x_\phi}, & s_x > 0 \\
K_{x_\phi}, & s_x < 0 
\end{cases}
\]

(26)

The same as the pitch subsystem, it can be proved that the rolling roller system can control the roll angle at zero angles under the controller \( M_{x_\phi} \). In the same way, we can suppose \( M_{y_\theta} = -K_{y_\theta} \frac{s_y}{s_x} \).

4. Simulation Analysis

The simultaneous equations are established by using the relative motion equation of the front host and the wingman, the system state equation and the controller equation as follows.

In order to verify the performance of the attitude controller designed in this paper, the dynamic parameters are as follows:

- The pitching inertia \( J_z = 891 \text{ kg} \cdot \text{m}^2 \), the yaw inertia \( J_y = 891 \text{ kg} \cdot \text{m}^2 \), the roll inertia \( J_x = 210 \text{ kg} \cdot \text{m}^2 \), the vehicle buoyancy \( B = 0 \), and the push force \( T = 789 \text{ N} \). In the simulation, a torque expressed by random function is added to the pitch motion to simulate the pitch force and the pitching moment generated by the cavitation device. The simulation mathematical model is as follows.

In the simulation, it is assumed that \( K_z = K_y = 1000 \text{ N} \cdot \text{m}, K_x = 200 \text{ N} \cdot \text{m}, c_z = c_y = c_x = 2, Rz = 200 \text{ N} \cdot \text{m}, \delta = 0.01 \). Assuming that the initial pitch angle, yaw angle and roll angle of the navigation body are zero, the instruction pitching angle and the yaw angle are respectively \( \theta_0 = 15^\circ, \psi_0 = 45^\circ \). Through using the Runge-Kutta method, we get the curve of the elevation, yaw and roll angle of the vehicle under the controlling of the controllers \( M_{z_\phi}, M_{y_\theta}, M_{y_\phi} \), as shown in Figure 2.
As shown in Figure 2, the attitude angle of the navigation body can quickly track the attitude angle of the instruction. The roll angle is controlled at zero angles and is not affected by the disturbance torque. It shows that the attitude controller designed in this paper is robust and can realize the decoupling control of the supercavitation vehicle attitude.

From Figure 3 and figure 5, the pitching moment and the yaw torque are smooth transition to the stable state, which not only eliminates the inherent buffeting of the variable structure control, but also ensures the stability. It shows that the controller designed in this paper has the feasibility of engineering.

It is shown in Figure 4 that there are some strong disturbances in the rolling torque; the main reason is that the requirements of the roll angle control at zero angle are more demanding. When the navigation body is maneuvering, there is a large coupling between the subsystems, and the existence of the interference torque causes the complex rolling motion of the navigation body. The strong disturbance of rolling torque will disappear if a small rolling is allowed in the project.

5. Conclusion
Through theoretical deduction and simulation, the attitude controller designed by the variable structure control strategy can control the attitude angle of the supercavitation vehicle steadily at a specified angle. Compared with the traditional control technology, the controller designed in this paper has the advantages of insensitivity to modeling errors and external disturbances, and realize the decoupling control between three channels that cannot be processed by the traditional control technology, which
provides technical support for improving the maneuverability and stability of the supercavitating vehicle.

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