Magnon squeezing enhanced entanglement in a cavity magnomechanical system

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Abstract

We investigate the generation of the entanglement in a cavity magnomechanical system, which consists of three modes: a magnon mode, a microwave cavity mode and a mechanical vibration mode, the couplings of the magnon-photon and the magnon-phonon are achieved by the magnetic dipole interaction and the magnetostrictive interaction, respectively. By introducing a squeezing of the magnon mode, the magnon-photon and the magnon-phonon entanglements are significantly enhanced compared with the case without inserting the magnon squeezing. We find that an optimal parameter of the squeezing exists, which yields the maximum entanglement. This study provides a new idea for exploring the properties of quantum entanglement in the cavity magnomechanical systems, and may have some potential applications in the quantum state engineering.

1. Introduction

Cavity magnomechanical system (CMM system) is becoming a new hotspot arising in recent years, this system consists of a microwave cavity and a yttrium-iron-garnet (YIG) sphere. Because of the high spin density and the strong spin-spin exchange interaction, the magnons which are embodied by a collective motion of the uniform of spins can be strongly coupled to cavity microwave photons in the YIG sphere\cite{1,2}, and the coupling strength between them can even achieve the ultrastrong coupling\cite{3}. Simultaneously, the magnon mode can also couple to a mechanical vibration mode with the magnetostrictive interaction (the radiation pressure like in the optomechanical systems), which realizes photon-phonon coupling. Comparing with the optomechanical systems\cite{4,5}, the CMM systems have the advantages of high adjustability and low loss. Hence this system can provide a promising vision for the realization of quantum information theory and macroscopic quantum states in hybrid quantum systems\cite{6,24}.

As a fundamental physical resource of the quantum information processing, the quantum entangled states are widely used in the quantum computing, quantum information processing and others, the study of entanglement emerges in endlessly in the mechanical systems. At the same time, people are also thinking about how to enhance this important physical resource\cite{25,26,27,28,29}. On the other hand, it is a prerequisite for observing various quantum effects in a mechanical system to make the quantum system cold to close to its ground state, and it is helpful for the generation of the entanglement\cite{30,31,32}. Li et al. have done a novel work\cite{33} and they found that the magnon squeezing can enhance the ground-state cooling in the CMM system. And the squeezing of the magnon mode can also use to enhance the nonlinearity and entanglement.

Here, We utilize a CMM system with the magnon squeezing to study the entanglement inside. The magnetostrictive interaction and magnetic dipole interaction mediate the magnon-phonon coupling and the photon-magnon coupling, respectively. We provide a protocol to enhance the continuous variable entanglement between the magnon mode and cavity mode (or the phonon mode) via the squeezing of the magnon mode, with the squeezing parameter \(\chi\) and the phase \(\theta\). We show the entanglement can be significantly enhanced, and the optimal values of the phase and the squeezing parameter corresponding to the maximum entanglement are given. Furthermore, the effect of temperature on entanglement is also discussed.

The paper is organized as follows. In Sec. II, we show the Hamiltonian and dynamical equations of the whole system. In Sec. III, we compare the entanglement in the cases with and without inserting the magnon squeezing. It shows that remarkable improvement of the entanglement can be achieved with the enhancement of nonlinear effect caused by the squeezing. Finally, a concluding summary is given in Sec. IV.

2. Theoretical model and dynamical equation

As shown in Fig. 1, we consider a three-mode CMM system, which consists of a magnon mode (the uniform-precession Kittel mode in a YIG sphere\cite{9}), a microwave cavity mode, and a mechanical vibration mode. When the frequency of the Kittel-mode magnons resonates with the frequency of the microwave photons, the magnon mode and optical mode can be strongly coupled. Simultaneously, the magnon mode can couple to the mechanical vibration mode by the magnetostrictive interaction. Because the size of the YIG sphere (the diameter of the YIG sphere is generally \(10^2\mu m\) to \(1mm\)) is much smaller than the wavelength of the microwave field, the photon-phonon coupling caused by the radiation pressure is ignored. Besides, an external microwave driving field is introduced to effectively enhance the magnon-phonon coupling\cite{10,11}. With a rotating
The assumption of the low-lying excitations, the Rabi frequency of the magnon mode, with the squeezing parameter $\chi$ noting that the last term in the total Hamiltonian is the squeezingless position and momentum quadrature of the mechanical YIG sphere is considered as a mechanical resonator. (b) The equivalent mode-coupling model.

The quantum Langevin equations (QLEs) are given by

$$\dot{\theta} = \omega_\theta p \tag{3}$$
$$\dot{p} = -\omega_\theta x - \gamma_\theta p - g_{mb} m^\dagger + \xi,$$

where $\gamma_\theta$ and $\eta_\theta$ represent the dissipation rates of magnon mode and mechanical mode, respectively. As the aim is to enhance entanglement, an optical effective gain $\Gamma = -\kappa_a + G_a$ (active optical cavity) is considering in this CMM system \(^{38}\), where $G_a$ is the gain of cavity mode and it is greater than the dissipation of the cavity field $\kappa_a$. This optical gain can be achieved by in many methods \(^{39,40}\). $a^\dagger, m^\dagger$ and $\xi$ are input noise operators of cavity, magnon and mechanical modes, with a Markovian approximation, the noise operators satisfy:

$$\langle a^{\dagger}(t)a(t') \rangle = (n_a + 1)\delta(t-t'),$$
$$\langle a(t)a^{\dagger}(t') \rangle = n_a\delta(t-t'),$$
$$\langle m^{\dagger}(t)m(t') \rangle = (n_m + 1)\delta(t-t'),$$
$$\langle m(t)m^{\dagger}(t') \rangle = n_m\delta(t-t'),$$
$$\langle \xi(t)\xi(t') \rangle = n_\xi\delta(t-t'),$$
$$\langle \xi(t)\xi^{\dagger}(t') \rangle = n_\xi\delta(t-t'),$$

with $n_a = (e^{\nu_\theta/\kappa_aT} - 1)/(\mu_a a, m, b), k_B$ is the Boltzmann constant and $T$ the environmental temperature.

3. Entanglement generation

According to the strong driving field, each Heisenberg operator can rewritten as a sum of the steady-state mean value and the corresponding quantum fluctuation, i.e., $O(t) = O_0 + \delta O(t)$ ($O = a, m, x, p$), then we can linearize this system. The linearized QLEs can be written in a compact form $\ddot{u}(t) = Au(t) + m(t)$, where quadratures of the quantum fluctuations is rewritten as $u(t) = [\delta X_1, \delta X_2, \delta Y_1, \delta Y_2, \delta x, \delta p]^T$, and we define $\delta X_1 = (\delta a + \delta a^\dagger)/\sqrt{2}, \delta X_2 = i(\delta a - \delta a^\dagger)/\sqrt{2}, \delta Y_1 = (\delta m + \delta m^\dagger)/\sqrt{2}$ and $\delta Y_2 = i(\delta m - \delta m^\dagger)/\sqrt{2}$. Defining the vector of noise $n(t) = [\sqrt{2\kappa_a}X_1(t), \sqrt{2\kappa_a}X_2(t), \sqrt{2\kappa_m}m(t), \sqrt{2\kappa_m}m(t), 0, \xi(t)]^T$ and the correlation matrix is given by

$$A = \begin{pmatrix}
\Gamma & \Delta_a & 0 & 0 & 0 & 0 \\
-\Delta_a & \Gamma & -g_{ma} & 0 & 0 & 0 \\
0 & g_{ma} & \mu_a & -G & 0 & 0 \\
-\Delta_m & 0 & g_{mb} & \mu_a & 0 & 0 \\
0 & 0 & 0 & 0 & \omega_{\theta} & 0 \\
0 & 0 & 0 & G & -\omega_{\theta} & -\gamma_\theta
\end{pmatrix},$$

where $G = i\sqrt{2\kappa_m}m_1$ is the coherent-driving-enhanced magnomechanical coupling strength, $\mu_a = -\kappa_m + 2\chi \cos \theta, \nu_a = \pm \Delta_m + 2\chi \sin \theta$ and $m_1$ is

$$m_1 = \frac{\epsilon_d(i\Delta_a - \Gamma)}{g_{ma} + i(\Delta_m - \Gamma)\beta},$$

with the effective magnon-drive detuning $\tilde{\Delta}_m = \Delta_m + g_{mb}q_\theta$ and $\beta = (i\Delta_m - 2\chi \sin \theta) + (\kappa_m - 2\chi \cos \theta)$.
For this system, due to the linearized dynamics of the system and the Gaussian input noise operators, the entanglement of magnon-cavity and magnon-phonon can be measure by the continuous variable (CV) entanglement, we choose a standard ensemble method (logarithmic negativity $E_N$) \cite{41}. When the stability conditions are satisfied, one gets the following equation for the steady-state correlation matrix (CM) \cite{42}:

$$AV + VA^T = -D, \quad (6)$$

with the diffusion matrix $D = \text{diag}[\kappa_n(2n_a + 1), \kappa_p(2n_a + 1), \kappa_m(2n_m + 1), \kappa_n(2n_m + 1), 0, \gamma_b(2n_b + 1)]$. In the CV case, the logarithmic negativity is defined as \cite{43}:

$$E_N = \max[0, -\ln 2\eta], \quad (7)$$

where $\eta = 2^{-1/2}[\Sigma(V) - (\Sigma(V))^2/4] = 4 \det V_s (1/2)^{1/2}$, with $\Sigma(V) = \det A + \det B - 2 \det C$. The matrix elements of the reduced $4 \times 4$ submatrix $V_s$ depend on the one of the bipartite entanglements: $E_{N,\text{am}}$ and $E_{N,\text{bm}}$, denote the photon-magnon and magnon-phonon entanglement, respectively. And $V_s$ can be written in the following form:

$$V_s = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}. \quad (8)$$

One can now characterize the entanglement through Eq. \cite{7}. In Fig. 2 we plot the bipartite entanglements $E_{N,\text{am}}$ and $E_{N,\text{bm}}$ versus the phase $\theta$, it can be seen that the optimal phase for the entanglement is $\theta = 0.44\pi$, the entanglement in $\theta \in [\pi, 2\pi]$ is much weak, which is not discussed here. Then the following experimentally achievable parameters in this work are given \cite{5, 11}: $\Delta_{\text{am}}/2\pi = \Delta_{\text{bm}}/2\pi = 10$MHz (blue detuning), $\omega_b/2\pi = 10$MHz, $\gamma_b/2\pi = 10Hz$, $\kappa_m/2\pi = 1$MHz, $\kappa_n/2\pi = 0.5$MHz, $g_{\text{am}}/2\pi = 0.2Hz$ and $s_{\text{d}} = 3.5 \times 10^{-4}$Hz, corresponding to the drive power $P = 0.84$mW. Fig. 3 shows that the squeezing of the magnon mode is helpful to enhance the bipartite entanglements. Compared with the case of without the squeezing ($\chi = 0$), the entanglement $E_{N,\text{am}}$ and $E_{N,\text{bm}}$ can be significantly enhanced, and it can be seen that the maximum of the entanglement with the squeezing increases by 320% for $E_{N,\text{am}}$ and 260% for $E_{N,\text{bm}}$. However, when the value of $\chi$ continues to increase and exceed the optimal value, the entanglement will decrease. Physically, the gradual increase of $\chi$ from 0 means that the nonlinearity of the system is enhanced, resulting in the increase of entanglement. However, when $\chi$ continues to increase and exceed the optimal value, the noise of magnons becomes a significant effective thermal bath for the mechanical mode, and its negative impact on the generation of entanglement exceeds the positive impact of the squeezing.

It is also important to address the effect of noise on the studied entanglement. This is investigated in Fig. 4 and Fig. 5, where the entanglements $E_{N,\text{am}}$ and $E_{N,\text{bm}}$ are plotted versus the temperature $T$, respectively. Both entanglements are sensitive to the temperature $T$ and the strong entanglements only exist at cryogenic temperatures. And the entanglements increase with the increase of $\chi$. Here we choose $\chi$ that are less than the optimal value in Fig. 2.

Lastly, we discuss the implementation of the experimental in the CMM system. Based on the development of the material science, we assume that an active cavity mode can be constructed by the doping active metamaterials with inherent enhancement in the cavity. This assumption is based on two
existing works: (1) the $\mathcal{PT}$-symmetric whispering-gallery microcavities \[45\]. In addition, the bipartite entanglements can be measured by the cavity field quadratures, which can be measured directly by the output of the cavity, and the magnon state can be measured indirectly. By an auxiliary optical cavity which is considered to couple the YIG sphere and the quadratures of the mechanical mode can be obtained \[10\]. Here, we set $G > \kappa_m$ to ensure that unwanted magnon Kerr effect is ignored in a strong driving field of magnon mode \[11\].

4. Conclusions

In summary, we have investigated the enhancement of the bipartite entanglements in a CMM system via the squeezing of the magnon mode. The squeezing leads to the enhancement of nonlinearity in the system, which leads to the increase of entanglement, especially for the case of the low temperature. And the enhancement is not a simple linear enhancement, there is an optimal parameter range, which corresponds to the optimal enhancement effect on the entanglement. Since a novel squeezing of the magnon mode is considered, we mainly focus on entanglement related to magnon mode, such as the photon-magnon and magnon-phonon entanglements. In addition, the experimental implementation is also discussed. We expect that the proposed scheme provides provides an alternative method to enhance entanglement in the CMM systems and it has potential applications in the study of macroscopic quantum state and quantum information network.

DISCLOSURES

The authors declare no conflicts of interest.

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