Spin dynamics in a tunnel junction between ferromagnets

Jonas Fransson1,2,3 and Jian-Xin Zhu1
1 Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA
2 Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, NM 87545, USA
E-mail: jkfransson@gmail.com and jxzhu@lanl.gov

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Abstract. The dynamics of a single spin embedded in the tunnel junction (quantum point contact) between ferromagnets is addressed. Using the Keldysh technique, we derive a quantum Langevin equation. As a consequence of the spin-polarization in the leads, the spin displays a rich and unusual dynamics. Parallel configured and equally strong magnetic moments in the leads yield an ordinary spin precession with a Larmor frequency given by the effective magnetic field. Unequal and/or non-parallel configured magnetization, however, causes nutation of the spin in addition to the precession. Our predictions may be directly tested for macroscopic spin clusters.

The interest in a number of techniques that allow one to detect and manipulate a single spin in the solid state remains tremendous both experimentally [1]–[4] and theoretically [5]–[11]. Being a crucial element in spintronics and spin-based quantum information processing, such studies are also of fundamental importance. So far, most of the efforts [6], [8]–[11] have been focused on understanding the mechanism for the tunneling current modulation, which is the hallmark of a single spin detection using scanning tunneling microscopy (STM) [1]–[3]. Recently, the coupling between a single spin and supercurrent in Josephson junctions has also been studied [12, 13].

In particular, a single spin nutation induced by an ac supercurrent in a dc biased Josephson junction was shown for the first time in [12]. From the viewpoint of single spin manipulation, such a kind of nutation provides a significant implication for the control of spin dynamics electrically. The question is whether the spin dynamics (or even switching) can be realized
in other types (i.e. non-superconducting) of leads, which are easily accessible experimentally. Recently, we have examined this issue in normal conducting leads \[14\]. It was found that the spin–flip process of tunneling electrons is important to manipulate the spin and the flip-rate determines the efficiency of spin manipulation. Studies of local spin dynamics in quantum dots between ferromagnetic leads were recently reported \[15, 16\], however, these studies were concerned with the time-dependent effects and the noise of a local spin under the influence of stationary external fields.

In this paper, we study the spin dynamics of a single spin embedded in a tunneling junction between two ferromagnetic leads. A quantum Langevin equation is derived for the single spin dynamics. Through the resulting equation, we show that the tunneling between the ferromagnetic leads converts the electric field, e.g. bias voltage, into an effective magnetic field. The resulting effective magnetic field, however, depends on the relative orientation between the magnetization in the two leads. Parallel (i.e. ferromagnet (FM)-type) alignment of equally strong magnetization in the two leads yields a shift of the Larmor frequency, which scales as the squared ratio between the induced and the external magnetic fields. The FM-type alignment but with unequal magnetization or anti-parallel (anti-ferromagnet (AFM)-type) alignment in the two leads, on the other hand, leads to nutations in the precession of the spin about the external field.

The model system we consider consists of two ferromagnetic leads coupled to each other by a single spin \(S\). We assume that the tunnel junction is formed by a quantum point contact between the leads, so that the magnetic fields generated at the tips of the magnetic leads can be neglected. We also neglect the direct interaction of the spin with the two leads. Then, the system Hamiltonian can be written as

\[
\mathcal{H} = \mathcal{H}_L + \mathcal{H}_R + \mathcal{H}_S + \mathcal{H}_T.
\]

(1)

The first two terms \(\mathcal{H}_{L/R} = \sum_{k\sigma \in L/R} \epsilon_{k\sigma} c^\dagger_{k\sigma} c_{k\sigma}\) describe the electrons in the leads, where an electron is created (annihilated) in the left (right) lead at the energy \(\epsilon_{k\sigma}\) by \(c^\dagger_{k\sigma}\) (\(c_{k\sigma}\)). Henceforth, we assign subscripts \(p (q)\) to electrons in the left (right) lead. The Hamiltonian for a free spin \(S\) in the presence of a magnetic field \(B\) is given by

\[
\mathcal{H}_S = -g \mu_B B \cdot S,
\]

(2)

where \(g\) and \(\mu_B\) are the gyromagnetic ratio and Bohr magneton, respectively. The two leads are weakly coupled via the tunneling Hamiltonian

\[
\mathcal{H}_T = \sum_{pq\alpha\beta} \left( c^\dagger_{pq\alpha} [T_{pq} \delta_{\alpha\beta} + T_1 S \cdot \sigma_{\alpha\beta}] c_{pq\beta} + \text{H.c.} \right).
\]

(3)

Here, \(\sigma_{\alpha\beta}\) is the vector of Pauli spin matrices, with spin indices \(\alpha, \beta\), whereas \(T_{pq}\) is the direct tunneling rate between the spin-polarized leads, and \(T_1\) is the tunneling rate between the leads modulated by the local spin. In this model, we neglect spin–flip transitions in the direct tunneling between the leads. For convenience we take the respective amplitudes to be momentum independent (although it is not required). Typically, from the expansion of the work function for tunneling \(T_1/T_{pq} \sim J/U\) \[7\], where \(J\) is the exchange coupling strength between the transport electrons and the local spin and \(U\) is a spin-independent tunneling barrier. We further allow a weak external magnetic field \(B \sim 10^2–10^4\) Gauss, which is applied along the \(z\)-direction in the \(z–x\) plane perpendicular to the electron tunneling direction (\(y\)-axis).

When a time-dependent voltage bias is applied across the tunneling barrier, such that \(V(t) = V_{dc} + V_{ac} \cos(\omega_0 t)\), where \(V_{dc}\) and \(V_{ac}\) are the dc and ac components, and \(\omega_0\) is the
frequency of the ac field, a dipole will be formed around the barrier region through the accumulation or depletion of electron charge. This process results in the time dependence of single-particle energies: \( E_p = \epsilon_p + W_L(t) \) and \( E_q = \epsilon_q + W_R(t) \), with the constraint \( W_L(t) - W_R(t) = eV_{ac} \cos(\omega_0 t) \). However, the occupation of each state in the respective contact remains unchanged and is determined by the distribution established before the time dependence is turned on. Therefore, the chemical potentials on the left \( \mu_L \) and on the right lead \( \mu_R \) differs by the dc component of the applied voltage bias, \( \mu_L - \mu_R = eV_{dc} \). The tunneling junction with the spin then has two time scales: the Larmor precession frequency of the spin \( \omega_L = g\mu_B B \) and the characteristic frequency \( \omega_0 \) of the ac field.

The model is gauge transformed by

\[
\hat{U} = e^{-iJ_0^L[\mu_L+W_L(t')]N_L}e^{-iJ_0^R[\mu_R+W_R(t')]N_R},
\]

with \( N_{L/R} = \sum_{k\sigma} e^{\dagger}_{k\sigma} c_{k\sigma} \), and we thus obtain the model \( K = K_L + K_R + K_S + K_T \), where \( K_{L/R} = \sum_{k\sigma} \xi_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma}, \xi_{k\sigma} = \xi_{k\sigma} - \mu_{L/R}, K_S = \mathcal{H}_S \), and

\[
K_T = \sum_{pq\alpha\beta} \left( c_{p\alpha}^\dagger \hat{T}_{\alpha\beta} c_{q\beta} e^{i\phi(t)} + \text{H.c.} \right).
\]

Here, we have introduced the notation \( \hat{T}_{\alpha\beta}(t) = T_a \delta_{\alpha\beta} + T_1 S(t) \cdot \sigma_{\alpha\beta} \) and \( \phi(t) = e \int_{t_0}^t [V_{dc} + V_{ac} \cos \omega_0 t'] \, dt' \).

We now derive the effective action via the Keldysh technique [17]. If all external fields are the same on both forward and backward branches of the Keldysh contour (C), then the partition function \( Z = \text{tr} \, T_C \exp \left[ -i \int_C K_T(t) \, dt \right] = 1 \), where the trace runs over both electron and spin degrees of freedom. We take the partial trace in \( Z \) over the lead electrons (bath) to obtain an effective spin action. In the present situation, this action represents the interaction of the magnetic spin with a non-equilibrium environment. The tunneling contribution to the resulting spin action reads

\[
\text{i} \delta S = -\frac{1}{2} \int_C \oint_T \langle C_T K_T(S(t), t) K_T(S'(t'), t') \rangle \, dt \, dt',
\]

much in the spirit of Eckern et al [18].

For brevity we put \( A_{\alpha\beta} = \sum_{pq} c_{p\alpha}^\dagger c_{q\beta} \). The tunneling Hamiltonian of a voltage biased junction then reads

\[
K_T(S(t), t) = \sum_{\alpha\beta} \langle \hat{T}_{\alpha\beta}(t) A_{\alpha\beta} c^\dagger_{\alpha} c_{\beta} \rangle + \text{H.c.}.
\]

For magnetic leads, the correction to the effective action for the spin dynamics is thus given by

\[
\text{i} \delta S = -\text{i} \oint C \oint T \hat{T}_{\alpha\beta}(t) D_{\alpha\beta}(t, t') \hat{T}_{\beta\alpha}(t') \, dt \, dt',
\]

where \( D_{\alpha\beta}(t, t') = -\text{i} \langle C_T A_{\alpha\beta}(t) A_{\beta\alpha}^\dagger(t') \rangle \).

Performing standard Keldysh manipulations, defining upper and lower spin fields \( S^{u/l} \) residing on the forward/backward contours and reducing the time ordered integral over Keldysh contour to the integral over forward running time at the cost of making the Green functions (GFs) \( D_{\alpha\beta} \) a \( 2 \times 2 \) matrix, for each spin combination \( \alpha\beta \). We then perform a rotation to the classical and quantum components

\[
S' \equiv (S^u + S^l)/2, \quad S^q \equiv S^u - S^l, \quad S^c \cdot S^q = 0, \quad (8)
\]
which makes the matrix GF uniquely determined in terms of the retarded/advanced component \( D_{\sigma\alpha}^{(a)}(t, t') = \mp i \theta(\mp t - t') \{ A_{\alpha\beta}(t) A_{\beta\alpha}^\dagger(t') \} \), and the Keldysh component \( D_{\sigma\alpha}^{K}(t, t') = -i \{ A_{\alpha\beta}(t) A_{\beta\alpha}^\dagger(t') \} \). The procedure leads to \( \delta S = \delta S_e + \delta S_q \), where

\[
\delta S_e = \int \left[ S_e^z(t) K_{z}^{(1)}(t, t') + S_e^\sigma(t) K_{ij}^{(2)}(t, t') S_e^\sigma(t') \right] dt dt'
\]

and

\[
\delta S_q = \int S_q^\sigma(t) K_{ij}^{(3)}(t, t') S_q^\sigma(t') dt dt',
\]

where summation over repeated indices \( i, j = x, y, z \), is understood. Here, the kernels are given by

\[
K_{z}^{(1)}(t, t') = -T_i \sum_{\sigma} \sigma_{z\sigma} T_{\sigma} \left[ D_{\sigma\alpha}^{(a)}(t, t') e^{i[\phi(t) - \phi(t')]} + D_{\alpha\sigma}^{a}(t', t) e^{-i[\phi(t) - \phi(t')]} \right] \]

\[
= -2T_i \theta(t - t') \sum_{pq\sigma} \sigma_{z\sigma} T_{\sigma} \left[ f(\xi_{pq}) - f(\xi_{q\sigma}) \right] \times \sin \left[ (\xi_{pq} - \xi_{q\sigma})(t - t') + \phi(t) - \phi(t') \right],
\]

\[
K_{ij}^{(2)}(t, t') = -T_i \sum_{\alpha\beta} \sigma_{\alpha\beta} \left[ D_{\alpha\beta}^{(a)}(t, t') e^{i[\phi(t) - \phi(t')]} + D_{\beta\alpha}^{a}(t', t) e^{-i[\phi(t) - \phi(t')]} \right] \sigma_{ij} \]

\[
= iT_i^2 \theta(t - t') \sum_{pq\alpha\beta} \sigma_{\alpha\beta} \left[ f(\xi_{pq\alpha}) - f(\xi_{q\beta}) \right] e^{i(\xi_{pq} - \xi_{q\beta})(t - t') + i[\phi(t) - \phi(t')] - i[\phi(t) - \phi(t')] - i[\phi(t) - \phi(t')] \sigma_{ij} \]

\[
K_{ij}^{(3)}(t, t') = -T_i^2 \sum_{\alpha\beta} \sigma_{\alpha\beta} D_{\alpha\beta}^{K}(t, t') \sigma_{ij} \]

\[
= iT_i^2 \sum_{pq\alpha\beta} \sigma_{\alpha\beta} \left[ f(\xi_{pq\alpha}) + f(\xi_{q\beta}) - 2f(\xi_{pq\alpha}) f(\xi_{q\beta}) \right] e^{i(\xi_{pq} - \xi_{q\beta})(t - t') + i[\phi(t) - \phi(t')]}. 
\]

Here \( f(\xi) \) is the Fermi distribution function \( f(\xi) = 1/\left[ \exp(\xi/k_B T) + 1 \right] \). It follows that the only non-vanishing components of \( K_{ij}^{(2)} \) are \( K^{(2)}_{yy} = K^{(2)}_{xx}, K^{(2)}_{zz}, \) and \( K^{(2)}_{xy} = -K^{(2)}_{yx} \).

To properly describe the dynamics of the spin, we employ the path integral representation for the spin fields. So in addition to the terms \( -\oint \mathcal{H}_B(t) dt \), the action for a free spin also contains a Wess-Zumino–Witten-Novikov (WZWN), \( S_{WZWN} \), which describes the Berry phase accumulated by the spin as a result of motion of the spin on the sphere. We generalize this action for non-equilibrium dynamics within the Keldysh contour formalism, which can be expressed as [12]

\[
S_{WZWN} = \frac{1}{S} \int dt \mathbf{S}^q \cdot \left( \mathbf{S}^e \times \partial_t \mathbf{S}^e \right).
\]

The total effective spin action is given by:

\[
S_{\text{eff}} = S_{WZWN} + g \mu_B \int dt \mathbf{B} \cdot \mathbf{S}^q(t) + \delta S_e + \delta S_q.
\]
As seen from equations (11) to (13), the first three terms on the right-hand side of equation (15) are real, which determine the quasi-classical equation of motion, while \( \delta S_q \) is imaginary, which stands for the fluctuations of the spin field \( S^q \). This means that the quantum effects have indeed been included even in the semi-classical approximation. We perform the Hubbard–Stratonovich transformation with an auxiliary stochastic field \( \xi(t) \) to decouple the quadratic term in \( \delta S_q \). The total effective action is rewritten as:

\[
S_{\text{eff}} = S_{WZWN} + g\mu_B \int [B + \xi(t)] \cdot S^q(t) \, dt \\
+ \int \left[ S^q(t) K^{(1)}_1(t, t') + S^q_1(t) K^{(2)}_{ij}(t, t') S^q_j(t') \right] \, dt \, dt',
\]

where the fluctuating random magnetic fields satisfy the correlation functions

\[
\langle g\mu_B \rangle^2 \langle \xi_i(t) \xi_j(t') \rangle = -2iK^{(2)}_{ij}(t, t').
\]

Equation (16) constitutes the central formula for the following analysis. Notice that the kernel \( K^{(2)}_{ij} \), which connects the quantum and classical spin fields (see equation (16)), represents the effects of electrons degrees of freedom. It has the following approximate behavior in time:

\[
K^{(2)}_{ij}(t, t') \propto \cos[\mu(t - t')] / [(t - t') + i\eta]^2,
\]

where \( \eta \) is an infinitesimal and \( \mu \) is the energy scale at the order of voltage bias. A similar type of approximate behavior has also been observed in the case of a mechanical oscillator coupled to two Luttinger liquids [19]. Equation (18) suggests that the kernel \( K^{(2)}_{ij} \) is peaked at \( t - t' = 0 \) while oscillates at larger times. The characteristic time is set by \( 1/\mu \), which also measures the width of the peak around \( t - t' = 0 \). Because of the oscillating nature of \( K^{(2)}_{ij} \) at the large timescale, the corresponding time integral on the right-hand side of equation (16) is dominated from the time range of \( 1/\mu \) around \( t - t' = 0 \). When \( 1/\mu \ll 1/\omega_L \) (i.e. \( \mu \gg \omega_L \)), we are in the regime where the spin dynamic processes are much slower than those of the conduction electrons. Since the energy associated with the spin dynamics, \( h\omega_L \sim 1 \mu \text{eV} \), while the typical energy for the electronic degrees of freedom at the order of 1 meV is routine, the above regime is easily accessible. Under this condition, it is reasonable for us to use the approximation

\[
S^{q}(t') \approx S^{q}(t) + (t' - t) \, dS^{q}(t) / dt.
\]

The variational equations \( \delta S_{\text{eff}} / \delta S^q(t) = 0 \) then yield

\[
\frac{dn}{dt} = \alpha(t) \frac{dn}{dt} \times n + g\mu_B \times \left[ B_{\text{eff}}(t) + \xi(t) \right],
\]

where we have put \( S^q(t) = S\eta(t) \), whereas \( \alpha(t) = S \int \xi^{(2)}(t, t')(t - t') \, dt' \), with the 3 \times 3 matrix \( \xi^{(2)}(t, t') = [K^{(2)}_{ij}(t, t')]_{i,j=x,y,z} \), and \( B_{\text{eff}}(t) = B(t) + B_{\text{ind}}^{(1)}(t) + B_{\text{ind}}^{(2)}(t) \). We expect that the Langevin term \( \langle \xi(t) \rangle \) is suppressed at frequencies much lower than the exchange interaction field in the leads.

First we note that \( \alpha(t) \sim 2T^2 g \omega_L / D \) at zero temperature, where \( 2D \) is the bandwidth in the leads. Hence, for large \( D \), which is reasonable for metals, \( \alpha \) is negligibly small such that the first term to the right in equation (19) drops out. It is reasonable to believe that this would be true also for finite temperatures. This result is different from the case of a dc biased superconducting tunnel junction, where \( \alpha(t) \) is finite leading to spin nutation [12].
The magnetic leads induce the field \( \mathbf{B}^{(1)}_{\text{ind}} = \mathbf{b}^{(1)}_{\text{ind}} \) with

\[
g\mu_B \mathbf{B}^{(1)}_{\text{ind}}(t) = \int K^{(1)}_{x}(t, t') \, dt'
\]

\[
= -2\pi T_1 \sum_{\sigma} \sigma \sigma_{\sigma} T_{\sigma} N_{L\sigma} N_{R\sigma} \sum_{nm} J_n(\sqrt{V_{ac}/\omega_0}) J_m(\sqrt{V_{ac}/\omega_0})(\sqrt{V_{dc} + m\omega_0})
\times \sin \omega_0(n - m)t,
\]

(20)

where \( N_{L\sigma}/N_{R\sigma} \) is the density of states of the spin \( \sigma \) sub-band in the left/right lead, whereas \( J_n(x) \) is the \( n \)th Bessel function. This induced magnetic field contributes to the spin dynamics whenever, at least, one of the leads is spin-polarized.

The second-induced magnetic field, \( \mathbf{B}^{(2)}_{\text{ind}}(t) \) is defined such that \( \mathbf{b} \times g\mu_B \mathbf{B}^{(2)}_{\text{ind}}(t) = \mathbf{b} \times \int \mathbf{K}^{(2)}(t, t') \, dt' \), and we find that this field can be written as

\[
g\mu_B \mathbf{B}^{(2)}_{\text{ind}}(t) = S \int \left[ K^{(2)}_{xy}(t, t')(n_x \hat{x} - n_y \hat{y}) - [K^{(2)}_{xx}(t, t') - K^{(2)}_{zz}(t, t')]n_z \hat{z} \right] dt'.
\]

(21)

Here, the longitudinal component is proportional to

\[
\int [K^{(2)}_{xx}(t, t') - K^{(2)}_{zz}(t, t')] \, dt' = 2\pi T_1^2 (N_{L\uparrow} - N_{L\downarrow})(N_{R\uparrow} - N_{R\downarrow}) \sum_{nm} J_n(\sqrt{V_{ac}/\omega_0}) J_m(\sqrt{V_{ac}/\omega_0})
\times \sin \omega_0(n - m)t,
\]

(22)

which thus contributes to the spin dynamics whenever both leads are spin-polarized. The transverse component of the induced field, which is proportional to

\[
\int K^{(2)}_{xy}(t, t') \, dt' = -2\pi T_1^2 (N_{L\uparrow} N_{R\downarrow} - N_{L\downarrow} N_{R\uparrow}) \sum_{nm} J_n(\sqrt{V_{ac}/\omega_0}) J_m(\sqrt{V_{ac}/\omega_0})
\times \cos \omega_0(n - m)t,
\]

(23)

vanishes for magnetic leads whenever \( N_{L\uparrow} N_{R\downarrow} = N_{L\downarrow} N_{R\uparrow} \), that is, when the magnetizations of the leads are equal and in parallel configuration. Thus, under those conditions, the resulting motion of the spin will be influenced by the magnetizations of the leads. More interestingly though, whenever the magnetizations of the leads are non-equal and/or in a non-parallel configuration (including anti-parallel), we expect the spin dynamics to become significantly modified, since then the equation of motion (19), contains also a non-trivial \( z \)-component which is induced by the non-vanishing transverse component of the induced magnetic field.

The contributions expressed in equations (20)–(23) are caused by the spin imbalance or spin-polarization in the leads and describe the effect of the current flow on the local spin dynamics. The field \( \mathbf{B}^{(1)}_{\text{ind}} \) arises whenever there is a spin imbalance in, at least, one of the leads. By its presence the local spin exerts a precession about its local direction. The \( z \)-contribution of the field \( \mathbf{B}^{(2)}_{\text{ind}} \), given in equation (22), is finite only when both leads are magnetic, and its effect on the local spin is analogous to that of \( \mathbf{B}^{(1)}_{\text{ind}} \). The transverse contribution from \( \mathbf{B}^{(1)}_{\text{ind}} \), given in equation (23), which is of principal interest in this paper, requires that the magnetic moments of the leads are non-parallel and gives maximal effect when they are anti-ferromagnetically (AFM) aligned. Caused by the misaligned magnetic moments of the leads, the tunneling electron has to undergo spin–flip transitions in order to tunnel between the leads. The spin–flipping tunneling electrons tend to change the local magnetic field in the vicinity of the local spin, such that its local direction is slightly altered. In the case with AFM aligned leads, while the local spin tends
Figure 1. The different qualitative behavior of the spin $S^c$ in the presence of an applied ac voltage bias when the magnetizations of the leads are equal and FM-type (dashed), and AFM-type (solid) aligned. (a) The polar displacements of the spin dynamics. Here, we have defined spin-polarized density of states $N_{L/R} = N_0(1 + \eta_{\sigma} p_{L/R})/2$ and $\eta_{\uparrow/\downarrow} = \pm 1$, where $T_1 N_0 \sim 0.1$, $p_L = 0.9$, $p_R = p_L$ (FM-type) and $p_R = -p_L$ (AFM-type), and $\omega_0 = 3$, which gives the induced field $g \mu_B B_{ind} \sim 0.01 eV_0$. (b) The resulting generic spin motion on the unit sphere.

to line up with the magnetic moment of the source lead, an ac component in the bias voltage tends to cause wobbling of the spin, as is shown below.

The classical equation of motion should be consistent with the parameterization of the spin on the unit sphere, $\vec{S} \vec{n} = S(\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta)$. Letting the external magnetic field $\vec{B}$ be oriented along the $z$-axis, we find the equations

$$\begin{align*}
\frac{d\phi}{dt} &= -g \mu_B [B(t) + B^{(1)}_{ind}(t)] + S \int [K^{(2)}_{xy}(t, t') - K^{(2)}_{zz}(t, t')] \, dt' \cos \theta, \\
\frac{d\theta}{dt} &= S \int K^{(2)}_{xy}(t, t') \, dt' \sin \theta.
\end{align*}$$

(24)

Defining $A_{nm}(V_{dc}, V_{ac})$ such that $S \int K^{(2)}_{xy}(t, t') \, dt' = \sum_{nm} A_{nm}(V_{dc}, V_{ac}) \cos \omega_0 (n - m) t$, and letting the spin initially at time $t_0 = 0$ be oriented at an angle $\theta_0$ relative to $\vec{B}$ we explicitly find the polar coordinate

$$\theta(t) = 2 \arctan \left[ \tan \frac{\theta_0}{2} \prod_{nm} \exp \left\{ A_{nm} \sin \omega_0 (n - m) t \right\} \right].$$

Generically, when a single spin is subjected to a uniform magnetic field, the spin azimuthally precesses with the Larmor frequency $\omega_L$. For the tunneling junction between FM, when a dc voltage bias is applied the spin precesses around an effective magnetic field $\vec{B}_{eff} = \vec{B} + B^{(1)} + B^{(2)}$ with a shifted Larmor frequency $\tilde{\omega}_L$. When an ac voltage bias is applied, equation (24) shows that the spin also exhibits polar ($\theta$) modulations, whenever the
magnetizations of the leads are non-equal and/or not in parallel configuration. In particular, when the external magnetic field \( \mathbf{B}(t) = 0 \), the effective field in equation (19) becomes \( \mathbf{B}_{\text{eff}}(t) = \mathbf{B}^{(1)}_{\text{ind}}(t) + \mathbf{B}^{(2)}_{\text{ind}}(t) \). Hence, we expect that the dynamics of the spin in the absence of external magnetic fields is analogous to that in the presence of external magnetic fields. In particular, when the external magnetic field \( \mathbf{B}(t) = 0 \), the effective field in equation (19) becomes

\[
\mathbf{B}_{\text{eff}}(t) = \mathbf{B}^{(1)}_{\text{ind}}(t) + \mathbf{B}^{(2)}_{\text{ind}}(t)
\]

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\[
\mathbf{B}_{\text{eff}}(t) = \mathbf{B}^{(1)}_{\text{ind}}(t) + \mathbf{B}^{(2)}_{\text{ind}}(t)
\]

In figure 1(a), we show the dynamics of the \( z \)-component of the spin, \( S^z \), when the magnetizations of the leads are aligned in FM-type (dashed) and AFM-type (solid) configuration. In the plot, we only illustrate the first harmonic \( \omega_0 \), e.g. \( n - m = 1 \) and \( \sum_{n-m=1} A_{nm} \) constant, for a pure ac voltage, since the complete spin dynamics is expected to be very complicated. Clearly, the polar angle \( \theta \) exhibits a modulation with a dominant \( \omega_0 \) harmonic. The resulting dynamics of the spin can to much extent be compared with that of a rotating rigid top, as schematically illustrated in figure 1(b). Analogous to a classical spinning top, the spin wobbles along the polar direction in addition to the azimuthal rotations. The spin-polarized current leads to a full non-planar gyroscopic motion (nutations) of the spin much like that generated by applied torques on a mechanical top, i.e. magnetically-induced nutations. Similar dynamics is expected to occur even for \( S = \frac{1}{2} \) since in a spin coherent state the Schrödinger equation is essentially classical. The effect found in the present system is very similar to that of a single spin embedded in a Josephson junction [12], although the origin is of different nature. In addition, the proposed system is experimentally more accessible because such challenges as extremely low temperatures and device engineering of a Josephson junction can be avoided here.

In conclusion, we find non-planar motion of a spin embedded in a tunnel junction between FMs in the presence of an alternating current, whenever the FMs are unequally strong and/or in non-parallel alignment. The characteristic frequency of the polar angle variation equals that of the ac field, whereas the Larmor frequency of the spin precession scales as the squared ratio between the external and induced magnetic fields in a dc case. The induced magnetic field could be probed with superconducting quantum interference devices.

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