NUCLEAR SHADOWING IN INCLUSIVE AND TAGGED
DEUTERON STRUCTURE FUNCTIONS AND EXTRACTION OF
$F^D_2 - F^n_2$ AT SMALL $x$ FROM ELECTRON-DEUTERON COLLIDER
DATA

L. FRANKFURT
School of Physics and Astronomy, Tel Aviv University,
69978, Tel Aviv, Israel
frankfur@lev.tau.ac.il

V. GUZEY
Institut für Theoretische Physik II, Ruhr-Universität Bochum,
D-44780 Bochum, Germany
vadimg@tp2.rub.de

M. STRIKMAN
Department of Physics, the Pennsylvania State University,
State College, Pennsylvania 16802, USA
strikman@phys.psu.edu

Received (Day Month Year)
Revised (Day Month Year)

We review predictions of the theory of leading twist nuclear shadowing for inclusive unpolarized and polarized deuteron structure functions $F^P_D$, $g^P_D$ and $b^P_D$ and for the tagged deuteron structure function $F^D_2(x,Q^2,p)$. We analyze the possibility to extract the neutron structure function $F^n_2$ from electron-deuteron data and demonstrate that an account of leading twist nuclear shadowing leads to large corrections for the extraction of $F^n_2$ from the future deuteron collider data both in the inclusive and in the tagged structure function modes. We suggest several strategies to address the extraction of $F^n_2$ and to measure at the same time the effect of nuclear shadowing via the measurement of the distortion of the proton spectator spectrum in the semi-inclusive $eD \to e'NX$ process. We address the issue of the final state interactions in the $eD \to e'NX$ process and examine how they affect the extraction of $F^n_2$.

Keywords: Nuclear shadowing; Electron-Ion Collider; deuteron structure functions
PACS Nos.: 16.60.Hb, 24.85.+p, 25.30.Dh

1. Introduction
The nuclear shadowing phenomenon is one of key elements in the modern understanding of the geometry of hadron-nucleus and nucleus-nucleus collisions. A reliable theory of nuclear shadowing in hard processes is important for establishing the
universality of hard QCD processes.

The initial and rather successful theory of this phenomenon was based on the optical (Glauber) approximation. However, at high energies, the picture of consecutive collisions becomes inapplicable as a consequence of the increase of distances involved in high energy processes with increasing energies.

In addition, the eikonal (optical) approximation to inelastic processes violates energy-momentum conservation. In the late 60’s, V. Gribov developed an approach to nuclear shadowing, which accounts for the specifics of high energy processes. This approach leads to the expression for the total hadron-nucleus cross section which is close to the Glauber approximation, but with an additional effect – inelastic shadowing. For the deuteron target, nuclear shadowing is unambiguously related to the cross section of diffraction in hadron-nucleon collisions.

The investigation of deep inelastic processes off nuclei is another challenge. In the case of scattering off the deuteron, a generalization of Gribov’s ideas gave a possibility to evaluate nuclear shadowing in terms of diffractive parton densities of the nucleon.

One of applications of the theory of nuclear shadowing is the possibility to extract the unpolarized $F_n^2$ and polarized $g_1^n$ neutron structure functions from electron-deuteron data.

The measurement of the non-singlet difference of the proton and neutron structure functions, $F_p^2 - F_n^2$, by using the deuteron beam and spectator tagging, is one of the important components of the planned physics program of the electron-ion collider (EIC). The main goal of the measurement is to study the flavor dependence of parton distribution functions (PDFs) in a wide kinematic region, including the low-$x$ region. In particular, the collider kinematics will allow to probe the values of Bjorken $x$, $x \approx 5 \times 10^{-4}$ for $Q^2 \geq 1$ GeV, which are a factor of ten smaller than in the available fixed-target data. In addition, the measurement of $F_p^2 - F_n^2$ is a unique way to investigate nuclear shadowing of non-singlet (valence) PDFs in the non-vacuum channel. Since at small $x$ the $F_p^2 - F_n^2$ difference is compatible to the predicted nuclear shadowing correction to the deuteron structure function $F_D^2$ (a few percent effect), the correct extraction of $F_p^2 - F_n^2$ from deuteron data requires an account for nuclear shadowing. We demonstrate the usefulness of the investigation of the deuteron tagged structure functions for the extraction of the nonsinglet flavor distributions.

The accurate measurement of nuclear shadowing in nuclear PDFs and understanding of its origins are of key importance for establishing the geometry of heavy-ion physics. This impacts the present and forthcoming RHIC data and especially the future LHC experiments. Therefore, it is crucial to determine the value of the nuclear shadowing correction, as well as its uncertainties, both to the inclusive structure function $F_D^2$ and to the tagged structure function, when the spectator proton (a proton with momentum $\leq 0.1$ GeV/c in the deuteron rest frame) is detected ensuring the kinematics maximally close to the scattering off a free nucleon.

This paper is organized as follows. In Sec. we discuss our predictions for the...
nuclear shadowing correction to the unpolarized $F_2^D$ and polarized $g_1^D$ deuteron structure functions. The roles of nuclear shadowing and final state interactions in the tagged unpolarized deuteron structure function are discussed in Sec. 3. We summarize and discuss our results in Sec. 4.

2. Leading twist shadowing and inclusive deuteron structure functions

In part, this review is based on Ref. 15. For the comprehensive review of hard processes with the deuteron, we refer the reader to Ref. 16. The leading twist theory of nuclear shadowing was developed in Ref. 6 and later applied to deep inelastic scattering (DIS) on nuclear targets in Refs. 17-20. The theory of nuclear shadowing of nuclear PDFs and structure functions is based on Gribov’s connection between the nuclear shadowing correction to the total hadron-deuteron cross section and the cross section of diffraction off a free nucleon, Collins’s factorization theorem for hard diffraction in DIS and QCD analyses of HERA data on hard inclusive diffraction.

On the qualitative level, Gribov’s connection of nuclear shadowing to diffraction can be understood as a consequence of the interference between the amplitudes for diffractive scattering of the projectile off the proton and off the neutron of the deuterium target. Such interference is possible for small $x$, $x \leq 5 \times 10^{-2}$, when the minimum momentum transfer to the nucleon, $\sim xm_N$, is smaller than the average nucleon momentum in the deuteron.

In graphical form, the imaginary part of the forward $\gamma^*\text{-deuteron}$ scattering amplitude, which is proportional to the deuteron structure function $F_2^D$, is presented in Figs. 1 and 2. The dashed vertical lines denote unitary cuts placing the cut lines and vertices on mass-shell. Figure 1 corresponds to the impulse approximation, when the shadowing correction to $F_2^D$ is neglected. Figure 2 depicts the interference diagram, which gives rise to the shadowing correction to $F_2^D$. In this figure, the zigzag lines denote the diffractive scattering (diffractive exchange).

One can demonstrate that the imaginary part of the interference graph decreases $F_2^D$ (gives the negative shadowing correction) using the Reggeon calculus cutting rules of Abramovskiǐ, Gribov and Kancheli (AGK), which relate the shadowing effects in the total and partial cross sections see also Ref. 25. The top left graph in Fig. 2 corresponds to the diffractive final state and is proportional to $(1 + \eta^2)(\text{Im}A)^2$, where $A$ is the amplitude for the photon-nucleon diffractive scattering and $\eta = \text{Re}A/\text{Im}A$. The top right and bottom left graphs correspond to the inelastic interaction with one of the deuteron nucleons and give the cumulative contribution proportional to $-4(\text{Im}A)^2$. Finally, the bottom right graph corresponds to the simultaneous inelastic interactions with two nucleons and is proportional to $2(\text{Im}A)^2$. Therefore, the sum of all four graphs in Fig. 2 is proportional to $-(1 - \eta^2)(\text{Im}A)^2 = -(1 - \eta^2)/(1 + \eta^2)F_2^{D(4)}$, where we expressed the diffractive amplitude in terms of the nucleon diffractive structure function $F_2^{D(4)}$. Therefore,
the AGK cutting rules demonstrate that after summing over all final states, the nuclear shadowing term (the imaginary part of the interference diagram) is simply
proportional to the contribution of the diffractive final state (the top left graph in Fig. 2).

The deuteron unpolarized inclusive structure function $F_D^2$ reads

$$F_D^2(x, Q^2) = F_p^2(x, Q^2) + F_n^2(x, Q^2) - \frac{2}{1 + \eta^2} \int_0^x \frac{d x_F}{x} \frac{d q_t^2}{q_t^2} F_D^{(4)}(\beta, Q^2, x_F, t) \rho_D(4 q_t^2 + 4(x_F m_N)^2),$$

where the first line corresponds to the impulse approximation (Fig. 1) and the second line corresponds to the interference term (Fig. 2). In this equation, $F_D^{(4)}$ is the nucleon diffractive structure function; $x_F$ is the fractional loss of the proton longitudinal momentum; $\beta \approx x/x_F$; $x_0 = 0.1$; $q = q_t + (x_F m_N) \epsilon_z$ is the momentum transferred to the proton; $\rho_D$ is the deuteron charge form factor; $|t| = q_t^2 + (x_F m_N)^2$. The deuteron charge form factor can be written as an overlap of the initial and final state deuteron wave functions

$$\rho_D(4 q_t^2 + 4(x_F m_N)^2) = \int d^3 p \left[ u(p) u(p + q) + w(p) w(p + q) \left( \frac{3}{2} \frac{\vec{p} \cdot (\vec{p} + \vec{q})}{p^2(p + q)^2} - \frac{1}{2} \right) \right],$$

where $u$ is the $S$-wave component of the deuteron wave function; $w$ is the $D$-wave component. Note the argument of the deuteron form factor, which is a consequence of the correct treatment of the deuteron center of mass. Since the $t$-dependence of $\rho_D$ is rather moderate (compared to heavier nuclei), the integral in Eq. (1) is sensitive to $F_D^{(4)}(t)$ up to $-t \leq 0.05$ GeV$^2$. The ratio of the real to imaginary parts of the diffractive scattering amplitudes $\eta$ can be calculated using dispersion relations over the energy or using the Regge-pole type parameterization $s^{\alpha_{IP}}$ for the energy dependence of the diffraction cross section, even though the energy dependence differs from that for soft QCD processes,

$$\eta = -\frac{\pi}{2} \frac{\partial \ln(\sqrt{F_D^2})}{\partial \ln(1/x_F P_F)} \approx \frac{\pi}{2} (\alpha_{IP}(t = 0) - 1),$$

where $\alpha_{IP}(0)$ is the intercept of the effective "Pomeron" trajectory, which differs from that of the actual Pomeron, which dominates soft QCD phenomena. Using $\eta \approx 0.32$, one readily observes that the correction for the real part of the diffractive scattering amplitude reduces nuclear shadowing by almost 20%. Note that in the Reggeon calculus derivation, it was assumed that $\eta = 0$, which is natural for the amplitudes slowly increasing with energy. This is not the case for DIS and, hence, the effect of $\eta$ should be taken into account. One should note that the simple final expression for $F_D^2$ in Eq. (1) is due to the used closure relation for the final nuclear states.

The use of the QCD factorization theorem for hard diffraction allows to extend Eq. (1) for the structure function $F_D^2$ to the deuteron parton distribution
functions $f_{j/D}$

$$f_{j/D}(x, Q^2) = f_{j/p}(x, Q^2) + f_{j/n}(x, Q^2) - 2 \frac{1 - \eta^2}{1 + \eta^2} \int_x^{x_0} dx_\perp q_\perp^2 f_{j/N}(\beta, Q^2, x_\perp, t) \rho_D(4q_\perp^2 + 4(x_\perp - m_N)^2).$$

(4)

Note that we use $x_0 = 0.1$ for quarks and $x_0 = 0.03$ for gluons, see the discussion in Ref. 18.

The results of the calculation of the ratio of the next-to-leading order (NLO) structure functions $F_2^D/(F_2^p + F_2^n)$ and the ratio of the NLO gluon PDFs $g_D/(2g_N)$ are presented in Fig. 3. The solid curves correspond to $Q = 2$ GeV; the dashed curves correspond to $Q = 5$ GeV; the dash-dotted curves correspond to $Q = 10$ GeV. The two sets of curves for $g_D/(2g_N)$ correspond to the two scenarios of nuclear shadowing for gluons, see the discussion below and Ref. 19 for the detailed discussion.

Fig. 3. The ratios $F_2^D/(F_2^p + F_2^n)$ and $g_D/(2g_N)$ as functions of $x$. The solid curve corresponds to $Q = 2$ GeV; the dashed curve corresponds to $Q = 5$ GeV; the dash-dotted curve corresponds to $Q = 10$ GeV. Two sets of curves for $g_D/(2g_N)$ reflect the uncertainty in the slope of the $t$-dependence of the gluon diffractive PDF (see details in the text).
As an input for our calculation, we used the H1 fit to the nucleon diffractive PDFs $f_{Dj/N}^D$, where the gluon distribution was reduced by the 0.75 factor in accordance with the most recent H1 data on hard inclusive diffraction at HERA. Even with this reduction, the gluon nucleon diffractive PDF is much larger than the quark diffractive PDFs. As a consequence, see Eq. (4), we predict that shadowing in the gluon channel is larger than that in the quark channel and for $F_{D2}^D/(F_{p2}^D + F_{n2}^n)$.

The $t$-dependence of the quark diffractive PDFs $f_{Dj/N}^D$ was chosen to be exponential with the slope $B = 7.2 \pm 1.1 \text{ GeV}^{-2}$. Since the $t$-dependence of the gluon PDF has not been measured, the slope of the gluon PDF could be different. In order to take into account the uncertainty in the value of the slope $B$ of the gluon diffractive PDF, we use two values: $B_g = 4 + 0.2 \ln(10^{-3}/x)$ GeV$^{-2}$ and $B_g = 6 + 0.25 \ln(10^{-3}/x)$ GeV$^{-2}$. The first one corresponds to the lower end of the values of the $J/\psi$ photoproduction slope reported at HERA, while the second one is close to the quark slope $B$ and to the $J/\psi$ slope reported in Ref. 30.

The used deuteron wave function corresponds to the Paris nucleon-nucleon potential.

Note that the results in Fig. 3 correspond to the leading twist component of nuclear shadowing because they are based on the leading twist analysis of diffraction at HERA. When one decreases $Q^2$ below $Q^2 \geq 4 \text{ GeV}^2$, for instance down to $Q^2 \leq 1 \text{ GeV}^2$, one expects a significant enhancement of the nuclear shadowing effect due to the enhancement of diffraction at small $Q^2$ by higher twist effects such as vector meson production. This will increase nuclear shadowing by approximately a factor of two.

Since the diffractive structure function $F_{D2}^{D(4)}$ is known with accuracy of approximately 20%, the accuracy of the calculation of the nuclear shadowing correction to the deuteron structure function $F_{D2}^D$ is $20 \times 0.03 = 0.6\%$. Correspondingly, the theoretical uncertainty for the ratio of $F_{D2}^D/F_{p2}^D$ extracted from the deuteron data will be $2 \times 0.6 = 1.2\%$, which is likely to be smaller than possible experimental systematic errors.

For completeness, we also list predictions for the polarized deuteron structure function $g_D^D$. Unlike the unpolarized case considered above when the shadowing correction was important for the extraction of $F_{D2}^D/F_{p2}^D$ from deuteron data because $F_{D2}^D$ and $F_{n2}^n$ are very close at small $x$, the polarized $g_D^D \approx -g_n^n$ at small $x$, which makes the shadowing effect a very small correction in the extraction of $g_n^n$ from polarized deuteron data, see e.g. Refs. 32 and 33. In almost complete analogy with the unpolarized case, the deuteron structure function $g_D^D$ can be written as a sum of the impulse approximation and nuclear shadowing (interference) contributions

$$g_D^D(x, Q^2) = \left(1 - \frac{3}{2} P_D\right) \left(g_p^p(x, Q^2) + g_n^n(x, Q^2)\right)$$

$$-2\frac{1 - P_D^2}{1 + P_D^2} \int_0^{x_{IP}} dx_{IP} dq_{IP}^2 \Delta F_{D2}^{D(4)}(3, Q^2, x_{IP}, t) \rho_D^{\frac{1}{2}} \left(4q_{IP}^2 + 4(x_{IP} m_N)^2\right),$$

where $1 - 3/2 P_D$ is the effective polarization of the proton and neutron in the
deuteron, which differs from unity due to the deuteron D-wave contribution \((P_D = 0.06\) for the Paris nucleon-nucleon potential); \(\rho_D^{11}\) is the electric form factor of the deuteron polarized in the longitudinal direction, which contains the charge and quadrupole form factor contributions. \(\Delta F^{D(4)} = F^{D(4)}_{1\uparrow \uparrow} - F^{D(4)}_{1\downarrow \downarrow}\) is the difference of the diffractive polarized nucleon structure functions. The first arrow stands for the helicity of the photon; the second arrow indicates the helicity of the nucleon.

The \(\rho_D^{11}\) form factor has the following representation in terms of the deuteron \(S\) and \(D\)-wave components:

\[
\rho_D^{11} \left(4q_t^2 + 4(x_Fm_N)^2\right) = \int d^3p \left[u(p)u(p+q) + \frac{u(p)w(p+q)}{\sqrt{2}} \left( \frac{3(p_z+q_z)^2}{2(p+q)^2} - \frac{1}{2} \right) + \frac{w(p+q)w(p)}{\sqrt{2}} \left( \frac{3p_z^2}{2q^2} - \frac{1}{2} \right) + w(p)w(p+q) \left( \frac{9}{2} \frac{(\vec{p}\cdot\vec{q}+\vec{q}\cdot\vec{p})}{p^2(p+q)^2} + \frac{3p_z^2}{4p^2} + \frac{3}{4} \frac{(p_z+q_z)^2}{(p+q)^2} - 1 \right) \right]. \tag{6}
\]

Since \(\Delta F^{D(4)}\) is a new and unmeasured quantity (it can be measured in polarized diffractive DIS on the nucleon), we cannot directly use the leading twist theory of nuclear shadowing to estimate the shadowing correction to \(g_1^D\). However, making an assumption that the relative strength of diffraction mediated by the non-vacuum exchange (responsible for the polarized structure function \(g_1\)) is the same as that of the exchange with vacuum quantum numbers (responsible for the unpolarized \(F_2\) at small \(x\)), one obtains that

\[
\frac{\Delta F^{D(4)}}{g_1^N} = 2 \frac{F_2^{D(4)}}{F_2^N}, \tag{7}
\]

where \(g_1^N = (g_1^p + g_1^n)/2\) and \(F_2^N = (F_2^p + F_2^n)/2\). This assumption allows one to express the shadowing correction to the ratio of the deuteron and nucleon spin structure functions in terms of unpolarized diffraction on the nucleon

\[
\frac{g_1^D(x,Q^2)}{2(1 - \frac{1}{2}P_D)g_1^N} = 1 - 4 \frac{1 - \eta^2}{1 + \eta^2} \frac{1}{(1 - \frac{1}{2}P_D)F_2^N(x,Q^2)} \frac{1}{x} \int_x^{x_0} dx_F \frac{d\eta^2}{d\beta} F_2^{D(4)}(\beta,Q^2,x_F,t) \rho_D^{11} \left(4q_t^2 + 4(x_Fm_N)^2\right). \tag{8}
\]

The assumption of Eq. (7) corresponds to the maximal shadowing correction. The additional factor of two is a source of the generic combinatoric enhancement of nuclear shadowing in polarized structure functions of few-nucleon nuclei compared to the unpolarized case.

Figure 4 presents the results of the calculation using Eq. (8). The solid curve corresponds to \(Q = 2\) GeV, the dashed curve corresponds to \(Q = 5\) GeV and the dot-dashed curve corresponds to \(Q = 10\) GeV.

As can be seen from Fig. 4 the shadowing correction to \(g_1^D\) could be as large as \(8\%\) at \(x \approx 10^{-5}\). Needless to say that in order to achieve such low values of Bjorken \(x\) simultaneously with \(Q^2 \geq 1\) GeV\(^2\), one needs a collider with the polarized deuteron.
Nuclear shadowing and extraction of $F_2^p - F_2^n$ at small $x$ from electron-deuteron collider data

Fig. 4. The ratio $g_1^D / [(1 - 3/2 P_D) g_1^N]$ as a function of $x$. The solid curve corresponds to $Q = 2$ GeV; the dashed curve corresponds to $Q = 5$ GeV; the dash-dotted curve corresponds to $Q = 10$ GeV.

It was noticed in Ref. 16 that for a spin-one target (deuteron), the cross section of DIS depends on the deuteron polarization even with the unpolarized beam. The associated asymmetry

$$T_{20} = \frac{\sigma^+ - \sigma^0}{\frac{1}{2}(\sigma^+ + \sigma^0)},$$

where $\sigma^{+,-0}$ denotes the $\gamma^*$-deuteron cross section and the superscript denotes the deuteron helicity, was estimated for $x > 0.1$ in the impulse approximation. Next, it was pointed out in Ref. 37 that nuclear shadowing in unpolarized DIS on deuterium leads to the values of the $T_{20}$ asymmetry at the level of one percent at small-$x$.

This can be estimated as follows. The definition (9) allows one to immediately write the expression for $T_{20}$ by replacing the deuteron charge form factor in Eq. (11)
by $\rho_{D}^{20}$,

$$T_{20}(x, Q^2) = \frac{2}{F_{2}^{D}(x, Q^2)} \frac{1 - \eta^2}{1 + \eta^2} \int_{x_{0}}^{x_{0}} dx_{g} \, dq_{1}^{2} \, F_{2}^{D}(4) (\beta, Q^2, x_{g}, t) \rho_{D}^{20} (4q_{1}^{2} + 4(xg\cdot m_{N})^{2})$$

where

$$\rho_{D}^{20} (4q_{1}^{2} + 4(xg\cdot m_{N})^{2}) = \frac{3}{2} \int d^{3}p \left[ \frac{u(p)w(p + q)}{\sqrt{2}} \left( 1 - \frac{3(p_{z} + q_{z})^{2}}{(p + q)^{2}} \right) + \frac{u(p)w(p)}{\sqrt{2}} \left( 1 - \frac{3p_{z}^{2}}{2} + \frac{p_{z}^{2} + q_{z}^{2}}{p^{2}} \left( \vec{p} \cdot (\vec{p} + \vec{q}) - 3p_{z}(p_{z} + q_{z}) \right) \right) \right].$$

(11)

Note that $T_{20}$ vanishes, if one ignores the $D$-wave component of the deuteron wave function or the nuclear shadowing correction.

This effect can be also formulated in terms of the third deuteron structure function, $b_{1}^{D}$, which has the following probabilistic interpretation in terms of the quark distributions

$$b_{1}^{D}(x) = \frac{1}{2} \sum_{q} e_{q}^{2} \left[ q^{0}(x) + \bar{q}^{0}(x) - \frac{1}{2} \left( q^{1}(x) + \bar{q}^{1}(x) + q^{-1}(x) + \bar{q}^{-1}(x) \right) \right],$$

(12)

where $q^{\lambda}$ is the unpolarized quark distribution in the deuteron with helicity $\lambda$. The connection between $b_{1}^{D}$ and $T_{20}$ is readily obtained using their definitions,

$$b_{1}^{D}(x, Q^2) = \frac{F_{2}^{D}(x, Q^2)}{2x} T_{20}(x, Q^2).$$

(13)

The factor $1/(2x)$ in Eq. (13) indicates that the often discussed $b_{1}^{D}$ structure function is a rather inappropriate quantity: even small values of the physically measured $T_{20}$ asymmetry correspond to huge values of $b_{1}^{D}$.

The results of the calculation of the tensor asymmetry $T_{20}$ and the deuteron structure function $b_{1}^{D}$ are presented in Fig. 5. The solid curve corresponds to $Q = 2$ GeV; the overlapping dashed and dash-dotted curves correspond to $Q = 5$ GeV and $Q = 10$ GeV.

As one can see from the left panel of Fig. 5, the obtained $T_{20}$ is at the level of 0.6-0.7%. This agrees with the the analyses of Refs. 32 and 37. At the same time, $b_{1}^{D}$ is large at small $x$, as can be seen from the right panel of Fig. 5. This is a purely kinematic effect due to the $1/(2x)$ factor in the definition of $b_{1}^{D}$ (13). The observation of surprisingly large $b_{1}^{D}$ at small $x$ was first presented in Refs. 32, 33 and 39.

The HERMES measurement (11) of $b_{1}^{D}$ indicates a (rapid) growth of $b_{1}^{D}$ when one decreases Bjorken $x$ from $x \approx 0.5$ down to $x \approx 10^{-2}$. However, the corresponding values of $Q^2$ are of the order of 1 GeV$^2$ and the values of Bjorken $x$ are not small enough to see the predicted dramatic rise of $b_{1}^{D}$ towards small $x$. Once again, the study of the behavior of $b_{1}^{D}$ at small $x$ will greatly benefit from the collider kinematics.
Fig. 5. The tensor asymmetry $T_{20}$ and the $b_{1D}^D(x, Q^2)$ structure function as functions of $x$. The solid curve corresponds to $Q = 2$ GeV; the overlapping dashed and dash-dotted curves correspond to $Q = 5$ GeV and $Q = 10$ GeV.

3. Nuclear shadowing and final state interactions in the tagged deuteron structure function

A strategy, which is complimentary to the inclusive measurement of $F_2^D$, is the use of the neutron and proton tagging. The scattering on the neutron of deuterium is then tagged by detecting a slow (spectator) proton. The usefulness of the tagged deuteron structure function for the extraction of the neutron $F_2^n$ at large $x$ was discussed in Ref. 41. In this work, we concentrate on the small-$x$ region of nuclear shadowing. We extend the analysis [15] by taking into account the final state interactions (FSI) between the final nucleons.

In the impulse approximation, the tagged deuteron structure function is given by the imaginary part of the left graph in Fig. 5.

The corresponding expression is

$$F_2^D(x, Q^2, \vec{p})|_{IA} = \left( 1 + \frac{p_z}{m_N} \right)^{\alpha_{FS}(0)} F_2^n(x, Q^2) \rho_D(p, p),$$

where $\vec{p}$ is the momentum of the spectator proton; $\alpha_{FS}(0)$ is the intercept of the ef-
Fig. 6. The impulse approximation and the final state correction to the tagged deuteron structure function $F_2^D$.

The effective “Pomeron” trajectory; $\rho_D(p,p) = u^2(p) + w^2(p)$, where $u(p)$ and $w(p)$ are the $S$-wave and the $D$-wave components of the deuteron wave function, is the unpolarized deuteron density matrix \cite{37} (with equal momenta). The factor $(1 + p_z/m_N)^{\alpha_{p}^{(0)}}$ comes from different invariant energies of the virtual photon-nucleus and the virtual photon-neutron interactions. Thus, the $(1 + p_z/m_N)^{\alpha_{p}^{(0)}}$ factor is the flux factor of the interacting neutron. Note that in the derivation of the $(1 + p_z/m_N)^{\alpha_{p}^{(0)}}$ factor we neglected $O(p^2/m_N^2)$ and higher corrections, which are ignored in our non-relativistic treatment of the deuteron wave function.

The impulse approximation receives corrections due to the final state interactions (the middle graphs in Fig. 6) and nuclear shadowing, which is also accompanied by the final state interactions (the right graph in Fig. 6). The shaded ovals in Fig. 6 denote the final state interaction. Since the FSI between two nucleons is largest at the small relative momentum, the FSI is accompanied by the diffractive scattering providing this condition.

In the case of the tagged structure function at small spectator nucleon momenta, $p \leq \sqrt{\epsilon m_N}$, where $\epsilon$ is the deuteron binding energy, it is legitimate to keep only the single and double scattering terms shown in Fig. 6. At larger spectator momenta, the additional contributions of triple and quadruple interactions with the target (not shown in Fig. 6) are not suppressed by the $p/\sqrt{\epsilon m_N}$ parameter and, hence, should be included. This will introduce a certain model dependence since those terms are not simply related to the elementary diffractive cross section. One should emphasize that this is only the case for the tagged structure function (differential cross section); the triple and quadruple interaction terms cancel in the inclusive structure function (total cross section), which is unambiguously expressed in terms of the nucleon diffractive structure function, see Eq. (1).
The complete expression for the tagged deuteron structure function reads

\[
F_2^D(x, Q^2, \vec{p}) = \left(1 + \frac{p_z}{m_N}\right)^{\alpha_{FSI}(0)} F_2^D(x, Q^2) \rho_D(p, p) \\
+ \int_{x_0}^{x_0} dx_{\vec{p}}' dq_{p} F_2^{D(4)}(\beta, Q^2, x_{\vec{p}}', t) \left[2Re \int d^3p' \left(F(p, p') \rho_D(p, p') \rho_D(p + q, p') \right)
+ \int d^3p' \left(d^3p'' F(p', p'') \rho_D(p', p'') \psi_{NN}^{FSI}(p - \frac{q}{2}; p' - \frac{q}{2}) \right) \psi_{NN}^{FSI}(p - \frac{q}{2}; p' - \frac{q}{2}) \right]
+ \int d^3p' \left(d^3p'' F(p', p'') \rho_D(p', p'') \psi_{NN}^{FSI}(p - \frac{q}{2}; p' - \frac{q}{2}) \right) \psi_{NN}^{FSI}(p - \frac{q}{2}; p' - \frac{q}{2}) \right),
\]

where \(\psi_{NN}^{FSI}\) is the continuum non-relativistic nucleon-nucleon wave function, which vanishes in the absence of the FSI. The arguments of \(\psi_{NN}^{FSI}\) denote the final and initial relative momenta of the involved proton and neutron. The \(\rho_D(p, p')\) denotes the deuteron unpolarized density matrix

\[
\rho_D(p, p') = u(p)u(p') + w(p)w(p') \left(\frac{3(p^2 + p'^2)}{2} - \frac{1}{2}\right).
\]

The factor \(F(p, p')\) is the generalization of the nucleon flux factor discussed above

\[
F(p, p') = \sqrt{\left(1 + \frac{p_z}{m_N}\right)^{\alpha_{FSI}(0)}} \left(1 + \frac{p_z}{m_N}\right)^{\alpha_{FSI}(0)}.
\]

The presence of the flux factor is typical for semi-inclusive cross sections. In total cross sections, the flux factor effects cancel in the impulse approximation and are of the order \(O(p^2/m_N^2)\) in the interference term. Therefore, they do not appear in Eqs. (1), (5) and (12).

By setting \(\psi_{NN}^{FSI} = 0\) in Eq. (15), we obtain the result of of Ref. 15. The second and third lines in Eq. (15) correspond to the FSI correction to the impulse approximation (the middle graph of Fig. 6), the rest of Eq. (15) corresponds to the shadowing correction, which also includes the FSI.

In our numerical analysis of Eq. (15), we make the following justified approximations. First, since the dominant diffractive exchange at small \(x\) has the vacuum quantum numbers, the isospin of the interacting proton-neutron pair is the same as in the deuteron, i.e. it is zero. Note that the approximation of the exchange with the vacuum quantum numbers is only justified for small \(x, x < 10^{-2}\). At larger \(x\), one should also take into account transitions with the isospin change, which makes an estimate of the final state interactions more involved. Second, an examination of the isospin-zero proton-neutron phase shifts reveals that, in the appropriate
kinematics (small kinetic energy in the laboratory frame), the only essential partial wave is the $^3S_1$-wave, i.e. the partial wave with $L = 0$ and $S = 1$. Therefore, this is the only partial wave included in our analysis of the FSI. This allows us to suppress all isospin and spin indices in Eq. (15).

The $L = 0$ relative orbital momentum allows for a simple expression of the nucleon-nucleon continuum wave function in the momentum space

$$
\psi_{NN}(k_1, k_2) = \delta(\vec{k}_1 - \vec{k}_2) + \psi_{FSI}^{NN}(k_1, k_2) = \delta(\vec{k}_1 - \vec{k}_2) + \left(\frac{e^{2i\delta_{01}} - 1}{4\pi^2|k_1|}\right),
$$

where $\delta_{01}$ is the phase shift of the $^3S_1$-wave partial wave. The wave function of Eq. (18) has the usual normalization of continuum wave functions in non-relativistic quantum mechanics

$$
\int d^3k_2 \psi_{NN}^*(k'_1, k_2) \psi_{NN}(k_1, k_2) = \delta(\vec{k}_1 - \vec{k}_1') - \frac{\epsilon}{|k_1|},
$$

(19)

Since $\delta_{01}$ is close to $90^0$ in the considered kinematics, $\psi_{NN}^{FSI}$ reduces to the following simple expression, which we used in our numerical analysis

$$
\psi_{NN}^{FSI}(k_1, k_2) \approx \frac{\cos(2\delta_{01}) - 1}{4\pi|k_1|} \delta(|k_1|^2 - |k_2|^2).
$$

(20)

Note that the continuum wave function of Eqs. (18) and (20) with $\delta_{01} \approx 90^0$ agrees with the classic Bethe-Piers result for the low relative momentum $k$ proton-neutron continuum wave function in the coordinate space

$$
\psi_{NN}^{FSI}(r) \propto \frac{\sin(kr + \delta_0)}{r},
$$

(21)

where $\cot \delta_0 = -\sqrt{\epsilon}/|k|$, $\epsilon = 2.15$ MeV being the deuteron binding energy. Assuming that the relevant momentum $k$ equals the root-mean-square momentum of the deuteron wave function, $k \approx 130$ MeV for the used Paris nucleon-nucleon potential, we find that the phase shift $\delta_0 \approx 110^0$, which is in agreement with the value used our analysis.

In our analysis, we used the parameterization of $\delta_{01}$ provided by the SAID program. Other applications of the non-relativistic nucleon-nucleon wave function to estimates of the FSI can be found in Refs. 47 and 48.

The coefficient in front of the nuclear shadowing correction term, $-2(1-\eta^2)/(1+\eta^2)$, is dictated by the AGK cutting rules. Indeed, in the considered case of the proton tagging, the shadowing correction is given by the sum of the two top graphs in Fig. 2 which enter with the coefficients 2 and $-4/(1 + \eta^2)$, respectively.

Equation (15) describes the modification of the spectrum of the produced protons by the FSI and nuclear shadowing. We characterize the modification by the ratio $R$ defined as

$$
R(x, Q^2, \bar{p}) = \frac{F_2^D(x, Q^2, \bar{p})}{F_2^D(x, Q^2, \bar{p}) \mid_{IA}}.
$$

(22)
The results of the numerical evaluation of the ratio $R$ at fixed $Q^2 = 4 \text{ GeV}^2$ and fixed $p_t$ and $p_z = 0$ as a function of $x$ are presented in Fig. 7. The top panel corresponds to the calculation without the FSI ($\psi_{NN} = 0$); the lower panel is the result of the full calculation. The solid curves correspond to $p_t = 0$; the short dashed curves correspond to $p_t = 100 \text{ MeV}$; the dash-dotted curves correspond to $p_t = 200 \text{ MeV}$.

As one can see from the top panel of Fig. 7, nuclear shadowing (without the FSI effects) suppresses the spectrum of the produced protons. The suppression is larger for larger $p_t$ and becomes as large as 20-30% at $p_t = 200 \text{ MeV}$ and $x = 10^{-5} - 10^{-4}$. This is much larger than the shadowing correction to the inclusive deuteron structure function $F_2^D$. This enhancement of nuclear shadowing is a common feature of semi-exclusive reactions with nuclei. Indeed, at large $p_t$, while the impulse ap-
proximation term is suppressed by the nuclear wave function, the rescattering term becomes increasingly prominent.

As can be seen from the bottom panel of Fig. 7, the large effect of nuclear shadowing on $R$ is mostly compensated by the final state interactions. A numerical analysis of Eq. (15) shows that the main contribution to the FSI effect comes from the terms proportional to $\psi_{FSI}^{NN}$ and that the terms containing $(\psi_{FSI}^{NN})^2$ can be neglected. The net result is the interplay between the large and negative contributions of $\psi_{FSI}^{NN}$ to $R$, which nearly cancel each other because of the opposite signs of the FSI correction to the impulse approximation and the nuclear shadowing term, and the shadowing term without the FSI.

From the experimental point of view, two strategies of the extraction of the neutron $F_2^n$ from the deuteron data by using the proton tagging will be possible. One would be to select only very low $p_t$ protons with a gross loss of statistics. According to our analysis, the distortion of the proton spectrum due to the final state interactions and FSI effects will be minimal. The other, more promising approach is to measure the $p_t$ dependence of the spectrum up to $p_t \sim 200$ MeV/$c$, which will allow to use most of the spectator protons. Provided good momentum resolution of the proton spectrometer, one would be able to make (longitudinal) momentum cuts to suppress/increase the shadowing effect and, thus, one would have an opportunity to independently study the enhanced nuclear shadowing. The tagged method will allow to extract $F_2^n$ from deuteron data with the accuracy at the level of a fraction of percent.

One can also use simultaneous tagging of protons and neutrons, when both neutron and proton are detected in the reactions $\gamma^*D \rightarrow nX$ and $\gamma^*D \rightarrow pX$. In this case, the effects of nuclear shadowing and FSI will cancel in the ratio $\sigma_{\gamma^*D\rightarrow nX}/\sigma_{\gamma^*D\rightarrow pX}$ and the main errors in the measurement of $F_2^n$ will be due to the determination of relative efficiencies of the proton and neutron taggers.

One could also try to obtain the ratio $F_2^n/F_2^p$ from the comparison of the rate of the tagged proton scattering events with the neutron spectator to inclusive $eD$ scattering. Such a strategy could also have certain merits as it avoids the issue of luminosity and does not require a leading proton spectrometer. The disadvantage of this strategy is the sensitivity to the nuclear shadowing and FSI effects and errors in the acceptance of the neutron detector. One possible way to deal with the latter problem will be to perform measurements at very small $x$ and large energies, where the $ep$ and $en$ cross sections are equal to better than a fraction of 1% and, hence, one would be able to cross-check the acceptances of the proton and neutron detectors.

Note also that taking proton data from an independent run will potentially lead to another set of issues such as relative luminosity, the use of different beam energies, etc., which is likely to be at the level of 1%.
4. Conclusions and discussion

In this brief review, we presented predictions of the theory of leading twist nuclear shadowing for inclusive unpolarized and polarized deuteron structure functions $F_2^D$ and $g_1^D$ and for the tensor polarization asymmetry $T_{20}$. The combined role of nuclear shadowing and final state interactions was analyzed for the tagged deuteron structure function $F_2^D(x, Q^2, \vec{p})$. The discussed effects are relevant for the collider kinematics of the future Electron-Ion Collider, $x < 10^{-2}$ and $Q^2 \geq 4 \text{ GeV}^2$.

The measurement of $F_2^D$ is relevant for the extraction of the neutron $F_2^n$ structure function from the deuteron data at small $x$. Since at small $x$, the proton and neutron $F_2$ structure functions are close, even a small correction to $F_2^D$ leads to large corrections to the extraction of the $F_2^p - F_2^n$ difference for the deuteron data. We make predictions for the shadowing correction to $F_2^D$, which enable one to extract $F_2^p/F_2^n$ from the deuteron data with an approximately 1% theoretical uncertainty.

At moderately small $x$, nuclear shadowing of structure functions of light nuclei is dominated by the double rescattering. Thus, the theoretical analysis of $F_2^A/F_2^D$, where $F_2^A$ denotes the structure function of a nucleus heavier than deuterium, presents another option for the extraction of the nuclear shadowing correction to $F_2^D$. This imposes additional constraints on the shadowing correction to $F_2^D$.

For completeness, we also consider polarized deuteron structure functions. While the shadowing correction to $g_1^D$ is approximately twice as large as the shadowing correction to $F_2^D$, its effect is totally negligible in the extraction of the neutron $g_1^n$ from the deuteron data since $g_1^n \approx -g_1^p$ at small $x$ and since the experimental errors in the measurement of $g_1^D$ are rather large. We find that the $T_{20}$ asymmetry at small-$x$ is at the one percent level so that one could try to observe it experimentally.

The strategy complimentary to the inclusive measurement of $F_2^D$ is the measurement of the proton tagging. We analyzed the deuteron tagged structure function and showed that the combined effect of nuclear shadowing and the final state interactions only insignificantly distorts the spectrum of final protons for $p_t \leq 200 \text{ MeV}$, which enables for a reliable extraction of $F_2^n$. A combined analysis of inclusive and semi-inclusive scattering off the deuteron coupled with a high resolution proton spectrometer will allow for the measurement of $F_2^n$ at small $x$ with the theoretical uncertainty at the level of 1%. It would be a challenge to reduce the experimental systematic errors to a comparable level. The measurement of the shape of the spectator spectrum would allow to determine nuclear shadowing in deuterium with precision by far exceeding that possible in the inclusive measurements.

This work was supported by the German-Israel Foundation (GIF), Sofia Kovalevskaya Program of the the Alexander von Humboldt Foundation (Germany) and the Department of Energy (USA).

References
1. R. J. Glauber, Phys. Rev. 100 (1955) 242.
2. E.L. Feinberg and I.Ia. Pomerancuk, Suppl. Nuovo Cimento III, 652 (1956).
3. V. N. Gribov, B. L. Ioffe and I. Y. Pomeranchuk, Sov. J. Nucl. Phys. 2 (1966) 549 [Yad. Fiz. 2 (1965) 768].
4. B. L. Ioffe, Phys. Lett. B 30 (1969) 123.
5. V. N. Gribov, Sov. Phys. JETP 29, 483 (1969) [Zh. Eksp. Teor. Fiz. 56, 892 (1969)].
6. L. Frankfurt and M. Strikman, Eur. Phys. J. A 5, 293 (1999) arXiv:hep-ph/9812322.
7. R. Holt et al., The Electron Ion Collider, BNL-68933, 2002.
8. M. Arneodo et al., Nucl. Phys. B 483, 3 (1997).
9. B. Badelek and J. Kwiecinski, Nucl. Phys. B 370, 278 (1992).
10. N. N. Nikolaev and B. G. Zakharov, Phys. Lett. B 260 (1991) 414.
11. W. Melnitchouk and A. W. Thomas, Phys. Rev. D 47, 3783 (1993) arXiv:nucl-th/9301016.
12. W. Melnitchouk and A. W. Thomas, Phys. Lett. B 317, 437 (1993) arXiv:nucl-th/9310005.
13. W. Melnitchouk and A. W. Thomas, Phys. Rev. C 52, 3373 (1995) arXiv:hep-ph/9508311.
14. G. Piller, W. Ratzka and W. Weise, Z. Phys. A 352, 427 (1995) arXiv:hep-ph/9504017.
15. L. Frankfurt, V. Guzey and M. Strikman, Phys. Rev. Lett., 91, 202001 (2003).
16. L. L. Frankfurt and M. I. Strikman, Phys. Rept. 76 (1981) 215.
17. L. Frankfurt, V. Guzey and M. Strikman, J. Phys. G, 27, R23 (2001) arXiv:hep-ph/0010248.
18. L. Frankfurt, V. Guzey, M. McDermott and M. Strikman, JHEP 0202, 027 (2002) arXiv:hep-ph/0201230.
19. L. Frankfurt, V. Guzey and M. Strikman, Phys. Rev. D 71, 054001 (2005) arXiv:hep-ph/0503022.
20. L. Frankfurt, V. Guzey and M. Strikman, Phys. Lett. B 586, 41 (2004) arXiv:hep-ph/0308189.
21. J. C. Collins, Phys. Rev. D 57, 3051 (1998) [Erratum-ibid. D 61, 019902 (2000)] arXiv:hep-ph/9709499.
22. ZEUS Collab., J. Breitweg et al., Eur. Phys. J. C 6, 43 (1999) arXiv:hep-ex/9807010.
23. H1 Collab., C. Adloff et al., Z. Phys. C 76, 613 (1997) arXiv:hep-ex/9708016.
24. V. A. Abramovskii, V. N. Gribov and O. V. Kancheli, Sov. J. Nucl. Phys. 18, 308 (1974) [Yad. Fiz. 18, 595 (1973)].
25. L. L. Frankfurt and M.I. Strikman, Phys. Lett. B 382, 6 (1996).
26. V. N. Gribov and A. A. Migdal, Sov. J. Nucl. Phys. 8, 583 (1969) [Yad. Fiz. 8, 1002 (1968)].
27. H1 Collab., C. Adloff et al., in Proceedings of ICHEP02, Amsterdam, 2002 (Elsevier Science, New York, 2003).
28. ZEUS Collab., J. Breitweg et al., Eur. Phys. J. C 1, 81 (1998).
29. H1 Collab., C. Adloff, Phys. Lett. B 483, 23 (2000).
30. ZEUS Collab., A. Bruni, in Proceedings of DIS 2000, Liverpool, 2000, edited by J.A. Gracey and T. Greenshaw (World Scientific, Singapore, 2000).
31. M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Cote, P. Pires and R. De Tourreil, Phys. Rev. C 21, 861 (1980).
32. J. Edelmann, G. Piller and W. Weise, Phys. Rev. C 57, 3392 (1997) arXiv:hep-ph/9709455.
33. G. Piller and W. Weise, Phys. Rept. 330 (2000) 1 arXiv:hep-ph/9908230.
34. L. L. Frankfurt, W. R. Greenberg, G. A. Miller, M. M. Sargsian and M. I. Strikman, Z. Phys. A 352, 97 (1995) arXiv:nucl-th/9501009.
35. L. Frankfurt, V. Guzey and M. Strikman, Phys. Lett. B 381, 379 (1996).
Nuclear shadowing and extraction of $F_2^p - F_2^n$ at small $x$ from electron-deuteron collider data

36. E. S. Ageev et al. [COMPASS Collaboration], Phys. Lett. B 612, 154 (2005) [arXiv:hep-ex/0501073].
37. M. Strikman, Proc. Workshop "Internal Spin Structure of the Nucleon": Semi-Inclusive Hadron Production in Polarized Lepton-Hadron Scattering, January 1994, pp. 153-166, World Scientific 1995, eds. V. H. Hughes and L. Cavata.
38. P. Hoodbhoy, R. L. Jaffe and A. Manohar, Nucl. Phys. B 312 (1989) 571.
39. W. Schafer, arXiv:hep-ph/9806297.
40. A. Airapetian et al. [HERMES Collaboration], arXiv:hep-ex/0506018.
41. W. Melnitchouk, M. Sargsian and M. I. Strikman, Z. Phys. A 359, 99 (1997) [arXiv:nucl-th/9609048].
42. L. Bertocchi and D. Treleani, J. Phys. G 3, 147 (1977).
43. R. A. Arndt, I. I. Strakovsky and R. L. Workman, Phys. Rev. C 62, 034005 (2000) [arXiv:nucl-th/0004039].
44. R. Machleidt, Phys. Rev. C 63, 024001 (2001) [arXiv:nucl-th/0006014].
45. H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. 8 (1936) 82.
46. SAID program [http://gwdac.phys.gwu.edu).
47. C. Ciofi degli Atti, L. P. Kaptari and D. Treleani, Phys. Rev. C 63 (2001) 044601 [arXiv:nucl-th/0005027].
48. M. M. Sargsian, T. V. Abrahamyan, M. I. Strikman and L. L. Frankfurt, Phys. Rev. C 71, 044614 and 044615 (2005) [arXiv:nucl-th/0406020] and 0501018].