Light Charged Higgs Bosons in Supersymmetric Models

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Abstract

We point out that present experimental limits from searches for neutral Higgs bosons at LEP already imply stringent lower bounds on the mass of the charged Higgs boson in the Minimal Supersymmetric Standard Model (MSSM); these bounds are especially severe for low values of $\tan \beta$ ($\tan \beta \leq 3$), where the $H^+ t b$ coupling is large. However, these indirect constraints are much weaker in simple extensions of the MSSM Higgs sector involving the introduction of an extra $U(1)$ gauge group or an extra $SU(2) \times U(1)_Y$ Higgs singlet field; in the latter case charged Higgs bosons can even be light enough to be pair produced at LEP.
The Higgs mechanism offers the theoretically best understood description of electroweak symmetry breaking, which is required to generate masses for $W$ and $Z$ gauge bosons as well as matter fermions (quarks and leptons). The minimal Standard Model (SM) predicts the existence of only a single physical Higgs boson, a neutral CP–even particle. However, in many extensions of the SM the Higgs sector is more complicated. In particular, in models with two or more Higgs doublets the spectrum of physical Higgs fields contains some charged Higgs bosons.

The currently best motivated extension of the SM involves the introduction of softly broken supersymmetry, which stabilizes the gauge hierarchy against radiative corrections \[1\]. Besides predicting the existence of superpartners of all known particles, realistic supersymmetric theories also contain \[2\] at least two Higgs doublet superfields, to allow for anomaly cancellation in the higgsino sector and to give masses to both hypercharge $Y = +1/2$ and $Y = −1/2$ matter fermions. This second Higgs doublet superfield also plays a crucial role in the unification of all known gauge interactions, which is natural in supersymmetric extensions of the SM, but leads to conflict with LEP data in the SM itself \[3\]. Finally, loops involving superpartners allow to cancel \[4\] potentially large, positive contributions from $t \to H^\pm$ loops to the partial width for radiative $b \to s\gamma$ decays, thereby avoiding stringent lower bounds \[5\] on the mass of charged Higgs bosons in non–supersymmetric models. Supersymmetry therefore now appears to be an (almost) necessary condition for the existence of charged Higgs bosons that are light enough to be produced in the decay of top quarks.

On the other hand, unsuccessful searches for neutral Higgs bosons at LEP have now reached a sensitivity that begins to impose nontrivial constraints on charged Higgs bosons in many supersymmetric models. The reason is that at least the simplest, and hence most attractive, supersymmetric models contain fewer free parameters in the Higgs sector than a general (non–supersymmetric) model with two Higgs doublets does. The purpose of this note is to explore these *indirect* lower bounds on the mass of the charged Higgs boson quantitatively, within the framework of three different supersymmetric models.

The first of these models is the minimal supersymmetric standard model (MSSM), which is a straightforward supersymmetrization of the SM. In particular, the gauge group is $SU(3) \times SU(2) \times U(1)_Y$, and the Higgs sector \[6\] consists of only two Higgs doublet superfields. The physical spectrum of Higgs bosons therefore contains five fields: Two neutral CP–even fields $h, H$; one neutral CP–odd field $A$; and charged Higgs bosons $H^\pm$. At the tree level the masses and interactions of these Higgs bosons are determined by just two free parameters, e.g. the mass $m_A$ of the CP–odd state and $\tan \beta$, the ratio of the vacuum expectation values (vev) of the neutral components of the two Higgs doublets.

However, the masses and couplings of the neutral CP–even states receive potentially large radiative corrections \[7\] from loops involving top quarks and their spin–0 superpartners, the stops. We treat these one–loop corrections using the effective potential method \[8\]. As pointed out in refs.\[9\], one can absorb the dominant two–loop QCD corrections by using a running (\textsc{MS} or \textsc{Dr}) top mass at appropriately chosen scale in the $m_t^4$ factors appearing in the expressions for the one–loop corrections. We take a pole top mass of 175 GeV, which corresponds to $m_t(m_t) \simeq 166$ GeV and $m_t(1$ TeV) $\simeq 151$ GeV. The size of these corrections increases logarithmically with the stop mass scale. We therefore conservatively take 1 TeV for the soft breaking masses of both the $SU(2)$ doublet and $SU(2)$ singlet stops $\tilde{t}_L$ and $\tilde{t}_R$, the largest value commonly accepted as being compatible with naturalness arguments. We also allow for mixing between the two stop current eigenstates. This can increase the corrections significantly, with $m_h$ becoming maximal \[10\] if $A_t + \mu \cot \beta = \sqrt{6} m_t$, where $A_t$ is a trilinear soft breaking parameter.
\[ m_{h}^{2} \leq M_{Z}^{2} \cos^{2}(2\beta) + \epsilon(m_{t}, m_{\tilde{t}}, A_{t}), \]  

(1)

where \( \epsilon \) parameterizes the effect of the radiative corrections described above. Note that \( \epsilon \) is approximately independent of \( \tan \beta \); for large \( m_{A} \), \( m_{t} = 175 \) GeV and \( m_{\tilde{t}} = 1 \) TeV it amounts to about \( 0.9M_{W}^{2} (1.6M_{h}^{2}) \) for no (maximal) stop mixing. It is important to note that the bound (1) can only be saturated for large \( m_{A} \). In the region of small \( \tan \beta \) the current LEP limits [11] on the masses of neutral Higgs bosons therefore already imply a quite stringent lower bound on \( m_{A} \). This in turn constrains the mass of the charged Higgs boson, which is given by

\[ m_{H^{\pm}}^{2} = M_{W}^{2} + m_{A}^{2} + \epsilon_{\pm}, \]  

(2)

where the radiative correction \( \epsilon_{\pm} \) is small and can be of either sign [1].

This is illustrated by the dotted curves in Fig. 1, which show the indirect lower bound on \( m_{H^{\pm}} \) that follows from LEP searches for neutral MSSM Higgs bosons; the upper (lower) curve is for no (maximal) stop mixing. The LEP search limits have been interpreted as implying the following constraints on tree–level Higgs production cross sections:

\[
\sigma(e^{+}e^{-} \rightarrow Zh) < 0.35 \text{ pb}; \quad (3a) \\
\sigma(e^{+}e^{-} \rightarrow hA) < 0.12 \text{ pb}, \quad (3b)
\]

at center–of–mass energy \( \sqrt{s} = 183 \) GeV. The bound (3a) corresponds to a lower limit of 88.7 GeV on the mass of the SM Higgs boson, while (3b) implies \( m_{h} \approx m_{A} \geq 75 \) GeV for \( \tan \beta \gg 1 \) in the MSSM. We have not attempted to combine the searches for \( Zh \) and \( hA \) production, since these final states have different backgrounds. The lower bound \( m_{A} > 75 \) GeV, which follows from (3b), implies \( m_{H^{\pm}} > 109 \) GeV; this explains the flat parts in the dotted curves in Fig. 1.

However, for low \( \tan \beta \), the constraint (3a) gives a stronger bound on \( m_{A} \), and hence on \( m_{H^{\pm}} \). This has important ramifications for charged Higgs searches at hadron colliders such as the Tevatron. The most promising searches [12] all rely on \( t \rightarrow H^{\pm}b \) decays. Since the relevant \( H^{\pm}tb \) couplings are proportional to \( m_{t} \cot \beta \pm m_{b} \tan \beta \), for a given value of \( m_{H^{\pm}} \) the branching ratio for such decays is large at small and at large \( \tan \beta \), but has a pronounced minimum at \( \tan \beta \approx \sqrt{m_{t}/m_{b}} \approx 7.5 \). Fig. 1 shows that already now LEP searches for neutral Higgs bosons imply that in the MSSM \( t \rightarrow H^{\pm}b \) decays are possible only for \( \tan \beta > 2.3 \) (1.4) for no (maximal) stop mixing.

We emphasize, however, that these constraints are entirely indirect, stemming from the search for neutral Higgs bosons. This motivated us to investigate the question to what extent these lower bounds on \( m_{H^{\pm}} \) can be relaxed by only modifying the neutral Higgs sector, keeping the charged Higgs sector unchanged. This implies that we restrict ourselves to models containing additional \( SU(2) \) singlet Higgs superfields and/or some new interactions.

To be specific, we studied two fairly modest extensions of the MSSM Higgs sector. The first of these models is a specific realization of the so–called superstring–inspired \( E(6) \) models [13].

*We will always assume \( \tan \beta > 1 \) here; this is required for the top Yukawa coupling to remain perturbative up to some high energy scale, and more generally seems indicated by the large ratio \( m_{t}/m_{b} \).
In general the Higgs sector of even the simplest such models \[13\] differs quite substantially from that of the MSSM. However, as pointed out in refs.\[13\], under certain assumptions one ends up with models that contain only one more parameter in the Higgs sector than the MSSM does. The first assumption is that the mass of the new neutral $Z'$ gauge boson present in these models is large compared to $M_Z$. In most cases current $Z'$ mass limits are in fact already so large \[13\] that this condition is automatically satisfied. This implies that the vev of the new $SU(2) \times U(1)_Y$ singlet Higgs field $N$ must be much larger than the vevs of the $SU(2)$ doublets. The second assumption is that the trilinear soft breaking term associated with the $N H_1 H_2$ term in the superpotential is not large; here $H_1$ and $H_2$ are the $Y = -1/2$ and $Y = +1/2$ Higgs doublets, respectively. Under these assumptions the singlet Higgs field $N$ is much heavier than the doublets, and does not mix with them. However, the trilinear scalar $N H_1 H_2$ interaction gets a large supersymmetric contribution $\propto \langle N \rangle$. Some $N$–exchange contributions to quartic Higgs couplings therefore remain even after $N$ is integrated out. Furthermore, the existence of a new $U(1)$ factor leads to new $D$–term contributions to the Higgs potential.

We refer the reader to refs.\[13\] for further details of these models. Here we merely state that the upper bound \[1\] on the mass of the lightest neutral CP–even state $h$ gets modified to

$$m_h^2 \leq M_Z^2 \cos^2(2\beta) + \frac{\lambda^2}{\sqrt{2} G_F} \left[ \frac{3}{2} + (2a - 1) \cos(2\beta) - \frac{1}{2} \cos^2(2\beta) - \frac{\lambda^2}{g_x^2} \right] + \epsilon. \quad (4)$$

Here, the radiative correction $\epsilon$ is the same as in eq.\[1\], $G_F$ is the Fermi constant, the constant $a$ depends on the $E(6)$ symmetry breaking pattern, $\lambda$ (called $f$ in refs.\[13\]) is the $NH_1 H_2$ superpotential coupling, and the coupling $g_x$ associated with the extra $U(1)$ can in most models be set equal to the standard hypercharge coupling $g_1$. As advertised, a given model only introduces a single new parameter $\lambda$. As an illustration we consider the so–called $\eta$–model, where $a = 0.2$. In this case consistency of the model requires \[13\] $\lambda \leq 0.35$.

The new, positive contribution in eq.\[1\] makes it easier to satisfy the Higgs search constraints from LEP, allowing for a reduced value of $m_A$ compared to the MSSM. Moreover, the relation between $m_A$ and $m_{H^+}$ also gets modified:

$$m_{H^+}^2 = m_A^2 + M_W^2 \left( 1 - \frac{2\lambda^2}{g_x^2} \right) + \epsilon, \quad (5)$$

where $g_2$ is the $SU(2)$ gauge coupling. This further reduces the lower bound on $m_{H^+}$. Note that the new contributions to eqs.\[1\] and \[3\] are maximized for different values of $\lambda$. The reduction of $m_{H^+}$ for fixed $m_A$ is obviously maximal for the largest allowed value of $\lambda$, $0.35$ for the $\eta$–model, while the new contribution to eq.\[1\] is maximal for $\lambda \simeq 0.27$ (for $a = 0.2$, $g_x = g_1 \simeq 0.35$). A numerical scan of the parameter space reveals that the absolute minimum of $m_{H^+}$ is reached for $\lambda \simeq 0.27$ if $\tan\beta \leq 2$, and for $\lambda = \lambda_{\text{max}} = 0.35$ for $\tan\beta > 2.5$.

The results of this scan are shown by the solid curves in Fig. 1; the upper (lower) curve again refers to no (maximal) stop mixing. Since this model contains exactly the same (potentially) light Higgs fields as the MSSM, the constraints \[3\] can be applied without any modification; in particular, for large $\tan\beta$ the CP–odd state is again nearly degenerate with one of the CP–even Higgs bosons.

In the region of small $\tan\beta$ the effects of this rather modest modification of the MSSM Higgs sector are quite dramatic. In particular, $t \to H^+b$ decays are now again allowed all the way down to $\tan\beta = 1.3$ even in the absence of stop mixing. If stop mixing is maximal, such decays are possible even for $\tan\beta = 1.0$; however, for such a low value of $\tan\beta$ the top
Yukawa coupling would have a Landau pole at an energy scale quite close to the weak scale. The modification of the lower bound on $m_{H^+}$ is more modest for $\tan\beta \geq 4$, where the limit (3b) provides the most stringent constraint. The reason is that the resulting bound on $m_A$ is essentially the same as in the MSSM, so the difference between the two bounds is entirely due to the new contribution to the charged Higgs mass in eq.(3).

The third model we investigate is the so-called next–to–minimal supersymmetric standard model (NMSSM). It differs from the MSSM only in the Higgs sector, where one postulates the existence of an $SU(2) \times U(1)_Y$ singlet superfield $N$. The model is therefore conceptually far simpler than the $E(6)$ models discussed above; nevertheless the modification of the Higgs sector is more extensive. Even if we restrict ourselves to purely cubic terms in the superpotential $f$, gauge symmetry allows one to introduce two different Higgs self–couplings:

$$f_{\text{Higgs}} = \lambda NH_1 H_2 - \frac{k}{3} N^3,$$

where we have used the notation of ref.[18]. Together with the corresponding soft breaking terms, there are six free parameters in the Higgs sector, even after we fix the sum of the squares of the vevs of the $SU(2)$ doublets to reproduce the known mass of the $Z$ boson. Moreover, the spectrum now contains three neutral CP–even fields $H_i$ and two CP–odd fields $A_i$ in addition to the charged Higgs field $H^\pm$.

Nevertheless one can still derive an upper bound on the mass of the lightest scalar Higgs field. After including radiative corrections, one has [20, 18]:

$$m_{H_1}^2 \leq M_Z^2 \cos^2(2\beta) + \frac{2\lambda^2 M_W^2}{g_2^2} \sin^2(2\beta) + \epsilon,$$

where $\epsilon$ is again the same as in eq.(1). Clearly this bound is only useful if an upper limit for $\lambda$ can be found. Such a limit can be derived [19, 18] from the requirement that all couplings of the model remain in the perturbative regime up to some very high energy scale, usually taken to be of the order of the GUT scale.

In Fig. 2 we show the resulting upper bound on $\lambda$ as a function of the value of the top Yukawa coupling $h_t$ at scale $M_Z$. We have used two–loop renormalization group equations [18] to derive this bound, with $\alpha_s(M_Z) = 0.120$. We have conservatively taken a rather low input scale ($M_Z$, rather than $m_t$) and a rather high value for the GUT scale ($3 \times 10^{16}$ GeV); the resulting bound only depends weakly on these scale choices. In the absence of sparticle loop corrections, the top Yukawa coupling is given by

$$h_t = \frac{g_2 m_t}{\sqrt{2 M_W \sin\beta}},$$

where $m_t$ is the running top mass; this gives $h_t(m_t) \geq 0.96$ for $m_t(\text{pole}) = 175$ GeV.\footnote{In general there can be substantial stop–gluino loop corrections to eq.(8) [21]. However, these will be small for large values of the stop masses, which maximize the radiative correction $\epsilon$.} The upper branch of the curve in Fig. 2 is determined by the requirement that $\lambda$ remains in the perturbative regime, while the sharp drop–off to the right comes from the requirement that $h_t$ remains perturbative. The absolute upper bound on $h_t$ corresponds to the well–known “fixed point” solution [22]; of particular interest to us is the corresponding lower bound on $\tan\beta$, which can be written in the form

$$\sin\beta \geq 0.84 \frac{m_t(\text{pole})}{175 \text{ GeV}}.$$
We remark that the bound (9) also applies to the MSSM if one requires \( h_t \) to remain perturbative up to scale \( M_X = 3 \times 10^{16} \) GeV; we have extended the dotted curves in Fig. 1 to lower values of \( \tan \beta \) in order to allow for possible intermediate scales, which could relax this bound (23).

The relation between the masses of charged and neutral CP–odd Higgs bosons also gets modified in the NMSSM [19, 24]:

\[
m^2_{H^+} = M^2_W \left( 1 - \frac{2\lambda^2}{g^2_2} \right) + m^2_{\lambda'} + \epsilon_+, \tag{10}
\]

where \( m^2_{\lambda'} \) is the mass of the neutral \( SU(2) \) doublet CP–odd state in the absence of doublet–singlet mixing. Note that this mixing can only reduce the mass of the lighter CP–odd state, i.e. \( m^2_{\lambda'} \geq m^2_A \geq 0 \). On the other hand, the \( \lambda^2 \)–term in eq.(10) obviously reduces the mass of the charged Higgs boson. Moreover, doublet–singlet mixing can also reduce the couplings of the light physical Higgs states to gauge bosons [23]; the bounds on \( m_{H^\pm} \) and \( m_{A_1} \) in the NMSSM are therefore much weaker [23, 24] than those on \( m_h \) and \( m_A \) in the MSSM.

We have interpreted these experimental constraints as follows. The LEP2 search limits (9) were taken to limit the sums \( \sum_i \sigma(e^+e^\to ZH_i) \) and \( \sum_{i,j} \sigma(e^+e^\to H_iA_j) \), respectively. However, unlike in the MSSM these bounds from searches at the highest available center–of–mass energy did not supersede the older LEP1 constraints completely, since the LEP2 constraints allow very light \( H_1 \), \( A_1 \) if they are dominantly \( SU(2) \) singlets. It turns out that the LEP1 constraints on the couplings of such light states to \( Z \) bosons are often stronger than those from higher energies. We have parametrized the ALEPH Higgs search limits from their lower energy data [27] as follows:

\[
(g_{ZZH_1})^2 \leq \left( \frac{g_2 M_Z}{\cos \theta_W} \right)^2 \cdot 1.6 \cdot 10^{-4} e^{m_{H_1}/8.2}; \tag{11a}
\]

\[
(g_{ZH_{H_1}A_1})^2 \leq \left( \frac{g_2}{2 \cos \theta_W} \right)^2 \cdot \begin{cases} 0.1, & m_{H_1} + m_{A_1} \leq 81 \text{ GeV} \\ 0.1(m_{H_1} + m_{A_1}) - 8.0, & m_{H_1} + m_{A_1} > 81 \text{ GeV} \end{cases}, \tag{11b}
\]

where all masses are in GeV. Note that eqs.(11) apply to individual couplings; we have not attempted any summation over related states, unlike in our NMSSM modification of the LEP2 bounds (3). However, we have checked that such a summation would not affect the derived lower limit on \( m_{H^+} \) significantly.

This limit is shown by the dashed curves in Fig. 1. Note that this bound shows little sensitivity to stop mixing even in the low \( \tan \beta \) region, unless \( \tan \beta \) lies just above the lower bound (8). More importantly, for \( 1.7 \leq \tan \beta \leq 2.5 \) the direct \( H^+ \) search limit from DELPHI [28], \( m_{H^+} \geq 53 \) GeV, can be saturated in the NMSSM, in sharp contrast to the other two models. We should mention that the indirect lower bound on \( m_{H^+} \) is not only determined by the upper bound on \( \lambda \) shown in Fig. 2 and the various LEP search limits described above, but also by the requirement that the desired minimum of the Higgs potential, where all three neutral Higgs fields have non-vanishing vev,

\[1\] is the absolute minimum of the potential. In particular, the allowed parameter space is constrained significantly by requiring that solutions where only \( H^0_2 \) or only \( N \) have non-vanishing vev should not be the absolute minimum.

The interplay of these constraints makes it difficult to give an analytical explanation for the behavior of the dashed curves in Fig. 1. It is clear from eq.(7), however, that the upper
bound on $m_{H_1}$ becomes independent of $\lambda$ for large $\tan\beta$, where it approaches the MSSM value. Our numerical scan of the parameter space finds that for $\tan\beta \leq 3.5$, $m_{H^+}$ takes its smallest possible value if $\lambda$ is at its maximum. The presence of intermediate scales, which could increase the upper bound on $\lambda$ [23], can therefore reduce the lower bound on $m_{H^+}$ even further in this region. Moreover, it would allow for a relative low $m_{H^+}$ even below the lower limit (9) on $\tan\beta$. However, for $\tan\beta > 4$, $m_{H^+}$ is minimized for $\lambda$ around 0.4. This optimal value of $\lambda$ is coincidently quite close to the maximal allowed $\lambda$ in the $E(6)$ $\eta$--model. As a result, for $\tan\beta > 4$ the indirect lower bound on $m_{H^+}$ that can be derived from neutral Higgs searches at LEP is quite close for these two models.

We have also used our program to search for NMSSM parameters that allow the decay chain $t \rightarrow H^+b \rightarrow W^+(H_1,A_1)b$. The main signature for such decays would resemble that for $H^+ \rightarrow W^+bb$ three–body decays [24], except that there would be a peak in the $bb$ invariant mass spectrum. The light neutral Higgs boson could also decay into $\tau^+\tau^-$ pairs, with branching ratio of order 10%. We found that in the NMSSM such scenarios can indeed be realized for small values of $\tan\beta$. For example, for $m_{H^+} = 150$ GeV and $\tan\beta = 1.6$, we found that the partial widths for $H^+ \rightarrow H_1W^+$ ($A_1W^+$) can exceed the sum of $H^+ \rightarrow c\bar{s}$ and $H^+ \rightarrow \tau^+\nu_\tau$ partial widths by a factor of more than 7.5 (150). Light CP–odd states can be produced more copiously in $H^+$ decays, since they cannot be produced singly at LEP, unlike neutral CP–even states; hence they can have much larger $SU(2)$ doublet components than CP–even states with the same mass. Such “unusual” $H^+$ decays allow one to evade [23] bounds on $t \rightarrow H^+b$ decays based on either direct searches for enhanced $\tau$ production, or on the reduction of $t\bar{t}e$ events containing one or two hard leptons (electrons or muons) [30].

Finally, we have attempted to assess the impact of future searches for neutral Higgs bosons at LEP on the lower bound on $m_{H^+}$. It now seems that the ultimate energy of LEP will be around $\sqrt{s} = 200$ GeV [31]. Using results of the LEP2 Higgs working group [32] we estimate that this could give lower limits of 107 GeV for an SM–like neutral Higgs boson, and of 93 GeV for degenerate CP–even and CP–odd states with full coupling to the $Z$ (as in the MSSM at large $\tan\beta$). These bounds assume that no indication of a signal is found; the discovery reach of LEP operating at this energy would be a few GeV lower. These possible future constraints can be implemented by requiring

\[
\sigma(e^+e^- \rightarrow Zh) < 0.16 \text{ pb}; \quad (12a)
\]
\[
\sigma(e^+e^- \rightarrow hA) < 0.025 \text{ pb}, \quad (12b)
\]

at center–of–mass energy $\sqrt{s} = 200$ GeV. In case of the NMSSM, we have again summed over $ZH_1$ and $H_iA_j$ final states when applying these constraints.

The resulting bounds are shown in Fig. 3, using the same notation as in Fig 1. In the MSSM, a failure to detect neutral Higgs bosons at LEP would lead to the absolute lower bounds $\tan\beta > 3$ (1.5) for no (maximal) stop mixing; charged Higgs bosons would then be accessible to top decays only for $\tan\beta > 4.75$ (2.6). Even in the $U(1)_{\eta}$ model a nontrivial lower bound on $\tan\beta$ would emerge unless stop mixing is substantial. On the other hand, in the NMSSM the charged Higgs boson could still be light enough to be produced in top decays even very close to the lower bound (1) on $\tan\beta$. However, even in this model a nontrivial absolute lower bound on $m_{H^+}$ of about 80 GeV could be derived. This is only slightly lower

\footnote{In this region $m_{H^+}$ is minimized in the NMSSM if the coupling $k$ takes its maximum value. However, the $k$--dependence of the bound is quite mild. We therefore simply require $|k| \leq 0.5$ for small and moderate values of $\lambda$, in agreement with results of refs. [3] [24].}
than the ultimate sensitivity of LEP for direct charged Higgs searches; in the absence of a signal for neutral Higgs boson production, searches for charged Higgs bosons at LEP could give additional constraints on parameter space only for $1.7 \leq \tan\beta \leq 2.7$.

Finally, all three models again give fairly similar indirect lower limits on $m_{H^+}$ for $\tan\beta \geq 7$. Indeed the ultimate LEP lower limit of 110–120 GeV on $m_{H^+}$ (Fig. 3) would hold throughout the region $\tan\beta \geq 7$. Thus there is plenty of room for direct $H^+$ search via $t \rightarrow H^+b$ decays in this region even in the MSSM. It may be noted here that a direct $H^+$ search via $t \rightarrow H^+b$ with the Tevatron collider data has recently given a lower limit of 100–120 GeV on $m_{H^+}$ for $\tan\beta = 40–50$ [33]. This result holds in the MSSM as well as its extensions discussed here. The main difference between them lies in the relatively low $\tan\beta$ region, where the negative result from neutral Higgs boson search at LEP implies a severe lower limit on $m_{H^+}$ in the former case but not the latter.

In summary, we have pointed out that experimental searches at LEP already impose significant indirect lower bounds on the mass of the charged Higgs boson in the MSSM. These bounds are particularly severe for low $\tan\beta$. This region is of special interest since here the $H^+tb$ coupling is large, while the partial widths of $H^+$ into light SM fermions is small, allowing other interesting decay modes of the charged Higgs boson to have sizable branching ratios.

However, we found that these indirect lower bounds on $m_{H^+}$ can be relaxed considerably in relatively modest extensions of the MSSM. Specifically, an extension based on $E(6)$ models with an extra $U(1)$ factor, which only adds one new parameter to the description of the low–energy Higgs sector, is sufficient to re-introduce the possibility of a substantial branching fraction for $t \rightarrow H^+b$ decays, although in these models the smallest allowed value of $m_{H^+}$ still lies beyond the region that can be covered by searches for $H^+H^-$ production at LEP. An even more dramatic reduction of the indirect lower bound on $m_{H^+}$ becomes possible in the NMSSM, where one adds one $SU(2) \times U(1)_Y$ singlet Higgs superfield to the MSSM. In this model the charged Higgs boson could still be light enough to be discovered at LEP, if $\tan\beta$ lies between 1.7 and 4, or even lower if one allows for intermediate scales. In this range of $\tan\beta$ the charged Higgs boson might dominantly decay into an on–shell $W$ boson and a light neutral Higgs boson, which would complicate the search for $t \rightarrow H^+b$ decays at hadron colliders. On the other hand, we found the indirect lower bound on $m_{H^+}$ to be less model–dependent for intermediate and large $\tan\beta$. For $\tan\beta > 4$ the bound in the $E(6)$ model or the NMSSM lies within 10 to 15 GeV of the MSSM value, which is itself quite modest.

If LEP experiments fail to observe a signal for neutral Higgs boson production after accumulating several hundred pb$^{-1}$ of data at $\sqrt{s} = 200$ GeV, even in the $E(6)$ models $t \rightarrow H^+b$ decays would become impossible for $\tan\beta \leq 2$, unless there is substantial mixing in the stop sector. However, even in this pessimistic scenario light Higgs bosons at small $\tan\beta$ could still be accommodated in the NMSSM, independent of stop mixing. We conclude that there remains plenty of parameter space for light charged Higgs bosons at small and moderate values of $\tan\beta$.

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Figure 1: The indirect lower bound on the mass of the charged Higgs boson that follows from the searches for neutral Higgs bosons at LEP. The dotted curves are for the MSSM, the solid ones for the $E(6)\eta$-model, and the dashed ones for the NMSSM. Radiative corrections to Higgs mass matrices have been included using $m_t$(pole) = 175 GeV and a common stop soft breaking mass of 1 TeV; the upper (lower) curve of a given pattern is for no (maximal) mixing between $SU(2)$ doublet and singlet stops, as described in the text.
Figure 2: The upper bound on the Higgs self coupling \( \lambda \) that follows from the requirement that all couplings remain in the perturbative regime up to scale \( M_X = 3 \cdot 10^{16} \) GeV is shown as a function of the top Yukawa coupling \( h_t \). This curve is valid if all other superpotential couplings are small. The bound on \( \lambda \) therefore becomes more stringent if \( \tan \beta \) is very large, in which case the bottom Yukawa coupling becomes sizable. We have assumed \( \alpha_s(M_Z) = 0.12 \).
Figure 3: The indirect lower bounds on $m_{H^+}$ that could be derived if LEP fails to find a signal for neutral Higgs bosons even after completing its run at the projected ultimate energy of $\sqrt{s} = 200$ GeV. The notation is as in Fig. 1.