Decomposition Principle in Control of Socio-Technical Systems

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Abstract. A general approach to decompose the interaction of agents in social, organizational and technical systems is proposed. Within this approach, the problems of reachability, sustainability and searching for an optimal trajectory can be solved independently for each agent.

1. Introduction
All issues on the reachability of a global goal by a given system with limited communication (coordination) between its parts are traditionally related to the problems of multi-agent systems control and network control. In addition, the decomposition of component-additive criteria is the subject of distributed optimization; see overviews in [1], [2]. However, in all the scientific areas mentioned above, the components of a system are passive and strictly follow the prescribed algorithms of behavior.

This short paper considers the model of a socio-technical system (STS) composed of active subjects: a single control authority (Principal) and n controlled subjects (agents). An active subject pursues his/her own interests and is capable of strategic behavior. Assume that the Principal and the agents play a hierarchical game \( \Gamma_2 \) with side payments. Recall that \( \Gamma_2 \) is a game with a fixed sequence of moves [3] as follows. The first move is made by the Principal, who chooses a control, and the second move is made by the agents, who choose their strategies simultaneously and independently of each other. The Principal’s control is a vector function of the agents’ strategies subsequently observed by him, and under this control, the agents play a normal-form game with each other [4].

As is well known [5], the calculation of a Nash equilibrium of an agents’ game is an NP-hard problem. In the model under consideration, this equilibrium depends on the desired control function; therefore, in the general case, a straightforward search for optimal control is analytically impossible and very computationally intensive. Hence, let’s try to decompose the control problem.

As will be demonstrated below, such a decomposition is possible: it yields a simple and practically interpretable analytical solution and, moreover, causes no loss in efficiency. Section 2 describes the model of a control system. Section 3 introduces the decomposition principle. Section 4 gives some examples.

2. Model of control system
Consider the set \( N = \{1, 2, \ldots, n\} \) consisting of a finite number \( n \) of agents. Agent \( i \in N \) chooses a strategy \( y_i \in A_i \subseteq \mathbb{R}^{m_i} \). His/her goal function \( f_i(y), f_i: A \rightarrow \mathbb{R}^1 \), where \( A = \prod_{j=1}^{n} A_j \) and
$y = (y_1, \ldots, y_n) \in A$, generally depends on the strategies vector of all agents \((\text{the strategy profile of the game described below})\). The Principal’s goal function $F(y)$ is $F: A \to \mathbb{R}^1$.

The essence of Principal’s control lies in establishing nonnegative side payments to the agents. (Non-negativity means that the Principal cannot penalize the agents.) In other words, the payoff functions of the Principal and of agent $i$ have the form $F(y) - \sum_{i \in N} \sigma_i(\cdot) + f_i(y) + \sigma_i(y)$, respectively.

The value $\sum_{i \in N} \sigma_i(\cdot)$ is called the Principal’s control costs. The vector of strategies chosen by the agents under a given control is called the strategy profile implemented by this control \([4]\) (or reachable under this control). The Principal and all agents are rational in the sense of maximizing their own goal functions.

Denote by $y_{-i} = (y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n) \in \prod_{j \in N \setminus \{i\}} A_j$ the opponents’ strategy profile for agent $i$.

Define the values $L_i(y_{-i}) = \max_{j \in A} f_j(y_i, y_{-i})$, $i \in N$, where all the functions and sets involved are such that the maxima and minima are assumed being achieved. For a given strategy profile $x \in A$, further referred to as a plan, construct the positional (feedback) control:

$$
\sigma_i(x_i, y) = \begin{cases} 
L_i(y_{-i}) - f_i(x_i, y_{-i}) + \varepsilon & \text{if } y_i = x_i, \\
0 & \text{if } y_i \neq x_i,
\end{cases}, \quad i \in N, 
$$

(1)

where $\{\varepsilon_i \geq 0\}$ are some constants. Also, let $\varepsilon = \sum_{i \in N} \varepsilon_i$.

Assume that complete awareness holds: the goal functions and the sets of admissible strategies of all agents are common knowledge for all system participants \([4]\).

Some practical interpretations include organizational, social and technical systems (STSs), distributed energy systems, and information and telecommunications systems. In such systems, “payments” can be financial, energy, information, and other resources. In a network system, any of the agents can be chosen as a virtual Principal, without any specific goal function.

3. Decomposition principle

Consider a set of propositions.

**Proposition 1.** The positional control \((1)\) implements a plan $x \in A$.

Proof of Proposition 1. Substitute the expression \((1)\) into the payoff function of agent $i$. In the case of plan fulfillment (choice of $y_i = x_i$), agent $i$ will obtain the payoff $\max_{j \in A} f_j(y_i, y_{-i}) + \varepsilon_i$. Choosing any other strategy, agent $i$ will obtain the payoff $f_i(y_i, y_{-i})$, which is not greater than the former under any (!) opponents’ strategy profile (and even strictly smaller than the former under strictly positive values $\varepsilon_i$).

Consider agent $i$ with a payoff function $g_i(\cdot)$, recall that his dominant strategy is $z_i \in A_i$ such that

$$
\forall y_i \in A_i, \forall y_{-i} \in \prod_{j \in N \setminus \{i\}} A_j: g_i(z_i, y_{-i}) \geq g_i(y_i, y_{-i}).
$$

(2)

If all agents have dominant strategies, the corresponding vector is called a dominant strategy equilibrium (DSE) \([4]\).

**Proposition 2.** Under the positional control \((1)\) applied by the Principal, $x \in A$ is a DSE in the agents’ normal form game. If $\varepsilon_i > 0$, $i \in N$, this DSE is unique.

For proving Proposition 2, just substitute the expressions \((1)\) into the payoff function of agent $i$ and then check the validity of conditions \((2)\).
The existence of a DSE indicates that the interaction of all agents is decomposed: each agent chooses his/her strategies independently of the other agents, and the Principal can control each of the agents separately. The application of positional controls like (1) in STSs is called the decomposition principle.

**Proposition 3.** With the positional control (1), the Principal implements a given plan \( x \in A \) as a DSE with the \( \varepsilon \)-minimum costs.

Proof of Proposition 3. Assume on the contrary that there exists a positional control \( \{ q(x) \geq 0 \} \) implementing a vector \( x \in A \) as a DSE in the agents’ game, and there exists an agent \( i \) such that

\[ q_i(x) < \sigma_i(x_i, x) - \varepsilon_i. \]

From (1) it follows that

\[ q_i(x) < L_i(x_{-i}) - f_i(x_i, x_{-i}) - \varepsilon_i. \tag{3} \]

Then, according to (2),

\[ \forall y_i \in A_i : f_i(x_i, x_{-i}) + q_i(x_i, x_{-i}) \geq f_i(y_i, x_{-i}) + q_i(y_i, x_{-i}). \tag{4} \]

In view of (4),

\[ q_i(x) \geq L_i(x_{-i}) - f_i(x_i, x_{-i}) - \varepsilon_i. \tag{5} \]

Inequality (5), together with the non-negativity of \( \sigma_i(\cdot) \), gives

\[ q_i(x) \geq L_i(x_{-i}) - f_i(x_i, x_{-i}). \tag{6} \]

The expressions (3) and (6), considered jointly, lead to the contradiction: \( \varepsilon_i < 0 \), and the conclusion follows. •

**Proposition 4.** The positional control (1) is \( \varepsilon \)-optimal, and the optimal plan has the form

\[ x^* = \arg \max_{x \in A} \left[ F(x) + \sum_{i \in N} f_i(x) - \sum_{i \in N} L_i(x_{-i}) \right]. \tag{7} \]

Proof of Proposition 4. The \( \varepsilon \)-optimality of the positional control (1) follows from the additive property of the Principal’s payoff function and Proposition 3. In turn, formula (7) can be obtained by substituting the expression (1) into the Principal’s payoff function. •

**Proposition 5.** Let the hypothesis of benevolence be satisfied. Then the positional control (1) with \( \varepsilon_i = 0, i \in N \), and the plan (7) is optimal.

(The hypothesis of benevolence states the following: if when calculating \( L_i(y_i) \), the agent’s payoff achieves maximum at several strategies, the agent will choose the strategy coinciding with his plan; see [4]).

Propositions 1–5 estimate constraints on the positional controls under which one or another vector of agents’ strategies is reachable.

The expression (7) can be interpreted as follows. The optimal plan is “Pareto efficient”: it maximizes the difference between the sum of the goal functions of all system participants (both the Principal and all agents) and the total losses of all agents from fulfilling the plan, compared to the choice of strategies that are most preferable for each of them under no control applied by the Principal. Indeed, in the right-hand side of the expression (7), the first term is the Principal’s goal function, the second is the sum of the goal functions of the agents, and the third term can be interpreted as “the costs of interests coordination.” Therefore, Propositions 4 and 5 claim that the optimal vector of agents’ strategies is the one maximizing the sum of the goal functions of all participants, taking into account the costs of coordinating their interests.

For the systems without the Principal, the optimal plan (7) takes the form

\[ x^* = \arg \min_{x \in A} \sum_{i \in N} \left[ \max_{y_i \in A_i} \left\{ f_i(y_i, x_{-i}) - f_i(x) \right\} \right], \tag{8} \]

and the value \( \sum_{i \in N} \left[ \max_{y_i \in A_i} \left\{ f_i(y_i, x_{-i}) - f_i(x^*) \right\} \right] \) is an estimate of the external resource required for implementing the optimal state. The computational complexity of the problems like (8) was estimated in the paper [6].
4. Examples

Example 1. Consider the problem of interests’ coordination in a multi-agent STS [4]. In this problem, 
\( A_i = \mathcal{R}_i^1 \), and the goal function of agent \( i \) is his/her costs taken with the minus sign: 
\( f(y) = -c(y) \), where \( c(y) \) is a nonnegative function that does not decrease in \( y \) and 
\( c(0, y, i) = 0 \) for any strategy profile \( y_i \).

Obviously, in this case, \( L(y, i) = 0 \) and the positional control (1) takes the form:

\[
\sigma_i(x_i, y) = \begin{cases} 
  c_i(x_i, y_i) + e_i & \text{if } y_i = x_i, \\
  0 & \text{if } y_i \neq x_i,
\end{cases}
\]

\( i \in N \).

The optimal plan (7) is 
\[ x^* = \arg \max_{x \in A} \left[ F(y) - \sum_{i \in N} c_i(x) \right]. \]

Example 2. Consider the problem of interests’ coordination in a multi-agent dynamic STS. In addition to the conditions of Example 1, let the Principal and agents be operating in a finite number \( T \) of time periods, 
\( A_i = \mathcal{R}_i^T \), \( L(y_i^{\tau_T}) = 0 \), and let the agent’s goal function be additive in time periods. In this case, the positional control (1) takes the form:

\[
\sigma_i^{\tau}(x_i^{\tau}, y_i^{\tau}) = \begin{cases} 
  c_i^{\tau}(x_i^{\tau}, y_i^{\tau}) + e_i^{\tau} & \text{if } \forall \tau = 1, \ldots, T: y_i^{\tau} = x_i^{\tau}, \\
  0 & \exists \tau: y_i^{\tau} \neq x_i^{\tau},
\end{cases}
\]

\( i \in N \).

(The optimal trajectory can be easily found using dynamic programming.)

5. Conclusions

In the book [4], some methods for decomposing the agents’ game in a two-level STS with a fan structure, in a dynamic STS, as well as in an STS with a network structure and in an STS with distributed control, were described. In the book [7], these results were generalized to the case of several agents interacting within a network and/or hierarchical structure and making decisions many times. Proposition 1 does not specify the type of agents’ interaction, and Propositions 2–5 are dealing with DSE. Hence, the above-mentioned results on the decomposition of the agents’ games, the periods of their operation, and the structure of OTSs immediately follow from Propositions 1–5.

Thus, the "promo version" of the obtained results sounds as follows: if the Principal in a STS can use side payments, then under complete awareness, the positional control (1), (7) will optimally decompose any interaction of agents.

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