Permittivity model for GNSS-R telemetry wetlands

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Abstract: The GNSS-R (Global Navigation Satellite System-reflectometry) wetland remote sensing is achieved by receiving a reflected signal. Based on the relationship between reflected energy and permittivity of the reflective medium, the inversion model is built for GNSS-R terrestrial humidity remote sensing, which is the terrestrial humidity dielectric model of the L-band satellite navigation signal left-polarized wave component model on the basis of the analytical expression of vertical polarization. This paper thereby provides theoretical and methodological support for the detection and inversion of GNSS-R soil moisture.

1. Introduction
With the development of spatial information technology, China has independently built, and operated Beidou satellite navigation system (COMPASS), which has become an important part of the global navigation satellite system. However, its further application in the field of detection still needs to be explored. COMPASS can transmit signals in the L-band and S-band (where the L-band B1 and B2 frequencies are allowed to transmit the navigation service signal), providing an open, highly stable source of microwave radiation for microwave detection. Global Navigation Satellite System Microwave Remote Sensing (GNSS-Reflections, GNSS-R) \cite{1} and its detection serves are an alternative solution.

At present, GNSS-R remote sensing is mainly focused on the detection of terrestrial humidity \cite{2-4}, snow cover, vegetation water content \cite{5}, forest cover, land surface imaging \cite{6} and so on. Meanwhile, the SNR multipath delay phase inversion model of soil moisture and the "non-parametric statistical estimation model" of the snow surface parameters \cite{7} aiming to obtain the surface parameters of the accurate snowfall weather have been put forward. In these models, the study between the permittivity and the reflected energy is the key to GNSS-R remote sensing.

The remote sensing of GNSS-R wetland is realized by capturing the reflected signals. The polarization of the reflected signals, the permittivity of the reflection medium and the height of the wave source are the main factors influencing the energy of the reflected signals. COMPASS launches the signals characterized by left-axis polarization in a right-handed way \cite{8}. In this paper, the permittivity model of the reflected wave of L-band satellite navigation signal is obtained by studying the correlations among the polarization of the signal, the permittivity of the reflection medium and the wave height.

2. The Reflectivity of GNSS-R Wave for Detection
Different echoes of different polarizations are the result of the interaction between electromagnetic waves and ground objects.
After the GNSS-R wave is reflected by the land surface, circularly polarized wave spins to reverse. Its reflection coefficient of the vertical and horizontal components is not the same, and the amplitude ratio and the phase difference of the reflected wave are changed before the elliptically polarized wave is synthesized. When the radiation source frequency is increased, the amplitude ratio and the phase difference are changed more significantly, as smooth surface reflection coefficient depends on the frequency, polarization, incident angle of wave and electrical properties of the surface [9].

GNSS-R wetland detection is to obtain the most important parameter, the reflection coefficient, by using the surface reflection signal data directly, and the reflection coefficient can be further associated with the complex permittivity, thus the correlation between the land reflection signal and complex permittivity can be achieved.

The surface reflectance of the GNSS reflected signal is the ratio of the total reflected power to the direct signal power, i.e., the total reflectivity is

$$\Gamma_{\text{total}} = \frac{P_{\text{t}}}{} = \frac{1}{2} \left( |R_{HH}|^2 + |R_{VV}|^2 \right) \tag{1}$$

Where $P_{\text{t}}$ is the total reflected power, $P_i$ is the incident power, namely, the direct power. $\Gamma_{\text{total}}$ is the total reflectivity.

The reflectivity is a physical quantity directly related to the reflection coefficient, which is a normalized parameter for the transmitter cross-sectional area of the irradiated surface area per unit area. In order to convert the reflectivity and the incident angle (the satellite grazing angle $\gamma = 90^\circ - \theta$) to the relationship with the satellite elevation angle (elevation angle $\theta$), the reflectivity in formula (1) can be expressed as:

$$\Gamma = |R(\theta)|^2 e^{-k \sin^2 \theta} \tag{2}$$

Here, $h = (4\pi / \lambda)\sigma$ is the roughness parameter, $\sigma$ is the standard deviation for the height of the table for flat surface of the land, according to Rayleigh criteria ($\sigma < \lambda / 8 \cos \theta$), $h$ is 0, namely:

$$\Gamma = |R(\gamma)|^2 = |R(\gamma)|^2 \tag{3}$$

For purely right circularly polarized GNSS signals, the vertical reflectance $\Gamma_{VV}$ and horizontal polarization reflectance $\Gamma_{HH}$ can be expressed as:

$$\Gamma_{VV} = R_{VV}^2 \tag{4}$$
$$\Gamma_{HH} = R_{HH}^2 \tag{5}$$

L-band frequency dispersion to water is very strong. For a purely circularly polarized GNSS wave, the left-axis component reflectivity can be expressed as:

$$\Gamma_{RL} = R_{RL}^2 = \frac{1}{4} (R_{VV} - R_{HH})^2 = \frac{(\varepsilon - 1)^2 \sin^2 \theta (\varepsilon - \cos^2 \theta)}{(\varepsilon \sin \theta + \sqrt{\varepsilon - \cos^2 \theta})^2 (\sin \theta + \sqrt{\varepsilon - \cos^2 \theta})^2} \tag{6}$$

And the left-handed circular component reflectivity can be expressed as:

$$\Gamma_{RR} = R_{RR}^2 = \frac{1}{4} (R_{VV} + R_{HH})^2 = \frac{(\varepsilon - 1)^2 \cos^4 \theta}{(\varepsilon \sin \theta + \sqrt{\varepsilon - \cos^2 \theta})^2 (\sin \theta + \sqrt{\varepsilon - \cos^2 \theta})^2} \tag{7}$$
Generally, the relative permittivity of wetlands varies with the volume of humidity, usually in a larger range of greater than 4. Fig.1 shows the relationship curve between the relative permittivity $\varepsilon \in [2,40]$ and the altitude angle at the altitude angle $\theta = \frac{5\pi}{12}$.

![Reflectivity Rate of Polarized Wave and Relative permittivity of Polarized Wave](image)

Fig. 1. $\varepsilon - \delta$ relationship curve of relative permittivity change

It can be seen that the reflectivity component tends to increase further when the relative permittivity ($\varepsilon$) increases at the larger altitude angle. Therefore, the reflection signal is obtained in a way to receive the main polarization component as its means.

3. Sensing Ground Surface Object Theory Based on GNSS-R Wave

3.1 Theoretical Derivation of Inversion Model

If the radiation is a linearly polarized wave, the reflected signal is dominated by the vertical polarization component under the condition of a large altitude angle, and the permittivity of the reflector can be detected by receiving the vertically polarized wave. By combining (4), the results are as follows:

$$\varepsilon^2 \sin^2 \theta \left(\frac{1-\Re}{1+\Re}\right)^2 - \varepsilon + \cos^2 \theta = 0 \quad (8)$$

The equation has two solutions. As formula (9), one solution can be taken to apply in the corresponding practical circumstances.
The estimation of soil moisture is mainly based on the ratio of the reflected wave to the direct wave-related power (the normalized power of the reflected signal). For the value of $\theta$, $VV\Re$ in (14), $VV\Re$ can be obtained from the direct wave and the reflected wave data output from the receiver, as well as the angle $\theta$ from the positioning data.

As with the right-handed polarization signal transmitted by BeiDou satellite, under the condition of high altitude of wave source, the reflected signal of the right-handed polarization signal is dominated by the left-handed polarization component, making it difficult to solve the relationship of $\varepsilon_{RL}$ directly from equation (6). Therefore, the relation between $VV\Re$ and $RL\Re$ needs to be discovered.

The following expressions are given by (7)

$$\frac{VV\Re}{RL\Re} = \frac{(\varepsilon \sin \theta - \sqrt{\varepsilon \cos^2 \theta})(\sin \theta + \sqrt{\varepsilon \cos^2 \theta})}{(\varepsilon - 1)(\sin \theta \sqrt{\varepsilon - \cos^2 \theta})} = 1 - \frac{\cos^2 \theta}{\sin \theta \sqrt{\varepsilon - \cos^2 \theta}} = 1 - \Delta(\theta) \quad (10)$$

Thus, it can be simplified as

$$VV\Re = (1 - \Delta(\theta))RL\Re \quad (11)$$

Here, $\Delta(\theta) = \frac{\cos^2 \theta}{\sin \theta \sqrt{\varepsilon - \cos^2 \theta}}$

By combining with (9), we can get the permittivity expression, as follows

$$\varepsilon = \frac{1 \pm \sqrt{1 - 4 \sin^2 \theta \times \cos^2 \theta \times (1 - (1 - \Delta(\theta))RL\Re) / (1 + (1 - \Delta(\theta))RL\Re)}}{2 \sin^2 \theta \times (1 - (1 - \Delta(\theta))RL\Re) / (1 + (1 - \Delta(\theta))RL\Re)} \quad (12)$$

This is the ground permittivity detection model of the GNSS-R circular polarization wave.

The models expressed by (9) and (12) are theoretical models related to the Fresnel reflection coefficient. And the actual detection is achieved by the relative power of the reflected signal (i.e., the reflectivity).

### 3.2 The Inversion Method of Soil Moisture Based on Permittivity

The complex permittivity of soil surface sand can be expressed as follow:

$$\varepsilon' = \varepsilon - j\omega\sigma = \frac{k}{\varepsilon_0} - j\frac{\sigma}{\omega\varepsilon_0} \quad (13)$$

Here, $\varepsilon'$ is the complex permittivity for the surface of the land, $\varepsilon$ is the real part of the relative complex permittivity, $k$ is the absolute permittivity, $\sigma$ is the conductivity, $\varepsilon_0$ is the permittivity of the free space, $\omega$ is the frequency of the electromagnetic, and $\varepsilon$ reflects refraction and reflection of the radio wave to the land surface.
The Beidou’s signal wavelength $\lambda = 0.19217m$, and the earth’s conductivity $\sigma$ changes within the range of $10^{-4} - 10^{-3}$. The relative complex permittivity of the imaginary part varies between 0.0011-0.011, which can be ignored compared to the real part. Thus:

$$\varepsilon' = \varepsilon = \frac{k}{\varepsilon_0}$$

(14)

There is a certain relationship between the permittivity of the terrestrial humidity and the reflected energy [9]. For 100% pure loess, the permittivity can be simplified as

$$\varepsilon = 2.863 + 3.462m_v + 119.639m_v^2$$

(15)

Here, $m_v$ is the sand water volume (i.e., humidity).

The greater the permittivity of the scattering medium is, the stronger the ability to reflect the microwave and the smaller the penetration effect could be. During the surface humidity microwave detection, the main factor affecting the electrical constant is the surface water content of the shallow surface. The electrical constant increases linearly with the target surface water content. So when the electromagnetic wave reflectivity is higher, the echo becomes stronger and the microwave penetration is weaker. Another factor that affects the electrical constant is the conductivity of the surface soil surface. The loss and attenuation of microwave energy is a function of the ground target conductivity and wavelength. When the water content of the ground target is constant, there would be higher frequency (the shorter the wavelength), greater energy attenuation in the soil and the lower transmittance. In conclusion, the difference in soil water content directly leads to changes in the permittivity, which in turn affects the reflectivity and transmittance of electromagnetic waves.

The permittivity of the reflection area is the ratio of the wave power obtained by the direct wave and by reflection received from the output of the receiver. The test model is established by obtaining the angle from the positioning data and the permittivity of the wetland when combined with the theoretical model[9].

$$\varepsilon = \Gamma^{-1}(\Gamma_0) = \Gamma^{-1}\left(\frac{P_r}{P_d}\right)$$

(16)

Here $\Gamma_0$ is the reflectance corresponding to the soil the permittivity $\varepsilon$, $P_r$ is the reflected signal power, and $P_d$ is the direct signal power.

Based on this test model, and combined with the inversion of soil-to-water volume, humidity, the reflectivity is the target parameter of the GNSS-R land-based receiver, and its value depends not only on the permittivity of the reflective medium, but also in relation to the geometry of the test equipment and the satellite position. These geometries can be converted into “Altitude angle”. In order to analyze the relationship between reflectivity and altitude angle under different permittivity, when the altitude angle $\theta = \frac{5\pi}{12}$ and the relative permittivity $\varepsilon \in [2,40]$, the $\Gamma - \varepsilon$ simulation curve is shown as Fig.2.
Fig. 2. $\Gamma - \varepsilon$ Relation curve under certain altitude angle

Fig.2 reflects that under a certain altitude angle, the greater the permittivity is, the more sensitive to the reflectivity it would be. When the reflectivity is large, a small error will easily lead to a large measurement error of the permittivity.

The Beidou/GPS usually emits a right-handed circularly polarized signal whose reflected signal is left-handed circularly polarized, which aims to detect the humidity of the earth. The relative permittivity varies with a wide range of humidity, and is related to the source location and the geometric distribution of the test system. In order to describe the corresponding relationship between the reflectivity of the left circularly polarized wave and the relative permittivity under the geometric distribution of different constellations and test systems, the relationship is shown in Fig.3. under the conditions of the altitude angle $\theta = \frac{\pi}{12}$, $\theta = \frac{\pi}{3}$, $\theta = \frac{5\pi}{12}$.

Fig. 3. Relations between right-handed rotating circular polarized wave reflection and permittivity under different heights
It can be seen from Fig.3 that, when the soil moisture is same (i.e., the permittivity is constant), the greater the altitude angle is, the larger left circularly polarized wave reflectivity is; when the reflectivity is the same, larger the incident angle (the altitude angle is smaller) represents larger permittivity. Moreover, when the altitude angle is over $\frac{\pi}{3}$, the effects of altitude angle on the relationship between the reflectivity and the permittivity are not significant. In order to take the number of visible satellites into account and to receive the reflected signal energy as much as possible, the height of the angle should be suitable for the value of the altitude angle.

4. Discussion

GNSS earth humidity detection is the inverse problem of navigation and positioning system. Based on the analysis of the influence of soil composition on the reflection Fresnel reflection coefficient, this paper proves that the distinguishing degree of signal reflectance between the surface of the loess and the sand is not obvious, and the humidity is the main factor affecting the strength of the reflected signal, which serves as the solid ground for simplifying the complex components of the soil and the rationality of the earth's surface humidity detection with "pure loess" instead of earth. Based on the polarization characteristics of the reflected signal in GNSS-R land-based remote sensing, this paper concludes that the polarization of the large-angle corners is mainly a vertical polarization, while polarization is mainly about left-handed circular components. Moreover, the reflectivity increases with increasing humidity at the same altitude angle.

When the simulated permittivity $\varepsilon = 4$ and altitude angle $\theta \geq \frac{\pi}{3}$, the main reflection energy and the main polarization wave ratio are more than 60%; when the altitude angle $\theta = \frac{5\pi}{12}$, $\delta$ is more than 90%. When the altitude angle is over $\frac{\pi}{3}$, the altitude angle has no significant effect on the relationship between the reflectivity and the permittivity.

5. Conclusion

In this paper, the theoretical model and test model of the permittivity of GNSS are established by the theoretical analysis of the relationship between permittivity, reflectance and the influence of wave source angle on permittivity. The model shows that the larger the permittivity is, the more sensitive the reflectivity is. And it is easy to cause a large permittivity measurement error. When the altitude angle is over a certain value, the altitude angle on the relationship between the reflectivity and permittivity is not significant. There is a suitable angle of incidence and altitude angle for different permittivity GNSS detection. Therefore, by preliminary estimation of the permittivity, the geometric layout of the detection equipment and satellite constellation selection can be made based on the model, which proves practical significance to improve the measurement accuracy.

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