1. INTRODUCTION

The Lyman-α (Lyα) emission line can be used to detect and study galaxies up to high redshifts (for reviews see, e.g., Barnes et al. 2014; Dijkstra 2014; Hayes 2015). Because Lyα is a resonant line, an Lyα photon will scatter frequently before reaching us. This implies that Lyα photons “sample” a wider region than merely that from where they were emitted. Lyα photons might thus contain unique information on the interstellar, circumgalactic, and even intergalactic medium (ISM, CGM, and IGM, respectively). However, it is currently unclear how well we can extract this information from observations.

The theory describing Lyα radiative transfer has been studied for decades and for a range of gas geometries. Neufeld (1990) performed an analytic study of Lyα transfer through a uniform, static, semi-infinite “slab” (later transferred to spherical and cubical geometries by Dijkstra et al. 2006 and Tasitsiomi 2006, respectively). Loeb & Rybicki (1999) also presented analytic solutions for Lyα transfer through a homogeneous, neutral, zero-temperature intergalactic medium. Monte-Carlo (MC) codes enabled studies of Lyα transfer through arbitrary gas configurations. These include—with increasing complexity—spherically symmetric clouds (Zheng & Miralda Escudé 2002; Dijkstra et al. 2006), an outflowing shell (Ahn et al. 2003; Verhamme et al. 2006; Schaerer et al. 2011), conical outflows (Dijkstra & Kramer 2012; Behrens et al. 2014; Zheng & Wallace 2014), clumpy ISM models (Neufeld 1991; Hansen & Oh 2006; Dijkstra & Kramer 2012; Laursen et al. 2013; Duval et al. 2014), and geometries from N-body and/or hydrodynamical simulations (e.g., Laursen & Sommer-Larsen 2007; Kollmeier et al. 2010; Zheng et al. 2011; Behrens & Braun 2014; Smith et al. 2015).

Lyα transfer on interstellar scales is a complex problem which depends sensitively on the distribution and kinematics of neutral gas. Observations indicate that outflows of neutral gas promote Lyα escape (Kunth et al. 1998; Atek et al. 2008; Steidel et al. 2010). The existence of cold (T ~ 10^4 K), neutral hydrogen gas in outflows with velocities of a few hundreds of km s^{-1} implies that these outflows likely were multiphase. Addressing the problem of modeling Lyα radiative transfer through the multiphase ISM is beyond current models, as it would require first-principle models for both the multiphase ISM and stellar feedback processes (e.g., Fujita et al. 2009 or Verhamme et al. 2012, who put an upper limit of ~20 pc on the required spatial resolution required for Lyα radiative transfer on hydrodynamical simulations). Both of these processes lie at the heart of understanding galaxy formation, and will be the subject of intense research for at least the next decade.

Lyα radiative transfer can be represented with simplified subgrid models for interstellar radiative transfer: the simple “shell model” reproduces a wide variety of observed spectra with only five parameters (e.g., Hashimoto et al. 2015; Martin et al. 2015; Verhamme et al. 2015; Yang et al. 2015). The simplicity of the shell model allows us to understand the radiative transfer process and the impact of each parameter on it in great detail. On the other hand, the shell model is clearly an over-simplification of the problem. In particular, the connection between the shell-model parameters and the actual physical properties of the scattering medium is not well understood. An alternative model is the “clump” model, which consists of cold, moving clumps of neutral hydrogen embedded within a static, hot inter-clump medium (ICM). The clump model is theoretically motivated by the expectation that the ISM is multiphase (Field et al. 1969; McKee & Ostriker 1977). Neufeld (1991) first introduced this model to explain Lyα escape from dusty media and to show that these models may enhance the Lyα equivalent width (the so-called “Neufeld effect,” see also Hansen & Oh 2006). This effect was later studied in more detail by Laursen et al. (2013), Duval et al. (2014), and Gronke & Dijkstra (2014), who all concluded that this effect is unlikely to happen in a more realistic environment due to the penetration of the Lyα photons in the (moving) clouds. Clumpy outflows have also been used to explain Lyα absorption and emission around star-forming galaxies (Steidel et al. 2010; Dijkstra & Kramer 2012), and...
there is increasing evidence for the existence of numerous cold, dense clumps of H\textsubscript{I} gas within the circumgalactic medium of massive galaxies (Cantalupo et al. 2014; Hennawi et al. 2015). In spite of the popularity of clump models, Ly\alpha radiative transfer studies have thus far focused mainly on the escape of Ly\alpha, but not on the spectral shape.\footnote{An exception to this is Duval et al. (2014), who touched upon Ly\alpha spectral shapes as well. However, as they restricted their analyses to “clumpy shells,” they only focused on a small subset of clumpy outflow models.} In this work, we focus in particular on three main points.

1. What range of Ly\alpha spectral shapes can we reproduce with the “clumpy” models?
2. Which of the model parameters predominantly affect the emergent Ly\alpha spectrum?
3. Can shell models reproduce spectra obtained from clumpy models? If so, then how do shell-model parameters relate to those of the clumpy model?

The paper is structured as follows. In Section 2, we describe our method. We present our results in Section 3 and subsequently discuss them in Section 4. We conclude in Section 5. Moreover, we show additional results in the Appendix, and present the correlation between the Ly\alpha and ionizing photon escape fractions for the same models as in Dijkstra et al. (2016b).

2. METHOD

2.1. MC Radiative Transfer

In this work, we used the MC radiative transfer code \texttt{tlac}, which was already used in Gronke & Dijkstra (2014) and Gronke et al. (2015). We refer the reader to these papers for details but here we show the basics of the calculation.

In a nutshell, MC radiative transfer simulations work by following the trajectories of individual photon packages in space and frequency-space, thus slowly converging toward the solution. The relevant steps included are as follow:

1. the emission of photons, where the photon is assigned an initial random direction and frequency according to some probability density function (PDF);
2. the propagation of a distance \( d \), where \( \tau = \int_0^d (n_\text{H,I} + n_\text{He,I}) \sigma d\nu \) is a random variable drawn from an exponential PDF: in the above equation, \( n_\text{H,I} \) and \( \sigma \) denote the number density and cross-section of neutral hydrogen (dust), respectively;
3. the interaction of the photon with a particle where either a new frequency and direction are assigned, or the photon is absorbed;
4. the output of the photons’ relevant quantities once it has escaped the simulation box.

One difference to the approach in Gronke & Dijkstra (2014) is the use of a partial core-skipping scheme. In particular, we do not use core skipping in the ICM (\( x_{\text{crit}} = 0 \)) and use dynamical core skipping (similar to the technique used by Smith et al. 2015) inside the clumps.\footnote{Core skipping is an acceleration technique which works by forcing the photon into the wing of the line by drawing the scattered atom’s velocity from a truncated Gaussian with cutoff \( x_{\text{crit}} \). Inside the clumps, we use \( x_{\text{crit}} = 0 \) for \( a \tau_{\text{H,I}} < 1 \), and \( x_{\text{crit}} = (\tau_{\text{H,I}})^{1/3} / 5 \) otherwise. Here, \( a = 4.7 \times 10^{-1} (T/10^5 \text{K})^{-1/2} \) denotes the Voigt parameter and \( \tau_{\text{H,I}} \approx \tau_{\text{H,I}} \rho_{\text{H,I}} (x = 0) (r_{\text{cl}} - r) \), where \( r \) is the distance from the photon’s position to the cloud center and \( r_{\text{cl}} \) is the cloud radius.} We verified that the results are indistinguishable from the runs without non-core skipping.

2.2. The Clumpy ISM Model

The clumpy ISM parametrization used in this work is adapted from Laursen et al. (2013) and has already been used in Gronke & Dijkstra (2014). We therefore refrain from describing the parameters again in great detail. A brief overview of the parameters is given below.

1. The geometry of the setup is described by the radius of the simulation sphere \( r_{\text{gal}} = 5 \text{kpc} \), the cloud radius \( r_{\text{cl}} \), and the covering factor \( f_{\text{cl}} \), which is the number of clouds on average passed from the center.
2. The content of the cold (hot) clumps (inter-clump medium, or ICM) is given by \( T_{\text{cl}} \), \( n_{\text{H,I}} \), \( T_{\text{ICM}} \), \( n_{\text{H,ICM}} \) for temperature\footnote{Using simplifying assumptions, this can be related to the metallicity via \( T_{\text{H}} \approx Z / Z_\odot \delta_p \), where \( Z_\odot \approx 1.58 \times 10^{-21} \text{cm}\text{H}^{-2} \) (Pei 1992; Laursen et al. 2009).} and the number density of hydrogen, respectively. We express the effect of dust in terms of dust cross-section per hydrogen atom, i.e., \( \sigma_d \equiv \tau_d / (n_\text{H,I} d) \), where \( \tau_d \) is the dust optical depth and \( d \) is the path-length considered.\footnote{One might argue that because of the isotropic initial direction of the photons, one should replace \( \tau_{\text{H,ICM}} \) with \( \tau_{\text{gal}} \) in the above expressions. However, as Ly\alpha turns out to escape via lower column densities, we choose to use Equation (1)—which can be seen as a lower limit of a hydrogen column density assigned to a clumpy ISM.} Furthermore, we choose to parametrize the dust content in the ICM as \( \zeta_d \equiv \sigma_d \text{ICM}/\sigma_d \text{cl} \). These number densities lead to a theoretical column density which a photon has to cross before escaping the simulation box, given by

\[ N_{\text{H,I}} = n_{\text{H,ICM}} r_{\text{gal}} - H_{\text{em}} + \frac{4}{3} \tau_{\text{cl}} (n_{\text{H,ICM}} - n_{\text{ICM}}), \]

where \( \tau_{\text{cl}} = f_{\text{cl}} \tau_{\text{gal}} - H_{\text{em}} \) is the reduced covering factor which reflects that Ly\alpha photons are emitted throughout the cloud, rather than in its center\footnote{The temperature given can be seen as an effective temperature with the effects of turbulence included.}, and \( H_{\text{em}} \) is the scale length of the emission radius as described below.

3. The emission properties of the photons are controlled with \( \sigma_i \) and \( P_{\text{cl}} \), which give the intrinsic width of the line and the probability that a photon is emitted within a cloud, respectively. The emission site is drawn randomly from \( r \sim \exp(-r/H_{\text{em}}) \), where \( r \) is the distance to the center of the simulation domain.

4. In addition, the clouds move with (i) an isotropic velocity component with each component drawn from a Gaussian with standard deviation \( \sigma_{\text{cl},i} \), and (ii) a radial velocity component given by

\[ v(r) = v_{\text{esc,cl}} \left\{ 1 - \left( \frac{r}{r_{\text{min}}} \right)^{-1} \right\}^{1/2} \]

for \( r > r_{\text{min}} = 1 \text{kpc} \), and otherwise zero.

This leaves us with 14 free parameters\footnote{Five might argue that because of the isotropic initial direction of the photons, one should replace \( v_{\text{gal}} \) with \( v_{\text{cl}} \) in the above expressions. However, as Ly\alpha turns out to escape via lower column densities, we choose to use Equation (1)—which can be seen as a lower limit of a hydrogen column density assigned to a clumpy ISM.} which are listed in Table 1. There we also provide our fiducial parameters which define our fiducial model. We chose these values to be centered on what Laursen et al. (2013) call “realistic parameters,” with the exception of the outflow velocity \( v_{\text{esc,cl}} \) for which Laursen et al. (2013) deliberately chose small values.

5 Note that the parameters given here differ slightly from what we used in Gronke & Dijkstra (2014). There we ignored the filling of the ICM since we were interested in (the enhancement of the) the Ly\alpha escape fraction.
Table 1
Overview of the Model Parameters

| Parameter          | Description                                          | Fiducial Value | Allowed Range | Units |
|--------------------|------------------------------------------------------|----------------|---------------|-------|
| \( v_{\text{cc,cl}} \) | Radial cloud velocity                                | 100.0          | [0.0, 800.0]  | km s\(^{-1}\) |
| \( r_{\text{cl}} \)  | Cloud radius                                         | 100.0          | [30.0, 200.0] | pc    |
| \( P_{\text{cl}} \)  | Probability to be emitted in cloud                   | 0.35           | [0.0, 1.0]    | ...   |
| \( H_{\text{exp}} \) | Emission scale radius                                 | 1000.0         | [500.0, 3.0 \times 10^4] | pc    |
| \( f_{\text{c}} \)   | Cloud covering factor                                | 3.5            | ...           | ...   |
| \( T_{\text{ICM}} \) | Temperature of ICM                                    | \( 10^6 \)     | [3.0 \times 10^5, 5.0 \times 10^7] | K     |
| \( n_{\text{H,ICM}} \) | \( ^{\dagger} \) H \text{I number density in ICM}  | \( 5.0 \times 10^{-4} \) | [10^{-12}, 10^{-6}] | cm\(^{-3}\) |
| \( \sigma_{\text{v}} \) | Width of emission profile                            | 50.0           | [5.0, 100.0]  | km s\(^{-1}\) |
| \( T_{\text{d}} \)   | Temperature in clouds                                 | \( 10^4 \)     | [5.0 \times 10^3, 5.0 \times 10^7] | K     |
| \( \beta_{\text{cl}} \) | Steepness of the radial velocity profile             | 1.5            | [1.1, 2.5]    | ...   |
| \( \delta_{\text{d,cl}} \) | Dust content in clumps                               | \( 3.2 \times 10^{-22} \) | [4.7 \times 10^{-24}, 1.6 \times 10^{-21}] | cm\(^2\) |
| \( \zeta_{\text{d}} \) | Ratio of ICM to cloud dust abundance                 | 0.01           | [10^{-4}, 0.1] | ...   |
| \( \sigma_{\text{d}} \) | Random cloud motion                                  | 40.0           | [5.0, 100.0]  | km s\(^{-1}\) |
| \( n_{\text{H,cl}} \) | \( ^{\dagger} \) H \text{I number density in clouds} | 0.35           | [0.03, 0.3]   | cm\(^{-3}\) |

Note. Variables marked with \(^{\dagger}\) were drawn in log-space (see Section 2.2).

Equipped with this model parametrization, we assembled a library of 2,500 spectra (using \(~10,000\) escaped photons each). We drew each parameter uniformly within the allowed range listed in Table 1, which is loosely based on the “extreme” range of Laursen et al. (2013). Note that \( n_{\text{H,ICM}}, n_{\text{d,ICM}}, T_{\text{ICM}}, f_{\text{c}}, \delta_{\text{d,cl}}, \) and \( \zeta_{\text{d}} \) were drawn in log-space (marked with \(^{\dagger}\) in Table 1).

### 2.3. Spectra Characterization

The simulation output is binned using the Freedman & Diaconis (1981) rule to optimize the relative error per bin without being too sensitive to outliers. We assigned to each bin the conservative error of 1.3 times the Poisson error with a minimum error in each bin of the mean error of the Poisson error of the spectrum. This choice can be interpreted as adding a systematic (instrumental) error to the spectrum and ensures that the fitting procedure (see Section 2.4) is not controlled by individual bins with a very small error (e.g., with no flux), and widens the set of “good fits” as well as the uncertainty on the model parameters. As both turn out to be quite small (see Section 3.3), we think the additional error is not overestimated.

The resulting Ly\(\alpha\) spectra are classified as single-, double-, or triple-peaked spectra (we did not find any spectra with more than three peaks in our data set) using a peak detecting algorithm. The algorithm\(^8\) flags a peak (a valley) if the following \( N_{\text{pd}} \) data points are at least a value of \( \delta_{\text{pd}} \) times the error in this region smaller (greater) than the candidate. For our purpose, we executed the algorithm for \( N_{\text{pd}} = (1, 2, 3, \ldots, 10) \) with the final result being the median of the detected number of peaks (under the constraint that the number of valleys found is one less than the number of peaks found). This procedure—which we verified visually using \(~10\%) of the spectra—ensures a robust characterization.

Once the peaks and valleys of a given spectrum are detected, the spectral shape can be characterized by a number of extracted parameters. Although a unique characterization does not exist, we will adapt the commonly used quantities \( F_{\text{blue}} \) (\( F_{\text{red}} \)) and FWHM\(_{\text{blue}} \) (FWHM\(_{\text{red}} \)) for the maximum flux and the FWHM of the blue-most (red-most) peak, respectively.

Furthermore, we analyze \( v_{\text{blue}} \) (\( v_{\text{red}} \)) and \( L_{\text{blue}} \) (\( L_{\text{red}} \)), which define the position and integrated flux of these peaks.

#### 2.4. Shell-Model Fitting

The shell model commonly used in Ly\(\alpha\) radiative transfer studies consists of a central, luminous source surrounded by an outflowing shell of hydrogen and dust. This geometry is described by five parameters: the width of the intrinsic emission line \( \sigma_{\text{v}} \), the outflow velocity \( v_{\exp} \), the (effective) temperature \( T \) (which includes a Doppler component due to turbulence), and the hydrogen column density and dust optical depth of the shell, \( N_{\text{H,I}} \) and \( \tau_{\text{d}} \), respectively. In order not to confuse these parameters with the ones from the clumpy model (presented in Section 2.2), we denote the shell-model parameters with a superscript \(^{\text{sm}}\) or \(^{\text{shell model}}\).

The shell model has been used to fit observed Ly\(\alpha\) spectra (e.g., recently by Patrício et al. 2015; Verhamme et al. 2015; Yang et al. 2015) with surprising success, given the simplicity of the model compared to the complexity of galactic structures. In the following, we use the improved pipeline described by Gronke et al. (2015) to fit shell-model spectra to Ly\(\alpha\) spectra obtained from the clumpy model. In short, the systematic pipeline consists of a library of 12,960 spectra covering the three discrete parameters \( T, N_{\text{H,I}}, v_{\exp} \) whereas the other parameters are modeled by weighting each photon individually (see Gronke et al. 2015, for details). In order to fit a spectra, two steps are performed: (i) the global maximum is found using a basin-hopping algorithm (Wales & Doye 1997) where we use the Jones et al. (2001) implementation, and (ii) to find the uncertainties as well as the degeneracies between parameters around that maximum, we use the affine-invariant MC sampler \texttt{emcee} (Goodman & Weare 2010; Foreman-Mackey et al. 2012).

Compared to the version described in Gronke et al. (2015), we improved the fitting pipeline in two ways. First, step (i) is now performed on the discrete and continuous parameters in an alternating fashion (i.e., the basin-hopping algorithm on the discrete parameters calls a second basin-hopping algorithm on

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\(^8\) In order to be able to better sample a multi-modal landscape, we also used the parallel tempered sampler. However, since the local maxima differ usually by \( \log p \lesssim 5 \), we returned to the vanilla sampler.
show the median and the difference to the 16th and 84th percentile to illustrate the asymmetric distribution). As expected from an outflow, the red peak is more pronounced than the blue peak, which is quantified with the ratios determined for the integrated fluxes $L_{\text{blue}}/L_{\text{red}} = 1 - 0.63^{+0.20}_{-0.39}$ and the maximum fluxes $F_{\text{blue}}/F_{\text{red}} = 1 - 0.62^{+0.40}_{-0.20}$. In addition, the location of the red peak is further away from line center than its blue counterpart ($792.32^{+455.62}_{-402.87}$ km s$^{-1}$ versus $-218.86^{+335.05}_{-224.15}$ km s$^{-1}$). This shift is also present in the single-peaked spectra where the peak position is $15.39^{+30.12}_{-740.22}$ km s$^{-1}$.

Figure 2 focuses on an interesting correlation of double-peaked spectra, namely, between the peak separation and the ratio of the mean fluxes of peaks and valleys. The one- and two-dimensional projections of the distribution reveal that clumpy models fill a large portion of the plane spanned by the two observables. This is particularly interesting as double-peaked shell-model spectra are mostly restricted to $F_{\text{valley}} \lesssim 0.1F_{\text{peaks}}$. We obtained this result by drawing 1000 random shell-model spectra from the parameter range described in Section 2.4 and show their features in Figure 2 for comparison. This difference might be useful to distinguish between the two models (also see Section 3.3).

3.2. Sensitivity of Spectral Shapes to Multiphase Medium

In order to analyze what correlations exist between the model parameters and the spectral shape parameters, we performed a “lasso” analysis (Tibshirani 1996) using the python library skikit-learn (Pedregosa et al. 2011).10 Figure 3 shows the summarized results of the “lasso” analysis with the darker color corresponding to a more relevant parameter for a given observable. This Figure shows, for example, that $f_{\text{cl}}, T_{\text{ICM}}$, and $n_{\text{ICM}}$ are the parameters that most

\footnote{The “lasso” works by adding to the usual linear minimization $1/(2N) \sum (y_i - \hat{y}_i)^2$ a penalty of $\alpha \sum_i |\hat{\beta}_i|$ where $|\hat{\beta}_i|$ denotes the L1-norm of the vector $\hat{\beta}$. This leads to the sparsity of $\hat{\beta}$, i.e., the reduction of relevant parameters. This analysis is repeated for a number of values for $\alpha$ and different sub-samples of the data. The measure of “relevance” is then the fraction of times a parameter was included in $\hat{\beta}$.}

9 Note that we omitted the “core-skipping” technique for models with $\alpha(T)N_{H_1}n_{H_1}(v = 0, T) < 200$ and re-computed the models required.

3. RESULTS

3.1. Characteristics of Spectra

Varying the parameters described in Section 2.2 gives rise to a wide variety of Ly$\alpha$ spectra. Figure 1 visualizes this fact by superimposing four particular spectra (in color) over 200 randomly drawn spectra. The four selected spectra feature the double-peaked fiducial spectrum (in red), a single-peaked spectrum (blue), an asymmetric triple-peaked spectrum (in green), and a wide double-peaked profile (in purple).

From our sample of $\sim2500$ spectra, we found that $77\%$ are single peaked, $\sim23\%$ double peaked, and $\lesssim1\%$ triple peaked. We caution, however, that the characterization can be very difficult in some cases. Even within the subset of double-peaked profiles, there is a big variation in the considered spectral parameters (see Section 2.3). For example, the peak separation $v_{\text{red}} - v_{\text{blue}}$ has a value of $1007.70^{+506.33}_{-488.24}$ km s$^{-1}$ (where we
The scores were computed using “lasso” analysis as described in Section 3.2 and reflect how strongly a parameter affects an observable.

Figure 3. Relevance score of the model parameters fitting the spectral features of the double-peaked spectra. The scores were computed using “lasso” analysis as described in Section 3.2 and reflect how strongly a parameter affects an observable.

Figure 4 shows examples of how parameters affect certain spectra characteristics. In particular, we plotted the position of the blue peak of the spectrum versus a certain model parameter. Note that for single-peaked spectra (marked with circles), the position of the blue peak is identical to the position of the only peak. In addition, we show the fraction of double-peaked spectra above the scatter plot. In the left panel of Figure 4, no correlation can be seen between the total column density (see Equation (1)) and the position of the blue peak as well as the spectral type. In this panel, we color-coded the value of $f_{cl}$ which affects $N_{H}$, most strongly. For comparison, the right panel shows an example of a nice correlation between the position of the blue peak and the content of the ICM. Clearly, a hotter ICM leads to a shift of the blue peak if $n_{H, ICM} \gtrsim 3 \times 10^{18} \text{ cm}^{-3}$ as well as an tremendous increase in double-peaked spectra. This shows visually the great relevance of $\log T_{ICM}$, $n_{H, ICM}$, and also the mixed term $\log T_{ICM} \times n_{H, ICM}$.

3.3. Shell-Model Fits

Figure 5 shows the results of the shell-model fitting (described in Section 2.4) for 60 randomly chosen spectra (of which 20 are double-peaked and 40 are single-peaked spectra). In the figure, the marker symbol stands for the spectral type, the color of the marker illustrates the quality of the fit, and in each panel we plot the recovered shell-model parameter (with its uncertainties) against a particular clumpy model parameter. Overall, we found that roughly one-third of the spectra were reasonably well recovered ($\rho(\chi^2) > 0.01$, in 18 cases).

In the upper left panel of Figure 5, we show the recovered shell-model column density against the calculated column density of the clumpy model (see Equation (1)). This shows that the correct column density is generally not recovered—indepedent of the type of spectrum or the quality of the fit. On average, $N_{H}$ is underestimated by 0.5–1 orders of magnitude. For individual spectra, however, the offset can be as large as 3 dex in either direction. Only in $\sim$15% of the cases was the correct value recovered.

The upper right panel of Figure 5 shows the recovered expansion velocity of the shell versus the $v_{sc,cl}$ parameter (see Equation (2)). Since $v_{exp}^{sm}$ is a fixed velocity for all of the hydrogen in the system whereas $v_{sc,cl}$ is the limiting bulk cloud velocity, we expected to find $v_{exp}^{sm} \lesssim v_{sc,cl}$. This is also the case for some of the analyzed spectra where we found values of $v_{exp}^{sm} \sim 0 \text{ km s}^{-1}$. Interestingly, this group consists only of double-peaked spectra. However, another big fraction of the spectra preferred very high values of $v_{exp}^{sm} \sim 500 \text{ km s}^{-1}$. Here, 500 km s$^{-1}$ is also the limit of our grid of shell models (see Section 2.4), and therefore we can expect even larger values for the best-fit values of $v_{exp}^{sm}$. In this group of models, we find single- as well as double-peaked profiles with input parameters covering the whole allowed range of $v_{sc,cl}$. Also, the fit quality is wide-spread—indepedent of the spectral type or the particular value of $v_{sc,cl}$.

The lower row of Figure 5 shows the temperature and the intrinsic spectral width relations (in the left and right panels, respectively). As is commonly known (see, e.g., Schaerer et al. 2011; Gronke et al. 2015), the shell-model spectral shapes are generally not very sensitive to the (effective) temperature of the shell, and hence the large uncertainties on $T_{eff}^{sm}$. The added value of the recovered temperatures and $\sigma_i$ is somewhat questionable as the geometries and emission sites are quite different. It is, however, interesting that (i) no clear clustering or correlation is visible, i.e., the spectral types and fit qualities seem to be well mixed, and (ii) the values of $\sigma_i$ are generally overestimated by a factor of a few. In particular, the latter point can be understood as emission within (randomly moving) clouds leading to an effective broadening of the intrinsic spectrum.

The red and blue rectangles drawn in Figure 5 highlight examples which are shown in Figure 6. In particular, Figure 6 shows the clumpy model spectra in red and the best-fit shell-model spectra in blue. In order to illustrate the uncertainty of the shell-model fits, we also display 25 spectra which are randomly drawn from the burnt-in MC, and thus distributed as given by the sampled likelihood function. We choose the two spectra because they help us to illustrate particular problems with shell-model fitting (see Section 4.2).
Figure 4. Position of the blue-most peak vs. the total column density (left panel) and vs. the number density of the ICM (right panel). In addition, the colors denote the values of $f_{cl}$ and $T_{ICM}$, respectively. Circles, squares, and triangles represent single-, double-, and triple-peaked spectra, respectively.

Figure 5. Results of the shell-model fitting as described in Section 3.3. On the x-axis, the (clumpy model) input parameters are given, and on the y-axis are the recovered shell-model parameters. The marker and errorbars denote the 50th, 16th, and 84th percentiles, respectively. Additionally, circles, squares, and triangles represent single-, double-, and triple-peaked spectra, and the color shows the quality of the fit. The gray dashed line marks the one-to-one relation and the gray arrows indicate if a data point lies outside the range displayed. Note that the spectra of the points surrounded by squares are shown in Figure 6.
In this section, we discuss the impact of our results. In particular, we focus on the set of spectral shapes achievable through our clumpy model parametrization in Section 4.1, we comment on the connection to the shell model in Section 4.2, and we transition to the observational side in Sections 4.3 and 4.4.

4.1. Sensitivity of the Lyα Spectrum to the Clump and ICM Properties

Our results show that the emergent Lyα spectrum is more sensitive to the ICM than the cloud parameters. To first order, this is already visible when comparing the left and right panels of Figures 8 and 9 in the Appendix, where we varied the temperatures and hydrogen contents in the clouds and ICM separately. The same result becomes apparent in the more detailed analysis presented in Section 3.2 (see Figures 3 and 4): $n_{H, ICM}$ and $T_{ICM}$ play a much more important role than $n_{H, cl}$ and $T_{cl}$, respectively. One should note, however, that this is only true for $n_{H, ICM} \geq 3 \times 10^{-8}$ cm$^{-3}$, as shown in Figure 4. This critical value of $n_{H, ICM}$ reflects the extent of the system (5 kpc) and the allowed ICM temperature range (see Table 1), which translate approximately to an optical depth at line center of $\tau_{\alpha} \approx 0.4$–5, i.e., where the ICM becomes optically thick for line-center photons. Naturally, this critical value also depends on other parameters, such as $f_{cl}$ or the cloud motion. Interestingly, this value lies within the “realistic” parameters of Laursen et al. (2013), and corresponds closely to the $n_{H, cl}$ of the hot ionized medium in the McKee & Ostriker (1977) picture (assuming collisional ionization equilibrium).12

Two notable exceptions to this rule are the cloud covering factor $f_{cl}$ and the dust content of the clouds $\bar{d}_{cl}$. The former plays a major—if not the most important—role in determining the spectral shape.

The parameter $f_{cl}$ enhances the total ICM column density encountered by Lyα photons in the ICM as $N_{H, ICM} = f_{cl} n_{H, ICM} (f_{cl} + 4/5)$ (see Hansen & Oh 2006). In addition, the movement of the clouds (random motions and/or ordered outflowing motions) provides an efficient way of transferring Lyα photons into the wings of the line profiles. We found this second effect to be most important by varying $f_{cl}$ from 0.8 to 8 in a static setup (i.e., $v_{\infty, cl} = c_{cl} = 0$) and found—in stark contrast to the left panel of Figure 12—only minor changes in the spectral shape.13

The dust content of the clumps is closely tied to the importance of $f_{cl}$. An increased value of $\bar{d}_{cl}$ shifts the weight of the emergent spectrum toward photons which did not experience many scattering events within clumps. Therefore, increasing $\bar{d}_{cl}$ provides an efficient way of decreasing the effective covering factor. This is apparent when comparing the right panels of Figure 10 and the left panel of Figure 12, where the increase of $\bar{d}_{cl}$ as well as the decrease of $f_{cl}$ lead to the disappearance of the extended red wing of the spectrum.

The result that the ICM plays such a strong role in shaping the emerging spectrum may be surprising as it apparently supports the model of Neufeld (1991) and Hansen & Oh (2006). In their picture, Lyα photons “reflect off” the surface of the clumps, which naturally minimizes the exposure to H$\alpha$ inside the cold clumps and maximizes the exposure to H$\alpha$ in the ICM. Laursen et al. (2013) refuted this model as, in a more realistic environment (mainly with moving clouds), the Lyα photons penetrate deeper into the clouds, scatter there many times, and are potentially destroyed. However, for the emergent spectrum, this does not play a role as the scatterings in the ICM

12 As the assumed HIM temperature of McKee & Ostriker (1977) is on the lower limit of our allowed range, one might conclude that we mostly overestimated the ICM hydrogen number density. However, two factors should be taken into account: (i) the ISM might be dominated by gas which is out of equilibrium (e.g., Walch et al. 2015), and (ii) the temperatures are effective temperatures with small-scale turbulence included, whereas the ionization relies on the absolute temperature.

13 Note that only movement of the clouds is needed, not structured movement. That is, in a second test where only $v_{\infty, cl} = 0$ but $\sigma_{cl}$ was left at its fiducial value the covering factor did affect the spectrum.
place the photon’s frequency far into the wing (in the clouds’ frame). Conversely, many scatterings in the cloud leave the photon still in the core (in the ICM frame). This effect can be illustrated by comparing the hydrogen column densities experienced by Lyα photons $\tilde{N}_{H_1}$ of two models with $n_{H_1,\text{cl}} = 0.1 \text{ cm}^{-3}$ and $1 \text{ cm}^{-3}$ (and otherwise fiducial parameters), which are $\log_{10}(\tilde{N}_{H_1}/\text{cm}^{-2}) = 20.3^{+0.7}_{-0.4}$ and $20.9^{+1.3}_{-0.6}$ (using the notation of Section 3.1), respectively. Also, the number of clouds intercepted are about the same ($12^{-7}_{+10}$ versus $11^{-6}_{+7}$), which supports our explanation.

4.2. The (Un)Usability of Shell-model Fitting

Section 3.3 shows that the shell model can reproduce some of the spectra of the clumpy model, but generally cannot be used to infer the physical parameters of the host system as the inferred shell-model parameters can be several orders of magnitudes off—indeed independent of the quality of the fit.

The tension between $N_{H_1}$ and $N_{H_1}^{\text{cl}}$ cannot be resolved if the latter is instead compared to the median of the actually experienced hydrogen column density $\bar{N}_{H_1}$, the analytically computed “experienced ICM column density” $\bar{N}_{H_1,\text{ICM}} = n_{H_1,\text{ICM}} f_\text{cl} (f_\text{g} + 4/5)^{14}$, or only the ICM column density $N_{H_1,\text{ICM}} = n_{H_1,\text{ICM}} f_\text{cl}$. The reason for these mismatches is well illustrated by the simplified model discussed in Section 4.1: the hydrogen column density experienced by the photons (or, the one that is given by the geometry) is dominated by the dense, cold, neutral medium, i.e., the clumps. However, the spectral shape is predominantly given by the (few) scattering events occurring in the hot, ionized medium (the ICM). This is in sharp contrast to the shell models which contain only one column density.

An immediate implication of this is the difference in the minimal flux between the peaks for double-peaked profiles. Figure 2 shows that this is $\sim 0$ for most shell models while clumpy models allow much greater values—due to escaping photons which experienced very little neutral hydrogen. As the right panel of Figure 6 shows, this is one of the reasons why the shell model cannot reproduce the symmetric, double-peaked profiles of the clumpy model.

Photons that encountered low column densities of H I can escape close to the line center (or even at line center). Behrens et al. (2014) already highlighted the importance of low-column-density channels for the escape of Lyα of galaxies (see also Figure 4 of Dijkstra et al. 2016a, for a simple demonstration of this effect). As these low-density escape routes—which govern the resulting shape of the Lyα spectrum—do not exist within the shell model, it may not be surprising that the actual properties (e.g., the column density) of the host system and those inferred from shell-model fitting do not match. Hence, one has to be cautious when assigning physical meaning to the unconverted shell-model parameters.

On the other hand, the power of the shell-model fitting should not be underestimated. As has been shown in numerous studies (e.g., Verhamme et al. 2012; Yang et al. 2015), the shell model can fit the majority of Lyα spectra observed—with relatively few free parameters to an astonishing accuracy. We have clearly demonstrated that there is no simple conversion from shell-model parameters to parameters describing the clumpy medium. The fact that shell models are so successful at reproducing data suggests that there may still be a connection, but at a more subtle level. We will explore this in future studies. However, it is already clear (as we laid out in Section 4.1) that this conversion must not ignore the crucially important hot ionized medium.

4.3. Comparison to Observations

As already pointed out several times in this paper, the shell model can reproduce observed Lyα spectra. In this work, we have shown that the shell model can also reproduce a subset of the clumpy model spectra. However, whether or not the clumpy model can reproduce observed spectra is still an outstanding question. Currently, we can only conjecture that this is the case—given the amount of free parameters and the established non-empty intersection with the set of shell-model spectra.

Yang et al. (2015) used the shell model to fit the Lyα spectra of the “Green peas” (high-redshift analogs at $z \sim 0.1$–0.3). They could reproduce 9 out of 12 spectra, while the remaining 3 have a valley position redward of the line center. The obtained expansion velocities (column densities) lie in the range $\sim 0$–350 km s$^{-1}$ ($\sim 10^{19}$–$10^{20}$ cm$^{-2}$). The shell-model fitting results of Hashimoto et al. (2015; 12 Lyα emitters at $z \sim 2.2$) show expansion velocities of $\sim 100$–200 km s$^{-1}$ and column densities of $\log_{10} N_{H_1}/\text{cm}^{-2} \sim 16$–20. These recent results, with high-quality Lyα spectra, confirm on the one hand the ability of the shell model to model observed spectral profiles remarkably well. This confirms the need to understand shell-model fitting in a broader picture. On the other hand, the spectra presented in Yang et al. (2015); and also see Kaluzny et al. 2012; Chonis et al. 2013) cannot be reproduced by the shell model might hint at the need for an extension of the modeling parameter space. Also, the shell model has difficulty reproducing the surface-brightness profiles of spatially extended Lyα sources (Barnes & Haehnelt 2010, but see Patrício et al. 2015).

A completely different interpretation of Lyα transfer was recently presented by Hagen et al. (2016), who interpreted their detection of 12 Lyα emitting galaxies (out of a sample of 63) in terms of an Lyα “opening angle.” In their model, this is the combined solid angle of holes in the ISM through which Lyα can escape easily. At first glance, this interpretation can be connected easily to clumpy outflows as the opening angle can be related to our properties via $\Omega_{\text{Lyα}} \approx 4\pi (1 - R_0) \exp (-f_\text{cl})$. However, we caution that the directional dependence of Lyα escape is weak: the directional dependence is set mostly by the last scattering event prior to escape, which causes the emerging radiation from clumpy models to be quite isotropic (see Gronke & Dijkstra 2014).

The fact that several observed Lyα spectra can be reproduced using shell-model spectra, and the fact that we found the overlap between the studied clumpy model and the shell model to not be very big, suggests that our current clumpy model parameterization is not sufficient to capture the full set of observer Lyα spectra. Possible extensions would be the introduction of a non-static ICM, temperature and density gradients in the ICM as well as in the clumps, a modification of...
the velocity profile, the introduction of one or several other phases, and the consideration of the galactic environment and the instruments (see Section 4.4 for the latter point). To conclude, the unification of the radiative transfer models and observations is still an outstanding issue.

4.4. Impact of the Galactic Environment and Instruments

As the setup we presented in this work represents a dusty, multiphase medium, the paths of actually observed Ly$\alpha$ photons differ in two main aspects from the spectra we simulated: (i) the photons have to pass through the immediate surrounding and the IGM before reaching us, and (ii) not all of the escaping flux of a galaxy is actually observed.

The impact of the IGM on spectra has been studied by several groups in the past (see, e.g., Dijkstra et al. 2007; Zheng et al. 2010; Laursen et al. 2011) with the conclusion that the blue side (up to $v \lesssim 100$ km s$^{-1}$) of the Ly$\alpha$ spectra can be strongly affected at redshifts $z \gtrsim 4$. However, this does not explain the different spectral shapes observed for lower redshifts (see Section 4.3).

In contrast to the IGM, the impact of the CGM on the Ly$\alpha$ spectra has not yet been studied systematically. This is partly due to the fact that the structure and kinematics of the CGM are highly complex and not fully understood. Observations show that neutral hydrogen can be found out to $\sim 300$ kpc for all galaxy types (Prochaska et al. 2011), and at least part of the CGM is in a multiphase state (Steidel et al. 2010). This picture is also supported by state-of-the-art hydrodynamical simulations (e.g., Shen et al. 2012) and allows us to put our work into the following context: (a) our simplistic, parametrized, multiphase medium can be seen as a sub-grid model for the ISM as well as (at least part) of the

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**Figure 7.** Impact of the environment and measurement apparatus on the emergent spectrum. The solid lines show the variation of the spectrum with the addition of a surrounding halo (using the ICM parameters), and the corresponding dashed lines show the spectrum with only the innermost 50% of the escaping flux. As examples, in the left panel, we show the fiducial spectrum (see Table 1), and in the right panel the double-peaked spectrum of Figure 6. See Section 4.4 for further details.

**Figure 8.** Impact of the gas temperature in the ICM (left panel) and in the clouds (right panel).

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CGM, and (b) possible further processing of the computed Ly$\alpha$ spectra might be necessary before comparing it to observations.

To mimic the effects of the galactic environment, we consider—as a first, crude approximation—a low-density, $\text{H}_1$ halo out to radius $r_{\text{gal}} + r_{\text{env}}$ filled with the same content as the
ICM. Figure 7 shows the changed fiducial spectrum (left panel) and double-peaked spectrum from Figure 6 (right panel) using these considerations. In this figure, the solid lines show the spectrum using all of the emergent photons and the dashed lines in corresponding colors show the spectra using only those photons within a certain impact parameter. Here, the cutoff was chosen so that half of the total escaped photons are used in the spectra. Note that the actual fraction varies and depends on (i) the redshift, (ii) physical properties of the object, and (iii) the instrument used (Steidel et al. 2010; Momose et al. 2016; Wisotzki et al. 2016). Figure 7 clearly shows that the galactic environment as well as the impact parameter cutoff have a (strong) effect on the spectrum. Possibly most interesting is the apparent suppression of the flux at the line center. This might be the key to the reconciliation of the clumpy model and the observed spectra. However, the environment considered here is over-simplified since in reality we expect density and temperature gradients (Pallottini et al. 2014; Suresh et al. 2015) as well as coherent gas motion (Bird et al. 2015). We leave this interesting topic to future studies.

5. CONCLUSION

We present a systematic study of Ly$\alpha$ spectra emerging from simplified models of multiphase outflows. While these models are well-motivated and have been used in Ly$\alpha$ radiative transfer studies before, spectra from these models have barely been analyzed.

Our main findings are the following.

1. Clumpy outflows give rise to a wide range of Ly$\alpha$ spectra, including spectra with high flux at the line center which are encountered less frequently with shell models (Section 3.1).

2. We demonstrate that in clumpy outflows, the key parameters that predominantly determine the emerging spectra are the covering factor $f_{cl}$ of clumps, the number density of HI in the hot ICM, and the temperature of the ICM. Interestingly, the radiative transfer process is less sensitive to the hydrogen content of the clumps. This result contrasts with the shell models where the average column density of the system is one of the most important parameters regulating outcome (Sections 3.2, 4.1).
3. We fit shell models to a subset of our clumpy models and find that, generally, the parameters of the best-fit shell models barely correlate with the physical parameters of the clumpy models (Sections 3.3, 4.2).

4. Shell models can fit only a small subset of clumpy outflow spectra well, which is partly because clumpy models allow for much more efficient escape of Lyα photons at the line center. These models agree better with the data if additional scattering in the CGM is invoked (Section 4.4).

This suggests that extracting physical information from shell-model parameter is less straightforward than previously thought. We therefore caution against overinterpreting the shell-model parameters until their physical meaning is understood better. We will be addressing this in future work.

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APPENDIX

IMPACT OF INDIVIDUAL PARAMETERS ON THE SPECTRAL SHAPE

Starting with our fiducial set of parameters (see Table 1, which we display in Figures 8–13 as black solid line), we varied each clumpy model parameter individually. The results are as follows.

1. Figure 8 shows the impact of the ICM and cloud temperatures (left and right panel, respectively). Whereas the temperature of the ICM has a strong impact on the spectral shape, the cloud temperature seems to play a minor role. Generally speaking, an increase in $T_{\text{ICM}}$ leads to a widening of the spectrum and a transition from a double- to a single-peaked spectrum.

2. We vary the hydrogen content of the ICM and in the clouds in Figure 9 (left and right panel, respectively). Above a value of $n_{\text{H}}_{1,\text{ICM}} \gtrsim 10^{-16}$ cm$^{-3}$, the ICM hydrogen number density affects the spectral shape. A greater value of $n_{\text{H}}_{1,\text{ICM}}$ leads to a deepening of the central valley and an increased peak separation. Contrary to that, $n_{\text{H}}_{1,\text{cl}}$ does not seem to affect the spectral shape.

3. We show the impact of dust in Figure 10. In the left panel, the dust content of the ICM was varied ($\zeta_{\text{d}} = \delta_{\text{d,ICM}}/\delta_{\text{g,cl}}$, which does not much affect the spectral shape. The dust content within the clumps, on the other hand, much more strongly affects the spectrum. With increasing dust optical depth, the spectrum becomes narrower and more symmetric, i.e., the red wing disappears. See Section 4.1 for a discussion of this effect.

4. Figure 11 shows the variation of the spectra due to the change in velocity parameters. All of them affect the emergent spectrum. The increase in random motion ($\sigma_{\text{cl}}$, left panel) results in a widening of the peaks (but not the peak separation). Having a stronger outflow ($v_{\text{c},\text{cl}}$, central panel) enhances the red peak and decreases the blue peak. It also widens the red peak. Similar effects can be observed when increasing $\beta_{\text{cl}}$ (right panel).

5. Figure 12 illustrates the impact of changing $f_{\text{cl}}$ (left panel) and the clouds’ radii ($r_{\text{cl}}$, right panel). Altering the covering factor has a major impact on the spectral shape (widening of the peaks with increased $f_{\text{cl}}$), whereas different values of $r_{\text{cl}}$ (while keeping the number of clouds constant) hardly change the spectra.

6. We display the spectra for different emission properties in Figure 13. None of them changes the spectrum significantly. We observe only a slight increase in the flux at line center for larger values of $H_{\text{em}}$ (right panel).

Although changing parameters individually is illustrative and the results are relatively easy to understand, we point out that these one-dimensional cuts through the 14-dimensional parameter space do not capture the full complexity of the problem. We therefore caution the reader not to overinterpret the results presented in this section.

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