Analysis of the vertices $D^*D^*P$, $D^*DV$ and $DDV$ with light-cone QCD sum rules

Zhi-Gang Wang
Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

Abstract

In this article, we study the vertices $D^*D^*P$, $D^*DV$ and $DDV$ with the light-cone QCD sum rules. The strong coupling constants $g_{D^*D^*P}$, $g_{D^*P}$, $f_{D^*DV}$, $f_{D^*D^*V}$, $g_{DDV}$ and $g_{D^*D^*V}$ play an important role in understanding the final-state interactions in the hadronic $B$ decays. They relate to the basic parameters $g$, $\lambda$ and $\beta$ in the heavy quark effective Lagrangian respectively. Our numerical values of the $g$, $\beta$ and $\lambda$ are much smaller than most of the existing estimations. If the predictions from the light-cone QCD sum rules are robust, the final-state interaction effects maybe overestimated in the hadronic $B$ decays.

PACS numbers: 12.38.Lg; 13.20.Fc

Key Words: Final-state interactions, light-cone QCD sum rules

1 Introduction

Final-state interactions play an important role in the hadronic $B$ decays [1, 2]. However, it is very difficult to take them into account in a systematic way due to the nonperturbative nature of the multi-particle dynamics. In practical calculations, we can resort to phenomenological models to outcome the difficulty. The one-particle-exchange model is typical (for example, one can consult Ref. [2]), in this picture, the soft interactions of the intermediate states in two-body channels with one-particle exchange make the main contributions. The phenomenological Lagrangian has many input parameters, which describe the strong couplings among the charmed mesons in the hadronic $B$ decays [2]. In the following, we write down the relevant phe-
nomenological Lagrangian \[2\],

\[
\mathcal{L} = -ig_{D^{*}DP} \left( D^{i} \partial^{\mu} P_{ij} D_{ij}^{\mu} - D_{ij}^{*} \partial^{\mu} P_{ij} D_{ij}^{\mu} \right) + \frac{1}{2} g_{D^{*}D^{*}P} \epsilon_{\mu\nu\alpha\beta} D_{i}^{\mu} \partial^{\nu} P_{ij} \left( \partial^{\alpha} - \overline{\partial}^{\alpha} \right) D_{j}^{*\beta} + ig_{DDV} D_{i} \left( \overline{\partial}_{\mu} - \partial_{\mu} \right) D_{j} V_{ij}^{\mu} + 2f_{D^{*}DV} \epsilon_{\mu\nu\alpha\beta} \partial^{\nu} V_{ij}^{\alpha\beta} \left( D_{i} \left( \overline{\partial}^{\mu} - \partial^{\mu} \right) D_{j}^{*\beta} - D_{i}^{*\beta} \left( \overline{\partial}^{\mu} - \partial^{\mu} \right) D_{j} \right) + ig_{D^{*}D^{*}V} \epsilon_{\mu\nu\alpha\beta} \left( \overline{\partial}_{\mu} - \partial_{\mu} \right) D_{j}^{\nu\mu} V_{ij}^{\mu} + 4i f_{D^{*}D^{*}V} D_{ij}^{*} \left( \partial^{\nu} V_{ij}^{\mu} - \partial^{\nu} V_{ij}^{\mu*} \right) D_{j}^{*\nu} ,
\]

(1)

\[
D^{*} = (D^{*0}, D^{*+}, D^{*}_{s}) ,
\]

\[
D = (D^{0}, D^{+}, D_{s}) ,
\]

\[
P = \begin{pmatrix}
\frac{\pi^{0}}{\sqrt{2}} + \frac{n}{\sqrt{6}} & \pi^{+} & K^{+} \\
-\frac{\pi^{0}}{\sqrt{2}} + \frac{n}{\sqrt{6}} & K^{0} & -\frac{\sqrt{2}}{2}\eta \\
\pi^{-} & K^{-} & \bar{K}^{0} \\
K^{0} & -\frac{\sqrt{2}}{2}\eta & \phi
\end{pmatrix} ,
\]

(2)

\[
V = \begin{pmatrix}
\frac{\rho^{0}}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^{+} & K^{*+} \\
-\frac{\rho^{0}}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} & \phi \\
\rho^{-} & K^{*-} & \bar{K}^{*0} \\
K^{*-} & -\frac{\sqrt{2}}{2}\eta & \phi
\end{pmatrix} ,
\]

where we take the convention $\epsilon_{0123} = 1$.

The strong coupling constants (for example, $g_{D^{*}DP}$, $g_{D^{*}D^{*}P}$, etc.) can be estimated with the heavy quark effective theory and chiral symmetry \[3\]. In the heavy quark limit, the strong coupling constants in the phenomenological Lagrangian can be related to the basic parameters $g$, $\lambda$ and $\beta$ in the heavy quark effective Lagrangian (one can consult Ref.\[3\] for the heavy quark effective Lagrangian and relevant parameters, here we neglect them for simplicity.),

\[
g_{D^{*}D^{*}P} = \frac{g_{D^{*}DP}}{\sqrt{M_{D^{*}}M_{D}}} = \frac{2}{f_{P}} g ,
\]

\[
f_{D^{*}DV} = \frac{f_{D^{*}D^{*}V}}{M_{D^{*}}} = \frac{\lambda g_{V}}{\sqrt{2}} ,
\]

\[
g_{D^{*}DV} = g_{D^{*}D^{*}V} = \frac{\beta g_{V}}{\sqrt{2}} ,
\]

(3)

where $g_{V} = 5.8$ from the vector meson dominance theory \[4\]. For existing estimations of the values of the $g$, $\lambda$ and $\beta$, one can consult Refs.\[5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\].

In previous work \[15\], we study the strong coupling constants of the $DDV$ and $D^{*}DV$ with the light-cone QCD sum rules, the numerical values of the $g_{DDV}$ and $f_{D^{*}DV}$ are much smaller than the existing estimations based on the vector meson
dominance theory [5]. In this article, we study the strong coupling constants $g_{D^*D^*P}$, $f_{D^*DV}$, $g_{DDV}$ with the light-cone QCD sum rules [3]. Furthermore, we analyze the corresponding parameters $g$, $\lambda$, $\beta$ in the heavy quark effective Lagrangian [3], and obtain the values of the strong coupling constants $g_{D^*D^*P}$, $f_{D^*DV}$ and $g_{DDV}$.

The light-cone QCD sum rules carry out operator product expansion near the light-cone, $x^2 \approx 0$, instead of short distance, $x \approx 0$, while the nonperturbative matrix elements are parameterized by the light-cone distribution amplitudes (which are classified according to their twists) instead of the vacuum condensates [16, 17]. The nonperturbative parameters in the light-cone distribution amplitudes are calculated with the conventional QCD sum rules and the values are universal [18].

The article is arranged as: in Section 2, we derive the strong coupling constants $g_{D^*D^*P}$, $f_{D^*DV}$ and $g_{DDV}$ with the light-cone QCD sum rules; in Section 3, the numerical result and discussion; and in Section 4, conclusion.

2 Strong coupling constants $g_{D^*D^*P}$, $f_{D^*DV}$ and $g_{DDV}$ with light-cone QCD sum rules

We study the strong coupling constants $g_{D^*D^*P}$, $f_{D^*DV}$ and $g_{DDV}$ with the two-point correlation functions $\Pi_{i\mu}^j(p,q)$, $\Pi_{i\mu}^j(p,q)$ and $\Pi_{ij}^l(p,q)$, respectively,

$$\Pi_{i\mu}^j(p,q) = i \int d^4x e^{-iqx} \langle 0 \mid T \{ J_\mu^i(0) J^{ij+}_{\nu}(x) \} \mid p_{ij}(p) \rangle, \quad (4)$$

$$\Pi_{ij}^l(p,q) = i \int d^4x e^{-iqx} \langle 0 \mid T \{ J_\mu^i(0) J^{ij+}_{\nu}(x) \} \mid V_{ij}(p) \rangle, \quad (5)$$

$$\Pi_{ij}^l(p,q) = i \int d^4x e^{-iqx} \langle 0 \mid T \{ J_\mu^i(0) J^{ij+}_{\nu}(x) \} \mid V_{ij}(p) \rangle, \quad (6)$$

$$J_\mu^i(x) = \bar{q}_i(x) \gamma_\mu c(x),$$

$$J^{ij+}_{\nu}(x) = \bar{q}_i(x) i\gamma_\nu c(x), \quad (7)$$

where the currents $J_\mu^i(x)$ and $J^{ij+}_{\nu}(x)$ interpolate the mesons $D^{*0}$, $D^{*+}$, $D_s^0$, $D^0$, $D^+$ and $D_s$, respectively. The $i$ denote the $u$, $d$ and $s$ quarks respectively. The external states $\pi$, $K$, $\rho$, $K^*$, $\phi$ have the four momentum $p_\mu$ with $p^2 = m^2_\pi$, $m^2_K$, $m^2_\rho$, $m^2_{K^*}$, $m^2_\phi$, respectively.

According to the basic assumption of current-hadron duality in the QCD sum rules [18], we can insert a complete series of intermediate states with the same quantum numbers as the current operators $J_\mu^i(x)$ and $J^{ij+}_{\nu}(x)$ into the correlation functions $\Pi_{i\mu}^j(p,q)$, $\Pi_{i\mu}^j(p,q)$ and $\Pi_{ij}^l(p,q)$ to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the mesons $D_i^*$

---

\[ In this article, we present the results for the strong coupling constants $f_{D^*DV}$ and $g_{DDV}$ which are originally obtained in Ref.[15] explicitly, and perform a comprehensive analysis of the strong coupling constants in Eq.(1). \]
and $D_i$, we get the following results,

$$\Pi^{\mu}(p, q) = \frac{f_{D_i} f_{D_j} M_{D_i} M_{D_j} g_{D_i D_j} \epsilon_{\mu \nu \alpha \beta} p^\alpha q^\beta + \cdots}{M_{D_i}^2 - (q + p)^2 \left\{ M_{D_j}^2 - q^2 \right\}}.$$  

(8)

$$\Pi^{ij}(p, q) = \frac{f_{D_i} f_{D_j} M_{D_i}^2 M_{D_j}^2 g_{D_i D_j} \epsilon_{\mu \nu \alpha \beta} \epsilon^{\mu \nu \alpha \beta} p^\alpha q^\beta + \cdots}{(m_i + m_j) \left\{ M_{D_i}^2 - (q + p)^2 \right\} \left\{ M_{D_j}^2 - q^2 \right\}}.$$  

(9)

$$\Pi_i^{ij}(p, q) = \frac{f_{D_i} f_{D_j} M_{D_i}^2 M_{D_j}^2 g_{D_i D_j} \epsilon_{\mu \nu \alpha \beta} \epsilon^{\mu \nu \alpha \beta} p^\alpha q^\beta + \cdots}{(m_i + m_j)(m_j + m_c) \left\{ M_{D_i}^2 - (q + p)^2 \right\} \left\{ M_{D_j}^2 - q^2 \right\}}.$$  

(10)

where the following definitions for the weak decay constants have been used,

$$\langle 0 | J_i^\mu(0) | D_i(p) \rangle = \frac{f_{D_i} M_{D_i}^2}{m_i + m_c},$$  

$$\langle 0 | J_i^\mu(0) | D_i^*(p) \rangle = \frac{f_{D_i} M_{D_i}^2}{\epsilon_{\mu \nu \alpha \beta} \epsilon^{\mu \nu \alpha \beta} p^\alpha q^\beta + \cdots}.$$  

(11)

In Eqs.(8-10), we have not shown the contributions from the high resonances and continuum states explicitly as they are suppressed due to the double Borel transformation. Non-conservation of the vector currents $J_i^\mu(x)$ can lead to non-vanishing couplings with the scalar mesons $D_0, D_0^*$ and $D_{s0}$,

$$\langle 0 | J_i^\mu(0) | D_{s0}(p) \rangle = f_{D_{s0}} \epsilon_{\mu \nu \alpha \beta} \epsilon^{\mu \nu \alpha \beta} p^\alpha q^\beta + \cdots.$$  

(12)

where the $f_{D_{s0}}$ are the weak decay constants. In this article, we choose the tensor structure $\epsilon_{\mu \nu \alpha \beta} p^\alpha q^\beta$ (or $\epsilon_{\mu \nu \alpha \beta} \epsilon^{\mu \nu \alpha \beta} p^\alpha q^\beta$) for analysis in Eqs.(8-9), the presence of the scalar mesons cannot result in contaminations. We have alternative choice to use the axial-vector currents $J_5^\mu(0)$ to interpolate the pseudoscalar mesons $D^0, D^*_0$ and $D_s$. However, the axial-vector currents $J_5^\mu(0)$ can also interpolate the axial-vector mesons $D_1^0, D_1^*$ and $D_{s1}$,

$$\langle 0 | J_5^\mu(0) | D_{s1}(p) \rangle = f_{D_{s1}} M_{D_{s1}} \epsilon_{\mu \nu \alpha \beta} \epsilon^{\mu \nu \alpha \beta} p^\alpha q^\beta + \cdots.$$  

(13)

where the $f_{D_{s1}}$ are the weak decay constants, we should be careful to avoid contaminations from the axial-vector mesons.

In the following, we briefly outline operator product expansion for the correlation functions $\Pi^{ij}_{\mu \nu}(p, q), \Pi^{ij}_{\mu}(p, q)$ and $\Pi^{ij}(p, q)$ in perturbative QCD theory. The calculations are performed at large spacelike momentum regions $(q + p)^2 \ll 0$ and $q^2 \ll 0$, which correspond to small light-cone distance $x^2 \approx 0$ required by validity of
the operator product expansion. We write down the propagator of a massive quark in the external gluon field in the Fock-Schwinger gauge first [19],

\[
\langle 0| T\{q_i(x_1) \bar{q}_j(x_2)\}|0 \rangle = i \int \frac{d^4k}{(2\pi)^4} e^{-ik(x_1-x_2)}\left\{ \frac{k + m}{k^2 - m^2} \delta_{ij} - \int_0^1 dv g_s G_{ij}^{\mu\nu}(vx_1 + (1-v)x_2) \right. \\
\left. \left[ \frac{1}{2} \frac{k + m}{(k^2 - m^2)^2} \sigma_{\mu\nu} - \frac{1}{k^2 - m^2} v(x_1 - x_2)_{\mu} \gamma_{\nu} \right] \right\}.
\]

(14)

The contributions proportional to \( G_{\mu\nu} \) can give rise to three-particle (and four-particle) meson distribution amplitudes with a gluon (or quark-antiquark pair) in addition to the two valence quarks, their corrections are usually not expected to play any significant roles. For examples, in the decay \( B \rightarrow \chi_{c0} K \), the factorizable contribution is zero and the non-factorizable contributions from the soft hadronic matrix elements are too small to accommodate the experimental data [20]; the net contributions from the three-valence particle light-cone distribution amplitudes to the strong coupling constant \( g_{D_sD^*K} \) are rather small, about 20% [21]. In this article, we observe that the contributions from the three-particle (quark-antiquark-gluon) light-cone distribution amplitudes are less than 5% for the strong coupling constants \( g_{D^*D^*P} \). The contributions of the three-particle (quark-antiquark-gluon) distribution amplitudes of the mesons are always of minor importance comparing with the two-particle (quark-antiquark) distribution amplitudes in the light-cone QCD sum rules. In our previous work, we also study the four form-factors \( f_1(Q^2) \), \( f_2(Q^2) \), \( g_1(Q^2) \) and \( g_2(Q^2) \) of the \( \Sigma \rightarrow n \) in the framework of the light-cone QCD sum rules approach up to twist-6 three-quark light-cone distribution amplitudes and obtain satisfactory results [22].

In a word, we can neglect the contributions from the valence gluons and make relatively rough estimations in the light-cone QCD sum rules. In this article, we take into account the three-particle light-cone distribution amplitudes of the pseudoscalar mesons, and neglect the three-particle light-cone distribution amplitudes of the vector mesons to avoid cumbersome calculations.

Substituting the above \( c \) quark propagator and the corresponding \( \pi, K, \rho, K^* \) and \( \phi \) mesons light-cone distribution amplitudes into the correlation functions \( \Pi_{ij}^{\mu\nu}(p,q) \), \( \Pi_{\mu}^{ij}(p,q) \) and \( \Pi^{ij}(p,q) \), respectively, and completing the integrals over the variables \( x \) and \( k \), finally we obtain the analytical results at the level of quark-gluon degrees of freedom. The explicit expressions are presented in appendix A.

In calculation, the two-particle \( \pi, K, \rho, K^*, \phi \) mesons and three-particle \( \pi, K \) mesons light-cone distribution amplitudes have been used [16, 19, 23, 24], the explicit expressions are given in appendixes B-C. The parameters in the light-cone distribution amplitudes are scale dependent and can be estimated with the QCD sum rules [16, 19, 23, 24]. In this article, the energy scale \( \mu \) is chosen to be \( \mu_c = \sqrt{M^2_D - m_c^2} \approx 1 \text{GeV} \).
Now we perform the double Borel transformation with respect to the variables $Q_1^2 = -(p + q)^2$ and $Q_2^2 = -q^2$ for the correlation functions $\Pi_{ij}^1, \Pi_{ij}^2$ and $\Pi_{ij}^3$ in Eqs.(8-10), and obtain the analytical expressions of the invariant functions in the hadronic representation,

$$B_{M_2}B_{M_1}\Pi_{ij}^1 = \frac{g_{D_1^*D_2^*} f_{D_1^*} f_{D_2^*} M_{D_1^*} M_{D_2^*}}{M_1^2 M_2^2} \exp \left[ -\frac{M_{D_1^*}^2}{M_1^2} - \frac{M_{D_2^*}^2}{M_2^2} \right] + \ldots,$$

$$B_{M_2}B_{M_1}\Pi_{ij}^2 = \frac{4 f_{D_1^*D_2^*} f_{D_1^*} f_{D_2^*} M_{D_1^*} M_{D_2^*}}{(m_c + m_j) M_1^2 M_2^2} \exp \left[ -\frac{M_{D_1^*}^2}{M_1^2} - \frac{M_{D_2^*}^2}{M_2^2} \right] + \ldots,$$

$$B_{M_2}B_{M_1}\Pi_{ij}^3 = \frac{2 g_{D_1^*D_2^*} f_{D_1^*} f_{D_2^*} M_{D_1^*} M_{D_2^*}}{(m_c + m_i)(m_c + m_j) M_1^2 M_2^2} \exp \left[ -\frac{M_{D_1^*}^2}{M_1^2} - \frac{M_{D_2^*}^2}{M_2^2} \right] + \ldots,$$

where we have not shown the contributions from the high resonances and continuum states explicitly for simplicity.

In order to match the duality regions below the thresholds $s_1^0$ and $s_2^0$ for the interpolating currents, we can express the correlation functions $\Pi_{ij}^1, \Pi_{ij}^2$ and $\Pi_{ij}^3$ at the level of quark-gluon degrees of freedom into the following form,

$$\Pi_{ij}^a = \int ds_1 \int ds_2 \frac{\rho_{ij}^a(s_1, s_2)}{s_1 - (q + p)^2} \frac{1}{s_2 - q^2},$$

where the $\rho_{ij}^a(s_1, s_2)$ ($a = 1, 2, 3$) are spectral densities, then perform the double Borel transformation with respect to the variables $Q_1^2$ and $Q_2^2$ directly. However, the analytical expressions of the spectral densities $\rho_{ij}^a(s_1, s_2)$ are hard to obtain, we have to resort to some approximations. As the contributions from the higher twist terms are suppressed by more powers of $\frac{1}{m_c^2 - (q + up)^2}$ (or $\frac{1}{M^2}$), the net contributions of the twist-3 and twist-4 terms are of minor importance, less than 10% for the strong coupling constants $g_{D^*D^*P}$, the continuum subtractions will not affect the results remarkably (for the strong coupling constants $G_S(D_s0D_s^*\phi)$ and $G_A(D_s1D_s\phi)$, the contributions are less than 20% [25]). The dominating contributions come from the two-particle twist-2 terms involving the $\phi(u), \phi_{\perp}(u)$ and $\phi_{\parallel}(u)$. We perform the same trick as Refs.[19, 26] and expand the amplitudes $\phi(u), \phi_{\perp}(u), \phi_{\parallel}(u)$ and $\phi_{\sigma}(u)$ in terms of polynomials of $1 - u$, for example,

$$\phi(u) = \sum_{k=0}^{N} b_k (1 - u)^k = \sum_{k=0}^{N} b_k \left( \frac{s_2 - m_c^2}{s_2 - q^2} \right)^k,$$

where the $b_k$ are coefficients, then introduce the variable $s_1$ and the spectral densities are obtained.

After straightforward calculations, we obtain the final expressions of the double Borel transformed correlation functions $\Pi_{ij}^1, \Pi_{ij}^2$ and $\Pi_{ij}^3$ at the level of quark-gluon
degrees of freedom. The masses of the charmed mesons are $M_D = 1.87\text{GeV}$, $M_{D^*} = 2.010\text{GeV}$ and $M_{D_s^*} = 2.112\text{GeV}$,

\[
\frac{M_D^2}{M_{D_s^*}^2} \approx 0.91, \quad \frac{M_{D^*}^2}{M_{D^*_s}^2} \approx 0.87, \\
\frac{M_{D_s}^2}{M_{D^*_s}^2} \approx 0.96, \quad \frac{M_{D^*_s}^2}{M_{D^*_s}^2} \approx 0.87, \\
\frac{M_{D_s^*}^2}{M_{D_s^*}^2} \approx 0.90, \quad \frac{M_{D_s^*}^2}{M_{D_s^*}^2} \approx 0.78, \tag{18}
\]

there exist overlapping working windows for the two Borel parameters $M_1^2$ and $M_2^2$, it is convenient to take the value $M_1^2 = M_2^2$,

\[
1 = \frac{M_1^2}{M_2^2} \approx \frac{M_{D_s^*}^2}{M_{D_s^*}^2} \approx \frac{M_{D^*_s}^2}{M_{D^*_s}^2} \approx \frac{M_{D^*_s}^2}{M_{D^*_s}^2}. \tag{19}
\]

We introduce the threshold parameters $s_0 = \max(s_1^0, s_2^0)$ ($s_1^0$ and $s_2^0$ corresponding to $M_1^2$ and $M_2^2$, respectively) and make the simple replacement,

\[
e^{-\frac{m_c^2 + u_0(1-u_0)n_{ij}^2}{M^2}} \rightarrow e^{-\frac{m_c^2 + u_0(1-u_0)m_{ij}^2}{M^2}} - e^{-\frac{s_{ij}^0}{M^2}}, \\
e^{-\frac{m_{D}^2 + u_0(1-u_0)M_{ij}^2}{M^2}} \rightarrow e^{-\frac{m_{D_s^*}^2 + u_0(1-u_0)m_{ij}^2}{M^2}} - e^{-\frac{s_{ij}^0}{M^2}}. \tag{20}
\]

for the correlation functions $\Pi_1^2_{ij}$, $\Pi_2^2_{ij}$ and $\Pi_3^2_{ij}$ respectively to subtract the contributions from the high resonances and continuum states [19].

Finally we obtain the sum rules for the strong coupling constants $g_{D^*D^*P}$, $f_{D^*DV}$ and $g_{DDV}$,
\[ g_{D_f^*D_l^*P_{ij}} f_{D_f^*} f_{D_l^*} M_{D_f^*} M_{D_l^*} \exp \left\{ - \frac{M_{D_f^*}^2}{M_f^2} - \frac{M_{D_l^*}^2}{M_l^2} \right\} \]

\[ = f_{P_{ij}} \left\{ M^2 \phi(u_0) + \frac{m_c m_{P_{ij}}^2 \phi_s(u_0)}{3(m_i + m_j)} \right\} \]

\[ + f_{P_{ij}} m_{P_{ij}}^2 \exp \left\{ - \frac{m_c^2 + u_0(1 - u_0)m_{P_{ij}}^2}{M^2} \right\} \left\{ \frac{1}{4} A(u_0) - \frac{m_c^2}{M^2} \right\} \]

\[ + \int_0^{u_0} d\alpha_j \int_{u_0 - \alpha_j}^{1 - \alpha_j} d\alpha_g A_\parallel (1 - \alpha_j - \alpha_g, \alpha_j) \left[ \frac{A_j}{\alpha_j} - \frac{V_\parallel}{2} + \frac{V_{\perp}}{2} \right] (1 - \alpha_j - \alpha_g, \alpha_j) \]

\[ + \int_0^{u_0} \frac{d}{du_0} \left\{ \int_0^{1 - \alpha_j} \alpha_g \int_{u_0 - \alpha_g}^{1 - \alpha_j} d\alpha_j \right. \]

\[ \left. \left. \left[ A_\perp + A_\parallel + \frac{V_\parallel}{2} + \frac{V_{\perp}}{2} \right] (1 - \alpha_j - \beta, \alpha) \right\} , \tag{21} \]

\[ 4 f_{D_f^*D_l^*V_{ij}} \frac{f_{D_f^*} f_{D_l^*} M_{D_f^*} M_{D_l^*}}{m_c + m_j} \exp \left\{ - \frac{M_{D_f^*}^2}{M_f^2} - \frac{M_{D_l^*}^2}{M_l^2} \right\} \]

\[ = f_{V_{ij}} M^2 \phi_\perp(u_0) \left\{ \exp \left[ - \frac{m_c^2 + u_0(1 - u_0)m_{V_{ij}}^2}{M^2} \right] \right\} \left\{ \frac{1}{4} A_\perp(u_0) - \frac{m_c m_{V_{ij}}^2}{M^2} \right\} \]

\[ + \exp \left[ - \frac{m_c^2 + u_0(1 - u_0)m_{V_{ij}}^2}{M^2} \right] \left\{ f_{V_{ij}} - f_{V_{ij}}^\perp \frac{m_i + m_j}{m_{V_{ij}}} \frac{m_c m_{V_{ij}}^2 g_{\perp}(u_0)}{2} \right\} \]

\[ - \frac{f_{V_{ij}} m_{V_{ij}} A_\perp(u_0)}{4} \left[ 1 + \frac{m_c^2}{M^2} \right] , \tag{22} \]
double Borel transformation with respect to the variables from the high resonances and continuum states \([26, 31]\). Firstly, we perform a replacement \(M_{ij}^2 \rightarrow 1/\sigma_1\), \(M_{ij}^2 \rightarrow 1/\sigma_2\) in above equation,

\[
2gD_{i}D_{j}V_{ij}(m_c + m_i)(m_c + m_j) \exp \left\{ -\frac{M^2_{Di}}{M_1^2} - \frac{M^2_{Dj}}{M_2^2} \right\} 
\]

\[
= f_{Vij}m_{Vij}M^2\phi_\parallel(u_0) \left\{ \exp \left[ -\frac{m^2_c + u_0(1 - u_0)m^2_{Vij}}{M^2} \right] - \exp \left[ -\frac{s^0_{Vij}}{M^2} \right] \right\} + \exp \left[ -\frac{m^2_c + u_0(1 - u_0)m^2_{Vij}}{M^2} \right] \left\{ \left[ f^\perp_{Vij} - f_{Vij} \frac{m_i + m_j}{m_{Vij}} \right] m_cm_{Vij}^2 h^{(s)}(u_0) \right. 
\]

\[
\left. - \frac{f_{Vij}m_{Vij}^3A(u_0)}{4} \left[ 1 + \frac{m^2_c}{M^2} \right] - 2f_{Vij}m_{Vij}^3 \int_0^{u_0} dt \int_0^\tau dC(t) \left[ 1 + \frac{m^2_c}{M^2} \right] \right\} \tag{23}
\]

where

\[
u_0 = \frac{M_1^2}{M_1^2 + M_2^2},
\]

\[
M^2 = \frac{M_1^2M_2^2}{M_1^2 + M_2^2}. \tag{24}
\]

Here we write down only the analytical results without the technical details, one can consult appendix D for some technical details.

In the following, we present another approach for subtracting the contributions from the high resonances and continuum states \([26, 31]\). Firstly, we perform a double Borel transformation with respect to the variables \(Q_1^2\) and \(Q_2^2\) respectively, and obtain the result,

\[
B_{M_2^2}B_{M_1^2} \int_0^1 du \frac{\Gamma(\alpha)f(u)}{\{m^2_c - (q + up)^2\}^\alpha} = \frac{M^{2(2-\alpha)}}{M_1^2 M_2^2} \exp \left\{ -\frac{m^2_c + u_0(1 - u_0)p^2}{M^2} \right\} f(u_0),
\]

\[
= \frac{1}{M_1^2 M_2^2} \int_{\Delta_1} ds_1 \int_{\Delta_2} ds_2 \exp \left\{ -\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2} \right\} \rho(s_1, s_2), \tag{25}
\]

where \(f(u)\) stand for the light-cone distribution amplitudes, \(\Delta_1 = \Delta_2 = m^2_c + u_0(1 - u_0)p^2\) and \(\rho(s_1, s_2)\) stand for the corresponding spectral densities. Then, we make a replacement \(M_1^2 \rightarrow 1/\sigma_1\), \(M_2^2 \rightarrow 1/\sigma_2\) in above equation,

\[
\int_{\Delta_1}^{s_1^0} ds_1 \int_{\Delta_2}^{s_2^0} ds_2 \exp \left\{ -s_1\sigma_1 - s_2\sigma_2 \right\} \rho(s_1, s_2)
\]

\[
= \frac{f(u_0)}{(\sigma_1 + \sigma_2)^{2-\alpha}} \exp \left\{ -\left[ m^2_c + u_0(1 - u_0)p^2 \right] (\sigma_1 + \sigma_2) \right\},
\]

\[
= \frac{f(u_0)}{\Gamma(2-\alpha)} \int_0^\infty d\lambda \lambda^{1-\alpha} \exp \left\{ -\left[ m^2_c + u_0(1 - u_0)m^2_\pi + \lambda \right] (\sigma_1 + \sigma_2) \right\}. \tag{26}
\]
Finally, we take a double Borel transformation with respect to the variables $\sigma_1$ and $\sigma_2$ respectively, the resulting QCD spectral density reads

$$\int_{s_1^0}^{s_1} ds_1 \int_{s_2^0}^{s_2} ds_2 \exp \left\{ -\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2} \right\} \rho(s_1, s_2)$$

$$= \frac{f(u_0)}{\Gamma(2-\alpha)} \int_{\Delta}^{s_0} ds \left\{ s - [m_c^2 + u_0(1-u_0)p^2] \right\}^{1-\alpha} \exp \left\{ -\frac{s}{M^2} \right\} ,$$

(27)

i.e.

$$B_{M_2^2}B_{M_2^2} \int_0^{1} du \frac{f(u)}{\left\{ m_c^2 - (q+up)^2 \right\}^\alpha}$$

$$= \frac{f(u_0)}{M_1^2M_2^2\Gamma(\alpha)\Gamma(2-\alpha)} \int_{\Delta}^{s_0} ds \left\{ s - [m_c^2 + u_0(1-u_0)p^2] \right\}^{1-\alpha} \exp \left\{ -\frac{s}{M^2} \right\}$$

$$+ \cdots .$$

(28)

For the twist-2 terms, $\alpha = 1$, the two subtracting approaches lead to the same results, the simple subtraction procedure we take in Eq.(20) is still reasonable.

3 Numerical result and discussion

The input parameters are taken as $m_s = (0.14 \pm 0.01)$GeV, $m_c = (1.35 \pm 0.10)$GeV, $m_u = m_d = (0.0056 \pm 0.0016)$GeV, $f_K = 0.160$GeV, $f_\pi = 0.130$GeV, $f_\rho = (0.216 \pm 0.003)$GeV, $f_{K^*} = (0.165 \pm 0.009)$GeV, $f_{K^{*+}} = (0.220 \pm 0.005)$GeV, $f_{K^{*0}} = (0.185 \pm 0.010)$GeV, $f_\phi = (0.215 \pm 0.005)$GeV, $f_{\phi^*} = (0.186 \pm 0.009)$GeV $[24]$, $m_K = 0.498$GeV, $m_\pi = 0.138$GeV, $m_\rho = 0.775$GeV, $m_{K^*} = 0.892$GeV, $m_\phi = 1.02$GeV, $M_D = 1.87$GeV, $M_{D_s} = 1.97$GeV, $M_{D_s^*} = 2.010$GeV and $M_{D_s^{*+}} = 2.112$GeV.

For the K meson: $\lambda_3 = 1.6 \pm 0.4$, $f_{K_3} = (0.45 \pm 0.15) \times 10^{-2}$GeV$^2$, $\omega_3 = -1.2 \pm 0.7$, $\omega_4 = 0.2 \pm 0.1$, $a_2 = 0.25 \pm 0.15$, $a_1 = 0.06 \pm 0.03$, $\eta_4 = 0.6 \pm 0.2$ $[16]$ $[19]$ $[23]$.

For the $\pi$ meson: $\lambda_3 = 0.0$, $f_{\pi_3} = (0.45 \pm 0.15) \times 10^{-2}$GeV$^2$, $\omega_3 = -1.5 \pm 0.7$, $\omega_4 = 0.2 \pm 0.1$, $a_2 = 0.25 \pm 0.15$, $a_1 = 0.0$, $\eta_4 = 10.0 \pm 3.0$ $[16]$ $[19]$ $[23]$.

For the $\rho$ meson: $a_1^\parallel = 0.0$, $a_1^\perp = 0.0$, $a_2^\parallel = 0.15 \pm 0.07$, $a_2^\perp = 0.14 \pm 0.06$, $\zeta_2^\parallel = 0.030 \pm 0.010$, $\tilde{\lambda}_3^\parallel = 0.0$, $\tilde{\omega}_3^\parallel = -0.09 \pm 0.03$, $\kappa_3^\parallel = 0.0$, $\omega_3^\parallel = 0.15 \pm 0.05$, $\lambda_3^\perp = 0.0$, $\kappa_3^\perp = 0.0$, $\omega_3^\perp = 0.55 \pm 0.25$, $\lambda_3^\perp = 0.0$, $\zeta_4 = 0.15 \pm 0.10$, $\zeta_4^\parallel = 0.10 \pm 0.05$ and $\tilde{\zeta}_4^\parallel = 0.10 \pm 0.05$ $[24]$.

For the $K^*$ meson: $a_1^\parallel = 0.03 \pm 0.02$, $a_1^\perp = 0.04 \pm 0.03$, $a_2^\parallel = 0.11 \pm 0.09$, $a_2^\perp = 0.10 \pm 0.08$, $\zeta_3^\parallel = 0.023 \pm 0.008$, $\tilde{\lambda}_3^\parallel = 0.035 \pm 0.015$, $\tilde{\omega}_3^\parallel = -0.07 \pm 0.03$, $\kappa_3^\parallel = 0.000 \pm 0.001$, $\omega_3^\parallel = 0.10 \pm 0.04$, $\lambda_3^\parallel = -0.008 \pm 0.004$, $\kappa_3^\perp = 0.003 \pm 0.003$, $\omega_3^\perp = 0.3 \pm 0.1$, $\lambda_3^\perp = -0.025 \pm 0.020$, $\zeta_4 = 0.15 \pm 0.10$, $\zeta_4^\parallel = 0.10 \pm 0.05$ and $\tilde{\zeta}_4^\parallel = 0.10 \pm 0.05$ $[24]$.

For the $\phi$ meson: $a_1^\parallel = 0.0$, $a_1^\perp = 0.0$, $a_2^\parallel = 0.18 \pm 0.08$, $a_2^\perp = 0.14 \pm 0.07$, $\zeta_3^\parallel = 0.024 \pm 0.008$, $\tilde{\lambda}_3^\parallel = 0.0$, $\tilde{\omega}_3^\parallel = -0.045 \pm 0.015$, $\kappa_3 = 0.0$, $\omega_3^\parallel = 0.09 \pm 0.03$, $\zeta_3^\perp = 0.024 \pm 0.008$, $\tilde{\lambda}_3^\perp = 0.0$, $\tilde{\omega}_3^\perp = -0.045 \pm 0.015$, $\kappa_3 = 0.0$, $\omega_3^\perp = 0.09 \pm 0.03$. 

10
Table 1: Numerical values of the ground state masses $M_{g^s}$ and the threshold parameters $\sqrt{s_0}$ from the QCD sum rules [32]. We denote the first excited state as $2S$ state, the values of the masses of the $2S$ states are taken from the predictions of the quark model [33].

|            | $M_{g^s}$ (GeV) | $M_{g^s}$ (GeV)(exp) | $\sqrt{s_0}$ (GeV) | $M_{2S}$ (GeV) |
|------------|-----------------|----------------------|--------------------|----------------|
| 0$^-$ (c$n$) | 1.90 ± 0.03     | 1.869                | 2.45 ± 0.15        | 2.589          |
| 1$^-$ (c$n$) | 2.00 ± 0.02     | 2.010                | 2.55 ± 0.05        | 2.692          |
| 0$^-$ (c$s$)  | 1.94 ± 0.03     | 1.969                | 2.50 ± 0.20        | 2.700          |
| 1$^-$ (c$s$)  | 2.05 ± 0.04     | 2.112                | 2.65 ± 0.15        | 2.806          |

Table 2: Threshold parameters for the strong coupling constants $g_{D^* D^* P}$, $g_{DDV}$ and $f_{D^* DV}$, respectively.

| $s_\pi^0$ (GeV$^2$) | 6.5 ± 0.5 | $g_{D^* D^* P}$ | 6.0 ± 0.5 | $g_{DDV}$ | 7.0 ± 0.5 | $f_{D^* DV}$ | 7.0 ± 0.5 |
|---------------------|-----------|-----------------|-----------|-----------|-----------|-------------|-----------|

The values of the decay constants $f_D$, $f_{D_s}$, $f_{D^*}$ and $f_{D^*_s}$ vary in a large range from different approaches, for example, the potential model, QCD sum rules and lattice QCD, etc [27, 28]. For the decay constant $f_D$, we take the experimental data from the CLEO Collaboration, $f_D = (0.223 ± 0.017)\text{GeV}$ [29]. If we take the value $f_{D_s} = (0.274\pm 0.013)\text{GeV}$ from the CLEO Collaboration, the $SU(3)$ breaking effect is rather large, $f_{D_s}/f_D = 1.23$, while most of the theoretical estimations indicate $f_{D_s}/f_D \approx 1.1$.

In this article, we take the value $f_{D_s}/f_D = 1.1$. For the decay constants $f_{D^*}$ and $f_{D^*_s}$, we take the central values from lattice simulation [30], $f_{D^*} = (0.23 ± 0.02)\text{GeV}$ and $f_{D^*_s} = (0.25 ± 0.02)\text{GeV}$,

\[
\frac{f_{D^*_s}}{f_{D^*}} \approx \frac{f_{D_s}}{f_D} = 1.1.
\] (29)

The duality threshold parameters $s_0$ are shown in Table 2, the numerical (central) values of $s_0$ are taken from the QCD sum rules for the masses of the pseudoscalar mesons $D^0$, $D^+$, $D_s$ and vector mesons $D^{*0}$, $D^{*+}$, $D_s^{*+}$, see Table 1 [32]. The threshold parameters $s_1^0$ and $s_2^0$ (corresponding to $M_1^0$ and $M_2^0$ respectively) are not equal in some channels in Eqs.(21-23), we choose the larger one i.e. $s_0 = \max(s_1^0, s_2^0)$.
to take into account all the contributions from the ground states, certainly, there
maybe some contaminations from the 2S state in the channel with smaller threshold
parameter i.e. \( \min(s_0^1, s_0^2) \), and impair the predictive power. The uncertainties of
the threshold parameters \( s_0^i \) are about \( \delta s_0 = (0.25 - 1.0)\text{GeV}^2 \), see Table.1, in this
article, we take \( \delta s_0 = 0.5\text{GeV}^2 \) for simplicity.

The Borel parameters are chosen as \( M_1^2 = M_2^2 \) and \( M^2 = (3 - 7)\text{GeV}^2 \), in those
regions, the values of the strong coupling constants \( g_{D^*D^*P}, g_{DDV} \) and \( f_{D^*DV} \) are
rather stable.

In the limit of large Borel parameter \( M^2 \), the strong coupling constants \( g_{D^*D^*P}, f_{D^*DV} \)
and \( g_{DDV} \) take up the following behaviors,

\[
\begin{align*}
g_{D^*_i D^*_j P_{i,j}} & \propto \frac{M^2 \phi(u_0)}{f_{D^*_i} f_{D^*_j}}, \\
f_{D^*_i D^*_j V_{i,j}} & \propto \frac{M^2 f_{V_{i,j}}^\perp \phi _\perp (u_0)}{f_{D^*_i} f_{D^*_j}}, \\
g_{D_i D_j V_{i,j}} & \propto \frac{M^2 f_{V_{i,j}} \phi ^\parallel (u_0)}{f_{D_i} f_{D_j}}.
\end{align*}
\]

It is not unexpected, the contributions from the twist-2 light-cone distribution am-
plitudes \( \phi (u), \phi ^\parallel (u) \) and \( \phi ^\perp (u) \) are greatly enhanced by the large Borel parameter
\( M^2 \), uncertainties of the relevant parameters presented in above equations have sig-
nificant impact on the numerical results.

Taking into account all the uncertainties, finally we obtain the numerical values
of the strong coupling constants \( g_{D^*D^*P}, f_{D^*DV} \) and \( g_{DDV} \), which are shown in
Figs.1-3, respectively,

\[
\begin{align*}
g_{D^*D^*\pi} &= (3.30 \pm 1.55)\text{GeV}^{-1}, \\
g_{D^*D^*K} &= (3.50 \pm 1.57)\text{GeV}^{-1}, \\
f_{D^*D^*\rho} &= (0.89 \pm 0.15)\text{GeV}^{-1}, \\
f_{D^*D^*\bar{K}^*} &= (1.01 \pm 0.20)\text{GeV}^{-1}, \\
f_{D^*_i D^*_j \phi} &= (0.82 \pm 0.16)\text{GeV}^{-1}, \\
g_{DD\rho} &= 1.31 \pm 0.29, \\
g_{DD\bar{K}^*} &= 1.61 \pm 0.32, \\
g_{D_i D_j \phi} &= 1.45 \pm 0.34.
\end{align*}
\]

The average values are about

\[
\begin{align*}
g_{D^*D^*P} &= (3.40 \pm 1.55)\text{GeV}^{-1}, \\
f_{D^*DV} &= (0.91 \pm 0.17)\text{GeV}^{-1}, \\
g_{DDV} &= 1.46 \pm 0.32.
\end{align*}
\]

The corresponding values of the basic parameters \( g, \lambda \) and \( \beta \) in the heavy quark
effective Lagrangian can be obtained from Eq.(3), and listed in Tables.3-5. From
Tables.3-5 or Eq.(3), we can obtain the values of the strong coupling constants $g_{D^*D\rho}^\ast$, $f_{D^*D\tau}^\ast$ and $g_{D^*D\tau}^\ast$, \[
\begin{align*}
  g_{D^*D\rho} & = 6.73 \pm 3.07, \\
  f_{D^*D\tau} & = 1.85 \pm 0.35, \\
  g_{D^*D\tau} & = 1.46 \pm 0.32.
\end{align*}
\] (33)

Taking the replacements $g_{DD\rho} \rightarrow g_{DD\rho}/2$ and $f_{D^*D\rho} \rightarrow f_{D^*D\rho}/4$ in Eq.(1), we can obtain the same definitions for the strong coupling constants in Ref.[31]. Our numerical values $g_{DD\rho} = 2.62 \pm 0.58$ and $f_{D^*D\rho} = (3.56 \pm 0.60)\text{GeV}^{-1}$ are compatible with the predictions $g_{DD\rho} = 3.81 \pm 0.88$ and $f_{D^*D\rho} = (4.17 \pm 1.04)\text{GeV}^{-1}$ in Ref.[31]. The value $g_{D^*D\rho} = 6.73 \pm 3.07$ obtained from the relation of heavy quark effective theory is different from the one obtained with the light-cone QCD sum rules $g_{D^*D\pi} = 12.5 \pm 1.0$ [19]. In Ref.[31], the authors take much smaller values for the decay constants of the charmed mesons than the present work. In Ref.[19], the authors take much smaller value of the decay constant $f_D$ and much larger value of the nonperturbative parameter $a_2(\mu)$ than the present work. It is not unexpected that the numerical values are different from each other, see Eq.(30). We can expect the relations in Eq.(3) work well.

The values of the $g$ vary in a large range from different approaches, see Table.3, the present prediction $g = 0.22 \pm 0.10$ is consistent with our previous calculation $g = 0.16^{+0.07}_{-0.05}$ with the light-cone QCD sum rules [14]. However, it is much smaller than most of the existing estimations, this maybe due to the shortcomings of the light-cone QCD sum rules.

The basic parameter $\lambda$ relates to the form-factor $V(q^2)$ of the hadronic transitions $\langle V | \bar{q}\gamma_\mu(1 - \gamma_5)b | B \rangle$ and $\langle V | \bar{q}\sigma_{\mu\nu}(1 + \gamma_5)b | B \rangle$, can be calculated with the light-cone sum rules approach and lattice QCD. With assumption of the form-factor $V(q^2)$ at $q^2 = q_{\text{max}}^2 = (M_B - M_V)^2$ is dominated by the nearest low-lying vector meson pole, we can obtain the values of the $\lambda$ [5, 6], which are presented in Table.4. The parameter $\beta$ can be estimated with the vector meson dominance theory, which is presented in Table.5, for technical details, one consult Ref.[15] . The large discrepancies maybe that the vector meson dominance theory overestimates the values of the $\beta g_V$ and $\lambda g_V$, the other possibility maybe the shortcomings of the light-cone QCD sum rules.

We can borrow some idea from the strong coupling constant $g_{D^*D_\pi}$, the central value ($g_{D^*D_\pi} = 12.5$ or $g_{D^*D_\pi} = 10.5$ with the radiative corrections are included in) from the light-cone QCD sum rules is too small to take into account the value ($g_{D^*D_\pi} = 17.9$) from the experimental data [9, 19, 34]. Naively, we can expect that the contributions from the radiative corrections cannot smear the discrepancies between our predictions and other estimations for the strong coupling constants $g_{D^*D_\pi}$, $f_{D^*D\tau}$ and $g_{DD\tau}$. It has been noted that the simple quark-hadron duality ansatz which works in the one-variable dispersion relation might be too crude for the double dispersion relation [35]. As in Ref.[34], we can postpone the threshold
| $g$       | Reference |
|-----------|-----------|
| 0.38 ± 0.08 | 3         |
| 0.34 ± 0.10 | 7         |
| 0.28       | 8         |
| 0.35 ± 0.10 | 9         |
| 0.50 ± 0.02 | 10        |
| 0.6 ± 0.1  | 11        |
| 0.59 ± 0.07 | 12        |
| 0.27^{+0.06}_{-0.03} | 13 |
| 0.16^{+0.07}_{-0.05} | 14         |
| 0.22 ± 0.10 | This work |

Table 3: Numerical values of the parameter $g$.

| $\beta$  | Reference |
|-----------|-----------|
| 0.9       | 5         |
| 0.36 ± 0.08 | This work |

Table 4: Numerical values of the parameter $\beta$.

| $|\lambda|(\text{GeV}^{-1})$ | Reference |
|---------------------------|-----------|
| 0.56                      | 5         |
| 0.63 ± 0.17               | 6         |
| 0.22 ± 0.04               | This work |

Table 5: Numerical values of the parameter $\lambda$. 
parameters $s_0$ to larger values to include the contributions from the radial excitations ($D'$ or $D^{**}$) to the hadronic spectral densities, with additional assumption for the values of the $g_{D^*P}, g_{D^*V}, f_{D^*V}, f_{D^*P},$ etc, we can improve the values of the $g_{D^*P}, g_{DDV}$ and $f_{D^*V}$, and smear the discrepancies between our values and other predictions. It is somewhat of fine-tuning.

From Tables.3-5, we can see that our numerical values are much smaller than most of the existing estimations (for example, the values taken in Ref.\cite{2}). Naively, we can expect that smaller values of the strong coupling constants $g_{D^*P}, g_{D^*P}, f_{D^*V}, f_{D^*P}, g_{DDV}$ and $g_{D^*P}$ lead to smaller final-state interaction effects in the hadronic $B$ decays. For example, the contributions from the rescattering mechanism in the decay

$$B \to D^*\rho \to D\pi$$

can occur through exchange of $D^*$ (or $D$) in the $t$ channel for the sub-precess $D^*\rho \to D\pi$ \cite{2}. The amplitude of the rescattering Feynman diagrams is proportional to

$$C_1g_{D^*D^*\pi}f_{D^*D\rho} + C_2g_{D^*D\pi}g_{DD}\rho \propto D_1g\lambda + D_2g\beta,$$

where the $C_i$ and $D_i$ are some coefficients.

4 Conclusion

In this article, we study the vertices $D^*D^*P$, $D^*D^*V$ and $DDV$ with the light-cone QCD sum rules. The strong coupling constants $g_{D^*D^*P}, g_{D^*D^*P}, f_{D^*D^*V}, f_{D^*D^*V}, g_{DDV}$ and $g_{D^*D^*V}$ play an important role in understanding the final-state interactions in the hadronic $B$ decays. They relate to the basic parameters $g, \lambda$ and $\beta$ respectively in the heavy quark effective Lagrangian. Our numerical values of the $g, \beta$ and $\lambda$
Figure 2: $g_{DD\rho}(A)$, $g_{DD,K^*}(B)$ and $g_{D_sD_s\phi}(C)$ with the Borel parameter $M^2$ after taking into account all the uncertainties.
Figure 3: $f_{D^*D^*_\rho}(A)$, $f_{D^*D^*_K}(B)$ and $f_{D^*_sD^*_s\phi}(C)$ with the Borel parameter $M^2$ after taking into account all the uncertainties.
are much smaller than most of the existing estimations. If the predictions from the light-cone QCD sum rules are robust, the final-state interaction effects maybe overestimated in the hadronic $B$ decays.

**Appendix A**

The explicit expressions of the $\Pi_{ij}^1$, $\Pi_{ij}^2$ and $\Pi_{ij}^3$ at the level of quark-gluon degrees of freedom,

$$\Pi_{ij}^1 = f_{P_i} \int_0^1 \frac{du}{AA} \frac{\phi(u)}{AA} + \frac{f_{P_i} m_c m_{P_i}}{3(m_i + m_j)} \int_0^1 \frac{du}{AA^2} \frac{\phi(u)}{AA}$$

$$- \frac{f_{P_i} m_{P_i}^2}{4} \int_0^1 \frac{du}{u} \frac{A(u)}{AA^2} \left[ \frac{1}{AA^2} + \frac{2m_c^2}{AA^3} \right]$$

$$- f_{P_i} m_{P_i}^2 \int_0^1 du \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_j \int_0^{\alpha_j} dv \frac{1}{du AA^2} \left| u = \alpha_j + (1-v) \alpha_g \right|$$

$$\left[ A_\perp + 2v A_\parallel + \frac{V_\parallel}{2} + \frac{V_\perp}{2} \right] (1 - \alpha_j - \alpha_g, \alpha_g, \alpha)$$

$$- f_{P_i} m_{P_i}^2 \int_0^1 du \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_j \int_0^{\alpha_j} dv \frac{1}{du AA^2} \left| u = \alpha_j + (1-v) \alpha_g \right|$$

$$\left[ A_\perp + A_\parallel + \frac{V_\parallel}{2} + \frac{V_\perp}{2} \right] (1 - \alpha - \alpha_g, \alpha_g, \alpha)$$

$$+ f_{P_i} m_{P_i}^2 \int_0^1 du \int_0^1 d\alpha_g \int_0^{\alpha_g} d\beta \int_0^{1-\beta} d\alpha \frac{1}{du AA^2} \left| u = 1 - \alpha | \alpha_g \right|$$

$$\left[ A_\perp + A_\parallel + \frac{V_\parallel}{2} + \frac{V_\perp}{2} \right] (1 - \alpha - \beta, \beta, \alpha) + \cdots ,$$

$$\Pi_{ij}^2 = f_{V_i} \int_0^1 \frac{du}{AA} \frac{\phi(u)}{AA} - \frac{f_{V_i} m_{V_i}^2}{4} \int_0^1 \frac{du}{AA^2} \frac{\phi(u)}{AA}$$

$$- \left[ f_{V_i} - \frac{f_{V_i}}{m_{V_i}} m_i + m_j \right] m_c m_{V_i} \int_0^1 \frac{du}{AA^2} \frac{g_{\perp}(u)}{AA^3}$$

$$\left[ f_{V_i} - \frac{f_{V_i}}{m_{V_i}} m_i + m_j \right] m_c m_{V_i} \int_0^1 \frac{du}{AA^2} \frac{g_{\perp}(u)}{AA^3}$$

$$\left[ f_{V_i} - \frac{f_{V_i}}{m_{V_i}} m_i + m_j \right] m_c m_{V_i} \int_0^1 \frac{du}{AA^2} \frac{g_{\perp}(u)}{AA^3}$$

$$\Pi_{ij}^3 = f_{V_i} m_{V_i} \int_0^1 \frac{du}{AA} \frac{\phi(u)}{AA} + \left[ f_{V_i} - \frac{f_{V_i}}{m_{V_i}} m_i + m_j \right] m_c m_{V_i} \int_0^1 \frac{du}{AA^2} \frac{h_{\perp}(u)}{AA^3}$$

$$- \left[ f_{V_i} - \frac{f_{V_i}}{m_{V_i}} m_i + m_j \right] m_c m_{V_i} \int_0^1 \frac{du}{AA^2} \frac{h_{\perp}(u)}{AA^3}$$

$$- 2 f_{V_i} m_{V_i} \int_0^1 \frac{du}{AA^2} \frac{\phi(u)}{AA}$$

$$\left[ f_{V_i} - \frac{f_{V_i}}{m_{V_i}} m_i + m_j \right] m_c m_{V_i} \int_0^1 \frac{du}{AA^2} \frac{h_{\perp}(u)}{AA^3}$$

where

$$AA = m_c^2 - (q + u p)^2.$$
Appendix B

The light-cone distribution amplitudes of the $K$ meson are defined by

\[
\langle 0 | \bar{u}(0) \gamma_\mu \gamma_5 s(x) | K(p) \rangle = i f_{Kp} \int_0^1 du e^{-iupx} \left\{ \phi(u) + \frac{m_K^2 x^2}{16} A(u) \right\} + \frac{i}{2} f_{Kp} \frac{x_\mu}{p \cdot x} \int_0^1 du e^{-iupx} B(u),
\]

\[
\langle 0 | \bar{u}(0) i \gamma_5 s(x) | K(p) \rangle = \frac{f_{Km_s}^2}{m_s + m_u} \int_0^1 du e^{-iupx} \phi_\sigma(u),
\]

\[
\langle 0 | \bar{u}(0) \sigma_{\mu\nu} \gamma_5 s(x) | K(p) \rangle = i (p_\mu x_\nu - p_\nu x_\mu) \frac{f_{Km_K}^2}{6(m_s + m_u)} \int_0^1 du e^{-iupx} \phi_\sigma(u),
\]

\[
\langle 0 | \bar{u}(0) \gamma_\mu \gamma_5 g s G_{\alpha\beta}(vx)s(x) | K(p) \rangle = f_{3K} \left\{ (p_\mu p_\alpha g_{\nu\beta}^\perp - p_\nu p_\alpha g_{\mu\beta}^\perp) - (p_\mu p_\beta g_{\nu\alpha}^\perp - p_\nu p_\beta g_{\mu\alpha}^\perp) \right\} \int \mathcal{D} \bar{\alpha}_\beta(\alpha_i) e^{-iup(x_{s + v\alpha})},
\]

\[
\langle 0 | \bar{u}(0) \gamma_\mu \gamma_5 g s G_{\alpha\beta}(vx)s(x) | K(p) \rangle = f_{Km_K}^2 f_{p\mu} \frac{p_\alpha x_\beta - p_\beta x_\alpha}{p \cdot x} \int \mathcal{D} \bar{\alpha}_i \bar{A}_\parallel(\alpha_i) e^{-iup(x_{s + v\alpha})} + f_{Km_K}^2 (p_\beta g_{\alpha\mu}^\perp - p_\alpha g_{\beta\mu}^\perp) \int \mathcal{D} \bar{\alpha}_i A_{\parallel}(\alpha_i) e^{-iup(x_{s + v\alpha})},
\]

\[
\langle 0 | \bar{u}(0) \gamma_\mu i g s \tilde{G}_{\alpha\beta}(vx)s(x) | K(p) \rangle = f_{Km_K}^2 f_{p\mu} \frac{p_\alpha x_\beta - p_\beta x_\alpha}{p \cdot x} \int \mathcal{D} \bar{\alpha}_i V_{\parallel}(\alpha_i) e^{-iup(x_{s + v\alpha})} + f_{Km_K}^2 (p_\beta g_{\alpha\mu}^\perp - p_\alpha g_{\beta\mu}^\perp) \int \mathcal{D} \bar{\alpha}_i V_{\parallel}(\alpha_i) e^{-iup(x_{s + v\alpha})},
\]

where $g_{\mu\nu}^\perp = g_{\mu\nu} - \frac{p_\mu x_\nu + p_\nu x_\mu}{p \cdot x}$, $\tilde{G}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} G^{\alpha\beta}$ and $\mathcal{D} \bar{\alpha}_i = d\alpha_u d\alpha_s d\alpha_v (1 - \alpha_u - \alpha_s - \alpha_v)$. 


The light-cone distribution amplitudes of the $K$ meson are parameterized as

\[
\phi(u, \mu) = 6u(1-u) \left\{ 1 + a_1 C_1^2(\xi) + a_2 C_2^2(\xi) \right\},
\]

\[
\phi_p(u, \mu) = 1 + \left\{ 30\eta_3 - \frac{5}{2} \rho^2 \right\} C_2^\frac{3}{2}(\xi)
+ \left\{ -3\eta_3\omega_3 - \frac{27}{20} \rho^2 - \frac{81}{10} \rho^2 a_2 \right\} C_4^\frac{3}{4}(\xi),
\]

\[
\phi_\sigma(u, \mu) = 6u(1-u) \left\{ 1 + \left[ 5\eta_3 - \frac{1}{2} \eta_3\omega_3 - \frac{7}{20} \rho^2 - \frac{3}{5} \rho^2 a_2 \right] C_2^\frac{3}{2}(\xi) \right\},
\]

\[
\phi_3(\alpha_i, \mu) = 360\alpha_u \alpha_s \alpha_g^2 \left\{ 1 + \lambda_3(\alpha_u - \alpha_s) + \omega_3 \frac{1}{2}(7\alpha_g - 3) \right\},
\]

\[
V_{||}(\alpha_i, \mu) = 120\alpha_u \alpha_s \alpha_g \left( v_{00} + v_{10} (3\alpha_g - 1) \right),
\]

\[
A_{||}(\alpha_i, \mu) = 120\alpha_u \alpha_s \alpha_g \alpha_{10} (\alpha_s - \alpha_u),
\]

\[
V_{\perp}(\alpha_i, \mu) = -30\alpha_g^2 \left\{ h_{00} (1-\alpha_g) + h_{01} [\alpha_g (1-\alpha_g) - 6\alpha_u \alpha_s] + h_{10} \left[ \alpha_g (1-\alpha_g) - \frac{3}{2} (\alpha_u^2 + \alpha_s^2) \right] \right\},
\]

\[
A_{\perp}(\alpha_i, \mu) = 30\alpha_g^2 (\alpha_u - \alpha_s) \left\{ h_{00} + h_{01} \alpha_g + \frac{1}{2} h_{10} (5\alpha_g - 3) \right\},
\]

\[
A(u, \mu) = 6u(1-u) \left\{ \frac{16}{15} \right. + \frac{24}{35} a_2 + 20\eta_3 + \frac{20}{9} \eta_4
+ \left[ -\frac{1}{15} + \frac{1}{16} - \frac{7}{27} \eta_3\omega_3 - \frac{10}{27} \eta_4 \right] C_2^3(\xi)
+ \left[ -\frac{11}{210} a_2 - \frac{4}{135} \eta_3\omega_3 \right] C_4^\frac{3}{2}(\xi)
\right\} + \left\{ \frac{18}{5} a_2 + 21\eta_4 \right\}
\left\{ 2u^3 (10 - 15u + 6u^2) \log u + 2\bar{u}^3 (10 - 15\bar{u} + 6\bar{u}^2) \log \bar{u}
+ u\bar{u}(2 + 13u\bar{u}) \right\},
\]

\[
g(u, \mu) = 1 + g_2 C_2^\frac{3}{2}(\xi) + g_4 C_4^\frac{3}{4}(\xi),
\]

\[
B(u, \mu) = g(u, \mu) - \phi(u, \mu),
\]

(39)
where

\[
\begin{align*}
    h_{00} &= v_{00} = -\frac{\eta_4}{3}, \\
    a_{10} &= \frac{21}{8} \eta_4 \omega_4 - \frac{9}{20} a_2, \\
    v_{10} &= \frac{21}{8} \eta_4 \omega_4, \\
    h_{01} &= \frac{7}{4} \eta_4 \omega_4 - \frac{3}{20} a_2, \\
    h_{10} &= \frac{7}{2} \eta_4 \omega_4 + \frac{3}{20} a_2, \\
    g_2 &= 1 + \frac{18}{7} a_2 + 60 \eta_3 + \frac{20}{3} \eta_4, \\
    g_4 &= -\frac{9}{28} a_2 - 6 \eta_3 \omega_3,
\end{align*}
\]

(40)

here \( \xi = 2u - 1 \), and \( C_2^1(\xi), C_4^1(\xi), C_3^1(\xi), C_2^3(\xi), C_4^3(\xi) \) are Gegenbauer polynomials, \( \eta_3 = \frac{f_K m_u + m_s}{m_K^2} \) and \( \rho^2 = \frac{(m_u + m_s)^2}{m_K^2} \) \cite{16, 19, 23}. The corresponding light-cone distribution amplitudes for the \( \pi \) meson can be obtained with a simple replacement of the nonperturbative parameters.
Appendix C

The light-cone distribution amplitudes of the $K^*$ meson are defined by

\[
\langle 0 | \bar{u}(0) \gamma_\mu s(x) | K^*(p) \rangle = p_\mu f_{K^*} m_{K^*} \frac{\epsilon \cdot x}{p \cdot x} \int_0^1 du e^{-iup \cdot x} \left\{ \phi_{\parallel}(u) + \frac{m_{K^*}^2 x^2}{16} A(u) \right\} 
+ \left[ \epsilon_\mu - p_\mu \frac{\epsilon \cdot x}{p \cdot x} \right] f_{K^*} m_{K^*} \int_0^1 du e^{-iup \cdot x} g_{\perp}^{(n)}(u) 
- \frac{1}{2} x_\mu \frac{\epsilon \cdot x}{(p \cdot x)^2} f_{K^*} m_{K^*}^3 \int_0^1 du e^{-iup \cdot x} C(u),
\]

\[
\langle 0 | \bar{u}(0) s(x) | K^*(p) \rangle = i \left[ f_{K^*}^\perp - f_{K^*} m_u + m_s \right] m_{K^*}^2 \epsilon \cdot x \int_0^1 du e^{-iup \cdot x} \left\{ \phi_{\perp}(u) + \frac{m_{K^*}^2 x^2}{16} A_{\perp}(u) \right\} 
+ i \left[ p_\mu x_\nu - p_\nu x_\mu \right] f_{K^*}^\perp m_{K^*}^2 \frac{\epsilon \cdot x}{(p \cdot x)^2} \int_0^1 du e^{-iup \cdot x} h_{\perp}^{(s)}(u) 
+ i \left[ p_\mu x_\nu - p_\nu x_\mu \right] f_{K^*} m_{K^*} \frac{1}{p \cdot x} \int_0^1 du e^{-iup \cdot x} B_{\perp}(u) 
+ i \frac{1}{2} \left[ \epsilon_\mu x_\nu - \epsilon_\nu x_\mu \right] f_{K^*} m_{K^*} \frac{1}{p \cdot x} \int_0^1 du e^{-iup \cdot x} C_{\perp}(u),
\]

\[
\langle 0 | \bar{u}(0) \gamma_\mu \gamma_5 s(x) | K^*(p) \rangle = - \frac{1}{4} \left[ f_{K^*}^\perp - f_{K^*} m_u + m_s \right] m_{K^*} \epsilon_{\mu \nu \alpha \beta} \epsilon^\nu p^\alpha x^\beta 
\int_0^1 du e^{-iup \cdot x} g_{\perp}^{(n)}(u). \tag{41}
\]
The light-cone distribution amplitudes of the $K^*$ meson are parameterized as

$$\phi_{\parallel}(u, \mu) = 6u(1-u) \left\{ 1 + a_{\parallel}^3 \xi + \frac{3}{2} \xi (5\xi^2 - 1) \right\},$$

$$\phi_{\perp}(u, \mu) = 6u(1-u) \left\{ 1 + a_{\perp}^3 \xi + \frac{3}{2} \xi (5\xi^2 - 1) \right\},$$

$$g_{\perp}^{(w)}(u, \mu) = \frac{3}{4} (1 + \xi^2) + a_{\perp}^3 \xi + \left\{ \frac{3}{7} a_{\perp}^3 + 5 \xi (3\xi^2 - 1) \right\} (3\xi^2 - 1)$$

$$+ \left\{ 5\kappa_3 \xi + \frac{15}{16} \lambda_3 + \frac{15}{8} \lambda_3 \right\} \xi (5\xi^2 - 3)$$

$$+ \left\{ \frac{9}{112} a_{\perp}^3 + \frac{15}{32} \omega_3 - \frac{15}{64} \zeta_3 \right\} (3 - 30\xi^2 + 35\xi^4),$$

$$g_{\perp}^{(a)}(u, \mu) = 6u \left[ 1 + \left\{ \frac{1}{3} a_{\perp}^3 + \frac{20}{9} \lambda_3 \right\} C^2_{\perp}(\xi) + \left\{ \frac{1}{6} a_{\perp}^3 + \frac{10}{9} \zeta_3 + \frac{5}{12} \omega_3 - \frac{5}{24} \zeta_3 \right\} C^2_{\perp}(\xi) + \left\{ \frac{1}{4} \lambda_3 - \frac{1}{8} \lambda_3 \right\} C^2_{\perp}(\xi) \right\} ,$$

$$h_{\parallel}^{(s)}(u, \mu) = 6u \left[ 1 + \left\{ \frac{1}{3} a_{\parallel}^3 + \frac{20}{9} \zeta_3 \right\} C^2_{\parallel}(\xi) + \left\{ \frac{1}{3} a_{\parallel}^3 + \frac{5}{3} \zeta_3 \right\} C^2_{\parallel}(\xi) + \left\{ \frac{1}{6} a_{\parallel}^3 + \frac{5}{4} \omega_3 - \frac{15}{16} \zeta_3 - \frac{15}{8} \omega_3 \right\} C^2_{\parallel}(\xi) \right\} ,$$

$$h_{\parallel}^{(t)}(u, \mu) = 3\xi^2 + \frac{3}{2} a_{\perp}^3 (3\xi^2 - 1) + \left\{ \frac{3}{2} a_{\perp}^3 \xi (5\xi^2 - 3) + \frac{5}{8} \omega_3 (3 - 30\xi^2 + 35\xi^4) \right\}$$

$$+ \left\{ \frac{15}{2} \kappa_3^3 - \frac{3}{4} \lambda_3^3 \right\} (5\xi^2 - 3),$$

$$g_3(u, \mu) = 1 + \left\{ -1 - \frac{2}{7} a_{\perp}^3 + \frac{40}{3} \zeta_3 - \frac{20}{3} \zeta_4 \right\} C^3_{\perp}(\xi)$$

$$+ \left\{ \frac{27}{28} a_{\perp}^3 + \frac{5}{4} \omega_3 - \frac{15}{16} \zeta_3 - \frac{15}{8} \omega_3 \right\} C^3_{\perp}(\xi),$$

$$h_3(u, \mu) = 1 + \left\{ -1 - \frac{3}{7} a_{\perp}^3 - 10 (\zeta_4^T + \frac{\zeta_4}{4}) \right\} C^3_{\parallel}(\xi) + \left\{ -\frac{3}{7} a_{\perp}^3 - \frac{5}{4} \omega_3 \right\} C^3_{\parallel}(\xi),$$

$$A(u, \mu) = 30u^2 \bar{a}^2 \left\{ \frac{4}{5} + \frac{4}{105} a_{\perp}^3 + \frac{8}{9} \zeta_3 + \frac{20}{9} \zeta_4 \right\} ,$$

$$A_{\perp}(u, \mu) = 30u^2 \bar{a}^2 \left\{ \frac{2}{5} + \frac{4}{35} a_{\perp}^3 + \frac{4 \zeta_4^T}{3} - \frac{8 \zeta_4^T}{3} \right\} ,$$

$$C(u, \mu) = g_3(u, \mu) + \phi_{\parallel}(u, \mu) - 2 g_{\perp}^{(w)}(u, \mu) ,$$

$$B_{\perp}(u, \mu) = h_{\parallel}^{(t)}(u, \mu) - \frac{1}{2} \phi_{\perp}(u, \mu) - \frac{1}{2} h_3(u, \mu) ,$$

$$C_{\perp}(u, \mu) = h_3(u, \mu) - \phi_{\perp}(u, \mu) ,$$

where $\xi = 2u - 1$, and $C^2_{\parallel}(\xi), C^2_{\perp}(\xi), C^2_{\parallel}(\xi), C^2_{\perp}(\xi), C^3_{\parallel}(\xi), C^3_{\perp}(\xi)$ are Gegenbauer polynomials. The corresponding light-cone distribution amplitudes for the $\rho$ and $\phi$ mesons can be obtained with a simple replacement of the nonperturbative parameters.
Appendix D

Here we present some technical details necessary in performing the Borel transformation which are not familiar to the novices,

\[
\int_0^1 dv \int_0^1 d\alpha_s \int_0^{1-\alpha_g} d\alpha_g f(v, \alpha_s, \alpha_g) \frac{d}{du} \exp \left[ -\frac{m_c^2 + u(1-u)m_K^2}{M^2} \right] \\
\delta(u - u_0) \bigg|_{u = \alpha_s + (1-v)\alpha_g} \\
= \int_0^1 du \int_0^1 dv \int_0^1 d\alpha_s \int_0^{1-\alpha_g} d\alpha_g f(v, \alpha_s, \alpha_g) \delta[u - \alpha_s - (1-v)\alpha_g] \\
\frac{d}{du} \exp \left[ -\frac{m_c^2 + u(1-u)m_K^2}{M^2} \right] \delta(u - u_0) \\
= \int_0^1 du \int_0^1 d\alpha_s \int_{1-\alpha_g}^{1-\alpha_g} d\alpha_g f(v, \alpha_s, \alpha_g) \frac{d}{du} \exp \left[ -\frac{m_c^2 + u(1-u)m_K^2}{M^2} \right] \delta(u - u_0) \\
= -\exp \left[ -\frac{m_c^2 + u_0(1-u_0)m_K^2}{M^2} \right] \frac{d}{du_0} \int_0^{u_0} d\alpha_s \int_{u_0-\alpha_s}^{1-\alpha_g} d\alpha_g f(v, \alpha_s, \alpha_g) \alpha_g \frac{\alpha_g}{\alpha_g},
\]

where the \( f(v, \alpha_s, \alpha_g) \) stand for the three-particle light-cone distribution amplitudes.

Acknowledgments

This work is supported by National Natural Science Foundation, Grant Number 10405009, and Key Program Foundation of NCEPU.

References

[1] P. Colangelo, G. Nardulli, N. Paver, Riazuddin, Z. Phys. C45 (1990) 575; M. Ciuchini, E. Franco, G. Martinelli, L. Silvestrini, Nucl. Phys. B501 (1997) 271; M. Ciuchini, R. Contino, E. Franco, G. Martinelli, L. Silvestrini, Nucl. Phys. B512 (1998) 3; Y. S. Dai, D. S. Du, X. Q. Li, Z. T. Wei and B. S. Zou, Phys. Rev. D60 (1999) 014014; C. Isola, M. Ladisa, G. Nardulli, T. N. Pham, P. Santorelli, Phys. Rev. D64 (2001) 014029; Phys. Rev. D65 (2002) 094005; M. Ablikim, D. S. Du and M. Z. Yang, Phys. Lett. B536 (2002) 34; P. Colangelo, F. De Fazio, Phys. Lett. B542 (2002) 71; M. Ladisa, V. Laporta, G. Nardulli, P. Santorelli, Phys. Rev. D70 (2004) 114025; X. Liu, B. Zhang, S. L. Zhu, Phys. Lett. B645 (2007) 185; C. Meng, K. T. Chao, Phys. Rev. D75 (2007) 114002.

[2] H. Y. Cheng, C. K. Chua, A. Soni, Phys. Rev. D71 (2005) 014030.
[3] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, G. Nardulli, Phys. Rept. **281** (1997) 145.

[4] M. Bando, T. Kugo and K. Yamawaki, Nucl. Phys. **B259** (1985) 493; Phys. Rept. **164** (1988) 217.

[5] C. Isola, M. Ladisa, G. Nardulli, P. Santorelli, Phys. Rev. **D68** (2003) 114001; P. Colangelo, F. De Fazio, T. N. Pham, Phys. Lett. **B597** (2004) 291.

[6] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, G. Nardulli, Phys. Lett. **B292** (1992) 371; R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, G. Nardulli, Phys. Lett. **B299** (1993) 139; R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, G. Nardulli, Phys. Lett. **B312** (1993) 315.

[7] P. Colangelo, F. De Fazio, Eur. Phys. J. **C4** (1998) 503.

[8] H. C. Kim, S. H. Lee, Eur. Phys. J. **C22** (2002) 707.

[9] A. Khodjamirian, R. Ruckl, S. Weinzierl, O. I. Yakovlev, Phys. Lett. **B457** (1999) 245.

[10] D. Melikhov, M. Beyer, Phys. Lett. **B452** (1999) 121.

[11] D. Becirevic, A. Le Yaouanc, JHEP **9903** (1999) 021.

[12] P. Colangelo, F. De Fazio, Phys. Lett. **B532** (2002) 193; A. Anastassov et al, Phys. Rev. **D65** (2002) 032003; P. Colangelo, F. De Fazio, T. N. Pham, Phys. Rev. **D69** (2004) 054023; F. De Fazio, Eur. Phys. J. **C33** (2004) S247.

[13] I. W. Stewart, Nucl. Phys. **B529** (1998) 62.

[14] Z. G. Wang, S. L. Wan, Phys. Rev. **D74** (2006) 014017.

[15] Z. G. Wang, [arXiv:0705.3720](http://arxiv.org/abs/0705.3720)[hep-ph].

[16] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. **B312** (1989) 509; V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. **B345** (1990) 137; V. L. Chernyak and A. R. Zhitnitsky, Phys. Rept. **112** (1984) 173; V. M. Braun and I. E. Filyanov, Z. Phys. **C44** (1989) 157; V. M. Braun and I. E. Filyanov, Z. Phys. **C48** (1990) 239.

[17] V. M. Braun, [hep-ph/9801222](http://arxiv.org/abs/hep-ph/9801222). P. Colangelo and A. Khodjamirian, [hep-ph/0010175](http://arxiv.org/abs/hep-ph/0010175).

[18] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B147** (1979) 385, 448; L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. **127** (1985) 1; S. Narison, QCD Spectral Sum Rules, World Scientific Lecture Notes in Physics **26** (1989) 1.
[19] V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Rückl, Phys. Rev. D51 (1995) 6177.

[20] L. Li, Z. G. Wang, T. Huang, Phys. Rev. D70 (2004) 074006; B. Melic, Phys. Lett. B591 (2004) 91.

[21] Z. G. Wang, J. Phys. G34 (2007) 753.

[22] Z. G. Wang, J. Phys. G34 (2007) 493.

[23] P. Ball, JHEP 9901 (1999) 010; P. Ball, R. Zwicky, Phys. Lett. B633 (2006) 289; P. Ball, R. Zwicky, JHEP 0602 (2006) 034; P. Ball, V. M. Braun, A. Lenz, JHEP 0605 (2006) 004.

[24] P. Ball, V. M. Braun, Nucl. Phys. B543 (1999) 201; P. Ball, V. M. Braun, hep-ph/9808229; P. Ball, V. M. Braun, Phys. Rev. D54 (1996) 2182; P. Ball, V. M. Braun, Y. Koike, K. Tanaka, Nucl. Phys. B529 (1998) 323; P. Ball, G. W. Jones, R. Zwicky, Phys. Rev. D75 (2007) 054004; P. Ball, G. W. Jones, JHEP 0703 (2007) 069.

[25] Z. G. Wang, Phys. Rev. D75 (2007) 034013.

[26] H. Kim, S. H. Lee and M. Oka, Prog. Theor. Phys. 109 (2003) 371.

[27] Z. G. Wang, W. M. Yang, S. L. Wan, Nucl. Phys. A744 (2004) 156; J. Bordes, J. Penarrocha, K. Schilcher, JHEP 0511 (2005) 014; L. Lellouch, C. J. David, Phys. Rev. D64 (2001) 094501; S. Narison, hep-ph/0202200.

[28] C. Albertus, E. Hernandez, J. Nieves, J. M. Verde-Velasco, Phys. Rev. D71 (2005) 113006; D. Ebert, R. N. Faustov, V. O. Galkin, Phys. Lett. B635 (2006) 93; G. L. Wang, Phys. Lett. B633 (2006) 492; H. M. Choi, Phys. Rev. D75 (2007) 073016.

[29] M. Artuso et al, Phys. Rev. Lett. 95 (2005) 251801; G. Bonvicini et al, Phys. Rev. D70 (2004) 112004; T. K. Pedlar, et al, arXiv:0704.0437[hep-ex].

[30] K. C. Bowler et al, Nucl. Phys. B619 (2001) 507.

[31] Z. H. Li, T. Huang, J. Z. Sun, Z. H. Dai, Phys. Rev. D65 (2002) 076005.

[32] A. Hayashigaki, K. Terasaki, hep-ph/0411285.

[33] M. Di Pierro, E. Eichten, Phys. Rev. D64 (2001) 114004.

[34] D. Becirevic, J. Charles, A. LeYaouanc, L. Oliver, O. Pene, J. C. Raynal, JHEP 0301 (2003) 009.

[35] A. Khodjamirian, AIP Conf. Proc. 602 (2001) 194.