Improved transient stability analysis method of power system with VSG inverter based on SMR

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Abstract. In order to reduce the effect of total inertia reduction on system stability after high penetration distributed power access, virtual synchronous generator technology (VSG) has become a research hotspot. In transient process, the inverter with VSG technology has similar dynamic behavior as synchronous machine. And traditional direct method has a certain conservatism in the analysis of transient stability. In this paper, an improved direct method based on SMR technology is proposed to analyze the transient stability of the power system with VSG inverter. By analyzing the virtual power angle transient characteristics of inverter with VSG strategy, a virtual power angle dynamic model is constructed, and the traditional direct method is improved by using SMR technology. Finally, the method is simulated by using the SMIB and IEEE three-machine nine-node systems. The results show that the improved direct method can correctly acquire the transient stability index and reduce the conservatism of traditional method.

1. Introduction
With the expanding scale of new energy, the number of power electronic converters, such as grid connected inverters, has also increased rapidly. The traditional control strategy of inverter leads to the reduction of the total inertia of the system, which is not conducive to the transient stability of the system. For this reason, people put forward the technology of virtual synchronous generator (VSG). When a large disturbance occurs in the power grid, its dynamic characteristics has similar to the traditional synchronous generator, so it is necessary to study its transient stability.

The methods of transient stability analysis in power system are mainly divided into two categories: time domain simulation method and direct method. Where, the direct method has the advantages of fast calculation speed and can give the stability margin quantificationally, so it has been developed rapidly. However, there are few researches on transient stability of inverter system with VSG strategy under direct method. The influence of virtual inertia of virtual synchronous generator on transient stability is mainly studied by EAC and Lyapunov function method. Reference [1] uses the Lyapunov function method to analyze the transient stability of inverter system with VSG strategy, and then uses bang-bang control strategy to change the virtual inertia adaptively, so as to improve the transient stability of system. Reference [2] uses EEAC method to analyze the potential and kinetic energy of the system in transient process, and obtains the relationship between three parameters of VSG controller, acceleration and deceleration area. It improves system’s transient stability by changing the inertia in
real time. And the transient stability of the inverter system with the VSG strategy is studied by the direct method, which does not account for the current saturation characteristic of inverter. Reference [3] makes the mechanism analysis of power angle instability, that is because the inverter fade into a current source as its current saturated when laugh disturbance, which makes the instability process more complex. However, the transient angle instability of VSG is analyzed qualitatively. The above transient energy function analysis method studies the conservatism of the stability of the inverter system under the VSG strategy [4].

To solve this problem, an improved direct method based on SMR technology is proposed in this paper. Firstly, by analyzing the virtual power angle transient characteristics of inverter under VSG strategy, a virtual power angle dynamic model of VSG inverter is constructed, and the improved Lyapunov method based on SMR technology is used to improve its conservatism. Finally, single machine infinite bus system(SMIB) and IEEE three-machine and nine-node systems are used to verify the effectiveness of the method.

2. Transient model of virtual power angle of inverter using VSG

2.1 Definition of the virtual power angle of inverter using VSG

1) The virtual power angle characteristic when the inverter is not saturated

When the inverter is not saturated, its output power expression is shown in formula (2-1) [5]:

$$P_c = \text{Re}(\bar{U}^\dagger) = \frac{\bar{E} \cdot \bar{U}}{X_S} \sin \delta'$$

(2-1)

Where, $\delta'$ is virtual power angle, it can also be considered as the angle between the inverter’s output voltage and the grid voltage. From the above formula, we find that the virtual power characteristic of VSG strategy when the inverter is not saturated is similar with traditional synchronous generator, and the virtual power angle curve is shown in the Figure 1.

![Figure 1. VSG virtual power angle curve](image)

2) The virtual power angle characteristic when the inverter is saturated

In order to prevent the excessive current flowing in the inverter from damaging the equipment, it is necessary to set the current limiter to protect the inverter during operation [6]. When $I_{\text{mag}} \leq I_{\text{max}}$, the inverter normally outputs. But when $I_{\text{mag}} > I_{\text{max}}$, its current limiting section start up, and the current is clamped at $I_{\text{max}}$. Therefore, it can be equivalent to the limit model shown in formula (2-2):

$$I = \begin{cases} I_{\text{max}}, & I < I_{\text{max}} \\ I_{\text{lim}}, & I_{\text{lim}} \leq I \leq I_{\text{max}} \\ I_{\text{max}}, & I > I_{\text{max}} \end{cases}$$

(2-2)

The presentation of the power angle characteristics when inverter’s current saturated is shown as formula (2-3):
According to formula (2-3), we can obtain the virtual power angle characteristic curve when the inverter’s current is saturated shown as Figure 1. Its’ cosine characteristic is no longer consistent with the sinusoidal power angle characteristic of the traditional synchronous generator.

2.2 Transient model of virtual power angle considering the inverter current saturation
We can establish the dynamic equivalent model of the inverter with VSG control strategy, and the rest of the grid can be seen as an infinite system [7]. Thus, this SMIB system is shown in Figure 2. And the relationship between electrical quantities can express as follows:

\[ E = U + jX_L \]

\[ I = \frac{E - U}{jX_L} = \frac{E\cos\delta - U + jE\sin\delta}{jX_L} = \frac{E\sin\delta}{X_L} - \frac{E\cos\delta - U}{X_L} \]  

(2-4)

Where, \( E \) is the terminal voltage of the Inverter, \( U \) is the infinity bus’s reference voltage, \( X_L \) is the line’s reactance, and \( I \) is the inverter’s output current.

Thus, the relation between current amplitude and power angle is shown in formula (2-5):

\[ I = \frac{\sqrt{(E\sin\delta)^2 + (E\cos\delta)^2 - 2EU\cos\delta + U^2}}{X_L} = \frac{\sqrt{E^2 - 2EU\cos\delta + U^2}}{X_L} \]  

(2-5)

That is,

\[ \cos\delta = \frac{E^2 + U^2 - (IX_L)^2}{2EU}, \quad I = \begin{cases} \frac{I_{\text{min}}}{I_{\text{max}}} & I_{\text{min}} < I < I_{\text{max}} \\ I_{\text{max}} & I > I_{\text{max}} \end{cases} \]  

(2-6)

Figure 2. SMIB System Model under VSG Strategy

3. Improved direct method based on SMR Technology

3.1 The basic principle of SMR Technology
We define \( V \) as a system’s rational Lyapunov function as follows[8]:

\[ V(x) = \frac{V_{\text{num}}(x)}{V_{\text{den}}(x)} \]  

(3-1)

Where, \( V_{\text{num}} \) and \( V_{\text{den}} \) are all polynomial sets, and the conditions shown in (3-2) to (3-3) are satisfied:
\[ \forall x \in D, \lim_{t \to +c} V(x) = \infty, \forall x \in D, V_{\text{den}}(x), \quad V_{\text{num}}(x) > 0, \text{and} V_{\text{num}}(0) = 0 \]  
(3-2)

\[ \dot{V}(x) < 0, \forall x \in D / \{0\} \]  
(3-3)

When getting system stability domains, we need firstly define a subset of \( V(x) \), that is \( \nu(c) = \{x \in \mathbb{R}^n : V(x) \leq c\} \). Our aim is to find the optimal Lyapunov function by solving the optimization model shown in (5-4), so as to reduce the conservative.

\[
\mu = \sup_{c} \rho(\nu(c))
\]
\[
\text{s.t.} \left\{ \begin{array}{l}
(3-2) - (3-3) \\
(3-4)
\end{array} \right.
\]

3.2 Improved direct method based on SMR technology

According to the above basic theory, the SMR technique introduced in this paper is mainly divided into the following steps:

1) Reconstruct the original system using Taylor series

The main idea in this step is to separate the polynomial function from the non-polynomial function and then approximate the non-polynomial part using the Taylor series. Specifically, we can express the system as follows[9].

\[ x(t) = h(x(t)) + \sum_{i=1}^{r} g_i(x(t))z_i(x(t)), x \in D \]  
(3-5)

Where, \( h(x(t)) \) and \( g(x(t)) \) are vector polynomial functions and belong to the polynomial set Pn. \( \zeta_i(x(t)), \ldots, \zeta_r(x(t)) \) represents an non-polynomial function. \( \zeta_i, i = 1, \ldots, r \) can be parsed in D. We assume the following formulas are set up:

\[ \alpha = \alpha_1 + \cdots + \alpha_n, \alpha! = \alpha_1! \cdots \alpha_n!, x^\alpha = x_1^{\alpha_1} \cdots x_n^{\alpha_n} \]  
(3-6)

where, \( x \in \mathbb{R}^n \) and \( \alpha = (\alpha_1, \ldots, \alpha_n)^T \in \mathbb{N}^n \) are dimension vectors. The derivatives can be expressed as (3-7):

\[ D^\alpha \zeta = \frac{\partial^{k} \zeta}{\partial x^{\alpha}} |_{x=0} \]  
(3-7)

Therefore, \( \zeta_i \) in Eq. (3-5) can be written as Taylor’s expansion as follows:

\[ \zeta_i(x) = \eta_i(x) + \sum_{|\alpha| = k+1} \xi_i \frac{x^\alpha}{\alpha!} \]  
(3-8)

Where, \( \xi_i \) is a bounded parameter, \( k \) represents the power series, \( \eta_i(x) \) is a \( k \)-order Taylor polynomial, as shown in equation (3-9):

\[ \eta_i(x) = \sum_{|\alpha| = k} D^\alpha \zeta |_{x=0} \frac{x^\alpha}{\alpha!} \]  
(3-9)

\( \xi_i \) is the value of the Taylor remainder value \( \zeta_i - \eta_i \), where \( \xi = (\xi_1, \ldots, \xi_r)^T \) satisfies:

\[ \Xi = \left[ r_1, \bar{r}_1 \right] \times \cdots \times \left[ r_r, \bar{r}_r \right], \quad r_i, \bar{r}_i \in \mathbb{R}, i = 1, \ldots, r \]  
(3-10)

2) Estimating the Stability Region Using Rational Lyapunov Function

In order to solve the problem (3-4), the most crucial step is to estimate the maximum stable region \( c_k \) using the rational Lyapunov function. The maximum stable region \( c_k \) is obtained by solving the (3-11) optimal problem.
\[ c_k = \sup c \]
\[ \begin{cases} -\psi(x, c, s(x), \xi) \in P_0^{\text{sos}} \\ \forall x \in \nu(c) / \{0\} \\ \forall \xi_i \in \text{ver}(\Xi), i = 1, ..., r \end{cases} \] (3-11)

Where \(-\psi(x, c, s(x), \xi) \in P_0^{\text{sos}} \) in \(-\psi(x, c, s(x), \xi) \) has the value of 0 at zero and the polynomial it is composed of the sum of the squares of each monomials (SOS). The composition method is as shown in Eqs (3-12) to (3-14):
\[
\sigma(x) = V_{\text{den}}(x) \nabla V_{\text{num}} - V_{\text{num}}(x) \nabla V_{\text{den}}, \quad r(x) = \sigma(x) \sum_{i=1}^{\deg} (g_i(x) \eta_i(x)) 
\] (3-12)
\[
q_i(x) = \sigma(x) g_i(x) \sum_{|\mathbf{p}|_{+1} = k} x^p \quad q(x) = (q_1(x), ..., q_r(x))^T 
\] (3-13)
\[
\psi(x, c, s(x), \xi) = r(x) + q(x) \xi + s(x)(cV_{\text{den}}(x) - V_{\text{num}}(x)) 
\] (3-14)

Where, \( \text{ver}(e) \) is the vertex set of \( e \). \( s(x) \in P_0^{\text{sos}} \)

3) Constructing Convex Optimization Model Using SMR Technology

SMR can be used to process the local SOS model, and the non-convex optimization model can be transformed into convex optimization model, so as to ensure the global optimum of the stable region of SMIB system[10]. SOS optimization model based on SMR technology is as follows:
\[
s(x) = (\psi)^T SP(n, d(q)), \psi(x, c, s(x), \xi) = (\psi)^T (\Psi(c, S, \xi) + L(\gamma)) \psi(n, d(\psi)) 
\] (3-15)
\[
u(x) = u_1(x) + u_2(x), \quad u_1(x) = -r(x) - q(x) \xi + s(x)V_{\text{num}}(x) 
\] (3-16)
\[
u_2(x) = s(x)\bar{V}(x), \quad \bar{V}(x) = V_{\text{den}}(x) + \lambda V_{\text{num}}(x) 
\] (3-17)

Where, \( R(\xi), W(S), U_2(S), \bar{V} \) are the SMR matrix of \(-r(x) - q(x) \xi + s(x)V_{\text{num}}(x), u_2(x), V(x) \). \( d(q) \) is the minimum integer not less than \( \deg(q)/2 \), \( \deg(q) \) is the highest degree of polynomial function \( q(x) \in P_0^{\text{sos}} \), \( \psi(n, d(q)) \) is a vector composed of variables with different powers, the power of the variables is less than or equal to the positive integer of \( d(q) \), and \( L(\gamma) \) is the affine space.
\[
\phi_q = \left\{ L(\gamma) \in \mathbb{R}^{v_k} : L(\gamma) = L'(\gamma', (*)^T L(\gamma) \phi(n, d(q)) = 0) \right\} 
\] (3-18)

According to the above method, the determination of the optimal rational Lyapunov function and the acquisition of the maximum stable domain \( c_k \) can be realized by (5-19).
\[
c_k = -\frac{\bar{e}}{1 + 4\bar{e}} 
\] (3-19)

Where \( \bar{e} \) is the solution to the GEVP problem shown in Eq. (3-20), the model is as follows:
\[
\begin{cases} \bar{e} = \inf e \\ S > 0 \\ \forall x \in \text{ver}(\Xi), i = 1, ..., r \end{cases} 
\] (3-20)

4) Finding the optimal Lyapunov function

The above procedure obtains the maximum stable region for rational Lyapunov function which uses SMR technique, while the result is not optimal. Therefore, based on the above theoretical analysis, the initial rational Lyapunov function \( V_0(x) \) is obtained by the following formula (3-21):
Where $V_{q}(x)$ is a quadratic Lyapunov function of the linear part of the system (3-5), $V_{a}(x)$ is an auxiliary polynomial function whose selection method is $(x^{T}x)(x^{T}P_{x})$, and our purpose is to expand the region enclosed by the region polynomial in the range seeking the optimal rational Lyapunov function, as shown in equation (3-22):

$$
\bar{\mu} = \sup_{\varepsilon} S(\varepsilon) \subseteq v(\varepsilon)
$$

s.t.

$$
\forall \xi_{i} \in \text{ver}(\Xi), i = 1, ..., r
$$

Where, $S(\varepsilon) = \{x \in R^{n}: \phi(x) \leq \varepsilon \}^{c}$, and $\phi(x)$ is a polynomial. We can choose $\phi(x) = \|x\|^{2}$. We use (3-23) to find the optimal Lyapunov function and obtain the maximum stable region.

$$
\bar{s} \in P_{SOS}, s \in P_{0_{SOS}}
$$

s.t.

$$
(c_{k}V_{den} - \bar{V}_{num}) - \bar{s}(\varepsilon - \phi) \in P_{SOS}
$$

$$
-\psi(x, c_{k}, s(x), \xi) \in P_{0_{SOS}}
$$

$$
\forall \xi_{i} \in \text{ver}(\Xi), i = 1, ..., r
$$

Figure 3. Flow chart which SMR technology improves the transient energy function

4. Realization of SMR improved direct method with VSG inverter

Taking SMIB and IEEE three-machine nine-node systems as examples, this paper illustrates the implementation of the improved direct method based on SMR technology.

4.1 SMIB system

According to the method below, the system equation of SMIB as shown in equation (4-1):

$$
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{i}_2 &= -\frac{D}{M} \frac{1}{M} f(x_i)
\end{align*}
$$
Where, \( x_1 = x = \delta - \delta_s \), \( x_2 = \omega - \omega_s \), \( f(x_i) = P_{em3} \sin(x_i + \delta_i) - P_m \). \( x_1 \) and \( x_2 \) represent virtual power angle and the rotor speed variation of inverter with VSG. Firstly, we use Taylor series to reconstruct the original system[11]. \( g_i = -1/M, \zeta_i = f(x_i) \), we choose to expand the maximum power \( k = 5 \), and then select the initial rational Lyapunov function as shown in Eq. (4-2)

\[
V = a_1x_1^2 + a_2x_2^2 + b_1x_1^4 + b_2x_1^2x_2^2 + b_3x_2^4 + c_1x_1^2 + c_2x_2^2 + d_1x_1^4 + d_2x_2^4 + d_3x_1x_2^3
\]

(4-2)

Where, \( a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2, d_3 \) are the coefficients to be optimized. Secondly, the polynomial \( \varphi(x_1, x_2) = mx_1^2 + px_1x_2 + nx_2^2 \) of the initial region shape is established, and the shape and size of the region are adjusted to approximate the boundary of the stable region by changing the polynomial structure under the premise to satisfy Lyapunov function conditions[12].

Finally, we establish a stable region boundary optimization model which is the local SOS optimization model, and get the global optimal coefficients by SMR technology, and then put them into equ.(4-2) to obtain the optimal rational Lyapunov function.

4.2 Three-machine nine-node system

The 3-machine system’s model is shown in Eq. (4-3), and we take the fourth node as reference node, that is \( \delta_4 = 0 \).

\[
\begin{align*}
\dot{\delta}_i &= \omega_i \\
M_i \dot{\omega}_i &= P_m - D_i \omega_i - \sum_{j=1}^{4} A_{ij} \sin(\delta_i - \delta_j), \quad A_{ij} = E_i E_j B_{ij}, i = 1, 2, 3
\end{align*}
\]

(4-3)

Where, \( \delta_1, \delta_2, \delta_3 \) and \( \omega_1, \omega_2, \omega_3 \) represent the virtual power angle and rotor speed variations of VSG1, VSG2 and VSG3. Firstly, we use of Taylor series reconstruction of the original system:

\[
\begin{align*}
g_1 &= 1, g_2 = -A_{13}, g_3 = 1, g_4 = -A_{23}, g_5 = 1, g_6 = A_{32} \\
\zeta_1 &= P_{m3} - A_{31} \sin(\delta_1 - \delta_3), \zeta_2 = \sin(\delta_1 - \delta_2), \zeta_3 &= P_{m2} - A_{21} \sin(\delta_2 - \delta_3), \\
\zeta_4 &= \sin(\delta_2 - \delta_4), \zeta_5 &= P_{m1} - A_{11} \sin(\delta_3 - \delta_1), \zeta_6 &= \sin(\delta_3 - \delta_2)
\end{align*}
\]

(4-4)

We still choose \( k = 5 \), then choose the initial rational Lyapunov function as follows:

\[
V = a_1x_1^2 + \cdots + a_6x_6^2 + b_1x_1^4 + b_2x_1x_2^3 + b_3x_2^4 + c_1x_1^2x_2^2 + c_2x_2^4 + c_3x_1^2 + d_1x_1^4 + d_2x_2^4 + d_3x_1x_2^3
\]

(4-5)

Where, \( a_1, \ldots, a_6, b_1, b_2, b_3, c_1, c_2, d_1, \ldots, d_5, e_1, \ldots, e_5 \) are the coefficients to be optimized. Secondly, establish the initial regional shape polynomial (4-3):

\[
\varphi(x_1, x_2) = \sum_{i=1}^{2n} a_i x_i^2 + \sum_{i=1, j=1}^{2n} b_{ij} x_i x_j + \sum_{i=1}^{2n} c_i x_i^4 + \sum_{i=1, j=1}^{2n} d_{ij} (x_i x_j)^2
\]

(4-6)

On the premise of satisfying the Lyapunov function conditions, the shape and size of the region are adjusted to approximate the stability domain boundary by changing the polynomial structure. Similarly, we establish the maximum stability domain boundary optimization model and use SMR technology to optimise.

5. Simulation and verification

5.1 Simulation conditions
In order to analyse the electromechanical transient stability under the VSG strategy, the droop control in the VSG control strategy is ignored. The basic data of SMIB and IEEE 3-machine 9-node systems are used in reference [14] and [15]. Their structural charts are shown in Figure 4:

5.2 Simulation Analysis
1) Simulation and Analysis of Improved direct method in SMIB
Substituting the simulation data into Eq. (3-22) gives Eq.(5-1) of single machine:
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -0.05x_2 - 0.5(1.223\sin(x_1 + 1.047) - 0.9)
\end{align*}
\]  
(5-1)

According to the method of selecting the initial rational Lyapunov function in the previous section, we obtain the equation (5-2):
\[
V_0 = \frac{x_1^2 + x_2^2 + x_1^2 - x_2^2 + x_1 - x_2}{1 + 2x_1 + 4x_2^2}
\]  
(5-2)

Then, select the region shape polynomial as shown in equation (5-3):
\[
\phi(x_1, x_2) = x_1^2 - 0.5x_1x_2 + x_2^2 + x_1 - x_2
\]  
(5-3)

Under the premise of satisfying the constraints, the steady-state equilibrium point of the system is changed to (1.047, 0) by changing the shape expansion region of the polynomial to approach the real stability region. The simulation results are as follows:

From the system equation can know \( g_1 = -0.6115, \zeta_1 = 0.45 - \sin(x_1 + 1.047) \), Taylor series expansion of \( \zeta_1 \) is:
\[
\zeta_1 = 0.45 - ((x_1 + 1.047) - 1/6(x_1 + 1.047)^3 + 1/120(x_1 + 1.047)^5 + \alpha(x_1 + 1.047)^5)
\]  
(5-4)

The optimal Lyapunov function is obtained by iterations of (3-21) and (3-23) is given by Eq. (5-5).
\[
\begin{align*}
R_1 &= 0.1765x_1^2 + 0.8524x_1x_2 + 1.4084x_2^2, \\
R_2 &= -0.7398x_1^2 + 0.8524x_1^2 + 0.1533x_1x_2^2 + 0.4084x_2^2, \\
R_3 &= 0.3726x_1^2 + 0.3780x_1^2 + 0.1533x_1^2 + 0.1818x_2^2, \\
R_4 &= 0.461x_1 - 0.3576x_2, \\
Q_1 &= -0.5364x_1^2 - 0.5079x_1x_2 + 0.0275x_2^2 \\
V &= \frac{R_1 + R_2 + R_3}{1 + Q_1 + Q_2}
\end{align*}
\]  
(5-5)

Figure 5 shows the stability domain of SMIB system based on the improved SMR technique. It can be clearly seen that the improved direct method introduced in this paper obtains a larger stable region and closer to the system stable region obtained by the time-domain simulation method.

Table 1 shows the critical cut-off time(CCT) of the system with different faulty lines obtained by
different transient analysis methods. Comparing with the traditional transient energy function method, the improved method introduced in this paper shows that the system criticality The resection time was longer and the result was in good agreement with the CCT obtained by time-domain simulation. This is mainly due to the fact that under the constraints of Lyapunov, the optimization model for obtaining the boundary of the stable region is established and the actual stable region is approximated by multiple iterations. Secondly, comparing the last two columns of Table 5-1, we can see that the improved method solves the conservative problem of the traditional transient energy function analysis method.

In order to further verify the superiority of the improved method introduced in this chapter, it is in line with the research thought in Chapter 4 that the simulation is carried out on the platform of the three-machine system. The simulation conditions adopt the data in Section 4.1 and substitute it into the system equation to get the expression as (5-6):

\[
\begin{align*}
\dot{\delta}_1 &= \omega_1 \\
\dot{\omega}_1 &= 0.2884 - 0.2\omega_1 - [0.2828\sin(\delta_1 - \delta_3) + 0.4034\sin(\delta_1 - \delta_3)] \\
\dot{\delta}_2 &= \omega_2 \\
\dot{\omega}_2 &= 0.2884 - 0.2\omega_2 - [0.2828\sin(\delta_2 - \delta_3) + 0.4034\sin(\delta_2 - \delta_3)] \\
\dot{\delta}_3 &= \omega_3 \\
\dot{\omega}_3 &= 0.2884 - 0.2\omega_3 - [0.2828\sin(\delta_3 - \delta_1) + 0.4034\sin(\delta_3 - \delta_2)]
\end{align*}
\]  

(5-6)

The stable equilibrium point of three machine system is \((-0.1704, 0, 0.5234, 0, 0.2083, 0\)), suppose

\[ x = (x_1, x_2, x_3, x_4, x_5, x_6)^T = (\delta_1 + 0.1704, \omega_1, \delta_2 - 0.5234, \omega_2, \delta_3, -0.2083, \omega_3)^T \]  

(5-7)

Use equation (5-7) to transform system equation (5-5) to get (5-8):

![Stability domain of SMIB system](image)
\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= 0.2884 - 0.2a_1 - (0.2828\eta_i + 0.4034\eta_j) \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= 0.4405 - 0.1a_1 - (-0.2828\eta_i + 1.701\eta_j) \\
\dot{x}_5 &= x_6, \\
\dot{x}_6 &= 0.2297 - 0.1a_1 - (-0.4034\eta_2 - 1.701\eta_j)
\end{align*}
\]  
(5-8)

Where \( \eta_i = \sin(x_i - x_3 - 0.6938), \eta_j = \sin(x_i - x_3 - 0.3787), \eta_3 = \sin(x_3 - x_5 + 0.3151) \).

According to step (4), the initial rational Lyapunov function is shown as equation (5-38):

\[
V_0 = \frac{x_1^2 + \ldots + x_n^2 + x_3^4 + x_4^4 + x_5^4 - 2x_1x_3x_5 - x_1^2x_5^2 - x_3^2x_5^2}{1 + x_1 + \ldots + x_n + x_1^2 + x_3^2 + \ldots + x_n^2} 
\]  
(5-9)

Then, select the area shape polynomial as shown in Equation (5-10):

\[
\varphi(x_1, x_2, x_3, x_4, x_5, x_6) = x_1^2 + x_2^2 + \ldots + x_6^2 - 0.5x_1x_3 + x_1x_3 + x_1x_5 + x_3x_5 
\]  
(5-10)

From the system equation, we can see:

\[
\begin{align*}
\zeta_1 &= 0.2884 - 0.2828\eta_i, \\
\zeta_2 &= 0.4405 + 0.2828\eta_i, \\
\zeta_3 &= 0.2884 + 0.2828\eta_j, \\
\zeta_4 &= 0.2297 + 0.4034\eta_j, \\
\zeta_5 &= \eta_3, \\
\zeta_6 &= \eta_3
\end{align*}
\]  
(5-11)

Among them, Taylor series expansions of \( \zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5, \zeta_6 \) are as follow in (5-12):

\[
\begin{align*}
\zeta_1 &= 0.2884 - 0.2828(x_1 - x_3 - 0.6938) - 1/6(x_1 - x_3 - 0.6938)^2 + 1/120(x_1 - x_3 - 0.6938)^3 + \ldots \\
\zeta_2 &= (x_1 - x_3 - 0.3787) - 1/6(x_1 - x_3 - 0.3787)^2 + 1/120(x_1 - x_3 - 0.3787)^3 + \ldots \\
\zeta_3 &= 0.4405 + 0.2828(x_1 - x_3 - 0.6938) - 1/6(x_1 - x_3 - 0.6938)^2 + 1/120(x_1 - x_3 - 0.6938)^3 + \ldots \\
\zeta_4 &= (x_1 - x_3 + 0.3151) - 1/6(x_1 - x_3 + 0.3151)^2 + 1/120(x_1 - x_3 + 0.3151)^3 + \ldots \\
\zeta_5 &= 0.2297 + 0.4034(x_1 - x_3 - 0.3787) - 1/6(x_1 - x_3 - 0.3787)^2 + 1/120(x_1 - x_3 - 0.3787)^3 + \ldots \\
\zeta_6 &= (x_1 - x_3 + 0.3151) - 1/6(x_1 - x_3 + 0.3151)^2 + 1/120(x_1 - x_3 + 0.3151)^3 + \ldots \\
\end{align*}
\]  
(5-12)

By the formula (3-21) and (3-23) iteration, the optimal rational Lyapunov function is shown as formula (5-13):

\[
R_n = 0.502x_1^2 + 0.3843x_1^3 + 0.4024x_1^4 + 0.6259x_1^5 + 0.3986x_1^6 + 0.2475x_6^2 + 0.8147x_2x_6 + 0.5019x_3x_6 + 0.9171x_4x_6 + 0.2279x_5x_6, \\
R_1 = 0.2395x_1^3 + 0.8574x_3x_1^2 + 0.4789x_4x_1^2 + 0.2484x_5x_1^2 + 0.3589x_6x_1^2 + 0.1469x_1x_2^2 + 0.1348x_1x_4^2 + 0.3256x_1^2 + 0.4121x_1^3 + 0.1185x_1^4 + 0.1886x_1^5, \\
R_2 = 0.096x_1^4 + 0.1653x_1x_2^2 + 0.2280x_1x_3^2 + 0.1653x_1^2 + 0.5381x_1^3 + 0.5129x_4^2 + 0.7056x_1^4 + 0.3450x_1^5, \\
Q_1 = 0.2659x_2 - 0.8002x_3 + 0.3679x_4 + 0.3638x_5 + 0.7519x_6 + 0.9920x_1, \\
Q_2 = -0.1258x_1^2 - 1.0369x_2 + 0.1758x_3^2 + 0.2916x_4^2 + 0.1153x_5^2 + 0.2144x_6^2 + 0.9181x_1^3 + 0.2720x_5x_6 + 0.2196x_5x_1 + 2388x_5x_1 + 1360x_5x_1. \\
V = \frac{R_2 + R_1}{1 + Q_1 + Q_2}
\]  
(5-13)

Table 2. System’s CCT when different line fault

| Number | Fault line | Time domain simulation | Critical excision time (CCT) | Non-improved TEF | ORLF |
|--------|------------|------------------------|-----------------------------|-----------------|------|
| 1      | 4'-5       | 0.32s ± 0.33s          | 0.322s                      | 0.329s          |
| 2      | 5'-4       | 0.41s ± 0.42s          | 0.413s                      | 0.417s          |
Table 2 shows the system’s CCT with different faults. It can also be seen that the CCT calculated by the improved method in this paper is closer to the time domain simulation results. Combined with the last two CCT values, it can be concluded that the improved method is also applicable to multi-machine systems.

6. Summary
In this paper, by introducing SMR technique, the quasi-convex optimization objective function is established to find the optimal Lyapunov function analysis method, so as to improve the traditional conservativeness of transient energy function. Firstly, the technical principle of SMR improvement is researched. Based on the four main steps, the improved Lyapunov function of SMIB and IEE 3-machine 9-node systems are respectively established, and it is verified by simulation. The results show that the SMR technique improves the direct method, and this method can satisfy transient stability analysis in both SMIB and Multi-machine system with the inverter by VSG.

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