$D - \bar{D}$ mixing and rare $D$ decays in the Littlest Higgs model with non-unitarity matrix

Chuan-Hung Chen$^{1,2}$, Chao-Qiang Geng$^{3,4}$ and Tzu-Chiang Yuan$^{3}$

$^1$Department of Physics, National Cheng-Kung University, Tainan 701, Taiwan
$^2$National Center for Theoretical Sciences, Hsinchu 300, Taiwan
$^3$Department of Physics, National Tsing-Hua University, Hsinchu 300, Taiwan
$^4$Theory Group, TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C. V6T 2A3, Canada

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Abstract

We study the $D - \bar{D}$ mixing and rare $D$ decays in the Littlest Higgs model. As the new weak singlet quark with the electric charge of $2/3$ is introduced to cancel the quadratic divergence induced by the top-quark, the standard unitary $3 \times 3$ Cabibbo-Kobayashi-Maskawa matrix is extended to a non-unitary $4 \times 3$ matrix in the quark charged currents and $Z$-mediated flavor changing neutral currents are generated at tree level. In this model, we show that the $D - \bar{D}$ mixing parameter can be as large as the current experimental value and the decay branching ratio (BR) of $D \to X_u \gamma$ is small but its direct CP asymmetry could be $O(10\%)$. In addition, we find that the BRs of $D \to X_u \ell^+ \ell^-$, $D \to X_u \nu \bar{\nu}$ and $D \to \mu^+ \mu^-$ could be enhanced to be $O(10^{-9})$, $O(10^{-8})$ and $O(10^{-9})$, respectively.
I. INTRODUCTION

As the observation of the $B_s - \bar{B}_s$ mixing in 2006 by CDF [1], all neutral pseudoscalar-antipseudoscalar oscillations ($P - \bar{P}$) in the down type quark systems have been seen. In the standard model (SM), the most impressive features of flavor physics are the Glashow-Iliopoulos-Maiani (GIM) mechanism [2] and the large top quark mass. The former results in the cancellation between the lowest order short-distance (SD) contributions of the first two generations to the mass difference $\Delta m_K$ in the $K^0$ system, while the latter makes $\Delta m_{B_q}$ ($q = d, s$) in the $B_q$ systems dominated by the SD effects [3]. In addition, these features also lead to sizable flavor changing neutral currents (FCNCs) from box and penguin diagrams, which contribute to the rare decays, such as $K \to \pi \nu \bar{\nu}$ and $B \to K^{(*)} \ell \bar{\ell}$. It is known that these processes could be good candidates to probe new physics effects [4, 5, 6]. However, it is clear that the new physics signals deviated from the SM predications for the $P - \bar{P}$ mixings and rare FCNC decays have to wait for precision measurements on these processes.

Unlike $K$ and $B_q$ systems, the SD contributions to charmed-meson FCNC processes, such as the $D - \bar{D}$ mixing [7] and the decays of $c \to u \ell^+ \ell^-$ and $D \to \ell^+ \ell^-$ [8], are highly suppressed due to the stronger GIM mechanism and weaker heavy quark mass enhancements in the loops. On the other hand, it is often claimed that the long-distance (LD) effect for the $D - \bar{D}$ mixing should be the dominant contribution in the SM. Nevertheless, because the nonperturbative hadronic effects are hard to control, the result is still inconclusive [9, 10, 11, 12]. Recently, BABAR [13] and BELLE [14, 15] collaborations have reported the evidence for the $D - \bar{D}$ mixing with

\[
x' = (-0.22 \pm 0.30 \pm 0.21) \times 10^{-3},
\]

\[
y' = (9.7 \pm 4.4 \pm 3.1) \times 10^{-3},
\]

respectively, where $x' = x \cos \delta + y \sin \delta$ and $y' = -x \sin \delta + y \cos \delta$ with the assumption of CP conservation and $\delta$ being the relative strong phase between the amplitudes for the
doubly-Cabbibo-suppressed $D \to K^+\pi^-$ and Cabbibo-favored $D \to K^-\pi^+$ decays\cite{16,17} and $y_{CP} = \tau(D \to K^-\pi^+)/\tau(D \to K^+K^-) - 1$. Moreover, no evidence for CP violation is found. The combined results of Eqs. (1) and (2) at the 68% C.L. are\cite{18}

$$x = (5.5 \pm 2.2) \times 10^{-3},$$  
$$y = (5.4 \pm 2.0) \times 10^{-3},$$  
$$\delta = (-38 \pm 46)^0.$$

Note that the upper bound of $x < 0.015$ at 95\% C.L. can be extracted from the BELLE data in Eq. (2)\cite{14,15}. The evidences of the mixing parameters by BABAR and BELLE collaborations reveal that the era of the rare charmed physics has arrived. The results in Eq. (3) can not only test the SU(3) breaking effects for the $D - \bar{D}$ mixing\cite{10,12}, but also examine new physics beyond the SM\cite{17,18,19,20,21}.

It is known that a straightforward way to enhance the rare $D$ processes is to include some new heavy quarks within the framework of the SM. For instance, if a new heavy quark with the electric charge of $-1/3$ is introduced, it could affect the $D$ system since the extra down type quark violates the GIM mechanism. However, the constraint on this heavy quark is quite strong as it could also lead to FCNCs for the down type quark sector at tree level, which are strictly limited by the well measured rare $K$ and $B$ decays, such as $K_L \to \mu^+\mu^-$ and $B \to X_s\gamma$\cite{22}. On the other hand, if the charge of the new heavy quark is $2/3$, it could generate interesting tree level FCNCs for the up type quark sector, for which the constraints are much weaker. In this paper, we will study $D$ physics based on a new weak singlet upper quark.

It has been known that in the framework of the Littlest Higgs model\cite{23}, there exists a new $SU(2)_L$ singlet vector-like up quark\cite{24}, hereafter denoted by $T$. Since the number of down type quarks is the same as that in the SM, the standard unitary $3 \times 3$ Cabibbo-Kobayashi-Maskawa (CKM) matrix is extended to a non-unitary $4 \times 3$ matrix in the charged currents. Moreover, $Z$-mediated FCNCs for the up quark sector are generated at tree level. In Ref.\cite{25}, it has been shown that the contributions of this new quark to the rare $D$ processes are small and cannot reach the sensitivities of future experiments\cite{25,26}. In this paper, we will demonstrate that by adopting some plausible scenario, the effects could not only generate a large $D - \bar{D}$ oscillation but also marginally reach the sensitivity proposed by BESIII for the rare $D$ decays\cite{27}. We note that the implication of the new data on the
$D - \bar{D}$ mixing in the Littlest Higgs model with T-parity has been recently studied in Ref. [21].

The paper is organized as follows. In Sec. II we investigate that when a gauge singlet $T$-quark is introduced in the Littlest Higgs model, how the non-unitary matrix for the charged current and the tree level $Z$-mediated FCNC are formed. By using the leading perturbation, the mixing matrix elements related to the new parameters in the Littlest Higgs model are derived. In addition, we study how to get the small mixing matrix element for $V_{ub}$, which describes the $b \to u(c)$ decays. In Sec. III we discuss the implications of the non-unitarity on the $D - \bar{D}$ mixing and rare $D$ decays by presenting some numerical analysis. Finally, we summarize our results in Sec. IV.

II. NON-UNITARY MIXING MATRIX IN THE LITTLEST MODEL

To study the new flavor changing effects in the Littlest Higgs model, we start by writing the Yukawa interactions for the up quarks to be [24, 25]

$$\mathcal{L}_Y = \frac{1}{2} l_{ab} f \epsilon_{ijk} \epsilon_{xy} \chi_{ai} \Sigma_{jx} \Sigma_{ky} u_b^c + l_0 f T T^c + h.c.,$$

where $\chi_T^1 = (d_1, u_1, 0)$, $\chi_T^2 = (s_2, c_2, 0)$, $\chi_T^3 = (b_3, t_3, T)$, $u'_b$ is the weak singlet and $\Sigma = e^{i\Pi/\Sigma_0} e^{i\Pi^T/f}$ with

$$\Sigma_0 = \begin{pmatrix} 1 & \mathbb{1}_{2 \times 2} \\ \mathbb{1}_{2 \times 2} & 1 \end{pmatrix}, \quad \Pi = \begin{pmatrix} h^+/\sqrt{2} & \phi^+ \\ h/\sqrt{2} & h^*/\sqrt{2} \\ \phi & h^T/\sqrt{2} \end{pmatrix}.$$ 

The scale $f$ denotes the global symmetry spontaneously breaking scale, which, as usual, could be around 1 TeV. Consequently, the $4 \times 4$ up-quark mass matrix is given by [25]

$$M_U = \begin{bmatrix} (i l_{ij} v)_{3 \times 3} & 0 \\ - & - & - & - & - \\ 0 & 0 & l_{33} f & l_0 f \end{bmatrix}.$$ 

We remark that the quadratic divergences for the Higgs mass from one-loop diagrams involving $t$ and $T$ get exactly cancelled as shown in Ref. [28]. Moreover, for other quarks other than the top quark, the one-loop quadratic divergent contributions do not necessitate fine-tuning the Higgs potential as the cutoff is around 10 TeV for $f \sim 1$ TeV due to the small
corresponding Yakawa couplings. That is why there is no need to introduce extra singlet states \( T \) \cite{28, 29}.

To obtain the quark mass hierarchy of \( m_t \gg m_c \gg m_u \), we can choose a basis such that the up-quark mass matrix is \cite{30}

\[
M_U = \begin{pmatrix}
\hat{m}_U & 0 \\
\mathbf{h} f & l_0 f
\end{pmatrix}
\]  

(7)

where \( \hat{m}_{Uij} = \delta_{ij}l_i v/\sqrt{2} \equiv m_i \) is diagonal matrix and \( \mathbf{h} = (h_1, h_2, h_3) \). The \( h_i \) is related to \( l_{33} \) by \( h_i = \tilde{V}_U^R l_{33} \) and \( \mathbf{h} \mathbf{h}^\dagger = |l_{33}|^2 \), in which \( \tilde{V}_U^R \) is the unitary transformation for the right-handed up quarks. We note that \( m_i \) are not the physical masses and in principle their magnitudes could be as large as the weak scale. In order to preserve the hierarchy in the quark masses, one expects that \( m_3 > m_2 > m_1 \). Furthermore, in terms of this basis, the charged and neutral currents, defined by

\[
\mathcal{L}^C = \frac{g}{\sqrt{2}} J^\mu_\mu W^{\mu} - \frac{g}{\sqrt{2} \tan \theta} J^\mu_\mu H^{\mu} + h.c.,
\]

\[
\mathcal{L}^N = \frac{g}{\cos \theta_W} (J^\mu_3 - \sin^2 \theta_W J^\mu_{em}) Z_{\mu} + \frac{g}{\tan \theta} J^\mu_3 H_{\mu} + h.c.,
\]

are expressed by

\[
J^\mu_\mu = \bar{U}_L \gamma'^\mu a_V D_L,
\]

\[
J^\mu_3 = \frac{1}{2} \bar{U}_L \gamma'^\mu \bar{V}_0 a_V \bar{V}_0^\dagger U_L - \frac{1}{2} \bar{D}_L \gamma'^\mu D_L,
\]

respectively, where \( U^T = (u_1, c_2, t_3, T) \), \( D^T = (d, s, b) \), \( a_V = \text{diag}(1, 1, 1, 0) \) and

\[
\bar{V}_0 = \begin{pmatrix}
(V_{CKM}^0)_{3 \times 3} & 0 \\
0 & 1
\end{pmatrix}
\]  

(10)

with \( V_{CKM}^0 V_{CKM}^{0\dagger} = \mathbb{1}_{3 \times 3} \). The null entry in \( a_V \) denotes the new \( T \)-quark being a weak singlet; and without the new \( T \)-quark, \( V_{CKM}^0 \) is just the CKM matrix. Since the down quark sector is the same as that in the SM, we have set the unitary transformation \( U^{DL} \) to be an identity matrix.

For getting the physical eigenstates, the mass matrix in Eq. (7) can be diagonalized by unitary matrices \( V_{UL,R} \) so that we have

\[
M_U^{\text{diag}} M_U^{\text{diag}} = V_{UL} M_U V_{UL}^\dagger
\]  

(11)
and
\[ M_U M_U^\dagger = \begin{pmatrix} \tilde{m}_U \tilde{m}_U^\dagger & \tilde{m}_U h f \\ h m_U^\dagger f & (|l_{33}|^2 + |l_0|^2)f^2 \end{pmatrix} \]  \quad (12)

Since \((|l_{33}|^2 + |l_0|^2)f^2\) is much larger than other elements, we can take the leading order of the perturbation in \(h_i m_i/f\) \((i = 1, 2, 3)\). According to Eq. (11), the leading expansion is given by
\[ M_U^{\text{diag}} M_U^{\text{diag}} = U^L M_U M_U^\dagger U^L \approx (1 + \Delta_L) M_U M_U^\dagger (1 - \Delta_L). \]  \quad (13)

By looking at the off-diagonal terms \((M_U^{\text{diag}} M_U^{\text{diag}})_{4(4i)}\), we can easily get
\[ \Delta_{Lij} \approx -\Delta_{Lij} = -\frac{h_i m_i f}{(|l_{33}|^2 + |l_0|^2)f^2 - m_i^2} \]  \quad (14)

with \(i \neq 4\). From the diagonal entries, if we set the light quark masses to be \(m_u \approx m_c \approx 0\), we obtain
\[ 0 \approx m_{a4}^2 \approx m_j^2 + 2\Delta_{Lj4}(M_U M_U^\dagger)_{4j}, \]
\[ \Delta_{Lj4} \approx -\frac{1}{2 h_j f} m_j \]  \quad (15)

with \(j = 1, 2\). To be consistent with Eq. (14), at the leading expansion the relation
\[ 2h_j^2 = (|l_{33}|^2 + |l_0|^2) \]  \quad (16)

should be satisfied. We emphasize that the choice of Eq. (16) is somewhat fine-tuned in order to have Eqs. (14) and (15) simultaneously. Since the top-quark is much heavier than other ordinary quarks, we have \(2h_3^2 \approx (1 - m_t^2/m_3^2)(|l_{33}|^2 + |l_0|^2)\) if \(f > m_3 > m_t\). Similarly, one obtains the flavor mixing effects for \(i \neq j \neq 4\) to be
\[ \Delta_{Lij} = \frac{h_i h_j m_i m_j}{m_i^2 - m_j^2} \frac{f^2[2(|l_{33}|^2 + |l_0|^2)f^2 - (m_i^2 + m_j^2)]}{(|l_{33}|^2 + |l_0|^2)(f^2 - m_j^2)} \]  \quad (17)

After diagonalization, the currents become
\[ J^\mu = U_L \gamma_\mu V^0 a_V D_L = \tilde{U}_L \gamma_\mu V^0 V^0 D_L, \]
\[ J_3^\mu = U_L \gamma_\mu V^0 a_V \tilde{V}^{0\dagger} V^{0\dagger} V^0 V^0 V^0 U_L = \tilde{U}_L \gamma_\mu V^0 V^0 V^0 V^0 V^0 U_L, \]
where \(U_T = (u, c, t, T)\),
\[ V^0 = \tilde{V}^{0\dagger} a_V = \begin{pmatrix} (V^0_{CKM})_{3\times 3} & 0 \\ 0 & 0 \end{pmatrix} \]  \quad (19)
and \(\text{diag}(V^0 V^0) = a_V\). Since the 4-th component of \(a_V\) is different from the first 3 ones, it is obvious that the matrix \(V \equiv V^U_L V^0\) associated with the charged current does not satisfy unitarity. In addition, \(V^U_L a_V V^U_L^\dagger\), which is associated with the neutral current, is not the identity matrix. As a result, \(Z\)-mediated FCNCs at tree level are induced. According to Eq. (18), we see that

\[ V V^\dagger = V^U_L a_V V^U_L^\dagger \]  

(20)

which is just the same as the effects of the \(Z\)-mediated FCNCs. Due to \(V\) being a non-unitary matrix, one finds

\[ (VV^\dagger)_{ij} = V_{i4} V_{j4}^*. \]  

(21)

Consequently, the interesting phenomena arising from non-unitary matrix elements are always related to \(V_{i4} V_{j4}^* = \Delta_{Li4} \Delta_{j4}\). We note that as we do not particularly address CP problem, in most cases, we set the parameters to be real numbers.

It has been known that enormous data give strict bounds on the flavor changing effects. In particular, the pattern describing the charged current has been fixed quite well. Any new parametrization should respect these constraints. It should be interesting to see the relationship with and without the new vector-like \(T\)-quark. From Eq. (18), we know that the new flavor mixing matrix for the charged current is given by \(V = V^U_L V^0\). At the leading order perturbation, one gets

\[ V = V^U_L V^0 \approx (1 + \Delta_L) V^0 = V^0 + \Delta_L V^0. \]  

(22)

If \(V_{tb}^0 \sim 1\) is taken, one finds that \(V_{ub} \approx V_{ub}^0 + \Delta_{L13}\) and \(V_{cb} \approx V_{cb}^0 + \Delta_{L23}\). In terms of Eq. (17) and \(h_1 = h_2 \approx h_3\), the relations \(\Delta_{L13} \approx -m_1/m_3\) and \(\Delta_{L23} \approx -m_2/m_3\) are obtained. Hence, in our approach, we have

\[ V_{us} \approx V_{us}^0 - \frac{m_1}{m_2}, \quad V_{ub} \approx V_{ub}^0 - \frac{m_1}{m_3}, \quad V_{cb} \approx V_{cb}^0 - \frac{m_2}{m_3}. \]  

(23)

From these results, it is clear that when the \(T\)-quark decouples from ordinary quarks, \(V_{us} \to V_{us}^0, V_{ub} \to V_{ub}^0\) and \(V_{cb} \to V_{cb}^0\), while \(m_1/m_2 \to m_u/m_c, m_1/m_3 \to m_u/m_t\) and \(m_2/m_3 \to m_c/m_t\), respectively. According to the observations in the decays of \(b \to u\ell \bar{\nu}_\ell\) and \(b \to c\ell \bar{\nu}_\ell\), the corresponding values have been determined to be \(|V_{ub}| = 3.96 \pm 0.09 \times 10^{-3}\) and \(|V_{cb}| = 42.21^{+0.10}_{-0.80} \times 10^{-3}\), respectively [16]. Since \(V_{ij}^0\) and \(m_i\) are free parameters, to
satisfy the experimental limits with interesting phenomena in low energy physics, it is reasonable to set the orders of magnitude for \(m_1/m_3\) and \(m_2/m_3\) \((m_1/m_2)\) to be \(O(10^{-2})\) and \(O(10^{-1})\), respectively. Consequently, the non-unitary effects on the rare charmed meson decays governed by \(V_{14}V_{24}^*\) could be as large as \(\Delta_{14}\Delta_{24} \sim O(10^{-4})\), which could be one order of magnitude larger than those in Ref. [25].

III. \(D - \bar{D}\) MIXING AND RARE \(D\) DECAYS

A. \(D - \bar{D}\) mixing

It is well known that the GIM mechanism has played an important role in the \(K - \bar{K}\) oscillation in the SM. In addition, due to the top-quark in the box and penguin diagrams, \(B_q - \bar{B}_q\) mixings are dominated by the SD effects, which are consistent with the data [16]. On the contrary, for the \(D - \bar{D}\) mixing the GIM cancellation further suppresses the mixing effect to be \(\Delta m_D \sim O(m_s^4/m_W^2 m_c^2)\) [7] and the bottom quark contribution actually is a subleading effect due to the suppression of \((V_{ub}V_{cb}^*)^2\). In the SM, the SD contribution to the mixing parameter is \(O(10^{-7})\) [34]. However, the LD contribution to the mixing is believed to be dominant. Due to the nonperturbative hadronic effects, the result is still uncertain with the prediction on the mixing parameter ranging from \(O(10^{-3})\) [9] to \(O(10^{-2})\) [10, 11, 12]. Nonetheless, the mixing parameters shown in Eq. (3) could arise from the LD contribution. Thus, it is important to have a better understanding of the LD effect. On the other hand, it is also possible that the mixings in Eq. (3) could result from new physics. In the following, we will concentrate on the Littlest Higgs model.

In the quark sector of the Littlest Higgs model due to the introduction of a new weak singlet, a direct impact on the low energy physics is the FCNCs at tree level. According to Eq. (18), the most attractive process with \(|\Delta C| = 2\) via the \(Z\)-mediated \(c - u - Z\) interaction, illustrated in Fig. 1, is given by

\[
\mathcal{H}(|\Delta C| = 2) = \frac{g^2}{4m_W^2} (V_{14}V_{24}^*)^2 \bar{u}\gamma_\mu P_L c \bar{u}\gamma_\mu P_L c, \\
= \frac{2G_F}{\sqrt{2}} (V_{14}V_{24}^*)^2 \bar{u}\gamma_\mu P_L c \bar{u}\gamma_\mu P_L c.
\]

(24)

In terms of the hadronic matrix element, defined by

\[
\langle \bar{D}|(\bar{u}c)_{V-A}(\bar{u}c)_{V-A}|D\rangle = \frac{8}{3} B_D f_D^2 m_D^2,
\]

(25)
FIG. 1: $Z$-mediated flavor diagram with $|\Delta C| = 2$.

the mass difference for the $D$ meson is \[25\]

$$\Delta m_D \approx \frac{\sqrt{2}}{3} G_F f_D^2 m_D B_D |(V_{14}V_{24}^*)^2|.$$ \hspace{1cm} (26)

If we assume no cancellation between new physics and SM contributions, by taking $\tau_D = 1/\Gamma_D = 6.232 \times 10^{11} \text{ GeV}^{-1}$, $f_D \sqrt{B_D} = 200 \text{ MeV}$ \[31, 32\] and $m_D = 1.86 \text{ GeV}$ and using Eq. (26), we obtain

$$\zeta_0 \equiv |V_{14}V_{24}^*| = |\Delta L_{14}\Delta L_{24}| = (1.47 \pm 0.29) \times 10^{-4},$$ \hspace{1cm} (27)

which is in the desirable range. In other words, the result in Eq. (27) demonstrates that the non-unitarity in the Littlest Higgs model could enhance the $D - \bar{D}$ mixing at the observed level. We note that the limit of $x < 0.015$ (95% C.L.) leads to

$$\zeta_0 < 2.5 \times 10^{-4}.$$ \hspace{1cm} (28)

In addition, we note that cancellation between the LD effect in the SM and the SD one from new physics could happen. In this case, the values in Eqs. (27) and (28) could be relaxed.

**B. $D \to X_u \gamma$ decay**

In the SM, the $D$-meson FCNC related processes are all suppressed since the internal fermions in the loops are all much lighter than $m_W$. For the decay of $D \to X_u \gamma$, without QCD corrections, the branching ratio is $O(10^{-17})$; and it becomes $O(10^{-12})$ when one-loop QCD corrections are included \[8\]. However, it is found that the two-loop QCD corrections can boost the BR to be as large as $3.5 \times 10^{-8}$ \[33\]. It should be interesting to see how large the non-unitarity effect on $c \to u \gamma$ is in the Littlest Higgs model.
To study the radiative decay of $c \rightarrow u \gamma$, we write the effective Lagrangian to be

$$
\mathcal{L}_{c \rightarrow u \gamma} = -\frac{G_F}{\sqrt{2}} V_{us} V_{cs}^* C_7 \frac{e}{4\pi^2} m_c \bar{u} \sigma_{\mu\nu} P_R \epsilon F^{\mu\nu},
$$

(29)

where $C_7 = C_7^{SM} + C_7^{NP}$ and $C_7^{SM} \approx -(0.007 + i0.02) = 0.021 e^{i\delta_s}$ with $\delta_s = 70.7^\circ$ [33], being the strong phase induced by the two-loop QCD corrections. In the extension of the SM by including a weak singlet particle, the flavor mixing matrix in the charged current is not unitary and the $Z$-mediated FCNC at tree level is generated as well. For $c \rightarrow u \gamma$, besides the QED-penguin diagrams induced by the $W$-boson displayed in Figs. 2a and 2b, the $Z$-mediated QED-penguin one in Fig. 2c will also give contributions. We note that the contributions from $W_H$ and $Z_H$ can be ignored as $m_W^2/m_W^2$ and $m_Z^2/m_Z^2$ are much less than one.

At the first sight, due to the light quarks in the loops, the contributions from Figs. 2a and 2b could be negligible. However, due to the non-unitarity of $(V V^\dagger)_{uc} = V_{14} V_{24}^* \neq 0$, even in the limits of $m_{d,s,b} \to 0$, the contributions from the mass independent terms do not vanish anymore and can be sizable. In terms of unitary gauge [22], we obtain

$$
C_7^W = \frac{11}{36} \frac{(VV^\dagger)_{12}}{V_{us} V_{cs}^*} = \frac{11}{36} \frac{V_{14} V_{24}^*}{V_{us} V_{cs}^*}.
$$

(30)

Furthermore, if we set $m_u \approx m_c = 0$, the contributions from Fig. 2c are given by

$$
C_7^Z = \frac{f_c^Z + f_u^Z}{V_{us} V_{cs}^*},
$$

$$
\frac{1}{2} e_u \left\{ \left( \frac{1}{2} - e_u \sin^2 \theta_W \right) [4\xi_0(0) - 6\xi_1(0) + 2\xi_2(0)]
\right.
\left. + e_u \sin^2 \theta_W [4\xi_0(0) - 4\xi_1(0)] \right\},
$$

$$
\frac{1}{4} e_u [2\xi_0(y_T) - 3\xi_1(y_T) + \xi_2(y_T)],
$$

(31)

where the functions $\xi_n(x)$ are defined by

$$
\xi_n(x) \equiv \int_0^1 \frac{z^{n+1} dz}{1 + (x - 1) z}
$$

(32)
and \( y_T = m_T/m_Z \). Numerically, the total contribution in Fig. 2 is

\[
C_7 = C_7^W + C_7^Z 
\approx \frac{0.53}{V_{cs}V_{us}} V_{14}V_{24}^* ,
\]

(33)

If we regard \( V_{14}V_{24}^* \) as an unknown complex parameter, i.e. \( V_{14}V_{24}^* = \zeta_0 e^{i\theta} \) with \( \theta \) being the CP violating phase, one can study the decay BR and direct CP asymmetry (CPA) of \( D \to X_{u\gamma} \) defined by

\[
\text{BR}(D \to X_{u\gamma}) = \frac{6\alpha_{\text{em}}|C_7|^2}{\pi|V_{cd}|^2} \text{BR}(D \to X_{d\mu\bar{\nu}_\mu}) ,
\]

\[
A_{CP} = \frac{\text{BR}(\bar{c} \to \bar{u}\gamma) - \text{BR}(c \to u\gamma)}{\text{BR}(\bar{c} \to \bar{u}\gamma) + \text{BR}(c \to u\gamma)} ,
\]

(34)

as functions of \( \zeta_0 \) and \( \theta \). In Fig. 3 the BR and CPA as functions of \( \zeta_0 \) are presented, where the solid, dotted, dashed and dash-dotted lines represent the CP violating phase at \( \theta = 0, 45^\circ, 90^\circ \) and \( 135^\circ \), respectively. From these results, it is interesting to see that \( \text{BR}(D \to X_{u\gamma}) \) is insensitive to the new physics effects, whereas the direct CPA could be as large as \( O(10\%) \). Explicitly, if we take \( \theta = 90^\circ \) and \( \zeta_0 = 1.5 \times 10^{-4} \), the CPA is about 3%. Note that this CPA vanishes in the SM.

FIG. 3: BR (in units of \( 10^{-8} \)) and CPA (in units of \( 10^{-2} \)) for \( D \to X_{u\gamma} \) as functions of \( \zeta_0 \), where the solid, dotted, dashed and dash-dotted lines represent the CP violating phase at \( \theta = 0, 45^\circ, 90^\circ \) and \( 135^\circ \), respectively.
C. \( D \to X_u \ell \bar{\ell} \) and \( D^0 \to \ell^+ \ell^- \) decays

Because the current experimental measurements in \( K \) and \( B \) \( q \) decays are all consistent with the SM predictions, it is inevitable that if we want to observe any deviations from the SM, we have to wait for precision measurements for \( K \) and \( B \) \( q \). SuperB factories or LHCb could provide a hope. However, the situation in \( D \) physics is straightforward. As stated before, unlike \( K \) and \( B \) \( q \) systems, due to no heavy quark enhancement in the \( D \) system, the rare \( D \)-meson decays, such as \( D \to X_u \ell \bar{\ell} (\ell = e, \mu, \nu) \), are always suppressed. Even by considering the long-distance effects, the related decays, such as \( D \to \mu^+ \mu^- \) and \( D \to X_u \nu \bar{\nu} \), get small corrections to the SD predictions on the BRs \[35\]. Therefore, these rare decays definitely could be good candidates to probe the new physics effects. Since the values in the SM are hardly reachable at \( D \) factories \[27\], if any exotic event is found, it must be a strong evidence for new physics. In the following analysis, we are going to discuss the implication of the Littlest Higgs model on the rare \( D \) decays involving di-leptons.

To study these decays, we first write the effective Hamiltonian for \( c \to u \ell^+ \ell^- (\ell = e, \mu) \) as

\[
\mathcal{H}(c \to u \ell^+ \ell^-) = -\frac{G_F \alpha_{\text{em}}}{\sqrt{2} \pi} V_{cs}^* V_{us} \left[ C_{\text{eff}}^9 O_9 + C_7 O_7 + C_{10} O_{10} \right],
\]

where the effective Wilson coefficients are given by

\[
C_{\text{eff}}^9 = \frac{2 \pi (VV^\dagger)^{14}}{\alpha_{\text{em}} V_{cs}^* V_{us}^c} c^\ell_V + (h(z_s, s) - h(z_d, s)) (C_2(m_c) + 3 C_1(m_c)),
\]

\[
C_{10} = -\frac{2 \pi (VV^\dagger)^{14}}{\alpha_{\text{em}} V_{cs}^* V_{us}^c} c_A,
\]

with \( s = q^2/m_c^2, z_i = m_i/m_c, c^\ell_V = -1/2 + 2 \sin^2 \theta_W, c_A = -1/2 \) and

\[
h(z, s) = -\frac{4}{9} \ln z + \frac{4}{27} + \frac{2}{9} x - \frac{1}{9} (2 + x) \sqrt{1 - x} \times \begin{cases} 
\ln \frac{\sqrt{1 - x^2}}{\sqrt{1 - x}} - i \pi, & \text{for } x \equiv 4 z^2/s < 1, \\
2 \arctan \frac{1}{\sqrt{x - 1}}, & \text{for } x \equiv 4 z^2/s > 1.
\end{cases}
\]

Here, we have neglected the small contributions from the penguin and box diagrams. We note that in the SM, the SD contributions are mainly from the term with \( h(z, s) \), induced
by the insertion of $O_2 = \bar{u}_L \gamma_\mu q_L \bar{q}_L \gamma_\mu c_L$ and mixing with $O_9$ at one-loop level [35, 36].

We note that the resonant decays of $D \to X_u V \to X_u \ell^+ \ell^-$ ($V = \phi, \rho, \omega$) would have large corrections to $c \to u\ell^+\ell^-$ at the resonant regions. However, in this paper we do not discuss these contributions as we only concentrate on the SD contributions. Moreover, these resonance contributions can be removed by imposing proper cuts in the phase space in dedicated searches.

From Eq. (35), the decay rate for $D \to X_u \ell^+ \ell^-$ as a function of the invariant mass $s = q^2/m_c^2$ can be found to be

$$
\frac{d\Gamma}{ds} = \frac{G_F^2 m_c^5 \alpha_{em}^2}{768\pi^5} |V_{us}V_{cs}|^2 (1 - s)^2 R(s),
$$

$$
R(s) = \left( |C_{\text{eff}}^9|^2 + |C_{10}|^2 \right) (1 + 2s) + 12Re(C_7^*C_{\text{eff}}^9) + 4 \left( 1 + \frac{2}{s} \right) |C_7|^2.
$$

In addition, by utilizing the lepton angular distribution, we can also study the forward-backward asymmetry (FBA), given by

$$
\frac{dA_{FB}}{ds} = \frac{\int_{-1}^{1} d\cos \theta d\Gamma/ds d\cos \theta \, \text{sgn}(\cos \theta)}{\int_{-1}^{1} d\cos \theta d\Gamma/ds d\cos \theta},
$$

$$
= -3 \frac{s}{R(s)} \text{Re} \left( C_{\text{eff}}^9 \frac{2}{s} C_7^* \right) C_{10}^*,
$$

where $\theta$ is the angle of $\ell^+$ related to the momentum of the $D$ meson in the $\ell^+\ell^-$ invariant mass frame. Since $C_{10}$ is small in the SM, $A_{FB}$ is negligible. With $m_c = 1.4$ GeV and the mixing parameter in Eq. (27), we get

$$
\text{BR}(D \to X_u e^+ e^-) = (4.18 \pm 0.91) \times 10^{-10},
$$

$$
\text{BR}(D \to X_u \mu^+ \mu^-) = (2.51 \pm 0.86) \times 10^{-10},
$$

comparing with the SM predictions of $\text{BR}(D \to X_u e^+ e^-)_{SM} = 2.1 \times 10^{-10}$ and $\text{BR}(D \to X_u \mu^+ \mu^-)_{SM} = 0.5 \times 10^{-10}$, respectively. Clearly, if some cancellation occurs between new physics and SM contributions in the $D - \bar{D}$ mixing, a larger value of $\zeta_0$ can be allowed. In Fig. 4, we show the tendency of the decay as a function of $\zeta_0$, where the negative horizontal values correspond to $-\zeta_0$. In addition, we present the differential decay BR [FBA] of $D \to X_u e^+ e^-$ as a function of $s = q^2/m_c^2$ in Fig. 5a [b], where the thick solid, dotted and dashed lines correspond to $\zeta_0 = 1.5, 3.0$ and $5.0$, while the thin ones denote the cases for $-\zeta_0$ except $\zeta_0 = 0$ for the thin solid line in Fig. 5a. From Fig. 5b, we see that the FBA
FIG. 4: BR (in units of $10^{-9}$) for $D \rightarrow X_u e^+ e^-$ as a function of $\zeta_0$.

FIG. 5: (a) [(b)] Differential BR (in units of $10^{-9}$) [FBA] for $D \rightarrow X_u e^+ e^-$ as a function of $s$, where the thick solid, dotted and dashed lines correspond to $\zeta_0 = 1.5, 3.0$ and 5.0, while the thin ones denote the cases for $-\zeta_0$ except $\zeta_0 = 0$ for the thin solid line in (a).

is only at percent level. In the Littlest Higgs model, this is because the $Z$ coupling to the charged lepton $c_\ell V = -1/2 + 2\sin^2 \theta_W$ appearing in $C_9^{\text{eff}}$ is much smaller than one. This is quite different from that in $b \rightarrow s \ell^+ \ell^-$ where the dominant effect in the SM for the FBA is from the box and QED-penguin diagrams.

Next, we discuss the decay of $D \rightarrow X_u \nu \bar{\nu}$. In the SM, the BR for $D \rightarrow X_u \nu \bar{\nu}$ is estimated
to be \(O(10^{-16}) - O(10^{-15})\) [35], which is vanishing small. In the Littlest Higgs model, by taking \(C_7 = 0\), \(C_9^{\text{eff}} = -C_{10} = -\pi (VV^\dagger)_{14}/(\alpha_{\text{em}} V_{us} V_{cs}^*)\), the effective Hamiltonian in Eq. (35) can be directly applied to \(c \to u\nu\bar{\nu}\). The decay rate for \(D \to X_u\nu\bar{\nu}\) as a function of \(s = q^2/m_c^2\) can be obtained as

\[
\frac{d\Gamma}{ds} = 3 \frac{G_F^2 m_c^5}{768\pi^5} (1 - s)^2 (1 + 2s) (2\pi^2 |(VV^\dagger)_{12}|^2),
\]

where the factor of 3 stands for the neutrino species. With \(\zeta_0 = 1.5 \times 10^{-4}\), we get \(\text{BR}(D \to X_u\nu\bar{\nu}) = 1.31 \times 10^{-9}\). However, if we relax the constraint on \(V_{14}V_{24}^\dagger\), the BR as a function of \(\zeta_0\) is shown in Fig. 6a. For a larger value of \(\zeta_0\), \(\text{BR}(D \to X_u\nu\bar{\nu})\) could be as large as \(O(10^{-8})\).

Finally, we study the decays of \(D \to \ell^+\ell^-\). It has been known that, in the SM, the SD contributions to \(D \to \mu^+\mu^-\) are \(O(10^{-18})\), while the LD ones are \(O(10^{-13})\) [35]. It is clear that any signal to be observed at the sensitivity of the proposed detector, such as BESIII, will indicate new physics effects. Since the effective interactions at quark level are the same as those in Eq. (35), one finds that

\[
\text{BR}(D \to \ell^+\ell^-) = \frac{G_F^2}{16\pi^2} \frac{\tau_D m_D m_\ell^2 f_D^2}{f_D^2} \sqrt{1 - \frac{4m_\ell^2}{m_D^2}} |\pi V_{14} V_{24}^*|^2.
\]

Here we have used equation of motion for the charged lepton so that \(\bar{\ell} p_D \ell = 0\). We also note that operators \(O_{7,9}\) make no contributions. With \(|V_{14} V_{24}^*| = \zeta_0 = 1.5 \times 10^{-4}\), the

![Figure 6](image-url)
predicted BR for \( D \rightarrow \mu^+\mu^- \) is \( 1.17 \times 10^{-10} \). In Fig. 6b, we present the BR as a function of \( \zeta_0 \). We see that \( \text{BR}(D \rightarrow \mu^+\mu^-) \) in the Littlest Higgs model could be as large as \( O(10^{-9}) \).

**IV. CONCLUSIONS**

We have studied the \( D - \bar{D} \) mixing and rare \( D \) decays in the Littlest Higgs model. In the model, as the new weak singlet vector-like quark \( T \) with the electric charge of \( 2/3 \) is introduced to cancel the quadratic divergence induced by the top-quark, the standard unitary \( 3 \times 3 \) CKM matrix is extended to a non-unitary \( 4 \times 3 \) matrix in the quark charged currents and \( Z \)-mediated flavor changing neutral currents are generated at tree level. We have shown that the effects on \( |\Delta C| = 2 \) and \( |\Delta C| = 1 \) processes are all related to \( V_{14}V_{24}^* \) in Eq. (21).

To avoid the scenario adopted by Ref. [25], in which \( l_0 \sim l_{33} \gg l_{ij} \) was assumed, we choose the basis such that the effective mass matrix for \( u_1, c_2 \) and \( t_3 \) is diagonal, while the corresponding masses \( m_1, m_2 \) and \( m_3 \) are free parameters and can be as large as the weak scale \( v \). Since the global symmetry breaking scale \( f \) is larger than \( v \), the mixing matrix relating physical and unphysical states could be extracted by taking the leading perturbative expansion. Accordingly, by using the approximation of \( m_u \approx m_c \approx 0 \), the explicit expressions for \( V_{14} \) and \( V_{24} \) have been obtained. In terms of the data for \( V_{ub} \) and \( V_{cb} \), we have found that the natural value for \( \zeta_0 \equiv |V_{14}V_{24}^*| \) is \( O(10^{-4}) \), which agrees with the observed parameter in the \( D - \bar{D} \) mixing but it is one order of magnitude larger than that in Ref. [25].

For the rare \( D \) decays, due to the non-unitarity effects in the model, \( \text{BR}(D \rightarrow X_u\ell^+\ell^-) \), \( \text{BR}(D \rightarrow X_u\nu\bar{\nu}) \) and \( \text{BR}(D \rightarrow \mu^+\mu^-) \) could be enhanced to be \( O(10^{-9}) \), \( O(10^{-8}) \) and \( O(10^{-9}) \), respectively, which could marginally reach the sensitivity proposed by BESIII [27].

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[1] A. Abulencia et al. [CDF Collaboration], Phys. Rev. Lett. 97, 242003 (2006) [arXiv:hep-ex/0609040].
[2] S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2, 1285 (1970).
[3] J. S. Hagelin, Nucl. Phys. B193, 123 (1981).
[4] S. R. Choudhury et al., Phys. Lett. B601, 164 (2004); J. Hubisz, S. J. Lee and G. Paz, JHEP 06, 041 (2006).
[5] M. Blanke et al., arXiv:hep-ph/0702136; M. Blanke et al., JHEP 01, 066 (2007); A. J. Buras et al., JHEP 11, 062 (2006); M. Blanke, JHEP 12, 003 (2006).
[6] C. H. Chen, Phys. Lett. B521, 315 (2001); J. Phys. G28, L33 (2002); C. H. Chen and C. Q. Geng, Phys. Rev. D66, 014007 (2002); Phys. Rev. D66, 094018 (2002); Phys. Rev. D71, 054012 (2005); Phys. Rev. D71, 115004 (2005); C. H. Chen and H. Hatanaka, Phys. Rev. D73, 075003 (2006).
[7] K. Niyogi and A. Datta, Phys. Rev. D20, 2441(1979); A. Datta and D. Kumbhakhar, Z. Phys. C 27, 515 (1985).
[8] G. Burdman, E. Golowich, J. Hewett and S. Pakvasa, Phys. Rev. D 52, 6383 (1995).
[9] H. Georgi, Phys. Lett. B297, 353 (1992); T. Ohl, G. Ricciardi and E. Simmons, Nucl. Phys. B403, 605 (1993); I. Bigi and N. Uraltsev, Nucl. Phys. B592, 92 (2001).
[10] A. A. Petrov, Int. J. Mod. Phys. A21, 5686 (2006).
[11] J. Donoghue et al., Phys. Rev. D33, 179 (1986); L. Wolfenstein, Phys. Lett. B164, 170 (1985); P. Colangelo, G. Nardulli and N. Paver, Phys. Lett. B242, 71 (1990); T. A. Kaeding, Phys. Lett. B357, 151 (1995); A. A. Anselm and Y. I. Azimov, Phys. Lett. B85, 72 (1979).
[12] A. F. Falk et al., Phys. Rev. D65, 054034 (2002); A. F. Falk, et al., Phys. Rev. D69, 114021 (2004).
[13] B. Aubert et al. (Babar Collaboration), Phys. Rev. Lett. 98, 211802 (2007) [arXiv:hep-ex/0703020].
[14] K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. 98, 211803 (2007) [arXiv:hep-ex/0703036].
[15] L. M. Zhang et al. (Belle Collaboration), arXiv:0704.1000 [hep-ex].
[16] Particle Data Group, W.M. Yao et al., J. Phys. G 33, 1 (2006).
[17] Y. Nir, arXiv:hep-ph/0703235.
[18] M. Ciuchini et al., arXiv:hep-ph/0703204.
[19] E. Golowich, S. Pakvasa and A. A. Petrov, arXiv:hep-ph/0610039.
[20] P. Ball, arXiv:hep-ph/0703245; arXiv:0704.0786 X. G. He and G. Valencia, arXiv:hep-ph/0703270.
[21] M. Blanke, A. J. Buras, S. Recksiegel, C. Tarantino and S. Uhlig, arXiv:hep-ph/0703254.
[22] C. H. Chang, D. Chang and W. Y. Keung, Phys. Rev. D 61, 053007 (2000).
[23] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B 513, 232 (2001) N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP 07, 034 (2002) arXiv:hep-ph/0206021.
[24] T. Han et al., Phys. Rev. D 67, 095004 (2003).
[25] J. Lee, JHEP 0412, 065 (2004).
[26] S. Fajfer and S. Prelovsek, Phys. Rev. D 73, 054026 (2006).
[27] H. B. Li, hep-ex/0605004.
[28] M. Perelstein, M. E. Peskin and A. Pierce, Phys. Rev. D 69, 075002 (2004); see also M. Perelstein, Prog. Part. Nucl. Phys. 58, 247 (2007).
[29] M. Schmaltz and D. Tucker-Smith, Ann. Rev. Nucl. Part. Sci. 55, 229 (2005).
[30] H. Fritzsch, Phys. Lett. B 73, 317 (1978); Phys. Lett. B 166, 423 (1986).
[31] M. Artuso et al. [CLEO Collaboration], Phys. Rev. Lett. 95, 251801 (2005).
[32] H. W. Lin, S. Ohta, A. Soni and N. Yamada, Phys. Rev. D 74, 114506 (2006).
[33] C. Greub, T. Hurth, M. Misiak and D. Wyler, Phys. Lett. B 382, 415 (1996); S. Prelovsek and D. Wyler, Phys. Lett. B 500, 304 (2001).
[34] E. Golowich and A. A. Petrov, Phys. Lett. B 625, 53 (2005).
[35] G. Burman, E. Golowich, J. Hewett and S. Pakvasa, Phys. Rev. D 66, 014009 (2002).
[36] S. Fajfer, P. Singer and J. Zupan, Eur. Phys. J. C 27, 201 (2003).