A FUZZY COMMITMENT SCHEME

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ABSTRACT
This paper attempt has been made to explain a fuzzy commitment scheme. In the conventional Commitment schemes, both committed string m and valid opening key are required to enable the sender to prove the commitment. However there could be many instances where the transmission involves noise or minor errors arising purely because of the factors over which neither the sender nor the receiver have any control.

The fuzzy commitment scheme presented in this paper is to accept the opening key that is close to the original one in suitable distance metric, but not necessarily identical. The concept itself is illustrated with the help of simple situation.

KEY WORDS
Cryptography, Error Correcting Codes, Fuzzy logic and Commitment scheme.

1. Introduction

The notion of Commitment scheme is at the heart of most the constructions of modern Cryptography protocols. Protocols are essentially a set of rules associated with a process or a scheme defining the process. Commitment schemes are the processes in which the interests of the parties involved in a process are safeguarded and the process itself is made as fair as possible.

Commitment protocols were first introduced by Blum [1] in 1982; many more Commitment Schemes were later developed with improved features [5, 6, 7, 8, 12, 13]. Moreover in the conventional Commitment schemes, opening key are required to enable the sender to prove the commitment. However there could be many instances where the transmission involves noise or minor errors arising purely because of the factors over which neither the sender nor the receiver have any control.

Our aim in this paper to describe commitment schemes, which use algorithms to counter possible uncertainness. Uncertainty leads to introduction of fuzzy sets and fuzzy logic[2] in to the protocol itself.

Fuzzy commitment scheme was first introduced by Juels and Martin [3], fuzziness also introduced later in [4,14,15] for generating cryptographic keys. They add new property called “fuzziness” in the open phase to allow, acceptance of the commitment using corrupted opening key that is close to the original one in appropriate metric or distance. In this paper we have attempted a more formal and mathematical definition of fuzzy commitment schemes. An overview of commitment schemes and description of related work is also incorporated. A brief introduction of error correcting codes, with real life situation to illustrate is attempted.

2. Crisp Commitment Schemes

In a conventional commitment scheme, one party, whom we denote the sender namely Alice, aim to entrust a concealed message m to the second party namely Bob. Intuitively a commitment scheme can be seen as the digital equivalent of a sealed envelope. If Alice wants to commit to some message m she just puts it into the sealed envelope, so that whenever Alice wants to reveal the message to Bob, she opens the envelope.

Clearly, such a mechanism can be useful only if it meets some basic requirements. First of all the digital envelope should hide the message from: Bob should be able to learn m from the commitment (this is often referred in the literature as the hiding property). Second, the digital envelope should be binding, meaning with this that Alice can not change her mind about m, and by checking the opening of the commitment one can verify that the obtained value is actually the one Alice had in mind originally (this is often referred to as the binding property).

Definition 1: A Commitment scheme is a tuple\(\{P, E, M\}\) Where \(M = \{0,1\}^n\) is a message space, \(P\) is a set of individuals, generally with three elements A as the committing party, B as the party to which Commitment is made and TC as the trusted party, \(E = \{ (t_i, e_i) \}\) are the events occurring at times
t,  i = 1,2,3 , as per algorithms e_i , i = 1,2,3. The scheme always culminates in either acceptance or rejection by A and B. The environment is setup initially, according to the algorithm Setupalg (e_i) and published to the parties A and B at time t_1. During the Commit phase, A uses algorithm Commitalg (e_i), which encapsulates a message m∊M, along with secret string S∊{0,1}^k into a string c. The opening key (secret key) could be formed using both m and S. A sends the result c to B at time t_2. In the Open phase, A sends the procedure for revealing the hidden Commitment at time t_3, and B uses this.

Openalg (e_i): B constructs c' using Commitalg, message m and opening key, and checks weather the result is same as the commitment c.

Decision making:
If ( c = c' ) Then A is bound to act as in m
Else he is free to not act as m

3 - Fuzzy Commitment Formally Defined:
When would a commitment scheme as in definition 1 become fuzzy? At the stage of decision making. This result of uncertainties that make crop up during transmission noise. We may formalize the whole process by properly defining it.

Definition 2:
A Fuzzy Commitment scheme is a tuple {P, E, M, f} Where M⊂{0,1}^n is a message space which consider as a code, P is a set of individuals , generally with three elements A as the committing party, B as the party to which Commitment is made and TC as the trusted party. f is error correction function (def. 5) and E = { ( t_i, e_i) } are called the events occurring at times t_i , i = 1,2,3 , as per algorithms e_i , i = 1,2,3. The scheme always culminates in either acceptance or rejection by A and B.

In the setup phase, the environment is setup initially and public commitment key CK generated, according to the algorithm Setupalg (e_i) and published to the parties A and B at time t_1. During the Commit phase, Alice commits to a message m∊M according to the algorithm Commitalg (e_i) into string c. In the Open phase, A sends the procedure for revealing the hidden Commitment at time t_3 and B use this.

Openalg (e_i): B constructs c' using Commitalg, message t(m) and opening key, and checks weather the result is same as the received commitment t(c), where t is the transmission function.

Fuzzy decision making:
If (nearest(t(c),f(c'))≤z_0) Then A is bound to act as in m
Else he is free to not act as m

4 - Numerical example:
Let P = { Alice, Bob} i.e. we consider a situation where there is not trusted party.
Message space: Let M = {0000, 1011, 0101, 1110, 1010, 1100, 1111} ⊂ {0,1}^4.
Message: let m = 1011

Encoding function: Let g: M→{0,1}^7 be one to one function defined as:
g(M) = C = {0000000 = g(0000), 0100101 = g(1011),
0010011 = g(0101), 1011010 = g(1010),
1101100 = g(1100), 1111111 = g(1111)} ⊂ {0,1}^7
The image set C under g is a code set, which is satisfies the closure property under XOR operation, an element of C is also called a codeword.

Setup phase: At time t_1, it is agreed between all that CK ⊆ XOR
f = nearest neighbour in set C.
z_0 = 0.20.

Commit phase: At time 2
Alice committed to her massage m=1011. She knows that g(m)=g(1011)=0100101 For sake of secrecy she selects S∊C at random, Suppose S=1011010.
Then her commitment c = Commitalg(CK, g(m), S) = g(m) XOR S = 1111111
Alice sends c to Bob, which Bob will receive as t(c), where t is the transmission function. Let the transmitted value t(c) = 1011111, which includes noise.

Open phase: At time t_3
Alice discloses the procedure g(m) and S to Bob to open the commitment.
Suppose Bob gets t(g(m))= 1100101 and t(s)=1011010. Bob compute c'=Commitalg(CK,t(g(m)),t(s))=t(g(m))XORt(S)=0111111.
Bob check that dist(t(c),c')=2, he will realize that there is an error occur during the transmission. Bob apply the error correction function f to c': f(c')=1111111 (the nearest neighbour of c'=0111111 is 1111111). Then Bob will compute nearness(t(c),f(c'))=dist(t(c),f(c'))/n = 1/7 =0.14. (def.6)
Since 0.14≤z_0=0.20.
Then FUZZ(f(c'≤0111111))=0 (def.7).
Bob accepted t(c)=f(c')=1111111.
Finally Bob calculate g'(1111111)=1011.

5- Error Correcting Codes:
Definition 3: A metric space is a set C with a distance function dist:C×C→R={0,∞}, which obeys the usual properties (symmetric, triangle inequality, zero distance between equal points).
Definition 4: Let C ⊂ {0,1}^n be a code set which consists of a set of codewords c_i of length n. The distance metric between any two codewords c_i and c_j in C is defined by

\[ dist(c_i,c_j) = \sum_{k=1}^{n} |c_{i_k} - c_{j_k}| \]

This known as Hamming distance[16]
Definition 5: An error correction function f for a code C is defined as
\[ f(c_i) = |c_i| \text{dist}(c_i,c_j) \text{ is the minimum, over C-{c_i}) \]
Here c_j= f(c_i) is called the nearest neighbor of c_i.
Definition 6: The measurement of nearness between two codewords c and c' is defined by
\[ \text{nearness}(c,c') = \text{dist}(c,c')/n, \]
it is obvious that 0≤ nearness(c,c')≤1.
Definition 7: The fuzzy membership function for a codeword \( c' \) to be equal to a given \( c \) is defined as

\[
\text{FUZZ}(c') = 0 \quad \text{if} \quad \text{nearness}(c, c') = z \leq z_0 < 1
\]

\[
= z \quad \text{otherwise}
\]

6- Real Life Situation : (Testament):
Alice wants to write a testament to declare she passes all her fortune to her son Bob after her death. Of course, the Alice's attorney is playing the role of the authority. Setup phase: at time \( t_1 \)

Attorney published to Alice and Bob an envelope as a public commitment key, error correction function \( f \) and \( z_0 \)

Commit phase: at time \( t_2 \)

Alice writes her testament \( m \) and put it in a sealed envelope (commitment \( c \)) and gives to her son Bob. During the time pass some letters of the testament corrupted we assume that it is \( t(c) \).

Open phase: at time \( t_3 \) (death time of Alice)

Attorney on behalf of Alice meet Bob and they calculate the envelope to obtain the testament \( m' \), and they calculate

- \( \text{nearness}(t(m), t(m')) \)
- If \( (\text{FUZZ}(m') = 0) \) Then \( m' = m \)
- Else \( m' \neq m \)

7- Fuzzy Commitment Schemes from

8- Concluding remarks:
We have attempted to formalize definition of a fuzzy commitment scheme by introducing a fuzzy membership function at the opening algorithm stage. Introduction of error correction function was introduced by many research workers earlier [16,17,19]. Introduction of the Fuzzy membership function makes the use of word fuzzy more explicit.

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