A study was carried out and the optimization process was carried out for one of the types of equipment for autonomous heat supply using renewable resources – a tubular pellet heater. The research is expedient, since there is no mathematical model of the unit operation for the pellet combustion unit, there is only a set of experimental results indicating the inconsistency of the criteria presented to it. As a result of the research, new algorithms have been obtained: firstly, an algorithm for selecting (multi-criteria optimization) the operating mode of the unit for burning pellets of tubular heaters, and secondly, algorithms for choosing, according to several criteria, the parameters of the heat exchange unit of a tubular heater with a screen. A set of algorithms for multicriteria optimization with binary selection ratios has been developed for tubular pellet heaters in full, including a pellet combustion unit and a heat exchange unit. Selection functions have been defined for a pellet combustion unit using dimensionless complexes based on experimental results. For a block of a tubular heat exchanger with a screen, a selection function is built taking into account the criteria of functioning and a mathematical model of the heater in the form of a system of nonlinear ordinary differential equations. The practical significance of the algorithm for selecting the operating mode for the pellet combustion unit lies in the possibility of obtaining the most preferable (optimal, taking into account many criteria) parameters in the entire range of permissible parameters, and not only among the experiments carried out. The practical significance of optimization algorithms for a heat exchange unit lies in the ability to select specific parameter values during design – the thermal power of the heater, air flow, the length of the tubular part and the screen, their diameters, taking into account several selection criteria.

Keywords: pellet heater, decision making, multiple criteria, selection function, evolutionary search.

1. Introduction

Tubular gas pellet heaters are now being designed and created only in single samples, although the practical feasibility of using wood pellets instead of natural gas is quite obvious, especially taking into account that pellets are renewable energy resources that do not introduce thermal disturbances to the temperature balance of the Earth. And, if the use of pellets in boilers is a developed practice in thermal power engineering, then tubular gas pellet heaters are just beginning to develop.

For the methodological support of the design of tubular gas heaters on pellets, of course, scientific results on the modeling and optimization of their operation are needed. Scientific results are necessary to select the optimal operating modes for tubular pellet heaters, taking into account the existing several criteria for fuel combustion, and also taking into account that only an increase in the volume of experimental research increases the costs of their implementation, but does not give confidence that the optimal parameters have been found. Results related to the optimization of the tubular part of the screen heater should ensure the selection of tubular heaters in environments where infrared gas tubular heaters cannot be used, such as in low rooms or in areas with limited radiant heat flow, such as greenhouses.

Undoubtedly, the determination of the best (optimal) parameters of pellet tubular heaters is of scientific and practical interest.
2. Literature review and problem statement

Pellet tube heaters (PTH) can be seen as a development of infrared gas tube heaters (IGTH). IGTH has a long history of development and use. The United States of America was engaged in advanced research in this direction in the middle of the twentieth century, and as a result, a methodology for calculating heat loss and heat release for a radiant heating system was developed [1]. The issues of energy saving when using radiant heating [2], as well as issues of designing infrared radiant heating systems in industry [3] were actively considered. These heaters are serially produced by a number of manufacturers in different countries, for example – ROBERTS GORDON [4]. The main components of such heaters are: an automatic gas burner, a tubular radiator, an infrared reflector and an exhaust or supply fan. Then, technical solutions appeared, in which, due to the change in the heat exchange part, the scope of application of gas tubular heaters expanded, which is reflected in [5]. Finally, pellet tube heaters appeared, in which the gas burner was replaced by a pellet one [6].

A schematic diagram of a pellet tubular heater is shown in Fig. 1 (pellet combustion unit PBU and heat exchange unit HEU).

The mathematical model of a gas-tube heater was formulated as a model of a hydraulic circuit with distributed parameters, first for a linear heater, and then for more complex structures [7]. The mathematical model of a pellet tube heater was formulated on the basis of a mathematical model for a gas heater.

The addition operator and the delete operator are proposed for the application of an evolutionary algorithm with an auxiliary choice strategy based on clustering is considered [17]. The addition operator and the delete operator are proposed to comprehensively account for diversity, both in the space of solutions and in the space of targets. The study [18] proposes a new multimodal multipurpose evolutionary algorithm using the strategy of two archives and recombination. The proposed algorithm first analyzes the properties of decision variables and the relationships between them to guide the evolutionary search. After that, to jointly solve these problems, a common structure is adopted using two archives, that is, archives of convergence and diversity.

In the above-mentioned works, the problem of making decisions in wide ranges of admissible values of parameters, for example, from a subset of the Euclidean space, was not posed. Similar problems were solved with multi-objective optimization. Having mathematical dependences for several output functions, it is possible to solve the decision-making problem as a multi-objective optimization problem [12]. There are a fairly large number of scientific results in the field of multipurpose optimization [13–15]. Most of the presented results refer to a situation when mathematical models exist for each of the output functions – Pareto optimization. But the question remains unresolved that in this case the adoption of the final decision from the set of Pareto-optimal ones is an additional procedure.

An alternative approach to solving this issue is the formulation of the optimization problem as an optimization problem with respect to the choice relation. Earlier, generalized problems of mathematical programming were formulated, for which methods of solution were proposed [16].

To solve multimodal multicriteria optimization problems, the application of an evolutionary algorithm with an auxiliary choice strategy based on clustering is considered [17]. The addition operator and the delete operator are proposed to comprehensively account for diversity, both in the space of solutions and in the space of targets. The study [18] proposes a new multimodal multipurpose evolutionary algorithm using the strategy of two archives and recombination. The proposed algorithm first analyzes the properties of decision variables and the relationships between them to guide the evolutionary search. After that, to jointly solve these problems, a common structure is adopted using two archives, that is, archives of convergence and diversity.

Almost all multiobjective optimization methods use objective functions to describe the optimization goal. It is convenient if it is possible to explicitly represent these objective functions, and if such a representation is impossible or difficult, then it is necessary to use other approaches to
finding optimal solutions with several criteria, for example, evolutionary or genetic algorithms.

Effective methods for solving optimization problems are developed on the basis of evolutionary search algorithms [19, 20]. Evolutionary algorithms are useful for solving generalized mathematical programming problems [21, 22] without the condition of convexity of the choice relation. A general approach to the construction of evolutionary algorithms for solving multicriteria problems with binary choice relations is presented in [23]. However, the solution of such real problems, in particular the problem of optimization of tubular pellet heaters, has not been previously realized, and, as a consequence, is of scientific interest.

It is expedient to conduct scientific research aimed at solving complex real problems, when part of the system is described by a system of differential equations, and the other part of the system is characterized by a set of experimental results. For this, it is necessary to develop new algorithms: an algorithm for the selection (multi-criteria optimization) of the operating mode of the unit for combustion of pellets of tubular heaters. For this, it is possible to use a limited set of experiments in multi-criteria conditions. Also, it is required to develop algorithms for selecting, according to several criteria, the parameters of the heat exchange unit of a tubular heater with a screen.

3. The aim and objectives of research

The aim of this research is to develop multicriteria optimization algorithms for tubular gas heaters on pellets, which include two structural units – a pellet combustion unit and a heat exchange unit of a tubular heater with a screen. Evolutionary search with binary choice relations was required as a basis for the development of algorithms. This approach will make it possible to optimize the design and operational parameters of tubular pellet heaters, including a pellet combustion unit and a heat exchange unit, in the presence of several decision criteria.

To achieve the aim of research, the following objectives were solved:

- for a pellet combustion unit, develop decision-making methods in the presence of several criteria for the operation of this unit, using only a limited set of experimental results of its operation in the form of dimensionless complexes (criteria);
- for the heat exchange unit of the heater, it is required to develop algorithms for multi-criteria optimization of the operation of this unit, using a mathematical model in the form of a system of differential equations describing the processes of motion and heat transfer in the tubular part of the heater.

4. Materials and methods of research

Tubular pellet heater is characterized by a set of parameters \( x_1 = (x_1^1, x_1^2, ..., x_1^n), x_1 \in \Omega_1 \) and \( x_2 = (x_2^1, x_2^2, ..., x_2^m), x_2 \in \Omega_2 \). There is a mathematical model for the parameter \( x_1 = (x_1^1, x_1^2, ..., x_1^n), x_1 \in \Omega_1 \) in the form of a system of ordinary nonlinear differential equations \( dx_i/dz = f_i(x_1, x_2, ..., x_n) \), \( i = 1, 2, ..., n \) and there is a set of criteria \( u = (u^1, u^2, ..., u^q) \) for parameters \( x_1 = (x_1^1, x_1^2, ..., x_1^n) \). It is necessary to obtain binary relations of choice \( R_{st} \) on the set \( \Omega_1 \) taking into account the set of criteria \( u = (u^1, u^2, ..., u^q) \).

There is a training set of experimental results: \( R_{sk} \) for \( q = 1, 2, ..., N_{sk} \) and the result of the expert assessment in the form of a correspondence matrix \( B = b_{ik} \), \( i = 1, 2, ..., N_{sk}, j = 1, 2, ..., N_{sk} \), which is obtained using the expert choice relation \( R \). It is required to find the choice function \( C \) for the entire set \( \Omega_2 \) with the binary relation \( R_{sk} \) in such a way that the binary relation \( R_{sk} \) corresponds to the relation of the expert choice \( R \).

As a result, it is necessary to obtain binary relations of the choice of \( R \) on the set \( \Omega_2 \), which is the final result for making decisions taking into account the binary relations of choosing \( R_{st} \) and \( R_{sk} \), and for this it is necessary to propose algorithms for making decisions.

In the evolutionary search for solutions, let’s use two choice functions: a choice function in the form of preference:

\[
S(x) = \left\{ x \in X \forall y \in [X \setminus S(x)], xR_y \right\},
\]

and a block selection function

\[
S^b(x) = \left\{ x \in X \forall y \in [X \setminus S^b(x)], yR_x \right\}.
\]

The search algorithm of RS – optimal solution can be represented with a choice function in the form of preference:

\[
X_k = S(G(X_{k-1})), k = 1, 2, ...
\]

4. Materials and methods of research

Pellet tubular heaters are considered [17]. The design parameters of tubular heaters (parameters of the input system)
are given below: burner area, \( S \); useful area for the passage of primary air, \( S_p \); primary air flow rate, \( L_p \); total air consumption, \( L \); burner power, \( W \).

There are criteria (output functions of the system – emissions) of the heater: ash carryover over time, \( Y_{A} \); CO concentration in waste gases, \( Y_{CO} \); concentration of NOx in waste gases, \( Y_{NOx} \). These parameters have dimensions. The experimental data of array 1 (parameters 1–3) is presented in Table 3. The experimental data of array 2 are presented in Table 4.

Table 1 experiments can see 5 input parameters – \( S, S_p, L, L_p, W \) and 3 output parameters – \( Y_A, Y_{CO}, Y_{NOx} \). These parameters have dimensions. The experimental data of array 1 and array 2 are presented in Tables 1, 2. Here all parameters are relative – referred to their maximum values. The experimental technique and characteristics of the experimental equipment are described in [6].

| No | Parameter 1 | Parameter 2 | Parameter 3 |
|----|-------------|-------------|-------------|
| 1  | 1.144       | 0.614       | 0.1982      |
| 2  | 1.144       | 0.632       | 0.1949      |
| 3  | 1.144       | 0.583       | 0.3407      |
| 4  | 0.643       | 0.932       | 0.9963      |
| 5  | 0.643       | 1.017       | 1.0636      |
| 6  | 0.643       | 0.861       | 0.1759      |
| 7  | 0.643       | 1.187       | 0.4505      |
| 8  | 0.254       | 0.428       | 0.1242      |
| 9  | 0.508       | 0.632       | 0.1825      |
| 10 | 0.508       | 0.677       | 0.2029      |
| 11 | 0.278       | 0.939       | 0.1551      |
| 12 | 0.287       | 1.126       | 0.3283      |
| 13 | 0.287       | 1.061       | 0.5365      |
| 14 | 0.572       | 0.537       | 0.1436      |
| 15 | 0.372       | 0.282       | 0.1505      |
| 16 | 0.286       | 0.535       | 0.1079      |
| 17 | 0.286       | 0.381       | 0.0741      |
| 18 | 0.286       | 0.625       | 0.1729      |
| 19 | 0.286       | 0.942       | 0.2383      |
| 20 | 0.286       | 0.631       | 0.1099      |
| 21 | 0.336       | 0.804       | 0.0912      |

Applying the methods of the theory of dimension and similarity, it is possible to reduce the modeling problem to modeling 3 dimensionless parameters (complexes):

\[
P_1 = \frac{S_p}{S},
\]

\[
P_2 = \frac{L_p}{L},
\]

\[
P_3 = \frac{W}{Q_c} / L / 10^3.
\]

The correspondence matrix \( B = \{b_{ij}\} \) is the ratio of the expert choice \( R \).

Matrix B represents the results of an expert comparison of each mode of operation of the tubular heater with every other mode, a value of 1 means that the mode under consideration is preferable (better) than the compared mode, a value of 0 – vice versa. Experimental data array 1 (parameters 1–3) is presented in Table 3. The experimental data of array 2 are presented in Table 4.
The result of the expert assessment in the form of a preference matrix $B = (b_{ij})$, $i=1, 2, \ldots, \text{Nobs}$, $j=1, 2, \ldots, \text{Nobs}$, following [25], for the experimental data in Table 1 is presented in Table 5, and for the experimental data in Table 2 is presented in Table 6.

### Table 5

| No. | $P_1$ | $P_2$ | $P_3$ |
|-----|------|------|------|
| 26  | 0.336| 1    | 0.1310|
| 27  | 0.336| 1.243| 0.2530|
| 28  | 0.336| 1.629| 0.4356|
| 29  | 0.336| 1.947| 0.5606|
| 30  | 0.336| 3.419| 0.3956|
| 31  | 0.336| 1.239| 0.2184|
| 32  | 0.336| 1.384| 0.2475|
| 33  | 0.336| 2.201| 0.3443|
| 34  | 0.336| 2.763| 0.4850|
| 35  | 0.336| 3.496| 0.3520|
| 36  | 0.336| 0.499| 0.0629|
| 37  | 0.336| 1.071| 0.1743|
| 38  | 0.336| 1.163| 0.1477|
| 39  | 0.336| 1.448| 0.3630|
| 40  | 0.336| 0.762| 0.1115|
| 41  | 0.336| 1.065| 0.1708|
| 42  | 0.336| 1.053| 0.2532|
| 43  | 0.336| 1.583| 0.3054|

Preference matrices (Tables 5, 6) for arrays of experimental data 1, 2 are the result of expert judgment, since their analysis is associated with the problem of finding the best compromise solution indicated above.

The mathematical model of the thermal and hydraulic regime for HEU is presented [7]:

\[ M = \rho \cdot wF = \text{const}, \]  \tag{15}

\[ p = \rho \cdot RT. \]  \tag{16}

\[ dp = \frac{\Delta \cdot \rho \cdot w^2}{2} \cdot dx + (\rho_s - \rho) \cdot g \cdot dh, \]  \tag{17}

\[ dQ_{\alpha} = \pi D dx \alpha (T - T_0), \]  \tag{18}

\[ dQ_{\beta} = \pi D dx \beta (T^4 - T_0^4) \cdot 10^{-4}, \]  \tag{19}

\[ dQ_s = dQ_{\alpha} + dQ_{\beta}. \]  \tag{20}

\[ dQ_{\beta} = \pi D dx \beta (T_{\beta} - T_0). \]  \tag{21}

\[ dQ_{\beta} = \pi D dx \beta (T_{\beta}^4 - T_{\beta}^4) \cdot 10^{-4}, \]  \tag{22}

\[ dQ_s = dQ_{\alpha} + dQ_{\beta}. \]  \tag{23}

\[ dQ_{\beta} = \pi D dx \beta (T_{\beta} - T_0). \]  \tag{24}

\[ dQ_{\beta} = \pi D dx \beta (T_{\beta}^4 - T_{\beta}^4) \cdot 10^{-4}, \]  \tag{25}

\[ dQ_s = dQ_{\alpha} + dQ_{\beta}. \]  \tag{26}

\[ dQ_s = dQ_{\alpha} = dQ_{\beta}. \]  \tag{27}

\[ d(\rho w FC_{\beta} T) = -dQ_{\alpha} + dQ_{\beta}, \quad \text{for } 0 < x < L_j, \]  \tag{28}

\[ d(\rho w FC_{\beta} T) = -dQ_{\beta}, \quad \text{for } x > L_j, \]  \tag{29}

\[ dQ_s / dx = Q_{\alpha} / S_j / 2 \pi \cdot y_j (x) \quad \text{for } 0 < x < L_j. \]  \tag{30}

A general approach has been developed for using model (15)–(30) when calculating the thermal and hydraulic conditions of heaters. The system of heat transfer equations (15)–(30) is closed, but nonlinear. The system of equations (15)–(30) is solved numerically and $dQ_s, dQ_{\alpha}, dQ_{\beta}, dQ_{\beta}, T_{\alpha}$. 
Energy-saving technologies and equipment

5. The results of the study of pellet tubular heaters to optimize design and operational parameters

5.1. Making decisions for the pellet combustion unit

It is required to find the choice function \( C \) for the entire set \( \Omega \) with the binary relation \( R_3 \) in such a way that the binary relation \( R_3 \) corresponds to the expert choice relation \( R \).

A choice rule \( \pi \) is formulated with a choice function \( C \) and a binary relation \( R_3 \), which is defined by a function \( G(x)=G(x_1,x_2,...,x_5) \) such that \( G(x_1)\geq G(x_2)\geq R_3x_1, R_3x_2 \), where, \( x_1, x_2, ..., x_5 \in \Omega \).

In the above search, the selection function was represented as a regression function. Possible regression relationships were presented in dimensionless form:

\[ G(x) = \psi(P_1, P_2, P_3). \]  

Specific expressions by the choice function in the form (35) are as follows:

\[ G(x) = a_{12} \cdot d_1 + a_{13} / d_2. \]  

The parameters \( a_{1}, a_{2}, ..., a_{13} \) were obtained as a result of the evolutionary search for the choice function for training data set 1 and tested for the data of the check array 2. The evolutionary search for the choice function is presented in Table 7.

5.2. The results of the study of pellet tubular heaters for estimating the pressure losses over the sections of the tubular heater; where the integral is understood as the algebraic sum of the direction of the medium.

\[ \int dp_i(x_i) + \Delta p_1 + \Delta p_0 = 0, \]  

where the integral is understood as the algebraic sum of the pressure losses over the sections of the tubular heater; \( x_i \) – coordinate of the length of section \( i, i=1, 2, 3, 4; p_i(x_i) \) – pressure distribution over section \( i \) of the tubular heater.

The presented system of equations can be solved numerically with known initial conditions.

The diagram of a linear tubular heater with a screen is shown in Fig. 3.

![Diagram of a linear tubular heater with a screen](image)

The diagram of a linear tubular heater with a screen determines the order of consideration of the main sections of the heater and its main structural elements, indicating the direction of the medium.

### Table 7: Evolutionary search for solutions for a function of choice.

| Evolutionary search iteration step | Error (relative) on training array 1 | Error (relative) on check array 2 |
|-----------------------------------|-------------------------------------|----------------------------------|
| 1                                 | 0.3136                              | 0.4444                           |
| 2                                 | 0.2976                              | 0.3210                           |
| 8                                 | 0.2592                              | 0.3210                           |
| 10                                | 0.2400                              | 0.2839                           |
| 165                               | 0.2304                              | 0.3025                           |
| 807                               | 0.2112                              | 0.3086                           |

### Table 8: Evolutionary search for solving a generalized mathematical programming problem

| Evolution iteration step | Maximum function \( G(x) \) Branch 1 of evolution | Maximum function \( G(x) \) Branch 2 of evolution | Maximum function \( G(x) \) Branch 3 of evolution |
|--------------------------|-----------------------------------------------------|--------------------------------------------------|--------------------------------------------------|
| 1                        | -1.788                                              | -3.319                                           | -1.854                                           |
| 2                        | -0.992                                              | -2.865                                           | -1.582                                           |
| 5                        | 0.737                                               | 0.606                                            | 4.78E–5                                          |
| 8                        | 0.961                                               | 0.991                                            | 0.967                                            |
| 10                       | 1                                                   | 1                                                | 0.998                                            |
| 11                       | 1                                                   | 1                                                | 0.999                                            |
| 13                       | 1                                                   | 1                                                | 1                                                |
| 14                       | 1                                                   | 1                                                | 1                                                |

### Table 9: The result of an evolutionary search for solving a generalized mathematical programming problem

| Evolution branch | Maximum function \( G(x) \) Branch 1 | Maximum function \( G(x) \) Branch 2 | Maximum function \( G(x) \) Branch 3 |
|------------------|---------------------------------------|--------------------------------------|--------------------------------------|
| Branch 1         | 0.01                                  | 0.1                                  | 0.08                                 |
| Branch 2         | 0.01                                  | 0.1                                  | 0.08                                 |
| Branch 3         | 0.01                                  | 0.1                                  | 0.08                                 |

Thus, using the selection function, the results of an evolutionary search were obtained for solving a generalized mathematical programming problem and for three branches.
of evolution for a pellet combustion unit of a linear tubular heater.

5.2. Making decisions for the heat exchanger unit
The operating diagram of the algorithm is shown in Fig. 4:

\[ \Omega_1, \Omega_2, \Omega_3, \Omega_4 \]

Fig. 4. Operator diagram of the algorithm

In the presented scheme, the following designations are adopted: \( \Omega_1 \) – start of search; \( \Omega_2 \) – generation of possible solutions \( x = \{ x^1, x^2, \ldots, x^N \} \); \( \Omega_3 \) – generation of a possible heater temperature value for the current integration step along the heater length; \( W_1^i \) – calculation of heat fluxes \( dQ_{1,1}, dQ_{1,2} \); \( W_2^i \) – calculation of heat fluxes \( dQ_{2,1}, dQ_{2,2} \); \( W_3^i \) – calculation of the screen surface temperature \( T_s \); \( W_4^i \) – calculation of heat fluxes \( dQ_{3,1}, dQ_{3,2} \); \( P_k \) – Has the required accuracy of the heat balance equations for the shield been achieved? If NO, then go to \( \Omega_5 \); \( \Omega_6 \) – start of integration of differential equations of motion; \( W_5^i \) – calculation of the parameters of movement and heat transfer at the beginning of the calculated zone; \( W_6^i \) – calculation of the parameters of movement and heat transfer at the end of the calculation section; \( P_8 \) – have you completed the integration process? If NO, then go to \( \Omega_5 \); \( W_9 \) – calculation of the objective function.

\[ E_1(x) = \eta \text{ -- dimensionless efficiency (efficiency), } E_2(x) = \text{generalized penalty function:} \]

\[ E_2(x) = E_{21}(x) + E_{22}(x), \quad \text{(40)} \]

where \( E_{21}(x) \) – dimensionless deviation between the pressure loss in the tubular heater and the maximum allowable pressure loss:

\[ E_{21}(x) = \left( \int \Delta p(x) + \Delta p_1 + \Delta p_2 + \Delta p_{\text{max}} \right) / \Delta p_{\text{max}}, \quad \text{(41)} \]

\( E_{22}(x) \) – dimensionless deviation between the temperature of the screen surface and the maximum allowable screen temperature:

\[ E_{22}(x) = \sum (\Delta T_s(x)) / T_{\text{max}}, \quad \text{(42)} \]

where

\[ \Delta T_s(x) = -(T_s(x) - T_{\text{max}}) \quad \text{if } T_s(x) \leq T_{\text{max}} \quad \text{(43)} \]

and

\[ \Delta T_s(x) = 0 \quad \text{if } T_s(x) > T_{\text{max}}, \quad \text{(44)} \]

\( W_{10} \) – selection of \( N_k \) preferred solutions for each branch of \( N_k \) evolution; \( P_{11} \) – have all possible solutions generated for each branch of evolution? If NO, then go to \( \Omega_5 \); \( W_{12} \) – calculation of search parameters for a new step of the evolution iteration; \( P_{13} \) – Have you reached the specified accuracy for determining the most preferred solution? If NO, then go to \( \Omega_5 \); \( \Omega_{14} \) – end of search.

The results of an evolutionary search for a solution to the problem for a linear tubular heater with a screen are shown in Fig. 4.

Thus, in the process of making decisions for the heat exchange unit of a linear pellet radiant heater with a screen, the operating scheme of the algorithm was determined and the dependences \( E_1(x) = \eta \) – dimensionless efficiency (efficiency), as well as \( E_2(x) \) – generalized penalty function.

The article outlines two directions for the development of decision-making algorithms for tubular pellet heaters, which are presented above. The first area relates to decision-making based on a limited set of experiments and is associated with decision-making in the presence of several criteria. This direction is presented in the form of the final result of optimization in Table 9.

In accordance with the Table 9 the solutions obtained matter:

\[ P_1 = S_p / S = 0.1, \quad \text{(45)} \]
\[ P_2 = L_p / L = 0.01, \quad \text{(46)} \]
\[ P_3 = W / Q_m / L / 10^3 = 0.08. \quad \text{(47)} \]

Fig. 5. Evolutionary search for a linear tubular heater with a screen for 2 criteria: \( a \) – the function of the dimensionless efficiency \( E_1 \) of the KIT iteration number; \( b \) – general penalty function \( E_2 \) of the KIT iteration number

Table 9 clearly shows that the solutions obtained coincide in the evolutionary search in three independent
branches of evolution, which indicates the existence of a single optimal solution with the constructed choice function and the adopted constraints. Obviously, knowing the values of dimensionless complexes, it is possible to determine the dimensional parameters.

The second direction relates to decision making in the presence of a mathematical model of the object, when the objective function and constraint conditions can be presented in an analytical form (14–34), (40–44).

When looking for a solution to the optimization problem for a heat exchanger as finding the most preferable solution, one can instead look for a blocking solution taking into account three criteria. In this case, following the approach [24], the binary choice relation can be written as:

\[ x_1 R_{12} y_1 \equiv [E_3(x_1) \leq 0 \text{ and } E_2(y_1) > 0] \]
\[ \text{and } [E_3(x_1) \geq E_1(y_1)]. \]

An evolutionary search for a solution to such an optimization problem in the presence of three criteria is shown in Fig. 5.

In the figures representing the optimization process, the points correspond to the found values of the optimized functions, on the abscissa axis – the number of iterations of the evolutionary search, the lines connecting the calculated points give a visual representation of the progress of the process of finding solutions. It also clearly shows the convergence of the evolutionary process for independent branches of evolution (here – 3 branches of evolution).

6. Discussion of the results of the study of optimization of heater parameters in the presence of several decision criteria

As a result of the study, it was established that both the first direction and the second direction of decision making are reduced to the formulation of generalized mathematical programming problems and their numerical solution, which indicates the fruitfulness of using evolutionary search algorithms with binary choice relations.

Let’s note that the algorithms used for the evolutionary search for the most preferable solutions are designed to solve a wide class of problems with binary choice relations. The presented results of numerical calculations indicate the advisability of using the proposed evolutionary search algorithms when making decisions for pellet tubular heaters.

To optimize the operating modes of the combustion unit, the following sequence of actions is proposed:

- use of dimensionless complexes (optimization criteria) (12)–(14) and the presentation of the entire set of experimental results in the form of dimensionless complexes (Tables 3, 4);
- use of expert judgment to build preference matrices (Tables 5, 6);
- construction of the decision selection function (Table 7), reflecting the preference matrix in the form of dependencies (36)–(39).

The end result of such actions is the formulation and solution of the generalized mathematical programming problem, presented in Tables 8 and 9.

Fig. 6. Evolutionary search for a linear tubular heater with a screen according to 3 criteria: a – dimensionless efficiency \( E_1 \); b – dimensionless difference \( E_2 \); c – dimensionless difference \( E_3 \)
To optimize the design parameters of the tubular part of the heater, the following sequence of actions is presented:
- use of a previously constructed mathematical model and expressions for the optimization criteria in analytical form (14)–(34), (40)–(44);
- formulation of the optimization problem using the binary choice relations (7)–(9).

The optimization results are presented in the graphs in Fig. 5, 6.

The advantages of this study are a unified approach to solving the problems of optimization of both the pellet combustion unit and the heat exchange unit for a tubular heater with a screen. The results of the optimization of the pellet combustion unit depend on the results of the expert judgment used to construct the preference matrices, which can be considered a disadvantage of a specific optimization, and the algorithmic part can also be used with other results of the expert assessment of the preferences of individual experimental results. It is advisable to develop the use of methods and results of expert evaluation to optimize the operation of tubular pellet heaters.

7. Conclusions

1. Methods for making decisions have been developed in the presence of several criteria and the absence of a mathematical model of operation for a pellet combustion unit, using only a set of experimental results of its operation with dimensionless parameters. To obtain these results, let’s use earlier experimental studies of the process with several criteria (functions) depending on its parameters. An expert assessment approach was used to construct a preference matrix for individual implementations. The constructed selection function is determined on the entire admissible range of input parameters, and not only on the set of experimental points. Thus, the selection mechanism covers the entire valid range of input parameters. The result obtained is a sequence of using a limited set of experiments, where there are several conflicting criteria and it is not possible to determine which parameters should be maintained in order to obtain an optimal solution. At the same time, it is proposed and substantiated the use of expert judgment to construct a matrix of preferences of individual experiments among themselves and the construction of a decision selection function, which makes it possible to extend the procedure of the most preferable choice to the entire set of admissible parameters, and not only to the set of performed experiments.

2. Algorithms for multicriteria optimization of work for the heat exchange unit of the heater have been developed using a mathematical model in the form of a system of differential equations describing the processes of motion and heat transfer in the tubular part of the heater. The difference between the results obtained is the integrity of the decision-making mechanism for the system based on inductive modeling of complex systems.

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