The misleading nature of the leaky box models in cosmic ray physics

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Abstract: Many experimental results around and above the energies where the solar modulation affects cosmic ion fluxes were quantified, conceptualized and debated using leaky box models. These models exploit the notion of equilibrium between creation and destruction processes of cosmic ions in an undifferentiated arbitrary volume representing the Galaxy, ignoring the galactic magnetic field, the size of the Galaxy, the position of the solar cavity, the spatial distribution of the sources, the space variation of the interstellar matter and other pertinent observations. Progress in the measurements of the quoted observational parameters renders obsolete the use of the leaky box models. Specific examples substantiating the inadequacy of the leaky box models are analyzed such as the conversion of the boron-to-carbon flux ratio into grammage and the residence times of cosmic ions in the Galaxy. The unphysical and misleading nature of the leaky box models is ascertained and illustrated at very high energy.

Introduction

Many experimental results on the primary cosmic radiation at very low energy gathered in the last five decades by balloon-borne instruments and satellite experiments have been expressed using simple theories of galactic cosmic rays referred to as Leaky Box Models. Two basic physical quantities, namely the gas column encountered by Galactic cosmic rays (grammage) and the related residence time ($T$), were believed to correctly interpret a number of measurements. Ritual fittings of the grammage above an adjustable energy $E_0$ (or the rigidity $R_0$) based on numerous measurements have the form:

$$g = Gr^{-\delta}$$

(1)

where $G$ is a constant grammage at the rigidity $R_0$, $\delta$ a constant and $r = R/R_0$, with $R$ the ion rigidity. Numerical values in classical interpolations are, for example: $G=10.8$ g/cm$^2$, $\delta=0.6$ and $R_0=4$ GV [19], or $G=24.0$ g/cm$^2$, $\delta=0.65$ and $R_0=5.5$ GV [12].

Grammage is converted into residence time by: $T = g/\rho v$ where $v$ is the ion velocity and $\rho$ the matter density. The extrapolation at very high energy of the residence time caused fictitious problems, as pointed out by Hillas [4, 20]. Figure 1 deliberately shows on a linear scale in energy the B/C flux ratio in the energy band 10 - 200 GeV/u indicating no compelling empirical evidence for a decrease of the grammage of the form $r^{-\delta}$ which, on the contrary, is well established in the region $1 - 4$ GeV/u. The recent measurements of the B/C flux ratio at 700 GeV/u by the Runjob Collaboration [18], along with the data shown in figure 1, exclude the functional form [19] above 20 GeV/u. The data in figure 1 and the B/C flux ratio [5] vividly testify to the inadequacy of the grammage fitting $v$ via $r^{-\delta}$ to physical reality in an area believed in past decades to be the realm of the Leaky Box Models.

The unjustified form of the cosmic-ray source power versus energy

In a suitable Galactic container, at a given energy $E$, two processes are at work to maintain cosmic-ray intensity at a constant value: escape from the container and nuclear interactions inside. The simplified equation describing these processes are:

$$dQ/dE = k(dN/dE)(1/\lambda + 1/f)$$

(2)

where $dQ/dE$ is the source power, $dN/dE$ is the differential intensity in a given location of the disc, $\lambda$ is the nuclear collision length, $f$ is the average escape length and $k$ a suitable normalization constant. It is an established result that the differential intensity at Earth, $dN/dE$, measured in many energy bands, obeys a power law i.e. $dN/dE = aE^{-\gamma}$ where $\gamma$ is the spectral index and $a$ a constant. Whenever one of the two terms $(1/\lambda)$ and $(1/f)$ dominates, the equation (1) simplifies further. At
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sufficient high energy, especially for light ions, the term $1/\lambda$ is negligible compared to $1/f$. Using the ad hoc hypothesis that the escape time has the form $E^{-\delta}$ it follows that $dQ/dE$ is proportional to $E^{-s}$ where $s = \gamma - \delta$. Therefore from the assumption $E^{-\delta}$ (disproved by the B/C data above 10 GeV/u) the unknown form of the source power versus energy of the cosmic radiation automatically transforms into a power law with a constant index $s$. The splitting of $\gamma$ in two constant parts, $s$ and $\delta$, in a large energy band remains unjustified taking into account, not only the B/C flux ratio at 160 and 700 GeV/u measured by the Runjob experiment [18], but also the classical data shown in figure 1.

The unphysical grammage extrapolated at high energy

From simple elaborations of the equation (1) it follows that the grammage, $g$, is related to the secondary-to-primary flux ratio of cosmic rays (for example, B/C but also $^3$He/$^4$He, subFe/Fe, Nitro-

gen/Oxygen and others) by the equation:

$$
\frac{g}{m} = \frac{1}{\sigma_{sp} N_s - \sigma_s}
$$

(3)

where the $N_s/N_p$ is the flux ratio observable by experiments at a given energy, $m$ the mean mass of the interstellar atom, $\sigma_p$ is the inelastic nuclear cross sections of the primary with the interstellar matter and $\sigma_{sp}$ secondary production rate. Equation (3) applies to a single parent nucleus generating a unique secondary while primary-to-secondary ratios resulting from many tributaries have more articulated formulæ.

In the limited energy range, 1 - 5 GeV/u, there is empirical evidence that $N_s/N_p$ ratio (i.e. the B/C ratio) is decreasing with energy. Consequently, since $\sigma_{sp}$ and $\sigma_p$ are smooth functions, the grammage has a decreasing trend with energy, often interpolated by $Gr^{-\delta}$ with a constant $\delta$. On the contrary, in the energy band 10 - 200 GeV/u, the classical measurements of the B/C flux ratio

Figure 1: Measurements of the boron-to-carbon flux ratios above 10 GeV/u up to the energy of 200 GeV/u displayed in a linear scale of energy. No decreasing trend of the B/C flux ratio is evident. Data below 10 GeV/u may be found elsewhere [22].

Figure 2: Interpolated grammage via $Gr^{-\delta}$ (thin lines a and b) for two sets of constants mentioned in the Introduction. It is evident that the slope $\delta$ tuned to the B/C flux ratio in the range 1 - 5 GeV/u is not in accord with the experimental data above 10 GeV/u. The computed grammage (c) is a tiny fraction of that displayed in figure 3.
Figure 3: The average distance of the sources from the Earth in the energy range $10^{10} - 10^{18}$ eV and the related grammage of galactic cosmic rays (He and Fe) intercepting the Earth, expressed in g/cm$^2$ in the same vertical axis. This calculation, unlike those based on Leaky Box Models, exploits numerous astronomical and radioastronomical observations (see Section 2 in [2]).

Additional calculations [3] of the grammage for the stable $^9$Be result in a grammage decrease of 30 per cent in the range 1 - 10 GeV/u both for the spiral and the circular galactic magnetic fields. Note that the mass and the charge of $^9$Be are similar to those of B and C, and consequently, the grammage profiles with energy are analogous. The measured B/C flux ratio decreases from 0.32 at 1 GeV/u to 0.20 at 10 GeV/u, exactly the same decrease of the computed $^9$Be grammage (see fig.2 [3]).

The unphysical walls reflecting back cosmic rays in the disc

The ad hoc assumption that the residence time of cosmic rays is compatible with a single value (for example, $15 \times 10^6$ years) is unphysical because cosmic rays originated in the Bulge resides longer than those populating the disc periphery, at 15 kpc from the galactic center. The volume where cosmic rays propagate is not specified and therefore the residence volume is undefined. The physical motion (migration, diffusion, convection, trapping or combinations of these classes of displacements) of cosmic rays...
mic rays is not defined. Since the magnetic field does not exist in Leaky Box Models and the ion motion is not specified, cosmic rays should travel freely in the undefined containment volume. But this free motion, a silent element of any variants of the Leaky Box Model, implies that a physical, real process, at some boundary of the disc, reverses the ion motion (reflection). Without the reflection at some boundary of the disc the grammage cannot accumulate to high values, because the free traversal of the disc without reflection entails a grammage of a few milligrams per cm\(^2\), some 4 orders of magnitude below the standard \(10\ g/cm^2\). What is the physical mechanism accomplishing this operation? To date (2007), it remains unknown.

Notice further that the matter density in the disc, adopted in Leaky Box Models for intrinsic calculation procedure to determine nuclear spallation rates, is also inconsistent with the matter density necessary to determine residence time of cosmic rays using radioactive clock measurements (\(^{10}\text{Be}/\text{Be}\) and others). A tangible sign of this embarrassing feature is that the mean gas density in the disc turns out to be in the range \(0.25 - 0.35\) atom/cm\(^3\) (see, for example, [17] for the data and [6] for the calculation procedure), a factor 3-4 below the average observed value of 1 atom/cm\(^3\).

A paradox is encountered by extrapolating the residence time at energies above \(10^{17}\) eV adopting \(Gr^{-\delta}\) with \(\delta=0.65\) [12]. The high energy galactic sources would populate a small volume in the disc, concentric to the Earth, and they all would reside in the solar system at energies above \(10^{23}\) eV.

References

[1] Codino A. and Plouin F. Galactic basins of helium and iron around the knee energy. INFN Report, INFN/TC-06/05, Frascati, Italia; reprinted in Astro-ph/0701498, Jan. 17th Jan. 2007.
[2] Codino A. and Plouin F. Astrophy. Jour., 639:173, 2006.
[3] Codino A. and Vocca H. Proc. 26th ICRC, Salt Lake City, Utah, 4:156, 1999.
[4] Hillas A.M. Proc. of VULCANO Conf., Soc. Italiana di Fisica, Bologna, Italia, pages 391–401, 1998.
[5] Strong A.W. and Moskalenko I.V. Proc. 26th ICRC, Salt Lake City, Utah, 4:255, 1999.
[6] Simpson J.A. DuVernois M.A. and Thayer M.R. Astron. Astrophys., 316:555, 1996.
[7] Juliiusson E. Astrophys. Jour., 191:331, 1974.
[8] Engelmann J.J. et al. Astrophys. Jour., 322:981, 1990.
[9] Orth C. D. et al. Astrophys. Jour., 226:1147, 1978.
[10] Ptuskin V.S. et al. Proc. 26th ICRC, Salt Lake City, Utah, 4:291, 1999.
[11] Simon M. et al. Astrophys. Jour., 239:712, 1980.
[12] Soutoul A. et al. Proc. 19th ICRC, La Jolla, California, 2:8, 1985.
[13] Weber W.R. et al. Proc. 26th ICRC, Salt Lake City, Utah, 4:222, 1999.
[14] Lezniai J.A. and Webber W. R. Astrophys. Jour., 223:676, 1978.
[15] Caldwell J.H. and Meyer P. Proc. 15th ICRC, Plovdiv, Bulgaria, 1:243, 1977.
[16] Chapell J.H. and Weber W.R. Proc. 17th ICRC, Paris, France, 2:59, 1981.
[17] Connell J.J. Proc. 25th ICRC, Durban, South Africa, 3:385, 1997.
[18] Areyma M. and (Runjob Coll.) Shibata T.A. Final results of runjob and related topics. Aspen, April 26th, 2005.
[19] Gupta M. and Webber W.R. Astrophys. Jour., 340:1124, 1989.
[20] Hillas A. M. Proc. 26th ICRC, Salt Lake City, Utah, 4:225, 1999.
[21] Brunetti M.T. and Codino A. Proc. 25th ICRC, Durban, South Africa, 3:277, 1997.
[22] Stephens S.A. and Streitmatter R.E. Astrophys. Jour., 505:266–277, 1998.