Domain Walls with Non-Abelian Clouds

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Phys.Rev.D77 (2008) 125008, [arXiv:0802.3135 (hep-th)],

1 Non-Abelian orientational moduli

BPS soliton has similarity to D-branes in string theory

Parameters of the Solution = Moduli

Moduli dynamics = Effective field theory of massless fields

Non-Abelian orientational moduli

Single instanton in $SU(N)$ gauge theory

$$A_\mu = U \left( A_\mu^{\text{BPST}}(x_0, \rho) \begin{pmatrix} 0 & 0 \\ 0 & 0_{N-2} \end{pmatrix} \right) U^\dagger, \quad U \in \frac{SU(N)}{SU(N-2) \times U(1)}$$

Non-Abelian clouds: non-Abelian orientational moduli (E.Weinberg)
Our purpose:

Study **Domain walls** with **non-Abelian orientational moduli**

Non-Abelian moduli occur when Higgs masses are (partially) **degenerate**

2 Models, BPS equations and the moduli matrix

**SUSY** \( U(N_C) \) **Gauge Theory with** \( N_F \) **Higgs fields**

Higgs fields \( H \) as an \( N_C \times N_F \) matrix, adjoint scalar \( \Sigma \)

\[
\mathcal{L} = \mathcal{L}_{\text{kin}} - V
\]

\[
\mathcal{L}_{\text{kin}} = \text{Tr} \left( -\frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{g^2} \mathcal{D}_\mu \Sigma \mathcal{D}^{\mu} \Sigma + \mathcal{D}^{\mu} H (\mathcal{D}_\mu H)^\dagger \right)
\]

\[
V = \text{Tr} \left[ \frac{g^2}{4} (c1 - HH^\dagger)^2 + (\Sigma H - HM)(\Sigma H - HM)^\dagger \right]
\]

Gauge coupling \( g \), Fayet-Iliopoulos parameter \( c \), diagonal mass matrix \( M \)

Bogomol’nyi bound (dependence on \( y \) only)

\[
E = \int_{-\infty}^{\infty} dy \text{Tr} \left[ (\mathcal{D}_y H - HM + \Sigma H)^2 + \frac{1}{g^2} (\mathcal{D}_y \Sigma - \frac{g^2}{2} (c1 - HH^\dagger))^2 \right. \\
+ \left. c \mathcal{D}_y \Sigma \right] \geq c \left[ \text{Tr} \Sigma(\infty) - \text{Tr} \Sigma(-\infty) \right]
\]
Lower bound is saturated if the **BPS equations** are satisfied

\[ \mathcal{D}_y H = HM - \Sigma H, \quad \mathcal{D}_y \Sigma = g^2 (c_1 - HH^\dagger) / 2 \]

**Solution of BPS equations**

\[ H = S^{-1}(y)H_0e^{My}, \quad \Sigma + iW_y = S^{-1}(y)\partial_y S(y) \]

**Moduli matrix** \( H_0 \): \( N_C \times N_F \) constant complex matrix of rank \( N_C \), contains all the moduli parameters \( \phi^i \)

Remaining BPS eq. (**Master eq.**): \( \Omega \equiv SS^\dagger, \quad \Omega_0 \equiv \frac{1}{c}H_0H_0^\dagger \)

\[ \partial_y (\Omega^{-1}\partial_y \Omega) = cg^2 \left(1_{NC} - \Omega^{-1}\Omega_0 \right) \]

3 **Non-Abelian Clouds in Abelian Gauge Theories**

**A simple example**: \( N_F = 4 \) with \( M = \text{diag} \left( m, m\epsilon/2, -m\epsilon/2, -m \right) \)

4 isolated vacua \( \langle 1 \rangle, \cdots, \langle 4 \rangle \), flavor symmetry \( U(1)^3 \) (\( \epsilon \neq 0 \))

Explicit solution at \( g^2 \to \infty \) limit: \( H = \frac{1}{\sqrt{\Omega_0}}H_0e^{My} \)

Moduli parameters are \( \varphi_1, \varphi_2, \varphi_3 \)

\[ H_0 = (1, e^{\varphi_1}, e^{\varphi_1+\varphi_2}, e^{\varphi_1+\varphi_2+\varphi_3}) = (1, \phi_2, \phi_3, \phi_4) \]
Domain wall trajectories in terms of $|h_i|^2$ of $H = \sqrt{c}(h_1, h_2, h_3, h_4)$ in the target space $\mathbb{C}P^3$ (as Toric diagram)

Wall configuration in terms of $\Sigma$

Well-separated walls: $\varphi_i$ is $i$-th wall position and phase difference

wave function is localized around the $i$-th wall

As $\epsilon$ decreases, $\varphi_2$ wave function spreads between 2 walls
Figure 1: Configuration of $\Sigma$ (first row) and density of the Kähler metric of $\varphi_1$, $\varphi_2$ and $\varphi_3$ (second row). Moduli parameters are $(\varphi_1, \varphi_2, \varphi_3) = (20, 0, -20)$ and $m = 1$.

→ Non-Abelian clouds

$\epsilon \to 0$ (degenerate mass) limit: enhanced flavor symmetry $U(1)^2 \times SU(2)$

2 isolated vacua $\langle 1 \rangle, \langle 4 \rangle$ and a degenerate vacuum $\langle 2 - 3 \rangle$

(with vacuum moduli $CP^1 \simeq SU(2)/U(1)$)

When domain walls are coincident,
all 6 massless modes are localized at the wall with identical wave functions

When domain walls are separated, 6 massless modes consist of positions of two walls: \(|\phi_2|^2 + |\phi_3|^2, |\phi_4|^2\) (1 NG, 1 qNG)

Localized at each wall

N-G bosons for \(U_1(1) \times U_2(1) \times SU(2)/U(1)\) (4 NG)

\(U_1(1), U_2(1)\): localized at each wall

\(SU(2)/U(1)\): spread between 2 walls

4 Conclusion

1. Domain walls in models with (partially) degenerate masses for Higgs scalars have normalizable non-Abelian Nambu-Goldstone (NG) modes, which are called Non-Abelian clouds.

2. When walls coincide, all the massless modes are localized at the walls with identical wave functions.

3. When walls separate, we find non-Abelian clouds which spread between two domain walls.

4. Effective Lagrangians are explicitly obtained.
5. When all the walls **coincide** in the $\mathbf{U}(N)$ gauge model, symmetry breaking $SU(N)_L \times SU(N)_R \times U(1) \rightarrow SU(N)_V$ gives $U(N)_A$ **NG modes**. In addition, there are $N^2 - 1$ **quasi-NG modes** besides 1 NG mode for broken translation. All these modes have identical wave function and are localized at the wall.

6. When $n$ walls **separate** in the $\mathbf{U}(N)$ gauge model, off-diagonal elements of $\mathbf{U}(n)$ NG modes have wave function **spreading between two separated walls** (**non-Abelian clouds**). Some **quasi-NG modes turn to NG modes** because of further symmetry breaking $\mathbf{U}(n)_V \rightarrow \mathbf{U}(1)^n_V$.

7. The **number of massless modes remain unchanged** as wall positions change.

8. In $4+1$-dimensions, we **dualize** the effective theory on the $3+1$ dimensional world-volume to the supersymmetric Freedman-Townsend model of **2-form fields** valued in $\mathcal{U}(N)$.

9. **Moduli matrix** approach is extremely useful to describe non-Abelian clouds of domain walls.