Boundary Effects in the Magnetic Catalysis of Chiral Symmetry Breaking

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The catalysis of chiral symmetry breaking by an applied constant magnetic field and in the presence of boundaries along third axis is investigated in the four-dimensional Nambu-Jona-Lasinio model. It is shown that in case of periodic boundary conditions for fermions the magnetic field breaks the chiral symmetry, generating a dynamical mass even at the weakest attractive interaction between fermions. For antiperiodic boundary conditions the effect of the finite third dimension is to counteract the chiral symmetry breaking.

It is a well established result that the global properties of the space-time, even if it is locally flat, can give rise to new physics. The seminal discovery in this direction is the so called Casimir effect, that is, the existence of an attractive force between neutral parallel perfectly conducting plates \([1]\). In this phenomenon the attractive force between the plates is mediated by the zero-point fluctuations of the electromagnetic field in vacuum. Thus, these Casimir forces are interpreted as a macroscopic manifestation of the vacuum structure of the quantized fields in the presence of domains restricted by boundaries or nontrivial topologies.

From the underlying physical mechanism, it is clear that the Casimir effect has a broad range of applications. Today, we can find research activity on this field in many different areas as statistical physics, condensed matter, elementary particles, cosmology, etc. \([2]\).

On the other hand, it has been recently found that a magnetic field can catalyze the dynamical chiral symmetry breaking in different quantum field systems \([3–5]\) (see, also \([6]\)). This is a universal phenomenon that can be understood as the generation, through the infrared dynamics of the fermion pairing in a magnetic field, of a fermion dynamical mass at the weakest attractive interaction between fermions. The essence of this phenomenon lies in the dimensional reduction of the electron dynamics when their energy is much less than the Landau gap \(\sqrt{\epsilon B}\) \((B\) is the magnitude of the magnetic field) \([3–5]\). In this case, the electrons are confined to the lowest Landau level, therefore having a (D-2)-dimensional dynamics. The lowest Landau level plays in this case a role similar to that of the Fermi surface in BCS superconductivity.

As it has been pointed out by several authors, the magnetic catalysis of dynamical chiral symmetry breaking can have important applications in condensed matter physics \([7]\), quantum chromodynamics \([8]\) and cosmology \([5,9]\).

Our main goal in this paper is to combine these two effects, that is, to investigate the chiral symmetry breaking in the presence of an external constant magnetic field for a fermion system in a space-time which is locally flat but which has a nontrivial topology represented by the domain \(R^3 \times S^1\) (i.e. a Minkowskian space with one of the spatial dimensions compactified in a circle \(S^1\) of finite length \(a\)).

The fact that the energy of the vacuum state of quantized matter fields in \(R^3 \times S^1\) is nonzero has been corroborated by several authors in different physical models \([11]\). Recently, the influence of a chemical potential \([12]\), as well as of an external magnetic field \([13]\), on the Casimir energy density of charged particles (bosonic and fermionic) has been also investigated.

We start considering the Lagrangian density of free fermions in the presence of a constant external magnetic field

\[
\mathcal{L} = \frac{1}{2} \bar{\psi} \left( i \gamma^\mu D_\mu - m \right) \psi , \quad \mu = 0, 1, 2, 3 ,
\]

(1)

where the covariant derivative \(D_\mu = \partial_\mu - i e A^\text{ext}_\mu\) depends on the external potential \(A^\text{ext}_\mu\), chosen in the Landau gauge

\[
A^\text{ext}_\mu = - \delta_{\mu 1} B x_2 .
\]

(2)

This potential corresponds to a constant magnetic field pointing in the \(OZ\) positive direction.

The nontrivial topology of the compactified space-time domain \(R^3 \times S^1\) is transferred into periodic boundary conditions \((PBC)\) for untwisted fermions or antiperiodic boundary conditions \((APBC)\) for twisted fermions \([11]\).

\(^{1}\)For a discussion of the role of the boundary conditions of the fermion fields on the dynamical symmetry breaking in flat space see, for example, ref. \([14]\). Curvature effects (as well as curvature in combination with an external magnetic field) on the symmetry breaking have also been studied thoroughly (for an excellent review and extended list of references see \([15]\)).
\[ \psi(t, x, y, z + a) = \pm \psi(t, x, y, z). \] (3)

In Eqs. (3) and (4) \( l \) and (3+1)-dimensions, when it is considered in a topologically trivial space-time, the momentum component in the direction of the compactified spatial coordinate.

In Eq. (8) the sum over the Landau levels was carried out, inversely on the compactified dimension length \( a \) in the direction of the compactified spatial coordinate.

In Eqs. (4) and (5) \( l \) represents the Landau level and \( n \) the discrete components of \( P_z \), the momentum component in the direction of the compactified spatial coordinate.

From Eq. (9) we see that for the PBC case there is no energy gap between the vacuum and the lowest Landau level \( (l = 0) \) at zero momentum \( (n = 0) \) in the \( m \to 0 \) limit. This is the same behavior found in this system in (2+1)- and (3+1)-dimensions, when it is considered in a topologically trivial space-time \( (a \to \infty) \). As in those cases, it should be expected here that the magnetic field catalyzes the dynamical breaking of chiral symmetry. The absence of an energy gap between the vacuum and the lowest Landau level at zero momentum makes possible the vacuum condensation of electron pairs which are interacting in the infrared band of the fermion spectrum.

For the APBC case (Eq. (9)) the situation is different. Even at \( m \to 0 \), an energy gap \( (\Delta E = \frac{\pi}{\rho}) \), which depends inversely on the compactified dimension length \( a \), exists between the vacuum and the lowest Landau level \( (l = 0) \) at zero momentum \( (n = 0) \).

In each case, to determine whether a chiral condensate \( \langle 0 \big| \bar{\psi} \psi \big| 0 \rangle \) catalyzed by the applied external field exists, we start from the fermion effective action in the presence of a constant magnetic field which in the Schwinger proper time formalism \( \{14\} \) is given by

\[
W = \frac{i}{2} \int_{1/\Lambda^2}^{\infty} ds s \text{Tr} e^{-is\mathcal{H}},
\] (6)

where \( 1/\Lambda^2 \) is a cutoff in the proper time \( s \), \( \text{Tr} \) means the trace over space-time and internal degrees of freedom, and the proper-time Hamiltonian density is

\[
\mathcal{H} = (P_\mu - e A^\text{ext}_\mu)^2 - \frac{e}{2} \sigma_{\mu\nu} F^{\mu\nu} + m^2
\] (7)

with \( P_\mu = -i \partial_\mu \) denoting the electron four-momentum and \( \sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu] \).

For the APBC case it was found in the second paper of Ref. \( \{13\} \) that the fermion effective action is given by

\[
W_{AP} = \frac{TAa}{8\pi^2} \int_{1/\Lambda^2}^{\infty} ds s^3 e^{-ism^2} \left[ 1 + 2 \sum_{n=1}^{\infty} (-1)^n e^{i(an)^2/4s} \right] \left[ 1 + iseBL(iseB) \right].
\] (8)

In Eq. (8) the sum over the Landau levels was carried out, \( V = TAa \) is the four-dimensional volume of the \( R^3 \times S^1 \) domain and \( L(\xi) = \coth \xi - \xi^{-1} \) is the Langevin function.

In case of periodic boundary conditions the only difference in the effective action \( W_P \) as compared to \( W_{AP} \) is the absence of the phase factor \( (-1)^n \) appearing in the sum of Eq. (8). Thus the effective action for the PBC case is

\[
W_P = \frac{TAa}{8\pi^2} \int_{1/\Lambda^2}^{\infty} ds s^3 e^{-ism^2} \left[ 1 + 2 \sum_{n=1}^{\infty} e^{i(an)^2/4s} \right] [1 + iseBL(iseB)].
\] (9)

The chiral condensate is found through the equation

\[
\frac{dW}{dm} \bigg|_{m=0} = - \langle 0 \big| \bar{\psi} \psi \big| 0 \rangle.
\] (10)
where \( W = \frac{W}{\pi a} \). From Eqs. (8) and (9) we have respectively

\[
\frac{dW_{AP}}{dm} = -\frac{m}{4\pi^2} \int_{1/A^2}^{\infty} \frac{ds}{s^2} e^{-sm^2} \theta^4 \left[ 0 \left| \frac{ia^2}{4\pi s} \right| [1 + seB(eB)] \right],
\]

(11)

\[
\frac{dW_P}{dm} = -\frac{m}{4\pi^2} \int_{1/A^2}^{\infty} \frac{ds}{s^2} e^{-sm^2} \theta^3 \left[ 0 \left| \frac{ia^2}{4\pi s} \right| [1 + seB(eB)] \right],
\]

(12)

where we have introduced the Jacobi \( \theta \)-functions \([17]\) to represent the series appearing in Eqs. (8) and (9), and made a Wick rotation to Euclidean space \((s \rightarrow -is)\).

Using the Jacobi imaginary transformations

\[
\theta_4 (0 \mid \tau) = \sqrt{\frac{i}{\tau}} \theta_2 (0 \mid \tau), \quad \theta_3 (0 \mid \tau) = \sqrt{\frac{i}{\tau}} \theta_3 (0 \mid \tau)
\]

(13)

with \( \tau = \frac{ia^2}{4\pi s} \), we can write Eqs. (11) and (12) as

\[
\frac{dW_{AP}}{dm} = -\frac{eB}{\pi^{3/2} a} \int_{m^2/A^2}^{\infty} \frac{ds}{s^{1/2}} \frac{1}{e} \sum_{n=1}^{\infty} e^{-s \frac{4\pi^2 (n+1/2)^2}{am^2}} \coth \left( \frac{seB}{m^2} \right),
\]

(14)

\[
\frac{dW_P}{dm} = -\frac{eB}{2\pi^{3/2} a} \int_{m^2/A^2}^{\infty} \frac{ds}{s^{1/2}} \frac{1}{e} \sum_{n=0}^{\infty} e^{-s \frac{4\pi^2 n^2}{am^2}} \coth \left( \frac{seB}{m^2} \right),
\]

(15)

where we have used again the series representation of the Jacobi functions \( \theta_2 \) and \( \theta_3 \). Evaluating Eqs. (14) and (15) at \( m = 0 \) we obtain for \( eB \neq 0 \)

\[
\langle 0 \mid \bar{\psi} \psi \mid 0 \rangle_{AP} = 0,
\]

(16)

\[
\langle 0 \mid \bar{\psi} \psi \mid 0 \rangle_{P} = -\frac{eB}{2\pi a}.
\]

(17)

Thus, from these results it can be seen that in the PBC case a chiral condensate is catalyzed by the magnetic field (Eq. (17)), while for the APBC case the condensate is absent (Eq. (16)).

To understand better the genesis of the different behavior of these two cases we should underline that the main difference between the APBC and the PBC configurations lies in the existence of a zero fermion mode \((n = 0)\) for the latter one. That is, for PBC the fermion spectrum accepts a zero mode, as discussed below Eq. (5). This zero mode contribution is essential to obtain a different from zero result (17).

We must also stress that the condensate (17) is in fact equal to the product of the condensate \((-\frac{eB}{2\pi a})\), found in a topologically trivial \((2+1)\)-dimensional domain in the presence of an external magnetic field \(B\), times \((1/a)\). The \(1/a\) factor here is a manifestation of an additional dimensional reduction (besides that introduced by the magnetic field \(B\)) associated to the nontrivial topology (i.e. the presence of the boundaries determined by the parallel plates).

When an external magnetic field is present, but the topology is trivial, the chiral condensate in the free fermions \((3+1)\)-dimensional case is given by \(\langle 0 \mid \bar{\psi} \psi \mid 0 \rangle \sim m \ln m\), and therefore the condensate is absent in the \(m \rightarrow 0\)-limit. From Eq. (17), we can see that when this model is considered within a nontrivial topology \(R^3 \times S^1\), the magnetic field catalyzes the chiral symmetry breaking, even though the fermions do not acquire a dynamical mass. Therefore, we can conclude that the magnetic catalysis of dynamical symmetry breaking in this case is specifically settled by the nontrivial topology.

To generate a fermion dynamical mass we must introduce fermion interactions. As shown below, in the PBC case a fermion dynamical mass, depending on the magnetic field \(B\) and the compactified dimension length \(a\), appears for weak fermion interactions.

Let us consider the Nambu-Jona-Lasinio (NJL) model \(18\)

\[
\mathcal{L}_{NJL} = \frac{1}{2} \left[ \bar{\psi} i\gamma^\mu D_\mu \psi \right] + \frac{G}{2N} \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} i\gamma^5 \psi)^2 \right],
\]

(18)
where $D_{\mu}$ is the covariant derivative introduced in Eq. (1) and the fermions carry an additional, "color", index $\alpha = 1, 2, \ldots, N$. It is known that introducing the composite fields

$$\sigma = -\frac{G}{N} \bar{\psi} \gamma_5 \psi, \quad \pi = -\frac{G}{N} \bar{\psi} \gamma_5 \gamma^5 \psi$$

the NJL Lagrangian density (18) can be rewritten as

$$\mathcal{L}_{NJL} = \frac{1}{2} \left[ \bar{\psi} i\gamma_{\mu} D_{\mu} \psi \right] - \bar{\psi} \left( \sigma + i\gamma_5 \pi \right) \psi - \frac{N}{2G} \left( \sigma^2 + \pi^2 \right).$$

Eqs.(19) are the Euler-Lagrange equations for the auxiliary fields $\sigma$ and $\pi$ in (20). Thus, with the constraints (19) the theories represented by the Lagrangian densities (18) and (20) are equivalent.

It is clear from Eq. (20) that if $\sigma$ gets a different from zero vacuum expectation value (vev) $\bar{\sigma}$, the fermions acquire mass. Then, to investigate the generation of a fermion dynamical mass we need to search for possible non-zero vev of $\sigma$. In the large $N$ limit the vacuum is determined by the stationary point of the effective action for the composite fields $\sigma$ and $\pi$, obtained by integrating over fermions in the path integral:

$$W(\sigma, \pi) = -\frac{N}{2G} \int d^4x (\sigma^2 + \pi^2) - i \text{Tr} \log \left[ i\gamma_{\mu} D_{\mu} - (\sigma + i\gamma_5 \pi) \right].$$

Since the vacuum should respect translational invariance, to find the vacuum solution we need to calculate the effective potential. Moreover, since the effective potential $V$ depends only on the chiral invariant $\rho^2 = \sigma^2 + \pi^2$, it is sufficient to consider a configuration with $\pi = 0$ and $\sigma$ constant. In the proper-time formalism we get

$$V_{AP}(\sigma) = \frac{N\sigma^2}{2G} + \frac{NeB}{8\pi^2} \int_{1/\Lambda^2}^{\infty} ds e^{-s\sigma^2} \left( \frac{0}{s} \frac{\text{coth}(eBs)}{4\pi s} \right),$$

$$V_{P}(\sigma) = \frac{N\sigma^2}{2G} + \frac{NeB}{8\pi^2} \int_{1/\Lambda^2}^{\infty} ds e^{-s\sigma^2} \left( \frac{0}{s} \frac{\text{coth}(eBs)}{4\pi s} \right),$$

for antiperiodic and periodic boundary conditions respectively. It is evident that in the limit $a \to \infty$, since the theta functions $\theta_3(0|ia^2/4\pi s)|_{a=\infty} = 1$, the two effective potentials are equal ($V_{AP} = V_{P}$) and coincide with the well known effective potential of the NJL model in the presence of an external magnetic field (2).

The dynamical mass $\bar{\sigma}$ for the PBC case is obtained as the solution of the gap equation $dV_{P}/d\sigma = 0$. This equation can be written separating the ultraviolet contribution (i.e. terms depending on $\Lambda$). In leading order in $1/\Lambda$ the PBC gap equation is given by

$$\left[ \frac{1}{G} - \frac{\Lambda^2}{4\pi^2} + \frac{\sigma^2}{4\pi^2} \left( \log \frac{\Lambda^2}{\sigma^2} + 1 - \gamma \right) - \frac{1}{4\pi^2} \int_{0}^{\infty} ds e^{-s\sigma^2} \left( \theta_3 \left( \frac{0}{s} \frac{\text{coth}(eBs)}{4\pi s} \right) - 1 \right) \right] = 0,$$

where $\gamma \approx 0.577$ is the Euler constant. The corresponding gap equation for APBC is obtained by replacing $\theta_3$ by $\theta_4$ in Eq. (24).

It is easy to see that under the condition $1/a \ll \sigma \ll \sqrt{eB}$, i.e. $a$ being the largest length scale in the problem, the gap equations for both cases (PBC and APBC) reduce to the following one

$$\sigma \left[ \frac{1}{G} - \frac{1}{G_c} \pm \left( \frac{2\sigma}{\pi a^2} \right)^{1/2} e^{-\sigma a} - \frac{eB}{4\pi^2} \log \frac{eB}{\pi \sigma^2} \right] = 0,$$

where $G_c = (4\pi^2/\Lambda^2)$ and $\pm$ refers to APBC and PBC, respectively. The solution of Eq. (25) in the $G \ll G_c$ approximation is
As expected, the solution \((26)\), which is nonanalytic in \(G\) as \(G \to 0\), coincides with the one found in \((3+1)\)-dimensions for \(B \neq 0\) and \(a = \infty\) \(4\).

Let us consider now the opposite limit of the small length \(a (\sigma, \sqrt{eB} \ll 1/a)\) which is the important one to study the effects of the compactified dimension. In this limit the gap equation \((24)\) for \(PBC\) reduces to

\[
\frac{\sigma^2}{2\pi^2} \left( \log \frac{\Lambda^2 a^2}{16\pi^2} + \gamma \right) - \frac{eB}{2\pi\alpha} \left( \sqrt{\frac{2}{eB}} \sigma \zeta \left( \frac{1}{2} \frac{\sigma^2}{2eB} + 1 \right) + 1 \right) = 0, \tag{27}
\]

where \(\zeta(\nu, x)\) is the generalized Riemann zeta function.

As \(B \to 0\), we recover the known gap equation \(22\) which admits a nontrivial solution only if the coupling is supercritical, \(G > G_c^a\), and the critical coupling \(G_c^a = (G_c^{-1} + 1/3a^2)^{-1}\). When an external magnetic field, \(B \neq 0\), is present a nontrivial solution exists at all \(G > 0\) and, in particular, at \(G \ll G_c^a\). Indeed, looking at the solution of Eq. \((27)\) satisfying \(\sigma \ll \sqrt{eB}\) we find

\[
m_{\text{dyn}}^p = \sigma \simeq \frac{eB}{2\pi\alpha} \frac{GG_c}{G_c^a - G} \tag{28}
\]

if the coupling \(G \ll G_c^a\). The condition \(G < G_c^a\) guarantees that \((28)\) is a minimum solution of the effective potential \(V_p\).

From the above result it is clear that the dynamical mass solution \(28\) exists in the weak coupling regime of the theory. The fact that there is no critical value of the coupling to produce chiral symmetry breaking is a characteristic feature of the catalysis of dynamical symmetry breaking by a magnetic field \(3\ 5\). It is remarkable that unlike the \(a = \infty\) case, where the dynamical mass has nonanalytical dependence on the coupling constant as \(G \to 0\) (see Ref. \(1\)), at finite \(a\) the dynamical mass \(28\) is an analytic function of \(G\) at \(G = 0\).

Note also the following point. Equation \((19)\) implies that \(m_{\text{dyn}} = \langle 0 | \sigma | 0 \rangle = -G \langle 0 | \bar{\psi} \psi | 0 \rangle / N\). From here and Eq. \((28)\) we find that the condensate \(\langle 0 | \bar{\psi} \psi | 0 \rangle \ll -N eB / 2\pi\alpha \) in leading order in \(G\); i.e. it coincides with the value of the condensate calculated in the problem of free fermions in a magnetic field (see Eq. \((17)\)). This point also explains why the dynamical mass \(m_{\text{dyn}}\) is an analytic function of \(G\) at \(G = 0\): indeed, the condensate already exists at \(G = 0\). As a result, we have big enhancement of the dynamical fermion mass generation in the presence of boundaries along the magnetic field direction and with periodic boundary conditions for the fermion fields comparing to the case of topologically trivial space-time (see Eqs. \((28)\) and \((29)\)).

As it is well known, the breaking of a continuous chiral symmetry is linked to the existence of Nambu-Goldstone (NG) bosons. The analysis of the NG-modes appearing in this problem will be published elsewhere.

Let us discuss now the \(APBC\) case. Following the same procedure we used for \(PBC\), it is easy to show that the \(APBC\) gap equation under conditions \(\sigma, \sqrt{eB} \ll 1/a\) does not have a nontrivial solution; on the other hand, when \(\sigma \ll \sqrt{eB}, 1/a\) it is reduced to

\[
\sigma \left[ \frac{1}{G} - \frac{1}{G_c} + \frac{1}{6a^2} - \frac{eB}{4\pi^2} \int_0^\infty ds \theta_4 \left( 0 \left| \frac{i}{4\pi s} \right. \right) \right] \left( \frac{\coth(eBa^2 s) - 1}{eBa^2 s} \right)
+ \frac{\sigma^2}{4\pi^2} \left( \log \frac{\Lambda^2 a^2}{\pi^2} + \gamma + eBa^2 \int_0^\infty ds \theta_4 \left( 0 \left| \frac{i}{4\pi s} \right. \right) \right) \left( \frac{\coth(eBa^2 s) - 1}{eBa^2 s} \right) = 0. \tag{29}
\]

From Eq. \((29)\) one can convince oneself that there is no nontrivial solution under the assumptions made if the coupling is weak \((G \to 0)\). For chiral symmetry breaking to take place, the coupling constant \(G\) must be larger than some critical value that depends on the magnitude of the magnetic field \(B\) and size \(a\). Indeed, Eq. \((29)\) can be simplified in the limiting case \(\sqrt{eB} a \gg 1\)

\[
\sigma \left[ \frac{1}{G} - \frac{1}{G_c} + \frac{1}{6a^2} - \frac{eB}{4\pi^2} \ln \frac{eBa^2}{\pi^3} + 2\gamma \right] + \frac{\sigma^2}{4\pi^2} \left( \log \frac{\Lambda^2 a^2}{\pi^2} + \frac{7\zeta(3)}{4\pi^2} eBa^2 + \gamma \right) = 0. \tag{30}
\]

From Eq. \((30)\) one can notice that the magnetic field is helping the symmetry breaking since the critical coupling is less than the one corresponding to the case with zero magnetic field.
On the other hand, the contribution of the compactified dimension length $a$ to the gap equation in the presence of a magnetic field has opposite sign for APBC (third term in Eq. (29)), as compared to PBC (third term in Eq. (27)). Consequently, the boundary effect in the APBC case is not enhancing the chiral symmetry breaking, but on the contrary, it is counteracting it; while in the PBC case the magnetic catalysis is substantially enhanced by the boundary.

The fact that the boundary effect in the APBC case is not enhancing the chiral symmetry breaking can be better understood if one realizes that due to the antiperiodic boundary conditions the quantity $1/a$ plays in the APBC case a role similar to temperature. We have seen above that the chiral symmetry breaking takes place at small $1/a$ with a corresponding dynamical mass given by Eq. (26), so we should expect that at $1/a$ larger than some critical value $1/a_c$ the chiral symmetry must be restored.

Such a critical value $1/a_c$ indeed exists and is determined from the condition that the second derivative of the effective potential at $\sigma = 0$ becomes positive. In fact, from Eq. (30) it is found to be

$$
\frac{1}{a_c} = \frac{e^7}{\pi} m_{\text{dyn}}
$$

with $m_{\text{dyn}}$ the dynamical mass (26). Therefore, we obtain that the inverse of the critical length, $1/a_c$, is of the order of $m_{\text{dyn}}$, a result equivalent to that found for the relationship between the critical temperature and the gap in BCS superconductivity. Also, it follows from Eq. (30) that the approach of $\bar{\sigma} = m_{\text{dyn}}$ to zero is given by the square root dependence $(a - a_c)^{1/2}$.

Finally, we should point out that the combined effect of an external magnetic field and periodic boundary conditions for fermions along third axis can find important applications to explaining a kink-like feature of thermal conductivity in the presence of a magnetic field in high-$T_c$ superconducting samples much below the critical temperature. Such an effect was observed in recent experiments by Krishana et al. [19]. These high-$T_c$ superconductors are known to possess a quasi-2D structure, and it has been suggested [13] that the magnetic field can induce a second phase transition in the superconducting state, leading to the opening of a gap at the nodes of a conventional $d$-wave gap.

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