Micro-analysis of slip differential heat of magnetorheological fluids based on micromechanics and microstructures

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Abstract

For describing the slip differential heat between two neighboring particles of magnetorheological fluids (MRFs) in shear and squeeze modes, a theoretical model was developed in this study based on micromechanics and microstructures. Firstly, the interaction force and the contact stress during the shear and squeeze deformation was analyzed based on the magnetic dipole theory and the Hertzian contact theory. And then the motion and the relative velocity of particles were analyzed by assuming the incline or bending of a single chain ordered in MRFs. Furthermore, the slip differential heat flow caused by the friction of neighboring particles in MRFs was analyzed based on the tribology theory. The model takes into account the effect of the main influencing factors on the slip differential heat flow between two neighboring particles in a single chain of MRFs in shear and squeeze modes, such as the magnetic induction intensity, the size of particles, the number of the particles in a single chain, the relative velocity between the upper and lower walls in shear mode and the squeeze velocity in squeeze mode. Using the proposed model, the individual effect of typical governing parameters on slip differential heat flow under the realistic conditions of the magnetorheological clutch and damper was investigated by the numerical simulation. The results indicate that these factors have great influences on the slip differential heat flow, and the proposed model can satisfactorily describe the main micro-characteristics of the slip differential heat of MRFs in shear and squeeze modes.

1. Introduction

Magnetorheological fluids (MRFs) are a kind of controllable intelligent suspension consisting of a nonmagnetic carrier liquid and dispersing ferromagnetic particles with micron size. When exposed to an external magnetic field, it demonstrates an instantaneous, reversible, and tunable change from a free-flowing liquid to a solid-like viscoplastic material, and exhibits dramatic differences in its macroscopic rheological characteristics [1], such as apparent viscosity and yield shear stress of MRFs which are precisely controlled by altering the applied magnetic field [2]. Based on these unique properties, MRFs have been attracting considerable attention during the past decades, and have been extensively used in the engineering applications, such as clutches, valves, shock absorbers and damper braking devices, etc [3–8]. However, high temperature has a great influence on the properties of MRFs, e.g., with the increase of temperature, the viscosity of the carrier liquid in MRFs will decrease, which causes yield shear stress of MRFs to fall. There are some main reasons which will result in the rise of temperature of MRFs, such as the slip differential heat between magnetic particles in MRFs which caused by friction between particles, current thermal effect of the excitation coil, friction between bearing and seal inside transmission devices and ambient temperature outside, where the slip differential heat is the maximum source [9]. Because the particles are micron and the slip differential heat is related to mechanics and kinematics, it is necessary to study micromechanics and microstructures of MRFs.

Great efforts have been made in micromechanical descriptions of MRFs. Bossis and Lemaire [10] proposed a dipolar sphere model to predict the yield shear stress of MRFs, but this model did not take into account the nonlinear magnetization. Ginder and Davis [11] calculated the interparticle forces and shear stress...
incorporating the nonlinearity and saturation of the particle magnetization. Jolly et al [12] developed a quasi-static, one-dimensional model to examine the mechanical and magnetic properties of magnetorheological materials, but this model did not comprise magnetic interactions. Ginder et al [13] described numerical and analytical models of MRFs phenomena, and these models took into account the effects of magnetic nonlinearity and saturation. Moreover, he derived approximate analytical expressions for the yield stress and shear modulus and carried out magnetorheological experiments to verify them. Tang et al [14] developed two- and three-dimensional analytical models for the yield stress of MRFs as a function of magnetic field and total volume fraction of particles, which are in accord with experimental data, but they have difficulties applying in practical problems. Bossis et al [15] presented the basic phenomena related to the interplay between interparticle magnetic forces and gave some analytical predictions for the yield stress. Peng and Li [16] formulated a micro–macro description for the constitutive behavior of MRFs under shear deformation based on the conventional magnetic-dipolar model and a statistical approach. Si et al [17] proposed a micromechanical model to describe the constitutive behavior of MRFs by making use of static magnetics. This model can take into account the effects of the intensity of the applied magnetic field, the particle size and particle volume fraction, the magnetic property of the particle, and the viscosity of the carrier fluid, on the mechanical properties of MR fluids. Based on the conventional magnetic-dipolar model, Yi et al [18] formulated a micro–macro description for the constitutive behavior of MRFs under shear deformation based on a more exact magnetic-dipolar model and a statistical approach. Later, Zhao et al [19] examined the validity of the two conventional micro-macro descriptions for MRFs based respectively on the exact dipole model and the simplified dipole model. Moreover, great efforts have been also made in heat transfer of MRFs. Dogruoz et al [20] presented a lumped system model for predicting the heat transfer from fail-safe magneto-rheological fluid dampers, and carried out the experiments of two automotive size dampers to evaluate the validity of the theoretical model. The results also indicate that both the mechanical and electrical power input contribute substantially to the temperature rise. Park et al [21] proposed a magnetorheological brake system and an optimum magnetorheological brake design with two rotating disks with finite element simulations involving magnetostatic, fluid flow and heat transfer analysis. Based on this, Karakoc et al [22] discussed design considerations for building an automotive magnetorheological (MR) brake, and performed a finite element analysis to analyze the resulting magnetic circuit and heat distribution within the MR brake configuration. Kavlicoglu et al [23] presented theoretical and experimental studies on heating of a high-torque, multi-plate MRFs limited slip differential clutch. And the results show that the transferred torque is insensitive to clutch temperature increase. Hou et al [9] established the thermal analysis model to obtain the influence of temperature field on characteristic of a magnetorheological transmission device, and analyzed the steady-state and transient temperature field of the transmission device by finite element method. Wang et al [24] presented a heat–flow coupling simulation method and theoretical basis for the simulation to pre-evaluate the cooling efficiency of a liquid-cooled magnetorheological clutch in high-power situations. Zhou et al [25] used the thermal lattice Boltzmann method with doubled-population to analyze the dilute MRFs flow and heat transfer performances in two-dimensional hot micro channels by regarding the magnetic particles (MPs) as a quasi fluid. However, little attention has been paid to micro-analysis of slip differential heat of MRFs, which caused the inability to solve the problem of high temperature of MRFs fundamentally. Therefore, to better understand the temperature increment of MRFs, it is necessary to establish a theoretical model which considers the combined effects of multi influencing factors on the slip differential heat of MRFs.

For formulating the model for slip differential heat between two neighboring particles of MRFs in shear and squeeze modes, in this paper, some necessary assumptions of the particles and the chains of MRFs are made. For shear and squeeze modes, micro working principles of chains subjected to an applied magnetic field and a shear motion are respectively introduced, and microscopic interaction force, contact stress and slip differential velocity between two neighboring particles of MRFs are respectively analyzed based on the magnetic dipole field and the Hertzian contact theory. Then, the expressions for slip differential heat flow between two neighboring particles are respectively obtained accurately based on tribology theory. Moreover, we made numerous numeral simulations, such as electromagnetic field analysis and contact analysis, to verify magnetic force and contact stress in shear mode. Finally, the results evaluation and analysis of related influencing factors are carried out.

2. Theoretical model of slip differential heat in shear mode

2.1. Assumptions

It is known that ferromagnetic particles aggregate from random distribution in a carrier liquid to chains when exposed to an external magnetic field. In order to formulate the theoretical model of slip differential heat in shear mode, we make the following assumptions to avoid complications:
All the particles are made of the same material and are of spheres with identical diameter.

The magnetized dipolar particles align in single straight chains, attaching the two walls by their ends [16].

All the chains and their changes are the same.

The friction between particles is assumed as the sliding friction.

The carrier liquid is a Newtonian fluid.

The other forces, such as Brownian force, gravity and Van der Waals’ force, etc, are so smaller than magnetic force that they can be negligible.

According to the experiment results [22], the heat caused by the internal friction of the carrier liquid and the friction between the carrier liquid and the particles is relatively much smaller than that caused by the friction between particles so that it can be negligible.

2.2. Introduction of generation of slip differential heat in shear mode

Ferromagnetic particles distribute randomly in a carrier liquid without applying an external magnetic field (figure 1(a)). When a magnetic field is applied, the particles aggregate into chains and arrange along the direction of magnetic field (figure 1(b)). In this process, because the aggregation of particles is instantaneous, the heat caused by the friction between two neighboring particles can be negligible. Then, according to the observation of the deformation of the chain [26], the stretching occurs between any two particles in a single chain (figure 1(c)). Meanwhile, all of neighboring particles except these two particles in a single chain rub against each other, which brings about the slip differential heat. With the increase of shear motion, the interaction between particles decreases and the breaking occurs between these two particles in a single chain (figure 1(d)). At the same time, the broken chains rebound to the direction of magnetic field, the new chains are formed, and a similar process is repeated (figure 1(e)). Therefore, the slip differential heat is mainly from friction between two neighboring particles in the process of stretching.

2.3. Theoretical study of slip differential heat in shear mode

The magnetic dipole theory plays an important role in the microscopic description of the mechanical characteristics of MRFs. According to the magnetic dipole theory, the ferromagnetic particles can be magnetized in a magnetic field, and each magnetized particle has two magnetic poles similar to N and S poles. Therefore, there is a magnetic attraction between two particles in the same magnetic field (figure 2), the expression of magnetic force between two neighboring particles is [27]

$$ F_{ij} = \sum_{j=i}^{n} \frac{4\pi \mu_0 \chi^2 H^2 R^6}{3r_{ij}^4} [(1 - 5 \cos^2 \theta_{ij})\hat{r}_{ij} + 2 \cos \theta_{ij}\hat{r}] $$

where $\mu_0 = 4\pi \times 10^{-7}$ A m$^{-1}$ is the permeability in free space, $\chi$ is the susceptibility, $H$ is the intensity of applied magnetic field, $R$ is the radius of the particle, $r_{ij}$ is the vector designating the relative position from dipolar particle $j$ to that of dipolar particle $i$, and $\|r_{ij}\|$ is the Euclidian norm of $r_{ij}$, $\theta_{ij}$ is the angle between the
inclined chain and the applied magnetic, \( \hat{r}_{ij} \) is the unit vector of the center of the particle \( i \) pointing to the center of the particle \( j \), and \( \hat{y} \) is the unit vector of the direction of the magnetic field.

The radial component of the magnetic force between two neighboring particles can be expressed as

\[
F_{mn} = \sum_{j=i}^{4\pi\mu_0\chi^2H^2R^6} \left(3 \cos^2 \theta_{ij} - 1 \right)
\]

(2)

Furthermore, two neighboring particles subjected to the attractive magnetic forces are pressed against each other. And then the slip differential heat will generate if they have a relative motion. According to the Hertzian contact theory, when the two spheres are pressed against each other, a circular interface appears at the contact location. Meanwhile, the distribution of the stress on interface conforms to the geometric law of the ellipse, and the maximum is at the center of the interface. The magnetic forces between the particles is approximately equivalent to the concentrated force \((2F_{mn})\) applied to the apex of one particle (figure 3) since the particles are micron.

Based on the expressions of the Hertzian contact radius and stress, the radius and the maximum contact stress between two neighboring particles can be determined as

\[
a = \sqrt[3]{\frac{3R(1 - \mu^2) \cdot 2F_{mn}}{4E}} = \sqrt[3]{\frac{(1 - \mu^2)\pi\mu_0\chi^2H^2R^3}{8E}}(3 \cos^2 \theta_{ij} - 1) ,
\]

(3)

\[
q_0 = \sqrt{\frac{6E^2 \cdot 2F_{mn}}{\pi^2 R^4 (1 - \mu^2)^2}} = \sqrt{\frac{\mu_0\chi^2H^2E^2}{\pi^2 (1 - \mu^2)}}(3 \cos^2 \theta_{ij} - 1) .
\]

(4)

where \( E \) and \( \mu \) are respectively the modulus of elasticity and Poisson’s ratio of the particles.
The frictional shear stress of the interface between two neighboring particles can be expressed as
\[ \tau = \mu_k q_k = \frac{2\mu_k}{3} \sqrt{\frac{\mu_k^3 H^2 E^2}{\pi^2(1 - \mu^2)^3}}(3\cos^2 \theta_{ij} - 1), \]
where \( \mu_k = 0.15 \) is the friction coefficient of the interface.

If the relative velocity between the upper and lower walls is \( v_\tau \), the relative velocity between any two neighboring particles can be written as
\[ v_{\tau0} = \frac{v_\tau}{N - 1}, \]
where \( N \) is the number of particles in a single chain.

According to the tribology theory, the slip differential heat flow rate between two neighboring particles during the shearing motion can be expressed as
\[ q_{cr} = FHTG \cdot \tau v_{\tau0} = \frac{2\mu_k v_\tau}{3(N - 1)} \sqrt{\frac{\mu_k^3 H^2 E^2}{\pi^2(1 - \mu^2)^3}}(3\cos^2 \theta_{ij} - 1), \]
where \( FHTG = 1 \) is the energy conversion coefficient.

Therefore, the slip differential heat flow between two neighboring particles during the shearing motion can be derived as
\[ J_r = q_{cr} s = \pi q_{cr} a^2 = \frac{\pi \mu_0 \mu_k^2 H^2 v_\tau R^2}{6(N - 1)}(3\cos^2 \theta_{ij} - 1). \]

3. Theoretical model of slip differential heat in squeeze mode

3.1. Assumptions and introduction of generation of slip differential heat in squeeze mode
In addition to the assumptions in shear mode, we also make the following assumptions:

(a) The bending of the chains is symmetrical and ordered \([28]\).

(b) The length of the chains is constant during the bending deformation.

(c) The relative velocity between the centers of two neighboring particles is approximately equivalent to that between two neighboring particles, because the micron size of the particles is relatively small.

(d) Researches on the viscous heat dissipation of fluid has been sufficient, and this part can be directly added to the macroscopic heat, so the micro viscous heat dissipation of the carrier fluid is not studied.

After the application of the magnetic field, the particles aggregate into the chains. When the chains are subjected to the pressure, they start to exhibit the bending deformation (Figure 4(a)). And then the friction between two neighboring particles leads to the slip differential heat. If the chains demonstrate a reciprocating vertical motion without breaking, they will generate heat continuously. However, if the chains break and form the new bundle chains (Figures 4(b) and (c)), they can provide the larger shear stress based on the combination of shear and squeeze modes. And this phenomenon is not studied in this paper.

3.2. Interaction force between two neighboring particles in squeeze mode
Compared to the situation in shear mode, the interaction force between two neighboring particles in squeeze mode includes the pressure in addition to the magnetic force. The bending deformation of a single chain is presented in Figure 5. The interaction force between the particles in the upper half chain is only analyzed, because

![Figure 4. Squeeze motion of particles. (a) Compression; (b) Breaking; (c) Re-formation.](image-url)
the bending of a single chain is symmetrical and ordered. The angles in figure 6 can be determined as

\[ \angle 1 = \frac{2R}{l} \]  
\[ \angle 5 = \frac{\pi}{2} - \frac{R}{l} \]  
\[ \angle 2 = \frac{(N - i)R}{l} \]  
\[ \angle 3 = \frac{\pi}{2} - \frac{(N - i)R}{l} \]

where \( l = \frac{R(N - 1)}{\beta} \) is the radius of the single curved chain.

Therefore, the angle between the line connecting two neighboring particles and the magnetic can be expressed as

\[ \alpha_{i(i+1)} = \angle 4 = \frac{(N - i - 1)\beta}{N - 1} \] (13)

where \( i = 1, 2, 3 \ldots \frac{N}{2} - 1 \).

The contribution which a single chain can provide to resist against bending deformation is derived as [28]

\[ F_{ms} = \sum_{i=1}^{N-1} \frac{\pi \mu_0 X^2 H^2 R^2}{24} \left[ (1 - \cos^2 \alpha_{i(i+1)}) - \frac{\chi}{24} (1 + 4 \cos^2 \alpha_{i(i+1)}) \right] \sin \beta \]
\[ = \sum_{i=1}^{N-1} \frac{\pi \mu_0 X^2 H^2 R^2}{24} \left[ 1 - \cos^2 \frac{(N - i - 1)\beta}{(N - 1)} \right] - \frac{\chi}{24} \left( 1 + 4 \cos^2 \frac{(N - i - 1)\beta}{(N - 1)} \right) \sin \beta \] (14)
And then the magnetic forces and the pressures of the particles from the first to the $\left(\frac{N}{2} - 1\right)$th can be expressed as (figures 7(a) and (b))

$$F_m^{(\frac{N}{2} - 1)(\frac{N}{2} - 1)} + F_m^{(\frac{N}{2} - 1)n} + P_{m1}^{(\frac{N}{2} - 1)} + P_{m2}^{(\frac{N}{2} - 1)} = 0$$  \tag{15}

$$F_m^{(\frac{N}{2} - 2)(\frac{N}{2} - 1)} + F_m^{(\frac{N}{2} - 2)n} + P_{m1}^{(\frac{N}{2} - 2)} + P_{m2}^{(\frac{N}{2} - 2)} = 0$$  \tag{16}

$$F_m^{(\frac{N}{2} - 2)(\frac{N}{2} - 1)} + F_m^{(\frac{N}{2} - 2)n} + P_{m1}^{(\frac{N}{2} - 2)} + P_{m2}^{(\frac{N}{2} - 2)} = 0$$  \tag{17}

where $F_m^{(\frac{N}{2} - 1)}$ is the magnetic force from the particle 2, $P_{m1}^{(\frac{N}{2} - 1)}$ is the pressure from the particle 2; $F_m^{(\frac{N}{2} - 2)}$ is the magnetic force from the particle 1, $F_m^{(\frac{N}{2} - 2)}$ is the magnetic force from the particle 1, $P_{m1}^{(\frac{N}{2} - 2)}$ is the pressure from the particle 1, $P_{m2}^{(\frac{N}{2} - 2)}$ is the pressure from the particle 3; $F_m^{(\frac{N}{2} - 2)(\frac{N}{2} - 1)}$ is the magnetic force from the particle $\left(\frac{N}{2} - 2\right)$, and $P_{m1}^{(\frac{N}{2} - 2)(\frac{N}{2} - 1)}$ is the pressure from the particle $\left(\frac{N}{2} - 2\right)$.

According to equations (15)–(17), the pressure between particle $i$ and particle $i + 1$ can be derived as

$$P_{i(i+1)} = P_{i+1(i)} = F_m - F_{m(i+i)}$$  \tag{18}

Therefore, the interaction force between two neighboring particles can be determined as

$$F_{pi(i+1)} = F_{i(i+1)} + F_{m(i+i)} = F_m + \pi \mu_0 \chi^2 H^2 R^2$$

where $i = 1, 2, 3, \ldots, \frac{N}{2} - 1$ and $\hat{y}$ the unit vector from particle $i$ to particle $i + 1$.

### 3.3. Relative velocity between two neighboring particles in squeeze mode

Noticing that the motion of the particles is irregular, and the velocities of the particles are assumed as uniform in unit time. And then the relative velocity is divided into the X-component and Y-component.

If the squeeze velocity is $v_p$, the displacement of particle 1 in unit time is

$$\Delta L = v_p$$  \tag{20}

Figure 8 shows the variation of the particles in unit time. The angles can be determined as

$$\psi = \frac{\beta(i - 1)}{N - 1} \quad \text{and} \quad \psi' = \frac{\beta(i - 1)}{N - 1}$$  \tag{21}

Because the length of the particle chain is constant, the arc length of the curved chain is

$$s = 2NR$$  \tag{22}

Moreover, the chord length corresponding to arc length of the curved chain is

$$L = 2l \sin \beta$$  \tag{23}

Therefore, the squeeze strain of the single particle chain is

$$\varepsilon = \frac{s - L}{s} = 1 - \frac{\sin \beta}{\beta}$$  \tag{24}

By incorporating a Taylor expansion approach and ignoring high-order small quantities, the equation (24) can be written as

\[\]
\[ \beta = \sqrt{6\varepsilon} \]  

(25)

So the angle \( \beta' \) can be derived as

\[ \beta' = \beta + \sqrt{6\Delta\varepsilon} = \beta + \sqrt{\frac{3\Delta L}{RN}} = \beta + \sqrt{\frac{3v_p}{RN}} \]  

(26)

Meanwhile, because the number of the particles is very large, the length between particle 1 and particle \( i \) can be approximated as

\[ L_1 \approx L'_1 \approx 2R(i - 1) \]  

(27)

Then, the X-component and Y-component of the velocities of particle \( i \) are respectively expressed as

\[ v_{xi} = \Delta L_x = L'_1 - L_2 \]  
\[ v_{yi} = \Delta L_y = L'_3 + \Delta L - L_3 \]  

(28)

Furthermore, the X-component and Y-component of the relative velocities between particle \( i \) and particle \((i + 1)\) can be respectively determined as

\[ v_{xi}(i+1) = v_{x(i+1)} - v_{xi} \]  
\[ v_{yi}(i+1) = v_{y(i+1)} - v_{yi} \]  

(29)

According to the equation (13), the relative velocity between particle \( i \) and particle \((i + 1)\) is derived as

\[ v_{pi(i+1)} = v_{xi(i+1)} \cos \alpha_{i(i+1)} - v_{yi(i+1)} \sin \alpha_{i(i+1)} \]

\[ \approx 2R \cos \left( \frac{(N - i - 1)\beta}{N - 1} \right) \left[ \sin \left( \frac{\beta + \sqrt{3v_p}}{RN} \right) i - \sin \frac{\beta}{N - 1} \right] \]

\[ + 2R \sin \left( \frac{(N - i - 1)\beta}{N - 1} \right) \left[ \cos \frac{\beta}{N - 1} - \cos \left( \frac{\beta + \sqrt{3v_p}}{RN} \right) i \right] \]  

(30)

where \( i = 1, 2, 3 \ldots \frac{N}{2} - 1 \).

3.4. Slip differential heat flow between two neighboring particles in squeeze mode

Substituting equations (19) into (3), the radius of the interface between the particle \( i \) and the particle \((i + 1)\) is written as
In order to simulate the characteristics of the particles in the MRFs and obtain the magnetic force, the electromagnetic field analysis which adopts the Maxwell equations in the module of Multiphysics in FE code ANSYS is carried out. It takes into account the effect of the nonlinear magnetization and can tackle more complex initial and boundary conditions, which should be more accurate. In this paper, a magnetic particles-air system was established (figure 9) and element Solid96 was adopted. Simulation parameters are presented in table 1. The magnetization curve of the particle is displayed in figure 10. And then we applied vertical boundary conditions of magnetic to the nodes on the upper and lower surfaces of the air body to obtain a uniform solution type was scalar RSP analysis of 3D static magnetic field, and kept the remaining boundaries the default parallel boundary conditions. Furthermore, the solution type was scalar RSP analysis of 3D static magnetic field.

Substituting equations (19), (30) and (31) into (8), the slip differential heat flow between the particle \( i \) and the particle \( i + 1 \) can be written as

\[
J_{p(i+1)} = \frac{2\mu_k \pi v_p(i+1) a_{p(i+1)}^2}{3} \sqrt{\frac{6E^2 \cdot 2F_{p(i+1)}}{\pi^2 R^2 (1 - \mu^2)^2}}
\]

\[
= \frac{2\mu_k \pi}{3} \left\{ \frac{12E^2}{\pi^2 R^2 (1 - \mu^2)^2} F_{mu} + \frac{2\mu_0 \chi^2 \mu_p^2}{\pi^2 (1 - \mu^2)^2} \left[ 3 \cos^2 \left( \frac{(N - i - 1)\beta}{N - 1} \right) \right] \right\} \frac{1}{2}
\]

\[
\times \left\{ \frac{3R(1 - \mu^2)}{2E} F_{mu} + \frac{\mu_0 \chi^2 \mu_p^2}{4E} \left[ 3 \cos^2 \left( \frac{(N - i - 1)\beta}{N - 1} \right) \right] \right\} \frac{1}{2}
\]

\[
\times \left\{ 2R \cos \left( \frac{(N - i - 1)\beta}{N - 1} \right) \sin \left( \frac{\beta + \sqrt{\beta^2 - 4\eta}}{2N} \right) i - \sin \left( \frac{\beta i}{N - 1} \right) \right\}
\]

\[
+ 2R \sin \left( \frac{(N - i - 1)\beta}{N - 1} \right) \cos \left( \frac{\beta i}{N - 1} - \cos \left( \frac{\beta + \sqrt{\beta^2 - 4\eta}}{2N} \right) i \right) \right\}
\]

where \( i = 1, 2, 3 \ldots \frac{N}{2} - 1 \).

4. FE simulation

4.1. Electromagnetic field analysis

In order to simulate the characteristics of the particles in the MRFs and obtain the magnetic force, the electromagnetic field analysis which adopts the Maxwell equations in the module of Multiphysics in FE code ANSYS is carried out. It takes into account the effect of the nonlinear magnetization and can tackle more complex initial and boundary conditions, which should be more accurate. In this paper, a magnetic particles-air system was established (figure 9) and element Solid96 was adopted. Simulation parameters are presented in table 1. The magnetization curve of the particle is displayed in figure 10. And then we applied vertical boundary conditions of magnetic to the nodes on the upper and lower surfaces of the air body to obtain a uniform magnetic field, and kept the remaining boundaries the default parallel boundary conditions. Furthermore, the solution type was scalar RSP analysis of 3D static magnetic field.

Figures 11(a) and (b) which were obtained from ANSYS codes reveal the computed vectorgraphs of the intensities of the magnetic field and the magnetic induction of two neighboring particles. It can be seen that, the intensity of the magnetic field inside the particles is almost uniform and far smaller than that in the left and right air body, but the intensity of the magnetic induction inside the particles is larger than that in the left and right air body. Also, the maximum of the intensity of the magnetic field and the magnetic induction is between two neighboring particles. The results and distributions agree with the literature data [19].

4.2. Contact analysis

In order to verify the maximum contact stress based on the Hertzian contact theory, the contact analysis was performed in the ANSYS code. Element Solid 185 was adopted and FE model of two neighboring particles was built (figure 12). We substituted the material properties of pure iron for the material properties of the particles, because the particles are ferromagnetic [16]. The material properties of pure iron are listed in table 2. There are some boundary conditions: (1) contact definition of the lower and upper hemispherical surfaces, (2) full constraint of the node displacements of the upper sphere in the x and z directions, (3) full constraint of the node displacements of the lower sphere, and (4) the application of magnetic force at the apex of the upper sphere.

The computed vectorgraphs of Y- component of displacement and stress are respectively shown in figures 13(a) and (b). It can be seen that, the Y-components of displacement and stress on interface are much
larger than that in the other area, and their maximums are all at the center of the interface. Also, the interface area is much smaller than the sphere area. Moreover, the maximum of the contact stress is \(1636.44 \text{ N m}^2\), \(-Y\) direction and the magnitude is \(1.6364 \times 10^6 \text{ Pa}\). At the same conditions, the maximum of

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Table 1. Simulation parameters.

| Parameter           | Value                   |
|---------------------|-------------------------|
| Length of air body  | \(2 \times 10^{-4} \text{ (m)}\) |
| Width of air body   | \(2 \times 10^{-4} \text{ (m)}\) |
| Height of air body  | \(6 \times 10^{-3} \text{ (m)}\) |
| Radius of the particle | \(5 \times 10^{-6} \text{ (m)}\) |
| Magnetic field      | \(5 \times 10^{5} \text{ (A m}^{-1})\) |
| Relative permeability of air body | 1 |
Figure 11. Vectorgraphs of magnetic field intensity and magnetic induction of two neighboring particles. (a) Vectorgraph of magnetic field intensity ($Am^{-1}$); (b) Vectorgraph of magnetic induction ($T$).

Figure 12. FE model of contact simulation.
the contact stress $q_0$ is about $1.7528 \times 10^9$ Pa based on the equation (5), and their relative error is only 7.11%. That meets the requirement basically.

5. Results and discussions

5.1. Slip differential heat flow between any two neighboring particles in shear mode

It is known that, when MRFs work in shear mode, they can be applied to a magnetorheological clutch or brake. Based to realistic conditions [29], the value of each influencing factor is converted. Given the magnetic field intensity $H = 5 \times 10^3$ A m$^{-1}$, the susceptibility $\chi = 3.3$, and the radius of the particle $R = 5 \times 10^{-6}$ m. If the working gap of the magnetorheological clutch or brake is 1 mm, i.e., the number of the particles in a single chain $N = 100$. If the radius and the relative velocity difference of a magnetorheological clutch or brake are 90 mm and 200 rad s$^{-1}$ respectively, the relative velocity between the upper and lower walls $v_r = 18$ ms$^{-1}$. The angle between the inclined chain and the applied magnetic $\theta_{ij}$ can be from $0^\circ$ to $45^\circ$, which can avoid the breaking of

Table 2. Material properties of pure iron.

| Parameter          | Value          |
|--------------------|----------------|
| Density            | 7860 (kg.m$^{-3}$) |
| Elastic modulus    | 80.65 (GPa)    |
| Poisson’s ratio    | 0.291          |

Figure 13. Vectorgraphs of displacement and stress of two neighboring particles. (a) Vectorgraph of Y-component of displacement; (b) Vectorgraph of Y-component of stress.
the chains. Therefore, the slip differential heat flow between two neighboring particles \( J_r \) is about from \( 2.44 \times 10^{-6} \) W to \( 6.1 \times 10^{-7} \) W, when \( \theta_{ij} \) varies from 0° to 45° (figure 14). It can be seen that \( J_r \) is the largest at \( \theta_{ij} = 0 \), it decreases with the increase of \( \theta_{ij} \). That is because the radial component of the magnetic force decreases.

5.1.1. Effect of the magnetic induction intensity \( B \)

It is known that the magnetic particles would be saturated as the magnetic field intensity increases. The intensity of magnetization can be expressed as \( M(H) = \chi(H)H \). Because the magnetization of particles is nonlinear and related to many influencing factors, the exact determination of the magnetic susceptibility \( \chi(H) \) is complicated. However, it can be approximately expressed as [11]

\[
\chi(H) = \frac{\chi_0 M_s}{\chi_0 H + M_s}
\]  

where \( M_s \) is the saturation intensity of magnetization and \( \chi_0 \) is the initial susceptibility when \( H \) tends to zero.

Substituting \( B = \mu_0(1 + \chi)H \) into equation (8) gives

\[
J_r = \frac{\pi \mu_0 \chi_0^2 B^2 R^2}{6(N-1)\mu_0 (1 + \chi)^2(3 \cos^2 \theta_{ij} - 1)}
\]  

For pure iron particles, it is known that \( \mu_0 M_s \approx 2.1 \) T, so that \( M_s \approx 1.67 \times 10^6 \) A m\(^{-1}\). Given \( \chi_0 = 1000 \) and \( \theta_{ij} = 22.5^\circ \), the variation of \( J_r \) is shown in figure 15(a). It can be seen that \( J_r \) is 0 at \( B = 0 \), it increases rapidly with the increase of \( B \), and then increases a little or even keeps constant after \( B \) approximately reaches 2.4 T. That is because the magnetic force initially increases with the increase of \( B \), but increases a little or even keeps constant after the magnetization is saturated. This tendency agrees qualitatively with the experimental result [16].

5.1.2. Effect of the radius of the particle \( R \)

When \( R \) is from \( 3 \times 10^{-6} \) m to \( 8 \times 10^{-6} \) m and \( \theta_{ij} \) is 22.5°, the variation of \( J_r \) is shown in figure 15(b). It can be seen that \( J_r \) is small at a small \( R \), it increases with the increase of \( R \), and the increasing speed of \( J_r \) is getting faster. It approximately agrees with the simulation result [19].

5.1.3. Effect of the relative velocity between the upper and lower walls \( v_r \)

When \( v_r \) is from 15 m s\(^{-1}\) to 25 m s\(^{-1}\) and \( \theta_{ij} \) is 22.5°, the variation of \( J_r \) is shown in figure 15(c). It can be seen that \( J_r \) is small at a small \( v_r \), it increases with the increase of \( v_r \), and \( J_r \) is proportional to \( v_r \). That is because the shear strain rate increases with the increase of \( v_r \), and this tendency agrees approximately with the research result [16].
5.1.4. Effect of the number of the particles in a single chain $N$

When $N$ is from 50 to 150 and $\theta_{ij}$ is 22.5°, the variation of $J_\tau$ is shown in figure 15 (d). It can be seen that $J_\tau$ is the largest at $N = 50$, it decreases with the increase of $N$, and the decreasing speed of $J_\tau$ is getting slower. That is because when $N$ increases, the relative velocity between any two neighboring particles decreases. This tendency agrees qualitatively with the experimental result [30], because $N$ is large at a large working gap.

5.2. Slip differential heat flow between two neighboring particles in squeeze mode

It is known that, when MRFs work in squeeze mode, they can be applied to a magnetorheological damper. Based to realistic conditions [31], the value of each influencing factor is converted. Similarly, given the magnetic field intensity $H = 5 \times 10^3$ Am$^{-1}$, the susceptibility $\chi = 3.3$, the radius of the particle $R = 5 \times 10^{-6}$ m and the velocity range of the magnetorheological damper $v_p$ is from $-20$ mm s$^{-1}$ to 20 mm s$^{-1}$. If the working gap of the magnetorheological damper is 50 mm, i.e., the number of the particles in a single chain $N = 5000$. Also, the wrap angle of a single chain $\beta$ can be from 0 to 70°, which can avoid the breaking of chains. Therefore, the slip differential heat flow $J_{\pi(i+1)}$ between two neighboring particles is from 0 to $3.3 \times 10^{-8}$ W, when $\beta$ is from 0 to 70°, $v_p = 20$ mm s$^{-1}$ and $i = 1000$ (figure 16(a)). It can be seen that $J_{\pi(i+1)}$ is 0 at $\beta$ tends to 0, it increases with the increase of $\beta$, and its increasing speed is getting slower. That is because with the increase of $\beta$, the interaction force between two neighboring particles $F_{\pi(i+1)}$ increases, but the relative velocity between two neighboring particles $v_{\pi(i+1)}$ decreases. Moreover, when $i$ is from 1 to 2499 and $\beta = 35^\circ$, $J_{\pi(i+1)}$ is shown in figure 16(b). It can be seen that $J_{\pi(i+1)}$ is small at $i = 1$, and it increases with the increase of $i$. In addition, is about proportional to $i$. That is mainly because with the increase of $i$, $v_{\pi(i+1)}$ increases and it conforms to this law of the variation of $J_{\pi(i+1)}$ versus $i$ during the bending deformation.

Figure 15. Variation of $J_\tau$ against influencing factors. (a) Variation of $J_\tau$ against $B$; (b) Variation of $J_\tau$ against $R$; (c) Variation of $J_\tau$ against $v_\tau$; (d) Variation of $J_\tau$ against $N$. 

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Also, it shows that the slip differential heat flow between two neighboring particles in shear mode is two orders of magnitude larger than that in squeeze mode under the realistic conditions. That is because the interaction force between two neighboring particles in shear mode is three orders of magnitude smaller than that in squeeze mode, but the relative velocity between two neighboring particles in shear mode is five orders of magnitude larger than that in squeeze mode.

5.2.1. Effect of the magnetic induction intensity $B$

Based on the approach in shear mode, we take into account the effect of saturation magnetization. Given $\beta = 35^\circ$, $i = 1000$ and $v_p = 20 \text{ mm s}^{-1}$, the variation of $I_{p(i+1)}$ is shown in figure 17(a). The variation is similar to that in shear mode. The tendency agrees approximately with the research result [28].

5.2.2. Effect of the radius of the particle $R$

When $R$ is from $3 \times 10^{-6}$ m to $8 \times 10^{-6}$ m, $\beta = 35^\circ$, $i = 1000$ and $v_p = 20 \text{ mm s}^{-1}$, the variation of $I_{p(i+1)}$ is shown in figure 17(b). The variation is also similar to that in shear mode. It also approximately agrees with the simulation result [19].

5.2.3. Effect of the squeeze velocity

When $v_p$ is from 0 to 40 mm s$^{-1}$, $\beta = 35^\circ$, and $i = 1000$, the variation of $I_{p(i+1)}$ is shown in figure 17(c). It can be seen that $I_{p(i+1)}$ is 0 at $v_p = 0$, it increases with the increase of $v_p$ and its increasing speed is getting slower. That is because $v_{p(i+1)}$ increases with the increase of $v_p$ and its increasing speed is getting slower according to the equation (30). The tendency agrees qualitatively with the experimental result [20].

5.2.4. Effect of the number of the particles in a single chain $N$

When $N$ is from 3000 to 5000, $\beta = 35^\circ$, $i = 1000$ and $v_p = 1 \text{ mm s}^{-1}$, the variation of $I_{p(i+1)}$ is shown in figure 17(d). The variation is similar to that in shear mode.

6. Summary

A theoretical model is formulated for slip differential heat between two neighboring particles of MRFs in shear and squeeze modes based on micromechanics and microstructures. The slip differential heat flow in different locations and between different two neighboring particles can be obtained. In addition, the proposed model takes into account the effects of the main influencing factors on the slip differential heat flow between two neighboring particles in a single chain of MRFs in shear and squeeze modes, such as the magnetic field intensity, the size of particles, the number of the particles in a single chain, the relative velocity between the upper and lower walls in shear mode and the squeeze velocity in squeeze mode. Furthermore, the individual effect of typical governing parameters on slip differential heat flow under the realistic conditions of the magnetorheological clutch and damper was investigated by the numerical simulation, and it shows that these factors have great influence on the heat flow.
influences on the slip differential heat flow. It is significance that the slip differential heat flow between two neighboring particles in shear mode is two orders of magnitude larger than that in squeeze mode under the realistic conditions. The proposed model can satisfactorily describe the main micro-characteristics of the slip differential heat of MRFs in shear and squeeze modes.

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