Realistic Type IIB Supersymmetric Minkowski Flux Models without the Freed-Witten Anomaly

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Abstract

We show that there exist supersymmetric Minkowski vacua on Type IIB toroidal orientifold with general flux compactifications where the RR tadpole cancellation conditions can be relaxed and the Freed-Witten anomaly can be cancelled elegantly. We present a realistic Pati-Salam like flux model without the Freed-Witten anomaly. At the string scale, we can break the gauge symmetry down to the Standard Model (SM) gauge symmetry, achieve the gauge coupling unification naturally, and decouple all the extra chiral exotic particles. Thus, we have the supersymmetric SMs with/without SM singlet(s) below the string scale. Also, we can explain the SM fermion masses and mixings. In addition, the unified gauge coupling and the real parts of the dilaton and Kähler moduli are functions of the four-dimensional dilaton. The complex structure moduli and one linear combination of the imaginary parts of the Kähler moduli can be determined as functions of the fluxes and the dilaton.

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I. INTRODUCTION

The great challenge in string phenomenology is the construction of realistic string models, which do not have additional chiral exotic particles at low energy and can stabilize the moduli fields. Employing renormalization group equations, we may test such models at the upcoming Large Hadron Collider (LHC). In particular, the intersecting D-brane models on Type II orientifolds \cite{1}, where the chiral fermions arise from the intersections of D-branes in the internal space \cite{2} and the T-dual description in terms of magnetized D-branes \cite{3}, have been very interesting during the last a few years \cite{4}.

In the beginning \cite{5}, a lot of non-supersymmetric three-family Standard-like models and Grand Unified Theories (GUTs) were constructed on Type IIA orientifolds with intersecting D6-branes. However, these models generically have uncanceled Neveu-Schwarz-Neveu-Schwarz (NSNS) tadpoles and the gauge hierarchy problem. Later, semi-realistic supersymmetric Standard-like models and GUT models have been constructed in Type IIA theory on the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold \cite{6,7,8} and other backgrounds \cite{9}. We emphasize that only Pati-Salam like models can realize all the Yukawa couplings at the stringy tree level. Moreover, Pati-Salam like models have been constructed systematically in Type IIA theory on the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold \cite{7,8}. Although the Standard Model (SM) fermion masses and mixings can be generated in one of these models \cite{10}, we can not stabilize the moduli fields and might not decouple all the chiral exotic particles. To stabilize the moduli via supergravity fluxes, the flux models on Type II orientifolds have also been constructed \cite{11,12,13,14,15,16,17,18,19,20}. Especially, for the supersymmetric AdS vacua on Type IIA orientifolds with flux compactifications, the Ramond-Ramond (RR) tadpole cancellation conditions can be relaxed \cite{18,20}. And then we can construct flux models that can explain the SM fermion masses and mixings \cite{20}. However, these models are in the AdS vacua and have quite a few chiral exotic particles that are difficult to be decoupled. Recently, on the Type IIB toroidal orientifold with the RR, NSNS, non-geometric and S-dual flux compactifications \cite{21,22,23}, we showed that the RR tadpole cancellation conditions can be relaxed elegantly in the supersymmetric Minkowski vacua \cite{24}. Unfortunately, the Freed-Witten anomaly \cite{25} can give strong constraints on model building \cite{24}, and the model in Ref. \cite{24} indeed has the Freed-Witten anomaly that might be cancelled by introducing additional D-branes \cite{11}.
In this paper, we revisit the Type IIB toroidal orientifold with the RR, NSNS, non-geometric and S-dual flux compactifications \cite{22}. We present supersymmetric Minkowski vacua where the RR tadpole cancellation conditions can be relaxed and the Freed-Witten anomaly free conditions can be satisfied elegantly. We construct a realistic Pati-Salam like flux model without the Freed-Witten anomaly. At the string scale, the gauge symmetry can be broken down to the SM gauge symmetry, the gauge coupling unification can be achieved naturally, and all the extra chiral exotic particles can be decoupled so that we obtain the supersymmetric SMs with/without SM singlet(s) below the string scale. The observed SM fermion masses and mixings can also be generated since all the SM fermions and Higgs fields arise from the intersections on the same two-torus. Moreover, the unified gauge coupling and the real parts of the dilaton and Kähler moduli are functions of the four-dimensional dilaton. And the complex structure moduli and one linear combination of the imaginary parts of the Kähler moduli can be determined as functions of the fluxes and the dilaton. The systematical model building and the detailed phenomenological discussions will be given elsewhere \cite{26}.

This paper is organized as follows: in Section II we review the Type IIB model building and study the supersymmetric Minkowski flux vacua. We construct a realistic Pati-Salam like flux model and discuss its phenomenological consequences in Section III. Discussion and conclusions are presented in Section IV.

II. TYPE IIB FLUX MODEL BUILDING

Let us consider the Type IIB string theory compactified on a $T^6$ orientifold where $T^6$ is a six-torus factorized as $T^6 = T^2 \times T^2 \times T^2$ whose complex coordinates are $z_i$, $i = 1, 2, 3$ for the $i^{th}$ two-torus, respectively. The orientifold projection is implemented by gauging the symmetry $\Omega R$, where $\Omega$ is world-sheet parity, and $R$ is given by

$$R : (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, -z_3).$$

(1)

Thus, the model contains 64 O3-planes. In order to cancel the negative RR charges from these O3-planes, we introduce the magnetized D$(3+2n)$-branes which are filling up the four-dimensional Minkowski space-time and wrapping $2n$-cycles on the compact manifold. Concretely, for one stack of $N_a$ D-branes wrapped $m^i_a$ times on the $i^{th}$ two-torus $T^2_i$, we turn on
$n_a^i$ units of magnetic fluxes $F_a^i$ for the center of mass $U(1)_a$ gauge factor on $T_2^i$, such that

$$m_a^i \frac{1}{2\pi} \int_{T_2^i} F_a^i = n_a^i,$$  \hspace{0.5cm} \text{(2)}$$

where $m_a^i$ can be half integer for tilted two-torus. Then, the D9-, D7-, D5- and D3-branes contain 0, 1, 2 and 3 vanishing $m_a^i$s, respectively. Introducing for the $i^{th}$ two-torus the even homology classes $[0_i]$ and $[T_2^i]$ for the point and two-torus, respectively, the vectors of the RR charges of the $a^{th}$ stack of D-branes and its image are

$$[\Pi_a] = \prod_{i=1}^3 (n_a^i[0_i] + m_a^i[T_2^i]),$$

$$[\Pi'_a] = \prod_{i=1}^3 (n_a^i[0_i] - m_a^i[T_2^i]),$$

respectively. The “intersection numbers” in Type IIA language, which determine the chiral massless spectrum, are

$$I_{ab} = [\Pi_a] \cdot [\Pi_b] = \prod_{i=1}^3 (n_a^i m_b^i - n_b^i m_a^i).$$  \hspace{0.5cm} \text{(4)}$$

Moreover, for a stack of $N$ D$(2n+3)$-branes whose homology class on $T^6$ is (not) invariant under $\Omega R$, we obtain a $(U(N)) USp(2N)$ gauge symmetry with three (adjoint) anti-symmetric chiral superfields due to the orbifold projection. The physical spectrum is presented in Table II.

The flux models on Type IIB orientifolds with four-dimensional $N = 1$ supersymmetry are primarily constrained by the RR tadpole cancellation conditions that will be given later, the four-dimensional $N = 1$ supersymmetric D-brane configurations, and the K-theory anomaly free conditions. For D-branes with world-volume magnetic field $F_a^i = n_a^i/(m_a^i \chi_i)$ where $\chi_i$ is the area of the $i^{th}$ two-torus $T_2^i$ in string units, the condition for the four-dimensional $N = 1$ supersymmetric D-brane configurations is

$$\sum_i (\tan^{-1}(F_a^i)^{-1} + \theta(n_a^i)\pi) = 0 \mod 2\pi,$$  \hspace{0.5cm} \text{(5)}$$

where $\theta(n_a^i) = 1$ for $n_a^i < 0$ and $\theta(n_a^i) = 0$ for $n_a^i \geq 0$. The K-theory anomaly free conditions are

$$\sum_a N_a m_a^1 m_a^2 m_a^3 = \sum_a N_a m_a^1 n_a^2 n_a^3 = \sum_a N_a n_a^1 m_a^2 n_a^3 = \sum_a N_a n_a^1 n_a^2 m_a^3 = 0 \mod 2.$$

(6)
TABLE I: General spectrum for magnetized D-branes on the Type IIB $T^6$ orientifold.

| Sector     | Representation                                      |
|------------|-----------------------------------------------------|
| $aa$       | $U(N_a)$ vector multiplet                            |
|            | 3 adjoint multiplets                                   |
| $ab + ba$  | $I_{ab} (N_a, N_b)$ multiplets                       |
| $ab' + b'a$| $I_{ab'} (N_a, N_b)$ multiplets                      |
| $aa' + a'a$| $\frac{1}{2}(I_{aa'} - I_{aO3})$ symmetric multiplets|
|            | $\frac{1}{2}(I_{aa'} + I_{aO3})$ anti-symmetric multiplets|

We turn on the NSNS fluxes $h_0$ and $a_i$, the RR fluxes $e_i$ and $q_i$, the non-geometric fluxes $b_{ii}$, and the S-dual fluxes $f_i$ [21, 22, 23, 26]. To avoid subtleties, these fluxes should be even integers due to the Dirac quantization. For simplicity, we assume

$$a_i \equiv a , \quad e_i \equiv e , \quad q_i \equiv q , \quad b_{ii} \equiv \beta_i . \quad (7)$$

We can show that the constraints on fluxes from the Bianchi identities are satisfied. The constraints on fluxes from the $SL(2, \mathbb{Z})$ S-duality invariance give

$$a_\beta i = qf_i . \quad (8)$$

The RR tadpole cancellation conditions are

$$\sum_a N_a n_a^1 n_a^2 n_a^3 = 16 - \frac{3}{2}aq ,$$
$$\sum_a N_a n_a^i m_a^j m_a^k = \frac{1}{2}q\beta_i ,$$
$$N_{NS7i} = 0 , \quad N_{I7i} = 0 , \quad (9)$$

where $i \neq j \neq k \neq i$, and the $N_{NS7i}$ and $N_{I7i}$ denote the NS 7-brane charge and the other 7-brane charge, respectively [22, 26]. Thus, if $aq < 0$ and $q\beta_i < 0$, the RR tadpole cancellation conditions are relaxed elegantly because $-aq/2$ and $-q\beta_i/2$ only need to be even integers. Moreover, we have seven moduli fields in the supergravity theory basis: the dilaton $s$, three
Kähler moduli $t_i$, and three complex structure moduli $u_i$. With the above fluxes, we can assume

$$u_1 = u_2 = u_3 \equiv u . \quad (10)$$

Then the superpotential becomes

$$W = 3ieu - 3qu^2 + s(ih_0 - 3au) - \beta_i t_i u - f_i st_i . \quad (11)$$

In addition, the holomorphic gauge kinetic function for a generic stack of D(2n+3)-branes is given by \[26, 27, 28\]

$$f_a = \frac{1}{\kappa_a} \left( n_a^1 n_a^2 n_a^3 s - n_a^1 m_a^2 m_a^3 t_1 - n_a^2 m_a^1 m_a^3 t_2 - n_a^3 m_a^1 m_a^2 t_3 \right) , \quad (12)$$

where $\kappa_a$ is equal to 1 and 2 for $U(n)$ and $USp(2n)$, respectively. And the Kähler potential for these moduli is of the usual no-scale form \[29\]

$$K = -\ln(s + \bar{s}) - \sum_{i=1}^{3} \ln(t_i + \bar{t}_i) - \sum_{i=1}^{3} \ln(u_i + \bar{u}_i) . \quad (13)$$

For the supersymmetric Minkowski vacua, we have

$$W = \partial_s W = \partial_t W = \partial_u W = 0 . \quad (14)$$

From $\partial_s W = \partial_t W = \partial_u W = 0$, we obtain

$$f_i t_i = ih_0 - 3au , \quad s = -\frac{q}{a} u , \quad (15)$$

then the superpotential turns out to be

$$W = \left( 3e - \frac{qh_0}{a} \right) iu . \quad (16)$$

Therefore, to satisfy $W = \partial_u W = 0$, we obtain

$$3ea = qh_0 . \quad (17)$$

Because $\text{Re}s > 0$, $\text{Re}t_i > 0$ and $\text{Re}u_i > 0$, we require

$$\frac{f_i \text{Re}t_i}{a} < 0 , \quad \frac{q}{a} < 0 . \quad (18)$$
In short, in our constructions, we have fixed a linear combination of the Kähler moduli $t_i$ and the complex structure moduli $u$ as follows from Eq. (15)

$$f_i \text{Re} t_i = \frac{3a^2}{q} \text{Re} s , \quad \text{Re} u = -\frac{a}{q} \text{Re} s ,$$

$$f_i \text{Im} t_i = h_0 + \frac{3a^2}{q} \text{Im} s , \quad \text{Im} u = -\frac{a}{q} \text{Im} s . \quad (19)$$

In general, this kind of D-brane models might have the Freed-Witten anomaly [11, 18, 25]. In the world-volume of a generic stack of D-branes we have a $U(1)$ gauge field whose scalar partner parametrizes the D-brane position in compact space. These $U(1)$'s usually obtain Stückelberg masses by swallowing RR scalar fields and then decouple from the low-energy spectra. In the mean time these scalars participate in the cancellation of $U(1)$ gauge anomalies through a generalized Green-Schwarz mechanism [30]. For the generic stack of D-branes, the $U(1)_a$ gauge field couples to the RR fields in four dimensions as follows

$$F^a \wedge N_a \sum_{I=0}^3 c^a_I C^{(2)}_I , \quad (20)$$

where $I = 0, 1, 2, 3,$ and

$$c^a_0 \equiv m_a^1 m_a^2 n_a^3 ; \quad c^a_1 \equiv m_a^1 n_a^2 n_a^3 ; \quad c^a_2 \equiv n_a^1 m_a^2 n_a^3 ; \quad c^a_3 \equiv n_a^1 n_a^2 m_a^3 , \quad (21)$$

where $C^{(2)}_0$ and $C^{(2)}_i$ are two-form fields that are Poincare duals to the Im$s$ and Im$t_i$ fields in four dimensions, respectively. In terms of them the couplings have a Higgs-like form

$$A^a_\mu \partial^\mu (c^a_0 \text{Im} s - c^a_1 \text{Im} t_1 - c^a_2 \text{Im} t_2 - c^a_3 \text{Im} t_3) . \quad (22)$$

Thus, certain linear combinations of the imaginary parts of the $s$ and $t_i$ fields obtain masses by combining with open string vector bosons living on the branes. In addition, these linear combinations of Im$s$ and Im$t_i$ fields will transform with a shift under $U(1)_a$ gauge transformations, like Goldstone bosons do. If the shift symmetry for any D-brane stack is violated by the flux potential, we shall have the Freed-Witten anomaly [25]. From the superpotential in Eq. (11), we obtain that the flux potential may fix Im$s$ and one linear combination of Im$t_i$. Thus, we derive the Freed-Witten anomaly free conditions

$$c^a_0 = 0 , \quad f_1 c^a_1 + f_2 c^a_2 + f_3 c^a_3 = 0 . \quad (23)$$

Or equivalently, we have

$$c^a_0 = 0 , \quad \beta_1 c^a_1 + \beta_2 c^a_2 + \beta_3 c^a_3 = 0 . \quad (24)$$
III. A REALISTIC MODEL

In this Section, we shall present a realistic model. We choose the following fluxes

\[ a = 8, \quad q = -2, \quad \beta_1 = 2, \quad \beta_2 = 6, \quad \beta_3 = 6, \]
\[ f_1 = -8, \quad f_2 = -24, \quad f_3 = -24, \quad h_0 = -12e, \]

where the flux \( e \) is not fixed. We present the D-brane configurations and intersection numbers in Table II and the resulting spectrum in Tables III and IV. One can easily check that our model satisfies the Freed-Witten anomaly free conditions in Eq. (23) or Eq. (24).

| Stack | \( N \) | \( (n_1, l_1) \) | \( (n_2, l_2) \) | \( (n_3, l_3) \) | \( A \) | \( S \) | \( b \) | \( c \) | \( d \) | \( d' \) | \( e \) | \( e' \) | \( f \) | \( g \) |
|-------|-------|-----------------|-----------------|-----------------|------|------|------|------|------|------|------|------|------|------|
| \( a \) | 4    | (2, 0) (1, -1) | (1, 1)          | 0(-1)           | 0    | 3-3  | 0(-3)| -2   | 1    | 2    | -2   | -2   | -2   | -2   |
| \( b \) | 2    | (1, -3) (1, 1) | (2, 0)          | 0(-3)           | 0    | -3   | 0(1) | 2    | -1   | 0    | 0    | 0    | 2    |
| \( c \) | 2    | (1, 3) (2, 0)  | (1, -1)         | 0(-3)           | 0    | -    | -    | 0    | 0    | -2   | 1    | 2    | 0    |
| \( d \) | 2    | (1, 1) (2, 0)  | (3, -1)         | 0(-2)           | 0    | -    | -    | -    | -1   | 0    | 2    | 0    | 0    |
| \( e \) | 2    | (1, -1) (3, 1) | (2, 0)          | 0(-2)           | 0(-1)| -    | -    | -    | -    | -    | -    | -    | -    |
| \( f \) | 1    | (0, 2) (0, -2) | (2, 0)          | -               | -    | -    | -    | -    | -    | -    | -    | -    | 0(-4) |
| \( g \) | 1    | (0, 2) (2, 0)  | (0, -2)         | -               | -    | -    | -    | 3\( \chi_1 = \chi_2 = \chi_3 \) |

TABLE II: D-brane configurations and intersection numbers where \( l_i \equiv 2m_i \). The complete gauge symmetry is \([U(4)_C \times U(2)_L \times U(2)_R]_{\text{Observable}} \times [U(2)^2 \times USp(2)^2]_{\text{Hidden}}, \) and the SM fermions and Higgs fields arise from the intersections on the first two-torus.

In our model, the anomalies from the global \( U(1) \)s of the \( U(4)_C, U(2)_L, U(2)_R, U(2)_d \) and \( U(2)_e \) gauge symmetries are cancelled by the generalized Green-Schwarz mechanism, and the gauge fields of the corresponding anomalous \( U(1) \)s obtain masses via the linear \( B \wedge F \) couplings. In addition, we can break the global \( U(1)_a, U(1)_L, U(1)_R, U(1)_d \) and \( U(1)_e \) gauge symmetries respectively of \( U(4)_C, U(2)_L, U(2)_R, U(2)_d \) and \( U(2)_e \) by giving the string-scale vacuum expectation values (VEVs) to \( S_a, \overline{S}_a, S_i^L, \overline{S}_L, S_i^R, \overline{S}_R, X_{de}, T_d, T_{d'}, S_{d}, \overline{S}_d, T_e, \overline{T}_e, S_i^e, \) and \( \overline{S}_e \). Without loss of generality, we can assume that their VEVs satisfy the D-flatness conditions for the global \( U(1)_a, U(1)_L, U(1)_R, U(1)_d \) and \( U(1)_e \) gauge symmetries. Thus, the effective gauge symmetry in the observable sector is indeed \( SU(4)_C \times SU(2)_L \times SU(2)_R \). In order to break the gauge symmetry down to \( SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L} \), on the first two-torus, we split the \( a \) stack of D-branes into \( a_1 \) and \( a_2 \) stacks with 3 and 1 D-branes, respectively, and split the \( c \) stack of D-branes into \( c_1 \) and \( c_2 \) stacks with 1 D-brane for each
one. We can break the $U(1)_{I_{LR}} \times U(1)_{B-L}$ gauge symmetry further down to the $U(1)_{Y}$ gauge symmetry by giving the string-scale VEVs to the vector-like particles with quantum numbers $(1,1,1/2,-1)$ and $(1,1,-1/2,1)$ under $SU(3)_{C} \times SU(2)_{L} \times U(1)_{I_{LR}} \times U(1)_{B-L}$ from $a_{2c}^{i}$ D-brane intersections. Similar to the discussions in Ref. [10], we can explain the SM fermion masses and mixings via the Higgs fields $H^{i}_{u}$, $H^{l}_{u}$, $H^{i}_{d}$ and $H^{l}_{d}$ because all the SM fermions and Higgs fields arise from the intersections on the first two-torus. Note that we give the string-scale VEVs to the fields $S^{i}_{L}, \overline{S}^{i}_{L}, S^{i}_{R}, \overline{S}^{i}_{R}, X_{de}, T_{d}, \overline{T}_{d}, S^{i}_{d}, \overline{S}^{i}_{d}, T_{e}, \overline{T}_{e}, S^{i}_{e}$, and

| Quantum Number | $Q_{1}$ | $Q_{2L}$ | $Q_{2R}$ | Field |
|----------------|--------|---------|---------|-------|
| $ab$           | $3 \times (4,2,1,1,1,1)$ | 1      | -1      | 0     | $F_{L}(Q_{L}, L_{L})$ |
| $ac$           | $3 \times (4,1,2,1,1,1)$ | -1     | 0       | 1     | $F_{R}(Q_{R}, L_{R})$ |
| $bc$           | $3 \times (1,2,2,1,1,1)$ | 0      | 1       | -1    | $\Phi_{1}(H^{u}_{u}, H^{d}_{u})$ |
| $ae'$          | $3 \times (4,1,2,1,1,1)$ | 1      | 0       | 1     | $X^{i}_{ac'}$ |
| $ac'$          | $3 \times (4,1,2,1,1,1)$ | -1     | 0       | -1    | $\overline{X}^{i}_{ac'}$ |
| $bd'$          | $1 \times (1,2,1,2,1,1)$ | 0      | 1       | 0     | $\Phi'(H^{u}_{u}, H^{d}_{u})$ |
| $bd$           | $1 \times (1,2,1,2,1,1)$ | 0      | -1      | 0     | $\overline{\Phi}'$ |
| $ae'           | $1 \times (1,1,2,1,1,1)$ | 2      | 0       | 0     | $S_{a}$ |
| $af$           | $2 \times (4,1,1,1,1,2)$ | 1      | 0       | 0     | $X^{i}_{af}$ |
| $ag$           | $2 \times (1,1,1,1,1,2)$ | -1     | 0       | 0     | $\overline{X}^{i}_{ag}$ |
| $bb'$          | $3 \times (1,1,1,1,1,1)$ | 0      | 2       | 0     | $S^{i}_{L}$ |
| $bc$           | $3 \times (1,1,1,1,1,1)$ | 0      | -2      | 0     | $\overline{S}^{i}_{L}$ |
| $bd$           | $2 \times (1,2,1,2,1,1)$ | 0      | 1       | 0     | $X^{i}_{bd}$ |
| $bd'$          | $2 \times (1,2,1,2,1,1)$ | 0      | -1      | 0     | $\overline{X}^{i}_{bd'}$ |
| $ce'$          | $3 \times (1,1,1,1,1,1)$ | 0      | 0       | 2     | $S^{i}_{R}$ |
| $ce$           | $3 \times (1,1,1,1,1,1)$ | 0      | 0       | -2    | $\overline{S}^{i}_{R}$ |
| $ce'$          | $2 \times (1,1,2,1,2,1)$ | 0      | 0       | -1    | $\overline{X}^{i}_{ce}$ |
| $cf$           | $2 \times (1,1,2,1,2,1)$ | 0      | -1      | 0     | $\overline{X}^{i}_{cf}$ |

TABLE III: The chiral and vector-like superfields in the observable sector, and their quantum numbers under the gauge symmetry $U(4)_{C} \times U(2)_{L} \times U(2)_{R} \times U(2)^{2} \times USp(2)^{2}$. 
In short, below the string scale, we have the supersymmetric SMs which may have zero, one or a few SM singlets from $S_L^i$, $\overline{S}_L^i$, $S_R^i$, and $\overline{S}_R^i$. Then the upper bound on the lightest

| Field | Quantum Number | $Q_4$ | $Q_{2L}$ | $Q_{2R}$ |
|-------|----------------|-------|-----------|-----------|
| $T_d$ | $1 \times (1,1,1,3,1,1,1)$ | 0 | 0 | 0 |
| $\overline{T}_d$ | $1 \times (1,1,1,\overline{3},1,1,1)$ | 0 | 0 | 0 |
| $S_d^i$ | $2 \times (1,1,1,1,1,1,1,1)$ | 0 | 0 | 0 |
| $\overline{S}_d^i$ | $2 \times (1,1,1,\overline{1},1,1,1,1)$ | 0 | 0 | 0 |
| $X_{de}$ | $1 \times (1,1,1,2,1,1)$ | 0 | 0 | 0 |
| $X_{df}$ | $2 \times (1,1,1,\overline{2},1,1,1)$ | 0 | 0 | 0 |
| $X_{ee^i}$ | $1 \times (1,1,1,1,3,1,1)$ | 0 | 0 | 0 |
| $X_{ea^i}$ | $1 \times (1,1,1,1,\overline{3},1,1)$ | 0 | 0 | 0 |
| $X_{ee^k}$ | $2 \times (1,1,1,1,1,1,1,1)$ | 0 | 0 | 0 |
| $X_{ea^k}$ | $2 \times (1,1,1,1,\overline{1},1,1,1)$ | 0 | 0 | 0 |

$S^i_c$, the chiral exotic particles can obtain masses around the string scale via the following superpotential from three-point and four-point functions

$$W \supset (T_d + \overline{S}_d^i)X^i_{ad}X_{ad'} + (T_e + S^i_e)X^i_{ae}X_{ae'} + X_{de}(X^i_{ad}X^j_{ae} + X^i_{ad'}X^j_{ae'})$$

$$+ (X^i_{fg} + \overline{X}^i_{fg})X^j_{af}X^k_{ag} + (T_d + S^i_d)X^i_{bd}X^j_{bd'} + \overline{S}_d^iX^j_{bg}X_{bk}$$

$$+ (T_e + \overline{S}_e^i)X^k_{ce}X_{ce'} + S^i_RX^i_{cf}X^k_{cf'} + (T_d + S^i_d)X^j_{df}X^k_{df'}$$

$$+ (T_e + \overline{S}_e^i)X^k_{eg}X^i_{eg} + \frac{1}{M_{St}}(S^i_L(T_d + S^i_d)X^k_{bd}X^l_{bd})$$

$$+ S^i_R(T_e + \overline{S}_e^i)X^k_{ce}X^i_{ce'} + (T_d + S^i_d)(T_e + \overline{S}_e^i)X^k_{de}X^i_{de}$$

where $M_{St}$ is the string scale, and we neglect the $O(1)$ coefficients in this paper. In addition, we can decouple all the Higgs bidoublets close to the string scale except one pair of the linear combinations of the Higgs doublets for the electroweak symmetry breaking at the low energy by fine-tuning the following superpotential

$$W \supset \Phi_i(\overline{S}_L^i\Phi' + S_R^i\overline{\Phi'}) + \frac{1}{M_{St}}(\overline{S}_L^iS_R^i\Phi_k\Phi_l + S_L^i\overline{S}_L^i\Phi'\Phi' + S_R^iS_R^i\Phi'\overline{\Phi'})$$

In short, below the string scale, we have the supersymmetric SMs which may have zero, one or a few SM singlets from $S^i_L$, $\overline{S}_L^i$, $S^i_R$, and $\overline{S}_R^i$. Then the upper bound on the lightest
CP-even Higgs boson mass in the minimal supersymmetric SM can be relaxed if we have the SM singlet(s) at low energy [31].

Next, we consider the gauge coupling unification and moduli stabilization. Note that $3\chi_1 = \chi_2 = \chi_3$ as given in Table I are derived from the supersymmetric D-brane configurations, we define

$$
\chi_1 \equiv \chi, \chi_2 = \chi_3 \equiv 3\chi . \tag{28}
$$

Thus, the real parts of the dilaton and Kähler moduli in our model are [26]

\begin{align*}
\text{Res} &= \frac{e^{-\phi_4}}{6\pi\chi\sqrt{\chi}}, \quad \text{Ret}_1 = \frac{3\sqrt{\chi}e^{-\phi_4}}{2\pi}, \\
\text{Ret}_2 &= \frac{\sqrt{\chi}e^{-\phi_4}}{2\pi}, \quad \text{Ret}_3 = \frac{\sqrt{\chi}e^{-\phi_4}}{2\pi}, \tag{29}
\end{align*}

where $\phi_4$ is the four-dimensional dilaton. From Eq. (12), we obtain that the SM gauge couplings are unified at the string scale as follows

\begin{align*}
g_{SU(3)c}^{-2} = g_{SU(2)L}^{-2} = \frac{3}{5}g_{U(1)_Y}^{-2} &= \frac{e^{-\phi_4}}{2\pi} \left( \frac{2}{3\sqrt{\chi}} + \frac{3\sqrt{\chi}}{2} \right). \tag{30}
\end{align*}

From the real part of the first equation in Eq. (15) or the first equation in Eq. (19), we obtain

$$
\chi = \frac{2}{3}. \tag{31}
$$

Therefore, $\chi_i$ are determined by the supersymmetric D-brane configurations and the conditions for the supersymmetric Minkowski vacua. Using the unified gauge coupling $g^2 \simeq 0.513$ in supersymmetric SMs, we get

$$
\phi_4 \simeq -1.61 . \tag{32}
$$

From Eq. (19), we obtain

\begin{align*}
\text{Re}u &= 4\text{Re} , \quad \text{Im}u = 4\text{Im} , \\
8\text{Im}t_1 + 24\text{Im}t_2 + 24\text{Im}t_3 &= -h_0 + 96\text{Im} . \tag{33}
\end{align*}

IV. DISCUSSION AND CONCLUSIONS

We showed that the RR tadpole cancellation conditions can be relaxed and the Freed-Witten anomaly can be cancelled elegantly in the supersymmetric Minkowski vacua on
the Type IIB toroidal orientifold with general flux compactifications. And we presented a realistic Pati-Salam like flux model in details. In this model, we can break the gauge symmetry down to the SM gauge symmetry, realize the gauge coupling unification, and decouple all the extra chiral exotic particles around the string scale. We can also generate the observed SM fermion masses and mixings. Furthermore, the unified gauge coupling and the real parts of the dilaton and Kähler moduli are functions of the four-dimensional dilaton. And the complex structure moduli and one linear combination of the imaginary parts of the Kähler moduli can be determined as functions of the fluxes and the dilaton.

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