Performances of a Compact Shielded Superconducting Magnet for Continuous Nuclear Demagnetization Refrigerator

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Abstract

We have successfully developed and tested a compact shielded superconducting (SSC) magnet with a FeCoV magnetic shield. This was developed for the PrNi5-based nuclear demagnetization refrigerator which can keep temperatures below 1 mK continuously (CNDR) (Toda et al. in J Phys Conf Ser 969:012093, 2018). The clear bore diameter, outer diameter, and total length of the SSC magnet are 22, 42, and 169 mm, respectively, and it produces the maximum field of 1.38 T at an electric current of 6 A. In order to realize both the compactness and the high shielding performance, we carefully chose material and optimized design of the magnetic shield by numerical simulations of the field distribution based on typical magnetization curves of several candidate materials with high permeability. We also measured the heat generated by sweeping the SSC magnet in vacuum to be 248 mJ per field cycle. This value agrees very well with an estimation from the measured magnetic hysteresis of the superconducting wire used to wind the magnet.

Keywords Superconducting magnet · Nuclear demagnetization refrigerator · Magnetic shield · Magnetic hysteresis

1 Introduction

The sub-mK temperature environment is important for studies of unique quantum phases and phase transitions in liquid and solid 3He and other condensed matters. It is also useful in even broader fields, for example, those in which higher sensitivities
in detection of X-ray or microwave are demanded, because thermal noises and heat capacities are extremely small at such temperatures. In order to realize much easier access to sub-mK temperatures for non-experts, we are developing a compact and continuous nuclear demagnetization refrigerator (CNDR) [1] using PrNi$_5$, a hyperfine enhanced nuclear magnet [2], as a coolant. CNDR has two independent PrNi$_5$ refrigeration units which are connected in series between the sample stage and the mixing chamber of dilution refrigerator through two superconducting heat switches. It can keep a constant temperature below 1 mK continuously with a cooling power larger than 10 nW.

To take full advantage of the design concept of CNDR, it is crucial to develop a compact, low heat dissipation, and high-performance shielded superconducting (SSC) magnet. Specifications required for the SSC magnet of our CNDR are the followings: (i) maximum magnetic field produced by a current of $I = 6$ A: $B_{\text{max}} \geq 1.2$ T, (ii) clear bore diameter: $D_{\text{bore}} = 22$ mm, outer diameter: $D_{\text{od}} = 42$ mm, and total length: $L = 169$ mm, (iii) fringe fields at positions 50 mm away from the shield surfaces: $B \leq 1$ mT, and (iv) heat generation rate when the sweep rate is 1 mT/s: $\dot{Q} \leq 1$ mW. It is noted that these requirements are rather arbitrary depending on practical constraints in each experiment such as the performance of superconducting wire and cooling powers of precooling stages. Some of them in our CNDR have been discussed elsewhere [1]. The requirement (iv) is closely related to the radiation heat from the magnet bobbin as discussed later. In this article, we describe design details and test results of the SSC magnet which meets all the requirements listed above. This magnet can be used not only for CNDRs in laboratory but also for other purposes where the size and weight of the magnet are severely restricted.

2 Design of the SSC Magnet

To reduce $\dot{Q}$, it is essential to wind the solenoid with a superconducting (SC) wire consisting of smaller diameter filaments (see later discussions). Also, to reduce the operation current, it is necessary to increase the coil constant by using a smaller overall diameter wire. Among commercial SC wires, we chose the 0.14-mm-diameter multifilamentary (54 filaments) NbTi wire with Cu clad (Cu:NbTi = 1.3:1) of Supercon, Inc (product No. 54S43). It was wound on a copper bobbin up to 26 complete layers of 140 mm length. The total length of the wire used is 2.1 km. The inner and outer diameters of the winding part are 24 and 31 mm, respectively.

To design the magnetic shield, we conducted numerical simulations of field distributions produced by solenoids wound by the above mentioned SC wire surrounded by two different types of cylindrical shields with the fixed dimensions ($D_{\text{bore}}$, $D_{\text{od}}$, and $L$) using the open source software of the finite element method (FEMM: Finite Element Method Magnetics by David Meeker). The results indicate that, when it is shielded by a cylinder made of a high permeability ($\mu$) and high saturation magnetization ($M_{\text{sat}}$) material [3], the coil constant can be twice as large as when shielded by an active shield coil [4, 5]. We thus chose the former shielding type with two end caps as shown in Fig. 1a. Dimensions of the SSC magnet are listed in Table 1. Note that the upper end cap has a 20-mm-diameter
open hole through which thermal links for the PrNi₅ rods extend, which degrades the shielding performance considerably.

Next, in order to select a material with suitably high $\mu$ and to optimize the shield thickness, we made numerical simulations of field distributions of SSC magnets with shields made of four candidate materials, i.e., Fe, FeSi, $\mu$-metal, and FeCoV, of various thicknesses. In the simulations, we used typical $\mu$ and $M_{\text{sat}}$ values preinstalled in FEMM. Figure 1b shows simulation results for the axial ($z$) and radial ($r$) distances from the magnet center, beyond which the fringe field $|B|$ exceeds 0.1 mT, are shown by the open (closed) symbols and plotted as a function of shield thickness. The results are obtained by numerical simulations for four different shield materials, i.e., Fe, FeSi, $\mu$-metal, and FeCoV. The dashed and dotted lines represent locations of the top and side surfaces of the shield. Larger deviations from the lines for the simulation data mean higher fringe fields or incomplete shielding. For all the materials, there is a critical thickness ($t_c$) below which such a deviation rapidly increases because of saturation of $M$ in the shield material. To reduce the weight of the magnet, it is better to use materials with lower $t_c$. So we decided to shield the solenoid by a $t = 4$ mm thick FeCoV cylinder (Fe-49 at.%, Co-49 at.%, V-2 at.%; Tohoku Steel Co., Ltd.), which meets all the requirements for our purpose. The total weight of the SSC magnet we made is 1.2 kg. Note that $t_c < 4$ mm for fringe fields at positions just outside of the closed bottom cap.

Table 1  Dimensions of the SSC magnet

|          | L1  | L2  | L3  | D1  | D2  | D3  | D4  |
|----------|-----|-----|-----|-----|-----|-----|-----|
| Length or diameter (mm) | 140 | 161 | 169 | 20  | 22  | 34  | 42  |

The definition of each dimension is given in Fig. 1a
3 Performances

3.1 Magnetic Field Profiles

We immersed the constructed SSC magnet in liquid $^4$He ($T = 4.2$ K) and measured its fringe fields with a Hall probe (OH002-2HR; Matsushita Electronics Industry Co., Ltd.). Figure 2a, b shows the measured on-axis and radial field profiles, respectively. The red closed and blue open circles represent the data with and without the FeCoV shield, respectively. The $B_{\text{max}}$ value with shield ($= 1.38$ T when $I = 6$ A) is 2% higher than that without shield, while it is reduced by a factor of two in the case of the active shield type (the dash-dot line in Fig. 2a). At least at $B \geq 1$ mT, the data agree well with the numerical simulations indicated by the solid (with shield) and the dashed (without shield) lines for both the axial and radial directions. At $z \geq 125$ mm (or $B \leq 1$ mT), the data are consistently larger than the simulations. This is presumably due to the stray field which was accidentally produced by an extra winding of the lead wire. The positions, beyond which the fringe field exceeds 1 mT, are 36 mm above the upper shield cap along the magnet center line and much closer than a few mm from the shield sidewall. These results meet the requirement (iii).

3.2 Estimation of Heat Generation from Magnetic Hysteresis

Magnetic hysteresis in the NbTi wire and the FeCoV shield is the main source of heat generation in the SSC magnet during sweep up and down. The hysteresis originates from trapping of fluxoids in type II superconductors and from frictional

![Fig. 2](image-url)
motions of magnetic domain walls in ferromagnetic materials. We measured the magnetization curve of a 6.5 mm long piece of the NbTi wire used in the present SSC magnet at $T = 2$ K in vacuum using magnetic properties measurement system (MPMS; Quantum Design, Inc.) (see Fig. 3a). A heat $Q$ generated during one hysteresis loop ($B : 0 \rightarrow 1.2$ T $\rightarrow 0$) can be calculated from

$$Q = \int MVdB,$$

(1)

where $M$ is the magnetization and $V$ is the volume of the superconductor. From this, we can estimate the heat generation of the whole solenoid per cycle to be $Q_{\text{solenoid}} = 211 \pm 2$ mJ, where 105 mJ is the heat generation during sweep up and 106 mJ during sweep down. We also estimated the heat generation of the whole FeCoV shield per cycle as $Q_{\text{shield}} = 10 \pm 1$ mJ by a similar procedure.

The eddy current heating of the copper bobbin per cycle is estimated to be $Q_{\text{bobbin}} = 27 \pm 3$ mJ based on the dimension and a measured electrical resistance of the bobbin. Then, the total heat generation of the SSC magnet per cycle is estimated to be $Q = Q_{\text{solenoid}} + Q_{\text{shield}} + Q_{\text{bobbin}} = 248 \pm 4$ mJ. In the continuous operation mode of CNDR, the SSC magnet will be swept at a maximum rate of 1 mT/s. Since the heat generation rate $\dot{Q}$ is proportional to $dM/dB$ under constant sweep rate, it becomes largest when $B$ returns to zero on sweeping down. Otherwise, $\dot{Q}$ is roughly constant and would be of the order of several tens $\mu$W. If this is the case, there is no problem to meet the requirement (iv).

In Fig. 3b, we show magnetization curves measured for four different types of SC wires listed in Table 2. It is known that $Q$ decreases with decreasing filament diameter [6]. This relation also holds in our data (the orange closed circles) but in a slightly different manner as shown in the inset.

![Fig. 3](image-url) Magnetization curve data of (a) the SC wire used in the SSC magnet (No. 1 in Table 2) and (b) of other types of wires (Nos. 2–4 in Table 2). The data were taken at $T = 2$ K in vacuum. (inset) $Q$ calculated from the magnetization curves shown in the main figure (orange closed circles) and $Q$ from the previous research [6] (black triangles) (Color figure online)
3.3 Direct Measurement of the Heat Generation Rate

We have made direct measurements of $\dot{Q}$ generated by sweeping the SSC magnet at $T \approx 4$ K in vacuum. The method is to monitor a temperature difference between the magnet and the thermal bath, which are connected to each other by a thermal link with a known thermal conductance. The setup for the measurement is shown in Fig. 4a, where the liquid helium bath is the thermal bath and the three 10-mm-long M3 screws made of stainless steel (SS304) are the thermal link. At the bottom of the shield end cap, a manganin heater (130 $\Omega$) and a resistance thermometer (Cernox CX-1050-SD, Lake Shore Cryotronics, Inc.), which measures the temperature of the magnet ($T_{\text{mag}}$), are attached. The bath temperature ($T_{\text{bath}}$) was determined by measuring the vapor pressure of $^4$He.

First of all, the thermal conductance $K$ of the SS screws was determined by measuring $T_{\text{mag}}$ and $T_{\text{bath}}$ at various constant heat flows $\dot{Q}_{\text{hf}}$. Then, the data were fitted to

$$\dot{Q}_{\text{hf}} + c = a(T_{\text{mag}}^b - T_{\text{bath}}^b),$$

where $a = 3.5 \pm 1.7 \mu W K^{-b}$, $b = 3.0 \pm 0.3$, and $c = 11.5 \pm 1.0 \mu W$. The fitting quality is sufficiently good as shown in Fig. 4b. Here, $c = \dot{Q}_{\text{RT}} - \dot{Q}_{\text{ex}}$, where $\dot{Q}_{\text{RT}}$ is a heat generation rate.

![Fig. 4](image) a Experimental setup to measure the heat generation of the SSC magnet. b Measured relation between the heater power $\dot{Q}$, the magnet temperature $T_{\text{mag}}$, and the bath temperature $T_{\text{bath}}$. The straight line is the fitting function (Eq. 2) for the data. c Thermal relaxation process after a constant heat flow $\dot{Q}_{\text{hf}} = 28 \mu W$ was suddenly applied. The process involves two different time constants (see text) (Color figure online)

### Table 2

|                      | No. 1 | No. 2 | No. 3 | No. 4 |
|----------------------|-------|-------|-------|-------|
| Number of filaments  | 54    | 18    | 54    | 1     |
| Filament diameter ($\mu m$) | 10    | 17    | 20    | 49    |
| Bare diameter (mm)    | 0.114 | 0.114 | 0.229 | 0.079 |
| Insulator diameter (mm) | 0.140 | 0.140 | 0.254 | 0.102 |
| Clad material         | Cu    | CuNi  | Cu    | CuNi  |
| Clad:NbTi ratio       | 1.3 : 1 | 1.5 : 1 | 1.3 : 1 | 1.5 : 1 |

No. 1 is the wire used in the present SSC magnet.
leak to the magnet from room temperature through lead wires or radiation, and \( \dot{Q}_{\text{cx}} \) is a heat leak to the liquid helium bath presumably through remnant \(^4\)He exchange gas. From this fitting, we determined \( K(\mu \text{W/K}) \) as \( K = abT^{(b-1)} = (10.5 \pm 5.2)T^{2.3} \). It is noted that this is slightly different from \( K = (31.5 \pm 2.2)T^{1.6} \) calculated from the previous report on the thermal conductivity of stainless steel 304 [7].

Figure 4c shows a thermal relaxation process when a constant heat flow \( \dot{Q}_{\text{hf}} = 28 \mu \text{W} \) was suddenly applied and then kept constant. Apparently, the process involves two time constants, i.e., \( \tau_1 = 1086 \pm 10 \text{ s} \) and \( \tau_2 = 2838 \pm 22 \text{ s} \). \( \tau_1 \) would be related to internal thermalization within the SSC magnet. \( \tau_2 \) should be associated with thermalization between the magnet and the thermal bath through the SS screws. This is because the time constant calculated from the estimated heat capacity of the magnet and the known \( K \) based on the two bath model agrees reasonably well with the measured \( \tau_2 \). Note that the data shown in Fig. 4b were taken after waiting for a much longer time than \( \tau_2 \).

After knowing the relation between \( \dot{Q}_{\text{hf}} \), \( T_{\text{mag}} \), and \( T_{\text{bath}} \) (Eq. 2), we measured time evolutions of \( T_{\text{mag}} \) and \( T_{\text{bath}} \) (see Fig. 5a) without applying heater power but while sweeping the magnetic field \( B \) of the SSC magnet in the following cycle (see Fig. 5b): (1) swept up from 0 to 1.38 T over 20 min, (2) kept constant at 1.38 T for the next 160 min until \( T_{\text{mag}} \) returns to the base temperature (\( = 4.28 \text{ K} \)), (3) swept back to 0 over the next 20 min, and (4) kept constant at 0 during the next 280 min. Figure 5c shows an instantaneous heat flow \( \dot{Q}_{\text{hf}} \) calculated from \( T_{\text{mag}} \) and \( T_{\text{bath}} \) assuming Eq. 2 (quasi-equilibrium assumption). Reflecting the asymmetric hysteresis loop (Fig. 3a), \( \dot{Q}_{\text{hf}} \) is also asymmetric between the up and down sweep.

The cycle (1)–(4) was repeated three times, successively. By averaging the heat integrated over processes (1) and (2) and over (3) and (4), we obtain the total heat generation by sweeping up and down as \( Q = 108 \pm 19 \text{ and } 118 \pm 11 \text{ mJ} \), respectively. These results are in excellent agreement with those estimated from the magnetic hysteresis. Such small amounts of heat can easily be absorbed by the still of...
the dilution refrigerator through an annealed silver thermal link which should have a much higher thermal conductance than the SS screws by several orders of magnitude. This means that \( T_{\text{mag}} \) can be kept around \( T = 0.8 \) K. Thus, the amount of thermal radiation from the SSC magnet to the PrNi\(_5\) nuclear stage should be approximately 0.1 nW, and this is much smaller than the expected cooling power (\( \approx 10 \) nW) of CNDR at \( T = 0.8 \) mK [1].

4 Conclusions

We have successfully designed and constructed a high-performance SSC magnet with a FeCoV magnetic shield, which is one of the key elements for development of the compact CNDR [1]. It can produce \( B_{\text{max}} = 1.38 \) T when \( I = 6 \) A with negligibly small fringe fields so that two SSC magnets can be located in close vicinity to each other in CNDR. The measured heat generation due to sweeping the field at a rate of 1 mT/s is of the order of several 10 \( \mu \)W, which is low enough to keep the magnet temperature at 0.8 K.

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