Investigation on the nonlinear dynamic response of a mass-spring system

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Abstract. This paper investigates the nonlinear dynamic response of a single-degree-of-freedom mass-spring system. The experimental modal analysis is carried out by three typical types of spectral testing which are burst random, sine testing and periodic chirp excitation to achieve the objective of this study. The experimental data from the burst random excitation is used to identify the natural frequency of the system. The nonlinear dynamic response of the system is detected by means of using the sine testing and periodic chirp excitation. However, from the sine testing, the results showed that the variations of the maximum peak of the vibration acceleration based on the different levels of excitation voltage that were applied to the input shaker. It was also found that the displacement values obtained from the periodic chirp excitation showed an inconsistent linear increment with the excitation level that applied to the mass-spring system.

Keywords. Nonlinear, dynamic response, vibration, experimental modal analysis, mass-spring system

1. Introduction

Structural dynamics analysis involves studying the vibration properties of engineering structures that is subjected to vibrating loads [1], [2]. Consideration on the dynamics analysis is essential in structural engineering, mechanical system and applied mechanics. Reliable and accurate prediction on the dynamic response of the mechanical system and structures can be obtained if the physical parameters and mathematical structure of the model is known [3], [4].

The approximation approach by assuming the linear dynamics model is mostly applied in structural dynamics analysis [5], [6]. However, the structure will exhibits nonlinear phenomenon when the structure operates in the extreme mechanical and environmental loading conditions [7], [8]. For instance, in aerospace applications, the typical nonlinearities in an aircraft system can be observed such as hardening nonlinearities in engine-to-pylon connections, nonlinear fluid–structure interaction, backlash and friction in control surfaces and joints [9], [10]. On top of that, the flexible wings of the aircraft that employ high-aspect-ratio to reduce weight and power for flight operation lead to the large structural deflections due to the nonlinear behaviour of the flexible wings [11].

High centrifugal vibration load experienced by wind turbine blades during operation lead to the geometrical and material nonlinearities that affect the structural mechanical integrity [12], [13]. The needs for more efficient and lightweight materials in industrial applications contribute to the
increasingly nonlinear structural dynamics analysis [14], [15]. The nonlinearities can change the dynamics behaviour [16]. As a consequence, the linear analysis is invalid for the mechanical systems and structures which have strong nonlinearities [17], [18]. Moreover, a linear design could underestimate the actual behaviour of the structure due to the external load [19], [20].

The nonlinearity is caused by several factors such as large amplitude of deformation when the vibration of the structures that are made from thin plates [21] and shells exceed the thickness [22], [23] and the nonlinear constitutive material properties of the structure and vibration absorber [24], [25]. Crack, looseness and friction on the structural joints [26], [27] also contribute to the structural nonlinearities. The dynamics behaviour of a mechanical system for any input type and any range of the output can be described by using experimental modal analysis through the extraction of the modal parameters (natural frequencies, mode shapes and damping ratios) [28], [29].

In modal analysis, the equation of motion for the dynamic response of a linear model to describe the structural motion [30], [31] as in equation (1).

\[
M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = F(t)
\]  

(1)

Where, \(M\), \(C\) and \(K\) are the mass, damping and stiffness matrices. \(\ddot{x}\), \(\dot{x}\) and \(x\) are the \(n \times 1\) acceleration, velocity and displacement vectors, respectively. \(F(t)\) is an external force vector [32]. For a nonlinear structure, the equation of motion [33], [34] consists of a nonlinear parameter as shown in equation (2).

\[
M\ddot{x}(t) + C\dot{x}(t) + Kx(t) + K_{NL} x(t) = F(t)
\]  

(2)

Where, \(K_{NL}\) is a non-linear stiffness term. For a hardening or softening nonlinear restoring force spring with cubic nonlinearities, the equation of motion is known as Duffing’s equation [35], [36] as shown in equation (3).

\[
M\ddot{x}(t) + C\dot{x}(t) + Kx(t) + K_{NL} x^3(t) = F(t)
\]  

(3)

The purpose of the present work is to investigate the dynamics response of the single-degree-of-freedom mass-spring system. Three types of spectral testing are used to measure the vibration response for further nonlinear dynamics analysis of the mass-spring system.

2. Methodology

2.1 Test Set-up

The experimental work consists of measuring the vibration response of the single-degree-of-freedom mass-spring system. The experimental setup is shown in figure 1 which served to simulate the nonlinear dynamics behaviour.

The system consists of a single mass that is connected by one spring. An electromagnetic shaker provides excitation at the top of the spring in the vertical direction. An accelerometer was mounted on the mass to measure vibration response of the mass-spring system. Figure 2 shows the schematic diagram of this experimental works. All the equipment involved are listed in table 1 while table 2 lists the property of the mass and spring.
Figure 1. The model experimental setup.

Figure 2. The schematic diagram of the experimental modal analysis.

Table 1. Specification of the equipment.

| Equipment           | Brand / Model          |
|---------------------|------------------------|
| Electromagnetic shaker | DATA PHYSICS V4/T4    |
| Force transducer    | PCB PIEZOTRONICS 208C03 |
| Accelerometer       | DYTRAN 3032A           |
| Data acquisition    | LMS                    |
Table 2. Property of the mass and spring.

| Parameter                | Value |
|--------------------------|-------|
| Mass (g)                 | 100   |
| Spring stiffness (N/mm)  | 105   |

2.2 Experimental measurement

Conventional modal testing (spectral test procedures) has been applied to the test system. This testing is used to describe the linear and nonlinear dynamic response of the test system. In figure 3, three different types of spectral testing were used in this study, respectively. The first test is burst random type with the purpose is to identify the natural frequency of the single-degree-of-freedom mass-spring system. However, in practice, the nonlinear dynamic response of the system is amplitude dependence to the input excitation. This nonlinear is more significant at higher levels of vibration rather than low levels.

Therefore, the sine testing was carried out to identify the nonlinear properties that is relate to the amplitude of the vibration acceleration. The time domain of the response was recorded using the data acquisition. The maximum peak of the vibration acceleration was extracted to obtain the relation between the input level and output acceleration response. The nonlinearity was identified based on the maximum peak value. For the second nonlinear investigation applied in this study, the periodic chirp test was performed to the single-degree-of-freedom mass-spring system. The purpose is to investigate the displacement value of the vibration response. The frequency response function of the test system was recorded. The variation of the displacement of the natural frequencies as a function of the input level were examined.

![Flow chart](image)

Figure 3. The flow chart of the experimental work.

3. Result and discussion

3.1. Natural Frequency Identification

The driving force as listed in table 3 was employed to excite the single-degree-of-freedom mass-spring system. The burst random excitation was utilized to determine the natural frequency. The frequency bandwidth is 16 Hz, the spectral line is 32 and the resolution value is 0.5. Medium voltage excitation value was chosen which is 0.5 volt. The excitation direction of the electromagnetic shaker is in negative Z direction. This excitation set-up is totally in the length of the spring component for stiffness investigation purposes.
Table 3. Experimental parameters for the burst random excitation.

| Parameter             | Value |
|-----------------------|-------|
| Excitation type       | Burst random |
| Bandwidth (Hz)        | 16    |
| Spectral line         | 32    |
| Resolution            | 0.5   |
| Voltage (V)           | 0.5   |
| Excitation DOF        | 1     |
| Excitation direction  | -Z    |

Figure 4 shows the experimental frequency response function measured at the mass of the system. Averages of runs were used during this test. It can be seen that this mass-spring system has a single natural frequency of 11.5 Hz. The damping properties is neglected since the damping ratio value obtained from this experiment is low, which is less than 1.0%.

**Figure 4.** Natural frequency of the mass-spring system.

Based on this natural frequency result, the nonlinear dynamic response can be investigated using two different types of spectral testing that will be explained in the next section.

3.2. Maximum Peak of the Vibration Acceleration

In extension to the burst random test, the sine testing using harmonic excitation force was performed in order to investigate the maximum peak of the vibration acceleration. The frequency of the sine excitation is at the natural frequency which is 11.5 Hz. The purpose is to investigate the vibration behaviour of the spring. Table 4 below summarize the experimental parameters for the sine testing excitation.

The time series of the vibration signal are shown in figure 5 to figure 10. It can be seen that the measured sinusoidal waveform response of the dynamical system. The maximum peak of the vibration acceleration for all excitation voltages has been extracted and listed in table 5.
Table 4. Experimental parameters for the sine testing excitation.

| Parameter                | Value                                      |
|--------------------------|--------------------------------------------|
| Excitation type          | Sine                                       |
| Frequency (Hz)           | 11.5                                       |
| Bandwidth (Hz)           | 16                                         |
| Spectral line            | 32                                         |
| Resolution               | 0.5                                        |
| Voltage (V)              | Random between 0.1 and 2.00                |
| Excitation DOF           | 1                                          |
| Excitation direction     | -Z                                         |

Figure 5. Vibration acceleration at 0.14 volt.

Figure 6. Vibration acceleration at 0.42 volt.
Figure 7. Vibration acceleration at 0.76 volt.

Figure 8. Vibration acceleration at 1.13 volt.

Figure 9. Vibration acceleration at 1.53 volt.
Figure 10. Vibration acceleration at 1.88 volt.

Table 5. Maximum peak of vibration acceleration.

| Voltage (volt) | Peak Amplitude |
|----------------|----------------|
| 0.10           | 0.0214         |
| 0.14           | 0.0334         |
| 0.20           | 0.0453         |
| 0.28           | 0.0605         |
| 0.30           | 0.0646         |
| 0.35           | 0.0718         |
| 0.40           | 0.0832         |
| 0.42           | 0.0904         |
| 0.50           | 0.0981         |
| 0.57           | 0.1212         |
| 0.60           | 0.1329         |
| 0.61           | 0.1267         |
| 0.70           | 0.1452         |
| 0.76           | 0.1676         |
| 0.80           | 0.1681         |
| 0.83           | 0.1798         |
| 0.90           | 0.1721         |
| 0.99           | 0.1991         |
| 1.00           | 0.1991         |
| 1.05           | 0.2021         |
| 1.10           | 0.1982         |
| 1.13           | 0.2283         |
| 1.20           | 0.2312         |
Table 5. Continued.

| Voltage (volt) | Peak Amplitude |
|---------------|----------------|
| 1.26          | 0.2280         |
| 1.30          | 0.2393         |
| 1.32          | 0.2464         |
| 1.40          | 0.2531         |
| 1.47          | 0.2702         |
| 1.50          | 0.2567         |
| 1.53          | 0.2579         |
| 1.60          | 0.2716         |
| 1.67          | 0.2735         |
| 1.70          | 0.2886         |
| 1.71          | 0.2766         |
| 1.80          | 0.2943         |
| 1.88          | 0.2906         |
| 1.99          | 0.2936         |
| 2.00          | 0.2895         |

Figure 11 shows the variation of the maximum peak of the vibration acceleration. The pattern of the maximum peak is increasing with the increase of the input excitation level that was applied to the mass-spring system. However, the inconsistent linear increment detected in this analysis result is an indicator to the nonlinear property in the system. The maximum peak is strongly dependent on the excitation level. It is shown that the zoomed plot within the figure reveals a clear nonlinear effect of the system. The low stiffness value of the spring used in this research (105 N/mm) investigation leads to the slight variations in the changes of maximum peak of the vibration acceleration.
3.3. Vibration Displacement
The periodic chirp excitation was carried out in order to investigate the vibration displacement. The effect of signal leakage for this type of excitation is non-existent when the steady state response achieved, making it suitable to be applied in this study. The excitation voltage was controlled from low to high frequency within the range of the frequency of interest. Table 6 lists the detail parameters of the testing. Based on the previous burst random testing measurement, the natural frequency of this single-degree-of-freedom mass-spring system is 11.5 Hz. Therefore, this sweeping up excitation is in the range of the natural frequency from 10 Hz to 12 Hz. The range selection for voltage amplitude is randomly between 0.50 volt and 2.10 volt.

| Parameter                  | Value                     |
|----------------------------|---------------------------|
| Excitation type            | Periodic chirp            |
| Sweep direction            | Up                        |
| Frequency sweep (Hz)       | 10 to 12                  |
| Bandwidth (Hz)             | 16                        |
| Spectral line              | 32                        |
| Resolution                 | 0.5                       |
| Voltage (V)                | Random between 0.50 and 2.10 |
| Excitation DOF             | 1                         |
| Excitation direction       | -Z                        |

Figure 12 shows the frequency response function obtained from the periodic chirp testing. The zooming plot on the x-axis of figure 12 is shown in figure 13. It can be seen that the natural frequencies shift to lower value when higher excitation voltages apply to the input shaker. This pattern founds that this spring component has a softening nonlinearity behaviour.
The response signal measured during the periodic chirp testing was analysed to obtain the vibration displacement. The measured displacement values for each excitation voltage is shown in table 7, as well as the natural frequencies and damping ratios. The investigation on the nonlinear dynamic behaviour of the mass-spring system was carried out by the two-dimensional plot of the displacement values and the excitation voltage.

**Table 7.** Natural frequency, damping ratio and displacement from the periodic chirp testing.

| Voltage (volt) | Natural frequency (Hz) | Damping ratio | Displacement (m) |
|---------------|------------------------|--------------|------------------|
| 0.57          | 11.51                  | 0.92         | 0.0509           |
| 0.67          | 11.567                 | 0.96         | 0.0554           |
| 0.70          | 11.517                 | 0.85         | 0.0516           |
| 0.76          | 11.504                 | 0.61         | 0.0490           |
| 0.80          | 11.511                 | 0.91         | 0.0552           |
| 0.83          | 11.499                 | 0.83         | 0.0538           |
| 0.90          | 11.503                 | 0.79         | 0.0529           |
| 0.99          | 11.494                 | 0.81         | 0.0544           |
| 1.00          | 11.518                 | 0.83         | 0.0546           |
| 1.05          | 11.494                 | 0.75         | 0.0530           |
| 1.13          | 11.493                 | 0.85         | 0.0542           |
| 1.20          | 11.512                 | 0.73         | 0.0539           |
| 1.26          | 11.509                 | 0.82         | 0.0558           |

**Figure 13.** Frequency response function between range 11.3 Hz and 11.7 Hz.
| Voltage (volt) | Natural frequency (Hz) | Damping ratio | Displacement (m) |
|---------------|------------------------|---------------|-----------------|
| 1.30          | 11.517                 | 0.89          | 0.0573          |
| 1.32          | 11.513                 | 0.84          | 0.0551          |
| 1.40          | 11.508                 | 0.87          | 0.0566          |
| 1.47          | 11.502                 | 0.76          | 0.0541          |
| 1.50          | 11.509                 | 0.71          | 0.0531          |
| 1.54          | 11.498                 | 0.78          | 0.0553          |
| 1.60          | 11.500                 | 0.77          | 0.0541          |
| 1.63          | 11.516                 | 0.85          | 0.0553          |
| 1.70          | 11.505                 | 0.79          | 0.0538          |
| 1.77          | 11.504                 | 0.78          | 0.0553          |
| 1.80          | 11.489                 | 0.80          | 0.0560          |
| 1.81          | 11.492                 | 0.73          | 0.0554          |
| 1.90          | 11.493                 | 0.73          | 0.0538          |
| 1.96          | 11.484                 | 0.85          | 0.0568          |
| 2.00          | 11.487                 | 0.79          | 0.0557          |
| 2.05          | 11.480                 | 0.76          | 0.0554          |
| 2.10          | 11.491                 | 0.81          | 0.0558          |

Figure 14 observes the changes regarding the variations of the vibration displacement values from the lowest excitation voltage to the highest excitation voltage. Even though the pattern of the result is increasing, it is noticed that a significant inconsistent linear increment exists in the plot. It is concluded that the ratio between the input (excitation voltage) and the output (displacement value) is not constant due to the nonlinear behaviour of this mass-spring system.
4. Conclusions

Experimental investigation on the nonlinear dynamics response of a spring component has been presented in this paper. A single-degree-of-freedom mass-spring system has been developed to analyses the dynamics property of the nonlinear stiffness of the spring. The results from the burst random excitation found the natural frequency of the system which is at 11.5 Hz. This natural frequency value has been used to investigate the nonlinear dynamic response of the system. The spectral sine testing with the excitation frequency of 11.5 Hz has been carried out and the results show that the variations of the maximum peak of the vibration acceleration is not in linear characteristic to the excitation voltage. By using the periodic chirp excitation test with the excitation frequency range between 10 Hz and 12 Hz, it has been analyzed that the measured displacement values show the inconsistent linear increment variations. Future works will focus on the numerical modelling and dynamics analysis of this mass-spring system.

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