On Magnetohydrodynamic Flow of Viscoelastic Nanofluids with Homogeneous–Heterogeneous Reactions

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Abstract: This article explores magnetohydrodynamic stretched flow of viscoelastic nanofluids with heterogeneous–homogeneous reactions. Attention in modeling has been specially focused to constitutive relations of viscoelastic fluids. The heat and mass transport process is explored by thermophoresis and Brownian dispersion. Resulting nonlinear systems are computed for numerical solutions. Findings for temperature, concentration, concentration rate, skin-friction, local Nusselt and Sherwood numbers are analyzed for both second grade and elastico-viscous fluids.

Keywords: viscoelastic fluids; nanoparticles; magnetohydrodynamics; heterogeneous–homogeneous reactions; numerical solution

1. Introduction

It is now acknowledged that non-Newtonian fluids in industrial, physiological and technological processes are more significant than viscous fluids. Few examples of such fluids may include silicon oils, printer ink, mud, ice cream, egg yolk, blood at low shear rate, shampoo, gypsum paste, polymer solutions, nail polish, sand in water, ketchup etc. Rheological properties of such fluids are different and thus all these cannot be explained employing one constitutive relationship between shear rate and rate of strain. The modelled expressions for the non-Newtonian liquids are more tedious and of higher order than Navier–Stokes expressions for viscous fluids. Researchers in the field face challenges in modelling, analysis and computations from different quarters. Through different non-Newtonian fluids, the objective here is to explore second grade and elastico-viscous fluids [1–8].

Nanofluids are described by carbon nanotubes (CNTs) [9–11], Buongiorno [12] and Tiwari and Das [13] models. Therefore, the information is very significant about flows involving thermophoresis aspects. Impact of slip in flow of copper-water nanoliquid over an extendable surface is examined by Pandey and Kumar [14]. Flow of couple stress nanomaterial bound by an oscillatory stretchable surface is analyzed by Khan et al. [15]. Turkyilmazoglu [16] discussed free and circular jets in view of single phase nanomaterial. Few relevant investigations for nanoliquids can be seen in studies [17–45]. According to previous literature, it is found that magnetohydrodynamic stretched flow of viscoelastic nanofluids with heterogeneous–homogeneous reactions has not been reported yet. Attention in modeling has been specially focused on constitutive relations of viscoelastic fluids. Heat and mass transport process is explored by thermophoresis and Brownian dispersion. Adequate transformations are considered to dimensionless the governing system. Numerical solutions of the resulting system are obtained by employing the shooting method. Contributions of numerous sundary variables on flow fields are interpreted through plots and numerical data.
2. Problem Formulation

Two-dimensional (2D) steady magnetohydrodynamic flow of incompressible viscoelastic nanoliquids by a linear stretchable surface with heterogeneous–homogeneous reactions is analyzed. Second grade and elastico-viscous liquids are considered. Attention in modeling has been specially focused on constitutive relations of viscoelastic fluids. Heat and mass transport process is explored by thermophoresis and Brownian dispersion. Let \( u_w(x) = cx \) denotes wall velocity along \( x \)-axis (see Figure 1). Homogeneous-reaction for cubic catalysis is [37]:

\[
A + B \rightarrow 3B, \text{ rate } = k_c ab^2. \tag{1}
\]

At catalyst surface heterogeneous-reaction is [37]:

\[
A \rightarrow B, \text{ rate } = k_s a. \tag{2}
\]

Figure 1. Flow configuration.

In above relations rate constants are described by \( k_s \) and \( k_c \) and chemical species \( B \) and \( A \) have concentrations \( b \) and \( a \) separately. Relevant equations for 2D flow satisfy [5,7]:

\[
\text{div } \mathbf{V} = 0, \tag{3}
\]

\[
\rho \frac{d\mathbf{V}}{dt} = \text{div } \mathbf{\sigma} + \rho \mathbf{b}. \tag{4}
\]

Cauchy stress tensor of second-order fluid is

\[
\mathbf{\sigma} = -p\mathbf{I} + \mu \mathbf{A}_1 + \alpha_1 \mathbf{A}_2 + \alpha_2 \mathbf{A}_1^2, \tag{5}
\]

in which \( \mathbf{A}_1 \) and \( \mathbf{A}_2 \) stand for 1st and 2nd Rivlin-Ericksen tensors respectively i.e.,

\[
\mathbf{A}_1 = (\text{grad } \mathbf{V})^* + (\text{grad } \mathbf{V}), \tag{6}
\]

\[
\mathbf{A}_2 = \frac{d\mathbf{A}_1}{dt} + (\text{grad } \mathbf{V})^* \mathbf{A}_1 + \mathbf{A}_1 (\text{grad } \mathbf{V}), \tag{7}
\]
where $a_1$ and $a_2$ stand for material constants, $b$ for body force, $D_t$ for material derivative and $p$ for pressure. Material moduli satisfy following relationships for second grade fluid:

$$a_1 \geq 0, \quad \mu \geq 0, \quad a_1 + a_2 = 0,$$

in which $\ast$ stands for matrix transpose and velocity distribution $\mathbf{V}$ is

$$\mathbf{V} = [u(x, y), v(x, y), 0].$$

The governing expressions for 2D stretching flow of viscoelastic nanofluids are \[5,7,37\]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - k_0 \left( u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial x^2 \partial y} \right) - \frac{\sigma B_k^2}{\rho} u,$$

$$\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + \frac{(\rho c)_f}{(\rho c)_p} \left( D_B \left( \frac{\partial^2 T}{\partial y^2} \right) + \frac{D_T}{T_{\infty}} \left( \frac{\partial T}{\partial y} \right)^2 \right),$$

$$\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_b \left( \frac{\partial^2 C}{\partial y^2} \right) + \frac{D_T}{T_{\infty}} \left( \frac{\partial^2 T}{\partial y^2} \right),$$

$$u \frac{\partial a}{\partial x} + v \frac{\partial a}{\partial y} = D_A \left( \frac{\partial^2 a}{\partial y^2} \right) - k_c a b^2,$$

$$u \frac{\partial b}{\partial x} + v \frac{\partial b}{\partial y} = D_B \left( \frac{\partial^2 b}{\partial y^2} \right) + k_c a b^2,$$

$$u = u_w(x) = c x, \quad v = 0, \quad T = T_w, \quad C = C_w, \quad D_A \frac{\partial a}{\partial y} = k_c a, \quad D_b \frac{\partial b}{\partial y} = -k_c a \text{ at } y = 0,$$

$$u \to 0, \quad T \to T_{\infty}, \quad C \to C_{\infty}, \quad a \to a_0, \quad b \to 0 \text{ as } y \to \infty.$$
\[
\frac{\delta}{Sc_b} h'' + fh' + K rh^2 = 0, 
\] (23) 
\[
f = 0, f' = 1, \theta = 1, \phi = 1, r' = K_s r, \delta h' = -K_s r \text{ at } \zeta = 0, 
\] (24) 
\[
f' \to 0, \theta \to 0, \phi \to 0, r \to 1, h \to 0 \text{ as } \zeta \to \infty. 
\] (25) 

Here \( k^*_v \) stands for viscoelastic parameter, \( \delta \) for ratio of mass diffusion coefficients, \( N_l \) for thermophoresis parameter, \( K_s \) for homogeneous-reaction strength, \( M \) for magnetic parameter, \( Sc \) for Schmidt number, \( Sc_b \) for Schmidt number (for heterogeneous–homogeneous reactions), \( N_b \) for Brownian motion parameter, \( K_s \) for heterogeneous-reaction strength and \( Pr \) for Prandtl number. We set these definitions as

\[
\begin{align*}
 k^*_v &= -k_0 \left( \frac{\zeta}{\delta} \right), 
 M^2 &= \frac{\sigma R^2}{k^0}, 
 Pr &= \frac{\nu}{\alpha}, 
 \delta &= \frac{D_B}{D_A}, 
 K &= \frac{k}{\nu \nu_0} , 
 K_s &= \frac{k}{\nu_0} \sqrt{\frac{3}{T}}, 
 Sc &= \frac{\nu}{D_A}, 
 Sc_b &= \frac{\nu}{D_A}, 
 N_b &= \frac{\rho c \nu_{D\nu}(C_{\text{w}}-C_{\infty})}{(\rho c)_T (T_{\infty}-T_{\infty})}, 
 N_l &= \frac{\rho c \nu_{D\nu}(T_{\infty}-T_{\infty})}{(\rho c)_T (T_{\infty}-T_{\infty})}. 
\end{align*}
\] (26)

Considering that \( D_A = D_B \) we have \( \delta = 1 \) and thus

\[
r(\zeta) + h(\zeta) = 1. 
\] (27)

Now Equations (22) and (23) give

\[
\frac{1}{Sc_b} r'' + fr' - K(1-r)^2 r = 0, 
\] (28)

with boundary conditions

\[
r'(0) = K_s r'(0), 
 r(\infty) \to 1. 
\] (29)

Coefficient of skin friction and local Sherwood and Nusselt numbers are

\[
\begin{align*}
 \text{Re}^{1/2} C_f &= (1-3k^*_v) f''(0), 
 \text{Re}^{-1/2} S h_x &= -\phi'(0), 
 \text{Re}^{1/2} N u_x &= -\theta'(0),
\end{align*}
\] (30)

in which \( \text{Re}_x = u_w x / \nu \) denotes the local Reynolds number.

3. Solution Methodology

By considering suitable boundary conditions on the system of equations, a numerical solution is developed using NDSolve in Mathematica. Shooting method is used via NDSolve. This method is very helpful in case of small step-size featuring negligible error. As a consequence, both \( x \) and \( y \) varied uniformly by a step-size of 0.01 [40].

4. Graphical Results and Discussion

Effects of magnetic parameter \( M \), homogeneous-reaction strength \( K \), Schmidt number \( Sc \), Schmidt number (for heterogeneous–homogeneous reactions) \( Sc_b \), thermophoresis parameter \( N_l \), heterogeneous-reaction strength \( K_s \), Prandtl number \( Pr \) and Brownian motion parameter \( N_b \) on concentration \( \phi(\zeta) \), concentration rate \( r(\zeta) \) and temperature \( \theta(\zeta) \) for both second grade and elastico-viscous fluids are sketched in Figures 2–12.

Figure 2 depicts impact of magnetic parameter \( M \) on temperature \( \theta(\zeta) \). Here \( M \neq 0 \) is for hydromagnetic flow situation and \( M = 0 \) corresponds to hydrodynamic flow case. Temperature \( \theta(\zeta) \) is higher for hydromagnetic flow in comparison to hydrodynamic flow for both second grade and elastico-viscous fluids. Physically magnetic parameter depends upon Lorentz force. Lorentz force is an agent which resists the motion of fluid and therefore temperature \( \theta(\zeta) \) enhances.

Figure 3 displays variations in temperature \( \theta(\zeta) \) for increasing Prandtl number \( Pr \). Temperature \( \theta(\zeta) \) decays for larger \( Pr \) for both second grade and elastico-viscous fluids. Physically Prandtl number
involves thermal diffusivity. Larger Prandtl number corresponds to weaker thermal diffusivity which produces a decay in temperature $\theta (\zeta)$. Figure 4 depicts impact of Brownian motion parameter $N_b$ on temperature $\theta (\zeta)$. Larger $N_b$ produces an increment in temperature $\theta (\zeta)$ for both second grade and elastico-viscous fluids. Larger Brownian motion parameter $N_b$ has stronger Brownian diffusivity and weaker viscous force which increased the temperature $\theta (\zeta)$.

Figure 5 shows that larger thermophoresis parameter $N_t$ leads to higher temperature $\theta (\zeta)$ for both second grade and elastico-viscous fluids. Larger $N_t$ causes strong thermophoresis force which tends to shift nanoparticles from hot to cold zone and therefore temperature $\theta (\zeta)$ increases.

Impact of magnetic parameter $M$ on concentration $\phi (\zeta)$ is displayed in Figure 6 Concentration $\phi (\zeta)$ is upgraded for increasing estimations of $M$ for both second grade and elastico-viscous fluids. Furthermore, the concentration $\phi (\zeta)$ shows similar trend for both second grade and elastico-viscous fluids. Figure 7 depicts that concentration $\phi (\zeta)$ is decreased for larger Schmidt number $Sc$ for both second grade and elastico-viscous fluids. Schmidt number $Sc$ has an inverse relation with Brownian diffusivity. Larger Schmidt number leads to weaker Brownian diffusivity which produces weaker concentration $\phi (\zeta)$.

Impact of Brownian motion $N_b$ on concentration $\phi (\zeta)$ is shown in Figure 8 Bigger $N_b$ produces a reduction in concentration $\phi (\zeta)$ for both second grade and elastico-viscous fluids. Physically Brownian force tries to push particles in opposite direction of concentration gradient and make nanofluid more homogeneous. Therefore, higher the Brownian force, lower the concentration gradient and more uniform concentration $\phi (\zeta)$.

Figure 9 displays that how thermophoresis $N_t$ affects concentration $\phi (\zeta)$. Here concentration $\phi (\zeta)$ is upgraded for higher estimations of $N_t$ for both second grade and elastico-viscous fluids. Furthermore, the concentration $\phi (\zeta)$ shows similar trend for both second grade and elastico-viscous fluids.

Table 1 displays skin-friction $-C_fRe^{1/2}$ subject to varying $k_t^*$ and $M$. Here skin-friction has higher estimations for larger $M$ for both second grade and elastico-viscous fluids. Table 2 depicts comparison for various estimations of $k_t^*$ with homotopy analysis method (HAM). Table 2 presents a good agreement of numerical solution with existing homotopy analysis method (HAM) solution in a limiting sense. Table 3 depicts local Nusselt number $Nu_xRe^{-1/2}$ subject to varying $k_t^*$, $N_b$ and $N_t$. Here larger $N_b$ and $N_t$ correspond to lower local Nusselt number for both second grade and elastico-viscous fluids. Table 4 shows local Sherwood number $Sh_xRe^{-1/2}$ subject to varying $k_t^*$, $N_b$ and $N_t$. Here larger $N_t$ produces lower local Sherwood number while opposite trend is noted via $N_b$ for both second grade and elastico-viscous fluids.

### Table 1. Skin-friction coefficient for various estimations of viscoelastic and magnetic parameters.

| $M$  | $k_t^* = 0.1$ | $k_t^* = -0.1$ |
|------|---------------|----------------|
| 0.0  | 0.7379        | 1.2395         |
| 0.2  | 0.7525        | 1.2640         |
| 0.5  | 0.8250        | 1.3858         |
Figure 2. Variations of temperature for magnetic parameter when $N_b = 0.2, N_l = 0.1$ and $Sc = Pr = 1.0$.

Figure 3. Variations of temperature for Prandtl number when $N_b = 0.2, N_l = 0.1, Sc = 1.0$ and $M = 0.2$.

Figure 4. Variations of temperature for Brownian motion parameter when $N_l = 0.1, Sc = Pr = 1.0$ and $M = 0.2$. 

$Pr = 0.6, 0.9, 1.2$

$M = 0.0, 0.6, 1.2$
Figure 5. Variations of temperature for thermophoresis parameter when $N_b = 0.2$, $Sc = Pr = 1.0$ and $M = 0.2$.

Figure 6. Variations of concentration for magnetic parameter when $N_b = 0.2$, $N_i = 0.1$ and $Sc = Pr = 1.0$.

Figure 7. Variations of concentration for Schmidt number when $N_b = 0.2$, $N_i = 0.1$, $Pr = 1.0$ and $M = 0.2$. 
Variations of concentration rate for Schmidt number (for heterogeneous–homogeneous reactions) when $K = 0.2$, $K_0 = 0.5$ and $M = 0.2$. 

Figure 8. Variations of concentration for Brownian motion parameter when $N_1 = 0.1$, $Sc = Pr = 1.0$ and $M = 0.2$. 

Figure 9. Variations of concentration for thermophoresis parameter when $N_b = 0.2$, $Sc = Pr = 1.0$ and $M = 0.2$. 

Figure 10. Variations of concentration rate for Schmidt number (for heterogeneous–homogeneous reactions) when $K = 0.2$, $K_0 = 0.5$ and $M = 0.2$. 

Figure 11. Variations of concentration rate for homogeneous-reaction strength when \( Sc_b = 1.0, K_s = 0.5 \) and \( M = 0.2 \).

Figure 12. Variations of concentration rate for heterogeneous-reaction strength when \( K = 0.2, Sc_b = 1.0 \) and \( M = 0.2 \).

Table 2. Comparative data of skin-friction coefficient for various estimations of viscoelastic parameter when \( M = 0 \).

| \( k^*_i \) | \(-C_fRe^{3/2}\) | Numerical | HAM [5] |
|-----------|----------------|-----------|---------|
| 0.0       | 1.0000         | 1.00000   |
| 0.1       | 0.7379         | 0.73786   |
| 0.2       | 0.4472         | 0.44721   |
| 0.3       | 0.1195         | 0.11952   |
Table 3. Local Nusselt number for various estimations of viscoelastic, Brownian motion and thermophoresis parameters when $Sc = Pr = 1.0$ and $M = 0.2$.

| $N_b$ | $N_t$ | $Nu_x Re_x^{-1/2}$ for $k_1^* = 0.1$ | $Nu_x Re_x^{-1/2}$ for $k_1^* = -0.1$ |
|-------|-------|--------------------------------------|--------------------------------------|
| 0.1   | 0.1   | 0.5232                               | 0.5425                               |
| 0.2   | -     | 0.4970                               | 0.5153                               |
| 0.3   | -     | 0.4716                               | 0.4890                               |
| 0.2   | 0.1   | 0.4970                               | 0.5153                               |
| -     | 0.2   | 0.4827                               | 0.5002                               |
| -     | 0.3   | 0.4689                               | 0.4856                               |

Table 4. Local Sherwood number for various estimations of viscoelastic, Brownian motion and thermophoresis parameters when $Sc = Pr = 1.0$ and $M = 0.2$.

| $N_b$ | $N_t$ | $Sh_x Re_x^{-1/2}$ for $k_1^* = 0.1$ | $Sh_x Re_x^{-1/2}$ for $k_1^* = -0.1$ |
|-------|-------|--------------------------------------|--------------------------------------|
| 0.1   | 0.1   | 0.2100                               | 0.2300                               |
| 0.2   | -     | 0.3994                               | 0.4203                               |
| 0.3   | -     | 0.4622                               | 0.4834                               |
| 0.2   | 0.1   | 0.3994                               | 0.4203                               |
| -     | 0.2   | 0.2435                               | 0.2647                               |
| -     | 0.3   | 0.0982                               | 0.1205                               |

5. Conclusions

Magnetohydrodynamic flow of viscoelastic nanofluids bound by a linear stretchable surface with heterogeneous–homogeneous reactions are analyzed. Both concentration $\phi (\zeta)$ and temperature $\theta (\zeta)$ are enhanced via higher $M$. Larger Brownian motion $N_b$ displays opposite trend for concentration $\phi (\zeta)$ and temperature $\theta (\zeta)$. Larger thermophoresis number $N_t$ produces higher concentration $\phi (\zeta)$ and temperature $\theta (\zeta)$. Temperature $\theta (\zeta)$ is reduced when Prandtl number enhances. Prandtl number is considered to control the rate of heat transfer in engineering and industrial processes. The suitable value of Prandtl number is very essential to control the rate of heat transfer in engineering and industrial processes. Larger homogeneous-reaction $K$ depicts a reduction in concentration rate $r (\zeta)$. Larger heterogenous-reaction $K_s$ and Schmidt number $Sc_b$ lead to higher concentration rate $r (\zeta)$. Skin friction is enhanced for larger magnetic parameter $M$. Reverse trend of local Sherwood number is seen for $N_t$ and $N_b$. Local Nusselt number is decreased for thermophoresis $N_t$ and Brownian motion $N_b$ parameters. Furthermore, the present analysis is reduced to Newtonian fluid flow case when $k_1^* = 0$.

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Nomenclature

| Symbol | Description |
|--------|-------------|
| $u, v$ | velocity components |
| $x, y$ | coordinate axes |
| $A, B$ | chemical species |
| $k_c, k_s$ | rate constants |
| $\mu$ | dynamic viscosity |
| $\rho$ | density of base fluid |
| $a, b$ | concentrations of chemical species |
| $k_0$ | elastic parameter |
| $\nu$ | kinematic viscosity |
| $\sigma$ | electrical conductivity |
| $V$ | velocity distribution |
| $B_0$ | magnetic field strength |
| $o$ | Cauchy stress tensor |
| $b$ | body force |
| $a_1, a_2$ | material constants |
| $\frac{d}{dx}$ | material derivative |
| $A_1, A_2$ | first and second Rivlin-Ericksen tensors |
| $D_A, D_B$ | mass diffusion coefficients |
| $C$ | concentration |
| $T_{\infty}$ | ambient fluid temperature |
| $C_\infty$ | ambient fluid concentration |
| $T_w$ | surface temperature |
| $C_w$ | surface concentration |
| $(pc)_p$ | effective heat capacity of nanoparticles |
| $(pc)_f$ | heat capacity of fluid |
| $a$ | thermal diffusivity |
| $k$ | thermal conductivity |
| $u_w$ | surface velocity |
| $c$ | positive constant |
| $D_B^*$ | Brownian diffusion coefficient |
| $D_T$ | thermophoretic diffusion coefficient |
| $\zeta$ | similarity variable |
| $f'$ | dimensionless velocity |
| $\theta$ | dimensionless temperature |
| $\phi$ | dimensionless concentration |
| $r$ | dimensionless concentration rate |
| $k_1^*$ | viscoelastic parameter |
| $Sc$ | Schmidt number |
| $K$ | homogeneous-reaction strength |
| $Pr$ | Prandtl number |
| $N_b$ | Brownian motion parameter |
| $N_l$ | thermodiffusivity parameter |
| $K_s$ | heterogeneous-reaction strength |
| $Sc_b$ | Schmidt number |
| $C_f$ | skin friction coefficient |
| $Re_x$ | local Reynolds number |
| $Nu_x$ | local Nusselt number |
| $Sh_x$ | local Sherwood number |

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