The Problem of the Quark-Lepton Mass Spectrum

C.D. Froggatt

Department of Physics and Astronomy
Glasgow University, Glasgow G12 8QQ, Scotland

Abstract

We review some approaches to the quark-lepton mass problem. The ideas of mass-protection and approximate chiral flavour symmetries as a framework for resolving the mass hierarchy problem are presented. Dynamical calculations of the top quark and Higgs particle masses, based on infra-red fixed points and the so-called multiple point principle respectively, are discussed. We also consider mass matrix texture and the neutrino mass puzzle.

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Introduction

The charged fermion masses and mixing angles arise from the Yukawa couplings, which are arbitrary parameters in the Standard Model (SM). The masses range over five orders of magnitude, from $1/2$ MeV for the electron to 175 GeV for the top quark. Also the elements of the quark weak coupling matrix, $V_{CKM}$, range from $V_{ub} \approx 0.003$ to $V_{tb} \approx 1$. This constitutes the charged fermion mass and mixing problem. It is only the top quark which has a mass of the order of the electroweak scale $< \phi_{WS} > = 174$ GeV and a Yukawa coupling constant of order unity $y_t \approx 1$. It therefore seems likely that the top quark mass will be understood dynamically before those of the other fermions. All of the other Yukawa couplings are suppressed, suggesting the existence of physics beyond the SM. Furthermore the accumulating evidence for neutrino oscillations provides direct evidence for physics beyond the SM, in the form of non-zero neutrino masses.

A fermion mass term essentially represents a transition amplitude between a left-handed Weyl field $\psi_L$ and a right-handed Weyl field $\psi_R$. If $\psi_L$ and $\psi_R$ have different quantum numbers, i.e. belong to inequivalent irreducible representations of a symmetry group $G$ ($G$ is then called a chiral symmetry), the mass term is forbidden in the limit of exact $G$ symmetry and they represent two massless Weyl particles. $G$ thus “protects” the fermion from getting a mass. For example the $SU(2)_L \times U(1)$ gauge quantum numbers of the left and right-handed top quark fields are different and the electroweak gauge symmetry protects the top quark from having a mass, i.e. the mass term $\bar{t}_L t_R$ is not gauge invariant. It is only after the $SU(2)_L \times U(1)$ gauge symmetry is spontaneously broken that the top quark gains a mass $m_t = y_t < \phi_{WS} >$, which is consequently suppressed relative to the presumed fundamental (GUT, Planck...) mass scale $M$ by the symmetry breaking parameter $\epsilon = < \phi_{WS} > / M$. The other quarks and leptons have masses suppressed relative to $< \phi_{WS} >$ and it is natural to assume that they are protected by further approximately conserved chiral flavour charges [1], as we discuss further in section 5.

We first consider dynamical calculations of the top quark and the Higgs particle masses, using Infra-Red Quasi-Fixed Points in section 2 and the so-called Multiple Point Principle in section 3. Mass matrix ansätze with texture zeros are considered in section 4. Finally, the neutrino mass problem is briefly discussed in section 6.
2 Top and Higgs Masses from Infra-red Fixed Point

The idea that some of the SM mass parameters might be determined as infra-red fixed point values of renormalisation group equations (RGEs) was first considered [1] some time ago. It was pointed out that the three generation fermion mass hierarchy does not naturally arise out of the general structure of the RGEs, although it does seem possible in special circumstances [2]. However it was soon [3] realised that the top quark mass might correspond to a fixed point value or more likely a quasi-fixed point [4] at the scale $\mu = m_t$.

The SM quasi-fixed point prediction of the top quark mass is based on two assumptions: (a) the perturbative SM is valid up to some high (e.g. GUT or Planck) energy scale $M \simeq 10^{15} - 10^{19}$ GeV, and (b) the top Yukawa coupling constant is large at the high scale $g_t(M) \gtrsim 1$. Neglecting the lighter quark masses and mixings, which is a good approximation, the SM one loop RGE for the top quark running Yukawa coupling $g_t(\mu)$ is:

$$16\pi^2 \frac{dg_t}{d\ln \mu} = g_t \left( \frac{9}{2}g_1^2 - 8g_2^2 - \frac{9}{4}g_3^2 - \frac{17}{12}g_1^4 \right)$$

Here the $g_i(\mu)$ are the three SM running gauge coupling constants. The nonlinearity of the RGEs then strongly focuses $g_t(\mu)$ at the electroweak scale to its quasi-fixed point value. The RGE for the Higgs self-coupling $\lambda(\mu)$

$$16\pi^2 \frac{d\lambda}{d\ln \mu} = 12\lambda^2 + 3 \left( 4g_t^2 - 3g_2^2 - g_1^2 \right) \lambda + \frac{9}{4}g_2^4 + \frac{3}{2}g_3^2 g_1^2 + \frac{3}{4}g_1^4 - 12g_t^4$$

similarly focuses $\lambda(\mu)$ towards a quasi-fixed point value, leading to the SM fixed point predictions [4] for the running top quark and Higgs masses:

$$m_t \simeq 225 \text{ GeV} \quad m_H \simeq 250 \text{ GeV}$$

Unfortunately these predictions are inconsistent with the experimental running top mass $m_t \simeq 165 \pm 6$ GeV.

The corresponding Minimal Supersymmetric Standard Model (MSSM) quasi-fixed point prediction for the running top quark mass is [5]:

$$m_t(m_t) \simeq (190 \text{ GeV}) \sin \beta$$

which is remarkably close to the experimental value for $\tan \beta = 2 \pm 0.5$. Some of the soft SUSY breaking parameters are also attracted to quasi-fixed point values [6]. For example the trilinear stop coupling $A_t(m_t) \rightarrow$
−0.59m_{\text{gluino}}. For this low tan β fixed point, there is an upper limit on the lightest Higgs boson mass: $m_{h_0} \lesssim 100$ GeV. There is also a high tan β = 60 ± 5 fixed point solution \[6\], corresponding to large Yukawa coupling constants for the $b$ quark and $\tau$ lepton as well as for the $t$ quark, sometimes referred to as the Yukawa Unification scenario. In this case the lightest Higgs boson mass is $m_{h_0} \simeq 120$ GeV. The origin of the large value of tan β is of course a puzzle and also SUSY radiative corrections to $m_t$ are then generically large.

3 Top and Higgs Masses from Multiple Point Principle

According to the Multiple Point Principle (MPP) \[3\], Nature chooses coupling constant values such that a number of vacuum states have the same energy density. This principle was first used in the Anti-Grand Unification Model (AGUT) \[8, 9\], as a way of calculating the values of the SM gauge coupling constants. In the Euclidean (imaginary time) formulation, the theory has a phase transition with the phases corresponding to the degenerate vacua. The coupling constants then become dynamical, in much the same way as in baby-universe theory, and take on fine-tuned values determined by the multiple point. This fine-tuning of the coupling constants is similar to that of temperature in a microcanonical ensemble, such as a mixture of ice and water in a thermally isolated container.

Here we apply the MPP to the pure SM, which we assume valid up to the Planck scale. This implies \[11\] that the effective SM Higgs potential $V_{\text{eff}}(|\phi|)$ should have a second minimum degenerate with the well-known first minimum at the electroweak scale $\langle |\phi_{\text{vac}} 1 | \rangle = 174$ GeV. Thus we predict that our vacuum is barely stable and we just lie on the vacuum stability curve in the top quark, Higgs particle mass ($M_t, M_H$) plane. Furthermore we expect the second minimum to be within an order of magnitude or so of the fundamental scale, i.e. $\langle |\phi_{\text{vac}} 2 | \rangle \simeq M_{\text{Planck}}$. In this way, we essentially select a particular point on the SM vacuum stability curve and hence the MPP condition predicts precise values for the pole masses \[11\]:

$$M_t = 173 \pm 5 \text{ GeV} \quad M_H = 135 \pm 9 \text{ GeV} \quad (5)$$
By imposing symmetries and texture zeros on the fermion mass matrices, it is possible to obtain testable relations between the masses and mixing angles. The best known ansatz for the quark mass matrices is due to Fritzsch [12]:

\[
M_U = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix} \quad M_D = \begin{pmatrix} 0 & C' & 0 \\ C' & 0 & B' \\ 0 & B' & A' \end{pmatrix}
\]

(6)

It contains 6 complex parameters A, B, C, A', B' and C', where it is necessary to assume:

\[
|A| \gg |B| \gg |C|, \quad |A'| \gg |B'| \gg |C'|
\]

(7)
in order to obtain a good fermion mass hierarchy. Four of the phases can be rotated away by redefining the phases of the quark fields, leaving just 8 real parameters (the magnitudes of A, B, C, A', B' and C' and two phases \(\phi_1\) and \(\phi_2\)) to reproduce 6 quark masses and 4 angles parameterising \(V_{CKM}\).

There are thus two relationships predicted by the Fritzsch ansatz:

\[
|V_{us}| \approx \sqrt{\frac{m_d}{m_s}} - e^{-i\phi_1} \sqrt{\frac{m_u}{m_c}}, \quad |V_{cb}| \approx \sqrt{\frac{m_s}{m_b}} - e^{-i\phi_2} \sqrt{\frac{m_c}{m_t}}
\]

(8)
The first prediction is a generalised version of the relation \(\theta_c \approx \sqrt{\frac{m_d}{m_s}}\) for the Cabibbo angle, which originally motivated the two generation Fritzsch ansatz and is well satisfied experimentally. However the second relationship cannot be satisfied with a heavy top quark mass \(m_t > 100\) GeV and the original three generation Fritzsch ansatz is excluded by the data. Consistency with experiment can, for example, be restored by introducing a non-zero 22 mass matrix element. A systematic analysis of symmetric quark mass matrices with 5 or 6 texture zeros at the the SUSY-GUT scale \(M_X\) yields five solutions [13]. An example, in which the non-zero elements are expressed in terms of a small parameter \(\epsilon = \sqrt{\frac{m_c}{m_t}} = 0.058\), is described in Stech’s talk [14].

The minimal SU(5) SUSY-GUT relation (using a Higgs field in the 5 representation) for the third generation, \(m_b(M_X) = m_t(M_X)\), is successful. However it cannot be extended to the first two generations as it predicts \(m_d/m_s = m_c/m_\mu\), which fails phenomenologically by an order of magnitude. This led Georgi and Jarlskog [15] to introduce an ad-hoc coupling of the
second generation to a Higgs field in the $\mathbf{45}$ representation, giving mass matrices with the following texture:

\[
M_U = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix}, \quad M_D = \begin{pmatrix} 0 & F & 0 \\ F' & E & 0 \\ 0 & 0 & D \end{pmatrix}, \quad M_E = \begin{pmatrix} 0 & F & 0 \\ F' & -3E & 0 \\ 0 & 0 & D \end{pmatrix},
\]

(9)

and the successful mass relation $m_d/m_s = 9m_e/m_\mu$. This ansatz has been developed further in the context of an SO(10) SUSY-GUT effective operator analysis [16] to give a good fit to all the masses and mixing angles.

5 Mass Hierarchy from Chiral Flavour Charges

As we pointed out in section 1, a natural resolution to the charged fermion mass problem is to postulate the existence of some approximately conserved chiral charges beyond the SM. These charges, which we assume to be the gauge quantum numbers in the fundamental theory beyond the SM, provide selection rules forbidding the transitions between the various left-handed and right-handed quark-lepton states, except for the top quark. In order to generate mass terms for the other fermion states, we have to introduce new Higgs fields, which break the fundamental gauge symmetry group $G$ down to the SM group. We also need suitable intermediate fermion states to mediate the forbidden transitions, which we take to be vector-like Dirac fermions with a mass of order the fundamental scale $M_F$ of the theory. In this way effective SM Yukawa coupling constants are generated, which are suppressed by the appropriate product of Higgs field vacuum expectation values measured in units of $M_F$.

Consider, for example, an $SMG \times U(1)_f$ model, obtained by extending the SM gauge group $SMG = S(U(3) \times U(2)) \simeq SU(3) \times SU(2) \times U(1)$ with a gauged abelian flavour group $U(1)_f$. $SMG \times U(1)_f$ is broken to SMG by the VEV of a scalar flavour group $U(1)_f$. Then it is natural to expect the generation of a $b$ mass of order:

\[
\left( \frac{\langle \phi_S \rangle}{M_F} \right)^2 \langle \phi_{WS} \rangle,
\]

(10)

via the exchange of two $\langle \phi_S \rangle$ tadpoles, in addition to the usual $\langle \phi_{WS} \rangle$ tadpole, through two appropriately charged vector-like fermion intermediate
We identify $\epsilon_f = \langle \phi_S \rangle / M_F$ as the $U(1)_f$ flavour symmetry breaking parameter. In general we expect mass matrix elements of the form:

$$M(i,j) = \gamma_{ij} \epsilon_f^{n_{ij}} \langle \phi_{WS} \rangle, \quad \gamma_{ij} = O(1), \quad n_{ij} = |Q_f(\psi_{L_i}) - Q_f(\psi_{R_j})|$$

between the left- and right-handed fermion components. So the effective SM Yukawa couplings of the quarks and leptons to the Weinberg-Salam Higgs field $y_{ij} = \gamma_{ij} \epsilon_f^{n_{ij}}$ can consequently be small even though all fundamental Yukawa couplings of the “true” underlying theory are of $O(1)$. However it appears not possible to explain the fermion mass spectrum with an anomaly free set of flavour charges in an $SMG \times U(1)_f$ model. It is necessary to introduce SMG-singlet fermions with non-zero $U(1)_f$ charge to cancel the $U(1)_3^f$ gauge anomaly (as in $MSSM \times U(1)_f$ models which also use anomaly cancellation via the Green-Schwarz mechanism) or by extending the SM gauge group further (as in the AGUT model based on the $SMG^3 \times U(1)_f$ gauge group).

## 6 Neutrino Mass and Mixing Problem

Physics beyond the SM can generate an effective light neutrino mass term

$$\mathcal{L}_{\nu-mass} = \sum_{i,j} \psi_{i\alpha} \psi_{j\beta} \epsilon^{\alpha\beta} (M_\nu)_{ij}$$

in the Lagrangian, where $\psi_{i,j}$ are the Weyl spinors of flavour $i$ and $j$, and $\alpha,\beta = 1, 2$. Fermi-Dirac statistics mean that the mass matrix $M_\nu$ must be symmetric. In models with chiral flavour symmetry we typically expect the elements of the mass matrices to have different orders of magnitude. The charged lepton matrix is then expected to give only a small contribution to the lepton mixing. As a result of the symmetry of the neutrino mass matrix and the hierarchy of the mass matrix elements it is natural to have an almost degenerate pair of neutrinos, with nearly maximal mixing. This occurs when an off-diagonal element dominates the mass matrix.

The recent Super-Kamiokande data on the atmospheric neutrino anomaly strongly suggests large $\nu_\mu - \nu_\tau$ mixing with a mass squared difference of order $\Delta m_{\nu_\mu,\nu_\tau}^2 \sim 10^{-3} \text{ eV}^2$. Large $\nu_\mu - \nu_\tau$ mixing is given by the mass matrix

$$M_\nu = \begin{pmatrix} \times & \times & \times \\ \times & \times & A \\ \times & A & \times \end{pmatrix}$$
where $\times$ denotes small elements and we have $\Delta m_{23}^2 \ll \Delta m_{12}^2 \sim \Delta m_{13}^2$, $\sin^2 \theta_{23} \sim 1$. However, this hierarchy in $\Delta m^2$'s is inconsistent with the small angle (MSW) solution to the solar neutrino problem, which requires $\Delta m_{12}^2 \sim 10^{-5} \text{ eV}^2$. Also the theoretically attractive solution \cite{21} of the atmospheric and solar neutrino problems, using maximal $\nu_e - \nu_\mu$ mixing, seems to be ruled out by the zenith angular distribution of the Super-Kamiokande data. Hence we need extra structure for the mass matrix such as having several elements of the same order of magnitude. For example:

$$M_\nu = \begin{pmatrix} a & A & B \\ A \times & \times & \times \\ B \times & \times & \times \end{pmatrix}$$

(14)

with $A \sim B \gg a$. This gives

$$\frac{\Delta m_{12}^2}{\Delta m_{23}^2} \sim \frac{a}{\sqrt{A^2 + B^2}}.$$  

(15)

The mixing is between all three flavours and is given by the mixing matrix

$$U_\nu \sim \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} \cos \theta & \frac{1}{\sqrt{2}} \cos \theta & -\sin \theta \\ \frac{1}{\sqrt{2}} \sin \theta & \frac{1}{\sqrt{2}} \sin \theta & \cos \theta \end{pmatrix}$$

(16)

where $\theta = \tan^{-1} \frac{B}{A}$. So we have large $\nu_\mu - \nu_\tau$ mixing with $\Delta m^2 = \Delta m_{23}^2$, and nearly maximal electron neutrino mixing with $\Delta m^2 = \Delta m_{12}^2$. The atmospheric neutrino anomaly requires $\sin^2 2\theta \gtrsim 0.7$ or $1/2 \lesssim B/A \lesssim 2$. The solar neutrino problem is explained by vacuum oscillations, although whether it is an ‘energy independent’ or a ‘just-so’ solution depends on the value of the mass squared difference ratio in eq. (15).

There is also some difficulty in obtaining the required mass scale for the neutrinos. In models such as the AGUT the neutrino masses are generated via super-heavy intermediate fermions in a see-saw type mechanism. This leads to too small neutrino masses:

$$m_\nu \lesssim \frac{(\phi_{WS})^2}{M_F} \sim 10^{-5} \text{ eV},$$

(17)

for $M_F = M_{\text{Planck}}$ (in general $m_\nu$ is also supressed by the chiral charges). So we need to introduce a new mass scale into the theory. Either some intermediate particles with mass $M_F \lesssim 10^{15} \text{ GeV}$, or an $SU(2)$ triplet Higgs field $\Delta$ with $\langle \Delta^0 \rangle \sim 1 \text{ eV}$ is required. Without further motivation the introduction of such particles is ad hoc.
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