Monte Carlo simulations of the Ising and the Sznajd model on growing Barabási - Albert networks

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Abstract

The Ising model shows on growing Barabási - Albert networks the same ferromagnetic behavior as on static Barabási - Albert networks. Sznajd models on growing Barabási - Albert networks show an hysteresis like behavior. Nearly a full consensus builds up and the winning opinion depends on history. On slow growing Barabási - Albert networks a full consensus builds up. At five opinions in the Sznajd model with limited persuasion on growing Barabási - Albert networks, all odd opinions win and all even opinions loose supporters.

Keywords: Monte Carlo simulations, Sociophysics, Ising model, Sznajd model, limited persuasion, growing Barabási - Albert networks

1 The Ising model

First, we initiate a Barabási - Albert network \( \mathbb{H} \) with \( m \) nodes. Each node is connected to every other node with exactly one connection. At every time step we put one new node to the network. This new node builds up randomly

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$m$ connections to already existing nodes. The probability for a existing node to be chosen as a neighbor is proportional to the number of its neighbors. After putting a new node on the network, we go through the whole network and use the Ising model on every node. We say that we make one Ising rpts (run per time step).

The explanation of the classical Ising model and how to simulate its Glauber kinetics can be found in [2].

The simulations show the same ferromagnetic behavior as given in [3] for static Barabási - Albert networks. Even when we make more than one Ising rpts the results do not change.

## 2 The Sznajd model

We initiate the network as the Ising model. At every time step we put one new node to the network. After that we randomly chose one node and a few (none, one, or three, depending on the model used) of its neighbors. If all

![Figure 1: Sznajd model on a 25000 nodes network with one Sznajd run per time step, $m = 4$.](image)
these randomly chosen nodes have the same opinion (spin), they convince all their neighbors \[\text{[4]}\]. We are using the opinions \(+1\) and \(-1\).

Initially, every node has with probability \(0 \leq p \leq 100\) the opinion \(+1\) and with probability \(1 - p\) the opinion \(-1\).

For the number of randomly chosen nodes we have a few possibilities:

- two randomly chosen neighbors,
- four randomly chosen neighbors or
- one randomly chosen node

convince all their neighbors.

In Figure 2 we can see that the two- and four-nodes-convincing models come to the same results. In these models nearly a consensus builds up. The one-node-convincing model does not show the same behavior as the other models. We can see too that in the two- or four-nodes-convincing model no
sharp opinion change takes place. We have a area of coexistence of both opinions. Like in magnetic hysteresis, the final magnetization is close to $-1$ or $+1$, depending on history (initial configuration); averages over 100 samples give an average value far away from consensus ($+1$ or $-1$).

The results shown in Figure 1 are nearly equal for every value of $m$.

If we make more Sznajd runs per time step we will see, that a consensus builds up. This result is shown in Figure 2 for the two nodes convincing model. The results are equal to the ones of the four nodes convincing model. We can see too that the area of coexistence becomes smaller for more Sznajd runs per time step.

3 The Sznajd model with limited persuasion

Now I simulated a system with the five opinions 1, 2, 3, 4 and 5. Therefore I used the Sznajd model with limited persuasion on growing Barabási -
Albert networks. A new node has with probability $0 \leq p_i \leq 100$ the opinion $i$. The initial probabilities

- $p_1 = 34$,
- $p_2 = 29$,
- $p_3 = 14$,
- $p_4 = 13$ and
- $p_5 = 10$

have been used.

The procedure is mainly the same as for the classical Sznajd model but node $i$ can convince node $j$ only if $|\text{opinion}(i) - \text{opinion}(i)| = 1$.

For the simulations the two-nodes-convincing model has been used.

The main results can be seen in Figure 3. All even opinions loose supporters while all odd opinions win supporters. Opinion 1 wins the most supporters and has on a 25000 nodes network nearly 60%, depending on the value of $m$ at one Sznajd run per time step. Opinion 2 loosest the most supporters.

If we make more than one Sznajd run per time step the difference becomes even stronger. At small values of $m$ opinion 2 is nearly not existing.

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