Two loop Induced Dirac Neutrino Model and Dark Matters with
Global $U(1)'$ Symmetry

Hiroshi Okada$^{1,*}$

$^1$School of Physics, KIAS, Seoul 130-722, Korea

We propose a two loop induced Dirac type neutrino model at TeV scale. Subsequently, three types of dark matter particles; fermion and two bosons, are naturally introduced. Here we discuss to analyze two possibilities; two component dark matter scenario (Dirac fermion and complex boson) and single dark matter one (another real boson), comparing to current experimental data such as Planck/WMAP and LUX. We briefly mention the possibility to explain the discrepancy of the effective number of neutrino species reported by several experiments.

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$^*$Electronic address: hokada@kias.re.kr
I. INTRODUCTION

The nature of neutrinos is not yet known apparently on even whether Majorana type or Dirac type in spite of several experiments of lepton flavor violating processes such as neutrinoless double beta decay [1–5] as well as their own masses and their mixings [6, 7]. Furthermore, the nature of dark matter (DM) is not also known well in spite of many experiments such as direct detection searches (e.g., XENON100 [8] and LUX [9]), indirect detection searches (e.g., AMS-02 [10], PAMELA [11, 12], Fermi-LAT [13, 14], and XMN-Newton X-ray observatory [15, 16]), IceCube [17], and collider searches such as LHC [18]. These two issues could be the important tasks to be clarified in the future.

In a view of theoretical aspect, on the other hand, radiatively induced neutrino models are one of the elegant solutions to implement a DM candidate within TeV scale [19–55], [56–61]. To achieve such kind of models, an additional local or global symmetry is always required to stabilize the DM candidate. Notice here that the local symmetry requires a continuous symmetry, but the global one allows both a continuous and a discrete one. Once one selects the global (continuous) symmetry, we should take in account a phenomenology of the goldstone boson (GB), which could sometimes provide a promising explanation of the discrepancy of the effective number of neutrino species $\Delta N_{\text{eff}}$ [62]. When one chooses the local one, on the other hand, an exotic neutral gauged boson comes into our world, which gives us another phenomenological interests [63, 64]. In this paper, we show that introducing of a global continuous symmetry, which forbids some tree-level Yukawa Lagrangians, can be achieved in the framework of two loop induced Dirac type neutrino\(^1\). After the global symmetry breaking spontaneously, a remnant symmetry can naturally identify the DM candidate for some mediated particles in neutrino masses.

This paper is organized as follows. In Sec. II, we show our model building including neutrino mass. In Sec. III, we analyze DM nature. We summarize and conclude in Sec. VI.

\(^1\) For another types of Dirac type neutrino, see, i.e., Refs. [65–68], [69, 70], [71].
TABLE I: The particle contents and the charges for fermions.

| Particle | $L_L$ | $e_R$ | $S_L$ | $S_R$ | $N_R$ |
|----------|-------|-------|-------|-------|-------|
| $(SU(2)_L, U(1)_Y)$ | $(2, -1/2)$ | $(1, -1)$ | $(1, 0)$ | $(1, 0)$ | |
| $U(1)'$ | $-2S - N$ | $-2S - N$ | $S$ | $S$ | $N$ |

TABLE II: The particle contents and the charges for bosons.

| Particle | $\eta$ | $\Phi$ | $\chi_1$ | $\chi_2$ | $\Sigma$ |
|----------|--------|--------|----------|----------|----------|
| $(SU(2)_L, U(1)_Y)$ | $(2, 1/2)$ | $(2, 1/2)$ | $(1, 0)$ | $(1, 0)$ | $(1, 0)$ |
| $U(1)'$ | $3S + N$ | $0$ | $-2S - S - N$ | $2(S + N)$ | |

II. THE MODEL

A. Model setup

We discuss a two-loop induced radiative neutrino model. The particle contents are shown in Tab. I and Tab. II. We add three $SU(2)_L$ singlet vector like neutral fermions $S_L$ and $S_R$, three singlet Majorana fermions $N_R$. For new bosons, we introduce $SU(2)_L$ doublet scalar $\eta$ and singlet scalars $\chi_1$, $\chi_2$, and $\Sigma$ to the standard model (SM). We assume that only the SM-like Higgs $\Phi$ and $\Sigma$ have vacuum expectation values (VEVs). The global $U(1)'$ symmetry is imposed so as to restrict their interaction adequately and guarantee DM stability. Moreover, even after the $U(1)'$ symmetry is broken by the VEV of $\Sigma$, a remnant symmetry of $Z_2$ retains which assures the stability.

The quantum number in the tables can be driven as follows. Let us at first define the $U(1)'$ charge $N$ for $N_R$ and $S$ for $S_{L/R}$. Then one finds that all the terms are written in terms of those two charges. But several remarks are as follows:

1. $S + N \neq 0$ to forbid the term of $\bar{L}_L \Phi^\dagger N_R$,
2. $S \neq 0$ to forbid the term of $\Sigma^\ast N_R^c N_R$, $\Sigma \chi_1 (\chi_2)^2$,
3. $S + 2N \neq 0$ to forbid the term of $\Sigma \bar{N}_R^c N_R$,
4. $S \neq N$ to forbid the term of $\bar{L}_L \eta^\dagger N_R^c$.

$\bar{L}_L \eta^\dagger N_R^c$ does not affect to our model, but we assume to be zero for simplicity.
\[3S + 2N \neq 0\] to forbid the term of \(\Sigma^* \chi_1(\chi_2)^2\).

Notice here that our charge assignment does not conflict with these conditions. The five-dimensional term \(\Sigma^* \bar{L}_L \Phi^* N_R\) cannot be forbidden by any symmetries. However once the cut-off scale is taken to be GUT scale \(\Lambda_{\text{GUT}} \sim O(10^{16})\) GeV, its Dirac mass scale is \(\langle \Phi \rangle \langle \Sigma \rangle / \Lambda_{\text{GUT}} \leq O(0.01)\) eV (where we fix \(\langle \Phi \rangle = 246\) GeV and \(\langle \Sigma \rangle = 1000\) GeV), which can be naturally tiny than the active neutrino mass \(O(0.1-1)\) eV. We impose \(S + N = 1\) for our convenience, as we will discuss later.

The renormalizable Lagrangian for Yukawa sector and scalar potential under these assignments are given by

\[
\mathcal{L}_Y = y_L \bar{L}_L \Phi e_R + y_\eta \bar{L}_L \eta^c S_R + y_{\chi_1} \bar{S}_L \chi_1 + y_{\chi_2} \bar{S}_R \chi_2 + M_S \bar{S}_L S_R + \text{h.c.} \quad (\text{II.1})
\]

\[
\mathcal{V} = m_1^2 \Phi^\dagger \Phi + m_2^2 \eta^\dagger \eta + m_3^2 \Sigma^\dagger \Sigma + m_4^2 \chi_1^\dagger \chi_1 + m_5^2 \chi_2^\dagger \chi_2 + \mu [\Sigma(\chi_2)^2 + \text{h.c.}] + \lambda_0 (\Phi^\dagger \eta)(\chi_1 \chi_2) + \text{h.c.} \\
+ \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 (\eta^\dagger \eta)^2 + \lambda_3 (\Phi^\dagger \Phi)(\eta^\dagger \eta) + \lambda_4 (\Phi^\dagger \eta)(\eta^\dagger \Phi) \\
+ \lambda_6 (\Sigma^\dagger \Sigma)^2 + \sum_{i=1,2} \lambda_6^{(i)} (\chi_i^\dagger \chi_i)^2 + \sum_{i=1,2} \lambda_8^{(i)} (\Sigma^\dagger \Sigma)(\eta^\dagger \eta) + \lambda_8 (\Sigma^\dagger \Sigma)(\eta^\dagger \eta) + \sum_{i=1,2} \lambda_8^{(i)} (\chi^\dagger_i \chi_i)(\eta^\dagger \eta), \\
(\text{II.2})
\]

where the first term of \(\mathcal{L}_Y\) can generates the charged-lepton masses, and \(\mu\) and \(\lambda_0\) can be chosen to be real without any loss of generality by renormalizing the phases to scalar bosons. The couplings \(\lambda_1, \lambda_2, \lambda_6\) and \(\lambda_6^{(i)}\) have to be positive to stabilize the Higgs potential.

Inserting the tadpole conditions: \(m_1^2 = -\lambda_1 v^2 - \lambda_7 v'^2/2\) and \(m_3^2 = -\lambda_6 v'^2 - \lambda_7 v'^2/2\), the resulting mass matrix of the neutral component of \(\Phi\) and \(\Sigma\) defined as

\[
\Phi^0 = \frac{v + \phi^0(x)}{\sqrt{2}}, \quad \Sigma = \frac{v' + \sigma(x)}{\sqrt{2}} e^{iG(x)/v'}, \\
(\text{II.3})
\]

is given by

\[
m^2(\phi^0, \sigma) = \begin{pmatrix}
2 \lambda_1 v^2 & \lambda_7 v v' \\
\lambda_7 v v' & 2 \lambda_6 v'^2
\end{pmatrix} = \begin{pmatrix}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{pmatrix} \begin{pmatrix}
m_h^2 & 0 \\
0 & m_H^2
\end{pmatrix} \begin{pmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{pmatrix}, \\
(\text{II.4})
\]

\[3\text{ If this term exists, we can generate a Majorana mass term for } N_R \text{ at two-loop level. As a result, neutrino mass is generated at four-loop level. Such a model could include a little different feature from this model.}\]
where $v = 246$ GeV, $h$ implies SM-like Higgs and $H$ is an additional CP-even Higgs mass eigenstate. The mixing angle $\alpha$ is given by

$$\tan 2\alpha = \frac{\lambda_7 v v'}{\lambda_6 v'^2 - \lambda_1 v^2}.$$

The Higgs bosons $\phi^0$ and $\sigma$ are rewritten in terms of the mass eigenstates $h$ and $H$ as

$$\phi^0 = h \cos \alpha + H \sin \alpha,$$

$$\sigma = -h \sin \alpha + H \cos \alpha.$$  

A goldstone boson $G$ appears due to the spontaneous symmetry breaking of the global $U(1)'$ symmetry.

Each mass eigenstate for the inert Higgses is given as

$$m_{\eta^0}^2 \equiv m^2(\eta^\pm) = m_2^2 + \frac{1}{2} \lambda_3 v^2 + \frac{1}{2} \lambda_8 v'^2,$$

$$m_{\eta^0}^2 = m_2^2 + \frac{1}{2} \lambda_8 v'^2 + \frac{1}{2} (\lambda_3 + \lambda_4) v^2,$$

$$m_{\chi_1}^2 = m_4^2 + \frac{1}{2} \left( \lambda_6^{(1)} v'^2 + \lambda_7^{(1)} v^2 \right),$$

$$m_{\chi_{2R}}^2 = m_5^2 + \frac{1}{2} \left( \lambda_6^{(2)} v'^2 + \lambda_7^{(2)} v^2 + 2\sqrt{2} \mu v' \right),$$

$$m_{\chi_{2L}}^2 = m_5^2 + \frac{1}{2} \left( \lambda_6^{(2)} v'^2 + \lambda_7^{(2)} v^2 - 2\sqrt{2} \mu v' \right).$$

Notice here that $\eta^0$ and $\chi_1$ are complex scalar neutral bosons.

### B. Neutrino mass matrix

The Dirac neutrino mass matrix at two-loop level as depicted in the left hand side of Fig. 1 is given by

$$(m_\nu)_{ab} = \frac{\lambda_0 v}{4} \left[ \left( y_\eta^a (M_S)_c y_{\chi_1}^b (M_S)_d y_{\chi_2}^c \right) \right] \frac{1}{2\sqrt{2} (4\pi)^4(M_S^2_{\nu} - m_{\eta^0}^2)} \left[ F(x_{ia}) |_{\chi_{2R}} - F(x_{ia}) |_{\chi_{2L}} \right],$$

where the loop function $F$ is computed by

$$F(x_{ia}) = \int_0^1 dy \int_0^{1-y} -x_{3d} \ln(x_{3c}) + (\alpha - x_{3d}) \left( \frac{1-y-z-x_{3d} y-x_{3d} z}{y(1-z)^2-x_{1d} z} \right) \ln \left( \frac{\alpha (z-z')}{y-x_{2d} y-x_{1d} z} \right) \frac{1-y-z+x_{2d} y+\{x_{1d} + x_{3d} (z-1)\} z}{},$$

$$F(x_{ia}) = \int_0^1 dy \int_0^{1-y} -x_{3d} \ln(x_{3c}) + (\alpha - x_{3d}) \left( \frac{1-y-z-x_{3d} y-x_{3d} z}{y(1-z)^2-x_{1d} z} \right) \ln \left( \frac{\alpha (z-z')}{y-x_{2d} y-x_{1d} z} \right) \frac{1-y-z+x_{2d} y+\{x_{1d} + x_{3d} (z-1)\} z}{},$$

$$F(x_{ia}) = \int_0^1 dy \int_0^{1-y} -x_{3d} \ln(x_{3c}) + (\alpha - x_{3d}) \left( \frac{1-y-z-x_{3d} y-x_{3d} z}{y(1-z)^2-x_{1d} z} \right) \ln \left( \frac{\alpha (z-z')}{y-x_{2d} y-x_{1d} z} \right) \frac{1-y-z+x_{2d} y+\{x_{1d} + x_{3d} (z-1)\} z}{},$$

$$F(x_{ia}) = \int_0^1 dy \int_0^{1-y} -x_{3d} \ln(x_{3c}) + (\alpha - x_{3d}) \left( \frac{1-y-z-x_{3d} y-x_{3d} z}{y(1-z)^2-x_{1d} z} \right) \ln \left( \frac{\alpha (z-z')}{y-x_{2d} y-x_{1d} z} \right) \frac{1-y-z+x_{2d} y+\{x_{1d} + x_{3d} (z-1)\} z}{},$$

$$F(x_{ia}) = \int_0^1 dy \int_0^{1-y} -x_{3d} \ln(x_{3c}) + (\alpha - x_{3d}) \left( \frac{1-y-z-x_{3d} y-x_{3d} z}{y(1-z)^2-x_{1d} z} \right) \ln \left( \frac{\alpha (z-z')}{y-x_{2d} y-x_{1d} z} \right) \frac{1-y-z+x_{2d} y+\{x_{1d} + x_{3d} (z-1)\} z}{},$$

$$F(x_{ia}) = \int_0^1 dy \int_0^{1-y} -x_{3d} \ln(x_{3c}) + (\alpha - x_{3d}) \left( \frac{1-y-z-x_{3d} y-x_{3d} z}{y(1-z)^2-x_{1d} z} \right) \ln \left( \frac{\alpha (z-z')}{y-x_{2d} y-x_{1d} z} \right) \frac{1-y-z+x_{2d} y+\{x_{1d} + x_{3d} (z-1)\} z}{},$$

$$F(x_{ia}) = \int_0^1 dy \int_0^{1-y} -x_{3d} \ln(x_{3c}) + (\alpha - x_{3d}) \left( \frac{1-y-z-x_{3d} y-x_{3d} z}{y(1-z)^2-x_{1d} z} \right) \ln \left( \frac{\alpha (z-z')}{y-x_{2d} y-x_{1d} z} \right) \frac{1-y-z+x_{2d} y+\{x_{1d} + x_{3d} (z-1)\} z}{},$$

$$F(x_{ia}) = \int_0^1 dy \int_0^{1-y} -x_{3d} \ln(x_{3c}) + (\alpha - x_{3d}) \left( \frac{1-y-z-x_{3d} y-x_{3d} z}{y(1-z)^2-x_{1d} z} \right) \ln \left( \frac{\alpha (z-z')}{y-x_{2d} y-x_{1d} z} \right) \frac{1-y-z+x_{2d} y+\{x_{1d} + x_{3d} (z-1)\} z}{},$$
where $\alpha \equiv (M_{Sc}/M_{Sd})^2$, $x_{ia} \equiv (m_{\chi_i}/M_{Sa})^2$ with $\chi_3 \equiv \eta^0$ and the indices of $x$ are defined as $i = (1, 2, 3)$ and $a = (c, d)$ \footnote{One can find the original Zee–Babu type neutrino formula in the limit of $M_S \to 0$ and $x_3 \to x_2$ \cite{73}.}. One finds rather wide allowed range to explain the neutrino masses reported by Planck data \cite{77}; $m_\nu < 0.933$ eV, with the following parameters: $(y_{\eta} y_{\chi_1}^* y_{\chi_2}) = \mathcal{O}(0.1)$, $M_S = \mathcal{O}(500)$ GeV, $m_{\chi_1} = \mathcal{O}(500)$ GeV, $m_{\eta^0} = \mathcal{O}(1000)$ GeV, $\mu = \mathcal{O}(0.1)$ GeV, and $v' = \mathcal{O}(1000)$ GeV, and $\lambda_0 = \mathcal{O}(0.5)$.

Lepton Flavor Violations (LFVs): $\mu \to e\gamma$ process gives the most stringent bound. The upper limit of the branching ratio is given by $\text{Br} (\mu \to e\gamma) \leq 5.7 \times 10^{-13}$ at 95\% confidence level from the MEG experiment \cite{75}.

Our contribution to the $\mu \to e\gamma$ process only comes from the coupling of $y_\eta$ and its branching ratio can be computed as

$$\text{Br} (\mu \to e\gamma) = \frac{3\alpha_{em}}{64\pi G_F^2 m_\eta^4} \left| \sum_i (y_\eta)^i_{i\mu} (y_\eta)^*_{i\mu} F_2 \left( \frac{M_S^2}{m_\eta^2} \right) \right|^2, \quad (\text{II.14})$$

where $\alpha_{em} = 1/137$ is the fine structure constant, $G_F$ is the Fermi constant and $F_2(x)$ is the loop function defined in ref. \cite{76}.

Ad can be seen in these Eq. (II.12) and Eq. (II.14), we can avoid this constraint very easily by taking that $y_\eta$ is diagonal. This is because neutrino sector has a lot of free parameters such as $y_{\chi_1}$ or $y_{\chi_2}$, from which we could obtain observed mixings as well as active neutrino masses \cite{6}.
III. DARK MATTER

We have three DM candidates: the lightest one of three vector like fermions $S$, the lightest one of $\chi_1$ (complex scalar) and $\chi_2$ (real scalar), as a result of the remnant symmetry $Z_2$ after the breaking of $U(1)’$ symmetry. Here we consider $S$ and $\chi_1$ as multicomponent DM scenario, since they do not decay into SM particles at leading order. Also we consider $\chi_2$ as a single DM scenario\(^5\). Notice here that neutral $\eta$ component is ruled out by the direct detection through $Z$-boson particle, since it is a complex scalar. Here we discuss to analyze two cases: multicomponent DMs scenario ($S, \chi_1$) and single DM scenario $\chi_2$.

A. Multicomponent Dark Matter scenario

At first, we will discuss the relic density of DMs; $\Omega h^2 \approx 0.12$, reported by Planck \[77\]. The DM ($S$) can annihilate into the other DM ($\chi_1$), but cannot decay into the SM particles with the renormalizable interactions. We have to compute the set of Boltzmann equations in order to obtain the correct relic density of those two DMs. The set of Boltzmann equations is written as

\[
\frac{dn_S}{dt} + 3Hn_S = -\langle \sigma_S v_{\text{rel}} \rangle \left( n_S^2 - n_S^{eq} \right) + \langle \sigma_{\text{ex}} v_{\text{rel}} \rangle \left[ n_{\chi_1}^2 - \left( \frac{n_{\chi_1}^{eq}}{n_S^{eq}} \right)^2 n_S^2 \right], \tag{III.1}
\]

\[
\frac{dn_{\chi_1}}{dt} + 3Hn_{\chi_1} = -\langle \sigma_{\chi_1} v_{\text{rel}} \rangle \left( n_{\chi_1}^2 - n_{\chi_1}^{eq} \right) - \langle \sigma_{\text{ex}} v_{\text{rel}} \rangle \left[ n_{\chi_1}^2 - \left( \frac{n_{\chi_1}^{eq}}{n_S^{eq}} \right)^2 n_S^2 \right], \tag{III.2}
\]

where the time of universe is expressed by $t$, $n_S$ and $n_{\chi_1}$ are the number density of $S$ and $\chi_1$ respectively. The thermally averaged annihilation cross section into all channels is written as $\langle \sigma_S v_{\text{rel}} \rangle$ for $S$. For $\chi_1$, the total cross section into the SM particles is written by $\langle \sigma_{\chi_1} v_{\text{rel}} \rangle$. $\langle \sigma_{\text{ex}} v_{\text{rel}} \rangle$ is the cross section of the DM exchange process $SS \rightarrow \chi_1^* \chi_1$. Notice here that we assume $m_{\chi_1} \leq 2M_S$, otherwise $\chi_1$ can decay into $2S$.

Fermionic DM (S): The dominant cross section for $S$ is obtained through $t$-channel of $\eta$ in the limit of massless final state lepton pairs as follows:

\[
\langle \sigma v_{\text{rel}} \rangle(\bar{S}S \rightarrow \ell \bar{\ell}) \approx \frac{|y^\dagger \eta y_{\ell}|^2}{128\pi M_S^2 (1 + x_3)^2} \left[ 1 - \frac{1 - x_3^2 + 3x_3}{3(1 + x_3)^2} v_{\text{rel}}^2 \right], \tag{III.3}
\]

\(^5\) Since the property of $\chi_{2R}$ and $\chi_{2L}$ is the same, we focus on the $\chi_{2L}$ as a DM candidate taking positive sign of $\mu$. 

\[\]
FIG. 2: Allowed regions of DM masses to obtain the observed Relic density $\Omega h^2 \approx 0.12$, where $m_{\chi_1} \leq 2M_S$ is assumed to forbid the rapid decay between DMs.

where $M_{DM}$ is the mass of $S$, $1 \leq x_3 = m_\eta^2 / M_S^2$, and we assume $m_\eta = m_{\eta^0}$ for simplicity.

*Bosonic DM ($\chi_1$):* There are four final state annihilation modes at tree level: $\chi_1 \chi_1^* \rightarrow hh, ZZ, W^+W^-, f\bar{f}$. Each cross section is given in Ref. [52].

*Exchange contribution of $\bar{S}S \rightarrow \chi_1^*\chi_1$:* The DM exchange channel $\bar{S}S \rightarrow \chi_1^*\chi_1$ via t-channel is found as

$$
\sigma_{ex} v_{rel} (\bar{S}S \rightarrow \chi_1^*\chi_1) \approx \frac{|y_{\chi_1}|^4}{128\pi(m_{\chi_1}^2 - 2M_S^2)^2} \sqrt{1 - \frac{m_{\chi_1}^2}{M_S^2}} \times \\
\left[ -m_{\chi_1}^2 + M_S^2 - \frac{m_{\chi_1}^6 - 6m_{\chi_1}^4M_S^2 + 20m_{\chi_1}^2M_S^4}{24(m_{\chi_1}^2 - 2M_S^2)^2}v_{rel}^2 \right], \quad (III.4)
$$

*Parameter set as a solution of the relic density:* We simply show an allowed region to obtain a observed relic density $\Omega h^2 \approx 0.12$ under the following cross sections in Fig. 2

$$
a_S = 4 \times 10^{-9} \text{ GeV}^{-2}, \quad b_S = 2.5 \times 10^{-12} \text{ GeV}^{-2}, \quad a_{\chi_1} = 4 \times 10^{-9} \text{ GeV}^{-2}, \\
b_{\chi_1} = 4 \times 10^{-12} \text{ GeV}^{-2}, \quad a_{ex} = 4 \times 10^{-10} \text{ GeV}^{-2}, \quad b_{ex} = 0, \quad (III.5)
$$

where each of $a_i$ and $b_i$ is the the s-wave contribution and the p-wave one ($i = S, \chi_1, ex$).

As can be seen from Fig. 2 we obtain

$$
50 \text{ GeV} \lesssim M_S \lesssim 1000 \text{ GeV}, \quad 50 \text{ GeV} \lesssim m_{\chi_1} \lesssim 250 \text{ GeV}. \quad (III.6)
$$
**Direct detection:** Only $\chi_1$ DM candidate can contribute to the spin independent elastic cross section that can be obtained through neutral Higgses as

$$\sigma_p = \frac{C\mu^2 m_p^2}{\pi m_{\chi_1}^2 v^2} \left( \frac{\mu_{\chi h} \cos \alpha}{m_h^2} + \frac{\mu_{\chi H} \sin \alpha}{m_H^2} \right)^2,$$

where $\mu_\chi$ is reduced mass defined as $\mu_\chi = (m_{\chi_1}^{-1} + m_p^{-1})^{-1}$, $m_p = 938$ MeV is the proton mass and $C \approx 0.079$. The elastic cross section is constrained by LUX as $\sigma_p \lesssim \mathcal{O}(10^{-45})$ cm$^2$ at around the point $m_{\chi_1} \approx \mathcal{O}(100)$ GeV [9]. The cubic couplings $\mu_{\chi h}$ and $\mu_{\chi H}$ are the three point vertex of $\chi_1 \chi_1 h$ and $\chi_1 \chi_1 H$ with a mass dimension, and can be written as a function of $(\lambda_1, \lambda_6, \lambda_6^{(1)}, \lambda_6^{(n)}, \lambda_7, \lambda_7^{(1)}$, $v, v', \alpha)$, which are not proportional to the term of $\mu$. Hence it is easy to satisfy the constraint of the direct detection experiments, by controlling $\mu_{\chi h}$ and $\mu_{\chi H}$.

**B. Bosonic DM $\chi_{2I}$**

**Relic density.** The dominant contribution for the relic density comes from the GB boson final state through the four-point interaction, $s$-channel, and $t(u)$-channel due to the $\mu$ term. Its thermal averaged cross section can be then obtained as

$$\langle \sigma v_{\text{rel}} \rangle \approx \frac{M_{\chi_{2I}}^2 (m_{\chi_{2I}}^4 + 4M_{\chi_{2I}}^2 - 5m_{\chi_{2I}}^2 M_{\chi_{2I}}^2 - 4\sqrt{2}\mu' m_{\chi_{2I}}^2 - 4\sqrt{2}\mu v') M_{\chi_{2I}}^2)^2}{64\pi v'^4 (m_{\chi_{2I}}^2 + M_{\chi_{2I}}^2)^2 (m_{\chi_{2I}}^2 - 4M_{\chi_{2I}}^2)^2},$$

where $M_{\chi_{2I}}$ is the mass of $\chi_{2I}$, we abbreviate the $p$-wave due to the complicated form.

**Direct detection:** The spin independent elastic cross section can be obtained through neutral Higgses as

$$\sigma_p = \frac{C\mu^2 m_p^2}{\pi M_{\chi_{2I}}^2 v^2} \left( \frac{\mu_{\chi_{2I} h} \cos \alpha}{m_h^2} + \frac{\mu_{\chi_{2I} H} \sin \alpha}{m_H^2} \right)^2,$$

where $\mu_\chi$ is reduced mass defined as $\mu_\chi = (M_{\chi_{2I}} + m_p^{-1})^{-1}$, $m_p = 938$ MeV is the proton mass and $C \approx 0.079$. Here each of $\mu_{\chi_{2I} h}$ and $\mu_{\chi_{2I} H}$ is the three point vertex of $\chi_{2I} \chi_{2I} h$ and $\chi_{2I} \chi_{2I} H$ with a mass dimension, and can be written as a function of $(\mu, \lambda_6^{(2)}, \lambda_6^{(n(2)}, \lambda_7^{(2)}, v, v', \alpha)$. One finds that there exists wide allowed region to satisfy the observed relic density and the constraint of the direct detection experiments due to the similar property of $\lambda_{2I}$, using the same bench parameter set as those of neutrino sector.

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$^6$ $\chi_{2I}$, of course, has the same annihilation channels as the $\chi_1$. 
Thus these quartic couplings are required to be $O(0.5)$ in order to satisfy the constraint when $v' \sim 1$ TeV and $\sin \alpha \sim 1$. Due to the strong constraint from direct detection of DM, the annihilation cross section for the process $\chi_{2I} \chi_{2I} \rightarrow f \bar{f}$ via Higgs s-channel $^7$ is extremely suppressed, which is given by

$$\sigma v_{\text{rel}} = \frac{y_f^2}{2\pi} \left(1 - \frac{4m_f^2}{s}\right)^{3/2} \left|\frac{\mu_{\chi_{2I} h} \cos \alpha}{s - m_h^2 + im_h \Gamma_h} + \frac{\mu_{\chi_{2I} H} \sin \alpha}{s - m_H^2 + im_H \Gamma_H}\right|^2,$$

where $s \approx 4m_{\chi_{2I}}^2 (1 + v_{\text{rel}}^2/4)$, $\Gamma_h$ and $\Gamma_H$ are the decay width of $h$ and $H$. This is because GB final state is the dominant.

$\Delta N_{\text{eff}}$: The discrepancy of the effective number of neutrino species $\Delta N_{\text{eff}}$ has been reported by several experiments such as Planck $^{77}$, WMAP9 polarization $^{78}$, and ground-based data $^{79, 80}$, which tell us $\Delta N_{\text{eff}} = 0.36 \pm 0.34$ at the 68% confidence level. Such a deviation $\Delta N_{\text{eff}} \approx 0.39$ is achieved due to GB in our model, if the following condition can be satisfied $^{52}$:

$$\sin^2 2\alpha (m_h^2 - m_H^2)^2 m_\mu^2 m_{\text{pl}} \approx 1,$$

where $m_{\text{pl}} \approx 1.2 \times 10^{19}$ GeV is the Planck mass and $m_\mu \approx 105.7$ MeV is the muon mass. It implies that an extra neutral boson $H$ to be tiny $O(500)$ MeV, and $\alpha$ is small enough. As a result, the DM mass should be less than $O(5)$ GeV. This could be achieved by our scenario in a different parameter set $^{52}$, since the dominant relic density of our DM does not include such a light extra Higgs.

IV. CONCLUSIONS

We have constructed a two-loop induced Dirac neutrino model with a global $U(1)'$ symmetry, in which we have naturally introduced DMs; Dirac fermion and neutral scalar bosons. Due to several Yukawa couplings related to neutrinos, we can easily control such parameters to avoid any LFV processes like a $\mu \rightarrow e, \gamma$.

We have analyzed two possibilities of the DM candidate; two component scenario with $S$ and $\chi_1$, and single boson one $\chi_{2I}$. As for two component scenario, we have computed the Boltzmann equation explicitly depicted the figure of the observed relic density in terms of two DM masses with a fixed parameter set of the cross section in Fig. 2. We have also

$^7$ Notice here that $2Z$ or $W^\pm$ final state mode does not appear in the limit of $\alpha = 0$. 
discussed the direct detection, in which it is easy to satisfy the current bound due to some free parameters that are not related to the relic density. As for bosonic DM ($\chi_{2r}$), we have shown that there exists a solution to satisfy the observed relic density and the direct detection, since some parameters that are used to each main channel are separate.

Also we have briefly mentioned the possibility to explain the observed discrepancy of the effective number of neutrino species.

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