Vertical helicity flux as an index of interannual atmospheric variability

M V Kurgansky, L O Maksimenkov and O G Chkhetiani
A M Obukhov Institute of Atmospheric Physics, Russian Academy of Sciences, Moscow, Russia

E-mail: kurgansk@ifaran.ru

Abstract. As an index of interannual climatic variability of the atmosphere, it is proposed to calculate the weighted average over the hemisphere area (to the pole from latitude 20°) of the vertical helicity flux across the upper boundary of the planetary boundary layer, which is determined by the product of the Coriolis parameter and the square of the wind speed at this boundary. In practical calculations, the data of reanalyses ERA-Interim and ERA-20C on the wind speed at the isobaric level of 850 hPa were used. The statistical distribution of the Earth’s surface area on the helicity flux values, as well as the informational entropy of this distribution, are calculated. It is shown that the introduced index usefully characterizes the interannual climate variability of the atmosphere in both hemispheres.

1. Preliminaries
This paper discusses the application of the fundamental geophysical fluid dynamical concept of the flow helicity to the analysis of interannual atmospheric variability. For a completely general form of the turbulent friction force and with the only condition that the vector sum of the pressure gradient force, the Coriolis force and the turbulent friction force is equal to zero within the planetary boundary layer (PBL), the following general expression holds [1, 2, 3], see also Appendix,

$$ s = f V_g^2 $$

for the absolute value of the helicity destruction rate in an air column of unit cross section within the PBL. Here $f$ is the Coriolis parameter and $V_g$ is the vector of the geostrophic wind in the free atmosphere, above the PBL top. When deriving (1), the traditional approximation is made, i.e. the horizontal component of the angular velocity of the Earth rotation is neglected in the governing equations, and it is also assumed that the effect of baroclinicity (the thermal wind effect) is small, amounting to no more than a few percent.

In the steady equilibrium state, the destruction of helicity within the PBL is exactly compensated by the influx of helicity from the free atmosphere across the upper boundary of the PBL. The corresponding downward flux of helicity across a unit area of the upper boundary of the PBL is given by the same expression (1). This flux, which has a well-defined finite value (1) and is completely independent of the details of dissipative processes occurring within the PBL, characterizes the level of vortex activity in the free atmosphere and can be considered as a measure of the instability of large-scale atmospheric processes that constitute the general circulation of the atmosphere.
Based on quite different arguments, the expression (1) was later independently obtained by Deusebio and Lindborg [4], who further assumed that the helicity entering the planetary boundary layer from the free atmosphere undergoes then a nonlinear cascade and is transferred to turbulent motions of ever decreasing scales, down to the Kolmogorov microscale. However, direct field measurements of helicity in a real atmosphere [5] revealed a sign of helicity opposite to what was expected. The goal of the work by Chkhetiani et al. [6] was, among other things, to interpret the results of these measurements, using for this purpose the idea of superimposing an Ekman spiral on the local breeze circulation developing over the measuring polygon in Tsimlyansk, which provides a local left rotation of the wind velocity vector with height and explains the observed negative helicity values.

The problem of determining the characteristics of the turbulent helicity in the PBL can be approached differently if to take into account the coincidence of the physical units of the helicity flux density (1) and the buoyancy flux $B$ and, in the frame of the emerging analogy, to proceed from the definition of the Obukhov scale $L = -u'_r/(\kappa B)$, where $u_*$ is the friction velocity and $\kappa = 0.4$ is the von Karman constant. Making a replacement $2gBf \Rightarrow V$ in this expression, we can try to determine the characteristic scale of the “helicity-containing” vortices, $|L| = u'_r/(\kappa fV^2_g)$, and for $u_* \sim 0.3 \text{ ms}^{-1}$, $f \sim 10^{-4} \text{ s}^{-1}$, $V_g \sim 10 \text{ ms}^{-1}$, we get that $|L| \sim 7 \text{ m}$. The corresponding estimate for the magnitude of the turbulent helicity per unit volume is given by $\chi \sim fV^2_g/u_* \sim 3 \times 10^{-2} \text{ ms}^{-2}$. The obtained numbers closely correspond to both the dimensions of the experimental setup for measuring helicity (a rectangular tetrahedron with a side of 5 meters with four acoustic anemometers in its vertices) and the measured in [5] helicity absolute values themselves.

The total account for the degree of vortex activity over the hemisphere is given by the integral of (1) over the hemispheric area, i.e.

$$S = \iint \frac{fV^2_g}{\kappa} \mathrm{d}\sigma.$$  

For the same values of the integral $I = \iint V^2_g \mathrm{d}\sigma$, the $S$-values will be higher by magnitude if the circulation systems that make the main contribution to $I$ (in fact, cyclones in the “storm track” area) are shifted closer to the poles. Thus, the total flux of helicity emphasizes the processes occurring at high latitudes.

Since the derivation of (1) is essentially based on the assumption of quasi-geostrophicity of motions in the free atmosphere, all calculations of the helicity flux were carried out over the region of the corresponding hemisphere to the pole from geographical latitude $20^\circ$. In this case, the square of the geostrophic wind speed was replaced by the square of the actual wind speed taken at the isobaric level of 850 hPa. This relatively high level was chosen in order, for example, to avoid difficulties in calculating the helicity flux over Greenland and Antarctica. Control calculations for the 900 hPa level gave similar results.

Specific examples of the distribution of the helicity flux density $s$ over the Earth's surface are given in [7, 8]. Calculations of the statistical distribution of $s$-values over the Earth's surface show the nature of such a distribution very close to the negative exponent. This circumstance will be discussed in more detail below.

2. Analysis into interannual atmospheric variability

We will now focus on the analysis of the interannual variability of the helicity flux $S$. Specifically, the average value of the helicity flux density (1) over the hemispheric area was calculated,

$$\bar{S} = \iint \frac{fV^2_g}{\kappa} \mathrm{d}\sigma / \iint \mathrm{d}\sigma.$$  

(3)
ERA-Interim re-analysis data for the months of January and July 1980–2018 were used as the main data source. The spatial resolution of the data set is approximately 80 km (T255 spectral) on 60 vertical levels from the surface up to 0.1 hPa; the model-level fields are interpolated to 37 pressure levels. Additionally, as auxiliary, the data from the re-analysis ERA-20C for the same months of 1980-2010 were used; the horizontal resolution is approximately 125 km (spectral truncation T159), atmospheric data are also on 37 pressure levels (as in ERA-Interim). The calculation results are shown in figure 1.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Interannual variability of the hemispheric averaged helicity flux $\tau$ (in m$^2$ s$^{-3}$) across the surface of 850 hPa. Monthly-mean $\tau$ -values according to reanalyses Era-Interim (blue line) and ERA-20C (red line): NH, January (a); NH, July (b), SH, January (c), and SH, July (d). Red arrows indicate the years when “heat waves” were observed in Europe.

It is noteworthy that the calculations according to the ERA-20C, although they correlate well with the calculations according to the ERA-Interim, systematically lead to the smaller helicity flux magnitude. This is especially noticeable in the Southern Hemisphere, where there is also a significant trend in
ERA-20C-based results, which reflects an increase in the vortex activity in the atmosphere of the Southern Hemisphere in the beginning of 21st century compared to the last two decades of 20th century. In calculations based on ERA-Interim data, this trend also exists, although it is less pronounced. The question arises whether this trend reflects real changes in the atmosphere (in other words, in the “atmospheric projection” of the Earth climate system) or is, at least partially, a consequence of improving of re-analyses, especially ERA-20C, given the scarcity of observational data over the Southern Hemisphere. Interestingly, in the Northern Hemisphere such a trend is absent and in January the $\bar{s}$ -values calculated from the data of both re-analyses are very close; however, in July, in the context of a fairly good correlation, in general, the $\bar{s}$ values calculated from ERA-20C are systematically lower than those calculated from ERA-Interim.

It is noteworthy that while in the winter of the Northern Hemisphere $\bar{s}$ changes quite irregularly, in the summer of the Northern Hemisphere there are quite clear fluctuations with a period of about 4-5 years. We can hypothesize (cf. [8]) that the corresponding maxima are in some correlation with the “heat waves” in Europe (both Western and Eastern, including the European territory of Russia) in 2006, 2010, 2015 and 2018, as well as with the “heat wave in North America in 2011. This correlation can be explained by the fact that the long-lived atmospheric blocking that causes these “heat waves” is itself characterized by low helicity flux densities (1), but the blocking action is always accompanied by intense cyclonic activity over the hemisphere that supports and feeds these blocking structures. In particular, cyclones are observed, bypassing the blocking region from the north and thus making a particularly large contribution to the total helicity flux. Thus, in total, an increased value of the average helicity flux $\bar{s}$ is obtained. Unfortunately, the “heat wave” of 2003 over Western Europe does not fit into this scheme, and an explanation must be sought for this. On the other hand, it is known that year 2004 was characterized by the absence of “heat waves” over the Northern Hemisphere, and this fact is in good correlation with the absolute minimum of $\bar{s}$ in figure 1.

The situation is quite similar to that taking place on the Sun. There, during the period of increased solar activity and the mass appearance of sunspots, whose temperature is by 1,500 K lower than the normal temperature of the photosphere of the Sun, and which therefore appear dark, the total radiation from the Sun increases, since the sunspots are always accompanied by bright areas (solar faculae) with higher than normal temperature. According to the Stefan–Boltzmann law, radiation depends on the temperature to the fourth degree; therefore, the excess of radiation from hardly visible by eye bright regions exceeds the radiation deficit from well-visible dark sunspots. Something similar happens in the Earth’s atmosphere with respect to the helicity flux during persistent blocking events, although the non-linearity inherent in (2) is not of the fourth, but only the second degree. Also, in our case displacements of cyclones to the north due to the persistent blocking, i.e. in the direction of the Coriolis parameter increase, play the role.

An increase in the helicity flux over the hemisphere during blocking events well-correlates with the idea of a higher degree of instability of atmospheric circulation during such events, characterized by anomalously high values of finite-time largest Lyapunov exponents; see [9] and references therein.

The statistical distribution $\mu(s)$ of the helicity flux density $s$ over the Earth’s surface and the informational entropy $H = -\int \mu(s) \log \mu(s) ds$ of this distribution were calculated. The distribution $\mu(s)$ is normalized to unity and its first moment is $\bar{s}$. For both hemispheres, separately, the exponential distribution

$$\mu_b(s) = |\bar{s}|^{-1} \exp(-s/\bar{s}).$$

supplies the maximum of the information entropy $H$, provided $\bar{s} = -\int s \mu(s) ds = \text{const}$, and this maximum equals to $H_b = \log |\bar{s}| + \text{const}$. The difference $\Delta H = H_b - H > 0$ characterizes the degree of
closeness between \( \mu(s) \) and \( \mu_\mu(s) \) distributions. This is the measure of the amount of information that \( \mu(s) \) possesses in addition to the known \( \overline{s} \)-value; cf. [10].

![Figure 2](image_url)

**Figure 2.** Interannual variability of the factual (red line) and Boltzmann (blue line) informational entropy of the hemispheric area statistical distribution on the helicity flux \( s \) values across the surface of 850 hPa: NH, January (a); NH, July (b), SH, January (c), and SH, July (d). Red arrows indicate the years when “heat waves” were observed in Europe.

For practical calculations, which follow the methodology applied in [11], the \( \mu(s) \)-values were set in the form of a table \( \mu_i, \ i=1,\ldots,M \), such as \( \sum_{i=1}^{M} \mu_i = 1 \) and \( \sum_{i=1}^{M} s_i \mu_i = \overline{s} \). In our case \( M = 20 \).
Information entropy is calculated as $H = -(\ln M)^{-1} \sum_{i=1}^{M} \mu_i \log \mu_i$, and the exponential distribution corresponds to a geometric progression $\mu_i^B = \mu_i^B \alpha^{i-1}$ with a denominator $\alpha \approx (2\bar{x} \pm \Delta s)/(2\bar{x} \pm \Delta s)$, which at $M \gg 1$ is also determined by the chosen cell (bin) size $\Delta s$ on the $s$-axis. For the informational entropy of the exponential distribution, which we name herein the Boltzmann entropy, a working expression

$$H_B \approx \left\{ \frac{\ln M}{1-\alpha} \left[ (1-\alpha) \log(1-\alpha) + \alpha \log \alpha \right] \right\}$$

is thus obtained. The calculation results are shown in figure 2.

Figure 2 makes two points evident. First, the $\mu(s)$ and $\mu_B(s)$ distributions are, in fact, close to each other, the measure of proximity being the ratio $(H_B - H)/H$ that is on the order of 1/30 or even less. Second, according to (4), the $\bar{x}$-value completely determines $\mu_B(s)$. This, in virtue of point #1, explains a good correlation between the curves in figures 1 and 2.

We also compared the calculations based on (3) with those based on a more complete formula for the so-called nonlinear Ekman boundary layer [7], which takes into account the curvature of isobars in the free atmosphere

$$\bar{x} = \int \left( f + 2\nabla^2 \psi \left( \nabla \psi \right)^2 \right) \frac{d\sigma}{6} \int \frac{d\sigma}{6}, \quad (5)$$

where $\psi$ is the stream function in the free atmosphere. Formula (5) takes into account that the helicity flux density increases in cyclones, while in anticyclones, on the contrary, it decreases.

![Figure 3](image-url)  
**Figure 3.** The seasonal course of monthly-mean helicity flux $\bar{x}$ (in m$^2$ s$^{-3}$) in 2015 calculated using expression (3) (blue line) and expression (4) (red line): NH (a) and SH (b).
the suitability and sufficient accuracy of using the simpler formulation (3) in mass calculations of helicity fluxes for studying the interannual atmospheric variability.

3. Concluding remarks

It is shown that the helicity flux across the top of planetary boundary layer serves as an informative characteristic of the general atmospheric circulation over both hemispheres. It quantifies both the intensity and latitudinal position of large-scale circulation systems and is capable of revealing peculiarities in both seasonal and interannual climate variability of the atmosphere.

It is prospective to study in more detail an existing correlation between temporal changes in the total helicity flux and persistent summer-time blocking events over the Northern Hemisphere, and also a similar correlation with sea-ice-cover conditions in the Arctic.

Due to relative ease of the helicity flux calculation and interpretation, it is prospective to compute the helicity flux for model projections for 21 century, based e.g. on outputs of the ensembles of climate models CMIP5/CMIP6.

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Appendix

The time rate of helicity frictional destruction per unit volume is given by the general fluid dynamical expression $2\mathbf{\omega} \cdot \mathbf{F} \ [1, 2, 3]$, where $\mathbf{\omega}$ is the absolute vorticity vector and $\mathbf{F}$ the friction force vector.

Inside the planetary boundary layer, this general expression is approximated with good accuracy as

$$2\mathbf{\omega} \cdot \mathbf{F} \approx -2 \frac{\partial v}{\partial z} F + 2 \frac{\partial u}{\partial z} G . \quad (A1)$$

On the right-hand side of (A1), one finds the horizontal components $(u, v)$ of the wind velocity vector and the corresponding, but in all other respects absolutely arbitrary, horizontal components of the turbulent friction force $(F, G)$ inside the boundary layer; $z$ is the vertical coordinate (height). Using the equations of motion in the quasi-static approximation and assuming that the components of the pressure gradient force do not depend on the height within the boundary layer and are fully determined by the components $(u_g, v_g)$ of the geostrophic wind in the free atmosphere, we will have the governing equations

$$-f (v - v_g) = F, \quad f (u - u_g) = G . \quad (A2)$$

Substituting (A2) into (A1) one gets

$$2\mathbf{\omega} \cdot \mathbf{F} \approx 2f \frac{\partial}{\partial z} \left( \frac{(u - u_g)^2 + (v - v_g)^2}{2} \right) = 2f \frac{\partial}{\partial z} \left( \mathbf{V} - \mathbf{V}_g \right)^2 . \quad (A3)$$

Integrating (A3) over the entire height of the PBL, taking into account the fact that the wind velocity $\mathbf{V}$ vanishes on the Earth’s surface and $\mathbf{V} \to \mathbf{V}_g$ at the upper boundary of the PBL, we arrive at an expression $-f \mathbf{V}_g^2$ that is fully consistent with (1), since we are talking in (1) about the absolute value of the quantity in question.
The Ekman layer is, figuratively speaking, constantly ventilated by helicity, which is destroyed by
turbulent viscosity and is replenished by its influx from the free atmosphere. This ‘helical metabolism’
is indeed very fast. For the classical Ekman spiral, the total helicity of an air column of unit cross-
section is equal to $(1/2)\mathbf{V}_g^2 [1, 2, 3]$ and the helicity is completely renewed over a time-period of
$(2f)^{-1} \sim 1.5$ hours, which is more than an order of magnitude shorter than the characteristic time of
dissipation of the kinetic energy of large-scale atmospheric motions due to Ekman friction.

References

[1] Kurgansky M V 1989 Izv. Atmos. Ocean. Phys. 25(12), 979–81
[2] Kurgansky M V 1993 Introduction to Large-scale Atmospheric Dynamics (Adiabatic Invariants
and their Use) (Saint-Petersburg: Gidrometeoizdat) (in Russian)
[3] Kurgansky M V 2002 Adiabatic Invariants in Large-scale Atmospheric Dynamics (London and
New York: Taylor & Francis)
[4] Deusebio E and Lindborg E 2014 J. Fluid Mech. 755, 654–71
[5] Koprov B M, Koprov V M, Kurgansky M V and Chkhetiani O G 2015 Izv. Atmos. Ocean. Phys.
51(6), 565–75
[6] Chkhetiani O G, Kurgansky M V and Vazaeva N V 2018 Boundary-Layer Meteorol. 168, 361–
85
[7] Kurgansky M V 2017 Izv. Atmos. Ocean. Phys. 53(2), 127–41
[8] Kurgansky M V, Maksimenkov L O, Khapaev A A and Chkhetiani O G 2018 Doklady Earth
Sciences 479, Part 2, 477–81
[9] Lucarini V and Gritsun A 2020 Clim. Dyn. 54, 575–98
[10] Yaglom A M and Yaglom I M 1983 Probability and Information. 3d ed (Boston: Reidel)
[11] Kurgansky M V and Pismichenko I A 2000 J. Atmos. Sci. 57, 822–35