Instantons corrections to the effective action of the 
CHL string and its type I dual

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Abstract: In this letter, we investigate corrections to quartic gauge couplings in compactifications of string theories on a 2-torus without vector structure. First, we calculate the threshold corrections to $F^4$ terms in the heterotic CHL string. Then, using the CHL string/type I duality dictionary, we map these corrections to perturbative and non-perturbative effects in type I string compactified on 2-torus without vector structure. Comparing the perturbative terms provides a quantitative test of the S-duality conjecture. The non-perturbative contributions are due to D-strings wrapping the torus. A T-duality along one of the compactified directions allows us to compare these instanton corrections to the ones obtained in a standard type I compactifications. The most striking feature is that these non-perturbative couplings turn out to be identical for both models.

Keywords: Orientifolds, D-instanton corrections, CHL string
1. Introduction

A fruitful way to strengthen a non-perturbative conjectured duality that relates two different string theories is to compare the higher derivative terms which appear in the effective actions of both theories. In the context of the type I/heterotic $SO(32)$ equivalence [1, 2], this programme has been first settled in [3, 4, 5, 6] for a special class of corrections which are half-BPS saturated, and therefore, can receive corrections only from half-BPS states. On the heterotic side, these terms are related to anomaly cancelling couplings and appear only at one-loop, for compactifications on torii $T^d$ with $d<6$. Indeed, in this situation, the only half-BPS instantons are due to fundamental string world-sheets wrapped on the target space torus. Translated into a type I language, these terms become tree-level, perturbative and non-perturbative corrections due to wrapped D-strings. In particular, the one-loop open string amplitudes reproduce the complex moduli dependent term of the heterotic Tr$(F^4)$ couplings. These calculations have been extended to configurations with Wilson lines and/or to other half-BPS saturated couplings by [7, 8, 9, 10, 11].

In this article, we will be interested into the model considered in [8], namely compactification of heterotic string theory on a two-torus with a Wilson line along one circle which breaks the symmetry group $SO(32)$ to $SO(16) \times SO(16)$. After S-duality, this model has a natural description is term of the type $I'$ string, obtained after a
T-duality along the circle with the Wilson line. Indeed, as we will describe in details in this paper, in this picture, the R-R tadpoles are not only globally canceled, but also locally and the dilaton is an arbitrary constant. On the heterotic side, part of the one-loop threshold corrections - the complex modulus independent terms - to the $F^4$ and $R^4$ have been obtained in [8] and interpreted as the effect of euclidian D0-particles in the dual type I' theory. Here, we will review this result and also calculate the complex modulus dependent corrections for the Tr($F^4$) couplings. The mapping of these terms to perturbative corrections on the type I' side will provide a quantitative test of the duality.

One of the aims of [8] was to propose a way to understand the D-instantonic contributions from the matrix model point-of-view which describes D0-branes on top of an O8-plane and sixteen D8-branes [12, 13, 14, 15]. Indeed, in the context of the maximally supersymmetric type IIB string, the non-perturbative contributions to the $R^4$ term [16] have been successfully interpreted [17, 18, 19] from the matrix theory point-of-view [20]: these corrections, including the numerical factor, are reproduced by the calculation of the partition function for the matrix model of D0-branes. However, in less supersymmetric cases such as in type I string theory or on the effective action of D3-branes [21], the relation remains elusive for far.

Therefore, in order to understand the rules of the instanton calculus, it is useful to have at disposal other examples in which D-instanton corrections appear. A particularly interesting case, because of the relations it shares with the standard type I' string, is the compactification of type I string theory on a torus without vector structure [22, 23, 24]. T-dualizing along one direction leads to a compactification with a O8-plane and D8-branes and locally canceled R-R charge, on a skew torus. This theory is believed to have the CHL string [25] as S-dual which can also be seen as a compactification of $\mathcal{M}$-theory on a Moebius strip [26]. The aim of the present article is to calculate the one-loop threshold corrections to the Tr($F^4$) term in the CHL string and to relate them to perturbative and non-perturbative contributions on the type I side.

In the next section, we will review the calculation of [8] and extend it to the complex modulus dependent corrections, allowing us to test the duality relation in the presence of the Wilson line that breaks the gauge group to $SO(16) \times SO(16)$. Then, in section 3, we describe the CHL string construction and calculate the one-loop four-point function for four gauge fields. Using the duality map between CHL string and type I compactification without vector structure, we give the interpretation of these couplings in a compactification of string theory with O8-planes and D8-branes. In particular, we notice that the non-perturbative corrections are identical - up to an obvious substitution in the action of the euclidean D-particles - to the contributions obtained in the standard type I' theory in section 2. Finally, comparing the $U$-modulus dependent terms to one-loop corrections on the type I side put the conjectured duality
on a firmer ground.

We have collected, for the reader’s convenience, some useful theta function identities in appendix A and world-sheets integrations in appendix B.

2. Review of heterotic thresholds and of their S-dual interpretation

2.1 Heterotic thresholds in the presence of a Wilson line

We consider the $SO(16) \times SO(16)$ heterotic string theory compactified on a square 2-torus, which can be seen as the product of two circles of radius $R_1$ and $R_2$. The initial $SO(32)$ gauge group has been broken to $SO(16) \times SO(16)$ by putting a Wilson line $Y = (0^8, (1/2)^8)$ on one of the two circles of the torus, say the first. We also index the two gauge groups by a Greek letter: $\alpha = 1, 2$.

We will calculate the one-loop threshold corrections to the $\text{Tr} \alpha (F^4)$ using the gauging procedure [27]. To do this, we switch on a background gauge field in the Cartan sub-algebra of the gauge group: $F_I^\alpha = v^I_\alpha$. In this background, the one-loop partition function reads:

$$Z(v) = -\frac{Q(\bar{\tau})}{4\tau^4} \sum_{a, b = 0, 1} \prod_{I=1}^8 \frac{\vartheta [\tilde{a}]}{\eta^{24}(\tau)} (v_I^a | \tau) \vartheta \left[ \frac{a+n^1_m}{b+m^2_n} \right] (v_I^b | \tau) \sum_{m^i, n^i \in \mathbb{Z}} \Gamma_{2,2}^{T,U} \left[ \begin{array}{cc} n^1_m & n^2_m \\ m^1 & m^2 \end{array} \right] (\tau, \bar{\tau})$$

where we have defined the contributions of the right-moving modes:

$$Q(\bar{\tau}) = \sum_{\tilde{a}, \tilde{b} = 0, 1} (-)^{\tilde{a} + \tilde{b} + \tilde{a} \tilde{b}} \vartheta \left[ \frac{\tilde{a}}{\tilde{b}} \right] (0 | \bar{\tau}) \eta^4(\bar{\tau})$$

(2.1)

We have also introduced the following notation

$$Z_{2,2}(T, U, \tau, \bar{\tau}) \equiv \sum_{m^i, n^i \in \mathbb{Z}} \Gamma_{2,2}^{T,U} \left[ \begin{array}{cc} n^1_m & n^2_m \\ m^1 & m^2 \end{array} \right] (\tau, \bar{\tau})$$

$$= T_2 \sum_{m^1, n^1 \in \mathbb{Z}} e^{-\frac{\sum T_1}{2} (m^1 + n^1 \tau + (m^2 + n^2 \tau) U)^2 + 2i\pi T_1 (m^1 n^2 - m^2 n^1)}$$

to define the lagrangian representation of the lattice sum over the Kaluza-Klein momenta and windings of the closed string. The indices $T$ and $U$ in $\Gamma_{2,2}^{T,U}$ will be omitted when there is no ambiguity.

The one-loop four-point function with four gauge fields is a half-BPS saturated amplitude; therefore, it can receive contributions only from half-BPS states of the heterotic string, which, in eight dimensions, correspond to fundamental string states. The right-moving part of the amplitude provides the kinematic structure, namely the
well-known tensor $t_8$ which contracts the Lorentz indices of the gauge fields. The terms whose gauge structure is $\text{Tr}_\alpha(F^4)$ are obtained by taking derivatives with respect to appropriate $v^\alpha_a$ \cite{1, 27}. Hence, the one-loop thresholds are given by

$$T_{\text{het}}^{\text{het}} = \Delta_{\alpha}^{\text{het}}(T, U) t_8 \text{Tr}_\alpha(F^4),$$

with

$$\Delta_{\alpha}^{\text{het}}(T, U) = - \frac{N}{3} \frac{1}{2^5} \int d\tau_2 \frac{d^2\tau}{\tau_2^2} \left( \partial^{(4)}_{\text{v}_a} - 3 \partial^{(2)}_{\text{v}_a} \partial^{(2)}_{\text{v}_a} \right) Z(v)|_{v=0} \equiv - \frac{N}{3} \frac{1}{2^5} \int d\tau_2 \Xi(\tau),$$

and

$$\Xi(\tau) = \sum_{a, b=0,1} \frac{\partial^8 [a]}{\text{det}^4} \left( \frac{\partial''''[a]}{\partial [a]} \right)^2 (\tau) \sum_{m', n' \in \mathbb{Z}} \Gamma_{2, 1} \left[ \frac{n' n^2}{m'^2} \right] (\tau, \bar{\tau})$$

valid up to an infrared divergence due to the contributions of massless states circulating into the loop. In the following section, we will briefly review the type I interpretation of these couplings given in \cite{8}. Moreover, we will see how the $U$-dependent terms are reproduced by a one-loop calculation on the type I side.

### 2.2 Type I interpretation

The Kähler and complex structure moduli of the square torus are given by

$$T = B_{\text{NSNS}} + i R_1 R_2, \quad U = \frac{i R_2}{R_1}. $$
Heterotic/type I duality tells us that the coupling constants, the metric and the $B$ fields are related by

$$\lambda_{s}^{\text{het}} = 1/\lambda_{s}^{I}, \quad g_{\mu\nu}^{\text{het}} = g_{\mu\nu}^{I}/\lambda_{s}^{I}, \quad B_{\text{NSNS}}^{\text{het}} = B_{\text{RR}}^{I}.$$  

We will also consider a T-dual version of this type I string theory, which in the case of the CHL string, will provides a simple, geometrical explanation of the reduction of the group rank. Therefore, we T-dualize the type I string along the direction $x^1$ to obtain the type I’ theory whose compactification radii and coupling constant are related to the type I ones as

$$R_{I}'_{1} = \frac{1}{R_{I}^{1}}, \quad R_{I}'_{2} = R_{I}^{2}, \quad \lambda_{s}' = \frac{\lambda_{s}^{I}}{R_{I}^{1}}.$$  

The $B_{RR}$ field is mapped to the value of the RR 1-form, $A_{RR}$, along the direction $x^2$. Therefore, the heterotic moduli are identified to

$$T = A_{RR} + i\frac{R_{I}'_{2}}{\lambda_{s}^{p}}, \quad U = iR_{I}'_{1}R_{I}'_{2}$$  

Finally, the orientifold operator $\Omega$, which exchanges the left and right moving sectors of closed strings and whose action on the type IIB string theory gives the type I string, is transformed to a combinaison of $\Omega$ and of the reflexion along the T-dualized direction : $\Omega' \equiv \Omega R_{1}$. Under this T-duality, the O9-plane splits into two O8-planes, sitting at opposite fixed planes ($x^1 = 0$ and $x^1 = \pi R_{I}^{1}$) under the orientifold projection $\Omega'$. The sixteen D9-branes become D8-branes, whose positions along $x^1$ depend on their initial Wilson lines. The $SO(16) \times SO(16)$ point corresponds to a configuration where half of these D8-branes are sitting at $x^1 = 0$ on top of one of the O8-planes and the other half at $x^1 = \pi R_{I}^{1}$, on the other O8-plane. We also point out that this configuration is the only one where the tadpole of the R-R 9-form is locally cancelled and the dilaton is constant.

Using the duality map (2.3), we can expand at small coupling and finite $R_{I}'_{2}$ radius the $T$-dependent part of the threshold corrections (2.2)

$$\Delta_{\alpha}^{\text{het}}(T) = \frac{N}{3} \left( -\pi T_{2} + 4 \text{Re} \sum_{N} \sum_{m|N} \frac{1}{m} e^{4i\pi NT} - 2 \text{Re} \sum_{N} \sum_{m|N} \frac{1}{m} e^{2i\pi NT} \right)$$  

The linear term comes from the contributions of the Born-Infeld action which describes the low-energy dynamics of the D8-branes wrapped on the direction $x^2$ of the torus.

The exponentially vanishing terms should be attributed to contributions of D0-branes whose euclidian trajectories wrap the cycle $c_{2}$ of the 2-torus. However, the
derivation of these couplings from first principles, for example from the matrix model which describes the dynamics of D0-branes in this theory [12, 13, 14, 15], is still lacking.

The non-trivial test of heterotic/type I duality that we can perform here is to compare the $U$-dependent terms of (2.2) to perturbative corrections on type I side. On the type I' side, they will correspond to one-loop threshold corrections due to euclidian fundamental strings stretched between the D8-branes and wrapped on the direction $x^2$. To calculate these contributions, we use the background field method introduced in [29, 30]. We will label the D-branes with the couple of Chan-Paton indices $(\alpha, i)$, for $\alpha = 1, 2$ and $i = 1, \ldots, 16$, according to the $SO(16)$ group they realize. We switch on a constant background gauge field, for example along the directions $x^8, x^9$: $F_{89} = QB$ where $Q$ is a generator of the gauge group.

The oscillator frequencies of the complex coordinate $X^8 + iX^9$ of a open string ending on the D-branes labelled by the indices $(\alpha, i)$ and $(\beta, j)$ are shifted by an amount $\epsilon$, where

$$\epsilon = \arctan(\pi q_i^\alpha B) + \arctan(\pi q_j^\beta B).$$

$q_i^\alpha$ and $q_j^\beta$ are the eigenvalues of the gauge-group generator acting on the Chan-Paton factors at the endpoints of the string. The one-loop open string partition function, namely, the sum of annulus and of the Möbius strip diagrams, is modified in the presence of this background

$$\mathcal{A}(B) = \frac{iV^{(8)}}{3 \cdot 2^{13} \pi^4} \int_0^\infty \frac{dt}{t} \frac{1}{(2\pi^2 t)^4} \frac{\partial^1_1(0|\frac{it}{2})}{\vartheta_1(\frac{it}{2}|\frac{it}{2})} \frac{i}{2} \left( q_i^\alpha + q_j^\beta \right) Bt \sum_{m_1 \in \mathbb{Z} + a_\alpha + a_\beta, m_2 \in \mathbb{Z}} \Gamma_2[m_1 m_2] \left( \frac{it}{2} \right)$$

$$\times \sum_{a,b=0,1} \frac{1}{2} (-)^{a+b+ab} \frac{\vartheta^3[a][b](0|\frac{it}{2}) \vartheta[a][b](\frac{it}{2}|\frac{it}{2})}{\eta^{12}(\frac{it}{2})},$$

$$\mathcal{M}(B) = -\frac{iV^{(8)}}{3 \cdot 2^{13} \pi^4} \int_0^\infty \frac{dt}{t} \frac{1}{(2\pi^2 t)^4} \frac{\vartheta^1_1(0|\frac{it+1}{2})}{\vartheta_1(\frac{it}{2}|\frac{it+1}{2})} \left( i q_i^\alpha Bt \right) \sum_{m_1 \in \mathbb{Z} + 2a_\alpha, m_2 \in \mathbb{Z}} \Gamma_2[m_1 m_2] \left( \frac{it}{2} \right)$$

$$\times \sum_{a,b=0,1} \frac{1}{2} (-)^{a+b+ab} \frac{\vartheta^3[a][b](0|\frac{it+1}{2}) \vartheta[a][b](\frac{it}{2}|\frac{it+1}{2})}{\eta^{12}(\frac{it+1}{2})}.$$
To extract the Tr($F^4$), we expand these expressions to quartic order in $B$:

$$A|_{B^4} = -\frac{V^{(8)}}{3}2^{12}\pi^4 (q_i^\alpha + q_j^\beta)^4B^4 \int_0^\infty \frac{dt}{t} \sum_{m_1 \in \mathbb{Z} + a, m_2 \in \mathbb{Z}} \Gamma_2[m_1, m_2] \left( \frac{it}{2} \right),$$

$$\mathcal{M}|_{B^4} = \frac{V^{(8)}}{3}2^{12}\pi^4 (2q_i^\alpha)^4B^4 \int_0^\infty \frac{dt}{t} \sum_{m_1 \in \mathbb{Z} + a} \Gamma_2[m_1, m_2] \left( \frac{it}{2} \right).$$

Selecting the term which corresponds to the trace structure Tr($F^4$) in these amplitudes leads us to the following threshold corrections

$$\mathcal{I}_{\text{Tr}(F^4)} = \Delta_{\alpha}(U) t_8 \text{Tr}_\alpha(F^4)$$

with

$$\Delta_{\alpha}(U) = -\frac{V^{(8)}}{3}2^{12}\pi^4 \int_0^\infty \frac{dt}{t} \left( 2 \times 16 \sum_{m_1 \in \mathbb{Z} + a} \Gamma_2[m_2, m_2] + 2 \times 16 \sum_{m_1 \in \mathbb{Z} + a} \Gamma_2[m_1, m_2] \right) - 2^4 \Gamma_2[m_1, m_2] \left( \frac{it}{2} \right)$$

These integrals are given in the appendix and the result exactly reproduces the $U$-dependent part of (2.2).

3. One-loop Tr($F^4$) couplings in the CHL string

The SO(16) CHL string can be seen as a non-abelian orbifold of the SO(32) heterotic string theory. If we decompose the 32 world-sheet fermions which realize the affine algebra into two groups, $\chi_\alpha^I$, $\alpha = 1, 2$, the action of the orbifold reads:

$$g_1 : X_1 \rightarrow X_1 + \pi R_1, \quad \chi_1^I \rightarrow \chi_1^I, \quad \chi_2^I \rightarrow -\chi_2^I$$

$$g_2 : X_2 \rightarrow X_2 + \pi R_2, \quad \chi_1^I \rightarrow \chi_2^I, \quad \chi_2^I \rightarrow \chi_1^I$$

The first generator corresponds to a Wilson line $Y = (0^8, (1/2)^8)$ which breaks the gauge group SO(32) into SO(16) $\times$ SO(16) as in the model of the previous section while the second operator projects on the diagonal SO(16) gauge group.

The orbifold partition function [23, 31] can be written as the sum of the contributions of the different sectors of the orbifold. As above, we will use the background field method to calculate the amplitude for four gauge fields. Therefore, we switch on a
background gauge field in the Cartan torus of the diagonal gauge group. The diagonal
gauge field couples to the world-sheet fields as:

\[
S_{ws} = \frac{1}{2\pi} \int d^2\sigma \left( \partial X^\mu \partial X_\mu + B_{\mu\nu} \partial X^\mu \partial X^\nu + \psi^\mu \partial \psi_\mu \\
+ \bar{\chi}^I \partial \chi^I + i A^I_\mu \left( \bar{\chi}^I_1 \partial X^{\mu} \chi^I_1 + \bar{\chi}^I_2 \partial X^{\mu} \chi^I_2 \right) \right)
\]

where we have defined the coordinates on the torus: \( z = \frac{1}{2} (\sigma^1 + \tau \sigma^2) \), \((\sigma^1, \sigma^2) \in [0, 1]^2 \)
and the derivative \( \partial = i(\bar{\tau} \partial_1 - \partial_2)/\tau_2 \).

In this background, the \([1, 1], [1, g_1], [1, g_2]\) and \([1, g_1g_2]\) contributions to the one-loop partition function read:

\[
\begin{align*}
Z_{(1,1)}(v) &= \frac{Q(\bar{\tau})}{8\tau_2^4} \sum_{a,b=0,1} \prod_{I=1}^{8} \frac{\vartheta_2^2(\theta_b)}{\eta^{24}(\tau)} \sum_{m^4, n^4 \in \mathbb{Z}} \Gamma_{2,2} \left[ \frac{2n^2_{1}}{2m^2_{1}+1} \right] (\tau, \bar{\tau}) \\
Z_{(1,g_1)}(v) &= \frac{Q(\bar{\tau})}{4\tau_2^4} \prod_{I=1}^{8} \frac{\vartheta_3^2(\theta) \vartheta_4^2(\theta)}{\eta^{24}(\tau)} \sum_{m^4, n^4 \in \mathbb{Z}} \Gamma_{2,2} \left[ \frac{2n^2_{2}}{2m^2_{2}+1} \right] (\tau, \bar{\tau}) \\
Z_{(1,g_2)}(v) &= \frac{Q(\bar{\tau})}{4\tau_2^4} \prod_{I=1}^{8} \frac{\vartheta_2(\theta) \vartheta_4(\theta)}{\eta^{24}(\tau)} \sum_{m^4, n^4 \in \mathbb{Z}} \Gamma_{2,2} \left[ \frac{2n^2_{1}}{2m^2_{1}+1} \right] (\tau, \bar{\tau}) \\
Z_{(1,g_1g_2)}(v) &= \frac{Q(\bar{\tau})}{4\tau_2^4} \prod_{I=1}^{8} \frac{\vartheta(\theta) \vartheta(\theta)}{\eta^{24}(\tau)} \sum_{m^4, n^4 \in \mathbb{Z}} \Gamma_{2,2} \left[ \frac{2n^2_{1}}{2m^2_{1}+1} \right] (\tau, \bar{\tau})
\end{align*}
\]

The contributions \([g_i, 1]\) and \([g_i, g_i]\) to the partition function are obtained by doing the
modular transformations \( \tau \rightarrow -1/\tau \) and \( \tau \rightarrow (\tau - 1)/\tau \) on \( Z_{(1,g_i)} \).

Taking the appropriate derivatives of this partition function, we can extract the
one-loop gauge thresholds:

\[
\mathcal{I}^{\text{CHL}}_{\text{Tr}(F^4)} = \Delta^{\text{CHL}}(T, U) t_8 \text{Tr}(F^4),
\]

with

\[
\Delta^{\text{CHL}}(T, U) = -\frac{N}{32^5} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \left( \Xi_{(1,1)} + \Xi_{(1,g_1)} + \Xi_{(1,g_2)} + \Xi_{(1,g_1g_2)} + \text{orb.} \right)(\tau).
\]
and

\[\Xi_{(1,1)}(\tau) = 2 \sum_{\alpha=2,3,4} \frac{\vartheta^1_6}{\eta^4} \left( \frac{\vartheta''}{\vartheta'} - 3 \left( \frac{\vartheta''}{\vartheta'} \right)^2 \right)(\tau) \sum_{m',n'\in\mathbb{Z}} \Gamma_{2,2} \left[ \frac{2n^1}{2m^1} \frac{2n^2}{2m^2} \right] (\tau, \bar{\tau}),\]

\[\Xi_{(1,g_1)}(\tau) = 2 \sum_{\alpha=3,4} \frac{\vartheta^3_8}{\eta^4} \left( \frac{\vartheta''}{\vartheta'} - 3 \left( \frac{\vartheta''}{\vartheta'} \right)^2 \right)(\tau) \sum_{m',n'\in\mathbb{Z}} \Gamma_{2,2} \left[ \frac{2n^1}{2m^1+1} \frac{2n^2}{2m^2} \right] (\tau, \bar{\tau}),\]

\[\Xi_{(1,g_2)}(\tau) = 2 \sum_{\alpha=3,4} \frac{\vartheta^3_8}{\eta^4} \left( \frac{\vartheta''}{\vartheta'} - 3 \left( \frac{\vartheta''}{\vartheta'} \right)^2 \right)(\tau) \sum_{m',n'\in\mathbb{Z}} \Gamma_{2,2} \left[ \frac{2n^1}{2m^1} \frac{2n^2}{2m^2+1} \right] (\tau, \bar{\tau}),\]

\[\Xi_{(1,g1g_2)}(\tau) = \frac{1}{\eta^8(\tau)} \left\{ \sum_{\alpha=2,3} \frac{\vartheta^8}{\eta^8} \left( \frac{\vartheta''}{\vartheta'} - 3 \left( \frac{\vartheta''}{\vartheta'} \right)^2 \right) (2\tau) \sum_{m',n'\in\mathbb{Z}} \Gamma_{2,2} \left[ \frac{2n^1}{2m^1+1} \frac{2n^2}{2m^2+1} \right] (\tau, \bar{\tau}) \right\}.

\]

Then, one can use the formula (A.3) and (A.3) to simplify these expressions. The elliptic functions cancel out and the result is simply:

\[\Xi_{(1,1)}(\tau) = 3 \cdot 2^6 \sum_{m',n'\in\mathbb{Z}} \Gamma_{2,2} \left[ \frac{2n^1}{2m^1} \frac{2n^2}{2m^2} \right] (\tau, \bar{\tau}),\]

\[\Xi_{(1,g_1)}(\tau) = -2^6 \sum_{m',n'\in\mathbb{Z}} \Gamma_{2,2} \left[ \frac{2n^1}{2m^1+1} \frac{2n^2}{2m^2} \right] (\tau, \bar{\tau}),\]

\[\Xi_{(1,g_2)}(\tau) = -2^6 \sum_{m',n'\in\mathbb{Z}} \Gamma_{2,2} \left[ \frac{2n^1}{2m^1} \frac{2n^2}{2m^2+1} \right] (\tau, \bar{\tau}),\]

\[\Xi_{(1,g1g_2)}(\tau) = 2^6 \sum_{m',n'\in\mathbb{Z}} \Gamma_{2,2} \left[ \frac{2n^1}{2m^1+1} \frac{2n^2}{2m^2+1} \right] (\tau, \bar{\tau}).\]

Therefore, we can group these contributions and rewrite the one-loop thresholds (3.2) as:

\[\Delta^{CHL}(T, U) = -\frac{2N}{3} \int \frac{d^2\tau}{\tau^2} \sum_{m',n'\in\mathbb{Z}} \left( 4\Gamma_{2,2} \left[ \frac{2n^1}{2m^1} \frac{2n^2}{2m^2} \right] (\tau, \bar{\tau}) - 2\Gamma_{2,2} \left[ \frac{n^1}{m^1} \frac{n^2}{m^2} \right] (\tau, \bar{\tau}) \right.

\[- 2\Gamma_{2,2} \left[ \frac{2n^1}{2m^1+1} \frac{n^2}{m^2} \right] (\tau, \bar{\tau}) + \left( \Gamma_{2,2} \left[ \frac{2n^1}{m^1} \frac{2n^2}{m^2} \right] (\tau, \bar{\tau}) + \text{orb.} \right) \right),\]

which can be evaluated using the method of orbits. To calculate the contribution of the last term, we use the techniques described in details and a more general context in the appendix of [13]: first, the lattice sum

\[\sum_{m',n'\in\mathbb{Z}} \Gamma^{T,U}_{2,2} \left[ \frac{2n^1}{m^1} \frac{2n^2}{m^2} \right] (\tau, \bar{\tau}),\]

can be written as

\[\frac{1}{2} \sum_{m',n'\in\mathbb{Z}} \Gamma^{2T,U}_{2,2} \left[ \frac{n^1}{m^1} \frac{n^2}{m^2} \right] (2\tau, 2\bar{\tau}).\]
where we have reinstated explicitly the dependence on the moduli of the torus. This term is integrated over the extended fundamental domain $F_{2}^{-}$ of the index 2 subgroup $\Gamma_{2}^{-}$ of $SL(2, \mathbb{Z})$, generated by $T$ and $ST^{2}S$. We can change the integration variable to $\rho = 2\tau$:

$$\int_{F_{2}^{-}} \frac{d^{2}\tau}{\tau_{2}} \sum_{m',n' \in \mathbb{Z}} \Gamma_{2,2}^{T,U} \left[ \frac{2n'^{2}}{m'^{2} m^{2}} \right] (\tau, \bar{\tau}) = \frac{1}{2} \int_{F_{2}^{-}} \frac{d^{2}\rho}{\rho_{2}} \sum_{m',n' \in \mathbb{Z}} \Gamma_{2,2}^{2T,U} \left[ \frac{n^{1} n'^{2}}{m^{1} m'^{2}} \right] (\rho, \bar{\rho})$$

(3.3)

and unfold the domain of integration into the fundamental domain of $SL(2, \mathbb{Z})$

$$\int_{F_{2}^{-}} \frac{d^{2}\tau}{\tau_{2}} \sum_{m',n' \in \mathbb{Z}} \Gamma_{2,2}^{T,U} \left[ \frac{2n'^{2}}{m'^{2} m^{2}} \right] (\tau, \bar{\tau}) = \frac{3}{2} \int_{F_{2}} \frac{d^{2}\tau}{\tau_{2}} \sum_{m',n' \in \mathbb{Z}} \Gamma_{2,2}^{2T,U} \left[ \frac{n^{1} n'^{2}}{m^{1} m'^{2}} \right] (\tau, \bar{\tau})$$

(3.4)

Finally, reinstating the moduli dependence of the three first terms, the threshold corrections read:

$$\Delta_{CHL}^{\tau}(T,U) = -\frac{2N}{3} \int_{F} \frac{d^{2}\tau}{\tau_{2}} \sum_{m',n' \in \mathbb{Z}} \left( \Gamma_{2,2}^{4T,U} - \Gamma_{2,2}^{2T,2U} - \Gamma_{2,2}^{2T,U/2} + \frac{3}{2} \Gamma_{2,2}^{2T,U} \right) \left[ \frac{n^{1} n'^{2}}{m^{1} m'^{2}} \right] (\tau, \bar{\tau})$$

$$= \frac{N}{3} \left( 2|\log|\eta(4T)||^{4} - \log|\eta(2T)||^{4} \\
+ 5|\log|\eta(U)||^{4} - 2|\log|\eta(2U)||^{4} - 2|\log|\eta(U/2)||^{4} \right)$$

(3.5)

4. Interpretation in type I compactifications without vector structure

4.1 Type I compactification without vector structure, its T-dual and D-instanton corrections

The CHL string model is supposed to be S-dual to type I string theory compactified on a torus without vector structure [22, 23, 24]. This compactification has a discrete half integer NS-NS $B$ field flux on the torus. Thanks to periodicity properties, we can always choose $B_{NSNS} = \frac{1}{2}$ so the Kähler moduli of $T^{2}$ is

$$T^{I} = \frac{1}{2} + iR_{1}^{I}R_{2}^{I}.$$ 

The mapping of the CHL moduli $T$ and $U$ to type I moduli is identical to their standard heterotic and type I counterparts, namely

$$\tau^{I} = B_{RR} + iR_{1}^{I}R_{2}^{I}/\lambda_{s}^{I} \quad \text{and} \quad U^{I} = iR_{2}^{I}/R_{1}^{I}.$$ 

The rank reduction of the group has a clear geometrical interpretation if we perform a T-duality along one of the directions of the torus, say $x^{1}$. This transformation
exchanges the Kähler and the complex structure of $T^2$. Therefore, we obtain a tilted torus (fig. 1), whose complex structure is

$$U' = \frac{1}{2} + i R_1 R_2 = \frac{1}{2} + i \frac{R_2'}{R_1'}.$$ 

Figure 1: Type I' string on a tilted torus.

After T-duality, the orientifold projection is mapped to $\Omega R_1$ and the O9-plane is transformed into O8-planes. However, contrary to the compactification on a torus with vector structure, there is only one O8-plane, wrapped on the cycle $2c'_2 - c'_1$ of the torus as one can see on the figure 1. To cancel the unphysical R-R flux induced by the presence of this plane, one introduces D8-branes which, as it is easy to verify using the standard T-duality transformations on the boundary conditions for open strings ending on the D-branes, are oriented along the same cycle and, hence, wrapped twice the torus. Therefore, compared to the standard case, only half of these D8-branes are needed and the rank of the gauge group is divided by two. Rather than to have two systems of [1 O8-plane+8 D8-branes] wrapped once on a square torus, we have only one system wrapped twice on the skew torus with angle $\pi/3$.

This interpretation is consistent with the spacetime anomaly cancellation. Indeed, we know that gravitational anomalies are canceled due to the presence of Wess-Zumino gravitational couplings in the actions of the D-branes and of the orientifold planes [32]. Since the D8-branes and the O8-planes are wrapped twice on the torus, they contribute exactly twice and cancel the same amount of anomaly as two O8-planes and sixteen D8-branes of the standard type I' compactification.

We are now in position to interpret the CHL thresholds (3.5) in this type I picture. Similarly to the first section, the $T$-dependent part corresponds to a tree-level
contribution coming from the expansion of the Born-Infeld action and of D-instanton contributions due to euclidian D0-branes:

\[ \Delta_{\text{CHL}}(T) = \frac{N}{3} \left( -2\pi T_2 + 4 \text{Re} \sum_{m|N} \frac{1}{m} e^{8\pi N T} - 2 \text{Re} \sum_{m|N} \frac{1}{m} e^{4i\pi N T} \right) \]

One the type I' side, the parameter \( T \) becomes \( A_R + i R'_2 g_{(1)}^T \). It corresponds to the effective action of an euclidian D-particle whose world-line describes the cycle \( 2c'_2 - c'_1 \). We observe that these non-perturbative corrections are exactly the same as (2.24), except that \( T \) has been replaced by \( 2T \) as one expects from the double wrapping required for closure of the D-particle world-line on the tilted torus. It would be very interesting to understand this result from a non-perturbative calculation, such as in the matrix model perspective suggested by [8]. Indeed, the quantum mechanics which describes the D0-branes on top of the [O8-plane + 8 D8-branes] system is the same, only the boundary conditions on the fields are modified.

### 4.2 One-loop Tr\((F^4)\) couplings

To test the S-duality relation between CHL string and type I compactification on a torus without vector structure, we can repeat the one-loop calculation made in section two in the context of type I string with gauge group \( SO(16) \times SO(16) \). In type I compactifications without vector structure, the presence of a background field leads to the following modified open string partition functions on the annulus and on the Möbius strip:

\[
\mathcal{A}(B) = \frac{iV^{(8)}}{32^{13}\pi^4} \int_0^\infty dt \frac{1}{t (2\pi^2 t)^4} \frac{\partial_1'(0|\frac{it}{2})}{\partial_1(\frac{it}{2})} \left( \frac{it}{2} \right) \sum_{m_1 \in \mathbb{Z}} \Gamma_2[m_1, m_2] \left( \frac{it}{2} \right) \\
\times \sum_{a,b=0,1} \frac{1}{2} \left( -1 \right)^{a+b+ab} \frac{\theta^3_{[a]}(0|\frac{it}{2}) \theta_{[b]}(\frac{it}{2})}{\eta^{12}(\frac{it}{2})}, \\
\mathcal{M}(B) = \frac{-iV^{(8)}}{32^{13}\pi^4} \int_0^\infty dt \frac{1}{t (2\pi^2 t)^4} \frac{\partial_1'(0|\frac{it+1}{2})}{\partial_1(\frac{it+1}{2})} \left( \frac{it+1}{2} \right) \sum_{\epsilon_1=0,1 \atop \epsilon_2 \in \mathbb{Z}+\epsilon_1} \left( -1 \right)^{\epsilon_1\epsilon_2} \Gamma_2[m_1, m_2] \left( \frac{it}{2} \right) \\
\times \sum_{a,b=0,1} \frac{1}{2} \left( -1 \right)^{a+b+ab} \frac{\theta^3_{[a]}(0|\frac{it+1}{2}) \theta_{[b]}(\frac{it+1}{2})}{\eta^{12}(\frac{it+1}{2})},
\]

whose quartic expansions in \( B \) give

\[ \mathcal{A}|_{B^4} = \frac{-V^{(8)}}{32^{13}\pi^4} (q_i + q_j)^4 B^4 \int_0^\infty dt \sum_{m_1 \in \mathbb{Z}} \Gamma_2[m_1, m_2] \left( \frac{it}{2} \right), \]

\[ \mathcal{M}|_{B^4} = \frac{V^{(8)}}{32^{13}\pi^4} (2q_i)^4 B^4 \int_0^\infty dt \sum_{\epsilon_j=0,1 \atop m_1 \in \mathbb{Z}} \left( -1 \right)^{\epsilon_1\epsilon_2} \Gamma_2[2m_1 + \epsilon_1, 2m_2 + \epsilon_2] \left( \frac{it}{2} \right) \]
From these amplitudes, we can extract the threshold corrections to the $\text{Tr}(F^4)$ terms:

$$Z_{\text{Tr}(F^4)}^{1\text{-loop}} = \Delta^1(U) t_8 \text{Tr}(F^4)$$

with

$$\Delta^1(U) = -\frac{V^{(8)}}{3 \cdot 2^{12} \pi^4} \int_0^\infty \frac{dt}{t} \left( 2 \times 16 \sum_{m_1 \in \mathbb{Z}} \Gamma_2[m_1 \; m_2] \right)$$

$$- 2^4 \sum_{\epsilon_i = 0, 1} (\epsilon_1 \epsilon_2 \Gamma_2[2m_1 + \epsilon_1 \; 2m_2 + \epsilon_2]) \left( \frac{it}{2} \right)$$

$$= \frac{V^{(8)}}{3 \cdot 2^8 \pi^4} \left( 5 \log |\eta(U)|^4 - 2 \log |\eta(2U)|^4 - 2 \log |\eta(U/2)|^4 \right)$$

which is the complex structure dependent part of the CHL correction $\Delta^{\text{CHL}}(U)$. Therefore, this calculation provides a quantitative test of the S-duality which relates the CHL string to type I compactification without vector structure.

5. Conclusion

In this note, we have investigated half-BPS saturated couplings in the effective action of the CHL string and of its type I dual. Comparing part of the threshold corrections to the heterotic $\text{Tr}(F^4)$ terms to one-loop corrections on the type I side has provided a quantitative test of the duality between the CHL string and the type I string compactified on a torus without vector structure.

This calculation also predicts the existence of D-instanton corrections due to D-strings wrapping the torus of the type I string theory. After one T-duality, these corrections can be attributed to D-particles whose euclidean world-lines wrap twice the dual skew torus. A striking feature of these instantonic corrections is that there are identical (up to an obvious factor related to the length of the world-line of the D-particle) to the D-instanton corrections which appear in another compactification, namely type I string with two orientifeld 8-planes and sixteen D8-branes, with gauge group is $SO(16) \times SO(16)$. From the 11-dimensional point-of-view, these two type I compactifications can be seen respectively as $\mathcal{M}$-theory compactified on a Mœbius strip and on an annulus, the gauge groups $SO(16)$ living on the boundaries of these open Riemann surfaces. The coincidence that we have observed in this article seems to indicate that instanton corrections to the half-BPS saturated gauge couplings are only a “boundary effect”. It would be very interesting to recover these corrections from a non-perturbative description such as the matrix model describing D0-branes on top of one O8-plane and 8 D8-branes. We leave this question for future work.
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Appendix

A. Theta function identities

Definition
\[ \vartheta [a \ b] (v|\tau) = \sum_{n \in \mathbb{Z}} q^{(n-\frac{a}{2})^2} e^{2i\pi (v-\frac{b}{2})(n-\frac{a}{2})} \]  
(A.1)

Jacobi identity
\[ \sum_{a, b = 0, 1} (-)^{a+b+ab} \vartheta [a \ b] (v_1|\tau) \vartheta [a \ b] (v_2|\tau) \vartheta [a \ b] (v_3|\tau) \vartheta [a \ b] (v_4|\tau) \]
\[ = -2\vartheta_1 (v_1'|\tau) \vartheta_1 (v_2'|\tau) \vartheta_1 (v_3'|\tau) \vartheta_1 (v_4'|\tau) \]  
(A.2)

with
\[ v_1' = \frac{1}{2} (-v_1 + v_2 + v_3 + v_4), \quad v_2' = \frac{1}{2} (v_1 - v_2 + v_3 + v_4), \]
\[ v_3' = \frac{1}{2} (v_1 + v_2 - v_3 + v_4), \quad v_4' = \frac{1}{2} (v_1 + v_2 + v_3 - v_4), \]

Derivatives of theta functions
We use a prime to denote the derivative of a theta function with respect to \( v \). The Jacobi theta functions verify the relation
\[ (i\pi \partial_v^{(2)} - \partial_\tau) \vartheta [a \ b] (v|\tau) = 0. \]  
(A.3)

The relevant formula for the calculations of this article are given by
\[ \frac{\vartheta_2''}{\vartheta_2} = \frac{1}{12} (\hat{E}_2 + \vartheta_3^4 + \vartheta_4^4), \quad \frac{\vartheta_2'''}{\vartheta_2} - 3 \left( \frac{\vartheta_2''}{\vartheta_2} \right)^2 = -\frac{1}{8} \vartheta_3^4 \vartheta_4^4, \]
\[ \frac{\vartheta_3''}{\vartheta_3} = \frac{1}{12} (\hat{E}_2 + \vartheta_2^4 - \vartheta_4^4), \quad \frac{\vartheta_3'''}{\vartheta_3} - 3 \left( \frac{\vartheta_3''}{\vartheta_3} \right)^2 = \frac{1}{8} \vartheta_2^4 \vartheta_4^4, \]
\[ \frac{\vartheta_4''}{\vartheta_4} = \frac{1}{12} (\hat{E}_2 - \vartheta_2^4 - \vartheta_3^4), \quad \frac{\vartheta_4'''}{\vartheta_4} - 3 \left( \frac{\vartheta_4''}{\vartheta_4} \right)^2 = -\frac{1}{8} \vartheta_2^4 \vartheta_3^4. \]  
(A.4)

Doubling formula
\[ \eta(2\tau)\eta_4(2\tau) = \eta^2(\tau). \]  
(A.5)
B. World-sheets integrations

The purpose of this appendix is to perform the relevant integrations over the moduli which parameterize the one-loop world-sheets of the closed and open strings.

**Torus**

For the toroidal partition function, we have to calculate a modular integral over the fundamental domain of $SL(2, \mathbb{Z})$. To do this, one unfolds this domain using the method of orbits [28]. This technics leads to the result:

$$
\int \mathcal{F} \frac{d^2 \tau}{\tau_2} \sum_{m^1, n^1 \in \mathbb{Z}} \Gamma_{T,U}^{2,2} \left[ \frac{n^1 n^2}{m^1 m^2} \right] (\tau, \bar{\tau}) = -\log \left( |\eta(T)|^4 |\eta(U)|^4 \right) .
$$

(B.1)

Actually, the integral has a logarithmic infrared divergence due to massless string modes circulating into the loop that has not been written in the above formula.

**Annulus and Möbius strip**

The calculation of open string amplitudes is more subtle since there is no modular invariance to cut off the quadratic ultraviolet divergence of the integrals over the open string world-sheets. However, when tadpoles are canceled, the ultraviolet divergences of the annulus and of the Möbius strip cancel each other. This is always the case for the models considered in this paper. For each diagram, the divergence is given by the zero windings sector after a double Poisson resummation of the open-strings Kaluza-Klein momenta to the closed channel windings.

Therefore, the formula needed in this article is:

$$
\int_0^{\infty} \frac{dt}{t} \left( \sum_{m_i \in \mathbb{Z}} \Gamma_2[m_1 \ m_2] \left( \frac{it}{2} - \frac{\pi T_2}{t} \right) \right) = -\log |\eta(U)|^4
$$

(B.2)

where we have explicitly subtracted the quadratic ultraviolet divergence part. These terms cancel out each other after adding the annulus and Möbius strip amplitudes. Moreover, we have also overlooked the logarithmic infrared divergence due to massless open-string states running into the loop. Finally, integrals of lattice sums with shifts of 1/2 on the momenta can be obtained from this formula as sum of terms with different
complex structure dependences:

\[
\int_0^\infty \frac{dt}{t} \sum_{m_1 \in \mathbb{Z}+\alpha_1, m_2 \in \mathbb{Z}+\alpha_2} \Gamma_2[m_1 m_2] \left( \frac{it}{2} \right)
\]

\[
= \log|\eta(U)|^4 - \log|\eta(U/2)|^4 \quad \text{for} \quad (\alpha_1, \alpha_2) = \left( \frac{1}{2}, \frac{1}{2} \right),
\]

\[
= \log|\eta(U)|^4 - \log|\eta(2U)|^4 \quad \text{for} \quad (\alpha_1, \alpha_2) = \left( 0, \frac{1}{2} \right),
\]

\[
= \log|\eta(U/2)|^4 + \log|\eta(2U)|^4 - 2\log|\eta(U)|^4 \quad \text{for} \quad (\alpha_1, \alpha_2) = \left( \frac{1}{2}, \frac{1}{2} \right). \quad (B.3)
\]

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