On the Intrinsic Parity of Black Holes

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Abstract

We investigate the intrinsic parity of black holes. It appears that discrete symmetries require the black hole Hilbert space to be larger than suggested by the usual quantum numbers M (mass), Q (charge) and J (angular momentum). Recent results on black hole production in trans-Planckian scattering lead to gravitational effects which do not decouple from low-energy physics. Dispersion relations incorporating these effects imply that the semi-classical black hole spectrum is similar in parity even and odd channels. This result can be generalized to other discrete and continuous symmetries.
1 Black hole quantum numbers

No-hair theorems [1] suggest that black holes are classified by quantum numbers such as mass $M$, charge $Q$ and angular momentum $J$. (There are more exotic possibilities, depending on the details of the matter content of the model [2].) In this note we investigate the intrinsic parity of black holes.

We adopt the viewpoint [3] that black holes can be described by quantum mechanics and have a unitary S-matrix. Given this assumption there are two possibilities: either the Hamiltonian commutes with parity, or not. Consider the following gedanken process. Let $N$ massive pseudoscalar particles undergo adiabatic, S-wave collapse to form a black hole. The parity of the initial state is

$$|i\rangle = (-1)^N |i\rangle. \quad (1)$$

If semi-classical gravitational interactions are parity invariant ($[P, H] = 0$), we have

$$|f\rangle = e^{-iHT} |i\rangle$$

$$P|f\rangle = e^{-iHT} P|i\rangle = (-1)^N |f\rangle \quad (2)$$

The final state $|f\rangle$ is a black hole characterized by its mass $M$, charge $Q$ and angular momentum $J$ (zero in this case), but with an intrinsic parity determined by the number of pseudoscalars $N$.

This suggests that the usual quantum numbers must be supplemented by a $\pm$ parity of the black hole wavefunction under spatial inversion $\vec{x} \rightarrow -\vec{x}$. This additional quantum number doubles the size of the black hole Hilbert space:

$$|M, J, Q\rangle \rightarrow |M, J, Q, \pm\rangle. \quad (3)$$

Interestingly, the Kerr metric has $+$ parity for all angular momenta $J$, whereas we expect that a generic quantum object has parity $\pm (-1)^J$, where $\pm$ reflects intrinsic parity. Unlike other forms of hair, intrinsic parity does not seem to have a classical counterpart in exterior black hole solutions.

However, it isn’t clear that $[H, P] = 0$. Consider the evaporation of a black hole. One can make a convincing argument that the semi-classical evaporation of a black hole violates continuous global symmetries such as baryon number. Because there is no classical hair associated with global symmetries, the evaporation process is independent of global charge, and hence the final state might have different charge than the black hole. In the case of a continuous symmetry an arbitrary amount of charge can be hidden in a large black hole, and the last moment of quantum evaporation cannot compensate for a large difference between the charges of the hole and of the semi-classical radiation.
In the case of discrete symmetries like C, P and T this argument is not conclusive: one could imagine each black hole storing a single bit of information representing the net parity of everything radiated, and then compensating for this in the final quantum part of the evaporation. Indeed, discrete symmetries might be the remnant of a local symmetry and therefore conserved by gravity \footnote{This is true of parity in some string models).}

Let us consider the implications of \([H,P] \neq 0\) due to gravitational effects. An immediate consequence is that the Hamiltonian cannot be diagonalized on the subspace of definite \(\pm\) parity, but rather has the form

\[
H = \begin{pmatrix} A & C^* \\ C & B \end{pmatrix},
\]

where A and B are real and C is complex. Each are functions of M, J and Q. Energy eigenstates are of the form: \(|\alpha|^2 + |\beta|^2 = 1\)

\[
|1\rangle = \alpha |+\rangle + \beta |-\rangle,
|2\rangle = \beta |+\rangle - \alpha |-\rangle.
\]

Note that for every M, J and Q there are now two energy eigenstates, generally with \(E_1 \neq E_2\). The states \(|1\rangle\) and \(|2\rangle\) are further distinguishable if they decay with different probabilities to plus or minus parity final states.

Neither \(E_1\) nor \(E_2\) need be equal to the classical black hole energy as a function of M, J and Q. The part of the Hamiltonian that violates parity must produce a mass splitting. Is there a way to avoid these strange results? One interesting (albeit speculative) possibility is to take

\[
H = \begin{pmatrix} 0 & E \\ E & 0 \end{pmatrix}.
\]

so that the eigenvalues are \(\pm E\). If we reinterpret the negative eigenvalue as a kind of black hole antiparticle energy, we obtain equal energies for each M, J and Q at the cost of introducing a new type of (anti) black hole. The energy eigenstates are

\[
|1\rangle = \frac{1}{\sqrt{2}} (|+\rangle + |-\rangle),
|2\rangle = \frac{1}{\sqrt{2}} (|+\rangle - |-\rangle).
\]

and appear equally likely to decay into plus or minus parity final states.

Whether or not \([H,P] = 0\), we seem to be led to a larger Hilbert space of black hole states. Similar conclusions hold for discrete symmetries such as C and T.
What are the observable consequences of intrinsic parity? Can we give a more general argument that black holes of both parities exist? Below we consider black hole production from the collision of particles with known parity. Recent work reveals a range of (trans-Planckian) energies and impact parameters where the quantum amplitude can be reliably computed, and the cross section is geometrical. The black holes are large compared to the Planck length $L_{\text{Planck}}$ and are themselves semi-classical. Seemingly, they can be produced in both plus and minus parity channels.

We investigate this using dispersion relations, which, due to analyticity, allow us to relate low energy correlators to black hole production at high energy. We study dispersion relations involving correlators of opposite parity. The correlators themselves are calculable at low energy, and through the dispersion relations imply the existence of large, semi-classical black holes in both $\pm$ parity channels.

Our analysis is similar in spirit to that of Peskin and Takeuchi [5], who used precision electroweak data to constrain the spectrum of states in technicolor models. In this case, we use low energy physics (low means relative to the Planck scale!) to constrain the black hole spectrum.

## 2 Black hole production from collisions

Eardley and Giddings [6] have analyzed classical solutions in general relativity which describe the collision of ultra-relativistic particles at non-zero impact parameter. They demonstrate the existence of a closed trapped surface for any collision with sufficiently small impact parameter (at fixed center of mass energy). The lower bound on the critical impact parameter is of order the radius

$$R_s \sim GE,$$

where $G$ is Newton’s constant and $E$ the center of mass energy. This leads to a geometrical classical cross section for black hole production in agreement with Thorne’s hoop conjecture [17].

In a classical collision which produces a black hole the maximum angular momentum is $J_{\text{max}} \sim R_s E \sim Gs$. The geometrical cross section for fixed $J$ is $\sigma_J(E) \sim J/s$, and the total cross section is

$$\sigma_{\text{total}} = \int_0^{J_{\text{max}}} dJ \sigma_J(E) \sim J_{\text{max}}^2/s \sim R_s^2,$$

(9)

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1For previous work on high energy scattering in general relativity, see [7]-[11]; for recent work on black hole production in particle collisions, see [12]-[16].
The relationship between classical and quantum black hole production has been investigated recently in [18] using a path integral representation of the S-matrix [19]-[20]. Although there are only two colliding particles, there are many gravitons in the initial state due to the strong gravitational fields. Thus, from the gravitational point of view the initial state is semi-classical, and from this perspective the subsequent evolution should not depend on whether the initial two particle state is in a parity even or odd state.

One would expect that the semi-classical approximation holds as long as \( GE^2 >> 1 \) and the impact parameter is larger than the Planck length \( L_{\text{Planck}} \). To express this in terms of angular momentum, note that a semi-classical initial state must have \( \Delta p << p \) and \( \Delta x << x \). Hence, using the uncertainty principle, we have \( J \sim px >> 1 \).

We can obtain a more restrictive condition by requiring that the curvature is small everywhere up to the formation of the closed trapped surface [18]. This is only possible if we leave the Aichelburg-Sexl limit of the metric, which describes an infinitely boosted particle of zero size [9], and instead consider particles of finite size \( r \) [18, 21]. In this case the maximum curvature in Planck units is given by

\[
\mathcal{R} \sim \left( \frac{R_s}{r} \right)^2 \left( \frac{L_{\text{Planck}}}{r} \right)^2.
\]

This leads to the requirement that \( E << M_{\text{Planck}}^2 J^{2/3} \), or equivalently that the range of validity of the semi-classical approximation for fixed \( J \) is

\[
M_{\text{Planck}}^2 << s << M_{\text{Planck}}^2 J^{4/3}.
\]

Clearly, only processes with large \( J \) can be described semi-classically.

Let us briefly comment on regions of \( s \) outside of (11) for scattering at fixed \( J \). When \( s \sim M_{\text{Planck}}^2 \) the resulting black hole is intrinsically quantum mechanical and stringy effects are expected. When \( s \approx M_{\text{Planck}}^2 J^{4/3} \) the resulting black hole is large and classical, however the initial state is not semi-classical. Roughly speaking, fixing \( J \) as \( s \to \infty \) requires that the impact parameter goes to zero. Eventually the impact parameter is smaller than the Planck length, and the uncertainty in position \( \Delta x \) is larger than \( x \). In this regime we expect black hole production, but cannot reliably calculate its cross section. If the geometrical result continues to hold, the dispersive integrals we consider below will get a logarithmically divergent contribution from production at asymptotically large \( s \), thereby violating unitarity. However, as discussed above, there is no reason to expect the semi-classical result to continue to hold in this region. We expect that the actual cross section is parametrically smaller than the geometric one at asymptotic energies, leading to convergent integrals.
3 Dispersive analysis

Analyticity allows us to relate the low- and high-energy behavior of quantum amplitudes. For example, given a vector current \( J^\mu(x) \), we define its correlator

\[
(q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi_V(q^2) = i \int d^4x \ e^{ixq} \langle 0 | T(J^\mu_V(x)J^\mu_V(0)) | 0 \rangle .
\]  

(12)

Using Cauchy’s theorem and some general properties of the singularity structure of a correlator in the complex \( q^2 \) plane, we obtain the following relation between the correlator and an integral over its imaginary part:

\[
\Pi(q^2) = \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im}\Pi(s)}{s - q^2 - i\epsilon} .
\]  

(13)

The imaginary part is also known as the spectral function, and describes the production of resonances or multi-particle final states with the same quantum numbers as the current.

Before proceeding further, let us discuss whether dispersion relations like (13) apply in the hyper-Planckian region, \( s > M^2_{\text{Planck}} \). Naively, one might think that quantum gravity governs the behavior of the integrand in this region. However, a quantum treatment of gravity is only necessary for \textit{curvatures} larger than the Planck scale. It is easy to see that trans-Planckian energies alone do not require quantum gravity, since any collision of macroscopic objects (for example, billiard balls) could involve energies larger than the Planck scale. As discussed in the previous section, there is a region in \( s \) (given in (11)) in which the amplitude is dominated by semi-classical configurations of low curvature.

More fundamentally, the derivation of (13) only requires unitarity and analyticity of scattering amplitudes, both properties which we expect to hold even at high energies (for example, in string theory). Indeed, in our use of (13) the left hand side is a forward scattering amplitude involving particles of low energy and in nearly flat space time. The relation of this amplitude to its imaginary part only requires analytic properties which are expected to be valid everywhere in the complex plane, even very large \( s \). Black holes appear as poles near the real axis in the complex \( s \) plane, and there is a branch cut (associated with a threshold for black hole production at \( s_0 \sim M^2_{\text{Planck}} \)) on the real axis. The part of the integral given in (11) only involves semi-classical black holes of low curvature.

Consider operators which produce states with large angular momentum \( J \), whose correlators receive contributions from semi-classical black hole formation. Define \( S_J \) to be the difference of correlators of currents with opposite parity:

\[
S_J(q^2) = \Pi_\Gamma(q^2) - \Pi_P^\Gamma(q^2) ,
\]  

(14)

where we extract the dimensionless scalar function \( \Pi_\Gamma(q^2) \) from the angular momentum \( J \) component of \( \langle 0 | T(J_\Gamma(x)J_\Gamma(0)) | 0 \rangle \). \( \Pi_\Gamma(q^2) \) is a function of \( q^2 \) only, and we can always...
rescale by $q^2$ to obtain something dimensionless. The current is defined as $J_\Gamma = \bar{\psi} \Gamma \psi$, with $\Gamma \sim \Pi_j \hat{\partial}_i$, where $\hat{\partial}$ denotes a unit vector. We obtain the angular momentum $J$ component by symmetrizing and subtracting the appropriate traces. In $\Pi^P_\Gamma$ we replace $\Gamma$ by $\gamma_5 \Gamma$.

By taking $q^2$ large and spacelike, we can ensure that short distance contributions dominate. In an asymptotically free theory, (14) can then be reliably computed, and shown to cancel up to corrections suppressed by powers of $1/q^2$. If there are no low-energy condensates that appear in the operator product expansion of the correlator, this cancellation is exact. Note, we keep $q^2 \ll M_{\text{Planck}}^2$, so that quantum gravity effects remain negligible. In perturbation theory, the leading contribution to each correlator comes from a one loop diagram. The leading contribution to $\Pi_\Gamma(q^2)$ is of the form

$$(-)^J \text{Tr} \left[ \mathcal{S} \Gamma^\dagger \mathcal{S} \Gamma \right],$$

where $\mathcal{S}$ is a fermion propagator. The leading contribution to the opposite parity correlator $\Pi^P_\Gamma(q^2)$ is of the form

$$(-)^{J+1} \text{Tr} \left[ \mathcal{S} \gamma_5 \Gamma^\dagger \mathcal{S} \Gamma \gamma_5 \right].$$

So, $\Pi_\Gamma(q^2) = \Pi^P_\Gamma(q^2)$ and $S_J(q^2)$ vanishes at leading order.

However, for sufficiently large $J$, there will be a trans-Planckian region of $s$ (see (11)) in the dispersion integral where the semi-classical approximation applies, and the black hole production cross section is given by $\sigma_J(s) \sim J/s$. This cross section implies constant $\text{Im} \Pi(s)$ in this region, with $s \gg q^2$. If black holes only exist with a single (e.g., either + or −) intrinsic parity, then the integrand is zero in one of the channels and the dispersion integral yields a correction

$$\Delta S_J \sim J \log J$$

for large $J$. Since the dispersion integral (13) runs to infinity, this result is independent of the Planck scale, and remains even if we take $M_{\text{Planck}} \to \infty$. $\Delta S_J$ can be made arbitrarily large by considering $J \to \infty$, which contradicts the result that (14) must vanish at large $q^2$.

The region $s > M^2_{\text{Planck}} J^{4/3}$ gives another unbalanced contribution to $\Delta S_J$ if there are not black holes of both parities, although we cannot calculate its size. It is highly unlikely that these contributions are cancelled by stringy processes in the region $s \sim M_{\text{Planck}}^2$. This would require that string scattering be highly parity asymmetric. Further, the string cross sections would have to be larger than geometric in order to cancel the contributions at larger energies.

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2Strictly speaking, correlators of operators separated by less than a Planck length may be ill-defined in quantum gravity. However, one can avoid this problem by considering the dispersion relation (13) as a relation between scattering amplitudes and their imaginary parts, without reference to operators or correlators. As discussed, the scattering amplitudes of interest are well-defined even at trans-Planckian energy.
The most plausible resolution is that black holes states appear in both parity channels, leading to a cancellation if the spectral functions are the same. As discussed, this can be the case whether or not $[H, P] = 0$.

This conclusion is quite general, and does not depend in detail on the matter content of the model. It is straightforward to generalize this result to other discrete symmetries, or even to continuous symmetries. If the symmetry can be applied to the low-energy amplitude (i.e. at energies small compared to $M_{\text{Planck}}$, where quantum gravity is negligible), then the corresponding dispersion relation requires that it apply to the black hole production process in that channel. Hence we conclude that black holes exist in each of the channels related by the symmetry.
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