A spherically-symmetric charged-dS solution in f(T) gravity theories

Gamal G.L. Nashed

Centre for Theoretical Physics, The British University in Egypt
Sherouk City 11837, P.O. Box 43, Egypt

e-mail:nashed@bue.edu.eg

A tetrad field with spherical symmetry is applied to the charged field equations of \( f(T) \) gravity theory. A special spherically-symmetric charged-dS solution is obtained. The scalar torsion of this solution is a vanishing quantity. The spacetime of the derived solution is rewritten as a multiplication of three matrices: The first matrix is a special case of Euler's angle “so(3)”, the second matrix represents a boost transformation, while the third matrix is the square root of the spherically-symmetric charged-dS metric. It is shown that the boost matrix is important because it plays an essential role in adjusting the spacetime to become a solution for \( f(T) \) theory.

§1. Introduction

Extensions of General Relativity (GR) aim to demonstrate the late-time acceleration of the universe and Dark Energy (DE). Recent observations suggest the presence of fluids as well as the known pressureless matter terms which permit our universe to exhibit a late-time acceleration [1, 2, 3]. At present, the physical existence of such fluids is unknown [4]. An intensive discussion has emerged to understand any form of fluids that can cause the observed cosmic acceleration [5]. Although its physical nature is not known, it displays an Equation of State (EoS) with negative pressure [6, 7, 8]. This EoS supplies an antigravity impact that balances the pull of gravity acting on standard matter [9, 10, 11]. This hypothetical fluid, usually named as DE, is representing more than 70% of all the universe energy budget [12].

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1Mathematics Department, Faculty of Science, Ain Shams University, Cairo, 11566, Egypt.

Egyptian Relativity Group (ERG) URL: http://www.erg.eg.net.
Recently, many efforts have been devoted to the amended theories of gravity that unify the method of early-time inflation and late-time acceleration of the universe. Such theories possibly supply another framework that features our understanding of DE. The gravitational action in such theories should apply at the low-energy limit of fundamental quantum gravity. These gravitational theories differ from GR (for example $f(R)$), where $R$ is the Ricci scalar in the Einstein-Hilbert action [13, 14]. In GR and its amendments the metric $g_{\mu\nu}$ and the quantities derived from it characterize the gravitational field.

The basic philosophy of the amended theories of gravity is that GR must be viewed as a special case of a wider theory derived from basic principles [15]-[23]. The underlying idea is that the standard Einstein-Hilbert action is changed by adding degrees of freedom. These include extra curvature-invariant corrections to scalar fields and Lorentz-violating terms.

Most recently, $f(T)$ gravitational theories were constructed as an alternative to the cosmological constant, to give an explanation to the accelerated expansion of the universe [24]. By analogy with the $f(R)$ gravitational theories, modified teleparallel equivalent of general relativity theories differ from GR by a function $f(T)$ in the Lagrangian, where $T$ is called the torsion scalar. This set of theories has gained a lot of attention in the last few years [25]-[38]. The $f(T)$ gravitational theories are particularly attractive since their field equations are of second order, rather than fourth order as in $f(R)$. The aim of this study is to find an analytic, spherically-symmetric charged-dS solution within the framework of $f(T)$ gravity theory.

$f(T)$ gravitational theories represent a category of models, which takes into account the influences related to the torsion scalar, $T$. The feature of $f(T)$ gravity is that the curvature tensor identically vanishes using the Weitzenböck connection. $f(T)$ model can be viewed as expansion of “teleparallel gravity” [25]. Therefore, one can amend the Lagrangian density by adding a torsion function $f(T)$.

In §2, a survey of the $f(T)$ gravitational theory is supplied. A non-diagonal, spherically-symmetric tetrad field is provided.

Application of charged equations of motion, of $f(T)$ gravitational theory, to the non-diagonal tetrad field is given in §3. The special analytic, non-vacuum, spherically-symmetric charged-dS solution is derived with two constants of integration.

In §4, the internal properties of the derived solution is demonstrated.

In the last section, a discussion of the main results is given.
§2. Brief review of $f(T)$

In a spacetime having absolute parallelism, the tetrad field $h^\mu_i$ [39] defines the nonsymmetric affine connection:

$$\Gamma^\mu_{\nu\lambda} \overset{\text{def.}}{=} h^\mu_i h^i_{\nu\lambda},$$  \hspace{1cm} (1)

where $h^i_{\nu\lambda} = \partial_\lambda h^i_\nu$. \hspace{1cm} (4)

The curvature tensor defined by $\Gamma^\mu_{\nu\lambda}$, given by Eq. (1), is identically vanishing. The metric tensor $g_{\mu\nu}$ is defined by:

$$g_{\mu\nu} \overset{\text{def.}}{=} \eta_{ij} h^i_{\mu} h^j_{\nu},$$  \hspace{1cm} (2)

with $\eta_{\mu\nu} = (+1, -1, -1, -1)$ denoting the metric of Minkowski spacetime. The torsion components and the contortion are given, respectively, as:

$$T^\mu_{\nu\lambda} \overset{\text{def.}}{=} \Gamma^\mu_{\lambda\nu} - \Gamma^\mu_{\nu\lambda} = h^\mu_i \left( \partial_\nu h^i_\lambda - \partial_\lambda h^i_\nu \right),$$

$$K^\mu_{\nu\lambda} \overset{\text{def.}}{=} -\frac{1}{2} \left( T^\mu_{\nu\lambda} - T^\nu_{\mu\lambda} - T^\lambda_{\mu\nu} \right),$$  \hspace{1cm} (3)

where the contortion equals the difference between Weitzenböck and Levi-Civita connection, i.e., $K^\mu_{\nu\lambda} = \Gamma^\mu_{\nu\lambda} - \{^\mu_{\nu\lambda}\}$.

The tensor $S^\mu_{\nu\lambda}$ is defined as:

$$S^\mu_{\nu\lambda} \overset{\text{def.}}{=} \frac{1}{2} \left( K^\nu_{\lambda\mu} + \delta^\nu_\mu T^3_{\beta\lambda} - \delta^\lambda_\mu T^3_{\beta\nu} \right).$$  \hspace{1cm} (4)

The torsion scalar is defined as:

$$T \overset{\text{def.}}{=} T^\mu_{\nu\lambda} S^\mu_{\nu\lambda}.$$  \hspace{1cm} (5)

Similar to the $f(R)$ theory, one can define the action of $f(T)$ theory as

$$\mathcal{L}(h^a_\mu) = \int d^4x h \left[ \frac{1}{16\pi} (f(T) - 2\Lambda) + \mathcal{L}_{em} \right],$$

where $h = \sqrt{-g} = \det (h^a_\mu)$,

$\mathcal{L}_{em}$ is the Lagrangian of electromagnetic field

and $\Lambda$ is the cosmological constant.

We are going to use the units in which $G = c = 1$ and $\mathcal{L}_{em} = -\frac{1}{4} F \wedge^* F = -\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}$ is the Maxwell Lagrangian, while\(^3\) $F = dA = A_{[\mu,\nu]}[dx^\mu \wedge dx^\nu]$, with $A = A_\mu dx^\mu$, is the electromagnetic potential 1-form [32]. Similar to the $f(R)$ theory, one can define the action of

\(^1\)We use the Greek indices $\mu, \nu, \ldots$ for local holonomic spacetime coordinates and the Latin indices $i, j, \ldots$ to label (co)frame components.

\(^3\)We will denote the symmetric part by $( )$, for example, $A_{(\mu\nu)} = \frac{1}{2}(A_{\mu\nu} + A_{\nu\mu})$ and the antisymmetric part by the square bracket $[ ]$, $A_{[\mu\nu]} = \frac{1}{2}(A_{\mu\nu} - A_{\nu\mu}).$
$f(T)$ theory as a function of the fields $h^{a}_{\mu}$ and, by putting the variation of the function with respect to the tetrad field $h^{a}_{\mu}$ to be vanishing, we can obtain the following equations of motion:

$$S^{\rho \nu}_{\mu \rho} f(T)_{TT} + \left[ h^{-1} h^{a}_{\mu} \partial_{\rho} (h h^{a}_{\alpha} S^{\rho}_{\alpha \nu}) - T^{\alpha \lambda \mu \nu} S^{\alpha \lambda \nu} \right] f(T)_{TT} - \frac{\delta_{\mu}^{\nu} (f(T) - 2\Lambda)}{4} = -4\pi T^{em}_{\mu \nu}, \quad (7)$$

where

$$T_{\rho} = \partial f(T) \partial x^{\rho}, \quad f(T)_{T} = \frac{\partial f(T)}{\partial T}, \quad f(T)_{TT} = \frac{\partial^{2} f(T)}{\partial T^{2}}.$$

and

$$T^{em}_{\mu \nu} = g_{\rho \sigma} F^{\mu \rho} F^{\nu \sigma} - \frac{1}{4} g^{\mu \nu} g^{\lambda \sigma} F_{\lambda \rho} F_{\sigma \nu},$$

is the energy momentum tensor for electromagnetic field.

§3. Spherically symmetric solution in $f(T)$ gravity theory

Assuming that the manifold possesses a stationary and spherical symmetry, the tetrad field has the form:

$$\begin{pmatrix}
A_{1}(r) & A_{2}(r) & A_{3}(r) & A_{4}(r) \\
B_{1}(r) \sin \theta \cos \phi & B_{2}(r) \sin \theta \cos \phi & B_{3}(r) \cos \theta \cos \phi & B_{4}(r) \sin \phi \sin \theta \\
C_{1}(r) \sin \theta \sin \phi & C_{2}(r) \sin \theta \sin \phi & C_{3}(r) \cos \theta \sin \phi & C_{4}(r) \cos \phi \sin \theta \\
D_{1}(r) \cos \theta & D_{2}(r) \cos \theta & D_{3}(r) \sin \theta & D_{4}(r) \cos \theta
\end{pmatrix}, \quad (9)$$

where $A_{i}(r)$, $B_{i}(r)$, $C_{i}(r)$ and $D_{i}(r)$, $i = 1 \cdots 4$, are sixteen unknown functions of the radial coordinate, $r$. The metric of spherically-symmetric has the form [40]:

$$ds^{2} = A(r)dt^{2} - \frac{1}{A(r)}dr^{2} - r^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2}). \quad (10)$$

To create metric (10) from tetrad field (9) we must have

$$A_{3} = A_{4} = D_{4} = 0, \quad B_{3} = -B_{4} = C_{3} = -D_{3} = C_{4} = r, \quad B_{2} = C_{2} = D_{2}, \quad B_{1} = C_{1} = D_{1}. \quad (11)$$

Constraints (11) result from the use of spherical symmetry. Using equation (11) in tetrad field (9) we get:

$$\begin{pmatrix}
A_{1}(r) & A_{2}(r) & 0 & 0 \\
B_{1}(r) \sin \theta \cos \phi & B_{2}(r) \sin \theta \cos \phi & r \cos \theta \cos \phi & -r \sin \theta \sin \phi \\
B_{1}(r) \sin \theta \sin \phi & B_{2}(r) \sin \theta \sin \phi & r \cos \theta \sin \phi & r \sin \theta \cos \phi \\
B_{1}(r) \cos \theta & B_{2}(r) \cos \theta & -r \sin \theta & 0
\end{pmatrix}. \quad (12)$$
The metric of tetrad field (12) takes the form

\[ ds^2 = \eta_{ij} \left( h^i_\mu \right)_1 \left( h^j_\nu \right)_1 \, dx^\mu \, dx^\nu = \left[ A_1^2(r) - B_1^2(r) \right] dt^2 - \left[ A_2^2(r) - B_2^2(r) \right] dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \]

Using Eq. (7) and tetrad field (12) one can obtain

\[ h = \det(h^a_\mu) = r^2 \sin \theta (A_1 B_2 - B_1 A_2). \]

With the use of equations (3) and (4), we obtain the torsion scalar and its derivatives in terms of \( r \) as:

\[ T(r) = -\frac{4}{r^2(A_1 B_2 - B_1 A_2)^2} \left[ r(A_1 B_2 - B_1 A_2) \left[ (B_2 - 1)A_1 - B_1 A_2 \right] \right. \]
\[ \left. + \frac{1}{2} \left[ (B_2 - 1)A_1 - B_1 A_2 - (B_2 - 1)A_2 + B_1 A_1 \right] \right], \]

where

\[ A_1' = \frac{\partial A_1(r)}{\partial r}, \quad A_2' = \frac{\partial A_2(r)}{\partial r}, \quad B_1' = \frac{\partial B_1(r)}{\partial r}, \quad \text{and} \quad B_2' = \frac{\partial B_2(r)}{\partial r}. \]

The equations of motion (7) can be rewritten in the form

\[ 4\pi T_0^0 = -\frac{f_{TT} T'(B_2 - 1)A_1^2 - A_1 A_2 B_1 + B_2^2}{r(A_1 B_2 - B_1 A_2)^2} - \frac{f_T}{r^2(A_1 B_2 - B_1 A_2)^3} \left( rA_1' \{ A_1(B_2 - 1)(B_2 A_1 - 2B_1 A_2) \} \right) \]
\[ + \frac{B_1^2(A_2^2 - B_2) - rB_1'(A_1^2 A_2 + A_2 B_1^2 - 2A_1 B_2 + B_2) + r(A_1 B_2' - A_1 B_1' + B_2' A_1^2 - A_1 A_2 B_1 + B_1^2)}{2} \]
\[ + (A_1 B_2 - B_1 A_2) \left[ (B_2 - 1)A_1^2 - A_1 A_2 B_1 + B_2^2 \right] + \frac{f - 2\Lambda}{4}, \]

\[ 4\pi T_0^1 = -\frac{2B_1 f_T \sin^2 \theta \{ A_1^2 + A_1 A_2 B_1 - B_1^2 + rB_1 B_1' - B_2 A_2^2 + rA_1 A_1' \}}{r^2(A_2^2 - B_2 A_2)^2}, \]

\[ 4\pi T_1^0 = \frac{f_{TT} T'(B_1 A_2^2 - A_1 (B_2 - 1) - B_1 B_2)}{r(A_1 B_2 - B_1 A_2)^2} + \frac{f_T [ A_1 A_2 B_1 - B_1 A_2 B_1' + A_1 B_1' - B_1 A_1' ]}{r(A_1 B_2 - B_1 A_2)^3}, \]

\[ 4\pi T_1^1 = -\frac{f_T \{ B_1^2 - A_1 A_2 B_1 + (B_2 - 1)A_1^2 + 2rB_1 B_1' + r[(B_2 - 2)A_1 - B_1 A_2]A_1' \}}{r^2(A_1 B_2 - B_1 A_2)^2} + \frac{f - 2\Lambda}{4}, \]

\[ 4\pi T_2^0 = \frac{-2f_T \sin 2\theta \{ A_1^2 - A_1 A_2 B_1 - B_1 A_2 - rB_1 B_1' + A_1 A_2 A_1' \}}{r(A_1 B_2 - B_1 A_2)^2}. \]

*For brevity, we write \( A_i(r) \equiv A_i, \quad B_i(r) \equiv B_i, \quad C_i(r) \equiv C_i \quad \text{and} \quad D_i(r) \equiv D_i. \)
$4\pi T_2^2 = 4\pi T_3^3 = \frac{f_{TT} T'(r[A_1 A'_1 - B_1 B'_1] - [B_2 - 1] A^2 + A_1 A_2 B_1 - B^2)}{2r(A_1 B_2 - B_1 A_2)^2}$

\[+ \frac{f_T}{2r^2(A_1 B_2 - B_1 A_2)^3} \left( r^2(A_1 B_2 - B_1 A_2)[A_1 A'_1 - B_1 B'_1] - r^2 B_2 A_2 A'_2 - r A'_1 \left[ rB'_1 (B_1 B_2 + A_1 A_2) \right. \right. \]

\[\left. \left. - r A_1 (A_1 B'_2 - B_1 A'_2) + A^2 B_2 (3 - 2 B_2) - 4 A_1 A_2 B_1 (1 - B_2) + (B_2 - 2 A^2) B^2 \right] \right) \]

\[- r^2 A_1 A_2 B_2 B'_1 - r B'_1 \left( r A_1 B'_2 - r B^2 A'_2 + A^2 A_2 - 4 A_1 B_1 B_2 + 3 A_2 B^2 \right) + r (A_1 B'_2 - B_1 A'_2) (B^2 - A^2) \]

\[- (A_1 B_2 - B_1 A_2) [(B_2 - 1) A_1 - (A_2 - 1) B_1] [(B_2 - 1) A_1 - (A_2 + 1) B_1] + \frac{f - 2\Lambda}{4}. \] (20)

When $f(T) = T$, i.e., $f_{TT} = 0$ and $f_T = 1$, Eqs. (13)–(19) will be identical with the teleparallel equivalent of GR. From equations (15)–(20), it is clear that $A_1 B_2 \neq B_1 A_2$. Equations (14)–(20) cannot be easily solved. This is because of the existence of the two terms $f_T$ and $f_{TT}$. If these two terms are vanishing then equations (15)–(20) can be easily solved. Therefore, to find an exact solution of equations (15)–(20), we put the following constraints:

\[
T' = 0, \\
A^2 + A_1 A_2 B_1 - B^2 - r B_1 B'_1 - B_2 A'_2 + r A_1 A'_1 = 0, \\
B_2 A'_1 - A_2 B'_1 + A_1 B'_2 - B_1 A'_2 = 0, \\
B_1 A^2 - A_1 A_2 B_2 - B_2 B_1 - r B_2 B'_1 + A_1 A_2 + r A_2 A'_1 = 0.
\] (21)

Constraints (21) are resulting from the fact that we put the derivatives of the scalar torsion to be vanishing, to attain the removed of $f_{TT}$ terms from the field equations (7). The other constraints in Eq. (21) come from the coefficient of $f_T$ in the off-diagonal components of Eqs. (16), (17) and (19). The constraints in Eq. (21) are adequate to secure the existence of special analytic solution to Eqs. (15)–(20), as will be show below. Equations (21) are four differential equations in four unknown functions $A_1$, $A_2$, $B_1$ and $B_2$. The only solution that can satisfy equations (21) is the following$^\text{6}$.

\[
A_1 = 1 + \frac{3 c_2^2 - 6 c_1 r - r^4 \Lambda}{6 r^2}, \quad A_2 = \frac{3 c_2^2 - 6 c_1 r - r^4 \Lambda}{2 \Delta}, \quad B_1 = 1 - A_1, \\
B_2 = - \frac{6 r^2 + [3 c_2^2 - 6 c_1 r - r^4 \Lambda]}{2 \Delta},
\] (22)

where $\Delta$, is defined by:

\[
\Delta = -3 r^2 + 3 c_2^2 - 6 c_1 r - r^4 \Lambda.
\] (23)

Here $c_1$ and $c_2$ are two constants of integration.

The tetrad field (12), after using equation (22), is axially-symmetric. This means that it is invariant under the transformation:

\[
\tilde{\phi} \rightarrow \phi + \delta \phi, \quad h^0 \rightarrow h^0, \quad h^1 \rightarrow h^1 \cos \delta \phi - h^2 \sin \delta \phi, \\
\tilde{h}_\mu \rightarrow h^1 \sin \delta \phi + h^2 \cos \delta \phi, \quad \tilde{h}^3 \rightarrow h^3 \mu.
\] (24)

$^6$These calculations were checked using Maple “15” software.
Using equations (12) and (22) in equation (5) we get a vanishing value of the scalar torsion, \( T \), because of constraints (21). Substituting (12) and (22) in Eq. (7), we get an exact non-vacuum solution to the field equations of \( f(T) \) given by equations (7) and (8), provided that 
\[
f(0) = 0, \quad f_T(0) = 1, \quad f_{TT} \neq 0, \quad A_t = -\frac{c_2}{r\sqrt{2\pi}}, \quad F_{tr} = \frac{c_2}{r^2\sqrt{2\pi}},
\]
\[
T_0^0 = T_1^1 = -T_2^2 = -T_3^3 = -\frac{c_2^2}{4r^4\pi}.
\] (25)

In summary: Solution (22) is a special analytic solution to the field equations of \( f(T) \) whatever the form of \( f(T) \) is provided that Eqs. (25) are satisfied. Equation (25) reduces to the vacuum case when the constant \( c_2 = 0 \) \[41\]. To understand the physical meaning of the constants of integration that appear in solution (22) we discuss the physics related to this solution. First, the metric associated with the tetrad field given by equation (12) (after using solution (22) in Eq. (13)) has the form:
\[
\alpha dt^2 - \frac{1}{\alpha} dr^2 - r^2 d\Omega^2, \quad \text{with} \quad d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2, \quad \alpha = \left( 1 - \frac{2c_1}{r} + \frac{c_2^2}{r^2} - \frac{r^2\Lambda}{3} \right).
\] (26)

This is similar to Reissner Nordström-dS spacetime provided that \( c_1 = M \) and \( c_2 = q \). When \( \Lambda \rightarrow 0 \), Eq. (26) reduces to Reissner Nordström spacetime. Further, when \( \Lambda \rightarrow 0 \) and \( c_2 = 0 \) then Eq. (26) reduces to Schwarzschild spacetime and, finally, when \( c_2 = 0 \) then Eq. (26) gives Schwarzschild-dS spacetime. All these special cases can be verified from solution (22).

§4. Internal properties

In this section, we rewrite the tetrad field (12) using equation (22) in terms of a special case of Euler’s angle “so(3)” and a boost transformation. For this purpose, we use the fact that any tetrad field can be written as:
\[
(h^i_{\mu}) \overset{\text{def.}}{=} (\Lambda^i_j) \left( h^j_{\mu} \right)_1,
\] (27)
where \((\Lambda^i_j)\) is a local Lorentz transformation satisfying
\[
(\Lambda^i_j) \eta_{jk} (\Lambda^k_l) = \eta_{il},
\] (28)
and \((h^i_{\mu})_1\) is another tetrad field which reproduces the same metric that is produced by the tetrad field \((h^i_{\mu})\). Now we use Eq. (22) to rewrite the tetrad field (12) as:
\[
(h^i_{\mu}) = (\Lambda^i_j) \left( \Lambda^j_k \right)_1 (h^k_{\mu})_d,
\] (29)
where
\[
(\Lambda^i_j) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \phi \\
0 & \sin \theta \sin \phi & \cos \theta \sin \phi & \cos \phi \\
0 & \cos \theta & -\sin \theta & 0
\end{pmatrix},
\] (30)
\[
\left( \Lambda^i_j \right)_1 = \begin{pmatrix}
\frac{2r^2 - 2Mr + q^2 - \frac{\Lambda r^4}{3}}{2r \sqrt{r^2 - 2Mr + q^2 - \frac{\Lambda r^4}{3}}} & \frac{2Mr - q^2 + \frac{\Lambda r^4}{3}}{2r \sqrt{r^2 - 2Mr + q^2 - \frac{\Lambda r^4}{3}}} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & \sqrt{r^2 - 2Mr + q^2 - \frac{\Lambda r^4}{3}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sqrt{r^2 - 2Mr + q^2 - \frac{\Lambda r^4}{3}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & r \sin \theta \\
\end{pmatrix},
\]

and
\[
\left( h^i_{\mu} \right)_d = \begin{pmatrix}
\frac{\sqrt{r^2 - 2Mr + q^2 - \frac{\Lambda r^4}{3}}}{r} & 0 & 0 & 0 \\
0 & \frac{r}{\sqrt{r^2 - 2Mr + q^2 - \frac{\Lambda r^4}{3}}} & 0 & 0 \\
0 & 0 & r & 0 \\
0 & 0 & 0 & r \sin \theta \\
\end{pmatrix}.
\]

The matrices (30) and (31) give local Lorentz transformations, i.e., they satisfy Eq. (28). We call matrix (32) the diagonal form of the tetrad field (12). Finally, equation (31) is the boost of tetrad field (12). Matrix (31) shows that the boost transformation depends on the gravitational mass and the charge parameters as well as the cosmological constant. When the charge parameter is vanishing, the boost transformation depends on the mass and the cosmological constant and when charge parameter and cosmological constant are vanishing then the boost transformation depends on the gravitational mass only.

§5. Main results and discussion

The \( f(T) \) gravitational theory is an amendment of the teleparallel equivalent of general relativity, which attempts approaching some new observation problems. To find exact solutions within this theory is not an easy task. We analyzed the non-vacuum case of \( f(T) \) gravitational theory, with the equations of motion applied to a non-diagonal spherically symmetric tetrad field. Four nonlinear differential equations were obtained.

To solve these differential equations, we imposed four constraints (in four unknown functions). The only solution that is compatible with these constraints contains two constants of integration as well as a cosmological constant. An important property of this solution is that it has a vanishing scalar torsion, i.e. \( T = 0 \), and satisfies the field equations of \( f(T) \), provided that Eq. (25) is satisfied.

In addition, this solution has an axial-symmetry, i.e., the components of its tetrad field are invariant under the change of the angle \( \phi \). We showed that the two constants of integration related to the gravitational mass and to the charge parameter. The associated metric of this solution gave the Reissner Nordström-dS spacetime.
To understand the internal properties of the derived solution we rewrote it as two local Lorentz transformations and a diagonal tetrad\(^\ast\ast\). It was shown that one of the local Lorentz transformations is related to Euler’s angle under certain conditions. This local Lorentz transformation is the same as the one studied in [42, 43]. The other local Lorentz transformation depends on the two parameters, the gravitational mass and the charge parameter, as well as the cosmological constant.

To recapitulate, any GR solution remains valid in f(T) theories having \( f_T(0) = 1 \) whenever the geometry admits a tetrad field with vanishing scalar torsion, \( T \) [41].

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