1. Introduction

Forecasting the dynamics of electric load, as well as power consumption, using its fractal properties has become common in energy engineering. Using the Hurst exponent makes it possible to detect and numerically assess such fundamental characteristics of time series as the existence of a long-term memory and its depth, persistence, and others. The application of these criteria allows building predictive models that prove to be efficient when classic prediction methods turn out to be ineffective.
information about the electric loading on civilian objects, to identify periods of unstable state in order to adequately forecast it and to effectively manage power consumption.

2. Literature review and problem statement

An analysis of research papers on the topic of our study has revealed that the existing methods for forecasting electric load face certain difficulties in their practical application. For example, the mathematical models that forecast electric load must take into consideration the a priori information about a process, fluctuations in a day light duration, the influence of external factors, seasonality in a network load and schedule of people daily activities, which are difficult to describe mathematically. All this predetermines the relevance of constructing new methods, which would reflect and predict complex dependences that are accompanied by a fuzzily formalized task. These new methods include the theory of fractals, which makes it possible to alternatively consider the process of forecasting power consumption and to efficiently solve the specified problems [1]. Studies apply fractals, a time series analysis, and nonlinear dynamic models, to explain the behavior and understanding of fluctuations in market prices [2]. Based on the theories of functions of fractal dimensionality and fractal interpolation, they employ correlation algorithms to the model for a short-term forecasting of loading the energy system. The attractor is derived by using an advanced deterministic algorithm based on a fractal interpolation function. Forecasting is based on a 3-day a priori information about a loading, which does not necessarily provide for a high accuracy [3]. Researchers address the issues on improving the efficiency of managing power consumption by devising a method and the models for operative forecasting of electric load on the systems of power consumption, at industrial enterprises, exploiting their fractal properties [4]. Conventional methods have a series of drawbacks, such as a vertical scaling factor that is hard to calculate, low accuracy, complexity of application. Therefore, researchers propose a preliminary analysis of the a priori information about electrical load using a wavelet, to be followed by using a value for the Hurst parameter in order to calculate coefficients of vertical scaling. Forecasting is performed in two aspects, namely the fractal interpolation and fractal extrapolation. Extrapolating the trend, as well as a regression model, are widely used in long-term forecasting of power load, however, such models cannot take into consideration the phenomenon of chaos in systems [5]. In practice, the use of multidimensional probability densities is quite complicated, as well as their determination. Random nonstationary load diagrams often demonstrate peculiarities that simplify their analysis and modelling. Such features include the repeatability of technological or peculiarities that simplify their analysis and modelling. Despite the unquestionable importance of the above types of studies, in many cases one has to abandon them given the non-stationarity of the process. It is possible to consider an algebraic approach the construction of mathematical models of processes based on the theory of fuzzy sets, artificial neural networks. This makes it possible to build a model of an object or a process under conditions of small and non-stationary samples, as well as to formalize the expert estimates by specialists. It is the mathematical predictive models, which are a combination of statistical and deterministic models, that makes it possible to provide for the best possible accuracy of prediction. However, such models do not always reproduce the complex dependences that are accompanied by a fuzzy formalized task [6]. Study [10] reported the software implementation of an artificial neural network to predict the technical energy losses along power transmission lines, but without separation of types of electricity consumers into industrial and civilian. Paper [12] assessed the risks for an integrated power system calculated by performing a temperature-enhanced probabilistic flow of electric load by using Monte Carlo simulation. The a priori information on electricity generation, on ambient temperature and the power of electrical load, collected over twelve specific time moments for the past five years, is previously handled by using three models of a linear regression in order to model uncertainty, which does not always warrant a high accuracy. Paper [13] applies a wavelet decomposition of power electric load into a series of low-frequency and high-frequency signals, which are predicted by different models. In order to find the optimal parameters for prediction, the authors propose an algorithm for checking the segmentation of the a priori information on load according to the time sequence.

However, the unsolved part of the issue is the actual lack of in-depth research into prediction of energy consumption, specifically by civil objects, in contrast to such a research on power systems at industrial enterprises.

3. The aim and objectives of the study

The aim of this study is to improve the efficiency of forecasting and managing power consumption by civilian objects using the method of a short-term and long-term prediction of electrical load diagrams employing their fractal properties.

To accomplish the aim, the following tasks have been set:
- to arrange, and to obtain from the automated system of commercial electric power accounting (ASCEPA), the retrospective information on electric load, on power consumption by multi-story residential buildings within a city’s neighborhood;
- to simulate the daily, weekly, and monthly dynamics of electric load, the power consumption by multi-story residential buildings within a city's neighborhood based on information from ASCERA;
- to explore the fractal properties of electric load on civilian objects and to estimate the depth of memory, trend resistance, crisis periods, based on retrospective information;
- to implement the fractal properties of electric load on civilian facilities, identified by the current research, in order to adequately predict and effectively manage power consumption.

4. The object and methods to study the fractal properties of electric load diagrams of civilian objects

4.1. The object of the study and the characteristics of consumers of electricity used in the experiment

The object of this study is the process of forming the structure of daily, weekly, monthly electrical load diagrams for apartments, residential buildings within a city's neighborhood. Their dynamics are random, depending on the presence of a set of electricity receivers, as well as daily activities of people, which change significantly over 24 hours, depending on a season. The shape of power consumption diagrams is affected by ambient temperature, the duration of daily light hours, cloudy sky, the speed and humidity of wind, etc. [14]. All this creates difficulties for their forecasting. Determining and prediction of electrical loads are the basis for designing internal networks in buildings, urban systems of electricity supply, for planning the volume of electricity for people. The dynamics of energy consumption at residential buildings are defined by two groups of electricity receivers. The first group includes lighting devices, household appliances. The second group is composed of general-purpose home appliances that light floor areas, stairwells, and fan areas, elevators, pumps for cold and hot water, etc. The magnitude of electrical power consumption is affected by the most powerful electricity receivers for everyday use. These include electric stoves, boilers, air conditioners, electric appliances, washing machines and dishwashers. Results from surveys of apartment houses dwellers have shown that the installed power for lighting the flats was from 0.34 to 0.65 kW, and much less with a gradual application of modern lighting devices. The surveyed flats have about 50 % of electric kettles and 95 % of tv sets per 100 families. Refrigerators are used by 100 % of dwellers, washing machines – by 75 %, the number of boilers, air conditioners are continuously increasing from 30 %. Families use other household appliances whose share in general use is insignificant. General-purpose loads on high-rise buildings, due to a special mode of operation throughout the year, account for 3–5 % of the total estimated load on a residential building.

4.2. A method to study the fractal properties of electrical load diagrams at civilian objects

The method for studying the fractal properties of electrical load diagrams, the power consumption at civilian objects, is based on the results from analyzing them using the Hurst exponent. In this case, we use an analogy between constructing a Koch's Curve and the principle for forming power consumption diagrams using an example of electricity consumption at multi-story residential buildings within a city's neighborhood [1]. As is known, to build a Koch's Curve, one needs to divide a section into three equal parts, to construct an equilateral triangle in the middle one. Each side of the formed shape must be again divided into three equal parts, to build equilateral triangles on the middle ones, etc. The result of this process is the Koch's Curve, whose construction in five iterations is shown in Fig. 1.

4. The object and methods to study the fractal properties of electric load diagrams of civilian objects

The Koch's Curve is self-similar: it consists of four parts, similar to the entire curve of infinite length at coefficient of similarity 1/3. The fractal dimensionality of a time series, or the accumulated changes at random walking, equals 1.5, of the curve line – equals 1, and the geometric line that fills the plane equals 2. Therefore, the fractal dimensionality of random walk is between a curve line and the plane. Thus, fractal dimensionality $D$ is the critical dimensionality that characterizes the way an object or a time series fill up the space. Dimensionality describes the structure of an object at a change in the coefficient of magnification, or when one changes the scale of the object. The statistics of a time series with fractal dimensionality other than 1.5 is significantly different from the Gaussian statistics and is not necessarily located within the normal distribution. The Hurst exponent $H$ can be transformed to fractal dimensionality $D$ using the following formula:

$$D = 2 - H.$$

Therefore, if $H = 0.5$, then $D = 1.5$. Both quantities characterize the independent random system. Magnitude $1 < H < 0.5$ will match the fractal dimensionality that is closer to the curve line. By the Hurst terminology, this is a persistent time series, which yields a smoother, less edged line, than a random walk. The anti-persistent magnitude $H (0 < H < 0.5)$ yields, accordingly, a higher fractal dimensionality and a more intermittent line than a random walk and, respectively, characterizes a system that is more susceptible to changes. This is in perfect agreement with an anti-persistent time series. Fractal dimensionality is a measure of complexity of the load diagram. By analyzing the alternating sections with a different fractal dimensionality and the way the system is affected by external and internal factors, one can learn how to predict the behavior of the system and, most importantly, to diagnose and forecast unstable states. The essential point in the proposed approach is the availability of a critical value for the fractal dimensionality of a diagram, approaching which leads to that the system loses stability and enters an unstable state. The parameters increase or decrease rapidly, depending on the trend in a given time. That is, the fractal dimensionality of a power consumption diagram can be used as an indicator for crisis. In addition, the magnitude for a fractal dimensionality could be an indicator for the number of factors that affect the system of power consumption. At fractal dimensionality less than 1.4, the system is affected by one or several forces that move the system in the same direction. At about 1.5, the forces acting on the system are multi-directional, but more or less compensate for each other. The system's behavior in this case is stochastic and is well described by classic statistical methods. At significantly
larger than 1.6, the system becomes unstable and is ready to enter a new state.

During sufficient stable periods and slow climbs, the fractal dimensionality of a time series remains quite low, while in periods of crises the total fractal dimensionality grows. When an abnormal magnitude for \( H \) is derived, the question arises whether the estimate for such a magnitude is justified. It is possible to check validity of the results by mixing data, the result of which is the completely different order of observations compared to the original series. Because the observations remain the same, their frequency distribution also remains unchanged. The Hurst exponent for these mixed data is then calculated. If a series is truly independent, then the Hurst exponent does not change, because the effect of a long-term memory was missing, that is, the correlation between observations. In this case, the mixing of data does not affect the qualitative characteristics for the data. If there is the effect of a long-term memory, then the order of data on electrical loads is very important. Mixing data thus breaks the structure of the system. The estimate \( H \) in this case is much lower, approaching 0.5, even if the frequency distribution of observations does not change.

The dimensionless ratio using the division of spread \( R \) on the standard deviation of observations \( S \) is commonly referred to as the method of rescaled range (\( R/S \)) analysis [4].

A fractal is the geometric shape that can be split into parts, each of which is a smaller version of the whole. This feature can be applied to constructing the electricity consumption diagram at residential houses, which will consist of the electrical load on each apartment, the consumption at individual houses.

As an example, Fig. 2, 3 show the diagrams of a daily load on apartments and a daily energy consumption at a residential building over a week.

Fig. 2. Diagrams of a daily electric load on an apartment over a week

Fig. 3. Diagrams of a daily power consumption at a residential building over a week

The diagrams of a weekly and a monthly electric load on multi-story residential buildings can be represented as the trend with a noise. We shall formalize an algorithm to determine the Hurst exponent by modern methods of a fractal analysis in the following order [4].

Fig. 4 shows the diagrams of a monthly energy consumption at a residential building over a year.

According to information from ASCEPA, electric load on an apartment building in the general form is as follows:

\[
P_i \in \{1, 2, ..., n\}.
\]

We shall consistently identify its initial sections within it

\[
P_\tau = p_1, p_2, ..., p_\tau,
\]

where \( \tau = 3, 4, ..., n \), for each of which we shall compute its current average

\[
\bar{P}_\tau = \frac{1}{\tau} \sum_{i=1}^{\tau} P_i.
\]

For each fixed \( P_\tau, \tau = 3, 4, ..., n \) we compute the accumulated deviation in the length sections

\[
t \cdot X_{t, \tau} = \sum_{i=1}^{\tau} (p_i - \bar{P}_\tau), t = 1, \tau.
\]

Compute a difference between the maximum and minimum accumulated deviation

\[
R = R(\tau) = \max(P_\tau, t) - \min(P_\tau, t)
\]

at \( 3 \leq \tau \leq n \).

This spread is normalized, that is, it is represented in the form of ratio \( R/S \), where \( S = S(\tau) \) is the standard deviation for the section of a time series \( P_\tau, 3 \leq \tau \leq n \).

The Hurst exponent \( H = H(\tau) \), which characterizes the fractal dimensionality of the assigned time series and the color of noise that matches it, is derived from the following ratio

\[
R/S = (a \cdot \tau)^{H}, \quad H = H(\tau).
\]

We shall take a logarithm for both sides of this equality, and, by accepting \( a = 1/2 \), we derive Cartesian coordinates \((x_m, y_m)\) for the points along the \( H \)-trajectory, the ordinates and abscissas of which will equal:

\[
y = H(\tau) = \frac{\log(R(\tau)/S(\tau))}{\log(\tau/2)} , \quad x = \tau, \tau = 3, 4, ..., n.
\]

The \( R/S \)-trajectory, required for the series fractal analysis, is represented in the Cartesian logarithmic coordinates...
by a sequence of points whose abscissas are: \( xt = \log(\tau / 2) \), and the ordinates are \( y_{R/S} = \log[R(\tau) / S(\tau)] \).

By connecting the adjacent points \((xt, y_{R/S}) i (xt+1, y_{R/S}+1)\), \( \tau = 3, 4, ..., n - 1 \) with a section line, we can derive a graphic mapping of the \( R/S \)-trajectory (H-trajectory) in logarithmic coordinates (Fig. 5).

![Fig. 5. H-trajectory and R/S-trajectory along a section in the daily diagram of electric load on a residential building](image)

By using the obtained \( R/S \)-trajectory and H-trajectory along a segment in the daily diagram of electric load on a residential building, one can numerically estimate such time series characteristics as the existence of a long-term memory and its depth, trend resistance, and other indicators.

### 5. Results of research and estimation of the fractal properties of electrical load diagrams at civilian objects

According to the research results, a segment from \( \log(\tau) - 0.176 \) to \( \log(\tau) = 0.653 \) (Fig. 5) is the best option for predicting electrical loads, because the value for \( H > 0.6 \), which is characterized by the “black color” of noise in accordance with the values for the Hurst exponent (Table 1).

An analysis of the fractal properties of annual power consumption diagrams at civilian objects is considered taking into account the fractal structure and the existence of a long-term dependence, inherent to self-similar stochastic processes. Feature of this is that the same process may include different types of a time dependence. Detecting the presence of a memory type enables the \( R/S \) analysis of change in the Hurst exponent \( H \) depending on the length of a time series, by deriving a point estimate for this parameter, which characterizes the degree of a long-term dependence.

In this case, the Hurst exponent \( H \) is regarded to be a function of the number of count of a time series \( H(n) \), whose behavior enables the identification of the important characteristics for a time series. These include the independence of random data, the presence of cyclical components, and the average length of a non-periodic cycle, the presence of a short-term and a long-term dependence.

The typical dependences \( \log(R/S) \) on the length of a series for processes with different types of memory are shown in Fig. 6.

![Fig. 6. Typical dependences log(R/S) on the series length](image)

The dotted line marks the theoretical values \( \log(R/S) \) for independent random data at \( H = 0.5 \) (curve 2). For the case of a long-term dependence (curve 1), the values for \( \log(R/S) \) would be above the dotted line, and for the case of anti-persistence – below (curve 3).

An analysis of the Hurst exponent behavior makes it possible to determine the values for a time interval, starting from which the process alters the properties for a long-term dependence. By using the \( R/S \) analysis, one can reveal the cyclical nature of the process and determine the average length of non-periodic cycles, typical of chaotic systems.

Because in this case the dynamics of a system is limited by the attractor (periodic or chaotic), starting from a certain period the Hurst exponent values \( H \) (a slope of the \( \log(R/S) \) curve) would stop changing. This period characterizes the average length of a cycle. Of particular importance in the study of fractal processes is the identification and elimination of the short-term dependence, which is characteristic of autoregressive processes. A linear dependence increases the value for the Hurst exponent and demonstrates the effect of a long-term memory.

| \( H \) value | Noise color | Pattern description |
|---------------|-------------|---------------------|
| 0–0.1         | Brown noise | It corresponds to the maximum fractal dimensionality of a time series and to the complete uncertainty regarding predictability, or matches a Brownian random process, for which the memory effects are missing or within which there is no any trend |
| 0.3±0.1       | Pink noise  | It is characterized by anti-persistence, that is, it does not support the current trend |
| 0.4±0.1       | White noise | It corresponds to the chaotic behavior of a time series and, accordingly, to the lowest reliability of prediction or the least predictability. The series that demonstrate the properties of «white noise» are characterized by «complete unpredictability», the cyclical character is inherent to them, as well as a frequent change in trends, accompanied by a loss of persistence |
| 0.6–1        | Black noise | The larger the value for \( H \), the greater the trend resistance, characteristic of the appropriate section of a time series. At \( H > 0.5 \), the examined time series is persistent or trend resistant, meaning that it supports the current trend (if the series increases over a certain period, then it is quite likely that it will keep this trend for some time in the future). Such a trend resistance in behavior increases when \( H \) approaches 1.0. When \( H \) approaches 1.0, the series becomes less noisy and has more consecutive observations with the same sign. Continuity of synergetic and classical statistical methods is provided at \( H = 0.9 \) |

Table 1: Correspondence of values for the Hurst exponent to the color of noise
To eliminate a short-term dependence, it is required that the value for a time series of process \( S(t) \) should be regressed as a variable value relative to \( S(t−1) \). Next, one has to derive a linear dependence between them and to perform the \( R/S \) analysis of the remaining \( X(t)−S(t)−(α+bS(t−1)) \). If the original series had a long memory, the dependence would be preserved, while a short-term dependence is eliminated.

To identify the properties of a fractal in the electrical load diagrams, we investigated a series of weekly power consumption at a 216-apartment building over the period from October 2, 2017 to April 8, 2018, which is composed of 1,680 observations (Fig. 7).

![Fig. 7. Dynamics of a weekly power consumption at a residential building over the period from October 2, 2017 to April 8, 2018](image)

To apply a fractal analysis in order to study a time series (the dynamics of power consumption), one must use significance criteria and methods for data preparation. We shall analyze, by using the electrical consumption diagrams at a residential building, the autoregressive \( AR(1) \)-differences over the specified period of time. These differences are used to eliminate or minimize a linear dependence. A linear dependence can shift the Hurst exponent such that it appears significant (when there are no long-term trends), that is, cause a first kind error. By using the autoregressive \( AR(1) \)-differences, we reduce the shift to a minimum. Such a process is called preliminary bleeding or removal of trends. For the case of an \( R/S \) analysis, the removal of trends would eliminate a serial correlation, or a short-term memory.

We begin studying the structure of a series of energy consumption at a residential building by constructing a series of the first difference \( D(–1) \).

The procedure of deriving the first difference is similar in value to removing an autoregressive dependence and a linear trend. This often gives a possibility to obtain a stationary, in a wide sense, series, which would be examined on stationarity based on the criteria for series, inversions, and turning points. Our study has demonstrated the absence of a trend within the time series (Fig. 8).

The behavior of the \( R/S \)-trajectory over the time from October 2, 2017 to April 8, 2018, shown in Fig. 9, testifies to the chaotic pattern in a time series (corresponds mostly to white noise), which is characterized by a frequent change in the trend of power consumption.

Results from an \( R/S \) analysis of power consumption diagrams and determining the Hurst exponent improve the effectiveness of forecasting and control over power consumption during the period of the identified chaotic behavior of a time series. These periods will see a frequent change in the trend of energy consumption, which would require a special approach to placing orders for supply of the predicted volume of electricity when compiling contracts between actors in the energy market – energy generating and energy supplying companies and objects that serve civilian purposes. Ignoring such an alignment among subjects can lead to accidents, deactivation of individual consumers from a power grid due to overload of lines, power transformers. The consequences could include damage to the electrical equipment at ESS, damage to individual households, which depend on the duration of interruptions in electricity supply, etc. When crises are identified, appropriate management activities aimed at the subjects in energy market could help avoid such emergencies by predicting an increase in the volume of electricity supply to consumers or by forced disconnection of parts of objects from power supply.

![Fig. 8. Autocorrelation function for \( D(–1) \) for a series of energy consumption](image)

![Fig. 9. Result from an \( R/S \) analysis of a building’s power consumption diagram](image)

### 6. Discussion of results of studying the power consumption diagrams at civilian objects

Existing methods for predicting electric load on civilian objects face certain difficulties during a period of the identified chaotic behavior of a time series, because mathematical models of the process must take into consideration numerous factors that affect its dynamics. The theory of fractals makes it possible to accurately (within 2.5 %) solve these tasks by means of a fractal analysis of time series, which improves the adequacy of forecasting through an in-depth analysis of causes that give rise to emergencies, for example, weeks 8, 9...27–31 (Fig. 9). Results of such an analysis would contribute to improving the efficiency of control over power supply, the criterion to which is the speed of making a relevant decision, both by an energy generating company (automatically, by using a software-hardware complex, by increasing/decreasing the generation of electricity) and by a consumer (given the required automatic control over electricity consumption).

Failure to respond to such a situation leads to a decrease in the quality of power supply or to an accident. However,
these dangerous periods require further careful study of actual factors that cause them in a given region over the examined periods, which are rather difficult to acquire and to organize, as well as operative alignment of joint management activities by subjects in the energy market, aimed at minimizing their negative impact on the quality of power supply.

7. Conclusions

1. In the course of this research, we have organized and acquired, by using ASCEPA, the retrospective information on electrical load at residential buildings within a city’s neighborhood. The systematization of daily, weekly, monthly data has made it possible to construct a database for the fractal analysis of diagrams’ structure. Obtaining a prehistory of load over six months has provided opportunities for performing a qualitative analysis into the formation of structure of diagrams.

2. The constructed models, based on the accumulated database about energy consumption at residential buildings within a city’s neighborhood, have revealed the specificity in compiling daily, weekly, monthly diagrams that are inherent to fractals. Specifically, the resulting monthly diagram of electric load at residential buildings is based on the dynamics of a weekly diagram, the weekly diagram – on the dynamics of a daily load at residential houses, which, in turn, is formed on the basis of the dynamics of load at each building, based on a load diagram at each apartment.

3. Based on the results from fractal analysis, we have determined the existence and estimated the critical value for a fractal dimensionality of a diagram, approaching which leads to that the system loses stability and enters an unstable state. The calculated fractal dimensionality of a diagram can be used in forecasting as an indicator for a crisis. This is of practical significance for substantiating the measures taken to avoid emergencies when supplying electricity to facilities that serve civilian purposes. According to the results of the study, such periods of crisis in the supply of residential buildings will occur in the morning and evening hours and at other hours, determined by the Hurst index. During this time, there is an increase in the likelihood of emergency disconnection in the hours of peak loads on a power grid, in the case of inconsistency of actions to regulate power consumption at objects by energy supply companies.

4. The fractal properties of electric load on civilian objects, established by the current research, were implemented in order to adequately forecast and to efficiently control power consumption during a period of the chaotic behavior of a time series, determined based on the Hurst exponent. Avoiding these critical periods is possible by aligning joint management activities by subjects in the energy market, aimed at minimizing their negative impact on the quality of power supply.

References

1. Freder J. Fractals. New York, 1988. 283 p. doi: https://doi.org/10.1007/978-1-4899-2124-6
2. Peters E. Fractal Market Analysis: Applying Chaos Theory to Investment and Economics. Boston, 1994. 336 p.
3. Jian-Kai L., Cattani C., Wan-Qing S. Power Load Prediction Based on Fractal Theory // Advances in Mathematical Physics. 2015. Vol. 2015. P. 1–6. doi: https://doi.org/10.1155/2015/827238
4. Lezhniuk P. D., Shullie Yu. A Operatyve prohnozuvannia elektrychnykh navantazhen system elektrospozhyvannya z vrakhuvannya yikh fraktalnykh vlastyvostei. Vinnytsia: VNTU, 2015. 104 p.
5. Zhai M.-Y. A new method for short-term load forecasting based on fractal interpretation and wavelet analysis // International Journal of Electrical Power & Energy Systems. 2015. Vol. 69. P. 241–245. doi: https://doi.org/10.1016/j.ijepes.2014.12.087
6. Bunn D. W., Farmer E. D. Comparative models for electrical load forecasting. New York: Jone & Sons, 1985. 200 p.
7. Multifactor modeling and analysis of electrical load of the power system using the data of long-term prehistory / Chernenko P., Martyniuk O., Zaslavsky A., Miroshnyk V. // Tekhnichna Elektrodynamika. 2018. Issue 1. P. 87–93. doi: https://doi.org/10.15407/technel2018.01.087
8. Box D., Jenkins G. Analysis of Time Series. Forecast and Management. New Jersey, 1977. 729 p.
9. Varetskyi Yu. O., Karach L. V. Operatyvno-dyspetcherske keruvalia elektroener-hetychnymy systemamy. Lviv: Lvivska politekhnika, 2002. 160 p.
10. Stakhiv P., Kozak Yu., Hoholyuk O. Increasing of effectiveness of algorithms to create macromodels of electric power systems objects // Tekhnichna elektrodynamika. 2014. Issue 5. P. 29–31.
11. Hovorov P. P., Bukulevskyi V. L. Improvement of Mathematical Model of Calculation and Power Loss Forecasting on the Basis of Neural Networks // Visnyk Vinnyts’koho politekhnichnoho instytutu. 2018. Issue 2. P. 14–19.
12. Prusty B. R., Jen A. An over-limit risk assessment of PV integrated power system using probabilistic load flow based on multi-time instant uncertainty modeling // Renewable Energy. 2018. Vol. 116. P. 367–383. doi: https://doi.org/10.1016/j.renene.2017.09.077
13. The power load’s signal analysis and short-term prediction based on wavelet decomposition / Wang H., Ouyang M., Wang Z., Liang R., Zhou X. // Cluster Computing. 2017. doi: https://doi.org/10.1007/s10586-017-1316-3
14. Bondarchuk A. S. Vnutrishnobudynekove elektropostachannia. Kyiv: Osвитa Ukrainy, 2015. 480 p.