Candy Crush is NP-hard

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Abstract

We prove that playing Candy Crush to achieve a given score in a fixed number of swaps is NP-hard.

Keywords: computational complexity, NP-completeness, Candy Crush.

1. Introduction

Candy Crush Saga is currently the most popular game on Facebook. It has been installed half a billion times on Facebook, and on iOS and Android devices. Candy Crush is a variant of match-three games like Bejeweled and Diamond Mine. These games themselves have also been very popular. For example, PopGap Games sold over 75 million copies of Bejeweled. But what makes Candy Crush (and its ancestors) so addictive? In this paper, we suggest one answer. Namely, part of its addictiveness may be that Candy Crush is a computationally hard puzzle to solve. It thus joins the ranks of other computationally intractable puzzles that have fascinated us like Minesweeper [Kay00], Sudoku [KPS08], and other matching problems like Tetris [DHLN03], and KPlumber [KMS+04]. It raises a number of open questions. For example, is the infinite version Turing-complete? How do we generate Candy Crush problems which are truly puzzling?

To provide a formal result, we need a precise description of the puzzle. We focus on the early rounds of Candy Crush Saga where a player has to achieve a given score with a fixed number of swaps. A swap interchanges two neighbouring candies to create a chain of identical candies. Such chains are deleted from the board and the candies above drop down into their place. We also focus on simple game play which creates chains of three identical candies. In all the boards we consider, chains of more than three identical candies cannot be formed. As in previous work [TW06], we consider a generalized version of Candy Crush in which the board size is not fixed.
**Name:** Candy Crush problem.

**Input:** An $a$ by $b$ board filled with one of six coloured candies, a number $k$ of swaps and a score $s$ to be achieved or beaten. The score equals the number of chains of 3 identical candies deleted.

**Question:** Is there a sequence of $k$ swaps which obtains a score of $s$ or more?

In case multiple chains are formed simultaneously, we assume that chains are deleted from the bottom of the board to the top as they appear, and the candies above immediately drop down.

2. Result

**Theorem 1.** The Candy Crush problem is NP-complete.

**Proof:** A witness is the sequence of $k$ swaps that results in a score of $s$ or more deletions of chains. The input requires $O(ab)$ bits to specify the board, $O(\log(k))$ bits to specify $k$, and $O(\log(s))$ bits to specify $s$. However, specifying the board dominates the size of the input as both $k$ and $s$ have to be smaller than $ab$. Each swap can be specified by giving the coordinates of a square on the board and a cardinal direction in which to swap it. The witness is thus $O(k \cdot \log(ab))$ bits in size which is less than the square of the input size. Hence, the problem is in NP.

To show NP-hardness, we reduce an instance of 3-SAT in $n$ variables and $m$ clauses to the Candy Crush problem. The reduction constructs a “circuit” using gadgets and “wires”. We set $k = n$. We require only 5 of the 6 colours in the standard Candy Crush problem. In addition, we only ever form chains of 3 candies of the same colour. We therefore do not need any special candies like the “wrapped” or “striped” candy generated when longer chains are formed. We first show how to construct a neutral background that will never result in any chains of 3 candies of the same colour. In even columns, we alternate red jelly beans and orange lozenges. In odd columns, we alternate yellow lemon drops and green chiclets. We introduce various gadgets into this neutral background made from purple clusters. These are inserted in between the red/orange and yellow/green sequences. The only chains ever formed will be of purple clusters. For example, consider the following part of the wire gadget made up of purple clusters:

```
. . .
. P P
P . .
```
Suppose we insert this into the neutral background:

\[
\begin{array}{cccc}
    r & y & r & y \\
    o & g & o & g \\
    r & y & r & y \\
    o & g & o & g \\
    r & y & r & y \\
\end{array}
\]

We then obtain the following arrangement of candies:

\[
\begin{array}{cccc}
    r & y & r & y \\
    o & g & o & g \\
    r & y & p & p \\
    o & p & r & y \\
    r & g & o & g \\
\end{array}
\]

This arrangement has the property that, even if we swap candies around so we get a chain of 3 purple clusters which then disappears, the background colours can never form a chain of three equal colours. For compactness, we describe each gadget using only a few rows. However, we can separate apart the different gadgets and even different parts of a gadget with neutral rows.

We next outline the reduction. The board has two parts. In the left half of the board, the user makes choices in setting the variable gadgets. This corresponds to setting the respective variables true or false. In the right half of the board, we have clause gadgets which decide if each clause is satisfied or not. We suppose the \(i\)th variable gadget from the left represents the truth value of \(x_i\). The variable gadget contains two columns of purple clusters. The user can swap the second candy down with one horizontally to the left. This constructs a vertical chain of three purple clusters. As a result, the middle column of candies moves down three rows. This corresponds to setting the variable to false:

\[
\begin{array}{cccc}
    \cdot & \cdot & \cdot & \cdot \\
    \cdot & p & \cdot & \cdot \\
    \cdot & p & \cdot & \cdot \\
    \cdot & p & \cdot & \cdot \\
    \cdot & \cdot & \cdot & \cdot \\
\end{array}
\Rightarrow
\begin{array}{cccc}
    \cdot & \cdot & \cdot & \cdot \\
    \cdot & p & \cdot & \cdot \\
    \cdot & p & \cdot & \cdot \\
    \cdot & p & \cdot & \cdot \\
    \cdot & \cdot & \cdot & \cdot \\
\end{array}
\]

Alternatively, the user can swap the third purple cluster down with one horizontally to the right. This also gives a vertical chain of three purple
clusters. As a result, the rightmost column of candies moves down three rows. This corresponds to setting the variable to true.

\[
\begin{array}{cccc}
\cdot&\cdot&\cdot&\cdot \\
\cdot&p&\cdot&\cdot \\
\cdot&p&\cdot&\cdot \\
\cdot&\cdot&\text{right swap} &\Rightarrow &\cdot&p&\Rightarrow &\cdot \\
\cdot&p&\cdot&\cdot &\cdot &p &\cdot &\cdot \\
\cdot&\cdot&\cdot&\cdot &\cdot &\cdot&\cdot&\cdot \\
\end{array}
\]

In both cases, exactly one vertical chain of 3 purple clusters is created. All our gadgets except for the clause gadget are constructed so that the same number of chains of 3 purple clusters are created whatever setting of truth values are chosen by the user. In this way, the biggest change that can be made to the final score will be by satisfying clauses. The maximum final score will require us to satisfy all the clause gadgets. In some cases, we pair gadgets together so that, irrespective of their inputs, they construct together the same number of chains of 3 purple clusters. All gadgets, except the clause gadgets, are 3 columns wide.

Next, we describe a “wire”. This will transmit information across the board. Initially the input and output contain candies from the neutral background. If a purple cluster is placed at the input, then a purple cluster appears shortly after at the output.

\[
\begin{array}{cccc}
\cdot&\cdot&\cdot&\cdot \\
\cdot&p&\cdot&\cdot \\
\cdot&\cdot&\cdot&\cdot \\
\end{array}
\] \Rightarrow
\[
\begin{array}{cccc}
\cdot&\cdot&\cdot&\cdot \\
\cdot&p&\cdot&\cdot \\
\cdot&\cdot&\cdot&\cdot \\
\end{array}
\] \Rightarrow
\[
\begin{array}{cccc}
\cdot&\cdot&\cdot&\cdot \\
\cdot&p&\cdot&\cdot \\
\cdot&\cdot&\cdot&\cdot \\
\end{array}
\]

Suppose a purple cluster is introduced at the input. Then the board goes through the following changes:

\[
\begin{array}{cccc}
\cdot&\cdot&\cdot&\cdot \\
\cdot&p&\cdot&\cdot \\
\cdot&\cdot&\cdot&\cdot \\
\end{array}
\] \Rightarrow
\[
\begin{array}{cccc}
\cdot&\cdot&\cdot&\cdot \\
\cdot&p&\cdot&\cdot \\
\cdot&\cdot&\cdot&\cdot \\
\end{array}
\] \Rightarrow
\[
\begin{array}{cccc}
\cdot&\cdot&\cdot&\cdot \\
\cdot&p&\cdot&\cdot \\
\cdot&\cdot&\cdot&\cdot \\
\end{array}
\]

4
We can glue wires together to communicate information across longer distances. Note that we can add any number of neutral rows into the middle of such wire gadgets. Note also that when a wire communicates information, two purple clusters are deleted in each column. Wires always come in pairs: one representing a literal and the other its negation. By construction, each pair of wires then deletes two rows from the table. The gadgets above therefore all come down in unison by two rows. Hence they remain connected in the same way as in the initial board.

We also need a connector gadget that connects a wire to the columns above the variable gadget. For example, suppose we want to connect a wire to the columns above the $i$th variable to represent the literal $x_i$. The following connector gadget creates an output bit that can act as the input bit to a wire gadget.

\[
\begin{array}{cccccccc}
\cdot & \cdot & p & . & . & . & . & . \\
\cdot & \cdot & . & . & . & . & . & . \\
\cdot & \cdot & p & . & . & . & . & . \\
\cdot & \cdot & . & . & . & . & . & . \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\end{array}
\]

This sits above a variable gadget. A dual construction is used to connect a wire to represent the literal $\neg x_i$.

\[
\begin{array}{cccccccc}
\cdot & \cdot & . & . & . & . & . & . \\
\cdot & \cdot & . & . & . & . & . & . \\
\cdot & \cdot & p & . & . & . & . & . \\
\cdot & \cdot & . & . & . & . & . & . \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\end{array}
\]

As such connectors come in pairs, we have different numbers of purple clusters deleted from the different columns. Any connector gadgets above therefore need additional neutral candies to compensate.

We also have wires that cross columns containing variable gadgets. This requires modifying the wire gadget to deal with the differing shifts of the three columns. Suppose we pass a wire above the variable gadget but beneath any of connectors. Our modified wire gadget is as follows:

\[
\begin{array}{cccccccc}
\cdot & \cdot & p & . & . & . & . & . \\
\cdot & \cdot & . & . & . & . & . & . \\
\cdot & \cdot & p & . & . & . & . & . \\
\cdot & \cdot & . & . & . & . & . & . \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\end{array}
\]
The modification has added additional rows to the wire gadget to ensure that, whatever truth value is selected, when the appropriate three clusters have been deleted in the variable gadget, we have purple clusters in all the positions of the original wire gadget. Similar modifications of the wire gadgets are required for wires that pass above connectors above a variable gadget. As before, we modify the wire gadget to include additional neutral candies to compensate for the different number of deleted candies.

In the right hand part of the board, we have a clause section. Each clause is represented by \( m \) clause gadgets. This repetition of clause gadgets is required to ensure the user gains the maximum score by setting variable gadgets and not indirectly by setting individual wires leading to a single one of these clause gadgets. Each clause gadget occupies a block of rows not occupied by any other clause gadget. The clause gadgets are arranged diagonally from top left to bottom right. In this way, no wires need pass over the top of any clause gadget. We therefore do not need to worry about the number of purple clusters deleted in each column of the clause gadget.

Suppose we have the clause \( x_3 \lor \neg x_5 \lor x_{19} \). From top to bottom, we have six wires coming from the variable section of the board representing the signal \( \neg x_3, x_3, x_5, \neg x_5, \neg x_{19}, x_{19} \). We denote these wires by \( \overline{i_{n_3}}, i_{n_3}, \overline{i_{n_2}}, i_{n_2}, \overline{i_{m_1}}, i_{n_1} \) respectively. We connect these wires to a clause gadget. The bottom part of the clause gadget is described in Figure 1.

When one or more of the input wires (\( i_{n_1}, \overline{i_{n_2}}, \) or \( i_{n_3} \)) is set, two chains of three purple clusters are created within the clause gadget. This occurs when
Figure 1: Bottom half of the clause gadget.
the wires represent a truth assignment that satisfies the clause. As a result, the fourth column from the left in the clause gadget drops down one row if and only if the clause is satisfied by the truth assignment set on the input wires. In the top half of the clause gadget, when the fourth column drops, the score is increased greatly. The increase in the score is larger than any other score that might be achieved. In particular, satisfying the clause creates \( ma \) chains of purple clusters in the top of the clause gadget. As each clause gadget is repeated \( m \) times, satisfying all the clause gadgets by assigning variables out scores anything else that the user can do. For example, the user can set wires directly, even set a wire and its negation. However, this will score less than just setting the variable gadgets provided this satisfies all the clause gadgets. The top half of the clause gadget has \( ma \) copies of the following purple clusters stacked above each other:

\[
\begin{array}{ccccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & p & & & & \\
\cdot & \cdot & p & p & & &
\end{array}
\]

Directly setting a wire will create less than \( a \) chains of purple clusters as this is the length of the longest possible wire. This is less productive than satisfying a clause. In fact, the only way we can get the maximum achievable score is to set only variable gadgets with an assignment that satisfies all the clause gadgets. Hence, deciding if we can achieve the (given) maximum achievable score determines the satisfiability of the original 3-SAT problem. Finally, we note that the reduction is polynomial. The board is \( O(n + m^2) \) wide and \( O(m^3(n + m^2)) \) high. Q.E.D.

Note that the proof only creates chains of 3 identical candies. Other closely matching problems in the same family like (generalized versions of) Bejeweled and Diamond Mine are therefore also NP-complete. The proof required a board that was unbounded in width and height. When we bound the width (or height) of the board, is the problem fixed parameter tractable. Another interesting open question is if we can approximate the problem easily. For instance, can we get within a constant factor of the optimal score? Or can we solve a problem within few additional swaps?

Note also that the whole board was visible and playable. In many levels of Candy Crush, only the bottom part of the board is visible to the player. The problem of playing Candy Crush remains NP-hard with partial information (since a special case of partial information is when there are no scores from
candies that fall down from the hidden part of the board into the playable and visible section). Finally, puzzles like Candy Crush may not be very puzzling if they have many solutions. Deciding if a Candy Crush problem has an unique solution is co-NP-hard.

**Theorem 2.** The Unique Candy Crush problem is co-NP-hard.

**Proof:** We reduce from the complement of the unique SAT problem. The unique SAT problem is itself co-NP-hard. We use a similar reduction as in the last proof. We modify the variable gadgets so that each only works when an input wire is set and so that each variable gadget sets an output wire when the user has selected a variable assignment. We then wire the variable gadgets up from left to right. In this way, we ensure that the variable gadgets must be set in sequence from left to right. This ensures that we cannot permute the order in which we set the variable gadgets. The SAT instance has an unique solution if and only if there is an unique sequence of variable assignments achieving the (given) maximum score. Q.E.D.

3. Conclusions

We have shown that the generalized version of Candy Crush is NP-hard to play. This result suggests a number of interesting future directions. For example, NP-hardness is only a worst-case concept. How do we generate Candy Crush problems that are hard in practice? Can we identify a “phase transition” in hardness? Phase transitions have been seen in many other NP-hard problems and are now frequently used to benchmark new algorithms [CKT91, GW94, GW96a, GW96b, Wal11]. How does the structure of Candy Crush problems influence their hardness [GW06]? Finally, it would be interesting to see if we can profit from the time humans spend solving Candy Crush problems. Many millions of hours have been spent solving Candy Crush. Perhaps we can put this to even better use by hiding some practical NP-hard problems within these puzzles?

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