Research Article

Effects of Particle Shapes and Sizes on the Minimum Void Ratios of Sand

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The minimum void ratio is an important parameter for evaluating soil properties. It is closely related to the compressive properties, permeability, and shear strength of soil, and it is affected by particle size distributions and particle shapes. However, existing research generally focuses on modeling the minimum void ratio with the effect of particle size distributions, ignoring the influences of particle shapes on the minimum void ratio. This paper analyzes the influences of particle size distributions and particle shapes on the minimum void ratio using four types of sand and alternative materials. The experiments showed that the minimum void ratio first decreased and then increased with the increase of the fines content. The minimum void ratio reached a minimum value when the proportion of fines content was approximately 40%. The more irregular the particle shapes, the more complicated the contact between particles, the more the void existed between the particles, and the larger the minimum void ratio. Based on the experimental data, a relational model between the minimum value of the minimum void ratio and the particle sizes ratio was derived with binary mixtures of different particle sizes and shapes. This proposed model required only one parameter \( T \), which was closely related to the sphericity of the particles, to predict the minimum value of the minimum void ratio with various fines contents. The experiment results showed that the predicted value was very close to the actual measured value.

1. Introduction

The granular soil is a mixture of particles with different sizes, and the particle size distribution controls the structural form of the soil, which affects the mechanical properties of the soil (e.g., [1–5]). Particle size distributions are widely used in industrial productions such as concrete mixes [6], ceramics processing [7], and powder metallurgy [8]. As an important parameter reflecting the particle size distribution of soil in geotechnical engineering, the minimum void ratio \( e_{\text{min}} \) is closely related to the compressive properties, permeability, and shear strength of soil.

It is generally accepted that the fines content is the main factor affecting \( e_{\text{min}} \) [9–14]. Kezdi [15] proposed an analytical method for estimating \( e_{\text{min}} \) of a mixture of two particle sizes, but this method is only suitable for fillers with very small particles.

Cubrinovski and Ishihara [16] proposed a set of empirical equations for the effect of fines content on \( e_{\text{min}} \) by analyzing a large amount of test data for silt. Chang et al. [17–19] established a model with only two parameters to predict \( e_{\text{min}} \) of sand-silt mixtures with a dominant particle structure network concept. This model reflected a close correlation between particle size and \( e_{\text{min}} \). The Furnas model [20] is only suitable for estimating the packing density of binary powder compacts, and it has not yet been examined for use with the packing density of sand-silt mixtures with different particle sizes.

It is generally accepted that another important factor is particles shapes, which affect \( e_{\text{min}} \) factor and thus affect the shear resistance of granular soils. Using a triaxial compression test of atomized stainless steel powder, Shinohara et al. [21] found that the internal friction angle increased with the increase of the grain edge angle and the initial compactness. Ashmawy et al. [22] analyzed the effect of particle shapes on liquefaction with a reciprocating loading undrained test. Sallam and Ashmawy [23] used the discrete element method to simulate the stress-strain relationship of
flat and narrow element assemblies with different shapes, and they pointed out that the dilatancy angle was also largely restricted by the particle shapes. Different particle shapes can significantly change the integrity and shear resistance of granular soils [24–28]. Cho et al. [10] and Cherif Taiba et al. [29] already proposed that increasing particle irregularity caused a decrease in the stiffness but a heightened sensitivity to the state of stress.

Scholars have mainly studied the effect of particle size distributions on $e_{\text{min}}$ of soils and proposed corresponding analytical methods to predict $e_{\text{min}}$ for soil mixtures. However, very few studies on the effect of particle shapes on $e_{\text{min}}$ have been carried out. In order to better study the distribution law of $e_{\text{min}}$, four types of sand from different origins were selected, and steel balls [11] and steel cylinder particles were introduced as alternative materials to further analyze the influence of particle shapes and particle size distributions on $e_{\text{min}}$.

2. Experimental Method and Conditions

2.1. Sand Used for Experimental Testing. The sand used in the experiment was from four different origins: Nanjing River Sand (abbreviated as NS), Dongting Lake Sand (DS), Yizheng Mountain Sand (YS), and Fujian Standard Sand (FS). The properties of these types of sand are presented in Table 1. The gradation curves of the four types of source sand before and after the compaction test are shown in Figure 1. The grain sizes ranged from 0.075 mm to 5 mm.

2.2. Shape Parameters of the Four Types of Source Sand. To take the influence of the particle shapes into consideration, a large number of representative particles of the four types of source sand were photographed with a Dino-Lite microscope, and then the two-dimensional images were binarized with Photoshop. The binarized images were analyzed with Image-ProPlus to get the primary parameters of the particle shapes, such as length $L$ and width $B$. The secondary shape parameters such as the flatness, sphericity, and angularity were calculated using the primary parameters [30, 31]. The mathematical expressions and the physical meanings of the secondary shape parameters are listed in Table 2.

The two-dimensional microscope image of the DS of the 0.5–1 mm grain group is shown in Figure 2(a), and the image after binarization is shown in Figure 2(b). The secondary shape parameters of the four types of experimental sand are listed in Table 3.

It could be seen by comparing the parameters from Table 3 that the sequence of the irregularities of the particle shapes from high to low was NS, DS, and FS. YS was not considered for comparison because of its fragility, resulting in the discreteness of the shape parameters.

2.3. Alternative Materials. Steel balls and steel cylindrical particles were used as substitute materials for sand to investigate the effects of particle shapes. The two-dimensional images and shape parameters of the eight particles are shown in Table 3. It should be noted that the grains of B1 and C1, B2 and C2, B3 and C3, and B4 and C4 were equal in volume.

| Sand  | $d_{30}$(mm) | $d_{60}$(mm) | $d_{90}$(mm) | $C_u$  | $C_C$  | $G_s$ |
|-------|-------------|-------------|-------------|-------|-------|-------|
| NS    | 0.13        | 0.23        | 0.76        | 5.85  | 0.54  | 2.64  |
| DS    | 0.16        | 0.32        | 0.71        | 4.44  | 0.90  | 2.64  |
| YS    | 0.12        | 0.21        | 0.81        | 6.75  | 0.45  | 2.54  |
| FS    | 0.13        | 0.22        | 0.80        | 6.15  | 0.47  | 2.62  |

Note: $C_u = d_{30}/d_{60}$, $C_C = d_{90}^2/(d_{90} \times d_{10})$, and $G_s$: specific gravities of the solid particles.

Figure 1: Particle size distribution curves of the test materials.

The $e_{\text{min}}$ value of sand is usually converted from the maximum dry density, which is measured using the vibration hammering test method [21]. The vibratory hammering method generally uses two kinds of compaction buckets whose volumes are 250 ml and 1000 ml. The two compaction buckets have the same height of 18 cm, but their corresponding inner diameters are 5 cm and 10 cm. It has been found that the inner diameter of the 250 ml compaction bucket is so small that the energy generated by the hammering is not easily diffused, which makes particles become crushed more easily. Particle breakage will result in a series of obvious changes in the initial properties of sand such as the changes in sand gradation, the increase in maximum dry density, and the decrease in $e_{\text{min}}$. Therefore, the 1000 ml compaction bucket was used in this experiment to minimize particle breakage.

The minimum void ratios of a single particle group for DS, NS, YS, FS, and alternative materials were measured. The minimum void ratios of the binary mixtures were also measured by mixing two different particles under different fines contents.

(1) Measuring the $e_{\text{min}}$ value of single-grain sand

The four types of source sand, i.e., DS, NS, YS, and FS, were screened into five particle groups, with each particle size ranging from 1 mm to 2 mm, 0.5 mm to 1 mm, 0.25 mm to 0.5 mm, and 0.1 mm to 0.25 mm. The $e_{\text{min}}$ value of each group was measured to explore the distribution law of $e_{\text{min}}$ of sand under a single particle group.
Measuring the $e_{\min}$ value of the binary mixture

First, the coarse particles (defined here as particle sizes between 2 mm and 5 mm) were mixed with the fine particles, whose particle sizes ranged from 0.5 mm to 1 mm, 0.25 mm to 0.5 mm, and 0.1 mm to 0.25 mm, in different mass ratios. The coarse particles (defined here as particle sizes between 2 mm and 5 mm) were mixed with the fine particles whose particle sizes ranged from 0.5 mm to 1 mm, 0.25 mm to 0.5 mm, and 0.1 mm to 0.25 mm in different mass ratios. The coarse particles (defined here as particle sizes between 2 mm and 5 mm) were mixed with the fine particles whose particle sizes ranged from 0.5 mm to 1 mm, 0.25 mm to 0.5 mm, and 0.1 mm to 0.25 mm in different mass ratios. Finally, the $e_{\min}$ values of the binary mixtures were measured to investigate the effects of fines content and particle shapes.

Measuring the $e_{\min}$ values of steel balls and steel cylindrical particles

First, the minimum void ratios of the steel balls and steel cylindrical particles were measured. Then the $e_{\min}$ values of the binary mixtures were measured by mixing two particles of different sizes with the same shapes or different shapes. Finally, the results of the different steel particles and types of sand were compared to verify the laws of $e_{\min}$ of binary mixtures via changing the particle sizes and particle shapes.

3. Results and Analysis

3.1. Variation Laws of $e_{\min}$ under Single-Particle Groups

The $e_{\min}$ values of the four types of sand and steel balls of different sizes under single-particle groups were measured. The particles of FS were excluded from the experiment due to the lack of 2–5 mm particles. According to the experimental data, the relationship between the average particle sizes of the particle group and $e_{\min}$ is shown in Figure 3. The average particle sizes of each particle group were calculated by taking the average values of the upper and lower grain diameters of the group.

From the geometric model of a single spherical particle, it can be found that $e_{\min}$ is independent of the size of the sphere. However, each group of sand obtained from the screening experiment was not of a certain particle size but rather lay within a particle size range, and the pores between the coarse particles were filled with fine particles. Therefore, the larger the particle size and the larger the difference between the coarse particle size and the fine particle size, the larger the $e_{\min}$.
between upper and lower limits of particle sizes in the same
group, the easier it was for two particles to fill each other and
the smaller the $e_{\text{min}}$ was. Conversely, the variation of $e_{\text{min}}$ of
spherical particles with a certain size was the opposite of the
above rule. Since the steel balls were of a certain particle size
and there was no filling with each other, the void between the
spherical particles and the inner wall of the compaction
bucket became larger with the increase of the particle sizes,
which led to a slight increase in the $e_{\text{min}}$.

It was obvious that the sequence of $e_{\text{min}}$ of single groups
from high to low was NS, DS, FS, and YS, as shown in
Figure 3. $e_{\text{min}}$ of the first three types of sand (NS, DS, and FS)
is closely related to the particle shape, and it tended to
decrease as the $S$ value increased. However, the $e_{\text{min}}$ and $S$
values of the YS did not satisfy the above rules. The reason
for this phenomenon was that YS was easier to crush during
the test. This could be validated by the gradation curve of
four types of sand after compaction, as shown in Figure 1.
Higher fines content in the YS will result in smaller voids and
thus a smaller $e_{\text{min}}$.

3.2. Effects of the Fines Content on the Minimum Void Ratio.
$e_{\text{min}}$ of the binary mixtures of the NS, DS, YS, and FS was
measured, while the grains in the groups of 2–5 mm and 1–
2 mm were designated as coarse grains and mixed with other
fine grains. Figure 4 shows schematically how $e_{\text{min}}$ varied
with the percentage of fines content given by the weight of
the mixtures. As the content of fines increased, the $e_{\text{min}}$ value
of the binary mixture exhibited a “V”-type tendency of first
decreasing and then increasing, and the mixture had a
minimum value ($e_{\text{min}}$)min.

It can be seen in Figure 4 that all the curves of $e_{\text{min}}$ of
fines content had minimum values, and the $e_{\text{min}}$ value first
decreased and then increased with the increase of the fines
content. Moreover, the value of $e_{\text{min}}$ decreased with the
increase of the grain size difference when the percentage of
the fines content was approximately between 0% and 80% in
the binary mixtures (i.e., the curves of the 2–5 mm and 1–
2 mm mixtures were above the curves of other mixtures
when the fines content was less than approximately 80%).
However, when the fines content was approximately greater
than 80%, the sequence of $e_{\text{min}}$ curves presented a reverse
trend with the increase in grain size difference (i.e., the
curves of the 2–5 mm and 1–2 mm mixtures were below the
curves of the other mixtures when the fines content was
more than approximately 80%).

The reason for this phenomenon is illustrated in Figure 5; with a small amount of fines content, the minimum void ratio decreased as the fines content increased. This occurred because the smaller particles filled the voids among
the larger particles. There was a critical value of fines content
when the voids among the larger particles were eventually
fully occupied, and thus, the minimum void ratio reached a
minimum value. After this, the curves showed a reverse
trend in which the $e_{\text{min}}$ values decreased as the fines content
increased. The reason for this was that smaller particles
became dominant in the mixture, while larger particles were
embedded into the smaller particles as isolated inclusions.
Moreover, the greater the particle size difference, the more
easily the fines filled the pores of the coarse particles, and the
smaller the $e_{\text{min}}$ Values. This occurred when the fines content
was approximately less than 80%. After the fines content
increased to more than 80%, the entire system was domi-
nated by fines, and thereby the order of curves was the same
as that of single particle groups.

3.3. Effects of Particle Shapes on $e_{\text{min}}$. To study the influences
of particle shapes on the minimum void ratio, the rela-
tionship between the minimum void ratios and the content
of fine grains for three different binary mixtures of sand was
analyzed, as shown in Figures 6–8.

It can be found in Figures 6–8 that the sequence of the
$e_{\text{min}}$ values of the binary mixtures from high to low was NS,
DS, and FS, which was the same as that of single particle

![Figure 3: The minimum void ratios corresponding to single particle groups of different sand types.](image-url)
FIGURE 4: Continued.
groups. This shows that the minimum void ratio of the sand was closely related to the shapes of the particles, and the smaller the $S$ value of the particle shape parameter, the larger the $e_{\text{min}}$ value.

To further explore the relationship between the $e_{\text{min}}$ values and the $S$ values of the particles, several tests were carried out with alternative materials. In these tests, the steel balls (SB) were mixed with the smaller sized steel balls (SB) and steel cylinders (SC). The $e_{\text{min}}$ values of the binary mixtures of the alternative materials are shown in Figure 9.

As illustrated in Figure 9, the $e_{\text{min}}$ value changed significantly when changing the shape of the fine particles in the binary mixtures. By comparing the results, it can be seen that $e_{\text{min}}$ was smaller when the fine particles were steel balls. This phenomenon further confirms that as $S$ of binary mixtures increased, $e_{\text{min}}$ decreased.

### 3.4. Preliminary Model for Estimating the Minimum Value of the Minimum Void Ratio

The corresponding changes in $e_{\text{min}}$ with the percentage of the fines content are shown in the above figures. It can be seen that the $e_{\text{min}}$ values decreased in the course of the filling-of-voids process and reached their minimum values when the fines content was about 40% of the mixture. Furthermore, it can be noted that the $(e_{\text{min}})_{\text{min}}$ values of all the binary mixtures were closely related to the particle size differences.
In order to better explore the effect of the particle size differences for \( \varepsilon_{\min} \), the experiments with mixed steel cylinders were performed, and the data were obtained as shown in Figure 10. The fitting curves of \( \varepsilon_{\min} \) and \( d/D \) for the various types of bimixtures are plotted in Figure 11.

It was obvious that when \( d/D \) was near zero, the pores formed by coarse particles were completely filled by the fine particles in an extremely fine state, and \( \varepsilon_{\min} \) took an extreme value of zero in ideal status. Therefore, the curve starts from the origin. As shown in Figure 11, the curves for \( \varepsilon_{\min} \) and \( d/D \) fit the significant relationships of exponential functions for both sand and alternative material mixtures. The fitting results show that the proposed model for \( \varepsilon_{\min} - d/D \) was in good agreement with the actual data, with a small deviation.

Equation (1) for \( \varepsilon_{\min} \) and \( d/D \) was obtained from fitting the curves in Figure 11 using Origin16 software:

\[
\varepsilon_{\min} = A \exp\left( -\left( \frac{d/D}{T} \right) \right) + y. \tag{1}
\]

Table 4 presents the different values of the parameters \( A \), \( T \), and \( y \) in the above equation for various types of sand and steel shots.

From Table 4, we can see that the coefficient of determination \( (R^2) \) was very high, which shows equation (1) can reflect the actual law well. By comparing the values of the three parameters, it can be found that the different values of \( A \) and \( y \) of different particles were obviously close. Therefore, the parameters \( A \) and \( y \) can be substituted for by taking the averages of their values for the five kinds of particles. Therefore, equation (1) can be represented by equation (2), as follows:

\[
\varepsilon_{\min} = 0.54 \times \left\{ 1 - \exp\left[ -\left( \frac{(d/D)}{T} \right) \right] \right\}. \tag{2}
\]

In order to verify the accuracy of the regression equation (2), the \( \varepsilon_{\min} \) values of the different mixtures estimated with equation (2) were compared with the measured values.
in Table 5, as shown in Figure 12. It can be found that the difference between the estimated value and the measured value was small, with the average discrepancy being approximately 3%. Therefore, the new regression equation (2) had a higher accuracy. By combining the particle shape parameters in Tables 3 and 6, it was found that $T$ increased significantly as $S$ of the mixture particles increased. To establish the relationship between $T$ and $S$, the values of $S$ for different particles were compared, as listed in Tables 3, 5, and 6. In Table 7, the sphericity $S$ of the mixture of substituted particles was weighted according to the respective contents of the two particles. Since $(e_{\min})_{\min}$ was obtained when the percentage of fines content was approximately 40%, the percentage of fines content was set to 40% in Table 7.

![Figure 9: Measured minimum void ratios versus fines content for alternative material mixtures (SC and SC).](image)

![Figure 10: Measured minimum void ratios versus fines content for alternative material mixtures (SC and SC).](image)
Figure 11: The relationship curves of \( (e_{\text{min}})_{\text{min}} \) and \( d/D \) for the binary mixtures.

Table 4: The fitting results of the relationship curve between \( (e_{\text{min}})_{\text{min}} \) and \( d/D \) values.

| Particle | NS     | DS     | FS     | SC     | SB & SC | SB     |
|----------|--------|--------|--------|--------|---------|--------|
| \( Y \)  | 0.54517| 0.55037| 0.53147| 0.53555| 0.54283 | 0.54684|
| \( A \)  | -0.53234| -0.53633| -0.53179| -0.53459| -0.54198| -0.5454|
| \( T \)  | 0.06913| 0.07206| 0.11679| 0.18705| 0.21647 | 0.24166|
| \( R^2 \)| 0.9563 | 0.95368| 0.98339| 0.99397| 0.99607 | 0.99403|

Table 5: The predicted and the measured values of \( (e_{\text{min}})_{\text{min}} \).

| Sands | Mixtures (mm) | \( d \) (mm) | \( D \) (mm) | Equation (2) | Measured | Predicted |
|--------|---------------|---------------|---------------|--------------|----------|-----------|
| NS     | 2–5~0.1–0.25  | 0.175         |               | \( (e_{\text{min}})_{\text{min}} = 0.54 \times \{1 - \exp((-d/d)/0.0691)} \) | 0.305    | 0.278     |
|        | 2–5~0.25–0.5  | 0.375         | 3.5           |              | 0.426    | 0.425     |
|        | 2–5~0.5–1     | 0.75          |               |              | 0.502    | 0.516     |
|        | 2–5~1–2       | 1.5           |               | \( (e_{\text{min}})_{\text{min}} = 0.54 \times \{1 - \exp((-d/d)/0.0691)} \) | 0.550    | 0.539     |
|        | 1–2~0.1–0.25  | 0.175         |               |              | 0.425    | 0.440     |
|        | 1–2~0.25–0.5  | 0.375         | 1.5           |              | 0.517    | 0.525     |
|        | 1–2~0.5–1     | 0.75          |               |              | 0.537    | 0.540     |
|        | 2–5~0.1–0.25  | 0.175         |               |              | 0.355    | 0.270     |
|        | 2–5~0.25–0.5  | 0.375         | 3.5           |              | 0.425    | 0.418     |
|        | 2–5~0.5–1     | 0.75          |               |              | 0.505    | 0.512     |
|        | 2–5~1–2       | 1.5           |               | \( (e_{\text{min}})_{\text{min}} = 0.54 \times \{1 - \exp((-d/d)/0.0721)} \) | 0.531    | 0.539     |
|        | 1–2~0.1–0.25  | 0.175         |               |              | 0.435    | 0.433     |
|        | 1–2~0.25–0.5  | 0.375         | 1.5           |              | 0.496    | 0.523     |
|        | 1–2~0.5–1     | 0.75          |               |              | 0.528    | 0.539     |
|        | 2–5~0.1–0.25  | 0.175         |               |              | 0.346    | 0.342     |
|        | 2–5~0.25–0.5  | 0.375         | 3.5           | \( (e_{\text{min}})_{\text{min}} = 0.54 \times \{1 - \exp((-d/d)/0.1168)} \) | 0.450    | 0.476     |
|        | 2–5~0.5–1     | 0.75          |               |              | 0.503    | 0.533     |
|        | 1–2~0.1–0.25  | 0.175         |               |              | 0.346    | 0.342     |
|        | 1–2~0.25–0.5  | 0.375         | 1.5           | \( (e_{\text{min}})_{\text{min}} = 0.54 \times \{1 - \exp((-d/d)/0.2165)} \) | 0.450    | 0.476     |
|        | 1–2~0.5–1     | 0.75          |               |              | 0.503    | 0.533     |
| FS     | 3~0.9         | 0.9           |               |              | 0.402    | 0.405     |
| B1 & C4| 3~1.5         | 1.5           | 3             |              | 0.482    | 0.486     |
| B1 & C3| 3~2.1         | 2.1           |               | \( (e_{\text{min}})_{\text{min}} = 0.54 \times \{1 - \exp((-d/d)/0.2165)} \) | 0.538    | 0.519     |
| B2 & C3| 2.1~1.5       | 1.5           | 2.1           |              | 0.525    | 0.520     |
| B2 & C4| 2.1~0.9       | 0.9           | 2.1           |              | 0.452    | 0.466     |
The fitting curve of $S$–$T$, as shown in Figure 13, further confirmed that the values of the $T$ and $S$ parameter were closely related. Therefore, it was feasible to obtain the $T$ parameter from the $S$–$T$ curve. Consequently, it was feasible to obtain the $T$ parameter from the $S$–$T$ curve. Then the $(e_{\text{min}})_{\text{min}}$ values of the binary mixture could be estimated using the $d/D$ values of the two particles and the $T$ value obtained from the $S$–$T$ curve using equation (2). However, to determine the accurate numerical relationship of the $T$ and $S$ parameters, more particle shapes and particle sizes still needed to be introduced since few types of the particle shapes were included in the test.

4. Conclusion

In this experiment, four types of sand and steel shots from different regions were used to explore the relationship between $e_{\text{min}}$ values and particle properties. It was found that for different binary mixtures, the relationships between the $e_{\text{min}}$ values with fines content were similar. Additionally, when the $e_{\text{min}}$ value reaches a minimum, the percentage of corresponding fines content was approximately 40%. The $e_{\text{min}}$ value for the binary sand mixture was significantly related to the particle size ratio and the particle shape, and $e_{\text{min}}$
decreased with the increase in the particle size difference and the $S$ values of the particles. These conclusions were further verified by the experiments with spherical and cylindrical steel particles.

Based on the experimental data, the functional relationship between $(e_{\text{min}})_{\text{min}}$ and $d/D$ was derived for binary mixtures with different particle sizes and shapes. This function could capture the curve shapes of $(e_{\text{min}})_{\text{min}}$ versus $d/D$ of soil mixtures well. This proposed model required only one parameter $T$ to predict the $(e_{\text{min}})_{\text{min}}$ values for various fines contents. The predicted trends were in good agreement with the measured values. It was found that the $T$ parameter was related to the $S$ value of the constituents of soil mixtures particles. Moreover, the relationship between $T$ and $S$ was preliminarily established. However, both the particle shapes that were available in the experiments and the quantitative descriptions of particle shape data in this essay were relatively inadequate. Therefore, the numerical relationship between $T$ and $S$ needs to be further verified.

**Table 7:** The weighted sphericity versus the minimum value of the minimum void ratio for the alternative material mixture.

| Coarsest Fine particle | Weighted sphericity | $T$   |
|------------------------|---------------------|-------|
| Ball1                  | Ball2               | 0.894 | 0.242 |
| Ball1                  | Ball3               | 0.891 | 0.242 |
| Ball1                  | Ball4               | 0.892 | 0.242 |
| Ball2                  | Ball3               | 0.884 | 0.242 |
| Ball2                  | Ball4               | 0.886 | 0.242 |
| Ball1                  | Cylinder2           | 0.790 | 0.216 |
| Ball1                  | Cylinder3           | 0.788 | 0.216 |
| Ball1                  | Cylinder4           | 0.786 | 0.216 |
| Ball2                  | Cylinder3           | 0.783 | 0.216 |
| Ball2                  | Cylinder4           | 0.781 | 0.216 |
| Cylinder1              | Cylinder2           | 0.632 | 0.187 |
| Cylinder1              | Cylinder3           | 0.630 | 0.187 |
| Cylinder1              | Cylinder4           | 0.629 | 0.187 |
| Cylinder2              | Cylinder3           | 0.626 | 0.187 |
| Cylinder2              | Cylinder4           | 0.624 | 0.187 |
| Cylinder3              | Cylinder4           | 0.621 | 0.187 |

**Notation**

- $e_{\text{min}}$: Minimum void ratio
- $(e_{\text{min}})_{\text{min}}$: Minimum value of minimum void ratio
- $C_u$: Uniformity coefficient
- $C_C$: Coefficient of curvature
- $G_s$: Specific gravity of solid particles
- NS: Nanjing River Sand
- DS: Dongting Lake Sand
- YS: Yizheng Mountain Sand
- FS: Fujian Standard Sand
- SB: Steel ball
- SC: Steel cylinder
- $d$: Fine particle size
- $D$: Coarse particle size
- $d/D$: Particle size ratio
- $A$, $y$, $T$: The parameters are obtained by fitting different curves of the minimum value of minimum void ratio and particle size ratio and the parameter $T$ is closely related to the sphericity of the particle
- $R^2$: Coefficient of determination.

**Data Availability**

The data shown in the tables of the article used to support the findings of this study are included within the article. The data used for the graphs in this article to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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**Figure 13:** The fitting curve of $S-T$. 

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**Table 7:** The weighted sphericity versus the minimum value of the minimum void ratio for the alternative material mixture.

| Coarsest Fine particle | Weighted sphericity | $T$   |
|------------------------|---------------------|-------|
| Ball1                  | Ball2               | 0.894 | 0.242 |
| Ball1                  | Ball3               | 0.891 | 0.242 |
| Ball1                  | Ball4               | 0.892 | 0.242 |
| Ball2                  | Ball3               | 0.884 | 0.242 |
| Ball2                  | Ball4               | 0.886 | 0.242 |
| Ball1                  | Cylinder2           | 0.790 | 0.216 |
| Ball1                  | Cylinder3           | 0.788 | 0.216 |
| Ball1                  | Cylinder4           | 0.786 | 0.216 |
| Ball2                  | Cylinder3           | 0.783 | 0.216 |
| Ball2                  | Cylinder4           | 0.781 | 0.216 |
| Cylinder1              | Cylinder2           | 0.632 | 0.187 |
| Cylinder1              | Cylinder3           | 0.630 | 0.187 |
| Cylinder1              | Cylinder4           | 0.629 | 0.187 |
| Cylinder2              | Cylinder3           | 0.626 | 0.187 |
| Cylinder2              | Cylinder4           | 0.624 | 0.187 |
| Cylinder3              | Cylinder4           | 0.621 | 0.187 |
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