Influence of the curvature correction to the surface tension on nuclear fragmentation at intermediate energy head-on heavy ion collisions

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Abstract. Hydrodynamic model of the "doughnut"-like structures formation in the head-on heavy ion collisions at intermediate energies is analyzed. Modification of the model is suggested to include the curvature correction to the surface tension coefficient. It is shown that accounting for the curvature correction changes the overall reaction time, maximum size of the system and the number of fragments. The results obtained show good correspondence with the results of the transport theory calculations and experimental data for the number of fragments and spreading size. The conducted analysis suggests that accounting for the curvature correction to the surface tension coefficient is essential when studying the dynamics of the system in the heavy ion collisions.

1. Introduction

During recent decades nuclear fragmentation in the heavy ion collisions was intensively studied by different scientific groups. The field covers a wide range of energies starting from about 3-4 MeV per nucleon up to the relativistic energies. There exist a number of different theoretical models of the multifragmentation phenomena in low energy collisions (e.g. statistical multifragmentation model [1], spinodal decomposition model [2], etc.) as well as different computational models [3]. Similarly for the relativistic energies one can find sophisticated theoretical and computational hydrodynamic models of collisions allowing one to describe the dynamics of the system and predict the number of the fragments formed [4, 5]. For the intermediate energies of about 50-100 MeV per nucleon the number of existing theoretical models is not that big. Collisions in the above energy range are studied mostly with the help of computer simulations. At the same time, studies of the intermediate energy collisions can be important as intermediate mass fragments produced in such reactions may provide a unique probe to study hot nuclear matter properties [6] while the process can still involve both statistical and dynamic mechanisms. Therefore, it should be possible to have a deep insight into the properties of nuclear matter and, hence, obtain the valuable information needed for developing the proper equation of state (EoS) that is one of the still unsolved fundamental questions in nuclear physics [7].

Among the existing studies dealing with intermediate energies it is interesting to look at head-on collisions. In the 1990s there appeared in the literature a number of microscopic transport
theory calculations \[8, 9\] predicting torroidal and bubble structure formation in the head-on heavy ion collisions (HIC) at intermediate energies. Later, the signatures of the “doughnut”-like structures with production of similar size intermediate mass fragments (IMF) were observed in central HIC experimentally \[10\]. It is important to mention that the possibility for the hollow structures to be formed in the nuclei was also predicted by different approaches \[11, 12, 13\]. Such phenomena seem to be quite important as the topology of the system is related to the compressibility of the nuclear matter \[9\] that is an important ingredient for the nuclear matter EoS. Recently, there was proposed the hydrodynamic model of the “doughnut”-like structures formation in the head-on collisions \[14\]. Within that model the reaction time, spreading size and number of fragments calculated with no adjusting parameters appeared to be in accord with the experimental data and with the results of the transport theory calculations. Among the important parameters of the model one can mention nuclear incompressibility, EoS and surface tension coefficient. The last is important during the system expansion stage and formation of the pre-fragments. Up to date there exist different methods to define the surface properties of the nuclear matter that give slightly different values for the surface tension coefficient. At the same time, during the expansion and pre-fragment formation stages the surface to volume ratio suggests that the surface effects can be dominant. More than that, at those stages at some parts of the system the curvature of surface is quite high that can make it important to account for the curvature correction to the surface tension coefficients. Recently, the model allowing to calculate the curvature correction to the surface tension from the nuclear matter EoS was suggested \[15\]. It was shown \[16\] that the average value of the Tolman length for the nuclear matter is about 0.5 fm that makes it important for small nuclear systems with high curvatures as in the studied case.

Therefore, our focus in this work is on modifying the earlier suggested hydrodynamic model of the head-on HIC at intermediate energies to account for the surface tension coefficient dependence on the curvature.

2. Theoretical model

Within the four stages hydrodynamic model of the head-on HIC at intermediate energies the system dynamics during the expansion stage is described by the equation (see Eq. (29) in Ref. \[14\]):

\[
\frac{d^2R}{dt^2} + 4 \frac{dR_t}{dt} \frac{(B - t)}{((B - t)^2 + A)} + R_t \frac{6(B - t)^2 - 2A}{((B - t)^2 + A)^2} = 0, \tag{1}
\]

with

\[
B = \tau_2 + \sqrt{\frac{2\rho_s}{\sigma}} \sqrt{\frac{\Omega}{\pi}}, \quad A = \frac{2\rho_s}{\sigma} \left( 2R_{bd}(\tau_2) d(\tau_2) R_{jet} - \frac{\Omega}{\pi} \right), \tag{2}
\]

were \(R_t\) is the lamella radius, \(\rho\) is the system density, \(\sigma\) is the surface tension coefficient, \(\Omega\) is the volume of the colliding nuclei, \(\tau_2\), \(R_{bd}(\tau_2)\), \(d(\tau_2)\), \(R_{jet}\) are the initial parameters defined during the previous stages of the collision. The process described by Eq. (1) is the linearly damped oscillatory motion of the lamella radius with time-dependent frequency and damping factor originating from the continuous transfer of momentum from the lamella to the rim. In that case pre-fragment formation is due to the Rayleigh-Plateau instability. The wave number of the most unstable perturbation \(k_f^{\max}\) that defines the size of the fragments and the associated timescale of the capillary instability is given by

\[
k_f^{\max} \approx \frac{0.7}{R_{rim}}, \quad \tau_{inst} \approx 2.91 \sqrt{\frac{\rho_s R_{rim}^3}{\sigma_{rim}}} \tag{3}
\]
Table 1. Saturation properties and surface characteristics of the studied Skyrme parameterizations. Surface tension coefficient of the semi infinite matter is calculated within the ETF approach (except for the MSkA parameterization).

| Parameter | SII [20] | MSkA [21] | PRC45 [22] | SV-min [23] |
|-----------|---------|---------|---------|---------|
| $\rho_0$, (fm$^{-3}$) | 0.148 | 0.153 | 0.145 | 0.161 |
| $E_0$, (MeV) | -15.99 | -15.99 | -15.82 | -15.91 |
| $K$, (MeV) | 341.40 | 313.33 | 367.58 | 221.76 |
| $m^*$, (M$^*/M$) | 0.58 | 0.79 | 1.00 | 0.95 |
| $\sigma$, (MeV/fm$^2$) | 1.105 | 0.972 [24] | 1.141 | 1.010 |
| $\delta$, (fm) [16] | -1.72 | -0.59 | -0.40 | -0.50 |

The above Eqs. (3) are obtained from the dispersion relation for the perturbation amplitude that describes the instability of the inviscid cylinder [17]. At the same time, for the curved interfaces the surface tension coefficient differs from one for the semi infinite layer and can be found from the Tolmans’ equation [18]:

$$\sigma(R) = \sigma_\infty \left(1 - \frac{2\delta}{R} + \cdots \right),$$

with $\delta = R_{em} - R$ being the distance between the equimolar surface $R_{em}$ and the surface of tension $R$ at the interphase boundary. Earlier, the model that links the Tolmans’ length $\delta$ with the EoS of nuclear matter has been suggested [15]. Within that model one can calculate $\delta$ as:

$$\delta = \frac{2}{3}\frac{1}{\rho_0^2} \times \frac{-33\Theta_{to} - 160W \rho_0^{-1/3} + t_3(1+\alpha)\rho_0^{\frac{1}{12}}(7(3\alpha+6)-3(3\alpha+6)^2)}{(15t_0 + \frac{1}{12}t_3(1+\alpha)((3\alpha+6)-3(3\alpha+6)^2))} \sigma_\infty,$$

where

$$W = \frac{h^2}{10m} \left(\frac{3}{8\pi g}\right)^{\frac{3}{2}} \left(5 - \frac{3m_s}{m_m}\right).$$

Substitute Eq. (5) into Eq. (6), one obtains the result that allows calculating the surface tension coefficient of the curved interface in case of the symmetric nuclear matter at $T = 0$, and at normal density $\rho_0$. The equation of state used is the nuclear matter EOS at low-temperature and high-density limit with Skyrme force parameterization that employs isospin-independent effective mass and has the form given in Ref. [19]. In this work we suggest instead of using the surface tension coefficient of the semi infinite matter in Eqs. (2)-(3) to use the one defined by Eqs. (4) - (6) that accounts for the curvature correction. In this work we do not dwell on calculations of the precise values of the reaction parameters that require accounting for the time dependence of the rim radius and lamella radius throughout the process. We rather adopt some average values for the curvature in order to have the qualitative picture with quantitative estimates that show the influence of the curvature correction inclusion on system dynamics.

3. Results and discussion

Similar to Ref. [14] we study the central $^{93}$Nb + $^{93}$Nb collision at $E = 60$ MeV/nucleon. We use four EoS with different Skyrme energy functionals to calculate the $\delta$-correction. Namely, we study the SII, PRC45, MSkA and SV-min parameterizations (see Table 1 for details). The first three parameterizations are representatives of the “stiff” EoS but are not consistent with...
a number of nuclear matter constraints [25]. At the same time, the SV-min parameterization produces a “soft” EoS but opposite to the first three it satisfies the nuclear matter constraints studied in Ref. [25]. All the parameterizations but SII give realistic values for the Tolman δ-correction [16]. The surface tension coefficient of the semi infinite matter at $T = 0$ for the MSkA parameterization is adopted to be $\sigma = 0.972$ Mev/fm$^2$ [24] when for all the other forces it is calculated within the restricted extended Thomas-Fermi (ETF) approach [26] for the symmetric case with $\rho_n = \rho_p = \frac{1}{2}\rho$ (without Coulomb interaction). Terms up to the fourth order are considered. All the calculations include the effective mass and the spin-orbit contribution.

The system radius dynamics for all four EoS during the expansion stage is shown at Figs. 1-4.

![Figure 1. Lamella radius dependence on time for the SII force. Dashed navy is without curvature correction; Solid red is for the case with the curvature correction.](image1)

![Figure 2. Lamella radius dependence on time for the PRC45 force. Dashed navy is without curvature correction; Solid red is for the case with the curvature correction.](image2)

It can be seen from the figures that the qualitative changes in the system dynamics are quite similar for all the four forces including the “soft” EoS with the SV-min force. Therefore, one can conclude that it is the surface tension but not compressibility that plays the leading role in the system behavior during the expansion and fragmentation stages. This fact suggests that the fragments characteristics in the intermediate energy HIC can give valuable information regarding the surface properties of the nuclear matter. The corresponding parameters of the reaction are given in Table 2. One can see that for both parameterizations inclusion of the curvature correction to the surface tension leads to the decrease in the reaction time, the number of fragments and the spreading size. At the same time, the rim radius at the moment of prefragment formation increases. In the case of the SII force the characteristic reaction times obtained are quite different from those obtained in the transport theory calculations. This result does not seem to be unexpected as the SII force fails to satisfy many of the nuclear matter constraints [25] and shows poor performance in describing the surface properties of the nuclear matter [16]. For the case of the MSkA force inclusion of the curvature correction to the surface tension coefficient improves the agreement of the model results with the results of the BUU calculations and with the available experimental data.
Figure 3. Lamella radius dependence on time for the MSkA force. Dashed navy is without curvature correction; Solid red is for the case with the curvature correction.

Figure 4. Lamella radius dependence on time for the SV-min force. Dashed navy is without curvature correction; Solid red is for the case with the curvature correction.

Table 2. Quantitative characteristics of the central $^{93}$Nb + $^{93}$Nb collision at $E = 60$ MeV/nucleon with different EoS parameterizations. Those marked “+δ” include curvature correction to the surface tension coefficient.

|                  | SII   | SII+δ | MSkA | MSkA+δ | [8, 9] | [10]    |
|------------------|-------|-------|------|--------|--------|---------|
| Reaction time τ₃, fm/c | 110   | 69    | 118  | 95     | 120-160|         |
| Spreading size $R_{\text{max}}$ | 2.5   | 2.2   | 2.6  | 2.4    | ~2     |         |
| Rim radius at τ₃, fm   | 1.9   | 2.1   | 1.9  | 2.0    |        |         |
| Number of similar mass IMF | 5.4   | 4.3   | 5.6  | 5.0    | 3-6    | 4-6     |

4. Conclusions
In the current work the modified hydrodynamic model of the head-on heavy ion collisions at intermediate energies that accounts for the curvature correction to the surface tension coefficient is developed. Characteristic parameters of the system dynamics are calculated in case of different Skyrme force parameterizations being used to construct the equation of state of nuclear matter. The analysis of the quantitative results obtained shows strong dependence of the system dynamics on the surface tension coefficient. At the same time, not all Skyrme parameterizations can be used to adequately describe the system behavior but rather those that have been shown to be able to correctly describe the surface properties of the nuclear matter. The number of fragments as well as the maximum expansion calculated within the introduced model is in accord with the data available in the literature on the head-on heavy ion collisions in the energy range studied.

The results obtained suggest that accounting for the curvature correction to the surface tension coefficient is essential when studying the system dynamics and fragments formation in the intermediate energy heavy ion collision. The strong correlation in between the experimental observables and the surface tension coefficient makes it possible to use the intermediate energy head-on heavy ion collisions as the probe for revealing the properties of the nuclear surface.
Further studies are necessary to account for the time dependence of the surface tension coefficient in order to have the precise quantitative results.

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