Light Scalar Decay in Diquark Chiral Effective Theory

Deog Ki Hong\textsuperscript{1,*} and Chaejun Song\textsuperscript{1,†}

\textsuperscript{1}Department of Physics, Pusan National University, Busan 609-735, Korea

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Abstract

We calculate the decay rate of light scalar mesons, using a diquark chiral effective theory, recently proposed to describe exotic hadrons. In the effective theory the light scalar mesons are postulated to be bound states of diquark and anti-diquark. We find our results are in good agreement with experimental data. The axial couplings of diquarks with pions and kaons are found to be quite small and the perturbation is reliable. It shows that the diquark picture captures the correct physics of light scalar mesons.

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\textsuperscript{*}Electronic address: dkhong@pusan.ac.kr
\textsuperscript{†}Electronic address: chaejun@charm.phys.pusan.ac.kr
I. INTRODUCTION

Recent discovery of exotic baryons [1] revived the interest in hadron spectroscopy, especially that of exotic hadrons. Among several models for exotic baryons, promising one is the diquark model [2, 3], which assumes a strong correlation of two quarks in the color antitriplet channel. In the Jaffe-Wilczek (JW) diquark model [2], the exotic baryons are bound states of two scalar diquarks and one antiquark. The salient features of JW model are the occurrence of low-dimensional multiplets and small mass difference among the members in the multiplet, which were soon supported by the NA49 experiment [4]. Furthermore, the extremely narrow decay width of exotic baryons was naturally explained in the diquark picture of exotic baryons, where the decay process is suppressed by tunnelling among two diquarks [5].

As the existence of hadrons containing quarks more than three is logically possible, its absence in the low-lying hadronic spectroscopy has been a puzzle since the advent of the quark model [6]. In late seventies, Jaffe suggested that the positive-parity scalar mesons of mass less than 1 GeV might be bound states of multi quarks, forming a $SU(3)$ nonet [7]. In his scheme, the scalar nonet is postulated to be the non-exotic components of 4-quark states ($Q_i \bar{Q}_i$). However, the quark content of light scalar mesons has been controversial and still is [8]. Recently, it was argued that $\sigma$, the lightest member in the scalar nonet, is not a pure $Q \bar{Q}$ state but mostly a four-quark state, using the unitarity argument in the large $N_C$ analysis [9]. (See also a recent paper for the diquark picture of scalar mesons [10].)

In this paper, we assume that the scalar nonet is a bound state of a diquark and an antidiquark and calculate the decay width of scalar nonet to compare with the experimental data [11, 12], using the diquark chiral effective theory [5].

II. DIQUARK CHIRAL EFFECTIVE THEORY

The success of quark model in hadron spectroscopy is understood in the framework of the chiral quark effective theory ($\chi$QET), derived by Georgi and Manohar [13]. The $\chi$QET is an effective theory of quantum chromodynamics (QCD) below the chiral symmetry breaking scale $\Lambda_{\chi SB}$ but above the confinement scale, $\Lambda_{\text{QCD}}$. The relevant degrees of freedom of the effective theory therefore contains quarks, gluons and the Nambu-Goldstone (NG) bosons of spontaneously broken $SU(3)_L \times SU(3)_R$ chiral symmetry. The chiral quark effective Lagrangian is given as

$$\mathcal{L}_{\chi Q} = \bar{\psi} \left( i \slashed{D} + \mathcal{V} \right) \psi + g_A \bar{\psi} \left( \gamma_5 \psi \right) A_{\alpha} - m \bar{\psi} \psi + \frac{1}{2} f^2 \text{tr} \partial^\mu \Sigma^\dagger \partial^\mu \Sigma - \frac{1}{2} \text{tr} F_{\mu\nu} F^{\mu\nu} + \cdots, \quad (1)$$

where $D_\mu = \partial_\mu + ig_A G_\mu$ is the QCD covariant derivative,

$$V_\mu = \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger \right), \quad A_\mu = \frac{1}{2} i \left( \xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right) \quad (2)$$

and $f = F_\pi = 93$ MeV is the pion decay constant,

$$\Sigma = e^{2i\pi/f} = \xi^2. \quad (3)$$

The Nambu-Goldstone fields $\pi = \pi_a T_a$ with $\text{tr} \left( T_a T_b \right) = 1/2 \delta_{ab}$. In $\chi$QET the higher-order terms or explicit chiral symmetry breaking terms are suppressed by powers of $\Lambda_{\chi SB} \simeq 4\pi f$. 
The typical size of symmetry breaking terms is
\[ \frac{m^2}{\Lambda^2_{\chi_{SB}}} \approx 0.1. \] (4)

Therefore, they make only small contributions to the naive quark model. Furthermore, by matching with QCD, one finds the axial coupling and the strong coupling to be small as well, \( g_A \approx 0.75 \) and \( \alpha_s \approx 0.28 \) so that the perturbation is reliable. This is why the naive quark model is so successful in describing non-exotic hadrons [13].

Similarly, the success of diquark picture for exotic hadrons may be explained by the diquark chiral effective theory (D\(\chi\)ET), proposed recently by Sohn, Zahed, and one of the authors [5]. Since the diquarks have mass around 450 MeV [2, 5], they fit naturally in the range where the \( \chi\)QET is applicable. Hence, in addition to the relevant degrees of freedom of \( \chi\)QET, the diquark chiral effective theory involves one more degree of freedom, namely a diquark field, defined as
\[ \varphi_i^j(x) = \lim_{y \to x} \frac{|y - x|^{\gamma_m}}{\kappa^2} e^{ijk} \epsilon_{\alpha\beta\gamma} \bar{\psi}_{c\beta}(x) \gamma_5 \psi_{c\alpha}(y), \] (5)

where \( \kappa \) is a mass scale for the diquark field, \( \gamma_m \) is the anomalous dimension of the diquark correlator and the Greek indices denote colors, while the Latin indices \( i, j, k = 1, 2, 3 \) denote flavors. \( \psi_c \equiv C \bar{\psi}^T \) is a charge conjugated field of a quark field, \( \psi \), and \( C = i\gamma_2\gamma_0 \). (Here we consider only scalar diquarks, since tensor diquarks [3] do not contribute to light scalar mesons.)

The diquark chiral effective Lagrangian is described by
\[ \mathcal{L} = \frac{1}{2} |(D_\mu + iV_\mu) \varphi_i^j|^2 - \frac{1}{2} M^2 |\varphi_i|^2 - \left( g \varphi_i^\alpha e^{ijk} \epsilon_{\alpha\beta\gamma} \bar{\psi}_{c\beta} \gamma_5 \psi_{c\alpha} \right) + h.c. \]

where higher-order interactions of diquarks and quarks is denoted as \( \mathcal{L}_{\text{int}} \), which may contain the (chirally) covariant derivatives. In this paper, we will take \( M = 420 \) MeV from the random instanton model [14] and \( g^2 = 3.03 \), the best value fitting the scalar nonet mass [5].

The Yukawa coupling splits diquark masses, \( M_{ud} = 450 \) MeV and \( M_{us} = M_{ds} = 630 \) MeV. And the binding energy between a diquark and an antidiquark in scalar nonet is \( B = 280 \) MeV. We list these parameters in Table I.

| \( M_{qs} \) (MeV) | \( M_{ud} \) (MeV) | \( B \) (MeV) | \( g^2 \) |
|-----------------|-----------------|--------------|---------|
| 630             | 450             | 280          | 3.03    |

TABLE I: The best-fit parameters in [5]. They are obtained from the random instanton model and the scalar nonet masses.

### III. DIQUARK-ANTIDIQUARK TO GOLDSTONE BOSONS

In general, it is very hard to deal with bound states of strongly interacting particles, especially with broad resonances. However, we will attempt to calculate the decay width
FIG. 1: The annihilation process, $\varphi + \varphi^* \to \pi + \pi$. The double line denotes diquarks and the dashed line denotes pions and kaons.

of strongly interacting scalar nonet, utilizing the diquark chiral effective theory. We will follow the scheme of nonrelativistic positronium decay, assuming the diquark and NG boson couplings are not large. We will see later indeed the couplings are small and the naive diquark model of Jaffe and Wilczek is reliable. In the effective theory, the scalar mesons are bound states of diquark and anti-diquark. The decay process occurs as annihilation of diquark with anti-diquark into NG bosons (see Fig. 1). The decay width is then given as

$$\Gamma_{S\to\pi\pi} = \lim_{v_2\to0} v_2 \sigma (\varphi + \varphi^* \to \pi + \pi) |\Psi(0)|^2$$

where $v_2$ is the velocity of anti-diquark in the rest frame of the target diquark and $\Psi(0)$ is the 1S wave function of the diquark-antidiquark bound state at the origin. The differential cross section for the annihilation process is then

$$d\sigma = \frac{(2\pi)^4 |\mathcal{M}|^2}{4\sqrt{(p_1 \cdot p_2)^2 - M_1^2 M_2^2}} d\Phi(p_1 + p_2; k_1, k_2),$$

where $\mathcal{M}$ is the annihilation amplitude for $\varphi^* + \varphi \to \pi + \pi$. The phase space is given as

$$d\Phi(p_1 + p_2; k_1, k_2) = \delta^4(p_1 + p_2 - k_1 - k_2) \frac{d^3k_1}{(2\pi)^32E_3} \frac{d^3k_2}{(2\pi)^32E_4}.$$  

Integrating over the phase space, we get

$$\Gamma_{S\to\pi\pi} = \frac{1}{16\pi} \frac{|\mathcal{M}|^2}{M_1 M_2} \frac{|\vec{k}_1|}{\sqrt{\vec{k}_1^2 + m^2_A + \sqrt{\vec{k}_1^2 + m^2_B}}} |\Psi(0)|^2,$$

where $M_1$ is anti-diquark mass, $M_2$ diquark mass, $m_A$ the mass of pion $A$, $m_B$ the mass of pion $B$, and $\vec{k}_1$ (or $-\vec{k}_1$) is the final momentum of pions in the CM frame.

To the order we are interested in

$$\mathcal{M} = \mathcal{M}_{\text{tree}} + \mathcal{M}_{1\text{loop}}.$$  

4
A. Tree contributions

In $D\chi ET$, the leading order contribution to the amplitude in $1/\Lambda_{\chi\text{SB}}$ expansion, shown in Fig. 2, is given as

$$M_{\text{tree}} = \langle \pi^A(k_1), \pi^B(k_2) \mid \mathcal{L}_1 \mid \varphi_a(p_1), \varphi_b^*(p_2) \rangle,$$

where

$$\mathcal{L}_1 = \text{Tr} V^\mu (\varphi^* \partial_\mu \varphi - \varphi \partial_\mu \varphi^*) + h_A \text{Tr} \left( \varphi^* \varphi A_\mu^2 \right) + h'_A \varphi^* \varphi \text{Tr} A_\mu^2.$$

Note that because of the parity invariance the leading coupling of (scalar) diquark with pions occurs at $1/\Lambda_{\chi\text{SB}}^2$. For the manifest $SU(3)$ covariance, we rewrite the incoming state in the basis of $SU(3)$ octet and singlet:

$$| \varphi_a(p_1), \varphi_b^*(p_2) \rangle = 2 T^C_{ab} | S^C(p_1, p_2) \rangle + \frac{1}{3} \delta_{ab} | S^0(p_1, p_2) \rangle.$$

Then, the tree amplitudes for octet and singlet become, respectively,

$$M_{\text{tree}}^{C \to AB} = -\frac{ig^2}{16 \pi^2 f^2} \int \frac{dx}{x} \int \frac{dy}{y} \left( x p_1 - y p_2 \right) \log \left[ \frac{(x p_1 - y p_2)^2 - x p_1^2 - y p_2^2 + m^2}{\mu^2} \right].$$

B. One-loop contributions

The vector-current of pions (Fig. 3a) contributes in the modified minimal subtraction scheme as

$$M_{\text{1-loop}} = -\frac{6ig^2}{16 \pi^2 f^2} \frac{f^{ABC}}{k_2} [(k_2 - k_1) \cdot (p_1 - p_2) I_1 + \cdots],$$

where $I_1$ is defined as

$$I_1 = \int_0^1 dx \int_0^{1-x} (4 - 3x + 3y) \log \left[ \frac{(xp_1 - yp_2)^2 - x p_1^2 - y p_2^2 + m^2}{\mu^2} \right].$$
in the SU(3) limit. The axial-current contribution (Fig. 3b) becomes

\[ M_{\text{1 loop}} = \frac{6g^2g_A^2}{16\pi^2 f^2} (k_1 \cdot k_2) [\epsilon^{alm} \epsilon^{bon} T_{ln}^{AB} T_{mo} T_2 - \{ \delta^{AB} \delta_{ab} - 2(T^{AB})_{ba} \} I_3 + \cdots ] \]  

(19)

where

\[ I_2 = 6 \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \log \left[ \frac{(xp_1 + yp_1 - yk_1 + zk_2)^2 - xp_1 - y(p_1 - k_1)^2 - zk_2^2 + m^2}{\mu^2} \right] \]

\[ I_3 = 2 \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \log \left[ \frac{(xk_1 + yk_2 + kp_1 - xk_1 - y(k_1 + k_2)^2 - zp_1^2 + m^2}{\mu^2} \right]. \]

\[ \Gamma_{S \to \pi^A \pi^B} = \frac{1}{16\pi M_1^* M_2^*} \frac{1}{\sqrt{k_1^2 + m_A^2 + \sqrt{k_2^2 + m_B^2}}} |\Psi(0)|^2, \]  

(20)

where \( M_1^* = M_1 - BM_1/(M_1 + M_2) \), \( M_2^* = M_2 - BM_2/(M_1 + M_2) \), and

\[ k_1^2 = \frac{((M_1^* + M_2^*)^2 + m_B^2 - m_A^2)^2}{4(M_1^* + M_2^*)^2} - m_B^2. \]  

(21)

By a naive dimensional analysis we find

\[ |\Psi(0)|^2 = c(MB)^{3/2} \]  

(22)

where \( c \) is a constant and \( M \) is the reduced diquark mass. From the analogy with the 1S wavefunction of positronium, we take \( c = 1/\pi \) here.

IV. TETRAQUARK SCALAR DECAY WIDTHS

A. Decays into two nonstrange Nambu-Goldstone bosons

In our constituent diquark picture, the light scalar mesons have a binding energy of 280 MeV, which is not negligible in calculating the decay width. Because of the large binding energy, Eq. (10) is far from the proper energy-momentum conservation. (The velocity \( v_2 \) does not also go to zero. However, it factors out in the leading order.) To take into account of the binding energy in the energy-momentum conservation we modify Eq. (10) as

\[ \Gamma_{S \to \pi^A \pi^B} = \frac{1}{16\pi M_1^* M_2^*} \frac{1}{\sqrt{k_1^2 + m_A^2 + \sqrt{k_2^2 + m_B^2}}} |\Psi(0)|^2, \]  

(20)

where \( M_1^* = M_1 - BM_1/(M_1 + M_2) \), \( M_2^* = M_2 - BM_2/(M_1 + M_2) \), and

\[ k_1^2 = \frac{((M_1^* + M_2^*)^2 + m_B^2 - m_A^2)^2}{4(M_1^* + M_2^*)^2} - m_B^2. \]  

(21)

By a naive dimensional analysis we find

\[ |\Psi(0)|^2 = c(MB)^{3/2} \]  

(22)

where \( c \) is a constant and \( M \) is the reduced diquark mass. From the analogy with the 1S wavefunction of positronium, we take \( c = 1/\pi \) here.
The lightest scalar $\sigma$ is believed to have no or just little strangeness in it. So we assume the ideal mixing between $\sigma$ and $f_0$. In our calculation, the (anti)diquark in $\sigma$ is nonstrange and the (anti)diquark in $f_0$ includes one strange (anti)quark. We take $g_A = 0.75$, the experimental value in the quark model [13, 15], and set the quark mass $m = 400$ MeV [16]. We then determine the couplings $h_A$ and $h'_A$ by fitting the experimental values for the decay widths of scalar mesons in the dominant modes. The results are shown in Table II [17]. We find our couplings are of order 1 for the decay widths in the range allowed by experiments. Since the expansion parameter is $h^2_A/4\pi$ or $h'^2_A/4\pi$ rather than the coupling itself, the higher-order corrections are quite suppressed and the perturbation is reliable.

| set | $h_A$ | $h'_A$ | $\Gamma_{\sigma \to 2\pi}$ (MeV) | $\Gamma_{f_0 \to 2\pi}$ (MeV) | $\Gamma_{a_0 \to \eta\pi}$ (MeV) |
|-----|-------|--------|-------------------------------|-----------------------------|-------------------------------|
| 1   | 1.46  | -1.66  | 317                           | 67                          | 83                           |
| 2   | 1.20  | -1.38  | 234                           | 71                          | 57                           |

TABLE II: Two fitting parameters and the decay widths of the light scalar mesons in dominant modes.

Once we determine the axial couplings $h_A$ and $h'_A$, we calculate in the next section the decay width of $f_0$ in its subdominant mode to see if D\chiET works.

B. $f_0$ decay into two kaons

Since the mass of two kaons is larger than $f_0$ mass, we cannot obtain the decay width with Eq. (20). The broadening of $f_0$ makes the decay into two kaons possible. So the broadening should be considered in order to calculate the decay mode into two kaons. Keeping the basic picture, we introduce the Breit-Wigner form to revise Eq. (20);

$$
\Gamma_{S \to \pi^A\pi^B} = N \int_{s_{\min}}^{s_{\max}} ds \frac{|M|^2}{4\pi^2 s} \frac{|\vec{k}_f|}{\sqrt{\vec{k}_f^2 + m_A^2} + \sqrt{\vec{k}_f^2 + m_B^2}} \frac{m_S \Gamma_S |\Psi_s(0)|^2}{(s - m_S^2)^2 + m_S^2 \Gamma_S^2} \tag{23}
$$

with scalar mass $m_S$ and total decay width $\Gamma_S$. $\sqrt{s_{\max}}$ is $m_S + \Gamma_S$ and $\sqrt{s_{\min}}$ is chosen to be the larger one between the threshold value for the decay mode and $m_S - \Gamma_S$. So $\sqrt{s_{\min}} = m_K + m_{\bar{K}}$ for $f_0 \to K + \bar{K}$ mode. The wavefunction at the origin and the final momentum are modified as

$$
|\Psi_s(0)|^2 \approx \left( (M_1 + M_2 - \sqrt{s})M \right)^{3/2} / \pi \tag{24}
$$

and

$$
\vec{k}_f^2 = \frac{(s + m_B^2 - m_A^2)^2}{4s} - m_B^2 \tag{25}
$$

in (23). The normalization constant $N$ is adjusted by

$$
\frac{1}{N} = \int_{s_{\min}}^{s_{\max}} ds \frac{1}{\pi} \frac{m_S \Gamma_S}{(s - m_S^2)^2 + m_S^2 \Gamma_S^2} \tag{26}
$$
Since $f_0 \to 2\pi$ is dominant among $f_0$ decay modes, we will use $\Gamma_{f_0 \to 2\pi}$ in Table III for $\Gamma_S$ in Eq. (23). Then we obtain 75 MeV with set 1 and 59 MeV with set 2 for $\Gamma_{f_0 \to 2K}$. Experimentally $\Gamma_{f_0 \to 2K}/(\Gamma_{f_0 \to 2\pi} + \Gamma_{f_0 \to 2K})$ is $0.14 \sim 0.32$ and the full decay width of $f_0$ is $\Gamma = 40 \sim 100$ MeV [12]. We find our result is a few times larger than the experimental value, $\Gamma_{f_0 \to 2K} = 5.6 \sim 32$ MeV. Considering the experimental uncertainty and the crude approximation we made for the bound state wavefunction, our result is not too bad.

V. CONCLUSION

We calculate the decay width of light scalar mesons in the framework of the diquark chiral effective theory, which was introduced recently to describe exotic hadrons like pentaquarks. Using the parameters fixed previously by the mass of scalar mesons, we determine the axial couplings of diquarks with pions and kaons from the experimental data for the decay width of scalars in the dominant decay modes. The axial couplings turned out to be small, showing that the naive diquark model of Jaffe and Wilczek is reliable. Then, we calculate the decay width of scalar mesons for the subdominant channel and find that our result is not far off from the experimental data.

To conclude, in the diquark chiral effective theory, based on the spontaneously broken chiral symmetry and the diquark picture of exotic hadrons, we find the decay width of scalar mesons in good agreement of experimental data. It shows that the diquark chiral effective theory captures correct physics of exotic hadrons.
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[16] We assume SU(3) symmetric mass. The SU(3) breaking mass terms contribute less than 10 percent.

[17] Since the pions in the final state carry large momenta, a fraction of $\Lambda_{\chi_{SB}}$, one has to include the re-scattering effect of pions in the final state. However, we find that such effect is about a few percents and thus we neglect it in our analysis.