Investigation of Isospin Conservation Effects on Pre-Equilibrium Nuclear Reactions for Some Nuclei

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Abstract. The pre-equilibrium emission spectra have been studied and calculated using the exciton model. The calculations took different reactions (nucleon − nucleon, nucleon − cluster, cluster − nucleon) at low and high incident energies to investigate and study the influence of adding the isospin conservation effects into the nuclear level density on the energy spectra. The isospin conservation effect added to the state density showed no significance variation for mixed and conserved isospin calculations for all reactions at different incident energies. The results were obtained by depending on the PRECO-2006 code of C. Kalbach.

1. Introduction

One of the important quantities that must be considered in most nuclear reactions is isospin, due to its contribution in predicting the states of the composite nucleus [1, 2]; and, in terms of exciton model, keeps exciton number (n) degrees of freedom conserved [3]. The including of isospin dependence in nuclear reactions came from the fact that the nuclear force does not distinguish between protons and neutrons. Since the distinguishing between protons and neutrons is considered in the two-component exciton model. Isospin is introduced to conserve the particle − hole number when $\Delta n = 0$ transition occur; i.e. when a proton or a neutron convert to one another [3, 4] which specifies the number of excited states.

In nuclear reactions, isospin may be conserved or mixed (non-conserved) quantity. But, most of pre-equilibrium calculations assume isospin to be a good quantum number shows a good agreement with experimental results for light nuclei at low excitation energies [5]. In order to include the isospin effects into calculations, the good isospin correcting factors must be added to state density particle − hole configurations [4]. There are two changes will be made on the state density: one is to the threshold energy ($E_{\text{th}}$) by taking into account the symmetry energy ($E_{\text{sym}}$) resulted from the difference between neutrons and protons binding energies, and the other is a weighting factor $f_{\text{iso}}$ that depend on the number of particles and holes resulted after neutron and proton interchange and the new isospin state as consequences of that interchange.

Then the threshold energy becomes [6]:

$$E_{\text{th}}(p, p_n, T, T_z) = E_{\text{th}}(p, p_n) + E_{\text{sym}}(p, p_n, T, T_z)$$

(1)

where $p$ and $p_n$ are particles and neutron particles numbers, $T$ the total isospin quantum number of the states and $T_z = (N - Z)/2$ is its $z$ − component.

The correction function $f_{\text{iso}}(p, p_n, T, T_z)$ has different forms depending on isospin flip transitions that occur when the neutrons and protons are converting to each other. For nuclei with excess neutrons, the
conversion of neutrons into protons is taken place. In the ground state (i.e. when there is no conversion) 
\( T = |T_z| \) where \( T_z = T_0 \), and when the conversion occurs, the state would be higher isospin 
\( T = |T_z| + 1 \) and \( T = |T_z| + 2 \) where \( T_z = T_0 - 1 \).

Due to the existence of passive particle and hole near the Fermi energy, the isospin \( T \) is replaced by 
the effective value \( T_e \) to consider only flips that affect the emission process. This is because the fact that 
passive particles and holes have no sufficient excitation energy to be excited (be active ones) and thus 
they not considered as degrees of freedom. So, the effective excitation energy can be given as:

\[
E_i = E - E_{sym}(|T_z| + i, T_z)
\]

where \( i = 0,1,2, \ldots \) which accounts for isospin flips.

The symmetry energies are given by Meyers and Swiatecki mass formula [4]:

\[
E_{sym}(T,T_z) = \frac{112MeV}{A} - \frac{133MeV}{A^{4/3}} (T^2 - T_z^2)
\]

where \( A \) is the mass number.

According to these considerations, the correction function \( f_{iso}(p, p_\pi, T, T_z) \) would has three different 
forms differ in order of excitation energy and isospin state. These forms are derived in details in 
references [4, 6].

Then, the density of states in terms of isospin is [7]:

\[
\omega_{ESM}(p, p_\pi, E, T) = \frac{(g_{p0})^{\nu_p}(g_{n0})^{\nu_n}[E - A(p, p_\pi, E, T)]^{n-1}}{p_\pi! \nu_p! p_\nu! \nu_n!} f_{iso}(p, p_\pi, T, T_z)
\]

where \( E \) is the system excitation energy, \( g_{p0} \) is the primary single particle level density for proton, \( g_{n0} \) 
is the primary single particle level density for neutron, and \( A(p, p_\pi, E, T) \) is the two component Pauli 
correction function, which is given by:

\[
A(p, p_\pi, E, T) = A(p, p_\pi, E) + E_{sym}(T, T_z)
\]

2. Results and Discussion

Most of the previous pre-equilibrium calculations ignored isospin effects because it is found that the 
energy spectra for reactions with light particles in the entrance or exit channels are not affected by 
whether isospin is considered to be conserved or mixed. But Kalbach calculations [5] showed this is not 
valid for some reactions. Her calculations took low energies (18 MeV – 25 MeV) and high energies 
(60 MeV – 90 MeV) for small \( N-Z \) targets and neutron rich targets. The large effects of conserved 
isospin consideration were found for \( (p, xp) \) reactions at low energies but for other reactions calculations 
appeared that isospin consideration doesn’t make a difference if it mixed or conserved but at high 
energies it is favored to be mixed. The PRECO-2006 Code, used to calculate energy spectra, takes these 
considerations into account and operates with either mixed or conserved isospin at low energies but for 
high energies > 50 MeV it operates with mixing only. PRECO-2006 [6] is a two-component exciton 
model code for the calculation of double differential cross sections of light particle nuclear reactions. The 
code, written in FORTRAN-77, runs on a PC and calculates the emission of particles up to mass four, 
including separate subroutines for nucleon transfer processes, knockout and inelastic scattering involving 
complex particles, and collective state excitation (both discrete and giant resonance states).

At high incident energies, above 60 MeV, isospin is assumed to be mixed for most reactions due to the 
fact that the state densities and the energy width increase with excitation energies. The same argument is
valid for reactions involve targets with large mass numbers $A$ because the symmetry energy increases as the difference $N-Z$ increases. This resulting in a large single particle density of states.

The comparisons between the present work and measurements for the previously mentioned reactions are given in figures below, which are the differential cross section $\frac{d\sigma}{d\epsilon}$ as a function of the emission energy ($\epsilon$). To study the isospin effects in the pre-equilibrium reactions the calculations were performed with isospin fully mixed and fully conserved. The results were compared with the experimental measurements [9-13] to elucidate what assumption is closer to it (experimental). The calculations were made for $^{14}N$, $^{59}Co$, $^{90}Zr$, $^{143}Nd$ and $^{208}Pb$ targets with $(n,n), (n,p), (n,\alpha), (p,p), (p,n), (p,\alpha), (\alpha,n)$ and $(\alpha,p)$ reactions at low (14.7 MeV, 18 MeV, 18.2 MeV, 23.1 MeV, 24.02 MeV and 25 MeV) and high (62.9 MeV and 96 MeV) energies. At low energies the results indicate that the energy spectra insensitive to whether isospin is assumed to be conserved or mixed but at high energies the dominant assumption is always mixing isospin.

When isospin assumed to be conserved during the incident protons reactions, the composite nucleus will be in the ground-state isospin ($T = |T_x|$) or at higher state isospin ($T = |T_x| + 1$) produced from the coupling of the protons [5]. The calculated spectra for such reactions are the summation of these two states spectra.

For low excitation energies spectra, which illustrate in figures 1 – 4, it is obvious that the consideration of conserved isospin does not make any contribution in enhancing the calculated spectra for all reactions taken into account. However, the differences between mixed and conserved isospin spectra are very small even for reactions with two isospin states. In figures 5 and 6 only the mixed isospin was taken into accounts because of the high excitation energies that make no sense for considering conserved isospin.

3. Conclusion

We have presented calculations for the isospin dependence pre-equilibrium reactions. Our calculations showed the importance of secondary pre-equilibrium at low and high energies. the including of isospin conservation doesn’t make a remarkable difference to the calculated spectra. There were very small differences between spectra with mixed and conserved isospin even for reaction with incident protons which have a property of coupling and produce a higher isospin state when they enter the target nucleus. The similarity between conserved isospin spectra in ground state and the higher state refer to the small probability of emission at the second state since it made no difference when it was compared with mixed isospin spectra. Thus, the calculations for whether isospin must be mixed or conserved to the taken into account reactions have no sensitive effects when compared to experimental data.

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Figure 1: A comparison between the calculated spectra, with conserved and mixed isospin (blue line and red dashed line), and experimental spectrum (black circles) for the reactions: (a) $^{14}$N(n,$\alpha$)$^{11}$B at $E_n=14.7$ MeV [8] and (b) $^{59}$Co(,$\alpha$)$^{62}$Ni and $E_\alpha=18$ MeV [9].
Figure 2: As figure 1 but for reactions: (a) $^{90}\text{Zr}(p,n)^{90}\text{Nb}$ at $E_p = 18$ MeV[10] and (b) $^{143}\text{Nd}(n,\alpha)^{140}\text{Ce}$ at $E_n = 18.2$ MeV[11].

Figure 3: As Figure 1 but for the reactions: (a) $^{143}\text{Nd}(p,\alpha)^{140}\text{Pr}$ at $E_p = 23.1$ MeV and (b) $^{90}\text{Zr}(p,\alpha)^{87}\text{Y}$ at $E_p = 24.02$ MeV[12].
Figure 4: As figure 1 but for the reactions: (a) $^{59}\text{Co}(p,n)^{59}\text{Ni}$ and (b) $^{90}\text{Zr}(p,n)^{90}\text{Nb}$ at $E_p = 25\text{ MeV}$[10].
Figure 5: A comparison between the calculated spectrum, with mixed isospin (blue line), and experimental spectrum (black circles) for the reactions: (a) $^{59}\text{Co}(p,n)^{59}\text{Ni}$ at $E_p = 62.9 \text{ MeV}$ and (b) $^{208}\text{Pb}(p,p)X$ at $E_p = 62.9 \text{ MeV}$ [13].

Figure 6: As figure 5 but for the reaction $^{208}\text{Pb}(n,n)X$ at $E_n = 96 \text{ MeV}$ [14].