Adiabatic invariant behavior of dynamical moment of inertia of superdeformed bands in the nucleus $^{194}Tl$

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Abstract.

The behavior of dynamical moment of inertia $J^{(2)}$ for super deformed (SD) bands of $^{194}Tl$ in $A=190$ mass region has been studied. The behavior of dynamical moment of inertia $J^{(2)}$ is found to increase in steps with rotational frequency $\hbar\omega$. We make an analogy here by taking a lead from classical Hamilton-Jacobi theory and find that the behavior of $J^{(2)}$ is adiabatic invariant.
1. Introduction

An adiabatic invariant is a physical quantity that remains constant when the parameters of system are slowly varied. Frank S. Crawford [1] examined the simple one-dimensional systems subject to adiabatic perturbations. For example, a simple pendulum of mass \( m \) and length \( L \) may be oscillating with energy \( E \) and period \( \tau \). Thus, \( E\tau \) is said to be adiabatic invariant for the pendulum if \( L \) is gradually decreased by putting the string up. In that case, \( E \) increases and \( \tau \) decreases but product \( E\tau \) remains constant [2]. In the old quantum theory of Niels Bohr, Max Born, Paul Ehrenfest, Arnold Sommerfeld and others, before the discovery of the Schrödinger equation, Ehrenfest was the first one to show that adiabatic invariance played an important role in old quantum theory.

The presence of superdeformation was predicted by Strutinsky [4] and the first experimental evidence of superdeformation was given by Twin et al. [5]. A large number of superdeformed (SD) bands have been observed in the mass regions \( A = 60, 80, 130, 150, 190 \) [6, 7]. Also, Ideguchi et al. [8, 9] has observed the superdeformation in \( A=40 \) mass region. Recently, One of the most intriguing difference between the properties of superdeformed (SD) nuclei in the \( A=150 \) and \( A=190 \) region is the behavior of dynamical moment of inertia \( J^{(2)} \) as a function of rotational frequency \( \hbar \omega \). A vast majority of super deformed bands near \( A = 190 \) display a pronounced increase of \( J^{(2)} \) with \( \hbar \omega \). Somewhat steeper slope of \( J^{(2)} \) has been observed in an even-even SD nuclei as compared to odd-even SD nuclei. So, the study of odd-odd SD nuclei should be very significant. It has been shown that the occupation of specific high-N intruder orbital cannot account for this observed rise [10, 11, 12]. Instead it has been suggested that quasi-particle alignments and the resulting changes in pairing might play an effective role [13, 14, 15]. One can extract the dynamical moment of inertia \( J^{(2)} \) by using the experimental intraband E2 transition energies [6, 7] as follows

\[
J^{(2)}(I)/\hbar^2 = \frac{4}{E_\gamma(I + 2 \rightarrow I) - E_\gamma(I \rightarrow I - 2)}. \tag{1}
\]

The dynamical moment of inertia is used to study the dependence of nuclear structure on the rotational frequency \( \hbar \omega \).

2. Analogy

The classical action \( S \) for a periodic system with one degree of freedom \( x \) with corresponding canonical momentum \( p \) is given by

\[
S = \int p\,dx, \tag{2}
\]

where the integral is over one complete cycle of motion. Ehrenfest, by using classical Hamilton-Jacobi theory proved that \( S \) is an adiabatic invariant [3]. He then postulated that \( S \) is to be quantized i.e. have allowed values given by

\[
S = (n + n_0)\hbar, \tag{3}
\]
where \( h \) is Planck’s constant, \( n=0,1,2,3,... \), and \( n_0 \) is a constant adjusted to agree with the experiment so as to give the correct ground state. The idea was that if a physical quantity is going to make all or nothing quantum jumps, it should make no jump at all, if the system is perturbed gently and adiabatically, and therefore any quantized quantity should be an adiabatic invariant like \( S \). Equation (3) with \( n_0 = \frac{1}{2} \) correspond to Bohr-Sommerfeld-Wilson quantization rule of the old quantum theory.

As we know that action angle variables are useful parameters for those systems which are periodic, conservative and orthogonally decomposable. By taking a lead from equation (2), we can write
\[
J^{(1)} = \int p\,dq,
\]
where \( J^{(1)} \) is kinematic moment of inertia and cyclic integral is performed over a full periodic variation in \( q \). Also, the classical definition of dynamical moment of inertia is given by
\[
\frac{\partial J^{(1)}}{\partial E} = J^{(2)},
\]
\[
J^{(2)} = \int \frac{\partial p}{\partial E} \,dq + \int p \frac{\partial (dq)}{\partial E}.
\]

The Hamiltonian for a given periodic system can be expressed as
\[
p^2 + q^2 = H,
\]
which sets the Hamilton-Jacobi equation as
\[
p^2 + q^2 = E,
\]
\[
p = \sqrt{(E - q^2)}.
\]
On differentiating equation (9), we get
\[
\frac{\partial p}{\partial E} = \frac{1}{2} \frac{1}{\sqrt{(E - q^2)}},
\]
because \( \frac{\partial (dq)}{\partial E} = 0 \), (\( q \) having no dependence on \( E \)). By putting the value of \( \frac{\partial p}{\partial E} \) in equation (6), we get
\[
J^{(2)} = \int_{\sqrt{E}} \sqrt{E} \, dq = \frac{\sqrt{E}}{2} \sqrt{(E - q^2)},
\]
\[
J^{(2)} = \text{Sin}^{-1}(1).
\]
\[
J^{(2)} = \frac{\pi}{2} \text{= adiabatic invariant}.
\]
Quantization of the action gives, according to equation (3),
\[
J^{(2)} = \frac{\pi}{2} (n_1) h
\]
where \( n_1 = (n + n_0) \) = is a real number.
3. Results and Discussion

We have calculated the dynamical moment of inertia $J^{(2)}$ for SD-2, SD-3, SD-4 and SD-5 of $^{194}\text{Tl}$ in A=190 mass region by using equation (2) and have plotted against $\hbar \omega$ as shown in Fig. 1, Fig. 2, Fig. 3 and Fig. 4 respectively. The experimental data has been taken from Ref. [6, 7]. We find that the value of $J^{(2)}$ rise in steps which are pointed out by red lines. In Fig. 1, the value of $J^{(2)}$ rises approximately in three steps. The difference between the value of $J^{(2)}$ between the alternating two steps are 1.59, 3.34 and 3.57 respectively. If we fit these values of differences with analogy made from equation (13), then value of $n_1$ becomes 1.01 for first step, 2.12 for second step and 2.27 for third step in $^{194}\text{Tl}$(SD-2) which are real numbers.

Similarly, in Fig. 2 the value of $J^{(2)}$ rises approximately in two steps and the difference between the value of $J^{(2)}$ between the alternating two steps are 3.72 and 7.37 respectively. The value of $n_1$ for $^{194}\text{Tl}$(SD-3) becomes 2.37 for first step and 4.69 for second step which are also real numbers.

In Fig. 3 the value of $J^{(2)}$ rises approximately in two steps and the difference between the value of $J^{(2)}$ between the alternating two steps are 5.23 and 2.66 respectively. The corresponding value of $n_1$ for $^{194}\text{Tl}$(SD-4) becomes 3.33 for first step and 1.69 for second step respectively which are also real numbers.

Similarly in Fig. 4 the value of $J^{(2)}$ rises approximately in three steps and the difference between the value of $J^{(2)}$ between the alternating two steps are 2.26, 2.67 and 1.85 respectively. The value of $n_1$ for $^{194}\text{Tl}$(SD-3) becomes 1.44 for first step, 1.77 for second step and 1.18 for third step which are again real numbers. Hence, the value of $J^{(2)}$ for SD-2, SD-3, SD-4 and SD-5 of $^{194}\text{Tl}$ rises in steps and we find that the behavior...
Figure 2. The plot of dynamic moment of inertia $J^{(2)}$ versus $\hbar \omega$ for $^{194}Tl$ (SD-3). The steps like behavior is pointed out by red lines.

Figure 3. The plot of dynamic moment of inertia $J^{(2)}$ versus $\hbar \omega$ for $^{194}Tl$ (SD-4). The steps like behavior is pointed out by red lines.

of $J^{(2)}$ is adiabatic invariant.
4. Conclusions

In the present work, we have studied the behavior of dynamical moment of inertia $J^{(2)}$ for super deformed (SD) bands of $^{194}Tl$ in $A=190$ mass region and find that as the rotational frequency $\hbar \omega$ increases the value of $J^{(2)}$ increase in steps. By taking a lead from classical Hamilton-Jacobi theory we have made an analogy which represents that the behavior of $J^{(2)}$ remains adiabatic invariant.

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