Capacitated lot-sizing problem with production carry-over and set-up splitting: mathematical models

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This work proposes mathematical models (MMs) for the capacitated lot-sizing problem with production carry-over and set-up splitting, which can handle two scenarios, namely (1) situation/scenario where the set-up costs and holding costs are product dependent and time independent, and with no backorders or lost sales, and (2) situation where the set-up costs and holding costs are product dependent and time dependent, and with no backorders or lost sales. Previously, in an existing study the authors had developed a MM for the same problem and situation where the set-up costs and holding costs are product dependent and time independent, i.e. our Scenario 1. We compare our proposed models with the model in the existing study that appears to be incorrect.

Keywords: capacitated lot-sizing problem; set-up splitting; production carry-over

1. Introduction and problem definition

The capacitated lot-sizing problem (CLSP) is a lot-sizing model in which the production of multiple products are allowed within a time period on a single machine, with a condition that the entire demand for a product within that period should be met from the production in that period and/or the inventory carried from the previous periods, without any backorders or lost sales. Finding a minimum cost production plan that satisfies all the demand requirements without exceeding the capacity limits of a period is the main objective of the CLSP. Research has been carried out in the area of CLSP and is extended to include production carry-over across periods.

When the capacity of a machine has to be utilised efficiently, the idle time present in a period should also be utilised judiciously. Therefore, there are three ways by which a machine can be set up for producing a product. They are: (i) a machine is completely set up for product \( i \) anywhere in period \( t \), and the production starts in period \( t \) itself and the production may be continued to the period(s) thereafter (this aspect is referred to as production carryover in our paper); (ii) when there is enough capacity left at the end of a period, it can be utilised in making a complete set-up for a product, followed by its production in time period \( t + 1 \); and (iii) the set-up started in period \( t \) can be split between the periods \( t \) and \( t + 1 \), followed by its production in time period \( t + 1 \) (this aspect is referred to as set-up splitting in our paper). These are the three ways of setting up a machine for production. This CLSP with production carry-over and set-up splitting (with no backorders or lost sales) was addressed by Mohan et al. (2012).

2. Literature review

Haase (1998), Gopalakrishnan, Miller, and Schmidt (1995), Sox and Gao (1999), Gopalakrishnan (2000) and Suerie and Stadtler (2003) are some of those who addressed the CLSP with production carry-over by proposing mathematical models (MMs). Further, researchers also proposed several approaches to solve the CLSP with production carry-over by developing algorithms and heuristics. Some of them are due to Gopalakrishnan et al. (2001), Suerie and Stadtler (2003), Karimi, Fatemi Ghomi, and Wilson (2006), Nascimento and Toledo (2008), Caserta et al. (2009), Sahling et al. (2009), Caserta, Ramirez, and Voß (2010), Goren, Tunali, and Jans (2012), Wu et al. (2012) and Caserta and Voß (2013). All the aforementioned attempts basically assume that while the production of a product can be carried over across periods, they have assumed that the set-up cannot be performed across time periods; i.e. set-up, once started, needs to be completed within the same time period, and the spill-over to the next time period is not allowed. Mohan et al. (2012) addressed the CLSP with production carry-over and set-up splitting (with no backorders or lost sales). During the

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second review process of this paper, one of the reviewers brought to our attention a related recent paper (Belo-Filho, Toledo, and Almada-Lobo 2014) that addresses the set-up carry-over across consecutive multiple periods, especially present in process industries.

In this paper, we address the CLSP with production carry-over and set-up splitting by developing mathematical models (called MM and alternate mathematical model [AMM] in our paper) with the consideration of a possible production carry-over over successive periods (see Mohan et al. 2012). In real-life situations, the cost for set-up may vary from one time period to another time period – see the well-known Wagner–Whitin algorithm for dynamic lot-sizing problem (Wagner and Whitin 1958) for such a scenario. In view of these, we develop MMs for the CLSP with the consideration of a possible set-up splitting and a possible production carry-over over successive periods which can handle two scenarios, namely (1) situation/scenario where the set-up costs and holding costs of all the products are product dependent and time independent, and with no backorders or lost sales (Section 3.1), and (2) situation where the set-up costs and the holding costs of all the products are product dependent and time dependent, and with no backorders or lost sales (Section 3.2). We compare Scenario 1 of our MM with the previously developed model by Mohan et al. and observe that the model formulated by them appears to be incorrect by yielding infeasible solutions.

3. MM for the CLSP with production carry-over and set-up splitting

The basic assumptions are the same as those presented in Mohan et al. (2012).

- A single machine is considered in the problem.
- Multiple products can be produced on the single machine and each product is made up of a single-level.
- Time unit is discrete and the time horizon considered is finite.
- Each product is associated with a set-up cost when set up on the machine, and it consumes time for set-up.
- Backorders and lost sales are not permitted.
- The capacity of the machine during a given period is assumed in time units and it may vary from period to period.
- The capacity of the machine per period is consumed by the set-up time and the production time of the products. Idle time on the machine can also be present.
- If excess capacity is left over on the machine in a period after production, it may be used to set up the product to be produced in the next period. If the set-up is not over, this set-up may be continued to the next period. In this work, this aspect is called set-up splitting.
- The excess quantity produced of a product can be stored and this incurs a holding cost, except in the last period where all the units in the inventory have to be consumed.
- Production of a product may extend over any number of periods subject to demand and capacity constraints. In this work, this aspect is called production carry-over.

3.1 Scenario 1: MM when set-up costs and holding costs are product dependent and time independent

Parameters

- $N$ number of products
- $T$ number of time periods
- $t$ time period
- $i$ product
- $SC_i$ set-up cost for product $i$
- $h_i$ holding cost per period per unit of product $i$
- $ST_i$ set-up time for product $i$
- $a_i$ number of time units required for producing one unit of product $i$
- $C_t$ capacity of the machine in period $t$
- $d_{i,t}$ demand for product $i$ in period $t$
- $M$ a large value
- $\varepsilon$ a small positive real number

Decision variables

- $\delta_{i,t}$ an indicator (binary) variable that takes value 1 if a complete set-up is done for product $i$ in period $t$ with the production starting in period $t$; 0 otherwise
\( \Delta_{t,t'} \) an indicator (binary) variable that takes value 1: it corresponds to the production carry-over from period \( t \) to period \( t' \) (\( t' = t + 1, t + 2, \ldots, T \)), due to the set-up of product \( i \) completely done in period \( t \), and with no intermittent set-up of any product before the production completion of product \( i \); 0 otherwise

\( \delta_{i,t}^{**} \) an indicator (binary) variable that takes value 1 if a set-up for product \( i \) is completed by the end of period \( t \) followed by its production starting in period \( t + 1 \); */ this is called end-of-period set-up in our paper */ 0 otherwise

\( \Delta_{i,i'}^{**} \) an indicator (binary) variable that takes value 1: it corresponds to the production carry-over from period \( t' \) to period \( t' + 1 \) (\( t' = t + 1, t + 2, \ldots, T \)), due to the set-up of product \( i \) completed by the end of period \( t \), followed by its production in period \( t' \), with no intermittent set-up of any product before the production completion of product \( i \); 0 otherwise

\( \delta_{i,t}^{***} \) an indicator (binary) variable that takes value 1 if the set-up for a product \( i \) is commenced in period \( t \) and is completed in period \( t + 1 \), followed by its production starting in period \( t + 1 \); */ this is called set-up splitting in this paper */ 0 otherwise

\( \Delta_{i,i'}^{***} \) an indicator (binary) variable that takes value 1 when it corresponds to the production carry-over from period \( t' \) to period \( t' + 1 \) (\( t' = t + 1, t + 2, \ldots, T \)), due to the set-up of product \( i \) starting in period \( t \) and completed in period \( t + 1 \), with no intermittent set-up of any product before the production completion of product \( i \); 0 otherwise

\( I_{i,t} \) inventory of product \( i \) at the end of period \( t \).

\( s_{i,t}^{**} \) set-up time for product \( i \) in period \( t \) that takes the value of \( ST_i \) when \( \delta_{i,t}^{**} = 1 \); 0 otherwise

\( s_{i,t}^{***} \) set-up time for product \( i \) in period \( t \) that takes the value of \( ST_i \) when \( \delta_{i,t}^{***} = 1 \); 0 otherwise

\( s_{i,t+1}^{***} \) set-up time for product \( i \) in period \( t + 1 \) when \( \delta_{i,t}^{***} = 1 \); 0 otherwise

Note: \( s_{i,t}^{***} + s_{i,t+1}^{***} = ST_i \).

\( X_{i,t'} \) production quantity for product \( i \) in period \( t' \) (due to its set-up completely done in period \( t \)), with \( 1 \leq t \leq T \) and \( t \leq t' \leq T \)

\( X_{i,t,t'}^{**} \) production quantity for product \( i \) in period \( t' \) (due to its set-up started in period \( t \) and completed by the end of that period), with \( 1 \leq t \leq T - 1 \) and \( t + 1 \leq t' \leq T \)

\( X_{i,t,t'}^{***} \) production quantity for product \( i \) in period \( t' \) (due to its set-up starting in period \( t \) and ending in period \( t + 1 \)), with \( 1 \leq t \leq T - 1 \) and \( t + 1 \leq t' \leq T \)

**Mathematical model**

**Objective function:**

\[
\text{Min } Z = \sum_{i=1}^{N} \sum_{t=1}^{T} SC_i \delta_{i,t}^{**} + \sum_{i=1}^{N} \sum_{t=1}^{T-1} SC_i \delta_{i,t}^{***} + \sum_{i=1}^{N} \sum_{t=1}^{T-1} SC_i \delta_{i,t}^{**} + \sum_{i=1}^{N} \sum_{t=1}^{T} h_i I_{i,t}, \tag{1}
\]

subject to the following:

/* Constraints (2) and (3) represent the conditions for the set-up */

\[
\sum_{i=1}^{N} \left( \delta_{i,t}^{**} + \delta_{i,t}^{***} \right) \leq 1, \quad t = 1, \ldots, T - 1, \tag{2}
\]

\[
\left( \delta_{i,t}^{**} + \delta_{i,t}^{***} + \delta_{i,t}^{**} \right) \leq 1 \quad \forall i \text{ and } t = 1, \ldots, T - 1. \tag{3}
\]

/* Constraints (4)–(11) capture a possible complete set-up within period \( t \), followed by the commencement of its production in the same period */
\[ \Delta_{i,t}^* = \delta_{i,t}^* \quad \forall i \text{ and } \forall t, \]
(4)

\[ \Delta_{i,t'}^* \geq \Delta_{i,t'+1}^* \quad \forall i, t = 1, \ldots, T - 1 \text{ and } t' = t, t + 1, \ldots, T - 1, \]
(5)

\[ \sum_{\ell=1}^{N} (d_{i,t-1}^* + d_{i,t' - 1}^* + d_{i,t'+1}^*) \leq M \left( 1 - \Delta_{i,t'}^* \right) \quad \forall i, t = 1, \ldots, T - 2 \text{ and } t' = t + 2, t + 3, \ldots, T, \]
(6)

\[ X_{i,t,t'}^* \leq M \Delta_{i,t,t'}^* \quad \forall i, t, t' = t, t + 1, \ldots, T, \]
(7)

\[ X_{i,t,t'}^* \geq e - M(1 - \Delta_{i,t,t'}^*) \quad \forall i, t, t' = t, t + 1, \ldots, T, \]
(8)

\[ a_t X_{i,t,t'}^* \leq C_t \Delta_{i,t,t'}^* \quad \forall i, t, t' = t, t + 1, \ldots, T, \]
(9)

\[ s_{i,t}^* \leq s_{i,t + 1} \quad \forall i \text{ and } \forall t, \]
(10)

\[ s_{i,t}^* \geq s_{i,t+1}^* - M(1 - \delta_{i,t}) \quad \forall i \text{ and } \forall t. \]
(11)

/* Constraints (12)–(19) correspond to a possible end-of-period set-up in period \( t \), followed by the commencement of its production in period \( t + 1 \) */

\[ \Delta_{i,t}^{**} = \delta_{i,t}^{**} \quad \forall i, t = 1, \ldots, T - 1 \text{ and } t' = t + 1, \]
(12)

\[ \Delta_{i,t,t'}^{**} \geq \Delta_{i,t,t'+1}^{**} \quad \forall i, t = 1, \ldots, T - 2 \text{ and } t' = t + 1, t + 2, \ldots, T - 1, \]
(13)

\[ \sum_{\ell=1}^{N} (d_{i,t-1}^{**} + d_{i,t' - 1}^{**} + d_{i,t'+1}^{**}) \leq M \left( 1 - \Delta_{i,t,t'}^{**} \right) \quad \forall i, t = 1, \ldots, T - 2 \text{ and } t' = t + 2, t + 3, \ldots, T, \]
(14)

\[ X_{i,t,t'}^{**} \leq M \Delta_{i,t,t'}^{**} \quad \forall i, t = 1, \ldots, T - 1 \text{ and } t' = t + 1, t + 2, \ldots, T, \]
(15)

\[ X_{i,t,t'}^{**} \geq e - M(1 - \Delta_{i,t,t'}^{**}) \quad \forall i, t = 1, \ldots, T - 1 \text{ and } t' = t + 1, t + 2, \ldots, T, \]
(16)

\[ a_t X_{i,t,t'}^{**} \leq C_t \Delta_{i,t,t'}^{**} \quad \forall i, t = 1, \ldots, T - 1 \text{ and } t' = t + 1, t + 2, \ldots, T, \]
(17)

\[ s_{i,t}^{**} \leq s_{i,t + 1}^{**} \quad \forall i \text{ and } t = 1, \ldots, T - 1, \]
(18)

\[ s_{i,t}^{**} \geq s_{i,t+1}^{**} - M(1 - \delta_{i,t}) \quad \forall i \text{ and } t = 1, \ldots, T - 1. \]
(19)

/* Constraints (20)–(29) represent a possible set-up being split between periods \( t \) and \( t + 1 \), followed by the commencement of its production in period \( t + 1 \) */

\[ \Delta_{i,t}^{***} = \delta_{i,t}^{***} \quad \forall i, t = 1, \ldots, T - 1 \text{ and } t' = t + 1, \]
(20)

\[ \Delta_{i,t,t'}^{***} \geq \Delta_{i,t,t'+1}^{***} \quad \forall i, t = 1, \ldots, T - 2 \text{ and } t' = t + 1, t + 2, \ldots, T - 1, \]
(21)

\[ \sum_{\ell=1}^{N} (d_{i,t-1}^{***} + d_{i,t' - 1}^{***} + d_{i,t'+1}^{***}) \leq M \left( 1 - \Delta_{i,t,t'}^{***} \right) \quad \forall i, t = 1, \ldots, T - 2 \text{ and } t' = t + 2, t + 3, \ldots, T, \]
(22)

\[ X_{i,t,t'}^{***} \leq M \Delta_{i,t,t'}^{***} \quad \forall i, t = 1, \ldots, T - 1 \text{ and } t' = t + 1, t + 2, \ldots, T, \]
(23)

\[ X_{i,t,t'}^{***} \geq e - M(1 - \Delta_{i,t,t'}^{***}) \quad \forall i, t = 1, \ldots, T - 1 \text{ and } t' = t + 1, t + 2, \ldots, T, \]
(24)

\[ a_t X_{i,t,t'}^{***} \leq C_t \Delta_{i,t,t'}^{***} \quad \forall i, t = 1, \ldots, T - 1 \text{ and } t' = t + 1, t + 2, \ldots, T, \]
(25)
Constraints (37) and (38) represent the inventory balance constraints:

\[ s^a_{i,t} + s^d_{i,t+1} \geq M(1 - \delta^a_{i,t}) \quad \forall i \text{ and } t = 1, \ldots, T - 1, \]

\[ s^a_{i,t+1} + s^d_{i,t} \geq M(1 - \delta^d_{i,t}) \quad \forall i \text{ and } t = 1, \ldots, T - 1, \]

\[ s^a_{i,t} + s^d_{i,t+1} \leq ST\delta^a_{i,t} \quad \forall i \text{ and } t = 1, \ldots, T - 1, \]

\[ s^a_{i,t} + s^d_{i,t} \geq ST\delta^d_{i,t} - M(1 - \delta^d_{i,t}) \quad \forall i \text{ and } t = 1, \ldots, T - 1. \]

/* Constraint (30) represents the condition for a production carry-over of only one product*/

\[ \sum_{i=1}^{N} \sum_{t=1}^{T-1} (\Delta_{i,t}^a + \Delta_{i,t}^d + \Delta_{i,t}^{a*} + \Delta_{i,t}^{d*}) \leq 1, \quad t = 2, \ldots, T. \]  

/* Constraints (31)–(33) represent capacity constraints */

\[ \sum_{i=1}^{N} (a_i X_{i,1}^* + s^a_{i,1} + s^d_{i,1} + s^{a*}_{i,1}) \leq C_1, \]

\[ \sum_{i=1}^{N} (s^a_{i,t} + s^d_{i,t} + s^{a*}_{i,t} + s^{d*}_{i,t}) + \sum_{i=1}^{N} \sum_{t=1}^{T} a_i X_{i,t'}^* + (a_i X_{i,t'}^* + a_i X_{i,t'}^{a*}) \leq C_t, \quad t = 2, \ldots, T - 1, \]

\[ \sum_{i=1}^{N} (s^a_{i,T} + s^{a*}_{i,T}) + \sum_{i=1}^{N} a_i X_{i,T}^* + (a_i X_{i,T}^{a*} + a_i X_{i,T}^{d*}) \leq C_T. \]

/* Constraints (34)–(36) represent demand constraints */

\[ X_{i,1,1}^* \geq d_{i,1} \quad \forall i, \]

\[ \sum_{t'=1}^{t} \sum_{t''=2}^{t} X_{i,t''}^{e*} + \sum_{t'=2}^{t} \sum_{t''=1}^{t-1} (X_{i,t''-1}^{e*} + X_{i,t''}^{a*}) \geq \sum_{t'=1}^{t} d_{i,t'} \quad \forall i \text{ and } t = 2, \ldots, T - 1. \]

\[ \sum_{t'=1}^{T} \sum_{t''=2}^{T-1} (X_{i,t''}^{e*} + X_{i,t''}^{e*}) = \sum_{t'=1}^{T} d_{i,t'} \quad \forall i. \]

/* Constraints (37) and (38) represent the inventory balance constraints*/

\[ X_{i,1,1}^* - d_{i,1} = I_{i,1} \quad \forall i, \]

\[ \sum_{t'=1}^{t} \sum_{t''=2}^{t} X_{i,t''}^{e*} + \sum_{t'=2}^{t} \sum_{t''=1}^{t-1} (X_{i,t''-1}^{e*} + X_{i,t''}^{a*}) - \sum_{t'=1}^{t} d_{i,t'} = I_{i,t} \quad \forall i \text{ and } t = 2, \ldots, T. \]

\[ \delta^a_{i,t}, \delta^d_{i,t}, \Delta^a_{i,t'}, \Delta^d_{i,t'}, \Delta^a_{i,t}, \Delta^d_{i,t} \text{ and } \Delta^{a*}_{i,t}, \Delta^{d*}_{i,t} \text{ are binary variables, and all other variables are } \geq 0. \]

Note the following:

\[ \delta^a_{i,t} = \delta^d_{i,t} = 0 \quad \forall i; \quad \Delta^a_{i,t'}, \Delta^d_{i,t'} = 0 \quad \forall i, \forall t; \quad X_{i,t'}^{*} = X_{i,t'}^{**} = 0 \quad \forall i, \forall t; \quad I_{i,0} = I_{i,T} = 0 \quad \forall i. \]

The objective function shown in Equation (1) is the minimisation of the set-up cost and holding cost of all products across all time periods. Constraint (2) indicates that there can be either an end-of-period set-up or a set-up split for at most one product in every period \(t\). Constraint (3) indicates that a product can be produced using either a complete set-up or an end-of-period set-up or a split set-up given in a period, in order to avoid setting up the machine more than once for the same product in the given period. Constraints (4), (12) and (20) indicate that when there is a set-up of product \(i\) in period \(t\), it means that a production is carried out in period \(t'\) corresponding to the set-up in \(t\). Constraints (5), (13) and (21) ensure that a production carry-over in period \(t'+1\) is not possible without a corresponding production carry-over in period \(t'\). Constraints (6), (14) and (22) indicate that a production can be carried over to period \(t'\) due to the set-up in period \(t\), only when there is no set-up of any product in between the set-up and the corresponding produc-
tion of product $i$. Constraints (7), (8), (15), (16), (23) and (24) are the production constraints which indicate that variables indicating the production quantity, i.e. $X_{i,j,f}, \Delta_{i,j,f}^*, \Delta_{i,j,f}^{***}$ exist only when the variables $\Delta_{i,j,f}^*, \Delta_{i,j,f}^{**}, \Delta_{i,j,f}^{***}$ exist. Constraints (10), (11), (18), (19) and (26)–(29) ensure that when product $i$ is set up in period $t$ (i.e. when $\delta_{i,t}, \delta_{i,t}^{**}$, and $\delta_{i,t}^{***}$ exist), the total set-up time $ST_i$ required by product $i$ for set-up is consumed. Constraint (30) ensures that only one production carry-over can occur between adjacent periods for at most one product. Constraints (9), (17), (25) and (31)–(33) denote the capacity constraints, with the consideration of the corresponding possible set-up time and production time of all products. Constraints (34)–(36) indicate the demand constraints with no backorders or lost sales. Constraints (37) and (38) represent the inventory balance constraints. It is to be noted that the parameter $\varepsilon$ is set to a small positive real value. Also, the set-up time of any product is less than or equal to the capacity of the machine in every period (i.e. $\max_i(ST_i) \leq C_i, \forall t$).

Looking at the literature in Section 2, we find that the papers dealing with CLSP with production carry-over across periods, deal mainly with three decision variables in their mathematical modelling. They are binary variable indicating whether or not a set-up occurs for a particular product in a period; variable indicating the production quantity of a product in a period; variable indicating the quantity of inventory carried between adjacent periods for a product; and binary variable indicating whether a production of an item is continued from period $t$ to $t$. Mohan et al. (2012) were the first to develop a model for the CLSP with production carry-over across periods, with the consideration of a possible set-up split across two consecutive time periods. In Mohan et al.'s model, apart from the decision variables stated above, the model includes a binary variable to indicate whether a set-up is split between two periods. Our modelling approach is different from the modelling approach by Mohan et al., especially in terms of addressing the time varying set-up cost of a product across time periods. To elaborate, in our MM, we have used three binary variables to indicate the set-up of a product in a period which are: $\delta_{i,t}, \delta_{i,t}^{**}$, and $\delta_{i,t}^{***}$. We use three of these variables to bring in the consideration of time-dependent set-up costs (see the supplemental online material of this paper, for more details with respect to the time-dependent cost structure). Corresponding to the three binary variables indicating set-up, we introduce binary variables $\Delta_{i,j,f}^*, \Delta_{i,j,f}^{**}, \Delta_{i,j,f}^{***}$ in order to track the starting and ending periods of production of the product. It is therefore evident that our modelling approach is different from that of Mohan et al. – our model is a step forward in comparison to the model of Mohan et al. by addressing the time-dependent set-up cost, as well as tracking the starting and ending periods of production.

The above MM represents the model of the CLSP with production carry-over and set-up splitting, considered by Mohan et al. Another mathematical model (called AMM) for the same problem is presented in the supplemental online material of this work. Both models give the same value of $Z$ for every possible instance (see Tables 1 and 2; a discussion on the tables follows in the next section); however they differ in respect of their approaches to modelling.

| Table 1. Sample problem instance. |
|----------------------------------|
| **Product** | **Set-up time ($ST_i$):** (time units) | **Number of time units required per unit of production of product $i$ ($a_i$):** (time units/production of one unit of product $i$) | **Set-up cost ($SC_i$):** (mu/set-up) | **Holding cost ($h_i$):** (mu/period/unit product carried over) |
| 1 | 10 | 1 | 50 | 2 |
| 2 | 10 | 1 | 10 | 3 |
| 3 | 20 | 1 | 30 | 1 |
| 4 | 20 | 1 | 20 | 1 |

| Demand ($d_{i,t}$) |
|-------------------|
| **Period ($t$)** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| **Product ($i$)** | 1 | 0 | 0 | 0 | 70 | 0 | 0 | 0 | 85 | 0 | 0 | 0 |
| 2 | 0 | 40 | 40 | 100 | 0 | 10 | 20 | 95 | 0 | 40 | 40 | 100 |
| 3 | 0 | 50 | 30 | 0 | 0 | 50 | 30 | 0 | 0 | 40 | 30 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 10 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Capacity ($C_i$): time units** | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
Table 2. Solution generated by our model with the consideration of Scenario 1 (corresponding terms in our work are used here).

| $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_5$ | $t_6$ | $t_7$ | $t_8$ | $t_9$ | $t_{10}$ | $t_{11}$ | $t_{12}$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------|----------|
| $\delta_1^* = 1$ | $\delta_2^* = 1$ | $\delta_{2,3}^* = 1$ | $\Delta_{2,3,4}^* = 1$ | $\delta_{15}^* = 1$ | $\delta_{26}^* = 1$ | $\delta_{78}^* = 1$ | $\delta_{89}^* = 1$ | $\delta_{10}^* = 1$ | $\delta_{11}^* = 1$ | $\Delta_{2,11,12}^* = 1$ |
| $\delta_{11}^* = 1$ | $\Delta_{2,12}^* = 1$ | $\delta_{2,3}^* = 1$ | $X_{2,3,4}^* = 100$ | $\delta_{15}^* = 1$ | $\delta_{26}^* = 1$ | $\delta_{78}^* = 1$ | $\delta_{89}^* = 1$ | $\delta_{10}^* = 1$ | $\delta_{11}^* = 1$ | $X_{2,11,12}^* = 100$ |
| $\Delta_{1,1,1} = 1$ | $\Delta_{2,2,2} = 1$ | $\Delta_{2,3,4} = 1$ | $\Delta_{2,5} = 1$ | $\Delta_{2,6,6} = 1$ | $s_i = 10$ | $s_j = 10$ | $s_k = 10$ | $s_l = 10$ | $s_m = 20$ | $s_n = 20$ |
| $s_i = 20$ | $s_j = 30$ | $s_k = 40$ | $s_l = 50$ | $s_m = 70$ | $s_n = 70$ | $s_o = 95$ | $s_p = 10$ | $s_q = 20$ | $s_r = 20$ | $s_s = 20$ |
| $X_{1,1,2} = 70$ | $X_{2,2,2} = 50$ | $X_{2,3,3} = 20$ | $X_{2,5,5} = 70$ | $X_{2,7,7} = 20$ | $X_{2,9,9} = 85$ | $X_{2,10,10} = 40$ | $X_{2,11,11} = 40$ | $X_{3,10,10} = 40$ | $X_{3,11,11} = 40$ | $X_{3,12,12} = 40$ |
| $X_{2,3,3} = 30$ | $X_{2,3,3} = 30$ | $X_{2,3,3} = 30$ | $X_{2,3,3} = 30$ | $X_{2,3,3} = 30$ | $X_{2,3,3} = 30$ | $X_{2,3,3} = 30$ | $X_{2,3,3} = 30$ | $X_{2,3,3} = 30$ | $X_{2,3,3} = 30$ | $X_{2,3,3} = 30$ |

$Z = 350$ mu
3.2 Scenario 2: MM when set-up costs and holding costs are product dependent and time dependent

The MM and the AMM can also be extended to a situation where the set-up cost (cost of resource used for the set-up) and holding cost of product $i$ may vary from period to period. This set-up cost for product $i$ may vary from period to period in view of different costs associated with the resources used for set-up over different time periods.

We present two variants of the proposed model (MM) for the situation where the set-up costs and holding costs are product dependent and time dependent, and with no backorders or lost sales. In this situation, when there is a splitting of set-up between periods $t$ and $t+1$ for product $i$, there may arise two possibilities (Variants (1) and (2), respectively) in a manufacturing system: (1) the manufacturer pays for the resource before utilising that resource for set-up (the cost for the set-up is incurred in period $t$ when the set-up is initiated), or (2) the manufacturer pays for the resource after utilising that resource for set-up (the cost for the set-up is incurred in period $t+1$ when the set-up ends).

3.2.1 Scenario 2 – Variant (1): MM when set-up costs and holding costs are product dependent and time dependent, and the set-up cost is calculated with respect to period $t$ when the set-up is initiated

In this model, the set-up costs and holding costs are assumed to be both product and time dependent apart from the other assumptions. When there is a set-up, the cost for the set-up is calculated with respect to the period when the set-up is initiated. The objective function for this model is shown in Equation (39).

Objective function:

\[ \text{Min } Z = \sum_{i=1}^{N} \sum_{t=1}^{T} SC_{it} \delta_{it} + \sum_{i=1}^{N} \sum_{t=1}^{T-1} SC_{it} \delta_{it} + \sum_{i=1}^{N} \sum_{t=1}^{T-1} SC_{it} \delta_{it} + \sum_{i=1}^{N} \sum_{t=1}^{T} h_{it} f_{it}, \]

subject to all constraints (i.e. Constraints (2)–(38)).

In the above, $SC_{it}$ indicates the cost for the set-up of product $i$ in period $t$, and likewise for $h_{it}$. For example, Table 3 shows different set-up costs for different products across different time periods. Here, if we consider set-up cost of product 3 in time period 2, we have $SC_{3,2} = 5$ monetary units (mu); and in time period 3, we have $SC_{3,3} = 10$ mu.

Table 3. Data pertaining to the problem instance with product- and time-dependent set-up costs and holding costs.

| Product | Set-up time ($ST_i$) (time units) | Number of time units required per unit of production of product $i$ ($a_i$) (time units/production of one unit of product $i$) |
|---------|----------------------------------|------------------------------------------------------------------------------------------------------------------|
| 1       | 10                               | 1                                                                  |
| 2       | 10                               | 1                                                                  |
| 3       | 20                               | 1                                                                  |
| 4       | 20                               | 1                                                                  |

Demand ($d_{it}$), set-up cost ($SC_{it}$) (mu/set-up) and holding cost ($h_{it}$) (mu/period/unit product carried over) data

| Product ($i$) | Period ($t$) |
|---------------|--------------|
|               | 1  2  3  4  5  6  7  8  9  10  11  12 |
| 1             | 70 0 0 0 0 70 0 0 0 0 85 0 0 0 |
|               | 10 10 10 10 5 10 10 10 20 10 10 10 |
|               | 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 |
| 2             | 0 40 40 40 100 0 10 20 95 0 40 40 100 |
|               | 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 |
|               | 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 |
| 3             | 0 50 30 0 0 50 30 0 0 40 30 0 |
|               | 10 5 10 10 10 5 30 30 30 30 30 30 |
|               | 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 |
| 4             | 0 0 0 0 0 0 10 0 0 0 0 0 0 0 0 0 0 0 |
|               | 40 40 40 40 40 40 40 40 40 40 40 40 40 40 40 40 40 |
|               | 1 1 1 1 1 1 1 1 1 1 1 1 |

Capacity ($C_i$): time units

|                | 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 |

Notes: \( \{a, b, c\} \) in a cell: in the above, $a$ refers to demand ($d_{it}$) data, $b$ refers to data pertaining to set-up cost ($SC_{it}$) (mu/set-up) and $c$ refers to data pertaining to holding cost ($h_{it}$) (mu/period/unit product carried over).
3.2.2 Scenario 2 – Variant (2): MM when set-up costs and holding costs are product dependent and time dependent, and the set-up cost is calculated with respect to the period when the set-up is completed

In this model, the set-up costs and holding costs are assumed to be product and time dependent, along with the other assumptions in the main work. When there is a set-up, the cost for the set-up is incurred with respect to the period when the set-up is completed. The objective function for this model is shown in Equation (40).

Objective function:

\[
\text{Min } Z = \sum_{i=1}^{N} \sum_{t=1}^{T} SC_i \delta_{ij} + \sum_{i=1}^{N} \sum_{t=1}^{T-1} SC_i \delta_{ij}^* + \sum_{i=1}^{N} \sum_{t=1}^{T-1} SC_{i,t+1} \delta_{ij}^* + \sum_{i=1}^{N} \sum_{t=1}^{T} h_i I_{i,t},
\]

subject to all constraints presented before.

4. Numerical illustrations and discussion

The illustration of the MMs and the results are presented in this section. The example considered shows the demand data \((d_{i,t})\) for four products across twelve time periods, along with their time-independent set-up costs \((SC_i)\) and holding costs \((h_i)\), number of time units required for producing one unit of product \(i (a_i)\) and set-up time \((ST_i)\) in Table 1. First, we illustrate the model of Mohan et al. and compare their results with those of Scenario 1 of our MM, i.e. the set-up costs and holding costs are product dependent and time independent, and with no backorders or lost sales (Section 4.1). In Section 4.2, we illustrate Scenario 2 of our MM, i.e. when the set-up costs and holding costs are product dependent and time dependent, and with no backorders or lost sales. Section 4.3 explains observations on our MM when modifications with respect to the set-up and production of a given product are considered in our model, and Section 4.4 presents further remarks on our model.

4.1 Observation on Mohan et al.'s model and comparison with Scenario 1 of our MM: the set-up costs and holding costs are product dependent and time independent, and with no backorders or lost sales

Our proposed model and Mohan et al.'s model are illustrated using one problem instance. First, we illustrate Mohan et al.'s model and compare the results obtained with Scenario 1 of our model (MM). When the model by Mohan et al. is executed using a set of initial and terminal (boundary) conditions, the solution obtained is infeasible and misleading as seen from the Gantt chart shown in Figure 1. Their model yields an objective function value equal to 320 mu.

Basically, their model does not give proper time tabling (in relation to the decision variables) and also results in a wrong computation of \(Z\). The solution values for the corresponding Gantt chart are given in the supplemental online

![Figure 1. Gantt chart](image-url)

Figure 1. Gantt chart (for the problem instance given in Table 1, and the solution given in Table 4 provided in the supplemental online material of this work) generated by the model of Mohan et al. (2012) (corresponding terms in that paper are used here). The value of \(Z = 320\) mu (see periods 3, 7 and 11).

Notes: Time period is denoted along the X-axis (see Figures 1–1 for the respective GANTT charts). The entries in the chart denote the product. The shaded region denotes the set-up of a product. The product which is set up is indicated inside the shaded region with the set-up time denoted in brackets. The unshaded region denotes the production of the product set-up, with the production time denoted in brackets. Idle time of a machine is denoted as ‘Idle’ with the idle time indicated in brackets. Values of some variables are shown in the figure for the sake of understanding.
material of this work. Here we observe that a product is produced in a period without a corresponding set-up in the same period, with the same product having \( v_{i,t} \) variable set to 1 in an earlier period, thereby resulting in an infeasible solution. This observation can be seen in Figure 1 which are presented as three cases below:

- **Case (i):** There appears a production carryover of product 3 from time period 2 to time period 3 due to an end-of-period set-up of product 3 in time period 1 (see: \( v_{3,2} = 1 \)), with product 2 being set up in between, which clearly indicates that product 3 is shown to produce without any corresponding set-up.
- **Case (ii):** a production carry-over of product 3 from time period 6 to time period 7 (due to an end-of-period set-up of product 3 in time period 5 (see: \( v_{3,6} = 1 \))) takes place with two other products (2 and 4) being set up in between.
- **Case (iii):** a production of product 3 is carried out in period 11 (due to a set-up split between periods 9 and 10 (see: \( v_{3,10} = 1 \))), with another product 2 set up in between the production of product 3.

Figure 2 shows the Gantt chart yielded by our model with the solution mentioned in Table 2, for the same example. Though our model has yielded a higher value of objective function, i.e. 350 mu, the solution is feasible and optimal. It is evident that while our model produces a feasible and optimal schedule, the model by Mohan et al., does not yield a feasible schedule (see Cases (i)–(iii)).

All the problem instances have been executed on a Windows 7 workstation using CPLEX v9.0 which runs on Pentium 3.10 GHz with 2.00 GB RAM. While using CPLEX, the values of the mixed integer optimality gap tolerance and integrality tolerance were set to zero in order to obtain accuracy in results and maintain the integrality with respect to binary variables.

The CPU time required for executing our model for some product and time period combinations is shown below. The computational time indicated here was obtained by executing our model on a Windows 7 workstation using CPLEX v12.4.0 which runs on Pentium 3.10 GHz with 4.00 GB RAM. The value of \( M \) was set to 500 in the model. It is to be noted that we can use parameters such as capacity and demand to tighten the value of \( M \) in various constraints.

| Number of periods | Computational time (s) |
|-------------------|------------------------|
|                   | Number of products     |
|                   | 5     | 10     | 15    |
| 5                 | 0.03  | 0.05   | 0.02  |
| 10                | 0.22  | 0.81   | 1.05  |
| 15                | 2.93  | 7.39   | 11.47 |
4.2 Observation on our MM for Scenario 2: the set-up costs and holding costs are product dependent and time dependent, and with no backorders or lost sales

For the two variants discussed in Sections 3.2.1 and 3.2.2, a single numerical example is used. The numerical example shows the demand data \((d_{i,t})\) for four products across twelve time periods along with the time- and product-dependent set-up costs \((h_{0i})\) and holding costs \((SC_{0i})\) written one below the other in Table 3. The values of the set-up time \((ST_i)\) and number of time units required per unit of production of product \(i\) \((a_i)\) for all the products are also given in Table 3.

For Scenario 2 – Variant (1) discussed in Section 3.2.1 of this work, (i.e. the set-up cost is calculated with respect to period \(t\) when the set-up is initiated) the objective function’s value is 240 mu. The resulting Gantt chart is shown in Figure 3 with its corresponding solution values provided in the supplemental online material of this work. For Scenario 2 – Variant (2) discussed in Section 3.2.2 of this work, (i.e. the set-up cost is calculated with respect to the period when the set-up is completed) the objective function’s value is 250 mu. The resulting Gantt chart shown in Figure 4 with its corresponding solution values provided in the supplemental online material of this work.

4.3 An observation on our proposed MM

In our proposed MM, we assume that if there is a set-up of a product, then at least a small (non-zero) unit of production has to take place in every production period when there is a production/production carry-over.

If we relax the above assumption in our proposed model, we need to modify constraint (8) (with the modified constraint shown in (41)); and remove constraints (16) and (24) from the MM in Section 3.1; and add the constraints (42)–(44):

![Figure 3. Gantt chart generated (for the problem instance given in Table 3, and the solution given in Table 5 provided in the supplemental online material of this work) by our model with the consideration of Scenario 2 – Variant (1).](image-url)

![Figure 4. Gantt chart (for the problem instance given in Table 3, and the solution given in Table 6 provided in the supplemental online material of this work) generated by our model with the consideration of Scenario 2 – Variant (2).](image-url)
are compared with the previous model of Mohan et al. (2012). The main contribution of this work is to propose new

5. Conclusion

The solution values corresponding to the Gantt chart are given (in Table 7) in the supplemental online material of this paper. From the example, we observe that idle-time periods may exist during the course of the production of a given product, with the changes mentioned in this section.

4.4 Further remarks

When the set-up costs and holding costs are product dependent and time independent, it may be enough to use two binary variables  and , and hence we can set  and  for . In addition, we remove constraints (26) and (27) from the model and execute the model. When we deal with some real-life situations where the set-up costs and holding costs are product dependent and time dependent, we need to include all the three binary variables; in such a situation, the use of two binary variables as stated above and removing constraints (26) and (27) may lead to an incorrect computation of Z value (more details are provided in the supplemental online material of this paper). More details on other extensions of the model are given in the supplemental online material provided in this work in order to save space in this paper.

5. Conclusion

In this work, we have presented new MMs for the CLSP with production carry-over and set-up splitting. Our models are compared with the previous model of Mohan et al. (2012). The main contribution of this work is to propose new
and correct models for this problem, as opposed to the model by Mohan et al. that appears to be incorrect. Our MMs are comprehensive in the sense that they can address two scenarios, namely (1) situation/scenario where the set-up costs and holding costs are product dependent and time independent, and with no backorders or lost sales (Scenario 1), and (2) situation where the set-up costs and holding costs are product dependent and time dependent, and with no backorders or lost sales (Scenario 2). It is evident that our approach in the proposed MMs is quite different from that of the existing model. We have also addressed a possible extension to our proposed models which is present in detail in the supplemental online material of this paper.

This work can be extended further to include set-up crossover across multiple periods. In addition, it is evident that in some manufacturing and process industries, production once started should be carried without interruption; moreover, production has to start immediately after the set-up without delay. Such situations arise in industries such as hot rolling, sugar, cement and pharmaceuticals. All these aspects, especially related to such manufacturing and process industries, provide motivation and scope for future research.

**Supplemental Material**

Supplemental data for this article can be accessed [http://dx.doi.org/10.1080/00207543.2015.1076942](http://dx.doi.org/10.1080/00207543.2015.1076942).

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