Semileptonic $B_c$ to P-Wave Charmonia ($X_{c0}, X_{c1}, h_c$) Transitions within QCD Sum Rules

K. Azizi$^{1,†}$, H. Sundu$^{2,‡}$, M. Bayar$^{2,+}$

$^1$Department of Physics, Middle East Technical University, 06531 Ankara, Turkey
$^2$Physics Department, Kocaeli University, 41380 Izmit, Turkey

$^†$e-mail: e146342@metu.edu.tr
$^‡$e-mail: hayriye.sundu@kocaeli.edu.tr
$^+$e-mail: melahat.bayar@kocaeli.edu.tr

The form factors of the semileptonic $B_c \to S(AV)\ell\nu$ ($\ell = \tau, \mu, e$) transitions, where $S$ and $AV$ denote the scalar $X_{c0}$ and axial vector ($X_{c1}$, $h_c$) mesons, are calculated within the framework of the three-point QCD sum rules. The heavy quark effective theory limit of the form factors are also obtained and compared with the values of the original transition form factors. The results of form factors are used to estimate the total decay widths and branching ratios of these transitions. A comparison of our results on branching ratios with the predictions of other approaches is also presented.

PACS numbers: 11.55.Hx, 13.20.He

I. INTRODUCTION

The $B_c$ meson is the only known meson composed of two heavy quarks of different flavor and charge; a charm quark and a bottom antiquark. It was discovered by the collider detector at Fermilab (CDF Collaboration) in $p\overline{p}$ collision via the decay mode $B_c \to J/\psi l^\pm\nu$ at $\sqrt{s} = 1.8$ TeV [1]. Discovery of the $B_c$ meson has demonstrated the possibility of the experimental study of the charm-beauty system and has created considerable interest in its spectroscopy [2, 3, 4, 5, 6]. When the large hadron collider (LHC) runs, a plenty number of the charm-beauty system and has created considerable interest in its level, the tree level elements, leptonic decay constant as well as the origin of the CP and T violation. Secondly, the interests in many respects.

Among the B mesons, the $B_c$ carries a distinctive signature and has reached great interest recently for the following reasons: Firstly, the $B_c$ meson decay channels are expected to be very rich in comparison with other B mesons, so investigation of such type of decays can be used in the calculation of the Cabibbo-Kabayashi-Maskawa (CKM) matrix elements, leptonic decay constant as well as the origin of the CP and T violation. Secondly, the $B_c$ meson, because of containing the heavy quarks, provides more accuracy and confidence in the understanding of the QCD dynamics.

The $B_c$ meson can decay via the $b \to u, d, c$ and also the $c \to u, d, s$ transitions. Among those transitions at quark level, the tree level $b \to c$ transition, governs the $B_c$ to P-wave charmonia, plays a significant role, because this is the most dominant transition. In the literature, there are several studies on the $B_c$ mesons in different models. Some possible $B_c$ meson decays such as $B_c^+\to \pi^+\gamma$, $B_c^0\to \rho^0\gamma$, $B_c^+\to K^{*+}\gamma$ and $B_c^0\to K_{s1}^{*+}\gamma$, $B_c^0\to B_{u}^{*\gamma}$, $B_c^0\to D^{\ast\pm}\omega$, $B_c^0\to D(D_s)\ell^\pm l^\mp l^{-}\nu$, $B_c\to P(D,D_s)\ell^\pm l^\mp l^{-}\nu$, $B_c\to X_{c0,1}\ell^\pm l^{-}\nu$, $B_c\to D^{\ast d}\ell^\pm l^{-}\nu$, $B_c\to X_{c0,1}\nu\nu$ and $B_c\to D_{c}\gamma$ have been studied in the frame of light-cone QCD and three-point QCD sum rules [9, 14, 11, 12, 13, 14, 15, 16]. The weak productions of new charmonium in semileptonic decays of $B_c$ were also studied in the framework of light cone QCD sum rules in [18]. In [19], a larger set of exclusive nonleptonic and semileptonic decays of the $B_c$ meson were investigated in the relativistic constituent quark model. Weak decays of the $B_c$ meson to charmonium and D mesons in the relativistic quark model have been discussed in [20, 21]. Moreover, the $B_c^0\to (D^\ast,J_{c})\nu\tau$ transitions were also studied within the relativistic constituent quark model in [22].

Present work is devoted to the study of the $B_c^0\to S(AV)\ell\nu$. The long distance dynamics of such transitions can be parameterized in terms of some form factors which play fundamental role in analyzing such transitions. For evaluation of the form factors, the QCD sum rules as nonperturbative approach based on the fundamental QCD Lagrangian is used. The obtained results for the form factors are used to estimate the total decay rate and branching fractions for the related transitions. The heavy quark effective theory (HQET) limit of the form factors are also calculated and compared with their values. In these transitions, the main contribution comes from the perturbative part since the heavy quark condensates are suppressed by inverse of the heavy quark masses and can be safely omitted and two-gluon condensate contributions are very small and we will ignore them. Note that, the $B_c$ to P-wave charmonia transitions have also been investigated in the framework of covariant light-front quark model (CLQM), the renormalization group method (RGM), relativistic constituent quark model (RCQM) and nonrelativistic constituent quark model (NRCQM) in [23, 24, 25, 26, 27]. For more about those transitions see also [28, 29, 30, 31, 32, 33, 34, 35].
The outline of the paper is as follows: In Section II, the some rules for the transition form factors relevant to the $B_c \to S(\text{AV})\ell\nu$ decays are obtained. Section III encompasses the calculation of the HQET limit of the form factors and, section IV is devoted to the numerical analysis of the form factors and their HQET limits, decay rates, branching ratios, conclusion and comparison of our results with the other approaches.

II. SUM RULES FOR THE $B_c \to S(\text{AV})\ell\nu$ TRANSITION FORM FACTORS

The $B_c \to X_{cd} (X_{c1}, \ h_c)\ell\nu$ decays proceed via the $b \to c$ transition at the quark level. The effective Hamiltonian responsible for these transitions can be written as:

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} \overline{\tau} \gamma_\mu (1 - \gamma_5) l \ \overline{\tau} \gamma_\mu (1 - \gamma_5) b.$$  \hspace{1cm} (1)

We need to sandwich Eq. (1) between initial and final meson states in order to obtain the matrix elements of $B_c \to S(\text{AV})\ell\nu$. Hence, the amplitude of this decay is written as follows:

$$M = \frac{G_F}{\sqrt{2}} V_{cb} \overline{\tau} \gamma_\mu (1 - \gamma_5) l < S(\text{AV})(p') \mid \overline{\tau} \gamma_\mu (1 - \gamma_5) b \mid B_c(p) >.$$ \hspace{1cm} (2)

It is necessary to calculate the matrix elements $< S(\text{AV})(p') \mid \overline{\tau} \gamma_\mu (1 - \gamma_5) b \mid B_c(p) >$ appearing in Eq. (2). In the $S$ case in final state, the only axial-vector part of the transition current, $\overline{\tau} \gamma_\mu (1 - \gamma_5) b$, contribute to the matrix element stated above. However, in the $AV$ case, both vector and axial-vector parts have contributions. Considering the parity and Lorentz invariances, the aforementioned matrix element can be parameterized in terms of the form factors in the following way:

$$< S(p') \mid \overline{\tau} \gamma_\mu \gamma_5 b \mid B_c(p) > = f_1(q^2) P_\mu + f_2(q^2) q_\mu,$$ \hspace{1cm} (3)

$$< AV(p', \varepsilon) \mid \overline{\tau} \gamma_\mu \gamma_5 b \mid B_c(p) > = i f_V(q^2) \varepsilon_{\mu \nu \alpha \beta} \varepsilon^{\ast \nu} p^\alpha p'^\beta,$$ \hspace{1cm} (4)

$$< AV(p', \varepsilon) \mid \overline{\tau} \gamma_\mu b \mid B_c(p) > = i \left( f_0(q^2)(m_{B_c} + m_{X_{c1}}) \varepsilon_\mu^* - \frac{f_+(q^2)}{(m_{B_c} + m_{X_{c1}})} (\varepsilon^* p) P_\mu - \frac{f_-(q^2)}{(m_{B_c} + m_{X_{c1}})} (\varepsilon^* p) q_\mu \right),$$ \hspace{1cm} (5)

where $f_1(q^2)$, $f_2(q^2)$, $f_V(q^2)$, $f_0(q^2)$, $f_+(q^2)$ and $f_-(q^2)$ are transition form factors and $P_\mu = (p + p')_\mu$, $q_\mu = (p - p')_\mu$.

From the general philosophy of the QCD sum rules, we see a hadron from two different windows. First, we see it from the outside, so we have a hadron with hadronic parameters such as its mass and leptonic decay constant. Second, we see the internal structure of the hadron namely, quarks and gluons and their interactions in QCD vacuum. In technique language, we start with the main object in QCD sum rules so called the correlation function. The correlation function is calculated in two different ways: From one side, it is saturated by a tower of hadrons called the phenomenological or physical side. On the other hand, the QCD or theoretical side, it is calculated in terms of quark and gluons interacting in QCD vacuum by the help of the operator product expansion (OPE), where the short and long distance effects are separated. The farmer is calculated using the perturbation theory (perturbative contribution), however, the latter is parameterized in terms of vacuum condensates with different mass dimensions. In the present work, there is no light quarks which the non-perturbative contributions mainly come from their vacuum condensates and the heavy quark condensate contributions are suppressed by inverse of the heavy quark mass and can be safely removed. The two-gluon contributions are also very small and here, we will ignore those contributions. Hence, the only contribution comes from the perturbative part. Equating two representations of the correlation function and applying double Borel transformation with respect to the momentum of the initial and final states to suppress the contribution of the higher states and continuum, sum rules for the physical quantities, form factors, are obtained. To proceed, we consider the following correlation functions:

$$\Pi_\mu(p^2, p'^2) = i^2 \int d^4x d^4y e^{-ipx} e^{ip'y} < 0 \mid T[J_S(y)J^{\text{AV}}_\mu(0)J_{B_c}(x)] \mid 0 >,$$ \hspace{1cm} (6)

$$\Pi_{\mu\nu}(p^2, p'^2) = i^2 \int d^4x d^4y e^{-ipx} e^{ip'y} < 0 \mid T[J_{\text{AV}}(y)J^{\text{AV}}_\mu(0)J_{B_c}(x)] \mid 0 >.$$

where \( J_S(y) = \tau c, J_{AV}(y) = \tau b \gamma_5 c, J_{B_c}(x) = \bar{b} \gamma_5 c \) are the interpolating currents of the S, AV and \( B_c \) mesons, respectively and \( J_{V}^\mu(0) = \tau \gamma_\mu b, J_{A}^\mu = \tau \gamma_\mu \gamma_5 b \) are the vector and axial-vector parts of the transition current. In order to calculate the phenomenological or physical part of the correlator given in Eq. (6), two complete sets of intermediate states with the same quantum numbers as the interpolating currents \( J_S, AV \) and \( B_c \) are inserted. As a result, the following representations of the above-mentioned correlators are obtained:

\[
\Pi_\mu(p^2, p'^2) = \frac{<0 | J_S(0) | S(p') > < S(p') | J^\mu_{AV}(0) | B_c(p) > < B_c(p) | J_{B_c}(0) | 0 >}{(p'^2 - m_S^2)(p^2 - m_{B_c}^2)} + \cdots,
\]

(8)

\[
\Pi_{\mu\nu}(p^2, p'^2) = \frac{<0 | J_{AV}^\mu(0) | AV(p', \varepsilon) > < AV(p', \varepsilon) | J^\nu_{AV}(0) | B_c(p) > < B_c(p) | J_{B_c}(0) | 0 >}{(p'^2 - m_{AV}^2)(p^2 - m_{B_c}^2)} + \cdots,
\]

(9)

where \( \cdots \) represents the contributions coming from higher states and continuum. The vacuum to the hadronic state matrix elements in Eq. (5) can be parameterized in terms of the leptonic decay constants as:

\[
<0 | J_S | S(p') > = -i f_S, \quad < B_c(p) | J_{B_c} | 0 > = -i \frac{f_{B_c} m_{B_c}^2}{m_b + m_c}, \quad <0 | J_{AV} | AV(p', \varepsilon) > = f_{AV} m_{AV} \varepsilon'.
\]

(10)

Using Eqs. (3-10), the final expressions of the phenomenological side of the correlation functions are obtained as:

\[
\Pi_\mu(p^2, p'^2) = - \frac{f_S}{(p^2 - m_S^2)(p'^2 - m_{B_c}^2)} \frac{f_{B_c} m_{B_c}^2}{m_b + m_c} \left[ f_1(q^2) P_\mu + f_2(q^2) q_\mu \right] + \text{excited states},
\]

\[
\Pi_{\mu\nu}(p^2, p'^2) = \frac{f_{B_c} m_{B_c}^2}{(m_b + m_c)(p^2 - m_{B_c}^2)} \frac{f_{AV} m_{AV}}{(p'^2 - m_{AV}^2)} f_0(q^2) g_{\mu\nu} (m_{B_c} + m_{AV}) - \frac{f_+(q^2) P_\mu P_\nu}{(m_{B_c} + m_{AV})} - \frac{f_-(q^2) q_\mu q_\nu}{(m_{B_c} + m_{AV})}
\]

\[+ \varepsilon_{\alpha\beta\mu\nu} p^\alpha p^\beta \left[ f_V(q^2) \frac{m_{B_c}^2}{m_{B_c} + m_{AV}} \right] + \text{excited states},
\]

(11)

where, we will choose the structures \( P_\mu, q_\mu, \varepsilon_{\mu\nu\alpha\beta} p^\alpha p^\beta, g_{\mu\nu} \) and \( \frac{1}{2} (p_\mu p_\nu \pm p'_\mu p'_\nu) \) to evaluate the form factors \( f_1, f_2, f_V, f_0 \) and \( f_\pm \), respectively.

On the QCD side, the aforementioned correlation functions can be calculated by the help of the OPE in the deep space-like region where \( p^2 \ll (m_b + m_c)^2 \) and \( p'^2 \ll (2m_c)^2 \). As we mentioned before, the main contributions to the theoretical part of the correlation functions come from bare-loop (perturbative) diagrams. To calculate those contributions, the correlation functions are written in terms of the selected structures as follows:

\[
\Pi_\mu = \Pi_1^\text{per} P_\mu + \Pi_2^\text{per} q_\mu,
\]

\[
\Pi_{\mu\nu} = \Pi_V^\text{per} \varepsilon_{\mu\nu\alpha\beta} p^\alpha p^\beta + \Pi_0^\text{per} g_{\mu\nu} + \frac{1}{2} \Pi_+^\text{per} (p_\mu p_\nu + p'_\mu p'_\nu) + \frac{1}{2} \Pi_-^\text{per} (p_\mu p_\nu - p'_\mu p'_\nu),
\]

(12)

where, each \( \Pi_i^\text{per} \) function is written in terms of the double dispersion representation in the following way:

\[
\Pi_i^\text{per} = \frac{-1}{(2\pi)^2} \int ds \int ds' \frac{\rho_i(s, s', q^2)}{(s - p^2)(s' - p'^2)} + \text{subtraction terms},
\]

(13)

where, the functions \( \rho_i(s, s', q^2) \) are called the spectral densities. Using the usual Feynman integral for the bare loop diagram, the spectral densities can be calculated with the help of Cutkosky rules, i.e., by replacing the quark propagators with Dirac delta functions:

\[
\frac{1}{p^2 - m^2} \rightarrow -2\pi \delta(p^2 - m^2),
\]

which implies that all quarks are real. After some straightforward calculations, the spectral densities are obtained as follows:

\[
\rho_1(s, s', q^2) = N_c I_0(s, s', q^2) \left[ 2(m_b - 3m_c)m_c + 2A\{(m_b - m_c)m_c - s\} + 2B\{(2m_b - m_c)m_c - s'\} \right],
\]

\[
\rho_2(s, s', q^2) = N_c I_0(s, s', q^2) \left[ -2(m_b + m_c)m_c + 2A\{(m_b - m_c)m_c + s\} - 2B\{(2m_b - m_c)m_c + s'\} \right],
\]

\[
\rho_3(s, s', q^2) = N_c I_0(s, s', q^2) \left[ -2(m_b - m_c)m_c + 2A\{(m_b - m_c)m_c - s\} + 2B\{(2m_b - m_c)m_c + s'\} \right],
\]

\[
\rho_4(s, s', q^2) = N_c I_0(s, s', q^2) \left[ 2(m_b - 3m_c)m_c + 2A\{(m_b - m_c)m_c + s\} - 2B\{(2m_b - m_c)m_c - s'\} \right],
\]

where \( I_0 \) is the form factor of the quark propagators.

\[
I_0(s, s', q^2) = \frac{1}{(s - p^2)(s' - p'^2)} + \text{subtraction terms}.
\]
\[
\rho_V(s, s', q^2) = 4N_c I_0(s, s', q^2) \left[ (m_c - m_b)A + 2m_c B + m_c \right],
\]
\[
\rho_0(s, s', q^2) = 2N_c I_0(s, s', q^2) \left[ (m_c^2 - C)(m_c - m_b) + m_c u + \{m_c(4s + u) - m_b u\}A + 2[-m_b s' + m_c(s' + u)]B \right],
\]
\[
\rho_+(s, s', q^2) = 2N_c I_0(s, s', q^2) \left[ -m_c + (m_b - 3m_c)A - 2m_c B + 2(m_b - m_c)D + 2(m_b - m_c)E \right],
\]
\[
\rho_-(s, s', q^2) = 2N_c I_0(s, s', q^2) \left[ m_c - (m_c + m_b)A + 2m_c B + 2(m_b - m_c)D + 2(m_b - m_c)E \right],
\]

where
\[
I_0(s, s', q^2) = \frac{1}{4\lambda^{1/2}(s, s', q^2)}
\]
\[
\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ac - 2bc - 2ab,
\]
\[
\Delta = m_b^2 - m_c^2 - s,
\]
\[
\Delta' = -s',
\]
\[
u = s + s' - q^2,
\]
\[
A = \frac{1}{\lambda(s, s', q^2)}(\Delta' u - 2\Delta s'),
\]
\[
B = \frac{1}{\lambda(s, s', q^2)}(\Delta u - 2\Delta'),
\]
\[
C = \frac{1}{2\lambda(s, s', q^2)}[\Delta^2 s + \Delta^2 s' - \Delta\Delta' u + m_c^2(-4ss' + u^2)],
\]
\[
D = \frac{1}{\lambda(s, s', q^2)}[6\Delta\Delta's'u + \Delta^2(2ss' + u^2) + 2s'(3\Delta^2s' + m_c^2(-4ss' + u^2))],
\]
\[
E = \frac{1}{\lambda(s, s', q^2)}[-3\Delta^2 s'u + 2\Delta\Delta'(2ss' + u^2) - u(3\Delta^2 s + m_c^2(-4ss' + u^2))].
\]

(14)

and \(N_c = 3\) is the number of colors. The integration region for the perturbative contribution in the Eq. (13) is determined requiring that the arguments of the three \(\delta\) functions vanish simultaneously. Therefore, the physical region in the \(s\) and \(s'\) plane is described by the following non-equality:
\[
-1 \leq f(s, s') = \frac{2ss' + (s + s' - q^2)(m_b^2 - s - m_c^2)}{\lambda^{1/2}(m_b^2, s, m_c^2)\lambda^{1/2}(s, s', q^2)} \leq +1.
\]

(16)

Equating the coefficient of the selected structures from the phenomenological and the OPE expressions and applying double Borel transformations with respect to the variables \(p^2\) and \(p'^2\) \((p^2 \rightarrow M^2, p'^2 \rightarrow M'^2)\) in order to suppress the contributions of the higher states and continuum, the QCD sum rules for the form factors \(f_1(q^2)\) and \(f_2(q^2)\) for the \(B_c \rightarrow X_c\ell\nu\) decay can be acquired:
\[
f_{1,2}(q^2) = \frac{(m_b + m_c)}{f_{B_c} m_{B_c}} \frac{1}{f_{X_c\ell\nu}} e^{m_{B_c}} / M^2 e^{m_{X_c\ell\nu}} / M'^2 \left\{ \right. \begin{array}{l}
\frac{1}{(2\pi)^2} \int_{(m_b + m_c)^2}^{s_0} ds \int_{(m_c)^2}^{s_0} ds' \rho_{1,2}(s, s', q^2) \theta[1 - f_2(s, s')][e^{-s/M^2} e^{-s'/M'^2}].
\end{array} \right\}
\]

(17)

The form factors \(f_V, f_0, f_+\) and \(f_-\) for \(B_c \rightarrow AV\ell\nu\) decays are also obtained as:
\[
f_i(q^2) = \frac{\kappa (m_b + m_c)}{f_{B_c} m_{B_c}} \frac{\eta}{f_{AV} m_{AV}} e^{m_{B_c}/M^2} + m_{AV}/M^2 \times \left[ \frac{1}{(2\pi)^2} \int_{(m_b + m_c)^2}^{s_0} ds \int_{(m_c)^2}^{s_0} ds' \rho_i(s, s', q^2) \theta[1 - f_2(s, s')][e^{-s/M^2} - e^{-s'/M'^2}]. \right]
\]

(18)
where \( i = V, 0, \pm \), and \( \eta = m_{B_c} + m_{AV} \) for \( i = V, \pm \) and \( \eta = \frac{1}{m_{B_c} + m_{AV}} \) for \( i = 0 \) are considered. Here \( \kappa = +1 \) for \( i = \pm \) and \( \kappa = -1 \) for \( i = 0 \) and \( V \). In the above equations, the \( s_0 \) and \( s'_0 \) are continuum thresholds in \( s \) and \( s' \) channels, respectively.

In order to subtract the contributions of the higher states and continuum, the quark-hadron duality assumption is used, i.e., it is assumed that

\[
\rho_{\text{higher states}}(s, s') = \rho_{\text{OPPE}}(s, s') \theta(s - s_0) \theta(s' - s'_0).
\]

(19)

Note that, the double Borel transformation used in calculations is written as:

\[
\frac{1}{(p^2 - m_i^2)^{m} (p^2 - m_j^2)^{n}} \rightarrow (-1)^{m+n} \frac{1}{\Gamma(m)} \frac{1}{\Gamma(n)} e^{-m_i^2/M_1^2} e^{-m_j^2/M_2^2} \left( \frac{M_1^2}{M_2^2} \right)^{n-1}. \]

(20)

Now, we would like to explain our reason for ignoring the contributions of the gluon condensates to the QCD side of the correlation function. These contributions for the related form factors are obtained as the following orders:

\[
\begin{align*}
& f^{(1, 2)}_{1,2} \sim \frac{\alpha_s}{\pi} G^2 \frac{m_{B_c} m_{D}}{M_1^2 M_2^2}, & n_1 + m_1 = 2k_1 + 2l_1, \\
& f^{(2)}_{V, +, -} \sim \frac{\alpha_s}{\pi} G^2 \frac{m_{B_c} m_{D}}{M_1^2 M_2^2}, & n_2 + m_2 = 2k_2 + 2l_2 - 1, \\
& f^{(0)}_0 \sim \frac{\alpha_s}{\pi} G^2 \frac{m_{B_c} m_{D}}{M_1^2 M_2^2}, & n_3 + m_3 = 2k_3 + 2l_3 + 1,
\end{align*}
\]

(21)

where, \( \alpha_s \) is the strong coupling constant and \( M_1^2 \) and \( M_2^2 \) are Borel mass parameters. Recalling the magnitude of the \( \langle \frac{4}{3} G^2 \rangle = 0.012 \text{GeV}^4 \) and considering the working region of the Borel parameters (see numerical analysis section), the gluon condensate contributions become very small and here, we ignore those small contributions (maximum contribution is obtained for \( f^{(0)}_0 \), which is not more than few percent).

At the end of this section, we would like to present the differential decay rates of the \( B_c \rightarrow S(\ell\nu) \) in terms of the transition form factors. The differential decay width for \( B_c \rightarrow S\ell\nu \) is obtained as follows:

\[
\frac{d\Gamma}{dq^2} = \frac{1}{192\pi^3 m_{B_c}^3} |G_F|^2 |V_{cb}|^2 \lambda^{1/2} \left( \frac{m_{B_c}^2}{m_{S}^2} \right) \left( q^2 - m_{S}^2 \right)^2 \left( -\frac{1}{2} |q^2 + m_{S}^2| \left( |f_1(q^2)|^2 + |f_2(q^2)|^2 \right) \right) + \frac{1}{2} (m_{B_c}^2 - m_{S}^2) \Re \left( f_1(q^2) f_2(q^2)^* \right) + \frac{1}{2} (m_{B_c}^2 - m_{S}^2) \Re \left( f_1(q^2) f_2(q^2)^* \right) + \frac{1}{2} (m_{B_c}^2 - m_{S}^2) \Re \left( f_1(q^2) f_2(q^2)^* \right) + \frac{1}{2} (m_{B_c}^2 - m_{S}^2) \Re \left( f_1(q^2) f_2(q^2)^* \right)
\]

(22)

and also, the differential decay width corresponding to \( B_c \rightarrow AV\ell\nu \) decays are acquired as:

\[
\frac{d\Gamma}{dq^2} = \frac{1}{16\pi^4 m_{B_c}^3} |G_F|^2 |V_{cb}|^2 \left( 4 \left( (2A_1 + A_2 q^2)|f_1|^2 (2m_{B_c}^2 + 2m_{S}^2 - q^2) \right) + \left( 2m_{B_c}^2 - m_{S}^2 \right) \Re \left( (2m_{B_c}^2 + 2m_{S}^2 - q^2) f_1 \right) \right)
\]

(23)
where

\[ |p^2| = \frac{\lambda^{1/2}(m_B^2, m_{AV}^2, q^2)}{2m_B}, \]

\[ A_1 = \frac{1}{12q^2}(q^2 - m_f^2)^2 I_0', \]

\[ A_2 = \frac{1}{6q^2}(q^2 - m_f^2)(q^2 + 2m_f^2) I_0', \]

\[ I_0' = \frac{\pi}{2}(1 - \frac{m_f^2}{q^2}). \]

(24)

III. HEAVY QUARK EFFECTIVE THEORY LIMIT OF THE FORM FACTORS

In this section, we calculate the heavy quark effective theory (HQET) limits of the transition form factors for \( B_c \to S(1^+ \Lambda) \ell \nu \). For this aim, following references [37, 38, 39, 40], we use the parameterization

\[ y = \nu\nu' = \frac{m_B^2 + m_{S(1^+ \Lambda)}^2 - q^2}{2m_B m_{S(1^+ \Lambda)}}, \]

(25)

where \( \nu \) and \( \nu' \) are the four-velocities of the initial and final meson states, respectively. Next, we try to find the \( y \) dependent expressions of the form factors by taking \( m_b \to \infty, m_c = \frac{m_b}{\sqrt{2}} \), where \( z \) is given by \( \sqrt{z} = y + \sqrt{y^2 - 1} \). In this limit, the new Borel parameters \( T_1 = M_B^2/2(m_b + m_c) \) and \( T_2 = M_B^2/4m_c \) are defined. The new continuum thresholds \( \nu_0 \), and \( \nu'_0 \) are also parameterized as:

\[ \nu_0 = \frac{s_0 - (m_b + m_c)^2}{m_b + m_c}, \quad \nu'_0 = \frac{s_0' - 4m_c^2}{2m_c}. \]

(26)

and the new integration variables take the following form:

\[ \nu = s - (m_b + m_c)^2, \quad \nu' = \frac{s' - 4m_c^2}{2m_c}. \]

(27)

The leptonic decay constants are rescaled:

\[ \hat{f}_{B_c} = \sqrt{m_b + m_c}f_{B_c}, \quad \hat{f}_{S(1^+ \Lambda)} = \sqrt{2m_c}f_{S(1^+ \Lambda)}. \]

(28)

To evaluate the form factors in HQET, we also need to redefine the form factors in the following form:

\[ f'_{1,2} = \frac{f_{1,2}}{m_B + m_S} \]

\[ f'_{V,0,+,--} = \frac{f_{V,0,+,--}}{m_B + m_{AV}}. \]

(29)

After standard calculations, we obtain the \( y \)-dependent expressions of the form factors for \( B_c \to S(1^+ \Lambda) \ell \nu \) transition as follows:

\[ f'_1 = \frac{3(-1 + y^2)\left[3 + z + y(-3 - 2\sqrt{z} + z)\right]}{8\sqrt{2}\pi^2 f_S f_{B_c} z^{13/4}(1 + y)\sqrt{1 + \frac{1}{\sqrt{z}}} \left[\left(-1 + y^2\right)(1 + \sqrt{z})\right]^{3/2}} e^{\frac{\sqrt{z} + \sqrt{z}}{2} + \frac{\sqrt{z} - \sqrt{z}}{2}} \left\{ \int_0^{\nu_0} dv \int_0^{\nu'_0} dv' e^{-\frac{\nu}{\sqrt{z}}} \frac{\nu' - \nu}{\sqrt{z}} \right\} \times \theta[1 - \lim_{m_b \to \infty} f^2(v, v')], \]

(30)
$$f_2^\prime = \frac{-3(-1 + y^2) \left[ -1 + y(1 + \sqrt{z})^2 + 4\sqrt{z} + z \right]}{8\sqrt{2\pi^2} f \bar{f} B_c z^{13/4}(1 + y) \sqrt{1 + \frac{1}{\sqrt{z}}} \left[ \frac{(-1+y^2)(1+\sqrt{z})^2}{z^2} \right]^{1/2}} e^{(\frac{\Lambda}{\sqrt{2}} + \frac{c}{\sqrt{2}})} \left\{ \int_0^{\nu_0} d\nu \int_0^{\nu_0} d\nu' e^{-\frac{\nu}{\sqrt{2}} - \frac{\nu'}{\sqrt{2}}} \right\} \times \theta[1 - \text{lim}_{m_b \to \infty} f^2(v, v')] ,$$

(31)

and for $B_c \to AV \ell \nu$ decay, the y-dependent expressions of the form factors are acquired as:

$$f_\nu' = \frac{3(3 + \sqrt{z}) \left[ -1 + y + \sqrt{z} + y\sqrt{z} \right]}{8\sqrt{2\pi^2} f \bar{f} B_c z^{5/4}(1 + y) \sqrt{1 + \frac{1}{\sqrt{z}}} \left[ \frac{(-1+y^2)(1+\sqrt{z})^2}{z^2} \right]^{1/2}} e^{(\frac{\Lambda}{\sqrt{2}} + \frac{c}{\sqrt{2}})} \left\{ \int_0^{\nu_0} d\nu \int_0^{\nu_0} d\nu' e^{-\frac{\nu}{\sqrt{2}} - \frac{\nu'}{\sqrt{2}}} \right\} \times \theta[1 - \text{lim}_{m_b \to \infty} f^2(v, v')] ,$$

(32)

$$f_0' = \frac{3(-1 + y) \left[ 1 + 2y(1 + \sqrt{z}) + 3\sqrt{z} \right]}{8\sqrt{2\pi^2} f \bar{f} B_c z^{5/4}(1 + y)(3 + \sqrt{z}) \sqrt{1 + \frac{1}{\sqrt{z}}} \left[ \frac{(-1+y^2)(1+\sqrt{z})^2}{z^2} \right]^{1/2}} e^{(\frac{\Lambda}{\sqrt{2}} + \frac{c}{\sqrt{2}})} \left\{ \int_0^{\nu_0} d\nu \int_0^{\nu_0} d\nu' e^{-\frac{\nu}{\sqrt{2}} - \frac{\nu'}{\sqrt{2}}} \right\} \times \theta[1 - \text{lim}_{m_b \to \infty} f^2(v, v')] ,$$

(33)

$$f_+^\prime = \frac{3(-1 + y^2)(3 + \sqrt{z}) \left[ 2 + 2y^2(1 + \sqrt{z})^2 + 5y(-1 + z) - 10\sqrt{z} \right]}{32\sqrt{2\pi^2} f \bar{f} B_c z^{13/4}(1 + y)^2 \sqrt{1 + \frac{1}{\sqrt{z}}} \left[ \frac{(-1+y^2)(1+\sqrt{z})^2}{z^2} \right]^{1/2}} e^{(\frac{\Lambda}{\sqrt{2}} + \frac{c}{\sqrt{2}})} \left\{ \int_0^{\nu_0} d\nu \int_0^{\nu_0} d\nu' e^{-\frac{\nu}{\sqrt{2}} - \frac{\nu'}{\sqrt{2}}} \right\} \times \theta[1 - \text{lim}_{m_b \to \infty} f^2(v, v')] ,$$

(34)

$$f_-^\prime = \frac{-3(-1 + y^2)(3 + \sqrt{z}) \left[ -2 + 2y^2(1 + \sqrt{z})^2 + y(3 + 8\sqrt{z} + 5z) + 10\sqrt{z} \right]}{32\sqrt{2\pi^2} f \bar{f} B_c z^{13/4}(1 + y)^2 \sqrt{1 + \frac{1}{\sqrt{z}}} \left[ \frac{(-1+y^2)(1+\sqrt{z})^2}{z^2} \right]^{1/2}} e^{(\frac{\Lambda}{\sqrt{2}} + \frac{c}{\sqrt{2}})} \left\{ \int_0^{\nu_0} d\nu \int_0^{\nu_0} d\nu' e^{-\frac{\nu}{\sqrt{2}} - \frac{\nu'}{\sqrt{2}}} \right\} \times \theta[1 - \text{lim}_{m_b \to \infty} f^2(v, v')] ,$$

(35)

where $\Lambda = m_{B_c} - (m_b + m_c)$ and $\bar{\Lambda} = m_{S(AV)} - 2m_c$. 
IV. NUMERICAL ANALYSIS

This section is devoted to the numerical analysis of the form factors, their HQET limit and branching ratios. The sum rules expressions for the form factors depict that they mainly depend on the leptonic decay constants, continuum thresholds $s_0$ and $s'_0$, and Borel parameters $M_1^2$ and $M_2^2$. In calculations, the quark masses are taken to be $m_c(\mu = m_c) = 1.275 \pm 0.015 \text{ GeV}$, $m_b = (4.7 \pm 0.1) \text{ GeV}$ [41] and the meson masses are chosen as: $m_{B_c} = 6.286 \text{ GeV}$, $m_{B_s} = 3.52528 \text{ GeV}$, $m_{X_{c0}} = 3.41476 \text{ GeV}$, $m_{X_{c1}} = 3.51066 \text{ GeV}$ [42]. For the values of the leptonic decay constants, we use $f_{B_c} = (400 \pm 40) \text{ MeV}$ and $f_{X_{c0}} = f_{X_{c1}} = f_{h_c} = (340^{+139}_{-101}) \text{ MeV}$ [23]. The two-point QCD sum rules are used to determine the continuum thresholds $s_0$ and $s'_0$. These thresholds are not completely arbitrary and they are related to the energy of the exited states. The result of the physical quantities, form factors, should be stable with respect to the small variation of these parameters. Generally, the $s_0$ are obtained to be $(m_{hadron} + 0.5)^2$ [36]. Here, we use $s_0 = (45 \pm 5) \text{ GeV}^2$ and $s'_0 = (16 \pm 2) \text{ GeV}^2$. Since the Borel parameters $M_1^2$ and $M_2^2$ are not physical quantities, the form factors should not depend on them. The reliable regions for the Borel parameters $M_1^2$ and $M_2^2$ can be determined by requiring that not only the contributions of the higher states and continuum are effectively suppressed, but the contribution of the operator with the highest dimension be small. As a result of the above-mentioned requirements, the working regions are determined to be $15 \text{ GeV}^2 \leq M_1^2 \leq 35 \text{ GeV}^2$ and $10 \text{ GeV}^2 \leq M_2^2 \leq 20 \text{ GeV}^2$. The numerical values of the form factors at $q^2 = 0$ for $B_c \to X_{c0} \ell \nu$ and $B_c \to AV \ell \nu$ transitions are given in the Tables I and II respectively.

| $f_1(0)$ | $f_2(0)$ |
|----------|----------|
| $B_c \to X_{c0} \ell \nu$ | $0.673 \pm 0.195$ | $-1.458 \pm 0.437$ |

TABLE I: The values of the form factors for the $B_c \to X_{c0} \ell \nu$ decay at $M_1^2 = 25 \text{ GeV}^2$, $M_2^2 = 15 \text{ GeV}^2$ and $q^2 = 0$.

| $f_0(0)$ | $f_1(0)$ | $f_2(0)$ | $f_3(0)$ |
|----------|----------|----------|----------|
| $B_c \to X_{c1} \ell \nu$ | $0.084 \pm 0.025$ | $0.949 \pm 0.261$ | $0.211 \pm 0.061$ | $-0.586 \pm 0.179$ |
| $B_c \to h_c \ell \nu$ | $0.084 \pm 0.025$ | $0.954 \pm 0.282$ | $0.211 \pm 0.061$ | $-0.588 \pm 0.181$ |

TABLE II: The values of the form factors for the $B_c \to AV \ell \nu$ decays at $M_1^2 = 25 \text{ GeV}^2$, $M_2^2 = 15 \text{ GeV}^2$ and $q^2 = 0$.

| $f_1$, $f_2$ & $f_{fit}$ |
|----------------|----------------|
| $f_1(B_c \to X_{c0} \ell \nu)$ | $0.218$ | $0.455$ | $5.043$ |
| $f_2(B_c \to X_{c0} \ell \nu)$ | $-0.721$ | $-0.738$ | $4.492$ |

TABLE III: Parameters appearing in the form factors of the $B_c \to X_{c0} \ell \nu$ decay at $M_1^2 = 25 \text{ GeV}^2$ and $M_2^2 = 15 \text{ GeV}^2$.

In order to estimate the decay width of the $B_c \to S(AV) \ell \nu$ transitions, we need to know the $q^2$ dependent form factors in the whole physical region, $m_{B_c}^2 \leq q^2 \leq (m_{B_c} - m_{S(AV)})^2$. Our form factors are truncated at about $q^2 = 4 \text{ GeV}^2$. To extend our results to the full physical region, we search for parameterization of the form factors in such a way that in the region $0 \leq q^2 \leq 4 \text{ GeV}^2$, this parameterization coincides with the sum rules predictions. The following fit parameterization is chosen for the form factors with respect to $q^2$:

$$f_1(q^2) = \frac{a}{(1 - \frac{q^2}{m_{fit_1}^2})} + \frac{b}{(1 - \frac{q^2}{m_{fit_2}^2})^2},$$

(36)

where, the values of the parameters $a$, $b$ and $m_{fit}$ for the $B_c \to X_{c0} \ell \nu$ and $B_c \to (X_{c1}, h_c) \ell \nu$ are given in the Tables III and IV respectively.
TABLE IV: Parameters appearing in the form factors of the $B_c \rightarrow X_{c1}^{\pm} l \nu$ and $B_c \rightarrow h_{c}^{\pm} l \nu$ decays at $M_1^2 = 25$ GeV$^2$ and $M_2^2 = 15$ GeV$^2$.

| $q^2$(GeV$^2$) | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    |
|----------------|------|------|------|------|------|------|------|------|
| $y$            | 1.1920 | 1.1687 | 1.1454 | 1.1221 | 1.0988 | 1.0755 | 1.0522 | 1.0289 |
| $f_1$ (HQET)   | 0.6735 | 0.7204 | 0.7732 | 0.8326 | 0.9001 | 0.9771 | 1.0656 | 1.1680 |
| $f_2$ (HQET)   | -1.1594 | -1.5760 | -1.7102 | -1.8658 | -2.0480 | -2.2636 | -2.5218 | -2.8354 |

TABLE V: Values of the form factors and their HQET limits for the $B_c \rightarrow X_{c0} l \nu$ at $M_1^2 = 25$ GeV$^2$, $M_2^2 = 15$ GeV$^2$, $T_1 = 2.09$ GeV and $T_2 = 2.94$ GeV.

To calculate the numerical values of the form factors at HQET limit, the values of $\Lambda = 0.31$ GeV and $\overline{\Lambda} = 0.86$ GeV (0.96 GeV) are used for $B_c \rightarrow S \ell \nu$ ($B_c \rightarrow A \nu$) transitions, respectively (see [43, 44]). In Tables, VI and VII, we compare the values of the form factors and their HQET limits for considered transitions in the interval $0 \leq q^2 \leq 7$ and corresponding values of the $y$. Comparing the form factors and their HQET values in those Tables, we see that all form factors and their HQET limits have the same behavior with respect to the $q^2$, i.e., they both grow or fail by increasing the values of $q^2$. The HQET limit of the form factors are comparable with their original values and in large $q^2$, those form factors and their HQET values become very close to each other. The results presented at Tables, VI and VII also indicate that the form factors and their HQET limits for $B_c \rightarrow X_{c1}^{\pm} l \nu$ and $B_c \rightarrow h_{c}^{\pm} l \nu$ have values very close to each other since the $X_{c1}$ and $h_c$ mesons are both axial vectors, i.e., $J^P = 1^+$ and have nearly the same mass.

At the end of this section, we would like to calculate the values of the branching ratios for those decays. Taking into account the $q^2$ dependency of the form factors and performing integration over $q^2$ from the differential decay rates in Eqs. (22, 23) in the interval $m_{B_c}^2 \leq q^2 \leq (m_{B_c} - m_{S(AV)})^2$ and also using the total life-time of the $B_c$ meson $\tau_{B_c} = 0.46 \pm 0.07 \times 10^{-12}$ s [42], we obtain the branching ratios of the related transitions as presented in Table VIII. This Table also contain the predictions of the other approaches such as covariant light-front quark model (CLQM), renormalization group method (RGM), relativistic constituent quark model (RCQM) and nonrelativistic constituent quark model (NRCQM) [22, 24, 25, 26, 27]. These results can be tested in the future experiments.

In conclusion, using the QCD sum rules approach, we investigated the semileptonic $B_c \rightarrow S(AV) l \nu$ decays. The $q^2$ dependencies of the transition form factors were calculated. The HQET limits of the form factors were also evaluated and compared with original form factors. The obtained results were used to estimate the total decay widths and branching ratios of these transitions. A comparison of the results for branching fractions was also presented.

V. ACKNOWLEDGMENT

The authors would like to thank T. M. Aliev and A. Ozpineci for their useful discussions. K. A. would like to thank TUBITAK, Turkish Scientific and Research Council, for their partial financial support.
TABLE VI: Values of the form factors and their HQET limits for the $B_c \to X_{c,1} \ell \nu$ at $M_2^2 = 25$ GeV$^2$, $M_1^2 = 15$ GeV$^2$, $T_1 = 2.09$ GeV and $T_2 = 2.94$ GeV.

| $q^2$(GeV$^2$) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---------------|---|---|---|---|---|---|---|---|
| $y$           | 1.1745 | 1.1518 | 1.1292 | 1.1065 | 1.0838 | 1.0612 | 1.0385 | 1.0159 |
| $f_0$         | 0.0841 | 0.0823 | 0.0800 | 0.0770 | 0.0732 | 0.0684 | 0.0624 | 0.0548 |
| $f_0$(HQET)   | 0.0683 | 0.0675 | 0.0664 | 0.0652 | 0.0641 | 0.0628 | 0.0604 | 0.0559 |
| $f_V$         | 0.9506 | 1.0171 | 1.0925 | 1.1784 | 1.2771 | 1.3915 | 1.5252 | 1.6833 |
| $f_V$(HQET)   | 0.4739 | 0.5421 | 0.6331 | 0.7566 | 0.9054 | 1.0872 | 1.1543 | 1.3421 |
| $f_-$         | -0.5862 | -0.6309 | -0.6828 | -0.7442 | -0.8176 | -0.9071 | -1.0185 | -1.1612 |
| $f_-(HQET)$   | -0.2954 | -0.3264 | -0.3682 | -0.4488 | -0.5412 | -0.6518 | -0.7839 | -1.0173 |
| $f_+$         | 0.2108 | 0.2207 | 0.2312 | 0.2424 | 0.2539 | 0.2654 | 0.2761 | 0.2841 |
| $f_+(HQET)$   | 0.1032 | 0.1157 | 0.1302 | 0.1545 | 0.1771 | 0.1998 | 0.2152 | 0.2305 |

TABLE VII: Values of the form factors and their HQET limits for the $B_c \to h_c \ell \nu$ at $M_1^2 = 25$ GeV$^2$, $M_2^2 = 15$ GeV$^2$, $T_1 = 2.09$ GeV and $T_2 = 2.94$ GeV.

| $q^2$(GeV$^2$) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---------------|---|---|---|---|---|---|---|---|
| $y$           | 1.1745 | 1.1518 | 1.1292 | 1.1065 | 1.0838 | 1.0612 | 1.0385 | 1.0159 |
| $f_0$         | 0.0842 | 0.0824 | 0.0800 | 0.0770 | 0.0732 | 0.0684 | 0.0625 | 0.0550 |
| $f_0$(HQET)   | 0.0692 | 0.0683 | 0.0665 | 0.0653 | 0.0641 | 0.0629 | 0.0604 | 0.0561 |
| $f_V$         | 0.9545 | 1.0213 | 1.0970 | 1.1833 | 1.2824 | 1.3970 | 1.5310 | 1.6890 |
| $f_V$(HQET)   | 0.4781 | 0.5483 | 0.6383 | 0.7627 | 0.9061 | 1.0922 | 1.1633 | 1.3948 |
| $f_-$         | -0.5891 | -0.6332 | -0.6845 | -0.7448 | -0.8167 | -0.9038 | -1.0114 | -1.1477 |
| $f_-(HQET)$   | -0.2983 | -0.3291 | -0.3704 | -0.4457 | -0.5404 | -0.6487 | -0.7671 | -1.0102 |
| $f_+$         | 0.2117 | 0.2217 | 0.2322 | 0.2433 | 0.2547 | 0.2659 | 0.2758 | 0.2823 |
| $f_+(HQET)$   | 0.1043 | 0.1166 | 0.1314 | 0.1557 | 0.1783 | 0.2017 | 0.2163 | 0.2314 |

TABLE VIII: Branching ratios of the semileptonic $B_c \to (X_{c,0}, X_{c,1}, h_c) \ell \nu$ ($\ell = e, \mu, \tau$) transitions in different approaches.
[1] F. Abe et al., CDF Collaboration, *Phys. Rev.* **D 58** 112004 (1998).
[2] A. Adb El-Hady, M. A. K. Lodhi and J. P. Vary, *Phys. Rev.* **D 59** 094001 (1999).
[3] L P Fulcher, *Phys. Rev.* **D 60** 074006 (1999).
[4] D. Ebert, R. N. Faustov and V. O. Galkin, *Phys. Rev.* **D 67** 014027 (2003).
[5] S. Godfrey, *Phys. Rev.* **D 70** 054017 (2004).
[6] A. K. Rai, P. C. Vinodkumar, *Pramana* **66**, 953 (2006), [hep-ph/0606194v1].
[7] S. Stone, to appear in proceedings of “Heavy Flavor Physics: A Probe of Nature’s Grand Design”, Varenna, Italy, July 1997, [hep-ph/9709500].
[8] D.S. Du, Z. Wang, *Phys. Rev.* **D 39** 1342 (1989); C.H. Chang, Y.Q. Chen, ibid. **48** (1993) 4086; K. Cheung, *Phys. Rev. Lett.* **71** 3413 (1993); E. Braaten, K. Cheung, T. Yuan, *Phys. Rev.* **D 48** R5049 (1993).
[9] T. M. Aliev, M. Savci, *Phys. Lett.* **B 434** (1998) 358.
[10] T. M. Aliev, M. Savci, J. Phys. **G 24** (1998) 2223.
[11] T. M. Aliev, M. Savci, *Eur. Phys. J.* **C 47** (2006) 413.
[12] T. M. Aliev, M. Savci, *J. Phys.* **G 24** (1998) 2223.
[13] T. M. Aliev, M. Savci, *Eur. Phys. J.* **C 47** (2006) 413.
[14] T. M. Aliev, M. Savci, *Phys. Lett.* **B 480** (2000) 97.
[15] A. Adb El-Hady, M. A. K. Lodhi and J. P. Vary, *Phys. Rev.* **D 59** 094001 (1999).
[16] L P Fulcher, *Phys. Rev.* **D 60** 074006 (1999).
[17] D. Ebert, R. N. Faustov and V. O. Galkin, *Phys. Rev.* **D 67** 014027 (2003).
[18] A. K. Rai, P. C. Vinodkumar, *Pramana* **66**, 953 (2006), [hep-ph/0606194v1].
[19] S. Stone, to appear in proceedings of “Heavy Flavor Physics: A Probe of Nature’s Grand Design”, Varenna, Italy, July 1997, [hep-ph/9709500].
[20] D.S. Du, Z. Wang, *Phys. Rev.* **D 39** 1342 (1989); C.H. Chang, Y.Q. Chen, ibid. **48** (1993) 4086; K. Cheung, *Phys. Rev. Lett.* **71** 3413 (1993); E. Braaten, K. Cheung, T. Yuan, *Phys. Rev.* **D 48** R5049 (1993).
[21] T. M. Aliev, M. Savci, *J. Phys.* **G 24** (1998) 2223.
[22] T. M. Aliev, M. Savci, *Eur. Phys. J.* **C 47** (2006) 413.
[23] T. M. Aliev, M. Savci, *Phys. Lett.* **B 480** (2000) 97.
[24] A. K. Rai, P. C. Vinodkumar, *Pramana* **66**, 953 (2006), [hep-ph/0606194v1].
[25] S. Stone, to appear in proceedings of “Heavy Flavor Physics: A Probe of Nature’s Grand Design”, Varenna, Italy, July 1997, [hep-ph/9709500].
[26] D. Ebert, R. N. Faustov, V. O. Galkin, *Phys. Rev.* **D 59** 094001 (1999).
[27] D. Ebert, R. N. Faustov, V. O. Galkin, *Eur. Phys. J.* **C 32** (2003).
[28] D. Ebert, R. N. Faustov, V. O. Galkin, *Phys. Rev.* **D 68** 094020 (2003).
[29] D. Ebert, R. N. Faustov, V. O. Galkin, *Eur. Phys. J.* **C 32** (2003).
[30] A. F aessler, Th. Gutsche, M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij, *Eur. Phys. J.* **direct C 4** (2002).
[31] Xiao-Xia Wang, Wei Wang, Cai-Dian L, [hep-ph/0901.193v1].
[32] C. H. Chang, Y. Q. Chen, G. L. Wang and H. S. Zong, *Phys. Rev.* **D 65**, 014017 (2002).
[33] M. A. Ivanov, J. G. Korner and P. Santorelli, *Phys. Rev.* **D 73**, 054024 (2006).
[34] M. A. Ivanov, J. G. Korner and P. Santorelli, *Phys. Rev.* **D 71**, 094006 (2005) [Erratum-ibid. **D 75**, 019901 (2007)].
[35] E. Hernandez, J. Nieves and J. M. Verde-Velasco, *Phys. Rev.* **D 74**, 074008 (2006) [arXiv:hep-ph/0607150].
[36] T. Becher, H. Boos, E. Lunghi, *JHEP* **0712**:062 (2007).
[37] U. Aglietti, L. D. Giustino, G. Ferrera, A. Renzaglia, G. Ricciardi, L. Trentadue, *Phys. Lett.* **B 653** 38-52 (2007).
[38] P. Urquijo, Belle Collaboration, *Proceedings of the XXXIII International Conference of the High Energy Physics (ICHEP’06), Moscow* (2006), arXiv: hep-ex/0611049.
[39] K. Abe, Belle Collaboration, *Proceedings of the XXXIII International Conference of the High Energy Physics (ICHEP’06), Moscow* (2006), arXiv: hep-ex/0609013.
[40] C. Schwanda, Belle Collaboration, *Phys. Rev.* **D 75** 032005 (2007).
[41] Chao-Hsi Chang, Yu-Qi Chen, Guo-Li Wang, Hong-Shi Zong, *Phys. Rev.* **D 65**, 014017 (2002).
[42] P. Colangelo, F. De Fazio, T.N. Pham, *Phys. Rev.* **D 69**, 054023 (2004).
[43] Chao-Hsi Cahng, *XXXVIIth Rencontres de Moriond on QCD and High Energy Hadronic Interactions*, [arXiv:hep-ph/0205112].
[44] K. Abe, Belle Collaboration, *Proceedings of the XXXIII International Conference of the High Energy Physics (ICHEP’06), Moscow* (2006), arXiv: hep-ex/0609013.
[45] C. Schwanda, Belle Collaboration, *Phys. Rev.* **D 75** 032005 (2007).
[46] Ming Qiu Huang, *Phys. Rev.* **D 69**, 014017 (2002).
[47] P. Colangelo, F. De Fazio, T.N. Pham, *Phys. Rev.* **D 69**, 054023 (2004).
[48] Chao-Hsi Cahng, *XXXVIIth Rencontres de Moriond on QCD and High Energy Hadronic Interactions*, [arXiv:hep-ph/0205112].
[49] M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, Nucl. Phys. **B 147** (1979) 385.
[50] Ming Qiu Huang, *Phys. Rev.* **D 69** (2004) 114015.
[51] M. Neubert, Phys. Rep. **245** (1994) 259.
[52] T. M. Aliev, K. Azizi, A. Ozpineci, Eur. Phys. J. **C 51** (2007) 593.
[53] K. Azizi, M. Bayar, *Phys. Rev.* **D 78**, 054011 (2008).
[54] B. L. Ioffe, Prog. Part. Nucl. Phys. **56** (2006) 232.
[55] C. Amsler et al., Particle Data Group, *Phys. Lett.* **B 667** 1 (2008).
[56] T. Huang, C. W. Luo, *Phys. Rev.* **D 50** (1994) 5775.
[57] Y. B. Dai, C. S. Huang, C. Liu, and S. L. Zhu, *Phys. Rev.* **D 68**, 114011 (2003).