Research on Coordinated Optimization of Active and Reactive Power in Active Distribution Network

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Abstract. Traditional optimization for distribution network usually uses reactive power optimization to ensure the safe operation of the distribution network, and uses the distributed power and various adjustable resources of active power adjustment to ensure the economic operation of the distribution network. However, in the distribution network, the impedance ratio of the line is large and the coupling of active and reactive power is strong. Unilateral active or reactive optimization for distribution network is not comprehensive. In addition, the uncertainty of Distributed Generation and load poses a great challenge to optimization for active distribution network, the control strategy formulated by traditional deterministic optimization might inaccurate. In this paper, the mathematical model of active and reactive power coordinated optimization for active distribution network is given. Various algorithms dealing with the uncertainty of distribution are introduced, and advantages and disadvantages of various algorithms are analysed.

1. Introduction

With the access of high-permeability Distributed Generation (DG), flexible load, and Energy Storage System (ESS) equipment, the traditional distribution network is gradually transferred to the Active Distribution Network (AND)\textsuperscript{1}.

The line impedance ratio of the distribution network is large, and the active/reactive coupling is strong. From the perspective of operating economy, active optimization reduces power generation costs, and reactive power adjustment reduces network losses, which can increase economic benefits. From the perspective of operating safety, traditional distribution network optimization guarantees voltage stability by regulating reactive power equipment\textsuperscript{[2]}. With the large-scale DG and ESS decentralized access, its active and reactive power output have become important factors that affect the voltage level and power flow distribution. Therefore, no matter from the perspective of safety or economy, it is of great significance to simultaneously optimize active and reactive power in the distribution network.

In addition, due to frequent changes in wind speed, light, and load fluctuations, there is a large error in the predicted values of distributed power output and load, which leads to the deviation of the optimization strategy from actual demand, which may cause problems such as frequent voltage fluctuations and trigger related protection devices to limit the capability of DG connected to the distribution network. Therefore, the uncertainties of DG and load need to be considered when formulating the control strategy.
2. Deterministic Mathematical Model

Compared with traditional distribution network, active distribution network has many controllable devices and control strategies. A simple structure of the active distribution network is demonstrated in Fig. 1, where a variety of devices are contained such as On-Load Tap Changer (OLTC), SVC, switch, ESS, wind power generation and photovoltaic power generation.

\[ f_{\text{loss}} = \sum_{t=1}^{T} \sum_{i,j \in N_B} G_{ij} \left( U_{i,t}^2 + U_{j,t}^2 - 2U_{i,t}U_{j,t} \cos \theta_{ij} \right) \]  

In the formula, \( f_{\text{loss}} \) is the system network loss; \( N_B \) is the number of nodes in the system; \( U_{i,t} \) is the voltage value of the node \( i \); \( \theta_{ij} \) is the voltage phase angle difference between the node \( i \) and the node \( j \).

\[ f_{\Delta U} = \sum_{t=1}^{T} \sum_{i=1}^{N_A} \left( \frac{U_{i,t} - U_e}{U_{i,\max} - U_{i,\min}} \right)^2 \]  

In the formula, \( f_{\Delta U} \) is the voltage deviation; \( U_{i,\max} \) and \( U_{i,\min} \) respectively represent the upper and lower limits of the node voltage; \( U_e \) represent the expected value of the node voltage.

2.2. Constraints

2.2.1. Power balance constraints.
\[
\begin{align*}
\begin{cases}
P_{i,t}^{DG} + P_{\text{dis},i,t}^{ESS} - P_{\text{ch},i,t}^{ESS} - P_{i,t}^{L} = U_{i} \sum_{j=1}^{N_{p}} U_{j,t} \left( G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij} \right) \\
Q_{i,t}^{DG} + Q_{\text{dis},i,t}^{ESS} + Q_{\text{ch},i,t}^{SVC} - Q_{i,t}^{L} = U_{i} \sum_{j=1}^{N_{p}} U_{j,t} \left( G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij} \right)
\end{cases}
\end{align*}
\]  

(3)

In the formula, \( P_{i,t}^{DG} \) and \( Q_{i,t}^{DG} \) are the active and reactive power output of DG; \( P_{i,t}^{L} \) and \( Q_{i,t}^{L} \) are the active and reactive power consumed by the load; \( Q_{i,t}^{SVC} \) is the reactive power output value of SVC; \( P_{\text{ch},i,t}^{ESS} \) and \( P_{\text{dis},i,t}^{ESS} \) are charge and discharge power of ESS; \( Q_{i,t}^{ESS} \) is the reactive power output of ESS; \( G_{ij} \) and \( B_{ij} \) are conductance and susceptance on the branch \( ij \).

2.2.2. Node voltage constraints.

\[ U_{\text{min}} \leq U_{i}(t) \leq U_{\text{max}} \]  

(4)

In the formula, \( U_{\text{min}} \) and \( U_{\text{max}} \) are the lower limit and upper limit of each node voltage.

2.2.3. Network structure constraints.

During the dynamic reconfiguration of the distribution network, the network structure must maintain a radial form, and it must be ensured that no islands will be generated.

2.2.4. OLTC constraints.

\[ T_{j,\text{min}} \leq T_{j,t} \leq T_{j,\text{max}} \quad \forall j \in B^{\text{OLTC}} \]  

(5)

In the formula, \( B^{\text{OLTC}} \) is the set of nodes containing OLTC; \( T_{j,\text{max}} \) and \( T_{j,\text{min}} \) are Upper and lower limits of the tap position of OLTC.

2.2.5. SVC constraints.

\[ Q_{j,\text{min}}^{SVC} \leq Q_{j,t}^{SVC} \leq Q_{j,\text{max}}^{SVC} \quad \forall j \in B^{\text{SVC}} \]  

(6)

In the formula, \( Q_{j,\text{min}}^{SVC} \) and \( Q_{j,\text{max}}^{SVC} \) are the minimum and maximum value of SVC reactive power output respectively; \( B^{\text{SVC}} \) is the set of nodes containing SVC.

2.2.6. Distributed power operating constraints.

Distributed generation mainly includes wind power generation, photovoltaic power generation, small diesel engine and micro turbine, and their grid-connected methods are shown in Table 1.

| DG                | Grid-connected method     | Capacity(W) |
|-------------------|---------------------------|-------------|
| Small diesel engine | Synchronous generator     | 10^3~10^6   |
| Micro turbine     | AC/AC converter           | 10^3~10^6   |
| Photovoltaic power generation | DC/AC converter | 1~10^3   |

Table 1. Grid-connected methods of DG.
Wind power generation

| Asynchronous generator | 1–10⁶ |

Distributed power operating constraints can be expressed as follow:

\[
\begin{align*}
0 & \leq P_{j,t}^{DG} \leq P_{j,t}^{DG,\text{pre}} \\
\left( P_{j,t}^{DG} \right)^2 + \left( Q_{j,t}^{DG} \right)^2 & \leq \left( S_{j}^{DG,\text{max}} \right)^2 \quad \forall j \in B^{DG}
\end{align*}
\]

(7)

In the formula, \( B^{DG} \) is the set of nodes containing DG; \( P_{j,t}^{DG,\text{pre}} \) is the predicted active value of DG at period \( t \).

2.2.7. Energy storage system operating constraints.

\[
\begin{align*}
0 & \leq P_{\text{ch},j,t}^{ESS} \leq P_{\text{ch},j,t}^{ESS,\text{max}} \\
0 & \leq P_{\text{dis},j,t}^{ESS} \leq P_{\text{dis},j,t}^{ESS,\text{max}} \\
\left( P_{\text{ch},j,t}^{ESS} \right)^2 + \left( P_{\text{dis},j,t}^{ESS} \right)^2 & \leq \left( S_{j}^{ESS} \right)^2 \quad \forall j \in B^{ESS}
\end{align*}
\]

(8)

In the formula, \( B^{ESS} \) is the set of nodes containing ESS; \( E_{j,t} \) is the total energy of the energy storage connected to node \( j \) at time \( t \); \( \eta_{\text{ch},j,t} \) and \( \eta_{\text{dis},j,t} \) are the charge and discharge efficiency of the energy storage; \( S_{j}^{ESS} \) is the limit of the energy storage capacity connected to node \( j \); \( P_{\text{ch},j,t}^{ESS,\text{max}} \) and \( P_{\text{dis},j,t}^{ESS,\text{max}} \) are respectively The upper limit of the charge and discharge power of the energy storage connected to node \( j \); \( \sigma_{\text{ch},j,t}^{ESS} \) and \( \sigma_{\text{dis},j,t}^{ESS} \) are respectively the charge and discharge state of the energy storage connected to node \( j \).

3. Strategy of dealing with uncertainty in distribution network

Due to the frequent changes in wind speed and light, the output of wind power, photovoltaic and other uncontrollable distributed power sources fluctuates randomly. As a result, the optimization of distribution networks including distributed generation has become a typical uncertainty optimization. At present, the related researches on the randomness of distributed power output are mainly stochastic optimization, robust optimization and distributionally robust optimization.

3.1. Stochastic optimization

Stochastic optimization is to treat uncertain parameters as random variables that obey a certain distribution function, express uncertain models as expected period models or chance constrained programming models, and then use intelligent algorithms to solve them, so as to obtain a control scheme that can make constraints and objective functions satisfy a certain confidence level[6-7]. Because the random optimization problem considers the distribution of uncertain data, the smaller data disturbance may destroy the feasibility of the optimal solution of the original problem, but it is also within an acceptable range. Although the stochastic optimization problem has the above advantages, a great challenge it faces is the availability of probability distribution of random variables. In many literatures of stochastic optimization, it is usually assumed that the strategists have accurate information about the probability distribution of random variables. However, it is unrealistic to make such an assumption in many practical applications. Firstly, according to the Scenario method, it may
be difficult to obtain the true probability distribution and generate a large number of samples, that is to say, the accurate probability distribution is often not obtained, which leads to the inability to apply such models. Secondly, even if the accurate probability distribution has been obtained, the expectation and probability involved in the problem cannot be calculated effectively in many cases.

3.2. Robust optimization

The purpose of robust optimization is to find a solution that satisfies all possible constraints and makes the function value of the objective function in the worst case optimal. This model does not consider the probabilistic significance of uncertain data, but limits its possible values to a certain range, and sets this range as an uncertainty set. Obviously, in practical problems, the uncertainty set is easier to obtain than the probability distribution. The worst-case scenario defined is that given a feasible solution, and then finding a set of parameters from the uncertainty set, the objective function value is the worst. Because each feasible solution corresponds to a function value, it can be regarded as a function that takes the feasible solution as its independent variable, and then the optimal value of this function is solved[8-9]. It can be seen from this that as long as the uncertain parameters are perturbed within the defined uncertainty set, the optimal solution will not fluctuate, which proves the robustness of the model.

3.3. Distributionally robust optimization

Distributionally robust optimization combines the stochastic optimization method and the robust optimization, considers the objective function expectation, and introduces the uncertainty set to consider the uncertainty of the probability distribution of the uncertainty, and finds the worst probability within the range of uncertainty set [10].

Distributionally robust optimization can be expressed as follow:

\[
\min \max \mathbb{E} \left( f(x, \eta) \right) \\
\text{s.t. } h(x, \eta) \leq 0
\] (9)

In the formula, \( x \) is the control variable; \( \eta \) is the random variable; \( f(x, \eta) \) is the objective function; \( \mathcal{D} \) is an uncertainty set constructed from samples of random variables.

The key to distributionally robust optimization lies in the construction of uncertainty set. At present, uncertainty set mainly includes uncertainty set based on moment information[11] and uncertainty set based on probability distances[12-14].

3.3.1. Uncertainty Set Based on Uncertain Moment Information.

The uncertainty set based on the probability distance can be expressed as follow:

\[
\mathcal{D} = \{ \eta : \mathbb{E}[\eta] \leq \mu, \mathbb{E}[\eta^T\eta] \leq \Sigma \}
\] (10)

It is generally difficult to deal with the distributionally robust optimization of uncertainty set based on moment information, which only uses the moment information of the data, ignoring the effective information that other data can provide, and the calculation is more complicated.

3.3.2. Uncertainty set based on probability distance.

The uncertainty set based on the probability distance is a spherical probability space centred on empirical probability and radius \( \rho \), which can be expressed as follow:

\[
\mathcal{D} = \{ P : d(P, P_m) \leq \rho \}
\] (11)
Where, $P_m$ the empirical distribution, $d$ is a measure of the similarity of the two distributions. Kullback-Leibler divergence[12], Hellinger distance[13], and Wasserstein distance[14] are usually selected as the metric $d$.

4. Conclusion

With the construction and development of distribution automation and the research on key technologies of active distribution networks, the structure of the distribution network is becoming more and more complex, unilateral active or reactive optimization is not comprehensive enough. It is necessary to make a decision from the perspective of active and reactive coordinated optimization. The randomness of the power distribution and load changes increase the uncertainty in the distribution network, which brings great challenges to the economic and safe operation of the switching network. Forecasting the optimization of the dispatching network needs to consider the uncertainty in the switching network. In recent years, scholars from various countries have done a lot of work by regarding DG, SVC, switch reconfiguration, OLTC as control methods and establishing an uncertain model of active and reactive power coordinated optimization to cope with the uncertainty in the distribution network. It will be of great significance to the economic and safe operation of the distribution network.

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