Comment on “Scientific collaboration networks. II. Shortest paths, weighted networks, and centrality”

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(Dated: January 27, 2022)

In this comment, we investigate a common used algorithm proposed by Newman [M. E. J. Newman, Phys. Rev. E 64, 016132(2001)] to calculate the betweenness centrality for all vertices. The inaccuracy of Newman’s algorithm is pointed out and a corrected algorithm, also with $O(MN)$ time complexity, is given. In addition, the comparison of calculating results for these two algorithms aiming the protein interaction network of Yeast is shown.

PACS numbers: 89.75.Hc, 89.65.-s, 89.70.+c, 01.30.-y

Betweenness centrality, also called load or betweenness for simplicity, is a quite useful measure in the network analysis. This conception is firstly proposed by Anthonisse and Freeman and introduced to physics community by Newman. Newman proposed a very fast algorithm taking only $O(MN)$ time to calculate the betweenness of all vertices, where $M$ and $N$ denote the number of edges and vertices, respectively. The whole algorithm processes are as follows.

1. Calculate the distance from a vertex $s$ to every other vertex by using breadth-first search.
2. A variable $b^s_i$, taking the initial value 1, is assigned to each vertex $v$.
3. Going through the vertices $v$ in order of their distance from $s$, starting from the farthest, the value of $b^s_i$ is added to corresponding variable on the predecessor vertex of $v$. If $v$ has more than one predecessor, then $b^s_i$ is divided equally between them.
4. Go through all vertices in this fashion and records the value $b^s_i$ for each $v$. Repeat the entire calculation for every vertex $s$, the betweenness for each vertex $v$ is obtained as

$$B(v) = \sum_s b^s_i.$$  

Since to a vertex $v$’s betweenness $B(v)$, the contributions of its predecessors are not equal, it is not proper to divide $b^s_i$ equally between them. Clearly, if the vertex $v$ has $n$ predecessors labelled as $u_1, u_2, \cdots, u_n$ and $\sigma_{uv}$ different shortest paths to vertex $s$, then we have

$$\sigma_{sv} = \sum_{i=1}^{n} \sigma_{suj}.$$
The different shortest paths from $s$ to $v$ are divided into $n$ sets $G_1, G_2, \ldots, G_n$. The number of elements in $G_i$, that is also the number of different shortest paths from $s$ to $u_i$, gives expression to the contribution of the predecessor $u_i$ to $v$’s betweenness. Therefore, the vertex $v$’s betweenness, induced by the given source $s$, should be divided proportionally to $\sigma_{su}$, rather than equally between its predecessors. The corrected algorithm is as follows.

1. Calculate the distance from a vertex $s$ to every other vertex by using breadth-first search, taking time $O(M)$.

2. Calculate the number of shortest paths from vertex $s$ to every other vertex by using dynamic programming, taking time $O(M)$ too. The processes are as follows. (2.1) Assign $\sigma_{ss} = 0$. (2.2) If all the vertices of distance $d(d \geq 0)$ is assigned (Note that the distance from $s$ to $s$ is zero), then for each vertex $v$ whose distance is $d + 1$, assign $\sigma_{sv} = \sum_u \sigma_{su}$ where $u$ runs over all $v$’s predecessors. (2.3) Repeat from step (2.1) until there are no unassigned vertices left.

3. A variable $\beta_v^s$, taking the initial value 1, is assigned to each vertex $v$.

4. Going through the vertices $v$ in order of their distance from $s$, starting from the farthest, the value of $\beta_v^s$ is added to corresponding variable on the predecessor vertex of $v$. If $v$ has more than one predecessor $u_1, u_2, \ldots, u_n$, $\beta_v^s$ is multiplied by $\sigma_{su_i}/\sigma_{sv}$ and then added to $\sigma_{su_i}$.

5. Go through all vertices in this fashion and records the value $\beta_v^s$ for each $v$. Repeat the entire calculation for every vertex $s$, the betweenness for each vertex $v$ is obtained as

$$B(v) = \sum_s \beta_v^s. \quad (4)$$

Clearly, the time complexity of the corrected algorithm is $O(MN)$ too. Besides, one should pay attention to a more universal algorithm proposed by Brandes [5], which can be used to calculate all kinds of centrality based on shortest-paths counting for both unweighted and weighted networks.

These two algorithms, Newman’s and the corrected one, will give the same result if the network has a tree structure. However, when the loops appear in the networks, the diversity between them can be observed. Figure (1) exhibits two examples, the first one is copied from the Ref. [2], and the second is the minimal network that can illuminate the difference between Newman’s and the corrected algorithms. The comparisons between these two algorithms are shown in table (1) and (2). The two algorithms produce different results even for networks of very few vertices.

In addition, we compare with the performances of these two algorithms on the protein interaction network of Yeast [3]. This network has 2617 vertices, but only its maximal component containing 2375 vertices is taken into account. Figure 2(a) and 2(b) report the absolute diversity and relative diversity between Newman’s and the accurate (obtained from the corrected algorithm) results, respectively. The departure is distinct and can not be neglected. Fortunately, the statistical features may be similar. Although the details of the Zipf plot of the top-100 vertices are not the same, both the two curves obey power-law form with almost the same exponent. We also have checked that the scaling law of betweenness distribution in Barabási-Albert networks is kept, while the power-law exponents are slightly changed.

The measure of betweenness is now widely used to detect communities/modules structures and to analysis dynamics upon networks. Since the statistical characters of betweenness distributions obtained by Newman’s and the corrected algorithm are almost the same, some researchers may have found the difference between these two algorithm but have not paid attention to it. However, many previous works have demonstrated that a few nodes’ betweennesses rather than the overall betweenness distribution, may sometimes, determine the key features of dynamic behaviors on networks. Examples are numerous: these include the network traffics, the synchronization, the cascading dynamics, and so on. In figure 2(b), one can find that for many nodes the relative diversities between those two algorithms exceed 10%, and even nearly 30% for a few nodes. Therefore, the difference can not be neglected especially in analyzing the networks dynamics.

Although Newman’s algorithm does not agree with the definition of betweenness, it may be more practical especially for the large-scale communication sys-
tems wherein the routers do not know how many shortest paths there are to the destination. Even if they can save the information of all the successors' weights, to implement the biased choices may bring additional costs in economy and technique. Hence just to choose with equal probability at each branch point may be more natural, which is in accordance with Newman’s algorithm.

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