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FISSION FRAGMENT ORIENTATION AND
\( \gamma \) RAY EMISSION ANISOTROPY

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Experimental data on angular distributions of γ rays emitted from binary and ternary spontaneous fission of $^{252}$Cf are analyzed. Their difference indicates that the alignment of fragments is higher in ternary fission than in binary one. The consequences of possible relation between the mechanism of ternary fission and the excitation of collective modes during the saddle - to - scission stage are discussed.
1 Introduction

The descent of the fissioning nucleus from saddle to scission point is of interest as the fragment mass, charge, excitation energy and spin distributions are formed at this stage. It is well known that the fragment spins are relatively high ($\langle J \rangle \sim 7 - 8$) even for the spontaneous fission of zero spin nucleus like $^{252}$Cf. Besides the fragment spins are aligned in the plane perpendicular to the fission axis causing an anisotropy of $\gamma$ rays emitted from fragments. The most probable reason for fragment spins and their alignment is the excitation of collective vibrational modes like bending or wriggling at the saddle - to - scission stage. Although such modes are being discussed for a long time [1, 2] their possible influence on the formation of mass and energy distributions of fragments is not really taken into account (see, e.g., Refs. [3, 4]). Similarly, vibrational modes are ignored in the A.Bohr model [5] for fission fragment angular distribution since they violate the axial symmetry of fissioning nucleus at the descent stage and thus can perturb the distribution of $K$ quantum number formed at the saddle point.

Recently the new experimental data on angular distributions of prompt $\gamma$ rays emitted from spontaneous fission of $^{252}$Cf have been published by Pilz and Neubert [6]. For the first time the results were obtained for ternary fission of $^{252}$Cf which probability is $\sim 1/300$ of binary fission [7]. Earlier only the angular anisotropy $w^{(t)}(0^0)/w^{(t)}(90^0) = 1.015 \pm 0.022$ for ternary fission of $^{252}$Cf had been measured [8]; this result was interpreted as the evidence for the destruction of fragment alignment owing to the $\alpha$ particle emission. Although the value of angular anisotropy in ternary fission was confirmed in Ref. [5] the measured angular distributions were found to be anisotropic! Whereas the angular distributions $w^{(b)}(\theta)$ in binary fission have the maximums at $0^0$ and $180^0$ with respect to the light fragment momentum
the distributions $w^{(t)}(\theta)$ in ternary fission reveal maximums at $20 - 40^0$ and $140 - 160^0$. The distributions $w^{(b)}(\theta)$ and $w^{(t)}(\theta)$ are presented in Ref. [6] for 8 groups of $\gamma$ rays with energies from 151-242 keV to 978-1208 keV, but the energy dependence of angular distributions is quite week.

Pilz and Neubert interpreted their results as the evidence for the tilting of the fragment alignment by the recoiling $\alpha$ particle causing the shift of the maximums of $\gamma$ ray angular distributions from $0^0$ and $180^0$ to $20 - 40^0$ and $140 - 160^0$. It seems, however, that the accidental tiltings would widen the maximums but not shift them. The purpose of this work is to demonstrate the possibility of more consistent explanation of the results obtained in Ref. [6] based on the standard description of the angular distribution of $\gamma$ rays emitted from aligned nuclei.

2 Helicity representation in fission

The reason for angular anisotropy of prompt $\gamma$ rays is the alignment of fragment spins. Choosing the light fragment momentum for the direction of the $z$ axis we can write in the center of mass system the wave function of two separated light and heavy fragments with spins $J_1$ and $J_2$ in the form of the superposition

$$\psi_{J_1J_2} = \sum_{K_1K_2} g(K_1, K_2) \psi_{J_1K_1} \psi_{J_2K_2}, \quad \sum_{K_1K_2} |g(K_1, K_2)|^2 = 1,$$

where $K_1$ is the projection of spin $J_1$ to the $z$ axis or to the momentum $\vec{p}_1$ of the light fragment, that is the helicity of the light fragment; $K_2$ is the projection of spin $J_2$ to the $z$ axis or to the momentum $\vec{p}_1 = -\vec{p}_2$, that is the helicity of the heavy fragment with the opposite sign (for simplicity we shall name $K_2$ the helicity of the heavy fragment); and $g(K_1, K_2)$ are the amplitudes of fission in helicity representation introduced in Ref. [10]. After
the averaging over the ensemble of fissioning nuclei and summation over helicity of additional fragment the spin state of light fragment is described by the density matrix

$$\rho_{K_1K'_1} = \sum_{K_2} g(K_1, K_2)g(K'_1, K_2)^*, \quad \sum_{K_1} \rho_{K_1K_1} = 1, \quad (2)$$

or by the set of spin-tensors of orientation

$$\tau_{Qq}(J_1) = \sum_{K_1K'_1} C_{J_1K_1K'_1Qq}^{J_1K'_1} \rho_{K_1K'_1}, \quad \tau_{00}(J_1) = 1. \quad (3)$$

Due to the axial symmetry with respect to the fission direction for spontaneous fission of $^{252}\text{Cf}$ with zero spin we obtain for spin-tensors

$$\tau_{Qq}(J_1) = \tau_{Qq}(J_1)\delta_{q0}. \quad (4)$$

The spin-tensors of fragment orientation for the fission of preliminary oriented nuclei were calculated in Ref. [10]. As the consequence of parity conservation

$$|g(K_1, K_2)|^2 = |g(-K_1, -K_2)|^2, \quad (5)$$

thus $\tau_{Q0}(J_1) \neq 0$ only for $Q = 0, 2, 4\ldots (Q < 2J_1)$. By the same way the set of spin-tensors $\tau_{Q0}(J_2) \neq 0$ for $Q = 0, 2\ldots 2J_2$ describes the spin state of the heavy fragment.

The total helicity $K_1 + K_2$ is nothing else but the projection $K$ of the spin $J$ of fissioning nucleus to the fission axis. Indeed, the total angular momentum $\vec{J}$ of the fissioning system remains unvarying, thus we have after the scission

$$\vec{J} = \vec{J}_1 + \vec{J}_2 + \vec{L}, \quad (6)$$

where $\vec{L}$ is the fragment orbital angular momentum. Projecting this equation to the fission axis and taking into account
that the angular momentum \( \vec{L} \) is perpendicular to the momenta \( \vec{p}_1 = -\vec{p}_2 \), we get \( K = K_1 + K_2 \). The dependence of fission probability on the total helicity

\[
\beta_K = \sum_{K_1} |g(K_1, K - K_1)|^2, \quad \sum_K \beta_K = 1
\]  

(7)

is the significant characteristic of the process since its shape determines the form of the fission fragment angular distribution \[11\] (see also Refs.\[12, 10\])

\[
w(\vec{n}_f) = \sum_Q \left( \frac{2Q + 1}{4\pi} \right)^{\frac{1}{2}} \tau_{Qq}(J) \alpha_Q(J) Y^*_{Qq}(\vec{n}_f)
\]

\[
\oint d\Omega_f w(\vec{n}_f) = 1,
\]

(8)

\[
\alpha_Q(J) = \sum_K C_{JQ0}^{JK} \beta_K,
\]

(9)

here \( \vec{n}_f = \vec{p}_1/p_1 \) is the unit vector along the fission axis. For the first time the same expression for angular distribution \( w(\vec{n}_f) \) had been obtained by A.Bohr \[5\] on the assumption that it coincides with the distribution of orientation of nuclear deformation axis at the saddle point or on the assumption that the distribution \( \beta_K \) of total helicity forms at the saddle point and remains unvarying during the descent to the scission point. In reality as it was noted above the dependence \( \beta_K' \) of the fission probability on the projection \( K' \) of the spin \( J \) to the deformation axis at the saddle point may be distorted at the descent stage owing to the nonaxiality of the exciting collective modes, therefore

\[
\beta_K = \beta_K' + \sum_{K'} f_{KK'} \beta_{K'},
\]

(10)

where \( f_{KK'} \) are the distortion factors.
In the spontaneous fission of $^{252}$Cf with spin $J = 0$ the total helicity $K$ is equal to zero thus there is no problem of conservation of distribution $\beta_K$ at the descent stage. The helicity distributions of the light and heavy fragments

$$\gamma_{K_1} = |g(K_1, -K_1)|^2, \quad \gamma_{K_2} = |g(-K_2, K_2)|^2, \quad \sum_{K_j} \gamma_{K_j} = 1 \quad \text{(11)}$$

coincide with each other (although the spins $J_1$ and $J_2$ may differ by the value of orbital angular momentum $L$) and are determined completely by the manner of nuclear motion at the saddle-to-scission stage. We have for spin-tensors of second and fourth ranks the explicit expressions

$$\tau_{20}(J) = \left( \frac{J(J+1)}{(2J-1)(2J+3)} \right)^{\frac{1}{2}} \left( 3 \frac{\langle K_2^2 \rangle}{J(J+1)} - 1 \right), \quad \text{(12)}$$

$$\tau_{40}(J) = \left( \frac{J^3(J+1)^3}{(2J-3)(2J-2)(2J-1)(2J+3)(2J+4)(2J+5)} \right)^{\frac{1}{2}} \cdot \left( 35 \frac{\langle K_4^1 \rangle}{J^2(J+1)^2} - 30 \frac{\langle K_2^2 \rangle}{J(J+1)} \left( \frac{5}{6J(J+1)} \right) + 3 \left( 1 - \frac{2}{J(J+1)} \right) \right), \quad \text{(13)}$$

where

$$\langle K^n \rangle = \sum_K K^n \gamma_K. \quad \text{(14)}$$

The bending mode leads obviously to the distributions $\gamma_{K_j}$ grouped around low values of $K_j$ (the spins $J_j$ are aligned at the plane perpendicular to the fission axis), therefore

$$\langle K_j^2 \rangle < \frac{J_j(J_j + 1)}{3}, \quad \tau_{20}(J_j) < 0, \quad j = 1, 2, \quad \text{(15)}$$
but the twisting mode would populate the substates with high helicities (the spins \( J_j \) are oriented along the fission axis), therefore

\[
\langle K_j^2 \rangle > \frac{J_j(J_j+1)}{3}, \quad \tau_{20}(J_j) > 0, \quad j = 1, 2. \quad (16)
\]

The magnitude of spin-tensor \( \tau_{40}(J_j) \) of fourth rank is determined by the more subtle characteristics of the distribution \( \gamma_{K_j} \).

### 3 \( \gamma \) ray emission from aligned nuclei

According to the standard formalism [9] the angular distribution of \( \gamma \) rays of multipolarity \( L \) emitted from the aligned nucleus with spin \( J_i \) in its transition to the state with spin \( J_f \) is of the form

\[
w(\vec{n}_\gamma) = \frac{1}{4\pi} \sum_{Q=0,2,\ldots} (2Q + 1) C_{L1Q0}^{L1} U(J_f L J_i Q, J_i L) \cdot \tau_{Q0}(J_i) P_Q(\cos \theta_\gamma),
\]

\[
\oint d\Omega_\gamma w(\vec{n}_\gamma) = 1,
\]

where \( \theta_\gamma \) is the angle between the unit vector \( \vec{n}_\gamma \) along the \( \gamma \) ray momentum and the axis of nuclear alignment, \( U(\vec{a} \vec{b} \vec{c} \vec{d}, \vec{e} \vec{f}) \) is the normalized Racah function [13], \( P_Q(\cos \theta_\gamma) \) are Legendre polynomials

\[
P_2(\cos \theta) = \frac{1}{2}(3\cos^2 \theta - 1),
\]

\[
P_4(\cos \theta) = \frac{1}{8}(35\cos^4 \theta - 30\cos^2 \theta + 3), \ldots \quad (18)
\]

The excited fission fragments emit mainly \( \gamma \) rays of multipoarities \( e1, m1 \) and \( e2 \), so \( L < 2 \) and, therefore, the angular
distribution \( w(\vec{n}_\gamma) \) consists only of the terms corresponding to \( Q = 0, 2 \) and 4.

The mean number of \( \gamma \) rays emitted in spontaneous fission of \( ^{252}\text{Cf} \) is \( \langle N_\gamma \rangle = 9.35 \) \cite{14}. The deexcitation process of fission fragment after neutron emission is going via two stages (see Fig.1). At first the nucleus emits statistical \( \gamma \) rays of \( e1 \) or \( m1 \) multipolarities and falls in one of the yrast-line states, then it descents to the ground state in series of transitions between the yrast-line states. In each transition \( J_i \to J_f \) the spin-tensors of nuclear orientation decrease

\[
\tau_{Q0}(J_f) = U(LJ_fJ_iQ, J_iJ_f)\tau_{Q0}(J_i). \tag{19}
\]

The factor of decrease, for example, for quadrupole transition \( J_i \to J_f = J_i - 2 \) is

\[
U(2J-2JQ, J J-2) = \frac{1}{4J(J-1)(2J-1)} \left( \frac{A(J, Q)B(J, Q)}{(2J-3)(2J+1)} \right)^{\frac{1}{2}},
\]

\[
A(J, Q) = (2J+Q+1)(2J+Q)(2J+Q-1)(2J+Q-2),
\]

\[
B(J, Q) = (2J-Q)(2J-Q-1)(2J-Q-2)(2J-Q-3). \tag{20}
\]

It is easy to show that the angular anisotropy is caused mainly by the stretched \( e2 \) transitions (experimental evidences for this fact were obtained in Ref.\cite{15}). The Racah functions entering in Eq.17 for \( L = 2 \) and \( J_f = J_i - 2 \) are positive and are of the form (\( Q = 2, 4 \))

\[
U(J_f2J_i2, J_i2) = \left( \frac{2(J_i + 1)(2J_i + 3)}{7J_i(2J_i - 1)} \right)^{\frac{1}{2}}, \tag{21}
\]

\[
U(J_f2J_i4, J_i2) = \frac{1}{6} \left( \frac{2(J_i + 1)(J_i + 2)(2J_i + 3)(2J_i + 5)}{7J_i(J_i - 1)(2J_i - 1)(2J_i - 3)} \right)^{\frac{1}{2}}. \tag{22}
\]
Figure 1: The scheme of deexcitation of even-even deformed fragment.
At the same time the statistical dipole transitions go to the states with spins $J_f = J_i, J_i \pm 1$, for which we have ($Q = 2$)

$$U(J_f 1, J_i 2, J_i 1) = \frac{1}{(10J_i(J_i + 1)(2J_i + 3)(2J_i - 1))^2} \cdot \begin{cases} 
J_i(2J_i - 1), & \text{if } J_f = J_i + 1; \\
(3 - 4J_i(J_i + 1)), & \text{if } J_f = J_i; \\
(J_i + 1)(2J_i + 3), & \text{if } J_f = J_i - 1; 
\end{cases}$$

so the angular anisotropy in the transition $J_i \rightarrow J_i$ is opposite by the sign to the angular anisotropy in the transitions $J_i \rightarrow J_i \pm 1$, inasmuch as for $J_i \geq 1$

$$U(J_i 1 J_i 2, J_i 1) < 0, \quad U(J_i \pm 1 J_i 2, J_i 1) > 0.$$  

This sign variability of Racah function is due to the identity

$$\sum_{J_f} (2J_f + 1) U(J_f L J_i Q, J_i L) = \delta_{Q0}(2L + 1)(2J_i + 1)$$

for any values of $L, J_i$ and $Q$.

It is interesting that the angular distributions of $\gamma$ rays emitted in the series of stretched $e2$ transitions along the yrast line are the same for all transitions [16] notwithstanding that the spin-tensors of nuclear orientation decrease. Indeed, if the $\gamma$ ray angular distribution in the transition $J_i \rightarrow J_f = J_i - 2$ is described by the Eq.17, thus taking into account Eq.19 we obtain for the $\gamma$ ray angular distribution in the following transition $J_i' = J_i - 2 \rightarrow J_f' = J_i - 4 (L = 2)$

$$w(\vec{\gamma}) = \frac{1}{4\pi} \sum_{Q=0,2,4} (2Q + 1) C_{L1Q0}^{L1} U(J_i-4 L J_i-2 Q, J_i-2 L) \cdot U(L J_i-2 J_i Q, J_i J_i-2) \tau_{Q0}(J_i) P_Q(\cos \theta_{\gamma}).$$

(26)
Using the explicit expressions for Racah functions for $L = 2$ and $Q = 2, 4$ we get

$$U(J_i - 4 \ L \ J_i - 2 \ Q, \ J_i - 2 \ L) \ U(L \ J_i - 2 \ J_i \ Q, \ J_i \ J_i - 2) =$$
$$= U(J_i - 2 \ L \ J_i \ Q, \ J_i \ L), \quad (27)$$

therefore the angular distributions corresponding to the sequential transitions $J_i \rightarrow J_i - 2 \rightarrow J_i - 4$ coincide.

## 4 Estimate of $\gamma$ ray anisotropy

A great number of $e2$ transitions between the yrast-line states of the fragments from spontaneous fission of $^{252}\text{Cf}$ was investigated in Ref.\[15\]. Among the even-even fragments with high yields the nuclei $^{104}\text{Mo}$ and $^{144}\text{Ba}$ have typical spectra. The transitions $8^+ \rightarrow 6^+ \rightarrow 4^+ \rightarrow 2^+ \rightarrow 0^+$ correspond to the $\gamma$ ray energies 606 keV, 520 keV, 369 keV, and 193 keV in the first nucleus and 511 keV, 432 keV, 331 keV, and 199 keV in the second one. The above discussed “conservation” of the $\gamma$ ray angular anisotropy in the stretched yrast-line transitions gives the natural explanation of week dependence of angular distributions on the $\gamma$ ray energies found at Ref.\[6\] (see also Ref.\[14\], where the energy dependence of $\gamma$ ray angular distributions was investigated for the binary spontaneous fission of $^{252}\text{Cf}$).

Now we go to the analysis of the differences between the $\gamma$ ray angular distributions in binary and ternary fission. I assume that the shift of maximums from $0^0$ and $180^0$ to $20^0$–$40^0$ and $140^0$–$160^0$ with respect to the fission axis is due to the term proportional to $P_4(\cos \theta_\gamma)$. This requires the high value of spin-tensor $\tau_{40}(J_i)$. To study the sensitivity of $\gamma$ ray angular distribution to the fragment alignment we take the fragment helicity distribution in the Gaussian form (the spins are aligned perpendicular to the fission...
Figure 2: The spin-tensors $\tau_{20}(J)$ (solid line) and $\tau_{40}(J)$ (dashed line) of orientation of nucleus with spin $J=8$ versus parameter $\sigma$ of Gaussian helicity distribution.

The spin-tensors $\tau_{20}(J)$ and $\tau_{40}(J)$ of orientation of nucleus with spin $J=8$ as functions of parameter $\sigma$ are shown in Fig.2. The populations $\gamma_K$ of helicity states are presented in Fig.3 for $\sigma = 0.5, 2.5$ and $5.0$. The angular distributions of $\gamma$ rays emitted in quadrupole transition $J_i = 8 \rightarrow J_f = 6$ from the nucleus whose alignment is determined by the same values $0.5, 2.5$ and $5.0$ of parameter $\sigma$ are shown in Fig.4 together with the fragment of experimental data [6].

We see that the difference between
Figure 3: The populations $\gamma_K$ of substates with helicity $K$ for nucleus with spin $J = 8$ corresponding to $\sigma = 0.5$ (circles and solid line), 2.5 (squares and dashed line) and 5.0 (triangles and dotted line).
Figure 4: a. The calculated $\gamma$ ray angular distributions in the transition $J_i=8 \rightarrow J_f=6$ corresponding to $\sigma = 0.5$ (solid line), 2.5 (dashed line) and 5.0 (dotted line). b. The measured in Ref.[3] angular distributions of $\gamma$ rays with energies 540-656 keV from binary (crosses) and ternary (circles) spontaneous fission of $^{252}$Cf.
binary and ternary fission seems to follow from the difference between initial alignments of fission fragments. The angular distributions in the subsequent transitions $6 \to 4 \to 2 \to 0$ are the same as presented in Fig. 4 but the distributions of populations differ from that corresponding to $J = 8$. Fig. 5 represents the evolution of the initial Gaussian helicity distribution corresponding to $\sigma = 0.5$; we see that the change of populations is less than one might expect from the decrease of spin-tensors.

5 Conclusions

The data obtained in Ref. [6] demonstrate that $\alpha$ particle emission in ternary fission does not destroy the fragment alignment, but on the contrary, ternary fission strongly correlates with high alignment of fission fragments. This result may be understood on the assumption that the mechanism of ternary fission is closely related with the excitation of collective modes during the saddle-to-scission stage. For example, let us suppose that the $\alpha$ particle emission occurs only if the scission of bending fragments happens just at the moment when both fragments have the maximal angular velocities; in this case the $\alpha$ particle emission should correlate with high and well aligned fragment spins.

The proposed mechanism should lead to the increase of yield of fragments with high spins in ternary fission compared with binary one. This assumption may be checked by comparison between the yields of high spin isomers in ternary and binary fission (see, e.g., Ref. [17], where the relation between the yields of isomers and the initial fragment spins was established).

The angular distributions measured in Ref. [6] consist of the amounts from a great number of fragments with different level schemes. The interpretation of data on angular distributions of specific $\gamma$ rays emitted from even-even fragments (see Ref. [15]).
Figure 5: The populations $\gamma_K$ of substates with helicity $K$ for nucleus undergoing sequential quadrupole transitions between the states $J = 8 \rightarrow J = 6 \rightarrow J = 4 \rightarrow J = 2$ provided that the initial helicity distribution ($J = 8$) is of the Gaussian form corresponding to $\sigma = 0.5$. 
would be much more reliable. The results of calculations presented in Fig.4 correspond really to the transitions in a separated even-even fragment.

Recently the enhancement of ternary fission probability for uranium fissioning isomers was found [19]. In the framework of the suggested hypothesis this enhancement may be caused by the increase of probability of excitation of collective modes at the descent stage due to some peculiarities of isomer structure. By the same way the irregularities in total kinetic energy of fragments, observed in neutron induced fission near vibrational resonances [19], may be explained, but in this case the correlation between these irregularities and probability of ternary fission should be directly investigated.

The final remark concerns the fission fragment angular distribution. As it was claimed in introduction the nonaxial collective modes like bending or wriggling would distort the $K$ distribution formed at the saddle point. The comparison between angular distributions of fragments from binary and ternary fission of aligned nuclei may make this distortion evident if really the role of collective modes is more significant in ternary fission than in binary one.

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