Super electron acoustic propagations in critical plasma density

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\textbf{ABSTRACT}

The propagation of some new electrostatic cnoidal, super-solitons, super-periodic and shocklike waves in Earth magneto-tail electron acoustic (EA) plasma at critical density has been investigated via MKP equation. These MKP solutions are obtained by the method of expansion Jacobian elliptic function (EMJEF). Numerous of the given solutions are significant on the observations of broadband electrostatic nonlinear noise forms in layers of magneto-tail.

\section{1. Introduction}

The existence of EA electrostatic excitations in laboratories has been perceived \cite{1,2}. Various surveillance in space plasma assured the electrostatic EA propagation in heliospheric shock, auroral-zone, magnetospheres, geomagnetic tail and broadband noises \cite{3–10}. The EA generations were reported as an acoustic-kind having inertia specified by cold electron mass and restoring forces given by hot electrons pressure \cite{11,12}. Abdelwahed et al. studied the characteristic modulations of EAs in vortex-like electrons plasma by introducing time-fraction parameter which modified the soliton structures in nonlinear modified fraction equation \cite{13}. Pakzad investigated the hot nonextensive electron effects on EA properties using geometric coordinates as spherical and cylindrical EAs \cite{14}. It was advised that the spherical EA amplitudes become larger than the cylindrical EA amplitudes. In another study, the geometric EA characteristics were examined in critical nonlinear attitude by Gardner modified equations \cite{15}. Several EA electrostatic applications in many space systems have been explored \cite{16–20}.

The effect of variable magnetic field on the dissipative EA propagation without using sources of dissipation has been studied. It was reported that the backward oscillatory shocks were formed in plasmas. Also, the properties of these waves were studied using numerical real data \cite{21}. Fahad et al. studied the dispersion features of EA waves in non-relativistic quantum recoil with isotropic non-degeneracy system. The coefficients of absorption for Landau damping were discussed for laser-produced plasmas \cite{22}. Also, Yahia et al. investigated the head-on EA collisions in quasi-elastic case in magnetized nonthermal plasma. They studied some parameter effects on both the solitary EA solitons and collision phase in auroral zone of the Earth \cite{23}.

Recently, there is a huge development in analytical methods for getting solutions for NPDEs, see \cite{24–36} and references therein. So, the aim of this work is to use one of these methods to introduce new solution forms applicable for auroral zone observations.

\section{2. Physical model}

By considering a fluid plasma includes cold electron fluid and two temperature thermal ions which is governed by normalized equations \cite{17}:

\begin{equation}
\frac{\partial n_{ec}}{\partial t} + \frac{\partial}{\partial x}(n_{ec} u_{ecx}) + \frac{\partial}{\partial y}(n_{ec} u_{ecy}) = 0, \tag{1}
\end{equation}

\begin{equation}
\frac{\partial u_{ecx}}{\partial t} + u_{ecx} \frac{\partial}{\partial x}(u_{ecx}) + u_{ecy} \frac{\partial}{\partial y}(u_{ecy}) - \frac{\partial \phi}{\partial x} = 0, \tag{2}
\end{equation}

\begin{equation}
\frac{\partial u_{ecy}}{\partial t} + u_{ecx} \frac{\partial}{\partial x}(u_{ecy}) + u_{ecy} \frac{\partial}{\partial y}(u_{ecy}) - \frac{\partial \phi}{\partial y} = 0, \tag{3}
\end{equation}

\begin{equation}
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} - \left( n_{ec} - \mu \exp \left( -\frac{\phi}{\mu + v_\beta} \right) \right) - \gamma \exp \left( \frac{-\beta \phi}{\mu + v_\beta} \right) = 0, \tag{4}
\end{equation}

where $T_{hi}(T_i)$ is high (low) temperature ion at density $\mu$, $\gamma$ with $\beta = \frac{T_i}{T_h}$. Using stretched coordinates $\xi = e(x - \lambda t)$, $\tau = e^3 t$, $\eta = e^2 y$, where $e$ is a small value and $\lambda$ is
the EA speed, the model reaches critical density $\mu = \mu_c$ which means that the nonlinear coefficient vanished. So, the modified-KP equation (mKP) reads:

$$\frac{\partial}{\partial \xi} \left( \frac{\partial \phi}{\partial t} + G \phi^2 \frac{\partial \phi}{\partial \xi} + R \frac{\partial^3 \phi}{\partial \xi^3} \right) + Q \frac{\partial^2 \phi}{\partial \eta^2} = 0$$

(5)

with

$$\mu_c = \frac{-\lambda^4 + \beta^2 \lambda^4 \pm (\beta - 1) \lambda^2}{2(-3\beta(\beta - 2) - 3)},$$

(6)

$$G = \frac{1}{2} \left( -\frac{3\nu \beta^2}{2(\mu + \nu \beta)^2} - \frac{3}{\lambda^4} \right),$$

$$Q = \frac{\lambda}{2}, \quad R = \frac{\lambda^3}{2}.$$  

(7)

By using a similarity transformation in the form

$$\chi = L \xi + M \eta - \tau (\nu_1 + \nu_2),$$

$$\phi(\chi) = \phi(x, y, t)$$

(8)

$$\tau = \tau,$$

(9)

the transformed MKP equation to the ODE form is

$$3(S - v) \phi + \delta \phi^3 + 3\sigma \frac{d^2 \phi}{d \chi^2} = 0,$$

(11)

$$S = \frac{M^2 Q}{L} - u, \quad \delta = GL, \quad \sigma = RL^3,$$

(12)

where $u$ and $v$ are travelling speeds in both directions.

### 3. Closed-form solutions

Now we give the closed form solutions for equation

$$\Lambda_1 \Xi'' + \Lambda_2 \Xi^3 + \Lambda_3 \Xi = 0,$$

(13)

based on the Jacobian elliptic functions [37, 38]. Balancing $\Xi''$ and $\Xi^3$, gives $m = 1$. Thus the solution of Equation (13) takes the form:

$$\Xi = a_0 + a_1 \text{sn}(\eta) + b_1 \text{cn}(\eta),$$

(14)

where $a_0, a_1$, and $b_1$ are parameters. From Equation (14), we get

$$\Xi' = a_1 \text{cn}(\eta) \text{dn}(\eta) - b_1 \text{sn}(\eta) \text{dn}(\eta),$$

$$\Xi'' = -m^2 \text{sn}(\eta) a_1 + 2a_1 \text{sn}(\eta)^2 m^2$$

$$+ 2m^2 \text{sn}(\eta)^2 \text{cn}(\eta) b_1 - a_1 \text{sn}(\eta) - b_1 \text{cn}(\eta).$$

(15)

(16)

Superposing Equations (14)–(16) into Equation (13) and setting all coefficients of $\text{sn}^3, \text{sn}^2 \text{cn}, \text{sn}^2, \text{scn}, \text{sn}, \text{cn}, \text{cn}^{0}$ to zero, gives a system of algebraic equations. Solving these equations, yields:

Case 1.

$$a_0 = 0, \quad a_1 = \pm \sqrt{-\frac{2\Lambda_1}{\Lambda_2}} m, \quad b_1 = 0,$$

$$\Lambda_3 = \Lambda_1(1 + m^2).$$

Hence the first family solution is

$$\Xi_1(x, t) = \pm \sqrt{-\frac{2\Lambda_1}{\Lambda_2}} m \text{sn}(\eta).$$

(17)

When $m \to 1$, Equation (17) becomes

$$\Xi_1(x, t) = \pm \sqrt{-\frac{2\Lambda_1}{\Lambda_2}} \text{tanh}(\eta).$$

(18)

Case 2.

$$a_0 = 0, \quad a_1 = \pm \sqrt{-\frac{\Lambda_1}{2\Lambda_2}} m, \quad b_1 = -\sqrt{-\frac{\Lambda_1}{2\Lambda_2}} m,$$

$$\Lambda_3 = \frac{1}{2} \Lambda_1(2 - m^2).$$

Hence the second family solution is

$$\Xi_2(x, t) = \pm \sqrt{-\frac{\Lambda_1}{2\Lambda_2}} m \text{sn}(\eta) - \sqrt{-\frac{\Lambda_1}{2\Lambda_2}} m \text{cn}(\eta).$$

(19)

When $m \to 1$, Equation (19) becomes

$$\Xi_2(x, t) = \pm \sqrt{-\frac{\Lambda_1}{2\Lambda_2}} \text{tanh}(\eta) - \sqrt{-\frac{\Lambda_1}{2\Lambda_2}} \text{sech}(\eta).$$

(20)

Case 3.

$$a_0 = 0, \quad a_1 = \pm \sqrt{-\frac{\Lambda_1}{2\Lambda_2}} m, \quad b_1 = \sqrt{\frac{\Lambda_1}{2\Lambda_2}} m,$$

$$\Lambda_3 = \frac{1}{2} \Lambda_1(2 - m^2).$$

Hence the third family solution is

$$\Xi_3(x, t) = \pm \sqrt{-\frac{\Lambda_1}{2\Lambda_2}} m \text{sn}(\eta) + \sqrt{-\frac{\Lambda_1}{2\Lambda_2}} m \text{cn}(\eta).$$

(21)

When $m \to 1$, Equation (21) becomes

$$\Xi_3(x, t) = \pm \sqrt{-\frac{\Lambda_1}{2\Lambda_2}} \text{tanh}(\eta) + \sqrt{-\frac{\Lambda_1}{2\Lambda_2}} \text{sech}(\eta).$$

(22)

Case 4.

$$a_0 = 0, \quad a_1 = 0, \quad b_1 = \pm \frac{2\Lambda_1}{\Lambda_2} m,$$

$$\Lambda_3 = \Lambda_1(1 - 2m^2).$$

Hence the fourth family solution is

$$\Xi_4(x, t) = \pm \sqrt{\frac{2\Lambda_1}{\Lambda_2}} m \text{cn}(\eta).$$

(23)

As long as $m \to 1$, Equation (23) becomes

$$\Xi_4(x, t) = \pm \sqrt{\frac{2\Lambda_1}{\Lambda_2}} \text{sech}(\eta).$$

(24)
4. Results and discussion

In this section, we employed the closed-form solution to solve Equation (5). Comparing this equation with Equation (13) yields $\Lambda_1 = 3\sigma$, $\Lambda_2 = \delta$ and $\Lambda_3 = -3(\upsilon - s)$. Thus the solutions of Equation (11) are:

Case 1. The first family of solution is

$$\phi_1(x, t) = \pm \sqrt{-\frac{6\sigma}{\delta}} m \text{sn}(x - (s - \sigma(1 + m^2))t), \ (25)$$

when $m \to 1$, the solutions of Equation (25) become

$$\phi_1(x, t) = \pm \sqrt{-\frac{6\sigma}{\delta}} \tanh(x - (s - 2\sigma)t). \ (26)$$

Case 2. The second family of solution is

$$\phi_2(x, t) = \pm \sqrt{-\frac{3\sigma}{2\delta}} m \text{sn}\left(x - \left(s - \frac{1}{2}\sigma(2 - m^2)\right)t\right)$$

$$- \sqrt{\frac{3\sigma}{2\delta}} m \text{cn}\left(x - \left(s - \frac{1}{2}\sigma(2 - m^2)\right)t\right). \ (27)$$

When $m \to 1$, the solutions of Equation (27) become

$$\phi_2(x, t) = \pm \sqrt{-\frac{3\sigma}{2\delta}} \tanh\left(x - \left(s - \frac{1}{2}\sigma\right)t\right)$$

$$- \sqrt{\frac{3\sigma}{2\delta}} \text{sech}\left(x - \left(s - \frac{1}{2}\sigma\right)t\right). \ (28)$$

Case 3. The third family of solutions is

$$\phi_3(x, t) = \pm \sqrt{-\frac{3\sigma}{2\delta}} m \text{sn}\left(x - \left(s - \frac{1}{2}\sigma(2 - m^2)\right)t\right)$$

$$+ \sqrt{\frac{3\sigma}{2\delta}} m \text{cn}\left(x - \left(s - \frac{1}{2}\sigma(2 - m^2)\right)t\right). \ (29)$$

When $m \to 1$, the solutions of Equation (29) become

$$\phi_3(x, t) = \pm \sqrt{-\frac{3\sigma}{2\delta}} \tanh\left(x - \left(s - \frac{1}{2}\sigma\right)t\right)$$

$$+ \sqrt{\frac{3\sigma}{2\delta}} \text{sech}\left(x - \left(s - \frac{1}{2}\sigma\right)t\right). \ (30)$$

Case 4. The fourth family of solutions is

$$\phi_4(x, t) = \pm \sqrt{\frac{6\sigma}{\delta}} m \text{cn}(x - (s - \sigma(1 - 2m^2))t). \ (31)$$

When $m \to 1$, the solutions of Equation (31) become

$$\phi_4(x, t) = \pm \sqrt{\frac{6\sigma}{\delta}} \text{sech}(x - (s + \sigma)t). \ (32)$$

In thermal plasma mode, two dimension EA solitons are described by the MKP equation (5). The EA solutions have been examined using parameters for Earth magneto-tails [16, 17]. At $\mu = \mu_0$, MKP solutions can depict the system features using EMJEF method which allows many solution groups. Equation (25) is a solution group $\phi_c = q_1(x; t)$ which have solutions depend on the model constants and parameters as shown in Figures 1–4. The first solution is cnoidal wave potential plotted for varying $\chi$ and $L$ as shown in Figures 1 and 2. It was founded that the cnoidal amplitude increases by $L$ with positive phase change for $\lambda = 1$ and negative phase change for $\lambda = -1$. The second solution introduces a shock wave as in Figures 3 and 4. It was shown that the shock amplitude increases by $L$ with negative phase change for $\lambda = 1$ and positive phase
change for $\lambda = -1$. Equations (27) and (29) are various solutions $\phi_c = q_2(x; t)$ and $\phi_c = q_3(x; t)$ that distinguish diverse solitary waveforms. On the other hand, an important solitary forms $\phi_c = q_4(x; t)$ Equation (31) which have three solutions. The potential change with $\chi$ and $L$ is shown in Figures 5–10. Figures 5 and 6 represent the soliton train. Moreover, super-solitary and soliton structures are given in Figures 7–10. Consequently, for all structures of Equation (31), it was assured that the direction parameter $L$ increases the train, super-solitary and soliton amplitudes with negative changes in wave phase for $\lambda = 1$ and positive changes in wave phase for $\lambda = -1$ as depicted in Figures 7–10.

5. Conclusion

Using EMJEF technique, several wave generations such as solitons, cnoidal, shocks, solitary trains and super-solitons are characterized by the MKP equation that defined EA progress in thermal plasma mode. It was confirmed that the $L$ direction can raise wave amplitude in plasma system, Otherwise, EA speed $\lambda$ is controlled by
the wave phase. So, the obtained results may apply to study the EA electrostatic observations and broadband magneto-tails.

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