QUANTUM HALL EFFECT AT FINITE TEMPERATURES

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Recent work on the temperature-driven delocalization in the quantum Hall regime is reviewed, with emphasis on the correlation properties of disorder and the role of electron-electron interactions.

1 Introduction

The underlying physics of the quantum Hall effect (QHE) is understood in terms of the Anderson localization in two dimensions (2D): a weakly disordered 2D electron gas experiences a series of metal-insulator transitions with increasing magnetic field $B$. The field-induced transitions are degenerate in the sense that the metallic phase occurs only at some particular values of $B$. At zero temperature, the dissipative conductivity $\sigma_{xx}(B)$ vanishes everywhere except at a discrete set of the critical points, where it exhibits peaks of zero width. My purpose is to summarize recent work showing how this “ideal” picture of QHE evolves with lowering $T$. Let us first go over some key features of the localization in QHE.

At $T = 0$, the crossover to the QHE regime should occur, for weakly interacting electrons, at $\lambda/l \sim 1$, where $\lambda$ is the magnetic length, $l$ the mean free path. Yet, making contact with most of experiments, we are accustomed to treating QHE in the extreme of high $B$, where disorder-broadened Landau levels are well separated from each other. The number of conducting phases, which arise in succession as the Fermi level sweeps through a single Landau level, is determined by competition between electron-electron interactions and disorder. Since, to date, there exists no reliable theory of localization in the fractional QHE, I restrict the discussion to the integer QHE with weak enough interactions. Then there is only one extended state per Landau level, and its energy $E_c$ coincides with the center of the level. Within this framework, the localization length $\xi(E)$ diverges as $|E - E_c|^{-\gamma}$, where $\gamma \approx 2.3$.

Naturally, in a close vicinity of $E_c$, the critical exponent $\gamma$ should not depend on the correlation radius of disorder, $d$; however, the range of the universal scaling behavior shrinks if $\lambda \ll d$. For the most interesting case of the lowest Landau level, $\xi(E)$ then scales as $\xi(\Delta_t)(\Delta_t/|E - E_c|)^{\gamma}$, only in a narrow band of the width $\Delta_t = \Gamma(\lambda/d)^2$ around $E_c$, $\Gamma$ being the width of the Landau level. I refer to this range of $E$ as the “tunneling band”, for within it the percolating classical trajectories, which are closed equipotentials in the high-$B$ limit, are strongly coupled via tunneling through saddle points.
of the random potential. Outside this band, the localization remains classical in the sense that one may neglect the coupling between the critical percolating trajectories, the characteristic radius of which behaves as \( R(E) \sim d(\Gamma/|E - E_c|)^{4/3} \). These trajectories almost touch each other at the critical saddle points so as to form the percolation network. However, provided \(|E - E_c| \gg \Delta_t\), the gaps that separate them are typically much wider than \( \lambda \) (to generate the power-law scaling, the critical saddle points at given \( E \) should have energies of the order of \(|E - E_c|\)), and so the tunneling plays no role. At the crossover point between the regimes of the quantum and classical localization, \( \xi(\Delta_t) \sim d(d/\lambda)^{8/3} \). Clearly, this picture rules out any relation between \( \gamma \) (which is close to \( 7/3 \)) and the critical exponent of \( R(E) \) (exactly equal to \( 4/3 \)). In the tunneling-dominated regime, the conducting network is essentially represented by that of the Chalker-Coddington model with the characteristic size of the elementary cell \( \xi(\Delta_t) \), independent of \( E \). A weak tunneling through chains of non-critical saddle points does not affect the topology of the network.

2 Temperature Scaling: Interactions vs Disorder

In this section, I present an outline of the problem of how \( \sigma_{xx} \) peaks broaden with temperature in the limit of short-range disorder. Since the localization at \( E \to E_c \) originates from the interference of multiple scattered waves on the large scale of \( \xi(E) \), the problem may be analyzed in terms of the phase breaking. The width of the energy band within which the localization fails to develop, \( \Delta_c(T) \), then obeys the simple scaling relation \( \xi(\Delta_c)/L_\phi \sim 1 \), where \( L_\phi \) is the dephasing length at the critical energy. The notion of the critical broadening of the \( \sigma_{xx} \) peaks implies that the inelastic scattering is strong enough, namely \( \Delta_c(T)/T \gg 1 \).

In trying to apply, after simple modifications, the standard theory of the phase breaking in dirty metals to the metallic phase in the QHE limit, one can immediately see that the Fermi liquid picture may only be marginally valid. Specifically, it appears that the crucial parameter \( \bar{\hbar}/T \tau_\phi \sim 1 \), \( \tau_\phi \) being the phase-breaking time associated with electron-electron interactions. In this circumstance, we are only able to say what is the scaling behavior of \( L_\phi \):

\[
L_\phi(T) \sim \lambda(\Gamma/T)^{1/2}.
\]  

Doing so, however, we run into a difficulty: for the critical exponent of \( \Delta_c \propto T^\kappa \), Eq. (1) gives \( \kappa = 1/2\gamma \), which is approximately half of the thoroughly measured, in samples with short-range disorder, value \( \kappa \simeq 0.4 \). To fix this flaw, we argue that there exists a crossover temperature \( T_c \sim U^2/\Gamma \), below which the Coulomb interaction between electrons cannot be treated perturbatively, however weak it is in comparison with disorder. Here \( U \) characterizes the strength of the interaction and scales as \( e^2/\varepsilon \lambda \), \( \varepsilon \) being the dielectric constant. We have
to assume the condition $U/\Gamma \ll 1$, in order to ensure that the interaction does not break down the integer QHE.

We first make the elementary observation that the Maxwell relaxation of a charged wave packet, built up from eigenstates with energies close to $E_c$, is characterized, in the quasi-2D geometry, by the constant velocity of the charge spreading $v_s = 2\pi \sigma_{xx}^p/\varepsilon$. Here $\sigma_{xx}^p \sim e^2/h$ stands for the peak value of the conductivity in the metallic phase. It follows that the dynamics of charged excitations on scales larger than $L_c \sim D_0/v_s \sim \lambda\Gamma/U$ is no longer diffusive ($D_0 \sim \lambda^2\Gamma/h$ is the bare diffusion coefficient, $L_c$ has the meaning of a screening radius); instead, it is governed by the interactions. In particular, the spectral function for the screened density-density correlator in the limit of small $q$ takes the form

$$S(\omega, q) = \left(\frac{\varepsilon q}{2\pi e^2}\right) \times \left[\frac{v_s q}{\pi \left(\omega^2 + v_s^2 q^2\right)}\right]$$

at criticality, the condition of small $q$ depends on $\omega$: $q$ should be small as compared to both $L_c^{-1}$ and $\omega/v_s$; if $qv_s/\omega \gg 1$, the effective velocity $v_s$ in Eq. (2) becomes a function of the ratio $q/\omega$ (a similar renormalization of the effective diffusion constant at $U = 0$ was studied in Ref. 8); note also that we deal with spatial scales smaller than $\xi(E)$ at all $E$ involved (cf. Ref. 9). This contrasts with the dynamical properties of non-interacting electrons, which are controlled, at the critical point, by the dynamical susceptibility depending on the single parameter $q^2D_0/\omega$. Eq. (2) describes the Coulomb correlations between charged density fluctuations (originating from the dynamical screening). The relation $\hbar/T\tau_\phi \sim 1$ still holds; however, the Coulomb correlations should affect the dynamical scaling at criticality, unless $L_\phi/L_c \ll 1$, i.e. $T/T_c \gg 1$. Given that in the interaction-dominated regime the dynamical scaling can be explained in terms of the charge-spreading velocity, $L_\phi$ at lower $T$ obeys the relation

$$L_\phi(T) \sim v_s \tau_\phi \sim e^2/\varepsilon T$$

(borrowing the terminology of thermally driven phase-transitions, one can say that the dynamical critical exponent $z = 1$). This last expression looks universal since $\sigma_{xx}^p$ does not depend on the strength of disorder. It is worth noting that the assumption of the long-range interaction is absolutely crucial in the picture under discussion. Hence we find that at $T/T_c \ll 1$

$$\Delta_c \sim \Gamma(T/U)^{1/\gamma},$$

which is in decent agreement with the low-temperature data on samples with short-range disorder. In the high-$T$ limit, one should expect a crossover to the regime of broadening governed by electron-phonon interactions.

Now for the localization-length exponent $\gamma$, which we keep the same as for non-interacting electrons. To see how it comes about that the interaction
strongly influences the dynamical behavior of electrons at criticality, and yet does not change \( \gamma \), we recall that, at \( T = 0 \), even a weak Coulomb interaction between localized electrons makes the one-particle density of states \( g(E) \) vanish at the Fermi level \( E_F \). In two dimensions, \( g(E) = c|E - E_F|^{\varepsilon^2/e^4} \), where \( c \sim 1 \) is a universal constant. At this point we can already identify \( T_c \) with the width of the gap in \( g(E) \) deep in the insulating phase. The concept of the Coulomb gap was developed for classical electrons, i.e., point charges. It is evident, however, that the same line of argument applies near the metal-insulator transition as well, only in the range \( |E - E_F| \ll e^2/\varepsilon \xi(E) \). Hence, rather remarkably, \( g(E_F) \) vanishes however small \( |E_F - E_c| \) is. As a consequence, whatever \( E_F \), the Coulomb energy on the scale of the one-particle localization length \( \xi(E_F) \) is of the order of the characteristic energy spacing \( \delta_c \sim 1/g(E_F + \delta_c)\xi^2(E_F) \) on the same scale. Within the Coulomb glass approach, this naturally implies that the long-range interaction merely leads to the repulsion of the levels of the one-particle states, but it cannot affect the critical behavior of \( \xi(E_F) \). This conclusion can be reached on more phenomenological grounds, though essentially by a similar dimension counting. The scaling behavior of \( \xi(E_F) \) has been observed by numerical simulation within the Hartree-Fock scheme. It is worth emphasizing that the range of energies

\[
|E - E_F| \ll e^2/\varepsilon \xi(E_F) ,
\]

where the above arguments about the stability of \( \xi(E) \) are true, shrinks as \( E_F \to E_c \). Thus the question as to the localization properties of the excitations at \( E_F = E_c \) remains open. Particularly, it is not clear what kind of the gap in \( g(E) \) is observed in the numerical simulations, when \( E_F \) is tuned to be precisely \( E_c \). It is possible that the gap at \( E_F = E_c \) is a reminiscence of the sharper “polaronic” gap, which pertains to clean systems. Yet, there is no doubt that the true Coulomb gap survives in the range given by Eq. (5), if \( E_F \neq E_c \). We conclude, therefore, that it is legitimate to exploit the idea of the one-particle localization at \( |E_F - E_c| \sim \Delta_c(T) \), since \( |E - E_F| \) does not exceed \( e^2/\varepsilon \xi(\Delta_c) \) within the range of the temperature smearing of the Fermi distribution.

Thus, near the metal-insulator transition, the notion of the Coulomb gap matches that of the charge spreading. More specifically, one can introduce the scale-dependent diffusion coefficient \( D(L) \sim (e^2/\hbar \varepsilon)L \) [see Eq. (2)], such that the one-particle density of states on the scale of \( \xi(E_F) \) obeys the relation \( \hbar g D(\xi) \sim 1 \). We recognize the latter as a familiar localization criterion (cf. Ref. 17). It is worthwhile to notice that, in this formula, both the diffusion coefficient and the one-particle density of states are renormalized by the interactions, whereas the Einstein relation links \( \sigma^{xx}_0 \) and the bare coefficient \( D_0 \) via the thermodynamic density of states. At \( T \gg T_c \), the Coulomb gap is full to the brim and the Coulomb correlations do not control \( L_\phi(T) \) any more (one can then say \( z = 2 \)).
The same behavior of $\Delta_c(T)$ is recovered if we use $L_h(T)/\xi(E)$ as a scaling variable instead of $L_\phi(T)/\xi(E)$, $L_h \sim [\xi(E)L_\phi]^{1/2}$ [with $L_\phi$ given by Eq. (3)] being the typical hopping length in the insulating phase. According to this approach, the conductivity may be represented as $\sigma_{xx}(x) = (e^2/h)F(x)\exp(-x)$, where $F(x)$ is a dimensionless power-law function of the single parameter $x = L_h(T)/\xi(E_F)$. The advantage here is that the dependence $\ln \sigma_{xx} \sim -(e^2/\varepsilon\xi(E_F)T)^{1/2}$ can be microscopically grounded in very general terms for the variable-range hopping regime. It is clear, however, that this scaling form of $\sigma_{xx}$ explicitly implies the strong coupling limit, in the sense that it is only valid for $\hbar/T\tau_\phi \sim 1$. Otherwise, say if the delocalization would come from weak interactions with phonons, $\sigma_{xx} \ll \sigma_{pxx}$ at $x \sim 1$, which means that there exists an intermediate, power-law hopping regime between the metallic phase and that of the variable-range hopping.

In the critical broadening regime, $\sigma_{pxx}(T)$ displays only a weak temperature dependence; however, it might be interesting in its own right. We first notice that $\sigma_{pxx}$ is a poorly defined quantity in coherent samples at zero $T$. Indeed, the conductance should show strong, sample specific fluctuations of the order of $e^2/h$ (p. 252 in Ref. 1). Thus the widely accepted notion that the conductivity $\sigma_{pxx}$ at $T \to 0$ has a universal value may only be relevant if $L_\phi$ is much shorter than the sample size. We take this limit so as to deal with the self-averaging $\sigma_{pxx}$. Then one can identify two contributions to the temperature deviation $\delta\sigma_{pxx}(T)$. One is related to the temperature smearing of the Fermi distribution and is apparently $\sim -(e^2/\hbar)(T/\Delta_c(T))^2$. It is negative and scales as $T^{2(1-\kappa)}$. The other is similar to the weak-localization correction, with the difference that the expansion in terms of the small parameter $\lambda/L_\phi$ should be done around the critical point. We obtain

$$\delta\sigma_{pxx} \sim (e^2/h)(\lambda/L_\phi)^D \propto T^x, \quad x = \kappa\gamma D_2,$$

(6)

where $D_2$ may be defined as a generalized dimension of the critical eigenstates. For non-interacting electrons $D_2 = 2 - \eta$, where $\eta \simeq 0.4$ is the critical exponent of eigenfunction correlations. The problem is more delicate in the interaction-dominated regime ($T \ll T_c$): within the framework of the above approach, however, the fractal dimension $D_2$ in Eq. (6) is just half that for non-interacting electrons. Thus in both cases $x = 1 - \eta/2$. The scaling arguments imply that the “weak delocalization” correction dominates at $T \to 0$. Strictly speaking, near the critical point, the sign of the correction cannot be obtained within the power-counting analysis. According to numerical simulations, however, the correction in the non-interacting case should be negative for the lowest Landau level $N = 0$. Yet, $\delta\sigma_{pxx}$ is likely to be positive for $N \geq 1$ (since $\sigma_{pxx}$ exceeds the SCBA value only at $N = 0$). Note that $\delta\sigma_{pxx}$ for higher Landau levels may be much larger than $e^2/h$, so that at $\delta\sigma_{pxx} \sim e^2/h$ a crossover to the logarithmic dependence on $T$ should occur (in the high-$T$ regime, the
"conventional" weak-localization correction is $h\sigma_{xx}^p/e^2$ times smaller than that originating from the electron-electron interaction).

We might be tempted to conclude that the above concept of the phase breaking due to the electron-electron interaction completely accounts for the broadening of $\sigma_{xx}$ peaks at low $T$. In fact, it is by no means obvious that at finite $T$ the interaction alone is able to delocalize electrons in the QHE regime. The subtlety is that we treat the problem of the phase breaking self-consistently, on the assumption that the inelastic scattering occurs due to electromagnetic fluctuations produced by delocalized excitations (Nyquist noise). This approach works perfectly in the weak-localization theory, but in the QHE it still constitutes a challenging problem because of its non-perturbative nature.

3 Long-Range Disorder: Brownian Motion in a Stream

So far we have dealt with the phase-breaking effects in a short-range random potential, in which case $\Delta_c(T)/T \gg 1$. As argued in the introduction, increasing the correlation radius of disorder $d$ brings the new energy scale $\Delta_t = \Gamma(\lambda/d)^2$ into play. The ratio $\Delta_c(T)/\Delta_t$ then becomes relevant; specifically, $\Delta_c(T)$ exhibits the universal scaling behavior in terms of $\lambda/L_\phi$ only as long as $\Delta_c(T)/\Delta_t \ll 1$. At $T \ll \Delta_t$, $\Delta_c(T)$ either obeys the relation $\xi(\Delta_c)/L_\phi \sim 1$ or saturates at the level of $\Delta_t$. The crucial observation is that the range of $E$ within which the inelastic scattering prevents the localization, $|E - E_c| \sim \Delta_c(T)$, cannot exceed $T$ if $\Delta_c(T)$ becomes wider than the tunneling band. For this reason, the width of the $\sigma_{xx}$ peak $\Delta \nu(T) \sim \max\{\Delta_c(T), T\}/\Gamma$, $\nu$ being the filling factor, grows linearly with $T$ at $T \gg \Delta_t$, irrespective of any particular mechanism of the inelastic scattering. A sample dependent behavior of $\Delta \nu(T)$, with a crossover to higher $\kappa$’s as $T$ is increased, indeed was observed in samples with long-range potential fluctuations.

In the classical regime, the following Fokker-Planck equation typifies the entire problem:

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial \rho} \left[ \mathbf{v} f - D_t \left( \frac{\partial f}{\partial \rho} + \frac{\hbar}{\lambda^2} \frac{\partial f}{\partial E} \right) \right] = 0 \, ,$$

where $\mathbf{v}(\rho)$ is the drift-velocity field, $\partial \mathbf{v}/\partial \rho = 0$, $\mathbf{b}$ is the unit vector along the magnetic field. Despite having a simple form, the equation does not allow for any perturbative treatment of the inelastic scattering diffusion, characterized by the temperature-dependent coefficient $D_t(T)$. The point is that, right at the percolation transition, the critical trajectories are strongly coupled by arbitrarily small $D_t$. The width of the conducting band then satisfies the self-consistent equation $\Delta_c \sim \Gamma[D_t/\Omega(\Delta_c)^2]^{1/2}$, $\Omega(E)$ being the inverse period of the critical trajectories with the energy $E$. The fractal dimensionality of the percolating trajectories is known to be $7/4$, which gives $\Delta_c \sim \Gamma(D_t/D_0)^p$. 

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with \( p = 3/13 \) (in the case of smooth disorder, \( D_0 \) can be re-expressed as \( D_0 \sim <v^2>^{1/2}d \)). Accordingly, \( \sigma_{xx}^p \) falls off with increasing \( T \) as
\[
\sigma_{xx}^p \sim (e^2/h)(\Gamma/T)[D_i(T)/D_0]^p.
\] (8)

The self-consistent treatment is necessary only as long as \( \Delta_c(T)/T \ll 1 \). In the opposite limit, \( 1 \gg D_i/D_0 \gg (T/\Gamma)^{1/p} \), the height of the peak is determined by the conductivity of the percolation network built up from trajectories with \( |E - E_c| \sim T \); apparently, this yields \( \sigma_{xx}^p \sim e^2/h \). Interestingly, \( D_i \) should be large enough for this regime to occur, yet neither \( \sigma_{xx}^p \) nor \( \Delta \nu \) depends on \( D_i \) in this case. Whatever \( D_i/D_0 \) is, in the classical limit \( \Delta \nu \sim T/\Gamma \). A noteworthy feature of the classical delocalization is also the strong enhancement of \( \sigma_{xx}^p \) as compared to the bare value \( e^2D_i/\lambda^2T \). Notice that the problem can be equivalently formulated in terms of inhomogeneous local conductivities. The above picture of percolation implies a sharp Fermi distribution for percolating particles; however, there remains a challenging question: To what extent is the self-consistent approach adequate in the strongly correlated electron system with long-range disorder?

4 Plateau Regime

In the case of short-range disorder and weak electron-phonon interactions, one can hardly expect any fascinating features of \( \sigma_{xx}(T) \) deep in the insulating phase at \( |E_F - E_c| \gg \Gamma \gg T \). Indeed, \( \sigma_{xx} \) then behaves as \( \sigma_0 \exp(-|E_F - E_c|/T) \) due to activation to the energy level \( E_i(T) \), such that \( |E_i(T) - E_c| \sim \Gamma \ln^{1/2}(T/\Gamma) \). The pre-exponential factor \( \sigma_0 \) is small in comparison with \( e^2/h \) and depends on the electron-phonon coupling constant, though weakly. That is why a lively debate arose concerning the universal activated behavior \( \sigma_{xx} = (e^2/h)\exp(-\Delta/T) \), which was reported in a number of experiments. The universality of \( \sigma_0 \) seemed to be intriguing since the activated conductivity occurs solely due to supposedly weak interactions with phonons. A theory that explained this feature was forthcoming in terms of the classical dynamics in a long-range random potential. If \( d \gg \lambda \), \( \sigma_{xx} \) is governed by the activation already at \( |E_F - E_c| \gg T \gg \Delta_i \), once the Fermi level leaves the tunneling band. Notice, since both the activation exponent and the tunneling one are linear functions of \( E \), the necessary condition of that the activation dominates is simply \( T \gg \Delta_i \). Again, we identify the crucial parameter \( \Omega(T)\tau_T \), where \( \tau_T \) is the time it takes for an electron to change its energy by \( T \). The parameter is small provided \( D_i/D_0 \gg (T/\Gamma)^{1/p} \). In this limit \( \Delta_c \) scales as \( T \). The essential idea of this approach is to represent the electron system as a random network of thermal reservoirs connected via ballistic contacts (critical saddle points) with energies \( V_i \) scattered around \( E_c \) in the tail of the Fermi distribution. This parallels the Landauer-Büttiker formalism, with the difference that the
latter was developed for the edge states. The ballistic conductances of the contacts, \( G_i = (e^2/h) \exp(-|E_c - E_F + V_i|/T) \), exhibit strong asymmetric fluctuations. However, since \( \ln G_i \) are distributed randomly, the conductivity of the network satisfies the exact relation \( \sigma_{xx} = \exp(\langle \ln G_i \rangle) \) (valid in 2D only), which immediately gives \( \sigma_0 = \frac{e^2}{h} \). If \( \Omega(T)\tau_T \gg 1 \), the conducting band \( \Delta_c(T) \) gets narrower than \( T \). To put it simply, electrons with \( \Delta_c \ll E - E_c \) are now out of play as they have no time to tune their electrochemical potential so that it equals the potential of the adjacent thermal reservoir. Apparently, \( \sigma_0 = \sigma_{xx}^p \) in the extreme of high \( T \) [see Eq. (8)].

5 Overlapped Levels

According to the Drude formula, \( \sigma_{xx} \) grows with increasing overlap of Landau levels. For this reason, \( \xi(E) \) in the middle between adjacent \( \sigma_{xx} \) peaks acquires an exponentially large factor, such that \( \ln \xi \) scales as \( (\hbar \sigma_{xx}/e^2)^2 \). This quickly breaks down QHE at any reasonable \( vT \) (unless the \( \sigma_{xx} \) peaks are due to the classical percolation, which is probably the case in high-mobility samples). What happens if the number of the coupled levels is limited to two? Let two Zeeman levels be strongly overlapped in the sense that \( \Gamma \gg \Delta_s \), where \( 2\Delta_s \) is the difference of the critical energies \( E_{\pm c} \) corresponding to two spin projections. Then turning on a spin-orbit (SO) interaction drives two systems of electrons with opposite spin into a new quantum Hall phase with an internal degree of freedom. We present arguments that the SO coupling is able to greatly facilitate the inelastic-scattering-induced delocalization.

In the case of short-range disorder, \( \Delta_s \) obeys the relation

\[
\xi(E_0)/R_{so} \sim 1,
\]

where \( E_0 = \frac{1}{2}(E_c^+ + E_c^-) \), \( R_{so} \) is the spin-flip scattering length. It follows that, for the weak SO interaction, \( \Delta_s \) is far larger than the local SO splitting. Numerical simulations carried out in this limit support the conclusion that the critical behavior of \( \xi(E) \) at \( E \rightarrow E_{\pm c} \) remains the same as for spinless electrons. Hence, whatever the ratio \( \Delta_c(T)/\Delta_s \) is, if disorder is short ranged, the SO coupling does not lead to any principal change in the critical broadening of the \( \sigma_{xx} \) peaks. It merely splits the critical point [though from an experimental point of view, it might be of great importance that the SO coupling does yield a sharp numerical growth of \( \xi(E) \)]. At \( d \gg \lambda \), however, the “coherent” contribution to \( \Delta_s \) cannot exceed \( \Delta_1 \) (the limit \( \Delta_s/\Delta_1 \sim 1 \) then corresponds to the generalized Chalker-Coddington model with strong spin-flip scattering). In the extreme of smooth disorder, \( \Delta_s \) may be described in completely classical terms as a result of splitting of percolating classical trajectories. Let us formulate an auxiliary percolation problem for particles with spin: Given two sets of closed equipotentials \( V(\vec{r}) \pm \Delta_s = E \), the particles are allowed to change the classical trajectories if two equipotentials come within a distance \( \delta \) of each other. The system undergoes a percolation transi-
tion at $\delta = \delta_c(E)$, such that $\delta_c(\pm \Delta_s)$ vanishes. The crucial observation is that $\delta_c(E) \sim d\Delta_s/\Gamma$ for $|E| < \Delta_s$, but rapidly grows as $\delta_s \sim d(|E| - \Delta_s)/|\Gamma|^{1/2}$ outside this band, since at $|E| > \Delta_s$ the percolation occurs only due to the coupling across the critical saddle points. This simple game leads us to an interesting result: the strong spin-flip scattering destroys the classical localization within the energy band $|E| < \Delta_s$. Consequently, in the limit of long-range disorder $(\Delta_\phi/\Delta_s \to 0)$, when the tunneling through the saddle points may be neglected, the SO coupling makes $\sigma_{xx}(E_F)$ exhibit a boxlike behavior, namely $\sigma_{xx} \sim e^2/h$ at $|E_F| \leq \Delta_s$, otherwise $\sigma_{xx} \to 0$. Thus, provided $T \ll \Delta_s$ and $R(\Delta_s)$ exceeds both $L_\phi$ and $R_{so}$ (the latter was evaluated in Ref. 31), two $\sigma_{xx}$ peaks merge and form the boxlike one. At lower $T$, when $L_\phi$ is still large in comparison with $R(\Delta_s)$, there are two peaks, but they are strongly asymmetric. Specifically, $\sigma_{xx}$ between the peaks falls off with decreasing $T$ only in a power-law manner: $\sigma_{xx} \sim (e^2/h)R(\Delta_s)/L_\phi(T)$. This power-law hopping occurs in a wide range of $T$ and goes over into the variable-range hopping only at very low $T$. The crucial point is that the strong spin-flip scattering changes the character of localization in the energy band limited by $E_c^+$ and $E_c^-$: it gives rise to the Anderson localization (instead of the classical one) with $\xi(E_0)/R(\Delta_s) \sim 1$. We conclude that, in the case of smooth disorder, the SO coupling is capable of strongly changing the conventional picture of QHE.

6 Conclusion

We have discussed inelastic broadening of $\sigma_{xx}$ peaks in the low-$T$ limit, stressing (i) the crucial role of electron-electron interactions in the integer QHE; (ii) the classical aspects of electron dynamics in samples with long-range disorder.

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