Clumps and triggered star formation in ionised molecular clouds

S. Walch1*, A.P. Whitworth2, T.G. Bisbas3, R. Wünsch4, D.A. Hubber5,6

1Max-Planck-Institute for Astrophysics, Karl-Schwarzschild-Str. 1, 85741 Garching, Germany
2School of Physics & Astronomy, Cardiff University, 5 The Parade, Cardiff CF24 3AA, Wales, UK
3Department of Physics & Astronomy, University College London, Gower Place, London WC1E 6BT, UK
4Astronomical Institute, Academy of Sciences of the Czech Republic, Bocni II 1401, 141 31 Prague, Czech Republic
5Technical University Munich, Excellence Cluster Universe, Boltzmannstr. 2, 85748 Garching, Germany
6University Observatory Munich, Department of Physics, Ludwig-Maximilians-University Munich, Scheinerstr.1, 81679 Munich, Germany.

ABSTRACT

Infrared shells and bubbles are ubiquitous in the Galaxy and can generally be associated with H\textsc{ii} regions formed around young, massive stars. In this paper, we use high-resolution 3D SPH simulations to explore the effect of a single O7 star emitting photons at $10^{50}$ s$^{-1}$ and located at the centre of a molecular cloud with mass $10^4 M_\odot$ and radius 6.4 pc; the internal structure of the cloud is characterised by its fractal dimension, $D$ (with $2.0 \leq D \leq 2.8$), and the variance of its (log-normal) density distribution, $\sigma_n^2$ (with $0.36 \leq \sigma_n^2 \leq 1.42$). Our study focuses on the morphology of the swept-up cold gas and the distribution and statistics of the resulting star formation. If the fractal dimension is low, the border of the H\textsc{ii} region is dominated by extended shell-like structures, and these break up into a small number of massive high-density clumps which then spawn star clusters; star formation occurs relatively quickly, and delivers somewhat higher stellar masses. Conversely, if the fractal dimension is high, the border of the H\textsc{ii} region is dominated by a large number of pillars and cometary globules, which contain compact dense clumps and tend to spawn single stars or individual multiple systems; star formation occurs later, the stellar masses are somewhat lower, and the stars are more widely distributed.

Key words: Galaxies: ISM - ISM: nebulae - H\textsc{ii} regions - bubbles - Hydrodynamics - Stars: formation

1 INTRODUCTION

Infrared shells and bubbles are ubiquitous in the Galaxy and can often be associated with H\textsc{ii} regions around young, massive stars (Deharveng et al. 2010; Simpson et al. 2012), which emit ionising radiation having $E_\gamma > 13.6$ eV. The ionised gas is heated to $\sim 10^4$ K, and the resulting pressure increase causes the H\textsc{ii} region to expand, sweeping up and compressing the much colder ($\sim 10$ to 30 K) surrounding molecular cloud. Since molecular clouds typically have a complicated, clumpy internal structure, the ionising radiation penetrates to different depths in different directions, producing highly irregular ionisation fronts. Thus, evolved H\textsc{ii} regions have diverse morphologies, sometimes appearing as perfectly round shells like RCW 120 (Deharveng et al. 2009), sometimes filamentary and/or clumpy with large holes through which the ionising radiation can escape, like Carina (Smith et al. 2014).

In Walch et al. (2012, hereafter W12) we show that all of these features can be reproduced by invoking different values for the fractal dimension, $D$, of the molecular cloud into which the H\textsc{ii} region expands. Low $D$ (i.e. $D \leq 2.2$) corresponds to clumpy clouds in which the density sub-structure is dominated by large-scale fluctuations. As $D$ is increased, small-scale fluctuations become increasingly more important. As a result, W12 report a morphological transition from shell-dominated H\textsc{ii} regions for low $D$, to pillar-dominated H\textsc{ii} regions for large $D$. In this paper we investigate the formation of cold clumps at the boundaries of H\textsc{ii} regions, and the triggering of star formation in them. We show that the statistical properties of the cold clumps, and of the stars that they spawn, are both correlated with the fractal dimension of the initial molecular cloud. In this context, two distinct modes of triggered star formation have traditionally been defined and contrasted: Collect-and-Collapse and Radiation-Driven Implosion.

The Collect-and-Collapse mode (hereafter C&C; Elmegreen & Lada 1977; Whitworth et al. 1994a; Dale et al. 2007, 2009; Wünsch et al. 2010) presupposes a rather homogeneous ambient medium; the expanding H\textsc{ii} region then sweeps up this medium into a dense shell, which eventually becomes sufficiently massive to fragment gravitationally and form a new generation of stars (e.g. Wünsch et al. 2010). One interesting feature of the C&C mode is that it is expected to produce quite massive fragments, and therefore the possibility exists that these will spawn a new generation of massive stars, so that the process can repeat itself.

* E-mail: walch@mpa-garching.mpg.de
2.1 Generation of fractal molecular clouds

It appears that the internal structure of molecular clouds is broadly self-similar over four orders of magnitude, from \( \sim 0.1 \) pc to 500 pc (e.g. Bergin & Tafalla 2007, Sánchez et al. 2010), and that it is approximately fractal, with dimension in the range 2.0 \( \leq D \leq 2.8 \) (e.g. Falgarone et al. 1991, Elmegreen & Falgarone 1996, Stutzki et al. 1998, Vogelstra & Wakker 1994, Lee 2004, Sánchez et al. 2005, Schneider et al. 2011, Mouville-Deschênes et al. 2010). To construct a fractal cloud we consider a \( 2 \times 2 \times 2 \) cubic computational domain, and specify three parameters: the fractal dimension, \( D = 2.0, 2.2, 2.4, 2.6, 2.8 \) (or equivalently \( k \approx \kappa \approx 4.0, 3.6, 3.2, 2.8, 2.4 \), where \( n \) relates to the 3D density power spectrum, \( P_k \propto k^{-n} \), \( k \) is wavenumber, and \( k \approx 1 \) corresponds to the linear size of the cubic domain, i.e. \( \lambda(k = 2/k) \); a random seed, \( \mathcal{R} \), which allows us to generate multiple realisations; and a density-scaling parameter, \( \rho_\text{ref} \). We populate all modes having integer \( k_x, k_y \), and \( k_z \) in (1, 128), with random phases and amplitudes drawn from the power-spectrum. Next we perform an FFT to evaluate the function \( \rho_\text{ref}(x, y, z) \).
We use the SPH code (Shadmehri & Elmegreen 2011). SPH particles are then distributed randomly in each cell of the grid according to its density, and SPH particles that fall outside a unit-radius sphere are culled. The resulting sphere can then be re-scaled to arbitrary total mass, \( M_\text{c} \), and arbitrary radius, \( R_\text{c} \).

With the above procedure, the random seed, \( R \) completely determines the pattern of the density field. The fractal dimension determines the distribution of power: for small \( D \) (large \( n \)), the power is on large scales, so the density field is dominated by extended structures; conversely, for large \( D \) (small \( n \)), there is more power on small scales, and so the density field is dominated by smaller structures. The scaling parameter, \( \rho_0 \), determines the density contrast, and hence the width of the (approximately log-normal) density PDF; increasing \( \rho_0 \) decreases the density contrast and therefore reduces the width of the PDF.

### 2.2 Numerical method

We use the SPH code seren (Hubber et al. 2011), which is well-tested and has already been applied to many problems in star formation (e.g. Walch et al. 2011; Bisbas et al. 2011; Stamatellos et al. 2011). We employ the SPH algorithm of Monaghan (1992) with a fixed number of neighbours, \( N_{\text{GRID}} = 50 \). The SPH equations of motion are solved with a second-order Leapfrog integrator, in conjunction with an hierarchical block time-stepping scheme. Gravitational forces are calculated using an octal spatial decomposition tree (Barnes & Hut 1986), with monopole and quadrupole terms and a Gadget-style opening-angle criterion (Springel et al. 2001). We use the standard artificial viscosity prescription (Monaghan & Gingold 1983), moderated with a Balsara switch (Balsara 1995).

The ionizing radiation is treated with an HEALPix-based, adaptive ray-splitting algorithm, which allows for optimal resolution of the ionization front in high-resolution simulations (see Bisbas et al. 2009). Along each HEALPix ray, the radiative transfer is evaluated at discrete points, \( j \). These points are separated by \( f_j h_j \), where \( f_j = 0.6 \) is an accuracy parameter and \( h_j \) is the local smoothing length (adjusted to enclose ~50 SPH neighbours). A ray is split if the linear separation of neighbouring rays is greater than \( f_j h_j \), where \( f_j = \) the angular resolution parameter. Typically, good results are achieved for \( 1.0 \leq f_j \leq 1.3 \); here we use \( f_j = 0.8 \) to further increase the angular resolution of the ray tracing scheme. We allow for a maximum number of \( N_{\text{MAX}} \approx 11 \text{ HEALPix levels corresponding to } 12 \times 4^{l_{\text{MAX}}} \approx 5 \times 10^3 \text{ rays if the whole sphere were refined to } l_{\text{MAX}}. \) In the simulations presented here, most directions only require \( \leq 8 \) levels of refinement, and there are typically \( \sim 10^5 \) rays in total.

The gas is assumed to be either fully molecular with a mean molecular weight of \( \mu_{\text{ION}} = 2.38 \), or fully ionized with \( \mu_{\text{ION}} = 0.7 \). The temperature of ionized gas is set to \( T_{\text{ION}} = 10^4 \text{ K} \). The temperature of neutral gas is given by a barotropic equation of state, \( T(\rho) = T_{\text{NEUT}} \left[ 1 + (\rho/\rho_{\text{CRIT}})^{(\gamma-1)/\gamma} \right]^\gamma \), where \( T_{\text{NEUT}} = 30 \text{ K}, \rho_{\text{CRIT}} = 10^{-13} \text{ g cm}^{-3}, \) and \( \gamma = 5/3 \). The use of \( T_{\text{NEUT}} = 30 \text{ K} \) may influence the fragmentation properties of the final cloud.

### Table 1. Simulation parameters

| ID | \( D \) | Seed | \( \rho_{\text{ION}} \) | \( \sigma_\text{T}^2 \) | \( t_1 \) | \( t_{15} \) |
|----|------|-----|-----------------|--------|-------|--------|
| \( D2.0/O7(1) \) | 2.0 | 1 | 1.23 | 1.08 | 0.46 | 0.66 |
| \( D2.0/O7(2) \) | 2.0 | 2 | 1.17 | 1.42 | 0.50 | 0.61 |
| \( D2.0/O7(3) \) | 2.0 | 3 | 1.17 | 1.10 | 0.56 | 0.66 |

Clumps and triggered star formation in ionised molecular clouds
afforded by multiple realisations to investigate the characteristics of the cold, swept-up clumps bordering the H\textsc{ii} region and the subsequent triggered star formation, as a function of $\mathcal{D}$.

### 3.2 Clump formation in shells and pillars

There is strong observational evidence that H\textsc{ii} regions are usually surrounded by shell-like structures that contain dense, molecular clumps, and that these clumps are often the sites of new star formation (e.g. [Zavagno et al. 2010; Deharveng et al. 2010]). However, it is still unclear how these clumps form, i.e. what is the relative importance of RDI, C&C, PAGI, EDGI, etc. (see Section 1). In some regions the clump mass function (CMF) appears to be rather similar to the one found in low-mass star forming regions ([Wünsch et al. 2012], whereas in other regions the presence of more massive clumps suggests that the C&C mechanism may have been at work ([Deharveng et al. 2003]).

To investigate the statistics of the cold clumps formed in our simulations, we first identify all the SPH particles having density $\rho_p > 6 \times 10^{-19} \text{ g cm}^{-3}$ (i.e., for molecular gas, $n_H > 1.5 \times 10^5 \text{ cm}^{-3}$). This density is sufficiently high for the gas to couple thermally to the dust, and a significant proportion of it should be destined to form stars. The free-fall time for a lump of gas with a uniform density of $6 \times 10^{-19} \text{ g cm}^{-3}$ is ~0.1 Myr. The positions of the SPH particles selected in this way are plotted in Fig. [3] for all the simulations performed with the first random seed; SPH particles having density $\rho_p > 6 \times 10^{-19} \text{ g cm}^{-3}$ are plotted in red, and – for comparison – those having density $6 \times 10^{-20} \text{ g cm}^{-3} \leq \rho_p < 6 \times 10^{-19} \text{ g cm}^{-3}$ in black. Individual clumps are clearly picked up more reliably with the higher density threshold.

We identify individual clumps by applying the Friends-of-Friends (hereafter FoF) algorithm to the selected subset of high-density SPH particles, using a linking length of $\ell = 0.05$ pc. Thus a clump represents a collection of high-density SPH particles, all of which are no further than 0.05 pc from at least one other member of this collection; any value of $\ell$ in the range (0.01, 0.1) pc delivers broadly similar clumps statistics.

Fig. 4 illustrates the dynamical evolution of the clumps in the simulations $\mathcal{D}2.0/\mathcal{O}7(1)$ and $\mathcal{D}2.8/\mathcal{O}7(1)$; positions are projected on the $z = 0$ plane, time is colour-coded, and the width of the symbol encodes the mass of the clump. In the case with low fractal dimension (left-hand frame, $\mathcal{D} = 2.0$), the plot is dominated by a small number of massive clumps, which are distributed very anisotropically with respect to the ionising star. In the case with high fractal dimension (right-hand frame, $\mathcal{D} = 2.8$), there are many more clumps, but they are much less massive, and they are distributed much more isotropically. All the clumps are being driven outwards by the rocket effect ([Kahn 1953; Oort & Spitzer 1955]), at speeds up to $\sim 10 \text{ km s}^{-1}$.

Fig. 5 illustrates the time evolution of clump masses. Different $\mathcal{D}$ are represented by different colours. For a given $\mathcal{D}$, the solid line shows the total mass in clumps; the dotted line shows the mass of the most massive clump; and the dashed line shows the mean clump mass. As $\mathcal{D}$ is increased, clumps start forming earlier, but ultimately the total mass in clumps and the masses of individual clumps are lower. To make the plot easier to read we only present results for $\mathcal{D} = 2.0, 2.4$ and 2.8, but the in-between values show the same trends; for each of these $\mathcal{D}$-values, we have combined the results from all three realisations.

The growth of a clump is not driven by gravity (as, for example, in Bondi accretion; [Bondi 1952], and there is also no strictly oligarchic growth (as e.g. inferred by [Dale & Bonnell 2006])).
Figure 2. False-colour column density images of all simulations at $t_{15}$ (see Table I). The ionising source is located at the center of each panel of size $14 \times 14$ pc. From top to bottom $D$ increases from $D = 2.0$ (top) to $D = 2.8$ (bottom). Each column represents clouds generated with the same random seed, and hence an initial density field with the same pattern. Sink particles are marked as turquoise dots; many of the sinks are in close (unresolved) multiple systems.
Figure 3. The positions of high-density SPH particles, projected onto the $z=0$ plane, for all the simulations set up with the first seed (i.e. $D=2.0/07(1)$ (left) to $D=2.8/07(1)$ (right) at $t_1$. Particles, $p$, with density $6 \times 10^{-20} \text{ g cm}^{-3} < \rho_p < 6 \times 10^{-19} \text{ g cm}^{-3}$ are plotted in black, and those with density $\rho_p > 6 \times 10^{-19} \text{ g cm}^{-3}$ are plotted in red. The red particles are the 'core' particles used to determine the core statistics.

Figure 4. The dynamical evolution of clump masses and positions projected on the $z=0$ plane. False colour encodes time (see colour bar), and track width encodes clump mass. The left frame shows a low fractal dimension case ($D=2.0/07(1)$), and the right frame a high fractal dimension case ($D=2.8/07(1)$).

Rather, in the first instance, matter is driven into a clump wherever the shock waves that precede the expanding ionisation front converge. Later on the ionisation front will start to erode the clump, but at the same time the clump will be driven outwards by the rocket effect, and sweep up material from further out in the cloud, like a snowplough. The evolution of its mass is then a competition between these two effects.

3.3 Clump statistics

Fig. 6 displays clump mass functions (CMFs) for the different $D$ values, at times $t = 0.40, 0.50, 0.60$ and $0.66$ Myr; $0.66$ Myr is the last time reached by all simulations. The bin size is $\Delta \log_{10}(M/M_\odot) = 0.2$, and the plot is logarithmic (so that the Salpeter stellar IMF would have slope $m = -1.35$). Typically, the mass function of cold clumps – which are not necessarily gravitationally bound – is somewhat flatter than Salpeter, with slopes $m \sim -0.7$ being commonly reported (e.g. Kramer et al. 1998, Wünsch et al. 2012). However, a steeply decreasing power law is not recovered in all molecular clouds; for example, in Orion, Li et al. (2007) even report an increasing power law of $m = +0.15$ over the mass range $0.1 M_\odot \lesssim M \lesssim 10 M_\odot$.

At early times all CMFs appear quite similar, but as time advances, there are three main trends. First, more clumps are formed, more clumps are formed,
Figure 6. CMFs at times, \( t = 0.4, 0.5, 0.6 \) and 0.66 Myr (reading from top left to bottom right). For each \( D \), we have combined the clump masses from the three different realisations. At \( t = 0.66 \) Myr we determine the power law slopes of the CMFs by means of \( \chi^2 \)-minimisation (dashed lines).

As more gas is swept up by the expanding H\textsc{ii} region. Second, the CMFs extend to higher mass, as a few of them have trajectories that cause them to intercept and sweep up dense material as they plough outwards. Third, the CMFs are shallow, almost flat, for \( D = 2.0 \), and become increasingly steep with increasing \( D \). We fit the CMFs at \( t = 0.66 \) Myr with a power-law, using \( \chi^2 \)-minimisation; the resulting fits are shown as dashed lines on Fig. 6 and have the following slopes:

\[
\begin{align*}
D &= 2.0, 2.2, 2.4, 2.6, 2.8 \\
m &= -0.18, -0.32, -0.37, -0.47, -0.91
\end{align*}
\]

The principal cause of this difference is that low \( D \) delivers coherent, extended density enhancements in the initial cloud, and this promotes the growth of high-mass clumps within the shells bordering the expanding H\textsc{ii} region; although these large clumps are eroded by the ionisation front on the side facing the star, they are also pushed outwards sweeping up the large amounts of neutral gas on the side facing away from the ionising star. In contrast, high \( D \) delivers smaller structures in the initial density field, and these develop into cometary globules and pillars; the ionisation front wraps round them, and so they are eroded from many different directions and having small cross-sections they do not sweep up so much extra mass as they plough outwards.
3.4 Internal velocity dispersion

On Fig. 7 (top panel) we plot the velocity dispersion inside each clump, $\sigma_{\text{CLUMP}}$, against its linear size, $L_{\text{CLUMP}}$, at time $t = 0.66$ Myr. The velocity dispersion of a clump is computed by adding in quadrature the contribution from non-thermal motions (i.e. the velocity dispersion of the constituent SPH particles) and the contribution from thermal velocity dispersion. The linear size of a clump is simply its maximum extent. Many of the clumps conform approximately to Larson’s Scaling relation (Larson 1981),

$$\sigma_{\text{CLUMP}} / \text{km/s} = 1.1 \left( L_{\text{CLUMP}} / \text{pc} \right)^{0.38}.$$ (3)

However, the clumps that are forming stars (hereafter “star-forming clumps”, represented by symbols containing a filled circle) all lie well above Larson’s Scaling Relation. Their velocity dispersion is higher because they are being accelerated and compressed by the H$\alpha$ region, and because they contain high-velocity flows onto forming protostars.

Heyer et al. (2009) have suggested that the velocity dispersions in molecular clouds might be fitted more accurately if one assumes approximate virialisation, i.e. $\sigma_{\text{CLUMP}} \approx (\pi \Sigma_{\text{CLUMP}} R_{\text{CLUMP}} / 5)^{1/2}$, where $\Sigma_{\text{CLUMP}}$ and $R_{\text{CLUMP}}$ are, respectively, the surface-density and radius of a clump (the triggered clumps typically have $10^2 \lesssim \Sigma_{\text{CLUMP}} \lesssim 10^4$ $M_\odot$/pc$^2$). Putting $\Sigma_{\text{CLUMP}} = M_{\text{CLUMP}} / \pi R_{\text{CLUMP}}^2$, and $R_{\text{CLUMP}} = L_{\text{CLUMP}} / 2$, this reduces to

$$\sigma_{\text{CLUMP}} \approx \left( \frac{2GM_{\text{CLUMP}}}{5L_{\text{CLUMP}}} \right)^{1/2}. \quad (4)$$

In the lower panel of Fig. 7 we plot $\sigma_{\text{CLUMP}}$ against $(M_{\text{CLUMP}} / L_{\text{CLUMP}})^{1/2}$. We find that all clumps lie above the Heyer Scaling Relation (Eqn. 4), but those that are forming stars have higher $\sigma$-values, more than a factor $\sim 10$ higher.

4 SPATIAL DISTRIBUTION AND INTRINSIC STATISTICS OF STARS

All the star formation in the simulations presented here is triggered. This has been demonstrated unequivocally in Walch et al. (2012), where, for comparison, we evolve the same fractal clouds without a central ionising star, and show that spontaneous star formation does not occur until long after the evolution time considered here. To characterise the consequences of triggered star formation, we calculate the statistical properties of the sink particles, as a function of $\mathcal{D}$. We employ the improved sink particle algorithm of Hubber et al. (2013), which has been demonstrated to improve the robustness of sink particle properties against numerical effects. All simulations are advanced until at least 15 sink particles have formed, and their properties are evaluated at this time ($t_{15}$).

4.1 The location of stars, relative to the ionising star

Fig. 8 shows how the formation radius, $R_{\text{FORM}}$ (i.e. the distance from the ionising star to a sink when it is first created), varies with the sink formation time, $t_{\text{FORM}}$. As $\mathcal{D}$ increases, the sinks form later and at larger radii. We derive a linear fit to the distribution of sinks in the $(R_{\text{FORM}}$, $t_{\text{FORM}})$-plane, using a $\chi^2$-minimization method. The best fit has a slope of 7.1 km/s (see dashed line in Fig. 8). This velocity is comparable to the radial velocity of the ionisation front, and shows that stars are progressively triggered by the expansion of the H$\alpha$ region.

4.2 The location of stars, relative to the ionisation front

Fig. 9 shows two histograms of the number of sinks as a function of the square-root of the clump mass-to-size ratio, $\sqrt{M_{\text{CLUMP}} / L_{\text{CLUMP}}^2}$. The relationship derived for galactic molecular clouds is indicated by the lowest dotted line (Heyer et al. 2009, see Eq. 4), the following dotted lines indicate the Heyer et al. (2009) relation multiplied by a factor of 2, 5, and 10.

Figure 7. Top: The internal velocity dispersion $\sigma$ of a clump plotted against its maximum extent $L$, at $t = 0.66$ Myr. Star-forming clumps are marked with an additional filled circle in the same colour. We also show Larson’s line-width-size relation (dashed line). Bottom: Velocity dispersion $\sigma$ as a function of the square-root of the clump mass-to-size ratio, $\sqrt{M_{\text{CLUMP}} / L_{\text{CLUMP}}^2}$. The relationship derived for galactic molecular clouds is indicated by the lowest dotted line (Heyer et al. 2009, see Eq. 4). The following dotted lines indicate the Heyer et al. (2009) relation multiplied by a factor of 2, 5, and 10.
density (see Thompson et al. 2012). For simplicity, we are assuming that a 2-dimensional projection of the system along an arbitrary line of sight would on average yield the same ratio $R_{15}/R_{\text{g}}$. In the top panel, we compile all sinks formed in all simulations into one histogram. There is a clear over-density of triggered stars at, or close to, $R_{15}$. In the bottom panel, we repeat the same analysis but distinguish the distributions for different $D$. For low $D$, the sinks typically stay ahead of the ionisation front, because the gas in the clumps from which these sinks are formed has been accelerated by the rocket effect. Therefore the sinks that condense out of it have significant radial velocities, whereas the expansion of the ionisation front is slowing down. Conversely, for high $D$, the sinks form in the heads of pillars and are frequently left behind in the H ii region, thus leading to a flat distribution of $R_{15}/R_{\text{g}}$.

Overall, the derived distribution of triggered stars compares remarkably well with the observational findings of Thompson et al. (2012), who study the over-density of young stellar objects around Spitzer bubbles. However, they find a significantly enhanced source density at small radii ($R_{15}/R_{\text{g}} < 0.5$), which is not present in our analysis. One reason for this is the fact that they use projected positions of stars, whereas their estimate of the radius of the H ii region is determined by its lateral extent. Thus, for a star on the far or near side of the H ii region, close to the line of sight through the ionising star(s), $R_{15}/R_{\text{g}}$ will appear much smaller than the true three-dimensional value.

### 4.3 The masses of stars

Observational results from the Milky Way project (Kendrew et al. 2012) and Deharveng et al. (2010) indicate that approximately 20% of young, massive star formation may have been triggered. This estimate is also in agreement with Thompson et al. (2012), who derive 14% - 30%.

Fig. 10 (top panel) shows how the sink mass, $M_{15}$, varies with the three-dimensional radial distance from the center $R_{15}$ at $t_{15}$. Massive sinks ($M > 8M_\odot$) are quite common for low $D$ (shell-dominated morphology), and much rarer for high $D$ (pillar-dominated morphology). The sinks seem to be aligned in vertical stripes, which are caused by two effects. First, $t_{15}$ and therefore the mean ionisation front radius varies for the three different realisations of every $D$, which causes preferential triggering at different radii. Second, in some cases multiple arcs form around the H ii region and therefore the sinks can be clustered about different radii.

In the bottom panel of Fig. 10 we plot the mean mass accretion rates onto sinks up to $t_{15}$, $\dot{M}$, as a function of their masses, $M_{15}$. Above and to the right of the dotted lines a sink that continues to accrete at the observed rate for 0.1, 0.2 or 0.3 Myr will exceed 8$M_\odot$, and therefore would be classified as a massive star. We see that for low $D$ a significant fraction of sinks either are already, or will soon be, massive in this sense, whereas for higher $D$ fewer of them are destined to be massive. The simulations with low-$D$ also appear to produce more low-mass stars, i.e. a bigger range of masses at both extremes is produced. The mass accretion rates are generally quite high ($\dot{M} \sim 5 \times 10^{-5} M_\odot/\text{yr}$), which is not unexpected for these early stages of star formation. However, they decline as soon as the surrounding cold material has been accreted onto a sink, or ablated by the ionising radiation.

With respect to triggered star formation, the major limitation...
of the simulations presented here is that radiative and mechanical feedback from newly-formed stars is not included. Therefore, the quoted mass accretion rates are upper limits and we probably overestimate the number of massive stars formed. At \( t_\text{s} \), the results are still credible, since the percentage of sinks with \( M \geq 8 \, M_\odot \) is only 6.7%. If all sinks were to continue accreting at their measured rate after \( t_\text{s} \), by \( t_\text{s} + 0.1 \) Myr the percentage of sinks with \( M \geq 8 \, M_\odot \) would be \( \sim 25\% \). If feedback from newly-formed stars were included, it is not clear whether such a high percentage of massive stars would be able to form.

### 4.4 The clustering of stars

For each simulation, at \( t_{15} \), we perform a Minimum Spanning Tree (MST) analysis. To construct an MST, we project the star positions onto a plane, and then identify the system of straight lines (“edges”) with minimum total length that links all the stars together; for an ensemble of \( N_\star \) stars, there are \( N_\star - 1 \) edges, and no closed loops. Having done this, we analyse the distribution of edge lengths, \( \ell \). To improve the statistics, we project the star positions onto each of the fundamental Cartesian planes, and we consider all three realisations, so we end up with \( N_\ell = 9 (N_\star - 1) \) edge-lengths. If we define the kth moment about the mean, for the ensemble of edge-lengths,

\[
m_k = \overline{(\ell - \mu_\ell)^k},
\]

the standard deviation of the ensemble is \( \sigma_\ell = m_{1/2} \), and the skewness of the ensemble is \( \gamma_\ell = m_3/\sigma_\ell^3 \); the standard deviation is a measure of the width of the distribution, and the skewness is a measure of the asymmetry of the distribution. Fig. 11 shows the cumulative distribution of edge-lengths, for the different \( D \) values (top plot), and the skewness plotted against the mean (bottom plot). These plots demonstrate that with low \( D \) the stars are strongly clustered, whereas with high \( D \) they are more uniformly distributed.

### 5 CONCLUSIONS

In this paper, we use high-resolution 3D SPH simulations to explore the effect of a single \( O7 \) star emitting photons at \( 10^{39} \text{s}^{-1} \) and located at the centre of a molecular cloud with mass \( 10^4 M_\odot \) and radius 6.4 pc. We focus on the statistics of dense clumps and triggered star formation, as a function of the initial fractal dimension, \( D \), of the molecular cloud into which the \( \text{H} \text{II} \) region expands. We find that most properties show a clear correlation with \( D \).

Cold clumps form due to the sweeping up of gas by the \( \text{H} \text{II} \) region. The clumps are pushed outward by the rocket effect and grow in mass by collecting material in a snowplough manner. Thus, large clumps cover a bigger surface area and may accrete faster, even though the growth is not caused by self-gravity.

- For low \( D \leq 2.2 \) (shell-dominated regime), we find a small number of massive clumps, whereas high \( D \geq 2.6 \) (pillar-dominated regime) results in many low-mass clumps.
- The clumps have trans- to super-sonic internal velocity dispersions. For non-star-forming clumps the internal velocity dispersion increases with clump size following Larson’s Relation. For star-forming clumps the internal velocity dispersion is significantly higher than predicted by Larson’s Relation.
- The resulting CMFs are well fitted by power-laws, with the slope increasing with increasing \( D \). Typically observed CMF slopes of \(-0.7\) are recovered for intermediate \( D \).

The statistical properties of triggered stars are also well correlated with \( D \). On average, clouds with lower \( D \)

- form stars earlier in the simulation and at smaller distances from the ionising source (these stars are mostly located within the
dense shell-like structures present for lower D; for higher D most stars sit in the tips of pillar-like structures;
• are more prone to massive star formation;
• form mainly small star clusters, whereas for higher D star formation occurs in small-N multiple systems spaced at large distances from one another.

Stars are strongly concentrated near the ionisation front \((R_{\text{if}} / R_{\text{if}} = 1)\), but stars that form in pillars (high D) tend to be left behind within the H\textsc{ii} region.

ACKNOWLEDGMENTS

We thank the anonymous referee for helpful comments and suggestions, which helped us to improve the paper. SKW thanks D. Kruĳssen for useful and interesting discussions on the manuscript, and the Deutsche Forschungsgemeinschaft (DFG) for funding through the SPP 1573 ‘The physics of the interstellar medium’. SKW and AW further acknowledge the Marie Curie ITN Constellation. RW acknowledges the support of the Czech Science Foundation grant 209/12/1795 and by the project RVO:67985815. The simulations have been performed on the Cardiff axcca Cluster. T.G.B. acknowledges support from STFC grant ST/J001511/1.

REFERENCES

Arthur S. J., Henney W. J., Mellema G., de Colle F., Vázquez-Semadeni E., 2011, MNRAS, 414, 1747
Balsara D. S., 1995, Journal of Computational Physics, 121, 357
Barnes J., Hut P., 1986, Nature, 324, 446
Bergin E. A., Tafalla M., 2007, ARAA, 45, 339
Bertoldi F., 1989, ApJ, 346, 735
Bisbas T. G., Wünsch R., Whitworth A. P., Hubber D. A., 2009, A&A, 497, 649
Bisbas T. G., Wünsch R., Whitworth A. P., Hubber D. A., Walch S., 2011, ApJ, 736, 142
Bondi H., 1952, MNRAS, 112, 195
Dale J. E., Bonnell I., 2011, MNRAS, 414, 321
Dale J. E., Bonnell I. A., Whitworth A. P., 2007, MNRAS, 375, 1291
Dale J. E., Ercolano B., Bonnell I. A., 2012, MNRAS, 427, 2852
Dale J. E., Wünsch R., Whitworth A., Palouš J., 2009, MNRAS, 398, 1537
Deharveng L., Lefloch B., Kurtz S., Nadeau D., Pomarès M., Caplan J., Zavagno A., 2008, A&A, 482, 585
Deharveng L., Lefloch B., Zavagno A., Caplan J., Whitworth A. P., Nadeau D., Martin S., 2003, A&A, 408, L25
Deharveng L., Schuller F., Anderson L. D., Zavagno A., Wyrowski F., Menten K. M., Bronfman L., Testi L., Walmsley C. M., Wienen M., 2010, A&A, 523, A6
Deharveng L., Zavagno A., Schuller F., Caplan J., Pomarès M., De Breuck C., 2009, A&A, 496, 177
Ekström S., Georgy C., Eggenberger P., Meynet G., Mowlavi N., Wytenbach A., Granada A., Decressin T., Hirschi R., Frischknecht U., Charbonnel C., Maeder A., 2012, A&A, 537, A146
Elmegreen B. G., Falgarone E., 1996, ApJ, 471, 816
Elmegreen B. G., Lada C. J., 1977, ApJ, 214, 725
Erbolano B., Gritschneder M., 2011, MNRAS, 413, 401
Falgarone E., Phillips T. G., Walker C. K., 1991, ApJ, 378, 186
Gritschneder M., Burkert A., Naab T., Walch S., 2010, ApJ, 722, 971
Gritschneder M., Naab T., Burkert A., Heitsch F., 2009, ApJ Letters, 694, L26
Haworth T. J., Harries T. J., 2012, MNRAS, 420, 562
Heyer M., Krawczyk C., Duval J., Jackson J. M., 2009, ApJ, 699, 1092
Hubber D. A., Batty C. P., McLeod A., Whitworth A. P., 2011, A&A, 529, A27+
Hubber D. A., Walch S., Whitworth A. P., 2013, ArXiv e-prints
Kahn F. D., 1954, Bull. astron. Inst. Netherlands, 12, 187
Kendrew S., Simpson R., Bressert E., Povich M. S., shelter, R., Lintott C. J., Robitaille T. P., Schawinski K., Wolf-Chase G., 2012, ApJ, 755, 71
Kessel-Deynet O., Burkert A., 2003, MNRAS, 338, 545
Kramer C., Stutzki J., Rohrig R., Corneliussen U., 1998, A&A, 329, 249
Krumholz M. R., Matzner C. D., 2009, ApJ, 703, 1352
Krumholz M. R., Stone J. M., Gardiner T. A., 2007, ApJ, 671, 518
Lee Y., 2004, Journal of Korean Astronomical Society, 37, 137
Li D., Velusamy T., Goldsmith P. F., Langer W. D., 2007, ApJ, 655, 351
Mac Low M.-M., Toraskar J., Oishi J. S., Abel T., 2007, ApJ, 668, 980
Mackey J., Lim A. J., 2011, MNRAS, 412, 2079
Mellema G., Arthur S. J., Henney W. J., Iliev I. T., Shapiro P. R., 2006, ApJ, 647, 397
Miao J., White G. J., Nelson R., Thompson M., Morgan L., 2006, MNRAS, 369, 143
Miville-Deschênes M.-A., Martin P. G., Abergel A., Bernard J.-P., Boulanger F., Lagache G., Anderson L. D., André P., et al., 2010, A&A, 518, L104
Monaghan J. J., 1992, ARA&A, 30, 543
Monaghan J. J., Gingold R. A., 1983, Journal of Computational Physics, 52, 374
Murray N., Quataert E., Thompson T. A., 2010, ApJ, 709, 191
Oort J. H., Spitzer Jr. L., 1955, ApJ, 121, 6
Peters T., Banerjee R., Klessen R. S., Mac Low M.-M., 2011, ApJ, 729, 72
Peters T., Mac Low M.-M., Banerjee R., Klessen R. S., Dullemond C. P., 2010, ApJ, 719, 831
Sánchez N., Añez N., Alfaro E. J., Crone Odekon M., 2010, ApJ, 720, 541
Sánchez N., Alfaro E. J., Pérez E., 2005, ApJ, 625, 849
Sánchez N., Alfaro E. J., Pérez E., 2007, ApJ, 656, 222
Sandford II M. T., Whitaker R. W., Klein R. I., 1982, ApJ, 260, 183
Schneider N., Bontemps S., Simon R., Ossenkopf V., Federrath C., Klessen R. S., Motte F., André P., Stutzki J., Brunt C., 2011, A&A, 529, A1
Shadmehri M., Elmegreen B. G., 2011, MNRAS, 410, 788
Simpson R. J., Povich M. S., Kendrew S., Lintott C. J., Bressert E., Arvidsson K., Cyganowski C., Maddison S., Schuwin K., Sherman R., Smith A. M., Wolf-Chase G., 2012, MNRAS, 424, 2442
Smith N., Povich M. S., Whitney B. A., Churchwell E., Babler B. L., Meade M. R., Bally J., Gehrz R. D., Robitaille T. P., Stassun K. G., 2010, MNRAS, 406, 952
Springel V., Yoshida N., White S. D. M., 2001, New Astronomy, 6, 79
Stamatellos D., Whitworth A. P., Hubber D. A., 2011, ApJ, 730, 32
Stutzki J., Bensch F., Heithausen A., Ossenkopf V., Zielińsky M., 1998, A&A, 336, 697
Thompson M. A., Urquhart J. S., Moore T. J. T., Morgan L. K., 2012, MNRAS, 421, 408
Toalá J. A., Arthur S. J., 2011, ApJ, 737, 100
Vogelaar M. G. R., Wakker B. P., 1994, A&A, 291, 557
Walch S., Whitworth A., Bishas T., Hubber D. A., Wuensch R., 2011, ArXiv e-prints
Walch S., Whitworth A. P., Girichidis P., 2012, MNRAS, 419, 760
Weaver R., McCray R., Castor J., Shapiro P., Moore R., 1977, ApJ, 218, 377
Whitworth A. P., Bhattacharya D., Chapman S. J., Disney M. J., Turner J. A., 1994a, A&A, 290, 421
Whitworth A. P., Bhattacharya D., Chapman S. J., Disney M. J., Turner J. A., 1994b, MNRAS, 268, 291
Wünsch R., Dale J. E., Palouš J., Whitworth A. P., 2010, MNRAS, 407, 1963
Wünsch R., Jáchym P., Sidorin V., Ehlerová S., Palouš J., Dale J., Dawson J. R., Fukui Y., 2012, A&A, 539, A116
Zavagno A., Russeil D., Motte F., Anderson L. D., Deharveng L., Rodón J. A., Bontemps S., Abergel A., Baluteau J.-P., Sauvage M., André P., Hill T., White G. J., 2010, A&A, 518, L81