Observations of animal contests pose a question whether the decision to quit the fight depends on the state of the competitor, to decide whether to flee or keep fighting for territory or a mating opportunity, for review see [1]. Much uncertainty, however, exists about the evolutionary justification of self-assessment only during a fight[2]. Evolution of strategies with advanced information processing and developed memory abilities is a major area of interest within the field of Evolutionary game theory[2–11]. Evolutionary game theory models, to the best of our knowledge, cannot answer the question whether observed fights without mutual assessment represent an optimal evolutionary strategy to compete or an arbitrary constraint on information exchange between specimens of the same species[12].

Additional knowledge regarding the opponent may reduce, rather than increase, Darwinian fitness. For instance, consider a competition of two cars that are racing towards each other either until a crash or until one of the competitors quits. It is a game of conflict escalation: the player’s dilemma is either to quit the race in the beginning or to keep competing for a prize with a high risk of death if both players decide to escalate the conflict[13]. Knowledge of the opponent’s intentions, on the other hand, may lead to the development of an intelligent strategy to compete. What was the preference of evolution?

Consider a population of agents that compete in an everlasting tournament of a game from generation to generation[14][16]. The lifetime score in this game is the agent’s Darwinian fitness. For any game, one can find an Evolutionary Stable Strategy (ESS)[14] that, if adopted by all host agents h, makes the survival of any mutant impossible. The payoff (fitness) of the ESS host hESS is:

\[ G(h_{\text{ESS}}, h_{\text{ESS}}) > G(m, h_{\text{ESS}}), \]

for any mutant strategy m, where G(p,q) is the average gain of an agent with strategy p against an agent with strategy q. To model the evolution of information assessment during a competition, let us consider three relevant approaches to define a binary game and its strategy space: the war of attrition, together with iterated and non-iterated strategies[5, 17].

War of attrition[14, 18, 19], in its classical form, is a model for conflict escalation without the assessment of the opponent’s state. In this game, a player at each time t decides between two options: whether to keep fighting or to quit. The probability to keep fighting is a function of time t and independent of the opponent’s strategy. This model is quite successful to fit the observed data of real fights between specimens of a species[1], though some alternative views exist[20]. War of attrition can be modified to include assessment of the opponent’s state[21].

In a binary game, a player adopts either of the two distinct behavior roles. We will use a defector D and a cooperator C as standard notations of the roles. There are four possible outcomes for a single fight (CC, CD, DC, DD) with corresponding payoffs \( \vec{W} = (P, S, T, R) \). According to the payoffs, the game may be viewed as conflict escalation, for example of two competing cars with the worst outcome for DD or social dilemma with the worst case of cooperation against the defector CD. The individual payoff is:

\[ G(p, q) = \hat{\Omega}(p, q)\vec{W}, \]
where $\Omega(p,q) = (\Omega_{CC}, \Omega_{CD}, \Omega_{DC}, \Omega_{DD})$ is statistics of interaction between strategies $p$ and $q$ and the components of the vector $\Omega$ are probabilities for the corresponding interactions.

Strategies to play binary games can be either iterated or non-iterated. A one-step memory (memory-one) iterated strategy [9, 13, 15, 22, 23], for instance, comprises four independent probabilities $(\rho_{CC}, \rho_{CD}, \rho_{DC}, \rho_{DD})$ to cooperate after the previous fight’s outcomes $(CC, CD, DC, DD)$, respectively.

Non-iterated strategies consist of conditional probabilities for choosing a specific role as a function of the opponent’s state. For instance, $P_{CI|D}$ is a probability to cooperate against a defector, where the standard notation of conditional probability $P_{A|B}$ for $A$ under condition of $B$ is used. Non-iterated strategies were discussed in the context of animal combat with sequential assessment of the opponent’s state [5, 24, 17] and social influence [23]. The advantage of iterated strategies is a detailed description of the interaction mechanism including information assessment. For instance, an iterated strategy can be reactive—it may depend only on the opponent’s state $\rho_{CC} = \rho_{DC}, \rho_{CD} = \rho_{DD}$ [22]. In analogy with self-assessment, one can define the passive iterated strategy $\rho_{CC} = \rho_{CD}, \rho_{DC} = \rho_{DD}$ that is independent of the opponent’s state. Evolutionary stable self-assessment, therefore, can be detected if the population includes only passive strategies.

To study the evolution of mutual assessment, it is tempting to find a way to combine the advantages of iterated and non-iterated strategies. Iterated strategies describe the assessment mechanism in detail, but cannot be applied to non-iterated mating combats in nature. Non-iterated strategies fit the observations better, but lack any description of the mechanism of assessment.

The main proposal of this work is an ability to bring iterated and non-iterated strategies under a single framework using zero-determinant (ZD) strategies [9]. Recently, a family of iterated ZD strategies was discovered that makes controlling the opponent’s payoff possible regardless of the opponent’s strategy. The surprising possibility of controlling the opponent’s payoff regardless of his will lead to an extended discussion of a possible evolutionary implication [6, 10, 11, 20, 34]. Coercive ZD strategies impose a specific value on the payoff of the opponent. This work will develop and utilize the similarities of coercive ZD and evolutionary stable strategies in any other strategy space.

In this article, we show that self-assessment of the opponent’s state is evolutionary stable for conflict escalation, e.g., the snowdrift game or war of attrition. It supports the use of the no information assessment assumption of the war of attrition model. An immediate implication is that no information assessment should be used in binary games for snowdrift payoffs in evolutionary studies. Technically, we show that non-iterated ESS sometimes possesses a corresponding indistinguishable iterated coercive ZD strategy. Indistinguishable strategies preserve the statistics of interactions between different players. Thus, an external observer cannot distinguish these strategies.

We also claim that specimens of a species do not assess each other’s state during a competition if the observed statistics of interactions $\Omega$ can be explained by a fight model (not necessarily ESS) without mutual state assessment. Thus there cannot be indistinguishable passive and non-passive strategies. The proof is to the contrary—if competitors assess each other’s state and use this information to decide their own state, then the corresponding statistics of interactions $\Omega$ cannot be described using a strategy that neglects this information.

An example of conflict escalation in nature is the mating combat of male bowl and doily spiders [32]. According to S. Austad, upon reaching maturity, specimens of male bowl and doily spiders compete with each other for access to a female’s web in order to fertilize her eggs. The fight between two males proceeds as a series of aggressive grapples until one of the competitors either receives a severe injury and dies or flees to search for other mating opportunities. The winner gains access to the web. During the mating period, the spiders do not eat: Darwinian fitness is well-represented by the amount of inseminated eggs.

S. Austad collected substantial data regarding the combat of male bowl and doily spiders. The data includes statistics on fatal injuries, the number of eggs in a female nest, the value of the female nest for a winner and his less fortunate opponent (by second insemination), the average lifetime reproductive success, and the duration of the fights.

In addition, S. Austad demonstrated that the observations fit the war of attrition model with deviations. War of attrition with the cost of fight rising linearly with time predicts the fight duration that decreases exponentially with time. The distribution of fight duration for the spiders of the same size, contrary to the predictions of the model, does not decay exponentially with time. This discrepancy indicates either a deviation from the war of attrition model’s central assumption of no state assessment of the opponent [24] or the cost of fight rising non-linearly with time [15]. To justify self-assessment during war of attrition and during conflict escalation in general, non-iterated strategies are used.

This work addresses only the fights between the spiders of similar sizes. The ratio of fights that end with a severe injury depends on the difference in the competitors’ sizes: a smaller spider generally flees from a larger opponent.

To calculate the ESS of non-iterated strategies in a binary game, this work introduces mean field assumption (the average probability of an individual to be in state $D$ is equal to the average abundance of state $D$ in the population). This method predicts the accurate ESS statistics of interactions for the combat of male bowl and doily spiders of similar sizes.
equal values of $P$ of the agents with this strategy. Contour lines correspond to between the individual probability $P$ cooperators $P$ of defectors in a population. The population includes only non-iterated strategy (FIG. 1: Average probability $P$ in round $N$ of state $D$ as a function of the non-iterated strategy $(P_{CD}, P_{CC})$ in a population composed of the agents with this strategy. Contour lines correspond to equal values of $P_D$. The mean field approach ensures equality of the individual probability $P_D$ and the average ratio of defectors in a population. The population includes only cooperators $P_D = 0$ or defectors $P_D = 1$ if $P_{CC} = 1$ or $P_{CD} = 0$, respectively. It is important to note that even in these cases, the agents preserve their ability either to defect unconditional cooperation $P_{CD} = 0$ or to cooperate with unconditional cooperation $P_{CC} = 0$.

I. MODELS

A. ESS with not iterated strategies.

To interpret the combats of male bowl and doily spiders as a binary game and to define non-iterated strategies, we will use modified ideas of sequential assessment game, see ref. [5] (page 403) and Kirman recruiting strategies [17].

To introduce non-iterated strategies, consider a population of agents that can be at two states, $C$ or $D$. The agents meet at random. During the meeting, the first player preserves its state while the second can either preserve or change its state. Each player has an equal chance to be either the first or the second. The second player becomes $C$ with the conditional probability $P_{CC}$ if the state of the opponent is $C$, and becomes $C$ with the conditional probability $P_{CD}$ if the opponent is in state $D$. The notation is similar to the standard notation of conditional probabilities $P(A|B)$ for possible values of $A$ if the value of $B$ is known.

Each fight of two agents belongs to one of the four types of interactions: $(CC, CD, DC, DD)$. The corresponding statistics of the interactions $\Omega = (\Omega_{CC}, \Omega_{CD}, \Omega_{DC}, \Omega_{DD})$ is observable. In an evolutionary stable population, statistics $\Omega$ depends on the payoffs for the corresponding interactions.

In the competitions $(CC, CD, DC, DD)$, the payoffs are $(R, S, T, P)$ and $(R, T, S, P)$ for the first and the second competitors, respectively. The payoffs define the game which follows the notation of social dilemmas. In a social dilemma, $C$ stands for cooperation and $D$ for defection. Prisoner’s Dilemma punishes $C$ for $CD$ interaction $R > T > P > S$. Snowdrift game punishes $DD$ $R > T > S > P$ - in conflict escalation, mutual boldness claims a heavy price from both sides.

To calculate the ESS statistics of interactions for non-iterated strategies, we assume that a first-to-respond agent preserves its average probability to be in $D$ state. The validity of this mean field assumption will be discussed later.

Consider multiple interactions of agents $i$ and $j$ with the strategies $(P_{CD}^i, P_{CC}^i)$ and $(P_{CD}^j, P_{CC}^j)$, respectively. If agent $i$ is the first to respond then, after $N$ rounds, the average probability $P_D^i$ of agent $j$ is to be in state $D$ is:

$$P_D^i = \frac{1}{N} \sum_{k=1}^{N} \left[ (1 - P_{CD}^i) \delta_{S^i(k), D} + (1 - P_{CC}^i) \delta_{S^i(k), C} \right].$$ (3)

where $S^i(k)$ is the state ($C$ or $D$) of the agent $i$ against agent $j$ in round $k$. The average probability $P_D^i$ of agent
i to be in state $D$ is:

$$\frac{1}{N} \sum_{k=1}^{N} \delta _{S^{ij},D} = P_{ij}^D,$$  \hspace{1cm} (4)

Under the assumption that an agent possesses the same probability to be in state $D$ while responding as the first or as the second, eqs. (3) and (4) become:

$$P_{ij}^D = (1 - P_{C|D}^i) P_{ij}^D + (1 - P_{C|C}^j)(1 - P_{ij}^D),$$

$$P_{ji}^D = (1 - P_{C|D}^i) P_{ji}^D + (1 - P_{C|C}^j)(1 - P_{ji}^D),$$  \hspace{1cm} (5)

The same equations (5) can be justified by symmetry consideration without addressing multiple interactions. The system (5) can be solved for $P_{ij}^D$ and $P_{ji}^D$:

$$P_{ij}^D = \frac{(1 - P_{C|C}^j) - (1 - P_{C|C}^j)(P_{C|D}^i - P_{C|C}^j)}{1 - (P_{C|D}^i - P_{C|C}^j)(P_{C|D}^j - P_{C|C}^j)},$$  \hspace{1cm} (6)

and

$$P_{ji}^D = \frac{1 - P_{C|C}^j - (1 - P_{C|C}^j)(P_{C|D}^i - P_{C|C}^j)}{1 - (P_{C|D}^i - P_{C|C}^j)(P_{C|D}^j - P_{C|C}^j)},$$  \hspace{1cm} (7)

where $(P_{C|D}^i, P_{C|C}^j)$ and $(P_{C|D}^j, P_{C|C}^i)$ are the strategies of the competitors.

Here are the probabilities $(\Omega_{CC}, \Omega_{CD}, \Omega_{DC}, \Omega_{DD})$ of interactions of types $(CC, CD, DC, DD)$ between the agents $i$ and $j$ under the condition that agent $i$ is the first, and agent $j$ the second to respond:

$$\Omega_{CC} = P_{C|C}^j(1 - P_{ij}^D),$$

$$\Omega_{CD} = (1 - P_{C|C}^j)(1 - P_{ij}^D),$$

$$\Omega_{DC} = P_{C|D}^j P_{C|D}^i, $$

$$\Omega_{DD} = P_{C|D}^i(1 - P_{C|D}^j),$$  \hspace{1cm} (8)

where $P_{ij}^D$ and $P_{ji}^D$ are the probabilities of the agents $j$ and $i$ to be in state $D$, respectively. If agent $i$ is the second to respond, expressions (5) are valid with a swap of the indices $i \leftrightarrow j$.

The average payoff over multiple interactions is:

$$G = R\Omega_{CC} + S\Omega_{CD} + T\Omega_{DC} + P\Omega_{DD},$$  \hspace{1cm} (9)

Thus, the payoff of agent $i$ if it is the first:

$$G_i^1 = P_{C|D}^i T + (1 - P_{C|C}^i)(1 - P_D^i)S + P_{C|C}^i(1 - P_D^i),$$

and if it is the second:

$$G_i^2 = P_{C|D}^i S + (1 - P_{C|C}^i)(1 - P_D^i)T + P_{C|C}^i(1 - P_D^i).$$

Under the condition that an agent possesses equal probabilities to be the second or the first, the total gain is:

$$G^{ij} = \frac{G_i^1 + G_i^2}{2},$$  \hspace{1cm} (12)

where $G^{ij}$ is a function of the strategies of both competitors.

Replicator dynamics formalism is used to find an evolutionary stable strategy $(P_C^{ESS}, P_C^{ESS})$. The population is presented as a point in space that moves with velocity $\vec{V} = (V_{C|D}, V_{C|C})$. The direction and absolute value of velocity correspond to the optimal gradient of a mutant’s fitness:

$$V_{C|D} = \frac{\partial G^{ij}}{\partial P_{C|D}} \bigg|_{P_{C|D}=P_{C|D}^{i}, P_{C|C}=P_{C|C}^{i}},$$

$$V_{C|C} = \frac{\partial G^{ij}}{\partial P_{C|C}} \bigg|_{P_{C|D}=P_{C|D}^{i}, P_{C|C}=P_{C|C}^{i}}$$

when mutation steps are assumed to be small. The stable points of flow field $\vec{V}$ correspond to the ESS states. It corresponds to condition (11) under the constraint of small mutation steps.

To get the ESS strategies as functions of the payoffs, it is convenient to reduce the payoffs to two parameters by transformation:

$$\vec{W} = \frac{\vec{V} - \vec{P}}{R - \vec{P}} = (1, S, T, 0),$$  \hspace{1cm} (14)

This transformation does not affect the ESS condition (11) because it does not change the relative gains (9) taking into account that $\Omega_{CC} + \Omega_{CD} + \Omega_{DC} + \Omega_{DD} = 1$.

Replicator dynamics (11) converges to a steady homogeneous population of identical individuals $(P_{C|D}^i, P_{C|C}^i) = (P_{C|D}^i, P_{C|C}^i)$. In this population, the probability of each agent to be at state $D$ reduces to:

$$P_{ij}^D = \frac{1 - P_{C|C}^j}{1 + P_{C|D}^i - P_{C|C}^j},$$  \hspace{1cm} (15)

following (6) and (7), see Figure 1. Abundance of state $D$ can take any value $0 \leq P_D \leq 1$ as a function of a strategy $(P_{C|D}, P_{C|C})$.

Replicator dynamics for the snowdrift game and Prisoner’s Dilemma together with the ESS states will be presented in the Results section.

B. Iterated coercive ZD strategies subspace

Let us calculate a subspace of coercive zero-determinant strategies in a non-iterated strategy space $(P_{C|D}, P_{C|C})$. Coercive ZD strategies are a subset of memory-one iterated strategies

$$\vec{p} = (p_{CC}, p_{CD}, p_{DC}, p_{DD}),$$  \hspace{1cm} (16)

where the components of vector $\vec{p}$ are the probabilities $p_t$ to cooperate in an iterated competition if the previous round was of type $t \in (CC, CD, DC, DD)$. 


The average probability of the interactions \( \tilde{\Omega}^{st} = (\Omega^{st}_{CC}, \Omega^{st}_{CD}, \Omega^{st}_{DC}, \Omega^{st}_{DD}) \), follows from the relation:

\[
M\tilde{\Omega}_{N-1} = \tilde{\Omega}_N,
\]

(17)

are the probabilities of interaction \( i \) after interaction \( j \), where each index goes over the \( (CC, CD, DC, DD) \) space.

The average statistics of interactions \( \Omega^{st} \) is an eigenvector of matrix (18):

\[
\hat{M}\tilde{\Omega}^{st} = \tilde{\Omega}^{st},
\]

(19)

with an eigenvalue 1, in the case of large number of iterations \( N \to \infty \) The average gain of agent \( p \) against an agent \( q \) is:

\[
G(p,q) = \tilde{W}\tilde{\Omega} = 1 \times \Omega^{st}_{CC} + S \times \Omega^{st}_{CD} + T \times \Omega^{st}_{DC} + 0 \times \Omega^{st}_{DD}
\]

(20)

where \( \tilde{W} = (1,S,T,0) \) is a vector of payoff for interactions \( (CC,CD,DC,DD) \).

Press and Dyson showed that if a strategy (16) fits linear relations:

\[
\begin{align*}
-1 + p_{CC} & = \alpha R + \beta R + \gamma, \\
-1 + p_{CD} & = \alpha S + \beta T + \gamma, \\
p_{DC} & = \alpha T + \beta S + \gamma, \\
p_{DD} & = \alpha P + \beta P + \gamma,
\end{align*}
\]

(21)

then the payoffs fit the linear relations:

\[
aG(p,q) + \beta G(q,p) + \gamma = 0,
\]

(22)

for any strategy \( q \).

Coercive ZD strategies correspond to \( \alpha = 0 \) in (22). In this case, strategy \( p \) imposes payoff \(-\gamma/\beta\) on any strategy \( q \). The ZD coercive strategies possess two independent variables \((p_{CC}, p_{DD})\). The probabilities \( p_{CD} \) and \( p_{DC} \):

\[
\begin{align*}
p_{CD} & = \frac{P_{p_{CC}} - (1 + p_{DD})R + (1 - p_{CC} + p_{DD})T}{P - R}, \\
p_{DC} & = \frac{P(-1 + p_{CC}) + S - p_{CC}S + p_{DD}(-R + S)}{P - R},
\end{align*}
\]

(23)

follow from (21).

A population composed of agents with non-iterated strategies \((P_{CD}, P_{DC})\) possesses the same statistics of interactions \( \tilde{\Omega}^{st} \) as a population composed of iterated strategies \( \tilde{p} = (p_{CC}, p_{CD}, p_{DC}, p_{DD}) \) if:

\[
P_{CD} = \frac{(1 + p_{CC} - p_{DD})p_{DD}}{(1 - p_{CC} + p_{DD})(p_{DD}(1 + 2S(-1 + T) - 2T) + T + p_{CC}T + S(1 + p_{CC} + 2T - 2p_{CC}T))},
\]

(24)

This result follows from the solution of (19) for \( \tilde{\Omega}^{st} \) and the definition of non-iterated strategies:

\[
P_{CD} = \frac{\Omega^{st}_{CD}}{\Omega^{st}_{CD} + \Omega^{st}_{DD}},
\]

(25)

\[
P_{DC} = \frac{\Omega^{st}_{DC}}{\Omega^{st}_{DC} + \Omega^{st}_{CC}}.
\]
as conditional probabilities.

To derive a coercive ZD subspace in the space of non-iterated strategies, one can scan all the possible values of iterated strategies $0 \leq p_{CC}, p_{CC}, p_{CC}, p_{CC} \leq 1$ under the constraint (24) to calculate the corresponding non-iterated strategies (24). The boundaries of the subspace can be derived analytically both for the zero-dominant game and Prisoner’s Dilemma (see Appendix).

The Results section includes a comparison of non-iterated ESS and the corresponding iterated ZD coercive strategy if it exists. We are particularly interested in passive ZD coercive strategies that do not assess the state of the opponent ($p_{CC} = p_{CD} \cdot p_{DC} = p_{CD}$). The $p_2 \rightarrow (1, 1, 0, 0)$ strategy is the only passive strategy of ZD coercive. It is the only solution of (23) under the constraint $p_{CC} = p_{CD}$ and $p_{DD} = p_{DC}$. For the sake of this comparison and estimating the opponent’s state assessment, we introduce the notion of indistinguishable strategies.

### C. Conditions for indistinguishable non iterated and iterated strategies

The ESS strategy $s^E_{ESS}$ in a binary game strategy space $K$ possesses an indistinguishable ZD coercive iterated strategy $s^*_{ZDc}$ in two conditions hold true. First, that there exists a strategy $s^*_{ZDc}$ in the ZD coercive space that predicts the same statistics of interactions as $s^E_{K}$ in a competition of agents with the same strategies:

$$
\Omega_{ZDc}(s^*_{ZDc}, s^*_{ZDc}) = \Omega_{K}(s^E_{K}, s^E_{K}), \tag{26}
$$

where $\Omega_{ZDc}$ and $\Omega_{K}$ are statistics of interaction for ZD coercive and $K$ strategies, respectively. Second, that the derivative of payoff $G_K$ due to the mutant strategy $s^E_{K}$ vanishes at $s^E_{K}$:

$$
\nabla_{s^E_{K}} G_K(s^E_{K}, s^E_{K})|_{s^m_{K} = s^E_{K}} = 0. \tag{27}
$$

This condition (27) generally holds true because at ESS the payoffs $G(s^E_{K}, s^E_{K}) \leq G(s^E_{K}, s^E_{K})$ and $G(s^E_{K}, s^E_{K}) \rightarrow G(s^E_{K}, s^E_{K})$ as $s^E_{K} \rightarrow s^E_{K}$. The ESS state that resides on the boundary of $K$, however, does not necessarily fit (27).

Consider two strategies $s^*_{ZDc} = (p^*_{CC}, p^*_{DD})$ and $s^E_{K} = (p^E_{CD}, p^E_{CC})$ that fit the conditions (26) and (27). Then, there exists a linear transformation $T_S$:

$$
\begin{pmatrix}
\Delta p_{CC} \\
\Delta p_{DD}
\end{pmatrix}
= T_S \begin{pmatrix}
\Delta P_{CD} \\
\Delta P_{CC}
\end{pmatrix}, \tag{28}
$$

where $T_S$ is a $2 \times 2$ matrix, which preserves the statistics of interaction:

$$
\begin{pmatrix}
\Delta p_{CC} + \Delta P_{CC} \cdot p^*_{DD} + \Delta P_{DD} \cdot p^*_{CC} \\
\Delta p_{DD} \cdot p^*_{CC} + \Delta P_{CC} \cdot p^*_{DD}
\end{pmatrix}
= T_K \begin{pmatrix}
\Delta P_{CD} + \Delta P_{CC} \cdot p^E_{DD} + \Delta P_{DD} \cdot p^E_{CC} \\
\Delta P_{CC} \cdot p^E_{DD} + \Delta P_{DD} \cdot p^E_{CC}
\end{pmatrix}. \tag{29}
$$

under the assumption of small changes in mutants’ strategies, $\Delta p^*/p^*, \Delta P/P < 1$.

Taking into account (26), the changes in the statistics of interaction (29) are:

$$
\Delta \tilde{\Omega}_{ZDc} = \begin{vmatrix}
\frac{\partial \tilde{\Omega}}{\partial p^m_{1}} & \frac{\partial \tilde{\Omega}}{\partial p^m_{2}} & \frac{\partial \tilde{\Omega}}{\partial p^m_{1}} & \frac{\partial \tilde{\Omega}}{\partial p^m_{2}}
\end{vmatrix}
\begin{pmatrix}
p^m_{1} & p^m_{2} & p^m_{1} & p^m_{2}
\end{pmatrix}
\Delta p = \tilde{A} \Delta p_1 + \tilde{B} \Delta p_4, \tag{30}
$$

and:

$$
\Delta \tilde{\Omega}_{K} = \begin{vmatrix}
\frac{\partial \tilde{\Omega}}{\partial P_{CD}} & \frac{\partial \tilde{\Omega}}{\partial P_{CC}} & \frac{\partial \tilde{\Omega}}{\partial P_{CD}} & \frac{\partial \tilde{\Omega}}{\partial P_{CC}}
\end{vmatrix}
\begin{pmatrix}
P^m_{CD} & P^m_{CC} & P^m_{CD} & P^m_{CC}
\end{pmatrix}
\Delta P = \tilde{C} \Delta p_1 + \tilde{D} \Delta p_4, \tag{31}
$$

where the vectors $\tilde{A}, \tilde{B}, \tilde{C}$ and $\tilde{D}$ are the vectors in the $\tilde{\Omega}$ space.

All the vectors $\tilde{A}, \tilde{B}, \tilde{C}$ and $\tilde{D}$, see (30) and (31), are perpendicular to the payoff vector $\tilde{W} = (1, T, S, 0)$:

$$\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D} \perp \tilde{W}, \tag{32}$$

because:

$$
\begin{align*}
\nabla_{s^*_{ZDc}} \left( \tilde{W} \tilde{\Omega}_{ZDc}(s^m_{ZDc}, s^*_{ZDc}) \right)|_{s^z_{ZDc} = s^*_{ZDc}} & = 0, \\
\nabla_{s^*_{K}} \left( \tilde{W} \tilde{\Omega}_{K}(s^E_{K}, s^E_{K}) \right)|_{s^m_{K} = s^E_{K}} & = 0.
\end{align*} \tag{33}
$$

In the K space (33) follows from (27) and (2). In the ZD coercive space also:

$$
\nabla_{s^z_{ZDc}} G_{ZDc}(s^m_{ZDc}, s^*_{ZDc})|_{s^z_{ZDc} = s^z_{ZDc}} = 0, \tag{34}
$$

because any strategy $s^z_{ZDc}$ determines the payoff of an opponent regardless of his strategy:

$$
G_{ZDc}(s^m_{ZDc}, s^z_{ZDc}) = G_{ZDc}(s^m_{ZDc}, s^z_{ZDc}), \tag{35}
$$

for any $s^z_{ZDc}$ and $s^z_{ZDc}$ including $s^m_{ZDc} = s^m_{ZDc} = s^z_{ZDc}$.

There is a linear transformation:

$$
\begin{pmatrix}
A \\
B
\end{pmatrix}
= \tilde{T}_V \begin{pmatrix}
C \\
D
\end{pmatrix}, \tag{36}
$$

where $\tilde{T}_V$ is a $2 \times 2$ because according to (32), all the vectors $\tilde{A}, \tilde{B}, \tilde{C}$ and $\tilde{D}$ are confined to a single two-dimensional plane. The vectors are three-dimensional as
\( \vec{\Omega} \). Despite having four components, the vectors \( \Omega \) are three-dimensional because the sum of the components is 1. The constraint to be perpendicular to \((1, T, S)\) brings all the vectors to a single plane.

The transformation of the strategies:

\[ \vec{T}_S = \vec{T}_V, \quad (37) \]

makes the strategy indistinguishable for an external observer. It may break down only if the vectors \( \vec{C} \) and \( \vec{D} \) are degenerate in the space of non-iterated strategies.

II. RESULTS

Figure 2 presents the main findings of this work. Replicator dynamics \((13)\) of the snowdrift game in a non-iterated strategy space possesses an ESS population that is indistinguishable of a passive ZD coercive strategy and, therefore, corresponds to self assessment. In addition, this ESS fits the observations of bowl and doily spiders.

In the case of the snowdrift game, there are two ESS regions (marked by red in Figure 2):

\[ \left( P_{SDESS}^{ESS} = S/T, P_{SDESS}^{SD} = 0 \right), \quad (38) \]

and the segment:

\[ P_{C|C} = 1, \quad 0 \leq P_{C|D} \leq P_{C|D}^{ESS}, \quad (39) \]

where:

\[ P_{C|D}^{ESS} = \frac{-1 + S + 3T - \sqrt{9 - 10S + S^2 - 14T + 6ST + 9T^2}}{2(-1 + S + T)}. \quad (40) \]

Implications of multiple ESSs will be discussed later.

To demonstrate that the ESS of the snowdrift game \((38)\) fits the observations of bowl and doily spiders, let us present the mating combat of bowl and doily spiders as a binary game. Each spider decides between two roles: to flee \( C \) or to fight until death \( D \). Under the assumption of a single fight per life, the payoffs \((R, S, T, P)\) are:

\[ R = \frac{V_F}{2} + V_{rest, life}, \]

\[ T = \left(1 - \delta\right)V_F + V_{rest, life}, \quad (41) \]

where \( V_F \) is an amount of unfertilized eggs in the female’s nest, \( V_{rest, life} \) is an estimate of amount of eggs to fertilize after the first fight. Payoff \( P \) corresponds to a \( CC \) fight where each player gets half of the nest and all the value of the future amount of eggs. In a \( CD \) competition, the \( D \) gets \( T \) equal to almost the total value of the nest \( (1 - \delta)V_F \) together with the value of the future amount of eggs, while \( C \) gets \( S \) equal to \( \delta V_F \) by the second insemination together with the value of the future amount of eggs. Payoff \( R \) corresponds to \( DD \) fight with equal probability to die and get 0 or get the total value of the nest together with the value of the future amount of eggs.

In the case of bowl and doily spiders, the value of payoffs \((41)\) can be estimated from the data collected by S. Austad \([32]\). The lifetime payoff of a spider without the fighting cost is \( V_L = 16.2 \) eggs. The average value of a female’s nest is \( V_F = 10 \) eggs. Then we can estimate that a spider that manages to flee uninjured from the first fight can expect \( V_L - V_F/2 < V_{rest, life} < V_L \). Thus \( V_{rest, life} \approx 13.5 \) eggs. This value of future amount of eggs is in agreement with the previous estimations \([24]\). The spider that flees from the competitor gets \( \delta \approx 5\% \) of the nest value by the second insemination.

Substituting the values from the previous paragraph to \((41)\) and applying a normalization procedure \([14]\), one gets:

\[ \vec{W} = (R = 1, S = 0.34, T = 1.66, P = 0). \quad (42) \]

The corresponding predicted ESS \((38)\) is:

\[ P_{C|D}^{ESS} = 0.207, P_{C|C}^{ESS} = 0. \quad (43) \]

The spiders play snowdrift (Chicken) game because mutual escalation of the conflict \( DD \) results in minimum payoff \( P \).

Statistics of the fight outcomes for bowl and doily spiders is estimated as:

\[ \vec{\Omega} = (\Omega_{CC}, \Omega_{CD}, \Omega_{DC}, \Omega_{DD}) = (0.0, 0.165, 0.165, 0.67) \quad (44) \]

In the case of bowl and doily spiders \([32]\), 0.67 of the fights between opponents of similar sizes result in the death of a participant. Thus \( \Omega_{DD} = 0.67 \), taking into account that the death of a competitor occurs only in a \( DD \) type competition, because both in \( CD \) and in \( DC \) the competitor flees and, therefore, stays alive. Mutual cooperation \( CC \) never happens \( \Omega_{CC} = 0 \). Then result \((44)\) follows because \( \Omega_{CC} + \Omega_{DC} + \Omega_{CD} + \Omega_{DD} = 1 \) and \( \Omega_{CD} = \Omega_{DC} \) due to symmetry.

Following \((14)\) and \((15)\), the observations of bowl and doily spiders correspond to a non-iterated strategy \((P_{SDESS}^{SD} = 0.198, P_{SDESS}^{SD} = 0)\). These results are surprisingly close to the prediction of this work \((43)\).

The ESS state \((P_{SDESS}^{SD} = S/T, P_{SDESS}^{SD} = 0)\) possesses an indistinguishable passive iterated ZD coercive strategy \( \vec{p}^{SD} \rightarrow (1, 1, 0, 0) \). The strategy is indistinguishable because conditions \([26]\) and \([33]\) hold true: the ESS belongs to the ZD coercive subspace (colored region) and fluxes of replicator dynamics vanish at the ESS point. The colors from brown to blue indicate the distance of the corresponding iterated strategy from \( \vec{p}^{SD} \), the only passive one among ZD coercive strategies. The correspondence of non-iterated ESS and \( \vec{p}^{SD} \) valid for all payoffs of the snowdrift game.
There is a discontinuity in iterated ZD coercive strategies at $\bar{p}^{ZD} \rightarrow (1, 1, 0, 0)$. The substitution of $\bar{p}^{ZD}$ in (44) does not result in (43). Statistics converges to the correct limit $\bar{p}^{ZD} \rightarrow (1, 1, 0, 0)$ under the constraint (23), see Figure 2 and the (Appendix). Therefore, this discontinuity does not affect the predictions of this work.

The indistinguishable iterated passive strategy indicates no assessment of the opponent’s state. It supports the zero opponent’s state assessment assumption of the war of attrition model. The cost of fight as a function of time $g(t)$, therefore, should fit the distribution of fight duration (see (Appendix)):

$$\int_0^T P(t)dt = 1 - \exp^{-2g(T)}, \quad (45)$$

where $P$ is the probability of a fight to take time $t$ and $V$ normalization. In the case of bowl and doily spiders, the obtained price of fight is highly correlated with the probability to get a fatal injury, see Figure 4.

Prediction of self-assessment is a unique property of snowdrift payoffs. For instance, in the case of Prisoner’s Dilemma, an ESS state possesses an indistinguishable iterated strategy only for specific values of the payoffs.

Prisoner’s Dilemma possesses two ESS regions:

$$P_{C|C} = 0,$$
$$P_{C|C}^{ESS} = \frac{-1 + S - T + \sqrt{9S^2 + (1 + T)^2 + 2S(-5 + 3T)}}{2(-1 + S + T)},$$

when $P_{C|D} = 0$ and:

$$P_{C|D} = 0,$$
$$P_{C|D}^{ESS} = \frac{-1 + S + 3T - \sqrt{9 - 10S + S^2 - 14T + 6ST + 9T^2}}{2(-1 + S + T)},$$

when $P_{C|C} = 1$. The first segment is the same as in the case of the snowdrift game.

The ZD coercive subspace in a non-iterated strategy space for the payoffs of Prisoner’s Dilemma takes either of two forms, see Figure 3 for the payoffs $S < -T$ and $S > -T$, respectively, as well as the (Appendix). In both cases, the passive iterated strategies correspond to a line between the points $(0, S/(1-S))$ and $(1/T, 0)$. Replicator dynamics fluxes vanish at the point $(0, P_{C|C}^{ESS}^D)$ and $(1, P_{C|D}^{ESS}^D)$, see (46) and (47). These points possess an indistinguishable passive strategy only if $S = 1 - T$. It differs from the snowdrift game in that it possesses the ESS and indistinguishable passive strategy for the entire range of allowed payoffs.

There are intuitive and counter-intuitive results. On one hand, one of the predicted ESSs for snowdrift fits the observations of bowl and doily spiders. On the other hand, the ESS of homogeneous cooperation is predicted both for snowdrift and Prisoner’s Dilemma. Prisoner’s Dilemma, however, possesses another ESS with homogeneous defection that is more intuitive. Surprisingly, at this ESS, agents preserve their ability to cooperate against an opponent with unconditional cooperation. Let us discuss the results in light of the obtained predictions, previous studies, and observations in nature.

### III. DISCUSSION

The current study claims that no information assessment of the opponent’s state during conflict escalation is an inevitable evolutionary development. This result emerges as a mathematical peculiarity while comparing similar iterated and non-iterated strategies to play a binary snowdrift game. Eventually, the non-iterated evolutionary stable strategy to play the snowdrift game possesses an indistinguishable iterated strategy that lacks assessment of the opponent’s state. On these grounds, we conclude that a non-iterated ESS strategy should also lack the opponent’s state assessment.

We use a hypothesis: The amount of information shared by the opponents should be preserved across indistinguishable strategies. Indistinguishable strategies result in the same payoffs and statistics of interactions in a population. The statistics of interaction depends on the information shared by the opponents, e.g. in the case of no common information, the responses of the opponents are arbitrary and independent of each other. This limits the possible statistics of interactions to the form:

$$\vec{\Omega}_{arb} = (x_1x_2, x_1(1 - x_2), (1 - x_1)x_2, (1 - x_1)(1 - x_2))$$

where $x_1$ and $x_2$ are probabilities to cooperate $C$ for the first and the second players, respectively. Any other statistics $\vec{\Omega}$ requires shared information and, therefore, the assessment of the opponent’s state.

Important to note that no assessment of the opponent’s state occurs only during actual fighting. Bowl and doily spider, as any other player of war of attrition game, recognizes a surrendering opponent. It results in statistics of responses $\vec{\Omega}$ which can not be explained by arbitrary responses [48]. e.g see statistics of bowl and doily spiders [44]. Thus one should distinguish assessment of the opponent’s state from mere recognition of the surrendering opponent.

Both iterated and non-iterated strategies are essential to analyze information assessment in an evolutionary stable population. For instance, in the case of bowl and doily spiders, non-iterated strategies predict the ESS in agreement with previous observations. Non-iterated strategies, however, lack any information regarding the opponent’s state assessment. Iterated strategies do not predict the same ESS. Moreover, iterated ZD coercive strategies cannot predict the ESS because the payoffs are indistinguishable across all mutants. An iterated stra-
egy nevertheless accurately describes the mechanism of opponent’s state assessment mechanism.

The interpretation of spiders’ fights as a binary game with non-iterated strategies is supported by the accurate prediction of bowl and doily spiders’ mating combat statistics as a function of the payoffs. This description, however, includes the approximation of a single fight per life. The prediction, therefore, may become less accurate in the case of multiple competitions per life.

Non-iterated strategies predict evolutionary stable population of cooperators both for snowdrift game and prisoner’s dilemma, see Figures 2 and 3. Non-iterated strategies, however, may predict a non-physical ESS. Non-iterated strategies lack the microscopic description of a fighting mechanism. Thus, the validity of the predicted ESS requires a valid description of fight mechanism[8], such as the war of attrition model for \((P_{C|D} = S/T, P_{C|C} = 0)\) ESS in the snowdrift game. Cooperative populations may be an artifact of small mutation steps in replicator dynamics or a consequence of non-iterated strategies ability to include an effective memory of the opponent’s behavior. These two phenomena promote evolutionary stable cooperation[7][11][33][36].

War of attrition is a mechanism behind \((P_{C|D} = S/T, P_{C|C} = 0)\) ESS of the snowdrift game. It unifies the war of attrition and snowdrift games in a single framework. Both of these models are associated with conflict escalation. Only the war of attrition, however, includes a built-in constraint of self assessment. The implication of this work is that assumption of self assessment should be considered in the case of evolutionary applications of the iterated snowdrift game.

On the other hand, the social Prisoner’s Dilemma does not necessarily prevent the emergence of mutual information assessment. Indistinguishable passive strategies exist only for a small subset of the possible payoffs.

It is important to note that the population converges to either of multiple ESSs as a function of initial state of a population: e.g. choose a point \((P_{C|D}, P_{C|C})\) in Figure 2 and follow the flux line until convergence. The choice of an initial condition cannot be restricted in general. A reasonable assumption is that the population starts at line \(P_{C|D} = P_{C|C}\) which represents arbitrary responses to be at state \(C\) and \(D\) with probabilities \(1 - P_{C|C}\) and \(P_{C|C}\), respectively, independent from the state of the opponent. Both for the snowdrift game and for Prisoner’s Dilemma, it leads to convergence to a more realistic (non-cooperative) ESS.

This work shares with previous studies the definition of non-iterated strategies[5][17], behavior roles with different fight activities[11][33][35], replicator dynamics[39] with small evolutionary steps to find ESS, and focus on the opponent’s state assessment[14][32][40–44] and mating combats’ statistics of bowl and doily spiders[24][62] to verify the theoretical predictions of the proposed model. The main differences include a mean field assumption to find the ESS of non-iterated strategies, the construction of an indistinguishable iterated strategy using iterated ZD coercive strategies, and the estimation of information assessment during the fight using properties of the corresponding indistinguishable strategy.

To conclude, this work extends the applicability of evolutionary game theory to predict invisible parameters of animal competitions, for instance an assessment of the opponent’s state. The propositions are corroborated by an accurate prediction of the behavior of specimens of bowl and doily spiders during mating combats. Zero assessment of the opponent’s state during conflict escalation (e.g. snowdrift game or war of attrition) is predicted with the help of recently discovered iterated zero-determinant strategies. Iterated strategies are used to analyze complex phenomena like policy-making and the nature of altruism. Zero-determinant strategies were pointed out as a promising route to tackle mind-like phenomena. Following this research line, conflict escalation is an evolutionary path to avoid complex decision-making that considers information about the opponent.

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strategy:
\[0 \leq p_{CC}, p_{CD}, p_{DC}, p_{DD} \leq 1,\] (49)

where \(p_{CD}\) and \(p_{DC}\) are defined by \(p_{CC}\) and \(p_{DD}\) by eq. [23]. Then extreme values are substituted in (24) of non-iterated strategies. The calculations are performed with the help of Wolfram Mathematica and verified using a direct scan of the possible \(p\) values [40].

1. Snowdrift Boundaries

The ZD coercive subspace is confined by the straight line (mostly from the right):

\[P_{C|D} = \frac{(S + P_{C|C}(1 - S))}{T},\] (50)

and three segments from the left, see Figure 2. The low segment is:

\[P_{ZDc}^{ZDc}(p_{CC}, (-S + p_{CC}S)/(-1 + S), T, S),\]
\[P_{ZDc}^{ZDc}(p_{CC}, (-S + p_{CC}S)/(-1 + S), T, S)\] (51)

where \((-1 + T)/(-S + T) \leq p_{CC} \leq 1\). The middle segment is:

\[P_{ZDc}^{ZDc}(p_{CC}, (1 - T + p_{CC}T)/(-1 + T), T, S),\]
\[P_{ZDc}^{ZDc}(p_{CC}, (1 - T + p_{CC}T)/(-1 + T), T, S)\] (52)

where \((-1 + T)/(-S + T) \leq p_{CC} \leq (-2 + 2T)/T\). The upper segment is:

\[P_{ZDc}^{ZDc}(p_{CC}, 1, T, S),\]
\[P_{ZDc}^{ZDc}(p_{CC}, 1, T, S)\] (53)

where \((-2 + 2T)/T \leq p_{CC} \leq 1\).

There are two ESS regions. The point:

\[P_{C|C}^{ESS} = 0,\]
\[P_{C|D}^{ESS} = \frac{S}{T},\] (54)

and [39]. It holds for all Snowdrift payoffs.

The replicator dynamics fluxes vanish at the point ESS and at the edge of ESS segment.

Near point ESS left boundary of ZD coercive space can be expanded as:

\[P_{C|D, ESS}^{ESS} = \frac{(-1 + P_{C|C})S}{P_{C|C}(-1 + S + T) - T} \] (55)

The right boundary corresponds to [50].

The point ESS [51] corresponds to a passive strategy in the ZD coercive subspace.

IV. APPENDIX

A. Subspace of iterated strategies

We found the boundaries by solving a system of inequalities for probabilities of an iterated ZD coercive strategy:
2. Prisoner’s Dilemma Boundaries

The ZD coercive boundaries are different for the cases $S < -T$ and $S > -T$.

The first case $S > -T$, see Figure 3 The bottom boundary is:

$$ P_{ZDc}^{C/D}(pcc, (1 - T + pccT)/(-1 + T), T, S), $$

$$ P_{ZDc}^{C/D}(pcc, (1 - T + pccT)/(-1 + T), T, S), $$

where $(-1 + T)/T < pcc \leq (-2 + 2T)/(-S + T)$. The left boundary includes two straight lines:

$$ P_{ZDc}^{C/D} = 0, $$

$$ 0 \leq P_{ZDc}^{C/D} < \frac{S}{-1 + S}, $$

(57)

where together with the line between the points $(0, S/(-1 + S))$ and $(1, 1/T)$:

$$ P_{ZDc}^{C/D} = -T P_{ZDc}^{C/D} + S \frac{-1 + S}{-1 + S}, $$

(58)

where $0 < P_{ZDc}^{C/D} \leq (1/(-1 + S))$. Right boundary is:

$$ P_{ZDc}^{C/D}(pcc, (-1 - S + pccS)/(-1 + S), T, S), $$

$$ P_{ZDc}^{C/D}(pcc, (-1 - S + pccS)/(-1 + S), T, S), $$

(59)

where $(-2 + 2T)/(-S + T) \leq pcc \leq 1$. The upper boundary is a segment:

$$ P_{ZDc}^{C/D} = 1, $$

$$ 1 \leq P_{ZDc}^{C/D} \leq 1, $$

(60)

between points $(1/T, 1)$ and $(1, 1)$.

The second case $S < -T$, see Figure 3 The left boundary consists of two segments:

$$ P_{ZDc}^{C/D} = -T P_{ZDc}^{C/D} + S \frac{-1 + S}{-1 + S}, $$

(61)

between the points $(0, S/(-1 + S))$ and $(1, 1/T)$ and

$$ P_{ZDc}^{C/D}(pcc, 0, T, S), $$

$$ P_{ZDc}^{C/D}(pcc, 0, T, S), $$

(62)

where $(1 + S)/S \leq pcc \leq 1$. The lower boundary is:

$$ P_{ZDc}^{C/D}(pcc, (-1 - S + pccS)/(-1 + S), T, S), $$

$$ P_{ZDc}^{C/D}(pcc, (-1 - S + pccS)/(-1 + S), T, S), $$

(63)

where $(1 + S)/S \leq pcc \leq 1$. The upper boundary is a segment:

$$ P_{ZDc}^{C/D} = 1, $$

$$ 1 \leq P_{ZDc}^{C/D} \leq 1, $$

(64)

between points $(1/T, 1)$ and $(1, 1)$.

A passive strategy corresponds to the points $(0, S/(-1 + S))$ and $(1, 1/T)$. These points coincide with $(0, P_{ESS}^{PD})$ and $(P_{ESS}^{C/D}, 1)$ (ESS points with zero replicator dynamics fluxes (condition [27]) see eqs. [46] and [47]) only when $S = 1 - T$.

B. ESS distribution of fight durations for war of attrition, eq. 45

Let us calculate the ESS of war of attrition following [15]. The ESS behavior in war of attrition is calculated under the assumption of no information assessment of the opponent’s state. According to this model, two players engage in a fight. They choose times $t_1$ and $t_2$ to stay in the fight according to distributions $p_1(t)$ and $p_2(t)$, respectively. The winner receives prize $V$ which is independent of time. The cost of fight as a function of time is $g(t)$, where $g(t)$ is a monotonic function of time in the units of $V$. Payoffs of the winner and the less fortunate opponent in a fight with duration $t$ are $V - g(t)$ and $-g(t)$, respectively. War of attrition is a type of conflict escalation like the snowdrift game: escalation of the fight for long durations $t$ results in minimum payoff for each player.

Let us find the probability of fight duration as a function of $V$ and $g(t)$. The first player wins if $t_1 > t_2$ with probability:

$$ \int_{t_1 > t_2} p_1(t_1)p_2(t_2)dt_1dt_2. $$

(65)

Thus, the payoff of the first player is:

$$ \int_{t_1 > t_2} (V - g(t_2))p_1(t_1)p_2(t_2)dt_1dt_2 - \int_{t_1 < t_2} g(t_1)p_1(t_1)p_2(t_2)dt_1dt_2, $$

(66)

It can be rewritten as:

$$ \int_0^\infty dt_1 \int_0^{t_1} dt_2 (V - g(t_2))p_1(t_1)p_2(t_2) - \int_0^\infty dt_1 g(t_1) \int_0^{\infty} dt_2 p_1(t_1)p_2(t_2), $$

(67)

Strategy $p(t)$ that slightly deviates from ESS $p_1 = p + \delta p_1$ bring no advantage to the first player if $p_2 = p$:

$$ \int_0^\infty dt_1 \delta p_1(t_1) $$

(68)

$$ \left[ \int_0^{t_1} dt_2 (V - g(t_2))p_2(t_2) - g(t_1) \int_0^{\infty} dt_2 p_2(t_2) \right] = 0, $$

for any $\delta p_1$. Following [68]:

$$ \int_0^t (V - g(t))p(t)dt - g(t) \int_t^\infty p(t)dt = 0. $$

(69)
is the integral equation for the ESS strategy \( p(t) \).

The solution of (69) is:

\[
p = \frac{g'}{\alpha} \exp \left( -\frac{g(t)}{\alpha} \right),
\]

(70)

where \( p \) is the ESS probability to keep fighting at time \( t \).

The probability of a fight to take time \( T \) is:

\[
P(T) = 2p(T) \int_T^\infty p(t) \, dt,
\]

(71)

Taking into account (70), the probability (71) becomes:

\[
P(T) = 2 \frac{g'}{\alpha} \exp \left( -\frac{2g}{\alpha} \right),
\]

(72)

The probability of fight duration (72) decays exponentially with time \( t \) if cost of fight grows linearly with time \( g(t) = Kt \).

Sometimes, it is more convenient to work with cumulative probability of fight to take less than time \( T \), see (45). Both (72) and (45) can be checked against an experiment.
FIG. 3: Prisoner’s Dilemma: replicator dynamics fluxes (blue arrows) in the space of non-iterated strategies ($P_{CD}, P_{CC}$). (A) The payoffs $S > -T$. Similar to the snowdrift game, there are two ESS regions (red lines). The corresponding populations are either cooperative $P_{CC} = 1, P_{CD} = 0$ or defective $P_{CC} = 0, P_{CD} = 1$, see Figure 1. The color map is a subspace of coercive zero-determinant strategies. Colors from dark green to brown show the distance of the corresponding iterated strategy ($p_{CC}, p_{CD}, p_{DC}, p_{DD}$) from $(1, 1, 0, 0)$. Conditions for indistinguishable iterated and non-iterated strategies occur only if $S = -T$ at the strategies $(0, S/(-1 + S))$ and $(1/T, 1)$. Thus, contrary to the snowdrift game, Prisoner’s Dilemma lacks the prediction of self-assessment for the entire ESS region. (B) The payoffs $S < -T$. The difference is only in the shape of ZD coercive subspace.
FIG. 4: Normalized cost of fight as a function of time (blue) multiplied by factor 2 \((2g(t)/V)\) and the probability of fatal injury (red) for the combat of male bowl and doily spiders. The data of probability to suffer a fatal injury is taken according to Figure 4 from reference [32] for combatants of the similar size. The cost of fight is calculated from the distribution of fight duration with eq. (45) and Figure 6 of the same reference. The probability approaches its maximum value 1 after 200 seconds. Until this time, the probability of fatal injury (red markers) and twice the cost of the fight (blue markers) fit each other. Factor two corresponds to the probability that a fight results in an injury of one of the competitors is twice the probability of a competitor getting an injury.