Criterion for Combined Mode I-II of Brittle Fracture

Hideo AWAJI, Toshiya KATO,* Sawao HONDA and Tadahiro NISHIKAWA
Nagoya Institute of Technology, Gokiso-cho, Showa-ku, Nagoya-shi 466-8555
*Toyota A. L. W. Ltd., Toyoda-cho, Kariya-shi 448-0848

The Griffith concept of energy equilibrium for brittle mode I crack propagation was extended to a combined mode I-II crack extension. First, the maximum energy release rate for an infinitesimally kinked crack extension under combined mode I-II loading, analyzed by other researchers, was successfully expressed with a combined mode stress intensity factor. Second, the fracture energy rate was estimated from the area of the frontal process zone estimated from the area enclosed by the isostress contours of both the maximum principal stress and the maximum shear stress. Third, these considerations led to the estimated $K_{IC}/K_{II}$ ratio of 1.20, which was close to the experimental results of 1.1 to 1.3 obtained previously. Lastly, the combined mode I-II fracture criterion was formulated by adopting the same procedure, and the predicted criterion coincided well with Shetty's empirical criterion. The fracture toughness values of mode I and mode II, and the combined mode I-II criterion for polycrystalline alumina, float glass and PMMA (polymethyl methacrylate) were also estimated experimentally using a disk method and a rectangular test. The estimated results of the $K_{II}/K_{i}$ ratio using the disk method ranged from 1.1 to 1.3, which was very close to the predicted value of 1.20, whereas the rectangular test specimens exhibited considerable stable crack growth at the crack tip and yielded a smaller $K_{II}/K_{i}$ ratio, which was related to the mode transformation and stress shielding.

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1. Introduction

Although a fluid withstands only normal stress under static conditions, a solid body has resistance to both normal and shear stresses. A crack in a solid under a multiaxial stress state thus yields a mode I stress intensity factor caused by tensile stress, and mode II and mode III stress intensity factors caused by shear stresses. Three critical values of the stress intensity factors, representing the resistance of the solid materials to crack extension, can thus be defined, i.e., mode I, mode II and mode III fracture toughness, corresponding to each mode. All of these critical values are considered to represent physical properties of materials, which lead to the result that the ratio of mode II to mode I fracture toughness, $K_{IC}/K_{i}$, should be a constant, as far as brittle crack propagation is concerned.

A number of testing techniques for ceramic materials have been proposed to estimate the mode II fracture toughness and combined mode I-II fracture criterion, using several specimen configurations, such as a rectangular plate, a disk with a thick through-slit, a disk with a chevron-shaped notch, a disk with a Knoop crack, an asymmetric four-point-bending specimen, and others. However, the ratios of $K_{IC}/K_{i}$, measured for several ceramics by other researchers, show considerable variation in the range from 0.5 to 2.

Crack resistance in polycrystalline ceramics with rising $R$-curve behavior under static loading consists of two parts, the intrinsic fracture toughness and the extrinsic increase of fracture resistance,

$$K_R(\Delta a) = K_i + \Delta K_R(\Delta a)$$

where $K_R(\Delta a)$ is the fracture toughness of a material with $R$-curve behavior, $K_i$ the intrinsic or initial fracture toughness, and $\Delta K_R(\Delta a)$ the increase of fracture toughness after a certain extension, $\Delta a$. The intrinsic toughening mechanism involves the actual creation of a damaged or phase-transformed microstructure, and produces a frontal process zone (frontal process zone toughening mechanism), as shown schematically in Fig. 1, whereas $\Delta K_R$ is caused by stress shielding effects due to interlocking and bridging in a process zone wake after several instances of crack growth (process zone wake toughening mechanism). The latter is an extrinsic or apparent increase of crack resistance and is not a unique function of $\Delta a$ because $\Delta K_R$ depends on the specimen configuration or the loading condition used. Therefore, the value of $K_R$ should be treated as $K_{IC}$, mode I fracture toughness as a unique material property.

Under mode II or combined mode I-II loading, a crack extension in brittle materials occurs in a non-coplanar direction, as shown in Fig. 2. In this case, the stress shielding mechanism due to bridging will behave in a different manner from that of mode I crack extension, which has been discussed by Tong et al. and Li and Sakai. Also, after a certain extension of the kinked crack under mode II or...
as mode I loading, Griffith’s energy equilibrium is expressed by the fracture energy rate required for crack growth, \( R = \frac{dW}{da} \), where strain energy and additional crack growth, equals the energy provided by the released strain energy. The surface must be supplied by released strain energy. Ac-tion is that the energy needed to create a new fracture surface must be supplied by released strain energy.

The superior point of the Griffith concept is that the equation for the energy equilibrium is composed of two discrete parts. One part is the critical energy release rate, as expressed in the left-hand side of Eq. (2). The energy release rate is not dependent on the nonlinear stress state in the process zone area in front of a crack tip. In other words, the energy release rate is precisely determined by the stress intensity factor that depends on only the outer boundary conditions, namely, the applied loading and specimen geometry, as long as the condition of small scale yielding is applicable. The second part is the fracture energy rate, as expressed in the right-hand side of Eq. (2), which is directly related to the damaged process zone, which represents the resistance of the material that must be overcome for crack growth to occur and should be a material constant.

Therefore, there is an essential difference between the Griffith concept and other criteria such as the maximum hoop stress theory and the minimum strain energy density theory. The maximum hoop stress theory, proposed by Erdogan and Sih, postulates that a crack will extend along the radial direction in which the hoop stress becomes maximum and when the maximum hoop stress at the crack tip reaches a critical value. On the other hand, the minimum strain energy density theory, which is credited to Sih, states that crack extension occurs in the direction of the region around the crack tip with the minimum strain energy density factor and when the minimum strain energy density factor in the region reaches a critical value. The critical values of these two theories, however, must be calculated under the assumption of a linear elastic stress field even in the nonlinearly damaged zone in the vicinity of the crack tip.

2.2 Maximum energy release rate

During combined mode I–II loading, crack extension in isotropic and brittle materials occurs in non-coplanar directions. The schematic for a kinked crack is shown in Fig. 2. The energy release rate for an infinitesimally kinked crack extension under combined mode I–II loading was analyzed by Kageyama and Okamura. The maximum energy release rate is expressed by the approximated equation:

\[
G_{\text{max}} = \frac{1}{2\pi} \left( K_{\text{II}}^2 + \alpha K_{\text{II}}^2 + K_{\text{I}} \sqrt{K_{\text{II}}^2 + 2\alpha K_{\text{II}}^2} \right)
\]

where \( \alpha = 3.042 \) and \( E' = \frac{E}{E/(1-v^2)} \) for plane stress and \( E' = \frac{E}{E/(1-v^2)} \) for plane strain.

A nondimensional form of the maximum energy release rate is defined as:

\[
G_{\text{max}}^* = \frac{G_{\text{max}}}{G_0}
\]

where \( G_{\text{max}}^* \) is the nondimensional maximum energy release rate, and \( G_0 \) is the energy release rate in the coplanar crack extension (\( \theta = 0 \)) under the combined mode and is defined as:

\[
G_0 = \frac{K_{\text{I}}^2 + K_{\text{II}}^2}{E'}
\]

Fig. 2. Combined mode I–II loading and a combined mode stress intensity factor. The dotted line indicates the initial direction of the kinked crack, and the solid line represents the path of the additional extension.

combined mode I–II loading, the mode of the stress intensity factor at the kinked crack tip changes drastically from the original mode of the initial crack tip. These indicate that the initial mode II fracture toughness, \( K_{\text{II}} \), is an intrinsic fracture resistance which relates to the energy required to create the frontal process zone, therefore, \( K_{\text{II}} \) should be \( K_{\text{II}} \), the mode II fracture toughness as a unique material property. Also, the mode II fracture toughness of a material should be measured at the very beginning of crack extension in order to prevent mode transformation due to the subsequent crack growth, using a notched specimen with no bridging to avoid the shielding effects.

Recent theoretical work on the combined mode fracture criterion presented by Awaji, and Awaji and Kato clarified that an estimated \( K_{\text{IIc}}/K_{\text{IC}} \) ratio based on the Griffith energy equilibrium was 1.20 for an ideally brittle material. This value is close to the experimental results of 1.1 to 1.3 for several inorganic materials, as obtained previously by Awaji and Sato, and Shetty et al. using the disk method with a machined thick through-slit. These experimental results also imply that \( K_{\text{IIc}} \) must be a unique material property like \( K_{\text{IC}} \).

The aim of this paper is twofold. First, the Griffith energy criterion for brittle fracture, which was originally formulated for mode I crack extension, is extended to combined mode I–II crack propagation. The mode II fracture toughness of brittle materials will be shown to be a unique material property, and a fracture criterion for combined mode I–II crack propagation will be formulated analytically for the situation in which a crack propagates into highly brittle solids such as ceramics. Second, the combined mode criterion is estimated experimentally. The disk method proposed by Awaji and coworkers will be used for polycrystalline alumina, float glass and PMMA (polymethyl methacrylate) to estimate mode II fracture toughness and the experimental combined mode I–II criterion. A rectangular plate with a slanted crack will also be applied to float glass and PMMA, and considerable toughness and the experimental combined mode I–II toughness of brittle materials will be shown to be a unique material property. Also, the Griffith concept and other criteria such as the maximum hoop stress theory and the minimum strain energy density theory. The maximum hoop stress theory, proposed by Erdogan and Sih, postulates that a crack will extend along the radial direction in which the hoop stress becomes maximum and when the maximum hoop stress at the crack tip reaches a critical value.

2. Theory

2.1 Energy equilibrium

Griffith postulated that a necessary condition for crack extension is that the energy needed to create a new fracture surface must be supplied by released strain energy. Accordingly, when the energy release rate, \( G = dU/da \), equals the fracture energy rate required for crack growth, \( R = dW/da \), crack growth can occur. For mode I loading, Griffith’s energy equilibrium is expressed as:

\[
\frac{K_{\text{IIc}}^2}{E'} = 2G_0
\]
Table 1. Relationship among $2\gamma_k^{*}, K_k$, $K_{IC}/K_{IC}$ and $K_{II}/K_{II}$ for Various $K_{II}$ and $K_{I}$ Ratios

| $\beta$=K_{II}/K_{I} cot^{1}\beta$ [degree] | $\theta_m$ [degree] | 2 $\gamma^{*}$ | $K_{I}$ | $K_{II}$ | $K_{IC}/K_{IC}$ | $K_{II}/K_{II}$ |
|--------------------------------------------|---------------------|------------|--------|--------|----------------|-------------|
| 0                                          | 90                  | 0          | 1.00   | 1.00   | 1.00          | 1.00        |
| 0.176                                       | 80                  | -19.1      | 1.04   | 1.02   | 0.974         | 0.172       |
| 0.364                                       | 70                  | -34.4      | 1.14   | 1.06   | 0.912         | 0.332       |
| 0.577                                       | 60                  | -45.4      | 1.30   | 1.13   | 0.833         | 0.481       |
| 0.839                                       | 50                  | -53.4      | 1.49   | 1.20   | 0.741         | 0.622       |
| 1.19                                        | 40                  | -59.6      | 1.69   | 1.27   | 0.635         | 0.757       |
| 1.73                                        | 30                  | -64.6      | 1.89   | 1.32   | 0.511         | 0.885       |
| 2.75                                        | 20                  | -68.9      | 2.04   | 1.36   | 0.365         | 1.002       |
| 5.67                                        | 10                  | -72.6      | 2.14   | 1.38   | 0.195         | 1.107       |
| $\infty$                                    | 0                   | -76.0      | 2.19   | 1.38   | 0             | 1.202       |

Fig. 3. Relationships among the maximum energy release rate, $G_{max}$, the combined mode stress intensity factor, $K_{k}^{*}$, and an angular function, $f(\theta_{m})$, where $\beta=K_{II}/K_{I}$.

For convenience in formulating the combined mode fracture criterion, a stress intensity factor under combined mode I-II loading, $K_{s}$, is introduced, which is called a "combined mode stress intensity factor," as shown in Fig. 2.

$$K_{s} = \sqrt{2\pi\sigma_{o}}$$

$$= K_{I} \cos \theta \cdot \left( \cos \frac{\theta}{2} - \frac{3}{2} \sin \theta \right)$$

$$= K_{I} g(\theta)$$

Here, $\sigma_{o}$ represents hoop stress defined in the vicinity of a crack tip, and $\beta=K_{II}/K_{I}$. The nondimensional form of the combined mode stress intensity factor, $K_{s}^{*}$, can also be defined as

$$K_{s}^{*} = \frac{K_{s}}{\sqrt{K_{I}^{2} + K_{II}^{2}}}$$

The maximum energy release rate under pure mode I loading is well known to be

$$G_{max} = \frac{K_{I}^{2}}{E'}$$

Referring to the form of Eq. (8), it is expected that the maximum energy release rate during combined mode loading will be related to the combined mode stress intensity factor as

$$G_{max} = \frac{f(\theta_{m})K_{k}^{*}}{E'}$$

where $K_{k}^{*}$ is the value of $K_{s}$ in the $\theta_{m}$ direction, and $f(\theta_{m})$ is a function described only by $\theta_{m}$. The relationship expressed in Eq. (9) can also be rewritten in the nondimensional form as

$$G_{max} = \frac{f(\theta_{m})K_{k}^{*}}{E'}$$

where $K_{k}^{*}$ is the nondimensional form of the $K_{s}$ defined by Eq. (7) for $\theta=\theta_{m}$.

Figure 3 shows the relationships among the values of $G_{max}$, $K_{k}^{*}$ and $f(\theta_{m})$. The horizontal axis in the figure indicates the value of cot$^{-1} \beta$. Clearly, the maximum energy release rate during combined mode loading can be expressed by the term for the combined mode stress intensity factor, and there exist interesting relationships among them, as follows.

For mode I crack extension

$$G_{max}^{*} = K_{k}^{*} f(\theta_{m}) = 1$$

and for mode II crack extension

$$G_{max}^{*} = 1.521 = 1.15^{3}, \quad K_{k}^{*} f(\theta_{m}) = 1.15^{3}.$$}

The energy equilibrium for mode II crack extension is then derived from Eq. (9) as

$$\frac{f(\theta_{m})K_{k}^{*}}{E'} = 2\gamma_{II}$$

2.3 Fracture energy rate

In general, ceramic materials intrinsically have ionic and/or covalent bonding, and the plastic deformation of structural ceramics due to dislocation movement is considered to be extremely limited compared to metals. Therefore, the frontal process zone in front of the crack tip in ceramics is formed mainly by microcracks[22] rather than by dislocations as in metals. If the fracture energy equals the energy required to form the frontal process zone including the microcracks, then it may be considered to be proportional to the size of the critical process zone area in the vicinity of the crack tip.

The frontal process zone area near the crack tip is simply estimated based on the area enclosed by the isostress contours of both the maximum principal stress and the maximum shear stress. These contours under mode I and mode II loading have been shown previously.[14,15] The calculated values of the areas enclosed by the contours are shown in Table 2, where the area enclosed by the maximum principal stress during mode I loading is designated to be unity.
The ratio of the fracture energy for the two modes, $\gamma_{II}/\gamma_I$, is considered to be equal to the ratio of the critical process zone areas, which in turn approximately equals the ratio of the process zone areas estimated from the areas enclosed by the isostress contours. The area ratio calculated using values in Table 2 is 2.19. This calculation was based on the assumption that the process zone area is restricted in size, as in the case with ceramic materials, so that the approximation of the linear elastic stress field is applicable even in the vicinity of a crack tip. Consequently, the ratio of the fracture energy for the two modes can be estimated as

$$\frac{\gamma_{II}}{\gamma_I} = 2.19 \quad (13)$$

Based on Eq. (12), the ratio of mode II to mode I fracture toughness can be calculated, independently of the materials considered as

$$K_{IC} \over K_{II} = 1.20 \quad (14)$$

This value of 1.20 is close to the experimental results in the range of 1.1 to 1.3 reported previously for several inorganic materials$^{23)}$ and to the recent result of 1.28 for float glass$^{16)}$ measured by the disk method.

### 2.4 Combined mode energy equilibrium

The extended Griffith’s energy equilibrium under combined mode I–II loading can be expressed in the nondimensional form using Eq. (10) as

$$f(\theta_m)(K_{IC})^2 = 2\gamma^* \quad (15)$$

where $2\gamma^*$ is the nondimensional fracture energy rate under combined mode loading and $K_{IC}$ is the nondimensional critical value of $K_{IC}$. The nondimensional fracture energy rate for mode I crack extension can be derived using Eq. (5) as

$$2\gamma_I = \frac{2\gamma^*}{g(\theta_m)} \quad (16)$$

and the nondimensional fracture energy rate for mode II crack extension can also be derived as

$$2\gamma_{II} = \frac{2\gamma^*}{g(\theta_m)} \quad (17)$$

Then, the nondimensional fracture energy rate for combined mode crack extension can be expressed, considering the fraction rates of $K_I$ and $K_{II}$, as

$$2\gamma^* = 2\gamma_I \frac{K_I^2}{K_{IC}^2 + K_{II}^2} + 2\gamma_{II} \frac{K_{II}^2}{K_{IC}^2 + K_{II}^2} \quad (18)$$

and the ratio of the combined mode and the mode I fracture energy per unit area of a crack can then be expressed as

$$\frac{\gamma_{II}}{\gamma_I} = \frac{1 + (\gamma_{II}/\gamma_I)\beta^2}{1 + \beta^2} \quad (19)$$

The ratio of the combined mode and the mode I fracture toughness, therefore, can be calculated from Eqs. (15) and (2) as

$$\frac{K_{II}}{K_{IC}} = \frac{1 + (\gamma_{II}/\gamma_I)\beta^2}{1 + \beta^2} \quad (20)$$

### Table 2. Relative Magnitude of the Area Enclosed by Isostress Contours

| Maximum principal stress | Maximum shear stress |
|--------------------------|----------------------|
| Mode I                   | $1.63 \times 10^{-2}$|
| Mode II                  | $3.23 \times 10^{-1}$|

Table 1 indicates the calculated results for $2\gamma^*$, $K_{II}/K_{IC}$, $K_{IC}^2/K_{IC}$ and $K_{II}^2/K_{IC}$ vs. $\beta$ from $\beta = 0$ to $\infty$. The estimated $K_{II}/K_{IC}$ vs. $K_{IC}^2/K_{IC}$ envelope is shown in Fig. 4 with other criteria such as the maximum energy release theory (conventional$^{26)}$ and the maximum hoop stress theory.$^1$ Palaniswamy and Knauss$^{26)}$ derived the following equation for the maximum energy release rate theory, based on the assumption that the fracture energy rate is independent of the loading mode.

$$2(1 - K_I) = 3K_{II}^2 \quad (22)$$

Singh and Shetty$^6)$ modified the above equation using an empirical parameter, $C$, to

$$\frac{K_I}{K_{IC}} + \left(\frac{K_{II}}{K_{IC}}\right)^2 = 1$$

$$C = \frac{K_{II}}{K_{IC}} \quad (23)$$

Singh and Shetty’s criterion (Shetty’s empirical equation), where $C=1.20$, is also shown in Fig. 4. It should be noted that the criterion presented, based on Griffith’s energy equilibrium, is close to Shetty’s empirical criterion, where $C=1.20$, which indicates that the conventional maximum energy release rate theory must be modified so that the frac-
ture energy is dependent on the loading modes.

3. Experiments

3.1 Specimens

To confirm the theoretical prediction of the combined mode fracture criterion, we used the disk method shown in Fig. 5, and the rectangular plate tension test (rectangular test) shown in Fig. 6. A commercially obtained float glass plate (2 mm thick), polycrystalline alumina (1 mm thick) (MA995, Mitsui Mining Material Co., Ltd.) and a PMMA plate (3 mm thick) were used as specimens, where float glass and alumina exhibit stress corrosion cracking in air and PMMA exhibits stable crack growth due to crazing at room temperature. These three materials were used for the disk test while float glass and PMMA were used for the rectangular test, too. The circular disk-shaped specimen was 40 mm in diameter for float glass and PMMA, and 30 mm in diameter for alumina. The central artificial crack was made by machining a slit with Chevron-shaped notches at the tips, using a thin round diamond saw of 13 mm in diameter and 0.15 mm in thickness. The notches in alumina and PMMA were then extended using a thin razor blade with diamond paste to make a sharp V-shaped notch, and the precracks in float glass were made by thermal stress using a copper soldering technique. The crack length ratio, \( c/R \), was approximately 0.4, where \( c \) is the half-length of the crack and \( R \) is the radius of the disk specimen. The radii of the artificial notch tips were approximately 20 \( \mu m \) for alumina and PMMA.

Fig. 5. Disk (diametral compression) test.

Fig. 6. Rectangular (rectangular plate tension) test.

3.2 Disk test

The disk test shown in Fig. 5 was conducted in air using a material testing machine (Autograph-5kND, Shimadzu Corp.), and the cross-head speed (CHS) was set at 0.1 mm/min. Slow crack growth phenomena prior to unstable crack propagation were inspected at the crack tip during the test using a magnifier.

The stress intensity factors for a central crack in a disk-shaped specimen are expressed as

\[
K_{I,II} = F_{I,II} \sqrt{\frac{P}{RB}} \sqrt{\frac{c}{\pi}}
\]

where \( P \) represents the load, \( B \) the thickness of the disk, and \( F_{I} \) and \( F_{II} \) the nondimensional mode I and mode II stress intensity factors, respectively. The values of \( F_{I} \) and \( F_{II} \) for each inclined angle were analyzed by Awaji and Sato.

3.3 Rectangular test

The rectangular test shown in Fig. 6 was performed using the material testing machine mentioned above. Each rectangular specimen was pasted onto soft PVC (polyvinyl chloride) sheets on both the top and bottom to reduce stable crack growth and misalignment of the loading system. The testing conditions were selected as follows. All tests for PMMA were performed in air, and two CHSs were selected, 3 and 10 mm/min, to examine the influence of the stable crack growth phenomenon. For float glass, all tests were performed in air with CHS = 3 mm/min, and in dry N\(_2\) gas with CHS = 10 mm/min.

The stress intensity factors for a central slant crack in the plate are expressed as

\[
K_{I,II} = F_{I,II} \sqrt{\frac{P}{2WB}} \sqrt{\frac{c}{\pi}}
\]

where \( W \) and \( B \) are the width and thickness of the plate, respectively. The values of \( F_{I} \) and \( F_{II} \) for each slant angle are shown in Ref. 25.

4. Results and discussion

The combined mode fracture resistance for alumina obtained by the disk test is shown in Fig. 7. All four specimens were subjected to the test with inclined angles of 5, 10, 15 and 20°, and ten specimens were used for the mode I test (the inclined angle is 0°) and nine specimens for the mode II test (the inclined angle is 25.2° for \( c/R = 0.4 \)). Fairly good coincidence between the experimental data and the presented criterion is observed.

The results for float glass tested with the disk test and rectangular test are summarized in Fig. 8, where the solid
lines correspond to the Shetty's equations, Eq. (23), with different C values. It is observed that the $K_{IC}/K_{IC}$ value obtained by the disk test is the highest, whereas the value obtained by the rectangular test in air, calculated using Shetty's equation, is the lowest; the value in the rectangular test in dry $N_2$ gas is estimated to be in between, which suggests that, due to the stable crack growth occurring by stress corrosion cracking in air and even in dry $N_2$ gas, the value of $K_{IC}$ may be estimated as lower than the actual value due to the mode transformation and the roughness of the stable extended surface. Figure 9 shows the stable crack growth phenomenon at the crack tip in float glass tested in air. Stable crack growth was observed in all rectangular specimens of float glass tested in air, however in some specimens tested in dry $N_2$ gas, it was difficult to recognize the stable crack extension using a magnifier.

Figure 10 shows a summary of the results for PMMA. At lower CHSs, considerable stable crack growth due to crazing could be observed, the lowest $K_{IC}/K_{IC}$ value was exhibited. At higher CHSs, stable crack growth decreased and a moderate value of $K_{IC}/K_{IC}$ was obtained. In contrast, the value of $K_{IC}/K_{IC}$ measured by the disk method exhibited the highest value of 1.11, where there was no stable crack extension.

The results of the mode I and mode II fracture toughnesses are summarized in Table 3, where the values of the mode II fracture toughness in the rectangular test were calculated using Shetty's equation. It was determined that the values of $K_{IC}/K_{IC}$ obtained by the disk method range from 1.1 to 1.3, and that these values are very close to the predicted value of 1.20. On the other hand, the values of $K_{IC}/K_{IC}$ estimated by the rectangular test show much scatter, and all data, except for the data measured in dry $N_2$ gas are less than 1. This is because considerable stable crack growths are observed at the crack tip of the rectangular specimens of float glass and PMMA, whereas there is no stable crack growth in the disk specimens irrespective of the materials used. We believe that the reason for the unstable crack growth in the circular disk specimen under a compressive load is that the energy release rate at the crack tip may increase sharply during crack propagation.

The results presented above indicate that the mode II fracture toughness should be measured at the very beginning of crack extension to prevent mode transformation, using a smooth surface precrack such as a machined notch that has no stress shielding effects.

The disk method, using a machined thick through-slit, has several advantages over other methods. First, these tests of mode I, mode II, and combined mode I-II fracture toughnesses can be performed under the same test configurations. Second, stable crack growth at the initial crack tip prior to unstable crack propagation is strictly limited to this technique. Third, there is no stress shielding on the machine-notched surface. Lastly, the test results of the disk method are almost independent of the researchers performing it. Singh and Shetty compared the results of the disk method with a thick through-slit obtained by Awaji and
5. Conclusions

Griffith’s concept of energy equilibrium for brittle crack propagation was extended to combined mode I-II crack extension in the present study. The maximum energy release rate for an infinitesimally kinked crack extension under combined mode I-II loading, analyzed by Kageyama and Okamura, was expressed with a combined mode stress intensity factor. The fracture energy rate was estimated from the areas of the frontal process zone in front of the crack tip under mode I and mode II loads. The fracture toughness values for mode I and mode II, and the combined mode I-II criterion for alumina, float glass and PMMA were also estimated experimentally using the disk method and the rectangular test. The results can be summarized as follows.

(1) The predicted $K_{IC}/K_{IC}$ ratio based on the energy equilibrium was 1.20, which is close to the experimental results in the range of 1.1 to 1.3 obtained previously for several inorganic materials, as well as the present results obtained using the disk method.

(2) The combined mode I-II fracture criterion was formulated using Griffith’s concept. The predicted criterion coincided well with Singh and Shetty’s empirical criterion, where $C=1.20$.

(3) The experimentally obtained combined mode fracture criterion was also closely related to the one predicted using the disk method.

(4) Considerable stable crack growth was observed in the rectangular specimen, which reduced the mode II fracture toughness.

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