Turbulent charge burning in SI engines

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Abstract. The article describes turbulent combustion in the front of a spherical flame of the fuel-air mixture in a spark-ignition engine. The paper considers two mathematical models of the workflow: a well-known “entrainment or eddy burning model” and a “laminar model of turbulent combustion” in the authors’ version of this paper. Both models are based on the Damköhler hypothesis (Damköhler G., 1940) on the turbulent combustion of gas, but have a significant difference in the mathematical formulation. Although both models contain empirical parameters, the authors’ model is much simpler and clearer. In numerical simulation, both mathematical models produce similar results. The authors also propose a “combined mathematical model of frontal turbulent combustion”.

1. Introduction

The problems of burning a homogeneous charge in internal combustion engines (ICE) with spark ignition have occupied researchers for many decades [1–3]. However, the process of fuel combustion in most of them is described by empirical formulas. In a number of mathematical models, the process of propagation of a turbulent combustion wave in a flame front in a combustion chamber is considered [4–6]. Such models reflect the physical picture of the process much better than the models with a probabilistic description of turbulent combustion [7, 8]. Although considerable experimental and theoretical material in the field of turbulent combustion of the mixture in spark-ignition engines (SI engines) has been accumulated, the development of a fully satisfactory model for describing turbulent processes in the flame front in the engine has not yet been completed [9–11].

2. Thermodynamic and kinematic equations in a two-zone model

The equations of conservation of the operating volume \( V_u \), charge mass \( m_u \), and mass fractions of unburned mixture \( x_u \) and combustion products \( x_b \equiv x \) will be written as

\[
V = V_u + V_b, \quad m_u = m_u + m_b, \quad x_b = x, \quad x_u = 1 - x.
\]

where the subscripts “\( u \)”, “\( b \)”, and “\( a \)” refer to the unburned mixture, combustion products, and the closing moment of the intake valve, respectively.

The unburned mixture and combustion products represent an ideal gas.
\[ pV_u = m_u R_u T_u = m_u (1 - x) R_u T_u, \quad pV_b = m_b R_b T_b = m_a x R_b T_b \] (5), (6)

where \( R_u = R / M_u \); \( R_b = R / M_b \); \( M_u, M_b \) are average molecular masses of gases; \( R \) is the universal gas constant; \( p \) is the pressure; \( T \) is the temperature.

Energy equations for the unburned mixture and combustion products are

\[ m_a (1 - x) c_{pu} T_u = Q_u + V_u p, \quad m_a x c_{pb} T_b = Q_b + V_b p + m_a x (c_{pu} T_u - c_{pb} T_b) \] (7), (8)

where \( c_{pu}, c_{pb} \) are mass heat capacities at constant pressure; \( Q_u, Q_b \) are the heat exchange with the environment; \( T_u = dT / d\phi \), \( T_b = dT_b / d\phi \), \( p = dp / d\phi \), \( x = dx / d\phi \).

The equation of the dynamics of the operating volume (for axial crank mechanism) is

\[ V = \frac{dV}{d\phi} = V_c \frac{\varepsilon - 1}{2} \sin \phi \left( 1 + \frac{\cos \phi}{\sqrt{1/\lambda^2 - \sin^2 \phi}} \right), \] (9)

where \( V_c \) is the combustion chamber volume; \( \varepsilon = V_{\text{max}} / V_c = 1 + r\pi D^2 / 2V_c \) is the geometric compression ratio; \( D \) is the piston diameter; \( \lambda = r/l \) is the ratio of the crank radius to the connecting rod length; \( \phi = 2\pi n \) is the crank angle (CA, deg); \( n \) is the rotational speed (1/s).

3. Charge burning in the front of a turbulent flame

As it is shown in figure 1A, the combustion process in a SI engine occurs after the charge ignition (air-fuel mixture) in a small volume in the vicinity of the spark plug (at the ignition point). The result is a spherical flame front, propagating in the engine operating volume, bounded by walls of the cylinder, the combustion chamber, and the cover.

According to the Borghi diagram (Borghi, 1984; Poinsot et al., 1991; Candel, 1994) [12–14], charge burning in a SI engine can be attributed to the turbulent frontal flame propagation with surface
burning (with the normal velocity of a laminar flame) at the curved continuous surface of a turbulent flame and the combustion centers [15].

Depending on the degree of charge turbulence in the engine, characterized by the turbulent transfer rate (root-mean-square pulsating speed) $u_T$, the thickness of the turbulent front of a spherical flame $L_f$ can be different. The minimum value is equal to the thickness of the zone of a laminar flame

$$L_{T\text{min}} = \delta_f = \kappa_b/S_u.$$  

The maximum value is close to the maximum size of the turbulent vortex $L_{f\text{max}} \approx 2l_T$. The following designations are used here: $S_u$ is the normal velocity of a laminar flame; $\kappa_b$ is the coefficient of thermal diffusivity; $l_T$ is the vortex size.

For any (not too small) degree of charge turbulence, three smooth spheres can be built from the ignition point. These are spheres of an external front (along the outer boundary of the flame centers with the radius $r_f$, the volume $V_f$, and the surface area $F_f = \partial V_f/\partial r_f$), the internal front (along the near boundary of the flame centers with the radius $r_c$, the volume $V_c$, and the surface area $F_c = \partial V_c/\partial r_c$), and an intermediate sphere of products (built as a smooth sphere, containing all combustion products with the radius $r_b$, the volume $V_b$, and the surface area $F_b = \partial V_b/\partial r_b$).

The combustion zone with the volume $V_{comb} = V_f - V_c \approx 2(V_f - V_b)$ is limited by radiuses $r_f$ and $r_c$. Therefore, the maximum size of the burning center is $L_{f\text{max}} \approx 2l_T = r_f - r_c \approx 2(r_f - r_b)$.

4. Laminar model of turbulent combustion in the engine

The laminar model of turbulent combustion assumes the minimum possible value of the flame front thickness $\delta_f = \kappa_b/S_u$ (from the thermal theory of flame propagation). This value is usually about $(3 \div 60) \cdot 10^{-6}$ m. Therefore, the volume of burning gas $V_{comb} = F_T \delta_f$ is negligible $V_{comb} << V_b, V_u$. The process of burning a homogeneous charge occurs on the curved surface of a laminar flame, which is formed by the turbulent transfer of matter with the rate $u_T$. The result is the formation of a turbulent flame surface $F_T$.

It is assumed that “... the action of turbulent pulsations is basically reduced to the curvature of the flame front surface or its fragmentation into separate burning volumes. Moreover, on each element of such a highly developed surface, combustion propagates with the laminar flame rate $S_u$ (so-called “laminar model of turbulent combustion”) [16].” In this case, the mass burning rate, proportional to the turbulent flame rate $S_T$, is determined by the total surface $F_T$ of the curved laminar flame: the area of the turbulent flame according to the proportional ratio $S_T/S_u = F_T/F_b$. This expression follows directly from the Damköhler hypothesis (Damköhler G., 1940) [15] on the laminar flame curvature, according to which the mass burning rate $dm_b/dt$ is equal to the equation (10)

$$\rho u S_u F_T = \rho u S_T F_b,$$

$$S_T \equiv S_u + u_T \quad (10), (11)$$

Therefore, on the basis of (10), it is possible to say alternatively that combustion takes place at the normal rate of a laminar flame $S_u$ on the area of a turbulent flame $F_T$ or at the turbulent rate $S_T$ on a spherical smooth flame surface with area $F_b$, limiting the volume of combustion products $V_b$. From the ratio of the rates of turbulent and normal flames $\chi_T = S_T/S_u = 1 + u_T/S_u$ (flame turbulence
factor) [9, 17], there follows the formula (11). It is widely used in modeling papers [15], where \( u_T \) is the turbulent transfer rate (root-mean-square pulsation velocity).

The equation (11) is approximate and does not quite correspondence to the physical combustion picture in an engine. Alternative models of addition of velocities \( S_u \) and \( u_T \) are not excluded. Imagine the surface of a turbulent flame \( F_T \) in the form of a large number of adjacent to each other cones, forming a saw-tooth line in cross section (Figure 1 B). The base diameter of these cones is equal to the average scale vortex (characteristic size) \( l_T \), and the height \( h = r_f - r_c \) is proportional to the turbulent transfer rate \( u_T \) (root-mean-square pulsation rate). In such a formulation of the problem, the height of the cone is equal to \( h = u_T l_T / 2S_u \). Consequently, the ratio \( S_T / S_u \) is equal to the ratio of the lateral surface of the cone to the area of its base \( \sqrt{1+(2h/l_T)^2} \) [16]

\[
S_T / S_u = \sqrt{1+(u_T / S_u)^2}.
\]

Hereby, instead of the very widespread equation (11), formula (12) is proposed. This formula has been recently used by the authors of this paper [18–20]. Taking into account (12), the mass burning rate of the charge is \( dm_b / dt = \rho_u F_b S_T \), and the mass fraction of the burnt charge is

\[
\frac{dx}{dt} = \frac{d(m_b / m_a)}{dt} = \frac{1-x}{V-V_b} F_b \sqrt{S_u^2 + u_T^2}.
\]

The normal rate of a laminar (normal) flame \( S_u \) in the combustion equation (13) is usually [9, 17]

\[
S_u = S_{u0} \left( \frac{p}{p_0} \right)^n \left( \frac{T_b}{T_{b0}} \right)^m \equiv S_{u0} \left( \frac{p}{p_0} \right)^\varepsilon.
\]

where \( S_{u0} \) is the flame rate in the stoichiometric mixture under normal conditions (\( p_0 = 101.325 \) kPa, \( T_0 = 273.15 \) K); \( \varepsilon = n + m(1-1/\gamma) \) is the thermobaric exponent; \( \gamma \) is the Poisson’s ratio (for the methane-air mixture \( S_{u0} = 0.338 \) m/s, \( \varepsilon = 0.26 \)) [7, 21–24].

The turbulent transfer rate \( u_T \) is due to the initial turbulence of the charge, caused by filling the cylinder (until closing of the inlet valve \( \varphi = \varphi_a \)) at the average gas rate \( \bar{u}_i = \eta_V (F_p / F_{IV}) \bar{u}_p \) and the average piston rate \( \bar{u}_p = 2LSn = 4rn \). According to experimental data [15], they are related by the dependence \( 1.05 \bar{u}_p = 0.19 \bar{u}_i \). Here, the following designations are used: \( \bar{u}_i \) is the average gas rate at the cylinder filling; \( \eta_V \) is the cylinder filling ratio; \( F_p = \pi D^2 / 4 \) is the piston area; \( F_{IV} \) is the effective section of the open intake valve; \( \bar{u}_p \) is the average piston speed; \( L_S = 2r \) is the piston stroke; \( r \) is the crank radius; \( n \) is the crankshaft rotational speed (1/s). In this regard, the turbulent transfer rate is

\[
u_T = k_i \bar{u}_i = k_p \bar{u}_p.
\]

5. Entrainment or eddy burning model in the engine
For over 30 years, an empirical model of turbulent combustion, the so-called “entrainment or eddy burning model”, has been popular among researchers who simulate combustion processes in SI engines. Its harbingers are apparently papers [4, 5]. In the finished look, the model was developed by J. C. Keck with colleagues (Keck J. C. et al., 1982&1983) [6, 9].

This charge combustion model as well as the “laminar model of turbulent combustion” (described above) also provides propagation of a turbulent flame in spherical sections of a cylinder and combustion chamber (Figure 1A). The basic balance equations of thermodynamics and kinematics have the usual form (1)–(9).

The similarity with the “laminar model of turbulent combustion” is as follows.
1) The known Damköhler formula on the mass burning rate is written in the form (10).
2) If to digress from averaging of cycles and convection, then it is possible to draw the sphere with radius $r_f$, volume $V_f$, and surface area $F_f = \partial V_f / \partial r_f$ along the external borders of burning (Figure 1B).
3) The normal velocity of a laminar flame $S_u$ is described by an equation similar to (14).
4) In contrast to determining the characteristic size of a turbulent vortex as $l_T \equiv (r_f - r_b)$, this model postulates the formula (16) of the vortex size $l_T$, which determines the characteristic burning time $\tau_b$ of a single vortex at the normal flame rate $S_u$ by equation (17)

$$l_T = \left( V_f - V_b \right) / \left( F_f - F_f \right), \quad \tau_b = S_u l_T. \quad (16), (17)$$

The combustion process of the mixture in the “entrainment or eddy burning model” is described by a system of two equations [6, 9]

$$dm_b / dt = \rho_u F_f S_u + \mu / \tau_b, \quad d\mu / dt = \rho_u F_f u_T - \mu / \tau_b \quad (18), (19)$$

where $\mu$ is the auxiliary variable mass (parametric mass); $u_T$ is the characteristic transfer rate; $\tau_b$ is characteristic burning time of the parametric mass (ensemble of vortices). It should be noted that the authors of papers [6, 9] do not identify the value $u_T$ in the equation (19) with the turbulent transfer rate and designate it simply as the characteristic rate.

According to experimental data, the effective rate $S_T$ (11) is close to the normal flame rate $S_T \equiv S_u + u_T \approx S_u$ at the beginning of the formation of the flame center and then, during time $t / \tau_b \geq (3 + 5)$, it increases to a value close to the turbulent rate $S_T \equiv S_u + u_T$. Therefore, the authors of papers [6, 9] add an empirical equation (20) for the variable thickness of the combustion zone $R_f$

$$R_f = r_f - r_b = L_T \left[ 1 - \exp \left( - \left( r_b / L_T \right)^2 \right) \right], \quad L_T = u_T \tau_b. \quad (20), (21)$$

which contains some characteristic size $L_T$ by the formula (21). Apparently from the equation (21), the effective thickness of the combustion zone $R_f = r_f - r_b$ (Figure 1B) with increasing flame radius $r_b$ tends to the characteristic size $L_T$ (21).

The “entrainment or eddy burning model” is currently considered quite reasonable and widely used in the special literature [10, 11, 25–28]. From the authors’ analysis of this paper (which is not given here), it follows that this model is practically equivalent to the simpler “laminar model of turbulent combustion”. Note that the “entrainment or eddy burning model” is unnecessarily complicated by introducing additional empirical parameters $(S_u, u_T, l_T = S_u \tau_b)$ and main variables
\((r_f, F_f, V_f, r_b, F_b, V_b)\) into consideration while main parameters in the “laminar model of turbulent combustion” are \((S_u, u_T)\) and main variables are \((r_b, F_b, V_b)\).

6. Comparison of turbulent combustion models in the engine
A spherical flame in an SI engine propagates from a spark plug, located, for example, on the cylinder axis in the cover at the level of its plane (Figure 1A). For such an ignition point location, the surface area of a smooth spherical flame \(F_b\) depends on the geometry of the operating volume. The surface area \(F_b\) (or \(F_f\)), which separates the combustion products volume \(V_b\) from the unburned mixture volume \(V_u\), is found using surface and volume integrals for specific values of the flame front radius \(r_b\) (or \(r_f\)) and piston coordinates \(z = 4V/\pi D^2\) (Fig. 1A) in view of \(F_b = \partial V_b / \partial r_b\) and \(F_f = \partial V_f / \partial r_f\).

Numerical simulation of the workflow of a gas engine with spark ignition was carried out to compare the “laminar model of turbulent combustion” (“Model 1”) in the authors’ version of this paper and the “entrainment or eddy burning model” (“Model 2”) [6, 9]. Results of modeling are partially shown in Figures 2 and 3. As it is seen, the simulation results of the turbulent combustion process according to “Model 1” and “Model 2” almost coincide. In different versions of calculation, parameters \(u_T\) and \(l_T\) are varied in the range \(u_T = (0.4 \pm 1.2)\mu_p\), \(l_T = (0.2 \pm 0.8)\) mm.

7. Combined model of turbulent frontal combustion
Since the “laminar model of turbulent combustion” in the authors’ version of this paper and the well-known “entrainment or eddy burning model” [6, 9] for SI engines are mathematically equivalent and almost identical in numerical modeling, we consider it possible to propose a “combined model of turbulent frontal combustion”.

The mathematical notation of the new model besides (1)–(9) and (14) as well as solutions of volume and surface integrals for finding the flame front radius \(r_b\) in view of \(F_b = \partial V_b / \partial r_b\) (which are not given in this paper) includes the following equations (where \(u_T = k_i u_i = k_p u_p\); \(l_T = \text{const}\).
\[ U_T = u_T \left[ 1 - \exp \left( \frac{-\left( \frac{\eta b S_u}{l_T u_T} \right)^2}{2} \right) \right], \quad \frac{dm_b}{dt} = \rho_u F_b \sqrt{S_u^2 + U_T^2}. \quad (22), (23) \]

**Conclusion**

The authors of this article reviewed turbulent combustion in the front of a spherical flame of a fuel-air mixture in SI engines, including two mathematical models of the workflow: the widely known “entrainment or eddy burning model” and the “laminar model of turbulent combustion” in the authors’ version of this paper. Both models are based on the Damköhler hypothesis (Damköhler, G., 1940) on the turbulent burning of gas, but have a significant difference in the mathematical formulation. Although both models contain empirical parameters, the authors’ model is much simpler and clearer. In numerical simulation, both mathematical models lead to similar results. The authors also proposed the “combined mathematical model of frontal turbulent combustion”.

**Acknowledgement**

This research was supported by the Vladimir Potanin Fellowship Program (contract No. ГСГК-21/18, 2018/19), the Russian President Scholarships (2017/19), and the V. I. Vernadsky Foundation Scholarship (2018/19) for PhD students.

**References**

[1] Eichelberg G 1939 *Engineering* **148** 463–547
[2] Daniel W A 1956 *Flame quenching at the walls of an internal combustion engine*. Sixth Symposium (International) on Combustion
[3] Ramaswamy M C and Gupta M C 1979 *Archiwum termodynamiki i spalania* **18** (3) 405–18
[4] Blizard N C and Keck J C 1974 *SAE Paper* 740191 846–64
[5] Tabaczynski R J, Trinker F H and Shannon B A S 1980 *Combustion and Flame* **39** (2) 111–21
[6] Keck J C 1982 *Nineteenth Symposium (International) on Combustion* (MIT, Cambridge, MA) **19** (1) 1451–66
[7] Kavtaradze R Z 2008 *Theory of Piston Engines. Special Chapters* (MSTU) p 720 (in Russian).
[8] Spalding D B 1971 *Thirteenth Symposium (International) on Combustion* **13** (1) 649–57
[9] Beretta G P, Rachidi M and Keck J C 1983 *Combustion and Flame* **52** (3) 217–45
[10] Bayraktar H and Durgun O 2003 *Energy Sources* **25** (7) 651–66
[11] Sezer I and Bilgin A 2008 *Proc. of Royal Society A* **464** (2100) 3107–28
[12] Borghi R B, Bruno C and Casci C 1984 *Recent advances in aeronautical science* (London: Pergamon)
[13] Poinsot T, Veynante D and Candel S 1991 *Journal of Fluid Mechanics* **228** 561–606
[14] Candel S, Veynante D, Lacas F, Darabiha N and Rolon C 1994 *Combustion Science and Technology* **98** (4–6) 245–64
[15] Varnatz J, Maas U and Dibble R W 2006 *Combustion: physical and chemical fundamentals, modeling and simulations, experiments, pollutant formation* (Berlin: Springer-Verlag)
[16] Voinov A N 1977 *Combustion in high-speed piston engines* (Moscow: Mashinostroenie) (In Russian)
[17] Senachin P K, Babkin V S and Borisenko A V 1997 *Combustion Explosion and Shock Waves* **33** (6) 631–9
[18] Senachin P K and Senachin A P 2011 *Proc. of Samara Sc. Center of the RAS* **13** (1) 487–91 (RU).
[19] Senachin A P 2012 Modeling the formation of toxic substances in a spark-ignition engine *Polzunovskii Vestnik* No 3/1 140–149 (in Russian)
[20] Briutov A A and Senachin P K 2013 Simulation and optimization of the workflow of the gas engine. *Proc. of Intern. Academy of Agrar. Educ.* 16(4), 50–55 (St Petersburg: RB IAAE) (In Russian)

[21] Lewis B and Elbe G 1987 *Combustion, flames and explosions of gases* (NY: Academic Press)

[22] Metghalchi M and Keck J C 1980 *Combustion and Flame* 38 143–54

[23] Metghalchi M and Keck J C 1982 *Combustion and Flame* 48 191–210

[24] Altin I, Sezer I and Bilgin A 2009 *Energy & Fuels* 23 (4) 1825–31

[25] Kaul B C 2008 *Addressing nonlinear combustion Instabilities in highly dilute spark ignition engine operation* (PhD Dissertation Missouri S&T Rolla)

[26] Bayraktar H and Durgun O 2005 *Energy Conversion and Management* 46 (13–14) 2317–33

[27] Curto-Risso P L, Medina A, Hernandez A C, Guzman-Vargas L and Angulo-Brown F 2013 *Entropy* 15 (8) 3277–96

[28] Medina A, Curto-Risso P L, Hernandez A C, Guzman-Vargas L, Angulo-Brown, F and Sen A K 2014 *Quasi-dimensional simulation of SI engines* (Spriger: London, Heidelberg, NY)