Color-flavor locked strange matter and strangelets at finite temperature

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Abstract

It is possible that a system composed of up, down and strange quarks consists the true ground state of nuclear matter at high densities and low temperatures. This exotic plasma, called strange quark matter (SQM), seems to be even more favorable energetically if quarks are in a superconducting state, the so-called color-flavor locked state. Here are presented calculations made on the basis of the MIT bag model considering the influence of finite temperature on the allowed parameters characterizing the system for stability of bulk SQM (the so-called stability windows) and also for strangelets, small lumps of SQM, both in the color-flavor locking scenario. We compare these results with the unpaired SQM and also briefly discuss some astrophysical implications of them. Also, the issue of strangelet’s electric charge is discussed. The effects of dynamical screening, though important for non-paired SQM strangelets, are not relevant when considering pairing among all three flavor and colors of quarks.

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I. INTRODUCTION

Three decades ago it was proposed that systems composed of an unconfined Fermi liquid of up, down, and strange quarks could be absolutely stable [? ? ? ?]. In its simplest Fermi liquid picture, stability depends on whether it would be possible or not to lower the energy of a system composed of quarks $u$ and $d$ by converting (through weak interactions) approximately one third of its components into the much more massive strange quark due to the introduction of a third Fermi sea.

Within the well-known MIT Bag Model [? ], it has been shown that this stability may be realized for a wide range of parameters of strange quark matter (SQM) in bulk [? ]. Other calculations also indicate that SQM can be absolutely stable within different frameworks, e.g. shell model [? ].

The attractive force between quarks that are antisymmetric in color tend to make quarks near the Fermi surface to pair at high densities, breaking the color gauge symmetry, and causing the phenomena of color superconductivity. Recently, studies have indicated that the color-flavor locked (CFL) state, in which all the quark modes are gapped, seems to be even more favorable energetically, widening the stability window [? ? ? ? ? ?].

If strange quark matter or CFL matter is indeed the ground state of cold and dense baryonic matter there would be some important astrophysical implications. For instance, all neutron stars would actually have their interiors composed only of exotic matter [? ? ? ? ? ? ], see also [? ? ? ] for recent reviews. The existence of strange stars would likely imply the presence of strangelets among cosmic ray primaries. A few injection scenarios have been considered as likely sites: the merging of compact stars (though not addressed in full detail yet) [? ? ], strange matter formation in type II supernova [? ], and acceleration from strange pulsars [? ]. Several cosmic ray events have been tentatively identified with primary strangelets (mainly the Centauro and Price events, and more recently data from the HECRO-81, ET event, and AMS01 experiments [? ? ? ? ? ? ] for the data obtained indicate a high penetration of the particle in the atmosphere, low charge-to-mass ratio, and exotic secondaries. New experiments are being designed that could identify these exotic primaries with the purpose to definitively test the validity of the Bodmer-Witten-Terazawa conjecture [? ? ? ].

For the description of these finite size lumps of strange matter, (termed strangelets) a
few terms have to be added to the bulk one in the free energy (see [?] and [?] for details). Large lumps will have essentially the same structure than bulk matter, with a small depletion of the massive strange quarks near the surface resulting in a net positive charge, a feature also expected for smaller chunks [? ],[? ] which thus resemble heavy nuclei.

Strangelets without pairing at finite temperature have been first analyzed by Madsen [?] in the $m_s = 0$ approximation. A more complete description has been given by He et al. [? ], in which energy, radius, electric charge (unscreened), strangeness fraction and minimum baryon number were presented.

CFL strangelets at $T = 0$ were discussed in Refs. [? ? ]. More recently, a finite-temperature analysis using pertubative QCD appeared [? ]. We address in this paper the issues of surface and curvature energies at $T > 0$, which are potentially important for fragmentation of CFL SQM in astrophysical environments among other things.

This paper is structured as follows: in section II we describe the theoretical approach used to determine the parameters characterizing CFL SQM at finite temperature for the construction of the windows of stability in bulk and strangelets; in section III, we present the numerical results for CFL SQM and compare then with unpaired SQM; in section IV, we present our final discussion and conclusions.

II. WINDOWS OF STABILITY

A. Bulk matter

Unpaired SQM in bulk contains $u$, $d$, $s$ quarks and also electrons to maintain charge neutrality. The chemical balance is maintained by weak interactions and neutrinos assumed to escape from the system. If SQM is in a CFL state in which quarks of all flavors and colors near the Fermi surface form pairs, an equal number of flavors is enforced by symmetry and the mixture is automatically neutral [? ]. In this case, the condition is that the Fermi momentum for the three quarks are equal, so that $3\mu = \mu_u + \mu_d + \mu_s$ and the common Fermi momentum is $\nu = 2\mu - (\mu^2 + m_s^2/3)^{1/2}$.

For bulk CFL SQM, the thermodynamical potential for the system to order $\Delta^2$ is [? ? ]

$$\Omega_{CFL} = \sum_i \Omega_i - \frac{3}{\pi^2} \Delta^2 \mu^2 + B \quad (1)$$
where $\Delta$ is the pairing energy gap and the term associated with this parameter is the binding energy of the diquark condensate. The term $\Omega_{\text{free}} = \sum_i \Omega_i$, mimics an unpaired state where all quarks have a common Fermi momentum, and $i$ stands for quarks $u, d, s$ and gluons (there is no electrons in the CFL state).

On the basis of the MIT bag model with $\alpha_s = 0$, the thermodynamic bulk potentials for each component of the unpaired “toy model” system are given by

$$\Omega_i = \pm T \int_0^{\infty} dk g_i \frac{k^2}{2\pi^2} \ln \left[ 1 \pm \exp \left( - \frac{\epsilon_i(k) - \mu_i}{T} \right) \right]$$

(2)

where the upper sign corresponds to fermions and the lower to bosons, $\mu$ and $T$ are the chemical potential and temperature, $k$ and $\epsilon_i$ the momentum and energy of the particle, respectively, and the factor $g_i$ is the statistical weight for quarks and gluons (6 for quarks and antiquarks and 16 for gluons). The limit of expression (2) for $T \rightarrow 0$ is the one given in [?], when the integral is made for momenta ranging from zero to the Fermi one, since the Fermi-Dirac distribution at $T = 0$ for the unpaired state presents a sharp cutoff at the Fermi momentum being useless to perform the integration to $k \rightarrow \infty$. At finite temperature, however, the broadening of the Fermi-Dirac distribution occurs, hence the integration has to be extended as well.

With these quantities we obtain the particle density given by $n_i = -\partial \Omega_i / \partial \mu_i$ (which accounts for the influence of the pairing condensate binding energy), and the total energy density, $E = \sum_i (\Omega_i + n_i \mu_i) - 3 \Delta^2 \mu^2 / \pi^2 + B + TS$, where $S = - (\partial \Omega / \partial T)_{V,\mu}$ is the entropy.

In spite that most of the analysis is performed using a constant value for $\Delta = \Delta_0$, the pairing gap is actually dependent of the temperature of the system. Following the studies of superconductivity in quark matter, we used for this dependence

$$\Delta(T) = 2^{-1/3} \Delta_0 \sqrt{1 - \left( \frac{T}{T_c} \right)^2}$$

(3)

$$T_c = 0.57 \Delta(T = 0) \times 2^{1/3} \equiv 2^{1/3} \times 0.57 \Delta_0$$

(4)

where $T_c$ is the critical temperature of the superconducting system, above which the system can no longer support pairing between quarks.
B. Strangelets

As stated above, for the description of strangelets, it is necessary to add surface and curvature contributions to the thermodynamical potentials of bulk matter

$$\Omega_i = \mp T \int_0^\infty dk \frac{dN_i}{dk} \ln \left[ 1 \pm \exp \left( - \frac{\epsilon_i(k) - \mu_i}{T} \right) \right]$$

(5)

In the multiple reflection expansion [? ? ] the density of states is given by

$$\frac{dN_i}{dk} = g_i \left\{ \frac{1}{2\pi^2} k^2 \mathcal{V} + f_S^{(i)} \left( \frac{m_i}{k} \right) k \mathcal{S} + f_C^{(i)} \left( \frac{m_i}{k} \right) \mathcal{C} \right\}$$

(6)

where $\mathcal{V}, \mathcal{S}$ and $\mathcal{C}$ stand for the volume, surface area and curvature of the strangelet, respectively.

The surface term for quarks is given by [? ]

$$f_S^{(q)} \left( \frac{m_q}{k} \right) = -\frac{1}{8\pi} \left[ 1 - \frac{2}{\pi} \arctan \left( \frac{k}{m_q} \right) \right]$$

(7)

For the curvature contribution, the following ansatz [? ] for massive quarks is adopted

$$f_C^{(q)} \left( \frac{m_q}{k} \right) = \frac{1}{12\pi^2} \left\{ 1 - \frac{3k}{2m_q} \left[ 2 - \arctan \left( \frac{k}{m_q} \right) \right] \right\}$$

(8)

while for gluons [? ], $f_C^{(g)} = -1/6\pi^2$.

The energy is obtained as $E = \sum_i (\Omega_i + N_i \mu_i) - 3\Delta^2 \mu^2/\pi^2 V + BV + TS$ and the mechanical equilibrium condition for a strangelet with vacuum outside is given by $B = -\sum_i \partial \Omega_i / \partial V$.

The relation obtained for strangelets without pairing [? ], $\mu_u = \mu_d = \mu_s$, found by minimizing the free energy with respect to the net number of quarks of each species and subjected to the constraint $A = 1/3 \sum_i N_i$, is here substituted by $\mu_u = \mu_d$ and $\mu_s = \sqrt{\mu_u^2 + m_s^2}$ which is actually a second constraint imposed for pairing to hold. The value of the common chemical potential is then obtained numerically by imposing the mechanical equilibrium condition at a given set of $B$, $\Delta$ and $m_s$.

The issue of Debye screening of the electric charge for strangelets without pairing is of major importance in determining the total charge of these particles [? ]. In the expression for energy density, there is a term proportional to $A_0^2/\lambda_D^2$, where $A_0$ is the gauge field for
the massless gauge boson and $\lambda_D$ is the Debye screening length. In this way, the general expression of the Debye screening length may be written as

$$\lambda_D^{-2} \propto \frac{\partial^2 \text{Energy density}}{\partial \mu_e^2}$$

(9)

where $\mu_e$ is the chemical potential for electric charge. It means that $\lambda_D$ is related to the response of a medium to a change in $\mu_e$.

In CFL matter the massless gauge boson is the rotated or $\tilde{Q}$ photon and the $\tilde{Q}$-charge of all the Cooper pairs forming the condensate is zero. Since all quasiparticles are gapped, which is due the unbroken $\tilde{U}(1)_{em}$ gauge symmetry in the ground state, the CFL phase is not an electromagnetic superconductor but a $\tilde{Q}$-insulator [?].

In CFL matter, the relevant electric chemical potential is $\mu_{\tilde{Q}}$ and the root mean square of (9) is zero [?], therefore the electric field is not screened and the charge of a strangelet in the CFL state will be defined by finite size effects only.

III. NUMERICAL RESULTS

The so-called “windows of stability” (regions in the plane $m_s - B$) for CFL matter in the framework of the MIT Bag Model are shown in Fig. I. The minimum value for $B^{1/4}$ is 145 MeV, because a lower $B$ would cause the spontaneous decay to non-strange matter ($u$ and $d$ quarks). As expected, the matter becomes less bound at finite temperature as can be seen both in the constant pairing gap approximation and when this parameter is temperature dependent.

The influence of considering the more realistic case of $\Delta$ depending on the temperature, $\Delta = \Delta(T)$, shows an destabilization of the system due to an effective reduction in the gap parameter. This conclusion holds even if the system is quite close to the critical temperature for pairing between quarks. The system, then, tends to approach the curves for $\Delta = 0$. But it does not exactly match the curves for $\Delta = 0$ due to the existence of the extra term in the entropy $(\partial[-3\mu^2/\pi^2\Delta(T)^2]/\partial T) \approx 3\mu^2/\pi^2T_c/0.41$, when $T = T_c$) for the temperature dependent scenario.
FIG. 1: Stability windows, regions bounded by the vertical line at $B^{1/4} = 145$ MeV and the curves of $E/A = 939$ MeV (shown for different temperatures and $\Delta$), for CFL SQM. On the top left panel, at $T = 0$ (following reference [?]) and on the top right panel, for CFL SQM at $T=10$ MeV and values of $\Delta$ as indicated. The full lines represent the calculations made considering $\Delta$ constant with temperature and the dashed lines for $\Delta = \Delta(T)$ (refer to text for details). On the bottom panels, stability window for CFL SQM at finite temperature and $\Delta = 100$ MeV. The solid line is for null temperature, the dashed line for $T = 10$ MeV, and the dotted line, $T = 30$ MeV. On the left, the curves where obtained considering a fixed $\Delta$ and on the right, $\Delta = \Delta(T)$. All the curves presented are calculated for fixed $E/A = 939$ MeV labelled with the corresponding value of $\Delta$, when necessary. The vertical line is the minimum $B$ value for stability.

We have also calculated the structure of spherical strangelets, numerically, with the results shown in Figs. 2 and 5 for the total energy of these particles as a function of different parameters characterizing them.
Just as in the case of bulk matter, there is a competition when considering $\Delta = \Delta(T)$ between the lowering of the effective pairing parameter and the raising of the chemical potentials in the CFL quark matter and the extra term in the volumetric entropy when compared to the case of a constant pairing gap parameter. For finite size drops of SQM, the additional terms of surface and curvature contributing in the thermodynamic potential with opposite sign to the volumetric term are affected only by the changes in $\mu$ (higher in the $\Delta(T)$ case) and on the strangelet radius (lower but not significantly affected, the difference being less than 1%). The overall result is that the stability for a given set of $m_s$, $B$, $\Delta_0$, $A$, and temperature is disfavored in the dependent delta scenario, as it is in the bulk case.

The behavior of the total energy per baryon number is to decrease when increasing the pairing gap and the strangelet’s baryon number, and increase with increasing $B$, $m_s$ and $T$. These can be understood with a comparison with the behavior of the stability windows of SQM show in Fig. 1.

The calculations show that the coefficient $R_0$, defining the strangelet radius as in $R = R_0 A^{1/3}$, decreases with increasing $A$ and $B$ but increases with $m_s$ and the $\Delta$ parameter (holding other parameters fixed on each comparison). It is also higher whenever there is an increase in the temperature for the thermic energy of quarks and gluons also increases. These behaviors are easy to understand: increasing the strangelet’s baryon content, its parameters get closer to the bulk ones, resulting in a decrease in $R/A^{1/3}$. Also, when increasing the bag constant, the vacuum pressure on the strangelet’s content is higher, explaining the radius dependence on this parameter. With increasing strange quark mass, the strange quark content decreases and so, with fixed $A$, the radius increases to maintain the constraint $A = \sum_i N_i$, the same reasoning applying to the raise in the pairing gap.

In the following figures, the dependence of the surface and curvature energies, defined as the coefficient that appears multiplying $A^{2/3}$ and $A^{1/3}$ in the expression for the total energy, respectively, on these parameters is also shown.
FIG. 2: Energy per baryon number as a function of the baryonic number, bag constant, strange quark mass, pairing energy gap, and temperature, from left to right, top to bottom, respectively. The values of the fixed constants are indicated for each plot. The first four plots are performed for $T = 0$ (full curves), $T = 15$ MeV (dashed curves) and $T = 30$ MeV (dotted curves). The last plot is performed for $A = 100$ (full curve) and $A = 1000$ (dashed curve).
FIG. 3: Surface and curvature energies of CFL strangelets, from left to right, respectively, as a function of $m_s$. The value of the fixed constants are $T = 0$ (full curve), $T = 15$ MeV (dashed curves), and $T = 30$ MeV (dotted curve), $A = 100$ MeV, $B^{1/4} = 145$ MeV, and $\Delta = 100$ MeV.

FIG. 4: Surface and curvature energies of CFL strangelets, from left to right, respectively, as a function of $\Delta$. The value of the fixed constants are $T = 0$ (full curve), $T = 15$ MeV (dashed curves), and $T = 30$ MeV (dotted curve), $A = 100$, $m_s = 150$ MeV, and $B^{1/4} = 145$ MeV. The critical temperature for $\Delta \lesssim 20$ MeV is below fifteen MeV and for $\Delta \lesssim 40$ MeV is below thirty MeV and that is why the curves at finite temperature are plotted starting at different values of the pairing gap energy.

The surface and curvature contributions decrease at higher temperatures, a feature also seen in ordinary nuclear matter for the surface energy (see, for example [7, 8] and references therein). The dependence of the surface energy with the strange quark mass shows a
FIG. 5: Energy per baryon number of CFL strangelets as a function of \( A \) calculated for \( \Delta = \Delta(T) \). The value of the fixed constants are \( T = 0 \) (full curve), \( T = 15 \) MeV (dashed curves), and \( T = 30 \) MeV (dotted curve), \( m_s = 150 \) MeV, \( B^{1/4} = 145 \) MeV, and \( \Delta = 100 \) MeV.

maximum at \( m_s \approx 150 \) MeV and goes to zero for massless quarks, and additionally shows a decrease for high values of the strange quark mass due to a depletion of this very massive component.

Simple numerical fits for the surface and curvature energy of strangelets at finite temperature were also obtained (to second order in the temperature \( T \)) for \( \Delta = \Delta_0 = 100 \) MeV (that is, for a gap parameter independent of temperature), \( B^{1/4} = 145 \) MeV, and \( m_s = 150 \) MeV and are presented here

\[
\sigma_{CFL}(T, A) = (81.09 + 0.013 T - 0.026 T^2) \times (0.96 + 0.17 e^{-\frac{A}{24.6}} + 0.053 e^{-\frac{A}{393.9}}) \tag{10}
\]

\[
C_{CFL}(T, A) = (163.85 + 0.003 T - 0.093 T^2) \times (0.98 + 0.082 e^{-\frac{A}{24.6}} + 0.026 e^{-\frac{A}{393.9}}) \tag{11}
\]

As expected, the behavior of the parameters characterizing strangelets for CFL matter is qualitatively the same when compared to SQM without pairing, i. e., the system at finite temperature is less stable than at absolute zero but the surface and curvature contributions decrease, a feature well known for nucleon systems. One interesting point is that for \( \Delta = 100 \) MeV, the chemical potential for the \( s \) quark is very close to the common chemical potential for stable strangelets without pairing at the same temperature and with the same value of
$B^{1/4} = 145$ MeV, $m_s = 150$ MeV, and $\Delta = 100$. $B$ and $m_s$, but the chemical potential for the light quarks is much lower. As a consequence the surface energy (determined only by the massive strange quark) is almost equal for the two scenarios, but the curvature energy is much lower for CFL strangelets. It means that for values of the pairing gap lower than 100 MeV, the surface energy is higher in the CFL state than without pairing. Meanwhile the curvature energy is always lower for CFL strangelets, regardless the value of $\Delta$.

The behavior of the electric charge for a strangelet with fixed $T$, $B$, $m_s$, and $\Delta = \Delta_0$ as a function of the baryon number is shown in Fig. 6. The electric charge grows with temperature for large baryon number strangelets driven by the dependence on the number density of quarks with $T$. For the massless quarks, a non-zero temperature slightly favor their increase in number through the term $\mu T^2 V$. But for the massive $s$ quark, the effect is the opposite since $N_s = \int_0^\infty dN_s/dk(1 + \exp((\sqrt{k^2 + m_s^2} - \mu_s)/T))dk$. So $Z/A^{2/3}$ deviates from being close to a constant behavior, expected from the suppression of massive strange quarks near the surface at $T = 0$ \[9], due to the higher importance of volumetric imbalance on the number density of different quark flavors at higher temperatures.
IV. CONCLUSIONS

CFL strange quark matter at $T = 0$ is more stable than SQM without pairing [? ? ] when one considers the strange quark mass, strong coupling constant and bag constant fixed. This result has been extended and quantified for $T > 0$ in the present work. Even when the temperature is as high as to be close to the critical temperature for pairing there is still room for (meta-) stability, depending on parameters choice. This suggests that the process of transition from a neutron star to a strange star could proceed right after its formation and the system might even skip the neutron star stage, if conditions for conversion of ordinary nuclear matter to the CFL state are met in the interior of these compact objects. As a general result, it is possible to observe that a finite temperature has always the consequence of destabilizing the system even when considering the dependence of $\Delta$ with $T$ which causes SQM to be slightly more disfavored than its “constant $\Delta$” version.

We also notice a very distinctive feature between strangelets with and without pairing concerning the existence of the critical baryon number, $A_{\text{crit}}$. This quantity represents the minimum baryon number to which strangelets are stable against neutron decay. The effects of surface and curvature tend to destabilize strange matter at low baryon number. As a result, the energy for creating small lumps of SQM increases as the baryon number decreases till it reaches a value above the neutron decay threshold, i.e., till it is above $\sim 930$ MeV. As shown in [? ] and [? ], the critical baryon number exists even for null temperature. It is known (see [? ] and [? ]) that the lower the value of the bag constant (of course, respecting the limit $B^{1/4} \geq 145$ MeV) and the higher the value of the pairing gap, the more stable strange quark matter in bulk is. In the case of CFL strangelets with high values of $\Delta$ and relatively low values of the bag constant, performing the analysis within the MIT bag model, the existence of $A_{\text{crit}}$ is not clear. It must be noted, however, that the liquid drop model does not provide a good description for low baryon number, being less reliable than shell models filling explicitly the quark states [? ].

Another important feature is that although for high baryon number CFL strangelets seem to be absolutely stable even for temperatures of order 30 MeV when $\Delta = 100$ MeV, values of the pairing constant much above a few hundreds of MeV are not expected to describe these systems. In being so, the critical temperatures for pairing of quarks inside strangelets are not expected to be higher than 70-100 MeV. For temperatures above this value, the quarks
inside the strangelet would no longer be paired and the gain in energy for this state compared to non-superconducting strangelets would vanish. Since strangelets without pairing are not stable for temperatures as high as the maximum critical temperatures expected, the strange quark matter stability would vanish above $T_{\text{crit}}$ if the pairing gap is too high.

We have also found that strangelets are more favorable in the CFL state, as expected. In particular, the curvature energy for CFL strangelets is lower than for “normal” strange quark matter, which may influence the fragmentation process of bulk CFL SQM, an important issue when considering the possible presence of these particles among cosmic rays and also when considering strangelet production in heavy ion collisions, although the very high temperatures disfavor the production of stable strangelets in these environments [? ].

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[1] When considering a finite strong coupling constant, $\alpha_c$, it has been shown that it can in fact correspond to an effective reduction on the value of the MIT bag constant $B$ [? ].

[2] The CFL state does not actually present a sharp Fermi surface. See [? ] for more details.