Super-Eddington growth of black holes in the early Universe: effects of disk radiation spectra

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ABSTRACT

We investigate the properties of accretion flows onto a black hole (BH) with a mass of $M_{\text{BH}}$ embedded in an initially uniform gas cloud with a density of $n_{\infty}$ in order to study rapid growth of BHs in the early Universe. In previous work, the conditions required for super-Eddington accretion from outside the Bondi radius were studied by assuming that radiation produced at the vicinity of the central BH has a single-power-law spectrum $\nu^{-\alpha}$ at $h\nu \gtrsim 13.6$ eV ($\alpha \sim 1.5$). However, radiation spectra surely depend on the BH mass and accretion rate, and determine the efficiency of radiative feedback. Here, we perform two-dimensional multi-frequency radiation hydrodynamical simulations taking into account more realistic radiation spectra associated with the properties of nuclear accretion disks. We find that the critical density of gas surrounding the BH, above which a transitions to super-Eddington accretion occurs, is alleviated for a wide range of masses of seed BHs ($10 \lesssim M_{\text{BH}}/M_\odot \lesssim 10^6$) because photoionization for accretion disk spectra are less efficient than those for single-power-law spectra with $1 \lesssim \alpha \lesssim 3$. For disk spectra, the transition to super-Eddington is more likely to occur for lower BH masses because the radiation spectra become too hard to ionize the gas. Even when accretion flows are exposed to anisotropic radiation, the effect due to radiation spectra shrinks the ionized region and likely leads to the transition to a wholly neutral accretion phase. Finally, by generalizing our simulation results, we construct a new analytical criterion required for super-Eddington accretion; $(M_{\text{BH}}/10^5 M_\odot)(n_{\infty}/10^4 \text{ cm}^{-3}) \gtrsim 2.4 \left(\langle\epsilon\rangle/100 \text{ eV}\right)^{-5/9}$, where $\langle\epsilon\rangle$ is the mean energy of ionizing radiation from the central BH.

Key words: accretion, accretion discs – black hole physics – (galaxies:) quasars: supermassive black holes – cosmology: theory

1 INTRODUCTION

Observations of bright quasars led by accreting supermassive black holes (SMBHs) with masses of $\gtrsim 10^9 M_\odot$ at high redshift $z \gtrsim 6$ (or $\lesssim 1$ Gyr from the Big Bang) require rapid growth of black holes (BHs) in the early Universe (e.g. Fan et al. 2004; Mortlock et al. 2011; Wu et al. 2015; Bañados et al. 2018). SMBHs are expected to play crucial roles on the history of the Universe such as via co-evolution with their host galaxies (e.g. Silk & Rees 1998; King 2003; Murray et al. 2005; Kormendy & Ho 2013), but their formation processes are still unclear.

A possible origin of high-$z$ SMBHs is highly-accreting stellar-mass BH seeds with $\sim 100 M_\odot$ (e.g., Madau & Rees 2001; Haiman & Loeb 2001; Volonteri et al. 2003; Li et al. 2007; Alvarez et al. 2009; Alexander & Natarajan 2014a), which are remnants of massive Population III stars (Pop III) (e.g., Yoshida et al. 2008; Hosokawa et al. 2011; Stacy et al. 2012; Hirano et al. 2014; Hosokawa et al. 2016). Accreting gas forms an accretion disk which radiates with a luminosity of $L = \eta M c^2$, where $\eta$ is the radiation efficiency, $M$ is the accretion rate, and $c$ is the speed of light. For a rapidly accreting BH, the radiation luminosity would exceed the Ed-
dington value \( L_{\text{Edd}} \), above which the radiation force due to electron scattering overcomes the BH gravity. Thus, the accretion rate could be limited at \( M \lesssim L_{\text{Edd}}/(\eta c^2) \). As result of this, the BH growth timescale from light seeds becomes significantly longer than the age of the Universe when high-\( z \) SMBHs already exist (\( \gtrsim 1 \) Gyr) in cases with \( \eta \approx 0.1 \) (Soltan 1982; Yu & Tremaine 2002).

Another possibility is more massive BH seeds with \( \sim 10^5 - 10^6 \ M_\odot \) formed by direct collapse of supermassive stars in protogalaxies (e.g., Loeb & Rasio 1994; Oh & Haiman 2002; Bromm & Loeb 2003; Begelman et al. 2006; Regan & Haehnelt 2009a,b; Hosokawa et al. 2012, 2013; Inayoshi et al. 2014; Vishal et al. 2014; Inayoshi & Tanaka 2015; Inayoshi et al. 2015; Chon et al. 2016; Regan et al. 2016a,b; Hirano et al. 2017; Inayoshi et al. 2018) and run-away stellar collisions (e.g., Omukai et al. 2008; Devecchi & Volonteri 2009; Katz et al. 2015; Yajima & Khochfar 2016; Stone et al. 2017; Sakurai et al. 2017; Reinoso et al. 2018). Even for such heavy seeds, we need to require a high duty cycle of BH growth at the Eddington accretion rate.

The possibility of super-Eddington accretion has been investigated by many authors. Because of two-dimensional radiation hydrodynamical simulation, Ohsuga et al. (2005) have revealed that super-critical accretion is realized as long as sufficient gas is supplied at the vicinity of the central BH. Ohsuga & Mineshige (2007) have concluded that trapping of diffusive photons in the optically-thick accretion disk and anisotropic radiation are crucial to realize super-Eddington accretion (see also Begelman 1979; Ohsuga et al. 2009; Ohsuga & Mineshige 2011; Jiang et al. 2014; Sadowski et al. 2015; Takahashi et al. 2016; Kitaki et al. 2018). On the other hand, gas supply from larger scale can be significantly suppressed due to photoionization heating and radiation momentum (e.g., Ciotti & Ostriker 2001; Milosavljevic et al. 2009a,b; Alvarez et al. 2009; Ciotti et al. 2009; Park & Ricotti 2011, 2012). Since the connection between BH feeding and feedback has been understood poorly yet, previous works with semi-analytical models adopted various prescriptions for BH accretion rates in the assembly history of dark matter halos (e.g. Volonteri & Rees 2005; Tanaka & Haiman 2009; Alexander & Natarajan 2014b; Mudan et al. 2014; Pacucci & Ferrara 2015; Valiante et al. 2016; Pezzulli et al. 2016, 2017; Valiante et al. 2018).

Recently, Inayoshi et al. (2016) found the conditions for super-critical accretion in a spherically symmetric system exposed to intense radiation from the BH with \( L \approx L_{\text{Edd}} \). When the size of an ionized region \( \eta_0 \) surrounding the accreting BH is smaller than the Bondi radius \( \eta_0 \), the ionized region collapses due to intense inflows of neutral gas and thus the accretion system transits to an isothermal \( (\approx 8000 \, K) \) Bondi accretion solution with a high accretion rate of \( \gtrsim 5000 \, L_{\text{Edd}}/c^2 \). The transition criterion is written as,

\[
M_{\text{BH}} \times n_0 \gtrsim 10^7 \, M_\odot \, \text{cm}^{-3} \left( T_0/c_0^4 \right)^{3/2} K^{3/2},
\]

where \( n_0 \) and \( T_0 \) are density and temperature of the ambient gas. Even with super-Eddington radiation feedback \( (L > L_{\text{Edd}}) \), the above criterion does not change significantly (Sakurai et al. 2016). Moreover, the criterion for super-critical accretion is alleviated under anisotropic radiation fields (Sugimura et al. 2017; Takeo et al. 2018). We note that the transition criterion is characterized by a quantity of \( M_{\text{BH}} \times n_0 \) because \( \eta_0^2/\eta_0 \propto (M_{\text{BH}} \times n_0)^{-2/3} \). Here \( \eta_0 \) is estimated by the Str"omgren radius

\[
r_{\text{strm}} = \left( \frac{3N_{\text{ion}}}{4\pi n_0^2 \sigma_B} \right)^{1/3},
\]

where \( N_{\text{ion}} \) is the emission rate of ionizing photons, \( n_0 \) is the number density of ionized gas, and \( \sigma_B \) is the case B radiative recombination rate.

The previous work assumed that radiation from the central region has a single-power-law (hereafter PL) spectrum with \( L_\nu \propto \nu^{-\alpha} \), where the spectral index \( \alpha \) is often set to 1.5 for any \( \nu \) (see also §4). However, the shape of the disk radiation spectrum would be more complicated. According to the analytic solutions of accretion disks, the disk surface temperature is described as \( T_\text{disk}(R) \propto R^{-p} \) (e.g., Mineshige et al. 1994; Kato et al. 2008), where \( R \) is the distance from the central BH. Assuming the disk surface locally emits blackbody radiation with \( T_\text{disk}(R) \), the spectrum is written as \( L_\nu \propto \nu^{-3-p} \), \( L_\nu \propto \nu^{-1} \) in the standard disk case (Shakura & Sunyaev 1973), and \( L_\nu \propto \nu^{-1} \) in the slim disk case (Abramowicz et al. 1988) (see §2.3 for more details). Moreover, since the disk temperature reaches \( \sim 10^4 \, K \), the maximum energy of the continuum spectrum is as high as \( \sim 1 \, keV \) (e.g., Watarai 2006), which is much higher than the mean photon energy of the PL spectrum \( \sim 40.8 \, eV \). This fact implies that radiative feedback effects for disk spectra are less efficient than that for the PL spectrum because the cross section to bound-free absorption of hydrogen atoms is \( \sigma_{\text{bf}} \propto \nu^{-3} \) (e.g., Draine 2011). On the other hand, electrons primarily produced by X-ray ionization are energetic enough to ionize the ambient gas (e.g. Shull 1979; Shull & van Steenberg 1985; Ricotti et al. 2002).

In this paper, we investigate the conditions for super-Eddington accretion under radiation with disk spectra associated with the standard and slim accretion disk model. We performed two-dimensional hydrodynamical simulations, including one-dimensional multi-frequency radiation transfer and primordial chemical reaction networks. We first conduct simulations under isotropic radiation from the central accretion disk. We construct an analytical formula for the criterion required for super-Eddington accretion under isotropic radiation and show that the conditions are alleviated for a wide range of BH masses, compared to the cases with single-PL spectra. Next, we perform simulations of accretion flows exposed to anisotropic radiation and investigate effects of the disk spectrum onto the inflow rate and conditions for the transition to a wholly neutral accretion phase.

The rest of this paper is organized as follows. In Section 2, we describe the methodology of our numerical simulations. In Section 3, we show our simulation results and give the conditions required for super-Eddington accretion. In Section 4, we discuss the analytical formula for the transition, the stability of highly-accreting system after the transition, and caveats of our simulation setups. In Section 5, we summarize the conclusion of this paper.
2 METHODS

Our goal is to study conditions for super-Eddington accretion led by gas supply from larger scales. Gas accretion begins from a critical radius, the so-called Bondi radius, defined by
\[ r_B = \frac{GM_{BH}}{c^2} \simeq 1.97 \times 10^{14} \frac{M_{BH}}{[M_{\odot}]} T_{\infty,4}^{-1} \text{ cm}, \] (3)
and the Bondi accretion rate for isothermal gas is given by
\[ M_B = \pi c^2 \rho_{\infty} \frac{G^2 M_{BH}^2}{c^6}, \] (4)
where \( M_{BH} \equiv M_{BH}/M_{\odot}, T_{\infty,4} \equiv (T_{\infty}/10^4 \text{ K}), c_{\infty} \equiv \sqrt{\gamma R T_{\infty}/\mu} \) is the sound speed, \( \gamma \) is the specific heat ratio, \( R \) is the gas constant, and \( \mu \) is the mean molecular weight. Note that the Bondi radius and rate as reference values are calculated by setting \( \gamma = 1, \mu = 1.23 \) and \( T_{\infty} = 10^8 \text{ K} \).

2.1 The code

We perform two-dimensional hydrodynamical simulations of axisymmetric flows with one-dimensional radiation transfer and chemical reaction networks (Takeo et al. 2018). Here we adopt the hydrodynamical simulation code developed in Takahashi & Ohsuga (2013). The advection terms for the ideal fluid are computed using the Harten-Lax-van Leer Riemann solver (Harten et al. 1983), and the second order accuracy in space and time are ensured (van Leer 1977). We adopt the spherical coordinates of \( (r, \theta, \phi) \) with the polar axis \( (\theta = 0 \text{ and } \pi) \) perpendicular to the disk plane. We add the radiation and chemical codes taken from Inayoshi et al. (2016) with necessary modifications.

2.2 Basic equations

The basic equations of the hydrodynamical part are the equation of continuity
\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0, \] (5)
the equations of motion
\[ \frac{\partial (\rho v_r)}{\partial t} + \nabla \cdot (\rho v_r v) = -\frac{\partial p}{\partial r} + \rho \left( \frac{v_r^2}{r} + \frac{v_\theta^2}{r} \right) - \rho \frac{\partial \phi}{\partial r} + f_{\text{rad}}, \] (6)
\[ \frac{\partial (\rho v_\theta)}{\partial t} + \nabla \cdot (\rho v_\theta v_r) = -\frac{\partial p}{\partial \theta} + \rho v_r^2 \cot \theta, \] (7)
\[ \frac{\partial (\rho v_\phi \sin \theta)}{\partial t} + \nabla \cdot (\rho v_\phi v_r \sin \theta) = 0, \] (8)
and the energy equation
\[ \frac{\partial e}{\partial t} + \nabla \cdot [(e + p) v] = -\frac{GM_{BH} \rho}{r^2} - \Lambda + \Gamma. \] (9)
where \( \rho \) is the gas density, \( v = (v_r, v_\theta, v_\phi) \) is the velocity, \( p \) is the gas pressure, and \( f_{\text{rad}} \) is the radiation force. We consider the gravity of the central BH \( (r = 0) \) and neglect the gas self-gravity. Since the general relativistic effect is negligible, the gravitational potential is given by \( \psi = -GM_{BH}/r \). The total energy per volume is defined as \( e \equiv e_{\text{int}} + \rho |v|^2/2, e_{\text{int}} \) is the gas internal energy density, \( \Lambda \) is the cooling rate per volume, and \( \Gamma \) is the radiative heating rate. We assume the equation of state of ideal gas as \( p = (\gamma - 1) e_{\text{int}} \) for \( \gamma = 5/3 \).

We solve the multi-frequency radiative transfer equation
\[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 F_r \right) = -\rho_k c E_r, \] (10)
where \( F_r \) is the radiation flux, \( E_r \) is the radiation energy density, and \( \rho_k \) is the absorption opacity. The radiation field is assumed to be steady because the light crossing time is much shorter than the hydrodynamical timescale. The frequency range is set to \( h \nu_{\text{min}} = 13.6 \text{ eV} \leq h \nu \leq h \nu_{\text{max}} = 100 \text{ keV} \), where \( h \) is the Planck constant. We note that only the radial component of the radiation flux is calculated because non-radial components produced by radiative recombination is negligible (see §4 in Takeo et al. 2018). Since the ionized gas is optically thin to electron scattering, we assume \( F_r = c E_r \) on the right-hand-side of Eq. (10).

We consider cooling processes associated with H, He, He\(^+\) atoms and free-free emission (Glover & Jappsen 2007), assuming the optically-thin cooling rates. In order to estimate their rates, we solve chemical reaction networks including six species of H, H\(^+\), He, He\(^+\), He\(^{++}\), and e\(^-\). The abundance of He nuclei relative to H nuclei is set to 8.33 \times 10^{-2}. Here we consider photoionization, collisional ionization and radiative recombination (Abel et al. 1997; Glover & Jappsen 2007), including effects of the secondary ionization (see below). Photoionization due to diffusive recombination photons is neglected, i.e., the case B recombination rate is adopted instead of the case A rate. The cooling/heating term in the energy equation (Eq. 9), the chemical reaction, and the radiative transfer equation (Eq. 10) are updated with an implicit method in order to solve them stably and save computation time. We set the time steps by setting the Courant number to 0.4.

The ionization rate coefficients and photoionization heating rates are calculated with the photon-conserving method (Whalen & Norman 2006). The primary ionization rates \( k_{\text{ph},i}^p \) \((i = \text{H}, \text{He}, \text{and He}^+)\) are estimated as
\[ k_{\text{ph},i}^p = \int_{\nu_{\text{min}}}^{\nu_{\text{max}}} \frac{dV}{h \nu} F_{\nu} \sigma_{\text{bf},i} \Phi(\nu, E_i, \chi_{\text{bf}}), \] (11)
where \( \sigma_{\text{bf},i} \) is the bound-free cross section. Since the energy of electrons produced by primary ionization is higher than the ionization potential energy, the electrons further ionize neutral hydrogen nearby (e.g. Shull 1979; Shull & van Steenberg 1985). The secondary ionization rates for species \( i = \text{H}, \text{He} \) and He are
\[ k_{\text{ph},j}^s = \sum_{i=\text{H, He}} \int_{\nu_{\text{min}}}^{\nu_{\text{max}}} \frac{dV}{h \nu} F_{\nu} \sigma_{\text{bf},i} \Phi(\nu, E_i, \chi_{\text{bf}}) \frac{\chi_j}{\chi_j}, \] (12)
where \( x_j \) is the abundance of species \( j \), \( \Phi(\nu, E_i, \chi_{\text{bf}}) \) is the fraction of secondary ionization of species \( j \) per primary electron of energy \( E_i \equiv h \nu - I_i \), and \( I_i \) is the ground state ionization potential energy of the species \( i \). The total photoionization rate is given by the sum of primary and secondary ionization rates. The photoionization heating rate \((i = \text{H, He, and He}^+)\) is
\[ \Gamma_i = \int_{\nu_{\text{min}}}^{\nu_{\text{max}}} \frac{dV}{h \nu} F_{\nu} \sigma_{\text{bf},i} E_i(\nu, \chi_{\text{bf}}), \] (13)
$E_h$ is the energy of primary electrons deposited as heat. We adopt the functional forms of $\phi(H), \phi(He)$, and $E_h$ (Ricotti et al. 2002). Note that secondary ionization of He$^+$ is negligible (Shull & van Steenberg 1985). The radiation force caused by electron scattering and bound-free absorption is given by

$$f_{\text{rad}} = \frac{n_e}{c} \int_{r_{\text{in}}}^{r_{\text{out}}} \sigma_{\text{eff}} F_{\nu} \, d\nu + \frac{4\pi}{c},$$  \hspace{1cm} (14)

where $\sigma_{\text{eff}}$ is the sum of the heating rates due to primary ionization of H, He, and He$^+$ atoms.

### 2.3 Disk radiation models

In order to study the effect of radiation produced from the nuclear accretion disk, we adopt models for the radiation spectra. Since the accretion rate through the disk we consider is as high as $\dot{m} \equiv M_\text{Edd}/M_\text{Edd} \gtrsim 10^{-2}$, where $M_\text{Edd} \equiv L_\text{Edd}/c^2$ and $L_\text{Edd}$ is the Eddington luminosity, the disk emission can be approximated as multi-color blackbody spectra (e.g., Kato et al. 2008). The specific radiation luminosity is calculated as

$$L_\nu = 2 \int_{R_\text{in}}^{R_\text{out}} 2\pi R B_\nu[T_{\text{eff}}(R)],$$ \hspace{1cm} (15)

where $B_\nu(T_{\text{eff}})$ is the Black body intensity with an effective temperature of $T_{\text{eff}}$ and $R_\text{in(out)}$ is the inner (outer) radius of the disk ($R$ is the radius of the cylindrical coordinate). The disk outer radius is set to $R_\text{out} = 10^7 R_{\text{Sch}}$ as our fiducial value, where $R_{\text{Sch}} \equiv 2GM_{\text{BH}}/c^2$ is the Schwarzschild radius. Note that we discuss the dependence of our results on the choice of $R_{\text{out}}$ in §4. For $\dot{m} < 10$, we set the inner disk radius to the inner most stable circular orbit (ISCO) for a non-spinning BH ($R_{\text{in}} = 3 R_{\text{Sch}}$). In the slim disk cases with $\dot{m} > 40$, we set $R_{\text{in}} = 1.1 R_{\text{Sch}}$ because the gas is optically thick even inside the ISCO. For $10 \leq \dot{m} \leq 40$, we estimate $R_{\text{in}}$ with a linear interpolation in the plane of $\log \dot{m} - \log R_{\text{in}}$. The radial structure of the effective temperature is given by Watarai (2006) as

$$T_{\text{eff}}(R) = 2.5 \times 10^7 \, K \, f^{1/8} \left(\frac{M_{\text{BH}}}{10^7}\right)^{-1/4} \left(\frac{R}{R_{\text{Sch}}}\right)^{-1/2} \mathcal{F}(R, \dot{m}),$$ \hspace{1cm} (16)

where

$$\mathcal{F}(R, \dot{m}) = \begin{cases} 1 - \sqrt{3} R_{\text{Sch}}/R, & \text{for } \dot{m} \leq 10, \\ 1 - \sqrt{3} R_{\text{in}}/R, & \text{for } 10 < \dot{m} < 40, \\ 1, & \text{for } \dot{m} \geq 40, \end{cases} \hspace{1cm} (17)
$$

and $f$ is a function of $R$ and $\dot{m}$ which connects the standard and slim disk solution smoothly. When the accretion rate is sufficiently high ($\dot{m} \gg 1$), the advection cooling timescale is shorter than the photon diffusion timescale within a characteristic radius, so-called the photon-trapping radius $R_\text{Tr} \equiv m R_{\text{Sch}}$, where $f \approx 1$ and $\mathcal{F} \approx 1$. At $R \gtrsim R_\text{Tr}$, optically-thick radiative cooling in the disk is dominant and $f \propto R^{-2}$. Thus, most of the radiation is produced within the trapping radius, and the bolometric luminosity is expressed as

$$\frac{L}{L_{\text{Edd}}} = 2 \left[1 + \ln \left(\frac{\dot{m}}{20}\right)\right],$$ \hspace{1cm} (19)

for $\dot{m} > 20$ (Watarai et al. 2000). We also model the angular dependence of radiation fields in the same way as in Takeo et al. (2018),

$$F_\nu(r = r_{\text{min}}, \theta) = \frac{(N + 1)L_\nu}{4\pi r_{\text{min}}^2} \cos^N \theta,$$ \hspace{1cm} (20)

where $r_{\text{min}}$ is the size of the inner-most grid (see §2.5) and $N$ characterizes the anisotropy of radiation fields. In this study, we explore both isotropic cases ($N = 0$) and anisotropic cases ($N > 4$).

Furthermore, we consider the photon redshift effect, i.e., the radiation intensity observed at infinity $I_{\nu\text{obs}}$ is connected with the intensity $I_{\nu\text{em}}$ at the photon emitting point $(R = R_{\text{em}})$ on the disk surface as $I_{\nu\text{obs}} = (\nu_{\text{obs}}/\nu_{\text{em}})^3 I_{\nu\text{em}}$, where we assume $\nu_{\text{obs}}/\nu_{\text{em}} = (1 - R_{\text{Sch}}/R)^{1/2}$ for simplicity.

### 2.4 Emergent spectra from the disk

In the top panel of Fig. 1, we show the spectral shape of radiation from an accretion disk around the central BH with $m_{\text{BH}} = 10$ at an accretion rate of $\dot{m} = 1$ (blue) and $\dot{m} = 10^3$
Table 1. Model parameters and results for isotropic radiation cases.

| Model          | $M_{\text{BH}}(M_\odot)$ | $n_{\infty}(\text{cm}^{-3})$ | transition | $t_{\text{tran-end}}(t_{\text{dyn}})$ |
|---------------|----------------|----------------|-------------|-------------------------------|
| 1e0M18N0      | 1              | $1 \times 10^3$ | N           | 5.2                           |
| 1e0M19N0      | 1              | $3 \times 10^3$ | N           | 5.2                           |
| 1e0M58N0      | 1              | $5 \times 10^6$ | Y           | 4.3                           |
| 1e1M19N0      | 1              | $1 \times 10^9$ | Y           | 1.8                           |
| 1e1M39N0      | 1              | $3 \times 10^9$ | Y           | 0.75                          |
| 1e1M37N0      | 10             | $3 \times 10^9$ | N           | 5.5                           |
| 1e1M18N0      | 10             | $1 \times 10^9$ | Y           | 2.9                           |
| 1e2M36N0      | $10^2$         | $3 \times 10^6$ | N           | 5.6                           |
| 1e2M17N0      | $10^2$         | $1 \times 10^7$ | Y           | 4.4                           |
| 1e2M37N0      | $10^2$         | $3 \times 10^7$ | Y           | 1.6                           |
| 1e5M14N0      | $10^5$         | $1 \times 10^4$ | N           | 6.8                           |
| 1e5M34N0      | $10^5$         | $3 \times 10^4$ | Y           | 6.1                           |
| 1e5M54N0      | $10^5$         | $5 \times 10^4$ | Y           | 2.7                           |
| 1e5M15N0      | $10^5$         | $1 \times 10^4$ | Y           | 1.3                           |

Column (1) model ID, (2) BH mass, (3) ambient gas density, (4) symbols Y (N) denoting that the transition occurs (does not occur) within the simulation time, and (5) the time when the transition occurs $t_{\text{tran}}$ (bold) and the duration $t_{\text{tran-end}}$ (thin, for models without transitions) in units of $t_{\text{dyn}}(\equiv v_B/c_0)$.

(38) For the standard disk case ($m = 1$), the spectrum is expressed by a multi-color Black body spectrum of

$$L_\nu^{\text{D}} = 5.7 \times 10^{13} \frac{n_{\infty}^4}{M_{\text{BH}}} h^{2/3} \nu^{1/3} \text{ erg s}^{-1} \text{ Hz}^{-1}$$

(21)

at the frequency range of $\nu_{\text{out}} < \nu < \nu_{\text{peak}}$, where

$$\nu_{\text{out}} = 3.16 \times 10^{18} \frac{n_{\infty}^{-1/4}}{M_{\text{BH}}} \frac{1}{R_{\text{out}}^{3/4}} \text{ Hz}$$

(22)

is the frequency of photons emitted from $R = R_{\text{out}}$ and

$$\nu_{\text{peak}} = 1.01 \times 10^{17} \frac{n_{\infty}^{1/4}}{M_{\text{BH}}} \text{ Hz},$$

(23)

is the peak frequency of the spectrum. The spectral shape is expressed by the Rayleigh-Jeans slope ($L_\nu \propto \nu^2$) at $\nu < \nu_{\text{out}}$, and has an exponential cutoff, the so-called Wien cutoff, at $\nu > \nu_{\text{peak}}$.

For the slim disk case ($m = 10^3$), the disk spectrum has an additional component associated with the modification of the effective temperature as described in Eq. (16).

$$L_\nu^{\text{D,slim}} = 5.15 \times 10^{38} \frac{n_{\infty}}{M_{\text{BH}}} \nu^{-1} \text{ erg s}^{-1} \text{ Hz}^{-1}.$$ (24)

at the frequency range of $\nu_{\text{tr}} < \nu < \nu_{\text{peak}}$, where

$$\nu_{\text{tr}} \equiv \frac{10^{17} (m_{\text{BH}}/10^{-1/4}) (M_{\odot}/10^3)^{-1/2}}{\text{Hz}},$$

(25)

corresponds to the frequency of photons emitted from $R_{\text{tr}}$.

In the bottom panel of Fig. 1, we compare three different radiation spectra: a single PL with an index of $\alpha = 1.5$ (black), and disk spectra with $M_{\text{BH}} = 10^5$ (red) and $10^3 M_{\odot}$ (blue) for $m = 10^3$. The disk spectrum with $m_{\text{BH}} = 10^5$ is harder than the PL spectrum; in the spectrum, the difference of the luminosities at $h\nu = 13.6$ Hz is the order of $10^5$. The disk spectrum becomes softer as the BH mass increases following $T_{\text{eff}} \propto m_{\text{BH}}^{-1}$.

2.5 initial and boundary conditions

We set a computational domain of $r_{\text{min}} \leq r \leq r_{\text{max}}$ and $0 \leq \theta \leq \pi$, where $(r_{\text{min}}, r_{\text{max}}) = (0.007 R_\odot, 6 R_\odot)$ for isotropic radiation, and $(r_{\text{min}}, r_{\text{max}}) = (0.07 R_\odot, 60 R_\odot)$ for anisotropic radiation. In the case with anisotropic radiation, we set a larger simulation box because the ionized region toward the bipolar directions tends to be larger than that in isotropic cases. We set logarithmically-spaced grids in the radial direction and uniformly-spaced grids in the polar direction. The number of the grid points is set to $(N_r, N_\theta) = (100, 120)$.

As our initial conditions, we set a neutral uniform and static ($v = 0$) gas cloud with a density $n_\infty$ and temperature $T_\infty = 10^4$ K. The BH mass is assumed to be constant throughout the simulations. In order to study the transition criterion, we explore a wide range of the ambient density and BH mass: $10^4 \leq n_\infty / \text{cm}^3 \leq 3 \times 10^9$ and $1 \leq M_{\text{BH}} / M_\odot \leq 10^5$. Our model setup with isotropic radiation is summarized in Table 1. We impose the absorption inner-boundary conditions which damp the gas density, the velocity, and gas pressure smoothly (e.g. Kato et al. 2004), and the free outer-boundary conditions for three components of the velocity and the specific entropy. We also fix the same gas density at $r = r_{\text{max}}$ as the initial value for grids with an inflow velocity i.e., $v_r(r = r_{\text{max}}) < 0$, otherwise the free boundary condition is imposed for the density. The reflection symmetry with respect to the polar axis is imposed for non-radial components of the velocity.

3 RESULTS

Fig. 2 presents the time evolution of accretion rates onto a BH with a mass of $M_{\text{BH}} = 10^5 M_\odot$ embedded in a gas cloud with different densities of $n_\infty = 10^4$ (black), $3 \times 10^4$ (red), $5 \times 10^4$ (green), and $10^5$ cm$^{-3}$ (blue). For high-density cases with $n_\infty \geq 3 \times 10^4$ cm$^{-3}$, the accretion rates dramatically increases to the corresponding Bondi rates. The transition epochs are marked by open circles. The dashed line shows the Eddington accretion rate ($m = 10$).
Figure 3. Radial structure of the gas density (top), temperature (middle), neutral fraction (bottom) at the equatorial plane. In the left panels, we present the profiles for Model 1e5M14N0, where the accretion occurs episodically without a transition to super-Eddington phases, at three different epochs during an oscillation: $t/t_{\text{dyn}} = 1.46$ (dotted), 1.55 (solid), and 1.69 (dashed). In the right panels, we show those for Model 1e5M15N0, where the accretion rate transits to a super-Eddington value, at $t/t_{\text{dyn}} = 0.334$ (dotted), 1.25 (solid) and 1.38 (dashed). For the case without the transition, the location of the ionization front $r_{\text{HII}}$ is outside the Bondi radius, while the ionized region is always confined inside the Bondi radius for the case with the transition.

of the oscillation is explained in what follows. In Fig. 3 (left panel), we present the radial structure of the gas density, temperature, and neutral fraction at different three epochs in an oscillation period. For this case, radiation associated with BH accretion propagates outward, and the gas outside the Bondi radius is ionized and heated up ($n_\text{HII} > n_B$, see phase I). Inside the ionized region, ionized gas within a new sonic radius at $\approx 0.1 \ r_B$ for the hot gas with $T \sim 10^5$ K can accrete onto the central region, while gas outside the radius flows outwards. As a result, a density cavity forms within the ionized region, where the outward and inward gas pressure forces are balanced. When the ionized gas is depleted from the ionized region, a density bump forms inside the ionization front because pressure inside decreases (phase II). This density bump provides a positive pressure gradient ($\partial p/\partial r > 0$) and accelerate gas accretion ($n_t = 7.46$, phase III). This episodic behavior has been studied in detail in previous studies (e.g. Ciotti & Ostriker 2001; Milosavljević et al. 2009a; Park & Ricotti 2011, 2012). The time-averaged accretion rate results in as small as $\langle M \rangle \approx 1.6 \ M_{\text{Edd}}$

For cases with higher densities ($n_\infty \geq 3 \times 10^4 \text{ cm}^{-3}$), the episodic accretion behavior ceases unlike the lowest density case. Instead, the accretion rate has a big jump to a very high value (red, green and blue curves in Fig. 2). Open circles indicate the epochs when transitions to super-Eddington accretion occur. In Fig. 3 (right panel), we present the radial structure of the gas density, temperature, and neutral fraction for the highest density (Model 1e5M15N0) at different three epochs of $t/t_{\text{dyn}} = 0.334$ (phase 1), 1.25 (phase 2), and 1.38 (phase 3). At the beginning, an ionized region forms and the gas is heated up to $T \sim 10^5$ K as in the lowest density case. However, because of the higher density, the size of the ionized region never becomes larger than the Bondi radius ($r_{\text{HII}} < r_B$). As a result of this, a dense shell forms at $r_{\text{HII}} \lesssim r \lesssim r_B$ and pushes the ionized gas inward (phase 2). During the transition, the ionized region shrinks and disappears because of efficient radiative recombination. Thus, the accretion flow settles down to an isothermal Bondi accretion solution with $T \approx 8000$ K (phase 3).

In Fig. 4, we summarize our results for different values of $M_{\text{BH}}$ and $n_\infty$ under isotropic radiation with spectra associated with accretion disks. Each circle symbol indicates whether the transition to super-Eddington accretion occurs (blue) or the accretion rate behaves episodically without the transition (orange). For the latter cases, we follow the simulations over $t > 5 \ t_{\text{dyn}}$, which is long enough to confirm the result. The red solid (black solid) line presents the transition criterion under the disk (single PL) spectrum (see
The transition conditions are well explained by the radiation spectral effect. As discussed in Inayoshi et al. (2016), the transition conditions are well explained by the transition criterion is alleviated from that for the power-law spectrum and the subsequent super-Eddington accretion is stable.

3.1 Analytic arguments: the transition criterion

We here give a simple analytic argument for the conditions required for super-Eddington accretion, taking into account the radiation spectral effect. As discussed in Inayoshi et al. (2016), the transition conditions are well explained by the comparison of the Bondi radius and the size of the ionized region (see Eqs. 2 and 3). For disk spectra, the number rate of ionizing photons absorbed by neutral hydrogen within the ionization front is estimated as

$$\dot{N}_{\text{ion,D}} \approx \int_{\nu_{\text{min}}}^{\infty} d\nu \frac{L^\text{D,st}_\nu}{\nu} \approx 1.22 \times 10^{16} m_{\text{BH}}^{5/4} \mathcal{N}_\nu^{3/4} \text{ s}^{-1}, \quad (26)$$

where $\dot{N}_{\text{ion,D}}$ is the ionizing photon number flux (in units of s$^{-1}$) and $L^\text{D,st}_\nu$ is the specific luminosity of radiation produced from a standard accretion disk because the BH accretion rate is sub-Eddington value ($m \approx 10$) before the transition. In the last expression, we neglect the contribution from ionizing photons with $\nu \geq \nu_{\text{peak}} = 10^{17} m_{\text{BH}}^{-1/4} \mu_{1/4} \text{ Hz}$. Note that this approximation causes at most 20 – 40% differences from the numerically integrated values for lower BH masses with $m_{\text{BH}} \lesssim 10^5 M_\odot$. Therefore, we obtain the ratio of the two radii

$$r_{\text{H}}/r_B \propto (\dot{N}_{\text{ion,D}})^{1/3} \mathcal{N}_\nu^{-2/3} m_{\text{BH}}^{-1} \propto (M_{\text{BH}}^{2/8} n_{\infty})^{-2/3}, \quad (27)$$

which nicely agrees to the transition criterion shown in Fig. 4 (red). When the mean photon energy of the radiation spectrum is harder, the ionizing photon number flux becomes smaller for a given bolometric luminosity. Since the intrinsic radiation spectrum is harder for the lower BH mass, the photon absorption rate $\dot{N}_{\text{ion,D}}$ becomes lower, and the size of the ionized region becomes relatively smaller. Therefore, the transition to super-Eddington accretion is more likely to occur.

Ionizing photons at frequencies of $\nu \gtrsim 10^{17}$ Hz are hardly absorbed by neutral gas even outside the ionization front because of the steep frequency dependence of absorption cross section of neutral hydrogen ($\sigma_H \propto \nu^{-3}$).
It is worthy comparing the transition criteria for disk spectra to those for PL spectra. Since the ionizing photon number flux for a PL spectrum with $\alpha = 1.5$ is estimated as

$$N_{\text{ion,PL}} \approx 2.46 \times 10^{47} \, \frac{m_{\text{BH}}}{\eta} \, \text{cm}^{-2} \, \text{s}^{-1},$$

(28)

where $L/L_{\text{Edd}} \approx 0.1$ at $\eta \lesssim 20$, therefore we obtain the ratio of the two photon number fluxes

$$\frac{N_{\text{ion,D}}}{N_{\text{ion,PL}}} = 4.96 \times 10^{-2} \, \frac{m_{\text{BH}}^{1/4}}{\eta^{-1/4}}.$$

(29)

Note that the above scaling relation is valid for lower BH masses with $M_{\text{BH}} \lesssim 10^5 \, M_\odot$ (see black dashed in Fig. 5). Since the peak energy for higher BH masses becomes as low as the ionization threshold energy, the ionization photon number sharply drops for higher BH masses (see red curve Fig. 5), and thus the radiation feedback effect is significantly reduced.

We briefly mention the effects of secondary ionization on our results. In Fig. 6, we show the efficiency of secondary ionization of H atoms per electron produced by primary ionization with a energy of $E_{\text{H}} = h \nu - I_H$ for different electron fractions of $10^{-3} \leq x_{\text{H}} \leq 1.0$ (from the top to the bottom). The horizontal dashed line presents $\Phi_{\text{H}}(E_{\text{H}}, x_{\text{H}}) = 1$, above which secondary ionization becomes more effective than primary ionization. Since the photons causing primary ionization are at $\nu \lesssim 10^{17} \, \text{Hz}$, the primary electrons hardly contribute to secondary ionization until the ionization degree increases to $\sim 0.4$. As a result, secondary ionization can enhance the ionization degree near the ionization front, but does not expand the size of the ionization region.

4 DISCUSSION

4.1 Mean photon energy and transition criterion

As described in §3.1, the transition criterion for super-Eddington accretion depends on the shape of radiation spectra. Here, we generalize the criterion and rewrite the critical value of $M_{\text{BH}} \times n_{\text{esc}}$ as a function of the mean photon energy. In Fig. 7, we summarize our simulation results for isotropic radiation. Red circles present the results for disk spectra for different BH masses of $M_{\text{BH}} = 1, 10, 10^2$, and $10^5 \, M_\odot$. By setting the accretion rate to $\eta = 10$, we obtain the relation $\langle \epsilon \rangle = 7.7 \times 10^2 \, m_{\text{BH}}^{-0.23} \, \text{eV}$. Thus, the transition criterion is expressed as

$$M_{\text{BH}} \times n_{\text{esc}} \gtrsim 2.4 \times 10^3 \, M_\odot \, \text{cm}^{-3} \left( \frac{\langle \epsilon \rangle}{100 \, \text{eV}} \right)^{-5/9}.$$

(30)

Note that this equation is no longer valid for $\langle \epsilon \rangle \lesssim 70 \, \text{eV}$, and the critical value sharply drops at $\langle \epsilon \rangle \lesssim 20 \, \text{eV}$. In addition, blue circles present the critical values for PL spectra ($L \propto \nu^{-\alpha} \mid 1.1 \leq \alpha \leq 3.0$), where the mean photon energy is $\langle \epsilon \rangle = h \nu_{\text{min}} \alpha/(\alpha - 1)$ for $\alpha > 1$, independent of both $M_{\text{BH}}$ and $\eta$ (see also Park & Ricotti 2012). In the cases, the transition criterion is expressed as

$$M_{\text{BH}} \times n_{\text{esc}} \gtrsim 3.2 \times 10^3 \, M_\odot \, \text{cm}^{-3} \left( \frac{\langle \epsilon \rangle}{100 \, \text{eV}} \right)^{-1/2},$$

(31)

which corresponds to the dashed line in Fig. 7. Note that the PL spectrum is normalized as $\eta n L_{\text{Edd}} = \int_{\nu_{\text{min}}}^{\nu_{\text{max}}} \nu L_{\nu} \, d\nu$, where $\eta = 0.1$.

Radiation spectra we observed in BH accreting systems are complex more than we considered in this paper. In some cases, radiation spectra consist of two components: thermal emission from the nuclear disk and non-thermal emission with PL spectra produced by Compton up-scattering in a hot corona (e.g., Haardt & Maraschi 1991; Svensson & Zdziarski 1994; Liu et al. 2002, 2003; Done & Kubota 2006). For super-Eddington accreting systems such as ultraluminous X-ray sources (ULXs), a PL component is produced by a radiation-pressure driven, hot ($\sim 10^5 - 10^8 \, \text{K}$) outflow where soft photons from the accretion disk are hardened by both thermal and bulk Comptonization (e.g., Kawashima et al. 2009, 2012; Narayan et al. 2017; Kitaki et al. 2017). In the following, we discuss three effects changing the mean photon energy from disk spectra.
Table 2. The photon number flux and mean photon energy affected by Comptonization.

| $M_{BH}/M_\odot$ | $N'_{abs}/N_{in,D}$ | $(e')/eV$ | $(\epsilon_{D})/eV$ |
|------------------|----------------------|------------|-------------------|
| 10               | $8.4 \times 10^{-2}$ | $2.0 \times 10^3$ | $4.0 \times 10^2$ |
| $10^2$           | $8.8 \times 10^{-2}$ | $1.6 \times 10^3$ | $2.4 \times 10^2$ |
| $10^3$           | $9.3 \times 10^{-2}$ | $1.2 \times 10^3$ | $1.5 \times 10^2$ |
| $10^4$           | 0.11                 | 7.2 $\times 10^2$ | 94                |

The number flux of ionizing photons absorbed by neutral hydrogen is calculated by $N'_{abs} = \int_{\nu_{min}}^{\nu_{max}} d\nu L_{\nu}' (1 - e^{-\tau_{nu}})/(\nu h\nu)$, where the spectral shape of $L_{\nu}'$ is taken from the results of Kitaki et al. (2017), and the optical depth is estimated at the Bondi radius as $\tau_{nu} = n_u r_{Sch}/\nu$. The mean photon energies of $(e')$ and $(\epsilon_{D})$ are estimated by taking the data from Kitaki et al. (2017) and by assuming the disk spectra, respectively. The accretion rate is set to $\dot{m} = 10^3$.

Recently, Kitaki et al. (2017) studied radiation spectra of super-Eddington accretion flows onto a BH with $10 \leq M_{BH}/M_\odot \leq 10^4$ under a mass inflow rate of $\dot{m} = 10^3$ at $R = 10^5$ $r_{Sch}$. With Monte Carlo radiation transfer calculations, they found that a significant excess in the spectrum is produced at $\nu \gtrsim 2$ kHz due to Comptonization. In Table 2, we summarize the number flux of ionizing photons absorbed by neutral hydrogen and mean photon energy estimated by taking the data from Kitaki et al. (2017) (lower panels of their Fig. 5). Compared to the cases assuming disk spectra, the mean energies are boosted by a factor of 5 – 8. Since such hard X-rays with $\nu \gtrsim 1$ keV are hardly absorbed even by neutral hydrogen, the numbers of absorbed photons are reduced by one order of magnitude ($N'_{abs}/N_{in,D} \approx 0.1$).

Therefore, the transition criterion is alleviated by a factor of 5.8.

The size of the nuclear accretion disk would affect the feedback efficiency because ionizing photons with $13.6$ eV $\lesssim h\nu \lesssim 1$ keV are produced from larger disk radii. In Fig. 8, we demonstrate the dependence of radiation spectra on the choice of the disk outer edge: $R_{out} = 10^4$ $r_{Sch}$ (black) and $10^5$ $r_{Sch}$ (red) for $M_{BH} = 10 M_\odot$ and $\dot{m} = 10$. For $R_{out} = 10^4$ $r_{Sch}$, the spectrum is no longer expressed as a multi-color blackbody spectrum but by the Rayleigh-Jean's law. Thus, the number flux of ionizing photons at $\nu \lesssim 3 \times 10^{17}$ Hz is significantly reduced and the mean photon energy increases to $(\epsilon) \approx 1.3$ keV from $(\epsilon) \approx 450$ eV. As a result, the critical value for the transition would be reduced by a factor of $\approx 2$.

In addition, the existence of dust grains in accretion flows significantly change the spectral shape due to UV attenuation caused by dust absorption and thus alleviate the criterion for super-Eddington accretion significantly. Recent work by Toyouchi et al. (2018) has found that rapid accretion of metal-polluted gas is allowed as long as $Z \lesssim 10^{-2} Z_\odot$, because ionizing radiation from the central BH is absorbed and reemitted to infrared lights with lower energies ($h\nu \ll 13.6$ eV).

4 The choice of $R_{out}$ depends on the angular momentum of inflowing gas from the Bondi radius. When the gas is optically thin to LyC lines, a quasi-hydrostatic dense torus with a constant temperature of $T = 8000$ K forms around the centrifugal radius of $r_{cent}$ ($< r_{Sch}$). As long as the angular momentum is so small that $r_{cent} < 0.03 r_{Sch}$ is satisfied, the rate of accretion driven by viscosity can be comparable to the Bondi rate (Sugimura et al. 2018).

5 In a partially ionized region, the strong dependence of opacity on gas temperature leads to a thermal-ionization instability (e.g., Meyer & Meyer-Hofmeister 1981; Kato et al. 2008). Though the location of the photosphere $r_{ph}$ results in time-dependent, our order-of-magnitude estimate is not significantly changed.
As shown in Fig. 4, super-Eddington accretion for higher BH masses ($M_{\text{BH}} \gtrsim 10^2 M_\odot$) satisfy the stability condition after the transition occurs.

### 4.3 Cases with anisotropic radiation

We also examine cases with anisotropic disk radiation spectra. Under anisotropic radiation with a PL spectrum, hot ionized gas expands towards the bipolar directions, and the neutral warm gas with $T \approx 8000$ K accretes through the equatorial plane at a rate of $\dot{m} \approx \dot{m}_{\text{Edd}} \sin \Theta$ (Sugimura et al. 2017; Takeo et al. 2018), where $\Theta$ is the half angle of the neutral region measured from the equator. Moreover, the transition to efficient accretion where the entire region is covered by neutral gas occurs when $M_{\text{BH}} \times n_\infty \gtrsim 5 \times 10^{10} M_\odot$ cm$^{-3}$ is satisfied (Takeo et al. 2018). We mention how those features are affected by disk spectra in cases with/without transitions, respectively.

In cases without transitions, we compare a quantity of $\sin \Theta$ for the two cases. Fig. 9 presents two-dimensional distribution of the gas density (left panels) and temperature (right panels) under the disk spectrum (top) and the PL spectrum (bottom) in cases with $1 M_\odot$, $10^9$ cm$^{-3}$, and $N = 4$. The elapsed time is $t = 1.7 \times 10^2$ yr = 20 $t_{\text{dyn}}$.

![Figure 9](image)

**Figure 9.** Two-dimensional distribution of the gas density (left panels) and the temperature (right panels) under the disk spectrum (top panels) and the PL spectrum (bottom panels) in cases with $1 M_\odot$, $10^9$ cm$^{-3}$, and $N = 4$. The elapsed time is $t = 1.7 \times 10^2$ yr = 20 $t_{\text{dyn}}$.

and the angular dependence reflects the anisotropic radiation flux given in Eq. (20). Therefore, the ratio of the half opening angle for the two cases is evaluated as

$$\frac{\sin \Theta_D}{\sin \Theta_{\text{PL}}} = (\frac{N_{\text{ion},D}}{N_{\text{ion,PL}}})^{-1/N}. \quad (35)$$

For the disk spectrum, the ionizing photon number flux is given by

$$N_{\text{ion},D} \approx \int v_\nu \frac{L_{D,\text{sim}}}{\nu} v_\nu \simeq 3.0 \times 10^{48} \frac{m_{\text{BH}}^{5/4}}{M_\odot} \text{ s}^{-1}. \quad (36)$$

where we approximately estimate $N_{\text{ion},D}$ taking account of the slim disk component because the accretion rate is as high as $\dot{m} \sim O(10^5)$. For a single power-law spectrum, the photon flux is calculated as

$$N_{\text{ion,PL}} = 4.90 \times 10^{48} \frac{m_{\text{BH}}}{M_\odot} \left[ 1 + \ln \left( \frac{\dot{m}}{20} \right) \right] \text{ s}^{-1}. \quad (37)$$

where the luminosity is estimated by Eq. (19). Therefore, we obtain the analytical expression of the ratio of the half opening angles for $N = 4$

$$\frac{\sin \Theta_D}{\sin \Theta_{\text{PL}}} \approx 1.13 \frac{m_{\text{BH}}^{1/4}}{M_\odot} \left[ 1 + \ln \left( \frac{\dot{m}}{20} \right) \right]^{1/4}. \quad (38)$$

This analytical expression agrees with the numerical results within errors of $\lesssim 10\%$.

In cases with the transition to the wholly neutral phase, the critical conditions can be derived by equating $\eta_{\text{H}_2}/r_B$ at...
poles towards which the radiation flux is collimated,

\[
\frac{M_{\text{BH}} \times n_{\text{ion}}}{10^9 \, \text{M}_\odot \, \text{cm}^{-3}} \geq \sqrt[20]{\frac{7.1 [1 + \ln (n/20)]}{8.0 (m_{\text{BH}}/10)^{1/8}}} \quad \text{(PL)} \quad \text{(39)}
\]

We note that the criterion for \( N = 4 \) agrees with the numerical result shown in Takeo et al. (2018).

5 SUMMARY AND CONCLUSIONS

We investigate the properties of accretion flows onto a black hole (BH) with a mass of \( M_{\text{BH}} \) embedded in an initially uniform gas cloud with a density of \( n_{\text{ion}} \) in order to study rapid growth of BHs in the early Universe. In previous work, the conditions required for super-Eddington accretion from outside the Bondi radius were studied by assuming that radiation produced at the vicinity of the central BH has a single-power-law spectrum \( \nu^{-\alpha} \) at \( h\nu \geq 13.6 \, \text{eV} \) (\( \alpha - 1.5 \)). However, radiation spectra surely depend on the BH mass and accretion rate, and determine the efficiency of radiative feedback. Here, we perform two-dimensional multi-frequency radiation hydrodynamical simulations taking into account more realistic radiation spectra associated with the properties of nuclear accretion disks. We find that the critical density of gas surrounding the BH, above which a transitions to super-Eddington accretion occurs, is alleviated for a wide range of masses of seed BHs (\( 10 \lesssim M_{\text{BH}}/M_\odot \lesssim 10^8 \)) because photoionization for accretion disk spectra are less efficient than those for single-power-law spectra with \( \lesssim \alpha \lesssim 3 \). For disk spectra, the transition to super-Eddington is more likely to occur for lower BH masses because the radiation spectra become too hard to ionize the gas. Even when accretion flows are exposed to anisotropic radiation, the effect due to radiation spectra shrinks the ionized region and likely leads to the transition to a wholly neutral accretion phase. Finally, by generalizing our simulation results, we construct a new analytical criterion required for super-Eddington accretion,

\[
\left( \frac{M_{\text{BH}}}{10^9 \, \text{M}_\odot} \right) \left( \frac{n_{\text{ion}}}{10^4 \, \text{cm}^{-3}} \right) \gtrsim 2.4 \left( \frac{\epsilon}{100 \, \text{eV}} \right)^{-5/9} \quad \text{(40)}
\]

where \( \epsilon \) is the mean energy of ionizing radiation from the central BH. This conditions would be applicable as a sub-grid model characterizing BH growth in large-scale cosmological simulations which do not resolve the Bondi radius of the BH.

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