Does the Choice of the Multivariate GARCH Model on Volatility Spillovers Matter? Evidence from Oil Prices and Stock Markets in G7 Countries

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ABSTRACT

In this paper, we employ asymmetric multivariate GARCH approaches to examine their performance on the volatility interactions between global crude oil prices and seven major stock market indices. Insofar as volatility spillover across these markets is a crucial element for portfolio diversification and risk management, we also examine the optimal weights and hedge ratios for oil-stock portfolio holdings with respect to the results. Our findings highlight the superiority of the asymmetric BEKK model and the fact that the choice of the model is of crucial importance given the conflicting results we got. Finally, our results imply that oil assets should be a part of a diversified portfolio of stocks as they increase the risk-adjusted performance of the hedged portfolio.

Keywords: Asymmetry, Multivariate GARCH, Stock Market, Oil Price, Volatility Spillover

JEL Classifications: C32, F3, G15, Q4

1. INTRODUCTION

Over the past years, the stock markets and crude oil markets have developed a reciprocal relationship. Every production sector in the international economy depends on oil as an energy source. Based on such dependence, fluctuations in oil price and its volatility are likely to affect the production sector and the international economy in general. Mork (1989) and Hooker (1999) documented that there is a significant negative relationship between crude oil price increases and world economic growth. Given that negative relationship, one would expect that increases in crude oil market prices will affect the firms’ earnings and hence their stock price levels. Subsequently, the linkage between crude oil price volatility and stock markets seems to be quite evident. Many relevant studies such as Sadorsky (1999; 2001; 2006), Papapetrou (2001), Ewing and Thompson (2007) and Aloui and Jammazi (2009) conclude that a change in oil prices of either sign may affect stock price behavior. For this reason, investors should be aware of how shocks and volatility are transmitted across markets over time. Also, the increased financial integration between countries and the financialization of oil markets can enhance the ways of diversification of investors’ portfolios. In order to take advantage of these ways, investors require a better understanding of how financial and oil markets correlate. By modeling volatility, researchers can produce accurate estimates of correlation and volatility which are key elements in developing optimal hedging strategies (see, for example, Chang et al. (2011)). Supporters of investing in commodities (mostly in oil) claim that if commodities have low or even negative correlations with stocks then a portfolio that includes commodities should perform better than a portfolio that excludes commodities (Sadorsky, 2014). This suggests that adding oil to an equity portfolio may lead to higher returns and lower risk than just investing in equities.

Since the development of the univariate ARCH model by Engle (1982) and GARCH model by Bollerslev (1986), an important...
body of literature has focused on using these models to model the volatility of oil and stock market returns. Furthermore, in the last decade, with the generalization of the univariate into multivariate GARCH models, the literature has focused on the volatility spillovers between oil and stock markets.

This paper makes several important contributions to the literature. First, while existing papers investigate the volatility dynamics between stock prices and oil prices, most of this literature focuses on individually developed economies, the Gulf Cooperation Council (GCC) countries or the BRICS (see, for example, Malik and Hammoudeh (2007); Arouri et al. (2011b); Creti et al. (2013)). This paper is specifically focused on the volatility dynamics between the G7 stock market prices and the Brent which is the global oil benchmark for light, sweet crudes. The choice of these countries is based on their importance to the global economy. For example, in 2017, according to worldstopexports.com the U.S. accounted for 15.9% of total crude oil imports and summing these percentages, the G7 countries accounted for 36.9% of total crude oil imports. Moreover, among the G7 countries, Canada is considered as an oil-exporter, so a slight distinction between oil -importers and -exporters can be made, adding this paper to the limited studies which make that kind of distinction (see, for example, Park and Ratti (2008); Apergis and Miller (2009); Filis et al. (2011)). Second, this paper differs from previous studies by comparing the performance of three asymmetric multivariate GARCH models namely, the ABEKK model of Kroner and Ng (1998), the AVARMA-CCC-GARCH model of McAleer et al. (2009) and the AVARMA-CCC-GARCH model which is a combination of the AVARMA-GARCH model of McAleer et al. (2009) and the CCC model of Engle (2002) in order to study the volatility spillover effects between developed stock market prices and oil prices. These models can simultaneously estimate the volatility cross-effects for the stock market indices and oil prices under consideration. In addition, these models can capture the effect of own shocks and lagged volatility on the current volatility, as well as the volatility transmission and the cross-market shocks of other markets.

The aim of this paper is to investigate the joint evolution of conditional returns, the correlation and volatility spillovers between the crude oil returns, namely Brent and the stock index returns of the G7 countries, namely CAC40 (France), DAX (Germany), DJIA (U.S.), FTSE100 (U.K.), MIB (Italy), Nikkei225 (Japan) and TSX (Canada). The asymmetric bivariate GARCH models are estimated using weekly return data from January 14, 1998, to December 27, 2017. A complementary objective is to use the estimated results to compute the optimal weights and hedge ratios that minimize overall risk in portfolios of each G7 country. Our results are crucial for building an accurate asset pricing model and forecasting volatility in stock and oil market returns.

The remainder of the paper is organized as follows. Section 2 reviews the literature. Section 3 describes the three asymmetric multivariate GARCH models. Section 4 presents the data and descriptive statistics. Section 5 discusses the empirical results and provides the economic implications for optimal portfolios and optimal hedging strategies. Section 6 concludes the paper.
and negative spillovers from oil to the stock market. In the post-financial crisis period, bidirectional positive risk spillovers are reported. Khalifaoui et al. (2015) is one of the extremely limited studies focusing on G7 countries. They investigate the linkage of the crude oil market (WTI) and stock markets of the G7 countries using a combination of multivariate GARCH models and wavelet analysis. They find strong volatility spillovers between oil and stock markets and that oil market volatility is leading stock market volatility. Phan et al. (2016) examine the price volatility interaction between the crude oil (WTI) and equity markets in the U.S. (S&P500 and NASDAQ) using intraday data over the period 2009 and 2012. They claim that even in the future markets there are cross-market volatility effects. Ewing and Malik (2016) use univariate and multivariate GARCH models to investigate the volatility of oil prices (WTI) and U.S. stock market prices (S&P500). They use daily data over the period from July 1996 to June 2013 and take into account structural breaks. Their results show no volatility spillover between these markets when structural breaks are ignored. However, after accounting for breaks, they find a significant volatility spillover between oil prices and the U.S. stock market.

The next few studies are focused on oil-exporting and oil-importing countries. Park and Ratti (2008) use monthly data for 13 European countries and the U.S. over the period 1986:1-2005:12. They find that positive oil price shocks cause positive returns for the stock market of the oil-exporting country (Norway), however, the opposite occurs for the rest of the European countries but not for the U.S. (oil-importers). Apergis and Miller (2009) use monthly data for the G7 countries and Australia to conclude that major stock market (independently of oil-exporting or oil-importing) returns do not respond in oil market shocks. Filis et al. (2011) employ multivariate DCC-GARCH-GJR models to investigate the time-varying correlation between oil prices and stock prices of oil-exporting (Brazil, Canada, and Mexico) and oil-importing (U.S.A., Germany, and Netherlands) countries. They find, among others, that the time-varying correlation does not differ between oil-importing and oil-exporting countries. Maghsood et al. (2016) utilize 3 oil-exporting and 8 oil-importing countries over the period 2008-2015. Their findings support that oil price volatility is the significant transmitter of volatility shocks to stock market volatilities and that there is no difference between oil-importers and oil-exporters.

3. ECONOMETRIC METHODOLOGY

Since the objective of this paper is to investigate volatility interdependence and transmission mechanisms between stock and oil markets, multivariate frameworks such as the VARMA-CCC-GARCH model of McAleer et al. (2009), the VARMA-DCCGARCH and the ABEKK-GARCH model of Kroner and Ng (1998) are more relevant than univariate GARCH models. The first model assumes constant conditional correlations, while the last two accommodate dynamic conditional correlations. Combined with a vector autoregressive (VAR) model for the mean equation, they allow us to examine returns spillovers too. In what follows we present the bivariate framework of these three models.

The econometric specification has two components, a mean equation, and a variance equation. The first step in the bivariate GARCH methodology is to specify the mean equation. For each pair of stock and oil returns, we try to fit a bivariate VAR model. For example, a bivariate VAR(1) model has the following specification for the conditional mean:\[ r_t = \mu + \Psi r_{t-1} + u_t \] (1)

where \( r = (r_s, r_o) \) is the vector of returns on the stock and oil price index, respectively. \( \Psi \) refers to a \( 2 \times 2 \) matrix of parameters of the form \( \Psi = \begin{bmatrix} \psi_{ss} & \psi_{so} \\ \psi_{os} & \psi_{oo} \end{bmatrix} \). \( u = (u_s, u_o) \) is the vector of the error terms of the conditional mean equations for stock and oil returns, respectively.

The asymmetric BEKK model proposed by Kroner and Ng (1998) is an extension of the BEKK model of Engle and Kroner (1995). Their difference is one extra matrix that takes into account the asymmetries. Its equation has the following form:

\[ H_t = C C + A' u_{t-1} u'_{t-1} + B + B' H_{t-1} B + D' v_{t-1} v'_{t-1} D \] (2)

where \( u_t = H_t^{1/2} \eta_t \), \( \eta_t \sim iid N(0,1) \) and \( H_t = \begin{bmatrix} h_{ss,t} & h_{so,t} \\ h_{os,t} & h_{oo,t} \end{bmatrix} \) is the conditional variance-covariance matrix. The individual elements for C, A, B and D matrices of equation (2) in the bivariate case are given as:

\[ C = \begin{bmatrix} c_{ss} & c_{so} \\ 0 & c_{oo} \end{bmatrix}, \quad A = \begin{bmatrix} a_{ss} & a_{so} \\ a_{os} & a_{oo} \end{bmatrix}, \quad B = \begin{bmatrix} b_{ss} & b_{so} \\ b_{os} & b_{oo} \end{bmatrix} \]

\[ D = \begin{bmatrix} d_{ss} & d_{so} \\ d_{os} & d_{oo} \end{bmatrix} \] (3)

where \( C \) is a \( 2 \times 2 \) upper triangular matrix, \( A \) is a \( 2 \times 2 \) square matrix of coefficients and shows the extent to which conditional variances are correlated with past squared errors. \( B \) is also a \( 2 \times 2 \) square matrix of coefficients and reveals how current levels of conditional variances are related to past conditional variances. \( D \) is a \( 2 \times 2 \) matrix and \( v \) is defined as \( u \) if \( u \) is negative and zero otherwise. For example, a statistically significant coefficient on \( d_{ss} \) would indicate that the “bad” news of the first variable affects its variance more than the “good” news of the same magnitude. Moreover, it should be mentioned that if the D matrix is zero then the ABEKK model reduces to the simple BEKK model. The ABEKK model has the property that the conditional variance-covariance matrix is positive definite. However, this model suffers from the curse of dimensionality (for more details see McAleer et al. (2009)). The following likelihood function is maximized assuming normally distributing errors:

\[ L(\theta) = -\frac{T}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^{T} \log |H_t| + u'_{t} H_{t}^{-1} u_{t} \] (4)

1 The appropriate lag length of the VAR models was chosen on the basis of the Schwarz information criterion (SIC).
where \( T \) is the number of observations and \( \theta \) refers to the parameter vector to be estimated. Numerical maximization techniques were employed to maximize this log-likelihood function. As recommended by Engle and Kroner (1995) several iterations were performed with the simplex algorithm to obtain the initial conditions. Then, the Broyden (1970), Fletcher (1970), Goldfarb (1970) and Shanno (1970) algorithm (BFGS) was employed to obtain the estimate of the variance-covariance matrix and the corresponding standard errors.

We now shift our attention to another class of GARCH specifications that model the conditional correlations rather than the conditional covariance matrix \( H_t \). In order to take into account asymmetries and interdependencies of volatility across different markets, McAleer et al. (2009) proposed the AVARMA-CCC-GARCH(1,1) model which has the following specification in its bivariate form for the conditional variances-covariance:

\[
h_{s,t} = c_{s} + a_{s}u_{s,t-1}^2 + b_{s}h_{s,t-1} + a_{s}u_{o,t-1}^2 + b_{s}h_{o,t-1} + d_{s}L_{t}u_{s,t-1}^2
\]

(5)

\[
h_{o,t} = c_{o} + a_{o}u_{o,t-1}^2 + b_{o}h_{o,t-1} + a_{o}u_{s,t-1}^2 + b_{o}h_{s,t-1} + d_{o}L_{t}u_{o,t-1}^2
\]

(6)

\[
h_{so,t} = \rho \sqrt{h_{s,t}} \sqrt{h_{o,t}}
\]

(7)

where \( L_t \) is defined as follows:

\[
L_t = \begin{cases} 
0, & u_{s,t-1} > 0 \\
1, & u_{s,t-1} \leq 0
\end{cases}
\]

(8)

The volatility transmission between stock and oil markets over time is captured by the cross values of error terms (\( u_{s,t-1}^2 \) and \( u_{o,t-1}^2 \)) and the lagged conditional volatilities (\( h_{s,t-1} \) and \( h_{o,t-1} \)).

The error terms gauge the impact of direct effects of shock transmission, while the lagged conditional variances measure the direct effects of risk transmission across the markets. In other words, the conditional variance of the stock market depends not only on its own past values and its own innovations but also on those of the oil market and vice versa. Hence, this model allows shock and volatility transmission between the oil and stock markets under consideration. As it is clear if the \( d_i \) are simultaneously zero, then the AVARMA-CCC model reduces to a VARMA-CCC model and if the elements \( a_{ij} \) and \( b_{ij} \) (\( i \neq j \)) are also zero then the model becomes the simple Constant Conditional Correlation (CCC). Ling and McAleer (2003) proposed the quasi-maximum likelihood estimation (QMLE) to obtain the parameters of the above bivariate model, which is appropriate when, \( \eta_i \) does not follow a joint multivariate normal distribution.

Our last model is a combination of the AVARMA-GARCH model of McAleer et al. (2009) and the DCC model of Engle (2002). This model is estimated in two steps simplifying the estimation of the time-varying correlation matrix. In the first step, the AVARMA-GARCH(1,1) parameters are estimated. In the second step, the conditional correlations are estimated. It has the same equation as the AVARMA-CCC-GARCH(1,1) model with an exception that the conditional covariance is not constant.

\[
H_t = L_tR_tL_t
\]

(9)

In the bivariate form, \( H_t \) is a 2 × 2 diagonal conditional covariance matrix, \( L_t \) is a diagonal matrix with time-varying standard deviations on the diagonal and \( R_t \) is the conditional correlation matrix.

\[
L_t = \text{diag} \left( h_{s,t}^{1/2}, h_{o,t}^{1/2} \right)
\]

(10)

\[
R_t = \text{diag} \left( q_{z_{s,t}}^{-1/2}, q_{z_{o,t}}^{-1/2} \right) Q_t \text{diag} \left( q_{z_{s,t}}^{-1/2}, q_{z_{o,t}}^{-1/2} \right)
\]

(11)

The expressions \( h_{s,t} \) and \( h_{o,t} \) are univariate GJR models of Glosten et al. (1993) with VARMA specification which is equal to an AVARMA specification (see, equations (5) and (6)). \( Q_t \) is a symmetric positive definite matrix.

\[
Q_t = (1-\theta_1 - \theta_2) \tilde{Q} + \theta_1 \tilde{z}_{t-1} \tilde{z}_{t-1}^\top + \theta_2 Q_{t-1}
\]

(12)

\( \tilde{Q} \) is a 2 × 2 unconditional correlation matrix of the standardized residuals \( \eta_{ij,t} \) (\( \eta_{ij,t} = u_{ij,t} / \sqrt{h_{ij,t}} \)). The parameters \( \theta_1 \) and \( \theta_2 \) are non-negative. The model is mean-reverting as long as \( \theta_1 + \theta_2 < 1 \). The matrix \( Q_t \) does not replace \( H_t \), its purpose is to provide conditional correlations \( \rho_{so,t} \).

\[
\rho_{so,t} = \frac{q_{so,t}}{\sqrt{q_{s,t}q_{o,t}}}
\]

(13)

Hence, for the conditional covariance equation, we end up in the following expression

\[
h_{so,t} = \rho_{so} \sqrt{h_{s,t}} \sqrt{h_{o,t}}
\]

(14)

which is the only difference from the AVARMA-CCC-GARCH(1,1) model. The AVARMA specification on the CCC and DCC models allows for spillovers among the variances of the series, and also makes the DCC form almost identical to that used for the ABEKK model, allowing for direct comparisons of model performance (Efimova and Serletis, 2014). In addition, permitting for asymmetries in the models provides valuable information to policy-makers and financial market participants, on the existing differences between the impact of positive and negative news on stock and oil market price fluctuations. The fact that asymmetric effects are significant depicts potential misspecification if asymmetries are neglected.

### 4. DATA AND PRELIMINARY RESULTS

For this study, weekly data on the Wednesday closing prices for crude oil and stock indices were used. Crude oil includes one of the two global light benchmarks, namely the Europe
Brent. The series for oil prices were obtained from the Energy Information Administration (EIA). The stock market indices are Dow Jones Industrial Average (United States), CAC40 (France), DAX (Germany), FTSE MIB (Italy), Nikkei225 (Japan), FTSE100 (United Kingdom) and S&P/TSX (Canada). This data was obtained from Yahoo Finance. The data range spans from 07 January 1998 to 27 December 2017 for a total of 1043 observations. Wednesday closing prices were used because in general there are fewer holidays on Wednesdays than on Fridays. Any missing data on Wednesday closes was replaced with closing prices from the most recent successful trading session. The use of weekly data significantly reduces any potential biases that may arise such as the non-trading days, bid-ask effect etc. Consistent with other studies, our analysis focuses on the returns as the price series were non-stationary in levels. Stock market and oil price returns are computed as the first log-difference, i.e. \( r_t = 100 \times \ln \left( \frac{P_t}{P_{t-1}} \right) \), where \( P_t \) is the weekly closing price. The summary for the corresponding returns, as well as the unit root tests and the Ljung and Box (1978) statistics, are shown in Table 1.

All the series have a positive mean except for MIB and for each series, the standard deviation is larger than the mean value. As measured by the standard deviation, equity market return unconditional volatility is highest in Italy, followed by Germany, Japan, France, U.K., Canada, and the U.S., while the oil price volatility is the highest among them all. In terms of skewness, each series displays negative skewness and a large amount of kurtosis, a fairly common occurrence in high-frequency financial data which implies that the GARCH model of Bollerslev (1986) is adequate. In addition, the null hypothesis of normality is rejected for all return series by the Jarque and Bera (1980) test statistic at 1% level of significance. The (squared) \( Q \) -statistic of Ljung and Box (1978) statistics at 24 lags for serial correlation and conditional heteroskedasticity of the series is shown in Table 1.

Regarding the variance equations and the CAC40 index (Tables A1 and 2-4), we find that each model provides evidence of conditional GARCH (significant coefficients on new and squared) effects in stock and oil’s variance equations meaning that each current volatility

3 Indices’ codes in the corresponding database: U.S.-^DJII, France-^FCHI, Germany-^GDAXI, Italy-^FTSEMIB.MI, Japan-^N225, U.K.-^FTSE, Canada-^GSPFSE, Europe Brent spot price FOB-RBRTE.

4 Oil prices are measured in U.S. dollars per barrel, however stock prices are in national units.

### 5. EMPIRICAL RESULTS

This section reports on the empirical results obtained from the estimating bivariate GARCH models. Empirical results are presented for our three competitive models: ABEKK-GARCH(1,1), AVARMA-CCC-GARCH(1,1) and AVARMA-DCC-GARCH(1,1) in Tables A1-A7 (in Appendix). In order to compare their performance on the volatility spillover effects, we will interpret their estimates using Wald tests (Tables 2-4). We focus on statistical significance at the 5% level. Wald test is used to test the matrix elements of the volatility spillover effect, which is the joint test for the significance of the model coefficients (see, Beirne et al. (2010); Liu et al. (2017)). We test the following two set of hypotheses:

\[
H_0: a_{so} = b_{so} = 0 \text{ or there is no volatility spillover from oil to stock} \tag{15}
\]

\[
H_1: a_{so} \neq 0 \text{ or } b_{so} \neq 0 \text{ or there is volatility spillover from oil to stock} \tag{16}
\]

\[
H_0: a_{os} = b_{os} = 0 \text{ or there is no volatility spillover from stock to oil} \tag{17}
\]

\[
H_1: a_{os} \neq 0 \text{ or } b_{os} \neq 0 \text{ or there is volatility spillover from stock to oil} \tag{18}
\]

In addition, implications of the results on optimal weights and hedge ratios for oil-stock portfolio holdings are depicted in Table 5.

First, we have to determine the mean equations. As it is apparent from Table 6, the Schwarz information criterion indicates not to use a VAR framework. Hence, the mean equations for all pairs will consist of just a constant for each series. Therefore, we cannot seek for mean spillover effects among the markets.

Regarding the variance equations and the CAC40 index (Tables A1 and 2-4), we find that each model provides evidence of conditional GARCH (significant coefficients on new and squared) effects in stock and oil’s variance equations meaning that each current volatility
Table 2: Wald tests for volatility spillover effects with the ABEKK model

| H0 | CAC40 | DAX | DJIA | FTSE100 | MIB | Nikkei225 | TSX |
|----|-------|-----|------|---------|-----|-----------|-----|
| β_{so} = 0 | 7.951 | 11.086 | 10.544 | 11.084 | 9.813 | 6.211 | 6.680 |
| Wald | 0.019 | 0.004 | 0.005 | 0.004 | 0.007 | 0.045 | 0.035 |
| Sig. | | | | | | | |
| Conclusion | Spillover from Brent to CAC40 | No spillover from Brent to DAX | Spillover from Brent to DJIA | Spillover from Brent to FTSE100 | Spillover from Brent to MIB | Spillover from Brent to Nikkei225 | Spillover from Brent to TSX |

Table 3: Wald tests for volatility spillover effects with the AVARMA-CCC model

| H0 | CAC40 | DAX | DJIA | FTSE100 | MIB | Nikkei225 | TSX |
|----|-------|-----|------|---------|-----|-----------|-----|
| β_{so} = 0 | 2.481 | 2.444 | 2.314 | 0.376 | 2.348 | 4.947 | 1.934 |
| Wald | 0.289 | 0.295 | 0.314 | 0.829 | 0.309 | 0.884 | 0.380 |
| Sig. | | | | | | | |
| Conclusion | No spillover from Brent to CAC40 | No spillover from Brent to DAX | No spillover from Brent to DJIA | No spillover from Brent to FTSE100 | No spillover from Brent to MIB | No spillover from Brent to Nikkei225 | No spillover from Brent to TSX |

Table 4: Wald tests for volatility spillover effects with the AVARMA-DCC model

| H0 | CAC40 | DAX | DJIA | FTSE100 | MIB | Nikkei225 | TSX |
|----|-------|-----|------|---------|-----|-----------|-----|
| β_{so} = 0 | 1.022 | 1.704 | 4.663 | 1.243 | 3.308 | 8.446 | 2.864 |
| Wald | 0.600 | 0.427 | 0.097 | 0.537 | 0.191 | 0.015 | 0.239 |
| Sig. | | | | | | | |
| Conclusion | No spillover from Brent to CAC40 | No spillover from Brent to DAX | No spillover from Brent to DJIA | No spillover from Brent to FTSE100 | No spillover from Brent to MIB | Spillover from Brent to Nikkei225 | No spillover from Brent to TSX |

Table 5: Optimal portfolio weights and hedge ratios for pairs of oil and stock assets

| Portfolio | ABEKK | AVARMA-CCC | AVARMA-DCC |
|-----------|-------|-------------|-------------|
| CAC40/Brent | 0.2172 | 0.2160 | 0.2058 |
| DAX/Brent | 0.1311 | 0.1092 | 0.1327 |
| DJIA/Brent | 0.2518 | 0.2479 | 0.2368 |
| MIB/Brent | 0.1194 | 0.1078 | 0.1272 |
| Nikkei225/Brent | 0.1165 | 0.1257 | 0.1121 |
| TSX/Brent | 0.0741 | 0.0538 | 0.0715 |

The table reports average optimal weights of oil and hedge ratios for an oil-stock portfolio using the estimated conditional variances and covariance from the three models for each oil/stock pair: ABEKK-GARCH(1,1), AVARMA-CCC-GARCH(1,1) and AVARMA-DCC-GARCH(1,1).

is depending on its own past volatility. The same holds, only for the oil’s variance equations for the ARCH effects (significant coefficients on $\beta_{so}$) which means that the current volatility is affected by its own past shocks. In addition, bidirectional volatility
spillover between the French stock market and the Brent oil was found according to the Asymmetric BEKK model however, both the AVARMA models failed to detect any volatility transmissions between these markets.

In terms of the DAX index (Table A2), all three models show that the conditional variances of stock and oil markets are characterized by their own lagged conditional variances. The asymmetric BEKK model supports the presence of ARCH effects in both equations (significant coefficients on \( a_\sigma \) and \( a_\epsilon \)) however, once again, the AVARMA-CCC and AVARMA-DCC models fail to provide evidence of own past shocks regarding the stock markets’ equations (insignificant coefficients on \( a_\sigma \)), yet both models show that the current variance of the oil market is depending on its own past shocks. Furthermore, the ABEKK model reveals volatility spillover from DAX to Brent as indicated by the statistically significant coefficient of the Wald test (Table 2). In contrast, the results of the other two models agree on the absence of any volatility spillover effects between the two variables.

With regard to Table A3 and the American stock market, all three models present strong evidence of own short and long-term persistence (except for the AVARMA-CCC and AVARMA-DCC models in which the coefficients on \( a_\sigma \) are not significant). The ABEKK model uncovers bidirectional volatility transmission while the remaining models do not show any relation among the markets.

Turning our interest in the English index (Table A4) and regarding the ABEKK model, our findings show that the conditional variance of both indices is depending on its own past shocks and own past volatilities. The AVARMA-CCC model indicates that only the conditional variance of the Brent oil is affected by its own past volatility, while, the AVARMA-DCC model depicts that the stock market’s variance is affected only by its own shocks and that the current volatility of the oil market depends on its own past shocks and past volatility. Once again, the AVARMA models validate the absence of any volatility transmission between the stock and oil markets. Nevertheless, the ABEKK model yields evidence of a two-way causality in the variance.

From the Italian stock market and Table A5, we ascertain that regardless of the model, the current volatility of the oil is affected by its own shocks and past volatility and that the current volatility of the MIB index is depending on its own past volatility. In addition, the ABEKK model depicts evidence of ARCH effects in the stock’s equation. All three models reveal a unidirectional volatility transmission from the stock market to the oil market while the ABEKK model supports also the reverse direction of causality.

Particularly interesting results arise for the Japanese stock market (Table A6). First, while the ABEKK model indicates considerable evidence of own short persistence in the stock’s equation, the rest of the models support that only the own past volatility has an effect on the current volatility for both indices. Second, the ABEKK framework provides evidence of unidirectional volatility spillover from the Brent oil to the Japanese stock market while, regarding the results of the AVARMA-CCC model, we find a lack of any volatility spillover. In contrast, the AVARMA-DCC model agrees with the asymmetric BEKK model on the one-way causality from the oil market to the stock market.

Finally from Table A7, the findings for the stock market of our only oil-exporting country-Canada note that for all models, the conditional variances are depending on their own lagged volatility. Moreover, the ABEKK model provides evidence of short-term persistence in the stock equation. In addition, according to ABEKK results, there is a feedback volatility spillover. The AVARMA-CCC reveals a unidirectional volatility transmission from the TSX to the Brent oil market. Instead, the AVARMA-DCC model supports that the two markets are independent.

For each pair of crude oil and stock assets, the estimated coefficients on the constant conditional correlations from the AVARMA-CCC models are very low and statistically insignificant. Moreover, the significant coefficients on \( d_{\sigma \epsilon} \) and \( d_{\sigma \sigma} \) in almost all cases, propose that the “bad” news tends to increase the volatility of the indices more than the “good” news of the same magnitude. In addition, given the significant coefficient on \( d_{\epsilon \sigma} \) only for the case of TSX, the results support that the past shocks of the Canadian stock market have an asymmetric effect on oil volatility.

The asymmetric BEKK model outperforms the rest of the models based on the Log-Likelihood value, with an exception of the DAX and Nikkei225 (AVARMA-DCC fits better), indicating its superiority. Diagnostics tests on the standardized residuals show that only in the Japanese stock market, the mean equations were not enough to deal with autocorrelation. Nevertheless, the Q-test statistics of Ljung and Box (1978) on the squared standardized residuals and the ARCH test of Engle (1982) are not statistically significant, implying that the MGARCH models were adequate to eliminate the ARCH effects.

Overall, the results from the ABEKK model reveal plenty of interactions among the markets while, both the AVARMA models...
are more parsimonious in the relations of the volatility. Figure 1 summarizes the results of the volatility spillover effects of the three competitive models. As it is apparent from Figure 1, in the case of the ABEKK model, all indices affect, or are affected by the oil market, yet this is not the case for the AVARMA models. The AVARMA-CCC model uncovers interactions only from the Italian and the Canadian stock markets to the oil market and the AVARMA-DCC model proposes that the Italian stock market is able to affect the oil market as well as that the Japanese stock market is depending on the Brent market. Moreover, for each asset, the estimated coefficient on own long-term persistence is greater than the estimated coefficient on own short-term persistence. Interestingly, we can conclude that volatility spillover effects are highly dependent on the choice of the multivariate GARCH model.

The conditional volatility estimates can be used to construct hedge ratios as proposed by Kroner and Sultan (1993). A long position in a stock asset can be hedged with a short position in an oil asset. The hedge ratio between stock and oil assets can be written as:

\[ \beta_{so,t} = \frac{h_{so,t}}{h_{oo,t}} \]  

(19)

where \( h_{so,t} \) is the estimated covariance and \( h_{oo,t} \) is the estimated variance of the crude oil market. We compute the hedge ratios from our three models (ABEKK, AVARMA-CCC and AVARMA-DCC). Their graphs are presented in Figures A3 and A4 and show considerable variability across the sample period indicating that hedging positions must be adjusted frequently.

Again, the estimated conditional volatilities from the three models can be used to construct optimal portfolio weights. The optimal holding weight of oil in a one-dollar portfolio of oil/stock asset at time \( t \), according to Kroner and Ng (1998), can be expressed as:

\[ w_{so,t} = \frac{h_{st,t} - h_{so,t}}{h_{oo,t} - 2h_{so,t} + h_{ss,t}} \]  

(20)

under the condition that

\[ w_{so,t} = \begin{cases} 0 & w_{so,t} < 0 \\ 1 - w_{so,t} & 0 \leq w_{so,t} \leq 1 \\ 1 & w_{so,t} > 1 \end{cases} \]  

(21)

Hence, the weight of the stock market index in the oil/stock portfolio is equal to \((1 - w_{so,t})\). By using three multivariate GARCH models to compute the optimal portfolio weights and the hedge ratios enable us to discuss the results from a comparative perspective.

We report the average values of optimal weights \( w_{so,t} \) and hedge ratios \( \beta_{so,t} \) in Table 5. For example, the average value of the hedge ratio between CAC40 and Brent, according to the ABEKK model, is 0.1311 indicating that a 1$ long position in CAC40 can be hedged for 13.11 cents in the oil market. Similarly, the corresponding value of the hedge ratio under the AVARMA-CCC model is 0.1092 implying that a 1$ long position in CAC40 should be shorted by 10.92 cents of Brent oil. Overall, all models give suchlike results in each stock index that are low in values. Finally, we identify that investors operating in Italy, with relatively greater hedge ratios and thus higher hedging costs, require more oil assets than those operating in the other countries of the Group of Seven to minimize the risk.

Turning our interest in the optimal weights, Table 5 shows fairly similar results for all models in each stock index. The average

![Figure 1: Aggregated results of volatility spillover effects](image-url)

The diagrams are based on the Wald tests at 5% significance level. The arrows indicate the direction of the volatility spillover effects. When there are no arrows, it means that there are not any spillover effects between the indices.
weight for the CAC40/Brent portfolio, following the results of the ABEKK model, is 0.2172, implying that for a $1 portfolio, 21.72 cents should be invested in the Brent oil and 78.28 cents invested in the stock index. In the same way, for the AVARMA-CCC model and the CAC40/Brent portfolio, the average portfolio weight is 0.2160, meaning that for a $1 portfolio, 21.60 cents should be invested in Brent crude oil and the remaining 78.40 cents invested in French stock market index. On the whole, the average weights range from 0.0599 (TSX/Brent-AVARMA-DCC/CCC) to 0.2742 (MIB/Brent-ABEKK). This finding means that the oil risk is considerably greater for Canada than for Italy, and any fluctuation in the price of crude oil could lead to undesirable effects on the performance of hedged portfolios. Finally, given our results for optimal hedge ratios, oil assets should be a part of a diversified portfolio of stocks as they increase the risk-adjusted performance of the hedged portfolio.

6. CONCLUDING REMARKS

The main objective of this article was to investigate the performance of asymmetric multivariate GARCH models on the mean and volatility transmission between oil and the stock markets of the Group of Seven (G7). Employing asymmetric models such as the ABEKK, the AVARMA-CCC, and the AVARMA-DCC-GARCH, which permit volatility spillover; we find considerable volatility spillover effects among the markets according to the ABEKK results. However, based on the AVARMA models there are negligible interactions and mostly from the stock to the oil markets. This finding is crucial and implies that the results of the volatility spillovers are highly depending on the choice of the multivariate GARCH model. In addition, the consensus of our results shows that the ABEKK models outperform the rest of the models.

Our examination of optimal weights and hedge ratios implies that optimal portfolios in all countries of the Group of Seven should possess more stocks than oil assets and that stock investment risk can be hedged by taking a short position in the oil markets. Moreover, regardless of the multivariate GARCH model used, our findings indicate that optimally hedged oil/stock portfolios are performing better than portfolios containing only stocks.

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APPENDIX A. SUPPLEMENTARY MATERIAL

Figure A1: Plot of time-series

| Closing Prices | Returns |
|----------------|---------|
| CAC40          |         |
| DAX            |         |
| DJIA           |         |
| FTSE100        |         |
Kartsonakis-Mademlis and Dritsakis: Does the Choice of the Multivariate GARCH Model on Volatility Spillovers Matter? Evidence from Oil Prices and Stock Markets in G7 Countries

**Figure A2: Plot of time-series**

| Closing Prices | Returns |
|----------------|---------|
| **MIB**        |         |
| ![MIB Closing Prices](image) | ![MIB Returns](image) |
| **Nikkei225**  |         |
| ![Nikkei225 Closing Prices](image) | ![Nikkei225 Returns](image) |
| **TSX**        |         |
| ![TSX Closing Prices](image) | ![TSX Returns](image) |
| **Brent**      |         |
| ![Brent Closing Prices](image) | ![Brent Returns](image) |
Figure A3: Hedge ratios from ABEKK, AVARMA-CCC and AVARMA-DCC models

|          | ABEKK | AVARMA-CCC | AVARMA-DCC |
|----------|-------|------------|------------|
| CAC40    | ![Graph](image1) | ![Graph](image2) | ![Graph](image3) |
| DAX      | ![Graph](image4) | ![Graph](image5) | ![Graph](image6) |
| DJIA     | ![Graph](image7) | ![Graph](image8) | ![Graph](image9) |
| FTSE100  | ![Graph](image10) | ![Graph](image11) | ![Graph](image12) |
Figure A4: Hedge ratios from ABEKK, AVARMA-CCC and AVARMA-DCC models

|        | ABEKK | AVARMA-CCC | AVARMA-DCC |
|--------|-------|------------|------------|
| MIB    | ![Hedge ratios for MIB](image) | ![Hedge ratios for MIB](image) | ![Hedge ratios for MIB](image) |
| Nikkei225 | ![Hedge ratios for Nikkei225](image) | ![Hedge ratios for Nikkei225](image) | ![Hedge ratios for Nikkei225](image) |
| TSX    | ![Hedge ratios for TSX](image) | ![Hedge ratios for TSX](image) | ![Hedge ratios for TSX](image) |
Table A1: Multivariate GARCH results for France CAC40

| Parameters       | ABEKK |             | AVARMA-CCC |             | AVARMA-DCC |             |
|------------------|-------|-------------|------------|-------------|------------|-------------|
|                  | Coeff. | Sig.        | Coeff.     | Sig.        | Coeff.     | Sig.        |
| Mean equation    |        |             |            |             |            |             |
| \(\text{con}_s\) | 0.044  | 0.515       | 0.040      | 0.554       | 0.064      | 0.324       |
| \(\text{con}_o\) | 0.016  | 0.884       | 0.061      | 0.641       | 0.087      | 0.519       |
| Variance equation|       |             |            |             |            |             |
| \(c_{ss}\)      | 0.606  | 0.000       | 0.421      | 0.005       | 0.399      | 0.051       |
| \(c_{so}\)      | 0.109  | 0.475       | 0.191      | 0.270       | 0.205      | 0.210       |
| \(a_{ss}\)      | 0.052  | 0.684       | 0.006      | 0.830       | 0.010      | 0.780       |
| \(a_{so}\)      | -0.086 | 0.005       | 0.006      | 0.475       | 0.005      | 0.596       |
| \(a_{oo}\)      | -0.148 | 0.091       | 0.045      | 0.262       | 0.046      | 0.274       |
| \(b_{ss}\)      | 0.895  | 0.000       | 0.723      | 0.000       | 0.746      | 0.000       |
| \(b_{so}\)      | 0.007  | 0.381       | 0.014      | 0.262       | 0.012      | 0.392       |
| \(b_{oo}\)      | -0.037 | 0.089       | -0.027     | 0.543       | -0.030     | 0.467       |
| \(b_{so}\)      | 0.968  | 0.000       | 0.917      | 0.000       | 0.917      | 0.000       |
| \(d_{ss}\)      | 0.483  | 0.000       | 0.328      | 0.000       | 0.299      | 0.004       |
| \(d_{so}\)      | 0.026  | 0.376       | 0.006      | 0.424       | 0.007      | 0.001       |
| \(d_{oo}\)      | 0.080  | 0.406       | 0.018      | 0.000       | 0.039      | 0.044       |
| \(\rho\)        |        |             |            |             |            |             |
| \(\theta_1\)    |        |             |            |             |            |             |
| \(\theta_2\)    |        |             |            |             |            |             |

Residual diagnostics for independent series

|                          | CAC40 | BREN| CAC40 | BREN| CAC40 | BREN|
|--------------------------|-------|-----|-------|-----|-------|-----|
| LogLik.                  | -5550.73 |     | -5573.01 |     | -5553.56 |     |
| \(Q(24)\)               | 28.341   | 19.725 | 29.384   | 18.703 | 29.098   | 22.114 |
| \(Q^2(24)\)             | 29.970   | 19.204 | 30.191   | 21.941 | 29.750   | 17.735 |
| ARCH(10)                 | 0.968   | 0.465 | 0.779   | 0.383 | 0.790   | 0.326 |

*, **, *** indicate statistical significance at 1%, 5% and 10% respectively. LogLik. is the value of the logarithmic likelihood. ARCH(10) represents the F-statistics of the ARCH test of Engle (1982) at 10th lag. \(Q(24)\) and \(Q^2(24)\) are the Ljung and Box (1978) statistics for serial correlation and conditional heteroskedasticity of the series at 24th lag. \(\text{con}_s\) and \(\text{con}_o\) denote the constants in the mean equations of stock and oil, respectively.

Table A2: Multivariate GARCH results for Germany DAX

| Parameters       | ABEKK |             | AVARMA-CCC |             | AVARMA-DCC |             |
|------------------|-------|-------------|------------|-------------|------------|-------------|
|                  | Coeff. | Sig.        | Coeff.     | Sig.        | Coeff.     | Sig.        |
| Mean equation    |        |             |            |             |            |             |
| \(\text{con}_s\) | 0.161  | 0.017       | 0.134      | 0.095       | 0.155      | 0.031       |
| \(\text{con}_o\) | 0.046  | 0.706       | 0.066      | 0.604       | 0.075      | 0.488       |
| Variance equation|       |             |            |             |            |             |
| \(c_{ss}\)      | 0.633  | 0.000       | 0.532      | 0.017       | 0.533      | 0.009       |
| \(c_{so}\)      | 0.229  | 0.161       | 0.234      | 0.184       | 0.261      | 0.121       |
| \(a_{ss}\)      | 0.156  | 0.011       | 0.009      | 0.816       | 0.013      | 0.709       |
| \(a_{so}\)      | 0.083  | 0.093       | 0.008      | 0.424       | 0.007      | 0.415       |
| \(a_{oo}\)      | -0.112 | 0.086       | 0.057      | 0.164       | 0.058      | 0.084       |
| \(b_{ss}\)      | 0.155  | 0.000       | 0.034      | 0.010       | 0.032      | 0.011       |
| \(b_{so}\)      | 0.902  | 0.000       | 0.700      | 0.000       | 0.715      | 0.000       |
| \(b_{oo}\)      | -0.010 | 0.260       | 0.020      | 0.412       | 0.017      | 0.507       |
| \(b_{so}\)      | -0.028 | 0.212       | -0.040     | 0.274       | -0.044     | 0.148       |
| \(b_{oo}\)      | 0.963  | 0.000       | 0.913      | 0.000       | 0.914      | 0.000       |
| \(d_{ss}\)      | 0.452  | 0.000       | 0.324      | 0.016       | 0.303      | 0.023       |
| \(d_{so}\)      | -0.018 | 0.642       | 0.012      | 0.646       |           |             |
| \(d_{oo}\)      | 0.071  | 0.271       | 0.012      | 0.646       | 0.012      | 0.646       |

(Contd...)
### Table A2: (Continued)

| Parameters | ABEK | AVEARMA-CCC | AVEARMA-DCC |
|------------|------|-------------|-------------|
|            | Coef. | Sig.        | Coef. | Sig. | Coef. | Sig. |
| $\rho$     | 0.299 | 0.000       | 0.077 | 0.002 | 0.080 | 0.002 |
| $\theta_1$ | ----- | -----       | ----- | ----- | ----- | ----- |
| $\theta_2$ | ----- | -----       | ----- | ----- | 0.029 | 0.013 |

Residual diagnostics for independent series

|          | DJIA | BRENT | DJIA | BRENT | DJIA | BRENT |
|----------|------|-------|------|-------|------|-------|
| LogLik.  | -5640.91 |       | -5650.37 |       | -5630.98 |       |
| $Q^2$ (24) | 19.556 | 17.736 | 20.159 | 18.879 | 20.120 | 21.941 |
| ARCH(10) | 0.778  | 0.392  | 0.806  | 0.398  | 0.775  | 0.347  |

### Table A3: Multivariate GARCH results for U.S.A. DJIA

| Parameters | ABEK | AVEARMA-CCC | AVEARMA-DCC |
|------------|------|-------------|-------------|
|            | Coef. | Sig.        | Coef. | Sig. | Coef. | Sig. |
| Mean equation |       |             |       |     |       |     |
| $\con$    | 0.101 | 0.041       | 0.098 | 0.041 | 0.118 | 0.029 |
| $\con_o$  | 0.001 | 0.995       | 0.061 | 0.635 | 0.039 | 0.779 |
| Variance Equation |       |             |       |     |       |     |
| $c_{so}$  | 0.457 | 0.000       | 0.200 | 0.033 | 0.187 | 0.018 |
| $c_{oo}$  | 0.217 | 0.307       | ----- | ----- | ----- | ----- |
| $c_{ss}$  | 0.244 | 0.337       | 0.124 | 0.538 | 0.093 | 0.614 |
| $a_{so}$  | -0.019| 0.730       | -0.080| 0.001 | -0.076| 0.000 |
| $a_{oo}$  | 0.093 | 0.002       | 0.007 | 0.212 | 0.006 | 0.089 |
| $a_{ss}$  | 0.221 | 0.000       | 0.029 | 0.642 | 0.020 | 0.638 |
| $a_{so}$  | -0.040| 0.591       | 0.024 | 0.098 | 0.022 | 0.094 |
| $b_{so}$  | 0.871 | 0.000       | 0.761 | 0.000 | 0.787 | 0.000 |
| $b_{oo}$  | 0.002 | 0.814       | 0.006 | 0.548 | 0.005 | 0.442 |
| $b_{ss}$  | -0.089| 0.106       | 0.071 | 0.466 | 0.080 | 0.291 |
| $b_{so}$  | 0.970 | 0.000       | 0.913 | 0.000 | 0.914 | 0.000 |
| $d_{so}$  | 0.529 | 0.000       | 0.407 | 0.001 | 0.366 | 0.000 |
| $d_{oo}$  | 0.015 | 0.594       | ----- | ----- | ----- | ----- |
| $d_{ss}$  | 0.119 | 0.241       | ----- | ----- | ----- | ----- |
| $d_{so}$  | 0.322 | 0.000       | 0.080 | 0.001 | 0.087 | 0.001 |
| $\rho$    | ----- | -----       | 0.126 | 0.000 | ----- | ----- |
| $\theta_1$ | ----- | -----      | ----- | ----- | 0.039 | 0.006 |
| $\theta_2$ | ----- | -----      | ----- | ----- | 0.950 | 0.000 |

Residual Diagnostics for Independent Series

|          | DJIA | BRENT | DJIA | BRENT | DJIA | BRENT |
|----------|------|-------|------|-------|------|-------|
| LogLik.  | -5215.86 |       | -5240.68 |       | -5216.26 |       |
| $Q^2$ (24) | 29.930 | 21.408 | 30.625 | 21.681 | 30.339 | 24.338 |
| ARCH(10) | 0.667  | 0.283  | 0.570  | 0.299  | 0.548  | 0.264  |

Same as Table A1
Table A4: Multivariate GARCH results for U.K. FTSE100

| Parameters | ABEKK | A VARMA-CCC | A VARMA-DCC |
|------------|-------|-------------|-------------|
|            | Coeff. | Sig.        | Coeff. | Sig.        | Coeff. | Sig.        |
| Mean equation |       |             |       |             |       |             |
| con        | -0.005 | 0.932       | -0.024 | 0.678       | -0.002 | 0.962       |
| conialog    | -0.033 | 0.806       | 0.047  | 0.739       | 0.045  | 0.753       |
| Variance equation |       |             |       |             |       |             |
| c          | 0.487  | 0.000       | 0.300  | 0.488       | 0.242  | 0.017       |
| c            | 0.161  | 0.389       | -----  | -----       | -----  | -----       |
| c            | 0.341  | 0.005       | 0.140  | 0.591       | 0.100  | 0.597       |
| a            | 0.076  | 0.135       | -.057  | 0.417       | -.051  | 0.042       |
| a            | -.069  | 0.001       | 0.003  | 0.580       | 0.001  | 0.708       |
| a            | -.191  | 0.021       | 0.043  | 0.756       | 0.015  | 0.840       |
| a            | 0.136  | 0.003       | 0.030  | 0.099       | 0.026  | 0.064       |
| b            | 0.889  | 0.000       | 0.718  | 0.151       | 0.790  | 0.000       |
| b            | 0.007  | 0.218       | 0.013  | 0.812       | 0.007  | 0.458       |
| b            | -.034  | 0.302       | 0.021  | 0.838       | 0.045  | 0.557       |
| b            | 0.965  | 0.000       | 0.913  | 0.000       | 0.918  | 0.000       |
| b            | 0.515  | 0.000       | 0.421  | 0.419       | 0.342  | 0.002       |
| b            | 0.004  | 0.852       | -----  | -----       | -----  | -----       |
| d            | 0.087  | 0.435       | -----  | -----       | -----  | -----       |
| d            | 0.300  | 0.000       | 0.076  | 0.003       | 0.079  | 0.006       |
| d            | -----  | -----       | 0.200  | 0.000       | -----  | -----       |
| ρ            | -----  | -----       | 0.036  | 0.072       | 0.952  | 0.000       |
| θ1          | -----  | -----       | -----  | -----       | -----  | -----       |
| θ2          | -----  | -----       | -----  | -----       | -----  | -----       |

Residual diagnostics for independent series

| FTSE100 | BREAT | FTSE100 | BREAT | FTSE100 | BREAT |
|---------|-------|---------|-------|---------|-------|
| LogLik. | -5293.86 | -5315.00 | -5294.07 | 20.217 | 17.309 |
| Q (24)  | 20.217 | 17.309 | 20.808 | 20.319 | 20.589 | 21.677 |
| Q^2 (24) | 26.659 | 18.004 | 24.457 | 19.809 | 24.182 | 25.662 |
| ARCH(10) | 0.004 | 0.060 | 0.005 | 0.042 | 0.074 | 0.022 |

Same as Table A1

Table A5: Multivariate GARCH results for Italy MIB

| Parameters | ABEKK | A VARMA-CCC | A VARMA-DCC |
|------------|-------|-------------|-------------|
|            | Coeff. | Sig.        | Coeff. | Sig.        | Coeff. | Sig.        |
| Mean equation |       |             |       |             |       |             |
| con        | 0.048  | 0.558       | 0.057  | 0.497       | 0.081  | 0.319       |
| conialog    | 0.045  | 0.738       | 0.107  | 0.412       | 0.155  | 0.260       |
| Variance equation |       |             |       |             |       |             |
| c          | 0.413  | 0.001       | 0.257  | 0.092       | 0.243  | 0.099       |
| c            | -.216  | 0.089       | -----  | -----       | -----  | -----       |
| c            | 0.000  | 1.000       | 0.469  | 0.122       | 0.488  | 0.187       |
| a            | 0.193  | 0.002       | 0.073  | 0.144       | 0.077  | 0.058       |
| a            | 0.054  | 0.002       | 0.013  | 0.134       | 0.013  | 0.073       |
| a            | 0.194  | 0.000       | 0.094  | 0.054       | 0.101  | 0.027       |
| a            | -.097  | 0.034       | 0.043  | 0.014       | 0.040  | 0.041       |
| b            | 0.933  | 0.000       | 0.877  | 0.000       | 0.882  | 0.000       |
| b            | 0.005  | 0.319       | -.018  | 0.143       | -.018  | 0.082       |
| b            | -.016  | 0.174       | -.095  | 0.011       | -.093  | 0.005       |
| b            | 0.965  | 0.000       | 0.906  | 0.000       | 0.905  | 0.000       |
| b            | 0.341  | 0.000       | 0.075  | 0.057       | 0.070  | 0.032       |
| d            | -.005  | 0.871       | -----  | -----       | -----  | -----       |
| d            | -.0074 | 0.348       | -----  | -----       | -----  | -----       |
| d            | 0.349  | 0.000       | 0.071  | 0.004       | 0.071  | 0.013       |

(Contd...)
Table A5: (Continued)

| Parameters | ABEKK | AVARMA-CCC | AVARMA-DCC |
|------------|-------|------------|------------|
|            | Coeff.| Sig.       | Coeff.     | Sig.       | Coeff.     | Sig.       |
| \( \rho \)  | ----- | -----      | 0.194      | 0.000      | -----      | -----      |
| \( \theta_1 \) | ----- | -----      | -----      | -----      | 0.026      | 0.033      |
| \( \theta_2 \) | ----- | -----      | -----      | -----      | 0.968      | 0.000      |

Residual diagnostics for independent series

|             | MIB   | BRENT | MIB   | BRENT | MIB   | BRENT |
|-------------|-------|-------|-------|-------|-------|-------|
| LogLik.     | −5658.23 |       | −5678.93 |       | −5662.72 |       |
| \( Q(24) \) | 31.408 | 19.609 | 33.197 | 18.170 | 32.796 | 22.433 |
| \( Q^2(24) \) | 12.245 | 25.056 | 13.992 | 31.868 | 15.049 | 25.207 |
| ARCH(10)    | 0.331 | 0.578 | 0.453 | 0.346 | 0.480 | 0.356 |

Same as Table A1

Table A6: Multivariate GARCH results for Japan Nikkei225

| Parameters | ABEKK | AVARMA-CCC | AVARMA-DCC |
|------------|-------|------------|------------|
|            | Coeff.| Sig.       | Coeff.     | Sig.       | Coeff.     | Sig.       |
| \( \rho \)  | ----- | -----      | -----      | -----      | -----      | -----      |
| \( \theta_1 \) | ----- | -----      | -----      | -----      | -----      | -----      |
| \( \theta_2 \) | ----- | -----      | -----      | -----      | -----      | -----      |

Residual diagnostics for independent series

|             | Nikkei225 | BRENT | Nikkei225 | BRENT | Nikkei225 | BRENT |
|-------------|-----------|-------|-----------|-------|-----------|-------|
| LogLik.     | −5676.83  |       | −5682.39  |       | −5673.51  |       |
| \( Q(24) \) | 37.814**  |       | 37.263**  |       | 37.228**  |       |
| \( Q^2(24) \) | 15.071   | 29.824 | 15.964    | 29.662 | 15.888    | 26.600 |
| ARCH(10)    | 0.415     | 0.670 | 0.540     | 0.496 | 0.532     | 0.475 |

Same as Table A1
Table A7: Multivariate GARCH results for Canada TSX

| Parameters            | ABEKK     |          | AVARMA-CCC |          | AVARMA-DCC |          |
|-----------------------|-----------|----------|------------|----------|------------|----------|
|                       | Coeff.    | Sig.     | Coeff.     | Sig.     | Coeff.     | Sig.     |
| Mean equation         |           |          |            |          |            |          |
| $\omega$              | 0.108     | 0.020    | 0.117      | 0.008    | 0.125      | 0.025    |
| $\mu$                 | 0.016     | 0.893    | 0.085      | 0.447    | 0.101      | 0.524    |
| Variance equation     |           |          |            |          |            |          |
| $\alpha_{1}$          | 0.450     | 0.000    | 0.154      | 0.077    | 0.143      | 0.606    |
| $\alpha_{2}$          | 0.371     | 0.021    | -----      | -----    | -----      | -----    |
| $\alpha_{3}$          | 0.000     | 1.000    | 0.123      | 0.214    | 0.108      | 0.317    |
| $\alpha_{4}$          | 0.052     | 0.691    | 0.228      | 0.031    | 0.198      | 0.471    |
| $\alpha_{5}$          | 0.007     | 0.921    | 0.015      | 0.265    | 0.015      | 0.333    |
| $\beta_{1}$           | 0.868     | 0.000    | 0.656      | 0.000    | 0.724      | 0.451    |
| $\beta_{2}$           | 0.007     | 0.265    | 0.025      | 0.196    | 0.018      | 0.877    |
| $\beta_{3}$           | 0.106     | 0.016    | -0.204     | 0.194    | -0.147     | 0.677    |
| $\gamma_{1}$          | 0.981     | 0.000    | 0.949      | 0.000    | 0.945      | 0.000    |
| $\gamma_{2}$          | 0.470     | 0.000    | 0.280      | 0.005    | 0.229      | 0.738    |
| $\delta_{1}$          | 0.011     | 0.602    | -----      | -----    | -----      | -----    |
| $\delta_{2}$          | 0.230     | 0.004    | -----      | -----    | -----      | -----    |
| $\gamma_{3}$          | 0.284     | 0.000    | 0.057      | 0.005    | 0.057      | 0.134    |
| $\rho$                | -----     | -----    | 0.322      | 0.000    | -----      | -----    |
| $\theta_{1}$          | -----     | -----    | -----      | -----    | 0.020      | 0.263    |
| $\theta_{2}$          | -----     | -----    | -----      | -----    | 0.973      | 0.000    |

Residual diagnostics for independent series

|               | TSX |               | BRENT |               | TSX |               | BRENT |               | TSX |               | BRENT |
|---------------|-----|---------------|-------|---------------|-----|---------------|-------|---------------|-----|---------------|-------|
| LogLik.       | -5213.55 |               | -5233.95 |               | -5221.00 |               |       |               |       |               |       |
| $Q(24)$       | 29.321 | 24.687        | 29.316 | 28.649        | 29.737 | 28.765        |       |               |       |               |       |
| $Q^2(24)$     | 15.953 | 21.231        | 16.044 | 24.521        | 14.674 | 17.912        |       |               |       |               |       |
| ARCH(10)      | 0.639 | 0.480         | 0.530 | 0.309         | 0.478 | 0.238         |       |               |       |               |       |

Same as Table A1