A Non-stationary Random Vibration Analysis of Vehicle-bridge Coupling under Road Surface Coherence Excitation

Jian Shen*, Lang Wu

1School of Civil Engineering and Architecture, Jiangxi Science & Technology Normal University, Fenglin Avenue 605#, Nanchang, Jiangxi P.R. China 330013
*Corresponding author’s e-mail: 894282069@qq.com

Abstract: Based on the pseudo excitation method and the uniform excitation input model of road roughness, the 3D vehicle-bridge coupling non-stationary random vibration model of vehicle variable speed driving is established. The effects of vehicle initial velocity and acceleration on the vertical displacement, root mean square value of acceleration and spectrum characteristics of bridge mid-span are studied. Based on the road roughness power spectrum suggested by national standard GB7031, the pseudo excitation method is used to construct a 3D vehicle-bridge coupling random vibration model to study the influence of the spatial effect of the spectrum input excitation on vehicle-bridge coupling vibration, vehicle vibration and the spectrum characteristics of vehicle-bridge system.

1. Introduction
Road roughness is one of the main factors affecting vehicle-bridge coupling vibration. The road undulation excitation can be regarded as a Gaussian stationary random process [1]. When a vehicle travels on a bridge with variable speed, it will cause the coupling non-stationary random vibration [2-3]. In engineering practice, not only response standard deviation but also mean and response maximum are cared [4].

2. 3D vehicle-bridge coupling vibration model
The three-axle dump truck is selected as the research object, and the vehicle prototype is shown as figure 1. The vehicle vibration equation is expressed as formula (1).

\[ M_v \ddot{z} + C_v \dot{z} + K_v z = F_{\text{int}} \]  \hspace{1cm} (1)

\( M_v \) - mass matrix, \( C_v \) - damp matrix, \( K_v \) - stiffness matrix, \( F_{\text{int}} \) - Inertial load column vector, \( z = [z_1, z_2, z_3, z_4, z_5, z_6, \theta_b, \Phi]^T \) - Vehicle vibration freedom column vector.

In order to accurately analyze the vibration response of each key point, the bridge is simplified into a finite element model of the spatial plate and shell structure. The modal synthesis technique is introduced in bridge modeling to reduce the matrix dimension of vehicle-bridge coupling calculation. The bridge structure is discretized and analyzed by finite element method.

\[ M_b \ddot{y} + C_b \dot{y} + K_b y = -F_{\text{int}} - F_g \]  \hspace{1cm} (2)

\( F_g \) - load vector of each wheel acting on bridge deck caused by vehicle gravity, \( y \) - freedom vector of element node.
When the bridge structure has employed Rayleigh damping, formula (2) can be rewritten as formula (3) according to the mode decomposition method.

$$ I \cdot \ddot{q} + X \cdot \dot{q} + \Omega \cdot q = - \Phi^T (F_{v}^{int} - F_{g}) $$

$$ I = \begin{bmatrix} \ddots & \ddots \\ \ddots & 1 \end{bmatrix}_{r \times r},
X = \begin{bmatrix} \ddots \\ 2\xi_i \omega_i \\ \ddots \end{bmatrix}_{r \times r},
\Omega = \begin{bmatrix} \ddots \\ \omega_i^2 \\ \ddots \end{bmatrix}_{r \times r} $$

$\Phi$- modal vector matrix, $\xi_i$- frequency damping ratio of each order, $\omega_i$- natural vibration frequency of bridge

When the vehicle-bridge coupling vibration equation is established, the vehicle and the bridge are regarded as two independent subsystems respectively. Assuming that the contact between the wheel and the bridge deck is always maintained by satisfying two conditions, the displacement coordination of the contact between the wheel and the bridge deck and the equilibrium condition of interaction force, and considering the road undulation excitation, the inertial force of the vehicle acting on the bridge is expressed as formula (4).

$$ F_{bv}^{int} = \sum_{i=1}^{n_l} N_i \cdot F_{v}^{int} = \sum_{i=1}^{n_l} N_i \cdot (F_{v}^{int} + F_{g}) $$

The load vector of a single wheel acting on a bridge is indicated as formula (5).

$$ F_{v}^{int} = k_l (-N \cdot \Phi \cdot q - r + z_l) + c_l (-v \cdot N \cdot \Phi \cdot q - N \cdot \Phi \cdot z - \dot{z}_l + \dot{z}_l) + F_{g} $$

Without considering the influence of road undulation excitation, the vibration equation of vehicle-bridge coupling system is regarded as formula (6).

$$ M_{bv} \dddot{u} + C_{bv} \ddot{u} + K_{bv} u' = F_{g} $$

$M_{bv}$-generalized mass matrix, $C_{bv}$-generalized damp matrix, $K_{bv}$-generalized stiffness matrix, $u'$-generalized coordinate rigid displacement vector, $\ddot{u}'$-generalized coordinate vertical velocity vector, $\dddot{u}'$-generalized coordinate cartesian acceleration vector.

3. Non-stationary stochastic response solved by pseudo excitation method

When considering only the random excitation $F_{w}$ of road roughness, the vibration equation of the vehicle-bridge coupling time-varying system can be written as formula (7).

$$ M_{bv} \dddot{u}' + C_{bv} \ddot{u}' + K_{bv} u'' = F_{w} $$
\( F_w = F_{w1} + F_{w2} = \begin{bmatrix} T_{b0} r(t) \\ T_{v0} r(t) \end{bmatrix}_{(r+9)} + \begin{bmatrix} T_{b1} r(t) \\ T_{v1} r(t) \end{bmatrix}_{(r+9)} \) \tag{8}

\[
T_{b0} = \begin{bmatrix}
\phi_1 N_1 k_{t1} & \phi_1 N_2 k_{t2} & \ldots & \phi_1 N_6 k_{t6} \\
\phi_2 N_1 k_{t1} & \phi_2 N_2 k_{t2} & \ldots & \phi_2 N_6 k_{t6} \\
\phi_r N_1 k_{t1} & \phi_r N_2 k_{t2} & \ldots & \phi_r N_6 k_{t6} \\
\phi_1 N_1 c_{t1} & \phi_1 N_2 c_{t2} & \ldots & \phi_1 N_6 c_{t6} \\
\phi_2 N_1 c_{t1} & \phi_2 N_2 c_{t2} & \ldots & \phi_2 N_6 c_{t6} \\
\phi_r N_1 c_{t1} & \phi_r N_2 c_{t2} & \ldots & \phi_r N_6 c_{t6} 
\end{bmatrix}_{r \times 6}
\]

\[
T_{b1} = \begin{bmatrix}
k_{t1} & k_{t2} & \ldots & k_{t6} \\
r(t-t_1) & r(t-t_2) & \ldots & r(t-t_6) 
\end{bmatrix}_{6 \times 6}
\]

\[
T_{v0} = \begin{bmatrix}
c_{t1} & c_{t2} & \ldots & c_{t6} 
\end{bmatrix}_{6 \times 1}
\]

\[
T_{v1} = \begin{bmatrix}
c_{t1} & c_{t2} & \ldots & c_{t6} 
\end{bmatrix}_{6 \times 1}
\]

4. **pseudo excitation load structure of road surface spectrum coherence effect**

Considering the spatial effect of vehicle vibration, the random spectral density matrix of wheel coherent road surface can be expressed as formula (9).

\[
G_{q1}(\omega) = V^* S \rho S V \
\rho = QQ^T
\]

\[
G_{q1}(\omega) = V^* S Q Q^T S V = P^* P^T
\]

\[
P = VSQ
\]

\( V \) - wheel time effect function matrix, \( \rho \) - coherent function matrix between wheels, \( G_{q1}(\omega) \) - semidefinite Hermitan matrix, \( \rho \) - real symmetric positive definite matrix, \( Q \) - real matrix, \( Q^T \) - transposed matrix.

Under the input excitation of road roughness in the vehicle-bridge coupling vibration model, the input power spectrum amplitude of road roughness at each wheel point is equal at any circular frequency.

\[
P = \sqrt{S_{rr}(\omega)} \cdot V \cdot I \cdot Q = \sqrt{S_{rr}(\omega)} \cdot V \cdot Q \tag{10}
\]

The pseudo excitation loads of displacement and acceleration terms can be formed as formula (11).

\[
F_{w1}(\omega, t) = \begin{bmatrix} T_{b0} r(t) \\ T_{v0} r(t) \end{bmatrix}_{(r+dof) \times 1} V \cdot Q \cdot I_6 \sqrt{S_{rr}(\omega)} e^{i \omega t}
\]

\[
F_{w2}(\omega, t) = \begin{bmatrix} T_{b1} r(t) \\ T_{v1} r(t) \end{bmatrix}_{(r+dof) \times 1} V \cdot Q \cdot I_6 \sqrt{S_{rr}(\omega)} e^{i \omega t}
\]

The pseudo excitation loads can be constructed as formula (12).

\[
F_{w}(\omega, t) = F_{w1}(\omega, t) + i F_{w2}(\omega, t) = \begin{bmatrix} T_{b0} r(t) \\ T_{v0} r(t) \end{bmatrix} + i \omega \begin{bmatrix} T_{b1} r(t) \\ T_{v1} r(t) \end{bmatrix} V \cdot Q \cdot I_6 \sqrt{S_{rr}(\omega)} e^{i \omega t} \tag{12}
\]

5. **Verification of non-stationary random vehicle-bridge coupling vibration model**

A prestressed concrete simply supported t-beam bridge is taken as a prototype. The bridge superstructure consists of six T beams, each T beam being 2m high and 2.10m wide. The bridge deck paving layer is 10cm thick and made of C50 concrete. The ANSYS software is used to establish the 3D finite element model of the bridge. The slab unit is used to simulate the bridge deck pavement and the main beam is
simulated by the plate-shell solid element. The elastic modulus of concrete material for main beam section is 34.5GPa, the density is 2600Kg/m$^3$, and the Poisson's ratio is 0.167. The first ten natural vibration frequencies and modes of simply supported beam bridges are extracted for numerical calculation. A 33T dump truck is selected for numerical simulation to analyze the vertical vibration response of the assembled simply supported beam bridge. The uneven power spectral density curve of road roughness suggested by national standard (GB7031-2005) is adopted. The roughness curves of road at grade A, B and C are obtained by sine function superposition method. The analysis results are shown as Fig.2 and Fig.3.

Figure 2 The displacement and acceleration RMS curve of mid-span

Figure 3 The PSD of acceleration at bridge deck
With the increase of vehicle acceleration, the spatial frequency of both road excitation and vehicle-bridge coupling resonance is widened and the resonance power spectrum amplitude decreases. There is a deviation between the resonant frequency caused by uniform speed and that caused by uniform acceleration. The resonance frequency width of uniform driving is small and its peak value is more obvious than that of uniform acceleration driving. When the vehicle accelerates uniformly, the power spectrum of the acceleration under the beam does not show the resonance frequency peak at the low frequency. While the vehicle travelling at a uniform speed, a small resonance frequency peak appears at the bridge base frequency of 4Hz.

The power spectra of bridge panel and road surface show resonance at the low spatial frequency, being lower at uniform acceleration. In addition, the peak value of resonance spectrum is more intensive than that of beam bottom. When the vehicle accelerates uniformly, the acceleration at the bottom of the beam is resonated with the road frequency at 12Hz, and the vehicle resonates at 14Hz. The width and magnitude of coupling resonance frequency is directly affected by the vehicle acceleration.

6. Conclusion
Considering the spatial effect of the road undulation excitation input, the pseudo excitation method is used to establish a 3D non-stationary random vibration model of vehicle-bridge coupling when the vehicle is at variable speed.

After having studied the influence of its initial velocity and acceleration on the vibration response of bridge and vehicle caused by road roughness in the coupling vibration system, it is obtained that the maximum velocity of the vehicle is the main factor that determines the maximum displacement and acceleration response at midspan.

In addition, based on the analysis of the spatial effect of road surface spectrum and the influence of its coherence function on the vehicle-bridge coupling vibration response by taking a three-span prestressed concrete continuous beam bridge as the research object, it is found out that the spatial effect of each wheel’s input excitation cannot be ignored when the influence of vehicle-bridge coupling vibration caused by road roughness on vehicle body vibration is studied.

Acknowledgments
Science and Technology Foundation of Jiang’xi Educational Committee (GJJ170667)

References
[1] Fu C. (2016) Dynamic behavior of a simply supported bridge with a switching crack subjected to seismic excitations and moving trains. Engineering Structures, 110:59-69
[2] Zhai W. M., Wang K. Y. (2009) Fundamentals of vehicle-track coupled dynamics. Vehicle System Dynamics, 47(11):1349-1376
[3] Gonzalez. A., Owley. C. (2008) A general solution to the identification of moving vehicle forces on a bridge. International journal for numerical methods in engineering, 75:350-354
[4] Kong. X. Cai. C. S. (2016) Scour Effect on bridge and vehicle responses under bridge-vehicle-wave interaction. Jounal of bridge engineering, 21(4):04015083-16