Correlation function intercepts for $\tilde{\mu}, \tilde{q}$-deformed Bose gas model implying effective accounting for interaction and compositeness of particles

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In the recently proposed two-parameter $\tilde{\mu}, \tilde{q}$-deformed Bose gas model [Ukr. J. Phys. 58, 1171 (2013), arXiv:1312.1573] aimed to take effectively into account both compositeness of particles and their interaction, the $\tilde{\mu}, \tilde{q}$-deformed virial expansion of the equation of state (EOS) was obtained. In this paper we further explore the $\tilde{\mu}, \tilde{q}$-deformation, namely the version of $\tilde{\mu}, \tilde{q}$-Bose gas model involving deformed distributions and correlation functions. In the model, we explicitly derive the one- and two-particle deformed distribution functions and the intercept of two-particle momentum correlation function. The results are illustrated by plots, and the comparison with known experimental data on two-pion correlation function intercepts extracted in RHIC/STAR experiments is given.

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I. INTRODUCTION

Deformed Bose gas models based on a set of identical deformed (nonlinear) oscillators, or on deformed thermodynamic relations provide nonlinear extension of standard Bose gas model which finds applications to physical systems with one or more factors of non-ideality [1-5]. In general, the effective description or modeling of essentially nonideal (nonlinear) systems usually is performed by means of reexpressing of the physical quantities of the initial complicated system in terms of the analogous quantities of deformed model. Such two factors as composite structure of particles of a gas and the interaction between them are of main interest for us here. Concerning the compositeness of particles, let us mention the works [1, 6-8] where $q$-deformed oscillators were applied for effective description of composite particles (like nuclei, nucleons, mesons, excitons, cooperons, atoms, molecules). Their Bose-Einstein condensation was also studied [9]. It was shown in [10, 11] that two-fermionic (and two-bosonic) composite bosons can be algebraically realized on their Fock states by deformed oscillator algebra with the quadratically nonlinear deformation structure function (DSF) $\varphi_{\tilde{\mu}}(N) = (1 + \tilde{\mu}) N - \tilde{\mu} N^2$, with $N$ the number operator, and discrete deformation parameter $\tilde{\mu} = 1/m$ involved. On the other hand, $q$-deformation of Arr-Koon type [12] based on the DSF equal to the “$q$-bracket” $[N]_q = \frac{qN + 1}{q - 1}$ was used for the effective description [13] of thermodynamics aspects (e.g., virial expansion) of Bose gas with interaction. So far, two aspects were treated separately, adhering to different methods and contexts. However, the task naturally arises of treating jointly: i) compositeness of particles linked, through the realization, with quadratic or $\tilde{\mu}$-deformation; ii) the interaction between particles modeled by $q$-deformation. We may expect that combining these two types of deformation into single one will reproduce effectively, in a unified manner, some specific features inherent to the thermodynamic or statistical quantities of more realistic systems of particles possessing both interaction and compositeness. The simplest variant of such unification is their functional composition or $\tilde{\mu}, q$-deformation. Of course, at the moment the ascription of the meaning of deformation parameters $\tilde{\mu}$ and $q$ as responsible respectively for the compositeness and the interaction is rather formal, and the detailed consistent analysis providing a reformulation in the deformed model terms, including the relation with the parameters of interaction or compositeness, is not completed to sufficient extent.

First steps to the (microscopics of) effective taking of interaction and compositeness jointly into account are made by introducing the $\tilde{\mu}, q$-deformed Bose gas model [3, 4] based on deforming the thermodynamics. Namely, in [3] the $\tilde{\mu}, q$-deformed Bose gas model was realized through deforming the total mean number of particles or the partition function by means of the deformed analog of the derivative $\frac{d}{dz} (z$ is fugacity) and use of the “hybrid” (combined) DSF $\varphi_{\tilde{\mu}, q}(z) \equiv \varphi_{\tilde{\mu}}(D_q) = (1 + \tilde{\mu}) D_q - \tilde{\mu} D_q^2$. In [4] the deformation virial expansion was studied. In the sequel to [3], the relation of the obtained virial coefficients of the $(\tilde{\mu}, q)$-deformed model (dependent explicitly on the deformation parameters $\tilde{\mu}$ and $q$) with scattering data of some interaction was explored [4], and the arising unusual temperature dependence of $\tilde{\mu}$ and $q$ discussed and justified.

The version of $\tilde{\mu}, q$-deformed Bose gas model consid-

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er herein uses for its definition the same DSF \( \varphi_{\tilde{\mu},q} \) as in [3] (composed of quadratically nonlinear SF and the \( q \)-deformed one). However, this time the DSF is exploited as the operator function of \( \bar{N} \) for obtaining one- and two-particle distribution functions. The resulting model is also called \((\tilde{\mu}, q)\)-deformed model.

Below we focus on one- and two-particle distributions and the related momentum correlation function intercept, and calculate them. Let us quote some preceding activity [14–21] on the (intercepts of) correlation functions for various deformed Bose gas models. The knowledge of the correlation function intercept is useful for an application in the effective modeling of non-Bose like properties of data on pionic intercepts extracted in RHIC and LHC experiments [22–29]. That is, as a particular physical system for which the considered \( \tilde{\mu}, q \)-deformed intercepts can be applied we mean the \( \pi \)-mesonic gas created in relativistic heavy-ion collisions. Obviously this is the gas of composites (quark-antiquark bound states) which moreover undergo interaction. Remark however that there are some further complications: i) this is non-equilibrium system, ii) the mentioned “interaction” may not reduce to only simple \( \pi + \pi \)-interaction, and iii) the non-ideality factors may play the prevailing role at the stage of \( \pi \)-meson formation (memory effects etc.). Nevertheless, for the results of the studied model we will make a comparison with some of the available experimental data on \( \pi \)-mesonic correlation functions. Note that, using other deformed Bose gas models, the comparison with experimental data on the \( \pi \)-mesonic correlation function intercepts of 2nd (and 3rd) order was considered in some earlier papers, e.g. in [19, 29, 30].

II. DEFORMED BOSE GAS MODEL WITH \( \tilde{\mu}, q \)-DEFORMED PARTITION FUNCTION: VIRIAL EXPANSION OF EOS

In this section we give an overview of the results from [3] where basing on [10, 11] and on [13] the specially designed two-parameter \( \tilde{\mu}, q \)-deformed Bose gas model capable to effectively describe the interacting gas of composite bosons was constructed. As we are interested in the effective description of interaction and compositeness effects existing in realistic gases, the model from [3, 4] can serve as the “link” to the study of more microscopical aspects (including the involved parameters), especially in view of [4]. As mentioned, the corresponding DSF \( \varphi_{\tilde{\mu},q} \) in (1) which determines the deformed Bose gas model to be considered in this section is the combination of previously studied ones, see [13] and the works [10, 11]. For the \( \tilde{\mu}, q \)-deformed Bose gas model [3], the corresponding deformed virial expansion was obtained along with first five virial coefficients, and interpreted as the virial expansion accounting for both the interaction of (composite) bosons and the very their compositeness. The thermodynamic relations for the deformed Bose gas model including partition function and the equation of state (EOS), which were used in the process of derivation of its virial expansion, were obtained by using the \( \bar{\mu}, q \)-generalization (1) of the Jackson derivative, adjusted for the concerned unifying deformation (note that in [31] similar procedure of deformation was applied within differently motivated deformed model, the \( \bar{\mu} \)-Bose gas model).

a. Compositeness aspects. The creation and annihilation operators for composite bosons in the second quantization scheme are constructed [1, 10, 32] from two-fermion (or two-boson) operators as

\[
A^{\dagger}_{\alpha} = \sum_{\mu\nu} \Phi^{\mu\nu}_{\alpha} a^\dagger_{\mu} b^\dagger_{\nu}, \quad A_{\alpha} = \sum_{\mu\nu} \Phi^{\mu\nu}_{\alpha} b_{\mu} a_{\nu}.
\]

Here \( a^\dagger_{\mu}, b^\dagger_{\nu}, \) and \( a_{\mu}, b_{\nu} \) are the creation and annihilation operators for the constituents; the matrices \( \Phi^{\mu\nu}_{\alpha} \) determine the composite boson wavefunction. The operators \( A_{\alpha} \) and \( A^{\dagger}_{\beta} \) obey the relation \( [A_{\alpha}, A^{\dagger}_{\beta}] = \delta_{\alpha\beta} - \Delta_{\alpha\beta} \) with

\[
\Delta_{\alpha\beta} = \sum_{\mu\nu}(\Phi_{\beta}^{\mu\nu})^{n'\mu'\nu'} a^\dagger_{\mu'} a_{\mu} + \sum_{\mu\nu}(\Phi_{\beta}^{\mu\nu})^{n'\mu'\nu'} b^\dagger_{\nu'} b_{\nu}
\]

(\( \Delta_{\alpha\beta} \) reflects a deviation from pure bosonic case).

The many-body system of composite (two-fermion or two-boson) quasi-bosons with certain composite wave function can be realized at the operator level, see [10, 11], by a deformed Bose gas model with quadratic DSF \( (\tilde{\mu} \geq 0) \)

\[
\varphi_{\tilde{\mu}}(N) \equiv [N]_{\tilde{\mu}} = \begin{cases} (1+\tilde{\mu})N - \tilde{\mu} N^2 & \text{(two-fermion),} \\ (1-\tilde{\mu})N + \tilde{\mu} N^2 & \text{(two-boson),} \end{cases}
\]

(2)

involving the discrete deformation parameter \( \tilde{\mu} = 1/m, \) \( m = 1, 2, \ldots \). Then, the gas of composite bosons can be treated (at least on the states) as the corresponding gas of deformed bosons. Note that such a realization of composite bosons by deformed oscillators is of importance in quantum information theory, as it was demonstrated in [33, 34] where the characteristics of bipartite entanglement were expressed directly through the deformation parameter \( \tilde{\mu} \).

b. Deformed Bose gas accounting for compositeness of particles. So, the \( \tilde{\mu} \)-deformed bosons with the quadratic DSF of the form (2) do realize [10, 11] the two-fermion (or two-boson) composite Bose-like particles. For the deformed thermodynamics of \( \tilde{\mu} \)-Bose gas, the deformed virial expansion of the EOS has been derived [3] along with the first five virial coefficients, by using the \( \tilde{\mu} \)-deformed derivative (compare with (2)):

\[
D_{\bar{z}}(\tilde{\mu}) = ((1 + \kappa \mu) n - \kappa \mu n^2) z^{n-1}, \quad \kappa = \pm 1.
\]

In the \( \tilde{\mu} \)-deformed picture, similarly to [31], we obtain the mean number of particles \( N(\tilde{\mu}) \) (with \( Z \) denoting non-deformed partition function) as

\[
N(\tilde{\mu}) = \left[z \frac{d}{d\bar{z}} \right]_{\tilde{\mu}} \ln Z \equiv \bar{z} D(\tilde{\mu}) \ln Z = \frac{V}{\lambda^3} \sum_{n=1}^{\infty} [n]_{\tilde{\mu}} z^n
\]

(3)
or, using the notation \( v = \frac{N}{\lambda^3} \), as

\[
\frac{\lambda^3}{v} = \sum_{n=1}^{\infty} \frac{[n]_{\tilde{\mu}}}{n^{3/2}} \zeta^n.
\]

In (3), (4) and below, the \( \tilde{\mu} \)-bracket means: \( [X]_{\tilde{\mu}} \equiv (1 + \tilde{\mu})X - \tilde{\mu}X^2 \).

The \( \tilde{\mu} \)-deformed partition function \( Z^{(\tilde{\mu})} \) is then obtained from (3) by applying the inversion:

\[
\ln Z^{(\tilde{\mu})} = \left( \frac{d}{dz} \right)^{-1} N^{(\tilde{\mu})} = \left( \frac{d}{dz} \right)^{-1} D^z_{\tilde{\mu}} \ln Z.
\]

As result, the deformed EOS takes the form

\[
\frac{PV}{k_B T} = \ln Z^{(\tilde{\mu})} =
V \left( \frac{2[2]_{\tilde{\mu}}}{2!/2} z^2 + \frac{[3]_{\tilde{\mu}}}{3!/2} z^3 + \frac{[4]_{\tilde{\mu}}}{4!/2} z^4 + \frac{[5]_{\tilde{\mu}}}{5!/2} z^5 + \ldots \right).
\]

More informative is the virial expansion of EOS that involves the series in powers of \( \frac{\lambda^3}{v} \). The desired virial expansion [3] modeling that of (non-interacting) gas of composite bosons and depending on the parameter \( \tilde{\mu} \) reads

\[
\frac{PV}{k_B T} = \sum_{k=1}^{\infty} V_k(\tilde{\mu}) \left( \frac{\lambda^3}{v} \right)^{k-1} =
-1 - \frac{2[2]_{\tilde{\mu}}}{2!/2} \frac{\lambda^3}{v} + \left( \frac{2[3]_{\tilde{\mu}}}{3!/2} - \frac{2[3]_{\tilde{\mu}}}{3!/2} \right) \left( \frac{\lambda^3}{v} \right)^2 +
\left( -\frac{3[4]_{\tilde{\mu}}}{4!/2} + \frac{[2]_{\tilde{\mu}}[3]_{\tilde{\mu}}}{2!/2!} - \frac{5[3]_{\tilde{\mu}}}{5!/2} \right) \left( \frac{\lambda^3}{v} \right)^3 + \ldots
\]

Since the second virial coefficient \( V_2 = -\frac{2[2]_{\tilde{\mu}}}{2!/2} = -\frac{1}{\lambda^3} \) vanishes at \( \tilde{\mu} \to 1 \), such a nullifying can be interpreted as (due to) mutual compensation, at \( \tilde{\mu} = 1 \), of the compositeness effects against the quantum-statistical many-particle effects, so that the quantum gas of composite bosons then behaves like a classical gas of pointlike particles, at least to the first order in \( \lambda^3/v \). This fact, on the other hand, implies that the compositeness effects measured by \( \tilde{\mu} \) can be interpreted as another amount of effective interaction (between quasibosons) contributing to \( V_2 \) similarly, though with opposite sign, to the bosonic quantum statistical many-particle effects. Similar analysis may be applied to higher virial coefficients \( V_3, V_4 \), etc.

d. **Account for the interaction between (elementary) Bose particles.** The interpretation [13] of interacting many-boson systems in terms of \( q \)-deformed oscillators (\( q \)-bosons) is based on the assumption that a suitably chosen \( q \)-deformed thermodynamic or statistical relation for non-interacting structureless system can be applied, within some approximation, to model interacting many-boson system with certain interaction. In this sense, an effective description of the interacting gas of Bose particles was dealt with in [13] by means of \( q \)-deformation with the structure function \( \varphi_q(n) = [n]_{\tilde{\mu},q} \).

The effects of interaction between particles of the Bose gas were incorporated in such deformed model by means of \( q \)-deformed thermodynamic relations. For instance, the respective specific volume (as function of fugacity) was obtained in its \( q \)-deformed version.

Using the series expansion of the basic-number like operator \( [N]_{\tilde{\mu}} \) in terms of \( \epsilon \equiv q - 1 \), natural interpretation is got as the picture of incorporating the interparticle interaction, since the contributions due to interaction can be viewed either in terms of \( N, N^2, N^3, \ldots \) or in terms of \( \epsilon, \epsilon^2, \epsilon^3, \ldots \) In both cases the parameters characterizing interaction enter the terms (coefficients) depending on the deformation parameter \( q \) or \( \epsilon = q - 1 \).

The Hamiltonian for \( q \)-boson is taken as \( H_q(N) \) being some \( \epsilon \)-deformation of standard quantum oscillator Hamiltonian. The expansion of \( H_q \) in powers of \( \epsilon \) is interpreted in a similar fashion: the terms of the first and higher orders in \( \epsilon \) in the expansion are again viewed as those linked with interaction. That is, they alltogether constitute the interaction Hamiltonian, and this implies physical meaning.

So, the picture of \( q \)-deformed non-interacting (ideal) many-particle system is used as a model of non-deformed, but interacting system. The \( q \)-deformed virial expansion is written in the form [13]:

\[
\frac{PV}{k_B T} = \sum_{k=1}^{\infty} a_k(\epsilon) \left( \frac{\lambda^3}{v} \right)^{k-1},
\]

with the virial coefficients \( a_k(\epsilon) \) given, say up to \( \epsilon^3 \), as \( a_2(\epsilon) = -\frac{1}{4\sqrt{2}} \) and likewise for higher \( a_n(\epsilon) \). Similarly to the preceding interpretation, \( \epsilon \neq 0 \) terms suggest corrections to the standard virial coefficients of the ideal Bose gas viewed as those arising from some explicitly given (though unspecified) interaction described by certain potential in the Hamiltonian. In effect, the interacting many-particle system gets effectively described (and interpreted) in terms of non-interacting, deformed system.

c. **Account for the interaction between (elementary) Bose particles.** Above, due to the realizability of composite bosons by deformed bosons, we deformed the Bose gas model with (the quadratic) structure function \( \varphi_{\tilde{\mu}}(n) \) in order to find effective thermodynamic relations or functions for the ideal, non-interacting quantum gas of composite bosons, in particular the deformed (i.e. depending on \( \tilde{\mu} \)) virial expansion of the EOS. To take into account the interaction between particles jointly with their compositeness, the two DSFs \( \varphi_{\tilde{\mu},q}(n) \) and \( \varphi_q(n) \) are combined into a single one yielding the unified DSF

\[
\varphi_{\tilde{\mu},q}(n) = (1 + \tilde{\mu})[n]_{\tilde{\mu},q} - \tilde{\mu}([n]_{\tilde{\mu},q})^2 \equiv [n]_{\tilde{\mu},q} \quad \text{(9)}
\]

which will play basic role in our treatment. For modeling the effects of the interaction between particle jointly with their compositeness, in parallel to DSF (9) the corresponding \( (\tilde{\mu},q) \)-extension of the derivative was used,

\[
\lambda D^z_{\tilde{\mu},q} = \left( \frac{d}{dz} \right) q - \tilde{\mu} \left( \frac{d}{dz} \right) q^2.
\]
The two-parameter Hamiltonian $H_{\tilde{\mu},\epsilon}(N)$ of $\tilde{\mu}, q$-bosons (single-mode case) can be split into $H_0$ (non-deformed part) and the Hamiltonian $H_1(\epsilon, \tilde{\mu}; N)$ that depends on $N$ and is the double series in $\tilde{\mu}$ and $\epsilon = q - 1$.

Using DSF $\varphi_{\tilde{\mu},q}(n)$ and the $\tilde{\mu}, q$-derivative $P_{\tilde{\mu},q}$ similarly to (4)-(6), the virial expansion of the EOS results in the form [3]

$$\frac{P_v}{k_BT} = \sum_{k=1}^{\infty} V_k(\tilde{\mu}, q) \left( \frac{\lambda^3}{v} \right)^{k-1} - \left[ \frac{2\tilde{\mu}q}{2^{7/4}v} \right]^3 + \left[ \frac{2\tilde{\mu}q}{2^{7/4}v} - \frac{2[3\tilde{\mu}q]}{37/2} \right] \left( \frac{\lambda^3}{v} \right)^2 + \left( \frac{2[3\tilde{\mu}q]}{37/2} - \frac{5[2\tilde{\mu}q]}{217/2} \right) \left( \frac{\lambda^3}{v} \right)^3 + \ldots \tag{10}$$

It is tempting to interpret this virial expansion as the effective one corresponding to the interacting gas of composite particles. The information about interaction and the composite structure is respectively encoded in the deformation parameters $q$ and $\tilde{\mu}$. If $\tilde{\mu} = 0, q \neq 1$, the expansion (10) accounts solely for interaction between the particles; likewise, when $q = 1, \tilde{\mu} \neq 0$, formula (10) should be interpreted as accounting for the compositeness of particles. When both $q \neq 1$ and $\tilde{\mu} \neq 0$, expression (10) incorporates jointly the both mentioned factors of Bose gas non-ideality.

The explicit virial coefficients with their dependence on the deformation parameters $q$ and $\tilde{\mu}$ can in principle be related to the characteristic parameters linked directly/explicitly with the interaction between composite bosons as well as inside them (between their constituents), see [4].

Remark that in [3] alternative DSFs defining the deformed Bose gas model suitable for the effective description were also discussed. Those correspond to other ways of composing the $q$-deformed SF and the quadratic one, in particular such as the DSF $\varphi_{q}(\varphi_{\tilde{\mu}}(n))$.

### III. ONE- AND TWO-PARTICLE DISTRIBUTIONS

Our main goal is to calculate the intercept of the momentum correlation function of 2nd order for $\tilde{\mu}, q$-deformed Bose gas model defined by the DSF $\varphi_{\tilde{\mu},q}(n)$ in (9), with the above-given interpretation (when $\tilde{\mu}$ and $q$ are responsible resp. for the compositeness and interaction) this time without the appeal to virial expansions.

We start with the defining formula (see e.g. [35] for nondeformed case, and [18] for deformed one)

$$\chi^{(r)}(k) = \frac{\langle a_{\tilde{\mu}q}^\dagger a_k^\dagger \rangle^{(r)}}{\langle a_k^\dagger a_k \rangle^{(r)}} - 1 \tag{11}$$

for the intercepts of $r$th order momentum correlation functions at a given momentum $k$, taken the same for all the $r$ particles. The notation $\langle \ldots \rangle$ means statistical (thermal) average, and $a_k^\dagger$ resp. $a_k$ are the creation resp. annihilation operators for the $\varphi_{\tilde{\mu},q}$-deformed bosons which obey the following set of commutation relations given by the DSF $\varphi_{\tilde{\mu},q}$:

$$[N_k, a_k^\dagger] = \delta_{kk'}a_k^\dagger, \quad [N_k, a_k] = -\delta_{kk'}a_k^\dagger, \quad [a_k^\dagger, a_k^\dagger] = \varphi_{\tilde{\mu},q}(N_k + 1) - \varphi_{\tilde{\mu},q}(N_k), \quad a_k^\dagger a_k = \varphi_{\tilde{\mu},q}(N_k).
$$

From these relations it follows that (from now on the label $k$ of fixed mode will be omitted)

$$a_k^\dagger \varphi_{\tilde{\mu},q}(N) = \varphi_{\tilde{\mu},q}(N - 1)a_k^\dagger, \quad a_k \varphi_{\tilde{\mu},q}(N) = \varphi_{\tilde{\mu},q}(N + 1)a_k. \tag{12}$$

As seen, (11) involves both the $r$-th order (in numerator) and $r$th power of the first order (in denominator) deformed analogs of distribution functions.

Let us observe the nilpotency of creation/annihilation operators $a_k^\dagger$, $a_k$ for certain $\tilde{\mu}, q$, and, as a consequence, the partially discontinuous (either in $\tilde{\mu}$ or in $q$) set of deformation parameters ($\tilde{\mu}, q$) for $\mu, q$-deformed oscillators. The nilpotency of $a_k^\dagger, a_k$ is related to the possibility of nullifying or changing the sign of the structure function $\varphi_{\tilde{\mu},q}(n)$ at some positive $n$. The reasoning is as follows. The norm of the vector $(a_k^\dagger)^r|0\rangle$ squared, i.e.

$$|| (a_k^\dagger)^r |0\rangle ||^2 = \langle 0 | (a_k^\dagger)^r |0\rangle = \varphi(r)\varphi(r-1) \cdot \ldots \cdot \varphi(1) \equiv \varphi(r)! \tag{13}$$

should be nonnegative: either positive for all $r \geq 1$ (unbounded occupation numbers) or zero at some $r = N_{\max} + 1$ where $N_{\max}$ is maximal occupation number. The former requirement, for the DSF $\varphi_{\tilde{\mu},q}(n)$, is equivalent to the condition

$$\min_{n \geq 1} \varphi_{\tilde{\mu},q}(n) > 0 \quad \text{or} \quad \varphi_{\tilde{\mu},q}(2) > \left[ \frac{2\mu}{2} \right]^{1+|q|-|q-1|} \tag{14}$$

So, the set of parameters ($\tilde{\mu}, q$) is continuous inside the two-dimensional region given by inequalities in (14) (grey-colored region in Fig. 1). Otherwise, when $q > (1 + \tilde{\mu})^{-1}$ there should exist an integer $r = N_{\max} + 1$ such that

$$\varphi_{\tilde{\mu},q}(r) = \varphi_{\tilde{\mu},q}(N_{\max} + 1) = 0. \tag{15}$$

This equation can be equivalently rewritten as

$$[N_{\max} + 1]_q = 0 \quad \text{or} \quad \tilde{\mu}q[N_{\max}]_q = 1. \tag{16}$$

Its solutions ($\tilde{\mu}, q$) form a discrete set of curves, see Fig. 1, as they are parameterized by the integer $N_{\max}$. Thus, the possibilities of discrete $\tilde{\mu} = q^{-1}[N_{\max}]^{-1}$ in couple with continuous $q$, or of discrete $q = \tilde{\mu}, N_{\max}$ with continuous $\tilde{\mu}$ are included. However, the former seems more probable in view of discreeteness of the set of composite bosons’ bound states. Note that other intermediate variants e.g. when a certain function of $\tilde{\mu}, q$ is continuous are
also allowed. From (16) we deduce the maximal occupation number of a fixed mode for the $\tilde{\mu}, q$-deformed Bose gas model with $\tilde{\mu}, q$ belonging to the discrete curves:

$$N_{\max}(\tilde{\mu}, q) = \ln \left( 1 + \frac{q - 1}{\tilde{\mu} q} \right) / \ln q. \quad (17)$$

Remark that at fixed values of $q$ the corresponding set of deformation parameter $\tilde{\mu}$ values (discrete ones plus regions of continuum), see Fig. 1, can be associated with bound and unbound states. The dependence of energy $E(n)$ on the occupation number $n$ can be of interest for diverse values of the deformation parameters. The respective dependences for the typical Hamiltonian $H$ determined in (17). The first (discrete) type holds when $q[N_{\max}]_q = 1$, $N_{\max} \geq 1$ or when $q = -1$, $\tilde{\mu}$ is arbitrary, $N_{\max} = 1$; the second (continuous) type holds when $\varphi_{\tilde{\mu}, q}(2) > [2]^q_{\frac{1}{2}} \frac{|q - q_1|}{2}$. Also, we have:

$$\langle a^\dagger a \rangle = \frac{1}{z-q} \left[ \varphi_{\tilde{\mu}, q}(2) - [2]^q_{\frac{1}{2}} \right] \left( \frac{1}{z-q}(z-q^2) \right) = \frac{1}{z-q} + \frac{\delta \varphi_{\tilde{\mu}, q}(2)(1-R)}{(z-q)(z-q^2)}, \quad (19)$$

where

$$\delta \varphi_{\tilde{\mu}, q}(n) = \varphi_{\tilde{\mu}, q}(n) - [n]_q, \quad R \equiv R_{\tilde{\mu}, q}(z) = \frac{[N_{\max}+1]_z}{[N_{\max}+1]}$$

$$z = e^x, \quad x = \beta \hbar \omega, \quad \beta = (k_B T)^{-1}, \quad k_B$$

is Boltzmann constant, in the discrete case i.e. when $\tilde{\mu}q[N_{\max}]_q = 1$, and

$$\langle a^\dagger a \rangle = \frac{z + \varphi_{\tilde{\mu}, q}(2) - [3]^q_{\frac{1}{2}}}{(z-q)(z-q^2)}. \quad (20)$$

in the continuous case when $\varphi_{\tilde{\mu}, q}(2) > [2]^q_{\frac{1}{2}} \frac{|q - q_1|}{2}$. This is our first result.

Let us consider the $q \to 1$ and $\tilde{\mu} \to 0$ limits of $\langle a^\dagger a \rangle$. For $q \to 1$ and $\tilde{\mu} \to 0$ limits, the evaluation yields

$$\langle a^\dagger a \rangle \to \begin{cases} \frac{z-1-2\tilde{\mu}}{(z-1)^2} + \frac{1}{[N_{\max}+1]_z}, & \tilde{\mu} > N_{\max}, \\ \frac{z-1-2\tilde{\mu}}{(z-1)^2}, & \tilde{\mu} \leq N_{\max}. \end{cases}$$

On the other hand, for $\tilde{\mu} \to 0$ and $q > 1$ we have:

$$\langle a^\dagger a \rangle_{\tilde{\mu} \to 0} \simeq \frac{1}{e^x - q} + \theta(\tilde{\mu})q \frac{e^x - 1}{(e^x - q)(e^x - q^2)} \left| \tilde{\mu} - \frac{1}{m_q} \right|^{-1}$$

FIG. 1. Admissible deformation parameters $\tilde{\mu}$, $q$, and the maximum occupation number $N_{\max}$ (discrete subset).

FIG. 2. Energy $E(n) = \frac{1}{2} \hbar \omega(\varphi_{\tilde{\mu}, q}(n) + \varphi_{\tilde{\mu}, q}(n+1))$ versus occupation number $n$, for some values of deformation parameters $\tilde{\mu}$ and $q$. 

its label $k$ is dropped)

$$\langle a^\dagger a \rangle = \frac{1}{z-q} + \frac{(\varphi_{\tilde{\mu}, q}(2) - [2]^q_{\frac{1}{2}})(1-R)}{(z-q)(z-q^2)} \quad (19)$$

where

$$\delta \varphi_{\tilde{\mu}, q}(n) = \varphi_{\tilde{\mu}, q}(n) - [n]_q, \quad R \equiv R_{\tilde{\mu}, q}(z) = \frac{[N_{\max}+1]_z}{[N_{\max}+1]}$$

$$z = e^x, \quad x = \beta \hbar \omega, \quad \beta = (k_B T)^{-1}, \quad k_B$$

is Boltzmann constant, in the discrete case i.e. when $\tilde{\mu}q[N_{\max}]_q = 1$, and

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in the continuous case when $\varphi_{\tilde{\mu}, q}(2) > [2]^q_{\frac{1}{2}} \frac{|q - q_1|}{2}$. This is our first result.

Let us consider the $q \to 1$ and $\tilde{\mu} \to 0$ limits of $\langle a^\dagger a \rangle$. For $q \to 1$ and $\tilde{\mu} \to 0$ limits, the evaluation yields

$$\langle a^\dagger a \rangle \to \begin{cases} \frac{z-1-2\tilde{\mu}}{(z-1)^2} + \frac{1}{[N_{\max}+1]_z}, & \tilde{\mu} > N_{\max}, \\ \frac{z-1-2\tilde{\mu}}{(z-1)^2}, & \tilde{\mu} \leq N_{\max}. \end{cases}$$

On the other hand, for $\tilde{\mu} \to 0$ and $q > 1$ we have:

$$\langle a^\dagger a \rangle_{\tilde{\mu} \to 0} \simeq \frac{1}{e^x - q} + \theta(\tilde{\mu})q \frac{e^x - 1}{(e^x - q)(e^x - q^2)} \left| \tilde{\mu} - \frac{1}{m_q} \right|^{-1}$$
where \( \theta(\tilde{\mu}) \) is the Heaviside step function. Otherwise for \( |q| < 1 \) the latter limit yields
\[
\langle a^\dagger a \rangle \xrightarrow{\tilde{\mu} \to 0} \frac{1}{e^x - q},
\]
(23)

which is the familiar result for Arik-Coon type q-Bose gas (see e.g. [18]).

With account of the easily verified equality
\[
(a^\dagger)^r a^r = \varphi(N)\varphi(N-1) \cdots \varphi(N-r+1)
\]
(see (12)) we consider the 2-particle \( \tilde{\mu}, q \)-deformed distribution function: \( \langle (a^\dagger)^2 a^2 \rangle_{\tilde{\mu}, q} = \langle \varphi_{\tilde{\mu}, q}(N)\varphi_{\tilde{\mu}, q}(N-1) \rangle \). To calculate \( \langle (a^\dagger)^2 a^2 \rangle_{\tilde{\mu}, q} \) we use the relation
\[
\sum_{i=0}^{d} (-1)^i q^{(i+1)/2} \left\{ \left( \begin{array}{c} 4 \vspace{0.2cm} \end{array} \right) \right\} \varphi_{\tilde{\mu}, q}(N+4-i)\varphi_{\tilde{\mu}, q}(N+3-i) =
\sum_{i=0}^{d} (-1)^i q^{(i+1)/2} \left\{ \left( \begin{array}{c} 4 \vspace{0.2cm} \end{array} \right) \right\} \varphi_{\tilde{\mu}, q}(4-i)\varphi_{\tilde{\mu}, q}(3-i)
\]
(25)

where \( \left\{ \left( \begin{array}{c} 4 \vspace{0.2cm} \end{array} \right) \right\} \) denotes \( q \)-binomial coefficient. Taking averages (\( \langle \cdots \rangle \)) of both sides of (25), after some algebra we find (recall that \( z = e^x \)):
\[
\langle (a^\dagger)^2 a^2 \rangle = \frac{\varphi_{\tilde{\mu}, q}(2)}{(z-q)(z-q^2)} \left\{ 1 + \frac{\varphi_{\tilde{\mu}, q}(3) - [3]_q}{(z-q^2)(z-q)} \right\}
\]
\[
\cdot \left\{ \left( \begin{array}{c} 4 \vspace{0.2cm} \end{array} \right) \right\} \varphi_{\tilde{\mu}, q}(2) - \varphi_{\tilde{\mu}, q}(N+1)\varphi_{\tilde{\mu}, q}(N+3) + (z-q[4]_q) \cdot \left\{ \varphi_{\tilde{\mu}, q}(2) - \varphi_{\tilde{\mu}, q}(N+3)\varphi_{\tilde{\mu}, q}(N+2) \right\}
\]
(26)

The second part of this expression (beginning with \( [N_{\text{max}}+1]^{-1}_q \)) can be evaluated using the relations
\[
\varphi_{\tilde{\mu}, q}(N_{\text{max}}+l) = -q[l-1]_q [N_{\text{max}} + l]_q, \quad l = 2, 3, \ldots
\]
(27)

that yields
\[
\varphi_{\tilde{\mu}, q}(l-1)\varphi_{\tilde{\mu}, q}(l-1) - \varphi_{\tilde{\mu}, q}(N_{\text{max}}+l)\varphi_{\tilde{\mu}, q}(N_{\text{max}}+l+1) =
\]
\[
- q^{-l-1} [2]_q [q[l-1]_q [N_{\text{max}} + l]_q + q[l+1]_q - [l-1]_q] \tilde{\mu}^2
\]
\[
\cdot \left\{ q(q-1) + (2q^{l+1} + [2]_q) \mu + q(q[l]_q + [l-1]_q) \tilde{\mu}^2 \right\}
\]
After substituting these expressions in (26) we obtain
\[
\langle (a^\dagger)^2 a^2 \rangle = \frac{\varphi_{\tilde{\mu}, q}(2)}{(z-q)(z-q^2)} \left\{ 1 + \frac{\varphi_{\tilde{\mu}, q}(3) + q^2 - \varphi_{\tilde{\mu}, q}(2)}{(z-q^2)(z-q)} \right\}
\]
\[
\cdot \left\{ \left( \begin{array}{c} 4 \vspace{0.2cm} \end{array} \right) \right\} \varphi_{\tilde{\mu}, q}(2)(1-R) - q^2 [2]_q R \left\{ \left( \begin{array}{c} 4 \vspace{0.2cm} \end{array} \right) \right\} \varphi_{\tilde{\mu}, q}(2) \right\}
\]
(28)

Recall that expression (28) is valid for the discrete case. For the continuous case, take the limit \( N_{\text{max}} \to \infty \) to obtain
\[
\langle (a^\dagger)^2 a^2 \rangle = \frac{\varphi_{\tilde{\mu}, q}(2)}{(z-q)(z-q^2)} \cdot \left\{ 1 + \frac{\varphi_{\tilde{\mu}, q}(3) - [3]_q(q^2 - \varphi_{\tilde{\mu}, q}(2))}{(z-q^3)(z-q^2)} \right\}
\]
(29)

Formulas (28)-(29) constitute our second result.

Now consider the limit cases:

If \( \tilde{\mu} = N_{\text{max}}^{-1} \),
\[
\langle (a^\dagger)^2 a^2 \rangle \xrightarrow{q \to 1} 2(\tilde{\mu}^{-1} + 1)^{-1}(z-1)^{-4}\left\{ (1-\tilde{\mu})(z\tilde{\mu}^{-3} - 1) + (6\tilde{\mu}^2 - 4\tilde{\mu} + 2)(\tilde{\mu}^{-1} - z) \right\}
\]
+ \( (6\tilde{\mu}^2 + 5\tilde{\mu} + 1)(z\tilde{\mu}^{-1} - z^2) \);
If \( \tilde{\mu} < 0 \),
\[
\langle (a^\dagger)^2 a^2 \rangle \xrightarrow{q \to 1} \frac{[2]_q}{(z-1)^2} \left\{ 1 + \frac{[3]_q - 3(z-4)[2]_q^{-1}}{(z-q^{-2})} \right\}
\]
Similarly, to \( \langle a^\dagger a \rangle \), letting \( \tilde{\mu} \to 0 \) in \( \langle (a^\dagger)^2 a^2 \rangle \) we find:
\[
\text{If } |q| < 1, \quad \langle (a^\dagger)^2 a^2 \rangle \xrightarrow{\tilde{\mu} \to 0} \frac{[2]_q}{(e^x - q)(e^x - q^2)}
\]
\[
\text{If } q > 1, \quad \langle (a^\dagger)^2 a^2 \rangle \xrightarrow{\tilde{\mu} \to 0} \frac{[2]_q}{(z-q)(z-q^2)}
\]
\[
\text{Recall that expression (28) is valid for the discrete case.}
\]

IV. INTERCEPT OF TWO-PARTICLE CORRELATION FUNCTION

The substitution of (28) and (19) in (11) leads us to the following resulting expression for the intercept:
\[
\lambda^{(2)} = -1 + \frac{\varphi_{\tilde{\mu}, q}(2)(z-q)(z-q^2)}{(z-q^2 + (1 - R_{\tilde{\mu}, q}(z)) \delta_{\tilde{\mu}, q}(2)) \left\{ (z-q^3)(z-q^2) \right\}}
\]
\[
\left\{ (z-q^3)(z-q^4) + (1 - R_{\tilde{\mu}, q}(z)) \varphi_{\tilde{\mu}, q}(2)(z+q^2 - \varphi_{\tilde{\mu}, q}(2)) - q^2 [2]_q R_{\tilde{\mu}, q}(z) \left\{ \left( \begin{array}{c} 4 \vspace{0.2cm} \end{array} \right) \right\} \varphi_{\tilde{\mu}, q}(2) \right\}
\]
(32)

In the continuous case this expression reduces to
\[
\lambda^{(2)} = -1 + \frac{\varphi_{\tilde{\mu}, q}(2)(z-q)(z-q^2)}{(z-[3]_q + \varphi_{\tilde{\mu}, q}(2)) \left\{ (z-q^3)(z-q^4) \right\}}
\]
\[
\cdot \left\{ (z-q^3)(z-q^4) + (\varphi_{\tilde{\mu}, q}(3) - [3]_q)(z+q^2 - \varphi_{\tilde{\mu}, q}(2)) \right\}
\]
(33)

These expressions for $\lambda^{(2)}$ give main result of the paper.

Now, by substituting the obtained limits for $(a^\dagger a)$ and $(\langle a^\dagger a \rangle^2a^2)$ in (11) we find the limits for the intercept:

If $q \to 1$ (and either $\tilde{\mu} > 0$ or $\tilde{\mu} < 0$),

$$\lambda^{(2)}_{\tilde{\mu} > 0} \to \int (1 - 2\tilde{\mu})(z^{2\tilde{\mu}^{-1}+1}+1) + (12\tilde{\mu}^2 - 4\tilde{\mu} - 2)(z^{2\tilde{\mu}^{-1}+3} + z)$$

$$+ (8\tilde{\mu}^2 + 6\tilde{\mu} + 1)(z^{2\tilde{\mu}^{-1}+2} + z^2) - (12\tilde{\mu}^2 + 12\tilde{\mu} + 6)z^{\tilde{\mu}^{-1}+1}(z^2 + 1)$$

$$- (16\tilde{\mu}^2 - 24\tilde{\mu} - 12)z^{\tilde{\mu}^{-1}+2}/z^{\tilde{\mu}^{-1}+2 - 1} - (2\tilde{\mu} + 1)(z^{\tilde{\mu}^{-1}+1} - z))^2,$$

$$\lambda^{(2)}_{\tilde{\mu} < 0} \to \int (z - 1)^2 + (\langle 3\tilde{\mu} - 3 \rangle(z^{4}/2\tilde{\mu}) + 1) - 1;$$

If $\tilde{\mu} \to 0$ (and either $|q| \leq 1$ or $q > 1$),

$$\lambda^{(2)}_{-1 < q < 1} \to \lim_{\tilde{\mu} \to 0} \frac{e^{x^2 - 1}}{e^{x^2 - q^2}}$$

$$\lambda^{(2)}_{\mu > 0} \to \lim_{\tilde{\mu} \to 0} \frac{q^2 \exp(\tilde{\mu})}{q^2 - (z - 1)^2}$$

$$\lambda^{(2)}_{\mu < 0} \to \lim_{\tilde{\mu} \to 0} \frac{q^2 \exp(\tilde{\mu})}{q^2 - (z - 1)^2}$$

Strictly speaking, the version of $\tilde{\mu}$, $q$-deformed Bose gas model (explored in this and preceding sections) in which we have derived the one-, two-particle distribution functions and the intercept of two-particle correlation function, is not identical to the $\tilde{\mu}$, $q$-Bose gas model of [3, 4] and of the Section II. Between the two versions of $\tilde{\mu}$, $q$-Bose gas there is a kind of “duality relation”: the base of this relation lies both in (i) the usage of the same DSF (in the form of $\varphi_{\tilde{\mu},q}(z^{d/2})$ to deform thermodynamics, or in the form $\varphi_{\tilde{\mu},q}(N)$ to calculate deformed distribution and correlation intercepts), and in (ii) the required coincidence of one-particle deformed distributions. We mean the distribution $n^q_k$ in $\varphi$-deformed model defined like in Sec. II and [3, 4] (with $\varphi$-deformed expression for the total number of particles $N(\varphi)$ and the corresponding partition function) and recovered from $N(\varphi) = \sum_k n^q_k$, on the one hand, and the distribution $\tilde{n}^q_k \equiv (\tilde{\varphi}(N_k))$ defined by DSF $\tilde{\varphi}$ similarly to (19), on the other hand. For these distributions to coincide, the DSFs $\varphi$ and $\tilde{\varphi}$ should be properly related. This “duality”, in explicit terms and with concrete examples, will be the subject of a separate paper. Here we only mention that for the $\tilde{\mu}$-Bose gas model given by $\varphi_{\tilde{\mu},q}(n)|q=1 = (1 + \tilde{\mu})n - \tilde{\mu}n^2$ which is the $q = 1$ sector of the whole $\tilde{\mu}$, $q$-Bose gas model, the distributions and intercepts in the dual version are calculated using the “dual” structure function $\tilde{\varphi}_{\tilde{\mu},q}(n) = (1 + \tilde{\mu})n - \tilde{\mu}n^2$.

The dependence $\lambda^{(2)}(K)$ on the momentum $K = |k|$ for some values of deformation parameters $\tilde{\mu}$, $q$ and temperature $T$ is shown in Fig. 3, where we take $\hbar \omega = \sqrt{m^2 + K^2}$, and for $m$ the $\pi$-meson mass (139.5 MeV). In addition we give three-dimensional plot of the function $\lambda^{(2)}(K, q)$ with fixed $\lambda^{(2)}_{\mu = 0.1}$ in Fig. 4.

To confront our results with experimental data it is more convenient to work (instead of the couple $\tilde{\mu}$, $q$) with the asymptotic value $\lambda^{(2)}_{\mu = 0.1}$ of the intercept and one of the parameters $\tilde{\mu}$, $q$, say $q$. The asymptotics is

$$\lambda^{(2)}_{\mu = 0.1} = \varphi_{\tilde{\mu},q}(2)! - 1 = q[1 - \tilde{\mu}(1 + q)].$$

Expressing $\tilde{\mu}$ through $q$ and $\lambda^{(2)}_{\mu = 0.1}$ as $\tilde{\mu} = \frac{1 - \lambda^{(2)}_{\mu = 0.1}/q}{1+q}$ and substituting this in (32), we obtain the function $\lambda^{(2)}(q, \lambda^{(2)}_{\mu = 0.1}; K)$. The corresponding plots for some values

FIG. 3. Dependence of the two-pion intercept $\lambda^{(2)}(K)$ on momentum $K$, for the values $\tilde{\mu} = 0.1, 0.2, q = 0.85, 0.92$, and $T = 120, 180\ MeV$.

FIG. 4. Intercept $\lambda^{(2)}(q, \lambda^{(2)}_{\mu = 0.1}; K)$ vs. momentum $K$ and deformation parameter $q$ for temperature $T = 180\ MeV$, at fixed $\lambda^{(2)}_{\mu = 0.1} = 0.6$ so that $\tilde{\mu} = \frac{1 - \lambda^{(2)}_{\mu = 0.1}}{1+q}$. 
of deformation parameters $\tilde{\mu}$, $q$ and temperature $T$ are presented in Fig. 5. Therein, we also place some experimental points for $\pi$-meson intercepts from RHIC/STAR, see [26, 27]. As seen, there is a qualitative agreement of experimental curves with the data. At first sight it is the $\pi$-meson compositeness effects which should be more sensitive to the increase of collision energy of colliding ions. Therefore, under such a condition within our interpretation, parameter $\tilde{\mu}$ somewhat more notably varies than $q$ with the change of the collision energy for the same colliding ions. So, besides an independent variation of $\tilde{\mu}$ and $q$ involved in the extrapolation, the second panel of Fig. 5 contains also the two curves with the same $q = 0.896$ both for 62.4 GeV and 200 GeV Cu+Cu-collisions. All the plots in Fig. 5 correspond to the discrete case. The analogous fitting curves for the continuous case show some-worse agreement and not shown. As seen from the third panel of Fig. 5 the experimental dots for $\pi^+$-meson intercept in 200 GeV Au+Au collisions lie mainly slightly higher than those for $\pi^-$-mesons. This is presumably explained by different effective Coulomb interaction for $\pi^+$- and $\pi^-$-mesons, and can be associated with slightly differing values (1.49 versus 1.492) of parameter $q$ of the corresponding extrapolating curves. It is clear that more detailed experimental information is needed in order to make more univocal conclusions about advantages of this model over others.

V. CONCLUSIONS AND OUTLOOK

In this work, within the $\tilde{\mu}$, $q$-deformed Bose gas model based on the deformation structure function (9) taken as operator function of the number operator, we have calculated both one- and two-particle distribution functions from which obtained the expression for momentum correlation function intercept. It should be stressed that in this particular model, unlike other deformed models, see [16, 19, 29, 30] and some others, the deformation parameters $\tilde{\mu}$ and/or $q$ may take not only continuum values but also the discrete ones. Of course, that is presumably inherited from the particles’ compositeness picture [10, 11, 33, 34].

The version of $\tilde{\mu}$, $q$-deformed Bose gas model considered in Sec. III and IV of this paper, though differs from its dual $\tilde{\mu}$, $q$-deformed partner model from [3, 4] and Sec. II above, shares with it three things: the same form of DSF (as main ingredient of any deformed model); the coinciding one-particle distributions derived with differing, but strictly related (in a special way) DSFs in the two models; the same impact, or goal, of effective description of the two basic nonideality properties of realistic Bose like gases, mentioned in Introduction. More detailed analysis of the duality relation between the two partner $\tilde{\mu}$, $q$-deformed Bose gas models, with some other instances of dual pairs of deformed models, will be the subject of forthcoming paper, including the specifics of implications in the two dual approaches for the goal of effective description.

What concerns application of the obtained $\tilde{\mu}$, $q$-deformed two-particle correlation function intercept for an effective description of the observed non-Bose like behavior of two-pion correlation intercepts observed in RHIC and LHC experiments [22–28], we applied the results obtained above and made some comparison which shows a qualitative agreement. Of course, for more detailed comparison and ultimate conclusion about viability of the studied deformed Bose gas models, the knowledge of (both formulas and data on) the 3rd order distribution function and respective correlation intercept no doubt is desirable, also in view of the existing characteristic function $r^{(3)}$ introduced in [36] (with some deformed cases studied in [19, 30]), which is a special combination of the two- and three-particle correlation function

![FIG. 5. Intercept $\lambda^{(2)}(K)$ vs. momentum $K$, for different values of $q$, $\tilde{\mu}$ and $T$ chosen to fit experimental data. Exper. dots taken from [26, 27] are shown by boxes.](image-url)

- $q = 1.48$, $\tilde{\mu} = 0.27$, $T = 90$ MeV
- $q = 1.543$, $\tilde{\mu} = 0.255$, $T = 93$ MeV
- 200 GeV Au+Au [0-5]% ($\pi^+ + \pi^-$)
- 62.4 GeV Au+Au [0-5]% ($\pi^+ + \pi^-$)
- $q = 1.542$, $\tilde{\mu} = 0.255$, $T = 84$ MeV
- $q = 1.175$, $\tilde{\mu} = 0.239$, $T = 127$ MeV
- $q = 0.896$, $\tilde{\mu} = 0.158$, $T = 275$ MeV
- 62.4 GeV Cu+Cu [0-10]% ($\pi^+ + \pi^-$)
- $q = 1.488$, $\tilde{\mu} = 0.27$, $T = 90$ MeV
- 200 GeV Cu+Cu [0-10]% ($\pi^+ + \pi^-$)
- $q = 1.135$, $\tilde{\mu} = 0.258$, $T = 138$ MeV
- $q = 0.896$, $\tilde{\mu} = 0.185$, $T = 288$ MeV
- 200 GeV Cu+Cu [0-10]% ($\pi^+ + \pi^-$)
- $q = 1.49$, $\tilde{\mu} = 0.27$, $T = 91$ MeV
- 200 GeV Au+Au [0-5]% ($\pi^+ + \pi^-$)
- 200 GeV Au+Au [0-5]% ($\pi^+ + \pi^-$)
intercepts $\lambda^{(2)}$ and $\lambda^{(3)}$. The particular results for 3rd order correlation function intercept $\lambda^{(3)}$ and the function $r^{(3)}$ obtained for the $\bar{\mu}, q$-deformed Bose gas, along with confronting them with available experimental data for $\pi$-mesons (as quark-antiquark composites), produced and registered in relativistic heavy-ion collisions, will be presented elsewhere.

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[1] S. S. Avancini and G. Krein, J. Phys. A: Math. Gen. 28, 685 (1995).
[2] A. M. Scarfone and P. Narayana Swamy, J. Stat. Mech. 2009, P02055 (2009).
[3] A. M. Gavrilik and Yu. A. Mishchenko, Ukr. J. Phys. 58, 1171 (2013).
[4] A. M. Gavrilik and Yu. A. Mishchenko, (2014), to appear in Phys. Rev. E, arXiv:1409.3423.
[5] A. Rovenchak, Phys. Rev. A 89, 052116 (2014).
[6] D. Bonatsos, J. Phys. A: Math. Gen. 25, L101 (1992).
[7] K. D. Sviratcheva et al., Phys. Rev. Lett. 93, 152501 (2004).
[8] Y.-X. Liu, C. P. Sun, S. X. Yu, and D. L. Zhou, Phys. Rev. A 63, 023802 (2001).
[9] S. S. Avancini, J. R. Marinelli, and G. Krein, J. Phys. A: Math. Gen. 36, 9045 (2003).
[10] A. M. Gavrilik, I. I. Kachurik, and Yu. A. Mishchenko, J. Phys. A: Math. Theor. 44, 475303 (2011).
[11] A. M. Gavrilik, I. I. Kachurik, and Yu. A. Mishchenko, Ukr. J. Phys. 56, 948 (2011).
[12] M. Arik and D. D. Coon, J. Math. Phys. 17, 524 (1976).
[13] A. M. Scarfone and P. Narayana Swamy, J. Phys. A: Math. Gen. 41, 275211 (2008).
[14] V. I. Man’ko et al., Phys. Lett. A 176, 173 (1993).
[15] M. Daoud and M. Kibler, Phys. Lett. A 206, 13 (1995).
[16] D. V. Anchishkin, A. M. Gavrilik, and N. Z. Iorgov, Eur. Phys. J. A 7, 229 (2000).
[17] Q. H. Zhang and S. S. Padula, Phys. Rev. C 69, 024907 (2004).
[18] L. V. Adamska and A. M. Gavrilik, J. Phys. A: Math. Gen. 37, 4787 (2004).
[19] A. M. Gavrilik and A. P. Rebesh, Eur. Phys. J. A 47, 55 (2011).
[20] A. M. Gavrilik and Yu. A. Mishchenko, Phys. Lett. A 376, 2484 (2012).
[21] A. M. Gavrilik and Yu. A. Mishchenko, (2014), arXiv:1410.0538.
[22] C. Adler et al. (STAR Collaboration), Phys. Rev. Lett. 87, 082301 (2001).
[23] I. G. Bearden et al., Phys. Lett. B 517, 25 (2001).
[24] J. Adams et al. (STAR Collaboration), Phys. Rev. Lett. 91, 262301 (2003).
[25] M. M. Aggarwal et al. (WA98 Collaboration), Phys. Rev. C 67, 014906 (2003).
[26] J. Adams et al. (STAR Collaboration), Phys. Rev. C 71, 044906 (2005).
[27] B. I. Abelev et al. (STAR Collaboration), Phys. Rev. C 80, 024905 (2009).
[28] B. Abelev et al. (ALICE Collaboration), Phys. Rev. C 89, 024911 (2014).
[29] D. V. Anchishkin, A. M. Gavrilik, and S. Y. Panitkin, Ukr. J. Phys. 49, 935 (2004).
[30] A. M. Gavrilik, SIGMA 2, 074 (2006).
[31] A. P. Rebesh, I. I. Kachurik, and A. M. Gavrilik, Ukr. J. Phys. 58, 1182 (2013).
[32] M. C. Tichy, P. A. Bouvrie, and K. Mølmer, Phys. Rev. A 88, 061602 (2013).
[33] A. M. Gavrilik and Yu. A. Mishchenko, Phys. Lett. A 376, 1596 (2012).
[34] A. M. Gavrilik and Yu. A. Mishchenko, J. Phys. A: Math. Theor. 46, 145301 (2013).
[35] S. Chapman and U. Heinz, Phys. Lett. B 340, 250 (1994).
[36] U. Heinz and Q. H. Zhang, Phys. Rev. C 56, 426 (1997).