Photonic and plasmonic transition radiation from graphene

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Abstract

We theoretically study the transition radiation in the framework of full Maxwell equations, when a swift electron crosses a monolayer graphene. Based on the Sommerfeld integration, we demonstrate in the frequency domain the spatial distribution of this free-electron radiation, which clearly shows the broadband excitation of both photons and graphene plasmons. Moreover, the radiation spectra for photons and graphene plasmons are analytically derived. We find that the excitation of photons and graphene plasmons favors different particle velocities. To be specific, a higher particle velocity gives rise to the excitation of photons with better directivity and higher intensity, while a lower particle velocity enables the efficient excitation of graphene plasmons in a broader frequency range. Our work indicates that the interaction between swift charged particles and various 2D materials or van der Waals heterostructures is promising for the design of terahertz on-chip radiation sources.

Keywords: free-electron radiation, graphene, surface plasmon polaritons, terahertz technology

(Some figures may appear in colour only in the online journal)

1. Introduction

Transition radiation, as a typical free-electron radiation process, was first theoretically proposed by Ginzburg and Frank in 1945 and later experimentally observed in 1959 [1, 2]. Transition radiation would occur, once a charged particle moves across a spatially inhomogeneous region, such as an interface. Remarkably, the occurrence of transition radiation has no specific requirements on the particle velocity \(v\). This is drastically different from another well-known phenomenon of free-electron radiation, namely Cherenkov radiation [3–5], whose emergence requires the particle velocity larger than the Cherenkov threshold (i.e. the phase velocity of light in the host material) [6, 7]. Furthermore, the usage of periodic photonic nanostructures or periodic electron bunches can lead to the formation of coherent or resonant transition radiation [8–11], which would further enhance the radiation intensity and directivity. As a result, transition radiation has wide applications for the design of radiation sources, ranging from microwave, THz to x-ray regimes [12–15]. On the other hand, for highly relativistic charged particles, the total energy of photons emitted in transition radiation per interface is proportional to \(\gamma\) [15], where \(\gamma = \sqrt{1 - v^2/c^2}\) is the Lorentz factor and \(c\) is the speed of light in free space. Such a unique feature makes transition radiation attractive as an underlying mechanism for high-energy particle detectors, namely the transition radiation detector [16–18]. The transition radiation detector is useful for the identification of particles at extremely high energies where other existing technologies such as Cherenkov detectors [19, 20] are not accessible. In addition, since the electron bunches can be focused to a nano-spot, the interaction between fast electrons and 2D materials with the precise spatial control has been widely exploited for the exploration...
of many emerging properties of 2D materials [21–29], for example, through the electron energy loss spectroscopy. In turn, due to the active tunability of their optical properties, these 2D materials [30–46], such as graphene, show the promise to provide new degrees of freedom for the flexible control of transition radiation, which is worthy of further studies.

Here, we systematically study the transition radiation from a monolayer graphene, by extending Ginzburg and Frank’s theory of transition radiation to graphene physics in the framework of classic electromagnetic wave theory. We theoretically investigate in the frequency domain the spatial distribution and energy spectrum of graphene plasmons excited by a swift electron perpendicularly impacting a monolayer graphene. The similar analysis is also performed on the associated light emission. We find that both graphene plasmons and photons can be excited from graphene, since the thickness of graphene is much smaller than the electron wavelength. This makes the graphene plasmons and photons a new degree of freedom for the flexible control of many emerging properties of 2D materials [32, 34]. Such a 2D model for graphene is widely adopted in the study of interactions between swift electrons and graphene [26–31]. Here we consider a monolayer graphene parallel to the interface between medium 1 (z < 0) and medium 2 (z > 0). To explore the influence of graphene on the transition radiation, we set both nonmagnetic media to be free space. That is, both the incident and scattered waves should have a same component of wavevector parallel to the interface, and the subscript \( z = \bar{z}v \) is the induced surface current density of graphene plasmons if \( \bar{z}v \) sign should be used for the forward radiation in the frequency domain of plane waves with different waves of \( \bar{z}v \) and \( \omega \). From equations (1) and (2), we readily get \( j_{\omega,\bar{z}v} = \frac{q}{(2\pi)^2} e^{i\bar{z}v} \). Moreover, from the Maxwell equations in the frequency domain, we can further obtain the following equation for \( E_{\omega,\bar{z}v} \) (i.e. the \( z \) component of the electric field \( E_{\omega,\bar{z}v} \)),

\[
\frac{\partial^2}{\partial z^2}(\varepsilon E_{\omega,\bar{z}v}) + \varepsilon_\perp \frac{\omega^2}{c^2} E_{\omega,\bar{z}v} = \frac{i\omega \mu_0 q}{(2\pi)^2} \left( \varepsilon - \frac{c^2}{\omega^2} \right) e^{i\bar{z}v}. \tag{5}
\]

To solve equation (5), we may express \( E_{\omega,\bar{z}v} \) in each medium as a sum of the charge field \( E_{\omega,\bar{z}v}^R \) and the radiation field \( E_{\omega,\bar{z}v}^R \). If we solve equation (5) in a homogeneous medium, we can directly obtain the charge field in each medium, that is,

\[
E_{\omega,\bar{z}v}^R = \frac{-iq}{\omega \varepsilon (2\pi)^2} \frac{1 - \frac{c^2}{\omega^2}}{\varepsilon - \frac{c^2}{\omega^2}} e^{i\bar{z}v}. \tag{6}
\]

The charge field in equation (6) acts equivalently as the incident waves, while the radiation field can be understood as the scattered waves. Due to the momentum conservation, both the incident and scattered waves should have a same component of wavevector parallel to the interface (i.e. \( \bar{z}v \)). Then without loss of generality, we may further assume the radiation field in each medium with the following form.

\[
E_{\omega,\bar{z}v}^R = \frac{-iq}{\omega \varepsilon (2\pi)^2} \hat{a} \cdot e^{i\bar{z}v} \sqrt{\varepsilon - \frac{c^2}{\omega^2}}. \tag{7}
\]

Since the radiation field propagates away from the boundary, the + sign should be used for the forward radiation in medium 2, and the − sign should be adopted for the backward radiation in medium 1. Note that the radiation field in equation (7) corresponds to the excited photons if \( \bar{z}v \leq \sqrt{\varepsilon \omega}/c \), while it corresponds to the excited graphene plasmons if \( \bar{z}v > \sqrt{\varepsilon \omega}/c \).

The unknown parameter \( a \) in the radiation fields can be solved by enforcing the electromagnetic boundary conditions for the total fields, that is,

\[
\hat{n} \times (E_{1\perp} - E_{2\perp}) \big|_{z=0} = 0, \quad \hat{n} \times (H_{1\perp} - H_{2\perp}) \big|_{z=0} = \hat{J}_s = \sigma_s E_{1\perp} \big|_{z=0}. \tag{8}
\]

In the above, \( \hat{n} = -\hat{z} \), \( \hat{J}_s \) is the induced surface current density due to the existence of the monolayer graphene at the interface, and the subscript \( \perp \) stands for the field component parallel to the interface. Here the surface conductivity \( \sigma_s \) of graphene can be readily calculated from the Kubo formula [32]. That is, \( \sigma_s(\omega) = \sigma_s(\text{intra} \,(\omega)) + \sigma_s(\text{inter} \,(\omega)) \).

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where \( \sigma_{\text{intra}} (\omega) = \frac{\omega^2}{\pi \hbar c} \left( \frac{\mu_s}{e^2} + 2 \ln \left( e^{-\mu_s/\hbar c} + 1 \right) \right) \) considers the intraband contribution, and \( \sigma_{\text{inter}} (\omega) = \frac{\omega^2}{\pi \hbar c} \int_0^\infty \frac{f_0(x) - f_i(x)}{\omega^2 + (\omega^2 + 4k^2/\hbar^2)^2} \, dx \) considers the interband contribution, and \( f_0(x) = \left( e^{-\omega^2/\hbar^2} + 1 \right)^{-1} \) is the Fermi–Dirac distribution. Without specific specification, we set the chemical potential of graphene to be \( \mu_c = 0.4 \) eV, the relaxation time to be \( \tau = 0.5 \) ps and the temperature \( T = 300 \) K in this work [32].

From these boundary conditions, we obtain the amplitude \( a_1 \) for the backward radiation in medium 1 and the amplitude \( a_2 \) for the forward radiation in medium 2 as follows

\[
a_1 = -a_2 = a = \frac{\nu c \epsilon_r - \frac{k_0^2 \nu^2 c^2}{c^2}}{\left( 1 - \frac{\nu^2 c^2}{c^2} \epsilon_r + \frac{k_0^2 \nu^2 c^2}{c^2} \right)^2 \left( 2 \epsilon_r + \frac{2 \epsilon_r}{c^2} \sqrt{\epsilon_r - \frac{k_0^2 \nu^2 c^2}{c^2}} \right)} \tag{9}
\]

Note that the above radiation fields are solved in the Cartesian coordinates. Actually due to the rotational symmetry of our studied structure, all fields can be expressed in the cylindrical coordinates. After some calculations, we have,

\[
E_m^e (\hat{r}, t) = E_m^e (\hat{r}, t) + \tilde{E}_m^e (\hat{r}, t) \quad \text{and} \quad \tilde{H}_m^e (\hat{r}, t) = H_m^q (\hat{r}, t) + \tilde{H}_m^q (\hat{r}, t) \tag{10}
\]

\[
\tilde{E}_m^e (\hat{r}, t) = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{d\omega}{8 \pi \epsilon_0 \epsilon_m} \left( \frac{\omega^2}{c^2} - \frac{\omega^2}{c^2} \right) H_0^{(1)} \times \left( \rho \sqrt{\frac{\omega^2}{c^2} - \omega^2} \right) \tilde{\phi} (\hat{z} - \omega t) + \tilde{\rho} \int_{-\infty}^{+\infty} \frac{d\omega}{8 \pi \epsilon_0 \epsilon_m} \left( \frac{\omega^2}{c^2} - \omega^2 \right) \tilde{\phi} (\hat{z} - \omega t) \tag{11}
\]

\[
\tilde{H}_m^q (\hat{r}, t) = \tilde{\phi} \int_{-\infty}^{+\infty} \frac{dq}{8 \pi} \sqrt{\frac{\omega^2}{c^2} - \omega^2} H_1^{(1)} \times \left( \rho \sqrt{\frac{\omega^2}{c^2} - \omega^2} \right) \tilde{\phi} (\hat{z} - \omega t) \tag{12}
\]
The radiation field, 

$$E_m^R (\vec{r}, t) = \frac{e^{-i \omega t}}{i c} \int_{-\infty}^{+\infty} d\omega \int_{0}^{+\infty} d\kappa_{\perp} \cdot \frac{iq}{\omega \varepsilon(2\pi)} a_m \kappa_{\perp} (2\pi J_0 (\kappa_{\perp} \rho))$$

where 

$$a_m (\kappa_{\perp} \rho) = \frac{1}{\kappa_{\perp}} \int_{0}^{\infty} \frac{dq}{q} \frac{1}{\sqrt{\varepsilon_{mr} - \frac{q^2 c^2}{\omega^2}}} \times \left\{ - \varepsilon_{mr} \frac{\kappa_{\perp}^2 c^2}{\omega^2} \right\}$$

and 

$$\kappa_{\perp} = \sqrt{\frac{\varepsilon_{mr} - \frac{\kappa_{\perp}^2 c^2}{\omega^2}}{\varepsilon_{mr} - \frac{q^2 c^2}{\omega^2}}}$$

is the polariton pole singularity on the real \(\kappa_{\perp}\) axis. This way, if the whole system is lossless, we have 

$$\kappa_{\perp} = \sqrt{\frac{\varepsilon_{mr} - \frac{q^2 c^2}{\omega^2}}{\varepsilon_{mr} - \frac{q^2 c^2}{\omega^2}}}$$

as a real number, and there will be a polariton pole singularity on the real \(\kappa_{\perp}\) axis, rendering the integral in equations (13) and (14) well defined. To overcome this ambiguity, the material loss should be assumed for the monolayer graphene (i.e. with a finite value of relaxation time). Then the polariton pole singularity is now located off the real \(\kappa_{\perp}\) axis, and the integral in equations (13) and (14) becomes unambiguous. Mathematically, the numerical integration of the radiation fields in equations (13) and (14) can be readily carried out through the application of the Sommerfeld integration path [50, 51].

3. Near-field analysis of the transition radiation from graphene

Figure 2 shows the radiation field in the region close to the monolayer graphene. For conceptual demonstration, we set \(\nu = 0.8c\), namely the kinetic energy of electrons is 340 keV, which can be achieved in modern electron microscope systems [5]. To investigate the excitation of graphene plasmons, we plot in figure 2(a) the radiation field along a specific line, which is parallel to the graphene plane and only has a vertical distance of 10 nm to the graphene plane. From figure 2(a), we can clearly see the broadband excitation of graphene plasmons. Meanwhile, as the frequency increases, the wavelength of graphene plasmons decreases in a rapid way. Such a feature can also be seen in figures 2(b)–(g), where we plot the near-field radiation field at four different THz frequencies. Especially, at a higher frequency (e.g. 4 THz in figure 2(e)), the wavelength of graphene plasmons is much smaller than that of the excited photons. From figures 2(b)–(g), the spatial confinement of graphene plasmons rapidly decreases if the frequency increases. Moreover, figures 2(b)–(g) shows that both the graphene plasmons and photons are excited during the transition radiation process. While the excited photons can freely propagate to the far field, the excited graphene plasmons will decay during the propagation, due to the finite loss of graphene.

The graphene plasmons excited during the transition radiation process can also be analyzed in a quantitative way. That is, we can approximately calculate the spectrum for the excited graphene plasmons. To facilitate the analytical calculation, all material losses are neglected in this part. Such a simplified treatment (i.e. neglecting the material loss) is widely adopted in the spectrum calculation of surface plasmons excited by swift electrons; see for example in [2, 21]. When considering the material loss, the spectrum of excited surface plasmons has to be calculated numerically [24], which is rather tedious and time consuming. Under this simplified treatment, the energy of excited graphene plasmons is equivalent to the energy of the fields carried by the graphene plasmons at the time of \(t \rightarrow \infty\).

In other words, we can express the electromagnetic energy of the excited graphene plasmons as

$$W_n = \frac{\varepsilon_{mr} c^2}{\omega} \int_{-\infty}^{+\infty} d\vec{r} \int_{-\infty}^{+\infty} d\vec{r}' \left\{ E_n^R (\vec{r}, t) \cdot \frac{\partial}{\partial t} E_n^R (\vec{r}, t) \right\}$$

which is in accordance with Ginzburg and Frank’s theory of transition radiation for the calculation of excited surface plasmons [2].

By substituting equations (13) and (14) into (15) and symmetrizing the result through the replacement of \(\omega \rightarrow -\omega'\) and

$$W_n = \frac{\varepsilon_{mr} c^2}{\omega} \int_{-\infty}^{+\infty} d\vec{r} \int_{-\infty}^{+\infty} d\vec{r}' \left\{ E_n^R (\vec{r}, t) \cdot \frac{\partial}{\partial t} E_n^R (\vec{r}, t) \right\}$$

Since the charge field is the evanescent field and it will not contribute to any radiation, only the radiation field will contribute to the excited graphene plasmons. This way, we only have the radiation field in equation (15), which is in accordance with Ginzburg and Frank’s theory of transition radiation for the calculation of excited surface plasmons [2].
with the time. Since \( \omega \epsilon \) of electric fields of excited waves at 1 THz. The distribution of magnetic fields of excited waves at (b) 0.5 THz, (c) 1 THz, (d) 2 THz and (e) 4 THz. Here, \( \parallel \) and close to the graphene plane with a vertical distance of 10 nm; see the schematic illustration in the inset. (b)–(e) Near-field distribution of magnetic fields of excited waves at (b) 0.5 THz, (c) 1 THz, (d) 2 THz and (e) 4 THz. Here, \( v = 0.8c \). (f) and (g) Distribution of electric fields of excited waves at 1 THz.

\( \bar{\kappa} \parallel = -\bar{\kappa} \parallel \), we further have

\[
\begin{align*}
W^{\kappa} &= \lim_{t \to \infty} 2\pi \left\{ \frac{1}{\bar{\kappa} \parallel} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega' f(\omega, \omega') e^{-(\omega - \omega') t} \right. \\
& \quad \times \left[ \epsilon_{\omega, \kappa, \parallel}(z, \omega) - \epsilon_{\omega', \kappa, \parallel}(z, \omega') \right] E_{\omega, \kappa, \parallel}(z) E_{\omega', \kappa, \parallel}(z) \\
& \quad + \frac{\epsilon_{\omega, \kappa, \parallel}(z, \omega) - \epsilon_{\omega', \kappa, \parallel}(z, \omega')}{\omega - \omega'} E_{\omega, \kappa, \parallel}(z) E_{\omega', \kappa, \parallel}(z) \\
& \quad + \frac{\epsilon_{\omega, \kappa, \parallel}(z, \omega) - \epsilon_{\omega', \kappa, \parallel}(z, \omega')}{\omega - \omega'} E_{\omega, \kappa, \parallel}(z) E_{\omega', \kappa, \parallel}(z) \\
& \left. \right\}. \\
& \quad \quad \quad \quad \text{(16)}
\end{align*}
\]

The fact of \( \bar{H}_{\parallel} = 0 \) from equation (14) is used in equation (16). As shown in figure 1, we have the permittivity \( \epsilon_{\omega, \kappa, \parallel}(z) = \epsilon_{\omega, \kappa, \parallel}(z < 0) + \epsilon_{\omega, \kappa, \parallel}(z = 0) + \epsilon_{\omega, \kappa, \parallel}(z > 0) \) in equation (16). If \( \omega' \to \omega \), \( \omega' \epsilon_{\omega, \kappa, \parallel}(z, \omega') \) is equivalent to \( \partial \omega \epsilon_{\omega, \kappa, \parallel}(\omega) / \partial \omega \), and \( \omega \epsilon_{\omega', \kappa, \parallel}(z, \omega) \) is equivalent to \( \partial \omega \epsilon_{\omega, \kappa, \parallel}(\omega) / \partial \omega \). Note that in equation (16), the term with \( \omega' \neq \omega \) would oscillate quickly with the time. Since \( \omega \to \infty \), only the terms corresponding to \( \omega' = \omega \) should be kept in the calculation of equation (16). On the other hand, the integration of \( \bar{\zeta} \) in equation (16) would generate three parts of energy, which are below, within and above the monolayer graphene, respectively. Since \( \epsilon_{\omega, \kappa, \parallel} = \frac{\kappa_{\parallel}^2}{\omega} \delta(z) \) and \( \epsilon_{\omega, \kappa, \parallel} \) is a finite constant for the monolayer graphene, only the field components parallel to graphene plane would contribute to the calculation of the field energy inside the graphene.

Equation (16) can be simplified by using the knowledge of graphene plasmons. For transverse magnetic graphene plasmons, we have their dispersion as

\[
\begin{align*}
\epsilon_{\omega, \kappa, \parallel} \sqrt{\frac{\kappa_{\parallel}^2 c^2}{\omega^2} + \epsilon_{\omega, \kappa, \parallel} \sqrt{\frac{\kappa_{\parallel}^2 c^2}{\omega^2}}} + \frac{\kappa_{\parallel}^2 c^2}{\omega^2} \epsilon_{\omega, \kappa, \parallel} \sqrt{\frac{\kappa_{\parallel}^2 c^2}{\omega^2}} &= 0. \\
& \quad \quad \quad \quad \text{(17)}
\end{align*}
\]

To facilitate the calculation, we can define a quantity \( \bar{\zeta}_{\omega, \kappa, \parallel} \) according to equation (17) as
\[
\bar{\zeta}_{\omega,\kappa_\perp} = \varepsilon_{1r} \sqrt{\frac{\kappa_\perp^2 c^2}{\omega^2}} - \varepsilon_{2r} + \varepsilon_{2r} \sqrt{\frac{\kappa_\perp^2 c^2}{\omega^2}} - \varepsilon_{1r} \\
+ \frac{i \sigma_s \omega}{c \varepsilon_{0} \varepsilon_{r}} \sqrt{\frac{\kappa_\perp^2 c^2}{\omega^2}} - \varepsilon_{1r} \sqrt{\frac{\kappa_\perp^2 c^2}{\omega^2}} - \varepsilon_{2r}.
\]

As such, we can re-express the amplitudes \( a_1 \) and \( a_2 \) with the usage of equation (18) as

\[
a_2 = -i \frac{\omega}{|\omega|} b_{2\omega,\kappa_\perp} \quad a_1 = -i \frac{\omega}{|\omega|} b_{1\omega,\kappa_\perp}
\]

Note that for the symmetric case discussed in this work, we set \( \varepsilon_{1r} = \varepsilon_{2r} = \varepsilon_r \). Accordingly, we actually have \( a_1 = -a_2 = a \), \( \bar{\zeta}_{\omega,\kappa_\perp} = 2 \varepsilon_r + \frac{i \sigma_s \omega}{c \varepsilon_{0} \varepsilon_{r}} \sqrt{\frac{\kappa_\perp^2 c^2}{\omega^2}} - \varepsilon_r \), and \( b_{1\omega,\kappa_\perp} = -b_{2\omega,\kappa_\perp} = b_{\omega,\kappa_\perp} = \frac{\omega^2}{1 - \frac{2 \varepsilon_r}{c \omega} + \frac{\varepsilon_r^2}{c^2 \omega^2}} \). This way, the plasmon energy in equation (16) can be expressed in terms of \( b_{\omega,\kappa_\perp} \), that is,

\[
W^a = \lim_{r \to \infty} \frac{q^2}{32 \pi \varepsilon_0} \int_0^\infty d\omega \int_{-\infty}^{+\infty} d\omega \int_{-\infty}^{+\infty} d\omega' \frac{e^{-(\omega-\omega')^2}}{\bar{\zeta}_{\omega,\kappa_\perp} \zeta_{\omega',\kappa_\perp}} |b_{\omega,\kappa_\perp}|^2 \\
\times \left\{ \frac{\partial \omega (\omega, \omega')}{\partial \omega} \left( 2 - \frac{\varepsilon_r (\omega) \omega^2}{\kappa_\perp^2 c^2} \right) \right\} + \omega^2 \left( \frac{2 \varepsilon_r (\omega)}{\kappa_\perp^2 c^2} - \varepsilon_r (\omega) \right)
\]

We highlight that the term in the bracket of equation (20) is an even function about \( \omega \) and \( \kappa_\perp \). If \( f(\omega) \) is an even function, that is, \( f(\omega) = f(-\omega) \), we have

\[
\lim_{r \to \infty} \int_{-\infty}^{\infty} d\omega f(\omega) \int_{-\infty}^{+\infty} \frac{d\omega}{\bar{\zeta}_{\omega,\kappa_\perp}} \zeta_{\omega',\kappa_\perp} = 8 \pi^2 \int_{0}^{\infty} d\omega f(\omega) \\
\times \frac{\delta (\bar{\zeta}_{\omega,\kappa_\perp})}{(\partial / \partial \omega) \bar{\zeta}_{\omega,\kappa_\perp}}.
\]

From equation (21), we can further simplify equation (20) to the following

\[
W^a = \int_{0}^{\infty} w_{\text{plasmon}} (\omega) d\omega = \frac{q^2}{4 \pi \varepsilon_0} \int_{0}^{\infty} \frac{d\omega}{\omega^2} \int d\kappa_\perp^2 \\
\times |b_{\omega,\kappa_\perp}|^2 \delta (\bar{\zeta}_{\omega,\kappa_\perp}) \frac{\partial \omega (\omega, \omega')}{\partial \omega} \left( 2 - \frac{\varepsilon_r (\omega) \omega^2}{\kappa_\perp^2 c^2} \right) + \omega^2 \left( \frac{2 \varepsilon_r (\omega)}{\kappa_\perp^2 c^2} - \varepsilon_r (\omega) \right)
\]

where \( w_{\text{plasmon}} (\omega) \) is the spectrum of the excited graphene plasmons during the process of transition radiation.

After the integration of \( d\kappa_\perp^2 \), we can finally obtain the energy spectrum of graphene plasmons as

\[
w_{\text{plasmon}} (\omega) = \frac{q^2 \varepsilon_0}{\pi \varepsilon_r} \left[ \frac{2 \varepsilon_r^2}{c} \left( 2 \varepsilon_r (\omega) \right)^2 \right]^{\frac{1}{2}} - \frac{\varepsilon_r}{c} \left[ 2 \varepsilon_r - \frac{\omega^2}{c^2} \right]^{\frac{1}{2}}.
\]

Figure 3 shows the energy spectrum of excited graphene plasmons according to equation (23). We first discuss the influence of the particle velocity on the excitation of graphene plasmons in figure 3(a). From figure 3(a), a lower particle velocity can lead to the more efficient excitation of graphene plasmons in a broader frequency range. This is mainly because the particle with a smaller velocity can have a longer time of interaction with the monolayer graphene. This way, more energy can be extracted from the swift electron and transformed into the graphene plasmons. Moreover, the peak-intensity frequency for the excited graphene plasmons increases if the particle velocity decreases. As such, it is reasonable to conclude that the more excitation of graphene plasmons favors a lower particle velocity.

Correspondingly, we can further define the plasmon yield as

\[
N_{\text{plasmon}} = \int_{0}^{\omega_0} \frac{w_{\text{plasmon}} (\omega)}{\hbar \omega} d\omega
\]

where \( \hbar \omega_0 = 0.2 \text{ eV} \) is used in the calculation. The plasmon yield is relatively high during the process of transition radiation. For example, we have \( N_{\text{plasmon}} = 0.18 \) in figure 3(b), if \( v = 0.05c \).

On the other hand, we discuss the influence of the chemical potential of graphene on the excitation of graphene plasmons at a fixed particle velocity in figure 3(c). The chemical potential of graphene can be flexibly tuned, for example, by the electrostatic gating [52–58]. From figure 3(c), a higher chemical potential leads to the more efficient excitation of graphene plasmons at higher frequencies. Meanwhile, the peak-intensity frequency for the excited graphene plasmons also increases with the chemical potential.

4. Far-field analysis of the transition radiation from graphene

Figure 4 shows the spatial distribution of excited waves in the far field when a swift electron perpendicularly crosses a graphene monolayer. To be specific, figure 4 studies the influence of the particle velocity on the transition radiation from graphene in the far field. From figure 4, when we increase the particle velocity, the intensity of excited photons becomes stronger, along with a better radiation directivity. Especially, when \( v \to c \) (figure 4(f)), most of the excited photons would propagate along a direction almost perpendicular to the graphene plane. The underlying reason is that for a
Figure 3. Spectrum of the excited graphene plasmons. The structural setup is the same as figure 1. (a) Influence of the electron velocity $v$ on the spectrum of excited graphene plasmons. (b) Plasmon yield, namely $N_{\text{plasmon}} = \int_0^{\infty} \frac{n_{\text{plasmon}}(\omega)}{\hbar \omega} d\omega$, as a function of electron velocity $v$. Here we set $\hbar \omega_0 = 0.2$ eV in the calculation. We have $\mu_c = 0.4$ eV in (a) and (b). (c) Influence of $\mu_c$ on the spectrum of excited graphene plasmons. We set $v = 0.1c$ in (c).

larger particle velocity, a more spatially-extended charge disturbance or polarization current is induced in the graphene plane in the neighborhood of the charge trajectory.

To quantitatively discuss the far-field radiation, below we analytically calculate the backward radiation energy $W_1$ of excited photons, namely the energy of photons emitted into medium 1. In principle, the backward radiation energy can be obtained by integrating the field energy density over all space in medium 1 at $t \to \infty$. At $t \to \infty$, the emitted radiation field, which is a photonic pulse in the time domain, is already far away from the boundary and well separated from the charge field. Moreover, if we move the coordinate origin along the particle trajectory into the position having the emitted photonic pulse, the integration with respect to $z$ can be performed from $-\infty$ to $+\infty$, since the field of excited photons is attenuated in both directions away from the central position of the photonic pulse.

For the excited photons, the electric and magnetic energy densities are equal in free space [1, 2, 59]. This way, the backward radiation energy can be readily expressed as

$$W_1 = \int \mathrm{d}x \mathrm{d}y \int_{-\infty}^{+\infty} \mathrm{d}z \cdot \varepsilon_1 |E_1^\mathbf{R}(\hat{\mathbf{r}}, t)|^2$$

(25)
Figure 4. Far-field distribution of the excited waves when a swift electron perpendicularly crosses a monolayer graphene. (a)–(f) Spatial distribution of the magnetic field $H^R_\phi(\bar{r})$ with (a) $v = 0.7c$, (b) $v = 0.8c$, (c) $v = 0.9c$, (d) $v = 0.95c$, (e) $v = 0.99c$ and (f) $v = 0.998c$. Here the working frequency is 10 THz.

\[
\left| E^R_\perp(r,t) \right|^2 = \frac{\omega}{c} \int E^R_{\perp,\omega}(\hat{r},t) \cdot E^R_{\perp,\omega'}(\hat{z}) \times e^{i[(\kappa_\perp - \kappa'_\perp)\pi - (\omega - \omega')d\kappa_\perp d\kappa'_\perp d\omega d\omega']}
\]

By substituting equation (26) into equation (25) and integrating the integration over $d\kappa_\perp$ and $d\omega'$, we obtain

\[
W_1 = 2 \int_0^{+\infty} d\omega \int_0^\infty |a_1|^2 \left( \frac{q}{\omega\epsilon_0(2\pi)^3} \right)^2 \times \omega^2 \epsilon_\perp \sqrt{\epsilon_{1R}} \sqrt{1 - \frac{\kappa_\perp^2}{\omega^2\epsilon_{1R}}(2\pi)^3 d\kappa_\perp}
\]

For the emitted photons in medium 1, we have $\kappa_\perp^2 < \omega^2\epsilon_{1R}$. As such, the integration over $d\kappa_\perp$ in equation (27) is operated in the range $\kappa_\perp^2 < \omega^2\epsilon_{1R}$. We can further express $\kappa_\perp = \frac{\pi}{\sqrt{2}\epsilon_{1R}} \sin \theta$, by defining $\theta$ to be the angle between the wavevector of excite photons and $-\hat{v}$; see the schematic illustration in figure 4(a). Then we have $2\pi\kappa_\perp d\kappa_\perp = 2\pi \left( \frac{\pi}{\sqrt{2}\epsilon_{1R}} \right) \sin \theta \cos \theta d\theta$. By substituting this relation into equation (27), we obtain

\[
W_1 = \int_0^{+\pi/2} U_1(\omega,\theta,\beta) \cdot (2\pi\sin \theta) d\theta d\omega = \frac{\sqrt{\pi q^2 \beta^2 \cos^2 \theta \sin^2 \theta}}{4\pi^3 \epsilon_0 (1 - \epsilon_1 \beta^2 \cos^2 \theta)^2} \left[ \frac{\frac{\pi}{\sqrt{2}\epsilon_{1R}} \sin \theta \cos \theta}{2\sqrt{\epsilon_\perp + \frac{\epsilon_0}{\epsilon_{1R}} \cos \theta}} \right]^2
\]

Here $U_1(\omega,\theta,\beta) = \frac{\sqrt{\pi q^2 \beta^2 \cos^2 \theta \sin^2 \theta}}{4\pi^3 \epsilon_0 (1 - \epsilon_1 \beta^2 \cos^2 \theta)^2} \left[ \frac{\frac{\pi}{\sqrt{2}\epsilon_{1R}} \sin \theta \cos \theta}{2\sqrt{\epsilon_\perp + \frac{\epsilon_0}{\epsilon_{1R}} \cos \theta}} \right]^2$ is the angular spectral energy density of the backward radiation. By substituting the expression for $a_1$ in equation (9), we can write the backward angular spectral energy density explicitly as

\[
U_1(\omega,\theta,\beta) = \frac{\sqrt{\pi q^2 \beta^2 \cos^2 \theta \sin^2 \theta}}{4\pi^3 \epsilon_0 (1 - \epsilon_1 \beta^2 \cos^2 \theta)^2} \left[ \frac{\frac{\pi}{\sqrt{2}\epsilon_{1R}} \sin \theta \cos \theta}{2\sqrt{\epsilon_\perp + \frac{\epsilon_0}{\epsilon_{1R}} \cos \theta}} \right]^2
\]

In addition, if we define $W_1 = \int_0^{+\pi/2} U_1(\omega,\theta,\beta) = (2\pi\sin \theta) d\theta d\omega$, the energy spectrum of backward radiation can be derived as

\[
w_1(\omega,\beta) = \int_0^{+\pi/2} U_1(\omega,\theta,\beta) \cdot (2\pi\sin \theta) d\theta.
\]
Figure 5. Spectrum of the excited photons. The structural setup is the same as figure 1. (a) Angular spectral energy density of the backward radiation as a function of the radiation angle $\theta$ and the electron velocity $v$ at 10 THz. The peak-intensity angle is highlighted by the dashed line, and it approaches to zero if the value of $v$ approaches to $c$. (b) Spectrum of the excited photons as a function of the frequency and $v$.

The calculation of the energy spectrum for the forward radiation $w_2(\omega)$ can be performed by following a similar calculation procedure of $w_1(\omega)$. Then the total spectrum of emitted photon can be written as $w_{\text{photon}}(\omega) = w_1(\omega) + w_2(\omega)$. Since $\varepsilon_{1r} = \varepsilon_{2r} = \varepsilon_r$ in this work, we have $w_1(\omega) = w_2(\omega)$ and $w_{\text{photon}}(\omega) = 2w_1(\omega)$.

Figure 5(a) shows the backward angular spectral energy density of excited photons as a function of the radiation angle $\theta$ and the particle velocity at a fixed frequency of 10 THz, according to equation (29). From figure 5(a), the energy of emitted photons increases with the particle velocity. Meanwhile, we find that the angular spectral energy density has a smaller peak-intensity angle for a larger electron velocity (figure 5(a)). Particularly, under the condition of $\varepsilon_r = 1$, the peak-intensity angle appears approximately at $\theta = \sqrt{1-\beta^2}\sqrt{2\beta}$ if $\beta \to 1$, according to equation (29). These results are in accordance with the far-field distribution of excited waves in figure 4.

Figure 5(b) shows the spectrum of excited photons. From figure 5(b), a faster electron can give rise to the more efficient excitation of photons in a wider frequency range. In other words, the excitation of photons prefers a larger particle velocity, which is distinct from the excitation of graphene plasmons in figure 3. Last but not least, we highlight that from the comparison between figures 3 and 5, most of the electron energy loss is converted into the graphene plasmons during the transition radiation process. For example, if $v = 0.05c$, we have $\int w_{\text{photon}}(\omega) d\omega \int [w_{\text{plasmon}}(\omega) + w_{\text{photon}}(\omega)] d\omega > 99\%$. 

\[
\int w_{\text{plasmon}}(\omega) d\omega \int [w_{\text{plasmon}}(\omega) + w_{\text{photon}}(\omega)] d\omega > 99\%.
\]
5. Conclusion

In conclusion, we have systematically investigated the transition radiation from a monolayer graphene in the frequency domain. From the spatial distribution of the radiation field, both the graphene plasmons and photons can be excited in a broad bandwidth. From the analytical spectra of the excited photons and plasmons, we find that the excitation of graphene plasmons prefers a lower particle velocity, while the excitation of photons favors a higher particle velocity. Our results might provide theoretical guidance for the design of novel on-chip radiation sources [6, 7, 60–62] without any specific requirement on the particle velocity, especially for those with a low particle velocity. Note that while the on-chip radiation sources based on a low particle velocity are important both in fundamental science and practical applications, their realization remains challenging. Up to this day, one reported way for this goal is related to the Cherenkov radiation in a hyperbolic metamaterial, where the particle velocity can be down to 0.03c [6, 7]. On the other hand, we highlight that graphene is just a typical example of 2D materials. Due to the abundance of 2D materials and emerging physics (e.g. twisted optics [37, 40–43, 63]) in van der Waals heterostructures [30, 31], our work may ignite more fundamental researches about the electromagnetic radiation from the interaction between swift charged particles and various van der Waals materials or heterostructures, such as the in-plane anisotropic black phosphorous [36, 37], the uniaxial and hyperbolic hexagonal boron nitride [30, 31], and the biaxial and hyperbolic α-phase molybdenum trioxide [40–44], and twisted atomic bilayers [45, 46].

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