Coulomb effects on growth of instabilities
in asymmetric nuclear matter

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Abstract

We study the effects of the Coulomb interaction on the growth of unstable modes in asymmetric nuclear matter. In order to compare with previous calculations we use a semiclassical approach based on the linearized Vlasov equation. Moreover, a quantum calculation is performed within the R.P.A.. The Coulomb effects are a slowing down of the growth and the occurrence of a minimal wave vector for the onset of the instabilities. The quantum corrections cause a further decrease of the growth rates.

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Very recently there has been a growing number of studies on the properties of highly asymmetric nuclear matter. This can be mainly ascribed to recent experimental investigations on processes involving neutron–rich nuclei [1].

Under certain kinematic conditions, a nuclear collision can produce physical situations in which bulk or surface instabilities take place. Several theoretical models have been proposed in order to study such situations [2–4]. Among microscopic approaches, that based on the Boltzmann–Langevin equation is one of the most suitable to describe the large fluctuations observed in fragment formation [3–4]. In this scheme the instabilities of the self–consistent mean field can play a crucial role in enhancing mass and charge fluctuations [5].

In reactions between neutron-rich nuclei regions of high charge asymmetry, like the neck zone in semi–peripheral collisions, can be produced. In these regions the chemical ( or diffusive ) instability can be the most effective [8].

The thermodynamics of phase separation in asymmetric nuclear matter was first treated in studies concerning the structure of neutron stars [9]. More recently thermodynamic approaches have been developed with specific attention to critical situations that can be reached in collisions of heavy ions [8–10].

From thermodynamics one can obtain both the equilibrium configurations and the conditions for the onset of the chemical instability, i. e. the critical values of neutron and proton densities and of temperature for a given equation of state. Thus it is possible to draw the borders of the instability in the pressure–density plane for instance. The result is that for asymmetric nuclear matter the region of mechanical instability is embedded in the region of chemical instability [8]. This means that in asymmetric nuclear matter the effective spinodal region is defined by the chemical instability.

In this report, rather than with the thermodynamics of phase separation, we are concerned with the dynamical development of the instabilities in asymmetric nuclear matter. The growth rates of the various unstable modes in asymmetric nuclear matter have been already studied in Refs. [11,12] using a Skyrme force for the effective nucleon–nucleon interaction. Here we want to evaluate also the effects of the Coulomb force on the growth of
instabilities. It is well known that the Coulomb force gives a divergence in the energy density of infinite nuclear matter. Thus it cannot be taken into account in thermodynamic studies of such a physical system. However we will see that within a mean–field treatment of nuclear excitations, the contribution from the Coulomb force does not give rise to divergences. Moreover, we assume that the unperturbed initial state is uniform and homogeneous. In this case the mean field of the initial state does not appear in calculations explicitly. Only the temperature, the density and the asymmetry of the initial state actually occur in the relevant equations.

In the study of the nuclear dynamics by means of kinetic equations, the self–consistent mean field is usually treated in semiclassical approximation \[5–7\]. In order to compare with previous calculations, here we study the unstable modes of asymmetric nuclear matter by using the linearized Vlasov equation. This equation is a semiclassical approximation of the RPA, valid in the long–wavelength limit. However, we also evaluate the quantum corrections to the semiclassical results.

Since the RPA is a linear approximation, we expect that the validity of these calculations is limited to times close enough to the onset of instabilities. An estimate of the time interval, in which the RPA can be considered valid, is given in Ref. [12]. There, the numerical solutions of the non linear Vlasov equation have been compared to the analytical results of the linear approximation. The two procedures give quite similar results for times \[t \lesssim 150 \, fm/c\].

In our calculations we have used a Skyrme–like form for the nuclear part of the nucleon–nucleon effective interaction. We start from the following simplified functional for the density of the nuclear potential energy:

\[
E^{(N)}(\varrho, \varrho_A) = \frac{A}{2} \varrho^2 \varrho_{eq} + \frac{B}{\sigma + 2} \varrho^{\sigma + 2} \varrho_{eq} + \frac{D}{2} (\nabla \varrho)^2 + \frac{C}{2} \varrho_A^2 \varrho_{eq} - \frac{D'}{2} (\nabla \varrho_A)^2, \tag{1}
\]

where \(\varrho = \varrho_1 + \varrho_2\) is the total density, \(\varrho_1\) and \(\varrho_2\) are the proton and neutron densities respectively, \(\varrho_{eq}\) is the density of symmetric nuclear matter at saturation and \(\varrho_A = \varrho_2 - \varrho_1\). For the parameters in Eq. (1) we take the values:

\[A = -356.8 \, MeV, \quad B = 303.9 \, MeV, \quad \sigma = \frac{1}{6}\]
\[ C = 32 \text{ MeV}, \quad D = 130 \text{ MeV} \cdot \text{fm}^5, \quad D' = 40 \text{ MeV} \cdot \text{fm}^5. \]

The values of \( A, B \) and \( \sigma \) reproduce the binding energy (15.75 MeV) and give an incompressibility modulus of 201 MeV for symmetric nuclear matter at saturation with \( \rho_{eq} = 0.16 \text{ fm}^{-3} \). For the values of \( D \) and \( D' \) we follow the prescriptions of Ref. [13] and Ref. [9] respectively. Finally this value of the parameter \( C \) gives a symmetry energy coefficient of 28 MeV in the Bethe–Weizsäcker mass formula. The energy density of Eq. (1) coincides with that used in Ref. [12], except for the symmetry energy term for which we prefer the simplest required form.

Concerning the Coulomb energy density, we use the expression given by the Hartree–Fock approximation, with the Fock term evaluated in the local density approximation:

\[
E^{(C)}(r) = \frac{e^2}{2} \varrho_1(r) \int d r' \frac{\varrho_1(r')}{|r - r'|} - \frac{3}{4} \left( \frac{3}{\pi} \right)^{\frac{1}{3}} \frac{1}{r} e^{3r} \varrho_1^3. \tag{2}
\]

Within the density functional theory the potential energy functional

\[
E^{(pot)}[\varrho_1, \varrho_2] = \int d r \left( E^{(N)} + E^{(C)} \right) \tag{3}
\]

is the quantity, which allows to define an effective nucleon–nucleon interaction for R.P.A. calculations [14]. The interaction is given by the double functional derivative of \( E^{(pot)}[\varrho_1, \varrho_2] \)

\[
V_{i,j}(r, r') = \frac{\delta^2 E^{(pot)}}{\delta \varrho_i(r) \delta \varrho_j(r')} \bigg|_0 \tag{4}
\]

calculated in the unperturbed initial state. As expected, the Hartree term of the Coulomb energy yields the bare Coulomb force, whereas the remaining terms of Eq. (1) and the exchange Coulomb term give rise to zero–range forces.

The R.P.A. equations for the oscillations of the proton and neutron densities, \( \delta \varrho_1(r, t) \) and \( \delta \varrho_2(r, t) \), around their unperturbed values, can be obtained by standard procedures (e.g. see Ref. [12]). Here we report the resulting equations for their space and time Fourier transforms. For a wave of frequency \( \omega \) and wave–vector \( k \), the following two coupled equations are obtained
\[
\left(1 - \Phi_1^{(T)}(k, \omega)\right) \left(F_V + k^2(D + D') + \frac{C}{\rho_{eq}} + \frac{4\pi e^2}{k^2} + V_C^{(ex)}\right) \delta \rho_1(k, \omega) \\
- \Phi_1^{(T)}(k, \omega) \left(F_V + k^2(D - D') - \frac{C}{\rho_{eq}}\right) \delta \rho_2(k, \omega) = 0 ,
\]
(5a)

\[
- \Phi_2^{(T)}(k, \omega) \left(F_V + k^2(D - D') - \frac{C}{\rho_{eq}}\right) \delta \rho_1(k, \omega) \\
+ \left(1 - \Phi_2^{(T)}(k, \omega)\right) \left(F_V + k^2(D + D') + \frac{C}{\rho_{eq}}\right) \delta \rho_2(k, \omega) = 0 ,
\]
(5b)

where

\[
F_V = \frac{A}{\rho_{eq}} + (\sigma + 1)B \frac{\rho_0}{\rho_{eq}^{\sigma + 1}}
\]
(6)

is the volume term of the effective nuclear interaction and

\[
V_C^{(ex)} = -\frac{1}{3} \left(\frac{3}{\pi}\right)^{\frac{1}{3}} e^2 \rho_{01}^{\frac{2}{3}}
\]
(7)

is the exchange term of the Coulomb interaction. The quantities \(\rho_0\) and \(\rho_{01}\) denote the total and proton densities in the unperturbed initial state. In Eqs. \(\Phi^{(T)}(k, \omega)\) represents the free particle–hole propagator, which in the long–wavelength limit takes the form

\[
\Phi_i^{(T)}(k, \omega) = -\int d\epsilon_p \Phi(p, k, \omega) \frac{\partial n_i}{\partial \epsilon_p} ,
\]
(8)

where

\[
n_i = \frac{2}{e^{(\epsilon_p - \tilde{\mu}_i)/T} + 1}
\]
(9)

is the mean occupation number of protons or neutrons in the state with kinetic energy \(\epsilon_p = p^2/2m\) and

\[
\Phi(p, k, \omega) = \frac{1}{(2\pi)^3} \int d\Omega_p \frac{\mathbf{v} \cdot \mathbf{k}}{\omega + i\eta - \mathbf{v} \cdot \mathbf{k}}
\]
(10)

\(\mathbf{v} = p/m\) is the nucleon velocity. We use units such that \(\hbar = 1, c = 1\). The effective chemical potential \(\tilde{\mu}_i\) in the distribution \(n_i\), is the chemical potential measured with respect to the uniform mean field acting on the nucleons in the unperturbed initial state. For
a given temperature $T$ it is completely determined by the nucleon densities. Therefore the ingredients of Eqs. (3) related to the unperturbed initial state are only the neutron and proton densities, besides the temperature. Moreover, we remark that the product
$$
\Phi_1^{(T)}(k, \omega) 4\pi e^2 / k^2
$$
does not diverge when $k \to 0$.

The dispersion relation $\omega = \omega(k)$ is obtained by equating to zero the determinant of the set of equations (3). We look for solutions of Eqs. (3) for values of $\rho_0$, $T$ and the asymmetry parameter $\alpha = (\rho_{02} - \rho_{01})/\rho_0$ inside the region of instability [8]. In general we find that the frequency $\omega(k)$ is or purely imaginary or purely real. The growth rate of the instability is given by $\Gamma(k) = \text{Im} \omega(k)$.

In Figs. 1 and 2 the calculated growth rate is displayed as a function of $k$ for $\rho_0 = 0.4 \rho_{eq}$ and for various values of the parameters $\alpha$ and $T$. Figures 1 and 2 show that the Coulomb force causes an overall decrease of the growth rate $\Gamma(k)$. This decrease depends only slightly on the temperature ($\sim 10\%$) and it is almost independent of the asymmetry. Moreover we observe that, when the Coulomb force is included, the wave vector $k$ must exceed a certain value $k_{\text{min}}$ in order to get solutions of Eqs. (3) with $\Gamma(k) \neq 0$. Below $k_{\text{min}}$ the solutions of Eqs. (3) correspond to undamped plasmon–like oscillations, which practically involve the proton density alone. In the present case $k_{\text{min}}$ is about $1/5$ of the Fermi momentum of the symmetric nuclear matter ($\alpha = 0$). These two effects (decrease of $\Gamma(k)$ and appearance of $k_{\text{min}}$) are due to the competition between the Coulomb and nuclear forces (responsible for the chemical instability). The Coulomb force pushes the protons towards regions of lower density, the nuclear forces instead push the neutrons in the same direction. Explicit calculations, with and without the Coulomb force, confirm that the proton and neutron densities oscillate according to this picture.

So far we have presented results of calculations performed in the semiclassical limit of R.P.A.. For infinite systems this limit essentially coincides with the Vlasov equation. However, Figs. 1 and 2 show that the values of the wave vector of the most unstable modes is not a small fraction of the proton and neutron Fermi momenta (for $\alpha = 0$, $k/p_F \sim 0.6$). Therefore, an evaluation of the quantum corrections to the Vlasov equation is of interest.
Moreover, since for increasing values of $\alpha$ the Fermi momentum of the protons decreases appreciably, the importance of the quantum corrections can depend on the asymmetry degree of the nuclear matter. For infinite systems and for the nucleon–nucleon interaction used here, it is a rather simple matter to perform full quantum RPA calculation. It is sufficient to substitute in Eqs. (8) the semiclassical version of the free particle–hole propagator given by Eq. (8) with its full quantum expression. This can be obtained by a straightforward generalization to finite temperature of the expression for $T = 0$ given in Ref. [16].

In Figs. 3 and 4 we show the effects of the quantum corrections on the growth rates of the unstable modes. We observe a further quenching of the dispersion curves $\Gamma(k)$. To be more specific, the maxima of $\Gamma(k)$ are lowered by a factor of $\sim 0.85$ for all the values of temperature and asymmetry parameter $\alpha$ shown.

For symmetric nuclear matter, a similar evaluation of quantum effects has been done in Ref. [17], without the Coulomb interaction and with a different nuclear interaction. Neglecting the Coulomb interaction, for $\alpha = 0$ we obtain results in qualitative agreement with those of Ref. [17].

In conclusion, for the growth rate of the unstable modes the net result of the Coulomb effects and of the quantum corrections can be summarized by a suppression factor of $\sim 0.7$, which does not significantly change in the range of temperature and asymmetry considered here. Moreover, the long range character of the Coulomb force inhibits the formation of chemical instabilities of wavelength $\lambda > 2\pi/k_{\text{min}}$ ( $\approx 32 \, \text{fm}$ for $\rho_0 = 0.4\rho_{\text{eq}}$ ).

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FIGURES

FIG. 1. Growth rates of the unstable modes calculated in the semiclassical approximation (Eqs. (5)) for $T = 0$ and for three different values of the asymmetry parameter $\alpha$. From top to bottom $\alpha = 0.0, 0.3, 0.6$. The density of nuclear matter is $\varrho_0 = 0.4\varrho_{eq}$. Results with the complete interaction (solid lines) and without the Coulomb interaction (dashed lines).

FIG. 2. The same as Fig. 1, but for $T = 5\, MeV$

FIG. 3. Growth rates of the unstable modes, calculated with the quantum corrections (solid lines) and in the semiclassical approximation (dashed lines). The values of the parameters are the same as in Fig. 1.

FIG. 4. The same as Fig. 3 but for $T = 5\, MeV$. 
Fig. 1
Fig. 3
