We study the mechanism of transverse energy ($E_T$) production in Au+Au collisions at RHIC. The time evolution starting from the initial energy loss to the final $E_T$ production is closely examined in transport models. The relationship between the experimentally measured $E_T$ distribution and the maximum energy density achieved is discussed.

1 Introduction

The Relativistic Heavy Ion Collider (RHIC) will provide us with a unique opportunity to study matter under extreme conditions. The transition from hadronic degrees of freedom to quark and gluon degrees of freedom is expected from collisions at these energies. The transverse energy spectrum will be among the very first results from RHIC. One important question we would like to answer from the day-one physics at RHIC is the maximum energy density reached in central Au+Au collisions. Lattice QCD calculations give us the critical energy density for the transition to quark-gluon plasma. In this talk, we explore the relationship between the transverse energy spectrum and the energy density, and discuss the possibility of experimentally determine the maximum energy density.

A general framework for computing and analysing energy-momentum tensor is introduced. As the first part of this study, we construct the energy-momentum tensor for simulated events from transport model RQMD. We also find the transverse energy, the equation of state, and the energy density for these events. Currently we are using our techniques to study the parton cascade model VNI as well as other event generators.

2 Energy-Momentum Tensor in Transport Models

In transport model, the energy-momentum tensor

$$T^{\mu\nu}(\vec{x}, t) = \sum_i \int d^3p \frac{p^\mu p^\nu}{p^0} f_i(\vec{x}, \vec{p}, t),$$

(1)
and the particle number current
\[ j^\mu(\vec{x}, t) = \sum_i \int \frac{d^3p}{p^\mu} p^\mu f_i(\vec{x}, \vec{p}, t). \]  

(2)

where \( f_i(\vec{x}, \vec{p}, t) \) is the distribution functions for particle type \( i \).

In a relativistic cascade, each particle is represented by a point in both position and momentum space, and the distribution function is an ensemble average of the \( \delta \)-functions,
\[ f_i(\vec{x}, \vec{p}, t) = \left\langle \sum_k \delta^3(\vec{x} - \vec{r}_k(t))\delta^3(\vec{p} - \vec{p}_k(t)) \right\rangle. \]  

(3)

The energy-momentum tensor and the particle current become
\[ T^{\mu\nu}(\vec{x}, t) = \lim_{V \to 0} \frac{1}{V} \left\langle \sum_{k} \frac{\vec{x}_k(t) \in V}{p_k^\mu p_k^{\nu'}} \frac{p_k^{\mu'} p_k^{\nu}}{p_k^\mu} \right\rangle, \]  

(4)

and
\[ j^\mu(\vec{x}, t) = \lim_{V \to 0} \frac{1}{V} \left\langle \sum_{k} \frac{\vec{x}_k(t) \in V}{p_k^\mu} \frac{p_k^\mu}{p_k^\mu} \right\rangle. \]  

(5)

At any given space-time location, \( T^{\mu\nu} \), being a symmetric tensor, has ten independent elements from which we can obtain ten physical quantities: local energy density \( \epsilon \), local pressures \( P_1, P_2, P_3 \), the flow velocity \( \vec{v}_f \), and the orientation of the principal momentum axes. Following the convention of Landau and Lifshitz, the local rest frame is defined as the frame in which
\[ T^{\mu\nu} = \begin{pmatrix} \epsilon & 0 \\ 0 & T^{ij} \end{pmatrix}. \]  

(6)

The Lorentz boost of \( T^{\mu\nu} \) to the above form gives us the flow velocity. The momentum tensor \( T^{ij} \) can be diagonalized by performing a rotation. After the boost and the rotation
\[ T^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & P_1 & 0 \\ 0 & 0 & P_2 \end{pmatrix}. \]  

(7)

The local particle density
\[ \rho \equiv j^\mu u_\mu. \]  

(8)
where $u^\mu = (\gamma, \gamma \vec{v})$ is the velocity four vector. We further define

$$\mathcal{P} \equiv (\mathcal{P}_1 + \mathcal{P}_2 + \mathcal{P}_3)/3 \quad \text{and} \quad T \equiv \mathcal{P}/\rho .$$

If there is local thermal equilibrium, then $\mathcal{P}$ would be the pressure and $T$ would correspond to the temperature.

### 3 Event Generator Studies

There are, in general, two stages in the evolution of $E_t$. Initially, $E_t$ increases due to the transverse excitations from interactions. Afterwards, the longitudinal expansion results in a decreasing $E_t$. What we measure in the experiment is the final $E_t$ at the end of this expansion. The details of the $E_t$ evolution, like all other observables in the relativistic heavy ion collisions, is model dependent. In order to get a reasonable estimate of the energy density through measurement, we need a systematic study of all plausible models. The transverse energy $E_t$ and the energy density $\epsilon$ from the transport model RQMD is shown in Figure 1.
Figure 2. For the same set of collisions from RQMD v2.4 as in Figure 1: (a) The ratio of local pressure, defined by Eq. (9), to local energy density as a function of time at various positions ($z = 0$ and $r_t = 0, 4, 8$ fm); (b) Flow velocities for $\pi, K$, and for other particles as a function of time at $z = 0$ and $r_t = 6$ fm.

A total of 144 central ($b = 0$) Au+Au events, with full event histories, at RHIC energy (100 GeV/A+100 GeV/A) was generated using RQMD version 2.4. The energy-momentum tensor and the particle current is computed according to Eqs. (4) and (5), where the small volume $V$ is chosen to be a sphere of an 1 fm radius. The local energy density $\epsilon$, pressure $P$, and the flow velocity $v_f$ are obtained from Eqs. (6)-(9). Figure 2 shows the variation of $P/\epsilon$ and the development of transverse flow velocity.

In the RQMD events, during the early stage, the longitudinal pressure is higher than the transverse pressure. After about 7 fm/c, the system is approximately isotropic, but it is still not in a local thermal equilibrium. Figure 2(b) displays one such nonthermal behaviour in the flow velocity distribution. Particles with lighter masses, e.g. pions and Kaons, have greater collective velocities than heavier particles, i.e. there is no common flow velocity.

Clearly, the system in Figures 1 and 2, can not be described by a simple Bjorken expansion picture. There are expansions along both the longitudinal and the transverse directions, and the system is not in a local thermal equilib-
rium. The time evolution of $\epsilon$ and $E_t$, in Figure 1, are not exactly correlated. So the measurement of $E_t$ by itself is not sufficient to determine $\epsilon$.

The initial state in RQMD is not partonic, so it may not be applicable for RHIC. But the model may still have a reasonable parametrisation for stopping and for initial transverse energy production, and it has a well tested hadronic final state after-burner. Models like VNI, HIJING, VENUS, and UrQMD have quite different descriptions of the initial stages at RHIC. By coupling these initial conditions with a common hadronic after-burner, we will have a better understanding of the relationship between the observables and the maximum energy density.

4 Remarks

An estimate of the maximum energy density might be possible if, in addition to transverse energy, we also have a good measurement of several collective observables such as the radial and elliptic flows, the system size from HBT, and other correlations. This kind of estimate would still be model dependent, so we need to make a survey of all models.

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