Fuzzy PID Control of the Magnetic Levitation Ball System
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Abstract. The magnetic levitation ball system has nonlinear and open loop instability. The traditional PID control system has good stability. But when the system is disturbed by the outside factor, the control performance of the system is limited because of the fixed PID parameters. In this paper, a fuzzy PID controller which could adjust the PID parameters on-line to enhance the robustness of the system was designed. The experimental study was carried out on the dSPACE platform. The experimental results show that the proposed control strategy exhibits better control character than that of the traditional PID control. The fuzzy PID control can reduce the overshoot of the system and has better anti-interference capability.

Introduction
Magnetic levitation ball system is a single degree of freedom and open-loop unstable nonlinear system, in which the parameter perturbation and external uncertain interference factors will influence the stability of system and other control performance. The traditional PID [1] control system has good stability. But when the system is subject to the parameter variation or outside interference, the use of fixed controller parameters will make the control system performance worse.

Aiming at the defects of traditional PID, a lot of new PID algorithms such as neural network PID [2-4], fuzzy PID [5-11], fuzzy neural PID [12], sliding PID [13], internal PID [14] are proposed and applied to the practical industrial process control to obtain good control effect. In this paper, the fuzzy control combined with PID control was designed for magnetic levitation ball system. The fuzzy PID controller with adaptive ability revised PID parameters constantly by using fuzzy control to make the system a certain adaptive ability, which could enhance both the control performance of the system in each stage and the robustness to external interference as well as its parameter perturbation. The experimental results based on the dSPACE platform show that the fuzzy PID can significantly reduce the overshoot of the system, and have better anti-jamming performance.

Mathematical Model of the System
The control structure of the magnetic levitation ball system is shown in Fig. 1.

In the picture, \( m \) is the steel ball quality, \( g \) is the gravity acceleration, \( x \) is the distance between the center of steel ball and the electromagnet pole and \( f \) is the electromagnetic force. \( u \) provides the...
control voltage for the external circuit and \( u_e \) is the sensor output voltage corresponding to the steel ball position.

Ignoring the influence of other external interference factors, the small ball is attracted by electromagnetic force \( F(i,x) \) and the impact of its own gravity \( mg \) in the vertical direction. By analyzing the mechanism of the magnetic levitation ball system, the following mathematical model can be established:

Kinetic equation: \[ m \frac{d^2x(t)}{dt^2} = mg - F(i,x) \] (1)

Electromagnetic mechanical equation: \[ F(i,x) = K \left( \frac{i}{x} \right)^2 \] (2)

Electrical equation: \[ U(t) = Ri(t) + L \frac{di(t)}{dt} \] (3)

In the above formula, \( x \) is the distance between the center of steel ball and the electromagnet pole, \( i \) is the current in the electromagnetic coil, \( m \) is the mass of the steel ball, \( g \) is the gravity acceleration, \( R \) is the resistance of the electromagnetic coil, and \( L \) is the inductance of the electromagnetic coil.

When the steel ball is in a stable suspension as \( x=x_0 \), the current in the electromagnetic coil is \( i=i_0 \), the equation of the ball at equilibrium point is

\[ F(i_0,x_0) = mg \] (4)

In order to facilitate the design of the controller, the system is linearized at the equilibrium position \((i_0, x_0)\). The \( F(i,x) \) in the Eq. 1 is carried out by the Taylor series expansion, and the high order term is neglected and Laplace transform is performed. The result is

\[ \frac{X(s)}{I(s)} = \frac{-1}{(i_0/2g)s^2 - i_0/x_0} \] (5)

In the actual control, the controlled variable is the input voltage of the power amplifier, and the relationship between the output of the sensor \( u_e \) and the input voltage of the power amplifier \( u \) is:

\[ G(s) = \frac{U_x(s)}{U(s)} = \frac{-(K_x/K_a)}{(i_0/2g)s^2 - i_0/x_0} \] (6)

where \( K_x \) is sensor gain, \( K_a \) is control voltage to coil current gain.

By the Eq. 6, it is known that the magnetic levitation system is an unstable second-order object. Therefore, it is necessary to combine the closed-loop control with the controller to make the controlled ball reach the stable suspension state.
The Design of Fuzzy PID Controller

\[ u_k = K_p e_k + K_i \sum_{j=0}^{k} e_j + \frac{K_d}{T}(e_k - e_{k-1}) \]  

(7)

in which \( u_k \) is the output of the controller at time \( k \), \( e_k \) is the input signal of the controller, \( K_p \), \( K_i \) and \( K_d \) are proportional, integral and differential coefficient respectively and \( T \) denotes the system sampling time.

When PID control parameters are regulated, their respective impact on system performance and mutual interconnected relationship at different times must be considered. Typically, proportional control can speed up the system response speed, integral control is used to eliminate the steady-state error of the system and differential link is used to restrain the change of deviation.

The structure of fuzzy PID controller is shown as Fig. 2. The input parameters of fuzzy PID controller are error \( e \) and error change rate \( e_c \). The output value is gotten after defuzzification and fuzzy inference. PID controller adjusts its parameters according to the output value of the fuzzy control.

The introduction of fuzzy control is to detect the response error \( e \) and change rate \( e_c \) at each sampling time, and then the correction of PID parameter is drawn according to the good fuzzy rules. So PID controller adjusts its parameter size according to the change of the system response to enhance both dynamic response performance and robustness to external disturbance.

The design of fuzzy PID controller includes the fuzzification of input and output variables, fuzzy rules, fuzzy reasoning and defuzzification.

**The fuzzification of input and output variables**

The displacement deviation \( e \) of ball and its change rate \( e_c \) are selected as the input variables of the fuzzy controller. The fuzzy variables \( E \) and \( E_c \) are obtained after the effect of quantitative factor. The output fuzzy variables are \( \Delta k_p \), \( \Delta k_i \) and \( \Delta k_d \). The fuzzy domain and membership function of the input and output are shown in Fig. 3 and Fig. 4 respectively.

**The determination of the fuzzy rules**

According to the influence of PID parameters on the system performance and the experimental parameter adjustment experience, the fuzzy PID control parameter tuning principle that enables the system to obtain the best response performance is shown in Table 1 - 3.
Table 1. Fuzzy rule table of $\Delta k_p$.

| $e$   | NB | NM | NS | ZO | PS | PM | PB |
|-------|----|----|----|----|----|----|----|
| NB    | NB | NB | NB | NB | NB | NB | NB |
| NM    | NM | NM | NM | NM | NM | NM | NM |
| NS    | NS | NS | NS | NS | NS | NS | NS |
| ZO    | ZO | ZO | ZO | ZO | ZO | ZO | ZO |
| PS    | PS | PS | PS | PS | PS | PS | PS |
| PM    | PM | PM | PM | PM | PM | PM | PM |
| PB    | PB | PB | PB | PB | PB | PB | PB |

Table 2. Fuzzy rule table of $\Delta k_i$.

| $e$   | NB | NM | NS | ZO | PS | PM | PB |
|-------|----|----|----|----|----|----|----|
| NB    | NB | NB | NB | NB | NB | NB | NB |
| NM    | NM | NM | NM | NM | NM | NM | NM |
| NS    | NS | NS | NS | NS | NS | NS | NS |
| ZO    | ZO | ZO | ZO | ZO | ZO | ZO | ZO |
| PS    | PS | PS | PS | PS | PS | PS | PS |
| PM    | PM | PM | PM | PM | PM | PM | PM |
| PB    | PB | PB | PB | PB | PB | PB | PB |

Table 3. Fuzzy rule table of $\Delta k_d$.

| $e$   | NB | NM | NS | ZO | PS | PM | PB |
|-------|----|----|----|----|----|----|----|
| NB    | NB | NB | NB | NB | NB | NB | NB |
| NM    | NM | NM | NM | NM | NM | NM | NM |
| NS    | NS | NS | NS | NS | NS | NS | NS |
| ZO    | ZO | ZO | ZO | ZO | ZO | ZO | ZO |
| PS    | PS | PS | PS | PS | PS | PS | PS |
| PM    | PM | PM | PM | PM | PM | PM | PM |
| PB    | PB | PB | PB | PB | PB | PB | PB |

Fuzzy reasoning and defuzzification

According to the fuzzy rules, the control input $e$ at each sampling time and change rate $e_c$ are fuzzied $E$ and $E_c$. Through fuzzy reasoning and defuzzification the corresponding fuzzy outputs $\Delta k_p$, $\Delta k_i$, $\Delta k_d$ can be drawn.

The membership degree of the first fuzzy rule corresponding to $\Delta k_p$ is as follows:

$$\mu_{\Delta k_p} = \mu_{NB}(E) \star \mu_{NB}(E_c)$$

where the type of operator ‘$\star$’ is the representation of small value, i.e.

$$\mu_{\Delta k_p} = \min \{\mu_{NB}(E), \mu_{NB}(E_c)\}$$

Order by analogy, the membership degree of all fuzzy rules of the output $\Delta k_p$ corresponding to different deviation and change rate can be obtained. By the method of gravity defuzzification of each fuzzy rule, the fuzzy value of available $\Delta k_p$ is as follows:

$$\Delta k_p = \frac{\sum_{j=1}^{49} \mu_{k_{pj}}(\Delta k_p) \Delta k_{pj}}{\sum_{j=1}^{49} \mu_{k_{pj}}(\Delta k_p)}$$

where $\Delta k_{pj}$ is the real value on the domain $\Delta k_p = [-6, 6]$ and $\mu_{k_{pj}}$ is the activation of corresponding fuzzy rules.

Similarly, the fuzzy output values of $\Delta k_i$ and $\Delta k_d$ for each sampling period can be obtained. However, these values are still the corresponding fuzzy values in their domain, so the actual output values $\Delta k_p$, $\Delta k_i$, and $\Delta k_d$ can be gotten after the fuzzy values respectively multiplied by their respective proportion factor. The adjustment algorithm of controller parameters updated corresponding to the system deviation $e$ and its change rate $e_c$ is as follows:

$$k_p = k_{p0} + \Delta k_p$$
$$k_i = k_{i0} + \Delta k_i$$
$$k_d = k_{d0} + \Delta k_d$$

where $k_{p0}$, $k_{i0}$ and $k_{d0}$ are the initial values of $k_p$, $k_i$ and $k_d$.

The Experimental Study

The parameters of the physical system considered are depicted as follows:

$$i_0 = 0.6105 A, \quad x_0 = 0.02 m$$
\[ K_a = 5.8929 \text{A/V}, \quad K_x = -458.7156 \text{V/m} \quad (12) \]

\[ g = 9.8 \text{m/s}^2 \]

With these parameters, the mathematical model of magnetic suspension system is

\[ G(s) = \frac{2499.1}{s^2 - 980} \quad (13) \]

A position command is the voltage signal corresponding to the suspension position. PID controller and fuzzy PID controller are respectively applied to the system. \( K_p, K_i \) and \( K_d \) of PID control is respectively taken as 1.6, 10 and 0.045. \( k_{p0}, k_{i0} \) and \( k_{d0} \) of fuzzy PID controller is 1.6, 10 and 0.045. The quantification factor of \( e \) and \( e_c \) respectively taken as 1.5/3 and 6/40. The proportion factor of \( \Delta k_p, \Delta k_i \) and \( \Delta k_d \) are respectively 0.2/6, 1/6 and 0.006/6. The sampling period \( T \) is 0.001s.

In this paper, the experiment is done based on the dSPACE hardware platform. The real time control platform of magnetic levitation ball system includes industrial control computer, hardware interface circuit, and magnetic suspension experiment device and so on. This platform is shown in Fig. 5.

![Real-time control platform of magnetic levitation ball system](image)

In the experiment, the given position voltage is -3V, and the manual interference is added after the magnetic levitation ball is stable for 15s. When the conventional PID control and fuzzy PID control are used respectively, the real-time position of the system is shown in Fig. 6. If the overshoot \( \sigma \), adjustment time \( t_s \) and transition time \( t_t \) after interference are chosen as the performance index of the system, the performance of the two control methods is shown in Table 4.

![Ball position curves of magnetic levitation ball system](image)

(a) The result of PID control

(b) The result of fuzzy PID control

Figure 6. Ball position curves of magnetic levitation ball system.
Table 4. Performance comparison of different controller.

| Control method | Tracking set value | Transition time after disturbance |
|----------------|--------------------|----------------------------------|
|                | $\sigma$ | $t_s$ | $t_t$ |
| PID            | 7.7%    | 2.5s  | 2.1s  |
| Fuzzy PID      | 0       | 1.3s  | 0.6s  |

From Fig. 6 and Table 4, we can see that the overshoot and adjusting time of the system can be reduced and the system has better anti jamming performance compared with the conventional PID control when the fuzzy PID control algorithm is applied to the magnetic suspension system.

Summary

Based on the analysis of conventional PID control limitations and aimed at the magnetic levitation ball system nonlinear and open-loop unstable, the fuzzy PID control algorithm is used for the magnetic levitation system. In order to enhance both the control performance of the system in each stage and the robustness to external interference as well as its parameter perturbation, PID parameters are constantly revised by using the fuzzy control to have a certain adaptive ability. The experimental results show that fuzzy PID has better performances than the conventional PID in the process of the whole dynamic response.

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