Improving the Performance of Layered Iterative minsum Approximation Message Passing Decoding Algorithm for 5G NR LDPC Codes

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Abstract. In order to cater to the needs of the 5G standard, one of the prominent solutions in terms of high coding gain, high throughput and low power dissipation is the 5G NR LDPC (New Radio Low Density Parity Check) codes which have been adopted by 5G Standard. LDPC codes are the successful Third Generation Forward Error Correcting codes. An Efficient decoder is the key to the success of any Error Correcting Code. LDPC codes are one whose parity check matrix have a very few non zero entries. 5G NR LDPC codes are constructed by Protographs or base graphs and are block structured. This paper improves the performance of Layered Iterative minsum Approximation Message Passing decoding Algorithm using Rate Matching, Fixed Point Quantization and Offset minsum techniques. Rate Matching is to maintain the exact number of encoded blocks with in the TTI. Simulation of this algorithm with BPSK encoding over AWGN channel shows that BER of 10^{-3} gives Eb/N0 of ~2 dB for the block length of 10^5.

1. Introduction

The noise is expected in all communication systems. In digital communication, this same noise is labelled as error bits. In late 1940’s coding theory have been evolved to detect and correct these error bits [1] [2]. Since FEC (Forward Error Correcting) codes performs better than Automatic Repeat Request (ARQ) in supporting real time applications, FEC mechanism became popular in higher data rate applications. Now a days the increasing data rate applications leads to an efficient third generation FEC Codes SISO LDPC of with very few non zero elements in the parity check matrix. LDPC codes are one of the codes which are represented by Parity Check Matrix. In 1962, Robert Gallager first introduced these LDPC codes in his thesis [3]. After three decades (in 1997), the same codes were re-invented by Mackay and Neal [4]. LDPC codes are having high coding gain because it gives improved error performance (High BER) at a low Eb/No dB. A regular LDPC (k,n) code is constructed by Parity Check matrix of n-k x n. Where k is the number of message bits and n is the number of code word bits. Here n-k is the number rows of parity check bits.H matrix can be represented by bipartite graph by Tanner [5]. Because of parallel architecture in quasi cyclic LDPC codes [6], have been implemented.

Introducing Iterative decoding approach in a SISO decoder was a great break point. To improve belief or Predictions about the received bit, Iterations were implemented in the decoding repeatedly [7, 8]. BP (Belief Propagation) algorithm has been introduced in designing of LDPC decoder [9]. But still
design complexity is the cost of this BP algorithm implementation [10]. Minsum approximation using logarithmic scale can be used to overcome this design complexity significantly [11]. Now In 2017, The new 5G NR LDPC encoding schemes were introduced by the recent release 15 5G new radio access technology standards by 3rd Generation Partnership Project (3GPP) [12]. This 3GPP Technical report, specifies 3 main usage scenarios – enhanced Mobile Broad Band (eMBB) which needs high data rate and coverage area, Ultra Reliable and Low Latency Communications (URLLC) which involves remote surgery and autonomous vehicles, and the other one is massive Machine Type Communications (mMTC) which supports Internet of Things (IoT) applications. LDPC codes have been adopted by 5G standard, this new version is called as 5G NR LDPC codes.

This iterative minsum message passing decoding algorithm (IMMPDA) [13] has been implemented in 5G NR LDPC codes for both BG1 and BG2. Simulation results are computed and achieved approximately 9dB coding gain for BER of $10^{-6}$ [13]. Then, Layered Approach has been implemented for LDPC decoder [14] which reduces the decoding complexity. Modified (IMMPDA) using layered approach has been introduced for 5G NR LDPC codes [15]. Physical Layer data has been released by 3GPP standard in 2018[17]. Rate matching concept is being adopted for 5G LDPC codes according to this standard [19]. Also Fixed Point quantization and offset minsum are explained in the 5G NR LDPC codes to improve the performance of the decoding algorithm [20]. This paper includes rate matching, Fixed Point Quantization [21] and offset minsum [22] to improve the decoding performance.

The rest of the paper is organized as follows: Section II describes the 5G NR LDPC codes. Section III presents an information of Improved Layered minsum iterative message passing algorithm. Section IV analyses the simulation results. Section V discusses the conclusion of the work.

2. 5G NR LDPC Codes

3GPP Technical report [12], Specifies 3 main usage scenarios – enhanced Mobile Broad Band (eMBB) which needs high data rate and coverage area, Ultra Reliable and Low Latency Communications (URLLC) which involves remote surgery and autonomous vehicles, and the other one is massive Machine Type Communications (mMTC) which supports Internet of Things (IoT) applications. In all the above scenarios the significant speed, low latency, high throughput and low power dissipation are required. To fulfill these requirements, 5G NR (New Radio) uses advanced channel coding Techniques [16].Parity Check Matrix is constructed by expanding these Base graphs or base matrices [18].

2.1. Base Graph Selection

**Step 1:** Obtain the base graph BG1 or BG2 for the given K (Transport Block) and R (Code Rate) [16,18] LDPC base graph selection.

As per the specification [16], if $K \leq 3824$ and $R = 0.67$ then BG2 is selected. If $K \leq 292$ then BG2 is selected. If $R \leq 0.25$ then BG2 is selected. Else BG1 is selected which is described in step 1.

**Step 2:** Determine the value of $K_b$ for the given K (Transport Block) and R (Code Rate) $K_b$ denotes the number of information bit columns for the lifting size $Z_c$. As per Specification [16]

For LDPC BG1, $K_b = 22$

For LDPC BG2, if $K > 640$ then $K_b = 10$; if $K$ is between 560 < $K$ <= 640 then $K_b = 9$; if $K$
is between $192 < K <= 560$ then $K_b = 8$; If $K$ is $<= 192$ then $K_b$ is $= 6$

Base matrix can be denoted by type of base graph, index $i$ and the expansion factor $Z_c$. For Ex. $(2\_6\_52)$

Double diagonal structured $E$ matrix provides an efficient way of computing single parity check code [20]. Hence in the 5G NR LDPC codes, SISO decoder for SPC code is the heart of decoder designing [18][13].

2.2. Decoding [20]

BPSK over AWGN channel is used, SISO decoder[13]:

Log Likelihood Ratio (LLR) of the received $i^{th}$ bit is calculated as in[13]:

$$P(C_1=0/r_1)/P(C_1=1/r_1) = f(r_1/C_1=0)/f(r_1/C_1=1).$$

for $C_1 = 0$, the symbol is +1; then $r_1 = 1+N(0,\sigma^2)$ and

for $C_1 = 1$, the symbol is -1; then $r_1 = -1+N(0,\sigma^2)$

$$L_1 = (2/\sigma^2) * r_1$$

3. Improving Layered Minsum Iterative Message Passing Algorithm (ILMIMPA)

In (ILMIMPA), column operations are being done more aggressively. No need to wait for all the rows to be completed. Since 5G NR LDPC codes are block structured, each expanded row by naturally are supporting layered architecture.

3.1 Algorithm [13]

Group rows of $H$ Matrix into one layer, after finishing that layer do the column operation. Column weight of the each layer is one (1 block row in 5G NR LDPC codes).

Step 1: Initialization (incoming received vector is being stored in $H$ matrix in 1’s places alone).

Step 2: Start with this received vector and start going down the parity check matrix row by row. For each

Process the receiving vector is updated.

Step 3: minsum approximation for that layer alone is implemented as per [13].

Step 4: Adding this layer with the received vector, now the receiving vector is updated.

Step 5: this updated value is used for the next layer (replaced in the nonzero entry places)

Step 6: Subtracting these values with the incoming beliefs.

Step 7: Repeat the process till it converges.

Step 8: Decision – if incoming belief ($S_j > 0$),

Then bit $j = 0$; If ($S_j < 0$), then bit $j = 1$
Since each layer is updated with the incoming beliefs, this algorithm converges fast and it needs less number of iterations.

3.2. Rate Matching, Shortening and Puncturing

According to the Specification [16], two base graphs have been proposed with standard size. But in real time Rate Matching is needed to maintain the exact number of encoded blocks with in the given TTI [17, 19]. To achieve this Puncturing and Shortening Techniques are used. Blocking first two columns of base graphs from transmission is called puncturing. But these punctured bits are to be decoded. Set zero to all extra message blocks is called shortening [20].

**Rate Matching for different code rates for BG1 [19]**

| Expansion Factor = Zc |
|-----------------------|
| K = messages bits     |
| N= Total Code word bits |
| R = Code Rate         |
| BG1 has 68 Column & 48 Rows |
| First two columns are always punctured = 2Zc bits |

As per specification for BG1 systematic/message bits "K" = 22Zc bits (including the first two punctured columns)
- Number of total possible CodeWord bits "N" = 68Zc bits
- Since first two columns are punctured total possible CW Bits "N" = 66Zc bits
- Hence minimum Code Rate "R" supported by BG1 = K/N = 22/66 = 1/3 = 0.333
- 20Zc are message bits & 46Zc are Parity check bits & 2Zc are punctured.

**BG1: Rate Matched to code rate 1/2**

| Systematic/message bits "K" = 22Zc bits (including the first two punctured columns) |
| Number of total possible Code word bits "N" = 44Zc bits |
| First 44Zc bits are transmitted, 20Zc is message and 24Zc are parity bits. |

Hence Code Rate "R" = K/N = 22/44 = 1/2 = 0.5 (at the decoder even though the first two columns are punctured, they have to be decoded so all 22Zc bits need to be considered for Code Rate).

For Rate match 1/2 only the LDPC encoded bit blocks between Column 44 and Row 24 of the base graph are transmitted, First two columns message bits are punctured so they are not transmitted, however at the decoder side they are retrieved and decoded.

Similarly for BG2 minimum supported code rate is:
| Expansion Factor = Zc |
|-----------------------|
| K = messages bits     |
| N= Total Code word bits |
| R = Code Rate         |
| BG2 has 52 Column & 42 Rows |
| First two columns are always punctured = 2Zc bits |

As per specification for BG2 systematic/message bits "K" = 10Zc bits (including the first two punctured columns)
- Number of total possible Code word bits "N" = 52Zc bits
- Since first two columns are punctured total possible CW Bits "N" = 50Zc bits
- Hence minimum Code Rate "R" supported by BG2 = K/N = 10/50 = 1/5 = 0.2

Expansion factor = Zc (each bit is further expanded to the factor of Zc to get the bit block)

Total # of Message Bit Blocks = 22Zc

Total # Code word bit blocks = 66 (Since first two columns are always punctured so 68 -2)

If all the 66 blocks are transmitted then in BG1 Minimum code rate = 22/66 = 1/3

Usually there may not be those many message blocks that need to be transmitted so further rate matching is done via Shortening and puncturing.
If \(Z_c\) is the expansion factor and message blocks are 22 then 22\(Z_c\) are the message bits.

In Actual transmission there may not be 22\(Z_c\) bits, so for example IF there are only 18\(Z_c\) bits that need to be transmitted.

Then 22\(Z_c\) – 18\(Z_c\) = 4\(Z_c\) message bits are set to "Zero" this is called shortening (as shown in above picture column 19 to 22 are shortened message blocks).

\# of parity bits = 46\(Z_c\).

If Total \# of transmitted Bits are "A".

Then \# of parity bits to be transmitted will be \(A-(18Zc+2Zc)\).

If A= 42\(Z_c\) then \# of Parity bits to be transmitted = 42 - (18+2) = 22\(Z_c\).

Out of the 46\(Z_c\) Parity bits only 22\(Z_c\) parity bits are transmitted and rest 24\(Z_c\) bits are punctured (not transmitted).

Shortened bits are represented by zeros.

Punctured bits are ignored or not transmitted.

3.3. Fixed Point Quantization and offset minsum [20]

Since BPSK and AWGN Channel is assumed, the received vectors will be a real values. To deal these real values in an efficient manner one of the proposed techniques is Fixed Pont Quantization. Of course the quantization may give same received value that will be some sub optimal but still it seems to be a good technique to improve the performance of the decoder.

Step 1: \(r\) is clipped as \(r'\)

\[
\begin{align*}
    r' &= r & \text{if } -r_{\text{max}} < r < r_{\text{max}} \\
    &= -r_{\text{max}} & \text{if } r < -r_{\text{max}} \\
    &= r_{\text{max}} & \text{if } r > r_{\text{max}}
\end{align*}
\]

Where \(r\) – received vector in real value

\(r_{\text{max}}\) - maximum quantized value

Step 2: allocating these in the rows

Step 3: Row Quantization

\[
\begin{align*}
    rq &= \text{floor}(r/r_{\text{max}}*\text{maxqr}); \\
    rq&(rq>\text{maxqr}) = \text{maxqr}; \\
    rq&(rq<-(\text{maxqr}+1)) = -(\text{maxqr}+1);
\end{align*}
\]

Step 4: Subtract the total LLR – the values already in the rows.

Step 5: minsum in the row

\[
\begin{align*}
    \text{min1} &= \text{min1- offset} \\
    \text{if } \text{min1} < 0 \text{ then } \text{min1} &= 0 \\
    \text{min2} &= \text{min2- offset} \\
    \text{if } \text{min2} < 0 \text{ then } \text{min2} &= 0
\end{align*}
\]

Step 6: Add the values back to the Total LLR and Quantization.

Hence there are two quantizations, one is row quantization the typical value could be 3 or 4 and the other one is Total LLR quantization typical value could be 127. The number bits required to represent \(r_{\text{max}}\) value is 6 (1 bit for representing sign and 5 bits for magnitude). Then the length of the Total LLR should be at least with 2 more bits.

4. Simulation Results
Similar simulation Environment is maintained as in [13] to validate the results which are shown in Fig.2. SPC SISO decoder is used for row operations and repetition code SISO decoder is used for column operations. For this simulation, BG 2 which is having index value of 6 and the expansion factor of 52 for the block length of $10^5$ and 20 iterations are used. Rate of the code is $\frac{1}{2}$. From the results it is observed that for BER of $10^{-3}$ with block length of $10^5$ with improved layered approach gives the Eb/N0 value of ~2 dB.

![Figure 2. Plot of BER Vs Eb/N0dB](image)

Since expansion factor is 52, each element in this base graph is expanded by 52 and if the code is (1000, 2000) then $k$ is 1000 $Z_c$ and check bits are 1000$Z_c$ where $Z_c$ is 52. From the results we can observe that number of blocks are upto 100000. Hence Total bits will be 100000*$Z_c$. Layered Approach is implemented and results have achieved with a less number of iterations.
Fig. 4 Shows that the performance of Improved Layered Iterative Minsum Message Passing Algorithm [15] in terms of BER Vs Eb/No dB. By Introducing Rate Matching to the required code value, Puncturing of first two columns and the shortening of excess bits improved the performance of Layered Iterative Message Passing Algorithm gives the better results as shown.

Fig. 5 Shows that the improved performance of proposed algorithm in terms of FER Vs Eb/N0dB. Since we use number of blocks as a metric then the Frame Error Rate is also an important to validate the performance of the 5G NR LDPC codes.

5. Conclusion

In this paper, the Layered Minsum Iterative Message Passing Algorithm [15] has been improved with Rate matching, Fixed Point Quantization and offset minsum for BG (2_6_52) with block length of $10^5$ and the algorithm converges in 20 iterations gives the Eb/N0 dB value of ~2dB and comparably
less than the Layered Iterative minsum Message Passing decoding algorithm [13]. Since BG2 of size (42x52) is used, mb is 42 and nb is 52. Then the number of message bits are (nb-mb)xZc where Zc is the Expansion factor. If Base matrix of high expansion factor is being used the code will perform comparatively better. Also the same can be implemented for BG1 of various expansion factors and results can be observed. Coding gain can also be calculated using the above results.

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