Conditions for Regional Frequency Stability in Power Systems—Part II: Applications

Luis Badesa, Member, IEEE, Fei Teng, Member, IEEE, and Goran Strbac, Member, IEEE

Abstract—In Part I of this paper we have introduced the closed-form conditions for guaranteeing regional frequency stability in a power system. Here we propose a methodology to represent these conditions in the form of linear constraints and demonstrate their applicability by implementing them in a frequency-secured Stochastic Unit Commitment (SUC). We consider the Great Britain system, characterised by two regions that create a non-uniform distribution of inertia: England in the South, where most of the load is located, and Scotland in the North, containing significant wind resources. Through several case studies, it is shown that inertia and frequency response cannot be considered as system-wide magnitudes in power systems that exhibit inter-area oscillations in frequency, as their location in a particular region is key to guarantee stability. In addition, securing against a medium-sized loss in the low-inertia region proves to cause significant wind curtailment, which could be alleviated through reinforced transmission corridors. In this context, the proposed constraints allow to find the optimal volume of ancillary services to be procured in each region.

Index Terms—Power system dynamics, inertia, frequency stability, unit commitment.

NOMENCLATURE

Acronyms
CCGT Combined Cycle Gas Turbine.
COI Centre Of Inertia.
FR Frequency Response.
GB Great Britain.
MILP Mixed-Integer Linear Program.
OCTG Open Cycle Gas Turbine.
PFR Primary Frequency Response.
RES Renewable Energy Sources.
RoCoF Rate-of-Change-of-Frequency.
SUC Stochastic Unit Commitment.

Indices and Sets
\( g, G \) Index, Set of generators.
\( i, j \) All-purpose indices.
\( n, N \) Index, Set of nodes in the scenario tree.

Constants and Parameters
\( \Delta \tau(n) \) Time-step corresponding to node \( n \) (h).
\( \Delta f_{\text{max}} \) Maximum admissible frequency deviation (Hz).
\( \Delta f_{\text{max}}^\text{qs} \) Maximum admissible frequency deviation at quasi-steady-state (Hz).
\( \pi(n) \) Probability of reaching node \( n \).
\( \omega_i \) Angular frequency of inter-area oscillations for region \( i \) (rad/s).
\( A_i \) Amplitude of inter-area oscillations for region \( i \) (Hz).
\( c_{\text{LS}} \) Value of lost load (£/MWh).
\( c_m^g \) Marginal cost of generating units \( g \) (£/MWh).
\( c_{\text{nl}}^g \) No-load cost of generating units \( g \) (£/h).
\( c_{\text{st}}^g \) Start-up cost of generating units \( g \) (£).
\( D_i \) Load damping factor in region \( i \) (%/Hz).
\( H_g \) Inertia constant of generating units \( g \) (s).
\( H_i \) Inertia constant of the outaged generator (s).
\( m_i \) Regression weights.
\( p_i^D \) Total demand in region \( i \) (MW).
\( p_{\text{max}}^g \) Maximum power output of units \( g \) (MW).
\( p_{\text{ming}}^g \) Minimum stable generation of units \( g \) (MW).
\( P_{\text{max}}^g \) Rated power of the largest generator (MW).
\( P_{\text{max}}^L \) PFR capacity of generators \( g \) (MW).
\( R_{\text{slope}}^g \) Proportion of headroom that can contribute to PFR.
\( \text{RoCoF}_{\text{max}} \) Maximum admissible RoCoF (Hz/s).
\( V_i \) Voltage magnitude in bus \( i \) (kV).
\( X_{i,j} \) Reactance of the transmission line connecting buses \( i \) and \( j \) (Ω).

Decision Variables
\( N^g_n \) Number of units \( g \) that start generating in node \( n \).
\( N^g_n^{\text{op}} \) Number of online generating units of type \( g \).
\( P_g \) Power produced by generating units \( g \) (MW).
\( P_L \) Largest power infeed (MW).
\( P_{\text{LS}} \) Load shed (MW).
\( R_g \) PFR provision from generating units \( g \) (MW).

Linear Expressions (linear combinations of decision variables)
\( C_g(n) \) Operating cost of units \( g \) at node \( n \) in the SUC (£).
\( H_i \) System inertia in region \( i \) (MW-s).
\( H \) Total system inertia (MW-s).
\( R_i \) PFR from all providers in region \( i \) (MW).
\( R \) Total system PFR (MW).

Functions and Operators
\[ \| \cdot \|_2 \text{-norm.} \]
\( \Delta f(t) \) Time-evolution of post-fault frequency deviation from nominal state in region \( i \) (Hz).
\( t_{\text{nadir}} \) Time when the frequency nadir occurs (s).

I. INTRODUCTION

In order to assure a secure operation of the system from a frequency-performance point of view, system operators must procure certain frequency services that would only come into play in the event of a frequency drop. Frequency services are any type of ancillary service that helps comply with frequency regulation, that is, that helps restore a power equilibrium after a generation/demand outage. These services are: inertia (i.e. the kinetic energy stored in the rotating masses of synchronous generators), frequency response (a power injection from different devices that is
activated after a loss), load damping (typically provided by frequency-responsive loads that reduce their consumption after a frequency drop) and the size of the largest possible loss of generation or demand (corresponding to the N-1 reliability criterion).

These ancillary services increment the operating cost of a power system, since the level of system inertia depends on the number of thermal units committed, while FR is a function of the headroom in online plants. The main providers of frequency services are generators and loads, therefore these services are inherently related to energy provision. In order to optimise the provision of inertia and FR, several works have focused on constraining Optimal Power Flow and Unit Commitment problems to explicitly respect frequency security. The authors in [1] enforced a minimum threshold of FR in a scheduling algorithm, a threshold driven by the requirement for frequency to return to its nominal value following a power outage, (i.e. the frequency steady-state limit). References [2]–[4] used various modelling approaches to demonstrate the importance of considering the level of system inertia when scheduling frequency response. Recently, the works in [5]–[7] have highlighted the benefits from accounting for fast frequency response to reduce ancillary services costs.

In the present paper, a scheduling model is constrained to guarantee frequency stability in every region of the power system, therefore moving for the first time beyond the uniform frequency model considered in previous works. To highlight the implications of such model, several case studies are run considering the Great Britain power system, characterised by two regions that create a non-uniform distribution of inertia: England in the South, where most of the load is located, and Scotland in the North, containing significant wind resources. The results presented demonstrate that inertia and frequency response cannot be considered as system-wide magnitudes in power systems that exhibit inter-area oscillations in frequency, as their location in a particular region is key to guarantee stability. In this context, the proposed constraints allow to find the optimal volume of ancillary services to be procured in each region.

The rest of this paper is organised as follows. Section II demonstrates the applicability of the proposed frequency-stability conditions to be implemented in Mixed-Integer Linear Programming (MILP) optimisation problems. Section III presents fundamental insight, gained through several relevant case studies, on how to guarantee stability in systems exhibiting inter-areas oscillations in frequency. Finally, Section IV provides the conclusion and proposes future lines of work.

II. APPLICABILITY OF THE CONSTRAINTS FOR REGIONAL FREQUENCY STABILITY

In Part I of this paper we have deduced analytical constraints for guaranteeing that RoCoF and nadir stay within pre-defined limits in the event of any credible contingency. Here we propose a numerical method to formulate those constraints as linear expressions, a method consisting on using linear regressions on samples obtained from dynamic simulations of the power system. We also demonstrate that the resulting constraints represent the actual stability boundary accurately, showing only a small degree of conservativeness.

Algorithm 1 Numerical estimation of terms ‘m’ in eq. (2)

Input: range of operating conditions for the power system

Output: estimation of $A_i$ and $\omega_i$ for each condition

1: for several feasible values of $P_i$ do
2: for several splits of $D$ among the regions do
3: $H_{\text{total}} = P_i/(2 \cdot \text{RoCoF}_{\text{max}})$
4: $R_{\text{total}} = H_{\text{total}}/k^*$
5: if $R_{\text{total}} < P_i - D \cdot P_D \cdot \Delta f_{\text{max}}$ then
6: $R_{\text{total}} = P_i - D \cdot P_D \cdot \Delta f_{\text{max}}$
7: for several splits of $R_{\text{total}}$ among the regions do
8: for several splits of $H_{\text{total}}$ among the regions do
9: run dynamic simulation (e.g. Simulink)
10: while RoCoF in the region $> \text{RoCoF}_{\text{max}}$ do
11: $H_{\text{total}} = H_{\text{total}} + \text{slight increase}$
12: run dynamic simulation
13: estimate $A_i$, $\omega_i$ (e.g. ‘estimate_oscillation.m’)
14: record features: system state ($H_1$, $H_2$, $P_i$, ...)
15: record labels: estimated ‘$A_i$, $\omega_i$’
16: regression with features & labels (‘Rocof_regression.m’)

A. RoCoF constraint: numerical estimation

The analytical deduction of the RoCoF constraint for each region, presented in Part I of this paper, gives the following inequality:

$$|\text{RoCoF}| = \frac{P_i}{2(H_1 + H_2)} + A_i \cdot \omega_i \leq \text{RoCoF}_{\text{max}} \quad (1)$$

Here we propose a numerical-estimation method for the parameters of the inter-area oscillations appearing in eqs. (1), i.e. parameters $A_i$ and $\omega_i$. This method allows to obtain linear RoCoF constraints for each region, in the form:

$$\frac{P_i}{2(H_1 + H_2)} + \frac{m_1 H_1 + m_2 H_2 + m_3 P_i + m_4 D_1 P_D}{2(H_1 + H_2)} + \frac{m_5 D_2 P_2 + m_6 R_1 + m_7 R_2 + m_8}{2(H_1 + H_2)} \leq \text{RoCoF}_{\text{max}} \quad (2)$$

Eq. (2) is obtained by estimating term ‘$A_i \cdot \omega_i$’ in (1) using a linear combination of every system magnitude (i.e. $H_1$, $H_2$, $P_i$, ...) divided by term ‘$2(H_1 + H_2)$’, so as to obtain a linear formulation for the RoCoF constraint. The procedure for estimating the oscillation parameters $A_i$ and $\omega_i$ numerically (i.e. finding the optimal value for the constant terms ‘m’) is described in Algorithm 1, while an implementation of this algorithm using MATLAB and Simulink is freely available in [8].

Algorithm 1 generates a number of feasible operating points for the power system, that exactly respect the regional RoCoF limit: dynamic simulations are run for a number of operating points, and samples that just barely respect the RoCoF requirement are recorded. Finally, a regression is run on these samples to find the right values for terms ‘m’ in eq. (2). Some further explanation on Algorithm 1:

- Algorithm 1 starts by considering a system operating point that just barely respects the COI frequency stability (i.e. that neglects the inter-area oscillations), as described in lines 3 through 6. Parameter ‘$k^*$’ in line 4 refers to...
the condition for respecting the COI nadir requirement, deduced in [4] as \( H \cdot R \geq k^* \) (where \( k^* \) is a function of the system operating state).

- The reason for starting from a system operating point that just barely respects the COI frequency stability is to iteratively reach an operating point precisely on the RoCoF security boundary, using the loop in lines 10 through 12. Using only samples at the RoCoF security boundary allows to obtain a more accurate regression for \( \mathbf{A}_i \cdot \omega_i \), as explained later in this section and illustrated in Fig. 1.

- The several ‘for’ loops included in the algorithm have the goal of generating samples that consider several possible splits of the system magnitudes (\( H, R, D \)) among the regions. This allows to obtain a wide range of system operating points, appropriately covering the space of feasible operating points of the power system.

The last line in Algorithm 1 applies a regression to estimate the values of \( \mathbf{A}_i \cdot \omega_i \), as this is the expression that needs to be estimated in eq. (1). The system operating points are used as features for the regression, which then will provide the values of terms ‘m’ in eq. (2). A conservative regression is used, so as to guarantee that the resulting constraints will lead to a frequency-stable operating point in every circumstance, with the tradeoff of a tighter frequency-security region if the security boundary is far from being linear. Nevertheless, as will be demonstrated in Section II-B, this tradeoff is not significant for practical purposes in a power system.

The conservative regression can be computed solving the following constrained optimisation:

\[
\begin{align*}
\min_{\theta} & \quad \frac{1}{2} \left\| \mathbf{X} \cdot \theta - \mathbf{y} \right\|^2 \\
\text{s.t.} & \quad \mathbf{X} \cdot \theta \geq \mathbf{y}
\end{align*}
\]

Where \( \mathbf{y} \) are the regression labels, \( \mathbf{X} \) are the regression features and \( \theta \) is the vector of the regression parameters (in this case, \( \theta = [m_1, \ldots, m_8] \) for eq. (2). The above optimisation problem simply defines a constrained least-squares regression, which forces the resulting regression to be above all training samples contained in vector \( \mathbf{y} \); in this case, the regression is forced to overestimate the regional RoCoF caused by the inter-area oscillation, i.e. by term \( \mathbf{A}_i \cdot \omega_i \) in eqs. (1).

As explained before, Algorithm 1 only uses samples close to the frequency-security boundary to train the conservative regression (i.e. samples that barely respect the RoCoF\(_{\text{max}}\) limit within the region). To illustrate the importance of training the conservative regression using only samples that fall very close to the frequency-security boundary, consider Fig. 1: as a conservative above-all-samples regression is used, training it only with samples close to the security boundary avoids an overly-conservative RoCoF constraint. Note that this Fig. 1 is for illustration purposes only, as it has been generated by considering the COI nadir limit (\( H \cdot R \geq k^* \), as deduced in [4]): since the analytical expression for that limit can be obtained, it is possible to visually understand the advantage of using appropriate samples when estimating the boundary through a regression.

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Fig. 1: Estimation of the COI frequency-stability boundary through a regression, using samples obtained from dynamic simulations.

**B. Conservativeness of the resulting RoCoF constraint**

In this section we demonstrate that the resulting RoCoF constraints do indeed respect the RoCoF\(_{\text{max}}\) limit in each region, while achieving a very small degree of conservativeness.

In order to assess how the resulting RoCoF constraints for each region perform, a number of system operating states was generated that exactly meet the RoCoF constraint in each region (i.e. that exactly meet constraint 2). Then, each system operating point was fed into a dynamic simulation to compute the actual RoCoF that would occur for that operating point. The comparison of the computed RoCoF and the predicted RoCoF using constraint (2) in each region is presented in Table I.

The results in Table I were obtained from 270 samples of system operating points in the ranges \( H \in [110, 170] \text{GW} \cdot \text{s} \), \( R \in [1.6, 3] \text{GW} \), \( P_L \in [1.2, 1.8] \text{GW} \) and \( P_D \in [20, 45] \text{GW} \), ranges within the GB 2030 system discussed in Section III-B. The RoCoF limit in both regions was set to RoCoF\(_{\text{max}}\) = 1Hz/s and the damping factor to \( D = 0.5\% / \text{Hz} \), while the transmis-
sion corridor considered had a line reactance of $X_{1,2} = 50\,\Omega$ and voltages of $V_1 = V_2 = 400\,kV$.

As shown in Table I, the proposed RoCoF constraints are almost perfect for region 2, while they achieve a small degree of conservativeness for region 1 (they overestimate RoCoF by 0.05Hz/s in the worst case). These results are consistent with the analytical deduction of the RoCoF constraint presented in Part I of Section IV-A: the proposed RoCoF constraint neglects the effect of the attenuation of inter-area oscillations, which entails no approximation for the faulted region but a certain approximation for other regions. This fact is illustrated in Fig. 1 of Part I, which shows that the maximum RoCoF in the faulted region occurs at the very instant of the fault, when there is no attenuation of oscillations present; however, the maximum RoCoF in the non-faulted region 1 occurs a few instants afterwards, so neglecting the attenuation entails some conservativeness.

In conclusion, these RoCoF constraints are suitable to be implemented in any frequency-secured optimal scheduling or market clearing routine, guaranteeing to respect frequency stability in every region of the power system while entailing a small degree of conservativeness.

C. Nadir constraint: numerical estimation

Analytical constraints to guarantee that the frequency nadir is above $\Delta f_{\text{max}}$ in every region have been deduced in eqs. (36) and (37) in Part I. In order to obtain a linear formulation for these constraints, a numerical estimation must be applied to the integrals of post-fault frequency deviation appearing in eqs. (36) and (37) in Part I. In a similar way as done in Section II-A for the RoCoF constraint, here we use a linear combination of every system magnitude to estimate these integrals:

\[
\int_{t_{\text{start}}}^{t_{\text{end}}} \left[ f_{\text{system operating state}}(\tau) \right] d\tau \text{ estimated by } \\
\int_{0}^{t} \left[ \Delta f_{1}(\tau) - \Delta f_{2}(\tau) \right] d\tau dt \text{ estimated by } \\
\int_{0}^{t} \left[ \Delta f_{1}(\tau) - \Delta f_{2}(\tau) \right] d\tau dt
\]

Using this constrained regression guarantees that the frequency nadir will be above the stability threshold of $\Delta f_{\text{max}}$ in every circumstance.

D. Conservativeness of the resulting nadir constraint

In order to assess how the resulting nadir constraints for each region perform, a number of system operating states was generated that exactly meet the nadir constraint in each region, i.e. eqs. (36) and (37) in Part I. A dynamic simulation was then run for each of these states, so as to compute the actual nadir that would occur for that operating point. Table II presents the comparison of the computed nadirs from the dynamic simulations and the predicted nadirs using the proposed constraint in each region.

The results in Table II were obtained from 300 samples of system operating points in the ranges $H \in [60, 125] \, GW \cdot s$, $R \in [3, 5.2] \, GW$, $P_L \in [1.2, 1.8] \, GW$ and $P_D \in [20, 45] \, GW$, ranges within the GB 2030 system discussed in Section III-B. The nadir limit in both regions was set to $\Delta f_{\text{max}} = 0.8Hz$ and the damping factor to $D = 0.5\%/Hz$, while the transmission corridor considered had a line reactance of $X_{1,2} = 50\,\Omega$ and voltages of $V_1 = V_2 = 400\,kV$. The results show that nadir is on average overestimated by around 0.02Hz in either region (i.e. a 2.5% conservativeness), while the maximum overestimation is just 0.09Hz.

III. CASE STUDIES

The simultaneous optimisation of energy and ancillary services for frequency support can be achieved by appropriately constraining a scheduling algorithm to guarantee frequency stability. In this section we demonstrate, through several case studies, the implications of distinct regional frequencies for the procurement of ancillary services.

A. Frequency-secured stochastic scheduling model

The tool used here for conducting simulations of a power system scheduling is a two-stage Stochastic Unit Commitment (SUC) model. The Unit Commitment is a problem of particular interest when studying the provision of frequency services, since the commitment state of synchronous generators determines the level of system inertia, the key driver of post-fault frequency evolution.

This SUC solves an optimisation problem in which the decision variables are the commitment status (on/off) of each generator and the power output of online generators. The objective is to minimise the system’s expected operational cost, subject to several system constraints, such as generation-demand balance, and numerous inter-temporal constraints such as start-up times or maximum ramp rates of thermal units. The SUC is solved with a 24h lookahead, for which it schedules
the system’s dispatchable generation with an hourly resolution. In order to account for the uncertainty in renewable generation, the SUC considers a number of possible realisations of the stochastic variable in the system, i.e. RES generation. The infinite set of possible realisations is discretised into a representative scenario tree, in which each scenario is related to a certain probability of occurrence. The constraints for guaranteeing regional frequency stability deduced in this paper are implemented in the SUC developed in [9]. Therefore, the frequency-constrained SUC allows to consider two main operational challenges of systems with high penetration of non-synchronous RES: uncertainty and low inertia.

This SUC is formulated as a Mixed-Integer Linear Program (MILP). The objective function of the SUC is the expected operation cost over all nodes in the scenario tree:

$$\min \sum_{n \in N} \pi(n) \left( \sum_{g \in G} C_g(n) + \epsilon^L \cdot \Delta \tau(n) \cdot P^LS(n) \right)$$

(7)

The sum of the operating cost $C_g$ of all thermal units at each node $n$, plus the penalties paid for load shedding, is weighted by the probability of reaching that node, $\pi(n)$. The operating cost of generating units $g$ is given by:

$$C_g(n) = c^u_g \cdot N^{up}_g(n) + \Delta \tau(n) \left( c^m_g \cdot N^{mp}_g(n) + c^l_g \cdot P_g(n) \right)$$

(8)

Note that generating units with the same characteristics are clustered in the SUC to reduce the computational burden. The objective function (7) is subject to several constraints, in order to correctly model the behaviour of a power system, which are omitted here as they can be found in [9].

By implementing frequency constraints into the SUC, the model assures post-fault frequency stability by optimally scheduling inertia and FR along with energy. Note that it is not necessary to add an explicit cost for these frequency services in the SUC: the cost of frequency services is implicitly considered by the lower efficiency of part-loaded generators. The cost per MWh of operating a thermal unit always decreases with increasing loading level of the generator, being rated power $P_g$ the most economically-efficient operating point. This implies that part-loading thermal units in order to provide FR always increases the operational cost of the system, even if no explicit cost for FR is considered, since the FR provision from a generator is constrained by its headroom:

$$R_g \leq R_{g,max}$$

(9)

$$R_g \leq R_{g,slope} \cdot \left( N_g^{up} \cdot P_{g,max} - P_g \right)$$

(10)

The same reasoning applies for inertia: bringing online a higher number of part-loaded thermal units in order to increase system inertia, increases the per MWh cost of providing energy when compared to the same amount of energy provided by a lower number of fully-loaded generators. Note that the inertia contribution of a generator depends only on its commitment state, not on its power output:

$$H = \sum_{g \in G} H_g \cdot P_g \cdot N^{up}_g - H_L \cdot P_{L,max}$$

(11)

Which considers the inertia lost from the outaged generator.

Note that there is an additional implicit cost associated to the provision of ancillary services during low net-demand conditions: when RES output is high, covering most if not all of the demand at a given time, a number of thermal units must be kept online to provide inertia and PFR. As these plants cannot operate below their minimum stable generation point $P_{g,min}$, the energy generated by these plants displaces energy that could be generated from RES. This RES curtailment is purely due to guaranteeing system stability in the event of a contingency, and therefore the subsequent increase in energy costs effectively constitute the cost of ancillary services during these periods.

In the simulation results presented in this Section III, the SUC is run for one year of operation of the system. A scenario tree with the aforementioned 24h lookahead is built in every hour of the year. Only the scheduling decision for the current time-step is recorded, with all other decisions for the future 24h being discarded before building the new scenario tree. By simulating the operation of the power system during a full year, it is possible to understand the need for ancillary services required to guarantee frequency stability while accounting for the different RES and demand levels throughout a year.

B. Great Britain 2030 power system

The power system considered to conduct simulations in this paper is a feasible representation of the Great Britain system by 2030, which considers a partially-decarbonised generation mix characterised by: 1) a nuclear fleet providing baseload and driving the $N-1$ reliability requirement for frequency services, due to the large power rating of some of these nuclear units (such as the 1.8GW-capacity Hinkley Point C power station [10], expected to be commissioned by 2025); 2) a fleet of gas-fired power plants including Combined-Cycle Gas Turbines (CCGTs) and Open-Cycle Gas Turbines (OCGTs); and 3) a high increase in renewable capacity from current levels, with 60GW of wind power considered in this paper (corresponding to the average wind capacity of the four scenarios envisioned for 2030 by the British system operator [11]). The parameters defining the operational characteristics of the thermal fleet are included in Table III.

Electricity demand ranges from 20GW to a peak of 60GW, accounting for daily and seasonal variations, with 90% of the load located in England and 10% in Scotland [12]. Wind capacity is split by 50%-50% between England and Scotland [13], and the transmission corridor connecting these two regions has a voltage of 400kV, line reactance of 50Ω and thermal limit of 7.5GW [14].

Regarding post-fault frequency limits, the values defined in GB are [15]:

- RoCoF must be below 0.125Hz/s at all times to avoid the tripping of RoCoF-sensitive protection relays (i.e. $\text{RoCoF}_{\text{max}} = 0.125\text{Hz/s}$).
- The frequency nadir must never be below 49.2Hz to prevent the activation of Under Frequency Load Shedding (i.e. $\Delta f_{\text{max}} = 0.8\text{Hz}$).
- Frequency must recover to be above 49.5Hz within 60s after the outage, referred to as the frequency quasi-steady-state requirement (i.e. $\Delta f_{\text{as,max}} = 0.5\text{Hz}$).
Due to increasing penetration of non-synchronous RES in GB, which would significantly increase the procurement cost of frequency services needed to comply with this regulation, the RoCoF limit is in the process of being relaxed to 1Hz/s in the whole network [16], therefore this paper considers a value of RoCoF_{max} = 1Hz/s.

C. Uniform-frequency model vs. Regional-frequency model

We first analyse the consequences of guaranteeing frequency stability in the system scheduling, and study the extra need for ancillary services in the presence of inter-area oscillations in frequency. The SUC is secured against a generation loss of 1.8GW occurring in England, i.e. \( P_L = 1.8GW \).

Fig. 2 compares three cases: 1) an energy-only scheduling, obtained from running the SUC with no frequency constraints implemented (therefore the scheduling solution does not guarantee frequency stability in the event of an outage); 2) a frequency-secured SUC, considering the uniform frequency model proposed in [4]; and 3) a frequency-secured SUC, considering the constraints for regional frequency stability proposed in this paper. In the first case, referred to as ‘No stability guarantee’ in Fig. 2, the inertia present is simply a by-product of energy, provided by nuclear plants and gas-fired plants used during periods of low wind generation. The volume PFR procured in this case is 0MW, since PFR is never a by-product of energy, as running part-loaded thermal plants would unnecessarily increase the cost of energy. On the other hand, both inertia and PFR volumes increase for cases that guarantee frequency stability. The results show that accounting for the regional variations in frequency would not significantly increase these volumes as compared to the ‘Uniform frequency’ case, which neglects the inter-area oscillations in frequency: the inertia procured is around 7GW·s higher (i.e. 5% higher), while PFR increases by 150MW (that is, just above 5% higher). In following sections of this paper, we however demonstrate that the location of inertia and PFR in a particular region is key to guarantee stability.

It is also insightful to visualize the carbon intensity of the system, presented in Fig. 3 for all three cases. The results show an increase of 40gCO_{2}/kWh from case ‘No stability guarantee’ compared to case ‘Uniform frequency’: since thermal plants must be committed to provide inertia and response, this increase in emissions is associated purely with stability actions that would have to be taken by the system operator. An additional 8.5gCO_{2}/kWh would be needed for guaranteeing stability in the presence of inter-area oscillations between England and Scotland.

D. Where to procure inertia and response?

This section has the aim of answering the following question: depending on the cost of inertia and response in each region, where should these services be procured? To do so, we run a sensitivity analysis by adding an explicit cost penalty to inertia and response in each region: although no explicit cost for these services is needed in the SUC, as explained in Section III-A, adding an explicit cost in this section allows to isolate the effect of a higher cost for inertia or PFR on the eventual volumes procured in each region. In the base case this penalty is of £250/MW for PFR and £5/MW·s for inertia, which roughly correspond to the cost of procuring these services during periods of low net-demand. Then, this base case penalty is doubled in one of regions to understand how the SUC solution changes. Again, the SUC is secured against a generation loss of 1.8GW occurring in England.

The first sensitivity considered is the penalty for inertia in each region, with the results presented in Figs. 4 and 5. These results show that inertia is mostly located in England, since the 1.8GW fault takes place there and therefore inertia located in that region is the most effective means to contain RoCoF in England. Fig. 4 also illustrates that, when inertia is twice as expensive in England, the volume of inertia procured in England decreases, and this decrease is compensated by a higher volume of inertia procured in Scotland. Note that the
Fig. 4: Average inertia procured in each region, for a 1.8GW loss occurring in England and a sensitivity analysis for the cost of inertia in each region.

Fig. 5: Average PFR procured in each region, for a 1.8GW loss occurring in England and a sensitivity analysis for the cost of PFR in each region.

Fig. 6: Average PFR procured in each region, for a 1.8GW loss occurring in England and a sensitivity analysis for the cost of PFR in each region.

Fig. 7: Average inertia procured in each region, for a 1.8GW loss occurring in England and a sensitivity analysis for the cost of PFR in each region.

Fig. 8: Average inertia procured in each region, for a 1.8GW loss occurring in England and a sensitivity analysis for the cost of inertia in each region.

The results in Figs. 4 through 7 show that the volumes of inertia and PFR procured in Scotland simply do not drop below the minimum values to contain the frequency drop in that region. The location of the fault has a clear impact on where the frequency services are needed, as the results presented in this Section have shown that most inertia and response must be located in England if a large generation outage occurs in that region. In the next section, we further analyse the impact of the fault location by considering a fault in Scotland, the low-inertia region given its excess wind generation.

E. Impact of fault location: fault in the low-inertia region

Here we analyse the implications of a generation loss taking place in the low-inertia region. We consider a 0.8GW loss in Scotland, corresponding to half of the capacity of the double-circuit HVDC interconnector named North Sea Link, expected to be commissioned in 2021 [17]. Three cases are considered: 1) the uniform frequency model for frequency stability from [4], with a thermal limit between England and Scotland of 7.5GW; 2) the model for guaranteeing regional frequency stability proposed in this paper, with the same thermal limit; and 3) the model for regional frequency stability, but removing the thermal limit to understand the implications in has on procurement of frequency services in each region.

The average inertia scheduled by the SUC in each region is included in Fig. 8, for each of these three cases. By comparing the uniform-frequency model solution with the solution considering regional frequency stability, it is clear that some thermal plants must be committed in Scotland to provide inertia, as otherwise even this medium-size fault in that region could lead to violation of the RoCoF limit. Also, by comparing the solutions respecting regional frequency stability with and without enforcing the thermal limit in the
Frequency by RES generation, as the benefits are not only in terms of value of strong transmission corridors in systems dominated makes it unavoidable to procure some inertia in that region, inertia in Scotland could be operating simultaneously to the thermal limit was not binding, wind curtailment could respecting regional frequency stability in Scotland aggravates this effect, as the thermal plants that must be online in that region to provide inertia displace wind generation so that the energy-export limit can be generated in Scotland so that the energy-export limit is respected. This has the effect of causing wind curtailment in Scotland, even in the ‘uniform frequency’ case where no thermal generation is committed in Scotland. However, respecting regional frequency stability in Scotland aggravates this effect, as the thermal plants that must be online in that region to provide inertia displace wind generation so that the thermal limit is respected. The last column in Fig. 9 shows that, if the thermal limit was not binding, wind curtailment could be significantly reduced, since the thermal plants providing inertia in Scotland could be operating simultaneously to the wind generation in that region.

In conclusion, a medium-sized fault in the low-inertia region makes it unavoidable to procure some inertia in that region, with the associated wind curtailment. This demonstrates the value of strong transmission corridors in systems dominated by RES generation, as the benefits are not only in terms of enhanced energy sharing across the system, but also from allowing a higher degree of freedom in the scheduling of ancillary services in each region.

**IV. Conclusion and Future work**

This paper has demonstrated the applicability of the frequency-stability conditions proposed in Part I to be implemented in optimisation problems formulated as MILPs. The Great Britain system has been used as the platform to test the proposed frequency-secured framework, demonstrating that it is key to procure inertia and response appropriately among the different regions of the system, and that medium-sized faults can have significant impacts if they occur in low-inertia regions. While the qualitative findings summarised here apply to any system with high penetration of non-synchronous generation located in isolated or weakly interconnected regions. It is credible that, to a certain degree, that will be the case for several other systems in the world as they become decarbonised to meet emissions targets.

Future lines of work on this topic should explore how to include probabilistic metrics for the provision of frequency response (so as to consider in a realistic way the response from distributed providers) and how to simultaneously guarantee transient stability in low-inertia systems after a generation outage (since the post-fault activation of response must not cause a phase-angle separation that could potentially lead to loss of synchronism).

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