The properties of the three-nucleon system with the dressed-bag model for \( NN \) interaction. II: Coulomb and CSB effects

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Abstract. Coulomb and charge-symmetry breaking effects in the \(^3\)He ground state within the dressed dibaryon model developed recently for \( 2N \) and \( 3N \) forces are examined in detail. Particular attention has been paid to the Coulomb displacement energy \( \Delta E_C = E_B(\text{\(^3\)H}) - E_B(\text{\(^3\)He}) \) and rms charge radii of \(^3\)H and \(^3\)He. A new scalar \( 3N \) force between the third nucleon and dibaryon is found to be very important for a correct description of the Coulomb energy and rms charge radius in \(^3\)He. In view of the new results for \( \Delta E_C \) obtained here, the role of the effects of charge symmetry breaking in the nuclear force is discussed.

1. Introduction

The problem of accurate description of Coulomb effects in \(^3\)He in the current \( 3N \) approach of the Faddeev or variational type has attracted much attention for last three decades (see e.g. [1, 2] and the references therein to the earlier works). The \( \Delta E_C \) problem dates back to the first accurate \( 3N \) calculations performed on the basis of the Faddeev equations in the mid-1960s [3]. These calculations first exhibited a hardly removable difference of ca. 120 keV between the theoretical prediction for \( \Delta E_C^{th} \approx 640 \) keV and the respective experimental value \( \Delta E_C^{exp} \approx 760 \) keV. In subsequent 35 years, numerous accurate \( 3N \) calculations were performed over the world using many approaches, but this puzzle was still generally unsolved. The most plausible quantitative explanation (but yet not free of serious questions) for the puzzle has been recently suggested by Nogga et al. [2]. They have observed that the difference in the singlet \( ^1S_0 \) scattering lengths of \( pp \) (nuclear part) and \( nn \) systems (originating from the effects of charge symmetry breaking (CSB)) can increase the energy difference between \(^3\)H and \(^3\)He binding energies and thus contribute to \( \Delta E_C \). Using some realistic, currently accepted, \( a_{pp} \) and \( a_{nn} \) values and other small corrections, Nogga et al. [2] were able to virtually remove the
gap of 120 keV between the conventional 3N approaches (which neglect CSB effects) and the experimental value. However, this success depends crucially on the accepted $a_{nn}$ value, which is not very reliable up to date (see the details in Section IV). For another admissible $a_{nn}$ value, the explanation given for the gap in ref. [2] is invalid. Thus, one should look for another, alternative, explanation for the puzzle.

In this paper, we give such an alternative explanation of the $\Delta E_C$ puzzle and other Coulomb effects in $^3$He without any free parameter on the basis of the new model recently developed for the 2N and 3N forces by our joint Moscow-Tübingen group [4, 5]. The model includes an inevitable dibaryon in the intermediate state dressed with $\pi$, $\sigma$, $\omega$, and $\rho$ fields, together with the traditional Yukawa $\pi$ and $2\pi$ exchanges, which describe the peripheral part of the NN interaction. Being embedded into three- and many-body systems, this specific two-body mechanism generates an inevitable scalar three-body force induced by $\sigma$-meson exchange between the dressed dibaryon and surrounding nucleons [6].

In the preceding paper [6], we formulated a new model for the 3N force of the scalar nature and tested it in 3N calculations. As was demonstrated in [6], this 3N force is so strong that can explain not only the 3N binding energy but also other important characteristics of the 3N system. This scalar 3N force is closely associated with the generation of an intermediate dressed dibaryon in the fundamental NN interaction [4, 5]. The contribution of the above scalar 3N force between the dressed dibaryon and third nucleon to the total 3N binding energy is much higher than that of the conventional 3N force associated with the generation of intermediate $\Delta$-isobars and two-pion exchanges between three nucleons (see e.g. the review paper [7] and the references therein to earlier works). Thus, this scalar 3N force should primarily determinate the properties of the 3N nuclei, e.g., the rms radii of matter and charge distributions, the probability of the $D$ state, the constants of the asymptotic normalization in the $S$- and $D$-wave components, etc. We also established that the complicated interplay between 2N and 3N forces in the new model is primarily responsible for Coulomb effects in $^3$He and the Coulomb displacement energy $\Delta E_C$.

This paper is devoted to the Coulomb and CSB effects in $^3$He, which provide an independent and important check for the consistency and adequacy of the new force model. We found that all basic Coulomb effects can be quite naturally explained in the framework of the new force model without any additional parameter. Thus, this independent check provides quite strong additional support for the force model used here.

The paper is organized as follows. In Sect. II, we present some theoretical framework for treating Coulomb effects within the new force model using the isospin formalism. Section III is devoted to calculation results for the $^3$He ground state. The detailed discussion of the results obtained is presented in Sect. IV, while Sect. V incorporates the concluding remarks of the study.
2. Theoretical framework

In this section, we present a necessary formalism for our variational calculations of 3N systems within the framework of the multicomponent dressed-bag model (DBM). In addition, we discuss here the details concerning the inclusion of Coulomb effects and determination of observables in the 6qN channel. The DBM described in detail in [4, 5] differs from the traditional OBE-type NN interaction models primarily by the presence of non-nucleonic components in the nuclear wave function. Contrary to numerous hybrid quark-nucleon models (popular in the 1980s), it explicitly involves mesonic degrees of freedom inside the dressed bags. Hence, we consider the 3N system, where the traditional 3N channel is supplemented by three other channels involving the dressed dibaryon (dressed six-quark bag) interacting with the third nucleon. In the case of 3He, a large scalar force appears due to σ-meson exchanges between the dibaryon and extra nucleon, and the additional Coulomb force arises because the bag and rest nucleon can have an electric charge. This new Coulomb three-body force is responsible for a significant part of the total 3He Coulomb energy (this three-body Coulomb force has been missed fully in previous 3N calculations within hybrid 6qN models [8]). It should be emphasized here that the contribution of this three-body Coulomb force to the total three-body binding energy is (as will be demonstrated below) quite significant (∼ 100 keV) and makes it possible to explain, in essence, the experimental ∆EC value.

The second feature of the interaction model used here is the absence of the local NN short-range repulsive core. The role of this core is played by the condition of orthogonality to the confined 6q states forbidden in the NN channel. (These states can be identified, e.g., with locked colour 6q states having the tetraquark-diquark structure [9].) This orthogonality requirement imposed on the relative-motion NN wavefunction is responsible for the appearance of some inner nodes in this wavefunction (very stable under variation of the NN-channel energy†) and respective short-range loops. These short-range nodes and loops lead to numerous effects and general consequences for the nuclear structure (see below). One of these consequences is a rather strong overestimation of the Coulomb contribution when using the Coulomb interaction between point-like nucleons. Thus, it is necessary to take into account the finite radius of the nucleon charge distribution.

2.1. Construction of a 3N variational basis and the wavefunction of the 6qN component

Here, we give the form of the basis functions used in this work and the corresponding notation for the quantum numbers. The total wavefunction of the 3N channel, Ψ3N, can be written in the antisymmetrized basis as a sum of the three components:

\[ Ψ_{3N} = Ψ_{3N}^{(1)} + Ψ_{3N}^{(2)} + Ψ_{3N}^{(3)}, \]

† The fact that these stationary nodes play role of the repulsive core in traditional NN-force models has long been established [10].
where the label (i) enumerates one (of three) possible set of the Jacobi coordinates \((r_i, \rho_1, \rho_2, \rho_3)\). Every component in eq. (1) takes the form

\[
\Psi_{3N}^{(i)} = \sum_{\gamma} \sum_{n=1}^{N_{\gamma}} C_{\gamma n} \Phi_{\gamma n}^{(i)}.
\]

The basis functions \(\Phi_{\gamma n}^{(i)}\) are constructed from Gaussian functions and the corresponding spin-angular and isospin factors:

\[
\Phi_{\gamma n}^{(i)} = \exp\left\{ -\alpha_{\gamma n} r_{\gamma i}^2 - \beta_{\gamma n} \rho_{\gamma i}^2 \right\} F_{\gamma n}^{(i)}(\hat{r}_i, \hat{\rho}_i; \{\xi\}_i) T_{\gamma n}^{(i)},
\]

where the composite label \(\gamma^{(i)} = \{\lambda_i l_i L S_{jk} S t_{jk}\}\) represents the respective set of the quantum numbers for the basis functions [3]: \(\lambda_i\) is the orbital angular momentum of the \(jk\) pair; \(l_i\) is the orbital angular momentum of the third nucleon \((i)\) relative to the center of mass of the \(jk\) pair; \(L\) is the total orbital angular momentum of the \(3N\) system; \(S_{jk}\) and \(t_{jk}\) are the spin and isospin of the \(jk\) pair, respectively; and \(S\) is the total spin of the system. We omit here the total angular momentum \(J = 1/2\) and its \(z\)-projection \(M\), as well as the total isospin of the system \(T = 1/2\) and its projection \(T_z\) (in this work, we neglect the very small contribution of the \(T = 3/2\) component). The spin-angular and isospin parts of the basis functions are taken in the form

\[
F_{\gamma}^{(i)} = |\{\lambda_i l_i : L\}\} s_j s_k (S_{jk}) s_i : S\rangle : JM\}
\]

\[
T_{\gamma}^{(i)} \equiv T_{\gamma i} = |t_j t_k (t_{jk}) t_i : TT_z\}
\]

The nonlinear parameters of the basis functions \(\alpha_{\gamma n}\) and \(\beta_{\gamma n}\) are chosen on the Chebyshev grid which provides the completeness of the basis and fast convergence of variational calculations [11]. As was demonstrated earlier [12], this few-body Gaussian basis is very flexible and can represent rather complicated few-body correlations. Therefore, it leads to quite accurate eigenvalues and eigenfunctions. The formulas for matrix elements of Hamiltonian (for local \(NN\) interactions) with antisymmetrized Gaussian basis are given in paper [13].

Having the three-nucleon component \(\Psi_{3N}\) found in the variational calculation, one can construct the \(6qN\)-channel wavefunction \(\Psi_{6qN}^{(i)}\), which depends on the coordinate (or momentum) of the third nucleon and the \(\sigma\)-meson momentum and includes the bag wavefunction (see eq.(33) of ref.[6]). Integrating the modulus squared of this function with respect to the meson momentum and inner variables of the bag, one obtains the density distribution of the third nucleon relative to the bag in the \(6qN\) channel. This density can be used to calculate all observables whose operators depend on the variables of the nucleons and bag. However, it is much more convenient and easier to deal with the quasi-wavefunction of the third nucleon in the \(6qN\) channel, which has been introduced by eq.(39) of ref. [6].

To calculate matrix elements of 3BF Coulomb and OPE forces one needs the spin-isospin part of \(6qN\) components of the total wavefunction. Here we give them explicitly.
The potential form factors in nucleon channels $\varphi_{L_i}^{J_L^M_i}$ now include the spin-isospin part with quantum numbers of the dressed bag:

$$\varphi_{L_i}^{J_L^M_i} = \phi_{J_iL_i}(p_i) \mathcal{Y}_{L_i}^{M_i}(\hat{r}_i) \mathcal{T}_{td},$$

where $S_d$ and $t_d$ denote the bag spin and isospin respectively, and $J_i$ and $L_i$ are the total and orbital angular momenta, respectively, referring just to the vertex form factor (the additional letter “i” is introduced to distinguish these quantum numbers from the respective total angular momentum $\mathbf{J}$ and orbital angular momentum $\mathbf{L}$ of the whole system). Since the present version of the DBM involves bag states with zero orbital angular momentum (although a more general treatment can also include dressed-bag components with $L_d \neq 0$), we have $J_i = S_d$, while the bag spin and isospin are opposite to each other: $t_d + S_d = 1$. The isospin part of the form factor is

$$\mathcal{T}_{td} = \left| \frac{1}{2} \right| : t_d t_d .$$

The total overlap function (with its spin-isospin part)

$$\chi_{L_i}^{J_L^M_i}(i) = \langle \varphi_{L_i}^{J_L^M_i} | \Psi_{3N} \rangle$$

(7)

can be written, e.g., as (cf. eq.(35) in ref. [6])

$$\chi_{L_i}^{J_L^M_i}(q_i) = \sum_{\lambda_i} \Phi_{L_i}^{J_L^M}(q_i) \langle \mathcal{J} M \mathcal{J} J_{M_i} | JM \rangle \mathcal{Y}_{L_i}^{J_{M_i}}(q_i) \langle t_d t_d \frac{1}{2} t_{z_i} | T T_z \rangle \mathcal{T}_{t_{z_i}}^{\frac{1}{2}}.$$ (8)

Here, $J$ and $M$ are the total angular momentum of the $3N$ system and its projection, $\lambda_i$ and $\mathcal{J}$ are the orbital and total angular momenta of the third (i-th) nucleon respectively, $\mathcal{T}_{t_{z_i}}$ is isospinor corresponding to the third nucleon. In the present calculations of the ground states of $^3$H and $^3$He ($J = 1/2$), we considered only the two lowest even partial waves ($S$ and $D$) in $3N$ wavefunctions. Therefore, $\lambda_i$ can take only two values 0 or 2. Moreover, the total angular momentum of third nucleon $\mathcal{J}$ is uniquely determined by value of $\lambda_i$: $\mathcal{J} = 1/2$ at $\lambda_i = 0$ and $\mathcal{J} = 3/2$ at $\lambda_i = 2$. So, we did not include the sum over $\mathcal{J}$ in eq.(8).

Now we redefine the quasi-wavefunction of the third nucleon in $6qN$ channel, including in it spin-isospin part of the bag:

$$\tilde{\Psi}_{6qN}^{J_iL_i} = \sum_{M_i t_{d_i} \lambda_i} \sqrt{-\frac{d}{dE} \lambda_{L_i}^{J_i} (E - \frac{q_i^2}{2m})} \chi_{L_i}^{J_L^M_i} (J_i M_i) \mathcal{T}_{t_{d_i} t_{d}} ,$$

(9)

where $|J_i M_i \rangle$ and $\mathcal{T}_{t_{d} t_{d}}$ are spin and isospin parts of the bag function respectively.

It is easy to see that the three form factors $\varphi_{L_i}^{J_i}$ used in this work ($\varphi_0^0$, $\varphi_1^1$, and $\varphi_2^1$) determine five radial components of the overlap function $\Phi_{L_i}^{J_L^M}(q_i)$ and five respective components of the quasi-wavefunction for the $6qN$ channel. To specify these components it is sufficient to give three quantum numbers, e.g. $S_d$, $\lambda_i$ and $L_i$, and we will use notation $\Psi_{6qN}^{J_iL_i \lambda_i L_i}(q_i)$ for these radial components:

$$\Psi_{6qN}^{J_iL_i \lambda_i L_i}(q_i) : (J_i = S_d = 0, t_d = 1, L_i = 0, \lambda_i = 0, \mathcal{J} = \frac{1}{2})$$
$$\Psi_{6qN}^{J_iL_i \lambda_i L_i}(q_i) : (J_i = S_d = 1, t_d = 0, L_i = 0, \lambda_i = 0, \mathcal{J} = \frac{1}{2})$$
$$\Psi_{6qN}^{J_iL_i \lambda_i L_i}(q_i) : (J_i = S_d = 1, t_d = 0, L_i = 0, \lambda_i = 2, \mathcal{J} = \frac{3}{2})$$
$$\Psi_{6qN}^{J_iL_i \lambda_i L_i}(q_i) : (J_i = S_d = 1, t_d = 0, L_i = 2, \lambda_i = 0, \mathcal{J} = \frac{1}{2})$$
$$\Psi_{6qN}^{J_iL_i \lambda_i L_i}(q_i) : (J_i = S_d = 1, t_d = 0, L_i = 2, \lambda_i = 2, \mathcal{J} = \frac{3}{2})$$
At last, we give the formula for the total quasi-wavefunction of $6qN(i)$ component, separating explicitly its spin-angular and isospin parts:

$$
\Psi_{6qN}^{(i)} = \sum_{\lambda_i, S_d} \sum_{L_i} \Psi_{S_d, \lambda_i, L_i}^{6qN}(q_i) \left\{ |\lambda_i \frac{1}{2}(J) S_d : J M \rangle | t_{d \frac{1}{2}} : T T_z \rangle \right\}.
$$

The explicit dependence of this function on the isospin projection $T_z$ is important for calculation of Coulomb matrix elements and rms charge radius.

The interaction matrix elements include the overlap integrals of the potential form factors with the basis functions $\Phi_{\gamma,n} = \Phi_{\gamma,n}^{(1)} + \Phi_{\gamma,n}^{(2)} + \Phi_{\gamma,n}^{(3)}$, where all five above components of the overlap function enter into the matrix elements independently (certainly, some of the matrix elements can vanish). The explicit formulas for the above overlap functions and detailed formulas for the matrix elements of all DBM interactions will be published elsewhere. However, when calculating both the normalization of the $6qN$ component and observables, the $6qN$ components distinguishing only by their radial parts, i.e. by only $L_i$, can be summed. Thus, only three components of the $6qN$ wavefunction (orthogonal due to their spin-angular parts) remain: $S$-wave singlet one ($S_d = 0$):

$$
\Psi_{6qN}^{00} \equiv \Psi_{6qN}^{00,0},
$$

and two triplet ($S_d = 1$) ones:

$$
\Psi_{6qN}^{10} = \Psi_{6qN}^{10,0} + \Psi_{6qN}^{10,2},
\Psi_{6qN}^{12} = \Psi_{6qN}^{12,0} + \Psi_{6qN}^{12,2}.
$$

The total weight of each of three $6qN(i), (i=1,2,3)$ components is equal to

$$
P_{6qN}^{(i)} = \| \Psi_{6qN}^{00} \|^2 + \| \Psi_{6qN}^{10} \|^2 + \| \Psi_{6qN}^{12} \|^2; \quad i = 1, 2, 3.
$$

Now, let us introduce the relative weights of individual $6qN$ components:

$$
P_{6qN}^{S0} = \frac{\| \Psi_{6qN}^{00} \|^2}{P_{6qN}^{(i)}}, \quad P_{6qN}^{S1} = \frac{\| \Psi_{6qN}^{10} \|^2}{P_{6qN}^{(i)}}, \quad P_{6qN}^{D} = \frac{\| \Psi_{6qN}^{12} \|^2}{P_{6qN}^{(i)}}.
$$

After renormalization of the total four-component wavefunction, the total weight of all $6qN$ components is equal to

$$
P_{6qN} = \frac{3P_{6qN}^{(i)}}{1 + 3P_{6qN}^{(i)}},
$$

Here we assume that the $3N$ component of the total wavefunction, $\Psi_{3N}$, obtained from the variational calculation is normalized to unity while the total weight of the three-nucleon component $\Psi_{3N}$ is equal to

$$
P_{3N} = \frac{1}{1 + 3P_{6qN}^{(i)}} = 1 - P_{6qN}.
$$

The total weight of the $D$-wave component in $^3He$ (and $^3H$ wavefunctions with allowance for non-nucleonic components is also changed:

$$
P_D = P_{3N}^D(1 - P_{6qN}) + P_{6qN}^D P_{6qN}.
$$
Numerical values of all above probabilities for $6qN$ and $3N$ components are given below in Table 2. The total weight of all $6qN$ components $P_{6qN}$ in the $3N$ system, as was demonstrated in [6], is rather large and approaches or even exceeds 10%. Furthermore, taking into account the short-range character of these components, the more hard nucleon momentum distribution (closely associated with the first property) for these components, and very strong scalar three-body interaction in the $6qN$ channel, one can conclude that these non-nucleonic components are very important for the properties of nuclear systems.

### 2.2. “Smeared” Coulomb interaction

The Gaussian charge distribution that has the rms charge radius $r_c$ and is normalized to the total charge $z$:

$$\rho(r) = z \left(\frac{\alpha}{\pi}\right)^{3/2} e^{-\alpha r^2}, \quad \alpha^{-1} = \frac{2}{3}r_c^2.$$  \hspace{1cm} (17)

The Coulomb potential for the interaction between such a charge distribution $\rho(r)$ and a point-like charged particle has the well-known form

$$V(R; \alpha) = \int \frac{d\rho(r)}{|R - r|} = \frac{z}{R} \text{erf}(R\sqrt{\alpha})$$

One can derive a similar formula for the Coulomb interaction between two charges $Z_1$ and $Z_2$ with Gaussian distributions with different widths $\alpha_1$ and $\alpha_2$ and rms radii $r_{1c}$ and $r_{2c}$, respectively:

$$V(R; \alpha_1, \alpha_2) = \frac{z_1 z_2}{R} \text{erf}(R\sqrt{\bar{\alpha}}), \quad \bar{\alpha} = \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}.$$

In our calculations, we used the following charge radii for the nucleon and dibaryon:

$$(r_c)_p = 0.87 \text{ fm},$$

$$(r_c)_{6q} = 0.6 \text{ fm}.$$  \hspace{1cm} (18)

These values lead to the “smeared” Coulomb interactions of the form

$$V_{NN}^{\text{Coul}}(r) = \frac{e^2}{r} \text{erf}(r\sqrt{\alpha_{NN}}), \quad \alpha_{NN}^{-1/2} = 1.005 \text{ fm};$$

$$V_{6qN}^{\text{Coul}}(\rho) = \frac{e^2}{\rho} \text{erf}(\rho\sqrt{\alpha_{6qN}}), \quad \alpha_{6qN}^{-1/2} = 0.863 \text{ fm};$$  \hspace{1cm} (19)

$\S$ This value is simply the rms charge radius of the six-quark bag with the parameters given in ref. [4]. The neutral $\sigma$ field of the bag changes this value only slightly. The evident difference between the charge radii of the nucleon and dibaryon can be well understood as follows: the charge radius of the $3q$ core of the nucleon is taken usually as $r_{3c}^2 \approx 0.5 \div 0.55$ fm, while remaining $0.3$ fm is assumed to come from the charge distribution of the $\pi^+$ cloud surrounding the $3q$ core in the proton. In contrast, the meson cloud of the dibaryon in our approach is due to the neutral scalar-isoscalar $\sigma$ meson, so that the dibaryon charge distribution is characterized only by the charge radius of the bare $6q$ core. However in more complete model, one should incorporate also $\pi$-meson (and also $\rho$- and $\omega$-meson) dressing of the dibaryon. So that, the charge radius of the $(pn)$ and $(pp)$ dibaryons will be a bit larger than the value $0.6$ fm accepted here.
2.3. Matrix elements of the three-body Coulomb force

The Coulomb interaction between the charged bag and third nucleon in the $3N$ channel is determined by the three-particle operator with the separable kernel (see eq.(40) in ref.[6]):

$$3\text{BF}_{\text{Coul}} V^{(i)}(p_i, p'_i; q_i, q'_i) = \sum_{J_i M_i L_i, J'_{i'}} \varphi^{J_i M_i}_{L_i}(p_i) \text{Coul}^{J'_{i'}}_{L_{i'}}(q_i, q'_i; E) \varphi^{J_i M_i}_{L_i}(p'_i) \frac{1 + \tau_3^{(i)} + 2 \tau_3^{(j)} + \tau_3^{(k)}}{2}, \quad (20)$$

where $p_i$ is the relative momentum in the $jk$ pair, $q_i$ is the third nucleon momentum, $E$ is the total energy of the $3N$ system. The kernel function $\text{Coul}^{J'_{i'}}_{L_{i'}}$ for the point-like Coulomb interaction can be taken for the OSE interaction from eq.(53) of ref.[1] with the substitution $m_o \to 0$ and $-g_s^2 NN \to e^2$. Variational calculations require only the matrix elements (m.e.) of the interaction operator between the basis functions chosen. It is evident that the m.e. of the operator (20) can be expressed in terms of the integrals of the overlap functions $\chi^{J_i M_i}_{L_i}(q_i)$:

$$\langle \text{Coul} 3\text{BF} \rangle_p = e^2 \sum_{J, L, J', L'} \chi^{J}_{L, L'}(0)(1+a) \frac{1}{1 + t_d} \int \frac{\chi^{J}_{L, L'}(q)}{E - E_0 - \frac{q^2}{2m}} \frac{1}{(q-q')^2} \frac{\chi^{J}_{L, L'}(q')}{E - E_0 - \frac{q^2}{2m}} dq dq', \quad (21)$$

where $t_d$ is the isospin of the bag (we remind that $S_d = J_i$, $s_d + t_d = 1$). For brevity, we omitted here the labels of the basis functions. After partial-wave decomposition (cf. eq.(8))

$$\frac{\chi^{J}_{L, L'}(q)}{E - E_0 - \frac{q^2}{2m}} = \sum_{\lambda} \psi^{J}_{\lambda L}(q) Y_{\lambda}(q), \quad (22)$$

integral (21) reduces to a sum of integrals of the form

$$\langle V^p_{\text{Coul}} \rangle^\lambda = \int \psi^{\lambda}_{\lambda}(q) V^p_{\text{Coul}}(q, q') \psi^{\lambda}_{\lambda}(q') q^2 dq q'^2 dq', \quad (23)$$

where

$$V^p_{\text{Coul}}(q, q') = \frac{2}{\pi} \int j_\lambda(qr) \frac{1}{r} j_\lambda(q' r) r^2 dr = \frac{2}{\pi} \frac{1}{2qq'} Q_\lambda(z); \quad z = \frac{q^2 + q'^2}{2qq'}. \quad (24)$$

Here, $Q_\lambda(z)$ is the Legendre function of the second kind. Now, we will replace the Coulomb potential $1/r$ between the point-like charges in eqs.(23) with the corresponding potential between the “smeared” charges:

$$\langle V^p_{\text{Coul}}(\alpha) \rangle^\lambda = \int \psi^{\lambda}_{\lambda}(q) \left[ \frac{2}{\pi} \int j_\lambda(qr) \text{erf}(r\sqrt{\alpha}) r^2 dr \right] \psi^{\lambda}_{\lambda}(q') q^2 dq q'^2 dq'. \quad (25)$$

It is necessary to comment the calculations of such integrals. In the momentum representation, the integrals include the Coulomb singularity. Thus, they must be carefully integrated numerically. In practice, it is much more convenient to treat them in the coordinate space especially in the case of the “smeared” Coulomb interaction. However, the presence of the propagators $(E - E_0 - q^2/2m)^{-1}$ in our case requires the use of the momentum representation from the beginning. Hence, we calculated the
above Coulomb integrals as follows. Taking into account that the overlap functions 
\( \chi(q) \) in the Gaussian basis reduce to a sum of Gaussians, we approximated the above 
propagators by a sum of few Gaussians. Then, \( \psi(\lambda) \) entering into eq. (22) takes the 
form
\[
\psi(\lambda) = \sum_{i=1}^{\lambda+n} C_{\lambda}^i q^i e^{-\beta_i q^2},
\]
where the additional degrees of \( q \) arise due to the use of an antisymmetrized basis. Now, 
the integral (25) reduces to a sum of terms involving only one-dimensional integrals:
\[
I_{\mu}(\beta, \alpha) = \int \frac{\text{erf}(r\sqrt{\alpha})}{r} r^{2+2\mu} e^{-\beta r^2} dr = \frac{1}{2} \frac{1}{\alpha^{\mu+1}} \sum_{k=1}^{\mu} \frac{k!}{(\frac{\beta}{\alpha})^{k+1}} \frac{(2\mu - 2k + 1)!!}{2^{\mu-k}(\frac{\beta}{\alpha} + 1)^{\mu-k+1/2}},
\]
where \( C_{\mu}^k \) are the binomial coefficients. Thus, this technique reduces the whole 
calculation of the three-body Coulomb interaction matrix to completely analytical 
formulas, which considerably simplify the variational calculation.

2.4. Rms matter and charge radii

In the 3N channel, the rms radii of the proton \( (r_p) \) and neutron \( (r_n) \) distributions are 
defined by the standard way:
\[
r_p^2 = \frac{3}{N_p} \langle \Psi_{3N} | 1 + \tau^{(1)}_3 | r_1^2 | \Psi_{3N} \rangle,
\]
\[
r_n^2 = \frac{3}{N_n} \langle \Psi_{3N} | 1 - \tau^{(1)}_3 | r_1^2 | \Psi_{3N} \rangle,
\]
where \( r_1 = (2/3)\rho_1 \) is the distance between particle (1) and the system center of mass, 
\( N_p = 3/2 + T_z \) and \( N_n = 3/2 - T_z \) are the numbers of protons and neutrons, respectively. 
\( N_p \) is equal to the total charge of the system \( Z \). Then, the rms matter radius is equal to
\[
r_m^2 = \frac{N_p r_p^2 + N_n r_n^2}{3} = \langle \Psi_{3N} | r_1^2 | \Psi_{3N} \rangle
\]
The rms charge radius in the 3N sector is also defined conventionally:
\[
\langle r_{ch}^2 \rangle_{3N} = r_p^2 + R_p^2 - \frac{N_n}{N_p} R_n^2,
\]
where \( R_p^2 = 0.7569 \text{ fm}^2 \) and \( R_n^2 = -0.1161 \text{ fm}^2 \) are the squared charge radii of the 
proton and neutron, respectively.

Further, we define the rms charge radius in 6qN channels as
\[
\langle r_{ch}^2 \rangle_{6qN} = \frac{1}{ZP_{6qN}^{(1)}} \left( \langle \Psi_{6qN}^{(1)} | 1 + \tau^{(1)}_3 | r_1^2 + R_p^2 \rangle + \frac{1 - \tau^{(1)}_3}{2} R_n^2 \right.
\]
\[\left. + (1 + t^2_3) r_d^2 + \sum_{t,t_2} \Gamma_{t,t_2} R_d^2(t, t_2) \right| \Psi_{6qN}^{(1)} \rangle,
\]
(31)
where
\[ r_1 = -\frac{m_d}{m_d + m_N} \rho_1 \equiv \alpha \rho_1 \] and
\[ r_d = -\frac{m_N}{m_d + m_N} \rho_1 = -(1 - \alpha) \rho_1 \]
are the coordinates of the third nucleon (with number 1) and the bag in the c.m.s., \( \hat{t}_d^3 \) is the operator of the third component of the bag isospin, \( \Gamma_{tt_z} \) is the projector into isospin state of the bag with definite values of its isospin \( t \) and \( z \)-projection \( t_z \). The \( R_d(t, t_z) \) is the value of charge radius of the bag in the specific isospin state. These values, in general, are different, but their difference should be rather small and, thus, can be safely ignored in subsequent calculations keeping in mind the relatively low probability of the bag-component. Thus we take value \( R_d = 0.6 \) fm for the mean charge radius of the bag in all isospin states, except the state with \( t_z = -1 \) corresponding to \( (nn) \)-bag. For latter (uncharged) state we put \( R_d^2(1, -1) = 0 \).

The total rms charge radius of the 3N system with allowance for both the three-nucleon and dibaryon-nucleon components thus takes the form
\[ \langle r_{ch}^2 \rangle = (1 - P_{6qN}) \langle r_{ch}^2 \rangle_{3N} + P_{6qN} \langle r_{ch}^2 \rangle_{6qN}. \] (32)

Similarly to eq. (28), one can define the rms radius of the proton (neutron) distribution in the 6qN channel as
\[ \langle r_{(p_n)}^2 \rangle_{6qN} = \frac{\langle \Psi_{6qN}^{(1)} \mid \frac{1}{2} (1 \pm \tau_3) r_1^2 \mid \Psi_{6qN}^{(1)} \rangle}{\langle \Psi_{6qN}^{(1)} \mid \frac{1}{2} (1 \pm \tau_3) \mid \Psi_{6qN}^{(1)} \rangle}, \] (33)
where the denominator determines the average number of protons \( N_{6qN}^{p} \) (or neutrons \( N_{6qN}^{n} \)) in the 6qN channel:
\[ N_{6qN}^{p(\ell)} = \frac{\langle \Psi_{6qN}^{(1)} \mid \frac{1}{2} (1 \pm \tau_3) \mid \Psi_{6qN}^{(1)} \rangle}{P_{6qN}^{(1)}}. \] (34)
One can note that, if to neglect the difference between isosinglet and isotriplet wavefunctions in the 6qN channel, then these numbers are equal \( N_p = 1/3 \) and \( 2/3 \) (\( N_n = 2/3 \) and \( 1/3 \)) for \( ^3\text{H} \) and \( ^3\text{He} \) respectively.

The total rms radii of the nucleon distributions including both the 3N and three 6qN components are
\[ \langle r_{(p_n)}^2 \rangle = (1 - P_{6qN}) r_{(p_n)}^2 + P_{6qN} \langle r_{(p_n)}^2 \rangle_{6qN}. \] (35)

2.5. Role of the \( pn \) mass difference

To accomplish calculation for the accurate Coulomb displacement energy \( \Delta E_C = E_B(^3\text{H}) - E_B(^3\text{He}) \), one should take into consideration some tiny effects associated with the mass difference between the proton and neutron. It is well known [2] that the above mass difference makes rather small contribution to the difference between \( ^3\text{He} \) and \( ^3\text{H} \) binding energies. Therefore, it is usually taken into account in the perturbation approach. However, since the average kinetic energy in our case is twice the energy in conventional force models, this correction is expected to be also much larger in our case.
Hence, we present here the explicit exact evaluation for such a correction term without usage of the perturbation theory.

It should be added that in our model involving various $6qN$ components the similar effect originated from the mass difference of dibaryons in different charge states with $Z = 0, 1$ and 2 should also be taken into account. This mass difference is equal about $\Delta M_d^C \sim 3$ MeV. || The latter effect seems to yield a negligible correction to the $\Delta E_C$ value, because the total probability of all $6qN$ components does not exceed $10 - 11\%$, while the nucleon mass difference is half the $\Delta M_d^C$ value, i.e. ca. 1.5 MeV, at the probability of the $3N$ channel ca. 90\%.

In the conventional isospin formalism, one can consider that the $^3H$ and $^3He$ nuclei consist of the equal-mass nucleons:

$$m = \frac{m_p + m_n}{2},$$

so that $m_p = m + \Delta m/2$, $m_n = m - \Delta m/2$, where $\Delta m = m_p - m_n$. The simplest way to include the correction due to the mass difference $\Delta m$ is to assume that all particles in $^3H$ have the average mass

$$\bar{m}_H = \frac{2m_n + m_p}{3} = m - \frac{1}{6}\Delta m,$$

while they have the different average mass

$$\bar{m}_{He} = \frac{2m_p + m_n}{3} = m + \frac{1}{6}\Delta m$$
in $^3He$. In spite of smallness of parameter $\Delta m/m$, the perturbation theory in this parameter does not work. So we used the average mass $\bar{m}_H$ in calculation of $^3H$ and $\bar{m}_{He}$ in calculation of $^3He$. The results of these corrections are given in fifth row of Table 2.

3. Results of calculations

Here, we present the results of the $3N$ bound-state calculations based on two variants of the DBM.

(i) In first version developed in [4], the dressed-bag propagator includes three loops, two of them are of the type shown in Fig. 2 of ref. [4], in which each loop was found with the $^3P_0$ model. The third loop consists of two such vertices and a convolution of the $\sigma$-meson and $6q$-bag propagators (see the Fig. 2 in ref [4]).

(ii) In the second version, we replaced two above loops with the effective Gaussian form factor $B(k)$, which describes the direct $NN \rightarrow 6q + \sigma$ transition, i.e., the direct transition from the $NN$ channel to the dressed-dibaryon channel.

Both versions have been fitted to the $NN$ phase shifts in low partial waves up to an energy of 1 GeV with almost the same quality. Therefore, they can be considered on equal footing. However, version (ii) has one important advantage. Here, the energy

|| The mass difference between baryons with different $ST$ values is already included in our force model.
dependence arising from the convolution of the two propagators involved into the loop, i.e., the propagators of the $\sigma$-meson and bare dibaryon, describes (with no further correction) just the energy dependence of the effective strength of the $NN$ potential $\lambda^{(2)}(E)$, which is thereby taken directly from the above loop integral. In contrast, in the first version of the model, two additional $qq\pi\pi\sigma$ loops give a rather singular three-dimensional integral for $\lambda^{(1)}(E)$, where the energy dependence at higher energies should be corrected by a linear term. The main difference between the results for both versions is that the energy dependence of $\lambda(E)$ for the second version is much weaker than that for the first variant. In addition, this energy dependence leads both to the decrease in the contribution of the $6qN$ component to all $3N$ observables and thus to the increase in the contribution of the two-body force as compared to the three-body force.

Table 1 presents the calculation results for the two above versions for the following characteristics: – the weights of the $6qN$ channels and $D$ wave in the total $3N$ function, as well as the weight of the mixed-symmetry $S'$ component (only for the $3N$ channel); – the averaged individual contributions from the kinetic energy $T$, two-body interactions $V^{(2N)}$ plus the kinetic energy $T$ and three-body force ($V^{(3N)}$) due to one-sigma and two-sigma exchanges to the total Hamiltonian expectation.

|       | $E$ | $P_D$ % | $P_{S'}$ % | $P_{6qN}$ % | Individual contributions to $H$, MeV |
|-------|-----|---------|------------|-------------|--------------------------------------|
|       |     |         |            |             | $T$ | $T + V^{(2N)}$ | $V^{(3N)}$ |
| $^3\text{H}$ |     |         |            |             |     |                |            |
| DBM(I) $g = 9.577^*$ | -8.482 | 6.87 | 0.67 | 10.99 | 112.8 | -1.33 | -7.15 |
| DBM(II) $g = 8.673^*$ | -8.481 | 7.08 | 0.68 | 7.39 | 112.4 | -3.79 | -4.69 |
| AV18+UIX$^1$ | -8.48 | 9.3 | 1.05 | - | 51.4 | -7.27 | -1.19 |
| $^3\text{He}$ |     |         |            |             |     |                |            |
| DBM(I) | -7.772 | 6.85 | 0.74 | 10.80 | 110.2 | -0.90 | -6.88 |
| DBM(II) | -7.789 | 7.06 | 0.75 | 7.26 | 109.9 | -3.28 | -4.51 |
| AV18+UIX$^1$ | -7.76 | 9.25 | 1.24 | - | 50.6 | -6.54 | -1.17 |

$^*)$ These values of $\sigma NN$ coupling constant in $^3\text{H}$ calculations have been chosen to reproduce the exact binding energy of $^3\text{H}$ nucleus. The calculations for $^3\text{He}$ have been carried out without any free parameters.

$^1)$ The values are taken from [15].

To compare with the respective results for the conventional $NN$ potential models, Table 1 also presents the results of recent calculations with the Argonne potential AV18 and Urbanna-Illinois three-body force UIX [14]. The Coulomb displacement energies $\Delta E_C$, together with the individual contributions to the $\Delta E_C$-value, are presented in Table 2. The rms radii of the charge and proton distributions in $^3\text{H}$ and $^3\text{He}$ found in the impulse approximation, as well as the respective experimental values and results obtained for AV18 + UIX $NN$ forces, are presented in Table 3. To demonstrate
Table 2. Contribution of various terms (in keV) of the interaction to the $^3$H - $^3$He mass difference.

| Contribution                  | DBM(I) | DBM(II) | AV18+UIX |
|-------------------------------|--------|---------|----------|
| point Coulomb 3N only         | 598    | 630     | 677      |
| point Coulomb 3N+6qN          | 840    | 782     | -        |
| smeared Coulomb 3N only       | 547    | 579     | 648      |
| smeared Coulomb 3N+6qN        | 710    | 692     | -        |
| np mass difference            | 46     | 45      | 14       |
| nuclear CSB$^1$               | 0      | 0       | -65      |
| magnetic moments & spin-orbit$^2$ | 17    | 17      | 17       |
| Total                         | 773    | 754     | 754      |

$^1$ See Table 4.

$^2$ Here we use the value for this correction from ref. [2]

Table 3. Rms proton, $r_p$, and charge, $r_{ch}$, radii (in fm) in DBM approach

| model     | $^3$H       | $^3$He       |
|-----------|-------------|--------------|
|           | $r_p$      | $r_{ch}$    | $r_p$    | $r_{ch}$    |
| DBM(I)    | 3N         | 1.625       | 1.779     | 1.805     | 1.989      |
|           | 6qN         | 1.608       | 1.188     | 1.854     | 1.412      |
|           | total       | 1.625       | 1.724     | 1.807     | 1.935      |
| DBM(II)   | 3N         | 1.613       | 1.769     | 1.795     | 1.980      |
|           | 6qN         | 1.573       | 1.171     | 1.829     | 1.396      |
|           | total       | 1.613       | 1.732     | 1.796     | 1.944      |
| AV18 + UIX| 1.59$^*$   | 1.76$^*$    |           |           |
| Experiment| 1.60$^*$    | 1.755       | 1.77$^*$  | 1.95       |

$^*$ These values are taken from [15]. The “experimental” values of point proton radii $r_p$ have been obtained there from charge radii by removing the proton and neutron charge radii 0.743 fm$^2$ and -0.116 fm$^2$ respectively.

the separate contributions of the three-nucleon and dibaryon-nucleon channels to these observables, we also present the values calculated separately with only nucleonic and 6qN parts of the total wavefunction.

We present here a few comments concerning the results in Tables 1-3.

Comments to Table 1.

(i) It is seen that an admixture of the mixed-symmetry $S'$ component in our 3N wavefunction is almost half that for the conventional force model (e.g., AV18+UIX). This difference can be attributed to the fact that the relative contribution of two-body interactions to the total 3N binding energy in our approach is much less than that in the conventional force models (this follows from the results presented in the
seventh and eighth columns of Table 1). While it is well known that the weight of the \(S'\) component is proportional to the difference between two-body spin-singlet and spin-triplet \(NN\) interactions, the leading contribution of three-body forces in our approach comes from the scalar-isoscalar 3BF that is completely insensitive to the above difference. Therefore, as a result of this redistribution of various force components, the weight of the \(S'\) component decreases by almost half. A similar but weaker diminution with respect to conventional force models is also seen in the weight of the \(D\)-wave component \(P_D\). This decrease has the same origin, viz. the suppression of the two-body force contribution and the large increase in the scalar three-body force contribution.

(ii) Another remarkable distinction from the conventional force models is the large increase in the average kinetic (and potential) energy, viz. 112 keV vs. 50 keV in conventional force models (see the sixth column in Table 1). This increase is caused by the appearance of the short-range radial nodes and respective loops in the radial \(3N\) wavefunctions (see Fig. 5 in the preceding paper). This large enhancement in nucleon velocities will strengthen all the effects associated with the nucleon currents, relativistic effects, meson-exchange contributions to electromagnetic observables, etc.

Comments to Table 2. Here, we emphasize three important points.

(iii) First, it is seen quite a large contribution from the Coulomb three-body force (cf. the differences between the entries “Coulomb 3\(N\) only” and “Coulomb 3\(N\) + 6\(qN\)” in this table). The second and third rows correspond to the Coulomb interaction between point-like charges, while the fourth and fifth rows include results for the Coulomb interaction between properly smeared charge distributions. In both cases, the contribution of the three-body Coulomb interaction (which has been completely overlooked in previous works) is as large as ca. 110 – 240 keV and, along with other minor effects, can quantitatively explain the Coulomb displacement energy of \(^3\)He.

(iv) The second point, which is closely interrelated to the first one, is rather high sensitivity of all above Coulomb contributions to the smearing of the charges (both for the proton and 6\(q\) dibaryon) with the Gaussian distribution. Table 2 shows that the inclusion of the smeared charge distribution reduces the Coulomb two-nucleon force contribution by 51 keV for both versions of the model. It should be compared to a difference of 29 keV in the Coulomb interactions between point-like and smeared nucleons for the AV18 + UIX force model. Smearing the charges leads also to significant reducing of the three-body Coulomb contribution: from 242 to 163 keV for version I and from 152 to 113 keV for version II. However, even in the minimum scenario, we obtain an additional contribution of 113 keV from the three-body (smeared) Coulomb force.

(v) The third interesting feature, which is distinguished from the conventional model result, is a quite large effect of the (small) \(np\) mass difference on the \(3N\) Coulomb displacement energy. This effect is about twice the respective contribution for the
AV18 + UIX force model. This enhancement is attributed to the much increased average kinetic energy in our approach. Thus, the variation of this energy due to the np mass difference should be also much larger.

We also add a small correction due to electromagnetic interactions and spin-orbit electromagnetic interaction (as in conventional models, we take a value of 17 keV for this correction). Including all these corrections, we obtain the total value $\Delta E_C^I = 773$ keV for version I and $\Delta E_C^{II} = 754$ keV for version II. Thus, we found a quite small space for nuclear CSB effects: -9 keV for DMB(I) and +10 keV for DBM(II). These values should be compared to a significant value of 65 keV for the AV18 + UIX force model. The more detailed discussion of CSB effects in our approach in the next section further corroborates this important conclusion: the admissible value of CSB effects in the DBM is noticeably smaller than that in the conventional force models.

Comments to Table 3.

(vi) The rms radii of the proton and charge distributions are presented in Table 3 for two versions of our model in comparison with the results for the A18 + UIX force model. As is seen in Table 3, the rms charge radii for the $6qN$ component in both $^3H$ and $^3He$ are much smaller than those for the $3N$ component (as could be expected in advance). On the other hand, the rms charge radii for the $3N$ component turn out to be larger than the respective experimental values in both $^3H$ and $^3He$. Thus, it is the contribution of the $6qN$ component to the total wavefunction of the $3N$ system that provides quite good agreement of the rms charge radii with the respective experimental values.

A small underestimation of charge radii (especially for $^3H$) can be due to too small value for charge radius of the bag (0.6 fm) accepted in our calculations. This value is, in fact, the quark-core radius of the bag, but our estimate shows that the pion cloud will increase its charge radius up to $0.65 \div 0.68$ fm. This will lead to increase of charge radius of $6qN$ component and, therefore, to some increase of the total charge radius. Besides that, there is a contribution to charge radius from the model-dependent two-nucleon current operator. As it shown in ref. [16], this contribution for AV18+UIX force model is about 0.014 fm for $^3H$ and 0.009 fm for $^3He$.

4. Discussion

Here, we will discuss the main results found in the work in the general context of few-body physics and compare them with the respective results based on conventional force models. Particular attention will be paid to some general conclusions that can be derived from the results presented here. Let us begin with the results for the Coulomb displacement energy $\Delta E_C = E_B(^3H) - E_B(^3He)$. We emphasize three important points, where our results differ from those for conventional models.

(i) First, we found a serious difference between conventional and our approaches in the
short-range behaviour of wavefunctions even for the nucleon channel. Conventional 3N wavefunctions are strongly suppressed along all three interparticle coordinates \( r_{ij} \) due to the short-range local repulsive core, while our wavefunctions (in the 3N channel) have stationary nodes and short-range loops along both all \( r_{ij} \) and the third Jacobi coordinates \( \rho_k \). Such a node along the \( \rho \) coordinate presents also in the \( 6qN \) relative-motion wavefunction (see Fig.5 in ref. [6]). This very peculiar short-range behaviour of our wavefunctions leads to a strong enhancement of the high-momentum components of nuclear wavefunctions, which is indicated by various modern experiments, e.g., \( ^3\text{He}(e,e'pp) \) [17] or \( pp \to pp\gamma \) etc. where high momentum transfers appear. On the other hand, these short-range radial loops lead to significant errors when using the Coulomb interaction between point-like particles within our approach. Hence, we must take into account the finite radii of charge distributions in the proton and \( 6q \) bag. Otherwise, all Coulomb energies are overestimated.

(ii) Another important effect following from our calculations is a quite significant contribution of the \( 6qN \) component to \( \Delta E_C \). In fact, just this interaction, which is completely missing in conventional nuclear force models, makes the main contribution (ca. 100 keV) to filling the gap in \( \Delta E_C \) between conventional 3N calculations and experiment.

The large magnitude of this 3N Coulomb force contribution is explained by two factors: first, a rather short average distance \( \langle \rho^2 \rangle \) between the \( 6q \) bag and third nucleon (which enhances the Coulomb interaction in the \( 6qN \) phase) and, second, a significant weight of the \( 6qN \) components where the bag has the charge +1 (i.e., it is constructed from an \( np \) pair). This specific Coulomb repulsion in the \( 6qN \) channel should appear also in all other nuclei where the total weight of such components is about 10% and higher. Therefore, it should strongly contribute to the Coulomb displacement energies over the entire periodic table and could somehow explain the long-term Nollen-Schiffer paradox [18] in this way.

(iii) The third specific effect that has been found in this study and contributes to the quantitative explanation of \( \Delta E_C \) is a strong increase in the average kinetic energy \( \langle T \rangle \) of the system. This increase in \( \langle T \rangle \) has been already discovered in the first early 3N calculations with the Moscow \( NN \) potential model [19] and results in a similar nodal wavefunction behaviour along all interparticle coordinates but without any non-nucleonic component.

The increase in \( \langle T \rangle \) leads to the proportional increase in the \( np \) mass difference correction to \( \Delta E_C \). As is seen in Table 2, this correction in our case is not very small and contributes significantly to \( \Delta E_C \). Many other effects attributed to increasing the average kinetic energy of the system will arise in our approach, e.g., numerous effects associated with the Fermi motion of nucleons in nuclei.

It is worth also to estimate here the possible contribution of \( 9q \)-bag component in \( ^3\text{He} \) to the \( \Delta E_C \), which has been omitted in our present calculations. If one assumes that the radius of the \( 9q \)-bag is near to that for \( 6q \)-system, i.e. \( r_{9q} \sim 0.6 \text{ fm} \), the
Coulomb and CSB effects in the three-nucleon system

Coulomb energy of this bag (with the charge $Z = 2$) $Ze^2/r_{9q}$ is about 3 MeV. The probability of the $9q$-bag is expected to be around $0.1 - 0.2 \%$. So that the additional Coulomb energy contribution to $\Delta E_C$ should be about $3 \cdot (0.1 \div 0.2) \cdot 10^{-3} \sim 3 \div 6$ keV, i.e. a quite small value as compared to the quantity 120 keV coming from $6qN$ component. It can also be expected that the corrections to kinetic energy difference due to contribution of $9q$ component would be very small.

The best explanation for the $\Delta E_C$ value in the framework of conventional force models published up to date [2] is based on the introduction of some CSB effect, i.e., the difference between $nn$ and $pp$ strong interactions. At present, two alternative values of the $nn$ scattering length are assumed:

$$a_{nn}^{(1)} = -18.7 \text{ fm, and } a_{nn}^{(2)} = -16.3 \text{ fm.}$$

(36)

The first value has been extracted from the previous analysis of experiments $d(\pi^-, \gamma)nn$ [20] (see also ref. [21] and refs. therein) and is used in all current $NN$ potential models, while the second value in [36] has been derived from numerous three-body breakup experiments $n + d \rightarrow nnp$ done for the last three decades. In recent years, such breakup experiments are usually treated in the complete Faddeev formalism, which includes most accurately both two-body and three-body forces [22]. Thus, this $a_{nn}$ value is considered as quite reliable. However, the quantitative explanation for the $\Delta E_C$ value in conventional force models uses just the first value of $a_{nn}$ as an essential point of all the construction. At the same time, the use of the second value $a_{nn} = -16.3 \text{ fm}$ (which is not less reliable than the first one) fails completely the above explanation!

Therefore, in order to understand the situation more deeply and to determine the degree of sensitivity of our prediction for $\Delta E_C$ to variation in $a_{nn}$, we made also our $3N$ calculations with two possible values of $a_{nn}$ from eq. (36). These exact calculations have been carried out with the effective values of the singlet-channel coupling constant corresponding to the $V_{NqN}$ part of the $NN$ force:

$$\lambda_{He}^{1S_0} = \frac{1}{3} \lambda_{pp} + \frac{2}{3} \lambda_{np};$$

(37)

$$\lambda_{H}^{1S_0} = \frac{1}{3} \lambda_{nn} + \frac{2}{3} \lambda_{np}.$$  

(38)

In the above calculations, we use the value $\lambda_{np} = 328.9 \text{ MeV}$ that provides the accurate description of the $1S_0 np$ phase shifts and the experimental value of the $np$ scattering length $a_{np} = -23.74$ fm [4]. Here, we employ the value $\lambda_{pp} = 325.523 \text{ MeV}$ fitted to the well-known experimental magnitude $a_{pp} = -8.72$ fm and two $\lambda_{nn}$ values corresponding to two available alternative values of the $nn$ scattering length: $a_{nn}^{(1)} = -16.3 \text{ fm and } a_{nn}^{(2)} = -18.9 \text{ fm.}$ The calculation results are presented in Table 4.

As is seen in Tables 2 and 4, within the DBM (version I) one has $\Delta E_C = 773-18 = 754$ keV, so that the version I of DBM can reasonably reproduce the experimental Coulomb displacement energy $\Delta E_C$ with the lower (in modulus) value $a_{nn} = -16.3 \text{ fm}$, while this model overestimates $\Delta E_C$ by 54 keV with the larger (in modulus) value
Table 4. Contribution of charge symmetry breaking effects to the $^3$H - $^3$He mass difference.

| $a_{nn}$, fm | $\Delta E_C$, keV |
|-------------|------------------|
| -16.3       | -18, -39         |
| -18.9       | +45, +26         |

$a_{nn} = -18.9$ fm. Thus, the DBM approach, in contrast to the conventional force models, prefers the lower (in modulus) possible value -16.3 fm of the $nn$ scattering length.

Now, let us discuss shortly the magnitude of CSB effects in our model. The difference between $a_{nn}$ and so-called “pure nuclear” $pp$ scattering length $a^N_{pp}$ is usually considered as the measure of CSB effects at low energies. The value $a^N_{pp}$ is extracted from $pp$ scattering data when the Coulomb potential is disregarded. The model dependence of the latter quantity was actively discussed in the 1980s [23, 24, 25]. However, the majority of modern $NN$ potentials fitted to the experimental value $a_{pp} = -8.72$ fm give the value $a^N_{pp} = -17.3$ fm when the Coulomb interaction is discarded. It is the value that is adopted now as an “empirical” value of the $pp$ scattering length [26]. Thus, the difference between this value and $a_{nn}$ is usually considered as the measure of CSB effects. However, our model (also fitted to the experimental value $a_{pp} = -8.72$ fm) gives a quite surprising result for pp-scattering length when the Coulomb effects are removed:

$$a^N_{pp}(DBM) = -16.57 \text{ fm}, \quad (39)$$

which differs significantly from the above conventional value (by 0.8 fm) due to the explicit energy dependence of the $NN$ force in our approach.

Thus, if the difference $a^N_{pp} - a_{nn}$ is still taken as the measure of CSB effects, the smallness of this difference obtained in our model testifies to a small magnitude of the CSB effects, which is remarkably smaller than the values derived from conventional OBE models for the $NN$ force.

Now, let us pass to the data from Table 3 for the radii of the charge and proton distribution in $^3$H and $^3$He. It is seen that both our versions (DBM(I) and DBM(II)) give quite similar values for all radii. The most interesting point here is the importance of $6qN$ component contributions. In fact, the contribution of the $6qN$ channel shifts all radii, i.e., $r_{ch}$ and $r_p$ in $^3$H and $^3$He, predicted with pure nucleonic components, much closer to the respective experimental values.

Thus, the dibaryon-nucleon component also works in a right way in this aspect. It is interesting to note that, in general, the predictions of our two-phase model are quite close to those of the conventional single-phase AV18 + UIX model. This means that (at least for many static characteristics) our multi-channel model is effectively similar to a conventional purely nucleonic model. However, this similarity will surely hold only for the characteristics that are sensitive mainly to low momentum transfers, while the properties and processes involving high momentum transfers will be treated in two alternative approaches in completely different ways.
It is worth here to add a few remarks about role of the Coulomb effects in 3N- and 4N-continuum and its interpretation on the basis of the present model. There are a few long-standing puzzles in the field which still cannot be resolved within conventional models of 2N and 3N forces. Here are:

(i) The values of $A_y$ and $T_{11}$ in $nd$ and $pd$ radiation capture at low energies are strongly underpredicted by conventional theoretical approach [27].

(ii) The pronounced discrepancy for the fore-aft asymmetries in mirror reactions $^3\text{H}(\gamma,n)^2\text{H}$ and $^3\text{He}(\gamma,p)^2\text{H}$ at energies a few MeV above the thresholds and also for inverse capture processes [28, 29].

(iii) The ratio of cross sections $(\gamma,p)$-to-$(\gamma,n)$ for $^4\text{He}$ at energies a few MeV above the thresholds is as large as 1.7 while the conventional theory predicts only the values ca. 1.3 - 1.4 [30].

The discrepancy in (i) can be reduced somehow by inclusion of the strong two-body meson-exchange currents incorporating $\Delta$-current with (unnaturally) high cut-off parameter $\Lambda_{\pi NN}^\Delta$ and $\Lambda_{\pi NN}^{\Delta\Delta}$ [31, 32]. The puzzle in (iii) could be explained or reduced by assuming a strong charge-symmetry breaking force component which is in an evident contradiction with results of other experiments and also with conclusions of the present force model.

It should be stressed that all three above-mentioned puzzles are interrelated to contribution of $P$-states in $nd$ and especially $nT$ systems. For example, as has been found in recent 4N calculations [33] the $P$-wave peak in $nT$ elastic scattering at energies ca. 3 - 4 MeV cannot be explained by the fully realistic 4N-calculations within the conventional force model. In addition, the issue (iii) can be explained by an enhancement of the Coulomb effects in $p-^3\text{H}$ exit channel.

As follows from the results of the present and preceding works, our force model predicts inevitably an enhancement of the $P$-wave contributions in $Nd$ and especially $NT$ near continuum (due to additional strong scalar 3N-force) and also an enhancement of the Coulomb effects in $pd$ and $p^3\text{H}$ near continuum. So the present approach could remove or reduce noticeably the above discrepancies in 3N and 4N low-energy continuum.

5. Conclusion

Here, we will summarize the main results of this work. In the previous work, we fixed the only coupling constant, $g_{\sigma NN}$, to obtain the experimental value of the triton binding energy. Then, all other calculations in both previous and this works did not include any fitting parameter. Thus, their results can be considered as a stringent test for the proposed new model for 2N and 3N forces.

First, we point to the precise value obtained for the Coulomb displacement energy $\Delta E_C$ of the $A=3$ system in the developed model. It should be emphasized that, contrary to other studies based on conventional force models (using the 2N and 3N...
forces generated via the meson-exchange mechanism), this explanation does not require any noticeable CSB effect, although our model is still compatible with such effects. However, these CSB effects do not contribute remarkably to $\Delta E_C$ in our approach. Two basic sources of this contribution, which differ from conventional force models, should be indicated here:

– the three-body Coulomb energy of the interaction between the dressed bag and third nucleon; and

– quite significant correction to the kinetic energy of the system due to the $np$ mass difference and high average kinetic energy.

The second general point that must be emphasized is a rather large admixture of dibaryon-nucleon components in both $^3$H and $^3$He, which has been calculated in a completely consistent way. Closely associated with the above $6qN$ components, it is a specific energy dependence of the two-body force in a three- (and many-) body system. This energy dependence strongly reduces the contribution of two-body force when a strong attractive three-body force is added to the system Hamiltonian. This is a manifestation of a very specific new interplay between two- and three-body forces: the stronger the three-body force, the smaller the total contribution of the two-body force to the nuclear binding energy! By this way, a very natural density dependence of nuclear interactions appears from the beginning. Thus, the general properties of the $3N$ system, where forces so much differ from any conventional model force, should appear also much differ from the predictions of any conventional model and, hence, from experiment.

It was very surprising to find that the characteristics of the $3N$ system in our case turned out to be very close to the predictions of the modern force model (such as AV18 + UIX) and thus to experiment. This gives us a good test of the self-consistency and accuracy of the new force model. However, predictions of the present $2N$- and $3N$-force model in other aspects will strongly deviate from those for conventional models. First, these are the properties determined by the high-momentum component of nuclear wavefunctions. The point is that the system described by our multi-component wavefunctions explicitly including dibaryon components can easily absorb quite high momentum transfers, which can hardly be absorbed by the system described by traditional multi-nucleon wavefunctions. Therefore, to fit the experimental data corresponding to large momentum transfers ($\sim 1$ GeV/c), many types of meson-exchange and isobar currents are often introduced to theoretical frameworks. However, these currents are often unrelated to the underlying force model. Hence, it is rather difficult to check the self-consistency of such calculations, e.g., the validity of gauge invariance etc.

Thus, the alternative description given here by the new force model can be more self-consistent and straightforward. One aspect of this new picture is evident – the present model applied to any electromagnetic process on nuclei automatically leads to a consistent whole picture of the process: single-nucleon currents at low momentum transfers, meson-exchange currents (including new meson currents) at intermediate
momentum transfers, and quark counting rules at very high momentum transfers, because the model wavefunction explicitly includes multinucleon, meson-exchange and multiquark components.

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