Holographic Techni-dilaton *

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Techni-dilaton, a pseudo-Nambu-Goldstone boson of scale symmetry, was predicted long ago in the Scale-invariant/Walking/Conformal Technicolor (SWC-TC) as a remnant of the (approximate) scale symmetry associated with the conformal fixed point, based on the conformal gauge dynamics of ladder Schwinger-Dyson (SD) equation with non-running coupling. We study the techni-dilaton as a flavor-singlet bound state of techni-fermions by including the techni-gluon condensate (tGC) effect into the previous (bottom-up) holographic approach to the SWC-TC, a deformation of the holographic QCD with \( \gamma_m \approx 0 \) by large anomalous dimension \( \gamma_m \approx 1 \). With including a bulk scalar field corresponding to the gluon condensate, we first improve the Operator Product Expansion of the current correlators so as to reproduce gluonic \( 1/Q^4 \) term both in QCD and SWC-TC. We find in QCD about 10% (negative) contribution of gluon condensate to the \( \rho \) meson mass. We also calculate the oblique electroweak \( S \)-parameter in the presence of the effect of the tGC and find that for the fixed value of \( S \) the tGC effects dramatically reduce the flavor-singlet scalar (techni-dilaton) mass \( M_{TD} \) (in the unit of \( F_\pi \)), while the vector and axial-vector masses \( M_\rho \) and \( M_{a_1} \) are rather insensitive to the tGC, where \( F_\pi \) is the decay constant of the techni-pion. If we use the range of values of tGC implied by the ladder SD analysis of the non-perturbative scale anomaly in the large \( N_f \) QCD near the conformal window, the phenomenological constraint \( S \approx 0 \) predicts the techni-dilaton mass \( M_{TD} \sim 600 \text{ GeV} \) which is within reach of LHC discovery.

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I. INTRODUCTION

The origin of mass is the most urgent issue of the particle physics today and is to be resolved at the LHC experiments. In the standard model, all masses are attributed to a single parameter of the vacuum expectation value (VEV) of the hypothetical elementary particle, the Higgs boson, which is simply transferred from the input mass parameter \( \mu \) tuned as tachyonic \((\mu^2 < 0)\) in an ad hoc manner. As such the standard model does not explain the origin of mass.

Technicolor (TC) [2] is an attractive idea to account for the origin of mass without introducing Higgs boson and tachyonic mass parameter in analogy with QCD: The mass arises dynamically from the condensate of the fermion-antifermion pair like Cooper pair in the Bardeen-Cooper-Schrieffer theory of the superconductor, picking up the intrinsic mass scale \( \Lambda_{TC} \) generated by the scale anomaly [3] through quantum effects in the gauge theory which is scale-invariant at classical level for massless flavors. Actually in QCD, the running of the coupling constant \( \alpha(\mu) \) implies existence of the intrinsic mass scale \( \Lambda_{QCD} \). The original version of TC, just a simple scale-up of QCD, however, is plagued by the notorious problems: Excessive flavor-changing neutral currents (FCNCs), and excessive oblique corrections of \( O(1) \) to the Peskin-Takeuchi \( S \) parameter [4] compared with the typical experimental bound about 0.1.

The FCNC problem was resolved long time ago by the scale-invariant/walking/conformal TC (SWC-TC) [5–7] initially dubbed as “scale-invariant TC” with the prediction of a “techni-dilaton”, a pseudo Nambu-Goldstone boson of the spontaneous breaking of the (approximate) scale invariance [8]. The theory was based on the strong coupling solution of Maskawa-Nakajima [9] for the ladder Schwinger-Dyson (SD) equation with non-running (conformal) gauge coupling \( \alpha > \alpha_{cr} = O(1) \). It was found [8] (for reviews, see [10]) that the solution implies a large anomalous dimension \( \gamma_m \approx 1 \) of the techni-fermion condensate operator \( \bar{T}T \) at \( \alpha \approx \alpha_{cr} \), namely the enhanced condensate...
In the case of \( \mu > m \) behaves as ordinary asymptotically free theory as QCD for \( \mu < m \) and get decoupled from the beta function for like an SWC-TC: \( \Lambda_{TC} \). \( \gamma_m \) is quantum mechanically induced by the scale anomaly as an analogue of \( \Lambda_{IRFP} \).

It is this (approximate) conformal symmetry that is responsible for the naturalness of the SWC-TC to guarantee the large hierarchy \( m \ll \Lambda \) in such a way that the coupling is almost non-running (conformal) over the wide energy range \( m < \mu < \Lambda \). Moreover, there also exists a possibility [12, 13] that the \( S \) parameter can be reduced in the case of SWC-TC.

An explicit gauge dynamics [14, 15] of such an SWC-TC is based on the Caswell-Banks-Zaks infrared fixed point [16, 17] (CBZ-IRFP) \( \alpha_* = \alpha_0(N_c, N_f) \) which appears in the two-loop beta function of the “large \( N_f \) QCD”, QCD with the number of massless flavors \( N_f (< 11N_c/2) \) larger than a certain value \( N_f^* \). Note that \( \alpha_* \to 0 \) when \( N_f \to 11N_c/2 \), and hence there exists a certain region \( (N_f^*) < N_f^* < N_f < 11N_c/2 \) (“conformal window”) such that \( \alpha_* < \alpha_{cr} \), where \( \alpha_{cr} \) is the critical coupling for the spontaneous chiral symmetry breaking and hence the chiral symmetry gets restored in this region. Here \( \alpha_{cr} \) may be evaluated as \( \alpha_{cr} = \pi/3C_2(F) \) in the ladder approximation [18], in which case \( \alpha_* = \alpha_0(N_c, N_f^*) \). \( \alpha_{cr} \) determines \( N_f^* \) as \( N_f^* \approx 4N_c \) [14]. Related to the conformal symmetry, this phase transition (“conformal phase transition” [15]) has unusual nature that the order parameter changes continuously but the spectrum does discontinuously at the phase transition point \( \alpha_* = \alpha_{cr} \) when we change \( \alpha_* \) (or \( N_f/N_c ) \) continuously.

When we set \( \alpha_* \) slightly larger than \( \alpha_{cr} \) (slightly outside of the conformal window), the walking coupling \( \alpha(\mu)(< \alpha_*) \) becomes larger than the critical coupling in the wide infrared region, we have a condensate or the dynamical mass of the techni-fermion \( m \), which is much smaller than the intrinsic scale of the theory \( \Lambda_{TC} \). Such an intrinsic scale \( \Lambda_{TC} \) is quantum mechanically induced by the scale anomaly as an analogue of \( \Lambda_{QCD} \) in QCD and the theory behaves as ordinary asymptotically free theory as QCD for \( \mu > \Lambda_{TC} \) (Region I in Fig. 1). Although the CBZ-IRFP \( \alpha_* \) actually disappears (then would-be IRFP) at the scale \( \mu \sim m \) where the fermions have acquired the mass \( m \) and get decoupled from the beta function for \( \mu < m \) (Region III in Fig. 1), the coupling is still walking due to the remnant of the CBZ-IRFP conformality in a wide region \( m < \mu < \Lambda_{TC} \) (Region II in Fig. 1). Then the theory acts like an SWC-TC: \( \Lambda_{TC} \) plays a role of cutoff \( \Lambda \) identified with the ETC scale: \( \Lambda_{ETC} \). It develops a large anomalous dimension \( \gamma_m \approx 1 \) for Region II to solve the FCNC problem [14, 15].

Existence of these two largely separated scales, \( m \) and \( \Lambda_{TC} \) such that \( m \ll \Lambda_{TC} \), is the most important feature of SWC-TC, in sharp contrast to the ordinary QCD with small number of flavors (in the chiral limit) where all the mass parameters like dynamical mass of quarks are of order of the single scale parameter of the theory \( \Lambda_{QCD} \). The intrinsic scale \( \Lambda_{TC} \) is related with the scale anomaly corresponding to the perturbative running effects of the coupling, with the ordinary beta function \( \beta(\alpha) \) in the Region I, in the same sense as in QCD [3].

\[
\langle \bar{T}_\mu \rangle = \langle \bar{T}_\mu \rangle = 4\langle \bar{T}_\mu \rangle = \frac{\beta(\alpha)}{4\alpha^2} (\alpha G_{\mu\nu}^2) = \mathcal{O}(\Lambda_{TC}^2),
\]

which implies that all the techni-glue balls have mass of \( \mathcal{O}(\Lambda_{TC}) \). On the other hand, the scale \( m \) is related with totally different scale anomaly due to the dynamical generation of \( m \) which does exist even in the idealized case

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\#1 In the case of \( N_c = 3 \), this value \( N_f^* \approx 4N_c = 12 \) is somewhat different from the lattice value \( 6 < N_f^* \approx 7 \), but is consistent with more recent lattice results [20].

\#2 There is another possibility for the SWC-TC with much less \( N_f \) based on the higher TC representation [21], although explicit ETC model building would be somewhat involved.
with non-running coupling \(\alpha(\mu) \equiv \alpha > \alpha_{ct}\) such as the Maskawa-Nakajima solution [3], as was discussed some time ago [22]. Such an idealized case well simulates the dynamics of Region II [14,15], with anomalous dimension \(\gamma_m \approx 1\) and \(m \ll \Lambda_{TC}\) in the numerical calculations [23] with the walking coupling constant in the most of the Region II being slightly larger than \(\alpha_{ct}, \alpha_\ast > \alpha(\mu) > \alpha_{ct}\). The coupling \(\alpha \equiv \alpha_\ast\) in the “idealized Region II” actually runs non-perturbatively according to the essential-singularity scaling (Miransky scaling [24]) of mass generation, with the non-perturbative beta function \(\beta_{NP}(\alpha)\). Then the non-perturbative scale anomaly reads [15]

\[
\langle \partial^\mu D_\mu \rangle = \langle \theta_{\mu}^\nu \rangle = 4\langle \theta_0^\nu \rangle = \frac{3\beta_{NP}(\alpha)}{4\alpha^2} \langle \alpha G^2_{\mu\nu} \rangle = O(m^4),
\]

which vanishes when we approach the conformal window from the broken phase \(\alpha_\ast \searrow \alpha_{ct} (m \to 0)\) where \(\alpha_\ast\) is the would-be CBZ-IRFP (See Eqs.[4.3] and [4.4]). All the techni-fermion bound states have mass of order of \(m\) when there are no light bound states in the symmetric phase (conformal window) \(\alpha_\ast < \alpha_{ct}\), a characteristic feature of the conformal phase transition [15]. The techni-dilaton is associated with the latter scale anomaly and should have mass on order of \(m(\ll \Lambda_{TC})\).

More concretely, the mass of techni-dilaton or scalar bound state in the SWC-TC was estimated in various methods: The first method was based on the assumption of partially conserved dilatation current (PCDC) [25] combined with the ladder SD equation for the gauged Nambu-Jona-Lasinio model which well simulates [14,15] the conformal phase transition in the large \(N_f\) QCD. The result indicates

\[
M_{TD} \approx \sqrt{2}m,
\]

which coincides with other methods in the ladder SD equation without use of the PCDC [26]. Also a straightforward calculation [23,27] of scalar bound state mass as well as the \(S\) parameter [13] was made in the vicinity of the CBZ-IRFP in the large \(N_f\) QCD, based on the coupled use of the SD equation and the Bethe-Salpeter (BS) equation in the ladder approximation:

\[
M_{TD} \sim 1.5\, m < M_\rho, M_{a_1},
\]

where \(M_\rho, M_{a_1}\) are masses of (techni-)\(\rho\) and (techni-)\(a_1\) mesons, respectively, which is consistent with Eq.(1.3) in contrast to the ordinary QCD where the scalar mass is larger than those of the vector mesons (“higgsless”) within the same framework of ladder SD/BS equation approach.

The SWC-TC, however, has a calculability problem, since its non-perturbative dynamics is not QCD-like at all, and hence no simple scaling of QCD results would be available. The best thing we could do so far has been a straightforward calculation based on the SD equation and (inhomogeneous) BS equation in the ladder approximation [13], which is however not a systematic approximation and is not very reliable in the quantitative sense.

Of a late fashion, based on the so-called AdS/CFT (anti-de-Sitter space/conformal field theory) correspondence, a duality of the string in the anti-de Sitter space background-conformal field theory [28], holography gives us a new method which may resolve the calculability problem of strongly coupled gauge theories [29]. Use of the holographic correspondence enables us to calculate Green functions in a four-dimensional strongly coupled theory from a five-dimensional weakly coupled theory. For instance, QCD can be reformulated based on the holographic correspondence either in the bottom-up approach [30,31] or in the top-down approach [32]. In both approaches we end up with the five-dimensional gauge theory for the flavor symmetry, whose infinite tower of Kaluza-Klein (KK) modes describe nicely a set of the massive vector/axial-vector mesons as the gauge bosons of Hidden Local Symmetries (HLSs) [33, 34], or equivalently as the Moose [35]. Although a holographic description is valid only for large \(N_c\) limit, several observables of QCD have been reproduced within 30% errors in both approaches. Moreover, through the high-energy behavior of current correlators in operator product expansion (OPE), some consistency with the QCD has been confirmed in the bottom-up approach.

Recently the \(S\) parameter in the SWC-TC was calculated [36,39] as an application of the above technique of bottom-up holographic QCD (hard-wall model) [30,31] to the holographic SWC-TC. In the previous work [39], based on the holographic correspondence in the bottom-up approach, we calculated the \(S\) parameter in the SWC-TC, treating the anomalous dimension \(\gamma_m\) as a free parameter as \(0 < \gamma_m \lesssim 1\), varying continuously from the QCD monitor value \(\gamma_m \approx 0\) through the one of the SWC-TC \(\gamma_m \approx 1\). We obtained \(S = 1\) as an explicit function of \(F_\pi/M_\rho\) in entire region, which turns out to be a positive and monotonically increasing function such that \(S\) continuously goes to zero when \(F_\pi/M_\rho \to 0\), where \(F_\pi\) and \(M_\rho\) are the (techni-)pion decay constant and the (techni-)\(\rho\) meson mass, respectively.

In this paper, we extend the previous paper [39] on the hard-wall-type bottom-up holographic SWC-TC by including effects of (techni-)gluon condensation, \(\Gamma\), through the bulk flavor/chiral-singlet scalar field \(\Phi_X\), in addition to the conventional bulk scalar field \(\Phi\) dual to the chiral condensate. For definition of \(\Gamma\), see text. The techni-dilaton, a flavor-singlet scalar bound state of techni-fermion and anti-techni-fermion, will be identified with the lowest KK mode.
from the bulk scalar field φ, not ΦX. Thanks to the additional explicit bulk scalar field ΦX, we naturally improve the matching with the OPE of the underlying theory (QCD and SWC-TC) for current correlators so as to reproduce gluonic 1/Q^4 term, which is clearly distinguished from the same 1/Q^4 terms from chiral condensate in the case of SWC-TC with γm ≃ 1. Our model with γm = 0 and Nf = 3 well reproduces the real-life QCD (See Table I). We find that the QCD ρ meson mass Mρ includes a (negative) contribution about 10% from the gluon condensate.

We analyze a generic case with 0 < γm ≃ 1 and calculate masses of the techni-ρ meson (Mρ), the techni-α1 meson (Mα1), and the flavor-singlet scalar meson, techni-dilaton (MTD), as well as the S parameter. We discuss the general tendency of the dependence of the meson masses relative to Fπ, (Mρ/Fπ, Mα1/Fπ, MTD/Fπ) on γm, S, and Γ. We find a characteristic feature of the techni-dilaton mass related to the conformality of SWC-TC. For fixed S and γm, (Mρ/Fπ) and (Mα1/Fπ) are not sensitive to Γ, while (MTD/Fπ) substantially decreases as Γ increases. Actually, in the formal limit Γ → ∞, we would have MTD/Fπ → 0 (This does not imply the existence of the isolated true massless NG boson of the scale symmetry, since in our case the decay constant Fπ diverges and the techni-dilaton gets decoupled in that limit, see text.) For fixed S and Γ, again (Mρ/Fπ) and (Mα1/Fπ) are not sensitive to γm, while (MTD/Fπ) substantially decreases as γm increases.

Particularly for the case of γm = 1, we study the dependence of the S parameter on (Mρ/Fπ) for typical values of Γ. It is shown that the techni-ρ gluon contribution reduces the value of S maximally about 10% in the region of ̂S ≃ 0.1, although the general tendency is similar to the previous paper [32] without techni-ρ gluon condensation: ̂S decreases monotonically with respect to (Fπ/Mρ) to continuously approach zero. This implies (Mρ/Fπ) necessarily increases when ̂S is required to be smaller.

To be more concrete, we consider a couple of typical models of SWC-TC with γm ≃ 1 and Nf TC = 2, 3, 4 based on the CBZ-IRFP in the large Nf QCD. Using some specific dynamical features of the conformal anomaly indicated by the analysis based on the ladder SD equation (Eqs. (4.3) and (4.4)), we find the relation of Γ to (ΛETC/Fπ): In the case of Nf TC = 3 (Nf = 4Nf TC) and S ≃ 0.1, we have Γ ≃ 7 for (ΛETC/Fπ) = 10^{-4}−10^{-5} (required by the FCNC constraint). Thanks to the large anomalous dimension γm and large techni-gluon condensation Γ, we obtain a relatively light techni-dilaton mass MTD ≃ 600 GeV compared with Mρ ≃ Mα1 ≃ 3.8 TeV, consistently with the perturbative unitarity of W L W L scattering. Note that Mρ and Mα1 are essentially determined by the requirement of S ≃ 0.1 fairly independently of techni-gluon condensation. The essential reason for the large Γ is due to the existence of the wide conformal region Fπ < μ < ΛETC with (ΛETC/Fπ) = 10^{-4}−10^{-5}, which yields the smallness of the beta function through the factor (ln 4ΛETC/m)^{-3} (see Eq. (4.3)) and hence amplifies the techni-gluon condensation compared with the ordinary QCD with Γ = 1. In the idealized (phenomenologically non-interesting) limit ΛETC/Fπ → ∞ we would have Γ → ∞ and hence MTD/Fπ → 0 in conformity with the general tendency mentioned above. (However, the would-be “massless” techni-dilaton is actually decoupled, see text. Indeed, spontaneous breaking of the scale symmetry is always accompanied with its explicit breaking.) The predicted mass ≃ 600 GeV of the holographic techni-dilaton (“conformal Higgs”) is within reach of LHC discovery.

This paper is organized as follows: In Sec. II we present our model which is an extension of our previous holographic SWC-TC model based on hard-wall-type bottom-up approach [32] by including effects of (techni-) quark condensation through the bulk flavor/chiral-singlet scalar field. Formulas for masses of mesons (techni-ρ, -α1, and -dilaton) and current correlators including S parameter are given. We show that our model reproduces gluonic 1/Q^4 terms in the OPE of vector/axial-vector current correlators. In Sec. III we estimate effects on meson masses and S parameter coming from the (techni-)gluon condensation Γ as a free parameter in a generic TC with 0 < γm ≃ 1 involving the case of QCD with γm = 0. In Sec. IV, to specify the value of Γ relevant to the actual model-building of SWC-TC, we consider a matching of our holographic model with typical models of SWC-TC based on the CBZ-IRFP in the large Nf QCD.

II. A HOLOGRAPHIC TECHNIColor MODEL WITH TECHNI-GluON CONDENSATION

In this section, we propose a holographic model dual to a generic class of technicolor (TC) with 0 < γm ≃ 1 including the degree of freedom of (techni-)gluon condensation, where γm denotes the anomalous dimension of (techni-)fermion chiral condensate (TT).

Following a bottom-up approach of holographic-dual of QCD [30, 31] with γm ≃ 0 and that of SWC-TC [37–39] with γm ≃ 1, we consider a five-dimensional gauge theory having SU(Nf) L × SU(Nf) R gauge symmetry. We will not consider the extra U(1) A that involves the anomaly. The theory is defined on the five-dimensional anti-de-Sitter space (AdS5) with L, the curvature radius of AdS5, described by the metric ds^2 = gMN dx^M dx^N = (L/z)^2 (ηµνdx^µdx^ν−dz^2) with ηµν = diag[1, −1, −1, −1, −1]. The fifth direction z is compactified on an interval extended from the ultraviolet (UV) brane located at z = ε to the infrared (IR) brane at z = zm, i.e., ε ≪ z ≪ zm. In addition to the bulk left- (L_M) and right- (R_M) gauge fields, we introduce a bulk scalar field Φ which transforms as bifundamental representation under the SU(Nf) L × SU(Nf) R gauge symmetry so as to deduce the information concerning the chiral condensation-operator.
The mass-parameter $m_\Phi$ is then related to $\gamma_m$ as $m_\Phi^2 = -(3 - \gamma_m)(1 + \gamma_m)/L^2$, where $\gamma_m = 0$ corresponds to QCD and QCD-like TC and $\gamma_m \approx 1$ is the case of SWC-TC. This is the same setup as in Refs. [37-39].

In order to incorporate effects from techni-gluon condensation, here we introduce an additional bulk scalar field $\Phi_X$ dual to techni-gluon condensate $\langle \alpha G_{\mu\nu}^2 \rangle$, where $\alpha$ is related to the TC gauge coupling $g_{TC}$ by $\alpha = g_{TC}^2/(4\pi)$. Since $\langle \alpha G_{\mu\nu}^2 \rangle$ is singlet under the chiral $SU(N_f)_L \times SU(N_f)_R$ symmetry and $U(1)_V$ symmetry, the dual-bulk scalar field $\Phi_X$ has to be a real field. We take $\dim(\alpha G_{\mu\nu}^2) = 4$ and the corresponding bulk-mass parameter $m_{\Phi_X}^2 = 0$.

The form of interaction terms involving the new bulk field $\Phi_X$ still remains undetermined. In the present work, we shall adopt a “dilaton-like” coupling, such that all the fields couple to $\Phi_X$ in the exponential form like $e^{i\Phi_X(z)/\alpha}$ (see Eq.(2.19)).

Thus the five-dimensional action employed in the present paper takes the form:

$$ S_5 = \int d^4x \int_{z_e}^{z_m} dz \sqrt{-g} \frac{1}{4g_5^2} e^{\alpha G_{\mu\nu}^2/\alpha} (\Phi_X(z)^2) \left( -\frac{1}{4} \text{Tr} [L_{MN}L_{MN} + R_{MN}R_{MN}] ight. $$

$$ + \text{Tr} \left[ D_M \Phi^D M \Phi - m_{\Phi_X}^2 \Phi^D M \Phi + \frac{1}{2} \partial_M \Phi_X \partial^M \Phi_X \right], \quad (2.1) $$

where $L_M(R_M) = L^a_M(R^a_M)T^a$ with the generators of $SU(N_f)$ normalized by $\text{Tr}[T^aT^b] = \delta^{ab}$; $L(R)_{MN} = \partial_M L(R)_N - \partial_N L(R)_M - i[L(R)_M, L(R)_N]$, $g = \det[g_{MN}] = -(L/z)^{10}$, $g_5$ denotes the gauge coupling in five-dimension and $c$ is the dimensionless coupling constant. The covariant derivative acting on $\Phi$ is defined as $D_M \Phi = \partial_M \Phi + iL_M \Phi - i\Phi R_M$.

We parametrize the bulk scalar fields $\Phi$ and $\Phi_X$ as follows:

$$ \Phi(x,z) = \frac{1}{\sqrt{2}} (v(z) + \sigma(x,z)) \exp[i\pi(x,z)/v(z)], \quad (2.2) $$

$$ \Phi_X(z) = v_X(z), \quad (2.3) $$

with the vacuum expectation values (VEVs), $v(z) = \sqrt{2}\langle \Phi \rangle$ and $v_X(z) = \langle \Phi_X \rangle$, respectively. In Eq.(2.3) we ignored Kaluza-Klein (KK) modes of $\Phi_X$ (including the lowest mode) which are identified with massive glueballs with mass of order $O(\Lambda_{TC})$ which is much larger than the electroweak scale, $\Lambda_{TC} \gg F_\pi$, in the case of SWC-TC with $\gamma_m \approx 1$. The techni-dilaton, a flavor-singlet scalar bound state of techni-fermion and anti-techni-fermion, will be identified with the lowest KK mode of $\sigma(x,z)$, but not of $\Phi_X$.

We choose the boundary condition for $v(z)$ as

$$ \alpha M = \lim_{z \to 0} Z_m \left( \frac{L}{z} v(z) \right) \bigg|_{z = \epsilon}, \quad Z_m = Z_m (L/z) = \left( \frac{L}{z} \right)^{\gamma_m}, \quad (2.4) $$

$$ \xi = L v(z) \bigg|_{z = z_m}, \quad (2.5) $$

where $M$ stands for the current mass of techni-fermions and $\xi$ is related to the techni-fermion condensate $\langle \bar{\Phi}\Phi \rangle$ as will be clarified later (See Eq.(2.27)). The parameter $\alpha$ has been introduced which can arise from ambiguity of definition of the current mass $M$. Here we take $\alpha = \sqrt{3}$, which turns out to be consistent with the operator product expansion (OPE) for the scalar current correlator in QCD [41].

For $v_X$, we impose the following boundary condition:

$$ M' = \lim_{z \to 0} L v_X(z) \bigg|_{z = \epsilon}, \quad G = L v_X(z) \bigg|_{z = z_m}, \quad (2.6) $$

where $M'$ becomes the external source for the techni-gluon condensation-operator $\langle \alpha G_{\mu\nu}^2 \rangle$ and $G$ is associated to the techni-gluon condensate $\langle \alpha G_{\mu\nu}^2 \rangle$ as we will see later (See Eq.(2.20)). We define the techni-gluon condensate in such a way that it does not include a trivial perturbative contribution, namely,

$$ \langle \alpha G_{\mu\nu}^2 \rangle \equiv \langle \alpha G_{\mu\nu}^2 \rangle^{\text{full}} - \langle \alpha G_{\mu\nu}^2 \rangle^{\text{perturbation}}. \quad (2.7) $$

We then see that $G$ in Eq.(2.6) is related only to the non-perturbative breaking of the conformal/scale invariance, while $M'$ in Eq.(2.6) serves as its source and itself plays a role of the explicit breaking of the conformal symmetry just like the current mass $M$ in the case of the chiral symmetry.

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#3 This should not be confused with so-called soft-wall model [10] where $1/z_m = 0$ in contrast to our case with $1/z_m \neq 0$. 

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We next introduce the five-dimensional vector and axial-vector gauge fields $V_M$ and $A_M$ defined by $V_M = (L_M + R_M)/\sqrt{2}$ and $A_M = (L_M - R_M)/\sqrt{2}$. The UV boundary values of $V_\mu$ and $A_\mu$, then play the role of the sources ($v_\mu$, $a_\mu$) for the vector and the axial-vector currents externally coupled to TC sector. Working in $V_z = A_z \equiv 0$ gauge, we choose their boundary conditions as

$$
\left. \partial_z V_\mu(x, z) \right|_{z = z_m} = \left. \partial_z A_\mu(x, z) \right|_{z = z_m} = 0,
\left. V_\mu(x, z) \right|_{z = \epsilon} = v_\mu(x), \quad \left. A_\mu(x, z) \right|_{z = \epsilon} = a_\mu(x).
$$

Once the boundary conditions (2.4), (2.5), (2.6), and (2.8) are given, the equations of motion for the bulk fields can be solved at the classical level. By substituting those solutions into the action (2.1), the effective action is expressed as a certain functional of the UV boundary values, $S$, $M$, $M'$, $v_\mu$, and $a_\mu$, i.e., $S_5^{\text{eff}} = S_5^{\text{eff}}[M, M', v_\mu, a_\mu]$. From the familiar AdS/CFT dictionary, this $S_5^{\text{eff}}$ corresponds to the generating functional $W$ in TC written in terms of the external sources $M, M', v_\mu$, and $a_\mu$. One can then readily calculate the two-point Green functions in the usual way:

$$
\frac{\delta^2 W[v_\mu]}{\delta \hat{v}_\mu(q) \delta \hat{v}_\mu(-q)} \bigg|_{v_\mu = 0} = i \int d^4x e^{iqx} \langle J_\mu^v(x) J_\mu^v(0) \rangle = -\delta^{ab} \left( \eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \Pi_V(-q^2),
$$

$$
\frac{\delta^2 W[a_\mu]}{\delta \hat{a}_\mu(q) \delta \hat{a}_\mu(-q)} \bigg|_{a_\mu = 0} = i \int d^4x e^{iqx} \langle J_\mu^a(x) J_\mu^a(0) \rangle = -\delta^{ab} \left( \eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \Pi_A(-q^2),
$$

$$
\left. \lim_{\epsilon \to 0} i \frac{\delta W[M]}{\delta M} \right|_{M = 0} = \langle \bar{T}T \rangle,
$$

$$
\left. \lim_{\epsilon \to 0} i \frac{\delta W[M']}{\delta M'} \right|_{M' = 0} = \langle aG^2_{\mu\nu} \rangle,
$$

where $\hat{v}_\mu(q)$ and $\hat{a}_\mu(q)$ respectively denote the Fourier component of $v_\mu(x)$ and $a_\mu(x)$. To facilitate the later discussions, as done in Ref. [39], we introduce $S$ as the $S$ parameter per each techni-fermion doublet, $S = S/(N_f/2)$, which is expressed by the vector and axial-vector current correlators $\Pi_V$ and $\Pi_A$ as

$$
\dot{S} = -4\pi \frac{d}{dQ^2} \left[ \Pi_V(Q^2) - \Pi_A(Q^2) \right]_{Q^2 = 0},
$$

where $Q \equiv \sqrt{-q^2}$. We also introduce the (techni-)pion decay constant defined as

$$
F_\pi = \Pi_V(0) - \Pi_A(0).
$$

### A. Condensates: $\langle \bar{T}T \rangle$ and $\langle aG^2_{\mu\nu} \rangle$

In the following, evaluating the equations of motion for the bulk fields explicitly, we shall present the formulas for the condensates, $\langle \bar{T}T \rangle$ and $\langle aG^2_{\mu\nu} \rangle$, the $S$ parameter and the decay constant $F_\pi$, and masses of vector mesons, axial-vector mesons, and scalar mesons.

The action (2.1) leads to the following equation of motion for $v_X(z)$:

$$
\partial_z \left( \frac{1}{z^3} \partial_z \chi(z) \right) + \frac{cg_5^2}{4L^2} \chi(z) L^2 \text{Tr} \left[ -\frac{1}{z^3} (\partial_z v(z))^2 + \frac{(3 - \gamma_m)(1 + \gamma_m)}{z^5} v^2(z) \right] = 0,
$$

where we have defined

$$
\chi(z) = \exp \left[ \frac{c}{2} g_5^2 v_X(z) \right].
$$

The boundary condition for $\Phi_X$ given in Eq. (2.6) is now rewritten in terms of $\chi$ as

$$
\chi(z) \big|_{z = \epsilon} = \exp \left[ \frac{c}{2} g_5^2 v_X(\epsilon) \right] = \exp \left[ \frac{c}{2} g_5^2 M' \right],
$$

$$
\chi(z) \big|_{z = z_m} = \exp \left[ \frac{c}{2} g_5^2 v_X(z_m) \right] = \exp \left[ \frac{c}{2} g_5^2 G \right] \equiv G + 1.
$$
We solve the equation of motion (2.15) keeping only the first term of the left hand side in Eq. (2.15). This assumption will be justified later in determining the size of $[(c/2)(g_5^2/L)]^2$, which turns out to be $\approx 10^{-4}$ (See Eq. (3.2)). Equation (2.15) is now easily solved to give the solution

$$\chi(z) = \exp \left[ \frac{c}{2} g_5^2 v(z) \right] = c_1^\chi + c_2^\chi \left( \frac{z}{L} \right)^4,$$  \hspace{1cm} (2.19)

where $c_1^\chi$ and $c_2^\chi$ are determined by Eqs. (2.17) and (2.18) in the limit $\epsilon \to 0$ as

$$c_1^\chi = e^{\frac{M^2}{2} M'}, \quad c_2^\chi = \left( \frac{L}{z_m} \right)^4 (G + 1 - c_1^\chi).$$  \hspace{1cm} (2.20)

Note that the solution in Eq. (2.19) gives rise to the induced metric for the vector and axial-vector gauge fields (See Eq. (2.29)).

We next turn to the equation of motion for $v(z)$ which is read off from the action (2.1) as follows:

$$\partial_z \left( \frac{1}{2} \chi^2(z) \partial_z v(z) \right) + \chi^2(z) \frac{(3 - \gamma_m)(1 + \gamma_m)}{z_5^5} v(z) = 0.$$  \hspace{1cm} (2.21)

Substituting Eq. (2.19) into Eq. (2.21) and taking $M' = 0$, we find the solution in the limit $\epsilon \to 0$

$$v(z) = \frac{1}{1 + G \left( \frac{z}{L} \right)^4} \left[ c_1 \left( \frac{z}{L} \right)^{\gamma_m+1} + c_2 \left( \frac{z}{L} \right)^{3-\gamma_m} \right],$$  \hspace{1cm} (2.22)

where $c_1$ and $c_2$ are determined by the boundary condition in Eqs. (2.1) and (2.5) as

$$c_1 = \sqrt{3} M,$$  \hspace{1cm} (2.23)

$$c_2 = \frac{\xi (1 + G)}{L} \left( \frac{L}{z_m} \right)^{3-\gamma_m} - \left( \frac{L}{z_m} \right)^{2(1-\gamma_m)} c_1.$$  \hspace{1cm} (2.24)

Note that in Eqs. (2.22) and (2.24) (techni-)gluon condensation effects are included: When $G = 0$ in Eqs. (2.22) and (2.24), we get back to the previous results without (techni-)gluon-condensation effects [30, 31, 37–39].

Putting the classical solutions, Eqs. (2.19) and (2.22), into the action (2.1), we are left with the four-dimensional boundary term which is holographically dual to the generating functional $W[M, M']$ in TC,

$$W[M, M'] = \int d^4x \frac{L^3}{2g_5^2} \left[ -\frac{1}{3} \left( \frac{4}{g_5^2} \right)^2 \partial_z \chi(z) \cdot \chi(z) - \frac{1}{z^3} \chi^2(z) \text{Tr} [\partial_z v(z) \cdot v(z)] \right]_{\epsilon} z_m.$$  \hspace{1cm} (2.25)

Using Eqs. (2.21) and (2.22) and performing the functional derivative with respect to the sources $M$ and $M'$, the techni-gluon condensate $\langle \alpha G_{\mu\nu}^2 \rangle$ and the techni-fermion condensate $\langle \bar{T} T \rangle$ are respectively expressed in terms of the five-dimensional gauge theory as

$$\langle \alpha G_{\mu\nu}^2 \rangle = -\frac{L^2}{g_5^2} \frac{1}{z_m^{\gamma_m}} G,$$  \hspace{1cm} (2.26)

$$\langle \bar{T} T \rangle_{1/L} = -\sqrt{3} \frac{L}{g_5^2} \frac{(3 - \gamma_m)}{z_m^{\gamma_m}} (1 + G) \frac{\xi}{Z_m^{-1}},$$  \hspace{1cm} (2.27)

where $Z_m = (L/z_m)^{\gamma_m}$.

### B. Vector, axial-vector current correlators, decay constant $F_*$, and $S$ parameter

Let us next focus on the vector and axial-vector sectors. The relevant action under the gauge-fixing $V_z = A_z \equiv 0$ reads

$$S_5 \equiv -\frac{1}{2g_5^2} \int d^4x \int_0^{z_m} dz \ w(z) \left( \text{Tr} \left[ \frac{1}{2} V_{\mu\nu} V^{\mu\nu} - \partial_z V_{\mu} \partial_z V^\mu + \frac{1}{2} A_{\mu\nu} A^{\mu\nu} - \partial_z A_{\mu} \partial_z A^\mu \right] - 2 \left( \frac{L}{z} \right)^2 \text{Tr} [v^2(z) A_{\mu} A^{\mu}] \right),$$  \hspace{1cm} (2.28)
where \( V(A)_{\mu\nu} = \partial_\mu V(A)_\nu - \partial_\nu V(A)_\mu \) and the induced metric \( w(z) \) is given by the solution in Eq. (2.19) as

\[
w(z) = \frac{L}{z} \chi^2(z) = \frac{L}{z} \left( 1 + G \left( \frac{z}{z_m} \right)^4 \right)^2.
\]  

(2.29)

In arriving at the last equality we have used Eqs. (2.19), (2.20), and set \( M' = 0 \). (When \( M' = 0 \) and \( \epsilon = 0 \) the explicit breaking of conformal/scale invariance only comes from \( 1/z_m \neq 0 \).) The action in Eq. (2.28) takes the same form as in Refs. [30, 31, 37–39] except that \( w(z) = L/z \) has been replaced by the one given in Eq. (2.29). Our induced metric determined by the equation of motion for the bulk scalar \( \Phi_X \) is compared with the effective metric of Ref. 42 lacking the bulk scalars \( \Phi \) and \( \Phi_X \), where the form of the effective metric was simply assumed to reproduce the OPE for the vector and axial-vector current correlators \( \Pi_{V,A} \).

We solve the equations of motion for the transversely polarized components of the gauge fields \( \nu(x, z) \) and \( \mu(x, z) \). The corresponding equations of motion are immediately read off from the action (2.28) as

\[
\left[ q^2 + w(z)^{-1} \partial_z w(z) \partial_z \right] V_\mu(q, z) = 0, \\
\left[ q^2 + w(z)^{-1} \partial_z w(z) \partial_z - \frac{L}{z} v^2(z) \right] A_\mu(q, z) = 0,
\]

(2.30)

(2.31)

where \( V_\mu(q, z) \) and \( A_\mu(q, z) \) denote the Fourier transforms of \( \nu(x, z) \) and \( \mu(x, z) \), respectively. It is convenient to decompose \( V_\mu(q, z) \) and \( A_\mu(q, z) \) into the external sources \( (\nu_\mu(q), \dot{a}_\mu(q)) \) and the remainders \( (\nu_\mu(q, z), A_\mu(q, z)) \), such as \( V_\mu(q, z) = \nu_\mu(q) V(q, z) \) and \( A_\mu(q, z) = \dot{a}_\mu(q) A(q, z) \). Using the equations of motion (2.30) and (2.31) together with the boundary conditions in Eq. (2.8), we rewrite the action (2.28) to get the four-dimensional UV boundary term which is holographically dual to the generating functional \( W[\nu_\mu, a_\mu] \) in TC,

\[
W[\nu_\mu, a_\mu] = \frac{1}{2} \int \frac{d^4q}{(2\pi)^4} \frac{1}{g_5^2} w(e) \text{Tr} \left[ \nu_\mu(-q) \partial_z V(q, \epsilon) \cdot \nu^\mu(q) + \dot{a}_\mu(-q) \partial_z A(q, \epsilon) \cdot \dot{a}^\mu(q) \right],
\]

(2.32)

where \( V(q, z) \) and \( A(q, z) \) respectively satisfy the boundary conditions,

\[
V(q, \epsilon) = 1, \quad \partial_z V(q, z_m) = 0,
\]

(2.33)

\[
A(q, \epsilon) = 1, \quad \partial_z A(q, z_m) = 0.
\]

(2.34)

From Eqs. (2.9) and (2.10), the vector and axial-vector current correlators \( \Pi_V \) and \( \Pi_A \) are now expressed in terms of the five-dimensional gauge theory as

\[
\Pi_V(Q^2) = \frac{w(e)}{g_5^2} \partial_z V(Q^2, \epsilon), \quad \Pi_A(Q^2) = \frac{w(e)}{g_5^2} \partial_z A(Q^2, \epsilon),
\]

(2.35)

where we have rewritten \( V(q, z) = V(Q^2, z) \) and \( A(q, z) = A(Q^2, z) \). We emphasize that, thanks to the introduction of the bulk scalar field \( \Phi_X \) dual to the role of the gluon condensation, the present model reproduces \( (\alpha G_{\mu\nu}^2)/Q^4 \) term in the OPE of the current correlators \( \Pi_V \) and \( \Pi_A \), which was missing in the previous approach without \( \Phi_X \) [30, 31, 37–39]. Leaving the details of the derivation in Appendix A, we will here just show the resultant expression of the high-energy expansion of \( \Pi_{V,A}(Q^2) \) in the large Euclidean-momentum region \((1/z_m)^2 \ll Q^2 \ll (1/\epsilon)^2\),

\[
\Pi_{V,A}(Q^2) \bigg|_{(1/z_m)^2 \ll Q^2 < (1/\epsilon)^2} = Q^2 \left[ \frac{L}{2g_5^2} \ln Q^2 + \frac{2g_5^2}{3L} \left( \alpha G_{\mu\nu}^2 \right)/Q^4 + \mathcal{O} \left( \frac{1}{Q^{6-2z_m}} \right) \right].
\]

(2.36)

Furthermore, by introducing some extra higher-dimensional interaction terms (See Eq. (A.8)), the present model exactly reproduces the high-energy behavior up to terms suppressed by \((1/Q^8)\), consistently with the form of the OPE,

\[
\Pi_{V,A}(Q^2) \bigg|_{(1/z_m)^2 \ll Q^2 < (1/\epsilon)^2} = Q^2 \left[ \frac{L}{2g_5^2} \ln Q^2 + \frac{2g_5^2}{3L} \left( \alpha G_{\mu\nu}^2 \right)/Q^4 + C_{6}^{V,A} L^{2z_m} \frac{(TT)^2}{Q^2(3-\gamma_m)} + \mathcal{O} \left( \frac{1}{Q^8} \right) \right],
\]

(2.37)

where the couplings \( C_{6}^{V,A} \) come from the higher-dimensional interaction terms (See Eq. (A.14)). It should also be stressed that such extra interaction terms do not affect all of our results shown in the later sections.
Our model is sharply contrasted with the approach in Ref. 42 where the effective metric was assumed so as to produce $1/Q^4$ term which, however, could be confused with the chiral condensation term $(\bar{T}T)^2/Q^6-2\gamma_m \approx (\bar{T}T)^2/Q^4$ in the case of SWC-TC with $\gamma_m \approx 1$ in the absence of the bulk scalars $\Phi$ and $\Phi^\dagger$. In our case which explicitly includes the bulk scalar field $\Phi_X$ dual to the gluon condensate, we are able to obtain not just the form behaving as $(1/Q^4)$ but the whole expression $(\alpha G^2_{\mu \nu})/Q^4$ involving the gluon condensate $(\alpha G^2_{\mu \nu})$, and hence clearly distinguish from the $(\bar{T}T)^2/Q^4$ term arising due to the bulk scalar $\Phi$.

In order to obtain the formulas for the decay constant $F_\pi$ and the $S$ parameter, we shall expand $\Pi_{V,A}(Q^2)$ perturbatively in powers of $Q^2$ as $\Pi_{V,A}(Q^2) = \Pi_{V,A}(0) + Q^2\Pi_{V,A}(0) + \mathcal{O}(Q^4)$, where $\Pi_{V,A}(0) = \pm \Pi_{V,A}(Q^2)/\partial Q^2\bigg|_{Q^2=0}$.

Then $\Pi_{V,A}(0)$ and $\Pi_{V,A}^\prime(0)$ are expressed as

$$
\Pi_V(0) = 0, \quad \Pi_V^\prime(0) = -\frac{L}{g_5^2} \int_\epsilon^{z_m} \frac{dz'}{z'} \chi^2(z'),
$$

$$
\Pi_A(0) = \frac{L}{g_5^2} \epsilon \chi^2(\epsilon) \partial_z A(0, z) \bigg|_{z=\epsilon}, \quad \Pi_A^\prime(0) = -\frac{L}{g_5^2} \int_\epsilon^{z_m} \frac{dz'}{z'} \chi^2(z') A^2(0, z'),
$$

where $A(0, z)$ is given as a solution to Eq. (2.31) with $q^2 = 0$. From Eqs. (2.14) and (2.13), we find that $F_\pi$ and $\hat{S}$ are expressed in terms of the five-dimensional gauge theory as

$$
F_\pi^2 = -\frac{L}{g_5^2} \epsilon \chi^2(\epsilon) \partial_z A(0, z) \bigg|_{z=\epsilon},
$$

$$
\hat{S} = 4\pi \frac{L}{g_5^2} \int_\epsilon^{z_m} \frac{dz}{z} \chi^2(z)(1-A^2(0, z)).
$$

C. Vector, axial-vector, and flavor-singlet scalar meson masses: $M_{V_n}$, $M_{A_n}$, and $M_{\sigma_n}$

A set of vector meson masses $\{M_{V_n}\}$ arises as an infinite tower of eigenvalues of normalizable solutions $\{V_n(z)\}$ satisfying Eq. (2.30) with $q^2$ replaced by $\{M_{V_n}^2\}$,

$$
[M_{V_n}^2 + w(z)^{-1} \partial_z w(z) \partial_z] V_n(z) = 0,
$$

with the boundary condition $V_n(\epsilon) = 0$ and $\partial_z V_n(z_m) = 0$. The lowest eigenvalue is identified as the techni-$\rho$ meson mass, $M_{V_1} = M_\rho$.

Similarly for axial-vector meson masses $\{M_{A_n}\}$, the eigenvalue equation for a set of normalizable modes $\{A_n(z)\}$ is obtained by taking $q^2 = M_{A_n}^2$ in Eq. (2.31):

$$
[M_{A_n}^2 + w(z)^{-1} \partial_z w(z) \partial_z - 2 \left(\frac{L}{g_5^2}\right)^2 v^2(z)] A_n(z) = 0,
$$

with the boundary condition $A_n(\epsilon) = 0$ and $\partial_z A_n(z_m) = 0$. The lowest eigenvalue is regarded as the techni-$\alpha_1$ meson mass, $M_{A_1} = M_{\alpha_1}$.

The equation of motion for the flavor-singlet scalar field $\sigma(x, z)$ is decomposed into the eigenvalue equations for a set of the KK modes $\sigma^{(n)}(x)$ arising as $\sigma(x, z) = \sum_{n=1}^{\infty} \sigma^{(n)}(x)\sigma_n(z)$. By taking into account Eq. (2.2) and replacing the momentum-squared $q^2$ with the mass-squared $M_{\sigma_n}^2$, the equation of motion for the wave function $\sigma_n(z)$ is read off from the action (2.1) as

$$
[M_{\sigma_n}^2 + \left(\frac{w(z)}{z^2}\right)^{-1} \partial_z \left(\frac{w(z)}{z^2}\right) \partial_z - \frac{3 - \gamma_m}{z^2} \left(\frac{3 - \gamma_m + 1 + \gamma_m}{z^2}\right)] \sigma_n(z) = 0,
$$

where the normalizable solution $\sigma_n(z)$ should satisfy the UV boundary condition, $\lim_{z \to 0} \sigma_n(\epsilon) = 0$, so as to make the action finite at $z = \epsilon$ in the limit $\epsilon \to 0$. The solution to Eq. (2.44) is then given as

$$
\sigma_n(z) = c_\sigma \left(\frac{z}{z_m}\right)^2 J_{1-\gamma_m}(M_{\sigma_n}z),
$$

where $c_{\sigma}$ is a constant.
where we have put \( M' = 0 \) for simplicity. One can easily see that, with respect to \( \xi \), \( V[\xi] \) is minimized at \( \xi = 0 \), which readily leads to \( \langle TT \rangle = 0 \) through Eq. (2.27) and hence to no spontaneous breaking of chiral symmetry. In order to avoid this problem, similarly to a procedure proposed in Ref. [41], we may introduce the following IR potential:

\[
L_{\text{IR}} = -\left( \frac{L}{z} \right)^4 \chi^2(z) V(\phi) \bigg|_{z_m},
\]

where the potential parameters \( m_b^2 \) and \( \lambda \) are taken to be positive. By adding this IR potential, the vacuum is now realized at \( \xi \neq 0 \):

\[
\xi^2 = \frac{1}{\lambda} \left( L^2 m_b^2 - \frac{L}{g_5^2} \left( 3 - \gamma_m - 4 \frac{G}{1 + G} \right) \right),
\]

where \( \xi^2 \) is tuned to be positive by adjusting \( m_b^2 \). The IR boundary condition for \( \sigma_n(z) \) is now assigned in a way that the total IR boundary term with respect to \( \sigma^n(z) \) is canceled in the quadratic order:

\[
\left[ \partial_z + 2 \left( \frac{L}{z_m} \right) g_5^2 m_{n5}^2 \right] \sigma_n(z) \bigg|_{z_m} = 0,
\]

where

\[
m_{n5}^2 = \frac{1}{L^2} \left[ \lambda \xi^2 - \frac{1}{2} \frac{L}{g_5^2} \left( 3 - \gamma_m - 4 \frac{G}{1 + G} \right) \xi^2 \right].
\]

Substituting the solution in Eq. (2.45) into the IR boundary condition (2.49), we thus obtain the eigenvalue equation (2.49):

\[
2\lambda \xi^2 g_5^2 \frac{L}{L} J_{1 - \gamma_m}(M_{sn} z_m) = M_{sn} z_m \cdot J_{2 - \gamma_m}(M_{sn} z_m).
\]

The lowest eigenvalue is identified as the techni-dilaton mass, \( M_{\sigma_1} = M_{\text{TD}} \).

### III. ANALYSIS ON GLUONIC-CONTRIBUTIONS

In this section, we shall discuss effects on observables coming from the gluon condensation in a generic TC with \( 0 \lesssim \gamma_m \lesssim 1 \) involving the case of QCD with \( \gamma_m \approx 0 \). Among observables, we particularly focus on the \( S \) parameter and the masses of the lowest KK modes for the vector, axial-vector, and flavor-singlet scalar mesons (\( M_p, M_{\sigma_1}, M_{\text{TD}} \)). To this end, we first pay our attention to the parameters describing the present five-dimensional model and momentarily discuss how they can be fixed by considering the holographic-dual of the generic TC with \( 0 \lesssim \gamma_m \lesssim 1 \). The parameters are following ten:

\[
\frac{L}{g_5^2}, \quad z_m, \quad \epsilon, \quad \gamma_m, \quad \xi, \quad M, \quad G(\text{or } \mathcal{G}), \quad M', \quad \epsilon, \quad \lambda.
\]

The UV brane position \( \epsilon \) is treated as the cutoff scale \( 1/\epsilon \) of the five-dimensional theory and is usually set to be 0 after all calculations are done. From a point of view of a typical TC scenario, on the other hand, the UV cutoff \( 1/\epsilon \) can be replaced by an ETC scale \( 1/\epsilon = \Lambda_{\text{ETC}} \). As for the IR brane position \( z_m \), similarly, it can
play a role of the IR cutoff scale of the theory associated with the chiral symmetry breaking or confinement, and hence \((1/z_m)\) can be related to a typical meson mass scale, say, \(m\), in TC. Since \((m/\Lambda_{ETC}) \ll 1\), we may simply put \((\epsilon/z_m) = (m/\Lambda_{ETC}) = 0\). Then we see that the \(S\) parameter does not depend on either \(\epsilon\) or \(z_m\) since it is a dimensionless quantity. Note, however, that other dimensionful quantities, such as \(F_\pi\), \(M_\rho\), \(M_\pi\), and \(M_{TD}\), still have a certain \(z_m\)-dependence which can be completely factorized by defining dimensionless ones like \(\tilde{F}_\pi = z_m F_\pi\), and so on.

The parameter \(M'\) is the external source of the techni-gluon condensation-operator \(\alpha G_{\mu\nu}^2\), and hence is regarded as the explicit breaking source of the conformal/scale symmetry associated with the dilatation current anomaly characterized by the intrinsic scale of order \(\Lambda_{ETC} \gg 1/z_m\) for \(\gamma_m \simeq 1\) in which we are not interested. Here we take \(M' = 0\). (Even when \(M' = 0\) and \(\epsilon = 0\), we have the explicit breaking of conformal/scale invariance due to \(1/z_m \neq 0\).)

The parameters \((L/g_5^2)\) and \(c\) are determined by comparing the high-energy behavior of the current correlators \(\Pi_{V,A}\) to those obtained by the OPE: \((L/g_5^2)\) from the \(\ln Q^2\) term and \(c\) from the \(\langle \alpha G_{\mu\nu}^2 \rangle /Q^4\) term. In the case of SWC-TC with \(\gamma_m \simeq 1\), \(\langle TT \rangle^2/Q^4\) term has the same \(Q^2\)-dependence as that of \(\langle \alpha G_{\mu\nu}^2 \rangle /Q^4\) term. As was discussed in Sec. [113] it is possible to clearly distinguish those two terms in our approach. We will leave the detailed calculation in Appendix [A] and here just quote the result on the OPE-matching:

\[
\frac{L}{g_5^2} = \frac{N_{TC}}{12\pi^2}, \quad c = -\frac{N_{TC}}{192\pi^3}.
\]

(3.2)

The parameter \(\lambda\) has been introduced so as to minimize the bulk scalar potential with non-zero \(\xi\). In order to know more about \(\lambda\), let us take a look at the two terms in the square bracket of Eq.(2.30). One then finds that the first term \((\lambda \xi^2)\) should be proportional to \(N_{TC}\) because of Eq.(3.22). Furthermore, from Eqs.(2.24) and (6.2) and taking into account \((TT) \propto N_{TC}\), we see that \(\xi \propto N_{TC}\) and hence \(\lambda \sim N_{TC}\). Supposing that the coupling \(\lambda\) is expected to be generated at one-loop level (through techni-fermion loops), we may totally write \(\lambda = \kappa \times N_{TC}/(4\pi)^2\) with \(O(1)\) parameter \(\kappa\):

\[
\lambda = \frac{N_{TC}}{(4\pi)^2}, \quad \kappa = 1.0 \pm 0.3.
\]

(3.3)

which reproduces the mass of the flavor-singlet scalar bound-state (two-quark state), \(f_0(1370)\), in QCD as the lightest KK mode of the flavor-singlet scalar, \(M_{f_0}\). Actually in QCD, there are other two candidates for the light flavor-singlet scalar particles other than \(f_0(1370)\), which are \(f_0(600)\) (so-called \(\sigma\)) and \(f_0(980)\). The following is the reason why we have adopted \(f_0(1370)\) to fit the value of \(M_{f_0}\): Since we are interested in application to a generic TC, we need to carefully select a certain appropriate flavor-singlet scalar bound-state realized in a generic strongly coupled dynamics with arbitrary \(N_c\) and \(N_f\). Thinking about the other two candidates from this point of view, one finds that those bound-states can be considered as four-quark states due to a specific property arising only in the case of real life QCD with \(N_f = N_c = 3\) (See, for example, Ref. [43]), so they are excluded from candidate of two-quark state. We further notice that similar characteristic features in real life QCD would cause mixing between two-quark states and four-quark states, which could make the observed mass of the two-quark state \(f_0(1370)\) lifted up. Without such a mixing, the mass of \(f_0(1370)\) is expected to be around 1.1-1.2 GeV [44] #4. Thus we have determined the values of \(\kappa\) as in Eq.(3.3) \((0.7 \leq \kappa \leq 1.3)\) using a set of QCD-fit values (See Eq.(3.3)) so as to reproduce the allowed range of the mass of \(f_0(1370)\) without the mixing.

Thus we are now left with the four undetermined parameters,

\[
z_m, \quad \xi, \quad G, \quad \gamma_m (0 \lesssim \gamma_m \lesssim 1),
\]

(3.4)

with \(z_m\) being the only dimensionful parameter.

### A. QCD case with \(\gamma_m \simeq 0\)

We shall first consider the case of QCD with \(N_c = 3\) and \(\gamma_m \simeq 0\). In the case of QCD, the parameters \(\xi, G,\) and \((1/z_m)\) can be fixed in such a way that Eqs.(2.40), (2.42), and (2.26) respectively reproduce the experimental

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#4 Actually in Ref.[44], the mixing between \(a_0(980)\) and \(a_0(1445)\) is discussed. It is known, however, that in the large \(N_c\) limit \(f_0(1370)\) and \(a_0(1445)\) are degenerate, so we can apply the similar argument against the mixing between \(f_0(980)\) and \(f_0(1370)\).
One might think that walking/near conformal dynamics is characterized by not 
resulting in good agreement with experiment as shown in Table. I. 

Other dimensionful quantities, \( f_\nu \) values \([45]\) and \( \kappa \) values of 
\( M_{\sigma_1} \) correspond to the mass estimated without mixing with a four quark state \( f_0(980) \) (See discussion below Eq. (3.3)).

values \([45]\) \( f_\pi \simeq 92.4 \, \text{MeV} \), \( M_\rho \simeq 775 \, \text{MeV} \), and a typical empirical value \([46]\) \( \frac{1}{\pi}(\alpha G_{\mu\nu}^2) \simeq 0.012 \, \text{GeV}^4 \): 
\[ \xi \simeq 3.1, \quad G \simeq 0.25, \quad \frac{1}{z_m} \simeq 347 \, \text{MeV}. \] (3.5)

Using these values, we calculate the values of \( M_{\sigma_1} \) (from Eq. (2.43)), \( M_{\sigma_1} \equiv M_{f_0(1370)} \) (from Eq. (2.51) and using the values of \( \kappa \) in Eq. (3.3)), \( \hat{S} = -16\pi L_{10} \) (from Eq. (2.41)), and the quark condensate \( \langle \bar{q}q \rangle \) (from Eq. (2.31)), which results in good agreement with experiment as shown in Table. I.

In our model, meson masses have some contributions from the gluon condensation. The effect of the gluon condensation on \( M_\rho \) can be analytically estimated for \( G \simeq 0.25 \) by expanding Eq. (2.42) perturbatively in \( G \):
\[ M_\rho \simeq \frac{2.41}{z_m} \times [1 - 0.388 G] \simeq 347 \, \text{MeV}, \] (3.6)

where in reaching the last line we have used \( (1/z_m) \simeq 347 \, \text{MeV} \) in Eq. (3.5). Equation (3.6) implies that the gluon condensation negatively contributes about 10% to the \( \rho \) meson mass in QCD. Similar expression was obtained in somewhat different approach \([42]\).

**B. Generic TC case with \( 0 \leq \gamma_m \leq 1 \)**

We next discuss the case of a generic TC with \( 0 \leq \gamma_m \leq 1 \) involving SWC-TC with \( \gamma_m \simeq 1 \) \#5 and evaluate gluonic-contributions to the \( S \) parameter \( S = \hat{S} \cdot (N_f/2) \), masses of techni-\( \rho \) \( (M_\rho) \), techni-\( a_1 \) \( (M_{a_1}) \), and the lightest flavor-singlet scalar, techni-dilaton \( (M_{TD} = M_{\sigma_1}) \).

As we noted in Eq. (3.4), the dimensionless quantity \( \hat{S} \) in Eq. (2.41) is given as a function of the three dimensionless parameters \( \xi, \gamma_m, \) and \( G \),
\[ \hat{S} = \hat{S} (\gamma_m, \xi, G). \] (3.7)

Other dimensionful quantities, \( F_\pi, M_\rho, M_{a_1} \), and \( M_{TD} \) are respectively expressed as follows:
\[ F_\pi = z_m^{-1} \cdot \bar{F}_\pi (\gamma_m, \xi, G), \] (3.8)
\[ M_\rho = z_m^{-1} \cdot \bar{M}_\rho (G), \] (3.9)
\[ M_{a_1} = z_m^{-1} \cdot \bar{M}_{a_1} (\gamma_m, \xi, G), \] (3.10)
\[ M_{TD} = z_m^{-1} \cdot \bar{M}_{TD} (\gamma_m, \xi, G). \] (3.11)

As in QCD, we will take the value of \( \kappa \) in Eq. (3.3), \( \kappa = 1.0 \pm 0.3 \). Absence of explicit dependence of \( \xi \) and \( \gamma_m \) in \( M_\rho \) can be seen in Eq. (2.42). Explicit \( \xi \)- and \( \gamma_m \)-dependences for \( M_{a_1} \) enter in the \( v^2(z) \) term in Eq. (2.43). \( M_{TD} \) has no explicit \( G \)-dependence as seen in Eq. (2.51). Note that every dimensionful quantities scales with the parameter \( (1/z_m) \). Hereafter we shall consider dimensionless quantities \( (M_\rho/F_\pi), (M_{a_1}/F_\pi) \), and \( (M_{TD}/F_\pi) \) which are free from \( z_m \).

---

\#5 One might think that walking/near conformal dynamics is characterized by not \( \gamma_m = 1 \) but \( \gamma_m \simeq 1 \), as in a typical example of SWC-TC based on the Caswell-Banks-Zaks infrared fixed point \([16, 17]\) in the large \( N_f \) QCD. However, as was clarified in Ref. \([39]\), in the present holographic approach, there exists no discontinuity between \( \gamma_m = 1 \) and the limit \( \gamma_m \to 1 \), so that both cases give the same result. In the present work, therefore, we have explicitly taken \( \gamma_m = 1 \).
The value of the upper bound, 0.05, can be estimated as follows: Consider a conservative upper bound of the $S/N$ parameter, $S \lesssim 0.1$ and look at the relationship with $\hat{S}$, $(\hat{S}/N_{TC}) = (N_f/2) \cdot (\hat{S}/N_{TC})$. Since $N_{TC} \geq 2$ and $N_f \geq 2$ (i.e. the number of techni-doublets $N_{TD} \geq 1$), one then finds that $S \lesssim 0.1$ leads to $(\hat{S}/N_{TC}) \lesssim (0.1/|N_{TC}|_{\min}) \times (2/|N_f|_{\min}) = 0.05$.
Actually, the phenomenological bound for the $S$ parameter is $S \lesssim 0.1$, not $\hat{S}(= S/(N_f/2)) \lesssim 0.1$. Both constraints would coincide only when $N_f = 2$ (minimal flavors). More detailed discussion including the flavor-dependence will be given in the next section.
very sensitive to \( \Gamma \), rapidly decreasing as \( \Gamma \) increases. In fact \( M_{\text{TD}}/F_\pi \to 0 \) in the formal limit \( \Gamma \to \infty \). This tendency is still operative even if \( S \) is much smaller, although the decreasing rate of \( M_{\text{TD}}/F_\pi \) becomes somewhat milder. In the next section we will discuss a matching with a couple of concrete models of SWC-TC in which the value of \( \Gamma \) is related to \( \Lambda_{\text{ETC}}/F_\pi \) in such a way that \( \Gamma \simeq 6 - 8 \) for \( \Lambda_{\text{ETC}}/F_\pi = 10^4 - 10^5 \). (\( \Gamma \to \infty \) for \( \Lambda_{\text{ETC}}/F_\pi \to \infty \).)

IV. MATCHING WITH TYPICAL MODELS OF SWC-TC BASED ON LARGE \( N_f \) QCD

In this section, we consider typical models of SWC-TC based on the Caswell-Banks-Zaks IR fixed point \[ 16, 17 \] \( \alpha_\ast \) of the two-loop beta function in the large \( N_f \) QCD, QCD with massless flavors \( 3 \ll N_f < 11 N_{\text{TC}}/2 \). It has been suggested that there exists a certain region \( \left(N_f^\ast < N_f < N_f^\text{cr} < 11 N_{\text{TC}}/2 \right) \) (which is called “conformal window”) such that \( \alpha_\ast < \alpha_{\text{cr}} \), where the critical coupling \( \alpha_{\text{cr}} \) for the spontaneous chiral symmetry breaking may be estimated as \( \alpha_{\text{cr}} = \pi/(3C_2(F)) \) based on the Schwinger-Dyson (SD) equation in the ladder approximation. Equating \( \alpha_\ast = \alpha_{\text{cr}} = \pi/(3C_2(F)) \), we find \( N_f^\text{cr} \simeq 4 N_{\text{TC}} \) \[ 14 \] for the conformal phase transition point. There are many studies about the existence of the conformal window and the value of \( N_f^\ast \) (if exists) in various non-perturbative methods including lattice gauge theories \[ 48 \]. Here we simply assume the existence of the conformal window and tentatively use the value of \( N_f^\ast \) from the perturbative two-loop beta function and ladder SD equation.

It was argued \[ 13 \] that the conformal phase transition is characterized by scaling of the essential singularity (Miransky scaling \[ 24 \])

\[
m \simeq 4 \Lambda \exp \left(-\frac{\pi}{\sqrt{\alpha/\alpha_{\text{cr}} - 1}}\right), \quad (4.1)
\]

where \( m \) is the dynamical mass of techni-fermion and \( \Lambda \) an intrinsic scale (\( \Lambda_{\text{TC}} \)) which may be identified with an ETC scale \( \Lambda_{\text{ETC}} \) in the actual model building:

\[
\Lambda = \Lambda_{\text{TC}} \simeq \Lambda_{\text{ETC}} \gg m. \quad (4.2)
\]

We can arrange a large hierarchy \( m \ll \Lambda \) in terms of (approximately) conformal symmetry by tuning the theory close to the conformal fixed point as \( \alpha \simeq \alpha_\ast \to \alpha_{\text{cr}} \) in the broken phase \( \alpha_\ast > \alpha_{\text{cr}} \), in such a way that the coupling constant is almost non-running over the wide range \( m < \mu < \Lambda \) (see Fig. 4). (There still exists a remnant of the conformal symmetry due to the IR fixed point \( \alpha_\ast \), although the IR fixed point \( \alpha_\ast \) actually disappears because techni-fermions with mass \( m \) decouple from the beta function for \( \mu < m \).)

In the SWC dynamics near the conformal window, the explicit breaking of the conformal symmetry manifest as the conformal anomaly is due to the generation of the dynamical mass of techni-fermions \( m \) which arises from the spontaneous breaking of the conformal and chiral symmetry. Thus the techni-gluon condensation for the conformal anomaly relevant to the dynamics near the conformal window is directly related to \( m \) but not the intrinsic scale \( \Lambda_{\text{TC}} \gg m \). The conformal anomaly for this dynamical generation takes the following form \[ 22 \]:

\[
\langle \partial^\mu D_\mu \rangle = \langle \theta^\mu_\mu \rangle = 4 \langle \theta^0_0 \rangle = \lim_{\Lambda \to \infty} \frac{\beta_{\text{NP}}(\alpha)}{4\alpha^2} (\alpha G^2_{\mu\nu}), \quad (4.3)
\]

FIG. 5: Plots of \( (M_\rho/F_\pi), (M_{\alpha_1}/F_\pi), \) and \( (M_{\text{TD}}/F_\pi) \) as a function of \( \Gamma \) for \( \hat{S} = 0.1 \) in a generic SWC-TC with \( N_{\text{TC}} = 3 \) and \( \gamma_m \simeq 1 \). The dotted (red), dashed (yellow), and solid (blue) curves respectively correspond to \( (M_\rho/F_\pi), (M_{\alpha_1}/F_\pi), \) and \( (M_{\text{TD}}/F_\pi) \).
where $D_\mu$ and $\theta_\mu^i$ are the dilatation current and the energy-momentum tensor, respectively, and $\beta_{NP}(\alpha)$ denotes the non-perturbative beta function of the gauge coupling $\alpha$ related to the generation of $m$ in Eq.(4.1):

$$\beta_{NP}(\alpha) \equiv \frac{\partial \alpha}{\partial \ln \Lambda} = -\frac{2}{3C_2(F)} \left( \frac{\alpha}{\alpha_{cr}} - 1 \right) = -\frac{2\pi^3}{3C_2(F)} \left( \ln \frac{\Lambda}{m} \right)^{-3},$$  \hspace{1cm} (4.4)

in which we have used $\alpha_{cr} = \pi/(3C_2(F))$. Straightforward calculation of the effective potential at two-loop order yields the vacuum energy $\langle \theta_0^0 \rangle$:

$$\langle \theta_0^0 \rangle = -\frac{N_fN_{TC}}{\pi^4}m^4. \hspace{1cm} (4.5)$$

From Eqs.(4.3) and (4.5), we obtain

$$\langle \alpha G_{\mu\nu}^2 \rangle = -\frac{16}{\pi^4} \frac{\alpha^2}{\beta_{NP}(\alpha)} N_fN_{TC} m^4. \hspace{1cm} (4.6)$$

Note that Eq.(4.3) takes the same form as a conventional conformal anomaly obtained in the all order perturbation theory [3], where we naturally expect $\langle \alpha G_{\mu\nu}^2 \rangle = \mathcal{O}(\Lambda_{TC}^4) \gg \mathcal{O}(m^4)$. Here we ignore techni-glueball dilaton with mass of this order $\Lambda_{TC}$ associated with the perturbative anomaly and the running effect of the coupling for $\mu > \Lambda_{TC}$. In the ordinary QCD with $N_c = N_f = 3$, we have $\langle \alpha G_{\mu\nu}^2 \rangle = \mathcal{O}(\Lambda_{QCD}^4) = \mathcal{O}(m^4)$: There is no such a large hierarchy $m \ll \Lambda_{QCD}$ and conformal region $m < \mu < \Lambda_{QCD}$ where the coupling constant is almost non-running $\alpha(Q) \approx \text{constant}$. In contrast, our techni-glueon condensate $\langle \alpha G_{\mu\nu}^2 \rangle$ in Eq.(4.3) is responsible for the conformal anomaly induced by the dynamical generation of mass $m$ and hence $\langle \alpha G_{\mu\nu}^2 \rangle = \mathcal{O}(m^4) \ll \mathcal{O}(\Lambda_{TC}^4)$ and our techni-dilaton is a bound state of techni-fermions with mass $m$ which breaks the conformal symmetry near the conformal window. Also note that the conformal region with the almost non-running coupling is realized only when we arrange $N_f \approx N_{TC}$ in such a way that the fermionic dynamics and the gluonic dynamics cooperate intimately. Thus in Eq.(4.6) we have $\langle G_{\mu\nu}^2 \rangle \sim \mathcal{O}(N_fN_{TC}) \sim \mathcal{O}(N_{TC}^2)$ in accord with large $N_{TC}$ counting relevant to holographic models.

From Eqs.(4.1), (4.4), and (4.6), the techni-glueon condensate $\langle \alpha G_{\mu\nu}^2 \rangle$ is expressed in terms of $m$ and $\Lambda$ as

$$\langle \alpha G_{\mu\nu}^2 \rangle |_{\alpha = \alpha_{cr}} \propto \lim_{\Lambda \to \infty} \frac{8}{3C_2(F)\pi^2} N_fN_{TC} \left( \ln \frac{4\Lambda}{m} \right)^3 \left( 1 + \left( \ln \frac{4\Lambda}{m} \right)^{-2} \right)^2 \hspace{1cm} (4.7)$$

Comparing this to Eq.(2.26) with Eq.(3.2) taken into account, we arrive at a relationship between $G$, $(z_m m)$, and $(\Lambda_{ETC}/m)$,

$$G = C \cdot (z_m m)^4 \left( \ln \frac{4\Lambda_{ETC}}{m} \right)^3 \left( 1 + \left( \ln \frac{4\Lambda_{ETC}}{m} \right)^{-2} \right)^2 \hspace{1cm} (4.8)$$

where

$$C = \frac{1}{2\pi} \frac{N_fN_{TC}}{N_{TC}^2 - 1}. \hspace{1cm} (4.9)$$

In terms of $m$, the techni-fermion condensate $\langle TT \rangle_m$ renormalized at $\mu = m$ can be evaluated as #9

$$\langle TT \rangle_m = Z_m \cdot \langle TT \rangle_\Lambda = -t \cdot \frac{N_{TC}}{4\pi^2} m^3 \hspace{1cm} \text{with } t \simeq 2, \hspace{1cm} (4.10)$$

where the mass renormalization constant is $Z_m = Z_m(m/\Lambda) = m/\Lambda$. We regard the condensate in Eq.(2.27) as the one renormalized at $\mu = (1/L)$ following the procedure suggested in Ref. [39]. We then find the parameter $\xi$ is related to $G$ together with $(z_m m)$ as follows:

$$\xi = \frac{\sqrt{3}}{1 + G \cdot (z_m m)^2}, \hspace{1cm} (4.11)$$

---

#9 Numerically $t$ coincides with the prefactor in Eq.(2.27), $3 - \gamma_m$, for $\gamma_m \approx 0, 1, 2$. In the case of QCD with $\gamma_m \approx 0$, this implies $m \approx 453 \text{ MeV}$ for the value of $(qq) \approx -(277 \text{ MeV})^3$ in accord with the conventional constituent quark mass $m \approx 350 \text{ MeV}$ and with $m \approx 420 \text{ GeV}$ from the Pagels-Stokar (PS) formula. For details see Appendix. [3]
where we have used Eqs. (3.2). From Eqs. (3.10) and (3.11), we see that the two parameters \( \xi \) and \( G \) are now replaced by \((z_m m)\) and \((\Lambda_{ETC}/m)\) involving the quantities concerning the SWC-TC, the dynamical mass \( m \) and the ETC scale \( \Lambda_{ETC} \).

To be concrete for our analysis, we will assume the value of \( N_f^\xi \) as that from the two-loop beta function and ladder SD equation \( N_f^\xi \approx 4 N_{TC} \) bearing in mind the large \( N_{TC} \) limit in accord with holographic setup.

Let us now recall Eqs. (3.7) and (3.11) which imply that all the quantities given in those equations (say, \( S = S(\xi, G) \)) and \( F_\pi = z m \tilde{f}_\pi(\xi, G) \) are determined once we fix the one dimensionful parameter \( z_m \) and the two dimensionless parameters \( \xi, G \) which are now rephrased by \((z_m m)\) and \((\Lambda_{ETC}/m)\). To fix \( \xi \) or \( G \), we may use a certain value of the \( S \) parameter (e.g. \( S = 0.1 \)) as a phenomenological input in such a way as was done in the previous section. To determine the size of \( z_m \), we can use the familiar formula

\[
F_\pi = 246/\sqrt{N_f^{\text{EW}}/2 \text{GeV}},
\]

where \( N_f^{\text{EW}} \) is the number of techni-fermions belonging to the doublet of \( SU(2)_L \) symmetry in the standard model (SM) and may be different from the number of techni-fermion flavors \( N_f \) which participate in the SWC-TC dynamics. Furthermore, the ETC scale \( \Lambda_{ETC} \) may be constrained to be in a range, \( 10^3 \lesssim \Lambda_{ETC} \lesssim 10^4 \) TeV:

\[
10^4 \lesssim \Lambda_{ETC}/F_\pi \lesssim 10^5,
\]

so as to accommodate the realistic light quark masses without suffering from the flavor changing neutral current (FCNC) syndrome. Use of these inputs now fixes the values of the parameters \( \xi, G, 1/z_m \). In Table II we list these values for each \( N_{TC} = 2, 3, 4 \), and \( (\Lambda_{ETC}/F_\pi) = 10^4, 10^5 \) in the case of \( N_f = N_f^{\text{EW}} = 4 N_{TC} \) and \( S = \tilde{S}(N_f^{\text{EW}}/2) = 0.1 \).

Using the values of the parameters given in Table II, we calculate the masses of the techni-\( \rho \) \((M_\rho)\), techni-\( a_1 \) \((M_{a_1})\), and techni-scalar \((M_{TD})\) mesons to obtain Table III. The values of \( M_{TD} \) are estimated by varying the value of \( \kappa \) from 0.7 to 1.3 (around 1.0 with 30\% error) in Table III. Note that \( M_\rho \) and \( M_{a_1} \) are almost degenerate to be \( M_\rho \approx M_{a_1} \approx 3.7-3.9 \text{ TeV} \) for every \( N_{TC} = 2, 3, 4 \), in accord with the general tendency of the model with large techni-gluon condensate in section III where the degeneracy was not linked to the smallness of the \( S \) parameter and would have a new phenomenological implications. In contrast, the techni-dilaton mass \( M_{TD} \approx 500-800 \text{ GeV} \) (when

| \( N_{TC} \) | \( \log_{10}(\Lambda_{ETC}/F_\pi) \) | \( M_{TD} \) [GeV] | \( M_\rho \) [TeV] | \( M_{a_1} \) [TeV] | \( \Gamma \) | \( m \) [TeV] | \( m/m_{PS} \) | \( R \) |
|-----|----------------|--------|--------|--------|----|--------|----------|-------|
| 2   | 4              | 777\(^{+106}_{-125}\) | 3.75   | 3.82   | 5.93 | 1.08   | 1.49     | 1.11  |
| 2   | 5              | 613\(^{+85}_{-99}\)    | 3.69   | 3.74   | 7.26 | 1.13   | 1.57     | 1.16  |
| 3   | 4              | 681\(^{+94}_{-110}\)   | 3.86   | 3.90   | 6.26 | 0.84   | 1.74     | 1.48  |
| 3   | 5              | 556\(^{+70}_{-92}\)    | 3.80   | 3.83   | 7.57 | 0.87   | 1.80     | 1.53  |
| 4   | 4              | 597\(^{+82}_{-96}\)    | 3.93   | 3.95   | 6.58 | 0.71   | 1.97     | 1.84  |
| 4   | 5              | 505\(^{+70}_{-82}\)    | 3.87   | 3.89   | 7.88 | 0.73   | 2.02     | 1.88  |

TABLE II: Values of the model-parameters fitted to the SWC-TC with \( N_{TC} = 2, 3, 4 \) and \( N_f = N_f^{QCD} = 4 N_{TC} \) based on the large \( N_f \) QCD. Here use has been made of \( S = \tilde{S}(N_f^{QCD}/2) = 0.1 \).

TABLE III: Estimates of \( M_\rho \), \( M_{a_1} \), and \( M_{TD} \) for \( S = 0.1 \) in the SWC-TC with \( N_{TC} = 2, 3, 4 \) and \( N_f = N_f^{QCD} = 4 N_{TC} \) based on the large \( N_f \) QCD. The range of the values of \( M_{TD} \) come from varying the value of \( \kappa \) in the range 0.7 ≤ \( \kappa \) ≤ 1.3, where the smallest values of \( M_{TD} \) correspond to the cases with \( \kappa = 0.7 \) while the largest values \( \kappa = 1.3 \). \( m_{PS} \) and \( R \) are defined in the text.
\[ \kappa = 1 \] is much lighter than \( M_\rho \) and \( M_{a_1} \) also in accord with the generic analysis for large \( \Gamma \) in the previous section, although \( \tilde{S} = S/(N_f^{EW}/2) = 0.1/(2N_{TC}) \) is somewhat smaller than \( \tilde{S} = 0.1 \) used in the generic analysis in Fig. 3. Indeed, in the present case we have \( \Gamma \approx 6-8 \) as is seen from Table III. The essential reason for the large \( \Gamma \) is due to the existence of the wide conformal region \( F_\pi < \mu < \Lambda_{ETC} \) with \( (\Lambda_{ETC}/F_\pi) = 10^4-10^5 \), which yields the smallness of the beta function through the factor \((\ln 4\Lambda/F_\pi)^{-3}\) in Eq.(4.14) and hence amplifies the techni-gluon condensation in Eq.(4.10) compared with the ordinary QCD with \( \Gamma = 1 \). Note that in the idealized (phenomenologically uninteresting) limit \( \Lambda_{ETC}/F_\pi \rightarrow \infty \) we would have \( \Gamma \rightarrow \infty \) and hence \( M_{TD}/F_\pi \rightarrow 0 \). (This does not mean that techni-dilaton becomes a true (exactly massless) NG boson, since its decay constant diverges, \( F_{TD}/F_\pi \rightarrow \infty \), in such an idealized limit and hence the techni-dilaton gets decoupled. See later discussions.)

Thus we would expect the techni-dilaton as a composite Higgs boson near the conformality of SWC-TC with mass

\[ M_{TD} \approx 600 \text{ GeV}, \]

(4.14)

while \( M_\rho \) and \( M_{a_1} \) are generally heavy:

\[ M_\rho \approx M_{a_1} \approx 3.8 \text{ TeV}. \]

(4.15)

The values of the ratio \( (m/m_{PS}) \) are also listed in Table III, where \( m_{PS} \) denotes the dynamical mass estimated based on the PS formula:

\[ F_\pi^2 \approx \frac{N_{TC}}{4\pi^2} m_{PS}^2 \cdot I, \quad \text{with} \quad I = \int_0^\infty dx x^2 \frac{\Sigma(x) - \frac{x}{4} \frac{d}{dx} \Sigma(x)}{(x + \Sigma(x))^2}, \]

(4.16)

in which \( \Sigma(x) \equiv \Sigma(Q^2)/m \) with \( \Sigma(Q^2) \) being the mass function. When we use a simple-minded parametrization for \( \Sigma(x) \), \( \Sigma(x) = x^{(\gamma_m-1)/2} \), for \( x = Q^2/m^2 > 1 \) and \( \Sigma(x) = 1 \) for \( x < 1 \), we get \( I \approx 1 \) for SWC-TC with \( \gamma_m \approx 1 \) while \( I \approx 0.6 \) for QCD with \( \gamma_m \approx 0 \) (For details see Appendix B). It is interesting to note that \( m/m_{PS} > 1 \). This can be understood by considering a PS formula appropriate for the present holographic analysis [39]:

\[ F_\pi^2 \approx \frac{N_{TC}}{4\pi^2} m^2 \cdot (z_mm)^{4-2\gamma_m}, \]

(4.17)

which is satisfied for \( (1/z_m) > m \) and \( 0 \lesssim \gamma_m \lesssim 1 \). Using Eqs.(4.10) and (4.17), certainly we find that \( m/m_{PS} \approx (1/z_m) > 1 \) for \( \gamma_m \approx 1 \).

Also listed is the ratio \( R \) defined as

\[ R^3 = \sqrt{\frac{N_{TC}}{3} \frac{\langle \bar{T}T \rangle m/F_\pi^3}{(\langle \bar{q}q \rangle/f_\pi^2)_{QCD}}, \]

(4.18)

which is slightly larger than 1 reflected by the result that \( m/m_{PS} > 1 \), while it would be smaller than 1 (\( R \approx 0.69 \)) if the PS formula were used. Note that the enhancement of \( R \) has nothing to do with that of \( \langle \bar{T}T \rangle \Lambda \) caused by \( Z_{\pi}^{-1}(\Lambda/m) \).

Let us now compare the present holographic model having the explicit techni-gluon contribution with the previous model [39] without the techni-gluon condensation. Were it not for the matching with the ladder SD analysis (namely without using Eqs. (4.11), (4.12), and (4.10)), the model of Ref. [39] would be simply the \( \Gamma = 0 \) limit of the present model. Actually, the values of the meson masses lying on the \( \Gamma = 0 \) line in Fig. 4 are the results for \( \tilde{S} = 0.1 \) in the previous model of Ref. [39]. Here we compare the two models for the same value of \( S \) as in Table III. In Table IV we show the values of the meson masses for \( S = 0.1 \) and \( N_{TC} = 2, 3, 4 \) in the holographic SWC-TC without techni-gluon condensation [39], where we have used \( \kappa = 1.0 \) in estimating the values of \( M_{TD} \). Comparing the values in Tables III and IV (present model with \( \Gamma \approx 6-8 \)) and IV (previous model), one hardly sees differences in \( M_\rho \) and \( M_{a_1} \), in accord with the general analysis in the previous section (see Fig. 2), while there is a substantial difference in \( M_{TD} \) arising from the non-zero techni-gluon condensation as seen in Fig. 2. Note that the relatively smaller \( M_{TD} \) compared with \( M_\rho \) and \( M_{a_1} \) is due to the large anomalous dimension as was mentioned in Eq.(3.18) and also seen from Fig. 3.

To summarize the characteristic feature of our holographic model, we may compare our result for \( (M_\rho/F_\pi, M_{a_1}/F_\pi, M_{TD}/F_\pi) \) of the SWC-TC in Table III with that of the ordinary QCD \( (N_c = N_f = 3) \) in Table I. See Table V where we show the case of \( N_{TC} = 3 \) and \( (\Lambda_{ETC}/F_\pi) = 10^4 \) as a representative case of our SWC-TC model (the first row) and the ordinary QCD with the input value of \( (M_\rho/f_\pi) \) (the second row), together with the \( S = 0.1 \) case of the previous holographic SWC-TC model without techni-gluon condensation \( \Gamma = 0 \) [39] (the third row). Table V shows that the values of \( (M_\rho/F_\pi) \) and \( (M_{a_1}/F_\pi) \) tend to become larger as the value of \( S \) gets smaller as noted in the generic analysis (See Eq.(3.14)). This tendency is actually almost independent of the values of \( \Gamma \) and \( \gamma_m \) as seen in Figs. 2 and 3. Notably, although \( M_{TD}/F_\pi \) has the same tendency with respect to \( S \), it receives other
important effects: One is from the large anomalous dimension \( \gamma_m \approx 1 \) as in Fig. 3 and Eq. (3.13), while the other from the large techni-gluon condensation \( \Gamma = 7.57 \) as one can see from Figs. 2 and 5.

It is also worth comparing with the result of the straightforward calculation of the large \( N_f \) QCD with \( N_{TC} = 3 \) based on the Bethe-Salpeter (BS) equation combined with the SD equation in the ladder approximation [13, 23, 27]. The straightforward calculation of current correlators using the SD and BS equations has shown \( \hat{S} \approx 0.17 \) [13] near the conformal phase transition point, for \( \alpha_s/\alpha_{ct} \approx 1.13 \), or equivalently, \( \Lambda_{ETC}/F_\pi \approx 10^{4.6} \). To make a direct comparison, we take the same value of \( \hat{S} \) and \( \Lambda_{ETC}/F_\pi \) between three SWC-TC models with \( N_{TC} = 3 \) without techni-gluon condensation \( \Gamma = 0 \) [39]. Looking at Table VI, one can see a curious coincidence among the values of \( M_{TD}/F_\pi \) and \( M_{TD}/F_\pi \) from different cases. This agreement does not depend so much on whether gluonic-contribution is turned off or not, as expected from Fig. 2. On the other hand, \( M_{TD}/F_\pi \) is somewhat smaller than the value calculated from the ladder BS equation with the SD equation.

| \( N_{TC} \) | \( M_{TD} \) [TeV] | \( M_\rho \) [TeV] | \( M_{a_1} \) [TeV] |
|---|---|---|---|
| 2 | 1.58 | 3.99 | 4.13 |
| 3 | 1.08 | 4.03 | 4.09 |
| 4 | 0.81 | 4.04 | 4.08 |

**TABLE IV:** Estimates of the meson mass in the holographic SWC-TC without techni-gluon condensation [39] for \( S = 0.1 \) with \( N_{TC} = 2, 3, 4 \) and \( N_f = 4N_{TC} \). In calculating the values of \( M_{TD} \) the value of \( \kappa = 1.0 \) has been used.

| Holographic models with \( N_{TC}(or N_c) = 3 \) | \( M_\rho/F_\pi \) | \( M_{a_1}/F_\pi \) | \( M_{TD}/F_\pi \) |
|---|---|---|---|
| SWC-TC with \( \gamma_m \approx 1 \) \( (\Gamma = 7.57, S = 0.1, \Lambda_{ETC}/F_\pi = 10^{4.6}) \) | 37.8 | 38.1 | 5.5 |
| QCD with \( \gamma_m \approx 0 \) \((\Gamma = 1, S = 0.31)\) | 8.4 | 13.7 | 12.4 |
| SWC-TC with \( \gamma_m \approx 1 \) \((\Gamma = 0, S = 0.1)\) | 40.1 | 40.7 | 10.7 |

**TABLE V:** Comparison of \((M_\rho/F_\pi, M_{a_1}/F_\pi, M_{TD}/F_\pi)\) between several holographic models with \( N_{TC}(or N_c) = 3 \). In calculating the values of \((M_{TD}/F_\pi) \) \( \kappa = 1 \) has been used.

| SWC-TC models with \( \gamma_m \approx 1 \), \( N_{TC} = 3 \) and \( \hat{S} = 0.17 \) [13] | \( M_\rho/F_\pi \) | \( M_{a_1}/F_\pi \) | \( M_{TD}/F_\pi \) |
|---|---|---|---|
| Holographic SWC-TC with \( \Gamma = 5.09 \) \((\Lambda_{ETC}/F_\pi = 10^{4.6})\) | 11.2 | 11.5 | 2.3 |
| Ladder BS with SD [23, 27] | 11.0 | 11.5 | 4.2 |
| Holographic SWC-TC with \( \Gamma = 0 \) | 11.6 | 13.9 | 8.9 |

**TABLE VI:** Comparison of the values of the meson masses normalized by \( F_\pi \) between three SWC-TC models with \( \gamma_m \approx 1 \) and \( N_{TC} = 3 \). In calculating the values of \((M_{TD}/F_\pi) \) in the holographic models \( \kappa = 1 \) has been used. \( F_\pi \) in Ref. [23, 27] is the value from the PS formula.

So far our analysis has been done for a specific value of \( S, S = 0.1 \). As we already noted in Eq. (3.14), \((M_\rho/F_\pi, M_{a_1}/F_\pi, M_{TD}/F_\pi)\) increase as \( S \) decreases. Here we show how our result changes for different values of \( S \) \((S \leq 0.1)\). In Fig. 4 we show the values of \( M_\rho \) and \( M_{TD} \) for \( \kappa = 1 \) in the SWC-TC with \( N_{TC} = 3 \) \((N_f = 4N_{TC} = 12)\). The shaded area is the region obtained for \( 10^4 \leq \Lambda_{ETC}/F_\pi \leq 10^6 \), so as to satisfy the phenomenological bound from the FCNC and \( S \leq 0.1 \). Figure 4 indicates that, even when \( 0.07 \leq S \leq 0.1\), \( M_{TD} \) can still lie in a region less than 1 TeV which is still in the discovery region at the Large Hadron Collider (LHC).

Finally, we calculate the decay constant of techni-dilaton, \( F_{TD} \), following a hypothesis of the partially conserved

\footnote{In Ref. [13] the straightforward calculation of current correlators in the large \( N_f \) QCD actually gives \( \hat{S} \approx 0.25 \) for \( N_{TC} = 3 \). However, since it is known that the ladder SD and BS method has a tendency to overestimate \( \hat{S} \) in QCD, which could be understood as scale ambiguity, the actual value near the conformal phase transition point with \( \gamma_m \approx 1 \) has to be re-scaled by a factor about (2/3) so as to fit the QCD value properly in extending to the case of QCD. Then we obtain the re-scaled value of \( \hat{S}, \hat{S}_{re-scal} \approx 0.17 \).}
dilatation current (PCDC)#11 together with using the formula of the conformal anomaly given in Eq. (4.3). In this framework, the decay constant $F_{TD}$ is given by

$$F_{TD}^2 = -\frac{4}{M_{TD}^2} \frac{(\theta_\mu^\mu)}{\pi^4} \frac{m^4}{M_{TD}^2},$$

(4.19)

where we have used Eq. (4.5). This implies that $F_{TD}/F_\pi \sim m/M_{TD} \to \infty$ (decoupled techni-dilaton!) in the idealized (phenomenologically uninteresting) limit $\Gamma \to \infty$ ($A_{ETC}/F_\pi \to \infty$) where the techni-dilaton would be a true (exactly massless) Nambu-Goldstone boson, $M_{TD}/F_\pi \sim M_{TD}/m \to 0$. In our framework the conformal symmetry is always broken explicitly as well as spontaneously. Table VII shows a list of the values of $F_{TD}$ for each $N_{TC} = 2, 3, 4$ with $N_f = 4 N_{TC}$, $(A_{ETC}/F_\pi) = 10^4$, and $\hat{S} = 0.1$ fixed. The value of $F_{TD}$ may be compared with that obtained based on the linear $\sigma$ model (LSM) approach which gives a formula between $F_{TD}$ and $F_{LSM}$ as

$$F_{TD}^{LSM} = (3 - \gamma_m) \sqrt{N_f/2} F_\pi = (3 - \gamma_m) \cdot 246 \text{ GeV} = 2 \times 246 \text{ GeV}$$

(4.19)

the third column of Table VII. The discrepancy between $F_{TD}$ and $F_{TD}^{LSM}$ is not surprising since the SWC-TC dynamics is near the conformal phase transition where the Ginzburg-Landau/Gell-Mann-Levy effective theory (linear $\sigma$ model) breaks down [15].

We can estimate the Yukawa coupling constant of techni-dilaton by $F_{TD}$ as

$$g_{TD}^Y = \sqrt{2}(3 - \gamma_m) m_f/F_{TD},$$

(4.20)

which is smaller than the Yukawa coupling in the SM, $g_{SM}^Y = \sqrt{2} m_f/(246 \text{ GeV}) = \sqrt{2}(3 - \gamma_m) m_f/F_{TD}^{LSM}$, as $g_{TD}^Y/g_{SM}^Y = (F_{TD}/F_{TD}^{LSM})^{-1} \approx 0.18 - 0.20$. Such a difference may be useful for experimental search for the techni-dilaton in LHC.

| $N_{TC}$ | $F_{TD}$ [TeV] | $F_{TD}/F_{TD}^{LSM}$ | $g_{TD}^Y/g_{SM}^Y$ |
|----------|---------------|-----------------------|-------------------|
| 2        | 2.42          | 4.91                  | 0.20              |
| 3        | 2.49          | 5.07                  | 0.20              |
| 4        | 2.71          | 5.52                  | 0.18              |

TABLE VII: Estimates of the values of $F_{TD}$ for each $N_{TC} = 2, 3, 4$ with $(A_{ETC}/F_\pi) = 10^4$ and $\hat{S} = 0.1$ fixed. The values of the ratios $F_{TD}/F_{TD}^{LSM}$ and $g_{TD}^Y/g_{SM}^Y$ are also displayed.

#11 We could calculate $F_{TD}$, not invoking the PCDC hypothesis, straightforwardly from the scalar-current correlator based on the holographic principle. Such an alternative (but direct) calculation would be pursued in future publications.
V. SUMMARY AND DISCUSSIONS

In this paper, we have proposed a holographic SWC-TC including the bulk flavor/chiral-singlet scalar field $\Phi_X$ corresponding to (techni-) gluon condensation, based on deformation of a hard-wall-type bottom-up holographic QCD by adjusting the anomalous dimension $\gamma_m$. Thanks to the additional explicit bulk scalar field $\Phi_X$, we naturally reproduced gluonic $1/Q^4$ terms in the OPE of the underlying theory (QCD and SWC-TC) for current correlators, in such a way as to clearly distinguish them from the same $1/Q^4$ terms due to the chiral condensate in the case of SWC-TC with $\gamma_m \approx 1$. We have analyzed a generic case with $0 \lesssim \gamma_m \lesssim 1$ and calculated the masses of the techni-$\rho$ meson ($M_\rho$), the techni-$/a_1$ meson ($M_{a_1}$), and the flavor-singlet scalar meson, techni-dilaton ($M_{TD}$), as well as the $S$ parameter.

We have shown that our model with $\gamma_m = 0$ and $N_f = 3$ well reproduces the real-life QCD (See Table VII), with an indication that the QCD $\rho$ meson mass $M_\rho$ includes a (negative) contribution about 10% from the gluon condensate (See Eq.(4.14)).

For the case of $\gamma_m = 1$, we studied the dependence of the $S$ parameter on $(M_\rho/F_\pi)$ for several values of the techni-gluon condensation $\Gamma$ (Eq.(4.12)), $\Gamma = 0, 5, 10$ (Fig. 4): $\hat{S}$ decreases monotonically with respect to $(F_\pi/M_\rho)$ to continuously approach zero. This implies $(M_\rho/F_\pi)$ necessarily increases when $\hat{S}$ is required to be smaller (Eq.(4.14)).

It was also shown that, in the region of $\hat{S} \lesssim 0.1$, the techni-gluon condensation reduces the value of $S$ maximally about 10%, compared with the previous analysis without techni-gluon contribution [39].

In the generic TC case with $0 \lesssim \gamma_m \lesssim 1$, we discussed how the meson masses relative to $F_\pi$, $(M_\rho/F_\pi, M_{a_1}/F_\pi, M_{TD}/F_\pi)$ change by varying $\gamma_m$, $\hat{S}$, and the techni-gluon condensation $\Gamma$. For fixed value of $\hat{S}(= 0.31)$ (QCD value, Fig. 2) and $\hat{S} = 0.1$ (minimal requirement for a realistic TC, Fig. 5), $(M_\rho/F_\pi)$ and $(M_{a_1}/F_\pi)$ are sensitive to neither $\Gamma$ nor $\gamma_m$, although the degeneracy between $(M_\rho/F_\pi)$ and $(M_{a_1}/F_\pi)$ takes place for somewhat larger $\Gamma$. In contrast, the techni-dilaton mass has a characteristic feature related to the conformality of SWC-TC: $(M_{TD}/F_\pi)$ substantially decreases as $\Gamma$ and/or $\gamma_m$ increases when $\hat{S}$ is fixed. Particularly, $M_{TD}/F_\pi \to 0$ in the formal limit $\Gamma \to \infty$.

To specify the value of $\Gamma$, we considered a couple of typical models of SWC-TC with $\gamma_m \approx 1$ and $N_{TC} = 2, 3, 4$ based on the Caswell-Banks-Zaks infrared fixed point in the large $N_f$ QCD. Using some specific dynamical features of the conformal anomaly indicated by the analysis based on the ladder SD equation, we found the relation of $\Gamma$ to $(\Lambda_{ETC}/F_\pi)$: In the case of $N_{TC} = 3$ ($N_f = 4N_{TC}$) and $S = (N_f/2) \cdot \hat{S} \approx 0.1$, we have $\Gamma \simeq 7$ for $(\Lambda_{ETC}/F_\pi) = 10^3 - 10^5$ (required by the FCNC constraint). Thanks to the large anomalous dimension $\gamma_m$ and large techni-gluon condensation $\Gamma$, we had a relatively light techni-dilaton with mass $M_{TD} \approx 600$ GeV compared with $M_\rho \simeq M_{a_1} \approx 3.8$ TeV (Table VII). Note that large values of $M_\rho$ and $M_{a_1}$ are essentially determined by the requirement of $S = 0.1$ fairly independently of the techni-gluon condensation $\Gamma$, though the degeneracy between them is due to the largeness of $\Gamma$. Such large values of $M_\rho$ and $M_{a_1}$ might make the standard signatures through these techni-hadrons quite invisible at LHC. Note however that the largeness of the spectra in our model simply comes from the constraint from the $S$ parameter evaluated by the TC sector alone. If we found other effects to reduce the $S$ parameter such as the ETC effects implementing the mass of the SM fermions, we could pull down the overall scale of the whole techni-hadron spectra, in which case the degenerate techni-$\rho$ and techni-$a_1$ as well as the lighter techni-dilaton in our model would have much impact in the LHC phenomenology.

The essential reason for the large $\Gamma$ is due to the existence of the wide conformal region $F_\pi < \mu < \Lambda_{ETC}$ with $(\Lambda_{ETC}/F_\pi) = 10^{3} - 10^{5}$, which yields the smallness of the beta function through the factor $(\ln 4\Lambda_{ETC}/m)^{-3}$ (see Eq.(4.14)) and hence amplifies the techni-gluon condensation in Eq.(4.11) compared with the ordinary QCD with $\Gamma = 1$. In the idealized (phenomenologically uninteresting) limit $\Lambda_{ETC}/F_\pi \to \infty$ we would have $\Gamma \to \infty$ and hence $M_{TD}/F_\pi \to 0$. The would-be “massless” techni-dilaton is actually decoupled, since its decay constant $F_{TD}$ becomes divergent in that limit as is implied in Eq.(4.19). There exists no isolated massless techni-dilaton in contrast to the chiral symmetry breaking: The scale symmetry is broken both spontaneously and explicitly.

The predicted mass of holographic techni-dilaton (“conformal Higgs”), 600 GeV, lies in the discovery region at LHC. The size of Yukawa coupling of the techni-dilaton was also estimated through the PCDC relation, which turned out to be somewhat smaller than that of the SM Higgs (Table VII). More detailed analysis on intrinsic signals at LHC concerning such a holographic techni-dilation/conformal Higgs will be explored in future publications.

Before closing this section, several comments are in order:
A. Perturbative unitarity-bound and partially EW gauged SWC-TC

Since the vector meson masses predicted from our analysis are of order of a few TeV and hence irrelevant to the unitarity (See Table II), it would be the light techni-dilaton that is responsible for the perturbative unitarity of \( W_L W_L \) scattering. The perturbative unitarity-bound on \( M_{TD} \) can then be estimated through a formula,

\[
M_{TD} \lesssim \Lambda_{uni} = \sqrt{8\pi F_\pi} = \sqrt{8\pi} \cdot \left( \frac{246 \text{ GeV}}{\sqrt{N_{EW}^f/2}} \right),
\]

where \( N_{EW}^f \) denotes the number of the techni-fermions charged under the electroweak (EW) gauge. In the case of the SWC-TC models with \( N_{EW}^f = N_f = 4N_{TC} \) listed in Table III however, the unitarity bound on the values of \( M_{TD} \) may be estimated as

\[
M_{TD} \Big|_{\text{Table III}} \lesssim (617, 503, 436) \text{ GeV}, \quad \text{for } N_{TC} = 2, 3, 4.
\]

Looking at the values of \( M_{TD} \) listed in Table III we see that, for every case of \( N_{TC} \), some masses of the techni-dilaton are somewhat heavier than that required by the unitarity bound above.

The situation with the perturbative unitarity would be much improved in a class of models (so-called partially gauged model) considered for other purpose [50]: Only a part of techni-fermion flavors carries the EW charges, while other fermion flavors are EW-singlets introduced only to achieve the SWC behavior of technicolor dynamics. Here the idea is to relax our condition \( N_{EW}^f = N_f = 4N_{TC} \) such that \( N_{EW}^f < N_f(= 4N_{TC}) = N_{EW}^f + N_{EW}^f - \text{singlet} \), which increases \( \hat{S} = S \cdot (2/N_{EW}^f) \) for the same \( S (= 0.1) \) compared with the analysis in Sec. IV. Actually, \( M_{TD}/\Lambda_{uni} = \frac{1}{\sqrt{2}} (M_{TD}/F_\pi) \) decreases when \( \hat{S} \) increases as shown in Figs. 3, 4 and Eq. (5.1). In Table VIII we give two examples of the partially gauged models with \( N_f = 4N_{TC} \) (one-doublet models with \( N_{EW}^f = 2 \) and one-family models with \( N_{EW}^f = 8 \)), where the unitarity condition is fulfilled, \( M_{TD}/\Lambda_{uni} \lesssim 1 \).

| \( N_{TC} \) | \( \log_{10}(\Lambda_{ETC}/F_\pi) \) | \( M_{TD} \) [GeV] | \( M_\rho \) [TeV] | \( M_{\omega_L} \) [TeV] |
|---|---|---|---|---|
| 2 | 4 | 777 \( \pm 106 \) | 3.75 | 3.82 |
| 2 | 5 | 613 \( \pm 99 \) | 3.69 | 3.74 |
| 3 | 4 | 725 \( \pm 99 \) | 3.81 | 3.86 |
| 3 | 5 | 581 \( \pm 81 \) | 3.74 | 3.78 |
| 4 | 4 | 686 \( \pm 94 \) | 3.84 | 3.88 |
| 4 | 5 | 557 \( \pm 77 \) | 3.78 | 3.81 |

**TABLE VIII:** Estimates of the meson masses in one-family and one-doublet partially gauged models based on the large \( N_f \) QCD with \( S = 0.1 \) fixed. The values of \( M_{TD} \) are estimated varying the value of \( \kappa \) in the range \( 0.7 \leq \kappa \leq 1.3 \), where the smallest values of \( M_{TD} \) correspond to the cases with \( \kappa = 0.7 \), while the largest values \( \kappa = 1.3 \).
B. Possible scaling behaviors of $S$ and the conformal phase transition/chiral restoration

The analyses in this paper were made for fixed values of $\hat{S}$ or $S$ ($\neq 0$) and $m/\Lambda_{TC} \simeq F_\pi/\Lambda_{TC} = 10^{-4} - 10^{-5}$. The simplest extension of our analysis would imply that $S \sim (F_\pi/M_\rho)^2 \to \text{constant} \neq 0$ as $m/\Lambda_{TC} \to 0$, which, as indicated in Table VI, seems to be in accord with the straightforward calculation based on ladder SD and BS equation [13]: $\hat{S} \simeq \text{constant} \simeq 0.17$ near $\alpha_*/\alpha_{cr} = 1.13$ ($F_\pi/\Lambda_{TC} = 10^{-4.6}$) and $\hat{S} \to \text{constant} \neq 0$ in the extrapolation toward $\alpha_*/\alpha_{cr} \to 1$ ($F_\pi/\Lambda_{TC} \to 0$).

Here we shall mention different possibilities for the scaling of $S$, other than $\hat{S} \to \text{constant}$, arising as $m/\Lambda_{TC} \to 0$ near the conformal phase transition. In the case of conformal phase transition, all the dimensionful parameters of order $O(m) \ll O(\Lambda_{TC})$ are expected to become zero at the critical point, $m/\Lambda_{TC} \to 0$. Therefore, vector meson mass also goes to zero, $M_\rho/\Lambda_{TC} \to 0$. Suppose first that $M_\rho/m = \text{constant}$ in the limit $m/\Lambda_{TC} \to 0$. Dimensionless parameter $S$ is written in terms of two independent dimensionless parameters, say, $\xi$ and $G$, once we fix $\gamma_m$. A set of $\xi$ and $G$ is converted to another set of dimensionless parameters $M_\rho/m$ and $\Lambda/m$ through Eqs. (4.8) and (4.11). If we fix $M_\rho/m = \text{constant}$, then $S$ is given as a function of $\Lambda/m$. In Fig. 7, we show the plot of $\hat{S}/N_{TC}$ as a function of $\Lambda/m$ for $\gamma_m = 1$ and $M_\rho/m = 2$. Actually, it turns out that $\hat{S}$ goes to zero with the chiral restoration, i.e., $\Lambda/m \to \infty$, whatever the constant value of $M_\rho/m$ is taken to be. (If $M_\rho$ were taken to be bigger than $2m$, $\hat{S}$ would become smaller.) This would imply $F_\pi/m \sim F_\pi/M_\rho \to 0$ and hence $\langle \bar{T}T \rangle_m/F_\pi^3 \sim m^3/F_\pi^3 \to \infty$ at the conformal phase transition point. This is the behavior somewhat different from that expected from the PS formula [39].

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Appendix A: Holographic matching to the operator product expansion of current correlators

We start with Eq. (2.30), the equation of motion for $V_\mu(q, z)$, and rewrite it in terms of $V(q, z)$ defined as $V_\mu(q, z) = \tilde{v}_\mu(q)V(q, z)$ as follows:

$$[Q^2 + w^{-1}(z)\partial_z w(z)\partial_z]V(q, z) = 0,$$

(A.1)
where \( Q^2 = -q^2 \) denotes a Euclidean momentum-squared and the induced metric \( w(z) \) is given (in the limit \( M' = 0 \)) as

\[
w(z) = \frac{L}{z} \left( 1 + G \left( \frac{z}{z_m} \right)^4 \right)^2. \tag{A.2}
\]

From Eq. (2.26) we note that

\[
\frac{G}{z_m^4 Q^4} = -\frac{1}{8} \frac{e g_4^4 \langle \alpha G^2_{\mu\nu} \rangle}{L^2 Q^4}. \tag{A.3}
\]

Using this relation, we express the induced metric \( w(z) \) as

\[
w(z = y/Q) = QL y \left( 1 - \frac{1}{8} \frac{e g_4^4 \langle \alpha G^2_{\mu\nu} \rangle}{L^2 y^4 Q^4} \right)^2, \tag{A.4}
\]

where the variable \( z \) has been replaced with \( y = Q z \).

Consider a large Euclidean momentum region \( (1/z_m)^2 \ll Q^2 < (1/\epsilon)^2 \). Expanding Eq. (A.1) in powers of \( Q^2 \) and using Eq. (2.35), we then obtain an asymptotic form of the vector current correlator \( \Pi_V(Q^2) \),

\[
\Pi_V(Q^2) \bigg|_{(1/z_m)^2 \ll Q^2 < (1/\epsilon)^2} = Q^2 \left[ \frac{L}{2 g_5^2} \ln Q^2 + \frac{2 g_5^2 \langle \alpha G^2_{\mu\nu} \rangle}{3 L} Q^4 + \mathcal{O}(1/Q^8) \right]. \tag{A.5}
\]

This expression may be compared with the form of the operator product expansion (OPE) for arbitrary \( \gamma_m \),

\[
\Pi_V(Q^2) \bigg|_{\text{OPE}} = Q^2 \left[ \frac{N_{TC}}{24 \pi^2} \log \left( \frac{Q^2}{\mu^2} \right) - \frac{1}{24 \pi} \frac{\langle \alpha G^2_{\mu\nu} \rangle}{Q^4} + 2 \pi \frac{\alpha \langle \bar{T} T \rangle g_2^2}{Q^2 (3-\gamma_m)} \right]. \tag{A.6}
\]

Then we find the following matching conditions:

\[
\frac{L}{g_5^2} = \frac{N_{TC}}{12 \pi^2}, \quad c = \frac{1}{16 \pi} \frac{L}{g_5^2} = \frac{N_{TC}}{192 \pi^3}. \tag{A.7}
\]

Similar discussion on the axial-vector current correlator \( \Pi_A \) provides the same matching conditions for \( (L/g_5^2) \) and \( c \) as it should.

The high-energy expansion form in Eq. (A.3) does not include the chiral condensate term behaving as \( 1/Q^{2(3-\gamma_m)} \) as in Eq. (A.6). In order to incorporate this missing term, we shall introduce higher dimensional interaction terms constructed from the bulk scalar field \( \Phi \) and the left- and the right-gauge fields \( L_M \) and \( R_M \).

We consider the following dimension-six operators which are invariant under the five-dimensional \( SU(N_f)_L \times SU(N_f)_R \) gauge symmetry:

\[
\Delta L_5 = \frac{L^2}{g_5^2} e^{g_5^2 \Phi} \left( \frac{C_{LL} L}{2} \text{Tr} \left[ \Phi^4 \Phi \right] \text{Tr} \left[ L_{MN} L_{MN} + R_{MN} R_{MN} \right] + C_{LR} \text{Tr} \left[ \Phi^4 L_{MN} \Phi R_{MN} \right] \right), \tag{A.8}
\]

where \( C_{LL} \) and \( C_{LR} \) are the dimensionless coupling constants. The Lagrangian \( \Delta L_5 \) gives shifts to the kinetic terms of the vector \( (V_M \equiv (L_M + R_M)/\sqrt{2}) \) and axial-vector \( (A_M \equiv (L_M - R_M)/\sqrt{2}) \) gauge fields as follows:

\[
\frac{L^2}{g_5^2} e^{g_5^2 \Phi} \left( \frac{C_V}{2} \frac{1}{2} v^2(z) \text{Tr} \left[ V_{MN} V_{MN} \right] + \frac{C_A}{2} \frac{1}{2} v^2(z) \text{Tr} \left[ A_{MN} A_{MN} \right] \right), \tag{A.9}
\]

where \( C_{V,A} = C_{LL} \pm \frac{1}{2} C_{LR} \).

Let us focus on the vector sector taking \( C_A = 0 \) for simplicity. Then we see that the \( C_V \) term in Eq. (A.9) modifies the induced metric \( w(z) \) in Eq. (A.2) as

\[
w(z) \rightarrow \tilde{w}(z) = \frac{L}{z} \left( 1 + G \left( \frac{z}{z_m} \right)^4 \right)^2 \left( 1 - C_V L^2 v^2(z) \right). \tag{A.10}
\]
It should be noted from Eqs. (2.27) and (2.22)-(2.24) that $v^2(z)$ in Eq. (A.10) is expressed (in the chiral limit $M = 0$) as

$$v^2(z = y/Q) \simeq \frac{y^{2(3-\gamma_m)}}{3(3-\gamma_m)^2} \left(\frac{g_5^2}{L}\right)^2 \frac{L^2 \langle T T \rangle^2_{1/L} L^{2\gamma_m-2}}{Q^2(3-\gamma_m)},$$  \hspace{1cm} (A.11)$$

where we have neglected higher order terms in $1/Q^2$ expansion. Thus we see that the modified-induced metric $\tilde{w}(z)$ now includes the desired $\bar{T}T$ term:

$$\tilde{w}(z = y/Q) \equiv \frac{Q}{y} \left[ -C_V \left(\frac{g_5^2}{L}\right)^2 \frac{y^{2(3-\gamma_m)}}{3(3-\gamma_m)^2} \frac{L^{2\gamma_m} \langle T T \rangle^2_{1/L}}{Q^2(3-\gamma_m)} \right].$$  \hspace{1cm} (A.12)$$

Combining this with Eq. (A.4), we obtain the total expression of the high-energy expansion for $\Pi_V$:

$$\Pi_V(Q^2) \bigg|_{(1/z_m)^2 \ll Q^2 < (1/y)^2} = Q^2 \left[ \frac{L}{2 g_5^2} \ln Q^2 + \frac{2 g_5^2}{3 L} \frac{(\alpha G_{\mu\nu})}{Q^4} + C_V L^{2\gamma_m} \langle T T \rangle^2_{1/L} + O\left(\frac{1}{Q^8}\right) \right],$$  \hspace{1cm} (A.13)$$

where

$$C_V = \frac{\sqrt{\pi} \alpha L}{6} \langle T T \rangle \left(\frac{1}{2 - \gamma_m}\right)^3 \frac{g_5^2}{L}.$$  \hspace{1cm} (A.14)$$

Comparing Eq. (A.13) with the $\langle T T \rangle^2$ term in Eq. (A.6), we determine the coefficient $C_V$ as

$$C_V = 12 \sqrt{\pi} \alpha \left(\frac{1}{3 - \gamma_m}\right)^3 \frac{L}{g_5^2}.$$  \hspace{1cm} (A.15)$$

For $C_A \neq 0$, one can similarly perform the high-energy expansion of $\Pi_A$. As a consequence of matching with the OPE for $\Pi_A$, we get

$$C_A = -12 \sqrt{\pi} \alpha \left(\frac{1}{3 - \gamma_m}\right)^3 \frac{L}{g_5^2} \left(\frac{1}{3 - \gamma_m} - \frac{1}{2 - \gamma_m}\right).$$  \hspace{1cm} (A.16)$$

Thus, it is shown that the present model with the higher dimensional terms in Eq. (A.8) added completely reproduces the OPE for the current correlators up to terms suppressed by $1/Q^8$.

**Appendix B: The Pagels-Stokar formula and chiral condensate**

1. Relationship between decay constant and dynamical fermion mass

The Pagels-Stokar formula relates the mass function $\Sigma(Q^2)$ with $F_\pi$ as

$$F_\pi = \frac{N_{TC}}{4 \pi^2 m_{PS}^2} \int_0^{(\Lambda^2/m_{PS}^2) \rightarrow \infty} dx x \frac{\Sigma^2(x) - \frac{x}{4 \pi^2} \Sigma^2(x)}{(x + \Sigma^2(x))^2},$$  \hspace{1cm} (B.1)$$

where $\Sigma(x) = \Sigma(Q^2)/m_{PS}$ which may be parametrized as

$$\Sigma(x) = \begin{cases} x^{2m-1} & \text{for } x > 1 \\ 1 & \text{for } x < 1 \end{cases}. \hspace{1cm} (B.2)$$

By using Eq. (B.2), Eq. (B.1) is calculated as \#12

$$\frac{4 \pi^2}{N_{TC} m_{PS}^2} \int_0^1 dx \frac{x}{(x + 1)^2} + \frac{1}{2} \left(\frac{3 - 2m}{2}\right) \int_1^\infty dx \frac{x^{2m-1}}{(x + x^{2m-2})^2}.$$  \hspace{1cm} (B.3)$$

\#12 One may neglect $\Sigma^2(x) = x^{\gamma_m-2}$ in the denominator of the integrand of Eq. (B.1) since in the integration dominant contributions come from the UV region where $\Sigma^2(x) = x^{\gamma_m-2} \ll x$ for $0 \ll \gamma_m \ll 1$. Then the form of Eq. (B.3) would be changed to $\frac{4 \pi^2}{N_{TC} m_{PS}^2} \int_0^1 dx \frac{x}{(x + 1)^2} + \frac{1}{2} \left(\frac{3 - 2m}{2}\right) \int_1^\infty dx \frac{x^{2m-1}}{(x + x^{2m-2})^2}$.
\[
\frac{4\pi^2}{N_{\text{TC}} m_{\text{PS}}} \sim \begin{cases} 
0.63 & \text{for } \gamma_m = 0 \\
1.00 & \text{for } \gamma_m = 1
\end{cases}
\]  
(B.4)

and for \( \gamma_m = 2 \) we have \( \frac{4\pi^2 \, m_{\text{PS}}^2}{N_{\text{TC}}} \sim \ln (\Lambda/m)^2 \).

## 2. Relationship between chiral condensate and dynamical fermion mass

The chiral condensate \( \langle \bar{T}T \rangle \) evaluated at a UV scale \( \Lambda \) is given as

\[
\langle \bar{T}T \rangle_{\Lambda} = \frac{N_{\text{TC}}}{4\pi^2} \int_{0}^{(\Lambda^2/m^2 \to \infty)} dx \frac{x \Sigma(x)}{x + \Sigma^2(x)}.
\]  
(B.5)

One can calculate Eq. (B.5) using the following relation derived based on analysis of the ladder SD equation \#13

\[
\int_{0}^{(\Lambda^2/m^2 \to \infty)} dx \frac{x \Sigma(x)}{x + \Sigma^2(x)} = \frac{d}{dx} \left( \lambda(x)/x \right) \bigg|_{x=(\Lambda/m)^2},
\]  
(B.7)

where \( \lambda(x) = \frac{1}{\alpha_{\text{cr}} x} \) with \( \alpha_{\text{cr}} = \pi/(3C_2(F)) \).

In the case of SWC-TC with \( \gamma_m \simeq 1 \) where \( \alpha(\simeq \alpha_*) \simeq \alpha_{\text{cr}} \), we have \( \lambda = 1/4 \). Using Eqs. (B.2) and (B.7) we then evaluate Eq. (B.5) as

\[
- \frac{4\pi^2}{N_{\text{TC}} m^3} \bigg|_{\text{SWC-TC}} \langle \bar{T}T \rangle_{\Lambda} \simeq 2 \cdot Z_m^{-1},
\]  
(B.8)

where \( Z_m = (\Lambda/m)^{-1} \) is the mass renormalization constant.

In the case of QCD with \( \gamma_m \simeq 0 \), \( \Sigma(x) \) and \( \lambda(x) \) are expressed as

\[
\Sigma(x) = \frac{1}{x} \left[ 1 + \frac{1}{2A} \ln x \right]^{A/2-1},
\]  
(B.9)

\[
\lambda(x) = \frac{A/2}{\ln x + 2A},
\]  
(B.10)

where \( A = 1/(b\alpha_{\text{cr}}) \) with \( b \) being the coefficient of QCD beta function at one-loop order, \( b = \frac{1}{6 \pi} (11N_c - 2N_f) \).

Substituting Eqs. (B.9) and (B.10) into Eq. (B.7), we have

\[
- \frac{4\pi^2}{N_{\text{C}}} \langle \bar{T}T \rangle_{\Lambda} \bigg|_{\text{QCD}} \simeq \frac{4}{(2A)^{A/2}} \cdot Z_m^{-1}
\]  
(B.11)

\[\#13\] Neglecting the \( \Sigma(x) \) in the denominator, which is not dominant in the integral, and substituting Eq. (B.2) into Eq. (B.7), we may evaluate Eq. (B.5) to reach a form,

\[
- \frac{4\pi^2}{N_{\text{TC}} m^3} \langle \bar{T}T \rangle_{\Lambda} \bigg|_{\text{QCD}} \simeq \left[ 1 - \ln 2 + \frac{2}{\gamma_m} \left( \frac{\Lambda}{m} \right)^{\gamma_m} - 1 \right],
\]  
(B.6)

which turns out to be good approximation and gives the same numbers as those in Eqs. (B.8) and (B.12), except for the QCD case with \( \gamma_m = 0 \).
where \( Z_m = (\ln(A^2/m^2))^{-A/2} \). For the case of constant mass \( \Sigma(x) = 1 \) which corresponds to the case with \( \gamma_m = 2 \) as in the Nambu-Jona-Lasinio (NJL) model, we can straightforwardly calculate Eq. (B.5) to get

\[
- \frac{4\pi^2 \langle \bar{T}T \rangle_\Lambda}{N_{TC} m^4} = 1 \cdot Z_m^{-1},
\]

(B.12)

where \( Z_m = (\Lambda^2/m^2)^{-1} \). Finally using \( \langle \bar{T}T \rangle_m = Z_m \langle \bar{T}T \rangle_\Lambda \), we thus reach a result

\[
- \frac{4\pi^2 \langle \bar{T}T \rangle_m}{N_{TC} m^4} \sim \begin{cases} 3 & \text{for } \gamma_m = 0 \\ 2 & \text{for } \gamma_m = 1 \\ 1 & \text{for } \gamma_m = 2 \end{cases}.
\]

(B.13)

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