Interacting Dark Energy and Dark Matter: observational Constraints from Cosmological Parameters

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Abstract

Several observational constraints are imposed on the interacting holographic model of Dark Energy and Dark Matter. First we use the age parameter today, as given by the WMAP results. Subsequently, we explained the reason why it is possible, as recently observed, for an old quasar to be observed in early stages of the universe. We discuss this question in terms of the evolution of the age parameter as well as in terms of the structure formation. Finally, we give a detailed discussion of the constraints implied by the observed CMB low \( \ell \) suppression. As a result, the interacting holographic model has been proved to be robust and with reasonable bounds predicts a non vanishing interaction of Dark Energy and Dark Matter.

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I. INTRODUCTION

The host of results brought by the continuing operation of WMAP assures us about the validity of the basis of the Standard Cosmological Model [1, 2, 3]. While in the eighties optical observations led us to information concerning the cosmos with high uncertainties, the present status is based on observations which deserve the denomination of precision cosmology.

One of the most tantalizing results connected with the WMAP observations as well as with the new available Supernova data [4] is the fact that 97% of the Universe consists of an unknown state of matter, from which 2/3 is the so called Dark Energy (DE), responsible for an unexpected cosmological acceleration and roughly 1/3 is a Dark Matter (DM), a gravitationally interacting form of non-baryonic matter.

Such forms of energy constitute a major puzzle of modern cosmology. They attracted a lot of attention and much effort has been spent to understand them in the last few years. Until now, the nature and origin of Dark Matter and Dark Energy are still the source of much debate [5].

Despite the theoretical difficulties in understanding Dark Energy, independent observational evidence for its existence is impressively robust. We have three largely independent types of observational arguments for dark energy: the supernova Hubble diagram [4], the dynamical evidence for low matter density, namely the fact that according to inflation there is a missing component in the energy balance of the universe [3] and the age of the universe [6]. In addition, a great success has been scored in high precision measurements of CMB anisotropy, as well as in galaxy clustering, the Ly$\alpha$ forest and gravitational lensing [7]. Along with these observations, the age of the universe is one of the most pressing pieces of data disclosing information about dark energy. Dark energy influences the evolution of the universe, thus any limit on the age of the universe during its evolution with redshift will reveal its nature. As estimated from globular clusters [8] and CMB measurements [3], the total expanding age at $z = 0$ is $t_0 \sim 13$Gyr. Such an astrophysical constraint on the age of the universe at $z = 0$ is useful to limit the equation of state of dark energy [9]. However, different dark energy models may lead to the same age of an expanding universe at $z = 0$. This degeneracy can be lifted by examining the age of the universe at different stages of its evolution and comparing with age estimates of high-redshift objects. Such a procedure constrains the age at different stages, being a powerful tool to test the viability of different models [10].

On the other hand, Dark Matter is also well established, not only by the long standing observations of rotation curves in galaxies [11] but also as a result of CMB observations by WMAP [3]. Although the interpretation of Dark Energy and Dark Matter as completely independent objects
is possible, as signalized by the CMB results alone which are compatible with the ΛCDM model, it is a rather strange and unnatural approach to the question \[12, 13\]. Indeed, observationally, the ΛCDM model fits the CMB data alone \[3\]. However, the only sensible way such a new state of matter can be understood at present is by means of a standard introduction of new fields in the framework of a Quantum Field Theory in a curved space-time. Even the introduction of a cosmological constant to explain the acceleration suffers from severe problems to explain its actual size \[14\]. Moreover, it does not explain the fact that today Dark Energy and Dark Matter density fractions are of the same order of magnitude, the so called coincidence problem \[12\]. Thus, one might argue that an entirely independent behavior of DE and DM is very special. Studies on the interaction between DE and DM have been carried out \[13\]. It is worthwhile mentioning that the interacting holographic model was shown to be consistent with the golden SN data \[15\] and can accommodate the transition of the dark energy equation of state from \(\omega_D > -1\) to \(\omega_D < -1\) \[13, 16\] as recently revealed from extensive data analysis \[17, 18\].

The motivation of the present paper is to analyse in detail up to what point an interaction of the two sectors, namely Dark Energy and Dark Matter, in the background of the holographic model, is compatible with observations. We thus begin by briefly explaining the interacting holographic model and some of its consequences. Subsequently, we compute the age parameter today and in the past, giving constraints to the interaction coefficient in order to explain the existence of an old astrophysical structure, the quasar APM 0879+5255. Next, as an important complementary piece of information, we show that such a structure can be generated more naturally in the interacting model. We further discuss in detail, by means of a fine numerical analysis and use of the cmbfast code, the probability that the model explains the low \(\ell\) CMB data as a function of the phenomenological parameters and arrive at the interesting conclusion that, together with the previous analysis and with a confidence estimated to be of the order of 90%, that the dark energy and the dark matter do interact. In the end we draw conclusions and try to foresee further developments along this line.

II. THE HOLOGRAPHIC DARK ENERGY MODEL

Standard Quantum Field Theory in a curved background has been successfully used in the description of the cosmos. On the other hand, quantum gravity has never been thoroughly and successfully included in a description of a unified theory of all interactions. Nevertheless, some ideas, such as holography \[19\], derived from a semi classical description of gravity together with
further general assumptions, have found applications in the cosmological setup \[20\].

Concerning cosmology, in particular, the energy content of the universe, we can set up a relation inspired by the fact that the whole energy content of the cosmos cannot exceed the mass of a Black Hole with the same size of the universe, which we call \(L\). We thus suppose that

\[
\rho_D = \frac{3c^2 M_p}{L^2}
\]

where a phenomenological constant \(c\) has been conveniently introduced, characterizing a free parameter of our model while \(M_p\) is the Planck mass.

Such a formulation is known in the literature as the holographic hypothesis \[21\]. In order not to violate the second law of thermodynamics in the event horizon, it has been argued that \(c \geq \sqrt{\Omega_\Lambda}\), or in general \(c \geq 1\) \[22\]. Since the thermodynamics in the event horizon may be problematic \[23\] and the IR regulator might not be simply related to the future event horizon \[21\] \[25\] \[26\], we suppose that there is a lower bound for \(c\) but try not to specify it strictly. We shall see however that a lower bound is natural (see also \[15\]). In most of the paper we suppose that \(c = 1\). As we shall see later, bounds in \(c\) are not very restrictive.

As far as energy conservation is concerned, we suppose that the interaction is described by the (separately non conserving) equations

\[
\dot{\rho}_m + 3H\rho_m = +Q \\
\dot{\rho}_D + 3H(1 + \omega_D)\rho_D = -Q
\]

where \(Q\) is some interaction. For the moment we take for granted that the interaction is the one proposed on general grounds in \[24\], that is,

\[
Q = 3b^2 H(\rho_m + \rho_D)
\]

where \(b^2\) is a second phenomenological constant coupling DE and DM. It has to be fit by observational data. The model has been further discussed in \[15\] for a flat universe and \[16\] for the closed universe. The question has been further discussed in \[25, 26\].

We just consider the interaction between DE and DM, but neglect the interaction between DE and Baryonic matter. The minimal coupling between DE and Baryonic matter is usually assumed in order to avoid testable violations of the equivalence principle \[27\] and a possible conflict with observational constraints on long-range forces \[28\]. In \[29\], it was found that good consistency with the observational bounds for a variety of parameter combinations does not depend on whether or
not the baryons are included in the interaction. We thus assume that baryonic matter is minimally coupled in our study.

Finally, we should mention that in general we admit that the energy pressure relation $\omega_D = p_D/\rho_D$ can be redshift (time) dependent, which is natural from the point of view of the two fluids interacting, as described in [2]. This is also consistent with the recent data analysis [17, 18].

III. AGE CONSTRAINTS

In this section we consider the age of the universe, as influenced by the holographic model interaction.

To begin with, we consider the simple cosmic expansion described by the Friedmann equation,

$$H^2 = H_0^2 [\Omega_{m0}(1+z)^3 + \Omega_{D0}f(z) - \Omega_{k0}(1+z)^2], \quad (4)$$

where $\Omega_{m0}, \Omega_{D0}$ and $\Omega_{k0}$ refer to densities of matter, dark energy and curvature at the present day in units of the critical density. The function $f(z)$ is related to the equation of state of dark energy by $f(z) = e^{3\int_0^z \frac{1+\omega(z')}{{1+z'}}dz'}$. The age of the universe at redshift $z$ is

$$t(z) = \int_z^\infty \frac{dz'}{(1+z')H(z')} \quad (5).$$
WMAP three year results \[3\] tell us that at \(z = 0\), \(t_0 = 13.73^{+0.13}_{-0.17}\) Gyr and that the current value of the Hubble parameter is \(H_0 = 73.4^{+2.8}_{-3.8}\) km/s/Mpc, which are compatible with direct age estimates from globular clusters \[8\] and HST measurements \[30\] for the Hubble parameter. Putting in \(H_0\)-independent terms one finds \(0.9646 \leq H_0 t_0 \leq 1.0794\). Using this WMAP range of the dimensionless age parameter at present, we can put constraints on the dark energy. By considering the constant equation of state in Fig. 1, we have shown the age constraints on the dark energy equation of state for flat space, closed and open spaces with curvatures allowed by WMAP observations \[3\]. For the flat universe, \(\Omega_{k_2} = 0\), we obtain the allowed range \(\omega_D \in [-1.9099, -0.8939]\); for a closed universe with \(\Omega_{k_1} = 0.019\) we get \(\omega_D \in [-1.7221, -0.8427]\); finally, for a flat universe with \(\Omega_{k_3} = -0.006\), \(\omega_D \in [-1.9777, -0.9111]\). Current universe age constraints are all consistent with the combination of WMAP, large scale structure and supernova data, that is, \(\omega_D = -1.06^{+0.13}_{-0.08}\) \[3\].

We analyse now the old quasar APM 0879+5255, inquiring about the constraints such data can bring to the question of Dark Energy. This quasar was discovered at a redshift \(z = 3.91\) and its age has been estimated to be 2.1 Gyr \[10, 31\]. Employing the WMAP determination of the Hubble parameter, the age of the quasar is in the dimensionless interval \(0.148 \leq T_g \leq 0.162\). A viable cosmological model should predict a considerably older universe at that high redshift in order to be compatible with the existence of this object.

Figs. 2(a)-2(d) show the dimensionless age parameter of a flat universe as a function of the redshift with different values of equation of state of dark energy allowed by the observations. In order to assure the robustness of our result, we have adopted in our computation the upper and lower limits of \(H_0\) and \(\Omega_{D_0}\) according to the WMAP results \[3\]. We found that all curves cross the shadowed bar at \(z = 3.91\), thus yielding an age parameter smaller than the value 2.1 Gyr required by the quasar APM 0879+5255. This result also holds for a universe with a small positive curvature, permitted by WMAP observations. Therefore, the simple cosmological model with dark energy is not compatible with the age estimate of the old quasar. The same result was also found for other dark energy models by using the HST constraint on the Hubble parameter \[10\].

We consider now the interacting holographic dark energy model proposed in \[15, 16\]. With the interaction, neither dark energy nor dark matter can conserve and evolve separately. For the closed universe, the evolution behavior of the dark energy was obtained as \[16\],

\[
\frac{\Omega_D (1 - \Omega_k - \Omega_D)}{\Omega_D} \left( \frac{2c_{osy}}{c \sqrt{\Omega_D}} + \frac{1}{\Omega_D} + \frac{\Omega'_k}{\Omega_D (1 + \Omega_k - \Omega_D)} - \frac{3b^2 (1 + \Omega_k)}{\Omega_D (1 + \Omega_k - \Omega_D)} \right),
\]

where \(b^2\) and \(c\) have been defined in section 2. The prime denotes the derivative with respect to
Figure 2: Dimensionless age parameter as a function of redshift for simple dark energy models.

We have taken the following parameters: \( H_0 = 76.2 \text{Km/s/Mpc}, \Omega_D = 0.76 \) in (a), \( H_0 = 69.6 \text{Km/s/Mpc}, \Omega_D = 0.76 \) in (b), \( H_0 = 76.2 \text{Km/s/Mpc}, \Omega_D = 0.68 \) in (c) and \( H_0 = 69.6 \text{Km/s/Mpc}, \Omega_D = 0.68 \) in (d). All curves cross the shadowed area yielding an age parameter smaller than the value 2.1Gyr required by the quasar APM 08279+5255.

\[ x = \ln a \quad \text{and} \quad \cos y = \sqrt{1 - \frac{c^2 \Omega_k}{\Omega_D}}. \]

The equation of state of dark energy was expressed as

\[ \omega_D = -\frac{1}{3} - \frac{2\sqrt{\Omega_D}}{3c} \cos y + \frac{b^2(1 + \Omega_k)}{\Omega_D}. \]  (7)

The evolution of the Hubble parameter was derived as well,

\[ \frac{H'}{H} = -\frac{3\Omega_D(1 + \omega_D + r)}{2} + \Omega_k, \]  (8)

where \( r = \frac{1 + \Omega_k - \Omega_D}{\Omega_D} \) is the ratio of the energy densities. Appropriately choosing the coupling between dark energy and dark matter, this model can also accommodate the transition of the dark energy equation of state from \( \omega_D > -1 \) to \( \omega_D < -1 \) [15, 16], which is in agreement with the recent analysis of the type Ia supernova data [17, 18].
Figure 3: Age of the universe as a function of the redshift for interacting holographic dark energy models. The shadowed region is the total age at \( z = 0 \) observed from WMAP [3]. In (a) we considered \( \Omega_k = 0 \); in (b) \( \Omega_k = 0.019 \).

We now want to limit this interacting holographic dark energy model from age considerations. Employing the age of the expanding universe at \( z = 0 \), we have shown our results in Fig.3(a) and Fig.3(b) respectively for \( \Omega_k = 0 \) and \( \Omega_k = 0.019 \), the closed universe allowed by WMAP. For \( \Omega_k = 0 \), the limit on the age of the universe from WMAP [3] puts the constraint on the coupling between dark energy and dark matter in the interval \( 0.01 \leq b^2 \leq 0.068 \) for \( c = 1 \). For \( \Omega_k = 0.019 \), the allowed coupling between dark energy and dark matter can be got at \( 0.018 \leq b^2 \leq 0.075 \) for \( c = 1 \) from the present age of the universe.

The estimated age of an old quasar APM 0879+5255 at redshift \( z = 3.91 \) has also been used to test the viability of the interacting holographic dark energy model. By adopting the lower and upper bounds of values of Hubble parameter and dark energy density respectively from WMAP [3], \( H_0 = 69.6, \Omega_D = 0.72 \), we have shown in Fig.4(a) and Fig.4(b) that the interacting holographic dark energy model with appropriate coupling between dark energy and dark matter is compatible with the estimated age for the APM 0879+5255. For \( \Omega_k = 0 \), the appropriate coupling is required as \( b^2 > 0.053 \) for \( c = 1 \) and for \( \Omega_k = 0.019 \), \( b^2 > 0.059 \) for \( c = 1 \) in order to accommodate the existence of the old quasar. Thus we have shown that the interacting holographic dark energy model is viable from the age constraints.

Combining with the requirement that \( \omega_D \) crosses -1 and the age constraints from \( z = 0 \) and \( z = 3.91 \), we have obtained the allowed parameter space of \( b^2 \) and \( c \) in Fig.5. Within the black region of parameter space, the interacting holographic dark energy model is compatible with the transition behavior of \( \omega_D \) and the age constraints from observation. It can describe the accelerated
Figure 4: Dimensionless age parameter as a function of redshift for interacting holographic dark energy models. In (a) we considered $\Omega_k = 0$, in (b) $\Omega_k = 0.019$. We see that for appropriately coupling between dark energy and dark matter, the interacting holographic dark energy model can accommodate the existence of APM08279+5255 system.

expansion of our universe happened before the present era.

IV. STRUCTURE FORMATION

In a model with interaction we certainly expect that structure formation has a different fate as compared with the non interacting case. In the model defined by \(\beta\) dark matter is continuously being fed at the expense of dark energy and the clumping properties have obviously to change.

In such a framework we shall discuss the formation of a structure, such as the quasar previously discussed, in the young universe. We thus have to consider the matter density perturbation

$$\delta \equiv \frac{\delta \rho_m}{\rho_m}$$

and its time evolution. We do not consider dark energy perturbations, assuming that it does not clump sufficiently to help forming structures. Rather we assume that hadronic matter just follows the general dark matter pattern. Thus, within general relativity the evolution equation of dark matter inhomogeneities is given by

$$\ddot{\delta} + 2H \dot{\delta} - \frac{a\delta}{2} \frac{d(H^2 \Omega_m)}{da} = 0.$$  \hspace{1cm} (10)

If there is no interaction, $b^2 = 0$ and we know that $\Omega_m = \frac{\rho_m}{3H^2} = \frac{\Omega_m H^2}{H^2 a^2}$. In that case the equation above becomes the usual matter density perturbation equation in a simple cosmological set up \cite{32},

$$\ddot{\delta} + 2H \dot{\delta} - \frac{3}{2} H^2 \Omega_m \delta = 0.$$  \hspace{1cm} (11)
Figure 5: The constrained parameter space of $b^2$ and $c$. The dark grey is the constraint from the total age at $z = 0$ from WMAP [3], the light grey is the constraint from the old quasar APM08279+5255 [10, 31] and the dark region is the parameter space compatible with the $w_D$ crossing $-1$ [18] and the age constraints. We have taken $\Omega_k = 0, \Omega_D = 0.76$ in (a), $\Omega_k = 0, \Omega_D = 0.68$ in (b), $\Omega_k = 0.019, \Omega_D = 0.76$ in (c) and $\Omega_k = 0.019, \Omega_D = 0.68$ in (d).

Changing variables from $t$ to $x = \ln a$, Eq. (10) can be rewritten as

$$\frac{d^2 \delta}{dx^2} H^2 + \frac{d \delta}{dx} [2H^2 + \frac{dH}{dx} H] + \frac{\delta}{2} \frac{d(H^2 \Omega_m)}{dx} = 0 .$$

(12)

Defining the growth variable $G = \delta/a$, Eq(12) can be further rearranged as

$$\frac{d^2 G}{dx^2} + (4 + \frac{1}{H} \frac{dH}{dx}) \frac{dG}{dx} + (3 + \frac{1}{H} \frac{dH}{dx} + \frac{1}{2H^2} \frac{d(H^2 \Omega_m)}{dx}) G = 0 .$$

(13)

Normalizing the density perturbation in terms of the present amplitude, we show the solution to the growth variable in Fig.6(a). Plotting the matter density perturbation evolution in Fig.6(b), we found that with strong coupling between dark energy and dark matter, the matter density
perturbation is stronger during the universe evolution till today, which shows that the interaction between dark energy and dark matter enhances the clustering of dark matter perturbation compared to the noninteracting case in the past. This phenomenon was also observed by studying quintessence interacting with dark matter [33]. The strong clustering of dark matter accounts for the old quasar appearing in the young universe with interacting dark energy. From Fig.6a we see that for stronger coupling between dark energy and dark matter, the growth decreases faster. This is due to the fact that for stronger coupling between dark energy and dark matter, dark energy will reveal itself earlier, the universe will evolve earlier into the accelerated expansion, and \( \omega_D \) can even cross \(-1\) and stay at \( \omega_D < -1 \) earlier [15, 16]. Thus the stronger repulsive pressure from dark energy in the early time for bigger \( b^2 \) implies a faster decrease of \( \delta/a \). If we observe the matter density perturbation in the future, the stronger coupling between dark energy and dark matter will weaken the clustering of the dark matter perturbation and it will be more difficult for the structure to be formed.

With interaction Dark Energy and Dark Matter follow one another, as displayed in figure (7). This means that in the recent history of the universe dark energy is being transformed into dark matter and the fluctuations do get more effective in the past according to figure 6. Therefore, in the beginning there were the same fluctuations, then dark matter was enhanced leading to higher fluctuations, to finish now with the same structures, but with more structures in the past, such as the case of the old quasar. Note that the stronger the interaction, the more effectively structures will have been formed in the past. We thus conclude that we must have \( b^2 \gtrsim 0.05 \) in this model.

The structure formed as the quasar APM 0879+5255 is thus a further support of the interaction
Figure 7: Evolution of Dark Energy and Dark Matter, with or without the interaction term. The line descending quickest corresponds to dark matter without interaction. The other lines, by order of steepness, are dark matter with interaction, dark energy with interaction and dark energy without interaction, respectively. The same type of lines describe the corresponding evolutions for corresponding Ω’s (as a matter of fact, we are using here the parametrization I, given below, with $b^2 = 0.18$. The qualitative results do not depend on this assumption).

between these rather puzzling objects that may open an immense avenue for the study of the universe.

V. NUMERICAL ANALYSIS OF LOW $\ell$ CMB SPECTRUM

In our previous analysis for a flat universe [15] as well as in a closed universe [16] we obtained several constraints on the parameters of the interacting holographic model. In this section we consider the consequences of the detailed analysis of the low $\ell$ suppression of the CMB power spectrum revealed by COBE/WMAP for the phenomenological constants that have been defined. The question of small $\ell$ suppression was investigated for flat [25] as well as closed [26] universes.

Here we will perform a more detailed numerical analysis in order to probe how essential is
the coupling between DE and DM. We are going to show that with a 90% confidence level the interaction is non vanishing, which constitutes a further support of the statements collected in the previous sections. This opens up a wide road for models of DE/DM. It also signalizes that a pure cosmological constant possibly cannot describe such dynamics of those essential parts of the universe.

As already discussed in [15, 16, 25, 26] we interprete the infrared cutoff as the maximum possible wavelenght, \( \lambda_c = 2L \). Since \( \Omega_\Lambda = \rho_\Lambda/\rho_c \) and \( \rho_c 3M_p^2 H^2 \), we have

\[
 k_c = \frac{\pi}{c} H^0 \sqrt{\Omega_\Lambda^0}. \tag{14}
\]

The above expression represents the minimum wave number allowed for the computation of the power spectrum. Thus we obtain, for the spectral coefficient,

\[
 C_\ell = (4\pi^2) \int_{k_c}^\infty k^2 dk P_\Psi(k) |\Delta T_\ell(k, \tau = \tau_0)|^2, \tag{15}
\]

where \( P_\Psi(k) \) is the initial power spectrum and \( \Delta T_\ell(k, \tau_0) \) is the \( \ell \)th coefficient.

We use the CMBFAST programm version 4.5.1, modified to take into account the cutoff \( k_c \).

Since we are lack of the knowledge of the perturbation theory in including the interaction between DE and DM, in fitting the WMAP data by using the CMBFAST we will first estimate the value of \( c \) without taking into account the coupling between DE and DM. The interaction becomes important for values of \( a \) near 1 (today). Considering the equation of state of DE is time-dependent as disclosed by recent data analysis [17, 18], we will adopt two extensively discussed DE parametrization models,

\[
 \omega^I(z) = \omega_0 + \omega_1 \frac{z}{z + 1}, \tag{16}
\]

\[
 \omega^{II}(z) = \omega_0 + \omega_1 \frac{z}{(z + 1)^2}. \tag{17}
\]

From (2), we know that the ratio of energy densities \( r = \rho_m/\rho_D \) obeys (in this section we only consider the flat case)

\[
 \dot{r} = 3b^2 H(1 + r)^2 + 3Hr \omega_D. \tag{18}
\]

Using the Friedmann equation \( \Omega_m + \Omega_D = 1 \) (valid for a flat universe) as well as \( \dot{r} = -\dot{\Omega}_D/\Omega_D^2 \), we arrive at

\[
 \omega_D = -\frac{\Omega_D'}{3\Omega_D(1 - \Omega_D)} - \frac{b^2}{\Omega_D(1 - \Omega_D)}. \tag{19}
\]
After some algebra, we get

$$\omega_D = -\frac{1}{3} - \frac{2\sqrt{\Omega_D}}{3c} - \frac{b^2}{\Omega_D}.$$  \hspace{1cm} (20)

Using the modified CMBFAST 4.5.1 (given in www.cmbfast.org) \cite{34}, we can again study the effect of the holographic model on the small $\ell$ CMB spectrum. We require that the equation of state of the holographic model persists with the same behavior as that of the two parametrizations above respectively.

We want to find a set of cosmological parameters maximizing the likelihood function $L$. However, since the cut parameter $c$ only affects the spectral region of low multipoles ($l < 10$), such a parameter has negligible correlation with the other relevant parameters, thus we seek at finding the value of $c$ maximizing the function $L(1/c)$.

For parametrization I we used the parameters given in table 1 \cite{34}.

Table 1: Cosmological parameters for a flat universe with equation of state I

| Parameter       | Value                  |
|-----------------|------------------------|
| $\Omega_\Lambda$| 0.715$^{+0.023+0.045+0.066}_{-0.024-0.047-0.070}$ |
| $100\Omega_b h^2$ | 2.33$^{+0.10+0.20+0.32}_{-0.09-0.17-0.25}$ (*) |
| $A$             | 0.837                  |
| $n_s$           | 0.978$^{+0.028+0.058+0.084}_{-0.022-0.041-0.059}$ |
| $H_0$           | 70.7$^{+2.4+4.9+7.4}_{-2.3-4.6-6.6}$ |
| $\tau$         | 0.152$^{+0.067+0.127+0.146}_{-0.056-0.101-0.136}$ |
| $\omega_0$      | $-0.981^{+0.193+0.38+0.57}_{-0.193-0.37-0.52}$ |
| $\omega_1$      | $-0.05^{+0.65+1.13+1.38}_{-0.83-1.92-2.88}$ |

* $\Omega_b = 0.0466$.

The value we obtained for $1/c$ with uncertainties given respectively by 68.26%, 95.44% and 99.73%, is

$$\frac{1}{c} = 0.56^{+0.12+0.28+0.41}_{-0.20-0.48-0.55}.$$  

For equation of state II we used the parameters given in table 2.

Table 2: Cosmological parameters for equation of state II
The table below lists the values of various cosmological parameters:

| Parameter | Value |
|-----------|-------|
| $\Omega_\Lambda$ | $0.76^{+0.03+0.6}_{-0.13-0.20}$ |
| $\Omega_b h^2$ | $0.0266^{+0.0004}_{-0.0006} (*)$ |
| $A$ | 1.22 |
| $n_s$ | $1.1^{+0.0}_{-0.2}$ |
| $H_0$ | $73^{+5}_{-12}$ |
| $\tau$ (reionization depth) | $0.35^{+0.04}_{-0.33}$ |
| $\omega_0$ | $-1.48^{+0.78+1.05}_{-0.19-0.45}$ |
| $\omega_1$ | $3.86^{+0.29+1.01}_{-4.6-6.36}$ |

*(Note: $\Omega_b = 0.05$.*

The value we obtained for $1/c$ with uncertainty given by 68.26%, 95.44% and 99.73%, respectively, is

$$\frac{1}{c} = 0.42^{+0.06+0.16+0.27}_{-0.24-0.39-0.42}$$

The two sets of values for $1/c$ above are already compatible at level 1 $\sigma$. Since the parameter $c$ affects only the region of small multipoles the calculation above could be performed with some simplifying assumptions concerning the other cosmological parameters, which are defined by the region with high multipoles. The corresponding likelihood functions are given in figure (8) ($1/c$ was used because for numerical precision and normalization).

After the determination of $c$ for the two equations of state for dark energy, as described above, we estimated the coupling between dark energy and dark matter — $b^2$ by comparing (20) with the parametrizations (16) and (17). We have to find the maximum of the likelihood function $L(b^2, 1/c, \Omega_\Lambda, \Omega_b, H_0, n_s, \frac{dn_s}{dk}, \tau, \omega_0, \omega_1, A)$ in order to find the best value of $b^2$ and its error bars.

For the first parametrization model, we have

$$-\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda(z)}}{3c} - \frac{b^2}{\Omega_\Lambda(z)} \simeq \omega_0^I + \omega_1^I \frac{z}{1+z}. \quad (21)$$

For the second parametrization, we get

$$-\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda(z)}}{3c} - \frac{b^2}{\Omega_\Lambda(z)} \simeq \omega_0^{II} + \omega_1^{II} \frac{z}{(1+z)^2}. \quad (22)$$

Today ($z = 0$) the above two equations boil down to

$$-\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda(0)}}{3c} - \frac{b^2}{\Omega_\Lambda(0)} \simeq \omega_0 \quad. \quad (23)$$
Figure 8: Likelihood functions for $1/c$. The first figure corresponds to equation of state I and the second to equation of state II. Note that in the case of the equation of state II $c$ has to be larger, $c_{\text{min}} \sim 1.4$.

Isolating $b^2$ we have

$$b^2 \simeq -\Omega_{\Lambda}(0) \left( \frac{1}{3} + \frac{2\sqrt{\Omega_{\Lambda}(0)}}{3c} \right).$$

We can now evaluate $b^2$ and the respective uncertainty. The likelihood function for $b^2$ as well as for $b^2_{\text{max}}$ defined below appear in figure (9) for parametrization I.

The results obtained for $b^2$ using the two parametrizations with uncertainties 68.26%, 95.44% and 99.73% are given below,

$$b^2 = 0.25^{+0.16}_{-0.15}^{+0.32}_{-0.30}^{+0.48} \text{ (eq. of state 1)},$$

and

$$b^2 = 0.7^{+0.1}_{-0.6}^{+0.5}_{-0.9}^{+0.8} \text{ (eq. of state 2)}.$$

We can impose a further constraint in $b^2$, using the condition given in equation (8) of [15], that is, that $\Omega_{\rho}'$ is always positive, namely that the dark energy has an increasing role with the flow of
Figure 9: Likelihood functions for $b^2$ and $b_{\text{max}}^2$. These figures correspond to equation of state I.

time,

$$b^2 < b_{\text{max}}^2 \frac{1 - \Omega_{\Lambda}}{3} (1 + 2 \sqrt{\frac{\Omega_{\Lambda}}{c}}) .$$

(25)

The values of $b_{\text{max}}^2$ using parametrization 1, with the respective intervals of uncertainty 68.26%, 95.44% and 99.73% are

$$b_{\text{max}}^2 = 0.191^{+0.027+0.052+0.075}_{-0.022-0.047-0.075} .$$

Combining $b^2$ and $b_{\text{max}}^2$,

$$0.10 < b^2 < 0.22 \quad (1 \sigma),$$

$$-0.05 < b^2 < 0.24 \quad (2 \sigma),$$

$$-0.20 < b^2 < 0.27 \quad (3 \sigma).$$

For parametrization 2, we obtain

$$b_{\text{max}}^2 = 0.14^{+0.06+0.10+0.15}_{-0.03-0.08-0.12} .$$
Combining $b^2$ and $b_{max}^2$ for parametrization 2 we get

$$0.1 < b^2 < 0.2 \ (1 \sigma) \ ,$$

$$-0.2 < b^2 < 0.24 \ (2 \sigma) \ ,$$

$$-0.6 < b^2 < 0.29 \ (3 \sigma) \ .$$

As a conclusion we see that the parameter $b^2$ is non zero with a confidence of 90%. Using the prior that the dark energy always increases, we have, for the probability the $0.05 < b^2 < 0.2$ is of the order of 75%.

We computed the probability of finding a non-positive value of $b^2$ according to the above data and found it to be of the order 4.5%.

Finally, let us comment on the improvement of the model when comparing it to the $\Lambda$CDM model. We have first to call attention to the fact that the first modification, namely the holographic input, leads us out of the $\Lambda$CDM case, requiring modifications in the equations thus used. Nevertheless, comparing the low $\ell$ part of the spectrum only, the $\chi^2$ comes down to roughly one half (the full $\chi^2$ shows little difference due to the broadness of the problem). The values for the spectrum corresponding to the first eight (8) multipoles are

$$\chi^2_{\Lambda CDM} = 8.3 \ ,$$

$$\chi^2_{\text{par 1}} = 5.2 \ ,$$

$$\chi^2_{\text{par 2}} = 2.8 \ .$$

Such values imply a nice improvement with respect to the $\Lambda$CDM model, completing our picture which tried, and in our view succeeded, in arguing that interaction between Dark Energy and Dark Matter are essential on the top of natural in a Quantum Field Theory description.

VI. CONCLUSIONS AND OVERVIEW

We discussed several observational constraints on the two parameter space of holographic dark energy/dark matter model. We can confidently conclude that the interaction must be nonvanishing in order that we explain all available data at the same time.

Concerning the holography coefficient $c$, we know that it must have a lower limit due to the second law of thermodynamics as well as from the low $\ell$ CMB data. We actually have considered the holographic model in all possible scenarios, including the $\Lambda CDM$ and the likelihood function always leads to a $c$ larger than unit.
The age constraint provided by the old quasar leads to a lower limit of the \( b \) parameter, namely \( b^2 > 0.05 \), which seems to be an outcome which should be respected as foreseen by all observations we analyzed. For the likelihood function discussed in section 5, we found that \( b^2 \) is larger than 0.05 with probability 0.9 and with the prior \( b < b_{\text{max}} \) with probability of the order 0.75. The age limit also puts maximum values for the same parameter. For the age today we got a maximum \( b \) of the order \( b^2 < 0.2 \), which is also the range obtained in section 5 in case we take into account the requirement of an always increasing dark energy, or \( b^2 \lesssim b^2_{\text{max}} \approx 0.2 \).

Therefore all observational constraints point to the same range \( 0.05 < b^2 < 0.2 \).

For the case of the \( c \) parameter the bounds are not so tight. The CMB data show very confidently that \( c > 1 \) in order that not too high values of \( \ell \) are suppressed in the power spectrum, an independent check of the result already obtained from more formal arguments. However, the likelihood function is very spread from unity to infinity, and we are not in position to say much more. We thus worked most of the time with the limiting value \( c = 1 \).

Although extremely simple, the model opens up wide possibilities towards new physics: 30% of the energy is composed of a new state of matter, interacting with the rest 67% yet unknown, a fascinating and probably extremely complex universe!

Certainly new kinds of interaction have to be considered, especially in case they can be computed from first principles and more well defined methods of theoretical physics and quantum field theory. Work in this direction is under way \[20, 36\]. We thus conclude pointing out that, in our opinion, interaction shoud be an essential worry in the description of the dark sector of the Universe.

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