Design of Diagnostic strategy for Radar System Based on Rollout Algorithm

Xiaoshuai Du\textsuperscript{1,2,a}, Bing Hu\textsuperscript{1,b}

\textsuperscript{1}Air Force Early Warning Academy, Wuhan, Hubei, 430010, China
\textsuperscript{2}The No. 94005th Troop of PLA, Jiuquan, Gansu, 735000, China
\textsuperscript{a}email: 819877910@qq.com, \textsuperscript{b}email: hubwh@126.com

*Corresponding author’s e-mail: afewa@kjld1000.onexmail.com

Abstract. To solve the problem of how to detect and isolate the fault of radar system with low test cost, a design method of diagnostic strategy for radar system based on rollout algorithm is proposed. This method takes the greedy search method based on information heuristic as the benchmark strategy, and uses the rollout strategy to iterate on the benchmark strategy, so as to overcome the problem of poor optimality of greedy search method. Taking the radar system dependency model as an example, this paper uses rollout algorithm to generate the diagnostic strategy of the radar system, which verifies the effectiveness of the method.

1. Introduction

With the development and application of high-tech, the structure and function of radar system become more and more complex and refined, and the problems of fault diagnosis and isolation during its use become more and more prominent. To solve this problem, it is necessary to conduct testability design in the design stage of the system\cite{1}. Diagnostic strategy design, as an important part of testability design and analysis, can be used to guide the process of system fault diagnosis and maintenance\cite{2}, is the key to reduce the cost of fault diagnosis and improve the efficiency of fault diagnosis.

Diagnostic strategy is a method to optimize system fault diagnosis by combining constraints, objectives and other related factors. Its purpose is to construct a test sequence and quickly isolate the system fault state with less test cost. Pattipati proved that the diagnostic strategy problem is an NP-complete problem\cite{3}. Although the enumeration method can obtain the optimal solution, the computation time increases exponentially with the complexity of the problem. In order to reduce computational complexity, domestic and foreign scholars apply heuristic search method to diagnostic strategy optimization problem. At present, the most widely used method is the greedy search method based on information heuristic, in which the mutual information of fault state test is the basis for determining the test\cite{4}. However, the greedy algorithm only considers the information obtained from the single-step test when select the test, but does not consider the impact of the selected test on the overall cost and subsequent tests. Therefore, the optimality of the diagnostic strategy obtained by the greedy algorithm is poor.

In view of the problem mentioned above, this paper studies the design method of diagnostic strategy with less test cost based on the radar system dependency model. On the basis of greedy algorithm, this paper proposes the design method of diagnostic strategy for radar system based on rollout algorithm, expounds the implementation process of the algorithm, and constructs the fault diagnostic strategy of radar system.
2. Problem formulations

2.1. Diagnostic strategy problem
In order to illustrate the diagnostic strategy problem, a five-tuple $\langle F, P, T, C, D \rangle$ is introduced as follows:[7]:

1) $F = \{f_0, f_1, ..., f_m\}$ represents the fault state set of the system, including all the fault states of the system, where $f_0$ is refers to the fault-free state, and $f_i (i = 1, 2, ..., m)$ refers to the fault state when only the $i$th fault occurs.

2) $P = \{P(f_0), P(f_1), ..., P(f_m)\}$ is the set of prior probabilities corresponding to the $m + 1$ fault states of the system, where $P(f_0)$ is the prior probability of fault-free state of the system and occupies a large proportion in $P$, and $P(f_i)$ is the prior probability of $f_i$.

3) $T = \{t_1, t_2, ..., t_n\}$ is the test set of the system. It is stipulated that the test can only pass or fail, and the test results are reliable.

4) $C = \{c_1, c_2, ..., c_n\}$ is the set of test costs, where $c_j$ represents the cost of executing test $t_j$. The test cost can refer to the measure of different categories, such as test execution time, manpower, material resources, etc. The test cost is set as a dimensionless physical quantity.

5) $D = [d_{ij}]_{(m+1) \times n}$ is the fault-test dependency matrix, which is a $(m + 1) \times n$ dimensional Boolean matrix, used to represent the logical relationship or dependency between the fault and the test. Elements in the matrix represent the dependency between the test and the fault state. $d_{ij} = 1$ means that the test $t_j$ can detect the fault $f_i$, that is, when the fault occurs, the test fails.

$\textbf{2.2. Inference mechanism of fault diagnosis}$

According to the order of test execution of the diagnostic strategy, after each test is executed, the fault conclusion of the equipment is inferred according to the results. Assuming that the fault state set of the system is $X$, test $t_j$ is used to detect $X$. There are two outputs of pass ($d_{ij} = 0$) and fail ($d_{ij} = 1$), denoted as $X_{j0}$ and $X_{j1}$ respectively. Inference mechanism is the reasoning method of logical inference of system fault state. The inference method for the process from $X$ to $X_{j0}$ and $X_{j1}$ is the inference mechanism:

\begin{align*}
X_{j0} &= \{ f_i | b_{ij} = 0, \forall f_i \in X \}, \quad (1) \\
X_{j1} &= \{ f_i | b_{ij} = 1, \forall f_i \in X \}. \quad (2)
\end{align*}

Moreover:

\begin{align*}
X_{j0} \cup X_{j1} &= X, \quad (3) \\
X_{j0} \cap X_{j1} &= \emptyset. \quad (4)
\end{align*}

$X$ is divided into two subsets $X_{j0}$ and $X_{j1}$ by test $t_j$:

\begin{equation}
P(X_k) = \sum_{f_i \in X_k} P(f_i), k = 0, 1. \quad (5)
\end{equation}

2.3. Optimization objective

The objective of the optimal diagnostic strategy is to use tests according to the given logic and sequence when a system failure occurs, to isolate the fault quickly and accurately, and to minimize the expected cost of the test. The calculation equation is[3]:

\begin{equation}
P(X_k) = \sum_{f_i \in X_k} P(f_i), k = 0, 1. \quad (5)
\end{equation}
represents the diagnostic strategy with lowest expected cost that isolates all faults without ambiguity; \( D_{\{D\}} \) represents the test sequence in the diagnosis tree that isolates fault \( f_i \), \( |D_{\{D\}}| \) represents the length of that sequence; \( c_{D_{\{D\}}[h]} \) represents the cost of test \( h \) in the sequence \( D_{\{D\}} \).

3. Design of diagnostic strategy based on rollout algorithm

3.1. The basic idea of rollout algorithm
Rollout algorithm[8] is a one-step look-ahead backtracking algorithm based on greedy search method, which can optimize the benchmark strategy through backtracking iteration of test cost. When constructing the diagnosis tree, the rollout algorithm uses two steps to determine a test. The first step is to extend the diagnosis tree from top to bottom, that is, to construct the diagnosis tree with each alternative test as the vertex by using the benchmark strategy[9]. The second step is test optimization. By calculating and comparing the test cost of each diagnosis tree, the first test corresponding to the diagnosis tree with the lowest test cost is selected as the current optimal test.

3.2. Benchmark strategy
According to the knowledge of information theory, if \( X \) and \( Y \) are discrete random variables, then the information entropy of \( X \) is:

\[
H(X) = - \sum_{i=1}^{m} P(X_i) \log_2 P(X_i),
\]  

(7)

The conditional entropy of \( X \) with respect to \( Y \) is:

\[
H(X|Y) = - \sum_{i=0}^{m} \sum_{j=0}^{n} P(Y_j) P(X_i|Y_j) \log_2 (X_i|Y_j),
\]  

(8)

The amount of information provided by \( Y \) about \( X \), that is, the mutual information between \( X \) and \( Y \) is:

\[
I(X;Y) = H(X) - H(X|Y),
\]  

(9)

\[
H(X) = - \sum_{i=0}^{m} P(X_i) \log_2 P(X_i).
\]  

(10)

In the diagnostic strategy design problem, the more information provided by the test, the more beneficial it is to isolate the fault, and the impact of the test cost should be considered. In the greedy search method based on information heuristic, the ratio of the mutual information of fault and test to the test cost is taken as the basis to select the test, and the heuristic evaluation function is:

\[
k^* = \arg \max_j \left\{ \frac{IG(X; t_j)}{c_{t_j}} \right\},
\]  

(11)

\[
IG(X; t_j) = -P(X_0) \log_2 P(X_0) - P(X_1) \log_2 P(X_1),
\]  

(12)

\( IG(X; t_j) \) represents the mutual information between test \( t_j \) and the fault state fuzzy set \( X \), that is, the information obtained by using test \( t_j \) to diagnose the fault state fuzzy set to be isolated.

The benchmark strategy selects the test that optimizes the heuristic evaluation function each time, isolated faults according to the test results, classifies fuzzy subsets of faults, and continues to expand further. The detailed steps are as follows:

Step 1: Initial failure state set \( X = F \), test set \( t = T \).
Step 2: For fault state set $X$, test $t_j$ in test set is selected in turn. According to the test results, fault state set $X$ is divided into two subsets $X_{j0}$ and $X_{j1}$. The total probability of each subset is calculated according to equation (5).

Step 3: Equations (11) and (12) are used to calculate the heuristic function values of each test. The test that minimizes the value of the heuristic function is selected as the current optimal test, denoted as $t_a$.

Step 4: Test $t_a$ is used to divide the fault state $X$ into two subsets $X_{a0}$ and $X_{a1}$, and the following equation is used to update the probability of the fault state in each subset:

$$P(f_i) = P(f_i)/P(X_{ja}), f_i \in X_{ja}, k = 0, 1.$$  

(13)

Step 5: Re-take $X$ as each subsets, and $j$ is the set after deleting $t_a$ from the original test set. Repeat 2~5 until the number of test subset elements is no more than one.

3.3. The specific steps of rollout algorithm

The basic steps of rollout algorithm are as follows:

Step 1: Initialize the five-tuple, system failure state set $X = F$, test set $t = T$.

Step 2: For fault state set $X$, test $t_q$ in test set is selected in turn. According to the test results, fault state set $X$ is divided into two subsets $X_{q0}$ and $X_{q1}$. The total probability of each subset is calculated according to equation (5). According to equation (13), the failure probability of each subset is updated.

Step 3: The benchmark strategy is used to get the test sequence of each subset. Calculate the expected test cost of each test sequence:

$$h(X_{qk}) = \sum_{i=1}^{m_q} P(f_i) \left( \sum_{k=1}^{[v_{qk}]} c_{D(0)[v_k]}, k = 0, 1 \right).$$  

(14)

Step 4: Calculate the expected test cost for the diagnosis tree corresponding to test $t_q$

$$J(t_q) = c_q + P(X_{q0}) h(X_{q0}) + P(X_{q1}) h(X_{q1}).$$  

(15)

Select the test in the test set that minimizes equation (15), denoted as $t_a$.

Step 5: Test $t_a$ is used to divide the fault state into two subsets $X_{a0}$ and $X_{a1}$, and equation (13) is used to update the probability of the fault state in each subset.

Step 6: Re-take $X$ as each subsets, and $j$ is the set after deleting $t_a$ from the original test set. Repeat 2~6 until the number of test subset elements is no more than one.

4. Example analysis and verification

4.1. Dependency matrix of radar system

Taking radar receiving processing and waveform generation subsystem as an example, the concrete process of diagnostic strategy construction is explained, and the application effect of rollout algorithm and benchmark strategy is analyzed and compared. The multi-signal model of radar receiving processing and waveform generation subsystem is shown in figure 1.
Figure 1. Radar receiving processing and waveform generation subsystem.

The system has a total of 11 components and 14 fault modes, among which, \( s_{10} (G) \) and \( s_{11} (G) \) are a group of fuzzy set, and the combination of the two faults is denoted as \( s_{10} (G) \). Through the selection of test points and tests, 8 test points and 12 tests were finally determined. By analysing the model, the dependency matrix is obtained as shown in table 1.

Table 1. Dependency matrix.

| Fault  | \( t_1 \) | \( t_2 \) | \( t_3 \) | \( t_4 \) | \( t_5 \) | \( t_6 \) | \( t_7 \) | \( t_8 \) | \( t_9 \) | \( t_{10} \) | \( t_{11} \) | \( t_{12} \) |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| \( s_1 (G) \) | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| \( s_2 (G) \) | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| \( s_2 (F) \) | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| \( s_3 (G) \) | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| \( s_4 (G) \) | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| \( s_4 (F) \) | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| \( s_5 (G) \) | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| \( s_6 (G) \) | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| \( s_7 (G) \) | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| \( s_8 (G) \) | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| \( s_8 (F) \) | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| \( s_9 (G) \) | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| \( s_{10} (G) \) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

The set of prior probabilities corresponding to all fault states is \{0.879, 0.005, 0.015, 0.009, 0.007, 0.018, 0.013, 0.018, 0.003, 0.005, 0.012, 0.011, 0.003, 0.002\}. The probability of fault is obtained from the reliability data of the component, where the probability of fault-free state is 0.879. Assuming that
all test costs is 1. The dependency matrix in table 1 is analyzed and calculated by using the benchmark strategy, and the diagnostic strategy is shown in figure 2.

Figure 2. Diagnostic strategy based on greedy algorithm.

According to equation (6), the expected test cost of the diagnostic strategy is 3.172.

4.2. Diagnostic strategy based on rollout algorithm

Process the dependency matrix according to the specific steps of the rollout algorithm. Start by selecting the first test. The 12 tests in the test set divided the fault state set into two subsets respectively, and the greedy algorithm was applied to construct the diagnostic strategy for each subset and calculate the test cost. For example, \( t_5 \) divided the test set into two subsets, \( \{ f_6, f_5, f_8, f_{13} \} \) and \( \{ f_{17}, f_{12}, f_{13}, f_{13} \} \). The greedy algorithm is used to construct the diagnostic strategy of the two subsets, and the expected test costs are obtained as 2 and 13.581, and the corresponding expected cost is calculated as 3.135 by using equation (15). Calculate the expected cost for all tests in the test set successively. After comparison, the expected cost of \( t_9 \) is minimal, so \( t_9 \) is selected as the first test. Repeat the above process for the subsets divided by \( t_9 \) to obtain the final diagnostic strategy as shown in figure 3.
According to equation (6), the expected test cost of the diagnostic strategy is 3.129, which is less than the test cost constructed by the greedy algorithm.

5. Conclusions
The purpose of diagnostic strategy design is to construct a set of test sequences to isolate the fault state of the system with less test cost. In this paper, the design method of radar system diagnostic strategy based on rollout algorithm is proposed. Based on greedy algorithm, the optimization test of rollout strategy is carried out to improve the optimality of the results. Taking the radar system dependency model as the research object, the radar system diagnostic strategy is generated by using the rollout algorithm. The results show that the method has good comprehensive performance and has important application value to reduce the cost of radar system fault diagnosis.

Acknowledgments
The authors express their sincere appreciation to the editor and reviewers for their suggestions on this paper. This study was supported by Military Postgraduate Funding Projects (No. JY2020C236).

References
[1] Lyu, Y., Song, J.Y., Kuang, C.T. (2016) Circle tests selection method based on β parameter. Computer Engineering and Design., 37: 237–241.
[2] Tian, Z., Shi, J.Y. (2003) System Testability Design, Analysis and Verification. Beijing University of Aeronautics and Astronautics Press, Beijing.
[3] Pattipati, K.R., Alexandrisis, M.G. (1990) Application of heuristic search and information theory to sequential fault diagnosis. IEEE Transactions on System, Man, and Cybernetics., 20 (4): 872–887.

[4] Huang, Y.F., Jing, B., Ru, C.J. (2011) Optimal method of diagnosis strategy in multi-value attribute system based on information entropy. Chinese Journal of Scientific Instrument., 32: 1003–1008.

[5] Guo, J.H., Shi, X.J., Wang, K. (2019) Diagnostic strategy optimization method based on information entropy theory. Ordnance Industry Automation., 38 (6): 29–32.

[6] Huang, Y.F., Jing, B., Xia, Y., (2010) Optimal strategy of test point selection for circuit based on information entropy. Application Research of Computer., 27 (11): 4149–4151.

[7] Shakeri, M., Raghavan, V., Pattipati, K.R., et al. (2000) Sequential testing algorithms for multiple fault diagnosis. IEEE Transactions on System, Man, and Cybernetics–Part A: Systems and Humans., 30 (1): 1–14.

[8] Tu, F., Pattipati, K.R. (2003) Rollout strategies for sequential fault diagnosis. IEEE Transactions on System, Man, and Cybernetics–Part A: Systems and Humans., 33(1):86–99.

[9] Liu, Y.H., Liu, J.M., Feng, F.Z., et al. (2015) Fault diagnostic strategy based on rollout information heuristic algorithm. Computer Engineering., 41: 291–295.