Single spin measurement in the solid state: a reader for a spin qubit

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We describe a paradigm for measuring a single electron spin in the solid state. This technique can be used to “read” a spin qubit relatively non-invasively in either a spintronic quantum gate or a spintronic quantum memory. The spin reader can be self assembled by simple electrochemical techniques and can be integrated with a quantum gate.

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Measuring single electron spin in a solid is a fundamental problem in condensed matter physics. Recently, it has assumed additional importance in view of the many spintronic proposals for scalable solid state quantum computers that advocate encoding a qubit in a single electron spin \[1, 2, 3, 4, 5, 6\]. In all of these proposals, it will be necessary to measure a single electron’s spin in order to execute quantum algorithms. This is a difficult challenge since unlike charge (which can be easily measured with electrometers, including the exquisitely sensitive single electron transistor electrometers), determining a single spin in a solid is a formidable challenge.

In quantum computer or memory applications, the requirement is to determine a target electron’s spin by relatively non-invasive means that are “conservative”, meaning that the electron should not be lost to a contact (electron reservoir) irretrievably. A basic idea might be the following: a target spin is coaxed into tunneling to a region where its wavefunction will overlap with that of a control spin. It is assumed that the control spin’s orientation is known. Since the Pauli principle forbids the tunneling event if the two spins are parallel, the presence or absence of a tunneling current provides a measurement of the target electron’s spin. The difficulty with this approach is knowing with certainty the orientation of the control spin (only a highly localized magnetic field confined to within perhaps 10 nm of space around the control spin can orient the spin deterministically without affecting the target spin. The target qubit has to be sufficiently close (in space) to the control qubit for tunneling to occur and there is no known technology to shield a magnetic field over this small distance). To our knowledge, there is no report of any successful attempt to demonstrate this reading scheme.

There have been proposals to use a single electron transistor to discriminate between a singlet state and a triplet state of a two interacting electrons in a solid \[7\]. Recently, such a discrimination (using a scheme different from that of ref. \[7\]) was demonstrated experimentally in a coupled quantum dot system \[8\]. However, discrimination between singlet and triplet states merely tells us if the spins are parallel or antiparallel. It does not tell us which electron has which spin, and is therefore not good enough.

In this paper, we describe a paradigm to measure a single electron spin without losing that electron to a reservoir irretrievably. The target spin is read via a “scout spin” whose
orientation is always known and which interacts directly with the measuring device (contacts). The target spin never interacts directly with the contacts. Additional advantages are that the measuring configuration is completely compatible (and hence can be integrated) with an existing model for a quantum gate [6].

Consider a penta-layered quantum wire structure as shown in Fig. 1(a). The transverse dimension of this wire is $\sim 10$ nm and the thicknesses of the semiconducting and insulating layers are also shown. Initially, the ferromagnetic contacts are not magnetized, so that the electrons in the ferromagnetic contacts are not spin polarized.

Unpolarized spin: The equilibrium energy band diagram along the length of the wire is shown in Fig. 1(b). We neglect band bending in the semiconductor and insulator because of resident charges, since such bending may cause small quantitative changes, but no qualitative change to the discussion that follows.

The insulating layers are thin enough to be at least translucent to electrons, but the semiconductor layer is too thick to allow tunneling through it.

If a small potential $V_{SD}$ is applied between the source and drain contacts shown in Fig. 1(b), a current will flow only if an electron can jump from the source to the lowest subband in the quantum dot, and thence to the drain. For this discussion, we assume that the temperature is zero ($kT = 0$) and we also neglect all weak virtual processes, so that such a transition is not possible as long as the lowest subband in the semiconductor dot is above the Fermi level in the source contact.

Let us now pull the lowest subband in the dot to the Fermi level in the source contact by applying a positive potential to the semiconductor, while maintaining $V_{SD} = 0$. This potential can be applied with a wrap-around gate (in much the same way as in ref. [8]) or with a remote gate in the configuration we will propose later. The gate potential does not affect the ferromagnetic metals (because they are “metals” which screen the gate field), but affects the energy states in the semiconductor (and insulator). We label the gate potential $V_g$ and the corresponding potential shift that it causes in the semiconductor conduction band states is called $V_g'$. The energy band diagram corresponding to the situation when the lowest
subband in the dot aligns with the Fermi level is shown in Fig. 1(c). For this case, the gate potential shift $V'_g = \phi_{ms} + \Delta$. We call this value of $V'_g$ ($V_g = V_{g1}$). When $V_g = V_{g1}$, it becomes energetically possible for an electron to jump into the lowest subband in the dot from either contact. The dot occupancy now changes from 0 to 1.

Once an electron occupies the dot, it repels a second electron from coming into the dot because of Coulomb interaction. For the second electron, the lowest available energy state appears to be the level shown by the broken line in Fig. 1(c) which is $e/2C$ ($C =$ capacitance of the dot) above the lowest subband energy. We have to increase $V'_g$ by an additional $e/2C$ to pull the levels down enough so that the broken line is aligned with the Fermi level in the source as shown in Fig. 1(d). We call this value of $V'_g = V'_{g2}$ and the corresponding $V_g = V_{g2}$. Obviously, $V'_{g2} = \phi_{ms} + \Delta + e/2C$. When $V_g = V_{g2}$, a second electron can enter the dot and occupy it. Pauli Exclusion Principle dictates that this electron must have its spin anti-parallel to that of the first electron since both electrons are occupying the same lowest subband of the quantum dot.

A plot of electron occupancy versus the gate voltage is shown in Fig. 2(a). If we increase the gate voltage further, beyond $V_{g2}$, ultimately, we will pull the second subband level below the Fermi level. Thereafter, more than two electrons can occupy the dot, but we shall not explore that region.

**Polarized spin**: Now assume that the ferromagnetic contacts are taken to their saturation magnetization so that the electrons in them are spin polarized. Furthermore, assume that the spin polarization is 100% (the ferromagnets are essentially half-metallic). Hence, every electron that enters the dot from the contact has the same spin.

The first electron still enters the dot at $V_g = V_{g1}$, but the second electron cannot enter the dot at $V_{g2}$. This is because this electron has the same spin as the first, and hence cannot co-exist in the lowest subband with the first electron because of Pauli Exclusion. In fact, the second electron can enter the dot only when the gate voltage shift $V'_g = \phi_{ms} + \Delta + \Delta' + e/2C$. We call this value of $V'_g$ ($V_g = V_{g3}$). The energy band diagram for this situation is shown in Fig. 1(e). The charging diagram (or dot occupancy versus gate voltage) for spin polarized
electrons is shown in Fig. 2(b).

**Transport: unpolarized spins** So far, we have assumed that no potential is applied across the source and drain contacts ($V_{SD} = 0$), so that no current flows. We were merely changing the occupancy of the dot by a gate potential. Now, let us assume that we apply a small voltage $V_{SD}$ between source and drain contacts to induce transport, so that a non-zero current can flow.

The energy band diagrams at different gate voltages $V_{ga}$, $V_{gb}$, $V_{gc}$, $V_{gd}$ and $V_{ge}$ are shown in Fig. 3 for a fixed value of $V_{SD}$.

When $V_g = V_{ga}$, no current can flow since the intermediate state in the quantum dot is not energetically accessible from the source.

When $V_g = V_{gb}$, a current will flow at any $V_{SD}$ because the intermediate state in the dot has become accessible from the source. Furthermore, the drain is also accessible from the intermediate state. This is true for any non-zero value of $V_{SD}$.

Now consider the situation in Fig. 3(c) when $V_g = V_{gc}$. The intermediate state is accessible from the source, but the drain is not accessible from this intermediate state unless $V_{SD} \geq V_t$. If $V_{SD} < V_t$, then the Fermi level in the drain is above the subband level in the dot and therefore electron cannot flow from the dot to the drain. Consequently, there will be a threshold behavior in the current-voltage characteristic. The current voltage characteristic for $V_{gb}$ and $V_{gc}$ are shown in Fig. 4(a).

Then, if we increase $V_g$ beyond $V_{ge}$, ultimately the Coulomb repelled level (shown by the broken line) will align with the Fermi level in the source contact (see Fig. 3(d)). We call this value of $V_g = V_{gd}$. Now the second electron can come in from the source into the dot and escape to the drain no matter how small $V_{SD}$ is. The first electron is still blocked for $V_{SD} < V_t$, but this does not matter since the second electron (and all following electrons) can cause current flow. Thus, the threshold behavior disappears at $V_{gd}$.

It is also obvious that the maximum value of $V_t$ is $e/2C$ and is reached just before the gate voltage reaches $V_{gd}$. The dependence of the threshold voltage on the gate voltage is shown in Fig. 4(b).
Fig. 4(b) is valid when the two electrons have opposite spins. Pauli Exclusion would have prevented the second electron from coming into the dot at \( V_g = V_{gd} \), if both electrons had the same spin.

**Transport: spin polarized electrons**. Next, we consider the situation when both electrons have the same spin. This corresponds to the case when the ferromagnets are magnetized and only one kind of spin can enter from the contacts. In this case, electron 1 and electron 2 will have the same spin (unless one flips a spin by scattering or because of user intervention).

When both electrons have parallel spins, the threshold behavior will not disappear until the gate voltage is much larger than \( V_{gd} \), and is, in fact, equal to \( V_{ge} \) as shown in Fig. 3(e). At this gate voltage, the second subband level aligns with the Fermi level in the source so that a second electron of the same spin as the first can enter the dot. This electron comes into the second subband since the first subband is not available by virtue of the Pauli Exclusion Principle.

The dependence of \( V_t \) on \( V_g \) for the same spin case is shown in Fig. 4(c). It is obvious that the maximum value of \( V_t \) in this case is \( \Delta'/\epsilon + e/2C \).

**Measurement of single spin**: One can now see how it is possible to measure a single electron spin. Our target spin is that of electron 1 and our “scout spin” is that of electron 2. The scout spin comes in from a magnetized ferromagnetic contact and hence its spin is known. By measuring the threshold behavior and discriminating between the cases shown in Fig. 4(b) and 4(c), we can tell if the target spin is parallel or anti-parallel to the scout spin. Hence we can determine the orientation of the target spin. Note that the target spin remains trapped in the quantum dot and the scout spin goes out to the contact to cause a current. Thus, we can determine the target spin somewhat non-invasively without pushing it out to the contact where it will be lost irretrievably.

**Candidate system**: One can synthesize the penta-layered structure of Fig. 1(a) by sequentially electrodepositing Fe, ZnSe, GaAs, ZnSe and Fe within the pores of a nanoporous
alumina film produced by the anodization of aluminum in sulfuric acid [9]. We have pro-
duced such structures in the past. Absorption and Raman spectroscopy have independently
determined that the subband spacing in these dots is about 500 meV [10, 11]. Coulomb
blockade experiments in these structures have shown that the capacitance of the semicon-
ductor layer can be about 0.5 aF, leading to a single electron charging voltage $e/2C = 160$
mV [12]. Therefore, we can attain the condition $\Delta' > e/2C > kT/e$ at $T = 77$ K. One
can also selectively contact a few (about 10) wires by relatively large area contacts of 100
$\mu$m $\times$ 100 $\mu$m by exploiting a feature of electrochemical synthesis that results in wires of
non-uniform height [12]. Therefore using 30 $\mu$m $\times$ 30 $\mu$m sized contact pads (easily made
by standard photolithography), one can hope to contact a single wire and make the mea-
surements described in this paper. The gate potential can be applied by a remote gate that
is located far away from the source and drain contacts. This structure is in fact identical to
the structure proposed for a universal quantum gate in ref. [6]. Hence, it can be easily used
as a reader of qubit in that structure.

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**Fig. 1:** (a) A penta-layered quantum wire structure of transverse dimension $\sim 10$ nm. The thicknesses of the constituent layers are shown. FM = ferromagnet, I = insulator and SC = semiconductor. (b) Equilibrium energy band diagram along the length of the wire. $\phi_{ms}$ is the metal semiconductor work function difference, $\Delta$ is the quantization energy of the first subband and $\Delta'$ is the difference between the quantization energies of the second and first subband. (c) Energy band diagram when a gate potential $V_{g1}$ pulls the semiconductor (and insulator) conduction band down to make the lowest subband level in the semiconductor quantum dot align with the Fermi level. A single electron can now occupy the dot. (d) Energy band diagram when the gate potential $V_{g2}$ pulls the quantum dot levels down by an additional $e/2C$ ($C$ = dot capacitance). Now two electrons can occupy the dot *if they have opposite spins*. (e) Energy band diagram when the second subband is pulled down flush with the Fermi level by gate potential $V_{g3}$. Now two electrons of the *same spin* can occupy the dot.

**Fig. 2:** Charging diagram (dot occupancy versus gate voltage) for (a) spin-unpolarized electrons and (b) spin-polarized electrons.

**Fig. 3:** Electron energy band diagram at different gate voltages (a) $V_g = V_{ga}$ when no current can flow since the intermediate dot state is not accessible from the source. (b) $V_g = V_{gb}$ when the first subband lines up with the Fermi level in the source. Now the intermediate state is accessible from the source and the drain is accessible from this intermediate state. Hence, a current can flow at *any* value of $V_{SD}$. (c) $V_g = V_{gc}$ when the first subband level dips below the Fermi level at the source. At this point the intermediate state is accessible from the source, but the drain is not accessible from this intermediate state unless $V_{SD}$ exceeds a certain threshold value. (d) $V_g = V_{gd}$ when the Coulomb repelled level (broken line) lines up with the Fermi level in the source. In this case, current can flow when $V_{SD} > e/2C$ so that the threshold voltage $V_t = e/2C$. This is the maximum value of the threshold voltage. If we increase the gate voltage further, the second electron can come in from the source and conduct current at any $V_{SD}$ thereby collapsing the threshold behavior. This will happen only if the two electrons have opposite spins so that Pauli blockade is not operative. If the
second electron has the same spin as the first, it cannot come into the dot because of Pauli exclusion and hence the threshold behavior will continue and not collapse. (e) $V_g = V_{ge}$ when the second subband level lines up with the Fermi level in the source. Now the second electron can enter the dot even if it has the same spin as the first because it is not occupying the same subband. At this point, the threshold behavior will collapse even if the electrons have parallel spins.

**Fig. 4:** (a) Current voltage characteristic showing a threshold behavior. Threshold voltage $V_t$ versus gate voltage $V_g$ when (b) the electrons have anti-parallel spins, and (c) the electrons have parallel spins.
Gate voltage ($V_g$)

Electron number

(a)

(b)
\[ V_g = V_{ga} \]

(a)

\[ V_g = V_{gb} \]

(b)

\[ V_g = V_{gc} \]

(c)

\[ V_g = V_{gd} \]

(d)

\[ V_g = V_{gd} \]

(e)

\[ V_g = V_{ge} \]

(f)

for polarized spin

for unpolarized spin
