Using the finite element method and dimension analysis in modelling laser shock processing of titanium alloys with shape memory

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Abstract. The technology of laser shock processing of titanium alloys with the shape memory effect is simulated using the finite element method and dimensional analysis. In total, 16 independent dimensionless parameters characterizing the mechanical behavior of shape-memory NiTi alloys are analyzed. The dependences of the influence of the dimensionless duration of the laser pulse and the dimensionless peak pressure in the shock wave on the dimensionless depth of the plastic zone are obtained.

Keywords: finite element simulation, shape memory alloys, laser shock processing, dimension analysis, plastic zone depth

1. Introduction
The shape memory effect in alloys is based on thermoelastic diffusion-free martensitic transformations. This effect is not the only unusual feature of that these alloys. In addition to the shape memory effect (SME), the phenomenon of pseudoeelasticity (superelasticity) is also of the greatest practical interest, in which plastic deformation, reaching tens of percent, is restored after the load is removed [1]. The general scheme of the laser-shock-processing (LSP) technology used in this work and the occurrence of compressive residual stresses (CRS) are shown in Figure 1. The surface to be treated (Fig. 1a, item 5) is covered with an absorbing layer with a low evaporation temperature: black paint, metal foil or tape (3). On top is a transparent layer (2), the role of which is most often performed by water or glass. The energy of the laser pulse (1) is absorbed by an absorbing layer, which leads to its heating, evaporation and the formation of high-temperature plasma, limited on one side by the surface of the material under study, and on the other by a transparent layer that does not allow the plasma to propagate. Due to the limited volume, the pressure increases sharply to high values and quickly propagates into the material in the form of a shock wave (4), which contributes to the occurrence of compressive residual stresses in the material. If these stresses exceed the elastic limit of Hugonio, the material is deformed plastically [5]. In more detail, the physical and mechanical processes occurring in LSP are described in [4].
Laser shock processing of materials (LSP) is a technology with great potential for processing shape memory alloys (SPA) and creating localized deformation-induced martensitic structures in them for specific practical applications [2].

2. Problem Statement
In this paper, we study the urgent scientific problem id est the analysis of the physical processes occurring in NiTi alloys with the shape memory effect (SME) subjected to LSP, using the finite element method and the method of dimension analysis. The main parameters that control all the main processes during LSP are the peak pressure in the shock wave $P_m$, the pulse duration $\tau$, and the radius of the laser spot $R$ [2].

3. Theory
To analyze the physical processes occurring in NiTi alloys with the shape memory effect subjected to LSP, the method of dimension analysis was used [3]. The main parameters that control all the main processes during LSP are the peak pressure in the shock wave $P_m$, the pulse duration $\tau$, and the radius of the laser spot $R$ [2].

It is known [1] that there are only 11 basic parameters that completely determine the mechanical behavior of the shape-memory alloy material. It is the elastic modulus of austenite $E_A$, the elastic modulus of martensite $E_M$, material density $\rho$, Poisson's ratio $\nu$, the initial strain of the direct martensitic transformations $\sigma_{SA}^{AM}$, the final stress of the direct martensitic transformation $\sigma_{SA}^{AM}$, the initial stress of the reverse transformation $\sigma_{MA}^{SM}$, the final stress of the reverse transformation $\sigma_{MA}^{SM}$, module $E_r$ of the strain hardening, the yield strength of martensite $\sigma_{SM}^{AM}$ and flexibility modulus $E_f$.

As a rule, during LSP, the thickness of the test sample is quite large compared to the LSP zone of influence, and, accordingly, the thickness can be considered infinitely large. Therefore, we can assume that the main influence of the LSP on the SME is characterized by the depth of the plastic zone along the axis of the laser beam $L_p$ and the surface residual stress in the center of the LSP $\sigma_m$, which, in turn,
are functions of the control parameters that characterize the laser effect and the characteristics of the material:

\[ L_P = f_1(P_m, \tau, R, E_A, E_M, E_{tr}, \rho, \nu, \sigma_s^{AM}, \sigma_f^{AM}, \sigma_s^{MA}, \sigma_f^{MA}, \sigma_s^M, \sigma_f^M, \gamma, b_A, c_A, b_M, c_M) \] (1)

\[ \sigma_m = f_2(P_m, \tau, R, E_A, E_M, E_{tr}, \rho, \nu, \sigma_s^{AM}, \sigma_f^{AM}, \sigma_s^{MA}, \sigma_f^{MA}, \sigma_s^M, \sigma_f^M, \gamma, b_A, c_A, b_M, c_M) \] (2)

In the finite element LSP simulation for the SPF, a special user-defined subroutine (VUMAT), available in ABAQUS/Explicit, was used, which allows the use of the generalized Lubliner-Auricchio plastic model [4]. The characteristics of the shape memory alloy under study are given in Table 1 [5].

**Table 1.** Physical and mechanical characteristics of NiTi alloys with shape memory

| Material characteristics, (unit) | Value |
|----------------------------------|-------|
| Young's austenite modulus, \( E_A \) (hPa) | 80 |
| Young's martensite modulus, \( E_M \) (hPa) | 40 |
| Poisson's ratio, \( \nu \) | 0.33 |
| Density, \( \rho \) (g / cm\(^3\)) | 6.45 |
| Initial stress of direct martensitic transformation, \( \sigma_s^{AM} \) (MPa) | 500 |
| Final stress of direct martensitic transformation, \( \sigma_f^{AM} \) (MPa) | 550 |
| Initial stress of reverse martensitic transformation, \( \sigma_s^{MA} \) (MPa) | 300 |
| Final stress of reverse martensitic transformation, \( \sigma_f^{MA} \) (MPa) | 250 |
| Modulus of strain hardening, \( E_{tr} \) (hPa) | 2.86 |
| Yield stress of martensite, \( \sigma_f^M \) (MPa) | 1100 |
| Modulus of plasticity, \( E_p \) (hPa) | 4.0 |
| Speed of sound in austenite, \( c_A \) (m / s) | 5.12 \( \cdot 10^3 \) |
| Speed of sound in martensite, \( c_M \) (m/s) | 3.56 \( \cdot 10^3 \) |
| The Mi-Gruneisen constant of austenite, \( b_A \) | -3.88 |
| The Mi-Gruneisen martensite constant, \( b_M \) | 4.87 |
| Adiabatic exponent, \( \gamma \) | 2.0 |

The Mi-Gruneisen coefficients are taken from [1]. The corresponding dimensionless parameters remain constant in numerical simulation; they are shown in Table 2.

**Table 2.** Dimensionless parameters of NiTi alloys with shape memory

| Dimensionless parameter | \( E_A \) \( / \) \( E_M \) | \( E_{tr} \) \( / \) \( E_M \) | \( E_{tr} \) \( / \) \( E_M \) | \( \sigma_s^{AM} \) \( / \) \( E_A \) | \( \sigma_f^{AM} \) \( / \) \( E_M \) | \( \sigma_s^{MA} \) \( / \) \( E_M \) | \( \sigma_f^{MA} \) \( / \) \( E_A \) | Value |
|------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-------|
| Value                  | 2.00            | 0.071           | 0.71            | 6.25 \( \cdot 10^3 \) | 1.38 \( \cdot 10^2 \) | 7.50 \( \cdot 10^3 \) | 3.12 \( \cdot 10^3 \) |       |
| Dimensionless parameter | \( \sigma_s^{AM} \) \( / \) \( E_M \) | \( c_A \) \( / \) \( \sqrt{E_A / \rho} \) | \( c_M \) \( / \) \( \sqrt{E_M / \rho} \) | \( \gamma \) | \( b_A \) | \( b_M \) | \( \nu \) | Value |
| Value                  | 2.75 \( \cdot 10^2 \) | 1.45            | 1.43            | 2.0             | -3.88          | 4.87           | 0.33           |       |
3.1. The finite element model

To simulate LSP process, a two-dimensional axisymmetric finite element model is developed for the SPF, which uses dimensionless coordinates $r^* = r/R$ and $y^* = y/R$, where $R$ is the radius of the laser spot, and $P$ is the pressure in the shock wave (Fig. 2). Since LSP is a highly localized process and the size of the laser-treated area is very small compared to the size of the sample under study itself, it is natural that smaller finite elements (1) are used to model the LSP zone than for the rest of the sample (2), where, as usual, rather coarse elements are used. The size of the finer mesh depends on the size of the laser spot. Usually, researchers choose the size of the laser spot, which is from 2 to 3.5 times larger than the size of the element, for a smaller cell area [6]. In this paper, the triple size of the laser spot is used. The size of the large-cell area is selected to be eight times the size of the laser spot. The model uses elements of (continuous axisymmetric 4-node reduced integrated).

Figure 2. A two-dimensional axisymmetric finite element model of LSP. 1 – fine mesh, 2 – coarse mesh, 3 – axis of symmetry, 4 – rigid fixing, 5 – contact zone of the sample with a material with high damping characteristics for the dissipation of shock wave energy.

The test SME alloy sample is exposed to a dimensionless pressure with a uniform distribution $p(t)/\sigma_y^M$ over the entire LSP area. Usually in LSP, the peak pressure of the laser shock wave is at the level of several hPa, and the duration of the shock wave is measured on a nanosecond scale. Although the pressure-time dependence usually has a Gaussian distribution profile, it is very close to a triangular shape due to the very short pulse duration (Fig. 3). Therefore, in this paper, it is assumed that the pressure has a triangular profile. The pressure in the shock wave increases linearly to the peak (maximum) pressure $P_m$ and then decreases linearly to zero over the period $\tau$. 
Figure 3. Graph of the dependence of the pressure in the shock wave on the time at LSP

4. Obtained simulation results and discussion
Let us analyze the effect of the laser pulse duration and the peak pressure in the shock wave on the depth of the plastic zone. Figure 4a shows the relationship between the dimensionless depth of the plastic zone \( \left( \frac{L_P}{R} \right) \) and the dimensionless duration of the laser radiation \( \left( \frac{\tau}{\sqrt{\frac{E_M}{\rho}}} \right) \) at various dimensionless peak pressures in the shock wave \( \left( \frac{P_m}{\sigma_M} \right) \). As we can see, while the peak pressure is less than the dimensionless value 7.27, the depth of the plastic zone increases linearly with increasing pulse duration (this pattern stays the same at different peak pressures), which is consistent with similar trends in traditional metals [7]. But when the peak pressure exceeds the value of 7.27, the depth of the plastic zone with an increase in the pulse duration manifests weakly nonlinear characteristics.

![Graph showing the relationship between dimensional depth of plastic zone and pulse duration](image)

**Figure 4.** (a) The effect of the dimensionless duration of the laser pulse on the dimensionless depth of the plastic zone at different peak pressures in the shock wave:

\[ P_m/\sigma_M = 2.73 (\bullet), 3.64 (\bullet), 4.09 (\triangle), 4.55 (\downarrow\triangledown), 5.45 (\triangle), 7.27 (\blacklozenge), 9.09 (\blacklozenge); \]

(b) The effect of the dimensionless peak pressure in the shock wave \( P_m/\sigma_M \) on the dimensionless depth of the plastic zone \( L_P/\left( \sqrt{E_M/\rho} \cdot \tau \right) \) at different dimensionless pulse durations

\[ \xi = \frac{\tau}{\left( \sqrt{E_M/\rho} \right)}; \]

\[ \xi = 0.0025 (\bullet), 0.0142 (\bullet), 0.0225 (\triangle), 0.0450 (\downarrow\triangledown), 0.0632 (\blacklozenge), 0.0825(\blacklozenge), 0.1000 (\blacklozenge). \]

To the left of the dotted line (where \( P_m/\sigma_M = 7.27 \)) we have an almost linear dependence, to the
right – a nonlinear one. The dimensionless material parameters stay constant; they are presented in Table 2.

Figure 4b shows the relationship between the dimensionless depth of the plastic zone and $L_P/\left(\sqrt{\frac{E_M}{\rho}} \cdot \frac{\tau}{\sigma_y}\right)$ the dimensionless peak pressure in the shock wave $P_m/\sigma_y^N$ at different dimensionless pulse durations. It can be seen that the dimensionless plastic zone $L_P/\left(\sqrt{\frac{E_M}{\rho}} \cdot \frac{\tau}{\sigma_y}\right)$ increases almost linearly with an increase in the dimensionless peak pressure, until the peak pressure in the shock wave is less than 7.27, and then the dependences become strongly nonlinear.

5. Conclusion

It is shown that the depth of the plastic zone increases linearly with the increase in the pulse duration and the peak pressure in the shock wave, as long as the peak pressure is less than the dimensionless value 7.27. When the peak pressure exceeds 7.27, the depth of the plastic zone shows non-linear characteristics. Presumably, such two-stage characteristic can be caused by the interaction of compressive waves arising under LSP and release waves reflected from the edges of the sample. These features should be taken into account in experimental studies during the application of LSP to SME alloys.

6. References

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