THERMODYNAMIC PROPERTIES OF THE PHASE TRANSITIONS IN
A CLASS OF SPIN-TRIPLET FERROMAGNETIC
SUPERCONDUCTORS

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Abstract

Magnetic susceptibility, entropy and specific heat are calculated at the equilib-
rium points of phase transition to a phase of coexistence of ferromagnetic order
and superconductivity in a new class of spin-triplet ferromagnetic superconductors.
The results are discussed in view of application to metallic ferromagnets as UGe$_2$,
ZrZn$_2$, URhGe.

1. Introduction

Recently, a spin-triplet superconducting phase has been discovered in a class of new
ferromagnetic superconductors (UGe$_2$ [1], ZrZn$_2$ [2], URhGe [3]). In these metallic com-
ounds the superconductivity exists only in the domain of stability of the ferromagnetic
order. Moreover, the ferromagnetic order enhances the superconductivity because of the
unconventional Cooper pairing with spin S = 1 [4-7]. A phenomenological model [7] of
Ginzburg–Landau (GL) type has been used to describe the possible ordered phases and
the phase diagram of these metallic compounds [8,9].

In this paper we present the results of our calculation of magnetic susceptibility, entropy
and specific heat near the phase transitions in ferromagnetic superconductors with a
spin-triplet superconductivity. For our aims we shall use the GL free energy introduced
in Ref. [7] and results published in Refs. [8,9].

2. Model and phase diagram

We consider the GL free energy [7-9] $F = \int d^3 x f(\psi, \vec{M})$, where

$$f = \frac{\hbar^2}{4m} (D_\mu \psi)^* (D_\mu \psi) + a_s |\psi|^2 + \frac{b}{2} |\psi|^4 + a_f \vec{M}^2 + \frac{\beta}{2} \vec{M}^4 + i\gamma_0 \vec{M} \cdot (\psi \times \psi^*) \quad . \quad (1)$$
In Eq. (11), $D_j = (\nabla - 2ieA_j/\hbar c)$, and $A_j$ ($j = 1, 2, 3$) are the components of the vector potential $\vec{A}$ related with the magnetic induction $\vec{B} = \nabla \times \vec{A}$. The complex vector $\psi = (\psi_1, \psi_2, \psi_3) \equiv \{\psi_j\}$ is the superconducting order parameter, corresponding to the spin-triplet Cooper pairing and $\vec{M} = \{\mathcal{M}_j\}$ is the magnetization. The coupling constant $\gamma_0 = 4\pi J > 0$ is given by the ferromagnetic exchange parameter ($J > 0$). Coefficients $a_s = \alpha_s(T - T_s)$ and $a_f = \alpha_f(T - T_f)$ are expressed by the positive parameters $\alpha_s$ and $\alpha_f$ as well as by the superconducting ($T_s$) and ferromagnetic ($T_f$) critical temperatures in the decoupled case, when $\mathcal{M}_i\psi_j\psi_j$-interaction is ignored; $b > 0$ and $\beta > 0$ as usual.

We assume that the magnetization $\mathcal{M}$ is uniform, which is a reliable assumption outside a quite close vicinity of the magnetic phase transition but keep the spatial ($\vec{x}$-) dependence of $\psi$. The reason is that the relevant dependence of $\psi$ on $\vec{x}$ is generated by the diamagnetic effects arising from the presence of $\mathcal{M}$ and the external magnetic field $\vec{H}$ [8,9] rather than from fluctuations of $\psi$ (this effect is extremely small and can be ignored). First term in (1) will be still present even for $\vec{H} = 0$ because of the diamagnetic effect created by the magnetization $\vec{M} = \vec{B}/4\pi > 0$. As we shall investigate the conditions for the occurrence of the Meissner phase where $\psi$ is uniform, the spatial dependence of $\psi$ and, hence, the first term in r.h.s. of (11) will be neglected. This approximation should be valid when the lower critical field $H_{cl}(T)$ is greater than the equilibrium value of the magnetization $\mathcal{M}$ in the phase of coexistence of superconductivity and ferromagnetism.

We take an advantage of the symmetry of model (11) and avoid the consideration of equivalent thermodynamic states that occur as a result of the respective continuous symmetry breaking at the phase transition point but have no effect on thermodynamics of the system. That is why we shall assume for concreteness of our calculation that the magnetization vector is along the $z$-axis: $\vec{M} = (0, 0, \mathcal{M})$, where $\mathcal{M} \geq 0$. This concrete choice of the direction of the magnetization vector does not restrict the generality of the present analysis and leads to the same structure of the ordered phases as previously predicted and discussed on the basis of general symmetry group considerations in Ref. [7].

We find convenient to use the following notations: $\varphi_j = b^{1/4}\psi_j$, $\varphi_j = \phi_j\exp(\theta_j)$, $M = \beta^{1/4}\mathcal{M}$, $\gamma = \gamma_0/(b^2\beta)^{1/4}$, $r = a_s/\sqrt{\beta}$, $t = a_f/\sqrt{\beta}$, $\phi^2 = (\phi_1^2 + \phi_2^2 + \phi_3^2)$, and $\theta = (\theta_2 - \theta_1)$. We consider the stable homogeneous phases in a zero external magnetic field ($\vec{H} = 0$) that are described by uniform order parameters $\mathcal{M}$ and $\psi$. These phases are three: (i) the normal phase ($\phi = \mathcal{M} = 0$), hereafter referred as “N”, (ii) the ferromagnetic phase ($\phi = 0, \mathcal{M} > 0$), hereafter referred as “FM”, and (iii) the phase of coexistence of superconductivity and ferromagnetism ($\phi > 0, \mathcal{M} > 0$), which will be called “FS”-phase. The standard N-FM phase transition occurs at $T_f$. The N-FM, N-FS and FM-FS phase transitions are shown in Fig. 1 [8,9]. The r-axis above the point $C$ coincides with the line of the N-FM phase transition of second order. The dashed lines correspond to phase transitions of second order: FM-FS transition for $t < (-\gamma^2/4)$ and a N-FS transition for $t > \gamma^2/2$. The solid lines $BC$ and $CA$ describe first order FM-FS and N-FS transitions, respectively. The line of circles $BCA$ defines the borderline of stability and existence of
Figure 1: The phase diagram in the plane $(t, r)$ with two tricritical points ($A$ and $B$) and a triple point $C$; $\gamma = 1.2$. Second order transition lines are dashed, first order transition line is solid.
FS. In the domain confined between the line of circles and the dashed line on the left of the point $B$ the stability condition for FS is satisfied but the existence condition is broken. The phase diagram contains two tricritical points ($A$ and $B$) and a triple point ($C$). In the domain between the solid lines $BC$ and $CA$ and the line of circles $BCA$, FS can be overheated. Domains of overcooling of phases FM and N below the solid lines $BC$ and $CA$ also exist [8,9]. For example, the domain below the solid line $CA$ and above the positive axes is a region of overcooling of the N-phase.

3. Magnetic susceptibility

The magnetic susceptibility $\chi = (\chi/V) = d^2 f(M)/dM^2$ per unit volume ($V$) in a zero external magnetic field is obtained in the general form

$$
\chi^{-1} = 2t + 6M^2 - \left(\frac{\partial \phi^2}{\partial M}\right)^2,
$$

(2)

where the equilibrium magnetization $M$ should be taken for the respective equilibrium phase: (a) $M = 0$ in the N-phase, (b) $M = \sqrt{|t|}$ in FM, and (c) in FS, $M$ is given as the maximal nonnegative root of the equation [8,9].

$$
\frac{\gamma r}{4} = \left(\frac{\gamma^2}{2} - t\right) M - M^3.
$$

(3)

We shall compare the known paramagnetic ($\chi_P = 1/2t; t > 0$) and the ferromagnetic ($\chi_F = 1/4|t|; t < 0$) susceptibilities with that corresponding to FS. Note, that both $\chi_P$ and $\chi_F$ are divergent on the N-FM phase transition line. While the magnetic susceptibility in phases N and FM and the behaviour of the same quantity at the N-FM phase transition do not depart from the standard predictions [6] we shall see that the phase transitions from FS to N or FM show quite special magnetic properties. As $\chi$ cannot be analytically calculated for the whole domain of stability of FS, we shall consider the close vicinity of the N-FS and FM-FS phase transition lines.

Near the second order phase transition line on the left of the point $B$ ($t < -\gamma^2/4$), the magnetization has a smooth behaviour and the magnetic susceptibility does not exhibit any singularities (jump or divergence). For $t > \gamma^2/2$, the magnetization is given by $M = (s_- + s_+)$, where

$$
s_\pm = \left\{-\frac{\gamma r}{4} \pm \left[\frac{(t - \gamma^2/2)^3}{27} + \left(\frac{\gamma r}{4}\right)^2\right]^{1/2}\right\}^{1/3}.
$$

(4)

For $r = 0$, $M = 0$, whereas for $|\gamma r| \ll (t - \gamma^2/2)$ and $r = 0$ one may obtain $M \approx -\gamma r/(2t - \gamma^2) \ll 2t$. This means that in a close vicinity ($r < 0$) of $r = 0$ along the second order phase transition line ($r = 0, t > \gamma^2$) the magnetic susceptibility is well described by the paramagnetic law $\chi_P = (1/2t)$. For $r < 0$ and $t \to \gamma^2/2$, we obtain $M = -(\gamma r/2)^{1/3}$ which yields

$$
\chi^{-1} = 6 \left(\frac{\gamma |r|}{2}\right)^{2/3}.
$$

(5)
On the phase transition line \( AC \) we have [8,9]

\[
M_{eq}(t) = \frac{1}{2\sqrt{2}} \left[ \gamma^2 - 8t + \gamma (\gamma^2 + 16t)^{1/2} \right]^{1/2}
\]

and, hence,

\[
\chi^{-1}(t) = -4t - \frac{\gamma^2}{4} \left[ 1 - 3 \left( 1 + \frac{16t}{\gamma^2} \right)^{1/2} \right]. \tag{7}
\]

At the tricritical point \( A \) this result yields \( \chi^{-1}(A) = 0 \), whereas at the triple point \( C \) with coordinates \((0, \gamma^2/4)\) we have \( \chi(C) = (2/\gamma^2) \). On the line \( BC \) we obtain \( M = \gamma/2 \) [8,9] and, hence,

\[
\chi^{-1}(t) = 2t + \frac{\gamma^2}{2}. \tag{8}
\]

At the tricritical point \( B \) with coordinates \((-\gamma^2/4, \gamma^2/2)\) this result yields \( \chi^{-1}(B) = 0 \).

The comparison of Eqs. (7) and (8) with \( \chi_P \) corresponding to the N-phase and with \( \chi_F \) corresponding to FM shows that the magnetic susceptibility undergoes finite jumps at the first order N-FS and FM-FS transitions. The finite jumps \( \Delta \chi_P \) and \( \Delta \chi_F \) can be easily calculated with the help of \( \Delta \chi_{P,F} = (\chi - \chi_{P,F}) \) and Eqs. (7) and (8). In particular, the jump of \( \chi \) at the triple point \( C \) is infinite because of the divergency of \( \chi_F \) in the limit \( t \to 0^+ \).

4. Entropy and specific heat

The entropy \( S(T) \equiv \langle \tilde{S} / V \rangle = -V \partial \langle f / \partial T \rangle \) and the specific heat \( C(T) \equiv \langle \tilde{C} / V \rangle = T \langle \partial S / \partial T \rangle \) per unit volume \( V \) are calculated in a standard way [6]. We are interested in the jumps of these quantities on the N-FM, FM-FS, and N-FS transition lines. The behaviour of \( S(T) \) and \( C(T) \) near the N-FM phase transition and near the FM-FS phase transition line of second order on the left of the point \( B \) (Fig. 1) is known from the standard theory of critical phenomena (see, e.g., Ref. [6] and for this reason we focus our attention on the phase transitions of type FS-FM and FS-N for \( t > -\gamma^2/4 \), i.e., on the right of the point \( B \) in Fig. 1.

Using the equations for the order parameters \( \psi \) and \( M \) [8,9] and applying the standard procedure for the calculation of \( S \), we obtain the general expression

\[
S(T) = -\frac{\alpha_s}{\sqrt{b}} \phi^2 - \frac{\alpha_f}{\sqrt{\beta}} M^2. \tag{9}
\]

The next step is to calculate the entropies \( S_{FS}(T) \) and \( S_{FM} \) of the ordered phases FS and FM. Note, that we work with the usual convention \( F_N = V f_N = 0 \) for the free energy of the N-phase and, hence, we must set \( S_N(T) = 0 \).

Consider the second order phase transition line \((r = 0, t > \gamma^2/2)\). Near this line \( S_{FS}(T) \) is a smooth function of \( T \) and has no jump but the specific heat \( C_{FS} \) has a jump at \( T = T_s \), i.e. for \( r = 0 \). This jump is given by

\[
\Delta C_{FS}(T_s) = \frac{\alpha_s^2 T_s}{b} \left[ 1 - \frac{1}{1 - 2t(T_s)/\gamma^2} \right]. \tag{10}
\]
The jump $\Delta C_{FS}(T_s)$ is higher than the usual jump $\Delta C(T_c) = T_c \alpha^2/b$ known from the Landau theory of standard second order phase transitions [6].

The entropy jump $\Delta S_{AC}(T) \equiv S_{FS}(T)$ on the line $AC$ is obtained in the form

$$\Delta S_{AC}(T) = -M_{eq} \left\{ \frac{\alpha_s \gamma}{4 \sqrt{b}} \left[ 1 + \left( 1 + \frac{16t}{\gamma^2} \right)^{1/2} \right] - \frac{\alpha_f \sqrt{\beta} M_{eq}}{\gamma} \right\},$$

where $M_{eq}$ is given by Eq. (6). From (6) and (11), we have $\Delta S(t = \gamma^2/2) = 0$, i.e., $\Delta S(T)$ becomes equal to zero at the tricritical point $A$. Besides we find from (6) and (11) that at the triple point $C$ the entropy jump is given by

$$\Delta S(t = 0) = -\frac{\gamma^2}{4} \left( \frac{\alpha_s}{\sqrt{b}} + \frac{\alpha_f}{\sqrt{\beta}} \right).$$

On the line $BC$ the entropy jump is defined by $\Delta S_{BC}(T) = [S_{FS}(T) - S_{FM}(T)]$. We obtain

$$\Delta S_{BC}(T) = \left| t \right| \left( \frac{\gamma^2}{4} \right) \left( \frac{\alpha_s}{\sqrt{b}} + \frac{\alpha_f}{\sqrt{\beta}} \right).$$

At the tricritical point $B$ this jump is equal to zero as it should be. The calculation of the specific heat jump on the first order phase transition lines $AC$ and $BC$ is redundant for two reasons. Firstly, the jump of the specific heat at a first order phase transition differs from the entropy by a factor of order of unity. Secondly, in caloric experiments where the relevant quantity is the latent heat $Q = T \Delta S(T)$, the specific heat jump can hardly be distinguished.

5. Concluding remarks

The results obtained in this paper can be used in interpretations of magnetic and caloric experiments in various classes of magnetic superconductors with a spin-triplet Cooper pairing. Our general consideration is irrespective of the ratio between $T_s$ and $T_f$. But one should be aware that depending on the concrete choice of the substance, either $T_s < T_f$ or $T_s \geq T_f$ may happen. For the new class of ferromagnetic compounds, mentioned in Sec. 1, $T_s \ll T_f$, and this must be taken in mind when the present theory is compared with the experiments performed with these metallic compounds. The condition $T_f > T_s$ leads to a reduction of the possible phases and phase transitions. For $T_f > T_s$, the diagram shown in Fig. 1 is reduced to the domain in the plane $(t, r)$ where the inequality $r > (\alpha_s/\alpha_f)t$ is fulfilled. Experimental data for the density of states and, hence, for the parameters $\alpha_s$ and $\alpha_f$ are needed in order to locate the respective part of the plane $(t, r)$. According to the current literature [7] the respective experimental data for the metallic compounds enumerated in the Sec. 1 are not yet available. It is, however, obvious that the metallic compounds with $T_f \gg T_s$ are described by the phase transition lines in the fourth quadrant of the $(t, r)$ plane and, therefore, one may observe second or first order FM-FS transitions depending on the ratios $\alpha_s/\alpha_f$ and $T_s/T_f$.

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