The use of generalized models in the variational formulation of the prediction tasks defining characteristics of composite materials

E L Gusev $^{1,2}$ and V N Bakulin $^3$

$^1$Institute of Oil and Gas Problems of Siberian Branch of the Russian Academy of Sciences, Oktyabrskaya str. 1, 677980, Yakutsk, Russia
$^2$Institute of Mathematics and Informatics of the North-East Federal University, Belinskaya str. 58, 677013, Yakutsk, Russia
$^3$Institute of Applied Mechanics of the Russian Academy of Sciences, Leningradsky pr 7, 125040, Moscow, Russia

Abstract. Within the framework of variational refined statements of inverse problems of forecasting, the questions of development of effective, refined methods of forecasting resource, strength, reliability, durability of structures made of polymeric, composite materials are investigated. Formulated the principle of a plurality of prediction models, whose inclusion in the refined formulation of the inverse task of forecasting opens new and promising possibilities to improve the efficiency of solving problems of forecasting the defining characteristics of structure composites.

In recent decades, in the development of various structures, machines and mechanisms, considerable attention is paid to the problem of creating reliable methods for quantifying the performance of structures made of polymer and composite materials [1-9]. One of the effective approaches for creation of mathematical models of forecasting of durability of composite materials are variational methods.

An effective solution to the problem of long-term forecasting of the determining characteristics of composite materials under the influence of extreme environmental factors is possible if the results of short-term experiments can be identified stable qualitative patterns of behavior of composite materials under these influences. Such stable qualitative regularities are determined by the peculiarities of micro-and macrostructure of specific types of composite materials.

The identification and investigation of such stable qualitative regularities is the basis of a new approach to improving the efficiency of methods for predicting the determining characteristics of composites. The proposed approach is based on more accurate statements of forecasting problems and is developed within the framework of variational principles for solving inverse problems, which for the first time include an assessment of the accuracy of predicted solutions, which allowed to obtain new scientifically sound results and satisfy the required accuracy of the forecast.

Composite materials, as a rule, are constantly under the influence of static and dynamic loads, which are further influenced by extreme environmental factors. In accordance with this, the problem of developing mathematical models, mathematical methods for solving inverse problems of predicting
changes in the durability of composite materials under the influence of operating loads and extreme environmental factors is of considerable relevance.

As a rule, the solution of the problems of resource forecasting, durability of complex composites, is carried out in simplified formulations within the framework of direct forecasting problems, that is with calculations based on the models of durability known up to the parameters. In the framework of the currently solved simplified statements, it is not possible to justify the correctness of the obtained predicted solutions, which may differ significantly from the real dependencies.

Different formulations of forecasting problems can be classified into two large classes: direct and inverse forecasting problems. Direct forecasting problems are associated with calculations based on known up to parameters models of durability. The inverse problem of prediction associated with the development of requirements, conditions of the forecasting problem, to the conditions of the experiments, develop the requirements, conditions to develop models of durability, in which you can ensure the development of the theoretical prediction with the predetermined accuracy is not below the minimum acceptable forecast accuracy \( \gamma^\text{max}_R \).

**Definition 1.** Under the maximum permissible accuracy of the forecast \( \gamma^\text{max}_R \) we understand the maximum permissible deviation of the predicted dependence \( \hat{R}(t) \) of the defining property \( R \) from the real dependence \( R^*(t) \) on the predicted period of time \([T_{\text{min}},T_{\text{max}}]\). In accordance with the introduced definition, the maximum permissible accuracy of the forecast \( \gamma^\text{max}_R \) satisfies the condition

\[
\| \hat{R}(\cdot) - R^*(\cdot) \|_{C} \leq \gamma^\text{max}_R .
\]  

In these designations:

\[
\| \hat{R}(\cdot) - R^*(\cdot) \|_{C} = \max_{t_{\text{min}}, T_{\text{max}}} | \hat{R}(t) - R^*(t) |.
\]  

In accordance with the introduced maximum permissible prediction accuracy, the inverse prediction problem can be formulated in the following statement:

On the basis of short-term tests, conducted on the time interval \([0, T_{\text{min}}]\), that is on the basis of information on the values of the defining property of \( R \) - \( R_1, R_2, ..., R_m \), measured at times \( t_1, t_2, ..., t_m \), to predict the change in the defining property of the composite structure on the interval \([T_{\text{min}}, T_{\text{max}}]\), with an error not exceeding the predetermined maximum permissible accuracy of the forecast \( \gamma^\text{max}_R \) (1).

In accordance with this, the actual problem is the development of effective, refined methods of forecasting resource, reliability, strength, durability of structures made of polymer, composite materials, based on modern achievements in the field of mathematical and computer modeling.

As noted in the works of member-corrrespondent, RAS Yu. S. Urzhumtsev, an effective solution to the problem of forecasting is possible if the results of short-term experiments can be identified stable qualitative patterns of behavior of polymer, composite materials, under the influence of extreme external factors [1]. Such stable qualitative regularities are determined by the features of micro-and macrostructure of specific types of polymer, composite materials. The development of methods for the identification and study of such stable qualitative patterns is the basis for the development of a new approach to improving the efficiency of methods for predicting the determining characteristics of polymer composites.

On the basis of the developed approach in the framework of inverse forecasting problems with the inclusion in the formulation of the forecasting problem of assessing the accuracy of the forecast, scientifically based predictable solutions can be developed within the required accuracy of the forecast. The inclusion of the established new qualitative regularities of the influence of micro-and macrostructural features of polymer, composite materials on the nature of changes in the residual life, durability, in the formulation of the problem allows:

- Significantly clarify the formulation of the forecasting problem ;
- Significantly improve the efficiency and reliability of predicting the residual life, strength, durability of polymer, composite materials.

In General, polymer, composite materials, composite structures can be affected in various combinations simultaneously by several different factors \( F_j \) associated with hardening processes, exposure to solar radiation, moisture saturation, exposure to ultraviolet radiation, exposure to extreme factors, the impact of operational loads both cyclic and non-cyclic nature, etc.

Then, under the assumption that various physical factors have an impact on the polymer composite, independent of the impact of other factors, and the changes caused in the composite are summed up, it can be assumed that a generalized model describing the simultaneous impact of several factors can be represented as:

\[
R = R_0 + \sum_{j=1}^{p} F_j \left( u_{j,1}, u_{j,2}, \ldots, u_{j,k_j}; t \right).
\]

Each of the functions \( F_j \left( u_{j,1}, u_{j,2}, \ldots, u_{j,k_j}; t \right) \), \( j = 1, 2, \ldots, p \), describing the effect of the \( j \)-th factor on the polymer composite can be represented as a series expansion in a certain system of basic functions \( \psi_{kj}(\beta_{kj};t) \), \( k = 1, 2, 3, \ldots \), which most fully characterize the features of the process of increasing the damage to the material under the influence of extreme environmental factors

\[
F_j = \sum_{k=0}^{\infty} \alpha_{kj} \left( u_{j,1}, \ldots, u_{j,k_j}; t \right) \psi_{kj}(\beta_{kj};t),
\]

\( j = 1, 2, \ldots, p \).

In these designations:

\( \alpha_{kj} \left( u_{j,1}, \ldots, u_{j,k_j}; t \right) \), \( \beta_{kj} \left( u_{j,1}, \ldots, u_{j,k_j}; t \right) \), \( j = 1, 2, \ldots, p ; k = 0,1,2, \ldots \)

undefined model parameters describing the impact of the \( j \)-th factor. In the framework adopted as the basic system of basic functions on which to build a decomposition in a number of forecasting models, and introduced a parametric family of models \( \{ R^N \}_{N=1}^{\infty} \) is built the prediction model of optimal complexity, that is, the model with the optimal number of components allowing to solve the problem of prediction with desired accuracy. The problem of constructing a model of optimal complexity is reduced to solving the following extreme problem:

\[
\max_{\gamma_{\max}, R^N(t)} \left| R^N \left( u^N; t \right) - R^* \left( t \right) \right| = \min_{\max_{R^N(t)} \left| R^N \left( u^N; t \right) - R^* \left( t \right) \right|} \max_{\gamma_{\max}, R^N(t)} \left| R^N \left( u^N; t \right) - R^* \left( t \right) \right|.
\]

In this notation: \( N^* \) is the optimal number of model parameters to predict the optimum complexity \( R^{N^*} \); \( u^{N^*} \) - the optimal vector of parameters of the optimal forecasting model of the parametric family corresponding to the parameter \( N \), \( \gamma_{\max} \) the maximum permissible required accuracy of the solution of the prediction problem; \( R^*(t) \) - the real time dependence of the determining properties of the composite (residual life, durability, etc.).

The developed approach is based on the principle of multiplicity of forecasting models. In accordance with this principle, it is assumed that the choice of a multiparameter family of forecasting
models is carried out in such a way that in this multiparameter family there is a model that is most adequate to the real predicted time dependence $R^*(t)$.

$$M = \{ R_n(u^*;t), u^* = (u_1^*, u_2^*, ..., u_n^*) \in U_n, T_{ma} \leq t \leq T_{max} \}_{n=1}^\infty$$  \hfill (6)

The principle of multiplicity of prediction models: If in a given multiparameter family of prediction models $M$ (6) there is a model $R_n^*( (u^*)^*; t )$, the most adequate to the real predicted time dependence $R^*(t)$, then this model in the considered parametric family is a model of optimal complexity:

$$J \left( R_n^* \left( (u^*)^*; t \right) \right) = \min_{1 \leq n \leq \infty} J \left( R_n \left( u^*; t \right) \right).$$  \hfill (7)

In these designations:
- $u^n$ - the parameter vector of the prediction model of the parametric family corresponding to the parameter value $n$: $u^n = (u_1^n, u_2^n, ..., u_n^n)$;
- $J \left( R_n(u^*;t) \right)$ - estimation of the degree of deviation of this model $R_n(u^n;t)$, a multiparameter family corresponding to the value of the parameter $n$ ($1 \leq n < \infty$), from the real time dependence of the defining property $R^*(t)$;
- $R_n^*(u^*;t)$ - prediction model of a multiparameter family of optimal complexity;
- $(u^*)^*$ - the parameter vector of the prediction model of optimal complexity corresponding to the optimal number of parameters equal to $n^*$.

On the basis of the developed generalized forecasting models in the refined variational formulations of inverse forecasting problems, on the basis of the introduced principle of multiplicity of forecasting models, the estimation of the forecast error allowed when using residual resource forecasting models, durability with a small number of parameters, such as the V. N. Bulmanis model (the number of parameters does not exceed four) is carried out [9]. On the basis of computational experiments it is shown that the use of forecasting models with a small number of parameters may not be adequate to the complexity of the problem and lead to significant prediction errors.

Thus, the application of the principle of multiplicity of forecasting models and models of optimal structure and optimal complexity in the framework of refined variational formulations of inverse forecasting problems, can significantly increase the accuracy of the solution of the forecast problem.

Summary.
Questions of development of effective, refined methods of forecasting of a resource, durability, reliability, durability of designs from polymeric, composite materials based on modern achievements in the field of mathematical and computer modeling are investigated. - The principle of multiplicity of forecasting models is formulated, the inclusion of which in the refined formulation of inverse forecasting problems opens up new opportunities in improving the efficiency of solving forecasting problems of determining characteristics of composites.

Within the framework of the formulated principle of multiplicity of forecasting models, the development of effective methods for constructing models of optimal complexity corresponding to the optimal number of model parameters $n^*(1 \leq n^* < \infty)$ can significantly improve the accuracy of the forecast in comparison with the use of forecasting models with a fixed number of parameters.

The solution accuracy of prediction tasks solved in the refined productions, including the principle of a plurality of prediction models associated with building forecasting models, the optimal
complexity can greatly exceed the accuracy of prognostication tasks, focused on applying predictive models with a fixed number of parameters.

- The application of the developed forecasting methods within the framework of the formulated refined variational formulations of inverse forecasting problems can significantly improve the accuracy of solving forecasting problems in comparison with existing approaches.

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