Phenomenological determination of polarized quark distributions in the nucleon

Jan Bartelski\textsuperscript{1} and Stanislaw Tatur\textsuperscript{2}

\textsuperscript{1}Institute of Theoretical Physics, Warsaw University, Hoża 69, 00-681 Warsaw, Poland.

\textsuperscript{2}N. Copernicus Astronomical Center, Polish Academy of Science, ul. Bartycka 18, 00-716 Warsaw, Poland.

Abstract:
We present a fit to spin asymmetries which gives polarized quark distributions. These functions are closely related to the ones given by the newest Martin, Roberts and Stirling fit for unpolarized structure functions. The integrals of polarized distributions are discussed and compared with the corresponding quantities obtained from neutron and hyperon $\beta$-decay data. We use the combination of proton and neutron spin asymmetries in order to determine the coefficients of our polarized quark distributions. Our fit shows that phenomenologically there is no need for taking gluonic degrees of freedom into account.

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The new data taken in experiments at CERN [1, 2] and SLAC [3] renewed the interest in the spin structure of the nucleon. We have now at our disposal relatively precise CERN data for proton spin asymmetries from Spin Muon Collaboration (SMC) [2] and very recent E143 experiment results from SLAC [4]. We have also the asymmetries measured from \(^{3}\text{He}\) in E142 experiment at SLAC [3] whereas the data from deuterium target come from SMC [1]. Together with an old SLAC [5] and EMC [6] data for proton one has considerable amount of information which can be used to study nucleon spin structure and in particular to determine the polarized quark distributions.

The unpolarized quark distributions in the nucleon are known due to several fits [7-10]. Martin, Roberts and Stirling (MRS) [7] gave a complete fit using an existing experimental data in order to determine parton (i.e. quark and gluon) distributions. Quite recently an improved version of such densities has been presented by them [8].

Our preliminary discussion how to determine the polarized quark distributions using the older version \(D'\) of the MRS fit was given in [9].

In this paper we would like to get polarized quark parton distributions starting from the unpolarized ones and using existing data for proton, neutron (made on \(^{3}\text{He}\)) and deuteron spin asymmetries. We will consider two different models of possible \(x\) behaviour (more and less singular at \(x \to 0\)) of the sea contribution. The calculated values of octet axial-vector couplings \(a_{3}\) and \(a_{8}\) obtained from the fit are compared with the experimental values gotten from nucleon and hyperon \(\beta\)-decays modified for QCD corrections. These quantities are used to differentiate between various fits (together with their \(\chi^2\) values) which were made for different combinations of the spin asymmetry data (e.g. proton+neutron versus proton+deuteron data). Putting the additional restriction for \(a_{8}\) (in order to stabilize the fits) we determine polarized quark distributions from the combination of proton and neutron data. We have also tried to include polarized gluons contributing in the way proposed in Ref. [11]. It does not lead to any substantial improvement (\(\chi^2\) per degree of freedom is worse) in the fit. In addition the sign of the gluon contribution is opposite to the one which is expected theoretically. The inclusion of newest SLAC data for proton asymmetry from the E143 experiment is also discussed.

Let us start with the formulas for unpolarized quark parton distributions (at \(Q^2 = 4\text{ GeV}^2\)) given by Martin, Roberts and Stirling [8]. We shall consider their fit called MRS(A) with rather singular behavior of sea distribution at
small $x$ values (which, however agrees with the results from HERA \cite{12}). We have for the valence quarks distributions:

$$
\begin{align*}
  u(x) &= 1.996x^{-0.462}(1 - x)^{3.96}(1 - 0.39\sqrt{x} + 5.13x), \\
  d(x) &= 0.296x^{-0.670}(1 - x)^{4.71}(1 + 5.03\sqrt{x} + 5.56x),
\end{align*}
$$

(1)

and for the sea ones:

$$
\begin{align*}
  \bar{u}(x) &= 0.392M(x) - \delta(x), \\
  \bar{d}(x) &= 0.392M(x) + \delta(x), \\
  \bar{s}(x) &= 0.196M(x), \\
  \bar{c}(x) &= 0.020M(x).
\end{align*}
$$

(2)

In eq. (2) one has for singlet:

$$
M(x) = 0.411x^{-1.3}(1 - x)^{9.27}(1 - 1.15\sqrt{x} + 15.6x),
$$

(3)

and isovector part:

$$
\delta(x) = 0.099x^{-0.462}(1 - x)^{9.27}(1 + 25.0x).
$$

(4)

The unpolarized gluon distribution is given by:

$$
G(x) = 0.775x^{-1.3}(1 - x)^{5.3}(1 + 5.2x).
$$

(5)

In analogy to the unpolarized case we assume that the polarized quark distributions are of the form: $x^\alpha(1 - x)^\beta P_2(\sqrt{x})$, where $P_2(\sqrt{x})$ is a second order polynomial in $\sqrt{x}$ and the asymptotic behavior for $x\to0$ and $x\to1$ (i.e. the values of $\alpha$ and $\beta$) are the same as in unpolarized case. The unpolarized parton distribution is the sum of helicity distributions along and opposite to parent nucleon helicity whereas the polarized one is the difference of such functions. Hence, our idea is just to split the numerical constants (coefficients of $P_2$ polynomial) in eqs.(1,3,4) in two parts in such a manner that the distributions are positive defined. Our expressions for $\Delta q(x) = q^+(x) - q^-(x)$ ($q(x) = q^+(x) + q^-(x)$) are:

$$
\begin{align*}
  \Delta u(x) &= x^{-0.462}(1 - x)^{3.96}(a_1 + a_2\sqrt{x} + a_3x), \\
  \Delta d(x) &= x^{-0.670}(1 - x)^{4.71}(b_1 + b_2\sqrt{x} + b_3x), \\
  \Delta M(x) &= x^{-0.800}(1 - x)^{9.27}(c_1 + c_2\sqrt{x}), \\
  \Delta \delta(x) &= x^{-0.462}(1 - x)^{9.27}d(1 + 25.0x).
\end{align*}
$$

(6)
At this stage we will not take into account gluonic degrees of freedom. In
the case of \( \Delta M \), i.e. total sea polarization, we assume that there is no term
behaving like \( x^{-1.3} \) at small \( x \) (we assume that \( \Delta M \) and hence all distributions
are integrable), which means that coefficient in this case have to be splitted
into equal parts in \( M^+ \) and \( M^- \). The next term \( (x^{-0.8}) \) is relatively singular
in comparison to valence quark distributions. That means that for \( x \to 0 \) the
sea contribution dominates and hence, proton and neutron spin asymmetries
will behave in the same way in this regime. That is not the trend that is
observed in the existing experimental data. Looking at the data points we
see that proton asymmetry is positive while neutron negative for small \( x \) values. Nevertheless, we shall consider two kinds of models: the first (later
abbreviated by I) with more singular \( (\Delta M \sim x^{-0.8}) \) behaviour for \( x \to 0 \)
and the second (II) with less singular \( (\Delta M \sim x^{-0.3}) \). In the second case the
coefficient in front of \( x^{-0.8} \) is also equally divided between \( M^+ (x) \) and \( M^- (x) \).

We would like to stress that such behaviour is not a Regge type extrapolation
in this region. There are some problems in MRS(A) fit with positivity of
quark sea distributions and the condition that \( q^+ \) and \( q^- \) distributions have
the same minimal non positive term determines \( d \) in eq.(6) and reduces the
number of parameters from 8 to 7 (or from 9 to 8 when \( c_1 \neq 0 \)).

To find the unknown parameters in the expressions for polarized quark
distributions we fit our formulas for \( A_p^1, A_n^1 \) and \( A_d^1 \) (with eight or seven
parameters) to the experimental data on spin asymmetries, which are given
by:

\[
A_p^1(x) = \frac{4\Delta u_v(x) + \Delta d_v(x) + 2.236\Delta M(x) - 3\Delta \delta(x)}{4u_v(x) + d_v(x) + 2.236M(x) - 3\delta(x)}(1 + R),
\]

\[
A_n^1(x) = \frac{\Delta u_v(x) + 4\Delta d_v(x) + 2.236\Delta M(x) + 3\Delta \delta(x)}{u_v(x) + 4d_v(x) + 2.236M(x) + 3\delta(x)}(1 + R),
\]

\[
A_d^1(x) = \frac{5\Delta u_v(x) + 4.472\Delta M(x)}{5u_v(x) + 4.472M(x)}(1 - \frac{3}{2}p_D)(1 + R).
\]

The ratio \( R = \sigma_L/\sigma_T \), which vanishes in the Bjorken limit, is taken from
\([13]\), whereas \( p_D \) is a probability of D-state in deuteron wave function (equal
to 5.8\%). Spin structure function \( g_1^p \) is given by:

\[
g_1^p(x, Q^2) = \frac{4\Delta u_v(x) + \Delta d_v(x) + 2.236\Delta M(x) - 3\Delta \delta(x)}{18}/18. \tag{8}
\]

In this paper we assume that the spin asymmetries do not depend on
\( Q^2 \) what is suggested by the experimental data \([1, 3]\) and phenomenological
analysis [14]. We have made fits using different assumptions about the sea contributions (I and II) and also for different sets of experimental data (i.e.: proton+neutron (pn), proton+deuteron (pd) and proton+neutron+deuteron (pnd)). We do this because it is known that neutron and deuterons spin asymmetries lead to different values of an integral: $\Gamma_1^p(Q^2) = \int_0^1 g_1^p(x, Q^2) dx$. We have also tried to include gluons along the line of ref.[11] by considering effective polarized quark distributions $\Delta q_{\text{eff}} = \Delta q - \frac{\alpha_s}{2\pi} \Delta \hat{G}$ and fitting the constant in front of gluonic distribution.

The obtained polarized quark distributions $\Delta u(x)$, $\Delta d(x)$, $\Delta M(x)$ and $\Delta \delta(x)$ can be used to calculate first moments. For a given $Q^2$ we can write the relations:

$$ \Gamma_1^p = \frac{4}{18} \Delta u + \frac{1}{18} \Delta d + \frac{1}{18} \Delta s, $$

$$ \Gamma_1^n = \frac{1}{18} \Delta u + \frac{4}{18} \Delta d + \frac{1}{18} \Delta s, $$

where $\Delta q = \int_0^1 \Delta q(x) dx$.

Other combinations (octet and singlet ones) of first moments of quark polarizations are:

$$ a_3 = \Delta u - \Delta d, $$

$$ a_8 = \Delta u + \Delta d - 2\Delta s, $$

$$ \Delta \Sigma = \Delta u + \Delta d + \Delta s, $$

The results for the integrated quantities (calculated at 4 GeV$^2$) after taking into account known QCD corrections (see e.g. Ref.[13]) could be compared with axial-vector coupling constant $g_A$ and $g_8$ known from neutron $\beta$-decay and hyperon $\beta$-decays (the last one with the help of SU(3) symmetry). We can express the combination of $\Gamma_1^p(Q^2)$ and $\Gamma_1^n(Q^2)$:

$$ \Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = c_{NS}(Q^2)g_A/6, $$

where $c_{NS}(Q^2)$ describes QCD corrections for non-singlet quantities [13] and $g_A = 1.2573 \pm 0.0028$ (see Ref.[16]) is obtained from the neutron $\beta$-decay. We get $a_3(4 \text{ GeV}^2) = c_{NS}(4 \text{ GeV}^2)g_A/6 = 1.11$ and with this value we shall compare $a_3$ calculated from our fits. Another combination of $\Gamma_1^p(Q^2)$ and $\Gamma_1^n(Q^2)$ is equal to:

$$ \Gamma_1^p(Q^2) + \Gamma_1^n(Q^2) = 5a_8(Q^2)/18 + 2\Delta s(Q^2)/3 $$

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with $a_8 = c_{NS}(Q^2)g_8$ and where $g_8 = 0.58 \pm 0.03$ [16] is obtained from the hyperon $\beta$-decays. Knowing $c_{NS}(Q^2)$ we can calculate $a_8(4\text{ GeV}^2) = 0.515$ and with this number we shall compare the results obtained from our fit. If we have had very precise experimental data and in the whole $x$ range there would be no problems with determination of polarized quark distributions. Unfortunately that is not the case yet. Actually, from the experiment we have information on $\Gamma_{1}^{p}$ and $\Gamma_{1}^{n}$. The combination $\Gamma_{1}^{p} - \Gamma_{1}^{n}$ is directly connected to $g_A$ experimental quantity modified by QCD corrections. On the other hand $\Gamma_{1}^{p} + \Gamma_{1}^{n}$ is the combination of $a_8$ and $\Delta s$ and it comes out that the fits are not sensitive enough to determine $a_8$ and $\Delta s$ separately in a stable way. The value of $a_8$ and $\Delta s$ are different for models I and II and different subsets of data. To stabilize the determination of parameters we assume in addition that $a_8 = 0.515$ with 0.1 as artificial theoretical error. We will not present the results of all our fits. Some examples are given in the Table 1.

**Table 1**

The first moments of polarized distributions (see eqs.(9) and (10)). The strange sea polarization $\Delta s$ is connected to the total sea polarization by the relation: $\Delta s = 0.196\Delta M$. We present figures for two type of models: I and II. We have made our fits taking different spin asymmetries, namely: for proton (p) (P stands for all proton data with the inclusion of newest SLAC E143 points), neutron (n) and deuteron (d) target.

|       | I(p,n,d) | II(p,n,d) | II(p,d) | II(p,n) | II(P,n) |
|-------|----------|-----------|---------|---------|---------|
| $\Gamma_1^p$ | 0.156    | 0.146     | 0.145   | 0.148   | 0.144   |
| $\Gamma_1^n$ | -0.045   | -0.055    | -0.076  | -0.038  | -0.042  |
| $g_A$   | 1.205    | 1.201     | 1.330   | 1.114   | 1.112   |
| $g_8$   | 0.508    | 0.512     | 0.515   | 0.523   | 0.478   |
| $\Delta \Sigma$ | 0.384    | 0.297     | 0.204   | 0.373   | 0.348   |
| $\Delta M$ | -0.210   | -0.366    | -0.529  | -0.255  | -0.221  |

It would be natural to use all the existing data for the determination of polarized quark distributions. We see that in this case fits I and II lead to similar results but formally fit I has one parameter more and higher $\chi^2$ per degree of freedom. From the Table 1 we see how integrated quantities: $\Gamma_1^p$, $\Gamma_1^n$, $a_3$, $a_8$, $\Delta \Sigma$ and $\Delta M$ depend on usage of different subsets of data. It seems that the closest value to the expected theoretical value for $a_3 = 1.11$ we get
when we use the combination of all proton and neutron SLAC data. In the ratio $a_8/a_3$ QCD corrections cancel so it can be expressed directly through the quantities known from low energy decay experiments i.e. $g_8/g_A = 0.46 \pm 0.02$. For the combination of proton and neutron SLAC data the ratio calculated from the fit ($a_8/a_3 = 0.47$) is closest to the value expected from experiment. We get for this fit $\chi^2 = 20.47$. If we add to this value the figure 6.93 corresponding to deuteron contribution (from the SMC experiment) we get the value 27.4 what is very close to the the $\chi^2$ value obtained from the fit to all (proton, neutron and deuteron) data, namely 27.1. If one starts, on the other hand, with the fit to proton and deuteron data and adds to the $\chi^2$ value the number corresponding to neutron contribution one gets $\chi^2 = 37.1$. It seems that it is possible to get a satisfactory deuteron asymmetry using proton+neutron data and the result is not as good when we start with the combination of proton and deuteron data (in this case $a_3 = 1.33$, the value which is far too big). So we have decided to use combination of proton and neutron ($^3He$) spin asymmetry data to calculate the parameters of the polarized quark distributions. We present in Figs.(1a,1b,2 and 3) the comparison of our fit $\Pi(p,n)$ with the experimental asymmetries for proton (1a,1b) neutron (2) and deuterium (3) target. In the last figure our curve should be treated as a prediction, because we do not take deuteron data into account in this fit. The parameters given in the first row of the Table 2 correspond to the considered model, namely the one from the fifth column of Table 1.

**Table 2**

The coefficients of polarized distributions (see eqs.(6)) for type II models. The second row corresponds to the figures gotten in a fit with the inclusion of the new proton data from the E143 experiment at SLAC.

|       | $a_1$ | $a_2$ | $a_3$ | $b_1$ | $b_2$ | $b_3$ | $c_2$ | $d$  |
|-------|-------|-------|-------|-------|-------|-------|-------|------|
| $\Pi(p,n)$ | 0.924 | -3.237| 11.30 | -0.029| -0.163| -1.644| -0.993| -0.015|
| $\Pi(P,n)$ | 0.929 | -3.005| 10.24 | -0.066| -0.064| -1.644| -0.861| -0.013|

The obtained quark distributions lead to the following integrated quantities: $\Delta u = 0.91$ ($\Delta u_v = 1.04$), $\Delta d = -0.33$ ($\Delta d_v = -0.18$) and $\Delta s = -0.07$. It gives the amount of sea polarization $\Delta M = -0.26$. We see that our integrated quantities ($\Gamma^p_1$ and $\Gamma^n_1$) differ slightly from the values quoted by the
experimental groups. The experimental figures are calculated directly from the experimental points with the assumption of Regge type behaviour at small $x$. On the other hand our polarized quark distributions satisfy all the constraints taken implicitly into account in fits to unpolarized data.

We have also tried to consider gluonic contributions which were proposed as a solution of spin “crisis”. We have taken into account gluons by considering effective polarized quark distributions $\Delta q^{eff} = \Delta q - \frac{\alpha_s}{2\pi} \Delta G$ where

$$\Delta G(x) = f x^{-0.3}(1 - x)^{5.3},$$

(13)

with a new $f$ constant which was fitted. In the basic fit (II(p,n)) we have got $\alpha_s \Delta G/2\pi = -0.08$ (which corresponds to $\Delta G = -1.84$ for $\alpha_s = 0.28$). This means that the sign of the gluonic contribution is opposite to what is expected theoretically and effective $\Delta \Sigma^{eff}$ is bigger than $\Delta \Sigma$ coming from quark distributions. We conclude that from the point of view of fitting we do not need gluonic contribution.

The Fig.(4) shows the comparison of $g_1^p(x)$ calculated from our fit with experimental points. We observe the substantial growth of $g_1^p$ for small $x$ values in our model.

The last column of Table 1 corresponds to the inclusion of recent data for spin asymmetries from the experiment E143 at SLAC. We see that the inclusion of new data changes integrated quantities only slightly what we consider as a positive fact. In the case of the good fit the additional experimental data should not change parameters in a drastic way. The parameters corresponding to this fit are given in the second row of Table 2 whereas the fitted curve is presented in Fig.(5) together with the one for fit II(p,n). We also would like to point out that with the new data from E143 included the fit of the type I model coincides with that of type II, which means that the more singular behaviour of the sea is eliminated by fitting the formulas to the experimental data. However, one should be aware of the fact that in the last fit we have much more proton than neutron data points.

Now, we would like to make some comments about the $x \to 1$ behaviour of valence quark distributions. Looking at the data points for proton spin asymmetry the value close to 1 at $x \sim 1$ is preferred, whereas for neutron and deuteron case values close to 0 seem to be natural (such observation is fragile due to the big experimental errors in this $x$ region). In our approach we can give predictions for the behaviour of polarized quark distributions and spin asymmetries in the $x \to 1$ limit. In our fit we get $A_1^p = A_1^n = A_1^d = 0.89$. 

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Starting from the new, improved version of the MRS fit to the unpolarized deep inelastic data we have made a fit to proton, neutron ($^3He$) and deuteron spin asymmetries in order to obtain polarized quark parton distributions. We have discussed two kinds of models with more and less singular sea contribution at small $x$ (both models being a possible consequence of the unpolarized distributions) and different combination of proton, neutron and deuteron spin asymmetries data. To stabilize the fits we add the experimental information on octet quantity $a_8$. We have calculated the parameters of the polarized quark distributions from the combination of proton and neutron spin asymmetries data (without the SMC deuteron data). The inclusion of the new proton SLAC data from E143 experiment modifies our fit only slightly, which is a positive feature. We do not need gluonic contributions to be taken into account, i.e. the fit with gluons is worse. The new consistent data for spin asymmetries (mainly for deuteron and neutron) could help us to determine polarized quark distributions and to resolve the doubts about the $x \to 0$ and $x \to 1$ behaviour of such functions.
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Figure captions

Figure 1a The comparison of spin asymmetry on protons (data points are from SLAC (E80, E130) and CERN (EMC,SMC) experiments) with the curve gotten from our fit II(p,n) (eqs.(6,7) and the first row of Table 2).

Figure 1b The same as in figure 1a but with $x$ in logarithmic scale.

Figure 2 The comparison of spin asymmetry on neutrons (SLAC E142 data) with the curve gotten from our fit.

Figure 3 Our prediction for deuteron asymmetry compared with the SMC data.

Figure 4 The data for $g_1^p(x)$ structure function with the curve gotten using the parameters of fit II(p,n).

Figure 5 The same as in Fig.(1b) but with the new experimental points (E143) from SLAC included. The dashed curve corresponds to the fit II(P,n), which takes into account the E143 experiment data.
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http://arxiv.org/ps/hep-ph/9502271v1
Figure 1a

Data points from SLAC and CERN are plotted against the variable $x$. The points are accompanied by error bars, indicating the uncertainty in the measurements. A smooth curve fits the data points, suggesting a trend across the range of $x$ values from 0 to 1.
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Figure 1b

SLAC
CERN
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Figure 3
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Figure 5

SLAC
CERN
SLAC (E143)