QCD isospin breaking ChPT low-energy constants from the instanton vacuum

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In the framework of the instanton vacuum model we evaluate the Chiral Perturbation Theory (ChPT) low-energy constants \( h_3, l_7 \). We found that in the instanton vacuum model the constant \( l_7 \) is very sensitive to the shape of the instanton and the instanton vacuum parameters. We evaluated the constant \( l_7 \) for two different zero-mode profiles and as a function of the average instanton size \( \rho \) and inter-instanton distance \( R \). Our result agrees with an old “order of magnitude” estimate of this constant from [1]. The obtained value of \( l_7 \) implies that the pure QCD contribution to the pion mass difference is small, \( \sim 1\% \) of the observed experimental value.

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I. INTRODUCTION

The spontaneous breaking of chiral symmetry (S\( \chi \)SB) is one of the most important phenomena of hadron physics. It defines the properties of all the light mesons and baryons. Using the general idea of chiral symmetry, it was proposed in [1] to use a phenomenological lagrangian, which has a form of the infinite series in the pion momenta \( p^2 \) and mass \( M_\pi^2 \). The low-energy constants of the series expansion (LEC’s) are the free parameters which encode the low-energy physics in a model-independent way. Up to now they were extracted phenomenologically from the experimental data, or from the lattice calculations ((MILC, ETM, JLQCD, RBC/UKQCD, PACS-CS) [2–5] within so-called Chiral Perturbation Theory (ChPT)).

One of the low-energy constants \( l_7 \) is particularly interesting since it encodes the “pure QCD” part of the \( SU(2) \) isospin symmetry breaking (i.e. part which is due to \( u^- \) and \( d^- \) quark current mass difference, \( m_u - m_d \)). For example, the QCD part of the pion mass difference \( m_{\pi^+} - m_{\pi^0} \) has a form [1]

\[
(m_{\pi^+}^2 - m_{\pi^0}^2)_{QCD} = \frac{2B^2}{F^2} l_7 (m_u - m_d)^2 ,
\]

where \( B \) and \( F \) are the leading order parameters in the chiral lagrangian, and \( m_u, m_d \) are the current quark masses. While experimentally the isospin breaking effects are known to a very high precision, separation of these effects on the “pure QCD” and electromagnetic parts has ambiguities and has been a subject of intensive debates [1, 6–8]. From phenomenology the constant \( l_7 \) is known only with an “order of magnitude” estimate [1],

\[
l_7 \sim 5 \times 10^{-3}.
\]

For this reason it makes sense to estimate this contribution in the framework of a reliable model.

QCD instanton vacuum model, often referred to as the instanton liquid model, provides a very natural nonperturbative explanation of the S\( \chi \)SB [10–21]. It provides a consistent framework for description of the pions and thus may be used for evaluation of the low energy constants. Due to instanton-induced nonlinear interaction all the quark and meson loop integrals are regularized by the natural scale \( \mu \sim \rho^{-1} \sim 600 \text{ MeV} \) in Pauli-Villars scheme [13], where \( \rho \) is the average size of the instanton. This means that all the scale-dependent quantities, such as the quark condensate \( \langle \bar{q}q \rangle \equiv \langle \bar{u}u \rangle + \langle \bar{d}d \rangle \) and the difference \( \delta \langle \bar{q}q \rangle \equiv \langle \bar{u}u \rangle - \langle \bar{d}d \rangle \), are given at the scale \( \mu \). Remarkably, the constant \( l_7 \) does not depend on the scale \( \mu \). Recently [22] it has been shown that this approach is able to give results consistent with phenomenological and lattice estimates for the constants \( l_3, l_4 \), providing current quark mass dependencies of the pion mass \( m_\pi \) and pion decay constant \( F_\pi \).
In this paper we would like to apply the instanton vacuum model for the evaluation of the constant $l_7$. We extract the constant $l_7$ from the correlator $\langle P^3(x)P^0(0) \rangle$ using the relation

$$\int d^4x e^{iqx} \langle P^3(x)P^0(0) \rangle = \frac{G_\pi \tilde{G}_\pi}{m_\pi^2 - q^2} + \mathcal{O}(q^2) = \frac{8B^3(m_u - m_d)}{q^2 - m_\pi^2} l_7 + \mathcal{O}(m, q^2). \quad (3)$$

From the Eqn. (3) we may see that evaluations may be done in the limit $m \equiv \frac{m_u + m_d}{2} \to 0$, and make only expansion over

$$\delta m \equiv (m_u - m_d). \quad (4)$$

The paper is organized as follows. In Section II we discuss the general framework used for evaluation and write out the next-to-leading order (NLO) gap equation in the presence of the current mass split $\delta m$, which are needed for evaluation of the dynamical mass split $\delta M \equiv M_u - M_d$. In Section IV we write out explicit expressions for the quark and meson propagators. In Section V we evaluate the effects of the mass split $\delta m$ on the quark condensate, $\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_u - \langle \bar{q}q \rangle_d$, and extract the constant $h_3$. In Section VI we evaluate the correlator $\langle P^3(x)P^0(0) \rangle$ and extract the constant $l_7$. In Section VII we discuss obtained results, their uncertainty limits and draw conclusions.

II. INSTANTON VACUUM MODEL

The instanton vacuum model is based on the assumptions that the QCD vacuum may be considered as a dilute gas of instantons and antiinstantons, and the number of colors $N_c$ is asymptotically large, $N_c \to \infty$ (see the reviews [13, 24]). While in general the sizes and local density of the instanton gas may be arbitrary, inter-instanton interaction stabilize these parameters. As it has been discussed in [22], the $1/N_c$-suppressed corrections due to the finite size distribution are indeed quite small, even for $N_c = 3$. Phenomenological, variational and lattice estimates lead to average instanton size $\rho \sim 0.3 \text{ fm}$ and inter-instanton distance $R \sim 1 \text{ fm}$ [12].

The partition function in the field of external scalar and pseudoscalar currents $s = (s_0 + \bar{s}r)$ and $p = (p_0 + \bar{p}r)$ has a form [22]

$$Z_N[s_0, \sigma, s_0, \bar{p}, \bar{s}, p_0] = \int d\lambda \exp (-\Gamma_{eff}[s_0, \lambda, \sigma, s_0, \bar{p}, \bar{s}, p_0, \bar{\sigma}, \eta, \bar{\eta}, \bar{u}]), \quad (5)$$

$$\Gamma_{eff} = S + \Gamma_{mes}^{eff}, \quad (6)$$

$$S = \frac{N}{V} \ln \lambda + 2 \int d^4x \sum \Phi^2(x) - Tr \ln \left( \frac{\hat{p} + is_0 + \bar{s}r + p_0\gamma_5 + i\bar{p}r \cdot \vec{\gamma}_5 + icF \Phi \cdot \Gamma F}{\hat{p} + is_0 + \bar{s}r + p_0\gamma_5 + i\bar{p}r \cdot \vec{\gamma}_5} \right), \quad (7)$$

The nonlocal formfactors $F(p)$ in the meson-quark interaction vertices come from the instanton-induced nonlocal interactions. Together with the factor $(\hat{p} + is_0 + \bar{s}r + p_0\gamma_5 + i\bar{p}r \cdot \vec{\gamma}_5)$ in denominator, which subtracts the divergent high-frequency modes, they guarantee finite results for all the observables in the instanton vacuum model. As it was discussed in [12, 13], the divergent high-frequency modes are responsible for renormalization of the parameters of the model. In what follows, we will fix them at the scale $\mu \sim \rho^{-1} \approx 600 \text{ MeV}$ in the Pauli-Villars scheme [12, 13].

The meson-loop correction $\Gamma_{mes}^{eff}$ to the effective action is given as

$$\Gamma_{mes}^{eff}[m, \lambda, \sigma] = \frac{1}{2} Tr \ln \left( 4\delta_{ij} + \frac{1}{\sigma^2} Tr \left( \frac{c(\lambda) F^2(p)}{\hat{p} + is_0 + \bar{s}r + p_0\gamma_5 + i\bar{p}r \cdot \vec{\gamma}_5 + icF \Phi \cdot \Gamma F} \Gamma_{i\times j} \right) \right), \quad (8)$$

$$\Phi \cdot \Gamma = \left( \sigma + i\gamma_5 \bar{r} \phi + i\bar{r} \bar{\sigma} \gamma_5 \eta \right), \quad (9)$$

where $c(\lambda) = \frac{(2\pi)^2\sqrt{s}}{2g^2}, g^2 = \frac{(N_c^2 - 1)2N_c}{2N_c - 1}$ is a color factor, $\Gamma = \{1, \gamma_5, i\bar{r}, i\bar{r} \gamma_5 \}$ is a set of matrices corresponding to quantum numbers of mesons present in the model, and we will use for the corresponding components of the field $\Phi$ the notations $\Phi = \{\sigma, \eta, \bar{\sigma}, \phi \}$. In contrast to NJL model, the variable $\lambda$ is a dynamical degree of freedom but not the parameter of the lagrangian. The current masses of the quarks come into play via constant external currents, viz.

$$s_0 = \frac{m_u + m_d}{2}, \quad (10)$$

$$s_3 = \frac{m_u - m_d}{2}. \quad (11)$$
Notice that with respect to chiral transformations, the mesons may be separated onto two independent doublets \((\sigma, \tilde{\sigma})\) and \((\eta, \tilde{\eta})\). The first doublet \((\sigma, \tilde{\sigma})\) corresponds to the pion field \(U = (u_0, \tilde{u})\) in the notations of [1], and the second doublet \((\eta, \tilde{\eta})\) is an additional degree of freedom which is absent in the chiral lagrangian. Now we are going to demonstrate explicitly on the example of the constant \(\ell_7\) that this additional degree of freedom \((\eta, \tilde{\eta})\) gives an essential contribution to the constant \(\ell_7\). As usual, the external currents \((s_0, \tilde{s}, p_0, \tilde{p})\) generate nonzero vacuum averages of the fields \(\langle \tilde{\sigma}_v \rangle = \tilde{\sigma}_v\), \(\langle \tilde{\eta}_v \rangle = \tilde{\eta}_v\) and \(\langle \sigma_v, \tilde{\sigma}_v \rangle = U = (u_0, \tilde{u})\).

Due to the chiral symmetry expansion of the \(\Gamma_{\text{eff}}\) yields the general structure

\[
\Gamma_{\text{eff}}[\lambda, \sigma, \tilde{\sigma}, \eta, \tilde{\eta}, u_i] = \Gamma_{\text{eff}}[m, \lambda, \sigma, \tilde{\sigma}, \eta, \tilde{\eta} = 0, u_i = 0] + A \left( \partial u_0 \right)^2 + B (s_0 u_0 + \tilde{p} \tilde{u}) + C (s_0 \eta_0 + \tilde{p} \tilde{\eta}_0)^2 + D (s_0 \eta_0 + \tilde{p} \tilde{\eta}_0)^2 + a \left( p_0^2 + s^2 \right) + b (p_0 \eta_0 + \tilde{\sigma}_v) + c (\eta_0^2 + \tilde{\eta}_0^2) + d (u_0 p_0 + \tilde{u} \tilde{s}) (u_0 \eta_0 + \tilde{u} \tilde{\eta}_0) + f (u_0 \eta_0 + \tilde{u} \tilde{\eta}_0)^2 + O \left( s^6, p^6 \right),
\]

where we omitted the terms containing derivatives of the fields, since the external currents are constants, the constants \(A - D, a - f\) should be evaluated with account of NLO corrections. The vacuum equations which follow from (12) are

\[
\frac{\partial \Gamma_{\text{eff}}[m, \lambda, \sigma, \tilde{\sigma}, \eta, \tilde{\eta}, u_i]}{\partial \lambda} = \frac{\partial \Gamma_{\text{eff}}[m, \lambda, \sigma, \tilde{\sigma}, \eta, \tilde{\eta}, u_i]}{\partial \sigma_v} = \frac{\partial \Gamma_{\text{eff}}[m, \lambda, \sigma, \tilde{\sigma}, \eta, \tilde{\eta}, u_i]}{\partial \eta_v} = \frac{\partial \Gamma_{\text{eff}}[m, \lambda, \sigma, \tilde{\sigma}, \eta, \tilde{\eta}, u_i]}{\partial u_i} = 0
\]

The coefficients \(A, B\) are relevant for the 2-point correlators with intermediate pions and \(A \sim F^2\) and \(B \propto M_\pi^2\) in \(m_u = m_d\) limit. The constants \(B, C, D\) are irrelevant to our problem since they are constants in front of the term with chiral doublet \(\chi = (\tilde{s}, \tilde{p})\) which we put to zero in the current paper.

The Eqs (13) are responsible for the dynamical mass generation and will be discussed in the next section. The Eqs (14) may be explicitly written as

\[
\frac{\partial \Gamma_{\text{eff}}[m, \lambda, \sigma, \tilde{\sigma}, \eta, \tilde{\eta}, u_i]}{\partial \eta_v} = \frac{\partial \Gamma_{\text{eff}}[m, \lambda, \sigma, \tilde{\sigma}, \eta, \tilde{\eta}, u_i]}{\partial \eta_v} = \frac{\partial \Gamma_{\text{eff}}[m, \lambda, \sigma, \tilde{\sigma}, \eta, \tilde{\eta}, u_i]}{\partial \sigma_v} = \frac{\partial \Gamma_{\text{eff}}[m, \lambda, \sigma, \tilde{\sigma}, \eta, \tilde{\eta}, u_i]}{\partial u_i} = 0
\]

Multiplying Eqn (15) on \(u_0\) and Eqn. (16) on \(\tilde{u}\) and adding results, we may find:

\[
u_0 \eta_0 + \tilde{u} \tilde{\eta}_0 = - \frac{b + c}{2(c + f)} (u_0 p_0 + \tilde{u} \tilde{s}).
\]

Repeating the same trick with \(p_0\) and \(\tilde{s}\), we may get

\[
\eta_0 \tilde{s} + \tilde{u} \tilde{\eta}_0 = - \frac{1}{2c} \frac{b^2}{2} \left( b \left( p_0^2 + s^2 \right) + e \left( f \frac{b + e}{c + f} \right) (u_0 p_0 + \tilde{u} \tilde{s}) \right),
\]

and repeating the same trick with \(\eta_0\) and \(\tilde{\sigma}_v\), we may get

\[
\eta_0^2 + \tilde{\sigma}_v^2 = - \frac{b^2}{2c \left( b \left( p_0^2 + s^2 \right) + e \left( f \frac{b + e}{c + f} \right) (u_0 p_0 + \tilde{u} \tilde{s}) \right)}
\]

Combining results (17), (18), (19), we may get for the effective action

\[
\Gamma_{\text{eff}}[\lambda, \sigma, \tilde{\sigma}, \eta, \tilde{\eta}, u_i] = \ldots + a \left( \tilde{\sigma}_v^2 \right) - \frac{b}{2c} \left( b \left( p_0^2 + s^2 \right) + e \left( f \frac{b + e}{c + f} \right) (u_0 p_0 + \tilde{u} \tilde{s}) \right)^2
\]

\[
- \frac{b^2}{4c} \left( b \left( p_0^2 + s^2 \right) + e \left( f \frac{b + e}{c + f} \right) (u_0 p_0 + \tilde{u} \tilde{s}) \right)^2
\]

\[
+ \frac{d}{(u_0 p_0 + \tilde{u} \tilde{s})^2} - e \left( f \frac{b + e}{2(c + f)} (u_0 p_0 + \tilde{u} \tilde{s}) \right)^2 + f \left( \frac{b + e}{2(c + f)} (u_0 p_0 + \tilde{u} \tilde{s}) \right)^2
\]

\[
= \ldots + \left( a - \frac{b^2}{4c} \right) \left( p_0^2 + s^2 \right) + \left( d - \frac{b}{4c} \left( e \left( f \frac{b + e}{c + f} \right) - e \left( f \frac{b + e}{4(c + f)} \right) \right) \right) \left( u_0 p_0 + \tilde{u} \tilde{s} \right)^2 + O \left( \chi, \chi^4 \chi \right),
\]
where we omitted the terms which are proportional to the chiral doublet χ. The terms shown in (20) are explicitly chiral invariant and correspond to the terms \( (\bar{\chi} U) \) and \( (\bar{\chi}^T U)^2 \) in the chiral lagrangian [1]. Respectively, for the constant \( l_7 \) we may deduce

\[
l_7 = \frac{d - \frac{b}{4c}(e - f \frac{b + e}{c + f}) - e \frac{b + e}{4(c + f)}}{4B^2}
\]  

(21)

Thus we can see that in addition to the term \( d \) in numerator there are three other terms which correspond to contributions of additional mesons. As we will see from the following sections, these contributions have different signs and approximately the same order of magnitude as the term \( d \). The formula (21) proves that we have to consider correlators instead of direct comparison of the terms in the expansion of the lagrangian.

Below we will not evaluate the constants \( A - D, a - f \), but instead evaluate the correlators directly.

### III. GAP EQUATION

The next-to-leading order (NLO) gap equations which follow from the effective action (6) have a form

\[
\sigma \frac{\partial S}{\partial \sigma} = 4\sigma^2 - \frac{1}{V} Tr \left( iM(p)\tilde{S}(p) \right) - \frac{1}{\sigma^2} \int \frac{d^4q}{(2\pi)^4} \sum V_2^{(ij)}(q)\Pi_{ij}(q) = 0,
\]

(22)

\[
\sigma_3 \frac{\partial S}{\partial \sigma_3} = 4\sigma_3^2 - \frac{1}{V} Tr \left( i\delta M(p)\tau_3\tilde{S}(p) \right) - \frac{1}{\sigma^2} \int \frac{d^4q}{(2\pi)^4} \sum \tilde{V}_3^{(ij)}(q)\Pi_{ij}(q) = 0
\]

(23)

\[
\chi \frac{\partial S}{\partial \chi} = N \frac{1}{V} Tr \left( \tilde{S}(p) (iM(p) + i\tau_3\delta M(p)) \right) + \frac{1}{2\sigma^2} \int \frac{d^4q}{(2\pi)^4} \sum \left( V_2^{(ij)}(q) - \tilde{V}_3^{(ij)}(q) \right)\Pi_{ij}(q) = 0,
\]

(24)

where we used notations

\[
V_2^{(gap)(ij)}(q) = \frac{1}{\sigma^2} \int \frac{d^4p}{(2\pi)^4} Tr \left( \frac{M(p)}{p + i\mu(p) + i\tau_3\delta \mu(p)} \frac{M(p + q)}{\bar{p} + \bar{q} + i\mu(p + q) + i\tau_3\delta \mu(p + q)} \Gamma_j \right),
\]

(25)

\[
V_3^{(gap)(ij)}(q) = \frac{i}{\sigma^2} \int \frac{d^4p}{(2\pi)^4} Tr \left( \frac{M(p)}{\bar{p} + i\mu(p) + i\tau_3\delta \mu(p)} \right)^2 \frac{M(p + q)}{\bar{p} + \bar{q} + i\mu(p + q) + i\tau_3\delta \mu(p + q)} \Gamma_j \right),
\]

(26)

\[
\tilde{V}_3^{(gap)(ij)}(q) = \frac{i}{\sigma^2} \int \frac{d^4p}{(2\pi)^4} Tr \left( \frac{M(p)\delta M(p)\tau_3}{(p + i\mu(p) + i\tau_3\delta \mu(p))^2} \right) \frac{M(p + q)}{\bar{p} + \bar{q} + i\mu(p + q) + i\tau_3\delta \mu(p + q)} \Gamma_j \right),
\]

(27)

explicit expressions for the vertices (25-27) are given in Appendix [A] and the propagators used for evaluation are written out in Section [IV] In general, equations (22-24) can be solved only numerically.

#### A. Expansion over δm

For the special case when \( \delta m \) is small, it is possible to solve the equations (23) making a systematic expansion over the small parameter \( \delta m \). For our purpose it suffices to keep just the first order corrections. From the first and the third gap equations in (23) and the structure of the vertices (25-27) we may conclude that the vacuum expectation values for \( \langle \sigma \rangle, \langle \chi \rangle \) get corrections only in the second order over \( \delta m \), thus in the first order they remain the same as for \( \delta m = 0 \). The equation for the \( \langle \sigma_3 \rangle \) has a form

\[
\sigma_3 \frac{\partial S}{\partial \sigma_3} \approx -4e^2\langle \sigma \rangle^2 - 8\epsilon N_c \int \frac{d^4p}{(2\pi)^4} \frac{(\mu^2 - \mu^2(p)) M(p)(\delta m + \epsilon M(p))}{(p^2 + \mu^2(p))^2} \]

(28)

\[
\frac{1}{\sigma^2} \int \frac{d^4q}{(2\pi)^4} \sum \tilde{V}_3^{(ij)}(q)\Pi_{ij}(q) = 0,
\]

(29)

where \( \epsilon = \frac{i\langle \sigma_3 \rangle}{\langle \sigma \rangle} \),
\[ V^{(ij)}_3 \Pi_{ij}(q) \approx \frac{\epsilon}{2\sigma^2} \int \frac{d^4p}{(2\pi)^4} M^2(p) M(p + q) \times \{ \]

\[
\left( \Pi^{(0)}_{\sigma\sigma}(q) - \Pi^{(0)}_{\sigma_1\sigma_2}(q) \right) T_r [S_+(p)S_+(p)S_-(p+q) - S_-(p)S_-(p)S_-(p+q)]_{O(\delta m)} + \\
2\Pi_{\sigma_1\sigma_2} T_r [S_+(p)S_+(p)S_+(p+q) + S_-(p)S_-(p)S_-(p+q)]_{\delta m=0} - \\
\sum_{i=1,2} \Pi^{(0)}_{\sigma_i}(k) T_r [S_+(p)S_+(p)S_-(p+q) - S_-(p)S_-(p)S_+(p+q)]_{O(\delta m)} - \\
\left( \Pi^{(0)}_{m\phi}(k) - \Pi^{(0)}_{\phi_1\phi_2}(k) \right) T_r [S_+(p)S_+(p)S_+(p+q) - S_-(p)S_-(p)S_+(p+q)]_{O(\delta m)} - \\
2\Pi_{\phi_1\phi_2} T_r [S_+(p)S_+(p)S_-(p+q) + S_-(p)S_-(p)S_+(p+q)]_{\delta m=0} + \\
\sum_{i=1,2} \Pi^{(0)}_{\phi_i}(k) T_r [S_+(p)S_+(p)S_-(p+q) - S_-(p)S_-(p)S_+(p+q)]_{O(\delta m)} \}
\]

the superscript \((0)\) on the propagators and subscripts on \(T_r[...],_a\) indicates that the proper propagator is to be taken in the limit \(\delta m = 0\) or just collecting the first \(O(\delta m)\)-correction. One can notice that \((28)\) has a form

\[ \epsilon (X\epsilon + Y\delta m) = 0, \]

where the coefficients \(X, Y\) should be evaluated with account of \(O(1/N_c)\)-corrections. A trivial nonzero solution is \(\epsilon = -\delta m Y/X\), which corresponds to

\[ \delta \mu(p) = \delta m \left(1 - M f^2(p) Y/X\right). \]

While in general case the explicit expression for the formfactor has a form

\[ f(p) = -x \frac{d}{dx} (I_0(x)K_0(x) - I_1(x)K_1(x)) \big|_{x = \frac{2p}{\sigma}}, \]

in order to speed up the evaluations here and below we consider two parameterizations of the formfactor. The first one is a simple “dipole” parameterization \((13)\) of the form

\[ f(p) = \frac{2}{2 + p^2/\sigma^2}, \]

which coincides with \((32)\) in the region of small \(p \lesssim 2/\sigma\). The second parameterization has a form

\[ f(p) = \frac{1}{\sqrt{1 + a_1 x^2 + a_2 x^4 + a_3 x^6}} \bigg|_{x = \frac{2p}{\sigma}}, \]

where the free parameters \(a_1...a_3\) are fitted to \((32)\), and \((34)\) agrees with \((32)\) both for the small and asymptotically large \(p\). We will refer to \((33)\) and \((34)\) as dipole and quasibessel parameterizations respectively. Direct comparison of the two close parameterizations is important in order to demonstrate that the results of this paper are very sensitive to the shape of the instanton.

We summarize results obtained for the constants \(X, Y\) with different parameterizations of formfactor in Table \(1\). As we can see, the \(1/N_c\)-corrections \(X_{NLO}\) and \(Y_{NLO}\) are small compared to \(X_{LO}\) and \(Y_{LO}\) respectively, so the \(1/N_c\)-expansion works very well here. It is important to note that both in the leading order and next-to-leading order the mass \(\delta \mu(p)\) changes sign for \(p \sim 0.5\) GeV, so we have a compensation of the small-\(p\) and large-\(p\) regions.

**IV. PROPAGATORS**

In this section we would like to discuss the propagators of the quarks and mesons in presence of the mass split \(\delta m\).
|               | $X_{LO} \times 10^{3}$ | $Y_{LO} \times 10^{2}$ | $(Y/X)_{LO}$ | $X_{NLO} \times 10^{3}$ | $Y_{NLO} \times 10^{2}$ | $(Y/X)_{LO+NLO}$ |
|---------------|------------------------|------------------------|--------------|------------------------|------------------------|------------------|
| Dipole        | -8.50                  | -3.53                  | -4.16        | 0.80                   | 0.61                   | -3.80            |
| QuasiBessel   | -9.04                  | -3.33                  | -3.68        | 1.02                   | 0.69                   | -3.30            |

Table I: Parameters of mass split in different parameterizations of formfactor. Dipole corresponds to (33). QuasiBessel corresponds to (34). Dimensions: $[X_{LO,NLO}] = [\text{GeV}^4]$, $[Y_{LO,NLO}] = [\text{GeV}^3]$.

**A. Quark propagator**

Here we consider only the leading order quark propagator, the NLO corrections to the quark propagator will be considered as separate meson loop corrections to proper correlators. Since the operator $\hat{p} + i\mu(p) + i\delta\mu(p)\tau_3$ is diagonal in the flavour space, its inversion is quite straightforward, with

$$\hat{S}(p) \equiv \frac{1}{\hat{p} + i\mu(p) + i\delta\mu(p)\tau_3} = \frac{1 - \tau_3}{2} S_-(p) + \frac{1 + \tau_3}{2} S_+(p),$$

(35)

$$S_\pm(p) = \frac{1}{\hat{p} + i\mu_\pm(p)},$$

(36)

where $\mu_\pm(p) = \mu(p) \pm \delta\mu(p)$.

**B. Meson propagator**

For evaluations in this paper we have to evaluate the meson propagator with account of $1/N_c$-corrections. However it is important to note that NLO evaluations are needed only for $\Pi_{\eta\eta}(0), Res_{q^2= -m_\pi^2}\Pi_{\phi\phi}(0),$ all the other components and expressions for $q \neq 0$ may be evaluated in leading order, which significantly simplifies the task. Due to $\delta m \neq 0$ the propagator is nondiagonal in indices $(ij)$–we get additional transitions $\sigma \leftrightarrow \bar{\sigma}$ and $\eta \leftrightarrow \bar{\phi}$. Inversion of the propagator is trivial and gives:

$$\Pi_{00} = \frac{\langle \Pi^{-1} \rangle_{33}}{\langle \Pi^{-1} \rangle_{00} \langle \Pi^{-1} \rangle_{33} - \langle \Pi^{-1} \rangle_{03} \langle \Pi^{-1} \rangle_{30}}, \quad \Pi_{33} = \frac{\langle \Pi^{-1} \rangle_{00}}{\langle \Pi^{-1} \rangle_{00} \langle \Pi^{-1} \rangle_{33} - \langle \Pi^{-1} \rangle_{03} \langle \Pi^{-1} \rangle_{30}},$$

(37)

$$\Pi_{03} = -\frac{\langle \Pi^{-1} \rangle_{30}}{\langle \Pi^{-1} \rangle_{00} \langle \Pi^{-1} \rangle_{33} - \langle \Pi^{-1} \rangle_{03} \langle \Pi^{-1} \rangle_{30}}, \quad \Pi_{30} = -\frac{\langle \Pi^{-1} \rangle_{03}}{\langle \Pi^{-1} \rangle_{00} \langle \Pi^{-1} \rangle_{33} - \langle \Pi^{-1} \rangle_{03} \langle \Pi^{-1} \rangle_{30}},$$

(38)

$$\Pi_{ij} = \frac{\delta_{ij}}{\langle \Pi^{-1} \rangle_i}, \quad (i, j) \neq 3$$

(39)

where we used a shorthand notation $(0, 3) = (\sigma, \sigma_3)$ for positive parity mesons, and $(0, 3) = (\eta, \phi_3)$ for negative parity mesons.

1. Leading order

In the leading order for the components $\langle \Pi^{-1} \rangle_{ij}$ we have

$$\langle \Pi^{-1} \rangle_{ij} = 4\delta_{ij} + \frac{1}{\sigma^2} Tr \left( \hat{Q}(p)\Gamma_i\hat{Q}(p + q)\Gamma_j \right)$$

(40)

where

$$\hat{Q}(p) = iM(p)\hat{S}(p) \equiv \frac{iM(p)}{\hat{p} + i\mu(p) + i\delta\mu(p)\tau_3},$$

(41)

and explicit expressions for the components are given in Appendix A.
2. **NLO correction**

As it was discussed earlier, we need a few values for propagators in the next-to-leading order. Since the NLO evaluations are numerically slow, from the very beginning we will concentrate on evaluation of the following quantities:

\[
\lim_{q \to 0, m \to 0, \delta m \to 0} \Pi_{\eta\gamma}^{-1}(q), \quad \lim_{q \to 0, m \to 0, \delta m \to 0} \frac{\Pi_{\eta\gamma}^{-1}(q)}{\delta m}, \quad Re_{s}\Pi_{\phi\phi}(q).
\]

All the terms which do not contribute to one of these limits will be omitted. For the sake of brevity, below we use notation

\[
Q(p) = \frac{Q_{+}(p) + Q_{-}(p)}{2} \approx \frac{iM(p)}{p + i\mu(p)} + O(\delta m^2)
\]

(42)

For the pion propagator \(\Pi_{\phi\phi}(q)\), we may use the chiral limit and put \(m, \delta m\) to zero. The NLO expression for the pion propagator has a form

\[
\Pi_{\phi\phi}^{(ab)\gamma}(q) = \left[ 4\delta^{ab} + \frac{1}{\sigma^2} Tr_{p} (Q(p)i\gamma_5 \tau^a Q(p + q)i\gamma_5 \tau^b) \right] + \\
+ \frac{1}{\sigma^2} \int \frac{d^4k}{(2\pi)^4} \Pi_{ij}(k) \left( 2Tr_{p} (Q(p)i\gamma_5 \tau^a Q(p + q)\Gamma_i Q(p + q + k)\Gamma_j Q(p + q)i\gamma_5 \tau^b) + \\
+ Tr_{p} (Q(p)i\gamma_5 \tau^a Q(p + q)\Gamma_i Q(p + q + k)i\gamma_5 \tau^b Q(p + q + k)\Gamma_j) \right) \\
- \frac{4}{\sigma^2} \int \frac{d^4k}{(2\pi)^4} \Pi_{ij}(k) \Pi_{jk}(k + q) Tr_{p} (Q(p)i\gamma_5 \tau^a Q(p + q)\Gamma_i Q(p + q + k)\Gamma_j) \times \\
Tr_{p} (Q(p)i\gamma_5 \tau^b Q(p - q)\Gamma_i Q(p - q - k)\Gamma_j). \\

\]

In complete analogy, for the \(\eta\)-meson propagator \(\Pi_{\eta\eta}(0)\) we have

\[
\Pi_{\eta\eta}^{-1}(0) = \left[ 4\delta^{ab} + \frac{1}{\sigma^2} Tr_{p} (Q(p)\gamma_5 Q(p)\gamma_5) \right] + \\
+ \frac{1}{\sigma^2} \int \frac{d^4k}{(2\pi)^4} \Pi_{ij}(k) \left( 2Tr_{p} (Q(p)\gamma_5 Q(p)\Gamma_i Q(p + k)\Gamma_j Q(p)\gamma_5) + \\
+ Tr_{p} (Q(p)\gamma_5 Q(p)\Gamma_i Q(p + k)\gamma_5 Q(p + k)\Gamma_j) \right) \\
- \frac{4}{\sigma^2} \int \frac{d^4k}{(2\pi)^4} \Pi_{ij}(k) \Pi_{jk}(k) Tr_{p} (Q(p)\gamma_5 Q(p)\Gamma_i Q(p + k)\Gamma_j) \times \\
Tr_{p} (Q(p)\gamma_5 Q(p)\Gamma_i Q(p - k)\Gamma_j),
\]

and again we can make evaluations in the chiral limit.

The nondiagonal matrix element \(\Pi_{\eta\phi}(0)\) is \(O(\delta m)\), so we will extract explicitly \(\delta m\) and after that the evaluation of the constant will be done in the chiral limit. Evaluation is quite tedious since LO propagators have nondiagonal components. The corresponding expression has a form

\[
\Pi_{\eta\phi}^{-1}(0) = \Pi_{\eta\phi}^{(LO)\gamma}(0) + \Pi_{\eta\phi}^{(1-meson)\gamma}(0) + \Pi_{\eta\phi}^{(2-meson)\gamma}(0),
\]

(43)

where

\[
\Pi_{\eta\phi}^{(LO)\gamma}(0) = \left[ \frac{1}{2\sigma^2} Tr_{p} (Q_{+}(p)i\gamma_5 Q_{+}(p)\gamma_5) - \frac{1}{2\sigma^2} Tr_{p} (Q_{-}(p)i\gamma_5 Q_{-}(p)\gamma_5) \right],
\]

(44)

\[
\Pi_{\eta\phi}^{(1-meson)\gamma}(0) = \int \frac{d^4q}{(2\pi)^4} \sum_{ij} \Pi_{ij}(q) \Gamma_{ij}^{(1-meson, \eta\phi)}(q) = \int \frac{d^4q}{(2\pi)^4} \sum_{ij} \Pi_{ij}(q) \frac{i}{2\sigma^4} \int \frac{d^4p}{(2\pi)^4} \times \\
Tr_{p} (2Q(p)\Gamma_{ij} Q(p)\Gamma_q Q(p)\Gamma_i Q(p + q)\Gamma_j + Q(p)\Gamma_{ij} Q(p)\Gamma_i Q(p + q)\Gamma_q Q(p + q)\Gamma_j),
\]

(45)
| QuasiBessel | LO | Mass Shift | Mass Split | Meson | All NLO | Total |
|-------------|-----|------------|------------|-------|---------|-------|
| $\Pi_{n\phi}^{-1}(0)$ | $5.65 \times 10^{-3}$ | $9.16 \times 10^{-3}$ | 0 | $-8.88 \times 10^{-3}$ | $2.86 \times 10^{-3}$ | $5.93 \times 10^{-3}$ |
| $-i\Pi_{n\phi}^{-1}(0)$ | $-4.73 \times 10^{-3}$ | $0.88 \times 10^{-3}$ | $0.19 \times 10^{-3}$ | $-2.30 \times 10^{-3}$ | $-1.23 \times 10^{-3}$ | $-1.89 \times 10^{-3}$ |
| $F_S^2$ | $1.24 \times 10^{-2}$ | $-0.59 \times 10^{-2}$ | 0 | $0.11 \times 10^{-2}$ | $-0.49 \times 10^{-2}$ | $0.76 \times 10^{-2}$ |
| $\langle \bar{q}q \rangle$ | $2.03 \times 10^{-2}$ | $-0.77 \times 10^{-2}$ | 0 | $0.31 \times 10^{-2}$ | $-0.45 \times 10^{-2}$ | $1.58 \times 10^{-2}$ |
| $B$ | $1.64$ | — | 0 | — | — | $2.09$ |

Table II: In this table we give the numbers obtained for propagators and other relevant constants. $F_S^2$ is used for evaluation of Res $(\Pi_{\phi})$, $\langle \bar{q}q \rangle$ is used for extraction of constant $B$.

$$
\Pi_{n\phi}^{(2-mes)}(0) = -\frac{4}{\sigma^0} \int \frac{d^4q}{(2\pi)^4} \Pi_{ij}(q)\Pi_{kl}(q)V^{(q)}_{ik}(q)V^{(\phi)}_{jl}(q),
$$  

$$
V^{(q)}_{ik}(q) = [Tr_p (Q(p)\gamma_5 Q(p)\Gamma_i Q(p+q)\Gamma_k)],
$$  

$$
V^{(\phi)}_{ik}(q) = [Tr_p (Q(p)i\gamma_5 \tau^3 Q(p)\Gamma_i Q(p+q)\Gamma_k)],
$$

and explicit expressions for the verices contributing to $\Pi_{n\phi}^{-1}(0)$ are given in Appendix A.

Numerical results of evaluation are presented in Table II. As we can see, even in the leading order (LO) there is a strong sensitivity of the propagator $\Pi_{n\phi}^{-1}(0)$ to the shape of the instanton (formfactor $F(p)$). This dependence is discussed in more detail in Section VII.

V. QUARK CONDENSATE

Due to the mass split $\delta m$ there is a flavour difference for the quark condensate $\delta \langle \bar{q}q \rangle = \langle \bar{u}u \rangle - \langle \bar{d}d \rangle$. In the leading order this split is

$$
\delta \langle \bar{q}q \rangle_{LO} = \frac{i}{2} Tr (\tau_3 S(p)) = 4N_c \int \frac{d^4p}{(2\pi)^4} \left( \frac{\mu_+(p)}{p^2 + \mu_+^2(p)} - \frac{\mu_-(p)}{p^2 + \mu_-^2(p)} \right).
$$  

In the NLO evaluation is also quite straightforward, with

$$
\delta \langle \bar{q}q \rangle_{meson} = \int \frac{d^4q}{(2\pi)^4} \sum_{ij} \Pi_{ij}(q)V^{(\delta \bar{q}q)}_{ij}(q),
$$  

$$
V^{(\delta \bar{q}q)}_{ij}(q) = -\int \frac{d^4p}{(2\pi)^4} M(p)M(p+q)Tr \left( \frac{i\tau_3}{2} S(p)\Gamma_i S(p+q)\Gamma_j S(p) \right).
$$

for meson corrections plus corrections from mass shift and mass split ($1/N_c$ corrections to $M_0$ and $M_3$), and explicit expression for $h_3$ is given in Appendix A.

Results of numerical evaluation are presented in the table III. As one can see, due to the large NLO corrections to the mass split $M_e(p) - M_d(p)$, the NLO corrections are larger than the LO result.

Using formula (11.3) from I, it is possible to get for the constant $h_3$ an estimate [28]:

$$
h_3 = \left( \frac{\langle \bar{u}u \rangle - \langle \bar{d}d \rangle}{\delta m} \right)_1^{LO+NLO} = \frac{0.10 \delta m}{4B^2 \delta m} \approx 5.48 \times 10^{-3}.
$$
Table III: Different contributions to $\langle \bar{u}u - \bar{d}d \rangle$. LO: Leading order result. Mass Split: Contribution due to NLO correction to mass split $M_u(p) - M_d(p)$. Mesons: Contribution of mesons. All NLO: sum of contributions of mesons and mass shift. LO+NLO–final result.

|                | LO  | Mass Shift | Mass Split | Mesons | All NLO | LO+NLO |
|----------------|-----|------------|------------|--------|---------|--------|
| Dipole        | -0.20 | $6.07 \times 10^{-2}$ | $1.26 \times 10^{-2}$ | $1.09 \times 10^{-2}$ | $7.45 \times 10^{-2}$ | -0.13  |
| QuasiBessel   | -0.18 | $6.11 \times 10^{-2}$ | $1.29 \times 10^{-2}$ | $2.99 \times 10^{-3}$ | $7.70 \times 10^{-2}$ | -0.10  |

VI. EVALUATION OF THE CONSTANT $l_7$

According to [1], it is possible to evaluate the constant $l_7$ from the correlator $\langle P^3 P^0(0) \rangle$ as

$$P_2(q) = \int d^4xe^{iqx}\langle P^3(x)P^0(0) \rangle = \frac{G_\pi}{M_\pi^2 - q^2} + O\left(q^2\right) = \frac{8B^3}{q^2 - m_u^2}l_7 + O\left(m, q^2\right), \quad (50)$$

where the constant $B$ is one of the phenomenological parameters of the chiral lagrangian (see Table II), the mass of the pion $m_\pi = 0$ in the limit $m \to 0$ and $m_u, m_d$ are the current quark masses. Since we are interested only in the residue of the correlator, we should consider only 1-particle reducible diagrams with pion in the intermediate state.

In the leading order, there are two diagrams shown in the Figure 1. Obviously, only the diagram on the right-hand side contributes to the residue, yielding

$$P_2^{LO}(q) = \sum_{i,j=\eta,\phi} L_i^{LO}(q)R_j^{LO}(q)\Pi_{ij}(q) = L_\eta(q)L_\phi(q)(\Pi_{\eta\eta}(q) + \Pi_{\phi\phi}(q)) + (L_\eta^2(q) + L_\phi^2(q))\Pi_{\eta\phi}(q), \quad (51)$$

where

$$L_\eta^{LO}(q) = -\frac{1}{2} \int \frac{d^4p}{(2\pi)^4}Mf(p)f(p+q)\left[Tr\left(S_+(p)\gamma_5S_+(p+q)\gamma_5\right) + Tr\left(S_-(p)\gamma_5S_-(p+q)\gamma_5\right)\right] = \quad (52)$$

$$= 4iN_c\int \frac{d^4p}{(2\pi)^4}Mf(p)f(p+q)\left[p^2 + p\cdot q + \mu_+(p)\mu_+(p+q)\right]\left[p^2 + p\cdot q + \mu_-(p)\mu_-(p+q)\right], \quad (53)$$

$$L_\phi^{LO}(q) = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4}Mf(p)f(p+q)\left[Tr\left(S_+(p)\gamma_5S_+(p+q)\gamma_5\right) - Tr\left(S_-(p)\gamma_5S_-(p+q)\gamma_5\right)\right] =$$

$$= -4N_c\int \frac{d^4p}{(2\pi)^4}Mf(p)f(p+q)\left[p^2 + p\cdot q + \mu_+(p)\mu_+(p+q)\right]\left[p^2 + p\cdot q + \mu_-(p)\mu_-(p+q)\right], \quad (54)$$

and we used identities

$$R_\eta^{LO}(q) = L_\phi^{LO}(q), \quad (55)$$

In the next-to-leading order there are seven diagrams shown in the Figure 2. Obviously, only the diagrams 3-6 from the second and the third row contribute to the residue in pion pole. The explicit expressions for the corresponding diagrams are given in Appendix A. Using (54) one may immediately get

$$Res\Pi_{\phi\phi}(q) \approx -\pi^{-1}_{\eta\phi}(0)\Pi_{\eta}(0)Res\Pi_{\phi\phi}(q) + O\left(\delta m^3, m\right) \quad (56)$$
Figure 2: Contribution to the \((P^3 P^0)\)-correlator in the next-to-leading order.

| QuasiBessel | LO  | Mass shift | Mass split | Meson | All NLO | Total         |
|-------------|-----|------------|------------|-------|---------|---------------|
| \(-i L_0\)  | -4.07 \times 10^{-2} | -1.53 \times 10^{-2} | 0          | -2.33 \times 10^{-2} | -3.87 \times 10^{-2} | 1.94 \times 10^{-2} |
| \(L_\phi\)   | -0.15 \times 10^{-3} | -8.03 \times 10^{-3} | -2.34 \times 10^{-3} | -6.93 \times 10^{-3} | -1.73 \times 10^{-2} | -1.74 \times 10^{-2} |
| \(l_7\)      | 0.17 \times 10^{-4} | -                     | -         |        |         | 1.198 \times 10^{-4} |
| Dipole       | -4.35 \times 10^{-2} | -1.45 \times 10^{-2} | 0          | -2.38 \times 10^{-2} | -3.83 \times 10^{-2} | 5.22 \times 10^{-2} |
| \(-i L_0\)  | 6.71 \times 10^{-3} | -1.06 \times 10^{-2} | -2.10 \times 10^{-3} | -1.70 \times 10^{-2} | -2.97 \times 10^{-2} | -2.30 \times 10^{-2} |
| \(l_7\)      | 0.34 \times 10^{-3} | -                     | -         |        |         | 1.00 \times 10^{-3} |

Table IV: Evaluation of the residue \(\text{Res}_{q^2=-m_2^2} \langle \bar{P}_3 P_0 \rangle\). See Eq. (57) for more details on meaning of \(L_\eta, L_\phi\). The first column is the LO result, columns 2-5 are NLO corrections, column 6 is the total result. In columns 7-8 we give results for \(l_7\) in LO and NLO (See the Table III for numbers used in evaluation).

So in evaluation of the residue \(\text{Res}_{q^2=0} \langle \bar{P}_3 P_0 \rangle \approx \text{Res}_{q^2=0} \langle \bar{P}_3 P_0 \rangle\) one has to keep only the terms

\[
\langle \bar{P}_3 P_0 \rangle = \sum_{i,j=\eta,\phi} L_i^{LO}(q)R_j^{LO}(q)\Pi_{ij}(q) = L_\eta(q)L_\phi(q)\Pi_{\eta\phi}(q) + (L_\eta^2(q) + L_\phi^2(q))\Pi_{\eta\phi}(q) + \text{non-singulars},
\]

all the other terms which are not written out explicitly do not contribute to the residue.

Results of numerical evaluation are presented in Table IV As one can see, the model is extremely sensitive to the change of formfactor. The reasons of such strong dependence will be discussed in Section VII.

VII. CONCLUSION

In this paper we evaluated the effects of the current quark mass split on the dynamical mass, quark condensate and correlator \((P_3 P_0)\). From these data we extracted the low energy constants \(h_3, l_7\). We found that the dynamical quark mass \(\delta M\) is negative, so as one can see from the left pane of the Figure there is a momentum-dependent mass \(\delta \mu(p) \equiv \delta m + \delta M^2(p)\) has different signs for small and large momenta. Due to cancellation of these contributions, we get very strong sensitivity of all quantities discussed in this paper to the details of the instanton vacuum model, such as the shape of instanton (which comes via the formfactor) and instanton parameters. In the right pane of the Figure we demonstrate explicitly this fast dependence on the example of the leading-order integrand of \(L_\phi(0)\). As it was explained above, due to different signs of large and small-\(p\) contributions, we have partial (solid line) or almost complete (dashed line) cancellation, which leads to the strong dependence on parameters of the model. Similar behaviour is observed for all quantities where the dynamical mass split \(\delta M(p)\) contributes, both in the leading and in the next-to-leading orders.

It is necessary to note that the instanton vacuum model contains chiral doublet \((\eta, \bar{\sigma})\)-additional degree of freedom which is absent in the chiral lagrangian, and the cancellation of the different contributions is due to the dynamics of the field. If we set \(-\gamma/\lambda = 0\) in and thus effectively eliminate the contribution of the \(\sigma_3\), we can see that the dynamical mass split \(\delta \mu(p)\) is constant for all momenta \(p\), and cancellation of different regions does not happen.
Figure 3: Left: dependence of the dynamical mass split $\delta \mu(p) \equiv \delta m + \delta M f^2(p)$ on the quark momentum $p$. Right: Instanton shape dependence of the integrand of $L_\phi(0)$ in the leading order. $g(p)$ is the integrand of the Eqn.\ref{eq:53}.

Figure 4: [Color online] Dependence of the constant $l_7$ on the instanton vacuum parameters $\rho$ and $R$

One of the consequences of the above-mentioned sensitivity of $l_7$ to model details is that uncertainty of the instanton vacuum parameters (average instanton size $\rho$ and inter-instanton distance $R$) leads to increased uncertainty in the final prediction for $l_7$. As it has been discussed in \cite{22}, different methods estimate the model parameters are in the range $\rho \sim 0.32 - 0.35$ fm, $R \sim 0.8 - 1$ fm. While the uncertainty in $\rho$, $R$ is just $\sim 10\%$ and is unimportant for most evaluations, for the constant $l_7$ it leads to sizeable uncertainty in the final result. Using the Figure\ref{fig:4} we may get for $l_7$ an estimate

$$l_7 \sim (6.6 \pm 2.4) \times 10^{-4}.$$ \hfill (58)

The result \cite{58} agrees with a phenomenological estimate \cite{2} within uncertainty limits. Using \cite{11}, we may obtain for the pure QCD contribution to the pion mass difference

$$(m_{\pi^+}^2 - m_{\pi^0}^2)_{QCD} \sim 1.4 \times 10^{-5} GeV^2,$$ \hfill (59)
i.e. \(\sim 1\%\) of the experimentally observed difference. This result does not contradict the well-known fact that the pion mass difference has electromagnetic origin [25–27].

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Appendix A: Explicit expressions for some vertices

In this section for the sake of completeness we would like to present some explicit expressions for the meson-quark interaction vertices which are used in this paper. For the quark-meson vertices [25, 27] which come into the gap Eqs. (22–24) we may get

\[
V^{(gap)(ij)}_2(q)\Pi_{ij}(q) = \frac{1}{\sigma^2} \int \frac{d^4p}{(2\pi)^4} Tr \left( \frac{M(p)}{\bar{p} + \gamma + i\mu(p) + i\tau_3\delta\mu(p)} \Gamma_i \times \frac{M(p + q)}{\bar{p} + \gamma + i\mu(p + q) + i\tau_3\delta\mu(p + q)} \Gamma_j \right) = \frac{1}{2\sigma^2} \int \frac{d^4p}{(2\pi)^4} M(p)M(p + q) \times \left\{ \Pi_{\sigma\sigma}(q) - \Pi_{\sigma_3\sigma_3}(q) \right\} Tr \left[ S_+(p)S_+(p + q) + S_-(p)S_-(p + q) \right] + 2\Pi_{\sigma_3\sigma_3} \sum_{i, j = 1, 2} \Pi_{ij}(k) \left( \bar{p} + \gamma + i\mu(p + q) + i\tau_3\delta\mu(p + q) \right) + \left( \Pi_{\sigma\sigma}(q) - \Pi_{\phi_3\phi_3}(q) \right) \sum_{i, j = 1, 2} \Pi_{ij}(k) \left( \bar{p} + \gamma + i\mu(p + q) + i\tau_3\delta\mu(p + q) \right) \left( \bar{p} + \gamma + i\mu(p + q) + i\tau_3\delta\mu(p + q) \right) \right\}
\]

\[
V^{(gap)(ij)}_3(q)\Pi_{ij}(q) = \frac{i}{\sigma^2} \int \frac{d^4p}{(2\pi)^4} Tr \left( \frac{M(p)}{\bar{p} + \gamma + i\mu(p) + i\tau_3\delta\mu(p)} \Gamma_i \times \frac{M(p + q)}{\bar{p} + \gamma + i\mu(p + q) + i\tau_3\delta\mu(p + q)} \Gamma_j \right) = \frac{1}{2\sigma^2} \int \frac{d^4p}{(2\pi)^4} M^2(p)M(p + q) \times \left\{ \Pi_{\sigma\sigma}(q) - \Pi_{\sigma_3\sigma_3}(q) \right\} Tr \left[ S_+(p)S_+(p + q) + S_-(p)S_-(p + q) \right] + 2\Pi_{\sigma_3\sigma_3} \sum_{i, j = 1, 2} \Pi_{ij}(k) \left( \bar{p} + \gamma + i\mu(p + q) + i\tau_3\delta\mu(p + q) \right) + \left( \Pi_{\sigma\sigma}(q) - \Pi_{\phi_3\phi_3}(q) \right) \sum_{i, j = 1, 2} \Pi_{ij}(k) \left( \bar{p} + \gamma + i\mu(p + q) + i\tau_3\delta\mu(p + q) \right) \left( \bar{p} + \gamma + i\mu(p + q) + i\tau_3\delta\mu(p + q) \right) \right\}
\]
For the components of the leading order meson propagator (40) we may get the following explicit expressions [29]

\[ \Gamma_{3}^{(q\alpha p)(ij)} \Pi_{ij}(q) = \frac{i}{\sigma^{2}} \int \frac{d^{4}p}{(2\pi)^{4}} \text{Tr} \left( \frac{M(p)\delta M(p)\tau_{5}}{(\bar{\rho} + i\mu(p) + i\tau_{3}\delta\mu(p))^{2}} \right) \Gamma_{i} \times \]

\[ = \frac{1}{2\sigma^{2}} \int \frac{d^{4}p}{(2\pi)^{4}} M(p)\delta M(p)M(p + q) \times \{ \]

\[ (\Pi_{\sigma\sigma}(q) - \Pi_{\sigma_{3}\sigma_{3}}(q)) \text{Tr} [S_{+}(p)S_{+}(p)S_{+}(p + q) - S_{-}(p)S_{-}(p)S_{-}(p + q)] + \]

\[ 2\Pi_{\sigma_{3}\sigma_{3}} \text{Tr} [S_{+}(p)S_{+}(p)S_{+}(p + q) + S_{-}(p)S_{-}(p)S_{-}(p + q)] - \]

\[ \sum_{i_{\perp} = 1, 2} \Pi_{i_{\perp}}(k) \text{Tr} [S_{+}(p)S_{+}(p)S_{-}(p + q) - S_{-}(p)S_{-}(p)S_{+}(p + q)] - \]

\[ (\Pi_{\eta\eta}(k) - \Pi_{\phi_{3}\phi_{3}}(k)) \text{Tr} [S_{+}(p)S_{+}(p)S_{-}(p + q) - S_{-}(p)S_{-}(p)S_{+}(p + q)] - \]

\[ 2\Pi_{\phi_{3}\phi_{3}}(k) \text{Tr} [S_{+}(p)S_{+}(p)S_{-}(p + q) + S_{-}(p)S_{-}(p)S_{-}(p + q)] + \]

\[ \sum_{i_{\perp} = 1, 2} \Pi_{i_{\perp}}(k) \text{Tr} [S_{+}(p)S_{+}(p)S_{+}(p + q) - S_{-}(p)S_{-}(p)S_{+}(p + q)] \}

\]  

For the components of the leading order meson propagator [40] we may get the following explicit expressions [29]

\[ (\Pi^{-1})_{\sigma\sigma} = 4 + \frac{1}{2\sigma^{2}} \text{Tr} (Q_{+}(p)Q_{+}(p + q)) + \frac{1}{2\sigma^{2}} \text{Tr} (Q_{-}(p)Q_{-}(p + q)) \]

\[ (\Pi^{-1})_{\sigma_{3}\sigma_{3}} = 4 - \frac{1}{2\sigma^{2}} \text{Tr} (Q_{+}(p)Q_{+}(p + q)) - \frac{1}{2\sigma^{2}} \text{Tr} (Q_{-}(p)Q_{-}(p + q)) \]

\[ (\Pi^{-1})_{\sigma_{3}\sigma_{3}} = 4 \frac{1}{2\sigma^{2}} \text{Tr} (Q_{+}(p)Q_{+}(p + q)) - \frac{1}{2\sigma^{2}} \text{Tr} (Q_{-}(p)Q_{-}(p + q)) \]

\[ (\Pi^{-1})_{\eta\eta} = 4 \frac{1}{2\sigma^{2}} \text{Tr} (Q_{+}(p)Q_{+}(p + q)) - \frac{1}{2\sigma^{2}} \text{Tr} (Q_{-}(p)Q_{-}(p + q)) \]

\[ (\Pi^{-1})_{\phi_{3}\phi_{3}} = 4 \frac{1}{2\sigma^{2}} \text{Tr} (Q_{+}(p)Q_{+}(p + q)) + \frac{1}{2\sigma^{2}} \text{Tr} (Q_{-}(p)Q_{-}(p + q)) \]

\[ (\Pi^{-1})_{i_{\perp}} = 4 \frac{1}{2\sigma^{2}} \sum_{i=\pm} \text{Tr} (Q_{\alpha}(p)\Gamma_{\alpha_{i}}Q_{\alpha}(p + q)\Gamma_{i}) , \ (i, j) \neq (0, 3) \]

where \( Q_{\pm}(p) = \frac{iM(p)}{p + i\mu(p) \pm \delta\mu(p)} \), \( \tilde{Q}_{\pm}(p) = -\gamma_{5}Q_{\pm}(p)\gamma_{5} = \frac{iM(p)}{p - i\mu(p) \pm \delta\mu(p)} \).
The 1-loop correction to the propagator $\Pi^{(1-mes)}(0)$ has a form

$$
\Pi_{\eta\phi}^{(1-mes)}(0) = \int \frac{d^4q}{(2\pi)^4} \sum_{ij} \Pi_{ij}(q)V_{ij}(q) =
$$

$$
\int \frac{d^4q}{(2\pi)^4} \sum_{ij} \Pi_{ij}(q) i\frac{1}{2\sigma} \int \frac{d^4p}{(2\pi)^4} Tr \left( 2Q(p)\Gamma_\eta Q(p)\Gamma_\phi Q(p) + Q(p)\Gamma_\eta Q(p)\Gamma_\phi Q(p) \right) =
$$

$$
\frac{i}{2\sigma} \int \frac{d^4p}{(2\pi)^4} M^2(p)M(p + q) \times \left\{ 
\Pi^{(0)}_{\sigma\sigma} - \Pi^{(0)}_{\phi_1\phi_2}(q) \right\} Tr \left( -2M(p) \left( \bar{S}_+(p)S_+(p)S_+(p)S_+ - \bar{S}_-(p)S_-(p)S_-(p)S_+ + S_+(p)S_+(p)S_+(p)S_+ - \bar{S}_-(p)S_-(p)S_-(p)S_+ \right) \right) +
\right.

$$

$$
\sum_{i=1,2} \Pi^{(0)}_{\phi_i}(q) Tr \left( -2M(p) \left( \bar{S}_+(p)S_+(p)S_+(p)S_+ - \bar{S}_-(p)S_-(p)S_-(p)S_+ \right) \right).
$$

The two-loop correction to $\Pi_{\eta\phi}(0)$ has a form

$$
\Pi_{\eta\phi}^{(2-mes)}(0) = -4 \sigma^2 \int \frac{d^4q}{(2\pi)^4} \Pi_{ij}(q)V_{ik}(q)V_{jl}(q), \quad (A2)
$$

where

$$
V_{ik}(q) = \left[ Tr_p \left( Q(p)\gamma_5 Q(p)\bar{S}_+(p)S_+(p)S_+(p)S_+ - \bar{S}_-(p)S_-(p)S_-(p)S_+ \right) \right],
$$

$$
V_{ij}(q) = \left[ Tr_p \left( Q(p)i\gamma_5\gamma_3 Q(p)\bar{S}_+(p)S_+(p)S_+(p)S_+ - \bar{S}_-(p)S_-(p)S_-(p)S_+ \right) \right].
$$

In explicit form, with account of $O(\delta m)$-counting, $\[A2\]$ has a form

$$
\sum_{ijkl} \Pi_{ij}(q)V_{ik}(q)V_{kl}(q)V_{jl}(q) = \quad (A3)
$$

$$
+ V^{(\eta)}_{\sigma_1\sigma_2} \Pi^{(\phi)}_{\sigma_1\sigma_2} + V^{(\phi)}_{\sigma_1\sigma_2} \Pi^{(\eta)}_{\sigma_1\sigma_2} + V^{(\phi)}_{\sigma_1\sigma_2} \Pi^{(\eta)}_{\sigma_1\sigma_2} + V^{(\phi)}_{\sigma_1\sigma_2} \Pi^{(\eta)}_{\sigma_1\sigma_2},
$$

$$
+ V^{(\eta)}_{\phi_1\phi_2} \Pi^{(\phi)}_{\phi_1\phi_2} + V^{(\phi)}_{\phi_1\phi_2} \Pi^{(\eta)}_{\phi_1\phi_2} + V^{(\phi)}_{\phi_1\phi_2} \Pi^{(\eta)}_{\phi_1\phi_2} + V^{(\phi)}_{\phi_1\phi_2} \Pi^{(\eta)}_{\phi_1\phi_2}.
$$

$$
+ V^{(\phi)}_{\sigma_1\sigma_2} \Pi^{(\phi)}_{\sigma_1\sigma_2} + V^{(\phi)}_{\sigma_1\sigma_2} \Pi^{(\phi)}_{\sigma_1\sigma_2} + V^{(\phi)}_{\sigma_1\sigma_2} \Pi^{(\phi)}_{\sigma_1\sigma_2} + V^{(\phi)}_{\sigma_1\sigma_2} \Pi^{(\phi)}_{\sigma_1\sigma_2},
$$

$$
+ 2V^{(\eta)}_{\phi_1\phi_2} \Pi^{(\phi)}_{\phi_1\phi_2} + 2V^{(\phi)}_{\phi_1\phi_2} \Pi^{(\phi)}_{\phi_1\phi_2} + 2V^{(\phi)}_{\phi_1\phi_2} \Pi^{(\phi)}_{\phi_1\phi_2}.
$$

(A4)

(A5)

(A6)
where the vertices have an explicit form:

\[ V^{(q)}_{\sigma \delta}(q) = \text{Tr}_p \left( Q(p) \gamma_5 Q(p) Q(p + q) i\tau_3 \gamma_5 \right) = \]

\[ \frac{i}{2} \int \frac{d^d p}{(2\pi)^d} M^2(p) M(p + q) \text{Tr}_p \left( \tilde{S}_+(p) S_+(p) S_+(p + q) - \tilde{S}_-(p) S_-(p) S_-(p + q) \right) \mathcal{O}(\delta m) \]  
(A7)

\[ V^{(\phi)}_{\sigma \phi}(q) = \text{Tr}_p \left( Q(p) i\gamma_5 r^\phi Q(p) Q(p + q) i\tau_3 \gamma_5 \right) = \]

\[ \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} M^2(p) M(p + q) \text{Tr}_p \left( \tilde{S}_+(p) S_+(p) S_+(p + q) + \tilde{S}_-(p) S_-(p) S_-(p + q) \right)_{\delta m=0} \]  
(A8)

\[ V^{(\eta)}_{\sigma \eta}(q) = \text{Tr}_p \left( Q(p) \gamma_5 Q(p) \gamma_5 Q(p + q) i\tau_3 \right) = \]

\[ \frac{i}{2} \int \frac{d^d p}{(2\pi)^d} M^2(p) M(p + q) \text{Tr}_p \left( S_+(p) \tilde{S}_+(p) S_+(p + q) + S_-(p) \tilde{S}_-(p) S_-(p + q) \right) \mathcal{O}(\delta m) \]  
(A9)

\[ V^{(\eta)}_{\sigma \phi}(q) = \text{Tr}_p \left( Q(p) i\tau_3 \gamma_5 Q(p) + q) i\tau_3 \gamma_5 \right) = \]

\[ \frac{i}{2} \int \frac{d^d p}{(2\pi)^d} M^2(p) M(p + q) \text{Tr}_p \left( S_+(p) S_+(p + q) S_+(p + q) \right)_{\delta m=0} \]  
(A10)

\[ V^{(\eta)}_{\sigma \phi}(q) = \text{Tr}_p \left( Q(p) \gamma_5 Q(p) \gamma_5 Q(p + q) i\tau_3 \gamma_5 \right) = \]

\[ \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} M^2(p) M(p + q) \text{Tr}_p \left( \tilde{S}_+(p) S_+(p) S_+(p + q) + \tilde{S}_-(p) S_-(p) S_-(p + q) \right) \mathcal{O}(\delta m) \]  
(A11)

\[ V^{(\eta)}_{\sigma \phi}(q) = \text{Tr}_p \left( Q(p) \gamma_5 Q(p) \gamma_5 Q(p + q) i\tau_3 \gamma_5 \right) = \]

\[ \frac{i}{2} \int \frac{d^d p}{(2\pi)^d} M^2(p) M(p + q) \text{Tr}_p \left( S_+(p) \tilde{S}_+(p) S_+(p + q) + S_-(p) \tilde{S}_-(p) S_-(p + q) \right) \mathcal{O}(\delta m) \]  
(A12)

Using the last four equations \((A15, A18)\), the two-loop contribution \((A3)\) may be cast into the form

\[ \sum_{ijkl} \Pi_{ij}(q) \Pi_{kl}(q) V^{(\eta)}_{ijkl}(q) V^{(\phi)}_{ijkl}(q) = \]

(A19)
\[
\sum_{ij} \Pi_{ij}(q)V_{ij}^{(\delta \eta \eta)}(q) \approx \frac{i e}{2 \sigma^2} \int \frac{d^4p}{(2\pi)^4} M(p)M(p+q) \times \{
\left( \Pi_{\sigma \sigma}^{(0)}(q) - \Pi_{\sigma^3 \sigma^3}^{(0)}(q) \right) T_r [S_+(p)S_+(p + q) - S_-(p)S_-(p + q)]_{\sigma(\delta m)} + \\
2\Pi_{\sigma^3}(q) T_r [S_+(p)S_+(p + q) + S_-(p)S_-(p + q)]_{\sigma(\delta m)} - \\
\sum_{i=1,2} \Pi_{\sigma_i}(q) T_r [S_+(p)S_+(p + q) + S_-(p)S_-(p + q)]_{\sigma(\delta m)} - \\
\left( \Pi_{\eta \eta}^{(0)}(q) - \Pi_{\phi^3 \phi^3}^{(0)}(q) \right) T_r [S_+(p)S_+(p + q) - S_-(p)S_-(p + q)]_{\sigma(\delta m)} - \\
2\Pi_{\phi^3}(q) T_r [S_+(p)S_+(p + q) + S_-(p)S_-(p + q)]_{\sigma(\delta m)} + \\
\sum_{i=1,2} \Pi_{\phi_i}(q) T_r [S_+(p)S_+(p + q) + S_-(p)S_-(p + q)]_{\sigma(\delta m)}
\}
\]

In complete analogy we may evaluate the diagrams shown in the Figure \[2\] and get the following explicit vertices

\textbf{a. Diagram \#3}

Left part:

\[
L^{(3)}_\eta = 4 N_c \int \frac{d^4p}{(2\pi)^4} M^2 f(p)f(p + q)f^2(p + k)f^2(p + q + k) \times \{
\left( \Pi_{\sigma \sigma}(k) - \Pi_{\sigma^3 \sigma^3}(k) \right) T_r \left[ \bar{S}_+(p)S_+(p + k)S_+(p + q + k)S_+(p + q) + \bar{S}_-(p)S_-(p + k)S_-(p + q + k)S_-(p + q) \right] + \\
2\Pi_{\sigma^3}(k) T_r \left[ \bar{S}_+(p)S_+(p + k)S_+(p + q + k)S_+(p + q) - \bar{S}_-(p)S_-(p + k)S_-(p + q + k)S_-(p + q) \right] - \\
\sum_{i=1,2} \Pi_{\sigma_i}(k) T_r \left[ \bar{S}_+(p)S_-(p + q + k)S_-(p + q + k)S_+(p + q) + \bar{S}_-(p)S_+(p + k)S_+(p + k)S_+(p + q + k)S_+(p + q) \right] + \\
\left( \Pi_{\eta \eta}(k) - \Pi_{\phi^3 \phi^3}(k) \right) T_r \left[ S_+(p)S_+(p + q + k)S_+(p + q + k)S_+(p + q) + S_-(p)S_-(p + k)S_-(p + q + k)S_-(p + q) \right] + \\
2\Pi_{\phi^3}(k) T_r \left[ S_+(p)S_+(p + q + k)S_+(p + q + k)S_+(p + q) - S_-(p)S_-(p + k)S_-(p + q + k)S_-(p + q) \right] - \\
\sum_{i=1,2} \Pi_{\phi_i}(k) T_r \left[ S_+(p)S_-(p + q + k)S_-(p + q + k)S_+(p + q) + S_-(p)S_+(p + k)S_+(p + k)S_+(p + q + k)S_+(p + q) \right]
\}
\]

Right part has been evaluated in \[\text{\ref{fig:diagram1}}\] and \[\text{\ref{fig:diagram2}}\].
b. Diagram #4

Left part has been evaluated in \([52, 53]\).

Right part:

\[
R_{\eta}^{(4)}(q) = L_{\phi}^{(3)}(q)
\]

\[
R_{\phi}^{(4)}(q) = L_{\eta}^{(3)}(q)
\]

Left part:

\[
L_{\eta}^{(5)}(q) = -8 N_c \int \frac{d^4 p}{(2\pi)^4} M^3 f^4(p) f^2(p+k) f(p+q) \times \{ \quad (\Pi_{\sigma\sigma}(k) - \Pi_{\sigma_3\sigma_3}(k)) \quad T r \left[ S_+(p) S_+(p+k) S_+(p) S_+(p+q) + S_-(p) S_-(p+k) S_-(p) S_-(p+q) \right] + \\
2 \Pi_{\sigma\sigma_3}(k) \quad T r \left[ S_+(p) S_+(p+k) S_+(p) S_+(p+q) - S_-(p) S_-(p+k) S_-(p) S_-(p+q) \right] - \\
\sum_{i=1,2} \Pi_{\sigma_i}(k) \quad T r \left[ S_+(p) S_-(p+k) S_+(p) S_+(p+q) + S_-(p) S_+(p+k) S_-(p) S_-(p+q) \right] - \\
(\Pi_{\eta\eta}(k) - \Pi_{\phi_3\phi_3}(k)) \quad T r \left[ S_+(p) S_+(p+k) S_+(p) S_+(p+q) - S_-(p) S_-(p+k) S_-(p) S_-(p+q) \right] - \\
2 \Pi_{\eta\phi_3}(k) \quad T r \left[ S_+(p) S_+(p+k) S_+(p) S_+(p+q) - S_-(p) S_-(p+k) S_-(p) S_-(p+q) \right] + \\
\sum_{i=1,2} \Pi_{\phi_i}(k) \quad T r \left[ S_+(p) S_-(p+k) S_+(p) S_+(p+q) + S_-(p) S_+(p+k) S_-(p) S_-(p+q) \right] \}
\]

Right part has been evaluated in \([54, 55]\).

c. Diagram #5

Left part:

\[
L_{\phi}^{(5)}(q) = -8 N_c \int \frac{d^4 p}{(2\pi)^4} M^3 f^4(p) f^2(p+k) f(p+q) \times \{ \quad (\Pi_{\sigma\sigma}(k) - \Pi_{\sigma_3\sigma_3}(k)) \quad T r \left[ S_+(p) S_+(p+k) S_+(p) S_+(p+q) - S_-(p) S_-(p+k) S_-(p) S_-(p+q) \right] + \\
2 \Pi_{\sigma\sigma_3}(k) \quad T r \left[ S_+(p) S_+(p+k) S_+(p) S_+(p+q) + S_-(p) S_-(p+k) S_-(p) S_-(p+q) \right] - \\
\sum_{i=1,2} \Pi_{\sigma_i}(k) \quad T r \left[ S_+(p) S_-(p+k) S_+(p) S_+(p+q) - S_-(p) S_+(p+k) S_-(p) S_-(p+q) \right] - \\
(\Pi_{\eta\eta}(k) - \Pi_{\phi_3\phi_3}(k)) \quad T r \left[ S_+(p) S_+(p+k) S_+(p) S_+(p+q) - S_-(p) S_-(p+k) S_-(p) S_-(p+q) \right] - \\
2 \Pi_{\eta\phi_3}(k) \quad T r \left[ S_+(p) S_+(p+k) S_+(p) S_+(p+q) + S_-(p) S_-(p+k) S_-(p) S_-(p+q) \right] + \\
\sum_{i=1,2} \Pi_{\phi_i}(k) \quad T r \left[ S_+(p) S_-(p+k) S_+(p) S_+(p+q) - S_-(p) S_+(p+k) S_-(p) S_-(p+q) \right] \}
\]

Right part has been evaluated in \([52, 53]\).

d. Diagram #6

Left part has been evaluated in \([52, 53]\).

Right part:

\[
R_{\eta}^{(6)}(q) = L_{\phi}^{(5)}(q)
\]

\[
R_{\phi}^{(6)}(q) = L_{\eta}^{(5)}(q)
\]
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[28] Note that the transition form Minkowsky to Euclid requires to change the signs of all quark condensates, \((\bar{q}q) \rightarrow -\langle \bar{q}q \rangle\)
[29] Notice that off-diagonal component \(\Pi_{43}^1\) is real, not imaginary. This is related to our previous choice of imaginary \(\langle \sigma_3 \rangle = -i|\langle \sigma_3 \rangle|\)