Non-Gaussian Error Distributions of Galactic Rotation Speed Measurements

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We construct the error distributions for the galactic rotation speed ($\Theta_0$) using 137 data points from measurements compiled in De Grijs et al.\cite{1}, with all observations normalized to the galactocentric distance of 8.3 kpc. We then checked (using the same procedures as in works by Ratra et al.) if the errors constructed using the weighted mean and the median as the estimate, obey Gaussian statistics. We find using both these estimates that they have much wider tails than a Gaussian distribution. We also tried to fit the data to three other distributions: Cauchy, double-exponential, and Students-t. The best fit is obtained using the Students-t distribution for $n = 2$ using the median value as the central estimate, corresponding to a $p$-value of 0.1. We also calculate the median value of $\Theta_0$ using all the data as well as using the median of each set of measurements based on the tracer population used. Because of the non-gaussianity of the residuals, we point out that the subgroup median value, given by $\Theta_{med} = 219.65 \text{ km/sec}$ should be used as the central estimate for $\Theta_0$.

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I. INTRODUCTION

Recently, de Grijs and Bono \cite{1} (hereafter G17), compiled a list of 162 galactic rotation speed measurements (denoted as $\Theta_0$) using data from all the published literature starting from 1927 right up to 2017. Two main goals of this meta-analysis was to look for evidence for publication bias and to check how close is the central estimate from all these measurements to the IAU recommended value of $\Theta_0 = 220 \text{ km/sec}$ \cite{2}. Although, no evidence for such a bias was seen, G17 found evidence for systematic biases in the measurements of $\Theta_0$ between the different tracer populations. The estimated value for Galactic rotation speed obtained in G17 using all the post-1985 measurements is given by $\Theta_0 = 225 \pm 3\text{(stat)} \pm 10\text{(syst)} \text{ km/sec}$, after positing a galactocentric distance of 8.3 kpc.

In the last decade, Ratra and collaborators have used a variety of astrophysical datasets to test the non-Gaussianity of the error distributions from these measurements. The datasets they explored for this purpose include measurements of $H_0$ \cite{3}, Lithium-7 measurements \cite{4} (see also \cite{5}), distance to LMC \cite{6}, distance to galactic center \cite{7}. Evidence for non-Gaussian errors has also been found in HST Key project data \cite{8}. For each of these datasets, they have fitted the data to a variety of probability distributions. For all of these studies, they have found the error distributions to be non-Gaussian. As a consequence they have argued that median statistics should be used for central estimates of these parameters instead of the weighted mean. Median statistics has therefore been used to obtain central estimates of $H_0$ [9–11], $G$ [11], mean matter density [12] and other cosmological parameters [13].

Inspired by these works, we revisit the issue of getting the best estimate of $\Theta_0$ from the catalog compiled in G17. The first step in this analysis is to check for non-Gaussianity of the residuals. The importance of checking for non-Gaussianity of the measurement errors for a large suite of astrophysical measurements has been stressed in a number of works in astrophysics \cite{3–9}. Most recently, its importance in other fields such as nuclear and particle physics, medicine and toxicology has also been elucidated by Bailey \cite{14}.

It is common practice to assume that datasets are Gaussian. However, this is not always the case. By carrying out Gaussianity tests on error distributions of measurements, several insights can be obtained. One application is the possibility of reduced significance of discrepancies between observed and expected values. Usually, when a central estimate for a physical quantity is needed, one typically calculates a weighted average of all the measurements. One assumption implicitly made herein is that the weighted mean error distributions have a Gaussian distribution. If this is not the case, then one cannot directly use weighted mean estimates or $\chi^2$ analysis for parameter estimation. One then needs to resort to median statistics [9, 11], which does not invoke the measurement errors and is not affected by the non-Gaussianity [7]. If the residuals are non-Gaussian, one possibility is that the errors are underestimated and there are additional unaccounted systematic errors , which could be “known unknowns” or “unknown unknowns”. Another possible reason could be due to outliers, which may arise due to egregious measurements. These outliers could potentially bias any estimates. Conversely, if the tails in a distribution are narrower than a Gaussian, it implies that the different measurements are correlated. For this reason a large number of studies in astrophysics and other fields have investigated and found evidence for non-Gaussianity for a diverse suite of measurements.

The galactocentric velocity is of tremendous importance in both galactic astrophysics as well as cosmology,
and hence in order to obtain a robust estimate of its central value, one needs to check for non-Gaussianity of errors.

The outline of this paper is as follows. The dataset used for our analysis is described in Sect. II. Our analysis procedure and results are described in Sect. III. The corresponding analysis on each sub-group of measurements is discussed in Sect. IV. We conclude in Sect. V.

II. DATASET USED

We briefly review the data on the galactic rotation speed measurements compiled by G15. More details can be found in their paper [1]. The main goal of their paper (intended as a follow-up to a series of papers [15–18] looking for similar effects in other observables) was to undertake a meta-analysis of all the measurements of $\Theta_0$ from published literature in order to look for intrinsic differences between the different categories of measurements of $\Theta_0$. They also wanted to see if there is evidence for publication bias or “bandwagon” effect.

The previous comprehensive review of literature on galactic rotation velocities was carried out by Kerr and Lynden-Bell [2], which explained the reasoning behind the IAU recommended value of 220 km/sec at the solar circle. G17 searched the NASA/Astrophysics Data System (ADS) by looking for articles referring to the Milky Way and using one of the following keywords in the abstract search: ‘rotation curve’, ‘kinematics’ (including its variants), ‘dynamics’, and ‘Oort’. They found a total of 9,690 articles starting from Oort’s original papers in 1927 [19, 20] until the end of June 2017. They data mined all these papers looking for new values of Galactic rotation constants. These papers either provided a direct measurements of the galactic rotation speed or the Oort constants $A$ and $B$ [19, 20], from which $\Theta_0$ is given by $(A - B)R_0$, where $R_0$ is the Galactocentric distance. Since majority of the $\Theta_0$ measurements hinge on the determination of $R_0$, G17 homogenized all measurements of $\Theta_0$ to a common value of $R_0 = 8.3$ kpc, based on recommendations from their previous set of studies [18].

In all, they found a total of 162 measurements. These consist of seven different types of stellar population tracers. All these measurements along with their statistical uncertainties have been uploaded on the internet. We note that no systematic errors have been included in the analysis. In addition to these measurements compiled by G17, we used two additional measurements compiled by Salucci et al [21, 22], which are not listed in G17. In one of them [21], $\Theta_0/R_0$ has been estimated to be $30.3 \pm 0.9$ km/sec/kpc. In Ref. [21], $\Theta_0$ has been estimated to be $239 \pm 7$ km/sec. We also found that one measurement by Glushkova et al [23] was incorrectly tabulated on the website. At the time of writing, the website reported a measurement of $277 \pm 3$ km/sec at a distance of 7.4 kpc. However, this is a typographical error on the website and the correct measurement reported in the paper is $204 \pm 15$ km/s. For our analysis, we used the correct measurement reported in the paper.

Out of these 164 measurements, we omitted 26 for which no errors were provided. We also left out the measurement in Ref. [24], corresponding to a value of 198 km/sec at 10 kpc. This value corresponds to a 23.5σ discrepancy compared to the weighted mean value. One possible reason for this low value of the rotation speed [24] is because of the simplified ansatz they used for the Galactic gravitational potential, viz. a spherical power law. We also normalized all the remaining 137 measurements of $\Theta_0$ and their associated errors, which are degenerate with galactocentric distance to a $R_0$ value of 8.3 kpc. Only five $\Theta_0$ measurements (four of them discussed in G17 and one from Salucci [22]) were not scaled, as they were independent of $R_0$. We note that for their analysis estimates of central values, G17 used only the post-1985 measurements.

III. ANALYSIS

A. Error Distribution and distribution functions

Similar to Ratra et al (4), we calculated the estimates using two methods: median statistics and weighted mean estimates. The weighted mean ($\Theta_M$) is given by [25]:

$$\Theta_M = \frac{\sum_{i=1}^{N} \Theta_i/\sigma_i^2}{\sum_{i=1}^{N} 1/\sigma_i^2},$$

(1)

where $\Theta_i$ indicates each measurement of the rotation and $\sigma_i$ indicates the total error in each measurement. The total weighted mean error is given by $\sigma_M = \frac{1}{\sum_{i=1}^{N} 1/\sigma_i^2}$. The goodness of fit is parameterized by $\chi^2$, which is given by

$$\chi^2 = \frac{1}{N-1} \sum_{i=1}^{N} (\Theta_i - \Theta_m)/\sigma_i^2$$

(2)

The $\chi^2$ defined in Eq. 2 is also referred to as reduced $\chi^2$. For a reasonably good fit, this $\chi^2$ has to be close to 1.

An alternate method to determining the central estimate is using median statistics. The main advantage of median statistics-based estimate is that it is robust against outliers and does not make use of individual error bars. The median estimate is also expected to be more robust if the error bars are not Gaussian [9–11]. More details on median statistics-based estimates and its applications to a various astrophysical datasets are

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1 See http://astro-expat.info/Data/pubbias.html for a compilation of all these measurements.
reviewed in Refs [9–13] and references therein. Using the dataset constructed by G17, we calculate the median central estimate of Θ₀ and its 68% confidence intervals, using the same prescription as in Ref. [10]. The weighted mean estimate is found to be Θ_{Mean} = 227.07 ± 0.70 km/sec and the median estimate is calculated to be Θ_{Med} = 237.58 ± 3.24 km/sec. We note that one difference between these estimates and those in G17, is that G17 did the calculations for only the post-1985 measurements, whereas we have included all the measurements tabulated in G17.

For both the estimates of Θ₀, we calculate N_{σ}, where

\[ N_{σ} = \frac{Θ_i - Θ_{CE}}{σ_{i} + σ_{CE}}. \]  

(3)

In the above equation, σ_{CE} is the error in the central estimate (which could be the median or weighted-mean based estimate). We now fit the histogram of N_{σ}, to various probability distributions as described in the next section.

B. Fits to probability distributions

We have used N_{σ} (defined in Eq. 3) for each data point, to construct a histogram for the error distributions using both the weighted mean and median central estimates. We also construct a corresponding histogram for |N_{σ}|, where the distributions were symmetrized around the central value, in the same way as in Ref. [13]. These histograms of |N_{σ}| and N_{σ} are shown in Fig. 1.

In terms of |N_{σ}|, with mean as the central estimate, only 44.53% of total distribution lies in the range |N_{σ}| ≤ 1, and 75.91% of the error distribution lies in the range |N_{σ}| ≤ 2. When we use the median as the central estimate for the error distribution, 54.02% of the distribution lies in range |N_{σ}| ≤ 1 and 79.56% of the total error distribution lies in the range |N_{σ}| ≤ 2. However, according to the Gaussian probability distribution, 68.3% of the total measurements should lie within |N_{σ}| ≤ 1 and 95.4% should lie within |N_{σ}| ≤ 2. Hence, we can conclude that the error distribution in this case deviates from a Gaussian probability distribution to a good extent. To further elucidate the discrepancy from normal distribution, we plot in Figure 2 the distribution of |N_{σ}| with a bin size of |N_{σ}| = 0.1 for both the mean and the median as central estimates compared to the normal distribution. The solid black line shows the expected Gaussian probabilities given by \( P(N_{σ}) = \frac{1}{\sqrt{2π}} \exp\left(-|N_{σ}|^2/2\right). \) The discrepancy from a normal distribution is conspicuous in both the weighted mean and median. We also find a total of five outliers with |N_{σ}| > 7.

So, as our next step, we have taken a few well-known non-Gaussian probability distribution functions into consideration, such as Cauchy, Double exponential, and Student-t distributions, in order to ascertain the probability distribution function which fits well to our error distribution.

We therefore fit the histograms of N_{σ} and |N_{σ}| to four different distributions: Gaussian, Cauchy, double-exponential, and Students-t distribution. We performed the fit using the stats.fit functionality in Scipy for each of these probability distribution functions. Figure 3 shows the Cauchy, Double exponential and Students-t probability distribution functions, fitted to the N_{σ} histograms using both the mean and median as the central estimates. To evaluate the best fit, we use the distribution-free Kolmogorov-Smirnoff (K-S) test [26]. We note that the Students-t distribution has an additional free parameter \( n \). We varied the values of \( n \) for this distribution until the p-value was maximized. The K-S probabilities for these distributions both with and without binning, and also with weighted mean/median as central estimates are shown in Table I.

From the corresponding p-values for each of the distribution functions, it can be concluded that the Student’s t distribution function (with \( n = 2 \)) fits comparatively better to our error distribution, when median is taken as the central estimate. All the remaining distribution functions give very poor fits.

C. Examination of outliers

We now briefly discuss some of the measurements, which are the cause of outliers in the |N_{σ}| distribution in Fig. 2, to see if a simple explanation can be found for these. When considering the weighted mean, we have five such measurements with |N_{σ}| > 7, corresponding to N_{σ} values of 10.7 [27], -9.7 [28], -8.6 [29], -8.4 [30], 7.3 [31].

The largest outlier comes from Branham [27], corresponding to Θ₀ = 298 ± 7 km/sec at R₀=8.15 kpc. This measurement has been made using the kinematics of OB stars using data from the Hipparcos satellite and by solving for 14 unknowns [27]. Their value of 298 km/sec was obtained after positing a linear model to simplify the equations. When a non-linear model is used, then a value close to our central estimate is obtained. For the outlier at 9.7σ [28], the website in G17 reports a measurement of 252 ± 2 km/s for R₀ = 10 kpc. This paper reports a measurement of galactic rotation curve as a function of galacto-centric distance from the UMASS StonyBrook CO survey. However, on closer examination of this paper, we find that the galactic rotation curve was plotted for two different values of Θ₀ and R₀, of which one is the value documented by G17. We note however, that no independent estimate of Θ₀ has been made from the observations. Two ad-hoc values for Θ₀ of 252 and 220 km/sec have been assumed for obtaining the galactic rotation curves. Therefore, this measurement should have been omitted from the database compiled by G17. The next outlier is at Θ₀ = 180 ± 6 km/sec [29], corresponding to 8.6σ. This measurement comes from the southern galactic plane CO survey carried out using the 4-m Epping telescope. Their value of 180 km/sec is obtained from the slope of terminal velocity curve as a function
FIG. 1: Histograms of the error distributions in half standard deviation bins. The top (bottom) row uses the weighted mean (median) of the 137 measurements as the central estimate. The left (right) column shows the signed (absolute) deviation. In the left column plots, positive (negative) \( N \sigma \) represent a value that is greater (less) than the central estimate. The dotted-line curve is the best fitting Gaussian probability distribution function in all cases. We note that this histograms are normalized so that the sum of the total number of events is equal to 1.

| Function        | Un-binned probability | Binned probability |
|-----------------|------------------------|--------------------|
| Gaussian        | mean \( 8.12 \times 10^{-9} \) median 0.0084 | mean \( 6.13 \times 10^{-12} \) median 0.0001 |
| Cauchy          | mean \( 9.68 \times 10^{-5} \) median 0.05 | mean \( 4.89 \times 10^{-7} \) median 0.022 |
| Double exponential | mean \( 7.84 \times 10^{-8} \) median 0.016 | mean \( 5.63 \times 10^{-11} \) median 0.0005 |
| Students-t      | mean \( (n=1)9.68 \times 10^{-5} \) median 0.1 | mean \( (n=1)4.89 \times 10^{-7} \) median 0.022 |

TABLE I: K-S test probability for various functional fits to \( N \sigma \) reconstructed from the rotation velocity data obtained from the compilation in G17. As we can see, only the Students-t distribution provides a reasonable \( p \) value when median estimate of \( \Theta_0 \) is used.

| Tracers                  | No. of Observations* | Median (km/s) | 68% c.l. (km/s) | p-value (median) | p-value (mean) |
|--------------------------|-----------------------|----------------|-----------------|------------------|----------------|
| Field stars              | 30                    | 223.99         | 3.53            | 0.23             | 0.15           |
| Young tracers            | 64                    | 245.48         | 2.49            | 0.62             | 0.13           |
| Galactic Mass Modeling   | 14                    | 214.82         | 5.74            | 0.57             | 0.44           |
| Intermediate/old age tracers | 6                   | 202.89         | 49.08           | 0.58             | 0.95           |
| Sgr A                    | 10                    | 244.46         | 4.72            | 0.11             | 0.21           |
| Others                   | 12                    | 215.31         | 1.95            | 0.17             | 0.28           |

*Observations which have null values for errors are omitted.

TABLE II: Median and 68.3% confidence interval around median for various tracer distributions along with the K-S test probability for Gaussian distribution fit (considering both mean and median as central estimate) for \( N \sigma \) reconstructed from the rotation velocity data obtained from the compilation in G17.
of Θ
of 0.0347 predicted by analytical models of Milky way.

is a consequence of a simplified model been used in the
ble reason for the outliers in the other two measurements.

Two others come from CO measurements. The possibility could be that there is a degeneracy between the
radial velocity Experiment (RAVE) stellar survey

southern Milky Way galaxy survey [30], which estimated
a value of Θ
and another parameter defined as
z
= 209 ± 2 km/sec. One assumption made
for this work was that the galactic rotation curve is com-
pletely flat and there is no variation with galactic lon-
gitude. The last outlier corresponds to Θ
= 232 ± 1.7
km/sec (N
= 7.3) [31]. This value was obtained from
the Radial Velocity Experiment (RAVE) stellar survey
using the Shu distribution function. Although it is hard
to discern a specific reason for this high value, one pos-
sibility could be that there is a degeneracy between the
value of Θ
and another parameter defined as α
z,
which denotes the vertical dependence of the circular speed.
The value they obtained for α
z,
 disagrees with the value
of 0.0347 predicted by analytical models of Milky way potential.
For smaller values of α
z,
the estimated values
of Θ
would also decrease.

Therefore to summarize, we find that one outlier me-
asurement is a consequence of an incorrect tabulation.
Two others come from CO measurements. The possible
reason for the outliers in the other two measurements
is a consequence of a simplified model been used in the
fitting procedure or due to degeneracy between the rota-
tion speed and another astrophysical parameter.

IV. ANALYSIS OF SUBSAMPLES

In order to understand the underlying cause of non-
Gaussianity when analyzing the full dataset, we under-
take a similar analysis on each subsample of data af-
after grouping the measurements according to the method
used. This will help us determine if there are unknown
systematic errors within each group. The classification
of Θ
measurements has already been done in G17, who
divided the measurements according to the stellar pop-
ulation tracer used. The entries were grouped into field
stars; young tracer populations; old and intermediate age
tracers; kinematics of Sgr A* near the galactic center; and
galactic mass modeling using HI as well as CO radio ob-
servations. In G17, central estimates using the weighted
mean values of each group has already been calculated,
including a discussion of which of these deviate from the
IAU recommended values. G17 have found differences
among the Θ
and Θ
/R
values between the different
tracer populations. They have found that young tracers
and kinematic measurements of Sgr A* near the galac-
tic center imply a significantly larger rotation speed at
the solar circle compared to the field stars and HI/CO
measurements. Here, we examine the non-gaussianity of
errors in each subset to see if there is any underlying un-
accounted systematics in each subset of measurements.

For each subset, we carry out the same analysis as in
Sect. III. We obtain the central estimate using both the
weighted mean and median and then construct N
histograms using each of these and fit these to a Gaussian
distribution. We check for Gaussianity using the p-values
resulting from K-S test. The results can be found in Ta-
ble II. To complement the analysis in G17, we show the
group-wise medians along with 60% c.l. ranges obtained
using the method by Chen and Ratra [10]. We can see
that our median-based estimates for each tracer popula-
tion agree with the weighted means by G17, except for
the intermediate and old age tracer population, for which
we get a value 10 km/sec more than the one by G17.

From Table II, we find that the p-value for a Gaussian
distribution fitting the data is greater than 0.1 for all the
subsets, using both the mean as well as the central esti-
mates. Therefore, there is no unknown systematic error
or egregious measurement within each group of measure-
ments. The underlying cause of non-gaussianity for the
full dataset is probably caused by combining the data
across the tracers, in addition to outliers.

Finally, similar to previous works on median statistics
estimates of astrophysical and cosmological parameters,
we obtain a central estimate by calculating median of this
group-wise median estimates. This central estimate from
the median of all these medians is given by Θ
=219.65
km/sec. The total number of measurement categories is
too small to get a robust 68% confidence level error bar
on this value.

Given the non-Gaussianity of the residuals from the
full dataset, this median value of 219.65 km/sec should
be used as the central estimate of Θ
. We note that

FIG. 2: Histogram of the error distributions in |Nσ| = 0.1
bins . The solid black line represents the expected Gaussian
probabilities for 137 measurements and the dot-dashed blue
(dashed red) line is the number of |Nσ| values in each bin for
the weighted mean (median). All the histograms are normal-
ized so that integral of the distribution over all data points is
equal to one.
FIG. 3: The left (right) plot in the each row denotes the histogram for the error distributions with mean (median) as the central estimate. The dotted line curve in the top, middle and bottom rows are the best fitting Cauchy, Double Exponential, and Student’s t probability distribution functions respectively. In each case, we consider mean (median) as the central estimate in the left (right) column.

this value is closer to the IAU recommended value of 220 km/sec and differs from the recommendation in G17 of 225 km/sec inspite of using the same galactocentric distance of 8.3 kpc.

V. CONCLUSIONS

We have used a compilation of 137 measurements of Galactic rotation speed and their corresponding errors from G17 [1] and two additional measurements (not included in G17), in order to gain a better insight of the non-Gaussianity of the residuals and to obtain a central estimate. We first scaled all the measurements, which were degenerate with galactocentric distance to a common value of 8.3 kpc. The error distributions were analyzed (following the same prescription as in the previous works by Ratra et al [3, 4, 6, 7]) and plotted using both the weighted mean as well as the median value as the central estimate. We note that the central estimates for the weighted mean and median used all the measurements unlike those in G17, which used only the post-1985 measurements.

We conclude from our observations that the error distribution for the galactic rotation speed measurements using both these estimates is inherently non-Gaussian. The deviation from Gaussian distribution motivated us to check the fit for other prominent non-Gaussian proba-
bility distribution functions. We have taken into consideration the Cauchy, Double exponential and Students-t probability distribution functions. The results show that with median as the central estimate, the error distribution have comparatively better fits with Student’s t probability distribution functions for $n = 2$, corresponding to a p-value of 0.1. All other distributions display poor fits with both mean and median values as central estimates.

We then redid the same analysis after grouping the measurements according to the tracers used. We find that the residuals within each subsample follow the Gaussian distribution. This implies that the non-Gaussianity of the error bars is caused by combining the measurements from different categories, in addition to outliers.

Finally, since the residuals are not Gaussian, instead of the weighted mean, the median value when grouped according to the measurement type should be used as the central estimate for $\Theta_0$. This group-wise median value is equal to 219.65 km/sec and is close to the IAU recommended value of 220 km/sec. This is inspite of using a galactocentric distance of 8.3 kpc.

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