IMPRINTS OF A HEMISPHERICAL POWER ASYMMETRY IN THE SEVEN-YEAR WMAP DATA DUE TO NON-COMMUTATIVITY OF SPACE-TIME

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ABSTRACT

Non-commutative geometry at inflation can give rise to parity violating modulations of the primordial power spectrum. We develop the statistical tools needed for investigating whether these modulations are evident in the Cosmic Microwave Background (CMB). The free parameters of the models are two directional parameters ($\theta, \phi$), the signal amplitude $A_*$, and a tilt parameter $n_*$ that modulates correlation power on different scales. The signature of the model corresponds to a kind of hemispherical power asymmetry. When analyzing the 7-year WMAP data we find a weak signature for a preferred direction in the Q-, V-, and W bands with direction $(l, b) = (-225^\circ, -25^\circ) \pm (20^\circ, 20^\circ)$, which is close to another previously discovered hemispherical power asymmetry. Although these results are intriguing, the significance of the detection in the W-, V- and Q-bands are nonzero at about $2\sigma$, suggesting that the simplest parameterization of the leading correction represents only partially the effects of the space-time non-commutativity possibly responsible for the hemispherical asymmetry. Our constraints on the presence of a dipole are independent of its physical origin and prefer a blue-tilted spectral index $n_* \approx 0$ with the amplitude $A_* \approx 0.18$.

Subject headings: cosmic microwave background — cosmology: observations — methods: numerical

1. INTRODUCTION

During recent years, studies of the cosmic microwave background (CMB) have greatly improved our understanding of the early universe. Observations of the CMB anisotropies, such as those obtained by the Wilkinson Microwave Anisotropy Probe (WMAP) experiment (Bennett et al. 2003, Hinshaw et al. 2007), have provided us with a deep insight on the composition of structure and energy in our universe, giving rise to the $\Lambda$CDM model. The $\Lambda$CDM model requires that the universe has undergone an epoch of rapid accelerated expansion. This epoch is named inflation, and is thought to have been driven by a single or several scalar fields (Guth et al. 1981). In addition, the model of inflation establishes a highly successful theory for the formation of primordial density perturbations, providing the required seeds for the large-scale structures (LSS). These large scales structures are observed in the CMB today (Guth et al. 1982; Mukhanov & Chibisov 1981; Starobinsky et al. 1982; Linde et al. 1983, 1994; Smoot et al. 1992; Ruhl et al. 2003; Ryuan et al. 2003; Scott et al. 2003).

Inflation can explain why the observed universe should be nearly isotropic on large scales. However, anomalies found in the CMB during the recent years (de Oliveira-Costa et al. 2003; Vielva et al. 2003; Eriksen et al. 2004a; Groeneboom et al. 2010; Hottilt et al. 2009) suggest that some anisotropy could be present at inflation too.

Several theoretical possibilities were put forward to explain such anomalies (Ackerman et al. 2007; Kanno et al. 2008; Dimopoulos 2009; Yokoyama and Soda 2008; Barrow and Hervik 2009; Esposito-Farese, Pitrou and Uzan 2009). One of the possibilities is to introduce a vector field that breaks the rotational invariance in the early universe. The presence of such vector fields lead to quadrupole modulations of the CMB anisotropy, which for vector perturbations was shown by Durrer et al. (1998). Recently, it was also suggested by Groeneboom et al. (2010) that the 5-year WMAP data contains a significant signal of a primordial vector field which would break rotational invariance, corresponding to a $\theta\sigma$ detection in the W-band. However, several authors have since then claimed that the signal is due to systematic effects, as the rotational axis is aligned with the rotational axis of the satellite. A likely candidate for the source of the signal is therefore asymmetric instrumental beams (Hanson et al. 2010; Komatsu et al. 2010, Bennett et al. 2010) and several other sorts of instabilities (Himmetoglu, Contaldi and Peloso 2009a, 2009b, 2009c).}

Unfortunately, such scenarios seem to be plagued with ghosts (Himmetoglu, Contaldi and Peloso 2009a, 2009b, 2009c), but various generalizations and alternatives can also be considered (Boehmer and Mota 2008; Koivisto et al. 2008; Jimenez et al. 2009, 2009a; ArmendarizPicon 2007).

Several of the anomalies, such as the axis of evil or the hemispherical asymmetry in the power spectrum, raise the question whether there could exist odd-parity modulations in the CMB. It was shown by Koivisto & Mota (2008) that the effects of space-time non-commutativity at inflation can generate modulations. The leading contribution, as generically in any parity-violating model, is a dipole modulation, but up to now it hasn’t been explic-
The parity violating effect on the primordial power spectrum studied in the present paper can be generated by quantum effects during or shortly after inflation. Such effects have yet to be considered due to the popular assumption that the power spectrum should be invariant under spatial inversion. In principle there are no physical arguments that exclude them as long as the physical observables generated from the theory are consistent, so one should therefore test this class of models against WMAP data and assess their viability. The odd-parity effects have yet to be considered due to the popular assumption, in both approaches one may predict, a power spectrum of the form $$\langle R(k)R^*(k') \rangle = \delta^3(k-k') \frac{2\pi^2 \sqrt{4\pi}}{k^3} \sum_{L,M} A_{L,M} \left( \frac{k}{k_0} \right)^{n_{L,M} - 1} Y_{L,M}(\hat{k})$$ where the parameterisation by Armendariz-Picon & Pekowsky (2009) is employed. The sum over $L$ could in principle cover the entire spectrum of multipoles and $M$ runs from $-L$ to $L$. The resulting signal covariance is written as

$$S_{\ell m; \ell' m'} = \frac{\delta_{\ell - \ell'}^2}{2\pi^2} \sum_{L,M} A_{L,M} \xi_{\ell m; \ell' m'} I_{\ell\ell'}^M$$

where the spectral-index $n_{L,M}$ that parameterises the scale-dependence is included in the integrated contribution over all scales,

$$I_{\ell\ell'}^M = \int_0^\infty \frac{dk}{k} \left( \frac{k}{k_0} \right)^{n_{L,M} - 1} \Theta_\ell(k) \Theta_{\ell'}(k).$$

The multipole moments of the sources $\Theta_\ell(k)$ are computed using a modified version of CAMB (Lewis et al. 2000) with the spectral index as an input parameter. The geometrical factors $\xi_{\ell m; \ell' m'}$ in eq. (4) are provided by Koivisto & Mota (2010). The non-commutative nature of the fields responsible for the perturbations give rise to non-hermitian signal covariance $S_{\ell m; \ell' m'; \ell', m'} = -S_{\ell m; \ell' m'}$ for...
the dipole \((L = 1)\) contribution which is the only term in the expansion that is considered in the present paper. The naïve expectation from both theory and observations is that higher order terms are suppressed. This make this assumption here but it should be examined more thoroughly. The anisotropy in (4) is then added to the isotropic signal, corresponding to the \(L = 0\) term. One obtains a direct interpretation of \(\mathcal{A}_{00} \equiv \mathcal{A}\) as the primordial scalar amplitude in the canonical expression for the isotropic matter spectrum. Furthermore

\[
\hat{A}_{1M} = A_{1M} \hat{r}_M
\]

where \(A_{1M}\) is the amplitude which we are estimating. The constant of proportionality in all three cases is \(i/\sqrt{3} r_0 A\) where \(r = |r_0| = H \theta\). Here \(\sqrt{\theta}\) is the non-commutative lengthscale, \(r_0\) is the pivot-scale. The unit vectors appearing in eq. (6) are the spherical vectors parameterising the direction of the anisotropy,

\[
\hat{r}_\pm = \mp \left( \frac{\hat{r}_x \pm i \hat{r}_y}{\sqrt{2}} \right), \quad \hat{r}_0 = \hat{r}_z.
\]

In the case \(\theta\) with \(\theta\) constant in the comoving frame, we have \(\theta = \bar{\theta}\) and \(n_{1M} = 2\), but as argued above this need not be the case for general models. Thus we will check this particular case separately but otherwise keep both the spectral index and the amplitude as free parameters. So in all the model contains six unknown parameters, the three spectral indices \(n_{1M}\), the non-commutative length-scale \(\mu^{-1}\), and a direction \((\theta, \phi)\) contained in the unit vectors of eq. (7). However, typically the slope of the modulation does not depend upon the azimuthal orientation. Then the number of parameters is reduced to four: the direction \((\theta, \phi)\), a coupling \(\mathcal{A}_s\), and the reduced spectral index \(n_s\) for the \(L = 1\) case.

\section{Methods}

We now discuss the method for mapping out the desired posterior. The method for obtaining the posterior is similar to the method presented in Groeneboom & Eriksen (2009); Groeneboom et al. (2009), with the exception of a new covariance matrix and a new parameter. CMB data observations can be modeled as:

\[
d = As + n,
\]

where \(d\) represents the observed data, \(A\) denotes convolution by an instrumental beam, \(s(\theta, \phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta, \phi)\) is the CMB sky signal represented in either harmonic or real space and \(n\) is instrumental noise. It is generally a good approximation to assume both the CMB and noise to be zero mean Gaussian distributed variables, with covariance matrices \(S\) and \(N\), respectively. In harmonic space, the signal covariance matrix is defined by \(S_{\ell m, \ell' m'} = \langle a_{\ell m} a^*_{\ell' m'} \rangle\). In the isotropic case, this matrix is diagonal. The connection to cosmological parameters \(\omega\) is made through this covariance matrix. Finally, for experiments such as WMAP, the noise is often assumed uncorrelated between pixels, \(N_{ij} = \sigma^2 \delta_{ij}\), for pixels \(i\) and \(j\), and noise RMS equals to \(\sigma_i\).

Let \(\omega\) denote a set of cosmological parameters. Our goal is to compute the full joint posterior \(P(\omega|d)\), which is given by \(P(\omega|d) \propto P(d|\omega)P(\omega) = \mathcal{L}(\omega) P(\omega)\), where \(\mathcal{L}(\omega)\) is the likelihood and \(P(\omega)\) a prior. For a Gaussian data model, the likelihood is expressed as:

\[
\mathcal{L}(\omega) \propto e^{-\frac{1}{2}d^T C^{-1}(\omega)d}/\sqrt{|C(\omega)|}.
\]

where \(C = S + N\) is the total covariance matrix.

\subsection{The Gibbs sampler}

The problem of extracting the cosmological signal \(s\) and \(\omega\) from the full signal by Gibbs sampling was addressed by Jewell et al. (2004), Wandelt et al. (2004) and Eriksen et al. (2004). The CMB Gibbs sampler is an exact Monte Carlo Markov chain (MCMC) method that assumes prior knowledge of the conditional distributions in order to gain knowledge of the full joint distribution. A significant fraction of the CMB data is completely dominated by galactic foreground, and about 20\% of the data needs to be removed. This might sound trivial, but in reality it complicates processes as the spherical harmonics no longer are orthogonal. The Gibbs sampler solves this problem intrinsically, as the galaxy mask becomes a part of the framework (Groeneboom 2009).

The main motivation for introducing the CMB Gibbs sampler is the drastically improvement in scaling. With conventional MCMC methods, one needs to sample the angular power spectrum, \(C_\ell = \langle a_{\ell m} a^*_{\ell m} \rangle\), from the distribution \(P(C_\ell|d)\), which scales as \(O(N_{\text{pix}}^3)\), where \(N_{\text{pix}}\) is the size of the covariance matrix. For a white noise case, the Gibbs sampler reduces this to \(O(N_{\text{pix}}^{1.5})\). In other words, the Gibbs sampler enables effective sampling in the high-\(\ell\) regime.

\subsection{Sampling scheme}

In order to sample from the full joint distribution \(P(C_\ell, \omega|s, d)\) using the Gibbs sampler, we must know the exact conditional distributions \(P(s|C_\ell, \omega, d)\) and \(P(C_\ell, \omega|s)\). The Gibbs sampler then proceeds by alternating sampling from each of these distributions:

\[
(C_\ell, \omega)^{t+1} \leftarrow P(C_\ell, \omega|s^t, d)
\]

\[
s^{t+1} \leftarrow P(s|C_\ell, \omega)^{t+1}, d)\).\]

The first conditional distribution is expressed as:

\[
P(C_\ell, \omega|s, d) \propto e^{-\frac{1}{2}S(s)^{-1}s}
\]

and is distributed according to an inverse Gamma function with \(2\ell + 1\) degrees of freedom. The remaining conditional distribution is

\[
P(s|C_\ell, \omega, d) \propto e^{-\frac{1}{2}(s-\bar{s})^T (S(\omega)^{-1} + N^{-1})(s-\bar{s})}
\]

where \(\bar{s} = N^{-1}d\). In other words, \(P(s|C_\ell, \omega, d)\) is a Gaussian distribution with mean \(\bar{s}\) and covariance \((S(\omega)^{-1} + N^{-1})^{-1}\). Numerical methods for sampling from these distributions were discussed by Groeneboom (2009).

\section{Model Properties and Predictions}

In this section, we review the numerical setup of the analysis. Most of the framework is similar to the one employed by Groeneboom & Eriksen (2009). However, we
The diagonal case \( L = 0 \) represents the power spectrum \( C_{\ell} \), where we assume the tilt is equal to the spectral index \( n_{\ell m} = n_s \) from the best-fit seven-year WMAP spectral index [Larson et al. 2010]. We introduce the parameter \( n_s \) to replace \( n_{\ell m} \) for the \( L = 1 \) case as such:

\[
I_{\ell'}^{1} = \int_{0}^{\infty} \frac{dk}{k} \left( \frac{k}{k_0} \right)^{n_s - 1} \Theta_{\ell'}(k) \Theta_{\ell'}(k)
\]

where we have chosen \( k_0 = 0.05 \text{Mpc}^{-1} \) as the tilt scale. Three examples of \( I_{1}^{1} \) for \( n_s = 0.5, 1.0 \) and 1.5 are presented in Figure 2. The integral in equation (15) needs to be pre-computed in numerical software such as CAMB [Lewis et al. 2000]. It is difficult to implement CAMB into the existing Gibbs sampler framework, so we utilize a different scheme. First, we pre-compute 10 000 integrals for \( n_s \) in an interval \([-1, 4]\) and store the data in a binary file. This interval is large enough to allow for all types of scale-dependence in the anisotropy, and is our prior for \( n_s \). For all purposes we consider \( n_s \) to be continuous. The precomputed binary file is then loaded into the anisotropic MCMC framework such that \( n_s \) can be treated as a free parameter.

When simulating CMB maps for this model, the connection to a hemispherical dipole asymmetry becomes imminent. This is depicted the top-left frame of Figure 3, where we present a simulated CMB map with \( A_\ell = 3.0 \) and \( n_s = 1.0 \) with direction \((l, b) = (224^\circ, 22^\circ)\). In order to verify that this really is a signal similar to a dipole modulation, we simulate a map with \( A_\ell = 0.0 \) and divide the \( A_\ell = 1.0 \) map with the \( A_\ell = 0.0 \) map. The resulting dipole structure can be seen in the remaining tree maps in Figure 3, where we have included simulations with tilt parameter \( n_s = 0.5 \) and \( n_s = 1.5 \). Note that from the integral in Figure 2 \( n_s = 0.5 \) corresponds to a large-scale dipole modulation while \( n_s = 1.5 \) contains more small-scale modulations.

4.2. Analyzing simulated data

We verify code by performing both a brute-force and MCMC analysis on a noise-free, unconvolved simulated CMB map with no sky cut. In order to build the joint two-dimensional distribution of \( n_s \) and \( A_\ell \), we need an increased amount of samples as compared to the case by Groeneboom & Eriksen [2009]. In addition, adding a new parameter will in general decrease the significance of the results if the parameters are correlated. As \( A_\ell \) and \( n_s \) can be expected to be correlated, we choose to normalize the integrals such that the area under each graph for all \( n_s \) are the same. The degeneracy is broken, as can be seen in Figure 4. Here, we create a noiseless, unconvolved CMB map with \( n_{\text{side}} = 128 \) and \( \ell_{\text{max}} = \ell_{\text{max}}^{\text{cutoff}} = 150 \) adopting the best-fit \( \Lambda \)CDM model determined from the seven-year WMAP data [Larson et al. 2011]. The model input parameters are \( n_s = 1.0 \), \( A_\ell = 1.5 \) and \( (l, b) = (224^\circ, 22^\circ) \). The joint and marginal posteriors are presented in Figure 4. Note how the posterior of \( n_s \) is distributed similarly to a \( \chi^2 \)-alike distribution, and that \( A_\ell \) and \( n_s \) are not degenerate. The distribution is always symmetric around \( A_\ell = 0.0 \), due to negative amplitude \( A_\ell \) in a direction \((l, b)\) corresponds to a positive amplitude in the opposite direction \((-l, -b)\). This
Fig. 3.— The effect of the dipole-modulating model on simulated CMB maps. Top left: A simulated CMB map with a large dipole contribution. The remaining maps show simulated dipole-modulated maps divided by a non-dipole-modulated map. Note how different values of the correlation tilt $n_*$ induce large-scale or small-scale dipole correlations.

Fig. 4.— Results from a simple analysis of simulated data without foregrounds, noise or convolution. The input parameters were $A_*=1.5, n_*=1.0$ with direction $(l,b) = (224^\circ, -22^\circ)$. Top left: The directional posterior from a brute-force run. Lower left: The joint posterior $P(A_*,n_*|d)$. Note that these parameters are not degenerate. Lower right: the marginal posteriors $P(A_*|d)$ and $P(n_*|d)$.

We continue by creating realistic V-band differencing assembly (DA) simulations with $n_{\text{side}}=512, A_*=0.25, n_*=0.25$ with direction $(l,b) = (224^\circ, -22^\circ)$. The maps are produced using the V-band beam and noise properties. In addition, we add synchrotron, free-free and thermal dust foreground templates as described by Gold et al. (2010). The V1 simulation is depicted in Figure 5. The analysis is performed using the Gibbs sampler for $l_{\text{cutoff}}=400$ where we impose the KQ85 mask (Gold et al. 2009), which removes 18% of the sky. The analysis successfully reproduce the input parameters, as is seen in Figure 5. However, note that the distributions are wider than in the noiseless, perfect case, and that tail of the marginal posterior $P(A_*|d)$ merges with the positive values of $P(-A_*|d)$ near zero.

5. WMAP ANALYSIS
In this paper, we consider the seven-year WMAP temperature sky maps of Jarosik et al. (2010), and analyze the V-, W and Q (61, 94 and 41 GHz, respectively). The V- and W-bands are believed to be the cleanest WMAP bands in terms of residual foregrounds. We adopt the template-corrected, foreground reduced maps recommended by the WMAP team for cosmological analysis, and impose the KQ85 mask (Gold et al. 2009). Point source cuts are imposed in all cases. We analyze the data frequency-by-frequency, and consider the combinations V1+V2, Q1+Q2 and W1 through W4. The noise RMS patterns and beam profiles are taken into account for each DA individually. The noise is assumed uncorrelated. For details on joint Gibbs analysis of multifrequency data, see Eriksen et al. (2004b). All data used in this analysis are available from LAMBDA.

The angular resolutions of the V-, W- and Q bands are 0.35°, 0.22° and 0.53°, respectively. The sky maps are pixelized at a HEALPix resolution of $N_{\text{side}} = 512$ with 7° pixels. We adopt a harmonic space cutoff of $\ell_{\text{max}} = 800$ for the two data sets, probing partly into the noise dominated regime. However, we do not consider multipoles at $\ell > 400$ for the anisotropic part of the signal covariance matrix, in order to minimize the chance of systematic effects such as residual point source contributions, beam uncertainties or noise mis-estimation to affect our results.

5.1. Results

We present the marginal posteriors for the dipole model obtained from the seven-year WMAP temperature sky maps, as computed with the method described in Section 3. First, in the top row of Figure 6 we show the preferred direction posteriors, $P(\ell, b | \mathbf{d})$ for the W- and V band data. The joint posterior $P(\mathcal{A}_* n_* | \mathbf{d})$ is depicted in the middle row, while the bottom row displays the marginal posteriors, $P(\mathcal{A}_* | \mathbf{d})$, and $P(\mathcal{n_* | \mathbf{d}}$. The results are listed in Table 1.

The direction of the dipole amplitude is located at about $(\ell, b) = (225°, -20°) \pm (20°, 20°)$ in all the bands. This corresponds to the direction of the previously discovered hemispherical power asymmetry by Hoftuft et al. (2003) at $(\ell, b) = (225°, -27°)$, and suggests that the signal has a cosmological origin. The tilt parameter $n_*$ is found consistently around 0, implying that the dipole effects are mostly concentrated on large scales for $\ell < 50$. The coupling strength of the dipole contribution $\mathcal{A}_*$ for various bands are listed in Table 1. Note the value of $\mathcal{A}_*$ is close to zero, there are several contaminating sources that contribute to the posterior. One source is the fact that there are degeneracies between $\mathcal{A}_*$ and $n_*$ for low values of $\mathcal{A}_*$, so the marginal posterior have noise-related contributions close to 0. In addition, when $\mathcal{A}_*$ is close to zero, there is a contribution from the symmetrical posterior from negative $\mathcal{A}_*$ that spills over to positive values. When keeping this in mind, it should altogether be clear that when inspecting the posteriors that the amplitude parameter $\mathcal{A}_*$ is nonzero at about a 2.0σ significance in the W-, V- and Q bands.

Since the canonical form of non-commutativity, given by Eq. (1) with θ a constant in the comoving frame, is of specific interest, we have tested this case separately. Basically this amounts to fixing the spectral index to the theoretical prediction $n_*=2$ and varying only three parameters. In that case we that the data is consistent with a vanishing anisotropic contribution and bounds $\mathcal{A} \lesssim 0.05$. This translates into a couple of orders of magnitude looser bounds than obtained by Akford et al. (2009) from the power spectrum alone, since the effect is at small scales which we cut out of the analysis $\ell > 400$.

6. Conclusion

In this paper, we have developed a numerical method for investigating traces of non-commutative geometry derived from field-theoretical implementations of microcausality violation in the seven-year WMAP data. The deformation of Lorentz symmetry relevant at inflation induces parity-violating modulations of the primordial power spectrum, which give rise to a dipole-modulation effect in the CMB. The dipole modulation has certain similarities to the previously detected hemispherical power asymmetry, and not surprisingly, we reproduced the direction of the hemispherical power asymmetry at $(\ell, b) = (-225°, -25°) \pm (20°, 20°)$ when analyzing the combined seven-year data sets. In addition, both the direction and amplitude are stable and nonzero at a 2σ level in the W-, V- and Q bands. The tilt parameter $n_*$ is firmly located around zero, indicating that the seven-year WMAP data prefers the dipole modulations to occur on large scales $\ell < 50$. While these results are intriguing, the significance is still too low to be considered a clear detection. This could be due to the fact our parameterization of the leading order effect may not capture the underlying physics to the full effect. In addition, one should take into account the higher order multipoles and vary also the cosmological parameters.

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Fig. 6.— The main results from the seven-year W (left) and V (right) band WMAP data. Note that the directions are stable in all bands (including the Q-band), and that the anisotropy amplitude $A_*$ is nonzero at $2\sigma$ in all cases. The direction of the previously described hemispherical power asymmetry is marked with a red circle.

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