MICROLENSING OF SUB-PARSEC MASSIVE BINARY BLACK HOLES IN LENSED QSOs: LIGHT CURVES AND SIZE–WAVELENGTH RELATION

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ABSTRACT

Sub-parsec binary massive black holes (BBHs) have long been thought to exist in many QSOs but remain observationally elusive. In this paper, we propose a novel method to probe sub-parsec BBHs through microlensing of lensed QSOs. If a QSO hosts a sub-parsec BBH in its center, it is expected that the BBH is surrounded by a circumbinary disk, each component of the BBH is surrounded by a small accretion disk, and a gap is opened by the secondary component in between the circumbinary disk and the two small disks. Assuming such a BBH structure, we generate mock microlensing light curves for some QSO systems that host BBHs with typical physical parameters. We show that microlensing light curves of a BBH QSO system at the infrared–optical–UV bands can be significantly different from those of corresponding QSO system with a single massive black hole (MBH), mainly because of the existence of the gap and the rotation of the BBH and its associated small disks) around the center of mass. We estimate the half-light radii of the emission region at different wavelengths from mock light curves and find that the obtained half-light radius versus wavelength relations of BBH QSO systems can be much flatter than those of single MBH QSO systems at a wavelength range determined by the BBH parameters, such as the total mass, mass ratio, separation, accretion rates, etc. The difference is primarily due to the existence of the gap. Such unique features on the light curves and half-light radius–wavelength relations of BBH QSO systems can be used to select and probe sub-parsec BBHs in a large number of lensed QSOs to be discovered by current and future surveys, including the Panoramic Survey Telescope and Rapid Response System, the Large Synoptic Survey telescope, and Euclid.

Key words: accretion, accretion disks – black hole physics – galaxies: formation – gravitational lensing: micro – quasars: general – relativistic processes

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1. INTRODUCTION

Massive binary black holes (BBHs) on sub-parsec scales are unavoidable products of mergers of galaxies if each galaxy contains a central massive black hole (MBH; e.g., Begelman et al. 1980; Yu 2002; Milosavljević & Merritt 2005). In the ΛCDM cosmology, big galaxies are formed through hierarchical mergers of small galaxies; therefore, BBHs may exist in the centers of many galaxies. In gas-rich mergers, the consequent sub-parsec BBHs may be surrounded by a circumbinary disk, within which a gap is opened by the inspiraling secondary MBH, and each component of the BBH may be associated with a small accretion disk within the gap, fed by material infalling from the inner edge of the circumbinary disk (e.g., Artyomowicz & Lubow 1994; Ivanov et al. 1999; Escala et al. 2005; Hayasaki et al. 2007, 2008; Dotti et al. 2007; Cuadra et al. 2009; D’Orazi et al. 2012; Rafikov 2013; Farris et al. 2013). These active BBHs may also emerge as QSOs but may have some unique features distinguishable from single active MBH systems.

Observational searching for and identifying BBHs is important because they are not only probes to the current galaxy formation model but also main sources for gravitational waves. Recent observations have already revealed a few active MBH pairs (dubbed as dual active galactic nuclei) on kiloparsec scales through various techniques (e.g., Komossa et al. 2003; Wang et al. 2009; Comerford et al. 2009; Liu et al. 2010; Shen et al. 2011; Fu et al. 2011; Rosario et al. 2011; Yu et al. 2011), which are presumably the precursors of sub-parsec BBHs. However, spatially resolving and identifying sub-parsec BBHs (both active and inactive ones) is still beyond the capabilities of the present-day telescopes (e.g., Yu 2002). A number of methods have thus been proposed and applied to probe the BBH systems indirectly. Such methods include the double-peaked or asymmetric broad emission line (e.g., Boroson & Lauer 2009; Tsalmantza et al. 2011; Eracleous et al. 2012; Ju et al. 2013; Shen et al. 2013), flux ratio of different emission lines (Montuori et al. 2011, 2012), periodic variation of the QSO light curves (e.g., Valtonen et al. 2008; Sesana et al. 2012; Gültekin & Miller 2012; Hayasaki et al. 2013), double broad relativistic Fe Kα lines (e.g., Yu & Lu 2001; Sesana et al. 2012; McKernan et al. 2013), and kinematic signature (Meiron & Laor 2013), etc. Most of these methods depend on the orbital motions of BBHs. A number of sub-parsec BBH candidates have been reported, but few of them were confirmed because of various complications in these methods, which make the differences of the properties of BBH systems from those of its alternatives ambiguous. It is therefore imperative to devise new a method/approach to unambiguously reveal sub-parsec BBHs/binary QSOs.

It has been demonstrated that the microlensing magnification of lensed QSOs (e.g., Wambsganss et al. 1990; Witt et al. 1995) is a powerful tool to probe the size and even temperature structure of accretion disks, as the Einstein radius of a single star in the foreground lens galaxies is comparable to the disk size of background QSOs, and the QSO flux variation is determined by the ratio of the Einstein radius to the size of the emission region (e.g., Wambsganss 2006). This method has
been applied to estimate the disk sizes for more than a dozen lensed QSOs, and the results show that the estimated disk temperature profile is consistent with the thin disk model though the estimated disk size may be a factor of ∼4–5 larger than the standard thin disk prediction (e.g., Wambsganss & Paczynski 1991; Mortonson et al. 2005; Eigenbrod et al. 2008; Poinrdexter et al. 2008; Morgan et al. 2010; Blackburne et al. 2011; Jiménez-Vicente et al. 2012). Dexter & Agol (2011) and Abolmasov & Shakura (2012) suggest that alternative disk structure models, such as inhomogeneous disk or super-Eddington accretion disk, may be required to explain the microlensing results. Long-term monitoring of microlensing of lensed QSOs are helpful to identify the detailed structure of accretion disks associated with these lensed QSOs.

The structure of disk accretion onto BBHs is fundamentally different from that onto a single MBH. The most distinct feature of an active BBH system should be the gap opened by the inspiraling secondary MBH (e.g., Artymowicz & Lubow 1994; Ivanov et al. 1999; Dotti et al. 2007; Hayasaki et al. 2007, 2008; Cuadra et al. 2009). Direct detection of such a feature is not possible currently. In this paper, we propose a novel method to probe sub-parsec BBHs/binary QSOs by detecting signatures of the gap through the microlensing of lensed QSOs.

This paper is organized as follows. In Section 2, we first illustrate the configuration of an active BBH system and calculate the surface brightness distribution of such a system at the given UV–optical–infrared bands by adopting a simple thin disk accretion model. In Section 3, mock microlensing light curves are generated by convolving the brightness distribution of background sources, either an active MBH or an active BBH system, with the foreground magnification maps. Adopting the Monte Carlo microlensing analysis method developed by Kochanek (2004), we demonstrate that the size of emission region can be estimated for a BBH system through the mock light curves by assuming a single standard accretion disk model in Section 4.

As shown in Section 4.2, the expected relationship between disk size and wavelength for a BBH system show some apparent anomaly at some wavelength compared with that for a single MBH system. We propose that this anomaly can be taken as a signature of BBHs and should be useful for selecting BBH candidates from lensed QSOs with current and future facilities. Discussions and conclusions are given in Sections 5 and 6.

2. CONFIGURATION OF ACCRETION ONTO MASSIVE BINARY BLACK HOLES

In order to calculate the microlensing signature of BBH systems, it is necessary to first know the structure of accretion onto these BBHs. In this section, we adopt a simple disk accretion model for BBH systems and then calculate the brightness distribution of these BBH sources. By convolving the background source brightness distributions with foreground magnification maps due to lens galaxies, we then obtain mock light curves for these sources.

2.1. Geometry

Considering a BBH system, the semimajor axis of the BBH is $a_{\text{BBH}}$ and the masses of the primary and secondary MBHs are $M_1$ and $M_2$, respectively. As revealed by a number of numerical simulations (e.g., Artymowicz & Lubow 1994; Ivanov et al. 1999; Dotti et al. 2007; Hayasaki et al. 2007, 2008; Cuadra et al. 2009; Farris et al. 2013), if initially the primary MBH is surrounded by a massive disk, the secondary MBH may quickly settle down to the disk and open a gap in the disk with a width of $\sim 2R_{\text{RL,BBH}}$, where $R_{\text{RL}}$ is the Roche radius and roughly given by

$R_{\text{RL}}(x) = 0.49a_{\text{BBH}}x^{2/3}/[0.6x^{2/3} + \ln(1 + x^{1/2})]$.  

where $x = M_2/M_1$ and $q = M_2/M_1$. For simplicity, we only consider those cases where BBHs are on circular orbits and their associated triple disks are coplanar. More complicated cases, such as eccentric BBHs and non-coplanar disks, etc., will be further discussed in Section 5.

2.2. A Simple Model for Accretion onto MBH/BBH Systems

2.2.1. Thin Disk Model for Accretion onto a Single MBH

For simplicity, we adopt the standard thin disk model to describe the radiatively efficient QSO accretion process around a single MBH (e.g., Shakura & Sunyaev 1973; Novikov & Thorne 1973). In such a model, the emission from an annulus

Figure 1. Schematic diagram for a sub-parsec BBH system. The BBH system is assumed to be on a circular orbit with a semimajor axis of $a_{\text{BBH}}$. The two MBHs are surrounded by a circumbinary disk (disk1), which is truncated at the edge of an inner gap opened by the secondary MBH. Within the gap, each component of the BBH also has its own small accretion disk truncated at half of the mean Roche radius (disk1 and disk2).

If we assume the disk size is the same as the Roche radius, we find that our results on the half-light radius–wavelength relation are not significantly changed qualitatively.
where $G$ is the gravitational constant, $\sigma_B$ is the Stefan–Boltzmann constant, $M_*$ and $M_{\text{acc}}$ are the mass and accretion rate of the MBH, $r_{\text{in}}$ is the radius of the disk inner edge, $\epsilon$ is the radiative efficiency, and $f_E$ is the Eddington ratio. The Eddington ratio here is defined as $f_E = M_{\text{acc}}/M_{*,E}$, where $M_{*,E} = 4\pi m_p G M_*/c^2 \epsilon$ is the Eddington accretion rate, $c$ is the speed of light, $m_p$ is the proton mass, and $\sigma_T$ is the Thompson scattering cross section. The infrared–optical–UV radiations in the following calculation come from disk regions far away from the MBH event horizon, and the general relativistic effects due to the central MBH and its spin are not significant. Observations on QSOs and MBHs have shown that $\epsilon \sim 0.1$ (e.g., Yu & Tremaine 2002; Marconi et al. 2004; Yu & Lu 2004, 2008; Shankar et al. 2009, 2013; Wu et al. 2013), which corresponds to $r_{\text{in}} \sim 3.5 r_g$ if adopting the standard thin accretion disk model for Kerr black holes, where $r_g = G M_*/c^2$ is the gravitational radius. Therefore, we adopt $r_{\text{in}} = 3.5 r_g$ and ignore the general relativistic corrections to the temperature profile in the innermost disk region in this study. Our results are not significantly affected if adopting the relativistic thin disk model by Novikov & Thorne (1973).

According to Equation (2), the monochromatic specific intensity at wavelength $\lambda$, as a function of radius $r$, is given by

$$B_\lambda(r) = \frac{2hc^2/\lambda^5}{\exp[hc/\lambda k_B T_{\text{eff}}(r)] - 1}, \quad (3)$$

where $h$ is the Planck constant and $k_B$ is the Boltzmann constant. For a narrow band filter with central wavelength at $\lambda$, the half-light radius of the disk ($r_{1/2}$), within which half of the light is contained, can be estimated by integrating the specific intensity at $\lambda$ over the disk radii, and it is given by

$$r_{1/2} = 2.4 \cos^{1/2} i \left[ \frac{45G^2 M_*^2 m_p f_E \lambda^4}{16\pi^4 h c^3 \sigma_T } \right]^{1/3} \approx 5.1 \times 10^{15} \text{ cm} \left( \frac{M_*}{10^8 M_\odot} \right)^{2/3} \left( \frac{f_E}{0.3} \right)^{1/3} \left( \frac{\lambda}{\mu \text{m}} \right)^{4/3},$$

$$\approx 340 r_g \left( \frac{M_*}{10^8 M_\odot} \right)^{-1/3} \left( \frac{f_E}{0.3} \right)^{1/3} \left( \frac{\lambda}{\mu \text{m}} \right)^{4/3}, \quad (4)$$

where $i$ is the inclination angle of the disk to the line of sight. The factor $\cos^{1/2} i$ is included to account for the inclination of the disk to our line of sight. For simplicity, we set all the disks to be face on so that $\cos i = 1$ ($i = 0^\circ$) in this study. The half-light radius scales with the rest frame wavelength as $\propto \lambda^{4/3}$ and MBH mass as $\propto M_*^{2/3}$.

### 2.2.2. Simple Model for Accretion onto Massive BBH Systems

For each of the three disks associated with a BBH system (see Figure 1), we assume its temperature profile still follows that given by the standard thin disk model (Equation (2)), except that there is a truncation at either the inner or outer disk. According to the assumptions made in Section 2.2.1, the disks associated with the primary and the secondary MBHs are truncated at an outer radius of $(1/2) R_{\text{E},1}(1/q)$ and $(1/2) R_{\text{E},2}(q)$, and their inner edges are $3.5 G M_*/c^2$ and $3.5 G M_{*,2}/c^2$, respectively. The circumbinary disk is truncated at an inner radius of $a_{\text{BBH}}/(1 + q) + R_{\text{E},1}(q)$. For the circumbinary disk, the mass of the central accretor is assumed to be the sum of the two BBH components, i.e., $M_* + M_{*,2}$.

Adopting the above single/triple-disk model, the brightness distribution of an active MBH/BBH system can be obtained. Figure 2 shows the brightness distribution maps at a given time and three different wavelengths, i.e., $0.16 \mu \text{m}, 0.63 \mu \text{m}$ and $3.16 \mu \text{m}$, for one single MBH system and eight BBH systems (denoted as S0, B1, B2, B3, B4, B5, B6, B7, and B8), respectively. Table 1 lists the basic parameters that define these systems, such as the BBH semimajor axis ($a_{\text{BBH}}$), the total mass ($M_*$) and mass ratio ($q$) of the BBH, and the Eddington ratio of the disk accretion onto each MBH (i.e., $f_{E,1}$, $f_{E,2}$, $f_{E,c} = f_{E,1}/(1+q) + q f_{E,2}/(1+q)$, etc.). As seen from Figure 2, the larger the semimajor axis of a BBH, the larger the size of the inner disks (disk1 and disk2) and the larger the inner edge of the circumbinary disk (disk3). For the surface brightness maps of those BBH systems, the part within the gap should rotate with time because of the periodic motions of the BBH systems.

In the standard thin disk accretion model, the short wavelength photons come mainly from the inner disk region, while the long wavelength photons from the outer disk region. In the case of a BBH system, a gap is opened in the primary disk by the secondary MBH (see Figure 1). When comparing with the radiation from a single MBH system, therefore, there should be some deficiency of the radiation of photons from a BBH system in a certain wavelength range, determined by the inner and outer boundaries of the gap. Figure 3 shows the spectral energy distributions (SEDs) for all those BBH systems listed in Table 1 (i.e., B1, B2, B3, B4, B5, B6, B7, and B8), according to the simple single/triple-disk model described above. For those BBH systems shown in Figure 3, we can clearly see the deficits of radiation in the optical to near-infrared bands, relative to the corresponding single disk systems. The deficits of thermal emission in a certain wavelength range have been proposed by

| Model | $M_*$ | $q$ | $f_{E,1}$ | $f_{E,2}$ | $f_{E,c}$ | $a_{\text{BBH}}(r_g)$ |
|-------|------|----|---------|---------|---------|----------------|
| S0    | $10^8 M_\odot$ | ... | ... | ... | ... | 0.3 |
| B1    | $10^8 M_\odot$ | 0.25 | 0.3 | 0.01 | 0.242 | 500 |
| B2    | $10^8 M_\odot$ | 0.25 | 0.3 | 0.01 | 0.242 | 1000 |
| B3    | $10^8 M_\odot$ | 0.25 | 0.3 | 0.01 | 0.242 | 2000 |
| B4    | $10^8 M_\odot$ | 0.25 | 0.3 | 0.01 | 0.242 | 3000 |
| B5    | $10^8 M_\odot$ | 0.25 | 10^{-4} | 0.3 | 0.06 | 500 |
| B6    | $10^8 M_\odot$ | 0.25 | 0.3 | 0.3 | 0.3 | 500 |
| B7    | $10^8 M_\odot$ | 1.0 | 0.3 | 0.3 | 0.3 | 500 |
| B8    | $10^8 M_\odot$ | 0.1 | 10^{-4} | 0.3 | 0.027 | 500 |

**Notes.** Basic parameters that define the example single/triple disk systems. For the S0 system, $M_*$ and $f_{E,E}$ represent the mass of the central MBH and Eddington ratio of the accretion disk, respectively; for other BBH systems, $M_*$, $q$, $a_{\text{BBH}}$, $f_{E,1}$, $f_{E,2}$, and $f_{E,c}$ represent the total mass, the mass ratio, the semimajor axis, the Eddington ratios of the disks associated with the primary component, secondary component, and the circumbinary disk of each BBHs, respectively.
The gap in a BBH system, and correspondingly the deficit of radiation in the gap region, could result in an anomaly in the half-light radius—wavelength relation (hereafter denoted as the \( r_{1/2} - \lambda \) relation), which may be significantly different from the smooth \( r_{1/2} - \lambda \) relation obtained for the standard thin accretion disk. This difference may offer an important way to probe BBHs among the lensed QSOs.

In order to demonstrate that microlensing light curves and the \( r_{1/2} - \lambda \) relation are useful for selecting/probing BBH systems, if any, in the distant lensed QSOs, we first generate mock light curves for a number of assumed BBH systems in Section 3. Then, we use these mock light curves to extract the \( r_{1/2} - \lambda \) relation and compare it with that obtained for the corresponding single disk systems to reveal the signature of BBHs in Section 4.

### 3. MOCK LIGHT CURVES

The variation of a macroimage magnitude of a lensed point source due to the microlensing effect can be represented by a magnification map projected to the source plane, of which the value of each pixel represents the difference between the macroimage’s magnitude measured by a distant observer when the source at a particular point in the map and the average macroimage magnitude. This original magnification map is determined by the properties of the lens (including the surface stellar mass density and the shear) and also the distances of the source and the lens to the observer (e.g., Kochanek 2004).

We generate the original magnification maps in the source planes by using the ray-shooting method (e.g., Wambsganss 2006, 1990; Kasyer et al. 1986; Paczynski 1986; Wambsganss et al. 1990). For simplicity and demonstration purposes, we adopt fixed values for the mean convergence (surface mass density; \( \kappa \)) and shear (\( \gamma \)) for all the systems listed in Table 1, and the values of \( \kappa, \gamma \) are arbitrarily set to be \((0.394, 0.395), (0.375, 0.390), (0.743, 0.733), \) and \((0.635, 0.623)\) for the images A, B, C, and D of each system, respectively, similar to the case for Q2237 + 0305 in Kochanek (2004). With these settings, images A and B have positive parity, while images C and D have negative parity. We set the fraction contributed by stars to the convergence as \( k_*/\kappa = 0.25 \), where \( k_* \) represents the stellar surface density. The positions of stars are randomly generated in the model. In principle, the mass of each star \( (m_*) \) could be randomly drawn from a distribution function, e.g., the Salpeter initial mass function \( \propto m_*^{-2.35} \), over a finite mass range \( m_*,1 < m_* < m_*,2 \). For simplicity and demonstration purposes, we set \( m_* = (m_* \pm m_*,1 = m_*,2 = 0.3 \, M_\odot) \), where \( m_* \) is the mean mass of the stars. The Einstein radius of a star in the source plane is fixed to be \( (R_{\text{E}}) = 3.0 \times 10^{16}(m_*/0.3 \, M_\odot)^{1/2} \, \text{cm} \), which is typical for the lensed QSOs listed in Mosquera & Kochanek (2011).

As lensed QSOs are diffuse sources on the accretion disk scales, it is necessary to obtain convolved magnification maps by convolving the original magnification maps with the surface brightness distribution maps of the sources. The rotation of the BBHs introduces further complications to the calculation of the convolved magnification maps. The orbital period of a BBH system is \( t_\text{orb} = 2\pi(a_{\text{BBH}}/GM)^{1/2} = 3.1(M_*/10^8 \, M_\odot)(a_{\text{BBH}}/10^2 r_g)^{3/2} \, \text{yr} \). The timescale of the variations due to the microlensing is determined by the source length scale and the effective relative moving velocity \( v_e \) of the source with respect to the lens. In the following analysis, the light curves of mock lensed QSOs are “monitored” over a period of about \( (R_\text{E})/v_e \), where \( v_e = \tilde{v}_e(m_*/0.3 \, M_\odot)^{1/2} \), \( \tilde{v}_e \) is the scaled velocity of the source relative to the lens. The source surface brightness map of a BBH system may rotate significantly within the monitoring period depending on the value of \( v_e \). If \( v_e \gtrsim 10^4 \, \text{km} \, \text{s}^{-1} \), the microlensing caustic passes through the BBH system in a very short time period \(~ a_{\text{BBH}}/v_e \lesssim 0.5y G(M_*/10^8 \, M_\odot)(a_{\text{BBH}}/10^2 r_g)^{1/2} < t_\text{orb} \) for \( M_*> 10^8 \, M_\odot \) and \( a_{\text{BBH}} \sim 10^2 r_g \), and thus the rotation of the BBH has little effect on the microlensing light curve within the monitoring period; however, if \( v_e \lesssim 1000 \, \text{km} \, \text{s}^{-1} \), the rotation of the BBH affects the light curve significantly. For those currently known lensed QSOs, some of them may have large \( v_e \) (e.g., Kochanek 2004) and some may have small \( v_e \) (see Mosquera & Kochanek 2011). Therefore, we consider two cases below: (1) static surface brightness distribution of the BBH sources, in which we ignore the rotation of the BBH systems during the monitoring time period; and (2) rotating surface brightness distribution of the BBH sources, in which the rotation of the BBHs introduces further complications to the calculation of the convolved magnification maps.
BBH systems is simultaneously considered for a typical value of $v_c \approx 10^4 \text{ km s}^{-1}$.

### 3.1. The Case of Static Surface Brightness Distributions

Assuming that the surface brightness distributions of the sources are static, we generate the convolved magnification maps in the source planes for all the systems listed in Table 1 by convolving the original magnification maps with the static surface brightness maps shown in Figure 2. Figure 4 shows the central part of each convolved magnification map for image A of each system (see Table 1) at wavelengths 3.16 (top panels), 0.63 (middle panels), and 0.16 $\mu$m (bottom panels), respectively.

We assign a trajectory for each image (A, B, C, or D) of each source system listed in Table 1 (e.g., the black line, representing the trajectory (a) in each panel of Figure 4 for image A). For different systems, we assign the same trajectory with respect to the disk (disk1) associated with the primary MBH. The length of the trajectory is set to be one Einstein radius. The effective velocity of the source with respect to the lens is set to be $v_c = 10^4 \text{ km s}^{-1}$. The length of an assigned trajectory thus corresponds to a period of $\Delta t = t_0 = R_E/v_c \approx 1$ yr. According to these trajectories, we can calculate the light curves for each image at each given wavelength (i.e., $\lambda = 0.16, 0.25, 0.40, 0.63, 1.0, 1.78,$ and $3.16 \mu$m). We record and measure each light curve for 50 times uniformly distributed over the 1 yr period. We add a random error to each “measurement” by assuming that this error follows a Gaussian distribution with a standard deviation of 0.05 mag (see Kochanek 2004). Note that the “measurement” error may scale with the magnitude according to the Poisson noise, but this scaling is ignored in this paper for simplicity. The background sources could also have some intrinsic variations, and normally the intrinsic emission of a QSO can vary on a timescale of within a day (see also Kochanek 2004). As the light curve is recorded every 7 days, substantially longer than a day, the intrinsic variation of the sources should appear as incoherent and can be treated as a random error. We also add the intrinsic variation to each “measurement” according to a Gaussian distribution with a standard deviation of 0.05 mag (see also Kochanek 2004). Then the “measured” magnitude at each epoch $i$ is given by

$$m_i = -2.5 \log f_i - 2.5 \log f_m + m_{int} + (\delta m/\sigma_1) + (\delta m/\sigma_2), \quad (5)$$

where $f_i$ is the mean amplification value due to microlensing at epoch $i$, $f_m$ is the amplification value due to macrolensing, $m_{int}$ is the mean intrinsic magnitude, and $(\delta m/\sigma_1)$ and $(\delta m/\sigma_2)$ are two quantities to represent the intrinsic variation and the measurement error, randomly set by Gaussian distributions with deviations of $\sigma_1 = 0.05$ and $\sigma_2 = 0.05$, respectively.

Figure 5 shows the resulting mock light curves for the trajectory (a) in the image A of each system listed in Table 1 at three different wavelengths, i.e., $\lambda = 0.16, 0.63,$ and $3.16 \mu$m, respectively. These light curves show the deviations of the measured magnitude $m_i$ at each epoch $i$ from the mean magnitude $\langle m_i \rangle$, i.e.,

$$\delta m_i = m_i - \langle m_i \rangle = -2.5 \log (f_i/\bar{f}) + (\delta m/\sigma_1) + (\delta m/\sigma_2), \quad (6)$$

where $\bar{f}$ is the mean of all $f_i$, $\sum_i f_i/N$, $i = 1, \ldots, N$ and $N$ is the total number of pixels in the magnification map. By this definition, $\delta m_i$ reflects mainly the effect of microlensing. The light curve of a system at a short wavelength (e.g., 0.16 $\mu$m or 0.63 $\mu$m) is more significantly affected by the microlensing event than that at a long wavelength (e.g., 3.16 $\mu$m; see the blue, red, and green lines in each panel of Figure 5). The maxima in the light curves are resulted from caustic crossing events. The heights of these maxima are sensitive to the ratio of the caustic size to the size of the emitting region, i.e., the larger the value of this ratio, the larger the height. The width of the caustic crossing event increases with increasing wavelength $\lambda$ and decreasing effective relative velocity $v_c$.

Figure 5 also shows the difference between the light curves obtained for the BBH systems ($\delta m_{BBH}$) and those obtained for the corresponding single disk systems ($\delta m_0^s$), i.e.,

$$\Delta \delta m_i = \delta m_i^{BBH} - \delta m_i^s, \quad (7)$$

where $\delta m_{BBH}$ is the light curve obtained for a BBH system, and $\delta m_0^s$ is the light curve obtained for a corresponding single disk system, which has a single central MBH with mass $M_s = M_{s1} + M_{s2}$ and accretion rate $f_E = f_{E0}$. For system S1, $\Delta \delta m_i$ is the difference between $\delta m_{BBH}$ and $\delta m_0^s$, where $\delta m_{BBH}$ and $\delta m_0^s$ are generated by different random numbers; therefore, $\Delta \delta m_i$ for system S0 only represents the contribution from the Gaussian intrinsic variations of the lensed QSO and the measurement errors. As seen from Figure 5, the difference is significant during the caustic crossing event at $\lambda = 0.16 \mu$m and 0.63 $\mu$m for most of the BBH systems except the BBH systems B3 and B4. The light curves for the BBH systems B3 and B4 do not significantly deviate from those of the corresponding single disk systems.
Figure 4. Convolved magnification maps in the source planes obtained by assuming static surface brightness distributions of the sources. Columns from left to right show the convolved magnification map for image A of systems S0, B1, B2, B3, B4, B5, B6, B7, and B8, respectively (assuming the convergence and shear are the same as those for the lensed QSO 2237+0305). Red colors represent regions with high magnification and consequently small $\delta m = -2.5 \log f$, see Equation (6)), while blue colors represent regions with low magnification, as indicated by the side bar. The top, middle, and bottom rows show the maps at wavelength 3.16 $\mu$m, 0.63 $\mu$m, and 0.16 $\mu$m, respectively. The source map of each convolved magnification map is correspondingly shown in Figure 2. The black solid lines (denoted as trajectory (a) in each map) and the white lines (denoted as (b), (c), and (d) in the maps for S0 and B7) show the trajectories that are adopted in calculating the mock light curves, and each asterisk symbol represents the starting point of a trajectory. Only the central parts of the convolution maps for each system are shown here, respectively. The side length of each map shown in this figure is four times of the Einstein radius in the source plane, and the map has 204 by 204 pixels.

Figure 5. Mock light curves obtained for different systems by assuming static surface brightness distributions of the sources. Panels with labels show the mock light curves for image A of systems S0, B1, B2, B3, B4, B5, B6, B7, and B8, respectively; a smaller panel below shows the differences between the mock light curves of the labeled system and that for a corresponding single MBH system (with the same mass as the total mass of the BBH and the accretion rate the same as that of the circumbinary disk in the BBH system). In each panel, points with errorbars in blue, red, and green are for the mock light curves at wavelengths 0.16, 0.63, and 3.16 $\mu$m, respectively, by considering the measurement errors; the solid curves in blue, red, and green are for the intrinsic mock light curves at wavelengths 0.16, 0.63, and 3.16 $\mu$m, respectively, without considering the measurement errors.

because the emission region for photons at $\lambda = 0.16 \mu$m and 0.63 $\mu$m are well within the gap. For all BBH systems, the differences are not significant at $\lambda = 3.16 \mu$m, because the emission region for most photons at this wavelength is well outside of the gap, and the size of this region is substantially larger than the caustic size.

For a BBH system, if the brightness of the secondary (or primary) disk is negligible compared with that of the primary (or secondary) disk, then the gap may have a significant effect on limiting the width of the spike in the light curve caused by the caustic crossing event. For system S0, for example, the width of the spike in the light curve at $\lambda = 0.16 \mu$m is narrower than
Figure 6. Mock light curves obtained for the S0 and B7 systems shown in Figure 4. Top panels show the mock light curves obtained for the trajectories (a), (b), (c), and (d) marked for the S0 system in Figure 4, respectively. Bottom panels show the mock light curves obtained for the trajectories (a), (b), (c), and (d) marked for the B7 system in Figure 4, from left to right, respectively. Symbols and lines are the same as those in Figure 5.

that at $\lambda = 0.63 \, \mu m$ (see Figure 5), while the width of the spike at $\lambda = 0.16 \, \mu m$ is almost the same as that at $\lambda = 0.63 \, \mu m$ for the system B1. The main reason for the latter case is that the gap in the disk limits the size of the emitting region of $0.63 \, \mu m$ photons.

If the brightness of the secondary disk is comparable to that of the primary disk, the light curves resulting from the BBH system could be more complicated. In system B7, the two disks associated with the two BBH components have comparable brightness. For some trajectories, each lens may pass through two sources one after the other, and the resulting light curves may thus have two or more spikes. To illustrate these special cases, we assign three more trajectories for system B7 (white lines from left to right in panel B7 of Figure 4, denoted as trajectories (b), (c), and (d), respectively), in addition to the trajectory assigned for all the systems. Correspondingly, we assign three more trajectories in the case of the single disk system S0 (white lines from left to right in panel S0 of Figure 4, also denoted as trajectories (b), (c), and (d), respectively). As shown in Figure 6, the mock light curves obtained from these trajectories can have more complicated structure, e.g., two or three spikes, as expected.

3.2. The Case of Rotating Surface Brightness Distributions

In a more general case, the surface brightness distribution of a BBH system changes with time, particularly at the region within the gap because of the rotation of the BBH and its associated disks around the center of mass. In this section, we assume $v_c \simeq 10^3 \, km \, s^{-1}$, and $\Delta t = f_0 = R_\odot/v_c \simeq 10 \, yr \gg t_{\text{orb}}$ for $M_\odot \simeq 10^6 M_\odot$ and $a_{\text{BBH}} \sim 1000 R_\odot$, and thus the light curves of these BBH systems can be significantly affected by the BBH motions (see discussion in Section 3.1). At each monitoring time, the surface brightness distribution of the BBH source is different from that at a previous time. Therefore, we do not have a fixed convolved magnification map for a system. We assign trajectories to the original magnification map for each system listed in Table 1, similar to that shown in Figure 4, and then we perform the convolution of the original magnification map with the surface brightness distribution map at each monitoring time to obtain the light curves for each system. We also add the intrinsic variations and the measurement errors to the mock light curves in the same way as that described in Section 3.1.

Figures 7 and 8 show the obtained light curves by considering the BBH rotation, which correspond to those shown in Figures 5 and 6 for the case of static surface brightness distributions. By comparing with the light curves shown in Figure 5 (or 6), we find that those light curves shown in Figure 7 (or 8) have much more complicated structure because of the BBH rotation. For example, there are six peaks in the light curves at $\lambda = 0.16 \, \mu m$ and $0.63 \, \mu m$ for system B7, mainly caused by the BBH rotation. During the caustic crossing event, the disk1 (or disk2) first crosses the caustic and moves away from it, following that the disk2 (or disk1) crosses the caustic again after a time roughly half of the BBH period, and then the disk1 comes back to cross the caustic again at a time roughly of one BBH period. These quasi-periodic caustic crossing events of disk1 and disk2 finish until the caustic crosses the whole gap region, which may consequently result in many peaks in the light curves as the emission from disk1 and disk2 are comparable. The number of the peaks depends on the ratio of the BBH orbital period to the time needed for the caustic crossing the whole gap region. (For similar rotation effects in Galactic microlensing, see Penny et al. 2011.) For system B8, there are only four peaks in the light curves (at $\lambda = 0.16 \, \mu m$ and $0.63 \, \mu m$), which is different from that for system B7. The reason is that the emission from the disk1 is negligible (even when amplified by microlensing) and the disk2 crosses the caustic again roughly one BBH period after a previous crossing. Similarly, the structures of the light curves for other systems shown in Figures 7 and 8 can also be understood.
Figure 7. Mock light curves obtained for different systems by considering the BBH rotation. Legends are similar to those for Figure 5.

Figure 8. Mock light curves obtained for the BBH system B7 by considering the BBH rotation. Legends are similar to those for the bottom panels of Figure 6.

In summary, we conclude that the rich structures in the microlensing light curves of lensed BBH QSOs shown in Figures 5–8 may help to probe and reveal the nature of BBH systems in lensed QSOs. For BBH systems that show strong rotation effects, even a single-color microlensing light curve is already highly suggestive of their existence.

4. DISK HALF-LIGHT RADIUS–WAVELENGTH RELATION ESTIMATED FROM THE MOCK LIGHT CURVES

4.1. Fitting Method

The size of the emitting region of the photons at a given wavelength can be extracted from the mock light curves of each lensed QSO generated in Section 3 (see Kochanek 2004). In reality, it is not clear at all which observed lensed QSO is due to a BBH system. Therefore, we first adopt the single disk model described in Section 2.2.1 to fit the mock light curves, although most of them are generated by assuming a background BBH source. We then check whether the $r_{1/2} - \lambda$ relationship estimated from the light curves of a BBH system is significantly different from that expected from a single MBH system and whether the difference, if any, can be used to select BBH candidates from the lensed QSOs.

According to the standard thin accretion disk model, the surface brightness profile at any given narrow band with a central wavelength $\lambda$ can be described by

$$I(r) \propto \exp((r/r_s)^{3/4}(1 - \sqrt{r_{in}/r})^{1/4})^{-1}, \quad (8)$$

where $r$ is the distance to the central MBH, $r_s = (\lambda k_B/\hbar c)^{1/3}(3G^2m_H/M_e^2f_{E/2}/2\epsilon\sigma_T\sigma_B)^{1/3} \approx 0.41r_{1/2}$, and $r_{1/2}$ is the half-light radius (see Equation (4)). Photons in the optical to near infrared bands are mainly emitted from the disk region with $r \gg r_{in}$, for which the term $(1 - \sqrt{r_{in}/r})^{1/4}$ in Equation (8) can be approximated as a constant. Therefore, the brightness distribution at a given wavelength (or a narrow band filter) of the disk is roughly determined by the single parameter $r_s$ or equivalently $r_{1/2}$. We choose 30 different brightness profiles for each source, and these profiles are described by Equation (8).
by setting $r_{1/2}$ to be in the range of 10 to $10^{-3} R_E$ with equal logarithmic intervals.

For the lens, we set the convergence $\kappa$ and shear $\gamma$ to be the same as those used to obtain the mock light curves in Section 3. The reason is that $\kappa$ and $\gamma$ can be well constrained by observations on the macro-lensing of the background QSO. The contribution of stars to the convergence $\kappa_*$ may also be constrained by detailed observations on the lens galaxy. However, it is practically difficult to constrain the effective relative velocity $v_e$, through observations other than the microlensing. For simplicity, nevertheless, we set $\kappa_*$ and $v_e$ the same as the initial input values to generate the mock light curves in the following Bayesian fitting to obtain the size of the emitting region, if not otherwise stated. In principle, $\kappa_*$ and $v_e$ can be simultaneously obtained from the fitting of the light curves, which will be discussed in Section 4.3. For general use, we keep $\kappa_*$ and $v_e$ as free parameters in the following formalization of the Bayesian fitting.

According to the above settings, we generate four different original magnification maps for the lens. (1) For the case of static surface brightness distributions, we obtain 120 convolved magnification maps by convolving with the 30 brightness profiles set above, each of which is stored in $2048 \times 2048$ arrays with a side length of $40(R_E)$ and a pixel size of $\sim 0.02(R_E)$. We note that the magnification maps here are generated by using random numbers different from that set to obtain the mock light curves in Section 3. We can then randomly choose trajectories (both the starting points and directions) in each of the convolved magnification maps, and the trajectory length is determined by the effective relative velocity and the “monitoring” period of the mock light curves. (2) For the case of rotating surface brightness distributions, we randomly choose trajectories in the original magnification map and obtain $\delta m_i^j$ at each monitoring time by convolving the original magnification map with the surface brightness distribution of the source at that time. Similar to that in Section 3 for obtaining the mock light curves, a model light curve $\delta m_i^j$ can be obtained for each trajectory. We use these light curves to fit the mock light curves by using the standard $\chi^2$ statistics, i.e.,

$$\chi^2 = \sum_i \frac{(\delta m_i - \delta m_i^{j})^2}{\sigma_i^2 + \sigma_1^2},$$

where $\sigma_1$ is set to be $\sigma_2$. By searching for the minimum of the $\chi^2$ value, we may then obtain a constraint on the half-light radius at any given wavelength for each system as follows.

Using the Bayesian theorem, the posterior probability distribution of the parameters involved in the fitting for a given set of data $\{D\}$ is

$$P(\hat{r}_i, \kappa_*, \hat{v}_e, \xi_\ell|D) \propto P(D|\hat{r}_i, \kappa_*, \hat{v}_e, \xi_\ell)P(\hat{r}_i)P(\kappa_*) \times P(\hat{v}_e)P(\xi_\ell),$$

where $P(\hat{r}_i)$, $P(\kappa_*)$, $P(\hat{v}_e)$, and $P(\xi_\ell)$ describe the prior probability estimates for the physical parameters $(\hat{r}_i, \kappa_*, \hat{v}_e)$, and the trajectory variables $(\xi_\ell)$, respectively, and $P(D|\hat{r}_i, \kappa_*, \hat{v}_e, \xi_\ell) = P(\chi^2) \propto \frac{N_{\text{dof}} - 2}{2} \left[ \frac{X^2}{2f_0^2}\right].$

By adopting a Gaussian profile to fit $P(\hat{r}_i|D)$, we obtain the most probable size of the emitting region and its uncertainty as the peak location of the Gaussian profile and its dispersion. Similar to the source size, the posterior probability distribution of the stellar convergence $\kappa_*$ (or the effective relative velocity $\hat{v}_e$) may also be obtained by marginalizing $P(\hat{r}_i)$ over other remaining parameters (see discussions in Section 4.3).

Equation (9) in Kochanek 2004). All the Bayesian parameter estimates are normalized by the requirement that the total probability is unity, i.e.,

$$\int d\hat{r}_i d\kappa_* d\hat{v}_e d\xi_\ell P(\hat{r}_i, \kappa_*, \hat{v}_e, \xi_\ell|D) = 1.$$

The starting points and directions of trajectories, $\xi_\ell$, are randomly chosen in the convolved (or original) magnification map, which are nuisance variables. The posterior joint probability distribution of the physical parameters $(\hat{r}_i, \kappa_*, \hat{v}_e)$ can be obtained by integrating Equation (10) over $\xi_\ell$, i.e.,

$$P(\hat{r}_i, \kappa_*, \hat{v}_e|D) \propto \int P(\hat{r}_i, \kappa_*, \hat{v}_e, \xi_\ell|D)d\xi_\ell.$$

We adopt logarithmic priors for the size parameter $r_i$ involved in Equation (10), i.e., $P(\hat{r}_i) \propto 1/\hat{r}_i$.

To speed up the fitting process, we also adopt a method similar to that in Kochanek (2004): we first try to find $10^3$ trial trajectories, of which the corresponding model light curves satisfy $\chi^2 \leq \chi^2_{\text{max}} = 3N_{\text{dof}}$, and then we locally optimize the starting points and directions of trajectories in the vicinity of each trial trajectory to search for the minimum value of $\chi^2$. Mock light curves at short wavelengths are more complicated than those at long wavelengths; many more test trajectories are needed to find $10^3$ trial trajectories for the mock light curves at short wavelengths than for that long wavelengths. We adopt $\sigma_1$ the same as the input ones initially if $10^3$ trial trajectories with $\chi^2 \leq 3N_{\text{dof}}$ can be found among the search of no more than $10^6$ trajectories and a good fit can be obtained; otherwise, we increase $\sigma_1$ until $10^5$ trial trajectories can be found among $10^7$ trajectories.

It has been assumed that four images (A, B, C, and D) are generated by the macrolensing for each system, and correspondingly four light curves, independent from each another, are generated for these images. Therefore, we combine the results obtained from each light curve together by multiplying the probability, i.e.,

$$P_{\text{tot}}(\hat{r}_i, \kappa_*, \hat{v}_e|D) = \prod_j P(\hat{r}_i, \kappa_*, \hat{v}_e|D)_j,$$
mainly due to the existence of a gap in the BBH systems, which limits the emitting area of photons with the wavelength in a certain range. From B1 to B4, this feature moves toward longer wavelengths and becomes less prominent mainly because the gap moves to larger radii. At longer wavelength (>1 \( \mu m \)), the half-light radii for each BBH system jump up to be even larger than that expected for the corresponding single disk system, which is due to the main emitting region jump from the inner disk(s) to the outer circumbinary disk and the lack of emission in the gap region.

We also show the residuals for the \( r_{1/2}-\lambda \) relation, i.e., the difference between the data points and the dotted line in each panel with a label, in a small panel below each labeled panel in Figure 9, respectively. As seen from the residual curves, there is a dip in each curve for a BBH system because the gap opened by the secondary MBH limits the emitting area of the photons in a certain wavelength range. Out of this wavelength range, the effect of the gap is negligible and the \( r_{1/2}-\lambda \) relation of a BBH system is similar to that for a single MBH system. With increasing separation of the two MBHs, the dip moves toward longer wavelength (see Figure 2 for systems B1, B2, B3, and B4) because the gap moves outward and the short wavelength photon emitting region is correspondingly less affected by the gap. According to panels B5, B6, B7, and B8 in Figure 9, the feature of a dip in the residual curve is maintained if choosing somewhat different mass ratio \( q \) and Eddington ratios \( f_E \) for \( f_{E,1} \) and \( f_{E,2} \). These dips are unique features of those BBH systems with physical parameters of some certain ranges and can be taken as indicators of the existence of BBHs in the background lensed QSOs.

For some light curves, e.g., B7 (b) in Figure 6 and B7 (a) in Figure 8, a good fit cannot be obtained if we set \( \sigma_1 \) the same as the input ones to generate the mock light curves. To solve this problem, we increase \( \sigma_1 \) until a good fit is obtained and then take the resulting \( r_{1/2} \) as the best fit (see Section 4.1). Figure 10 also shows the \( r_{1/2}-\lambda \) relation obtained for the light curves only for the image A shown in Figure 6 (for trajectories (b), (c), and (d) in panels S0 and B7 of Figure 4). For example, \( \sigma_1 \) is set to be 0.05 for the light curves at short wavelengths in panels (a) and (c) in Figure 6 for the system B7, while it is set to be 0.1
for the light curves at a short wavelength in panels (b) and (d) in Figure 6 for the same system. As seen from Figure 10, there are also clear dips in the residuals for the $r_{1/2} - \lambda$ relation estimated from the light curves, corresponding to trajectories (b), (c), and (d), for the BBH system B7, similar to those shown in Figure 9. These dips can also be taken as indicators of the existence of a BBH in the background lensed QSO system.

4.2.2. The Case of Rotating Surface Brightness Distributions

We adopt the same method as that for the case of static surface brightness distributions in Section 4.2.1 to fit the light curves obtained for the case of rotating surface brightness distributions (see Figures 7 and 8). Figure 11 shows the $r_{1/2} - \lambda$ relations estimated from the light curves shown in Figure 7, of which the rotation of BBHs and their associated disks are considered. These estimated $r_{1/2} - \lambda$ relations look very similar to those shown in Figure 9 for the case of static surface brightness distributions, and the main feature of a dip is still present in each residual curve for BBH systems, which suggests that this feature can be always taken as an indicator of the existence of a BBH in a lensed QSO. Figure 12 shows the $r_{1/2} - \lambda$ relations obtained from the light curves only for image A shown in Figure 4. As seen from Figure 12, the estimated $r_{1/2}$ for the light curves corresponding to trajectories (b), (c), and (d) are larger than that expected from the corresponding single disk systems by a factor of ~2–4. These cases might be responsible for some of the sources that have flatter $r_{1/2} - \lambda$ relations and relatively larger disk sizes compared with the expectations from the standard thin disk model, as discovered by Blackburne et al. (2011).

4.2.3. Exploration of the Parameter Space

We further explore the parameter space for BBH systems by varying the values of $a_{BBH}$, $M_*$, $q$, $f_{E,1}$, and $f_{E,2}$. As demonstrated above, the basic feature of the estimated $r_{1/2} - \lambda$ relation for BBH systems by considering the BBH rotation is similar to that without considering BBH rotation. For simplicity, therefore, we only consider the case of static surface brightness distributions in this section. The trajectories adopted in the convolved magnification maps are the same as the trajectory (a) shown in Figure 4.

Figure 13 shows the residuals of the $r_{1/2} - \lambda$ relation for BBH systems with different total mass (i.e., $M_* = 3 \times 10^7$, $10^8$, $3 \times 10^8$, and $10^9 M_\odot$) at different separations (i.e., $a_{BBH} = 500$, 1000, 2000, and 3000$^2$) but the same mass ratio $q = 0.25$ and Eddington ratios ($f_{E,1} = 0.3$, $f_{E,2} = 10^{-4}$). As seen from Figure 13, the $r_{1/2} - \lambda$ residual is more prominent for BBH systems with smaller $M_*$ and smaller $a_{BBH}$, and the wavelength of the dip in the $r_{1/2} - \lambda$ residual increases with increasing $M_*$ and increasing $a_{BBH}$. For BBH systems with $M_* = 10^8 M_\odot$ and $a_{BBH} \gtrsim 3000 r_g$ (or $M_* = 10^9 M_\odot$ and $a_{BBH} \gtrsim 5000 r_g$), the dip moves to the infrared band in the QSO rest frame. Considering that many lensed QSOs are at redshift $z \sim 1–2$, the dip can be observed in the optical bands only for those systems with small BBH mass ($M_* \lesssim 10^8 M_\odot$) and/or small separations ($a_{BBH} \lesssim 1000 r_g$).

Figure 14 shows the $r_{1/2} - \lambda$ residuals for BBH systems with different total mass (i.e., $M_* = 3 \times 10^7$, $10^8$, $3 \times 10^8$, and $10^9 M_\odot$) and different mass ratio $q$ but the same separation $a_{BBH} = 500 r_g$ and Eddington ratio $f_{E,1} = 0.3$, $f_{E,2} = 0.3$. As seen from Figure 14, the dips in the $r_{1/2} - \lambda$ residual curves remain more or less at the same location for different choices of the mass ratio, but the width of the dip shrinks slightly with decreasing $q$.

Figure 15 shows the $r_{1/2} - \lambda$ residuals for BBH systems with different Eddington ratios ($f_{E,1}$, $f_{E,2}$) but the same total mass $M_* = 10^8 M_\odot$, mass ratio $q = 0.25$, and separation $a_{BBH} = 500 r_g$. In panel (a), the black points (solid curve), red points (dashed curve), blue points (dotted curve), and green points (dot-dashed curve) represent the case with ($f_{E,1}$, $f_{E,2}$) = (0.3, 0.3),...
Figure 13. Difference between the $r_{1/2} - \lambda$ relation obtained for BBH systems and that for the corresponding single MBH systems. Panels (a), (b), (c), and (d) show the BBH system with total mass $3 \times 10^7$, $10^8$, $3 \times 10^9$, and $10^9 M_\odot$, respectively. The BBH systems shown in each panel have the same mass ratio ($q = 0.25$) and Eddington ratios ($f_{E_{1}, f_{E,2}} = (0.3, 0.01)$ but different separation $a_{BBH} = 500 r_g$ (black symbols and line), $1000 r_g$ (red symbols and dashed line), $2000 r_g$ (green symbols and dot-dashed line), and $3000 r_g$ (blue symbols and dotted line), respectively. Black, red, green, and blue vertical arrows mark the locations of the gap expected from Equation (4) if assuming that the half-light radius $r_{1/2} = (1/2)(a_{BBH} + (1/2) R_{E}(1/q) - R_{E}(q))) - q a_{BBH}/1 + q$ (i.e., the distance from the center of the gap to the mass center of the BBH) for each system.

(A color version of this figure is available in the online journal.)

(0.3, 0.1), (0.3, 0.01), and (0.3, 0.001), respectively, while they represent $(f_{E_{1}}, f_{E,2}) = (0.1, 0.3), (0.01, 0.3), (0.001, 0.3)$, and $(10^{-4}, 0.3)$ in panel (b), respectively. As seen from panel (a) in Figure 15, the location of the dip is insensitive to the Eddington ratio of the secondary disk as the emission is dominated by the primary disk. The dip may move toward a slightly smaller wavelength if the emission from the secondary disk becomes dominant as shown in panel (b). The reason is that the mass of the secondary MBH is smaller than the primary ones (see Equation (4)).

4.3. Stellar Convergence and Effective Relative Velocity

In the above Bayesian fitting processes, we adopt fixed $\kappa_*$ and $\nu_\ast$. In principle, these two parameters can also be simultaneously fitted, which may introduce some degeneracy in the fitting parameters. To test this, we choose four different values for the stellar convergence (i.e., $\kappa_* = 0.125\kappa, 0.25\kappa, 0.5\kappa, \kappa$) and five different values for the effective relative velocity (i.e., $\nu_\ast = 10^{-1.5}, 10^{-1.25}, 10^{-1}, 10^{-0.75}, 10^{-0.5} R_{E}/yr$) to simultaneously obtain estimates of $\kappa_*$ and $\nu_\ast$ by using the mock light curves.

According to the best fits to the mock light curves of the single MBH system in Section 4.2, we find that the best-fit values of the stellar convergence $\kappa_*$ and the effective relative velocity $\nu_\ast$ are consistent with the input ones. For the BBH systems, we find that majority of the best fits, especially those cases at short wavelength, can recover the input $\kappa_*$ and $\nu_\ast$ well, while some of the best fits for those cases (especially for system B7) at a long wavelength may give a $\kappa_*$ or $\nu_\ast$ larger than the input ones. At a short wavelength, the stellar convergence and effective relative velocity can be well recovered from the mock light curves mainly because sharp structures in the mock light curves, due to caustic crossing events, encode sufficient information of both quantities; at a long wavelength, however, the mock light curves are flat because of relatively less significant microlensing effect.

(A color version of this figure is available in the online journal.)

Figure 14. Difference between the $r_{1/2} - \lambda$ relation obtained for BBH systems and that obtained for the corresponding single MBH systems. Panels (a), (b), (c), and (d) show the BBH systems with total mass $3 \times 10^7$, $10^8$, $3 \times 10^9$ and $10^9 M_\odot$, respectively. The BBH systems shown in each panel have the same separation ($a_{BBH} = 500 r_g$) and Eddington ratios ($f_{E_{1}, f_{E,2}} = (0.3, 0.3)$ but different mass ratio $q = 1$ (black symbols and solid line), $0.5$ (red symbols and dashed line), $0.25$ (green symbols and dot-dashed line), and $0.1$ (blue symbols and dotted line), respectively.

(A color version of this figure is available in the online journal.)

Figure 15. Difference between the $r_{1/2} - \lambda$ relation obtained for BBH systems and that obtained for the corresponding single MBH systems. The BBH systems shown here have the same total mass ($M_\ast = 10^6 M_\odot$), mass ratio ($q = 0.25$), and separation ($a_{BBH} = 500 r_g$) but different Eddington ratios. In panel (a), black, red, green, and blue symbols (solid, dashed, dot-dashed, and dotted curves) represent $(f_{E_{1}}, f_{E,2}) = (0.3, 0.3), (0.3, 0.1), (0.3, 0.01)$, and $(0.3, 0.001)$, respectively, while they represent $(0.1, 0.3), (0.01, 0.3), (0.001, 0.3)$, and $(10^{-4}, 0.3)$ in panel (b), respectively.

(A color version of this figure is available in the online journal.)
and thus offer less capability of putting accurate constraints on the two quantities (see Figure 5). However, we find that the constraints on the disk size and especially the behavior of the $r_{1/2} - \lambda$ relations are not significantly affected by the uncertainties in the constraints of $\kappa_*$ and $v_\phi$.

In system B7, the disk associated with the secondary MBH has a luminosity comparable to that associated with the primary MBH. For some trajectories in the convolved magnification map (see Figure 4), a star may pass through the two disks one after the other, which leads to two peaks in the mock light curves. Similar light curves, with two peaks over a similar observational time period, can also be produced by a background source composed of a single disk but a lens with higher $\kappa_*$ and/or higher $v_\phi$, as two stars may pass through the background single disk in the observational period. As discussed in Section 4.2, the amplitude of the fluctuations in the light curve is mainly determined by the source size but not $\kappa_*$ and $v_\phi$. The estimates of the source size in Section 4.2 are not affected much by the degeneracy existed in the fitting. We note here that the degeneracy described above may be broken up if more than one caustic crossing events are observed, as the two peaks due to the two disks are always associated with each other and come together, but the two peaks due to large $\kappa_*$ and $v_\phi$ are independent.

5. OTHER COMPLICATIONS AND DISCUSSIONS

For demonstration purposes, a number of simplifications for the BBH systems are made in the above calculations. Below, we will discuss the possible effects of these simplifications on the estimated $r_{1/2} - \lambda$ relation and probing the signals of BBH systems.

1. Eccentric BBH orbit. We have assumed that the BBH systems are on circular orbits. However, the eccentricity of BBH systems may be excited/de-excited to a moderate value during the orbital decay of BBHs in gaseous disks according to some numerical simulations (e.g., MacFadyen & Milosavljević 2008; Hayasaki et al. 2007; Roedig et al. 2011). If the orbits of sub-parsec BBH systems are really significantly eccentric, the gap opened by the secondary MBH may be larger (or smaller) than that in the corresponding case with a circular BBH orbit if the BBH orbit is prograde (or retrograde; e.g., Roedig & Sesana 2013). By comparing with that of a circular BBH, we find that the disk associated with each component in the case of an eccentric BBH should be smaller and the inner edge of the circumbinary disk should be larger, thus the anomalies in the $r_{1/2} - \lambda$ relations of the BBH systems become more (or less) prominent.

2. Periodical variations of the accretion rates due to the BBH dynamics. In the above calculations, the light emitted from each BBH system was assumed to not vary significantly. However, as suggested by a number of recent studies (e.g., MacFadyen & Milosavljević 2008; Haiman et al. 2009; Sesana et al. 2012; Hayasaki et al. 2013; D’Orazio et al. 2012), the accretion onto active eccentric BBH systems may vary on a timescale on the order of the orbital periods because of the BBH dynamics. This variation is periodic and may be substantial and is totally different from the intraday intrinsic variation assumed in Section 3. The variation due to the BBH dynamics could be mixed with the variation because of the microlensing effect. The rotation of BBHs and their associated disks within the gap may also cause quasi-periodic variation during the caustic crossing events, but this quasi-periodic variation finishes after the caustic crosses the whole gap region as discussed in Section 3.2. If the period of a BBH system is sufficiently long compared with the caustic crossing effect, then there is not much effect caused by the BBH dynamics on the microlensing light curve. If the period of the BBH systems and the time for the caustic to cross the whole gap region is relatively short compared to the light curve period, then variations due to the BBH dynamics and the microlensing effect can be separated by modeling the light curves.

3. Non-coplanar disks associated with BBH systems. It is possible that the BBH orbital plane is misaligned with the disk plane. In a misaligned BBH system, it may be difficult to open a gap in the primary disk by the secondary MBH (see Hayasaki et al. 2013); therefore, the difference between the $r_{1/2} - \lambda$ relation of a misaligned BBH system and that of a single disk system may thus be not significant. However, the exact structure of disks associated with BBH systems is still not fully understood. Future high resolution simulations, following the dynamical evolution of gas and BBHs from the galactic to disk scale, may be able to answer whether sub-parsec active BBH systems have coplanar or non-coplanar disk structures.

4. Disk model. A standard thin disk model is adopted in the above calculations. In reality, a thin disk model may be too simple even for the single disk case. In order to check the effect on the resulting $r_{1/2} - \lambda$ relation by the usage of different disk models, we also adopt the TLUSTY model to estimate the $r_{1/2} - \lambda$ relation for the systems listed in Table 1. In the TLUSTY model, the relativistic effects, the disk vertical structure, and the radiative transfer in the disk are simultaneously considered (for details, see Hubeny et al. 2000). We find that the estimated size at a given optical wavelength by adopting the TLUSTY model is slightly larger than that obtained by adopting the standard thin disk model, but the general shapes of the $r_{1/2} - \lambda$ relations estimated by adopting the two different disk models are the same. If the area of emitting region increases rapidly with increasing wavelength in any adopted disk model, the anomaly in the $r_{1/2} - \lambda$ relation of BBH systems always exists as a reflection of the gap, a unique and generic property of accretion flows onto BBH systems.

In this study, we adopt the standard thin disk model with a constant accretion rate for each of the triple disks associated with those BBH systems (see Section 2). It is possible, however, that the temperature structures of and radiation from these disks may deviate from that of the thin disk model because of several factors: BBH torques (e.g., Rafikov 2013), accretion stream(s) connecting the outer circumbinary disk with the inner disk(s), and the associated (magneto)hydrodynamical shock(s) at both the inner edge of the outer circumbinary disk and the outer edges of the inner disks. Compared with the thin disk, these disks might emit more power at the inner edge of the circumbinary disk and at the outer edge of the disks associated with the two BBH components. This complication certainly needs further investigation, though it may not significantly affect the anomaly in the $r_{1/2} - \lambda$ relation found in this study because of the existence of the gap.

5. Disk orientation. All the disk systems are assumed to be face on in the above calculations. In reality, the background QSOs should not be exactly face on. As the lensed QSOs are normally type 1 QSOs, they have an inclination
angle $i$, i.e., the angle between the disk normal and the line of sight, roughly in the range of $\cos i \sim 0.5–1$ (e.g., Krolik 1999). Considering this orientation effect, the surface brightness distributions for each disk in the non-face on cases are elliptic-like. Comparing with the surface brightness distributions shown in Figure 2 for the face-on cases, those for non-face on cases shrink at one direction by $|\cos i|$ while maintaining the same at the other, orthogonal direction. This change in the surface brightness distributions leads to corresponding changes in the microlensing light curves. For illustration purposes, we assume two BBH systems, which are similar to B5 and B8 except their disks are non-face on. Our calculations show that the heights and separations of the several magnification peaks in their light curves are quite different from those shown in Figure 7, mainly because of the more significant asymmetric distribution of the surface brightness in the non-face on cases. These differences may help us to simultaneously constrain the disk orientation and the BBH nature. We also note that the projected area of the emitting region for a disk system, with an inclination angle of $i$, is a factor of $\cos i (\sim 0.5–1)$ smaller than that of a face on system. This difference only leads to a slightly underestimation of the source size but does not affect the shape of the $r_{1/2} - \lambda$ relation and thus does not introduce any anomaly in the $r_{1/2} - \lambda$ relation.

6. Reconstruction of BBH systems through their microlensing light curves. In Section 4, we estimate the $r_{1/2} - \lambda$ relation from the mock light curves of BBH QSO systems by adopting a single disk model. In principle, we may reconstruct BBH systems once they are selected as BBH candidates on the basis of their light curves and $r_{1/2} - \lambda$ relations. By this reconstruction, we may obtain strong constraints on the physical parameters of the BBHs, such as $a_{BBH}$, $M_*$, $q$, $f_{d1}$, and $f_{d2}$, etc. We defer the feasibility study of the reconstruction of those BBH systems through their microlensing light curves to a future study.

6. CONCLUSIONS

In this paper, we propose a novel method to probe sub-parsec BBH QSOs through the microlensing of lensed QSOs. If a QSO hosts a sub-parsec BBH in its center, it is expected that the BBH is surrounded by a circumbinary disk at outside region, each component of the BBH is surrounded by a small accretion disk, and a gap is opened by the small component in between the circumbinary disk and the two small disks. Assuming such a structure for some hypothetical BBH systems, we generate mock microlensing light curves for BBH QSO systems with typical physical parameters. We show that the microlensing light curves of a BBH QSO system at some given bands can be significantly different from those of a single MBH QSO system because of the existence of the gap and the rotation of the BBH and its associated small disks around the center of mass. We estimate the half-light radius-wavelength relations from those mock light curves and find that the obtained half-light radius-wavelength relations of BBH QSO systems can be much flatter than that of single MBH QSO systems at a wavelength range determined by the BBH parameters, such as the total mass, mass ratio, separation, etc., which is primarily due to the existence of the gap. Such a unique feature of BBH QSO systems can be used to select and probe sub-parsec BBHs in a large number of lensed QSOs to be discovered in the near future.

Note that hints for the existence of BBHs in some QSO systems may be revealed by their SEDs (Gültekin & Miller 2012, see Section 2.2) and other spectral features (see the introduction), but these features may be model dependent and somewhat obscure because of complications in the emission processes. The new method proposed in this paper can provide stronger physical constraints on BBH systems and is less model dependent, as the structure of the disk accretion associated with BBH systems can be well resolved by microlensing. The signatures are particularly striking when the rotation effects are present (see Figures 7 and 8).

Many of the lensed QSOs may be single MBH systems, but some of them might be accretion systems with BBHs in their centers. The occurrence rate of BBHs in QSOs is an important quantity to determine the efficiency of using microlensing to probe sub-parsec BBHs in lensed QSOs. In principle, the BBH occurrence rate is determined by: (1) the active level of the two MBHs within the gap and (2) the residence time of BBHs at the separation that has the most significant signals (i.e., $\sim 500–1000 \, r_g$). Most recent numerical simulations suggest that significant accretion can be induced onto the two MBH components although a low-density cavity (gap) at the center of the circumbinary disk is maintained (e.g., Hayasaki et al. 2007; MacFadyen & Milosavljevic 2008; Roedig et al. 2011; Noble et al. 2012; Shi et al. 2012; D’Orazio et al. 2012). In this paper, we assume that the level of activities of sub-parsec BBH systems embedded in circumbinary disks are more or less the same as that of normal bright QSOs; therefore, the occurrence rate of BBH systems in QSOs is mainly determined by the BBH residence time. For those BBHs with total mass in the range of $10^2 - 10^6 \, M_\odot$, mass ratio $q \sim 0.1–1$, and separation $\sim 500–1000 \, r_g$, the residence time is roughly in the range from a few $10^2$ to a few $10^6$ yr, according to Haiman et al. (2009, see Figures 1 and 3 therein). The lifetime of normal QSOs is estimated to be in the range of a few times $10^7–10^8$ yr from various methods (e.g., Haiman & Hui 2001; Martini & Weinberg 2001; Jakobsen et al. 2003; Yu & Lu 2004, 2008; Marconi et al. 2004; Worseck & Wisotzki 2006; Worseck et al. 2007; Gonçalves et al. 2008; Shankar et al. 2009, 2013; Lu & Yu 2011). Thus, the occurrence rate of BBHs in lensed QSOs should be in the range of a few thousandth to a few percent.

The total number of currently known lensed QSOs is $\sim 100$ (Kochanek 2006; http://www.cfa.harvard.edu/castles; http://www-utap.phys.s.u-tokyo.ac.jp/~sdss/sqls/). According to the BBH occurrence rate estimated above, at most, several of these lensed QSOs could host BBH systems with separation $\lesssim 1000 \, r_g$, which may be detectable through the microlensing event(s). Ground-based surveys, such as the Panoramic Survey Telescope and Rapid Response System (Pan-STARRS; which is already operating), the planned Large Synoptic Survey telescope (LSST), and Euclid, will monitor a large area of sky in multi-bands from UV, optical to infrared for multiple times over a long period (e.g., 10 yr). It is expected that the Pan-STARRS and the LSST will discover $\sim 5000–8000$ luminous QSOs that are gravitational lensed into multiple images by foreground galaxies (Abell et al. 2009; Oguri & Marshall 2010). It is possible that tens to one hundred of these lensed QSOs contain BBHs in their centers according to the BBH occurrence rate estimated above. Long-term monitoring of these lensed QSOs by the Pan-STARRS, LSST, Euclid, and other future facilities may provide a powerful and efficient way to discover sub-parsec BBHs unambiguously through the anomaly in the $r_{1/2} - \lambda$ relation estimated from the QSO light curves.
Figure 16. rms of the magnification histogram at different wavelengths obtained from the magnification maps for image A of the BBH systems shown in Figure 4 (asterisks) and those of corresponding single MBH systems (solid line).

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APPENDIX

ROOT MEAN SQUARE HISTOGRAM OF THE CONVOLVED MAPS AND THE HALF-LIGHT RADIUS

Mortonson et al. (2005) have demonstrated that the half-light radius of a source can be well determined by the dispersion (rms) of the convolved magnification histogram (see Figure 9 in Mortonson et al. 2005). Note that the magnification histogram represents the probability distribution of the pixels in the convolved magnification map that has a certain magnitude shift of the macroimage’s flux with respect to its mean. In order to better understand the results on the $r_{1/2} - \lambda$ relations obtained above for the BBH systems through the Bayesian fitting method, we also adopt the rms method given by Mortonson et al. (2005) to infer $r_{1/2}$, i.e., we first obtain the dispersions (rms) of magnification histograms for all BBH systems and their corresponding single MBH systems and then use rms to infer $r_{1/2}$. The magnification histogram of a BBH system does not depend on whether we consider the rotation of the BBH and its associated disk around the center of mass or not.

Figure 16 shows the dispersion of the magnification histogram (rms) for the BBH systems obtained from the convolved magnification maps shown in Figure 4 and those of the corresponding single MBH systems. As seen from Figure 16, rms estimated for the BBH systems (asterisk symbols) obviously deviates from that for its corresponding single disk systems (solid line). If we adopt the relation between rms and $r_{1/2}$ of the single disk system to estimate the size of the emitting region of BBH systems at a given wavelength, then the resulting $r_{1/2}$ may also deviate from that expected for the single disk model (see Equation (4)).

Figure 17 shows the resulting relationship between the half-light radius and the wavelength for each BBH system listed in Table 1 and its residual with respect to the relation expected from the corresponding single MBH system, which are similar to that shown in Figure 7. The consistency of these two results suggests that the magnification histograms can be used to reveal the difference between the $r_{1/2} - \lambda$ relation of a BBH system and that of the corresponding single MBH system, and this difference offers an easy way to explore the parameter space of the BBH systems.

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