Precise predictions for multi-TeV and PeV energy neutrino scattering rates

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The scattering rate of multi-TeV and PeV energy neutrinos is fast becoming an interesting topic in (astro)particle-physics. This is due to experimental progress at Neutrino Telescopes such as IceCube which have begun to gain sensitivity to the flux of neutrinos in this energy range. In view of this, a precise calculation of the scattering rate of neutrinos upon atoms is presented. The two main components of the calculation are the differential cross-section predictions for neutrino scattering upon an atomic nucleus (such as that of water), as well as upon atomic electrons. In the first case, the predictions for neutrino-nucleus cross-sections in charged- and neutral-current scattering are refined by including resonant contributions generated within the photon field of the nucleus, which alter the considered distributions by $\approx 2\%$. In the latter case, radiative corrections are provided for all $2 \rightarrow 2$ scattering processes of the form $\bar{\nu}_e e^- \rightarrow f \bar{f}'$. For antineutrino energies of $E_{\bar{\nu}_e} \approx 6 \text{ PeV}$, where these processes become resonantly enhanced (the Glashow resonance) and dominate the total cross-section, these corrections amount to $\approx -10\%$.

INTRODUCTION

The measurement of a flux of ultra high energy (UHE) neutrinos$^1$ at detectors on Earth is extremely important for understanding potential sources of cosmic ray accelerators in our universe. As neutrinos are weakly interacting, they propagate through the Universe without being deflected by magnetic fields or scattering on background photon radiation. The detection of these neutrinos within large volume detectors such as IceCube therefore provides information on their source of production, which in turn can help to identify the source(s) of UHE cosmic rays in our Universe which are expected to generate such a flux. The potential of this physics program has been recently demonstrated with the identification of the Blazar TXS 0506+056 as a source of high energy neutrinos$^1$.$^2$.

The IceCube experiment has already become sensitive to the flux of neutrinos with multi-TeV and PeV energies$^3$, which has enabled various measurements of the cross-sections and event characteristics induced by neutrinos within this energy range$^4$.$^5$. A measurement of the event rate of PeV-energy neutrinos is of particular interest as the scattering process of electron antineutrinos upon atomic electrons becomes resonantly enhanced for centre-of-mass (CoM) energies of $\sqrt{S} \approx 2m_eE_{\bar{\nu}_e} \approx m_W^2$ (the Glashow resonance$^6$), which is achieved for $E_{\bar{\nu}_e} \approx 6 \text{ PeV}$. As this process is directly sensitive to the flux of electron antineutrinos at Earth, its measurement provides flavour separation which is invaluable for understanding the production mechanisms which source UHE neutrinos$^8$.$^9$. As more data is collected at IceCube, as well as other large volume detectors such as KM3NeT$^{10}$ and Baikal-GVD$^{11}$, it is anticipated that a measurement of this process will become feasible.

In anticipation of this experimental progress, it is the purpose of this work to re-visit the corresponding theoretical predictions for the scattering rates of multi-TeV and PeV energy neutrinos on atomic targets. Technical details of the calculation are given in the following Section, before applying the calculations to study the following observables: the total inclusive cross-section; the mean inelasticity distribution in muon production; as well as the inclusive cross-section for quark production in $\bar{\nu}_e + e^-$ collisions (with focus on the resonance region).

DETAILS OF CALCULATION

The computation of the scattering rate of a high energy neutrino with a stationary atom is performed by separately considering the processes where the neutrino scatters either upon an atomic nucleus or electron, the details of which are summarised below.

Neutrino-nucleus scattering. In the case of a nuclear target, the dominant contributions to the cross-section arise from either the charged-current (CC) or neutral (NC) processes, where an interaction between the incident neutrino and the constituents (partons) of the target nucleus is mediated by either W- or Z-boson exchange respectively. The differential cross-section for this process can be conveniently written in terms of the Deep Inelastic Structure (DIS) functions $F_i$ and kinematic variables which characterise the scattering process (see for example Eq. (2.2) and (2.3) of Ref. [14]). These $F_i$ are a function of the (negative) squared four-momentum $Q^2$ of the exchanged gauge-boson and the variable $x$ (which in the parton model of the nucleus, corresponds to the fraction of the nucleon’s momentum carried by the struck parton). They describe the underlying dynamics of the nucleus as

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1 Unless specifically stated, 'neutrinos' will collectively refer to both neutrinos and antineutrinos.
probed by the exchanged gauge-boson, and consequently their description is process dependent with respect to the type of interaction (CC/NC), projectile ($\nu/\bar{\nu}$), and target nucleon (bound/free proton/neutron). A prediction for the relevant $F_i$ can be performed perturbatively by convoluting process dependent coefficient functions with a set of parton distribution functions (PDFs) for the target nucleus. The resultant predictions for the cross-section provided in this way can be made differential with respect to the outgoing lepton kinematics.

The CC and NC processes describe the class of partonic sub-processes where the gauge-boson is exchanged in the $t$-channel, and the four-momentum of the exchanged gauge-boson is time-like. In addition to this, there are also partonic subprocesses generated within the photon field of the nucleus which take the form $\bar{\nu}_e \gamma \rightarrow f \bar{f} \ell$. These subprocesses (and charge conjugate versions) receive a resonant enhancement when $m_{f,\ell} \approx m_W$ (on-shell W-boson exchange), and additionally receive an enhancement due to collinear splittings of the form $\gamma \rightarrow \ell \bar{\ell}$ (on-shell lepton exchange) [15]. While the contribution of these subprocesses is (naively) suppressed by $O(\alpha^2)$ as compared to leading-order (LO) CC and NC predictions, they can (as a result of these enhancements) still impact the total cross-section by several percent [16] and should therefore be included.

To consistently take into account all contributions discussed above, a PDF set based on the NNLO NNPDF3.1uxQED analysis [17] (which uses the luxQED formalism [18, 19]) is generated with the program APFEL [20]. This PDF set is modified at the input scale $Q_0 = 1.65$ GeV to include non-zero (anti)electron and (anti)muon PDFs according to the fixed-order ansatz assumed in Ref. [21] (see Eq. 2.6). This PDF set is then evolved including both NNLO QCD and complete LO QED corrections to the DGLAP equations [21].

The predictions for the $F_i$ are obtained by convoluting the resultant PDFs with coefficient functions computed at the same order. Heavy quark mass effects (via FONLL-B,C schemes [22, 23]) and nuclear corrections (from the EPPS16 analysis [24, 25]) are included for the CC and NC predictions as in Ref. [24].

The computation of the resonantly enhanced contributions is straightforward in this setup, and is obtained by convoluting the evolved lepton PDFs with the LO partonic cross-section $\bar{\nu}_e \ell \rightarrow f \bar{f} \ell$. This approximate calculation includes an all-orders resummation of the collinear logarithms (which dominate the cross-section) via the PDF evolution. It was checked, by directly computing the $O(\alpha)$ correction, that the finite (non-logarithmically enhanced) corrections are numerically unimportant and have been neglected in the results presented here.

(Anti)neutrino-electron scattering. The scattering processes described above provide the dominant contributions to the cross-section for most neutrino energies. An exception occurs for the scattering of electron antineutrinos upon atomic electrons, where the process $\bar{\nu}_e e^- \rightarrow f f'$ becomes resonantly enhanced for $E_{\bar{\nu}_e} \approx 6$ PeV.

To provide an accurate prediction of the scattering rate in this energy range, the complete QCD and Electroweak (EW) corrections have been evaluated for all processes of the form $\nu_e e^- \rightarrow f f'$ (including non-factorisable corrections). To account for the finite width effects of the W-boson at $O(\alpha)$, the calculation is performed in the Complex-Mass-Scheme (CMS). The masses of fermions are neglected (wherever possible) throughout the calculation, with the exception that $b$- and $t$-quark masses are retained throughout. The results are obtained with the aid of FeynArts [27] and FormCalc [28] and presented in terms of complex scalar one-loop integrals. To provide differential cross-section predictions, the calculation has been implemented in a flexible FORTRAN code, where one-loop integrals are evaluated numerically with OneLOop [29, 30]. The integration over the relevant two- and three-body phase spaces for virtual and real corrections is performed numerically with the VEGAS algorithm implemented in CUBA [31], and the technique of Dipole-Subtraction is used to regularise the implicit and explicit infrared divergences present in the differential calculation [32] (see also [33] for massless QED calculations). Further refinements to the calculation are also made by including the effect of higher-order initial state radiation (ISR) corrections to the incoming electron state. These corrections are included by applying the structure function approach [34]. For the numerical results shown in this work, the leading logarithmic (LL) corrections up to $O(\alpha^3)$ as well as the impact of soft exponentiation [35] are included [36-39] (see for example Eqns. (5.2)-(5.6) of Ref. [40]) — results including these corrections will contain the label ‘+LL’.

**NUMERICAL RESULTS**

In this Section, numerical predictions are provided for a few specific scattering processes encountered in the collision of (anti)neutrinos with an $H_2O$ molecule. These results are obtained with the following numerical inputs: $\alpha_0 = 1/137$, $\alpha_s = 0.118$, $M_W^{\text{pole}} = 80.385$ GeV, $\Gamma_W^{\text{pole}} = 2.085$ GeV, $M_Z^{\text{pole}} = 91.1876$ GeV, $\Gamma_Z^{\text{pole}} = 2.4952$ GeV, $m_h = 4.5$ GeV, $m_t = 173$ GeV, $m_b = 125.0$ GeV, $m_\tau = 0.511$ MeV. The following fermion masses are also used in the evaluation vacuum polarisation $\Delta\alpha$ (which are evaluated perturbatively at one-loop): $m_u = m_d = 50$ MeV, $m_s = 150$ MeV, $m_c = 1.5$ GeV, $m_t = 105$ MeV, $m_\tau = 1.78$ GeV. For numerical comparisons, the following value of Fermi’s constant is used $G_F = 1.16638 \times 10^{-5}$ GeV$^{-2}$. The calculation is performed in the CMS where a complex value for the weak mixing angle $s_{\omega}$ is derived according to the relation $s_{\omega}^2 = 1 - \mu_W/\mu_Z$, with
\(\mu_V = m_V^2 - i\Gamma V M_V\). The \(\alpha_0\)-scheme is used as default throughout the calculation.

**Total inclusive cross-section.** This study focusses on the total (summed over all final states) inclusive cross-section obtained for antineutrinos collisions with either a nucleus (\(\bar{\nu} + N\)) or electron (\(\bar{\nu} + e^-\)) target. The predictions are produced specifically for electron antineutrino projectiles within the energy range of \(E_{\bar{\nu}} \in [0.05, 50]\) PeV. The total cross-section in \(\nu + N\) collisions has also been produced, and is available upon request. The cross-sections obtained for either neutrino or muon/tau antineutrinos incident upon an electron target are numerically unimportant in the considered energy range.

The results of this study are shown in Fig.1. In the upper plot, distributions for the total \(\bar{\nu}_e + e^-\) cross-section (summed over all \(f f^\dagger\) final states), the total \(\bar{\nu} + N\) cross-section (the sum of CC, NC, and resonant contributions), and the resonant-only contribution to the \(\bar{\nu}_e + N\) cross-section are shown. In the lower plot, each of these distributions are shown normalised with respect to total \(\bar{\nu} + N\) cross-section. The theoretical uncertainties of each prediction is typically a few \% (relatively), and have been omitted in this Figure. It should be noted that the atomic cross-section for \(\text{H}_2\text{O}\) is obtained by multiplying \(\bar{\nu}_e + N\) and \(\bar{\nu}_e + e^-\) cross-sections by a factor of 18 and 10 respectively, corresponding to the mass and atomic number of the atom.

![Figure 1: The total electron antineutrino cross-section on atomic electrons and a H2O-nucleon as a function of incident antineutrino energy. In the case of \(\bar{\nu}_e + N\) collisions, the resonant contribution generated within the photon field of the nucleus is shown separately.](image1)

As shown in Fig.1 all contributions are necessary to provide a precise (%-accurate) prediction of the total cross-section. In the multi-TeV range, the resonant contributions amount to \(\approx (2 - 3)\%\) and should therefore be taken into account. At higher energies, these contributions are relatively less important as the CC and NC cross-sections grow more quickly. This behaviour is a consequence of the faster growth of the quark PDFs at small-\(x\) as compared to the leptons PDFs (the quark PDFs are additionally enhanced at small-\(x\) due to the singular behaviour of the \(g \rightarrow gg\) splitting function \(P_{gg}\)). Within the energy range of \(E_{\bar{\nu}} \in [4, 10]\) PeV, the \(\bar{\nu} + e^-\) contribution dominates the total cross-section. It should also be noted that this process receives large corrections (\(\approx\) factor of two) for \(\sqrt{2m_{\bar{\nu}}E_{\bar{\nu}}} \gtrsim m_W\). The impact of radiative corrections to this process within the resonance region will be considered towards the end of this Section.

Finally, as an additional check, the CC and NC cross-sections obtained in this work have been compared to those presented in Ref. [41] obtained at NNLO+NLL\(x\) accuracy, and lead to consistent results within PDF uncertainties. The latter predictions were obtained with the NNPDF3.1sx PDF set [41] which were extracted with the same input data set. The main differences, due to the treatment of small-\(x\) resummation and the constraints provided by the LHCB D-hadron data considered in Ref. [14], are not relevant for the PeV energy regime considered here.

**Inelasticity distribution.** While the study of the total inclusive cross-section is theoretically instructive, it is experimentally more relevant to study exclusive final states. As a first example, the production of a charged muon in neutrino-nucleon collisions is considered. This process leads to an experimental signature composed of a cascade of hadrons and a muon, an event topology which is often referred to as a starting track. The IceCube Collaboration has recently performed a measurement of the mean inelasticity in events of this type [6], which is a measure of the fractional energy transfer of the incident neutrino to hadrons. The inelasticity is equivalent to \(y = 1 - E_\mu/E_\nu\) for the CC DIS process. A comparison to the available data for \(\langle y \rangle\) is presented in Fig.2

![Figure 2: The mean inelasticity in muon production, as a function of the incident (anti)neutrino energy. In addition to the total prediction, the central values obtained when either resonant contributions or nuclear effects are excluded are also shown. For reference, the distributions obtained in either \(\nu + N\) or \(\bar{\nu} + N\) collisions are also shown.](image2)
The total prediction is obtained by including contributions from both $\mu^+$ and $\mu^-$ production to provide a consistent comparison with the experimental measurement [9]. The uncertainty of the theoretical prediction is obtained by adding in quadrature the $1\sigma$ CL uncertainties of the free PDFs and nuclear corrections, which are then combined linearly with the uncertainty due to scale variation. The scale variation uncertainty is obtained by varying renormalisation and factorisation scales by a factor of two around the nominal scale $Q$, with the additional constraint $1/2 < \mu_F/\mu_R < 2$. In addition to this, the central values obtained when either the nuclear corrections or the resonant contributions are excluded are also shown.

The nuclear corrections lead to a suppression of $\langle y \rangle$ by up to (2-3)% within the PeV range. This is primarily due to the shadowing of the gluon PDF at small-$x$ values, which can lead to a suppression of CC and NC cross-sections by $\approx 5\%$ [14]. This suppression is largest at small-$x$ values, which at fixed-$Q^2$ corresponds to large-$y$, and thus leads to a reduced $\langle y \rangle$. It was checked that similar behaviour is observed with nNPDF1.0 nuclear corrections obtained in Ref. [12].

The resonant contributions lead an enhancement of this distribution, which almost cancels the nuclear effect discussed above. While the contribution to the total cross-section from this process is small as compared to the CC DIS contribution (below 1%), there is still an impact on the $\langle y \rangle$ distribution due to the large inelasticity of these type of events. As an additional note, the prediction in Fig. 2 is obtained using the relation $y = 1 - E_\mu/E_\nu$ which does not strictly correspond to the fractional energy transfer to hadrons for the resonant contributions, as missing energy is transferred to an outgoing neutrino. This (small) effect is not accounted for in the measurement, and is also neglected here.

It is worth mentioning that secondary lepton production (e.g. via heavy-quark production [6][13]) may also lead to an apparent starting track type event, and subsequently alter the observed inelasticity distribution. The impact of these types of contributions is best assessed by the experimental collaborations where the impact of detector response is included.

**Inclusive quark cross-section.** The most promising channel for observing resonantly enhanced events in $\bar{\nu}_e + e^-$ collisions is the quark final-state, which leads to an experimental signature of hadronic showers. If the corresponding scattering process occurs within the detector, and the resultant hadronic showers are contained, a resonant peak can be constructed in the visible energy spectrum [14].

Assuming a combined (and equal) flux of electron neutrino and antineutrinos of $E^2 \Phi_{\nu_e, \bar{\nu}_e} = 1 \times 10^{-8}$ GeV cm$^{-2}$ s$^{-1}$ sr$^{-1}$, the IceCube Collaboration has estimated that 0.9 contained hadronic shower events may be detected per year [14]. The event rate prediction for this process is proportional to the cross-section, flux of electron antineutrinos, and the effective detector volume. A precise prediction for this cross-section is therefore necessary to allow a reliable estimation of the flux of PeV energy electron antineutrinos at Earth.

Predictions for the inclusive cross-section are presented in Fig. 3 as a function of the incident antineutrino energy. The central values obtained in the $a_0$-scheme at various accuracies are shown, and these results are compared to the LO prediction obtained when the cross-section is computed in terms of $G_F$ [45] via the replacement $\pi\alpha/s_w^2 \to \sqrt{2}G_F\mu_W$. In the lower plot (where the distributions are normalised to “LO ($G_F$)”), the theoretical uncertainties of the NLO + LL prediction are also shown. The quoted uncertainty is the envelope of the uncertainty due scale variation, which is applied to predictions obtained when the QCD corrections are included in either a multiplicative or additive to the EW-corrected cross-section. The uncertainty due to factorisation scale dependence (which results from including higher-order_ISR corrections via structure functions) is also assessed when changing the reference scale from $\mu = \sqrt{s}$ and $\mu = \sqrt{-t}$, where $t = (p_e - p_\nu)^2$. The central value of the NLO + LL prediction is obtained with $\mu_F = \sqrt{-t}$ and the multiplicative prescription for the QCD corrections. In addition to the distributions in Fig. 3, the inclusive cross-section in bins of $E_\nu$ is also provided in Table I.

![Figure 3: The inclusive cross-section for quark production in $\bar{\nu}_e + e^-$ collisions as a function of the incident antineutrino energy. The theoretical predictions are computed in the $a_0$-scheme, and compared to the LO cross-section parameterised in terms of $G_F$.](image)

The higher-order corrections have an important impact on the line-shape around the resonance region. As compared to LO ($G_F$), the NLO + LL prediction receives a correction of $\approx -10\%$ in the peak-region which will be relevant for the interpretation data (even with small statistics). While naively using $G_F$ as an input is sensible (it naturally includes some of the universal higher-order EW effects), this scheme provides a poor approximation.
Table I: The inclusive cross-section (in nb) for quark production in $\bar{\nu}_e + e^- \rightarrow f f'$ for several bins of $E_{\nu_e}$. The predictions are obtained in the $a_0$-scheme, except those labelled as "LO ($G_F$)" as described in the text.

| $E_{\nu_e}$ [PeV] | LO ($G_F$) | LO | NLO | NLO + LL |
|-----------------|------------|----|-----|---------|
| [5.8, 6.0]      | 42.02      | 39.16 | 38.32 | 39.04$^{+0.4}_{-0.3}$ |
| [6.0, 6.2]      | 120.8      | 112.5 | 107.1 | 109.5$^{+1.2}_{-1.0}$ |
| [6.2, 6.4]      | 288.0      | 268.4 | 253.5 | 258.1$^{+2.7}_{-2.3}$ |
| [6.4, 6.6]      | 158.4      | 147.6 | 153.3 | 154.3$^{+0.8}_{-0.9}$ |
| [6.6, 6.8]      | 55.91      | 52.10 | 63.10 | 61.41$^{+0.4}_{-0.3}$ |

to the total cross-section as the dominant ISR corrections are absent. These corrections receive a logarithmic enhancement of the form $\alpha/\pi \ln [m_W/m_e]$ (which multiplies the splitting function $P_{f\bar{f}}$), and amount to $\approx -15\%$ of the LO cross-section. The uncertainties of the NLO + LL predictions are sufficiently small enough (below 1%) that they can be ignored as compared to the expected precision of the data.

DISCUSSION AND CONCLUSIONS

In this work, predictions for multi-TeV- and PeV-energy neutrino scattering rates on both nuclear and electron targets have been revisited.

For neutrino-nucleus scattering, these predictions have been improved by including the resonant contributions which are generated within the photon field of nucleus. These contributions have been included by generating leptons PDFs within the nucleus, using a PDF set based on the luxQED formalism as a boundary condition, and are found to impact the considered distributions by $\approx 2\%$. These predictions are consistent with previous determinations based on an equivalent photon approximation for inclusive $W$-boson production. The main differences are that the predictions are now made available for exclusive final states ($f f'$) differential in the outgoing fermion kinematics, and additionally include a resummation of the leading collinear logarithms.

A precise calculation of $\bar{\nu}_e + e^- \rightarrow f f'$ scattering rates has also been presented, which is NLO-accurate and additionally includes the impact of universal higher-order ISR corrections. For quark final states, the inclusive cross-section receives a correction of $\approx -10\%$ in the resonance region and will be relevant for the interpretation of PeV-energy event rates at Neutrino Telescopes.

While only a select number of (mostly inclusive) observables have been presented in this work, the calculations have been implemented in such a way that fully differential predictions can be produced. It is anticipated that these calculations can also be useful as a tool for the experimental collaborations. In particular, interfacing the fixed-order calculations presented in this work with a fully exclusive Parton Shower would allow the experimental collaborations to have a more accurate modelling of both QCD and QED radiation in event simulations, which may lead to improved sensitivity of the experimental measurements. This is foreseen for future work.

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