Self-energy of strongly interacting Fermions in Medium: a Holographic Approach

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We consider the self-energy of strongly interacting fermions in the medium using gauge/gravity duality of D4/D8 system. We study the mass generation of the thermal and/or dense medium and the collective excitation called plasmino, by considering the spectral function of fermion and its dispersion relation. Our results are very different from those of the hard thermal loop method: for zero density, there is no thermal mass or plasmino in any phase. Plasmino in deconfined phase is not allowed in D4/D8 set up. In the confined phase, there is plasmino modes only for a window of density.

I. INTRODUCTION

The study of fermion self-energy in medium has a long history due to its fundamental importance in studying electronic as well as nuclear matter system. When the excitations are strongly interacting, perturbative field theory method cannot give a reliable result since the diagrams should be truncated to the ladder or rainbow types, which can not be justified in strong coupling. Furthermore in the presence of chemical potential, the lattice technique is not much useful due to the sign problem. Therefore it is worthwhile to utilize the gauge/gravity duality for this tantalizing problem. The gauge/gravity duality was used to study the fuzzy fermi surface [1] and the non-fermi liquid nature [2–4] of the strongly interacting system, for recent developments, we refer to [5–8].

The weakly interacting field theory (QED or QCD) results for the fermion self energy in medium can be summarized by the existence of the plasmino mode [9] and thermal mass generation of order $gT$. Plasmino is a collective mode whose dispersion curve has a minimum at finite momentum. However, recent study of the thermal field theory [10, 11], by solving Schwinger-Dyson equation numerically, showed that the thermal mass is reduced as the coupling grows. It raises an interesting questions what happens to the thermal mass and more generally to the hard and soft momentum scales and magnetic mass scale, $T, gT, g^2T$ respectively, in the strong coupling limit.

In this letter we study the dispersion relation and thermal mass generation using gauge/gravity duality and report a feature of plasmino in strongly coupled system. We consider D4/D8 model [12] and turn on a fermion field on the flavor brane world-volume with finite quark/baryon number density [13, 14, 15]. In deconfined phase, the fermions represent the fundamental strings, connecting the horizon of black branes and the probe D8. In the zero ’t Hooft coupling limit, it is a bi-fundamental representation. In the large ’t Hooft coupling limit, D4-branes disappear and the color index of the fermions should disappear also. In confined phase, to describe a baryon we need to introduce Witten’s baryon vertex [16]. $N_c$ strings are connecting it to the probe D8/D8. The dynamics of connecting open strings define that of the baryon vertex as well as the D8. Since each string gives fermionic mode, total vibrational dynamics should be described by the composite operator coming from the product of all $N_c$ fermions. This is the baryon in confined phase which is fermion if $N_c$ is odd. We replace this composite operator by a massive fermion field. So our construction for baryon is rather phenomenological. We consider just one flavor brane case mostly. Previously fermions on a probe brane in adjoint representation, called mesino, were studied in [17, 18], which are different from ours.

By solving the Dirac equations in each phase, we obtain dispersion relations for the fermionic excitations in medium. Our results show that for zero density, there is no thermal mass generation and no plasmino in any phase, which is sharply different from weakly coupled field theory result. To get plasmino in deconfined phase we need to add quark mass as well as quark density. In D4/D8 set up, the quark mass is not allowed so plasmino is forbidden in deconfined phase. In the confined phase, there is a plasmino mode only for a certain window of density.

II. PLASMINO IN FIELD THEORY

We begin our discussion by giving a brief discussion of fermionic collective excitations in a plasma. The fermion propagator is written as

$$G(p) = \frac{1}{\gamma \cdot p - m - \Sigma(p)},$$

(1)

where $\Sigma = \gamma_{\mu}\Sigma^\mu$ is self-energy. The gauge invariant result is available in the hard (high temperature) thermal loop approximation (HTL) where fermion mass $m$ can be ignored since it is small compared with $T$ or $\mu$. The most noticeable effect of the medium is effective mass generation. The HTL result of effective mass is given by [19]

$$m^2_F = \frac{1}{8} g^2 C_F \left( T^2 + \frac{\mu^2}{\pi^2} \right),$$

where $C_F = 1$ for electron and $C_F = 4/3$ for quark. There are two branches of dispersion curves $\omega = \omega_{\pm}(p)$ whose asymptotic forms are given
Example text
The retarded Green function in boundary field theory is the motion for the final work of Sakai-Sugimoto model. The induced metric effective 5 dimensional world volume following the original work of Sakai-Sugimoto model. We put a probe fermion field for by replacing where \( \phi = (\mathbf{v} + m_5, \mathbf{v} - m_5) \).

Following the procedure in [3], we rewrite the Dirac equation for \( \Psi \) can be given by
\[
\sqrt{g_{rr}/g_{rr}}(\partial_{rr} - m_5\sqrt{g_{rr}})\Psi + iK_{\mu}\Gamma_{\mu}\Psi = 0 ,
\]
where \( K_{\mu} = (-v(r), k_i) \), \( v(r) = (\omega + q\alpha_0)\sqrt{-g_{ii}/g_{tt}} \).

Following the procedure in [3], we rewrite the Dirac equation in terms of two component spinors
\[
(\partial_{rr} + m_5\sqrt{g_{rr}} \sigma^3)\Phi_\alpha = \sqrt{g_{rr}/g_{ii}}(i\sigma^2 v(r) + (-1)^{\alpha} k\sigma^1)\Phi_\alpha ,
\]
where \( \sigma^i \) are Pauli matrices and \( \alpha = 1, 2 \) denoting the up and down two spinor respectively. Further decomposing \( \Phi_1 = (y_1, z_1)^T, \Phi_2 = (y_2, z_2)^T \), we get equations of motion for \( y_\alpha \) and \( z_\alpha \). When \( \alpha = 2 \) we have
\[
(\partial_{rr} + m_5\sqrt{g_{rr}})y_2(r) = \sqrt{g_{rr}/g_{ii}}(v(r) + k)z_2(r),
\]
\[
(\partial_{rr} - m_5\sqrt{g_{rr}})z_2(r) = \sqrt{g_{rr}/g_{ii}}(-v(r) + k)y_2(r).
\]

By replacing \( k \) by \( -k \), we obtain the equations of motion for \( y_1 \) and \( z_1 \). We would like to define the following new variables \( G_1(r) := y_1(r)/z_1(r), G_2(r) := y_2(r)/z_2(r) \). The retarded Green function in boundary field theory is obtained as
\[
G_{1,2}^R = \lim_{\epsilon \to 0} e^{-8m_5Rr^{1/4}} G_{1,2}(r)|_{r=1/\epsilon} ,
\]
where \( G_1 \) and \( G_2 \) satisfy the following equations
\[
\sqrt{g_{ii}/g_{rr}} G_\alpha + 2m_5\sqrt{g_{ii}} G_\alpha = (-1)^{\alpha} k + v(r) + ((-1)^{\alpha} - 1 + v(r)) G_\alpha^2 .
\]

Now the remain task is to solve \([15]\) by imposing proper boundary conditions.

**Confined phase** In the confined phase of Sakai-Sugimoto model, \( v(r) \) function appearing in the Dirac equation is given by \( v(r) = \omega + q\alpha_0 r \). where the \( U(1) \) flux solution on flavor brane is obtained from DBI action as
\[
a_0(r) = \mu + \int_r^\infty d\rho \left( \frac{f(\rho)^{-1} D^2 D^2}{\rho^2 + D^2} \right)^{1/2} .
\]

Notice that \( g_{rr} = R^2 r^{-3/2} f(r)^{-1} \) which diverges at \( r_0 \). The boundary conditions for flow equation \([15]\) can be found by requiring \([15]\) regular at \( r = r_0 \). They are given by
\[
G_\alpha(r_0) = -\frac{mR + \sqrt{m^2 R^2 + k^2 - \omega^2}}{(-1)^{\alpha} k - \omega} ,
\]
where \( \omega = \omega + m \). And we define a 4 dimensional vacuum mass \( m := m_5\sqrt{g_{ii}}|_{r=r_0} \). Notice that imposing the boundary condition for retarded (advanced) Green function corresponds to \( \omega \to \omega + i\epsilon \) (\( \omega \to \omega - i\epsilon \)).

**Deconfined phase** In the deconfined phase, \( v(r) \) function is given by \( v(r) = \frac{\omega + q\alpha_0 r}{\sqrt{f}} \) and the electric flux is given by
\[
a_0(r) = \mu + \int_r^\infty d\rho \left( \frac{D^2}{\rho^2 + D^2} \right)^{1/2} .
\]

Due to the black hole horizon, we imposing the in-falling boundary condition at \( r = r_H \). The boundary condition at \( r = r_H \) is obtained as
\[
G_{1,2}(r_H) = i .
\]

The fermion dispersion relation can be obtained by searching for poles of spectral function, which is the imaginary part of retarded Green function. We solve \([15]\) numerically with IR boundary conditions \([17]\) in confined phase and \([19]\) in deconfined phase.

V. NUMERICAL RESULTS

Now we discuss the results of fermionic spectral function. First in the deconfined phase, the self-energy term gets some imaginary part due to the in-falling boundary condition. If the imaginary part is not large we can read off the dispersion relation from the positions of the maxima of spectral function. The 3D plot of spectral function with zero density is shown in Figure 2. Since the fermions here are deconfined quarks, we set \( m = 0 \).
Our main question here is whether thermal mass can be generated by finite temperature effect in strong coupling limit. In Figure 2 the dispersion curve passes through the origin and this feature is independent of temperature although it is illustrated for $T = 1$. As a result, no thermal mass is generated and there is no plasmino in deconfined phase with zero density. The absence of thermal mass is actually one of the most drastic differences compared with result of vanishing thermal mass is actually consistent with a recent claim made in [11] by numerical study of Dyson-Schwinger equation in the strong coupling region.

If we turn on finite density in the system, density effect can generate effective mass. However for massless fermion we have an exact Green function $G_R = i$ at $k = 0$, so it is impossible to observe peak structure near $k = 0$. We can not observe plasmino mode in finite density for the massless fermion.

What happen if we added a finite bulk fermion mass for curiosity? We find that density effect can generate effective mass as well as plasmino mode for large enough chemical potential.

Now we turn to the confined phase. In this phase, all the peaks of spectral function are delta function-like peaks. In general the Fermi momentum is defined by $\omega(k_F) = 0$. We plot dispersion relations $\omega = \omega_\pm(k)$ in Figure 3.

Observation of plasmino We observe a plasmino dispersion relation characterized by the presence of the minimal energy at finite momentum. As we change chemical potential, the slope of $\omega_-$ at $k = 0$ changes. We plot the slope $\alpha(\mu_0)$ as a function of density in Figure 4. In HTL approximation, the value of slope at $k = 0$ is $-\frac{1}{3}$ independent of density or temperature.

The high density behavior of the dispersion curve is complex and we will report it elsewhere. We restrict ourselves to the density range where traditional plasmino mode exists. We could observe plasmino only in a chemical potential window $\mu_1 < \mu_0 < \mu_2$. When $m = 0.1$, $\alpha = \frac{d\omega}{dk}$ at $k = 0$. The curve is plotted only in the density window $\mu_1 < \mu_0 < \mu_2$ where there is plasmino. Parameter $m = 0.1$. Notice that $\alpha = \frac{d\omega}{dk}$ at $k = 0$ has a constant value $-1/3$ for the weakly coupled field theory.

FIG. 2. Spectral functions of $G_R^2$ at zero density: the dispersion curve passes through the origin and thermal mass vanishes. Top view and side view of 3D plot of Im$G_R^2$ with $T = 1$, $m = 0$. True range of both $\omega$ and $k$ is $[-5, 5]$.

FIG. 3. Dispersion relations in confined phase. The upper and lower branches describe the normal fermion $\omega_+$ and plasmino $\omega_-$ respectively. Dotted line denotes light cone.

FIG. 4. Dispersion relations of $G_R^2$ in confined phase. As chemical potential increases, dispersion curve moves down. Parameter $m = 0.1$.

FIG. 5. $\mu_0$ dependence of $\alpha = \frac{d\omega}{dk}$ at $k = 0$. The curve is plotted only in the density window $\mu_1 < \mu_0 < \mu_2$ where there is plasmino. Parameter $m = 0.1$. Notice that $\alpha = \frac{d\omega}{dk}$ at $k = 0$ has a constant value $-1/3$ for the weakly coupled field theory.
the window is given by \( \mu_1 = 0.69 \), \( \mu_2 = 1.94 \). As \( m \) increases, this window gets wider. Inside the window, as density becomes larger, dispersion curve moves down and bends more and more as shown in Figure 6. This may be compared with field theory result in weak coupling, where effective mass and plasmino are generated for any density.

We can read off baryon mass in medium defined by \( m_* := \bar{\omega}(0) \). We will show \( m_* \) is a monotonically increasing function of chemical potential. The mass shift in medium is defined as \( \delta m := m_* - m \). The normalized mass shift quantity \( \chi(\mu_0) := \delta m/m \) is plotted in figure 6. Notice that for massless case, there is no mass correction at all.

FIG. 6. a) \( \mu_0 \) dependence of \( \chi \). b) \( \mu_0 \) dependence of \( k_F \). Parameter \( m = 0.1 \).

If we turn off both charge and bulk mass in confined phase, we have exact solution \( G_2(r) = \sqrt{\frac{k_B}{k_B^2 + k^2}} \), which is independent of radial direction.

**Fitting dispersion relation** Now we try to fit the dispersion curve of plasmino. We write the Green function as

\[
G^R_2(\omega, k) = \frac{Z}{\omega - \delta m - \frac{\kappa k}{k + B}}. \tag{21}
\]

Although the numerator \( Z \) is some undetermined function, the pole information is assumed to be contained only in the denominator. Parameter \( B \) and \( C \) can be written in terms of \( \delta m = m_* - m \), \( k_F \) and \( v_F \) as follows

\[
B = \frac{k_F^2(1 - v_F)}{\delta m + k_F v_F}, \quad C = \frac{\delta m^2 + 2\delta m k_F + k_F^2 v_F}{\delta m + k_F v_F}, \tag{22}
\]

where \( \delta m \) is mass shift, \( k_F \) is Fermi momentum and \( v_F \) is Fermi velocity. \( v_F \) is defined as the slope at \( k = k_F \). Expanding (21) near Fermi momentum, we get

\[
G^R_2(\omega, k) \sim \frac{Z}{\omega - v_F(k - k_F) - \Sigma}, \tag{23}
\]

where the self-energy near Fermi momentum is obtained as

\[
\Sigma = \delta m + \frac{(1-v_F)(\delta m + k_F v_F)}{k_F(\delta m + k_F)}(k-k_F)^2 + O((k-k_F)^3). \tag{24}
\]

The vanishing mass shift limit \( \delta m = 0 \) corresponds to the massless fermion in confined phase. In this limit, self-energy becomes

\[
\Sigma = \frac{(1 - v_F)v_F}{k_F} \cdot (k - k_F)^2 + O((k - k_F)^3). \tag{25}
\]

**VI. CONCLUSION AND DISCUSSION**

In this paper we discuss our observations on the characters of fermion’s self-energy in the dense medium. By using gauge/gravity dual, we recover normal and plasmino branch. In deconfined phase, we showed that for zero density, there is no thermal mass generation. In the weakly coupled thermal gauge theory, \( T, g T \) and \( g^2 T \) provide three well separated scales, with physical interpretations: hard momentum, thermal mass for fermion(or electric screening mass for gluon), magnetic mass respectively. These masses play the role of the infrared regulator of different scales. Such separation does not happen for large coupling \( \sim O(1) \), and we believe that we can not define any such physical scale either for strong coupling. In the confined phase, plasmino excitations are present only for a window of the density. The group velocity of the plasmino mode at zero momentum turns out to be density dependent rather than a constant, \(-1/3\), showing the deviation of the HTL approximation. It is also worthwhile to notice that without medium effect, there is no mass correction. We found a simple empirical formula for plasmino dispersion relations. We expect that the phenomena we discovered here is common in many holographic models, so that it is a universal character.

There is an issue whether \( m_0 \) can be calculated in a way following [18], where one starts with massless fermion in 10 dimension and performs the KK reduction from 10 dim to 5 dim. The answer is negative because D4 background does not have a direct product structure. To see this, let us consider

\[
(\gamma.D_x + \gamma.D_y - m_{10})\Psi(x,y) = 0 , \tag{26}
\]

where \( x \) represents the non-comact directions and \( y \) does the compact directions. For D3 background, where manifold is direct product \( AdS_5 \times S^5 \), the \( \gamma.D_y \) term produces \( \sim (l + 2)/r_{ads} \) which can be interpreted as a mass term. While in our model, D4 background is not a direct product. Therefore the KK reduction in our case generates term \( \sim (l + 2)/r \) which is a potential rather than a mass. One may call such potential term as a “mass function” which depends on the radius. We will study the effect of mass function in future work.

**Note added**: In the closing period of this work, an observation on two branches of dispersion relation appeared in [33] with emphasis on the spin physics.

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