Models for structure formation attempt to predict the power spectrum of density perturbations in the present universe from the initial power spectrum and the nature of dark matter. Observational constraints on the power spectrum at different scales in the present epoch can, therefore, be used to eliminate (or choose between) different theoretical models. Such a comparison is fairly easy at large scales (at which linear theory is valid), and one can use observations like the MBR anisotropy, large scale streaming motions etc to constrain the models. But to discriminate between the models effectively, it is necessary to constrain the power spectrum at small scales. The most reliable constraints on the power spectra at small scales come from the predicted abundance of bound systems which can be estimated reasonably accurately using Press-Schechter (or similar) methods. In the past, this method has been used in conjunction with the quasar abundance and cluster abundance. We show here that the abundance of damped Lyman alpha systems (DLAS, hereafter), provides a far stronger constraint on the models for structure formation. Models with a mixture of hot and cold dark matter (which are consistent with large scale observations) are strongly ruled out by the DLAS constraints while models with cosmological constant are marginally inconsistent. It is also possible to combine the constraints from the abundance of clusters, DLAS and QSO’s to obtain model-independent bounds on the power spectrum at the nonlinear scales. These bounds are to be respected by any viable model for structure formation.

The damped Lyman $\alpha$ systems have been studied extensively in the recent years. The spectroscopic surveys for DLAS have shown that the absorbers - found between
redshifts 1.7 to 4 - have an average HI column density $< \bar{N} > \sim 10^{21}\text{cm}^{-2}$. The most striking property of the DLAS comes from a determination of their abundance characterised by the mean number density of lines per unit redshift, $dN/dz$. Lanzetta et al (1991) find the number of damped Ly$\alpha$ systems per unit redshift interval with $\bar{N} \gtrsim 2 \times 10^{20}\text{cm}^{-2}$ is

$$0.16 \pm 0.03 < \frac{dN_{\text{damp}}}{dz} < 0.25 \pm 0.04 \quad \text{at} \quad < z > = 2.5.$$  \hspace{1cm} (1)

Here $< z >$ refers to the average redshift of these systems and the two bounds are derived from two distinct subsamples of the data. Their data is also consistent with there being no evolution in the absorber properties. The number density of the damped systems exceeds that expected from present day galaxies by a factor of about $3 - 5$ (see ref. 16). This has been interpreted to indicate that the cross section of the absorbers is larger than that associated with galaxies.

More importantly, the observed value of $dN_{\text{damp}}/dz$, used in conjunction with the mean column density allows one to estimate the density of neutral hydrogen contributed by the DLAS. The mean mass density of neutral gas contributed by the damped Ly$\alpha$ systems at their mean redshift $< z >$ is $\rho_{\text{damp}}(< z >) = \mu m_p < \bar{N} > (dN/cdt)$. Here $\mu = 1.4$ is the mean molecular weight of the gas and $m_p$ is the proton mass. As the universe expands this average mass density would have decreased by a factor $(1+< z >)^3$. Comparing the resulting density with the present day critical density one gets the current density parameter of the HI making up the DLAS to be

$$\Omega_{\text{damp}} = \mu m_p < \bar{N} > \frac{dN}{dz} |_{< z >} \frac{(1 + 2q_0 < z >)^{1/2}}{\rho_c(1+< z >)} \left( \frac{H_0}{c} \right)$$ \hspace{1cm} (2)

(This result can be easily be modified to take into account a cosmological constant). To obtain the total mass associated with the DLAS we have to include two factors: Suppose $f_N$ is the HI fraction of the gas in the absorbers and $f_b$ is the baryonic fraction in the universe. Then the density parameter contributed by the total mass associated with the DLAS, which has collapsed to form bound objects is

$$\Omega_{\text{DLAS}} = \frac{\Omega_{\text{damp}}}{f_N f_b} \approx \frac{0.352}{(1+z)^{1/2}h^{-1}} \left( \frac{dN/dz}{0.2} \right) \left( \frac{< \bar{N} >}{10^{21}\text{cm}^{-2}} \right) \left( \frac{f_N}{0.5} \right)^{-1} \left( \frac{f_b}{0.015h^{-2}} \right)^{-1}$$ \hspace{1cm} (3)

for a $\Omega = 1$ universe. At an average redshift of 2.5 one gets $\Omega_{\text{DLAS}} = 0.094$ if $h = 0.5$.

It is not easy to estimate the mass associated with the individual DLAS directly. However, there are several indications to suggest that these systems have masses in the range of $10^{11-12}M_\odot$. If we make this assumption that DLAS are galactic scale objects, then we see from (3) that their abundance will provide the most stringent constraint on models of structure formation: any such model should have enough power on galactic scales to be consistent with a collapsed fraction of about 10 percent, at the redshifts associated with DLAS. We elaborate on this point in greater detail below after considering briefly the evidence that the damped systems indeed represent galactic mass objects.

For $dN/dz = 0.2$, a typical value indicated by (1), one infers $\Omega_{\text{damp}} = 1.5h^{-1} \times 10^{-3}$ for a flat universe. By comparison the mass density of the stars in present day galaxies
are thought\(^{17}\) to account for \(\Omega_{\text{vis}} \sim 3h^{-1} \times 10^{-3}\). This similarity of \(\Omega_{\text{damp}}\) and \(\Omega_{\text{vis}}\) first led to the suggestion that the damped Ly\(\alpha\) systems may be the progenitors of present day luminous galaxies observed before the bulk of their stars formed, when the gas fraction was high. An alternate interpretation\(^{18}\) is that these systems may represent a population of gas rich dwarf galaxies, which had smaller cross section but were much more abundant in the past. Direct information on the size and mass of the damped Ly\(\alpha\) absorbers is limited to only a few cases at present: Study of the 21 cm absorption line\(^{19}\) caused by the \(z = 2.04\) DLAS in PKS 0458-02, indicates that the absorber must extend more than \(8h^{-1}\)kpc across the line of sight. Spatial imaging\(^{20}\) of the Ly\(\alpha\) emission at \(z = 2.811\) towards the QSO 0528-250, shows that the DLAS at \(z = 2.811\) could be a compact group of gas rich galaxies or a single large cloud of diameter \(\sim 100\) kpc. Detection of CO emission lines\(^{21}\) from the \(z = 2.14\) DLAS in the same QSO indicates that the absorber may be a group of galaxies each galaxy having a gas mass \(\sim 2 \times 10^{11}M_\odot\) and virial dimension \(\sim 40\) kpc. Deep imaging of the field around the QSO 0000-263 has revealed a candidate galaxy associated with the \(z = 3.34\) DLAS seen along the line of sight.\(^{22}\) In view of these observations, it appears likely that the damped Ly\(\alpha\) systems are gas rich massive galaxies rather than dwarfs. This is also consistent with the finding\(^{23}\) that luminous galaxies, rather than dwarfs, are responsible for producing the Mg\(\text{II}\) absorption systems, of which the damped systems are a subset, at \(z = 0.3 - 0.9\). We shall explore the consequences of this point of view below.

To do this we have to compute the theoretically expected value of \(\Omega_{\text{theory}}(M, z)\) contributed by bound systems with mass greater than \(M\) at any given redshift \(z\). This can be done using the Press-Schechter formalism and we get the result that:

\[
\Omega_{\text{theory}}(> M, z) = \text{erfc} \left[ \frac{\delta_c(1+z)^2}{\sqrt{2\sigma_0(M)}} \right]
\]

where \(\text{erfc}\) denotes the complementary error function, \(\sigma_0(M)\) is the power spectrum evaluated at \(z = 0\) by the linear theory and \(\delta_c\) is the linearly extrapolated density contrast needed for collapse. In the simple spherical top hat model one finds that \(\delta_c = 1.68\). Given a theoretical model, we know \(\sigma_0(M)\) and hence can estimate \(\Omega_{\text{theory}}\). For a model to be viable we need \(\Omega_{\text{DLAS}} \leq \Omega_{\text{theory}}\), with the equality implying that every collapsed object is hosting a DLA system. This restriction, it turns out, can constrain the models very effectively.

In figure 1 we have plotted \(\Omega_{\text{theory}}(10^{12}M_\odot, z)\), the fraction of the mass which has collapsed into objects with \(10^{12}M_\odot < M < 10^{13}M_\odot\), for different theoretical models and compared them with the \(\Omega_{\text{DLAS}}(z)\) inferred from the data of Lanzetta et al.\(^{16}\), using (3). The data points of Lanzetta et al. have been given by Turner and Ikuechi\(^{24}\) as a table of \(dN/dz\) at different redshifts. The data points with the error bars are for an \(\Omega = 1\) universe. If \(\Omega_{\text{matter}} < 1\), due to the presence of a non-zero cosmological constant, then the data needs to be shifted up. For \(\Omega_{\text{vac}} = 0.8\), the centers of the data need to be shifted to the location marked by the crosses.

All the theoretical models in figure 1 are based on a primordial power spectrum of the form \(P(k) \propto k\). The solid curve is for the pure CDM model with the BBKS transfer function; the broken curve is for a model with CDM and cosmological constant (called V+CDM model hereafter) with \(\Omega_{\text{vac}} = 0.8, \Omega_{\text{cdm}} = 0.2\); the dotted curve is for the CDM
model with a bias factor of 2.5 (such a model, of course, cannot reproduce large scale observations including the COBE results unless the bias factor is scale dependent; we have included it only for the sake of comparison). The dot-dash curve is for a mixed H+CDM model with \( \Omega_{\text{hdm}} = 0.7, \Omega_{\text{cdm}} = 0.3 \). All models (except, of course, the b=2.5 one) are normalised so as to reproduce the MBR quadrupole anisotropy measured by COBE. We have also taken \( h=0.5 \) for all cases except for the V+CDM model for which \( h=0.8 \). These parameters are chosen so that the models are consistent with the large scale observations.

The comparison shows that the mixed dark matter models are strongly ruled out by the abundance of DLAS. The disparity is so high that the theoretical and observational uncertainties in the analysis is unlikely to be of any use in saving the model. The V+CDM model also falls short of producing the required abundance (note that in this case the data points are shifted up; it is the location of the crosses which one has to compare with the theoretical curves). However, the uncertainties in the data as well as some freedom which is available in theoretical modelling can be utilised to make the disparity lower. We feel that the data does rule this model also out though not as firmly and conclusively the H+CDM model. The pure CDM with COBE normalisation, of course, has no difficulty in explaining the abundances since it has a lot of small scale power; however this model is known to be inconsistent with other observations (like results from galaxy surveys). Finally, the (once popular) CDM model with \( b=2.5 \) also falls short of explaining the DLAS abundances.

In the past, these models were tested as regards the observed quasar abundance. The constraints posed by the abundance of DLAS are far more stringent. To see this, let us calculate the \( \Omega_{\text{quasar}} \), the density contributed by the mass associated with the quasars, along the following lines: The basic strategy is to relate the observed quasar abundance to the abundance of host galaxies of quasars. Quasars are thought to be powered by accretion onto black holes. Unless special models using super Eddington luminosities are involved, one can infer a characteristic black hole mass by assuming that the observed luminosity of the quasar corresponds to the Eddington luminosity. The mass of the host galaxy associated with a quasar of blue magnitude \( M_B \) can then be estimated to be:

\[
M_G \approx c_1 \times 10^{12} M_\odot \times 10^{0.4[-M_B-26]} \tag{5}
\]

where

\[
c_1 = 1.2 \left( \frac{f_{\text{bol}}}{10} \right) \left( \frac{f_b}{0.06} \right)^{-1} \left( \frac{f_{\text{hole}}}{0.01} \right)^{-1}. \tag{6}
\]

The three f-factors arise as follows: To convert the blue magnitude to the bolometric magnitude we have to use a correction factor, \( f_{\text{bol}} \), which depends on the quasar spectrum but is in general of order of about 10. Secondly, only a fraction \( f_b \) of matter in the universe may be in baryonic form. And, finally, only a fraction \( f_{\text{hole}} \) of the baryons in a collapsing host galaxy be able to form the compact central object. We find from (5) that a bright quasar with \( M_B = -26 \) is typically hosted by a galaxy of mass of about \( 10^{12} M_\odot \).

The density contributed by the quasars can now be written as

\[
\Omega_{\text{quasar}}(> M, z) = \rho^{-1} \int_{-\infty}^{M_B(M)} \tau M_G(M_B) \Phi(M_B, z) dM_B \tag{7}
\]
where \( M_B(M) \) is got by inverting (5), \( \Phi \) is the quasar luminosity function and \( \rho \) is the mass density of the universe, at present. Further \( \tau = \text{max}(1, t(z)/t_Q) \), takes into account the fact that only a fraction of order \( t_Q/t(z) \) of galaxies will display quasar activity if the quasar lifetime is smaller than than the age of the universe \( t(z) \) at redshift \( z \). For the quasar luminosity function we shall use the form derived by Boyle et al.\(^{25} \) for a \( \Omega = 1, h = 0.5 \) universe. We shall also assume that their luminosity function, which is unevolving beyond \( z = 1.9 \), can be used up to \( z = 4 \) (see Ref. 4). Consider the abundance of collapsed halos required to explain the most luminous quasars, with \( M_B < -26 \). One can evaluate the integral in (7) numerically to find

\[
\Omega_{\text{quasar}}(> c_1 10^{12} M_\odot, z > 1.9) = 1.4 c_1 \times 10^{-3}
\]

(8)

For a particular model of structure formation to successfully explain the abundance of quasars it must give \( \Omega_{\text{theory}}(> M, z) > \Omega_{\text{quasar}}(> M, z) \). In figure 1 we have also plotted this line for \( 1.9 < z < 4.0 \). While this also rules out H+CDM model, it is clear that the constraint arising from this consideration is weaker than the one from DLAS.

The abundance of objects like DLAS, quasars (and clusters) can be used to obtain bounds on the power spectrum \( \sigma_0(M) \) at different scales. This is shown in figure 2 which is a plot of \( \Omega \) versus \( \sigma \) for different values of \( z \). This plot is clearly independent of the power spectrum in the model and depends only on the validity of Press-Schecter analysis. The observed abundances of different objects at different redshifts can be used to mark out regions in this graph, thereby constraining the value of \( \sigma \) at the mass scales corresponding to the objects. For example, the abundance of Abell clusters at \( z \simeq 0 \) gives \( \Omega \simeq (0.001 - 0.02) \); from the plot (marked by short horizontal lines in the \( z = 0 \) curve) we see that this leads to the tight constraint of \( \sigma \simeq (0.5 - 0.7) \) at cluster scales [which turns out to be around \( 8h^{-1}\text{Mpc} \)]. Similarly, the quasar abundance (marked by a straight horizontal line) gives \( \sigma > (2 - 2.5) \) at galactic scales. The DLAS give \( \sigma > (3 - 4) \) at the same scale (see the shaded region). Any model for structure formation should provide sufficient power at these galactic and cluster scales in order to be compatible with the observations.

Similar constraints on the models for structure formation (from the abundance of DLAS) has been reached independently in ref. 26.

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Figure 1: The density contributed by collapsed objects with mass in the range of $10^{12} - 10^{13}$ in various theoretical models is compared with observations. All the theoretical models are based on a primordial power spectrum of the form $P(k) \propto k$. The solid curve is for the pure CDM model with the BBKS transfer function; the broken curve is for a model with CDM and cosmological constant (called V+CDM model hereafter) with $\Omega_{\text{vac}} = 0.8, \Omega_{\text{cdm}} = 0.2$; the dotted curve is for the CDM model with a bias factor of 2.5. The dot-dash curve is for a mixed H+CDM model with $\Omega_{\text{hm}} = 0.7, \Omega_{\text{cdm}} = 0.3$. All models (except, of course, the b=2.5 one) are normalised so as to reproduce the MBR quadrupole anisotropy measured by COBE. We have also taken $h=0.5$ for all cases except for the V+CDM model for which $h=0.8$. The data points with the error bars are based on the observed abundance of DLAS and are for an $\Omega = 1$ universe. If $\Omega_{\text{matter}} < 1$, due to the presence of a non-zero cosmological constant, then the data needs to be shifted up. For $\Omega_{\text{vac}} = 0.8$, the centers of the data need to be shifted to the location marked by the crosses. The horizontal line is based on the abundance of quasars.

Figure 2: The $\Omega$ contributed by collapsed objects is plotted against the linearly extrapolated density contrast $\sigma_0$. The curves are parametrised by the redshifts $z = 0, 1, 2, 3, 4$ from top to bottom. The observed abundances of different objects at different redshifts are used to mark out regions in this graph, thereby constraining the value of $\sigma$ at the mass scales corresponding to the objects. The abundance of Abell clusters at $z \simeq 0$ (marked by short horizontal lines in the $z = 0$ curve) gives $\Omega \simeq (0.001 - 0.02)$ and leads to the tight constraint of $\sigma \simeq (0.5 - 0.7)$ at cluster scales [which turns out to be around $8h^{-1}\text{Mpc}$]. Similarly, the quasar abundance (marked by a straight horizontal line) gives $\sigma > (2 - 2.5)$ at galactic scales. The DLAS give $\sigma > (3 - 4)$ at the same scale (see the shaded region) thereby providing the most stringent constraint.