180\textsuperscript{Ta} production in the classical s-process

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The production and survival of the quasi-stable isomer 180\textsuperscript{Ta} during the stellar nucleosynthesis has remained a matter of discussion for years. A careful analysis of the available experimental data and theoretical calculations enabled us to reproduce the observed solar abundance of 180\textsuperscript{Ta} even in the classical s-process ($kT = 28$ keV - $33$ keV).

180\textsuperscript{Ta} is the only nucleus present in nature in an $9^{-}$ isomeric state ($^{180}\textsuperscript{Ta}^{m}$) at an energy of 75.3 keV. For its half-life an experimental lower limit of $1.2 \times 10^{15}$ y was obtained [1]. The $1^{+}$ ground state of 180\textsuperscript{Ta} decays to 180\textsuperscript{Hf} and 180\textsuperscript{W} (half-life 8.15 h). 180\textsuperscript{Ta} has a very small abundance: Only 0.012\% of natural Ta consists of 180\textsuperscript{Ta} m, the rest is 181\textsuperscript{Ta}. The solar abundance of 180\textsuperscript{Ta} m (normalized to Si with the solar abundance of 10\textsuperscript{12}) is 2.48 (cf. solar abundances of neighbouring isotopes: 4.20 \times 10\textsuperscript{12} for 178\textsuperscript{Hf}, 2.10 \times 10\textsuperscript{14} for 179\textsuperscript{Hf}, 5.4 \times 10\textsuperscript{14} for 180\textsuperscript{Hf}, and 173 for 180\textsuperscript{W}) [2].

To explain the production of 180\textsuperscript{Ta} m during the nucleosynthesis, two types of production processes were proposed.

I. Explosive processes occur in supernovae:

1. The β-process leading to an isomer in 180\textsuperscript{Lu} decay-
ing to an 8\textsuperscript{-} isomer in 180\textsuperscript{Hf} at 1.14 MeV, further
decaying partially to 180\textsuperscript{Ta} m [3,4]: The amount of
180\textsuperscript{Ta} m produced in the r-process relative to the
amount of 180\textsuperscript{Hf} can be obtained from [5]:

$$\frac{N_r(180^{\text{Ta}^m})}{N_r(180^{\text{Hf}})} = f^{180}_{m} \cdot f^{m}_{\beta^{-}}, \quad (1)$$

where $f^{180}_{m}$ and $f^{m}_{\beta^{-}}$ are the branching factors for
the $\beta^{-}$ or $\beta^{-} + \gamma$ transitions 180\textsuperscript{Lu} \rightarrow 180\textsuperscript{Hf} and
180\textsuperscript{Hf} \rightarrow 180\textsuperscript{Ta} m, respectively. For $f^{m}_{\beta^{-}}$, a value of $(0.29 \pm 0.08)\%$ was found [4]. The laboratory value
can also be used for the r-process. Corrections due
to the high degree of ionization of 180\textsuperscript{Hf} m can be
neglected since the r-process duration ($\leq 100$ s) is
short compared to the isomer half-life (5.5 hr) and
the temperature and density are greatly diminished
when the decay of 180\textsuperscript{Hf} m occurs. Values of (0.46 ± 0.15)\% [3] or (0.005 ± 0.018)\% [6] were found for $f^{180}_{m}$. These two values of $f^{180}_{m}$ lead to (9 ± 4)\% of
the observed 180\textsuperscript{Ta} m solar abundance or to a
negligible 180\textsuperscript{Ta} m production.

2. In the rapid p-process, 180\textsuperscript{Ta} m can be produced in the
181\textsuperscript{Ta}(\gamma,n) reaction (a negligible production [7]
or an overproduction comparable to that of neighboring
nuclides [8]).

3. In the $\nu$-process during the supernova-core collapse
into a neutron star, 180\textsuperscript{Ta} m can be produced in the
181\textsuperscript{Ta}(\nu,\nu'n) reaction [9,10].

II. Non-explosive processes can be summarized as follows:

1. The s-process branching in 180\textsuperscript{Hf}: In the s-process
via 179\textsuperscript{Hf}(n,\gamma), the 8\textsuperscript{-} isomer in 180\textsuperscript{Hf} partially
decaying to 180\textsuperscript{Ta} m (with the branching factor $f^{m}_{\beta^{-}}$)
is populated with the branching factor $Hf^{fm}_{\beta^{-}}$ or $B$
[11]. We assume that the classical s-process lasts
more than 1 year, the temperature lies between
$kT = 28$ keV and 33 keV [12], the neutron density
$n_{n} = (4.1 \pm 0.6) \times 10^{8}$ cm\textsuperscript{-3} [13], the electron
density $n_{e} = 5.4 \times 10^{26}$ cm\textsuperscript{-3} [14], and $kT$, $n_{n}$, and
$n_{e}$ remain constant during the s-process. For
the relative abundance of 180\textsuperscript{Ta} m one then obtains:

$$\left[ \frac{N_s(180^{\text{Ta}^m})}{N_s(179^{\text{Hf}})} \right]_{1} = \left( \frac{\sigma(179^{\text{Hf}}+\gamma)}{\sigma(180^{\text{Ta}^m}+\gamma)} \right) Hf^{fm}_{\gamma} \cdot f^{m}_{\beta^{-}}, \quad (2)$$

where $(\sigma(179^{\text{Hf}}+\gamma)$ and $(\sigma(180^{\text{Ta}^m}+\gamma)$ are
Maxwellian averaged neutron-capture cross sections taken from
[5] (Table VIII) and [15,16] (Table 9), respectively.
For $(\sigma(179^{\text{Hf}}+\gamma)$ = $(991 \pm 30)$ mb, $(\sigma(180^{\text{Ta}^m}+\gamma)$ =
$(1465 \pm 100)$ mb ($kT = 30$ keV), $Hf^{fm}_{\gamma} =
\sigma^{m}(179^{\text{Hf}})/\sigma(179^{\text{Hf}}) = (1.24 \pm 0.06)\%$ [5], and
$f^{m}_{\beta^{-}}$ = 0.7\% [5], this s-process branching can
account for only (16 ± 3)\% of the observed 180\textsuperscript{Ta} m
solar abundance.

2. The s-process branching in 179\textsuperscript{Hf}: Excited states in
179\textsuperscript{Hf} decaying to 179\textsuperscript{Ta} can be thermally populated.
180\textsuperscript{Ta} m is then produced in the 179\textsuperscript{Ta}(n,\gamma)
reaction [17] with the branching factor $Ta^{fm}_{\gamma}$. For
the relative abundance of 180\textsuperscript{Ta} m one then obtains:

$$\left[ \frac{N_s(180^{\text{Ta}^m})}{N_s(178^{\text{Hf}})} \right]_{2} = \left( \frac{\sigma(178^{\text{Hf}}+\gamma)}{\sigma(180^{\text{Ta}^m}+\gamma)} \right) Ta^{fm}_{\gamma} \cdot f^{180}_{\beta^{-}} \cdot \left[ \frac{\lambda(179^{\text{Ta}})\EC}{\lambda(179^{\text{Hf}})\beta^{-}} \right] + 1 \right)^{-1}, \quad (3)$$

where $f^{180}_{\beta^{-}}$ is the branching factor for neutron
captures at 178\textsuperscript{Hf} leading to 180\textsuperscript{Ta},

$$f^{180}_{\beta^{-}} = \frac{\lambda(179^{\text{Ta}})\EC}{\lambda(179^{\text{Hf}})\beta^{-}} \cdot \left[ \frac{\lambda(179^{\text{Ta}})\EC}{\lambda(179^{\text{Hf}})\beta^{-}} + 1 \right] \right)^{-1} \quad (4)$$
and $\lambda$ are transition rates. For $kT = 30$ keV, $(\sigma)_{180}\text{Hf} + n = 310$ mb [5], $T_{\text{f}m}^{(\lambda)} \approx (4.3 \pm 0.8)\%$ [18]. The other parameters are taken from [19] (an error of $\pm 30\%$ assumed) and interpolated for the $n_e$ and $T$. For $kT = 30$ keV this s-process branching yields $(190 \pm 40)\%$ of the $^{180}\text{Ta}^{(\lambda)}$ solar abundance.

It should be noted that all $^{180}\text{W}$ can be produced in the s-process via the decay of the $^{180}\text{Ta}$ ground-state. For the relative abundance of $^{180}\text{W}$ one can write

$$N_s(^{180}\text{W}) = \langle \sigma \rangle_{178}\text{Hf} + n \cdot (1 - Ta f_m) \cdot f_{180} \cdot f_{\beta-},$$

where $f_{\beta-}$ is the branching factor for the $^{180}\text{Ta}$ ground-state decay to $^{180}\text{W}$ (numerical values taken from [18,19]). For $kT = 30$ keV this process can account for $(95 \pm 36)\%$ of the $^{180}\text{W}$ solar abundance.

3. In the s-process during the He shell burning in the AGB phase of 1.5 to 3$M_\odot$ mass stars about 85% of $^{180}\text{Th}^{(\lambda)}$ can be produced [16].

4. The p-process in highly evolved massive stars: During the presupernova phase under temperatures $T > 10^9$ K, thermal photons can induce the reaction $^{181}\text{Ta}(\gamma,n)$ populating $^{180}\text{Th}^{(\lambda)}$ [20].

5. $^{180}\text{Th}^{(\lambda)}$ production in the cosmic radiation: Protons from the low-energy component of the galactic cosmic radiation can produce $^{180}\text{Th}^{(\lambda)}$ via the $(p,yp xn)$ reaction on s-process or r-process nuclei in the interstellar medium [21].

The total relative s-process abundance of $^{180}\text{Th}^{(\lambda)}$ can be calculated from:

$$\frac{N_s(^{180}\text{Th}^{(\lambda)})}{N_s(^{178}\text{Hf})} = \frac{N_s(^{179}\text{Hf})}{N_s(^{178}\text{Hf})} \cdot \left[ \frac{N_s(^{180}\text{Th}^{(\lambda)})}{N_s(^{179}\text{Hf})} \right]_1 \right. + \left[ \frac{N_s(^{180}\text{Th}^{(\lambda)})}{N_s(^{178}\text{Hf})} \right]_2$$

From Fig. 1 one can see that for $kT = 30$ keV an amount of $^{180}\text{Th}^{(\lambda)}$ ($2.0 \pm 0.5$) times larger than the observed one can be produced in the classical s-process.

![FIG. 1. The total s-process abundance of $^{180}\text{Th}^{(\lambda)}$ (solid line) and $^{180}\text{W}$ (long-dashed line) calculated relative to the $^{178}\text{Hf}$ abundance as a function of the thermal energy $kT$ or temperature $T$. Error bands are depicted by dotted lines. The s-process temperature window [12] is marked by dot-dashed lines.](image)

This overproduction is reduced by intermediate states (IS) in $^{180}\text{Ta}$: In the s-process site thermal photons may excite higher-lying levels which then decay back either to the $1^+$ ground state or to the $9^-$ isomer. To find these levels Belic et al. [10,23] used the Stuttgart Dynamitron facility with both enriched (5.6%) and natural Ta targets. Irradiations were performed for bremsstrahlung endpoint energies $E_0 = 0.8$–3.1 MeV. Depopulation of the isomer was observed down to $E_0 \approx 1.01$ MeV. This means that the lowest IS may have an excitation energy $E_{IS} = 1.085$ MeV (above the ground state). Assuming that $E_{IS}$ is the excitation energy of the lowest IS, the experimental total integrated depopulation cross section $I_{\text{p}}$ turns out to be $(5.7 \pm 1.2)$ eV fm$^2$. Then the effective lifetime of the IS for the depopulation into the ground state $\tau_{\text{eff}}$ is roughly equal to $6 \cdot 10^{-11}$ s.

The IS (IS) decays via a $\gamma$-cascade, and lifetimes and energies of the decay states $|k|$ are not known. Klay [24] showed that multi-step transitions from |IS| to the ground-state |g| can be substituted by a direct transition from |IS| to |g| if the states |k| possess short enough lifetimes $\tau_k$, i.e.

$$\tau_k \ll \tau_{\text{eff}} \cdot \frac{2I_k + 1}{2I_{IS} + 1} \cdot \exp \left( \frac{|E_{IS} - E_k|}{kT} \right)$$

![FIG. 2. Internal and external population and depopulation possibilities in the three-level system described by means of transition rates $\lambda$ [22].](image)
where $E_{IS}$ and $I_{IS}$, $E_k$ and $I_k$ are the energies and spins of the IS and the states $|k\rangle$, respectively. It can be shown that the condition (7) is fulfilled in $^{180}$Ta and transitions from the isomer $|m\rangle$ to the ground-state $|g\rangle$ can be studied in the three-level system (see Fig. 2).

The population of the three-level system can be described by the following coupled inhomogeneous linear differential equations:

$$\frac{dN_m}{dt} = \lambda_{pm} N_s - (\lambda_{mIS} + \lambda_{md}) N_m + \lambda_{ISm} N_{IS}$$
$$\frac{dN_{IS}}{dt} = \lambda_{pIS} N_s + \lambda_{gIS} N_g + \lambda_{mIS} N_m - (\lambda_{ISg} + \lambda_{ISm} + \lambda_{ISd}) N_{IS}$$
$$\frac{dN_g}{dt} = \lambda_{pg} N_s - (\lambda_{gIS} + \lambda_{gd}) N_g + \lambda_{ISg} N_{IS},$$

where $\lambda_{px}$ ($\lambda_{xd}$) are the population (depopulation) transition rates of the state $|x\rangle$, $\lambda_{xy}$ are the transition rates between the states $|x\rangle$ and $|y\rangle$, $N_s$ is the number of nuclei in the state $|x\rangle$, $N_s(t=0) = 0$, and $N_s$ is a constant number of seed nuclei ($^{178}$Hf in our case).

In previous analyses, e.g. [23,25], the effect of an IS on the survival of $^{180}$Ta$^m$ in the presence of a stellar photon bath was calculated by solving coupled differential equations for the three-level system isomer $\leftrightarrow$ IS $\leftrightarrow$ ground-state without taking into account the population of the three levels due to the s-process simultaneously, i.e. $\lambda_{pm} = \lambda_{pIS} = \lambda_{pg} = 0$ was assumed.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Survival ratio $R$ of $^{180}$Ta$^m$ as a function of $kT$ or $T$ for the IS at $E_{IS} = 1.085$ MeV. Error bands are marked by dotted lines, the s-process temperature window by dot-dashed lines.}
\end{figure}

For $t \to \infty$ the solution of (8) approaches the equilibrium solution. Since the maximum equilibrium relaxation time $\tau_{max}$ ($\tau_{max} < 100$ days for $kT$ between 15 keV and 45 keV) is smaller than the s-process duration ($> 1$ year) the exact solution of (8) can be approximated by the equilibrium solution. In the case of existing IS one can define the survival ratio $R$ of $^{180}$Ta$^m$ (depicted in Fig. 3):

$$R = \frac{N_m(t \to \infty)}{N_s(178\text{Hf})} \frac{N_s(180\text{Ta}^m)}{N_s(178\text{Hf})}$$

where $N_s(178\text{Hf}) = N_s$ in (8), $N_s(180\text{Ta}^m)/N_s(178\text{Hf})$ represents the relative abundance for no IS and is obtained from (6). The survival ratio $R$ is larger then 0.2 in the whole temperature interval. For $kT \leq 21$ keV the coupling between the isomer and the ground state via the IS is negligible. Note that $R > 1$ for $kT > 40$ keV.

It can be shown that for $kT = 30$ keV only dominant transition rates $\lambda$ must be taken into account and that the equilibrium solution can be approximated by

$$\frac{N_m(t \to \infty)}{N_s(178\text{Hf})} = \frac{N_s(180\text{Ta}^m)}{N_s(178\text{Hf})}$$

where

$$P = \frac{2I_m + 1}{2I_g + 1} \exp \left( -75.3 \text{ keV}/kT \right) \approx 0.5,$$

$$\lambda_{md} = \lambda(180\text{Ta}^m+n) = n_n \cdot \left( \frac{2kT}{m_n} \right)^{1/2} \langle \sigma \rangle_{180\text{Ta}^m+n} \cdot$$

$$\lambda_{md} \approx 10^{-7} \text{ s}^{-1}, m_n \text{ is the neutron mass}, \lambda_{gd} = \lambda(180\text{Ta})_{\beta^-} + \lambda(180\text{Ta})_{EC} = 4.2 \cdot 10^{-6} \text{ s}^{-1} [19],$$

$$\lambda_{pm} + \lambda_{pg} + \lambda_{pIS} \approx \lambda(178\text{Hf}+n) \cdot \left( f_{180\text{Hf}} \cdot f_{180} + f_{178} \cdot f_{\beta^-} \right),$$

where $\lambda(178\text{Hf}+n)$ can be calculated from $\langle \sigma \rangle_{178\text{Hf}+n}$. The two terms in (13) originate in the two s-process branchings via $179$Hf and $180$Hf that can produce $^{180}$Ta.

As a consequence of (10) the approximate solution does not depend on properties of the IS like its spin, energy, lifetime and transition rates to isomer and ground state.

In Fig. 4 the relative abundance of $^{180}$Ta$^m$ as a function of $kT$ or $T$ is depicted. The equilibrium solution of (8) with the IS at 1.085 MeV is compared to the case of no IS (cf. Fig. 1). Note that for $kT \geq 29$ keV the approximate (10) and the equilibrium solution are indistinguishable. As can be seen in Fig. 4 exactly 100% of the solar $^{180}$Ta$^m$ abundance can be reproduced in the middle of the s-process temperature window. Other IS that may be found in the future will not change this result (cf. a possible IS below 737 keV [26]).

If the IS at 1.085 MeV in $^{180}$Ta is taken into account the relative abundance of $^{180}$Ta$^5$ must be corrected:

$$\frac{N_s(180\text{W})}{N_s(178\text{Hf})} \approx \frac{\langle \sigma \rangle_{178\text{Hf}+n}}{\langle \sigma \rangle_{180\text{W}+n}} \cdot \left( 1 - Ta f_{180} R \right) \cdot f_{180} \cdot f_{\beta^-}.$$

(14)
Since the survival ratio $R \leq 1$ for the s-process temperature window (see Fig. 3) and $^{180}\text{Ta}^{m} = 4.3\%$ the maximum change of the relative abundance of $^{180}\text{W}$ due to the IS represents about 4\% and can be neglected.

![Diagram](image)

FIG. 4. Comparison of the total s-process abundances of $^{180}\text{Ta}^{m}$ calculated relative to the $^{178}\text{Hf}$ abundance as a function of the thermal energy $kT$ or temperature $T$: for the IS at 1.085 MeV the equilibrium solution of (8) is denoted by a bold solid line, and the approximate solution (10) by a bold long-dashed line, and the total abundance for no IS from (6) by a bold dashed line. Dotted lines correspond to error bands, dot-dashed lines to the s-process temperature window.

We have shown that $^{180}\text{Ta}^{m}$ could be produced in the classical s-process dominantly via the branching in $^{179}\text{Hf}$. The production of $^{180}\text{Ta}^{m}$ is then connected to the production of $^{180}\text{W}$. For the temperature window $kT = (28 \pm 33) \text{ keV}$ we obtain $(50 \pm 20)\% - (220 \pm 80)\%$ of $^{180}\text{W}$ produced in the s-process via $^{180}\text{Ta}$. Assuming sufficiently low temperatures in the star evolution phases following the s-process and the IS at $E_{IS} \leq 1.085 \text{ MeV}$ as observed in the Stuttgart photoactivation experiment [23,10], the s-process production of $^{180}\text{Ta}^{m}$ ranges from $(30 \pm 10)\%$ to $(230 \pm 80)\%$ of its observed solar abundance for the same temperature window. For the middle of the temperature window ($kT = 30.5 \text{ keV}$) we obtain $(90 \pm 30)\%$ of $^{180}\text{Ta}^{m}$ (see Fig. 4) and $(110 \pm 40)\%$ of $^{180}\text{W}$. It should be noted that $^{180}\text{W}$ is often listed among p-process nuclei [8]. An upper limit for its production in the s-process provided by an improved model of the p-process would have direct consequences for the $^{180}\text{Ta}^{m}$ production in the classical s-process via the branchings in $^{178}\text{Hf}$ and $^{180}\text{Hf}$.

ACKNOWLEDGMENTS

The authors are grateful to Dr. F. Käppeler and D. Brandmaier for valuable discussions.

[1] J.B. Cumming, D.E. Alburger, Phys. Rev. C 31, 1494 (1985).