Magnetic dipole moments for composite dark matter

Alfredo Aranda,\textsuperscript{a,b} Luis Barajas\textsuperscript{a,c} and Jose A.R. Cembranos\textsuperscript{a,b,d}

\textsuperscript{a}Facultad de Ciencias — CUICBAS, Universidad de Colima, Bernal Díaz del Castillo 340, C.P. 28045, Colima, México
\textsuperscript{b}Dual CP Institute of High Energy Physics, Bernal Díaz del Castillo 340, C.P. 28045, Colima, México
\textsuperscript{c}Department of Physics, University at Buffalo, The State University of New York, 239 Fronczak Hall, Buffalo, NY 14260-1500, U.S.A.
\textsuperscript{d}Departamento de Física Teórica I, Facultad Ciencias Físicas, Universidad Complutense de Madrid, Ciudad Universitaria, E-28040 Madrid, Spain

E-mail: fefo@ucol.mx, luisedua@buffalo.edu, cembra@fis.ucm.es

Received December 24, 2015
Accepted February 29, 2016
Published March 18, 2016

Abstract. We study neutral dark matter candidates with a nonzero magnetic dipole moment. We assume that they are composite states of new fermions related to the strong phase of a new gauge interaction. In particular, invoking a dark flavor symmetry, we analyze the composition structure of viable candidates depending on the assignations of hypercharge and the multiplets associated to the fundamental constituents of the extended sector. We determine the magnetic dipole moments for the neutral composite states in terms of their constituents masses.

Keywords: dark matter experiments, dark matter theory, particle physics - cosmology connection

ArXiv ePrint: 1511.02805
1 Introduction

Many different observational evidences prove the existence of Dark Matter (DM). Galaxy clusters and dynamics, structure formation, big-bang nucleosynthesis, and the cosmic microwave background show that baryons can only account for a small part of the total matter density of the Universe. Many extensions of the Standard Model (SM) provide viable DM candidates, however no clear evidence for any particular extension has been found. This fact motivates the analysis of DM properties from a broader approach.

DM is typically assumed to have negligible direct couplings to photons but there are other interesting possibilities [1–6] such as Magnetic DM (MDM), i.e. DM particles with a nonzero magnetic dipole moment ($\mu_{\text{DM}}$). This has been explored in different scenarios with some interesting results. For example in ref. [7, 8] one finds a general study of its phenomenological signatures and constraints. Ref. [9] studies direct detection in experimental observations to constraint different types of MDM. In [10] the authors analyze the situation particularly for the CoGeNT data for a DM mass of several GeVs. Ref. [11] remarks similar results for the DAMA signature. The works in [12] and [13] also analyze direct detection results for MDM, but they compare their conclusions with other constraints from indirect searches and colliders. The indirect detection of MDM is studied in ref. [14] as a possible explanation of the 130 GeV line observed by Fermi-LAT. The constraining power of supernova SN 1987A data in order to restrict the viability of light MDM is shown in [15]. A similar analysis for the beam dump experiment E613 is done in ref. [16].

A particle with a permanent $\mu_{\text{DM}}$ must have a nonzero spin. In this work we only consider fundamental spin-1/2 Dirac fermions $\psi_{\text{DM}}$, since a Majorana fermion cannot have
this type of moments. In contrast with the electric dipole moment, the magnetic dipole moment is an axial vector and can couple to the spin without violating time-reversal and parity symmetries. In contrast to charged particles or neutral particles with electric dipole moments [17], particles provided with a $\mu_{\text{DM}}$ do not have the ability to form atom-like bound states with other charged particles or with each other. This fact changes completely the phenomenology of MDM.

The magnetic dipole moment is expected to be small enough to satisfy perturbative constraints: $\mu_{\text{DM}} \lesssim e m_{\text{DM}}^{-1}$, where $m_{\text{DM}}$ is the mass of the DM particle. A more rigorous bound can be imposed by unitarity arguments. Indeed, the total s-wave annihilation cross section must be $\sigma \lesssim 4\pi/m_{\text{DM}}^2$ [18]. By using the expression for two photons annihilation [7, 8, 12], it is possible to find $\mu_{\text{DM}} m_{\text{DM}} \lesssim 20 (m_e/m_{\text{DM}})$.

The viability of MDM can be divided in three different regions depending on its mass: if $m_{\text{DM}} \lesssim 10 \text{ MeV}$, the constraints on additional relativistic degrees of freedom from big-bang nucleosynthesis (BBN) introduce the important restrictions. However MDM can decouple before the QCD phase transition and evade these bounds [7, 8]. In any case, it is difficult to find a production mechanism associated to this light MDM in order to account for the total amount of DM. In addition, there are more constraining bounds for very light MDM even if it just constitutes part of the total non-baryonic matter. For example, the energy-loss analysis of stellar objects in globular clusters constraints dipole moments more strongly for masses $m_{\text{DM}} \lesssim 5 \text{ keV}$ [19]. Similar bounds can be found by taking into account the data from the supernova 1987A [15], but in this case, it can be extended up to masses of order $m_{\text{DM}} \lesssim 10 \text{ MeV}$ or even $100 \text{ MeV}$ depending on different assumptions about the thermal properties of the supernova.

For the middle region, with $10 \text{ MeV} \lesssim m_{\text{DM}} \lesssim 1 \text{ GeV}$, the experimental and observational constraints may be satisfied for a larger value of $\mu_{\text{DM}}$. In this case, the most robust constraints come from precision measurements and, in particular, from the contribution of the MDM to the running of the fine-structure constant, which modifies the mass of the $W^{\pm}$ boson predicted in the SM [7, 8]. Similar constraints can be placed by MDM direct production in particle accelerators [20–28]. The cleaner environment makes the single photon channel data at LEP slightly more constraining than mono-jet signatures at the Tevatron or at the LHC [12].

For values close to the above bound, MDM can achieve the abundance by the classical thermal freeze-out mechanism in order to account for the total DM density [7, 8, 12]. However, this type of DM suffers the constraints associated to general light WIMPs, and it is difficult to think that DM with masses below $10 \text{ GeV}$ can constitute the total missing matter (due to restrictions coming from observations of cosmic-ray positrons, cosmic-ray antiprotons and radio observations [29]).

Finally, for MDM heavier than $\sim 1 \text{ GeV}$, the constraints on the value of $\mu_{\text{DM}}$ are even more important due to direct detection experiments. However, in this and the former case, the DM abundance can be produced by the thermal freeze-in mechanism. Within the standard inflationary framework, the preferred value for $\mu_{\text{DM}}$ depends on the reheat and maximum temperatures with respect to the MDM mass [30].

By the definition of MDM, the magnetic interaction with photons is its leading interaction with SM particles. However, the possible values for the magnetic moment commented above can be different if we assume a more involved cosmological setup, for example the relic abundance can be larger if exotic processes increase the expansion rate during freeze-out [31], or if there is a particle-antiparticle asymmetry for the MDM. In general, a particle with a non-
zero magnetic dipole, it is not its own anti-particle and this fact can play an important role
in order to compute the relic abundance of this type of DM. Indeed, an asymmetry between
the abundance of particles and anti-particles generally increases the amount of DM. This
improves the viability of MDM since the constraints commented in the introduction typically
restrict severely the abundance of this type of DM. There are different possibilities for the pro-
duction of the initial asymmetry. In general, a linear combination of the SM baryon number
and the new baryon number has to be broken. It means that one of them can be preserved,
but not both. There are also different mechanisms for the creation of this asymmetric DM:
the mentioned freeze-out and freeze-in processes are modified in this context, they can also be
produced by the decays of heavy particles out of equilibrium, or the Affleck-Dine mechanism
or even a spontaneous genesis can be the responsible of the relic density of asymmetric DM [32].

Other laboratory constraints, as the one coming from the Lamb shift [33, 34] or the
targeted experiment at SLAC [35], are also subdominant. Astrophysical analyses related
to the stability of the Galactic disk, annihilations in the solar neighborhood, or lifetimes of
compact objects, are not competitive either [7, 8, 36–43]. The same situation was found
by [7, 8] for the constraints derived from Large-Scale Structure or the Cosmic Microwave
Background. In contrast, different conclusions can be found for the indirect signatures of
MDM as we have already commented, although there are important uncertainties involved
in these studies associated with different assumptions [7, 8, 44–47].

One motivated way of having MDM arises for composite DM [48–50]. If that is the
case, it is possible that the constituents that form these dark hadrons might have non zero
electric charge and thus contribute to a non zero dipole magnetic moment for the bound
or composite state. Motivated by the situation in QCD, where the use of an SU(3)_F flavor
symmetry at low energies facilitates a description of the different mesons and baryons, we
consider a similar situation for DM, where new fundamental fermionic degrees of freedom are
introduced and are assumed to interact strongly through an unspecified new interaction present
at a high energy scale. At lower energies, a flavor symmetry is assumed to exist that allows
us to consider the different composite states to be analyzed in terms of their possible values
for µ_{DM}. The new fundamental fermionic particles, denoted by q in analogy to quarks, can
be electrically charged and thus contain SU(2)_L × U(1)_Y quantum numbers.

Following the example of QCD, we think of the new strong interaction as a scaled-up
version of it and thus consider, at low energies, a situation where a SU(3)_D dark flavor sym-
mmetry is present for three new dark quarks that transform in its fundamental representation 3. With respect to the Standard Model (SM) gauge group, they are singlets of SU(3)_C and
might transform non-trivially under SU(2)_L × U(1)_Y. We then have the following possibili-
ties: they can form a triplet of SU(2)_L with one hypercharge Y = y_1; two of them can form
doubles and the third one a singlet with hypercharges Y = y_1, and Y = y_2, respectively;
or they can all be singlets with independent hypercharges. We analyze each case separately.

The paper is organized as follows: in section 2 we present how the three new fermions
lead to composite states associated to the dark flavor symmetry SU(3)_D. After these are
presented, in section 3 we explore the different possibilities for the SU(2)_L × U(1)_Y quantum
numbers of the new states in order to determine if there are neutral composite states that
can play the role of DM. Section 4 shows the expressions for µ_{DM} in all cases considered. In
section 5, we discuss a generalized version of the Gell-Mann-Nishijima relation for our case,
and finally, we conclude in section 6.
2 Composite states from SU(3)$_D$

First, we describe how three fundamental fermions in the 3 of SU(3)$_D$ can form the composite states. Using the fact that they belong to the fundamental representation, the new elementary particles can be characterized by their $T_3^D$ and $Y_D$ “quantum numbers”, corresponding to the eigenvalues of the two diagonal generators of SU(3)$_D$ (see appendix A for the QCD case). If we denote each fundamental state by $q_i = q(Y_D, T_3^D)$, then we have (see figure 1)

$$q_1 = q\left(\frac{1}{3}, \frac{1}{2}\right), \quad q_2 = q\left(\frac{1}{3}, -\frac{1}{2}\right), \quad q_3 = q\left(-\frac{2}{3}, 0\right). \quad (2.1)$$

Composite states made up of three constituents are obtained in the triple product of the fundamental representation $3$, (see appendix B for the description of the spin wavefunctions used in the analysis),

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10. \quad (2.2)$$

The octet states are denoted by $D_i(q_k, q_l, q_m) = D(Y_D, T_3^D)$, with $i = 1, 2, \ldots, 8$ and $k, l, m = 1, 2, 3$. From this notation, we have (see figure 2)

$$D_1(q_1, q_1, q_2) = D\left(1, \frac{1}{2}\right), \quad D_2(q_1, q_2, q_2) = D\left(1, -\frac{1}{2}\right), \quad D_3(q_1, q_3, q_3) = D\left(-1, \frac{1}{2}\right),$$

$$D_4(q_2, q_3, q_3) = D\left(-1, -\frac{1}{2}\right), \quad D_5(q_1, q_2, q_3) = D(0, 0), \quad D_6(q_1, q_2, q_3) = D(0, 0),$$

$$D_7(q_1, q_2, q_3) = D(0, -1), \quad D_8(q_1, q_1, q_3) = D(0, 1). \quad (2.3)$$

In the same way we can denote the decuplet states shown in figure 3 as $D^*_i(q_k, q_l, q_m) = D^*(Y_D, T_3^D)$:

$$D^*_1(q_1, q_1, q_2) = D^*\left(1, \frac{1}{2}\right), \quad D^*_2(q_1, q_2, q_2) = D^*\left(1, -\frac{1}{2}\right), \quad D^*_3(q_1, q_3, q_3) = D^*\left(-1, \frac{1}{2}\right),$$

$$D^*_4(q_2, q_3, q_3) = D^*\left(-1, -\frac{1}{2}\right), \quad D^*_5(q_1, q_2, q_3) = D^*(0, 0),$$

$$D^*_6(q_1, q_2, q_3) = D^*(0, 0), \quad D^*_7(q_1, q_1, q_3) = D^*(0, -1), \quad D^*_8(q_1, q_1, q_3) = D^*(0, 1).$$
These are the composite states that we consider in this work. The next step is to explore the different possibilities emanating from the different SU(2)\textsubscript{L} \times U(1)\text{Y} assignments of the new fermions \(q_i\) in order to determine the neutral composite states. Note that we label one of the states of the decuplet with \(D^*_{11}\) instead of \(D^*_{5}\). In this way, we can relate the particles \(D_i\) of the octet with the \(D^*_i\) associated with the decuplet since they are formed by the same constituents, as we will discuss.

### 3 Charge assignments and neutral states

Since we are interested in the electrically neutral composite states, we want to know the conditions under which these states will be neutral for each of the three different charge assignments mentioned above, i.e. whether the three new fundamental fermions form a triplet, a doublet plus a singlet or three singlets of SU(2)\textsubscript{L}. The electric charge of the fermion \(f_i\) is then determined using the relation \(Q_i = T_3 + Y/2\), where \(T_3\) and \(Y\) correspond to the third component of isospin and hypercharge of the fermion \(f_i\), respectively.

Note that \(D_i\) and \(D^*_i\) are made up of the same particles for \(i = 1, \ldots, 8\) and so, when specifying the neutral states, we only do it for \(D_i\), \(D^*_9\), \(D^*_10\) and \(D^*_11\).

It is important to note that at this level of discussion, we are assuming that there is a flavor symmetry SU(3)\textsubscript{D} that is present in the new sector at low energies. Although in
principle, and as a first and natural extension, we do have in mind a scaled-up version of QCD, we do not explicitly consider the strongly interacting gauge group that is assumed to be present at high energy.

3.1 Triplet

Let the three new fermions $q_i$ form a triplet of SU(2)$_L$ with $Y = y_1$. A priori, we have the freedom of assigning any $q_i$ in the SU(2)$_L$ triplet to any position in the SU(3)$_D$ triplet, however, as we will see later, the different combinations can be related and so, we use the simplest one in which they occupy the same position.

The electric charges of the fundamental particles $q_i$ are then given by

$$Q_1 = 1 + \frac{y_1}{2}, \quad Q_2 = \frac{y_1}{2}, \quad Q_3 = -1 + \frac{y_1}{2}. \quad (3.1)$$

Using these charges we find that the composite states will be neutral in the following situations:

1. $D_1$ is neutral for $y_1 = -\frac{4}{3}$.
2. $D_4$ is neutral for $y_1 = \frac{4}{3}$.
3. $D_2$ and $D_8$ are neutral for $y_1 = -\frac{2}{3}$.
4. $D_3$ and $D_7$ are neutral for $y_1 = \frac{2}{3}$.
5. $D_5$, $D_6$ and $D^*_10$ are neutral for $y_1 = 0$.
6. $D^*_5$ is neutral for $y_1 = -2$. 

Figure 3. Decuplet states.
7. $D_{11}^*$ is neutral for $y_1 = 2$.

Note that $D_9^*$, $D_{10}^*$, and $D_{11}^*$ are made up of three neutral fundamental particles and therefore they do not have magnetic moment (in a model where only the valence contribution is considered).

### 3.2 Doublet and singlet

Consider now the case where we have a SU(2)$_L$ doublet and a singlet. Let the hypercharge of the doublet be $y_1$, and the one of the singlet be $y_2$. The corresponding electric charges are given by

$$Q_1 = \frac{1}{2} + \frac{y_1}{2}, \quad Q_2 = -\frac{1}{2} + \frac{y_1}{2}, \quad Q_3 = \frac{y_2}{2}. \quad (3.2)$$

The composite states will be neutral in the following situations:

1. $D_1$ is neutral if $y_1 = -\frac{1}{3}$, independently of $y_2$.
2. $D_2$ is neutral if $y_1 = \frac{1}{3}$, independently of $y_2$.
3. $D_3$ is neutral if $y_1 = -(1 + 2y_2)$.
4. $D_4$ is neutral if $y_1 = 1 - 2y_2$.
5. $D_5$ and $D_6$ are neutral if $y_1 = -\frac{y_2}{2}$.
6. $D_7$ is neutral if $y_1 = 1 - \frac{y_2}{2}$.
7. $D_8$ is neutral if $y_1 = -(1 + \frac{y_2}{2})$.
8. $D_9^*$ is neutral if $y_1 = -1$, independently of $y_2$.
9. $D_{10}^*$ is neutral if $y_1 = 1$, independently of $y_2$.
10. $D_{11}^*$ is neutral if $y_2 = 0$, independently of $y_1$.

### 3.3 Singlets

Let the hypercharges of the three SU(2)$_L$ singlets be $y_1$, $y_2$, $y_3$, respectively. Their electric charges are given by

$$Q_1 = \frac{y_1}{2}, \quad Q_2 = \frac{y_2}{2}, \quad Q_3 = \frac{y_3}{2}. \quad (3.3)$$

Now we find that the composites states will be neutral if at least one of the following conditions is satisfied:

1. $D_1$ is neutral if $y_1 = -\frac{y_2}{2}$, independently of $y_3$.
2. $D_2$ is neutral if $y_1 = -2y_2$, independently of $y_3$.
3. $D_3$ is neutral if $y_1 = -2y_3$, independently of $y_2$.
4. $D_4$ is neutral if $y_3 = -\frac{y_2}{2}$, independently of $y_1$.
5. $D_5$ and $D_6$ are neutral if $y_1 + y_2 + y_3 = 0$.
6. $D_7$ is neutral if $y_2 = -\frac{y_3}{2}$, independently of $y_1$. 

---

- 7 -
7. $D_8$ is neutral if $y_1 = -\frac{y_3}{2}$, independently of $y_2$.

8. $D_9^*$, $D_{10}^*$ or $D_{11}^*$ are neutral if $y_1 = 0$, $y_2 = 0$ or $y_3 = 0$, respectively. However we are not interested in these conditions, since the states are made up of three neutral particles and therefore their magnetic moments is zero.

Each one of the conditions in 1-4, 7 and 8 can be considered individually or combined with one of the others (1 with 4, 2 with 3, and 7 with 8) so that we get

$$y_i = y_j = -\frac{y_k}{2} \neq 0,$$

for $i \neq j \neq k$. Then, depending on the values of $y_1$, $y_2$ and $y_3$ it is possible to obtain two, three or four neutral states.

4 Magnetic moments

The expressions for the magnetic moments of the octet and decuplet states, in terms of the magnetic moments of their constituents, are the same as those obtained in the Quark Model using the spin-flavor wavefunctions given in appendix B. For the octet states the results are

$$
\begin{align*}
\mu_{D_1} &= \frac{1}{3} (4\mu_1 - \mu_2), \\
\mu_{D_2} &= \frac{1}{3} (4\mu_2 - \mu_1), \\
\mu_{D_3} &= \frac{1}{3} (4\mu_3 - \mu_1), \\
\mu_{D_4} &= \frac{1}{3} (4\mu_3 - \mu_2), \\
\mu_{D_5} &= \mu_3, \\
\mu_{D_6} &= \frac{2}{3} (\mu_2 + \mu_1) - \frac{1}{3}\mu_3, \\
\mu_{D_7} &= \frac{1}{3} (4\mu_2 - \mu_3), \\
\mu_{D_8} &= \frac{1}{3} (4\mu_1 - \mu_3),
\end{align*}
$$

(4.1)

and for the decuplet states we get

$$
\begin{align*}
\mu_{D_1^*} &= 2\mu_1 + \mu_2, \\
\mu_{D_2^*} &= 2\mu_2 + \mu_1, \\
\mu_{D_3^*} &= 2\mu_3 + \mu_1, \\
\mu_{D_4^*} &= 2\mu_3 + \mu_2, \\
\mu_{D_5^*} &= \mu_1 + \mu_2 + \mu_3, \\
\mu_{D_6^*} &= 3\mu_1, \\
\mu_{D_{10}^*} &= 3\mu_2, \\
\mu_{D_{11}^*} &= 3\mu_3.
\end{align*}
$$

(4.2)

We see that, except for $D_5$ and $D_6$, there are symmetries under the exchanges $i \leftrightarrow j$, $j \leftrightarrow k$, $k \leftrightarrow i$, and $i \rightarrow j \rightarrow k \rightarrow i$: the same magnetic moments are obtained among different states. Consider for example the case $1 \leftrightarrow 2$ and note that $\mu_{D_1} \leftrightarrow \mu_{D_2}$. The origin of these relations can be traced to the symmetries of the spin-wavefunctions and this, in fact, is the reason of why we obtain the same magnetic moments (though corresponding to different symmetry-related states), independently of how we assign the order between the components in the SU(2)$_L$ and SU(3)$_D$ multiplets.

Defining the mass ratios $r_{ij} \equiv m_i/m_j$, we can express all the magnetic moments in units of $\frac{e\hbar}{2m_1}$. We obtain the results shown in tables 1, 2 and 3 for the magnetic moments of the neutral dark hadrons when the constituents are in a SU(2)$_L$ triplet, doublet plus singlet and three singlets respectively. The first column presents the hypercharges leading to the neutral composite states shown in the second column. The third column displays the expressions for the magnetic moments in units of $\frac{e\hbar}{2m_1}$. The last three columns are added for curiosity: they show specific values for degeneracies in the constituents masses. Note that the zero magnetic moments in these three last columns are accidental since they exist only in the case of exact mass degeneracy.
Hypercharge \((y_i)\) Charge \((Q_1, Q_2, Q_3)\) Dark hadron \(\mu_{D_i, D_i^*} \left( \frac{e \hbar}{2m_1} \right)\) \(r_{12} = 1\) \(r_{13} = 1\) \(r_{12} = r_{13}\)

| \(y_1 = -4/3\) | \(1/3, -2/3, -5/3\) | \(D_1\) | \(\frac{2}{9} \left( 2 + r_{12} \right)\) | \(2/3\) | \(-\) | \(-\) |
| \(y_1 = 4/3\) | \(5/3, 2/3, -1/3\) | \(D_4\) | \(\frac{-2}{9} \left( 2r_{13} + r_{12} \right)\) | \(-\) | \(-\) | \(-\) |
| \(y_1 = -2/3\) | \(2/3, -1/3, -4/3\) | \(D_2\) | \(\frac{-2}{9} \left( 2 + 2r_{12} \right)\) | \(-\) | \(-\) | \(-\) |
| \(y_1 = 2/3\) | \(4/3, 1/3, -2/3\) | \(D_6\) | \(\frac{2}{9} \left( r_{13} + 2r_{12} \right)\) | \(-\) | \(-\) | \(-\) |
| \(y_1 = 0\) | \(1, 0, -1\) | \(D_5\) | \(-r_{13}\) | \(-\) | \(-\) | \(-\) |
| \(y_1 = -2\) | \(0, -1, -2\) | \(D_6^*\) | \(1 - r_{13}\) | \(-\) | \(-\) | \(-\) |
| \(y_1 = 2\) | \(2, 1, 0\) | \(D_{11}^*\) | \(0^*\) | \(-\) | \(-\) | \(-\) |

Table 1. Magnetic moments of the neutral states if the three constituents are in a SU(2)\(_L\) triplet. The last three columns correspond to specific cases where there are mass degeneracies among the different constituents \(r_{ij} \equiv m_1/m_j\). Note that the cases where the magnetic moment is zero fall in two different categories: those that are zero because their constituents are neutral (denoted by 0*) and those where a degeneracy in constituents mass is present (denoted by 0**).

Recall that there is an ordering symmetry among the components in the multiplets of SU(2)\(_L\) and SU(3)\(_D\) except for \(D_5\) and \(D_6\). In this case the way we relate the components of the multiplets matters. We show this in tables 4, 5, and 6, where we use the following notation: call \(\tilde{q}_i\) the components of the SU(2)\(_L\) multiplet and \(q_i\) those of the SU(3)\(_D\) one (our previous analysis and results in tables 1, 2 and 3 corresponds to the case \(\tilde{q}_i = q_i\) for the SU(2)\(_L\)). The first column of each of these tables contains the relation among the multiplets, followed by the expressions for the magnetic moments.

5 Gell-Mann-Nishijima formula

The electric charge of a particle is given by

\[ Q = T_3 + \frac{1}{2} Y_W. \]
| Hypercharge \((y_i)\) | Charge \((Q_1, Q_2, Q_3)\) | Dark hadron | \(\mu_{D_i, D'_i} \left( \frac{e \hbar}{2m_i} \right)\) | \(r_{12} = 1\) | \(r_{13} = 1\) | \(r_{12} = r_{13}\) |
|-----------------|-------------------------|------------|-------------------------------------------------|----------------|----------------|----------------|
| \(y_1 = -1/3, \ y_2\) \((1/3, -2/3, y_2/2)\) | \(D_1\) \(D'_1\) | \(\frac{2}{3} (2 + r_{12})\) \(\frac{2}{3} (1 - r_{12})\) | 2/3 | 0** | – |
| \(y_1 = 1/3, \ y_2\) \((2/3, -1/3, y_2/2)\) | \(D_2\) \(D'_2\) | \(-\frac{2}{3} (1 + 2r_{12})\) \(\frac{2}{3} (1 - r_{12})\) | -2/3 | 0** | – |
| \(y_1 = -(1 + 2y_2)\) \((-y_2, -(1 + y_2), \frac{y_2}{2})\) | \(D_3\) \(D'_3\) | \(\frac{2y_2}{3} (1 + 2r_{13})\) \(-y_2 (1 - r_{13})\) | – | \(y_2\) | 0** |
| \(y_1 = (1 - 2y_2)\) \((1 - y_2, -y_2, y_2/2)\) | \(D_4\) \(D'_4\) | \(\frac{2y_2}{3} (2r_{13} + r_{12})\) \(y_2 (r_{13} - r_{12})\) | – | – | \(y_2r_{13}\) 0** |
| \((2y_2/4, -(2+y_2), y_2/2)\) | \(D_5\) \(D'_5\) | \(\frac{2y_2}{3} r_{13}\) \(\frac{1}{3} (1 - r_{12})\) \(-\frac{2y_2}{6} (1 + r_{12} + r_{13})\) \(\frac{1}{2} (1 - r_{12})\) \(-\frac{2y_2}{4} (1 + r_{12} - 2r_{13})\) | – | \(y_2/2\) | – |
| \((1 - \frac{y_2}{4}, -\frac{y_2}{4}, \frac{y_2}{2})\) | \(D_7\) \(D'_7\) | \(-\frac{2y_2}{6} (r_{13} + 2r_{12})\) \(\frac{2y_2}{3} (r_{13} - r_{12})\) | – | – | \(-y_2r_{13}/2\) 0** |
| \((-\frac{y_2}{4}, -(1 + \frac{y_2}{4}), \frac{y_2}{2})\) | \(D_8\) \(D'_8\) | \(-\frac{2y_2}{6} (2 + r_{13})\) \(-\frac{2y_2}{6} (1 - r_{12})\) | – | \(-y_2/2\) | 0** |
| \((-\frac{y_2}{4}, -\frac{1+y_2}{4}, \frac{y_2}{2})\) | \(D'_9\) \(D'_9\) | 0* | – | – | – |
| \((-y_2/4, y_2/4, -y_2/2)\) | \(D_{10}\) \(D_{10}\) | 0* | – | – | – |
| \((y_2 + 1, y_2 - 1, 0)\) | \(D_{11}\) \(D_{11}\) | 0* | – | – | – |

Table 2. Magnetic moments of the neutral states when the new fermions form a \(SU(2)_L\) doublet and a singlet. The last three columns correspond to specific cases where there are mass degeneracies among the different constituents. \(r_{ij} \equiv m_i/m_j\). Note that the cases where the magnetic moment is zero fall in two different categories: those that are zero because their constituents are neutral (denoted by 0*), and those where a degeneracy in constituents mass is present (denoted by 0**).

where \(T_3\) and \(Y_W\) are the third component of isospin and the hypercharge, associated to the generators of the gauge groups \(SU(2)_L\) and \(U(1)_Y\), respectively.

In the Quark Model, where it is considered a flavor \(SU(3)_F\) symmetry, there are two group diagonal generators, \(I_3\) and \(Y_F\). As it turns out, because the diagonal generators in this case correspond to \(SU(2)\) and \(U(1)\) symmetries, the specific assignment of the light quarks under the \(SU(3)_F\) as a fundamental allows us to express electric charge in terms of the \(SU(3)_F\) generators through the well known Gell-Mann-Nishijima formula

\[
Q = I_3 + \frac{1}{2} Y_F. \tag{5.2}
\]
| Hypercharge $(y_i)$ | Dark hadron | $\mu_{D_i,D_i^*}\left(\frac{\epsilon h}{2m_i}\right)$ | $r_{12} = 1$ | $r_{13} = 1$ | $r_{12} = r_{13}$ |
|-------------------|-------------|---------------------------------|--------------|--------------|-----------------|
| $y_1 = -y_2/2, y_3$ | $D_1$ | $-\frac{y_2}{6} (2 + r_{12})$ | $-y_2/2$ | $-$ | $-$ |
| $(-y_2/4, y_2/2, y_3/2)$ | $D_1^*$ | $-\frac{y_2}{6} (1 - r_{12})$ | $0^{**}$ | $-$ | $-$ |
| $y_1 = -2y_2, y_3$ | $D_2$ | $\frac{y_2}{3} (1 + 2r_{12})$ | $y_2$ | $-$ | $-$ |
| $(-y_2, y_2/2, y_3/2)$ | $D_2^*$ | $-y_2 (1 - r_{12})$ | $0^{**}$ | $-$ | $-$ |
| $y_1 = -2y_3, y_2$ | $D_3$ | $\frac{y_3}{3} (1 + 2r_{13})$ | $-$ | $y_3$ | $-$ |
| $(-y_3, y_2/2, y_3/2)$ | $D_3^*$ | $-y_3 (1 - r_{13})$ | $0^{**}$ | $-$ | $-$ |
| $y_1, y_3 = -y_2/2$ | $D_4$ | $-\frac{y_2}{6} (r_{12} + 2r_{13})$ | $-$ | $-$ | $-y_2r_{13}/2$ |
| $(y_1/2, y_2/2, -y_2/4)$ | $D_4^*$ | $\frac{y_2}{6} (r_{12} - r_{13})$ | $-$ | $-$ | $0^{**}$ |
| $y_1 + y_2 + y_3 = 0$ | $D_5$ | $y_3r_{13}/2 - y_2(r_{12} - 1) - y_1(2 + r_{13})$ | $-$ | $y_3/2$ | $-$ |
| $(-y_2 + y_3, y_2, y_3/2)$ | $D_5^*$ | $-y_2(r_{12} - 1) - y_1(1 - r_{13})$ | $-$ | $-$ | $-$ |
| $y_1, y_3 = -2y_2$ | $D_7$ | $\frac{y_2}{3} (2r_{12} + r_{13})$ | $-$ | $-$ | $-y_2r_{13}$ |
| $(y_1/2, y_2/2, -y_3)$ | $D_7^*$ | $y_2 (r_{12} - r_{13})$ | $-$ | $-$ | $0^{**}$ |
| $y_1 = -y_3/2, y_2$ | $D_8$ | $-\frac{y_2}{3} (2 + r_{13})$ | $-$ | $-y_3/2$ | $-$ |
| $(-y_3/4, y_2/2, y_3/2)$ | $D_8^*$ | $-\frac{y_2}{3} (1 - r_{13})$ | $-$ | $0^{**}$ | $-$ |
| $y_1 = 0, y_2, y_3$ | $D_9$ | $0^*$ | $-$ | $-$ | $-$ |
| $(0, y_2/2, y_3/2)$ | | | | | |
| $y_1, y_2 = 0, y_3$ | $D_{10}$ | $0^*$ | $-$ | $-$ | $-$ |
| $(y_1/2, 0, y_3/2)$ | | | | | |
| $y_1, y_2, y_3 = 0$ | $D_{11}^*$ | $0^*$ | $-$ | $-$ | $-$ |
| $(y_1/2, y_2/2, 0)$ | | | | | |

**Table 3.** Magnetic moments of the neutral states if each of the three constituents is a SU(2)$_L$ singlet. The last three columns correspond to specific cases where there are mass degeneracies among the different constituents ($r_{ij} \equiv m_i/m_j$). Note that the cases where the magnetic moment is zero fall in two different categories: those that are zero because their constituents are neutral (denoted by 0*) and those where a degeneracy in constituents mass is present (denoted by 0**).

Since in our model we consider three fundamental particles with arbitrary electric charge, it is of interest to determine those cases where it is possible to define the electric charge in terms of the flavor generators $T_3^D$, $Y^D$ of the group SU(3)$_D$, as in the Quark Model, with a generalization of the Gell-Mann-Nishijima formula $Q = c_T T_3^D + c_Y Y^D$. This of course depends crucially on the relation between $\tilde{q}_i$ and $q_i$ and the different SU(3)$_D$ representations used above. We now discuss each case separately.

### 5.1 Triplet

When the generalization can be used, the charges are given by

$$Q_i = \frac{c_T}{2} + \frac{c_Y}{3}, \quad Q_j = -\frac{c_T}{2} + \frac{c_Y}{3}, \quad Q_k = -\frac{2}{3}c_Y, \quad i \neq j \neq k. \quad (5.3)$$
\begin{center}
\begin{tabular}{|c|c|}
\hline
Relation among multiplets & Magnetic moments $\mu_{D_{5,6}} \left( \frac{e\hbar}{2m_1} \right)$ \\
\hline
$\tilde{q}_1 = q_2, \tilde{q}_2 = q_1, \tilde{q}_3 = q_3$ & $\mu_{D_5} = -r_{13}$ \\
& $\mu_{D_6} = \frac{1}{3} (2r_{12} + r_{13})$ \\
$\tilde{q}_1 = q_3, \tilde{q}_2 = q_2, \tilde{q}_3 = q_1$ & $\mu_{D_5} = r_{13}$ \\
& $\mu_{D_6} = -\frac{1}{3} (2 + r_{13})$ \\
$\tilde{q}_1 = q_1, \tilde{q}_2 = q_3, \tilde{q}_3 = q_2$ & $\mu_{D_5} = 0^*$ \\
& $\mu_{D_6} = \frac{2}{3} (1 - r_{12})$ \\
$\tilde{q}_1 = q_2, \tilde{q}_2 = q_3, \tilde{q}_3 = q_1$ & $\mu_{D_5} = 0^*$ \\
& $\mu_{D_6} = -\frac{2}{3} (1 - r_{12})$ \\
$\tilde{q}_1 = q_3, \tilde{q}_2 = q_1, \tilde{q}_3 = q_2$ & $\mu_{D_5} = r_{13}$ \\
& $\mu_{D_6} = -\frac{1}{3} (2r_{12} + r_{13})$ \\
\hline
\end{tabular}
\end{center}

Table 4. Magnetic moments of the states $D_5$ and $D_6$ if the three constituents are in a SU(2)$_L$ triplet for different relations among the multiplet components (see text for definition of $\tilde{q}_i$). The case $\tilde{q}_i = q_i$ is included in table 1. $r_{ij} \equiv m_1/m_j$.

\begin{center}
\begin{tabular}{|c|c|}
\hline
Relation among multiplets & Magnetic moments $\mu_{D_{5,6}} \left( \frac{e\hbar}{2m_1} \right)$ \\
\hline
$\tilde{q}_1 = q_2, \tilde{q}_2 = q_1, \tilde{q}_3 = q_3$ & $\mu_{D_5} = y_2 r_{13}/2$ \\
& $\mu_{D_6} = -\frac{(1-r_{12})}{3} - \frac{y_2 (1+r_{12}+r_{13})}{6}$ \\
$\tilde{q}_1 = q_3, \tilde{q}_2 = q_2, \tilde{q}_3 = q_1$ & $\mu_{D_5} = \frac{(2 - y_2) r_{13}/4}{12}$ \\
& $\mu_{D_6} = \frac{- (r_{13} + 2r_{12}) + y_2 (4 + r_{13} - 2r_{12})}{6}$ \\
$\tilde{q}_1 = q_1, \tilde{q}_2 = q_3, \tilde{q}_3 = q_2$ & $\mu_{D_5} = - \frac{(2 + y_2) r_{13}/4}{12}$ \\
& $\mu_{D_6} = \frac{(2 + r_{13}) + y_2 (r_{13} + 4r_{12} - 2)}{6}$ \\
$\tilde{q}_1 = q_2, \tilde{q}_2 = q_3, \tilde{q}_3 = q_1$ & $\mu_{D_5} = - \frac{(2 + y_2) r_{13}/4}{12}$ \\
& $\mu_{D_6} = \frac{(2 + r_{13}) + y_2 (r_{13} + 4r_{12} - 2)}{6}$ \\
$\tilde{q}_1 = q_3, \tilde{q}_2 = q_1, \tilde{q}_3 = q_2$ & $\mu_{D_5} = \frac{(2 - y_2) r_{13}/4}{12}$ \\
& $\mu_{D_6} = \frac{(3 + r_{13}) + y_2 (r_{13} + 4r_{12} - 2)}{6}$ \\
\hline
\end{tabular}
\end{center}

Table 5. Magnetic moments of the states $D_5$ and $D_6$ if the three constituents are in a SU(2)$_L$ doublet plus singlet for different relations among the multiplet components (see text for definition of $\tilde{q}_i$). The case $\tilde{q}_i = q_i$ is included in table 2. $r_{1j} \equiv m_1/m_j$.

Note that here there are only two independent relations while in (3.1) the three relations are dependent. If we want to express the charge in terms of the flavor generators, the system of equations is only consistent for $y_1 = 0$. For this particular value of the hypercharge we obtain four neutral states, $D_5, D_6, D_6^*$ and $D_{10}^*$.

- For $\tilde{q}_1 = q_i, cY = \frac{3}{2}$ and $c_T = 1$.
- For $\tilde{q}_1 = q_2, \tilde{q}_2 = q_1$, and $\tilde{q}_3 = q_3, cY = \frac{3}{2}$ and $c_T = -1$.
- For $\tilde{q}_1 = q_3, \tilde{q}_2 = q_2$, and $\tilde{q}_3 = q_1, cY = -\frac{3}{2}$ and $c_T = -1$.
- For $\tilde{q}_1 = q_1, \tilde{q}_2 = q_3$, and $\tilde{q}_3 = q_2, cY = 0$ and $c_T = 2$. 

- 12 -
| Relation among multiplets | Magnetic moments $\mu_{D_5,6}$ ($\frac{e\hbar}{2m}$) |
|---------------------------|---------------------------------------------|
| $\tilde{q}_1 = q_2$, $\tilde{q}_2 = q_1$, $\tilde{q}_3 = q_3$ | $\mu_{D_5} = y_3 r_{13}/2$  \[\mu_{D_6} = y_2 (1 - r_{12}) - y_1 (2 r_{12} + r_{13})\] |
| $\tilde{q}_1 = q_3$, $\tilde{q}_2 = q_2$, $\tilde{q}_3 = q_1$ | $\mu_{D_5} = - (y_2 + y_3) r_{13}/2$  \[\mu_{D_6} = y_2 (2 r_{12} + r_{13}) + y_1 (2 + r_{13})\] |
| $\tilde{q}_1 = q_3$, $\tilde{q}_2 = q_3$, $\tilde{q}_3 = q_2$ | $\mu_{D_5} = y_2 r_{13}/2$  \[\mu_{D_6} = - y_2 (2 + r_{13}) + y_1 (1 - r_{12})\] |
| $\tilde{q}_1 = q_2$, $\tilde{q}_2 = q_3$, $\tilde{q}_3 = q_2$ | $\mu_{D_5} = y_2 r_{13}/2$  \[\mu_{D_6} = - y_2 (2 r_{12} + r_{13}) + y_1 (1 - r_{12})\] |
| $\tilde{q}_1 = q_3$, $\tilde{q}_2 = q_1$, $\tilde{q}_3 = q_2$ | $\mu_{D_5} = - (y_2 + y_3) r_{13}/2$  \[\mu_{D_6} = y_2 (2 r_{12} + r_{13}) + y_1 (2 r_{12} + r_{13})\] |

Table 6. Magnetic moments of the states $D_5$ and $D_6$ if the three constituents are singlets of SU(2)$_L$ for different relations among the multiplet components (see text for definition of $\tilde{q}_i$). The case $\tilde{q}_i = q_i$ is included in table 3. $r_{ij} \equiv m_1/m_j$.

- For $\tilde{q}_1 = q_2$, $\tilde{q}_2 = q_3$, and $\tilde{q}_3 = q_1$, $c_Y = 0$ and $c_T = -2$.
- For $\tilde{q}_1 = q_3$, $\tilde{q}_2 = q_1$, and $\tilde{q}_3 = q_2$, $c_Y = -\frac{3}{2}$ and $c_T = 1$.

5.2 Doublet and singlet

From the charges given in equation (3.2), the generalization can be used for two relations between $y_1$ and $y_2$:

1. $y_2 = -2y_1$ can be used with the following relations between $\tilde{q}_i$ and $q_i$:
   - $\tilde{q}_i = q_i$, $c_Y = \frac{3}{2}y_1$ and $c_T = 1$.
   - $\tilde{q}_1 = q_2$, $\tilde{q}_2 = q_1$ and $\tilde{q}_3 = q_3$, $c_Y = \frac{3}{2}y_1$ and $c_T = 1$.
   - $\tilde{q}_1 = q_3$, $\tilde{q}_2 = q_2$ and $\tilde{q}_3 = q_1$, $c_Y = -\frac{3}{4}(y_1 + 1)$ and $c_T = -\frac{1}{2}(3y_1 - 1)$.
   - $\tilde{q}_1 = q_1$, $\tilde{q}_2 = q_3$ and $\tilde{q}_3 = q_2$, $c_Y = -\frac{3}{4}(y_1 - 1)$ and $c_T = \frac{1}{2}(3y_1 + 1)$.
   - $\tilde{q}_1 = q_3$, $\tilde{q}_2 = q_1$ and $\tilde{q}_3 = q_2$, $c_Y = -\frac{3}{4}(y_1 + 1)$ and $c_T = \frac{1}{2}(3y_1 - 1)$.

Note that $y_2 = -2y_1$ is consistent with almost all the necessary conditions to obtain neutral states, except for $D_7$ and $D_8$ when $\tilde{q}_i = q_i$.

2. $y_2 = y_1 + 1$ works when $\tilde{q}_1 = q_2$, $\tilde{q}_2 = q_3$ and $\tilde{q}_3 = q_1$, $c_Y = -\frac{3}{4}(y_1 - 1)$ and $c_T = -\frac{1}{2}(3y_1 + 1)$. This relation between the hypercharges is consistent with all the conditions for neutral states.

5.3 Singlets

Here we have that each particle $\tilde{q}_i$ has a charge given by $\frac{y_i}{2}$, so independently of the relation between $\tilde{q}_i$ and $q_i$, we obtain the same relations between the three hypercharges, up to a change of the form $i \rightarrow j$, $j \rightarrow k$, $k \rightarrow i$.
For the particular relation $\tilde{q}_i = q_i$, it is found that the generalization can be used when

$$
c_Y = \frac{3}{4} (y_1 + y_2), \quad c_T = \frac{1}{2} (y_1 - y_2) = \frac{1}{2} (y_2 - y_1),
$$

this is

$$
y_1 = y_2, \quad y_3 = -2y_1, \quad c_Y = \frac{3}{2} y_1, \quad c_T = 0.
$$

These relations are satisfied by the conditions for the neutral states $D_5$, $D_6$, $D_7$ and $D_8$. The conditions for the remaining states could also be satisfied, but this takes place only for $y_1 = y_2 = y_3 = 0$.

\section{Conclusions}

MDM posses an interesting and useful possibility that broadens up the spectrum of candidates and scenarios for the problem of DM. In this work, we consider DM candidates that can have a magnetic dipole moment due to the fact that they are composite states coming from a high energy strongly interacting sector.

Three additional elementary fermions have been introduced in addition to the SM particle content. These new fermions are singlets under $SU(3)_C$ but can have non-trivial representations under the electroweak gauge group. Since there are three of them, then only three different possibilities for their $SU(2)_L$ transformation exist: triplet, doublet plus singlet or three singlets.

Assuming there is a low energy $SU(3)_D$ dark flavor symmetry, and in analogy with low energy QCD, we construct the composite states and determine those that can be neutral. We find that there are several possibilities for each of the three cases above. We have assumed that the lightest hidden state is neutral since it is needed for having a viable DM candidate. This is a strong assumption and it could be unnatural in some cases, but it is completely necessary. On the other hand, even within a specific model, it is not simple to compute the spectrum of the multiplets. Indeed, there may be additional constraints that we have not analyzed in our work. For instance, if the value of two hypercharges are the same in the singlet case, there is an unbroken $SU(2)_L \times SU(2)_R$ global chiral symmetry, which will be dynamically broken by the strong $SU(3)_D$ interaction. It implies the presence of electrically neutral Nambu-Goldstone bosons, which will be stable due to the unbroken diagonal $SU(2)$. This type of models suffer very important experimental restrictions.

The results for $\mu_{DM}$ are presented in terms of the constituents masses and hypercharges, and cases where they are zero are singled out for two different scenarios: i) when the constituents themselves are electrically neutral and ii) when there is a particular mass degeneracy among some of the elementary constituents masses.

\section*{Acknowledgments}

This work has been supported in part by SNI and CONACYT CB-2011-01-167425 (Fondo Sectorial de Investigación para la Educación - México). J.A.R.C. acknowledges financial support from the MINECO (Spain) projects FIS2014-52837-P, FPA2014-53375-C2-1-P, Programa Becas Iberoamérica funded by Santander Universidades (Spain) 2015, and Consolider-Ingenio MULTIDARK CSD2009-00064.
A QCD

QCD is a particular case for the charge assignment corresponding to one $SU(2)_L$ doublet and one singlet with $\bar{q}_1 = q_1$, $y_1 = \frac{1}{3}$, $y_2 = -2y_1 = -\frac{2}{3}$, $c_T = \frac{3}{2}y_1 = \frac{1}{2}$ and $c_T = 1$. Here the three fundamental particles $q_i$ correspond to the three light quarks

$$q_1 \rightarrow u, \quad q_2 \rightarrow d, \quad q_3 \rightarrow s,$$

which carry a charge

$$Q_u = \frac{1}{2} + \frac{y_1}{2} = \frac{2}{3}, \quad Q_d = -\frac{1}{2} + \frac{y_1}{2} = -\frac{1}{3}, \quad Q_s = \frac{y_2}{2} = \frac{1}{3}.$$  \hspace{1cm} (A.2)

For the octet states the correspondence is

$$D_1 \rightarrow p, \quad D_2 \rightarrow n, \quad D_3 \rightarrow \Xi^0, \quad D_4 \rightarrow \Xi^-,$$

$$D_5 \rightarrow \Lambda, \quad D_6 \rightarrow \Sigma^0, \quad D_7 \rightarrow \Sigma^-, \quad D_8 \rightarrow \Sigma^+;$$

while for the decuplet states we have

$$D_1^* \rightarrow \Delta^+, \quad D_2^* \rightarrow \Delta^0, \quad D_3^* \rightarrow \Xi^{*0}, \quad D_4^* \rightarrow \Xi^{*-}, \quad D_5^* \rightarrow \Sigma^{*0},$$

$$D_6^* \rightarrow \Sigma^{*-}, \quad D_7^* \rightarrow \Sigma^{*+}, \quad D_8^* \rightarrow \Delta^{++}, \quad D_9^* \rightarrow \Delta^{+-}, \quad D_{10}^* \rightarrow \Omega^-.$$ \hspace{1cm} (A.4)

B Spin-flavor wavefunctions

If we consider the spin as an internal degree of freedom, then each fundamental particle $q_i$ is in the spin-flavor group product $SU(3) \otimes SU(2)$. So the composite states are obtained in the product

$$(3 \otimes 3 \otimes 3) \otimes (2 \otimes 2 \otimes 2).$$ \hspace{1cm} (B.1)

The decomposition of this product leads to singlets, octects and decuplets with spin 1/2 and 3/2. The spin-flavor wavefunctions of the octet states are [51]

$$|D_{11} \rangle = \frac{1}{\sqrt{18}} \left( 2 |q_1 q_1 q_2_1 \rangle + 2 |q_1 q_2 q_1 \rangle + 2 |q_2 q_1 q_1 \rangle - |q_1 q_1 q_2 \rangle - |q_1 q_2 q_1 \rangle - |q_2 q_1 q_1 \rangle \right) + \text{perms},$$

$$|D_{21} \rangle = \frac{1}{\sqrt{18}} \left( 2 |q_1 q_1 q_2 \rangle - |q_1 q_1 q_2 \rangle - |q_1 q_1 q_2 \rangle + \text{perms} \right),$$

$$|D_{31} \rangle = \frac{1}{\sqrt{18}} \left( 2 |q_1 q_3 q_1 \rangle - |q_1 q_3 q_1 \rangle - |q_1 q_3 q_1 \rangle + \text{perms} \right),$$

$$|D_{41} \rangle = \frac{1}{\sqrt{18}} \left( 2 |q_1 q_3 q_2 \rangle - |q_1 q_3 q_2 \rangle - |q_1 q_3 q_2 \rangle + \text{perms} \right),$$

$$|D_{51} \rangle = \frac{1}{\sqrt{12}} \left( |q_2 q_1 q_1 \rangle + |q_2 q_1 q_1 \rangle - |q_1 q_2 q_1 \rangle - |q_1 q_2 q_1 \rangle + \text{perms} \right).$$

$$|D_{61} \rangle = \frac{1}{6} \left( 2 |q_1 q_1 q_1 \rangle + 2 |q_1 q_2 q_1 \rangle - |q_2 q_1 q_1 \rangle - |q_1 q_1 q_2 \rangle - |q_1 q_1 q_2 \rangle + \text{perms} \right),$$

$$|D_{71} \rangle = \frac{1}{\sqrt{18}} \left( 2 |q_2 q_2 q_1 \rangle - |q_2 q_2 q_1 \rangle - |q_2 q_2 q_1 \rangle + \text{perms} \right),$$

$$|D_{81} \rangle = \frac{1}{\sqrt{18}} \left( 2 |q_1 q_1 q_1 \rangle - |q_1 q_1 q_1 \rangle - |q_1 q_1 q_1 \rangle + \text{perms} \right).$$  \hspace{1cm} (B.2)
In a similar way, for the decuplet states the wavefunctions are given by

\begin{align}
D^*_1 &= \frac{1}{\sqrt{3}} (|q_1,q_2,q_3\rangle + \text{permutations}), \\
D^*_2 &= \frac{1}{\sqrt{3}} (|q_2,q_1,q_3\rangle + \text{permutations}), \\
D^*_3 &= \frac{1}{\sqrt{6}} (|q_3,q_1,q_2\rangle + |q_2,q_1,q_3\rangle + \text{permutations}), \\
D^*_4 &= \frac{1}{\sqrt{3}} (|q_1,q_3,q_2\rangle + \text{permutations}), \\
D^*_5 &= \frac{1}{\sqrt{3}} (|q_1,q_2,q_3\rangle + \text{permutations}), \\
D^*_6 &= \frac{1}{\sqrt{3}} (|q_2,q_1,q_3\rangle + \text{permutations}), \\
D^*_7 &= \frac{1}{\sqrt{6}} (|q_3,q_2,q_1\rangle + |q_2,q_3,q_1\rangle + \text{permutations}), \\
D^*_8 &= \frac{1}{\sqrt{3}} (|q_1,q_3,q_2\rangle + \text{permutations}), \\
D^*_9 &= |q_1,q_2,q_3\rangle, \\
D^*_{10} &= |q_2,q_3,q_1\rangle, \\
D^*_{11} &= |q_3,q_1,q_2\rangle,
\end{align}

where permutations indicates the change 1 → 3 and 2 → 3 over each of the previous kets.

References

[1] J.A.R. Cembranos, A. Dobado and A.L. Maroto, *Brane world dark matter*, Phys. Rev. Lett. 90 (2003) 241301 [hep-ph/0302041] [INSPIRE].

[2] J.A.R. Cembranos, A. Dobado and A.L. Maroto, *Cosmological and astrophysical limits on brane fluctuations*, Phys. Rev. D 68 (2003) 103505 [hep-ph/0307062] [INSPIRE].

[3] J.A.R. Cembranos, A. Dobado and A.L. Maroto, *Dark geometry*, Int. J. Mod. Phys. D 13 (2004) 2275 [hep-ph/0405165] [INSPIRE].

[4] A.L. Maroto, *The nature of branon dark matter*, Phys. Rev. D 69 (2004) 043509 [hep-ph/0310272] [INSPIRE].

[5] A.L. Maroto, *Brane oscillations and the cosmic coincidence problem*, Phys. Rev. D 69 (2004) 101304 [hep-ph/0402278] [INSPIRE].

[6] J.A.R. Cembranos, A. de la Cruz-Dombriz, A. Dobado and A.L. Maroto, *Is the CMB cold spot a gate to extra dimensions?*, JCAP 10 (2008) 039 [arXiv:0803.0694] [INSPIRE].

[7] K. Sigurdson, M. Doran, A. Kurylov, R.R. Caldwell and M. Kamionkowski, *Dark-matter electric and magnetic dipole moments*, Phys. Rev. D 70 (2004) 083501 [Erratum ibid. D 73 (2006) 089903] [astro-ph/0406355] [INSPIRE].

[8] K. Sigurdson, *How dark is ‘dark’? electromagnetic interactions in the dark sector*, astro-ph/0412671 [INSPIRE].

[9] E. Masso, S. Mohanty and S. Rao, *Dipolar dark matter*, Phys. Rev. D 80 (2009) 036009 [arXiv:0906.1979] [INSPIRE].

[10] V. Barger, W.-Y. Keung and D. Marfatia, *Electromagnetic properties of dark matter: dipole moments and charge form factor*, Phys. Lett. B 696 (2011) 74 [arXiv:1007.4345] [INSPIRE].

[11] T. Banks, J.-F. Fortin and S. Thomas, *Direct detection of dark matter electromagnetic dipole moments*, arXiv:1007.5515 [INSPIRE].

[12] J.-F. Fortin and T.M.P. Tait, *Collider constraints on dipole-interacting dark matter*, Phys. Rev. D 85 (2012) 063506 [arXiv:1103.3289] [INSPIRE].

[13] E. Del Nobile, C. Kouvaris, P. Panci, F. Sannino and J. Virkjarvi, *Light magnetic dark matter in direct detection searches*, JCAP 08 (2012) 010 [arXiv:1203.6652] [INSPIRE].

[14] J.M. Cline, A.R. Frey and G.D. Moore, *Composite magnetic dark matter and the 130 GeV line*, Phys. Rev. D 86 (2012) 115013 [arXiv:1208.2685] [INSPIRE].
[15] K. Kadota and J. Silk, Constraints on light magnetic dipole dark matter from the ILC and SN 1987A, Phys. Rev. D 89 (2014) 103528 [arXiv:1402.7295] [inSPIRE].
[16] S. Mohanty and S. Rao, Detecting dipolar dark matter in beam dump experiments, arXiv:1506.06462 [inSPIRE].
[17] E. Fermi and E. Teller, The capture of negative mesotrons in matter, Phys. Rev. 72 (1947) 399 [inSPIRE].
[18] K. Griest and M. Kamionkowski, Unitarity limits on the mass and radius of dark matter particles, Phys. Rev. Lett. 64 (1990) 615 [inSPIRE].
[19] G. Raffelt, Stars as laboratories for fundamental physics, University of Chicago Press, Chicago U.S.A. (1996).
[20] J. Alcaraz, J.A.R. Cembranos, A. Dobado and A.L. Maroto, Limits on the brane fluctuations mass and on the brane tension scale from electron positron colliders, Phys. Rev. D 67 (2003) 075010 [hep-ph/0212269] [inSPIRE].
[21] L3 collaboration, P. Achard et al., Search for branons at LEP, Phys. Lett. B 597 (2004) 145 [hep-ex/0407017] [inSPIRE].
[22] J.A.R. Cembranos, A. Dobado and A.L. Maroto, Brane skyrmions and wrapped states, Phys. Rev. D 65 (2002) 026005 [hep-ph/0106322] [inSPIRE].
[23] J.A.R. Cembranos, A. Dobado and A.L. Maroto, Branor direct searches in hadronic colliders, Phys. Rev. D 70 (2004) 096001 [hep-ph/0405286] [inSPIRE].
[24] J.A.R. Cembranos, A. Dobado and A.L. Maroto, Branon radiative corrections to collider physics and precision observables, Phys. Rev. D 73 (2006) 035008 [hep-ph/0510399] [inSPIRE].
[25] J.A.R. Cembranos, A. Dobado and A.L. Maroto, Dark matter clues in the muon anomalous magnetic moment, Phys. Rev. D 73 (2006) 057303 [hep-ph/0507066] [inSPIRE].
[26] J.A.R. Cembranos, A. Dobado and A.L. Maroto, Some model-independent phenomenological consequences of flexible brane worlds, J. Phys. A 40 (2007) 6631 [hep-ph/0611024] [inSPIRE].
[27] J.A.R. Cembranos, J.L. Diaz-Cruz and L. Prado, Impact of DM direct searches and the LHC analyses on branon phenomenology, Phys. Rev. D 84 (2011) 083522 [arXiv:1110.0542] [inSPIRE].
[28] J.A.R. Cembranos, R.L. Delgado and A. Dobado, Brane-worlds at the LHC: branons and KK-gravitons, Phys. Rev. D 88 (2013) 075021 [arXiv:1306.4900] [inSPIRE].
[29] T. Bringmann, M. Vollmann and C. Weniger, Updated cosmic-ray and radio constraints on light dark matter: implications for the GeV gamma-ray excess at the galactic center, Phys. Rev. D 90 (2014) 123001 [arXiv:1406.6027] [inSPIRE].
[30] A. Aranda and J.A.R. Cembranos, The right generations, arXiv:1412.4836 [inSPIRE].
[31] M. Kamionkowski and M.S. Turner, Thermal relics: do we know their abundances?, Phys. Rev. D 42 (1990) 3310 [inSPIRE].
[32] K. Petraki and R.R. Volkas, Review of asymmetric dark matter, Int. J. Mod. Phys. A 28 (2013) 1330028 [arXiv:1305.4939] [inSPIRE].
[33] S. Davidson, S. Hannestad and G. Raffelt, Updated bounds on millicharged particles, JHEP 05 (2000) 003 [hep-ph/0001179] [inSPIRE].
[34] S. Davidson, B. Campbell and D.C. Bailey, Limits on particles of small electric charge, Phys. Rev. D 43 (1991) 2314 [inSPIRE].
[35] A.A. Prinz et al., Search for millicharged particles at SLAC, Phys. Rev. Lett. 81 (1998) 1175 [hep-ex/9804008] [inSPIRE].
[36] G.D. Starkman, A. Gould, R. Esmailzadeh and S. Dimopoulos, *Opening the window on strongly interacting dark matter*, *Phys. Rev. D* **41** (1990) 3594 [inSPIRE].

[37] S. Rudaz and F.W. Stecker, *Cosmic ray anti-protons, positrons and gamma-rays from halo dark matter annihilation*, *Astrophys. J.* **325** (1988) 16 [inSPIRE].

[38] J.A.R. Cembranos, J.L. Feng, A. Rajaraman and F. Takayama, *SuperWIMP solutions to small scale structure problems*, *Phys. Rev. Lett.* **95** (2005) 181301 [hep-ph/0507150] [inSPIRE].

[39] J.A.R. Cembranos, J.L. Feng and L.E. Strigari, *Resolving cosmic gamma ray anomalies with dark matter decaying now*, *Phys. Rev. Lett.* **99** (2007) 191301 [arXiv:0704.1658] [inSPIRE].

[40] J.A.R. Cembranos, J.L. Feng and L.E. Strigari, *Exotic collider signals from the complete phase diagram of minimal universal extra dimensions*, *Phys. Rev. D* **75** (2007) 036004 [hep-ph/0612157] [inSPIRE].

[41] J.A.R. Cembranos and L.E. Strigari, *Diffuse MeV gamma-rays and galactic 511 keV line from decaying WIMP dark matter*, *Phys. Rev. D* **77** (2008) 123519 [arXiv:0801.0630] [inSPIRE].

[42] J.A.R. Cembranos, *The Newtonian limit at intermediate energies*, *Phys. Rev. D* **73** (2006) 064029 [gr-qc/0507039] [inSPIRE].

[43] J.A.R. Cembranos, *Dark matter from $R^2$-gravity*, *Phys. Rev. Lett.* **102** (2009) 141301 [arXiv:0809.1653] [inSPIRE].

[44] J.A.R. Cembranos, A. de la Cruz-Dombriz, V. Gammaldi and A.L. Maroto, *Detection of branon dark matter with gamma ray telescopes*, *Phys. Rev. D* **85** (2012) 043505 [arXiv:1111.4448] [inSPIRE].

[45] J.A.R. Cembranos, V. Gammaldi and A.L. Maroto, *Possible dark matter origin of the gamma ray emission from the galactic center observed by HESS*, *Phys. Rev. D* **86** (2012) 103506 [arXiv:1204.0655] [inSPIRE].

[46] J.A.R. Cembranos, V. Gammaldi and A.L. Maroto, *Spectral study of the HESS J1745-290 gamma-ray source as dark matter signal*, *JCAP* **04** (2015) 039 [arXiv:1503.08749] [inSPIRE].

[47] G. Jungman, M. Kamionkowski and K. Griest, *Supersymmetric dark matter*, *Phys. Rept.* **267** (1996) 195 [hep-ph/9506380] [inSPIRE].

[48] J.L. Diaz-Cruz, *Holographic dark matter and Higgs*, *Phys. Rev. Lett.* **100** (2008) 221802 [arXiv:0711.0488] [inSPIRE].

[49] O. Antipin, M. Redi and A. Strumia, *Dynamical generation of the weak and dark matter scales from strong interactions*, *JHEP* **01** (2015) 157 [arXiv:1410.1817] [inSPIRE].

[50] O. Antipin, M. Redi, A. Strumia and E. Vigiani, *Accidental composite dark matter*, *JHEP* **07** (2015) 039 [arXiv:1503.08749] [inSPIRE].

[51] W. Thirring, *Electromagnetic properties of hadrons in the static SU(6) model*, *Acta Phys. Austriaca Suppl.* **2** (1965) 205 [inSPIRE].