The heat transfer enhancement mechanisms of tubes with different arrays in the transitional flow

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Abstract. Forced convection heat transfer from multiple circular cylinders in tandem, in-line and staggered configurations has been studied using. The numerical simulations are performed with a developed finite volume code based on the compound grid system, which simulates accurately the information around cylinders. This simulation is performed for the Prandtl numbers (Pr) of 0.7 at the Reynolds numbers (Re) of 200. The streamwise center-to-center distance ratio is set at 2, 3, 4, 5, 6 and 8. The time-mean Nusselt number on cylinders was presented to understand the heat transfer performance. By investigating the time-mean velocity and the velocity-temperature cross-correction, it can be found that the recovery effect of velocity and the instability effect of vortex can enhance heat transfer.

1. Introduction
Study of the cross-flow over circular cylinders in different arrangements has received considerable attention from engineering applications, including groups of chimney stacks, tubes in heat exchangers, overhead power-line bundles, bridge piers, chemical reaction towers, offshore platforms, adjacent skyscrapers, etc. Furthermore, flow over circular cylinders leads a series of different fundamental fluid dynamic phenomena, such as: flow separation, reattachment, vortex impingement and vortex shedding. Lots of problems resulting from phenomenon deteriorate the performance of engineering applications. Therefore, further physical insights into the nature of the flow or heat transfer need to be revealed to explain the formation of the phenomenon for a better design of applications.

At first, the attention of researchers was on features of the development and separation of boundary layer as well as the vortex shedding on a single circular cylinder, which is one of the elementary shapes of structures or structural components used in engineering practice. This provides a possibility to analyze the heat transfer performance by fundamental flow characteristics. Eckert and Soehngen [1] and Krall and Eckert [2] reported the experimental values of the local Nusselt number on the surface of the cylinder for the flow of air at a wide range of $10 \leq Re \leq 400$. More recently, Bharti et.al [3] researched the forced convection heat transfer from an unconfined circular cylinder in the steady cross-flow regime using a finite volume method. He found that the rate of heat transfer increases with an increase in the Reynolds or Pr.

When the fluid flows around multiple circular cylinders, the interactions of shear layer and the features of vortex shedding involve a more complicity to produce different types of flow pattern which
are important to understand heat transfer features. The typical forms of multiple circular cylinders are the tandem, side-by-side, in-line and staggered. The scope of this paper is to numerically investigate the flow and heat transfer characteristics in the fundamental forms of the tandem, in-line and staggered configurations, which were reviewed primarily.

For the tandem configuration, a number of investigations resulted in the identification of different types of flow pattern. Igarashi [4], Zdravkovich [5] and Sumner et al. (2000) [6] proposed classifications of flow pattern by investigating diverse interference regimes in flow visualization experiments. Jester and Kallinderis [7] numerically investigate the incompressible flow about fixed cylinder pairs in tandem, side-by-side, and staggered configurations at a range of $80 \leq Re \leq 1000$. They found that an extensive series of staggered simulations in experimentally classified interference regimes has been performed. However, the heat transfer characteristics in tandem configuration have not been studied extensively. Buyruk [8] numerically investigated the heat transfer from two isothermal cylinders in tandem arrangement at $Re=400$. He obtained local Nusselt number and temperature contours over the entire cylinder surface. Juncu [9] and Mahir and Altac [10] numerically studied the forced convection heat transfer around two isothermal circular cylinders in tandem arrangement at a range of $1 \leq Re \leq 30$ and $100 \leq Re \leq 200$, respectively. Harimi and Saghaifian [11] numerically studied the forced convection heat transfer from two and three isothermal circular cylinders in tandem arrangement. The numerical simulations are performed with a developed finite volume code using the overset grid method.

For the studies of the in-line configurations, Sayers [12] and Sayers [13] measured the lift and drag coefficient and St on a single cylinder in a group of four equally spaced cylinders in an open-jet wind tunnel. Lama [14] studied two and three-dimensional numerical simulations of cross-flow around four cylinders in an in-line square configuration using a finite-volume method. The 2-D studies reveal three distinct flow patterns: a stable shielding flow, a wiggling shielding flow and a vortex shedding flow. Wang and Georgiadis [15] studied the steady forced convection heat transfer in a laminar flow field over an infinite (periodic) in-line array of cylinders. They found two truncated domains, including a single cylinder cell in developed regime and a five-cylinder cell in the developing regime. Li et.al [16] studied numerically mass transfer at the shell side of an in-line hollow fiber array subjected to cross-flow in the laminar flow of $Re=10–200$.

For the studies of the staggered configurations, Zdravkovich [5] suggested three primary categories of flow behavior: proximity interference, wake interference, and combined proximity and wake interference. Akbaril [17] studied the flow patterns around two staggered circular cylinders in cross-flow for the low subcritical $Re$ regime ($Re=800$). They found five distinct flow regimes, depending on the geometrical arrangement of the cylinders. In a more recent investigation by Sumner et al. [6], the flow over a pair of staggered cylinders was classified into nine categories. Li and Sumner [18] found that the closely spaced staggered finite cylinders were characterized by the same Strouhal number measured behind both cylinders, an indication of single bluff-body behaviour. However, the heat transfer characteristics of staggered cylinder were studied relatively fewer. Buyruk [8] studied three isothermal cylinders in staggered arrangement and four isothermal cylinders in an in-line square configuration at $Re=80$, 120 and 200. He obtained local Nusselt number and temperature contours over the entire cylinder surface and reported good agreement with other published data.

In this paper, the finite volume method was adopted to simulate numerically the flow and heat transfer of circular cylinders in tandem, in-line and staggered configurations using the compound grid system, which is similar computation method with literature [11] to process the information around circular cylinders. Pr was set as 0.7 and Re was set as 200. The streamwise center-to-center distance ratio ($Ps/D$) was at 2, 3, 4, 5, 6, 8 in all configurations and while the normal center-to-center distance ratio ($Pt/D$) was set at 2.5 in in-line and staggered configurations. The time-mean Nusselt number ($Nu$) on all cylinders, the flow fields as well as the time-mean velocity in $X$ direction ($U_m$) and the velocity-temperature cross-correction ($V\theta$) in a cross-section were computed as an attempt to illuminate the features of wake to explain the heat transfer enhancement.
2. Numerical model description

In the present study, a two-dimensional numerical computation is carried out for unsteady flow and temperature fields. The governing equations of the flow and temperature fields are as follows:

**Continuity equation**

\[ \frac{\partial}{\partial X} (U) + \frac{\partial}{\partial Y} (V) = 0 \]  

(1)

**Momentum equations**

\[ \rho \frac{\partial}{\partial t} (U) + \rho \frac{\partial (U^2)}{\partial X} + \rho \frac{\partial (UV)}{\partial Y} = -\frac{\partial P}{\partial X} + \mu \left( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right) \]  

(2)

\[ \rho \frac{\partial}{\partial t} (V) + \rho \frac{\partial (UV)}{\partial X} + \rho \frac{\partial (V^2)}{\partial Y} = -\frac{\partial P}{\partial Y} + \mu \left( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right) \]  

(3)

**Energy equation**

\[ \rho \frac{\partial}{\partial t} (C_P T) + \rho \frac{\partial (C_P U T)}{\partial X} + \rho \frac{\partial (C_P V T)}{\partial Y} = \lambda \frac{\partial T}{\partial X} + \lambda \frac{\partial T}{\partial Y} \]  

(4)

where \( U \) and \( V \) are the instantaneous velocity components along the X-direction and Y-direction, respectively. \( T \) is the fluid temperature and \( P \) is the pressure. \( \rho \), \( \mu \), \( \lambda \) and \( C_P \) are the density, the viscosity, the thermal conductivity and the specific heat at constant pressure of the fluid, respectively.

Figure 1 shows the two-dimensional computational domain of tandem (a), in-line (b) and staggered (c) configurations. \( X \) and \( Y \) are the streamwise and normal coordinate respectively. \( D \) denotes the cylinder diameter (0.007m). \( Ps/D \) and \( Pt/D \) represent the streamwise center distance ratio and normal center distance ratio between two cylinders, respectively. \( Ps/D \) ranges from 2 to 8 in all configurations and \( Pt/D=2.5 \) in in-line and staggered configurations as the calculation conditions.

The boundary conditions can be stated as:

- For the inlet in all configurations, \( U=U_0 \), \( V=0 \), \( T=T_0 \),
- For the outlet in all configurations, \( \frac{\partial U}{\partial X} = 0 \), \( \frac{\partial V}{\partial X} = 0 \), \( \frac{\partial T}{\partial X} = 0 \),
- For the top and bottom in tandem configuration, \( \frac{\partial U}{\partial Y} = 0 \), \( V=0 \), \( \frac{\partial T}{\partial Y} = 0 \),
- For the top and bottom in in-line and staggered configurations, the periodic boundary,
- For the cylinders walls, \( U=0 \), \( V=0 \), \( T=T_w \),

where \( U_0 \) and \( T_0 \) (10\(^{\circ}\)C) are the free-stream velocity and temperature, and \( T_w \) is the cylinder wall temperature which is constant of (40\(^{\circ}\)C).

There are other parameters as follows:

\[ -\lambda \frac{\partial T}{\partial n} \bigg|_{w} = h_0 (T_w - T_0) \]  

(5)

where \( h_0 \) is heat local heat transfer coefficient.

\[ Nu= \frac{\lambda h_0 D}{\lambda}, \quad Nu = \frac{1}{2\pi} \frac{1}{\tau} \int_0^{2\pi} \int_0^{\tau} \text{Nu} \ d\theta \]  

(6,7)

where \( \tau \) is the total time of ten cycles.

\[ \omega = \frac{\partial V}{\partial Y} - \frac{\partial U}{\partial X} \]  

(8)

where \( \omega \) is the vorticity.

\[ U_m = \frac{1}{\tau_0} \int_0^{\tau} U \ d t \]  

(9)

\[ V' = V - \bar{V}, \quad \theta' = T - \bar{T}, \quad \bar{V} = \frac{VT - \bar{V} \bar{T}}{U_0 T_0} \]  

(10,11,12)

Where \( \bar{V} \) and \( \bar{T} \) are the time-mean normal velocity and temperature of ten cycles.
In order to simulate the features of flow and temperature around the cylinders, a compound grid system with two different grid systems was adopted in this paper. As shown in Figure 2, one using rectangular coordinate system is for the whole computational domain (Main-domain) having an inner boundaries (red line) around body surfaces, and the other using cylindrical coordinate system is for the near-body region (Sub-domains) having an outer boundaries. The inner boundaries (red line) of Main-domain are always fixed in the region enclosed by the outer boundaries of Sub-domains. At each iteration step, computation is firstly carried out in Main-domain and then in Sub-domains in turn. The values of outer boundaries of Sub-domains are obtained by interpolation in the Main-domain.

Fully implicit forms of the finite-difference equations are adopted. In finite-differencing the governing equations, a central finite-differencing scheme was used for the diffusion terms in the compound system. For the convection terms, the quick scheme was applied to the finite difference equations to be relaxed in the Main-domain and the Sub-domains. The finite-difference equations of fully implicit form were solved step by step along the time axis. In one time step, one cycle of iterative procedure was repeated five times to effectively relax the solution. For the evaluation of pressure, the SIMPLE algorithm by Patankar and Spalding [19] was employed. The magnitude of time step was determined in such a way that the Courant number for the smallest grid cell evaluated with the cross-sectional average velocity was equal to unity. The independence of grid of a single cylinder at Re=200 was validated and the heat transfer coefficient in Re=120 and 218 were also agree with the published literature [20].
3. Results and discusses

Figure 3 shows $\bar{Nu}$ of upstream cylinder (a) and downstream cylinder (b) in different $Ps/D$ at $Re=200$ in configurations of tandem, in-line and staggered. In Figure 3 (a), the curve of $\bar{Nu}$ in tandem is similar to that of in-line and the values of $\bar{Nu}$ in different $Ps/D$ in in-line are larger than that of tandem. The main reason is that the fluid between the upstream cylinders in the in-line configuration is accelerated to thin the velocity boundary layer in this region. This is called block effect. Another feature is that $\bar{Nu}$ increases rapidly from $Ps/D=3$ to $Ps/D=4$ and this is caused by the structure of flow field which will be investigated in the next in detail. The $\bar{Nu}$ in staggered configuration in different $Ps/D$ changes a little. Compared $\bar{Nu}$ of tandem and in-line, the heat transfer on the back of face of the upstream cylinder in staggered configuration is enhanced by the accelerated fluid. Therefore, $\bar{Nu}$ in $Ps/D=2$ and 3 in staggered are larger than that of tandem and in-line.

In Figure 3 (b), it can be found a more drastic change with the increase of $Ps/D$ compared with that of the upstream cylinder. For the tandem and in-line configuration, $\bar{Nu}$ in $Ps/D=2$ and 3 also increases rapidly because of the flow instability of vortex $\bar{Nu}$ in different $Ps/D$ in in-line are larger than that of tandem because of the block effect. For the staggered configuration, $\bar{Nu}$ in $Ps/D=2$ and 3 are large due to the accelerated fluid generating from the block effect of the upstream cylinder and then increases in $Ps/D=4$ because of the flow instability and finally decreases with increase of $Ps/D$ because of recession of flow instability.

Figure 4. Flow field in different $Ps/D$ in the tandem configuration
Figure 4～6 show the flow field in different Ps/D of tandem, in-line and staggered, respectively.

In order to illustrate the influence between the upstream cylinders in in-line and staggered configurations, the vorticity field of double region were processed. The blue regions represent the negative vortex which leads an anticlockwise motion and the red regions represent the positive vortex which leads a clockwise motion. The interface between negative vortex and the positive vortex presents a state of wave.

In Figure 4 and Figure 5, the flow fields between the two cylinders in Ps/D=2 and 3 is steady and in fact contain a motion of low velocity and counter flow. Therefore, a weak heat transfer enhancement effect is formed on the back face of the upstream cylinder and the front face of the downstream cylinder. This behavior contributes to weak $\text{Nu}$ of the two cylinders. When Ps/D=4, an obvious feature of flow field between the two cylinders is the formation of the vortex to enhance the back face of the upstream cylinder and especially the front face of the downstream cylinder. Because of vortex, the amplitude of the interface between negative vortex and positive vortex increases with moving downstream to accelerate the recovery of velocity and fluid temperature. This behavior of the recovery improves the effective Re and the temperature difference in the front of the downstream cylinder to enhance the heat transfer here. With a further increase of Ps/D, the flow instability in the front of the downstream cylinder decreases to weaken the heat transfer here.

Compared with the tandem or in-line two cylinders, it is essential that the front face of downstream cylinder in staggered is struck directly by the main flow. In Figure 6, the wake of the upstream cylinder in Ps/D=2 and 3 belongs to laminar state and is squeezed by the block effect of the
downstream cylinder to thin the boundary layer of back face of the upstream cylinder. This fact explains why $Nu$ in $Ps/D=2$ and $3$ in staggered are larger than that of tandem and in-line. When $Ps/D=4$, the wake behind the upstream cylinder is displayed by the form of vortex. The vortex weakens the impact effect on the front face of the downstream cylinder but enhances the washing effect [21] on the down or up face of the downstream cylinder. Therefore, both of the impact effect and the washing effect are relatively strong to enhance the heat transfer of the downstream cylinder. However, with a further increase of $Ps/D$, the intensity of vortex decreases to weaken the impact effect and the washing effect to reduce $Nu$ of the downstream cylinder.

Figure 7. $U_m$ (a) and $V' \theta'$ (b) in the cross-section at the upstream 1D to the center of the downstream cylinders in tandem configuration

Figure 8. $U_m$ (a) and $V' \theta'$ (b) in the cross-section at the upstream 1D to the center of the downstream cylinders in in-line configuration

Figure 9. $U_m$ (a) and $V' \theta'$ (b) in the cross-section at the upstream 1D to the center of the downstream cylinders in staggered configuration
It is worthy to further investigating the influence of block effect and the flow instability in the front of the downstream cylinders. In order to measure the results from these two effects, the $U_m(a)$ and $V_\theta(b)$ in the cross-section at the upstream 1D to the center of the downstream cylinders in different $Ps/D$ were obtained as shown in Figure 7. In Figure 7 (a), $U_m$ near $Y/D=0$ in $Ps/D=2$ and 3 are negative, which is an important fact to prove that the flow field between the two cylinders is low velocity and counter flow. With a further increase of $Ps/D$, $U_m$ near $Y/D=0$ increases to generate an effective $Re$ to the downstream cylinder. In Figure 7 (b), $\overline{V_\theta}$, which represents a intensity of the flow instability, in $Ps/D=2$ and 3 are mostly zero. Nevertheless, $\overline{V_\theta}$ in $Ps/D=4$ is large especially at $Y/D=1.1$ and contributes to temperature mixing including bringing the cold fluid from the main flow into the wall of cylinder and pumping the hot fluid out from the main flow. This behavior strengthens the temperature mixing between the cylinder and the main flow. With a further increase of $Ps/D$, $\overline{V_\theta}$ decreases to reduce the heat transfer enhancement from the flow instability. The above features from the analyzation of Fig.7 are similar with that of the in-line configuration as shown in Figure 8. The difference point is that the recession of $\overline{V_\theta}$ in in-line is faster than that of in tandem, which is caused by the block effect of the upstream cylinder.

Figure 9 shows $U_m(a)$ and $\overline{V_\theta(b)}$ in the cross-section at the upstream 1D to the center of the downstream cylinders in staggered configuration in different $Ps/D$ at $Re=200$. In Figure 9 (a), when $Ps/D=2$ and 3, $U_m$ near $Y/D=-0.625$ and $Y/D=1.875$ are large due to the accelerated main flow caused by the block effect. When $Ps/D=4$ and 5, $U_m$ at similar location decreases because of the vortex behind the upstream cylinder. Another feature is that $U_m$ in $Ps/D=8$ in the front of the downstream cylinder decreases with a further increase of $Ps$, which reduce the effective $Re$ to deteriorate the heat transfer of the downstream cylinder. In Figure 9 (b), it is found similar changing rules compared with that of the tandem or in-line. The difference is that the positive or negative of $\overline{V_\theta}$ in the front of the cylinder in staggered are opposite compared with that of tandem or in-line. This is a very interesting phenomenon which denotes a totally different mechanism of the heat transfer on the front face of cylinder.

4. Conclusions
In this paper, three computational models with the tandem, in-line and staggered configurations in different $Ps/D$ at $Re=200$ was carried out by using a compound grid system. The heat transfer enhancement performances caused by flow features in the three models were compared and some conclusions were summarized as follows.

1. In the in-line and staggered configurations, the block effect caused by the small pitch of the upstream cylinders thins the boundary layer here and especially accelerates the development of vortex behind the upstream cylinders to enhance the heat transfer enhancement on the downstream cylinder.

2. In the tandem and in-line configurations, the flow fields between the two cylinders in $Ps/D=2$ and 3 contain a motion of low velocity and counter flow to deteriorate the back face of the upstream cylinder and the front face of the downstream cylinder.

3. In the tandem and in-line configurations, an obvious feature of flow field between the two cylinders in $Ps/D=4$ is the formation of the vortex to produce the flow instability, which contributes $Nu$ of the front face of the downstream cylinder.

4. In the staggered configuration, it is essential that the front face of downstream cylinder in staggered is struck directly by the main flow and the location affected by the flow instability is different with the front face in the tandem and in-line configurations.

Acknowledgements
The authors are grateful to the National Natural Science Foundation of China for providing financial support of this work through Award Number No. 51476080.

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