A note on covers of fibred hyperbolic manifolds

Jérôme Los, Luisa Paoluzzi, and António Salgueiro

February 19, 2016

Abstract

For each surface $S$ of genus $g > 2$ we construct pairs of conjugate pseudo-Anosov maps, $\varphi_1$ and $\varphi_2$, and two non-equivalent covers $p_i : \tilde{S} \rightarrow S$, $i = 1, 2$, so that the lift of $\varphi_1$ to $\tilde{S}$ with respect to $p_1$ coincides with that of $\varphi_2$ with respect to $p_2$.

The mapping tori of the $\varphi_i$ and their lift provide examples of pairs of hyperbolic 3-manifolds so that the first is covered by the second in two different ways.

AMS classification: Primary 57M10; Secondary 57M50; 57M60; 37E30.

Keywords: Regular covers, mapping tori, (pseudo-)Anosov diffeomorphisms.

1 Introduction

Given a finite group $G$ acting freely on a closed orientable surface $\tilde{S}$ of genus larger than 2 one considers the space $X$ of the orbits for the $G$-action on $\tilde{S}$. The projection $\tilde{S} \rightarrow X$ is a regular cover and $X$ is again a surface, of genus $g \geq 2$, whose topology is totally determined by the order of $G$. Assume now that $G$ contains two normal subgroups, $H_1$ and $H_2$, non isomorphic but with the same indices in $G$. In this situation one can construct the following commutative diagram of regular coverings:

\[
\begin{array}{ccc}
\tilde{S} & \xleftarrow{\pi_1} & S_1 = \tilde{S}/H_1 \\
\downarrow & & \downarrow \\
X & \cong & S_2 = \tilde{S}/H_2 \end{array}
\]

$X = \tilde{S}/G$

\[\text{\textsuperscript{*}Partially supported by ANR project 12-BS01-0003-01}\]

\[\text{\textsuperscript{†}Partially supported by the Centre for Mathematics of the University of Coimbra – UID/MAT/00324/2013, funded by the Portuguese Government through FCT/MEC and co-funded by the European Regional Development Fund through the Partnership Agreement PT2020.}\]
We are interested in the following:

**Question.** Is there a pseudo-Anosov diffeomorphism \( \varphi \) of \( X \) which lifts to pseudo-Anosov diffeomorphisms \( \varphi_1, \varphi_2 \) and \( \tilde{\varphi} \) of \( S_1, S_2 \) and \( \tilde{S} \) respectively such that there is a diffeomorphism \( g : S_1 \to S_2 \) conjugating \( \varphi_1 \) to \( \varphi_2 \), i.e. \( \varphi_2 = g \circ \varphi_1 \circ g^{-1} \)?

The aim of the present note is to provide explicit constructions of surface coverings and pseudo-Anosov diffeomorphisms satisfying the above properties. This will be carried out in the next sections. More explicitly, we prove:

**Theorem 1.** For each closed oriented surface \( S \) of genus greater than 2, there exists an infinite family of pairs \((\varphi_1, \varphi_2) : S \to S\) of conjugate pseudo-Anosov maps and two non-equivalent coverings \( p_i : \tilde{S} \to S \) such that a lift of \( \varphi_1 \) with respect to \( p_1 \) and a lift of \( \varphi_2 \) with respect to \( p_2 \) are the same map \( \tilde{\varphi} : \tilde{S} \to \tilde{S} \).

Here, the expression *infinitely many pairs of diffeomorphisms* means that there is an infinite family of pairs so that if \( \varphi_i \) and \( \varphi'_j \) belong to different pairs then no power of \( \varphi_i \) is a power of \( \varphi'_j \), for \( i, j = 1, 2 \), up to conjugacy.

A positive answer to our initial question implies the existence of hyperbolic 3-manifolds with interesting properties. By considering the mapping tori of the four diffeomorphisms \( \varphi, \varphi_1, \varphi_2, \) and \( \tilde{\varphi} \), one gets four hyperbolic 3-manifolds \( N, M_1, M_2, \) and \( \tilde{M} \) respectively. The covers of the surfaces \( \tilde{S}, \tilde{S}_1, \tilde{S}_2 \) and \( X \) induce covers of these manifolds:

\[
\begin{array}{c}
\tilde{M} \\
\downarrow \\
M_1 \quad M_2 \\
\uparrow \\
N
\end{array}
\]

Since \( \varphi_1 \) and \( \varphi_2 \) are conjugate, we see that \( M_1 \) and \( M_2 \) are homeomorphic (and hence isometric by Mostow’s rigidity theorem [Mo]). It follows that \( \tilde{M} \) is a regular cover of a manifold \( M \cong M_1 \cong M_2 \) in two different ways.

**Corollary 2.** There exists an infinite family of pairs of hyperbolic 3-manifolds \((\tilde{M}, M)\), such that there exist two non-equivalent regular covers \( p_1, p_2 : \tilde{M} \to M \) with non isomorphic covering groups. Moreover, for each \( k \in \mathbb{N} \), there is a 3-manifold \( M \), which belongs to at least \( k \) distinct such pairs \((\tilde{M}, M_\ell)\), \( 1 \leq \ell \leq k \).

The existence of hyperbolic 3-manifolds with this type of behaviour was already remarked in [RS] but our examples show that one can moreover ask for the manifolds to fibre over the circle and for the two group actions to preserve a fixed fibration (see also Section 3 for other comments on the two types of examples).
2 Main construction

In this section we answer in the positive to a weaker version of our original question, where the diffeomorphisms involved are not required to be pseudo-Anosov.

2.1 Symmetric surfaces

For every pair of integers \( n, m \geq 1 \) we will construct a closed connected orientable surface of genus \( nm + 1 \) admitting a symmetry of type \( G = \mathbb{Z}/n \times \mathbb{Z}/m \).

Let \( n \) and \( m \) be fixed. Consider the torus \( T = \mathbb{R}^2/\mathbb{Z}^2 \) and the following \( G \)-action: the generator of \( \mathbb{Z}/n \) is \( (x, y) \mapsto (x + 1/n, y) \) and that of \( \mathbb{Z}/m \) is \( (x, y) \mapsto (x, y + 1/m) \), where all coordinates are thought mod 1.

The union of the sets of lines \( L_x = \{ (i/n, y) \in \mathbb{R}^2 \mid i \in \mathbb{Z}, y \in \mathbb{R} \} \) and \( L_y = \{ (x, j/m) \in \mathbb{R}^2 \mid j \in \mathbb{Z}, x \in \mathbb{R} \} \) maps to a \( G \)-equivariant family \( \mathcal{L} \) of simple closed curves of \( T \): \( n \) meridians and \( m \) longitudes, as in Figure 1.

Consider a standard embedding of \( T \) in the 3-sphere \( S^3 \subset \mathbb{C}^2 \) so that the \( G \) action on the torus is realised by the \( (\mathbb{Z}/n \times \mathbb{Z}/m) \)-action on \( S^3 \) defined as \( (z_1, z_2) \mapsto (e^{2i\pi/n}z_1, z_2) \) and \( (z_1, z_2) \mapsto (z_1, e^{2i\pi/m}z_2) \). A small \( G \)-invariant regular neighbourhood of \( \mathcal{L} \) in \( S^3 \) is a handlebody \( \tilde{H} \) of genus \( nm + 1 \). Its boundary is the desired surface \( \tilde{S} \).

2.2 The normal subgroups \( H_1 \) and \( H_2 \)

Notation 1. Let \( n \in \mathbb{N} \).

- We denote by \( \Pi(n) \) the set of all prime numbers that divide \( n \).
- For any \( P \subset \Pi(n) \) we denote by \( n_P \in \mathbb{N} \) the divisor of \( n \) such that \( \Pi(n_P) = P \) and \( \Pi(n/n_P) = \Pi(n) \setminus P \).

Definition 1. Let \( A \) and \( B \) be two finite sets of prime numbers such that

- \( A \cap B = \emptyset \);
- \( A \cup B \neq \emptyset \).

Let \( n, m \in \mathbb{N}, n, m \geq 2 \). We say that \( (n, m) \) is admissible with respect to \( (A, B) \) if the following conditions are verified:

- \( A \cup B \subset \Pi(n) \cap \Pi(m) \);
- \( \frac{n_{A \cap B}}{m_{A \cap B}} \) is an integer strictly greater than 1.

In this case we let \( C = \Pi(n) \setminus (A \cup B) \) and \( D = \Pi(m) \setminus (A \cup B) \).

We note that, since \( \frac{n_{A \cap B}}{m_{A \cap B}} \) is an integer greater than one, then \( m_{A \cap B} = m \neq n_{A \cap B} = n_{A \cup B} \).

Remark 1. If \( \gcd(n, m) = d > 1 \) and at least one between \( \gcd(d, n/d) \) and \( \gcd(d, m/d) \) is not 1, then there is a choice of sets \( A, B \) such that \( (n, m) \) is admissible with respect to \( (A, B) \). Note that this choice may not be unique. In fact, for each \( k \in \mathbb{N}^* \) there is a pair \( (n, m) \) such that one has at least \( k \) choices...
of sets \((A, B)\) for which \((n, m)\) is admissible. Let \(p_1, \ldots, p_k\) be \(k\) distinct prime numbers and consider \(n = p_1^2 \cdots p_k^2\) and \(m = p_1 \cdots p_k\) so that \(n = m^2\). For each \(1 \leq \ell \leq k\) let \(A_\ell = \{p_\ell\}\) and \(B_\ell = \emptyset\), then for each \(\ell\) the pair \((n, m)\) is admissible with respect to \((A_\ell, B_\ell)\).

We consider the \(G = \mathbb{Z}/n \times \mathbb{Z}/m\) actions on the torus, where \((n, m)\) is admissible with respect to some choice of \((A, B)\) as in Definition 1. Of course we have \(\mathbb{Z}/n \cong \mathbb{Z}/n_A \times \mathbb{Z}/n_B \times \mathbb{Z}/n_C\) and \(\mathbb{Z}/m \cong \mathbb{Z}/m_A \times \mathbb{Z}/m_B \times \mathbb{Z}/m_D\).

The two subgroups of \(G\) we shall consider are:

\[
H_1 = (\mathbb{Z}/n_A \times \mathbb{Z}/n_C) \times (\mathbb{Z}/m_B \times \mathbb{Z}/m_D)
\]

and

\[
H_2 = (\mathbb{Z}/(n_A/m_A) \times \mathbb{Z}/n_B \times \mathbb{Z}/n_C) \times (\mathbb{Z}/m_A \times (\mathbb{Z}/(m_B/n_B) \times \mathbb{Z}/m_D)
\]

which are obviously normal (since \(G\) is abelian) and of the same order:

\[
nm/(n_Bm_A) = n_Am_Bn_Cm_D \geq n_Am_B > 1,
\]

since \(A \cup B \neq \emptyset\). Clearly the two subgroups \(H_1\) and \(H_2\) depend on the choice of \((A, B)\).

\[\text{Figure 1: The set } \mathcal{L} \text{ of simple closed curves of } T, \text{ with 6 meridians and 4 longitudes, and the action of two subgroups } H_1 = \mathbb{Z}/3 \times \mathbb{Z}/4 \text{ and } H_2 = \mathbb{Z}/6 \times \mathbb{Z}/2 \text{ of } G = \mathbb{Z}/6 \times \mathbb{Z}/4. \text{ In this case, } A = \emptyset, B = \{2\}.\]

**Lemma 3.** The two subgroups \(H_1\) and \(H_2\) are not isomorphic but their quotients \(G/H_1\) and \(G/H_2\) are.

**Proof:** Since, according to Definition 1, \(n_A/m_A\) and \(m_B/n_B\) cannot be both equal to 1, there is a prime \(p \in A \cup B\) such that the Sylow \(p\)-subgroup of \(H_1\) is cyclic but not that of \(H_2\). Finally, we observe that \(G/H_1 \cong \mathbb{Z}/n_B \times \mathbb{Z}/m_A \cong \mathbb{Z}/m_A \times \mathbb{Z}/n_B \cong G/H_2\), that is, both quotients are cyclic of order \(n_Bm_A\), since \(A \cap B = \emptyset\). \(\square\)
2.3 Lifting diffeomorphisms on the different covers.

An easy Euler characteristic check shows that $X = S/G$ is a surface of genus 2 bounding a handlebody $H_X = \tilde{H}/G$. Similarly, one can verify that $H_i = \tilde{H}/H_i$ is a handlebody of genus $n_Bm_A + 1$.

We analyse now how the regular coverings $S_i \to X$ are built. Consider the following composition of group morphisms

$$\pi_1(X) \to \pi_1(H_X) \to H_1(H_X) \cong \mathbb{Z}^2$$

where the first map is induced by the inclusion of $X$ as the boundary of $H_X$. Note that $\pi_1(H_X)$ is a free group of rank 2 generated by the images $\mu$ and $\lambda$ of a meridian and a longitude of the original torus $T$. Of course, these two curves can be pushed onto the boundary $X$ of $H_X$. We can also assume that they have the same basepoint $x_0 \in X$. Let us denote by $[\mu]$ and $[\lambda]$ the classes of $\mu$ and $\lambda$ respectively in $H_1(H_X)$. There are two natural morphisms from $H_1(H_X) \cong \mathbb{Z}^2$ to $\mathbb{Z}/n_Bm_A \cong \mathbb{Z}/m_A \times \mathbb{Z}/n_B$: the first one maps $[\mu]$ to a generator of $\mathbb{Z}/m_A$ and $[\lambda]$ to a generator of $\mathbb{Z}/n_B$ while the second one exchanges the roles of the two elements and maps $[\mu]$ to a generator of $\mathbb{Z}/n_B$ and $[\lambda]$ to a generator of $\mathbb{Z}/m_A$.

The two coverings $S_i \to X$ are determined by the composition of these two group morphisms:

$$\pi_1(X) \to \pi_1(H_X) \to H_1(H_X) \cong \mathbb{Z}^2 \to \mathbb{Z}/n_Bm_A \cong \mathbb{Z}/m_A \times \mathbb{Z}/n_B$$

that is, the fundamental groups $\pi_1(S_i)$ correspond to the kernels of the two morphisms just constructed.

**Lemma 4.** The two coverings $S_i \to X$, $i = 1, 2$ are conjugate. More precisely there is a diffeomorphism $\tau$ of order 2 of $X$, inducing a well-defined element $\tau_\ast \in Aut(\pi_1(X, x_0))$ such that $\tau_\ast$ exchanges $\pi_1(S_1)$ and $\pi_1(S_2)$.

**Proof:** The diffeomorphism $\tau$ is the involution with two fixed points, $x_0$ and $y_0$ pictured in Figure 2. Note that $\tau$ fixes $x_0$ and $y_0$. The fact that $\tau$ defines an element of $Aut(\pi_1(X, x_0))$ (and not just $Out(\pi_1(X, x_0))$) follows from the fact that $\tau(x_0) = y_0$.

We are interested in diffeomorphisms $f$ of $X$ which commute with $\tau$ and fix both $x_0$ and $y_0$. We have the following easy fact.

**Lemma 5.** A diffeomorphism $f$ of $X$ commutes with $\tau$ and fixes both $x_0$ and $y_0$ if and only if it is the lift of a diffeomorphism of the torus fixing two points $\bar{x}_0$ and $\bar{y}_0$.

**Proof:** Observe that the orbifold quotient $X/\tau$ is a torus with two cone points of order 2. Clearly, any diffeomorphism $f$ that commutes with $\tau$ and fixes $x_0$ and $y_0$ induces a map of $X/\tau$ which fixes the two cone points. Vice-versa, given a diffeomorphism of the torus which fixes two points $\bar{x}_0$ and $\bar{y}_0$ we can lift it to $X$ once we chose an identification of the torus with $X/\tau$ such that $\bar{x}_0$ and $\bar{y}_0$ are mapped to the two cone points.

We are interested in diffeomorphisms of $X$ which commute with $\tau$ and lift to the covers $S_i \to X$, $i = 1, 2$, and $\bar{S} \to X$.  


Figure 2: The action of $\tau$ on $X$ and the quotient $X/\tau$.

**Lemma 6.** Let $f$ be a diffeomorphism of $X$ which commutes with $\tau$ and fixes $x_0$ and $y_0$. One can choose $k \in \mathbb{N}$ such that $f^k$ lifts to diffeomorphisms of $S_1$, $S_2$, and $\tilde{S}$ which fix pointwise the fibres of $x_0$.

**Proof:** The diffeomorphism $f$ fixes $x_0$ and so induces an automorphism $f_*$ of $\pi_1(X, x_0)$. Choose $x_1$, $x_2$ and $\tilde{x}$ points of $S_1$, $S_2$, and $\tilde{S}$ respectively which map to $x_0$. Since $\pi_1(X, x_0)$ is finitely generated, there is a finite number of subgroups of $\pi_1(X, x_0)$ with a given finite index. Since $\pi_1(S_1, x_1)$, $\pi_1(S_2, x_2)$, and $\pi_1(\tilde{S}, \tilde{x})$ have finite index in $\pi_1(X, x_0)$ then there is a power of $f_*$ which leaves $\pi_1(S_1, x_1)$, $\pi_1(S_2, x_2)$, and $\pi_1(\tilde{S}, \tilde{x})$ invariant. As a consequence, the corresponding power of $f$ lifts to $S_1$, $S_2$, and $\tilde{S}$. Since each lift acts by leaving the fibre of $x_0$ invariant, up to possibly passing to a different power, we can assume that the lifts fix pointwise the fibre of $x_0$. Note moreover that for this to happen it suffices that the fibre of $x_0$ in the covering $\tilde{S} \to X$ is pointwise fixed.

**Remark 2.** The argument of the above lemma shows that one can choose a power of $f$ which lifts, as in the statement of the lemma, to any covering of $X$ corresponding to a subgroup $K$ such that $\pi_1(\tilde{S}, \tilde{x}) \subset K \subset \pi_1(X, x_0)$. Recall that each such $K$ is normal in $\pi_1(X, x_0)$, since $G \cong \pi_1(X, x_0)/\pi_1(\tilde{S}, \tilde{x})$ is abelian.

Let $f$ be a diffeomorphism of $X$ commuting with $\tau$ and fixing $x_0$ and $y_0$, and let $\varphi$ be a power of $f$ satisfying the conclusions of Lemma 6. Denote by $\tilde{\varphi}$ the lift of $\varphi$ to $\tilde{S}$ and by $\varphi_1$ and $\varphi_2$ its projections to $S_1$ and $S_2$ respectively. Note that in principle the lift $\tilde{\varphi}$ of $\varphi$ is not unique: two possible lifts differ by composition with a deck transformation. In this case, however, since we require that $\tilde{\varphi}$ fixes pointwise the fibre of $x_0$ while the group $G$ of deck transformations acts freely on it, we can conclude that our choice of $\tilde{\varphi}$ is unique.

**Proposition 7.** The maps $\varphi_1$ and $\varphi_2$ are conjugate.
Proof: By construction, the involution $\tau$ of $X$ lifts to a map $g$ between $S_1$ and $S_2$ conjugating a lift of $\varphi$ on $S_1$ to a lift of $\varphi$ on $S_2$. Since two different lifts differ by composition with a deck transformation, reasoning as in the remark above we see that $g$ conjugates $\varphi_1$ to $\varphi_2$ since both $\varphi_1$ and $\varphi_2$ are the only lifts of $\varphi$ that fix every point in the fibre of $x_0$. \qed

3 Proofs of Theorem 1 and Corollary 2, and some remarks on commensurability

In this section we use the construction detailed in Section 2 to prove our main result. We will then discuss some consequences for 3-dimensional manifolds.

3.1 Proof of Theorem 1

By Proposition 7, it is sufficient to show that a pseudo-Anosov $f : X \to X$ which fixes $x_0$ and $y_0$, and commutes with $\tau$, does exist. According to Lemma 5, any such $f$ is the lift of a diffeomorphism $\bar{f}$ of the torus that fixes two points $\bar{x}_0$ and $\bar{y}_0$. Let $A$ be an Anosov diffeomorphism of the torus. Since $A$ has infinitely many periodic orbits (see [Si] for instance), we can choose a power $\bar{f}$ of $A$ which fixes two points on the torus. Let $f$ denote the lift of $\bar{f}$ to $X$. We need to show that $f$ is pseudo-Anosov, that is we need to exclude the possibilities that $f$ is finite order or reducible. The following argument is standard (see [FLP] exposé 13). Clearly $f$ cannot be periodic since its quotient $\bar{f}$ has infinite order. Since, by assumption, $\bar{f}$ is an Anosov map, it admits a pair of invariant foliations $(\mathcal{F}^+, \mathcal{F}^-)$. These lift to invariant foliations $(\tilde{\mathcal{F}}^+, \tilde{\mathcal{F}}^-)$ for $f$. Note also that $x_0$ and $y_0$, which are lifts of the two fixed points of $\bar{f}$, are singular points for the foliations $(\tilde{\mathcal{F}}^+, \tilde{\mathcal{F}}^-)$. If $f$ were reducible then at least one leaf $\tilde{\gamma}$ of $\tilde{\mathcal{F}}^+$ or of $\tilde{\mathcal{F}}^-$ would be fixed by $f$ and connect one singularity between $x_0$ or $y_0$ either to itself or to the other one. Such a leaf would project to a leaf of either $\mathcal{F}^+$ or $\mathcal{F}^-$ satisfying the analogous property. This however cannot happen for an Anosov map.

This shows that any $f$ which is the lift of an Anosov map is a pseudo-Anosov map. Any nonzero power $\varphi$ of a pseudo-Anosov map $f$ is again pseudo-Anosov, and, reasoning as above, so are its lifts $\varphi_1$, $\varphi_2$, and $\tilde{\varphi}$.

It remains to prove that infinitely many choices of $\varphi_i$’s do not share common powers. This follows readily from the fact that there exist infinitely many primitive Anosov maps on the torus. \qed

3.2 Hyperbolic fibred 3-manifolds

The aim of this part is to prove Corollary 2 and compare the examples constructed here to those given in [RS].

For each choice of conjugate pseudo-Anosov maps $\varphi_1$ and $\varphi_2$ and common lift $\tilde{\varphi}$ as in Theorem 1, we can consider the associated mapping tori $M_1$, $M_2$, and $\tilde{M}$ respectively. The 3-manifolds thus obtained are hyperbolic according to Thurston’s hyperbolization theorem for manifolds that fibre over the circle (see [E]). By construction, the mapping tori $M_1$ of $\varphi_1$ and $M_2$ of $\varphi_2$ are homeomorphic, i.e. $M_1 = M_2 = \tilde{M}$ since $\varphi_1$ and $\varphi_2$ are conjugate. Moreover, by construction, the mapping torus $\tilde{M}$ of $\tilde{\varphi}$ covers $M$ in two non-equivalent ways.
According to Remarks 1 and 2, for each $k$ one can find pseudo-Anosov maps $\tilde{\varphi}$ which cover at least $k$ pairs of conjugate pseudo-Anosov maps in the fashion described in Theorem 1. This proves the last part of the corollary.

It remains to show that there are infinitely many pairs of hyperbolic manifolds $(\tilde{M}, M)$ such that the first covers the second in two non-equivalent ways. Note that the fact that Theorem 1 provides infinitely many choices is not sufficient to conclude, since a hyperbolic manifold can admit infinitely many non-equivalent fibrations (see [Th]).

The existence of infinitely manifolds follows from the following observation. Up to isomorphism, there are infinitely many groups $G$ to which our construction applies. Each of these groups acts by hyperbolic isometries on some closed $\tilde{M}$. Since the group of isometries of hyperbolic 3-manifold is finite, we can conclude that there are infinitely many pairs of manifolds $(\tilde{M}, M)$ up to hyperbolic isometry and hence, because of Mostow’s rigidity theorem [Mo], up to homeomorphism.

Another way to reason is the following. Given $\varphi_1$, $\varphi_2$, and $\tilde{\varphi}$ as above we can consider the mapping tori $M_1^{(k)}$, $M_2^{(k)}$, and $\tilde{M}^{(k)}$ of $\varphi_1$, $\varphi_2$, and $\tilde{\varphi}$ respectively, for $k \geq 1$. All the manifolds thus obtained are commensurable, and volume considerations show that the manifolds $\tilde{M}^{(k)}$ are pairwise non-homeomorphic. Indeed, given a pseudo-Anosov $f$ of $X$, for any choice of $G$ and of $\varphi_1$, $\varphi_2$, and $\tilde{\varphi}$, all the mapping tori obtained are commensurable to the mapping torus of $f$. More precisely, all these manifolds are fibred commensurable according to the definition of [CSW], that is, they admit common fibred covers such that the covering maps preserve the fixed fibrations.

This latter observation shows that we can construct infinitely many distinct pairs $(\tilde{M}, M)$ such that the first covers the second in two non-equivalent ways. Unfortunately, we do not know whether the manifolds we construct belong to infinitely many distinct commensurability classes as well. The result in [RS] shows that it is possible to find infinitely many pairs of manifolds $(\tilde{M}, M)$ such that the first covers the second in two non-equivalent ways and the manifolds $\tilde{M}$ are pairwise non-commensurable.

References

[CSW] D. Calegari, H. Sun, and S. Wang, On fibered commensurability, Pacific J. Math. 250, (2011), 287-317.

[FLP] A. Fathi, F. Laudenbach, and V. Poenaru, Travaux de Thurston sur les surfaces, Astérisque 66-67, 1979.

[Mo] G. D. Mostow, Strong rigidity of locally symmetric spaces, Annals of Math. Studies 78, 1973.

[O] J. P. Otal, Le théorème d’hyperbolisation pour les variétés fibrées de dimension 3, Astérisque 235, 1996.

[RS] A. Reid and A. Salgueiro, Some remarks on group actions on hyperbolic 3-manifolds, preprint.

[Si] Y. G. Sinai, Markov partitions and $C$-diffeomorphisms, Funct. Anal. Appl. 2, (1968), 64-89.

[Th] W. P. Thurston, A norm for the homology of 3-manifolds, Mem. Amer. Math. Soc. 339, (1986), 99-130.

Aix-Marseille Université, CNRS, Centrale Marseille, I2M, UMR 7373, 13453 Marseille, France
jerome.los@univ-amu.fr
