Stochastic Loss Aversion for Random Medium Access

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Abstract

We consider a slotted-ALOHA LAN with loss-averse, noncooperative greedy users. To avoid non-Pareto equilibria, particularly deadlock, we assume probabilistic loss-averse behavior. This behavior is modeled as a modulated white noise term, in addition to the greedy term, creating a diffusion process modeling the game. We observe that when player’s modulate with their throughput, a more efficient exploration of play-space (by Gibbs sampling) results, and so finding a Pareto equilibrium is more likely over a given interval of time.

I. INTRODUCTION

The “by rule” window flow control mechanisms of, e.g., TCP and CSMA, have elements of both proactive and reactive communal congestion control suitable for distributed/information-limited high-speed networking scenarios. Over the past ten years, game theoretic models for medium access and flow control have been extensively explored in order to consider the effects of even a single end-user/player who greedily departs from such prescribed/standard behaviors [1], [6], [9], [13]–[16], [22]–[24], [27]. Greedy end-users may have a dramatic effect on the overall “fairness” of the communication network under consideration. So, if even one end-user acts in a greedy way, it may be prudent for all of them to do so. However, even end-users with a noncooperative disposition may temporarily not practice greedy behavior in order to escape from sub-optimal (non-Pareto) Nash equilibria. In more general game theoretic contexts, the reluctance of an end-user to act in a non-greedy fashion is called loss aversion [7].

In this note, we focus on simple slotted-ALOHA MAC for a LAN. We begin with a noncooperative model of end-user behavior. Despite the presence of a stable interior Nash equilibrium, this system was shown in [13], [14] to have a large domain of attraction to deadlock where all players’ transmission probability is one and so obviously all players’ throughput is zero (here assuming feasible demands and throughput based costs). To avoid non-Pareto Nash equilibria, particularly those involving zero throughput for some or all users, we assume that end-users will probabilistically engage in non-greedy behavior. That is, a stochastic model of loss aversion, a behavior whose aim is long term communal betterment.

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We may be able to model a play that reduces net-utility using a single “temperature” parameter $T$ in the manner of simulated annealing (e.g., [12]); i.e., plays that increase net utility are always accepted and plays that reduce net utility are (sometimes) accepted with probability decreasing in $T$, so the players are (collectively) less loss averse with larger $T$. Though our model of probabilistic loss aversion is related that of simulated annealing by diffusions [10], [28], even with a free meta-parameter ($\eta$ or $\eta w$ below) possibly interpretable as temperature, our modeling aim is not centralized annealing (temperature cooling) rather decentralized exploration of play-space by noncooperative users.

We herein do not model how the end-users will keep track of the best (Pareto) equilibria previously played/discovered. Because the global extrema of the global objective functions (Gibbs exponents) we derive do not necessarily correspond to Pareto equilibria, we do not advocate collective slow “cooling” (annealing) of the equivalent temperature parameters. Also, we do not model how end-user throughput demands may be time-varying, a scenario which would motivate the “continual search” aspect of the following framework.

The following stochastic approach to distributed play-space search is also related to “aspiration” of repeated games [3], [8], [18], where a play resulting in suboptimal utility may be accepted when the utility is less than a threshold, say according to a “mutation” probability [17], [25]. This type of “bounded rational” behavior been proposed to find Pareto equilibria, in particular for distributed settings where players act with limited information [25]. Clearly, given a global objective $L$ whose global maxima correspond to Pareto equilibria, these ideas are similar to the use of simulated annealing to find the global maxima of $L$ while avoiding suboptimal local maxima.

This paper is organized as follows. In Section II we formulate the basic ALOHA noncooperative game under consideration. Our stochastic framework (a diffusion) for loss aversion is given in Section III; for two different modulating terms of the white-noise process, the invariant distribution in the collective play-space is derived. A two-player numerical example is used to illustrate the performance of these two approaches in Section IV. We conclude in Section V with a discussion of future work.

II. A DISTRIBUTED SLOTTED-ALOHA GAME FOR LAN MAC

Consider an idealized ALOHA LAN where each user/player $i \in \{1, 2, ..., n\}$ has (potentially different) transmission probability $v_i$. For the collective “play” $v = (v_1, v_2, ..., v_n)$, the net utility of player $i$ is

$$V_i(v) = U_i(\theta_i(v)) - M\theta_i(v),$$

(1)

where the strictly convex and increasing utility $U_i$ of steady-state throughput

$$\theta_i := v_i \prod_{j \neq i} (1 - v_j)$$

The players could, e.g., alternate between (loss averse) greedy behavior to discover Nash equilibrium points, and the play dynamics modeled herein for breadth of search (to escape non-Pareto equilibria).

We herein do not consider physical layer channel phenomena such as shadowing and fading as in, e.g., [16], [24].
is such that $U_i(0) = 0$, and the throughput-based price is $M$. So, the throughput-demand of the $i$th player is

$$y_i := (U')^{-1}(M).$$

This is a quasi-stationary game wherein future action is based on the outcome of the current collective play $\mathbf{v}$ observed in steady-state [5].

The corresponding continuous Jacobi iteration of the better response dynamics is [13], [14], [26]: for all $i$

$$\frac{d}{dt}v_i = \prod_{j \neq i} (1 - v_j) - v_i =: -E_i(\mathbf{v}),$$

(2)

cf. (6). Note that we define $-E_i$, instead of $E_i$, to be consistent with the notation of [28], which seeks to minimize a global objective, though we want to maximize such objectives in the following.

Such dynamics generally exhibit multiple Nash equilibria, including non-Pareto equilibria with significant domains of attraction. Our ALOHA context has a stable deadlock equilibrium point where all players always transmit, i.e., $\mathbf{v} = \mathbf{1} := (1, 1, ..., 1)$ [13], [14].

III. A DIFFUSION MODEL OF LOSS AVERSION

Generally in the following, we consider differently loss-averse players. Both examples considered are arguably distributed (information limited) games wherein every player’s choice of transmission probability is based on information knowable to them only through their channel observations, so that consultation among users is not required. In particular, players are not directly aware of each other’s demands ($y$).

A. Model overview

We now model stochastic perturbation of the Jacobi dynamics (2), allowing for suboptimal plays despite loss aversion, together with a sigmoid mapping $g$ to ensure plays (transmission probabilities) $\mathbf{v}$ remain in a feasible hyperrectangle $D \subset [0, 1]^n$ (i.e., the feasible play-space for $\mathbf{v}$): for all $i$,

$$du_i = -E_i(\mathbf{v})dt + \sigma_i(v_i)dW_i$$

(3)

$$v_i = g_i(u_i)$$

(4)

where $W_i$ are independent standard Brownian motions. An example sigmoid is

$$g(u) := \gamma(\tanh(u/w) + \delta),$$

(5)

where $1 \leq \delta < 2$ and $0 < \gamma \leq 1/(1 + \delta)$. Thus, $\inf_u g(u) = \inf v = \gamma(-1 + \delta) \geq 0$ and $\sup_u g(u) = \sup v = \gamma(1 + \delta) \leq 1$. Again, to escape from the domains of attraction of non-Pareto equilibria, the deterministic Jacobi dynamics (i.e., $-E_i(\mathbf{v})dt$ in (3)) have been perturbed by white noise ($dW_i$) here modulated by a diffusion term of the form:

$$\sigma_i(v_i) = \sqrt{\frac{2h_i(v)}{f_i(v)}}.$$
where
\[ f_i(v_i) := g_i'(g_i^{-1}(v_i)). \]
For the example sigmoid (5),
\[ f(v) = \frac{\gamma}{w} \left( 1 - \left( \frac{v}{\gamma} - \delta \right)^2 \right). \]
In the following, we will consider different functions \( h_i \) leading to Gibbs invariant distributions for \( v \).

Note that the discrete-time \((k)\) version of this game model would be
\[
\begin{align*}
    u_i(k+1) - u(k) &= -E_i(v(k))\varepsilon + \sigma_i(v(k))N_i(k) \\
    v_i(k+1) &= g_i(u_i(k+1)),
\end{align*}
\]
(6)
where the \( N_i(k) \) are all i.i.d. normal \( N(0, \varepsilon) \) random variables.

The system just described is a variation of E. Wong’s diffusion machine [28], the difference being the introduction of the term \( h \) instead of a temperature meta-parameter \( T \). Also, the diffusion function \( \sigma_i \) is player-\( i \) dependent at least through \( h_i \). Finally, under the slotted-ALOHA dynamics, there is no function \( E(v) \) such that \( \partial E/\partial v = E_i \), so we will select the diffusion factors \( h_i \) to achieve a tractable Gibbs stationary distribution of \( v \), and interpret them in terms of player loss aversion.

Note that in the diffusion machine, a common temperature parameter \( T \) may be slowly reduced to zero to find the minimum of a global potential function (the exponent of the Gibbs stationary distribution of \( v \)) [20], [21], in the manner of simulated annealing. Again, the effective temperature parameter here (\( \eta \) or \( \eta w \)) will be constant.

**B. Example diffusion term** \( h_i \) **decreasing in** \( v_i \)

In this subsection, we analyze the model when, for all \( i \),
\[
    h_i(v_i) := \eta y_i(1 - v_i)^2.
\]
(7)
with \( \eta > 0 \) a free meta-parameter (assumed common to all players). So, a greedier player \( i \) (larger \( y_i \)) will generally tend to be less loss averse (larger \( h_i \)), except when their current retransmission play \( v_i \) is large.

**Theorem 3.1:** The stationary probability density function of \( v \in D \subset [0, 1]^n \), defined by (4) and (3), is
\[
p(v) = \frac{1}{Z} \exp \left( \frac{\Lambda(v)}{\eta Y} - \log H(v) \right),
\]
(8)
where: the normalizing term

\[ Z := \int_D \exp \left( \frac{\Lambda(v)}{\eta Y} - \log H(v) \right) \, dv, \]

\[ D := \prod_{i=1}^{n} (\gamma_i(-1 + \delta_i), \gamma_i(1 + \delta_i)) \]

\[ \Lambda(v) := \prod_{i=1}^{n} \frac{y_i}{1 - v_i} - \sum_{j=1}^{N} \left( \frac{v_j}{1 - v_j} + \log(1 - v_j) \right) \prod_{i \neq j} y_i \]

\[ H(v) := \prod_{j=1}^{n} (1 - v_i)^2, \]

\[ Y := \prod_{j=1}^{n} y_j. \]

**Remark:** \( \Lambda \) is a Lyapunov function of the deterministic (\( \sigma_i \equiv 0 \) for all \( i \)) Jacobi iteration [13], [14].

**Proof:** Applying Ito's lemma [19], [28] to (3) and (4) gives

\[ dv_i = g'_i(u_i)du_i + \frac{1}{2} g''_i(u_i)\sigma_i^2(v)dt \]

\[ = [-f_i(v_i)E_i(v) + \frac{1}{2} g''_i(g_{i}^{-1}(v_i))\sigma_i^2(v)]dt \]

\[ + f_i(v_i)\sigma_i(v)dW_i, \]

where the derivative operator \( z' := \frac{d}{dv_i} z(v_i) \) and we have just substituted (3) for the second equality. From the Fokker-Planck (Kolmogorov forward) equation for this diffusion [19], [28], we get the following equation for the time-invariant (stationary) distribution \( p \) of \( v_i \): for all \( i \),

\[ 0 = \frac{1}{2} \partial_i (f_i^2 \sigma_i^2 p) - [-f_i E_i + \frac{1}{2} (g''_i \circ g_{i}^{-1})\sigma_i^2 p], \]

where the operator \( \partial_i := \frac{\partial}{\partial v_i} \).

Now note that

\[ f_i^2(v_i)\sigma_i^2(v) = 2h_i(v_i)f_i(v_i) \quad \text{and} \]

\[ g''_i(g_{i}^{-1}(v_i))\sigma_i^2(v) = h_i(v_i)g''_i(g_{i}^{-1}(v_i))/f_i(v_i) \]

\[ = h_i(v_i)f'_i(v_i). \]

So, the previous display reduces to

\[ 0 = \partial_i(h_i f_i p) - (-E_i f_i + h_i f'_i)p \]

\[ = (h_i \partial_i p + h'_i p + E_i p) f_i, \]
where the second equality is due to cancellation of the $h_i f'_i$ terms. For all $i$, since $f_i > 0$,

$$\frac{\partial_i p(v)}{p(v)} = \partial_i \log p(v) = -\frac{E_i(v)}{h_i(v_i)} \frac{h'_i(v_i)}{h_i(v_i)} = \frac{1}{\eta Y} \partial_i \Lambda(v) + \frac{2}{1 - v_i}. \quad (9)$$

Finally, (8) follows by direct integration. \(\square\)

Unfortunately, the exponent of $p$ under (7),

$$\hat{\Lambda}(v) := \Lambda(v) - \eta Y \log H(v),$$

and both its component terms $\Lambda$ and $-\log H$, remain maximal in the deadlock region near 1.

C. Example diffusion term $h_i$ increasing in $v_i$

The following alternative diffusion term $h_i$ is an example which is instead increasing in $v_i$, but decreasing in the channel idle time from player $i$’s point-of-view \[2\], \[11\].

$$h_i(v) := \frac{\eta v_i}{\prod_{j \neq i}(1 - v_j)}. \quad (10)$$

That a user would be less loss averse (higher $h$) when the channel was perceived to be more idle may be a reflection of a “dynamic” altruism \[2\] (i.e., a player is more courteous as s/he perceives that others are). The particular form of (10) also leads to another tractable Gibbs distribution for $v$.

**Theorem 3.2:** Using (10), the stationary probability density function of the diffusion $v$ on $[0, 2\gamma]^n$ is

$$p(v) = \frac{1}{W} \exp(\Delta(v)) \quad (11)$$

where

$$\Delta(v) = \sum_{i=1}^n \left( \frac{y_i}{\eta} - 1 \right) \log v_i + \frac{1}{\eta} \prod_{i=1}^n (1 - v_i), \quad (12)$$

and $W$ is the normalizing term.

**Proof:** Following the proof of Theorem 3.1, the invariant here satisfies also satisfies (9):

$$\partial_i \log p(v) = -\frac{E_i(v)}{h_i(v_i)} \frac{h'_i(v_i)}{h_i(v_i)} = \frac{y_i}{\eta v_i} - \frac{1}{\eta} \prod_{j \neq i}(1 - v_j) - \frac{1}{v_i}. \quad (10)$$

Substituting (10) gives:

$$\partial_i \log p(v) = \left( \frac{y_i}{\eta} - 1 \right) \frac{1}{v_i} - \frac{1}{\eta} \prod_{j \neq i}(1 - v_j).$$
So, we obtain (12) by direct integration.

Note that if \( \eta > \max_i y_i \), then \( \Delta \) is strictly decreasing in \( v_i \) for all \( i \), and so will be minimal in the deadlock region (unlike \( \hat{\Lambda} \)). So the stationary probability in the region of deadlock will be low. However, large \( \eta \) may result in the stationary probability close to 0 being very high. So, we see that the meta-parameter \( \eta \) (or \( \eta w \)) here plays a more significant role (though the parameters \( \delta \) and \( \gamma \) in \( g \) play a more significant role in the former objective \( \hat{\Lambda} \) owing to its global extrema at 1).

IV. NUMERICAL EXAMPLES

A. Using (7)

For an \( n = 2 \) player example with demands \( y = (8/15, 1/15) \) and \( \eta = 1 \), the two interior Nash equilibria are the locally stable (under deterministic dynamics) at \( v^*_a = (2/3, 1/5) \) and the (unstable) saddle point at \( v^*_b = (4/5, 1/3) \) (both with corresponding throughputs \( \theta = y \) [13], [14]. Again, \( 1 \) is a stable deadlock boundary equilibrium which is naturally to be avoided if possible as both players’ throughputs are zero there, \( \theta = 0 \). Under the deterministic dynamics of (2), the deadlock equilibrium \( 1 \) had a significant domain of attraction including a neighborhood of the saddle point \( v^*_b \).

The exponent of \( p, \hat{\Lambda} \), for this example is depicted in Figure 1. \( \hat{\Lambda} \) has a similar shape as that the Lyapunov function \( \Lambda \), but without interior local extrema or saddle points. The extreme mode at \( 1 \) is clearly evident.

![Gibbs distribution for n = 2 players with demands y = (8/15, 1/15) under (7)](image)

When we took \( \gamma_i, \delta_i \) in (5) so that \( 0.05 \leq v_i = g(u_i) \leq 0.85 \) for all players \( i \), the stationary probability of the
“good” region containing the two interior Nash equilibria is
\[ P(\bar{v} \in [0.65, 0.82] \times [0.18, 0.35]) \approx 0.18, \]  
(13)
as computed using (8). This probability does not appreciably improve by varying \( \eta \) from 1 (both \( \Lambda \) and \( \log H \) diverge at \( \frac{1}{2} \)), though it does dramatically decrease as \( \sup_u g(u) \uparrow 1 \), e.g., if we take \( \sup_u g(u) = 0.9 \) then (13) decreases to about \( 10^{-54} \) (essentially zero, of course) which is consistent with Figure 1. Also, the largest contribution of this probability is the region around the saddle point which is part of the deadlock domain of attraction of \( \frac{1}{2} \), the global maximum of \( \Lambda \).

To reiterate, the fundamental advantage of stochastic loss aversion is seen by comparing the large domain of attraction of the deadlock equilibrium of the deterministic dynamics [14], with positive probability of presence near the interior Nash equilibrium points where both players’ demands \( y \) are satisfied. This advantage is born out more clearly in the following example.

B. Using (10)

For the same two-player example with \( \eta = 4.5/15 = (y_1 + y_2)/2 \), and \( \sup_u g(u) = 0.9 \), the probability in (13) was 0.05, compared to essentially zero for the same parameter range for (7). The exponent of \( p, \Delta \), for this example is depicted in Figure 1. \( \Delta \) is dissimilar to the function \( \tilde{\Lambda} \) (or \( \Lambda \) or \( H \)) without significant modes in the range of \( \eta \) close to the demands \( y \), and a much smaller overall range of values (and likewise for the stationary density \( p \) of \( v \)). Though the performance under (10) was more sensitive to \( \eta \) (temperature) than under (7), using (10) clearly resulted in more effective searching of the play-space \( D \) and was far less sensitive to the parameters \( \delta, \gamma \) defining it.

V. Conclusions and Future Work

The diffusion term (10) was clearly more effective than (7) at exploring the play-space, and in so doing, was dramatically less sensitive to the choice of the parameters \( \delta \) and \( \gamma \) governing the range of the play-space \( D \).

In future work, we plan to explore other diffusion factors \( h \) (numerically if they do not lead to a Gibbs stationary distribution \( p \)) with a goal to reduce the stationary probability that \( v \) occupies the boundary regions. Also, we will consider a model with power based costs, \( i.e., \mathcal{M}v \) instead of \( \mathcal{M}\theta \) in the net utility (1). Finally, we will study the effects of asynchronous and/or multirate play among the users [2], [4], [15].

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Fig. 2. The Gibbs distribution for $n = 2$ players with demands $y = (8/15, 1/15)$ under (10).
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