Phenomenology with Supersymmetric Flipped SU(6)

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Abstract

The supersymmetric flipped $SU(6) \times U(1)$ gauge symmetry can arise through compactification of the ten dimensional $E_8 \times E_8$ superstring theory. We show how realistic phenomenology can emerge from this theory by supplementing it with the symmetry $R \times U(1)$, where $R$ denotes a discrete ‘$R$’-symmetry. The well-known doublet-triplet splitting problem is resolved to ‘all orders’ via the pseudo-Goldstone mechanism, and the GUT scale arises from an interplay of the Planck and supersymmetry breaking scales.

The symmetry $R \times U(1)$ is also important for understanding the fermion mass hierarchies as well as the magnitudes of the CKM matrix elements. Furthermore, the well known MSSM parameter $\tan \beta$ is estimated to be of order unity, while the proton lifetime $(\tau_p \sim 10^2 \tau_{psu(5)})$ is consistent with observations. Depending on some parameters, $p \to K\mu^+$ can be the dominant decay mode.

Finally, the observed solar and atmospheric neutrino ‘anomalies’ require us to introduce a ‘sterile’ neutrino state. Remarkably, the $R \times U(1)$ symmetry protects it from becoming heavy, so that maximal angle $\nu_\mu$ oscillations into a sterile state can explain the atmospheric anomaly, while the solar neutrino puzzle is resolved via the small angle $\nu_e - \nu_\tau$ MSW oscillations. The existence of some ($\sim 15\text{-}20\%$ of critical energy density) neutrino hot dark matter is also predicted.

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1 Introduction

It is a curious fact that the most well known grand unified theories (GUTs) \( SU(5) \) and \( SO(10) \) do not readily arise within the framework of the simplest superstring theories. On the contrary, compactification of the ten dimensional \( E_8 \times E_8 \) heterotic superstring theory say on a Calabi-Yau manifold leads one to a variety of subgroups of \( E_6 \), such as \( SU(3)^3 \) and \( SU(6) \times U(1) \) (or flipped \( SU(6) \)). The group \( SU(3)^3 \) as a grand unified symmetry has attracted a certain amount of attention, but the flipped \( SU(6) \) case has so far been more or less ignored. In this paper we hope to remedy this situation by discussing how a realistic ‘low energy’ phenomenology can emerge from flipped \( SU(6) \).

The fact that \( SU(5) \) and \( SO(10) \) do not readily appear from superstrings may be a blessing in disguise. Consider, for instance, the doublet-triplet (DT) splitting problem. A number of mechanisms for resolving this thorny problem have been proposed. These include the missing partner and the missing VEV mechanisms. However, their implementation results in a grand unified framework which is far from ‘simple’. A much more attractive possibility of realizing a pair of light electroweak doublets is provided by the pseudo-Goldstone mechanism. Here the light doublets emerge as the pseudo-Goldstone modes from an ‘accidental’ and larger global symmetry of the Higgs superpotential. It turns out that this idea is hard (if not impossible) to realize in \( SU(5) \) and \( SO(10) \), but is readily implemented in GUTs such as \( SU(6) \) and \( SU(3)^3 \).

In this paper we will see that the pseudo-Goldstone mechanism for resolving the DT splitting problem can be neatly realized within the framework of flipped \( SU(6) \). Moreover, it also becomes possible to understand how the GUT scale can emerge from an interplay of the Planck and supersymmetry breaking scales. We also study fermion masses and mixings in this scheme. It turns out that the well known \( SU(5) \) relation \( m_b = m_\tau \) holds in the flipped \( SU(6) \) scheme presented here. An important role is played by the symmetry \( R \times U(1) \) that we impose in addition. This symmetry, among other things, helps implement the pseudo-Goldstone mechanism and distinguishes the families so that the fermion masses and mixings (especially the observed hierarchies) can be explained.

The flipped \( SU(6) \) scheme has a number of testable predictions. The well-known MSSM parameter \( \tan \beta \) is estimated to be of order unity. One expects the dominant proton decay mode to be \( p \to K^0 \mu^+ \), with a rate that is suppressed relative to the dominant \( SU(5) \) mode \( (p \to K^+ \nu_\mu) \) by about two orders of magnitude. It is worth noting that the symmetry \( R \times U(1) \) plays an important role in the suppression of all dimension five Planck scale induced operators.

Finally, the neutrino sector of flipped \( SU(6) \) turns out to be quite interesting and unique. It turns out that in order to explain the recent Superkamiokande results on atmospheric neutrinos as well as the solar neutrino puzzle, one is led to introduce one sterile neutrino state which is kept ‘light’ thanks to the presence of the \( R \times U(1) \) symmetry!
One finds that the atmospheric neutrino anomaly is explained via $\nu_\mu -\nu_s$ oscillations with maximal mixing, while the resolution of the solar neutrino puzzle relies on the small angle MSW oscillations of $\nu_e$ into $\nu_\tau$. It is worth emphasizing that this scheme implies the existence of some hot dark matter ($\sim$ 15-20% of critical energy density).

The paper is organized as follows: In section 2 we describe the salient features of flipped $SU(6)$, the symmetry breaking pattern, and details of how the pseudo-Goldstone mechanism is realized in this scheme. Section 3 is devoted to understanding the fermion masses and mixings, especially the hierarchies. In section 4 we discuss proton decay including suppression of Planck scale induced dimension five operators. The discussion about neutrinos is contained in section 5 and the conclusions are presented in section 6.

2 Flipped $SU(6) \times U(1)$ Model

The flipped $SU(6)$ models are perhaps best motivated from the compactification of the ten dimensional heterotic $E_8 \times E_8$ superstring theory [3], [11]-[13], on a suitable Calabi-Yau manifold $K$. The compactification process leads to a four dimensional theory with $E_6$ gauge symmetry and $N = 1$ supersymmetry, a certain number of left-handed superfields belonging to the 27 and $\overline{27}$ representations of $E_6$, and a group of discrete symmetries (the isometries of $K$) [14]. With a non-simply connected $K$ it is possible induce the breaking of $E_6$ with Wilson loops to a subgroup $H$. It turns out that the simplest constructions lead to $H = SU(3)^3$ or $H = SU(6) \times U(1)$ [2].

It is perhaps fair to state that so far there does not exist a single example of a string derived four dimensional theory which provides a satisfactory explanation of the most important issues in phenomenology. These include the gauge hierarchy problem, proton stability, fermion mass hierarchies and mixings, etc. A route chosen by many is the so-called ‘string inspired’ approach, in which the choice of the underlying gauge symmetry is dictated by some string theory, which is then supplemented by additional symmetries so that a realistic ‘low energy’ scenario can be realized, hopefully with some predictions that can be experimentally tested. Our approach here follows this philosophy and is similar to the one pursued in the $SU(3)^3$ case [1, 2]. For earlier works on flipped $SU(6)$ see refs. [4], [13], while $SU(6) \times SU(2)_{L,R}$ has been discussed in ref. [14].

2.1 Pseudogoldstone Mechanism and GUT Scale in Flipped $SU(6)$

Under the $SU(6) \times U(1)$ gauge symmetry, the chiral 27-plet of $E_6$ transforms as $15_0 + 6_1 + 6_{-1}$, where the subscripts refer to the $U(1)$ charge. More explicitly,
\[ 15_0 = (q, d^c, \nu^c, g, L)_0, \]
\[ \bar{6}_1 = (u^c, l, N)_1, \]
\[ \bar{6}_{-1} = (\bar{g}, \bar{L}, e^-)_{-1}. \]

(1)

Decomposition under the $SU(5) \times U(1)'$ yields:

\[ 15_0 = 10_1 + 5_{-2}, \quad \bar{6}_1 = 5_{-3} + 1_0, \]
\[ \bar{6}_{-1} = 5_2 + 1_5, \]

(2)

which is just the chiral content of flipped $SU(5)$, supplemented by the singlet $1_0$ and the pair $5_2 + 5_{-2}$ of vector states. We will focus in this section on the Higgs sector of the theory, the implementation of the pseudo-Goldstone mechanism through the introduction of an additional symmetry $R \times U(1)$, and also show how the GUT scale can arise from an interplay of $M_P$ and $m_{3/2}$, the two basic scales in the theory.

For the breaking of $SU(6) \times U(1)$ to $G_{SM}$ ($\equiv SU(3)_C \times SU(2)_W \times U(1)_Y$) it is enough to introduce the following Higgs supermultiplets:

\[ \Psi + \bar{\Psi} \]
\[ \bar{H} + H \]

(3)

The $\Psi + \bar{\Psi}$ and $\bar{H} + H$ contain the same fragments as supermultiplets of the chiral matter of $15_0$ and $\bar{6}_1$ (see (1)). The VEVs of the fragments $\bar{\Psi}$ and $\nu^c_{\bar{\Psi}}$ from $\bar{\Psi}$ and $\Psi$ break $SU(6) \times U(1)$ to $SU(4) \times SU(2)_W \times U(1)_1$, while the VEVs of the fragments $N_H$, $\bar{N}_\bar{H}$ from $H$, $\bar{H}$ break $SU(4) \times SU(2)_W \times U(1)_1$ down to $G_{SM}$. The VEVs in the group space have the following directions:

\[ \langle \Psi_{mn} \rangle = \langle \bar{\Psi}_{mn} \rangle = \frac{V}{\sqrt{2}} (\delta_{4m} \delta_{5n} - \delta_{4n} \delta_{5m}) , \]
\[ \langle H_m \rangle = \langle \bar{H}_m \rangle = v \delta_{m6} , \]

(4)

where $m, n$ are $SU(6)$ indices, the indices 4, 5 correspond to the $SU(2)_W$ group, and the index 6 is a broken degree of freedom of $SU(6)$.

Due to the $SU(6) \times U(1)$, at the renormalizable level there are no couplings between the $\Psi + \bar{\Psi}$ and $\bar{H} + H$ superfields. Thus, the renormalizable superpotential has an $SU(6)^2 \times U(1)$ global symmetry\(^\text{3}\). The existence of this global symmetry leads to the possibility of the realization of the pseudo-Goldstone mechanism. Let us assume that the scalar

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\(^\text{3}\)The factor $U(1)$ appears only in the first power because the $\bar{\Psi}, \Psi$ superfields do not carry the $U(1)$ charges.
part of the superpotential has $SU(6)^2 \times U(1)$ global symmetry up to some desired level of the nonrenormalizable terms. Without considering the detailed form of the scalar superpotential and through simple counting of the numbers of broken generators of the local $SU(6) \times U(1)$ group and Goldstone modes which correspond to the breakdown of the $SU(6)^2 \times U(1)$ global symmetry of the superpotential, one can easily make sure that one $SU(2)_W$ doublet-antidoublet pair emerge as a pseudo-Goldstone mode and its lightness is guaranteed by SUSY and Goldstone theorem.

As far as the decoupled and unphysical states are concerned the necessary terms which must be included in the superpotential are:

$$W_1 = \frac{\lambda_1}{12\sqrt{2}} \Psi^3 + \frac{\lambda_2}{12\sqrt{2}} \overline{\Psi}^3.$$  (5)

Along the directions (4) this superpotential is flat and the values of $V$ and $v$ are not fixed. Substituting in (4) the VEVs of $\Psi + \overline{\Psi}$ superfields the mass terms will have the form:

$$W_m = \lambda_1 V d_\Psi^c \cdot g_\Psi + \lambda_2 V \overline{d}_\Psi^c \cdot \overline{g}_\Psi.$$  (6)

So, the triplets (antitriplets) are decoupled after symmetry breaking. The states $\overline{q}_\Psi + q_\Psi$ and $u_\Psi^c + \overline{u}_\Psi^c$ are the Goldstone modes. In addition, one superposition of doublets (antidoublets) $L_\Psi$ and $l_H$ ($\overline{L}_\Psi$ and $\overline{l}_H$) is genuine Goldstone:

$$h_d^G = \frac{vh_\Psi + \sqrt{2V}h_\Psi^H}{\sqrt{v^2 + 2V^2}}, \quad h_u^G = \frac{vh_\Psi + \sqrt{2V}h_H}{\sqrt{v^2 + 2V^2}},$$  (7)

The massless pseudo-Goldstone states are given by:

$$h_d = \frac{vh_\Psi - \sqrt{2V}h_\Psi^H}{\sqrt{v^2 + 2V^2}}, \quad h_u = \frac{vh_\Psi - \sqrt{2V}h_H}{\sqrt{v^2 + 2V^2}}.$$  (8)

In order to guarantee the DT hierarchy we have to exclude the dangerous higher order mixing terms between the $\Psi$ ($\overline{\Psi}$) and $H$ ($\overline{H}$) fields which do not respect the global $SU(6)^2 \times U(1)$ symmetry and spoil the hierarchy. This is readily achieved by introducing suitable discrete or continuous symmetries. Our task then is to prescribe some transformation properties to the scalar supermultiplets in such a way as to obtain ‘all order’ hierarchy while preserving the terms of (4). In order to suitably accommodate the ‘matter’ sector (described in the next section), we introduce two additional singlet states $X$ and $Z$ and also $R(Z) \times U(1)$ symmetry ($R(Z)$ is a discrete $R$ symmetry) under which the superfields transform as:

$$\phi_i \rightarrow e^{iQ_e^i} \phi_i,$$  (9)

\footnote{Discussions of these and other relevant issues can be found in the original works \cite{7} and in ref. \cite{8} as well.}
Table 1: $\mathcal{R}(\mathcal{Z}) \times \mathcal{U}(1)$ charges of the scalar superfields and the superpotential. $\alpha = \frac{2\pi}{7}$ and $R$ is an undetermined phase.

|       | $W$ | $\Psi, \bar{\Psi}$ | $H$ | $\bar{H}$ | $X$ | $Z$ |
|-------|-----|---------------------|-----|-----------|-----|-----|
| $\mathcal{R}(\mathcal{Z})$ | $3\alpha$ | $\alpha$ | $\frac{\alpha}{2}$ | 0 | $-\frac{\alpha}{2}$ | 0 |
| $\mathcal{U}(1)$ | 0 | 0 | 0 | $-\frac{17}{14}R$ | $\frac{39}{14}R$ | $R$ |

where $Q_{\phi_i}$ denotes the $\mathcal{R}(\mathcal{Z})$ and $\mathcal{U}(1)$ charges of the $\phi_i$ superfield. The transformation properties of the scalar superfields and superpotential are presented in Table 1. This prescription of the charges will guarantee 'all order' hierarchy, generation of scale of GUT and, as we will see in section 3, give top Yukawa coupling of order one. We have taken one of the simple choice, of the $Q_i$ charges of appropriate superfields, which gives solution of these fundamental problems, however other choices are also possible.

Including the lowest order 'nonflat' terms the superpotential $W$ allowed by $\mathcal{R}(\mathcal{Z}) \times \mathcal{U}(1)$ symmetry is given by:

$$W = \Psi^3 + \bar{\Psi}^3 + M_P^3 \left( \frac{\Psi \bar{\Psi}}{M_P^2} \right)^5 + M_P^3 \left( \frac{H \bar{H}}{M_P^2} \right)^{10} \left( \frac{X}{M_P} \right)^4 \frac{Z}{M_P}. \quad (10)$$

We easily observe that along the directions (4) the SUSY conserving minima of the potential is obtained for $V = v = 0$. After SUSY breaking a’ la $N = 1$ minimal supergravity theory, the soft SUSY breaking mass terms which enter in the Lagrangian have the form:

$$V_{SSB}^m = m_{3/2}^2 \left( |\Psi|^2 + |\bar{\Psi}|^2 + |H|^2 + |\bar{H}|^2 + |X|^2 + |Z|^2 \right), \quad (11)$$

where $m_{3/2}$ is the gravitino mass. Together with (10), one finds nonzero solutions for $V$ and $v$ with the following magnitudes:

$$V \sim M_P \left( \frac{m_{3/2}}{M_P} \right)^{1/8}, \quad \langle X \rangle \sim \langle Z \rangle \sim v \sim M_P \left( \frac{m_{3/2}}{M_P} \right)^{1/23}. \quad (12)$$

5The terms which contain $\Psi^3$ or $\bar{\Psi}^3$ are ‘flat’ and therefore do not affect the masses of the doublets and also do not take part in the fixing of the VEVs of the scalar fields. Because of this we do not take them into the account during the investigation of the higher order terms.
For $m_{3/2} = 10^3$ GeV and $M_P = 2.4 \cdot 10^{18}$ GeV (reduced Planck mass), we have:

$$\epsilon \equiv \frac{V}{M_P} \sim \left(\frac{m_{3/2}}{M_P}\right)^{1/8} \sim 10^{-2},$$

$$\epsilon_X \equiv \frac{v}{M_P} \sim \frac{\langle X \rangle}{M_P} \sim \frac{\langle Z \rangle}{M_P} \sim \left(\frac{m_{3/2}}{M_P}\right)^{1/23} \sim 0.2. \quad (13)$$

Thus, at a scale of $4.8 \cdot 10^{17}$ GeV the $SU(6) \times U(1)$ group breaks to $SU(5) \times U(1)'$ group through the VEVs of the $H, \bar{H}$ fields. The VEVs ($V \sim 10^{16}$ GeV) of the $\Psi, \bar{\Psi}$ fields reduce the $SU(5) \times U(1)'$ symmetry to $SU(3)_C \times SU(2)_W \times U(1)_Y$.

The lowest order $SU(6)^2 \times U(1)$ global symmetry violating operator $(\bar{\Psi} \Psi)^3 (\bar{H} H)^9 X Z^6$ which is permitted by the $R \times U(1)$ symmetry gives negligible ($\sim 1$ keV) contribution to the $\mu$-term. We have therefore obtained an ‘all order’ solution of the gauge hierarchy problem and even an understanding of the origin of the GUT scale, whose magnitude is given by an interplay of the two ‘fundamental’ scales, the Planck and the SUSY breaking scales. Let us note that the properties of the scalar content of the $SU(3)^3$ gauge theory also permits one to understand the origin of the GUT scale $\[I\]$.

While the VEVs of the scalar superfields obey the hierarchy $v \gg V$, taking into the account (7), (8) and (13) we will see that the physical doublet-antidoublets reside in the $\bar{\Psi} + \Psi$ and $\bar{H} + H$ superfields respectively with the following weights:

$$\bar{\Psi} \supset h_u, \quad H \supset \frac{\epsilon}{\epsilon_X} h_u, \quad \frac{v}{M_P} \sim \frac{\langle X \rangle}{M_P} \sim \frac{\langle Z \rangle}{M_P} \sim \left(\frac{m_{3/2}}{M_P}\right)^{1/23} \sim 0.2. \quad (13)$$

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### 3 Charged Fermion Masses and Mixings

In this section we will describe the pattern of charged fermion masses and mixings in our model. Together with the chiral supermultiplets $(15 + \bar{6}_{-1} + \bar{6}_1)^{(i)}$ ($i$ is a family index), we introduce one pair of $20_i + \bar{20}_i$, which is necessary for obtaining the top quark Yukawa coupling of order unity. Using this 20-plet pair the top quark mass is generated by the heavy particle exchange mechanism $\[E\]$.

Before considering all three generations let us demonstrate how the $b - \tau$ unification occurs in our scheme. The relevant couplings for down-type quark and lepton for the third family are:

$$W_Y(b, \tau) = \frac{1}{4\sqrt{2}} A 15^{(3)} 15^{(3)} \Psi + \sqrt{2} B \bar{6}_{-1}^{(3)} \bar{6}_1^{(3)} \Psi + \sqrt{2} C 15^{(3)} \bar{6}_1^{(3)} H. \quad (15)$$
Here and below we use the proper normalization of the appropriate Yukawa couplings. Substituting the VEVs of the GUT and doublet fields the corresponding mass matrices will be:

\[
\begin{pmatrix}
\tilde{g}^{(3)} & d^{(3)} \\
g^{(3)} & 0
\end{pmatrix}
\begin{pmatrix}
Cv & AV \\
Ah_d & 0
\end{pmatrix},
\begin{pmatrix}
e^{(3)} \\
\tilde{L}^{(3)}
\end{pmatrix}
\begin{pmatrix}
0 & B_h d \\
Cv & BV
\end{pmatrix}.
\]

Assuming that \(AV, BV \gg Cv\), the \((22)\) elements of these matrices can be integrated out and we will have:

\[
\lambda_b = \lambda_\tau = CvV.
\]

Thus, in the framework of the flipped \(SU(6)\) model we can obtain \(b-\tau\) unification, which does not hold in flipped \(SU(5)\) models.

Assuming \(C \sim 10^{-3}\) and for the values of \(v\) and \(V\) presented in (13), we will have \(\lambda_b = \lambda_\tau \sim 10^{-2}\), which suggests the regime \(\tan \beta \sim 1\). This value of \(\tan \beta\) is also preferred for the pseudo-Goldstone scenario [16] and proves useful for nucleon stability. The ‘smallness’ of the \(C\) parameter is explained below by replacing it with the fourth power of the ratio \(Z/M_P\).

Returning to three generations, together with the pair of 20-plets, the Yukawa superpotential is given by:

\[
W_Y = \frac{1}{4\sqrt{2}} A_{ij} 15^{(i)} 15^{(j)} \Psi + \sqrt{2} B_{ij} \tilde{6}_{-1}^{(i)} \tilde{6}_1^{(j)} \Psi + \sqrt{2} C_{ij} 15^{(i)} \tilde{6}_1^{(j)} \tilde{H} + 2D_{ij} 15^{(i)} \tilde{6}_1^{(j)} \tilde{\overline{H}} M_P + \frac{1}{2\sqrt{3}} F_{15^{(i)} 20_{1} H} + \frac{1}{2\sqrt{3}} G_{6_1^{(i)} 20_{-1} \overline{\Psi}} + X_{20_{-1} 20_{1}}.
\]

As mentioned above, the first three terms are responsible for generating the masses of down quarks and leptons, while the remaining terms are relevant for the up type quarks. Let us note, that in our scheme we have to still impose the ‘ordinary’ family independent matter parity in order to guarantee nucleon stability.

For a reasonable explanation of the hierarchy of the Yukawa couplings we will assume that the matrices \(A, \ldots, D\), as well as the couplings \(F\) and \(G\) are not constants but operators which depend on powers of ratios \(\tilde{H} H/M_P^2\). To avoid the unacceptable asymptotic relations \(\lambda_s = \lambda_\mu\) and \(\lambda_d = \lambda_e\) in our scheme, the appropriate entries of the \(B\) matrix are dependent on the ratio \(\tilde{H} H/M_P^2\). The transformation properties under \(R \times U(1)\) of the various matter superfields are presented in Table 2. The first four couplings of (18) can schematically be written as:

\[\text{In the framework of the ‘ordinary’ } SU(6)\text{ pseudo-Goldstone scenario, in the 3rd - 5th papers of ref. [8] suggested ways of accommodating the fermion masses and mixings.}\]
Table 2: \( R \) charges of the fermionic superfields under the \( \mathcal{R}(\mathcal{Z}) \) and \( U(1) \) symmetries.

| \( \mathcal{R}(\mathcal{Z}) \) | 15\(^{(1)}\) | 15\(^{(2)}\) | 15\(^{(3)}\) | \( 6_{-1}^{(1)} \) | \( 6_{-1}^{(2)} \) | \( 6_{-1}^{(3)} \) | \( 6_1^{(1)} \) | \( 6_1^{(2)} \) | \( 6_1^{(3)} \) | 20\(_1\) | 20\(_{-1}\) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| \( \alpha \) | \( \frac{3}{2} \alpha \) | \( \alpha \) | \( \frac{5}{2} \alpha \) | \( \frac{1}{2} \alpha \) | \( \frac{3}{2} \alpha \) | \( \frac{1}{2} \alpha \) | \( \frac{3}{2} \alpha \) | \( \frac{1}{2} \alpha \) | \( 2 \alpha \) | \( \frac{3}{2} \alpha \) |
| \( U(1) \) | \(-2R\) | \(-\frac{20}{14}R\) | 0 | \(-\frac{111}{14}R\) | \(\frac{17}{14}R\) | \(-\frac{30}{14}R\) | \(-\frac{30}{14}R\) | \(\frac{20}{14}R\) | 0 | \(-\frac{30}{14}R\) |

\[
\begin{pmatrix}
\frac{15^{(1)}}{M_P^4} & \frac{15^{(2)}}{M_P^2} & \frac{15^{(3)}}{M_P^2} \\
\frac{XZ^2}{M_P} & \frac{Z^2}{M_P} & \frac{Z^2}{M_P} \\
\frac{Z^2}{M_P} & \frac{Z^2}{M_P} & \frac{Z^2}{M_P} \\
\end{pmatrix}
\cdot \Psi 
\]

\[
\begin{pmatrix}
\frac{6_1^{(1)}}{M_P^4} & \frac{6_1^{(2)}}{M_P^2} & \frac{6_1^{(3)}}{M_P^2} \\
\frac{ZHH}{M_P^2} & \frac{Z}{M_P} & \frac{Z}{M_P} \\
\frac{ZHH}{M_P^2} & \frac{Z}{M_P} & \frac{Z}{M_P} \\
\end{pmatrix}
\cdot \frac{\Psi}{\tilde{H}} 
\]

\[
\begin{pmatrix}
\frac{6_1^{(1)}}{M_P^2} & \frac{6_1^{(2)}}{M_P^2} & \frac{6_1^{(3)}}{M_P^2} \\
\frac{XZ^2}{M_P} & \frac{X}{M_P} & \frac{X}{M_P} \\
\frac{XZ^2}{M_P} & \frac{X}{M_P} & \frac{X}{M_P} \\
\end{pmatrix}
\cdot \frac{\Psi}{\tilde{H}} 
\]

The \( \mathcal{F}, \mathcal{G} \) terms have the form:

\[
20_1 \left( \frac{Z^2}{M_P^2} 15^{(1)} + \frac{X}{M_P} 15^{(2)} + 15^{(3)} \right) H + 20_{-1} \left( \frac{Z^3}{M_P^3} \bar{6}_1^{(1)} + \frac{X^2}{M_P^2} \bar{6}_1^{(2)} + \bar{6}_1^{(3)} \right) \Psi \ .
\]
Without loss of generality one can choose the basis in which only \(15^{(3)}\) and \(\bar{6}^{(3)}\) states participate in (22). Through the choice of this basis the couplings of (19)-(21) will be the same. So, for estimates we use couplings:

\[
\frac{F}{2\sqrt{3}} 15^{(3)} H + \frac{G}{2\sqrt{3}} \bar{6}^{(3)} \Psi.
\]

(23)

These couplings are relevant for the generation of the top quark mass and should be taken into account during the analyses of the neutrino masses as well. For \(\epsilon\) and \(\epsilon_X\) (see (13)) we will take the values \(10^{-2}\) and 0.2 respectively.

Starting with the masses of the charged leptons the mass matrix relevant for the three families will have the form:

\[
\hat{M}_L = \begin{pmatrix} L^{(1)} & L^{(2)} & L^{(3)} & l^{(1)} & l^{(2)} & l^{(3)} \\
0 & 0 & 0 & b_1^{11} \epsilon_X h_d & 0 & 0 \\
0 & 0 & 0 & b_1^{11} \epsilon_X h_d & b_2^{22} \epsilon_X h_d & 0 \\
c_{11} \epsilon_X^9 & c_{21} \epsilon_X^8 & c_{31} \epsilon_X^7 & b_3^{11} \epsilon_X h_d & b_3^{22} \epsilon_X h_d & b_3^{33} h_d \\
0 & 0 & 0 & b_3^{11} \epsilon_X h_d & b_3^{22} \epsilon_X h_d & b_3^{33} h_d \\
c_{13} \epsilon_X^6 & c_{23} \epsilon_X^5 & c_{33} \epsilon_X^4 & b_3^{11} \epsilon_X h_d & b_3^{22} \epsilon_X h_d & b_3^{33} h_d \end{pmatrix}
\]

(24)

where coefficients with primes appear because there are several possibilities of the convolution of the gauge indices in the corresponding entries of the second ‘matrix term’ of (13). One can easily see that for values of the constants \(c_{21} \sim 1/3\) and \(c_{31} \sim 1/10\) the see-saw limit works well and the states which correspond to the \(3 \times 3\) block of the lowest right side of the (24) matrix could be integrated out. For the ‘light’ charged leptons we obtain the following mass matrix:

\[
\hat{m}_e = \begin{pmatrix} e^{c(1)} & e^{c(2)} & e^{c(3)} \\
L^{(1)} & L^{(2)} & L^{(3)} \end{pmatrix} = \begin{pmatrix} l_1^{(1)} & l_2^{(2)} & l_3^{(3)} \\
1 & 0 & s_{13}^{l} \\
0 & 1 & s_{23}^{l} \end{pmatrix} \begin{pmatrix} \frac{h_d}{c} \epsilon \end{pmatrix}
\]

(25)

A biunitary transformation

\[
R_e^{\dagger} \hat{m}_e L_e = \hat{m}_e^{\text{diag}}, \quad L_e = \begin{pmatrix} 1 & 0 & s_{13}^{l} \\
0 & 1 & s_{23}^{l} \\
-s_{13}^{l} & -s_{23}^{l} & 1 \end{pmatrix},
\]

(26)
where
\[ s_{13}^l = \frac{c_{13}}{c_{33}} \epsilon_X^2, \quad s_{23}^l = \frac{c_{23}}{c_{33}} \epsilon_X, \]
(27)
transforms the matrix in (25) into the diagonal form with eigenvalues:
\[ \lambda_e = c_{11} b_{11}^{10} \epsilon_X^{10}, \quad \lambda_\mu = c_{22} b_{22}^{7} \epsilon_X^7, \quad \lambda_\tau = c_{33} \epsilon_X^5, \]
(28)
In estimating quark mixings and nucleon decay we will deal with \( L \) matrices which rotate the basis of the left handed fields. In obtaining (28) the smallness of \( c_{21} \) and \( c_{31} \) with respect to the other couplings has been taken into account.

Following the same strategy the mass matrix which leads to the masses of down quarks will have the form:
\[
\hat{M}_D = \begin{pmatrix}
\hat{g}^{(1)} & \hat{g}^{(2)} & \hat{g}^{(3)} \\
0 & 0 & 0 \\
0 & 0 & 0 \\
c_{11} \epsilon_X^4 & 0 & c_{13} \epsilon_X^4 \\
c_{21} \epsilon_X^6 & c_{22} \epsilon_X^6 & c_{23} \epsilon_X^6 \\
c_{31} \epsilon_X^8 & c_{32} \epsilon_X^8 & c_{33} \epsilon_X^8 \\
\end{pmatrix}
\begin{pmatrix}
d_e^{(1)} \\
d_e^{(2)} \\
d_e^{(3)} \\
0 \\
da_{11} \epsilon_X^4 h_d & a_{12} \epsilon_X^3 h_d & a_{13} \epsilon_X^2 h_d \\
a_{12} \epsilon_X^3 h_d & a_{22} \epsilon_X^2 h_d & a_{23} \epsilon_X h_d \\
a_{13} \epsilon_X^2 h_d & a_{23} \epsilon_X h_d & a_{33} h_d \\
0 & a_{11} V \epsilon_X^4 & a_{12} V \epsilon_X^3 & a_{13} V \epsilon_X^2 \\
0 & a_{12} V \epsilon_X^3 & a_{22} V \epsilon_X^2 & a_{23} V \epsilon_X \\
0 & a_{13} V \epsilon_X^2 & a_{23} V \epsilon_X & a_{33} V \\
\end{pmatrix}.
\]
(29)
and, after integrating out the appropriate heavy states, this matrix reduces to:
\[
\hat{m}_d = \begin{pmatrix}
d_1^e \\
d_2^e \\
d_3^e \\
0 \\
da_{11} \epsilon_X^4 h_d & a_{12} \epsilon_X^3 h_d & a_{13} \epsilon_X^2 h_d \\
a_{12} \epsilon_X^3 h_d & a_{22} \epsilon_X^2 h_d & a_{23} \epsilon_X h_d \\
a_{13} \epsilon_X^2 h_d & a_{23} \epsilon_X h_d & a_{33} h_d \\
0 & a_{11} V \epsilon_X^4 & a_{12} V \epsilon_X^3 & a_{13} V \epsilon_X^2 \\
0 & a_{12} V \epsilon_X^3 & a_{22} V \epsilon_X^2 & a_{23} V \epsilon_X \\
0 & a_{13} V \epsilon_X^2 & a_{23} V \epsilon_X & a_{33} V \\
\end{pmatrix}.
\]
(30)
By a transformation:
\[
L_d^\dagger \hat{m}_d R_d = \hat{m}_d^{\text{diag}}, \quad L_d = \begin{pmatrix}
1 & 0 & s_{13}^d \\
0 & 1 & s_{23}^d \\
-s_{13}^d & -s_{23}^d & 1 \\
\end{pmatrix},
\]
(31)
where
\[
s_{13}^d = \frac{c_{13}}{c_{33}} \epsilon_X^2, \quad s_{23}^d = \frac{c_{23}}{c_{33}} \epsilon_X.
\]
(32)
the matrix in (30) is diagonalized with eigenvalues given by:

\[ \lambda_d = c_{11} \frac{\epsilon_X^{10}}{\epsilon}, \quad \lambda_s = c_{22} \frac{\epsilon_X^7}{\epsilon}, \quad \lambda_u = c_{33} \frac{\epsilon_X^5}{\epsilon}. \]  

(33)

Turning to the masses of the up type quarks from the couplings (21), (23) and also taking into account the last term of (18), the matrix which is responsible for generation of the masses of the up quarks will have the form:

\[
\hat{M}_U = \begin{pmatrix}
q^{(1)} & u^{c(1)} & u^{c(2)} & u^{c(3)} & \bar{q}_{20} \\
q^{(2)} & 0 & d_{12} \epsilon_X^4 h_u & 0 & 0 \\
q^{(3)} & d_{21} \epsilon_X^4 h_u & d_{22} \epsilon_X^3 h_u & d_{23} \epsilon_X h_u & 0 \\
q_{20} & 0 & d_{32} \epsilon_X^3 h_u & 0 & Fv \\
& 0 & 0 & 0 & G h_u \langle X \rangle
\end{pmatrix}
\]  

(34)

Integrating out the corresponding heavy fragments from the 20-plets we obtain the following up quark mass matrix:

\[
\hat{m}_u = \begin{pmatrix}
q^{(1)} & u^{c(1)} & u^{c(2)} & u^{c(3)} \\
q^{(2)} & 0 & d_{12} \epsilon_X^4 h_u & 0 \\
q^{(3)} & d_{21} \epsilon_X^4 h_u & d_{22} \epsilon_X^3 h_u & d_{23} \epsilon_X h_u \\
q_{20} & 0 & d_{32} \epsilon_X^3 h_u & \lambda_t \langle X \rangle
\end{pmatrix} h_u
\]  

(35)

Rotating the basis of the left-right fields

\[
L_u^\dagger \hat{m}_u R_u = \hat{m}_u^{\text{diag}}, \quad L_u = \begin{pmatrix}
1 & s_{12}^u & 0 \\
-s_{12}^u & 1 & s_{23}^u \\
0 & -s_{23}^u & 1
\end{pmatrix},
\]  

(36)

where

\[
s_{12}^u = \frac{d_{12}}{\lambda_c} \epsilon_X^4 \sim \epsilon_X, \quad s_{23}^u = \frac{d_{23}}{\lambda_t} \epsilon_X .
\]  

(37)

we will have:

\[
\lambda_t = -FG \frac{v}{\langle X \rangle} \sim 1, \quad \lambda_c \sim \epsilon_X^3, \quad \lambda_u \sim \epsilon_X^5.
\]  

(38)

We see that by imposing the \( \mathcal{R} \times \mathcal{U}(1) \) horizontal symmetry we can obtain a reasonable description of fermion mass hierarchy. The \( b-\tau \) unification still holds, while the unwanted relations \( \lambda_s = \lambda_\mu \) and \( \lambda_d = \lambda_c \) are avoided. The masses of the electron and \( \mu \) meson are proportional respectively to \( \frac{b_{11}'}{b_{11}} \) and \( \frac{b_{22}'}{b_{22}} \), while the down quark mass matrix depends only on the \( c_{ij} \) couplings (see (30)). Since appropriate couplings (see (30)) are built through the nonrenormalizable operators and there exist several ways of the convolution of the
gauge indices, the $b, b'$ constants are completely arbitrary and taking $b_{11}' \sim b_{22}' \sim 1/3$ we will have

$$m_s \sim \frac{1}{3} m_\mu, \quad m_s/m_d \sim \frac{1}{9} \frac{m_\mu}{m_e} \sim 20,$$

which are desirable asymptotic relations.

Although the masses of quark-lepton families do not depend on the $a_{ij}$ coefficients, limits on their magnitudes could be obtained from some physical considerations. These coefficients could not be taken very small because the masses of the decoupled (heavy) states crucially depend on their magnitudes. Taking them too small the see-saw limit in which the matrices (25) and (30) were obtained does not work, and formulae (28) and (33) are invalid. The masses of the (decoupled) three generations of triplet and doublet states are respectively:

$$m_T^{(3)} = a_{33} V, \quad m_D^{(3)} = b_{33} V,$$

$$m_T^{(2)} = (a_{22} - \frac{a_{22}^2}{a_{33}}) \epsilon_X^2 V, \quad m_D^{(2)} = b_{22} \epsilon_X^3 V,$$

$$m_T^{(1)} = (a_{11} - \frac{a_{13}^2}{a_{33}} - \frac{a_{12}^2}{a_{22} - a_{23}^2/a_{33}}) \epsilon_X^4 V, \quad m_D^{(1)} = b_{11} \epsilon_X^6 V.$$

From these formulae and for values of the parameters:

$$a_{33} \sim b_{11} \sim b_{22} \sim b_{33} \sim 1,$$

$$a_{11} \sim \epsilon_X^2, \quad a_{22} \sim \epsilon_X$$

$$a_{12} \sim (\epsilon_X)^{3/2}, \quad a_{13} \sim \epsilon_X, \quad a_{23} \sim (\epsilon_X)^{1/2},$$

we will have

$$m_T^{(i)} \simeq m_D^{(i)},$$

and the successful unification of the three gauge couplings will be unchanged in the one loop level. For values of the coefficients in (41) the see-saw limit still works if $c_{13} \sim 2 \cdot 10^{-2}$ and $c_{23} \sim 0.2$. Note that the picture will not be changed if the coefficients $a_{12}, a_{13}$ and $a_{23}$ are taken to be less than the values in (41). As we will see in the next section, the proton decay rate in our model crucially depends on these parameters and we can say more about them from the requirement of proton stability as demanded by experiments.

To conclude this section, let us summarize the results which we have obtained. For the Yukawa couplings and hierarchies in the down quark and lepton sectors we have:

$$\lambda_b = \lambda_\tau \sim \frac{\epsilon_X^5}{\epsilon}, \quad \tan \beta \sim 1.$$
\[ \lambda_d : \lambda_s : \lambda_b \sim \epsilon_X^5 : \epsilon_X^2 : 1, \]
\[ \lambda_e : \lambda_{\mu} : \lambda_{\tau} \sim \epsilon_X^5 : \epsilon_X^2 : 1, \] \tag{43}

while for the up quarks:

\[ \lambda_t \sim 1, \]
\[ \lambda_u : \lambda_c : \lambda_t \sim \epsilon_X^3 : 1, \] \tag{44}

In the notation used above (see (31) and (36)) the CKM matrix is:

\[ \hat{V} = L_T^u L_d^* \] \tag{45}

with the matrix elements

\[ \hat{V}_{us} = \frac{d_{12}}{\lambda_c} \epsilon_X^4 \sim \epsilon_X, \]
\[ \hat{V}_{ub} = \left( c_{12}^* \epsilon_X - \hat{V}_{us} c_{23}^* \right) \frac{\epsilon_X}{c_{33}}, \]
\[ \hat{V}_{cb} = \left( c_{23}^* \frac{c_{23}}{c_{33}} - \frac{d_{23}}{\lambda_t} \right) \epsilon_X. \] \tag{46}

With \( c_{23} \sim d_{23} \sim 1/5, \ c_{13} \sim 2 \cdot 10^{-2} \) and the other participating couplings of order unity, the CKM matrix elements all have acceptable values.

4 Nucleon Decay

Dimension five nucleon decay occurs in our model through exchange of the colored Higgsinos from the \( \Psi \) and \( \overline{\Psi} \) superfields. The colored states from the \( H + H \) superfields are goldstones ‘eaten’ by the appropriate gauge fields and are irrelevant for nucleon decay. In our model there exist insertions (see (6)) between triplet states which come from the same superfields (from \( \Psi \) and \( \overline{\Psi} \)). Because of this the \( d = 5 \) operator which is related to the left handed fields can emerge only from the first coupling in (18). Its decomposition into components which are relevant for nucleon decay is:

\[ \frac{1}{4\sqrt{2}} A_{ij} 15(ij) 15(j\Psi) \rightarrow \frac{1}{2} A_{ij} q_i q_j g_4 + A_{ij} q_i L_j d_\psi. \] \tag{47}

From the mass matrix in (24) we have seen that left handed states of the lepton doublets mainly come from the \( L_i \) states. Taking into account (26), (31) and (36) in the mass eigenstate basis the couplings (47) take the form:

\[ \text{As we will see in the next section for proton stability a better value for } c_{23} \text{ is } \sim 1/12. \] In this case \( V_{ub} \) and \( V_{cb} \) can still have the desired values since they also contain other entries.
\[ u(L_u^\dagger A L_d^*) d g_\psi + u(L_u^\dagger A L_e) e d_\psi - d(L_u^\dagger A L_d) \nu d_\psi \, , \] (48)

After integrating out the heavy triplet states and dressing the \( d = 5 \) operators by the wino states, we will obtain the four fermion operators for nucleon decay. The neutrino decay channel will occur through the operator:

\[ \mathcal{O} = x \cdot (u^a d_m^b) (d^c_j \nu_i) \varepsilon_{abc} \] (49)

where \( a, b, c \) are color indices and

\[
x = \alpha \left( -(L_d^\dagger A L_e)_{ji} (L_u^\dagger A L_d^*)_{ln} \hat{V}_{lm} \hat{V}_{n1}^{\dagger} + (L_u^\dagger A L_d^*)_{lm} (L_u^\dagger A L_d^*)_{ln} \hat{V}_{ij} \right. \\
- \left. (L_d^\dagger A L_u^*)_{jl} (L_u^\dagger A L_d^*)_{mi} \hat{V}_{lm} \hat{V}_{n1}^{\dagger} - (L_d^\dagger A L_u^*)_{mi} (L_u^\dagger A L_d^*)_{li} \hat{V}_{ij} \right) \] (50)

where \( \alpha \) is a family independent factor.

For nucleon decay the first two generations of quarks and leptons occur in the external lines, while the contribution of the third generation through internal loops are somewhat suppressed. For estimates we will assume that the family indices can be 1 and/or 2. Since for two generations the only mixing term of the CKM matrix is the Cabbibo angle, which is not renormalized between the GUT and electroweak scales \([17]\), we can replace \( \hat{V} \) in (50) using formula (45). After substituting \( \hat{V} \) and summing over the repeated indices we see that the first two terms of (50) exactly cancel out, while the sum of the remaining two terms gives:

\[ x = -2\alpha (L_d^\dagger A L_d^*)_{mj} (L_u^\dagger A L_e)_{li} \] (51)

Similarly the four fermion operator which corresponds to nucleon decay into charged leptons is given by:

\[ \mathcal{O}' = x' \cdot (u^a d_j^c) (u^c e_i) \varepsilon_{abc} \] (52)

with

\[
x' = \alpha \left( -(L_u^\dagger A L_d^*)_{lj} (L_u^\dagger A L_e)_{mi} \hat{V}_{ml}^{\dagger} \hat{V}_{n1}^{\dagger} + (L_u^\dagger A L_d^*)_{lm} (L_u^\dagger A L_d^*)_{nj} \hat{V}_{mj} \hat{V}_{n1}^{\dagger} \\
+ (L_u^\dagger A L_d^*)_{lm} (L_u^\dagger A L_d^*)_{ni} \hat{V}_{mj} \hat{V}_{n1}^{\dagger} + (L_u^\dagger A L_d^*)_{mi} (L_u^\dagger A L_d^*)_{nj} \hat{V}_{mj} \hat{V}_{n1}^{\dagger} \right) \] (53)

After some simplifications we find:

\[ x' = 2\alpha (L_u^\dagger A L_u^*)_{1j} (L_d^\dagger A L_e)_{ji} \] (54)

From (49) and (51), the \( p \to K \nu_\mu \) decay width normalized with respect to the \( SU(5) \) case \([18]\) is:

\[
\Gamma(p \to K \nu_\mu) \Gamma_{SU(5)}(p \to K \nu_\mu) = \left| \frac{a_{12}(a_{22} \sin \theta + a_{12} \varepsilon_X) \varepsilon_X^5}{\lambda_s \lambda_c \sin^2 \theta} \right|^2 . \] (55)
Table 3: Proton lifetime in units of $\tau(p \to K\nu_\mu)_{SU(5)}$.

| | (i) | (ii) | (iii) |
|---|---|---|---|
| $\tau(p \to K\nu_\mu)$ | $\sim 10^2$ | $\gtrsim 10^4$ | $\sim 6 \cdot 10^2$ |
| $\tau(p \to K\nu_e)$ | $\sim 2 \cdot 10^4$ | $\gtrsim 8 \cdot 10^5$ | $\sim 2 \cdot 10^4$ |
| $\tau(p \to \pi\nu_\mu)$ | $\sim 7 \cdot 10^3$ | $\sim 7 \cdot 10^3$ | $\sim 3 \cdot 10^4$ |
| $\tau(p \to \pi\nu_e)$ | $\sim 8 \cdot 10^5$ | $\sim 4 \cdot 10^6$ | $\sim 8 \cdot 10^5$ |
| $\tau(p \to K\mu)$ | $\sim 2 \cdot 10^2$ | $\sim 2 \cdot 10^2$ | $\sim 5 \cdot 10^3$ |
| $\tau(p \to Ke)$ | $\sim 3 \cdot 10^4$ | $\sim 10^5$ | $\sim 10^5$ |
| $\tau(p \to \pi\mu)$ | $\sim 3 \cdot 10^4$ | $\sim 10^4$ | $\sim 10^4$ |
| $\tau(p \to \pi e)$ | $\sim 3 \cdot 10^5$ | $\sim 3 \cdot 10^5$ | $\sim 2 \cdot 10^6$ |

Similarly, the decay width for the reaction $p \to K\mu$ is:

$$\frac{\Gamma(p \to K\mu)}{\Gamma_{SU(5)}(p \to K\nu_\mu)} = 0.12 \cdot \left[ \frac{a_{22}(a_{11}\epsilon_X^2 - 2a_{12}\epsilon_X \sin \theta + a_{22} \sin^2 \theta)\epsilon_X^4}{\lambda_s\lambda_c \sin^2 \theta} \right]^2,$$

(56)

where the factor 0.12 arises from the difference between proton-neutrino and proton-charged lepton hadronic matrix elements [18].

As we mentioned earlier, there is some freedom in the choice of the $a_{ij}$ parameters. None of them should exceed the values presented in (41) in order to keep unification of three gauge coupling constant at $M_{GUT}$. For appropriate $a_{ij}$ given by (41) the decay widths in (55) and (56) will have the values:

$$\frac{\Gamma(p \to K\nu_\mu)}{\Gamma_{SU(5)}(p \to K\nu_\mu)} \sim 7.7 \cdot 10^{-3}, \quad \frac{\Gamma(p \to K\mu)}{\Gamma_{SU(5)}(p \to K\nu_\mu)} \sim 5 \cdot 10^{-3},$$

(57)

which are well suppressed relative to the dominant $SU(5)$ decay mode. However, as we will see below, there are parameter choices for which the charged lepton decay mode is dominant.

Since (55), (56) are sensitive to the couplings $a_{12}$ and $a_{22}$ we will study proton decay by varying $a_{12}$ and $a_{22}$, keeping the other $a_{ij}$ unchanged. Three cases, which give different phenomenological implications for proton decay, will be relevant:

(i) $a_{22} \sim \epsilon_X$, $a_{12} \sim \epsilon_X^{3/2}$;  (ii) $a_{22} \sim \epsilon_X$, $a_{12} \lesssim 0.3 \cdot \epsilon_X^{3/2}$;

(iii) $a_{22} \lesssim \epsilon_X^{3/2}$, $a_{12} \sim \epsilon_X^{3/2}$.

(58)

As we have seen, (i) gives the same proton decay rate in the $K\nu_\mu$ and $K\mu$ channels.
However, in case (ii) of (58) the reaction $p \to K\mu$ dominates, while case (iii) has dominant decay $p \to K\nu\mu$. In Table (3) we present the proton lifetimes in the units of $\tau(p \to K\nu\mu)_{SU(5)}$. As we see the other decay channels are more suppressed for all three cases.

In conclusion, we expect the proton to be quite long lived ($\tau_p \approx 10^{33\pm 2}$ yr.).

4.1 Suppression of Planck Scale Induced $d = 5$ Operators

Even in the framework of the minimal SUSY standard model Planck-scale physics may generate $d = 5$ operators

$$\lambda \frac{q \cdot q \cdot q \cdot l}{M_P},$$

which are permitted by the matter $R$-parity, and could induce rapid nucleon decay if the dimensionless coupling $\lambda$ is not less than $10^{-8}$ or so. It is therefore desirable to have some mechanism which naturally suppress such operators. One possibility to forbid them is to employ discrete gauge symmetries [20]. Other ways include a prescription, where the matter fields have family dependent transformation properties under some flavor group [21], or to have a ‘redefined’ $R$-symmetry. The last possibility was suggested recently [22] where all baryon number violating terms were forbidden.

Since in our $SU(6) \times U(1)$ model the matter superfields carry family dependent $R \times U(1)$ charges the suppression of the Planck scale $d = 5$ operators can be checked directly.

The relevant terms include:

$$\frac{1}{M_P^2} \hat{\Gamma}_{ijmn} \cdot 15(i) \cdot 15(j) \cdot 15(m) \cdot 15(n) \cdot \bar{\Psi},$$

$$\frac{1}{M_P^2} \hat{\Gamma}'_{ijmn} \cdot 15(i) \cdot 15(j) \cdot 15(m) \cdot 6(n) \cdot \bar{X} \cdot H,$$

where $i, \ldots, n$ are family indices and $\hat{\Gamma}, \hat{\Gamma}'$ are tensors which depend on powers of $\bar{\Psi}\Psi$, $\bar{H}H$, $X$ and $Z$ superfields, and are chosen in such a way as to respect the $R \times U(1)$ symmetry.

The elements of $\hat{\Gamma}$ which correspond to the processes $p \to K\nu_{e,\mu,\tau}$ respectively are:

$$\hat{\Gamma}_{1112} = \frac{\bar{H}H}{M_P^2} \left( \frac{\bar{\Psi}\Psi}{M_P^2} \right)^2 \left( \frac{Z}{M_P} \right)^{10}, \quad \hat{\Gamma}_{1122} = \frac{\bar{H}H}{M_P^2} \frac{X}{M_P} \left( \frac{\bar{\Psi}\Psi}{M_P^2} \right)^2 \left( \frac{Z}{M_P} \right)^8,$$

$$\hat{\Gamma}_{1123} = \frac{\bar{H}H}{M_P^2} \left( \frac{\bar{\Psi}\Psi}{M_P^2} \right)^2 \left( \frac{Z}{M_P} \right)^8.$$

Refs. [19] also discuss models which also predict nucleon decay with emission of the charged lepton.
Substituting the VEVs of the scalar superfields and taking (60) into the account, we see that the appropriate \( d = 5 \) operators:

\[
\frac{1}{M_P} \epsilon^5 \epsilon_{\lambda}^{10} q_1 q_2 (\epsilon_X^2 l_1 + \epsilon_X l_2 + l_3)
\]

(63)

are strongly suppressed. Building all the other \( \hat{\Gamma} \) ‘coefficients’ we can check that they all contain at least a factor \( \sim \epsilon^4 \), which is already enough for the required suppression.

In (61), the \( SU(2)_W \) doublet state should be extracted from \( \bar{6}_1^{(n)} \). The latter are the decoupled heavy states and contain the light \( l_i \) doublets given by the weights \( \epsilon_{X_i} / \epsilon \), where \( k_1 = k_2 = 4, k_3 = 5 \). As far as the \( \hat{\Gamma}'s \) are concerned, they contain at least the \( \overline{\Psi} \Psi \) combination, which makes nucleon decay unobservable.

Finally, note, that in building the fermion mass sector we introduced a pair of 20-plets for the generation of the top quark mass. Since the \( \overline{20} \) state contains a \( q_3 \) state of weight 1, one should look for terms which could induce the relevant \( d = 5 \) operators. Writing down all possible operators:

\[
\frac{1}{M_P} \hat{\Omega}_{ijm} \cdot \overline{20} \cdot 15^{(i)} \cdot 15^{(j)} \cdot \left( \frac{\bar{6}_1^{(m)}}{M_P} \right) \Phi H
\]

(64)

\[
\frac{1}{M_P^2} \hat{\Upsilon}_{ij} \cdot \overline{20} \cdot 2 \cdot 15^{(i)} \cdot \left( \frac{\bar{6}_1^{(j)}}{M_P} \right) \Phi H
\]

(65)

\[
\frac{1}{M_P^3} \hat{\Sigma}_i \cdot \overline{20} \cdot 3 \cdot \left( \frac{\bar{6}_1^{(i)}}{M_P} \right) \Phi H
\]

(66)

one can simply verify that in addition to suppression from the CKM matrix elements, the \( R \times U(1) \) symmetry also provides a strong suppression.

5 Neutrino Masses in the Minimal Scheme

Turning now to neutrino masses, at first glance it seems from (21) that the neutrinos have large ‘Dirac’ masses \( \nu_L^i N h_0 \). However, we also have the term \( V \nu^c N \), which is crucial for the suppression of the left handed neutrino mass by the see-saw mechanism [23].

The mass matrix which include three families of the appropriate matter fields and is relevant for neutrino masses is \( 15 \times 15 \) dimensional and is schematically written as:
\[ \hat{M}_\nu = \begin{pmatrix} \nu_L^L & \nu_L^c & \nu_L^c & \nu^c & N \\ 0 & C_v & \hat{\delta} & 0 & \hat{m}' \\ C^T_v & 0 & \hat{B}^T V & 0 & 0 \\ \hat{\delta} & \hat{B}^T V & 0 & \hat{m}_u^T & 0 \\ 0 & 0 & \hat{m}_u & 0 & \hat{M} \\ \hat{m}'^T & 0 & 0 & \hat{M}^T & 0 \end{pmatrix}, \] (67)

where, according to (19) and (20),

\[ \hat{B} = \begin{pmatrix} b_{11} \epsilon_X^6 & 0 & 0 \\ 0 & b_{22} \epsilon_X^3 & 0 \\ b_{31} \epsilon_X^3 & b_{32} \epsilon_X^2 & b_{33} \end{pmatrix}, \quad \hat{C} = \begin{pmatrix} c_{11} \epsilon_X^6 & 0 & c_{13} \epsilon_X^6 \\ c_{21} \epsilon_X^3 & c_{22} \epsilon_X^6 & c_{23} \epsilon_X^5 \\ c_{31} \epsilon_X^3 & 0 & c_{33} \epsilon_X^4 \end{pmatrix}, \] (68)

\[ \hat{m}_u \text{ is given by (35), and} \]

\[ \hat{\delta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\lambda_t/V \end{pmatrix} \begin{pmatrix} V(h_u^0)^2 \end{pmatrix}, \quad \hat{m}' = \begin{pmatrix} d' - d & 0 \\ 0 & (d' - d)_{21} \epsilon_X^4 & (d' - d)_{22} \epsilon_X^3 & (d' - d)_{23} \epsilon_X^2 \\ 0 & 0 & (d' - d)_{32} \epsilon_X^3 & -\left(\frac{V}{v}\right)^2 \lambda_t \end{pmatrix} \cdot h_u^0. \] (69)

\[ \hat{M} = \begin{pmatrix} 0 & d'_{12} \epsilon_X^4 & 0 \\ d'_{21} \epsilon_X^4 & d'_{22} \epsilon_X^3 & d'_{23} \epsilon_X^2 \\ 0 & 0 & d'_{32} \epsilon_X^3 \end{pmatrix} \cdot V. \] (70)

In the second matrix of (69) the expression \((d' - d)_{ij}\) stands for \(d'_{ij} - d_{ij}\). The (3,3) elements of the matrices (69) and (70) are obtained after integrating out the \(\nu_{20}\) and \(\nu_{20}^c\) states (which come from \(\nu_{20}\) and \(\nu_{20}^c\) respectively).

Integrating out the states which correspond to the (4,5) and (5,4) blocks of the matrix (67), we obtain the reduced mass matrix for the neutrino masses:

\[ \hat{M}'_\nu = \begin{pmatrix} \nu_L^L & \nu_L^c & \nu^c \\ 0 & C_v & \hat{\Omega} \\ C^T_v & 0 & \hat{B}^T V \end{pmatrix}, \] (71)

where

\[ \hat{\Omega} = \begin{pmatrix} 0 & \epsilon_X^4 & 0 \\ \epsilon_X^3 & \epsilon_X^3 & \epsilon_X^2 \\ 0 & \epsilon_X^2 & 0 \end{pmatrix} \cdot \frac{(h_u^0)^2}{V}. \] (72)
After the second stage of integration of the (2,3) and (3,2) blocks of the matrix in (71), the mass matrix for the light neutrinos will be:

$$\hat{m}_\nu = \begin{pmatrix} 0 & \rho \epsilon^2_X & 0 \\ \rho \epsilon^2_X & \sigma \epsilon_X & 1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \frac{\epsilon^6_X (h^0)'^2}{\epsilon V},$$

(73)

where $\rho$ and $\sigma$ are couplings of order unity. This matrix is diagonalized through the transformation

$$L^\dagger_\nu \hat{m}_\nu L_\nu = \hat{m}_\nu^{\text{diag}}, \quad L_\nu = \begin{pmatrix} 1 & \frac{\rho \epsilon^2_X}{\sqrt{2}} & \frac{\rho \epsilon^2_X}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{\rho \epsilon^2_X}{2} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

(74)

and the corresponding eigenvalues are

$$m_{\nu_1} = 0, \quad m_{\nu_2} \sim m_{\nu_3} \sim \frac{\epsilon^6_X (h^0)'^2}{\epsilon V} \sim 10^{-5} \text{ eV}.$$ 

(75)

With these values of neutrino masses and mixings it is not possible it seems to explain the atmospheric and solar neutrino deficit, so that some extension of the model is needed for accommodating the present experimental data [9].

5.1 Accommodating Atmospheric and Solar Neutrino Data

To provide an explanation of the solar and atmospheric neutrino deficit in the framework of our model we invoke one of the mechanisms described in refs. of [24]. For our model the most natural scenario is one in which the atmospheric anomaly is resolved through maximal $\nu_\mu - \nu_\tau$ oscillations, while the solar neutrino puzzle is explained by the small angle $\nu_e - \nu_\tau$ MSW oscillations.

Introducing the sterile neutrino $\nu_s$ and right handed neutrino $N'$, with the following transformation properties under the $\mathcal{R} \times U(1)$ symmetry

$$\mathcal{R}(Z) : \quad R_{\nu_s} = \frac{1}{2} \alpha, \quad R_{N'} = \frac{3}{2} \alpha$$
$$U(1) : \quad R_{\nu_s} = -\frac{185}{14} R, \quad R_{N'} = -\frac{39}{14} R,$$

(76)

the relevant couplings will be:

$$W_{N'\nu_s} = \kappa \left( \frac{X Z^2}{M_P^2} 15_1 + \frac{X^2}{M_P} 15_2 + \frac{X}{M_P} 15_3 \right) N' \Psi^+$$
where \( \kappa \) is a dimensionless constant.

Substituting in (77) the VEVs of appropriate fields and integrating out the heavy \( N' \) state, the light neutrino mass matrix will have the form

\[
m_\nu = \begin{pmatrix}
\nu_e & \nu_\mu & \nu_\tau & \nu_s \\
\nu_e & m' & m' & m' \\
\nu_\mu & m' & m' & m' \\
\nu_\tau & m' & m' & m' \\
\nu_s & m & m' & m'
\end{pmatrix},
\]

where

\[
m = \epsilon_X h_u, \quad m' = \frac{\kappa^2 h_u^2}{M_P \epsilon_X}.
\]

For \( \epsilon_X \simeq 0.2 - 0.22 \), \( \kappa \sim 1/3 \), from (78) and (79) we see that \( \nu_\mu \) form a quasi-degenerate massive state with \( \nu_s \) (\( m_{\nu_\mu} \simeq m_{\nu_s} \simeq m \sim 1 - 5 \text{ eV} \)), while the mass of the active neutrino state is \( m_{\nu_3} \simeq m' \sim 3 \cdot 10^{-3} \text{ eV} \). The sterile neutrino state is kept light by the symmetry \( R \times U(1)^9 \). For atmospheric and solar neutrino oscillation parameters we obtain respectively

\[
\Delta m_{\nu_1 \nu_2}^2 \simeq 3 m m' \epsilon_X^2 \simeq 10^{-3}\text{eV}^2, \\
\sin^2 2\theta_{\mu s} = 1 - O(\epsilon_X^2),
\]

\[
\Delta m_{31}^2 \simeq m_3^2 \simeq 10^{-5}\text{eV}^2, \\
\sin^2 2\theta_{e\tau} \sim 4\epsilon_X^4 \simeq 6 \cdot 10^{-3},
\]

which are in good agreement with the latest atmospheric [9] and solar neutrino data [26]. In (80) we have taken \( m \simeq 3 \text{ eV} \). One active neutrino with mass in this range can contribute roughly 15% to the critical energy density of the universe. Models of structure formation with cold and hot dark matter [27] are in good agreement with the observations.

Integration of the heavy \( N, \nu^c \) states will not change this picture if we introduce states \( N_1, N_2 \), with proper transformation properties, and include couplings \( N_1 \nu_3 \bar{H} \) and \( N_2 \nu_1 \bar{H} \) in the theory. Then it is easy to verify that after decoupling of heaviest states, the matrix (78) will not be affected and the results (80), (81) will still hold.

\footnote{For obtaining light sterile states the \( R \) symmetry was also used in ref. [25].}
6 Conclusions

Inspired by superstring theories, we have discussed how a realistic ‘low energy’ phenomenology can arise from a supersymmetric $SU(6) \times U(1)$ gauge theory. In order to realize this goal, one must supplement the gauge symmetry with additional symmetries, and we have provided one example of this, to wit, the symmetry $\mathcal{R} \times U(1)$, where $\mathcal{R}$ denotes a discrete $R$-symmetry (it is likely that the continuous $U(1)$ symmetry can be replaced by some discrete symmetry which ‘effectively’ behaves as $U(1)$). The model we have presented has several interesting features. The gauge hierarchy problem is resolved by the pseudo-Goldstone mechanism, and the proton lifetime is consistent with observations. The parameter $\tan \beta$ turns out to be of order unity. Finally, the $\mathcal{R} \times U(1)$ symmetry also enables us to explain the observed fermion mass hierarchies and mixings, and it plays an essential role in providing a ‘light’ sterile neutrino that is needed here for a simultaneous explanation of the solar and atmospheric neutrino puzzles. The scenario predicts that neutrino hot dark matter constitutes roughly 15-20% of the critical energy density.

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