We describe the many pathways to generate Majorana and Dirac neutrino mass through generalized dimension-5 operators a la Weinberg. The presence of new scalars beyond the Standard Model Higgs doublet implies new possible field contractions, which are required in the case of Dirac neutrinos. We also notice that, in the Dirac neutrino case, the extra symmetries needed to ensure the Dirac nature of neutrinos can also be made responsible for stability of dark matter.
I. INTRODUCTION

The discovery of neutrino oscillations [1, 2] represents a milestone in particle physics, with far-reaching implications. Indeed, the existence of neutrino mass provides a fundamental guide for the nature of the new physics that may lie “beyond the desert” [3]. Given their charge and color neutrality, massive neutrinos are generally expected to be Majorana [4] irrespective of the nature of the mechanism engendering their mass. However, the characteristic signature [5] of Majorana neutrinos, namely, the observation of neutrinoless double beta decay ($0\nu\beta\beta$), has so far remained elusive [6–8].

In this paper we sketch the landscape of theories where neutrino mass arises at the dimension-5 operator level within a broader perspective. As an introduction, we revisit the simplest case of Majorana neutrinos where, as shown by Weinberg [9], this operator is unique if only Standard Model scalar fields are considered. Its simplest high energy completion requires new messenger particles. For example, the tree-level exchange of heavy singlet fermions induces small neutrino masses through what is now generically called “type-I seesaw mechanism” [4, 10–14]. The heavy messengers can also be extra scalar Higgs bosons, such as scalar triplets in the “type-II seesaw” realization [4]. The resulting theoretical pathways to Majorana seesaw were nicely synthesized in Ref. [15].

The purpose of this paper is to provide a wider roadmap for tree level neutrino mass generation within Weinberg’s approach. In doing so we encounter yet unexplored Majorana seesaw realizations. In addition, we discuss the possibilities for the case of Dirac seesaw [1]. We note that Dirac neutrinos can indeed arise in many symmetry–based scenarios for generating neutrino mass in generalized versions of the seesaw mechanism [16, 17]. Motivated by the growing interest in theories with Dirac neutrinos [16, 17], we generalize Weinberg’s approach to this case. We sketch the architecture of neutrino mass generation through the seesaw mechanism when the neutrino mass is of the Dirac type. The methodology is the same as in the Majorana case, but there are many more possibilities, even at the lowest, dimension-5 level. The reason is that more Higgs multiplets, beyond that of the Standard Model are necessarily required, leading to many possible field contractions. Since in order to obtain Dirac neutrinos we need extra symmetries beyond those of the Standard Model, we also examine the possibility to utilize them to stabilize dark matter as well.

The plan of this paper is as follows. In Section II we revisit the well known case of Majorana neutrino mass generation through the Weinberg operator and its possible seesaw completions. In addition, we discuss new ways to obtain Majorana neutrino mass through “generalized Weinberg” operators involving new scalar fields. These can arise in contexts where the standard Weinberg operator is forbidden by symmetry. We also briefly discuss some of their possible seesaw realizations. In Section III we move on to the case of Dirac neutrinos. In this case, with Standard Model fields only, there is no dimension-5 possibility for Dirac neutrinos. Within the minimal Standard Model Higgs sector, seesaw Dirac neutrino masses can only be realized at the dimension-6 level or higher, as shown in Sec. IIIA. In Section IIIB we discuss the generation of Dirac neutrino mass from generalized Weinberg operators which involve other scalars beyond the Standard Model Higgs doublet. We also discuss the various possibilities for Dirac seesaw completion of such generalized Weinberg operators. In order for neutrinos to be Dirac particles, an additional symmetry is required to protect its Dirac nature. In Section IV we show that, quite generally, such symmetries can be used to stabilize dark matter [20].

II. MAJORANA NEUTRINOS

The aim of this section is to provide a brief introduction to seesaw constructs and to set up our notations. We start our task by summarizing the procedure to induce neutrino mass at the operator level, for the case of Majorana
neutrino masses. First we take the simplest case where the theory only contains Standard Model fields and then generalize to the case where new fields are present.

A. Only Standard Model fields

It is a well known fact that, for the case of Majorana neutrinos, if only Standard Model fields are present, there is a unique dimension-5 operator that gives rise to neutrino masses, the well-known Weinberg operator \[ \left( \begin{array}{c} \frac{1}{\Lambda} L^c \otimes \Phi \otimes \Phi \otimes L \\ \end{array} \right) \]

where \( L \) and \( \Phi \) denote the lepton and Higgs doublets, and \( \Lambda \) represents the cutoff scale. Above the cutoff scale the Ultra-Violet (UV) complete theory is at play, involving new “messenger” fields, whose masses lie close to the scale \( \Lambda \).

In order to set the stage for our discussion, we start by recalling the basic features of the operator in (1). There are three different ways of contracting the relevant fields, associated to the tree-level exchange of different messengers. The UV completions of these three different contraction possibilities correspond to the so-called type I, II and III seesaw mechanism:

\[ \left( \begin{array}{c} \frac{1}{1} L^c \otimes \Phi \otimes \Phi \otimes L \\ \end{array} \right), \quad \left( \begin{array}{c} \frac{1}{3} L^c \otimes \Phi \otimes \Phi \otimes L \\ \end{array} \right), \quad \left( \begin{array}{c} \frac{1}{3} L^c \otimes \Phi \otimes \Phi \otimes L \\ \end{array} \right) \]

In the underbrace denotes a \( SU(2)_L \) contraction of the fields involved. The number under the brace denotes the \( SU(2)_L \) transformation of the contracted fields. Although not explicitly written, the global contraction should always be a singlet for the operator to be allowed by \( SU(2)_L \) gauge symmetry. The UV complete realization of these contractions results into the three well known seesaw variants as shown in a diagramatical way in Figure 1.

Figure 1. Feynman diagram generating Majorana masses in Type I, II and III seesaw mechanism.

Figure 1 illustrates three UV–complete seesaw realizations of the same Weinberg operator of (2), differing from each other in the nature of the messenger fields involved. In the left–panel diagram of Fig. 1 corresponding to type-I seesaw, the field \( N_R \) is a heavy fermion which transforms as a singlet under \( SU(2)_L \) and carries no \( SU(3)_C \) or \( U(1)_Y \) charge. In the middle diagram corresponding to type-II seesaw, the field \( \Delta^0 \) is the neutral component of a heavy scalar multiplet transforming as triplet under \( SU(2)_L \). In the right–panel diagram corresponding to type-III seesaw, \( \Sigma^0 \) is the neutral component of the heavy fermion multiplet transforming as triplet under \( SU(2)_L \) symmetry.

Notice that the three possible messenger fields and their \( SU(2)_L \) transformation properties arise from the different possibilities of field contractions of the Weinberg operator as shown in (2). The possibility where both \( L^c \otimes L \) and \( \Phi \otimes \Phi \) contract to a singlet is forbidden, since \( \Phi \otimes \Phi \) is symmetric, while the singlet contraction is antisymmetric, and therefore vanishes. Even in the presence of another Higgs doublet, the singlet contraction would vanish due to electric charge conservation.\(^3\)

\(^3\) Such messengers may, however, be used in radiative schemes of neutrino mass generation.\[47\].
Weinberg’s dimension-5 operator is the lowest one which can generate Majorana neutrino masses. One can easily generalize the above discussion to higher dimensional operators (a similar discussion for radiative mass generation was given in [18]). In general, using only Standard Model fields, i.e. a single Higgs doublet $\Phi$, it is easy to show that the $SU(2)_L$ symmetry implies that Majorana masses can only be generated by odd-dimensional operators. This means that they can only arise from operators involving even number of Higgs doublets. The general operator allowed by $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry is

$$\frac{1}{\Lambda^{2n+1}} \bar{L}^c \Phi^2 (\Phi^\dagger \Phi)^n L, \quad n \in \{0, 1, 2, 3, \ldots\}$$

(3)

where $\Lambda$ is the cutoff scale, i.e. the scale at which the new physics associated to the messengers required for UV completion comes into play.

B. Models with new Higgs fields

Weinberg’s operator and its higher dimensional siblings only involve Standard Model fields. However, the new physics responsible for generating neutrino mass might be such that the operators involving only Standard Model Higgs are forbidden. Such a scenario can also occur in many contexts. One well known example is provided by models where supersymmetry is the origin of neutrino mass [19] through the spontaneous breaking of R-parity [50–52]. In this case the messengers are supersymmetric states. As an alternative example consider the possibility of global Lepton number $(U(1)_L)$ symmetry, broken to its $Z_n$ subgroups [43]. Where $U(1)_L$ breaks to a $Z_2$ subgroup, the Weinberg operator is in principle allowed. However, in scenarios where $U(1)_L$ breaks to higher $Z_n$ symmetries then Weinberg’s operator can be easily forbidden [20, 23, 34]. As a simple example, consider the scenario where $U(1)_L$ is explicitly broken to a $Z_3$ subgroup by messenger field mass terms. In such a case for the lepton doublet transforming non-trivially under $Z_3$ say $L \sim \omega$, with $\omega^3 = 1$ and the Standard Model Higgs being neutral under $Z_3$ it is easy to see that the Weinberg operator as well as other higher dimensional ones involving only Standard Model fields are all forbidden. One can still generate Majorana masses for neutrinos if the new physics involved also contains additional scalars (carrying nontrivial $U(1)_L$ or $Z_n$ charges) beyond Standard Model Higgs. In this section we briefly discuss such possibilities, most of which to the best of our knowledge, have not yet been explored in the literature.

These new possibilities arise in the presence of new scalars carrying a nonzero vacuum expectation value (vev), such as a scalar $\chi \sim 1$, singlet under the $SU(2)_L$ symmetry, or the field $\Delta \sim 3$, triplet under $SU(2)_L$ [4]. In order to generate Majorana masses some (or all) of these new vev-carrying scalars must also be charged under the symmetry that forbids the Weinberg operator, such as $Z_3$ in the simple example considered above.

The tree level coupling between $\bar{L}^c \otimes L$ (dimension 3) is forbidden by $U(1)_Y$. For the same reason, the dimension-4 operator $\bar{L}^c \otimes \chi \otimes L$ is also forbidden, unless $\chi$ is charged under $U(1)_Y$. In such case electric charge conservation prevents it from having a vev. However, one could in principle write the dimension 4 operator with a triplet $\Phi$.

$$\bar{L}^c \Delta L.$$  

(4)

Though in principle allowed, this would require a tiny vev for the triplet $\Delta$, in order to account for the smallness of neutrino masses. An appealing possibility is to have this term forbidden by symmetry, and to consider instead, “Weinberg like” dimension 5 operators of the form

$$\frac{1}{\Lambda} \bar{L}^c \otimes X \otimes Y \otimes L$$

(5)

involving two different color singlet scalar fields $X$ and $Y$, transforming as multiplets of $SU(2)_L$ and with appropriate $U(1)_Y$ charges so as to make $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ invariant. Of course, if we take $X \equiv Y \equiv \Phi$, such

4 Here we restrict to singlets and triplets, although in principle one can also consider scalar fields in higher $SU(2)_L$ multiplets.

5 Note that higher $SU(2)_L$ scalar multiplets can not couple to $L$ at the dimension 4 level, due to $SU(2)_L$ symmetry.

6 Note that the same symmetry forbidding the dimension 4 term may also forbid the conventional dimension 5 Weinberg operator.
generalized Weinberg operator in (5) reduces to the standard one in (1). Note that the simplest possibility for generalized Weinberg operator where $X = Y = \chi$ where $\chi$ is a $SU(2)_L$ singlet, is forbidden by charge conservation. However, in general the $X$ and $Y$ fields can be distinct and this opens up more possibilities. In particular, if $X$ transforms as the “$n$-th multiplet” under $SU(2)_L$, then $Y$ has to transform as $n - 2$, $n$ or $n + 2$ multiplet of $SU(2)_L$ with appropriate $U(1)_Y$ charges. For example, if the $X$ field is a $SU(2)_L$ singlet, $X \equiv \chi$, then the $Y$ field must be either a $SU(2)_L$ singlet or triplet. Therefore, the first operator allowed by gauge invariance would be $\frac{1}{2} \langle \Delta \rangle \nu \nu L$.

This operator can be opened up:

\[
\begin{align*}
\langle \chi \rangle & \quad \langle \Delta \rangle \\
\frac{1}{2} \langle \chi \rangle \nu \nu L & \quad \frac{1}{3} \langle \chi \rangle \nu \nu \nu \nu L \\
\text{Type I like} & \quad \text{Type II like}
\end{align*}
\]

As before, the underbrace denotes the relevant field contraction, and the number under the brace denotes the $n$-plet transformation of the contracted fields under $SU(2)_L$. In order to preserve the gauge invariance, the global contraction should be an $SU(2)_L$ singlet. The first type-I like possibility in (6) requires vector-like heavy leptons $E_L, E_R$ transforming as $SU(2)_L$ doublet, as shown in the left diagram in Fig. 2. The second, type-II like, requires the presence of an additional $SU(2)_L$ triplet scalar $\Delta'$ carrying an induced vev through its coupling with the $\chi, \Delta$ fields, see the right diagram in Fig. 2.

Figure 2. The Feynman diagrams for the two seesaw realizations for the generalized Weinberg operator in (6).

There are other possible choices for the $X$ and $Y$ scalars. For example, in case of $X \equiv \Phi$ i.e. a $SU(2)_L$ doublet, the $Y$ field can only transform as a doublet or as a quadruplet ($\Xi$) of $SU(2)_L$. In this case, in addition to the standard Weinberg operator $\frac{1}{2} \langle \chi \rangle \nu \nu L$, we can also have the generalized one $\frac{1}{2} \langle \chi \rangle \nu \nu \nu \nu L$. On the other hand, if $X \equiv \Delta$ i.e. a $SU(2)_L$ triplet, the $Y$ field can only transform as singlet, triplet or quintuplet ($\Omega$) under $SU(2)_L$. In this case, in addition to $\frac{1}{2} \langle \chi \rangle \nu \nu L$, already discussed, one has two other generalized Weinberg operators $\frac{1}{3} \langle \chi \rangle \nu \nu \nu \nu L$ as well as $\frac{1}{4} \langle \chi \rangle \nu \nu \nu \nu \nu \nu L$. These latter cases seem not to have been explored in the literature. However, we will not develop the detailed analysis of the new fields required for their UV–completion in this paper.

Note also that the dimension 5 operator of (5) can easily be generalized to other higher dimensional operators as

\[
\frac{1}{\Lambda^{n-1}} \langle \chi \rangle \nu \nu L \otimes X_1 \otimes \cdots \otimes X_n \otimes L
\]

where $X_i$, $i = 1, \ldots, n$ are scalar fields, at least some of which lie in different $SU(2)_L$ multiplets, and carry appropriate $U(1)_Y$ charges. Again, we will not explore such possibilities any further.

III. DIRAC NEUTRINOS

With the above brief recap of the familiar case of Majorana neutrinos, we now move on to the possibility of naturally small Dirac neutrino masses arising from dimension 5 generalized “Weinberg-type” operators. Indeed, there has been a renewed interest in Dirac neutrino mass generation mechanisms in the recent literature [18–46]. Some of these works have also considered several possibilities of generation of neutrino masses using operator methods like ours [35–39]. However, a systematic classification of all possibilities at a given dimension for generalized Weinberg
operators is still lacking. In this paper and a follow up work we aim to consider all possible generalized Weinberg operators at dimension 5 and dimension 6 level. Since there are actually infinite such possibilities, we will restrict our discussion only to operators involving scalars which transform either as singlet, doublet or triplet under $SU(2)_L$ symmetry. Furthermore we will only consider the seesaw high energy completions of these operators and will not consider the various loop realizations of these operators. Our method illustrated here can easily be generalized to include higher $SU(2)_L$ multiplets as well as loop realizations of the operators.

Before beginning our discussion on possible Dirac seesaw completions of generalized Weinberg operators, let us mention certain generic conditions which must be satisfied in order to have Dirac neutrinos. By definition, a Dirac fermion can be viewed as two chiral fermions, one left handed and other right handed, having exactly degenerate masses [4]. Thus in order to have massive Dirac neutrinos one must extend the Standard Model particle content by adding the right handed partners of the known neutrinos, $\nu_R$, being singlets under $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry. Second, owing to the color and electric charge neutrality of neutrinos, an additional exactly conserved symmetry beyond the Standard Model gauge symmetries is required to protect the Diracness of neutrinos. Several ways have been proposed to protect the Dirac nature of neutrinos, involving symmetries such as an extra $U(1)$ lepton number symmetry [28, 32] or its discrete $Z_n$ subgroups [19, 33]. In our discussion here we will take lepton quarticity, a discrete $Z_4$ lepton number symmetry, as a benchmark symmetry used to protect the Dirac nature of neutrinos [20, 23, 34]. As we will see in Sec. IV, such symmetry has an additional virtue of ensuring the stability of dark matter and is thus a very attractive possibility.

### A. Only Standard Model fields

Here we consider the possibility of having just the minimal Standard Model Higgs doublet augmented by three right handed singlet neutrinos. The presence of a lepton quarticity symmetry under which both the lepton doublet $L$ and the right handed neutrinos $\nu_R$ transform as $z$, with $z^4 = 1$, ensures that the neutrinos remain Dirac particles provided this symmetry remains exact [20, 23, 34]. This implies that all scalar fields which develop vev should be neutral under the $Z_4$ quarticity symmetry. This requirement has profound implications for the stability of dark matter, as we will discuss in section IV. With only Standard Model Higgs the simplest possibility to generate Dirac mass is through the dimension 4 Yukawa interaction

$$ y_\nu \bar{L} \Phi^c \nu_R $$

where $y_\nu$ is the Yukawa coupling constant. Although this possibility is allowed on theory grounds, it leaves the smallness of neutrino masses unexplained, implying the need for a tiny Yukawa coupling for neutrinos ($y_\nu \sim O(10^{-13})$).

A more attractive possibility would be to forbid this term by an adequate symmetry and to obtain naturally small neutrino masses through generalized Weinberg operators or their higher dimensional counterparts. Such a scenario can easily arise in many ways, for example, from a simple $Z_2$ symmetry [20], from flavor symmetries [23, 34] or from an unconventional $U(1)_{B-L}$ symmetry [28, 32].

In general, using only the Standard Model Higgs doublet $\Phi$, the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry implies that the only allowed dimensions for the operators that can induce Dirac neutrino masses are even, i.e. operators involving odd number of Higgs doublets, namely

$$ \frac{1}{\Lambda^{2n}} \bar{L} \Phi^c (\Phi^\dagger \Phi)^n \nu_R, \quad n \in \{0, 1, 2, 3, 4, \ldots \} $$

Therefore, after the dimension 4 Yukawa term of (8), the next allowed operator involving only Standard Model Higgs would be of dimension 6. Here we restrict ourselves to the discussion only of dimension 5 operators, and hence we will not develop this possibility and its various UV–completions in detail.
B. Models with new Higgs fields

As argued before, at the dimension 5 level, Dirac neutrino masses can only arise from generalized Weinberg operators involving additional scalars beyond the Standard Model Higgs. As before, we will restrict our discussion to generalized Weinberg operators involving singlet (χ), doublet (Φ) and triplet (∆) fields only. However, our analysis can easily be generalized to the cases of scalars transforming as higher SU(2) multiplets. The generalized dimension 5 Weinberg operator for Dirac neutrinos is given by

$$\frac{1}{\Lambda} \bar{L} \otimes X \otimes Y \otimes \nu_R$$

where X and Y are scalar fields transforming as some n-plets of SU(2) with appropriate U(1)Y charges.

Invariance of (10) under SU(2)L symmetry implies that, if X transforms as a n-plet under SU(2)L, then Y must transform either as a n + 1-plet, or a n − 1-plet under SU(2)L symmetry. For example, if we take X to be a singlet then Y should be a doublet. If we take X to be a doublet then Y can only be a singlet (equivalent to the previous case) or a triplet.

Each of these cases leads to different SU(2)L contractions which, as we will see shortly, will lead to different seesaw UV completions. For example, for the case X = χ i.e. a singlet under SU(2)L symmetry and Y = Φ i.e. a doublet under SU(2)L symmetry, we have the following possible contractions, which can be viewed as Dirac analogues of the type I, II and III Majorana seesaw mechanism,

$$\begin{align*}
\bar{L} \otimes \Phi^c \otimes \chi \otimes \nu_R, & \quad \text{(Type I analogue)} \\
\bar{L} \otimes \nu_R \otimes \Phi^c \otimes \chi, & \quad \text{(Type II analogue)} \\
\bar{L} \otimes \chi \otimes \Phi^c \otimes \nu_R, & \quad \text{(Type III analogue)}
\end{align*}$$

where again the underbrace denotes a contraction of the fields involved and the number under the brace denotes the n-plet contraction of SU(2) to which the fields contract. Note that invariance under U(1)Y requires that Φc should appear in this operator. The global contraction should be a singlet in order that the operator is allowed by SU(2)L.

The seesaw completion of these operators will lead to three different possibilities. These diagrams are shown in figure 3.

Figure 3. Feynman diagrams representing the Dirac Type I, II and III seesaw analogues.

As in the previous cases, the fields N_L and N_R are heavy fermions transforming as singlets under SU(2)_L, σ^0 is the neutral component of a SU(2)_L doublet scalar, and E^0_L and E^0_R are the neutral components of the heavy vector like fermions transforming as doublets under SU(2)_L. Note that the new SU(2)_L doublet scalar σ must be a new scalar, and cannot be identified with the Standard model Higgs Φ, as such an identification will also imply presence of a tree-level Dirac neutrino mass term we assumed to be forbidden by symmetries.

Some of the corresponding UV–complete theories have been discussed in the literature, while others have not. For example, explicit models employing type I Dirac seesaw have already been realized in [20, 23, 28, 34], while explicit models for type II Dirac seesaw where considered in [18, 19, 21, 22]. In contrast, to the best of our knowledge, a full–fledged UV–complete theory using the Dirac type III seesaw has so far not been explicitly developed.
Going beyond singlets and doublets opens up still more possibilities. For example, taking $X = \Phi$ and $Y = \Delta$ yields

$$\begin{align*}
\bar{L} \otimes \nu_L \otimes \Phi \otimes \Delta, & \quad \text{Type II like} \\
\bar{L} \otimes \Delta \otimes \Phi \otimes \nu_R, & \quad \text{Type III like} \\
\bar{L} \otimes \phi \otimes \Delta \otimes \nu_R, & \quad \text{Type III like}
\end{align*}$$

(12)

The underbrace denotes, as before, field contraction, and the number under the brace denotes the $n$-plet contraction of $SU(2)_L$ to which the fields reduce. The global contraction should be an $SU(2)_L$ singlet. Note that for this operator we have two possibilities for $U(1)_Y$ charge of $\Delta$. Apart from the operator in (12) (which has $\Delta$ with $U(1)_Y = -2$) another operator namely $\bar{L} \Phi^c \Delta_0 \nu_R$ with $\Delta_0$ carrying $U(1)_Y = 0$ is also possible. The diagrams for this case will be identical to those discussed here but the hypercharges of the intermediate fields will be different. Note that one can always induce the vevs of either $\chi$ or $\Delta$ with the coupling to a pair of $\Phi$’s. Such operators will have dimension 6 and will be discussed in a follow-up work. The diagrams leading to the seesaw completion of (12) are shown in Figure 4.

![Figure 4. Feynman diagrams for Type-II and Type-III Dirac seesaw mechanism.](image)

where, as before, the field $\Delta^0$ is the neutral component of the $SU(2)_L$ triplet, $\Sigma^0_L$ and $\Sigma^0_R$ are the neutral components of the heavy $SU(2)_L$ triplet fermions, $\sigma^0$ is the neutral component of an $SU(2)_L$ doublet scalar $\sigma$, while $E^0_L$ and $E^0_R$ are the neutral components of the heavy vector–like fermions transforming an $SU(2)_L$ s doublet. As before, owing to the symmetry requirements, $\sigma$ must be a new $SU(2)_L$ doublet, distinct from the Standard model Higgs $\Phi$. There have been so far no dedicated study of UV–complete theories in literature corresponding to these new possibilities.

Going yet to higher multiplets of $SU(2)_L$ will open up novel ways to generate Dirac neutrino mass at the dimension 5 level. These can easily be realized following our procedure in a straightforward way, so here we skip the details. Furthermore, as already discussed, Dirac neutrino masses can also be generated at higher dimensions. The general operator for such a scenario involves several scalar fields $X_i$; $i = 1, \cdots, n$ which can be different $SU(2)_L$ multiplets, carrying appropriate $U(1)_Y$ charges, as follows

$$\frac{1}{\Lambda^{n-1}} \bar{L} \otimes X_1 \otimes \cdots \otimes X_n \otimes \nu_R$$

(13)

Note that some of the $X_i$ may coincide. In this paper we will not develop such possibilities, which we plan to do as a follow-up work.

**IV. DIRAC NEUTRINOS AND DARK MATTER STABILITY**

As stated before, if neutrinos are Dirac particles an additional symmetry is required to protect their Dirac nature. Various types of additional symmetries have been used in many different new physics scenarios. However one of the simplest possibilities for such a symmetry is a discrete residual $\mathbb{Z}_4$ subgroup arising from the breaking of the usual Standard Model global $U(1)_Y$ symmetry, by the new physics. Such discrete lepton number symmetries can indeed ensure that the Dirac nature of neutrinos can be consistently preserved. As an example, the discrete $\mathbb{Z}_4$ symmetry, called lepton quarticity, has been employed in actual UV–complete models in Refs. [20] [23] [34].

An attractive feature of such constructions is that the same symmetry which ensures the Dirac nature of neutrinos can also provide stability to the dark matter particle, thus linking dark matter stability with Dirac nature of neutrinos in an intimate way [20].
In this section we argue that this connection may be quite deep. The Dirac nature of neutrinos and dark matter stability can be accomplished by the same symmetry, irrespective of the details of the particular mass model, and of the nature of their UV–completion. For illustration, take the case of $Z_4$ lepton quarticity symmetry. The $Z_4$ group admits only one–dimensional irreducible representations, conveniently represented by the fourth roots of unity $z, z^4 = 1$. Such quarticity symmetry in context of Dirac neutrinos may arise as a residual subgroup of $U(1)_{B-L}$. Under the quarticity symmetry the lepton doublets $L$ and right handed neutrinos $\nu_R$ transform as $z, z^7$. Since the quarticity symmetry is preserved, no scalar fields which obtain a non-zero expectation value should be charged under $Z_4$. Thus we have:

\[ \text{If } \langle X_i \rangle \neq 0, \text{ then } X_i \sim 1 \text{ under } Z_4 \]
\[ \text{If } \zeta_i \sim 1 \text{ under } Z_4, \text{ then } \langle \zeta_i \rangle = 0. \]

where $X_i, \zeta_i; i = 1, \cdots n$ denote the scalar fields. The above constraints have a profound effect in a completely unexpected direction. Consider for example, the generalized Weinberg operator for Dirac neutrinos of (10). Here, the scalar fields $X, Y \sim 1$ under $Z_4$ to preserve the Diracness of neutrinos.

Consider now another scalar field $\zeta$, singlet under Standard Model gauge symmetry, but transforming as $\zeta \sim z$ under the $Z_4$, hence carrying no vev i.e. $\langle \zeta \rangle = 0$. Its interactions with the other fields are severely restricted by $Z_4$. Indeed, notice that the Yukawa coupling of $\zeta$ with any fermion, as well as the cubic couplings with the scalars $X_i$, i.e. $X_i^\dagger X_i \zeta$, which would lead to its decay, are all forbidden by the $Z_4$.

In order to make sure that the $Z_4$ symmetry stabilizing the $\zeta$ field has indeed a (discrete) lepton number nature, one needs a “messenger” field $\eta$ connecting the $Z_4$ charges of neutrinos with that of $\zeta$ [20]. This is easily accomplished by having a new scalar field $\eta$, singlet under the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge group, but transforming as $z^2$ under $Z_4$ symmetry. Since $\eta$ has nontrivial $Z_4$ charge, we must require $\langle \eta \rangle = 0$. Owing to its $Z_4$ charge, $\eta$ has a Yukawa coupling to right handed neutrinos, $\bar{\nu}_R \nu_R \eta$, as well as a cubic $\eta \zeta \zeta$ coupling term, as seen in Fig. 5. Both terms are invariant under the $Z_4$ as well as the Standard Model gauge group. The $\eta$ field thus connects neutrinos to dark matter, through the diagram shown in the Fig. 5.

Notice that the $Z_4$ symmetry remains conserved even after $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry breaking takes place. Hence the stability of the $\zeta$ field is also ensured after electroweak symmetry breaking, making it a "bona fide" dark matter candidate, whose stability emerges from the same quarticity symmetry responsible for protecting the Dirac nature of neutrinos. Due to the presence of extra singlet scalar bosons, its relic density is somewhat model–dependent, due to new diagrams contributing to the dark matter annihilation. Nevertheless, dark matter relic abundances are typically WIMP-like.

Concerning its interaction with Standard Model particles, which determine its nuclear recoil direct detection cross section, it proceeds through the so-called Higgs portal and has been discussed in [20], see Fig. 4. Recent direct WIMP dark matter searches in LUX [53], PandaX [54] and Xenon1T [55] lead to constraints on dark matter masses versus coupling strength which are slightly improved in comparison to analysis of [20]. For the low mass Higgs–portal dark matter region, below half of the Higgs boson mass, there are somewhat improved constraints on the Higgs invisible decay width which come from recent searches at the LHC [56].

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7 If such quarticity symmetry is a remnant of the $U(1)_{B-L}$ symmetry, then the quarks can also transform as $z$ under quarticity.
Figure 5. Coupling between right handed neutrinos $\nu_R$ and the WIMP scalar dark matter particle $\zeta$, mediated by the scalar $\eta$.

As a final comment we stress that the above argument can also be extended to higher dimensional Weinberg operators, such as those in [13]. Hence the connection between the Dirac nature of neutrinos and dark matter stability can be made quite generic, irrespective of the dimensionality of the operators leading to Dirac neutrino mass generation, as well as the details of the particle content involved in the UV–completion.

V. SUMMARY AND DISCUSSION

Here we have described the various ways to induce neutrino masses through generalized dimension-5 Weinberg operators, both for the case of Majorana as well as Dirac neutrinos. In both cases, the need for extra scalar multiplets beyond the Standard Model Higgs doublet, implies new possible field contractions. These are absent in the simple case of the Majorana seesaw mechanism obtained from the UV–completion of the conventional Weinberg operator. We have identified many possibilities which have already been employed within specific models, as well as novel ones. We have also noticed that, for the case of Dirac neutrinos, the extra symmetries required for “Diracness” can, rather generically, play a double role in also ensuring the existence of a stable WIMP dark matter candidate.

As a final remark, we have stressed that the connection with WIMP dark matter that we have proposed can be made quite general, irrespective of model realizations. We stress that our proposed connection between neutrino masses and dark matter is rather different from most of the existing ones, such as the “scotogenic” approach [57], where the WIMP dark matter particle is a radiative neutrino mass messenger [58, 59], and whose simplest realization yields Majorana neutrinos, instead of Dirac neutrinos.

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