Quantum radiation by electrons in lasers and the Unruh effect

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In addition to the Larmor radiation known from classical electrodynamics, electrons in a laser field may emit pairs of entangled photons – which is a pure quantum effect. We investigate this quantum effect and discuss why it is suppressed in comparison with the classical Larmor radiation (which is just Thomson backscattering of the laser photons). Further, we provide an intuitive explanation of this process (in a simplified setting) in terms of the Unruh effect.

I. INTRODUCTION

With the availability of strong and stable lasers in a very clean background, we are rapidly approaching a regime in which relativistic quantum effects start to play a role. As a simple example, let us consider an electron (of mass \( m \) and charge \( q \)) in a laser field (which we approximate by a plane wave). Here we assume that the laser has a small (e.g., optical) frequency \( \omega \) and a large (but not too large) Keldysh adiabaticity parameter (i.e., small electric field strength \( E \) when normalized w.r.t. \( m \))

\[
\omega \ll m, \quad \gamma = \frac{m \omega}{qE} \gg 1. \tag{1}
\]

In this case, the electron oscillates non-relativistically and nearly harmonically and thus emits radiation of the laser frequency – which is just the Larmor radiation known from classical electrodynamics. Alternatively, one can view this classical process as Thomson (i.e., low-energy Compton) backscattering of the laser photons. The lowest-order Feynman diagrams of this process are depicted in Fig. 1. Due to \( \omega \ll m \), the recoil of the electron can carry away momentum, but basically no energy \( p_{\text{out}}^2/m \ll \omega \) and thus we can directly read off the classical resonance condition \( |k| = \omega \) (energy conservation).

However, in addition to these lowest-order Feynman diagrams in Fig. 1 there are also further (next-to-leading-order) Feynman diagrams such as the one sketched in Fig. 2. They correspond to the emission of two photons with momenta \( k_1 \) and \( k_2 \). With the same argument as before, the resonance condition (energy conservation) now reads \(|k_1| + |k_2| = \omega\), i.e., the sum of the energies of the two emitted photons equals the laser frequency. However, if the electron oscillates with frequency \( \omega \), the emission of two photons with, say, half the energy each \(|k_1| = |k_2| = \omega/2\) cannot be explained within classical electrodynamics. It is, therefore, a pure quantum effect. This interesting observation poses two major questions:

- How can this effect be understood?
- One would expect that this quantum effect is suppressed compared to the classical process in Fig. 1. If this is correct, how does this suppression work?

In the following, we shall discuss these two questions.

II. CLASSICAL RADIATION

Let us first discuss the classical process, which is just Larmor radiation known from classical electrodynamics and can be understood as Thomson (or Compton) scattering of the incoming laser photons by the electron. We employ standard notation with \( a \cdot p = a_\mu p^\mu \) and \( \phi = a_\mu \gamma^\mu \) etc. Here \( p_{\text{in}} \) and \( p_{\text{out}} \) are the initial and final four-momenta of the electron, respectively, and \( u_{\text{in}} \) as well as \( u_{\text{out}} \) are the associated spinor solutions. The amplitude and momentum of the laser photon are denoted by \( a_L \) and \( k_L \), whereas \( a_1 \) and \( k_1 \) describe the emitted photon. The first Feynman diagram in Fig. 1 generates the amplitude

\[
A_{\text{class}} = u_{\text{in}} \phi_L \frac{1}{p_{\text{in}} \cdot k_L - m} q q_{\text{out}}. \tag{2}
\]

Since \( p_{\text{in}} \) and \( k_L \) are on-shell, the square of the denominator in the electron propagator yields \( 2p_{\text{in}} \cdot k_L \). In the following, we go into the (initial) rest frame of the electron \( p_{\text{in}} = (m, 0) \) which yields \( 2p_{\text{in}} \cdot k_L = 2m\omega_L \). Furthermore, we perform an expansion into (inverse) powers of the electron mass \( m \) where \( \sim \) denotes the scaling in leading order

\[
A_{\text{class}} \sim \frac{q^2}{m\omega_L} u_{\text{in}} \phi_L (p_{\text{in}} + k_L + m) q_{\text{out}} \tag{3}
\]
Using the Clifford algebra $\gamma_L \gamma^+_\text{in} + \gamma^+_\text{in} \gamma_L = 2p_{\text{in}} \cdot a_L$ and $u_{\text{in}}(\gamma^+_\text{in} - m) = 0$, we get

$$A_{\text{class}} \sim \frac{q^2}{m \omega L} u_{\text{in}}(\gamma_L \gamma^+_L \gamma^+_1 + 2p_{\text{in}} \cdot a_L \gamma^+_1)u_{\text{out}}.$$ \hspace{1cm} (4)

In leading order in $1/m$, we may neglect the recoil of the electron $p_{\text{out}} \approx p_{\text{in}}$ and set $u_{\text{out}} \approx u_{\text{in}}$ which leads to

$$A_{\text{class}} \sim \frac{q^2}{m \omega L} \text{Tr} \left\{ (\gamma_L \gamma^+_L \gamma^+_1 + 2p_{\text{in}} \cdot a_L \gamma^+_1)(\gamma^+_\text{in} + m) \right\}. \hspace{1cm} (5)$$

In temporal (radiation) gauge, we have $p_{\text{in}} \cdot a_1 = 0$ as well as $p_{\text{in}} \cdot a_2 = 0$, which yields the final result

$$A_{\text{class}} \sim q^2 a_1 \cdot a_L + \mathcal{O}\left(\frac{1}{m}\right). \hspace{1cm} (6)$$

This expression reproduces the well-known result that the emitted photon has the same polarization as the laser field, i.e., in the direction of the electron trajectory. (One obtains the same result for the other lowest-order diagram in Fig. [1])

![Fig. 2](image)

**FIG. 2:** One of the (six) lowest-order Feynman diagrams for quantum radiation. The notation is the same as in Fig. [1]

III. QUANTUM RADIATION

Now, having discussed the classical process, let us turn to the quantum radiation, cf. Fig. [2] In this case, we have two emitted photons with momenta $k_1$ and $k_2$ as well as amplitudes $a_1$ and $a_2$. The Feynman diagram in Fig. [2] then generates the amplitude

$$A_{\text{quant}} \sim q^3 \bar{u}_{\text{in}} \gamma^+_1 \bar{p}_{\text{in}} - \bar{k}_1 - m \bar{p}_{\text{out}} - \bar{k}_2 - m \gamma^+_1 u_{\text{out}}.$$ \hspace{1cm} (7)

With manipulations analogous to those in the previous Section, we find the leading-order behavior

$$A_{\text{quant}} \sim \frac{q^3}{m^2 \omega_1 \omega_2} \bar{u}_{\text{in}}(2p_{\text{in}} \cdot a_1 - \gamma^+_1 k_1)\gamma^+_L \times \gamma^+_L \gamma^+_2\gamma^+_2 u_{\text{out}}.$$ \hspace{1cm} (8)

Remembering $p_{\text{in}} \cdot a_1 = \mathcal{O}(m^0)$ and $p_{\text{out}} \cdot a_2 = \mathcal{O}(m^0)$, we see that the amplitude for quantum radiation is suppressed by a factor of $1/m$. Again neglecting the recoil of the electron and setting $u_{\text{out}} \approx u_{\text{in}}$, we get

$$A_{\text{quant}} \sim \frac{q^3}{m^2 \omega_1 \omega_2} \text{Tr} \left\{ (2p_{\text{in}} \cdot a_1 - \gamma^+_1 k_1)\gamma^+_L \times \gamma^+_L \gamma^+_2\gamma^+_2(\gamma^+_\text{in} + m) \right\}. \hspace{1cm} (9)$$

The trace of the products of $\gamma^+_\text{in}$-matrices yields various combinations of scalar products of the four-vectors involved. The leading terms (in $1/m$) are obtained when $\gamma^+_\text{in}$ is combined with $k_1$ or $k_2$ which yields $p_{\text{in}} \cdot k_1,2 = m \omega_1,2 + \mathcal{O}(m^0)$. Considering the case where $\gamma^+_\text{in}$ is combined with $k_1$, the remaining contributions are scalar products of the four-vectors $a_1, a_2, a_L, k_2$. Thus we obtain a polarization entangled part

$$A_{\text{entangl}} \sim \frac{q^3}{m} (a_1 \cdot a_2)(k_1 \cdot a_L) + 1 \leftrightarrow 2, \hspace{1cm} (10)$$

plus a remaining part with non-entangled polarizations, which contains contributions such as $(a_1 \cdot a_L)(a_2 \cdot p_{\text{out}})$ as well as the interchanged terms $1 \leftrightarrow 2$.

In the polarization entangled part $A_{\text{entangl}}$, the polarizations of the two emitted photons are equal, whereas the polarization of each one is undetermined, i.e., we obtain an EPR-like state in the polarization

$$|\text{EPR}\rangle = \frac{\left|\uparrow\downarrow\right> + \left|\downarrow\uparrow\right>}{\sqrt{2}}.$$ \hspace{1cm} (11)

In the remaining contributions, the polarizations of the two outgoing photons are determined separately by the laser field and the momenta involved. If both photons propagate in the same direction as the electron (given by laser polarization $a_L$), these remaining terms vanish and only the entangled part $A_{\text{entangl}}$ survives.

IV. UNRUH EFFECT

With the above estimates, we can already answer the second question posed in the introduction, i.e., the suppression of the quantum effect in Fig. [2] compared with the classical Larmor radiation, cf. Fig. [1]. Of course – as one can simply infer from the Feynman diagrams in Figs. [1] and [2] by counting the vertices – the probability $P_{\text{quant}} \sim |A_{\text{quant}}|^2$ of the quantum effect contains one factor of $\alpha_{\text{QED}} \approx 1/137$ (i.e., the fine-structure constant)
more than the classical probability $P_{\text{class}} \sim |A^2_{\text{class}}|$. However, the power of $\alpha_{\text{QED}}$ is not sufficient for distinguishing classical and quantum effects: As a counter-example, consider the process of two-photon scattering sketched in Fig. 2, which scales with $\alpha^4_{\text{QED}}$ but is still a purely classical effect. Within our approach, the expected suppression of the quantum effect manifests itself in the small ratio $\omega/m$, i.e.,

$$\frac{P_{\text{quant}}}{P_{\text{class}}} = O \left( \alpha_{\text{QED}} \left( \frac{\hbar \omega}{mc^2} \right)^2 \right) \ll 1. \quad (12)$$

Here, we inserted $\hbar$ and $c$ in order to illustrate that we are dealing with a relativistic quantum effect $\sim \hbar/c^2$. Intuitively speaking, the heavier the electron is, the more classically it behaves.

Having addressed the problem of suppression, let us now turn to the first question posed in the Introduction: How can we understand this effect? As a first approach, let us simplify the situation by assuming that the two emitted photons propagate in the same direction as the electron, i.e., $k_1|k_2|e_L$. In this case, we have an effectively 1+1-dimensional situation, where the two polarizations can be treated as two independent scalar fields $\phi_\omega(t, x)$ and $\phi_{\bar{\omega}}(t, x)$. As a further simplification, we replace the harmonic oscillation of the electron by a uniform acceleration $a$. In this case, an observer sitting on the accelerated electron will experience the Minkowski vacuum as a thermal bath with the Unruh temperature $T_{\text{Unruh}} = \frac{\hbar a}{2\pi k_B c}$. \( (13) \)

In view of the finite cross section $\sigma \propto q^4/m^2$ of the electron for Thomson scattering, this uniformly accelerated observer would conclude that there is a finite probability for scattering a photon out of this thermal bath back into another mode with the same polarization. This scattering event would change the thermal bath experienced by the accelerated observer since it removes a photon from one mode and adds it to another mode. However, since this thermal bath in the accelerated frame is just our Minkowski vacuum in the inertial frame, this modification must also change the Minkowski vacuum: The removal of a thermal photon in the accelerated frame corresponds to the emission of a real photon in the inertial frame (since the Minkowski vacuum is the ground state, changing this state can only be done via an excitation) and the re-insertion of the thermal photon into another mode does also correspond to the emission of a real photon in the inertial frame. As a result, this scattering event in the accelerated frame translates to the emission of two real photons in the inertial frame. The expected scaling behavior from the Thomson cross section

$$P_{\text{quant}} \sim \sigma \times f(T_{\text{Unruh}}) \sim \frac{q^4}{m^2} \times f(qE) \quad (14)$$

fits this simple picture. In this way, we can understand the relativistic quantum effect sketched in Fig. 2 in terms of the Unruh effect — after some very strong simplifications.

In order to facilitate the discussion of angular momentum conservation, it is more convenient to use circular polarizations $\bigcirc$ and $\bigcirc$. In this basis, the two fields $\phi_{\bigcirc}(t, x)$ and $\phi_{\bigcirc}(t, x)$ are roughly equivalent to the carriers of positive and negative charge, respectively, and the EPR state in Eq. (11) reads

$$|\text{EPR}\rangle = \frac{|\bigcirc\bigcirc\rangle - |\bigcirc\bigcirc\rangle}{\sqrt{2}}. \quad (15)$$

Within this picture, the entanglement of the above Bell state can be understood as a consequence of the vacuum entanglement, which is responsible for the thermal nature of the Unruh effect: The uniformly accelerated observer can only access a part of the full Minkowski space-time and hence experiences the pure state of the Minkowski vacuum as a mixed state (thermal density matrix).

V. CONCLUSIONS AND OUTLOOK

In summary, we have studied the process described by the Feynman diagrams sketched in Fig. 2 and concluded that it is a pure (relativistic) quantum effect, which cannot be explained within classical electrodynamics. As one would expect, it is suppressed (in the parameter region under consideration) in comparison with the classical Larmor radiation (cf. Figs. 1 and 3). This suppression manifests itself by an additional power of $1/m$ (rather than $\alpha_{\text{QED}}$, cf. Fig. 3). As one would expect, heavier particles behave more classically than light ones (if all the other parameters remain the same).

Furthermore, we have provided an intuitive explanation of this quantum process in terms of the Unruh effect using a very simplified 1+1 dimensional geometry, i.e., in forward direction $k_1|k_2|e_L$. In other directions, there are also non-entangled contributions and the situation becomes more complicated. One reason for this lies in the fact that the polarization can no longer be treated as a charge which is independent of other quantum numbers. The question of whether the full 3+1 dimensional quantum effect can also be understood in terms of the Unruh effect or is due to a different quantum mechanism requires further study.

Interestingly, there is a paper more than twenty years old by Zel’dovich et al., in which they discuss the same major idea and conclude that a scattering event in the accelerated frame corresponds to the emission of two real photons in the laboratory frame (even though they did not discuss their entanglement). In addition, they estimated the pair creation rate in analogy to Eq. (14). However, apparently this paper did not receive as much attention as it deserved and in a later paper, Chen and Tajima discussed the signatures of the Unruh effect without noting its two-photon nature.

It should also be noted that the Feynman diagrams sketched in Fig. 2 have been studied extensively before...
in connection with double Compton scattering \[7\], but mainly from a different point of view. For example, many of these calculations are based on an average over the photon polarizations – whereas here we are specifically interested in the entanglement in polarization and its relation to the quantum nature of the process\[11\]. It should also be mentioned here that spontaneous parametric down-conversion (with x-rays \[8\] or in quantum optics) is based on the same diagrams as in Fig. 2. However, in those cases the Unruh picture based on a nearly classical electron trajectory with a specific acceleration can no longer be applied and additional aspects (such as spatial phase matching) start to play a role.

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[9] Nevertheless, even for non-uniform accelerations, the accelerated observer would still experience excitations – though no longer in a stationary thermal state – so the basic picture remains correct.
[10] We would like to stress that this remark is not meant to blame Chen and Tajima for not knowing the paper by Zel’dovich *et al* – we also just discovered it recently.
[11] It might be adequate to add a few comments regarding the recent paper [arXiv:0809.1505v1](http://arxiv.org/abs/0809.1505) by F. Bell where several interesting ideas are discussed. First of all, we would like to stress that the Unruh effect is not a phenomenon beyond QED, but rather a way of understanding quantum effects via the non-trivial comparison of inertial and non-inertial frames. Furthermore, the paper by F. Bell is based on calculations (see, e.g., [3]) where the polarizations are averaged over from the very beginning. Therefore, it cannot address the polarization entanglement which played a crucial role in our analysis. Indeed, expectation expressed at the end of [arXiv:0809.1505v1](http://arxiv.org/abs/0809.1505), namely that the polarizations of the two created photons are always identical to the laser polarization (which in fact would mean no polarization entanglement), is not correct.