Nuclear PDFs from neutrino deep inelastic scattering

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We study nuclear effects in charged current deep inelastic neutrino-iron scattering in the framework of a $\chi^2$ analysis of parton distribution functions. We extract a set of iron PDFs and show that under reasonable assumptions it is possible to constrain the valence, light sea and strange quark distributions. Our iron PDFs are used to compute $x_{Bj}$-dependent and $Q^2$-dependent nuclear correction factors for iron structure functions which are required in global analyses of free nucleon PDFs. We compare our results with nuclear correction factors from neutrino-nucleus scattering models and correction factors for $\ell^\pm$-iron scattering. We find that, except for very high $x_{Bj}$, our correction factors differ in both shape and magnitude from the correction factors of the models and charged-lepton scattering.

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I. INTRODUCTION

The high statistics measurements of neutrino deeply inelastic scattering (DIS) on heavy nuclear targets has generated significant interest in the literature since these measurements provide valuable information for global fits of parton distribution functions (PDFs) \[1\]. The use of nuclear targets is unavoidable due to the weak nature of the neutrino interactions, and this complicates the extraction of free nucleon PDFs because model-dependent corrections must be applied to the data.

Additionally, these same data are also useful for extracting the nuclear parton distribution functions (NPDFs): for such an analysis, no nuclear correction factors are required. Due to the limited statistics available for individual nuclear targets with a given atomic number \( A \) the standard approach is to model the \( A \)-dependence of the fit parameters, and then combine the data sets for many different target materials in the global analysis \[2, 3, 4, 5, 6\]. However, the high statistics NuTeV neutrino–iron cross section data (\( > 2000 \) points) offer the possibility to investigate the viability of a dedicated determination of iron PDFs \[8\].

With this motivation, we will perform a fit to the NuTeV neutrino–iron data and extract the corresponding iron PDFs. Since we are studying iron alone and will not (at present) combine the data with measurements on different target materials, we need not make any assumptions about the nuclear corrections; this side-steps a number of difficulties \[9, 10, 11\].

While this approach has the advantage that we do not need to model the \( A \)-dependence, it has the drawback that the data from just one experiment will not be sufficient to constrain all the parton distributions. Therefore, other assumptions must enter the analysis. The theoretical framework will roughly follow the CTEQ6 analysis of free proton PDFs \[12\]; this will be discussed in Sec. IV.

In Sec. III we present the results of our analysis, and compare with nuclear PDFs from the literature. In Sec. IV we extract the nuclear correction factors from our iron PDFs and compare with a SLAC/NMC parameterization taken from the \( e^\pm - \)Fe DIS process \[12\] and also with the parameterization by Kulagin & Petti \[14, 15\]. Finally, we summarize our results and conclusions in Sec. V.

II. THEORETICAL FRAMEWORK

A. Basic formalism

For our PDF analysis, we will use the general features of the QCD-improved parton model and the \( \chi^2 \) analyses as outlined in Ref. \[12\]. Here, we will focus on the issues specific to our study of NuTeV neutrino–iron data in terms of nuclear parton distribution functions. We adopt the framework of the recent CTEQ6 analysis of proton PDFs where the input distributions at the scale \( Q_0 = 1.3 \) GeV are parameterized as \[12\]

\[
xf_i(x;Q_0) = \begin{cases} 
A_0 x^{A_1} (1-x)^{A_2} e^{A_3 x}(1+e^{A_4 x})^{A_5} & : i = u, d, s, \bar{s}, \bar{v}, \bar{d}, v \\
A_0 x^{A_1} (1-x)^{A_2} + (1+e^{A_4 x})(1-x)^{A_4} & : i = d/\bar{u}, 
\end{cases}
\]

where \( u_r \) and \( d_r \) are the up- and down-quark valence distributions, \( \bar{u}, \bar{d}, s, \bar{s} \) are the up, down, strange and antistrange sea distributions, and \( g \) is the gluon. Furthermore, the \( f_i = f_i^{p/A} \) denote parton distributions of bound protons in the nucleus \( A \), and the variable \( 0 \leq x \leq A \) is defined as \( x := Ax/A \) where \( x_A = Q^2/2p_A \cdot q \) is the usual Bjorken variable formed out of the four-momenta of the nucleus and the exchanged boson. Equation (1) is designed for \( 0 \leq x \leq 1 \) and we here neglect\[1\] the distributions at \( x > 1 \). Note that the condition \( f_i(x > 1, Q) = 0 \) is preserved by the DGLAP evolution and has the effect that the evolution equations and sum rules for the \( f_i^{p/A} \)

are the same as in the free proton case.\[2\]

The PDFs for a nucleus \( (A, Z) \) are constructed as

\[
f_i^A(x, Q) = \frac{Z}{A} f_i^{p/A}(x, Q) + \frac{(A - Z)}{A} f_i^{n/A}(x, Q)
\]

where we relate the distributions inside a bound neutron, \( f_i^{n/A}(x, Q) \), to the ones in a proton by assuming isospin symmetry. Similarly, the nuclear structure functions are

\[2\] While the quark number and momentum sum rules for the nuclear case are satisfied as in the proton, there is no requirement that the momentum fractions carried by the PDF flavors be the same. A recent analysis at low \( Q^2 \) found the Cornwall-Norton moments to be the same in iron as in deuterium formed from a free proton and a free neutron to within 3\% \[10\].

\[1\] While the nuclear PDFs can be finite for \( x > 1 \), the magnitude of the PDFs in this region is negligible for the purposes of the present study (cf., Refs. \[2, 3, 4, 5, 6\]).
given by
\[ F_i^A(x, Q) = \frac{Z}{A} F_i^{p/A}(x, Q) + \frac{(A - Z)}{A} F_i^{n/A}(x, Q) \] (3)
such that they can be computed in next-to-leading order as convolutions of the nuclear PDFs with the conventional Wilson coefficients, i.e., generically
\[ F_i^A(x, Q) = \sum_k C_{ik} \otimes f_i^A. \] (4)

In order to take into account heavy quark mass effects we calculate the relevant structure functions in the ACOT scheme in NLO QCD. Finally, the differential cross section for charged current (anti-)neutrino–nucleus scattering is given in terms of three structure functions:

\[
\frac{d^2 \sigma}{dx \ dy} \bigg|_{\nu A} = \frac{G^2 M \pi}{2} \left[ (1 - y - \frac{M x y}{2 E}) F_2^{\nu A} \right. \\
+ \left. \frac{y^2}{2} 2x F_1^{\nu A} \pm y(1 - \frac{y}{2}) x F_3^{\nu A} \right],
\] (5)

where the '+' ('−') sign refers to neutrino (anti-neutrino) scattering and where \( G \) is the Fermi constant, \( M \) the nucleon mass, and \( E \) the energy of the incoming lepton (in the laboratory frame).

### B. Constraints on PDFs

We briefly discuss which combinations of PDFs can be constrained by the neutrino–iron data. For simplicity, we restrict ourselves to leading order, neglect heavy quark mass effects (as well as the associated production thresholds), and assume a diagonal Cabibbo-Kobayashi-Maskawa (CKM) matrix. The neutrino–iron structure functions are given by (suppressing the dependence on \( x \) and \( Q^2 \)):

\[
F_1^{\nu A} = d^A + s^A + \bar{u}^A + \bar{c}^A + \ldots, \qquad F_2^{\nu A} = 2x F_1^{\nu A}, \qquad F_3^{\nu A} = 2 \left[ d^A + s^A - \bar{u}^A - \bar{c}^A + \ldots \right].
\] (6)

The structure functions for anti-neutrino scattering are obtained by exchanging the quark and anti-quark PDFs in the corresponding neutrino structure functions:

\[
F_{1,2}^{\bar{\nu} A} = +F_{1,2}^{\nu A}[^q \leftrightarrow \bar{\bar{q}}], \\
F_3^{\bar{\nu} A} = -F_3^{\nu A}[^q \leftrightarrow \bar{\bar{q}}].
\] (7)

Explicitly this gives

\[
F_1^{\bar{\nu} A} = u^A + c^A + \bar{d}^A + \bar{s}^A + \ldots, \quad F_2^{\bar{\nu} A} = 2x F_1^{\bar{\nu} A}, \quad F_3^{\bar{\nu} A} = 2 \left[ u^A + c^A - \bar{d}^A - \bar{s}^A + \ldots \right].
\] (8)

It is instructive to compare this with the parton model expressions for the structure function \( F_2 \) in \( l^\pm A \) scattering, where \( l^\pm \) denotes a charged lepton:

\[
\frac{1}{x} F_2^{lA} = 4 \left[ \frac{1}{9} \left( u^A + \bar{u}^A \right) + \frac{5}{9} \left( d^A + \bar{d}^A \right) + \frac{1}{9} \left( s^A + \bar{s}^A \right) + \frac{4}{9} \left( c^A + \bar{c}^A \right) + \ldots \right].
\] (9)

Using the Callan–Gross relations in Eqs. \( 7 \) and \( 12 \), and neglecting the proton mass, the differential cross section Eq. \( 5 \) can be simplified in the form

\[
d\sigma \propto (1 - y + y^2/2) F_2^{\nu A} + y(1 - \frac{y}{2}) x F_3^{\nu A}
\] (10)

with the limiting cases:

\[
\begin{align*}
\frac{1}{x} F_2^{\nu A} &\rightarrow \left\{ \begin{array}{l}
\frac{1}{2} F_2^{\nu A} + \frac{1}{2} x F_3^{\nu A} & \text{(for } y \rightarrow 1) \\
F_2^{\nu A} & \text{(for } y \rightarrow 0) \end{array} \right. \\
\frac{1}{x} F_3^{\nu A} &\rightarrow \left\{ \begin{array}{l}
\frac{1}{2} F_2^{\nu A} + \frac{1}{2} x F_3^{\nu A} & \text{(for } y \rightarrow 1) \\
F_3^{\nu A} & \text{(for } y \rightarrow 0) \end{array} \right.
\end{align*}
\] (11)

The latter form of \( d\sigma \) shows that the (anti-)neutrino cross section data naturally encodes information on the four structure function combinations \( F_2^{\nu A} \pm x F_3^{\nu A} \) and \( F_2^{\nu A} \) in separate regions of the phase space.

If we assume \( s^A = \bar{s}^A \) and \( c^A = \bar{c}^A \), the structure functions \( F_2^{\nu A} \) constrain the valence distributions \( d_v^A = d^A - \bar{d}^A, u_v^A = u^A - \bar{u}^A \) and the flavor-symmetric sea \( \Sigma^A := \bar{u}^A + \bar{d}^A + \bar{s}^A + \bar{c}^A + \ldots \) via the relations:

\[
\begin{align*}
\frac{1}{x} F_2^{\nu A} &\rightarrow \left\{ \begin{array}{l}
\frac{1}{2} [d_v^A + \Sigma^A], \\
\frac{1}{2} [u_v^A + \Sigma^A] \end{array} \right. \\
\frac{1}{x} F_3^{\nu A} &\rightarrow \left\{ \begin{array}{l}
\frac{1}{2} [d_v^A + \Sigma^A], \\
\frac{1}{2} [u_v^A + \Sigma^A] \end{array} \right.
\end{align*}
\] (12)

Furthermore, we have

\[
\begin{align*}
\frac{1}{x} F_2^{\nu A} + F_3^{\nu A} &\rightarrow 4(d^A + s^A), \\
\frac{1}{x} F_2^{\nu A} - F_3^{\nu A} &\rightarrow 4(\bar{d}^A + \bar{s}^A).
\end{align*}
\] (13)

Since we constrain the strange distribution utilizing the dimuon data,\(^4\) the latter two structure functions are useful to separately extract the \( d^A \) and \( \bar{d}^A \) distributions.

For an isoscalar nucleus we encounter further simplifications. In this case, \( u^A = d^A \) and \( u^A = d^A \) which implies \( u_v^A = d_v^A = \bar{v}^A = \bar{d}_v^A \). Hence, the independent quark distributions are \( \{ v^A, \bar{v}^A, s^A = \bar{s}^A, c^A = \bar{c}^A, \ldots \} \). It is

\(^4\) Note that these equations are known not to be exact as the DGLAP evolution equations at NNLO generate an asymmetry even if one starts with \( s = \bar{s} \) or \( c = \bar{c} \) at some scale \( Q^2 \). However, these effects are tiny and far beyond the accuracy of our study.

\(^5\) See Refs. [21, 22, 23, 24, 25, 26] for details.

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\( ^3 \) All these effects are properly included in our calculations.
instructive to introduce the parameter $\Delta := 1/2 - Z/A$ which describes the degree of non-isoscalarity. This allows us to write the PDFs in a way which makes deviations from isoscalarity manifest:

\[
\begin{align*}
    u_v^A &= v^A - \Delta[u_p^p/A - d_p^p/A] \\
    d_v^A &= v^A + \Delta[u_p^p/A - d_p^p/A] \\
    \bar{u}^A &= \bar{q}^A - \Delta[\bar{u}^p/A - \bar{d}^p/A] \\
    \bar{d}^A &= \bar{q}^A + \Delta[\bar{u}^p/A - \bar{d}^p/A]
\end{align*}
\]

in terms of an averaged nuclear valence distribution $v^A = (u_p^p/A + d_p^p/A)/2$ and an averaged nuclear sea distribution $\bar{q}^A = (\bar{u}^p/A + \bar{d}^p/A)/2$. Recall, $f_i^{p/A}$ represents the distribution for a bound proton in the nucleus $A$; hence, the nuclear effects are encoded in these terms. Notice that non-isoscalar targets ($\Delta \neq 0$) therefore provide information on the difference between the valence distributions $(u_v^A - \bar{u}^A)$ and the light quark sea distribution $(\bar{u}^p/A - \bar{d}^p/A)$ in the nucleus. Unfortunately, the data are often corrected for non-isoscalar effects and this information is lost.

C. Methodology

The basic formalism described in the previous sections is implemented in a global PDF fitting package, but with the difference that no nuclear corrections are applied to the analyzed data; hence, the resulting PDFs are for a bound proton in an iron nucleus. The parameterization of Eq. (22) provides enough flexibility to describe current data sets entering a global analysis of free nucleon PDFs; given that the nuclear modifications of the $x$-shape appearing in this analysis are modest, this parameterization will also accommodate the iron PDFs.

Because the neutrino data alone do not have the power to constrain all of the PDF components, we will need to impose some minimal set of external constraints. For example, our results are rather insensitive to the details of the gluon distribution with respect to both the overall $\chi^2$ and also the effect on the quark distributions. The nuclear gluon distribution is very weakly constrained by present data, and a gluon PDF with small nuclear modifications has been found in the NLO analysis of Ref. [7]. We have therefore fixed the gluon input parameters to their free nucleon values. For the same reasons the gluon is not sensitive to this analysis, fixing the gluon will have minimal effect on our results. Furthermore, we have set the $d/\bar{u}$ ratio to the free nucleon result assuming that the nuclear modifications to the down and up sea are similar such that they cancel in the ratio. This assumption is supported by Fig. 6 in Ref. [7].

Because we have limited the data set to a single heavy target (iron), the $\chi^2$ surface has some parameter directions which are relatively flat. To fully characterize the parameter space, we perform many “sample fits” starting from different initial conditions, and iterate these fits including/excluding additional parameters. The result is a set of bands for fits of comparable quality ($\Delta \chi^2 \sim 50$ for 2691 data points) which provide an approximate measure of the constraining power of the data.

III. ANALYSIS OF IRON DATA

A. Iron Data Sets

We determine iron PDFs using the recent NuTeV differential neutrino (1371/1170 data points) and antineutrino (1146/966 data points) DIS cross section data [8] where the quoted numbers of data points refer to the two different combinations of kinematic cuts introduced below. In addition, we include NuTeV/CCFR dimuon data (174 points) [21] which are sensitive to the strange quark content of the nucleon.

There are other measurements of neutrino–iron DIS available in the literature from the CCFR [27, 28, 29, 30], CDHS [31] and CDHSW [32] collaborations; see, e.g., Ref. [33] for a review. There is also a wealth of charged lepton–iron DIS data including SLAC [34] and EMC [35, 36, 37] for the present study we limit our analysis to the NuTeV experiment alone; we will compare and contrast different experiments in a later study.

B. Fit results

The results of our fits to the NuTeV iron cross section and dimuon data are summarized in Table I. The cross section data have been corrected for QED radiation effects, and the non-isoscalarity of the iron target [37]; correspondingly, we have used $A = 56, Z = 28$ in Eqs. (23). $^6$ For the Durham HEP Databases for a complete listing: http://www-spires.dur.ac.uk/hepdata/

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline

| Scheme | Cuts | Data | # points | $\chi^2/pts$ |
|--------|------|------|-----------|--------------|
| ACOT   | $Q > 1.3$ GeV $\nu + \bar{\nu}$ | no $W_{cut}$ | 2691 | 3678 | 1.37 | A |
|        | $\nu$ | 1459 | 2139 | 1.47 | A$\nu$ |
|        | $\bar{\nu}$ | 1232 | 1430 | 1.16 | A$\bar{\nu}$ |
| ACOT   | $Q > 2$ GeV $\nu + \bar{\nu}$ | $W > 3.5$ GeV $\nu$ | 2310 | 3111 | 1.35 | A2 |
|        | $\nu$ | 1258 | 1783 | 1.42 | A2$\nu$ |
|        | $\bar{\nu}$ | 1052 | 1199 | 1.14 | A2$\bar{\nu}$ |
| MS     | $Q > 1.3$ GeV $\nu + \bar{\nu}$ | no $W_{cut}$ | 2691 | 3732 | 1.39 | M |
|        | $\nu$ | 1459 | 2205 | 1.51 | M$\nu$ |
|        | $\bar{\nu}$ | 1232 | 1419 | 1.15 | M$\bar{\nu}$ |
| MS     | $Q > 2$ GeV $\nu + \bar{\nu}$ | $W > 3.5$ GeV $\nu$ | 2310 | 3080 | 1.33 | M2 |
|        | $\nu$ | 1258 | 1817 | 1.44 | M2$\nu$ |
|        | $\bar{\nu}$ | 1052 | 1201 | 1.14 | M2$\bar{\nu}$ |
\hline
\end{tabular}
\caption{Fits to NuTeV cross section and dimuon data.}
\end{table}
Note, for an iron target the isoscalar correction factors are small and do not exceed the few % level. We have performed fits to the combined data as well as to the neutrino- and anti-neutrino data sets separately. Furthermore, two different cuts in the kinematic plane have been examined: a) $Q > 1.3$ GeV, no cut on the hadronic invariant mass $W$ and b) $Q > 2$ GeV and $W > 3.5$ GeV, cf., Table II. The NLO QCD calculation was performed in both the $\overline{\text{MS}}$ and ACOT schemes. The ACOT scheme calculation takes into account the heavy quark mass effects, whereas the $\overline{\text{MS}}$ scheme assumes massless partons. The dominant target mass effects have been incorporated and the isoscalar correction factors are small and do not exceed the few % level.

The $\chi^2$ values have been determined taking into account the full correlations of the data employing the effective $\chi^2$ function given in Eq. (B.5) of Ref. [12]. The numbers for the $\chi^2/\text{pts}$ are roughly on the order of 1.4 for both the ACOT and the $\overline{\text{MS}}$ schemes. Furthermore, the fits to the anti-neutrino data have considerably better $\chi^2$ values; however, we will see below that this is at least partly due to the larger uncertainties of these data.

1. PDF Reference Sets

For the purposes of this study, we use two different reference sets of free-proton PDFs which we denote ‘Base-1’ and ‘Base-2’.

Since we focus on iron PDFs and the associated nuclear corrections, we need a base set of PDFs which are essentially free of any nuclear effects; this is the purpose of the Base-1 reference set [10]. Therefore, to extract the Base-1 PDFs we omit the CCFR and NuTeV data from our fit so that our base PDFs do not contain any large residual nuclear corrections. The absence of such nuclear effects will be important in Sec. IV when we extract the nuclear corrections factors.

The Base-2 PDFs are essentially the CTEQ6.1M PDFs with a modified strange PDF introduced to accommodate the NuTeV dimuon data. In the manner of the CTEQ6.1M PDF’s, the Base-2 fit does not apply any deuteron corrections to the data; this is in contrast to the Base-1 PDFs. Also, the Base-2 fit does include the CCFR data that has been corrected to a free nucleon using charged-lepton correction factors; the Fermilab CCFR experiment is the predecessor of NuTeV with comparable statistics as those from NuTeV [38]. The CCFR results in the large-$x$ region ($x > 0.4$) are consistently lower than those from NuTeV, and various sources contributing to the difference have been identified [8, 14]. One third of the discrepancy has been attributed to a mis-calibration of the magnetic field map of the muon spectrometer, i.e., to the muon energy scale in the CCFR analysis. About another third comes from model differences (cross section model, muon and hadron energy smearing models). A comparison of NuTeV and CCFR data can be found in Ref. [8].

By comparing the free-proton PDF ‘Base-1’ and ‘Base-2’ sets with the iron PDF sets of Table I we can gauge the size of the nuclear effects. Furthermore, differences between observables using the ‘Base-1’ respectively the ‘Base-2’ reference sets will indicate the uncertainty due to the choice of the free-proton PDF.

The fit provides a good description of the data which are distributed around unity for most of the bins. For reference, the results of fit ‘A’ (solid line) and Base-1

set as in the CTEQ6 analysis with the addition of the the NuTeV dimuon data. The changes to the strange sea induce only minor changes to the other fit parameters; this has a minimal effect on the particular observables ($d\sigma, F_2$) we examine in the present study.

All results have been computed with both Base-1 and Base-2 PDFs. Since the Base-2 PDFs use CCFR and NuTeV data, the resulting PDFs will depend on the nuclear corrections which we are trying to determine. Therefore, we will predominantly display the Base-1 PDFs for comparison in the following Sections.

Conversely, global analyses of nuclear PDFs tend to use loose kinematic cuts due to the lack of small-$x$ data and the interest in the very large-$x$ region.

7 We have checked that omitting the isoscalar correction factors and using $A = 56, Z = 26$ gives almost identical results.

8 Target mass effects (TMC) are expected to be relevant at large Bjorken-$x$ or small momentum transfers $Q^2 > 50$. For orders of higher orders and higher twist cf. Refs. [11, 39, 42, 43].

9 Fits to this same data neglecting the correlations between the errors and using the conventional $\chi^2$ function (cf. Eq. (B.1) in [12]), have smaller $\chi^2/\text{pts} \simeq 1$. While the uncorrelated errors are larger, the extracted parameters are similar.

10 We do retain the deuteron data as this has only a small correction over the central $x$-range, cf. Sec. V. The deuteron correction has been applied in the Base-1 fit. Also, for the Drell-Yan Cu data (E605), the expected nuclear corrections in this kinematic range are small (a few percent) compared to the overall normalization uncertainty (15%) and systematic error (10%).

11 These PDFs have been determined from a fit to the same data
PDFs (dotted line) are shown as well. For fit ‘A2’, the effect of the $Q > 2$ GeV cut is to remove data at low $y$ in the small-$x$ region, and the $W > 3.5$ GeV cut excludes low-$y$ data at large $x$. The effects of these cuts on the fit are visible by comparing the difference of the solid line (‘A’) from unity (‘A2’). For $x \gtrsim 0.045$, we observe minimal differences between the ‘A’ and ‘A2’ fits, and conclude the effect of the kinematic cuts ($Q > 2$ GeV and $W > 3.5$ GeV) are nominal in this region. In the lowest $x$ bin ($x \sim 0.015$), much of the data is eliminated by

FIG. 1: Representative comparison of fit ‘A2’ to the NuTeV neutrino and anti-neutrino cross section data. Shown are the data points for various $x$-bins versus the inelasticity $y$ for an energy of $E = 65$ GeV in a data-over-theory representation. For comparison, we also show results for the Base-1 PDFs (dotted) and the ‘A’ fit (solid); the fit ‘A2’ imposes more stringent cuts on $Q > 2$ GeV and $W > 3.5$ GeV.
the $Q > 2$ GeV cut such that fit ‘A2’ is only constrained by a few data points at large $y$ for the higher neutrino energies, cf. Fig. 3. Since both, fit ‘A’ and fit ‘A2’, have large uncertainties in this $x$-region the comparison of individual representatives is less significant—in particular at medium and low $y$ where no data points lie. In conclusion, we discern no relevant differences between the two classes of fits over the entire kinematic plane and will therefore mainly focus on fit ‘A2’ in the following sections.

3. Comparison of the Fits with Reference PDFs

The dotted curve in Figures 1-3 shows the cross sections obtained with Base-1 free-proton PDFs, inserted
FIG. 3: The same as in Fig. 1 for a neutrino energy of $E = 245$ GeV.

We observe that the Base-1 results at small-$x$ ($x \sim [0.045 \text{—} 0.08]$) are generally below unity (the ‘A2’ fit) in the $y$ region of the data points implying an enhancement due to nuclear effects. As discussed above, the results in the lowest $x$ bin ($x = 0.015$) are less clear as the uncertainties are larger since the kinematic cuts remove much of the data. Nevertheless, do not see a clear signal of shadowing in this region (cf., Fig. 3 at large $y$).

For intermediate $x \sim [0.125 \text{—} 0.175]$ the Base-1 (dotted
line) results are very similar to fit ‘A2’. For larger $x \sim [0.225 – 0.65]$ we observe a suppression of the nuclear cross sections qualitatively similar to what is known from charged lepton DIS. Finally, in the region $x \gtrsim 0.75$ the nuclear cross section is again enhanced—an effect usually attributed to the Fermi motion of the nucleons in the nucleus.

In conclusion, we observe the following pattern for the nuclear cross section compared to the free nucleon cross section: i) enhancement for $x \gtrsim 0.75$, ii) suppression for $x \sim [0.225 – 0.65]$, iii) equality for $x \sim 0.125$, and iv) slight enhancement for $x \sim [0.045 – 0.08]$. This is to be contrasted with the expectation from charged lepton DIS with the well-known pattern: i) enhancement for $x \gtrsim 0.75$ (Fermi motion), ii) suppression for $x \sim [0.3 – 0.8]$ (EMC effect), iii) enhancement for $x \sim [0.06 – 0.3]$ (Anti-shadowing), and iv) suppression for $x \lesssim 0.06$ (Shadowing). Thus, for $x \gtrsim 0.3$ our results are generally as expected. However, we find that the usual behavior at medium and small $x$ is modified. We will examine this further in the following sections.

C. Iron PDFs

Having established the quality of our fits, we now examine the nuclear (iron) parton distributions $f_i^A(x, Q^2)$ according to Eq. (2). Figure 4 shows the PDFs from fit ‘A2’ at our input scale $Q_0 = m_e = 1.3$ GeV versus $x$. For an almost isoscalar nucleus like iron the $u$ and $d$ distributions are very similar, see Eqs. (21 – 24). Therefore, we only show the $u_v$ and $\bar{u}$ partons, together with the strange sea. As explained above, the gluon distribution is very similar to the familiar CTEQ6M gluon at the input scale such that we don’t show it here. In order to indicate the constraining power of the NuTeV data, the band of reasonable fits is depicted. The fits in this band were obtained (as outlined above) by varying the initial conditions and the number of free parameters to fully explore the solution space. All the fits shown in the band have $\chi^2/DOF$ within 0.02, which roughly corresponds to a range of $\Delta \chi^2 \sim 50$ for the 2691 data points.

As can be seen in Figure 4, the $u_v$ distribution (Fig. 4a) has a very narrow band across the entire $x$-range. The up- and strange-sea distributions (Fig. 4b and Fig. 4c) are less precisely determined. At values of $x$ down to, say, $x \approx 0.07$ the bands are still reasonably well confined; however, they open up widely in the small-$x$ region. Cases where the strange quark sea lies above the up-quark sea are unrealistic, but are present in some of the fits since this region ($x \lesssim 0.02$) is not constrained by data. We have included the curves for our free-proton Base-1 PDFs (dashed), as well as the HKN04 (dot-
ted), the NLO HKN07 [4] (dotted-dashed), and DS [7] (dot-dashed) nuclear PDFs.

The comparison with the Base-1 PDFs is straightforward since the same theoretical framework (input scale, functional form, NLO evolution) has been utilized for their determination. Therefore, the differences between the solid band and the dashed line exhibit the nuclear effects, keeping in mind that the free-proton PDFs themselves have uncertainties.

For the comparison with the HKN04 distributions, it should be noted that a SU(3)-flavor symmetric sea has been used; therefore, the HKN04 strange quark distribution is larger, and the light quark sea smaller, than their Base-1 PDF counterparts over a wide range in $x$. Furthermore, the HKN04 PDFs are evolved at leading order.

In a recent publication, Eskola et al. [6] have extracted the iron nuclear PDFs from only iron data, we do not assume any particular form for the nuclear structure functions to deuteron structure functions is nontrivial and model-dependent.

In Fig. 6, we display the NMC data for $F_2^D/F_2^{p}$ and contrast this with the neutral current (NC) $\nu-A$ process (cf. Eq. (14)).

$$R_{CC}^\nu(F_2^D; x, Q^2) \simeq \frac{d^A + \bar{u}^A + \ldots}{d^0 + \bar{u}^0 + \ldots}$$

and compare this to a variety of data parameterizations (cf. Eq. (15)).

Clearly, the $R$-factors depend on both the kinematic variables and the factorization scale. Finally, we note that Eq. (25) is subject to uncertainties of both the numerator and the denominator.

We will now evaluate the nuclear correction factors for our extracted PDFs, and compare these with selected results from the literature [13, 14, 15]. Because we have extracted the iron PDFs from only iron data, we do not assume any particular form for the nuclear $A$-dependence; hence, the extracted $R[O]$ ratio is essentially model independent.

### A. Deuteron corrections for the $F_2^{Fe}/F_2^D$ ratio

The structure function ratio $F_2^{Fe}/F_2^D$ provides a common (and useful) observable to use to gauge the nuclear effects of iron. To construct the numerator, we will use our iron PDFs as extracted in fits ‘A’ and ‘A2.’ For the denominator, we will use the Base-1 and Base-2 free proton PDF; however, converting from free proton structure functions to deuteron structure functions is nontrivial and model-dependent.

In Fig. 6, we display the NMC data for $F_2^D/F_2^{p}$ and compare this to a variety of data parameterizations.
we compare the experimental results for the structure function ratio \( \frac{F_2^D}{F_2^p} \) computed using free-proton Base-2 PDFs. The dashed line shows the structure function ratio obtained using the parameterization of Ref. \[46\]. The solid line shows the structure function ratio obtained using ratios of \( F_2 \) computed from experiments at SLAC (SLAC-E049 \[51\], SLAC-E139 \[9\], SLAC-E140 \[34\]). Nor-

normalization uncertainties of the data have not been included.

\[ F_2^D/F_2^p \] at \( Q^2 = 5.47 \text{ GeV}^2 \) in comparison with the theory prediction for \( F_2^D/F_2^p \) computed using free-proton Base-2 PDFs. The dashed line shows the structure function ratio obtained using the parameterization of Ref. \[46\]. For comparison, we also show the parameterizations of Arneodo et al. \[47, 48\] and Tvaskis et al. \[47, 48\]. The dotted line (Arneodo) is the parameterization of Ref. \[46\], and the dot-dashed line (Tvaskis) is the parameterization of Ref. \[47, 48\]. We see that the range of discrepancies in the deuterium corrections are typically on the order of a percent or two except at large \( x \); while this correction cannot be neglected, it is small compared to the much larger iron nuclear corrections. To explore a range of possibilities (reflecting the underlying uncertainty) we have incorporated deuteron corrections into the Base-1 PDF, but not the Base-2 PDF; hence the spread between these two reference PDFs will, in part, reflect our ignorance of \( F_2^D \) and other uncertainties of proton PDFs at large-\( x \).

\[ F_2^D/F_2^p \] for neutral current (NC) charged lepton scattering

We will also find it instructive to compare our results with the \( F_2^D/F_2^p \) as extracted in neutral current charged-lepton scattering, \( \ell^+\ell^- \text{Fe} \). In Fig. 6 we compare the experimental results for the structure function ratio \( F_2^D/F_2^p \) for the following experiments: BCDMS-85 \[49\], BCDMS-87 \[50\], SLAC-E049 \[51\], SLAC-E139 \[9\], SLAC-E140 \[34\]. The curve (labeled SLAC/NMC parameterization) is a fit to this data \[13\]. While there is a spread in the individual data points, the parameterization describes the bulk of the data at the level of a few percent or better. It is important to note that this parameterization is independent of atomic number \( A \) and the energy scale \( Q^2 \); this is in contrast to the results we will derive using the PDFs extracted from the nuclear data. Additionally, we note that while this parameterization has been extracted using ratios of \( F_2 \) structure functions, it is often applied to other observables such as \( F_{1,3,5} \) or \( d \sigma/dx \). We can use this parameterization as a guide to judge the approximate correspondence between this neutral current (NC) charged lepton DIS data and our charged current (CC) neutrino DIS data.

**C. Correction Factors for \( d^2\sigma/dx dQ^2 \)**

We begin by computing the nuclear correction factor \( R \) according to Eq. (25) for the neutrino differential cross section in Eq. (5) as this represents the bulk of the NuTeV data included in our fit. More precisely, we show \( R \)-factors for the charged current cross sections \( d^2\sigma/dx dQ^2 \) at fixed \( Q^2 \) which can be obtained from Eq. (5) by a simple Jacobian transformation and we consider an iron target which has been corrected for the neutron excess, i.e., we use the PDFs in Eq. (2) (for the numerator) and Eq. (25) (for the denominator) with \( A = 56 \) and \( Z = 28 \). Our results are displayed in Fig. 7 for \( Q^2 = 5 \text{ GeV}^2 \) and a neutrino energy \( E_{\nu} = 150 \text{ GeV} \) which implies, due to the relation \( Q^2 = 2M_{\nu,xy} \), a minimal \( x \)-value

\[ 0.5 \] for large \( x \) and \( Q \) comparable to the proton mass the target mass corrections for \( F_2^D/F_2^p \) are essential for reproducing the features of the data; hence the \( Q \) dependence plays a fundamental role.
FIG. 7: Nuclear correction factor $R$ according to Eq. (26) for the differential cross section $d^2\sigma/dx dQ^2$ in charged current $\nu Fe$ scattering at $Q^2 = 5 \text{ GeV}^2$ and $E_\nu = 150 \text{ GeV}$. Results are shown using the ‘A2’ fit for the charged current neutrino (solid lines) and anti-neutrino (dashed lines) scattering from iron. The upper (lower) pair of curves shows the result of our analysis with the Base-2 (Base-1) free-proton PDFs. The correction factors shown here are for an iron target which has been corrected for the neutron excess.

of $x_{\text{min}} = 0.018$. The solid (dashed) lines correspond to neutrino (anti-neutrino) scattering using the iron PDFs from the ‘A2’ fit.

We have computed $R$ using both the Base-1 and Base-2 PDFs for the denominator of Eq. (26); recall that Base-1 includes a deuteron correction while Base-2 uses the CCFR data and does not include a deuteron correction. The difference between the Base-1 and Base-2 curves is approximately 2% at small $x$ and grows to 5% at larger $x$, with Base-2 above the Base-1 results. As this behavior is typical, in the following plots (Figs. 8 and Figs. 9) we will only show the Base-1 results. We also observe that the neutrino (anti-neutrino) results coincide in the region of large $x$ where the valence PDFs are dominant, but differ by a few percent at small $x$ due to the differing strange and charm distributions.

D. Correction Factors for $F_2^\nu(x, Q^2)$ and $F_2^\bar{\nu}(x, Q^2)$

We now compute the nuclear correction factors for charged current neutrino–iron scattering. The results for $\nu–Fe$ are shown in Fig. 8 and those of $\bar{\nu}–Fe$ are shown in Fig. 9. The numerator in Eq. (26) has been computed using the nuclear PDF from fit ‘A2’, and for the denominator we have used the Base-1 PDFs. For comparison we also show the correction factor from the Kulagin–Petti model \cite{Kulagin96} (dashed-dotted), and the SLAC/NMC curve (dashed) \cite{HKN07} which has been obtained from an $A$ and $Q^2$-independent parameterization of calcium and iron charged–lepton DIS data.

Due to the neutron excess in iron,\textsuperscript{19} both our curves and the KP curves differ when comparing scattering for neutrinos (Fig. 8) and anti-neutrinos (Fig. 9); the SLAC/NMC parameterization is the same in both figures. For our results (solid lines), the difference between

\textsuperscript{19} Note that the correction factors shown in Figs. 8 and 9 are valid for the case in which the data have not been corrected for the neutron excess in iron. For data that already have been corrected for the neutron excess one should, for consistency, compute the $R$-factors using $A = 56$, $Z = 28$ in equation Eq. (2). The magnitude of the difference between the $R$-factors in these two cases ($Z = 26$ vs. $Z = 28$) is typically a few percent.
the neutrino and anti-neutrino results is relatively small, of order 3% at $x = 0.6$. Conversely, for the KP model (dashed-dotted lines) the $\nu$–$\bar{\nu}$ difference reaches 10% at $x \sim 0.7$, and remains sizable at lower values of $x$.

To demonstrate the dependence of the $R$ factor on the kinematic variables, in Figs. 8 and Fig. 9 we have plotted the nuclear correction factor for two separate values of $Q^2$. Again, our curves and the KP model yield different results for different $Q^2$ values, in contrast to the SLAC/NMC parameterization.

Comparing the nuclear correction factors for the $F_2$ structure function (Figs. 8 and Fig. 9) with those obtained for the differential cross section (Fig. 7), we see these are quite different, particularly at small $x$. Again, this is because the cross section $d^2\sigma$ is comprised of a different combination of PDFs than the $F_2$ structure function. In general, our $R$-values for $F_2$ lie below those of the corresponding $R$-values for the cross section $d\sigma$ at small $x$. Since $d\sigma$ is a linear combination of $F_2$ and $F_3$, the $R$-values for $F_3$ (not shown) therefore lie above those of $F_2$ and $d\sigma$. Again, we emphasize that it is important to use an appropriate nuclear correction factor which is matched to the particular observable.

As we observed in the previous section, our results have general features in common with the KP model and the SLAC/NMC parameterization, but the magnitude of the effects and the $x$-region where they apply are quite different. Our results are noticeably flatter than the KP and SLAC/NMC curves, especially at moderate-$x$ where the differences are significant. The general trend we see when examining these nuclear correction factors is that the anti-shadowing region is shifted to smaller $x$ values and any turn-over at low $x$ is minimal given the PDF uncertainties. In general, these plots suggest that the size of the nuclear corrections extracted from the NuTeV data are smaller than those obtained from charged lepton scattering (SLAC/NMC) or from the set of data used in the KP model. We will investigate this difference further in the following section.

E. Predictions for Charged-Lepton $F_2^{\nu e}/F_2^{D}$ from iron PDFs

Since the SLAC/NMC parameterization was fit to $F_2^{\nu e}/F_2^{D}$ for charged-lepton DIS data, we can perform a more balanced comparison by using our iron PDFs to compute this same quantity. The results are shown in Fig. 10, where we have used our iron PDFs to compute $F_2^{\nu e}$, and the Base-1 and Base-2 PDFs to compute $F_2^{D}$. As with the nuclear correction factor results of the previous section, we find our results have some gross features in common while on a more refined level the magnitude of the nuclear corrections extracted from the CC iron data differs from the charged lepton data. In particular, we note that the so-called “anti-shadowing” enhancement at
where $x \sim [0.06 - 0.3]$ is not reproduced by the charged current (anti-)neutrino data. Examining our results among all the various $R(Q^2)$ calculations, we generally find that any nuclear enhancement in the small $x$ region is reduced and shifted to a lower $x$ range as compared with the SLAC/NMC parameterization. In fact, this behavior is expected given the comparisons of Figs. 1-8 which show that at $x \sim 0.1$ the cross sections obtained with the base PDFs are not smaller than the ‘A’ and ‘A2’ fitted cross sections. Furthermore, in the limit of large $x$ ($x \gtrsim 0.6$) our results are slightly higher than the data, including the very precise SLAC-E139 points; however, the large theoretical uncertainties on $F_2^D$ in this $x$-region (see Fig. 5) make it difficult to extract firm conclusions.

This discussion raises the more general question as to whether the charged current ($\nu$--Fe) and neutral current ($\ell^+--Fe$) correction factors are entirely compatible [3, 43, 53, 54, 55, 56]. There is a priori no requirement that these be equal; in fact, given that the $\nu$--Fe process involves the exchange of a $W$ and the $\ell^+--Fe$ process involves the exchange of a $\gamma$ we necessarily expect this will lead to differences at some level. To say definitively how much of this difference is due to this effect and how much is due to the uncertainty of our nuclear PDFs requires further study; in particular, it would be interesting to extend the global analysis of nuclear PDFs to include neutral current charged-lepton as well as additional charged current neutrino data. Here, the analysis of additional data sets such as the ones from the CHORUS experiment [57, 58] (neutrino-lead interactions) should help clarify these questions. We are in the processes of adding additional nuclear data sets to our analysis; however, this increased precision comes at the expense of introducing the “A” degree of freedom into the fit.

V. CONCLUSIONS

We have presented a detailed analysis of the high statistics NuTeV neutrino–iron data in the framework of the parton model at next-to-leading order QCD. This investigation takes a new approach to this problem by studying a single nuclear target (iron) so that we avoid the difficulty of having to assume a nuclear “A”-dependence. In this context, we have extracted a set of iron PDFs which are free of any nuclear model dependence. By comparing these iron PDFs with “free proton” PDFs, we can construct the associated nuclear correction factor $R$ for any chosen observable in any given $(x, Q^2)$ kinematic range.

While the nuclear corrections extracted from charged current $\nu$--Fe scattering have similar characteristics as the neutral current $\ell^+--Fe$ charged-lepton results, the detailed $x$ and $Q^2$ behavior is quite different. These results raise the deeper question as to whether the charged current and neutral current correction factors may be substantially different. A combined analysis of neutrino and charged-lepton data sets, for which the present study provides a foundation, will shed more light on these issues. Resolving these questions is essential if we are to reliably use the plethora of nuclear data to obtain free-proton PDFs.

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