Research Article

Analysis of Heat and Mass Transfer of Fractionalized MHD Second-Grade Fluid over Nonlinearly Moving Porous Plate

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This study investigates the heat and mass transfer in the MHD flow of fractionalized second-grade fluid induced by impulsively moved bottom porous plate with nonlinear velocity of the magnitude KTD. To acquire the fractionalized nondimensional set of flow administering differential equations, fractional calculus and dimensionless variables are considered. The solution process utilizes Laplace transform and results in the acquired outputs in terms of generalized functions. The exact solutions for concentration, temperature, velocity, and the shear stress are then reduced by certain limits into fractional/traditional second-grade and Newtonian fluids as per the special cases within and out of the magnetic and porous effects. It is observed that these special cases occur in the previous published literature which verify the results of this study. The results are pictorially visualized to perform the analysis for impacts of diverse physical parameters and dimensionless quantities on concentration, temperature, and velocity fields. It is learned from the analysis that magnitude of viscoelastic parameter is directly proportional to velocity whereas the porous and magmatic effects are inversely proportional. Increasing fractional parameter values reduce flow fields of velocity and temperature. Effects of dimensionless parameters for heat and mass transfer are analysed in detail.

1. Introduction

A number of technological developments and industrial applications of fluids require the complete information about their complex rheological flow, due to which the study of non-Newtonian fluids is carried out at large scale with a variety of enclosure limitations in different situations. These situations practically appear in many electrochemical, geophysical, biorheological, petroleum engineering, metallurgical, and other industrial practices like plastic and polymer melts, pulps, oil, and greases, etc. A second-grade submodel of a non-Newtonian differential type model suitably describes the complex rheological behavior of the above-mentioned fluids [1]. The analytic solutions of this type of fluid model are found in literature serving as a solution to such existing flow problem, and it supports a verification of different numerical schemes for extremely complex flows. Haq [2] investigated solutions of different flow forms of second-grade fluid with MHD Darcy’s law using an Caputo-Fabrizio derivative approach. Salman [3] calculated the values for movement of second-grade fluid over the plane having constant acceleration and presented the exact solutions as the collection of high magnitude time and transient exact solutions simply reducible to the similar solution for Newtonian fluid there by concluding that non-Newtonian effects diminish by time.

For a proper and accurate investigation of all the viscoelastic properties of a fluid, fractional calculus is adopted to acquire the fractionalized form of flow administering equations [4]. Like the rate type fluids, the second-grade
fluid model is passed through a modification for the purpose of generalization, that is, roughly speaking, nothing but a replacement of integer order differential operator of time derivative with one of the broadly applied fractional order differential operators as here we use the Caputo [5]. Flow produced by a heated plane moving impulsively having fractionalized second grade fluid is studied by Tassaddiq [6]; he concluded that the fractional approach is more useful for theory analysis of viscoelasticity. Riaz and Iftikhar [7] compared local and nonlocal derivative study to analyse heat transfer in MHD Maxwell fluid. Khan and Wang [8] investigated the generalized second-grade fluid flow enclosed by two perpendicular walls induced through the impulsive movement of surface over which fluid is supposed to be, and they verified their results with previous literature by vanishing the perpendicular bounding walls. Fahad et al. [9] analysed MHD second grade fluid, Newtonian heating, and Dufour effect over an infinite vertical plate with fractional mass diffusion and thermal transports using noninteger-order derivative Caputo Fabrizio (CF) with nonsingular kernel.

MHD flow incorporated with porous medium is reported in many processes of the biology and medicine industry. It is utilized in chemical industry for filtration, separation, and purification processes which involve the interaction of electric or magnetic field with hydrodynamic boundary layers. Analytical results including porous and MHD terms in second-grade fluid are acquired by Ali et al. [10], where there exists the magnetohydrodynamic fluctuating free convection flow of incompressible electrically conducted viscoelastic fluid in a porous medium in the presence of a pressure gradient. Salah [11] analysed the rotationally accelerated fluid and found that variable and constant accelerated MHD flow behave equivalently. Bajwa et al. [12] found that when the transpiration parameter approaches zero, the solution for the flow with transpiration tends to the solution corresponding to the case without perspiration. Influence of hall currents with MHD and porous effects is investigated in the second grade fluid with oscillations by Hussain et al. [13].

Boundary layer flow of viscoelastic non-Newtonian fluids is of fundamental importance in industrial and applied sciences. Sticking to the problem under study, heat transfer is observed due to both the exponential motion of plate and the buoyancy force that is generated by the difference of temperatures between the fluid and moving bottom plate. In practical applications, reduction of drag due to friction; paper production processes; and the cooling mechanisms of electronic, nuclear, chemical, and industrial processes involve such issues. A lot of research material is available over the flow in touch with the first layer of boundary and heat transfer for being found in much application of industrial interest. The presence stretching sheet with fluid flow of second-grade type is effected by heat of friction, heat absorbed or generated internally, and the heat in terms of deformation work is dealt by Vajravelu and Roper [14]. Tan and Masuoka [15] solved Stokes’ first problem over the high-temperature surface in a semipermeable space. They obtained the steady-state solution in the form of damping exponential function of distance to a hot plate. Concentration and chemical reaction are considered by Hayat et al. [16] with heat transfer in second-grade fluid thereby taking into account the HAM method. Hayat and Abbas [17] also obtained the solutions to same problem in the presence of MHD and porous effects. The movement of fluid of second-grade type in the presence of stretching sheet in unsteady form is discussed by Sajid et al. [18]; they addressed both processes of heating and the prescribed surface temperature, as well as prescribed surface heat flux, and obtained the analytic solutions valid for all times by using HAM method.

In recent years, Baoku et al. [19] adopted numerical approach to find the approximate solutions, where they applied the Runge-Kutta-Fehlberg of order five by shooting method strategy. Das [20] did the same in addition with heated sheet of stretchable form but utilized Nachtsheim-Swigert shooting method for sixth order Runge-Kutta scheme. El-Dabe et al. [21] analysed the second-grade fluid flow with nonlinear form of transfer of both the mass and heat and discussed the thermophoresis applications in detail. Khan [22] studied heat transfer in a thin film of the steady form of flow in touch with the first layer of boundary of porous material in the presence of second-grade fluid. Wakif analysed the analytical and numerical solution of fluid flow with convection heat transfer [23–25]. Bhatti et al. [26] analysed higher-order slip flow of Eyring-Powell nanofluid for Darcy-Forchheimer in the presence of bioconvection and nonlinear thermal radiation and proved that bioconvection Lewis number declines the microorganism profile while the increasing trend is noted for higher values of slip parameter. Bhatti et al. [27] also carried out group analysis and used a robust computational approach to examine mass transport and found successive linearization approach (SLM) more efficient.

This study investigates the heat and mass transfer in the MHD flow of fractionalized second-grade fluid induced by impulsively moved bottom porous plate with exponential velocity of the magnitude $K_2^d$. To acquire the fractionalized nondimensional set of flow administering differential equations, fractional calculus and dimensionless variables are considered. The solution process utilizes Laplace transform and results in the acquired outputs in terms of generalized functions. The solutions in the exact form for concentration, temperature, velocity, and stress are then reduced by certain limits into fractional second-grade and Newtonian fluids as per the special cases within and out of the magnetic and porous effects. The results are pictorially presented to perform the analysis for impacts of diverse physical parameters and dimensionless quantities. This study mainly focuses on the following research questions.

What is the compact form of analytical solutions of flow field in the presence of MHD and porous medium in terms of summation style and newly defined $M$ function?

What are the effects of nonlinear movement of plate over the fluid velocity, heat, and mass transfer?

How the shear stress profile behaves for nonlinear movement of porous surface in presence of MHD effect?
Differentiate the responses of the flow field between the linear and nonlinear/exponential movement of the porous surface (Table 1).

2. Flow Field Description and Its Solutions

The free convection heat and mass transfer in MHD unsteady flow of second-grade fluid over an infinite vertical porous plate in \((x,z)\) plane is considered, while \(y\)-axis being normal to the plate. Species concentration \(\zeta\), temperature distribution \(\gamma\), velocity \(v\) in \(x\)-direction, and shear stress are considered as function of only \(\tau\) and \(\xi\). Initially the plate is at rest and the fluid too with ambient fluid temperature \(\gamma_\infty\) (constant) and the constant concentration \(\zeta_\infty\). At the moment \(\tau = 0^+\), plate starts to move in its own plane with the nonlinear velocity \(K\gamma_\tau^d\) there by raising the temperature and concentration level of the plate to \(\gamma_w\) and \(\zeta_w\), respectively. A transverse magnetic field \(B_0\) with uniform strength is normally applied to the plate in the direction parallel to \(y\)-axis as shown in Figure 1, while assuming the negligible induced magnetic field as compared to transverse magnetic field due to very low Reynold’s number. The viscous dissipation, Soret and Duoffier effects are also neglected for low level of concentration.

From the above assumptions with constant Boussinesq approximation, the set of flow and transfer equations for heat and mass transfer in incompressible second-grade fluid is shown [28–30].

\[
\begin{align*}
\frac{\partial v(\xi, \tau)}{\partial \tau} &= \left(\nu + \alpha \frac{\partial}{\partial \tau}\right) \frac{\partial^2 v(\xi, \tau)}{\partial \xi^2} - \frac{\sigma B_0^2}{\rho} v(\xi, \tau) \\
&- \frac{\rho}{k} \left(\nu + \alpha \frac{\partial}{\partial \tau}\right) v(\xi, \tau) \\
&+ gB_1 \left(\gamma(\xi, \tau) - \gamma_\infty\right) + gB_2 \left(\zeta(\xi, \tau) - \zeta_\infty\right) \\
&- \zeta_\infty; \xi, \tau > 0,
\end{align*}
\]

\(L(\xi, \tau) = \left(\mu + \alpha \frac{\partial}{\partial \tau}\right) \frac{\partial v(\xi, \tau)}{\partial \xi} = 0, \xi, \tau > 0,\)

\(\rho C_p \frac{\partial \gamma(\xi, \tau)}{\partial \tau} = k \frac{\partial^2 \gamma(\xi, \tau)}{\partial \xi^2} - \frac{q_r(\xi, \tau)}{\partial \xi}; \xi, \tau > 0,
\]

subject to the following initial, boundary and natural conditions, respectively.

\[
\begin{align*}
v(\xi, 0) &= 0, L(\xi, 0) = 0, \gamma(\xi, 0) = \gamma_\infty, \zeta(\xi, 0) = \zeta_\infty, \xi \geq 0, \\
v(0, \tau) &= K\gamma_\tau^d, \gamma(0, \tau) = \gamma_w, \zeta(0, \tau) = \zeta_w; \tau > 0,
\end{align*}
\]

\[
\begin{align*}
\nu(\xi, \tau) &\rightarrow 0, \gamma(\xi, \tau) \rightarrow \gamma_\infty, \zeta(\xi, \tau) \rightarrow \zeta_\infty, \text{as} \xi \rightarrow \infty,
\end{align*}
\]

where \(v, \mu, \sigma, k, \rho, \phi, \kappa, g, B_0, B_1, B_2, C_p, q_r, D, \) and \(\alpha_i\) are kinematic viscosity, dynamic viscosity, fluid electric conductivity, fluid thermal conductivity, fluid density, porosity parameter, permeability of the porous medium, gravitational acceleration, magnetic field strength, volumetric heat transfer coefficient, volumetric mass transfer coefficient, specific heat capacity, heat flux radiation, mass diffusion coefficient, and second grade fluid parameter, respectively, whereas \(\alpha = \alpha_i/\rho\). By introducing the following dimensionless variables,

\[
\begin{align*}
\nu^* &= \frac{\nu}{(K\gamma_\tau^d)^{1/2 + d/1}}, \\
\gamma^* &= \frac{\gamma}{(K\gamma_\tau^d)^{1/2 + d/1}}, \\
\zeta^* &= \frac{\zeta}{(K\gamma_\tau^d)^{1/2 + d/1}}, \\
\ell^* &= \frac{\ell}{(K\gamma_\tau^d)^{2/2 + d/1}}, \\
\gamma^{* \tau} &= \frac{\gamma}{(K\gamma_\tau^d)^{1/2 + d/1}},
\end{align*}
\]

and considering

\[
\begin{align*}
v(\xi, \tau) &= \left(\mu + \alpha \frac{\partial}{\partial \tau}\right) \frac{\partial v(\xi, \tau)}{\partial \xi} \\
&- \frac{\rho}{k} \left(\nu + \alpha \frac{\partial}{\partial \tau}\right) v(\xi, \tau) \\
&+ gB_1 \left(\gamma(\xi, \tau) - \gamma_\infty\right) + gB_2 \left(\zeta(\xi, \tau) - \zeta_\infty\right) \\
&- \zeta_\infty; \xi, \tau > 0,
\end{align*}
\]

\(L(\xi, \tau) = \left(\mu + \alpha \frac{\partial}{\partial \tau}\right) \frac{\partial v(\xi, \tau)}{\partial \xi} = 0, \xi, \tau > 0,\)

\(\rho C_p \frac{\partial \gamma(\xi, \tau)}{\partial \tau} = k \frac{\partial^2 \gamma(\xi, \tau)}{\partial \xi^2} - \frac{q_r(\xi, \tau)}{\partial \xi}; \xi, \tau > 0,
\]
\[
\lambda = \left( \frac{K_\nu^{\beta} 2^{2/3} d^{-1} + 1}{\nu^2} \right)
\]

\[
M = \frac{\nu\alpha B_0^2}{(K_\nu^{\beta} 2^{2/3} d^{-1} + 1)}
\]

\[
\Psi = \frac{\nu^2 q}{(K_\nu^{\beta} 2^{2/3} d^{-1} + 1)}
\]

\[
G_r = \frac{\nu g B_1 (\gamma - \gamma_{\infty})}{(K_\nu^{\beta} 2^{2/3} d^{-1} + 1)}
\]

\[
G_m = \frac{\nu g B_2 (\gamma - \gamma_{\infty})}{(K_\nu^{\beta} 2^{2/3} d^{-1} + 1)}
\]

\[
P_r = \frac{\mu C_p}{K}
\]

\[
F = \frac{\nu^2 4I}{(K_\nu^{\beta} 2^{2/3} d^{-1} + 1)}
\]

\[
S_c = \frac{\nu}{D_b}
\]

where the dimensionless quantities \( M, G_r, G_m, P_r, S_c, \Psi, \) and \( F \) represent Hartmann number, thermal Grashof number, mass Grashof number, Prandtl number, Schmidt number, thermal radiation, and porosity parameter, respectively.

For the sake of brevity, we omit " * "; thus, the results are

\[
\frac{\partial v(\xi, \tau)}{\partial \tau} = \left( 1 + \lambda \frac{\partial}{\partial \tau} \right) \frac{\partial^2 v(\xi, \tau)}{\partial \xi^2} - Mu(\xi, \tau) - \Psi \left( 1 + \lambda \frac{\partial}{\partial \tau} \right)\]

\[
v(\xi, \tau) + G_r^2(\xi, \tau) + G_m^2(\xi, \tau),
\]

\[
L(\xi, \tau) = \left( 1 + \lambda \frac{\partial}{\partial \tau} \right) \frac{\partial v(\xi, \tau)}{\partial \xi}.
\]

\[
\frac{\partial \gamma(\xi, \tau)}{\partial \tau} = \frac{1}{P_r} \frac{\partial^2 \gamma(\xi, \tau)}{\partial \xi^2} - \frac{F}{P_r} \gamma(\xi, \tau),
\]

\[
\frac{\partial \sigma(\xi, \tau)}{\partial \tau} = \frac{1}{S_c} \frac{\partial^2 \sigma(\xi, \tau)}{\partial \xi^2},
\]

with imposed conditions

\[
v(\xi, 0) = 0, L(\xi, 0) = 0, \gamma(\xi, 0) = 0, \sigma(\xi, 0) = 0, \xi \geq 0,
\]

\[
v(0, \tau) = 1^d, L(0, \tau) = 1, \gamma(0, \tau) = 1, \sigma > 0,
\]

\[
\nu(\xi, \tau) \longrightarrow 0, \gamma(\xi, \tau) \longrightarrow 0, 2(\xi, \tau) \longrightarrow 0, as \xi \longrightarrow \infty.
\]

(16)

The fractionalized form of the governing equations is

\[
\frac{\partial \gamma(\xi, \tau)}{\partial \tau} = \frac{1}{P_r} \frac{\partial^2 \gamma(\xi, \tau)}{\partial \xi^2} - \frac{F}{P_r} \gamma(\xi, \tau),
\]

\[
\frac{D^\alpha_r \gamma(\xi, \tau)}{D^\tau} = \frac{1}{P_r} \frac{\partial^2 \gamma(\xi, \tau)}{\partial \xi^2},
\]

(17)

\[
\frac{D^{\beta}_r \gamma(\xi, \tau)}{D^\tau} = \frac{1}{P_r} \frac{\partial^2 \gamma(\xi, \tau)}{\partial \xi^2},
\]

(18)

\[
\frac{D^{\gamma}_r \gamma(\xi, \tau)}{D^\tau} = \frac{1}{P_r} \frac{\partial^2 \gamma(\xi, \tau)}{\partial \xi^2},
\]

(19)

where \( \beta, \gamma, \) and \( \gamma \) are the fractional parameters, and \( D^\alpha_r \) is the Caputo fractional operator defined by [23, 24].

\[
D^\alpha_r f(\tau) = \left\{ \begin{array}{ll}
\frac{1}{(1 - \alpha)} \int_0^\tau \exp \left( \frac{1 - \alpha}{\gamma - s} \right) f'(s) ds, & \alpha > 0;
\end{array} \right.
\]

(20)

As per the above system of fractional partial differential equations (17)–(20) with (14) in mind, we take Laplace of (15) and (16).

\[
\nu(0, q) = \frac{\Gamma(d + 1)}{q^{d+1}} \frac{\gamma(0, q)}{\chi}, \quad \frac{1}{q}, \frac{\gamma(0, q)}{\gamma},
\]

(22)

\[
\nu(0, q) \longrightarrow 0, \frac{\gamma(0, q)}{\gamma} \longrightarrow 0, \frac{\gamma(0, q)}{\gamma} \longrightarrow 0, as \xi \longrightarrow \infty,
\]

(23)

where \( q \) stands for transform parameter while image function of \( v(\xi, \tau) \) is represented by \( \nu(\xi, q) \).

2.1. Calculation of the Mass Concentration. Taking the Laplace transform of (20) and using the initial condition (14), we get

\[
\frac{\partial^2 \gamma(\xi, \tau)}{\partial \xi^2} = S_c q^\gamma \gamma(\xi, \tau) = 0.
\]

(24)

Solving (24) using the natural condition (23) and boundary condition (22), we obtained

\[
\gamma(\xi, q) = \exp \left[ - \frac{S_c q^\gamma 1/2}{} \right].
\]

(25)

In terms of series, the above equation can be written as
\[ \Psi(\xi, q) = \frac{1}{q} + \sum_{\gamma_1=1}^{\infty} \left( -\frac{\sqrt{S_\gamma}}{\gamma_1} \right)^2 q^{\gamma_1/2 - 1}. \] (26)

Applying the inverse discrete Laplace transform, we have
\[ \Psi(\xi, \tau) = 1 + \sum_{\eta_0=0}^{\infty} \left( -\frac{\sqrt{S_\eta}}{\eta_0/2} \right) \Gamma(-\eta_0/2 + 1) \] (27)
\[ \Psi(\xi, \tau) = \sum_{\eta_0=0}^{\infty} \left( -\frac{\sqrt{S_\eta}}{\eta_0/2} \right)^2 \Gamma(-\eta_0/2 + 1) \] (28)
In the form of general Wright function [31],
\[ \Psi(\xi, \tau) = W_{-\eta_0/2,1} \left( -\frac{\sqrt{S_\eta}}{\tau} \xi \right) \] (29)
where the general Wright function is defined as
\[ W_{\lambda, \mu}(\xi) = \sum_{k=0}^{\infty} \frac{\xi^k}{k!} \Gamma(\lambda k + \mu). \] (30)

2.2. Calculation of the Temperature Distribution. Taking the Laplace transform of (19) and using the initial condition (14), we have
\[ \frac{\partial^2 \psi(\xi, q)}{\partial q^2} - \left( -P_c q^\gamma + F \right) \psi(\xi, q) = 0. \] (31)
Solving (31) using natural condition (23) and boundary condition (22), we get
\[ \psi(\xi, q) = \exp \left[ -\left( -P_c q^\gamma + F \right)^{1/2} \xi \right] q \] (32)
In terms of series, the above equation can be given as
\[ \psi(\xi, q) = \frac{1}{q} + \sum_{\eta_0=0}^{\infty} \left( -\frac{\sqrt{S_\eta}}{\eta_0/2} \right)^\rho \Gamma(-\eta_0/2 + \rho_2) \left( -\frac{P_c}{P_r} \right)^\rho_2 q^{\rho_2/2 - \rho_2} - 1. \] (33)
Applying the inverse discrete Laplace transform, we have
\[ \psi(\xi, \tau) = 1 + \sum_{\eta_0=0}^{\infty} \left( -\frac{\sqrt{S_\eta}}{\eta_0/2} \right)^\rho \Gamma(-\eta_0/2 + \rho_2) \left( -\frac{P_c}{P_r} \right)^\rho_2 q^{\rho_2/2 - \rho_2} \] (34)
Presenting the above equation in more generalized way of M function,
\[ \Psi(\xi, \tau) = 1 + \sum_{\eta_0=0}^{\infty} \left( -\frac{\sqrt{S_\eta}}{\eta_0/2} \right)^\rho \Gamma(-\eta_0/2 + \rho_2) \left( -\frac{P_c}{P_r} \right)^\rho_2 q^{\rho_2/2 - \rho_2} \] (35)
where the newly formulated \( M \)-function with the help of Fox H-function [31–33] is explained by
\[ \Gamma \left[ \sum_{\gamma=0}^{\infty} (-z)^\gamma \left( \prod_{j=1}^{m} \Gamma \left( x_j + X_j \right) \right) \right] = M^{1m}_{n, n+1} \] (36)

2.3. Calculation of the Velocity. Initial conditions (14) are imposed after taking Laplace transform of (17), which yield the results
\[ \frac{\partial^2 \psi(\xi, q)}{\partial q^2} - \left[ \frac{(q + M)}{(1 + \lambda q^\gamma)} + \psi \right] \psi(\xi, q) = \frac{G_r}{q} \exp \left[ -\left( -P_c q^\gamma + F \right)^{1/2} \xi \right] \left( 1 + \lambda^\gamma q^\gamma \right) \] (37)
Solving (37) utilizing conditions (22) and (23), we get

\[
\varpi(\xi, q) = \left\{ \frac{\Gamma(d + 1)}{q^{d+1}} + \frac{G_r}{q} \left[ P_r q^r + F - \left((q + M)(1 + \lambda^d q^d) - 1 + \Psi \right) \right] \right\} + \frac{G_m}{q} \left[ \frac{S_m q^m - \left((q + M)(1 + \lambda^d q^d) - 1 + \Psi \right)}{q} \right] \exp \left\{ \left[ P_r q^r + F \right]/\xi \right\} \right. \\
\times \left. \exp \left\{ \left[(q + M)(1 + \lambda^d q^d) - 1 + \Psi \right]^{1/2} \xi \right\} - \frac{G_m}{q} \left[ S_m q^m - \left((q + M)(1 + \lambda^d q^d) - 1 + \Psi \right) \right] \exp \left\{ [S_m q^m]^{1/2} \xi \right\} \right.
\]

Presenting (38) in series form to easily produce \( \varpi(\xi, q) = L^{-1}[\varpi(\xi, q)] \) without prolix computational complexities of residuals and contours integrals,
Practicing the discrete inverse Laplace transform, we obtain

\[
\begin{aligned}
\psi(\xi, \tau) &= \tau^{b} + \Gamma (d + 1) \sum_{\varepsilon_{1} = 1}^{\infty} \frac{(-\xi)^{\varepsilon_{1}}}{\varepsilon_{1}!} \sum_{\varepsilon_{2} = 0}^{\infty} \frac{(-\omega)^{\varepsilon_{2}}}{\varepsilon_{2}!} \lambda^{\beta} (-\varepsilon_{1}^{2} + \varepsilon_{2}) \psi_{1}^{(\beta-1)} \psi_{3}^{(\beta-2) + \varepsilon_{1} + \varepsilon_{2}} \\
&\times \sum_{\varepsilon_{3} = 0}^{\infty} \frac{(-\rho)^{\varepsilon_{3}}}{\varepsilon_{3}!} \psi_{4}^{(\beta-1) (\varepsilon_{1} + \varepsilon_{2}) + \varepsilon_{3}} \\
&+ \frac{G_{r}}{\theta_{1} \theta_{3} \theta_{4}} \sum_{\varepsilon_{2} = 0}^{\infty} \frac{(-\xi)^{\varepsilon_{2}}}{\varepsilon_{2}!} \left( \frac{1}{P_{r}} \right)^{\varepsilon_{2} + 1} \frac{(-\omega)^{\varepsilon_{2}}}{\varepsilon_{2}!} \frac{(-\omega)^{\varepsilon_{2}}}{\varepsilon_{2}!} \lambda^{\beta} (-\varepsilon_{2}^{2} + \varepsilon_{3}) \\
&\times \Gamma ((\beta-1) (\varepsilon_{1} + \varepsilon_{2}) + \varepsilon_{3}) + \beta + \theta_{4} + \beta \theta_{5} + \gamma (\varepsilon_{2} + \varepsilon_{3} + 1) + 1) \\
&+ \frac{G_{m}}{\theta_{1} \theta_{3} \theta_{4}} \sum_{\varepsilon_{2} = 0}^{\infty} \frac{(-\xi)^{\varepsilon_{2}}}{\varepsilon_{2}!} \left( \frac{1}{S_{r}} \right)^{\varepsilon_{2} + 1} \frac{(-\omega)^{\varepsilon_{2}}}{\varepsilon_{2}!} \frac{(-\omega)^{\varepsilon_{2}}}{\varepsilon_{2}!} \lambda^{\beta} (-\varepsilon_{2}^{2} + \varepsilon_{3}) \\
&\times \Gamma (((\beta-1) (\varepsilon_{1} + \varepsilon_{2}) + \varepsilon_{3}) + \beta + \theta_{4} + \beta \theta_{5} + \gamma (\varepsilon_{2} + \varepsilon_{3} + 1) + 1) \\
&- \frac{G_{r}}{\theta_{1} \theta_{3} \theta_{4}} \sum_{\varepsilon_{2} = 0}^{\infty} \frac{(-\rho)^{\varepsilon_{2}}}{\varepsilon_{2}!} \left( \frac{1}{P_{r}} \right)^{\varepsilon_{2} + 1} \frac{(-\omega)^{\varepsilon_{2}}}{\varepsilon_{2}!} \frac{(-\omega)^{\varepsilon_{2}}}{\varepsilon_{2}!} \lambda^{\beta} (-\varepsilon_{2}^{2} + \varepsilon_{3}) \\
&\times \Gamma (((\beta-1) (\varepsilon_{1} + \varepsilon_{2}) + \varepsilon_{3}) + \beta + \theta_{4} + \beta \theta_{5} + \gamma (\varepsilon_{2} + \varepsilon_{3} + 1) + 1) \\
&- \frac{G_{m}}{\theta_{1} \theta_{3} \theta_{4}} \sum_{\varepsilon_{2} = 0}^{\infty} \frac{(-\omega)^{\varepsilon_{2}}}{\varepsilon_{2}!} \left( \frac{1}{S_{r}} \right)^{\varepsilon_{2} + 1} \frac{(-\omega)^{\varepsilon_{2}}}{\varepsilon_{2}!} \frac{(-\omega)^{\varepsilon_{2}}}{\varepsilon_{2}!} \lambda^{\beta} (-\varepsilon_{2}^{2} + \varepsilon_{3}) \\
&\times \Gamma (((\beta-1) (\varepsilon_{1} + \varepsilon_{2}) + \varepsilon_{3}) + \beta + \theta_{4} + \beta \theta_{5} + \gamma (\varepsilon_{2} + \varepsilon_{3} + 1) + 1).
\end{aligned}
\]
Rewriting the velocity expression using generalized M-function,

\[
u(\xi, \tau) = \tau^b + \Gamma(d + 1) \sum_{\xi_1 = 1}^{\infty} \frac{(-\xi)^{\xi_1}}{\xi_1!} \sum_{\xi_3 = 0}^{\infty} \frac{(-\Psi)^{\xi_3}}{\xi_3!} \frac{(-M)^{\xi_4}}{\xi_4!} \lambda^\beta (-\xi^{1/2} - \xi_1) \\
\times M_{3,5}^{1,4} \left[ \left( \frac{1}{1 + \xi} \right) \left( 1 + \xi_2 \right) \left( 1 + \xi_3 \right) \left( 1 + \xi_4 \right) \left( 1 + \xi_5 \right) \right] \\
\times M_{4,6}^{1,4} \left[ \left( \frac{1}{1 + \xi} \right) \left( 1 + \xi_2 \right) \left( 1 + \xi_3 \right) \left( 1 + \xi_4 \right) \right] \\
\times M_{3,5}^{1,4} \left[ \left( \frac{1}{1 + \xi} \right) \left( 1 + \xi_2 \right) \left( 1 + \xi_3 \right) \right] \\
+ G_r \left( \frac{1}{1 + \xi} \right) \left( 1 + \xi_2 \right) \left( 1 + \xi_3 \right) \left( 1 + \xi_4 \right) \left( 1 + \xi_5 \right) \\
\times M_{4,6}^{1,4} \left[ \left( \frac{1}{1 + \xi} \right) \left( 1 + \xi_2 \right) \left( 1 + \xi_3 \right) \right] \\
- G_r \left( \frac{1}{1 + \xi} \right) \left( 1 + \xi_2 \right) \left( 1 + \xi_3 \right) \left( 1 + \xi_4 \right) \left( 1 + \xi_5 \right) \\
\times M_{3,5}^{1,4} \left[ \left( \frac{1}{1 + \xi} \right) \left( 1 + \xi_2 \right) \left( 1 + \xi_3 \right) \right] \\
+ G_m \left( \frac{1}{1 + \xi} \right) \left( 1 + \xi_2 \right) \left( 1 + \xi_3 \right) \left( 1 + \xi_4 \right) \left( 1 + \xi_5 \right) \right] .
\]

2.4. Formulating Shear Stress. Initial conditions (14) are imposed after taking Laplace transform of Equation (18) yielding the results

\[
\Gamma(\xi, q) = (1 + \lambda^\beta q^\beta)^{-1} \frac{\partial \Theta(\xi, q)}{\partial \xi},
\]

where the Laplace transform of \( \Lambda(\xi, \tau) \) is given as \( \Lambda(\xi, q) \).

With the help of (38), the given expression can be rearranged as

\[
\Gamma(\xi, q) = \left\{ \frac{\Gamma(d + 1) (1 + \lambda^\beta q^\beta)^{-1}}{d!} + \frac{G_r}{q} \left[ \frac{1}{P_r} q^\gamma + F - (q + M) \left( \frac{1 + \lambda^\beta q^\beta)^{-1}}{1 + \lambda^\beta} + \Psi \right] \right\} \\
+ \left\{ \frac{1}{q} \left[ \frac{1}{S_c q^n - \left( (q + M) (1 + \lambda^\beta q^\beta)^{-1} + \Psi \right) \left( (q + M) (1 + \lambda^\beta q^\beta)^{-1} + \Psi \right)^{1/2} \right] \left[ \frac{1}{(q + M) (1 + \lambda^\beta q^\beta)^{-1} + \Psi} \right] \right\}
\]
Writing the given expression in the form of a series

\[
\mathcal{L}(\xi, q) = -\Gamma(d + 1) \sum_{i=0}^{\infty} \frac{(-\xi)^{\beta_i}}{\xi!} \sum_{\eta, \xi_1} \frac{(-\Psi)^{\beta_i}}{\eta^i!} \frac{(-M)^{\beta_i}}{\xi_1^i!} \lambda^\beta (-\eta + 1/2 - \eta, 1)
\]

\[
\times \exp \left\{ \left[ -\lambda^\beta (1 + \lambda^\beta q^\delta)^{-1} + \Psi \right] \xi \right\} + \frac{G_m}{q} \frac{[S q^\delta F]/[\xi + (q + M) (1 + \lambda^\beta q^\delta)^{-1} + \Psi]}{\xi^i!} \exp \left\{ -[S q^\delta F]/[\xi + (q + M) (1 + \lambda^\beta q^\delta)^{-1} + \Psi] \right\}
\]

\[
+ \frac{G_m}{q} \frac{[S q^\delta F]/[\xi + (q + M) (1 + \lambda^\beta q^\delta)^{-1} + \Psi]}{\xi^i!} \exp \left\{ -[S q^\delta F]/[\xi + (q + M) (1 + \lambda^\beta q^\delta)^{-1} + \Psi] \right\}.
\]

(43)

\[
\mathcal{L}(\xi, q) = -\Gamma(d + 1) \sum_{i=0}^{\infty} \frac{(-\xi)^{\beta_i}}{\xi!} \sum_{\eta, \xi_1} \frac{(-\Psi)^{\beta_i}}{\eta^i!} \frac{(-M)^{\beta_i}}{\xi_1^i!} \lambda^\beta (-\eta + 1/2 - \eta, 1)
\]

\[
\times \exp \left\{ \left[ -\lambda^\beta (1 + \lambda^\beta q^\delta)^{-1} + \Psi \right] \xi \right\} + \frac{G_m}{q} \frac{[S q^\delta F]/[\xi + (q + M) (1 + \lambda^\beta q^\delta)^{-1} + \Psi]}{\xi^i!} \exp \left\{ -[S q^\delta F]/[\xi + (q + M) (1 + \lambda^\beta q^\delta)^{-1} + \Psi] \right\}
\]

\[
+ \frac{G_m}{q} \frac{[S q^\delta F]/[\xi + (q + M) (1 + \lambda^\beta q^\delta)^{-1} + \Psi]}{\xi^i!} \exp \left\{ -[S q^\delta F]/[\xi + (q + M) (1 + \lambda^\beta q^\delta)^{-1} + \Psi] \right\}.
\]

(44)
Inverting the Laplace transform results,

\[
I(\xi, \tau) = -\Gamma(d + 1) \sum_{\xi_1=0}^{\infty} \frac{(-\xi)^{\beta_1}}{\xi_1!} \sum_{\xi_2, \xi_3, \xi_4, \xi_5=0}^{\infty} \frac{(-\Psi)^{\beta_3} (-M)^{\beta_4} \lambda^\beta (-\xi_1 + 1/2 + \xi_5)}{\xi_1^3 \xi_3! \xi_4!} \chi(\beta - 1)(\xi_1/2 - \xi_5 - 1/2)(\beta - 1 + \xi_4 + b)
\]

\[
\times \sum_{\xi_5=0}^{\infty} \frac{(-\xi)^{\beta_1}}{\xi_1!} \frac{(-\Psi)^{\beta_3} (-M)^{\beta_4} \lambda^\beta (-\xi_1 + 1/2 + \xi_5)}{\xi_1^3 \xi_3! \xi_4!} \chi(\beta - 1)(\xi_1/2 - \xi_5 - 1/2)(\beta - 1 + \xi_4 + b)
\]

\[
\times \sum_{\xi_5=0}^{\infty} \frac{(-\xi)^{\beta_1}}{\xi_1!} \frac{(-\Psi)^{\beta_3} (-M)^{\beta_4} \lambda^\beta (-\xi_1 + 1/2 + \xi_5)}{\xi_1^3 \xi_3! \xi_4!} \chi(\beta - 1)(\xi_1/2 - \xi_5 - 1/2)(\beta - 1 + \xi_4 + b)
\]

\[
\times \sum_{\xi_5=0}^{\infty} \frac{(-\xi)^{\beta_1}}{\xi_1!} \frac{(-\Psi)^{\beta_3} (-M)^{\beta_4} \lambda^\beta (-\xi_1 + 1/2 + \xi_5)}{\xi_1^3 \xi_3! \xi_4!} \chi(\beta - 1)(\xi_1/2 - \xi_5 - 1/2)(\beta - 1 + \xi_4 + b)
\]

\[
\times \sum_{\xi_5=0}^{\infty} \frac{(-\xi)^{\beta_1}}{\xi_1!} \frac{(-\Psi)^{\beta_3} (-M)^{\beta_4} \lambda^\beta (-\xi_1 + 1/2 + \xi_5)}{\xi_1^3 \xi_3! \xi_4!} \chi(\beta - 1)(\xi_1/2 - \xi_5 - 1/2)(\beta - 1 + \xi_4 + b)
\]

\[
\times \sum_{\xi_5=0}^{\infty} \frac{(-\xi)^{\beta_1}}{\xi_1!} \frac{(-\Psi)^{\beta_3} (-M)^{\beta_4} \lambda^\beta (-\xi_1 + 1/2 + \xi_5)}{\xi_1^3 \xi_3! \xi_4!} \chi(\beta - 1)(\xi_1/2 - \xi_5 - 1/2)(\beta - 1 + \xi_4 + b)
\]
In terms of M-function

\[
L(\xi, \tau) = -\Gamma (d + 1) \sum_{q=1}^{\infty} \frac{(-\xi)^q}{\xi_q!} \sum_{q=1}^{\infty} \frac{(-\Psi)^q}{\xi_q!} \frac{(-M)^q}{\xi_q!} \lambda^\beta (-\xi, \tau; q, i)
\]

\[
\times M_{3,5}^{1,3} \left[ \frac{\Gamma}{\Lambda^{q,1}} \left( \begin{array}{c} \frac{1}{P_r} \frac{1}{\lambda^{q,1}} \frac{(-\Psi)^q}{\xi_q!} \frac{(-M)^q}{\xi_q!} \frac{(-F/P)^q}{\xi_q!} \lambda^\beta (-\xi, \tau; q, i) \\ \end{array} \right) \right]
\]

\[
G_r \sum_{e_1, e_2, e_3, e_4 = 0}^\infty \left( \begin{array}{c} \frac{1}{P_r} \frac{1}{\lambda^{q,1}} \frac{(-\Psi)^q}{\xi_q!} \frac{(-M)^q}{\xi_q!} \frac{(-F/P)^q}{\xi_q!} \lambda^\beta (-\xi, \tau; q, i) \\ \end{array} \right)
\]

\[
G_m \sum_{e_1, e_2, e_3, e_4 = 0}^\infty \left( \begin{array}{c} \frac{1}{P_r} \frac{1}{\lambda^{q,1}} \frac{(-\Psi)^q}{\xi_q!} \frac{(-M)^q}{\xi_q!} \frac{(-F/P)^q}{\xi_q!} \lambda^\beta (-\xi, \tau; q, i) \\ \end{array} \right)
\]

\[
G_r \sqrt{P_r} \sum_{e_1, e_2, e_3, e_4 = 0}^\infty \left( \begin{array}{c} \frac{1}{P_r} \frac{1}{\lambda^{q,1}} \frac{(-\Psi)^q}{\xi_q!} \frac{(-M)^q}{\xi_q!} \frac{(-F/P)^q}{\xi_q!} \lambda^\beta (-\xi, \tau; q, i) \\ \end{array} \right)
\]

\[
G_m \sqrt{S_c} \sum_{e_1, e_2, e_3, e_4 = 0}^\infty \left( \begin{array}{c} \frac{1}{P_r} \frac{1}{\lambda^{q,1}} \frac{(-\Psi)^q}{\xi_q!} \frac{(-M)^q}{\xi_q!} \frac{(-F/P)^q}{\xi_q!} \lambda^\beta (-\xi, \tau; q, i) \\ \end{array} \right)
\]

\[
G_m \sqrt{S_c} \sum_{e_1, e_2, e_3, e_4 = 0}^\infty \left( \begin{array}{c} \frac{1}{P_r} \frac{1}{\lambda^{q,1}} \frac{(-\Psi)^q}{\xi_q!} \frac{(-M)^q}{\xi_q!} \frac{(-F/P)^q}{\xi_q!} \lambda^\beta (-\xi, \tau; q, i) \\ \end{array} \right)
\]

\[
G_m \sqrt{S_c} \sum_{e_1, e_2, e_3, e_4 = 0}^\infty \left( \begin{array}{c} \frac{1}{P_r} \frac{1}{\lambda^{q,1}} \frac{(-\Psi)^q}{\xi_q!} \frac{(-M)^q}{\xi_q!} \frac{(-F/P)^q}{\xi_q!} \lambda^\beta (-\xi, \tau; q, i) \\ \end{array} \right)
\]

\[
G_m \sqrt{S_c} \sum_{e_1, e_2, e_3, e_4 = 0}^\infty \left( \begin{array}{c} \frac{1}{P_r} \frac{1}{\lambda^{q,1}} \frac{(-\Psi)^q}{\xi_q!} \frac{(-M)^q}{\xi_q!} \frac{(-F/P)^q}{\xi_q!} \lambda^\beta (-\xi, \tau; q, i) \\ \end{array} \right)
\]

3. Some Interesting Particularized Cases

3.1. Ordinary MHD Second-Grade Fluid in Porous Material.

\[
\lambda(\xi, \tau) = W_{-1/2,1} \left( -\sqrt{S_c} \xi \right)
\]

\[
\tau(\xi, \tau) = 1 + \sum_{e_1, e_2, e_3, e_4 = 0}^\infty \left( \begin{array}{c} \frac{1}{P_r} \frac{1}{\lambda^{q,1}} \frac{(-\Psi)^q}{\xi_q!} \frac{(-M)^q}{\xi_q!} \frac{(-F/P)^q}{\xi_q!} \lambda^\beta (-\xi, \tau; q, i) \\ \end{array} \right)
\]

\[
\nu(\xi, \tau) = \tau^\beta + \Gamma (d + 1) \sum_{e_1, e_2, e_3, e_4 = 0}^\infty \left( \begin{array}{c} \frac{1}{P_r} \frac{1}{\lambda^{q,1}} \frac{(-\Psi)^q}{\xi_q!} \frac{(-M)^q}{\xi_q!} \frac{(-F/P)^q}{\xi_q!} \lambda^\beta (-\xi, \tau; q, i) \\ \end{array} \right)
\]
\[
+ G_r \sum_{\eta_1, \eta_2, \eta_3, \eta_4=0}^{\infty} \frac{(-\xi)^{\eta_1}}{\eta_1!} \left( \frac{1}{P_r} \right)^{\eta_4+1} \frac{(-\Psi)^{\eta_3}}{\eta_3!} \frac{(-M)^{\eta_4}}{\eta_4!} \frac{(-F/P_r)^{\eta_5}}{\eta_5!} \lambda (-\eta_1^{1/2}-\eta_3^{1/2}-\eta_4^{1/2})
\]
\[
\times M^{I,4}_{4,3} \left[ \left( \begin{array}{c}
(1+\eta_1/2-\eta_3,0), (1+\eta_2/2,0), (-\eta_1^{1/2},0), (1-\eta_1^{1/2},\eta_4^{1/2})
\end{array} \right) \right]
\]
\[
- G_r \sum_{\eta_1, \eta_2, \eta_3, \eta_4=0}^{\infty} \frac{(-\xi)^{\eta_1}}{\eta_1!} \left( \frac{1}{P_r} \right)^{\eta_4+1} \frac{(-\Psi)^{\eta_3}}{\eta_3!} \frac{(-M)^{\eta_4}}{\eta_4!} \frac{(-F/P_r)^{\eta_5}}{\eta_5!} \lambda (-\eta_1^{1/2}-\eta_3^{1/2}-\eta_4^{1/2})
\]
\[
\times M^{I,4}_{4,3} \left[ \left( \begin{array}{c}
(1+\eta_1/2-\eta_3,0), (1+\eta_2/2,0), (-\eta_1^{1/2},0), (1-\eta_1^{1/2},\eta_4^{1/2})
\end{array} \right) \right]
\]
\[
- G_m \sum_{\eta_1, \eta_2, \eta_3, \eta_4=0}^{\infty} \frac{(-\xi)^{\eta_1}}{\eta_1!} \left( \frac{1}{S_r} \right)^{\eta_4+1} \frac{(-\Psi)^{\eta_3}}{\eta_3!} \frac{(-M)^{\eta_4}}{\eta_4!} \frac{(-F/P_r)^{\eta_5}}{\eta_5!} \lambda (-\eta_1^{1/2}-\eta_3^{1/2}-\eta_4^{1/2})
\]
\[
\times M^{I,4}_{4,3} \left[ \left( \begin{array}{c}
(1+\eta_1/2-\eta_3,0), (1+\eta_2/2,0), (-\eta_1^{1/2},0), (1-\eta_1^{1/2},\eta_4^{1/2})
\end{array} \right) \right]
\]

and shear stress is

\[
\tau(\xi, \tau) = -\Gamma (d+1) \sum_{\eta_1=1}^{\infty} \frac{(-\xi)^{\eta_1}}{\eta_1!} \sum_{\eta_2=0}^{\infty} \frac{(-\Psi)^{\eta_2}}{\eta_2!} \frac{(-M)^{\eta_3}}{\eta_3!} \frac{(-F/P_r)^{\eta_4}}{\eta_4!} \lambda (-\eta_1^{1/2}-\eta_3^{1/2}-\eta_4^{1/2})
\]
\[
\times M^{I,4}_{4,3} \left[ \left( \begin{array}{c}
(1+\eta_1/2-\eta_3,0), (1+\eta_2/2,0), (-\eta_1^{1/2},0), (1-\eta_1^{1/2},\eta_4^{1/2})
\end{array} \right) \right]
\]
\[
- G_r \sum_{\eta_1, \eta_2, \eta_3, \eta_4=0}^{\infty} \frac{(-\xi)^{\eta_1}}{\eta_1!} \left( \frac{1}{P_r} \right)^{\eta_4+1} \frac{(-\Psi)^{\eta_3}}{\eta_3!} \frac{(-M)^{\eta_4}}{\eta_4!} \frac{(-F/P_r)^{\eta_5}}{\eta_5!} \lambda (-\eta_1^{1/2}-\eta_3^{1/2}-\eta_4^{1/2})
\]
\[
\times M^{I,4}_{4,3} \left[ \left( \begin{array}{c}
(1+\eta_1/2-\eta_3,0), (1+\eta_2/2,0), (-\eta_1^{1/2},0), (1-\eta_1^{1/2},\eta_4^{1/2})
\end{array} \right) \right]
\]
\[
- G_m \sum_{\eta_1, \eta_2, \eta_3, \eta_4=0}^{\infty} \frac{(-\xi)^{\eta_1}}{\eta_1!} \left( \frac{1}{S_r} \right)^{\eta_4+1} \frac{(-\Psi)^{\eta_3}}{\eta_3!} \frac{(-M)^{\eta_4}}{\eta_4!} \frac{(-F/P_r)^{\eta_5}}{\eta_5!} \lambda (-\eta_1^{1/2}-\eta_3^{1/2}-\eta_4^{1/2})
\]
\[
\times M^{I,4}_{4,3} \left[ \left( \begin{array}{c}
(1+\eta_1/2-\eta_3,0), (1+\eta_2/2,0), (-\eta_1^{1/2},0), (1-\eta_1^{1/2},\eta_4^{1/2})
\end{array} \right) \right]
\]
\[
+ G_r \sqrt{P_r} \sum_{\eta_1, \eta_2, \eta_3, \eta_4=0}^{\infty} \frac{(-\xi)^{\eta_1}}{\eta_1!} \left( \frac{1}{P_r} \right)^{\eta_4+1} \frac{(-\Psi)^{\eta_3}}{\eta_3!} \frac{(-M)^{\eta_4}}{\eta_4!} \frac{(-F/P_r)^{\eta_5}}{\eta_5!} \lambda (-\eta_1^{1/2}-\eta_3^{1/2}-\eta_4^{1/2})
\]
\[
\times M^{I,4}_{4,3} \left[ \left( \begin{array}{c}
(1+\eta_1/2-\eta_3,0), (1+\eta_2/2,0), (-\eta_1^{1/2},0), (1-\eta_1^{1/2},\eta_4^{1/2})
\end{array} \right) \right]
\]
3.2. Fractionalized MHD Second-Grade Fluid in Porous Medium with Constant Radiative Heat Flux. For the constant radiative heat flux along \( y \)-direction, \( q_r \) = constant or \( F \rightarrow 0 \), the flow results are

\[
\Gamma (\xi, \tau) = W^{\eta/2} \frac{1}{2} \left( -\sqrt[\tau]{\frac{S_c}{\eta} \xi} \right),
\]

\[
\Gamma (\xi, \tau) = \frac{P}{T^{\eta/2}} \left( -\sqrt[\tau]{\frac{S_c}{\eta} \xi} \right),
\]

\[
\tag{48}
\]

\[
\tag{49}
\]
with velocity

\[ v(\xi, \tau) = \tau + \Gamma (d + 1) \sum_{\xi = 1}^{\infty} \frac{(-\xi)^{\xi_i}}{\xi_i!} \sum_{\eta = 0}^{\infty} \frac{(-\Psi)^{\eta_i}}{\eta_i!} (-M)^{\eta_i} \xi^\beta (-\eta_i/2 - \eta_i) \]

\[ \times M_{1,3}^{(1)} \left[ \frac{\Gamma}{\xi} \left( 1 + \xi/2 - \eta_i, \beta \right) \right] \left( 1 + \xi/2 - \eta_i, \beta \right) \]

\[ \times G \sum_{\xi = 1}^{\infty} \frac{(-\xi)^{\xi_i}}{\xi_i!} \left( 1 - \Psi \right)^{\eta_i} \frac{(-M)^{\eta_i}}{\eta_i!} \xi^\beta (-\eta_i/2 + \eta_i, \beta) \]

\[ \times M_{1,3}^{(1)} \left[ \frac{\Gamma}{\eta_i} \left( 1 + \xi/2 - \eta_i, \beta \right) \right] \left( 1 + \xi/2 - \eta_i, \beta \right) \]

\[ \times G \sum_{\xi = 1}^{\infty} \frac{(-\xi)^{\xi_i}}{\xi_i!} \left( 1 - \Psi \right)^{\eta_i} \frac{(-M)^{\eta_i}}{\eta_i!} \xi^\beta (-\eta_i/2 + \eta_i, \beta) \]

\[ - G \sum_{\xi = 1}^{\infty} \frac{(-\xi)^{\xi_i}}{\xi_i!} \left( 1 - \Psi \right)^{\eta_i} \frac{(-M)^{\eta_i}}{\eta_i!} \xi^\beta (-\eta_i/2 + \eta_i, \beta) \]

\[ \times M_{1,3}^{(1)} \left[ \frac{\Gamma}{\xi} \left( 1 + \xi/2 - \eta_i, \beta \right) \right] \left( 1 + \xi/2 - \eta_i, \beta \right) \]

\[ \times G \sum_{\xi = 1}^{\infty} \frac{(-\xi)^{\xi_i}}{\xi_i!} \left( 1 - \Psi \right)^{\eta_i} \frac{(-M)^{\eta_i}}{\eta_i!} \xi^\beta (-\eta_i/2 + \eta_i, \beta) \]

\[ \times M_{1,3}^{(1)} \left[ \frac{\Gamma}{\eta_i} \left( 1 + \xi/2 - \eta_i, \beta \right) \right] \left( 1 + \xi/2 - \eta_i, \beta \right) \]

and the respective shear stress is

\[ L(\xi, \tau) = -\Gamma (d + 1) \sum_{\xi = 1}^{\infty} \frac{(-\xi)^{\xi_i}}{\xi_i!} \sum_{\eta = 0}^{\infty} \frac{(-\Psi)^{\eta_i}}{\eta_i!} (-M)^{\eta_i} \xi^\beta (-\eta_i/2 - \eta_i) \]

\[ \times M_{1,3}^{(1)} \left[ \frac{\Gamma}{\xi} \left( 1 + \xi/2 - \eta_i, \beta \right) \right] \left( 1 + \xi/2 - \eta_i, \beta \right) \]

\[ \times G \sum_{\xi = 1}^{\infty} \frac{(-\xi)^{\xi_i}}{\xi_i!} \left( 1 - \Psi \right)^{\eta_i} \frac{(-M)^{\eta_i}}{\eta_i!} \xi^\beta (-\eta_i/2 + \eta_i, \beta) \]

\[ \times M_{1,3}^{(1)} \left[ \frac{\Gamma}{\eta_i} \left( 1 + \xi/2 - \eta_i, \beta \right) \right] \left( 1 + \xi/2 - \eta_i, \beta \right) \]
3.3. Fractionalized Second-Grade Fluid in Porous Medium.

For $M \rightarrow 0$, in (41) and (46), the obtained velocity results are

$$v(\zeta, \tau) = r^b + \Gamma (d + 1) \sum_{\theta = 1}^{\infty} \frac{(-\xi)^{\theta_1}}{\theta_1!} \sum_{\theta = 0}^{\infty} \frac{(-\Psi)^{\theta_3}}{\theta_3!} \delta(-\zeta, 2^b \Psi_0)$$

(51)

$$\times M_{1.13}^{3.5} \left[ \begin{array}{c}
\frac{r^\theta}{M} \left( 1^{\theta_4}, -1^{\theta_1} \Psi_0, -1^{\theta_3} \Psi_0, -1^{\theta_4} \Psi_0 \right)
\end{array} \right]$$

$$\times M_{1.3}^{3.5} \left[ \begin{array}{c}
\frac{r^\theta}{M} \left( 1^{\theta_4}, -1^{\theta_1} \Psi_0, -1^{\theta_3} \Psi_0, -1^{\theta_4} \Psi_0 \right)
\end{array} \right]$$

$$\times M_{1.3}^{3.5} \left[ \begin{array}{c}
\frac{r^\theta}{M} \left( 1^{\theta_4}, -1^{\theta_1} \Psi_0, -1^{\theta_3} \Psi_0, -1^{\theta_4} \Psi_0 \right)
\end{array} \right]$$

$$\times M_{1.3}^{3.5} \left[ \begin{array}{c}
\frac{r^\theta}{M} \left( 1^{\theta_4}, -1^{\theta_1} \Psi_0, -1^{\theta_3} \Psi_0, -1^{\theta_4} \Psi_0 \right)
\end{array} \right]$$

(52)
and the corresponding shear stress is

$$L(\xi, \tau) = -\Gamma(d + 1) \sum_{q_1 = 1}^{\infty} \frac{(-\xi)^{q_1}}{q_1!} \sum_{q_3 = 0}^{\infty} \frac{(-\Psi)^{q_3}}{q_3!} \lambda^\beta (-e_{-1/2-q_3})$$

$$\times M_{2, 3}^{1, 2} \left[ \frac{\xi^\beta}{\lambda^\beta} \right]_{(0, 1)} \left( 1 - \frac{1}{\xi} \right) \sum_{q_1 = 1}^{\infty} \frac{(-\xi)^{q_1}}{q_1!} \left( \frac{1}{P_r} \right)^{q_1+1} \left( -\Psi \right)^{q_3} \lambda^\beta (-e_{-1/2-q_3})$$

$$- G_r \sum_{q_1, q_2, q_3 = 0}^{\infty} \frac{(-\xi)^{q_1}}{q_1!} \left( \frac{1}{S} \right)^{q_2+1} \left( -\Psi \right)^{q_3} \lambda^\beta (-e_{-1/2-q_3})$$

$$\times M_{3, 3}^{1, 2} \left[ \frac{\xi^\beta}{\lambda^\beta} \right]_{(0, 1)} \left( 1 - \frac{1}{\xi} \right) \sum_{q_1 = 1}^{\infty} \frac{(-\xi)^{q_1}}{q_1!} \left( \frac{1}{S} \right)^{q_2+1} \left( -\Psi \right)^{q_3} \lambda^\beta (-e_{-1/2-q_3})$$

$$- G_m \sum_{q_1, q_2, q_3 = 0}^{\infty} \frac{(-\xi)^{q_1}}{q_1!} \left( \frac{1}{S} \right)^{q_2+1} \left( -\Psi \right)^{q_3} \lambda^\beta (-e_{-1/2-q_3})$$

$$\times M_{4, 3}^{1, 2} \left[ \frac{\xi^\beta}{\lambda^\beta} \right]_{(0, 1)} \left( 1 - \frac{1}{\xi} \right) \sum_{q_1 = 1}^{\infty} \frac{(-\xi)^{q_1}}{q_1!} \left( \frac{1}{S} \right)^{q_2+1} \left( -\Psi \right)^{q_3} \lambda^\beta (-e_{-1/2-q_3})$$

$$+ G_r \sqrt{P_r} \sum_{q_1, q_2, q_3 = 0}^{\infty} \frac{(-\xi)^{q_1}}{q_1!} \left( \frac{1}{S} \right)^{q_2+1} \left( -\Psi \right)^{q_3} \lambda^\beta (-e_{-1/2-q_3})$$

$$\times M_{5, 3}^{1, 2} \left[ \frac{\xi^\beta}{\lambda^\beta} \right]_{(0, 1)} \left( 1 - \frac{1}{\xi} \right) \sum_{q_1 = 1}^{\infty} \frac{(-\xi)^{q_1}}{q_1!} \left( \frac{1}{S} \right)^{q_2+1} \left( -\Psi \right)^{q_3} \lambda^\beta (-e_{-1/2-q_3})$$

$$+ G_m \sqrt{S} \sum_{q_1, q_2, q_3 = 0}^{\infty} \frac{(-\xi)^{q_1}}{q_1!} \left( \frac{1}{S} \right)^{q_2+1} \left( -\Psi \right)^{q_3} \lambda^\beta (-e_{-1/2-q_3})$$

$$\times M_{6, 3}^{1, 2} \left[ \frac{\xi^\beta}{\lambda^\beta} \right]_{(0, 1)} \left( 1 - \frac{1}{\xi} \right) \sum_{q_1 = 1}^{\infty} \frac{(-\xi)^{q_1}}{q_1!} \left( \frac{1}{S} \right)^{q_2+1} \left( -\Psi \right)^{q_3} \lambda^\beta (-e_{-1/2-q_3})$$

$$3.4. \text{Fractionalized MHD Second-Grade Fluid after Vanishing Porosity.}$$

Substituting $\Psi \rightarrow 0$, in (41) and (46), the acquired velocity is

$$\nu(\xi, \tau) = t^\beta + \Gamma(d + 1) \sum_{q_1 = 1}^{\infty} \frac{(-\xi)^{q_1}}{q_1!} \sum_{q_3 = 0}^{\infty} \frac{(-M)^{q_3}}{q_3!} \lambda^\beta (-e_{-1/2-q_3})$$

$$\times M_{2, 4}^{1, 2} \left[ \frac{\xi^\beta}{\lambda^\beta} \right]_{(0, 1)} \left( 1 - \frac{1}{\xi} \right) \sum_{q_1 = 1}^{\infty} \frac{(-\xi)^{q_1}}{q_1!} \left( \frac{1}{P_r} \right)^{q_1+1} \left( -\Psi \right)^{q_3} \lambda^\beta (-e_{-1/2-q_3})$$

$$+ G_r \sum_{q_1, q_2, q_3 = 0}^{\infty} \frac{(-\xi)^{q_1}}{q_1!} \left( \frac{1}{P_r} \right)^{q_1+1} \left( -M \right)^{q_3} \lambda^\beta (-e_{-1/2-q_3})$$

$$\times M_{3, 4}^{1, 2} \left[ \frac{\xi^\beta}{\lambda^\beta} \right]_{(0, 1)} \left( 1 - \frac{1}{\xi} \right) \sum_{q_1 = 1}^{\infty} \frac{(-\xi)^{q_1}}{q_1!} \left( \frac{1}{P_r} \right)^{q_1+1} \left( -M \right)^{q_3} \lambda^\beta (-e_{-1/2-q_3})$$

$$+ G_m \sum_{q_1, q_2, q_3 = 0}^{\infty} \frac{(-\xi)^{q_1}}{q_1!} \left( \frac{1}{P_r} \right)^{q_1+1} \left( -M \right)^{q_3} \lambda^\beta (-e_{-1/2-q_3})$$

$$\times M_{4, 4}^{1, 2} \left[ \frac{\xi^\beta}{\lambda^\beta} \right]_{(0, 1)} \left( 1 - \frac{1}{\xi} \right) \sum_{q_1 = 1}^{\infty} \frac{(-\xi)^{q_1}}{q_1!} \left( \frac{1}{P_r} \right)^{q_1+1} \left( -M \right)^{q_3} \lambda^\beta (-e_{-1/2-q_3})$$
whereas the related shear stress is

\[
L(\xi, \tau) = -\Gamma (d + 1) \sum_{\xi = 1}^{\infty} \frac{(-\xi)^{\xi_i}}{\xi_i!} \sum_{\xi = 0}^{\infty} \frac{(-M)^{\xi_i}}{\xi_i!} \lambda^\beta (-\xi_i^{-1/2})
\]

\[
\times M_{1,4}^2 \left[ \left( 1 + \xi_i + 1/2 - \xi_i \right) \left( 1 - \xi_i - 1/2 \right) \right]
\]

\[
- G_r \sum_{\xi = 1}^{\infty} \frac{(-\xi)^{\xi_i}}{\xi_i!} \left( \frac{1}{P_r} \right)^{\xi_i+1} (-M)^{\xi_i} (-F/P_r)^{\xi_i} \lambda^\beta (-\xi_i+1/2 - \xi_i)
\]

\[
\times M_{1,3}^2 \left[ \left( 1 + \xi_i + 1/2 + \xi_i \right) \left( 1 - \xi_i - 1/2 \right) \right]
\]

\[
- G_m \sum_{\xi = 1}^{\infty} \frac{(-\xi)^{\xi_i}}{\xi_i!} \left( \frac{1}{S_c} \right)^{\xi_i+1} (-M)^{\xi_i} \lambda^\beta (-\xi_i+1/2 - \xi_i)
\]

\[
\times M_{1,4}^2 \left[ \left( 1 + \xi_i + 1/2 + \xi_i \right) \left( 1 - \xi_i - 1/2 \right) \right]
\]
3.5. Fractionalized MHD Newtonian Fluid in Porous Medium. By making $\lambda \longrightarrow 0$, in (38) and (43), proceed as in the previous section, we obtain the velocity of fractionalized Newtonian fluid:

$$
\mathbf{v}(\xi, \tau) = \mathbf{v}_0 + \Gamma (d + 1) \sum_{\zeta_1=1}^{\infty} \left( \frac{-\xi^{\zeta_1}}{\zeta_1!} \right) \sum_{\zeta_3=0}^{\infty} \left( \frac{-\mathbf{v}^{\zeta_3}}{\zeta_3!} \right)
\times \mathbf{M}_{2,4}^{1,2} \left[ \mathbf{M}_1 \left( \mathbf{1}^{\zeta_1/2+\zeta_3}, 0, \mathbf{1}^{\zeta_1/2+\zeta_3} \right), \left( \mathbf{1}^{\zeta_1/2+\zeta_3} \right) \right] 
\times \mathbf{G}_r \sum_{\zeta_1=1}^{\infty} \left( \frac{-\xi^{\zeta_1}}{\zeta_1!} \right) \left( \frac{1}{P_r} \right) \left( \frac{-\mathbf{v}^{\zeta_3}}{\zeta_3!} \right)
\times \mathbf{M}_{3,5}^{1,3} \left[ \mathbf{M}_1 \left( \mathbf{1}^{\zeta_1/2+\zeta_3}, 0, \mathbf{1}^{\zeta_1/2+\zeta_3} \right), \left( \mathbf{1}^{\zeta_1/2+\zeta_3} \right) \right]
\times \mathbf{G}_m \sum_{\zeta_1=1}^{\infty} \left( \frac{-\xi^{\zeta_1}}{\zeta_1!} \right)
\times \mathbf{M}_{3,5}^{1,3} \left[ \mathbf{M}_1 \left( \mathbf{1}^{\zeta_1/2+\zeta_3}, 0, \mathbf{1}^{\zeta_1/2+\zeta_3} \right), \left( \mathbf{1}^{\zeta_1/2+\zeta_3} \right) \right]
\times \mathbf{M}_{2,4}^{1,2} \left[ \mathbf{M}_1 \left( \mathbf{1}^{\zeta_1/2+\zeta_3}, 0, \mathbf{1}^{\zeta_1/2+\zeta_3} \right), \left( \mathbf{1}^{\zeta_1/2+\zeta_3} \right) \right],
$$

and the associated shear stress is

$$
\mathbf{t} (\xi, \tau) = -\Gamma (d + 1) \sum_{\zeta_1=1}^{\infty} \left( \frac{-\xi^{\zeta_1}}{\zeta_1!} \right) \sum_{\zeta_3=0}^{\infty} \left( \frac{-\mathbf{v}^{\zeta_3}}{\zeta_3!} \right)
\times \mathbf{M}_{2,4}^{1,2} \left[ \mathbf{M}_1 \left( \mathbf{1}^{\zeta_1/2+\zeta_3}, 0, \mathbf{1}^{\zeta_1/2+\zeta_3} \right), \left( \mathbf{1}^{\zeta_1/2+\zeta_3} \right) \right]
\times \mathbf{G}_r \sum_{\zeta_1=1}^{\infty} \left( \frac{-\xi^{\zeta_1}}{\zeta_1!} \right) \left( \frac{1}{P_r} \right) \left( \frac{-\mathbf{v}^{\zeta_3}}{\zeta_3!} \right)
\times \mathbf{M}_{3,5}^{1,3} \left[ \mathbf{M}_1 \left( \mathbf{1}^{\zeta_1/2+\zeta_3}, 0, \mathbf{1}^{\zeta_1/2+\zeta_3} \right), \left( \mathbf{1}^{\zeta_1/2+\zeta_3} \right) \right]
\times \mathbf{G}_m \sum_{\zeta_1=1}^{\infty} \left( \frac{-\xi^{\zeta_1}}{\zeta_1!} \right)
\times \mathbf{M}_{3,5}^{1,3} \left[ \mathbf{M}_1 \left( \mathbf{1}^{\zeta_1/2+\zeta_3}, 0, \mathbf{1}^{\zeta_1/2+\zeta_3} \right), \left( \mathbf{1}^{\zeta_1/2+\zeta_3} \right) \right]
\times \mathbf{M}_{2,4}^{1,2} \left[ \mathbf{M}_1 \left( \mathbf{1}^{\zeta_1/2+\zeta_3}, 0, \mathbf{1}^{\zeta_1/2+\zeta_3} \right), \left( \mathbf{1}^{\zeta_1/2+\zeta_3} \right) \right].
$$
4. Results and Discussions

Here, we analyse the flow behavior with the help of graphical illustrations after finding the exact solutions of temperature, concentration, velocity, and shear stress in the previous sections. All graphs are made in Mathcad software with SI units. The values of parameters of interest that are used in the study are provided in respective figures. All the imposed and boundary conditions for velocity and shear stress mentioned in (14)–(16) are satisfied. It can be observed in Figures 2 and 3 that natural and boundary conditions for mass concentration \( \xi(\xi, 0) = 0 \) and \( \xi(0, \tau) = \xi_1 \) are satisfied. Natural and boundary conditions for temperature \( \Psi(\xi, 0) = 0 \) and \( \Psi(0, \tau) = \Psi_1 \) are satisfied. From Figures 4–6 it can be easily analysed that natural and boundary conditions for mass concentration, temperature, and shear stress at \( \xi(\xi, 0) = 0 \) and \( \xi(0, \tau) = \xi_1 \) are satisfied. Natural and boundary conditions for velocity and shear stress at \( \nu(\xi, 0) = 0 \), \( L(\xi, 0) = 0 \) and \( \nu(0, \tau) = \nu_1 \) are well visualized in Figures 7 and 8, respectively.

Assurance of present work is manifested by constructing comparison with previous published literature by limiting cases in a section 3 of this study. These special cases are compared to those of Aman et al. [29] and Shahid [30]. Flow field results with ordinary derivatives and fractional derivatives are compared graphically, and the analytical results are the same as those of the previously published literature by vanishing few parameters.

In Figure 2 we have discussed the variations in mass concentration at different values of time and space above the plate. A natural behavior of decay in mass concentration is observed for higher position of the fluid over the plate. The significance of fractional parameter \( \eta \) and the effects of variations in Schmidt number over the dimensionless mass concentration are analysed in Figure 3. This reveals the facts of increase in mass concentration with increase in fractional parameter values. On the other hand, the opposite behavior is observed for increase in Schmidt number. Figure 4 reflects the temperature distribution with respect to increasing time and vertical space values. The thermal boundary layer thickens by increasing time while temperature values decrease by considering the increasing vertical position of fluid over plate. The effects of fractional parameter and Prandtl number on temperature of fluid over the moving plate are portrayed in Figure 5. Naturally, the viscosity of fluid gets minimized by increasing Prandtl number that ultimately yields the reduction in thermal boundary layer. Figure 6 depicts the influence of fractional parameter and thermal diffusivity on temperature distribution which implies that thermal boundary layer gets thinner with increasing thermal diffusivity whereas fractional parameter acts opposite to that of thermal diffusivity on temperature distribution.

Velocity fields along with shear stress behavior are given in Figure 7 for vertical space values above the moving plate which shows that velocity of the particles at the moving surface is higher to that of the particles away from plate, whereas shear stress profile behaves numerically opposite to it. It can be noted from Figure 8 that as the time passes, velocity of the fluid increases and the stress too, while the impact gets more intense with passage of time. Figure 9 shows the effects of increasing second-grade parameter \( \lambda \); it reduces the fluid velocity and the stress numerically which is due to the characteristics of second-grade fluid. The influence of magnetic and porous parameters is displayed in Figures 10 and 11, where both parameters have similar effect, and the speed and stress increase numerically by raising either the magnetic or porous parameter values.

To discuss the impacts of thermal Grashof number and modified Grashof number, we plot Figures 12 and 13, where velocity and stress mount numerically for the dimensionless numbers individually since it is buoyancy force that flows the fluid, whereas the impacts of modified Grashof number are more intense than those of the thermal Grashof number. The importance of Prandtl and Schmidt number can be observed in Figures 14 and 15; both the dimensionless numbers endorse their definition there by increasing the stress and velocity of the particles for increasing values of these numbers. Figures 16 points out the relation of thermal diffusivity to the stress and velocity of the particles. Increasing the thermal diffusivity parameter, velocity of the fluid reduces and the stress reduces too. Figures 17–19 are prepared to analyse the significance of fractional parameters on the flow field. It can be noted that the increase in fractional parameters reduces the stress and velocity of the particles. Thus, fractional derivatives are essential in studying the fluid flows more significantly and accurately.

Figure 20 displays increasing magnitude of stress and velocity of the particles in relation with increasing values of the exponent of the time factor in the boundary condition for velocity. A comparison of model flow is displayed graphically in Figure 21 which assures the influence of fractional parameters over the stress and velocity of the particles.

Flow characteristics of fractional model are higher in magnitude than those of the ordinary model. Stress and velocity of the particles second-grade particles are
Figure 3: Descriptions of the mass concentration $\mathcal{M}(\xi, \tau)$ given by (29), for $\tau = 2$ s, and different values of $S_c$ and $\eta$.

Figure 4: Descriptions of the temperature distribution $\mathcal{T}(\xi, \tau)$ given by (35), for $P_r = 0.6, F = 0.2$, and different values of $\xi$ and $\tau$. 
\( \gamma = 0.2 \)

\( \Pr = 0.6 \)

\( 1 \quad 0.79 \quad 0.58 \quad 0.37 \quad 0.16 \quad -0.05 \)

\( \mathcal{L} (\xi) \)

\( 0 \quad 0.8 \quad 1.6 \quad 2.4 \quad 3.2 \quad 4 \)

\( \gamma = 0.2 \)

\( \Pr = 0.6 \)

\( 1 \quad 0.79 \quad 0.58 \quad 0.37 \quad 0.16 \quad -0.05 \)

\( \mathcal{L} (\xi) \)

\( 0 \quad 0.8 \quad 1.6 \quad 2.4 \quad 3.2 \quad 4 \)

\( T = 0.10s \quad T = 0.25s \quad T = 0.60s \quad T = 2.00s \)

\( T = 0.10 \)

\( T = 0.25 \)

\( T = 0.60 \)

\( T = 2.00 \)

\( \gamma = 0.8 \)

\( \Pr = 0.5 \)

\( \Pr = 1.5 \)

\( \Pr = 3.0 \)

\( \Pr = 5.0 \)

\( T = 0.5 \)

\( T = 2 \)

\( \gamma = 0.8 \)

\( \Pr = 0.5 \)

\( \Pr = 1.5 \)

\( \Pr = 3.0 \)

\( \Pr = 5.0 \)

\( T = 0.5 \)

\( T = 2 \)

\( \gamma = 0.8 \)

\( \Pr = 0.5 \)

\( \Pr = 1.5 \)

\( \Pr = 3.0 \)

\( \Pr = 5.0 \)

\( T = 2 \)

\( \gamma = 0.8 \)

\( \Pr = 0.5 \)

\( \Pr = 1.5 \)

\( \Pr = 3.0 \)

\( \Pr = 5.0 \)

Figure 5: Descriptions of the temperature distribution \( \mathcal{L} (\xi, \tau) \) given by (35), for \( F = 0.2 \), for variational points of \( \tau \) and \( \Pr \).

Figure 6: Descriptions of the temperature distribution \( \mathcal{L} (\xi, \tau) \) given by (35), for \( \Pr = 2 \), for variational points of \( F \) and \( \eta \).
Figure 7: Descriptions of the velocity \( v(\xi, \tau) \) and the stress \( L(\xi, \tau) \) given by (41) and (46), for \( \lambda = 3, M = 0.5, \Psi = 0.2, \Gamma_r = 0.2, \Gamma_m = 0.6, S_\xi = 0.6, P_r = 0.6, F = 0.2, \eta = \beta = \gamma = 0.5, d = 1 \) for variational points of \( \xi \).

Figure 8: Descriptions of the velocity \( v(\xi, \tau) \) and the stress \( L(\xi, \tau) \) given by (41) and (46), for \( \lambda = 3, M = 0.5, \Psi = 0.2, \Gamma_r = 0.2, \Gamma_m = 0.6, S_\xi = 0.6, P_r = 0.6, F = 0.2, \eta = \beta = \gamma = 0.5, d = 1 \) for variational points of \( \tau \).
Figure 9: Descriptions of the velocity $v(\xi, \tau)$ and the stress $L(\xi, \tau)$ given by (41) and (46), for $M = 0.1, \Psi = 0.2, G_r = 0.2, G_m = 0.6, S_t = 0.6, P_r = 0.6, F = 0.2, \eta = \beta = \gamma = 0.5, d = 1, \tau = 0.5$ s for variational points of $\lambda$.

Figure 10: Descriptions of the velocity $v(\xi, \tau)$ and the stress $L(\xi, \tau)$ given by (41) and (46), for $M = 0.5, \Psi = 0.2, G_r = 0.2, G_m = 0.6, S_t = 0.6, P_r = 0.6, F = 0.2, \eta = \beta = \gamma = 0.5, d = 1, \tau = 0.5$ s for variational points of $M$. 
Figure 11: Descriptions of the velocity $\nu(\xi, \tau)$ and the stress $L(\xi, \tau)$ given by (41) and (46), for $M = 0.5, \Psi = 0.2, G_r = 0.2, G_m = 0.6, S_c = 0.6, P_r = 0.6, F = 0.2, \eta = \beta = \gamma = 0.5, d = 1, \tau = 0.5 s$ for variational points of $\Psi$.

Figure 12: Descriptions of the velocity $\nu(\xi, \tau)$ and the stress $L(\xi, \tau)$ given by (41) and (46), for $\lambda = 3, M = 0.5, \Psi = 0.2, G_m = 0.6, S_c = 0.6, P_r = 0.6, F = 0.2, \eta = \beta = \gamma = 0.5, d = 1, \tau = 0.2 s$ for variational points of $G_r$. 
Figure 13: Descriptions of the velocity \(v(\xi, \tau)\) and the stress \(L(\xi, \tau)\) given by (41) and (46), for \(\lambda = 3, M = 0.5, \Psi = 0.2, G_m = 0.6, S_c = 0.6, P_r = 0.6, F = 0.2, \eta = \beta = \gamma = 0.5, d = 1, \tau = 0.5s\) for variational points of \(G_m\).

Figure 14: Descriptions of the velocity \(v(\xi, \tau)\) and the stress \(L(\xi, \tau)\) given by (41) and (46), for \(\lambda = 3, M = 0.5, \Psi = 0.2, G_m = 0.6, S_c = 0.6, P_r = 0.6, F = 0.2, \eta = \beta = \gamma = 0.5, d = 1, \tau = 0.5s\) for variational points of \(S_c\).
Figure 15: Descriptions of the velocity \( \upsilon(\xi, \tau) \) and the stress \( \mathcal{L}(\xi, \tau) \) given by (41) and (46), for \( \lambda = 3, M = 0.5, \Psi = 0.2, G_m = 0.6, S_c = 0.6, P_r = 0.6, F = 0.2, \eta = \beta = \gamma = 0.5, d = 1, \tau = 0.5s \) for variational points of \( P_r \).

Figure 16: Descriptions of the velocity \( \upsilon(\xi, \tau) \) and the stress \( \mathcal{L}(\xi, \tau) \) given by (41) and (46), for \( \lambda = 3, M = 0.5, \Psi = 0.2, G_m = 0.6, S_c = 0.6, P_r = 0.6, F = 0.2, \eta = \beta = \gamma = 0.5, d = 1, \tau = 0.5s \) for variational points of \( F \).
Figure 17: Descriptions of the velocity $\upsilon(\xi, \tau)$ and the stress $L(\xi, \tau)$ given by (41) and (46), for $\lambda = 3, M = 0.5, \Psi = 0.2, G_m = 0.6, S_c = 0.6, P_r = 0.6, F = 0.2, \eta = \beta = \gamma = 0.5, d' = 1, \tau = 0.2s$ for variational points of $\eta$.

Figure 18: Descriptions of the velocity $\upsilon(\xi, \tau)$ and the stress $L(\xi, \tau)$ given by (41) and (46), for $\lambda = 3, M = 0.5, \Psi = 0.2, G_m = 0.6, S_c = 0.6, P_r = 0.6, F = 0.2, \eta = \beta = \gamma = 0.5, d' = 1, \tau = 0.5s$ for variational points of $\beta$. 

Figure 19: Descriptions of the velocity $\nu(\xi, \tau)$ and the stress $L(\xi, \tau)$ given by (41) and (46), for $\lambda = 3, M = 0.5, \Psi = 0.2, G_m = 0.6, S_\iota = 0.6, P_r = 0.6, F = 0.2, \eta = \beta = \gamma = 0.5, d = 1, \tau = 0.5s$ for variational points of $\gamma$.

Figure 20: Descriptions of the velocity $\nu(\xi, \tau)$ and the stress $L(\xi, \tau)$ given by (41) and (46), for $\lambda = 3, M = 0.5, \Psi = 0.2, G_m = 0.6, S_\iota = 0.6, P_r = 0.6, F = 0.2, \eta = \beta = \gamma = 0.5, d = 1, \tau = 2s$ for variational points of $d$. 
In this article, transfer and flow characteristics of fractional MHD second-grade fluid on the porous plate moving with nonlinear velocity is analysed. Solutions for velocity, temperature distribution, and mass concentration are acquired for both the second-grade and Newtonian-type fluids. The obtained results are compared with the analytical solutions of the flow field presented by the recent studies \cite{23–25} which are found to be more compact and simplified. The accuracy of the flow behavior is graphically analysed by setting three decimal places in numerical calculations. However, solutions are analytical, thus the numerical solutions and their comparison are beyond the scope of this study.

5. Conclusions

In this article, transfer and flow characteristics of fractional MHD second-grade fluid on the porous plate moving with nonlinear velocity is analysed. Solutions for velocity, temperature distribution, and mass concentration are acquired using Laplace transforms after nondimensionalising the system of differential equations. Obtained results satisfying all imposed natural initial and boundary limitations are graphically analysed for transfer and rheological parameters at different times and positions over the plate. The investigation reveals the following results *'. All solutions are represented in simpler forms in terms of new summation style and natural generalized function $M$ for such flows.
(i) Mass concentration rate increases with increasing Schmidt number and passage of time while it decreases with increasing fractional parameter values and position of fluid over the plate.

(ii) Temperature distribution of the fluid is the decreasing function of Prandtl number, thermal radiation parameter, and position of fluid over the plate whereas it increases with increasing time and fractional parameter values.

(iii) Velocity field and stress gets higher magnitude for increasing thermal Grashof number, mass Grashof number, Prandtl number, Schmidt number, time, second grade, magnetic (Hartmann), and porous and thermal radiation parameters.

(iv) Fractional operators significantly affect the stress and velocity of the particles there by reducing their magnitude for increasing fractional parameters.

(v) The nonlinearity parameter $d$ has direct effects on fluid motion. The large values of $d$ enhance the motion of the fluid.

(vi) It is also verified from graphical results that when fractional parameters approach to 1, the behavior of fractionalized second-grade fluid turns into the usual fluid.

**Data Availability**

No data were needed to perform this research.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

**Authors’ Contributions**

Muhammad Jamil formulated the problem and similarity transformation. Israr Ahmed solved the problem and obtained results. Umar Faryaz computed results, updated coding, and carried out revision. Mulugeta Andualem improved the model, used the software, did the revision, and carried out the discussion.

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