The Large $N_c$ Baryon-Meson $I_t=J_t$ Rule Holds for Three Flavors

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Abstract

It has long been known that nonstrange baryon-meson scattering in the $1/N_c$ expansion of QCD greatly simplifies when expressed in terms of $t$-channel exchanges: The leading-order amplitudes satisfy the selection rule $I_t=J_t$. We show that $I_t=J_t$, as well as $Y_t=0$, also hold for the leading amplitudes when the baryon and/or meson contain strange quarks, and also characterize their $1/N_c$ corrections, thus opening a new front in the phenomenological study of baryon-meson scattering and baryon resonances.

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I. INTRODUCTION

A classic series of papers by Adkins, Nappi, and Witten [1], written over two decades ago showed that a number of relations among baryon observables in chiral soliton models, particularly the Skyrme model, appear to be model independent and related to the large $N_c$ limit of QCD. An extensive body of literature followed, notably including work by the Siegen group and Mattis and Karliner [2, 3], finding linear relations between $\pi N$ scattering amplitudes in various isospin and angular momentum channels that hold at all energies. Developing this theme, Mattis and Peskin [4] found a remarkable group structure to be responsible for the relations: In the soliton language, the conserved underlying quantum number in $s$-channel scattering is the “grand spin” $K$, where $K=I+J$. Multiple observable scattering amplitudes arise as linear combinations of a smaller set of “reduced” scattering amplitudes labeled by $K$.

Donohue subsequently noted [5] the remarkable result that these linear relations simplify dramatically when expressed in the $t$ channel. Armed with this observation and SU(2) group theory identities to manifest the crossing symmetry, Mattis and Mukerjee (MM) proved underlying $K$-spin conservation for the 2-flavor system to be equivalent to the rule $I_t=J_t$ at large $N_c$ [6]. However, when attempting to extend their rule to include strangeness, MM obtained [6, 7] results not only that did not appear to support the $I_t=J_t$ rule, but moreover that they could not show to contain the 2-flavor results as a special case. Even so, using a 3-flavor Skyrme picture Donohue later found [8] that the number of independent amplitudes reduces substantially when the problem is expressed in the $t$ channel.

The problem then lay largely unnoticed until a few years ago, when Cohen and the present author revived the amplitude relation approach [9, 10, 11, 12, 13, 14, 15, 16, 17] in order to study baryon resonances in the $1/N_c$ expansion. In addition to a number of successes, such as an understanding of the large $N(1535)$ $\eta N$ coupling [9] and the large $N_c$ reason why the quark model produces a good but not perfect accounting of the resonance spectrum [10], we considered more formal issues as well. In particular, we noted that the SU(3) group theory for baryons at arbitrary $N_c$ required development both mathematically and in terms of the proper identification of baryon quantum numbers [15, 16, 17] and showed that the MM 3-flavor $s$-channel expression does indeed reduce to the appropriate 2-flavor one [17].

In this paper we show that, in fact, the $I_t=J_t$ rule holds for the 3-flavor case as well,
and also obtain a new selection rule $Y_t = 0$. We give a proof both using linear amplitude expressions similar to those obtained by MM, and also using the more recent operator approach \[18, 19\]. We further argue that corrections to the $I_t = J_t$ rule among processes of a given fixed strangeness go as $1/N_c^n$ for $|I_t - J_t| = n$, while corrections to the $Y_t = 0$ only fall off as $N_c^{-|Y_t|/2}$.

In Sec. II we rederive the MM $t$-channel master scattering expression, making appropriate corrections and modifications. The proofs of the $Y_t = 0$ and $I_t = J_t$ rules follow in Sec. III. The nature of the $1/N_c$ corrections are described in Sec. IV, and we make a few comments about phenomenological applications and conclude in Sec. V.

II. AMPLITUDES IN THE $t$ CHANNEL

We consider the baryon-meson scattering process $\phi(S, R, I, Y) + B(S_B, R_B, I_B, Y_B) \rightarrow \phi'(S', R', I', Y') + B'(S_B', R_B', I_B', Y_B')$, where $S$, $R$, $I$, and $Y$ stand, respectively, for the spin, SU(3) representation, isospin, and hypercharge of the mesons $\phi$ and $\phi'$ and the baryons $B$ and $B'$. Primes indicate final-state quantum numbers. We take the baryons to lie in the ground-state band, the arbitrary-$N_c$ analogue of the SU(6) 56, whose lowest states ($N$, $\Delta$, $\Sigma$, etc.) in the large $N_c$ limit are stable against strong decay. The relative angular momenta between the meson-baryon pairs are denoted by $L$ and $L'$. As shown in the original derivation \[14\], it is convenient to cross the quantum numbers of the process to consider instead $\phi + \phi' \rightarrow B^* + B'$. The amplitude is described in terms of $t$-channel angular momentum $J_t$, SU(3) representation $R_t$, isospin $I_t$, and hypercharge $Y_t$. In addition, multiple copies of $R_t$ may arise in the products $R_\phi \otimes R_\phi^*$ and $R_B \otimes R_B$, and the quantum numbers defined to lift this degeneracy are labeled, respectively, by $\gamma_t$ and $\gamma'_t$ (which need not be equal). In the physical amplitude, one of course sums coherently over all allowed $t$-channel quantum numbers. After a derivation following the methods of Ref. \[7\], we obtain the large $N_c$ master expression for such scattering amplitudes expressed in the $t$ channel:

$$S_{LL'S_B S_B' J_t J_t' r_t r_t' \gamma_t \gamma_t'} = \delta_{J_t J_t'} \delta_{J_t J_t' \gamma_t \gamma_t'} \delta_{R_t R_t'} \delta_{I_t I_t' \gamma_t \gamma_t'} \delta_{Y_t Y_t'} \times (-1)^{S_{\phi'} - S_{\phi} + J_t - J_t'} \times \left( \begin{array}{c|c} R_\phi & R_\phi^* \hline I_\phi & I_\phi' \end{array} \right) \left( \begin{array}{c|c} R_t & R_t' \hline I_t Y_t & I_t' Y_t' \end{array} \right)$$
\[
\times \left( \begin{array}{cc} R_B^* & R_{B'} \\ S_B, -\frac{N_c}{3} & S_{B'}, +\frac{N_c}{3} \end{array} \right) \left( \begin{array}{c} R_t^* \\ I_B, -Y_B \end{array} \right) \gamma_t' \left( \begin{array}{cc} R_B^* & R_{B'} \\ I_B, -Y_B & I_{B'} Y_{B'} \end{array} \right) \left( \begin{array}{c} R_t^* \\ I_t \end{array} \right) \gamma_t' \right)
\times \sum_{K K K'} (-1)^K \tilde{\chi}[K][\bar{K}][\bar{K}'][1/2] \left\{ \begin{array}{ccc} J_\phi & I & K \\ I' & J_{\phi'} & J_t \end{array} \right\} \left\{ \begin{array}{ccc} J_\phi & I & K \\ \bar{K} & S_\phi & L \end{array} \right\} \left\{ \begin{array}{ccc} J_{\phi'} & I' & K \\ \bar{K}' & S_{\phi'} & L' \end{array} \right\}
\times \tau^{(II'Y)}_{KKK'LL'} .
\]

The quantities containing double vertical bars are SU(3) isoscalar Clebsch-Gordan coefficients (CGC) \[15\], while those in braces are ordinary SU(2) 6j symbols. The notation [X] refers to the dimension of a given representation, whether X is labeled by I or J in SU(2), or by the actual dimension in SU(3) (i.e., [J = 1] = 3, but [R = 8] = 8). The quantities \(\tau\) are the reduced amplitudes, which represent the independent dynamical degrees of freedom in the large \(N_c\) limit; in the \(1/N_c\) expansion, Eq. (1) is corrected both by including \(O(1/N_c)\) corrections to the \(\tau\)'s, as well as by adding (as discussed in Sec. [IV]) terms with group-theoretical structures distinct from those in Eq. (1) times additional \(O(1/N_c)\) reduced amplitudes.

Equation (1) should be compared to the original result Eq. (8) of Ref. [6] or Eq. (15) of Ref. [7] (the latter of which provides details of the original derivation). We previously showed [16] in rederiving the corresponding s-channel expressions (Eq. (7) of [6] or Eq. (12) of [7]) that small but significant discrepancies arise, and the same comments hold for our rederivation of the t-channel results: First, Ref. [7] appears to average over baryons and mesons in the external states with all possible quantum numbers within the given SU(3) multiplets; if we do the same with Eq. (1), two of our SU(3) CGC are absorbed through an orthogonality relation (Eq. (11.3a) of Ref. [20]), matching the form of the older result. Second, their explicit unity values for the nonstrange baryon hypercharges must be modified to \(+\frac{N_c}{3}\), in light of the proper quantization [21] of the Wess-Zumino term for arbitrary \(N_c\); similarly, the baryon representations must be generalized to their proper arbitrary-\(N_c\) forms: For example, the literal SU(3) 8, becomes an \(\text{“8”} = [1, (N_c - 1)/2]\). Finally, we obtain a phase quite different [22] from the one in the original result.

The only \(N_c\)-dependent factors in Eq. (1) appear in the last two SU(3) CGC and the dimension factors \([R_B], [R_{B'}]\), which refer to the large baryon representations. Focusing only on these factors, one may use SU(3) CGC reflection properties (Eqs. (14.9) and (14.13) of Ref. [20]), augmented by a proper treatment of phase factors [22] for representations with
non-integer hypercharges, and Eq. (1) of Ref. [17] \([R_B] \rightarrow N_c^2 [S_B]/8\):

\[
\frac{\sqrt{[R_B][R_B']}}{[R_t]} \begin{pmatrix}
R_B^* & R_B' & R_t \gamma' \\
S_B, -\frac{N_c}{3} & S_B', +\frac{N_c}{3} & J_t 0
\end{pmatrix} \begin{pmatrix}
R_B^* & R_B' & R_t \gamma' \\
I_B, -Y_B & I_B' Y_B' & I_t Y_t
\end{pmatrix} = (-1)^{(I_{B'} - S_{B'}) - \frac{1}{2}(Y_B - \frac{N_c}{6}) - (t_t - J_t)} \times \frac{[S_B][I_B']}{[I_t][J_t]} \begin{pmatrix}
R_B & R_t & R_{B'} \tilde{\gamma} \\
S_B, +\frac{N_c}{3} & J_t 0 & S_B', +\frac{N_c}{3}
\end{pmatrix} \begin{pmatrix}
R_B & R_t & R_{B'} \tilde{\gamma} \\
I_B Y_B & I_t Y_t & I_B' Y_B'
\end{pmatrix}.
\]

Since the baryons with \(N_s\) strange quarks have \(Y_B = \frac{N_c}{3} - N_s\), all \(N_c\)-dependent factors are relegated to the two new CGC. Again, the complete amplitude requires a coherent sum over multiplicity factors, in this case \(\tilde{\gamma}\). We therefore seek to prove that, at \(O(N_c^0)\),

\[
\sum_{\tilde{\gamma}} \left( \begin{pmatrix}
R_B & R_t & R_{B'} \tilde{\gamma} \\
S_B, +\frac{N_c}{3} & J_t 0 & S_B', +\frac{N_c}{3}
\end{pmatrix} \begin{pmatrix}
R_B & R_t & R_{B'} \tilde{\gamma} \\
I_B Y_B & I_t Y_t & I_B' Y_B'
\end{pmatrix} \right) \propto \delta_{I_t J_t}.
\]

\section*{III. PROVING THE \(I_t = J_t\) RULE}

\subsection*{A. The \(Y_t = 0\) Rule}

We begin by recalling the theorem demonstrated in Ref. [17]: Let \(R_B = (2S_B, \frac{N_c}{2} - S_B)\) denote an SU(3) representation corresponding to baryons in the ground-state SU(6) \(\"56\"\) with spin \(S_B\), so that the top (nonstrange) row in the weight diagram has isospin \(I_{B,\text{top}} = S_B\) and \(Y_{B,\text{max}} = +\frac{N_c}{3}\), let \(R_\phi = (p_\phi, q_\phi)\) be an SU(3) (meson) representation with weights \(p_\phi, q_\phi = O(N_c^0)\), and let \(R_s \gamma_s \subset R_B \otimes R_\phi\), where \(Y_{s,\text{max}} = \frac{N_c}{3} + r\) and \(R_s = (2I_{s,\text{top}}, \frac{N_c}{2} + \frac{3r}{2} - I_{s,\text{top}})\), \(r = O(N_c^0)\). Then the SU(3) CGC satisfy

\[
\left( \begin{pmatrix}
R_B & R_\phi \\
I_B, \frac{N_c}{3} - m & I_\phi Y_\phi
\end{pmatrix} \begin{pmatrix}
R_s \gamma_s \\
I_s, \frac{N_c}{3} + Y_\phi - m
\end{pmatrix} \right) \leq O(N_c^0 |Y_\phi - r|/2),
\]

for all allowed \(O(N_c^0)\) values of \(m\), saturation of the inequality occurring for almost all CGC. In words: The \(O(N_c^0)\) CGC must have a meson hypercharge that equals the hypercharge difference between the tops of the two baryon representations.

The CGC in Eq. (3) have \(R_s = R_{B'}\), which lies in the ground-state \(\"56\"\), so that \(r = 0\). The first CGC in Eq. (3) is thus automatically of leading \([O(N_c^0)]\) order, while the second is of leading order only for \(Y_t = 0\). This result, derived using the same theorem Eq. (4), was
noted (in \(s\)-channel language) in Ref. \[17\]; since here we use explicit \(t\)-channel expressions, we call it the \(Y_t=0\) rule.

At the level of quark diagrams and combinatorics, the \(Y_t=0\) rule is perfectly sensible. Baryon-meson scattering diagrams all involve gluon and/or quark exchanges; the latter diagrams combine to produce the proper \(O(N_c^0)\) amplitude only when each of the \(N_c\) quarks in the baryon is permitted to be the one that is exchanged with the meson. Furthermore, strangeness-changing \((Y_t \neq 0)\) baryon-meson scattering can only proceed through such a (strange) quark exchange. Since the large-\(N_c\) counterparts of the physical baryons possess only \(O(N_c^0)\) strange quarks, such \(Y_t \neq 0\) processes require an exchange of the comparatively rare baryon \(s\) quarks, costing a factor of \(N_c^{1/2}\) in the amplitude (once the proper wave function normalization is taken into account). Thus, at \(O(N_c^0)\) one has \(Y_t=0\).

Alternately, using the familiar operator approach to baryonic matrix elements \[18\], one may observe that strangeness-changing operators with \(O(N_c^{1/2})\) matrix elements do indeed occur (such as those of the combined spin- \((i)\) flavor \((a)\) operator \(G^{ia}\) with \(a=4,5,6,7\)) when sandwiched between two states with \(O(N_c^0)\) strange quarks.

\section*{B. \(I_t=J_t\): Nonstrange Case}

We already possess from Ref. \[17\] the elements of a proof that the \(I_t=J_t\) rule holds for nonstrange baryon-meson scattering; in \[14\] we showed that the 3-flavor scattering amplitude expressed in the \(s\) channel reduces for nonstrange processes to the long-known 2-flavor result \[4\]. This, in turn, was the equation that MM used to prove \[6\] the \(I_t=J_t\) rule. Since the 3-flavor \(s\)-channel and \(t\)-channel results are necessarily equivalent—they obtain from the same source and use the same formalism—the \(I_t=J_t\) rule must directly follow as a result of the \(t\)-channel result restricted to 2 flavors. Notably, however, the authors of Ref. \[6\] state their inability to prove this step.

In fact, the missing ingredients in Ref. \[6\] are precisely those described in the last section, that \(N_c\)-dependent factors such as the sizes of baryon representations and their hypercharges must be treated correctly. Here we show that the nonstrange \(I_t=J_t\) rule follows directly from the 3-flavor \(t\)-channel expression Eqs. \[10\]–\[12\]. Our previous \(s\)-channel proof \[17\] mandates this result, but it is not merely instructional to prove the result using the \(t\)-channel expression: As we see in the next subsection, this exercise provides the necessary impetus.
to prove $I_t = J_t$ in the 3-flavor case. But first, the nonstrange case:

Consider only the two SU(3) CGC appearing in Eq. (3). We have already proved that $Y_t = 0$ for leading-order processes. For the nonstrange case, $Y_B = +\frac{N_c}{3} = Y_{B'}$, which specifies states in the singly-degenerate top row of their respective SU(3) multiplets, $R_B$ and $R_{B'}$, within the ground-state “56”. In particular, knowing $S_{B'}$ uniquely specifies $R_{B'} = (S_{B'}, \frac{N_c}{2} - S_{B'})$ among the “56” states.

Using Eq. (4) however, one can turn the argument around and show that $R_{B'}$ can only lie in the “56”. In the context of Eq. (4), we have $I_B = S_B$, $m = 0$, and $Y_\phi = 0$, meaning that the $O(N_c^0)$ CGC all have $r = 0$ and thus $Y_{s,\text{max}} = +\frac{N_c}{3}$ and $R_{B'} = R_s = (2I_{s,\text{top}}, \frac{N_c}{2} - I_{s,\text{top}})$, which are precisely the SU(3) representations lying in “56”, and therefore $I_{B'} = I_{s,\text{top}} = S_{B'}$. The combination of these two observations tells us that choosing $S_{B'}$ specifies one and only one $R_{B'}$; therefore, one may sum over $R_{B'}$ without changing Eq. (3). Moreover, the hypercharges in the kets of Eq. (3) are fixed to equal $+\frac{N_c}{3}$, but this entry may be replaced with a variable $\tilde{Y}$ and summed over without loss of generality; the CGC of Eq. (3) may thus be replaced by

$$
\sum_{R_{B'}, \tilde{\gamma}, \tilde{Y}} \left( \begin{array}{c|c} R_B & R_{B'} \tilde{\gamma} \\ \hline S_B, +\frac{N_c}{3} & J_t 0 \\ \end{array} \right) \left( \begin{array}{c|c} R_B & R_t \\ \hline S_B, +\frac{N_c}{3} & I_t 0 \\ \end{array} \right) = \delta_{I_t J_t}.
$$

The final equality, which is precisely the $I_t = J_t$ rule, is just a special case of the orthogonality relation Eq. (11.3b) of Ref. [20]. In light of our previous comments, the sums over $R_{B'}$ and $\tilde{Y}$ are unnecessary, but the sum over $\tilde{\gamma}$ is required; one may check this explicitly in cases where the relevant CGC are tabulated [15].

C. $I_t = J_t$: The 3-Flavor Case

Now we return to the CGC in Eq. (3), but note that $Y_t = 0$ still holds, so that $Y_{B'} = Y_B$. The second, but not the first, CGC in (3) depends upon quantum numbers corresponding to nonzero strangeness. We claim that one may obtain the second CGC from the first by means of repeated applications of recursion relations, and that no step in this recursion depends upon the value of $\tilde{\gamma}$. It then follows that the second CGC is proportional to the first, with the proportionality constant being a complicated function of all the quantum numbers in the two CGC except for $\tilde{\gamma}$. But then, these complicated prefactors simply pull through the sum on $\tilde{\gamma}$, and the sum reduces to the one that we obtained in Eq. (5).
Proving the 3-flavor $I_t = J_t$ rule therefore requires one only to one show that the first CGC in Eq. (3) uniquely sets the scale for all CGC of the form of the second CGC in (3), independent of $\tilde{\gamma}$. Of course, their absolute sizes are determined by unitarity.

The required recursion relations are none other than those for the strangeness-changing SU(3) ladder operators $U_\pm$ and $V_\pm$, which indeed were the ingredients used to prove Eq. (4). In general, such relations involve six SU(3) CGC (e.g., Eq. (2.5) of Ref. [15]). However, in the case of large $N_c$ baryon-meson couplings, the large square-root prefactors (analogues to the familiar SU(2) factors $[(I \mp I_z)(I \pm I_z + 1)]^{1/2}$ associated with operators $I_\pm$) that change meson hypercharge (and isospin) are relatively smaller by a factor $N_c^{-1/2}$ [17]. Incidentally, these suppressed factors are also the only ones that depend upon $R_t$.

Therefore, for large $N_c$, recursion relations based upon $U_\pm$ and $V_\pm$ connect only CGC with a fixed $I_t$ and $Y_t = 0$. Now it remains only to show that these recursion relations point uniquely back to the nonstrange CGC in Eq. (3),

$$\begin{pmatrix}
R_B & R_t & R_{B'} \tilde{\gamma} \\
S_B, +\frac{N_c}{3} & I_t 0 & S_{B'}, +\frac{N_c}{3}
\end{pmatrix}. \tag{6}$$

But this is not difficult to show, for consider an arbitrary CGC having the form of the second one in Eq. (3), with $Y_t = 0$. One may repeatedly apply recursion relations that increase $Y_B = Y_{B'}$ by one unit at each step until one reaches $Y_{B,\text{top}} = +\frac{N_c}{3}$. However, the top row of the baryon weight diagrams consists of singly-occupied sites of a unique isospin, and consequently the only CGC appearing at that level is the one given by Eq. (3). In most cases, this completes the proof.

But one special exception must be noted. Given an arbitrary allowed CGC having the form of the second one in Eq. (3), it can occur that the given value of $I_t$ satisfies the triangle rule $\delta(I_B I_t I_{B'})$ but not the nonstrange triangle rule $\delta(S_B I_t S_{B'})$, and hence the CGC Eq. (3) vanishes. Since $J_t$ by construction satisfies the triangle rule $\delta(S_B J_t S_{B'})$, it follows that this particular value of $I_t$ cannot equal $J_t$. But how does one then prove that the expression in Eq. (3) vanishes? In that case, one simply notes that the recursion process upwards in values of $Y_B$ leads eventually to a value of hypercharge where the quantum numbers are no longer allowed, and the CGC vanishes. But one may then reverse the process, applying hypercharge-lowering operators to such a disallowed CGC to obtain recursion relations for nominally allowed CGC of lower hypercharge, including the ones we start with. All such CGC must therefore vanish for large $N_c$ (but might survive for finite $N_c$), guaranteeing that
Eq. (3) vanishes. This mechanism, incidentally, is the origin of such CGC that are nonzero but do not saturate the bound given by Eq. (4).

In light of the $Y_t=0$ and $I_t=J_t$ rules, and using Eq. (2), the master expression Eq. (1) is most conveniently written (keeping factors originating as $I_t$ or $J_t$ distinct) as

\[ S_{LL'}^n S_{B'} B J_t J_t R_t \gamma_{1Y}^I Y_t = \delta_{J_t J_t'} \delta_{I_t I_t'} \delta_{R_t R_t} \delta_{I_t I_t'} \delta_{Y_t Y_t'} \delta_{I_t I_t} \delta_{Y_t Y_t} \]

\[ \times (-1)^{S_{B'}-S_B^T+I_0-J_0+(I_{B'}-S_B^T)-\frac{1}{2}(Y_B-\frac{N_c}{3})} ([S_B][I_B^T][J_{\phi'}][J_{\phi}]/[I_t][J_t])^{1/2} \]

\[ \times \sum_{I \in R \phi, I' \in R \phi'} \left( \begin{array}{cc} R_{\phi} & R_{\phi'}^* \\ I Y & I' Y' \end{array} \right) \left( \begin{array}{cc} R_{\phi} & R_{\phi'}^* \\ I_0 Y_{\phi} & I_0 Y_{\phi'} \end{array} \right) \]

\[ \times \sum_{K K' L L'} (\gamma_{KL}^T K [K][K'])^{1/2} \left( J_{\phi} I K \\ J_{\phi'} I' K' \right) \left( J_{\phi} I K \\ J_{\phi'} I' K' \right) \]

\[ \times \gamma_{K K' LL'}^T. \]  

IV. $1/N_c$ CORRECTIONS

The arguments of Refs. 19 that demonstrate the 2-flavor $I_t = J_t$ rule in baryon-baryon scattering using the operator approach 18 can be generalized, not only to the baryon-meson case 9, but to three flavors and to delineating the form of $1/N_c$ corrections as well.

The primary tool in the 2-flavor case is the observation that arbitrary $n$-body operators (i.e., having $n$ quark creation and destruction operators) can be written in terms of products of $n$ 1-body operators, and give matrix elements subleading in the $1/N_c$ expansion unless all of the 1-body operators are either of the form $\mathbb{I}$ (quark number operators) or $G^a$. Then, the operator reduction rules 18 indicate that contractions of indices among the $G$'s always lead to operators with matrix elements of lower order than $O(N_c^1)$ for each $G$, while uncontracted $G$'s may be symmetrized among their spin and flavor indices—one of each for every $G$—and therefore the leading-order operator has $I_t = J_t$. Each contraction or non-$G$, non-$\mathbb{I}$ 1-body operator (i.e., pure isospin $I^a$ or spin $J^i$) costs a relative factor $N_c$, and therefore operators with $|I_t - J_t| = n$ are suppressed by a relative factor $1/N_c^n$ 12, 13.

These arguments may be generalized to three flavors (as, indeed, was strongly suggested in Refs. 19). The only additional 1-body operator with $O(N_c^1)$ matrix elements on the
baryons with \( N_s = O(N_c^0) \) baryons is \( T^8 \), but its \( O(N_c^1) \) part is simply proportional to 1. Next, in principle the factors of \( G^a \) with \( a = 4, 5, 6, 7, 8 \) (i.e., not isovector) spoil the \( I_t = J_t \) rule. However, as observed in Subsec. \( \text{III} \) A the strangeness-changing components of \( G \) give only \( O(N_c^{1/2}) \) matrix elements on these baryon states, as do the strangeness-changing matrix elements of \( T^a \). Finally, \( G^8 \) on these states has only \( O(N_c^0) \) matrix elements \([18]\). Thus, only the \( I_t = J_t \) portions of \( G^a \) (\( a = 1, 2, 3 \)) contribute to the leading-order amplitudes; in fact, this can be taken as an alternate proof of our primary conclusion.

But this argument also indicates the nature of the \( 1/N_c \) corrections. For the \( N_s = O(N_c^0) \) states, each strangeness-changing operator costs a factor of \( N_c^{1/2} \), while each unit of \( |I_t - J_t| \), by the same arguments as before, costs a factor of \( N_c \). The \( Y_t = 0 \) rule has an additional \( O(N_c^{-1/2}) \) correction associated with each unit of strangeness change, while within a sector of fixed \( Y_t = 0 \), the \( I_t = J_t \) rule has an additional \( O(1/N_c) \) correction for each unit of difference between \( I_t \) and \( J_t \).

V. CONCLUSIONS

We have shown that baryon-meson scattering amplitudes, regardless of the strangeness content of the hadrons involved, satisfy two selection rules when expressed as \( t \)-channel exchanges: \( Y_t = 0 \) and \( I_t = J_t \). We have also explained how to characterize their \( 1/N_c \) corrections by means of their quantum numbers: \( N_c^{-|Y_t|/2} \) and \( N_c^{-|I_t - J_t|} \), respectively.

The phenomenological implications are immediate, but their detailed application will be reserved for another paper. For example, the process \( K^- p \rightarrow \pi^+ \Sigma^- \) is suppressed in cross section by \( 1/N_c \) compared to, say, \( K^- p \rightarrow K^- p \). More restrictive, however, will be relations among amplitudes with no strangeness exchange, such as \( K N \rightarrow K N \). Such constraints were implicitly used in studying possible pentaquark multiplets \([14, 16]\), and indeed can be considered as relations not relying on perfect SU(3) symmetry but only SU(2) \( \times \) U(1) symmetry along a line of fixed strangeness. Clearly, an SU(3)-derived selection rule that does not require SU(3) symmetry will provide robust information.

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[22] Part of this discrepancy is due to the fact that quantities such as $I_z + \frac{Y}{2}$ appearing in phases in the original derivation are integers for mesons, but not for baryons in the SU(3) representations $(p, q) = (2S_B, \frac{N_c}{2} - S_B)$; we repair this deficiency by replacing all such phase arguments by $I_z + \frac{Y}{2} - \frac{1}{3}(2p + q)$. Still other differences can arise through the chosen order of coupling angular momenta (e.g., $L + S$ vs. $S + L$), through flipping the sign of an integer-valued phase factor argument [i.e., $(-1)^n = (-1)^{-n}$], or through noting that fermionic baryons have $(-1)^{2S_B} = -1$. 

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