Forward-backward asymmetry on Z resonance in \( SO(5) \times U(1) \) gauge-Higgs unification

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Abstract

We find that the tree-level predictions of the forward-backward production asymmetries on the Z resonance for \( b \) and \( c \) quarks, \( A_{FB} \), in an \( SO(5) \times U(1) \) gauge-Higgs unification model are markedly close to the central values of the Particle Data Group data unlike the standard model. The decay width of Z boson is evaluated and the \( S \) and \( T \) parameters are discussed.
1 Introduction

The standard model of elementary particles describes physics well up to the weak scale. It is a gauge theory with quarks and leptons as particles of matter and with gauge bosons as particles of force. In addition to matter and force, the ground state is quantified in a unified framework. If the theory were in a symmetric phase, the quarks, leptons and gauge bosons would be all massless. To yield their masses, the gauge symmetry is broken. A single scalar field minimally represents an effective source of gauge symmetry breaking, though it is unknown even whether further microscopic structure of the symmetry breaking is possible. In the standard model, quarks, leptons, gauge bosons and even the Higgs boson are treated on an equal footing. This is a feasible unified model. However, the standard model has the gauge hierarchy problem. From an intuitive viewpoint, while fermions and gauge bosons are called particles of matter and force, respectively, it seems obscure what particle the physical Higgs boson should be called.

The gauge-Higgs unification scenario is to treat gauge fields and Higgs field literally in a unified way. The four-dimensional Higgs field is identified with a part of the extra-dimensional component of gauge fields in higher dimensions \([1, 2]\). Due to a physical Wilson line phase, the electroweak symmetry is dynamically broken \([3, 4, 5]\). The non-locality of the Wilson line phase yields a finite mass to the physical Higgs boson and it is a solution to the gauge hierarchy problem \([6]\).

In a gauge theory with group \(SO(5) \times U(1)\), the custodial symmetry of the standard model is applied to the gauge-Higgs unification scenario. It leads to the decisive prediction. The presence of top quark which is a part of fermions introduced in the vectorial representation of \(SO(5)\) in the bulk five-dimensional Randall-Sundrum spacetime dynamically induces the electroweak symmetry breaking where the effective potential for the Wilson line phase \(\theta_H\) is minimized at \(\theta_H = \pm \frac{1}{2} \pi\) \([7]\). The four-dimensional Higgs field \(H(x)\) corresponds to four-dimensional fluctuations of the Wilson line phase. The effective interactions are determined for definite matter content. The \(WWH\), \(ZZH\) and Yukawa couplings are suppressed by a factor \(\cos \theta_H\) compared with those in the standard model \([8, 9]\). At \(\theta_H = \pm \frac{1}{2} \pi\) for the potential minimum, the couplings with a single Higgs field vanish. If the annihilation rate of Higgs bosons is small, it can become a candidate of the dark matter in the universe \([10]\). From an intuitive viewpoint, the physical Higgs boson might be called a particle of dark matter.

Recently, an \(SO(5) \times U(1)\) gauge-Higgs unification model in the Randall-Sundrum spacetime has been generalized to inclusion of three generations of fermions (quarks and leptons) and gauge bosons, i.e., all particles of matter and force at the weak scale \([11]\). The effective chiral theory of the model is anomaly free. In the model which holds the distinctive prediction of dynamical gauge symmetry breaking and the stable Higgs boson, the \(W\), \(Z\) and electromagnetic currents have been determined. The electroweak currents depend on profiles of wave functions of \(W\), \(Z\) bosons and quarks and leptons both in the fifth-dimension and \(SO(5)\) group. Despite the highly nontrivial profiles, it has been found that the deviations of the couplings of fermions to gauge bosons from the standard model are less than 1% except for top quark. This small deviation raises the expectation that this \(SO(5) \times U(1)\) gauge-Higgs unification model may be regarded as a realistic model. Once these gauge couplings are indicated in an expression of observed quantities, they can be compared with experimental data rather than the values of the standard model. A representative measured quantity relevant to gauge couplings of fermions is the forward-backward asymmetry on the \(Z\) resonance, which is denoted as \(A_{FB}\). In the standard
model, $A_{FB}$ has been extensively studied including radiative corrections. For $b$ quark production, there seems to be a discrepancy between the standard model prediction and the experimental value. It is certain that $A_{FB}$ is a decisive indication to express the gauge couplings also in the $SO(5) \times U(1)$ model.

In this paper, we present the tree-level predictions of the forward-backward production asymmetries on the $Z$ resonance for quarks and leptons, $A_{FB}$, in the model given in Ref. [11]. It is found that the tree-level prediction for $b$ quark production gives $A_{FB}^b = 0.09952$, which is quite close to the central value of the experimental data $A_{FB}^b(\text{Exp.}) = 0.0992 \pm 0.0016$.[1] We find for $c$ quark production $A_{FB}^c = 0.07073$ which is also close to the central value of the experimental data $A_{FB}^c(\text{Exp.}) = 0.0707 \pm 0.0035$. For all fermions, the tree-level predictions of $A_{FB}$ are given, and it is shown that the values are not very sensitive to whether input parameters for quarks and leptons correspond to their running masses at the $m_Z$ scale or their pole masses. Because the gauge-Higgs unification scenario is a higher-dimensional theory, it is non-renormalizable and its radiative effects should be treated appropriately. We discuss quantum loop corrections and divergences for observed quantities. We also evaluate other electroweak quantities such as the decay width of $Z$ boson and the $S$ and $T$ parameters.

The paper is organized as follows. In Section 2, our model is summarized. In Section 3 we give the equations for the $Z$ boson couplings of fermions required for calculating $A_{FB}$. The numerical analysis of $A_{FB}$ is shown in Section 4. The decay width and the $S$ and $T$ parameters are evaluated in Section 5. Summary and discussions are given in Section 6. Details of the notation and the results for input parameters different than in the main text are given in Appendices A and B respectively. The values of the $S$ and $T$ parameters at one-loop in the standard model are summarized in Appendix C.

2 Model

We work on the model given in Ref. [11]. The model is defined in the Randall-Sundrum (RS) warped spacetime whose metric is given by [12, 13]

$$ds^2 = G_{MN}dx^Mdx^N = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2,$$

(2.1)

where $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$. $\sigma(y) = \sigma(y + 2L)$, and $\sigma(y) = k|y|$ for $|y| \leq L$. The fundamental region in the fifth dimension is given by $0 \leq y \leq L$. The Planck brane and the TeV brane are located at $y = 0$ and $y = L$, respectively. The bulk region $0 < y < L$ is an anti-de Sitter spacetime with the cosmological constant $\Lambda = -6k^2$.

We consider an $SO(5) \times U(1)_X$ gauge theory in the RS warped spacetime. The $SO(5) \times U(1)_X$ symmetry is broken to $SO(4) \times U(1)_X$ by the orbifold boundary conditions at the Planck and TeV branes. The symmetry is spontaneously broken to $SU(2)_L \times U(1)_Y$ by additional interactions at the Planck brane. Here we do not address a question of how the orbifold structure of spacetime appears with orbifold conditions.

The action integral consists of four parts:

$$S = S_{\text{gauge}}^{\text{bulk}} + S_{\text{scalar}}^{\text{brane}} + S_{\text{fermion}}^{\text{brane}} + S_{\text{fermions}}^{\text{brane}},$$

(2.2)

The bulk parts respect $SO(5) \times U(1)_X$ gauge symmetry. There are $SO(5)$ gauge fields $A_M$ and $U(1)_X$ gauge field $B_M$. The former are decomposed as $A_M = \sum_{I=1}^{10} A_M^I T^I = A_M^0 T^0 + A_M^I T^I$.

*The experimental value is quoted from the Particle Data Group data [18].
\[ \sum_{a_l=1}^{3} A_{a_l}^{aL}T^{aL} + \sum_{a_R=1}^{3} A_{a_R}^{aR}T^{aR} + \sum_{a=1}^{4} A_{a}^{a}T^{a}, \text{ where } T^{aL,aR}(a_L, a_R = 1, 2, 3) \text{ and } T^{a}(a = 1, \ldots, 4) \text{ are the generators of } SO(4) \sim SU(2)_L \times SU(2)_R \text{ and } SO(5)/SO(4), \text{ respectively.} \]

In a vectorial representation, the components of the generator are \( T^{aL, aR}_{ij} = -\frac{i}{2} \epsilon^{abc}(\delta_{i}^{a} \delta_{j}^{b} - \delta_{i}^{b} \delta_{j}^{a}) \) and \( T^{a}_{ij} = -\frac{i}{\sqrt{2}}(\delta_{i}^{a} \delta_{j}^{b} - \delta_{i}^{b} \delta_{j}^{a}) \), where \( i, j = 1, \ldots, 5 \) and \( \text{Tr}(T^{I}T^{J}) = \delta^{IJ}. \)

The action integral for pure gauge boson part is

\[ S_{\text{gauge}}^{\text{bulk}} = \int d^5x \sqrt{-g} \left[ -\text{tr}(\frac{1}{4} F^{(A)MN} F^{(A)^{MN}} + \frac{1}{2\xi}(f_{\text{gf}}^{(A)})^2 + \mathcal{L}_{\text{gh}}^{(A)}) \right. \]

\[ \left. -\frac{1}{4} F^{(B)MN} F^{(B)^{MN}} + \frac{1}{2\xi}(f_{\text{gf}}^{(B)})^2 + \mathcal{L}_{\text{gh}}^{(B)} \right], \quad (2.3) \]

where the gauge fixing and ghost terms are denoted as functionals with suffixes gf and gh, respectively. Here \( F_{MN}^{(A)} = \partial_{M}A_{N} - \partial_{N}A_{M} - ig_{A}[A_{M}, A_{N}] \) and \( F_{MN}^{(B)} = \partial_{M}B_{N} - \partial_{N}B_{M} \).

The orbifold boundary conditions at \( y_{0} = 0 \) and \( y_{1} = L \) for gauge fields are given by

\[
\begin{align*}
(A_{\mu}^{a}) (x, y_{j} - y) &= P_{j} \left( \begin{array}{c}
A_{\mu}^{a} \\
-A_{\mu}^{a}
\end{array} \right) (x, y_{j} + y) P_{j}^{-1}, \\
(B_{\mu}^{a}) (x, y_{j} - y) &= \left( \begin{array}{c}
B_{\mu}^{a} \\
-B_{\mu}^{a}
\end{array} \right) (x, y_{j} + y), \\
P_{j} &= \text{diag}(-1, -1, -1, -1, +1), \quad (j = 0, 1),
\end{align*}
\]

(2.4)

which reduce the \( SO(5) \times U(1)_{X} \) symmetry to \( SO(4) \times U(1)_{X}. \) A scalar field \( \Phi(x) \) on the Planck brane belongs to \((0, \frac{1}{2})\) representation of \( SO(4) \sim SU(2)_{L} \times SU(2)_{R} \) and has a charge of \( U(1)_{X}. \) With the brane action

\[ S_{\text{Pl. brane}}^{\text{scalar}} = \int d^{5}x \delta(y) \left\{ -(D_{\mu} \Phi)^{\dagger} D^{\mu} \Phi - \lambda_{\Phi}(\Phi^{\dagger} \Phi - w_{\Phi}^{2})^{2} \right\}, \]

(2.5)

\[ D_{\mu} \Phi = \partial_{\mu} \Phi - i \left( g_{A} \sum_{a_{R}}^{3} A_{a_{R}}^{a}T^{a} + \frac{g_{B}}{2}B_{\mu} \right) \Phi, \]

the \( SU(2)_{R} \times U(1)_{X} \) symmetry breaks down to \( U(1)_{Y}, \) the weak hypercharge in the standard model. The massless modes of \( A_{1}^{a_{R}}, A_{2}^{a_{R}} \) and \( A_{3}^{a_{R}} \) acquiring large masses. Here

\[
\left( \begin{array}{c}
A_{3R}^{a} \\
A_{M}^{a}
\end{array} \right) = \left( \begin{array}{cc}
c_{\phi} & -s_{\phi} \\
s_{\phi} & c_{\phi}
\end{array} \right) \left( \begin{array}{c}
A_{3R}^{a} \\
B_{M}^{a}
\end{array} \right), \quad c_{\phi} = \frac{g_{A}}{\sqrt{g_{A}^{2} + g_{B}^{2}}}, \quad s_{\phi} = \frac{g_{B}}{\sqrt{g_{A}^{2} + g_{B}^{2}}},
\]

(2.6)

The four-dimensional gauge coupling for electromagnetic interaction is

\[ e = \frac{g_{A}g_{B}}{\sqrt{g_{A}^{2} + 2g_{B}^{2}L}} = \frac{g_{A}s_{\phi}}{\sqrt{(1 + s_{\phi}^{2})L}}. \quad (2.7) \]

For \( e = g \sin \theta_{W} \) and \( g = g_{A}/\sqrt{L}, \) a relationship between the couplings and the weak mixing angle is give by \( s_{\phi}^{2} = \tan^{2} \theta_{W}. \) We assume that \( w \) is much larger than the KK mass scale, being \( \mathcal{O}(M_{\text{GUT}}) \) to \( \mathcal{O}(M_{\text{Planck}}). \) The net effect for low-lying modes of the KK towers of \( A_{1}^{a}, A_{2}^{a} \) and \( A_{3}^{a} \) is that they effectively obey Dirichlet boundary conditions at the Planck brane. This is a limit of the Dirichlet-Neumann mixed boundary condition. The effective orbifold boundary conditions are tabulated in Table 1. From the consistency requirement with the five-dimensional gauge transformation, the effective
Table 1: Boundary conditions for gauge bosons and bulk fermions: The effective Dirichlet condition made by brane dynamics is denoted as $D_{\text{eff}}$. For fermions $j = 1, \cdots, 4$. 

| $A_{\mu}$ | $A_{\mu}^R$ | $A_{\mu}^B$ | $A_{\mu}^Y$ | $A_{\mu}'$ | $B_{\mu}$ | $\psi_{a_1L}$, $\psi_{a_5R}$ |
|-----------|-------------|-------------|-------------|-------------|----------|-----------------------------|
| (N,N)     | (D, N)     | (D, N)     | (N, N)     | (N, N)     | (N, N)   | (N, N)                     |

Dirichlet condition in Table 1 is not allowed to be replaced by Dirichlet condition without brane dynamics [14, 15].

Bulk fermions for quarks and leptons are introduced as multiplets in the vectorial representation of $SO(5)$. In the quark sector two vector multiplets are introduced for each generation. In the lepton sector it suffices to introduce one multiplet for each generation to describe massless neutrinos, whereas it is necessary to introduce two multiplets to describe massive neutrinos. They are denoted by $\Psi_a = (\psi_{a_1}, \ldots, \psi_{a_5})^t$ where the subscript $a$ runs from 1 to 3 or 4 for each generation.

In the bulk the action integral is

$$ S_{\text{bulk}}^{\text{fermion}} = \int d^5x \sqrt{-G} \sum_{a=1}^{3} i\bar{\psi}_a D(c_a) \psi_a, $$

where the Dirac conjugate is $\bar{\psi} = i\psi\Gamma^0$ and Gamma matrices are given by

$$ \Gamma^\mu = \left( \begin{array}{cc} \bar{\sigma}^\mu & \sigma^\mu \\ -1 & -1 \end{array} \right), \quad \sigma^\mu = (1, \bar{\sigma}), \quad \bar{\sigma}^\mu = (-1, \bar{\sigma}). $$

The non-vanishing spin connection is $\omega_{\mu \nu 5} = -\sigma^\nu \sigma^\mu \delta_{\mu \nu}$, where $\delta_{\mu \nu}$ denotes a vierbein in the four-dimensional Minkowski spacetime. The $c_a$ term in Eq. (2.9) gives a bulk kink mass, where $\sigma'(y) = k \epsilon(y)$ is a periodic step function with a magnitude $k$. The dimensionless parameter $c_a$ plays an important role in the Randall-Sundrum warped spacetime. The orbifold boundary conditions are given by

$$ \Psi_a(x, y_j - y) = P_j \Gamma^5 \Psi_a(x, y_j + y). $$

With $P_j$ in Eq. (2.4) the first four component of $\Psi_a$ are even under parity for the 4D left-handed ($\Gamma^5 = -1$) components. An $SO(5)$ vector $\Psi$ can be expressed as the sum of $(\frac{1}{2}, \frac{1}{2})$ representation and a singlet $(0, 0)$ of $SU(2)_L \times SU(2)_R$. The $(\frac{1}{2}, \frac{1}{2})$ representation is written as

$$ \hat{\psi} = \left( \begin{array}{c} \hat{\psi}_{11} \\ \hat{\psi}_{21} \\ \hat{\psi}_{22} \end{array} \right), \quad \frac{1}{\sqrt{2}}(\psi_1 + i \bar{\psi} \cdot \bar{\sigma})i\bar{\sigma}_2 = -\frac{1}{\sqrt{2}} \left( \begin{array}{c} \psi_2 + i \psi_1 \\ \psi_4 - i \psi_3 \\ \psi_2 - i \psi_1 \end{array} \right). $$

The singlet $(0, 0)$ is $\psi_5$. The quarks in the third generation, for instance, are composed of bulk Dirac fermions of $SO(5)$ vectorial representation

$$ \Psi_1 \left( \begin{array}{c} \frac{2}{3} \\ \frac{1}{3} \end{array} \right) = \left[ Q_1 = \left( \begin{array}{c} T \\ B \end{array} \right), \quad q = \left( \begin{array}{c} t \\ b \end{array} \right), \quad t' \right], $$

$$ \Psi_2 \left( \begin{array}{c} -\frac{1}{3} \end{array} \right) = \left[ Q_2 = \left( \begin{array}{c} U \\ D \end{array} \right), \quad Q_3 = \left( \begin{array}{c} X \\ Y \end{array} \right), \quad b' \right]. $$
and boundary right-handed fermions of the \((\frac{1}{2}, 0)\) representation for \(SU(2)_L \times SU(2)_R\)

\[
\hat{\chi}_{1R} \left( \frac{7}{6} \right) = \left( \begin{array}{c} \hat{T}_R \\ \hat{B}_R \end{array} \right), \quad \hat{\chi}_{2R} \left( \frac{1}{6} \right) = \left( \begin{array}{c} \hat{C}_R \\ \hat{D}_R \end{array} \right), \quad \hat{\chi}_{3R} \left( -\frac{5}{6} \right) = \left( \begin{array}{c} \hat{X}_R \\ \hat{Y}_R \end{array} \right). \tag{2.15}
\]

For boundary fermions, the hypercharge \(Y/2\) is equal to the \(U(1)_X\) charge, \(Q_X\). The leptons in the third generation are composed of bulk Dirac fermions of \(SO(5)\) vectorial representation

\[
\Psi_3(-1) = \left[ \ell = \left( \begin{array}{c} \nu_r \\ \tau \end{array} \right), \quad L_1 = \left( \begin{array}{c} L_{1X} \\ L_{1Y} \end{array} \right), \quad \tau' \right], \tag{2.16}
\]

\[
\Psi_4(0) = \left[ L_2 = \left( \begin{array}{c} L_{2X} \\ L_{2Y} \end{array} \right), \quad L_3 = \left( \begin{array}{c} L_{3X} \\ L_{3Y} \end{array} \right), \quad \nu_r' \right], \tag{2.17}
\]

and boundary right-handed fermions of the \((\frac{1}{2}, 0)\) representation for \(SU(2)_L \times SU(2)_R\)

\[
\hat{\chi}_{1R}^{\ell} \left( -\frac{3}{2} \right) = \left( \begin{array}{c} \hat{L}_{1XR} \\ \hat{L}_{1YR} \end{array} \right), \quad \hat{\chi}_{2R}^{\ell} \left( \frac{1}{2} \right) = \left( \begin{array}{c} \hat{L}_{2XR} \\ \hat{L}_{2YR} \end{array} \right), \quad \hat{\chi}_{3R}^{\ell} \left( -\frac{1}{2} \right) = \left( \begin{array}{c} \hat{L}_{3XR} \\ \hat{L}_{3YR} \end{array} \right). \tag{2.18}
\]

The number in the parenthesis on the left-hand side for each fermion denotes the \(U(1)_X\) charge. The hypercharge and the electric charge are given by \(Y/2 = T^{3R} + Q_X\) and \(Q_E = T^{3L} + T^{3R} + Q_X\), respectively. The components \(B\) and \(t\) couple to \(t'\) through the vacuum expectation value \(\langle A^c_y \rangle \propto \theta_H T^{3L}\) with the Wilson line phase \(\theta_H\). Similarly, \(D\) and \(X\) couple to \(b'\) and for leptons \(\tau\) and \(L_{1X}\) couple to \(\tau'\). With \(\theta_H\) alone, there remain extra massless modes of fermions. In order to make them heavy, we introduce right-handed fermion \(\hat{\chi}_{aR}\) and \(\hat{\chi}_{aR}^{\ell}\) in the \((\frac{1}{2}, 0)\) representations of \(SU(2)_L \times SU(2)_R\) localized on the Planck brane \(y = 0\). The brane fermions \(\hat{\chi}_{aR}\) and \(\hat{\chi}_{aR}^{\ell}\) couple to the corresponding bulk fermions and the brane scalar \(\Phi\) in Eq. (2.5) through Yukawa couplings. After the \(\Phi\) develops the vacuum expectation value, the general brane action for \(\hat{\chi}_{aR}\) and \(\hat{\chi}_{aR}^{\ell}\) is given by

\[
S_{\text{fermion}}^\text{brane} = \int d^5x \sqrt{-G} i \delta(y)
\]

\[
\times \left\{ \sum_{a=1}^{3} \left[ \hat{\chi}_{aR}^{\ell} \bar{\sigma}^\mu D_\mu \hat{\chi}_{aR} - \mu_a (\hat{\chi}_{aR}^{\ell} Q_{aL} - Q_{aL}^{\ell} \hat{\chi}_{aR}) \right] - \bar{\mu} (\hat{\chi}_{aR}^{\ell} q_L - q_{aR}^{\ell} \hat{\chi}_{aR}) 
+ \sum_{a=1}^{3} \left[ \hat{\chi}_{aR}^{\ell} \bar{\sigma}^\mu D_\mu \hat{\chi}_{aR}^{\ell} - \mu_a^{\ell} (\hat{\chi}_{aR}^{\ell} L_{aL} - L_{aL}^{\ell} \hat{\chi}_{aR}) \right] - \bar{\mu}^{\ell} (\hat{\chi}_{aR}^{\ell} L_{aL} - L_{aL}^{\ell} \hat{\chi}_{aR}) \right\} \tag{2.19}
\]

where \(D_\mu\) in the kinetic term has the same form as in Eq. (2.5) with \(A^{\mu T aL}\) replaced by \(A^a_{\mu} T^{aL}\). The \(\mu\) terms mix bulk left-handed fermions and brane right-handed fermions. When each coupling \(\mu_a, \bar{\mu}, \mu_a^{\ell}\) and \(\bar{\mu}^{\ell}\) is taken as a matrix, a flavor mixing can be introduced \cite{16, 17}. These couplings have the dimension \([M]^{1/2}\) and are collectively denoted as \(\mu\). For simplicity, we adopt flavor-diagonal couplings. Then only the modest conditions, \(\mu^2 \gg m_{KK} \equiv \pi k/(z_L - 1)\), are necessary to be satisfied for the values of the couplings to get the desired low energy mass spectrum. The brane scalar \(\Phi\) is not required to be protected from having the quadratic divergent corrections. In the case neutrinos are massless, \(\Psi_4, \hat{\chi}_{3R}^{\ell}\) and \(\hat{\chi}_{3R}^{\ell}\) are unnecessary. The model is anomaly free with respect to \(SU(2)_L \times SU(2)_R \times U(1)_X\) independently of inclusion of these three fields.
### 3 Z boson couplings of quarks and leptons

In order to evaluate the forward-backward production asymmetry on the Z resonance for quarks and leptons, the four-dimensional gauge couplings of fermions with Z boson are needed. This derivation is given in Ref. [11]. In this section, we summarize the resulting Z boson couplings.

The four-dimensional Lagrangian terms for the Z boson coupling of $t$ quark are obtained as

$$\mathcal{L}_{Ztt} = \frac{g_A Z_\mu}{\cos \theta_W} (\mathcal{T}_L \bar{t} L \gamma^\mu t_L + \mathcal{T}_R \bar{R} R \gamma^\mu t_R),$$

with $\mathcal{T}_{L,R} = \frac{1}{2} \mathcal{T}_L^3 - \frac{2}{3} \mathcal{T}_L^Q \sin^2 \theta_W$. The quantities $\mathcal{T}_L^3$ and $\mathcal{T}_L^Q$ are given by

$$\mathcal{T}_L^3 = \int_1^{z_L} dz \left[ N_Z(z)(C_L(z; \lambda_t))^2 \left( a_L^2 - 2 \cos \theta_H a_{B+t} a_{B-t} \right) 
- 2 \sin \theta_H D_L(z) C_L(z; \lambda_t) S_L(z; \lambda_t) a_{B+t} a_{B-t} \right], \quad \mathcal{T}_L^Q = \int_1^{z_L} dz N_Z \left[ (C_L(z; \lambda_t))^2 \left( a_L^2 + a_{B+t}^2 + a_{B-t}^2 \right) + (S_L(z; \lambda_t))^2 \right],$$

Here $N(z)$ and $D(z)$ are the fundamental functions in mode expansion of Z boson, $C_L(z; \lambda_t)$ and $S_L(z; \lambda_t)$ are the fundamental functions in mode expansion of $t$ quark. In expressing mode function profiles, the conformal coordinate $z = e^{\sigma(y)}$ for the fifth dimension is employed, with which the metric becomes $ds^2 = z^{-2} \left\{ \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 / k^2 \right\}$. The fundamental region $0 \leq y \leq L$ is mapped to $1 \leq z \leq z_L = e^{kL}$. In the bulk region $0 < y < L$, one has $\partial_y = k z \partial_z$, $A_y = k z A_z$, $B_y = k z B_z$. The $a_L$, $a_{B+t}$, $a_{B-t}$ and $a_{B'}$ are the coefficients in mode expansion of $t$ quark. These explicit definitions are shown in Appendix A. For the right-handed $t$ quark, $\mathcal{T}_R^3$ and $\mathcal{T}_R^Q$ are given by $\mathcal{T}_L^3$ and $\mathcal{T}_L^Q$ with $(C_L, S_L)$ replaced by $(S_R, C_R)$, respectively.

The four-dimensional Lagrangian terms for the Z boson coupling of $b$ quark are

$$\mathcal{L}_{Zbb} = \frac{g_A Z_\mu}{\cos \theta_W} (\mathcal{B}_L \bar{b} L \gamma^\mu b_L + \mathcal{B}_R \bar{R} R \gamma^\mu b_R),$$

with $\mathcal{B}_{L,R} = -\frac{1}{2} \mathcal{B}_L^3 + \frac{1}{3} \mathcal{B}_L^Q \sin^2 \theta_W$, where $\mathcal{B}_L^3$ and $\mathcal{B}_L^Q$ are given by

$$\mathcal{B}_L^3 = \int_1^{z_L} dz \left[ N_Z(z)(C_L(z; \lambda_b))^2 \left( a_b^2 + 2 \cos \theta_H a_{D+X} a_{D-X} \right) 
+ 2 \sin \theta_H D_L(z) C_L(z; \lambda_b) S_L(z; \lambda_b) a_{D+X} a_{D-X} \right], \quad \mathcal{B}_L^Q = \int_1^{z_L} dz N_Z(z) \left[ (C_L(z; \lambda_b))^2 \left( a_b^2 + a_{D+X}^2 + a_{D-X}^2 \right) + (S_L(z; \lambda_b))^2 \right].$$

For the right-handed $b$ quark, $\mathcal{B}_R^3$ and $\mathcal{B}_R^Q$ are given by $\mathcal{B}_L^3$ and $\mathcal{B}_L^Q$ with $(C_L, S_L)$ replaced by $(S_R, C_R)$, respectively.

To describe a massless neutrino for each generation, we need to introduce only a vector multiplet $\Psi_3$ in Eqs. (2.16) and (2.17). The four-dimensional Lagrangian terms for the Z boson coupling of $\nu_\tau$ neutrino are obtained as

$$\mathcal{L}_{Z\nu\bar{\nu}} = \frac{g_A Z_\mu}{\cos \theta_W} N_L \bar{\nu}_\tau \gamma^\mu \nu_\tau L,$$
A formal equation for the forward-backward asymmetry on the initial electron, it is given by a simple formula

\[ N_L^3 = \int_1^{z_L} dz \left[ N_Z(z)(C_L(z; \lambda_r))^2 a_{\nu_r}^2 \right]. \]  

(3.8)

The four-dimensional Lagrangian terms for the Z boson coupling of τ lepton are

\[ \mathcal{L}_{Z_{\tau\tau}} = \frac{g_A Z_{\mu}}{\cos \theta_W} (\mathcal{X}_L^3 \bar{\tau}_L \gamma^\mu \tau_L + \mathcal{X}_R^3 \bar{\tau}_R \gamma^\mu \tau_R), \]  

(3.9)

with \( \mathcal{X}_{L,R} = -\frac{1}{2} \mathcal{X}_{L,R}^Q + \mathcal{X}_{L,R}^Q \sin^2 \theta_W \). The quantities \( \mathcal{X}_L^3 \) and \( \mathcal{X}_L^Q \) are given by

\[ \mathcal{X}_L^3 = \int_1^{z_L} dz \left[ N_Z(z)(C_L(z; \lambda_r))^2 (2 \cos \theta_H a_{\tau+L_{1X}} a_{\tau-L_{1X}}) + 2 \sin \theta_H D_Z(z) C_L(z; \lambda_r) S_L(z; \lambda_r) a_{\tau+L_{1X}} a_{\tau-L_{1X}} \right], \]  

(3.10)

\[ \mathcal{X}_L^Q = \int_1^{z_L} dz N_Z(z) [(C_L(z; \lambda_r))^2 (a_{\tau+L_{1X}}^2 + a_{\tau-L_{1X}}^2) + (S_L(z; \lambda_r))^2 a_{\nu_r}^2]. \]  

(3.11)

For the right-handed τ lepton, \( \mathcal{X}_R^3 \) and \( \mathcal{X}_R^Q \) are given by \( \mathcal{X}_L^3 \) and \( \mathcal{X}_L^Q \) with \( (C_L, S_L) \) replaced by \( (S_R, C_R) \), respectively.

To describe massive neutrinos one needs to introduce two multiplets \( \Psi_3 \) and \( \Psi_4 \). The structure is the same as in the quark sector. The quantities \( N_{L,R}^3 \) and \( \mathcal{X}_{L,R}^3 \) are obtained with use of the correspondence between leptons and quarks:

\[ (\nu_\tau, L_{2Y}, L_{3X}, \nu_\tau') \leftrightarrow (U, B, t, t'), \quad (\hat{L}_{3X}, \hat{L}_{2Y}) \leftrightarrow (\hat{U}, \hat{B}), \] 

(3.12)

\[ (L_{3Y} \tau, L_{1X}, \tau') \leftrightarrow (b, D, X, b'), \quad (\hat{L}_{3Y}, \hat{L}_{1X}) \leftrightarrow (\hat{D}, \hat{X}), \] 

\[ (\mu_1^\ell, \mu_2^\ell, \mu_3^\ell, \bar{\mu}^\ell) \leftrightarrow (\mu_3, \mu_1, \bar{\mu}_2). \]

The four-dimensional Lagrangian term for the Z boson coupling of the right-handed \( \nu_\tau \) neutrino should be added as

\[ \mathcal{L}_{Z_{\nu\bar{\nu}R}} = \frac{g_A Z_{\mu}}{\cos \theta_W} N_R \bar{\nu}_R \gamma^\mu \nu_R, \]  

(3.13)

with \( N_R = \frac{1}{2} N_{R}^3 \).

Typical numerical values for these Z couplings have been given in Ref. [11]. It has been shown that they deviate slightly from the standard model. In next section, we will calculate \( A_{FB} \) as an observed indication for the Z couplings.

### 4 Forward-backward asymmetry: numerical analysis

A formal equation for the forward-backward asymmetry on the Z resonance for the scattering \( e^+ e^- \rightarrow f \bar{f} \) is the same as in the standard model at tree level. For the unpolarized initial electron, it is given by a simple formula

\[ A_{FB}^f = \frac{3}{4} A_{LR}^f A_{LR}^f. \]  

(4.1)
Here the superscript denotes species of fermions such as $e$ as in $A_{LR}^{e}$. The polarization asymmetry for the decay $Z \rightarrow f \bar{f}$ is determined from the $Z$ boson couplings of fermions. The $A_{LR}^{f}$ is given by

$$A_{LR}^{f} = \frac{(g_{L}^{f})^{2} - (g_{R}^{f})^{2}}{(g_{L}^{f})^{2} + (g_{R}^{f})^{2}}.$$

Here $g_{L,R}^{f}$ are $T_{L,R}, B_{L,R}, N_{L,R}, \chi_{L,R}$ for $t$ and $b$ quarks and $\nu_{\tau}$ and $\tau$ leptons, respectively. In the standard model, at tree level $g_{L,R}^{f}$ are defined as quantities including integrals with respect to $z$ as given in the previous section.

The asymmetries $A_{LR}^{f}$ and $A_{FB}^{f}$ are obtained through a numerical calculation.

The model with massive neutrinos has 16 parameters: $c$ (whose number is 6) for three generations of quarks and leptons, $|\mu/\mu_{2}|$ (whose number is 3) for three generations of quarks, $|\mu_{3}/\bar{\mu}_{3}|$ (whose number is 3) for three generations of leptons, and $g_{A}, g_{B}, k, L$ except for the modestly-tuned parameters, $\mu^{2} \gg m_{KK}$, to make unwanted fields heavy. We take 16 inputs as 6 quark masses, 6 lepton masses, $\sin \theta_{W}, m_{Z}, k$ and $z_{L}$. If one treats neutrinos as massless approximately, the number of the parameters is reduced. The model with massless neutrinos has 13 parameters: $c$ (whose number is 6) for three generations of quarks and leptons, $|\bar{\mu}/\mu_{2}|$ (whose number is 3) for three generations of quarks, and $g_{A}, g_{B}, k, z_{L}$. For this case, the 13 inputs can be taken as 6 quark masses, 3 charged lepton masses, $\sin \theta_{W}, m_{Z}, k$ and $z_{L}$. The case with massless neutrinos can also be regarded as a limit of the case with massive neutrinos. The parameters $k$ and $z_{L}$ are inputs for warped geometry, $\sin \theta_{W}$ and $m_{Z}$ are inputs for the $Z$ boson and the others are assigned for masses of fermions.

For input parameters of warped geometry, we take $z_{L} = (10^{18}\text{GeV})/(1\text{TeV}) = 1.0 \times 10^{15}$ and $k = 4.7 \times 10^{17}$, where $k$ is chosen such that the value of $m_{W} = m_{W}(k, z_{L}, \theta_{H})$ is appropriately reproduced. For these values of $z_{L}$ and $k$, the Kaluza-Klein scale is given by $m_{KK} = \pi k z_{L}^{-1} = 1.48\text{TeV}$. For input parameters of the $Z$ boson, we take $m_{Z} = 91.1876\text{GeV}$ (the central values in Particle Data Group data [18]) and $\sin^{2} \theta_{W} = 0.2312$ (the $\overline{\text{MS}}$ value in Particle Data Group data [18]). The Wilson line phase is $\theta_{H} = \pi/2$. For fermion masses, we take three sets: The first set is the central values of the running masses at the $m_{Z}$ scale given in Ref. [19], which we refer to as XZZ. The second set is the central values of the running masses at the $m_{Z}$ scale given in Ref. [20], which we refer to

|     | $m_{u}$  | $m_{d}$  | $m_{s}$  | $m_{c}$  | $m_{b}$  | $m_{t}$  | $m_{e}$  | $m_{\mu}$  | $m_{\tau}$  |
|-----|----------|----------|----------|----------|----------|----------|----------|------------|-------------|
| XZZ | 1.27     | 2.90     | 55       | 619      | 2890     | 171700   | 0.486570161| 102.7181359 | 1746.24     |
| FK  | 2.33     | 4.69     | 93.4     | 677      | 3000     | 181000   | 0.48684727 | 102.75138   | 1746.69     |
| PDG | 2.4      | 4.75     | 104      | 1270     | 4200     | 171200   | 0.510998910| 105.658367  | 1776.84     |
as FK. The final set is the central values of the physical (pole) masses given in Ref. [18], which we refer to as PDG. The mass parameters of the three sets are tabulated in Table 2. In the sets XZZ and FK, neutrinos are taken as massless. In the set PDG, $m_{\nu_e} = 10^{-3}$ eV, and $m_{\nu_e} = m_{\nu_1}$, $m_{\nu_\mu} = m_{\nu_2}$, $m_{\nu_\tau} = m_{\nu_3}$ are assumed. The differences of masses squared are taken as $\Delta m^2_{21} = 8 \times 10^{-5}$ eV$^2$ and $\Delta m^2_{32} = 2.45 \times 10^{-3}$ eV$^2$ (the central values for the difference of masses squared in Ref. [18]). This corresponds to taking $m_{\nu_\mu} = 9 \times 10^{-3}$ eV and $m_{\nu_\tau} = 5.0309 \times 10^{-2}$ eV.

With the above input parameters, the polarization asymmetries for the decay $Z \to f \bar{f}$, $A_{LR}^f$ are numerically calculated. These values are tabulated in Table 3. For comparison, the values derived from Eq. (4.3) are shown as the tree-level values in the standard model. For neutrinos, $A_{LR}^\nu = 1$, where the difference $A_{LR}^\nu - 1$ is suppressed by large orders of magnitude. It is read from Table 3 that the values of $A_{LR}^f$ are close to the values of the standard model. On the other hand, the flavor universality is slightly violated except for $t$ quark. The non-universality between $A_{LR}^u$ and $A_{LR}^t$ is estimated as $A_{LR}^t/A_{LR}^u - 1 \sim -23\%$ for the set XZZ.

From the above values for $A_{LR}^f$ and the formula (4.1), the forward-backward asymmetries on the $Z$ resonance for $e^+e^- \to f \bar{f}$, $A_{FB}^f$ are calculated. The values of them are tabulated in Table 4. It is found that the tree-level predictions of $A_{FB}^e$, $A_{FB}^\mu$ and $A_{FB}^\tau$ are

| $f$ | Exp. | XZZ | Pull | FK | PDG | SM | Pull |
|-----|------|-----|------|----|-----|----|------|
| $u$ | —    | 0.07071 | 0.07071 | 0.07063 | 0.07500 |
| $c$ | 0.0707 ± 0.0035 | 0.07073 | 0.07073 | 0.07065 | 0.0738 | (-0.9) |
| $t$ | —    | 0.05425 | 0.05205 | 0.05431 | 0.10496 |
| $d$ | —    | 0.09950 | 0.09950 | 0.09939 | 0.1034 | (-0.5) |
| $s$ | 0.0976 ± 0.0114 | 0.09950 | -0.2 | 0.09950 | 0.09941 | (-2.5) |
| $b$ | 0.0992 ± 0.0016 | 0.09952 | -0.2 | 0.09952 | 0.1033 | (-2.5) |
| $e$ | 0.0145 ± 0.0025 | 0.01511 | -0.2 | 0.01511 | 0.01507 | 0.1677 | (0.01627) | (-0.7) |
| $\mu$ | 0.0169 ± 0.0013 | 0.01513 | 1.4 | 0.01513 | 0.01510 | (0.5) |
| $\tau$ | 0.0188 ± 0.0017 | 0.01515 | 2.1 | 0.01515 | 0.01511 | (1.5) |

Table 3: $A_{LR}^f$.

Table 4: $A_{FB}^f$. For comparison, the values derived with use of Eq. (4.3) are shown as the tree-level values in the standard model. In addition, the central values and the deviations of the standard model prediction given in Particle Data Group review [18] are shown in parentheses. For XZZ, the deviation between the prediction and the experimental value is denoted as Pull $\equiv [(\text{Central value})_{\text{Exp.}} - (\text{Prediction})]/(\text{Error})_{\text{Exp.}}$. Since $m_t > m_Z$, $A_{FB}^t$ should be interpreted as a mere reference value given in Eq. (4.1) with $A_{LR}^t$.
quite close to the central values of the experimental values given in Ref. [18]. This is a remarkable result of the present model, which is different from the standard model. As for the lepton sector, it seems that both of theory and experiment need improvements. The notable agreement in the quark sector is the case for all the three sets, XZZ, FK, PDG of fermion masses. That the $A^f_{\text{FB}}$ are not very sensitive to whether the fermion masses are running masses or physical masses is led to relaxing the dependence of the predictions on the values of $c$, $\mu_\alpha$, $\tilde{\mu}$. Hence, the model have made a realistic prediction with a moderate tuning as a whole.

The polarization asymmetry and the forward-backward asymmetry are not very sensitive to the values of input parameters for warped geometry. For a large warp factor $z_L = 1.0 \times 10^{17}$ and $k = 5.0 \times 10^{19}$GeV, $A^f_{\text{LR}}$ and $A^f_{\text{FB}}$ are given in Appendix [3]. For these input parameters, the predictions are also shown to be close to the central values of the experimental data.

5 Other electroweak quantities

In addition to the forward-backward asymmetry, the experimental measurement has been developed for other electroweak quantities. In this section, we present the tree-level prediction of the decay width of $Z$ boson and the $S$ and $T$ parameters. For various electroweak quantities, it has been proposed that there is a class of theory beyond the standard model which is applied to a global analysis [21]. Such an analysis may be useful when the treatment of the model with radiative corrections is as transparent as in the standard model. We will choose the values of the input parameters given in the previous section instead of searching a global fit. One reason is because the warp factor and the masses are fixed from the hierarchy and the experiments, respectively at the leading level. The other is because the parameter values for a global fit would be affected by radiative corrections. Our standpoint is that the tree-level analysis should be prioritized and that the quantitative results should be given.

5.1 Decay width

At tree level in the $SO(5) \times U(1)_X$ model, the decay width of $Z$ boson is given by

$$\Gamma(Z \rightarrow f \bar{f}) = \frac{m_Z \alpha L}{3 \sin^2 \theta_W \cos^2 \theta_W} \left[ \frac{(g^f_L)^2 + (g^f_R)^2}{2} + 2 g^f_L g^f_R \frac{m^2_l}{m^2_Z} \right] \sqrt{1 - \frac{4m^2_f}{m^2_Z}}. \quad (5.1)$$

where the couplings $g^f_L$ and $g^f_R$ have been used as in Eq. (4.2). The total width and the branching fraction are shown in Table 5. Table 5 includes the experimental values given in Particle Data Group data [18]. It is seen that the deviation of the tree level prediction from the experimental value is significantly large for leptonic decay modes and that fractions in the quark sector are not so different from the experimental data. This deviation in the lepton sector seems too large. We have find that a comparatively large deviation in the lepton sector arises also in the forward-backward asymmetry in the previous section and that $A^f_{\text{FB}}$, $A^f_{\text{FB}}$ and $A^f_{\text{FB}}$ in the quark sector are quite close to the central values of the experimental values.

On the other hand, the analysis here has been to estimate only the leading contribution. Since the lepton sector has the large flavor mixing, the mixing
Table 5: Z boson decay: the branching fraction and the total width. For the values of masses, the set XZZ is adopted and $\alpha = 1/128$. The invisible decay mode means the decay into $\nu_e\bar{\nu}_e + \nu_\mu\bar{\nu}_\mu + \nu_\tau\bar{\nu}_\tau$ in the model.

| $Z$ decay modes | Fraction ($\Gamma_i/\Gamma$) | Exp. | Pull |
|-----------------|------------------------------|------|------|
| $e^+e^-$ (%)    | 3.46173                      | 3.363 ± 0.004 | -25  |
| $\mu^+\mu^-$ (%)| 3.46097                      | 3.366 ± 0.007 | -14  |
| $\tau^+\tau^-$ (%)| 3.45545                     | 3.370 ± 0.008 | -11  |
| invisible (%)   | 20.5164                      | 20.00 ± 0.06 | -8.6 |

The effect might change the fraction in the lepton sector even at the tree level. This analysis will be left to future work.

5.2 $S$ and $T$ parameters

In electroweak physics, it is conventional to represent the effect of new physics for observables by the $S$ and $T$ parameters [23, 24] (see [25] for review). This is estimated for new physics after loop corrections in the standard model are taken into account. The general form of the Lagrangian associated with oblique corrections is $\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{new}}$, with $\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{SM}}(\vec{e}, \sin \theta_W, \vec{m}_Z, \vec{m}_W)$ and

$$\mathcal{L}_{\text{new}} = \frac{\Pi'_{\gamma\gamma}(0)}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\Pi'_{WW}(0)}{2} \tilde{W}_\mu^+ \tilde{W}^{-\mu} + \frac{\Pi'_{ZZ}(0)}{4} \tilde{Z}_\mu^+ \tilde{Z}^{-\mu} + \frac{\Pi'_{\gamma Z}(0)}{2} \tilde{F}_{\mu\nu} \tilde{Z}^{\mu\nu}$$

$$- \frac{\Pi_{WW}(0)}{2} \tilde{W}_\mu^+ \tilde{W}^{-\mu} - \frac{\Pi_{ZZ}(0)}{2} \tilde{Z}_\mu^+ \tilde{Z}^{-\mu}.$$  \hspace{1cm} (5.2)

where $\tilde{F}_{\mu\nu}$, $\tilde{Z}_\mu$ and $\tilde{W}_\mu^\pm$ are the field strengths for photon, Z boson and W boson, respectively and the quantities $\Pi'_{\gamma\gamma}(0)$, $\Pi'_{WW}(0)$, $\Pi'_{ZZ}(0)$, $\Pi_{WW}(0)$ and $\Pi_{ZZ}(0)$ are assumed to be small. For the Lagrangian $\mathcal{L}_{\text{eff}}$, the $S$ and $T$ parameters are given by

$$\alpha S = 4s^2 c^2 \left( \Pi'_{ZZ}(0) - \Pi'_{\gamma\gamma}(0) - \frac{c^2 - s^2}{c s} \Pi'_{\gamma Z}(0) \right), \hspace{1cm} \alpha T = \frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2},$$  \hspace{1cm} (5.3)

where $s = \sin \theta_W$ and $c = \cos \theta_W$. After the kinetic terms are canonically normalized, the gauge boson part of the effective Lagrangian becomes

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} W_\mu^+ W^{-\mu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - m_W^2 W_\mu^+ W^{-\mu} - \frac{1}{2} m_Z^2 Z_\mu^+ Z^{-\mu}$$

$$+ e \sum_i \tilde{f}_i \gamma^\mu Q_E f_i A_\mu + \left( \frac{\tilde{e}}{\sqrt{2}s} \left( 1 + \frac{\Pi_{WW}(0)}{2} \right) \sum_{ij} \tilde{V}_{ij} \tilde{f}_i \gamma^\mu P_L f_j W_\mu^+ + \text{h.c.} \right)$$

$$+ \frac{\tilde{e}}{s\tilde{c}} \left( 1 + \frac{\Pi_{ZZ}(0)}{2} \right) \sum_i \tilde{f}_i \gamma^\mu \left[ T^3_i P_L - Q_E s^2 + Q_E \tilde{s} c \Pi'_{\gamma Z}(0) \right] f_i Z_\mu.$$  \hspace{1cm} (5.4)
where \( T_i^3 \) is the eigenvalue of \( T_L^3 \) for a fermion \( f_i \). The physical masses and coupling are identified as \( m_W = (1 + \Pi_{WW}(0)/\bar{m}_W^2 + \Pi_{WW}(0)/2)\bar{m}_W, \) \( m_Z = (1 + \Pi_{ZZ}(0)/\bar{m}_Z^2 + \Pi_{ZZ}(0)/2)\bar{m}_Z \) and \( \epsilon = (1 + \Pi_{\gamma\gamma}(0)/2)\epsilon \). In Eq. (5.5), the effect of new physics is included in the charged and neutral currents. The Z boson coupling in Eq. (5.4) is written in terms of \( S \) and \( T \) as

\[
e(1 + \alpha T/2) \sum_i \bar{f}_i \gamma^\mu \left[ T_i^3 P_L - Q_E \left( s^2 + \frac{\alpha S}{4(c^2 - s^2)} - \frac{c^2 s^2 \alpha T}{c^2 - s^2} \right) \right] f_i Z^\mu, \tag{5.5}
\]

where \( s \) satisfies \( c^2/(s^2 c^2 m_Z^2) = \bar{c}^2(1 - \Pi_W W(0)/\bar{m}_W^2)/(\bar{c}^2 c^2 m_Z^2). \) If an effective Lagrangian is in the form of Eq. (5.4), the corresponding \( S \) and \( T \) can be estimated. The \( SO(5) \times U(1)_X \) model has gauge couplings of fermions different from the values in the standard model while the canonical kinetic terms are kept. We have found that this occurs at tree level and that this is the effect of extra-dimensional new physics. The point is that in the \( SO(5) \times U(1)_X \) model the gauge interactions are not universal with respect to the generation of fermions unlike Eq. (5.4). Due to this deviation, the tree-level currents in the \( SO(5) \times U(1)_X \) model may need some alternative parameters instead of the \( S \) and \( T \) parameters. However, we will not discuss this issue further. Our tree-level estimation is to find how large the values are if contributions of the \( S \) and \( T \) parameters rather than the alternative parameters were dominant for corrections to couplings. In other words, for the moment, we treat the case where corrections are characterized by the \( S \) and \( T \) parameters with a flavor fixed.

We formally estimate the tree-level values of the \( S \) and \( T \) parameters in the \( SO(5) \times U(1)_X \) model. For a fermion \( f \) with corrections dominated by the \( S \) and \( T \) parameters, comparing Eq. (5.5) with the Z couplings given in Section 3 yields

\[
\alpha S = -\frac{2}{Q_E} (c^2 - s^2)(g_L^f + g_R^f)\sqrt{L} - 8c^2 s^2
+ \frac{(g_L^f - g_R^f)\sqrt{L}}{T_f^3} \left[ \frac{2}{Q_E} (c^2 - s^2)(T_f^3 - 2Q_E s^2) + 8c^2 s^2 \right], \tag{5.6}
\]

\[
\alpha T = 2 \left( \frac{g_L^f - g_R^f}{T_f^3} - 1 \right), \tag{5.7}
\]

where \( Q_f \neq 0 \) and the couplings \( g_L^f \) and \( g_R^f \) have been employed as in Eq. (4.2). For electron, this evaluation leads to

\[
S(\text{electron}) = 2.2049, \quad T(\text{electron}) = 2.72176. \tag{5.8}
\]

The experimental data is \( S(\text{Exp.}) = -0.10 \pm 0.10 \) and \( T(\text{Exp.}) = -0.08 \pm 0.11 \) [18]. In the experimental constraint, loop corrections in the standard model are taken into account. The one-loop contribution in the standard model is given by

\[
S(\text{SM 1-loop}) = 0.247565, \quad T(\text{SM 1-loop}) = 1.25605, \tag{5.9}
\]

for \( m_h = 117 \text{GeV} \). The full equations for the \( S \) and \( T \) parameters in the standard model are summarized in Appendix [10]. The contribution (5.8) is large compared to the loop corrections in the standard model. We emphasize that Eq. (5.8) is a formal equation. The vacuum polarization in the gauge sector is expected to be flavor universal. It might
be proper to adopt the picture that tree-level corrections are flavor-violated and correspond to some alternative parameters except for \( S \) and \( T \) parameters and that one-loop vacuum polarization is the first contribution for \( S \) and \( T \) parameters. In such a case, the experimental constraint should be read for the alternative parameters as well as for the \( S \) and \( T \) parameters. Further investigation will be left to future work.

6 Summary and discussions

We have presented the tree-level prediction of the forward-backward production asymmetry on the Z resonance for quarks and leptons, \( A^f_{FB} \), in the \( SO(5) \times U(1)_X \) gauge-Higgs unification model given in Ref. [11]. It has been found that the tree-level prediction for \( b \) quark production gives \( A^b_{FB}(XZZ, FK) = 0.09952 \), which is quite close to the central value of the experimental data \( A^b_{FB}(\text{Exp.}) = 0.0992 \pm 0.0016 \). We have also found for \( c \) quark production \( A^c_{FB} = 0.07073(XZZ, FK) \) which is also close to the central value of the experimental data \( A^c_{FB}(\text{Exp.}) = 0.0707 \pm 0.0035 \). For all fermions, the tree-level predictions of \( A^f_{FB} \) have been given, and it has been shown that the values are not very sensitive to whether masses of quarks and leptons are taken as running masses or pole masses. We have also evaluated the \( Z \) decay width and the \( S \) and \( T \) parameters. As for these quantities, it has been shown that the effect of lepton mixing and the identification of relevant parameters are worth examining.

The small deviation from the experimental data is closely related to the left-right symmetry similar to the custodial symmetry in the standard model as shown in Ref. [11], according to a general discussion in Ref. [22], although a numerical analysis is required under the present understanding. The normalized coefficients of mode function for fermions are non-vanishing only for the part symmetric under the exchange of left and right isospin under the present understanding. The normalized coefficients of mode function for fermions according to a general discussion in Ref. [22], although a numerical analysis is required under the present understanding.

In such a case, the experimental constraint should be read for the alternative parameters as well as for the \( S \) and \( T \) parameters. Further investigation will be left to future work.

The scattering process \( e^+e^- \to f \bar{f} \) receives contributions from not only tree level but also quantum loop level. This must be treated appropriately. For example, one-loop corrections to couplings of heavy quarks to \( Z \) boson have been shown to be sizable in similar models [27]. In the standard model, radiative effects of heavy fields with masses much larger than \( m_Z \) are dominated by oblique corrections. The polarization asymmetry for the decay \( Z \to e^+e^- \) is corrected as

\[
A^e_{LR} = \frac{\left[ -\frac{1}{2} + s^2(q^2) \right]^2 - [s^2_0(q^2)]^2}{\left[ -\frac{1}{2} + s^2_0(q^2) \right]^2 + [s^2_0(q^2)]^2}, \tag{6.1}
\]

and the forward-backward asymmetry for \( b \) quark, for instance, is given by

\[
A^b_{FB} = 3 A^e_{LR} \frac{\left[ -\frac{1}{2} + \frac{1}{2}s^2_0(q^2) \right]^2 - [\frac{1}{2}s^2_0(q^2)]^2}{\left[ -\frac{1}{2} + \frac{1}{2}s^2_0(q^2) \right]^2 + [\frac{1}{2}s^2_0(q^2)]^2} \left( 1 - k_A \alpha_s / \pi \right), \tag{6.2}
\]

where QCD corrections are included in \( k_A \) and the strong coupling constant is \( \alpha_s \). In Eqs. (6.1) and (6.2), the weak mixing angle is replaced by the effective quantity,

\[
s^2_0(q^2) = \sin^2 \theta_0 + \frac{\alpha}{e^2 - s^2} \left( \frac{1}{4}S - s^2 c^2 T \right), \quad \sin 2\theta_0 \equiv \left( \frac{4\pi \alpha(m_Z)}{\sqrt{2}G_F m_Z^2} \right)^{1/2}, \tag{6.3}
\]
where the Fermi constant are denoted as $G_F$. When $\alpha(m_Z)$, $G_F$ and $m_Z$ are taken as input parameters, the effect of radiative corrections is accommodated in the $S$ and $T$ parameters which is rewritten in the standard model as

\begin{equation}
\alpha S = 4e^2 \left[ \Pi'_{3L3L}(0) - \Pi'_{1L,3L}(0) \right], \quad \alpha T = \frac{e^2}{s^2c^2m_Z^2} \left[ \Pi_{1L,1L}(0) - \Pi_{3L3L}(0) \right],
\end{equation}

where the vacuum polarizations $\Pi$ have the components for the electric charge $Q_E$ and the $SU(2)_L$ indices, $1_L, 2_L, 3_L$. The $S$ and $T$ parameters in the standard model are two finite linear combinations of vacuum polarizations. This finiteness is understood from a viewpoint that the symmetry of the theory should be recovered at large momentum, where $\Pi_{3L3L}|_{\text{div}} \sim \Pi_{1L,1L}|_{\text{div}}$ and $(\Pi_{3L3L}|_{\text{div}} - \Pi_{3L,3L}|_{\text{div}}) \sim q^2$-independent. In higher-dimensional theory, a strategy to extract radiative corrections would be to describe some observed quantities in terms of some other observed quantities as in the standard model. Then $A_{FB}$ should be described in terms of physical quantities such as running masses. Because the gauge-Higgs unification scenario is based on the gauge principle, it may be similar to identify finite combinations such as the $S$ and $T$ parameters, involving recovery of a symmetry at large momentum. Indeed, finite corrections of $S$ and $T$ parameters has been given in some extra-dimensional models [21]-[29]. Particularly, in an $SO(5) \times U(1)$-invariant formulation, the $S$ parameter seems to give a value much above the current experimental bounds for the Wilson line phase $\theta_H = \pm \pi/2$ [28]. The present model has $SO(4) \times U(1)$ multiplets on the Planck brane so that their results for the $S$ parameter cannot be directly applied to the present model. In addition, higher-dimensional theory includes couplings with negative mass dimension and a new feature appears differently in models with dimensionless parameters and masses. Even if there is only one interaction and usual kinetic energy terms, radiative corrections in higher-dimensional models lead to two point functions with multiple poles [30]. This make the treatment of loop corrections complicated. We leave investigation of these radiative corrections in the present $SO(5) \times U(1)_X$ model to future work.

While we have restricted our attention on the forward-backward asymmetry on $Z$ resonance, recently a large forward-backward asymmetry for $t$ quark has been observed [31]

\begin{equation}
A_{FB} = 0.193 \pm 0.065^{\text{stat.}} \pm 0.024^{\text{syst.}},
\end{equation}

at $\sqrt{s} = 1.96$TeV. There seems a discrepancy between this value and the standard model prediction. In this circumstance, there has been a possibility that the contributions of Kaluza-Klein excitations of gauge bosons account for the discrepancy in a warped extra-dimensional model [32]. Kaluza-Klein particles in the present $SO(5) \times U(1)_X$ model might also yield sizable contributions.

We have found that the violation of universality at the $Z$ boson vertices is crucial for obtaining the results concerning $A_{FB}$. If such a violation is arbitrarily large, flavor-changing neutral currents would also be large. In the present model, couplings of photon with fermions are just the normalization of the fermion wave functions and determined completely by the electric charges because the photon is a constant mode. At tree level, the only source of lepton-number violation is the mixing of the low-mass neutrinos. Then flavor-changing neutral current processes such as $\mu \rightarrow e\gamma$ are extremely small unobservable probabilities. This may also be affected by radiative corrections. In addition to the issue of the above radiative corrections, the model has to be examined in more detail.
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A Mode functions of Z boson and fermions

In this appendix, mode functions of Z boson and fermions and some related functions are summarized.

A.1 Z boson

The SO(5) gauge fields are split into classical and quantum parts \( A_M = A^c_M + A^q_M \), where \( A^c_M = 0 \) and \( A^q_y = (dz/dy)A^c_z = k z A^c_z \). With the gauge-fixing functional

\[
S_gf^{(A)} = z^2 \left\{ \eta^{\mu\nu} \mathcal{D}_\mu A^q_y + \xi k^2 z \mathcal{D}_z^c \left( \frac{1}{z} A^q_z \right) \right\},
\]

the quadratic action for the SO(5) gauge fields is

\[
S^\text{gauge}_{\text{bulk}} = \int d^4x \frac{dz}{k z} \left\{ \text{tr} \left\{ \eta^{\mu\nu} A^q_y (\Box + k^2 \mathcal{P}_4) A^q_y + k^2 A^q_z (\Box + k^2 \mathcal{P}_z) A^q_z \right\} \right\},
\]

for \( \xi = 1 \). Here \( A^c_\mu = 0 \) have been taken. The differential operators are \( \Box = \eta^{\mu\nu} \partial_\mu \partial_\nu, \mathcal{P}_4 = z \mathcal{D}_z (1/z) \mathcal{D}_z, \mathcal{P}_z = \mathcal{D}_z z \mathcal{D}_z (1/z) \), where \( \mathcal{D}_z A^c_N = \partial_M A^c_N - i g_A [A^c_M, A^c_N] \). The linearized equations of motion is

\[
\Box A^q_\mu + k^2 z \mathcal{D}_z \left( \frac{1}{z} \mathcal{D}_z A^q_\mu \right) = 0, \quad \Box A^q_z + k^2 \mathcal{D}_z z \mathcal{D}_z \left( \frac{1}{z} \mathcal{D}_z A^q_z \right) = 0. \tag{A.3}
\]

The SO(4) vector \( A^\theta_\mu \) which forms the \( SU(2)_L \)-doublet \( \Phi^\theta_H = (A^2_y + iA^3_y, A^3_y - iA^2_y) \) has zero modes. One can utilize the residual symmetry such that the zero mode of \( A^4_y \) yield a nonzero vacuum expectation value \( \langle A^\theta_y \rangle = v \delta^{a4} \). The Wilson line phase \( \theta_H \) is given by \( \exp \{ i \theta_H (2\sqrt{2}T^4) \} = \exp \{ i g_A \int dz \langle A_z \rangle \} \) so that \( \theta_H = \frac{1}{2} g_A v (z_L^2 - 1)/k \).

By a large gauge transformation which maintains the orbifold boundary conditions, \( \theta_H \) can be shifted to \( \theta_H + 2 \pi \). The gauge invariance of the theory implies that physics is periodic in \( \theta_H \) with a period \( 2 \pi \). With a gauge transformation, a new basis can be taken in which the background field vanishes, \( A^\theta_\mu = 0 \). The new gauge is called the twisted gauge as the boundary conditions are twisted. The new gauge potentials are related to the original ones by \( A_M = \Omega A_M^\theta \Omega^{-1}, B_M = B_M^\theta \). Here \( \Omega(z) = \exp \{ i \theta(z) \sqrt{2} T^4 \} \) and \( \theta(z) = \theta_H (z_L^2 - z^2)/(z_L^2 - 1) \). The equations of motion (A.3) become

\[
\Box \tilde{A}_\mu + k^2 \left( \partial^2 - \frac{1}{z} \partial_z \right) \tilde{A}_\mu = 0, \quad \Box \tilde{A}_z + k^2 \left( \partial^2 - \frac{1}{z} \partial_z + \frac{1}{z^2} \right) \tilde{A}_z = 0. \tag{A.4}
\]

The field \( \tilde{B}_M \) satisfies the same equations as \( \tilde{A}_M \).

The four-dimensional components of the SO(5) and U(1)_X gauge bosons contain W and Z bosons and photon as

\[
\tilde{A}_\mu(x, z) = W_\mu \left\{ h_W^L T^{-L} + h_W^R T^{-R} + h_W^c T^c \right\} + W^\dagger_\mu \left\{ h_W^L T^{+L} + h_W^R T^{+R} + h_W^c T^c \right\} + Z_\mu \left\{ h_Z^L T^{3L} + h_Z^R T^{3R} + h_Z^c T^c \right\} + A^\gamma_\mu \left\{ T^{3L} + T^{3R} \right\} + \cdots, \tag{A.5}
\]

\[
\tilde{B}_\mu(x, z) = Z_\mu h_Z^c + A^\gamma_\mu h^c_\gamma + \cdots. \tag{A.6}
\]

The wave functions \( h_W^L(z), h_W^R(z) \) and \( h_W^c(z) \) for \( W_\mu(x), h_W^L(z), h_W^R(z), h_Z^c(z) \) and \( h_Z^c(z) \) for \( Z(x) \) and \( h_\gamma(z) \) and \( h^c_\gamma(z) \) for \( A_\gamma(z) \) satisfy their own equations of motion and boundary
conditions. For the $Z$ boson tower, for instance, the boundary conditions at $z = 1$ is given as

\begin{align*}
0 &= s_\phi \left( \sin^2 \frac{\theta_H}{2} \partial_z h_L^Z + \cos^2 \frac{\theta_H}{2} \partial_z h_R^Z + \frac{1}{\sqrt{2}} \sin \theta_H \partial_z h_y^Z \right) + c_\phi \partial_z h_y^Z, \\
0 &= c_\phi \left( \sin^2 \frac{\theta_H}{2} h_L^Z + \cos^2 \frac{\theta_H}{2} h_R^Z + \frac{1}{\sqrt{2}} \sin \theta_H h_y^Z \right) - s_\phi h_y^Z.
\end{align*}

(A.7)  

(A.8)

The lowest mass modes for $W_\mu(x), Z_\mu(x)$ and $A_\gamma(x)$ are $W$ and $Z$ bosons and photon, respectively.

The wave functions for the bosons in the $Z$ boson tower are

\begin{align*}
Z_L^T(z) &= \frac{c_\phi^2 + \cos \theta_H (1 + s_\phi^2)}{2 \sqrt{1 + s_\phi^2}} N_Z(z; \lambda), \\
Z_R^T(z) &= \frac{c_\phi^2 - \cos \theta_H (1 + s_\phi^2)}{2 \sqrt{1 + s_\phi^2}} N_Z(z; \lambda), \\
Z_y^T(z) &= -\frac{\sin \theta_H}{\sqrt{2}} \left( 1 + s_\phi^2 \right) D_Z(z; \lambda), \\
Z_y^B(z) &= -\frac{s_\phi c_\phi}{\sqrt{1 + s_\phi^2}} N_Z(z; \lambda).
\end{align*}

(A.9)

Here $N_Z = 2b \sqrt{1 + s_\phi^2} C(z; \lambda)$ and $D_Z = 2b \sqrt{1 + s_\phi^2} (C(1; \lambda)/S(1; \lambda)) S(z; \lambda)$. The $C$ and $S$ functions are defined as

\begin{align*}
C(z; \lambda) &= \frac{\pi}{2} \lambda z L_1,0(\lambda z, \lambda z_L), \\
C'(z; \lambda) &= \frac{\pi}{2} \lambda^2 z L_0,0(\lambda z, \lambda z_L), \\
S(z; \lambda) &= -\frac{\pi}{2} \lambda z L_1,1(\lambda z, \lambda z_L), \\
S'(z; \lambda) &= -\frac{\pi}{2} \lambda^2 z L_0,1(\lambda z, \lambda z_L).
\end{align*}

(A.10)  

(A.11)

where a useful linear combination of Bessel functions is defined as

\begin{equation}
F_{\alpha,\beta}(u, v) = J_\alpha(u) Y_\beta(v) - Y_\alpha(u) J_\beta(v),
\end{equation}

(A.12)

and the prime denotes such as $C' = dC/dz$. A relation $CS' - SC' = \lambda z$ holds. From the normalization \( \int_1^{z_L} dz (kz)^{-1} \{ (h_L^Z)^2 + (h_R^Z)^2 + (h_y^Z)^2 + (h_y^B)^2 \} = 1 \) the coefficient $b$ is determined to be

\begin{equation}
b^{-2} = \int_1^{z_L} \frac{dz}{kz} 2(1 + s_\phi^2)^2 \left[ \frac{2}{1 + s_\phi^2} (C(z; \lambda))^2 \right.
\end{equation}

\begin{equation}
- \sin^2 \theta_H \left( (C(z; \lambda))^2 - \left( \frac{C(1; \lambda)}{S(1; \lambda)} \right)^2 (S(z; \lambda))^2 \right) \right].
\end{equation}

(A.13)

The equation (A.8) is automatically fulfilled with Eq. (A.9).

The mass spectrum of the $Z$ tower is determined by

\begin{equation}
2S(1; \lambda) C'(1; \lambda) + \lambda (1 + s_\phi^2) \sin^2 \theta_H = 0.
\end{equation}

(A.14)

The other boundary condition (A.7) is fulfilled with Eq. (A.14). The mass of the lightest mode, the $Z$ boson, is given by

\begin{equation}
m_Z \approx \frac{m_W}{\cos \theta_W}, \quad m_W \approx \frac{m_{KK}}{\pi \sqrt{kL}} | \sin \theta_H |,
\end{equation}

(A.15)

where $m_{KK} \approx \pi ke^{-kL}$.
A.2 Fermions

In terms of the rescaled fields $\tilde{\Psi}_a = z^{-2}\Omega(z)\Psi_a$ in the twisted gauge where

$$\Omega(z) = \begin{pmatrix} 1 & \cos \theta(z) & \sin \theta(z) \\ -\sin \theta(z) & \cos \theta(z) & 0 \end{pmatrix}, \quad \theta(z) = \frac{z^2 - z^2}{z^2 L - 1}, \quad (A.16)$$

the action for the fermions in the bulk region becomes

$$S^{\text{fermion}}_{\text{bulk}} = \sum_a \int d^4x \frac{dz}{k} i\tilde{\Psi}_a \left\{ \Gamma^\mu (\partial_\mu - ig_A A_\mu - ig_B Q_{Xa} \tilde{B}_\mu) + \Gamma^5 \sigma^i (\partial_z - ig_A \tilde{A}_z - ig_B Q_{Xa} \tilde{B}_z) - \frac{\sigma^0}{z} \right\} \tilde{\Psi}_a, \quad (A.17)$$

If there were no brane interactions, it would obey

$$\left\{ \left( \tilde{\sigma} \cdot \partial - k \begin{pmatrix} D_-(c_a) & D_+(c_a) \end{pmatrix} \right) \begin{pmatrix} \tilde{\Psi}_{aR} \\ \tilde{\Psi}_{aL} \end{pmatrix} \right\} = 0, \quad (A.18)$$

where $D_\pm(c) = \pm(d/dz) + (c/z)$. Neumann conditions for $\tilde{\Psi}_R$ and $\tilde{\Psi}_L$ are given by $D_-(c)\tilde{\Psi}_R = 0$ and $D_+(c)\tilde{\Psi}_L = 0$, respectively. The resulting second order differential equations are

$$\{\partial^2 - k^2 D_-(c)D_+(c)\} \tilde{\Psi}_{aL} = 0, \quad \{\partial^2 - k^2 D_+(c)D_-(c)\} \tilde{\Psi}_{aR} = 0. \quad (A.19)$$

The basic functions are given by

$$\begin{pmatrix} C_L \\ S_L \end{pmatrix}(z;\lambda,c) = \frac{\pm \pi}{2} \lambda \sqrt{z z_L} F_{c+\frac{1}{2},c+\frac{1}{2}}(\lambda z, \lambda z_L), \quad (A.20)$$

$$\begin{pmatrix} C_R \\ S_R \end{pmatrix}(z;\lambda,c) = \frac{\mp \pi}{2} \lambda \sqrt{z z_L} F_{c-\frac{1}{2},c-\frac{1}{2}}(\lambda z, \lambda z_L). \quad (A.21)$$

They satisfy

$$\{ D_+(c)D_-(c) - \lambda^2 \} \begin{pmatrix} C_R(z) \\ S_R(z) \end{pmatrix} = 0, \quad \{ D_-(c)D_+(c) - \lambda^2 \} \begin{pmatrix} C_L(z) \\ S_L(z) \end{pmatrix} = 0, \quad (A.22)$$

They satisfy the boundary conditions that $C_R = C_L = 1$, $D_-C_R = D_+C_L = 0$, $S_R = S_L = 0$ and $D_-S_R = D_+S_L = \lambda$ at $z = z_L$. Further $D_\pm$ links $L$ and $R$ functions by $D_+(C_L,S_L) = \lambda(S_R,C_R)$ and $D_-(C_R,S_R) = \lambda(S_L,C_L)$.

**Top quark**

In the quark sector we chose $c_1 = c_2 = c$. The top quark component $t(x)$ in four dimensions is contained in the form

$$\begin{pmatrix} \tilde{U}_L \\ (\tilde{B}_L \pm \tilde{t}_L)/\sqrt{2} \end{pmatrix}(x,z) = \sqrt{k} \begin{pmatrix} a_U C_L(z;\lambda,c) \\ a_U S_L(z;\lambda,c) \end{pmatrix} t_L(x), \quad (A.23)$$

$$\begin{pmatrix} \tilde{U}_R \\ (\tilde{B}_R \pm \tilde{t}_R)/\sqrt{2} \end{pmatrix}(x,z) = \sqrt{k} \begin{pmatrix} a_U S_R(z;\lambda,c) \\ a_U C_R(z;\lambda,c) \end{pmatrix} t_R(x), \quad (A.24)$$
of the coefficients $a$ are described in a similar way. We suppose that the scale of brane masses in much larger than the Kaluza-Klein scale: $\mu_1^2, \mu_2^2, \mu_3^2, \tilde{\mu}^2 \gg m_{\text{KK}}$. Then the ratios of the coefficients $a$ are given by

$$[a_U, a_{B-t}, a_{t'}] \simeq \begin{bmatrix} -\sqrt{2}\tilde{\mu} \\ \mu_2 \\ -c_H, -\frac{s_H C_L|_{z=1}}{S_L|_{z=1}} \end{bmatrix} a_{B+t}.$$  (A.25)

Here $c_H = \cos \theta_H$ and $s_H = \sin \theta_H$. The coefficient $a_{B+t}$ is determined by

$$a_{B+t}^{-1} = \int_1^{z_L} dz \left\{ 2 \left( \frac{\tilde{\mu}}{\mu_2} \right)^2 + 1 + c_H^2 \right\} \left(C_L(z) \right)^2 + s_H^2 \left( \frac{C_L|_{z=1}}{S_L|_{z=1}} \right)^2 \left(S_L(z) \right)^2 \right\}. \quad \text{(A.26)}$$

The top quark mass $m_t = k\lambda_t$ obeys

$$\tilde{\mu}^2 S_R C_L + \mu_2^2 C_L \left\{ S_R + \frac{s_H^2}{2S_L} \right\} |_{z=1, \lambda=\lambda_t} = 0. \quad \text{(A.27)}$$

This equation includes the ratio $\tilde{\mu}/\mu_2$ and $c$ as parameters. There is the corresponding equation for bottom quark mass $m_b$ which includes the same parameters.

**Bottom quark**

The bottom quark component $b(x)$ is contained in the form

$$\left( \frac{1}{\sqrt{2}}(\tilde{D}_L \pm \tilde{X}_L) \right)_{x, z} = \sqrt{k} \begin{bmatrix} a_b C_L(z; \lambda, c_1) \\ a_{D+X} C_L(z; \lambda, c_2) \\ a_b S_L(z; \lambda, c_2) \end{bmatrix} b_L(x), \quad \text{(A.28)}$$

$$\left( \frac{1}{\sqrt{2}}(\tilde{D}_R \pm \tilde{X}_R) \right)_{x, z} = \sqrt{k} \begin{bmatrix} a_b S_R(z; \lambda, c_1) \\ a_{D+X} S_R(z; \lambda, c_2) \\ a_b C_R(z; \lambda, c_2) \end{bmatrix} b_R(x). \quad \text{(A.29)}$$

The $d$ and $s$ quarks are described in a similar manner. The ratios of the coefficients are given by

$$[a_b, a_{D-X}, a_{t'}] = \left[ -\frac{\sqrt{2}\mu_2}{\tilde{\mu}}, c_H, -\frac{s_H C_L|_{z=1}}{S_L|_{z=1}} \right] a_{D+X}. \quad \text{(A.30)}$$

The coefficient $a_{D+X}$ is given by

$$a_{D+X}^{-2} = \int_1^{z_L} dz \left\{ 2 \left( \frac{\mu_2}{\tilde{\mu}} \right)^2 + 1 + c_H^2 \right\} \left(C_L(z) \right)^2 + s_H^2 \left( \frac{C_L|_{z=1}}{S_L|_{z=1}} \right)^2 \left(S_L(z) \right)^2 \right\}. \quad \text{(A.31)}$$

The mass $m_b = k\lambda_b$ obeys

$$\mu_2^2 S_R C_L + \tilde{\mu}^2 C_L \left\{ S_R + \frac{s_H^2}{2S_L} \right\} |_{z=1, \lambda=\lambda_b} = 0. \quad \text{(A.32)}$$

Combining Eqs. (A.27) and (A.32), one finds

$$\frac{\tilde{\mu}^2}{\mu_2^2} = -\left\{ 1 + \frac{s_H^2}{2S_L(1; \lambda_b, c)S_R(1; \lambda_b, c)} \right\}^{-1}.$$  \quad \text{(A.33)}
The value of $\theta_H$ is dynamically determined. In the present model $\theta_H = \pm \frac{1}{2} \pi$. Hence, given $k$, $m_t$, $m_b$, the parameters $c$ and $|\bar{\mu}/\mu_2|$ are determined. These values are not very sensitive on the value of $k$.

The wave functions $C_L$ and $S_L$ for the left-handed quarks $t_L$, $b_L$ are localized near the Planck brane, whereas $C_R$ and $S_R$ for the right-handed quarks $t_R$, $b_R$ are localized near the TeV brane.

**Tau lepton and tau neutrino**

We need to introduce only a vector multiplet $\Psi_3$ in Eqs. (2.16) and (2.17) to describe a massless neutrino for each generation. The $\tau$ lepton is contained in the form

$$\left( \frac{1}{\sqrt{2}} (\tilde{\tau}_L \pm \tilde{L}_{1XL}) \right) (x, z) = \sqrt{k} \left( \frac{a_{\tau+L_{1x}} C_L(z; \lambda, c_3)}{a_{\tau} S_L(z; \lambda, c_3)} \right) \tau_L(x),$$

$$\left( \frac{1}{\sqrt{2}} (\tilde{\tau}_R \pm \tilde{L}_{1XR}) \right) (x, z) = \sqrt{k} \left( \frac{a_{\tau+L_{1x}} S_R(z; \lambda, c_3)}{a_{\tau} C_R(z; \lambda, c_3)} \right) \tau_R(x). \tag{A.34}$$

The $e$ and $\mu$ leptons are described in a similar way. The ratios of the coefficients are given by

$$[a_{\tau-L_{1x}}, a_{\tau}] \simeq \left[ c_H, \frac{s_H C_L(z = 1)}{S_L(z = 1)} \right] a_{\tau+L_{1x}}, \tag{A.35}$$

for $(\mu_1^\ell)^2 \gg m_{\text{KK}}$. The coefficient $a_{\tau+L_{1x}}$ is given by

$$a_{\tau+L_{1x}}^2 = \int_{1}^{z_L} dz \left\{ (1 + c_H^2) (C_L(z))^2 + s_H^2 \left( \frac{C_L(z = 1)}{S_L(z = 1)} \right)^2 (S_L(z))^2 \right\}. \tag{A.36}$$

The mass $m_{\tau} = k \lambda_{\tau}$ is determined by

$$(\mu_1^\ell)^2 C_L \left( S_R + \frac{s_H^2}{2 S_L} \right) \bigg|_{z=1} = 0. \tag{A.37}$$

The $\nu_\tau$ neutrino is contained in the form

$$\tilde{\nu}_{\tau L}(x, z) = \sqrt{k} a_{\nu_{\tau}} C_L(z; 0, c) \nu_{\tau L}(x). \tag{A.38}$$

The $\nu_e$ and $\nu_\mu$ neutrinos are described in a similar way. For massless fermions, the $C$ function becomes $C_L(z; 0, c) = (z_L/z)^c$. The coefficient is written as a simple equation $a_{\nu_{\tau}} = \sqrt{(2c-1)/(z_L^c - z_L)}$.

To describe massive neutrinos one needs to introduce two multiplets $\Psi_3$ and $\Psi_4$. The structure is the same as in the quark sector. We choose $c_3 = c_4 = c$. Equations in the lepton sector are obtained with the correspondence between leptons and quarks:

$$(\nu_\tau, L_{2Y}, L_{3X}, \nu_\ell) \leftrightarrow (U, B, t, t'), \quad (\hat{L}_{3X}, \hat{L}_{2Y}) \leftrightarrow (\hat{U}, \hat{B}, \hat{R}),$$

$$(L_{3Y}, \tau, L_{1X}, \tau') \leftrightarrow (b, D, X, b'), \quad (\hat{L}_{3Y}, \hat{L}_{1X}) \leftrightarrow (\hat{D}, \hat{X}, \hat{R}),$$

$$(\mu_1^\ell, \mu_2^\ell, \mu_3^\ell, \mu_4^\ell) \leftrightarrow (\mu_3, \mu_1, \bar{\mu}, \mu_2). \tag{A.39}$$

The behavior of localization for wave functions in the lepton sector is similar to that in the quark sector.
B  The asymmetries for a large warp factor

In this appendix, we take \( z_L = 1.0 \times 10^{17} \) and \( k = 5.0 \times 10^{19} \text{GeV} \) as input parameters for warped geometry. For these values of \( z_L \) and \( k \), the Kaluza-Klein scale is given by \( m_{\text{KK}} = 1.57 \text{TeV} \). For the set XZZ, the polarization asymmetry \( A^{f}_{LR} \) and the forward-backward asymmetry \( A^{f}_{FB} \) are shown in Table 6. For this large warp factor, the tree-level predictions of the model are also close to the central values of the experimental data.

\[
\begin{array}{cccc}
 f & A^{f}_{LR} & A^{f}_{FB} & \text{Pull} \\
 u & 0.6649 & 0.07124 & \\
 c & 0.6650 & 0.07125 & -0.2 \\
 t & 0.5310 & 0.05690 & \\
 d & 0.9349 & 0.10017 & \\
 s & 0.9349 & 0.10018 & -0.2 \\
 b & 0.9351 & 0.10019 & -0.6 \\
 e & 0.1429 & 0.01531 & -0.3 \\
 \mu & 0.1431 & 0.01533 & 1.2 \\
 \tau & 0.1432 & 0.01534 & 2.0 \\
\end{array}
\]

C  The values of \( S \) and \( T \) parameters at one-loop in the standard model.

In this appendix, the equations for one-loop corrections for the \( S \) and \( T \) parameters in the standard model are summarized [33]-[35].

Each vacuum polarization diagram includes logarithmic divergence

\[
E = \frac{2}{\epsilon} - \gamma + \log \left( \frac{4\pi}{M^2} \right). \tag{C.1}
\]

The other part can be described with Feynman parameter integrals

\[
b_0(12X) = \int_0^1 dx \log \left( \frac{\Delta(M_1^2, M_2^2, q_X^2)}{M^2} \right), \tag{C.2}
\]

\[
b_1(12X) = \int_0^1 dx \, x \log \left( \frac{\Delta(M_1^2, M_2^2, q_X^2)}{M^2} \right), \tag{C.3}
\]

\[
b_2(12X) = \int_0^1 dx \, x(1-x) \log \left( \frac{\Delta(M_1^2, M_2^2, q_X^2)}{M^2} \right), \tag{C.4}
\]

where \( q_q^2 \equiv q^2 \) and \( \Delta(M_1^2, M_2^2, q^2) = xM_2^2 + (1-x)M_1^2 + x(1-x)q^2 \).

The top and bottom loops contribute

\[
S_{tb} = -\frac{1}{2\pi} \left( 1 + \frac{1}{3} \log \frac{m_b^2}{m_t^2} \right), \tag{C.5}
\]

\[
T_{tb} = \frac{3}{8\pi s^2 c^2 m_Z^2} \left( \frac{m_t^2 m_b^2}{m_t^2 - m_b^2} \log \frac{m_b^2}{m_t^2} + \frac{m_t^2 + m_b^2}{2} \right). \tag{C.6}
\]
The Higgs boson loops contribute
\[
S_h = \frac{1}{\pi} \left\{ E \left( \frac{-1}{12} \right) - \frac{1}{4} \left( m_Z^2 - m_h^2 \right) [2b'_1(hZ0) - b'_0(hZ0)] - \frac{1}{4} \left[ 4b_2(hZ0) - b_0(hZ0) \right] - m_Z^2 b'_0(hZ0) \right\}, \tag{C.7}
\]
\[
T_h = \frac{1}{4\pi s^2 c^2 m_Z^2} \left\{ E(m_W^2 - m_Z^2) - \frac{1}{4} \left[ 3m_W^2 m_h^2 \log \frac{m_W^2}{m_h^2} + 4m_W^2 \log \frac{m_W^2}{M^2} \right] - \frac{3m_Z^2 m_h^2}{m_Z^2 - m_h^2} \log \frac{m_Z^2}{m_h^2} - 4m_Z^2 \log \frac{m_Z^2}{M^2} - \frac{7}{2} \left( m_W^2 - m_Z^2 \right) \right\}, \tag{C.8}
\]
which depend on \( E \) and \( M \). The sum of \( S_h \) and the contribution to \( S \) from gauge boson loops is independent of \( E \) and \( M \). Also the sum of \( T_h \) and the contribution to \( T \) from gauge boson loops is independent of \( E \) and \( M \). Here
\[
b'_0(hZ0) = \frac{1}{m_Z^2 - m_h^2} \left[ \frac{1}{2} + \frac{m_Z^2}{m_Z^2 - m_h^2} - \frac{m_Z^2 m_h^2}{(m_Z^2 - m_h^2)^2} \log \frac{m_Z^2}{m_h^2} \right], \tag{C.9}
\]
\[
b'_1(hZ0) = \frac{1}{m_Z^2 - m_h^2} \left[ -\frac{1}{3} + \frac{m_Z^2}{2(m_Z^2 - m_h^2)} - \frac{m_Z^2 m_h^2}{(m_Z^2 - m_h^2)^2} + \frac{m_Z^4}{(m_Z^2 - m_h^2)^3} \log \frac{m_Z^2}{m_h^2} \right],
\]
\[
b_0(hZ0) = -1 + \log \frac{m_Z^2}{M^2} + \frac{m_Z^2}{m_Z^2 - m_h^2} \log \frac{m_Z^2}{m_h^2},
\]
\[
b_2(hZ0) = \frac{1}{6} \log \frac{m_Z^2}{M^2} - \frac{5}{36} + \frac{m_h^2}{3(m_Z^2 - m_h^2)} - \frac{m_h^4}{2(m_Z^2 - m_h^2)^2} \log \frac{m_Z^2}{m_h^2} + \frac{m_h^6}{3(m_Z^2 - m_h^2)^3} \log \frac{m_Z^2}{m_h^2}. \tag{C.9}
\]
The gauge boson loops contribute
\[
S_g = \frac{1}{\pi} \left\{ E \left( \frac{1}{12} \right) + (b_2 - \frac{1}{4} b_0)(WW0) \right\}, \tag{C.10}
\]
\[
T_g = \frac{1}{4\pi s^2 c^2 m_Z^2} \left\{ E(m_W^2 - m_Z^2) + (2c^2 + \frac{1}{4})(m_W^2 - m_Z^2)(2b_1 - b_0)(WZ0) - \frac{1}{2} \left( m_Z^2 - 3m_W^2 \right) b_0(WZ0) - 2m_W^2 b_0(WW0) + 2s^2 m_W^2 (2b_1 - b_0)(W00) \right\}, \tag{C.11}
\]
Here
\[
b_0(WW0) = \log \frac{m_W^2}{M^2}, \tag{C.12}
\]
\[
b_2(WW0) = \frac{1}{6} \log \frac{m_W^2}{M^2}, \tag{C.13}
\]
\[
b_0(WZ0) = \log \frac{m_Z^2}{M^2} - 1 + \frac{m_W^2}{m_Z^2 - m_W^2} \log \frac{m_Z^2}{m_W^2}, \tag{C.14}
\]
\[
b_1(WZ0) = \frac{1}{2} \log \frac{m_Z^2}{M^2} - \frac{1}{4} + \frac{1}{2} \frac{m_W^2}{m_Z^2 - m_W^2} - \frac{1}{2} \frac{m_W^4}{(m_Z^2 - m_W^2)^2} \log \frac{m_Z^2}{m_W^2}, \tag{C.15}
\]
\[
b_0(W00) = \log \frac{m_W^2}{M^2} - 1, \tag{C.16}
\]
\[
b_1(W00) = \frac{1}{2} \log \frac{m_W^2}{M^2} - \frac{3}{4}. \tag{C.17}
\]
For the input values given in Section 4, the $S$ and $T$ parameters become

\begin{align*}
S_{tb} &= 0.2742, \quad T_{tb} = 1.1853, \quad \text{(C.18)} \\
S_h + S_g &= -0.02666, \quad T_h + T_g = 0.07076, \quad \text{for } m_h = 117\text{GeV}, \quad \text{(C.19)} \\
S_h + S_g &= 0.04866, \quad T_h + T_g = 0.1701, \quad \text{for } m_h = 340\text{GeV}, \quad \text{(C.20)} \\
S_h + S_g &= 0.1108, \quad T_h + T_g = 0.3121, \quad \text{for } m_h = 1000\text{GeV}, \quad \text{(C.21)}
\end{align*}

where the Higgs boson mass in the standard model has been taken as an input parameter. The total contributions to the $S$ and $T$ parameters, $S_{tb} + S_h + S_g$ and $T_{tb} + T_h + T_g$, are

\begin{align*}
S(\text{SM 1-loop}) &= 0.2476, \quad T(\text{SM 1-loop}) = 1.2561, \quad \text{for } m_h = 117\text{GeV}, \quad \text{(C.22)} \\
S(\text{SM 1-loop}) &= 0.3229, \quad T(\text{SM 1-loop}) = 1.3554, \quad \text{for } m_h = 340\text{GeV}, \quad \text{(C.23)} \\
S(\text{SM 1-loop}) &= 0.3850, \quad T(\text{SM 1-loop}) = 1.4974, \quad \text{for } m_h = 1000\text{GeV} \quad \text{(C.24)}
\end{align*}
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