Investigation of the volumetric fractal structure shaping

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Abstract. On the basis of a specially developed algorithm, shaping of the fractal structures, the formation of which is regulated by the power index of the fractal, has been carried out. A sphere and an ellipsoid were used as the basic shapes. The formation analysis has made it possible to determine the characteristic principles of the fractal structure growth. The performed numerical experiment with respect to volumetric fractal structures of power 8 made it possible to determine the spectrum of frequencies and forms of natural oscillations. The offered objects of fractal architecture are analogs of unique high-rise and large-span buildings and structures. The application of the fractal shaping paradigm in combination with digital technologies and innovative materials will make it possible to create completely new objects of architecture.

Introduction
The variability in buildings and structures design expands significantly with the fractal shaping paradigm introduction. Fractal geometry is a comparative new theory, widely developed through the use of modern digital technologies. Volumetric fractal develops in space within the limits of mathematical equations and algorithms, is in constant motion and tends simultaneously to order and chaos [1-3].

Fractal structures can be used in the construction industry as a shell, a building frame, or as separate or groups of structures [4]. A feature of the fractal geometry use in the unique buildings and structures’ design is the need to use innovative materials and technologies that allow the complex structures implementation [5]. According to the authors, the most rational technology for creating volumetric fractal structures is 3D printing technology. The development of 3D printing makes it possible to create the objects that differ in unique geometry, properties and material [6-7].

The object of the research is the Mandelbrot volumetric fractal. The modeling of a three-dimensional Mandelbrot fractal and the study of the external fractal structure formation has been performed in this work.

Research methods and principles
3D Mandelbrot fractal is an analogue of the Mandelbrot set, created by Daniel White and Paul Nielander using hypercomplex algebra based on spherical coordinates. The study uses the concept of the power of a fractal or quasi-volume. The fractal power is taken as an analogue of the local dimension of the fractal set. For the power greater than three, the result is a three-dimensional Mandelbrot fractal [8].

Modeling a volumetric Mandelbrot fractal is carried out using a special program "External structure of a fractal set", developed by the authors of the study [9].

The purpose of the algorithm implemented in the program is to determine the points belonging to the outer surface of the Mandelbrot fractal with the subsequent construction of a finite element model in the PC SCAD.
Determination of the points' coordinates is carried out by checking their belonging to the surface of the fractal structure after a given number of iterations or generations. Points membership is checked in spherical coordinates, changing cyclically the distance from the center, then the horizontal and vertical angles. The result of the calculations is the coordinates of the points and sets of finite elements that form the external structure of the fractal set. There is a rational distribution of the points' density over the surface to create the most optimal mesh of finite elements.

To obtain a rational external fractal structure, it is necessary to correctly specify the initial data when calculating the shaping in the program "External structure of a fractal set". The initial data affect the quality of the finite element mesh and, accordingly, the accuracy of the stress-strain state calculations.

Modeling of fractal structures is implemented taking into account the introduction of 3D printing technology into the process. 3D printers create real objects based on a computer volumetric model using specialized methodologies. The advantages of 3D printers in construction are the low cost of the manufactured structure, efficiency and quality, reduced labor costs and reduced waste. A distinctive feature is the ability to erect the buildings with a unique frame in a short time. The development of robotics and the introduction of 3D printers that use innovative materials will contribute to the fractal architecture concepts' implementation.

Results
Complex modeling of the endostructure and exostructure of a fractal represents a new stage in the development of parametric design of fractal architecture objects. The proposed object of fractal architecture is not only based on a fractal form, but also repeats the natural algorithms.

Modeling of different forms of a fractal structure was carried out in the program "External structure of a fractal set" with the transfer of topological data to the PC SCAD. Models of the external fractal structure of the fourth, eighth and fourteenth powers with identical initial parameters have been developed (Figure 1). It should be noted that the fractal structure of the first power is a sphere that transforms into new types of structure with an increase in fractal power.

![Figure 1. Volumetric Mandelbrot fractal: a) power 4; b) power 8; c) power 14.](image)

The external structure of a bulk fractal includes an exobase, an f-quark surface, and a corona [10]. According to the authors, the fractal structure acquires its original and familiar form with a power value of 8. External structures of low power differ significantly from each other and have little in common with the classical form of the Mandelbrot volumetric fractal. Power 4 structure (Figure 1 (a)) has fundamental differences from power 8 and 14 structures (Figure 1 (b) and 1 (c), respectively). Low-power structures are characterized by shaping asymmetry. The higher powers of the fractal are distinguished by the symmetry of the forms.

During the study, it was found that the corona and exobase become more detailed with increasing power, but at the same time they occupy a smaller volume of the entire volume fractal [10]. The surface of f-quarks becomes pronounced, but with increasing power it occupies an ever-larger volume of the structure.
The program "External structure of a fractal set" makes it possible to change the structure of a three-dimensional Mandelbrot fractal by pulling it in different directions. For a more detailed study of the shaping, the developed program provides for a change in the initial parameters, which makes it possible to take the rotation ellipsoid as the first power. For the first time, the structures that are visually comparable with the concept of shaping of high-rise unique buildings, skyscrapers have been obtained.

Figure 2 shows the examples of fractal structures of 8 and 14 powers with development along the vertical axis. In the process of shaping, the relationship between the power value and the external structure components’ detail remains.

As part of the fractal structure formation study, a modal analysis of the exostructures of the power 8 fractal was performed in the SCAD software package (Figure 1 (b), Figure 2 (a)). It should be noted that the free oscillations spectrum gives an idea of the possible ways of the structure deformation in case when the structure would be a shell of a building or structure. Modal analysis gives an opportunity to establish resonant frequencies corresponding to the most dangerous bending-torsional modes of natural oscillations.

In a numerical experiment, a frequency spectrum with twenty modes of natural oscillations was obtained. The results are summarized in Table 1.

Table 1. Forms and frequencies of natural oscillations.

| Spherical Mandelbrot fractal, power 8 | Mandelbrot ellipsoidal fractal, power 8 |
|--------------------------------------|----------------------------------------|
| **The form, No.**                   | **Frequency Hz**                       | **The form, No.**                   | **Frequency Hz** |
| 1                                    | 0.0993663                              | 1                                    | 0.120958         |
| 2                                    | 0.0993663                              | 2                                    | 0.121018         |
| 3                                    | 0.215402                               | 3                                    | 0.299469         |
| 4                                    | 0.215402                               | 4                                    | 0.299624         |
| 5                                    | 0.224461                               | 5                                    | 0.301273         |
| 6                                    | 0.224461                               | 6                                    | 0.301327         |
| 7                                    | 0.290477                               | 7                                    | 0.30596          |
| 8                                    | 0.290477                               | 8                                    | 0.305993         |
| 9                                    | 0.362324                               | 9                                    | 0.353494         |

Figure 2. Mandelbrot ellipsoidal fractal: a) power 8; b) power 14.
It should be noted that for a volumetric fractal power 8 structure, the first two modes of oscillation are translational, the third is torsional (Figure 3, Figure 4).

|   |   |   |   |
|---|---|---|---|
| 10 | 0.48646 | 10 | 0.66468 |
| 11 | 0.498527 | 11 | 0.664875 |
| 12 | 0.498527 | 12 | 0.82499 |
| 13 | 0.547859 | 13 | 0.825148 |
| 14 | 0.547859 | 14 | 0.873489 |
| 15 | 0.762812 | 15 | 0.906053 |
| 16 | 0.762812 | 16 | 0.906104 |
| 17 | 0.769759 | 17 | 0.928005 |
| 18 | 0.781128 | 18 | 0.928083 |
| 19 | 0.78113 | 19 | 1.032162 |
| 20 | 0.874072 | 20 | 1.032211 |

**Figure 3.** Mandelbrot spherical fractal a) initial structure; b) the first form of oscillation; c) the fourth mode of oscillation.

The frequencies of the Mandelbrot ellipsoidal fractal are higher than the frequencies of the spherical fractal of the corresponding oscillation modes. In this case, the modes of natural oscillations from 3 to 8 are characterized by significant displacements of the structure in the middle zone of the f-quark surface in different directions. In most cases, the structure corona movements are minimal or equal to zero.

**Figure 4.** Mandelbrot ellipsoid fractal a) initial structure; b) the first form of oscillation; c) the fourth form of oscillation.
Discussion
The external fractal structure has features of existence and behavior, i.e., the fractals grow in a special way depending on the power and the iterative process. These features make it possible to form the structures of any type on the basis of a fractal structure, both long-span and high-rise. The unique buildings obtained as a result of shaping will have fractal properties, low weight, taking into account the use of innovative materials and create a comfortable interior space.

The design methodology for buildings and structures of fractal architecture can be significantly transformed with the help of mathematics, algorithms and digital technologies [11]. Application of the fractal shaping paradigm will make it possible to create completely new architectural objects that can be distinguished by their reliability and resistance to natural and man-made influences, aesthetics and a high degree of comfort of the endostructure.

Collaboration of fractal geometry, digital technologies and innovative materials will allow the construction industry to move to a completely new development level.

Summary
Models of volumetric fractal structures, which are the basis for further numerical experiments to study the stress-strain state of unique buildings, have been developed.

The external fractal structure formation study in the program "External structure of a fractal set" is carried out. For the first time, three-dimensional spherical and ellipsoidal fractal structures are obtained, which are analogous to high-rise and large-span buildings. External fractal structures were visualized using the SCAD software package and numerical experiments were carried out with the fractal power variation.

Dynamic calculation of the fractal architecture object made it possible to obtain a spectrum of twenty frequencies and modes of natural oscillations. The resonant frequencies corresponding to the most dangerous bending-torsional forms have been determined. An analysis of the exobase displacements, the surface of f-quarks and the corona is carried out for two models of the spherical and ellipsoidal Mandelbrot fractals. The results of the study allow us to conclude that fractal structures are resistant to natural and technogenic impacts. The structures are recommended to be implemented using innovative materials and 3D printing technology.

In conditions of dense urban development, it is necessary to use the principles of fractal shaping [12]. The architecture of modern large cities is a complex system with fractal properties that cannot be neglected when forming an urban network. Unique objects of fractal architecture will have cardinal differences from the existing buildings and structures, will be resistant to various types of external influences and will be able to harmoniously fit into the existing urban environment.

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