Effects of Parton $k_T$ in High-$p_T$ Particle Production

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We report on recent work concerning the phenomenology of initial-state parton-$k_T$ effects in direct-photon production and related processes in hadron collisions. After a brief summary of a $k_T$-smearing model, we present a study of recent results on fixed-target and collider direct-photon production, using complementary data on diphoton and pion production that provide empirical guidance on the required amount of $k_T$ broadening. This approach provides a consistent description of the observed deviations of next-to-leading order QCD calculations relative to the inclusive direct-photon and $\pi^0$ data. We also comment on the implications of these results for the extraction of the gluon distribution of the nucleon.

Introduction

Direct-photon production has long been viewed as an ideal process for measuring the gluon distribution in the proton [2]. The quark-gluon Compton scattering subprocess ($gq \rightarrow \gamma q$) provides a large contribution to inclusive $\gamma$ production for which the cross sections have been calculated to next-to-leading order (NLO) [3]. The gluon distribution in the proton is relatively well-constrained at $x < 0.1$ by deep-inelastic scattering (DIS) and Drell-Yan (DY) data, but less so at larger $x$. Consequently, direct-photon data from fixed-target experiments can, in principle, provide an important constraint on the gluon content at moderate to large $x$.

However, a pattern of deviations has been observed between the measured direct-photon cross sections and NLO calculations [4]. The discrepancy is particularly striking in the recently published higher-statistics data from E706 [5], for both direct-photon and $\pi^0$ cross sections. The final direct-photon results from UA6 [6] also exhibit evidence of similar, although smaller, discrepancies. The suspected origin of the disagreement is from effects of soft-gluon radiation. Such radiation generates transverse components of initial-state parton momenta, referred to in this discussion as $k_T$. (To be precise, $k_T$ denotes the magnitude of the effective transverse momentum vector, $\vec{k}_T$, of each of the two colliding partons.)

Evidence of significant $k_T$ has long been observed in production of muon, photon, and jet pairs. A collection of measurements of the average transverse momentum of the pairs ($\langle p_T \rangle_{\text{pair}}$) is displayed in Fig. 1, for a wide range of center-of-mass energies ($\sqrt{s}$). The values of $\langle p_T \rangle_{\text{pair}}$ are large, and increase slowly with increasing $\sqrt{s}$. The values of $\langle k_T \rangle$ per parton (estimated as $\approx \langle p_T \rangle_{\text{pair}}/\sqrt{2}$) indicated by these DY, diphoton, and

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1This work was done in collaboration with L. Apanasevich, C. Balázs, M. Begel, C. Bromberg, T. Ferbel, G. Günther, J. Huston, S. Kuhlmann, A. Maul, J. Owens, P. Slattery and W. K. Tung [1].
Fig. 1: $\langle p_T \rangle$ of pairs of muons, photons, and jets produced in hadronic collisions versus $\sqrt{s}$.  

dijet data, as well as the inclusive direct-photon and $\pi^0$ production data, are too large to be interpreted as “intrinsic” — i.e., due only to the finite size of the proton. (From the observed data, one can infer that the average $k_T$ per parton is about 1 GeV/$c$ at fixed-target energies, increasing to 3–4 GeV/$c$ at the Tevatron collider, while one would expect $\langle k_T \rangle$ values on the order of 0.3–0.5 GeV/$c$ based solely on proton size.) Perturbative QCD (PQCD) corrections at NLO level are also insufficient to explain the size of the observed effects, and, in fact, full resummation calculations are required to describe DY and W/Z [7, 8, 9], and diphoton [10, 11] distributions. Similar soft-gluon (or $k_T$) effects can be expected in all hard-scattering processes, such as the inclusive production of jets or direct photons [12, 13].

After reviewing a phenomenological model for $k_T$ effects in direct-photon production we will discuss its applications to data, as well as the implications for determining the gluon distribution. A more detailed presentation of these results can be found in [1].

$k_T$ Smearing Model

The Collins-Soper-Sterman resummation formalism [14] provides a rigorous basis for theoretical understanding of soft-gluon radiation effects. Despite recent progress [15, 16], no full treatment of inclusive direct-photon cross sections is yet available. In its absence, we use a PQCD-based model that incorporates transverse kinematics of initial-state partons to study the major consequences of $k_T$ for direct-photon production (and, by extension, for all hard-scattering processes).
In PQCD, the expression for the leading-order (LO) cross section for direct-photon production at large $p_T$ has the form:

$$\sigma(h_1 h_2 \rightarrow \gamma X) = \int dx_1 dx_2 f_{a_1/h_1}(x_1, Q^2) f_{a_2/h_2}(x_2, Q^2) \hat{\sigma}(a_1 a_2 \rightarrow \gamma a_3),$$

where $\hat{\sigma}$ is the hard-scattering matrix element, and $f_{a_1/h_1}$ and $f_{a_2/h_2}$ are the parton distribution functions (pdf) for the colliding partons $a_1$ and $a_2$ in hadrons $h_1$ and $h_2$, respectively. To introduce $k_T$ degrees of freedom, we extend each integral over the parton distribution functions to the $k_T$-space,

$$dx f_{a/h}(x, Q^2) \rightarrow dx d^2 k_T g(k_T) f_{a/h}(x, Q^2),$$

and take $g(k_T)$ to be a Gaussian (as justified, e.g., by E706 data on high-mass pairs),

$$g(k_T) = \frac{e^{-k_T^2/(\langle k_T^2 \rangle)}}{\pi \langle k_T^2 \rangle}.$$ 

Here, $\langle k_T^2 \rangle$ is the square of the 2-dimensional RMS width of the $k_T$ distribution for one parton and is related to the square of the average of the absolute value of $k_T$ of one parton through $\langle k_T^2 \rangle = 4\langle k_T \rangle^2/\pi$.

Since an exact treatment of the modified parton kinematics can be implemented in a Monte Carlo framework, but is more difficult in an analytic approach, it is convenient to employ Monte Carlo techniques in evaluating the cross sections according to the above prescription. In general, because the unmodified PQCD cross sections fall rapidly with increasing $p_T$, the net effect of $k_T$ smearing is to increase the expected yield. We denote the enhancement factor as $K(p_T)$.

A Monte Carlo program that includes such a treatment of $k_T$ smearing, and the LO cross section for high-$p_T$ particle production, has long been available [17]. The program provides calculations of many experimental observables, and can be used for direct photons, jets, and for single high-$p_T$ particles resulting from jet fragmentation (such as inclusive $\pi^0$ production). Unfortunately, no such program is available for NLO calculations, but one can approximate the effect of $k_T$ smearing by multiplying the NLO cross sections by the LO $k_T$-enhancement factor. Admittedly, this procedure involves a risk of double-counting since some of the $k_T$-enhancement may already be contained in the NLO calculation. However, we expect such double-counting effects to be small.

A complete treatment of soft-gluon radiation in high-$p_T$ production, should eventually predict the effective $k_T$ values expected for each process and $\sqrt{s}$. We will employ $\langle k_T \rangle$ values representative of those found in comparisons of kinematic distributions in high-mass pair data with the above described model.

**Applications of the $k_T$ Model to Data**

The experimental consequences of $k_T$ smearing are expected to depend on the collision energy. At the Tevatron collider, the smallest photon $p_T$ values probed by the
CDF and DØ experiments are rather large (10–15 GeV/c), and the $k_T$-enhancement factors modify only the very lowest end of the $p_T$ spectrum, where $p_T$ is not significantly greater than $k_T$. In the E706 energy range, large $k_T$-effects can modify both the normalizations and the shapes of the cross sections as a function of $p_T$. Consequently, E706 data provide a particularly sensitive test of the $k_T$ model. At lower fixed-target energies, the $k_T$ enhancements are expected to have less $p_T$ dependence over the range of available measurements, and can therefore be masked more easily by uncertainties in experimental normalizations and/or choices of theoretical scales. Nonetheless, the UA6 and WA70 data generally support expectations from $k_T$-smearing.

Comparisons to Tevatron Collider Data

At the Tevatron collider, the above model of soft-gluon radiation leads to a relatively small modification of the NLO cross section. In Fig. 2 we compare the CDF and DØ isolated direct-photon cross sections [18] to theoretical NLO calculations with and without $k_T$ enhancement. In the lower part of the plot we display the quantity $(Data–Theory)/Theory$, for NLO theory without the $k_T$-enhancement factor, and the expected effect from $k_T$ for $\langle k_T \rangle = 3.5$ GeV/c. This is the approximate value of $\langle k_T \rangle$ per parton measured in diphoton production at the Tevatron [18], and one expects a similar $\langle k_T \rangle$ per parton for single-photon production. (In the diphoton process, the 4-vectors of the photons can be measured precisely, providing a direct determination of the transverse momentum of the diphoton system, and thereby $\langle k_T \rangle$.)

As seen in Fig. 2, the $k_T$ effect diminishes rapidly with increasing $p_T$, and is essentially negligible above $\approx 30$ GeV/c. The trend of deviations of NLO calculations from the measured inclusive cross sections is described reasonably well by the expected $k_T$ effect. Some of the observed excess can be attributed to the fragmentation effects in the isolated direct-photon production [19], but this alone cannot account for the entire deviation of the theory from data.

Comparisons to E706 Data

The conventional ($\langle k_T \rangle=0$) NLO calculations yield cross sections that are significantly below the E706 direct-photon and $\pi^0$ measurements [5] (see Fig. 3). No choices of current parton distributions, or conventional PQCD scales, provide an adequate description of the data (for the presented comparisons all QCD scales have been set to $p_T/2$). The previously described $k_T$-enhancement algorithm was used to incorporate the effects of soft-gluon radiation in the calculated yields. That is, the theory results plotted in the figures represent the NLO calculations multiplied by $k_T$-enhancement factors $K(p_T)$.

As seen in Fig. 3, the NLO theory, when supplemented with appropriate $k_T$ enhancements, is successful in describing both the shape and normalization of the E706 direct-photon cross sections at both $\sqrt{s} = 31.6$ GeV and 38.8 GeV. As expected, the $k_T$-enhancement factors affect the normalization of the cross sections, as well as the shapes of the $p_T$ distributions. The values of $\langle k_T \rangle = 1.2$ GeV/c at $\sqrt{s} = 31.6$ GeV, and 1.3 GeV/c at $\sqrt{s} = 38.8$ GeV, provide good representations of the incident-proton data. Both $\langle k_T \rangle$ values are consistent with those emerging from a comparison of the
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Fig. 2: Top: The CDF and DØ isolated direct-photon cross sections, compared to NLO theory without $k_T$ (dashed) and with $k_T$ enhancement for $\langle k_T \rangle = 3.5$ GeV/c (solid), as a function of $p_T$. Bottom: The quantity (Data–Theory)/Theory (for theory without $k_T$ adjustment), overlaid with the expected effect from $k_T$ enhancement for $\langle k_T \rangle = 3.5$ GeV/c.

same PQCD Monte Carlo with E706 data on the production of high-mass $\pi^0\pi^0$, $\gamma\pi^0$, and $\gamma\gamma$ pairs [5]. Similar conclusions are reached from comparisons between calculations and E706 data for $\pi^-\text{Be}$ interactions at $\sqrt{s} = 31.1$ GeV (not shown).

For comparison, results of calculations using $\langle k_T \rangle$ values $\pm 0.2$ GeV/c relative to the central values are also shown in Fig. 3. These can be taken as an indication of uncertainties on $\langle k_T \rangle$, on the basis of several considerations. These include: (i) the range of $k_T$ values inferred from different distributions in the E706 high-mass pair data; (ii) differences observed between photon and $\pi^0$ results; (iii) comparisons of $\langle k_T \rangle$ values required in inclusive cross sections with those representing the properties of massive pairs at E706 and WA70/UA6 energies, and (iv) the differences between dimuon, diphoton, and dijet values of $\langle p_T \rangle_{\text{pair}}$ seen in Fig. 1. Examples of enhancement factors $K(p_T)$ for fixed-target experiments are displayed in Fig. 4.

It is interesting to note that, for the fixed-target energy range, the $k_T$ enhancement increases at the highest values of $p_T$. The shape of $K(p_T)$ can be understood through the following argument. At the low $p_T$ end of the measurements, a $\langle k_T \rangle$ of $\approx 1$ GeV/c is non-negligible in comparison to the $p_T$ in the hard-scattering, and the addition of $k_T$ smearing therefore increases the size of the cross section (and steepens the slope).
Fig. 3: The photon and $\pi^0$ cross sections from E706 at $\sqrt{s} = 31.6$ GeV (left) and 38.8 GeV (right) compared to $k_T$-enhanced NLO calculations.

Fig. 4: Left: The variation of $k_T$ enhancements, $K(p_T)$, relevant to the E706 direct-photon data for protons at $\sqrt{s} = 31.6$ GeV, for different values of $\langle k_T \rangle$. $K(p_T)$ used by the MRST group [20] is also shown. Right: The same for $\sqrt{s} = 23.0$ GeV (relevant to WA70 data).

At highest $p_T$ (corresponding to large $x$), the unmodified NLO cross section becomes increasingly steep (due to the rapid fall in parton densities), and hence the effect of $k_T$ smearing again becomes larger.

NLO calculations for $\pi^0$ production have a greater theoretical uncertainty than those
for direct-photon production since \( \pi^0 \) production involves parton fragmentation. However, the \( k_T \) effects in \( \pi^0 \) production can be expected to be similar to those observed in direct-photon production, and the \( \pi^0 \) data can be used to extend tests of the consequences of \( k_T \)-smearing. Figure 3 also shows comparisons between NLO calculations [21] and \( \pi^0 \) production data from E706, using BKK fragmentation functions (ff) [22]. The previously described Monte Carlo program was employed to generate \( k_T \)-enhancement factors for \( \pi^0 \) cross sections, and \( \langle k_T \rangle \) per parton values similar to those that provided good agreement for direct-photon data also provide a reasonable description of \( \pi^0 \) data. For \( \pi^0 \) production, an additional smearing of the transverse momentum expected from jet fragmentation has also been taken into account.

**Comparisons to WA70 and UA6 Data**

Both WA70 and UA6 have measured direct-photon production with good statistics, and their data have been included in recent global fits to parton distributions. WA70 measured direct-photon and \( \pi^0 \) production in \( pp \) and \( \pi^- p \) collisions at \( \sqrt{s} = 23.0 \text{ GeV} \) [23], and UA6 has recently published [6] their final results (with substantially reduced uncertainties) for direct-photon production in \( pp \) and \( \bar{p}p \) collisions at \( \sqrt{s} = 24.3 \text{ GeV} \). These center of mass energies are smaller than those of E706, and the \( \langle k_T \rangle \) values are therefore expected to be smaller (perhaps of order 0.7–0.9 \( \text{GeV}/c \), based on Fig. 1). WA70 has compared kinematic distributions observed in diphoton events (for \( \pi^- p \) interactions) to NLO predictions, and has found that smearing the NLO theory with an additional \( \langle k_T \rangle \) of 0.9±0.2 \( \text{GeV}/c \) provides agreement with their data [24]. We therefore use this \( \langle k_T \rangle \) as the central value for the \( k_T \)-enhancement factors for both experiments, and vary the \( \langle k_T \rangle \) by ±0.2 \( \text{GeV}/c \), as was done with E706. Over the narrower \( p_T \) range of WA70 and UA6 measurements, the effect of \( k_T \) is essentially to produce a shift in normalization, as illustrated on the right side of Fig. 4.

Comparisons of the WA70 direct-photon and \( \pi^0 \) cross sections with the \( k_T \)-enhanced NLO calculations are shown in Fig. 5 (using QCD scales of \( p_T/2 \)). The \( \pi^0 \) cross sections both for incident proton and \( \pi^- \) beams, and the photon data from incident \( \pi^- \) beam, all lie above the NLO calculations for \( \langle k_T \rangle \) = 0, and are in better agreement with the \( k_T \)-enhanced calculations; only the photon cross section for incident protons seems not to require a \( k_T \) correction.

The photon and \( \pi^0 \) cross sections from UA6 for \( pp \) and \( \bar{p}p \) scattering are shown in Fig. 6. The photon cross section for \( pp \) interactions lies clearly above the NLO calculation for \( \langle k_T \rangle = 0 \), but is consistent with \( k_T \)-adjusted calculations for \( \langle k_T \rangle \) in the range of 0.7–0.9 \( \text{GeV}/c \). The result for \( \bar{p}p \) interactions is also above the unmodified NLO calculation, but requires a smaller value of \( \langle k_T \rangle \). We note that the dominant production mechanisms for the two processes are different: quark-gluon Compton scattering dominates for \( pp \), and \( \gamma \bar{q} \) annihilation for \( \bar{p}p \) at the UA6 energy. As in the case of E706 and WA70, the UA6 \( \pi^0 \) cross sections are higher than the NLO calculation without \( k_T \), and can be described much better by introducing \( k_T \) enhancement.
Fig. 5: The photon and $\pi^0$ cross sections from WA70 at $\sqrt{s} = 23.0$ GeV for incident protons (left) and $\pi^-$ (right), compared to $k_T$-enhanced NLO calculations.

Fig. 6: The photon and $\pi^0$ cross sections from UA6 at $\sqrt{s} = 24.3$ GeV for incident protons (left) and antiprotons (right), compared to $k_T$-enhanced NLO calculations.

**Impact on the Gluon Distribution**

It is now generally accepted that the uncertainty in the gluon distribution at large $x$ is still quite large. Thus, it would appear important to incorporate further constraints on the gluon, especially from direct-photon data. To investigate the impact of $k_T$ effects...
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Fig. 7: A comparison of the CTEQ4M, MRST, and CTEQ4HJ gluons, and the gluon distribution derived from fits that use E706 data. The $g_1$ and $g_1'$ gluon densities correspond to the maximum variation in $\langle k_T \rangle$ that MRST allowed in their fits.

On determinations of the gluon distribution, we have included the E706 direct-photon cross sections for incident protons, along with the DIS and DY data that were used in determining the CTEQ4M pdfs, in a global fit to the parton distribution functions. The CTEQ fitting program was employed to obtain these results [25], using the NLO PQCD calculations for direct-photon cross sections, adjusted by the $k_T$-enhancement factors. However, the WA70, UA6, CDF, and DØ data were excluded from this particular fit. The resulting gluon distribution, shown in Fig. 7, is similar to CTEQ4M, as might have been expected, since the $k_T$-enhanced NLO cross sections using CTEQ4M provide a reasonable description of the data shown in Fig. 3.

The new MRST gluon distribution [20] (also shown in Fig. 7) is significantly lower than CTEQ4M at large $x$. While the MRST fit employs $k_T$-enhancements (obtained using an analytic integration technique), it attempts to accommodate the WA70 incident-proton direct-photon data, which does not exhibit an obvious $k_T$ effect. In addition, the MRST $k_T$-enhancements are larger than ours at large $p_T$ (see Fig. 4), resulting in a smaller gluon at large $x$ [26]. In contrast, the CTEQ4HJ gluon distribution [27], designed to improve the description of the high-$p_T$ jet data from CDF in Run IA, is much larger than CTEQ4M in the same $x$ range. This spread of the solutions for the gluon distribution at large $x$ is uncomfortably large, and additional theoretical effort is warranted to properly incorporate the available direct-photon data in the pdf fits.
Conclusions

We have described a phenomenological model in which $\langle k_T \rangle$ values used in the calculations of $k_T$-enhancements are derived from data. The results are remarkably successful in reconciling the data and theoretical calculations for a broad range of energies. The $k_T$-enhancements improve the agreement of PQCD calculations with E706, UA6, and $\pi^-$ beam WA70 direct-photon cross sections over the full $p_T$ range of measurements, as well as with the low-$p_T$ end of CDF and DO results. All discussed fixed-target $\pi^0$ measurements also agree much better with such $k_T$-enhanced calculations.

A definitive conclusion regarding the quantitative role of $k_T$ effects in hard scattering awaits a more rigorous theoretical treatment of soft-gluon radiation. Such theoretical progress is crucial for a more reliable determination of the gluon distribution, especially in the large-$x$ region, where significant uncertainties remain.

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