Non-equilibrium Josephson–like effects in mesoscopic S-N-S junctions

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Wide mesoscopic superconducting–normal-metal–superconducting (S-N-S) junctions exhibit Andreev bound states which carry substantial supercurrents, even at temperatures for which the equilibrium Josephson effect is exponentially small — the currents carried by different states can cancel each other. This cancellation is incomplete whenever the junctions are driven out of equilibrium, e.g., by a dc voltage. This leads to phenomena similar to the usual dc and ac Josephson effects, but dominated by the second harmonic of the Josephson frequency, which may explain some striking recent experiments. A simple description of these, in the spirit of the Resistively–Shunted–Junction model, is suggested.

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Mesoscopic superconducting–normal-metal–superconducting (S-N-S) junctions exhibit a spectrum of low–lying electronic “Andreev bound states”, which depends strongly on the phase–difference φ between the order parameters in S (see Fig. 1). This φ–dependence persists when the temperature T is raised, and the “normal–metal coherence length”, \( \xi_N = \sqrt{\hbar D/k_B T} \), becomes much smaller than the distance L between the two S electrodes (\( D \) is the diffusion constant in N). In this high–temperature regime the equilibrium Josephson coupling is exponentially weak. However, as the spectrum depends on φ, and hence on time t (the voltage \( V \) is proportional to \( d\phi/dt \)), non–equilibrium (NEQ) effects occur, including supercurrents which are the topic of this Letter.

Such junctions have attracted much attention. The φ–dependence of their conductance was studied extensively in recent years [1], and it was demonstrated that a naive Ginzburg–Landau description of proximity effects was insufficient. In these “Andreev interferometers” N is connected to external normal–metal electrodes, which thermalize the occupations [2]. In contrast, for an “isolated” N the electronic excitations are “bound”: an electron near the Fermi level in N cannot enter S because of the superconducting gap, except by “Andreev reflection” — it pairs with another electron from the Fermi sea, leaving behind a hole. Conversely, a hole may break up a Cooper pair and produce an electron. These processes coherently mix the electron and hole states in N, in a φ–dependent manner. As energy–relaxation is slow in the mesoscopic regime (\( \tau_\phi > L^2/D \), with \( \tau_\phi \) the single–particle dephasing time), NEQ situations naturally develop.

NEQ phenomena have been studied for both clean [3] and dirty [4] S-N-S junctions [5]. The progress made here is in (a) developing a simple but versatile model of NEQ phenomena, in the spirit of the resistively–shunted–junction (RSJ) model (at the price of a restriction to small voltages); and (b) using the unusual harmonic content of the NEQ supercurrents to understand a recent observation [6], which cannot be explained [7] in terms of previously proposed effects [8]. The simplest description of NEQ, with a constant energy–relaxation rate \( 1/\tau_E \) (with \( \tau_E \geq \tau_\phi \)), is used to discuss wide (or long) dirty junctions, in the high–temperature regime, \( E_C = \hbar D/L^2 \ll T \ll \Delta \). Here \( E_C \) is the Thouless energy (the correlation energy of the spectrum), \( k_B = 1 \), and \( \Delta \) is the gap in S. The single–particle level spacing in N, \( \delta \), is taken small (metallic limit): \( \delta \ll \hbar/\tau_E \ll E_C \).

The density of states \( \nu(\epsilon, \phi) \) can be calculated from the (disorder–averaged) Usadel equations [1][2]. It is convenient to define the energy \( E_n(\phi) \) of the nth level: \( n = \int_0^{E_n} \nu(\epsilon, \phi) \, d\epsilon \), see Fig. 2 (\( \epsilon = 0 \) is the Fermi level). We have assumed for simplicity that time–reversal symmetry is preserved, that the pair potential \( \Delta \) is constant in S and vanishes in N, and that the N-S boundaries are “perfect”, i.e. that no normal reflection occurs and the amplitude for Andreev reflections is of unit magnitude. A mini–gap is proximity–induced in N. Its size is

![Figure 1](image1.png)

**FIG. 1**: Energies of some of the Andreev bound states \( E_n \) (equally spaced in \( n \)), as a function of \( \phi \), for a diffusive S-N-S junction (see inset) with \( \delta \ll E_C \ll \Delta \) (and \( \tau_\phi \rightarrow \infty \)). Thick line: a representative used below, \( E_{rep}(\phi) \).
For our system, the curve of width $E_g$ for $\phi = 0$, and it closes when $\phi = \pi$ and reopens periodically (fluctuations in $E_g$ and $E_n$ are only $\sim \delta$). The spectrum is very sensitive to symmetry breaking — for example, a moderate amount of spin–flip scattering, with $\hbar/\tau_{sf} \sim E_C$, can close the mini–gap. However, for most S-N-S geometries it contains a band of width $\sim E_C$ of levels with a significant $\phi$–dependence.

For our system, the curve $E_{rep} \simeq 3.4 E_C \sqrt{1 + 0.7 \cos(\phi)}$ represents the low–lying part of the spectrum well, and the effective number of levels in this band is $N \simeq 10 E_C / \delta$, including a factor of 2 for spin [a simple shift in energy does not affect the contribution of a level $E_n(\phi)$]. As we will use only the fact that $E_{rep}$ is $\phi$–periodic, the results will be qualitatively applicable also to the experimentally–relevant clean case, with a mean–free–path of a few times $L$ (though perhaps not to the theoretical clean limit $\frac{4}{\sqrt{7}}$ which has a separable spectrum).

The states $E_n$ carry currents, $I = (2e/\hbar)(dE_n/d\phi)$ per occupied state, as seen by equating the spent and stored energies, $IVdt = \sum dE_n$ (neglecting any changes in the interaction energy), and using the Josephson relationship $d\phi = (2e/\hbar)Vdt$. For each $n$ there is also a state at $-E_n$, with the opposite current. The total supercurrent is thus

$$I_S = -\frac{2e}{\hbar} \sum_n dE_n d\phi (1-2f),$$

where $f$ is the occupation probability of $E_n$, and $1-f$ is that of $-E_n$ (Ref. 13; the spin index is included in $n$).

The fact that NEQ occupations often enhance supercurrents was demonstrated in the seventies [14], and follows from an elegant argument. In thermal equilibrium, $f = f_{eq} = 1/(1+\exp(\epsilon/T))$, and $I_S$ takes the form

$$I_{eq}(\phi) = \int_0^\infty d\epsilon j(\epsilon, \phi) \tanh(\epsilon/2T),$$

where $j(\epsilon)$ is the “Josephson current density”, $j(E_n, \phi) \propto \nu(E_n, \phi)(dE_n/d\phi)$. This $j(\epsilon)$ is the imaginary part of a “Green’s function” which is analytic in the upper half of the complex $\epsilon$ plane — a positive imaginary part of $\epsilon$ corresponds to a dephasing rate ($\hbar/\tau_\phi$) which would smooth out any singularities. Using contour integration (the integrand is even), one finds the well–known Matsubara sum: $I_{eq} = 2\pi T \sum j(i\omega_n)$, where $\omega_n = (2n-1)\pi T$ are the poles of the tanh factor. At high temperatures, even the smallest Matsubara frequency has a decay time $\hbar/\omega < L^2/D$, yielding an exponentially small $j(i\omega_n)$. Thus, for physical quantities of this form, thermal averaging is mathematically equivalent to dephasing.

In NEQ situations contour integration cannot be used, and the currents are not exponentially small. For Eq. (1) to give small results, $j(\epsilon)$ must oscillate: in our diffusive system, $j(i\omega)$ decays exponentially with $\sqrt{\omega/E_C}$ on the imaginary axis, and correspondingly $j(\epsilon)$ oscillates and decays rapidly with $\sqrt{\epsilon/E_C}$. The $E_n(\phi)$ curves in Fig. 1 thus change their character repeatedly at higher $\epsilon$, occasionally having shallow maxima at $\phi = \pi$, rather than minima. As can be seen from Eq. (2) below, the deviations of $f$ from equilibrium change sign in rhyme with these oscillations, and thus the integrand of the NEQ part of the supercurrent, $\int_0^\infty d\epsilon j(f_{eq} - f)$, does not change sign and cannot be affected by any cancellations.

The expression for $I_S$ may thus be approximated by

$$I_S \simeq I_{eq}(\phi) + 2\frac{2e}{\hbar} N \frac{dE_{rep}}{d\phi} (f-f_{eq}),$$

(3)

The errors incurred here [14] are probably smaller than those of the relaxation–time approximation,

$$\frac{df}{dt} = -\frac{1}{\tau_E} (f-f_{eq}),$$

(4)

which we shall also employ. For example, the latter ignores the effects of the mini–gap on the electron–electron and electron–phonon relaxation processes.

When a dc voltage is applied to the junction, $d\phi/dt = 2eV/\hbar = \text{const.}$, this model gives ac supercurrents, see Fig. 2 (the units $I_{eq}$ and $t_J$ are defined below). Here $T \gg E_C$ was used to take $I_{eq} = 0$ and $f_{eq} = \frac{1}{2} - E_{rep} / 4T$. The second harmonic dominates because $I_S$ of Eq. (3) is a product of two oscillatory factors, $dE_{rep}/d\phi$ and $(f-f_{eq})$. For junctions with $\tau_\phi \sim L^2/D$ one expects a much “softer” spectrum, with $E_{rep} \propto \cos(\phi)$, and $I_S$ would exhibit true frequency doubling, evolving from a $\sin^2(\phi)$ behavior to $-\sin(2\phi)$ as $V$ is increased.

In the limit of small voltages, we reproduce the results of Refs. 13, 14: $f-f_{eq} \approx \tau_E (2eV/\hbar)(dE_{eq}/d\phi)$, and

$$I_S = \left(\frac{2e}{\hbar}\right)^2 \frac{N \tau_E V}{2T} \left(\frac{dE_{rep}}{d\phi}\right)^2 + O(V^2).$$

(5)
This adds a phase–dependent term to the ohmic conduction of the junction. For a large dc voltage, we may approximate $f$ by a constant — the phase average of $f_{eq}$ — which yields a purely oscillatory $I_S$. As can be seen from the “Debye mechanism” of Ref. [5], which monitors the steady–state transfer of energy into the heat bath ($\propto 1/\tau_E$) rather than that into the electrons, the total power dissipates approaches a constant $P_{\text{max}}$ at large $V$, and so $f I dt$ decreases as $P_{\text{max}}/V$.

So far, we have ignored the normal current component, $I_N \approx G_N V$, where $G_N$ is the conductance of $N$ in the absence of proximity effects. Clearly, in the metallic component, the occupations can not evolve adiabatically (as in Ref. [5]). Instead the relevant Green’s functions become intertwined. (a) The probability density in $f$, for $\tau_E \to \infty$, is $p(f) \propto f d\phi \exp(-F/T_\phi)$; here $T_\phi = 0.25 E_{\text{neq}}$. (c) The effective potential, $\tilde{F}(\tilde{f}) = -T_\phi \log p$. When $T \ll T_\phi$, its minima “trap” the system.

$-1 \leq \tilde{E} \leq 1$ fixing $A$ and $B$ (in our model $A \approx 1.3 E_C$). Here $I_{\text{neq}} = (2e/h) N A^2/4T$, and the energy scale is $E_{\text{neq}} = (h/2e) I_{\text{neq}} \approx 4.1 E_C^2/\delta T$, only a factor of $E_C/3.5T$ smaller than the $T = 0$ Josephson coupling energy. The correlators $(I_F(t) I_F(0)) = 2T_c G \delta(t)$ and $(\dot{J}_F(t) J_F(0)) = (\tau_E T/E_{\text{neq}}) \delta(t)$ give the Gaussian fluctuations of $I_F$ and $J_F$. For thermal noise, $T_\phi = T$; high–frequency external noise can be described by $T_\phi > T$.

Eqs. (6) and (7) describe overdamped motion in a free–energy landscape $F(\phi, \dot{\phi}) = E_{\text{neq}}(\dot{\phi}^2 - \dot{\phi}) - (h/2e) \dot{\phi}$. See Fig. 4(a). This generalizes the tilted washboard potential of the RSJ model, $F(\phi) = - (h/2e) (I_C \cos \phi + I_0 \phi)$ (where $f$ need not be followed). For fixed $\phi$, the fluctuations induced by $J_F$ reproduce the $\sqrt{N}$ noise of thermal occupations (and result in increased current noise, $I_S \propto \partial F/\partial \phi \dot{\phi}$, Ref. [5]). Note that for $\tau_E \to 0$ (fast equilibration), $\dot{f}$ may be integrated out, which must reproduce the RSJ model. Here $I_0 = 0$, but the $\dot{\phi}$–dependence of the “bottom of the valley” re–emerges at lower $T$.

In the opposite limit, $\tau_E \to \infty$, the “fast variable” $\phi$ may be removed (see Fig. 5), giving a two–peaked probability density in $\dot{f}$ (one broad peak for large $T_\phi$). The corresponding effective potential, $\tilde{F}(\tilde{f})$, has valleys with a depth roughly $\sim T_\phi$, which trap the system if $T \ll T_\phi$ (Ref. [5]). This drastically reduces the rate of phase–slip: the $\phi$ variable remains near the extrema of $\tilde{E}$, and changing it with $\dot{f}$ fixed (because $\tau_E$ is large) entails overcoming a barrier of order $E_{\text{neq}}$. This surprising fact — that noise enhances conductance near $I = 0$ — is demonstrated numerically below.

The $I–V$ curves of this model, obtained with $I(t) = \text{const.}$ and no noise ($I_F = J_F = 0$), are displayed in Fig. 6 using units of $I_{\text{neq}}, V_{\text{neq}} = G_N I_{\text{neq}}$ and $\tau_{\text{neq}} = h/2e V_{\text{neq}}$ for current, voltage and time. The results for large $\tau_E$ lie remarkably close to the $I–V$ curves of the RSJ model [20], except for the finite zero–bias conductance, equal to $1 + \tau$, or $G_N + \tau E(2e/h) I_{\text{neq}}$. Re-
results with external noise (and a large \( \tau_E \)) are shown in Fig. 3(a). Such numerical calculations indicate that for finite \( \tau_E \) the scaled conductance grows at most as \( \tau^2 \), and not exponentially as in the \( \tau_E \to \infty \) case.

An ac drive, \( I_{ac} \cos(\omega t) \), produces Shapiro steps in the \( I-V \) curves — phase-locking of the ac Josephson oscillations to the external frequency \( \omega \), see Fig. 3(b). As in the RSJ model \( [8] \), closely-related results obtain for the simpler, voltage-biased case \( [9] \). For weak ac driving, the sub-harmonic step at \( \omega = \frac{\omega}{2} \) dominates. As observed in the experiments of Ref. \( [8] \), this step is the largest for \( T \gtrsim E_C \), but does not appear to be parametrically larger than the others (a mechanism with true frequency doubling would behave differently). The experimental steps were much smaller in size, which could indicate fast equilibration — the steps disappear quadratically for small \( \tau_E \), because to leading order Eq. \( (5) \) gives \( I = G(\phi)V \), which relates the time integrals of the current and voltage by a constant ratio, \( \int G(\phi)\,d\phi/2\pi \), and precludes any steps \( [8] \). A detailed modeling of the experiment requires additional measurements, which are in progress.

In summary, we have discussed NEQ effects very similar to both the dc (Fig. 3) and the ac (Fig. 3) Josephson effects, which decay with temperature only as \( I_{\text{neq}} \propto E_C/T \). The NEQ occupations can be produced by microwave irradiation, a noisy external circuit, or simply an applied voltage. The new effects have the signature of being dominated by the second harmonic of the Josephson frequency, and may have already been observed.

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