Non-perturbative determination of heavy quark action coefficients

Huey-Wen Lin

Department of Physics, Columbia University, New York, NY, 10027

We propose to determine the coefficients in the Fermilab heavy quark action by matching the finite-volume, off-shell, gauge-fixed propagator and vertex functions with those determined in the exact, relativistic theory. The matching relativistic amplitudes may be determined either from short-distance perturbation theory or from finite-volume step-scaling recursion. Tree-level matching and non-perturbative off-shell amplitudes are presented.

1. Introduction

The Fermilab action \([1]\) is the least restrictive framework permitting lattice methods to be used for heavy quarks when the quark mass is large compared to the inverse lattice spacing, \(m_{\text{quark}} a > 1\). However, at present the coefficients of this action have only been calculated using low order lattice perturbation theory. In order to have better control of errors for the physical quantities calculated on the lattice, we propose to determine these coefficients non-perturbatively. We plan to achieve this by imposing a matching condition between the off-shell quark propagator and quark-gluon vertex function determined from the Fermilab action, and the same propagator and vertex computed under more physical conditions.

We propose two ways to achieve this goal. The most naive method would be matching the propagator and vertex function computed in the continuum theory up to some order in perturbation theory with the RI-MOM NPR \([2]\) calculation on the lattice (Figure 1). This would be done through the “matching window” \([3]\): \(\Lambda_{\overline{\text{MS}}}^2 \ll m^2 - p^2 \ll 1/a^2\). The alternative will be purely non-perturbative, by connecting the original calculation with one performed at a smaller lattice spacing \(a' = a + \epsilon\) (as shown in Figure 2). A further advantage of such an entirely non-perturbative matching is that the kinematic “window” for performing such matching now has no lower bound since there is no requirement that the perturbation theory be accurate.

\[\text{Lattice} \quad \text{Continuum}\]

Figure 1. Proposal-I. Match the lattice result to corresponding continuum perturbation theory calculation to sufficient order, in the usual RI-MOM scheme.

\[\text{Lattice} \quad \text{Continuum}\]

Figure 2. Proposal-II. Match the lattice result, through a step-scaling technique, to finer non-perturbative, \(O(a)\) improved light quark calculations.

2. Off-shell Fermilab action

Extending the off-shell \(O(a)\) improvement of light quarks \([4]\) to heavy quark, we will adopt the approach of using an unchanged fermion action...
in the simulation:

\[ S = \sum_{n} \psi_n (D_{FL} + m) \psi_n \]

and improved quark fields of the form:

\[ \psi \rightarrow \tilde{\psi} = Z_q \frac{1}{2} [1 + ac^i (D_{FL} + m)] \psi \]

\[ \tilde{\psi} \rightarrow Z_q \frac{1}{2} \tilde{\psi} [1 + ac^i (-D_{FL} + m)] \psi \]

where \( D_{FL} \) is the Fermilab Dirac operator. Note: \( Z_q \) and \( c \)'s are all functions of \( ma \). In summary, we have eight parameters as a function of \( ma \) to be determined: \( \zeta, r_s, c_E, c_B \) (in the action) and \( Z_q \), \( c' \), \( c_{NGI} \) and \( c'_{NGI} \) (in the improved quark fields).

If we look at the low spatial momentum behavior of the Dirac structure we find that there are more than the six required conditions from \( \Lambda_\mu \) and the two from \( \hat{S}(p) \), and thus we are able to fix these eight parameters.

3. Tree-level coefficient calculations

Before we start the complicated computer calculations, we can test our proposal-I by starting with lattice perturbation theory in tree-level and matching to the corresponding continuum calculations, we can test our proposal-I by starting with lattice perturbation theory in tree-level and matching to the corresponding continuum calculations.

Taking the \( p_0 a \) and \( ma \ll 1 \) limit, we reproduce the known coefficients

\[ c_E = c_B = \zeta = r_s = 1, c_{NGI} = 0, \]

\[ c_q = 1/2, Z_q = 1 - 2ma \]

for the improved, light quark action.

4. Non-perturbative Green’s functions

We have studied connected 2-point Green’s functions (quark and gluon propagators) and have produced answers consistent with published papers. The accuracy achievable with moderate statistics is adequate to determine those improvement coefficients that are constrained by the 2-point functions. From now on, we will focus on the least known quantity: the quark-gluon vertex function.

The quark-gluon vertex function can be calculated from the following 3-point function:

\[ V^a_{\mu}(x, y, z)_{\alpha, \beta} = \langle S_{\alpha, \beta}(x, z) A^a_{\mu}(y) \rangle. \]

The Fourier transformed amputated vertex function in momentum space is

\[ \Lambda^{a,\text{lat}}_{\mu}(p, q)_{\alpha, \beta}^{ij} = \langle \hat{S}(p) \rangle^{-1} V_{\mu}^a(p, q)_{\alpha, \beta}^{ij} (\hat{S}(p + q))^{-1} (D(q)_{\mu \nu})^{-1} \]

We calculate the quantity \( \frac{1}{2} Im \sum_{\mu} Tr(\gamma_{\mu} A^{\text{lat}}_{\mu,R}) \) in the case where \( q = 0 \).

As a first step we plan to non-perturbatively determine the coefficients in light quark clover action, since only four parameters are involved: \( c_{sw} \) (in the action) and \( Z_q \), \( c' \), and \( c_{NGI} \) (in the improved quark fields). Our gauge configurations use the Wilson gauge action with \( \beta = 6.0, a^{-1} = 1.922 \text{ GeV}, 16^3 \times 32 \text{ lattice size}. \)

We use Landau gauge and proceed by two approaches: 1. fix to maximal axial gauge (MAG) before Landau gauge fixing; 2. fix only to Landau gauge. We found that the two different procedures agree for the gluon propagator, but slight effects on quark propagators (decreasing as the momentum increases) but give very different results in the vertex calculation, as shown in Figure 4. We believe the differences are caused by Gribov copies. From now on, we will follow procedure 2. for better control over which gauge orbit is selected by the Landau gauge fixing procedure, and for the cleaner signal.
Figure 3. Gluon “q = 0” vertex function calculation from the tadpole improved clover action with $\kappa = 0.137$ from 43 configurations.

Figure 4 shows the comparison of the unrenormalized $q = 0$ vertex functions from NP clover with $\kappa = 0.13445$ without off-shell improvement, and with domain wall fermions ($L_S = 16, M_5 = 1.8$). DWF is automatically $O(a)$ off-shell improved and the vertex nicely becomes independent of momenta in high momentum limit, as one would expect from a perturbative analysis. It is precisely this difference in behavior that will enable us to determine the coefficients to off-shell improve the clover action, first in the light quark limit, and later as a function of $ma$. The masses of the pseudoscalar meson calculated from these two actions are similar, around 500 MeV. Once we impose the $O(a)$ off-shell improved quark field clover action, the difference should be reduced.

The “$q = 0$” kinematics has cleaner signal than “$q \neq 0$” and would be a better candidate for a step-scaling proposal, while the “$q \neq 0$” would be required to implement proposal-I. This would require better control of the statistical errors. The large error bar at “$q \neq 0$” comes mainly from the weak correlations between the quark propagator and the gluon fields. We are currently working on using sources with definite momentum to achieve improved statistical accuracy.

5. Summary and Outlook

In this paper, we have proposed two methods for non-perturbatively determining the coefficients of the heavy quark action. We calculated the tree-level coefficients and reproduced the known light quark limit. It seems possible to determine the Fermilab coefficients with sufficient effort through either of our proposals. We will next look at other Dirac projections and momentum configurations. Once we have investigated the optimal method more thoroughly, we will start with non-perturbative determination of $c_{sw}$ for the light quark action and reproduce the Alpha collaboration result. We will then move on to the large quark mass case: the Fermilab action.

ACKNOWLEDGMENTS

I thank N. Christ and P. Boyle for physics discussions, C. Dawson for comparing RI-MOM quark propagator data, J. Skullerud for a vertex function calculation comparison, T. Bhattacharya for comments in the lattice conference and the RBC collaboration for the physics system and QCDSP machine time.

REFERENCES

1. A. X. El-Khadra et al., Phys.Rev. D55, 3933 (1999), hep-lat/9604004.
2. G. Martinelli et al., Nucl.Phys. B445, 81 (1995), hep-lat/9411010.
3. H. Lin, Nucl.Phys.Proc.Suppl 129, 429-431 (2004), hep-lat/0310060.
4. G. Martinelli et al., Nucl.Phys. B611, 311 (2001), hep-lat/0106003.
5. J. Skullerud et al., JHEP 0209, 013 (2002), hep-ph/0205318; JHEP 0304, 047 (2003), hep-ph/0303176.
6. H. Lin et al. in preparation.