Evolution of three-dimensional waves on vertically falling liquid films. Comparison between calculations and experiment

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Abstract. In this paper we present the results of a comparison of calculated and experimental data on the main regularities of the evolution of a three-dimensional wave on vertically falling liquid films in the range of film flow Reynolds number $5 < Re < 30$. Numerical calculation based on the Shkadov model equations shows both experimentally observed scenarios of the evolution of three-dimensional waves: propagation of a single wave or wave train evolution. Comparison of the wave shape shows good qualitative agreement between experiment and calculation for both scenarios.

1. Introduction
Liquid films flowing down different surfaces are widely encountered both in nature and technological processes, for example, heat exchangers, coolers, etc. As is known, the instability of a free surface leads to the formation of waves, which significantly affect the intensity of the transport processes. Therefore, a correct determination of the characteristics of the emerging waves is an important task. The modern level of development of computer systems allows for numerical modeling of processes in flowing films in a wide range of parameters. And modeling in some cases (for example, in hard-to-reach places) becomes more preferable in view of the difficulties or even the impossibility of carrying out experiments. Numerical investigation of the complete system of Navier-Stokes equations in the three-dimensional case is a very time-consuming procedure. Therefore, in practice, simplified equations are used.

The widely known model system of equations obtained by V. Ya. Shkadov on the basis of the assumption that the longitudinal velocity in the film has a self-similar semiparabolic profile. Despite the presence of shortcomings (for example, incorrect determination of the critical Reynolds number), the modeling on the basis of this system allows obtaining reliable results in the three-dimensional case over a wide range of $Re$ (for details see [1]). For example, the characteristics of stationary three-dimensional waves on vertical fluid films are in good agreement with the experimental data [2]. However, most of the waves on the surface of the films are evolving, and a detailed comparison of the theoretical and experimental data on the features of the evolution of three-dimensional waves has not been previously carried out. This is because evolution of waves is strongly affected by the initial conditions, and hence the direct quantitative comparison between experiments and calculations is difficult. Therefore, the aim of this paper is to compare the calculated and experimental data on the
main regularities of the development of three-dimensional waves on the surface of vertically falling liquid films.

2. Methods and formulation

Both the experimental investigation and numerical simulations were performed for the case of vertically falling liquid film in the range of Reynolds number of film flow $5 < Re < 30$. Water with Kapitza number $\gamma = \left( \frac{\sigma^3}{g \rho \gamma^2} \right)^{1/3} = 3660$ and water-glycerol solution (WGS) with $\gamma = 1170$ were used as working liquids. Here and later $\rho$ is the liquid density, $\nu$ is the kinematic viscosity, $\sigma$ is the surface tension, $g$ is the gravitational acceleration.

2.1. Calculation method

The evolution of the free surface of a thin liquid film was simulated using the three-dimensional Shkadov model (for details see [1 – 3]):

$$ h_t + q_x + w_z = 0, $$

$$ q_x + \frac{6}{5} \left( \frac{q^2}{h} \right)_x + \frac{6}{5} \left( \frac{qw}{h} \right)_z = \frac{3h}{\varepsilon Re} - \frac{3q}{\varepsilon Re h^2} + We c^2 h \left( h_{xx} + h_{zz} \right), $$

$$ w_x + \frac{6}{5} \left( \frac{w^2}{h} \right)_z + \frac{6}{5} \left( \frac{qw}{h} \right)_x = -\frac{3w}{\varepsilon Re h^2} + We c^2 h \left( h_{zz} + h_{xx} \right). $$

Here $h$ is the local thickness of the film, $q$ is the fluid flow rate in the $x$ coordinate, $w$ is the fluid flow rate in the $z$ coordinate, $Re = \frac{gh_N^2}{3\nu^2}$ is the Reynolds number (where $h_N$ is the Nusselt’s flat film thickness), $We = 3^{1/3} \gamma Re^{-5/3}$ is the Weber number, and the $c$ is the long-wave parameter.

To find the solutions of the Shkadov system of equations (1), the functions $h$, $q$ and $w$ are represented as a Fourier spatial series, which corresponds to solutions periodic in $x$ and $z$:

$$ h(x, z, t) = \sum_n \sum_m h_{n,m}(t) \exp(ikx) \exp(ikmz), $$

$$ q(x, z, t) = \sum_n \sum_m q_{n,m}(t) \exp(ikx) \exp(ikmz), $$

$$ w(x, z, t) = \sum_n \sum_m w_{n,m}(t) \exp(ikx) \exp(ikmz). $$

After substituting (2) into the system of equations (1), we obtain an infinite system of ordinary differential equations for the Fourier harmonics $h_{n,m}(t)$, $q_{n,m}(t)$ and $w_{n,m}(t)$. Assuming that all the $h_{n,m}(t)$, $q_{n,m}(t)$ and $w_{n,m}(t)$ with indices $n > N$ and $m > M$ are equal to zero, we arrive at its finite-dimensional analogue. The resulting system of differential equations is solved by the fourth-order Runge-Kutta method.

In this way, the evolution was calculated in time. The initial conditions were the Nusselt’s flat film with a single positive domed (Gaussian) perturbation.

2.2. Experimental techniques

Experiments were carried out on vertically freely falling liquid films. Part of the experimental results was previously obtained by the conductivity method and was described in [4] (they are presented here with the authors’ permission). Another part of experimental results was obtained with help of Laser-Induced Fluorescence technique described for example in [5]. For all experiments the waves were
generated by the a short impact of droplet of the working liquid by the film surface at the upper part of the flow. The excitation energy (mass and velocity of the droplet) varied in a wide range. Using as a working area the top region of the flow where natural waves have not been formed yet allowed us to eliminate the influence of other waves on the evolution of the excited waves.

3. Results
In experimental investigation two scenarios of wave evolution have been registered depending on the excitation energy, i.e. mass and velocity of the droplet. The basic scenario of wave evolution is the travelling solitary wave which is realized when excitation energy is higher than some threshold value. In that case well pronounced horseshoe-shaped wave with low-amplitude capillary precursor is observed. As it turned out in order to observe that scenario in calculation, the spatial spectrum of initial pulse should contain predominantly small wavenumbers. This is achieved by scaling the width of initial (Gaussian) pulse. As a result, the calculated shape (figure 1 (c)) of the wave and general evolution tendencies are in good agreement with experimental observations (compare figure 1 (a) and (b)).

![Figure 1](image-url)

**Figure 1.** Solitary wave evolution for water Re=10. (a) – experimental results, (b) calculation results. (c) – shape of calculated wave at T=110ms.

Here and after in figures axis X is the downstream distance is directed along the gravity, Y is the transversal coordinate tied to the center of the pulse (coincides with the central section of the computational domain), h is the dimensional film thickness, H=h/hN and T is the time from the impact (for experiments). Since the initial conditions of experiments and calculations are quite different, in order to achieve the same phase of evolution in the experiment and calculation, different time and, correspondingly, distance are required. Therefore beginnings of axis T and X are shifted on this difference for each regime.

The second scenario of evolution is the wave train formation (figure 2 (a)) which is observed in experiments for small excitation energies. As it turned out in order to observe that scenario in calculation the spatial spectrum of initial pulse should contain large wavenumbers. As is known on the surface of film only perturbations with wavenumbers lying within a restricted range can develop. Out of this region perturbations are damped. In calculation the initial pulse size was adjusted so it contains the whole range of undamped wavenumbers. The results of calculation using such initial pulse are shown in figure 2 (b).
Figure 2. Wave train formation for WGS Re=16. (a) – experimental result, (b) – calculation.

The main difference between solitary wave evolution and wave train formation is connected with the growth rate of the main hump. And for the case of wave train the evolution occurs in the following manner: Initially the shape of the evolving pulse is similar to the shape of the solitary wave (figure 3 (a) – (c)). Then such elements as capillary precursor and perturbation behind the dimple grow faster than main hump (figure 3 (d) – (f)). And at some point their height equalize (figure 3 (g) – (i)). After that no main hump can be marked up. At the same time another humps both in the front and in the back of the wave train grow up which leads to expanding of the wave train. As it can be seen in figures 2 and 3 such behavior agrees well with experimental observation. Small difference of the wavelength is the result of difference of the initial pulses. But all elements of the wave pattern are calculated correctly.

Additionally using different Gaussian wavelets the initial pulse was constructed with spatial spectrum containing the region of large wavenumbers without small ones. In that case evolution leads to wave train formation in contrast to the case of single wave propagation realized for initial pulse which spectrum contains only small wavenumbers.
Figure 3. Comparison of experiment with calculation for WGS Re=10. (a), (d), (g) – experimentally registered shape of waves at different distances. (b), (e), (h) – calculated shape of waves at appropriate positions. (c), (f), (i) – cross sections along and across the flow passing through a maximum of main peak; solid line – experimental profiles, dashed – calculation results.

Conclusions
Despite mentioned above difficulties of comparison between experiments and calculations the main trends of wave evolution calculated with help of Shkadov’s model are in good agreement with experimental results. So depending on spectral distribution of the initial pulse calculation results show wave evolution by one of two scenarios observed in experiments: solitary wave or wave train. And for appropriate initial pulse there exist even the quantitative agreement between calculated and experimental observations of evolving waves.
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