Advanced Fruit Fly Optimization Algorithm and Its Application to Irregular Subarray Phased Array Antenna Synthesis

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ABSTRACT In this paper, an advanced fruit fly algorithm (FOA) is proposed and applied in subarray phased array antenna synthesis. The proposed algorithm introduces orthogonal crossover, quantum selection and simulated annealing operations on the individuals, and then combines them by using an adaptive expansion-contraction factor. Accordingly, a linear generation mechanism of candidate solution based fruit fly algorithm (LGMS-FOA) is generated, in which individuals are selected in a highly balanced way, and the poor solutions are still accepted with a varying probability during the iteration. These mechanisms help the proposed algorithm enhance the population diversity and global searching capability but avoid falling into local optimum. Numerical classical unimodel benchmark functions are provided to test the proposed algorithm (OLFOA) in comparison with other advanced algorithms. In addition, to further validate its superiority, the proposed algorithm is applied to handle the subarray array synthesis of several tough planar and circular apertures with different array sizes and subarray shapes. Simulation results show that the proposed OLFOA can achieve better performance than other improved evolutionary algorithms.

INDEX TERMS Irregular subarray, fruit fly algorithm, orthogonal crossing, quantum behavior, simulated annealing, array synthesis.

I. INTRODUCTION

Recently, subarray technology has attracted considerable interests in radar and communication systems, because it can greatly reduce the cost and complexity of the system by reducing the number of antenna splitters and phase shifters [1]. Currently, the subarray partition and optimization problem remains to be a large barrier in the development of subarray technology. Various solutions have been explored to solve this problem. The subarray partitioning method based on genetic algorithm (GA) was proposed in [2]–[5]. These methods could solve the partitioning of irregular subarray, but got poor performance in solving the exact partition problem. Then, a method based on analytic schemata-driven optimization, which achieves exact partition in low-/medium-sized rectangular subarrays, was introduced in [6]. The X algorithm-based subarray tiling method, which exhibits great efficiency and accuracy in irregular subarray partitioning, was proposed in [7]. Although subarray technology applied in array antenna design helps to alleviate the high grating lobe to certain extent, exploring high-performance intelligent optimization algorithms for array synthesis, especially for large array or huge elements, is urgently desired.

Evolutionary algorithms have long been considered as efficient candidates to solve antenna synthesis problems. Genetic algorithm (GA) and particle swarm optimization (PSO) are two well-known optimization algorithms for array antenna synthesis [8], [9]. Simulated annealing algorithm (SA), ant colony optimization algorithm (ACO), and differential evolution algorithm (DE) are also widely used in array antenna optimization [10]–[12]. In addition, several recently proposed algorithms, such as invasive weed optimization
algorithm (IWO) [13] and artificial bee colony algorithm technique (ABC) [14], have been successfully implemented in antenna design and optimization. These algorithms appear to be less and less effective in solving complex problems with high dimension and large computation overhead.

In 2012, motivated by the behavior of fruit fly in seeking food, a new fruit fly optimization algorithm (FOA) [15] was proposed. As a new global evolutionary algorithm, FOA is well-known for its simple principle, few adjustable parameters, strong global optimization capability, and fast convergence speed. Similar to other evolutionary algorithms, FOA also shows the disadvantages of easily falling into the local optimum and limited performance when calculating multi-dimensional problems. To address these shortcomings, a modified fruit fly algorithm integrated with the linear generation mechanism of candidate solution (LGMS) was proposed in [16], showing that the algorithm performance was greatly improved without changing any natural concept. However, the shortcomings of the fruit fly algorithm were still not addressed completely.

In this study, an advanced LGMS-FOA, namely, OLFOA, is proposed. By introducing orthogonal cross [17] and quantum selection [19] operations on the individuals, the proposed OLFOA combines them with an adaptive expansion-contraction factor to improve the convergence performance in solving high-dimensional complex problems. On this basis, the simulated annealing [20] strategy is introduced to further enhance the diversity of the population. The superior performance of the proposed algorithm has been validated via both numeral benchmark function tests and high-dimensional subarray phased array antenna synthesis problems with different array sizes and subarray shapes.

This paper is organized as follows: Section II introduces the concepts of FOA and LGMS-FOA. Section III presents a detailed architecture of the proposed OLFOA. Section IV makes comparisons between OLFOA and several recently proposed algorithms. Section V describes the application of OLFOA in solving high-dimensional subarray phased array antenna synthesis problems. Section VI draws the conclusion.

II. FOA AND LGMS-FOA

A. FOA

Fruit fly algorithm is a global optimization method based on the food-finding behavior of the fruit fly population. In the process of searching for food in the search space, other fruit flies are randomly searched for a certain range from the current location of fruit fly closest to the food. The search process is repeated starting from the fruit fly that is currently closest to the food until the food is found. Fig. 1 illustrates the food-finding iterative process of the fruit fly swarm. The x- and y-axis are the search spaces for the fruit fly population. The circles indicate the fruit fly swarms. The solid arrows indicate the food-finding direction of the fruit fly population, whereas the dashed arrows indicate the food-finding direction of fruit fly individuals. \( x_{\text{best}}, y_{\text{best}} \) indicates the fruit fly that is closest to the food in the \( i \)th iteration. \( x_n, y_n \) indicates the location of \( n \)th fruit fly individual in the x- and y-axis, respectively.

The specific steps of the FOA algorithm are shown as follows:

**Step 1.** The maximum number of iterations (\( \text{genmax} \)), population size (\( \text{popsize} \)), range of random fruit flies (LR), and random direction of fruit fly (FR) in the fruit fly algorithm are initialized. Then, the location of the fruit fly population is defined.

\[
\begin{align*}
x_{\text{axis}} &= \text{rand}(LR) \\
y_{\text{axis}} &= \text{rand}(LR)
\end{align*}
\]

where the \( x_{\text{axis}} \) and \( y_{\text{axis}} \) represent the searching center of the fruit fly population in x- and y-axis, respectively.

**Step 2.** Each fruit fly is updated with a random search direction and distance by

\[
\begin{align*}
x_i &= x_{\text{axis}} + \text{rand}(FR) \\
y_i &= y_{\text{axis}} + \text{rand}(FR)
\end{align*}
\]

**Step 3.** The distance (Dist) between the location of the fruit fly and the origin and the taste decision value (S) of the fruit fly (S characterizes the distance from the food) are calculated as follows:

\[
\begin{align*}
\text{Dist}_i &= \sqrt{x_i^2 + y_i^2} \\
S_i &= \frac{1}{\text{Dist}_i}
\end{align*}
\]

**Step 4.** The taste decision value is substituted into the fitness function to calculate the corresponding fitness value (Smell).

\[
\text{Smell}_i = \text{fitness function}(S_i)
\]

**Step 5.** The optimal fitness value (bestsmell) and corresponding location (bestindex) of the fruit fly individual are obtained by

\[
[\text{bestsmell}, \text{bestindex}] = \min(\text{Smell})
\]

**Step 6.** The optimal fitness is set as the global optimal value (gsmell), and the corresponding location of fruit fly is used
as the searching center coordinate of the next iteration.

\[
\begin{align*}
g_{\text{smell}} &= \text{bestsmell} \\
x_{\text{axis}} &= x(\text{bestindex}) \\
y_{\text{axis}} &= y(\text{bestindex})
\end{align*}
\]

**Step 7.** Steps 2–6 are repeated until the terminal criterion is met or the maximum number of iterations is reached.

**B. LGMS-FOA**

From the description of the fruit fly algorithm, it is discovered that, given that the taste determination value \(S\) is always greater than zero, the fruit fly algorithm cannot express the negative number in the dimension level. Moreover, given that the taste judgment value cannot satisfy the uniform distribution, the searching of the fruit fly algorithm in the dimension is not uniform; thus, the fruit fly algorithm cannot effectively solve the complex optimization problem [16].

With this inspiration, the LGMS-FOA proposed by D. Shan in 2014 [16] expands the range of taste judgment values by introducing the LGMS mechanism. By doing so, the new taste judgment value satisfies the uniform distribution. The process of LGMS-FOA is shown as follows:

**Step 1.** The maximum number of iterations \((\text{genmax})\), population size \((\text{popsize})\), searching coefficient \((n)\), initial weight \((w_{0})\), and weight coefficient \((\alpha)\) are initialized. Then, the location of the fruit fly population is defined.

\[
x_{\text{axis}} = n \times \text{rand}(\text{domain})
\]

where \(x_{\text{axis}}\) represents the searching center of the fruit fly population.

**Step 2.** Each fruit fly is updated with a random search direction and distance by

\[
\begin{align*}
x_{i} &= x_{\text{axis}} + w \times \text{rand}(\text{domain}) \\
w &= w_{0} \times \alpha^{\text{gen}}
\end{align*}
\]

where \(\text{gen}\) represents the current number of iterations.

**Step 3.** The taste decision value \((S)\) of individual flies is calculated on the basis of LGMS.

\[
S_{i} = x_{i}
\]

**Step 4.** The taste decision value is substituted into the fitness function to calculate the corresponding fitness value \((\text{Smell})\).

\[
\text{Smell}_{i} = \text{fitness function}(S_{i})
\]

**Step 5.** The optimal fitness value \((\text{bestsmell})\) and corresponding location \((\text{bestindex})\) of the fruit fly individual are obtained by

\[
[\text{bestsmell}, \text{bestindex}] = \min(\text{Smell})
\]

**Step 6.** The optimal fitness is set as the global optimal value \((g_{\text{smell}})\), and the corresponding location of the fruit fly is used as the searching center of the next iteration.

\[
g_{\text{smell}} = \text{bestsmell}
\]

**Step 7.** Steps 2–6 are repeated until the terminal criterion is met or the maximum number of iterations is reached.

**III. OLFOA**

For the proposed OLFOA, on the basis of the LGMS, the orthogonal cross and quantum selection mechanism (OQSM) and simulated annealing strategies (SM) are employed to achieve good performance and highly stable convergence. The two mechanisms are presented below in details.

**A. OQSM**

1) QUANTUM BEHAVIOR MECHANISM

Quantum behavior mechanism [17] has been already used in quantum particle swarm optimization (QPSO) to improve the performance of the algorithm. In the original PSO, the velocity of the particles and the search space of each iteration are limited, which means that the searching area cannot cover the entire feasible searching space. Therefore, the PSO algorithm cannot guarantee the absolute convergence to the global optimal solution [18]. In the quantum space, particles with quantum motion characteristics can appear at any point in space with a certain probability and can reach the entire feasible solution space. As a result, the capability of the PSO algorithm to obtain the global optimum is greatly improved.

On the basis of the principle of quantum behavior, the locations of the particles are updated by

\[
X_{r,j}(\text{gen} + 1) = g_{\text{best}r,j} + \frac{W(\text{gen})}{2} \ln\left(\frac{1}{u(\text{gen})}\right) u \sim u(0, 1)
\]

where \(X\) means the location of particle, \(g_{\text{best}r,j}\) means the best location of the particle in the last iteration, \(\text{gen}\) means the number of current iteration, \(r\) and \(j\) represent the number of individuals and the dimension of the locations, respectively. \(W(\text{gen})\) means the probability that a particle will appear at a relative point and is expressed by the following formula:

\[
W(\text{gen}) = 2\beta \cdot |m_{\text{best}}(\text{gen}) - X_{r,j}(\text{gen})|
\]

where \(m_{\text{best}}\) means the mean value of the best position and is expressed by

\[
m_{\text{best}}(\text{gen}) = \frac{1}{\text{popsize}} \sum_{r=1}^{\text{popsize}} g_{\text{best}r}(\text{gen})
\]

where \(\text{popsize}\) represents the population size. \(\beta\) in (21) means expansion-contraction factor and is expressed by

\[
\beta = 0.5 \times (\text{genmax} - \text{gen})/\text{gen} + 0.5
\]

where \(\text{genmax}\) represents the maximum number of iterations.

The expansion-contraction factor is vital in the QPSO because it controls the convergence speed and search range of the particle. In QPSO, the expansion–contraction factor scales linearly and lacks flexibility. In this study, a weighting factor \(w_{f}\) with values between 0 and 4 is introduced in
OLFOA. The adaptive expansion–contraction factor value is determined as follows, which replaces (23) in OLFOA.

$$\beta = 0.5 \times ((\text{gen max} - \text{gen})/\text{gen max})^\alpha + 0.5$$  \hspace{1cm} (24)

Fig. 2 shows the adaptive expansion–contraction factor versus iterations for different values of wf. The corresponding expansion–contraction factors have different convergence trends due to the different values of wf. A small wf value results in a slow convergence speed and a wide searching range. On the contrary, a large wf value leads to a fast convergence speed.

2) ORTHOGONAL DESIGN MECHANISM

As a multi-factor optimized experimental design method, the orthogonal design [19] introduces a series of orthogonal arrays for different numbers of factors and levels. The level represents the variable that affects the experimental results, whereas the factor represents the value of these variables. Different level and factor combinations will generate different samples and obtain different experimental results. The orthogonal crossover mechanism shows particular efficiency in handling the optimization problem, which doesn’t require an exhaustive search over all the possible combinations to find the optimal solution.

By using orthogonal arrays, the levels and factors are rearranged in a certain order in accordance with the orthogonality, and only the most representative combinations are selected instead of calculating all possible combinations to find a suboptimal solution, thereby greatly reducing the computational burden. Specifically, the process of orthogonal crossover mechanism is established as follows:

I. Orthogonal array $L_M(Q^N)$ is selected for the orthogonal crossing, where $L$ denotes a Latin square, $Q$ and $N$ denote the numbers of levels and factors, which are used for orthogonal crossing, respectively. $M$ represents the number of combinations that is obtained by orthogonal crossing. Generally, a large number of $M$ and $Q$ will obtain good experiment results with an increase in the complexity. After numerical experiments, the $L_9(3^4)$ has been proven to be an efficient orthogonal array by considering performance and calculation complexity [19]. Table 1 shows the orthogonal array $L_9(3^4)$.

| Combination | Factor1 | Factor2 | Factor3 | Factor4 |
|-------------|---------|---------|---------|---------|
| 1           | 1       | 1       | 1       | 1       |
| 2           | 1       | 2       | 2       | 2       |
| 3           | 1       | 3       | 3       | 3       |
| 4           | 2       | 1       | 2       | 3       |
| 5           | 2       | 2       | 3       | 1       |
| 6           | 2       | 3       | 1       | 2       |
| 7           | 3       | 1       | 3       | 2       |
| 8           | 3       | 2       | 1       | 3       |
| 9           | 3       | 3       | 2       | 1       |

Table 1. Orthogonal array $L_9(3^4)$.

In Table 1, each row represents a combination of different factors and levels. For example, the sixth combination consists of the Factor 1 of Level 2, Factor 2 of Level 3, Factor 3 of Level 1, and Factor 4 of Level 2. The combinations are the most representative candidates for all possible combinations.

The orthogonality of the orthogonal array is referred to the following: (1) for the factor in any column, each level occurs for the same number of times; (2) for the two factors in any two columns, each combination of two levels occurs for the same number of times; and (3) the selected combinations are uniformly distributed over the entire space of all possible combinations [19].

II. The three individuals belonging to the same experiment are divided into four factors:

$$E_1 = (e_{1,1}, e_{1,2}, e_{1,3}, e_{1,4})$$  \hspace{1cm} (25)
$$E_2 = (e_{2,1}, e_{2,2}, e_{2,3}, e_{2,4})$$  \hspace{1cm} (26)
$$E_3 = (e_{3,1}, e_{3,2}, e_{3,3}, e_{3,4})$$  \hspace{1cm} (27)

where $E$ means the level of experiments and $e$ means the factor of experiments.

III. Three individuals are taken into the orthogonal array $L_9(3^4)$ and nine combinations are obtained.

$$O_d = (e_{md,1}, e_{md,2}, e_{md,3}, e_{md,4}) \quad 1 \leq d \leq 9$$  \hspace{1cm} (28)

where $O_d$ means the $d$th combination and $m_d$ means the parameter determined by the orthogonal array.

B. SM

The simulated annealing algorithm [20] is originally designed to search the metal state with the lowest energy during metal temperature dropping, i.e., metal annealing process. To avoid the situation that the optimal state cannot be reached due to rapid cooling, the simulated annealing algorithm accepts the solution, which is worse than the current with a certain probability.

In the proposed OLFOA, the similar mechanism named simulated annealing mechanism (SM) is introduced. Specifically, if the current solution is better than the previous one, the SM does not work; otherwise, the current solution will be
still taken as the global optimum with a certain probability determined by SM.

C. OLFOA STEPS

With the improved mechanisms, the process of the OLFOA is described as follows:

Step 1. Initialization

The population size \( \text{popsize} \), searching coefficient \( n \), annealing temperature \( T \), final temperature \( T_{\text{fin}} \), maximum iteration number \( \text{gen}_{\text{max}} \), and weighting factor \( \text{wf} \) are initialized. Then, the locations of the fruit fly swarm are initialized and updated as follows.

\[
x = n \times \text{rand} \text{(domain)}
\]  

(29)

Step 2. Evaluation of the swarm

In this step, the \( S \) parameters of the fruit fly individual, which are calculated by LGMS, will be substituted into the objective function to obtain the smell values of the swarm. The optimal smell value is selected as the global optimal value \( g_{\text{smell}} \), and the corresponding fruit fly location is denoted as \( g_{\text{best}} \).

\[
S_i = x_i
\]  

(30)

\[
\text{Smell}_i = \text{fitness function}(S_i)
\]  

(31)

\[
[g_{\text{smell}}, g_{\text{best}}] = \min(\text{Smell})
\]  

(32)

Step 3. OQSM update of the swarm

The locations for each individual of the fruit fly population are calculated three times on the basis of (20), and the three locations are set as levels of orthogonal array. Each level is divided into four parts and set as four factors. To describe the division process visually, its diagram is presented as follows:

![Diagram of division process.](image)

The \( S \) parameters of the individual flies are assumed to have 12 dimensions. Each black circle represents the \( S \)-parameter of each dimension of the fruit fly individuals. Each row represents a fruit fly individual, the vertical numbers represent the levels, and the horizontal numbers represent the factors respectively. As shown in Fig. 3, the taste decision value \( S \) of the three fruit fly individuals are divided into four equal parts (as equal as possible). Each of the divided parts is brought into Table 1 on the basis of its number of levels and factors, and nine new individuals are reconstituted after the crossover.

Then, the smell value of all nine combinations of fruit fly individuals can be obtained on the basis of the fitness function (31). The fruit fly individual corresponding to the best value of smell among the nine individuals will be selected as the new fruit fly individual. Then, this process is repeated until the entire fruit fly swarm are updated.

Step 4. SM update of global optimum

Calculate the smell value of the updated fruit fly swarm. The best smell value \( (\text{bestsmell}) \) and corresponding location of the specific fruit fly \( (\text{bestindex}) \) is found:

\[
[\text{bestsmell}, \text{bestindex}] = \min(\text{Smell})
\]  

(33)

The SM triggering conditions are defined as follows:

\[
\exp\left(-\left(\text{bestsmell} - g_{\text{smell}} \right)/T \right) > \text{rand}(0, 1)
\]  

(34)

If the current \( \text{bestsmell} \) is better than the \( g_{\text{smell}} \), or the \( g_{\text{smell}} \) and \( \text{bestsmell} \) meet the SM triggering conditions in (34), set \( \text{bestsmell} \) as the global optimal value \( g_{\text{smell}} \), and the corresponding location of specific fruit fly \( (\text{bestindex}) \) will be set as \( g_{\text{best}} \).

Then, \( T \) is updated by

\[
T = T - (T - T_{\text{fin}})/(\text{gen}_{\text{max}} - \text{gen})
\]  

(35)

Step 5. Termination

Steps 3–4 is repeated until the maximum number of iterations is reached or the optimal value is found.

IV. EXPERIMENTS

In this section, OLFOA will be evaluated via the classical benchmark functions. The test results are compared with those of other improved algorithms to show the superior performance of the proposed OLFOA algorithm. Table 2 lists the improved algorithms in the experiments.

| Algorithm | Method | Authors and reference |
|-----------|--------|-----------------------|
| FOA | Fruit fly optimization algorithm | Pan et al. [15] |
| IFFOA | Improved fruit fly optimization algorithm | Pan et al. [22] |
| CMFOA | Chaotic fruit fly optimization algorithm | L. Wu et al. [23] |
| MSFOA | Multi-scale cooperative mutation fruit fly optimization algorithm | Y. Zhang et al. [24] |
| CEFOA | Co-evolution fruit fly optimization algorithm | X. Lian et al. [25] |
| MFOA | Mixed modified fruit fly optimization algorithm | Pan et al. [26] |
| RMPSO | Repository and mutation-based particle swarm optimization | B. Jana et al. [27] |
| BLPSO | Biogeography-based learning particle swarm optimization algorithm | X. Chen et al. [28] |
| JADE | Adaptive differential evolution algorithm | J. Zhang et al. [29] |
| NGHS | Novel global harmony search algorithm | D. Zou et al. [30] |

A. EXPERIMENTAL RESULTS

In this subsection, the capability of OLFOA is compared with the algorithms via 16 classic benchmark functions presented in Table 3. These algorithms are implemented by MATLAB 2016a and run on an i5 3.3 GHz CPU with 8 GB memory.
TABLE 3. Benchmark functions.

| ID   | Function name       | Equation                                                                 | Global optimum                  | Domain          |
|------|---------------------|--------------------------------------------------------------------------|---------------------------------|-----------------|
| F1   | Exponential problem | $f(x) = -\exp(-0.5\sum x_i^2)$                                           | $x^* = 0$ and $f(x^*) = -1$     | $-1 \leq x_i \leq 1$ |
| F2   | Quatic              | $f(x) = \sum x_i^4 + \text{rand}()$                                      | $x^* = 0$ and $f(x^*) = 0$      | $-5.12 \leq x_i \leq 5.12$ |
| F3   | Schwefels           | $f(x) = \sum_{i=1}^n x_i \exp(-\|x_i\|)$                                | $x^* = 0$ and $f(x^*) = 0$      | $-100 \leq x_i \leq 100$ |
| F4   | Schwefels 2.21      | $f(x) = \max_j |x_j|, 1 \leq j \leq n$                                      | $x^* = 0$ and $f(x^*) = 0$      | $-100 \leq x_i \leq 100$ |
| F5   | Schwefels 2.22      | $f(x) = \sum_{i=1}^n |x_i| + \|x_i\| |x_i|$                               | $x^* = 0$ and $f(x^*) = 0$      | $-100 \leq x_i \leq 100$ |
| F6   | Sphere              | $f(x) = \sum x_i^2$                                                      | $x^* = 0$ and $f(x^*) = 0$      | $-100 \leq x_i \leq 100$ |
| F7   | Axis                | $f(x) = \sum_{i=1}^n x_i^2$                                              | $x^* = 0$ and $f(x^*) = 0$      | $-5.12 \leq x_i \leq 5.12$ |
| F8   | Shifted Sphere      | $f(x) = \sum_{i=1}^n x_i^2 + f_{bias}$                                   | $x^* = 0$ and $f(x^*) = 0$      | $-100 \leq x_i \leq 100$ |
| F9   | Shifted Schwefels 1.2 | $f(x) = \sum_{i=1}^n x_i^2 + f_{bias}$                                  | $x^* = 0$ and $f(x^*) = 0$      | $-100 \leq x_i \leq 100$ |
| F10  | Ackley              | $f(x) = -20 \exp\left(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}\right) - \exp\left(-\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i)\right) + 20 + e$ | $x^* = 0$ and $f(x^*) = 0$      | $-32 \leq x_i \leq 32$ |
| F11  | Alpine              | $f(x) = \sum_{i=1}^n |x_i| \sin x_i + 0.1x_i$                                    | $x^* = 0$ and $f(x^*) = 0$      | $-100 \leq x_i \leq 100$ |
| F12  | Griewank            | $f(x) = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{n}}\right) + 1$ | $x^* = 0$ and $f(x^*) = 0$      | $-600 \leq x_i \leq 600$ |
| F13  | Cosine Wave         | $f(x) = -\frac{1}{8} \sum_{i=1}^n \exp\left(-x_i^2 + x_i^2 + 0.5x_{i+1}\right)$ $\times \cos(4\sqrt{x_i^2 + x_{i+1}^2 + 0.5x_{i+2}})$ | $x^* = 0$ and $f(x^*) = 1-n$    | $-5 \leq x_i \leq 5$ |
| F14  | Neumaier 3 problem  | $f(x) = \sum_{i=1}^{n^2} (x_i - 1)^2 - \sum_{i,j} x_i x_{i+j}$          | $x^* = \pm(n+1-l)$ and $f(x^*) = 1-n$ | $-n^2 \leq x_i \leq n^2$ |
| F15  | Rastrign            | $f(x) = \sum_{i=1}^n (x_i^2 - 10 \cos(2\pi x_i) + 10)$                 | $x^* = 0$ and $f(x^*) = 0$      | $-5.12 \leq x_i \leq 5.12$ |
| F16  | Salomon             | $f(x) = 1 - \cos(2\pi \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}) + 0.1 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}$ | $x^* = 0$ and $f(x^*) = 0$      | $-100 \leq x_i \leq 100$ |

capacity. These algorithms are applied to minimize 16 benchmark functions with dimensions Dim = 30, 50. The population size is set as 80. In order to make a fair comparison, the maximum generation number is set to 300 for each function, which is consistent with the references. The experiment results are obtained by independently running 10 times for fairness. The control parameters of all improved algorithms are set as the same as in [15] and [22]–[30]. In OLFOA, $T$ is set as 900, $T_{fin}$ is set as $0.001 \times 10^{-10}$, and the weighting factor $w_f$ is set as 0.4.

Table 4 shows the mean values and standard deviations of different fruit fly algorithms evaluated via numerical
benchmark functions with dimensions of 30 and 50. OLFOA exhibits computational benefit and superior performance compared with FOA, MFOA, IFFO, and CMFOA. OLFOA performed better than MSFOA and CEFOA with a faster convergence speed for most of the benchmark functions (F1, F4, F6, F7, F8, F9, F11, and F15), whereas it does not show outstanding performance compared to MSFOA in F2 and F13. Compared with CEFOA, the OLFOA does not
work well in F3, F5, F10, F12, F14 and F16. Thus, conclusion can be drawn that the proposed OLFOA prevails over other algorithms for most of the benchmark functions. In the case with a dimension of 50, similar results have been acquired except both the mean value and standard deviation of all algorithms have certain deterioration.

**B. CONVERGENCE ANALYSIS**

To describe the convergence performance of the algorithms visually, Figs. 4 and 5 provide the average convergence results of randomly selected test functions (F2, F4, F5, F6, F11, and F15) after 300 iterations with dimensions of 30 and 50, respectively. The dB fitness value is used and displayed in the y-axis of the figures in the optimization of F2, F4, F5, and F11. Only for benchmark functions F2 and F5, OLFOA does not show outstanding performance compared to other algorithms in terms of the convergence and optima. However, for the other unimodel benchmark functions, OLFOA shows its obvious advantage in stability and efficiency in acquiring the global optimum, especially for benchmark functions F11 and F15. For benchmark functions F4 and F6, OLFOA obtained the best fitness value and showed a fast convergence speed. These results indicate that the proposed OLFOA algorithm outperforms numerous improved FOA methods (IFFO, CEFOA, CMFOA, MFOA, and MSFOA) and the original FOA in global optimum and convergence capability.
C. EXTRA EXPERIMENTAL RESULTS

Considering the array synthesis application of the proposed algorithm has a specific range of $[0, 1]$, it is necessary to carry out tests on benchmark functions in the specific range for further performance verification. In this subsection, OLFOA is compared with CEFOA, and the recently proposed improved FOAs in Table 5. The search space of fruit fly individuals are set as $[0, 1]$.

Table 6 shows the mean value and the standard deviation of those FOAs for different benchmark functions in the range of $[0, 1]$ with a dimension of 30. Conclusion can be drawn that the performance of CEFOA deteriorates severely when applied in such test functions, whereas the proposed OLFOA obtains better and more stable results when compared with other improved FOA algorithms.

D. COMPARISON WITH OTHER ALGORITHMS

In this subsection, OLFOA is compared with other different types of improved algorithms (RMPSO, BLP SO, JADE, and

TABLE 5. Comparison algorithms in extra tests.

| Algorithm | Method | Authors and reference |
|-----------|--------|-----------------------|
| A-F.gms-FOA | Averager engine linear generation mechanism of candidate solution of FOA | A. Darvish et al. [21] |
| IFOA | Improved fruit fly algorithm | Xu et al. [31] |
TABLE 6. Results of extra basic benchmark functions with dimension = 30.

| Dim | F1       | F2       | F3       | F4       |
|-----|----------|----------|----------|----------|
|     | Mean     | Std      | Mean     | Std      | Mean     | Std      |
| AE-LGMS-FOA | -1.04E-1 | 3.57E-2 | 3.31E-2 | 7.76E-2 | 4.49     | 3.36     | 7.95E-4 | 1.54E-4 |
| IPFOA | -2.19E-1 | 4.10E-2 | 3.54E-2 | 1.82E-2 | 3.18     | 2.68     | 6.68E-6 | 2.97E-6 |
| CEFOA | -0.51    | 3.27E-1 | 232.5    | 6.77E-1 | 7.54E5   | 8.82E-5 | 0.7     | 5.99E-4 |
| OLOFA | -1       | 0.00     | 2.9E-3   | 1.78E-3 | 9.37E-1  | 3.34E-1 | 5.27E-9 | 3.75E-8 |

| Dim | F5       | F6       | F7       | F8       |
|-----|----------|----------|----------|----------|
|     | Mean     | Std      | Mean     | Std      | Mean     | Std      |
| AE-LGMS-FOA | 5.96E-7  | 8.58E-6 | 5.46E-23 | 4.14E-23 | 3.33E-25 | 4.90E-25 | -4.49E2 | 9.12E-14 |
| IPFOA | 6.68E-6  | 2.85E-6 | 8.82E-26 | 2.65E-26 | 6.47E-27 | 3.43E-26 | -4.49E2 | 2.05E-17 |
| CEFOA | 20       | 2.79E-2 | 15       | 1.02E-15 | 4.23E3   | 5.87E-3 | -3.35E2 | 6.65E-24 |
| OLOFA | 1.87E-15 | 2.5E-14 | 0.00     | 0.00     | 0.00     | 0.00     | -450    | 0.00      |

| Dim | F9       | F10      | F11      | F12      |
|-----|----------|----------|----------|----------|
|     | Mean     | Std      | Mean     | Std      | Mean     | Std      |
| AE-LGMS-FOA | -4.49E2 | 7.14E-9 | 3.98E-7 | 9.66E-6 | 5.68E-11 | 4.89E-10 | 2.69E-23 | 6.77E-23 |
| IPFOA | -4.48E2 | 5.31E-12| 7.29E-9 | 4.47E-7 | 2.74E-12 | 2.78E-10 | 7.32E-21 | 5.02E-21 |
| CEFOA | -4.35E2 | 8.83E-30| 3.57     | 5.14E-1 | 4.04E-4 | 1.91E-4 | 5.45E-1 | 1.46E-2  |
| OLOFA | -450     | 0.00     | 3.64E-14| 1.13E-14| 5.74E-20 | 1.32E-18 | 0.00     | 0.00      |

| Dim | F13      | F15      | F16      |
|-----|----------|----------|----------|
|     | Mean     | Std      | Mean     | Std      | Mean     | Std      |
| AE-LGMS-FOA | -28.57  | 2.12E-3 | 4.98E-4 | 5.79E-3 | 4.63E-1 | 4.24E-1 |
| IPFOA | -27.71  | 1.78E-5 | 1.89E-4 | 2.18E-4 | 5.17E-2 | 2.98E-2 |
| CEFOA | -23.84  | 7.87E-9 | 4.06E2  | 8.00E1  | 6.28E-1 | 3.05E-1 |
| OLOFA | -29      | 0        | 1.95E-8 | 1.50E-7 | 3.13E-3 | 7.71E-3 |

NGHS) to justify its performance. The parameter settings are the same as those in Part A of Section IV.

Table 7 shows the mean value and standard deviation of those algorithms for different benchmark functions with dimensions of 30 and 50. Obviously, OLOFA exhibits its superior advantage in global optimum and convergence stability when compared with RMPSO, BLPSO, and JADE. When compared with NGHS, OLOFA performs better for most of the benchmark functions (F1, F2, F4, F5, F6, F7, F8, F9, F10, F11, F12, F13, F15, and F16) with fast and stable convergence speed, whereas OLOFA does not show competitive performance advantage compared to NGHS for F3 and F14.

From all these results and analysis above, conclusions can be drawn below:

1) OLOFA obtains much more superior values among all the algorithms in most cases. OLOFA can achieve excellent results with high stability in different dimensional and population problems.

2) OLOFA shows a strong local search capability and stable convergence rate among most of the algorithms.

3) OLOFA can demonstrate its competitive advantage in the benchmark tests in the range of [0, 1], which exactly corresponds to the application considered later in this paper.

V. OLOFA APPLICATION

In this section, OLOFA is applied into several different subarray phased array synthesis problems to prove its performance. In the simulation, the excitation of each subarray element is set as one dimension of the individual fruit fly, and the excitation of the entire subarray antenna element constitutes the location of individual fruit fly, the number of the swarm is set as 80, T is set as 900, Tfin is set as 0.001 × 10^{-10}, the weighting factor wf is set as 0.4, and the maximum iteration number is set as 100.

A. ANTENNA ARRAY NOTATION

Considering a planar array antenna with N rows and M columns of elements arranged in a rectangular grid in the x- and y-axis, then the radiation pattern of planar array can be expressed as:

\[
FF(\theta, \phi) = \sum_{n=1}^{N} \sum_{m=1}^{M} I_{nm}f_{nm}(\theta, \phi) \times e^{-j(kd_{xmn} \sin \theta \cos \phi + kd_{ymn} \sin \theta \sin \phi + \phi_{nm})} \tag{36}
\]

where \(I_{nm}\) is the element excitation current amplitude, \(\phi_{nm}\) is the element excitation current phase, \(f_{nm}(\theta, \phi)\) is the array element pattern, \(d_{xmn}\) and \(d_{ymn}\) are the distance from array element to axis origin in x- and y-axis respectively.

Put this array antenna into the subarray tiling and each subarray consists of C elements. Then the maximum number of subarrays (L) that can be used to tile the entire aperture is \(L = N \cdot M / C[2]\). If the shape of the subarray is the same, after rotating and folding, eight different shapes exist on the array aperture. Furthermore, if the subarray is symmetrical, the subarray shapes are reduced to four or even two [3].

In this study, the subarray tiling method based on X algorithm [7] is adopted to achieve the exact partition of subarrays on the array aperture. After the exact partition, the radiation
TABLE 7. Results of five algorithms on basic benchmark functions with dimension \(= 30, 50\).

| Dim | F1 | F2 | F3 | F4 |
|-----|----|----|----|----|
|     | Mean | Std | Mean | Std | Mean | Std | Mean | Std |
| RMPSO | 30 | 3.71E-1 | 5.48E-1 | 6.67E-1 | 1.44 | 5.78E-1 | 2.26E-1 | -4.41E2 | 3.72E-4 |
| 50 | 6.35 | 4.42 | 1.92 | 1.77 | 3.56 | 1.85 | -4.23E2 | 1.36E-4 |
| BLPSO | 30 | 1.12E-1 | 2.92E-2 | 1.78E-4 | 4.22E-4 | 2.88E-4 | 5.60E-3 | -4.40E2 | 1.41E-3 |
| 50 | 1.08E-1 | 6.19E-2 | 1.25E-4 | 3.59E-4 | 1.93E-4 | 3.11E-3 | -4.40E2 | 1.03E-3 |
| JADE | 30 | 1.12E-1 | 5.29E-2 | 1.78E-4 | 4.22E-4 | 2.88E-4 | 5.60E-3 | -4.43E2 | 3.89E-1 |
| 50 | 1.17E-1 | 3.51E-2 | 1.19E-4 | 3.08E-6 | 5.70E-5 | 2.03E-4 | -4.33E2 | 2.94E-1 |
| NGHS | 30 | 4.55E-7 | 3.24E-6 | 6.99E-8 | 3.18E-8 | 1.50E-6 | 8.62E-8 | -4.46E2 | 1.06E-1 |
| 50 | 2.44E-7 | 2.28E-6 | 6.14E-8 | 1.27E-8 | 9.12E-7 | 7.29E-8 | -4.40E2 | 1.01E-1 |
| OLFOA | 30 | 1.50E-7 | 2.12E-7 | 0.00 | 0.00 | 0.00 | -450 | 0.00 |
| 50 | 3.25E-6 | 1.89E-6 | 0.00 | 0.00 | 0.00 | -450 | 0.00 |

For engineering practicality, the optimization object of subarray antenna array synthesis problems is set as the subarray excitation current amplitude \(I_{sub}\), which ranges from 0 to 1. The optimization target is the maximum sidelobe level of the antenna pattern. The fitness function can be expressed as:

\[
\text{Fitness} = \frac{\text{abs}(\theta, \phi) \in FF}{\text{FF}_{max}(FF)}
\]

where \(FF_S\) represents the area of sidelobe.

B. RESULTS OF ARRAY SYNTHESIS

The first synthesis case is a 40-element array (5 \times 8) with half-wavelength spacing, and a rectangular subarray consisting of two connected elements is desired [6], as shown in Fig. 6. Fig. 7 presents the far-field radiation pattern of the arrays optimized after 100 iterations, and the corresponding PSL is \(-23.46\) dB, which is 4.57 dB lower than that in [6].

Arrays of 128 (8 \times 16) and 432 (18 \times 24), with inter-element spacing of half-wavelength, are considered in the second and the third synthesis cases, respectively.
A subarray with an L-octomino shape consisting of eight connected elements is desired. Fig. 8 and 10 show the acceptable exact partition of the aperture by using the X algorithm. Fig. 9 and 11 present the far-field radiation patterns of the arrays optimized by OLFOA after 100 iterations. The corresponding PSL for these two arrays are $-19.98$ and $-30.64$ dB, respectively.

The fourth case is a 349-element circular aperture array with half-wavelength spacing. The subarray consists of eight connected polyhex-shaped elements. As shown in Fig. 12, the array is divided into 55 subarrays by the X algorithm (including 42 eight-element polyhex-shaped subarrays and 13 independent elements) [7]. The subarray partition and the beam pointing of the array are set to be the same as in [7]. Fig. 13 displays the far-field radiation pattern of the arrays optimized after 100 iterations, and the corresponding PSL is $-30.76$ dB. This optimization result is $3.56$ dB lower than the maximum sidelobe achieved in [7], which indicates the
superiority of the proposed algorithm when applied in the design of irregular subarray antennas.

Table 8 presents the comparison results in terms of PSL optimized by those different algorithms under four irregular subarray cases. For fair comparison, the parameters of the other algorithms are set the same as those of the proposed algorithm. From Table 8, conclusion can be drawn that the proposed OLFOA shows the superior PSL performance when compared with the other algorithms for all these cases.

VI. CONCLUSION

In this paper, an advanced fruit fly optimization algorithm, namely, OLFOA has been proposed. Individuals are selected in a highly balanced way by adding the QOQM and SM mechanisms. The population diversity, global searching capability, and ability to escape from the local optimum of the algorithm are significantly improved. The OLFOA has been tested via numerous classical benchmark functions in comparison with other improved FOAs and advanced algorithms to demonstrate its capability. Then, OLFOA is applied to synthesize four different cases of subarray antenna array of planar and circular aperture. Simulation results show that the proposed algorithm presents a superior performance both in benchmark function test and the subarray antenna array synthesis problems.

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TABLE 8. Performance comparison of different subarray antennas synthesized by different methods of three algorithms.

| Synthesis case | Case 1 | Case 2 | Case 3 | Case 4 |
|----------------|--------|--------|--------|--------|
| Uniform excitation | -12.05 | -13.17 | -13.29 | -17.34 |
| LGMS-FOA [16] | -21.97 | -17.19 | -26.45 | -25.47 |
| AE-LGMS-FOA [21] | -22.63 | -18.68 | -28.28 | -27.89 |
| GA | -19.26 | -15.74 | -24.5 | -24.14 |
| DE | -18.13 | -16.61 | -25.1 | -25.43 |
| QPSO [17] | -18.64 | -16.79 | -24.4 | -23.85 |
| OLFOA | -23.46 | -19.98 | -30.64 | -30.76 |
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