Deformability of a Volume-Compressed Concrete

A L Krishan¹, V I Rimshin², M A Astafeva³

¹ Professor, Doctor of Technical Sciences, Head of Department of Building Design and Constructions, Nosov Magnitogorsk State Technical University, Uritsky, 11, Magnitogorsk 455000, Russia
² Corresponding member of Russian Academy of Architecture and Construction Sciences, Doctor of Technical Sciences, Professor, Head of the Institute of urban development of Russian Academy of Architecture and Construction Sciences, Moscow 107031, Russia
³ Post-Graduate Student of the Department of Industrial and Civil Engineering, Nosov Magnitogorsk State Technical University, Uritsky, 11, Magnitogorsk 455000, Russia

E-mail: kris_al@mail.ru

Abstract. This article is devoted to a theoretical study of the volume-compressed concrete deformability. The derivation of the formula for the concrete deformations determination with indirect reinforcement at maximum compressive stresses is given. This formula adequately takes into account the main factors affecting the amount of deformation unlike the known empirical formulas. The proposed formula use for designing will increase the reliability of calculations the compressed reinforced concrete and reinforced concrete structures with indirect reinforcement bearing capacity, performed on the nonlinear deformation model.

1. Introduction
For vertical load-bearing structures, reinforced concrete and steel-reinforced concrete structures are often used, where concrete works under volume compression conditions. Reinforced concrete columns with indirect reinforcement by grids or spirals, as well as concrete-filled steel tube columns are often used for these purposes [1-6]. Reinforced concrete structures with indirect reinforcement have high load-carrying capacity and survivability. The field of its practical application is constantly expanding. Therefore, it is necessary to improve the existing methods for its designing.

2. Relevance
The load-carrying capacity calculation of such structures is usually performed on limiting forces method. The current Russian and European design standards [7-9], taking into account the nature of these structures reinforcement, indicate to make calculations based on a nonlinear deformation model.

It is known that the practical implementation of deformation calculation for structures with volume-compressed concrete is difficult. One of them is the necessity of reliable determinations of deformations of concrete under conditions of triaxle compression. The most interesting things are deformations at maximum compressive stresses $\varepsilon_{cc1}$ and limiting deformations preceding the destruction of concrete $\varepsilon_{cc2}$ (figure 1). It is these deformations, along with the strength of the volume-
compressed concrete, determine the deformation diagram parameters necessary for the deformation calculation.

![Deformation Diagram](image)

**Figure 1.** The diagrams of deformation for uniaxial compressed (1) and volume-compressed (2) concrete.

There are many proposals for determining deformations $\varepsilon_{cc1}$ and $\varepsilon_{cc2}$ in the literature. Some of them are presented in [10-18]. Most researchers recommend using the following formula

$$\varepsilon_{cc1} = \varepsilon_{c1} \left( 1 + k \frac{\sigma_{cr}}{f_c} \right),$$  

(1)

where $\varepsilon_{c1}$ is the deformation at the maximum stress of uniaxial compressed concrete;

$f_c$ is the strength of uniaxial compressed concrete;

$\sigma_{cr}$ is lateral pressure on concrete due to the presence of indirect reinforcement;

$k$ is empirical coefficient.

The coefficient values are usually taken in the range from 17.5 [11] to 20.5 [14].

The magnitude of the limiting strain $\varepsilon_{cc2}$ is also determined by empirical coefficients, for example, according to the following formula

$$\varepsilon_{cc2} = 5.5 \varepsilon_{c2},$$  

(2)

where $\varepsilon_{c2}$ is the ultimate deformation of uniaxial compressed concrete.

In the case of circular pipe-shaped columns, European norms [8] suggest another formula

$$\varepsilon_{cc1} = \varepsilon_{c1} \left( \frac{f_{cc}}{f_c} \right)^2,$$  

(3)

where $f_{cc}$ is the strength of volumetric compressed concrete.

The main disadvantage of the above formulas is that they are all obtained from the results of the corresponding experiments. It greatly limits the scope of their application. Empirical formulas are reliable only for those types of structures, concrete, reinforcement, according to the results of experiments they are obtained. Nowadays effective designs of pipe-concrete columns with a pre-crushed concrete core, various cross-sectional shapes, and also with spiral reinforcement of the
concrete core have been developed [19, 20]. The empirical formulas obtained earlier are inappropriate for such columns.

Therefore, it seems very relevant to obtain universal formulas based on theoretical premises.

3. Statement of the problem
The main task is to obtain a formula for calculating the strain at the maximum compressive stresses \( \varepsilon_{\text{cc}1} \). The generic formula for its calculation is obtained on the phenomenological approach.

The diagram of concrete deformation for steel and concrete structures with indirect reinforcement can be obtained by numerical method with considering the force resistance of a short centrally compressed element. In this case, all structural and geometric (except length) parameters are taken by analogy with the designed structure [5].

There is no lateral pressure on the concrete core of structures with indirect reinforcement at low levels of the compressive load [6,20] or it is so small and can be neglected. It means that the initial section of the concrete deformation diagram (with small deformations) coincides with the diagram of uniaxial compressed concrete. There is lateral pressure on the concrete core at a certain load. It leads to the appearance of volume compression in concrete structure, its strength and deformation increase (figure 2). In the process of gradual increase in load, lateral pressure also increases. Consequently, each value of the lateral pressure will have its own concrete deformation diagram.

4. Theoretical part
The authors considered the diagram of volume-compressed concrete deformation (a curve 3 in figure 2) corresponding to the maximum reached stress and compared it with the uniaxial compressed concrete diagram. It follows from the above that the initial modulus of elasticity for both diagrams is the same.

The total deformation at the vertex of this diagram can be regarded as the sum of the elastic and plastic components

\[
\varepsilon_{\text{cc}1} = \varepsilon_{\text{el}} + \varepsilon_{\text{pl}},
\]

(4)

The elastic part \( \varepsilon_{\text{el}} \) of the total deformation \( \varepsilon_{\text{cc}1} \), taking into account the assumed equality of the volume-compressed and uniaxial compressed concrete modulus of elasticity, is determined with formula

\[
\varepsilon_{\text{el}} = \varepsilon_{\text{el}} \frac{f_{\text{cc}}}{f_{\text{c}}},
\]

(5)

where \( \varepsilon_{\text{el}} \) is the elastic part of the deformation \( \varepsilon_{\text{cc}1} \).

It is obvious that the plastic part of the total deformation \( \varepsilon_{\text{pl}} \) can also be expressed by corresponding component of the total deformation at the top of the uniaxial compressed concrete \( \varepsilon_{\text{pl}} \) and the ratio \( f_{\text{cc}} / f_{\text{c}} \). The volume-compressed concrete deformability is more intensive than the strength growth (see formula (2)), so the formula for determination is written in the following way:

\[
\varepsilon_{\text{pl}} = \varepsilon_{\text{pl}} \left( \frac{f_{\text{cc}}}{f_{\text{c}}} \right)^n,
\]

where \( n > 1 \) is a constant.
where $m$ is the exponent, $m > 1$.

The plastic part of the deformation at the top is determined by the formula

$$\varepsilon_{pl} = \varepsilon_{cl} - \frac{f_c}{E_c}. \quad (7)$$

Taking (7) into account, formula (6) takes the form:

$$\varepsilon_{pl} = \left( \varepsilon_{cl} - \frac{f_c}{E_c} \right) \left( \frac{f_{cc}}{f_c} \right)^m. \quad (8)$$

Thus, the total deformation of the volume-compressed concrete at the maximum stress is determined by the formula

$$\varepsilon_{cc1} = \varepsilon_{cl} \left( \frac{f_{cc}}{f_c} \right)^m + \frac{f_{cc}}{E_c} - \frac{f_c}{E_c} \left( \frac{f_{cc}}{f_c} \right)^m. \quad (9)$$

The performed statistical analysis showed that the best match with the results of the experiments corresponds to a value of $m = 2.5$. Therefore, the final formula for determining the longitudinal deformation at the top of the volume-compressed concrete deformation diagram is written as:

$$\varepsilon_{cc1} = \varepsilon_{cl} \left( \frac{f_{cc}}{f_c} \right)^{2.5} \left[ 1 - V_c + V_c \left( \frac{f_{cc}}{f_c} \right)^{-1.5} \right], \quad (10)$$

where $V_c$ is the coefficient of elasticity at the top of the diagram for uniaxial compressed concrete, determined by the formula

$$V_c = \frac{f_c}{\varepsilon_{cl} E_c}. \quad (11)$$
The analysis of the formula (10) shows that the strain value $\varepsilon_{cc1}$ depends not only on the value $\varepsilon_{c1}$ and the degree of growth of concrete strength under triaxle compression, but also on the compressive strength and elastic modulus of the initial uniaxial compressed concrete. This connection seems more logical and corresponds to modern concepts of concrete deformation characteristics.

Relative deformation at the end of the deformation diagram of a volume-compressed concrete core, according to the recommendations of [8], is determined by the formula

$$\varepsilon_{cc2} = \varepsilon_{cl} \frac{\varepsilon_{cc1}}{\varepsilon_{c1}},$$

where $\varepsilon_{c2} = 0.0035$ – the relative strain at the end of the deformation diagram for uniaxial compressed concrete.

5. Conclusions

The formula for determining the longitudinal deformation of a volume-compressed concrete at the apex of its deformation diagram was obtained phenomenologically. The main advantage of this dependence in comparison with the previously proposed empirical formulas is adequate accounting all the main factors affecting the amount of deformation. The proposed formula in the design practice will increase the reliability of bearing capacity of compressed reinforced concrete and reinforced concrete structures with indirect reinforcement calculations based on a nonlinear deformation model.

6. References

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