On the Systematic Errors in the Detection of the Lense-Thirring Effect with a Mars Orbiter

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We show here that the recent claim of a test of the Lense-Thirring effect with an error of 0.5 % using the Mars Global Surveyor is misleading and the quoted error is incorrect by a factor of at least ten thousand. Indeed, the simple error analysis of [1] neglects the role of some important systematic errors affecting the out-of-plane acceleration. The preliminary error analysis presented here shows that even an optimistic uncertainty for this measurement is at the level of, at least, \( \sim 3026\% \) to \( \sim 4811\% \), i.e., even an optimistic uncertainty is about 30 to 48 times the Lense-Thirring effect. In other words by including only some systematic errors we obtained an uncertainty almost ten thousand times larger than the claimed 0.5 % error.

1 Systematic Errors in the Measurement of the Lense-Thirring Effect with a Mars Orbiter: Introduction

Before discussing the error analysis related to the strange claim of the measurement of the Lense-Thirring effect with the Mars Global Surveyor discussed in [1, 2], we stress that the statement in section (1) and in the abstract of [1], that the measurement of the Lense-Thirring effect with the LAGEOS satellites “is still controversial” is simply wrong and misleading, in fact all the literature quoted by Iorio, apart from the literature signed by Iorio only, fully confirms the 10% error budget of this LAGEOS measurement.

On other hand, even the papers by Iorio simply claim that the error budget should have been twice larger. Therefore even if one considers the claims of Iorio to be right, claims that however have been shown to be wrong in several detailed papers not only by the authors of [3], the error would be
of about 20 % and of course this still is a measurement of the Lense-Thirring effect.

However, we show here that having some familiarity with error analysis and with data analysis, one can derive an error budget that differs from Iorio’s error budget by at least a factor of about ten thousand!

Furthermore, anybody can repeat the LAGEOS data analysis and the details of this LAGEOS analysis are clear to everybody since they have been published in a number of detailed papers, see, e.g., [4]; indeed, in a forthcoming paper it is shown how the results of different, independent, groups do confirm the previous 2004 measurement with the LAGEOS satellites. On the other hand, the analysis of other experiments such as Gravity Probe B could hardly be understood or repeated.

Finally, Iorio forgets to mention the relevant literature that led to the measurement of the Lense-Thirring effect using the LAGEOS satellites (see, e.g., I. Ciufolini, Phys. Rev. Lett., 1986 [5]; Int. Journ. of Mod. Phys., 1989 [6]; and Il Nuovo Cimento A, 1996 [7]).

In section (1) of [1] the equation \( \Delta N = a (1 + e^2)^{0.5} \sin i \Delta \Omega \) is displayed, this equation can be found in any text book of celestial mechanics and relates the shift of the nodal longitude \( \Delta \Omega \) with the out-of-plane shift \( \Delta N \). Therefore, any serious discussion of any measurement using this equation cannot avoid a treatment of all the errors that enter in the modeling of \( \Delta N \), i.e., a treatment of all the nodal uncertainties must be provided.

In the following, we just treat some of the main sources of error in the modeling of the out-of-plane acceleration. The orbital perturbations that we treat here are: (a) perturbations due to the main spherical harmonics of the Mars gravity field; (b) errors in the knowledge of the orbital parameters of the Mars Global Surveyor; (c) uncertainties in the modeling of solar radiation pressure and planetary radiation pressure. A discussion about the other error sources may be the subject of a following, more detailed, paper.

Here we shall compare directly the uncertainties in the nodal rates. Translating this figures into the mean normal shift \( \Delta N \) is just a trivial step.

2 Preliminary Error Analysis

2.1 Uncertainties in the gravitational perturbations

The nodal drift due to the even zonal harmonics is:
\[ \dot{\Omega}_{\text{class}} = -\frac{3}{2} n \left( \frac{R_\oplus}{a} \right)^2 \frac{\cos I}{(1 - e^2)^2} \left\{ J_2 + J_4 \left[ \frac{5}{8} \left( \frac{R_\oplus}{a} \right)^2 \times \right. \right. \\
\left. \left. \times \left( 7 \sin^2 I - 4 \right) \frac{\left( 1 + \frac{3}{2} e^2 \right)}{(1 - e^2)^2} \right] + \Sigma N_{2n} \times J_{2n} \right\} \]  

(1)

In the case of the Mars gravity field, a state-of-the-art model is for example the GGM1041c model [8]. In this recent, accurate, model we find that the value of the quadrupole coefficient \( C_{20} \) of Mars is \(-8.7450461309664714 \times 10^{-4}\), where its formal uncertainty is \(8.6998585172904000 \times 10^{-11}\). Even though this uncertainty may not include systematic errors we shall use it for our preliminary estimate of the total error. The corresponding uncertainty would however be optimistic; for example if we take the difference between the second degree even zonal coefficient of this Mars gravity field model and the one of the other recent Mars gravity model GMM-2B [9], we find \(8.57721772895559 \times 10^{-10}\), i.e., an uncertainty ten times larger; this last figure might indeed reflect the systematic errors in \( C_{20} \).

Similarly the GGM1041C value of the even zonal coefficient of Mars of degree four, \( C_{40} \), is: \(5.1227082083113746 \times 10^{-6}\), however its formal uncertainty is: \(7.6882432484647995 \times 10^{-11}\). Once again this is just a formal uncertainty but we shall use it in our optimistic error analysis.

By then inserting these two uncertainties in the equation (1) we find a nodal rate error of about 63 milliarcsec/yr due to the uncertainty \( \delta C_{20} \) and a nodal rate error of about 112 milliarcsec/yr due to \( \delta C_{40} \).

We have further calculated the uncertainty only up to degree ten, i.e., we have only included the first 5 even zonal harmonics uncertainties, once again this will produce a very optimistic error budget since the MGS spacecraft is orbiting at only about 400 km of altitude from the Mars surface and therefore higher even zonal harmonics uncertainties would be very important in the final error budget.

By only including in our calculation the first 5 even zonal harmonics, we found, in the hypothesis of non-independent errors, a total uncertainty of 707 milliarcsec/yr. Once again this figure is very optimistic since it neglects the large errors due to the harmonics of degree > 10.

Now, by taking the Lense-Thirring nodal rate of 33 milliarcsec/yr calculated by Iorio, by neglecting the important errors due to higher even zonal harmonics uncertainties and by just using the formal uncertainties to estimate the nodal rate error (due to the uncertainty in the even zonal harmonics up to degree 10) we then have an error 21.4 times the size of the
Lense-Thirring effect, that is:

**Error due to First 5 Mars Even Zonals = 2140 % the Lense-Thirring effect**

**Uncertainties in the GM of Mars**

In [11] several values of the GM of Mars, obtained with different techniques, are reported. From these values, it is possible to infer a relative uncertainty in the GM of Mars of about $2 \times 10^{-7}$. By plugging this uncertainty in equation (1) we then find an uncertainty in the nodal drift of MGS of about 66 milliarcsec/yr, that is about:

**Error due uncertainty in GM of Mars = 200 % the Lense-Thirring effect**

**2.2 Uncertainties in the Mars Global Surveyor orbital parameters**

In [1] an uncertainty of 15 cm in the semimajor axis of the Mars Global Surveyor is assumed. Even by assuming this uncertainty to be realistic, by plugging it in equation (1), describing the nodal drift of the Mars orbiter, we find an uncertainty of about 88 milliarcsec/yr, i.e.,

**Error due to uncertainty in semimajor axis of MGS = 267 % the Lense-Thirring effect**

Next, we estimate the error due to the uncertainty in the inclination of the Mars Global Surveyor. In order to provide a figure for this uncertainty, we assume an error in the inclination that at the MGS altitude is comparable in size with the error in the semimajor axis, i.e., 15 cm. By thus considering an error of 15 cm this translates into an error of about $3.9610^{-8}$ rad in the inclination. Finally, by plugging this uncertainty in equation (1) we find an error of about 504 milliarcsec/yr; this large uncertainty is due to the quasi-polar orbit of the Mars Global Surveyor which makes it very sensitive to any small change or modeling error in the inclination. We then have:

**Error due to uncertainty in inclination of MGS = 1527 % the Lense-Thirring effect**
For simplicity, we have neglected here the error due to the uncertainty in the eccentricity of MGS.

**Preliminary calculation of the uncertainty in modeling solar and planetary radiation pressure**

In [10], see also figure (3) of [10], the size of the solar radiation pressure acceleration on the Mars Global Surveyor is reported to be equal to about $7.1 \times 10^{-6} cm/s^2$; the planetary radiation pressure to be about $8.4 \times 10^{-7} cm/s^2$ and the MGS atmospheric drag $3.8 \times 10^{-7} cm/s^2$. On other hand, the Lense-Thirring acceleration is on MGS of the order of $10^{-9} cm/s^2$. Since the MGS spacecraft, contrary to the simple spherical shape of the LA-GEOS satellites, has a complex shape, the modeling of the non-gravitational accelerations is a rather difficult task. Even by assuming a 10% uncertainty in the modeling of the out-of-plane radiation pressure effects on MGS, even by optimistically neglecting the atmospheric drag on the MGS node due to its density variations and finally even by assuming an optimistic reduction of the overall nodal drift due to solar radiation pressure by a factor 0.0085, i.e., by a factor equal to the MGS eccentricity, and thus by neglecting phenomena such as spacecraft eclipses and thermal drag, we have that the unmodeled nodal acceleration due to solar and planetary acceleration is $0.1 \times 0.0085 \times (7.1 + 0.84) \times 10^{-6} cm/s^2$, that is $6.749 \times 10^{-9} cm/s^2$. Even though a precise calculation can be performed following the analysis of [7][12] it is clear that the size of the radiation pressure unmodeled effects is then at least:

$\text{Error in solar and planetary radiation pressure} = 675 \% \text{ the Lense-Thirring effect}$

### 3 Conclusions

In conclusion, for the gravitational perturbations we have:

$\text{Error due to Gravitational Perturbations} = 2342 \% \text{ the Lense-Thirring effect}$

for the uncertainties in the orbital parameters, we have:

$\text{Error due to MGS orbital parameters uncertainties} = 1794 \% \text{ the}$
Lense-Thirring effect

and for the uncertainties in the radiation pressure effects, we have:

Error in solar and planetary radiation pressure = 675 \% the Lense-Thirring effect

If we sum these errors we get an error of about 4811 \%. Even by taking the root-sum-square of these uncertainties we finally have a RSS error on MGS of 3026 \% of the Lense-Thirring effect! However, it must be noted that this figure is very optimistic since we have neglected a number of large perturbations such as the systematic errors in the uncertainties of the first five even zonal harmonics and the errors in the harmonics of degree higher than 10!

Considering that in [1] is reported an error of about 0.5 \% of the Lense-Thirring effect, our optimistic estimate of the real error is a factor about ten thousand larger than what quoted in [1].

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