Granular Motion in a Rotary Kiln:  
the Transition from Avalanching to Rolling

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Abstract

We report measurements of flow transitions, from avalanching to rolling, for granular material in rotary kilns. In the avalanching mode, the surface slips periodically; in the intervals between avalanches, all particles rotate with the kiln. In the rolling mode, the surface particles slide down continuously; the material underneath the surface rotates with the kiln.

Our measurements give Froude numbers ($Rw^2/g$) for transitions, which are significantly different for sand and TiO$_2$ powder.

For the avalanching mode, we measured cycle times and deduced $t_{12}$, the avalanche time; $t_{12}$ was also measured directly by video photography. For kilns of diameters 0.2–0.5 m, both methods give $t_{12}$, of order 1–2 sec and it appears to be proportional to $\sqrt{l}$, $l$ being the chord length of the granular bed, the maximum distance of fall for avalanche material.

Simple theory, assuming the avalanche particles slide down a frictional surface, gives fair estimates of $t_{12}$ and may be a basis for predicting avalanche-to-rolling transitions in large industrial kilns.

1. Introduction

A rotary kiln for processing granular material is usually a cylinder rotating slowly about its axis, inclined at a few degrees to the horizontal. Granular material is fed continuously at the top end of the cylinder. Within the cylinder, the material forms a bed which may occupy about one third of the cylinder volume. The bed surface is inclined to the horizontal at about the angle of repose for the granular material. A full-size industrial kiln is typically about 3 m diameter and 50 m long, and rotates once in 5–10 minutes. At these slow rotation speeds, the flow regime of the granular material may be in one of two alternative modes, as follows.

(1) The bed may be in "avalanching" mode, otherwise described as "slumping": for most of the time, the whole bed rotates at the same angular velocity, $\omega$, as the kiln, thus undergoing "solid body" rotation. Then there is an avalanche, starting when the bed surface reaches the static angle of repose $\gamma_r$: during the avalanche, material from the upper part of the bed surface slides rapidly down; when the avalanche stops, the bed surface is inclined at a lesser angle $\gamma_i$, the dynamic angle of repose. Solid body rotation of the whole bed ensues, until the surface inclination again reaches angle $\gamma_r$ when avalanching occurs and the cycle is repeated.

(2) At higher kiln speeds, the motion of near-surface particles is continuous, the "rolling" mode. Most of the bed rotates with the kiln in solid body rotation, but the surface and near-surface particles fall continuously down the slope, whose inclination is approximately $\gamma_r$.

This paper is concerned with the transition from avalanching to rolling. In all cases centrifugal accelerations are small, i.e., the Froude number $Fr << 1$, where $Fr = Rw^2/g$, $R$ being the kiln radius and $g$ the acceleration of gravity. The surface of the bed is essentially flat. The avalanching/rolling transition was addressed by Henein et al [1, 2] who proposed the Froude number as a suitable parameter to define the transition from avalanching to rolling. Their experiments showed that very low values of $Fr$, less than $10^{-5}$, are consistent with avalanching. Then there is a transition region where $10^{-5} < Fr < 10^{-4}$; values of $Fr$ above $10^{-4}$ are consistent with the rolling mode.

The use of Froude number is important for scale-up. If the transition from avalanching/slumping to rolling were uniquely described by $Fr$, then large
Kilns would have the transition at lower speeds than small kilns.

The work described here was a study of the transition from avalanching/slumping to rolling in drums of diameters 194, 288 and 500 mm, to test the effect of scale-up, albeit over a small range of R. Experiments were done with two granular materials, namely (i) sand in the size range 300–500 μm and (ii) titanium dioxide powder as discharged from a rotary kiln. The sand was chosen as a free-flowing granular material. The titanium dioxide powder represented the powder near the kiln discharge: it had a wide range of particle sizes, with a mean particle size of about 250 μm and a very wide size distribution with particle diameters up to about 10 mm; the powder was somewhat cohesive.

The results from observations of avalanching to rolling transitions for the three drum diameters and the two powders are presented in terms of Froude number.

There were also observations of cycle time for the avalanching mode, this being the time interval between one avalanche and the next, the form of the powder bed being identical at the beginning and end of each cycle: for example a cycle could be assumed to begin when the inclination of the powder bed is a maximum. Analysis of the relation between cycle time and rotation speed, ω, casts light on the mechanism of avalanching, particularly the time for an avalanche, which is finite. While this does not yet give a new criterion for scale-up, it suggests how an improved criterion might be developed.

2. Experimental

2.1 Apparatus

The experiments used a cylindrical steel drum of internal diameter 500 mm, length 300 mm, with a horizontal axis. The drum was flanged at one end: mounted on the flange was a flat transparent Perspex plate, diameter 600 mm, so that the granular bed in the drum could be viewed from the front. This plate contained a small opening port, with a Perspex lid forming a flush internal face; the port was used to load powder into the drum and to unload. The other end of the drum was vertical steel plate, welded to the cylindrical part and connected to a shaft at the back: this shaft supported the drum and was driven by a variable speed motor with speed control and interchangeable gearbox. Thus the drum could be rotated at speeds from 0.1 to 6 revolutions per minute. The cylindrical part of the drum was lined with sandpaper.

To study the effects of varying drum diameter, two cylindrical Perspex inserts were made, of diameters 194 and 288 mm. Each was closed at the front end by a 600 mm diameter Perspex plate that replaced the above mentioned 600 mm diameter plate on the drum. The two cylindrical Perspex inserts were each closed at the back end to give a (length/diameter) ratio = 3/5, as for the 500 mm drum, and the cylindrical parts were lined with sandpaper. When the drum operated with an inserted cylinder, the granular bed was contained entirely within the insert.

In this way, there were three alternative rotary drums of diameters 194, 288 and 500 mm with the capability of viewing the slumping or rolling granular bed through a transparent end plate.

2.2 Experiments: slumping/rolling observations

A number of experiments were performed to test the effect of the nature of the granular material and the percentage fill of the drum on the bed behaviour i.e., whether it was (i) slumping, (ii) transition or (iii) rolling. Sand of size range 300–500 μm and TiO₂ particles discharged from a kiln were used. These TiO₂ particles were in two forms namely (i) raw calciner discharge with a wide size range, up to 10 mm, or (ii) with the larger particles sieved out, so that there was still a wide size range, but up to 1 mm.

The following experiments were made with the objective of plotting bed behaviour diagrams, such as Figures 1 and 2, which were suggested by Henein et al. [1]. The steps in the experiments were as follows:

(i) An amount of granular material was introduced into the cylinder, which was then rotated to give a bed with a flat surface, albeit inclined. With the drum rotation stopped, the chord length of the surface was measured and from thence the percentage fill calculated; the percentage fill is the ratio (area of the crescent-shaped cross section of the bed)/πR².

(ii) With the drum rotating steadily, the angular speed was measured by timing one revolution. The bed behaviour was classified as slumping, transitional or rolling, using the following definitions:

(i) In the slumping or avalanching mode, there was a clearly cyclical motion: over a distinct period, the whole bed rotated with the drum, in solid body rotation; then there was an avalanche. The times for these separate parts of the cycle were the subject of further measurements, see below.

(ii) In the rolling mode, the surface layer of particles moved down continuously. Below this surface layer, which was only a few millimetres thick, the...
main part of the bed moved with the drum, in solid body rotation.

(iii) There was a rather ill-defined transition region, in which some parts of the surface were falling continuously; other parts were subject to cyclical motion. As will be evident from the results, the transitional region included an appreciable range of drum speeds.

2.3 Experiments: cycle time in the slumping mode

With the drum speed slow enough to give well-defined slumping, the cycle time was measured for a range of drum speeds, for each of the three drum diameters. Two methods of observation were used, as follows.

(i) The time for ten slump cycles was recorded. This measurement was repeated nine times, so there were ten recordings of the time for ten slump cycles. The drum rotation speed was measured by simply timing a revolution. These observations gave the total cycle time $t_{13}$. From analysis of these results, given below, the duration of the avalanche, $t_{12}$, could be deduced.

(ii) Direct measurements of the avalanche duration, $t_{12}$, were obtained from video pictures of the cyclical motion. The video camera was set up to observe the granular bed through the transparent front wall of the drum. The video picture included a digital stopwatch. The video was played back at a low framing rate: the tape was stopped at the beginning and end of an avalanche; from the stopwatch readings, the time $t_{12}$ for the avalanche was obtained. The kiln speed and the chord length of the particle bed were measured as before, giving the percentage fill. For each speed/percentage fill combination, the video photography and analysis were repeated five times, to give an average value of $t_{12}$.

3. Results and discussion

3.1 Bed behaviour diagrams: Froude number as a scaling parameter

Henein et al. [1] used the bed behaviour diagram, plotting either bed depth or percentage fill against Froude number, $Fr$, and delineating areas of the diagram as slumping, transition or rolling. Figures 1 and 2 show our data plotted in this form as percentage fill against $Fr$, the plotting points indicating transitions, measured in the way described above.

Figures 1 and 2 show bed behaviour diagrams for our data, respectively for sand and raw TiO$_2$ calciner discharge. For sand, Figure 1, it appears that the slumping/transition boundary is in the region $0.00002 < Fr < 0.00004$; the transition to rolling is less well defined, in the region $0.00006 < Fr < 0.0001$. Figure 2 shows the results for raw TiO$_2$ calciner discharge: here the slumping/transition boundary is in the region $0.0001 < Fr < 0.0002$; the transition to rolling is in the region $0.0006 < Fr < 0.0015$.

From these results, the following conclusions may be drawn:

(1) The Froude number gives a rough guide to transitions in the flow regimes. But the transition Froude numbers are markedly different between sand and raw TiO$_2$. Thus the boundary for slumping is about five times higher for TiO$_2$ as compared with sand; the boundary for transition to rolling is about ten times higher for TiO$_2$ as compared with sand.

(2) With regard to the effect of drum diameter, the data in Figure 1 show that the Froude numbers for
The two transitions tend to increase as the drum diameter decreases; the same effect is approximately true of the data in Figure 2.

The overall conclusion is that although the Froude number is a very rough guide to behaviour, there is certainly not a unique Froude number for each of the two observed transitions. There must therefore be grave doubts about the use of these transition Froude numbers for industrial sized rotary kilns.

With this in mind, the measurements of cycle times in the slumping mode were undertaken, with the objective of understanding the mechanism of the slumping/avalanching mode.

3.2 Cycle time results and analysis

Figure 3 shows the sequence observed at slow rotation speeds with clearly defined avalanching or slumping giving a sequence of bed states 1, 2 and 3 shown in the diagram. An avalanche starts at state 1, when the bed surface is at maximum angle \( \gamma_s \) (degrees), the static angle of repose. Then a finite quantity of granular material slides down the surface. At time \( t_{12} \), the avalanching material comes to rest, relative to the drum, when the inclination of the bed surface is \( \gamma_d \) (degrees), the dynamic angle of repose; this is condition 2 in Figure 3. Between conditions 2 and 3, the granular bed rotates with the drum in solid body motion at angular velocity \( \omega \). At condition 3, the bed is in the same state as at 1. Subsequently the cycle repeats itself, with a cycle time \( t_{13} \), measured in the experiments.

As the angular speed, \( \omega \), varies, it is assumed that the avalanche time \( t_{12} \) is constant, plausible because \( t_{12} \) is usually much less than \( t_{13} \), so the drum rotation during time \( t_{12} \) is small. During the 2–3 sequence, there is solid body rotation, so it follows that

\[
\gamma_s - \gamma_d = \omega(t_{13} - t_{12}) \frac{180}{\pi} \quad \text{and hence}
\]

\[
t_{13} = t_{12} + (\gamma_s - \gamma_d) \frac{\pi}{180\omega}
\]

Figures 4, 5 and 6 show our data for sand, plotted as

\[
t_{13} \text{ against } 1/\omega \text{ from equation (1), this should be a linear relationship for a given powder-in-drum combination. Each of Figures 4, 5 and 6 does indicate a linear relationship between } t_{13} \text{ and } 1/\omega; \text{ extrapolation to } 1/\omega=0 \text{ gives values of } t_{12}, \text{ the avalanche time, shown in Table 1. This analysis neglects the change in path length of the avalanche, } TB \text{ in Figure 3, as}
\]
the percentage fill changes. The effect of path length, TB, on avalanche time t₁₂ is considered further in Section 3.3 below.

Table 1 also shows the slope of the best fit line in each diagram, Figures 4, 5 and 6. From equation (1) the slope should be $(\gamma_5 - \gamma_d)\pi/180$, the difference between static and dynamic angles of repose. There appears to be a size effect, $(\gamma_5 - \gamma_d)$ diminishing as drum size increases. The bed inclinations (i) just before an avalanche and (ii) just after an avalanche, were measured for sand by Powell and Ramsay [5, 6], using photography. Their results suggested $\gamma_s = 33^\circ$ and $\gamma_d = 30^\circ$: thus $\gamma_s - \gamma_d = 3^\circ$, in good agreement with Table 1.

Figure 7 shows the corresponding data for TiO₂, both the raw calciner discharge (sizes up to 10 mm) and the calciner discharge with large particles sieved out, leaving sizes up to 1 mm. The data are much more scattered than for sand, but t₁₂ is still roughly linear against $1/\omega$, and the deduced values of $\gamma_s - \gamma_d$ are all of the same order as with sand. The slopes of the two fitted lines are: (i) $7.2^\circ$ for raw TiO₂ and (ii) $5.0^\circ$ for sieved TiO₂. These angles, representing $(\gamma_s - \gamma_d)$ the difference between static and dynamic angles of repose, are greater than the values for sand, given in Table 1. The higher values of $\gamma_s - \gamma_d$ for TiO₂ are entirely plausible, the TiO₂ powder being somewhat cohesive.

Henein et al. [1] report experimental measurements of slumping frequency $N$ (min⁻¹) against kiln rotation speed, $n$, in revolutions per minute (rpm) for two drum sizes and a variety of granular materials. Their most complete data set, for limestone particles, is shown in Figure 8. Now $1/t_{13}$ is the slumping frequency, and $\omega = 2\pi n/60$ so equation (1) can be transformed, with appropriate conversion of units, to give

$$N = 60/t_{12} + (\gamma_s - \gamma_d)/6n$$  \hspace{1cm} (2)$$

Curve fitting of Equation (2) to the data in Figure 8 gives values of $(\gamma_s - \gamma_d)$ and $t_{12}$; the value of $(\gamma_s - \gamma_d)$ is obtained from the slope at the origin, because $dN/dn = 360/\gamma_5 - \gamma_d)$ at $n=0$ and $t_{12}$ is obtained by curve fitting at finite $n$. This procedure gives $\gamma_s - \gamma_d = 6$ degrees for both kiln diameters, and $t_{12} = 1.13$ sec for kiln diameter $D = 0.4$ m and $t_{12} = 1.57$ sec for $D = 1.06$ m. These parameters were used to give the plots of Equation (2) shown in Figure 8: the curves are in remarkably good agreement with the data. Henein et al. [1] report values of static and dynamic angle of repose: for the limestone of Figure 8 they give $\gamma_s = 40.3^\circ$ and $\gamma_d = 39.6^\circ$ or $36.5^\circ$, so their values of $\gamma_s - \gamma_d$ range from $0.7^\circ$ to $3.8^\circ$. However, their values of $\gamma_d$ were for rolling motion in the kiln, whereas ours are values of $\gamma_d$ at the end of an avalanche, likely to be smaller than the value of $\gamma_d$ for continuous rolling motion.

3.3 Avalanche times $t_{12}$

Bird [3] and Herbert [4] plotted their data for sand as in Figures 4–6. For each drum diameter, they fitted a straight line to the data; by extrapolation to

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**Table 1** Avalanche times $t_{12}$ for sand from Figures 4–6 on the basis of Equation (1)

| Figure Number | Drum diameter (mm) | Avalanche time $t_{12}$ (sec) | $180\pi/\gamma_s - \gamma_d$ (degrees) |
|---------------|-------------------|-------------------------------|---------------------------------------|
| 4             | 194               | 0.6                           | 3.9                                   |
| 5             | 288               | 1.0                           | 3.4                                   |
| 6             | 300               | 1.2                           | 2.4                                   |

**Figure 7** Slump cycle time against $1/\omega$ (angular speed) for Titanium Dioxide powder of wide size range

**Figure 8** Slumping frequency data of Henein et al. [1]. The curves are from Equation (2) with $(\gamma_s - \gamma_d)$ and $t_{12}$ chosen to give best fit to the data.
These data are for a range of values of the frictional force on the particle, parallel to the slope. Hence a range of values of the friction coefficient, between the particle and the slope, to be \( \tan \gamma \); the dynamic friction is relevant because the particle is in motion. The inclination of the slope is \( \gamma \); the normal force between the slope and the particle, per unit mass, is \( g \cos \gamma \). We assume the friction coefficient, between the particle and the slope, to be \( \tan \gamma \); the dynamic friction is relevant because the particle is in motion. The frictional force on the particle, parallel to the slope, is thus \( g \tan \gamma \cos \gamma \); this acts against the gravitational force \( g \sin \gamma \). The net force causing acceleration is \( g(\sin \gamma - \tan \gamma \cos \gamma) \) in place of \( g \sin \gamma \) in deriving Equation (3). It follows that the flight time, with friction, is

\[
t_{12} = \frac{2l}{g(\sin \gamma - \tan \gamma \cos \gamma)^{1/2}}
\]  

Bird [3] and Herbert [4] used, for sand, \( \gamma_s=36^\circ \) and \( \gamma_t=34^\circ \), giving the line plotted in Figure 9, in fair agreement with most of the data. However, Powell [5] and Ramsay [6] give \( \gamma_s=33^\circ \), \( \gamma_t=30^\circ \) from video measurements, so \( \gamma_t-\gamma_s=3^\circ \); in fair agreement with the results in Table 1. Figure 9 shows a plot of Equation (4), using Powell and Ramsay's values of \( \gamma_s \) and \( \gamma_t \).

In spite of the fair agreement between Equation (4) and the data in Figure 9, there must be reservations about its validity for scale-up, as follows:

1. The range of values of \( \sqrt{l} \) in Figure 9 is small. Data are needed for much larger cylinders.
2. There is the unexplained difference between the data in Figure 9 from the video records compared with the data from Figures 4–6.
3. Attempts to plot \( t_{12} \) against \( \sqrt{l} \) for TiO₂, from Figure 7, gave very wide scatter, although the values of \( t_{12} \) were of the same order as those in Figure 8.

4. Conclusions

1. The Froude number \( Fr=R\omega^2/g \), provides a rough guide to flow regime transitions in a rotary kiln. But the Froude numbers for transition vary from one granular material to another. Our results for kiln diameters from 194 to 500 mm show
   (i) For sand, there is avalanching (otherwise described as slumping) for \( Fr<0.00002-0.00004 \), and rolling for \( Fr>0.00006-0.0001 \).
   (ii) For TiO₂ powder, calciner discharge with a wide size range, the corresponding values are: avalanching for \( Fr<0.00001-0.00002 \), rolling for \( Fr>0.00006-0.00015 \).

2. It is by no means certain that these transition Froude numbers are independent of kiln diameter. Hence the transitions for large industrial kilns may be at very different Froude numbers.

3. For the avalanching mode, the granular material moves cyclically: the bed rotates with the kiln in solid body motion till its surface reaches a maximum inclination, the static angle of repose \( \gamma_s \). Then there is an avalanche, of duration \( t_{12} \); when the avalanche terminates, the inclination is \( \gamma_t \), the dynamic angle of repose. The avalanche time was measured in two ways.

   (i) Measuring the total cycle time \( t_{13} \) and plotting it against \( 1/(\text{rotation speed } \omega) \) gave, by extrapolation to \( \omega=0 \), values of \( t_{12} \); these are of order 1–2 sec for our cylinders, 198–500 mm...
(iii) Video photography gave direct measurements of \( t_{12} \).

The values of \( t_{12} \) from method (i) are somewhat lower than from method (ii).

4. The avalanche time \( t_{12} \) is approximately proportional to \( \sqrt{l} \); here \( l \) is the chord length of the sloping granular bed, the maximum length of fall for avalanche material. The times \( t_{12} \) are much greater than the time for a particle to fall down a smooth slope of length \( l \). But \( t_{12} \) is reasonably well predicted by theory for a particle falling down a frictional slope of inclination \( \gamma_s \), the static angle of repose: between the falling particle and the slope, the coefficient of friction is \( \tan \gamma_d \), \( \gamma_d \) being the dynamic angle of repose. The results suggest that for sand, the values are: \( \gamma_s=36^\circ \) with \( \gamma_d=34^\circ \), or \( \gamma_s=33^\circ \) with \( \gamma_d=30^\circ \).

5. The cycle time analysis suggests that for sand, \( (\gamma_s-\gamma_d) \) is between 2.4 and 3.9°; for TiO\(_2\) particles \( (\gamma_s-\gamma_d) \) is about 5°–7°; for Henein et al's [1] limestone B, \( (\gamma_s-\gamma_d) \) is about 6°.

6. The analysis of cycle times and the mechanism of avalanching may lead to an improved scale-up method for predicting flow regimes in large kilns.

Acknowledgement

The authors are grateful to Huntsman Tioxide for loan of the apparatus for this work and for constructive discussions.

Nomenclature

| Symbol | Description | Unit |
|--------|-------------|------|
| D      | Kiln diameter | (m)  |
| Fr     | Froude number | \( \frac{R\omega}{g} \) |
| g      | Acceleration due to gravity | (m/s\(^2\)) |
| l      | Chord length of granular bed | (m) |
| R      | Kiln radius | (m) |
| N      | Slumping frequency | (min\(^{-1}\)) |
| n      | Kiln revolutions per minute | (min\(^{-1}\)) |
| \( t_{12} \) | Avalanche time | (sec) |
| \( t_{13} \) | Complete cycle time for avalanching bed | (sec) |
| \( t_{23} \) | Duration of solid body rotation | (sec) |
| \( \gamma \) | Angle of repose for granular material | (degrees) |
| \( \gamma_d \) | Dynamic angle of repose | (degrees) |
| \( \gamma_s \) | Static angle of repose | (degrees) |
| \( \omega \) | Angular speed of kiln rotation | (rad/sec) |

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The Chemical Engineering Tripos Part II is the final examination of the four year course. The Project Reports are available in the Library of the Department of Chemical Engineering.
Authors' short biography

J.F. Davidson
John Davidson was a Faculty Member in the Department of Chemical Engineering, Cambridge, from 1952 until he retired in 1993; he was Head of Department 1975–1993. Since 1993 he has worked in the Department on research, in collaboration with other Faculty Members, helping to supervise students working for the PhD degree and students undertaking research projects during the final year of the MEng degree course. The paper here given is the outcome of two such final year projects.

D.M. Scott
David Scott has been a Faculty Member in the Department of Chemical Engineering, Cambridge, from 1983. He received his B.A in Mathematics in 1970 and Ph.D. in Theoretical Physics in 1974, both from the University of Cambridge. From 1974 to 1983 he carried out research in Theoretical Physics at the Rutherford Laboratory, the University of Wisconsin-Madison, and in Cambridge. His research interests include granular materials, adsorption and fluid mechanics.

Paul Bird, Oliver Herbert, Andrew Powell and Helen Ramsay were students whose project work for the MEng course in Cambridge is reported here. Their work follows a long tradition in the Department: 40 years of research on fluidization began with MEng projects involving limited funding and no forward planning.