Grand Unification in High-scale Supersymmetry

Junji Hisano\textsuperscript{a,b}, Takumi Kuwahara\textsuperscript{a}, and Natsumi Nagata\textsuperscript{a,c}

\textsuperscript{a}Department of Physics, Nagoya University, Nagoya 464-8602, Japan
\textsuperscript{b}Kavli Institute for the Physics and Mathematics of the Universe (WPI), Todai Institutes for Advanced Study, the University of Tokyo, Kashiwa 277-8568, Japan
\textsuperscript{c}Department of Physics, University of Tokyo, Tokyo 113-0033, Japan

Abstract

A constraint on masses of superheavy gauge and Higgs multiplets at the grand unification (GUT) scale is obtained from the gauge coupling unification in the case of high-scale supersymmetry. We found that all of the particles may lie around a scale of $10^{16}$ GeV so that the threshold corrections to the gauge coupling constants at the GUT scale are smaller than those in the case of the low-energy supersymmetry. In addition, the GUT scale tends to be slightly lower when the gauginos are heavier and, thus, the proton decay rate via the $X$-boson exchange process is expected to be enhanced.
1 Introduction

Supersymmetric grand unified theories (SUSY GUTs) [1][2] are promising candidates of physics beyond the Standard Model (SM). Indeed, they are strongly motivated by an experimental observation which implies that the gauge coupling constants of the SM gauge groups are to be unified at a certain high-energy scale with good accuracy [3–7]. From the observation, the unified scale is estimated as $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV. In the GUTs, new superheavy particles are supposed to appear around the scale, which make the couplings run together above the scale. These particles are naturally expected to have masses of the order of $M_{\text{GUT}}$. They are, of course, beyond the reach of collider experiments, and there is little hope to search for them or to measure their masses directly.

In Refs. [8,9], a way of constraining the masses of superheavy particles indirectly by requiring the gauge coupling unification is discussed and limits on the masses are presented. Later, by applying the same method with more accurate gauge coupling constants, the authors in Ref. [10] derive a more stringent constraint on the masses of the color-triplet Higgs boson, the adjoint Higgs bosons, and the $X$ bosons in the context of the minimal SUSY SU(5) GUT [1][2]. They have found that while the masses of the adjoint Higgs and the $X$ bosons are around $10^{16}$ GeV, the mass of the color-triplet Higgs boson lies in the region of $3.5 \times 10^{14} \text{ GeV} \lesssim M_{H_C} \lesssim 3.6 \times 10^{15}$ GeV, which is significantly below the GUT scale. This implies that the threshold corrections to the gauge coupling constants are still not small, even in the SUSY GUTs. In fact, the analysis is quite sensitive to the mass spectrum in the intermediate scale, such as those of the SUSY particles. In Ref. [10], all of the particles except for gauginos are assumed to be at 1 TeV. The gaugino masses are set to be around the electroweak scale with the GUT relation for the gaugino masses being assumed.

Currently, on the other hand, the SUSY models with heavy sfermions have been widely discussed with a lot of attention [11–18]. Although such models originally have been discussed as a possible candidate of SUSY models from a theoretical point of view, now they are also supported by the latest results of the LHC experiments; no significant excess in the SUSY searches [19–21] and the discovery of the 126 GeV Higgs boson [22,23] both suggest that the SUSY breaking scale is somewhat higher than the electroweak scale. While this high-scale SUSY scenario is difficult to be probed at the collider experiments, it still has a lot of phenomenological consequences which might be checked in other experiments. Recent studies on the subject are given in Refs. [24–36].

As mentioned above, the GUT scale mass spectrum inferred from the indirect analysis presented in Refs. [8,9] is highly dependent on the SUSY mass spectrum. In particular, if the SUSY particles have masses much larger than $O(1)$ TeV, previous results are expected to be changed significantly. In this Letter, therefore, we revisit the analysis in the high-scale SUSY scenario. We carry out the calculation in the minimal SUSY SU(5) GUT with sfermions having masses well above the electroweak scale. We will see that while the constraint on the adjoint Higgs and $X$ boson masses only differs from previous ones slightly, that on the color-triplet Higgs mass is found to be changed by more than an order of magnitude and actually improved in the sense that it may also be around the GUT...
scale. Interestingly enough, this result again indicates that the high-scale SUSY scenario is rather supported than the traditional low-energy SUSY models.

This Letter is organized as follows: in Sec. 2 the high-scale SUSY model which we discuss below is presented, and its phenomenological aspects are briefly described. The mass spectrum of the superheavy particles in the minimal SUSY SU(5) GUT is also displayed there. In the subsequent section, we discuss the method of constraining the masses of the GUT-scale multiplets by means of the renormalization group equations (RGEs) as well as the threshold corrections of the gauge couplings. Then, we show some results in Sec. 4. Section 5 is devoted to conclusions and discussion.

2 Model and Spectrum

Let us begin by presenting a high-scale SUSY model discussed in this Letter. We consider the particle content of the minimal supersymmetric Standard Model (MSSM). Then, the only assumption which we adopt here is that there exists a dynamical SUSY-breaking sector with the Kähler potential having a generic structure. Then, all of the scalar bosons except the lightest Higgs boson acquire masses of the order of the gravitino mass $m_{3/2}$, while the lightest Higgs boson mass is fine-tuned to be $m_h \sim 126$ GeV. The higgsinos may in general have similar masses to the gravitino mass, though they might be suppressed if there are some extra chiral symmetries. The gaugino masses are, on the other hand, generated by the quantum effects [11,37] and, thus, suppressed by a loop factor compared with $m_{3/2}$. There are two kinds of contributions to the gaugino masses; one is the anomaly mediation effect [11, 37] and the other is from the higgsino-Higgs boson loop diagram. These two effects give rise to the gaugino masses as

$$
M_1 = \frac{33}{5} \frac{g_1^2}{16\pi^2} \left( m_{3/2} + \frac{1}{11} L \right),
M_2 = \frac{g_2^2}{16\pi^2} (m_{3/2} + L),
M_3 = -3 \frac{g_3^2}{16\pi^2} m_{3/2},
$$

with

$$
L = \mu_H \sin 2\beta \frac{m_A^2}{|\mu_H|^2 - m_A^2} \ln \frac{|\mu_H|^2}{m_A^2}
$$

representing the higgsino-Higgs boson loop contribution. Here, $M_a$ and $g_a$ $(a = 1, 2, \text{ and } 3)$ are the U(1)$_Y$, SU(2)$_L$, and SU(3)$_C$ gaugino masses and gauge coupling constants, respectively. We use the SU(5) normalization for the U(1)$_Y$ coupling, i.e., $g_1 \equiv \sqrt{5/3}g'$. Further, $\mu_H$ and $m_A$ denote the higgsino and the heavy Higgs boson masses, respectively, which are assumed to be the same order of magnitude as sfermion masses. $\tan \beta$ is the ratio of the vacuum expectation values (VEVs) of the Higgs fields; $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle$. Since the higgsino mass is presumed to be around the gravitino mass, the higgsino-loop contribution $L$ is also expected to be as large as $m_{3/2}$. The values of the gaugino masses
are, however, dependent on the relative phase between $\mu_H$ and $m_{3/2}$. In the following discussion we just assume the gaugino masses are lighter than the scalar masses by a loop factor, and regard them as free parameters.

As mentioned in Introduction, this model has a lot of fascinating features from a phenomenological point of view. They are originally discussed in Refs. [11–18], and recent development is given in Refs. [24–36]. In these works, a typical scale for sfermion masses is taken to be $\mathcal{O}(10^2–10^3)$ TeV, which explains the 126 GeV Higgs boson mass. With such heavy particles the SUSY flavor and CP problems [38] as well as the gravitino problem are considerably relaxed. In this case, the gaugino masses are $\mathcal{O}(1)$ TeV because of an one-loop factor. Note that from Eq. (1) it is found that with a mode rate value of $L$, wino becomes the lightest among gauginos, and thus the lightest SUSY particle in this model. It is quite interesting since the wino dark matter with a mass of 2.7–3 TeV is consistent with cosmological observations [39]. The dark matter might be searched directly [28,40,41] or indirectly [26,42,43] in future dark matter experiments.

Moreover, the gauge coupling unification in this high-scale SUSY model is found to be achieved as precisely as that in the MSSM. Thus the SUSY GUT is still promising in the case of high-scale supersymmetry. In the next section, we in turn require the gauge coupling unification and constrain the GUT scale mass spectrum by using the requirement.

Now, we briefly summarize the superheavy particles in the minimal SUSY SU(5) GUT, which we adopt as the working hypothesis in this paper. The SUSY SU(5) gauge theory includes twenty-four gauge superfields $V^A$ with $A = 1, \ldots , 24$. By using the SU(5) generators $T_A$ we define a $5 \times 5$ matrix representation of the vector superfields such that $V \equiv V^A T_A$, with the components written as

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} G - \frac{2}{\sqrt{30}} B & X^{11} & Y^{11} \\ X_1 & X_2 & X_3 & Y^{11} \\ Y_1 & Y_2 & Y_3 & Y^{12} \\ \frac{1}{\sqrt{2}} W^3 + \frac{3}{\sqrt{30}} B & W^- & -\frac{1}{\sqrt{2}} W^3 + \frac{3}{\sqrt{30}} B \end{pmatrix}. \quad (3)$$

We collectively call $X_\alpha$ and $Y_\alpha$ the $X$-bosons hereafter. The unified gauge group SU(5) is spontaneously broken by the VEV of the adjoint Higgs boson $\Sigma^A$ ($A = 1, \ldots , 24$) to SU(3)$_C \times$SU(2)$_L \times$U(1)$_Y$. Again, we define $\Sigma \equiv \Sigma^A T^A$ and its components by

$$\Sigma = \begin{pmatrix} \Sigma_8 & \Sigma_{(3,2)} \\ \Sigma_{(3^* , 2)} & \Sigma_3 \end{pmatrix} + \frac{1}{2\sqrt{15}} \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \Sigma_{24}. \quad (4)$$

The MSSM Higgs superfields are, on the other hand, incorporated into fundamental and anti-fundamental representations as follows:

$$H = \begin{pmatrix} H_C^1 \\ H_C^2 \\ H_C^3 \\ H_u^+ \\ H_u^0 \end{pmatrix}, \quad \tilde{H} = \begin{pmatrix} \tilde{H}_{C1} \\ \tilde{H}_{C2} \\ \tilde{H}_{C3} \\ \tilde{H}_d^- \\ -\tilde{H}_d^0 \end{pmatrix}. \quad (5)$$
where
\[ H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \quad (6) \]
are the MSSM Higgs superfields. \( H_C^a \) and \( \bar{H}_C^a \) are called the color-triplet Higgs multiplets.

The superpotential of the Higgs sector in the minimal SU(5) SUSY GUT is given by
\[ W = \frac{1}{3} \lambda_\Sigma \mathrm{Tr} \Sigma^3 + \frac{1}{2} m_\Sigma \mathrm{Tr} \Sigma^2 + \lambda_\Sigma \bar{H} \Sigma H + m_H \bar{H} H. \quad (7) \]

After the adjoint Higgs field gets the VEV, \( \langle \Sigma \rangle = V \cdot \mathrm{diag}(2, 2, 2, -3, -3) \) (\( V = m_\Sigma / \lambda_\Sigma \)), the SU(5) gauge group is broken to the SM SU(3)_C × SU(2)_L × U(1)_Y gauge groups without breaking the supersymmetry. Also the parameter \( m_H \) is fine-tuned as \( m_H = 3 \lambda_\Sigma V \) in order to realize the doublet-triplet mass splitting in \( H \) and \( \bar{H} \). With the condition, the mass of the color-triplet Higgs boson is \( M_{H_C} = M_{\bar{H}_C} = 5 \lambda_\Sigma V \). The \( X \)-boson mass is \( M_X = 5 \sqrt{2} g_5 V \) with \( g_5 \) the unified gauge coupling constant. As regards the adjoint Higgs multiplets, the components \( \Sigma_3 \) and \( \Sigma_8 \) have masses of \( M_\Sigma \equiv M_{\Sigma_3} = M_{\Sigma_8} = \frac{5}{2} \lambda_\Sigma V \) and \( \Sigma_{24} \) has \( M_{\Sigma_{24}} = \frac{1}{2} \lambda_\Sigma V \). The components \( \Sigma_{(3^*2)} \) and \( \Sigma_{(3,2)} \) become the longitudinal component of the \( X \)-bosons, and thus do not show up as physical states.

### 3 Renormalization Group Analysis

In this section, we present the RGEs of the gauge and Yukawa coupling constants, as well as the boundary conditions at each threshold. We use the \( \overline{\text{DR}} \) scheme [44] in this work. First, we write down the two-loop beta functions [45]. In the MSSM, the two-loop RGEs for the gauge coupling constants are given as
\[ \mu \frac{\partial g_a}{\partial \mu} = \frac{1}{16\pi^2} b_a^{(1)} g_a^3 + \frac{g_a^3}{(16\pi^2)^2} \left[ \sum_{b=1}^{3} b_{ab}^{(2)} g_b^2 + \sum_{i=t,b,\tau} c_{ai} y_i^2 \right], \quad (8) \]

where
\[ b_a^{(1)} = \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix}, \quad b_{ab}^{(2)} = \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix}, \quad (9) \]

and
\[ c_{ai} = \begin{pmatrix} 26/5 & 14/5 & 18/5 \\ 6 & 6 & 2 \\ 4 & 4 & 0 \end{pmatrix}, \quad (10) \]

with \( y_i \) (\( i = t, b, \tau \)) the top, bottom and tau Yukawa coupling constants, respectively. Since the Yukawa couplings enter into the two-loop level contributions to the gauge coupling RGEs, it is sufficient to consider the RGEs for the Yukawa couplings at one-loop level.
They are given as

\[
\frac{\mu}{\partial \mu} y_t = \frac{1}{16\pi^2} y_t \left[ 6y_t^2 + y_b^2 - \frac{13}{15} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2 \right],
\]

\[
\frac{\mu}{\partial \mu} y_b = \frac{1}{16\pi^2} y_b \left[ 6y_b^2 + y_t^2 + y_r^2 - \frac{7}{15} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2 \right],
\]

\[
\frac{\mu}{\partial \mu} y_r = \frac{1}{16\pi^2} y_r \left[ 3y_b^2 + 4y_r^2 - \frac{9}{5} g_1^2 - 3g_2^2 \right].
\]  

(11)

Below the SUSY breaking scale \((M_S)\), the squarks and sleptons, the higgsinos, and the heavy Higgs boson masses are decoupled so that the theory is regarded as the SM with gauginos. The contribution of gauginos and the SM particles to the coefficients of the beta functions is given as

\[
b_{a1} = \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix}_{\text{SM}}^{\text{gaugino}},
\]

\[
b_{ab} = \begin{pmatrix} 199/50 & 27/10 & 44/5 \\ 9/10 & 35/6 & 12 \\ 11/10 & 9/2 & -26 \end{pmatrix}_{\text{SM}} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 64/3 & 0 \\ 0 & 0 & 48 \end{pmatrix}_{\text{gaugino}},
\]

(13)

and

\[
c_{ai} = \begin{pmatrix} 17/10 & 1/2 & 3/2 \\ 3/2 & 3/2 & 1/2 \\ 2 & 2 & 0 \end{pmatrix}_{\text{SM}},
\]

(14)

where the subscripts “SM” and “gaugino” indicate that the contributions are of the SM particles and gauginos, respectively. The running of the Yukawa couplings in this case is given as follows:

\[
\frac{\mu}{\partial \mu} y_t = \frac{1}{16\pi^2} y_t \left[ \left( \frac{9}{4} y_t^2 + y_b^2 + y_r^2 - \frac{17}{20} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2 \right)_{\text{SM}} \right],
\]

\[
\frac{\mu}{\partial \mu} y_b = \frac{1}{16\pi^2} y_b \left[ \left( \frac{3}{2} y_t^2 + \frac{9}{2} y_b^2 + y_r^2 - \frac{1}{4} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2 \right)_{\text{SM}} \right],
\]

\[
\frac{\mu}{\partial \mu} y_r = \frac{1}{16\pi^2} y_r \left[ \left( \frac{3}{2} y_t^2 + y_b^2 + \frac{9}{2} y_b^2 - \frac{9}{4} g_1^2 - \frac{9}{4} g_2^2 \right)_{\text{SM}} \right].
\]  

(15)

Next, we consider the matching conditions at each threshold scale. At the GUT scale, the gauge coupling constants in the \(\text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y\) gauge theories are equated to the unified coupling constant \(g_5\) with the following threshold corrections at one-loop
level [16, 47]:

\[
\begin{align*}
\frac{1}{g_1^2(\mu)} &= \frac{1}{g_2^2(\mu)} + \frac{1}{8\pi^2} \left[ \frac{2}{5} \ln \frac{\mu}{M_{HC}} - 10 \ln \frac{\mu}{M_X} \right], \\
\frac{1}{g_2^2(\mu)} &= \frac{1}{g_3^2(\mu)} + \frac{1}{8\pi^2} \left[ 2 \ln \frac{\mu}{M_{\Sigma}} - 6 \ln \frac{\mu}{M_X} \right], \\
\frac{1}{g_3^2(\mu)} &= \frac{1}{g_2^2(\mu)} + \frac{1}{8\pi^2} \left[ \ln \frac{\mu}{M_{HC}} + 3 \ln \frac{\mu}{M_{\Sigma}} - 3 \frac{\mu}{M_X} \right].
\end{align*}
\]  

(16)

Here, the conditions do not include constant (scale independent) terms since we use the DR scheme for the renormalization [5, 48]. From the equations it immediately follows that:

\[
\begin{align*}
\frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)} - \frac{1}{g_1^2(\mu)} &= \frac{3}{10\pi^2} \ln \frac{\mu}{M_{HC}}, \\
\frac{5}{g_1^2(\mu)} - \frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)} &= \frac{3}{2\pi^2} \ln \frac{\mu}{M_{\Sigma} M_{X}},
\end{align*}
\]  

(17)

The relations allow us to evaluate the masses of the heavy particles, $M_{HC}$ and $M_{X} M_{\Sigma}$, from the gauge coupling constants determined in the low-energy experiments through the RGEs [8, 9]. While the couplings are well measured with high precision, the estimation is quite dependent on the spectrum in the intermediate region, especially on the masses of gauginos and higgsinos.

For the SUSY breaking threshold, we just equate the gauge couplings above and below the threshold, and change the beta functions appropriately for each region. This approximation is valid since the particles appearing at the scale are assumed to be degenerate in mass. In the case of gauginos, on the other hand, we need to consider the threshold corrections since the mass difference among gauginos might be sizable. The condition is:

\[
\begin{align*}
\frac{1}{g_1^2(\mu)_{SM}} &= \frac{1}{g_1^2(\mu)_{gaugino}}, \\
\frac{1}{g_2^2(\mu)_{SM}} &= \frac{1}{g_2^2(\mu)_{gaugino}} + \frac{1}{6\pi^2} \ln \frac{\mu}{M_2}, \\
\frac{1}{g_3^2(\mu)_{SM}} &= \frac{1}{g_3^2(\mu)_{gaugino}} + \frac{1}{4\pi^2} \ln \frac{\mu}{M_3},
\end{align*}
\]  

(18)

where $g_a(\mu)_{SM}$ are the couplings in the SM while $g_a(\mu)_{gaugino}$ are those above the gaugino threshold.

The Yukawa couplings are matched as usual, i.e., at the SUSY breaking scale, the Yukawa couplings $y_i(\mu)$ below the SUSY breaking scale are matched with the supersym-


metric ones, \( y_t(\mu)_{\text{MSSM}} \), as follows:

\[
y_t(M_S)_{\text{MSSM}} = \frac{1}{\sin \beta} y_t(M_S),
\]

\[
y_b(M_S)_{\text{MSSM}} = \frac{1}{\cos \beta} y_b(M_S),
\]

\[
y_\tau(M_S)_{\text{MSSM}} = \frac{1}{\cos \beta} y_\tau(M_S).
\] (19)

Before concluding this section, we solve the RGEs at one-loop level and, taking the threshold corrections into account, derive relations between the superheavy masses and the low-energy gauge coupling constants. Such relations reflect the dependence of \( M_{H_C} \) and \( M_X^2 M_\Sigma \) on the mass spectrum of the SUSY particles. By inserting to Eq. (17) the one-loop solutions of the RGEs for the gauge couplings, we have

\[
\frac{3}{\alpha_2(m_Z)} - \frac{2}{\alpha_3(m_Z)} - \frac{1}{\alpha_1(m_Z)} = \frac{1}{2\pi} \left[ \frac{12}{5} \ln \left( \frac{M_{H_C}}{m_Z} \right) - 2 \ln \left( \frac{M_S}{m_Z} \right) + 4 \ln \left( \frac{M_3}{M_2} \right) \right], \quad (20)
\]

\[
\frac{5}{\alpha_1(m_Z)} - \frac{4}{\alpha_2(m_Z)} - \frac{2}{\alpha_3(m_Z)} = \frac{1}{2\pi} \left[ 12 \ln \left( \frac{M_X^2 M_\Sigma}{m_Z^2} \right) + 4 \ln \left( \frac{M_2}{m_Z} \right) + 4 \ln \left( \frac{M_3}{m_Z} \right) \right]. \quad (21)
\]

From Eq. (20) we find that the mass of the color-triplet Higgs \( M_{H_C} \) gets larger as the SUSY breaking scale \( M_S \) is taken to be higher. This originates from the mass difference among the components of the fundamental Higgs multiplets, i.e., the triplet-Higgs, higgsinos, heavy Higgs bosons, and the lightest Higgs boson. Therefore, the behavior of \( M_{H_C} \) with respect to the SUSY breaking scale is universal in a sense. Further, \( M_{H_C} \) depends only on the ratio of \( M_2 \) and \( M_3 \). \( M_X^2 M_\Sigma \) is, on the other hand, independent of the SUSY braking scale \( M_S \) while dependent on the scale of the gauginos, not their ratio. This is because the right-hand side of Eq. (21) results from the mass difference in the gauge vector multiplets and the adjoint Higgs multiplet, a part of which is included as the longitudinal mode of the gauge multiplets. It is also found that \( M_X^2 M_\Sigma \) decreases when the gaugino masses are taken to be large values. This is owing to the opposite sign of the contribution of gauge fields to the gauge beta functions to those of matter fields. This feature is, therefore, again model-independent. In the subsequent section, we carry out a similar analysis using the two-loop RGEs.

### 4 Results

Now we present some results for the RGE analysis which we discuss in the previous section. As noted above, the running of the gauge couplings is computed at two-loop level and the threshold corrections are taken into account at one-loop level. The masses of sfermions, heavy Higgs bosons, and higgsinos are taken to be \( M_S \) for brevity. Gaugino masses are assumed to be lighter than \( M_S \) by an one-loop factor.
Figure 1: Predicted color-triplet Higgs mass $M_{H^c}$ as functions of the SUSY breaking scale $M_S$ (pink lines). Here, wino mass $M_2$ is fixed to be 3 TeV and $\tan\beta = 3$. Gluino-wino mass ratio, $M_3/M_2$, is set to be $M_3/M_2 = 3$, $9$, and $30$ from top to bottom, respectively. Theoretical errors coming from the strong coupling constant $\alpha_s(m_Z) = 0.1184(7)$ are also shown. Horizontal blue line shows a result in the case of low-energy SUSY ($M_S = 1$ TeV, $M_2 = 200$ GeV, and $M_3/M_2 = 3.5$).

Figure 2: Color-triplet Higgs mass $M_{H^c}$ as functions of gluino mass $M_3$ (pink lines). Here, $\tan\beta = 3$ and $M_S = 10^3$ TeV. Upper and lower lines correspond to $M_2 = 3$ TeV and 300 GeV, respectively. Error bars indicate the input error of the strong coupling constant $\alpha_s(m_Z) = 0.1184(7)$. Horizontal blue line shows a result in the case of low-energy SUSY ($M_S = 1$ TeV, $M_2 = 200$ GeV, and $M_3/M_2 = 3.5$).
Figure 3: GUT scale $M_{\text{GUT}} \equiv (M_X M_\Sigma)^{1/3}$ as functions of gluino mass $M_3$ (pink lines). Here, $\tan \beta = 3$ and $M_S = 10^3$ TeV. Upper and lower lines correspond to $M_2 = 300$ GeV and 3 TeV, respectively. Error bars indicate the input error of the strong coupling constant $\alpha_s(m_Z) = 0.1184(7)$ \cite{49}. Horizontal blue line shows a result in the case of low-energy SUSY ($M_S = 1$ TeV, $M_2 = 200$ GeV, and $M_3/M_2 = 3.5$).

First, we consider the color-triplet Higgs mass $M_{H_C}$. In Fig. 1 we plot the dependence of $M_{H_C}$ on the SUSY breaking scale $M_S$ in the pink lines. Here, the wino mass $M_2$ is fixed to be 3 TeV, which is favored from the thermal relic abundance, and $\tan \beta = 3$. The ratio of the gluino and wino masses, $M_3/M_2$, is set to be $M_3/M_2 = 3, 9,$ and 30 from top to bottom, respectively. Further, we show the error of the calculation coming from that of the strong coupling constant $\alpha_s(m_Z) = 0.1184(7)$ \cite{49}. The horizontal blue line shows a result in the case of low-energy SUSY ($M_S = 1$ TeV, $M_2 = 200$ GeV, and $M_3/M_2 = 3.5$) as a reference. In this case, we have $8.6 \times 10^{14} \leq M_{H_C} \leq 1.4 \times 10^{15}$ GeV. This figure well illustrates the feature read from the approximated expression given in Eq. (20); $M_{H_C}$ increases as the SUSY breaking scale grows while it decreases when the ratio $M_3/M_2$ becomes large. To see the latter feature more clearly, we show its dependence on the gluino mass $M_3$. Again, we set $\tan \beta = 3$, and the SUSY breaking mass is fixed to be $M_S = 10^3$ TeV. The upper and lower lines correspond to $M_2 = 3$ TeV and 300 GeV, respectively. These two figures show that $M_{H_C}$ is strongly dependent on $M_S$ and $M_3/M_2$. Therefore, to predict the mass with high accuracy, precise determination of the masses of gauginos as well as the SUSY breaking scale is inevitable. Any way, in the high-scale SUSY scenario it is found to be possible for the mass of the color-triplet $M_{H_C}$ to be around $\sim 2 \times 10^{16}$ GeV, which is expected by the gauge coupling unification.

Next, we discuss constraints on $M_X^2 M_\Sigma$ derived from the relation (21). From now on we define $M_{\text{GUT}} \equiv (M_X M_\Sigma)^{1/3}$ and refer to it as the GUT scale. The equation (21) tells us that the GUT scale depends on only the gaugino masses at the leading order, so
we express \( M_{\text{GUT}} \) as functions of the gaugino masses. In Fig. 3 we plot it as functions of gluino mass. Here again we fix \( \tan \beta = 3 \) and \( M_S = 10^3 \) TeV. The upper and lower lines correspond to \( M_2 = 300 \) GeV and \( 3 \) TeV, respectively. Again, the horizontal blue line shows a result in the case of low-energy SUSY with \( M_S = 1 \) TeV, \( M_2 = 200 \) GeV, and \( M_3/M_2 = 3.5 \), which gives \( M_{\text{GUT}} \simeq 1.9 \times 10^{16} \) GeV. The error bars indicate the input error of the strong coupling constant \( \alpha_s(m_Z) = 0.1184(7) \) [49], though the effect is negligible. We see that the GUT scale also has little dependence on the gaugino masses. In that sense, the prediction is robust compared with that for \( M_{H^C} \). However, as discussed in the previous section, the GUT scale \( M_{\text{GUT}} \) gets lower when the gaugino masses become larger \( (M_{\text{GUT}} \propto (M_3/M_2)^{-1/9}) \). This feature is quite interesting when one considers the proton decay via the X-boson exchange processes. Although the change in \( M_{\text{GUT}} \) is small, it might be significant since the proton decay lifetime scales as \( \propto M_X^4 \).

For instance, when \( M_X \simeq 0.8 \times 10^{16} \) GeV, the proton lifetime via the X-boson exchange reduces to around \( 5 \times 10^{34} \) years [50], which is slightly above the current experimental bound, \( \tau(p \to e^+\pi^0) > 1.29 \times 10^{34} \) yrs [51].

Finally, we briefly comment on the \( \tan \beta \) dependence of the results. Although the one-loop computation is not related with \( \tan \beta \), the two-loop results might be affected through the running of the Yukawa couplings. We have found, however, that the effects on the results are negligible.

5 Conclusions and Discussion

In this Letter, we have presented constraints on the masses of the GUT scale particles from the gauge coupling unification in the case of the high-scale SUSY scenario. To that end, we have used the two-loop RGEs for the gauge couplings with one-loop threshold corrections considered. As a result, the mass of the color-triplet Higgs multiplets \( M_{H^C} \) turns out to be considerably large compared with previous results in the traditional low-energy SUSY scenario, while the GUT scale is found to be slightly lower. These are generic features resulting from the mass difference among the components of the same supermultiplet of the SU(5) gauge group. Interestingly, all of the superheavy particles might be around \( 10^{16} \) GeV in the high-scale SUSY models.

The mass spectrum of the GUT scale particles predicted here stimulates us to reconsider the proton decay in the case of high-scale supersymmetry. As mentioned to above, the relatively low GUT scale enhances the proton decay rate via the X-boson exchanging process. If the enhancement is strong enough, the proton decay in the \( p \to e^+\pi^0 \) channel might be searched in future experiments. Furthermore, the experiments may also reach the proton decay through the color-triplet Higgs exchange. Since the decay process predicts too short lifetime [10], some suppression mechanism for the process has been assumed. Nevertheless, with \( M_{H^C} \) larger than those considered in the previous literature and sfermions much heavier than the electroweak scale, this process might evade the current experimental bound without any mechanism of limiting the color-triplet Higgs exchanging process. In such a case, it is possible for the \( p \to K^+\bar{\nu} \) mode, which is the
main decay mode in the case of the color-triplet Higgs exchange, to be searched in near future. A detailed analysis of this decay process will be given elsewhere [52].

Note that, in proposed models where the color-triplet Higgs exchange is suppressed by some mechanism, large threshold corrections to the gauge coupling constants at the GUT scale tend to appear. One of the examples is introduction of the Peccei-Quinn symmetry [53]. It was found that when the threshold corrections at the GUT scale are small, the suppression mechanism does not work [54,55]. In the SUSY SU(5) GUTs in higher dimensional space the U(1)_R symmetry forbids the dimension-five proton decay, though the Kaluza-Klein particles generate large threshold corrections to the gauge coupling constants [56]. The high-scale supersymmetry would be another solution for the proton decay problem, in which such large threshold corrections are not required.

Acknowledgments

The work of N.N. is supported by Research Fellowships of the Japan Society for the Promotion of Science for Young Scientists. The work of J.H. is supported by Grant-in-Aid for Scientific research from the Ministry of Education, Science, Sports, and Culture (MEXT), Japan, No. 20244037, No. 20540252, No. 22244021 and No. 23104011, and also by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan.

References

[1] S. Dimopoulos and H. Georgi, Nucl. Phys. B 193, 150 (1981).
[2] N. Sakai, Z. Phys. C 11, 153 (1981).
[3] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D 24, 1681 (1981).
[4] W. Marciano and G. Senjanović, Phys. Rev. D 25, 3092 (1982).
[5] M.B. Einhorn and D.R. Jones, Nucl. Phys. B 196, 475 (1982).
[6] U. Amaldi, W. de Boer and H. Furstenaup, Phys. Lett. B 260, 447 (1991).
[7] P. Langacker and M. -x. Luo, Phys. Rev. D 44, 817 (1991).
[8] J. Hisano, H. Murayama and T. Yanagida, Phys. Rev. Lett. 69, 1014 (1992).
[9] J. Hisano, H. Murayama and T. Yanagida, Nucl. Phys. B 402, 46 (1993).
[10] H. Murayama and A. Pierce, Phys. Rev. D 65, 055009 (2002).
[11] G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP 9812, 027 (1998).
[12] J. D. Wells, hep-ph/0306127.
[13] N. Arkani-Hamed and S. Dimopoulos, JHEP 0506, 073 (2005).
[14] G. F. Giudice and A. Romanino, Nucl. Phys. B 699, 65 (2004) [Erratum-ibid. B 706, 65 (2005)].
[15] N. Arkani-Hamed, S. Dimopoulos, G. F. Giudice and A. Romanino, Nucl. Phys. B 709, 3 (2005).
[16] J. D. Wells, Phys. Rev. D 71, 015013 (2005).
[17] L. J. Hall and Y. Nomura, JHEP 1003, 076 (2010).
[18] L. J. Hall and Y. Nomura, JHEP 1201, 082 (2012).
[19] G. Aad et al. [ATLAS Collaboration], arXiv:1208.0919 [hep-ex].
[20] [ATLAS Collaboration], ATLAS-CONF-2012-109.
[21] S. Chatrchyan et al. [CMS Collaboration], Phys. Rev. Lett. 109, 171803 (2012).
[22] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716, 1 (2012).
[23] S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716, 30 (2012).
[24] G. F. Giudice and A. Strumia, Nucl. Phys. B 858, 63 (2012).
[25] M. Ibe and T. T. Yanagida, Phys. Lett. B 709, 374 (2012).
[26] M. Ibe, S. Matsumoto and T. T. Yanagida, Phys. Rev. D 85, 095011 (2012).
[27] L. E. Ibanez and I. Valenzuela, arXiv:1301.5167 [hep-ph].
[28] J. Hisano, K. Ishiwata and N. Nagata, Phys. Rev. D 87, 035020 (2013).
[29] K. S. Jeong, M. Shimosuka and M. Yamaguchi, JHEP 1209, 050 (2012).
[30] R. Saito and S. Shirai, Phys. Lett. B 713, 237 (2012).
[31] R. Sato, S. Shirai and K. Tobioka, JHEP 1211, 041 (2012).
[32] B. Bhattacharjee, B. Feldstein, M. Ibe, S. Matsumoto and T. T. Yanagida, Phys. Rev. D 87, 015028 (2013).
[33] L. J. Hall, Y. Nomura and S. Shirai, JHEP 1301, 036 (2013).
[34] N. Arkani-Hamed, A. Gupta, D. E. Kaplan, N. Weiner and T. Zorawski, arXiv:1212.6971 [hep-ph].
[35] T. Moroi and M. Nagai, arXiv:1303.0668 [hep-ph].
[36] D. McKeen, M. Pospelov and A. Ritz, arXiv:1303.1172 [hep-ph].
[37] L. Randall and R. Sundrum, Nucl. Phys. B 557, 79 (1999).

[38] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B 477, 321 (1996).

[39] J. Hisano, S. Matsumoto, M. Nagai, O. Saito and M. Senami, Phys. Lett. B 646, 34 (2007).

[40] J. Hisano, K. Ishiwata and N. Nagata, Phys. Lett. B 690, 311 (2010).

[41] J. Hisano, K. Ishiwata and N. Nagata, Phys. Rev. D 82, 115007 (2010).

[42] J. Hisano, S. Matsumoto and M. M. Nojiri, Phys. Rev. Lett. 92, 031303 (2004).

[43] J. Hisano, S. Matsumoto, M. M. Nojiri and O. Saito, Phys. Rev. D 71, 063528 (2005).

[44] W. Siegel, Phys. Lett. B 84, 193 (1979).

[45] M. E. Machacek and M. T. Vaughn, Nucl. Phys. B 222, 83 (1983).

[46] S. Weinberg, Phys. Lett. B 91, 51 (1980).

[47] L. J. Hall, Nucl. Phys. B 178, 75 (1981).

[48] I. Antoniadis, C. Kounnas and K. Tamvakis, Phys. Lett. B 119, 377 (1982).

[49] J. Beringer et al. [Particle Data Group Collaboration], Phys. Rev. D 86, 010001 (2012).

[50] J. Hisano, D. Kobayashi and N. Nagata, Phys. Lett. B 716, 406 (2012).

[51] H. Nishino, K. Abe, Y. Hayato, T. Iida, M. Ikeda, J. Kameda, Y. Koshio and M. Miura et al., arXiv:1203.4030 [hep-ex].

[52] J. Hisano, D. Kobayashi, T. Kuwahara and N. Nagata, arXiv:1304.3651 [hep-ph].

[53] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38, 1440 (1977).

[54] J. Hisano, H. Murayama and T. Yanagida, Phys. Lett. B 291, 263 (1992).

[55] J. Hisano, T. Moroi, K. Tobe and T. Yanagida, Phys. Lett. B 342, 138 (1995).

[56] L. J. Hall and Y. Nomura, Phys. Rev. D 64, 055003 (2001).