Confronting nuclear equation of state in the presence of dark matter using GW170817 observation in relativistic mean field theory approach

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Abstract

We confront admixture of dark matter inside neutron star using gravitational wave constraints coming from binary neutron star merger. We consider a relativistic mean field model including $\sigma - \omega - \rho$ meson interaction with NL3 parameterization. We study fermionic dark matter interacting with nucleonic matter via Higgs portal mechanism. We show that admixture of dark matter inside the neutron star soften the equation state and lower the value of tidal deformability. Gravitational wave GW170817 observation puts an upper bound on tidal deformability of a binary neutron star with low spin prior at 90% confidence level, which disfavors stiff equation of state such as Walecka model with NL3 parameterization. However, we show that Walecka model with NL3 parameterization with a fermionic dark matter component satisfy the tidal deformability bound coming from the GW170817 observation.

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1. INTRODUCTION

Compact objects like neutron stars (NS) are nature’s laboratory which can shed light directly or indirectly on the different branches of physics such as low energy nuclear physics, QCD under extreme conditions, the general theory of relativity etc. A neutron star is one of the remnants of a dying star undergoing gravitational collapse. Gravitational collapse of stars with a mass range between $1.4-3.0M_\odot$ evolve into a neutron star. Neutron degeneracy pressure inside the neutron star makes it hydrostatically stable against the gravitational collapse. If the mass of a dying star is very large (beyond $10M_\odot$) then the stellar remnant will overcome the neutron degeneracy pressure and gravitational collapse will produce a black hole. Matter density inside the neutron star can be as large as few times nuclear saturation density ($n_B = 0.16\text{ fm}^{-3}$). Interior of a neutron star provides a situation to study the behavior of matter at extreme conditions. In this context, the possible equation of state (EoS) of infinite nuclear matter has been explored extensively (for a brief review see [1]). The main challenge in the description of matter at high densities inside neutron stars is to develop a model which not only describes matter at high densities, but also the properties of matter observed at normal densities [2, 3]. Valid nuclear EoS has to satisfy presently well accepted empirical and experimental constraints [4–6], e.g. ground state properties of spherical and deformed nuclei, saturation density, binding energy, symmetry energy, compression modulus etc. as well as constraints coming from infinite nuclear matter e.g., neutron star mass radius relation, tidal deformability etc. Rotating neutron stars or pulsars give important information about superfluid nature of nucleon inside the neutron star [7, 8]. Superfluidity of nucleons is important to explain timing irregularities (glitch) of pulsars. From the high energy nuclear physics point of view, neutron star provides an ideal condition where high density QCD matter (quark matter phase, color superconducting phase etc.) can exist [9–12]. Historically neutron star mass radius relationship coming from solving Tolman-Oppenheimer-Volkoff (TOV) equation have been studied extensively to put constraints on the nuclear EoS [1]. However recent observation of gravitational waves from neutron star mergers [13] opens up another dimension in the study of the nuclear EoS.

On August 17, 2017, the Advanced LIGO and Virgo observatories detected the gravitational waves (GW) from a merging binary NS [13]. GW170817 data open up a new way to understand the neutron star structure and the underlying EoS of dense matter. Details of the internal structure of the neutron stars in the binary mergers become important as the orbital separation become
comparable with the size of the bodies. For neutron star binary merger the tidal field of the companion induces a quadrupole moment to the other neutron star. The relation of the induced quadrupole moment to the external tidal field is proportional to the tidal deformability. Tidal deformability is sensitive to the mass, radius and tidal love number, which intern depends on the nuclear EoS. Using observed tidal deformability parameters of the neutron star merger one can put strong constraints on the neutron star EoS, for details and related studies see [14–18] and references therein. The GW170817 observation puts an upper bound on tidal deformability of the combined binary NSs at 90% confidence [13]. Consequently, this can be used to rule out certain equation of states of neutron stars.

Observations of the kinematics of self gravitating objects such as galaxies and clusters of galaxies give strong hint of the existence of dark matter (DM). Cosmological observations tell us that this invisible matter cannot consist of baryons, it must be a new kind of matter which interacts with the rest of the standard model particles very weakly. The exact nature of the dark matter, its coupling between standard model particles and the mass is still not known. However extensive studies on the particle physics dark matter models have put strong constraints on the coupling constant and mass of the dark matter particles (for a recent review on dark matter physics see [19]). Among different proposals of dark matter weakly interacting massive particle (WIMP) scenario has gained favor because it gives the correct prediction of the measured relic abundance of the dark matter today very naturally.

Presence of dark matter inside neutron star and its consequences have been discussed in the literature [20–31]. This discussions include dark matter capture by neutron star and heating of old neutron star in the galactic halo to a temperature detectable by upcoming infrared telescopes [20], trapped WIMPs inside the neutron star [21], charged massive dark matter particle and its effect on neutron star [22], heating of a neutron star due to dark matter annihilation [23–25], or the collapse of a neutron star due to accretion of non annihilating dark matter [26] etc. In [28] authors have considered possible effects of a self interacting dark matter core on the maximum mass of a neutron star, mass-radius relation and on the NS tidal deformability parameter. They have computed radial density and pressure profiles of the baryonic and dark matter components for different nuclear EoS and different dark matter fractions. In [31], the authors have considered Walecka relativistic mean field model including $\sigma – \omega$ interaction for the nucleonic part [2–5] and fermionic dark matter inside the neutron star. Using mean field approximation they have calculated
effective nucleon mass, variation of $\sigma$ field, EoS and the corresponding mass radius relation in this model. However, it is important to mention that the simple relativistic mean field model (RMF) model taken in this work is unsuccessful in producing nuclear saturation properties. This simple model is ruled out due to the fact that it gives high nuclear incompressibility ($\approx 500\text{MeV}$) and very low nucleon mass [32]. Keeping this limitation of the simple $\sigma-\omega$ in this work we have considered the a generalized Lagrangian for the nucleonic sector including $\sigma-\omega-\rho$ meson interaction with NL3 parameterization[33]. The EoS of this model is disfavoured by GW170817 tidal deformability upper bound limit. However, we show that if we consider fermionic dark matter interaction via Higgs portal mechanism, then it can evade the GWs tidal deformability upper bound constraint. We have also considered non gravitational interaction of the dark matter and the nucleon field in a relativistic mean field approach.

This paper is organized as follows: in Sec.(2) we discuss generalized Walecka model with NL3 parameterization. In Sec.(3), we discuss fermionic dark matter model and its interaction with nuclear matter. The EoS of total Lagrangian density of dark matter and nuclear matter is presented in Sec.(4). Constraint from GWs tidal deformability on EoS is discussed in Sec.(5). Finally, we conclude in Sec.(6).

2. WALECKA MODEL WITH NL3 PARAMETERIZATION

In this section, we briefly summarize the relativistic mean field model (RMF) [2–5], which is also known as Quantum Hadron Dynamics (QHD) [34]. In this framework, nucleons are quasiparticles with an effective medium dependent mass and baryon chemical potential. They move in the mean field of mesons. Simplest QHD model is known as $\sigma-\omega$ model. In this model nucleon-nucleon interaction is mediated by the exchange of $\sigma$ and $\omega$ mesons. Properties of symmetric nuclear matter has been studied in this model. In general $\sigma$ mesons give rise to a strong attractive central force and a spin-orbit nuclear force, on the other hand, $\omega$-mesons are responsible for the repulsive central force. However, this simple model does not reproduce nuclear saturation properties, e.g. compressibility [32]. More advanced versions of QHD includes $\rho$ meson exchange interaction between nucleons[35]. Since protons and neutrons only differ in terms of their isospin projections, $\rho$ mesons are included to distinguish between these baryons and to give a better account of the symmetry energy. These vector mesons are charged, hence the reaction between $\rho$ meson and proton will differ from the reaction between $\rho$ meson and neutron. In general one can also include photon field, however,
neutron star is assumed to be charge neutral, hence the contribution from the photon field can be neglected.

Lagrangian including nucleon field, σ, ω and ρ mesons and their interactions can be written as [35],

\[
\mathcal{L} = \bar{\psi} \left[ \gamma^\mu \left( i \partial_\mu - g_v V_\mu - g_\rho \tau \cdot b_\mu \right) - (M_n + g_s \phi) \right] \psi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_s^2 \phi^2 - \frac{1}{3} g_2 \phi^3 - \frac{1}{4} g_3 \phi^4 \\
- \frac{1}{4} V^{\mu\nu} V_{\mu\nu} + \frac{1}{2} m_V^2 V_\mu V^\mu - \frac{1}{4} b^{\mu\nu} \cdot b_{\mu\nu} + \frac{1}{2} m_\rho^2 b^{\mu} b_{\mu} \tag{1}
\]

In general one can also include terms quartic in ω meson and ω − ρ interactions. However in the present work we have used NL3 parameterization of the above Lagrangian. In this parameterization coupling of terms quartic in ω meson and ω − ρ interactions are taken to be zero. In the above equation ψ is nucleon doublet, φ, V_μ and b_μ denotes σ, ω, ρ meson field respectively. m_s, m_V and m_ρ are the masses of the mesons and M_n denotes the nucleon mass. g_s, g_v and g_ρ are the scalar, vector and isovector coupling constants respectively. Field strength tensor of the vector and isovector mesons are given by,

\[
V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu, \tag{2}
\]

and,

\[
b_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu \tag{3}
\]

ρ meson field can be written explicitly as, \( b^{\mu} = (b^{\mu}_1, b^{\mu}_2, b^{\mu}_3) \). \( b^{\mu}_3 \) represents neutral \( \rho^0 \) meson and \( \rho^\pm \) are the orthogonal linear superposition of \( b^{\mu}_1 \) and \( b^{\mu}_2 \).

\[
b^{\mu}_\pm = \frac{1}{\sqrt{2}} (b^{\mu}_1 \pm b^{\mu}_2). \tag{4}
\]

\( \tau = (\tau_1, \tau_2, \tau_3) \) are the Pauli matrices, these are also the isospin operators. Proton and neutron are the different projections of nucleon in isospin space. Operation of \( \tau_3 \) on neutron and proton is as follows,

\[
\tau_3 |p\rangle = +1 |p\rangle \quad \& \quad \tau_3 |n\rangle = -1 |n\rangle \tag{5}
\]

Numerical values of the all the parameter of the the Lagrangian are given in the following table[35],

\[
\text{Table}\]

5
3. INTERACTION LAGRANGIAN BETWEEN NUCLEAR MATTER AND DARK MATTER

We consider fermionic dark matter ($\chi$) which is uniformly distributed in the neutron star with fixed Fermi momentum. Here we consider the lightest neutralino which acts as a fermionic dark matter candidate [31, 36]. Dark matter is not directly coupled with the nucleons rather they are coupled to the Higgs field $h$. Coupling between the dark matter and the Higgs field is $y$. For neutralino mass ($M_\chi=200$ GeV), the value of $y$ varies from 0.001 - 0.1. We fix $y = 0.07$ for the rest of our analysis [31, 36]. The Higgs field is also coupled to the nucleons through effective Yukawa coupling $\frac{FM}{v}$, where $f$ the proton-Higgs form factor and its value has been estimated to be approximately 0.35 [37]. We have not considered $h^3$ and $h^4$ term in the Higgs potential, because in the mean field approximation the value of the $h$ is small and the only dominant term is the $h^2$. So, the dark sector Lagrangian and its interaction with the nucleons and Higgs field is given by [31]

$$L_{DM} = \bar{\chi} \left[ i\gamma^\mu \partial_\mu - M_\chi + y h \right] \chi + \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{1}{2} M_h^2 h^2 + \frac{FM}{v} \bar{\psi} h \psi. \quad (6)$$

Here two assumptions are taken: 1) inside the neutron star, the average number density of nuclear matter is $10^3$ times larger than the average dark matter density ($n_{DM}$), 2) Fermi momentum of the dark matter $k_F^{DM}$ is constant [31]. Using this we get the ratio between mass of the dark matter inside neutron star and mass of the neutron star as

$$\frac{m_{DM}}{M_{NS}} = \frac{4}{3} \frac{\pi r^3 \rho_{DM}}{\frac{4}{3} \pi r^3 \rho_{NS}} = \frac{n_{DM} \times M_\chi}{n_{DM} \times M_\chi + n_B \times M_n} \approx \frac{1}{6} \quad (7)$$

We know that nuclear saturation density $n_B \sim 0.16fm^{-3}$, so dark matter number density is $n_{DM} \sim 10^{-3} n_B \sim 0.16 \times 10^{-3}fm^{-3}$. Number density of dark matter $n_{DM} = \frac{(k_F^{DM})^3}{3 \pi^2}$, which gives $k_F^{DM} \sim 0.033$ GeV. In our calculations we have varied $k_F^{DM}$ from 0.02 GeV to 0.06 GeV. For these values of $k_F^{DM}$ corresponding dark matter density will be different.

| $M_n$ (MeV) | $m_\sigma$ (MeV) | $m_\rho$ (MeV) | $g_s$ | $g_\rho$ | $g_2$ (fm$^{-1}$) | $g_3$ |
|------------|-----------------|----------------|------|---------|-----------------|------|
| 939        | 508.194         | 782.501        | 763.000 | 10.217  | -10.431         | -28.885 |

TABLE I:
4. NEUTRON STAR EQUATION OF STATE AND BETA EQUILIBRIUM

The Euler Lagrange Equation of motion for nucleon doublet ($\psi$), scalar($\phi$), vector($V^\mu$), isovector($b^\mu$), DM particle ($\chi$) and Higgs boson ($h$) can be derived from Lagrangian densities Eq. (1) and Eq. (6) as,

\[
\gamma^\mu \left( i \partial_\mu - g_v V_\mu - g_\rho \tau_3 b_\mu \right) - \left( M_\chi + g_s \phi - \frac{f M_n}{v} h \right) \psi = 0,
\]

\[
\partial_\mu \partial^\mu \phi + m_\phi^2 \phi + 2g_2 \phi^2 + g_3 \phi^3 + g_s \bar{\psi} \psi = 0,
\]

\[
\partial_\mu V^{\mu\nu} + m_V^2 V^{\nu} = g_v \bar{\psi} \gamma^\nu \psi,
\]

\[
\partial_\mu b^{\mu\nu} + m_\rho^2 b^{\nu} = g_\rho \bar{\psi} \gamma^\nu \tau_3 \psi,
\]

\[
\left( i \gamma_\mu \partial_\mu - M_\chi + y h \right) \chi = 0,
\]

\[
\partial_\mu \partial^\mu h + M_h^2 h = y \bar{\chi} \chi + \frac{f M_n}{v} \bar{\psi} \psi,
\]

respectively. The DM particle mass and Higgs particle mass are denoted as $M_\chi$ and $M_h$, respectively. Applying standard relativistic mean field approximation we get,

\[
\phi_0 = \frac{1}{m_\phi^2} \left( -g_s \langle \bar{\psi} \psi \rangle - 2g_2 \phi_0^2 - 3g_3 \phi_0^3 \right),
\]

\[
V_0 = \frac{g_\rho}{m_V^2} \langle \bar{\psi} \tau^3 \psi \rangle = \frac{g_\rho}{m_V^2} \left( \rho_p + \rho_n \right),
\]

\[
h_0 = \frac{y \langle \bar{\chi} \psi \rangle + f M_n}{M_h^2} \langle \bar{\psi} \psi \rangle,
\]

\[
b_0 = \frac{g_\rho}{M_h^2} \langle \bar{\psi} \tau^3 \psi \rangle = \frac{g_\rho}{M_h^2} \left( \rho_p - \rho_n \right),
\]

\[
\left( i \gamma_\mu \partial_\mu - g_v \gamma^0 V_0 - g_\rho \tau_3 \gamma^0 b_0 - M_\chi^* \right) \psi = 0,
\]

\[
\left( i \gamma_\mu \partial_\mu - M_\chi^* \right) \chi = 0,
\]

where $M_n^*$ and $M_\chi^*$ are nucleon and dark matter effective mass respectively. $\rho_p$ and $\rho_n$ are the densities of proton and neutron respectively. The effective mass of nucleon and dark matter can be given as,

\[
M_n^* = M_n + g_s \phi_0 - \frac{f M_n}{v} h_0,
\]

\[
M_\chi^* = M_\chi - y h_0.
\]
The baryon density \( \rho \), scalar density \( \rho_s \) and dark matter scalar density \( \rho_{sDM} \) are

\[
\rho = \langle \bar{\psi} \psi \rangle = \frac{\gamma}{(2\pi)^3} \int_0^{k_F} d^3 k, \\
\rho_s = \langle \bar{\psi} \psi \rangle = \frac{\gamma}{(2\pi)^3} \int_0^{k_F} \frac{M_n^*}{\sqrt{M_n^* + k^2}} d^3 k; \\
\rho_{sDM} = \langle \bar{\chi} \chi \rangle = \frac{\gamma}{(2\pi)^3} \int_0^{k_{DM}^*} \frac{M_{\chi}^*}{\sqrt{M_{\chi}^* + k^2}} d^3 k,
\]

(11)

where \( k_F \) and \( k_{DM}^* \) are the Fermi momentum for the nucleonic matter and dark matter respectively. \( \gamma \) is the spin degeneracy factor of nucleon and \( \gamma = 2 \) for neutron and proton individually.

The masses for both nucleon and dark matter depends on baryon density for fixed values of dark matter Fermi momentum \( k_{DM}^* \) and coupling constants. These masses and coupling values are discussed in Table.(I) and Sec.(3). To get the density dependent profile for \( M_n^* \) and \( M_{\chi}^* \), one needs to solve numerically Eq. (11) together with the field equations Eq. (9) in self consistent manner. In this work, we have taken the average dark matter number density approximately 1000 time smaller than the average neutron number density. This implies the dark matter mass fraction with respect to the neutron star mass is \( \simeq 1/6 \). The expectation values of the energy-momentum tensor or the stress tensor provide the energy density and pressure of the system in static case i.e., the EoS, which is given by,

\[
\epsilon = \langle T^{00} \rangle \quad \text{and} \quad P = \frac{1}{3} \langle T^{ii} \rangle.
\]

(12)

The expression for the total energy density \( \epsilon \) and the pressure \( P \) can be obtained by combining the Lagrangian density Eq.(1) and Eq.(6):

\[
\epsilon = g_v V_0 (\rho_p + \rho_n) + g_\rho b_0 (\rho_p - \rho_n) + \frac{1}{\pi^2} \int_0^{k_F} d^3 k \frac{k^2 \sqrt{k^2 + (M_n^*)^2}}{2} \\
\quad + \frac{1}{\pi^2} \int_0^{k_n} d^3 k \frac{k^2 \sqrt{k^2 + (M_n^*)^2}}{2} + \frac{1}{\pi^2} \int_0^{k_{FM}^*} d^3 k \frac{k^2 \sqrt{k^2 + (M_{\chi}^*)^2}}{2} \\
\quad + \frac{1}{2} m_\phi^2 \phi_0^2 + \frac{1}{3} g_2 \phi_0^3 + \frac{1}{4} g_3 \phi_0^4 - \frac{1}{2} m_V^2 V_0^2 - \frac{1}{2} m_{\rho}^2 b_0^2 + \frac{1}{2} M h_0^2.
\]

(13)

\[
P = \frac{1}{3\pi^2} \int_0^{k_F} \frac{k^4 dk}{\sqrt{k^2 + (M_n^*)^2}} + \frac{1}{3\pi^2} \int_0^{k_n} \frac{k^4 dk}{\sqrt{k^2 + (M_n^*)^2}} + \frac{1}{3\pi^2} \int_0^{k_{DM}^*} \frac{k^4 dk}{\sqrt{k^2 + (M_{\chi}^*)^2}} \\
\quad - \frac{1}{2} m_\phi^2 \phi_0^2 - \frac{1}{3} g_2 \phi_0^3 + \frac{1}{4} g_3 \phi_0^4 + \frac{1}{2} m_V^2 V_0^2 + \frac{1}{2} m_{\rho}^2 b_0^2 - \frac{1}{2} M h_0^2.
\]

(14)

\( \rho_n \) and \( \rho_p \) are the neutron and proton number density with \( k_F^n \) and \( k_F^p \) are the corresponding Fermi momentum of neutron and proton, respectively. The number densities and corresponding Fermi
momenta are equal for the symmetric nuclear matter. The matter inside neutron star mainly composed of neutrons. However, the neutron will eventually $\beta^-$ decays as,

\begin{equation}
n \rightarrow p + e^- + \bar{\nu}, \tag{15}\end{equation}

\begin{equation}n + \nu \rightarrow p + e^- . \tag{16}\end{equation}

To maintain the neutron star matter charge neutral, muons ($\mu$) will appear when the chemical potential of the electrons reaches the muon rest mass ($m_\mu = 106$ MeV). For a given baryon density ($\rho = \rho_n + \rho_p$), the charge neutrality is given as,

\begin{equation}\rho_p = \rho_e + \rho_\mu \tag{17}\end{equation}

and the $\beta^-$ equilibrium condition is given as,

\begin{equation}\mu_n = \mu_p + \mu_e \quad \text{and} \quad \mu_e = \mu_\mu \tag{18}\end{equation}

Where, the chemical potentials $\mu_p$, $\mu_n$, $\mu_e$ and $\mu_\mu$ are given as,

\begin{equation}\mu_p = \frac{\partial \epsilon}{\partial \rho_p} = g_e V_0 + g_\rho b_0 + \sqrt{k^2_p + (M^*_n)^2} \tag{19}\end{equation}

\begin{equation}\mu_n = \frac{\partial \epsilon}{\partial \rho_n} = g_e V_0 - g_\rho b_0 + \sqrt{k^2_n + (M^*_n)^2} \tag{20}\end{equation}

\begin{equation}\mu_e = \sqrt{k^2_e + m^2_e} \tag{21}\end{equation}

\begin{equation}\mu_\mu = \sqrt{k^2_\mu + m^2_\mu} \tag{22}\end{equation}

The particle fractions of neutrons and protons will depend on both charge neutrality and the $\beta^-$ equilibrium condition as given above. The self consistent numerical solution of Eq.(17) and Eq.(18) will set the fraction of neutron, proton, electron and muon number density for a given baryon density. The total energy density and pressure of leptons are given as,

\begin{equation}\epsilon_l = \sum_{l=e,\mu} \frac{1}{\pi^2} \int_0^{k_l} \frac{k^2 \sqrt{k^2 + m_l^2} dk}{k^4} \tag{23}\end{equation}

\begin{equation}P_l = \sum_{l=e,\mu} \frac{1}{3\pi^2} \int_0^{k_l} \frac{k^4 dk}{\sqrt{k^2 + m_l^2}} \tag{24}\end{equation}

The total energy density and pressure for $\beta^-$ equilibrated neutron star matter are

\begin{equation}\epsilon_{NM} = \epsilon_l + \epsilon , \tag{25}\end{equation}

\begin{equation}P_{NM} = P_l + P. \tag{26}\end{equation}
In Fig.(1), we plot pressure \(P_{NM}\) as function of the total energy density \(\epsilon_{NM}\) for different dark matter Fermi momentum \(k_F^{DM} = 0.0 - 0.06\) GeV. Fermi momentum \(k_F^{DM} = 0.0\) GeV corresponds equation of state without dark matter. Increasing the value of \(k_F^{DM}\) from 0.02 GeV to 0.06 GeV, the EoS becomes softer, i.e. with increasing density of dark matter pressure reduces, which is consistent with earlier work [31]. This behaviour is evident from the expression of energy density and pressure from Eq.(13) and Eq.(14), i.e. with increasing \(k_F^{DM}\) the dark matter contribution in energy density increases much faster than the pressure.

Neutron star mass radius relationship can be obtained by solving TOV equation for a given nuclear matter EoS [38]. The EoS for the core is obtained from the Walecka Model with NL3 parameterization in the presence and absence of dark matter components. Crust EoS is modeled using the BPS EoS [39] for the range of density \(\rho \sim 4.8 \times 10^{-9}\) to \(2.6 \times 10^{-4}\) fm\(^{-3}\). We use the polytropic form \(P_{NM}(\epsilon_{NM}) = a_1 + a_2\epsilon_{NM}^{\gamma'}\) [40] to join the core and crust of the NS, where \(a_1\) and \(a_2\) are obtained by matching the edge of the core at one end with the inner edge of the outer crust at other end, and \(\gamma'\) is taken 4/3 [41]. In Fig.(2) we plot the mass radius of NS using the EoS as shown in Fig.(1). It is clear from the Fig.(1) that the equation state is softer in the presence of larger dark matter density. A softer equation of state gives a lower value of the maximum mass of neutron star. From Fig.(2), the maximum mass without dark matter \((k_F^{DM} = 0) \sim 2.8M_\odot\) and the
value of the maximum mass of NS decreases with increasing dark matter density.

FIG. 2: Mass radius relation of NS for different dark matter Fermi momentum $k_{\text{F}}^{\text{DM}} = 0.0 - 0.06\text{GeV}$. The maximum mass of NS without dark matter is $2.8M_\odot$ and $2.1M_\odot$ with dark matter Fermi momentum $k_{\text{F}}^{\text{DM}} = 0.06\text{GeV}$.

Tidal deformability also depends upon the compactness and equation of state. In the next section, we study the effect of dark matter on nuclear matter EoS using the tidal deformability constraint from GW170817 observation.

5. TIDAL DEFORMABILITY CONSTRAINT

One of the important measurable structural properties of a binary merger is the tidal deformability. In a coalescing binary NS system, during the last stage of inspiral, each NS develops a mass quadrupole due to the tidal gravitational field induced by the other NS forming the binary. The tidal deformability describes the degree of deformation of a NS due to the tidal field of the companion NS and is sensitive to the nature of the EoS. The tidal deformability is defined as,

$$\lambda = \frac{2}{3} k_2 R^5,$$

where $R$ is the radius of the NS. The value of $k_2$ is typically in the range $\simeq 0.05 - 0.15$ [14–16] for NSs and depends on the stellar structure. This quantity can be calculated using the following
expression [14]

\[ k_2 = \frac{8C^5}{5} (1 - 2C)^2 [2 + 2C (y_R - 1) - y_R] \times \left\{ 2C (6 - 3y_R + 3C(5y_R - 8)) + 4C^3 \left[ 13 - 11y_R + C(3y_R - 2) + 2C^2(1 + y_R) \right] + 3(1 - 2C)^2 [2 - y_R + 2C(y_R - 1)] \log (1 - 2C) \right\}^{-1}, \] (28)

where \( C = M/R \) is the compactness parameter of the star of mass \( M \). The quantity \( y_R = y(R) \) can be obtained by solving the following differential equation

\[ r \frac{dy(r)}{dr} + y(r)^2 + y(r) F(r) + r^2 Q(r) = 0 \] (29)

with

\[ F(r) = \frac{r - 4\pi r^3 (\epsilon(r) - p(r))}{r - 2M(r)}, \]

\[ Q(r) = \frac{4\pi r \left( 5\epsilon(r) + 9p(r) + \frac{\epsilon(r) + p(r)}{\partial \epsilon(r)/\partial \epsilon(r)} - \frac{6}{4\pi r^2} \right)}{r - 2M(r)} - 4 \left[ \frac{M(r) + 4\pi r^3 p(r)}{r^2 (1 - 2M(r)/r)} \right]^2, \]

along with TOV equation with proper boundary conditions [38, 42]. One can then define the dimensionless tidal deformability: \( \Lambda = \frac{2}{5} k_2 C^{-5} \).

Individual dimensionless tidal deformability of two stars, \( \Lambda_1 \) and \( \Lambda_2 \) cannot be extracted independently from the observed gravitational waveform. Instead, an effective dimensionless tidal deformability of the binary \( \tilde{\Lambda} \) can be extracted, which is a mass-weighted average of the individual dimensionless tidal deformability \( \Lambda_1 \) and \( \Lambda_2 \). The effective tidal deformability \( \tilde{\Lambda} \) of binary system in terms of \( \Lambda_1 \) and \( \Lambda_2 \) is defined as [43]

\[ \tilde{\Lambda} = \frac{16}{13} \frac{(m_1 + 12m_2)m_1^4 \Lambda_1 + (m_2 + 12m_1)m_2^4 \Lambda_2}{(m_1 + m_2)^5} \] (30)

where \( m_1 \) and \( m_2 \) are the masses of the neutron stars in the binary. Similarly, masses of the two companion neutron stars cannot be measured directly, rather the chirp mass, \( M_c = (m_1 m_2)^{3/5}(m_1 + m_2)^{-1/5} \), which is measured directly. By assuming low-spin prior which is consistent with the binary neutron star systems that have been observed in, GW170817 put an upper
bound on the NSs combined dimensionless tidal deformability and chirp mass with 90% confidence [13]. This analysis predicts that the combined dimensionless tidal deformability of the NS merger is $\tilde{\Lambda} \leq 800$. It is important to note that a reanalysis of GW170817 observation has been done assuming the same EoS for both stars and this puts an upper limit on the dimensionless tidal deformability, $\tilde{\Lambda} \leq 1000$ [44]. However lower bound on dimensionless tidal deformability can be put using the investigation of the UV/optical/infrared counterpart of GW170817 with kilonova models [45]. The lower bound of dimensionless tidal deformability is $\tilde{\Lambda} \geq 400$.

One of the important goals of this work is to study the structural properties of neutron stars in the presence of the dark matter component. For the sake of arguments it is important to understand the behavior of dimensionless tidal deformability and the tidal Love number of the companion neutron stars. Solving Eq.(29) and TOV equation with appropriate boundary conditions, we get $y_R$. Using the value of $y_R$ and compactness one can get $k_2$ using the expression Eq.(28). Left plot in Fig.(3) shows the dimensionless tidal deformability $\Lambda$ and the right plot shows tidal Love number $k_2$ as a function of the NS mass for our EoS with different dark matter density. The value of $k_2$ is of the range 0.09-0.13 for a typical neutron star of mass $1.5M_\odot$, which is expected [14, 15]. For a given neutron star mass (say around $1.5M_\odot$) EoS without the dark matter predicts larger radius and with increasing dark matter density radius decreases. Since the dimensionless tidal deformability is inversely proportional to the compactness ($C = M/R$), its value is larger in the absence of dark matter.

To study the tidal deformability constraint from the GW170817 observation on EoS of NS, we plot the combined tidal deformability of the binary system in $\Lambda_1, \Lambda_2$ plane in Fig.4. $\Lambda_1$ and $\Lambda_1$ are the individual dimensionless tidal deformability of the high mass $m_1$ and low mass $m_2$ neutron stars in a binary, respectively. The curves are corresponding to the EoS with different dark matter density and obtained by varying $m_1$ and $m_2$ independently. $m_1$ has been taken in the range $1.365 < m_1/M_\odot < 1.60$ and the range of the $m_2$ is determined by keeping the chirp mass $M_c$ fixed at $1.188M_\odot$. The dashed and the dot lines represent, respectively, the 90% and 50% confidence limits of the combined dimensionless tidal deformability obtained from the GW170817 for the low spin prior. One can see from this plot that EoS given by the NL3 parameterization without dark matter component can be excluded at 90% confidence level upper bound on tidal deformability of a binary system. However if we consider dark matter component in neutron stars, then NL3 parameterized EoS comes within the 90% confidence level. Hence a small component of
dark matter inside a neutron star can revive well known EoS, which otherwise might be excluded by the GW170817 observation.

FIG. 3: Dimensionless tidal deformability ($\Lambda$) of NS and Love number($k_2$) as a function of neutron star mass for different dark matter Fermi momentum is shown.

FIG. 4: Tidal deformability parameters of the low and high mass components of binary neutron star merger (GW170817 observation). Dashed line and dotted line indicates 90% and 50% confidence limit for low spin priors [13]. The diagonal solid line corresponds to $\Lambda_1 = \Lambda_2$ boundary. Walecka model with NL3 parameterization is disfavoured by GW observation at 90% C.L. in the absence of dark matter i.e. $k_F^{DM} = 0.0$ GeV. Note that low dark matter density, e.g. $k_F^{DM} = 0.02$ GeV is also disfavoured. However, Walecka model in NL3 parameterization with relatively higher DM density, e.g. $k_F^{DM} = 0.03 - 0.06$ GeV is allowed by 90% C.L. of the GW170817 observation.
6. CONCLUSIONS

We have confronted the neutron star equation of state in the presence of dark matter component using the gravitational wave constraint from the binary star merger. We have shown that for a uniformly distributed dark matter inside neutron star, the EoS becomes softer which eventually produces lower NS mass with increasing dark matter density. We have taken Walecka model with NL3 parameterization in the nuclear matter sector. Walecka model with NL3 parameterization without dark matter admixture gives rise to a maximum mass of NS $\sim 2.8M_\odot$. By increasing dark matter density (Fermi momentum) inside neutron star reduces the value of maximum mass. Value of the maximum mass of neutron star changes from $2.8M_\odot$ to $2.1M_\odot$ by increasing dark matter Fermi momentum from 0.0 GeV to 0.06 GeV. One of the striking results of our analysis is that the stiffer equation of states such as relativistic mean field model (Walecka model) with NL3 parameterization is ruled out at 90% C.L. using the GW170817 observation. However, in the presence of dark matter this constraint can be evaded and NL3 parameterization can be brought within 90% C.L..

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