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**Magnetohydrodynamics Mixed Convective Couple Stress Hybrid Nanofluid Darcy-Forchheimer Flow through a Rotating Porous Space**

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**Abstract**

In this study, we consider the magnetohydrodynamics mixed convective couple stress hybrid nanofluid Darcy-Forchheimer flow through a rotating porous space with velocity slip condition. The nonlinear thermal stratification and thermal radiation of Magnetohydrodynamics (MHD) are discussed in detail. For relative analysis, we have taken the nanoparticles samples of Aluminum oxides (\(\text{Al}_2\text{O}_3\)) and Titanium dioxide (\(\text{TiO}_2\)). The rotation in the disk is produces for the generation of the flow in the system. Furthermore, the variable permeability and porosity of porous space is regarded as Darcy-Forchheimer expression. The resulting nonlinear system of ODE’s are solved by Homotopy Analysis Method (HAM). The governing of several sundry parameters i.e. “Couple Stress, coefficient of inertia, radiation parameter, magnetic parameter, Prandtl number, heat source or sink parameter” are presented both graphically as well as in numerical tables. The behavior of the flow predicted that the increase of both mixed convection and couple stress parameters cause increase in the momentum profile. Temperature of the system rises for higher values of radiation parameter and magnetic parameter. The higher local heat transfer rate of Aluminum oxides (\(\text{Al}_2\text{O}_3\)) and Titanium oxide (\(\text{TiO}_2\)) or water is examined as compared to hybrid nanofluid.

**Keywords:**

Darcy-Forchheimer porous; Titanium oxide (\(\text{TiO}_2\)); Aluminum oxide (\(\text{Al}_2\text{O}_3\)); water; Hybrid nanofluid; variable porosity and permittivity; Couple stress fluid; Velocity slip
conditions; Nonlinear thermal radiation; Thermal stratification; Heat absorption/generation, Magnetohydrodynamics (MHD).

Nomenclature:

| Symbol          | Description                                      | Symbol          | Description                  |
|-----------------|--------------------------------------------------|-----------------|------------------------------|
| \((u, v, w)\)   | velocity components \((m/s)\)                    | \(\Omega\)     | constant angular velocity    |
| \(M\)           | Magnetic field parameter                         | \(T_w\)        | Surface temperature \((K)\)  |
| \(\rho_{inf}\)  | Density of hybrid nanofluid \((kgm^{-3})\)      | \(C_b\)        | drag coefficient             |
| \((\rho c_p)_{inf}\) | volumetric heat capacity of hybrid nanofluid \((m^2/s^2K)\) | \(d\)          | variable permeability        |
| \(\phi\)        | nanoparticle volume fraction                     | \(d^*\)        | variable porosity            |
| \(\theta\)      | Dimensionless temperature \((-)\)                | \(T_0\)        | Reference temperature \((K)\) |
| \(k_f\)         | Thermal conductivity \((Wm^{-1}K^{-1})\)         | \(\sigma^*\)   | Stefan Boltzmann constant    |
| \(G_r\)         | Grashof number                                   | \(K_x\)        | dimensional permeability     |
| \(q_r\)         | Rosseland approximation                          | \(\bar{\varepsilon}_x\) | dimensional porosity        |
| \(Pr\)          | Prandtl number                                   | \(k\)          | mean absorption coefficient   |
| \(C_f, C_g\)    | Skin friction coefficient                        | \(T_\infty\)   | Ambient temperature \((K)\)  |
| \(K\)           | coupled stress parameter                         | \(\mu_{inf}\)  | dynamic viscosity of hybrid nanofluid \((kg/ms)\) |
| \(Nu\)          | Nusselt number                                   | \(g\)          | acceleration due to gravity \((m/s^2)\) |
| \(\lambda\)     | mixed convection or buoyancy parameter           | \(Q\)          | volumetric rate of heat generation/absorption |
| \(\rho_f\)      | Density \((kgm^{-3})\)                           | \(v_f\)        | Kinematic viscosity \((m^2s^{-1})\) |
| \(f^*\)         | Dimensionless velocity \((-)\)                   | \(\mu_f\)      | Dynamic viscosity \((kgm^{-1}s^{-1})\) |
| \(\phi_1, \phi_2\) | volume fraction of first and second nanoparticles | \(w_1, w_2\)   | the first nanoparticle, the second nanoparticle |
| \(\theta_w\)    | Temperature ratio parameter                      | \(Re_r\)       | Reynolds number               |
| \((\rho c_p)_p\) | Effective heat capacity of nanoparticles \((m^2/s^2K)\) | \(\xi\)        | Similarity variable          |
| \(k_1\)         | Non-dimensional porosity parameter              | \(T\)          | Temperature of the fluid \((K)\) |
| \(\alpha\)      | heat source parameter                            | \(\gamma^*\)   | velocity slip parameter      |
| \(S_t\)         | thermal stratification parameter                 |                 |                              |

Introduction:
The study of nanofluids has gained a valuable attraction due to its vast applications in different fields of industry. In order to increase thermal conductivity of base fluid nanoparticle has been introduced. Because the simplest base fluids do not meet the required cooling in the industries. For this purpose the nanoparticles of very small size of (1nm to 100nm) can be included in both Newtonian and non-Newtonian models. The various forms of chemically stable nanoparticles are diamond, graphite, carbon nanotubes (CNTs) and fullerene. In the oxides of metals it consists on zirconia, alumina, titanium and silica. In metals it included gold (\(Au\)) and copper (\(Cu\)). \(AlN\) and \(SiN\) are metals nitrides, \(SiC\) metal carbide and functionalized nanoparticles. The two different types of hybrid nanoparticles can increase the thermal conductivity of single nanoparticle by hybridization known as hybrid nano-fluids. It is the next generation of nano-fluid having innovative thermophysical characteristics. The first mathematician was Choi [1] who introduced the nanoparticles in base fluid. During his experimental work he found that nanoparticles in base fluid increases efficient thermal conductivity of the common fluid. The literature review reveals that there have been numerous investigations into the boundary layer flow of Nano fluids on stretch surface. Kuznetsov et al. [2] described the simultaneous solutions over vertical plate of viscous nanofluids. The pioneer work of Khan et al. [3] represented his work on the topics concern to the nanofluids flow on stretching surfaces. The analytical solutions of steady laminar nanoliquids flow for suction or injection effects by Nadeem et al. [4]. Mehboob et al. [5] explored numerically the impact of heat transfer on boundary layer flow of nanofluid over nonlinearly elongating sheet. Exponentially flow of the nanofluid with the condition of convective boundary layer stream was explored by Mustafa et al. [6]. It has been observed that the geometry of particles strongly disturbs the nanoparticles thermal conductivity. Das and Tiwari et al. [7], providing numerical solutions of mixed convective flow of nanofluids. The improvement in heat transfer in the past on stretch surface flow of nonfluid has been analyzed by Vajravelu [8]. The stream of nanofluid in 3D under the condition of convective boundary layer over sheet elongating in two direction was explored by Khan [9]. The flow of mixed convective nanofluid composed of \(SiO_2 - H_2O\) past a spinning circular cylinder was explored by Selimfedigil [10]. Hayat and Nadeem et al [11] investigated the heat transfer enhancement in chemically reactive flow of silver and copper oxide (\(Ag - CuO\)/water) hybrid nanofluid. Rashad [12] presented in his work the natural convective stream of
Cu–AlO$_2$–water hybrid under the impacts of MHD and joule heating. Some developments in the study of hybrid nanofluids are consolate in refs: [13-15]. In industries Mixed convection boundary layers flows having the large number of real life uses such as electronic devices, heat exchanger, nuclear reactor, central solar receiver et al. [16]. In light of these numerous applications it is opening a new door for the researchers, to investigate the consequences of mixed convection boundary layer flow analytically as experimentally. Hayat [17] have discussed analytically viscoelastic mixed convective nanofluid through stretched cylinder. The transfer of heat through time dependent convective flow over an elongating belt was discussed by Turkyilmazoglu [18]. The heat transfer through time dependent convective boundary layer over shrinking/stretching surface by the spectral relaxation technique was analyzed by Haroun [19]. Behazadmehr [20] designed and studied convective flow of nanoliqiuds in tube. The heat-transfer properties of convective nano-fluids subjected to stretched sheet was investigated by Mahdi [21]. The branch of magnetohydrodynamics shortly known as MHD has incorporates ascending phenomenon when attractive fields is smeared to electrically engaged liquids. The research in the field of magnetohydrodynamics (MHD) has confirmed reckless in modern research “Davidson [22]”. There are applications in the method of accumulation in the exploration of the stream and heat-transfer of electrically conductive liquids in the vision of an attractive field over a hot surface. For example: cooling a metal plate, polymer explosion and atomic reactors etc. MHD of two-layer electro-osmotic flow with entropy generation over micro-parallel stations was considered by Xie and Jian [23]. Khan [24] studied the behavior of steady state flow in 3D with MHD for Powell–Eyring nanofluids through convective and nanoparticles mass flux situations. Ellahi [25] examined Heat transmission in a limit-layer stream with entropy generation and MHD impact. The MHD mixed convective flow of nanofluid composed of copper and water (Cu–H$_2$O) due to rotation of the circular cylinder across a trapezoidal vessel saturated by spongy medium was analyzed by Ali et al [26]. In this study they shown that the perpendicular magnetic field disparages the stream function by a large amount compared to inclined and horizontal. Thermal stability mentions the formation or accumulation of layers. It happens when thermal Stimulation of density gradient $>>$0 is not upset by convective flow. Because of this situation the system remains in stable balance. Thermal stratified fluid flow of nanofluids with MHD under solar radiation was explored numerically by Kadasamy et al. [27]. Zaib and Shafi et al. [28] studied electrically conducting and chemically
reactive fluid flow of viscous fluid with the generation of heat or thermally stratification and absorption. The doubly stratified flow of nanoliquid over a vertical plate was studied by Srinivasacharya and Surender et al. [29]. Sheremet [30] has explored the free convective stream of nanofluid by keeping the impact of thermal stratification inside the cavity with square shape. The purpose of this article to investigate the combined effect of Magnetohydrodynamics MHD numerically, the non-linear thermal radiation of Darcy-Forchheimer fluid flow of mixed convective couple stress of hybrid nanofluids using Titanium oxide ($\text{TiO}_2$), Aluminum oxide ($\text{Al}_2\text{O}_3$) nanoparticles along with thermal stratification. The velocity slip condition is also analyzed. The flow for this problem was generated by rotating disk. The governing system of nonlinear equations is solved by Homotopy Analysis Method HAM. The different emerging variables are examined graphically. In addition for all physical quantities the numeric data is interpreted is and tabularized. The graphs of for different governing variables are plotted and their results are discussed.

**Problem Formulation:**

Three dimensional i.e. 3D steady flow of mixed convective couple stress hybrid-nanofluid is produced by a rotating disk with magnetohydrodynamics MHD. Velocity slip condition for the flow is illustrated in Figure 1. The angular velocity $\Omega$ of the rotation of the disk is considered uniformly. The $x$, $y$ and $z$ component of velocity $(u,v,w)$ in the increasing direction $(r,\psi,z)$ which results the equation of the flow is satisfied.

\[
\begin{align*}
\frac{\partial}{\partial r} u + \frac{1}{r} u + \frac{\partial}{\partial z} w &= 0, \\
\frac{u}{r} \frac{\partial u}{\partial r} - \frac{v^2}{r} + w \frac{\partial u}{\partial z} &= \nu_{nf} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial^2 u}{\partial z^2} \right) - \nu_{nf} \varepsilon(z) \frac{K(z)}{\rho_{nf}} u - \frac{\sigma_{nf} \beta_0^2}{\rho_{nf}} u \\
- \frac{\eta_0}{\rho_{nf}} \frac{\partial^4 u}{\partial y^4} - \frac{C_{v} \varepsilon^2(z)}{K(z)} u \sqrt{u^2 + v^2} + \frac{\left[ (1 - \phi_1) \left( 1 - \phi_3 \rho_i \beta_f + \phi_1 \rho_f \beta_i \right) \right]}{\rho_{nf}} \phi_f \rho_f \beta_f} g (T - T_s),
\end{align*}
\]
\[
\begin{align*}
\frac{u}{r} \frac{\partial v}{\partial r} + \frac{w}{r} \frac{\partial u}{\partial z} &= v_{nf} \left( \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{1}{r} \frac{\partial^2 v}{\partial z^2} \right) - \frac{v_{nf} \varepsilon(z)}{K(z)} - \frac{\sigma_{nf} \beta_0}{\rho_{nf}} v \\
&\quad - \frac{\eta_0}{\rho_{nf}} \frac{\partial^4 v}{\partial y^4} - C_s \varepsilon^2(z) v \sqrt{u^2 + v^2} + \frac{(1 - \phi_2) \left( (1 - \phi_1) \rho_s \beta_1 + \phi_s \rho_s \beta_1 \right)}{\rho_{nf}} + \frac{\phi_s \rho_s \beta_2}{\rho_{nf}} g(T - T_\infty), \\
\frac{u}{r} \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} &= v_{nf} \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r} \frac{\partial^2 w}{\partial z^2} \right) - \frac{v_{nf} \varepsilon(z)}{K(z)} w - \frac{\sigma_{nf} \beta_0}{\rho_{nf}} w \\
&\quad - \frac{\eta_0}{\rho_{nf}} \frac{\partial^4 w}{\partial y^4} - C_s \varepsilon^2(z) w \sqrt{u^2 + v^2} + \frac{(1 - \phi_2) \left( (1 - \phi_1) \rho_s \beta_1 + \phi_s \rho_s \beta_1 \right)}{\rho_{nf}} + \frac{\phi_s \rho_s \beta_2}{\rho_{nf}} g(T - T_\infty), \\
\frac{u}{r} \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} &= \alpha_{nf} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r} \frac{\partial^2 T}{\partial z^2} \right) - \frac{1}{(\rho c_p)_{nf}} \frac{\partial q_r}{\partial r} + \frac{Q}{(\rho c_p)_{nf}} (T - T_\infty),
\end{align*}
\]

Figure: Schematic diagram of the 3D problem
\[
\begin{align*}
\frac{u}{L} &= \frac{\partial u}{\partial z}, v = r \left( \Omega + \frac{L}{r} \frac{\partial}{\partial z} u \right), w = 0, T = T_w = T_0 + Ar, \text{ at } z = 0 \\
\end{align*}
\]

\[
\begin{align*}
 u &\to 0, v \to 0, T \to T_w = T_0 + Br, \text{ at } z = \infty
\end{align*}
\]

(6)

Where

\[
K_e \left( 1 + \text{de} \right) = K, \text{ Where } K \text{ is function of } z
\]

(7)

\[
e_{e} \left( 1 + \text{de} \right) = e(z),
\]

(8)

\[
q_r = -\frac{4\sigma^* T^4}{3k} - \frac{16\sigma^* T^3}{3k} \left( \frac{\partial T}{\partial z} \right),
\]

(9)

Thus the energy equation becomes:

\[
\begin{align*}
&u \left( \frac{\partial}{\partial r} T \right) + w \left( \frac{\partial}{\partial z} T \right) = \alpha_{\text{hmf}} \left( \frac{\partial^2}{\partial r^2} T + \frac{1}{r} \frac{\partial}{\partial r} T + \frac{\partial^2}{\partial z^2} T \right) + \frac{1}{\left( \rho c_p \right)_{\text{hmf}}} \frac{16\sigma^*}{3k} \frac{\partial}{\partial z} \left( T^3 \frac{\partial T}{\partial z} \right) \\
&+ \frac{Q}{\left( \rho c_p \right)_{\text{hmf}}} (T - T_w),
\end{align*}
\]

(10)

The hybrid nanofluid Theoretical model is [13]

\[
\begin{align*}
\mu_{\text{hmf}} &= \frac{\mu_f}{(1 - \phi_1 - \phi_2)^{2.5}}, \nu_{\text{hmf}} = \frac{\mu_{\text{hmf}}}{\rho_{\text{hmf}}}, \rho_{\text{hmf}} = (1 - \phi_1 - \phi_2) (\rho)_f + \phi_1 (\rho) + \phi_2 (\rho), \\
\alpha_{\text{hmf}} &= \frac{k_{\text{hmf}}}{\left( \rho c_p \right)_{\text{hmf}}}, \left( \rho c_p \right)_{\text{hmf}} = (1 - \phi_1 - \phi_2) (\rho c_p)_f + \phi_1 (\rho c_p)_1 + \phi_2 (\rho c_p)_2, \\
k_{\text{hmf}} &= \frac{k_f}{k_f} \frac{k_1 \phi_1 + k_2 \phi_2 + 2\phi k_f + 2\phi (\phi k_1 + \phi k_2) - 2(\phi_1 + \phi_2)^2 k_f}{k_f} \\
\end{align*}
\]

(11)
\( \rho_1 \) and \( \rho_2 \) are densities of Titanium oxide \( \text{TiO}_2 \) and Aluminum oxide \( \text{Al}_2\text{O}_3 \) respectively. \( k_1 \), \( k_2 \) and \( k_f \) are thermal conductivities of Titanium oxide \( \text{TiO}_2 \), Aluminum oxide \( \text{Al}_2\text{O}_3 \) and base fluids respectively.

In Table 1 thermo-physical properties of nanoparticles and water are discussed and given as:

**Table 1:** Thermo-physical properties of nanoparticles characteristics and water.

| Physical properties | Base fluid | \( \text{TiO}_2 \) | \( \text{Al}_2\text{O}_3 \) |
|---------------------|------------|-----------------|-----------------|
| \( \rho \) (kg/m\(^3\)) | 9.97\times10\(^2\) | 4.23\times10\(^3\) | 4.0\times10\(^3\) |
| \( k \) (W/mK) | 6.13\times10\(^{-1}\) | 8.4 | 40 |
| \( c_p \) (J/kgK) | 4.179\times10\(^3\) | 6.92\times10\(^2\) | 7.73\times10\(^2\) |

Considering

\[
\begin{align*}
    u &= r\Omega f' (\xi), \\
    v &= \Omega g (\xi), \\
    w &= -\sqrt{2\Omega v_f} f (\xi), \\
    \theta (\xi) &= \frac{T - T_\infty}{T_w - T_0}, \\
    \xi &= \left( \frac{2\Omega}{v_f} \right)^{1/2} z,
\end{align*}
\]

(12)

And

\[
T = (T_w - T_0) \theta (\xi) + T_\infty
\]

Then Equation (1) is satisfied and Equations. (2 to 11) gives
\[
\frac{1}{(1-\phi_1-\phi_2)} \left(1-\phi_1-\phi_2 + \frac{\rho_1}{\rho_f} \phi_1 + \frac{\rho_2}{\rho_f} \phi_2 \right) \left(2f'' - \frac{1}{2k_i \text{Re}_r} \left(1 + d^* e^{-\xi} \right) f' \right) \]

(13)

\[-F_r \left( \frac{1 + d^* e^{-\xi}}{\sqrt{1 + d e^{-\xi}}} \right) \left( f'^2 + (0.5) g^2 \right) - \left( f'^2 \right) + \left( g^2 \right) + 2f f' - \frac{\sigma_{hnf}}{\rho_{hnf}} M_s' + 4 \frac{\rho_f}{\rho_{hnf}} K_f v + A_\lambda \lambda \theta = 0, \]

(14)

\[-F_r \left( \frac{1 + d^* e^{-\xi}}{\sqrt{1 + d e^{-\xi}}} \right) \left( g^2 + (0.5) f'^2 \right) g' - f^g + f^g' - \frac{\sigma_{hnf}}{\rho_{hnf}} M_s g + 4 \frac{\rho_f}{\rho_{hnf}} K g w + A_\lambda \lambda \theta = 0, \]

(15)

\[\frac{1}{(1-\phi_1-\phi_2)} \left(1-\phi_1-\phi_2 + \frac{\rho_1}{\rho_f} \phi_1 + \frac{\rho_2}{\rho_f} \phi_2 \right) \left( \left( \frac{k_{hnf}}{k_f} + \frac{4}{3} R \left( \frac{1}{\theta_w + s_r} \right) \theta + 1 \right)^3 \right) \theta' + Pr f \theta' \]

\[+ \frac{k_f}{k_{hnf}} \alpha \theta = 0, \]

(16)

\[f = 0, f'' = \frac{f'}{\gamma}, 1 - g + \gamma g' = 0, \quad \theta = 1 - S_r \quad \text{at} \; \xi = 0 \]

\[\theta \to 0, \; g \to 0, \; f' \to 0 \quad \text{at} \; \xi \to \infty \]

In the above equations

\[\gamma^* = \text{Slip velocity parameter}, \quad k_1 = \text{Porosity parameter}, \]

\[\text{Re}_r = \text{Local Reynolds number}, \quad F_r = \text{Local inertia parameter}, \]

\[Pe_r = \text{Peclet number}, \quad \theta_w = \text{Variable for temperature}, \]

\[M = \text{Magnetic field parameter}, \quad \lambda = \text{Mixed convection i.e. buoyancy parameter}, \]
$S_r =$ Thermal stratification parameter, $R =$ Radiation parameter,

$P_r =$ Prandtle number, and heat source parameter ($\alpha$)

These are defined by:

$$
\gamma^* = L \left( \frac{2\Omega}{v_f} \right)^{1/2},
\frac{k_1}{r^2 \xi},
\frac{Re_r}{\sqrt{K}},
\frac{F_r}{v_f},
\frac{C_p c_p^2}{\rho r \Omega},
\frac{\sigma \beta^2}{\rho r \Omega},
$$

$$
\theta_w = \frac{T_0}{Ar},
\frac{S_r}{B} = \frac{B}{A},
R = \frac{4\sigma T_\infty^3}{kk_f},
Pr = \frac{v_f}{\alpha_f},
Pe_r = Re_r Pr, 
\frac{\alpha}{\rho r} = \frac{Gr_r}{Re_r^2},
$$

$$
A_1 = A_2 \times \left( 1 - \frac{\rho_1 w_1 + \rho_2 w_2}{\rho_1 \rho_2} \right)
\left( \frac{1}{\rho_1 \rho_2 \rho_f} \frac{1}{\rho_1 \rho_2 w_3} \right),
$$

$$
A_2 = \left[ \frac{\rho_2 w_1 + \rho_2 w_2}{\rho_1 \rho_2} \right]
\left( 1 - \frac{\rho_2 w_1 + \rho_2 w_2}{\rho_1 \rho_2} \right),
$$

$$
A_3 = A_2 \left( \frac{\rho_2 \rho_f w_2}{\rho_2 \rho_f w_1 + \rho_2 \rho_f w_2 + \rho_2 w_f} \left( \frac{1}{\rho_1 \rho_2 \rho_f} \right) \right),
$$

$$
\left[ Re_r \right]^{1/2} C_f = \frac{1}{(1 - \phi_1 - \phi_2)^{2.5}} f''(0),
$$

$$
\left[ Re_r \right]^{1/2} C_g = \frac{1}{(1 - \phi_1 - \phi_2)^{2.5}} g'(0),
$$

$$
\frac{1}{2} \left[ Re_r \right]^{1/2} Nu = -\left( \frac{k_{inf}}{k_f} + \frac{4}{3} R \left( \frac{1}{\theta_w + S_r} + 1 \right) \right)^{3/2} \theta'(0),
$$
Solution by HAM

The solution of the problem in Equation (13 to 15) was found by using HAM under the boundary conditions in Equation (16).

\[
f'' = L_j(f'), \quad g'' = L_k(g'), \quad \theta'' = L_\theta(\theta),
\]

(20)

The linear operators for the problem are:

\[
L_j \left( e_1 + \eta \left( e_2 + e_4 \eta + e_6 \eta^2 + e_8 \eta^3 \right) \right) = 0,
\]

\[
L_k \left( e_6 + \eta \left( e_7 + e_9 \eta + e_{10} \eta^2 \right) \right) = 0,
\]

\[
L_\theta \left( e_{10} + e_1 \eta \right) = 0,
\]

(21)

The non-linear operator is chosen as \( N_j \), \( N_\theta \) and \( N_k \) identify in system:

\[
N_j \left( \frac{1}{(1 - \phi_1 - \phi_2)} \left( \frac{1}{1 - \phi_1 - \phi_2 + \frac{\rho_1}{\rho_f} \phi_1 + \frac{\rho_2}{\rho_f} \phi_2} \left( 2 f_{zzz} - \frac{1}{2k} \text{Re} \left( \frac{1 + d e^{-\xi}}{1 + d e^{-\xi}} \right) f_{\xi} \right) - F_r \left( \frac{1 + d e^{-\xi}}{\sqrt{1 + d e^{-\xi}}} \left( f_{\xi}^2 + \frac{1}{2} g^2 \right) - f_{\xi}^2 + 2 f_{\xi z}^2 \right) \right) \right)
\]

\[
+ \frac{\sigma_{\text{nfj}}}{\rho_{\text{nfj}}} M f_{\xi} + 4 \frac{\rho_f}{\rho_{\text{nfj}}} K f_{\xi z z} + A_\lambda \dot{\theta},
\]

\[
N_k \left[ \frac{1}{(1 - \phi_1 - \phi_2)} \left( \frac{1}{1 - \phi_1 - \phi_2 + \frac{\rho_1}{\rho_f} \phi_1 + \frac{\rho_2}{\rho_f} \phi_2} \left( 2 g_{zz} - \frac{1}{2k} \text{Re} \left( \frac{1 + d e^{-\xi}}{1 + d e^{-\xi}} \right) g \right) - F_r \left( \frac{1 + d e^{-\xi}}{\sqrt{1 + d e^{-\xi}}} \left( g^2 + \frac{1}{2} f_{\xi}^2 \right) - f_{\xi} g \right) \right) \right)
\]

\[
+ \frac{\sigma_{\text{nfk}}}{\rho_{\text{nfk}}} M g + 4 \frac{\rho_f}{\rho_{\text{nfk}}} K g_{zz} + A_\lambda \dot{\theta},
\]

(22)

(23)
\[ N_\theta \left[ \frac{\partial f(\xi; \zeta)}{\partial \zeta}, \frac{\partial f(\xi; \zeta)}{\partial \xi} \right] = \frac{1}{(1 - \phi_1 - \phi_2) \left( 1 - \phi_1 - \phi_2 + \frac{(\rho_{cp})_1}{(\rho_{cp})_f} \phi_1 + \frac{(\rho_{cp})_2}{(\rho_{cp})_f} \phi_2 \right)} \]

\[
\left[ \left( \frac{k_{naf}}{k_f} + \frac{4}{3} R \left( \frac{1}{(\theta_n + s_1)} \frac{\partial f}{\partial z} + 1 \right) \right) \frac{\partial f}{\partial \xi} \right] + Pr \left( \frac{\partial \theta}{\partial \xi} + \frac{k_f}{k_{naf}} \alpha \theta, \right)
\]

While BCs are

\[
\left. \left. \frac{\partial^2 f(\xi; \zeta)}{\partial \xi^2} \right|_{\xi = 0} = \gamma \frac{\partial^2 f(\xi; \zeta)}{\partial \zeta^2} \right|_{\xi = 0}, \left. \right|_{\xi = 0} = 0, \left. \frac{\partial g(\xi; \zeta)}{\partial \eta} \right|_{\eta = 0} = 1 + \gamma \left. \frac{\partial g(\xi; \zeta)}{\partial \eta} \right|_{\eta = 0}, \left. \frac{\partial \theta(\xi; \zeta)}{\partial \xi} \right|_{\xi = 0} = 0.
\]

Here, \(\zeta\) is the embedding parameter \(\zeta \in [0, 1]\), to standardize the convergence of the solution of \(h_j, h_k, h_\theta\) is used. By choosing \(\zeta = 0\) and \(\zeta = 1\) we have:

\[
\left. \frac{f(\xi)}{\xi = 0}, \frac{g(\xi)}{\xi = 0}, \frac{\theta(\xi)}{\xi = 0} \right|_{\zeta = 0} = 1, \left. \right|_{\zeta = 1}.
\]

Develop the Taylor’s series for \(f(\xi; \zeta), g(\xi; \zeta)\) and \(\theta(\xi; \zeta)\) about the point \(\zeta = 0\)

\[
\left. \frac{f(\xi; \zeta)}{\xi = 0} = \sum_{n=1}^{\infty} f_n(\xi) \zeta^n \right|_{\zeta = 0}, \left. \right|_{\zeta = 1} = 1, \left. \right|_{\zeta = 1}
\]

\[
\left. \frac{g(\xi; \zeta)}{\xi = 0} = \sum_{n=1}^{\infty} g_n(\xi) \zeta^n \right|_{\zeta = 0}, \left. \right|_{\zeta = 1} = 1, \left. \right|_{\zeta = 1}
\]

\[
\left. \frac{\theta(\xi; \zeta)}{\xi = 0} = \sum_{n=1}^{\infty} \theta_n(\xi) \zeta^n \right|_{\zeta = 0}, \left. \right|_{\zeta = 1} = 1, \left. \right|_{\zeta = 1}
\]
\[ f_n(\xi) = \frac{1}{n!} \left. \frac{\partial^n f(\xi, \zeta)}{\partial \zeta^n} \right|_{p=0}, \]
\[ g_n(\xi) = \frac{1}{n!} \left. \frac{\partial^n g(\xi, \zeta)}{\partial \zeta^n} \right|_{p=0}, \]
\[ \theta_n(\xi) = \frac{1}{n!} \left. \frac{\partial^n \theta(\xi, \zeta)}{\partial \zeta^n} \right|_{p=0}. \]  

While BCs are:

\[ f(0) = 0, f'(0) = \gamma f^*, g(0) = 1 + \gamma g^*(0), \theta(0) = 1 - S, \]
\[ f'(\infty) \to 0, g(\infty) \to 1, \theta(\infty) \to 0. \]  

**Results and Discussion:**

This section describes the behavior of emerging flow variables i.e. the porosity parameter \( k \), local inertial parameter \( F \), local Reynolds number, the variable parameter for temperature \( \theta \), mixed convection or buoyancy parameter, stratification parameter \( S \), radiation parameter, and Prandtl number and heat source parameter which is represented by \( \alpha \). The impact on velocity profile and temperature of these parameters is shown in figures 2-19. The impact of magnetic parameter \( M \) on \( f'(\xi), g(\xi) \) is shown in figures 2 and 3. From the figures it is obvious that magnetics parameter is inversely proportional to the velocity profile \( f'(\xi), g(\xi) \) which reduces corresponding boundary layer. Figures 4 and 5 show the impact of \( F \), local inertial parameter on velocity profiles \( f'(\xi), g(\xi) \). It is noticed that the stronger physical resistance is offered to the flow of the base fluid cause reduction in the characteristic of \( f'(\xi), g(\xi) \), which reduces the corresponding boundary layer. Figures 6 and 7 demonstrate that increasing values of porosity
factor enhance extra resistance to the system. Consequently the stronger frictional force results decay in \( f'(\xi), g(\xi) \) physically. Figures 8 and 9 show that enhancement in the mixed convection parameter insensibly yields to enhance dimensionless velocity distribution. It also gives sharp rise in the velocity profile \( \left( f'(\xi), g(\xi) \right) \) close to stretching wall. One the important feature of mixed convection parameter is that “in heat transfer mode in mixed convection for the problem the dimensionless flow field becomes coupled with the dimensionless temperature field. Figures 10 and 11 are showing the impact of Local Reynolds number \( \text{Re}_r \) on the velocity profile \( f'(\xi), g(\xi) \). It is obvious from these figures that the velocity profile is improved for greater estimation of Local Reynolds number \( \text{Re}_r \). Figures 12 and 13 also denotes the outcome of couple-stress parameter \( K \) on the \( f'(\xi), g(\xi) \). It can be comprehended from these figures that the \( f'(\xi), g(\xi) \) increases while increasing couple-stress parameter \( K \). However, the increase in velocity profiles will be not significant for large values of \( K \). That is to say that large values of \( K \) will lead to pure viscous fluid. Figure 14 displays the possessions of \( p_r \) versus \( \theta(\xi) \) temperature profiles. These figure show that the temperature profile showing decrease when increasing the value of Prandtl number \( p_r \). It is obvious from the figure the increment in Prandtl number \( p_r \) cause decrease in the thermal diffusion. Figure 15 discloses that the temperature for the nanofluid is raised due to the influence of \( R \) (thermal radiation parameter). Enhancement in radioactive heat flux fosters the molecular passage inside the base fluid, and thus numerous collisions between molecules translate in the form of thermal energy. Consequently, higher value of temperature distribution has been observed as a result of a rise in the value of thermal radiation parameter \( R \). It is remarkable to remember that being extremely viscous in kind; hyperbolic tangent nanofluids produce more heat than Newtonian nanofluid. The effect of heat sink for a very small i.e. \( \alpha < 0 \) or for a very large value i.e. \( \alpha > 0 \) parameter in the dimensionless temperature are illustrated in Figure 16 and 17. With increase in the strength of heat source \( \alpha > 0 \) fluid, temperature increases while increasing the thickness of thermal boundary layer. On the other hand increase in heat sink strength \( \alpha < 0 \), falls the temperature due to decrease in the thermal boundary layer thickness. Figure 18 shows relation between \( \theta(\xi) \)
(temperature) and \( \theta_w \). It is clear from the figure that \( \theta(\xi) \) (temperature) is a decreasing function of \( \theta_w \) for titanium oxide \( \text{TiO}_2 \) and Aluminum oxide \( \text{Al}_2\text{O}_3 \) / water. Figure 19 is plotted between \( (S_t) \) and temperature \( \theta(\xi) \). It describes that \( \theta(\xi) \) reduces for larger value of \( (S_t) \) for nanofluid and hybrid nanofluid.

Figure 2: Influence of \( M \) and \( f'(\xi) \) when \( K = 0.7, k_i = 0.3, F_r = 0.5, \lambda = 0.3, \text{Re}_r = 1.3 \).

Figure 3: Influence of \( M \) on \( g(\xi) \) when \( K = 0.7, k_i = 0.3, F_r = 0.5, \lambda = 0.3, \text{Re}_r = 1.3 \).
Figure 4: Influence of $F_r$ on $f'(\xi)$ when $K = 0.7, k_i = 0.3, M = 1.0, \lambda = 0.3, \text{Re}_\tau = 1.3$.

Figure 5: Influence of $F_r$ on $g(\xi)$ when $K = 0.7, k_i = 0.3, M = 1.0, \lambda = 0.3, \text{Re}_\tau = 1.3$. 

$k_1 = 0.5, 0.7, 0.9.$
Figure 6: Influence of $k_1$ on $f' (\xi)$ when $K = 0.7, F_\tau = 0.5, M = 1.0, \lambda = 0.3, \text{Re}_\tau = 1.3$. 

Figure 7: Influence of $k_1$ on $g(\xi)$ when $K = 0.7, F_\tau = 0.5, M = 1.0, \lambda = 0.3, \text{Re}_\tau = 1.3$. 

Figure 8: Influence of $\lambda$ on $f' (\xi)$ when $k_1 = 0.3, K = 0.7, F_\tau = 0.5, M = 1.0, \text{Re}_\tau = 1.3$. 
Figure 9: Influence of $\lambda$ on $g(\xi)$ when $k_1 = 0.3, K = 0.7, F_r = 0.5, M = 1.0, \text{Re}_r = 1.3$.

Figure 10: Influence of $\text{Re}_r$ on $f'(\xi)$ when $k_1 = 0.3, K = 0.7, F_r = 0.5, M = 1.0, \lambda = 0.3$. 
Figure 11: Influence of $Re_r$ on $g(\xi)$ when $k_i = 0.3, K = 0.7, F_r = 0.5, M = 1.0, \lambda = 0.3$.

Figure 12: Influence of $K$ on $f'(\xi)$ when $k_i = 0.3, F_r = 0.5, M = 1.0, \lambda = 0.3, Re_r = 1.3$. 
Figure 13: Influence of $K$ on $f' (\xi)$ when $k_i = 0.3, F_r = 0.5, M = 1.0, \lambda = 0.3, \text{Re}_r = 1.3$.

Figure 14: Influence of $P_r$ on $\theta (\xi)$ when $\theta_w = 1.1, S_i = 0.5, \alpha = 1.0, R = 0.8$. 
**Figure 15:** Influence of $R$ on $\theta(\xi)$ when $P_r = 7.0, \theta_w = 1.1, S_t = 0.5, R = 0.8$. 

**Figure 16:** Influence of $\alpha > 0$ on $\theta(\xi)$ when $P_r = 7.0, \theta_w = 1.1, S_t = 0.5, \alpha = -1.0, R = 0.8$. 

$R = 0.2, 0.4, 0.8$. 

$\alpha = 0.5, 1.0, 1.5$. 
Figure 17: Influence of $\alpha < 0$ on $\theta(\xi)$ when $P_r = 7.0, \theta_w = 1.1, S_i = 0.5, \alpha = 1.0, R = 0.8$.

$\alpha = -1.5, -1.0, -0.5.$

Figure 18: Influence of $\theta_w$ on $\theta(\xi)$ when $P_r = 7.0, S_i = 0.5, \alpha = 1.0, R = 0.8$.

$\theta_w = 1.1, 1.4, 1.8.$
Figure 19: Influence of $S_t$ on $\theta(\xi)$ when $P_r = 7.0, \theta_w = 1.1, \alpha = 1.0, R = 0.8$.

Table discussion:
Table 2 shows that $C_f$ and $C_g$ are increased when the values of $F_r, k, M$ are increased. They $C_f$ and $C_g$ are decreased, when the values of $K, Re, \lambda$ is increased. Table 3 shows that $Nu$ is increased when the values when the values of $R, \alpha, \theta_w, S_t$ are increased. The $Nu$ is decreased, when the values of $p_t$ is increased.

Table 2: Impact of different physical parameters over Skin friction

| $F_r$ | $k_1$ | $M$ | $K$ | $Re$ | $\lambda$ | $\frac{1}{(1-\phi_1-\phi_2)^{2.5}} f''(0)$ | $\frac{1}{(1-\phi_1-\phi_2)^{2.5}} g'(0)$ |
|-------|-------|-----|-----|------|----------|-----------------|-----------------|
| 0.1   | 0.5   | 0.5 | 0.4 | 1.0  | 0.6      | 1.2347805       | 1.4207392       |
| 0.3   |       |     |     |      |          | 1.3437309       | 1.5349731       |
| 0.5   |       |     |     |      |          | 1.5768929       | 1.7236894       |
|       | 0.5   |     |     |      |          | 1.1423802       | 2.0451896       |
|       | 0.7   |     |     |      |          | 1.2816925       | 2.1239842       |
|       | 0.9   |     |     |      |          | 1.3469237       | 2.2058903       |
|       | 0.5   |     |     |      |          | 1.2487189       | 1.5243892       |
|       | 1.0   |     |     |      |          | 1.4586316       | 1.7598134       |
|       | 1.5   |     |     |      |          | 1.6236892       | 1.9209678       |
|       | 0.4   |     |     |      |          | 2.4108219       | 1.2348123       |
|       | 0.8   |     |     |      |          | 2.3904527       | 1.1607837       |
|       | 1.0   |     |     |      |          | 2.1486349       | 1.1038031       |
Table 3: Impact of various physical parameters over Nusselt number

\[
\frac{1}{2} \left[ \frac{Re_e}{(Re_e)^2} \right] Nu = - \left( \frac{k_{inf}}{k_f} + \frac{4}{3} R \left( \frac{1}{\theta_w + S_t} + 1 \right)^3 \right) \theta'(0),
\]

| α   | Pr | R   | S_t | θ_w | \left( \frac{k_{inf}}{k_f} + \frac{4}{3} R \left( \frac{1}{\theta_w + S_t} + 1 \right)^3 \right) \theta'(0) |
|-----|----|-----|-----|-----|-------------------------------------------------|
| 0.5 | 7.0| 0.2 | 0.2 | 1.0 | 0.9216734                                       |
| 1.0 | 7.0| 0.2 | 0.2 | 1.0 | 0.6328432                                       |
| 1.5 | 7.0| 0.2 | 0.2 | 1.0 | 0.4637381                                       |
| 7.0 | 7.5| 0.2 | 0.2 | 1.0 | 0.5370358                                       |
| 7.5 | 8.0| 0.2 | 0.2 | 1.0 | 0.7195309                                       |
| 8.0 | 0.2| 0.2 | 0.2 | 1.0 | 0.9063276                                       |
|     | 0.4| 0.2 | 0.2 | 1.0 | 0.8021434                                       |
|     | 0.8| 0.2 | 0.2 | 1.0 | 0.8956873                                       |
|     | 0.5| 0.2 | 0.2 | 1.0 | 0.3984305                                       |
|     | 1.0| 0.2 | 0.2 | 1.0 | 0.3769196                                       |
|     | 0.5| 1.0 | 0.2 | 1.0 | 0.3415809                                       |
|     | 1.0| 1.0 | 0.2 | 1.0 | 0.8714792                                       |
|     | 1.5| 1.0 | 0.2 | 1.0 | 0.6438503                                       |
|     | 1.8| 1.0 | 0.2 | 1.0 | 0.3274239                                       |

Conclusions:

The Darcy-Forchheimer flow of mixed convective couple stress hybrid nanofluid with Magnetohydrodynamics (MHD), nonlinear thermal radiation and thermal stratification is investigated. The major outcomes are listed as follows:

- There is a decreasing in \( f'(\xi), g(\xi) \) for magnetic parameter \( M, (F_r) \) and \( (k_f) \).
- There is an increasing in \( f'(\xi), g(\xi) \) for couple-stress parameter \( K \) and mixed convection parameter.
- Augmentation in velocity profiles are analyzed through \( Re_e \).
• Increase in Pr decreases the temperature profile, while increase in R increases the temperature profile.
• Temperature profile exhibits opposite behavior for of heat sink (α < 0) or source (α > 0) parameter.
• Temperature of the nanofluid is a decreasing function of \((S_t)\) and \((\theta_w)\).

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Figure 1

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Figure 2

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Figure 3

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![Figure 3 Diagram]

$F_r = 0.1, 0.3, 0.5.$

Figure 4

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Figure 5

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Figure 6

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