Mathematical Model Using Discrete-Continuous Approach for Moisture Transfer in Enclosing Construction

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Abstract. The paper presents an unsteady-state calculation method for single-layer enclosing structures moisture regime using a discrete-continuous approach. A new formula allowing moisture potential distribution determination along enclosing structure thickness in any section, at any moment of time, under continuous control for temperature field is introduced. The proposed method was verified by finite difference method using an explicit difference scheme. Single-layer enclosing structure moisture regime calculation results obtained by various moisture potential methods are compared. Calculations using the proposed method give quantitative and qualitative results similar to Gagarin’s unsteady-state method, but are described by calculated expression. Results of this research can be used in practice by design engineers.

1. Introduction

Enclosing structure moisture behavior researches are actively developed at present. There are several principal directions of study: experimental studies to determine moisture flow coefficients [1,2], on-site inspections of buildings and structures for moisture distribution determination along enclosing structure thickness under operating conditions [3,4], new laboratory equipment development to increase experimental accuracy [5]. Moisture influence assessment on building thermal insulation [6,7] and on enclosing structure durability [8,9,10,11,12] are applied researches. Research on moisture content influence in enclosing structure thickness on electromagnetic shielding is worth noting separately [13].

Enclosing structure moisture behavior mathematical models development is one of the most important research problems [14,15,16,17,18]. Moisture behavior calculation methods based on theory of moisture potential are considered to be the most progressive ones [19]. Consider an example using Gagarin’s unsteady-state mathematical model.

Temperature distribution is set by differential heat transfer equation:

\[
\frac{\partial t}{\partial \tau} = a \cdot \frac{\partial^2 t}{\partial x^2}.
\]
\( t \) – temperature, \(^\circ\) C; \( \tau \) – time, s; \( x \) - coordinate, m; \( a \) – material temperature conductivity coefficient, \( m^2 / s \).

Moisture transport passes much slower than heat transport, thus, the steady-state problem of temperature distribution along enclosing structure thickness within the base month can be studied instead of (1) equation without considerable loss of accuracy:

\[
\frac{\partial^2 t}{\partial x^2} = 0. \tag{2}
\]

Gagarin’s differential equation describes moisture movement in enclosing structures:

\[
\frac{\partial F}{\partial \tau} = \frac{\partial}{\partial x} \left( \frac{10^3}{\mu} \beta + \frac{\partial \varphi}{\partial \nabla} E_i \right) \cdot \frac{\mu}{\gamma_0} \cdot \frac{\partial F}{\partial x}, \tag{3}
\]

where \( F \) – moisture potential, Pa; \( \beta \) – moisture conductivity coefficient, \( g/(m \cdot h \cdot \% ) \); \( \mu \) – vapor permeability coefficient \( mg/(m \cdot h \cdot Pa) \); \( \gamma_0 \) – enclosing structure dry material density; \( E_i \) – maximum water vapor tension, Pa.

The moisture potential \( F \) is a function of two variables: moisture and temperature. It is determined by the equation:

\[
F = E_i \cdot \varphi + \frac{10^3}{\mu} \int_0^\gamma \beta(\sigma)d\sigma. \tag{4}
\]

The solution of (2), (3), and (4) simultaneous equations allows to calculate moisture distribution in the enclosing structure using Gagarin’s unsteady-state mathematical model.

V.V. Kozlov proposed to simplify Gagarin’s moisture transfer mathematical model by setting moisture potential time derivative equal to zero in (3) equation. Thus, moisture potential distribution along enclosing structure thickness becomes steady-state:

\[
\frac{\partial^2 F}{\partial x^2} = 0. \tag{5}
\]

Kozlov’s method assumes the solution of (2), (4), (5) simultaneous equations under the assumption of steady-state temperature distribution and steady-state moisture potential distribution along enclosing structure during each month.

Discrete-analytical solutions for various construction problems have been developed in parallel with moisture regime theory development. In 2010, A.B. Zolotov, P.A. Akimov at al. proposed a method of heat transfer equation (1) solving using a discrete-continuous approach [20]:

\[
\widetilde{U} = e^{A \tau} \cdot \widetilde{U}_0 - A^{-1} \left( E - e^{A \tau} \right) \cdot \widetilde{S}. \tag{6}
\]

where \( \widetilde{U} \) – temperature distribution column vector; \( \widetilde{U}_0 \) – initial temperature distribution column vector; \( A \) – coefficient matrix; \( \widetilde{S} \) – boundary conditions column vector.

Later, this method was improved by V.N. Sidorov and S.M. Matskevich [20] by means of discrete-continuous approach extension in two-dimensional case on finite element method.

This work aims at development in moisture regime calculation method for single-layer enclosing structures.

2. The problem

Engineering unsteady-state method development for moisture regime calculation using discrete-continuous approach under continuous control for temperature distribution influence.

3. Materials and methods
Consider Gagarin’s modified moisture transfer equation:

\[
\frac{\partial F}{\partial \tau} = -0.024 \frac{\mu}{\gamma_0 \cdot \xi_{F_0}} \frac{\partial}{\partial x} \left( \frac{E_i}{E_i} \frac{\partial F}{\partial x} \right).
\]  

(7)

where \( \tau \) – time, day; \( \xi_{F_0} \) – material relative potential capacity coefficient, g/kg.

This transformation allowed to separate coefficient responsible for temperature distribution and coefficient responsible for humidity distribution in sorption wetting zone. Relative potential capacity is taken as constant in the equation (7) for calculation within a month. Equations (2), (4), (7) describe heat and moisture transport. Temperature and moisture distributions are related by equation obtained from Clausius–Clapeyron dependence:

\[
E_i = 1.84 \cdot 10^{11} \cdot \exp(-5330/(273 + t)).
\]  

(8)

On the boundaries of the enclosing structure, boundary conditions of the first kind are set. It is assumed that temperature boundary conditions do not depend on time within a month, moisture distribution boundary conditions are constant at enclosing structure inner side, and are set as a linear function at enclosing structure external boundary [Fig.1].

Discrete-continuous approximation of spatial-time domain was made to solve heat and moisture transport problem [Fig.2].

Equation (7) can be presented in matrix form taking into account boundary conditions given in [Fig.1]:

\[
\begin{bmatrix}
F_1'(	au) \\
F_2'(	au) \\
\vdots \\
F_{N+1}'(\tau)
\end{bmatrix}
= \begin{bmatrix}
-2 & 1 & 0 & 0 & 0 \\
1 & -2 & 1 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 1 & -2 & 1 \\
0 & 0 & 0 & 1 & -2
\end{bmatrix}
\begin{bmatrix}
F_1 \\
F_2 \\
\vdots \\
F_N \\
F_{N+1}
\end{bmatrix}

+ \begin{bmatrix}
\mu \\
\mu \\
\vdots \\
\mu \\
\mu
\end{bmatrix}
\begin{bmatrix}
\gamma_0 \cdot \xi_{F_0} \\
\gamma_0 \cdot \xi_{F_0} \\
\vdots \\
\gamma_0 \cdot \xi_{F_0} \\
\gamma_0 \cdot \xi_{F_0}
\end{bmatrix}
\begin{bmatrix}
\tau \\
\tau \\
\vdots \\
\tau \\
\tau
\end{bmatrix}

\begin{bmatrix}
F_1 \\
F_2 \\
\vdots \\
F_N \\
F_{N+1}
\end{bmatrix}

= \begin{bmatrix}
F_1'(	au) \\
F_2'(	au) \\
\vdots \\
F_{N+1}'(\tau)
\end{bmatrix}

\]  

(9)
losing structure in January –

-\right) \left( E_{0} \cdot A \right)^{-2} \cdot e^{E_{0} \cdot A \cdot t} - (E_{1} \cdot A)^{-1} - (E_{1} \cdot A)^{-2} \right) \cdot L + (E_{0} \cdot A)^{-1} \left( e^{E_{0} \cdot A \cdot t} - E \right) \cdot \vec{B} + e^{E_{0} \cdot A \cdot t} \cdot \vec{F}_{0}.

This formula (10) determines moisture potential distribution in any enclosing structure section at any time under continuous control for temperature distribution.

4. Results and discussion

Separate moisture potential theory methods for single-layer enclosing structure moisture potential are compared. These are methods by V.G. Gagarin, V.V. Kozlov, and the proposed discrete-continuous method. 0.6 m thick enclosing construction made of aerated concrete with outside and inside plastering is used as a basic example. The construction area is Moscow. It was assumed that constant temperature of 22°C and relative humidity of 60% has been maintained in the building during the year. Attainable moisture potential values were calculated by means of the obtained formula (10), by means of explicit finite-difference scheme using Gagarin’s method, and by means of Kozlov’s analytical formula (5). Then material moisture content in enclosing structure thickness was determined by formula (4) using calculated moisture potential and temperature potential values.

Above-mentioned moisture potential theory methods for moisture potential distribution along single-layer enclosing structure thickness in January is given in [Fig.3], and for moisture regime calculation of single-layer enclosing structure in January - in [Fig.4].

Figure 3. Comparison of moisture potential theory methods for moisture potential distribution along single-layer enclosing structure thickness in January (1 – maximum water vapor pressure, 2 – moisture potential distribution in enclosing structure according

Figure 4. Comparison of moisture potential theory methods for moisture regime calculation of single-layer enclosing structure in January (1 – moisture distribution in enclosing structure according to Gagarin’s unsteady-state method, 2 – moisture distribution in enclosing structure
The result analysis shows that moisture distribution obtained by means of unsteady-state discrete-continuous method gives quantitative and qualitative results similar to Gagarin’s unsteady-state method. The proposed method is advantageous as moisture distribution analytical expression for final formula is determined (10).

5. Conclusion
An engineering assessment method for single-layer enclosing structure moisture state is proposed. The method allows for moisture distribution determination on final formula, which, on the one hand, simplifies moisture regime calculation problem, and, on the other hand, allows design engineer to calculate unsteady-state processes in practice.

Unsteady-state discrete-continuous mathematical model of moisture transfer has been improved. This model allows determination of moisture content of material in any enclosing structure section, at any time, under continuous control for temperature distribution influence on the enclosing structure.

The considered unsteady-state discrete-continuous mathematical model of moisture transfer is verified by finite difference method using an explicit difference scheme. This model gives a result which is closer to moisture regime discrete calculation method based on Gagarin’s unsteady-state model as compared to well-known Kozlov’s analytic quasistationary model.

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