Double machine vibration synchronization system based on GA-PID

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Abstract: Aiming at the problem that it is difficult to realize the synchronization of motor speed under the electromechanical coupling condition of DC motor and vibrating mass in double machine drive vibration system, the PID parameters of DC motor are optimized by using master-slave control mode and genetic algorithm. Firstly, the electromechanical coupling model of the vibration system is established, and the differential equation of motion of the system is obtained by using Lagrange equation; Using master-slave control structure and genetic PID control algorithm, a controller based on master motor speed difference and slave motor phase difference is designed; The MATLAB / Simulink simulation model is established to verify the effectiveness of the control system. The experimental results show that the genetic PID algorithm based on master-slave control structure can better realize the synchronous control of dual machine driven vibration system. The research content can provide a theoretical basis for the design, control and application of this type of vibration system.

1. Introduction

Synchronization is a widespread phenomenon in nature. Synchronization of mechanical vibration system means that two or more moving parts in the vibration system operate at the same or similar speed or angular speed [1]. Since the 1960s, Blekhman [2-3], a scientist from the former Soviet Union, first studied the vibration synchronization of dual motors and put forward the corresponding theoretical research basis. Chinese scholar Wen Bangchun [4-7] introduced the small parameter average method in the research of self synchronization theory, studied the vibration synchronization of two motors in the same plane, and obtained the synchronization conditions and stability conditions of the system. Chen Xiaozhe [8] introduced the body arrangement from plane to space, and studied the self-synchronous vibration theory of dual machine coaxial arrangement. Tian Xiaochong et al. [9] conducted Synchronization Research and mechanical analysis on the vibrating screen driven by four motors. Liu Yunshan et al. [10] studied the vibration synchronization system driven by three motors and obtained the criteria of synchronization and stability when it realizes self synchronization. Chen Bang et al. [11] conducted synchronous analysis on rotors installed on different coupling bodies and found the relationship between coupling frequency and vibration torque. Fang pan et al. [12] studied the vibration synchronization system of three space machines, and obtained the effects of the arrangement position of two coaxial motors and the third motor and the rotor quality on the synchronization and stability.. Sun Zhenbao et al. [13] combined fuzzy algorithm with PID parameter tuning to design a system synchronization controller that can be optimized in real time. Shang Xianhe et al. [14] the PID adaptive control based on BP neural network has higher stability than the traditional PID control strategy and
improves the robustness of the system. Zhang Chenghui et al. [15] studied a fuzzy neural network synchronous controller to realize the synchronous control of four machine vibration system.

For some small vibration machinery (such as medical vibration massage instrument), due to the small design size, high requirements for safety and portability, multiple small DC motors are often driven by synchronous vibration. Traditional vibration synchronization mainly depends on gears and chains, which brings a series of problems: the mechanical structure is very complex, easy to wear and difficult to maintain, and easy to produce environmental noise[16]. Therefore, using the new intelligent control algorithm to realize the synchronous control of multiple vibration motors has a certain practical significance. Genetic Algorithm (GA) was first proposed by John Holland of the United States in the 1970s. The algorithm is designed and proposed according to the evolution law of organisms in nature. This paper combines genetic algorithm with DC PID parameter tuning, and designs a synchronous controller based on master motor speed and slave motor phase difference, which achieves the control effect that the motor speed and phase difference of dual machine driven mechanical vibration system are in a stable state.

2. System dynamics model

The dual machine drive vibration system is mainly composed of a rigid body, vibration exciter and support spring. Its dynamic model is shown in Figure 1. The excitation motors 1 and 2 are organized symmetrically in a plane, and the support spring is symmetrically installed on the fixed frame. When the system works, the excitation motor 1 and the excitation motor 2 rotate in reverse at the same speed. In order to facilitate the analysis, the excitation motor in Figure 1 is represented by an eccentric rotor. The mass of the eccentric rotors are \( m_1 \), \( m_2 \), the rotation centers are \( O_1 \), \( O_2 \), the rotation radius are \( r_1 \), \( r_2 \), the phases are \( \phi_1 \), \( \phi_2 \), and the distance from the rotation center of the eccentric rotor to the mass centers of the body are \( l_1 \), \( l_2 \). The included angle between the connecting line from the rotation center of the two exciters to the mass center of the body and the x-axis of the vibration system are \( \beta_1 = 2\pi / 3 \), \( \beta_2 = 4\pi / 3 \) respectively. The vibration system has three degrees of freedom, which are horizontal x motion, vertical y motion and \( \psi \) motion parallel to the oxy plane.

Figure 1. The dynamic model of the vibration system of the circularly symmetric four-vibrator

The kinetic energy of the system is:

\[
T = \frac{1}{2} m (\dot{\phi}_1^2 + \dot{\phi}_2^2) + \frac{1}{2} J_{m1} \dot{\phi}_1^2 + \frac{1}{2} J_{m2} \dot{\phi}_2^2 + \frac{1}{2} \sum_{i=1}^{4} m_i (\dot{x}_i^2 + \dot{y}_i^2) \tag{1}
\]

Where: \( m \) is the body mass; \( J_{m} \) is the moment of inertia of the machine body; \( J_{i} \) is the moment of inertia of the eccentric block of the exciter around their respective rotation centers; \( x_i, y_i \) are the horizontal and vertical coordinates of the eccentric block of the exciter in the oxy coordinate system.

\[
x_i = x + (l_1 \cos \beta_i + r \cos \phi_i) \cos \psi + (l_1 \sin \beta_i + r \sin \phi_i) \sin \psi
\]

\[
y_i = y - (l_1 \cos \beta_i + r \cos \phi_i) \sin \psi + (l_1 \sin \beta_i + r \sin \phi_i) \cos \psi \tag{2}
\]
The potential energy of the system is:

\[ V = \frac{1}{4} k_x \left[ (x + \psi L_x)^2 + (x - \psi L_x)^2 \right] + \frac{1}{4} k_y \left[ (y + \psi L_y)^2 + (y - \psi L_y)^2 \right] \]  
(3)

Where: \( k_x, k_y \) is the spring stiffness in X direction and Y direction respectively. \( L_x, L_y \) is the distance from the mass center of the body to the spring connection point in X direction and Y direction respectively.

The energy dissipation function of the system is:

\[ D = \frac{1}{4} f_x \left[ (\dot{x} + \omega L_x)^2 + (\dot{x} - \omega L_x)^2 \right] + \frac{1}{4} f_y \left[ (\dot{y} + \omega L_y)^2 + (\dot{y} - \omega L_y)^2 \right] + \frac{1}{2} \sum_{i=1}^{4} f_i (\psi_i + \psi) \]  
(4)

Where: \( f_x, f_y \) is the damping coefficient of spring in X direction and Y direction respectively; \( f_i \) represents the damping of the exciting motor \( i \) shaft respectively.

For the whole system, the Lagrange equation is:

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial U}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} = Q_j \]  
(5)

The generalized force of the system is:

\[ Q = \{ Q_x, Q_y, Q_{e_x}, Q_{e_y}, Q_{e_z} \}^T \]  
(6)

Where:

- \( Q_x = Q_y = Q_z = 0 \)
- \( Q_{e_x} = T_x, T_y \) is the electromagnetic torque of the excitation motor.

By substituting the expressions of the kinetic energy, potential energy and energy dissipation function of the system into the Lagrange equation, the differential equations of motion of the system in X, Y and directions and the rotation equations of two eccentric rotors can be obtained:

\[ M \ddot{x} + f_x \dot{x} + k_x x = m_i r_i \left( \dot{\phi}_i \cos \phi_i + \dot{\phi}_i \sin \phi_i \right) - \sum_{i=2}^{\psi} m_i r_i \left( \dot{\phi}_i \cos \phi_i + \dot{\phi}_i \sin \phi_i \right) \]

\[ M \ddot{y} + f_y \dot{y} + k_y y = m_i r_i \left( \dot{\phi}_i \sin \phi_i - \dot{\phi}_i \cos \phi_i \right) + \sum_{i=2}^{\psi} m_i r_i \left( \dot{\phi}_i \sin \phi_i - \dot{\phi}_i \cos \phi_i \right) \]

\[ J \ddot{\psi} + f_x \psi + k_x \psi = f_i (\dot{\phi}_i - \psi) + \sum_{i=2}^{\psi} m_i r_i \left( \dot{\phi}_i \sin (\phi_i - \beta_i - \psi) - \dot{\phi}_i \cos (\phi_i - \beta_i - \psi) \right) \]

\[ + \sum_{i=2}^{\psi} m_i r_i \left( \dot{\phi}_i \sin (\phi_i - \beta_i + \psi) - \dot{\phi}_i \cos (\phi_i - \beta_i + \psi) \right) \]

\[ = f_i (\dot{\phi}_i + \psi) \]

\[ (f_i + m r_i) \ddots \phi_i + f_i (\phi_i - \psi) = T_{a_i} - T_{\psi} - m r_i \left[ \dot{y} \cos \phi_i - \dot{x} \sin \phi_i \right] \]

\[ - l \psi \cos (\phi_i - \beta_i - \psi) - l \psi \sin (\phi_i - \beta_i - \psi) \] \( i = 1, 2 \)

Where:

- \( M = m + \sum_{i=1}^{\psi} m_i \) is the total mass of the body and the excitation motor; \( f_\psi = \frac{1}{2} (f_i l_i^2 + f_i l_i^2) \) is the rotational damping coefficient of the engine block; \( J = J_m + \sum_{i=1}^{\psi} m_i (\dot{x}_i^2 + \dot{y}_i^2) \) is the total moment of inertia of the body and the excitation motor; \( k_\psi = \frac{1}{2} (k_i l_i^2 + k_i l_i^2) \) is the rotational stiffness of the spring.

3. Design of PID master-slave controller based on genetic algorithm

3.1. Basic principle of genetic algorithm

Genetic algorithm (GA) is a computational model of biological evolution process simulating the natural selection and genetic mechanism of Darwin's biological evolution theory. It is a method to search the optimal solution by simulating the natural evolution process. Its main characteristic is that it directly operates the structural object, and there is no limitation of derivation and function continuity; It has inherent implicit parallelism and better global optimization ability; Using the probabilistic optimization
method, the optimized search space can be automatically obtained and guided without certain rules, and the search direction can be adjusted adaptively. It takes all individuals in a group as the object, and uses randomization technology to guide the efficient search of a coded parameter space. Among them, selection, crossover and mutation constitute the genetic operation of genetic algorithm; Parameter coding, initial population setting, fitness function design, genetic operation design and control parameter setting constitute the core content of genetic algorithm.

![Figure 2. Process diagram of genetic algorithm](image)

### 3.2. Genetic-Algorithm PID parameter tuning

PID parameter tuning is an important research content in the field of automatic control. The selection of system parameters determines the stability and rapidity of control, and can also ensure the reliability of the system. The traditional PID parameters are mostly optimized manually by trial and error, which is often time-consuming and difficult to meet the real-time requirements of control. In order to optimize control parameters and improve system performance, genetic algorithm can be used to optimize PID parameters.

![Figure 3. PID parameter optimization process of genetic algorithm](image)

### 3.3. Design of master-slave controller

The controller based on the speed difference of the master motor and the phase difference of the slave motor is used to realize the dual adjustment of the speed and phase of the system. The control system structure is shown in Figure 4. The motor speed in the main motor controller is compared with the preset value, and the difference is processed by the angular speed controller to output the control signal to the main motor; The phase difference obtained by integrating the speed of the master and slave motors is used as the input signal of the slave motor controller for control. The speed and torque signals of the master-slave motor communicate with the body of the dual machine drive vibration system. After the body is calculated, the movement of the system in x, y and ψ direction is obtained.
4. Control simulation of dual drive vibration system

In order to verify the rationality of the system control synchronization design, Simulink is used to build the simulation model of motor vibration system, as shown in Figure 2. Runge Kutta algorithm and variable step size control strategy are used to substitute the system parameters in Table 1 for simulation analysis. When the angular velocity and phase difference of motor 1 and 2 tend to be stable, the vibration system enters the synchronous state.

Table 1. The parameters of the vibrating system

| M/kg | (m₁~m₂)/kg | J/(kg · m²) | (J₁~J₂)/(kg · m²) | (r₁~r₂)/m | k₁ · φ | f/m |
|------|------------|-------------|-------------------|------------|--------|-----|
| 148  | 3.5        | 17          | 0.01              | 0.05       | 0.23   | 1.26 |

![Figure 5. simulation result](image)

- (a). Main motor speed
- (b). Slave motor speed
- (c). phase difference
- (d). Displacement in x direction
- (e). Displacement in y direction
- (f). Displacement in Ψ direction

It can be seen from figures 5(a) and (b) that the speed ω₁ of the main motor is stable near the preset value of 35 rad/s, the speed ω₂ of the slave motor is also quickly stable near 35 rad/s, and the phase difference between the two motors is finally stable at about -0.6°. The displacement in X and Y directions of the vibration system makes sinusoidal motion near 0, and the motion in Ψ direction also makes sinusoidal motion near 0. Based on the above situation, it can be seen that the vibration system realizes synchronous steady-state motion.
5. Conclusion
The PID master-slave controller based on genetic algorithm realizes the steady-state motion of angular velocity, phase and x, y, $\psi$ direction of the dual machine vibration synchronization system, and the master-slave motor can also maintain the synchronous motion state, which realizes a good control effect.

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