Rough Neutrosophic Digraphs with Application

Florentin Smarandache
Sidra Sayed
Nabeela Ishfaq
Muhammad Akram

Follow this and additional works at: https://digitalrepository.unm.edu/math_fsp

Part of the Business Administration, Management, and Operations Commons, Business Analytics Commons, Mathematics Commons, and the Strategic Management Policy Commons
Rough Neutrosophic Digraphs with Application

Sidra Sayed 1, Nabeela Ishfaq 1, Muhammad Akram 1,* and Florentin Smarandache 2

1 Department of Mathematics, University of the Punjab, New Campus, Lahore 54590, Pakistan; sidratulmuntha228@yahoo.com (S.S.); nabeelaishfaq123@gmail.com (N.I.)
2 Mathematics & Science Department, University of New Mexico, 705 Gurley Ave., Gallup, NM 87301, USA; fsmarandache@gmail.com
* Correspondence: m.akram@pucit.edu.pk; Tel.: +92-42-99231241

Received: 5 December 2017; Accepted: 15 January 2018; Published: 18 January 2018

Abstract: A rough neutrosophic set model is a hybrid model which deals with vagueness by using the lower and upper approximation spaces. In this research paper, we apply the concept of rough neutrosophic sets to graphs. We introduce rough neutrosophic digraphs and describe methods of their construction. Moreover, we present the concept of self complementary rough neutrosophic digraphs. Finally, we consider an application of rough neutrosophic digraphs in decision-making.

Keywords: rough neutrosophic sets; rough neutrosophic digraphs; decision-making

MSC: 03E72, 68R10, 68R05

1. Introduction

Smarandache [1] proposed the concept of neutrosophic sets as an extension of fuzzy sets [2]. A neutrosophic set has three components, namely, truth membership, indeterminacy membership and falsity membership, in which each membership value is a real standard or non-standard subset of the nonstandard unit interval \([0-, 1+\right]\), where \(0- = 0 - \epsilon, 1+ = 1 + \epsilon\), \(\epsilon\) is an infinitesimal number. To apply neutrosophic set in real-life problems more conveniently, Smarandache [3] and Wang et al. [4] defined single-valued neutrosophic sets which takes the value from the subset of \([0, 1]\).

Actually, the single valued neutrosophic set was introduced for the first time by Smarandache in 1998 in [3]. Ye [5] considered multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. Ye [6] also presented improved correlation coefficients of single valued neutrosophic sets and interval neutrosophic sets for multiple attribute decision making.

Rough set theory was proposed by Pawlak [7] in 1982. Rough set theory is useful to study the intelligence systems containing incomplete, uncertain or inexact information. The lower and upper approximation operators of rough sets are used for managing hidden information in a system. Therefore, many hybrid models have been built, such as soft rough sets, rough fuzzy sets, fuzzy rough sets, soft fuzzy rough sets, neutrosophic rough sets, andrough neutrosophic sets, for handling uncertainty and incomplete information effectively. Dubois and Prade [8] introduced the notions of rough fuzzy sets and fuzzy rough sets. Liu and Chen [9] have studied different decision-making methods. Broumi et al. [10] introduced the concept of rough neutrosophic sets. Yang et al. [11] proposed single valued neutrosophic rough sets by combining single valued neutrosophic sets and rough sets, and established an algorithm for decision-making problem based on single valued neutrosophic rough sets on two universes. Mordeson and Peng [12] presented operations on fuzzy graphs. Akram et al. [13–16] considered several new concepts of neutrosophic graphs with applications. Zafer and Akram [17] introduced a novel decision-making method based on rough fuzzy information. In this research study, we apply the concept of rough neutrosophic sets to graphs. We introduce rough neutrosophic digraphs and describe methods of their construction. Moreover,
we present the concept of self complementary rough neutrosophic digraphs. We also present an
application of rough neutrosophic digraphs in decision-making.

We have used standard definitions and terminologies in this paper. For other notations,
terminologies and applications not mentioned in the paper, the readers are referred to [18–22].

2. Rough Neutrosophic Digraphs

Definition 1. [4] Let \( Z \) be a nonempty universe. A neutrosophic set \( N \) on \( Z \) is defined as follows:

\[
N = \{ x : \mu_N(x), \sigma_N(x), \lambda_N(x), x \in Z \}
\]

where the functions \( \mu, \sigma, \lambda : Z \rightarrow [0,1] \) represent the degree of membership, the degree of indeterminacy and the
degree of falsity.

Definition 2. [7] Let \( Z \) be a nonempty universe and \( R \) an equivalence relation on \( Z \). A pair \((Z, R)\) is called an
approximation space. Let \( N^* \) be a subset of \( Z \) and the lower and upper approximations of \( N^* \) in the approximation
space \((Z, R)\) denoted by \( \underline{RN}^* \) and \( \overline{RN}^* \) are defined as follows:

\[
\underline{RN}^* = \{ x \in Z | [x]_R \subseteq N^* \},
\]

\[
\overline{RN}^* = \{ x \in Z | [x]_R \subseteq N^* \},
\]

where \( [x]_R \) denotes the equivalence class of \( R \) containing \( x \). A pair \((\underline{RN}^*, \overline{RN}^*)\) is called a rough set.

Definition 3. [10] Let \( Z \) be a nonempty universe and \( R \) an equivalence relation on \( Z \). Let \( N \) be a neutrosophic
set \( \text{NS}(\text{NS}) \) on \( Z \). The lower and upper approximations of \( N \) in the approximation space \((Z, R)\) denoted by \( \underline{RN} \) and
\( \overline{RN} \) are defined as follows:

\[
\underline{RN} = \{ x, \mu_{\underline{R}(N)}(x), \sigma_{\underline{R}(N)}(x), \lambda_{\underline{R}(N)}(x) : y \in [x]_R, x \in Z \},
\]

\[
\overline{RN} = \{ x, \mu_{\overline{R}(N)}(x), \sigma_{\overline{R}(N)}(x), \lambda_{\overline{R}(N)}(x) : y \in [x]_R, x \in Z \},
\]

where,

\[
\mu_{\underline{R}(N)}(x) = \bigwedge_{y \in [x]_R} \mu_N(y), \quad \mu_{\overline{R}(N)}(x) = \bigvee_{y \in [x]_R} \mu_N(y),
\]

\[
\sigma_{\underline{R}(N)}(x) = \bigwedge_{y \in [x]_R} \sigma_N(y), \quad \sigma_{\overline{R}(N)}(x) = \bigvee_{y \in [x]_R} \sigma_N(y),
\]

\[
\lambda_{\underline{R}(N)}(x) = \bigvee_{y \in [x]_R} \lambda_N(y), \quad \lambda_{\overline{R}(N)}(x) = \bigwedge_{y \in [x]_R} \lambda_N(y).
\]

A pair \((\underline{RN}, \overline{RN})\) is called a rough neutrosophic set.

We now define the concept of rough neutrosophic digraph.

Definition 4. Let \( V^* \) be a nonempty set and \( R \) an equivalence relation on \( V^* \). Let \( V \) be a NS on \( V^* \), defined as

\[
V = \{ x, \mu_V(x), \sigma_V(x), \lambda_V(x) : x \in V^* \}.
\]

Then, the lower and upper approximations of \( V \) represented by \( \underline{RV} \) and \( \overline{RV} \), respectively, are characterized
as \( \text{NSs} \) in \( V^* \) such that \( \forall x \in V^* \),

\[
\underline{RV} = \{ x, \mu_{\underline{R}(V)}(x), \sigma_{\underline{R}(V)}(x), \lambda_{\underline{R}(V)}(x) : y \in [x]_R \},
\]

\[
\overline{RV} = \{ x, \mu_{\overline{R}(V)}(x), \sigma_{\overline{R}(V)}(x), \lambda_{\overline{R}(V)}(x) : y \in [x]_R \},
\]

where,

\[
\mu_{\underline{R}(V)}(x) = \bigwedge_{y \in [x]_R} \mu_V(y), \quad \mu_{\overline{R}(V)}(x) = \bigvee_{y \in [x]_R} \mu_V(y),
\]

\[
\sigma_{\underline{R}(V)}(x) = \bigwedge_{y \in [x]_R} \sigma_V(y), \quad \sigma_{\overline{R}(V)}(x) = \bigvee_{y \in [x]_R} \sigma_V(y),
\]

\[
\lambda_{\underline{R}(V)}(x) = \bigvee_{y \in [x]_R} \lambda_V(y), \quad \lambda_{\overline{R}(V)}(x) = \bigwedge_{y \in [x]_R} \lambda_V(y).
\]
Let $E^* \subseteq V^* \times V^*$ and $S$ an equivalence relation on $E^*$ such that

$$((x_1, x_2), (y_1, y_2)) \in S \iff (x_1, y_1), (x_2, y_2) \in R.$$ 

Let $E$ be a neutrosophic set on $E^* \subseteq V^* \times V^*$ defined as

$$E = \{< xy, \mu_E(xy), \sigma_E(xy), \lambda_E(xy) >: xy \in V^* \times V^*\},$$

such that

$$\mu_E(xy) \leq \min\{\mu_{RV}(x), \mu_{RV}(y)\},$$

$$\sigma_E(xy) \leq \min\{\sigma_{RV}(x), \sigma_{RV}(y)\},$$

$$\lambda_E(xy) \leq \max\{\lambda_{RV}(x), \lambda_{RV}(y)\} \ \forall x, y \in V^*.$$

Then, the lower and upper approximations of $E$ represented by $\mathcal{SE}$ and $\mathcal{SE}$, respectively, are defined as follows

$$\mathcal{SE} = \{< xy, \mu_{\mathcal{SE}}(xy), \sigma_{\mathcal{SE}}(xy), \lambda_{\mathcal{SE}}(xy) >: wz \in [xy]_S, xy \in V^* \times V^*\},$$

$$\mathcal{SE} = \{< xy, \mu_{\mathcal{SE}}(xy), \sigma_{\mathcal{SE}}(xy), \lambda_{\mathcal{SE}}(xy) >: wz \in [xy]_S, xy \in V^* \times V^*\},$$

where,

$$\mu_{\mathcal{SE}}(xy) = \bigwedge_{wz \in [xy]_S} \mu_E(wz), \quad \mu_{\mathcal{SE}}(xy) = \bigvee_{wz \in [xy]_S} \mu_E(wz),$$

$$\sigma_{\mathcal{SE}}(xy) = \bigwedge_{wz \in [xy]_S} \sigma_E(wz), \quad \sigma_{\mathcal{SE}}(xy) = \bigvee_{wz \in [xy]_S} \sigma_E(wz),$$

$$\lambda_{\mathcal{SE}}(xy) = \bigvee_{wz \in [xy]_S} \lambda_E(wz), \quad \lambda_{\mathcal{SE}}(xy) = \bigwedge_{wz \in [xy]_S} \lambda_E(wz).$$

A pair $SE = (SE, SE)$ is called a rough neutrosophic relation.

**Definition 5.** A rough neutrosophic digraph on a nonempty set $V^*$ is a four-ordered tuple $G = (R, RV, S, SE)$ such that

(a) $R$ is an equivalence relation on $V^*$;
(b) $S$ is an equivalence relation on $E^* \subseteq V^* \times V^*$;
(c) $RV = (RV, RV)$ is a rough neutrosophic set on $V^*$;
(d) $SE = (SE, SE)$ is a rough neutrosophic relation on $V^*$ and
(e) $(RV, SE)$ is a neutrosophic digraph where $\mathcal{G} = (RV, \mathcal{SE})$ and $\mathcal{G} = (RV, \mathcal{SE})$ are lower and upper approximate neutrosophic digraphs of $G$ such that

$$\mu_{\mathcal{SE}}(xy) \leq \min\{\mu_{RV}(x), \mu_{RV}(y)\},$$

$$\sigma_{\mathcal{SE}}(xy) \leq \min\{\sigma_{RV}(x), \sigma_{RV}(y)\},$$

$$\lambda_{\mathcal{SE}}(xy) \leq \max\{\lambda_{RV}(x), \lambda_{RV}(y)\},$$

and

$$\mu_{\mathcal{SE}}(xy) \leq \min\{\mu_{RV}(x), \mu_{RV}(y)\},$$

$$\sigma_{\mathcal{SE}}(xy) \leq \min\{\sigma_{RV}(x), \sigma_{RV}(y)\},$$

$$\lambda_{\mathcal{SE}}(xy) \leq \max\{\lambda_{RV}(x), \lambda_{RV}(y)\} \ \forall x, y \in V^*.$$

**Example 1.** Let $V^* = \{a, b, c\}$ be a set and $R$ an equivalence relation on $V^*$

$$R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}. $$
Let $V = \{(a, 0.2, 0.3, 0.6), (b, 0.8, 0.6, 0.5), (c, 0.9, 0.1, 0.4)\}$ be a neutrosophic set on $V^*$. The lower and upper approximations of $V$ are given by,

$$
\under{R} = \{(a, 0.2, 0.1, 0.6), (b, 0.8, 0.6, 0.5), (c, 0.2, 0.1, 0.6)\},
\over{R} = \{(a, 0.9, 0.3, 0.4), (b, 0.8, 0.6, 0.5), (c, 0.9, 0.3, 0.4)\}.
$$

Let $E^* = \{aa, ab, ac, bb, ca, cb\} \subseteq V^* \times V^*$ and $S$ an equivalence relation on $E^*$ defined as:

$$
S = \begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1
\end{bmatrix}.
$$

Let $E = \{(aa, 0.2, 0.1, 0.4), (ab, 0.2, 0.1, 0.5), (ac, 0.1, 0.1, 0.5), (bb, 0.7, 0.5, 0.5), (ca, 0.1, 0.1, 0.3), (cb, 0.2, 0.1, 0.5)\}$ be a neutrosophic set on $E^*$ and $SE = (\over{S}, \under{S})$ a rough neutrosophic relation where $\over{S}$ and $\under{S}$ are given as:

$$
\over{S} = \{(aa, 0.1, 0.1, 0.5), (ab, 0.2, 0.1, 0.5), (ac, 0.1, 0.1, 0.5), (bb, 0.7, 0.5, 0.5),
ca, 0.1, 0.1, 0.5), (cb, 0.2, 0.1, 0.5)\},
\under{S} = \{(aa, 0.2, 0.1, 0.3), (ab, 0.2, 0.1, 0.5), (ac, 0.2, 0.1, 0.3), (bb, 0.7, 0.5, 0.5),
ca, 0.2, 0.1, 0.3), (cb, 0.2, 0.1, 0.5)\}.
$$

Thus, $G = (\under{R^{\ast}}, \over{SE})$ and $\over{G} = (\over{R^{\ast}}, \under{SE})$ are neutrosophic digraphs as shown in Figure 1.

![Figure 1. Rough neutrosophic digraph $G = (\over{G}, \over{G})$.](image)

We now form new rough neutrosophic digraphs from old ones.

**Definition 6.** Let $G_1 = (G_1, \over{G}_1)$ and $G_2 = (G_2, \over{G}_2)$ be two rough neutrosophic digraphs on a set $V^*$. Then, the intersection of $G_1$ and $G_2$ is a rough neutrosophic digraph $G = G_1 \cap G_2 = (G_1 \cap G_2, \over{G}_1 \cap \over{G}_2)$, where $G_1 \cap G_2 = (\under{R}_1 \cap \under{R}_2, \over{SE}_1 \cap \under{SE}_2)$ and $\over{G}_1 \cap \over{G}_2 = (\over{R}_1 \cap \over{R}_2, \under{SE}_1 \cap \over{SE}_2)$ are neutrosophic digraphs, respectively, such that
**Theorem 1.** The intersection of two rough neutrosophic digraphs is a rough neutrosophic digraph.

**Proof.** Let $G_1 = (\Gamma_1, \overline{\Sigma}_1)$ and $G_2 = (\Gamma_2, \overline{\Sigma}_2)$ be two rough neutrosophic digraphs. Let $G = G_1 \bowtie G_2 = (\Gamma_1 \cap \Gamma_2, \overline{\Sigma}_1 \cap \overline{\Sigma}_2)$ be the intersection of $G_1$ and $G_2$, where $G_1 \cap G_2 = (\Gamma V_1 \cap \Gamma V_2, SE_1 \cap SE_2)$ and $\overline{\Sigma}_1 \cap \overline{\Sigma}_2 = (\overline{RV}_1 \cap \overline{RV}_2, SE_1 \cap SE_2)$. To prove that $G = G_1 \bowtie G_2$ is a rough neutrosophic digraph, it is

$$(1) \quad \mu_{\Gamma V_1 \cap \Gamma V_2}(x) = \min\{\mu_{\Gamma V_1}(x), \mu_{\Gamma V_2}(x)\},$$

$$\sigma_{\Gamma V_1 \cap \Gamma V_2}(x) = \min\{\sigma_{\Gamma V_1}(x), \sigma_{\Gamma V_2}(x)\},$$

$$\lambda_{\Gamma V_1 \cap \Gamma V_2}(x) = \max\{\lambda_{\Gamma V_1}(x), \lambda_{\Gamma V_2}(x)\} \quad \forall x \in \Gamma V_1 \cap \Gamma V_2,$$

$$\mu_{S E_1 \cap S E_2}(xy) = \min\{\mu_{S E_1}(x), \mu_{S E_2}(y)\},$$

$$\sigma_{S E_1 \cap S E_2}(xy) = \min\{\sigma_{S E_1}(x), \sigma_{S E_2}(y)\},$$

$$\lambda_{S E_1 \cap S E_2}(xy) = \max\{\lambda_{S E_1}(x), \lambda_{S E_2}(y)\} \quad \forall xy \in S E_1 \cap S E_2.$$

**Example 2.** Consider the two rough neutrosophic digraphs $G_1$ and $G_2$ as shown in Figures 1 and 2. The intersection of $G_1$ and $G_2$ is $G = G_1 \bowtie G_2 = (\Gamma_1 \cap \Gamma_2, \overline{\Sigma}_1 \cap \overline{\Sigma}_2)$ where $G_1 \cap G_2 = (\Gamma V_1 \cap \Gamma V_2, SE_1 \cap SE_2)$ and $\overline{\Sigma}_1 \cap \overline{\Sigma}_2 = (\overline{RV}_1 \cap \overline{RV}_2, SE_1 \cap SE_2)$ are neutrosophic digraphs as shown in Figure 3.
enough to show that \( \overline{SE}_1 \cap \overline{SE}_2 \) and \( \overline{SE}_1 \cap \overline{SE}_2 \) are neutrosophic relation on \( RV_1 \cap RV_2 \) and \( RV_1 \cap RV_2 \), respectively. First, we show that \( \overline{SE}_1 \cap \overline{SE}_2 \) is a neutrosophic relation on \( RV_1 \cap RV_2 \).

Thus, from above it is clear that \( \overline{SE}_1 \cap \overline{SE}_2 \) is a neutrosophic relation on \( RV_1 \cap RV_2 \).

Similarly, we can show that \( \overline{SE}_1 \cap \overline{SE}_2 \) is a neutrosophic relation on \( RV_1 \cap RV_2 \). Hence, \( G \) is a rough neutrosophic digraph. □

**Definition 7.** The Cartesian product of two neutrosophic digraphs \( G_1 \) and \( G_2 \) is a rough neutrosophic digraph \( G = G_1 \times G_2 = (G_1 \times G_2, \overline{SE}_1 \times \overline{SE}_2) \), where \( G_1 \times G_2 = (RV_1 \times RV_2, SE_1 \times SE_2) \) such that

\[
(1) \quad \mu_{G V_1 \times G V_2}(x_1, x_2) = \min\{\mu_{G V_1}(x_1), \mu_{G V_2}(x_2)\}, \\
\sigma_{G V_1 \times G V_2}(x_1, x_2) = \min\{\sigma_{G V_1}(x_1), \sigma_{G V_2}(x_2)\}, \\
\lambda_{G V_1 \times G V_2}(x_1, x_2) = \max\{\lambda_{G V_1}(x_1), \mu_{G V_2}(x_2)\}, \quad \forall (x_1, x_2) \in RV_1 \times RV_2,
\]

\[
\mu_{SE_1 \times SE_2}(x, x)(x, y) = \min\{\mu_{SE_1}(x, x), \mu_{SE_2}(x, y)\}, \\
\sigma_{SE_1 \times SE_2}(x, x)(x, y) = \min\{\sigma_{SE_1}(x, x), \sigma_{SE_2}(x, y)\}, \\
\lambda_{SE_1 \times SE_2}(x, x)(x, y) = \max\{\lambda_{SE_1}(x, x), \mu_{SE_2}(x, y)\}, \quad \forall x \in RV_1, x_2 y_2 \in SE_2,
\]

\[
(2) \quad \mu_{RV_1 \times RV_2}(x_1, x_2) = \min\{\mu_{RV_1}(x_1), \mu_{RV_2}(x_2)\}, \\
\sigma_{RV_1 \times RV_2}(x_1, x_2) = \min\{\sigma_{RV_1}(x_1), \sigma_{RV_2}(x_2)\}, \\
\lambda_{RV_1 \times RV_2}(x_1, x_2) = \max\{\lambda_{RV_1}(x_1), \mu_{RV_2}(x_2)\}, \quad \forall (x_1, x_2) \in RV_1 \times RV_2,
\]

\[
\mu_{SE_1 \times SE_2}(x, x)(x, y) = \min\{\mu_{SE_1}(x, x), \mu_{SE_2}(x, y)\}, \\
\sigma_{SE_1 \times SE_2}(x, x)(x, y) = \min\{\sigma_{SE_1}(x, x), \sigma_{SE_2}(x, y)\}, \\
\lambda_{SE_1 \times SE_2}(x, x)(x, y) = \max\{\lambda_{SE_1}(x, x), \mu_{SE_2}(x, y)\}, \quad \forall x \in RV_1, x_2 y_2 \in SE_2,
\]
Example 3. Let \( V^* = \{a, b, c, d\} \) be a set. Let \( G_1 = (G_1, \overline{G}_1) \) and \( G_2 = (G_2, \overline{G}_2) \) be two rough neutrosophic digraphs on \( V^* \), as shown in Figures 4 and 5. The cartesian product of \( G_1 \) and \( G_2 \) is \( G = (G_1 \times G_2, \overline{G}_1 \times \overline{G}_2) \), where \( G_1 \times G_2 = (RN_1 \times RN_2, SE_1 \times SE_2) \) and \( \overline{G}_1 \times \overline{G}_2 = (\overline{RN}_1 \times \overline{RN}_2, \overline{SE}_1 \times \overline{SE}_2) \) are neutrosophic digraphs, as shown in Figures 6 and 7, respectively.

Figure 4. Rough neutrosophic digraph \( G_1 = (RN_1, SE_1) \).

Figure 5. Rough neutrosophic digraph \( G_2 = (RN_2, SE_2) \).
Theorem 2. The Cartesian product of two rough neutrosophic digraphs is a rough neutrosophic digraph.

Proof. Let $G_1 = (G_1, \overline{G}_1)$ and $G_2 = (G_2, \overline{G}_2)$ be two rough neutrosophic digraphs. Let $G = G_1 \times G_2 = (G_1 \times G_2, \overline{G}_1 \times \overline{G}_2)$ be the Cartesian product of $G_1$ and $G_2$, where $G_1 \times G_2 = (\mathcal{R}V_1 \times \mathcal{R}V_2, \mathcal{SE}_1 \times \mathcal{SE}_2)$ and $\overline{G}_1 \times \overline{G}_2 = (\overline{\mathcal{R}V}_1 \times \overline{\mathcal{R}V}_2, \overline{\mathcal{SE}}_1 \times \overline{\mathcal{SE}}_2)$. To prove that $G = G_1 \times G_2$ is a rough neutrosophic digraph, it is enough to show that $\overline{\mathcal{SE}}_1 \times \overline{\mathcal{SE}}_2$ and $\mathcal{SE}_1 \times \mathcal{SE}_2$ are neutrosophic relation on $\overline{\mathcal{R}V}_1 \times \overline{\mathcal{R}V}_2$ and $\mathcal{R}V_1 \times \mathcal{R}V_2$, respectively. First, we show that $\overline{\mathcal{SE}}_1 \times \overline{\mathcal{SE}}_2$ is a neutrosophic relation on $\overline{\mathcal{R}V}_1 \times \overline{\mathcal{R}V}_2$. 

Figure 6. Neutrosophic digraph $G_1 \times G_2 = (\mathcal{RN}_1 \times \mathcal{RN}_2, \mathcal{SE}_1 \times \mathcal{SE}_2)$.

Figure 7. Neutrosophic digraph $\overline{G}_1 \times \overline{G}_2 = (\overline{\mathcal{R}V}_1 \times \overline{\mathcal{R}V}_2, \overline{\mathcal{SE}}_1 \times \overline{\mathcal{SE}}_2)$. 

Axioms 2018, 7, 5  of 20
If \( x \in RV_1 \), \( x_2 y_2 \in SE_2 \), then
\[
\mu_{SE_1 \times SE_2}(x, x_2)(x, y_2) = \mu_{GV_1}(x) \wedge \mu_{SE_2}(x_2, y_2) \\
\leq \mu_{GV_1}(x) \wedge (\mu_{GV_2}(x_2) \wedge \mu_{GV_2}(y_2)) \\
= (\mu_{GV_1}(x) \wedge \mu_{GV_2}(x_2)) \wedge (\mu_{GV_1}(x) \wedge \mu_{GV_2}(y_2)) \\
= \mu_{GV_1 \times GV_2}(x, x_2) \wedge \mu_{GV_1 \times GV_2}(x, y_2)
\]
\[
\sigma_{SE_1 \times SE_2}(x, x_2)(x, y_2) = \sigma_{GV_1}(x) \wedge \sigma_{SE_2}(x_2, y_2) \\
\leq \sigma_{GV_1}(x) \wedge (\sigma_{GV_2}(x_2) \wedge \sigma_{GV_2}(y_2)) \\
= (\sigma_{GV_1}(x) \wedge \sigma_{GV_2}(x_2)) \wedge (\sigma_{GV_1}(x) \wedge \sigma_{GV_2}(y_2)) \\
= \sigma_{GV_1 \times GV_2}(x, x_2) \wedge \sigma_{GV_1 \times GV_2}(x, y_2)
\]
\[
\lambda_{SE_1 \times SE_2}(x, x_2)(x, y_2) = \lambda_{GV_1}(x) \vee \lambda_{SE_2}(x_2, y_2) \\
\leq \lambda_{GV_1}(x) \vee (\lambda_{GV_2}(x_2) \vee \lambda_{GV_2}(y_2)) \\
= (\lambda_{GV_1}(x) \vee \lambda_{GV_2}(x_2)) \vee (\lambda_{GV_1}(x) \vee \lambda_{GV_2}(y_2)) \\
= \lambda_{GV_1 \times GV_2}(x, x_2) \vee \lambda_{GV_1 \times GV_2}(x, y_2)
\]
\[
\lambda_{SE_1 \times SE_2}(x, x_2, x_2)(x, y_2) \leq \max(\lambda_{GV_1 \times GV_2}(x, x_2), \lambda_{GV_1 \times GV_2}(x, y_2)).
\]

If \( x_1 y_1 \in SE_1, z \in RV_2 \), then
\[
\mu_{SE_1 \times SE_2}(x_1, z)(y_1, z) = \mu_{SE_1}(x_1, y_1) \wedge \mu_{GV_2}(z) \\
\leq (\mu_{GV_1}(x_1) \wedge \mu_{GV_1}(y_1)) \wedge \mu_{GV_2}(z) \\
= (\mu_{GV_1}(x_1) \wedge \mu_{GV_2}(z)) \wedge (\mu_{GV_1}(y_1) \wedge \mu_{GV_2}(z)) \\
= \mu_{GV_1 \times GV_2}(x_1, z) \wedge \mu_{GV_1 \times GV_2}(y_1, z)
\]
\[
\sigma_{SE_1 \times SE_2}(x_1, z)(y_1, z) = \sigma_{SE_1}(x_1, y_1) \wedge \sigma_{GV_2}(z) \\
\leq (\sigma_{GV_1}(x_1) \wedge \sigma_{GV_1}(y_1)) \wedge \sigma_{GV_2}(z) \\
= (\sigma_{GV_1}(x_1) \wedge \sigma_{GV_2}(z)) \wedge (\sigma_{GV_1}(y_1) \wedge \sigma_{GV_2}(z)) \\
= \sigma_{GV_1 \times GV_2}(x_1, z) \wedge \sigma_{GV_1 \times GV_2}(y_1, z)
\]
\[
\lambda_{SE_1 \times SE_2}(x_1, z)(y_1, z) = \lambda_{SE_1}(x_1, y_1) \vee \lambda_{GV_2}(z) \\
\leq (\lambda_{GV_1}(x_1) \vee \lambda_{GV_2}(y_1)) \vee \lambda_{GV_2}(z) \\
= (\lambda_{GV_1}(x_1) \vee \lambda_{GV_2}(z)) \vee (\lambda_{GV_1}(y_1) \vee \lambda_{GV_2}(z)) \\
= \lambda_{GV_1 \times GV_2}(x_1, z) \vee \lambda_{GV_1 \times GV_2}(y_1, z)
\]
\[
\lambda_{SE_1 \times SE_2}(x_1, z)(y_1, z) \leq \max(\lambda_{GV_1 \times GV_2}(x_1, z), \lambda_{GV_1 \times GV_2}(y_1, z)).
\]

Thus, from above, it is clear that \( SE_1 \times SE_2 \) is a neutrosophic relation on \( RV_1 \times RV_2 \).

Similarly, we can show that \( SE_1 \times SE_2 \) is a neutrosophic relation on \( RV_1 \times RV_2 \). Hence, \( G = (G_1 \times G_2, G_1 \times G_2) \) is a rough neutrosophic digraph. \( \square \)

**Definition 8.** The composition of two rough neutrosophic digraphs \( G_1 \) and \( G_2 \) is a rough neutrosophic digraph \( G = G_1 \odot G_2 = (G_1 \odot G_2, G_1 \odot G_2) \), where \( G_1 \odot G_2 = (RV_1 \odot RV_2, SE_1 \odot SE_2) \) and \( G_1 \odot G_2 = (RV_1 \odot RV_2, SE_1 \odot SE_2) \) are neutrosophic digraphs, respectively, such that
\[
(1) \quad \mu_{GV_1 \odot GV_2}(x_1, x_2) = \min\{\mu_{GV_1}(x_1), \mu_{GV_2}(x_2)\},
\]
\[\sigma_{RV_1 \circ RV_2}(x_1, x_2) = \min\{\sigma_{RV_1}(x_1), \mu_{RV_2}(x_2)\},\]
\[\lambda_{RV_1 \circ RV_2}(x_1, x_2) = \max\{\lambda_{RV_1}(x_1), \mu_{RV_2}(x_2)\} \quad \forall (x_1, x_2) \in RV_1 \times RV_2,\]
\[\mu_{SE_1 \circ SE_2}(x, x_2)(x, y_2) = \min\{\mu_{SE_1}(x), \mu_{SE_2}(x_2, y_2)\},\]
\[\sigma_{SE_1 \circ SE_2}(x, x_2)(x, y_2) = \min\{\sigma_{SE_1}(x), \sigma_{SE_2}(x_2, y_2)\},\]
\[\lambda_{SE_1 \circ SE_2}(x, x_2)(x, y_2) = \max\{\lambda_{SE_1}(x), \lambda_{SE_2}(x_2, y_2)\} \quad \forall x \in RV_1, x_2y_2 \in SE_2,\]
\[\mu_{SE_1 \circ SE_2}(x_1, z)(y_1, z) = \min\{\mu_{SE_1}(x_1, y_1), \mu_{RV_2}(z)\},\]
\[\sigma_{SE_1 \circ SE_2}(x_1, z)(y_1, z) = \min\{\sigma_{SE_1}(x_1, y_1), \sigma_{RV_2}(z)\},\]
\[\lambda_{SE_1 \circ SE_2}(x_1, z)(y_1, z) = \max\{\lambda_{SE_1}(x_1, y_1), \lambda_{RV_2}(z)\} \quad \forall x_1y_1 \in SE_1, z \in RV_2,\]
\[\mu_{SE_1 \circ SE_2}(x_1, y_2)(y_1, y_2) = \min\{\mu_{SE_1}(x_1, y_1), \mu_{RV_2}(y_2)\},\]
\[\sigma_{SE_1 \circ SE_2}(x_1, y_2)(y_1, y_2) = \min\{\sigma_{SE_1}(x_1, y_1), \sigma_{RV_2}(y_2)\},\]
\[\lambda_{SE_1 \circ SE_2}(x_1, y_2)(y_1, y_2) = \max\{\lambda_{SE_1}(x_1, y_1), \lambda_{RV_2}(y_2)\} \quad \forall x_1y_1 \in SE_1, x_2y_2 \in RV_2, x_2 \neq y_2.\]

**Example 4.** Let \(V^* = \{p, q, r\}\) be a set. Let \(G_1 = (G_{11}, G_{12})\) and \(G_2 = (G_{21}, G_{22})\) be two RND on \(V^*\), where \(G_1 = (RV_1, SE_1)\) and \(G_2 = (RV_2, SE_2)\) are ND, as shown in Figure 8. \(G_2 = (RV_2, SE_2)\) and \(G_2 = (RV_2, SE_2)\) are also ND, as shown in Figure 9.

The composition of \(G_1\) and \(G_2\) is \(G = G_1 \circ G_2 = (G_{11} \circ G_{21}, G_{12} \circ G_{22})\) where \(G_1 \circ G_2 = (RV_1 \circ RV_2, SE_1 \circ SE_2)\) and \(G_1 \circ G_2 = (RV_1 \circ RV_2, SE_1 \circ SE_2)\) are NDs, as shown in Figures 10 and 11.
Figure 8. Rough neutrosophic digraph $G_1 = (\bar{G}_1, \bar{G}_1)$.

Figure 9. Rough neutrosophic digraph $G_2 = (\bar{G}_2, \bar{G}_2)$.

Figure 10. Neutrosophic digraph $G_1 \circ G_2 = (\bar{R}V_1 \circ \bar{R}V_2, \bar{S}E_1 \circ \bar{S}E_2)$. 
Theorem 3. The Composition of two rough neutrosophic digraphs is a rough neutrosophic digraph.

**Proof.** Let $G_1 = (G_1, \overline{T}_1)$ and $G_2 = (G_2, \overline{T}_2)$ be two rough neutrosophic digraphs. Let $G = G_1 \circ G_2 = (G_1 \circ G_2, \overline{T}_1 \circ \overline{T}_2)$ be the Composition of $G_1$ and $G_2$, where $G_1 \circ G_2 = (RV_1 \circ RV_2, SE_1 \circ SE_2)$ and $\overline{T}_1 \circ \overline{T}_2 = (RV_1 \circ RV_2, SE_1 \circ SE_2)$. To prove that $G = G_1 \circ G_2$ is a rough neutrosophic digraph, it is enough to show that $SE_1 \circ SE_2$ and $SE_1 \circ SE_2$ are neutrosophic relations on $RV_1 \circ RV_2$ and $RV_1 \circ RV_2$, respectively. First, we show that $SE_1 \circ SE_2$ is a neutrosophic relation on $RV_1 \circ RV_2$.

If $x \in RV_1, y_1 y_2 \in SE_2$, then

$$
\mu_{SE_1 \circ SE_2}(x, y_2) = \mu_{RV_2}(x) \land \mu_{SE_2}(y_2)
$$

$$
\leq \mu_{RV_2}(x) \land (\mu_{RV_2}(x) \land \mu_{RV_2}(y_2))
$$

$$
= (\mu_{RV_2}(x) \land \mu_{RV_2}(x)) \land (\mu_{RV_2}(x) \land \mu_{RV_2}(y_2))
$$

$$
= \mu_{RV_2}(x, x) \land \mu_{RV_2}(y_2)
$$

$$
\mu_{SE_1 \circ SE_2}(x, y_2) \leq \min\{\{\mu_{RV_2}(x, x), \mu_{RV_2}(x, y_2)\}\}
$$

$$
\sigma_{SE_1 \circ SE_2}(x, y_2) = \sigma_{RV_2}(x) \land \sigma_{SE_2}(y_2)
$$

$$
\leq \sigma_{RV_2}(x) \land (\sigma_{RV_2}(x) \land \sigma_{RV_2}(y_2))
$$

$$
= (\sigma_{RV_2}(x) \land \sigma_{RV_2}(x)) \land (\sigma_{RV_2}(x) \land \sigma_{RV_2}(y_2))
$$

$$
= \sigma_{RV_2}(x, x) \land \sigma_{RV_2}(y_2)
$$

$$
\mu_{SE_1 \circ SE_2}(x, y_2) \leq \min\{\{\sigma_{RV_2}(x, x), \sigma_{RV_2}(x, y_2)\}\}
$$

$$
\lambda_{SE_1 \circ SE_2}(x, y_2) = \lambda_{RV_2}(x) \vee \lambda_{SE_2}(x, y_2)
$$

$$
\leq \lambda_{RV_2}(x) \vee (\lambda_{RV_2}(x) \lor \lambda_{RV_2}(y_2))
$$

$$
= (\lambda_{RV_2}(x) \lor \lambda_{RV_2}(x)) \lor (\lambda_{RV_2}(x) \lor \lambda_{RV_2}(y_2))
$$

$$
= \lambda_{RV_2}(x, x) \lor \lambda_{RV_2}(x, y_2)
$$

$$
\lambda_{SE_1 \circ SE_2}(x, y_2) \leq \max\{\{\lambda_{RV_2}(x, x), \lambda_{RV_2}(x, y_2)\}\}
$$

If $x_1 y_1 \in SE_1, z \in RV_2$, then

$$
\mu_{SE_1 \circ SE_2}(x_1, z) = \mu_{SE_1}(x_1) \land \mu_{RV_2}(z)
$$

$$
\leq (\mu_{RV_1}(x_1) \land \mu_{RV_1}(y_1)) \land \mu_{RV_2}(z)
$$

$$
= (\mu_{RV_1}(x_1) \land \mu_{RV_1}(y_1)) \land (\mu_{RV_1}(y_1) \land \mu_{RV_2}(z))
$$

$$
= \mu_{RV_1 \circ RV_2}(x_1, z) \land \mu_{RV_1 \circ RV_2}(y_1, z)
$$

$$
\mu_{SE_1 \circ SE_2}(x_1, z) \leq \min\{\{\mu_{RV_1 \circ RV_2}(x_1, z), \mu_{RV_1 \circ RV_2}(y_1, z)\}\},
$$
\[ \sigma_{S_E \circ S_E}(x_1, z) = \min \{ \sigma_{S_E}(x_1, y_1), \sigma_{S_E}(z) \} \]

\[ \mu_{S_E \circ S_E}(x_1, z) = \max \{ \mu_{S_E}(x_1, y_1), \mu_{S_E}(z) \} \]

If \( x_1, y_1 \in S_E \), \( x_2, y_2 \in R V_2 \) such that \( x_2 \neq y_2 \),

\[ \sigma_{S_E \circ S_E}(x_1, y_1, y_2) = \sigma_{S_E}(x_1, y_1) \land \sigma_{S_E}(x_2, y_2) \land \sigma_{S_E}(y_2) \]

\[ \mu_{S_E \circ S_E}(x_1, y_1, y_2) = \mu_{S_E}(x_1, y_1) \land \mu_{S_E}(x_2, y_2) \land \mu_{S_E}(y_2) \]

Thus, from above, it is clear that \( S_E \circ S_E \) is a neutrosophic relation on \( R V_1 \circ R V_2 \).

Similarly, we can show that \( S_E \circ S_E \) is a neutrosophic relation on \( R V_1 \circ R V_2 \). Hence, \( G = (G_1 \circ G_2, \Phi_1 \circ \Phi_2) \) is a rough neutrosophic digraph.

**Definition 9.** Let \( G = (G, \Phi) \) be a RND. The complement of \( G \), denoted by \( G' = (G', \Phi') \) is a rough neutrosophic digraph, where \( G' = ((R V)'$, $(S E)'$) and \( G' = ((R V)'$, $(S E)'$) are neutrosophic digraph such that

1. \( \mu_{G'}(x) = \mu_{G}(x) \)
2. \( \sigma_{G'}(x) = \sigma_{G}(x) \)
3. \( \lambda_{G'}(x) = \lambda_{G}(x) \) \( \forall x \in V^* \)
4. \( \mu_{(S E)'}(x, y) = \min \{ \mu_{G}(x), \mu_{G}(y) \} - \mu_{S E}(x, y) \)
5. \( \sigma_{(S E)'}(x, y) = \min \{ \sigma_{G}(x), \sigma_{G}(y) \} - \sigma_{S E}(x, y) \)
6. \( \lambda_{(S E)'}(x, y) = \max \{ \lambda_{G}(x), \lambda_{G}(y) \} - \lambda_{S E}(x, y) \) \( \forall x, y \in V^* \).
(2) $\mu_{\mathcal{R}^\prime}(x) = \mu_{\mathcal{R}}(x)$, $\sigma_{\mathcal{R}^\prime}(x) = \sigma_{\mathcal{R}}(x)$,
$\lambda_{\mathcal{R}^\prime}(x) = \lambda_{\mathcal{R}}(x)$, $\forall x \in V^*$
$\mu_{\mathcal{SE}^\prime}(x, y) = \min\{\mu_{\mathcal{R}}(x), \mu_{\mathcal{R}^\prime}(y)\} - \mu_{\mathcal{SE}}(xy)$
$\sigma_{\mathcal{SE}^\prime}(x, y) = \min\{\sigma_{\mathcal{R}}(x), \sigma_{\mathcal{R}^\prime}(y)\} - \sigma_{\mathcal{SE}}(xy)$
$\lambda_{SE}(x, y) = \max\{\lambda_{\mathcal{R}}(x), \lambda_{\mathcal{R}^\prime}(y)\} - \lambda_{SE}(xy)$ $\forall x, y \in V^*$.

**Example 5.** Consider a rough neutrosophic digraph as shown in Figure 4. The lower and upper approximations of graph $G$ are $\mathcal{G} = (\mathcal{R}, \mathcal{SE})$ and $\mathcal{G} = (\mathcal{R}, \mathcal{SE})$, respectively, where

$\mathcal{R} = \{(a, 0.2, 0.4, 0.6), (b, 0.2, 0.4, 0.6), (c, 0.2, 0.5, 0.9), (d, 0.2, 0.5, 0.9)\}$,
$\mathcal{R} = \{(a, 0.3, 0.8, 0.3), (b, 0.3, 0.8, 0.3), (c, 0.5, 0.6, 0.8), (d, 0.5, 0.6, 0.8)\}$,
$\mathcal{SE} = \{(aa, 0.2, 0.3, 0.3), (ab, 0.2, 0.3, 0.3), (ad, 0.1, 0.3, 0.8), (bc, 0.1, 0.3, 0.8),
(bd, 0.1, 0.3, 0.8), (cd, 0.2, 0.4, 0.7), (dd, 0.2, 0.4, 0.7)\}$,
$\mathcal{SE} = \{(aa, 0.2, 0.4, 0.3), (ab, 0.2, 0.4, 0.3), (ad, 0.2, 0.4, 0.7), (bc, 0.2, 0.4, 0.7),
(bd, 0.2, 0.4, 0.7), (cd, 0.2, 0.4, 0.7), (dd, 0.2, 0.4, 0.7)\}$.

The complement of $G$ is $G^\prime = (\mathcal{G}^\prime, \mathcal{G}^\prime)$. By calculations, we have

$(\mathcal{R}^\prime) = \{(a, 0.2, 0.4, 0.6), (b, 0.2, 0.4, 0.6), (c, 0.2, 0.5, 0.9), (d, 0.2, 0.5, 0.9)\}$,
$(\mathcal{R}^\prime) = \{(a, 0.3, 0.8, 0.3), (b, 0.3, 0.8, 0.3), (c, 0.5, 0.6, 0.8), (d, 0.5, 0.6, 0.8)\}$,
$(\mathcal{SE}^\prime) = \{(aa, 0.2, 0.3, 0.3), (ab, 0.2, 0.3, 0.3), (ad, 0.1, 0.1, 0.1), (ba, 0.2, 0.4, 0.6), (bb, 0.2, 0.4, 0.6),
(bc, 0.1, 0.1, 0.1), (bd, 0.1, 0.1, 0.1), (ca, 0.2, 0.4, 0.9), (cb, 0.2, 0.4, 0.9), (cc, 0.2, 0.5, 0.9), (cd, 0.2, 0.5, 0.9),
(da, 0.2, 0.4, 0.9), (db, 0.2, 0.4, 0.9), (dc, 0.2, 0.4, 0.9), (dd, 0.2, 0.4, 0.9)\}$,
$(\mathcal{SE}^\prime) = \{(aa, 0.1, 0.4, 0), (ab, 0.1, 0.4, 0), (ac, 0.3, 0.6, 0.8), (ad, 0.1, 0.2, 0.1), (ba, 0.3, 0.8, 0.3), (bb, 0.3, 0.8, 0.3),
(bc, 0.1, 0.2, 0.1), (bd, 0.1, 0.2, 0.1), (ca, 0.3, 0.6, 0.8), (cb, 0.3, 0.6, 0.8), (cc, 0.5, 0.6, 0.8), (cd, 0.5, 0.6, 0.8),
(da, 0.3, 0.6, 0.8), (db, 0.3, 0.6, 0.8), (dc, 0.3, 0.2, 0.1), (dd, 0.3, 0.2, 0.1)\}$.

Thus, $G^\prime = (\mathcal{R}^\prime, \mathcal{SE}^\prime)$ and $\overline{G} = (\mathcal{R}^\prime, \mathcal{SE}^\prime)$ are neutrosophic digraph, as shown in Figure 12.

**Figure 12.** Rough neutrosophic digraph $\mathcal{G} = (\mathcal{G}, \mathcal{G})$.

**Definition 10.** A rough neutrosophic digraph $G = (\mathcal{G}, \mathcal{G})$ is self complementary if $G$ and $G^\prime$ are isomorphic, that is, $\mathcal{G} \cong \mathcal{G}^\prime$ and $\overline{\mathcal{G}} \cong \overline{\mathcal{G}^\prime}$. 
Example 6. Let $V^* = \{a, b, c\}$ be a set and $R$ an equivalence relation on $V^*$ defined as:

$$R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}. $$

Let $V = \{(a, 0.2, 0.4, 0.8), (b, 0.2, 0.4, 0.8), (c, 0.4, 0.6, 0.4)\}$ be a neutrosophic set on $V^*$. The lower and upper approximations of $V$ are given as,

$$RV = \{(a, 0.2, 0.4, 0.8), (b, 0.2, 0.4, 0.8), (c, 0.4, 0.6, 0.4)\},$$
$$RV = \{(a, 0.4, 0.6, 0.4), (b, 0.2, 0.4, 0.8), (c, 0.4, 0.6, 0.4)\}.$$

Let $E^* = \{aa, ab, ac, ba\} \subseteq V^* \times V^*$ and $S$ an equivalence relation on $E^*$ defined as

$$S = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

Let $E = \{(aa, 0.1, 0.3, 0.2), (ab, 0.1, 0.2, 0.4), (ac, 0.2, 0.2, 0.4), (ba, 0.1, 0.2, 0.4)\}$ be a neutrosophic set on $E^*$ and $SE = (SE, SE)$ a RNR where $SE$ and $SE$ are given as

$$SE = \{(aa, 0.1, 0.2, 0.4), (ab, 0.1, 0.2, 0.4), (ac, 0.1, 0.2, 0.4), (ba, 0.1, 0.2, 0.4)\},$$
$$SE = \{(aa, 0.2, 0.3, 0.2), (ab, 0.1, 0.2, 0.4), (ac, 0.2, 0.3, 0.2), (ba, 0.1, 0.2, 0.4)\}.$$

Thus, $G = (RV, SE)$ and $C = (RV, SE)$ are neutrosophic digraphs, as shown in Figure 13. The complement of $G$ is $G' = (G', C')$, where $G' = G$ and $C' = C$ are neutrosophic digraphs, as shown in Figure 13, and it can be easily shown that $G$ and $G'$ are isomorphic. Hence, $G = (G, C)$ is a self complementary RND.

![Figure 13. Self complementary RND $G = (G, C)$](image)

Theorem 4. Let $G = (G, C)$ be a self complementary rough neutrosophic digraph. Then,

$$\sum_{w,z \in V^*} \mu_{SE}(wz) = \frac{1}{2} \sum_{w,z \in V^*} (\mu_{RV}(w) \land \mu_{RV}(z)),$$
$$\sum_{w,z \in V^*} \sigma_{SE}(wz) = \frac{1}{2} \sum_{w,z \in V^*} (\sigma_{RV}(w) \land \sigma_{RV}(z)),$$
$$\sum_{w,z \in V^*} \lambda_{SE}(wz) = \frac{1}{2} \sum_{w,z \in V^*} (\lambda_{RV}(w) \land \lambda_{RV}(z)),$$
$$\sum_{w,z \in V^*} \mu_{SE}(wz) = \frac{1}{2} \sum_{w,z \in V^*} (\mu_{RV}(w) \lor \mu_{RV}(z)),$$
$$\sum_{w,z \in V^*} \sigma_{SE}(wz) = \frac{1}{2} \sum_{w,z \in V^*} (\sigma_{RV}(w) \lor \sigma_{RV}(z)),$$
$$\sum_{w,z \in V^*} \lambda_{SE}(wz) = \frac{1}{2} \sum_{w,z \in V^*} (\lambda_{RV}(w) \lor \lambda_{RV}(z)).$$
Proof. Let $G = (\overline{G}, \overline{T})$ be a self complementary rough neutrosophic digraph. Then, there exist two isomorphisms $g : V^* \rightarrow V^*$ and $\overline{g} : V^* \rightarrow V^*$, respectively, such that

$$
\mu_{RV} (g(w)) = \mu_{RV}(w),
\sigma_{RV} (g(w)) = \sigma_{RV}(w),
\lambda_{RV} (g(w)) = \lambda_{RV}(w), \; \forall \; w \in V^*
$$

and

$$
\mu_{SE} (\overline{g}(w)) = \mu_{SE}(w),
\sigma_{SE} (\overline{g}(w)) = \sigma_{SE}(w),
\lambda_{SE} (\overline{g}(w)) = \lambda_{SE}(w), \; \forall \; w \in V^*.
$$

By Definition 7, we have

$$
\mu_{SE} (g(w) \overline{g}(z)) = (\mu_{RV}(w) \wedge \mu_{RV}(z)) - \mu_{SE}(wz)
$$

$$
\mu_{SE}(wz) = (\mu_{RV}(w) \wedge \mu_{RV}(z)) - \mu_{SE}(wz)
$$

$$
\sum_{w,z \in V^*} \mu_{SE}(wz) = \sum_{w,z \in V^*} (\mu_{RV}(w) \wedge \mu_{RV}(z)) - \sum_{w,z \in V^*} \mu_{SE}(wz)
$$

$$
2 \sum_{w,z \in V^*} \mu_{SE}(wz) = \sum_{w,z \in V^*} (\mu_{RV}(w) \wedge \mu_{RV}(z))
$$

$$
\sum_{w,z \in V^*} \mu_{SE}(wz) = \frac{1}{2} \sum_{w,z \in V^*} (\mu_{RV}(w) \wedge \mu_{RV}(z))
$$

$$
\sigma_{SE} (g(w) \overline{g}(z)) = (\sigma_{RV}(w) \wedge \sigma_{RV}(z)) - \sigma_{SE}(wz)
$$

$$
\sigma_{SE}(wz) = (\sigma_{RV}(w) \wedge \sigma_{RV}(z)) - \sigma_{SE}(wz)
$$

$$
\sum_{w,z \in V^*} \sigma_{SE}(wz) = \sum_{w,z \in V^*} (\sigma_{RV}(w) \wedge \sigma_{RV}(z)) - \sum_{w,z \in V^*} \sigma_{SE}(wz)
$$

$$
2 \sum_{w,z \in V^*} \sigma_{SE}(wz) = \sum_{w,z \in V^*} (\sigma_{RV}(w) \wedge \sigma_{RV}(z))
$$

$$
\sum_{w,z \in V^*} \sigma_{SE}(wz) = \frac{1}{2} \sum_{w,z \in V^*} (\sigma_{RV}(w) \wedge \sigma_{RV}(z))
$$

$$
\lambda_{SE} (g(w) \overline{g}(z)) = (\lambda_{RV}(w) \vee \lambda_{RV}(z)) - \lambda_{SE}(wz)
$$

$$
\lambda_{SE}(wz) = (\lambda_{RV}(w) \vee \lambda_{RV}(z)) - \lambda_{SE}(wz)
$$

$$
\sum_{w,z \in V^*} \lambda_{SE}(wz) = \sum_{w,z \in V^*} (\lambda_{RV}(w) \vee \lambda_{RV}(z)) - \sum_{w,z \in V^*} \lambda_{SE}(wz)
$$

$$
2 \sum_{w,z \in V^*} \lambda_{SE}(wz) = \sum_{w,z \in V^*} (\lambda_{RV}(w) \vee \lambda_{RV}(z))
$$

$$
\sum_{w,z \in V^*} \lambda_{SE}(wz) = \frac{1}{2} \sum_{w,z \in V^*} (\lambda_{RV}(w) \vee \lambda_{RV}(z))
$$
Similarly, it can be shown that
\[
\sum_{w, z \in V^*} \mu_{SE}(wz) = \mu_{RV}^V(w) \wedge \mu_{RV}^V(z) \\
\sum_{w, z \in V^*} \sigma_{SE}(wz) = \sigma_{RV}^V(w) \wedge \sigma_{RV}^V(z) \\
\sum_{w, z \in V^*} \lambda_{SE}(wz) = \lambda_{RV}^V(w) \vee \lambda_{RV}^V(z).
\]

This completes the proof. \(\square\)

3. Application

Investment is a very good way of getting profit and wisely invested money surely gives certain profit. The most important factors that influence individual investment decision are: company’s reputation, corporate earnings and price per share. In this application, we combine these factors into one factor, i.e. company’s status in industry, to describe overall performance of the company. Let us consider an individual Mr. Shahid who wants to invest his money. For this purpose, he considers some private companies, which are Telecommunication company (TC), Carpenter company (CC), Real Estate business (RE), Vehicle Leasing company (VL), Advertising company (AD), and Textile Testing company (TT). Let \(V^* = \{TC, CC, RE, VL, AD, TT\}\) be a set. Let \(T\) be an equivalence relation defined on \(V^*\) as follows:

\[
T = \begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 0
\end{bmatrix}.
\]

Let \(V = \{(TC, 0.3, 0.4, 0.1), (CC, 0.8, 0.1, 0.5), (RE, 0.1, 0.2, 0.6), (VL, 0.9, 0.6, 0.1), (AD, 0.2, 0.5, 0.2), (TT, 0.8, 0.6, 0.5)\}\) be a neutrosophic set on \(V^*\) with three components corresponding to each company, which represents its status in the industry and \(TV = (TV, TV)\) a rough neutrosophic set, where \(TV\) and \(TV\) are lower and upper approximations of \(V\), respectively, as follows:

\[
TV = \{(TC, 0.1, 0.2, 0.6), (CC, 0.8, 0.1, 0.5), (RE, 0.1, 0.2, 0.6), (VL, 0.8, 0.6, 0.5), (AD, 0.1, 0.2, 0.6), (TT, 0.8, 0.6, 0.5)\},
\]

\[
\overline{TV} = \{(TC, 0.3, 0.5, 0.1), (CC, 0.8, 0.1, 0.5), (RE, 0.3, 0.5, 0.1), (VL, 0.9, 0.6, 0.1), (AD, 0.3, 0.5, 0.1), (TT, 0.9, 0.6, 0.1)\}.
\]

Let \(E^* = \{(TC, CC), (TC, AD), (TC, RE), (CC, VL), (CC, TT), (AD, RE), (TT, VL)\}\), be the set of edges and \(S\) an equivalence relation on \(E^*\) defined as follows:

\[
S = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}.
\]
Let \( E = \{(TC, CC), 0.1, 0.1, 0.1\}, (TC, AD), 0.1, 0.2, 0.1\), 
\((TC, RE), 0.1, 0.2, 0.1\), 
\((CC, VL), 0.8, 0.1, 0.5\), 
\((CC, TT), 0.8, 0.1, 0.5\), 
\((AD, RE), 0.1, 0.2, 0.1\), 
\((TT, VL), 0.8, 0.6, 0.1\) \}

be a neutrosophic set on \( E^* \) which represents relationship between companies and 
\( SE = (\overline{SE}, \overline{SE}) \)
a rough neutrosophic relation, where \( \overline{SE} \) and \( \overline{SE} \) are lower and upper upper approximations of \( E \), respectively, as follows:

\[
\overline{SE} = \{((TC, CC), 0.1, 0.1, 0.1), ((TC, AD), 0.1, 0.2, 0.1), ((TC, RE), 0.1, 0.2, 0.1), 
((CC, VL), 0.8, 0.1, 0.5), ((CC, TT), 0.8, 0.1, 0.5), ((AD, RE), 0.1, 0.2, 0.1), 
((TT, VL), 0.8, 0.6, 0.1)\},
\]

\[
\overline{SE} = \{((TC, CC), 0.1, 0.1, 0.1), ((TC, AD), 0.1, 0.2, 0.1), ((TC, RE), 0.1, 0.2, 0.1), 
((CC, VL), 0.8, 0.1, 0.5), ((CC, TT), 0.8, 0.1, 0.5), ((AD, RE), 0.1, 0.2, 0.1), 
((TT, VL), 0.8, 0.6, 0.1)\}.
\]

Thus, \( G = (TV, SE) \) and \( \overline{G} = (TV, \overline{SE}) \) is a rough neutrosophic digraph as shown in Figure 14.

![Figure 14. Rough neutrosophic digraph \( G = (\overline{G}, \overline{G}) \).](image)

To find out the most suitable investment company, we define the score values

\[
S(v_i) = \sum_{v_j \in E^*} \frac{T(v_j) + I(v_j) - F(v_j)}{3 - (T(v_i,v_j) + I(v_i,v_j) - F(v_i,v_j))},
\]

where

\[
T(v_j) = \frac{T(v_j) + T(v_j)}{2},
I(v_j) = \frac{I(v_j) + I(v_j)}{2},
F(v_j) = \frac{F(v_j) + F(v_j)}{2},
\]

and
of each selected company and industry decision is \( v_k \) if \( v_k = \max_i S(v_i) \). By calculation, we have 

\[
S(TC) = 0.4926, \quad S(CC) = 1.4038, \quad S(RE) = 0.0667, \quad S(VL) = 0.3833, \quad S(AD) = 0.1429 \quad \text{and} \quad S(TT) = 1.3529.
\]

Clearly, \( CC \) is the optimal decision. Therefore, the carpenter company is selected to get maximum possible profit. We present our proposed method as an algorithm. This Algorithm 1 returns the optimal solution for the investment problem.

**Algorithm 1** Calculation of Optimal decision

1. Input the vertex set \( V^* \).
2. Construct an equivalence relation \( T \) on the set \( V^* \).
3. Calculate the approximation sets \( TV \) and \( TV \).
4. Input the edge set \( E^* \subseteq V^* \times V^* \).
5. Construct an equivalence relation \( S \) on \( E^* \).
6. Calculate the approximation sets \( SE \) and \( SE \).
7. Calculate the score value, by using formula

\[
S(v_i) = \sum_{v_i,v_j \in E^*} \frac{T(v_j) + I(v_i) - F(v_i)}{3 - (T(v_i,v_j) + I(v_i,v_j) - F(v_i,v_j))},
\]

8. The decision is \( S(v_k) = \max_{v_i \in V^*} S(v_i) \).
9. If \( v_k \) has more than one value, then any one of \( S(v_k) \) may be chosen.

4. Conclusions and Future Directions

Neutrosophic sets and rough sets are very important models to handle uncertainty from two different perspectives. A rough neutrosophic model is a hybrid model which is made by combining two mathematical models, namely, rough sets and neutrosophic sets. This hybrid model deals with soft computing and vagueness by using the lower and upper approximation spaces. A rough neutrosophic set model gives more precise results for decision-making problems as compared to neutrosophic set model. In this paper, we have introduced the notion of rough neutrosophic digraphs. This research work can be extended to: (1) rough bipolar neutrosophic soft graphs; (2) bipolar neutrosophic soft rough graphs; (3) interval-valued bipolar neutrosophic rough graphs; and (4) neutrosophic soft rough graphs.

**Acknowledgments:** The authors are very thankful to the Editor and referees for their valuable comments and suggestions for improving the paper.

**Author Contributions:** Sidra Sayed and Nabeela Ishfaq conceived and designed the experiments; Muhammad Akram performed the experiments; Florentin Smarandache contributed reagents/materials/analysis tools.

**Conflicts of Interest:** The authors declare that they have no conflict of interest regarding the publication of the research article.

**References**

1. Smarandache, F. *A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic*; American Research Press: Rehoboth, NM, USA, 1999.
2. Zadeh, L.A. Fuzzy sets. *Inf. Control* 1965, 8, 338–353.
3. Smarandache, F. *Neutrosophy Neutrosophic Probability, Set, and Logic*; American Research Press: Rehoboth, NM, USA, 1998.
4. Wang, H.; Smarandache, F.; Zhang, Y.; Sunderraman, R. Single-valued neutrosophic sets. *Multispace Multistructure* 2010, 4, 410–413.

5. Ye, J. Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. *Int. J. Gen. Syst.* 2013, 42, 386–394.

6. Ye, J. Improved correlation coefficients of single valued neutrosophic sets and interval neutrosophic sets for multiple attribute decision making. *J. Intell. Fuzzy Syst.* 2014, 27, 2453–2462.

7. Pawlak, Z. Rough sets. *Int. J. Comput. Inf. Sci.* 1982, 11, 341–356.

8. Dubois, D.; Prade, H. Rough fuzzy sets and fuzzy rough sets. *Int. J. Gen. Syst.* 1990, 17, 191–209.

9. Liu, P.; Chen, S.M. Group decision making based on Heronian aggregation operators of intuitionistic fuzzy numbers. *IEEE Trans. Cybern.* 2017, 47, 2514–2530.

10. Broumi, S.; Smarandache, F.; Dhar, M. Rough neutrosophic sets. *Neutrosophic Sets Syst.* 2014, 3, 62–67.

11. Yang, H.L.; Zhang, C.L.; Guo, Z.L.; Liu, Y.L.; Liao, X. A hybrid model of single valued neutrosophic sets and rough sets: Single valued neutrosophic rough set model. *Soft Comput.* 2016, 21, 6253–6267, doi:10.1007/s00500-016-2356-y.

12. Mordeson, J.N.; Peng, C.S. Operations on fuzzy graphs. *Inf. Sci.* 1994, 79, 159–170.

13. Akram, M.; Shahzadi, S. Neutrosophic soft graphs with application. *J. Intell. Fuzzy Syst.* 2017, 32, 841–858.

14. Akram, M.; Sarwar, M. Novel multiple criteria decision making methods based on bipolar neutrosophic sets and bipolar neutrosophic graphs. *Ital. J. Pure Appl. Math.* 2017, 38, 368–389.

15. Akram, M.; Siddique, S. Neutrosophic competition graphs with applications. *J. Intell. Fuzzy Syst.* 2017, 33, 921–935.

16. Akram, M.; Sitara, M. Interval-valued neutrosophic graph structures. *Punjab Univ. J. Math.* 2018, 50, 113–137.

17. Zafer, F.; Akram, M. A novel decision-making method based on rough fuzzy information. *Int. J. Fuzzy Syst.* 2017, doi:10.1007/s40815-017-0368-0.

18. Banerjee, M.; Pal, S.K. Roughness of a fuzzy set. *Inf. Sci.* 1996, 93, 235–246.

19. Liu, P.; Chen, S.M.; Junlin, L. Some intuitionistic fuzzy interaction partitioned Bonferroni mean operators and their application to multi-attribute group decision making. *Inf. Sci.* 2017, 411, 98–121.

20. Liu, P. Multiple attribute group decision making method based on interval-valued intuitionistic fuzzy power Heronian aggregation operators. *Comput. Ind. Eng.* 2017, 108, 199–212.

21. Zhang, X.; Dai, J.; Yu, Y. On the union and intersection operations of rough sets based on various approximation spaces. *Inf. Sci.* 2015, 292, 214–229.

22. Bao, Y.L.; Yang, H.L. On single valued neutrosophic refined rough set model and its application. *J. Intell. Fuzzy Syst.* 2017, 33, 1235–1248.

© 2018 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).