Fairness-aware power allocation in downlink MIMO-NOMA systems

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Abstract
Non-orthogonal multiple access (NOMA) has attracted great attention due to its potential of providing high spectral efficiency and massive connectivity. Combining it with multiple-input multiple-output (MIMO) technology can further improve the spectrum efficiency. In this paper, the power allocation problem in downlink multi-cluster MIMO-NOMA systems is investigated for maximizing the fairness utility function. First, a long-term fairness function is considered and the optimization problem is formulated as a weighted sum-rate maximization problem. The problem is transformed into a convex form by introducing two sets of auxiliary variables, and propose an iterative algorithm to update the weight factors and the auxiliary variables. Then, an instantaneous fairness is considered and the optimization problem is formulated as a minimum data-rate maximization problem. It is transformed into a one-dimensional optimization problem based on an iterative algorithm and a closed-form power allocation expression is deduced. Simulation results illustrate that the proposed two fairness power allocation schemes have better performance of edge-user rate than the comparable schemes.

1 INTRODUCTION

Non-orthogonal multiple access (NOMA) has been viewed as one of the potential technologies for the future wireless communications. By applying superposition coding at transmitter and successive interference cancellation (SIC) at receiver, NOMA allows multiple users to transmit signals at the same time-frequency resource. It can fully exploit the spectrum resource to meet the requirements in terms of high spectral efficiency and massive connectivity [1, 2]. In addition, NOMA has therefore attracted increasing attention due to the advantages of fairness and flexibility [1–4].

Power allocation is an important issue to fully exploit the advantages of NOMA and has been widely studied. The basic criterion of power allocation in downlink NOMA systems is that the users with strong channel gains are allocated less power while the users with weak channel gains are allocated more power [5–7]. It is necessary to make a certain difference in power allocated to users, since it determines whether the receiver can perform the perfect SIC or not. The studies on designing power allocation with NOMA start with single-input single-output (SISO) systems. In [8–10], the performance analyses of SISO-NOMA systems are given. They prove that NOMA can provide higher capacity than orthogonal multiple access (OMA) systems. Then, to enhance the potential performance of the NOMA systems, several power allocation schemes have been proposed in [7, 11–15]. Particularly, the authors in [13] investigate the resource allocation problem based on different performance criteria in the SISO-NOMA system, such as sum-rate and energy efficiency. Wang et al. in [14] study a downlink two-user NOMA system and derive power allocation coefficients by the Karush–Kuhn–Tucker (KKT) conditions, which can ensure that the ergodic sum-rate is higher or as well as OMA schemes. The authors in [15] propose a dynamic power allocation method to minimize the outage probability by adopting an asymptotic approximation with the assumption of high signal-to-noise ratio. However, most of the above studies on power allocation focus on the single-antenna NOMA systems.
Multi-input multi-output (MIMO) technology allowing multi-antennas to send and receive messages simultaneously, has been widely adopted in wireless communication systems. The combination of MIMO with NOMA can further enhance system throughput and spectral efficiency based on the high spatial degree of freedom [16–18]. The authors in [19] study the ergodic capacity maximization problem in MIMO-NOMA systems with statistical channel state information at the transmitter and prove that its capacity significantly outperforms the traditional MIMO-OMA systems. Motivated by the potential performance of MIMO-NOMA, more and more researchers investigate the resource allocation problem to enhance the superior gain of MIMO-NOMA in different communication scenario [20, 21]. The authors in [22] propose a power allocation method for a two-user MIMO-NOMA scenario and it achieves higher sum-rate than OMA system. In [23], a joint beamforming and power allocation problem aiming at maximizing the sum-rate in multi-user systems is investigated. In addition, the heterogeneous network based on NOMA is presented in [24], where the authors investigate the trade-off between the data rate and the energy consumption, and it shows that the proper power allocation can increase energy efficiency. In [25] the average rate maximized problem in decode-and-forward relaying downlink network is investigated where a joint user-channel assignment and power allocation algorithm is proposed. In [26] and [27] the combination of millimetre and NOMA is introduced, in which optimal power allocation schemes for ideal beamforming are developed.

Fairness has always received wide attention in wireless communication networks. Many researchers treat the power allocation principle of the NOMA system that allocate more power to weak users as a method of ensuring users’ fairness, but it is not sufficient. Even so, unfortunately, the power allocation principle has not been achieved in most of the above work. For single-antenna NOMA systems, in [28] and [29], the weight factors are introduced to optimization problem to ensure users fairness, and power order constraints are taken into account to ensure the performance of weak users. In [30], a max-min rate proportional fairness problem is studied in two-user pairing NOMA system, in which the authors propose a fairness-aware NOMA-based scheduling. However, this method only applies to the two-user NOMA system, and it also assumed that the single antenna was placed at the transmitter and receivers. In [31], the authors analyse the fairness performance of the wireless powered communication network based on NOMA. In [32], a rate splitting method is proposed to guarantee the fairness in uplink single-input multiple-output (SIMO) NOMA system and it can achieve higher minimum data-rate. However there are very few works investigating the fairness in MIMO-NOMA systems.

In this paper, we investigate the fairness-aware power allocation problem in multi-cluster MIMO-NOMA systems. We formulate the power allocation problem as a utility function maximization problem. Then, we design two fairness schemes: the long-term fairness scheme and the instantaneous fairness scheme. In each case, we propose a power allocation algorithm to ensure the fairness. The main contribution of this paper can be summarized as follows:

- We propose two power allocation methods with considering user fairness in the downlink multi-cluster MIMO-NOMA system, where we maximize two types of fairness utility functions corresponding to long-term fairness and instantaneous fairness respectively.
- We formulate the long-term fairness power allocation problem as the optimization problem of maximizing weighted sum-rate. To solve this non-convex problem, we transform the original problem into a convex form by introducing a set of auxiliary variables. We then provide an iterative algorithm to update the users’ weight factors and the auxiliary variables.
- We formulate the instantaneous fairness power allocation problem as the optimization problem of maximizing minimum achievable rate. We transform the original problem as a one-dimension optimization problem, and then obtain the optimal power allocation based on an iterative algorithm.
- We provide comparisons of the proposed fairness power allocation schemes with the traditional methods. Simulation results show that the proposed schemes can achieve better cell-edge user performance.

The rest of this paper is organized as follows. In Section 2 the MIMO-NOMA system model is introduced and the fairness utility function maximization problem is formulated. In Section 3, the weighted sum-rate problem is studied and the power allocation algorithm with long-term fairness is proposed. In Section 4, the maximizing minimum achievable rate problem is studied and the power allocation algorithm with instantaneous fairness is proposed. In Section 5, simulation results are presented and analysed. Finally, Section 6 concludes the work.

2 | System Model and Problem Formulation

2.1 | System model

We consider a downlink multi-cluster MIMO-NOMA system, where a base station (BS) equipped with $N_r$ antennas is located at the centre of the cell. The BS can simultaneously serve $M$ clusters with $K$ users per cluster. Each user is equipped with $N_t$ antennas. We denote the set of clusters as $\mathcal{M} = \{1, \ldots, M\}$ and the set of users in cluster $m$ as $\mathcal{K}_m = \{1, \ldots, K\}$, and $\mathcal{K} = \mathcal{K}_1 \cup \ldots \cup \mathcal{K}_M$. The channel fading matrix from BS to user $k$ in cluster $m$ is expressed as $H_{n,k} \in \mathbb{C}^{N_t \times N_r}$. The BS transmits superposed signal in power domain to all users at the same slot with the NOMA principle. The superposed signal $\mathbf{s} \in \mathbb{R}^{K \times 1}$
can be expressed as
\[
S = \left[ \sqrt{P_{1,1}^k s_1} + \cdots + \sqrt{P_{K_1,K}^k s_K} \right],
\]
where \(P_{m,k}\) and \(s_{m,k}\) are the allocated power and the transmit signal to user \(k\) in cluster \(m\) respectively. Without loss of generality, we assume \(\mathbb{E}\{|s_{m,k}|^2\} = 1\).

The precoding matrix used by the BS is expressed as \(B \in \mathbb{C}^{N_t \times M}\), which is denoted by
\[
B = [B_1, B_2, \ldots, B_M],
\]
where \(B_m\) is the precoding vector to serve the users in the \(m\)th cluster. Therefore, the corresponding superposed signal transmitted from the BS can be expressed as
\[
X = BS.
\]

Thus, the received signal at user \(k\) in cluster \(m\) is given by
\[
y_{m,k} = H_{m,k} B_m \left( \sqrt{P_{m,1}^k s_1} + \cdots + \sqrt{P_{m,K}^k s_K} \right)
+ \sum_{n \neq m} H_{m,k} B_n s_n + n_{m,k},
\]
where \(n_{m,k} \in \mathbb{C}^{N_t \times 1}\) is the additive white Gaussian noise (AWGN) vector subjected to independent and identically distributed \(n_{m,k} \sim \mathcal{CN}(0, \sigma_m^2)\). We can observe from (4) that there is strong inter-cluster interference at each receiver. To avoid it, zero-forcing receiving is assumed to be applied at the users. The zero-forcing vector of user \(k\) in cluster \(m\) is defined as \(v_{m,k} \in \mathbb{C}^{N_t \times 1}\) which is normalized and follows \(v_{m,k}^H H_{m,k} B_m = 0\), \(\forall i \neq m\), where \(v_{i}^H\) represents the conjugate transpose of \(v_{i}\).

By applying zero-forcing receiving, we obtain the effective channel gain of user \(k\) in cluster \(m\) as \(|v_{m,k}^H H_{m,k} B_m|^2\). Without loss of generality, we assume the channel gains of all users in each cluster are ordered as
\[
|v_{m,1}^H H_{m,k} B_m|^2 \geq \cdots \geq |v_{m,k}^H H_{m,k} B_m|^2.
\]

At the receivers in each cluster, the user with higher channel gain can decode and remove weaker users’ signal based on the SIC principle. That is to say, user \(k\) can first decodes the later \(\{k + 1, \ldots, K\}\) users and then subtracts these signals to decode itself. By performing perfect SIC, the received signal at user \(k\) in cluster \(m\) can be simplified as
\[
y_{m,k} = v_{m,k}^H H_{m,k} B_m \sqrt{P_{m,k}^k s_{m,k}}
+ \sum_{i=1}^{k-1} v_{m,k}^H H_{m,k} B_m \sqrt{P_{m,i}^k s_{m,i}} + v_{m,k}^H n_{m,k}.
\]

In this case, the signal to interference plus noise ratio (SINR) of user \(k\) in cluster \(m\) is denoted as
\[
\text{SINR}_{m,k} = \frac{|v_{m,k}^H H_{m,k} B_m|^2 P_{m,k}^k}{\sum_{i=1}^{k-1} |v_{m,k}^H H_{m,k} B_m|^2 P_{m,i}^k + \sigma_{m,k}^2},
\]
where \(\sigma_{m,k}^2 = |v_{m,k}^H n_{m,k}|^2\) is the effective noise power. Thus, the data rate of user \(k\) in cluster \(m\) can be given by
\[
R_{m,k} = \log \left( 1 + \text{SINR}_{m,k}(P) \right),
\]
where \(P \in \mathbb{R}^{M \times K}\) is the power allocation matrix with the \((m, k)\)th element \(P_{m,k}\).

### 2.2 Problem formulation

In the existing MIMO-NOMA systems, user fairness has not been well studied for designing power allocation. Most of the work aimed at maximizing the sum-rate performance of the NOMA systems, which leads to that almost all the power is allocated to the user who has strong channel gains. In this paper, we consider the power allocation in the MIMO-NOMA system with users’ fairness. We denote the fairness utility function as \(U(P)\) and formulate the power allocation optimization problem as
\[
\max_P U(P)
\quad \text{s.t.}
\quad C1 : \sum_{m=1}^M \sum_{k=1}^K P_{m,k} \leq P_{\text{max}},
\quad C2 : P_{m,k} \geq 0, \ k \in K_m, m \in M,
\]
where \(P_{\text{max}}\) is the total power budget of the BS. Constraint C1 indicates that the total allocated power of all users is less than the budget. Constraint C2 denotes that the power allocated to each user is non-negative. In the following sections, we consider two fairness utility functions respectively.

User fairness is one of the key performance indicators for radio resource management (RRM), such as proportional fairness, max-min fairness and round robin [32]. In the design of the fairness algorithms, two strategies can be adopted. One strategy, known as instantaneous fairness, guarantees the user fairness at each time slot. It is a strict fairness requirement that might result in large loss of sum-rate performance. Due the large difference of channel conditions among the users, the performance of weak users are hardly guaranteed at all time slots, which makes more resource always be allocated to the weak users. Another strategy, known as long-term fairness, guarantees an average user fairness over time. In this case, the introduced user weights can be used to steer the resource allocation towards priority of users. For example, the user’s weight can be set to be the reciprocal of the average rate of the user prior to the current time slot, which approaches an approximate proportional fairness. Therefore, in the following sections, we first
consider a long-term fairness function and formulate the optimization problem as a weighted sum-rate maximization problem. Then we consider an instantaneous fairness and formulate the optimization problem as a minimum data-rate maximization problem.

3 LONG-TERM FAIRNESS POWER ALLOCATION

3.1 Optimization problem of maximizing weighted sum-rate

In this section, we consider the long-term fairness power allocation problem. We introduce a set of weight factors $w$ to ensure user fairness, and define the fairness utility function as the weighted sum-rate at each time slot, i.e.

$$U^L(P) = \sum_{m=1}^{M} \sum_{k=1}^{K} w_{m,k} R_{m,k}(t). \tag{10}$$

We design the weight factors by updating

$$w_{m,k}(t) = \frac{\tau_{m,k}}{R_{m,k}(t)}, \tag{11}$$

at time slot $t$, where $\tau_{m,k}$ is the long-term proportional fairness factor of user $k$ in cluster $m$. $R_{m,k}(t)$ is the average data-rate over $T$ time slots before $t$ (i.e. time window is $[t-T, t-T+1, ..., t-1]$), and it is updated by

$$\bar{R}_{m,k}(t) = \frac{1}{T} R_{m,k}(t) + \left(1 - \frac{1}{T}\right) \bar{R}_{m,k}(t-1). \tag{12}$$

The value of $w_{m,k}$ is updated after finding the optimal power allocation at each time slot. We can reformulate the optimization problem omitting the time index as

$$\max_P \sum_{m=1}^{M} \sum_{k=1}^{K} w_{m,k} R_{m,k} \quad \text{s.t.} \quad C1, C2, \quad C3 : w_{m,k} \geq 0, \ k \in K_m, m \in M. \quad \tag{13}$$

Problem (13) is a non-convex optimization problem and it is challenging to find the optimal power allocation solution directly.

Lemma 1. Given a non-negative function $F_s(x) = \frac{A_s(x)}{B_s(x)}, x \in \mathcal{X}$, if $\mathcal{X}$ is a non-empty constraint and $A_s(x)$ is a non-negative function of $x$, and $B_s(x)$ is a positive function of $x$, then the sum-of-logarithmic maximization problem

$$\max_x \sum_{s=1}^{N} \log \left(1 + F_s(x)\right), \quad \text{s.t.} \quad x \in \mathcal{X}, \quad \tag{14}$$

is equal to the following problem [34],

$$\max_x \sum_{s=1}^{N} f(x, \kappa), \quad \text{s.t.} \quad x \in \mathcal{X},\quad \tag{15}$$

where $f(x, \kappa)$ is denoted as

$$f(x, \kappa) = \log(1 + \theta_s) - \theta_s + \frac{(1 + \theta_s)^{-\kappa} A_s(x)}{A_s(x) + B_s(x)}, \quad \tag{16}$$

and $\theta_s$ is the introduced non-negative ancillary variable for each $F_s(x)$.

Proof. See Appendix 9.1.

According to Lemma 1, we can transform $U^L(P)$ into the following form with auxiliary variable $\theta_{m,k}$ as

$$U^L_o(P, \theta) = \sum_{m=1}^{M} \sum_{k=1}^{K} w_{m,k} \left(\log(1 + \theta_{m,k}) - \theta_{m,k}\right) \tag{17}$$

$$+ \frac{(1 + \theta_{m,k}) ||v_{m,k}^lf_{m,k}B_m||^2 ||P_{m,k}||}{\sum_{i=1}^{K} ||v_{m,k}^lf_{m,k}B_m||^2 P_{m,i} + \sigma_{m,k}} \tag{17}$$

We can derive the optimal value of $\theta_{m,k}$ by setting $\frac{\partial U^L_o(P, \theta)}{\partial \theta_{m,k}} = 0$, i.e.

$$\frac{w_{m,k}}{1 + \theta_{m,k}} - w_{m,k} + \frac{w_{m,k} ||v_{m,k}^lf_{m,k}B_m||^2 ||P_{m,k}||}{\sum_{i=1}^{K} ||v_{m,k}^lf_{m,k}B_m||^2 P_{m,i} + \sigma_{m,k}} = 0, \quad \tag{18}$$

$$\Rightarrow \tilde{\theta}_{m,k} = \frac{||v_{m,k}^lf_{m,k}B_m||^2 ||P_{m,k}||}{\sum_{i=1}^{K} ||v_{m,k}^lf_{m,k}B_m||^2 P_{m,i} + \sigma_{m,k}} \quad \tag{18}$$

In (18), we know that $\tilde{\theta}_{m,k}$ is the extreme point of $t^L_{o}(P, \theta)$. To prove that this extreme point is the maximum point, we check the sign of the coefficient of the second derivative. We find that

$$\frac{\partial^2 U^L_o(P, \theta)}{\partial \theta_{m,k}^2} = \frac{-1}{(1 + \tilde{\theta}_{m,k})^2} < 0, \quad \tag{19}$$

is always satisfied. Thus, the optimal value of $\theta_{m,k}$ can be given by

$$\theta^*_m = \tilde{\theta}_{m,k} = \frac{||v_{m,k}^lf_{m,k}B_m||^2 ||P_{m,k}||}{\sum_{i=1}^{K} ||v_{m,k}^lf_{m,k}B_m||^2 P_{m,i} + \sigma_{m,k}} \quad \tag{20}$$

Obviously, the third part of $U^L_o(P, \theta)$ is the multiple-ratio problem, where

$$A_{m,k}(P) = w_{m,k}(1 + \theta_{m,k}) ||v_{m,k}^lf_{m,k}B_m||^2 P_{m,k}, \quad \tag{21}$$

$$B_{m,k}(P) = \sum_{i=1}^{K} ||v_{m,k}^lf_{m,k}B_m||^2 P_{m,i} + \sigma_{m,k}. \quad \tag{21}$$
Meanwhile, from (21), we have that \( A_{m,k}(P) \geq 0 \) and \( B_{m,k}(P) > 0 \).

**Lemma 2.** Given a non-negative function \( F_m(x) = \frac{A_m(x)}{B_m(x)} \), \( x \in \mathcal{X} \), where all other things are equal to that in Lemma 1. The multiple-ratio problem [35],

\[
\max_{x} \sum_{i=1}^{N} F_m(x), \quad s.t. \ x \in \mathcal{X},
\]

is equivalent to

\[
\max_{x,Z} \sum_{i=1}^{N} f_m(x,Z), \quad s.t. \ x \in \mathcal{X},
\]

where

\[
f_m(x,Z) = \left( 2Z_m\sqrt{A_m(x)} - Z_m^2B_m(x) \right),
\]

and \( Z_m \) is an auxiliary variable to reformulate the form of sum-of-ratios.

**Proof.** See Appendix 9.2.

According to Lemma 2, the objective function \( U_{\theta,x,z}^L(P,\theta,Z) \) can be convert into \( U_{\theta,x,z}^L(P,\theta,Z) \) as shown at the top of next page.

\[
U_{\theta,x,z}^L(P,\theta,Z) = \sum_{i=1}^{M} \sum_{j=1}^{K} \frac{\sum_{k=1}^{Z} \left( \sum_{i=1}^{K} \left( \frac{w_{m,k}|1+\theta_{m,k}|(H_{m,k}B_{m})|^2P_{m,k}}{Z_m} + \sigma_{m,k}^2 \right) \right) + \text{Const}}{Z_m^2}.
\]

The dual function \( g(\lambda) \) is the maximum value of the Lagrangian over \( P \), i.e.

\[
g(\lambda) = \min_{\lambda \geq 0} \max_{P} L_r(P,\lambda).
\]

Since the dual function is the point-wise infimum of a family of affine functions of \( \lambda \), it is convex. Then the expression of optimal \( P_{m,k} \) is given by

\[
P_{m,k} = \frac{Z_m^2w_{m,k}|1+\theta_{m,k}|(H_{m,k}B_{m})|^2P_{m,k}}{\lambda + \left( \sum_{m,k} \frac{Z_m^2w_{m,k}|1+\theta_{m,k}|(H_{m,k}B_{m})|^2P_{m,k}}{\lambda} \right)^2},
\]

and the optimal value of \( \lambda \) can be updated by the sub-gradient method [36].

### 3.2 | Iterative algorithm for long-term fairness power allocation

We then propose an iterative algorithm to update expressions of the auxiliary variables, power allocation and weight factors, ensuring a long-term fairness of the MIMO-NOMA system. The update steps are summarized in Algorithm 1. We introduce an iteration index \( \iota \), where \( \theta^{(0)} \), \( Z^{(0)} \), and \( P^{(0)} \) represent the auxiliary variables and power allocation in the 0th iteration.
According to Algorithm 1, \( \Theta^{(i)} \) is updated by \( P^{(i)} \), \( Z^{(i)} \) is updated by \( \Theta^{(i-1)} \) and \( P^{(i-1)} \), and \( P^{(i)} \) is updated by \( Z^{(i)} \) and \( \Theta^{(i-1)} \) in sequence.

**Remark 1.** Algorithm 1 can guarantee to converge, and the objective function \( U_L(P) \) is monotonically non-decreasing after each iteration.

**Proof.** From Lemma 1 and Lemma 2, we have \( U_L(P) \geq U_L(P, \Theta) \), and the equivalence is established if only if \( \Theta \) satisfies the equation (20). Meanwhile, \( U_L(P, \Theta) \geq U_L(P, \Theta, Z) \), and the equivalence is established if only if \( Z \) satisfies Equation (26). We have

\[
U_L(P^{(i)}) = U_L(P^{(i)}, \Theta^{(i)}) \\
\geq U_L(P^{(i)}, \Theta^{(i-1)}) \\
\geq U_L(P^{(i)}, \Theta^{(i-1)}, Z^{(i)}) \\
\geq U_L(P^{(i-1)}, \Theta^{(i-1)}, Z^{(i)}) \\
\geq U_L(P^{(i-1)}, \Theta^{(i-1)}) \\
= U_L(P^{(i-1)})
\]

where Equation \( a \) and Inequality \( b \) are satisfied since \( \Theta^{(i)} \) is obtained by (20) for a given \( P^{(i)} \) in the \( i \)-th iteration; Inequality \( c \) is satisfied since only when \( Z \) is obtained in (26) with given \( P^{(i)} \), we have \( U_L(P^{(i)}, \Theta) = U_L(P^{(i)}, \Theta, Z) \), but \( Z^{(i)} \) is computed based on \( P^{(i-1)} \); In Equation \( d \) is determined by the optimal update of \( P \) in (30) for given \( \Theta^{(i-1)} \) and \( Z^{(i)} \); Equations \( e \) and \( f \) can be determined easily. Therefore the objective function \( U_L(P) \) is monotonically non-decreasing over iteration index. Since the value of \( U_L(P) \) is up bounded, Algorithm 1 can ensure convergence.

We then evaluate the computational complexity of Algorithm 1 at time \( t \). In each iteration, the computational complexity is \( \mathcal{O}(NK^2L_d) \), where \( L_d \) is iteration number of bisection method or subgradient method for updating dual variable \( \lambda \). For example, when the bisection method is adopted, it generally achieves a logarithmic convergence rate with complexity \( \mathcal{O}(\log_2(\lambda_{d-1}/\lambda_{d})) \), where \( \lambda_d \) and \( \lambda_{d-1} \) are the initial values and \( \varepsilon \) is the required accuracy. When the subgradient method is adopted, the iteration for updating \( \lambda \) converges to the optimal solution in polynomial time. Therefore, the computational complexity of Algorithm 1 is \( \mathcal{O}(NK^2L_dL_{out}) \), where \( L_{out} \) is the number of the outer iteration (less than 10 shown in Figure 1).

**4 | INSTANTANEOUS FAIRNESS POWER ALLOCATION**

**4.1 | Optimization problem of maximizing minimum user-rate**

In this section, we introduce another fairness power allocation in the MIMO-NOMA system, i.e., ensuring the instantaneous fairness of users. We formulate the fairness utility function as the minimum achievable rate of users as

\[
U_L(P) = \min_{m,k} \{ R_{m,k}, \forall m, k \}.
\]

Then, the optimization problem (9) is reformulated as

\[
\max_{P} \min_{m,k} \{ R_{m,k}, \forall m, k \} \quad \text{s.t.} \quad C1, C2.
\]

Obviously, this problem is also hard to solve directly. We introduce an auxiliary variable \( \tilde{R} \) which represents the minimum achievable data-rate among the \( MK \) users in the cell. Based on it, the optimize problem (33) can be rewritten by an equivalent epigraph form [56] as

\[
\max_{P, \tilde{R}} \tilde{R} \quad \text{s.t.} \quad C1, C2, \quad C4 : R_{m,k} \geq \tilde{R}, k \in K_{m}, m \in M.
\]

To solve it, we introduce \( y = 2^\tilde{R} - 1 \) for transforming the optimization variable. The optimal power allocation is given by the following theorem.
Theorem 1. When the minimum achievable rate $R_{m,k}$ is equal to $\bar{R}$, the optimal power allocation of the users in cluster $m$ is expressed as

$$P_{m,1} = \gamma \left( \frac{\|v_{m,1}^H n_{m,1}\|^2_2}{\|v_{m,1}^H H_{m,1} B_m\|^2_2} \right),$$

$$P_{m,2} = \gamma \left( P_{m,1} + \frac{\|v_{m,2}^H n_{m,2}\|^2_2}{\|v_{m,2}^H H_{m,2} B_m\|^2_2} \right),$$

$$\vdots$$

$$P_{m,K} = \gamma \left( \sum_{k=1}^{K-1} P_{m,k} + \frac{\|v_{m,K}^H n_{m,K}\|^2_2}{\|v_{m,K}^H H_{m,K} B_m\|^2_2} \right).$$

Proof. We first prove the existence of (35) and then prove the expression of the power allocation is optimal. Since the decoding order in each cluster follows the increasing order of channel gains, i.e. the first user suffers no inter-cluster interference, and the second user experiences the interference caused by the first user. Following this principle, we can obtain the power allocation with the expression in (35) if and only if $R_{m,k} = \bar{R}$ is satisfied.

Then, we prove the expression (35) is optimal. Assume $\bar{R}^*$ is the optimal minimum rate, and the optimal $\gamma^*$ can be expressed as $\gamma^* = 2^{\bar{R}^*} - 1$. The optimal power allocation is defined as $P^o_{m,k}$. Then assume we can obtain the another optimal solution of problem (34), which is given by

$$P^o_{m,k} = \gamma^* \left( \sum_{k=1}^{K} P^o_{m,k} + \frac{\|v_{m,k}^H n_{m,k}\|^2_2}{\|v_{m,k}^H H_{m,k} B_m\|^2_2} \right).$$

It is obvious that $P^o_{m,k} \geq 0$, thus constraint C2 is satisfied. Then we can prove $P^o_{m,k}$ is the optimal solution if and only if C1 and C4 are satisfied. Since $P^o_{m,k}$ is the optimal solution, we can find $P^o_{m,k} > P^o_{m,k-1}$ when $k = 1$. We assume $P^o_{m,k} > P^o_{m,k-1}, \ldots, P^o_{m,1} > P^o_{m,1}$ are established. So, when $k = n + 1$, according to (36) we have

$$P^o_{m,n+1} = \gamma^* \left( \sum_{k=1}^{n} P^o_{m,k} + \frac{\|v_{m,n+1}^H n_{m,n}\|^2_2}{\|v_{m,n+1}^H H_{m,n+1} B_m\|^2_2} \right)$$

$$\leq \gamma^* \left( \sum_{k=1}^{n} P_{m,k} + \frac{\|v_{m,n+1}^H n_{m,n}\|^2_2}{\|v_{m,n+1}^H H_{m,n+1} B_m\|^2_2} \right)$$

$$= P^o_{m,n+1}.$$

Thus, constraint C2 is satisfied. Substituting $P^o_{m,k}$ to (8), we have

$$R^o_{m,k} = \log \left( 1 + \frac{P^o_{m,k} \gamma}{\sum_{k=1}^{K-1} P^o_{m,k} + \frac{\|v_{m,k}^H n_{m,k}\|^2_2}{\|v_{m,k}^H H_{m,k} B_m\|^2_2}} \right)$$

$$= \log (1 + \gamma^*) = \bar{R}^*.$$
Algorithm 2 Instantaneous fairness power allocation in MIMO-NOMA

Initialization:
1. Initialize channel matrix $H_{m,k}, \forall m \in \mathcal{M}, \forall k \in \mathcal{K}_m$;
2. Initialize total transmit power $P_{\text{max}}$, search accuracy $\varepsilon$ and iteration index $n$;

Iterative Algorithm
3. Calculate zero-forcing vector $v$;
4. Calculate $\gamma^{(n)}$ by (45), (46) and set $\gamma_d^{(n)} = 0$;
5. while $|\gamma^{(n)} - \gamma^{(n)}_d| > \varepsilon$ do
6. Update $\gamma$ by $\gamma^{(n+1)} = (\gamma^{(n)} + \gamma^{(n)}_d)/2$;
7. Calculate the value of $P_{\text{sum}}(\gamma)$ by (39);
8. if $P_{\text{sum}}(\gamma) > P_{\text{max}}$ set $\gamma_{d}^{(n+1)} = \gamma^{(n+1)}$ and $\gamma_d^{(n+1)} = \gamma^{(n)}$;
9. else set $\gamma_{d}^{(n+1)} = \gamma^{(n+1)}$ and $\gamma_d^{(n+1)} = \gamma^{(n)}$;
10. end if
11. end while
12. end if
13. Return the optimal $\gamma$;
14. Calculate the optimal value of $P_{\text{max}}$ by (35).

\[ Q_m = \sum_{k=1}^{N_m} \frac{\gamma(1 + \gamma)^{n+1-k} \sigma^2_{m,k}}{G_{m,k}} + \gamma \frac{\sigma^2_{m,k}}{G_{m,k}} \]
\[ = \sum_{k=1}^{n+1} \frac{\gamma(1 + \gamma)^{n+1-k} \sigma^2_{m,k}}{G_{m,k}} \]
\[ = \sum_{k=1}^{n+1} \frac{\gamma(1 + \gamma)^{n+1-k} ||v_n^{H} H_{m,k} B_{m}||^2_2}{||v_n^{H} n_{m,k}||^2_2} \]  \hspace{1cm} (44)

Thus, the expression of $Q_m$ in (41) is true for any $m$. \hfill \Box

4.2 Iterative algorithm for instantaneous fairness power allocation

Problem (34) is converted to finding the optimal $\gamma$ of problem (40). Obviously, $\gamma$ is a single variable, and the constraint of problem (40) is a monotonically increasing function of $\gamma$ from (39) and (41). We can adopt bisection method to find the optimal value, summarized in Algorithm 2.

The optimal value of $\gamma$ can be found in the range of $[\gamma_d, \gamma_u]$, which satisfies the maximum power constraint. We set $\gamma_d = 0$, which means there is at least one user who does not interfere by others in theory. The maximum value of $\gamma$ can be obtained if all power is allocated to the user who has the strongest channel gain in each cluster. Then, $\gamma_u$ can be obtained by

\[ \gamma_u = \max \{\gamma_1, \gamma_2, \ldots, \gamma_M\} \]  \hspace{1cm} (45)

where the maximum $\gamma$ of each cluster can be given by

\[ \gamma_w = \frac{||v_n^{H} H_{m,k} B_{m}||^2_2}{||v_n^{H} n_{m,k}||^2_2}, \forall m \in \mathcal{M} \]  \hspace{1cm} (46)

Remark 2. The long-term fairness power allocation in Section 3 can achieve a trade-off between the data-rate of NOMA system and the fairness of users by adjusting $\tau_{m,k}$ in (11), which is equivalent to a long-term proportional fairness. Meanwhile, the instantaneous fairness power allocation can obtain higher fairness among the NOMA users.

Finally, we evaluate the computational complexity of Algorithm 2. The bisection method for searching for $\gamma$ achieves a logarithmic convergence rate with complexity $O(\log_2(\frac{\gamma_u-\gamma_d}{\varepsilon}))$. Therefore, the total computational complexity of Algorithm 2 is $O(KM \log_2(\frac{\gamma_u-\gamma_d}{\varepsilon}))$.

5 NUMERICAL SIMULATIONS

In this section, we evaluate the performance of our proposed long-term fairness power allocation algorithm (MIMO-NOMA-Pro-LF) and instantaneous fairness power allocation algorithm (MIMO-NOMA-Pro-IF). In the simulations, we consider a scenario that the BS is located at the centre of the cell, and users are assumed to be distributed with distance differences in the cell. For easy description, user $k$ in cluster $m$ is denoted as $UE_{m,k}$. Assume the channels compose of large-scale fading and Rayleigh fading, i.e. $H_{m,k} = \sqrt{\tau_{m,k}} h_{m,k}, \forall m, k$. The large-scale fading $L_{m,k}$ is modelled as $L_{m,k} = 114 + 38 \log(d)$, where $d$ is the distance between the BS and $UE_{m,k}$. For simplicity, we normalize the bandwidth of all users, and set the noise power as $\sigma^2 = -174$dBm.

5.1 Convergence performance

We investigate the convergence performance of the proposed power allocation algorithms. Without loss of generality, we consider 3 users in each cluster, denoted as $UE_{m,1}, UE_{m,2}$ and $UE_{m,3}, \forall m$. The distances between the BS and three users are ordered as $d_{m,1} < d_{m,2} < d_{m,3}$. According to the principle of NOMA, the users in each cluster should have the difference of channel gains to ensure that the receiver can perform a perfect SIC. We assume that the distances satisfy that $0.1km < d_{m,1} < 0.4km$, $0.4km < d_{m,2} < 0.7km$, and $0.7km < d_{m,3} < 1km$. We set the numbers of antennas equipped at BS and users as $N_t = 3$ and $N_r = 2$ respectively.

Figure 1 shows the weighted sum-rate versus iteration number with different transmit power at BS for the MIMO-NOMA-Pro-LF algorithm. In this figure, to verify the convergency of this algorithm, we consider different cases of the maximum transmit power of BS, i.e. 5, 10, 15W respectively. It can be observed that this algorithm converges within ten times iterations in each case. Meanwhile, when the transmit power of the BS becomes larger, the weighted sum-rate of the system increases.

Figure 2 depicts the maximized minimum achievable SINR of the users versus iteration index for the MIMO-NOMA-Pro-IF algorithm. We present the results of SINR performance since
the minimum data-rate is positively correlated with the minimum SINR. Here, we consider two transmit power parameters of BS as 5 and 15W respectively. From Figure 2 we find that the MIMO-NOMA-Pro-IF algorithm reaches to convergence within 12 iterations, which means it also has good convergence. In addition, we find that the minimum SINR of the user will become larger, as the transmit power of the BS becomes larger.

Figure 3 show the dual variable $\lambda$ versus iteration index for the MIMO-NOMA-Pro-LF algorithm with two transmit power parameters of BS as 5W and 15W respectively. From Figure 3 we find that the dual variable in MIMO-NOMA-Pro-LF algorithm reaches to convergence within 10 iterations.

5.2 | Sum-rate and fairness performance of power allocation algorithms

In this section, we compare the fairness of the proposed MIMO-NOMA-Pro-LF and MIMO-NOMA-Pro-IF algorithms with different numbers of the antennas. We also compare our proposed algorithms with the following schemes:

- MIMO-NOMA-Fixed: No fairness is considered in this power allocation scheme. The optimization objective is maximizing sum-rate of the MIMO-NOMA system (i.e. the weight factors of the NOMA users in (13) are fixed and the same). Note that, the optimal power allocation is solved by a similar transformation to the MIMO-NOMA-Pro-LF scheme.
- MIMO-OMA-ERA: The OMA method is applied in multi-user MIMO system as a comparison. Compared to MIMO-NOMA scheme where the users in each cluster are served in same frequency, all users are served through $MK$ different fractional frequencies in MIMO-OMA scheme. In this scheme, we also consider the fairness of users, thus we guarantee that all users are served with the same data-rate by the BS.
- MIMO-NOMA-EPA: Equal power allocation is adopted for all NOMA users, i.e. the power allocated to each user is $P_{m,k} = P_{\text{max}} / MK, \forall m, k$.
- MIMO-NOMA-SPA: Scaling factor based power allocation is designed for the MIMO-NOMA system based on [28]. The power is allocated to user $k$ in cluster $m$ as $P_{m,k} = \rho_k^{1-\frac{1}{M}}P_{\text{max}}/\left(\sum_{k'=0}^{K-1}\rho_k^{1-\frac{1}{M}}\right)$, which is a boundary case of the optimal solution to the sum-rate maximization problem for a given $\rho (\geq 1)$ in [28]. $\rho$ is the scaling factor that can guarantee SIC and user fairness. For the larger $\rho$, more power is allocated to the latter users.

In Figure 4, we demonstrate sum-rate performance achieved by the proposed MIMO-NOMA-Pro-LF and MIMO-NOMA-Pro-IF algorithms. It can be seen from Figure 3 the MIMO-NOMA-Fixed power allocation method has largest sum-rate, although it ignores the user fairness. In this scheme, the BS tends to allocate all power to the users with large channel gains to achieve higher sum-rate. The users with poor channel conditions will be hardly allocated the power. In addition, the proposed two fairness methods are superior to the MIMO-NOMA-EPA scheme and the MIMO-OMA-ERA scheme. This is because the MIMO-NOMA-EPA scheme allocates equal power to each user without considering the difference of channels between users, which does not rationally utilize the channel state information. The MIMO-OMA-ERA scheme reduces the available bandwidth of each user and greatly reduces the data-rate, although it eliminate user interference completely.

We can see that the proposed long-term fairness power allocation algorithm has a better performance than the instantaneous fairness power allocation algorithm in terms of sum rate. It is reasonable, because the MIMO-NOMA-Pro-LF scheme takes into account both sum-rate and user fairness, but the MIMO-NOMA-Pro-IF
scheme focuses on assigning more power to the user who has poor channel gain to ensure fairness and limits the overall data rate of the NOMA system.

Figure 5 illustrates the average data-rate of cell-edge users for different schemes versus maximum transmit power at BS. In this figure, the MIMO-NOMA-Pro-LF and MIMO-NOMA-Pro-IF schemes are superior to all the comparative schemes, which means the proposed schemes can guarantee the fairness of edge users. Particularly, the MIMO-NOMA-Pro-IF scheme has a better cell-edge performance than the MIMO-NOMA-Pro-LF scheme. In addition, the MIMO-NOMA-Fixed scheme has worst edge-user rate performance, because almost no power is allocated to the cell-edge users with poor channel conditions, which leads to its low data rate and fairness of cell-edge users. Moreover, we find that the MIMO-NOMA-EPA method is worse than that of the MIMO-OMA-ERA method. It is because that the MIMO-OMA-ERA method provides equal rate for OMA users (i.e. more power is allocated to the cell-edge user), but the MIMO-NOMA-EPA method just allocates equal power to each NOMA user and the users suffer the interference between each other in the same cluster although they have larger effective bandwidth. The MIMO-NOMA-SPA method can achieve a better cell-edge rate than the other traditional methods because we choose an appropriate scaling factor ($\rho = 1.4$) of power to obtain a trade-off between the sum-rate and the cell-edge rate [28, 29].

To further investigate the fairness of the two proposed power allocation algorithms, we introduce the fairness index as

$$\kappa = \frac{\left(\sum_{m=1}^{M} \sum_{k=1}^{K} R_{m,k}\right)^2}{MK \sum_{m=1}^{M} \sum_{k=1}^{K} (R_{m,k})^2}, \quad (47)$$

where $\kappa \in [0, 1]$ and a larger $\kappa$ indicates higher fairness among the users. The fairness indexes of the proposed two schemes for different transmit power levels are listed in Table 1. From the results in this table, we find that the MIMO-NOMA-Pro-IF scheme always has a higher $\kappa$ than the MIMO-NOMA-Pro-LF scheme for different simulation parameters, which shows that the MIMO-NOMA-Pro-IF scheme has a better performance in guaranteeing the fairness of users.

Table 1: Fairness index $\kappa$

| Simulation parameters | $P_{\text{max}}$ | MIMO-NOMA-Pro-LF | MIMO-NOMA-Pro-IF |
|-----------------------|------------------|------------------|------------------|
|                       |                  |                  |                  |
| $N_t = 3$             | $10W$            | 0.5840           | 1                |
| $N_r = 2$             | $20W$            | 0.6462           | 1                |
| $M = 2$               | $30W$            | 0.6712           | 1                |
| $K = 3$               |                  | 0.6858           | 1                |
|                       |                  | 0.6271           |                  |
| $N_t = 6$             | $40W$            | 0.7363           | 1                |
| $N_r = 6$             | $50W$            | 0.7837           | 1                |
| $M = 2$               | $60W$            | 0.8105           | 1                |

Figure 6 provides the results of power allocation to all users in the clusters for different schemes. As mentioned before, $UE_{1,1}$ and $UE_{2,1}$ are the users with strongest channel gains in cluster 1 and cluster 2, and $UE_{1,3}$ and $UE_{2,3}$ are the weakest users. From Figure 6(a) we can see that the power is almost only allocated to $UE_{1,1}$ and $UE_{2,1}$ in the MIMO-NOMA-Fixed scheme. In Figure 6(c), the MIMO-NOMA-EPA allocate the same power to all users. Particularly, we can observe that the proposed MIMO-NOMA-Pro-LF and MIMO-NOMA-Pro-IF schemes take into account both the channel conditions and the fairness of users, which allocates power more reasonably. The proposed
MIMO-NOMA-Pro-IF scheme (i.e. Figure 6(d)) strictly follows the principle of NOMA to allocate more power to the weak users, since it requests instantaneous fairness at each time slot. In addition, the MIMO-NOMA-Pro-LF algorithm (i.e. Figure 6(b)) can also basically guarantees fairness over a long time scale.

Figure 7 depicts the sum-rate versus maximum transmit power of BS. In this figure, the numbers of transmitting antennas and receiving antennas are set as $6 \times 6$. Compared with the results in Figure 4, we find that the sum-rate of both the proposed MIMO-NOMA-Pro-LF and MIMO-NOMA-Pro-IF schemes with $6 \times 6$ antennas are improved by almost 1.5 bps/Hz. Moreover, the sum-rates of all comparable schemes such as MIMO-NOMA-Fixed and MIMO-NOMA-EPA are also improved with the larger number of antennas. Obviously, the sum-rate performance gains come from the space diversity of the larger multi-antenna array.

Figure 8 displays the cell-edge users’ rate versus maximum transmit power with $6 \times 6$ antennas equipped at the transmitter and the receivers. We evaluate the influence of antennas on cell-edge users and fairness compared with Figure 5. We find that the cell-edge users’ rate of all schemes are improved with the increase of the number of transmitting and receiving antennas. Particularly, the gap of cell-edge user’s rate between the MIMO-NOMA-Pro-LF and MIMO-NOMA-Pro-IF schemes is decreased, which indicates that the fairness (cell-edge users’ rate) of the MIMO-NOMA-Pro-LF scheme can be improved as the number of antennas increases. A similar conclusion can be drawn by the results in Table 1. However, in MIMO-NOMA-Fixed, the cell-edge rate still is zero, since there is no any fairness mechanism considered in this method.
Figure 8: Edge user rate versus maximum transmit power $P_{\text{max}}$ ($N_t = 6, N_r = 6, M = 2, K = 3$)

Figure 9 provides the results of power allocation when the transmitting and receiving antennas are set as $6 \times 6$. Figure 9(a), (b) show the power allocations in the MIMO-NOMA-Pro-LF scheme and the MIMO-NOMA-Pro-IF scheme. We find that both the two proposed algorithms can ensure sufficient power being allocated to the cell-edge users and guarantee their demands of data-rate and the fairness.

Simplified case comparison: To further illustrate the performance of our algorithm, we compare it with the fairness-aware solution in [30]. However, the power allocation expression in [30] only applies to the single-cluster and two-user NOMA scenario, and it cannot be extended to more users and more clusters. For comparison, we simply consider a single-cluster two-user NOMA system ($M = 1$ and $K = 2$). To be fair, we also extend the single-antenna assumption in [30] to the multi-antenna ($N_t = 2$ and $N_r = 3$) where the precoding vector $\mathbf{B}$ and receiving matrix $\mathbf{V}$ are the same as those in our algorithm. Other parameter settings are the same as those in [30]. Figure 10 provides the sum-rate and edge-user rate performance of the two algorithms. The results show that our proposed algorithm can obtain higher sum-rate and edge-user rate.

5.3 User mobility

From the above analysis of convergence and complexity, we observe that our algorithms can be done within a normal transmission time interval (TTI) (e.g. the TTI of LTE system is in milliseconds) based on the computing capacity of general BSs. To further reduce the computational complexity, we then carry out the power allocation algorithm once during multiple time intervals. In other words, the power allocation remains unchanged in multiple TTIs.

Figure 9: Power allocation for users at each cluster ($N_t = 6, N_r = 6, M = 2, K = 3$)

Figure 10: Sum-rate and edge-user rate versus maximum transmit power $P_{\text{max}}$ ($N_t = 2, N_r = 3, M = 1, K = 2$)
TABLE 2 Performance loss of sum-rate

| $t$ | $\alpha = 0.001$ | $\alpha = 0.01$ | $\alpha = 0.1$ |
|-----|-----------------|-----------------|-------------|
| 1   | 7.61%           | 9.06%           | 9.82%       |
| 2   | 7.64%           | 9.08%           | 9.74%       |
| 3   | 7.70%           | 9.04%           | 9.69%       |
| 4   | 7.72%           | 9.07%           | 9.88%       |
| 5   | 7.78%           | 9.06%           | 9.82%       |

Finally, we test this algorithm performance and the influence of user mobility to the performance. To describe the user mobility, we model time-varying characteristics of the user channels by a first order Gaussian Markov model for the channel coefficients [37]. The small-scale fading coefficient at time $t$ is given by

$$h_{m,k}(t) = \sqrt{1 - \alpha^2} h_{m,k}(t-1) + \alpha \varphi_{m,k}(t),$$

where $\varphi_{m,k}(t) \sim CN(0,1)$ denotes the random noise satisfied $\varphi_{m,k}(t) \sim CN(0,1)$. The time correlation coefficient $\alpha \in [0,1]$ is determined by

$$\alpha = \sqrt{1 - f_d^2(2\pi f_d T_s)},$$

where $f_d(\cdot)$ is the first kind zero-order Bessel function, $T_s$ is the symbol period and $f_d$ is relative Doppler spread. We note that $f_d$ is determined by the relative mobile speed $v$ between the BS and the users. As the users moves faster, $\alpha$ becomes larger.

In Table 2 and Figure 11, we give the performance loss of sum-rate due to the unchanged power allocation. We compute the optimal power allocation $P^*(0)$ at $t = 0$ and then reuse $P^*(0)$ as the power allocation policy from $t = 1$ to $t = 10$. We can see that the performance loss is less than 10% when $\alpha < 0.1$ within the 10 time intervals. Bearing this loss, the power allocation just needs to be computed at the 11th time slots. Note that, as the coefficient $\alpha$ is lager, the performance loss becomes lager.

6 | CONCLUSION

In this paper, we have studied two fairness power allocation schemes in downlink multi-cluster MIMO-NOMA systems for maximizing the fairness utility function. To sufficiently investigate the fairness problem, we consider two fairness utility functions: long-term fairness and instantaneous fairness. We formulate them as the weighted sum-rate maximization problem and minimum achievable rate maximization problem respectively. We solve the optimization problems by adopting the convex transformations and the iterative algorithms to obtain the optimal power allocations. Simulation results show that both of the proposed fairness power allocation algorithms have good convergence and have better cell-edge user rate performance than the traditional schemes. The instantaneous fairness power allocation scheme can obtain a higher fairness than the long-term fairness power allocation scheme.

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APPENDICES A

A.1 Proof of Lemma 1

The Lagrangian dual transformation removes the ratio $F_n(x)$ out of the logarithm, which expresses the objective function as a linear form of the optimization variables. We observe that $f(x, \cdot)$ is a linear and differentiable function over $\hat{\theta}_n$ when the other variables are fixed. Through the derivations, we have

$$\frac{\partial f(x, \cdot)}{\partial \hat{\theta}_n} = \frac{1}{1 + \theta_n} - 1 + \frac{A_n(x)}{A_n(x) + \beta_n(x)}, \quad (A.1)$$

$$\frac{\partial^2 f(x, \cdot)}{\partial \hat{\theta}_n^2} = -\frac{1}{(1 + \theta_n)^2}. \quad (A.2)$$

When setting (A.1) to zero, we can obtain $\hat{\theta}_n = \frac{A_n(x)}{\beta_n(x)}$. Thus, $\hat{\theta}_n$ is the extreme point. Then, we can find that the second order derivative $\frac{\partial^2 f(x, \cdot)}{\partial \hat{\theta}_n^2}$ is always less than zero since $\frac{1}{(1 + \theta_n)^2}$ is strictly positive. Therefore, $\hat{\theta}_n$ is the maximum point. We have that $f(x, \cdot)$ obtains the optimal value if and only if $\hat{\theta}_n$ satisfies $\frac{1}{1 + \theta_n} - 1 + \frac{A_n(x)}{A_n(x) + \beta_n(x)} = 0$. By substituting $\hat{\theta}_n$ to $f(x, \cdot)$, the equivalence of the problems (14) and (15) is easily established.

A.2 Proof of Lemma 2

The optimization problem with multiple-ratio objective function $\sum_{n=1}^{N} f_n(x, Z)$ has the same optimal solution with the original problem with objective function $\sum_{n=1}^{N} F_n(x)$ if it satisfies the following constraints:

- The form of the new objective function $f_n(x, Z)$ is a quadratic function of $Z$.
- The new objective function is a convex function of $Z$ for any fixed $x$, i.e. the inequality $\frac{\partial^2 f_n(x, Z)}{\partial Z^2} \leq 0$ is always true.
- The auxiliary variable $Z$ and variable $x$ maximize the new function $f_n(x, Z)$, meanwhile $x$ maximizes $F_n(x)$. That is to say, if $Z = \arg \max_x f_n(x, Z)$, then $x = \arg \max_z F_n(x)$.

The first constraint makes sure that the new objective function is a quadratic function of the auxiliary variables. The second constraint makes sure the transformed function is a convex function. The third constraint ensures the new objective function $f_n(x, Z)$ is equivalent to the original function $F_n(x)$.
and these two functions have the same optimal solution. We observe that the new objective function $f_z(x, Z)$ is a quadratic function of $Z_n$ when the values of other variable are fixed, thus the first constraint is satisfied. Then, we have

$$\frac{\partial f_z(x, z)}{\partial Z_n} = 2\sqrt{A_n(x)} - 2Z_nB_n(x),$$  \hspace{2cm} (A.3)

$$\frac{\partial^2 f_z(x, z)}{\partial Z_n^2} = -2Z_nB_n(x) < 0.$$  \hspace{2cm} (A.4)

We find that the second derivative $-2Z_nB_n(x)$ is strictly negative since the denominator $B_n(x)$ is strictly positive, thus the second constraint is satisfied. According to (A.3), we can obtain the extreme point $Z_n = \frac{\sqrt{A_n(x)}}{B_n(x)}$ by setting $\frac{\partial f_z(x, z)}{\partial Z_n} = 0$. It is known that the maximum point of objective function $f_z(x, Z)$ is either located in the boundary or an extreme point. Since the coefficient of the quadratic term is negative, so the function is a convex function about $Z$. Thus the extreme point is the maximum point. Substituting $Z_n$ to $f_z(x, Z)$, the equivalence of (22) and (23) is obtained.