THE PROPERTIES OF LONG-PERIOD VARIABLES IN THE LARGE MAGELLANIC CLOUD FROM MACHO

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ABSTRACT

We present a new analysis of the long-period variables in the Large Magellanic Cloud (LMC) from the MACHO Variable Star Catalog. Three-quarters of our sample of evolved, variable stars have periodic light curves. We characterize the stars in our sample using the multiple periods found in their frequency spectra. Additionally, we use single-epoch Two Micron All Sky Survey measurements to construct the average infrared light curves for different groups of these stars. Comparison with evolutionary models shows that stars on the red giant branch (RGB) or the early asymptotic giant branch (AGB) often show non-periodic variability, but begin to pulsate with periods on the two shortest period–luminosity (P–L) sequences (3 & 4) when they brighten to \( K_s \approx 13 \). The stars on the thermally pulsing AGB are more likely to pulsate with longer periods that lie on the next two P–L sequences (1 & 2), including the sequence associated with the Miras in the LMC. The Petersen diagram and its variants show that multi-periodic stars on each pair of these sequences (3 & 4, and 1 & 2) typically pulsate with periods associated only with that pair. The periods in these multi-periodic stars become longer and stronger as the star evolves. We further constrain the mechanism behind the long secondary periods (LSPs) seen in half of our sample, and find that there is a close match between the luminosity functions of the LSP stars and all of the stars in our sample, and that these star’s pulsation amplitudes are relatively wavelength independent. Although this is characteristic of stellar multiplicity, the large number of these variables is problematic for that explanation.

Key words: galaxies: individual (LMC) – stars: AGB and post-AGB – stars: variables: other

Online-only material: color figures, machine-readable and VO tables

1. INTRODUCTION

Stellar pulsation of giant stars appears to be a ubiquitous and important phenomenon—RR Lyrae and Cepheid variables form the basis for the distance scales we use. Miras and other long-period variables (LPVs), however, are not as well understood, largely because their cool and tenuous atmospheres are dynamic environments with a great diversity of molecular species forming and disassociating as the star pulsates. In recent years, however, these stars have attracted increased attention as micro-lensing surveys of the Large Magellanic Cloud (LMC) and the Small Magellanic Cloud (SMC) (OGLE, Paczynski et al. 1994; OGLE II, Udalski et al. 1997; MACHO, Alcock et al. 1997) have produced large catalogs of LPVs. Well-sampled light curves and excellent photometry give us an opportunity to better understand both the mechanisms behind LPVs and the physical processes at work in the latest stages of stellar evolution.

Before the wealth of data from micro-lensing surveys, LPVs were traditionally classified by the amplitude and stability of their variability in the \( V \) band (e.g., The General Catalog of Variable Stars; Khlopopov et al. 1996). In this scheme, stars with well-defined pulsation are classified as Miras if the amplitude of their variability is greater than 2.5 mag in \( V \), and as semi-regular type a (SRA) stars if not. Those with multiple periods, or unstable periodicity, or poorly resolved periodicity, are classified as SRb stars. The MACHO survey of the LMC revealed five parallel sequences of LPVs in period–luminosity (P–L) space (Cooke et al. 1996), prompting a classification scheme that uses the period of pulsation as the primary discriminator. Wood et al. (1999) identified the cause of the first three P–L sequences (denoted A, B, and C) as pulsation, and suggested that the two longest period sequences (E and D) were due to binary systems. The stars in sequence E showed the characteristic light curves of contact binary systems, and sequence D stars—those with the longest periods—simultaneously exhibited at least one shorter period that was coincident with sequence B. This is likely to be the LMC equivalent to the “long secondary periods” described by Houk (1963) for Galactic LPVs, although the periods that comprise sequence D are on average three times shorter than the LSPs listed in Houk. Wood et al. (1999) proposed that these stars are composed of accreting binary systems, with the long period caused by partial eclipses due to an unseen, dust enshrouded companion.

In this work the LMC P–L sequences will be named in the manner of Fraser et al. (2005), from shortest to longest period: 4, 3, 2, 1, E, and D. We retain the names D and E from Wood et al. (1999), but rename his sequence C to sequence 1 and count up toward shorter periods. This approach allows a graceful way to accommodate additional short-period sequences. Indeed, the use of Two Micron All Sky Survey (2MASS) \( K_s \) magnitudes as the luminosity indicator resulted in the splitting of the original sequence B into two (producing sequences 2 and 3; Kiss & Bedding 2003). A fifth sequence was identified by Soszynski et al. (2004a) through the examination of all significant frequencies of these stars instead of just the strongest.

In general, stars brighten and redden as they evolve along the red giant branch (RGB) and the asymptotic giant branch (AGB). Since the typical \( J - K_s \) color of the stars in each sequence redden as we progress from the short period sequence 4 to the longer period sequence 1, this suggests that evolution proceeds from shorter periods toward longer periods, at least for sequences 1–4 (Fraser et al. 2005). In fact, the low luminosity bases of sequences 2, 3, and 4 are heavily populated by RGB stars (Ita et al. 2002, 2004; Kiss & Bedding 2003, 2004). Above the tip of the RGB, models of AGB stars that include the effects of mass loss confirm that stars continue their evolution to higher
Figure 1. The $W$ vs. $V - R$ CMD of the MACHO LMC Variable Star Catalog. $W = R - 4(V - R)$ is the Wesenheit reddening free magnitude (Alcock et al. 1995). The highlighted objects are those identified as LPVs in Paper I (Fraser et al. 2005), which used the SuperSmoother period and $K_s$ to determine variable type. Our sample is drawn from those stars with $V - R \geq 0.5$ and $W \leq 15$. This region surrounds the AGB and encompasses 98% of the LPVs from Paper I; the remaining 2%, which lie bluer than the present sample, are galactic foreground stars with long SuperSmoother periods. SuperSmoother failed to find a period for approximately half of the stars in this sample, while our technique succeeds 87% of the time.

In this work, we expand the stars with periods to 93% of our sample, as well as consider their multi-periodic properties. We also use our results to describe the characteristic variability at each of the stages of RGB and AGB evolution by comparison with population synthesis models, including the stars that show very weak or non-existent periodicity.

2. DATA

The MACHO project (Alcock et al. 1997) comprises eight years worth of observations of the LMC and SMC and the bulge of the Milky Way. Our sample is drawn from the MACHO LMC Variable Star Catalog (Alcock et al. 2003). Sources from the full MACHO database of several million objects were selected for the Variable Star Catalog if the central 80% of points in the object's light curve failed to fit a constant magnitude in a $\chi^2$-squared test. This criterion resulted in 207,632 candidate variables in the LMC.

MACHO data were taken simultaneously in two non-standard filters: red and blue. These can be transformed using the method of Alcock et al. (1999b) to Cousins $V$ and $R$, and then used to find the Wesenheit reddening free magnitude, $W = R - 4(V - R)$ (Alcock et al. 1995). The construction of $W$ allows very dim stars to enter our sample, so we have removed stars beyond MACHO's dim limits. Our LMC sample is defined by $V - R \geq 0.5$ and $W \leq 15$ (see Figure 1) and is composed of 56,453 stars. These luminosity and color limits encompass 98% of the LPVs from Paper I.

MACHO employed a nonparametric phasing technique known as the SuperSmoother Method (Reimann 1994) to phase all of the light curves in the Variable Star Catalog. This technique is robust against complex light curve morphology, but fails...
when presented with strongly multi-periodic behavior. Multi-
periodicity is common among LPVs, and SuperSmother failed to
find a period for nearly half of our sample.

We used the CLEANest algorithm of Foster (1995, 1996a,
1996b) to determine the frequency characteristics of the stars in
our sample. As implemented by Rorabeck (1997) and described
in Alcock et al. (1999a), CLEANest uses the robust date-
compensated discrete Fourier transform (DCDFT) algorithm
of Ferraz-Mello (1981), which finds accurate estimates of the
amplitudes of the Fourier spectrum for data with uneven time
sampling. The CLEANest algorithm iteratively finds the most
significant peak in the power spectrum from the DCDFT, adds
this frequency to those already known, determines the best fit by
allowing all known frequencies to vary slightly, and subtracts the
model light curve from the data. The algorithm exits when there
is no longer any statistically significant power5 in the frequency
spectrum. We verified our implementation by checking our
results against the 41 MACHO Beat Cepheids from Alcock et al.
(1995), and the test dataset from Rorabeck (1997). CLEANest
successfully found every frequency down to our significance
threshold.

In the final analysis of our LMC sample, we searched a
frequency space of 0.0003 day$^{-1}$ to 0.3 day$^{-1}$ (corresponding
to periods from 3.3 days to 3333 days) with a typical frequency
resolution of 0.00003 day$^{-1}$. The average number of frequencies
returned for a blue light curve was 13. Although the MACHO
light curves have a time span necessary to uncover frequencies as
low as 0.0003 day$^{-1}$, the presence of power at these frequencies
is likely caused by slow mean brightness changes. Our analysis
found an excess number of frequencies between 0.0003 day$^{-1}$
and 0.0006 day$^{-1}$ (periods between 1666 days and 3333 days).
We interpret this as due to mean brightness changes over the
time-span of the MACHO observations, and remove these
frequencies from further analysis.

We employ 2MASS (Cutri et al. 2003) $K_s$ measurements as
our luminosity indicator. The use of infrared luminosities splits
Wood’s sequence B into our sequences 2 and 3. We chose to take
all 2MASS matches within 2 arcsec of the MACHO source.6
This is the distance at which there is a 50% chance of a false
match. More than 95% of our sample has a match within this
radius.

Excluding light curves with fewer than 50 points (1% of
our sample), and stars where the CLEANest analysis failed to
converge to reasonable values (8% of our sample), the final
number of stars with good red and blue CLEANest periods is
48,990 (87% of our original sample). A comparison of the
frequencies found in both the red and blue light curves shows
that the error in our period estimates is within one-half of a
percent up to periods of 55 days, after which it grows to follow
the curve corresponding to approximately three times our typical
frequency resolution (or 0.00009 day$^{-1}$).

The SuperSmother and CLEANest analyses of our 56,453
star sample is published in its entirety in the online journal; a
portion is shown here as Table 1. Many of the columns have
already been described in this section. The table’s first column
is the MACHO field, tile, sequence identifier, followed by the
coordinates for that object. The next six columns present the
results of the SuperSmother analysis for both the red and
blue light curves. The primary Fourier period of each star, in
days, is the inverse of the strongest frequency found in the blue
light curve, $P_0 = 1/\chi^2_0$; $P_0$ Amp is the associated peak-to-peak
amplitude. The secondary Fourier period ($P_1$) and amplitude ($P_1$
Amp) are found in a similar manner from the second strongest
frequency found in the Blue light curve. This is followed by
photometric information from MACHO and 2MASS.

The final column is the classification on the Fourier period-luminosity
diagram as described in the next section and Section 3.

Plotting $K_s$ versus $\log_{10} P_0$ (insert of Figure 2) immediately
reveals the familiar period-luminosity sequences of the LPVs in the
LMC. The sequences are named in the manner of Fraser et al.
(2005), from shortest to longest period: 4, 3, 2, 1, E, and D.

2.1. Artifact Removal From the Period–Luminosity Diagram

A very strong vertical feature at periods of one year ($\log_{10} P = 2.56$) overlaps both sequences 1 and D in the Fourier
P–L diagram, shown in the inset of Figure 2. This feature is a
result of the annual observing schedule of the MACHO project
(Alcock et al. 1999b, Section 6.2), and is not an intrinsic prop-
erty of the star (a similar feature that corresponds to one month
is faintly visible in Figure 2 at $\log_{10} P_0 = 1.49$). Since
the strongest period of these stars is not due to the star itself, we
associate these 11,215 stars with the one-year artifact rather than
the sequences that they overlap in P–L space. Stars asso-
ciated with the one-year artifact are, by their inclusion in the
MACHO Variable Star Catalog, variable objects, but they are
not necessarily periodic. They certainly have no higher ampli-
dude periodicity in their frequency spectra than the weak signal
due to the annual schedule of earthbound telescopes.

In Paper I we simply masked all the stars in this region. In
this work we identify just the stars associated with the one-year
artifact by exploiting differences in the properties of the stars
belonging to the artifact with the stars nearby in the Fourier
P–L diagram. We identify stars as associated with the one-year
artifact if they lie in the region $2.53 > \chi^2_0 > 2.60$, and
according to the following rules:

- Sequence 1 is composed of the reddest stars in our sample.
  We identify stars as part of the one-year artifact if their
  luminosity places them in or above sequence 1 and they are
  bluer than $J - K_s = 1.5$.
- At luminosities dimmer than sequence 1 we find that the
  one-year artifact stars have poorly correlated red and blue
  light curves. We identify stars as part of the one-year artifact
  if their red and blue periods differ by more than 20%, or
  if the amplitudes corresponding to those periods differ by
  more than 90%.
- In the luminosity range between sequences 1 and D we find
  that we require the additional parameter of the $\chi^2$ statistic
  for a sine wave corresponding to $P_0$, which is in the range
  $2.53 > \log_{10} P_0 > 2.60$. We select stars with either blue
  amplitudes of less than 0.05 mag mean-to-peak, or those
  with $\chi^2 < 7 \times 10^4$.

Unfortunately the stars in the one-year artifact that are
dimmer than sequence D are difficult to differentiate from the
background, although certainly the great majority of stars in this
region should be identified with the one-year artifact. We chose
to simply identify stars as members of the one-year artifact if
they lie below sequence D and are in the normal period range of
$2.53 > \log_{10} P_0 > 2.60$.

The one-year artifact consists of 11,215 stars with very weak
or non-existent periodicity, or 24% of all the stars with good

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5 We use a limit of 2, in units of the DCDFT’s power, based on the work of
Rorabeck (1997) and the suggestion in Foster (1995) that power levels below 2
are “not even remotely significant.”

6 We used an updated astrometric solution for the LMC based on the UCAC
system (Zacharias et al. 2000).
Table 1
The SuperSmoother and CLEANest analyses of the LPV sample in the LMC

| fts        | α2000 | δ2000 | Period Red | Period Blue | Blue Red | Blue | P0 | P0 amp | P1 | P1 amp | V  | R  | W  | J  | H  | Ks | Type |
|------------|-------|-------|------------|-------------|---------|------|----|--------|----|--------|----|----|----|----|----|----|------|
| 10.3186.18 | 75.04337 | −70.25356 | 47.77 | 47.80 | −9.27 | −8.14 | 0.141 | 0.149 | 47.87 | 0.130 | 45.99 | 0.052 | 15.78 | 14.84 | 11.10 | 12.43 | 11.51 | 11.23 | −99 |
| 10.3186.19 | 75.03504 | −70.25077 | 47.40 | 47.42 | −9.14 | −7.96 | 0.113 | 0.146 | 47.45 | 0.085 | 46.78 | 0.059 | 15.96 | 14.98 | 11.09 | 12.46 | 11.54 | 11.25 | 4   |
| 10.3186.23 | 75.07184 | −70.24387 | 351.60 | 365.64 | −8.46 | −7.60 | 0.023 | 0.051 | 367.11 | 0.044 | 19.47 | 0.023 | 16.37 | 15.61 | 12.58 | 13.44 | 12.68 | 13.43 | 9   |
| 10.3187.11 | 75.04780 | −70.21822 | 526.02 | 523.26 | −9.54 | −7.57 | 0.589 | 1.073 | 518.94 | 0.732 | 262.33 | 0.347 | 16.22 | 14.73 | 8.74  | 11.78 | 10.51 | 9.73  | 1   |
| 10.3187.12 | 75.09056 | −70.21551 | 0.25  | 1.00 | −9.24 | −8.27 | 0.052 | 0.096 | 49.31 | 0.041 | 357.53 | 0.035 | 15.67 | 14.84 | 11.53 | 12.79 | 11.93 | 11.72 | 3   |
| 10.3187.18 | 75.01444 | −70.18326 | 0.51  | 0.50 | −8.34 | −7.42 | 0.037 | 0.059 | 380.37 | 0.063 | 459.14 | 0.039 | 16.53 | 15.74 | 12.56 | 13.75 | 12.88 | 12.64 | 9   |
| 10.3187.19 | 75.07648 | −70.19257 | 301.38 | 307.55 | −8.29 | −7.48 | 0.199 | 0.245 | 308.83 | 0.127 | 174.37 | 0.051 | 16.49 | 15.77 | 12.87 | 13.99 | 13.10 | 12.93 | −2  |
| 10.3187.20 | 75.04959 | −70.17938 | 0.96  | 1.00 | −8.36 | −7.53 | 0.026 | 0.087 | 355.37 | 0.056 | 31.82  | 0.036 | 16.44 | 15.70 | 12.75 | 13.84 | 12.99 | 12.82 | 9   |
| 10.3187.27 | 75.00127 | −70.18986 | 462.95 | 479.13 | −8.11 | −7.09 | 0.049 | 0.207 | 369.82 | 0.157 | 328.73 | 0.122 | 16.85 | 15.98 | 12.53 | 13.78 | 12.82 | 12.58 | −99 |
| 10.3188.12 | 75.02099 | −70.14119 | 0.97  | 35.47 | −8.82 | −7.68 | 0.100 | 0.154 | 35.49 | 0.089 | 369.82 | 0.049 | 16.24 | 15.29 | 11.50 | 12.93 | 12.01 | 11.75 | 4   |

(This table is available in its entirety in machine-readable and Virtual Observatory (VO) forms in the online journal. A portion is shown here for guidance regarding its form and content.)
red and blue CLEANest results and a 2MASS $K_s$ magnitude. This greatly outnumbers other stars in this narrow period range. Although our process likely does not isolate every star that is associated with the one-year artifact, it does serve to uncover what is below this distracting feature. The Fourier P–L diagram of the LMC with the one-year artifact removed is presented in the main panel of Figure 2.

2.2. Finding the Average Infrared Light Curves

Although the amplitude of LPVs in the infrared is much lower than in the visible, there is still intrinsic scatter in the 2MASS observations due to the measurement of each star at a random point in its light curve. Collections of stars with similar light curves can be corrected for this effect in the manner that Nikolaev et al. (2004) used for Cepheid variables. The Fourier P–L sequence in some band is fit with a relation that includes a correction term which is a function of $\phi$, the phase of the 2MASS observation with respect to the MACHO light curve.

$$m_i = \alpha \log_{10} P_i + \beta + \Omega(\phi_i).$$

The correction term, $\Omega(\phi_i)$, is the average infrared light curve for these stars. We chose a form for the correction function based on 2MASS light curves available for 46 stars from our sample. Light curves exist for stars in the 2MASS “calibration tiles”—fields which were observed multiple times each night to provide photometric calibration. Several tiles lie in the vicinity of the Magellanic Clouds, but only tile 90400 provided light curves of sufficient length to investigate the behavior typical of LPVs. An example 2MASS light curve phased to the period of the corresponding MACHO blue light curve is shown in Figure 3. For the 46 stars from our sample that match 2MASS sources from this tile, we found that a second-order Fourier series was an adequate model of the light curves in $J$, $H$, and $K_s$:

$$\Omega(\phi) = \sum_{j=1}^{2} A_j \cos(2\pi j \phi) + B_j \sin(2\pi j \phi).$$

The phase of the 2MASS observation, $\phi$, for each star $i$, is the fractional part of the difference between the time of maximum light, $T_{\text{max},i}$, for the primary Fourier period and $T_{\text{2mass},i}$, the time of the 2MASS observation, in units of the period:

$$\phi_i = \text{mod}\left(\frac{T_{\text{2mass},i} - T_{\text{max},i}}{P_i}\right).$$

The model of the infrared light curve, as based on the 46 2MASS light curves, is fit simultaneously with each P–L relationship in our full sample. The scatter about each of these relationships is not completely accounted for; for example, Ita et al. (2004) found different relations for RGB and AGB stars. For this reason we have fitted each sequence separately above

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7 In classical pulsators this is due to the black-body behavior of the atmosphere’s continuum emission. The large amplitudes of Mira light curves cannot be modeled by this behavior alone; it is also necessary to consider very strong effects from the formation of molecules in the photosphere of the star. See, for example, Reid & Goldston (2002).
Figure 3. Example optical and infrared light curves for the star with MACHO designation 55.3126.13. The infrared light curve is taken from the 2MASS calibration tile 90400, and both are phased to the CLEANest period. The infrared light curve shows a phase lag of approximately 10% as compared to the optical (the vertical dashed line indicates the approximate maximum of the infrared light curve). This star has an optical period of 64.03 days and lies in sequence 3. It is optical light curve is shown with the CLEANest model of the primary period; all other frequencies have been subtracted from the model. The average infrared light curve for sequence 3 was too low in amplitude to measure using our technique, so this star must have unusually strong variation.

(A color version of this figure is available in the online journal.)

3. RESULTS

The Fourier P–L diagram of the LMC with the one-year artifact removed is presented in Figure 2. The sparsely populated fifth sequence found by Soszynski et al. (2004a), which lies just to shorter periods of sequence 4, is not seen in this diagram. This sequence is only seen in the secondary periods ($P_1$), and we have plotted just the primary Fourier period ($P_0$) for each star. All of the stars that compose the fifth sequence have stronger periods elsewhere on the Fourier P–L diagram, primarily on sequence 4 or the one-year artifact.

Stars in this diagram were grouped into sequences based on the contour traced at a density of five stars per 0.05 in $\log_{10} P_0$ and 0.1 in $K_s$ (as shown in the inset of Figure 2). The stars in the high luminosity tip of sequence 1 are carbon stars (Groenewegen 2004), some of which are heavily self-extincted and fall into the gap between sequences 1 and D. Such stars are easy to recognize due to their heavy reddening (Nikolaev & Weinberg 2000), thus stars in the gap between sequences 1 and D with $J - K_s > 1.4$ are assigned to sequence 1.
Table 2
Fit Parameters of the Fourier P-L Sequences

| Sequence | Band | $\alpha$ (mag/log$_{10} P$) | $\beta$ (mag) | Amplitude* (Peak-to-Peak) | Optical-IR Phase Lag |
|----------|------|-----------------------------|--------------|---------------------------|---------------------|
| 4 RGB$^b$ | $W$  | $-3.8371$ | $17.5432$ | $0.08$ | ... |
|          | $J$  | $-2.2496$ | $16.7511$ | ... | ... |
|          | $H$  | $-2.5210$ | $16.2389$ | ... | ... |
|          | $K_s$ | $-2.6689$ | $16.2291$ | ... | ... |
|          |      |              |              |              |                     |
| 4 AGB$^b$ | $W$  | $-4.7744$ | $18.7935$ | $0.07$ | ... |
|          | $J$  | $-3.8972$ | $18.9384$ | ... | ... |
|          | $H$  | $-3.9638$ | $18.1385$ | ... | ... |
|          | $K_s$ | $-4.1457$ | $18.1713$ | ... | ... |
|          |      |              |              |              |                     |
| 3 RGB$^b$ | $W$  | $-3.0237$ | $16.9699$ | $0.08$ | ... |
|          | $J$  | $-2.0084$ | $16.7729$ | ... | ... |
|          | $H$  | $-2.3948$ | $16.4997$ | ... | ... |
|          | $K_s$ | $-2.4976$ | $16.4552$ | ... | ... |
|          |      |              |              |              |                     |
| 3 AGB$^b$ | $W$  | $-5.0891$ | $20.2000$ | $0.17$ | ... |
|          | $J$  | $-3.9127$ | $19.7273$ | ... | ... |
|          | $H$  | $-3.9762$ | $19.8294$ | ... | ... |
|          | $K_s$ | $-4.2055$ | $19.0728$ | ... | ... |
|          |      |              |              |              |                     |
| 2 RGB$^b$ | $W$  | $-2.5347$ | $16.4754$ | $0.17$ | ... |
|          | $J$  | $-1.9506$ | $16.9278$ | ... | ... |
|          | $H$  | $-2.2132$ | $16.5054$ | ... | ... |
|          | $K_s$ | $-2.2066$ | $16.2846$ | ... | ... |
|          |      |              |              |              |                     |
| 2 AGB$^b$ | $W$  | $-4.8399$ | $20.2378$ | $0.41$ | ... |
|          | $J$  | $-2.6364$ | $17.8648$ | $0.07$ | $14\%$ |
|          | $H$  | $-3.0089$ | $17.6513$ | $0.06$ | $12\%$ |
|          | $K_s$ | $-3.6511$ | $18.5875$ | $0.04$ | $14\%$ |

1 oxygen stars$^c$

amp $\leq 0.4$

|          | $W$  | $-3.9231$ | $19.5621$ | $0.22$ | ... |
|          | $J$  | $-2.9129$ | $19.0857$ | $0.09$ | $24\%$ |
|          | $H$  | $-3.0443$ | $18.4948$ | $0.07$ | $23\%$ |
|          | $K_s$ | $-3.2104$ | $18.5805$ | $0.06$ | $22\%$ |
|          |      |              |              |              |                     |
| 0.4 < amp < 1
|          | $W$  | $-5.7398$ | $23.5574$ | $0.64$ | ... |
|          | $J$  | $-3.3339$ | $20.1339$ | $0.04$ | $3\%$ |
|          | $H$  | $-3.4560$ | $19.5341$ | $0.06$ | $1\%$ |
|          | $K_s$ | $-3.6736$ | $19.7285$ | $0.07$ | $2\%$ |
|          |      |              |              |              |                     |
| amp $\geq 1$
|          | $W$  | $-5.1763$ | $22.5559$ | $1.96$ | ... |
|          | $J$  | $-3.2996$ | $20.1896$ | $0.31$ | $11\%$ |
|          | $H$  | $-3.4980$ | $19.7971$ | $0.31$ | $13\%$ |
|          | $K_s$ | $-3.8292$ | $20.2298$ | $0.26$ | $15\%$ |

1 carbon stars$^c$

amp $\leq 0.4$

|          | $W$  | $-3.509$ | $18.1327$ | $0.28$ | ... |
|          | $J$  | $-1.9225$ | $17.1136$ | $0.12$ | $3\%$ |
|          | $H$  | $-2.1524$ | $16.5671$ | $0.09$ | $2\%$ |
|          | $K_s$ | $-2.5586$ | $16.9922$ | $0.08$ | $16\%$ |
|          |      |              |              |              |                     |
| 0.4 < amp < 1
|          | $W$  | $-4.3239$ | $20.4330$ | $0.62$ | ... |
|          | $J$  | $-2.1333$ | $17.7452$ | $0.18$ | $0\%$ |
|          | $H$  | $-2.5860$ | $17.7218$ | $0.13$ | $1\%$ |
|          | $K_s$ | $-3.1615$ | $18.5259$ | $0.08$ | $1\%$ |
|          |      |              |              |              |                     |
| amp $\geq 1$
|          | $W$  | $-3.9039$ | $19.7674$ | $1.80$ | ... |
|          | $J$  | $-0.4514$ | $13.9507$ | $0.58$ | $8\%$ |
|          | $H$  | $-1.5353$ | $15.4351$ | $0.49$ | $12\%$ |
|          | $K_s$ | $-2.8910$ | $17.9953$ | $0.36$ | $13\%$ |

E

|          | $W$  | $-2.9765$ | $19.1364$ | $0.13$ | ... |
|          | $J$  | $-2.4438$ | $19.1903$ | $0.05$ | $11\%$ |
|          | $H$  | $-2.7049$ | $18.9063$ | $0.06$ | $6\%$ |
|          | $K_s$ | $-2.8198$ | $18.9370$ | $0.05$ | $4\%$ |

D amp $\leq 0.2$

|          | $W$  | $-2.9953$ | $19.7903$ | $0.09$ | ... |
|          | $J$  | $-2.4171$ | $19.6843$ | $0.05$ | $-11\%$ |
|          | $H$  | $-2.5615$ | $19.1820$ | $0.05$ | $-11\%$ |
|          | $K_s$ | $-2.7983$ | $19.5482$ | $0.05$ | $-12\%$ |
Sequence E is well known to merge with sequence D when plotted at its true orbital period (Soszynski et al. 2004b; Derekas et al. 2006), but we have chosen to plot sequence E at its Fourier period like the other stars on this diagram. This allows us to differentiate these stars from those at the bottom of sequence D, but it presents us with the problem of separating the two sequences. It is clear that sequence E in Figure 2 is poorly populated, and at the top of the sequence (K_s < 13) it is difficult to tell if sequence E exists among the background of stars between sequences 1 and D. We chose limits between these sequences such that sequence E would remain sparsely populated at its high luminosity end.

Compared with Paper I, where half of that sample had no period assigned by the SuperSmooth analysis, or a period assigned to one year or multiples of one day, we have added approximately 20,000 stars to the P–L diagram. The newly analyzed stars lie mostly on the sequences with the lowest amplitudes (D, 4, and to a lesser extent, 3) and are below the tip of the RGB (K_s = 12.3, Nikolaev & Weinberg 2000). An example is shown in Figure 4 (top panel). Sequence E stars were well represented in Paper I even though their light curves also have low amplitudes. Half as many stars are identified with the one-year artifact due to the careful identification of these stars, as opposed to masking all stars with in this region. Luminosity functions for many of the sequences are shown in Figure 5.

Sequence D as presented in Paper I was under-represented, accounting for only 9% of LPVs. It now represents 31% of the stars with good red and blue CLEANest results and a 2MASS K_s magnitude (Table 3). The right panel of Figure 5 shows the close match between the luminosity functions of
sequence D and our sample, suggesting that stars on sequence D are drawn from the entire population of LPVs in the LMC.

The amplitude of the light curves that are quoted in this work are the peak-to-peak amplitudes found by CLEANest for the $P_0$ term. CLEANest amplitudes underestimate the actual light variation since some of the power associated with this period is contained in harmonics and mixing terms. The amount by which the amplitudes are underestimated is found by comparing the CLEANest amplitudes with amplitudes listed in the MACHO Variable Star Catalog for stars with SuperSmoother periods. SuperSmoother amplitudes were calculated by finding the difference between the mean magnitudes of the closest points to the maximum and minimum of the SuperSmoother phased light curve. On average, the ratio of the SuperSmoother amplitude to the CLEANest amplitude is 1.5 for stars in sequence E and stars in the long-period edge of sequence 1, while the remaining stars on the Fourier P–L diagram tend to show ratios of 1.7. We have not applied a correction factor to our CLEANest amplitudes, and use them only for relative comparisons.

The relationship between the pulsation amplitude and $\log_{10} P_0$ is markedly different for the different sequences. Figure 6 shows that for stars in sequences 1–4 there is a correlation between increasing period, increasing amplitude, and increasing range in amplitudes. As compared to stars on sequences 3 and 4, stars on sequences 1 and 2 pulsate with much higher amplitudes. Sequence E, as expected for binary stars, does not show a strong dependence of amplitude on period. This sequence is a clear continuation of the lower amplitude group.
of sequence D stars, those related to the OSARGs. The moderate amplitude–luminosity correlation for stars in sequence D reported by Derekas et al. (2006) is not seen for the bulk of the small amplitude population of sequence D, using $\log_{10} P_0$ as a proxy for luminosity.

3.1. The Average Infrared Light Curves of LPVs

The comparison of the average infrared light curves with the corresponding optical light curve’s properties is a useful constraint on variability mechanisms in these stars. In many types of pulsating stars—including Miras—pulsation has a stronger effect in the optical than the infrared, while variation due to binary systems show light curves of similar amplitudes in all wavebands. Although the average infrared light curves’ amplitudes are smaller than the specific star’s amplitudes to which they are fit comparisons can be made in a relative sense. While Table 2 presents information for all of the infrared light curves, a summary of amplitude ratios and phase lags for the average blue and $K_s$ light curves is presented in Table 3 sequences for each of the sequences.

We find a clear division in the blue/$K_s$ amplitude ratios between sequences 1 and 2, and sequence E. As expected for binary systems, sequence E has a smaller amplitude ratio and
it has similar amplitudes among the $J$, $H$, and $K_s$ light curves. The pulsating stars in sequences 1 and 2 show higher ratios and decreasing amplitude with redder wavebands. Interestingly, only the low amplitude group of sequence D stars is similar to sequence E. Although the higher amplitude group in sequence D has similar amplitudes among the 2MASS bands, the average blue amplitude is much higher, leading to an amplitude ratio of 8, which is most similar to sequence 1.

Pulsation modes in LPVs can also be constrained using the phase-lag between the optical and the infrared light curves. Smith et al. (2006) searched for such phase-lags using data from the DIRBE instrument on the COBE satellite. In their sample of 21 stars, all of the Miras—including one carbon star—showed phase lags, while four of the five SR variables did not. They compared the stars in their sample to time-resolved dynamical models of oxygen-rich stars from the literature, and found that phase lags are predicted for fundamental-mode oxygen rich stars, but not for stars pulsating in the first overtone mode. The carbon star models available at the time did not consistently predict a phase lag. Our average infrared light curves show phase lags of 10–20% for both Miras and SR variables. The example light curve from the 2MASS calibration tiles, shown in Figure 3, demonstrates this phase lag. The average infrared light curves for the 0.4 $< \text{amplitude} < 1$ bins in sequence 1 are the exceptions to this rule. Neither the oxygen stars nor the carbon stars in this range show the typical phase lag seen among the other LPVs. However, the most notable exception is sequence D, both its low-amplitude and high-amplitude infrared light curves lead the optical light curves, unlike any of the other sequences.

### 3.2. Multi-Periodic Stars

Stars on sequence D are well known to be multi-periodic pulsators; Wood et al. (1999) found that sequence D stars exhibited a shorter period that fell on his sequence B (which is composed of our sequences 2 and 3). Figure 7 shows the secondary Fourier period ($P_1$) of all of the sequence D stars overlaid on the normal Fourier P–L diagram. We see that many of these periods do in fact lie on sequences 2 and 3 as found by Wood et al. (1999), although we also see that sequence 3 is favored. There are also a substantial number of secondary periods of sequence D stars on sequence 4 below the tip of the RGB, and some fall between sequences 1 and 2. However, many of these secondary periods still fall on sequence D, or have periods half as long as in sequence D. These stars with $P_0/P_1 \approx 1$ and $P_0/P_1 \approx 2$ are discussed below in the context of the Petersen diagram.

The Petersen diagram is the plot of the ratio of the two strongest periods versus the longer period in a multi-periodic star, and is presented in Figure 8. Multiple periods are found for the great majority of stars in our sample, so stars from all of the Fourier P–L sequences are represented here. The one-year artifact is visible in this plot since we only removed stars whose primary Fourier periods have low or non-existent periodicity. Stars with their secondary Fourier periods on the one-year artifact lie either in a vertical stripe at $P = \log_{10}(365 \text{ days})$ (when their secondary period is longer than the primary, since it is the longest period that defines placement on the horizontal axis) or along the curve $P_0/P_1 = P_0/(365 \text{ days})$. As observed in Wood et al. (1999), there is a wide locus of points centered around period ratios of 10; these represent stars with their primary or secondary Fourier periods lying on sequence D.

There is a great deal of structure lying at period ratios below 3, which we show in a variant of the Petersen diagram: Figure 9 plots the $P_0/P_1$ ratio for our stars with respect to the $\log_{10} P_0$ Fourier period. Note that $P_0$ is the strongest period, but not necessarily the longest period, as shown by the existence of many stars with period ratios less than 1. In this plot, stars with $P_1$ on the one-year artifact lie only along the curve $P_0/P_1 = P_0/(365 \text{ days})$. Aside from stars with periods on the one-year artifact, there are at least ten groups of stars visible with period ratios between 0 and 2.
Figure 8. Petersen diagram of the LMC: for the primary and secondary Fourier periods of a star, the ratio of the longer to the shorter versus the log of the longer period. Stars are color-coded according to the sequence on which their primary Fourier period lies, as shown in the inset. The structure in this diagram is discussed in Section 3.2.

(A color version of this figure is available in the online journal.)

Figure 9. $P_0/P_1$ vs. $\log_{10} P_0$ showing just the period ratios below 3. Stars are color-coded according to the sequence on which their primary Fourier period ($P_0$) lies, as shown in the inset. Note that $P_0$ is the strongest period, but not necessarily the longest period, as shown by the existence of many stars with period ratios between 0 and 1. Stars with secondary Fourier periods ($P_1$) on the one-year artifact lie along the curve $P_0/P_1 = P_0/(365 \text{ days})$. The structure in this diagram is discussed in Section 3.2.

(A color version of this figure is available in the online journal.)

The groups of stars with the smallest period ratios, and therefore the longest secondary periods ($P_1$) relative to their primary period ($P_0$), have their primary periods lying on one of the numbered sequences and their secondary periods lying on sequence D.

Many stars show period ratios around 1. These closely spaced frequencies are often taken to indicate non-radial pulsation among the least luminous stars from all of the numbered sequences; they have been seen before in both Galactic field stars (Kiss et al. 2000) and the Magellanic Clouds (Soszynski et al. 2004a). The solid lines indicate the region where the period ratio is exactly 1 within our estimated errors; arcs that are visible at the long-period extreme of this region are due to the limits of our frequency resolution.
These frequencies can be seen to produce beats, as seen in the light curve of a sequence 4 star in Figure 4 (middle panel). It is also possible that some of these closely spaced frequencies may be due to a single, slowing varying period. Templeton et al. (2005) found that approximately one-tenth of their sample of 547 AAVSO Mira light curves showed “meandering” periods that did not simply increase or decrease in length. An example Mira variable from sequence 1 that has closely spaced frequencies in its CLEANest spectrum, and a changing period, is shown in Figure 4 (bottom panel). It is overlaid with a sine curve corresponding to the primary Fourier period that CLEANest found: 168.55 days. If the light curve of this star is split into two at the 1760 day mark, we find that the Fourier period of the first half is 166.62 days while the second half has a Fourier period of 163.86 days.

Apart from the period ratios above, stars in sequences 1 and 2 and stars in sequences 3 and 4 tend to show secondary periods that correspond to the other Fourier P–L sequence in each group. The sequence 2 stars at a period ratio of 0.6 have their secondary periods on sequence 1, while the sequence 1 stars at ratios between 1.6 and 2.2 have their secondary periods on sequence 2. Likewise, stars on sequences 3 and 4 show groups at period ratios of 1.4 and 0.7. We do see stars whose periods cross-over between these two groups, for example, the sequence 4 stars at a period ratio of 0.5, which indicate a secondary period on sequence 2, and a few sequence 3 stars at a period ratio of 0.65, indicating a secondary period on sequence 1. There are even fewer stars that cross from sequences 1 or 2 to the shorter period sequences, but some sequence 2 stars are visible at period ratios around 2, which represent secondary periods on sequence 4, and a scattering of sequence 1 stars continue up to period ratios greater than 3, which represent secondary periods on sequence 3. The fact that the period ratios of stars on each pair of sequences, 3 and 4, and 1 and 2, lie at reciprocals of each other means that on a Petersen diagram these groups would substantially overlap, and that similar physical mechanisms are occurring in each case.

Visible at a period ratio of exactly 0.5, 1.5, and 2 are groups of stars from sequence E. The CLEANest periods found for these contact binaries represent only half of the orbital period of these systems; therefore, these period ratios represent the true orbital period, and the third and fourth harmonics, respectively. These stars often have minima of alternating depth as is characteristic of contact binaries (Figure 10).

The sequence D stars with $P_0/P_1 \approx 2$ have period ratios of exactly 2 within our estimated errors, indicating the prevalence of the second harmonic in these light curves. The cause for the closely spaced frequencies ($P_0/P_1 \approx 1$) in sequence D stars is at present unknown.

Finally, the sparsely populated fifth sequence (Soszynski et al. 2004a) is visible in the sequence 4 stars with ratios of 1.3.

### 3.3. Period Changes

Period changes are well known for Miras (Templeton et al. 2005) and semi-regular variables (Kiss et al. 2000). Groenewegen (2004) observed changes from historical periods in OGLE LMC LPVs that indicated stars had moved between sequences 1, 2, and D.\(^9\) It is likely that some of the stars in our sample underwent a period change during the eight years of the MACHO survey. To explore this possibility, we have split each of our light curves into two equal length halves and run our analysis on each half independently. The split light curves are almost four years long, which Lah et al. (2005) found was a sufficient time-span to resolve the familiar Fourier P–L sequences. This is the case for our split light curves as well, but instead of finding changing periods, this technique appears to find stars with multiple periods of almost equal power.

Fifty percent of this sample shows changes in log 10 period of greater than 0.1 dex, the approximate width of the sequences.\(^9\) Actually between sequences B, C, and D. We take their sequence B as sequence 2, but not sequence 3, because these stars lie on the long-period side of sequence B (Figure 8 from Groenewegen 2004).
We have tabulated these movements in Table 3 for all cases where more than 10% of the stars in one sequence move to another. Many move to sequence D, and we also see stars switching sequences between the pairs 3 and 4, and 1 and 2. These movements between the pairs of numbered sequences only happen for stars in the inside edge of each pair of sequences, as shown in Figure 11.

Considering the multi-periodic nature of these stars and the much smaller observed rate of period changes in Miras from Templeton et al. (2005), we do not believe these data indicate a flurry of period changes. Instead, we observe that the changes we see correspond very well to the multiple periods explored in Section 3.2. Examination of some of the light curves of these stars shows that both periods are typically present in the frequency spectrum of both halves of the light curve, but the relative amplitudes of these periods changes. For stars moving between a short period sequence (such as 3 or 4) and sequence D we see that the power in the high frequency component becomes split among multiple closely-spaced frequencies, which no longer have greater power (individually) than the stable low frequency component. It is unclear if this is an effect of the different time-sampling in each half of the light curve, or if it is evidence of some change in the pulsation on the star itself.

Although this analysis may not find period changes directly, the equivalence of these periods can be understood as a result of longer term period changes. Whitelock (1986) and Lattanzio & Wood (2003) argued that the P–L sequences could be understood as the result of different pulsation modes that are excited in turn as the star evolves. The stars that are close to the point where the dominant modes switch (and therefore the sequence too) would be expected to have multiple periods with similar amplitudes.

4. DISCUSSION

The analysis of the frequency spectra of variable stars has been used with great success for characterizing light curve morphology, identifying binary stars, and constraining observed pulsation modes in studies of Cepheids (Simon & Lee 1981), Beat Cepheids (Alcock et al. 1995), Type II Cepheids and RV Tauri stars (Alcock et al. 1998), RR Lyrae (Alcock et al. 2000), and LPVs (see Section 1). Here we discuss how these techniques contribute to the understanding of the origin of sequence D and the relationship between the different LPV stages and stellar evolution.

4.1. Sequence D

One-third of the stars in our sample exhibit the “long secondary period” phenomenon. Although the exact mechanism for sequence D is still unknown, there are many reasons to think that it is correlated with binary systems. In this paper, the evidence includes the similarity between the luminosity functions of sequence D and our entire sample (Figure 5, right panel), and the similar amplitudes of sequence D’s average infrared light curves across the 2MASS bands. Other evidence includes the smooth connection between sequence D and sequence E—when E is plotted at its orbital period (Soszynski et al. 2004b), the presence of ellipsoidal light curves (Soszyński 2007), and radial velocity studies of these stars (Adams et al. 2006). However, there are also several significant features of sequence D that are unique, or show differences from sequence E. Sequence D does not suffer from period halving as sequence E does, and sequence E stars often have the third and fourth harmonics present in their frequency spectra, while sequence D stars show the second harmonic instead (Section 3.2). Derekas et al. (2006) found that in an amplitude–luminosity plot, stars in
sequence D follow a different pattern than sequence E. We see that sequence E appears to be a continuation of only the small amplitude (<0.2 blue peak-to-peak) group of stars in sequence D, those stars that roughly correspond to the OSARGs of Soszynski et al. (2004a). This is consistent with the comparison of the average light curve amplitudes in blue and $K_s$, where only the lower amplitude sequence D stars were a good match to sequence E. Finally, sequence D’s infrared light curves lead the optical light curves by 10–15%, which is a feature unique to this sequence. These facts do not necessarily preclude a binary star mechanism for sequence D, but they are useful constraints for proposed mechanisms. We note, as an additional constraint, that stars associated with sequence 1 are far less likely to have a period lying on sequence D, and that sequence D is not observed in LMC stars dimmer than $K_s \approx 13.7$.

The model proposed by Soszyński (2007), based on the original model proposed by Wood et al. (1999), is that of a binary system in which the mass lost from the red giant is concentrated near the companion, and regularly obscures the red giant. The wide range of observed light curve amplitudes for sequence D stars, from 0.1 up to 5 mag in the MACHO blue filter (Fraser et al. 2005), can be readily explained by the projection effects of different inclination angles in a binary system.

If sequences E and D are truly composed of binary systems, then the population of binary stars in our sample includes stars with either their primary or secondary Fourier period lying within the boundaries of these sequences (e.g., the top panel of Figure 4 shows a star whose secondary Fourier period lies on sequence D). After removing periods identified with the one-year artifact, we find that 48% of the stars in our sample have variability associated with sequences E or D (see Table 3). For comparison, Duquennoy & Mayor (1991) found a binary fraction of approximately 40% for nearby solar-type stars, and Reid & Gizis (1997) found a fraction of approximately 35% for low-mass stars, a trend in mass which is discussed in Lada (2006). It is reasonable to assume that we cannot see pole-on binaries, and there is no reason to think that all binaries have periods shorter than four years, both of which imply that 48% is an underestimate of the total percentage of binaries seen in the LMC. This is a serious problem for any attempt that uses only binary systems to explain sequence D. However, it is very likely that a subset of these stars show variability due to binarity, perhaps the stars in one of the populations that can be separated by color or amplitude.

4.2. Comparison to Evolutionary Models

We can begin to characterize the evolution of LPVs by comparison in color–magnitude space to models. Marigo et al. (2003) produced a population synthesis model of the $K_s$ versus $J-K_s$ color–magnitude diagram (CMD) of the LMC using the RGB and early AGB evolutionary models of Girardi et al. (2000), and a preliminary version of their thermally pulsing AGB star models (published later in Marigo & Girardi 2007). Their Figure 12 illustrates the luminosities and colors corresponding to major phases in an LMC star’s evolution from RGB, through the early AGB, and finally to the thermally pulsing AGB (including a transformation to carbon-dominated atmospheres for some stars). For comparison, our Figure 12 shows the average $J-K_s$ color binned in magnitude for each of the sequences 1–4, E, D, as well as the one-year artifact and the background population of stars. The color of the “background” stars match Galactic disk turn-off stars and LMC intermediate-mass stars on the early AGB in the synthetic CMD of Marigo et al. (2003).

As shown in Figure 12, the $J-K_s$ color is more effective at distinguishing evolutionary stages in the AGB than in the RGB. However, with the additional information from the luminosity functions, we can also estimate the importance of RGB stars to each sequence. A distinct peak in the luminosity function at the tip of the RGB ($K_s = 12.3$; Nikolaev & Weinberg 2000) is widely taken to indicate that the majority of the stars dimmer
than this point are themselves on the RGB (Ita et al. 2002, 2004; Kiss & Bedding 2003, 2004; Fraser et al. 2005).

The stars in our sample that show very weak or non-existent periodicity (the 24% identified with the one-year artifact) are predominately RGB stars. The left two panels of Figure 5 show the luminosity functions of stars in the one-year artifact with sequences 3 and 4. Below the tip of the RGB the population of one-year artifact stars dominates, suggesting that very weak or non-periodic variability is common among RGB stars. Above the tip of the RGB there is a much closer correspondence between the one-year artifact and sequences 3 and 4. Thus it appears that stars at the dimmest luminosities in our sample vary aperiodically while on the RGB, but most begin to show periodic behavior when they brighten to \( K_s \approx 13 \).

After passing off the tip of the RGB, stars may pulsate with shorter periods and lower luminosity as RR Lyrae on the horizontal branch. They become LPVs again as they rise up the early AGB (i.e. prior to the first thermal pulse or helium shell flash). Soszynski et al. (2004a) used the slight offset in period between RGB and AGB stars to show that AGB stars pulsate alongside RGB stars below the tip of the RGB.

Evolution proceeds to brighter luminosities until the onset of thermal pulses, which begin at \( K_s \approx 12 \) on the synthetic CMD from Marigo et al. (2003). Stars primarily populate sequences 1 and 2 above this luminosity. Wagenhuber & Tuchman (1996) predict that thermal pulses will create large modulations in luminosity and the pulsation period (and mode) in AGB stars with timescales of thousands of years. The models of Marigo & Girardi (2007) substantially agree with these predictions, and show large changes in pulsation period due to mode switching as a direct result of a thermal pulse. Period changes in LPVs are well known (Templeton et al. 2005) but only a small percentage of stars at any one time should be undergoing a thermal pulse due to the short timescales of thermal pulses relative to the long inter-pulse period. The observed period changes of LPVs are not well explained by the effects of thermal pulses alone.

The middle two panels of Figure 5 compare the luminosity functions of the numbered sequences. The relative importance of the two giant branches shifts from the RGB to the AGB as we move to sequences 1 and 2. Additionally, the peak number of stars above the TRGB in each sequence is found at higher luminosity from sequence 4 to sequence 1. The OSARG versus SRV/Mira distinction of Soszynski et al. (2004a), by virtue of its definition, roughly corresponds to a division between two pairs of sequences: sequences 3 and 4, and sequences 1 and 2. This division is also clearly seen in the observed period ratios (Figure 9). Considering the synthetic CMD from Marigo et al. (2003), OSARGs are closely related to RGB and E-AGB stars, while the SRV/Mira stars are more closely related to TP-AGB stars.

At \( K_s \approx 11 \) the synthetic CMD predicts the formation of carbon stars, and the typical \( J - K_s \) colors of each sequence diverge (Figure 12). Only sequences 1, 2, and D redden to the expected \( J - K_s \) color of the carbon star tail of Marigo et al. (2003). Their models also show that these stars do not evolve in brightness after this stage, so the observed range of carbon star luminosities in our sample may be interpreted as a range of stellar masses. Marigo & Girardi (2007) show that only stars between 1.2 and 2.5 \( M_\odot \) undergo a dredge-up that can bring the results of nuclear burning to the atmosphere without quickly destroying it through hot bottom burning. Since Miras exist on sequence 1 at luminosities both above and below \( K_s = 11 \) (Fraser et al. 2005), not all large amplitude pulsators are carbon stars. Also, not all carbon stars have such red \( J - K_s \) colors: Groenewegen (2004) found carbon stars on the shorter period sequences—the popular cut-off at \( J - K_s = 1.4 \) appears more effective at segregating M stars, which are rarely this red.

Using \( J - K_s > 1.4 \) to select areas of only carbon stars, we see that many occupy the highest luminosities of both sequences 1 and 2, as also seen in Lebzelter & Wood (2007). Approximately 40% of the carbon stars lie on sequence 2, similar to the prediction of Marigo et al. (2003), who fitted the observed \( K_s \) versus \( J - K_s \) CMD by assuming a 50% mix of fundamental and first overtone pulsation among the carbon stars in their model. Figure 12 shows that the stars on sequence 1 evolve to redder \( J - K_s \) colors than on sequence 2, presumably due to increased mass-loss driven by fundamental mode pulsation (Marigo & Girardi 2007). Some stars on the long-period extreme of sequence 1 are underluminous for their color, which may be self-extinction due to dusty outflows.

Beyond this point, stars begin their rapid post-AGB evolution, and they quickly move out of the color–magnitude space of our sample.

4.3. Brief Summary of LPV Evolution

LMC stars on the RGB and AGB are characterized by the presence of multiple long periods that show increasing length and amplitude as these stars evolve. Apart from the presence of the long secondary period phenomenon, which appears in approximately half of the stars in our sample, stars initially vary non-periodically, and only later begin to pulsate with periods of 20–120 days on sequences 3 and 4. The Petersen diagram and its variants show that stars with their primary period on either one of these sequences often have their secondary period on the other sequence. Furthermore, the period changes between the first half and last half of the MACHO light curves suggest that the amplitudes of these pairs of periods are very similar for stars on the inside edges of these two sequences. Similar results are obtained for stars which have evolved to the luminosity at which thermal pulses begin on the AGB (\( K_s \approx 12 \)); these stars are usually found on sequences 1 and 2 with periods of 45–500 days. This supports the arguments of Whitelock (1986) and Lattanzio & Wood (2003) that different pulsation modes are excited in turn as the star evolves. At the points where the dominant modes switch we observe pulsation in multiple periods with near equal strengths. The increase in the luminosity of the maximum of the luminosity function above the tip of the RGB also lends support to this argument. The highest luminosity stars on sequences 1 and 2 have become carbon stars. After this point, the mass loss rate of these stars increases drastically and they rapidly evolve out of our sample of luminous red stars.

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