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Synchronization of Memristor-Based Delayed Neural Networks via Aperiodically Intermittent Control

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Abstract. This paper investigates the exponential synchronization problem for a class of memristor-based neural networks via novel aperiodically intermittent control approach. By using Lyapunov stability theory, several novel and useful synchronization criteria are obtained, which guarantee global exponential synchronization of memristive neural networks. Finally, a numerical example is given to illustrate the effectiveness of proposed method.

1. Introduction
In these years, it has been showed that memristor devices have many promising applications, such as associative memory and signal processing and so on. Therefore, the memristor has attracted widespread attentions. Due to the good properties of memristor, a variety of memristor-based neural networks(MNNs) have been constructed, and many scholar focused on the dynamic behaviours and circuit implementation of MNNs[1-4].

As is well known, synchronization phenomenon as one of most common and important collective dynamical behaviours, which has been played an extremely vital role in many fields of science. Unfortunately, the whole networks cannot synchronize by themselves in many case, hence, controllers should be designed and applied the force the networks to synchronize. Intermittent control is an important kind of discontinuous control in engineering fields, which has the advantage of reducing the amount of information required to be transmitted to achieve synchronization in networks. However, compared with traditional intermittent control schemes, each control interval includes only one switched period, and each switched period possesses the same time width[5] and same rates of control duration[6].Unfortunately, to the best of our knowledge, the results about aperiodically intermittent control with two switched periods for neural networks, especially the case of completely aperiodically has not yet appeared.

Motivated by the above discussion, this paper aims to investigate the synchronization problem for a class of neural networks via aperiodically intermittent control with two different switched periods.

2. Problem Formulation and Preliminaries
In this paper, we consider a class of memristor-based neural networks with time-vary delays as follows:

\[
\frac{dx_i(t)}{dt} = -C_i x_i(t) + \sum_{j=1}^{n} a_{ij} (x_j(t)) f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} (x_j(t-\tau_{ij}(t))) g_j(x_j(t-\tau_{ij}(t))) + I_i, \quad (1)
\]

\(x_i(t)\) is the state of \(i\)th neuron at time \(t\), \(C_i > 0\) is the \(i\)th neuron self-inhibitions. \(a_{ij} (x_j(t))\) and \(b_{ij} (x_j(t-\tau_{ij}(t)))\) are memristor synpctic connection weights, \(\tau_{ij}(t)\) corresponds to be transmission delay.
delays, and satisfy $\tau \leq \tau_j(t) \leq \tau$, $I_j$ is an external constants input. Then, through the typical feature of the memristor, we can have

$$a_y(x_j(t)) = \begin{cases} a_y^* & \text{if } x_j(t) \leq \Lambda_j \\ a_y^\infty & \text{if } x_j(t) > \Lambda_j \end{cases}$$

$$b_y(x_j(t-\tau_j(t))) = \begin{cases} b_y^* & \text{if } x_j(t-\tau_j(t)) \leq \Lambda_j \\ b_y^\infty & \text{if } x_j(t-\tau_j(t)) > \Lambda_j \end{cases}$$

Let $a_y = \max_{[i,j]} \{a_y^*, a_y^\infty\}$, $b_y = \max_{[i,j]} \{b_y^*, b_y^\infty\}$, the switch jumps $\Lambda_j > 0$, $a_y^*$, $a_y^\infty$, $b_y^*$ and $b_y^\infty$ are constants.

The initial values associated with system (1) are $x_i(t) = \phi_i(t) \in C([-\tau, 0], R^*)$, $i = 1, 2, \ldots, n$.

To establish the main result of this paper, the following assumption and definition are necessary.

**Assumption 2.1:** For the neuron activation functions $f_j(\pm \Lambda_j) = g_j(\pm \Lambda_j) = 0$ and there exist positive constants $1_{j_l}$, $2_{j_l}$ such that

$$|f_j(u) - f_j(v)| \leq l_j^f |u - v|$$

$$|g_j(u) - g_j(v)| \leq l_j^g |u - v|$$

**Definition 2.1:** Drive-response neural networks (2) and (3) are said to achieve exponential complete synchronization if there exist $\alpha \geq 1$ and $\varepsilon > 0$ such that

$$|y_i(t) - y_j(t)| \leq \alpha e^{-\varepsilon t}$$

for any $t \geq 0$. $\varepsilon$ is defined as the degree of exponential synchronization.

Through the theories of differential inclusions and set-valued maps, the memristor-based neural networks (1) can be written as

$$\frac{dx_i(t)}{dt} = -C_i x_i(t) + \sum_{j=1}^n c_{ij} \left[ a_y \left( x_j(t) \right) \right] f_j(x_j(t)) + \sum_{j=1}^n c_{ij} \left[ b_y \left( x_j(t-\tau_j(t)) \right) \right] g_j(x_j(t-\tau_j(t))) + I_j$$

Where

$$c_{ij} \left[ a_y \left( x_j(t) \right) \right] = \begin{cases} a_y^* & \text{if } x_j(t) \leq \Lambda_j \\ a_y^\infty & \text{if } x_j(t) > \Lambda_j \end{cases}$$

$$c_{ij} \left[ b_y \left( x_j(t-\tau_j(t)) \right) \right] = \begin{cases} b_y^* & \text{if } x_j(t-\tau_j(t)) \leq \Lambda_j \\ b_y^\infty & \text{if } x_j(t-\tau_j(t)) > \Lambda_j \end{cases}$$

Analogously, based on the theories of inclusion, we design the response system as follows

$$\frac{dy_j(t)}{dt} = -C_j y_j(t) + \sum_{i=1}^n c_{ij} \left[ a_y \left( y_j(t) \right) \right] f_j(y_j(t)) + I_j + \sum_{i=1}^n c_{ij} \left[ b_y \left( y_j(t-\tau_j(t)) \right) \right] g_j(y_j(t-\tau_j(t))) + u_j(t)$$

the initial values associated with systems (3) are $y_i(t) = \phi_i(t) \in C([-\tau, 0], R^*)$, $i = 1, 2, \ldots, n$.

Then, in order to analyze the problem of synchronization (2) and (3), we will define the system item $e_i(t) = y_i(t) - x_i(t)$, at the same time, the following aperiodically intermittent controller $u_i(t)$ will be designed:

$$u_i(t) = \begin{cases} -k e_i(t) & t \in E_i^+ \cup E_i^- \\ 0 & t \in E_i^0 \cup E_i^\infty \end{cases}$$

where $k > 0$ is control gain to be determined. So, subtracting (2) from (3) yields the following error system:

$$\frac{de_i(t)}{dt} = -C_i e_i(t) + \sum_{j=1}^n c_{ij} \left[ a_y \left( y_j(t) \right) \right] f_j(y_j(t)) - \sum_{j=1}^n c_{ij} \left[ a_y \left( x_j(t) \right) \right] f_j(x_j(t)) - k e_i(t)$$

$$+ \sum_{j=1}^n c_{ij} \left[ b_y \left( y_j(t) \right) \right] f_j(y_j(t-\tau_j(t))) - \sum_{j=1}^n c_{ij} \left[ b_y \left( x_j(t) \right) \right] f_j(x_j(t-\tau_j(t))) \quad t \in E_i^+ \cup E_i^-$$
\[
\frac{de_i(t)}{dt} = -C_i e_i(t) + \sum_{j=1}^n c_j \left[ a_i(y_j(t)) f_j(y_j(t)) - \sum_{j=1}^n c_j b_j(x_j(t)) f_j(x_j(t)) \right] - \sum_{j=1}^{n-1} a_i(x_j(t)) f_j(x_j(t)) \quad t \in E^m_i \cup E^n_i
\]

(6)

We denote that
\[
E^m_i = \left[ \sum_{j=1}^{n-1} T_i, \sum_{j=1}^{n-1} T_i + \theta_{m1} T_{m1} \right]
\]

is the control width in switched period $T_{m1}$;
\[
E^n_i = \left[ \sum_{j=1}^{n-1} T_i + \theta_{m1} T_{m1}, \sum_{j=1}^{n-1} T_i + \theta_{m2} T_{m2} \right]
\]

is the control width in switched period $T_{m2}$;
\[
E^m_i = \left[ \sum_{j=1}^{n-1} T_i + \theta_{m1} T_{m2} + \theta_{m2} T_{m2}, \sum_{j=1}^{n-1} T_i \right]
\]

is the non-control width in switched period $T_{m2}$.

**Lemma 2.1:** Let $W^m_1(t), W^m_2(t), W^m_3(t)$ and $W^m_4(t)$ be piecewise functions as defined by the following:
\[
W^m_1(t) = h \exp \left[ \gamma (1 - \theta_{m1}) \sum_{j=1}^{n-1} T_i \right] \quad t \in E^m_1
\]
\[
W^m_2(t) = h \exp \left[ \gamma \left( t - \theta_{m1} \left( \sum_{j=1}^{n-1} T_i + T_{m1} \right) \right) \right] \quad t \in E^m_2
\]
\[
W^m_3(t) = h \exp \left[ \gamma \left( 1 - \theta_{m2} \left( \sum_{j=1}^{n-1} T_i + T_{m1} \right) \right) \right] \quad t \in E^m_3
\]
\[
W^m_4(t) = h \exp \left[ \gamma \left( t - \theta_{m2} \sum_{j=1}^{n-1} T_i \right) \right] \quad t \in E^m_4
\]

where $h > 1, \gamma > 0, \theta_{m1} \geq \theta_{m2} \geq \cdots \geq \theta_{m} \geq \theta_{m2} \geq \cdots$, and let $\theta = \min_{m \in N} \{ \theta_{m1}, \theta_{m2} \}$. Then
\[
\cdots < W^m_4(t^n_1) \leq W^m_3(t^n_2) < W^m_2(t^n_3) \leq W^m_1(t^n_4) < \cdots
\]
i.e. the piecewise functions are increasing functions.

### 3. Main Result

**Theorem 3.1** Suppose that Assumption 2.1 holds, and we denote that $\beta = \max_{1 \leq i \leq n} \left\{ \sum_{j=1}^{n-1} b_j l_j^2 \right\}$, then, for positive vectors $a_i, a_i$ such that following conditions hold:
\[
a_i > \beta - a_i (1 - \theta) - a_i > 0
\]
\[
\frac{1}{2} \sum_{j=1}^{n} a_i l_j^2 + \frac{1}{2} \sum_{j=1}^{n} a_i l_j^2 + \frac{1}{2} \sum_{j=1}^{n} b_j l_j^2 < \inf_{k \in \mathbb{R}^+} \left\{ C_i + k_i, C_i + \frac{a_i}{2} \right\}
\]

where $\beta$ is the unique positive solution of equation $\beta - \alpha_i + \beta e^{\alpha_i} = 0$. Then memristor-based neural networks (2) with response system (3) can achieve globally exponential complete synchronization.

**Proof:** Choose the following Lyapunov functional candidate for synchronization error system (5) and (6) as:
\[
V(t) = \frac{1}{2} e^{-\alpha_i} \sum_{j=1}^{n} e_j^2(t)
\]

(9)
Taking the upper right Dini derivative with respect to \( t \) along the solution of error system, it can follow that for \( t \in E_1^* \cup E_2^* \)

\[
D^+ V(t) = -\alpha V(t) + \sum_{i=1}^{n} \left( e^{-\alpha \tau_i} \left| e_i(t) \right| \text{sgn}(e_i(t)) D^+ e_i(t) \right)
\]

\[
\leq -\alpha V(t) + \sum_{i=1}^{n} \left( e^{-\alpha \tau_i} \left| e_i(t) \right| \text{sgn}(e_i(t)) (-C_i e_i(t) + \sum_{j=1}^{m} a_{ij} e_j(t) + \sum_{j=1}^{m} b_{ij} e_j(t) \right) \right)
\]

\[
\leq -\alpha V(t) + e^{-\alpha \tau_i} \sum_{i=1}^{n} \left( (-C_i - k_i) \left| e_i(t) \right| + \frac{1}{2} \sum_{j=1}^{m} a_{ij} \left| e_j(t) \right| \right)
\]

\[
+ \frac{1}{2} \sum_{j=1}^{m} b_{ij} \left| e_j(t) \right|^2 + \frac{1}{2} \sum_{j=1}^{m} b_{ij} \left| e_j(t - \tau_j(t)) \right|^2 
\]

\[
< -\alpha V(t) + \beta V(t - \tau_i(t)) \quad (10)
\]

In a similar way, for \( t \in E_2^* \cup E_2^* \), we can have

\[
D^+ V(t) \leq (\alpha_2 - \alpha_1) V(t) + e^{-\alpha \tau_i} \sum_{i=1}^{n} \left( (-C_i - k_i) \left| e_i(t) \right| + \frac{1}{2} \sum_{j=1}^{m} a_{ij} \left| e_j(t) \right| \right)
\]

\[
+ \frac{1}{2} \sum_{j=1}^{m} b_{ij} \left| e_j(t) \right|^2 + \frac{1}{2} \sum_{j=1}^{m} b_{ij} \left| e_j(t - \tau_j(t)) \right|^2 
\]

\[
< (\alpha_2 - \alpha_1) V(t) + \beta V(t - \tau_i(t)) \quad (11)
\]

Next, we will show that error system (5) and (6) are global exponential synchronization.

Let \( M = \sup_{t \geq 0} V(t) \). We denote \( \overline{Q}(t) = \overline{V}(t) - hM \). It is easy to see that for any \( t \in [-\tau, 0] \), \( \overline{Q}(t) < 0 \) is true. In the next step, we will show that \( \overline{Q}(t) < 0 \) also holds for \( t \in E_1^* \), otherwise, \( \exists i \in E_1^* \) such that \( \overline{Q}(i) = 0 \). \( \overline{Q}(i) > 0 \) for \( t \in [-\tau, i] \).

\[
\overline{Q}(i) = \partial e^{\overline{Q}_1(t)} + e^{\overline{Q}_1} D^+ \overline{V}(i)
\]

\[
\leq \partial \overline{V}(i) + e^{\overline{Q}_1} (-\alpha V(t) + \beta V(t - \tau(t)))
\]

\[
< (\alpha_2 - \alpha_1) V(i) + \beta \overline{V}(t - \tau_i(t)) = 0
\]

This leads to a contradiction with \( \overline{Q}(i) > 0 \), hence, \( \overline{Q}(t) < 0 \) is true for \( t \in E_1^* \).

Now, based on the Lemma 2.1, we will prove that \( \overline{Q}(t) = \overline{V}(t) - hM \exp \left[ \alpha_2 (t - \theta_i T_i) \right] < 0 \) for \( t \in E_2^* \). Otherwise, \( \exists i \in E_2^* \) such that \( \overline{Q}(i) = 0 \), \( \overline{Q}(i) > 0 \) and \( \overline{Q}(t) < 0 \) for \( t \in [-\tau, i] \).

\[
\overline{Q}(i) = \partial e^{\overline{Q}_1(t)} + e^{\overline{Q}_1} D^+ \overline{V}(i)
\]

\[
\leq \partial \overline{V}(i) + e^{\overline{Q}_1} \left( (\alpha_2 - \alpha_1) V(i) + \beta V(i - \tau_i(t)) \right) - \alpha \overline{V}(i)
\]

\[
\leq (\alpha_2 - \alpha_1) V(i) + \beta \overline{V}(t - \tau_i(t)) = 0
\]
This leads to a contradiction with \( Q_1(t) \geq 0 \), hence, \( Q_1(t) < 0 \) is true for \( t \in E_1 \). Now we will prove that \( Q_1(t) = \bar{W}(t) - hM \exp(\alpha_1 (1 - \theta_1)T_1) \geq 0 \) for \( t \in E_1 \). Otherwise, \( \exists \iota_1 \in E_1 \) such that \( Q_1(t) = 0 \), \( Q_1(t) \geq 0 \) and \( Q_1(t) < 0 \) for \( t \in [\bar{T}, t_1] \).

\[
Q_1(t) = \delta e^{\beta t} V(t_1) + e^{\beta t} \bar{W}(t_1) \\
\leq \delta \bar{W}(t_1) + e^{\beta t} \left(-\alpha_1 V(t_1) + \beta V(t_1 - \tau(t_1))\right) \\
\leq (\theta - \alpha_1 + \beta e^{\beta t}) \bar{W}(t_1) = 0
\]

Hence, this leads to a contradiction with \( Q_1(t) \geq 0 \), so, \( Q_1(t) < 0 \) is true for \( t \in E_1 \).

Furthermore, we will show that \( Q_{11}(t) = \bar{W}(t) - hM \exp(\alpha_2 (t - \theta_2)T_1) \geq 0 \) for \( t \in E_1 \). Otherwise, \( \exists \iota_{11} \in E_1 \) such that \( Q_{11}(t) = 0 \), \( Q_{11}(t) \geq 0 \) and \( Q_{11}(t) < 0 \) for \( t \in [\bar{T}, t_1] \).

\[
Q_{11}(t) = \delta e^{\beta t} V(t_1) + e^{\beta t} \bar{W}(t_1) - \alpha_2 \bar{W}(t_1) \\
\leq \delta \bar{W}(t_1) + e^{\beta t} \left((\alpha_2 - \alpha_1) V(t_1) + \beta V(t_1 - \tau(t_1))\right) - \alpha_2 \bar{W}(t_1) \\
\leq (\theta - \alpha_1 + \beta e^{\beta t}) \bar{W}(t_1) = 0
\]

Obviously, this also leads to a contradiction with, hence, \( Q_{11}(t) < 0 \) is true for \( t \in E_1 \).

Similarly, we can prove the following results are also true in the \( (m-1)th \) control period:

\[
\bar{W}(t) < hM \exp(\alpha_1 (1 - \theta_1 \sum_{i=1}^{m-2} T_i)) \quad t \in E_1^{m-1}
\]

\[
\bar{W}(t) < hM \exp(\alpha_1 (t - \theta_1 \sum_{i=1}^{m-2} T_i)) \quad t \in E_2^{m-1}
\]

\[
\bar{W}(t) < hM \exp(\alpha_2 \left(1 - \theta_2 \sum_{i=1}^{m-2} T_i\right)) \quad t \in E_3^{m-1}
\]

\[
\bar{W}(t) < hM \exp(\alpha_3 \left(1 - \theta_3 \sum_{i=1}^{m-2} T_i\right)) \quad t \in E_4^{m-1}
\]

Then, form the Lemma 2.1, for any \( t \in [\bar{T}, \sum_{i=1}^{m-1} T_i] \), we can have

\[
\bar{W}(t) < hM \exp(\alpha_2 \left(t - \theta_1 \sum_{i=1}^{m-2} T_i\right)) < hM \exp(\alpha_2 (1 - \theta_1 \sum_{i=1}^{m-2} T_i))
\]

Hence, in the \( mth \) control period, we will continue to prove that

\[
Q_1(t) = \bar{W}(t) - hM \exp(\alpha_1 (1 - \theta_1 \sum_{i=1}^{m-1} T_i)) < 0 \quad t \in E_1^n
\]

Otherwise, \( \exists \iota_1 \in E_1^n \) such that \( Q_1(t) = 0 \), \( Q_1(t) \geq 0 \) and \( Q_1(t) < 0 \) for \( t \in [\bar{T}, \iota_1] \).

\[
Q_1(t) = \delta e^{\beta t} V(t_1) + e^{\beta t} \bar{W}(t_1) \\
\leq (\theta - \alpha_1 + \beta e^{\beta t}) \bar{W}(t_1) = 0
\]
Hence, this leads to a contradiction with $\hat{Q}^*_{m_1}(t_{\nu_1}) \geq 0$, then, $\hat{Q}^*_{m_1}(t) < 0$ is true for $t \in E_{m_1}$. Based on the similar proof, we can also obtain that

$$W(t) < h M \exp \left[ \alpha_1 \left( t - \theta_m \left( \sum_{i=1}^{m-1} T_i + T_{m_1} \right) \right) \right] \quad t \in E_{m_1}$$

$$\tilde{W}(t) < h M \exp \left[ \alpha_1 (1 - \theta_m) \left( \sum_{i=1}^{m-1} T_i + T_{m_1} \right) \right] \quad t \in E_{m_1}$$

$$W(t) < h M \exp \left[ \alpha_1 \left( t - \theta_m \sum_{i=1}^{m} T_i \right) \right] \quad t \in E_{m_1}$$

In summary, for any $t > 0$, $\exists m \in N^+$, and $t \in E_{m_1} \cup E_{m_2} \cup E_{m_3} \cup E_{m_4}$, that is, $\sum_{i=1}^{m-1} T_i \leq t < \sum_{i=1}^{m} T_i$, when $t \in E_{m_1} \Rightarrow \sum_{i=1}^{m} T_i \leq t \Rightarrow (1 - \theta_m) \sum_{i=1}^{m} T_i \leq (1 - \theta_m) t \leq (1 - \theta) t$. So, for $t \in E_{m_1}$, $W(t) < h M \exp \left[ \alpha_1 (1 - \theta) t \right]$ is true, i.e. $V(t) < h M \exp \left[ - \left( \theta - \alpha_1 (1 - \theta) t \right) \right]$. On the other hand, if $t \in E_{m_2}$, $t < \sum_{i=1}^{m-1} T_i + T_{m_1} \Rightarrow t - \theta_m \left( \sum_{i=1}^{m-1} T_i + T_{m_1} \right) < (1 - \theta_m) t \leq (1 - \theta) t$. So, for $t \in E_{m_2}$, we can also have $V(t) < h M \exp \left[ - \left( \theta - \alpha_2 (1 - \theta) t \right) \right]$. In a similar way, for $t \in E_{m_3} \cup E_{m_4}$, we can derive the following truth: $V(t) < h M \exp \left[ - \left( \theta - \alpha_3 (1 - \theta) t \right) \right] \quad t \geq 0$ which implied that

$$\|e(t)\| < 2 h M \exp \left[ - \frac{1}{2} \left( \theta - \alpha_1 (1 - \theta) - \alpha_2 \right) t \right] \quad t \geq 0$$

Then, from (7) of Theorem 3.1 and Definition 2.1, we can know that memristor-neural networks (2) is globally exponentially lag synchronized to the trajectory of response system (3). This proof is complete.

4. Example

In this section, a numerical simulation is given to demonstrate the effective of proposed synchronization schemes for synchronizing the memristor-based neural networks with aperiodically intermittent controller (4).

Example 1: Consider the following memristor-based chaos neural networks:

$$\frac{dx_i(t)}{dt} = -C_i x_i(t) + \sum_{j=1}^{2} a_{ij} \left( x_j(t) - f_j(x_j(t)) \right) + \sum_{j=1}^{2} b_{ij} \left( x_j(t - \tau_j(t)) - g_j(x_j(t - \tau_j(t))) \right)$$

where

$$a_{i1} \left( x_i(t) \right) = \begin{cases} -0.3 & |x_i(t)| \leq 1 \\ -1.2 & |x_i(t)| > 1 \end{cases} \quad a_{i2} \left( x_i(t) \right) = \begin{cases} 0.22 & |x_i(t)| \leq 1 \\ 1.7 & |x_i(t)| > 1 \end{cases} \quad a_{i3} \left( x_i(t) \right) = \begin{cases} 1.5 & |x_i(t)| \leq 1 \\ 0.4 & |x_i(t)| > 1 \end{cases}$$

$$a_{i2} \left( x_i(t) \right) = \begin{cases} -2.3 & |x_i(t)| \leq 1 \\ -0.32 & |x_i(t)| > 1 \end{cases} \quad b_{i1} \left( x_i(t - \tau_i(t)) \right) = \begin{cases} -0.2 & |x_i(t - \tau_i(t))| \leq 1 \\ -0.3 & |x_i(t - \tau_i(t))| > 1 \end{cases}$$

$$b_{i1} \left( x_i(t - \tau_i(t)) \right) = \begin{cases} 0.2 & |x_i(t - \tau_i(t))| \leq 1 \\ 0.1 & |x_i(t - \tau_i(t))| > 1 \end{cases} \quad b_{i2} \left( x_i(t - \tau_i(t)) \right) = \begin{cases} 0.8 & |x_i(t - \tau_i(t))| \leq 1 \\ 0.6 & |x_i(t - \tau_i(t))| > 1 \end{cases}$$
\[ b_{2i}(x_i(t-r_i(t))) = \begin{cases} -0.5 & |x_i(t-r_i(t))| \leq 1 \\ -0.1 & |x_i(t-r_i(t))| > 1 \end{cases} \]

and choosing \( C_1 = C_2 = 0.1 \), \( r_i(t) = \frac{e^t}{1+e^t} \), \( r_1(t) = 0.25(1-\cos t) \) and \( \theta = 0.2 \). Hence, by simple computing, we get \( \tau = 1, l_i^1 = l_i^2 = 1(i = 1,2) \) and \( \beta = 1.1 \). By using Matlab Toolbox and calculating inequalities (7) and (8), we can get the following set of feasible solutions:

\[ k_1 = 0.1831 \quad k_2 = 0.2152 \quad \alpha_1 = 0.1171 \quad \alpha_2 = 0.3110 \]

Moreover, by solving the equation \( \theta - \alpha_i + \beta e^{\varphi} = 0 \), we can obtain \( \theta = 0.9758 \), obviously, \( \theta - \alpha_i + \beta e^{\varphi} > 0 \), the condition of Theorem 3.1 hold. Hence, it shows that the proposed results in Theorem 3.1 are feasible.

**Remark 4.1:** In this paper, we discuss the problem of exponentially synchronization for a class of memristor-based neural networks via aperiodically intermittent control with different switched periods. Particularly, in order to get more general results, we assume the rates of control duration, every control periods and control width in each switched period are different, therefore, it has brought great challenges to our research work under these assumptions.

**Remark 4.2:** In this paper, Lemma 2.1 plays a very important role in analyzing our main results. Compared with [7], the results of this paper have been extended to include different switched periods by utilizing Lemma 2.1. Obviously, the main advantages of the proposed methods are easy to verify and also complement.

**5. Conclusions**

We have investigated, in this paper, the problem of exponential synchronization for a class of memristor-based neural networks with time-varying delays. By applied aperiodically intermittent control schemes, we have obtained less conservative criteria for the synchronization of such neural networks. A numerical simulation is provided to verify the effectiveness of our results. It is shown that, to some degree, the results in this paper have extended the existing results on intermittent control.

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