Reconstruction of $f(T)$ and $f(R)$ gravity according to ($m, n$)-type holographic dark energy

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Abstract: Motivated by earlier works on reconstruction of modified gravity models with dark energy components, we extend them by considering a newly proposed model of ($m, n$)-type of holographic dark energy for two models of modified gravity, $f(R)$ and $f(T)$ theories, where $R$ and $T$ represent Ricci scalar and torsion scalar respectively. Specifically we reconstruct the two later gravity models and discuss their viability and cosmography. The obtained gravity models are ghost free, compatible with local solar system tests and describe effective positive gravitational constant.

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I. INTRODUCTION

From observational data and different types of cosmological evidences we know that our universe is in acceleration expansion era due to an unknown reason [1]. Different kinds of theories have been introduced to explain and clarify this accelerated behavior of the universe. In the frame of classical theory of gravity based on Einstein equations, we label this acceleration to the dark energy with an unknown essence with a simple candidate as cosmological constant or vacuum energy of quantum fields spread in the whole space time [2], with two significant features as fine-tuning of different orders of vacuum energy and cosmic coincidence (for a review see [2]).

Inspired from string theory, holographic dark energy (HDE) scenario has been constructed to explain problems of cosmological constant and others [3]. In this scenario we supposed that the total energy of all matter fields in a system with physical size $L$ must be less than the total energy of a black hole with the same Schwarzschild radius, namely

$$L^3 \rho_V \leq LM_p^2,$$  
(1)

here $\rho_V$ denotes the holographic dark energy density as a function of the UV cut-off, $L$ as preferred IR cut-off and $M_p$ Planck mass. By saturating the above inequality (1.1) by choosing the largest length $L$ one identifies the holographic dark energy:

$$\rho_V = 3c^2 M_p^2 L^{-2},$$  
(2)

where $c$ is some constant \(^1\).

Since the model of holographic dark energy model emerges from one of the fundamental principles of physics, it has been extremely successful in explaining numerous cosmological puzzles and has been a leading candidate of cosmic acceleration problem. Specifically it can resolve the coincidence problem [4] and phantom crossing [5]. Besides the model has reasonable agreement with the astrophysical data of CMB, SNe Ia and galaxy redshift surveys [9]. On these accounts, the HDE paradigm has been extended via different cut-offs [7] and entropy corrections [8]. A recent extension of this idea is $(m, n)$-type holographic dark energy [14], where $m$ and $n$ are the parameters associated with the chosen IR cut-off at a phenomenological level (explicitly written in later sections). It is a generalization of the old models of HDE and agegraphic dark energy. Reconstruction of modified gravity theories according to HDE has been discussed in literature before. Independently Setare and Wu & Zu [10] studied correspondence of HDE and $f(R)$ gravity.

\(^1\) Here $c$ is not speed of light
Recently it has been investigated the relations between HDE and $f(T)$ gravity \cite{11}. In this article, we investigate the correspondence of $(m, n)$-type HDE in first modification of Einstein gravity in $f(R)$ and secondly in a recently proposed Weitzenbock model based on scalar torsion $f(T)$ gravity. The plan of this paper is as follows: In Sec-II, we study correspondence of $(m, n)$-type HDE in $f(T)$ gravity. In Sec-III, we obtain the reconstructed form of $f(T)$ under the assumption of power-law acceleration. In Sec-IV, we discuss the cosmography for the $f(T)$ model. In Sec-V, we study correspondence of $(m, n)$-type HDE in $f(R)$ gravity and discuss its viability in Sec-VI. We briefly discuss conclusions in Sec-VII. We adopt the natural gravitational units $c = 8\pi G = 1$.

### II. $f(T)$ Gravity According to the $(M,N)$-Type Holographic Dark Energy

In this section we summarize basics of generalized teleparallel gravity (GTEGR) of namely $f(T)$ in which we extend the TEGR Lagrangian $T$ (which is dynamically equivalent to GR) to a generic function $f(T)$ as we did for the generalization of GR to the modified $f(R)$ gravity. One form of the action of $f(T)$ gravity coupled minimally to matter $\mathcal{L}_m$ is given by \cite{12}

$$
S = \frac{1}{2} \int d^4x e [T + g(T) + \mathcal{L}_m],
$$

where $e = \text{det}(e^i_{\mu}) = \sqrt{-g}$ and $\mathcal{L}_m$ is the matter fields Lagrangian density. The TEGR Lagrangian $\mathcal{L}_{TEGR} = T$, is just the torsion scalar and can be written explicitly as the following:

$$
T = S^\mu_{\nu\rho} T^\rho_{\mu\nu},
$$

where

$$
T^\rho_{\mu\nu} = e^\rho_i(\partial_{\mu} e^i_{\nu} - \partial_{\nu} e^i_{\mu}),
$$

$$
S^\mu_{\nu\rho} = \frac{1}{2}(K^\mu_{\nu\rho} + \delta^\mu_{\nu} T^\theta_{\rho\theta}, -\delta^\nu_{\rho} T^\theta_{\mu\theta}),
$$

where $K^\mu_{\nu\rho}$ is the contorsion tensor

$$
K^\mu_{\nu\rho} = -\frac{1}{2}(T^\rho_{\nu\mu} - T^\rho_{\mu\nu} - T^\mu_{\nu\rho}).
$$

Here dynamical variable is vierbein tetrads $e^i_{\mu}$, consequently the field equations are derived by

$$
\frac{\delta S}{\delta e^i_{\mu}} = 0:
$$

$$
e^{-1} \partial_\mu (e S^\mu_{i\nu})(1 + g_T) - e^\lambda e^\rho_{\mu\nu} S^\rho_{i\mu} g_T + S^\mu_{i\nu\rho} \partial_\mu(T) g_{TT} - \frac{1}{4} e^\nu (1 + g(T)) = \frac{1}{2} e^\rho T^\rho_{i\nu},
$$
where $g_T = \frac{dg}{dT}, g_{TT} = \frac{d^2g}{dT^2}$, $\tau_{\rho\nu}$ is the stress tensor. Different aspects of this torsion theory have been investigated \cite{13}. We consider cosmology of a spatially-flat $k = 0$ universe obeys the modified Friedmann-Robertson-Walker (FRW) equations from the metric as:

$$ds^2 = dt^2 - a(t)^2 \sum_{i=1}^{3} (dx^i)^2,$$

where $a(t)$ stands for the scale factor and $t$ is the cosmic time. Because of isotropy and homogeneous assumptions, we assume that the spacetime contains perfect fluid matter fields, so the field equations (2.5), we can write

$$T = -6H^2,$$

$$3H^2 = \rho - \frac{1}{2}g - 6H^2 g_T,$$

$$-3H^2 - 2\dot{H} = p + \frac{1}{2}g + 2(3H^2 + \dot{H})g_T - 24\dot{H}H^2 g_{TT},$$

here $\rho$ and $p$ are the energy density and pressure of extra fields, $H = \frac{\dot{a}}{a}$ is the Hubble parameter.

We now propose the correspondence between the $(m, n)$--type holographic dark energy scenario and GTEGR $f(T)$ dark energy model. For $(m, n)$ holographic dark energy density $\rho_V$ we write:

$$\rho_V = \frac{3b^2}{L^2},$$

with $b = \text{constant}$ and $L = \text{generalized IR cut-off is defined by}$ \cite{14}

$$L = \frac{1}{a^m(t)} \int_0^t a^n(t')dt',$$

which, on using an appropriate transformation can be written as

$$L = \frac{1}{a^m(t)} \int_0^t a^{n-1}\frac{1}{H}da.$$

This model proposed in the level of a phenomenological model for dark energy. It is straightforward to show that the model mimics a kind of generalized agegraphic dark energy models. But it is an essential difference between the agegraphic model of dark energy and this new proposed alternative. The naive difference backs to the choice of the appropriate cut-off of the model. The cut-off is a physically meaningful and reachable scale of length. Here in this new proposed model as $(m, n)$ type, we select” the conformal-like age as the holographic characteristic size”. Another advantages of this model is that although the pair of the $(m, n)$ are real arbitrary constants, but for some reasonable values of $(m, n)$ this extended model of holographic dark energy suddenly passes the phantom cross line $\omega = 1$. The main motivation of this phantom cross line is that, in
spite of the usual scalar models of phantom dark energy, here there is no need to introducing an interaction function phenomenologically between a two component mix fluids of the dark energy and matter. This model also is applicable to introduce a new generalized future event horizon as the characteristic size of the model and consequently a new cut-off. We indicate here that in this model the pair of \((m, n)\) are arbitrary and at level of phenomenological they high energy models, need not be integers. In general about these parameters we can say, for age-like holographic models, in the case of \(m = n\) it seems that dark energy has the same time evolution as the dominant ingredient in the early epochs of the universe, implying that dark energy might be unified with dark matter, analogous to what happened in cosmological models with generalized Chaplygin gas. Also we mention here that particle horizon as the holographic characteristic scale in this model corresponds to \(m = n = 1\). Further we want to specify the values of \(m\) as a special case. It is easy to show that for \(m > 0\), then the HDE is equivalent to a phantom field. When \(m = 0\), then the holographic dark energy is just the cosmological constant. Also the new agegraphic dark energy model corresponds to \((n = 1, m = 0)\). Finally if \(1 < m < 0\), the HDE can drive the universe into an accelerating phase. Clearly we must fix this choice in favor of observational data. The pairs \((m, n) = (0, 1)\) and \((m, n) = (4, 3)\), are compatible with observation data.

Using the definition of critical energy density \(\rho_{cr} = 3H^2\), we can write the dimensionless dark energy parameter as

\[
\Omega_V = \frac{\rho_V}{\rho_{cr}} = \frac{b^2}{H^2 L^2}. \tag{13}
\]

Apply the definition of \(\Omega_V\) and \(\rho_{cr}\), we can write

\[
\dot{L} = \dot{R}_h = -mHL + a^{n-m}(t), \tag{14}
\]

\[
\dot{L} = \dot{R}_h = a^{n-m}(t) - \frac{mb}{\sqrt{\Omega_V}}. \tag{15}
\]

In dark energy dominated era it evolves according to the conservation equation

\[
\dot{\rho}_V + 3H(\rho_V + p_V) = 0. \tag{16}
\]

Differentiating (2.8) and using (2.13), we can write

\[
\dot{\rho}_V = -\frac{2}{L} \left[ a^{n-m}(t) - \frac{mb}{\sqrt{\Omega_V}} \right] \rho_V. \tag{17}
\]

So with this, Eq. (2.14) becomes

\[
-\frac{2}{L} \left[ a^{n-m} - \frac{mb}{\sqrt{\Omega_V}} \right] + 3H(1 + \omega_V) = 0, \tag{18}
\]
and we can obtain
\[ \omega_V = \frac{2}{3} \sqrt[3]{\frac{\omega}{b}} a^{n-m}(t) - \frac{2m + 3}{3} \] (19)

Now equations (2.7) can be rewritten:
\[ 3H^2 = \rho + \rho_V, \quad \rho_V = -\frac{1}{2}g - 6H^2 g_T, \] (20)
\[ -3H^2 - 2\dot{H} = p + p_V, \quad p_V = \frac{1}{2}g + 2(3H^2 + \dot{H})g_T - 24\dot{H}H^2 g_{TT}. \] (21)

Combining above equations (2.18) and (2.19) we get
\[ \rho_V + p_V = 2\dot{H}g_T - 24\dot{H}H^2 g_{TT}. \] (22)

While on using \( p_V = \omega_V \rho_V \) and the values of \( H \) and \( \dot{H} \) we can find the general form of the \( g(T) \) from the (22).

### III. POWER LAW ACCELERATION

We assume that the accelerated expansion of the whole universe is governed by a power law scale factor, like
\[ a(t) \sim t^p, \quad p > 1 \] (23)

Using this simple but physically reasonable assumption, we have the following quantities from (12), (13), (9), (10)
\[ L = \frac{t^{1+p(n-m)}}{pm+1}, \] (24)
\[ \Omega_V = \left[ \frac{b(pm+1)}{p} \right]^2 t^{2p(m-n)}, \] (25)
\[ T = \frac{-6p^2}{t^2}, \] (26)
\[ \rho_V = 3\left[ b(pm+1) \right]^2 \left[ -\frac{6p^2}{T} \right]^{1-p(m-n)}. \] (27)

Substituting these expressions in (22) and by a change of the variable to the \( x = -T \), we obtain
\[ g_{xx} + \frac{g_x}{2x} = -\frac{A}{2}x^{1-p(m-n)}, \] (28)

which gives us the following elementary solution
\[ g(T) = \frac{A}{(1 - p(m - n))(2p(m - n) - 1)}(-T)^{1-p(m-n)} + 2c_1\sqrt{-T} + c_2, \] (29)
where
\[
A = -2\sqrt{p\rho_0^V} \left[ -2 + 2p(m - n) \right] (6p^2)^{-\frac{3}{2} + p(m-n)}.
\]

This GTEGR model is a natural extension of our former model of generalized teleparallel gravity \cite{15}. If \( m = n \), then this model coincides with \cite{15}. From the cosmography \cite{16} and using data of BAO, Supernovae Ia and WMAP, we notice that in this case with \( m = n \) which is equivalence to the extended particle horizon as cut-off, if we choose

\[
A = \Omega_{m0}, \quad c_1 = \sqrt{6}H_0(\Omega_{m0} - 1), \quad c_2 = 0,
\]

**IV. COSMOGRAPHY**

But for general model, proposed in \cite{29} it is needed to check that the cosmography parameters. We need that

\[
g(T_0) = 6H_0^2(\Omega_{m0} - 1), \quad g'(T_0) = 0, \quad 6H_0^2(1 + g''(T_0)) = \frac{1}{2} - \frac{3\Omega_{m0}}{4(1 + q_0)}
\]

It’s better we write the \cite{29} in the following form

\[
g(T) = \alpha(-T)^\mu + 2c_1\sqrt{-T} + c_2, \quad (32)
\]

where

\[
\alpha = \frac{A}{(1 - p(m-n))(2p(m-n) - 1)}, \quad \mu = 1 - p(m-n).
\]

By substituting \cite{32} in \cite{31} we obtain

\[
\alpha = -3 \frac{H_0^2 \left( 24 H_0^2 q_0 + 3 \Omega_{m0} - 2 - 2 q_0 \right)}{6^\mu H_0^2 \mu \left( 2 \mu q_0 + 2 \mu - 1 - q_0 \right)} \quad (33)
\]

\[
c_1 = \frac{1}{2} \frac{H_0 \left( 24 H_0^2 q_0 + 3 \Omega_{m0} - 2 - 2 q_0 \right) \sqrt{6}}{2 \mu q_0 + 2 \mu - 1 - q_0} \quad (34)
\]

\[
c_2 = -\frac{\Gamma}{\mu \left( 2 \mu q_0 + 2 \mu - 1 - q_0 \right)}, \quad (35)
\]
\[ \Gamma = 3 H_0^2 (4 \mu^2 q_0 + 4 \mu^2 - 2 \mu - 2 \mu q_0 - 4 \Omega_m \mu^2 q_0 - 4 \Omega_m \mu^2 + 2 \Omega_m \mu + 2 \Omega_m \mu q_0 + 6 \mu \Omega_m) \]
\[ + 3 H_0^2 (48 \mu H_0^2 q_0 + 48 \mu H_0^2 - 4 \mu - 4 \mu q_0 - 24 (-1) \mu H_0^2 - 24 (-1) \mu H_0^2 q_0 - 3 (-1) \mu \Omega_m + 2 (-1) \mu + 2 (-1) \mu q_0) \]

V. RECONSTRUCTION OF \( f(R) \) GRAVITY ACCORDING TO THE \((M,N)\)-TYPE HOLOGRAPHIC DARK ENERGY

To reconstruction of FRW cosmology in \( f(R) \) (for general review, see [17] and [18]) we start by the following action:
\[ S = \int d^4x \sqrt{-g} \left( \frac{f(R)}{2} + \mathcal{L}_m \right) \tag{37} \]
The first FRW equation is:
\[ -\frac{f}{2} + 3 (H^2 + \dot{H}) f' - 18 (4H^2 \dot{H} + H \ddot{H}) f'' + \rho_V = 0 \tag{38} \]
Again by assumning the power law expansion \( a(t) \sim t^p, \quad p > 1 \) as the previous \( f(T) \) reconstruction, using
\[ R = \frac{6p(2p - 1)}{t^2}, \tag{39} \]
\[ \rho_V = 3 \left[ b(pn + 1) \right]^2 t^{2(-1+p(m-n))}. \tag{40} \]
So
\[ \rho_V = 3 \left[ b(pn + 1) \right]^2 \left( \frac{6p(2p - 1)}{R} \right)^{2(-1+p(m-n))}. \tag{41} \]
Thus
\[ -\frac{f}{2} + \left( \frac{p - 1}{2(2p - 1)} \right) R f' - \frac{1}{1 - 2p} R^2 f'' + 3 \left[ b(pn + 1) \right]^2 \left( \frac{6p(2p - 1)}{R} \right)^{2(-1+p(m-n))} = 0. \tag{42} \]
The solution for (42) reads
\[ f(R) = C_1 R^{\frac{1}{2}(-p^3 - \sqrt{p^2 + 26p - 7}) + C_2 R^{\frac{1}{2}(-p^3 + \sqrt{p^2 + 26p - 7}) - \frac{72 (pn + 1)^2 (p - 1/2)^2 \kappa^2 p}{2 + 4 (m - n) (m - 1/4 - n) p^2 + (-1 - 5 m + 5 n) b^2 p^2 R^{2 + (2 m + 2 n) p}. \tag{43} \]
To recovering the usual HDE model with particle horizon we consider the case \( m = n = 1 \), we have
\[ f(R) = C_1 R^{\frac{1}{2}(-p^3 - \sqrt{p^2 + 26p - 7}) + C_2 R^{\frac{1}{2}(-p^3 + \sqrt{p^2 + 26p - 7}) - \frac{72 (p + 1)^2 b^2 (p - 1/2)^2 p}{2 - p} \kappa^2 R^2. \tag{44} \]
VI. VIABILITY CONDITIONS OF (M,N)-TYPE HOLOGRAPHIC DARK ENERGY- 
\( f(R) \) THEORIES

Based on cosmological viable \( f(R) \) models tests in Ref. [19] the essential constraints and limitations [20] for action:
\[
S_g = \frac{1}{16\pi G} \int d^4x \sqrt{-g} f(R)
\]
to be consistent with gravitational (Solar system tests) and cosmological (Large scale behaviors) are:

1. \( f''(R) \geq 0 \) for \( R \gg f''(R) \) [21].

2. \( 1 + f'(R) > 0 \) to avoiding of ghost [22].

3. \( f'(R) < 0 \).

4. \( f'(R) \) must be very small in recent cosmological dark energy dominant era [23].

To examine these conditions (1-4) for a general \( f(R) \) model described in [24] we observe that validity conditions (1-4) depends on the parameters \( m \) and \( n \). For generality, we consider the only case \( m \neq n \).

1. \( f''(R) = R^{\delta_{-2}} \) for \( R \gg R_0 \). Thus this model is stable for ultra violent (UV) completion of GR with an effective gravitational constant \( G_{\text{eff}} \equiv \frac{G}{1 + \epsilon R^\delta_{-2}} \). \( \delta_{-} > 1 \). Obviously in the very early universe the curvature was very high when \( R >> R_0 \implies G_{\text{eff}} \simeq G \), \( -1 < \delta_{-} < 0 \).

2. To avoiding the guest problem in UV regime we must check \( R \gg R_0, 1 + f'(R) > 0 \). It means \( G_{\text{eff}} > 0 \) so theory is ghost free due to effects of modified gravity.

3. \( \frac{f(R)}{R} \mid_{R \to \infty} \to \infty \). Further \( f'(R) \to \infty \). So the model recovers GR in the begining UV regime.

4. For our model in UV regime \( f'(R) \ll 1, R \). So this model confirms the local solar system tests. Also we estimate it numerically to find:
\[
|R^{\delta_{-2}}| < 10^{-6} \implies |\delta_{-}| < \frac{\log(10^{-6})}{\log R} + 2, R \approx O(\Lambda)
\]

So our f(R) model reconstructed from HDE is a ghost free and observationally viable according to viability conditions.
FIG. 1: Variation of the $\delta_{\pm} = \frac{1}{4}(-p + 3 \pm \sqrt{p^2 + 26p - 7})$ for $p > 1$. Note that $\delta_- > 1$, $\delta_+ < -\frac{1}{2}$. So at late time, i.e. at $R >> R_0$, the power $\delta_-$ is dominant.

VII. CONCLUSION

In this paper, we studied a correspondence between a newly proposed model of $(m, n)$-type HDE model with two modified theories of gravity namely $f(R)$ and $f(T)$ gravity. This study generalizes some previous studies such as the papers cited in the abstracts. For our obtained $f(T)$ model, we calculated the cosmographic parameters while for the calculated $f(R)$ model, we investigated the stability and viability conditions. It is shown that our model of $f(R)$ recovers to general relativistic limit at early times, free from ghosts, and consistent with the solar system tests. So, we reconstructed a newly motivated model of holographic dark energy in the frame work of models of modified gravity, one in usual Riemannian form $f(R)$ and another with Weitzenbock connections in the frame work of modified Einstein-Cartan theory.
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