Compressive sensing compensation algorithm about nonlinear distortion of clipping for underwater acoustic OFDM systems

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Abstract. Orthogonal frequency division multiplexing (OFDM) is one of the most attractive multicarrier modulation schemes over underwater acoustic (UWA) communication due to its robustness against multipath channels. However, the multicarrier modulation technique also leads to a higher peak to average power ratio (PAPR). The nonlinear distortion is produced when the traditional clipping method was used to restrain PAPR. In this paper, we proposed a compensation algorithm based on compressive sensing (CS) about the nonlinear distortion. Combined with orthogonal matching pursuit (OMP) algorithm, pilot and null subcarriers are used to compensate the nonlinear distortion in the receiver. Theoretical analysis and simulation results show that the new algorithm can effectively reduce computational complexity and restrain the nonlinear distortion for UWA communication systems.

1. Introduction

There exists serious multipath delay spreading in UWA channel, which makes the OFDM technique be widely used in the UWA communication systems [1]. But the OFDM signal is composed of multiple subcarriers, so that the instantaneous power of the peak value will be greater than the average value when the phases of the multiple subcarriers are consistent [2]. The above high peak to average power ratio (PAPR) can drive a high power amplifier (HPA) into the nonlinear region which increases the out-of-band radiation, causes in-band noise, and reduces HPA efficiency. In addition, due to the large PAPR, OFDM will not be suitable for smaller power terminal, so it is very necessary for UWA OFDM communication systems that the suitable technique or algorithm should be applied to reduce the influence of high PAPR.

At present, many techniques and algorithms have been used in order to reduce the PAPR value. There are two typical methods, including clipping and selected mapping (SLM) [3]. The former distorts the signal and leads to a bit error rate (BER) degradation if the clipped signal part is not reconstructed at the receiver. Since the clipping distortion can be modeled as a sparse signal, this is a typical application for the theory of compressive sensing (CS). A change of perspective, the clipping operation caused by the distortion can be regarded as a new noise source. The noise is different from additive white Gaussian noise and it is a kind of negative impulse noise with sparse properties [4-5], known as the clipping noise.

Clipping is one of the simplest solutions for PAPR, and the nonlinear loss can be resolved through corresponding methods. There are some traditional solutions to restore the original data, including sender coding, recording clipping data and decision-aided reconstruction [6-7]. In a word, the above methods all need transferring auxiliary data to compensate the nonlinear distortion. As a result, this will influence the system effectiveness. Aimed at the problem existing in the traditional clipping
method, combined with sparse characteristics of clipping impulse noise data, this paper proposes a novel algorithm based on CS for compensating the nonlinear distortion. Specifically, pilot, null subcarriers and orthogonal matching pursuit (OMP) algorithm are used to tackle the nonlinear distortion with less computational complexity in the receiver. Theoretical analysis and simulation results all show that the new algorithm can effectively restrain and compensate the nonlinear distortion of clipping without additional auxiliary data.

The rest of the paper is organized as follows. OFDM system model and CS theory were reviewed briefly in the next section. In section 3, the compressive sensing compensation algorithm about nonlinear distortion of clipping for UWA OFDM systems were addressed in details. The simulation experiment results were given in section 4. Finally, the conclusion was presented in section 5.

2. System model

2.1. OFDM system model

In one OFDM symbol duration, let \( X(k) \) denote the complex information symbol to be transmitted on the \( k \)th subcarrier. After performing \( N \) points IDFT, we get the time domain expression

\[
x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi nk/N}, \quad n = 0, 1, \cdots, N-1
\]

and (1) can be expressed in vector-matrix form as [8]

\[
x = F^H X
\]

where \( x = [x(0), x(1), \cdots, x(N-1)]^T \), \( X = [X(0), X(1), \cdots, X(N-1)]^T \), and \( F^H \) denotes IDFT matrix with elements \( F_{m,n}^H = \frac{1}{N} e^{2\pi j(m-1)(n-1)/N} \), \( 1 \leq m \leq N, 1 \leq n \leq N \), \( H \) the conjugate transpose and \( T \) the matrix transpose.

Due to the amplitude of \( x(n) \) may be over HPA’s ability, clipping operation must be done before sending \( x(n) \) to HPA. The simplest clipping algorithm is that the envelope peak of the input signal was limited within a predetermined value. Thus the nonlinear distortion due to the saturated HPA will be reduced [9]. The above algorithm can be given by

\[
c(n) = \begin{cases} x(n) & |x(n)| \leq A \\ A e^{j \arg(x(n))} & |x(n)| > A \end{cases}
\]

where \( c(n) \) denotes data symbols after clipping, \( A \) threshold and \( \arg(x(n)) \) phase angle. So the difference of value between the original data and the clipped data can be expressed as

\[
i(n) = c(n) - x(n)
\]

To the UWA communication systems, the data can be viewed as a negative pulse noise in (4). In \( x(n) \), the probability of large amplitude data is smaller, so the impulse noise data \( i(n) \) has sparse characteristics in time domain. Based on the sparse features, companied with the corresponding algorithm, \( i(n) \) can be renewed through CS technique at the receiving end of the systems. Thereby, the nonlinear distortion caused by clipping can be compensated. Then the discrete data in baseband in the receiver can be acquired in the form

\[
y(n) = h(n) *[x(n) + i(n)] + g(n)
\]

where \( h(n) \) denotes the unit impulse response of UWA channel and \( g(n) \) additive white Gaussian noise. In order to express (5) using matrix, several \( N \)-dimensional column vector were defined as
following, the discrete time-domain received signal \( y = [y(0), y(1), \ldots, y(N-1)]^T \), the impulse response \( h = [h(0), h(1), \ldots, h(N-1)]^T \), \( i = [i(0), i(1), \ldots, i(N-1)]^T \), \( g = [g(0), g(1), \ldots, g(N-1)]^T \). Then we write (5) with matrix as below

\[
y = C(x + i) + g
\]

where \( C \) is \( N \times N \) cyclic matrix of the channel which role is to do convolution operation between the channel impulse response and time domain signal, and its form is as follow

\[
C = \begin{bmatrix}
h(0) & 0 & \cdots & 0 & h(L-1) & \cdots & h(1) \\
h(1) & h(0) & 0 & \cdots & 0 & h(L-1) & \vdots \\
\vdots & h(1) & h(0) & 0 & \cdots & 0 & h(L-1) \\
0 & h(L-1) & \cdots & h(1) & h(0) & 0 & \vdots \\
\vdots & 0 & h(L-1) & \cdots & h(1) & h(0) & 0 \\
0 & \cdots & 0 & h(L-1) & \cdots & h(1) & h(0)
\end{bmatrix}
\]

where \( N - L \) zero elements locates in each column and row, and \( L \) denotes the number of channel path. Then we can get the frequency domain model about (6) as

\[
Y = AX + FCI + G
\]

where \( A = FCF^H = \text{diag}(h(0), h(1), \ldots, h(N-1)) \), and \( G = Fg \).

### 2.2. Compressive sensing (CS) theory model

Sampling and compression to the sparse signal can be done at the same time through CS technique in the transmitter. Then the original signal can be restored with a high probability using some kind of optimization algorithm at the receiving end. CS mainly involves a problem of recovering a signal \( \theta \) from a relatively small number of its measurements in the form as follow [10]

\[
u = \Psi \theta
\]

where \( u \in \mathbb{R}^N \) is \( N \times 1 \) column vector and \( \Psi = [\psi_1, \psi_2, \ldots, \psi_N] \) the \( N \times N \) orthonormal basis. Specially, there exits \( K \) non-zero elements in vector \( \theta \) that \( K \ll N \). Next, \( M \) linear measurements can be acquired from a \( K \)-sparse signal \( u \) through using \( \Phi \)

\[
v = \Phi u = \Phi \Psi \theta
\]

where \( v \) denotes \( M \times 1 \) measurement vector and \( \Phi \) the \( M \times N \) measurement matrix. Knowing \( \Phi \) and \( \Psi \), the receiver can get \( \theta \) from \( v \) based on CS, and then the vector \( u \) can be obtained using (9). Because of \( M \ll N \), (10) is an underdetermined equation. So some appropriate algorithm should be applied to search the optimal solution in the solution space.

According to the related literature, generally there are two kinds of typical search optimization algorithm, the first is Basic Pursuit (BP) algorithm, and the second is the orthogonal matching pursuit (OMP) algorithm [11]. Especially to the latter, sparse recovery of \( \theta \) can intuitively be achieved by seeking the sparsest estimate of \( \theta \) via the constrained \( l_2 \) norm optimization problem

\[
\hat{\theta} = \arg \min \| \theta \|_{l_2} \quad \text{s.t.} \quad \| \Phi \Psi \theta - v \|_{l_2} \leq \varepsilon
\]

where \( \varepsilon \) is the \( l_2 \) norm error tolerance (residual error) [12].

Because of computational complexity, BP algorithm affects the real-time performance of the channel estimation greatly, especially for time-varying UWA channel. So the algorithm is hard to get the actual application in the underwater environment. But OMP is a greedy algorithm to solve the
sparse reconstruction problem. The OMP algorithm selects one most possible atom at each step and
updates the residual error based on all the chosen atoms [13]. And this makes the computation have
fallen sharply, so the OMP algorithm is more applicable to the time-varying UWA channel. OMP
algorithm process is as follows [14]

Input: \( M \times N \) measurement matrix \( \Phi \), \( M \times 1 \) measurement vector \( y \), residual error \( \epsilon \);

Output: approximation value \( \hat{x} \);

Initialize: residual error \( r_0 = y \), index set \( S_0 = \emptyset \), number of iterations \( t = 1 \);

Execute step 1-5:

Step 1: Find out the subscript of the maximum in the inner product of the residual error and each
column of the measure matrix, \( s_t = \arg \max_{j=1,2,\ldots,N} \langle r_{t-1}, \Phi_j \rangle \);

Step 2: Refresh the index set \( S_t = S_{t-1} \cup \{ s_t \} \), record \( M \times N \) matrix \( \Phi_t = [\Phi_{s_1}, \Phi_{s_2}, \ldots, \Phi_{s_t}] \);

Step 3: Get the approximation value \( \hat{x}_t = \arg \min_{x} \| y - \Phi_{s_t} \hat{x}_t \|_2 \) through Least Square algorithm;

Step 4: Refresh the residual error \( r_t = y - \Phi_{s_t} \hat{x}_t \), \( t = t + 1 \);

Step 5: Continue to from step 1 to 5 until the residual error satisfies \( \| r_t \| \leq \epsilon \);

The OMP algorithms assume that the sparse degree is unknown, so the residual error \( \epsilon \) needs to be
known. And this condition determines the time of iterations of the whole process of the operation. In
this paper, based on the LS algorithm, a novel algorithm is proposed through using the energy of noise
to set the threshold.

3. OMP compensation algorithm for clipping

3.1. Clipping compensation using null subcarriers based on CS

There are \( N \) subcarriers including \( (N - N_p - N_n) \) data carrier, \( N_p \) pilot and \( N_n \) null subcarriers in
OFDM systems. The signal energy is zero for null subcarriers before clipping, and a sparse impulse
noise is added to the null ones after clipping. Then the Gaussian white noise will appear due to the
UWA channel. Combining with the characteristics of the above two kinds of noise, using CS
algorithm, the nonlinear distortion can be estimated and compensated in the receiver. Let \( J = \{ j_1, j_2, \ldots, j_{N_n} \} \) denotes the set of \( N_n \) null subcarriers’ number for OFDM systems. Because of no
data in null subcarriers in the transmitting terminal, according to (8), the frequency domain data in null
subcarriers in the receiver will be showed as follows

\[
Y_N = F_N C_i + G_N
\]

where \( F_N \) is \( N_N \times N \) DFT matrix; the vector \( Y_N \) and \( G_N \) can be expressed as \( Y_N = [Y_{j_1}, Y_{j_2}, \ldots, Y_{j_{N_n}}]^T \),
\( G_N = [G_{j_1}, G_{j_2}, \ldots, G_{j_{N_n}}]^T \). According to CS theory, the measurement matrix can be defined as
\( \Phi_N = F_N \). Let \( i_c = C_i \), then (12) can be written as

\[
Y_N = \Phi_N i_c + G_N
\]

where the vector \( i_c \) denotes the time domain value of the sparse pulse noise for clipping after passing
the UWA channel. Combining (11) with(13), \( i_c \) can be estimated under the premise of knowing \( \epsilon \).
Then the nonlinear distortion of clipping can be compensated.

\[
\hat{i}_c = \arg \min \| \hat{i}_c \| \text{ s.t. } \| \Phi_N \hat{i}_c - Y_N \| \leq \epsilon
\]
3.2. The estimation of error tolerance values for OMP based on the LS algorithm

Unknowing the sparse degree of $i_c$, the value of $\varepsilon$ can determine the time of iterations of the whole process of OMP algorithms also. Now we will focus on discussing how to acquire the threshold of $\varepsilon$ using Gaussian noise based on the LS algorithm. Let $P = \{p_1, p_2, \cdots, p_N\}$ denote the set of position of pilot if considering the noise. Then in the receiver, using the LS algorithm, the valuation for the channel frequency response of pilot subcarriers can be acquired as

$$\hat{H}_p(p_m) = \frac{Y_p(p_m)}{X_p(p_m)} = H_p(p_m) + \frac{I_p(p_m)}{X_p(p_m)} + \frac{G_p(p_m)}{X_p(p_m)}, \quad m = 1, 2, \cdots, N_p \quad (15)$$

where $H_p, X_p, I_p, G_p$ denotes the element of following vector respectively

$$\begin{align*}
H_p &= [H_p(p_1), H_p(p_2), \cdots, H_p(p_N)]^T \\
X_p &= [X_p(p_1), X_p(p_2), \cdots, X_p(p_N)]^T \\
I_p &= F_p G_c I = [I_p(p_1), I_p(p_2), \cdots, I_p(p_N)]^T \\
G_p &= [G_p(p_1), G_p(p_2), \cdots, G_p(p_N)]^T
\end{align*} \quad (16)$$

where $F_p$ is $N_p \times N$ DFT matrix which column is compose of $p_1, p_2, \cdots, p_N$ row of $F$. Let $\hat{h}_p(n)$ be the sequence obtained by taking the $N_p$-IDFT of (15), as follows

$$\hat{h}_p(n) = \frac{1}{N_p} \sum_{k=0}^{N_p-1} \hat{H}_p(p_{k+1}) e^{\frac{2\pi in}{N_p}}, \quad n = 0, 1, \cdots, N_p - 1 \quad (17)$$

Introducing (15) into (17), we arrive at the following expression

$$\hat{h}_p(n) = \frac{1}{N_p} \left( \sum_{k=0}^{N_p-1} H_p(p_{k+1}) e^{\frac{2\pi in}{N_p}} \left\{ \sum_{k=0}^{N_p-1} I_p(p_{k+1}) e^{\frac{2\pi in}{N_p}} \right\}^m + \sum_{k=0}^{N_p-1} G_p(p_{k+1}) e^{\frac{2\pi in}{N_p}} \right) \quad (18)$$

where $n = 0, 1, \cdots, N_p - 1$, the first part of the right side represents the real value of pilot for the channel impulse response, the second part is the influence of impulse noise and the third part has something to do with Gaussian noise. Since the Gaussian noise samples of frequency are statistically independent, the noise is zero-mean with variance $\sigma^2$. In this paper, the model of pilot $X_p(p_n)$ are normalized, so the Gaussian noise samples of time domain should be with zero-mean and variance $\sigma^2/N_p$. The above theoretical analysis shows that the total energy of $N_p$ noise samples is $\sigma^2$ in an OFDM symbol period. In an OFDM symbol period, to the unit impulse response of the channel, the real total energy of pilot can be described as

$$E = \sum_{n=0}^{N_p-1} \left| \hat{h}_p(n) \right|^2 + E_{\sigma^2} \quad (19)$$

where $E_{\sigma^2}$ denotes the total energy related with impulse noise of $N_p$ pilot in a symbol period.

From what has been discussed above, for useful information in each sample, the average energy is $E/N_p$, the average energy of Gaussian noise is $\sigma^2/N_p$, and the average energy of the negative impulse noise is $E_{\sigma^2}/N_p$. Thus the threshold of zero and non-zero value used to determine the $h(n)$ is
given by \( \delta \approx \left( \frac{\sigma^2}{N_p} \right)^{1/2} - \left( \frac{E_i}{N_p} \right)^{1/2} \). Because of the sparse features of impulse noise, namely the smaller value of \( E_i \), thus \( \delta \) can be approximated as \( \delta \approx \left( \frac{\sigma^2}{N_p} \right)^{1/2} \). At the receiver, if \( \hat{h}_p(n) \) satisfied as

\[
\left| \hat{h}_p(n) \right| > \left( \frac{E}{N_p} \right)^{1/2}
\]

(20)

then the value of \( \hat{h}_p(n) \) may be considered nonzero, otherwise the value equals zero. Thus (20) provides a further basis for calculating the error tolerance values in (14). So based on CS and combined with statistical independence characteristic of Gaussian noise, the relationship between \( \varepsilon \) and \( \delta \) can be obtained as

\[
\varepsilon = N_p \delta = \left( N_p \sigma^2 \right)^{1/2}
\]

(21)

The value of \( \varepsilon \) can be obtained according to (21), and then the steps 1-5 can be completed based on OMP algorithm. So as to estimate \( i_k \), the nonlinear distortion produced by clipping will be compensated finally. For the specific process of the proposed method, as shown in Figure 1, the above algorithm is expressed in a system schematic diagram as follows

![Figure 1. Schematic diagram of compensation for clipping noise based CS](image)

4. Simulation results and discussing

In order to highlight the performance of the new algorithm in this paper, in addition to the transmitter and receiver, the additive white Gaussian noise is the only other factor that influences the algorithm in the UWA channel. Besides, suppose the time synchronization and frequency synchronization is perfect. The detailed system specifications used in the simulation are indicated in Table 1. The clipping threshold is set to 8 ~ 10 dB and the performance of the new algorithm will be measured mainly through BER curve in the simulation process.

| Parameter           | Value    | Parameter           | Value    |
|---------------------|----------|---------------------|----------|
| FFT size            | 512      | Bandwidth           | 12 kHz   |
| Data subcarriers    | 336      | Subcarrier bandwidth| 23.44Hz  |
| Pilot subcarriers   | 128      | Cyclic prefix duration| 10.7ms  |
| Null subcarriers    | 48       | Symbol duration with CP| 53.3ms  |
| Carrier frequency   | 24kHz    | Modulation order    | 8-QAM    |
For the sake of verifying the advantages of the new method, figure 2 shows BER curve comparison of results of the three methods, namely direct clipping algorithm, compression extension clipping algorithm and OMP algorithm. Due to the former two traditional clipping method can't remove the clipping noise at the receiving end, the BER performance of the system will degrade although the HPA can run in the course of nature. But the OMP algorithm proposed in this paper can meet the requirements of HPA and BER at the same time. According to the four BER curves in Figure 2, it is not difficult to verify the above conclusion.

Based on OMP algorithm, in this paper, pilot and null subcarriers are used to compensate the nonlinear distortion in the receiver. So the two kinds of subcarriers have an effect to the new algorithm inevitably. The following simulation will be done to analyse the influence of the above two kinds of subcarriers respectively using BER curve. First of all, Figure 3 shows that the system performance will decline with the rising frequency interval of pilot. This is mainly due to the frequency interval of pilot has a large influence on the LS algorithm which will affect the judgment to the threshold of $e$. Next, in Figure 4, different BER curve has been given in order to show that different number of null subcarriers will produce different effect to the system. It can be seen from Figure 4 that the larger the number of subcarriers is, the better the BER performance of the system has. In addition, the BER performance will get a smaller increase with the more quantity of null subcarriers, this is mainly on account of the sparse nature of the negative pulse noise and the smaller energy of the null subcarriers. In a word, more null subcarriers act little to the improvement of the BER performance of the system.
5. Conclusions
Based on CS theory, combined with pilot and null subcarriers, a novel algorithm is proposed to compensate the nonlinear distortion for the clipping in the article. The method can make the receiver restore accurately the original signal before clipping. The paper mainly takes advantage of the sparse nature of clipping impulse noise and the processing method of CS, which greatly reduces the clipped noise. And then the performance of the system can be improved effectively with the decrease of the nonlinear distortion to HPA. Algorithm analysis and simulation results show that the complexity of the algorithm is low, and the improvement effect is very obvious to reduce the clipping noise. As a whole, the improved algorithm is superior to the ordinary clipping algorithm, and has a practicability. Especially for high speed real-time OFDM UWA communication systems, the method has a certain reference value.

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