Research on parameter estimation methods of fatigue life distribution model of automotive chassis parts

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Abstract
The reliability assessment of fatigue life of automotive chassis parts is an important part of automobile safety. The fatigue endurance tests of automotive half-axles were carried out. The three-parameter Weibull distribution model was established according to the fatigue life distribution of samples. Using the right approximation method and the genetic algorithm to estimate three parameters of the distribution model and a comparative analysis was made. The results show that the initial life unreliability estimated by the median rank in the right approximation method had a large deviation from the true value, which may cause the iterative algorithm fall into the local optimal solution and affect the estimation accuracy. Therefore, a new parameter estimation method based on dual genetic algorithm was proposed. The newly proposed dual genetic algorithm not only improves the iteration speed, but also improves the accuracy of parameter estimation, which provides new ideas and theoretical basis for reliability assessment of mechanical components.

Keywords
Fatigue life, Weibull distribution, parameter estimation, genetic algorithm

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Introduction
There are many kinds of automotive chassis parts with complex structures so that if wear, fracture, or other problems occur, the operation of the entire chassis structure will be affected, then the whole vehicle system cannot work normally and lead to failure, even cause major traffic accidents and casualties.¹ Therefore, it is essential to predict the fatigue reliability of automotive chassis parts.

As for mechanical components, the manufacturing process, loading condition, the initial defects of components and the fatigue properties of materials are all random variables with uncertainty. Additionally, some researches show that a scatter factor may be demonstrated during the above conditions to affect the results of fatigue life of the same type of products.²,³ The fatigue life distribution of mechanical components can be analyzed statistically.⁴

Weibull distribution is the most commonly used distribution model for describing the life of mechanical products, such as bearings, gears, and numerical control machines, etc.⁵–⁷ The three-parameter Weibull

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distribution is widely used in the fatigue life distribution model of mechanical components because it can describe the minimum fatigue life of samples and has strong adaptability.

At present, the main methods of estimating parameters of Weibull distribution are statistical estimation method, gray estimation method, maximum likelihood method, genetic algorithm, and right approximation method. These methods have different adaptability to different sample sizes, but basically all of them have high requirements on the initial value of iteration and need to go through complex calculations.

The right approximation method (RA method) converts the target distribution function into a linear function based on a specific transformation method. Then the least square method is applied to fit the optimal parameters, and approximates the optimal target parameters from the right side through continuous iterative calculation, which can effectively improve the calculation non-convergence problem. However, in the process of iteration, the right approximation estimation method needs to use the median rank as the initial iteration value of the life unreliability. As an empirical parameter, the median rank may deviate greatly from the true value, which makes it easy for the iterative algorithm to obtain the local optimal solution rather than the global optimal solution, resulting in the inaccuracy of Weibull parameter estimation. The genetic algorithm (GA) is an adaptive and intelligent search technology with strong global optimization capabilities, which is widely used in complex nonlinear optimization problems. Generally, the genetic algorithm does not use external information during optimization iteration. It only uses the objective function value of each individual in the population to search based on the objective function. The iteration speed and accuracy of the algorithm are greatly affected by the initial population and constraint boundaries. Therefore, combining the GA with the RA method in the Weibull distribution parameter estimation can effectively avoid the unreasonable selection of iterative initial values.

In this study, the fatigue endurance bench tests of automotive half-axes were carried out. A three-parameter Weibull distribution model was established according to the fatigue life distribution of the samples. The fatigue sample data were expanded based on a set of given reference Weibull distribution parameters. The Weibull three parameters were estimated for the expanded fatigue data by the RA method and the GA, respectively. The estimated parameters were compared with the given reference Weibull parameters to study the influence of the initial iteration value on the estimation accuracy. Furthermore, combining the RA method and the GA, a dual genetic algorithm (DGA) for Weibull distribution parameter estimation was proposed, which improves the iterative speed of parameter estimation and the accuracy of the result, and provides a new method and theoretical basis for the fatigue reliability evaluation of automotive chassis parts.

Fatigue endurance bench tests for automotive half-axes

The fatigue endurance bench tests for automotive half-axes were carried out on the eight-channel hydraulic servo fatigue test system. The samples were fixed on the test bench and the repeated torque was applied on the ends of them. The assembly was shown in Figure 1. The procedure of the test was as follows: First, applied a sine wave load with a loading frequency of 3.0 Hz. When the torque was reached the maximum value of 3214 N·m, unloaded to the minimum torque of 292.2 N·m. At this time, a loading loop was complete. Then repeating the loading loop based on the above process until the failure cracks were observed on each sample. The failure situation of one specimen was shown in Figure 2, and the specimen exhibited shear
fatigue failure characteristic. The statistics of fatigue life of automotive half-axles were listed in Table 1.

The number of original test data of samples was so small that the data sample size needed to be expanded. The three-parameter Weibull distribution function was used to describe the fatigue life condition of the half-axles, and the probability density function expression was shown in equation (1). Where \( N_i \) was the fatigue life of the half-axles, \( \beta \) was the shape parameter, \( N_a \) was the scale parameter, and \( N_0 \) was the location parameter.

\[
f(N_i) = \frac{\beta}{N_a - N_0} \left( \frac{N_i - N_0}{N_a - N_0} \right)^{\beta-1} \exp \left[-\left( \frac{N_i - N_0}{N_a - N_0} \right)^{\beta}\right]^{15}
\]

(1)

Assume that the shape parameter was \( \beta = 2.0007 \), the scale parameter was \( N_a = 34.31 \) and the location parameter was \( N_0 = 15.47 \), generating 100 random data that conformed to the three-parameter Weibull distribution, as shown in Table 2. Subsequently, the fitting accuracy of different parameter estimation methods will be explored based on the parameter group of \( \{N_0, N_a, \beta\} \).

### Parameter estimation of Weibull distribution

#### Estimation of Weibull distribution parameters by the RA method

The RA method first converted the objective function into a linear function. Based on equation (1), the fatigue life distribution function of the structural component was given by equation (2), where \( F(N_i) \) was the life unreliability.

\[
F(N_i) = 1 - \exp \left[-\left( \frac{N_i - N_0}{N_a - N_0} \right)^{-\beta}\right]^{17}
\]

(2)

After taking the natural logarithm twice for the three-parameter Weibull distribution of equation (2), equation (3) was obtained as follows:

\[
\frac{1}{\beta} \ln \left[ 1 - F(N_i) \right]^{-1} = \ln (N_i - N_0) - \ln (N_a - N_0) \]

(3)

The median rank is the value that the true failure probability should have at the 50% confidence level when the \( i \) th sample fails in the \( N \) total sample size. It is a measurement that used for obtaining the estimated value of life unreliability.\(^{32,33} \) The value of initial unreliability in equation (3) was determined by the median rank \( \hat{F}_0^M \), which can be expressed as:

\[
\hat{F}_0^M = 1 - (i - 0.3)/(n + 0.4) \]

(4)

Assuming

\[
\begin{align*}
  x_i &= \ln \left[ \frac{1}{\hat{F}_0^M} \right] \\
  y_i &= \ln (N_i - N_0) \\
  a &= \frac{1}{\beta}, b &= \ln (N_a - N_0)
\end{align*}
\]

(5)

In equation (5), let \( \hat{F} = \hat{F}_0^M \). Equation (3) was transformed into a linear relationship in the form of equation (6) below, thereby simplifying the original objective function equation (2).

\[
y = ax + b \]

(6)

Note that it is only necessary to assign the value of \( \hat{N}_0 \) in equation (5) to obtain the corresponding \( \{\hat{x}_i, \hat{y}_i\} \), and then used the least squares method to perform linear regression based on equation (6). The fitting coefficient expression was as follows:

| Number | Fatigue life/1×10⁴ times |
|--------|-------------------------|
| 1      | 23.80                   |
| 2      | 45.14                   |
| 3      | 34.75                   |
| 4      | 32.85                   |
| 5      | 23.87                   |

### Table 1. The statistics of fatigue life of automotive half-axles.

| Number | Fatigue life/1×10⁴ times |
|--------|-------------------------|
| 1      | 23.80                   |
| 2      | 45.14                   |
| 3      | 34.75                   |
| 4      | 32.85                   |
| 5      | 23.87                   |

### Table 2. The statistics of fatigue life of automotive half-axles.

| Fatigue life/1×10⁴ times |
|-------------------------|
| 29.32                   |
| 30.15                   |
| 33.38                   |
| 26.80                   |
| 34.83                   |
| 25.89                   |
| 23.82                   |
| 24.61                   |
| 30.82                   |
| 30.39                   |
| 30.24                   |
| 27.89                   |
| 32.34                   |
| 44.43                   |
| 41.41                   |
| 33.63                   |
| 32.77                   |
| 19.77                   |
| 22.08                   |
| 44.69                   |
| 22.50                   |
| 30.17                   |
| 34.35                   |
| 30.58                   |
| 29.24                   |
| 27.09                   |
| 21.97                   |
| 35.367                  |
| 29.19                   |
| 43.35                   |
| 37.18                   |
| 26.24                   |
| 25.25                   |
| 30.47                   |
| 37.26                   |
| 26.63                   |
| 33.71                   |
| 27.367                  |
| 41.19                   |
| 42.05                   |
| 35.62                   |
| 30.64                   |
| 28.32                   |
| 22.75                   |
| 48.69                   |
| 32.48                   |
| 25.12                   |
| 32.57                   |
| 39.36                   |
| 27.20                   |
| 42.93                   |
| 16.97                   |
| 25.05                   |
| 31.49                   |
| 25.46                   |
| 52.81                   |
| 33.58                   |
| 23.51                   |
| 33.33                   |
| 31.26                   |
| 20.16                   |
| 38.69                   |
| 20.44                   |
| 33.66                   |
| 37.87                   |
| 35.27                   |
| 24.15                   |
| 25.13                   |
| 25.60                   |
| 39.75                   |
| 27.93                   |
| 43.69                   |
| 18.60                   |
| 27.36                   |
| 32.48                   |
| 32.91                   |
| 25.45                   |
| 40.65                   |
| 23.72                   |
| 31.26                   |
| 31.62                   |
| 43.46                   |
| 39.66                   |
| 25.78                   |
| 26.99                   |
| 37.01                   |
| 34.06                   |
| 22.73                   |
| 24.62                   |
| 41.51                   |
| 28.07                   |
| 46.72                   |
| 41.93                   |
| 30.70                   |
| 34.52                   |
| 39.47                   |
| 38.78                   |
| 17.37                   |
| 35.61                   |
| 47.54                   |
The correlation coefficient \( r \) reflects the degree of linear correlation between the independent variable and dependent variable. The larger the value of \( r \), the higher the degree of linear correlation between \( x_i \) and \( y_i \). Defining the initial value of the location parameter \( N_0 = \min(N_i) = 16.97 \) and setting the step interval to \( \Delta = 0.05N_0 \). Substituting \( N_0 = N_{01} - \Delta \), \( N_{02} = N_{01} - \Delta \), \( N_{03} = N_{02} - \Delta \), \( N_{04} = N_{03} - \Delta \) into equations (5) and (8) in turn to calculate the correlation coefficients of \( \rho = \{p_1, p_2, ..., p_k\} \). The variation of the correlation coefficient \( \rho \) with the location parameter \( N_0 \) was shown in Figure 3.

It can be found in Figure 3 that the correlation coefficient \( \rho \) increased first and then decreased with the increase of the location parameter \( N_0 \). The maximum value of \( \rho_j = \max\{p_k|k = 1, 2, ..., n\} \) appears in the middle of the curve, which was equal to 0.9935. The corresponding location parameter \( N_0 \) was \( N_0 = 13.57 \). Furthermore, the estimated values of the scale parameter and the shape parameter can be obtained as \( N_a = 34.09 \) and \( \beta = 2.6438 \), respectively. The Weibull distribution parameters of fatigue life estimated by the RA method were \( \{N_0 = 13.57, N_a = 34.09, \beta = 2.6438\} \).

The initial estimated value of the life unreliability based on the GA

Substituting the fatigue life data into equation (2), and setting the parameters in the three-parameter Weibull distribution as the initial given parameters of \( \{N_0 = 15.47, N_a = 34.31, \beta = 2.0007\} \), the reference value of life unreliability \( F^B \) can be obtained.

From equation (4), it is an empirical method of using the median rank to get the initial life unreliability \( F^M \). There may be a large error between the value of \( F^B \) and that of \( F^M \). The unreasonable selection of the initial iteration value will affect the accuracy of the parameter estimation results. Therefore, how to make the initial value of life unreliability \( F_0 \) closer to the reference value is a very critical point.

The genetic algorithm (GA) was used to optimize the initial value of life unreliability. The initial estimated value of the unreliability was denoted as \( F^{GA}_0 \), the expression of which was as follows, where the parameters \( \omega \) and \( \gamma \) were the undetermined coefficients by the GA.

\[
F^{GA}_0(i) = 1 - \frac{i + \omega}{n + \gamma} \tag{9}
\]

Using \( \theta_j = (\omega_j, \gamma_j)(j = 1, 2, ..., m) \) to represent the vector of the unknown parameters, and the mathematical model that estimated the unknown parameters \( \{\omega_j, \gamma_j\} \) can be established as equation (10), where \( g(\theta_j) \) represented the objective function of the GA.

\[
\min\{g(\theta_j)|j = 1, 2, ..., m\}
= \min\left\{\sum_{i=1}^{n} \left[F^B - \left(1 - \frac{i + \omega_j}{n + \gamma_j}\right)\right]^2 | j = 1, 2, ..., m\right\} \tag{10}
\]

If \( F^{GA}_0 \leq 1 \) was set in the restricted condition, it may lead to the local optimal solution at the beginning. Considering that the probability of the sample’s failure at the early stage is low, which means there were few data points with the value of \( F^{GA}_0 \) near one, the following constraints were set in the algorithm:

\[
\begin{align*}
\text{if } F^{GA}_0 > 1, & F^{GA}_0(i) = 1 \\
\text{if } F^{GA}_0 \leq 1, & F^{GA}_0(i) = 1 - \frac{i + \omega_j}{n + \gamma_j}
\end{align*} \tag{11}
\]

Substituting each individual into equations (10) and (11) to evaluate the fitness. In the GA, the population type was set as Double vector and the fitness scaling was set as Rank. Individuals were randomly selected by using roulette. In addition, the crossover was set as Scattered, and the genetic binary vectors were randomly generated.
which hybridized according to 0–1. The optimization algorithm was used for finding out a set of optimal \( f_v, g_j \) that minimized the value of the corresponding objective function \( g(u_j) \). The calculation results from the GA were shown in Figure 4. The minimum value of objective function is 0.03615 and the corresponding optimal parameters were 
\[
\begin{align*}
    f_v &= 24.479, \\
    g_j &= 8.086
\end{align*}
\]

The initial life unreliability \( \hat{F}_0^M \) obtained from the median rank and the initial life unreliability \( \hat{F}_0^G \) obtained from the GA were compared with the reference life unreliability \( F^R \), as shown in Figure 5. It can be seen from Figure 5 that the initial life unreliability curve obtained by the median rank was quite different from the reference unreliability curve, indicating that there was a large deviation between the life unreliability estimated by the median rank and the real value. Using this value as the initial iteration value may lead to the result that the iterative algorithm falling into a local optimal solution. While the initial life unreliability curve obtained by the GA was in good agreement with the reference one, which means that this method more reasonably corrected the initial estimated value of fatigue life reliability. It is more reasonable to use this value instead of the median rank as the initial value of the iterative algorithm.

**Determination of three parameters of Weibull distribution based on DGA**

In actual situation, only the failure values \( N_i \) of fatigue life of samples were obtained from the fatigue endurance test, and the initial estimation of life unreliability cannot be determined by using the GA in section The initial estimated value of the life unreliability based on the GA. It is noticed that the RA method will obtain a set of three parameters \( \{\hat{N}_0^R, \hat{N}_a^R, \hat{R}^R\} \) of Weibull distribution with a certain error. Therefore, based on the RA parameters \( \{\hat{N}_0^R, \hat{N}_a^R, \hat{R}^R\} \), the unknown parameters \( \{\omega_0, \gamma_0\} \) of initial life unreliability can be determined by using the GA in section The initial estimated value of the life unreliability based on the GA, which makes the
initial iteration value closer to the real value. And then a new objective function of the GA can be set to determine the three parameters of Weibull distribution. The GA were used twice in the process of determining the three parameters of Weibull distribution, so this parameter estimation method was called the dual genetic algorithm (DGA) in this study, which is convenient for the subsequent comparison of different parameter estimation methods. The calculation process of the DGA was shown in Figure 6.

Step 1: Obtaining the data of the sample’s fatigue life from the fatigue endurance bench test of automotive chassis parts.

Step 2: According to the expanded data from Table 2, get a set of Weibull distribution parameters \( \{ \hat{N}_0 = 13.57, \hat{N}_a = 34.09, \hat{b} = 2.6438 \} \) based on the RA method, and it would become the reference parameters of the GA objective function. In addition, if the number of the test data is sufficient, the data expansion step can be skipped.

Step 3: Subsisting the fatigue life data into equation (2) with the parameters of \( \{ \hat{N}_0 = 13.57, \hat{N}_a = 34.09, \hat{b} = 2.6438 \} \) to get the initial life unreliability \( \hat{F}_0 \). Then Substituting \( \hat{F}_0 \) into equation (10) and taking equation (10) as the objective function to establish a genetic optimization algorithm for parameter combination \( \{ \omega_j, \gamma_j \} \). The restricted requirements of the optimization algorithm were the same as those in section The initial estimated value of the life unreliability based on the GA. The results were shown in Figure 7. It can be found that the minimum value of the objective function was 0.04248, which corresponded to the parameters \( \{ \omega_j, \gamma_j \} = \{-29.448, -7.651\} \). Then the initial life unreliability \( \hat{F}_0^{GA} \) was obtained by equation (9). Compared with the initial life unreliability \( \hat{F}_0 \) determined by the median rank, \( \hat{F}_0^{GA} \) was closer to the true value, which provided input for the subsequent parameter estimation algorithm of Weibull distribution.

Step 4: The unknown parameters of genetic algorithm established in this step were \( \omega_j = \{ \hat{N}_0^{GA}, \hat{N}_a^{GA}, \hat{b}^{GA} \} \). In this genetic algorithm, on one hand, the empirical value \( \hat{F} \) of Weibull distribution function that calculated by \( \{ \hat{N}_0^{GA}, \hat{N}_a^{GA}, \hat{b}^{GA} \} \) should be close to the initial unreliability \( \hat{F}_0^{GA} \), which means meeting equation (12) to avoid divergence of iterative process. On the other hand, the independent variables and dependent variables \( \{ \hat{x}_i, \hat{y}_i \} \) which were transformed by Substituting \( \hat{F} \) into equation (5), should satisfy a highly linear correlation. It means that the value of correlation coefficient \( r \) should be as large as possible. In order to achieve the two objectives above, the mathematical model of the estimation method of the parameters \( \omega_j = \{ \hat{N}_0^{GA}, \hat{N}_a^{GA}, \hat{b}^{GA} \} \) was given by equation (13).

\[
\begin{align*}
    h_j &= \min \{ g(\theta_j) | j = 1, 2, ..., m \} \\
    &= \min \left\{ \sum_{i=1}^{n} \left[ F(N_i) - \hat{F}_0^{GA} \right]^2 | j = 1, 2, ..., m \right\} \quad (12) \\
    \min \{ \eta(\omega_j) | j = 1, 2, ..., m \} \\
    &= \min \left\{ h_j \cdot \frac{1}{p_j} | j = 1, 2, ..., m \right\} \quad (13)
\end{align*}
\]
Defining equation (13) as the objective function, obtaining the optimal parameters $^0\text{NGA}$, $^a\text{NGA}$, $^b\text{GA}$ by virtue of the optimal genetic algorithm where the corresponding value of the objective function $h(v_j)$ became minimum. Similarly, in the GA, the population type was set as Double vector, the fitness scaling was set as Rank and the crossover was set as Scattered. Individuals were randomly selected by using roulette and the genetic binary vectors were randomly generated with the range was 0–1. The calculation results of the GA were shown in Figure 8. It indicates that by restricting the value of initial life unreliability, the iterative process of the genetic algorithm converged rapidly and the calculated results were stable. The minimum value of the objective function was 0.0390, and the corresponding parameters were $^0\text{NGA}$, $^a\text{NGA}$, $^b\text{GA}$ = [16.44, 33.95, 2.2720].

**Comparison of results accuracy of the DGA and the RA method**

Calculating the life unreliability $F^B$ by $\{N_0 = 15.47, N_a = 34.31, \beta = 2.0007\}$ given in section Fatigue endurance bench tests for automotive half-axes as the reference value. The life unreliability obtained by the RA method and the DGA was denoted as $F^{RA}$ and $F^{DGA}$ respectively, and comparing them with the reference value $F^B$. Using equation (5) to transform non-linear $(N_i, F)$ into bilinear $(x_i, y_i)$, the comparison result was shown in Figure 9. It is found that the unreliability curve of the DGA fitted better to the reference curve, which means the parameter estimation result of the DGA is more accurate.

To further compare the estimation accuracy of each parameter, the error values of three parameters calculated by equation (14) were listed in Table 3, where $\{^0\text{N}_0, ^a\text{N}_a, ^b\text{DGA}\}$ were the estimated parameters and $\{N_0, N_a, \beta\}$ were the reference parameters.

$$
\begin{align*}
\text{error}_{N_0} &= \left| \frac{N_0 - ^0\text{N}_0}{N_0} \right| \cdot 100\% \\
\text{error}_{N_a} &= \left| \frac{N_a - ^a\text{N}_a}{N_a} \right| \cdot 100\% \\
\text{error}_{\beta} &= \left| \frac{\beta - ^b\text{DGA}}{\beta} \right| \cdot 100\%
\end{align*}
$$

(14)

It can be seen from Table 3 that the errors of both $N_0$ and $\beta$ based on the DGA were smaller than the RA method. Especially, the error of $N_0$ decreased from 12.28% to 6.27%, the error of $\beta$ decreased from 64.31% to 13.56%, the accuracy is improved significantly. Even though the error of $N_a$ slightly increased from 0.64% to 1.05%, overall the DGA is more accurate than the RA method. This is because the iterative initial value of the RA method deviates greatly from the real value, which may lead to that the algorithm falling into the local optimal solution. For the DGA, the closer initial fatigue life unreliability was obtained in the first genetic algorithm, which broke the
Figure 8. The result of parameters estimation by the DGA.

Figure 9. Comparison of life unreliability between the DGA and the RA method.

Table 3. Comparison of errors between the DGA and the RA method.

| Method         | $N_0$ | $N_a$ | $\beta$ | $\text{error}_{N_0}$ (%) | $\text{error}_{N_a}$ (%) | $\text{error}_{\beta}$ (%) |
|----------------|-------|-------|---------|---------------------------|--------------------------|--------------------------|
| Reference value| 15.47 | 34.31 | 2.0007  | –                         | –                        | –                        |
| RA method      | 13.57 | 34.09 | 2.6438  | 12.28                     | 0.64                     | 64.31                    |
| DGA            | 16.44 | 33.95 | 2.2720  | 6.27                      | 1.05                     | 13.56                    |
limitation of the local optimal solution. Additional, using two objective functions to restrict the second genetic algorithm can avoid the divergence of the iterative process, so the iteration speed and the precision of parameter estimation were improved at the same time.

Conclusion

In this study, the fatigue endurance bench tests of the automotive half-axles were carried out. The three-parameter Weibull distribution model was established according to the test data. The right approximation method and the genetic algorithm were applied respectively to estimate the parameters of Weibull distribution, and the effect on results accuracy of the initial iterative value was explored. Additionally, the right approximation method and genetic algorithm were combined to propose a new estimation method named dual genetic algorithm. The main conclusions are as follows:

1. In the right approximation method, using the median rank to estimate the life unreliability deviated greatly from the real value. Using this value as the iterative initial value may cause the iterative algorithm to fall into the local optimal solution and affect the estimation accuracy.

2. A new estimation method named dual genetic algorithm was proposed to estimate the three parameters of Weibull distribution. Taking the distribution parameters obtained by the right approximation method as the reference value, the initial unreliability of fatigue life that closer to the real value was obtained by the first genetic algorithm. By setting the constraint on double objective function, the final three parameters of Weibull distribution were determined by the second genetic algorithm.

3. The dual genetic algorithm based on reference parameters of right approximation method can overcome the problem of local optimal solution effectively. In addition, by setting the constraint of double objective function, the dual genetic algorithm avoided the divergence in the iteration process, which improved the accuracy of parameter estimation and improved the iteration speed. This method provides new ideas and theoretical basis for reliability assessment of mechanical components.

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