Method for Tuning the Parameters of Active Force Reducing Building Vibrations—Numerical Tests

Andrzej Dymarek 1,*, Tomasz Dzitkowski 1, Krzysztof Herbuś 1, Piotr Ociepka 1, Andrzej Niedworok 2, and Łukasz Orzech 2

1 Faculty of Mechanical Engineering, Silesian University of Technology, 44-100 Gliwice, Poland; tomasz.dzitkowski@polsl.pl (T.D.); krzysztof.herbus@polsl.pl (K.H.); piotr.ociepka@polsl.pl (P.O.)
2 Laboratory of Applied Tests, KOMAG Institute of Mining Technology, 44-101 Gliwice, Poland; aniedworok@komag.eu (A.N.); lorzech@komag.eu (Ł.O.)
* Correspondence: andrzej.dymarek@polsl.pl

Abstract: The paper formulates a method of active reduction of structure vibrations in the selected resonance zones of the tested object. The method ensures reduction of vibrations of the selected resonance zones by determining the parameters of the active force that meets the desired dynamic properties. The paper presents a method for determining the parameters of the active force by reducing the vibrations of the structure in its resonance zones to a given vibration amplitude. For this purpose, an analytical form was formulated, which will clearly define the dynamic properties of the tested object and the force reducing the vibrations in the form of a mathematical model. The formulated mathematical model is a modified object input function, which in its form takes into account the properties of the active force reducing the vibrations. In such a case, it is possible to use the methods of mechanical synthesis to decompose the modified characteristic function into the parameters of the system and the parameters of the force being sought. In the formulated method, the desired dynamic properties of the system and the vibration reducing force were defined in such a way that the determined parameters of the active force (velocity-dependent function) had an impact on all forms of natural vibrations of the tested system. Based on the formalized method, the force reducing the vibrations of the four-story frame to the desired displacement amplitude was determined. Two cases of determining the active force reducing the vibrations to the desired vibration amplitude of the system by modifying the dynamic characteristics describing the object together with the active force were considered. For both cases, the system’s responses to the oscillation generated by harmonic force of frequencies equal to the first two forms of natural vibrations of the tested system were determined. In order to verify the determined force reducing the vibrations of the object and to create a visualization of the analyzed phenomenon, the building structure dynamics were analyzed with the use of PLM Siemens NX 12 software. The determined force parameters were implemented into the numerical model, in which the tested system was modeled, and the response time waveforms were generated with regard to the considered story. The generated waveforms were compared with the waveforms obtained in the formalized mathematical model for determining the active force reducing the vibrations. The vibrations of the tested numerical model were induced by the kinematic excitation with the maximum amplitude equal to 100 mm, which corresponds to the vibration amplitude during the earthquake with a force equal to level 5 on the Richter scale.

Keywords: structure vibrations; resonance zones; numerical model; building

1. Introduction

Advanced construction methods and the desire to build impressive structures have made buildings increasingly taller. Unfortunately, an increase in height is accompanied by an increase in the probability of low natural frequencies occurring, hence these buildings are susceptible to vibrations. Vibrations can be caused by wind or earthquakes, which can
be very troublesome for residents. Vibrations of building structures can also be induced by
detonation of an explosive, i.e., by para-seismic vibrations. Para-seismic vibrations, such as
earthquakes or wind excitation, belong to the group of non-stationary signals, consisting
of a whole series of harmonic components, which means that an attempt to analyze the
impact of this type of vibration on the environment is a complicated process that requires
many factors that may affect their intensity.

Striving to meet the requirements for the protection of people against the negative
impact of vibrations resulting from the development of civilization has resulted in great
interest in reducing the vibrations. The dynamic impact on structures is reduced through
the structural control of the tested infrastructure. To ensure the effective functionality
of flexible structures, various modifications are possible, from alternative designs to the
use of additional control systems attached to the structure. These include active, passive,
and semi-active control systems. Passive reduction of vibrations in building structures is
realized by the introducing passive devices in the form of external dampers [1–3], damping
materials [4–7], or dynamic vibration eliminators (TMD) [8–15], which reduce building
vibrations. The use of passive devices is becoming popular all over the world, both in
new building constructions and modernizations in terms of seismic resistance [16–18]. The
physical properties to dissipate energy without the use of an additional energy source is the
advantage of passive devices. However, this approach does not always allow for reduction
of structural vibrations within a controlled range, therefore use of active and semi-active
control systems has begun. Contrary to passive systems, active and semi-active systems
use control devices that play a superior role in them.

Among active devices, the active dynamic vibration eliminator (ATMD) is popular, in
which a control force is generated by various control algorithms based on wavelet trans-
form [19], neural networks [20,21], particle swarm optimization [22,23], linear quadratic
controller [24], and the use of fuzzy logic drivers (FLC) [25–28]. Work on the use of active
devices using FLC in ATMD is concerned not only the control itself, but also the compar-
ison of the results with those obtained by other methods [26–31]. Another type of work,
which dealt with active devices used to reduce vibrations of building structures, concerned
testing the fixation place of such systems and their impact on the structure response [32,33].
However, the main work in this field concerns the use of various types of control algorithms
to mitigate the effects of vibrations in the structure caused by natural phenomena.

Therefore, it is necessary to define the structure searching criterion and parameters
of the model based on the knowledge of the dynamic properties of the real object. The
work uses the methods of synthesis which enable the identification of the parameters
of desired properties, such frequency and amplitude. Parameters of active elements are
generated in the systems subjected to vibration reduction on the basis of information about
the dynamic state of the system [34,35]. The reduction of vibrations comes down to the
determination of parameters of the control systems in the feedback loop that meet specific
vibration amplitude of a discussed material point. Conditions in which the structural and
parametric identification of the system [36] subjected to vibration reduction are realized
should meet the required dynamic properties in advance.

This paper presents a method for determining the parameters of the active force re-
ducing the vibrations of the structure in its resonance zones to a given vibration amplitude.
For this purpose, an analytical form was developed, in the form of a mathematical model,
which will clearly define the dynamic properties of the tested object and the force-reducing
vibrations. The formulated mathematical model is a modified object-input function, which
in its form considers the properties of the active force-reducing vibrations. In such a case, it
is possible to use the methods of mechanical synthesis to decompose the modified charac-
teristic function into the parameters of the system and the parameters of the force being
sought. The method is universal in the fact that the force parameters can be selected in
such a way that they affect all the resonance zones of the tested system. Thus, the dynamic
properties expressed in the form of a mathematical model reflect the conditions that the
sought active force should meet in order to ensure its optimal parameters in terms of the
amount of energy needed to generate it, the place of attachment, or the number of determined active forces. Despite many works in the field of vibration reduction of building structures, none of them deal with the reduction of the systems to the desired vibration amplitude. The authors use the methods of synthesis of mechanical systems to search for fully controlled vibration reduction methods using passive and active elements [14,34–37]. The presented paper is the next stage of the work using synthesis methods to reduce vibrations in building structures.

Two cases of determining the active force reducing the vibrations to the desired amplitude of the system, by modifying the dynamic characteristics describing the object together with the active force, were considered. The vibrations of the model of the tested structure were induced by a kinematic excitation simulating an earthquake equal to 5 degrees on the Richter scale. For both cases, the responses of the system for excitation by the harmonic force of frequencies equal to the first two forms of the natural vibrations of the tested system were generated. To verify the determined force reducing the vibrations of the object and to create a visualization of the analyzed phenomenon, the building structure was dynamically analyzed with the use of PLM Siemens NX 12 software [38–40]. The determined force parameters were implemented into a numerical model in which the tested system was modelled, and the response time waveforms were generated for each story. The generated waveforms were compared with the waveforms obtained based on the mathematical model used for determining the active force reducing vibrations.

2. Method of Active Reduction of Vibrations

The chapter presents two methods for active reduction of vibrations in the analyzed building model in the form of a sheared frame shown in Figure 1. Dynamic properties of the structure, described in the form of frequency transition functions, are the basic criteria used to determine the active forces. Assuming the desired dynamic properties and taking into account the damping of resonance zones in analyzed transition functions will allow avoidance of vibrations that are burdensome for people and affect the durability of the building structure. Determination of force parameters due to the desired dynamic properties will be presented in the paper based on original methods, included in the group of tasks for the synthesis of mechanical systems [14,34–37,41–43].

![Figure 1. Model of the sheared frame.](image-url)
In order to determine the vibration reduction parameters of the sheared frame model (Figure 1), the dynamic parameters of the tested system should be given in an analytical form. The method for determining the analytical form of dynamic characteristics, presented in the paper, consists in adopting a series of resonance and anti-resonance frequencies (poles and zeros of the dynamic characteristics sought) in the following form:

$$\omega_{b1} < \omega_{z1} < \omega_{b2} < \omega_{z2} < \ldots < \omega_{b(i-1)} < \omega_{b1}$$

where: $\omega_{b1}, \omega_{b2}, \ldots, \omega_{bi}$ — resonance frequencies (circular frequencies of natural vibrations in an undamped system); $\omega_{z1}, \omega_{z2}, \ldots, \omega_{zi}$ — anti-resonance circular frequencies; $i = 1, 2, 3, \ldots, n$. Based on the adopted dynamic properties, characteristic functions are determined as dynamic flexibility $Y(s)$ or dynamic stiffness $Z(s)$, defined by the following formula:

$$Y(s) = \frac{1}{Z(s)} = Y_{11}(s) = \frac{x_1(s)}{F_1(s)} = H \left( \frac{s^2 + \omega_{11}^2}{s^2 + \omega_{b1}^2} \right) \left( \frac{s^2 + \omega_{21}^2}{s^2 + \omega_{b2}^2} \right) \ldots \left( \frac{s^2 + \omega_{iz}^2}{s^2 + \omega_{b(i-1)}^2} \right),$$  

(1)

where: $s$—complex variable; $\varphi_1(s)$—Laplace transform of the linear displacement of the first inertial element established at zero initial conditions; $F_1(s)$—Laplace transform of the exciting force with respect to the first inertial element established at zero initial conditions; $H$—dimensionless proportionality coefficient.

The dynamic characteristics (1) should be modified by damping the vibration within the resonance zones for active reduction of vibration. Two methods of such a modification will be presented. Characteristic of the circular frequency of the damped natural vibrations $\omega_{ti}$ is used in the first method by introducing the damping coefficients $h_i$ into the following relationship:

$$h_i = \frac{\delta_i}{2\pi \omega_{ti}},$$  

(2)

where: $\delta_i$—logarithmic decrement of damping.

Then the values included in the characteristics of the natural vibration frequency $\omega_{ti}$ can be expressed in the form of the following relationships:

$$\omega_{ti} = \sqrt{\omega_{b1}^2 - h_i^2},$$  

(3)

while the dynamic susceptibility takes the following form:

$$Y2(s) = \frac{\prod_{i=1}^{n-1} (s^2 + \omega_{zi}^2)}{\prod_{i=1}^{n} (s^2 + 2h_i s + \omega_{bi}^2)},$$  

(4)

where: $h_i$—damping coefficient related to the $i$-th resonant frequency; $\omega_{zi}$—damping coefficient related to the $i$-th anti-resonance frequency.

Another method of modifying the characteristic also consists in damping the vibrations of the resonant zones by introducing the damping coefficients $h_i$. However, in this case, it is the resonant frequency that is the frequency of natural damped vibrations, and the dynamic susceptibility takes the following form:

$$Y3(s) = \frac{\prod_{i=1}^{n-1} (s^2 + \omega_{zi}^2)}{\prod_{i=1}^{n} (s^2 + h_i^2 s + \omega_{bi}^2)},$$  

(5)

It should be emphasized that in both cases of modification of the dynamic characteristics, consisting of damping the vibrations of the resonance zones, the modification can be done in relation to all resonance areas of the characteristics (in accordance with Equations (4) and (5)), as well as in relation only to the selected resonance areas. Selection of one of the options for damping the characteristic, as well as the number of damped areas, directly affects the form of the determined dynamic parameters in a form of active
force. Values of the damping coefficients \( h_{i1}, h_{i2}, \ldots, h_{in} \), introduced into the dynamic characteristics should be selected in such a way as to obtain the expected values of the forced vibration amplitudes of the analyzed system [34].

In order to dampen the vibrations of the resonant zones of the modified characteristics, the action of additional dynamic parameters is assumed in the form of the active force \( F_{Ai} \) applied to the \( n \)-th inertial element, the form of which may be as follows:

\[
F_{Ai}(t) = b_{ni}x_1(t) + b_{n2}x_2(t) + \ldots + b_{ni}x_n(t),
\]

or

\[
F_{Ai}(t) = k_{ni}x_1(t) + b_{11}x_1(t) + \ldots + k_{ni}x_n(t) + b_{ni}\ddot{q}_n(t),
\]

where: \( b_{ni} [\text{Ns/m}] \) — amplification factors of the active force applied to the \( n \)-th inertial element depending on the linear speed \( x(t) \), \( \dot{x}(t) = [\dot{x}_1(t), \dot{x}_2(t), \ldots, \dot{x}_n(t)]^T \) corresponding to the generalized coordinates of the points defining the position of the inertial elements; \( k_{ni} [\text{N/m}] \) — amplification factors of the active force applied to the \( n \)-th inertial element, dependent on linear displacements \( x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \), corresponding to the generalized coordinates of the points defining the position of inertial elements; \( i = 1, 2, \ldots, n \). The active force in the form (6) is generated, when the circular frequency of damped natural vibrations \( \omega_n \) is considered in relation to only one selected resonance frequency. In all other cases, the active force will have the form (7).

Next, the method of determining the values of the amplification coefficients of active forces is presented. Based on the specified dynamic parameters of the structure model of the system, the dynamic stiffness matrices of the system are determined in the following form:

\[
Z(s) = \begin{bmatrix}
m_1s^2 + c_1 + c_2 & -c_2 & \cdots & 0 \\
-c_2 & m_2s^2 + c_2 + c_3 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & m_ns^2 + c_n
\end{bmatrix},
\]

and using the relationship (6), the following matrix of active force coefficients \( F_A(s) \) is determined (for example, the force was applied to the last inertial element):

\[
F_A(s) = \begin{bmatrix}
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
b_{n1}s & b_{n2}s & \cdots & b_{ns}
\end{bmatrix},
\]

and using the relationship (7), the following matrix of active force coefficients \( F_A(s) \) (for example, the force was applied to the last inertial element) was determined:

\[
F_A(s) = \begin{bmatrix}
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
k_{n1} + b_{n1}s & k_{n2} + b_{n2}s & \cdots & k_{ns} + b_{ns}
\end{bmatrix}.
\]

The rows of the matrix (9 and 10) correspond to the active forces and the generalized coordinates of the points determining the position of the inertial elements of the analyzed models of systems subjected to vibration reduction to which the sought forces are applied. On the basis of the matrix \( F_A(s) \) and the stiffness matrix \( Z(s) \), the following polynomial is determined:

\[
\det(Z(s) + F_A(s)) = A_{2n}s^{2n} + A_{2n-2}s^{2n-2} + A_{2n-4}s^{2n-4} + \ldots + A_0.
\]
In order to calculate the active force parameters, the obtained polynomials should be divided by the $A_{2n}$ coefficient, and then compared to the polynomial characterizing the dynamic properties of the analyzed system in the form of resonance frequencies and damping coefficients related to all or selected resonance frequencies. Such equations take the following form:

$$\frac{\det(Z(s) + F_A(s))}{A_{2n}} = \prod_{i=1}^{n} (s^2 + 2h_i + \omega_{bi}^2).$$

(12)

or

$$\frac{\det(Z(s) + F_A(s))}{A_{2n}} = \prod_{i=1}^{n} (s^2 + h_i^2 + 2h_i + \omega_{bi}^2).$$

(13)

when comparing the coefficients at the same polynomial powers of Equations (12) or (13), a system of equations is built, on the basis of which the parameters of active forces reducing the vibrations of the identified system are determined.

3. Numerical Example of the Analyzed System

The analyzed model is a four-story wall frame shown in Figure 2, the dynamic parameters of which have been taken from work [33]. Parameters of the inertial and elastic elements of the tested system are as follows: $m_1 = 450,000$ [kg], $m_1 = m_2 = m_3 = 345,000$ [kg], $c_{40} = 18,050,000$ [N/m], $c_{34} = 326,000,000$ [N/m], $c_{23} = 285,000,000$ [N/m], and $c_{12} = 250,000,000$ [N/m].

![Figure 2. Model of the tested system.](image)

The system was excited by a force in the form of the maximum displacement $x_0 = x_1 + x_2 = 50 + 50 = 100$ mm corresponding to the amplitude of the force caused by ground movements equal to 5 on the Richter scale, simulating the para-seismic movements caused by human impact on the rock mass during mining operations. Moreover, it is assumed that the frequencies of the para-seismic vibrations are close to the first and second harmonics of the tested model: $x_1 \sin(\omega_1 t)$ and $x_2 \sin(\omega_2 t)$. 
Based on the dynamic values of the system, the characteristic function of the system is determined for the inertial element \( m_1 \) in the following form:

\[
Y(s) = H \frac{s^2(s^2 + \omega_2^2)(s^2 + \omega_3^2)(s^2 + \omega_4^2)}{(s^2 + \omega_1^2)(s^2 + \omega_2^2)(s^2 + \omega_3^2)(s^2 + \omega_4^2)}
\]

\[
= \frac{s^2(s^2 + 13.1376^2)(s^2 + 35.3303^2)(s^2 + 51.2401^2)}{(s^2 + 3.4096^2)(s^2 + 21.1928^2)(s^2 + 39.4225^2)(s^2 + 52.8776^2)}. \tag{14}
\]

In the case of the considered system, it is assumed that the vibrations of the resonance zones are reduced in relation to the first two resonance frequencies of the system by applying an active force acting on the last (fourth) story. Moreover, it was assumed that the system was to meet the required value of the forced vibration amplitude of the last inertial element, in the form of a linear displacement \( A_{bx1} \leq 0.04 \) m, with the frequency of the excitation force equal to the first and second resonance frequencies. The form of the modified characteristic functions of the system for such assumptions are as follows:

\[
Y_2(s) = \frac{s^2(s^2 + 13.1376^2)(s^2 + 35.3303^2)(s^2 + 51.2401^2)}{(s^2 + 2h_1 + 3.4096^2)(s^2 + 2h_2 + 21.1928^2)(s^2 + 39.4225^2)(s^2 + 52.8776^2)},
\]

or

\[
Y_3(s) = \frac{s^2(s^2 + 13.1376^2)(s^2 + 35.3303^2)(s^2 + 51.2401^2)}{(s^2 + 2h_1 + h_1^2 + 3.4096^2)(s^2 + 2h_2 + h_2^2 + 21.1928^2)(s^2 + 39.4225^2)(s^2 + 52.8776^2)}.
\]

The following damping coefficients were determined in relation to the assumed vibration amplitude:

- In the case of the first method for reduction of vibrations:
  \[ h_{b1} = 4.9 \text{ rad/s}, \quad h_{b2} = 1.2 \text{ rad/s}, \]

- In the case of the second method for reduction of vibrations:
  \[ h_{b1} = 4.2 \text{ rad/s}, \quad h_{b2} = 1.2 \text{ rad/s}. \]

Then the dynamic parameters of the active forces \( F_{A1} \) (inertial \( m_1 \)), determined on the basis of the relationships (8–13), assume the values listed in Table 1.

| Table 1. Determined dynamic parameters of active forces. |
|---------------------------------------------------------|
| \( i \) | I Method of Reduction | II Method of Reduction |
|        | \( k_{1i} \) [N/m] | \( b_{1i} \) [Ns/m] | \( k_{1i} \) [N/m] | \( b_{1i} \) [Ns/m] |
|--------|---------------------|---------------------|---------------------|---------------------|
| 1      | 18.03               | 9.35                | 30.08               | 8.28                |
| 2      | 6.57                | 8.09                | 19.38               | 7.09                |
| 3      | -7.31               | 6.50                | 6.31                | 5.60                |
| 4      | -21.10              | 6.97                | -2.98               | 5.91                |

In order to obtain the desired amplitude of the displacement of the element \( m_1 \), the obtained active force should be multiplied by the value of the inertial element, on which the exciting force acts. In the considered case, this value is equal to 450,000.

In the next step, using professional Matlab/Simulink software, calculations enabled checking the correctness of the results of active reduction of vibration in the analyzed system. The time waveform of the displacement of the first and last stories was generated, taking into account the action of the exciting force and active force on the system (see Figures 3–5).
Figure 3. The object response to the dynamic excitation caused by para-seismic vibrations with the frequency of the first and second harmonics of the system at a maximum amplitude of 100 mm.

Figure 4. The system response in the case of active force $F_{A1}$ action on the inertial elements (I method of reduction): (a) $m_4$; (b) $m_1$.

Figure 5. The system response in the case of active force $F_{A1}$ action on the inertial elements (II method of reduction): (a) $m_4$; (b) $m_1$. 
4. Description of the Numerical Model of the Analyzed Building

CAD/CAE software, PLM Siemens NX 12 software was used to create the virtual model of the building. Verification of the determined parameters of the forces reducing the vibrations of the object and the visualization of the analyzed phenomenon was carried out in the motion simulation module, and the “simcenter motion” solver was used for the calculations. At the first step, a 3D model of the building was created (Figure 6). Each story of the building was modelled in the form of plates of dimensions 15 m \( \times \) 20 m \( \times \) 0.5 m, and the load-bearing columns in the form of beams of a cross-section dimension 0.3 m \( \times \) 0.5 m and a height of 3.5 m. The dimensions of the plates were selected so that the masses of the story (assuming that they will be the plates and beams made of reinforced concrete) correspond to the parameters of the analyzed system.

Flexible link elements were used to create yielding system for the building supporting columns. These are the elements that may deform during dynamic analysis and are converted at each iterative step of the analysis using the FEM solver (NX Nastran). For this purpose, an initial modal analysis was performed using the FEM method, determining the frequencies and modes of natural vibrations of the supporting columns. At the first step, the supporting columns were discretized with finite elements of the CHEXA type (8), then appropriate boundary conditions were introduced, and plastic parameters were assigned. In order to obtain the appropriate stiffness of the supporting columns, while defining the plastic parameters, the equivalent Young’s modulus [33] was determined in accordance with the following relationship (15):

\[
E = \frac{c \cdot I^3}{12 \cdot J'}
\]

where: \( c \) — stiffness of the supporting, \( l \) — height of the column, \( J \) — geometric moment of the column cross-section inertia.

Figure 7 shows the FEM model of the analyzed supporting column, the introduced boundary conditions, and its first determined form of vibrations.

During the numerical tests, two cases were considered. The first concerned reaction of the system to paraseismic excitation \( x(t) \), while in the second case a vibration reducing force \( F_{A1}(\dot{x}, x, t) \) was additionally introduced in the model.
5. Tests and Analysis of the Results

In the first stage of the tests, a case (Figure 8), in which the building is affected only by a paraseismic excitation of a displacement applied to the base, with amplitude $A$ and frequencies equal to the first $\omega_1$ and the second $\omega_2$ of the natural frequency of the system, was considered.

\begin{equation}
    x(t) = A \cdot (\sin(\omega_1 \cdot t) + \sin(\omega_2 \cdot t))
\end{equation}

Figure 7. FEM model, the adopted boundary conditions and the first form of vibration of the supporting column [33].

Figure 8. Applied excitation in the first analyzed case.
where: \( A \) — vibration amplitude (in the analysis \( A = 50 \text{ mm} \) was assumed), \( \omega_1 \) — first vibration frequency (in the analysis \( \omega_1 = 3.4096 \text{ rad/s} \) was assumed), \( \omega_2 \) — second vibration frequency (in the analysis \( \omega_2 = 21.1928 \text{ rad/s} \) was assumed).

Figure 9 presents a diagram of vibration amplitude in time, in relation to the first story of the building (\( m_4 \)).

![Figure 9. Vibration amplitude in time for the first story recorded on the sensor Se_M4D.](image)

Analyzing the diagram (Figure 9), it can be observed that the building get into resonance (rumbling) and there is a large amplitude of building vibrations (up to 600 mm).

In the second stage of numerical tests, when active force \( F_{A1} \) reducing vibrations was added to the system (Figure 10), to exit the resonance zone, thus damping the structure vibrations generated by a given kinematic excitation (16) was analyzed.

![Figure 10. The applied excitation in the second analyzed case.](image)

Since the active \( F_{A1} \) force is a variable in time and depends on the displacements and their velocities, the values of which are time dependent; therefore, to simulate the process of active vibration damping in the analyzed building, a feedback had to be entered to the model by the use of markers and sensors. They allow to read the displacements...
and their velocities in each story in time and the excitation. As regards the analysis, the forces reducing the vibrations to the assumed amplitude were defined in the Siemens NX software, taking into account the previously determined dynamic parameters with respect to the I (17) and II (18) vibration reduction methods.

\[ F_{A1}(\ddot{x}, x, t) = -0.001 \cdot 450,000 \cdot (30.08 \cdot Se_{M1D} + 8.28 \cdot Se_{M1V} + 19.38 \cdot Se_{M2D} + 7.09 \cdot Se_{M2V} + 6.31 \cdot Se_{M3D} + 5.6 \cdot Se_{M3V} - 2.98 \cdot Se_{M4D} + 5.91 \cdot Se_{M4V}) \]  
(17)

\[ F_{A1}(\ddot{x}, x, t) = -0.001 \cdot 450,000 \cdot (18.03 \cdot Se_{M1D} + 9.35 \cdot Se_{M1V} + 6.57 \cdot Se_{M2D} + 8.09 \cdot Se_{M2V} - 7.31 \cdot Se_{M3D} + 6.5 \cdot Se_{M3V} - 21.1 \cdot Se_{M4D} + 6.97 \cdot Se_{M4V}) \]  
(18)

To measure the displacement and velocity of each story over time, sensors (Table 2) were introduced into the model that measure the distances and velocities between markers placed on the stories (A1–A4) and a marker placed on the base of the building A0 (Figure 11).

Table 2. Description of the sensors introduced to the model.

| Sensor Number | Sensor Name | Measured Quantity |
|---------------|-------------|-------------------|
| S1            | Se_{M1D}   | displacement between markers A1 and A0 in x axis, |
| S2            | Se_{M2D}   | displacement between markers A2 and A0 in x axis, |
| S3            | Se_{M3D}   | displacement between markers A3 and A0 in x axis, |
| S4            | Se_{M4D}   | displacement between markers A4 and A0 in x axis, speed of A1 marker movement in relation to marker A0 in x axis, |
| S5            | Se_{M1V}   | speed of A2 marker movement in relation to marker A0 in x axis |
| S6            | Se_{M2V}   | speed of A3 marker movement in relation to marker A0 in x axis |
| S7            | Se_{M3V}   | speed of A4 marker movement in relation to marker A0 in x axis |
| S8            | Se_{M4V}   | speed of A4 marker movement in relation to marker A0 in x axis |

Figure 12 shows curves of displacement for each story in response to a given excitation with use of the vibration reduction force determined according to the method I.

Figure 13 shows response curves of displacements for each story to a given excitation, taking into account the vibration reduction force determined according to the method II.

Comparing the results of both analyzes (Figure 4, Figure 5, Figure 12, and Figure 13), it can be observed that introduction of the force \( F_{A1}(\ddot{x}, x, t) \) caused the system to exit the resonance zone and significantly reduce the vibration amplitude of each story. Figures 14 and 15 show the graphic visualization of the results. Behavior of the building model without the active vibration damping force and with the use of the force \( F_{A1}(\ddot{x}, x, t) \), was compared.

Figures 14 and 15 show a graphic visualization of the results. The maximum displacements of the floor slabs and load-bearing columns in the case of resonance vibrations (Figure 14) and in the case of application of the proposed vibration damping method, are shown. The displacements of each story can be read from the diagrams presented in Figures 12 and 13. The enlargement (detail) shown in Figure 14 shows the range of displacements in relation to the floor slab m_1.
Figure 11. Arrangement of markers (A1–A4), and sensors (S1–S8).

Figure 12. The displacement in time depending on the story m1, m2, m3 and m4 with use of the vibration reducing force determined in accordance with method I using the PLM Siemens NX 12 software.
Figure 13. The displacements in time for stories m1, m2, m3, and m4 when using the vibration reduction force determined according to the method II using the PLM Siemens NX 12 software.

Figure 14. Maximum building deflection recorded during the simulation of a system without the use of vibration-reducing force.

Figure 15. Maximum building deflection recorded during the simulation of a system with the use of vibration-reducing force.
6. Conclusions

Earlier research work of the authors dealt with the reduction of system vibrations to the desired amplitude by determining the value of passive and active elements supporting the reduction. In the work [36], in the case of tests carried out on the same model, the determination of the parameters of the active force dependent on the displacement was concerned, and the reduction concerned one natural frequency of the system. On the other hand, in this work, the tests were aimed at formalizing a method that would allow the reduction of several natural frequencies of the system exposed to resonance, and the force sought was a function dependent on displacement and velocity. This approach reduced the amplitude of all modes of natural vibrations of the tested system and eliminated lack of stabilization of the desired amplitude on all stories [36]. In such a case [36], the amplitude of each story increases up to three times the value of the assumed amplitude in relation to the displacement of the story to which the determined active force is applied.

The main objective of the tests was to verify the correctness of the formalized method of selecting the parameters for active vibration reduction. The article discusses the issue of active vibration reduction of a building structure on the example of a four-story building. For this purpose, an active synthesis of determining the controlling force reducing the vibration amplitudes of the selected resonance frequencies, was used. The model of the discussed building was loaded with two harmonic forces with excitation frequencies coinciding with at least two modes of natural vibrations of the tested system. In addition, it was tested whether the active force reduces the vibrations of the system to a maximum of the desired amplitude when excited by several frequencies.

The considered 3D model was designed in the PLM Siemens NX 12 program. The model was subjected to vibration simulation in accordance with the obtained parameters and the conditions of the analysis of the 2D model. Based on the analysis of the system being a 2D model of the building and the simulation on the 3D model, the following conclusions can be drawn:

- The results of numerical analyses using PLM Siemens NX 12 software confirmed that the vibration reduction forces were determined correctly;
- The formalized method is effective for vibrations excited by forces of different frequencies;
- The obtained displacements of each story of the 3D model are within the accepted value of the vibration amplitude $A_{b1} \leq 0.04 m$;
- The max displacement of the stories are close to the assumed amplitude and occur on the extreme stories of the model;
- The obtained results suggest that the method can be used in the selection of parameters of the active force reducing the vibrations of structures caused by an earthquake, which are characterized by a wide frequency band.

The authors intend to extend the methods of synthesizing mechanical systems in reducing the vibrations of building structures by determining several active forces acting on each story. Such an approach will make it possible to replace active forces with passive or mixed passive–active elements. Determination of few active forces will increase the flexibility in the selection of elements and reduce the energy demand needed to supply the active forces reducing the vibrations of building structures. In addition, some tests are also planned to determine the sensitivity of the obtained settings of the active force with regard to the desired value of the vibration amplitude of the model.

**Author Contributions:** Conceptualization: A.D., T.D., K.H. and P.O.; methodology: A.D., T.D., K.H. and A.N.; software: T.D. and K.H.; validation: A.D. and Ł.O.; formal analysis: Ł.O.; investigation, K.H.; resources: P.O. and A.N.; data curation: P.O. and A.N.; writing—original draft preparation: A.D. and K.H.; writing—review and editing: A.N. and Ł.O.; visualization: A.D. and T.D.; supervision, A.D.; project administration: T.D. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research received no external funding.
Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Arroyo-Espinoza, D.; Teran-Gilmore, A. Strength reduction factors for ductile structures with passive energy dissipating devices. J. Earthq. Eng. 2003, 7, 297–325. [CrossRef]

2. Cardone, D.; Dolce, M.; Ponzo, F.C. The behaviour of sma isolation systems based on a full-scale release test. J. Earthq. Eng. 2006, 10, 815–842. [CrossRef]

3. Mirtaheri, M.; Zandi, A.P.; Samadi, S.S.; Samani, H.R. Numerical and experimental study of hysteretic behavior of cylindrical friction dampers. Eng. Struct. 2011, 33, 3647–3656. [CrossRef]

4. Brando, G.; D’Agostino, F.; De Matteis, G. Experimental tests of a new hysteretic damper made of buckling inhibited shear panels. Mater. Struct. 2013, 46, 2121–2133. [CrossRef]

5. Rai, D.C.; Annam, P.K.; Pradhan, T. Seismic testing of steel braced frames with aluminum shear yielding dampers. Eng. Struct. 2013, 46, 737–747. [CrossRef]

6. Chen, J.; Zhang, C.; Wu, M. The performance of low-yield-strength steel shear-panel damper with without buckling. Mater. Struct. 2015, 48, 1233–1242. [CrossRef]

7. Cho, S.G.; Chung, S.; Sung, D. Application of Tuned Mass Damper to Mitigation of the Seismic Responses of Electrical Equipment in Nuclear Power Plants. Energies 2020, 13, 427. [CrossRef]

8. Shen, Y.; Peng, H.; Li, X.; Yang, S. Analytically optimal parameters of dynamic vibration absorber with negative stiffness. Mech. Syst. Signal Process 2017, 85, 193–203. [CrossRef]

9. Wang, X.; Liu, J.; Shan, Y.; Shen, Y.; He, T. Analysis and optimization of the novel inerter-based dynamic vibration absorbers. IEEE Access 2018, 6, 33169–33182. [CrossRef]

10. Warburton, G.B. Optimum absorber parameters for various combinations of response and excitation parameters. Earthq. Eng. Struct. Dyn. 1982, 10, 381–401. [CrossRef]

11. Sadek, F.; Mohraz, B.; Taylor, A.W.; Chung, R.M. A method of estimating the parameters of tuned mass dampers for seismic applications. Earthq. Eng. Struct. Dyn. 1997, 26, 617–635. [CrossRef]

12. Barano, G.C.; Greco, R.; Chiaia, B. A comparison between different optimization criteria for tuned mass dampers design. J. Sound Vib. 2010, 329, 4880–4890. [CrossRef]

13. Bekdaş, G.; Nigdeli, S.M. Estimating optimum parameters of tuned mass dampers using harmony search. Eng. Struct. 2011, 33, 2716–2723. [CrossRef]

14. Dymarek, A.; Dzitkoñski, T. The use of synthesis methods in position optimisation and selection of tuned mass damper (TMD) parameters for systems with many degrees of freedom. Arch. Control Sci. 2021, 31, 185–211.

15. Lian, J.; Zhao, Y.; Lian, C.; Wang, H.; Dong, X.; Jiang, Q.; Zhou, H.; Jiang, J. Application of an eddy current-tuned mass damper to vibration mitigation of offshore wind turbines. Energies 2018, 11, 3319. [CrossRef]

16. Cheraghì, A.; Zahrî, S.M. Cyclic testing of multilevel pipe in pipe damper. J. Earthq. Eng. 2019, 23, 1695–1718. [CrossRef]

17. Formisano, A.; Lombardi, L.; Mazzolani, F.M. Perforated metal shear panels as bracing devices of seismic-resistant structures. J. Constr. Steel Res. 2016, 126, 37–49. [CrossRef]

18. Javanmardi, A.; Ibrahim, Z.; Ghadimi, H. Seismic isolation retrofitting solution for an existing steel frame. Energies 2021, 14, 8293.
27. Samali, B.; Al-Dawod, M. Performance of a five-storey benchmark model using an active tuned mass damper and a fuzzy controller. Eng. Struct. 2003, 25, 1597–1610. [CrossRef]
28. Samali, B.; Aldawod, M.; Kwok, K.; Naghdy, F. Active control of cross wind response of 76-story tall building using a fuzzy controller. J. Eng. Mech. 2004, 130, 492–498. [CrossRef]
29. Guclu, R.; Yazici, H. Vibration control of a structure with ATMD against earthquake using fuzzy logic controllers. J. Sound Vib. 2008, 318, 36–49. [CrossRef]
30. Aldawod, M.; Samali, B.; Naghdy, F.; Kwok, K.C. Active control of along wind response of tall building using a fuzzy controller. Eng. Struct. 2001, 23, 1512–1522. [CrossRef]
31. Alli, H.; Yakut, O. Fuzzy sliding-mode control of structures. Eng. Struct. 2005, 27, 277–284. [CrossRef]
32. Yaghin, M.A.; Reza, M.N.; Karimi, B.; Bagherti, B.; Balkanlou, V.S. Vibration control of multi degree of freedom structure under earthquake excitation with TMD control and active control force using fuzzy logic method at the highest and the lowest story of the building. IOSR J. Mech. Civ. Eng. 2013, 8, 7–12.
33. Nigdeli, S.M.; Bekdas, G. Performance comparison of location of optimum TMD on seismic structures. Int. J. Theor. Appl. Mech. 2018, 3, 99–106.
34. Dymarek, A.; Dzitkowski, T. Inverse task of vibration active reduction of mechanical systems. Math. Probl. Eng. 2016, 2016, 3191807. [CrossRef]
35. Dzitkowski, T.; Dymarek, A. Active reduction of identified machine drive system vibrations in the form of multi-stage gear units. Mechanika 2014, 20, 183–189. [CrossRef]
36. Dymarek, A.; Dzitkowski, T.; Herbuś, K.; Ociepka, P.; Sękala, A. Use of active synthesis in vibration reduction using an example of a four-storey building. J. Vib. Control. 2020, 26, 1471–1483. [CrossRef]
37. Dzitkowski, T.; Dymarek, A.; Margielewicz, J.; Gaśka, D.; Orzech, L.; Lesiak, K. Designing of drive systems in the aspect of the desired spectrum of operation. Energies 2021, 14, 2562. [CrossRef]
38. Dong, Z.H.; Ye, X.; Yang, F. Research on the construction of flexible multi-body dynamics model based on virtual components. IOP J. Phys. Conf. Ser. 2018, 1016, 012014. [CrossRef]
39. Herbuś, K.; Ociepka, P. Mapping of the characteristics of a drive functioning in the system of CAD class using the integration of a virtual controller with a virtual model of a drive. Appl. Mech. Mater. 2015, 809, 1249–1254. [CrossRef]
40. Herbuś, K.; Ociepka, P. Determining of a robot workspace using the integration of a CAD system with a virtual control system. IOP Conf. Ser. Mater. Sci. Eng. 2016, 145, 052010. [CrossRef]
41. Redfield, R.C.; Krishnan, S. Dynamic system synthesis with a bond graph approach. Part I: Synthesis of one-port impedances. ASME J. Dyn. Syst. Meas. Control 1993, 115, 357–363. [CrossRef]
42. Park, J.S.; Kim, J.S. Dynamic system synthesis in term of bond graph prototypes. KSME Int. J. 1998, 12, 429–440. [CrossRef]
43. Smith, M.C. Synthesis of mechanical networks: The inerter. IEEE Trans. Autom. Control 2002, 47, 1648–1662. [CrossRef]