Magnon-Mediated Dzyaloshinskii-Moriya Torque in Homogeneous Ferromagnets

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In thin magnetic layers with structural inversion asymmetry and spin-orbit coupling, a Dzyaloshinskii-Moriya interaction arises at the interface. When a spin wave current \( j \) flows in a system with a homogeneous magnetization \( m \), this interaction produces an effective field-like torque on the form \( T_{FL} \propto m \times (z \times j_m) \) as well as a damping-like torque, \( T_{DL} \propto m \times [(z \times j_m) \times m] \) in the presence of spin-wave relaxation (\( z \) is normal to the interface). These torques mediated by the magnon flow can reorient the time-averaged magnetization direction and display a number of similarities with the torques arising from the electron flow in a magnetic two dimensional electron gas with Rashba spin-orbit coupling. This magnon-mediated spin-orbit torque can be efficient in the case of magnons driven by a thermal gradient.

Recent developments in condensed matter physics have renewed the interest of the scientific community in the design and exploitation of materials accommodating large spin-orbit coupling. Topics such as spin Hall effect \( [1] \), topological insulators \( [2] \), or skyrmions \( [3] \), all taking advantage of relativistic effects in solid state, have profoundly challenged our understanding of spin transport lately and present tremendously rich opportunities for innovative expansion of the research in condensed matter systems. Utilizing spin-orbit coupling to control the electron flow and thereby enable the electrical manipulation of ferromagnets and magnetic textures has attracted a considerable amount of interest in the past few years \( [4, 6] \). The key mechanism, tagged spin-orbit torque, appears in ultrathin magnetic systems displaying inversion symmetry breaking such as (but not limited to) bilayers composed of noble metals and ferromagnets. The recent experimental results are interpreted in terms of Rashba \( [7] \) and spin Hall effect-induced torques \( [11] \) and the complexity of the spin transport in such systems is currently under intense investigations \( [8, 10] \). A major progress in the understanding of this field is the importance of Dzyaloshinskii-Moriya interaction (DMI) \( [11] \). DMI results from the spin-orbit coupling in systems with broken inversion symmetry and is responsible for the emergence of skyrmions and chiral spin textures \( [12, 14] \). Interestingly, DMI also arises from the interfacial spin-orbit coupling in ultrathin magnetic bilayers \( [15, 16] \) and results in chiral magnetic domain walls \( [14] \), providing an explanation to mysterious experimental behaviors such as current-induced domain wall motion against the electron flow \( [15, 17, 18] \).

In conjunction with electrically driven spin-orbit torques, another adjacent emerging topic aims at exploiting magnon flows and propagating spin waves instead of electrical carriers \( [19] \). Indeed, magnons can carry spin currents \( [20] \), transmit information \( [21] \) and even control the motion of magnetic domain walls \( [22, 23] \) and skyrmions \( [24] \). The magnon flow may be driven by radio-frequency (RF) magnetic field or temperature gradient \( [25] \), the latter being an important topic of the spin caloritronics field \( [26] \). Recently, it has been realized that DMI impacts the propagation of spin waves yielding to topological behaviors such as magnon Hall effect and edge currents \( [27] \). Thus just like spin-orbit coupling affects the electron flow, DMI affects the magnon flow. It was reported \( [28, 30] \) that the DMI effect on the spin wave dispersion is similar to the Rashba spin-orbit coupling effect on electron dispersion. Therefore, one anticipates that spin-orbit torque due to electron flow in Rashba spin-orbit coupled systems might have their counterpart due to magnon flow in systems displaying DMI.

In this Letter, we demonstrate that even in the absence of magnetic texture, a magnon flow generates torques if magnons are subject to DMI just as an electron flow generates torques when submitted to Rashba interaction, even when the magnetization is homogeneous \( [8, 9] \). A di-
rect consequence is the capability to control the magnetization direction of a homogeneous ferromagnet by applying a temperature gradient or local RF field to generate the magnon flow. We show that merging the spin-orbit torques with spin caloritronics is rendered possible by the emergence of DMI in magnetic materials and opens promising avenues in the development of chargeless information technology.

The magnon-induced torque arises both in in-plane and perpendicular magnetic anisotropy systems. For simplicity, we demonstrate it only for the in-plane magnetic anisotropy case. Let us consider a thin magnetic film with a magnetization aligned along the in-plane easy axis (x-axis) and subjected to an external ac magnetic field applied locally to make spin waves propagate along the x-axis, as displayed in Fig. 1. In this system, the magnetic energy reads

$$W = A \sum_i (\partial_i m)^2 - D m \cdot ((z \times \nabla) \times m)$$

$$+ 2\pi M_s^2 (m \cdot z)^2 - K (m \cdot x)^2,$$

where the first two terms are the symmetric (A) and anti-symmetric Dzyaloshinskii-Moriya (D) exchange energies, the last two terms are the demagnetizing (2\pi M_s^2) and the in-plane anisotropy (K) energies, and \(\nabla = (\partial_x, \partial_y, \partial_z)\). The form of the DMI we adopt here is derived for a cylindrically symmetric system with an interfacial inversion asymmetry along the normal z \[13\] \[15\]. All along the current study, we consider that the DMI is not too strong to make uniformly magnetized state energetically unstable \[29\].

Before going forward with the simulations and in order to get a semi-quantitative understanding of the physics at stake, we first analytically derive the magnon-mediated Dzyaloshinskii-Moriya torque. We treat the problem in cartesian coordinates for the sake of simplicity. For a magnetization initially (in the absence of magnon flow) aligned along the in-plane anisotropy axis ±x, we expect that the DM torque driven by the magnon flow induces small deviations, \(\Delta m_{y,z}\), transverse to x (|\(\Delta m_{y,z}\)| \(\ll 1\) - see Figs. 2 and 3). Then, the total magnetization reads

\(\mathbf{m} \approx (\epsilon, \Delta m_y + s_y, \Delta m_z + s_z)\),

where \(\epsilon = \pm 1\) accounts for the initial magnetization direction and \(s_y, s_z\) describes the spin wave (|\(s_y, s_z\)| \(\ll 1\)), oscillating in space and time. This definition holds as long as |\(\Delta m_y s_y + \Delta m_z s_z| \ll 1 \[31\]. Notice that the background magnetization, \(\mathbf{m}_0 = (\epsilon, \Delta m_y, \Delta m_z)\) is assumed to be time-independent since its dynamics is much slower (\(\sim ns\)) than the precession time of the spin wave (\(\lesssim 0.1\) ns). This allows us to treat the spin wave as quasi-particles flowing in an otherwise static magnetic profile. Injecting this form into the Landau-Lifshitz-Gilbert (LLG) equation, defined as

\(\partial_t \mathbf{m} = -\gamma \mathbf{m} \times \mathbf{H}_{eff} + \alpha \mathbf{m} \times \partial_t \mathbf{m}\),

and after averaging over time we obtain the differential equations describing the spatial variation of the deviations \[31\]

\[\partial_s^2 \Delta m_y = \frac{1}{\lambda_y^2} \Delta m_y + \epsilon \frac{D^*}{J} \langle s_y \partial_x s_z \rangle, \quad (2)\]

\[\partial_s^2 \Delta m_z = \frac{1}{\lambda_z^2} \Delta m_z + \epsilon \frac{D^*}{J} \langle s_y \partial_x s_z \rangle, \quad (3)\]

where \(\lambda = \sqrt{J/H_k}\) and \(\lambda_{y,z} = \sqrt{J/(H_k + H_d)}\) are the in-plane and out-of-plane characteristic lengths of the magnetic texture, with \(J = 2A/M_s\) and \(D^* = 2D/M_s\).

The right hand side of Eqs. (2) and (3) shows that the deviations \(\Delta m_{y,z}\) are driven by DMI, mediated by propagating spin waves. The propagating spin waves are described by the LLG equation keeping only the terms linear in \(s_y, s_z\) such that

\[\epsilon \partial_t s_y + \alpha \partial_x s_y = \gamma J \partial_s^2 s_y - \gamma (H_k + H_d) s_y, \quad (4)\]

\[\epsilon \partial_t s_z - \alpha \partial_x s_y = -\gamma J \partial_s^2 s_z + \gamma H_k s_y, \quad (5)\]

where the terms coupling \(\Delta m_{y,z}\) with \(s_y, s_z\) have been discarded \[31\]. Indeed, in order to describe the spatial profile of the (small) deviations, one needs only to describe the spin waves to the zero-th order in \(\Delta m_{y,z}\). This yields a spin wave, \(\psi_m = s_y + i s_z\), on the form

\[\psi_m = e^{-|x|/2\Lambda} (s_y^0 \cos(qx - \omega t) + e^{+|x|/2\Lambda} \sin(qx - \omega t)),\]

\[\omega^2 = \gamma^2 (H_k^2 + H_d^2) (J_k^2 + H_k^2), \quad \Lambda = \gamma J \omega / \sqrt{\Lambda^*},\]

where \(2\Lambda\) is the spin wave attenuation length. Note that the sign of the exponent of \(e^{-|x|/2\Lambda}\) is chosen to ensure the spin wave attenuation away from the spin wave source at \(x = 0\). Now, by inserting Eq. \[5\] into Eqs. \[2\] and \[3\], and taking the time average over a spin wave precession period (i.e., \(\langle s_y \partial_x s_z \rangle = -\frac{1}{2\Lambda} (s_y^0)^2 e^{-|x|/\Lambda}\) and \(\langle s_y \partial_x s_z \rangle = \frac{1}{2} (s_y^0)^2 e^{-|x|/\Lambda}\)), we can track the impact of this damped spin wave on the deviations \(\Delta m_{y,z}\). Acknowledging that the tilting vanishes at the source, \(\Delta m_{y,z}|_{x=0} = 0\), and away from the source, \(\Delta m_{y,z}|_{x \to \infty} = 0\), one finds that

\[\Delta m_y = -\frac{H_{\text{eff}}}{H_k} \frac{\Lambda^2}{\Lambda^2 - \lambda_y^2} (1 - e^{-|s|/\Lambda^*}), \quad (8)\]

\[\Delta m_z = -\frac{H_{\text{eff}}}{H_k + H_d} \frac{\Lambda^2}{\Lambda^2 - \lambda_z^2} (1 - e^{-|s|/\Lambda_z^*}), \quad (9)\]

where \(H_{\text{eff}} = D^* (s_y^0)^2 |e^{-|s|/\Lambda}|/2\) and \(H_{\text{eff}} = D^* (s_y^0)^2 |e^{-|s|/\Lambda}|/4\Lambda\) and \(\lambda_{y,z}^{-1} = \lambda_{y,z}^{-1}/\Lambda^{-1}\). Notice that \(\Delta m_y\) is even in magnetization direction, whereas \(\Delta m_z\) is odd. Therefore, Eqs. \[8\] and \[9\] clearly show that the magnon-mediated DM torque possesses two components, a Dzyaloshinskii-Moriya field-like torque (DM-FLT, \(\propto \mathbf{m} \times \mathbf{y}\)) inducing \(\Delta m_y\) and a Dzyaloshinskii-Moriya damping-like torque (DM-DLT, \(\propto \mathbf{m} \times (\mathbf{y} \times \mathbf{m})\)) inducing \(\Delta m_z\), in complete similarity with the Rashba torque \[8\] \[9\].

To get a better insight into the impact of propagating spin waves on the otherwise spatially homogeneous background magnetization, we now show micromagnetic
simulation results for a semi-one dimensional system (i.e., the system is discretized along the length direction with the unit cell size of 4 nm - total length of 16 μm -, but not along the width or the thickness direction). For the simulations, we solve the LLG equation with the magnetic energy function given in Eq. (1). We define the gyromagnetic ratio \( \gamma = 1.76 \times 10^7 \text{ Oe}^{-1} \text{s}^{-1} \), the saturation magnetization \( M_s = 800 \text{ emu/cm}^3 \), the exchange stiffness constant \( A = 1.3 \times 10^{-6} \text{ erg/cm} \), and vary the easy axis anisotropy field \( H_k (= 2K/M_s) \). The demagnetization field along the thickness direction \( D = 0.5 \) erg/cm\(^2\), and the damping constant \( \alpha \). To excite spin waves, we apply an ac field \( H_{ac}\cos(2\pi ft)y \) to two unit cells at the center of the model system (i.e. \( x = 0 \)) where \( H_{ac} \) is 100 Oe and \( f = 15 \text{ GHz} \). We consider the absorbing boundary condition \[32\] \[33\] at the system edges to suppress spin wave reflection.

Figure 2(a) shows the spatial distribution of the transverse projection of the magnetization direction \( m_y \) for different DMI coefficients \( D \). Note that in our configuration, the spin wave vector \( q \) is positive for \( x > 0 \) and negative for \( x < 0 \). For \( D = 0 \), the spatial distribution of \( m_y \) is symmetric with respect to the spin wave source (i.e. \( x = 0 \)) and described by \( m_y \propto 0 + s_y(x) \cos(qx - \omega t) \), where "0" represents the \( y \)-component of the background magnetization and the spin wave amplitude \( s_y(x) \) decays with growing \( |x| \) due to damping. For \( D \neq 0 \), on the other hand, the distribution becomes antisymmetric with respect to \( x = 0 \) and \( m_y \propto \Delta m_y(x) + s_y(x) \cos(qx - \omega t) \), according to Fig. 2(a). Thus the propagating spin wave modifies the \( y \)-component of the background magnetization from 0 to a nonzero time-averaged value \( \Delta m_y(x) \), which is shown in Fig. 2(b) for a background magnetization initially lying along \(+x\). Since \( \Delta m_y \) decays antisymmetrically with respect to \( x = 0 \), it clearly appears that the sign of \( \Delta m_y \) reverses with respect to the inversion of the wavevector \( q \) and its magnitude decays away from the spin wave source. Reversing the direction of the background magnetization along \(-x\) does not change the sign of \( \Delta m_y \) [see Fig. 2(c)]. Considering that \( \Delta m_y \) is time-independent, this numerical result demonstrates that a propagating spin wave generates an effective magnetic field along \( y \) or a torque along \( m \times y \), which corresponds to the DM-FLT uncovered by Eq. (8). Figure 2(b) and (c) show the deviation \( \Delta m_y \) for various values of \( \alpha \), where symbols are numerical results and lines are obtained from Eq. (8). We use \( s_{yz} \) obtained from numerical simulations at \( x = 0 \). The analytical expression reproduces the numerical results very well.

Another intriguing observation is the emergence of the DM-DLT \([x \times (y \times m)]\) that induces the deviation \( \Delta m_z \), as demonstrated by Fig. 3 and Eq. (9), and is proportional to the damping constant \( \alpha \) (since \( \Lambda \propto 1/\alpha \)). This feature echoes the non-adiabatic correction to the electronic spin torque in the presence of spin-flip relaxation, as proposed by Zhang et al. in magnetic textures and spin-valves [34]. In metallic systems, spin relaxation modifies the spin dynamics of the itinerant electrons which results in an additional torque component on the form \(-\beta m \times \tau \), where \( \tau \) is the torque in absence of spin relaxation and \( \beta \) is proportional to the spin relaxation rate [34]. In the present case of magnonic DM torque, the magnetic damping \( \alpha \) attenuates the spin wave current, as seen above, but also relaxes the spin polarization carried by the spin waves and therefore produces the additional damping-like torque \( H^{\text{DM}}_{\text{DMD}} \). The same effect is at the origin of non-adiabatic torque in electron-driven and magnon-driven magnetic excitations. Therefore, the spin wave relaxation has two distinct consequences: it produces a corrective DM-DLT and it renormalizes the effective tilting by the factor \( \Lambda^2/(\Lambda^2 - \Lambda_c^2) \). The latter originates from the competition between the decaying spin wave current (i.e. spatially dependent DM torque) and the magnetic stiffness \( A \). By tuning these two, on can then expect to enhance the DM-DLT. Figure 3(a) and (b) show the spatial distribution of the out-of-plane deviation \( \Delta m_z \) for different \( \alpha \) when the magnetization is initially aligned along \(+x\) and \(-x\), respectively. For clarity, we assume \( H_d = 0 \) to make \( \lambda_d = (\Delta m_z \) larger. This case is similar to what is expected in perpendicular-
FIG. 3. (Color online) Numerical results for magnon-mediated Dzyaloshinskii-Moriya damping-like torque. (a,b) Spatial distribution of the magnetization tilting $\Delta m_z$ for various damping constants $\alpha$ when the magnetization initially lies along $+x$ (a) and $-x$ (b) for $D=1.5$ erg/cm$^2$ calculated numerically (open symbols) and using Eq. (9) (solid line). Here we assume $H_d=0$.

differ from the presence of the s-d exchange term. Indeed, in contrast with electron spins, the magnon spin is by definition aligned on the local magnetization and its wavefunction $\psi_m$ is not a two-component spinor. Nevertheless, their similarity implies that properties of the Rashba system, such as current-induced Rashba field (also called inverse spin galvanic effect) on the form $\gamma H_R \approx \alpha_R z \times j_\sigma/\mu_B M_s$, ($j_\sigma$ being the flowing spin current) $[3]$, are at least partly enabled by the presence of DMI in magnonic systems. The propagating spin wave and background magnetization $m$ interact through the energy term $(\alpha_{DM}/\hbar)(\mathbf{p}_m \cdot (\mathbf{z} \times \mathbf{m}))$, where $\langle \ldots \rangle_m$ denotes the quantum average on the magnon state $\psi_m$. This interaction term yields a torque of the following form

$$T_{DLT} = \gamma m \times \partial_m \langle \dot{H}_m \rangle = -m \times \frac{\partial \Delta m_z}{\partial x} \times (\mathbf{z} \times \mathbf{p}_m) \tag{12}$$

The expression within the brackets $\{\ldots\}$ is nothing but the effective field of the DM-FLT, $H_{DMF}$, and $(\mathbf{p}_m)$ amounts to the spin wave current. The DM-DLT can be obtained qualitatively by considering the correction of the damping on the spin wave dynamics [see Eqs. (4)-(5)]. In a Landau-Lifshitz approach, the magnetic damping corrects the torque by adding a contribution on the form $\gamma m \times (m \times H_{DMF})$ that produces the DM-DLT term.

The simulations and discussion provided above demonstrate that flowing spin waves assisted by the DMI can generate a torque on the local magnetization and tilt its orientation. However, using RF excitations to generate spin waves has two major drawbacks: (i) the wavelength $2\pi/q$ of the spin wave is rather large (74 nm in the present study) and (ii) the magnon flow (hence, the DMI torque) vanishes away from the RF source over the attenuation length $\Lambda$. Therefore, thermal magnons driven by a uniform temperature gradient, $\nabla T$, are interesting candidates for the proposed effect. Following Eq. (12), these magnons exert a torque on the form $T_{HL}^{\text{th}} = \gamma m \times \partial_m \langle \dot{H}_m \rangle = (\alpha_{DM}/\hbar M_s)\sigma_T m \times (\mathbf{z} \times \nabla)k_BT$ on the magnetization, where $\sigma_T$ is the magnon conductivity which can be estimated using a phenomenological Boltzmann equation such as in Ref. [35]. In this case, the torque is exerted homogeneously over the region where the temperature gradient is applied and is not limited to the attenuation length $\Lambda$. In the presence of a non-equilibrium distribution of magnons, the effective deviations become $\langle \Delta m_y \rangle_{T}$ and $\langle \Delta m_z \rangle_{T}$, where $\langle \ldots \rangle_T$ denotes thermal averaging over the whole range of allowed frequencies. Using a free magnon dispersion ($\varepsilon_q \approx D_\sigma^2$), this thermal averaging results in an effective magnon wave vector such that $\langle q \rangle_T = \tau^*/\hbar(T/T_c)^{3/2}T^{3/2}\Gamma_{5/2}k_B T$. This expression is obtained exploiting Eq. (8) in Ref. [35] and depends on the temperature through the effective magnon relaxation time $\tau^*$ and the factor $\Gamma_{5/2}$ ($\approx 0.6$ when $T \approx T_c$). Therefore, while in the case of RF-excited spin waves, the DM torque is driven by coherent spin

where $\mathbf{m} = hM_s/4\gamma A$ is the magnon mass and $\alpha_{DM}/\hbar = 2\gamma D/M_s$ is Dzyaloshinskii-Moriya velocity for the spin waves. This equation instructively resembles Schrödinger’s equation of an itinerant electron spin in a homogeneous magnetic two dimensional electron gas in the presence of Rashba spin-orbit coupling $[7]$

$$i\hbar \partial_t \psi_m = \hat{H}_m \psi_m = \left( \frac{\mathbf{p}^2}{2m^*} + \frac{\alpha_R}{\hbar} \mathbf{p} \cdot (\mathbf{z} \times \mathbf{\sigma}) \right) \psi_m , \tag{10}$$

where $m^* = \hbar M_s/4\gamma A$ is the magnon mass and $\alpha_{DM}/\hbar = 2\gamma D/M_s$ is Dzyaloshinskii-Moriya velocity for the spin waves. This expression is obtained exploiting Eq. (8) in Ref. [35] and depends on the temperature through the effective magnon relaxation time $\tau^*$ and the factor $\Gamma_{5/2}$ ($\approx 0.6$ when $T \approx T_c$). Therefore, while in the case of RF-excited spin waves, the DM torque is driven by coherent spin

$$i\hbar \partial_t \psi_e = \left( \frac{\mathbf{p}^2}{2m} + \frac{\alpha_R}{\hbar} \mathbf{p} \cdot (\mathbf{z} \times \mathbf{\sigma}) + J_{\text{ex}} \mathbf{m} \cdot \mathbf{\sigma} \right) \psi_e , \tag{11}$$

where $\mathbf{p} = -i\hbar \nabla$ is the momentum operator, $\alpha_R$ is the Rashba spin-orbit coupling and $J_{\text{ex}}$ is the s-d exchange between itinerant electron spins $\mathbf{\sigma}$ and the local moments aligned along $\mathbf{m}$. Equations (10) and (11)
waves with a single wavevector \( q_0 \), the whole population of non-equilibrium thermal magnons contribute to the torque in the presence of a thermal gradient through \( \langle q \rangle \). Standard RF-excited spin waves, such as the one used in our simulations (\( f=10 \text{ to } 20 \text{ GHz} \)), possess a wavevector of \( q_0 \approx 70 \text{ to } 140 \mu \text{m}^{-1} \). In contrast, by using the parameters for YIG at room temperature, \( T_c =550\text{K} \), \( \partial_x T \approx 0.02 \text{ K/nm} \) and \( \tau^* =255 \text{ ps} \) [36], we obtain \( \langle q \rangle \approx 400 \mu \text{m}^{-1} \), which indicates that thermal magnons are more efficient in generating DM Torque \( \langle \Delta q \rangle / q_0 \approx 3 - 5 \).

In the present Letter, we demonstrated that Dzyaloshinskii-Moriya interaction mediated by spin waves can generate a torque on a homogeneous magnetization that resembles the Rashba torque, its electronic counterpart, displaying both field-like and damping-like components. The torque is expected to be much more efficient in the case of a magnon flow driven by a thermal gradient than for a standard RF-excited spin wave. It is important to stress out that our results are not limited to systems displaying interfacial DMI but can be also extended to materials accommodating bulk DMI since the energy functional needs only to display an antisymmetric exchange term \( \sum_{ij} D_{ij} \cdot \mathbf{S}_i \times \mathbf{S}_j \), such as in pyrochlore crystals [27] and chiral magnets [37]. The present results build up a bridge between spin-orbit transport, magnonics and spin caloritronics and is expected to be detectable in systems ranging from thin magnetic bilayers to skyrmion crystals.

The authors acknowledge fruitful discussions with M.D. Stiles and K.W. Kim. K. J. L. was supported by NRF (NRF-2013R1A2A2A01013188), KU-KIST School Joint Research Program, and Human Resources Development Program, MKE/KETEP (No. 2011-14010100640). H. W. L. was supported by NRF (NRF-2013R1A2A2A05006237 and 2011-0030046).

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