Research on Dynamic Response Test of Subgrade Vibrator for High-speed Railway

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Abstract. During subgrade dynamic response test for high-speed railway, light weight inertial vibration with high frequency excitation might lose impact force caused by suspension. A structural optimization approach for inertial vibration is proposed to solve the problem. The dynamic response model of subgrade vibrator is built by analyzing the influences of vibration frequency, static weight, vibration amplitude, phase and impact force. The result shows the ratio of static weight and eccentric vibration force is the main factor that influences the vibrator working performance. Through studying the relationships of the parameters, the recommended parameters for the vibrator with lighter weight are simulated and tested, which are valuable for optimal design of inertia vibrator.

1. Introduction
Dynamic response test of subgrade vibrator is essential for the site selection and acceptance of high-speed railway because its results determine the subgrade safety during high-speed rail working [1,2]. At present, subgrade vibrators are mostly composed of eccentric rotary vibration system controlled by hydraulic cylinder or hydraulic motor, which has the advantage of simple structure with wide vibration frequency range [3]. However, how to select the proper vibrator weight under the given vibrating frequency is a problem [4]. One the one hand, the overweight vibrator leads to more fuel consumption and inconvenient installation of the vibration system. On the other hand, the underweight vibrator leads to empty suspension and impact force loss [5]. Therefore, the research on dynamic response test of subgrade vibrator is very important.

2. Dynamic Response Test Model of Subgrade Vibrator
The vibrator structure is shown in Fig. 1. In the device, gears drive two sets of eccentric block to do symmetrical rotary motion. The horizontal components offset each other, and the resultant force of the vertical direction is simple harmonic excitation. For convenient, in the process of the analysis, subgrade soil is assumed to be linear elastic body and the viscous damping model, and it can absorb the all impact energy of the vibration device. The elastic coefficient of subgrade soil is k, the damping coefficient is c. Figure 2 shows the simplified dynamic model of inertia vibrator. The subgrade dynamic response of mechanical vibration of the mechanical system is simplified into a lumped parameter ‘quality-spring-damping vibration system’. In Fig. 2, M is the static weight of the vibrator, F(t) is resultant force, ω is rotating circular frequency of eccentric rotor, y is the instantaneous
displacement of vertical direction for the vibration plate and the vibrator body, the origin of coordinate is in the static equilibrium position of vibrator box.

1.Counterweight mass 2-box body 3-shaft 4-gears 5-eccentric rotor 6-hydraulic motor 7-vibration plate

Figure 1. Vibrator Structure

Figure 2. Dynamic Model of the Vibrator

According to Fig.2, a vibrator dynamics differential equation is established as Eq.(1), with which the relationship of the static weight of the vibrator, vibration frequency and vibration displacement, landing time and impact force can be analyzed.

\[ M \ddot{y} + c \dot{y} + k y = F(t) \]  

(1)

3. Vibration Displacement and Static Weight

As shown in Fig.2, amplitude \( F_0 \) of the vertical eccentric vibration force is:

\[ F_0 = 4 \pi^2 m_0 f^2 r \]  

(2)

Where \( m_0 \) is eccentric rotor’s mass (kg), \( f \) is vibration frequency (Hz), and \( r \) is eccentric distance (m). Therefore, the eccentric exciting force amplitude \( F_0 \) can be adjusted by vibration frequency. When the eccentric exciting force amplitude is greater than the gravity of the vibrator, the vibrator jumps up. When \( F_0 < Mg \), the vibrator does not jump up from subgrade surface. Define the vibration frequency that the vibrator just jumps up from subgrade surface as the critical frequency. Therefore, \( F_0(f_0) = Mg \), and the critical frequency \( f_0 \) is obtained as:

\[ f_0 = \sqrt{Mg / 4 \pi^2 m_0 r} \]  

(3)

In the process of vibration, low frequency excitation can simulate working condition that trains go through subgrade at a low speed. If the vibration frequency is lower than \( f_0 \), the eccentric vertical excitation force amplitude is less than the gravity of the vibrator. Then vibration plate keeps in contact
with the subgrade surface, and the vibration displacement is neglected as 0. When $F_0 \geq Mg$, the vibrator jumps up from subgrade surface. Then, the elasticity coefficient and damping coefficient of subgrade soil are 0. The vibrator goes through its initial unstable process. If the phase of eccentric rotor is in the third or fourth quadrant, and vibrator is parked in the subgrade surface, vibrator will be into the normal state of the vibration in the next cycle. The vibrator will vibrate at the setting vibration frequency. With the eccentric rotor rotating a cycle, movement status of the vibrator is shown in Fig. 3. In the movement process, when the eccentric rotor rotates into a critical angle at 1/4 cycle of the upper motion circle, the vibrator jumps up from subgrade surface, and then begins acceleration upwards. After a while, the vibrator begins deceleration upwards. The vibrator shocks subgrade surface, when it rotates into an angle at the bottom half of the motion circle. Then the vibrator stops to wait for next motion cycle.

In a cycle of motion, $k$ and $c$ in Eq. (1) are 0. Therefore, $F(t) = F_0 \sin \omega t - Mg$. The motion differential equation can be rewritten as:

$$M \ddot{y} = F_0 \sin \omega t - Mg$$  \hspace{1cm} (4)

The acceleration $a$ is obtained:

$$a = \ddot{y} = \frac{F_0}{M} \sin \omega t - g$$  \hspace{1cm} (5)

The initial state of takeoff is that upward eccentric exciting force is equal to the total gravity of the vibrator:

$$\dot{y} = \frac{F_0}{M} \sin \omega t_0 - g = 0$$

$$t_0 = \frac{\arcsin(Mg / F_0)}{\omega}$$

If speed $v$ equals 0 at time $t_0$, velocity can be obtained as:

$$v = \dot{y} = \int \ddot{y} dt = - \frac{F_0}{M \omega} \cos \omega t + c_1$$  \hspace{1cm} (6)

Where

$$c_1 = \frac{g}{\omega} \arcsin \frac{Mg}{F_0} + \frac{F_0}{M \omega} \sqrt{1 - \frac{M^2 g^2}{F_0^2}}$$

The displacement equation can be obtained through the velocity-time integral. The initial displacement is 0 at time $t_0$. 

Figure 3. Motion Cycle of the Vibrator
\[ y = \int y \, dt = -\frac{F_0}{M\omega} \sin \omega t - \frac{1}{2} gt^2 + c_1 t + c_2 \]  

(7)

Where

\[ c_1 = \frac{g}{\omega} \arcsin \frac{Mg}{F_0} - \frac{F_0}{M\omega} \sqrt{1 - \left(\frac{Mg}{F_0}\right)^2} \]

\[ c_2 = \frac{g}{\omega^2} - \frac{g}{2\omega^2} \left(\arcsin \frac{Mg}{F_0} \right)^2 + \frac{F_0}{M\omega} \sqrt{1 - \left(\frac{Mg}{F_0}\right)^2} \left(\arcsin \frac{Mg}{F_0} \right) \]

In Eqs. (4-6), \( M \) and \( F_0 \) are showed in ratio form. Therefore, a dimensionless parameter \( \xi = \frac{Mg}{F_0} \) can be defined, and \( \xi \) is a significant parameter to vibration state. Then, Eqs. (4-6) can be rewritten as:

\[ a = \dot{y} = \frac{1}{\xi} g \sin \omega t - g \]  

(8)

\[ v = \dot{y} = \int \dot{y} \, dt = -\frac{1}{\xi \omega} g \cos \omega t - gt + c_1 \]  

(9)

\[ y = \int \dot{y} \, dt = -\frac{1}{\xi \omega^2} g \sin \omega t - \frac{1}{2} gt^2 + c_1 t + c_2 \]  

(10)

Where,

\[ c_1 = \frac{g}{\omega} \arcsin \xi + \frac{1}{\xi \omega} g \cdot \sqrt{1 - \xi^2} \]

\[ c_2 = \frac{g}{\omega^2} - \frac{g}{2\omega^2} \left(\arcsin \xi \right)^2 + \frac{1}{\xi \omega^2} g \cdot \sqrt{1 - \xi^2} \cdot \left(\arcsin \xi \right) \]

Setting the displacement as 0, the initial value and the landing time can be calculated. According to Eq. (9), when the dimensionless parameter \( \xi \) is fixed, eccentric rotor has an identical phase at the landing time under different vibration frequencies. For example, \( \xi = 0.5 \), under the different vibration frequencies, the eccentric rotor of the vibrator lands at the same phase, 295.2°, 0.82 of a motion cycle, as shown in Fig. 4.

When \( m_0 \) and \( r \) are fixed, as the vibration frequency gets greater, the eccentric vibration force of the excitation amplitude gets greater. Therefore, at a high vibration frequency, the gravity of the vibrator may be lighter than the eccentric vibration force. It leads to the vibrator displacement unable to return to 0 in one cycle, then the vibrator can not work normally under the designed frequency. When designing a vibrator, the weight of vibration plate needs to be adjusted for different frequencies. The dimensionless parameter \( \xi \) should be adjusted to satisfy the motion shown in Fig. 3.
Compared with the vibration cycle, the impact contact time is short. Therefore, if the vibrator can satisfy the displacement curve which can go back to zero position in a cycle, shown in Fig. 3, the vibrator is supposed to be work normally under the given vibration frequency. Under different frequencies, as long as the $\xi$ is fixed, landing time phase will be the same. Setting the vibration frequency as 40 Hz, displace time curves, shown in Fig. 5, can be obtained under different $\xi$ in the range of 0.3 to 0.65, at interval of 0.05. In Fig. 5, from top to bottom, the curves have the value of $\xi$ from 0.3 to 0.65. Figure 5 shows that, in order to have normal vibration and avoid suspension, reserving 10% for allowance, $\xi \geq 0.4$ is recommended.

![Figure 5. Displacement-time Curve under Different $\xi$](image)

4. Conclusion
In this paper, dynamics model of an inertia vibrator is established. The influence of the ratio $\xi$ of vibrator’s static weight and eccentric vibration force on the working performance of the vibrator is analyzed. Unlike general model which only analyzes vibrator staying on subgrade surface, this model consists of two kinds of working condition: low frequency vibration that the vibrator does not jump up from subgrade surface and high frequency vibration that the vibrator jumps up from the subgrade surface. Based on the model, impact force loss caused by suspension is solved.

5. References
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