Time-varying Flexural Reliability Analysis of RC Beams Strengthened with SNSM CFRP under Environment Effects

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Abstract. In order to evaluate the influences of environmental effects on the reliability of reinforced concrete (RC) beams strengthened with side near surface mounted (SNSM) carbon fibre-reinforced polymer (CFRP), combined with the relevant research results and the actual parameters of strengthened beams, the time-varying functions of the flexural capacity of the strengthened beams considering environmental effect was established. Then, based on the JC methods, the time-varying flexural reliability of RC strengthened beams under different ratios of standard values of dead load and live load was calculated. The results showed that the flexural reliability of RC beams decreased slowly with service time in the early period, and decreased rapidly in the later period. Compared with unstrengthened beams, the reliability of strengthened beams had been improved to a certain extent. When the unstrengthened beams were serviced 70% of the base period $T$, and then it could continue to be serviced about 16% of the base period $T$ after been strengthened with SNSM CFRP. In general, the SNSM CFRP technique could improve the service time of RC beams. According to the analysis results, it was recommended that the RC beams should be strengthened in the early period to better improve the service time.

1. Introduction

Due to the environment or other factors such as fire, freezing, etc., RC structures have been facing damage problems since early period of service. When the damage accumulates to a certain extent, the resistance of the structure will decline, which will reduce the safety, suitability and durability of the RC structures. Therefore, it is necessary to take appropriate strengthening measures for RC structures with little damage to improve service performance. At present, FRP materials are widely used in the field of engineering strengthening because of their many advantages, such as corrosion resistance, high tensile strength and light weight. In recent years, the reliability analysis of RC structure strengthened with FRP has gradually been paid more attention. However, due to the many uncertainties of concrete materials, FRP materials and strengthening methods, there are many uncertainties in the reliability assessment of RC structures strengthened with FRP[1–2].

Considering the degradation factors of the existing RC structure due to corrosion of steel bars and FRP performance degradation under the influence of the environment, the reliability of the resistance partial factor of flexural capacity of T-beams strengthened with FRP was analyzed$^{[3]}$. Wieghaus$^{[4]}$strengthened the beams of rectangular and T-sections with FRP to improve its flexural performance, and then studied the influence of uncertain factors such as concrete, steel bars and FRP materials on the flexural reliability of RC beams. Shi$^{[5]}$analyzed that the main failure mode of externally
bonded CFRP-strengthened RC structure was intermediate crack-induced peeling failure. Based this failure mode, the reliability of the strengthened RC structure was analyzed. Xie Huibing established an uncertainty model of the bridge's flexural capacity and mechanical properties of FRP materials, and then evaluated the reliability of concrete structures strengthened with FRP. In 2018, Arteaga proposed a stochastic model, in which the combined effects of chloride-induced corrosion, climate change, and fatigue cyclic loading were considered. Based on this model, the reliability of RC T-section bridges subjected to cyclic loading under different environmental conditions was analyzed.

In the above research, the influence of relevant factors such as the geometrical dimensions of the structure, the characteristics of material variation, and the strength of the material were mainly considered, and some useful results of the flexural reliability of the RC structure were obtained. In this paper, the reliability analysis of monorail traffic track beams strengthened with SNSM CFRP technique was taken as the research background. Therefore, according to the research results of literature and relevant railway specifications, the reliability and service time of strengthened beams were analyzed and evaluated, respectively.

2. Establishment of RC structural reliability calculation model
During the service of the RC structures, the resistance and variable load changes with time, so the functional function also change with time. Let the functional function of structural reliability at time \( t \) be.

\[
Z(t) = \sum_{i=1}^{n} R_i(t) - \sum_{i=1}^{k} S_{Gi} - \sum_{i=1}^{m} S_{Qi}(t)
\]

(1)

Where: \( \sum_{i=1}^{n} R_i(t) \) is the combination of \( n \) structural resistances at time \( t \); \( \sum_{i=1}^{k} S_{Gi} \) is the combination of \( k \) dead load effect; \( \sum_{i=1}^{m} S_{Qi}(t) \) is the combination of \( m \) variable load effects at time \( t \).

The mean and standard deviation of the functional function can be calculated as follows\[9\].

\[
\begin{align*}
\mu_Z &= -\frac{1}{\alpha_T} \ln \left\{ \frac{1}{m} \sum_{i=1}^{m} \exp \left[ -\alpha_T \sum_{i=1}^{m} \mu_{Z_i} g_i(t) \right] \right\} \\
\sigma_Z &= \frac{\sum_{i=1}^{m} g_i(t) \exp \left[ -\alpha_T \sum_{i=1}^{m} \mu_{Z_i} g_i(t) \right]}{\sum_{i=1}^{m} \exp \left[ -\alpha_T \sum_{i=1}^{m} \mu_{Z_i} g_i(t) \right]} \tag{3}
\end{align*}
\]

Based on Eqs.(2) and (3), the reliability of the RC structures in different periods can be obtained according to the JC methods, whose the specific expressions are as follows\[10\].

\[
\beta = \frac{\mu_Z}{\sigma_Z} = \frac{g_x(x^*) + (x^* - x^*)^T \nabla g_x(x^*)}{\left\| \text{diag}[\sigma_x] \nabla g_x(x^*) \right\|} \tag{4}
\]

According to the uniform standard of structure, the variable load effect of structure obey the extreme type\( \text{Ⅰ} \) distribution. The mean \( u_{S_{Qi}} \) and standard deviation \( \sigma_{S_{Qi}} \) of the variable load effect in the period \( T \) can be expressed as.

\[
\begin{align*}
u_{S_{Qi}} &= u_{S_{Qi}} + \ln(\text{T}/T_D) / \alpha \\
\sigma_{S_{Qi}} &= \sigma_{S_{Qi}} \tag{6}
\end{align*}
\]
Where: $T_D$ is the design basis period; $\alpha = \pi \sqrt{6 \sigma_Q T_D}$; $u_{\sigma_Q}$ and $\sigma_{\sigma_Q}$ are the mean and standard deviation of the maximum value of the variable load effect during the design base period, respectively.

3. Selection of resistance expression function and related parameters

The RC beams strengthened with SNSM technique still follow the flat section assumption. There are two main modes to reach the limit state. First, the steel bars yield, and the concrete at the top of the beams reaches the limit strain. At this time, neither the steel bars nor the CFRP strain reaches the allowable limit state. Second, the steel bars yield, and the steel bars have reached the allowable limit state, but the concrete at the top of the beams have not reached the limit strain state.

(1) The function of the normal section flexural capacity of the RC beams under the limit state can be expressed as:

$$Z_3 = K_{P1} R_S - K_{s1} S_{GH} - K_{s2} S_{GF} - (1 + \mu_0) S_Q$$  \hspace{1cm} (7)

Where:
- $R_S$ is the resistance of the normal section of the RC beams;
- $K_{P1}$ is the uncertainty coefficient of the calculation model of the normal section resistance;
- $S_{GH}$ is dead load effect;
- $S_{GF}$ is the additional constant load effect;
- $S_Q$ is the live load effect; $(1 + \mu_0)$ is the dynamic coefficient. The calculation of dynamic coefficient can refer to the literature [11].

(2) The resistance function of RC beams

When the concrete compressive strain is between the compressive strain corresponding to the peak value of the concrete axial compression and the ultimate compressive strain, the steel bars strain is greater than its yield strain. Namely, $\epsilon_c \leq \epsilon_s \leq \epsilon_{cu}$, $\epsilon_s \geq \epsilon_y$. The expression of the flexural resistance of the RC beams are as follows.

(a) The resistance function of the unstrengthened beams is expressed as follows:

$$R_s(f_s, A_s, h_0, x_{cm}) = f_s A_s (h_0 - \frac{\beta_s}{2} x_{cm})$$  \hspace{1cm} (8)

Where: $x_{cm}$ is the height of the compression zone of the unstrengthened beams.

(b) The resistance function of RC beams strengthened with SNSM CFRP is as follows:

$$R_s(f_s, h_0, E_s f, T_p, x_{cs}, \cdots) = f_s A_s (h_0 - \frac{\beta_s}{2} x_{cs}) +$$

$$6E_s f_0 A_s (h - y_0 - \frac{\beta_s}{2} x_{cs}) + 2T_p (h - a - \frac{\beta_s}{2} x_{cs})$$  \hspace{1cm} (9)

Where: $x_{cs}$ is the height of the compression zone.

Reference value of each letter in the above equation can refer to the literature[8].

4. Reliability analysis of RC beams under ultimate conditions

It is assumed that the service time of the RC beams was $T_D=100a$, and the resistance of the structure decreases to 40% of the initial resistance after the base period $T_D$. A time variation function $\phi(t)=1-6 \times 10^{-7} t^3$ was selected as the comprehensive influence coefficient of the environment on the reduction of concrete beams resistance[13]. Because the magnitude of the absolute value of various load effects of RC beams in service were relatively large, it was difficult to analyze and calculate according to the actual absolute value of load effects. At present, the effect ratio of dead load and live load standard value was mainly used to analyze the reliability of the RC beams. For railway RC beams, in general, it could be considered that the ratio $\rho$ of the standard value of the effect of the dead load and the live load was between 0.2 and 1, and the ratio of dead load to additional dead load effect was in the range of 2 to 4[14]. In this paper, the ratio of the dead load to the additional dead load effect was taken as 2.

The geometric dimensions and mechanical properties of RC beams and CFRP were actually definite
values, but due to certain measurement errors, they were usually treated as random variables. However, in general, the variability of geometric dimensions was very small. To simplify the calculation, it was taken as a deterministic quantity. The relevant parameter values of the concrete beams are shown in Table 1.

| Variable | Standard value | Variable | Standard value | Variable | Standard value |
|----------|----------------|----------|----------------|----------|----------------|
| b        | 0.300m         | $f_y$    | 559MPa         | $A_s$    | 1473mm$^2$    |
| $h_0$    | 0.555m         | $T_p$    | 600MPa         | $A_{cf}$ | 28mm$^2$      |
| $a$      | 0.03m          | $f_c$    | 26.8MPa        | $\beta_n$| 0.813         |
| $d$      | 0.05m          | $E_{cf}$ | 147×10^3MPa    | $\beta_s$| 0.824         |

The probability distribution types and statistical parameters of random variables, such as the dead load, additional dead load and live load, are shown in Table 2.

| Random variables | Probability distribution | $k$ | $\delta$ | Random variables | Probability distribution | $k$ | $\delta$ |
|------------------|--------------------------|-----|----------|------------------|--------------------------|-----|----------|
| $K_{s1}$         | Normal distribution      | 1.020 | 0.022   | $S_Q$            | Extreme value distribution | 1.274 | 0.340   |
| $K_{s2}$         | Normal distribution      | 1.321 | 0.068   | $K_{Pj}$        | Normal distribution       | 1.050 | 0.0647  |

Note: $k$ indicates that the load statistical parameter is the ratio of the average value to the standard value; $\delta$ is the coefficient of variation of the load statistical parameter.

By substituting the above statistical parameters into Eq.(8) and Eq.(9), it could obtain the resistance expression of normal section of the RC beams, and then substitute it into Eq.(7) to obtain the functional function expression of the RC beams. Combining this functional function with Eq.(2) and Eq.(3), the flexural reliability of RC strengthened beams could be calculated using the JC methods. The reliability index of flexural ductility was selected as 5.2$^{[13]}$. Based on this reliability index, the time required to strengthen the RC beams and the service life after strengthening could be calculated.

Figure 1 presents that the reliability of flexural RC beams change with service time based on different ratios of dead load to variable load. As the value of $\rho$ increased, the reliability of RC beams gradually increased. When the value of $\rho$ was the same, the reliability of it decreased with the service time increasing. From the comparison of the reliability of unstrengthened and strengthened beams with service time, the change tends of the two were basically the same. The reduced reliability was relatively slow when it was within 50% of the base period $T_D$, and then the reduced rate of reliability was faster than that of the earlier period, because the influence of the later environmental effects and the attenuation of the material strength to resistance were intensified.

For the unstrengthened beams, when $\rho$ was between 0.2 and 1, the maximum reliability ranged from 3.26 to 7.71 in the early period. The value of the reliability was between 0.82 and 4.34 when the service time reached the base period $T_D$. For the beams strengthened with SNSM CFRP, the maximum reliability value ranged from 3.68 to 8.41 in the early period, and the final value ranged from 1.33 to 4.95.

In general, the reliability of strengthened beams was improved to a certain extent compared to unstrengthened beams. In the paper, the SNSM CFRP strengthening technique used can theoretically reduce the flexural failure probability of the RC beam.
Figure 1. Reliability changes with service time of flexural beams based on different ratios of dead load to variable load

(a) unstrengthened beams
(b) strengthened beams

Figure 2. The relation between load ratio and service time

As the load ratio increases, the service time of RC beams will increase.

When the load ratio is between 0.2-0.45, the calculated reliability of the RC beams is lower than the target reliability.

Figure 3. Reliability changes with service time of flexural strengthened beams

Figure 2 presents the curve of load ratio with service time for the strengthened beams. When the ratio of dead load to variable load $\rho$ was between 0.2 and 0.45, the calculated initial reliability was less than the target reliability, which indicated that the service time of the beams was approaching 0 due to the excessive variable load, which was obviously not in line with the actual situation. As $\rho$ gradually increased, the variable load gradually decreased, and the service time of the unstrengthened beams to achieve the target reliability gradually increased. When $\rho$ was between 0.45 and 0.65, the service time of unstrengthened beams increased rapidly from 0 to about 70% of the base period $T_D$. When $\rho$ was between 0.65 and 1, the service time of unstrengthened beams had increased from 70% to 90% of the base period $T_D$, and the increasing trend has slowed down significantly. With the change of the $\rho$ value, the service time of the RC beams showed a relatively obvious three-stage change. The $\rho$ values of 0.45 and 0.65 were the two inflection points of the curve.

Figure 3 shows the trend of the reliability with service time for the strengthened beams. The value of $\rho$ was selected as 0.65. The reliability of unstrengthened beams was close to the target reliability after about 70% of the base period $T_D$. At this time, the beams were strengthened with SNSM CFRP; it could continue to be served about 16% of the base period $T_D$. This showed that the service time of the strengthened beam had been extended to a certain extent. After that, the corresponding strengthening treatment needed to be continued to ensure that the reliability of the strengthened beams was not less
than the target reliability.

In this paper, the attenuation of structural resistance due to environmental effects was considered. However, the actual attenuation of the RC structures in service was not only affected by the environment, but also it was effected by fatigue loads. Therefore, the reliability calculation of RC beams needs to consider the influence of the dual factors of environment and fatigue.

5. Conclusion
In this paper, considering the environmental effect, the reliability of RC beams strengthened with SNSM CFRP was studied, and the service time of the strengthened beams was evaluated.

- Under environment effects, the reliability of the RC beams decreased slowly with service time in the early period, and then decreased rapidly in the later period. In general, the reliability of strengthened beams was improved to a certain extent compared to unstrengthened beams.

- When the reliability of unstrengthened beams was close to the target reliability after about 70% of the base period TD, the RC beams were strengthened with SNSM CFRP, and they could continued to be served about 16% of the base period TD. This showed that the service time of the strengthened beams had been extended to a certain extent.

- The research results can provide a reference for the reliability calculation and service time evaluation of monorail traffic track strengthened beams.

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References
[1] Okeil A M and El-Tawil S, 2002. Shahawy M. Flexural reliability of reinforced concrete bridge girders Strengthened with carbon fiber-reinforced polymer laminates. Journal of Bridge Engineering. 7(5):290-299.
[2] Atadero R A, Lee L and Karbhari V M, 2005. Consideration of material variability in reliability analysis of FRP strengthened bridge decks. Composite Structures. 70(4):430-443.
[3] Atadero R A and Karbhari V M, 2008. Calibration of resistance factors for reliability based design of externally-bonded FRP composites. Composites Part B: Engineering. 39(4):665-679.
[4] Wieghaus K T and Atadero R A, 2011. Effect of existing structure and FRP uncertainties on the reliability of FRP-based repair. Journal of composites for construction. 15(4):635–643.
[5] Shi J , Wu Z and Wang X, et al, 2014. Reliability analysis of intermediate crack-induced debonding failure in FRP-strengthened concrete members. Structure and Infrastructure Engineering .1-21.
[6] Xie H B, 2016. Reliability of FRP strengthened reinforced concrete simple supported T-beam bridge in flexure. Beijing: Beijing Jiaotong University PhD thesis.
[7] Bastidas-Arteaga E,2018. Reliability of reinforced concrete structures subjected to corrosion-fatigue and climate change. International Journal of Concrete Structures & Materials. 12(1):1-14.
[8] Zhu Z W and Zhu E Y , 2018. Flexural behavior of large-size RC beams strengthened with side near surface mounted (SNSM) CFRP strips. Composite Structures.201:178-192.
[9] Gong J X, 1998. Reliability Analysis for Deteriorating Structures.Journal of Building Structures. 19(5):43-51.
[10] Zhang M and Jin F, 2015. Structure Reliability.Beijing:Science Press.
[11] Li T F, 2006. Railway Bridge Reliability Design.Beijing:China Railway Press
[12] Q/CR 9300-2014. Interim code for design of limit state method for railway bridges and culverts .Beijing:Beijing:China Railway Press.
[13] Li G Q and Li Q S, 2001. Time-varying reliability theory of engineering structures and its application.Beijing:Science Press.
[14] Zhou H F, 2013. Study on Reliability of Reinforced Concrete Railway Bridge. *Dalian: Dalian University of Technology Thesis.*