Prognostics for structural health monitoring of historic masonry structures with a novel LL-GPR model

Keyang Sun a, Qiang Xu b, Qingzhen Yao and Na Li

School of Architecture and Civil Engineering, Liaocheng University, Liaocheng 252000, China

a sunkeyangde@163.com, b xuqiang@lcu.edu.cn

Abstract. Structural health monitoring plays a significant role in civil engineering. To achieve more accurate prognostic evaluation of historic masonry structures, this paper presents a novel local linearization-based Gaussian process regression (LL-GPR) model. Local linearization is used to characterize the state of health (SOH) values of adjacent data points. Gaussian process regression is employed to predict the approximate linear relations. The proposed model is validated through a case study. The results demonstrate that the prediction results of LL-GPR can well reflect the degradation trend of SOH. Moreover, compared with the published methods, more accurate SOH prediction results can be obtained under this model.

Keywords: structural health monitoring, state of health, historic masonry structures, local linearization, Gaussian process regression.

1. Introduction

Structural health monitoring can provide quantitative information for both structural safety evaluations and maintenance purposes [1]. Prognosis is the estimation of a system’s state of health (SOH), beyond which, corrective action is required [2]. Developing prognostic methodologies for gradually deteriorating historic masonry monuments and infrastructure affords the possibility of ensuring structural safety, reducing maintenance costs, and preventing secondary damage of such cultural heritage [3]. The advanced prognostics and health management strategies have been made possible in many fields; however, prognostic evaluation of masonry heritage structures is in its infancy [4]. With prognostic techniques fully developed and successfully applied to historic masonry monuments, timely condition-based maintenance and restoration efforts can be planned and the life of such heritage structures can be prolonged.

Masonry construction is prone to experience gradual degradations affecting structural integrity in two forms: material degradations resulting primarily from environmental impacts and structural degradations resulting primarily due to applied loads or movement of supports. Of the latter, differential support settlements are common in masonry structures because of the heavy weight of the construction and are particularly detrimental to the integrity of the structure because of the low tensile capacity of unreinforced masonry [5, 6]. Nondestructive inspection techniques with potential to be automated that provide an indication of the global (rather than local) structural integrity are desired for prognostic evaluation of historic masonry structures. Particularly, dynamic responses, such as time-dependent...
accelerations or quasi-static responses, such as strains, can supply a viable solution to providing a diagnostic assessment of the structure. Other nondestructive techniques, such as the acoustic impact method, the impact-echo method, and the ultrasonic wave propagation method \cite{7, 8}, typically provide a local assessment and thus are appropriate for prognostic evaluation of a component or region of the structure. Using strains as the health indicator (HI), Atamturktur \cite{4} has proposed two support vector regression (SVR) based methods to predict the SOH of a Historic Masonry Fort. However, the prediction accuracy still needs to be improved.

In this paper, a new approach called local linearization-based Gaussian process regression (LL-GPR) model is proposed to predict SOH of historic masonry structures. This model can well reflect the degradation trend and get more accurate prediction results.

2. Methodology

2.1. Gaussian Process Regression Model.

Gaussian process regression (GPR) model is a flexible non-parametric model. It has been widely applied to multi-step-ahead predictions in time series analysis \cite{9, 10}. A GP model is completely specified by the mean function \( m(x) \) and the covariance function \( k(x, x') \), where \( m(x) \) and \( k(x, x') \) are described as follows:

\[
m(x) = E(f(x))
\]

\[
k(x, x') = \text{cov}(f(x), f(x')) = E( (f(x) - m(x))(f(x') - m(x')))
\]

The GP regression function is expressed as

\[
f(x) \sim \text{GP}(m(x), k(x, x'))
\]

For any finite set of points, this process defines a joint Gaussian:

\[
p(f|X) = N(f|m(X), K)
\]

where \( K_{ij} = k(x_i, x_j) \), \( m(X) = (m(x_1), ..., m(x_N)) \).

Note that selection of the proper forms of mean and covariance functions could take a positive effect on the prediction performance. The SOH time series shows a non-stationary characteristic, a linear mean function is utilized to capture the decrease trend, which is represented by a linear combination of the input shown as

\[
m(x) = \sum_{i=1}^{h} \theta_i x_i + \theta_0
\]

Where \( h \) is the dimension of \( x \), and \( \theta = [\theta_0, \theta_1, ..., \theta_h]^T \) is unknown hyper-parameters in mean function.

In addition, the squared exponential covariance function applied in SOH prognostics is as follows:

\[
k(x, x') = \sigma_f^2 \exp\left(-\frac{1}{2l^2}(x - x')^2\right)
\]

where \( \sigma_f^2 \) controls the vertical scale of this function, and \( l \) is the length-scale parameter and governs the speed of the correlation decrease as the input data distance increases.
For training dataset \( D = \{(x_i, y_i) \mid i = 1, 2, \ldots, N\} \), the target \( y_i \) is described by
\[
y_i = f(x_i) + \epsilon_i, \quad i = 1, 2, \ldots, N
\] (7)
where \( \epsilon_i \sim N(0, \sigma^2) \). This means that \( \epsilon \) is assumed to be the additive independent identically distributed Gaussian noise with zero mean and variance \( \sigma^2 \). The prior on the noisy observations becomes
\[
\text{cov}(y_i, y_j) = \text{cov}(f(x_i), f(x_j)) + \sigma^2 \delta_{ij}
\] (8)

Or
\[
\text{cov}(y) = K(X, X) + \sigma^2 I
\] (9)

where \( \delta_{ij} \) is a Kronecker delta which is one if \( i = j \) and zero otherwise, and \( I \) is the identity matrix of same dimensions as \( K \).

The accuracy of SOH prediction depends on the unknown hyper-parameters in both mean and covariance functions \( \Theta = [\theta, \sigma, I]^T \). Therefore, these hyper-parameters need to be optimized. The training of GPR model is carried out by minimizing the negative marginal log-likelihood function given by
\[
J = -\log p(y \mid X, \Theta)
= \frac{1}{2} \log |K(X, X) + \sigma^2 I|
+ \frac{1}{2} (y - m(X))^T (K(X, X) + \sigma^2 I)^{-1} (y - m(X)) + \frac{N}{2} \log 2\pi
\] (10)

Where \( \frac{1}{2} \log |K(X, X) + \sigma^2 I| \) is the complexity penalty depending on the covariance function and the inputs. Note that the model complexity decreases with the increasing of length-scale. A conjugate gradient method is utilized to optimize the hyper-parameters in this study.

For new input \( x^* \), the prior distribution of the GP is described as
\[
\begin{bmatrix} y \\ f^* \end{bmatrix} \sim \mathcal{N}\left( \begin{bmatrix} m(X) \\ m(x^*) \end{bmatrix}; \begin{bmatrix} K(X, X) + \sigma^2 I \\ k(x^*, X) \\ k(x^*, x^*) \end{bmatrix} \right)
\] (11)

where \( k(x^*, X) = [k(x^*, x_1), k(x^*, x_2), \ldots, k(x^*, x_N)] \). The posterior distribution of \( x^* \) is
\[
p(f^* \mid X, y, x^*) = \mathcal{N}(\tilde{f}^*, \text{cov}(f^*))
\] (12)

Where
\[
\tilde{f}^* = m(x^*) + k(x^*, X)\left[ K(X, X) + \sigma^2 I \right]^{-1}(y - m(X))
\] (13)
cov(f*') = k(x*, x*) - k(x*, X)[K(X, X) + \sigma^2 I]^{-1}k^T (x*, X) \quad (14)

2.2. Local Linearization-based Gaussian process regression Model.
Although the degeneration of SOH usually shows a nonlinear trend, the current SOH value has a strong connection with the previous SOH values. In this paper, it is assumed that several adjacent data points show an approximate linear trend. This means that the overall degeneration of SOH consists of many line segments with different gradient. We assume that each segment contains M data points. Take the first N of the M data points as the training dataset. Also, the current data index is denoted as CI, and health indicator is denoted as HI. The flowchart of the LL-GPR method is shown in Figure 1.

![Flowchart of LL-GPR method](image)

Fig. 1 Flowchart of LL-GPR method

3. Experiments and Analysis

3.1. Fort Sumter degeneration datasets
The construction of the pentagonal-shaped clay masonry fort began in 1829 on an artificial island. The Fort Sumter was declared a national monument in 1948, and is considered the structure of national heritage today.

The finite element (FE) model of the single casemate is developed in Ansys by incorporating data from on-site inspections and evaluations discussed in detail in [11]. Material properties are obtained through laboratory tests conducted on core samples of the masonry and a masonry prism specimen from fallen debris. The three-dimensional (3D) laser scanning is performed to obtain the precise as-is geometry of the casemate with which the FE model is constructed preserving key geometrical features. Then, the FE model is used to simulate support settlement, and the strain at the two control point locations are chosen as the health indicators and monitored during settlement simulations. The results are plotted in Figure 2.
3.2. Experiments and Comparison

In the prognostic evaluation, 15 data points simulating the strain response of the casemate under settlement up to 40 mm, at which first cracks start to appear, are used. To determine the hyper-parameters of GPR, the 15 data point are used as the training dataset. After that, with the identified hyper-parameter, the GPR model is executed to forecast the next five data points (from 40 to 52.5 mm settlement). This process is repeated as new measurement data become available, and the hyper-parameters are updated during each iteration. As a result, a total of five iterations are completed to reach 100 mm settlement. The training data index and prediction data index are the same as Ref. [4], which is helpful for performance comparisons among different prediction methods.

For health indicator #1 and #2, the prediction results of LL-GPR model are shown in Figure 3 and Figure 4 respectively. Moreover, the results of the two prediction approaches reported in Ref. [4] are plotted in the figures to compare with this proposed model. As seen in Figure 3 and Figure 4, the prediction results of LL-GPR can reflect the degradation trend of SOH. Obviously, the predictions based on the LL-GPR model are much better than the basic SVR model and weighted SVR model.

The quantitative comparison and evaluation results are shown in Table 1. From Table 1, we can see that the LL-GPR method has much better prediction performance than the two published methods. Between the two published methods, weighted SVR has better prediction results. The prediction errors of weighted SVR on health indicator #1 and #2 are 0.0719 and 0.0057 respectively, while the prediction errors of the LL–SVR are 0.0477 and 0.0034 respectively. It can be concluded that the LL–SVR could effectively improve the prediction performance of the masonry fort SOH.
Fig. 3 Comparison of local linearization-based GPR model to other approaches using HI #1: (a) prediction response; (b) prediction error

Fig. 4 Comparison of local linearization-based GPR model to other approaches using HI #2: (a) prediction response; (b) prediction error

4. Conclusion
To obtain more accurate prediction results of historic masonry structures, we have proposed a LL-GPR model. Based on the evidence that the degeneration of adjacent data points shows an approximate linear trend, GPR is employed to predict the local linear relations. Case studies were carried out to evaluate the performance of the proposed model. The results showed that using the previous degeneration data, this model can achieve more accurate SOH prognostics.

| Table 1. Comparison of different prognostic methods for HI #1 and HI #2 |
|--------------------------------------------------|------------|------------|
| Health Indicator | #1 | #2 |
|SVR\(^a\) | 0.1898 | 0.0178 |
|Weighted SVR\(^a\) | 0.0719 | 0.0057 |
|LL-GPR | 0.0477 | 0.0034 |

a. Results of these methods are obtained from reference [4]
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