Effects of interactions in transport through Aharonov-Bohm-Casher interferometers

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We study the conductance through a ring described by the Hubbard model (such as an array of quantum dots), threaded by a magnetic flux and subject to Rashba spin-orbit coupling (SOC). We develop a formalism that is able to describe the interference effects as well as the Kondo effect when the number of electrons in the ring is odd. In the Kondo regime, the SOC reduces the conductance from the unitary limit, and in combination with the magnetic flux, the device acts as a spin polarizer.

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Advances in semiconductor technology have provided useful tools to test fundamental concepts of quantum physics, such as the superposition principle and the existence of topological phases [1]. Beautiful demonstrations of these are studies of the Aharonov-Bohm (AB) effect in mesoscopic rings, particularly with embedded quantum dots (QDs) [2, 3]. The effect of interactions in these systems is still a matter of debate [4]. Despite the enormous effort to describe transport through interacting regions [5], at present we do not have a unified procedure to extend the results of the single particle case to many-body cases. A serious shortcoming of the calculation of the conductance $G$ through an interacting ring is that even knowing the exact eigenstates of the ring, there is no simple procedure to calculate $G$. When the coupling $V$ of the ring to the conducting leads is small, Jagla and Balseiro (JB) used a perturbative expression in $V$ for $G$ that is exact for any $V$ in the non-interacting limit [5]. Similar equations were used recently, assuming that a Zeeman term destroys the Kondo effect in the system [6, 7]. Another expression in order $V^2$ was proposed last year [10]. Unfortunately these expressions are not valid in the Kondo regime, in which the number of electrons in the ring is odd, because the resulting Kondo physics cannot be described by perturbation theory in $V$. The ideal conductance in the Kondo regime was recovered by mapping the model into an impurity Anderson model, but in this formulation interference effects were lost [3].

Recently, the Aharonov-Casher (AC) effect [11], the charge-spin dual of the AB effect, has been demonstrated experimentally in semiconductor mesoscopic rings [12, 13]. The AC phases are originated due to the Rashba spin-orbit coupling (SOC) in the ring, resulting from electronic motion in the presence of an electric field normal to the plane of the ring. The interference between electrons of given spin travelling clockwise and anticlockwise produces a strong modulation of the electronic current through the device. Recent theoretical research [14] has successfully explained the modulation of the conductance in terms of non-interacting electrons. However, the single-electron picture, turns out to be inadequate to describe electronic transport in the strongly-interacting case, particularly in the Kondo regime, as we will show.

In this Letter, we describe a systematic procedure to calculate equilibrium conductance $G$ through a ring of an interacting system weakly coupled to conducting leads, that takes into account both the effects of interference and correlations in presence of a magnetic flux and SOC. Using a non-abelian gauge transformation (NAGT), we show that for on-site interactions, the SOC can be absorbed in opposite AC phases for spin up and down in an adequately chosen quantization axis. For a Hubbard model (that describes a ring of an even number of QDs), in absence of SOC $G$ vanishes when the magnetic flux amounts to half a flux quantum. For other fluxes in the Kondo regime, $G$ reaches the unitary limit (ideal conductance [1]). When the SOC is turned on, the ideal conductance is destroyed and $G$ shows a strong spin dependence in this regime.

Our first task is to derive the appropriate extension to the Hubbard model to include the SOC in an adequate representation that simplifies our subsequent calculations. To illustrate the procedure, it is easier to begin with non-interacting electrons in the continuum. The correct Hamiltonian for this case was derived by Meijer et al. [13]. The SOC is $H_{\text{SOC}} = \alpha' \vec{E} \times \vec{r}$, where $\alpha'$ is the Rashba constant and $\vec{E}$ is the electric field, which in our case is in the $z$ direction, perpendicular to the plane of the ring. Including SOC and the orbital effects of the magnetic field, but neglecting the Zeeman term (usually several orders of magnitude smaller than the Kondo energy scale in QDs [3]), the Hamiltonian can be written in the form [14] (a)

\begin{equation}
H_{\text{NI}} = \hbar \Omega \left[ -i \frac{\partial}{\partial \phi} - \frac{\phi}{\phi_0} + \frac{\omega_{so}}{2\Omega} \sigma_z(\phi) \right]^2,
\end{equation}

where $\Omega = \hbar/(2m^*r^2)$, $m^*$ is the effective electron mass, $r$ is the radius of the ring, $\omega_{so} = \alpha' \hbar r$, $\phi = B\pi r^2$ is the magnetic flux, $\phi_0 = \hbar/e$ is the flux quantum and $\sigma_z(\phi) = \sigma_x \cos \phi + \sigma_y \sin \phi$ is the Pauli matrix in the radial direction, and $\phi$ is the azimuthal angle (see Fig. 1). Although the Schrödinger equation $H_{\text{NI}} \chi(\phi) = E \chi(\phi)$ (where $\chi$ is a spinor) has been solved [14], we are interested in a simplification of this equation...
that can be extended to the interacting case. This can be achieved by a NAGT \( \chi(\varphi) = \hat{U}(\varphi)\chi'(\varphi) \), where the operator \( \hat{U}(\varphi) \) satisfies the differential equation

\[
i\frac{\partial}{\partial \varphi}\hat{U}(\varphi) = \begin{bmatrix} -\frac{\phi}{\phi_0} + \frac{\omega_{so}}{2\Omega} \sigma_T(\varphi) \\
\end{bmatrix} \hat{U}(\varphi). \tag{2}\]

It can be easily checked that in the transformed Hamiltonian, \( H'_{N\Omega} = \hat{U}^\dagger H_N \hat{U} = -\hbar \Omega \partial^2/\varphi^2 \) the magnetic flux and the SOC disappeared, and enter now in the boundary condition, since \( \chi(2\pi) = \chi(0) \) implies \( \chi'(2\pi) = \hat{U}'(2\pi)\chi'(0) \). The solution of Eq.\,(2) with \( \hat{U}(0) = 1 \) is

\[
\hat{U}(\varphi) = \exp \left[ -i \sigma_z \frac{\varphi}{2} \right] \exp \left[ i \hat{n}_0 \frac{\varphi'}{2} \right] \exp \left[ i \frac{\phi}{\phi_0} \varphi \right], \tag{3}\]

where \( \hat{n}_0 = (-\sin \theta, 0, \cos \theta) \), \( \theta = \arctan (\omega_{so}/\Omega) \), and \( \varphi' = \varphi \sqrt{1 + (\omega_{so}/\Omega)^2} \).

To construct the tight binding version of \( H'_{N\Omega} \), let us assume that we have \( N \) sites, lattice parameter \( a \) (with \( Na = 2\pi r \)) and site 0 at angle \( \varphi = 0 \). For simplicity we consider only hopping between nearest neighbors (NN). Then, we can take a constant hopping \( t \) between all NN, except between sites \( N - 1 \) and 0, in which the boundary condition should be included. The matrix \( \hat{U}(2\pi) \) is easily diagonalized in the quantization axis \( \hat{n} \) and its eigenvalues are \( \exp[\mu(\Phi_{AB} + \sigma \Phi_{AC})] \), where \( \sigma = 1 \) (1-1) for spin up (down) in this direction, \( \Phi_{AB} = 2\pi \phi/\phi_0 \), and \( \Phi_{AC} = \pi \left\{ [1 + (\omega_{so}/\Omega)^2]^{1/2} - 1 \right\} \). Therefore, destroying a particle with spin \( \sigma \) at site \( N - 1 \) and creating it at site 0 should be accompanied by the corresponding exponential factors. On-site interactions are not affected by the NAGT. Changing the phases of the second quantization operators to write the Hamiltonian in rotationally invariant form, the transformed Hubbard model in the ring becomes

\[
H'_r = \sum_{i=0,\sigma}^{N-1} t \left[ e^{i(\Phi_{AB} + \sigma \Phi_{AC})/N} d_{i+1\sigma}^\dagger d_{i\sigma} + \text{H.c.} \right] +
+ Ud_{i1}^\dagger d_{i1} d_{i1}^\dagger d_{i1}. \tag{4}\]

From the curvature of the dispersion relation at small wave vector \( t = \hbar^2/(2m^*a^2) \), and then \( \omega_{so}/\Omega = \alpha N/(2\pi a) \). Thus, the AC phase can be written as

\[
\Phi_{AC}/N = \sqrt{\left(\frac{\pi}{N}\right)^2 + \left(\frac{\alpha}{2ta}\right)^2} - \frac{\pi}{N}. \tag{5}\]

Therefore, for large \( \alpha \) or \( N \), the properties of the system are periodic with \( \alpha \) as observed experimentally [12,13].

The fact that the SOC can be gauged away in one dimension has been noted previously [14], but the explicit form of the transformation has not been derived. This transformation has important consequences. In the thermodynamic limit the boundary conditions are irrelevant and therefore the thermodynamic properties of the system should be identical to those of the Hubbard model without SOC. This is not obvious in alternative treatments [17]. In particular it seems that the opening of a spin gap in the system requires long-range interactions.

To study the conductance, we must consider the Hamiltonian of the complete system \( H = H_l + H'_r + H_V \), where with the appropriate quantization axis \( \hbar/2m^*a^2 \) and choice of phases \( H_l = t_c(\sum_{i=0,\sigma} c_{i-1,\sigma}^\dagger c_{i\sigma} + \sum_{i=1,\sigma} c_{i+1,\sigma}^\dagger c_{i\sigma} + \text{H.c.}) \), describes the non-interacting leads, and \( H_V = V(\sum_{\sigma} c_{0\sigma}^\dagger d_{N/2,\sigma} + c_{1\sigma}^\dagger d_{0,\sigma} + \text{H.c.}) \) is the coupling between the ring and leads. For simplicity we will focus here on the particular case \( N = 4 \), illustrated in Fig. 1. We assume that the leads are described by a constant density of states \( \rho_0 = 1/W \), and we take for the band width of the leads \( W = 60t \) (\( W \) is usually much larger than \( t \) in QD arrays). The Fermi level is set at \( \epsilon_F = 0 \). To control the charge in the ring we add to \( H'_r \) a term \( -V_g \sum_{\sigma} d_{4\sigma}^\dagger d_{4\sigma} \) that represents the effect of a gate voltage. Our approximations to calculate \( G \) amount to a truncation of the Hilbert space of \( H'_r \) and a slave boson mean-field approximation for the resulting generalized Anderson model (GAM). \( H'_r \) can be diagonalized exactly (numerically for not too large \( N \)). We retain only two neighboring charge configurations with \( n \) and \( n - 1 \) particles and we have chosen \( n = 4 \). Furthermore, we retain only the lowest lying singlet state for 4 particles \( \langle \psi_0^4 \rangle \) with energy \( E_0^4 \) and all doublets for 3 particles. This procedure is valid for small enough \( V \) [19]. Calculating the matrix elements of \( H_V \) in the truncated Hilbert space leads to a GAM

\[
H_{GAM} = H_l + \sum_{j,\sigma} E_{3j}^3 |\psi_{3j}^3 \rangle \langle \psi_{3j}^3 | + E_{0}^4 |\psi_{0}^{4} \rangle \langle \psi_{0}^{4} | +
+V \sum_{j,\sigma,\eta = 0,1} (\alpha_{j\sigma}^\eta |\psi_{j}^{3} \rangle \langle c_{j\sigma} + \text{H.c.}), \tag{6}\]

where \( |\psi_{j}^{3} \rangle \) and \( E_{3j}^3 \) denote the \( j \)-th eigenvector and eigenvalue of \( H'_r \) in the configuration with 3 particles with
In ascending order of energy and

$$\alpha_{ij}^3 = \langle \psi_0^3 | d_{i\sigma}^3 | \psi_0^3 \rangle, \quad \alpha_{ij}^{\bar{3}} = \langle \psi_0^{\bar{3}} | d_{i/2,\sigma}^{\bar{3}} | \psi_0^{\bar{3}} \rangle. \quad (7)$$

$H_{\text{GAM}}$ can be expressed exactly in terms of a slave-boson representation similar to that proposed by Coleman [20]:

$$| \Psi_j^3 \rangle \langle \Psi_j^3 | \rightarrow f_j^\dagger f_j, \quad | \Psi_j^4 \rangle \langle \Psi_j^4 | \rightarrow b_j^\dagger b_j | \Psi_j^4 \rangle \langle \Psi_j^4 | \rightarrow f_j^\dagger b_j \quad \text{and} \quad | \Psi_j^{\bar{3}} \rangle \langle \Psi_j^{\bar{3}} | \rightarrow b_j^\dagger f_j,$$

where the operators $b^\dagger$ and $f_j^\dagger$ create a boson and a fermion respectively and are subject to the constraint $\sum_j f_j^\dagger f_j + b_j^\dagger b_j = 1$, which is incorporated in the Hamiltonian with a Lagrange multiplier $\lambda$. We perform a saddle-point approximation in the bosonic degrees of freedom, which reproduces the Kondo physics at low temperatures and becomes exact in the limit of infinite degeneracy of the magnetic configuration, due to vanishing fluctuations around the mean-field value of the bosonic field $\lambda$. Thus, we can use the two-terminal Landauer formula to calculate the conductance, giving at zero temperature

$$G = \sum \sigma G_{\sigma},$$

$$\frac{G_{\sigma}}{G_0} = 2 (\pi \rho_0 V^2 b_0^2)^2 \sum_{ij} \alpha_{i,j}^3 \alpha_{i,j}^{\bar{3}} (\epsilon_F)^2, \quad (9)$$

where $G_0 = 2e^2/h$ and

$$g_{ij}^\sigma = g_{ij}^{0\sigma} + \frac{b_0^2 V^2 g_{ij}^{0\sigma}}{A_{01}^\sigma A_{00}^\sigma - A_{10}^\sigma A_{01}^\sigma} \sum_{\eta,\eta'} \alpha_{i,j}^{\bar{3}} A_{\eta'\eta}^\sigma,$$

where $g_{ij}^{0\sigma}(\omega) = |g_{ij}^{0\sigma}(\omega - E_0 - \lambda)|^{-1}$ is the propagator of the $j$-th state of 3 particles with spin projection $\sigma$ in the isolated ring, and the functions $A_{\eta'\eta}^\sigma$ are $A_{\eta'\eta} = 1 - b_0^2 V^2 g_{ij}^{0\sigma}(\omega - E_0 - \lambda)$, for $\eta = \eta'$ and $A_{\eta'\eta} = b_0^2 V^2 g_{ij}^{0\sigma}(\omega - E_0 - \lambda)$, for $\eta \neq \eta'$, where $g_{ij}^{0\sigma}(\omega)$ is the Green’s function at site $\eta$ of the corresponding isolated lead ($V = 0$).

In Fig. 2 we show $G$ as a function of magnetic flux in the non-magnetic regime $E_0^{(4)} < E_0^{(3)}$, for different values of $V$ and without electric field ($\alpha = 0$). The conductance is even with flux [21] and therefore, it is enough to show $G$ in the interval $0 \leq \phi \leq \phi_0/2$ (or $0 \leq \Phi_{AB} \leq \pi$). In this regime correlations play a minor role and one expects that the JB formula [21], which is exact in the non-interacting case, gives accurate values for $G$. Our results show the same qualitative behavior. In fact, for small $V$ it can be demonstrated that both approaches are equivalent in this regime. For $\phi \leq \phi_0/2$, $G$ vanishes due to destructive interference.

The difference $E_0^{(4)} - E_0^{(3)}$ can be reduced and turned negative applying a negative gate voltage. The most important results of this work are those obtained in this case i.e., when the ring is in the mixed valence or Kondo regime. Results for $\alpha = 0$ are presented in Fig. 3 (a). For small enough $\Delta/|E_0^{(3)} - E_0^{(4)}|$, where

$$\Delta = \pi \rho_0 V^2 (|\alpha_{0\sigma}^3|^2 + |\alpha_{0\sigma}^{\bar{3}}|^2),$$

charge fluctuations are frozen and a clear signature of Kondo physics is displayed in the characteristic plateau in $G$ at the ideal conductance $G_0$ (the unitary limit) [4]. This is shown in the figure for the smaller values of $V$ at small fluxes. The dependence of $G$ with flux, is related with the corresponding dependence of the energy levels and matrix elements with $\phi$. For larger $V$ and $\Phi_{AB} \sim \pi$, the system is in the intermediate valence regime, as reflected in Fig. 3 (b) in which the total occupancy of the configuration with three particles $n_f = \sum_{j=1}^3 f_j f_j^\dagger$ is shown. Therefore, the conductance deviates from the unitary limit.

Independently of the other parameters, $G$ vanishes at $\Phi_{AB} = \pi$. Within our formulation, at this point the states of the $n = 3$ configuration become doubly degen-
erate between states of different parity. The matrix elements $\alpha'_{\sigma'}^\alpha_{\sigma}$ entering Eq. 4 have the same modulus but different sign, therefore producing a complete destructive interference inside the absolute value. To our knowledge, there are no calculations so far showing at the same time this destructive interference and ideal conductance in the Kondo regime. The JB expression gives values below $0.1G_0$ for all $\Phi_{AB}$ and parameters of Fig. 3. Previous mappings to the Anderson model displayed the Kondo physics, but did not capture the interference [11].

The effect of the SOC on the total conductance is dramatic in the Kondo regime. The results presented in Fig. 4 show dips (additional to that of $\Phi_{AB} = \pi$) which are larger as $\alpha$ grows. The main difference with the case $\alpha = 0$ is that $n_f \neq n_f'$, therefore producing a partial destruction of the Kondo resonance, mimicking the effect of a Zeeman term. This effect is larger for lower $\Delta$ (when the system is deeper inside the Kondo regime), which for the parameters of Fig. 4 corresponds to $\Phi_{AB} \sim \pm 0.3\pi$. For $\Phi_{AB} = \pi$, complete cancellation is not achieved. This can be interpreted as an effect of quasiparticles acquiring relative phases differing slightly from $\pi$ due to the effect of the additional AC phase (see inset in Fig. 3).

Another important effect of the SOC in the Kondo regime is that it leads to currents with significant spin polarization. If a spin $\sigma$ (up or down) in the quantization direction $\vec{n}_0$ is injected in the ring at the right lead ($\varphi = 0$) it comes out at the left lead ($\varphi = \pi$) with spin $\sigma$ in the direction $\vec{n}'_0 = (\sin \theta, 0, \cos \theta)$ or vice versa [15]. The corresponding conductance $G_{\sigma}$ is spin dependent, as shown in Fig. 5. The ratio of the conductances can reach a factor 2 or larger with ideal $G_{\uparrow} (G_{\downarrow})$ for flux $\Phi_{AB} = 0.15\pi (-0.15\pi)$ and rather small $\alpha$ [21]. For these values, the $z$ component of the quantization axis for any $\varphi$ is larger than 0.99 [15].

In summary, we have presented an approach to calculate the conductance through a ring of interacting QDs threaded by a magnetic flux and with spin-orbit coupling $\alpha$ in the Kondo, mixed-valence and non-magnetic regimes. The effects of $\alpha$ are incorporated into Aharonov-Casher phases using a gauge transformation that leads to the Hubbard Hamiltonian Eqs. 4 and 5. Using a method based on a mapping of the relevant exact eigenstates of the ring onto an effective multilevel Anderson impurity and with the use of a slave-boson representation in the saddle-point approximation, we are able to describe the properties of the ring connected to the leads. The method is valid for small values of the coupling between rings and leads $V$ and small values of magnetic field $B$, such that the Zeeman energy is much less than $T_K$. When the ring is in the Kondo regime, we obtain ideal conductance for $\alpha = 0$ and magnetic flux far from half a flux quantum, for which there is complete destructive interference. The effect of a small non-vanishing $\alpha$ is to produce a progressive destruction of the Kondo effect, decreasing the conductance and leading to a strong spin dependence of it. Extensions to include the Zeeman term or other interacting systems with local interactions are straightforward.

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Figure 4: Conductance as a function of flux for $V = 3.5t$ and different values of $\alpha$. Other parameters as in Fig. 3.

Figure 5: Conductance for each spin as a function of flux for the same parameters as in Fig. 4 and different values of $\alpha$.

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