Charmonium Production in $e^+e^-$ Annihilation at $\sqrt{s} = 10.6\text{GeV}$

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We review the charmonium production in $e^+e^-$ annihilation at $\sqrt{s} = 10.6\text{GeV}$. NRQCD predictions for the $J/\psi$ production are compared with the recent measurements by BaBar and Belle. A NRQCD calculation for the $D$-wave charmonium production is reported to further test the color-octet mechanism and detect the $J^{PC} = 2^{--}$ state. The issue of double $c\bar{c}$ production in $e^+e^-$ annihilation observed by Belle is also discussed.

1. INTRODUCTION

The newly developed nonrelativistic QCD (NRQCD) factorization formalism \cite{1} allows the infrared safe calculation of inclusive heavy quarkonium production and decay rates. In the NRQCD production mechanism, a heavy quark-antiquark pair can be produced at short distances in a conventional color-singlet or a color-octet state, and then evolves into an observed quarkonium nonperturbatively. With this color-octet mechanism, one might explain the Tevatron data on the surplus production of $J/\psi$ and $\psi'$ at large $p_T$, but puzzles about their polarizations still remain (for a review see \cite{2} and references therein).

To further test the color octet mechanism, it may be interesting to study the charmonium production in $e^+e^-$ annihilation. The $J/\psi$ production in $e^+e^-$ annihilation has been investigated within the color-singlet model \cite{3,4,5} and the color-octet model \cite{6,7,8}. The angular distribution and energy distribution of color-singlet $J/\psi$ production at $\sqrt{s} = 10.6\text{GeV}$ have been discussed in \cite{5}. In \cite{5} it is noted that a clean signature of the color-octet mechanism may be observed in the angular distribution of $J/\psi$ production near the end point region. In \cite{5} contributions of the color-octet as well as color-singlet to the $J/\psi$ production cross sections are calculated in a wide range of $e^+e^-$ collider energies and it is found that with reasonable choices of both color-singlet and octet matrix elements the color-octet contribution will dominate. Moreover, the $J/\psi$ polarizations are predicted in \cite{5}. Recently, BaBar \cite{9} and Belle \cite{10} have measured the direct $J/\psi$ production in continuum $e^+e^-$ annihilations at $\sqrt{s} = 10.6\text{GeV}$. The total cross section and the angular distribution seem to favor the NRQCD calculation over the color-singlet model \cite{9}, but some issues still remain. The P-wave charmonium $\chi_{cJ}$ production in $e^+e^-$ annihilation has been discussed in \cite{11}. The total $\chi_{c1,2}$ cross sections are predicted to be dominated by the color-octet process. In addition, as a further test of the production mechanism, we will report a calculation for the $D$-wave charmonium production in $e^+e^-$ annihilation, to which the color-octet process will make substantial contributions \cite{14}, and these states may be detected in the future by BaBar and Belle.

2. $J/\psi$ PRODUCTION IN $e^+e^-$ ANNihilation

The leading order color-singlet contributions to direct $J/\psi$ production include the following processes

$$e^+e^- \rightarrow \gamma^* \rightarrow J/\psi gg, \quad (1)$$

$$e^+e^- \rightarrow \gamma^* \rightarrow J/\psi c\bar{c}, \quad (2)$$

$$e^+e^- \rightarrow \gamma^* \rightarrow g\bar{q}q^* \text{ with } g^* \rightarrow J/\psi gg. \quad (3)$$

Eq.\(1\) is the gluon process, and Eq.\(2\) is the quark process (the charm quark fragmentation), and they are shown respectively in Fig.1(a) and Fig.1(b). Eq.\(3\) is the gluon jet process with $q = u, d, s, c$ quarks.

At $\sqrt{s} = 10.6\text{GeV}$ the gluon process (1) dominates over the other two color singlet processes. The process (3) is negligible at low energies, but grows as the energy increases, and at high enough
energies ($\sqrt{s} \geq 250 GeV$) its contribution will dominate over processes (1) and (2).

In Fig.2, we display the energy distributions of the gluon process (1), the quark process (2), and their sum at $\sqrt{s} = 10.6 GeV$. The energy spectrum of these two processes are both flat and the gluon process contribution is much larger than the quark process (see Fig.2).

The leading order color-octet contributions to direct $J/\psi$ production in $e^+e^-$ annihilation include the following two processes:

\begin{align}
e^+ e^- &\rightarrow \gamma^* \rightarrow g + cc[8,^{2S+1}L_J], \quad (4) \\
e^+ e^- &\rightarrow \gamma^* \rightarrow q\bar{q} + c\bar{c}[8,^3S_1], \quad (5)
\end{align}

as shown respectively in Fig.1(c) and Fig.1(d). Here $^{2S+1}L_J$ denotes the states $^1S_0$ and $^3P_J$, $q$ represents the $u$, $d$, $s$, $c$ quarks. At low energies the dominant process is the process (4), whereas the process (5) will become dominant at $\sqrt{s} > 25 GeV$.

As shown above, with a reasonable range of the color-singlet and octet matrix elements, for
the color-singlet cross section the quark process (2) contributes 0.07-0.1 pb $\chi_{cJ}$, which is consistent with $\chi_{cJ}$, the gluon process (1) contributes 0.2-0.3 pb, and the total color-singlet cross section is 0.3-0.4 pb. The color-octet cross section contributes 0.6-0.7 pb. Including both the color-singlet and octet contributions, we get a total cross section of about 0.9-1.1 pb. BaBar gives the total cross section $\sigma(e^+e^+ \to J/\psi X) = 2.52 \pm 0.21 \pm 0.21 \text{ pb}$, and Belle gives $\sigma(e^+e^- \to J/\psi X) = 1.47 \pm 0.10 \pm 0.13 \text{ pb}$. Their values are much larger than the predicted color-singlet value of 0.3-0.4 pb. Moreover, for the $J/\psi$ angular distribution parameter $A$, at high momentum $p^*$, NRQCD predicts $0.6 < A < 1.0$ $\chi_{cJ}$ while the color-singlet model predicts $A \approx -0.8$, BaBar gives $A = 1.5 \pm 0.6$ for $p^* > 3.5 \text{ GeV}$, clearly favoring NRQCD.

However, there are some problems for the color-octet model. First, the expected peak at the upper end point in the $J/\psi$ momentum $p^*$ distribution in process (4) has not been observed. It is not clear whether this could be due to the smearing effect of soft gluon emission at the end point $\chi_{cJ}$. Second, the $J/\psi$ polarization observed by BaBar and Belle is mainly longitudinal, but the process (4) predicts the unpolarized $J/\psi$ $\chi_{cJ}$. The color-singlet process (1) predicts longitudinally polarized $J/\psi$ but it only gives a small portion of the cross section ($\sim 0.3$ pb). Again, the polarization problems remain as at the Tevatron for the NRQCD color-octet mechanism $\chi_{cJ}$. One of the possible solutions for the $J/\psi$ polarization puzzle is that the color $M_1$ transitions, which will flip the spin of the charm quark and give rise to longitudinal polarization, may not be unimportant as compared with the $E1$ transitions in the evolution of the charm quark pair.

3. $\chi_{cJ}$ PRODUCTION IN $e^+e^-$ ANNIHILATION

The P-wave charmonium $\chi_{cJ}$ production in $e^+e^-$ annihilation has been discussed in $\chi_{cJ}$. Because the C-parity forbids the process $e^+e^- \to c\bar{c}[3P_J]gg$, the color-singlet processes are $e^+e^- \to \chi_{cJ} + c\bar{c}$ and $e^+e^- \to \chi_{cJ} + \gamma$, and the color-octet processes are $e^+e^- \to (c\bar{c})[1S_0,2P_J] + g$. We get these cross sections as

$$\sigma(e^+e^- \to \chi_{cJ}) = 0.001, 0.012, 0.005 \text{ pb}, \sigma(e^+e^- \to \chi_{cJ} + c\bar{c}) = 0.005, 0.018, 0.008 \text{ pb}, \text{ and } \sigma_{\text{octet}}(\chi_{cJ}) = 0.022, 0.067, 0.112 \text{ pb for } J = 0, 1, 2 \text{ respectively}.$$

Then we get their total cross sections

$$\sigma(e^+e^- \to \chi_{cJ}X) = 0.028, 0.097, 0.125 \text{ pb},$$

which are consistent with $\chi_{cJ}$. The color-octet contributions are larger than the color-singlet by a factor of $2 \sim 10$. These total cross sections are smaller than the upper limit given by Belle.

4. D-WAVE CHARMONIA $\delta_J$ PRODUCTION IN $e^+e^-$ ANNIHILATION

In a recent study $\chi_{cJ}$ we find that the spin triplet D-wave charmonia $\delta_J(J = 1, 2, 3)$ production in $e^+e^-$ annihilation at $\sqrt{s} = 10.6 \text{ GeV}$ mainly comes from Fig.1(a) for the color-singlet process, and Fig.1(c) for the color-octet process. The contribution of color singlet process in Fig.1(b) to the $\delta_1$ cross section is found to be smaller by a factor of about 60 than that for the $J/\psi$ and therefore is negligible, which is consistent with $\chi_{cJ}$. Using the velocity scaling rule and relating the $\delta_J$ color-octet matrix elements to that of $\psi'$, we get the total cross sections

$$\sigma(\delta_1) = \sigma_1 + \sigma_8 \approx 0.027 + 0.016 = 0.043 \text{ pb},$$

$$\sigma(\delta_2) = \sigma_1 + \sigma_8 \approx 0.067 + 0.027 = 0.094 \text{ pb} \quad (7)$$

However, if we use a more radical choice of the $\delta_J$ color-octet matrix elements by relating them to that of $J/\psi$, we would get much larger values for the $\delta_J$ production cross sections, $\sigma(\delta_1) \approx 0.16 \text{ pb}$ and $\sigma(\delta_2) \approx 0.29 \text{ pb}$. Therefore the measurement of $\delta_J$ production in the future will be helpful to clarify the color-octet mechanism and the $\delta_J$ color-octet matrix elements.

In particular, for the $2^- \chi_{cJ}$ D-wave charmonium $\delta_2$, which is expected to be below the open charm $D\bar{D}^*$ threshold and forbidden to decay to $D\bar{D}$, its width is estimated to be $300 - 400 \text{ KeV}$ and its branching fraction of decay mode $J/\psi\pi^+\pi^-$ is $B(\delta_2 \to J/\psi\pi^+\pi^-) \approx 0.12 \chi_{cJ}$, which is only smaller than that of $B(\psi' \to J/\psi\pi^+\pi^-) = 0.324 \pm 0.026$ by a factor of 3. With more data available at the $B$ Factories in the future, it will be possible to detect this $2^- \chi_{cJ}$ D-wave charmo-
nium state. The $1^{--}$ D-wave state $\psi''(3770)$ could also be detected via $\psi'' \rightarrow D\bar{D}$ decay.

5. PUZZLE OF DOUBLE $c\bar{c}$ PRODUCTION IN $e^+e^-$ ANNIHILATION

Recently the Belle Collaboration has reported a measurement on the double $c\bar{c}$ production in $e^+e^-$ annihilation at $\sqrt{s} = 10.6\text{GeV}$, and found that a very large fraction of the produced $J/\psi$ is due to the double $c\bar{c}$ production in $e^+e^-$ annihilation\(^{[17]}\)

$$\sigma(e^+e^- \rightarrow J/\psi c\bar{c})/\sigma(e^+e^- \rightarrow J/\psi X) = 0.59^{+0.15}_{-0.13} \pm 0.12, \quad (8)$$

which corresponds to $\sigma(e^+e^- \rightarrow J/\psi c\bar{c}) \approx 0.9\text{pb}$.

This result is puzzling in terms of perturbative QCD calculations. The color-singlet process (2) (the charm quark fragmentation, see Fig.1(b)) will contribute only $\sim 0.1\text{pb}$ to the cross section. Moreover, the produced $J/\psi$ should be transversely polarized in process (2)\(^{[1]}\), in contrast to the observed $J/\psi$ being mainly longitudinally polarized\(^{[1]}\). The color-octet process (5) may also contribute to the double $c\bar{c}$ production with $q = c$, but again its cross section is too small. Whether the QCD radiative correction to process (2) can enhance the double $c\bar{c}$ production by a factor of $\sim 9$ remains interesting, but it may still be difficult to explain the observed $J/\psi$ polarization. So, we intend to conclude that it is very hard to explain the double $c\bar{c}$ production data observed by Belle based on NRQCD or, more generally, on perturbative QCD, and possible nonperturbative QCD effects have to be considered.

In summary, we find that charmonium production in $e^+e^-$ annihilation at $\sqrt{s} = 10.6\text{GeV}$ is interesting in testing the heavy quarkonium production mechanism in NRQCD. We expect that with more data collected at BaBar and Belle in the future, many theoretical issues will be further clarified, and we will reach a new step towards a better understanding for heavy quarkonium physics in QCD.

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