Interference and multi-particle effects in a Mach-Zehnder interferometer with single-particle sources

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(Dated: November 4, 2014)

We investigate a Mach-Zehnder interferometer fed by two time-dependently driven single-particle sources, one of them placed in front of the interferometer, the other in the centre of one of the arms. As long as the two sources are operated independently, the signal at the output of the interferometer shows an interference pattern, which we analyse in the spectral current, in the charge and energy currents, as well as in the charge current noise. The synchronisation of the two sources, allowing for collisions and absorptions of particles at different points of the interferometer, has a strong impact on the detected signals. It introduces new relevant time-scales and can even lead to a full suppression of the interference in some of the discussed quantities. The interpretation of this phenomenon in terms of spectral properties and tuneable two-particle effects is put forward in this manuscript.

PACS numbers: 72.10.-d, 73.23.-b, 73.23.Ad, 72.70.+m

I. INTRODUCTION

The coherent emission of single particles into a nano-electronic circuit can be realised by the time-dependent modulation of mesoscopic structures. Recently, the creation of Lorentzian current pulses carrying exactly one electron charge has been realised by periodically driven mesoscopic capacitors as single-particle sources by time-dependent gating, the emission of particles from quantum dots with surface-acoustic waves and from dynamical quantum dots have been intensively studied. Nano-electronic devices fed by these single-particle sources allow for the observation of controlled and tuneable quantum-interference and multiple-particle effects and the combination of both.

A useful tool to observe quantum-interference effects in an electronic system is a Mach-Zehnder interferometer (MZI) as sketched in Fig. 1 a.), which can be realised by edge states in Quantum Hall systems with the help of quantum point contacts (QPCs). It has been shown that the investigation of the output current of an MZI, when fed by a single-particle source (SPS), such as the one realised by Fève et al., allows for the extraction of an electronic single-particle coherence time. More generally, it carries interesting new features of coherence properties of the travelling particles. The combination of several of these sources makes it possible to study controlled two-particle effects, for example the electronic analogue of the Hong-Ou-Mandel effect which was realised experimentally by Bocquillon et al. and Dubois et al. The combination of several MZIs and SPSs is a possibility to create and detect time-bin entanglement. However, the impact of controlled multiple-particle effects on the interference pattern detected in electronic interferometers was studied only sparsely and leaves a number of open questions concerning the interplay of the two effects.

In this manuscript, we investigate an MZI into which particles are injected from an SPS, such that quantum interference effects can be detected at the interferometer output. The signal detected at the output shows intriguing features due to the energy-dependent transmission of the MZI. Subsequently, a second SPS is introduced...
injecting particles into one of the interferometer arms, only. The particle emission (and absorption) from this second source has a tunable impact on the interference effects obtained from the signal of the first SPS. We use this setup to carefully investigate the occurrence of tunable two-particle effects from synchronised SPSs in an electronic MZI, as shown in Fig. (1a). In order to do so, we study the spectral properties of the detected signal, the charge and energy currents, as well as the charge-current noise based on a Floquet scattering-matrix approach.

Importantly, the observables that we investigate theoretically in this manuscript, can be envisaged to be studied also in experiments. Indeed, the charge current and charge-current noise of SPSs in Quantum Hall devices was recently measured. Also interference effects in energy or heat currents were detected in a stationary superconducting interferometer.

The aim of our study is to scrutinise the role of the particle nature of injected electrons and holes, compared to the spectral properties of the related current pulses. Here, the spectral current allows us to characterise the energy-resolved interference, occurring in a so called channelled spectrum, which gives an insight into the behaviour of plane waves as the constituents of the complex signal of the MZI with one or two sources. With the averaged charge and energy current, we then consider two observables in which these spectral properties enter. At the same time, they reveal that the impact of the synchronised sources on the quantum interference can to a certain extent be explained by the occurrence of two-particle effects. In order to reliably investigate the impact of two-particle effects we move on to analyse the charge-current noise, obtained from a correlation function of two current operators, which is hence able to capture two-particle physics directly.

The paper is organised as follows. We introduce the system and the investigated observables, as well as the scattering matrix approach we employ in Sec. [II] The presentation of results starts with the spectral current, the charge and the energy current of the case of an interferometer fed by one SPS only, in Sec. [III] In Sec. [IV] this is followed by a study of the same quantities in an MZI where particles from two SPSs can collide or where particles can get absorbed. Finally results for the charge-current noise are shown in Sec. [V] In the Appendix all relevant analytic results which are not presented explicitly in the main text are summarised.

II. MODEL AND TECHNIQUE

A. Mach-Zehnder interferometer with two single-particle sources

The electronic analogue of an MZI, as sketched in Fig. (1a), can be realised in a two-dimensional electron gas in the quantum Hall regime. In these setups, transport takes place along spin-polarised, chiral edge states depicted as black lines in Fig. (1a), where arrows indicate their chirality. Two quantum point contacts, QPC, \( \ell = L, R \), with energy-independent transmission (reflection) amplitudes \( t_\ell (r_\ell) \) and the related transmission (reflection) probabilities \( T_\ell = |t_\ell|^2 \) and \( R_\ell = |r_\ell|^2 \) act as beam splitters. The incoming electronic signal is reflected or transmitted at QPC, into the upper arm (u) or the lower arm (d) of the interferometer, with the respective length \( L_u \) and \( L_d \). At QPC, the signal is finally reflected or transmitted into reservoir 3 or 4. Assuming a linear dispersion with the drift velocity \( v_D \), the traversal time of the interferometer arms is given by \( \tau_u = L_u/v_D \) and \( \tau_d = L_d/v_D \). The interferometer is penetrated by a magnetic flux \( \Phi_0 \). Therefore, the phase acquired by the electronic wave function due to the propagation along the upper and the lower arm is given by \( \phi_{u/d} = \Phi_{u/d} + E\tau_{u/d}/h \) with the energy-dependent dynamical phase \( E\tau_{u/d}/h \) and the energy-independent part, \( \Phi_{u/d} \), including the magnetic-flux contribution \( \Phi_0 \). The energy and charge currents observed at the detector are known to depend on the difference between the two phases, \( \Delta\phi(E, \Phi) = \Phi + E\Delta\tau/h \) with \( \Phi = \Phi_u - \Phi_d \) and the detuning, \( \Delta\tau = \tau_u - \tau_d \) of the traversal times of the interferometer, which is a measure of the imbalance of the interferometer. The electronic reservoirs, \( \alpha = 1, 2, 3, 4 \), are at temperature \( \theta \) and they are grounded at the equilibrium chemical potential \( \mu \), which we take as the zero of energy from here on.

Particles - electrons and holes - are injected into the MZI by means of a controllable single-particle source, SPS, situated at the channel incoming from reservoir 1. A second single-particle emitter, SPS, is placed at the lower arm at \( L_d/2 \). We take the SPSs to be mesoscopic capacitors which are time-dependently driven by periodic gate potentials as sketched in Fig. (1b), inspired by the experimental realisation by Fève et al. These SPSs, with \( k = A, B \), consist of a quantum dot with a discrete spectrum, weakly coupled to an edge state through a QPC. A periodically oscillating time-dependent gate voltage \( U_k(t) \), with period \( T = 2\pi/\Omega \) and frequency \( \Omega \), moves the energy levels of the respective quantum dot, such that one of the levels is subsequently driven above and below the electro-chemical potential \( \mu \). This triggers the emission of an electron from source \( k = A, B \) at time \( t_k^e \), during one half of the driving period, and the emission of a hole (which is equivalent to the absorption of an electron) at a time \( t_h^e \) during the other half of the period.

This particle emission from SPS leads to current pulses carrying one electron or one hole. The injection of current pulses from SPS into the MZI, results in an interference pattern in the detected observables at the output of the interferometer. This is in contrast to the current pulses emitted from SPS which travel along the lower arm only and therefore do not create an interference pattern on their own.

However, the synchronisation of the two sources, obtained by tuning the phase difference between the two
driving potentials $U_k(t)$, influences the interference pattern drastically. The synchronisation of the two sources results in collisions of particles (i.e. the overlap of current pulses carrying an electron each, respectively carrying a hole each) at SPS$_B$ or QPC$_R$ or in an absorption process (i.e. the overlap of a current pulse carrying an electron with a current pulse carrying a hole) at SPS$_B$. It has been shown in Ref. [20] that these collisions and absorptions add a non-trivial phase to the interference pattern in the time-resolved current at the detector at the output of the MZI, which can even lead to the full suppression of interference in the detected average charge current. Of particular relevance for these synchronised two-particle events are the two time-differences $\Delta t_d^{ij}$, $\Delta t_d^{ji}$. The first one is the difference between the time at which a particle $i = e,h$ emitted from SPS$_A$ travelling the lower arm arrives at SPS$_B$ and the emission time of a particle $j = e,h$ at SPS$_B$, $\Delta t_d^{ij} = t_A - t_B + \tau_d/2$. The second one is the difference between the time at which a particle $i$ emitted from SPS$_A$ travelling the upper arm arrives at QPC$_R$ and the time at which a particle $j$ emitted from SPS$_B$ arrives at QPC$_R$, $\Delta t_d^{ji} = t_A - t_B + \tau_u - \tau_d/2$.

B. Scattering matrix formalism

We describe the transport properties of the above introduced system with the help of a Floquet scattering matrix formalism. Due to the time-periodic modulation of the SPSs, coherent inelastic scattering can take place. Thus the scattering matrix elements $S_{\alpha\beta}(E_n, E_m)$, connect the incoming currents from reservoir $\beta$ at energy $E_m = E + mh\Omega$ to the outgoing currents at reservoir $\alpha$ at energy $E_n = E + nh\Omega$ differing from the incoming energy by an integer multiple $n - m$ of the energy quantum $h\Omega$ given by the driving frequency (Floquet quanta) [21]. These scattering matrices can be conveniently written in terms of the partial Fourier transforms,

$$S_{\alpha\beta}(E_n, E_m) = \int_0^T dt \frac{d}{dE} e^{-i(n-m)\Omega t} S_{in,\alpha\beta}(t, E_m),$$

$$S_{\alpha\beta}(E_n, E_m) = \int_0^T dt \frac{d}{dE} e^{i(n-m)\Omega t} S_{out,\alpha\beta}(E_n, t).$$

Here, $S_{in,\alpha\beta}(t, E_m)$ is the dynamical scattering amplitude for a current signal incoming from reservoir $\beta$ at energy $E_m$ to be detected at a time $t$ at reservoir $\alpha$, while $S_{out,\alpha\beta}(E_n, t)$ is the dynamical scattering matrix for a current signal incoming from reservoir $\beta$ at time $t$ to be found at energy $E_n$ at reservoir $\alpha$.

In this work, we are interested in the regime of adiabatic driving, namely where the dwell time of a particle in the mesoscopic capacitor constituting the SPS is much smaller than the modulation period $\mathcal{T}$ of the driving potential [22]. Note that this is an assumption on the timescales describing the SPSSs and their driving only, and does not concern the timescales describing the traversal of the interferometer which can be of arbitrary magnitude. In this regime, the dynamical scattering matrices describing the subsystem of an SPS, $S_k(t)$ for $k = A,B$, are energy independent on the scale of the driving frequency and $S_{in,k}(t, E) = S_{in,k}(t, \mu) = S_{out,k}(E, t) = S_{out,k}(\mu, t) \equiv S_k(t)$. For weak coupling and slow driving of the sources, these scattering matrices are given by [15]

$$S_k(t) = n_k^e \frac{t - t_k^p + i\sigma_k}{l - t_k^p - i\sigma_k} + n_k^h \frac{t - t_k^h - i\sigma_k}{l - t_k^h + i\sigma_k}.$$ (2)

The emission times of electrons and holes, $t_k^e$ and $t_k^h$, and the width of the emitted current pulses, $\sigma_k$, are directly related to the properties of the sources and are thus tunable [23]. We introduced the variables $n_k^e$ in order to distinguish whether the emission of an electron or of a hole is treated. This variable takes the value $n_k^e = 1$ if a time-interval where an electron/hole is emitted from source $k$ is considered, and $n_k^e = 0$ otherwise. We assume that electron and hole emission happen at times, which differ from each other by much more than the pulse width $\sigma_k$, $\lvert t_k^e - t_k^h \rvert \gg \sigma_k$, meaning that the different current pulses emitted from the same source are well separated. The scattering matrices of the full system including the MZI and SPSs are given in Appendix A.

C. Observables

In this paper, we study the impact of two-particle effects on the flux-dependence of the charge current, the energy current, and their spectral functions, as well as on the zero-frequency charge-current noise. In this section we introduce the studied observables.

We start from the time-resolved charge [24] and energy [25,26] current operators in lead $\alpha$, $I_{\alpha}(t)$ and $J_{\alpha}(t)$, defined as

$$\hat{I}_{\alpha}(t) = -\frac{e}{h} \int_{-\infty}^{t} dE e^{i(E-E')t/h} \hat{i}_{\alpha}(E, E'),$$

$$\hat{J}_{\alpha}(t) = \frac{1}{h} \int_{-\infty}^{t} dE e^{i(E-E')t/h} \times \left[ \frac{(E + E')}{2} \right] \hat{i}_{\alpha}(E, E').$$

Note that the energy current with respect to the electrochemical potential $\mu$ in this setup equals the heat current, since no voltages or temperature gradients are applied. Here, we introduced the operator $\hat{i}_{\alpha}(E, E') = [\hat{b}_{\alpha}^\dagger(E') \hat{a}_{\alpha}(E') - \hat{a}_{\alpha}^\dagger(E) \hat{a}_{\alpha}(E)]$, and the electron charge $-e$. The creation and annihilation operators, $\hat{b}_{\alpha}^\dagger(E)$ and $\hat{b}_{\alpha}(E)$, of particles incident in reservoir $\alpha$ are related to the respective operators for particles emitted from reservoir $\beta$ onto the scattering region, $\hat{a}_{\beta}^\dagger(E)$ and $\hat{a}_{\beta}(E)$, through the Floquet scattering matrix introduced in the
previous section by

\[ \hat{b}_\alpha^\dagger(E) = \sum_{\beta} \sum_{n=-\infty}^{\infty} S_{n,\beta}^*(E, E_n) \hat{a}_{\beta}^\dagger(E_n), \]

and equivalently for the annihilation operators.

We are interested in the time-averaged charge and energy currents, \( \bar{I}_\alpha \) and \( \bar{J}_\alpha \), which are given by the time integral over the expectation values of Eqs. (3) and (4).

\[ \bar{I}_\alpha = \int_0^T \frac{dt}{T} \langle \dot{I}_\alpha(t) \rangle \]

\[ \bar{J}_\alpha = \int_0^T \frac{dt}{T} \langle \dot{J}_\alpha(t) \rangle. \]

Here, \( \langle \ldots \rangle \) indicates a quantum-statistical average. The quantum-statistical average of particles incoming from the reservoirs is given by the Fermi function, namely the equilibrium distribution function of the reservoirs. Substituting Eq. (3) into Eqs. (3) and (4) and taking the time-average of the expectation values as given in Eqs. (6) and (7), we find

\[ \bar{I}_\alpha = \frac{-e}{h} \int_{-\infty}^{\infty} dE \ i_\alpha(E) \]

\[ \bar{J}_\alpha = \frac{1}{\hbar} \int_{-\infty}^{\infty} dE \ e_\alpha(E). \]

The excess-energy distribution function \( i_\alpha(E) \), which we also refer to as the spectral current, entering the two current expressions is given by

\[ i_\alpha(E) = \int_0^T \frac{dt}{T} \int_{-\infty}^{\infty} dE' e^{i(E-E')t/\hbar} \langle \dot{\hat{a}}_\alpha(E, E') \rangle \]

\[ = \sum_{\beta} \sum_{n=-\infty}^{\infty} |S_{n,\beta}(E, E_n)|^2 [f(E_n) - f(E)] \]

It describes the distribution of electron and hole excitations with respect to the Fermi sea incident in reservoir \( \alpha \). In the following, we focus on the zero-temperature regime. The Fermi functions are therefore replaced by sharp step functions, \( [f(E_n) - f(E)] \to [\Theta(-E_n) - \Theta(-E)] \).

Finally, we are interested in the zero-frequency charge-current noise, which is known to be sensitive to two-particle effects,

\[ \mathcal{P}_{\alpha,\beta} = \frac{1}{2} \int_0^T \frac{dt'}{T} \int_{-\infty}^{\infty} d(t - t') \]

\[ [\langle \hat{I}_\alpha(t) \hat{I}_\beta(t') + \hat{I}_\beta(t') \hat{I}_\alpha(t) \rangle - 2\langle \hat{I}_\alpha(t) \rangle \langle \hat{I}_\beta(t') \rangle]. \]

In the limit of zero temperature, the expression for the zero-frequency noise power assumes a rather compact form. Substituting Eq. (4) into Eq. (11), we find

\[ \mathcal{P}_{\alpha,\beta} = \frac{e^2}{2\hbar} \sum_{m=-\infty}^{\infty} \text{sign}(m) \int_m^0 \frac{dE}{T} \int_0^T \frac{dt}{T} \int_0^T \frac{dt'}{T} e^{i\Omega t - i\Omega t'} \]

\[ \sum_{\gamma,\delta} [S_{\alpha,\gamma}(t, E) S_{\alpha,\delta}(t, E_m) S_{\beta,\delta}(t', E_m) S_{\beta,\gamma}(t', E)]. \]

In what follows all currents are evaluated at the detector, situated at reservoir \( \alpha = 4 \). We thus suppress the reservoir index, taking \( \dot{i}_4(E) \equiv \dot{i}(E) \), \( \bar{I}_4 \equiv \bar{I} \), \( J_4 \equiv J \). Furthermore, we are interested in the cross-correlation function of charge currents, for which we have \( \mathcal{P}_{44} = \mathcal{P}_{4\dot{4}} \equiv \mathcal{P} \). Note that the time average over one period will always include \( \text{electron as well as hole contributions from the different time-dependently driven SPSs.} \)

We will in the next sections separate the contributions by adding superscripts \( e \) and \( h \) to the considered quantities and by using the variables \( n_{\alpha,\gamma}^e/h \), previously introduced in the context of Eq. (2), to highlight the origin of the different terms stemming from electron and hole contributions.

III. SINGLE-PARTICLE INTERFERENCE - WAVE PACKET PICTURE

It is instructive to first consider the situation, where SPS\(_B\) is switched off and the signal injected into the MZI from SPS\(_A\) leads to an interference pattern in the detected signal in reservoir 4. The excess-energy distribution function (or spectral current) at the detector reads

\[ i_{\text{MZI},A}(E, \Phi) = i_{\text{MZI},A}^e(E) + i_{\text{MZI},A}^\text{int}(E, \Phi) \]

where the classical part and the interference part are given by

\[ i_{\text{MZI},A}^e(E) = (R_L R_R + T_L T_R) [i_{\alpha}^e(E) + i_{\alpha}^h(E) ] \]

\[ i_{\text{MZI},A}^\text{int}(E, \Phi) = -2\gamma \cos \Delta \theta(E, \Phi) [i_{\alpha}^e(E) + i_{\alpha}^h(E) ] \]

Here, we have defined \( \gamma = t_u^e t_u^l t_R^e t_R^l = \sqrt{T_L T_R R_L R_R} \). The excess-energy distribution function contains both electron- and hole-like contributions from the emission of the different types of particles from SPS\(_A\). The particles injected by SPS\(_A\) into the edge states are described by the excess-energy distribution functions

\[ i_{\alpha}^e(E) = \Theta(E) n_{\alpha}^e 2\Omega A e^{-2E_{\alpha} \sigma / h} \]

\[ i_{\alpha}^h(E) = -\Theta(-E) n_{\alpha}^h 2\Omega A e^{2E_{\alpha} \sigma / h} \]

of electron-like and hole-like excitations, with contributions in the positive, respectively the negative, energy range, only. Note that, according to the definition given in Eq. (10), the excess-energy distribution function of the
hole-like excitations, $i_{\text{el}}^E_N(E)$, is always negative, which is consistent with the interpretation of a “hole” as a missing electron in the Fermi sea.

The term $i_{\text{MZI,A}}^{\text{el}}(E)$, see Eq. (13b), is of classical nature and it is given by the sum of contributions from particles reaching the detector after travelling the upper or the lower arm with a probability $R_L R_R$, respectively $T_L T_R$. In contrast, $i_{\text{MZI,A}}^{\text{int}}(E, \Phi)$, see Eq. (13c), shows the wave nature of the emitted signals. It is due to the interference between waves propagating along the upper and the lower arms. In the almost perfectly balanced case, $\Delta \tau \leq \sigma_A$, shown in Fig. 2 a.), we see the flux-dependent part $E \Delta \tau / \hbar$ of the phase $\Delta \phi(E, \Phi)$ starts to play an important role leading to exponentially damped, fast energy-dependent oscillations in the spectral current. This goes along with a phase shift between the different energy contributions. In Fig. 2 c.), where we show phase- and energy-dependent cuts through the plot in Fig. 2 b.), this behaviour is clearly visible.

The energy dependence of the interference part of the excess-energy distribution function is the electron analogue of the so-called channelled spectrum known from optics. This energy dependence leads to dramatic differences for the charge and energy currents – namely the energy-integrated quantities – between the case of a balanced and a strongly unbalanced interferometer. The analytic results for the time-averaged charge and energy currents, consisting of the sum of an electronic and a hole-like contribution, are given by

$$I_{\text{MZI,A}} = \left( R_L R_R + T_L T_R \right) \left( n_0^A - n_0^h \right)$$

$$-2 \gamma \Re \left( e^{-i \phi} \left( n_A - 2i \sigma_A \Delta \tau \right) - n_A^h \frac{2i \sigma_A}{\Delta \tau + 2i \sigma_A} \right)$$

$$J_{\text{MZI,A}} = \left( R_L R_R + T_L T_R \right) \left( n_0^A + n_0^h \right)$$

$$-2 \gamma \Re \left( e^{-i \phi} \left( n_A^c \frac{-2i \sigma_A}{\Delta \tau - 2i \sigma_A} \right)^2 + n_A^h \frac{2i \sigma_A}{\Delta \tau + 2i \sigma_A} \right).$$

FIG. 2: Electronic part of the excess-energy distribution function, $\bar{n}_{\text{MZI,A}}^E(\Phi, E)$ as a function of the energy $E$ in units of $\hbar / \sigma_A$ and the magnetic-flux-dependent phase $\Phi$. a.) Almost perfectly balanced interferometer, with $\Delta \tau = 0.01 \sigma_A$. b.) Unbalanced interferometer, with $\Delta \tau = 20 \sigma_A$. c.) Cuts through the 3D plot of b.) at different energies, $E$, and phases, $\Phi$. In all plots, the transmission probabilities are $T_L = T_R = 0.5$.

FIG. 3: Electronic part of the average charge current, $\bar{I}_{\text{MZI,A}}^E$, (full lines) and of the average energy current, $\bar{J}_{\text{MZI,A}}^E$, (dashed lines) as a function of the phase $\Phi$ for different values of the detuning $\Delta \tau$. The transmission probabilities are $T_L = T_R = 0.5$. 
These time-averaged charge and energy currents are obtained from the energy integral over the excess-energy distribution function. The equations show the sum of the electron and hole contributions, which are indicated by factors \( n_A^e \) and \( n_A^h \) stemming from different parts of the driving cycle. When considering a full period, both \( n_A^e \) and \( n_A^h \) are equal to one. Fig. 3 shows their electronic contributions only (a full 3D plot as function of \( \Delta \tau \) and \( \Phi \) is shown in Figs. 7 a.) and d.); equivalent results are found for the hole-like contributions). Importantly, when \( \Delta \tau \leq \sigma_A \), the interference contributions to charge and energy currents are strongly suppressed. This suppression of the flux dependence can be understood as an averaging effect of the phase-shifted contributions of the excess-energy distribution function at different energies.

On the other hand, this suppression of interference is also a manifestation of the particle nature of the injected signal, made of a sequence of well-separated current pulses carrying exactly one electron or one hole. It has been shown in Refs. 22 25 that the width in time of these current pulses, \( \sigma_A \), is directly related to the single-particle coherence time of electrons and holes. The latter can be read out by measuring the visibility of the current signal detected at the output of an MZI: whenever the detuning of the interferometer, characterised by \( \Delta \tau \), is much larger than the single-particle coherence time \( \sigma_A \), the interference in the charge (and energy) current is suppressed. In this case the current pulses travelling along the upper arm and the lower arm arrive at the detector in well separated time intervals and the signals from the two different paths are thus distinguishable.

The coexistence of these two interpretations is consistent with the idea that, in quantum mechanics, a particle is described by a wave packet, composed of a superposition of plane waves at different energies.

Furthermore, from Eqs. (10) and (17), we see that the contributions for electrons and holes have different weights for finite detuning \( \Delta \tau \). This is related to the different energies at which electron- and hole-like excitations occur and to the energy-filtering properties of the MZI. Consequently, as soon as the detuning is finite, the dc charge current at each of the two outputs is finite, even though the charge current injected by the SPS\(_A\) into the MZI sums up to zero. As an additional result of the finite detuning, a phase shift with respect to the \( \cos(\Phi) \)-dependence is introduced. The energy dependence of the excess-energy distribution function, namely the channelled spectrum, hence leads to charge and energy currents which are in general out of phase. The different dependence of this phase shift as well as of the suppression of the visibility for charge and energy currents as a function of the detuning can easily be seen by rewriting their interference contributions as

\[
\begin{align*}
J^{\text{int}}_{\text{MZI}A} & = -e/\hbar \\
& = \frac{-2\gamma}{\sqrt{\Delta \tau^2 + 4\sigma_A^2}} \left( n_A^e \cos(\Phi + \psi^f) + n_A^h \cos(\Phi - \psi^f) \right) \\
\end{align*}
\]

\[
J^{\text{int}}_{\text{MZI}A} = \frac{4\sigma_A^2}{\Delta \tau^2 + 4\sigma_A^2} \left( n_A^e \cos(\Phi + \psi^f) + n_A^h \cos(\Phi - \psi^f) \right) .
\]

The different phase shifts are (where for the energy current we here give the explicit form for small detuning, \( \Delta \tau < 2\sigma_A \))

\[
\begin{align*}
\psi^f = & \arctan \left( \frac{\Delta \tau}{2\sigma_A} \right) \\
\psi^f = & \arctan \left( \frac{4\sigma_A^2 \Delta \tau}{4\sigma_A^2 - \Delta \tau^2} \right) .
\end{align*}
\]

Only when \( \Delta \tau \to 0 \), the phase difference \( \Delta \phi \) becomes energy independent in Eq. (13c), and we find \( \psi^f = \psi^f = 0 \). Consequently, charge and energy currents are then in phase.

Since the energy current, \( J = h^{-1} \int_{-\infty}^{\infty} dE \ E \ i_\alpha(E) \), contains an additional factor \( E \) in the integrand with respect to the charge current, this quantity is more sensitive to the energy dependence of the distribution function. Thus, it is also more sensitive than the charge current to the variation of the interferometer imbalance showing interference suppression at smaller \( \Delta \tau \) values, see Fig. 3 for the electronic contributions to charge and energy currents. The visibility extracted from Eq. (18) for the charge current in the case of symmetric transmission of the QPCs, namely \( |J^{\text{int}}_{\text{MZI}A}/J^{\text{cl}}_{\text{MZI}A}| = 2\sigma_A/\sqrt{\Delta \tau^2 + 4\sigma_A^2} \), indeed decays slower with \( \Delta \tau \) than the visibility extracted from Eq. (19) for the energy current, namely \( |J^{\text{int}}_{\text{MZI}A}/J^{\text{cl}}_{\text{MZI}A}| = 4\sigma_A^2/(\Delta \tau^2 + 4\sigma_A^2) \).

An MZI fed by a non-adiabatically driven SPS has recently been studied by Ferraro et al. in the framework of Wigner functions. In that case the excess-energy distribution function of emitted particles, \( \nu^{\text{th}}_A(E) \), is approximated by a Lorentzian function. The system shows a qualitatively similar behaviour to the one described here. A closely related work by Hofer and Flindt focuses on the propagation of multi-electron pulses through a Mach-Zehnder interferometer.

IV. SYNCHRONISED PARTICLE EMISSION FROM TWO SOURCES

We now come to the main subject of our work, the influence of the particle emission from SPS\(_B\) on the interference pattern of the currents at the output of the
MZI. It has been shown in Ref. [20] that the interference pattern in the time-resolved current, \( \langle I(t) \rangle \), detected at the output of the MZI is subject to a phase-shift, which can take values between 0 and \( 2\pi \), depending on the emission time of electrons or holes from source B. This has as a consequence that the interference effects in the time-averaged current, \( \bar{I} \), detected at the output of the interferometer in every half period, get strongly suppressed when the emission of the particles is synchronised such that either particles emitted from SPS\(_A\) can be absorbed at SPS\(_B\) or that particles of the same kind can collide at QPC\(_R\). This synchronisation of particles occurs as a perfect overlap of the time-resolved wave packets emitted from the two sources. A full absorption thus can occur when \( t_{A}^{e} + \tau_{d}/2 = t_{B}^{h} \) (or \( t_{A}^{h} + \tau_{d}/2 = t_{B}^{e} \)), which corresponds to \( \Delta t_{d}^{eh} = 0 \) (or \( \Delta t_{d}^{he} = 0 \)), together with \( \sigma_{A} = \sigma_{B} \). A full collision of electrons (or holes) can occur when \( t_{A}^{e} + \tau_{u} = t_{B}^{h} + \tau_{d}/2 \) (or \( t_{A}^{h} + \tau_{u} = t_{B}^{e} + \tau_{d}/2 \)), which corresponds to \( \Delta t_{u}^{eh} = 0 \) (or \( \Delta t_{u}^{he} = 0 \)), together with \( \sigma_{A} = \sigma_{B} \).

Interestingly, the conditions for the averaging of the time-resolved currents, leading to a full suppression of the interference effects in the detected charge, allow for a particularly interesting interpretation, which has been put forward in Ref. [20]. Whenever the condition \( \Delta t_{d}^{eh} = 0 \) is fulfilled, no particle arrives at any of the outputs, when the electron emitted from source A takes the lower arm of the MZI and gets absorbed, while fluctuations occur when the particle emitted from A takes the upper arm. This leads to which-path information suppressing the interference effect. Equally, when the condition \( \Delta t_{u}^{eh} = 0 \) is fulfilled, two electrons emitted from SPS\(_A\) and SPS\(_B\) could collide at QPC\(_R\). When the electron emitted from source A travels along the upper arm of the MZI, the two electrons - being in the same state - would have to be scattered to the two opposite outputs of the MZI at QPC\(_R\) in the case that the particle emitted from A takes the lower arm of the MZI both particles can go to both outputs randomly. This again leads to which-path information leading to an interference suppression.

In the following, in addition to the charge current we will investigate also the spectral current and the energy current of the emitted signals as well as the charge-current noise with the aim to extend the understanding of the impact of these multi-particle effects on the MZI signal.

## A. Spectral properties

We start by considering the spectral currents for the case where one source emits an electron and one source emits a hole, allowing for the absorption of particles at SPS\(_A\), as well as the case where both sources, SPS\(_A\) and SPS\(_B\), emit the same kind of particles, allowing for possible collisions between particles in one half period. The synchronised emission from the two sources goes along with inelastic scattering processes leading to a deformation of the spectral distribution of the current as will be shown in the following.

![Energy-distribution function, \( i^{eh}(E, \Phi) \), shown for positive values of the energy \( E \) only, in the regime where absorptions of electrons emitted by A are possible through the emission of holes from B depending on the time difference \( \Delta t_{d}^{eh} \). Here we take \( \Delta t_{d}^{eh} = 0.1\sigma_{A} \) and show \( i^{eh}(E, \Phi) \) as a function of the energy \( E \) in units of \( \hbar/\sigma_{A} \) and the magnetic-flux-dependent phase \( \Phi \). The interferometer is almost perfectly balanced, \( \Delta \tau = 0.01\sigma_{A} \), the pulse widths are assumed to be equal, \( \sigma_{A} = \sigma_{B} \), and the transmission probabilities are given by \( T_{L} = T_{R} = 0.5 \).](#)

### 1. Absorption of particles

In the case where particles of opposite type emitted from the two sources are detected in the same half period, absorptions can occur at source B and the spectral current is given by


\[
\begin{align*}
\gamma^e(E, \Phi) &= R_L R_R \gamma_A^e(E) + R_L T_R \gamma_B^e(E) + T_L T_R \left( \gamma_B^e(E) + i \gamma_A^e(E) \right) \left( 1 - \frac{4 \sigma_A \sigma_B}{\Delta \tau_{eh}^2 + (\sigma_A + \sigma_B)^2} \right) \\
&- 2 \gamma_A^e(E) \Re \left\{ e^{-i \Phi} e^{-i E \Delta \tau / \hbar} \left( 1 - \frac{2i \sigma_B}{\Delta \tau_{eh}^2 + i (\sigma_A + \sigma_B)} \right) \right\}.
\end{align*}
\]

(22)

From now on, for observables calculated for the MZI with two sources, we drop the subscript indicating the presence of the MZI and the number of working sources, the latter being evident from the superscript \(ij\) for the type of particle \(i = e, h\) emitted from SPS\(_A\) and the type of particle \(j = e, h\) emitted from SPS\(_B\). Here, we show the case where SPS\(_A\) emits an electron and SPS\(_B\) a hole \((n_A^e = n_B^h = 1\) and \(n_A^h = n_B^e = 0)\); the opposite case is shown in Appendix B.1.

Far away from the condition, \(\Delta \tau_{eh} = 0\) and \(\sigma_A = \sigma_B\), the two particles are emitted independently, such that the electron emitted from SPS\(_A\) is not in the vicinity of SPS\(_B\), when a hole emission occurs at the latter. Then the expression given in Eq. (22) reduces to the sum of the separate contributions of the two sources, namely for the hole emitted from SPS\(_B\) and transmitted at QPC\(_R\), \(T_R \gamma_B^h(E)\), and the electron term containing interference effects, given in Eq. (13a).

The collision of an electron emitted from SPS\(_A\) and a hole emitted from SPS\(_B\) at the position of the latter source (which is equivalent to the absorption of electrons emitted from SPS\(_A\) at SPS\(_B\)) can occur when the time difference \(\Delta \tau_{eh}^d\) is of the order of the width of the associated time-resolved current pulses \(\sigma_A, \sigma_B\). It leads to a cancellation of the contribution of the current travelling along the lower arm in an energy-independent manner, depending only on how accurately the absorption conditions, \(\Delta \tau_{eh}^d = 0\) and \(\sigma_A = \sigma_B\), are fulfilled. Equally, the suppression of the interference part of the current takes place in a way which is independent of the energy \(E\). It becomes evident also from Fig. 2 where the electronic part of this spectral current is shown as a function of energy and of the magnetic-flux dependent phase. Indeed, the amplitude of the flux-dependent oscillations is suppressed with respect to the case where \(\Delta \tau_{eh}^d \gg \sigma/A, B\) – the latter being equivalent to the case of an emission from A only, while source B is switched off, see Fig. 2a.

2. Collision of particles of the same kind

In the case where particles of the same type emitted from both sources are detected in one half period, we find for the spectral current

\[
\begin{align*}
\gamma^{ec}(E, \Phi) &= R_L R_R \gamma_A^{ec}(E) + T_L T_R \gamma_A^{ec}(E) \Re \left\{ 1 + \frac{4 \sigma_A \sigma_B}{\Delta \tau_{ec}^2 + (\sigma_A - \sigma_B)^2} - 2i \sigma_B \frac{\Delta \tau_{ec} - i(\sigma_A + \sigma_B)}{\Delta \tau_{ec}^2 + (\sigma_A - \sigma_B)^2} e^{-i E(\Delta \tau_{ec} + i(\sigma_A - \sigma_B)) / \hbar} \right\} \\
&+ R_L T_R \gamma_B^{ec}(E) + T_L T_R \gamma_B^{ec}(E) \Re \left\{ 1 + \frac{4 \sigma_A \sigma_B}{\Delta \tau_{ec}^2 + (\sigma_A - \sigma_B)^2} - 2i \sigma_A \frac{\Delta \tau_{ec} - i(\sigma_A + \sigma_B)}{\Delta \tau_{ec}^2 + (\sigma_A - \sigma_B)^2} e^{-i E(\Delta \tau_{ec} + i(\sigma_B - \sigma_A)) / \hbar} \right\} \\
&- 2 \gamma_A^{ec}(E) \Re \left\{ e^{-i \Phi} e^{-i E \Delta \tau} \left[ 1 + \frac{2i \sigma_B}{\Delta \tau_{ec}^2 + i (\sigma_A - \sigma_B)} \right] \left( 1 - e^{-i E(\Delta \tau_{ec} + i(\sigma_A - \sigma_B)) / \hbar} \right) \right\},
\end{align*}
\]

(23)

where we here show the electron part, only; the hole contribution is given in Appendix B.1.

The classical part, \(\gamma^{ec, cl}(E)\), is given by the expression in the first two lines of Eq. (23). Again, it reduces to the sum of the single-particle contributions, namely the sum of \(T_R \gamma^h_{R}\) and of the expression in Eq. (13b), when \(\Delta \tau_{ec}^d \gg \sigma_A, \sigma_B\). The resulting exponential behaviour of the spectral current is represented by the black (dashed-dotted line) in Fig. 3. However, if the tuning of the emission times from SPS\(_A\) and SPS\(_B\) is such that particles could collide at SPS\(_B\), in other words, if there is an overlap of the time-resolved current pulses emitted from the two sources and the difference of the emission times, \(\Delta \tau_{ec}\), is of the order of the width of the current pulses, then energy-dependent oscillations occur in the classical part of the spectral current on a scale given by the inverse of the time difference, \(\hbar / \Delta \tau_{ec}^d\). This oscillation on top of the energy-dependent exponential decay of the spectral current is a result of the complex exponential factor in the last term of the first two lines of Eq. (23). Importantly, its amplitude gets suppressed for large time differences. Therefore the amplitude of the oscillations is largest close
to the collision condition $\Delta t_{d}^{ee} = 0$, while the frequency of the oscillations is reduced. This behaviour becomes apparent from the red (full) line in the plot shown in Fig. 6, where damped oscillations are visible. The oscillations of the blue (dashed) line are hardly visible due to the small oscillation frequency.

This behaviour is very different from the energy-independent suppression of parts of the spectral current in the regime of possible particle absorptions.

The interference contribution, $i^{\text{ee, int}}(E)$, is given by the third line of Eq. (23) and it is shown in Fig. 6. Far from the collision condition, this contribution stems from the signal emitted from source A only, where it equals Eq. (13c). When the particles from SPS_B are emitted such that collisions between them are possible at SPS_B, oscillations with two competing time-scales appear, namely the time-scale of the collision condition, $\Delta t_{d}^{ee}$, and the time-scale related to the detuning of the interferometer, $\Delta \tau$. Again, oscillations on the energy scale given by $\hbar/\Delta t_{d}^{ee}$ are suppressed for large time differences $\Delta t_{d}^{ee}$. This is however very different from the absorption case where the time-scale of the absorption condition enters in a fully energy-independent manner. For an almost perfectly balanced interferometer, $\Delta \tau \ll \sigma_{A}$, the interference contribution to the spectral current is shown as a function of the energy and the flux-dependent phase in Fig. 6 a.), exhibiting slow oscillations on the scale $\hbar/\Delta t_{d}^{ee}$, where we here chose the case close to the collision condition, $\Delta t_{d}^{ee} = 0.1\sigma_{A}$. In Fig. 6 b.) cuts through the three-dimensional plot of Fig. 6 a. are shown as a function of energy for different values of the phase, $\Phi$. We compare these curves with the case slightly farther away from the collision condition, where the modulation on the energy scale given by $\hbar/\Delta t_{d}^{ee}$ becomes more obvious. Interestingly, the areas enclosed by the curves below and above the energy-axis (indicated by the green dotted line in Fig. 6 b.) close to the collision condition, $\Delta t_{d}^{ee} = 0.1\sigma_{A}$, sum up to a value close to zero independently of the value of the magnetic flux entering the phase $\Phi$. We will see in the following section, Section IV B, that this leads to a suppression of the interference in the (energy-integrated) charge current, when the two sources are adequately synchronised. However, as soon as the time difference $\Delta t_{d}^{ee}$ is increased while keeping the interferometer balanced, $\Delta \tau \approx 0$, the sum of the enclosed areas becomes flux dependent, as can be seen from the dashed lines in Fig. 6 b.).
B. Charge current

The energy-dependent interference occurring in the previously studied spectral currents is equivalent to what is known as a channelled spectrum from optics. The behaviour of the charge end energy currents, which are given by the energy averages of the spectral currents multiplied by the charge, respectively the energy, see Eqs. (5) and (9), can therefore be understood based on the previous investigations. Here, we start with the presentation of the charge current which is found in one half period in which an electron emitted from SPS_A and a hole emitted from SPS_B are detected in reservoir 4 (namely taking $n_A^+ = n_B^+ = 1$ and $n_A^− = n_B^− = 0$), allowing for the absorption of particles if $\Delta \tau_{eh} \approx 0$. The charge current is then given by

$$\bar{I}_{eh} = -\frac{e}{\mathcal{T}} R_L R_R + T_L T_R - T_R$$

$$-2\gamma \text{Re} \left\{ e^{-i\Phi} \frac{-2i\sigma_A}{\Delta \tau - 2i\sigma_A} \left(1 - \frac{2i\sigma_B}{\Delta \tau_{eh} + i(\sigma_A + \sigma_B)}\right)\right\}. \quad (24)$$

We find that only the interference part of the charge current is affected by the synchronisation of the particle emission from the two sources. The dependence of the spectral current on the time-difference $\Delta \tau_{eh}$, see Eq. (22), thus cancels out in the classical part. The factor leading to the maximum of interference for a balanced MZI, $\Delta \tau \to 0$, in the absence of absorptions, and the factor suppressing the interference in case of absorptions, $\Delta \tau_{eh}$, are of very similar nature, both leading to a Lorentzian-type structure together with a phase shift at the maximum/minimum of their contribution. This similarity becomes also obvious when comparing Figs. a) and b.) which bring out the two effects.

The suppression of interference can be interpreted as the result of an energy average of the spectral current given in Eq. (22) on one hand, but also due to the impact of the possible absorptions on the fluctuations in the charge current, as explained in detail in Ref. 20. When the particle emitted from SPS_A takes the upper arm, the electron and the hole emitted from SPS_B can result in a zero average current at each of the reservoirs; however, the charge in the reservoirs fluctuates. In contrast an absorption can occur at SPS_B when the electron takes the lower arm suppressing thereby the fluctuations. This is shown in a detailed study of the noise in Sec. [7] and can be understood as a which-path information suppressing interference.

The charge current detected in the half period in which holes emitted from SPS_A can be absorbed at SPS_B behaves similarly as the one for the opposite case and it is given by

$$\bar{I}_{eh} = -\frac{e}{\mathcal{T}} R_L R_R - T_L T_R + T_R$$

$$+2\gamma \text{Re} \left\{ e^{-i\Phi} \frac{2i\sigma_A}{\Delta \tau + 2i\sigma_A} \left(1 - \frac{-2i\sigma_B}{\Delta \tau_{eh} - i(\sigma_A + \sigma_B)}\right)\right\}. \quad (25)$$

The difference with respect to Eq. (24), is given by a sign difference due to the contribution of oppositely charged particles and by a different phase which enters both through the factor stemming from the detuning properties of the MZI as well as from the factor describing the synchronisation of particles. As a consequence from this phase shift between hole and electron contribution, the charge current detected at reservoir 4 during a whole period, does not vanish (even though the total current injected into the MZI is zero) and it is given by

$$\bar{I}_{eh} = -\frac{e}{\mathcal{T}} R_L R_R + T_L T_R - T_R$$

$$-2\gamma \text{Re} \left\{ e^{-i\Phi} \frac{-2i\sigma_A}{\Delta \tau - 2i\sigma_A} \left(1 - \frac{-2i\sigma_B}{\Delta \tau_{eh} - i(\sigma_A + \sigma_B)}\right)\right\}. \quad (27)$$

Also here, the classical contribution is independent of the synchronisation of the two sources, in this case resulting from an energy-average of the synchronisation-dependent oscillations of $\bar{I}_{eh}(E)$, shown in Fig. 5. In contrast, the interference part of the time-averaged charge current is sensitive to the collision of particles, as a consequence of the fact that the spectral current, $\bar{I}_{eh}(E, \Phi)$, oscillates as a function of energy depending on the time-difference $\Delta \tau_{eh}$ as shown in Fig. 6. It is interesting to notice that the relevant time-scale for the interference suppression of the charge current is given by $\Delta \tau_{eh}$, meaning that the effect is largest when particles can collide at QPC. This is in contrast to the spectral current where the effect of two synchronised particle emissions enters through the time-scale $\Delta \tau_{eh}$, which leads to prominent effects when particles are emitted on top of each other at SPS_B. The reason for this difference is the interplay of features at
$\Delta t^{ee}_d$ due to the presence of two particles and of the detuning of the interferometer $\Delta \tau$ which - through an energy average of the spectral current - result in the occurrence of the time-scale $\Delta t^{ee}_d = \Delta t^{ee}_u + \Delta \tau$ in the charge current. The result is a suppression of interference in the case of possible full collisions at QPC$_R$, $\Delta t^{ee}_u = 0$, and a phase shift in the vicinity of $\Delta t^{ee}_u = 0$ as shown in Fig. 7 (c.).

The structure of the expression given in Eq. (27) is very similar to the one for the absorption case. It is however important to notice that the corresponding spectral currents have very different behaviours and consequently, in contrast to the case shown in Eq. (24), the relevant time-scale for the synchronisation of the SPSSs now becomes $\Delta t^{ee}_d$, after the energy integration of the spectral current.

This fact allows for two coexisting interpretations for the interference suppression, as it was already the case for the suppression due to detuning discussed in Sec. III. The suppression through the energy average of the energy-dependent interference pattern occurring due to the wave-like behaviour of electrons, can therefore at the same time be interpreted to result from the collision of electrons at QPC$_R$.

When the electron emitted from SPS$_A$ travels along the upper arm and the collision condition, $\Delta t^{ee}_u$ and $\sigma_A = \sigma_B$, is fulfilled, it will collide with the electron emitted from SPS$_B$ leading to the transmission of exactly one electron to each MZI output. When the electron emitted from SPS$_A$ takes the lower arm, the charge in the two MZI outputs fluctuates due to the probabilistic transmission at QPC$_R$. The question whether one particle arrives in each reservoir on average or whether it is indeed exactly one particle in each period, can however only be clarified by considering the noise, see Sec. V. This is an important question concerning the argument of which-path information based on fluctuations as it was put forward already in previous sections.

Also here, the case where the other type of particles is emitted from the SPSSs (namely a hole both from SPS$_A$ and from SPS$_B$) leads to a phase difference with respect to the case of two emitted electrons, yielding a finite current in the reservoirs also when considering the total current of one full period, if only $\Delta \tau$ or $\Delta t^{ee}_u$ are different from zero.

The full general expressions for the charge current in the case of collision and absorption are given in Appendix B.
C. Energy current

The results of the last section show the impact of absorptions and collisions on the charge current and how they can be explained either based on the structure of the spectral current or on the occurrence of two-particle effects. Both interpretations are clearly related to the energetic properties of the contributing current pulses, motivating the following discussion of the energy currents detected at the output of the MZI.

In the case where a particle emitted by SPS\(_A\) can possibly be absorbed at SPS\(_B\), the energy current in reservoir 4 is given by

\[
\bar{J}^{eh} = \frac{\hbar}{2\sigma_A T} (R_L R_R + T_L T_R) + \frac{\hbar}{2\sigma_B T} T_R + T_L T_R \left( \frac{\hbar}{2\sigma_A T} + \frac{\hbar}{2\sigma_B T} - \frac{4\sigma_A \sigma_B}{\Delta t^e_d + (\sigma_A + \sigma_B)^2} \right)
\]

We see that the synchronisation of the two particle sources affects both the classical as well as the interference part of the energy current. Let us start by considering the classical contribution: While the emission of independent electrons and holes leads to the emission of the same amount of energy related to the width of the current pulse, \(\hbar/2\sigma_{A/B}\), the absorption of a particle (which can occur when the particle emitted from SPS\(_A\) takes the lower MZI path) leads to an annihilation not only of the charge but also of the energy current.

The classical part of the energy current thus reduces to

\[
\frac{\hbar}{2\sigma_A T} R_L R_R h/2\sigma_A + R_L T_R h/2\sigma_B \text{ in the case of absorption in the lower arm, namely when } \Delta t^e_d = 0 \text{ and } \sigma_A = \sigma_B.
\]

In the same way we see that the interference is suppressed under the condition, \(\Delta t^e_d = 0\) and \(\sigma_A = \sigma_B\), because if the particle is absorbed along the lower path also the energy going along with it does not fluctuate any more at the output and the same coexisting interpretations as for the charge current can possibly be employed, based on the wave and the particle nature of the injected signal. Indeed, we find that the effect of the collision is the suppression of the factor \((-2i\sigma_A)^2/((\Delta\tau - 2i\sigma_A)^2\), which was found to be typical for the energy current in the interferometer, see Eq. (17). The energy current in the case of absorption is shown in Fig. 7 (e.) as compared to the case of an MZI with a single working source shown in Fig. 7 (d.). Results for the absorption of a hole, namely the synchronised emission of a hole from SPS\(_A\) and an electron from SPS\(_B\) are given in Appendix B.3

\[
\bar{J}^{ee} = \frac{\hbar}{2\sigma_A T} (R_L R_R + T_L T_R) + \frac{\hbar}{2\sigma_B T} T_R + T_L T_R \left( \frac{\hbar}{2\sigma_A T} + \frac{\hbar}{2\sigma_B T} - \frac{4\sigma_A \sigma_B}{\Delta t^e_d + (\sigma_A + \sigma_B)^2} \right)
\]

Also here we show the electronic contribution only; the general expression is given in Appendix B.3. The classical part of the energy current shows an enhancement when a particle from SPS\(_B\) is emitted on top of a particle emitted from SPS\(_A\) travelling along the lower arm, since the two particles can not occupy the same energy state. This enhancement occurs hence under the condition \(\Delta t^e_d = 0\) and \(\sigma_A = \sigma_B \equiv \sigma\) and leads to the classical energy current \((R_L + 4T_L T_R) h/2\sigma\). In contrast, the interference part of the heat current is not affected by this event.

However, like in the case of the charge current, the interference contribution to the heat current is sensitive to possible collisions at the interferometer output taking place if \(\Delta t^e_u \approx 0\). The interference term contains
two contributions: the first is suppressed when the two emitted particles can collide at QPC_R and one could be tempted to associate it to the corresponding amount of energy of the colliding particles. However, there is an additional term which appears in the vicinity of the collision condition, which stems from the additional oscillations of the spectral current related to the energy scale which can be associated to the time-scale of the particle emission synchronisation, see Eq. (23).

Intriguingly, the energy current for two particles of the same kind hence behaves rather differently from the charge current: it has features both at the condition $\Delta \xi^c = 0$ (classical part) and at the condition $\Delta \xi^c = 0$ (interference part) and the interference effects in the energy current do not get suppressed under the collision condition (neither at QPC_R nor at SPS_B). The collision at QPC_R rather introduces a phase shift only, which can be seen in Fig. 1(f). This behaviour has the following important implications.

The continued existence of the interference in the energy current in the case of possible collisions at QPC_R can obviously not be explained within a particle picture, as it was done for the suppression of interference due to collisions in the charge current. If a particle picture could be used then it would lead to an apparent separation of energy and charge of the particles, namely interference occurring in the energy current while the charge current is flux-independent. This “paradox” in the particle-interpretation of the energy-charge separation as well as its alternative description by quantum interference has recently been debated for spin-particle and polarisation-particle separation under the name ”quantum Cheshire cat” 13,23,41,50,51.

Finally, we notice that the enhancement of the energy current when collisions at SPS_B can occur could be considered as a which-path information. It however turns out that this does not influence the interference pattern neither in the charge current nor in the heat current. Consequently, we find that the coexistence of the interpretations of interference suppression due to phase averages and due to multi-particle effects is to be questioned when energy currents are taken into account.

V. TWO-PARTICLE EFFECTS FROM THE NOISE

In order to better understand true two-particle effects, it is useful to consider the current noise that occurs in the cases studied in the previous sections. Indeed, the noise carries clear signatures of collisions of particles, as it was shown theoretically 13,23,24,25 as well as experimentally 30–32 for the case of the two-particle collider. The collision of particles with the same energy at a beamsplitter leads to a full suppression of the partition noise, since the two colliding particles are not allowed to enter the same outgoing channel. Equally, the full suppression of the noise in the case of particle absorption in a two-sources setup has previously been shown 15.

A. Noise of an MZI with one source

We start by considering the current noise produced by the setup, when SPS_B is switched off and particles are injected into the MZI by SPS_A, only. The current noise, for the half period in which an electron emitted from SPS_A arrives at the MZI outputs, can then be written as

$$\mathcal{P}_{\text{MZI}}^{-} = T_R R_R + T_L R_L - 4\gamma^2 \quad (30)$$

$$= 2\gamma (T_L - R_L) (T_R - R_R) \Re \left\{ e^{-i\Phi} \frac{-2i\sigma_A}{\Delta \tau - 2i\sigma_A} \right\} - \left( 2\gamma \Re \left\{ e^{-i\Phi} \frac{-2i\sigma_A}{\Delta \tau - 2i\sigma_A} \right\} \right)^2 .$$

A similar expression is found for the hole contribution; see the full expression in Appendix C. Due to the product of current operators contributing to the noise, we here get contributions for the first as well as the second harmonic in the magnetic flux. Since only single particles are emitted into the interferometer per half period it is quite intuitive that we should be able to understand the noise as a simple product of currents. More precisely, it should be proportional to a product of transmission probabilities to the contacts at which the two currents are detected.

In order to show that, we consider the charge current in the detector, see Eq. (16), and rewrite it in terms of effective transmission probabilities, $T_{41}^{\text{eff},e}$ and $T_{41}^{\text{eff},h}$, for electrons and holes, $I_{\text{MZI}}^{\text{eff},e} = -e (n_A^{s} T_{41}^{\text{eff},e} + n_A^{h} T_{41}^{\text{eff},h} ) / T$, with

$$T_{41}^{\text{eff},e} = R_L R_R + T_L T_R - 2\gamma \Re \left\{ e^{-i\Phi} \frac{-2i\sigma_A}{\Delta \tau - 2i\sigma_A} \right\}$$

$$T_{41}^{\text{eff},h} = -R_L R_R - T_L T_R + 2\gamma \Re \left\{ e^{-i\Phi} \frac{2i\sigma_A}{\Delta \tau + 2i\sigma_A} \right\} .$$

Extracting in an equivalent manner effective transmission probabilities, $T_{31}^{\text{eff},e}$ and $T_{31}^{\text{eff},h}$, from the current in contact 3, we are indeed able to show that the noise of the MZI with a single source can simply be written as

$$\mathcal{P}_{\text{MZI}} = -\frac{e^2}{T} \left[ n_A^{s} T_{31}^{\text{eff},e} T_{31}^{\text{eff},e} + n_A^{h} T_{31}^{\text{eff},h} T_{31}^{\text{eff},h} \right] . \quad (31)$$

This product form of the noise, shown in Eq. (31), is clearly not expected to hold in the case where two particles are injected into the interferometer from different sources and two-particle effects will hence contribute to the noise. In order to better understand the impact of two-particle effects, as discussed in Sec. V B, the following interpretation of the classical part of the noise, given in Eq. (30), turns out to be useful. The classical part $T_R R_R + T_L R_L - 4\gamma^2 = (R_L R_R + T_L T_R)(R_L T_R + T_L R_R)$,
stemming from the product of the classical parts of the effective transmission probabilities, results in the partition noise of the left and the right QPC, $T_L R_L$ and $T_R R_R$, and a mixed contribution, $-4\gamma^2$. Furthermore this can be rewritten as $T_R R_R + T_l R_l - 4\gamma^2 = T_R R_R + T_l R_l (T_R - T_l)^2$. It means that the classical part of the noise is given by the partition noise of QPC$_R$, $T_R R_R$, on one hand, and the partition noise of QPC$_L$ in the presence of QPC$_R$, $T_l R_l (T_R - T_l)^2$, on the other hand. The latter shows that, in the absence of interference, QPC$_L$ only produces partition noise if QPC$_R$ is not symmetric. Indeed, if QPC$_R$ was symmetric, the probability of particles from SPS$_A$ to be scattered into the reservoirs 3 and 4 was one half each, independently of the transmission probability of QPC$_L$, and its partition noise would thus be invisible.

$$\frac{\mathcal{P}^{eh}}{-e^2/T} = R_L T_L - 4\gamma^2 + 2R_l T_l R_R + 2T_l T_l R_R \left( 1 - \frac{4\sigma_A \sigma_B}{\Delta \Delta \sigma^2 + (\sigma_A + \sigma_B)^2} \right) + \, (32)$$

$$2\gamma (T_l - R_l) (T_R - R_R) \Re \left\{ e^{-i\Phi} \frac{-2i\sigma_A \Delta t^{eh}}{\Delta e} + i (\sigma_A - \sigma_B) \right\} - \left( 2\gamma \Re \left\{ e^{-i\Phi} \frac{-2i\sigma_A \Delta t^{eh}}{\Delta e} + i (\sigma_A + \sigma_B) \right\} \right)^2 .$$

For the MZI with two sources, we again drop the subscript for the amount of working sources and the presence of the MZI. The classical part of the noise, shown in the first line of Eq. (32), is partly suppressed by the absorptions. In particular, if the particle from SPS$_A$ took the lower arm of the interferometer with probability $T_l$ and could hence get absorbed, the partition noise at the right barrier created by particles coming from SPS$_A$ and the opposite type of particle coming from SPS$_B$, $2T_l R_R$, is fully suppressed. What is then left from the classical part of the noise is given by $R_L T_L - 4\gamma^2 + 2R_l T_l R_R = 2R_l T_l R_R + T_l R_l (T_R - R_R)^2$. It equals the partition noise of the two particles at QPC$_L$ if the particle from SPS$_A$ took the upper arm, $2R_l T_l R_R$, and the additional noise of the particle from SPS$_A$ at QPC$_L$ in the presence of QPC$_R$, which can obviously not get affected by the absorptions happening behind QPC$_L$, $T_l R_l (T_R - R_R)^2$. Also the interference part of the noise gets fully suppressed by the factor $\frac{\Delta t^{eh} + i (\sigma_A - \sigma_B)}{\Delta t^{eh} + i (\sigma_A + \sigma_B)}$, in the case of absorptions. The result for the noise thus fully confirms that the absorption condition leads to a suppression of fluctuations at QPC$_R$, yielding information on the path that a particle emitted from SPS$_A$ took in the MZI.

Finally, we consider the case where an electron emitted each from SPS$_A$ and SPS$_B$ can reach the reservoirs in the same half period of the source operation. The charge-current noise takes the form

$$\frac{\mathcal{P}^{ee}}{-e^2/T} = R_L T_L - 4\gamma^2 + 2R_l T_l R_R + 2R_l T_l R_R \left( 1 - \frac{4\sigma_A \sigma_B}{\Delta \Delta \sigma^2 + (\sigma_A + \sigma_B)^2} \right) + \, (33)$$

$$+ 2\gamma (T_l - R_l) (T_R - R_R) \Re \left\{ e^{-i\Phi} \frac{-2i\sigma_A \Delta t^{ee}}{\Delta e} - i (\sigma_A - \sigma_B) \right\} - \left( 2\gamma \Re \left\{ e^{-i\Phi} \frac{-2i\sigma_A \Delta t^{ee}}{\Delta e} - i (\sigma_A + \sigma_B) \right\} \right)^2 .$$

Equivalently to the absorption case, the behaviour of the charge-current noise corroborates the interpretation of the suppression of interference effects in the charge current based on two-particle collisions. Indeed, only when the collision condition at QPC$_R$ is fulfilled, the classical part of the noise gets suppressed by the contributions stemming from the partition at QPC$_R$, when the particle took the upper arm, allowing for collisions at the output.

### B. Noise of an MZI with two sources

In the following, we will consider the impact of two-particle effects on the charge current noise. Let us again start to consider the case where possible absorptions might occur. This is the situation, where indeed the interpretation based on an averaging effect of the spectral current as well as the interpretation based on the absorption of particles, carrying charge and energy, could coexist to explain the occurrence or absence of interference effects even when considering energy currents. In that case the charge-current noise is given by
of the MZI. The remaining classical noise is then given by $2T_LT_RR_L + T_LR_R (R_L - R_R)^2$. At the same time also a full suppression of the interference part of the charge-current noise is found.

Again, the results for the absorption of holes by electrons emitted from SPS$_B$ and the collision of holes at QPC$_R$ are shown in the Appendix C.

VI. CONCLUSIONS

In this manuscript, we studied the charge current and charge-current noise as well as the spectral current and the energy current in an MZI which could be fed by either one or two single-particle sources. When the MZI is fed by only one source, SPS$_A$, interference effects occur in all four quantities. They are shown to be strongly influenced by the time-scale $\Delta \tau$ stemming from the detuning of the MZI. We thereby emphasise the role of the time-scale related to the interferometer detuning entering the spectral current; this shows the characteristics of the interferometer as an energy filter. It results in a finite dc charge current at each of the MZI outputs at finite detuning, even though the amount of injected electrons and holes is equal. We furthermore show that the suppression of interference in charge and energy currents for large detuning can be interpreted both as an averaging effect of the spectral currents as well as through the particle-like properties of the injected signal, namely by the limited single-particle coherence length.

In a second step, we investigate the impact of the synchronisation of two SPSs, one of them placed in the centre of the lower interferometer arm, on the quantum-interference effects. Also the synchronisation of the two sources is shown to introduce new relevant time-scales. These new time-scales lead to a suppression of the interference in the spectral current when the sources are tuned to allow for absorptions of particles, or even to the occurrence of additional energy-dependent oscillations when the possibility of collisions of particles of the same type is given. The occurrence of this new time-scale leads to the fact that two-particle effects are already visible in the dc charge current.

Both the absorption of particles at SPS$_B$, as well as the collision of particles at QPC$_R$ lead to a suppression of the interference in the charge current. This can both be interpreted as an averaging effect of the spectral properties influenced by a phase factor resulting from the time-dependently driven source SPS$_B$, as well as by two-particle effects resulting in a reduction of fluctuations. A characterisation of the noise properties corroborates the possibility of a particle-interpretation of the interference suppression by showing that the absorption and collision of particles indeed leads to a specific reduction of fluctuations.

However, this manuscript also shows that the particle-interpretation does not hold in the case of collisions, whenever the behaviour of the energy current is considered. We show that the energy current behaves fundamentally different from the charge current of electrons and holes showing interference when the charge current does not.

Acknowledgments

We thank Gwendal Fève and Patrick Hofer for useful comments on the manuscript. J. S and M. M. are grateful for the hospitality at the University of Geneva where part of this work was done. We acknowledge financial support from the Ministry of Innovation NRW, Germany. Furthermore, financial support from the Excellence Initiative of the German Federal and State Governments (J. S. and F. B.), and from the Knut and Alice Wallenberg foundation through the Wallenberg Academy Fellows program (J. S.) is acknowledged.

Appendix A: Scattering matrices of the MZI with two single-particle sources

In the regime in which the SPSs are adiabatically driven, the total dynamical scattering matrix for electrons/holes to be scattered from reservoir $\beta$ to reservoir $\alpha$ of the MZI, fed by the two sources as described in Section II A contains the following matrix elements

\begin{align}
S_{in,41}(t, E) &= S_A(t - \tau_u) r_L e^{i \phi_u(E)} r_R + S_A(t - \tau_d) t_L S_B(t - \frac{\tau_d}{2}) e^{i \phi_d(E)} t_R \\
S_{in,42}(t, E) &= t_L e^{i \phi_u(E)} r_R + r_L S_B(t - \frac{\tau_d}{2}) e^{i \phi_d(E)} t_R \\
S_{in,31}(t, E) &= S_A(t - \tau_u) r_L e^{i \phi_u(E)} t_R + S_A(t - \tau_d) t_L S_B(t - \frac{\tau_d}{2}) e^{i \phi_d(E)} r_R \\
S_{in,32}(t, E) &= t_L e^{i \phi_u(E)} t_R + r_L S_B(t - \frac{\tau_d}{2}) e^{i \phi_d(E)} r_R
\end{align}

All other matrix elements have no relevance for the quantities studied in this paper. Similar expressions are found for the corresponding elements of $S_{out,\alpha/\beta}(E, t)$. 

\[ \text{Appendix A: Scattering matrices of the MZI with two single-particle sources} \]
Appendix B: Synchronized two-particle emission - expressions for the spectral, charge and energy current

1. Spectral current

In Section [IV A] we present the spectral currents detected at the output of the MZI when both SPSs are working, leading to the collision of (or the absorption of) electrons. Here, we complement this discussion by presenting the analytic results for the spectral current in the case where a hole emitted from SPS_A encounters an electron emitted from SPS_B

\[ i^{he}(E, \Phi) = R_L R_R i^h_A(E) + R_L T_R i^h_B(E) + T_L T_R (i^e_B(E) + i^h_A(E)) \left( 1 - \frac{4\sigma_A \sigma_B}{\Delta t_{he}^2 + (\sigma_A + \sigma_B)^2} \right) \]

\[ -2\gamma i^h_A(E) \text{Re} \left\{ e^{-i\Phi} e^{-iE\Delta \tau/h} \left( 1 - \frac{-2i\sigma_B}{\Delta t_{he}^2 - i(\sigma_A + \sigma_B)} \right) \right\}. \]  

Furthermore, we find for the hole part of the spectral current in the case of possible collision of holes

\[ i^{hh}(E, \Phi) = R_L R_R i^h_A(E) + T_L T_R i^h_B(E) \text{Re} \left\{ 1 + \frac{4\sigma_A \sigma_B}{\Delta t_{hh}^2 + (\sigma_A - \sigma_B)^2} + 2i\sigma_B \frac{\Delta t_{hh}}{\Delta t_{hh}^2 + (\sigma_A - \sigma_B)^2} e^{-iE(\Delta t_{hh} - i(\sigma_A - \sigma_B))/\hbar} \right\} \]

\[ + R_L T_R i^h_B(E) + T_L T_R i^h_B(E) \text{Re} \left\{ 1 + \frac{4\sigma_A \sigma_B}{\Delta t_{hh}^2 + (\sigma_A - \sigma_B)^2} + 2i\sigma_A \frac{\Delta t_{hh}}{\Delta t_{hh}^2 + (\sigma_A - \sigma_B)^2} e^{-iE(\Delta t_{hh} - i(\sigma_A - \sigma_B))/\hbar} \right\} \]

\[ -2\gamma i^h_A(E) \text{Re} \left\{ e^{-i\Phi} e^{-iE\Delta \tau/h} \left( 1 - \frac{2i\sigma_B}{\Delta t_{hh}^2 - i(\sigma_A - \sigma_B)} \right) \right\}. \]  

In order to find the limit in which either SPS_A of SPS_B is switched off, it is enough to set \( \sigma_A \to 0 \) (respectively, \( \sigma_B \to 0 \)). The same applies for Eqs. (22) and (23).

2. Charge current

All expressions for the time-averaged charge current given in the main text in the regime where particles of opposite type arrive in the detector from the two SPSs can be obtained from the general expression

\[ \bar{I}^{+he} \equiv \frac{i^{+he}}{\epsilon/T} = R_L R_R (n^h_A - n^e_A) + R_L T_R (n^h_B - n^e_B) + T_L T_R \frac{\Delta t_{d}^2 + (\sigma_A - \sigma_B)^2}{\Delta t_{d}^2 + (\sigma_A + \sigma_B)^2} \left( n^h_A + n^h_B - n^e_A - n^e_B \right) \]

\[ - 2\gamma \text{Re} \left\{ e^{-i\Phi} \left( n^h_A \frac{2\sigma_A}{\Delta \tau + 2i\sigma_A} \left( 1 - n^e_A \frac{-2i\sigma_B}{\Delta t_{d} - i(\sigma_A + \sigma_B)} \right) - n^h_A \frac{-2i\sigma_A}{\Delta \tau - 2i\sigma_A} \left( 1 - n^h_B \frac{2i\sigma_B}{\Delta t_{d} + i(\sigma_A + \sigma_B)} \right) \right) \right\}. \]  

by setting the respective particle numbers \( n^\pm = 0, 1 \). Here, we assume that the time difference \( \Delta t_{d}^{\pm} = \Delta t_{d}^{e} = \Delta t_{d} \) is equal for electrons and holes. However, different collision conditions \( \Delta t_{d}^{\pm} \) can be obtained straightforwardly by adjusting them for each contribution \( n^\pm \). The result for the MZI with a single SPS_A is found by setting \( n^e_B = n^h_B = 0 \). Also \( \sigma_B \) equals zero if SPS_B is switched off.

The general expression for the charge current in the regime where particles of the same type arrive in the detector from both SPSs is

\[ \bar{I}^{+hh} \equiv \frac{i^{+hh}}{\epsilon/T} = R_L R_R (n^h_A - n^e_A) + R_L T_R (n^h_B - n^e_B) + T_L T_R \frac{\Delta t_{d}^2 + (\sigma_A + \sigma_B)^2}{\Delta t_{d}^2 + (\sigma_A - \sigma_B)^2} \left( n^h_A + n^h_B - n^e_A - n^e_B \right) \]

\[ - 2T_L T_R \frac{4\sigma_A \sigma_B}{\Delta t_{d}^2 + (\sigma_A - \sigma_B)^2} \left( n^h_A n^h_B - n^e_A n^e_B \right) - 2\gamma \text{Re} \left\{ e^{-i\Phi} \left( n^h_A \frac{-2i\sigma_A}{\Delta \tau + 2i\sigma_A} \frac{2i\sigma_B}{\Delta t_{d} + i(\sigma_A + \sigma_B)} - n^e_A n^e_B \frac{-2i\sigma_A}{\Delta \tau - 2i\sigma_A} \right) \right\}. \]  

Also here we took \( \Delta t_{d}^{eh} = \Delta t_{d}^{he} = \Delta t_{d} \) and \( \Delta t_{u}^{ee} = \Delta t_{u}^{hh} = \Delta t_{u} \) for simplicity.
3. Energy current

Similar to the case of the charge current, we only show a part of the different particle contributions to the energy current in the main text. In this appendix we report the full expressions, where the same considerations for the different contributing particles, $n^e_A$ and $n^h_A$, and the time differences characterising their synchronised emissions, $\Delta t^e_A$ and $\Delta t^h_A$, apply, as it was explained for the charge currents in Appendix B 2.

When the SPSs are tuned such that particles of different type emitted from the two sources arrive at the detector in the same half period and hence absorptions can possibly occur, the general expression for the energy current is

$$j^{\text{eh+he}} = \frac{\hbar}{2\sigma_A T} \left( n^e_A + n^h_A \right) (R_L R_R + T_L T_R) + \frac{\hbar}{2\sigma_B T} \left( n^e_B + n^h_B \right) T_R \quad \text{(B5)}$$

$$- T_L T_R \left( \frac{\hbar}{2\sigma_A T} + \frac{\hbar}{2\sigma_B T} \right) \left( n^e_A n^e_B + n^h_A n^h_B \right) \frac{4\sigma_A \sigma_B}{\Delta t^e_A + (\sigma_A + \sigma_B)^2}$$

$$-2\gamma \frac{\hbar}{2\sigma_A T} \text{Re} \left\{ e^{-i\Phi} \left[ n^e_A \left( \frac{-2i\sigma_A}{\Delta \tau - 2i\sigma_A} \right)^2 \left( 1 - n^e_B \frac{-2i\sigma_B}{\Delta t_d + i(\sigma_A + \sigma_B)} \right) \right] + n^h_A \left( \frac{2i\sigma_A}{\Delta \tau + 2i\sigma_A} \right)^2 \left( 1 - n^h_B \frac{-2i\sigma_B}{\Delta t_d - i(\sigma_A + \sigma_B)} \right) \right\}.$$ 

For the regime in which collisions between particles can occur, we find

$$j^{\text{eh+hh}} = \frac{\hbar}{2\sigma_A T} \left( n^e_A + n^h_A \right) (R_L R_R + T_L T_R) + \frac{\hbar}{2\sigma_B T} \left( n^e_B + n^h_B \right) T_R \quad \text{(B6)}$$

$$+ T_L T_R \left( \frac{\hbar}{2\sigma_A T} + \frac{\hbar}{2\sigma_B T} \right) \left( n^e_A n^e_B + n^h_A n^h_B \right) \frac{4\sigma_A \sigma_B}{\Delta t^e_A + (\sigma_A + \sigma_B)^2}$$

$$-2\gamma \frac{\hbar}{2\sigma_A T} \text{Re} \left\{ e^{-i\Phi} \left[ n^e_A \left( \frac{-2i\sigma_A}{\Delta \tau - 2i\sigma_A} \right)^2 \left( 1 - n^e_B \frac{-2i\sigma_B}{\Delta t_u - i(\sigma_A + \sigma_B)} \right) \right] + n^h_A \left( \frac{2i\sigma_A}{\Delta \tau + 2i\sigma_A} \right)^2 \left( 1 - n^h_B \frac{-2i\sigma_B}{\Delta t_u + i(\sigma_A + \sigma_B)} \right) \right\} + 2\gamma \frac{\hbar}{2\sigma_B T} \text{Re} \left\{ e^{-i\Phi} \left[ n^e_A n^h_B \frac{-2i\sigma_A}{\Delta \tau - 2i\sigma_A} \left( \frac{-2i\sigma_B}{\Delta t_u - i(\sigma_A + \sigma_B)} \right)^2 + n^h_A n^e_B \frac{2i\sigma_A}{\Delta \tau + 2i\sigma_A} \left( \frac{2i\sigma_B}{\Delta t_u + i(\sigma_A + \sigma_B)} \right)^2 \right] \right\}.$$ 

Appendix C: Analytic expressions for the noise

Finally, we consider the charge-current noise, stemming from the current-current correlator of the currents detected in reservoirs 3 and 4. If $SPS_B$ is switched off and particles are emitted into the MZI only from $SPS_A$, the total noise stemming from electrons and holes is given by

$$P_{\Delta Z L A} = \frac{e^z}{e^2 / T} \left( T_R R_R + T_L R_L - 4\gamma^2 \right) \left( n^e_A + n^h_A \right) \quad \text{(C1)}$$

$$+ 2\gamma (T_L - R_L) (T_R - R_R) \text{Re} \left\{ e^{-i\Phi} \left[ n^e_A \frac{-2i\sigma_A}{\Delta \tau - 2i\sigma_A} + n^h_A \frac{2i\sigma_A}{\Delta \tau + 2i\sigma_A} \right] \right\}$$

$$- n^e_A \left( 2\gamma \text{Re} \left\{ e^{-i\Phi} \frac{-2i\sigma_A}{\Delta \tau - 2i\sigma_A} \right\} \right)^2 - n^h_A \left( 2\gamma \text{Re} \left\{ e^{-i\Phi} \frac{2i\sigma_A}{\Delta \tau + 2i\sigma_A} \right\} \right)^2.$$ 

The noise for the case of a possible absorption of a hole emitted by $SPS_A$ by an emission of an electron from $SPS_B$.
Given by

$$\frac{\mathcal{P}^\text{he}}{-e^2/j^2} = R_L T_L - 4\gamma^2 + 2R_L T_R R_R + 2T_L T_R R_R \left(1 - \frac{4\sigma_A \sigma_B}{\Delta t_{\text{he}}^2 + (\sigma_A + \sigma_B)^2}\right) +$$

$$2\gamma (T_L - R_L) (T_R - R_R) \Re\left\{ e^{-i\phi} \frac{2i\sigma_A}{\Delta \tau} \frac{\Delta t_{\text{he}}}{\tau} - i (\sigma_A - \sigma_B) \right\}$$

For the noise in the case of the collision of two holes we find

$$\frac{\mathcal{P}^\text{hh}}{-e^2/j^2} = R_L T_L - 4\gamma^2 + 2T_L T_R R_R + 2R_L T_R R_R \left(1 - \frac{4\sigma_A \sigma_B}{\Delta t_{\text{hh}}^2 + (\sigma_A + \sigma_B)^2}\right) +$$

$$+ 2\gamma (T_L - R_L) (T_R - R_R) \Re\left\{ e^{-i\phi} \frac{2i\sigma_A}{\Delta \tau} \frac{\Delta t_{\text{hh}}}{\tau} + i (\sigma_A - \sigma_B) \right\}$$

$$- \left(2\gamma \Re\left\{ e^{-i\phi} \frac{2i\sigma_A}{\Delta \tau} \frac{\Delta t_{\text{hh}}}{\tau} + i (\sigma_A - \sigma_B) \right\} \right)^2.$$
D. Mailly, F. Pierre, W. Wegscheider, and P. Roche, Phys. Rev. Lett. 108, 256802 (2012).

E. Iyoda, T. Kato, K. Koshino, and T. Martin, Phys. Rev. B 89, 205318 (2014).

B. Gaury and X. Waintal, Nat. Commun. 5, 3844 (2014).

E. Weisz, H. K. Choi, I. Sivan, M. Heiblum, Y. Gefen, D. Mahalu, and V. Umansky, Science 344, 1363 (2014).

M. Moskalets, Phys. Rev. B 90, 155453 (2014).

T. Bautze, C. Süssmeier, S. Takada, C. Groth, T. Meunier, M. Yamamoto, S. Tarucha, X. Waintal, and C. Bäuerle, Phys. Rev. B 89, 125432 (2014).

A. A. Vyshnevyy, A. V. Lebedev, G. B. Lesovik, and G. Blatter, Phys. Rev. B 87, 165302 (2013).

A. A. Vyshnevyy, G. B. Lesovik, T. Jonckheere, and T. Martin, Phys. Rev. B 87, 165417 (2013).

P. Hofer and C. Flindt, arXiv:1410.0574 (2014).

P. N. Butcher, Journal of Physics: Condensed Matter 2, 4869 (1990).

Y. Blanter and M. Büttiker, Physics Reports 336, 1 (2000).

M. Moskalets and M. Büttiker, Phys. Rev. B 66, 035306 (2002).

F. D. Parmentier, E. Bocquillon, J.-M. Berroir, D. C. Glattli, B. Plaçais, G. Fève, M. Albert, C. Flindt, and M. Büttiker, Phys. Rev. B 85, 165438 (2012).

F. Giazotto and M. J. Martínez-Pérez, Nature 492, 401 (2012).

M. Büttiker, Physical Review B 46, 12485 (1992).

D. Sergi, Physical Review B 83, 033401 (2011).

F. Battista, M. Moskalets, M. Albert, and P. Samuelsson, Physical Review Letters 110, 126602 (2013).

M. F. Ludovico, J. S. Lim, M. Moskalets, L. Arrachea, and D. Sánchez, Physical Review B 89, 161306 (2014).

Y. Aharonov, S. Popescu, D. Rohrlich, and P. Skrzypczyk, New Journal of Physics 15, 113015 (2013).

T. Denkmayr, H. Geppert, S. Sponar, H. Lemmel, A. Matzkin, J. Tollaksen, and Y. Hasegawa, Nature Communications 5, 4492 (2014).

R. Corrêa, M. F. Santos, C. H. Monken, and P. L. Saldanha, arXiv:1409.0808 (2014).

W. M. Stuckey, M. Silberstein, and T. McDevitt, arXiv:1410.1522 (2014).

G. Fève, P. Degiovanni, and T. Jolicoeur, Phys. Rev. B 77, 035308 (2008).

The energy-resolved spectral current should not be confused with the time-resolved current pulses studied e.g. in Ref. [20].

When introducing the magnetic field, which determines the direction of propagation of the chiral edge states, as an additional variable to the excess-energy distribution function, the equality \( i_\alpha (E, B) = -i_\alpha (-E, -B) \) relates the excess-energy distribution function of electrons, \( i_\alpha \), to the one of holes, \( i_\alpha \).