Intrinsic nonlinearity of interaction of an electromagnetic field with quantum plasma and its research

A. V. Latyshev\textsuperscript{1} and A. A. Yushkanov\textsuperscript{2}

Faculty of Physics and Mathematics, Moscow State Regional University, 105005, Moscow, Radio str., 10A

Abstract

The analysis of nonlinear interaction of transversal electromagnetic field with quantum collisionless plasma is carried out. Formulas for calculation electric current in quantum collisionless plasma at any temperature are deduced. It has appeared, that the nonlinearity account leads to occurrence of the longitudinal electric current directed along a wave vector. This second current is orthogonal to the known transversal classical current, received at the classical linear analysis. The case of degenerate electronic plasma is considered. The concept of longitudinal-transversal conductivity is entered. The graphic analysis of the real and imaginary parts of dimensionless coefficient of longitudinal-transversal conductivity is made. It is shown, that for degenerate plasmas the electric current is calculated under the formula, not containing quadratures. In this formula we have allocated known Kohn’s singularities (W. Kohn, 1959).

Key words: collisionless plasmas, Schrödinger equation, Dirac, Fermi, degenerate plasma, electrical current, longitudinal-transversal conductivity.

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Introduction

Dielectric permeability in quantum plasma was studied by many authors {[1]} – {[11]}. Dielectric permeability is one of the major plasma characteristics.
This quantity is necessary for the description of skin-effect [12], for the analysis surface plasmons [13], for descriptions of process of propagation and attenuation of the transversal plasma oscillations [8], for studying of the mechanism of penetration electromagnetic waves in plasma [7], and for the analysis of other problems in the plasma physics [14] – [19].

Let us notice, that for the first time in work [1] the formula for calculation of longitudinal dielectric permeability into quantum plasma has been deduced. Then the same formula has been deduced and in work [2].

In the present work formulas for calculation electric current into quantum collisionless plasma at any temperature (at any degrees of degeneration of the electronic gas) are deduced.

Here the approach developed by Klimontovich Silin [1] is generalized.

At the solution of Schrödinger equation we consider and in expansion of distribution Wigner function, and in Wigner—Vlasov integral expansion the quantities proportional to square of potential of an external electromagnetic field.

It has appeared, that electric current expression consists of two summands. The first summand, linear on vector potential, is known classical expression of an electric current. This electric current is directed along vector potential electromagnetic field. The second summand represents itself an electric current, which is proportional to the square vector potential of electromagnetic fields. The second current it is perpendicular to the first and it is directed along the wave vector. Occurrence of the second current comes to light the spent account intrinsic nonlinear character interactions of an electromagnetic field with quantum plasma.

For the case of degenerate quantum plasma expression of the electric current, not containing quadratures, is received. At the deducing of this expression Landau’ rule for calculation singular integrals is used. At use of this rule calculation these integrals containing a pole on the real axis, it is carried out by means of integration on infinitesimal half-circles in
the bottom half-plane with the centre in this pole.

1. Kinetic equation for Wigner function

Let us consider Shrödinger equation which has been written down for a particle in an electromagnetic field on a density matrix $\rho$

$$i\hbar \frac{\partial \rho}{\partial t} = H \rho - H^\ast \prime \rho.$$  

Here $H$ is the Hamilton operator, $H^\ast$ is the complex conjugate operator to $H$, $H^\ast \prime$ is the complex conjugate operator to $H$, which operates on the shaded spatial variables $r'$. We believe that the scalar potential is equal to zero.

In work [16] it is shown that the Shrödinger equation under condition of calibration of potential of electromagnetic field

$$\nabla \cdot A = 0$$

will be transformed to the kinetic equation

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial r} = W[f],$$  \hspace{1cm} (1.1)

written down in regard to Wigner function

$$f(r, p, t) = \int \rho(r + \frac{a}{2}, r - \frac{a}{2}, t)e^{-ipa/\hbar}d^3a,$$

besides

$$\rho(R, R', t) = \frac{1}{(2\pi\hbar)^3} \int f(\frac{R + R'}{2}, p, t)e^{ip(R-R')/\hbar}d^3p.$$ 

Here $e$ is the electron charge, $m$ is the electron mass, $c$ is the speed of light.

Wigner integral [16] equals

$$W[f] = \int \int \left\{ \frac{e}{2mc} \left[A(r + \frac{a}{2}, t) + A(r - \frac{a}{2}, t) - 2A(r, t) \right] \nabla f + \right\}$$
\[ + \frac{ie}{mch} \left[ \mathbf{A}(\mathbf{r} + \frac{\mathbf{a}}{2}, t) - \mathbf{A}(\mathbf{r} - \frac{\mathbf{a}}{2}, t) \right] \mathbf{p}' f + \]

\[- \frac{ie^2}{2mc^2\hbar} \left[ \mathbf{A}^2(\mathbf{r} + \frac{\mathbf{a}}{2}, t) - \mathbf{A}^2(\mathbf{r} - \frac{\mathbf{a}}{2}, t) \right] f \right\} e^{i(p' - p)\mathbf{a}/\hbar} \frac{d^3\mathbf{a} d^3p'}{(2\pi\hbar)^3}.\]

Vector potential of an electromagnetic field we take orthogonal to direction of a wave vector \( \mathbf{k} (\mathbf{k} \cdot \mathbf{A} = 0) \) in the form running harmonious wave \( \mathbf{A}(\mathbf{r}, t) = A_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}. \)

We denote further

\[ f_\pm \equiv f(\mathbf{r}, \mathbf{p} \mp \hbar \mathbf{k}/2, t) \quad f_{\pm\pm} \equiv f(\mathbf{r}, \mathbf{p} \mp \hbar \mathbf{k}, t). \]

Now we transform the Wigner integral to the following form

\[ W[f] = \frac{e\mathbf{A}}{2mc} \left( \frac{\partial f_+}{\partial \mathbf{r}} + \frac{\partial f_-}{\partial \mathbf{r}} - 2 \frac{\partial f}{\partial \mathbf{r}} \right) + \frac{ie\mathbf{A}}{mch} \mathbf{p}(f_+ - f_-) - \]

\[- \frac{ie^2\mathbf{A}}{2mc^2\hbar} (f_{++} - f_{--}). \quad (1.2)\]

We will enter the locally equalibrium and absolute Fermi—Dirac distribution \( f^{(0)} \) and \( f_F, \)

\[ f^{(0)} = f^{(0)}(\mathbf{P}, \mathbf{r}, t) = [1 + \exp(C^2 - \alpha)]^{-1}, \]

and

\[ f_F = f_F(P) = [1 + \exp(P^2 - \alpha)]^{-1}, \]

Here

\[ C \equiv C(P, \mathbf{r}, t) = \frac{\mathbf{v}}{v_T} = \mathbf{P} - \frac{e}{c_p T} \mathbf{A}(\mathbf{r}, t), \quad \alpha = \frac{\mu}{k_B T}, \]

\( C \) is the dimensionless electron velocity, \( v_T = 1/\sqrt{\beta} \) is the thermal electron velocity, \( \beta = m/2k_B T, \) \( P = p/p_T \) is the dimensionless electron momentum, \( m \) is the electron mass, \( k_B \) is the Boltzmann constant, \( T \) is the plasma temperature, \( \mu \) is the chemical potential electronic gas, \( \alpha \) is the dimensionless chemical potential.
Let us operate with a method consecutive approximations. In square-law approach on vector potential $A$ $f$ in the first summand in (1.2) it is necessary to replace Wigner function on locally equilibrium distribution $f^{(0)}$, in the third summand — on absolute distribution $f_F$, and in second — on its linear approach found in [16], i.e. to put $f = f^{(1)}$, where

$$f^{(1)} = f^{(0)} - PA\left(\frac{2e}{c\rho_T}g(P) + \frac{ev_T}{ch} f^+_F - f^-_F\right), \quad (1.3)$$

where

$$g(P) = e^{P^2-\alpha}\left(1 + e^{P^2-\alpha}\right)^{-2}, \quad f^+_F = [1 + e^{P^2-\alpha}]^{-1},$$

$$P^2 = \left(\frac{P + \frac{\hbar k}{2\rho_T}}{2}\right)^2.$$

Let us notice, that in linear approximation

$$f^{(0)} = f_F + PA\frac{2e}{c\rho_T}g(P).$$

Hence, function $f^{(1)}$ is represented in the form

$$f^{(1)} = f_F(P) - \frac{ev_T}{ch}PA(r, t)\frac{f^+_F - f^-_F}{\omega - v_T kP}.$$

Let us show, that the first summand in Wigner integral (1.2) equally to zero. According to problem statement vector potential of electromagnetic field varies along an axis $x$. Hence, gradient of locally equilibrium distribution of Fermi–Dirac is proportional to the vector $k$: $\partial f^{(0)}_+/\partial r \sim k$, $\partial f^{(0)}_-/\partial r \sim k$. Therefore

$$A \left[\frac{\partial f^{(0)}_+}{\partial r} + \frac{\partial f^{(0)}_-}{\partial r} + 2\frac{\partial f^{(0)}}{\partial r} \right] \sim Ak = 0.$$

We notice that

$$f^+_F = f^-_F = f_F = f_F(P).$$

Therefore, Wigner integral (1.2) equals

$$W[f] = \frac{iev_T}{ch}PA\left[f^+_F - f^-_F - \frac{ev_T}{ch}PA(r, t)\frac{f^+_F + f^-_F - 2f_F}{\omega - v_T kP}\right].$$
\[-\mathbf{A}^2 \frac{ie^2}{2mc^2\hbar} \left( f_F^{++} - f_F^{--} \right). \tag{1.4} \]

Here
\[ f_F^{\pm\pm} = \left[ 1 + e^{P_{\pm\pm}^2 - \alpha} \right]^{-1}, \quad P_{\pm\pm}^2 = \left( \mathbf{P} \pm \frac{\hbar \mathbf{k}}{\rho_T} \right)^2. \]

Let us return to the decision of the equation (1.1) with Wigner integral (1.4). Let us search for Wigner function in the form, square-law concerning of vector potential \( \mathbf{A} = \mathbf{A}(\mathbf{r}, t) \):
\[ f = f_F(P) - \frac{ev_T}{c\hbar} \mathbf{P} \mathbf{A} \frac{f_F^{+} - f_F^{-}}{\omega - v_T k \mathbf{P}} + \mathbf{A}^2 h(\mathbf{P}), \]
where \( h(\mathbf{P}) \) is the new unknown function.

We receive the equation from which it is found
\[ \mathbf{A}^2 h(\mathbf{P}) = \frac{(ev_T)^2}{2(c\hbar)^2} [\mathbf{P} \mathbf{A}]^2 \frac{f_{F}^{++} + f_{F}^{--} - 2f_{F}}{(\omega - v_T k \mathbf{P})^2} + \frac{e^2}{4mc^2\hbar} \mathbf{A}^2 \frac{f_{F}^{++} - f_{F}^{--}}{\omega - v_T k \mathbf{P}}. \]

By means of last two equalities let us construct the Wigner function Вигнера in the second approximation on vector potential \( \mathbf{A}(\mathbf{r}, t) \):
\[ f = f^{(0)} - \mathbf{P} \mathbf{A}(\mathbf{r}, t) \left[ \frac{2e}{c\rho_T} g(P) + \frac{ev_T}{c\hbar} \frac{f_{F}^{+} - f_{F}^{-}}{\omega - v_T k \mathbf{P}} \right] + \frac{(ev_T)^2}{2(c\hbar)^2} [\mathbf{P} \mathbf{A}]^2 \frac{f_{F}^{++} + f_{F}^{--} - 2f_{F}}{(\omega - v_T k \mathbf{P})^2} + \frac{e^2}{4mc^2\hbar} \mathbf{A}^2 \frac{f_{F}^{++} - f_{F}^{--}}{\omega - v_T k \mathbf{P}}. \tag{1.5} \]

This function represents square-law decomposition of distribution function on vector potential \( \mathbf{A}(\mathbf{r}, t) \).

2. Density of electric current in quantum plasmas

By definition, the density of electric current is equal
\[ \mathbf{j}(\mathbf{r}, t) = e \int \mathbf{v}(\mathbf{r}, \mathbf{p}, t) f(\mathbf{r}, \mathbf{p}, t) \frac{2d^3p}{(2\pi\hbar)^3}. \tag{2.1} \]
Substituting in equality (2.1) explicit expression for velocity

\[ \mathbf{v}(\mathbf{r}, P, t) = \frac{\mathbf{p}}{m} - \frac{e\mathbf{A}(\mathbf{r}, t)}{mc} = \frac{p_T\mathbf{P}}{m} - \frac{e\mathbf{A}(\mathbf{r}, t)}{mc} = v_T\mathbf{P} - \frac{e\mathbf{A}(\mathbf{r}, t)}{mc}. \]

and, leaving linear and quadratic (square-law) expressions concerning vector potential of the field, we receive

\[ \mathbf{j}(\mathbf{r}, t) = -\frac{2e^2p_T^4}{(2\pi\hbar)^3mc} \int \mathbf{P}[\mathbf{PA}] \left[ \frac{2}{p_T} g(P) + \frac{v_T}{\hbar} \frac{f_F^+ - f_F^-}{\omega - v_Tk\mathbf{P}} \right] d^3P + \]

\[ + \frac{2e^3p_T^3}{(2\pi\hbar)^3mc^2} \mathbf{A} \int [\mathbf{PA}] \left[ \frac{2}{p_T} g(P) + \frac{v_T}{\hbar} \frac{f_F^+ - f_F^-}{\omega - v_Tk\mathbf{P}} \right] d^3P + \]

\[ + \frac{2e^3p_T^4}{(2\pi\hbar)^3mc^2\hbar} \int \mathbf{P} \left[ \frac{m v_T^2}{2\hbar} [\mathbf{PA}]^2 \frac{f_F^{++} + f_F^{--} - 2f_F}{(\omega - v_Tk\mathbf{P})^2} + \frac{A^2}{4} \frac{f_F^{++} - f_F^{--}}{\omega - v_Tk\mathbf{P}} \right] d^3P. \] 

(2.2)

The first summand in (2.2) is linear expression of the density of electric current,

\[ \mathbf{j}_{\text{linear}}(\mathbf{r}, t) = \]

\[ = -\frac{2e^2p_T^4}{(2\pi\hbar)^3mc} \int \mathbf{P}[\mathbf{PA}] \left[ \frac{2}{p_T} g(P) + \frac{v_T}{\hbar} \frac{f_F^+ - f_F^-}{\omega - v_Tk\mathbf{P}} \right] d^3P. \] 

(2.3)

found, in particular, in our previous work [16]. The second summand is the square-law amendment to the first. The third summand represents density of longitudinal electric current, unlike density of classical transversal electric current described first two summands.

Thus, in square-law approximation on vector potential of electromagnetic field it has appeared, that vector potential electromagnetic fields generates also the longitudinal electric current besides the transversal current (2.3).

Vector potential of the field we will direct along an axis \( y \): \( \mathbf{A} = A(x, t) \) \{0, 1, 0\}, \( A(x, t) = A_y e^{i(kx - \omega t)} \), and wave vector \( \mathbf{k} \) we direct along axis
$x$: $k = k\{1, 0, 0\}$. According to (2.2) the longitudinal equals $j_{\text{long}} = j_{\text{long}}(x, t)\{1, 0, 0\}$, where

$$j_{\text{long}}(x, t) = \frac{e^3 p_T^2 A^2(x, t)}{(2\pi\hbar)^3 mc^2 q^2} \times \left[ \int \frac{f_{F+}^+ + f_{F-}^- - 2f_F P_x P_y^2 d^3 P - q}{2} \int \frac{f_{F+}^+ - f_{F-}^-}{P_x - \omega/v_T k} P_x d^3 P \right]. \tag{2.4}$$

Here $q = k/k_T$, $k_T = mv_T/\hbar$.

Let us simplify the formula (2.4), having calculated internal integrals in planes $(P_y, P_z)$. We receive as the result, that

$$j_{\text{long}}(x, t) = \frac{e^3 p_T^2 A^2(x, t)}{(2\pi\hbar)^3 mc^2 q^2} \times \left[ \int_{-\infty}^{\infty} \frac{P_x L(P_x, \alpha) dP_x}{(P_x - \omega/v_T k)^2} - \frac{q}{2} \int_{-\infty}^{\infty} \ln \frac{1 + e^{-P_x^2 + \alpha}}{1 + e^{-P_x^2 + \alpha}} P_x dP_x \right],$$

where

$$L(P_x, \alpha) = \int_{0}^{\infty} \frac{(1 + e^{-P_x^2 + \alpha})^\beta (1 + e^{-P_x^2 + \alpha})^\rho}{(1 + e^{-P_x^2 + \alpha})^\rho} d\rho,$$

$$P_x^\pm = P_x \mp \frac{\hbar k}{2p_T}, \quad P_x^{\pm\pm} = P_x \mp \frac{\hbar k}{p_T}.$$  

At calculation "dispersing" integrals it is necessary to use known Landau rule, bypassing a pole on the real axis on the half-circle laying in the bottom half-plane, preliminary having executed integration in parts. It is equivalent to the following application of the formula Sokhotsky

$$\int_{a}^{b} \frac{\varphi(\tau)d\tau}{(\tau - x)^2} = \lim_{\varepsilon \to 0} \int_{a}^{b} \frac{\varphi(\tau)d\tau}{\tau - (x + i\varepsilon)^2} = \lim_{\varepsilon \to 0} \left[ -\frac{\varphi(\tau)}{\tau - (x + i\varepsilon)} \right]_{a}^{b} + \int_{a}^{b} \frac{\varphi'(\tau)d\tau}{\tau - (x + i\varepsilon)} \right] = -\frac{\varphi(\tau)}{\tau - x} \bigg|_{a}^{b} + i\pi \varphi'(x) + \int_{a}^{b} \frac{\varphi'(\tau)d\tau}{\tau - x}. \tag{2.5}$$

3. Degenerate plasmas
Let’s consider the case of degenerate plasmas.

In the formula (2.4) we will pass to the limit at $T \to 0$ and we will carry out replacement of one variable of integration

$$P_x \to \frac{v_F}{v_T} P_x,$$

where $v_F$ is the electron velocity on Fermi’ surface.

Let us notice, that in the limit zero temperature ($T \to 0$) $\mu \to \mathcal{E}_F = \frac{mv_F^2}{2}$ and $f_F \to \Theta(1 - P^2)$, where $\Theta(x)$ – Heaviside’ function,

$$\Theta(x) = \begin{cases} 
1, x > 0, \\
0, x < 0.
\end{cases}$$

Besides, at $T \to 0$ $f_{F\pm} \to \Theta_{\pm}$, where

$$\Theta_{\pm} = \Theta[1 - (P_x \mp \hbar \mathbf{k}/p_F)^2 - P_y^2 - P_z^2].$$

Thus, the formula (2.4) for degenerate electronic plasma it will be transformed to the following form

$$j_{\text{long}}(x, t) = \frac{e^3 p_F^2 A^2(x, t)}{(2\pi \hbar)^3 mc^2 q^2} \times$$

$$\times \left[ \int \frac{\Theta^{++} + \Theta^{--} - 2\Theta}{(P_x - \omega/v_T k)^2} P_x P_y^2 d^3 P - \frac{q}{2} \int \frac{\Theta^{++} - \Theta^{--}}{P_x - \omega/v_T k} P_x d^3 P \right], \quad (3.1)$$

Here, in (3.1) $q = k/k_F$, $k_F = mv_F/\hbar$ is the Fermi wave number.

We notice that

$$\frac{\omega}{kv_F} = \frac{k}{v_F k_F}, \quad \frac{k}{k} = \frac{\Omega}{q}, \quad \frac{\omega}{v_F k_F}, \quad \frac{\hbar k}{p_F} = q.$$

Let us put in (3.1) $P_x = \tau$, $x_0 = \omega/kv_F = \Omega/q$ and we will calculate entering into (3.1) integrals. For the first integral it is received

$$\int \frac{\Theta^{++} + \Theta^{--} - 2\Theta}{(P_x - \omega/kv_F)^2} P_x P_y^2 d^3 P =$$

$$= \int_{p^2 < 1} \left[ \frac{\tau + q}{(\tau + q - x_0)^2} + \frac{\tau - q}{(\tau - q - x_0)^2} - \frac{2\tau}{(\tau - x_0)^2} \right] P_y^2 d^3 P =$$
\[
\frac{\pi}{4} \int_{-1}^{1} \left[ \frac{\tau + q}{(\tau + q - x_0)^2} + \frac{\tau - q}{(\tau - q - x_0)^2} - \frac{2\tau}{(\tau - x_0)^2} \right] (1 - \tau^2)^2 d\tau.
\]

The second integral equals
\[
\int \frac{\Theta^{++} - \Theta^{--}}{P_x - \omega/v_F k} P_x d^3 P = 
\]
\[
= \pi \int_{-1}^{1} \left( \frac{\tau + q}{\tau + q - x_0} - \frac{\tau - q}{\tau - q - x_0} \right) (1 - \tau^2) d\tau.
\]

By means of two last equalities and the formula (2.5) for density transversal electric current (3.1) we receive expression through one-dimensional integrals
\[
j_{\text{long}}(x, t) = \sigma_{l,\text{tr}}^{(2)} E_{\text{tr}}^2(x, t).
\]

Here \(\sigma_{l,\text{tr}}^{(2)}\) is the quantity which it is natural to name longitudinal-transversal (nonlinear) conductivity of the second order,
\[
\sigma_{l,\text{tr}}^{(2)} = \Sigma_{l,\text{tr}}^{(2)} \frac{1}{\Omega^2 q^2} J(x_0, q),
\]
where
\[
\Sigma_{l,\text{tr}}^{(2)} = -\frac{e^3}{32\pi^2 \hbar mv_F^2},
\]
\(J(x_0, q)\) is the dimensionless coefficient of longitudinal – transversal (nonlinear) conductivity of the second order,
\[
J(x_0, q) = \int_{-1}^{1} \left[ \frac{[(\tau + q)(1 - \tau^2)]'}{(\tau + q - x_0)} + \frac{[(\tau - q)(1 - \tau^2)]'}{(\tau - q - x_0)} - \frac{2[\tau(1 - \tau^2)]'}{(\tau - x_0)} \right] d\tau - 
\]
\[
-2q \int_{-1}^{1} \left[ \frac{(\tau + q)(1 - \tau^2)}{\tau + q - x_0} - \frac{(\tau - q)(1 - \tau^2)}{\tau - q - x_0} \right] d\tau.
\]

Let us underline, that is longitudinal – transversal conductivity is caused that the transversal electromagnetic field leads to the longitudinal current.
The integrals entering in (3.3), we will calculate by means of equality (2.5).

The first integral is equal

\[ J_1 = 28x_0q^2 + [(x_0 - q)^2 - 1][5(x_0 - q)^2 + 4\tau_0(x_0 - q) - 1] \times \]
\[ \times \left[ \ln \left| \frac{x_0 - q - 1}{x_0 - q + 1} \right| + \begin{cases} i\pi, & |x_0 - q| < 1 \\ 0, & |x_0 - q| > 1 \end{cases} \right] + 
\[ + [(x_0 + q)^2 - 1][5(x_0 + q)^2 - 4(x_0 + q) - 1] \times \]
\[ \times \left[ \ln \left| \frac{x_0 + q - 1}{x_0 + q + 1} \right| + \begin{cases} i\pi, & |x_0 + q| < 1 \\ 0, & |x_0 + q| > 1 \end{cases} \right] - 
\[ -2(x_0^2 - 1)(5x_0^2 - 1) \left[ \ln \left| \frac{x_0 - 1}{x_0 + 1} \right| + \begin{cases} i\pi, & |x_0| < 1 \\ 0, & |x_0| > 1 \end{cases} \right]. \]

The second integral equals

\[ J_2 = 4x_0q + 
\[ -x_0[(x_0 - q)^2 - 1] \left[ \ln \left| \frac{x_0 - q - 1}{x_0 - q + 1} \right| + \begin{cases} i\pi, & |x_0 - q| < 1 \\ 0, & |x_0 - q| > 1 \end{cases} \right] + 
\[ + x_0[(x_0 + q)^2 - 1] \left[ \ln \left| \frac{x_0 + q - 1}{x_0 + q + 1} \right| + \begin{cases} i\pi, & |x_0 + q| < 1 \\ 0, & |x_0 + q| > 1 \end{cases} \right]. \]

The found integrals we will substitute in (3.3) and we will allocate in it the real and imaginary parts, 

\[ J(x_0, q) = J_1(x_0, q) - 2qJ_2(x_0, q) = R(x_0, q) + i\pi S(x_0, q), \quad (3.4) \]

believing, that \( x_0 = \Omega/q \).

We receive that 

\[ R(\Omega, q) = 20\Omega q + \left[ \left( \frac{\Omega}{q} - q \right)^2 - 1 \right] \left[ 5\frac{\Omega^2}{q^2} - 4\Omega + q^2 - 1 \right] \times \]
\[ \times \ln \left| \frac{\Omega - q^2 - q}{\Omega - q^2 + q} \right| + \left[ \left( \frac{\Omega}{q} + q \right)^2 - 1 \right] \left[ 5\frac{\Omega^2}{q^2} + 4\Omega + q^2 - 1 \right] \times \]
\[
\times \ln \left| \frac{\Omega + q^2 - q}{\Omega + q^2 + q} \right| - 2\left( \frac{\Omega^2}{q^2} - 1 \right) \left( \frac{5\Omega^2}{q^2} - 1 \right) \ln \left| \frac{\Omega - q}{\Omega + q} \right|,
\]
and

\[
S(\Omega, q) = \left[ \left( \frac{\Omega}{q} - q \right)^2 - 1 \right] \left[ \frac{5\Omega^2}{q^2} - 4\Omega + q^2 - 1 \right] \left\{ \begin{array}{ll} 1, & |\Omega - q^2| < |q| \\ 0, & |\Omega - q^2| > |q| \end{array} \right\} + \\
+ \left[ \left( \frac{\Omega}{q} + q \right)^2 - 1 \right] \left[ \frac{5\Omega^2}{q^2} + 4\Omega + q^2 - 1 \right] \left\{ \begin{array}{ll} 1, & |\Omega + q^2| < |q| \\ 0, & |\Omega + q^2| > |q| \end{array} \right\} - \\
-2\left( \frac{\Omega^2}{q^2} - 1 \right) \left( \frac{5\Omega^2}{q^2} - 1 \right) \left\{ \begin{array}{ll} 1, & |\Omega| < |q| \\ 0, & |\Omega| > |q| \end{array} \right\}.
\]

According to (3.2) and (3.4) for longitudinal – transversal conductivity we receive following expression

\[
\sigma_{l, tr}^{(2)} = \sum_{l, tr}^{(2)} \frac{R(\Omega, q) + i\pi S(\Omega, q)}{\Omega^2 q^2}.
\]

Let us underline, that in expression (3.5) are allocated Kohn’s singularities of the form \( X \ln X \). It means, that expression (3.5) has no singularities in zero of logarithms, i.e. in those points \((\Omega_0, q_0)\), in which \(X(\Omega_0, q_0) = 0\).

4. Conclusions

On Figs. 1 – 6 we will present behaviour of coefficients \( R \) and \( S \) depending on dimensionless frequency of oscillations of the vector potential \( \Omega \) and dimensionless wave number \( q \).

On Fig. 1 and 2 we will represent behaviour of coefficient \( R \) in dependence from frequency \( \Omega \) at various values \( q \). From these plots it is visible, as at small values \( \Omega \) and at values \( q \), comparable with unit, coefficient \( R \), proportional to the real part of the generated longitudinal current, has at first a minimum, and then a maximum, and a minimum lays near to a point \( \Omega = q \).
On Fig. 3 and 4 we will represent behaviour of coefficient $R$ in dependence from wave number $q$ at various values of frequency $\Omega$. In both cases at small values of frequency and at values frequencies near to unit the coefficient $R$ has at first a maximum, and then the minimum, and a minimum is found out nearby considered value

$$\Omega = q = 0.10, \quad 0.11, \quad 0.12.$$  

On Fig. 5 and 6 the behaviour of coefficient $S$ as functions wave number at various small values of oscillations frequency is represented. At small values of frequency of oscillations coefficient $S$ has a minimum. With increase $\Omega$ the coefficient $S$ can to have two minima and one maximum. This maximum vanishes with growth $\Omega$.

From Fig. 7 it is visible, that at $\Omega = 1$ function $S = S(\Omega, q)$ has in the point $q = 1$ a local maximum, and near to this point at the left and to the right of it has two more local minima. At $\Omega = 2$ function $S = S(2, q)$ in a point $\Omega = 2$ has the local maximum, and at the left and to the right of it has two more local minimum.

On Fig. 8 we observe the similar similar situation for three curves $S = S(0.7, q), S = S(1, q)$ and $S = S(1.3, q)$.

In the present work the account of nonlinear character of interaction electromagnetic field with quantum plasma is considered. It has appeared, that the account of nonlinearity of an electromagnetic field finds out generating of an electric current, orthogonal to a direction fields.
Fig. 1. Real part of coefficient $R(\Omega, q)$. Curves 1, 2, 3 correspond to values of dimensionless wave number $q = 0.1, 0.11, 0.12$.

Fig. 2. Real part of coefficient $R(\Omega, q)$. Curves 1, 2, 3 correspond to values of dimensionless wave number $q = 0.9, 1.0, 1.1$. 
Fig. 3. Real part of coefficient $R(\Omega, q)$. Curves 1,2,3 correspond to values of dimensionless frequency $\Omega = 0.1, 0.11, 0.12$.

Fig. 4. Real part of coefficient $R(\Omega, q)$. Curves 1,2,3 correspond to values of dimensionless frequency $\Omega = 0.7, 1.0, 1.3$. 
Fig. 5. Imaginare part of coefficient \( S(\Omega, q) \). Curves 1,2,3 correspond to values of dimensionless frequency \( \Omega = 0.01, 0.02, 0.03 \).

Fig. 6. Imaginare part of coefficient \( S(\Omega, q) \). Curves 1,2,3 correspond to values of dimensionless frequency \( \Omega = 0.3, 0.4, 0.5 \).
Fig. 7. Imaginare part of coefficient $S(\Omega, q)$. Curves 1 and 2 correspond to values of dimensionless frequency $\Omega = 1$ and $\Omega = 2$.

Fig. 8. Imaginare part of coefficient $S(\Omega, q)$. Curves 1,2,3 correspond to values of dimensionless frequency $\Omega = 0.7, 1.0, 1.3$. 
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