The superconducting proximity effect is a well known phenomenon and has drawn a lot of interest both from the fundamental and the practical points of view. In a high transmission contact between a superconductor and a normal metal the superconducting wave-function varies smoothly across the interface causing a suppression of the pair amplitude in the superconductor and an enhancement of pairing on the normal side. The characteristic distance in which superconductivity “leaks” into the normal region is the normal state coherence length \( \xi_N = \sqrt{\frac{\pi D}{2\pi k_B T}} \), where \( D \) is the diffusivity, and the length-scale at which superconductivity is suppressed in the superconducting side is the superconducting coherence length, \( \xi_S \).

In the Cooper limit [1] (in which both the superconductor and the normal metal layers are smaller than the characteristic coherence lengths \( \xi_S \) and \( \xi_N \) respectively) the behavior of an S-N bilayer is well described within the framework of the de Gennes model [2, 3]. The electrons experience an average pairing interaction (average between the two materials) and \( T_C \) can be described within the BCS weak coupling form adapted to the Cooper limit proximity effect in the following way:

\[
kT_C = 1.13\hbar\nu_D e^{-\frac{\nu_D}{\nu_D}}\]

\[
[N(0)V]_{S+N} = \frac{d_S[N(0)V]_S + d_N[N(0)V]_N}{d_S + d_N}
\]

where \( \nu_D \) is the debye frequency, \( N(0)V \) is the pairing interaction (\( N(0) \) being the density of states at the Fermi level and \( V \) the pairing interaction), and \( d_{S,N} \) is the thickness of the superconducting and normal metal films respectively. This model assumes that the debye frequencies in the two metals are similar and it is based on the understanding that the electrons spend \( \frac{N_N}{N_N + N_S} \) of their time in the normal metal and \( \frac{N_S}{N_N + N_S} \) in the superconductor (\( N_N \) and \( N_S \) are the density of states in the normal metal and in the superconductor respectively).

The above picture is modified in the case of strong coupled superconductors where \( T_C \) is given by the McMillan expression [4]:

\[
T_C = \frac{\omega_D}{1.45} \exp\left[-\frac{1.04(1+\lambda)}{\lambda - \mu^* (1 + 0.62\lambda)}\right]
\]

here \( \lambda \) is the dimensionless electron-phonon coupling coefficient and \( \mu^* \) is the effective coulomb repulsion. In a normal-metal/strong coupled superconductor within the Cooper limit one can expect \( \lambda \) in equation [2] to be replaced by an average electron-phonon coupling [5]:

\[
\lambda_{S+N} = \frac{\lambda_S d_S + \lambda_N d_N}{d_S + d_N}
\]

The proximity effect in the Cooper limit has been observed experimentally in numerous systems of superconductor/normal metal bilayers. In this paper we study systems of multilayers of superconductor/normal metal (Pb/Ag) in which the total thicknesses of the metals are comparable to the relevant coherence lengths. The transport and tunneling experimental results confirm that the basic idea of the Cooper limit notion, i.e. that the electrons experience an average pairing, is valid also in a multilayer sample. We observe good quantitative agreement with theory as long as the thickness of the layers is larger than \( \sim 30\AA \) while thinner layers deviate from the theoretical predictions. We suggest that the simple picture is modified in ultra-thin layers due to effects of screening of e-e interactions (the inverse proximity effect as previously reported [4]).

The Pb and Ag layers were prepared by quench condensation, i.e. evaporation on a cryogenically cooled substrate within the measurement apparatus [5]. This allows in-situ sequential depositions under UHV conditions and simultaneous transport and tunneling measurements on a single sample. This method has several essential advantages for the study of the superconducting proximity effect [5]. Since Pb and Ag are immiscible, the alloying of these two materials is very improbable...
even at room temperatures. Alloying becomes practically negligible when the samples are quench condensed and are kept below T=10K throughout the entire experiment (sample growth and measurement). In addition, the extremely clean environment of the experiment leads to very high quality interfaces between the superconducting layers and the normal metal layers. Such clean interfaces provide ideal conditions for proximity effects.

The geometry of the samples used in this study is shown in the insert of figure 1. We begin by evaporating two strips of Al onto a room temperature polished glass substrate. These strips are used both as voltage leads and as the base electrode for tunneling measurements. Then we allow a native oxide barrier to grow on the Al for 20 minutes. We connect metallic leads to the substrate and place it in an evaporation chamber which is then pumped out and immersed in liquid helium. A 10Å Ge adhesion layer is then quench condensed across the Al strips, permitting the evaporation of a continuous layer of Pb as thin as 5Å [7]. The thicknesses are monitored by a calibrated quartz crystal situated in the chamber. Multilayers, made of base units of Pb/Ag bilayers having different thicknesses, are quench condensed across the Al strips. This configuration allows the simultaneous measurements of transport properties of the multilayers (thus determining the critical temperature), and tunneling into the multilayers (for determination of the energy gap). We performed these measurements at incremental stages of the evaporation of each of the Pb or Ag layers. All measurement were performed in a screened room using standard AC methods.

In this paper we present data on three Pb/Ag systems having different base unit thickness:

- Sample #1 where \(d_{pb} = d_{Ag} = 32\) Å,
- Sample #2 where \(d_{pb} = d_{Ag} \approx 15\) Å, and
- Sample #3 where \(d_{pb} = d_{Ag} \approx 9\) Å.

Each of these unit bilayers is within the cooper limit; \(\xi_N\) of the Ag at low temperatures is of the order of 1 \(\mu m\) and \(\xi_S\) of the amorphous Pb (which is in the dirty limit) at low temperature is given by \(\xi' = \sqrt{\xi_0 l}\), where \(\xi_0\) is the clean limit coherence length (for Pb this is 800 Å) and \(l\) is the mean free path which is approximately an interatomic distance, about 2.7Å in our samples. These values yield a zero temperature coherence length, \(\xi' \approx 46\) Å, which is larger than each one of our Pb layers. Furthermore, as the temperature increases the coherence length increases until it diverges at \(T = T_C\).

We begin by presenting the results for the initial bilayer. Figure 1 shows the resistance versus temperature of sample #2 as a layer of 14.5 Å thick Ag is grown in steps of \(\approx 1.55\) Å on top of a 14.4Å thick initial Pb layer. Such measurements are used to extract \(T_C\) by taking the midpoint of the transition \((R_N/2)\). It is seen that the initial Pb layer has a \(T_C\) which is smaller than the bulk
value of 7.2K. This is typical for very thin superconducting layers which are known to exhibit a superconductor-insulator transition as a function of film thickness (and $R_{sq}$) $^{11,12,13}$. It has been argued $^{14,15}$ that as $R_{sq}$ (the disorder) increases, the electronic screening is reduced. Coulomb interactions are then enhanced leading to a weakening of the superconducting coupling. This explains the decrease of $T_C$ in the Pb films when the thickness is reduced from 30Å to 9Å as is seen in figure 2. The enhancement of Coulomb interactions for high $R_{sq}$ is also evident in figure 3 which depicts the $dI/dV-V$ measurements (proportional to the density of states) of a 9Å thick Pb film with increasing layers of Ag, taken at temperatures above $T_C$. It is seen that in the ultrathin Pb layer the density of states at the Fermi level is relatively low. The tunneling conductance exhibits a strong voltage (energy) dependence demonstrating that this is a strongly coulomb correlated system. The density of states dramatically increases when layers of Ag are evaporated on top of the Pb, presumably due to enhancement of screening and, hence, suppression of the Coulomb interactions $^{14}$.

As subsequent layers of Ag are added (and $R_{sq}$ is reduced as $1/d_{Ag}$) $T_C$ is decreased even further due to the proximity effect as evident in figure 1. Since the bilayer is in the cooper limit, $T_C$ is indeed expected to decrease as a function of the normal metal thickness. We note, however, that this is not always the case. We have shown $^6$ that when the initial Pb layer is extremely thin (thinner than that of sample 3) the first added Ag sub-layers cause $T_C$ to increase in what seems to be an inverse proximity effect. This behavior was interpreted as being due to the fact that the Ag layers enhance screening in the system thus suppressing the e-e interactions which are responsible for the $T_C$ suppression. This trend opposes the usual proximity effect and becomes more important in thinner superconducting films.

The evolution of $T_C$ as more Pb and Ag layers are added is shown in figure 2 for the three samples. It is seen that $T_C$ oscillates when we change between Pb and Ag depositions reaching a local minimum or maximum at the completion of each layer. The oscillation amplitude decreases with increasing number of bilayers and $T_C$ appears to approach a value of $\approx 3K$ in all three samples. The oscillation behavior is also observed in the energy gap measurements which are illustrated in figure 4 for sample #1. The $dI/dV$ curves were measured at $T=1.65K$ well below the $T_C$ of the multilayer. Note that despite that fact that tunneling is performed into the bottom Pb layer, the results are still affected by adding the 6th layer on top. This clearly demonstrates that within the cooper limit the tunneling electrons probe the entire sample and the superconducting parameters are determined by the mean properties of all the layers in the multilayer. Figure 4 also shows the ratio $2\Delta/k_BT_C$ as a function of the number of layers. The superconducting gap was evaluated from a classical temperature dependent BCS fit. It is seen that this ratio crosses over from a value of 4.7 (which is close to the strong coupling limit of $\sim 4.8$) to 3.6 (which is close to the weak coupling limit of $\sim 3.5$). The fact that this ratio is relatively constant as we add the last few layers gives us further confidence that all our multilayers are within the Cooper limit.

We have attempted to fit the curves in figure 2 to the proximity effect modified McMillan expression of equation $^2$ using an average $\lambda$ for the Cooper limit (equation $^3$) in the spirit of the de-Gennes model. These are shown in figure 2 were we used the known values for Pb: $\lambda_S = 1.55, \omega_D = 105K, \mu^* = 0.11$ and taking a low electron-phonon coupling for Ag, $\lambda_N = 0.2$, without any fitting parameters. It is seen that the experimental results are in excellent agreement with this model for sample #1 (where $d_{pb} = d_{Ag} \approx 32Å$). We note that the values for $\lambda, \omega_D$ and $\mu^*$ are not adjusted, but accepted values and so the agreement is quite impressive. However, when we used the same parameters for the other samples with thinner base layers, we get substantial deviations from the model for the first few layers (figure 2). In fact, in sample #1 (and in the theory) the oscillations are superimposed on a global decrease of $T_C$, in sample #2 the background is roughly constant and in sample #3 $T_C$ globally increases with the number of bilayers. We argue that this behavior is an extension of the inverse proximity effect observed in the ultrathin layers $^6$. The large Coulomb interactions characteristics of these thin samples are continuously weakened with increasing numbers of Pb/Ag bilayers, thus, as more layers are added, we approach the Cooper limit proximity effect theory.

In summary, we have shown that the de Gennes considerations regarding the Cooper limit are valid for the case of multilayers until the layers become very thin. Using
a simple model we were able to describe very accurately the variation of the critical temperature and the energy gap as a function of the number of layers. We showed that for very thin layers the model breaks down indicating that other physics is involved. We suggest that this breakdown is due to the effect of strong Coulomb interactions in the superconducting ultrathin films. These interactions are screened out as normal metal is added to the superconductor resulting in an inverse proximity effect which must also be taken into account in the thinnest layers.

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