The article contains a mathematical model of parametrized network of knowledge exchange. It considers a process of knowledge spreading among a group of people united by common professional interests (a company or its sub-unit). A network is described by a fuzzy graph where vertices stand for individuals and edges — for contacts between them. A fuzzy set of edges defines the process quality characteristics that have direct influence on the result: coincidence of professional interests of individuals (in the model — cognitive potential of knowledge transfer) and evaluation of communication activity between them (how often they communicate — communication intensity). The first parameter is defined on the base of knowledge structure of individuals (for particular knowledge domain). The second one — communication intensity — is counted on information of communication occurrence (one-on-one communication, in groups, round tables and other ways of communication are taken into account). On the base of the quality parameters of relation between members of a group the model define clusters in a network (connected components in the graph theory) — subgroups of individuals that have strong productive communication among them. The model has a set of overall process characteristics of the process of knowledge spreading and may be used for analysis of network states in different timestamps and as consequence of all that — for knowledge management support. Information for analysis can be retrieved for corporate social networks. The model itself and its instruments can be added as an add-on to corporate networks to sustain knowledge management in organization. The article contains an example of the model use (calculation, analysis, illustration of fuzzy graphs). It defines the model’s advantages and probable ways of improvement.

**Introduction.** In the modern world investment in human capital, on the one hand, and the development of infrastructure for knowledge exchange, on the other hand, determine possibility of process of employees innovation creativity and, as a result, a company's success in
market. Validity of this assertion, set out in the report «The Global Innovation Index 2014», has been proved theoretically and repeatedly reaffirmed with real business practices [1]. Thereby, applied research in the area of knowledge management in enterprises becomes a subject of top interest. An increased number of domestic and foreign scientific and practical essays shows the value of this trend [2].

In a broad sense, knowledge management implies the process of creation, accumulation and distribution of it among people [3]. All the components of this process are fundamentally important for the success of modern companies, but are significantly different from the viewpoint of possibility of scientific formalization, justification and implementation of particular management decisions. In this sense, a creative process of knowledge production is the most difficult part; the problem of knowledge accumulation can be easily solved considering the current level of information technologies. And, finally, the knowledge dissemination, which involves not only using of modern communication tools, but, mainly, the interaction of knowledge carriers is, in the authors' opinion, the cornerstone of the knowledge management process and is of the greatest scientific interest for them.

The basic form of distribution of «working» knowledge (mainly, implicit) at all times has been an exchange of knowledge through the direct communication between people. Today, all the geographical, time and bureaucratic communication barriers and, consequently, knowledge sharing barriers are almost faded [4]. And this positive fact is one of the reasons for the increased involvement of employees of advanced technology industries, scientific and research organizations in the global knowledge exchange process. As a consequence, the interest of business and science communities for tools of analysis and optimization of processes occurring during this exchange greatly increased.

The existence and practical significance of a number of qualitative conditions of the productive knowledge exchange is a difficulty for formalization and simultaneously a great challenge to academic community. At the stage of searching for different approaches to the description of this process the authors can not ignore the various properties of knowledge (knowledge can be explicit or implicit, knowledge is inexhaustible in its essence), exchange participants (each person has their own set of knowledge) and forms of the process (direct communication, attending conferences, participation in roundtables, etc.) [5].

Analysis of the existing scientific and methodological initiatives. The models presented in [6—8] can be distinguished as knowledge dissemination models. The selected models aim to justify and formalize the key factor in knowledge dissemination — cognitive distance between employees («remoteness» of individuals in knowledge aspect). However, these models have shortcomings, such as static character, lack of analytical justification and sufficient assessment of the qualitative aspects of the process. Simultaneously, these shortcomings determine the direction of their improvement.

Another group of models [9], used to represent knowledge exchange processes in organizations, is based on the mathematical tools for description and analysis of communication networks. In the models it simulates the space and time aspects of information exchange between the participants of social networks, which imposes certain limitations on possibility of using it for the formalization and analysis of knowledge exchange.

The endeavor to make a positive contribution to the critical remarks and to present own vision of knowledge distribution processes was the initial motive of writing this article. Moreover the existing mathematical apparatus provides opportunities to achieve the goal.

Commonality which unites the above mentioned models is that they are based on the ideology of graph theory, which is the most appropriate tool to reflect the relations between objects. In terms of this theory, objects are represented as graph with vertices \( v_i \in V \) and relations (links) between them either undirected edges \( e_j \in E \) or directed arcs of the graph \( a_j \in A \); graph in general — a tuple \( G = \langle V, E \rangle \) or \( D = \langle V, A \rangle \). Herewith, the nature and characteristics of the relations between objects in no way is taken into account: it is only important that the connection exists, and information about it is available.

In due time the development of classical graph theory was determined by networks — weighted graphs, which edges and arcs were credited with numerical parameters quantitatively characterizing the existing links: distance, time,
cost, etc. On this basis there were created and
developed networking models of process, which
are now widely used in transport planning,
organization of project work, etc.

A key and at the same time a non-numerical
nature aspects for knowledge exchange process
are the characteristics of relations between
people – the intensity of subject communication
and matching of degree of professional interests
(knowledge) in a particular area, defining the
very possibility of knowledge exchange. For a
quantitative image of these characteristics, we
suggest to use apparatus of fuzzy graphs.

For development of the classic graphs theory
positions the tuple \( G = \{V, E\} \) is called as fuzzy
undirected graph of the first kind, where \( V \) –
conventional (clear) set of vertices, \( E = \{\mu_E(v_i, v_j) / \{v_i, v_j\}\} \) – fuzzy set of edges,
where \( v_i, v_j \in V, \mu_E: E \to [0, 1] \) – membership
function, \( \mu_E(v_i, v_j) \) – membership function value
for the edge \( \{v_i, v_j\} \), which characterize in one
sense or another «quality» of connection between
vertices \( v_i \) and \( v_j \). There are no other formal
imposed restrictions on \( \mu \), so any function
defined on the set of edges and taking values
from the segment \([0, 1]\) can be interpreted as the
membership function [10].

The results of the implementation of the
announced approach to modeling the propagation
of knowledge are presented in this article.

**Parameterization of networks of knowledge
exchange.** Let us consider a process of
knowledge exchange among people united by
professional interests (enterprises and their
subdivisions). This process is defined by time and
space measures. Employees and professional
contacts between them represent a network of
knowledge dissemination.

The network is modeled by a fuzzy graph:
\( \tilde{G} = \{V, \tilde{E}\} \) – is a set of vertices that identify
employees and \( \tilde{E} \) – is a fuzzy set of edges that
describes professional contacts between
employees \( E = \{e_{ij}\} \) – is a fuzzy set carrier –
each element defines whether a professional
communication between two people takes place).

As a membership function for the fuzzy set
of the graph edges the model considers two
parameters that define the possibility and the
result of knowledge dissemination process –
density of communication between employees
and cognitive potential of knowledge exchange.

The cognitive potential of knowledge sharing
between two individuals defines to which extent
their professional knowledge coincide. It is
assumed that the exchange of knowledge between
two employees is possible when the participants
can easily speak the same «professional language»
and can communicate on the same topic.

As it is shown in [7] graph vertices are
determined by not just labels, but by a set of
knowledge elements, formally expressed as a
vector \( v_i = (((x_{ij})_m) \) where \( x \) – is a component
of a certain field of knowledge and takes value of
0 or 1 depending on whether an employee has
this knowledge or not, \( m = 1, 2, ..., M – \) subjects
of knowledge, \( l_m = 1, ..., L_m – \) fields of
knowledge, \( k_{lm} = 1, ..., K_{ml} – \) components
(elements) of knowledge.

Vector form of employee knowledge allows
evaluating of cognitive potential for knowledge
sharing between the two employees, numerically
determining the «angle» between the vectors
(disparity direction) of their knowledge.

Formally, the cognitive potential of
knowledge sharing on the certain field of
knowledge between the two network agents
(employees) \( i \) and \( j \) we offer to calculate using
the formula:

\[
\alpha_{ij}^{lm} = \frac{(v_i \cdot v_j)_m}{|v_i|_m \cdot |v_j|_m} = \frac{\sum_{k=1}^{K} x_k y_k}{\sqrt{\sum_{k=1}^{K} (x_k)^2} \cdot \sqrt{\sum_{k=1}^{K} (y_k)^2}}, \quad (1)
\]

where \( v_i = (((x_{ij})_l) \) and \( v_j = (((y_{ij})_l) \) –
knowledge vectors of employees with indexes \( i \)
and \( j \).

Here in the numerator – the amount of
paired products of the coordinates (scalar
product) and in the denominator – the product
of the lengths of these vectors.

The reasons for the proposed approach are
the following. In low-dimensional spaces, where
the vector is directed segments, a similar formula
calculates the cosine of the angle between the
vectors. At full coincidence directions vectors
cosine of the angle between them equals 1;
divergence of directions reduces it down to a
value of 0 (orthogonal vectors).
Extrapolation of this approach in a higher dimensional space allows meaningful interpretation of $\alpha_{ij}$ as an indicator of the degree of closeness of knowledge of employees. The fact that the parameter takes values in the interval $[0; 1]$ allows using it as a membership function in the fuzzy set determining the edges of the graph: $\hat{E} = \{a_g(v_i, v_j) / \{v_i, v_j\}\}$.

The second characteristic of the process of knowledge dissemination in the model is the intensity of communication which formally can be measured by the number of acts of communication for a certain time period. When describing the diffusion of knowledge it is reasonable not to limit it to only formal communication (trainings, round tables, etc.), but also include consideration of informal communication – as one of the key distribution channels of «working» tacit knowledge. As long as we are not limited by a form of knowledge transfer and talk about the impact of this communication in terms of knowledge dissemination, quantitative at first glance characteristic (how many times two employees talked) turns into qualitative one.

Given the latter, the authors propose to evaluate the relationship between the employees in terms of it to be «full-fledged» assuming that if the contact between individuals «full-fledged» the transfer of knowledge is considered to be working. Otherwise, communication between employees is considered partial.

In the model, this premise is expressed as follows:

Let $\delta_{ij} = \frac{t_{ij}}{T}$ be the amount of communication over a communication channel between employees with indexes $i$ and $j$ (number of acts of intercourse between individuals over the time period, $T$ – the length of the time period in days) and $\bar{\delta}_i = \frac{1}{|E|} \sum_{\delta_{ij} / \{v_i, v_j\} = F} \delta_{ij}$ – average number of pairs of communication between employees in the network.

Then the characteristic of communication intensity between individuals is defined as follows:

$$\beta(e_{ij}) = \begin{cases} 1, & \delta_{ij} \geq \bar{\delta}_i; \\ \delta_{ij} / \bar{\delta}_i, & \delta_{ij} < \bar{\delta}_i. \end{cases}$$

The parameter $\beta(e_{ij})$ also takes values in the interval $[0; 1]$, which allows us to interpret it as a membership function for the fuzzy defining of the edges of the graph: $\hat{E} = \{\beta_g(v_i, v_j) / \{v_i, v_j\}\}$.

Dissemination of knowledge in the parameterized network. To describe the process of knowledge dissemination in professional networks the model uses the concept of parameterized routes. A route in a graph is determined by a set of alternating edges and vertices in which any two adjacent elements incident. Routes can be interpreted as ways in which knowledge disseminates from its original owner to probable recipients.

Based on figures parametrizing knowledge networks (the cognitive potential and the communication intensity), the model identifies the characteristics of routes that have their own meaningful interpretation.

The theory of fuzzy graphs uses various characteristics of routes between nodes and each of them can be interpreted in terms of knowledge propagation process. The model presented in this paper uses three kinds of routes strength – conjunctive, disjunctive and cumulative. Conjunctive and disjunctive strength define edges with the lowest or highest cognitive potentials, respectively (in cognitive units). These characteristics can be used to analyze the strength of the range of the route, to identify strong and weak links in the path.

For a route $L(v_i, v_j)$ connecting vertices with numbers $i$ and $j$ conjunctive and disjunctive strength accordingly in the model defined as follows (examples of calculation formulas for cognitive units):

$$\eta_{\wedge}^c(L(v_i, v_j)) = \alpha \left\langle v_i, v_m \right\rangle \wedge \alpha \left\langle v_e, v_m \right\rangle, \quad (3)$$

$$\eta_{\vee}^c(L(v_i, v_j)) = \alpha \left\langle v_e, v_m \right\rangle \vee \alpha \left\langle v_e, v_m \right\rangle. \quad (4)$$

The cumulative strength of the route (in terms of cognitive and time units) characterizes the entire route as a whole, its overall «reliability» to transfer knowledge from one to another vertex. A need for such index occurs when a comparative characteristic of various knowledge flow directions from the initiator are required, as well as for construction of complex propagation characteristics in the knowledge
network. Formally, the cumulative strength of the route is defined as follows:

$$\eta^a_c(L(v_i, v_j)) = \prod_{\langle v_m, v_n \rangle \in L(v_i, v_j)} \alpha\{v_m, v_n\}. \quad (5)$$

In professional networks the cumulative maximum strength of the route can be interpreted as the most reliable direction of knowledge dissemination.

The parameter of maximum cumulative strength in the model is shown by a fuzzy reachability matrix of vertices (analog of reachability matrix in the classic graph theory), in which each element shows the most «strong» («reliable») route between the vertices.

Technically fuzzy reachability matrix is constructed in several stages as follows.

1. We construct a fuzzy adjacency matrix by the parameters of cognitive potential of knowledge for all pairs of vertices with specified level of intensity of their communication ($\lambda^b$):

$$\tilde{S}^a = (a_{ij} \mid \beta_{ij} \geq \lambda^b).$$

By their meaning, elements of these matrices correspond to paths of length 1 in the parameterized network. Routes of greater lengths represented by corresponding degrees of adjacency matrix.

2. The model uses the following rule of matrix multiplication.

Each element of the result matrix is the maximum of element-wise product of a line to a column that is different from zero, if the number of the element in the line does not match the item in the column:

$$\tilde{S}^a_1 \cdot \tilde{S}^a_1 = \tilde{S}^a_2 = C;$$

$$c_{ij} = \begin{cases} \max \{a_{ir} \cdot a_{rj} \}, & r = 1, \ldots, n, i \neq j, \\ 0, & i = j. \end{cases}$$

Fuzzy adjacency matrix raised to the powers from 2 to $n - 1$. Each of the resulting matrix contains in fact the strongest possible routes of length $k$ between two vertices: $\tilde{S}^a_2, \tilde{S}^a_3, \ldots, \tilde{S}^a_{n-1}$.

3. Final fuzzy reachability matrix by is defined element-wise comparison of matrices

$$\tilde{S}^a_1, \tilde{S}^a_2, \tilde{S}^a_3, \ldots, \tilde{S}^a_{n-1}.$$

$$d^a_{ij} = \max \{a_{ij}^k\}, \quad k = 2, \ldots, n - 1.$$

The resulting matrix contains elements that characterize the maximum strength of the path between two vertices in the graph in cognitive terms, taking into account the necessary intensity of communication between individuals.

**Clustering of parameterized knowledge dissemination network.** Characteristic feature of all communities (professional or interest) is the allocation of small subgroups in which there is the most intensive communication between their members. Companies and their subdivisions are not exception to this rule [11].

Theory of Graphs provides tools for searching connection components — sets of employees «united» with the same interests — vertices of a graph based on the relations between them. In the case of «simple» not parameterized network (as shown in [7]), the connection components will substantively mean disjoint classes with no connection in-between that is knowledge exchange is impossible. Network without any parameters allows only roughly estimate its internal division into isolated subgroups, limiting the ability of full analysis of its structure due to the fact that the factors of communication intensity and proximity of professional interests are not included.

Parameterized network based on coincidence of interests and intensity of employees’ communication, through selection of connection components is divided into clusters — cohesive groups of information exchange with an intensive productive communication within, but at the same time with the links to external vertices and groups. In contrast to classes (defined in the network with no parameters), division into clusters in the graph occurs within the parameters of individuals’ communication and enables structural analysis, evaluation of interest groups network coverage.

Interest groups of parameterized network are determined by introducing of minimum admissible strength coefficient — $\lambda^a$ ($\lambda^b$), that fixes required intensity of communication or cognitive capacity among the employees. Based on the fuzzy adjacency matrix of vertices and the minimum allowable bond strength index, it is determined in the model by a binary matrix of elements’ connection ($T^a_{mn}$), that is determinate analogue of fuzzy reachability matrix (for example, cognitive strength):

$$t^a_{ij} = \begin{cases} 1, & d^a_{ij} \geq \lambda^a; \\ 0, & d^a_{ij} < \lambda^a, \end{cases}$$
where $\lambda^* –$ the minimum acceptable level of strength of the way between the vertices in cognitive units.

The final matrix $T_{nn}$ ($t_{ij} = t^a_{ij} \cdot t^b_{ij}$) identifies substantively the «strong», «working» network links between agents considering cognitive potential and intensity of communication and it is interpreted by the authors as a connection matrix of an undirected graph vertices. According to the well-known in graph theory algorithm connection components are found on vertices connection matrix.

**Parametrized network characteristics.** Analysis and modeling of economic processes are focused on the development and justification of management decisions. The process of knowledge dissemination, covered in this article, and the suggested by the authors approach to its analysis is not an exception in this respect. In this paper we offer a set of indicators by means of which it is possible to analyze knowledge dissemination networks and some practical recommendations for improving the conductivity of new knowledge in networks are given also.

Configuration network characteristics (characteristics of elements) and processes of knowledge distribution characteristics are defined in the model.

**Characteristics of vertices in the parameterized network of knowledge dissemination.** Each vertex in the graph (individual in a professional network) can be characterized in terms of cognitive and time «distance» from their nearest neighbors:

$$b^a(v_i) = \alpha_{ij} = \frac{1}{|\Gamma(v_i)|},$$

where $\Gamma(v_i) –$ the set of vertices reachable from $v_i$ by a single step. The indicator shows the average cognitive «distance» of the individuals from their neighbors. The time analog of the parameter (average «distance» in time units) is calculated by: $b^b(v_i) = \bar{\beta}_{ij} = \frac{1}{|\Gamma(v_i)|}$.

The indicators can help to assess every employee involved in the process of knowledge sharing. Overall analysis of the individuals using time and cognitive «distance» parameters will identify active participants in the exchange of knowledge among the others and that need to be motivated to get new knowledge and spread it on in the community (they can be sent to conferences, trainings etc).

**Characteristics of knowledge dissemination process.** For each field of knowledge used in professional community (enterprise), the model suggests the following characteristics of the process of knowledge sharing that can be used to analyze the process itself and to identify its probable sore points in the terms of knowledge sharing:

1. Potential scale of knowledge dissemination – amount of employees that able to «absorb» new knowledge from a particular field:

$$w^*_{im} = \{v_i \mid (x_{kim})_{m} > 0\}. \quad (9)$$

2. Average strength of the route in terms of cognitive units:

$$\bar{w}^a_{im} = \frac{\sum_{i,j} d^a_{ij}}{n^2}. \quad (10)$$

3. Average strength of the route in terms of time units:

$$\bar{w}^b_{im} = \frac{\sum_{i,j} d^b_{ij}}{n^2}. \quad (11)$$

4. The average number of interest groups (connected components):

$$\bar{V} = \frac{|V_p|}{p}, \quad (12)$$

where $|V_p|$ – the amount of elements in the set.

**Illustration of the model.** Let us illustrate with a calculation example how the model works.

Suppose there is a group of 15 colleagues (employees of organization or department), associated with professional contacts and interests in the same area of expertise.

Fig. 1 shows a graph, illustrating the information exchange between the network members, where the edges are defined by the existence of contact between the vertices. Such network (and its model illustration) provides a first idea of how connections are established between individuals in the group.

There is the structure of the professional knowledge of each member of the group (Tab. 1) and the frequency of their communication (Tab. 2) per unit of time taken for one month. Professional knowledge of the area, which is used in a hypothetical group, divided into five components.
Fig. 1. Non-parameterized network of information exchanges in a team (vertices identify employees, edges — contacts between them on the principle of «individuals know each other personally»)

| $x_i$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $v_7$ | $v_8$ | $v_9$ | $v_{10}$ | $v_{11}$ | $v_{12}$ | $v_{13}$ | $v_{14}$ | $v_{15}$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $x_1$ | 1     | 0     | 0     | 0     | 1     | 1     | 1     | 1     | 0     | 1     | 1     | 1     | 0     | 1     | 0     |
| $x_2$ | 1     | 1     | 0     | 0     | 1     | 1     | 0     | 0     | 1     | 1     | 1     | 0     | 1     | 0     | 1     |
| $x_3$ | 0     | 1     | 0     | 1     | 1     | 1     | 0     | 0     | 1     | 1     | 1     | 0     | 1     | 1     | 1     |
| $x_4$ | 1     | 1     | 1     | 0     | 0     | 1     | 0     | 0     | 1     | 0     | 1     | 0     | 1     | 0     | 0     |
| $x_5$ | 0     | 1     | 0     | 0     | 1     | 1     | 0     | 1     | 1     | 0     | 1     | 1     | 1     | 1     | 1     |

Table 1

The structure of the employees’ knowledge

| $v_i$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $v_7$ | $v_8$ | $v_9$ | $v_{10}$ | $v_{11}$ | $v_{12}$ | $v_{13}$ | $v_{14}$ | $v_{15}$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $v_1$ | 1     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 2      | 0      | 0      | 0      | 0      | 0      |
| $v_2$ | 1     | 0     | 0     | 6     | 4     | 0     | 0     | 0     | 3     | 0      | 0      | 0      | 0      | 0      | 0      |
| $v_3$ | 0     | 0     | 1     | 0     | 0     | 11    | 0     | 0     | 0     | 0      | 0      | 0      | 0      | 0      | 0      |
| $v_4$ | 0     | 0     | 1     | 0     | 0     | 0     | 0     | 0     | 0     | 0      | 0      | 0      | 0      | 1      | 0      |
| $v_5$ | 0     | 0     | 1     | 0     | 0     | 1     | 0     | 0     | 15    | 0      | 0      | 0      | 0      | 0      | 0      |
| $v_6$ | 0     | 0     | 1     | 0     | 0     | 0     | 0     | 0     | 0     | 0      | 0      | 0      | 0      | 0      | 0      |
| $v_7$ | 0     | 0     | 1     | 0     | 0     | 1     | 0     | 0     | 3     | 0      | 18     | 0      | 0      | 0      | 0      |
| $v_8$ | 0     | 0     | 1     | 0     | 0     | 0     | 0     | 0     | 4     | 0      | 0      | 0      | 0      | 0      | 0      |
| $v_9$ | 0     | 0     | 1     | 0     | 0     | 0     | 0     | 0     | 0     | 0      | 0      | 0      | 0      | 0      | 0      |
| $v_{10}$ | 2    | 3     | 0     | 0     | 15    | 0     | 9     | 0     | 0     | 0      | 0      | 0      | 0      | 0      | 0      |
| $v_{11}$ | 0    | 0     | 0     | 0     | 0     | 0     | 1     | 0     | 0     | 0      | 0      | 0      | 0      | 1      | 0      |
| $v_{12}$ | 0    | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0      | 0      | 0      | 0      | 0      | 24     |
| $v_{13}$ | 0    | 0     | 0     | 1     | 0     | 0     | 0     | 0     | 0     | 0      | 0      | 24     | 0      | 0      | 0      |
| $v_{14}$ | 0    | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0      | 0      | 1      | 0      | 0      | 0      |
| $v_{15}$ | 0    | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0      | 0      | 0      | 0      | 11     | 0      | 0      |

Table 2

Amount of acts of communication between employees ($t_{ij}$)
By the formulas (1) and (2) the network parameters are defined – the indicator of communication intensity and the cognitive potential of knowledge exchange between individuals. Tab. 3 shows the indicators for each pair of employees, and Fig. 2 illustrates a parameterized knowledge exchange network.

The graph shown in Fig. 2 is an illustration of the derived model calculations. Each edge of the graph is painted in two colors – for the indicators of communication intensity and cognitive potential, respectively. For visualization of indicators we took gradation of colors, where the maximum intensity of the color corresponds to the highest value of indicator (each edge is marked with α and β symbols to designate correspondence between a component of the edge and its’ index).

**Table 3**

|   | vi | α  | β  | v1 | v2 | v3 | v4 | v5 | v6 | v7 | v8 | v9 | v10 | v11 | v12 | v13 | v14 | v15 |
|---|----|----|----|----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|-----|-----|
| v1| α  | 0.6| 0.2| 0.6| 0.6| 0.5| 0.5| 0.6| 0.8| 0.9| 0.9| 0.9| 0.9  | 0.9  | 0.9  | 0.8  | 0.7  | 0.7  |
| v2| β  | 0.2| 0.8| 0.7| 0.8| 0.8| 0.9| 0.9| 0.9| 0.9| 0.9| 0.8| 0.7  | 0.7  | 0.7  | 0.7  | 0.8  | 0.8  |
| v3| α  | 0.6| 0.5| 0.5| 0.5| 0.4| 0.4| 0.4| 0.4| 0.4| 0.4| 0.4| 0.4  | 0.4  | 0.4  | 0.4  | 0.4  | 0.4  |
| v4| β  | 0.0| 0.2| 0.2| 0.2| 0.2| 0.2| 0.2| 0.2| 0.2| 0.2| 0.2| 0.2  | 0.2  | 0.2  | 0.2  | 0.2  | 0.2  |
| v5| α  | 0.6| 0.8| 0.8| 0.8| 0.8| 0.8| 0.8| 0.8| 0.8| 0.8| 0.8| 0.8  | 0.8  | 0.8  | 0.8  | 0.8  | 0.8  |
| v6| β  | 0.9| 0.9| 0.9| 0.9| 0.9| 0.9| 0.9| 0.9| 0.9| 0.9| 0.9| 0.9  | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  |
| v7| α  | 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6  | 0.6  | 0.6  | 0.6  | 0.6  | 0.6  |
| v8| β  | 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  |
| v9| α  | 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6  | 0.6  | 0.6  | 0.6  | 0.6  | 0.6  |
| v10| β  | 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  |
| v11| α  | 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6  | 0.6  | 0.6  | 0.6  | 0.6  | 0.6  |
| v12| β  | 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  |
| v13| α  | 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6| 0.6  | 0.6  | 0.6  | 0.6  | 0.6  | 0.6  |
| v14| β  | 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  |
| v15| α  | 0.3| 0.3| 0.3| 0.3| 0.3| 0.3| 0.3| 0.3| 0.3| 0.3| 0.3| 0.3  | 0.3  | 0.3  | 0.3  | 0.3  | 0.3  | 0.3  |
| v16| β  | 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0| 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  |
Using the parameters of knowledge exchange in networks we can define routes and their characteristics that provide a value of each route and can be used to define conductivity properties of the network.

The presented approach to illustration of a parameterized network enables the primary analysis of the links in the network. Thus, for example, the edge between the vertices with the numbers 3 and 4 shows a very weak link for transfer of knowledge between the employees, in both cognitive and time units. In fact, this means that individuals do not have common interests and, moreover, communicate extremely seldom. In the model calculations parameters of the edge «3 – 4» are interpreted as conjunctive strength of routes passing through this edge. At the same time, exactly through this element knowledge exchange between a group of vertices with the numbers 12, 13, 15 and the rest of the team is possible that makes this connection essentially important in terms of the whole network conduction. The most appropriate management decisions in this situation will be a redefinition of relations between these two groups of employees (on one and on the other «side» of the «weak» in terms of knowledge exchange element).

Parameters of the edge lying between the vertices with the numbers 2 and 5 are defined as indicators of disjunctive strength for routes passing through this element. Substantially, this connection means a reliable knowledge dissemination channel, both in terms of time for distribution, and from the point of view of professional interests matching: individuals communicate frequently and use «the same language».

For each pair of connected vertices we calculated the cumulative strength of the routes, which are the elements of the fuzzy reachability matrix in cognitive and time units (\(\tilde{D}_{\text{n.c.}}^\alpha\) and \(\tilde{D}_{\text{n.c.}}^\beta\)). These indicators allow a comprehensive analysis of the conductivity of knowledge within the network based on two parameters that determine the quality of the dissemination of knowledge. Tab. 4 contains the results of calculations of cumulative strengths routes in the network in question.
The cumulative strength of routes between the vertexes in time and cognitive units

| $v_i$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $v_7$ | $v_8$ | $v_9$ | $v_{10}$ | $v_{11}$ | $v_{12}$ | $v_{13}$ | $v_{14}$ | $v_{15}$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------|----------|----------|----------|----------|
| $v_1$ | $\alpha$ | 0.58  | 0   | 0   | 0.58  | 0.52  | 0.38  | 0   | 0.52  | 0.67  | 0.17    | 0        | 0        | 0.11      | 0        |
|       | $\beta$ | 0.29  | 0.31 | 0.05 | 0.31  | 0.18  | 0.31  | 0.19 | 0.18  | 0.31  | 0.05    | 0.01     | 0.01     | 0.01      | 0.01     |
| $v_2$ | $\alpha$ | 0.58  | 0   | 0   | 0.75  | 0.89  | 0.4   | 0   | 0.89  | 0.65  | 0.18    | 0        | 0        | 0.11      | 0        |
|       | $\beta$ | 0.29  | 0.93 | 0.14 | 0.93  | 0.62  | 0.93  | 0.58 | 0.62  | 0.93  | 0.14    | 0.02     | 0.02     | 0.02      | 0.02     |
| $v_3$ | $\alpha$ | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0        | 0        | 0        | 0         | 0        |
|       | $\beta$ | 0.31  | 0.93 | 0.16 | 0.16  | 0.58  | 1   | 0.62 | 0.58  | 1   | 0.16    | 0.02     | 0.02     | 0.02      | 0.02     |
| $v_4$ | $\alpha$ | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0        | 0.2      | 0.45     | 0.12      |          |
|       | $\beta$ | 0.05  | 0.14 | 0.16 | 0.16  | 0.09  | 0.16 | 0.1  | 0.09  | 0.16  | 0.16    | 0.02     | 0.02     | 0.02      | 0.02     |
| $v_5$ | $\alpha$ | 0.58  | 0.75 | 0   | 0   | 0.67  | 0.5  | 0   | 0.67  | 0.87  | 0.22    | 0        | 0        | 0.14      |          |
|       | $\beta$ | 0.31  | 0.93 | 1   | 0.16 | 0.58  | 1   | 0.62 | 0.58  | 1   | 0.16    | 0.02     | 0.02     | 0.2       | 0.02     |
| $v_6$ | $\alpha$ | 0.52  | 0.89 | 0   | 0   | 0.67  | 0.45 | 0   | 1     | 0.58  | 0.2     | 0        | 0        | 0.13      |          |
|       | $\beta$ | 0.18  | 0.62 | 0.58 | 0.09 | 0.58  | 0.58 | 0.36 | 1     | 0.58  | 0.09    | 0.01     | 0.01     | 0.01      | 0.01     |
| $v_7$ | $\alpha$ | 0.38  | 0.4  | 0   | 0   | 0.5   | 0.45 | 0   | 0.45  | 0.58  | 0.45    | 0        | 0        | 0.28      | 0        |
|       | $\beta$ | 0.31  | 0.93 | 1   | 0.16 | 1     | 0.58 | 0.62 | 0.58  | 1     | 0.16    | 0.02     | 0.02     | 0.02      | 0.02     |
| $v_8$ | $\alpha$ | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0        | 0        | 0        | 0         | 0        |
|       | $\beta$ | 0.19  | 0.58 | 0.62 | 0.1  | 0.62 | 0.36 | 0.62 | 0.36  | 0.62  | 0.1     | 0.01     | 0.01     | 0.01      | 0.01     |
| $v_9$ | $\alpha$ | 0.52  | 0.89 | 0   | 0   | 0.67  | 1     | 0.45 | 0   | 0.58  | 0.2     | 0        | 0        | 0.13      | 0        |
|       | $\beta$ | 0.18  | 0.62 | 0.58 | 0.09 | 0.58  | 1     | 0.58 | 0.36  | 0.58  | 0.09    | 0.01     | 0.01     | 0.01      | 0.01     |
| $v_{10}$ | $\alpha$ | 0.67  | 0.65 | 0   | 0   | 0.87  | 0.58 | 0.58 | 0   | 0.58  | 0.58    | 0.26     | 0        | 0.16      | 0        |
|       | $\beta$ | 0.31  | 0.93 | 1   | 0.16 | 1     | 0.58 | 1   | 0.62 | 0.58  | 0.16    | 0.02     | 0.02     | 0.02      | 0.02     |
| $v_{11}$ | $\alpha$ | 0.17  | 0.18 | 0   | 0   | 0.22  | 0.2   | 0.45 | 0   | 0.2   | 0.26    | 0        | 0        | 0.63      | 0        |
|       | $\beta$ | 0.05  | 0.14 | 0.16 | 0.02 | 0.16 | 0.09 | 0.16 | 0.1  | 0.09  | 0.16    | 0.01     | 0.01     | 0.16      | 0        |
| $v_{12}$ | $\alpha$ | 0   | 0   | 0   | 0.2  | 0   | 0   | 0   | 0   | 0   | 0.45    | 0        | 0.58      | 0         | 0        |
|       | $\beta$ | 0.01  | 0.02 | 0.02 | 0.16 | 0.02 | 0.01 | 0.02 | 0.01  | 0.01  | 0.02    | 0        | 0         | 0         | 0        |
| $v_{13}$ | $\alpha$ | 0   | 0   | 0   | 0.45 | 0   | 0   | 0   | 0   | 0   | 0       | 0.45      | 0.26      | 0         | 0        |
|       | $\beta$ | 0.01  | 0.02 | 0.02 | 0.16 | 0.02 | 0.01 | 0.02 | 0.01  | 0.01  | 0.02    | 0.004     | 0         | 0         | 0        |
| $v_{14}$ | $\alpha$ | 0.11  | 0.11 | 0   | 0   | 0.14  | 0.13 | 0.28 | 0   | 0.13  | 0.16    | 0.63      | 0        | 0         | 0        |
|       | $\beta$ | 0.01  | 0.02 | 0.02 | 0.004| 0.02 | 0.01 | 0.02 | 0.01  | 0.01  | 0.02    | 0.16      | 0.0006    | 0         | 0        |
| $v_{15}$ | $\alpha$ | 0   | 0   | 0   | 0.12 | 0   | 0   | 0   | 0   | 0   | 0       | 0.56      | 0.26      | 0         | 0        |
|       | $\beta$ | 0.01  | 0.02 | 0.02 | 0.16 | 0.02 | 0.01 | 0.02 | 0.01  | 0.01  | 0.02    | 0        | 0.0006    | 0         | 0        |
At this phase of the knowledge conductivity analysis we can make a preliminary conclusion about the strength of the routes of the entire network. As seen from the table above, only a few routes have strengths exceeding the threshold $\alpha = 0.5$ and $\beta = 0.5$ (highlighted in color). There are only ten such routes of forty-two possible. Even now we can conclude uneven coverage of the network by «working» («strong») contacts, which makes the process of knowledge transfer chaotic and unorganized in the network in its current configuration.

On the basis of the cumulative strengths the model determines connected components that stand for groups of employees, in which there is the most intensive exchange of knowledge and competencies. In the example we settled the threshold levels of knowledge sharing to following values: $\alpha = 0.7$ and $\beta = 0.9$. The following connected components appeared as a result of calculation: $(v_2, v_5, v_{10})$ and $(v_6, v_{9})$. Fig. 3 illustrates the results obtained.

The final stage of the analysis involves consideration of the general knowledge conductivity characteristics of the network. The developed model indicators help to assess the current state of the network and make a comparative analysis after application of reorganization measures, directly or indirectly affecting the throughput performance of the network.

For a comparative analysis different states of the network, in the example of we have simulated management actions to reorganize the connections in the network in order to improve the conductivity of knowledge between employees. Based on the analysis of cognitive potential parameters of knowledge sharing between employees in the network we have partially established new contacts between those pairs of individuals for which the rate of cognitive potential of knowledge sharing exceeds 0.9. In fact, new edges were added to the initial graph (by indexes of the vertexes): $1 - 9, 1 - 6, 2 - 11, 2 - 13, 2 - 15, 5 - 6, 6 - 10, 6 - 11, 9 - 10, 9 - 11, 11 - 13, 11 - 15, 13 - 15, 14 - 15$. For each of the added links indicator of the intensity of communication was established at 0.9 (in practice it may be achieved by introducing regular round tables for the staff). Fig. 4 illustrates a new connection in the network (the values for the parameters are set only for «new» edges).

Tab. 5 contains the results of calculations of the knowledge conductivity main characteristics for both cases — the initial state and after the implementation of new connections between the employees.

As the table shows, the average strength indicators have improved markedly after the new routes determination between some pairs of employees. This gives grounds to conclude that the updated network communication channels between employees more reliable and stable. This in its turn makes it possible to assume that in the new network, there is an intense circulation of knowledge with greater reliability and, as a result, efficiency. An additional argument in favor of this
Table 5
Parameters of knowledge conductivity in the network for the initial state and after the reorganization of the network

| Parameter                              | Initial network | After the reorganization |
|----------------------------------------|-----------------|--------------------------|
| Average strength of the route          | 0.23            | 0.5                      |
| in terms of cognitive units            |                 |                          |
| Average strength of the route          | 0.32            | 0.75                     |
| in terms of time units                 |                 |                          |
| The amount of interest groups          | 2               | 1                        |
| (connected components)                 |                 |                          |
| The average number of interest groups  | 2.5             | 10                       |

conclusion is the indicators of the amount of interest groups and their average number. As soon as the model example considers one area of knowledge common to the entire group of 15 people, it is logically reasonable to assume that if there is a particularly cohesive internal team of ten people, in which there is active and productive communication, knowledge will be intensively and effectively spread between the employees. In the reverse situation, when interest groups numerically small (2, 3 persons) and relatively few in the whole team, as shown in the example before the reorganization of relations, knowledge spreading will take a relatively long time and the knowledge itself will remain virtually inaccessible to other members of the team.

The provided example helps to evaluate the application and use of the model and to identify possible directions for its further development. Thus, for any enterprise in which the main production resource is knowledge, the model provides an opportunity to assess how the organization has established communication between its employees and how these relationships contribute to the spread of knowledge. The model uses qualitative characteristics of the diffusion of knowledge, which is an adequate reflection of the properties of such a complex and difficult process, as the dissemination of knowledge.
Conclusion. Scientific discoveries and innovations usually accumulate practices of several generations of scientists and researchers. Knowledge in this or the other area, reaching a certain critical mass, embodies in a new knowledge and moves to a new form. The ability to implement existing and newly created knowledge into economic products and operate knowledge as a product of modern economy is considered to be the key to success for organizations of different structures and sizes—from start-ups to international corporations and enterprises.

Nowadays, these facts are recognized at all levels of administrative management, in both the public and private sector.

As a result, analysis and development of applied tools for assessing and modeling processes of knowledge dissemination is an urgent scientific problem. The concept of numbers at all times has been the basis of decision-making.

Within the framework of this article the authors present a model, developed for the analysis and evaluation of the knowledge dissemination process. The model can be used to support management decisions in important emerging stream of Management — Knowledge Management. Corporate social networks, which contain information about the knowledge and competencies of the employees and support electronic forms of communication, can serve as an information base for using the model in real companies. The authors see the following necessary directions for further research of this model:

- the introduction of fuzzy characteristics for describing the structure of the employees knowledge — this will bring a possibility to construct more precise assessment of staff knowledge and cognitive capacity among them;
- introduction of a time-dependent factor of knowledge assimilation by the individuals — this will allow to define the time characteristics of knowledge dissemination process, to consider dynamics of knowledge accumulation.

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