A quantum point contact as a (near) perfect spin polariser

Samuel Bladwell

School of Physics, University of New South Wales, Sydney 2052, Australia

In this paper, I present a simple method of obtaining spin-polarised current from a QPC with a large Rashba interaction. The origin of this spin polarisation is the adiabatic evolution of spin “up” of the first QPC sub-band, into spin “down” of the second QPC sub-band. Unique experimental signatures of this effect can be obtained using a magnetic focusing setup, with the characteristic “double” peak of spin-split magnetic focusing only present for conductances \( G > 2e^2/h \). These particular magnetic focusing features are present in recently published experimental results. Finally, I consider hole QPCs, where due to the particular kinematic structure of the Rashba interaction, the required parameters are much less favourable for experimental realisation.

PACS numbers:

The prototypical spintronic device is the spin field effect transistor (spin-FET), first proposed by Datta and Das in 1990. The device consists of a spin polarised injector, a gate controllable spin-orbit interaction, and a spin sensitive detector. There exist many variations on the spin-FET, but all maintain these same essential components. In Datta and Das original spin-FET proposal, ferromagnetic leads were suggested to achieve spin-polarised injection. More recently, experimental and theoretical effort has been devoted to an “all electric” spin polariser. To this end, elaborate gating potentials and many-body effects in quantum point contacts (QPCs) have been studied extensively, with the goal of opening many-body effects in quantum point contacts (QPCs) 

This paper examines an alternative single particle mechanism for generating spin polarised current from QPCs, without magnetic fields. The spin-polarisation emerges due to an anti-crossing between spin “up” and “down” states of sub-bands of differing parity, as shown in the left panel of Fig. 1. If the passage through the level crossing is perfectly adiabatic, all “up” states are converted to “down” states, and 100% spin polarised current is obtained. This effect was first proposed and studied numerically by Eto et al with a QPC formed in a quantum wire. By varying the length of the QPC constriction and the strength of the spin-orbit interaction spin-polarisation of up to 50% was found. Similar results have been obtained elsewhere considering analogous geometries. In Sec. I present a simple analytical model for this effect. In QPCs formed between two reservoirs, the shape of the QPC potential, combined with a large, but experimentally achievable, spin-orbit interaction yields (near) perfect polarisation.

The principal difficulty of experimental verification of this mechanism of spin polarisation is the absence of any features in conductance. One alternative to conductance measurements is to utilise transverse magnetic focusing, where strong spin-splitting in momentum results in a real space splitting and a “doubling” of the first focusing peak. The relative height of the spin-split peaks can be directly associated with polarisation of the constituent QPCs. I show, in Sec. II that when paired transverse magnetic focusing, this particular form of spin polarisation yields a clear experimental signature. This can be distinguished from other sources of polarisation by via the QPC conductance dependence. These unique experimental signatures are present in recently published results for TMF in InGaAs two dimensional electron systems.

Finally, in Sec. III I discuss the case of hole systems. While the spin-orbit interaction in heavy hole systems can exceed 30% of the Fermi energy, the dominant kinematic structure is cubic in momentum. This leads to a more complicated form for the spin-orbit interaction in the QPC channel, and spin-orbit parameter requirements that are less favourable.
is there no change to the transmission coefficient, This spin-orbit interaction modifies the one dimensional α*QPC is with the width

A quantum point contact consists of a narrow quasi one-dimensional constriction between two reservoirs. The conductance of a QPC occurs in steps of $2e^2/h$ and is defined by the adiabatic motion of the one dimensional states through the QPC constriction, a result of the smooth variation of the width of the channel. For simplicity, I consider an infinite square well as the confining potential,

$$\mathcal{H} = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + V(y)$$

$$V(y) = \begin{cases} 0 & \text{if } 0 < y < W(x) \\ \infty & \text{if otherwise} \end{cases}$$

with the width $W$ varying adiabatically along the channel; see Fig. 2. The minimum width of the channel of the QPC is $W_0 \approx \lambda_F/2$ where $\lambda_F$ is the Fermi wavelength in the reservoirs. In addition to the confinement potential, there is a spin-orbit interaction, with interaction strength $\alpha$,

$$\mathcal{H}_{so} = \alpha (\sigma_x p_y - \sigma_y p_x)$$

This spin-orbit interaction modifies the one dimensional states, splitting the spin-states in momentum. However, there is no change to the transmission coefficient, $T$, and hence no change to the conductance curves. The dispersion of the one-dimensional states, taking $\langle p_y \rangle = 0$, is

$$\varepsilon_n = \frac{\hbar^2 k_x^2}{2m} + \frac{\hbar^2 n^2 \pi^2}{2mW^2} \pm \alpha \hbar k_x$$

which is plotted in Fig. 1. At some characteristic $\alpha$ and $W$ there will be an anti-crossing between the spin `up' state of one band, $|1, \uparrow\rangle$ and the `down' state of the band above, $|2, \downarrow\rangle$. In principle, such band crossings occur for arbitrary small $\alpha$, and $\pm k$, however, for current injection, only the forward moving states are relevant, the blue components of the dispersion curves in Fig. 1. In a real device, the QPC states project onto the bulk states at a finite width, $W_{max}$, and the spin-orbit interaction must be sufficiently large that the width of the crossing is smaller than this. Experimental studies place this as being similar to the lithographic width of the device, $W_{max} \sim 200nm$, however, the exact point of decoupling is unknown.

From Eq. (3), this crossing point occurs at

$$k\text{cross} = \frac{3\hbar\pi^2}{4mW^2\alpha}$$

When $k_x > k\text{cross}$, the bands are inverted, and $|1, \uparrow\rangle$ is above $|2, \downarrow\rangle$. The states are well separated in energy far away from the crossing point, and in the crossing region, the remaining spin states of the 1st and 2nd sub-band of the QPC are distant. As a result, the crossing region can be described by an effective Hamiltonian acting on a basis of $|1, \uparrow\rangle$ and $|2, \downarrow\rangle$,

$$\mathcal{H} = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \alpha \sigma_z p_x - \alpha \sigma_x p_y$$

In this crossing region, the linear in $p_y$ term in the Rashba Hamiltonian becomes important, and leads to an avoided crossing between $|1, \uparrow\rangle$ and $|2, \downarrow\rangle$, as can be seen in Fig 1. The eigenvalues of the effective Hamiltonian, Eq. (5) are

$$\varepsilon_\pm = \frac{\hbar^2 k_x^2}{2m} + \frac{5 \pi^2 \hbar^2}{4 mW^2} \pm \Delta$$

$$\Delta = \sqrt{\left( \frac{3}{2} \frac{\pi^2 \hbar^2}{2mW^2} - \alpha \hbar k_x \right)^2 + \left( \frac{8 \hbar \alpha}{3W} \right)^2}$$

The problem of transitions between $|1, \uparrow\rangle$ and $|2, \downarrow\rangle$ is reduced to a standard Landau-Zener problem, with the transition probability,

$$P = e^{-2\pi \Gamma}$$

$$\Gamma = \frac{\Delta^2}{2L \frac{dW}{dx}}$$

where $P$ is the probability of non-adiabatic transitions. A critical point to note is the exponential dependence on $\Gamma$ for non-adiabatic transitions, with a pre-factor of $2e^2/h$. Finally, the factor $\Gamma$ can be determined at the level crossing itself to be

$$\Gamma = \frac{4 m\alpha}{3 \hbar k_x} \frac{dx}{dW}$$

In general $k_x < k_F$, where $k_F = \sqrt{2m\varepsilon_F}$ is the Fermi momentum in the reservoirs,

$$k_x \approx k_F \left( 1 - \frac{5 m\alpha}{3 \hbar k_F} \right)$$

$\Gamma$ depends on the shape of the QPC via $dW/dx$. While $dW/dx$ is unknown at the crossing point, estimates can be made of its magnitude. If the width varies exponentially,

$$dW/dx \approx \frac{W_{crossing}}{L} \ln \left( \frac{W_{max}}{W_0} \right)$$
where \( L \) is the length of the QPC. Collimation studies suggest the maximum width is comparable or slightly smaller than the lithographic width of the gates\(^\text{14}\). The length depends on the lithography of the sample, and for a square QPC it is comparable to the exit width. The crossing width, \( \text{W}_{\text{crossing}} \) is

\[
\text{W}_{\text{crossing}} \approx \lambda F \sqrt{\frac{k_F}{k_a}} \frac{\sqrt{3}}{4}
\]

for \( m_\alpha/hk_F \sim 0.1, \lambda_F < \text{W}_{\text{crossing}} < 3\lambda_F/2 \), while the minimum width is \( W_0 \geq \lambda_F/2 \). Taken together, this implies that \( dW/dx < 1 \) in the crossing region. Due to the pre-factor of 2\( \pi \) in the Landau-Zener transition probability, \( \Gamma \sim 0.5 \) is sufficient to yield near perfect (~90\%) spin polarisation.

II. EXPERIMENTAL SIGNATURES

It is important to note that the mechanism of spin-polarisation leaves no features in the conductance. This makes detection of spin polarisation from this mechanism a difficult exercise. One option is to connect the QPC to ferromagnetic leads; a QPC exhibiting spin-polarisation will show conductance signatures\(^\text{6,8,18}\).

Unique experimental signatures can be obtained using transverse magnetic focusing (TMF). A TMF experiment consists of an injector, and a detector, located in the plane with a weak magnetic field applied transverse to the plane of the 2DEG to ‘focus’ the electrons from the source to the detector\(^\text{16}\). When the two dimensional charge gas has a significant spin-splitting, the focusing peaks become spin split, with spin states separated in real space, with the splitting\(\text{17}\)

\[
\frac{\delta B}{B} = \frac{2\kappa}{k_F}
\]

Spin polarisations can be easily detected as variations in the height of the constituent peaks\(\text{19,20}\) while complete polarisation will result in the absence of one of the spin-split peaks\(\text{20}\).

What makes the TMF signature of adiabatic spin polarisation unique when compared to polarisation of the QPC due to electron-electron interactions or magnetic fields is the continued presence of spin-polarisation when the injector is tuned to \( G = 2e^2/h \). For conductances \( G > 2e^2/h \), the second TMF peak will gradually appear, with the peaks only being equal for \( G = 4e^2/h \), when the lower two bands of the QPC are fully occupied. If adiabatic spin-polarisation occurs for \( n > 1 \), the imbalance in the spin-states will persist for arbitrary conductance, with finite polarisation for arbitrary conductances. This is unlikely for a QPC with a horn-like shape, since \( dW/dx \) increases for larger \( W \), and the crossing occur at larger values of \( W \) for higher bands.

The adiabatic evolution of the spin states in the injector QPC means that the exit wave-functions contain both the first and second mode,

\[
|1, \downarrow\rangle \propto \sin \left( \frac{\pi y}{W} \right) \chi_\downarrow
\]

\[
|2, \downarrow\rangle \propto \sin \left( \frac{2\pi y}{W} \right) \chi_\downarrow
\]

where \( \chi_\downarrow \) is the spin state. While the parity of the states changes, the TMF spectrum is insensitive and the shape of the peaks varies only minimally. Using the method outlined in Ref.\(^\text{19}\) I have calculated the TMF spectrum, with the injector exhibiting 100\% spin polarisation, and the detector tuned to \( 4e^2/h \), thereby allowing both spin species from either of the sub-bands through. For this calculation, I use \( m_\alpha/hk_F = 0.125 \), with magnetic focusing length of \( l = 1500\mu m \), and QPC exit width of 200nm. These are approximately inline with the parameters of Ref.\(^\text{19}\). The plots of the resulting spectrum are presented in Fig.\(\text{3}\). If the injector is tuned to the second plateau, all sub-bands are occupied, and the double peak structure is restored. The TMF spectrum is insensitive to the particular mode structure of the constituent QPCs; while the angular distribution of the QPCs differs depending on the particular mode\(\text{20}\), the interference spectrum is dominated by trajectories close to the phase minimum\(\text{19}\).

Additional experimental evidence for this mechanism of spin polarisation could be obtained via this angular dependence, which is known to differ significant depending on the particular sub-band of the injecting QPC\(\text{21,22}\).

Recent experimental results from Chuang\ et al, employing transverse magnetic focusing in a two dimensional electron gas formed in InGaAs\(\text{23,24}\) present these characteristic features. The results of Chuang\ et al are reproduced in Fig.\(\text{4}\). From the splitting of the focusing peaks, \( m_\alpha/hk_F \approx 0.13 \). For these results, the side gates of the QPC were biased, resulting in an asymmetrical confinement, and an additional spin-orbit interaction \( \propto \sigma_\alpha p_x \). The effect of this additional spin-orbit
FIG. 4: The focusing setup with focusing length $l$. A large spin-orbit interaction leads to a splitting between the spin-states, with the first focusing peak doubled. Conductances $G \leq G_0$ display only a single peak, a clear signature of adiabatic spin polarisation. The “doubled” peak structure is only fully restored for $G \approx 4\hbar e^2/h$.

The focus length is $l \sim 1\mu m$. The focusing setup is shown in Fig. 5, with the two trajectories of injection angle $\theta_x$.

FIG. 5: Left Panel: Cartoon of the $n = 1$ and $n = 2$ sub-bands for heavy holes subject to a Rashba interaction in a QPC. Blue and red traces indicate the different spin states. The spin-states are inverted in the $n = 2$ sub-band compared to $n = 1$. Dashed black line indicates the location of the anti-crossing.

Right Panel: The strength of the Rashba splitting, Eq. (15), with varying width at a fixed energy is plotted, with the first sub-band in red, and the second sub-band in blue. The vertical dashed line indicates where the second sub-band ($n = 2$) is below the Fermi level. Criticality, at all values where the second sub-band is below the Fermi level, the sign of the interaction opposite the first sub-band. For comparison purposes, I have plotted the electron Rashba strength, from Eq. (23), for $n = 1$ ($n = 2$) in dashed red (blue).

III. HEAVY HOLE QPC SPIN POLARISATION

Two dimensional heavy hole gases can have very large spin-orbit interactions; the Rashba spin-orbit interaction can exceed $0.3\gamma\hbar$. Due to heavy holes having $J_z = \pm 3/2$, the dominant kinematic structure for the Rashba interaction is cubic in momentum,\[ \mathcal{H}_{RH} = \frac{i\gamma}{2} (p_x^a \sigma_- - p_y^a \sigma_+) \]

where $\sigma_\pm = \sigma_x \pm i\sigma_y$, and $p_\pm = p_x \pm ip_y$. Here $\sigma_i$ are the Pauli matrices acting on the pseudo-spin doublet $J_z = \pm 3/2$. With the confining potential of Eq. (2), the kinematic structure of the Rashba interaction is\[ \mathcal{H}_{RH} = \gamma (3 \langle p_y^a \rangle k_x - k_y^a) \sigma_y = B \cdot \sigma \]

I have introduced the effective magnetic field, $B$, which is convenient when discussing the orientation of the spin-states. The effective magnetic field, $B$, changes sign at $k_x = \sqrt{3 \langle p_y^a \rangle}$, shown in Fig. 5 by the vertically dashed line. The change in sign in $B$ occurs at different $k_x$ depending on the sub-band, and for the $n = 1$ sub-band, this coincides with the $n = 2$ sub-band first being below the Fermi energy. As a result, the effective magnetic field has opposite sign in $n = 1$ compared to $n = 2$ for a substantial range of widths, $W$.

Relative to the $n = 1$ sub-band, the orientation of the spin-states is “inverted” in the $n = 2$ sub-band. That is, $\lvert 1, \uparrow \rangle$ has higher energy than $\lvert 1, \downarrow \rangle$, while $\lvert 2, \uparrow \rangle$ has lower energy than $\lvert 2, \downarrow \rangle$. While the spin-states in the upper band are inverted, the anti-crossing is between $\lvert 1, \uparrow \rangle$ and $\lvert 2, \downarrow \rangle$, as shown in the left panel of Fig. 5. The inversion of the spin-states, and the sign change in the effective magnetic field also results in a flipping of the spin filtered states in TMF in heavy hole gases.

Anti-crossings between opposite spin states which lead to polarisation only occur for $k_x > 2\pi\sqrt{3}/W$, or $W > \sqrt{3}\lambda_F$, when the effective magnetic field in the second sub-band changes sign. According to Eq. (10), the increase in the crossing width results in a larger $dW/dx$. Since $k_x \sim 1/W$, $k_x$ increases at large widths. Together these increase the Landau-Zener velocity, and therefore suppresses $\Gamma$ in Eq. (5). As a result, spin-polarisation due to the evolution of $\lvert 1, \uparrow \rangle$ into $\lvert 2, \downarrow \rangle$ is considerably suppressed.
weaker in heavy hole gases subject to a Rashba spin-orbit interaction, compared to electron systems. To compensate for the unfavourable quasi-one dimensional spin-orbit interaction in Rashba heavy holes systems smoother QPC geometries could be used.

Other kinematic structures are more favourable. The heavy hole Dresselhaus interaction does not exhibit the same sub-band dependent sign change as Rashba interaction\(^ {22}\)

\[
\mathcal{H}_{D,H} = \beta \left( k_x^2 + p_y^2 \right) k_x \sigma_y
\]  

(16)

where \(p_y\) is the transverse momentum. The principle limitation in the use of the Dresselhaus interaction is the relatively small magnitude; in heterojunctions, \(\beta k_x^2 < 0.1 \varepsilon_F^{11}\). Typically Rashba is dominant in heavy hole systems with surface inversion asymmetry, and either highly symmetric quantum wells, or other compensating spin-orbit interactions would be required.

IV. SUMMARY AND ACKNOWLEDGEMENTS

In summary, a QPC with a strong spin-orbit interaction can function as a near perfect spin-polarised injector. The shape of a typical QPC, combined with a large spin-orbit interaction is sufficient for this to occur. Recent transverse magnetic focusing experiments have spectrums with clear evidence of this particular mechanism of QPC polarisation. While hole systems can have comparable, or even larger spin-orbit interactions, the kinematic structure of the spin-orbit interaction results in a sign change of the spin-orbit interaction for the quasi one dimensional states in the QPC. As a result, the parameters required for this mechanism of spin-polarisation in holes are not as experimentally feasible as electrons.

This research was partially supported by the Australian Research Council Centre of Excellence in Future Low-Energy Electronics Technologies (project number CE170100039) and funded by the Australian Government. SSRB would like to thank Alex Hamilton, Scott Lilles, and Matt Rendall for valuable comments, and Oleg Sushkov for a critical reading and many suggestions for this manuscript.

1 S. Datta and B. Das, Appl. Phys. Lett. 56, 665 (1990), ISSN 00036951, URL http://scitation.aip.org/content/aip/journal/apl/56/7/10.1063/1.102730.
2 P. Debray, S. Rahman, J. Wan, R. Newrock, M. Cahay, A. Ngo, S. Ulloa, S. Herbert, M. Muhammad, and M. Johnson, Nature Nanotechnology 4, 759 (2009).
3 M. Kohda, S. Nakamura, Y. Nishihara, K. Kobayashi, T. Ono, J.-i. Ohe, Y. Tokura, T. Mineno, and J. Nitta, Nature communications 3, 1082 (2012).
4 M. Eto, T. Hayashi, and Y. Kurotani, J. Phys. Soc. Japan 74, 1934 (2005), ISSN 0333-9015, URL http://journals.jps.jp/doi/abs/10.1143/JPSJ.74.1934.
5 M. Eto, T. Hayashi, Y. Kurotani, and H. Yokouchi, physica status solidi c 3, 4168 (2006).
6 A. Reynoso, G. Usaj, and C. Balseiro, Phys. Rev. B 75, 085321 (2007), ISSN 1098-0121, URL http://link.aps.org/doi/10.1103/PhysRevB.75.085321.
7 P. Silvestrov and E. Mishchenko, Physical Review B 74, 165301 (2006).
8 L. P. Rotkinson, V. Larkina, Y. B. Lyanda-Geller, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 93 (2004), ISSN 003468, 043645.
9 P. Chuang, S.-C. Ho, L. Smith, F. Sfigakis, M. Pepper, C.-H. Chen, J.-C. Fan, J. Griffiths, I. Farrer, H. E. Beere, et al., Nature nanotechnology 10, 35 (2015).
10 S.-T. Lo, C.-H. Chen, J.-C. Fan, L. W. Smith, G. L. Creeth, C.-W. Chang, M. Pepper, J. P. Griffiths, I. Farrer, H. E. Beere, et al., Nature Communications 8, 15979 EP (2017), URL http://dx.doi.org/10.1038/ncomms15979.
11 E. Marcellina, A. Hamilton, R. Winkler, and D. Culcer, Physical Review B 95, 075305 (2017).
12 B. Van Wees, H. Van Houten, C. Beenakker, J. G. Williamson, L. Kouwenhoven, D. Van der Marel, and C. Foxon, Physical Review Letters 60, 848 (1988).
13 D. K. R. S. Li Glazman, GB Lesovik, Jetp Letters 48, 238 (1988).
14 L. W. Molenkamp, A. A. M. Staring, M. Harmans, C. W. J. Beenakker, R. Eppenga, C. E. Timmering, J. G. Williamson, C. J. P. M. Harmans, and C. T. Foxon, Phys. Rev. B - Condens. Matter Mater. Phys. 41, 1274 (1990).
15 C. Zener, Proc. R. Soc. Lond. A 137, 696 (1932).
16 J. B. V. T. S. Tsui and P. Wyder, Reviews of Modern Physics 71, 1614 (1999).
17 S. Bladwell and O. Sushkov, Physical Review B 92 (2015).
18 S. Chesi, G. F. Giuliani, L. P. Rotkinson, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 106, 236601 (2011), ISSN 0031-9007, URL http://link.aps.org/doi/10.1103/PhysRevLett.106.236601.
19 S. Bladwell and O. Sushkov, Physical Review B 96 (2017).
20 M. Topinka, B. LeRoy, S. Shaw, E. Heller, R. Westervelt, K. Maranowski, and A. Gossard, Science 289, 2323 (2000).
21 D. S. Pradip Khatua, B Bansal, Physical Review Letters 112 (2014).
22 M. Topinka, B. LeRoy, R. Westervelt, S. Shaw, R. Fleischmann, E. Heller, K. Maranowski, and A. Gossard, Nature 410, 183 (2001).
23 M. Rendell, O. Klochan, A. Srinivasan, I. Farrer, D. A. Ritchie, and A. Hamilton, Semiconductor Science and Technology 30, 102001 (2015).
24 R. Winkler, Spin–Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems, vol. 191 of Springer Tracts in Modern Physics (Springer Berlin Heidelberg, Berlin, Heidelberg, 2003), ISBN 978-3-540-01187-3, URL http://www.springerlink.com/index/10.1007/
For example, spin-polarisation due to an applied magnetic field will be complete when the injector is tuned below or at \( G = e^2/h \), while the double peak will be restored when the injector conductance is \( G = 2e^2/h \). This feature of TMF was demonstrated experimentally by Rokhinson et al. A detailed discussion of this is presented in Ref. \(^\text{29}\).