Exclusive semileptonic decays of heavy mesons in quark model

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Semileptonic decays $D \to K, K^*, \pi, \rho, D_\Psi \to \eta, \eta', \phi$, and $B \to D, D^*, \pi, \rho$ are analyzed within the dispersion formulation of the quark model. The form factors at timelike $q$ are derived by the analytical continuation from spacelike $q$ in the form factors of the relativistic light–cone quark model. The resulting double spectral representations allow a direct calculation of the form factors in the timelike region. The results of the model are shown to be in good agreement with all available experimental data. From the analysis of the $B \to D, D^*$ decays we find $|V_{cb}| = 0.037 \pm 0.004$, and the $B \to \pi, \rho$ decays give $|V_{ub}| = 0.004 \pm 0.001$.

The interest in exclusive semileptonic decays of heavy mesons lies in a possibility to obtain the most accurate values of the quark mixing angles and test various approaches to the description of the internal hadron structure. The decay rates are expressed through the Cabbibo–Kobayashi–Maskawa (CKM) matrix elements and hadronic form factors of the weak currents which contain the information on hadron structure. New accurate data on $B \to D, D^*$ and first measurements of the $B \to \pi, \rho$ decays open a possibility to determine $V_{cb}$ and $V_{ub}$ with high accuracy and require reliable theoretical predictions on the form factors and decay rates. A nonperturbative theoretical study should give these form factors in the whole kinematical region of momentum transfers $0 \leq q^2 \leq (M_1 - M_2)^2$, $M_1$ and $M_2$ the initial and final meson masses, respectively.

The amplitudes of meson decays induced by the quark transition $q_i \to q_f$ through the vector $V_\mu = \bar{q}_f \gamma_\mu q_i$ and axial–vector $A_\mu = \bar{q}_f \gamma_\mu \gamma_5 q_i$ currents have the following structure

\[
\begin{align*}
< P(M_2, p_2)|V_\mu(0) |P(M_1, p_1)> &= f_+(q^2) P_\mu + f_-(-q^2) q_\mu, \\
< V(M_2, p_2, \epsilon)|V_\mu(0) |P(M_1, p_1)> &= 2 g(q^2) \epsilon_{\mu \alpha \beta \gamma} \epsilon^{* \nu \alpha \beta} p_1^\alpha p_2^\beta, \\
< V(M_2, p_2, \epsilon)|A_\mu(0) |P(M_1, p_1)> &= i \epsilon^{\gamma \alpha} \left[ f(q^2) g_{\mu \alpha} + a_+(q^2) p_1^\alpha P_\mu + a_-(q^2) p_1^\alpha q_\mu \right],
\end{align*}
\]

with $q = p_1 - p_2$, $P = p_1 + p_2$. We use the notations: $\gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3$, $\epsilon^{1234} = -1$, $S_{\mu \nu}(\gamma^5 \gamma^\mu \gamma^\nu \gamma^\alpha) = 4 i \epsilon^{\mu \nu \alpha \beta}$. In general all the form factors are independent functions to be calculated within a nonperturbative approach. Considerable simplifications occur when both the parent and the daughter quarks active in the weak transition are heavy. Due to the heavy quark symmetry (HQS) all the form factors which are in general functions of $q^2$ and the masses, depend on the one dimensionless variable $\omega = p_1 p_2 / M_1 M_2$. It is convenient to introduce dimensionless ‘heavy quark’ form factors

\[
\begin{align*}
f_\pm(q^2) &= \frac{M_2 \mp M_1}{2 \sqrt{M_1 M_2}} h_\pm(\omega), \\
g(q^2) &= \frac{1}{2 \sqrt{M_1 M_2}} h_g(\omega), \quad a_+(q^2) = - \frac{1}{2 \sqrt{M_1 M_2}} h_a(\omega), \quad f(q^2) = \sqrt{M_1 M_2} (1 + \omega) h_f(\omega).
\end{align*}
\]

In the leading $1/m_Q$ order all the form factors $h$ can be expressed through the single universal form factor, the Isgur–Wise function $\xi$ as follows

\[
\begin{align*}
h_+(\omega) = h_g(\omega) = h_a(\omega) = h_f(\omega) = \xi(\omega), \quad h_-(\omega) = 0.
\end{align*}
\]

The normalization of the Isgur–Wise function at zero recoil is known, $\xi(1) = 1$. In contrast to meson decays induced by the heavy–to–heavy quark transitions, the general case of the transitions between hadrons with arbitrary masses, and in particular meson decays induced by a heavy–to–light quark transitions are not well–understood. To date, theoretical predictions on semileptonic decays induced by heavy–to–light quark transitions coming from the quark model, QCD sum rules, and lattice calculations differ significantly. Recently, B. Stech noticed that new relations between the form factors of meson transition can be derived if use is made of the constituent quark picture. These relations are based on the observation that if the meson wave function in terms of its quark constituents is strongly peaked in the momentum space with the width of order $\Lambda \simeq 0.5$ GeV then for a heavy parent quark a small parameter $\Lambda / m_Q$ appears in the picture and one can derive the leading–order expressions for the form factors of interest which turn out to be independent of subtle details of the meson structure. Although these relations give a guideline for the analysis of the decay processes, they cannot substitute calculations of the form factors in a more detailed dynamical model. The kinematically accessible $q^2$–interval in the meson decay induced by the heavy–to–light transition is $O(m_Q^2)$ so a relativistic treatment is necessary.

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A relativistic light–cone quark model (LCQM) is an adequate framework for considering decay processes. The model is formulated at spacelike momentum transfers and the direct application of the model at timelike momentum transfer is hampered by pair–creation subprocesses which at \( q^2 > 0 \) cannot be killed by an appropriate choice of the reference frame. In the form factors at \( q^2 > 0 \) were obtained by numerical extrapolation from the region \( q^2 < 0 \). As the relevant \( q^2 \)–interval is large, the accuracy of such a procedure is not high. In the nonpartonic contribution of pair–creation subprocesses is neglected and only the partonic part of the form factor is calculated in the whole kinematical interval of \( q^2 \). Unfortunately, the nonpartonic contribution is under control only at \( q^2 = 0 \) where it vanishes.

The dispersion formulation of the LCQM proposed in overcomes these difficulties it allows the analytical continuation to the timelike \( q \). We apply this approach to calculating the transition form factors of the semileptonic decays.

The transition of the initial meson \( Q(m_2)\bar{Q}(m_3) \) with the mass \( M_1 \) to the final meson \( Q(m_2)\bar{Q}(m_3) \) with the mass \( M_2 \) induced by the quark transition \( m_2 \to m_1 \) is given by the diagram of Fig.1. The constituent quark structure of the initial and final mesons are described by the vertices \( \Gamma \) and \( \Gamma_2 \), respectively. The initial pseudoscalar meson has the spinorial structure \( \Gamma_1 = i\gamma_5 G_1/\sqrt{N_c} \); the final meson pseudoscalar state and the structure \( \Gamma_2 = [A \gamma_\mu + B(k_1 - k_3)\mu] G_2/\sqrt{N_c} \) for the vector state. The values \( A = -1, B = 1/(\sqrt{s_2} + m_1 + m_3) \) correspond to an \( S \)–wave vector meson, and \( A = 1/\sqrt{s_2}, B = [2/(\sqrt{s_2} + m_1 + m_3)]/[\sqrt{2}(s_2 + (m_1 + m_3)^2)] \) correspond to a \( D \)–wave vector particle.

At \( q^2 < 0 \) the form factors of the LCQM can be represented as double spectral representations over the invariant masses of the initial and final \( q\bar{q} \) pairs as follows

\[
f_\pm(q^2) = f_{21}(q^2) \int_{(m_1+m_3)^2}^{\infty} \frac{ds_2 G_2(s_2)}{\pi(s_2-M_2^2)} s_2^{-1}(s_2,q^2) \int_{s_1^{-1}(s_2,q^2)}^{\infty} \frac{ds_1 G_1(s_1)}{\pi(s_1-M_1^2)} \tilde{f}_\pm(s_1,s_2,q^2),
\]

where

\[
s_\pm(s_1,s_2,q^2) = \frac{s_2(m_1^2 + m_2^2 - q^2) + q^2(m_1^2 + m_3^2) - (m_1^2 - m_2^2)(m_1^2 - m_3^2)}{2m_1^2} \pm \frac{\lambda^{1/2}(s_2,m_3^2,m_2^2)\lambda^{1/2}(q^2,m_1^2,m_2^2)}{2m_1^2}
\]

and \( \lambda(s_1,s_2,s_3) = (s_1 + s_2 - s_3)^2 - 4s_1s_2 \) is the triangle function. Here \( f_{21}(q^2) \) is the form factor of the constituent quark transition \( m_2 \to m_1 \). In what follows we set \( f_{21}(q^2) = 1 \).

The double spectral densities \( f_\pm(s_1,s_2,q^2) \) of the form factors have the following form:

\[
\tilde{f}_+ = 4\lambda(m_1 m_2 a_1 - m_2 m_3 a_1 + m_1 m_3 (1 - a_1) - m_2^2 (1 - a_1) + a_2 s_2],
\]

\[
\tilde{f}_- = 4\lambda(m_1 m_2 a_2 - m_1 m_3 a_2 + m_2 m_3 (1 - a_2) - m_1^2 (1 - a_2) + a_1 s_1],
\]

\[
\tilde{g} = -4\lambda(m_1 m_2 a_1 + m_1 m_3 (1 - a_1) - m_2^2 (1 - a_1) - 4B),
\]

\[
\tilde{a}_+ + \tilde{a}_- = -4\lambda(2m_1 a_1 + 2m_2 a_2 + 2m_3 a_3) + 4B[C_2 a_1 + C_3 a_2],
\]

\[
\tilde{f}_{LC} = \frac{M_2}{\sqrt{s_2}} \tilde{f}_D + \left( \frac{1}{2\sqrt{s_2}} - \frac{M_1^2 - M_2^2 - s_3}{2M_2} \right) M_2 \tilde{a}_+.
\]

where

\[
\tilde{f}_D = -4\lambda(m_1 m_2 m_3 + \frac{m_2}{2}(s_2 - m_1^2 - m_2^2) + \frac{m_1}{2}(s_1 - m_2^2 - m_3^2) + \frac{m_3}{2}(s_1 - m_1^2 - m_2^2) + 2\beta(m_2 - m_3)] + 4BC_3\beta,
\]

\[
\alpha_1 = \left[ (s_1 + s_2 - s_3)(s_2 - m_1^2 + m_3^2) - 2s_2(s_1 - m_2^2 + m_3^2) \right] / \lambda(s_1,s_2, s_3),
\]

\[
\alpha_2 = \left[ (s_1 + s_2 - s_3)(s_1 - m_2^2 + m_3^2) - 2s_1(s_2 - m_2^2 + m_3^2) \right] / \lambda(s_1,s_2, s_3),
\]

\[
\beta = \frac{1}{2} \left[ 2m_3^2 - \alpha_1(s_1 - m_2^2 + m_3^2) - \alpha_2(s_2 - m_1^2 + m_3^2) \right],
\]

\[
\alpha_{11} = \alpha_1^2 + 4\beta\lambda(s_1,s_2, s_3), \quad \alpha_{12} = \alpha_1\alpha_2 - 2\beta(s_1 + s_2 - s_3) / \lambda(s_1,s_2, s_3),
\]

\[
C_1 = s_2 - (m_1 + m_3)^2, \quad C_2 = s_1 - (m_2 - m_3)^2, \quad C_3 = s_3 - (m_1 + m_2)^2 - C_1 - C_2.
\]

Let us underline that the representation with the spectral densities are just the dispersion form of the corresponding light–cone expressions from [3]. It is important that double spectral representations without subtractions are valid for all the form factors except \( f_+ \) which requires subtractions. In the LCQM the particular
form of such a representation for the form factor $f$ depends on the choice of the current component used for its determination and cannot be fixed uniquely. In II the behaviour of the form factors of the vector, axial–vector and tensor current has been studied in the limits of heavy–to–heavy and heavy–to–light quark transitions. The analysis of the behavior of the form factor $f$ in the case of a heavy–to–light quark transition suggests another expression II:

$$f^{HQ}(s_1, s_2, q^2) = f_D(s_1, s_2, q^2) + (M_1^2 - s_1 + M_2^2 - s_2)\hat{g}(s_1, s_2, q^2).$$

We shall use both of these prescriptions in the numerical analysis of semileptonic decays.

For a pseudoscalar or vector meson with the mass $M$ built up of the constituent quarks $m_q$ and $m_{\bar{q}}$, the function $G$ is normalized as follows [14]

$$\int \frac{G^2(s)ds}{\pi(s - M^2)^2} \frac{\lambda^{1/2}(s, m_q^2, m_{\bar{q}}^2)}{8\pi s} (s - (m_q - m_{\bar{q}})^2) = 1. \quad (18)$$

The quark–meson vertex $G$ can be written as [3]

$$G(s) = \frac{\sqrt{s - (m_1 - m_2)^2}}{\sqrt{s - (m_1 - m_2)^2}} \frac{s - M^2}{s^{3/4}} w(k), \quad k = \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{2\sqrt{s}} \quad (19)$$

where $w(k)$ is the ground–state $S$–wave radial wave function.

As the analytical continuation of the form factors [12] to the timelike region is performed, in addition to the normal contribution which is just the expression [3] taken at $q^2 > 0$ the anomalous contribution emerges. The corresponding expression is given in [13]. The normal contribution dominates the form factor at small timelike $q$ and vanishes as $q^2 = (m_2 - m_1)^2$ while the anomalous contribution is negligible at small $q^2$ and steeply rises as $q^2 \rightarrow (m_2 - m_1)^2$. It should be emphasized that we derive the analytical continuation in the region $q^2 \leq (m_2 - m_1)^2$. For the constituent quark masses used in the quark models this allows a direct calculation of the form factors of the $P \rightarrow V$ transitions in the whole kinematical decay region $0 \leq q^2 \leq (M_P - M_V)^2$, as $M_P - M_V < m_2 - m_1$. For the $P \rightarrow P'$ transition this is not the case: normally, $M_P - M_{P'} > m_2 - m_1$. For the $P \rightarrow P'$ decays we directly calculate the form factors in the region $0 \leq q^2 \leq (m_2 - m_1)^2$ and perform numerical extrapolation in $(m_2 - m_1)^2 \leq q^2 \leq (M_P - M_{P'})^2$. Numerical analysis shows the accuracy of this extrapolation procedure to be very high. We would like to notice that the direct calculation shows the derivative of the form factor $f_+$ to be positive at the point $q^2 = (m_2 - m_1)^2$. This suggests that the maximum of the form factor $f_+$ at $q^2 = (m_2 - m_1)^2$ observed in [12] is just an artifact of neglecting the nonpartonic contribution to the form factor.

For calculating the form factors of semileptonic decays we assume that the wave function $w$ can be approximated by a simple exponential function $w(k) = \exp(-k^2/2\beta^2)$ and adopt the numerical parameters of the ISGW2 model [24] shown in Table IV.

The results of calculating the form factors are fitted by the functions

$$f(q^2) = f(0)/[1 - \alpha_1 q^2 + \alpha_2 q^4]$$

with better than 0.5% accuracy, and for the form factor $f_+$ this formula is used for numerical extrapolation to the region $(m_1 - m_2)^2 \leq q^2 \leq (M_1 - M_2)^2$. The decay rates are calculated from the form factors via the formulas from [14]. Decay rate calculations are performed using the two prescriptions for the form factor $f$ given by the relations (15) and (17); the corresponding results are labelled as LC and HQ, respectively. The decay $D \rightarrow K, K^*$. These CKM–favoured decays extend the widest possibility for detailed verification of the model. The parameters of the fits to the form factors are given in Table IV. Using the value $V_{cs} = 0.975$ [25] the decay rates are found to be

$$\Gamma(D \rightarrow K) = 8.7 \times 10^{10} \text{ s}^{-1}$$

$$\Gamma(D \rightarrow K^*) = \begin{cases} 5.58 \times 10^{10} \text{ s}^{-1}, & \Gamma_L/\Gamma_T = 1.34 \quad (LC) \\ 5.38 \times 10^{10} \text{ s}^{-1}, & \Gamma_L/\Gamma_T = 1.34 \quad (HQ) \end{cases}$$

[1] The analytical continuation to $q^2 > (m_2 - m_1)^2$ is also possible. However one should be careful when applying the constituent quark model for such $q^2$: we approach the unphysical $q\bar{q}$ threshold $q^2 = (m_1 + m_2)^2$ which is obviously absent in the amplitudes of hadronic processes. This is a sign that we are coming to the region where the constituent quark picture is not adequate.
Table IV compares the results of the model with the HQ prescription for the form factor $f$ with the experimental data. One can observe perfect agreement with the data. The results of other approaches which give predictions for a wide set of the semileptonic decay modes are also shown.

**The decay $D \rightarrow \pi, \rho$.**

Table IV present the parameters of the fits to the calculated form factors. Using the value $V_{cd} = 0.22$ [23] yields the following decay rates:

$$
\Gamma(D^0 \rightarrow \pi^-) = 0.62 \times 10^{10} \text{ s}^{-1}
$$
$$
\Gamma(D^0 \rightarrow \rho^-) = \begin{cases} 
0.30 \times 10^{10} \text{ s}^{-1}, & \Gamma_L/\Gamma_T = 1.32 \quad (LC) \\
0.26 \times 10^{10} \text{ s}^{-1}, & \Gamma_L/\Gamma_T = 1.27 \quad (HQ)
\end{cases}
$$

(21)

Table V presents the rates for the HQ prescription of the model and the results of other approaches versus experimental data. The experimental results are obtained by combining the decay rates of the $D \rightarrow K, K^*$ transition with the following ratios measured by CLEO [23] $\text{Br}(D^0 \rightarrow \pi^- e^+ \nu)/\text{Br}(D^0 \rightarrow K^- e^+ \nu) = 0.103 \pm 0.039 \pm 0.013$ and E653 [24] $\text{Br}(D^0 \rightarrow \rho^- e^+ \nu)/\text{Br}(D^0 \rightarrow K^- e^+ \nu) = 0.888 \pm 0.062 \pm 0.028$. The calculated rates seem to be a bit small but nevertheless agree with the experimental values within large errors.

**The decay $D_s \rightarrow \eta, \eta'$, $\phi$.**

Table V presents the results on the form factors. The calculated decay rates depend on the content of $\eta$ and $\eta'$ mesons and with $|V_{cs}| = 0.975$ read

$$
\Gamma(D_s \rightarrow \eta) = 0.111 \sin^2(\varphi) \text{ ps}^{-1}
$$
$$
\Gamma(D_s \rightarrow \eta') = 0.030 \cos^2(\varphi) \text{ ps}^{-1}
$$
$$
\Gamma(D_s \rightarrow \phi) = \begin{cases} 
0.047 \text{ ps}^{-1}, & \Gamma_L/\Gamma_T = 1.30 \quad (LC) \\
0.040 \text{ ps}^{-1}, & \Gamma_L/\Gamma_T = 1.28 \quad (HQ)
\end{cases}
$$

(22)

Here $\varphi = \theta_R + \arcsin(2/\sqrt{3})$ [25]. The decay rate of $D_s \rightarrow \phi$ calculated with the (HQ) prescription agrees well with the results of the analysis [21] $\Gamma(D_s \rightarrow \phi e^+ \nu) = (0.035 \pm 0.005) \text{ ps}^{-1}$. Table VI compares the results on branching ratios with recent CLEO measurements [26] and the ISGW2 model. The results for all $\theta_R$ in the range $-18^\circ \leq \theta_R \leq -10^\circ$ compare favourably with the data, but the best agreement is observed for $\theta_R = -14^\circ$.

**The decay $B \rightarrow D, D^*$.**

This is a very interesting mode as it allows measuring corrections to the HQS limit. Table VII shows the fit parameters of the form factors and Table IV presents the parameters of the fit to the heavy–quark form factors in the form

$$
h_i(\omega) = h_i(1) \left[1 - h_i^2(\omega - 1) + \delta(\omega - 1)^2\right].
$$

(23)

We find $h_+(1) = 0.96$ and $h_-(1) = -0.04$ which compare favourably with the size of corrections to the HQS limit [26]. The values $h_f^{HQ}(1) = 0.94$ and $h_f^{LC}(1) = 0.9$ both agree with the estimate of Neubert [30] $h_f(1) = 0.93 \pm 0.03$. For the ratios of the heavy quark form factors $R_1 = h_g(1)/h_f(1)$ and $R_2 = h_{g^2}(1)/h_f(1)$ we obtain $R_1^{HQ} = 1.05$ [$R_1^{LC} = 1.1$] and $R_2^{HQ} = 0.84$ [$R_2^{LC} = 0.88$] to be compared with a recent CLEO result $R_1 = 1.18 \pm 0.15 \pm 0.16$ and $R_2 = 0.71 \pm 0.22 \pm 0.07$ and predictions of the ISGW2 model [21] $R_1 = 1.27$, $R_2 = 1.01$, Neubert [31] $R_1 = 1.35$, $R_2 = 0.79$, and Close and Wambach [23] $R_1 = 1.15$, $R_2 = 0.91$. CLEO [32] reported the value $|V_{cb}| h_f(1) = 0.0351 \pm 0.0019 \pm 0.0018 \pm 0.0008$. Combining this value with our result $h_f^{HQ}(1) = 0.94$ yields

$$
|V_{cb}| = 0.0373 \pm 0.0053 \quad \text{[lepton endpoint region in $B \rightarrow D^* l\bar{\nu}$]}
$$

For the decay rates we find

$$
\Gamma(B \rightarrow D) = 8.712 \times 10^{12} |V_{cb}|^2 \text{ s}^{-1}
$$
$$
\Gamma(B \rightarrow D^*) = \begin{cases} 
21.0 \times 10^{12} |V_{cb}|^2 \text{ s}^{-1}, & \Gamma_L/\Gamma_T = 1.17 \quad (LC) \\
23.2 \times 10^{12} |V_{cb}|^2 \text{ s}^{-1}, & \Gamma_L/\Gamma_T = 1.28 \quad (HQ)
\end{cases}
$$

(24)

Table V compares the calculated decay rates for the HQ prescription with other approaches. Combining our result with a CLEO measurement [27] $\Gamma(B \rightarrow D^* l\bar{\nu}) = [29.9 \pm 1.9 \text{(stat)} \pm 2.7 \text{(syst)} \pm 2.0 \text{(lifetime)}] \text{ns}^{-1}$ yields

$$
|V_{cb}| = 0.036 \pm 0.004 \quad \text{[decay rate $B \rightarrow D^* l\bar{\nu}$]}
$$

The branching ratio $\text{Br}(B^0 \rightarrow D^- l^+ \nu) = (1.9 \pm 0.5)\%$ and the $B^0$ lifetime $\tau_{B^0} = (1.56 \pm 0.06) \text{ ps}$ [25] give the experimental decay rate $\Gamma(B^0 \rightarrow D^- l^+ \nu) = (1.22 \pm 0.3) \times 10^{10} \text{ s}^{-1}$ and comparing with our (HQ) result yields

$$
|V_{cb}| = 0.038 \pm 0.004 \quad \text{[decay rate $B^0 \rightarrow D^- l^+ \nu$]}
$$
All the three estimates of \( V_{cb} \) agree with each other and the average value is found to be
\[
|V_{cb}|_{\text{excl}} = 0.037 \pm 0.004. \tag{25}
\]
This is in perfect agreement with the updated values \[3\] \( |V_{cb}|_{\text{excl}} = 0.0373 \pm 0.0045(\text{exp}) \pm 0.0065(\text{th}) \) and \( |V_{cb}|_{\text{incl}} = 0.0398 \pm 0.0008(\text{exp}) \pm 0.0004(\text{th}) \).

**The decay \( B \to \pi, \rho \).**

This mode allows a determination of \( |V_{ub}| \) and hence reliability of theoretical predictions is very important. The form factors are given in Table XI. For the decay rates we find
\[
\begin{align*}
\Gamma(B^0 \to \rho^-) & = 7.2 \times 10^{12} |V_{ub}|^2 \, s^{-1}, \\
\Gamma(B^0 \to \rho^+) & = 9.64 \times 10^{12} |V_{ub}|^2 \, s^{-1}, \\
\Gamma_L/\Gamma_T & = 0.95 \quad (\text{LC}) \\
\Gamma_L/\Gamma_T & = 1.13 \quad (\text{HQ})
\end{align*}
\]
(26)

This calculation is in agreement with our previous analysis of this decay mode using other sets of the quark model parameters \[19\]. The calculated decay rates are compared with other theoretical predictions and first measurements by CLEO \[34\] in Table XII. The experimental values are obtained by combining the CLEO results \[33\] \( |V_{cb}|_{\text{excl}} = 0.0373 \pm 0.0045(\text{exp}) \pm 0.0065(\text{th}) \) and \( \text{Br}(B^0 \to \rho^- \pi^0) = (1.8 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4} \) and \( \text{Br}(B^0 \to \rho^- l^+ \nu) = (2.5 \pm 0.4 \pm 0.7 \pm 0.5) \times 10^{-4} \) with the \( B^0 \) lifetime \( \tau_{B^0} = (1.56 \pm 0.06) \) ps \[22\]. Comparing our results with the experimental values gives
\[
|V_{ub}| = 0.004 \pm 0.001 \quad [B \to \pi] \\
|V_{ub}| = 0.00407 \pm 0.001 \quad [B \to \rho]
\]
A good agreement of these values with each other shows that we have predicted correctly the ratio of the branching fractions \( B \to \pi \) and \( B \to \rho \). The average value obtained from the two modes reads
\[
|V_{ub}| = 0.004 \pm 0.001.
\]
Taking the \( |V_{cb}| \) from (25) we find
\[
|V_{ub}/V_{cb}| = 0.108 \pm 0.02
\]
which is perhaps a bit large but nevertheless agrees with the PDG value \( |V_{ub}/V_{cb}| = 0.08 \pm 0.02 \).

In conclusion, we have analysed semileptonic decays of heavy mesons within dispersion formulation of the constituent quark model and found agreement with all available data. The extracted values of the CKM matrix elements \( V_{cb} \) and \( V_{ub} \) are also in agreement with estimates of other models. Nevertheless, we would like to briefly outline possible sources of uncertainties in the predictions of the model which should be taken into account in further analyses:

1. We have used the parameters of the ISGW2 quark model and a simplified exponential ansatz for the wave function. It should be noticed however that the ISGW2 model does not calculate the form factors through the wave functions; rather a special prescription for constructing the form factors is formulated. As the analysis of \[7\] shows, the form factors of the heavy-to-light transition can be rather sensitive to the wave function shape.
2. We have taken into account only the leading process and neglected the \( O(\alpha_s) \) corrections. Although the analysis of such corrections in the elastic pion form factor at low and high momentum transfers within the LCQM \[42\] found only a few \% contribution even at \( \omega \approx 10 - 20 \) a numerical consideration of this contribution is plausible.
3. We have neglected the constituent quark transition form factor which has a complicated structure at timelike momentum transfers. In particular the quark transition form factor should contain a pole at \( q^2 = M^2_{\text{res}} \) with \( M_{\text{res}} \) the mass of a resonance with appropriate quantum numbers.
4. We have identified the form factors obtained within the constituent quark model with the form factors of the full theory. However, the relationship between these two quantities is nontrivial: e.g. the form factors of the full theory acquire logarithmic corrections because of renormalization of the quark currents, which are absent in the quark model form factors. Analysing the \( 1/m_Q \) expansion Scora and Isgur \[21\] performed a special matching procedure for obtaining the form factors of the full theory from their quark-model form factors. Although the \( 1/m_Q \) behaviour of the LCQM form factors studied in \[20\] is better than that of the generically nonrelativistic form factors in the ISGW model \[3\], the relationship between the LCQM form factors and the form factors of the full theory should be studied in more detail.

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TABLE I. Parameters of the quark model.

| Ref. | $m_u$ | $m_c$ | $m_s$ | $m_b$ | $\beta_\pi$ | $\beta_K$ | $\beta_{\pi^0}\rho^0$ | $\beta_D$ | $\beta_B$ | $\beta_K^*$ | $\beta_{\pi^0}$ | $\beta_{D^*}$ |
|------|-------|-------|-------|-------|-------------|------------|-----------------------|---------|---------|-----------|----------------|-------------|
| ISGW2 [21] | 0.33  | 0.55  | 1.82  | 5.2   | 0.41        | 0.44       | 0.53                  | 0.45    | 0.43    | 0.30       | 0.33           | 0.37        |

TABLE II. Parameters of the fits to the calculated $D \to K, K^*$ transition form factors.

| $D \to K$ | $D \to K^*$ |
|-----------|-------------|
| $f_\perp$ | $g$         | $a_\perp$ | $f_{LC}$ | $f_{HQ}$ |
| $f(0)$    | 0.781       | 0.28      | -0.168   | 1.747    | 1.733     |
| $a_1 (GeV^{-2})$ | 0.201 | 0.24      | 0.189    | 0.0971   | 0.0767    |
| $a_2 (GeV^{-4})$ | 0.0086 | 0.0135   | 0.001    | 0.001    | 0.001     |
| $f(q_{max})$ | 1.2 | 0.35     | -0.205   | 1.92     | 1.86      |
### TABLE III. Decay rates for the $D \to K, K^*$ transition in $10^{10}$ s$^{-1}$ using $|V_{us}| = 0.975$

|                  | This work | WSB [2] | ISGW2 [21] | Jaus [2] | BBD [1] | Exp. [22] |
|------------------|-----------|---------|------------|----------|---------|-----------|
| $\Gamma(D \to K)$ | 8.7       | 7.56    | 10.0       | 9.6      | 6.5 ± 1.3 | 9.0 ± 0.5 |
| $\Gamma(D \to K^*)$ | 5.38     | 7.73    | 5.4        | 5.5      | 3.7 ± 1.2 | 5.1 ± 0.5 |
| $\Gamma(K^*)/\Gamma(K)$ | 0.62     | 1.02    | 0.54       | 0.57     | 0.57 ± 0.15 | 0.57 ± 0.08 |
| $\Gamma_L/\Gamma_T$ | 1.31     | —       | 0.94       | 1.33     | 0.86 ± 0.06 | 1.15 ± 0.17 |

### TABLE IV. Parameters of the fits to the calculated $D^0 \to \pi^-, \rho^-$ transition form factors.

|                  | $D \to \pi$ | $D \to \rho$ |
|------------------|-------------|-------------|
| $f(0)$           | 0.68        | 0.252       |
| $\alpha_3[GeV^{-2}]$ | 0.225     | 0.274       |
| $\alpha_2[GeV^{-4}]$ | 0.010     | 0.017       |
| $f(q_{max})$     | 1.63        | 0.36        |

### TABLE V. Decay rates for the $D^0 \to (\pi^-, \rho^-)e^+\nu$ transition in $10^{10}$ s$^{-1}$ using $|V_{ub}| = 0.22$.

|                  | This work | WSB [2] | ISGW2 [21] | Jaus [2] | Ball [10] | Exp. [22] |
|------------------|-----------|---------|------------|----------|-----------|-----------|
| $\Gamma(D \to \pi)$ | 0.62      | 0.68    | 0.24       | 0.8      | 0.39 ± 0.08 | 0.92 ± 0.45 |
| $\Gamma(D \to \rho)$ | 0.26      | 0.67    | 0.12       | 0.33     | 0.12 ± 0.03 | 0.45 ± 0.22 |
| $\Gamma(\rho)/\Gamma(\pi)$ | 0.41     | 0.98    | 0.51       | 0.41     | 0.3 ± 0.1 | 0.5 ± 0.35 |
| $\Gamma_L/\Gamma_T$ | 1.27      | 0.91    | 0.67       | 1.22     | 1.31 ± 0.11 | —         |

### TABLE VI. Parameters of the fits to the calculated $D_s \to s\bar{s}, \phi$ transition form factors.

|                  | $D_s \to s\bar{s}$ | $D_s \to \phi$ |
|------------------|---------------------|----------------|
| $f(0)$           | 0.800               | 0.266          |
| $\alpha_1[GeV^{-2}]$ | 0.192     | 0.246          |
| $\alpha_2[GeV^{-4}]$ | 0.008     | 0.015          |
| $f(q_{max})$     | 0.009               | —              |

### TABLE VII. Ratio of the decay rates for the $D_s \to \eta, \eta', \phi$ transitions.

|                  | $\theta_p = -10^\circ$ | $\theta_p = -14^\circ$ | $\theta_p = -20^\circ$ | $\theta_p = -10^\circ$ | $\theta_p = -20^\circ$ |
|------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| $\Gamma(\eta)/\Gamma(\phi)$ | 1.45                   | 1.24                   | 0.9                    | 1.2                    | 0.8                    |
| $\Gamma(\eta')/\Gamma(\phi)$ | 0.4                    | 0.46                   | 0.6                    | 0.5                    | 0.7                    |
| $\Gamma(\eta)/\Gamma(\eta')$ | 0.27                   | 0.37                   | 0.67                   | 0.35 ± 0.16            |

### TABLE VIII. Parameters of the fits to the calculated $B \to D, D^*$ transition form factors.

|                  | $B \to D$ | $B \to D^*$ |
|------------------|-----------|------------|
| $f(0)$           | 0.684     | −0.337     |
| $\alpha_1[GeV^{-2}]$ | 0.0386   | 0.039      |
| $\alpha_2[GeV^{-4}]$ | 0.00042  | 0.00038    |
| $f(q_{max})$     | 1.12      | −0.56      |
TABLE IX. Parameters of the heavy–quark form factors for the $B \to D, D^*$ transition.

|        | $B \to D$ | $B \to D^*$ |
|--------|-----------|------------|
| $h_+$  | $h_-$     | $h_+$      | $h_-$      | $h_f^{(e)}$ | $h_f^{(a)}$ |
| 0.96   | −0.04     | 0.99       | 0.79       | 0.92        | 0.90        | 0.94        |
| $\rho^2$ |           | 1.04       | 1.20       | 1.08        | 1.10        | 1.06        |
| $\delta$ |           | 0.54       | 0.63       | 0.50        | 0.55        | 0.53        |

TABLE X. Decay rates for the $B \to D, D^*$ transition in ps$^{-1}$.

|        | This work | WSB [2] | ISGW2 [21] | Jaus [3] | Exp. |
|--------|-----------|---------|------------|----------|------|
| $\Gamma(B \to D)$ | 8.7|$|V_{ub}|^2$ | 8.1|$|V_{ub}|^2$ | 11.9|$|V_{ub}|^2$ | 9.6|$|V_{ub}|^2$ | 1.27 ± 0.3 × 10$^{-2}$ [24] |
| $\Gamma(B \to D^*)$ | 23.2|$|V_{ub}|^2$ | 21.9|$|V_{ub}|^2$ | 24.8|$|V_{ub}|^2$ | 25.3|$|V_{ub}|^2$ | 2.99 ± 0.66 × 10$^{-2}$ [27] |
| $\Gamma(D^*)/\Gamma(D)$ | 2.65 | 2.71 | 2.08 | 2.64 | 2.35 ± 1.3 |
| $\Gamma_L/\Gamma_T$ | 1.28 | − | 1.04 | − | 1.24 ± 0.16 [28] |
| $\Gamma_L/\Gamma_T$ | 0.85 ± 0.45 [29] |

TABLE XI. Parameters of the fits to the calculated $\bar{B}^0 \to \pi^+, \rho^+$ transition form factors.

|        | $B \to \pi$ | $B \to \rho$ |
|--------|-------------|-------------|
| $f(0)$ | 0.2937      | 0.0356      |
| $g$ | 0.0511      | 0.0635      |
| $a_+$ | −0.0256     | 0.0567      |
| $f_{LC}$ | 1.025      | 0.032       |
| $f_{HQ}$ | 1.098      | 0.0316      |
| $f(q_{max})$ | 2.30      | 0.17        |
| $f(q_{max})$ | 2.19      | 2.12        |

TABLE XII. Decay rates for the $\bar{B}^0 \to (\pi^+, \rho^+)\bar{\nu}\bar{\nu}$ transition in ps$^{-1}$.

|        | This work | WSB [2] | ISGW2 [21] | Jaus [3] | Exp. [32] |
|--------|-----------|---------|------------|----------|----------|
| $\Gamma(B \to \pi)$ | 7.2|$|V_{ub}|^2$ | 7.4|$|V_{ub}|^2$ | 9.6|$|V_{ub}|^2$ | 10.0|$|V_{ub}|^2$ | 5.1 ± 1.1|$|V_{ub}|^2$ | 1.2 ± 0.6 × 10$^{-4}$ |
| $\Gamma(B \to \rho)$ | 9.64|$|V_{ub}|^2$ | 26.0|$|V_{ub}|^2$ | 14.2|$|V_{ub}|^2$ | 19.1|$|V_{ub}|^2$ | 12 ± 4(14.5 ± 4.5)$|V_{ub}|^2$ | 1.67 ± 1.0 × 10$^{-4}$ |
| $\Gamma(\rho)/\Gamma(\pi)$ | 1.34 | 3.5 | 1.48 | 1.91 | 2.35 ± 1.2 |
| $\Gamma_L/\Gamma_T$ | 1.13 | 1.34 | 0.3 | 0.82 | 0.06 ± 0.02 (0.52 ± 0.1) |

FIG. 1. One-loop graph for a meson decay.