Evolution of shifted cosmological parameter and shifted dust matter in a two-phase tachyonic field universe

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Abstract

We propose a model of the evolution of the tachyonic scalar field over two phases in the universe. The field components do not interact in phase I, while in the subsequent phase II, they change flavours due to relative suppression of the radiation contribution. In phase II, we allow them to interact mutually with time-independent perturbation in their equations of state, as Shifted Cosmological Parameter (SCP) and Shifted Dust Matter (SDM). We determine the solutions of their scaling with the cosmic redshift in both phases. We further suggest the normalized Hubble function diagnostic, which, together with the low- and high-redshift $H(z)$ data and the concordance values of the present density parameters from the CMBR, BAO statistics etc., constrains the strength of interaction, by imposing the viable conditions to break degeneracy in 3-parameter ($\gamma, \varepsilon, \dot{\phi}^2$) space. The range of redshifts ($z = 0.1$ to $z = 1.75$) is chosen to highlight the role of interaction during structure formation, and it may lead to a future analysis of power spectrum in this model vis a vis Warm Dark Matter (WDM) or $\Lambda$CDM models. We further calculate the influence of interaction in determining the age of the universe at the present epoch, within the degeneracy space of model parameters.

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1 Introduction

The observed accelerated expansion of the universe \[1, 2, 3\] is thought to be driven by some exotic field, called dark energy, with an equation of state (EOS) very close to \( w_\lambda = -1 \) at the present epoch \[4, 5, 6\]. A class of scalar fields is one of the promising candidates of dark energy \[7, 8, 9, 10, 11, 12\]. Among itself, tachyonic scalar field arising from string theory \[13\] (for different reasons though) appears more relevant than the conventional non-relativistic scalar field in form of quintessence, and it is widely used in literature \[14, 15, 16, 17\]. One of its reasons is that the Lagrangian adopted in tachyonic scalar field is relativistic which is more profound and appealing than its non-relativistic counterpart.

In this paper, we propose a model of the evolution of the tachyonic field universe over two phases— namely, non-interacting (phase I) and interacting (phase II)—, respectively. The general evolution, however, may span several such phases with different intervals of time distinguishing each other in terms of their respective interaction strengths. In Sec. 2 of our present work, we investigate phase I, when the tachyonic scalar field has two components— one is radiation while the other is an unknown stuff. This later component has negative pressure and thus mimics the cosmological constant that may have caused the inflation early on during this phase. It is further decomposed into two components— true cosmological constant and matter with negative energy density but zero EOS.

The same tachyonic scalar field manifests itself in form of two components in phase II. If we take one component as pressureless dark matter with \( w_m = 0 \), then other one is found to behave as the cosmological constant with \( w_\lambda = -1 \). The dark matter component includes the baryonic contributions having the same equation of state while any contributions arising from radiation would be almost negligible in a matter dominated universe. Several workers have investigated the cosmological behaviour of tachyonic scalar field having this composition \[14, 15, 16, 17\].

In Sec. 3 we study the behaviour of two new components. One component of tachyonic scalar field is cosmological constant with \( w \) slightly perturbed from \(-1\). This component is called Shifted Cosmological Parameter (SCP) with \( w = -1 + \varepsilon(t) \), where \( \varepsilon(t) \) is a small perturbation introduced in the equation of state(EOS) of the true cosmological constant. The other component is dark matter with \( w \simeq 0 \). Since it is not exactly a pressureless dust, we call it Shifted Dust Matter (SDM). Given the form of energy density and
EOS of the SCP we find its pressure. This further leads to the pressure and EOS of SDM.

The non-zero pressure of SDM contributes to the total kinetic energy of dark matter particles, and justifiably raises the status of cold dark matter to warm dark matter (WDM). This connection is also supported by the current work in favour of eV- (or perhaps keV-) mass sterile neutrino\textsuperscript{[18]} that elevates the energy of the cold dark matter, or of WDM \textsuperscript{[19]}. It is clearly seen that the perturbation introduced in EOS of SCP influences the EOS of the dark matter without involving any change in the nature of overall tachyonic scalar field. We further study the interaction among SCP and SDM which plays a major role in the post-recombination era in structure formation imprints on the power spectra of WDM \textsuperscript{[19]}, or Lambda Cold Dark Matter (ΛCDM) \textsuperscript{[20]} models. Interaction is further important around the present epoch in sharply reducing the value of SCP energy density, which suggests a possible solution to the cosmological constant problem, namely, why the present value of cosmological constant is merely $\approx 10^{-123}$ of its value in very early universe. This interaction emerges as a natural corollary of the effective predominance of dark energy through a long evolution of the universe wherein it seems hard to believe that this component, though drove the dynamics of the universe by accelerating it, nevertheless failed to interact with another major co-existing component in form of dark matter.

In Sec. 4, we introduce a normalized Hubble function $E(x)$ as a diagnostic to constrain the interaction strength in degeneracy space of the cosmological parameters in this model. We use the data of the Hubble parameter at different redshifts ($z = 0.1$ to $z = 1.75$) and the present concordance values of density parameters from Cosmic Microwave Background Radiation (CMBR) and Baryon Acoustic Oscillations (BAO) data sets in a spatially flat universe. We also discuss the cosmic age issue in Sec. 5 to highlight the role of interaction in determining it.

2 Phase I— Non-Interacting era—radiation and shifted cosmological parameter

This phase consists of the evolution of two main tachyonic field components in the universe with further resolution into three components. The complete
action given by \((\text{taking } c = 1 \text{ in this paper})\)

\[
\mathcal{A} = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi G} - V(\phi) \sqrt{1 - \partial^i \phi \partial_i \phi} \right)
\]

(1)
couples gravity to tachyon scalar field. Here, \(V(\phi)\) is potential of the field which can be determined for an \(a \text{ priori}\) form of evolution of the scale factor, such as the quasi-exponential expansion \([17]\). The corresponding field Lagrangian is given as

\[
\mathcal{L}(\phi, \partial^i \phi) = -V(\phi) \sqrt{1 - \partial^i \phi \partial_i \phi}.
\]

(2)
The energy-momentum stress tensor for the field defined as

\[
T^{ik} = \frac{\partial \mathcal{L}(\phi, \partial^i \phi)}{\partial \partial_i \phi} \partial^k \phi - g^{ik} \mathcal{L}(\phi, \partial^i \phi)
\]

(3)
yields energy density and pressure, respectively, as

\[
\rho(\phi) = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}
\]

(4)
and

\[
P(\phi) = -V(\phi) \sqrt{1 - \dot{\phi}^2}.
\]

(5)

For spatially homogeneous tachyonic scalar field, (4) and (5), respectively, become

\[
\rho(\phi) = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}; \quad P(\phi) = -V(\phi) \sqrt{1 - \dot{\phi}^2}
\]

(6)
where, and henceforth in this paper, an overdot implies the derivative with respect to time.

We break the energy density and pressure of tachyonic scalar field into two components \(A\) and \(B\), which do not interact mutually. Thus, \(P = P_A + P_B\) and \(\rho = \rho_A + \rho_B\). The first term in pressure is \(P_A = \frac{\dot{\phi}^2 V(\phi)}{\sqrt{1 - \dot{\phi}^2}}\) and we take it as
the radiation pressure. Then energy density of this component (radiation) is
\[ \rho_A = \frac{3\dot{\phi}^2 V(\phi)}{\sqrt{1-\dot{\phi}^2}} \]
with EOS \( w_r = 1/3 \). Next, for the second component, we have
\[ \rho_B = \frac{(1-3\dot{\phi}^2)V(\phi)}{\sqrt{1-\dot{\phi}^2}}; \quad P_B = -\frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} \tag{7} \]
with its EOS \( w_B = -1/(1-3\dot{\phi}^2) \).

It can be seen that as \( \dot{\phi}^2 \to 0 \), \( w_B \to -1 \), and therefore, we call the component \( B \) as ‘Shifted Cosmological Parameter’ (SCP) with perturbed EOS undergoing a shift \( \eta = -3\dot{\phi}^2/(1-3\dot{\phi}^2) \).

The SCP gets further decomposed into two components \( B_1 \) and \( B_2 \). Hence, \( \rho_B = \rho_{B_1} + \rho_{B_2} \) with \( \rho_{B_1} = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} \) and \( \rho_{B_2} = \frac{-3\dot{\phi}^2 V(\phi)}{\sqrt{1-\dot{\phi}^2}} \). The corresponding pressure components are given as \( P_{B_1} = -\frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} \) and \( P_{B_2} = 0 \), with total SCP pressure \( P_B = P_{B_1} + P_{B_2} \). Since the EOS of component \( B_1 \) is \( w_{B_1} = -1 \), therefore, we recognize it as the true cosmological constant with \( \rho_\lambda = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} \) and \( P_\lambda = -\frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} \). The second component \( B_2 \) is a pressure-less exotic matter with \( w_{B_2} = 0 \) and negative energy density \( \rho_{B_2} = \rho_m = \frac{-3\dot{\phi}^2 V(\phi)}{\sqrt{1-\dot{\phi}^2}} \).

In the absence of interaction, the laws of conservation of energy for each component—radiation, cosmological constant and exotic matter—are given, respectively, as
\[ \dot{\rho}_r + 3H(1+w_r)\rho_r = 0 \tag{8} \]
\[ \dot{\rho}_\lambda + 3H(1+w_\lambda)\rho_\lambda = 0 \tag{9} \]
\[ \dot{\rho}_m + 3H(1+w_m)\rho_m = 0 \tag{10} \]
where \( H(t) = \dot{a}(t)/a(t) \) is the Hubble parameter with \( a(t) \) as the scale factor at some epoch \( t \) lying in this phase. Using redshift \( z \) scaling as \( 1+z = a_0/a(t) \) with the present scale factor \( a_0 \), the solution of (8) with \( w_r = 1/3 \) is obtained as
\[ \rho_r = \rho_r^i \left( \frac{1+z}{1+z_i} \right)^4 \tag{11} \]
where $\rho^i_r$ and $z_i (\leq z)$ are the energy density of radiation and redshift, respectively, at the epoch $t_i$ when matter begins to interact with cosmological constant and the next phase starts.

Similarly, from (9)

$$\rho_\lambda = \rho^i_\lambda = \text{constant}$$

(12)

and from (10) we have $\rho_m$, with $w_m = 0$, scaling as

$$\rho_m = \rho^i_m \left( \frac{1 + z}{1 + z_i} \right)^3.$$  

(13)

In phase I, matter does not play effective role in the evolution of universe owing to the dominant counterpart of radiation. Thus, the Friedmann equation for spatially flat ($k = 0$) universe in the non-interacting phase is given as

$$H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} [\rho_r + \rho_m + \rho_\lambda].$$

(14)

In terms of (11), (12) and (13), it changes to the form

$$H^2(z) = H^2_i \left[ \Omega^i_r \left( \frac{1 + z}{1 + z_i} \right)^4 + \Omega^i_m \left( \frac{1 + z}{1 + z_i} \right)^3 + \Omega^i_\lambda \right]$$

(15)

where $H_i$, $\Omega^i_n (= \rho^i_n/\rho^i_c)$ and $\rho^i_c (= 3H^2_i/8\pi G)$ are, respectively, the Hubble parameter, density parameter (with subscript $n$ denoting each component) and critical energy density at the epoch $z_i$.

3 Phase II—Interacting era—shifted cosmological parameter and shifted dust matter

By the end of the non-interacting era at $t = t_i$, radiation loses strength and ceases to influence the large scale dynamics of the universe appreciably. In the subsequent interacting era (phase II), thus, the same tachyonic scalar
field breaks into two components (cosmological constant and matter). The energy density and pressure of cosmological constant are given as

\[ \rho'_\lambda = V(\phi)\sqrt{1 - \partial^i \phi \partial_i \phi}; \quad P'_\lambda = -V(\phi)\sqrt{1 - \partial^i \phi \partial_i \phi} \]  

with \( w'_\lambda = -1 \), (here and henceforth in this paper, a prime denotes the quantities pertaining to the interacting phase), and for matter as

\[ \rho'_m = V(\phi)\frac{\partial^i \phi \partial_i \phi}{\sqrt{1 - \partial^i \phi \partial_i \phi}}; \quad P'_m = 0 \]

with \( w'_m = 0 \).

If we introduce a small perturbation \( \varepsilon(t) \) in the EOS of cosmological constant, the new EOS becomes \( w'_\lambda = -1 + \varepsilon(t) \). Due to this change, the new incarnation of this component is called the shifted cosmological parameter (SCP). Although \( \varepsilon(t) \) is a function of time in general, yet here, we assume it to be a very small constant. Considering energy density of SCP, \( \rho'_\lambda = V(\phi)\sqrt{1 - \partial^i \phi \partial_i \phi} \), its pressure can be given as

\[ P'_\lambda = -V(\phi)\sqrt{1 - \partial^i \phi \partial_i \phi} + \varepsilon V(\phi)\sqrt{1 - \partial^i \phi \partial_i \phi}. \]

Thus, pressure of the second component, namely, shifted dust matter (SDM) is

\[ P'_m = -\varepsilon V(\phi)\sqrt{1 - \partial^i \phi \partial_i \phi} \]

whose EOS now becomes (with its energy density given in (17))

\[ w'_m = -\frac{\varepsilon}{\partial^i \phi \partial_i \phi} + \varepsilon \]

while the EOS of the total tachyonic scalar field is

\[ w_{\text{tach}} = (\partial^i \phi \partial_i \phi - 1). \]

Here, we consider spatially homogeneous scalar field, thus making for SCP, \( \rho'_\lambda = V(\phi)\sqrt{1 - \dot{\phi}^2} \) and \( P'_\lambda = (\varepsilon - 1)V(\phi)\sqrt{1 - \dot{\phi}^2} \), while for SDM, \( \rho'_m = V(\phi)\dot{\phi}^2(t)(1 - \dot{\phi}^2)^{-1/2} \) and \( P'_m = -\varepsilon V(\phi)(1 - \dot{\phi}^2)^{1/2} \) with \( w'_m = -\varepsilon/\dot{\phi}^2 + \varepsilon \).

In order to have negative pressure in the SCP component the perturbation condition \( \varepsilon < 1 \) must be satisfied. This is because, if \( \varepsilon \geq 1 \), dark energy will lose its effective role in causing acceleration. Thus, the following inequalities must hold with \( \dot{\phi}^2 << 1 \).

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(i) If \(0 < \varepsilon < 1\) then \(P'_m < 0\) with \(w'_m < 0\), and \(P'_\lambda < 0\) with \(w'_\lambda < 0\).

(ii) Alternatively, if \(\varepsilon < 0\) then \(P'_m > 0\) and \(P'_\lambda < 0\) (since \(w'_\lambda < -1\) in this case, the SCP behaves like phantom energy).

In case (i) above, it can be seen that with \(\varepsilon > 0\) both SCP and SDM act gravitationally in the similar way, while in case (ii), with \(\varepsilon < 0\), their respective roles in the dynamics of the universe are mutually opposite.

We have two simple justifications for calling upon the role of interaction among these components. First, we emphasize that it seems highly unlikely that the cosmological constant, even though, now occupies a “larger than life” status (with its present value of density parameter \(\approx 0.73\)) and also admittedly drives the acceleration in form of dark energy, and yet, it must lie dormant without falling into interaction with its close counterpart (matter).

Second, it is worthwhile to recall here that the cosmological coincidence problem is hitherto one of the major unsolved issues in modern cosmology. Since their densities grow with evolution of universe at different rates, there should be some fine-tuning among the components. To solve this issue, several authors have attempted to consider the possible interaction between dark energy and dark matter \([21, 22, 23, 24, 25, 26, 27]\). Recently, it has also been proposed that the interaction can sharply cut down the value of the cosmological constant during a middle phase of the universe, sandwiched between two non-interacting phases, and thus solve the cosmological constant problem \([28]\). As another dividend of such decaying cosmological constant, a mechanism has been suggested to generate dark matter \([29]\) to siphon off energy. We study below a possible form of interaction between SCP and SDM, and the consequent scaling of their energy densities as the universe evolves.

The laws of conservation of energy for SDM and SCP, allowing the interaction between them in phase II, are given as

\[
\dot{\rho}'_m + 3H(1 + w'_m)\rho'_m = Q
\]  

\[
\dot{\rho}'_\lambda + 3H(1 + w'_\lambda)\rho'_\lambda = -Q
\]

respectively, where the interaction term \(Q\) is assumed to behave as

\[
Q = \gamma \dot{\rho}'_m
\]
with $\gamma$ as the constant of proportionality. Using (24) in (22) we obtain

$$\frac{\dot{\rho}_m}{\rho_m} = -\frac{3}{1 - \gamma} \left(1 - \frac{\dot{\varepsilon}}{\dot{\phi}^2} + \varepsilon\right) \frac{\dot{a}}{a}.$$  \hspace{1cm} (25)

The present behaviour of the accelerating universe appears very similar to inflation in its early stage. Therefore, we can take $\dot{\phi}^2 \approx 0$ (constant), just like the early inflationary era which was dominated by cosmological constant (or slow-roll of a scalar field with $\dot{\phi}^2 \ll V(\phi)$) giving rise to an exponential expansion.

Solving (25), we get

$$\rho'_m = \rho'_m \left(\frac{a_0}{a}\right)^{\alpha}$$  \hspace{1cm} (26)

with

$$\alpha = \frac{3}{1 - \gamma} (1 + \varepsilon - \varepsilon/\dot{\phi}^2).$$  \hspace{1cm} (27)

Here, we see that as $\varepsilon \to 0$, $\rho'_m = \rho'_m \left(\frac{a_0}{a}\right)^{\frac{3}{1 - \gamma}}$, as may be obtained in case of $w_m = 0$ (interacting but normal dust matter). In the similar way, we calculate $\rho'_\Lambda$ from (23) by re-writing it as

$$\dot{\rho}'_{\Lambda} = -\frac{\dot{a}}{a} \left[3\varepsilon \rho'_\Lambda - \gamma \rho'_m \alpha \left(\frac{a_0}{a}\right)^{\alpha}\right].$$  \hspace{1cm} (28)

Since $w'_m + w'_\Lambda = -1 - \varepsilon/\dot{\phi}^2 + 2\varepsilon$, we have the inequality $w'_m + w'_\Lambda \neq w_{tach}$. Thus, $w'_m + w'_\Lambda = w_{tach} - \delta$ with $\delta = \varepsilon/\dot{\phi}^2 + \dot{\phi}^2 - 2\varepsilon$.

For the SCP, solution of (28) is given as

$$\rho'_\Lambda = \rho'_\Lambda x^{3\varepsilon} - \gamma \rho'_m \frac{(1 - \varepsilon/\dot{\phi}^2 + \varepsilon)(x^\alpha - x^{3\varepsilon})}{1 - \varepsilon/\dot{\phi}^2 + \gamma \varepsilon}$$  \hspace{1cm} (29)

where $x = a_0/a = 1 + z$, (with $0 \leq z \leq z_i$ in phase II) and, particularly, in the absence of perturbation ($\varepsilon \to 0$), we have $\rho'_\Lambda \to \rho'_\Lambda - \gamma \rho'_m (x^{3/1 - \gamma} - 1)$. This, further, gives $\rho'_\Lambda = \rho'_\Lambda$ in the absence of interaction ($\gamma = 0$) as expected. The scaling behaviour of energy density of SCP indicates a possible solution to the cosmological constant problem stating as to why the present value of this constant is very small compared to that at the very early epoch following the big-bang.
The (00) Friedmann equation in this phase, as (15) in phase I, for spatially flat universe \((k = 0)\) becomes

\[
H^2(x) = H_0^2 \left[ \Omega_m^0 x^\alpha + \Omega_\Lambda^0 x^{3\varepsilon} + \frac{\Omega_m^0 \gamma (1 - \varepsilon/\dot{\phi}^2 + \varepsilon)}{1 - \varepsilon/\dot{\phi}^2 + \gamma \varepsilon} (x^{3\varepsilon} - x^\alpha) \right]
\] (30)

with \(H_0, \Omega_m^0\) and \(\Omega_\Lambda^0\) refer to the present values of the Hubble parameter, and density parameters for matter and cosmological constant respectively. We can re-write (26) and (29) in terms of redshift and determine the epoch of equality \(z_{eq}\) at which \(\rho'_m = \rho'_\Lambda\) from

\[
z_{eq} = \left[ \frac{\Omega_\Lambda^0 + \beta \Omega_m^0}{\Omega_m^0 (1 + \beta)} \right]^{\frac{1}{3\varepsilon}} - 1
\] (31)

where

\[
\beta = \frac{\gamma (1 - \varepsilon/\dot{\phi}^2 + \varepsilon)}{1 - \varepsilon/\dot{\phi}^2 + \gamma \varepsilon}.
\] (32)

\(\alpha\) is given by (27), and \(\Omega_\Lambda^0\) and \(\Omega_m^0\) are the present values of density parameters of cosmological constant and matter, respectively. Here, we take \(\Omega_\Lambda^0 = 0.73\) and \(\Omega_m^0 = 0.27\) or \(\Omega_\Lambda^0/\Omega_m^0 \approx 2.7037\) and the values of \(\gamma\) and \(\varepsilon\) can be calculated from observations discussed next in Sec. 4. In the absence of perturbation and interaction both, \(z_{eq} \approx 0.3932\). On the other hand, if we consider a particular \(z = z'\) when the values of \(\dot{\rho}'_\Lambda\) and \(\dot{\rho}'_m\) are equal, then we get

\[
\frac{\dot{\rho}'_m}{\dot{\rho}'_\Lambda} = \frac{\varepsilon (1 - \gamma)}{(1 + \gamma)(1 - \varepsilon/\dot{\phi}^2 + \varepsilon)}.
\] (33)

From (33), we see that in the absence of perturbation \((\varepsilon = 0)\) the ratio \(\rho'_m/\rho'_\Lambda = 0\) which is un-physical. This implies that in case of unperturbed EOS, the rates of fall of energy densities of these components cannot become equal. Alternatively, along with \(\gamma < 1\), both the components behave like SCP and SDM in this phase.
4  Diagnostics and calculation of interaction strength in the three-parameter space $(\gamma, \varepsilon, \dot{\phi}^2)$

Recently, Sahni et al \[31\] introduced the redshift dependent function

$$O_m(x) = \frac{E^2(x) - 1}{x^3 - 1}$$  \hspace{1cm} (34)

where $E(x) = H(x)/H_0$ is the normalized Hubble function as derived from (30). We attempt to find an approach to constrain the interaction strength between cosmological constant and matter by using $E(x)$. Squaring it, we have

$$E^2(x) = \Omega_m^0 \varepsilon x^\alpha + \Omega_\Lambda^0 x^3\varepsilon + \frac{\Omega_m^0 \gamma (1 - \varepsilon/\dot{\phi}^2 + \varepsilon)}{1 - \varepsilon/\dot{\phi}^2 + \gamma \varepsilon} (x^3\varepsilon - x^\alpha)$$  \hspace{1cm} (35)

Using (35), (34) becomes

$$O_m(x) = \frac{(1 - \gamma)(1 - \varepsilon/\dot{\phi}^2)\Omega_m^0(x^\alpha - x^3\varepsilon) + (1 - \varepsilon/\dot{\phi}^2 + \varepsilon)(x^3\varepsilon - 1)}{(1 - \varepsilon/\dot{\phi}^2 + \varepsilon)(x^3 - 1)}$$  \hspace{1cm} (36)

It is clear that as $\varepsilon \to 0$ and $\gamma \to 0$, $O_m(x) \to \Omega_m^0$ which is just as expected.

From the normalized Hubble function (35), we calculate the difference $\Delta E^2(x_i, x_j) = E^2(x_i) - E^2(x_j)$ for the pair of two different redshifts $x_i$ and $x_j$ as

$$\Delta E^2(x_i, x_j) = \Psi(x_i^\alpha - x_j^\alpha) + \left( \Omega_\Lambda^0 + \frac{\gamma \Omega_m^0 (1 - \varepsilon/\dot{\phi}^2 + \varepsilon)}{1 - \varepsilon/\dot{\phi}^2 + \gamma \varepsilon} \right) (x_i^3\varepsilon - x_j^3\varepsilon)$$  \hspace{1cm} (37)

where

$$\Psi = \frac{\Omega_m^0 (1 - \gamma)(1 - \varepsilon/\dot{\phi}^2)}{1 - \varepsilon/\dot{\phi}^2 + \gamma \varepsilon}.$$  \hspace{1cm} (38)

In case $\varepsilon \to 0$, we obtain

$$\Delta E^2(x_i, x_j) = (1 - \gamma)\Omega_m^0 (x_i^{3/1-\gamma} - x_j^{3/1-\gamma}).$$  \hspace{1cm} (39)

For a given pair of redshifts, the values of $\Delta E^2(x_i, x_j)$ can be calculated using data set of the Hubble parameter observations at high- and low-z.
epochs, both. The present value of the matter density parameter $\Omega_m^0$ is ascertained from the concordance analysis of the Cosmic Microwave Background Radiation (CMBR) power spectrum and Baryon Acoustic Oscillation (BAO) data using the condition of spatial flatness. The redshift pairs used in these calculations must, however, be sufficiently spaced over a wide range so that the key-role of $\gamma$, and thus of $Q$ having the form [24], in structure formation could be understood well. The reason is that the form and strength of interaction between the components in phase II of the universe, mainly cosmological constant and matter, substantially influence the growth of structures, and thus may be easily constrained by matching with the available structure surveys and power spectra of the WDM or $\Lambda$CDM models vis a vis our approach adopted here. The data about the precise measurements of $H_0$ will further break the degeneracy in the cosmological parameter space [32].

Therefore, with our choice of the set of values of $H(z)$ at four epochs: at $z = 0.1$, $0.4$, $1.3$, and $1.75$, the values of $H(z) = 69 \pm 12$, $95 \pm 17$, $168 \pm 17$ and $202 \pm 40$ km s$^{-1}$ Mpc$^{-1}$ respectively [33, 34]. The present value of Hubble parameter is used as $H_0 = 73.8 \pm 2.4 \approx 73.8$ km s$^{-1}$ Mpc$^{-1}$ [35] along with the present value of the density parameter of matter component $\Omega_m^0 = 0.272$ [30]. Thus, correspondingly we have $x_1 = 1.1$, $x_2 = 1.4$, $x_3 = 2.3$ and $x_4 = 2.75$ for these redshifts, and the squared normalized Hubble function may be calculated as:

$$E^2(x_1) = 0.874, \quad E^2(x_2) = 1.657, \quad E^2(x_3) = 5.182, \quad E^2(x_4) = 7.492$$

with their differences

$$\Delta E^2(x_1, x_2) = -0.783, \quad \Delta E^2(x_1, x_3) = -4.308,$$

$$\Delta E^2(x_1, x_4) = -6.618, \quad \Delta E^2(x_2, x_3) = -3.525,$$

$$\Delta E^2(x_2, x_4) = -5.835 \text{ and } \Delta E^2(x_3, x_4) = -2.310$$

These data lead to determine six values of the proportionality constant $\gamma$ through following expressions

$\Psi [(1.1)^\alpha - (1.4)^\alpha] + (1 - \Psi)[(1.1)^{3\alpha} - (1.4)^{3\alpha}] = -0.783,$

$\Psi [(1.1)^\alpha - (2.30)^\alpha] + (1 - \Psi)[(1.1)^{3\alpha} - (2.30)^{3\alpha}] = -4.308,$

$\Psi [(1.1)^\alpha - (2.75)^\alpha] + (1 - \Psi)[(1.1)^{3\alpha} - (2.75)^{3\alpha}] = -6.618,$

$\Psi [(1.4)^\alpha - (2.30)^\alpha] + (1 - \Psi)[(1.4)^{3\alpha} - (2.30)^{3\alpha}] = -3.525,$

$\Psi [(1.4)^\alpha - (2.75)^\alpha] + (1 - \Psi)[(1.4)^{3\alpha} - (2.75)^{3\alpha}] = -5.835,$
\[ \Psi [(2.30)^{\alpha} - (2.75)^{\alpha}] + (1 - \Psi)[(2.30)^{3\varepsilon} - (2.75)^{3\varepsilon}] = -2.310. \]

The solution of the above set can be attained by further imposing viable conditions to break degeneracy in the three-parameter space \((\gamma, \varepsilon, \dot{\phi}^2)\) in order to extract information about the single parameter \(\gamma\).

5 Role of interaction in cosmic age

The age of universe at a redshift \(z\) can be calculated by integrating

\[ dt = -\frac{dz}{(1 + z)H(z)} \]  

between the corresponding limits of redshifts \([z, \infty]\). In a two-phase model discussed in this paper, the total age of universe at the present epoch is the sum of duration of the non-interacting phase and that of the interacting one. Therefore, we may find the time duration of universe \(t_i\) in phase I, from (15) and (40) as

\[ t_i = \int_{z_i}^{\infty} \frac{dz}{(1 + z)H_i \left[ \Omega_m^i \left( \frac{1+z}{1+z_i} \right)^3 + \Omega_r^i \left( \frac{1+z}{1+z_i} \right)^4 + \Omega_\lambda^i \right]^{1/2}} \]  

and in phase II, from (30) (with the total age at present taken to be \(t_0\)), as

\[ t_0 - t_i = \int_{0}^{z_i} \frac{dz}{(1 + z)H_0 \left[ \Omega_m^0 (1 + z)^\alpha + \Omega_r^0 \beta \left\{ (1 + z)^{3\varepsilon} - (1 + z)^\alpha \right\} + \Omega_\lambda^0 (1 + z)^{3\varepsilon} \right]^{1/2}} \]

Thus, the total age of the universe at the present epoch depends on three parameters, \(\gamma, \varepsilon\) and \(\dot{\phi}^2\), that is, \(t_0 = f(\gamma, \varepsilon, \dot{\phi}^2)\). The parameters \(\gamma\) and \(\varepsilon\) can be obtained from observations, while \(\dot{\phi}^2\) is not directly fixed by them. Clearly, the age is directly influenced by interaction, and with the unperturbed EOS of the components in this phase, it is given as

\[ t_0 = \int_{z_i}^{\infty} \frac{dz}{(1 + z)H_i \left[ \Omega_m^i \left( \frac{1+z}{1+z_i} \right)^3 + \Omega_r^i \left( \frac{1+z}{1+z_i} \right)^4 + \Omega_\lambda^i \right]^{1/2}} + \int_{0}^{z_i} \frac{dz}{(1 + z)H_0 \left[ (1 - \gamma)\Omega_m^0 (1 + z)^{3/1-\gamma} + \gamma\Omega_m^0 + \Omega_\lambda^0 \right]^{1/2}} \]
6 Conclusion

In this paper, we have resolved the tachyonic scalar field into its possible components, and studied their evolution in the expanding universe spanned over two phases, that is, the non-interacting phase and the interacting one. In the first phase I, the field breaks into two components, namely radiation and the shifted cosmological parameter (SCP). The SCP is further found to consist of two components—true cosmological constant and some form of exotic pressure-less matter with negative energy density. In this phase, these three components do not interact mutually, and, therefore in Sec. 2, we find their specific evolution with respect to the redshift of the large scale cosmic expansion.

In the following phase II, as examined in Sec. 3, radiation component descending from phase I is suppressed, and the tachyonic field is dominated by two components—matter and true cosmological constant. We introduce a small constant perturbation $\varepsilon$ (indeed, it may be time-dependent, in general, and we would take up such form elsewhere) into the EOS of cosmological constant, which in turn changes the EOS of the matter component as well. These components are then, respectively, known as the shifted cosmological parameter (SCP) and the shifted dust matter (SDM) of phase II, and we allow them to interact by choosing a specific form (24), so as to determine the role of this interaction on their cosmic evolution, and in turn on the dynamics of the universe. We also determine the redshift $z_{eq}$ when the SCP and SDM have equal energy densities and obtain a condition that applies when the corresponding rates of their scaling become equal. This highlights mainly the role of $\gamma$ and $\varepsilon$ in determining the evolution of dark energy in our model.

In Sec. 4, we attempted to frame a normalized Hubble function, earlier used in $O_{m}(x)$ diagnostic, to constrain the interaction between the components of phase II. The low- and high-redshift observations for Hubble parameter and the concordance values of the density parameters from CMBR, BAO data-sets lead us to construct equations for determining the strength of interaction. However, in the three-parameter space ($\gamma, \varepsilon, \dot{\phi}^2$), further conditions must be imposed to extract information about interaction alone. We will attempt to devise such reasonable conditions to this end in future. We also emphasize that the interaction substantially modifies the growth of structures in past, and so, the observations must be included over a sufficiently large range of redshifts in the post-recombination era, co-extensive with the
structure formation, to enable us to examine the distinguishing imprints on the power spectrum. With this view, in a future study, our model may be further developed and compared with the detailed analysis of the power spectra of WDM or ΛCDM models.

Finally, we proceeded to find in Sec. 5 the influence of interaction in phase II in determining the present age of the universe. This was calculated by including the time spans of both phases. Again, re-affirming our findings in Sec. 4, the total cosmic age too exhibits a functional form in the three-parameter space, only to be constrained with appropriate observational and/or a priori conditions. We would endeavour to address these issues in our future work.

7 Acknowledgments

The authors thankfully acknowledge the financial support F. No. 37-431/2009 (SR) received from UGC, New Delhi regarding this work.

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