Network model of passenger railway rolling stock scheduling

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Abstract. Passenger rolling stock scheduling to the passenger train timetable is based on the same type and number of carriages, which is the standard way of passenger trainset assignment. The possibility of train scheduling with a single carriage, a group of carriages or a railcar is topical issue for each carrier engaged in rail passenger transport in order to reduce the required rolling stock fleet. The paper presents an original network model developed for rolling stock scheduling for trainsets. The model makes possible to determine the optimum distribution of rolling stock in trainsets minimizing downtime at stations through the out-of-kilter algorithm. The model developed is applied to the scheduling of trainsets in part of the railway network in Bulgaria served by BDZ Passenger Transport Enterprise at the Plovdiv region. The use of the model allows to save passenger rolling stock and increases the efficiency of passenger rolling stock operation.

1. Introduction

The number of passengers at different times in a 24-hour period is different, so the number of carriages required for each train is determined in compliance of demands. The sample scheduling is performed on the basis of one and the same number and type of carriages. According to the traffic volume served, the so-called Plan of Train Formation (PTF) presets trains that consist of 2-10 carriages, one or two multiple units with different numbers of seats. On the other hand, although two trains can be of one and the same number of carriages, their type could vary depending on the train category. In practice is used train scheduling with one and the same type and number of carriages, which is the standard way to operate passenger carriages and trainsets. This way of train scheduling is not always rational because of long downtimes existing in practice.

It is why train scheduling that considers a carriage, a group of carriages or multiple units is topical for each carrier engaged in passenger transportation by rail in order to reduce the rolling stock fleet needed for train operation.

2. Overview of references

The references on passenger railway transport describe a variety of ways to perform train scheduling. The model in [1] does not consider the number of carriages required while the main problem discussed in [2] is how to reduce the number of trains depending on demands for passenger seats. The method suggested is based on calculating the number of carriages of each type for each train. Another model [3] defines rolling stock scheduling on a single-line for one day aiming to minimize the number of trainsets needed for the Train Timetable (TTT).

The approach using linear programming for scheduling trainsets [4] is complimented in [5] with specificity of train at stations. The two-step optimization model of rolling stock scheduling in [6] concerns the operation of passenger trains with existence of delays and large disturbances in the rail
system, which means that it has been designed in a way to deal with delays. The two-step optimization model is formulated as a Mixed Integer Linear Programming model (MILP).

The optimization framework of railway rolling stock scheduling described in paper [7] is based on a two-step approach that combines the scheduling tasks related to train services, short-term maintenance operations and empty runs. Paper [8, 9] presents an analytical comparison between two approaches within the operational and real-time planning phase. The first one is known as Mixed Integer Linear Program (MILP) while the second one is a new method, an extension of a recently introduced MILP, which uses a column and row generation approach. The authors have validated the theoretical methods developed in this paper on the railway networks of Denmark and the Netherlands.

The problem of scheduling in the references examined above is solved by using linear programming. However, this paper examines the rail transport network where passenger train traffic and rolling stock scheduling take place through a structure of respective graphs that give a possibility to use the well-developed mathematic apparatus of the graph theory and especially the so-called out-of-kilter algorithm [10, 11].

3. **Model of rolling stock scheduling to trains**

There are two mathematical models appropriate to perform rolling stock scheduling to trains: the maximum flow through a network with minimum cost and out-of-kilter algorithm. Both of them allow to find different options depending on the constraints with choosing optimality criteria, which after being matched can determine the optimal solution. The model of rolling stock scheduling developed on the basis of out-of-kilter algorithm passes through the following steps (figure 1):

![Figure 1. Algorithm of rolling stock scheduling to trains.](image)

The number of stations, where scheduling is performed, is labelled with „G” in figure 1 and determined in step 1. The iterative procedure of scheduling at each current station „g” involving steps 2-5 is performed until all stations included in specified number „G” are considered - g=1…G.

3.1. **Determination of stations to perform scheduling**

The setting of this task in practice requires to allocate rolling stock to trains with the given TTT and PTF. It is therefore necessary to define stations G where rolling stock scheduling is performed. G stations can be considered a subset of the set of all stations in the polygon examined where at least one of the following conditions is met: to be terminal or starting stations for one or more trains or a place where carriages are attached to or detached off one or more trains. In cases of scheduling multiple units (multiple units), the algorithm is applied to the whole network, and in cases of scheduling carriages it is applied to railway lines or sections. By the distribution of trains depending on grouping carriages, the task is reduced to solving a number of subtasks with decreased dimensionality as each of them would give a solution of rolling stock scheduling to the trains that serve a certain section.
3.2. **Determination of the balance of rolling stock units**

Depending on scheduling, a unit of rolling stock is understood as a railcar, a group of carriages or a carriage. Passenger rolling stock (carriages or multiple units) does not have dead mileage, which is taken into account when designing the TTT and respective PTF, so the number of arriving carriages and multiple units is equal to that of departing ones. Unlike the number of rolling stock units, the number of arriving and departing trains at a given station can be different.

3.3. **Determination of minimal “downtime” technologically required to change the direction of trainset running**

To ensure the real application of scheduling, it is necessary that the values of technologically needed minimal “downtimes” are greater than any predetermined minimal permissible value depending on the station type and rolling stock handling, namely: the downtime is only for scheduling (according to standards), cleaning and equipment of trains at the station or technical maintenance and arrangement of carriages and multiple units at the technical station. This is achieved by introducing additional restrictive condition assumed in the form of:

\[
T_{stij} \min = T_{ij}(a,T_{st}),
\]

where: \( T_{stij} \min \) is the estimated value of required downtime of trainset from the moment of its release from train \( i \) to the moment of its inclusion to train \( j \); \( T_{ij}(a,T_{st}) \) – a predetermined value of the downtime of trainset depending on the size of carriage group \( a \) and the type of station \( T_{st} \) and technological operations.

3.4. **Determination of mandatory scheduling**

For example, in relatively short intervals between the arrival and departure of a suburban passenger train to a given destination, it is desirable to use electric multiple units for a number of train pairs, which facilitates the specialization of station tracks and organization of their service. Such a type of scheduling leads to higher technology performance in train timetable implementation and can be achieved by setting zero downtimes in scheduling to certain trains or by artificially raising the values of arrival-departure interval of trains.

3.5. **Applying the out-of-kilter algorithm for scheduling trains with carriages**

The advantage of this algorithm in comparison to other network algorithms is that it is more generalized, more computerized and allows to solve problems with a large number of variables and constraints. Since the balance of rolling stock is provided in advance for the station, the carriages or multiple units of arriving train \( i \) can participate entirely or in groups of carriages (multiple units) in the departing trains \( j \).

The criterion function has the following form:

\[
\min T = \sum_{i=1}^{n} T_{k} \rightarrow \min
\]

\[
\frac{\min T = \sum_{k=1}^{k} T_{k}}{T_{k} = \sum_{i=1}^{n} T_{k} \rightarrow \min}
\]

where \( T \) is total carriage hours of downtime of all rolling stock units involved in scheduling;

\( F_{kis}, F_{ij}, F_{kjt}, F_{kts} \) – number of rolling stock units on links \( s-i, i-j, j-t, t-s \);

\( T_{kis}, T_{kij}, T_{kjt}, T_{kts} \) – downtime of rolling stock units on links \( s-i, i-j, j-t, t-s \);

\( n \) – number of rolling stock units arriving with trains;

\( m \) – number of rolling stock units departing with trains;

\( k \) – type of rolling stock unit involved in scheduling at a given station.

In this case, due to the balance provided per unit of rolling stock for each station, \( n = m \).

The group of constraints related to keeping the flow at each node:
\[ F_{ksi} = \sum_{j=1}^{m} F_{kij}, \text{ for each fixed node } i, \]  
\[ \sum_{i=1}^{n} F_{kij} = F_{ksi}, \text{ for each fixed node } j, \]  
\[ F_{kzs} = \sum_{i=1}^{n} F_{ksi}, \text{ for node } s, \]  
\[ \sum_{j=1}^{m} F_{kjt} = F_{kts}, \text{ for node } t. \]  

The group of constraints related to capacity of links:

\[ L_{ksi} \leq F_{kxi} \leq U_{ksi}, \text{ with } F_{kxi} > 0, \]  
\[ L_{kij} \leq F_{kij} \leq U_{kij}, \text{ with } F_{kij} > 0, \]  
\[ L_{kjt} \leq F_{kjt} \leq U_{kjt}, \text{ with } F_{kjt} > 0, \]  
\[ L_{kts} \leq F_{kts} \leq U_{kts}, \text{ with } F_{kts} > 0, \]  

where \( U_{ksi}, U_{kij}, U_{kjt}, U_{kts} \) – maximum capacity of links \( s-i, i-j, j-t, t-s \); \( L_{ksi}, L_{kij}, L_{kjt}, L_{kts} \) – minimum capacity of links \( s-i, i-j, j-t, t-s \).

The constraint related to the downtime of the \( k \)-th type of rolling stock unit:

\[ T_{ksi} = 0, T_{kij} = 0, T_{kts} = 0, T_{kjt} \geq T_{ijmin}, \]  

for each \( i, j \) where \( T_{ijmin} \) is minimum necessary downtime of a rolling stock unit with scheduling from train \( i \) to train \( j \), min.

The downtime of carriages is defined as a difference between the departure time \( t_{dst} \) of any type of rolling stock unit at a given station and the time of arrival \( t_{at} \) of any type of rolling stock unit at the same station by one of the following formulas:

\[ T_{ij} = t_{dst} - t_{at}, \text{ min, with } t_{dst} > t_{at}; \]  
\[ T_{ij} = 1440 - (t_{at} - t_{dst}), \text{ min, with } t_{dst} \leq t_{at}. \]

This downtime is graphically shown in figure 2 (a) when the train arrives earlier than the departing one and in figure 2 (b) when the train arrives later than the departing one.

**Figure 2.** Determination of downtime \( T_{ij} \) per rolling stock unit.
While checking if the condition in formula 7 is satisfied, the following cases are possible:

- with $T_{ij} \geq T_{ij\text{min}}$ the so-specified interval remains;
- with $T_{ij} < T_{ij\text{min}}$ the interval for a given train $T_{ij} = 1440 + (t_{dtj} - t_{ati})$, min, figure 2 (c).

The use of out-of-kilter algorithm requires to make a graph structure (figure 3) in the following sequence:

- A fictitious main manufacturer node $S$, and a fictitious main user node $t$ are introduced. All trains arriving at the station are represented as nodes manufacturers of rolling stock units $- P_1, P_2, \ldots, P_n$. All trains departing from the station are represented as nodes users of rolling stock units $- K_1, K_2, \ldots, K_m$.
- Directed links are constructed as follows:
  1. From node $S$ to all nodes manufacturers with parameters: maximum capacity $U_{ksi} = 1$ equal to the number of the rolling stock units arriving by the train represented with the respective node, $L_{ksi} = 0$, $T_{ksi} = 0$ and $F_{ksi} = 0$ (figure 3, link $s-P_i$);
  2. From all nodes manufacturers to all nodes users with parameters: maximum $t$ capacity $U_{kij} = \infty$, minimum capacity $L_{kij} = 0$, downtime $T_{kij}$ equal to the value of $T_{ij}$ for this pair of trains and flow $F_{kij} = 0$;
  3. From all nodes users to nodes with parameters: maximum $U_{kjt}$ throughput, equal to the necessary number of carriages to service the corresponding train for the given node, $L_{kij} = 0$, $T_{kjt} = 0$ and $F_{kjt} = 0$ (figure 3, link $K_1-t$);
  4. Closing link from node $L$ to node $S$ with parameters: maximum capacity equal to the total production of rolling stock units, minimum capacity $L_{kts}$ equal to the total consumption of a unit of rolling stock, $T_{kts}$ and $F_{kts} = 0$ (figure 3, link $t-S$). Since the task is balanced, the values of the maximum and minimum capacities of closing link must be equal to each other.

With using the out-of-kilter algorithm in excel, $(U_{kij}, L_{kij}, T_{kij})$ are introduced as input and $F_{kij}$ is determined.

3.6. Construction of scheduling

The final result of applying the out-of-kilter algorithm is optimal allocation of carriages between the trains at each station and these are the links where the flows are different from zero. To construct scheduling, it is necessary to walk around all these nodes at different stations in one or several cycles and to determine the sequence of operation of carriages (multiple units) included in these cycles.
3.7. Further analysis and determination of the indicators of scheduling obtained

The problem solution obtained by applying the out-of-kilter algorithm is not the only one possible. In order to enable comparison of the final results obtained to the initial option or of alternative solutions to each other, it is necessary to determine the values of the following quantitative indicators: the total number of rolling stock /carriages or multiple units/ involved in scheduling, the average vehicle mileage and the coefficient of their use in running.

4. Implementation of the scheduling model with diesel multiple unit (DMU) and electric multiple unit (EMU) rolling stock

For comparison, the described methods will be applied to the actual TTT and PTF in the period from December 2018 to December 2019 in the BDZ Passenger Transport Enterprise in Plovdiv, which has available 5 DMU of series 10 with 123 seats, 5 EMU of series 30 with 3 carriages and 190 seats and 5 EMU of series 31 with 4 carriages and 254 seats. The trains set in the TTT and PTF to be provided with multiple units are 119.

The stations where scheduling will take place are sixteen as shown in figure 4. These stations are of different category [12] related to the service of trains and the carriages they consist.

![Figure 4. Map of stations where multiple units are scheduled.](image-url)

The number of arriving and departing trains at most stations is equal and consists of one multiple unit. The station of Plovdiv is an exception because train 10141 departing at 6:10 has two multiple units and train 10144 arriving in Plovdiv at 19:44 is also of 2 multiple units.

The necessary minimal downtime for equipment and cleaning is 50 minutes determined as work [13] and the minimal downtime to turn the direction of running is 5 minutes.

The downtime for obligatory scheduling at Plovdiv station has to be set 0 for the trains serving suburban section Plovdiv-Asenovgrad and this will be the first multiple unit 31 in the samples schedule. The obligatory DMU scheduling will be set for the trains in sections Plovdiv-Peshtera, Plovdiv-Panagyurishte and Dimitrovgrad-Podkova because they are not electrified.

The use of the out-of-kilter algorithm requires to form a graph structure for each station as nodes manufacturers are equal to the number of arriving DMUs and EMUs, and nodes users are equal to the number of departing units. A procedure to create the excel files of graph structure has been developed. These files are used as input data in excel procedure for the out-of-kilter algorithm and the scheduling is built on the basis of the obtained result.

To make scheduling, it is necessary to walk around all trains in several cycles and in succession starting from the earliest departure time from one of the main passenger stations. Each multiple unit of the train is presented as a node of departure and a node of arrival. As shown in figure 5, DMU multiple
unit 1 is scheduled to trains 81021, 81202, 81203, 18206 and 18207 and finally to 81206. Its turnover begins and ends in Panagyurishte and the current technical inspection takes place in Plovdiv from 13.00-15.00 PM. In this way, by a number of walking around cycles, the sequence of multiple unit operation is determined on the basis of scheduling developed at different stations and the respective sample schedule is built. The hours of 24-four period are put on the horizontal of sample schedule and multiple units needed to provide scheduling to TTT are shown on the vertical (figure 5). The figure shows the scheduling of DMU 10 and EMU 30 where there is a change in regard to the sample schedules of BDZ PPE. The turnovers of multiple units are given as background in two colours and it can be seen that there are 4 turnovers lasting 24 hours, 2 with a duration of 48 hours.

Figure 5. Sample schedule of multiple units.

Using the TTT and sample schedules of the turnover of multiple units – real and optimized, the parameters to evaluate the two models of scheduling are determined: work in km carriage and usage coefficient, which are given in table 1.

As it can be seen from the parameters determined in sample schedule, the optimized model is significantly better since there are 2 spare units left in reserve. The reserve can be used in case of an accident or breakdown, thus reducing train delays due to broken scheduling. There is also an increase of all three indicators used to assess the quality of rolling stock performance – number, mileage and coefficient of use in traffic.

Table 1. Indicators of implementation of MU scheduling.

|                      | Active scheduling | Optimized scheduling |
|----------------------|-------------------|----------------------|
| Multiple units, number | 15                | 13                   |
| Average mileage, kmwagon$^{-1}$ | 357.62            | 412.63               |
| Usage coefficient in traffic | 0.38            | 0.46                 |
5. Conclusion

The developed model of passenger train scheduling based on the first come, the first served principle where each carriage/multiple unit or main group of carriages is individually assigned gives better results than the standard scheduling of whole trainsets, which has been proved in practice. The model allows to determine the optimal scheduling of trainsets by minimizing the carriage/hours of downtime using the out-of-kilter algorithm.

In order to implement the model, it is necessary to provide database for TTT and PTF for the respective timetable and to predefine the minimal necessary downtime at stations according to the number of carriages and operations carried out at the station depending on its category. It is also necessary to determine the balance of carriages for each station, to establish scheduling mandatory needed for passenger trains depending on the station track development and destinations served; to build the graph structure and apply the optimized model.

The practical implementation of rolling stock scheduling involves the trains along three main lines and their deviations provided with rolling stock by the BDZ Passenger Transport Enterprise in Plovdiv region. The following options are compared: the actual train scheduling for the TTT being in force from 12.12.2018 to 11.12.2019, which is based on one and the same number and type of carriages, and scheduling performed on principle the first come, the first served using the optimization model based on the out-of-kilter algorithm. The timetable includes 119 trains with scheduling at 16 stations, so the railway carrier needs totally 15 DMUs and EMUs. The application of the proposed model has shown that only 13 multiple units are needed for scheduling of the same number of trains. Thus the model leads to railway rolling stock saving and increasing the indicators of its effective usage.

References

[1] Peeters M and Kroon L 2009 Circulation of railway rolling stock: a branch-and-price approach Erasmus University Rotterdam (ERIM Research Report) 32
[2] Schrijver A 1993 Minimum circulation of railway stock CWI Quarterly 6 205–17
[3] Maroti G and Kroon L 2004 Maintenance routing for train units: the transition model Transportation Science 39(4)
[4] Cordeau J, Soumis F and Desrosiers J 2000 A benders decomposition approach for the locomotive and car assignment problem Transportation Science 34 133–49
[5] Cordeau J, Soumis F and Desrosiers J 2006 Simultaneous assignment of locomotives and cars to passenger trains European J. of Operational Research 174 1281–97
[6] Cacchiani V, Caprara A, Galli L, Kroon L, Marotti G and Toth P 2012 Railway rolling stock planning: robustness against large disruptions Transportation Science 46(2) 217
[7] Giacco G, Carillo D, D’Ariano B, Pacciarelli D and Marinc A 2014 Short-term rail rolling stock rostering and maintenance scheduling Transportation Research Proc. 3 651–9
[8] Haahr J, Wagenaar J, Veelenturf L and Kroon L 2016 A comparison of two exact methods for passenger railway rolling stock (re) scheduling Transportation Research Part E: Logistics and Transportation Review 91 15–32
[9] Nielsen L 2011 Rolling Stock Rescheduling in Passenger Railways (Erasmus Universiteit Rotterdam, ERIM Electronic Series Portal) p 284
[10] Ahuja R, Magnati T and Orlin J 1993 Network Flows: Theory, Algorithms and Applications (Prentice Hall Upper Saddle River N.J.)
[11] Phillips D and Garcia-Diaz AI 1981 Fundamentals of Network Analysis (Prentice Hall Englewood Cliffs NJ) p 474
[12] Stoilova S. 2019 Methodology for classification sections of railway network using cluster analysis Proc. 18th Int. Sc. Conf. Engineering for Rural Development (Jelgava Latvia) 886–97
[13] Todorova M 2019 Gert model of a technical station for determining the passenger trainset delay Transport Problems 14(1) 81–94