A fast and efficient method for producing partially coherent sources

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Abstract
A fast, flexible and efficient method for generating partially coherent sources is presented. It is shown that the Schell-model (uniformly correlated) and non-uniformly correlated sources can be produced quickly using a fast steering mirror and low-actuator-count deformable mirror, respectively. The statistical optics theory underpinning the proposed technique is presented and discussed. Simulation results of two Schell-models and one non-uniformly correlated source are presented and compared to the theory to test the new approach.

Keywords: coherence, Fourier optics, random sources

1. Introduction
The ability to precisely control beam shape and polarization just by manipulating spatial coherence has motivated much work into developing methods to generate partially coherent sources. A quick review of the literature reveals numerous approaches; the three most popular and relevant are summarized below.

The first technique exploits the Van Cittert–Zernike theorem (VZT) [1] by using a spatially incoherent source—commonly produced using a coherent source in combination with a rotating ground glass diffuser—to produce the desired partially coherent beam via propagation (typically achieved using lenses) and amplitude filtering [2–5]. Because of the fast-moving diffuser, partially coherent sources can be produced quickly enough to be used in the real-time reduction of atmospheric-turbulence-induced scintillation or in biological applications. However, using the VZT approach, one is generally limited to producing uniformly correlated, or Schell-model sources [6].

The second approach uses an acousto-optic deflector (AOD) driven by an RF modulated signal to produce partially coherent fields [7–9]. In general, the light diffracted from the AOD is the superposition of weighted, Doppler-frequency-shifted components propagating in slightly different directions centered on the Bragg angle. Each component’s weight, frequency, and direction is determined by the spectrum of the applied AOD signal. Since each component propagates at a different temporal frequency, the components add incoherently, producing a partially coherent source. The AOD technique is extremely fast—RF modulations in the MHz are easily possible; however, the technique as presented in [7–9] is limited to producing one-dimensional, Schell-model sources. The authors do comment that two-dimensional Schell-model sources can be produced by using crossed AODs [8, 9].

The third and most popular approach is to use a spatial light modulator (SLM) to actively manipulate the optical field [6, 10–16]. Using this method, one can generate uniformly correlated as well as non-uniformly correlated (NUC) sources. Unfortunately, because of the relatively slow speeds of current SLMs, partially coherent sources cannot be generated quickly enough to be used in the applications mentioned above.
In this paper, a new approach for generating partially coherent sources is presented. The new technique permits any uniformly correlated or Schell-model source to be produced using only a fast steering mirror. The technique also permits NUC sources to be produced using a deformable mirror with very few actuators. In this way, the new method combines the benefits of the VZT, AOD and SLM approaches, namely speed and flexibility.

In the next section, the theory underpinning the new method is detailed. Three example sources—two Schell-model and one NUC source—are presented to show how to apply the technique. Lastly, section 3 presents the simulation results and compares those results to the theoretical predictions to validate the new approach.

2. Methodology

The sufficient condition for a genuine cross-spectral density (CSD) function \( W \) is

\[
W(p_1, p_2) = \int \int_{-\infty}^{\infty} p(v) H(p_1, v) H^*(p_2, v) \, dv,
\]

(1)

where \( H \) is an arbitrary kernel and \( p \) is a non-negative function [17]. Following Gori and Santarsiero [17], let

\[
H(p, v) = \tau(p) \exp[-j2\pi v \cdot g(p)],
\]

(2)

where \( \tau \) is, in general, a complex function and \( g \) is an arbitrary, real vector function. Substituting (2) into (1) and assuming that \( p \) is Fourier transformable yields

\[
W(p_1, p_2) = \tau(p_1) \tau^*(p_2) \hat{p}[g(p_1) - g(p_2)],
\]

where \( \hat{p} \) is the Fourier transform of \( p \). A very popular form for \( W \) is

\[
W(p_1, p_2) = \sqrt{S(p_1)} \sqrt{S(p_2)} \mu(p_1, p_2),
\]

(4)

where \( S \) is the spectral density and \( \mu \) is the spectral degree of coherence [6]. The correspondence of the terms in (3) and (4) is quite clear.

2.1. Random field instance

The goal here is to generate phase screens that produce partially coherent fields of the form in (3). Let a single instance of a partially coherent field be

\[
U_\rho(p) = \sqrt{S(p)} \exp[jg(p) \cdot \phi(p)],
\]

(5)

where \( \phi \) is a vector phase screen. Taking the autocorrelation of (5) yields

\[
(U_\rho(p_1)U_\rho^*(p_2)) = \sqrt{S(p_1)} \sqrt{S(p_2)} \chi(p_1, p_2).
\]

(6)

\( \chi \) is the joint characteristic function of the random variables \( \phi \) at \( p_1 \) and \( p_2 \) and is clearly linked to \( \mu \) and \( \hat{p} \).

2.2. Schell-model sources

A general Schell-model source can be produced by letting \( g = \rho \), where \( \rho = \hat{x} + \hat{y} \) [17]. Substituting \( g \) into (3) reveals

\[
\hat{p}(\rho_1, \rho_2) = \mu(\rho_1, \rho_2) = \chi(\rho_1, \rho_2).
\]

(7)

Expressing \( \chi \) in integral form yields

\[
\mu(\rho_1 - \rho_2) = \int \int \int \int e^{i\rho_1 \phi_1 e^{-i\rho_2 \phi_2}} P(\phi_1, \phi_2) \, d\phi_1 \, d\phi_2,
\]

(8)

where \( \phi_1 = \hat{x} \phi_1 + \hat{y} \phi_2 \) (likewise for \( \phi_2 \)) and \( P \) is the joint probability density function (PDF) of \( \phi_1 \) and \( \phi_2 \).

Since \( \hat{p} \) and \( \mu \) are functions of \( \Delta \rho = \rho_1 - \rho_2 \) so must \( \chi \); therefore, \( \phi_1 = \phi_2 \).

\[
\mu(\Delta \rho) = \int \int \int \int e^{i\Delta \rho \phi} P(\phi) \, d\phi.
\]

(9)

Note that

\[
\mu(0) = \int \int P(\phi) \, d\phi = 1.
\]

(10)

One quickly recognizes (9) as an inverse spatial Fourier transform. Applying the forward Fourier transform to both sides yields an expression for the PDF, namely

\[
P(\phi) = \frac{1}{(2\pi)^2} \int \int \mu(\Delta \rho) e^{-j\Delta \rho \phi} \Delta \rho.
\]

(11)

By the Wiener–Khintchine theorem [1], (11) states that the joint PDF of \( \phi_1 \) and \( \phi_2 \) is equal to the spatial power spectrum of the desired random source.

The field instance that produces a general Schell-model source is given by

\[
U(\rho) = \sqrt{S(\rho)} \exp[j(x\phi_1 + y\phi_2)].
\]

(12)

Note that \( \phi_1 \) and \( \phi_2 \) are random numbers drawn from the joint PDF given in (11). They do not vary spatially as in traditional SLM phase screen approaches [6, 14, 15, 18–21].

The form of (12) implies that a Schell-model source can be produced by incoherently summing many independent randomly tilted waves. This means that any uniformly correlated source can be generated using only a fast steering mirror. This observation that Schell-model sources can be produced from just tilts is consistent with that made by other researchers [8, 22]. Two examples are presented below to elucidate these concepts.

2.2.1. Gaussian Schell-model (GSM) source. Let \( \tau = \sqrt{S} = \exp[-\rho^2/(4\sigma^2)] \) and \( \hat{p} = \mu = \exp[-\Delta \rho^2/(2\delta^2)] \), where \( \sigma \) and \( \delta \) are the source and spatial coherence radii, respectively. Substituting these expressions into (3) or (4) yields

\[
W(p_1, p_2) = \exp\left(-\frac{\rho_1^2 + \rho_2^2}{4\sigma^2}\right) \exp\left(-\Delta \rho^2/2\delta^2\right).
\]

(13)

which is quickly recognized as a GSM source [6, 23]. The joint
PDF of \(\phi_1\) and \(\phi_2\) is found via (11) and is given by

\[
P(\phi) = \frac{\delta^2}{2\pi} \exp \left[ -\frac{\delta^2}{2} (\phi_1^2 + \phi_2^2) \right]
\]

\[
= \frac{\delta}{\sqrt{2\pi}} \exp \left[ -\frac{\delta^2}{2} \phi_1^2 \right] \frac{\delta}{\sqrt{2\pi}} \exp \left[ -\frac{\delta^2}{2} \phi_2^2 \right].
\]  

(14)

Thus, \(\phi_1\) and \(\phi_2\) can be produced very simply by generating sequences of independent, zero-mean, \(1/\delta^2\)-variance Gaussian random numbers. The field instance that produces a GSM source is given in (12).

2.2.2. Bessel–Gaussian-correlated Schell-model (BGSM) source. Let \(\tau = \sqrt{S} = \exp[-\rho^2/(4\sigma^2)]\) and \(\hat{\rho} = \rho = J_0(\beta/\delta\Delta\rho)\exp[-\Delta\rho^2/(2\delta^2)]\), where \(J_0\) is a zeroth-order Bessel function of the first kind and \(\beta\) is a real constant. Substituting these expressions into (3) or (4) yields

\[
W(\rho_1, \rho_2) = \exp \left[ -\frac{\rho_1^2 + \rho_2^2}{4\sigma^2} \right] J_0 \left( \beta/\delta \Delta\rho \right) \exp \left[ -\frac{\Delta\rho^2}{2\delta^2} \right].
\]  

(15)

which is recognized as the CSD function for a BGSM source [6, 24]. The joint PDF of \(\phi_1\) and \(\phi_2\) is found by Fourier transforming \(\mu\) resulting in

\[
P(\phi) = \frac{\delta^2}{2\pi} \exp \left[ -\frac{\delta^2}{2} (\phi_1^2 + \phi_2^2) \right] J_0(\beta\delta\phi),
\]  

(16)

where \(J_0\) is a zeroth-order modified Bessel function of the first kind. Again, the field instance that produces a BGSM source is given in (12).

In contrast to the GSM source discussed above, \(\phi_1\) and \(\phi_2\) for BGSM sources are not independent. Indeed, for general Schell-model sources, \(\phi_1\) and \(\phi_2\) will not be independent. The problem then becomes generating random numbers from an arbitrary joint PDF. One approach is to discretize the two-dimensional PDF—forming the probability mass function (PMF) —and to represent it as a vector. The row of the PMF vector corresponds to a unique combination of \(\phi_1\) and \(\phi_2\); the value in that row is the probability. The cumulative distribution function is then computed and numerically inverted. Instances of \(\phi_1\) and \(\phi_2\) are then produced using the inverse transformation method [25].

2.3. NUC sources

A NUC source has a position-dependent \(\mu\), i.e., \(\mu\) depends on \(\rho_1\) and \(\rho_2\) and not just on \(\rho_1 - \rho_2\). Unfortunately, forming NUC sources requires \(g\) that are nonlinear; therefore, the approach described above for Schell-model sources cannot be used. In fact, the approach to generate NUC source is the opposite of that taken for Schell-model sources—one must first choose a form for the joint PDF of \(\phi\) and then proceed.

Returning to (6) and expressing \(\chi\) in its most general form yields

\[
\rho[g(\rho_1) - g(\rho_2)] = \mu[g(\rho_1) - g(\rho_2)] = \chi[g(\rho_1) - g(\rho_2)]
\]

\[
= \int_{-\infty}^{\infty} e^{i\varphi \cdot \xi} \Phi(\varphi) d\varphi,
\]  

(17)

where \(g_1 = \tilde{x}g_x(\rho_1) + \tilde{y}g_y(\rho_1)\) and similarly for \(g_2\). Note that because \(\hat{\rho}\) and \(\mu\) are functions of \(g_1 - g_2\), \(\chi\) must be as well. This means that \(\phi_1\) must equal \(\phi_2\).

Although \(P\) cannot be found directly, the near Fourier transform relationship between \(P\) and \(\mu\) can provide insight into \(P\)’s form. Here, for convenience, it is assumed that \(\phi\) is Gaussian distributed and zero mean. \(P\), in this case, is the joint Gaussian PDF and the above integral (i.e., \(\chi\)) is quite easy to evaluate [26]:

\[
\chi(\rho_1, \rho_2) = \exp \left[ -\frac{1}{2} \Delta G^T (\Phi \Phi^T) \Delta G \right].
\]  

(18)

where \(\Delta G\) and \(\Phi\) are

\[
\Delta G = \begin{bmatrix} g_1(\rho_1) - g_1(\rho_2) \\ g_2(\rho_1) - g_2(\rho_2) \end{bmatrix}, \quad \Phi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}.
\]  

(19)

By assuming that \(\phi\) is Gaussian distributed, it is quite clear that \(\hat{\rho}\) must take a Gaussian form, namely

\[
\hat{\rho}(s) = \exp \left[ -\frac{s^2}{2\delta^2} \right].
\]  

(20)

Substituting \(g_1 - g_2\) into the expression for \(\hat{\rho}\) and simplifying produces

\[
\hat{\rho}(g_1 - g_2) = \exp \left[ -\frac{(g_1 - g_2)^2}{2\delta^2} \right] = \exp \left[ -\frac{g_1^2 - 2g_1g_2 + g_2^2}{2\delta^2} \right].
\]  

(21)

Expanding (18) and comparing the result to (21) reveals the following:

(1) The diagonal elements of \(\Phi \Phi^T\) must be equal, i.e.,

\[
\langle \phi_1 \phi_1 \rangle = \langle \phi_2 \phi_2 \rangle = \sigma_0^2,
\]

where \(\sigma_0\) is the standard deviation of \(\phi_0\).

(2) The off-diagonal elements of \(\Phi \Phi^T\) are 0 implying that

\[
\phi_1\]  and \(\phi_2\) are statistically independent.

(3) \(\sigma_0 = 1/\delta\).

2.3.1. Lajunen and Saastamoinen’s NUC source. As an example, the NUC source originally introduced by Lajunen and Saastamoinen [27] is generated here. Let \(g = a \rho - \gamma|\gamma|^2\) and \(\tau = \sqrt{S} = \exp[-\rho^2/(4\sigma^2)]\). Here, \(\gamma\) is a real-valued vector and \(a = x, y\). Substituting these relations into (3) and (21) yields

\[
W(\rho_1, \rho_2) = \exp \left[ -\frac{\rho_1^2 + \rho_2^2}{4\sigma^2} \right] \times \exp \left[ -\frac{(\rho_1 - \gamma)|\gamma|^2 - (\rho_2 - \gamma)|\gamma|^2)^2}{2\delta^2} \right].
\]  

(22)

Using (5), the field that produces this NUC source is

\[
U(\rho) = \exp \left[ -\frac{\rho^2}{4\sigma^2} \right] \exp [i \rho - \gamma|\gamma|^2 \phi_0].
\]  

(23)
Just like for the GSM source, $\phi_g$ is a zero-mean, $1/\delta^2$-variance Gaussian random number.

Interestingly, the form of (23) implies that Lajunen et al.’s NUC source can be produced by incoherently summing many independent randomly focused Gaussian beams. This is an important finding and to the authors’ knowledge, has never been reported before. In prior work, NUC sources were generated using the Cholesky factorization.

| Table 1. GSM, BGSM, and NUC source parameters. |
|-----------------------------------------------|
|     | GSM | BGSM | NUC |
| $A$  | 1   | 1    | 1   |
| $\sigma$ | 0.50 mm | 0.50 mm | 0.50 mm |
| $\delta$ | 0.20 mm | 0.80 mm | 0.063 mm² |
| $\beta$ | N/A | 20   | N/A |
| $\gamma$ | N/A | N/A | 0.35 mm |

Figure 1. Theoretical and simulation results for the GSM, BGSM, and NUC sources—(a) GSM $S^\text{thy}$, (b) GSM $S^\text{sim}$, (c) $y = 0$ slice of GSM $S$ theory versus experiment, (d) BGSM $S^\text{thy}$, (e) BGSM $S^\text{sim}$, (f) $y = 0$ slice of BGSM $S$ theory versus experiment, (g) NUC $S^\text{thy}$, (h) NUC $S^\text{sim}$ and (i) $y = 0$ slice of NUC $S$ theory versus experiment.
method [28, 29]. The Cholesky approach is very computationally intensive requiring a great deal of memory to store the four-dimensional $\mu$ and significant time to compute the Cholesky factors of the resulting correlation matrix—the number of operations is $O(n^3)$ [29]. With the analysis presented above, the same NUC source can be produced by generating sequences of phase screens with random amounts of focus reducing computation time and memory usage by several orders of magnitude. More importantly, the above analysis shows that a general NUC source can be produced quickly using a low-actuator-count deformable mirror.

3. Validation

In this section, simulation results of GSM and BGSM sources as well as Lajunen et al’s NUC source are presented. The simulation results are compared to theory to validate the above methodology.

For the GSM and BGSM simulations, the source plane was discretized using 512 points per side with a 30 $\mu$m sample spacing. The $\lambda = 1 \mu$m random field instances were propagated to the far zone using a two-dimensional fast Fourier transform (FFT) [30]. The propagation distance $z$ was arbitrarily chosen to be 1 m. 20 000 independent instances of the far-zone field were incoherently summed/averaged to form the spectral density $S$. The theoretical expression for the far-zone BGSM $S$ is

$$S_{\text{BGSM}}(\rho, z) = \frac{k^2\sigma^2}{2wz^2} \exp\left(-\frac{\beta^2}{4wz^2}\right) \left(\frac{k^2\sigma^2}{4wz^2}\right) f_0\left(\frac{\beta k \rho}{2wz}\right),$$

where $w = 1/(8\sigma^2) + 1/(2\beta^2)$. Note that the far-zone GSM $S$ expression can be obtained by setting $\beta = 0$ in (24).

Table 1 reports the GSM and BGSM source parameters.

For the NUC source simulation, the source plane was again discretized using 512 points per side with a 30 $\mu$m sample spacing. The $\lambda = 1 \mu$m random field instances were numerically propagated $z = 17$ cm to match Lajunen et al using the Fourier transform form of the Fresnel integral [30, 31]. Like the GSM and BGSM sources, 20000 independent instances of the field were incoherently summed to form the NUC $S$. The theoretical expression for the near-zone NUC $S$ is

$$S_{\text{NUC}}(\rho, z) = \frac{k^2\sigma^2}{2z^2} e^{\gamma z} \int_0^\infty \exp\left(-\frac{2\pi^2 \beta^2 v^2}{2\sigma^2 w^2(v)}\right) \frac{k^2 \rho^2}{2z^2} e^{-2\pi\gamma v} dv,$$

where $w = 1/(4\sigma^2) + [2\pi v - k/(2z)]^2$. The integral in (25) was computed numerically using adaptive quadrature.

Figure 1 shows the results. The rows of the figure show the results for the GSM, BGSM and NUC source, respectively. The columns show the theoretical $S_{\text{BGSM}}$ and the $y = 0$ slice of $S$ theory versus simulation. The agreement between the theoretical predictions and simulated results is excellent. These results validate the partially coherent field synthesis approach presented here.

4. Conclusion

A new fast and flexible method for generating uniformly correlated and NUC partially coherent sources was presented. The approach permitted any Schell-model source to be produced using only a fast steering mirror; NUC sources could be generated using a low-actuator-count deformable mirror. Section 2 detailed the theory necessary to implement the method. Three examples—a GSM, BGSM, and NUC source—were presented to show how to apply the technique. Section 3 presented optical wave propagation simulation results of a GSM, BGSM and NUC source. These results were compared to the theoretical predictions to validate the approach. The agreement between simulation and theory was excellent. Future work will include implementing the proposed approach in the laboratory.

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