Mathematical model of snake-type multi-directional wave generation

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Abstract. Research on extreme wave generation is one intensive research on water wave study because the fact that the occurrence of this wave in the ocean can cause serious damage to the ships and offshore structures. One method to be used to generate the wave is self-correcting. This method controls the signal on the wavemakers in a wave tank. Some studies also consider the nonlinear wave generation in a wave tank by using numerical approach. Study on wave generation is essential in the effectiveness and efficiency of offshore structure model testing before it can be operated in the ocean. Generally, there are two types of wavemakers implemented in the hydrodynamic laboratory, piston-type and flap-type. The flap-type is preferred to conduct a testing to a ship in deep water. Single flap wavemaker has been explained in many studies yet snake-type wavemaker (has more than one flap) is still a case needed to be examined. Hence, the formulation in controlling the wavemaker need to be precisely analyzed such that the given input can generate the desired wave in the space-limited wave tank. By applying the same analogy and methodology as the previous study, this article represents multi-directional wave generation by implementing snake-type wavemakers.

1. Introduction
In ocean, an extreme wave can occur unpredictably, but its effect can cause serious damage to the ships and offshore structures (see [1–8]). Therefore any information about waves is very important for the design of ships and offshore structures. This information can be acquired by studying wave generation. The wave can be generated by a wavemaker. The wavemaker has been used a hundred years in a wave tank for ship testing [9]. It is really important to do testing to a model in constructing any maritime structures [10]. A wave tank in a hydrodynamic laboratory is a facility where maritime structures and ships can be tested on a model scale.

Study on extreme wave itself has been actively conducted. Marwan [11] studied the deformation of propagating wave which is initially bichromatic signal based on the third order solution of Korteweg-de Vries (KdV) equation. The wave is reported experiencing amplitude amplification. The extent of the study was presented in [12] where the fifth order solution of KdV with bichromatic signal gave higher amplification. Ramli [13] applied the same signal to the Boussinesq equation which yielded even higher amplitude amplification compared to the both previous results. An extreme wave
generation also studied on the basis of Benjamin-Bona-Mahony (BBM) equation in [14] and the results suggested that the amplitude amplification is affected by the initial amplitude and frequency difference between the supports of the bichromatic signal. Signal in trichromatic form also trigger an extreme wave. This phenomenon stated in [15]. Extreme wave can also be studied via the wave group envelope. It has been studied that the wave groups of some nonlinear wave equations satisfying Nonlinear Schrodinger equation, such as KdV [16, 17] and BBM [18].

Signaling problem is an important factor in generating wave. A signal is given to the wavemaker such that the desired wave profile will appear in the wave tank. The wave tank has a wavemaker at one side and a wave absorber that act as the beach at the other side [19]. According to O’Dea and Newman [9], wavemakers installed in testing tanks today are typically either hinged flaps (rotating about a pivot point below the still water level), or translating pistons. Furthermore they said that hinged flaps are more commonly found in deep-water tanks (water depth large compared to typical wave lengths), while pistons are more commonly found in shallow tanks used to study coastal waves, Tsunamis, etc.

Generally, there are two types of wavemaker which are widely used in hydrodynamic laboratories, piston and flap type wavemakers [19], see figure 1. Beside that there is also snake type wavemaker. This wavemaker has many part that can move like a flap or piston. Single flap wavemaker has been explained in many studies (see [19–21]). In this study, we will consider specifically the snake type of wavemaker (three dimensionals) which moves like a flap with extended boundary condition to the system of the water motion in the tank, see figure 2.

![Figure 1. Wavemaker type: (a) Flap, (b) Piston [9].](image)

![Figure 2. Snake type wavemaker [9].](image)
2. Wavemaker Formulation
Assume that the wavemaker in this case looks like figure 3.

![Figure 3. Scheme of snake type wavemaker](image)

In this case, we consider the governing equation for the fluid motion under a periodic water wave is the Laplace equation in linear form for a single flap.

\[
\frac{\partial^2 X}{\partial x^2} + \frac{\partial^2 Y}{\partial y^2} + \frac{\partial^2 Z}{\partial z^2} = 0 - h \leq z \leq \eta(x,t), 0 \leq y \leq L, 0 \leq x \leq P. \tag{1}
\]

Dean and Dalrymple [13] stated that the boundary conditions for this case are:

- **Bottom boundary condition for horizontal bottom (no flow condition/assumed no leak),**
  \[-\frac{\partial \phi}{\partial z} = 0 \text{ at } z = -h. \tag{2}\]

- **Dynamic free surface boundary condition,**
  \[\eta - \frac{1}{g} \frac{\partial \phi}{\partial t} = 0, \text{ at } z = 0. \tag{3}\]

- **Kinematic free surface boundary condition,**
  \[-\frac{\partial \phi}{\partial z} = \frac{\partial \eta}{\partial t}, \text{ at } z = 0. \tag{4}\]

- **Lateral boundary condition at the wavemaker,**
  \[U(z) \cos(\lambda y - \sigma t) = -\frac{\partial \phi}{\partial x}, \text{ at } x = 0. \tag{5}\]

where \(\phi\) is the velocity potential of the propagating wave, \(\eta\) is the elevation above the still water level, \(x, y, z\) are spaces dimension, \(t\) is time dimension, \(h\) is the water depth, \(\lambda\) is a wave number and \(g\) is gravitational acceleration.

Now in this study we will consider one more condition. Assumed that, there is no leak at the wall of the wave tank and neglect reflective wave of the wall, we can write:

- **Wall boundary conditions**
  \[\frac{\partial \phi}{\partial y} = 0 \text{ at } y = L\frac{\partial \phi}{\partial x} = 0 \text{ at } x = 0. \tag{6}\]
while \( L \) and \( P \) represents the wide and the length of the wave tank respectively. Assumed that the solution to equation (1) take form,

\[
\phi(x, y, z, t) = X(x)Y(y)Z(z)T(t),
\]

where \( X(x) \) is some function that depends only on \( x \), \( Y(y) \) depends only on \( y \), \( Z(z) \) depends only on \( z \), and \( T(t) \) varies only with time.

Using separation variable method we got,

\[
Z = Ae^{-k_p x} + Be^{k_p x},
\]

\[
Y = e^{i\lambda y} + Fe^{-i\lambda y},
\]

\[
X = Ce^{-i\sqrt{k_p^2 - \lambda^2}x} + De^{i\sqrt{k_p^2 - \lambda^2}x},
\]

where \( A, B, C, D, E, F, k_p, \) and \( \lambda \) are constants. Now substitute equations (7), (8), and (9) to equation (6) we get,

\[
\phi = (Ae^{-k_p x} + Be^{k_p x}) \left( Ce^{-i\sqrt{k_p^2 - \lambda^2}x} + De^{i\sqrt{k_p^2 - \lambda^2}x} \right) \left( e^{i\lambda y} + Fe^{-i\lambda y} \right) T(t),
\]

\( \phi \) must be periodic in time by the lateral boundary conditions, so \( T(t) \) specify as \( T(t) = \sin \omega t \). Equation (10) can be written as

\[
\phi = (Ae^{-k_p x} + Be^{k_p x}) \left( Ce^{-i\sqrt{k_p^2 - \lambda^2}x} + De^{i\sqrt{k_p^2 - \lambda^2}x} \right) \left( e^{i\lambda y} + Fe^{-i\lambda y} \right) \sin \omega t.
\]

Now equation (11) by using the bottom boundary condition (equation (2)) takes form,

\[
\phi = G \cosh k_p (h + z) \left( Ce^{-i\sqrt{k_p^2 - \lambda^2}x} + De^{i\sqrt{k_p^2 - \lambda^2}x} \right) \left( e^{i\lambda y} + Fe^{-i\lambda y} \right) \sin \omega t,
\]

with \( G = 2Ae^{k_p h} \) a new constant. Now by applying the wall boundary conditions (equation (5)), \( \phi \) takes final form,

\[
\phi = A_p \cosh k_p (h + z) \cos \left( \sqrt{k_p^2 - \lambda^2} (P - x) \right) \cos \lambda (y - L) \sin \omega t.
\]

Here we got a different equation from Dean and Dalrymple [13], because in this study we use the wall boundary conditions (equation (5)).

Elevation calculated by using dynamic free surface boundary condition (equation (3)),

\[
\eta = \frac{1}{g} \left. \frac{\partial \phi}{\partial t} \right|_{z=0},
\]

\[
\eta = \left[ A_p \cosh k_p h \right] \cos \left( \sqrt{k_p^2 - \lambda^2} (P - x) \right) \cos \lambda (y - L) \cos \omega t.
\]
Now by using kinematic free surface boundary condition (equation (4)) \( \sigma \) will be determined,

\[
- A_p k_p \sinh k_p h \cos \left( \sqrt{k_p^2 - \lambda^2(P - x)} \right) \cos \lambda(y - L) \sin \sigma t
\]

\[
= - \frac{A_p \sigma^2}{g} \cosh k_p h \cos \left( \sqrt{k_p^2 - \lambda^2(P - x)} \right) \cos \lambda(y - L) \sin \sigma t
\]

\[
\sigma^2 = g k_p \tanh k_p h.
\]

For a complete solution, \( A_p \) need to be determined. This is evaluated by the lateral boundary condition at the wavemaker.

\[
U(z) \cos(\lambda y - \sigma t) = - \frac{\partial \phi}{\partial x} |_{x=0},
\]

\[
U(z) \cos(\lambda y - \sigma t) = -A_p \sqrt{k_p^2 - \lambda^2} \cosh k_p (h + z) \sin \left( \sqrt{k_p^2 - \lambda^2} p \right) \cos \lambda(y - L) \sin \sigma t. \quad (13)
\]

To find \( A_p \), equation (13) is multiplied by \( \cosh k_p (h + z) \) and integrated from \(-h\) to 0. Therefore we get,

\[
A_p = - \frac{4k_p \cos(h \gamma - \sigma t) \int_{-h}^{0} U(z) \cosh(k_p(h + z))dz}{\sqrt{k_p^2 - \lambda^2} \left( \cosh(k_p h) - 1 - h \right) \sin \left( \sqrt{k_p^2 - \lambda^2} p \right) \cos \lambda(y - L) \sin \sigma t}. \quad (14)
\]

Now if we introduce \( \theta \) as a directional angle made by the wave orthogonal to \( x \) axis (figure 4), where \( \lambda \) is the wave number in the \( y \) direction and \( \sqrt{k_p^2 - \lambda^2} \) is the wave number in the \( x \) direction. \( k_p \) represents the wave number in the propagation direction.

![Figure 4](image)

**Figure 4.** Definition for \( \theta \) [13].

According to figure 4, \( \sqrt{k_p^2 - \lambda^2} = k_p \cos \theta \) and \( \lambda = k_p \sin \theta \), substituted to equation (12), now velocity potential can be written,

\[
\phi(x, y, z, t) = A_p \cosh k_p (h + z) \cos \left( k_p \cos \theta (P - x) \right) \cos k_p \sin \theta (y - L) \sin \sigma t, \quad (15)
\]

where \( A_p \) is given by equation (14). Equation (15) shows the solution for a single flap wavemaker. Now due to linearity of the problem we can superimposed numerous single flap wavemaker motions and become a snake type wavemaker. So velocity potential can be written,
\[ \sum_{i=1}^{n} \phi_{n}(x, y, z, t) = \sum_{i=1}^{n} A_{p} \cosh k_{p}(h + z) \cos \left( k_{p} \cos \theta (P - x) \right) \cos \left( k_{p} \sin \theta (y - L) \right) \sin \sigma(t + \gamma_{n}). \]

with \( n=1,2,3,... \) is the number of the flap and \( \gamma_{n} \) is phase of flaps. Equation above is proposed as model for snake-type multi-directional wave generation.

3. Conclusion
We have considered snake type wavemaker by linear theory approximation. It has been shown that the wave produced by snake type wavemaker can be considered as linear superposition of single flap wavemaker motion. Mathematical model of snake-type multi-directional wave generation can be determined reasonably well from linear wave theory. This study takes into account only the mathematical model of snake type wavemaker.

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