Investigation of tool-workpiece contact rate and milling force in elliptical ultrasonic vibration-assisted milling

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Abstract
Elliptical ultrasonic vibration-assisted milling (EUVAM) is widely used as an efficient processing method for hard-to-machining materials such as titanium alloy, superalloy, and hard-brittle materials. To uncover the mechanism of the intermittent cutting characteristics in EUVAM, the tool-workpiece contact rate model is developed by combining with the kinematic relationship between the tool edge and the workpiece in the process. According to the analysis of the contact rate model, the phenomenon that the contact rate increases rapidly with the time-varying tooth position angle in one-dimensional ultrasonic vibration assisted milling can be improved in EUVAM. In addition, considering the variation of window function and undeformed cutting thickness, a force model is established. And the experiment of EUVAM is performed to verify the model of ultrasonic milling force, and the influence of process parameters (amplitude, cutting speed, feed rate and cutting depth) on ultrasonic milling force is also analyzed.

Keywords Elliptical ultrasonic vibration-assisted milling · Tool workpiece contact rate · Ultrasonic milling force model

1 Introduction
As an important structural metal, titanium alloy has the advantages of high strength, high temperature resistance, and corrosion resistance. Therefore, it is widely used in aviation impellers and blades. Due to serious problems in traditional processing of titanium alloys (e.g., cutting force, high cutting heat, and poor surface quality), ultrasonic vibration assisted milling (UVAM) as an efficient processing method is used in difficult-to-process materials. At present, the existing research on the UVAM has confirmed that it has obvious advantages in reducing cutting force, improving cutting heat and improving surface quality. On this basis, the elliptical ultrasonic vibration device is designed to further enrich the ultrasonic vibration forms; the elliptical ultrasonic contact rate model was established to quantify the intermittent processing characteristics in ultrasonic milling; the ultrasonic milling force model is established to explain the reason of the reduction of ultrasonic milling force. In this paper, the blank areas of EUVAM are studied to help its better application in the production process.

According to the working principle of vibration, two-dimensional vibration assisted cutting device is divided into resonance type and non-resonance type. In the aspect of resonant device, Li et al. developed a single excitation ultrasonic elliptical vibration transducer [1], which uses the radial asymmetric structure of the horn to form a longitudinal bending compound vibration mode. Guo et al. proposed and designed a new type of ultrasonic vibrator for elliptical vibration deformation, which uses the flexure hinge structure to complete the coupling of two-dimensional vibration, and verified the feasibility of the device through experiments [2]. Due to the low vibration intensity of non-resonant devices, they are mainly used in the low frequency field, and focus on the design of flexible vibration structures [3–5]. In terms of ultrasonic assisted milling device, at present, more research focuses on the development of ultrasonic vibration tool holder to realize ultrasonic milling [6–12]. Although this idea can be more effectively combined with actual production, for scientific research purposes, using ultrasonic vibration device to drive...
workpiece vibration to realize ultrasonic milling can undoubtedly save the cost of scientific research. At present, the research of this kind of device mainly focuses on one-dimensional vibration [13–17]. In principle, UVAM is to add high-frequency vibration on the basis of common milling. According to the different types of additional vibration, it can be divided into different types, and the specific processing effect is not the same. Because kinematics is the basis of dynamics and machining surface quality, the corresponding research content is more substantial. Nath et al. first proposed the concept of tool-workpiece contact rate model in ultrasonic vibration assisted turning, and explained the reason why tool vibration frequency, tool vibration amplitude and workpiece cutting speed can have an important impact on cutting force through the model [18]. Zou et al. proposed a kinematics model of longitudinal torsional ultrasonic assisted spiral milling which is closer to the real situation [19]. Yang and Ni proposed a one-dimensional tool-workpiece contact rate model in the field of ultrasonic grinding and ultrasonic milling respectively, which can effectively describe the intermittent machining characteristics in grinding and side milling processes [20, 21]. For the UVAM, kinematics is considered as a basic part in the research of UVAM by most scholars. The separation characteristics under various specific conditions have been studied completely, but there is a lack of more comprehensive and holistic quantitative calculation of separation degree.

Milling force is a very important parameter in the milling process, which directly affects the stability and surface quality in the milling process, especially for the titanium alloy impeller with complex surface shape and thin wall thickness. Therefore, the milling force in the research process becomes particularly important. Through a series of slot milling experiments, Shen et al. verified that ultrasonic vibration assisted milling can reduce the average milling force and further optimize the processing parameter combination [22]. Ding et al. proposed a three-dimensional mechanical model of two-dimensional vibration assisted micro milling [23]. At the same time, they also proposed a surface integrated model of two-dimensional vibration assisted micro milling [24]. Hu et al. built a two-dimensional numerical simulation model for ultrasonic vibration assisted milling of aluminum alloy materials [25]. Huang et al. created a finite element simulation analysis of Ti-6Al-4V in ultrasonic vibration assisted milling. It was found that cutting force and cutting temperature were reduced after ultrasonic vibration was applied [26]. Lu et al. systematically studied the surface texture generation mechanisms including ultrasonic vibration end milling (UVEM) and ultrasonic vibration peripheral milling (UVPM) [27]. Li et al. established ultrasonic grinding force models for different brittle and hard materials [28, 29]. Ultrasonic machining force is the core part of ultrasonic machining research, but most of the research focuses on the characteristics and performance of ultrasonic machining force, and lacks the mechanism analysis of its changes.

The paper is organized into five sections. In Section 2, the separation time and re-contact time in EUVAM are analyzed, and the tool-workpiece contact rate model in elliptical ultrasonic vibration is established. Section 3 establishes the ultrasonic milling force model by modifying the undeformed cutting thickness in the common milling force model. In Section 4, the elliptical ultrasonic vibration device is designed and tested, the processing experimental platform is built. Section 5 throughs the experimental data to verify the ultrasonic milling force model, and analyzes the influence of various parameters on the ultrasonic milling force. Some conclusions are obtained in Section 6.

2 Tool-workpiece contact rate model in EUVAM

In the research of ultrasonic vibration-assisted cutting technologies, most of them take kinematics as the starting point, because the most important characteristic of ultrasonic vibration assisted cutting is its intermittence in the machining process, and the research of kinematics can show this characteristic in a more direct form. Therefore, it is necessary to establish a more accurate tool-workpiece contact rate model to quantify the intermittent machining characteristics in UVAM. The existing contact rate model is limited to one-dimensional ultrasonic vibration and side milling with small radial cutting depth. This paper presents a tool-workpiece contact rate model which can be applied to any ultrasonic milling including EUVAM and groove milling.

2.1 Tool-workpiece separation time in EUVAM

In the milling process, because the spindle drives the tool to rotate, the direction of the cutting speed changes with the rotation of the tool. When the ultrasonic vibration effects on the workpiece, the direction of the ultrasonic vibration is fixed, so the separation condition is different from that of the ultrasonic turning. Because the feed motion is in the same plane, the feed velocity should also be considered. As can be seen from Fig. 1, there is an angle between ultrasonic vibration and cutting motion, which is named time-varying tooth position angle θ, with the change of the angle, the separation condition will inevitably change. The existing separation condition of one-dimensional ultrasonic milling is to project the cutting speed in the vibration direction, and then the projection value is substituted into the separation conditions of ultrasonic turning for calculation:

$$V_c \sin \theta + 2\pi f \cos(2\pi ft) + V_f = 0$$

(1)
When angle $\theta$ is near angle $\pm \pi/2$, the separation condition can better reflect the actual process, however, when angle $\theta$ is near 0, the component of cutting velocity perpendicular to the vibration direction is large, while the component of cutting velocity parallel to the vibration direction is small. Therefore, if only the component parallel to the vibration direction is considered in the separation condition, it will lead to a great difference from the actual vibration process. In view of this situation, another solution of separation condition is put forward in this paper, the ultrasonic vibration velocity is decomposed orthogonally along the cutting velocity direction. When the vector sum of the ultrasonic vibration velocity component and the cutting velocity vector is zero, the milling-tool and the workpiece just separate. For one-dimensional UVAM:

$$V_c + 2\pi af \cos(2\pi ft) \sin\theta + V_f \sin\theta = 0 \quad (2)$$

It can be seen from Eq. (2) that when $V_c \leq (2\pi af + V_f) \cdot \sin\theta$, the intermittent cutting will not occur in ultrasonic milling. Only the cutting velocity $V_c$ meets the condition that $V_c > (2\pi af + V_f) \cdot \sin\theta$, the equation has real solutions, the separation time can be solved analytically:

$$t_{se} = \frac{\arccos[(V_c/\sin\theta + V_f)/2\pi af]}{2\pi f} \quad (3)$$

For EUVAM, because the component of vibration velocity in the direction parallel to the cutting velocity is no longer a simple sinusoidal change, it is difficult to deduce the formula to express the specific moment when the vector sum of cutting velocity and vibration component is zero, so the analytic geometry method is used to solve the problem.

In any period, the separation time condition is that the sum of the vibration velocity component in the cutting velocity direction and the cutting velocity vector is zero, that is $V_c + \cos(2\pi ft) \cdot \sin\theta = 0$. The directional change and relationship between cutting velocity and vibration velocity are shown in Fig. 1 (b), the components of elliptical vibration velocity parallel and vertical cutting velocity in the figure are represented by dotted lines. Since the ultrasonic vibration frequency is up to 20kHz and the vibration period is very short, it is assumed that the cutting velocity is constant in a vibration period, the cutting
velocity to each vibration period can be expressed by a certain vector, and the linear equation corresponding to the vector is as follows:

\[ y = -(\cos \theta / \sin \theta)x \]  \hspace{1cm} (4)

When the tool edge moves to the positive direction of x-axis and \( V_c \) is the velocity vector along Y direction, define \( \theta = 0 \), so the diagram discusses any case of \(-\pi/2 < \theta < 0\). In order to find out the vibration velocity OB with the component of -\( V_c \) in the direction parallel to the cutting velocity, a vertical line AB is drawn through point B. The corresponding analytical equation of AB is as follows:

\[ y = \tan \theta(x - V_c \sin \theta) - V_c \cos \theta \]  \hspace{1cm} (5)

The intersection of AB and one-dimensional vibration velocity vector is C, the intersection of AB and ellipse vibration velocity vector is A. Therefore, the vector expression of the vibration velocity corresponding to the separation time is OC and OA, the Eq. (5) is combined with the elliptic equation of vibration velocity:

\[ x^2/(2\pi af)^2 + y^2/(2\pi bf)^2 = 1 \]  \hspace{1cm} (6)

After the coordinate of intersection, \( A(x_{sc}, y_{sc}) \) is determined, the separation time \( t_{sc} \) is obtained by using the angle relation:

\[ 2\pi ft_{sc} + \arccos \left( V_c / \sqrt{(x_{sc}^2 + y_{sc}^2)} \right) - \theta = \pi \]  \hspace{1cm} (7)

### 2.2 Tool-workpiece re-contacting time in EUVAM

When the tangential separation occurs in the UVAM process, the tool workpiece re-contact will occur within a vibration cycle. Similar to separation condition, analytical geometry is used to solve the re-contact condition of UVAM. As shown in Fig. 1 (c), the vibration displacement is decomposed in X and Y directions for obtaining the vector coordinates of vibration displacement(\( \Delta x, \Delta y \)). Since the vibration velocity used in the previous calculation is equivalent to the milling cutter, the negative value should be taken when calculating the vibration displacement of the workpiece:

\[ -\Delta x = a \cos(2\pi ft_{sc}) - a \cos(2\pi ft_{sc}); \]
\[ -\Delta y = b \sin(2\pi ft_{sc}) - b \sin(2\pi ft_{sc}); \]  \hspace{1cm} (8)

The displacement produced by the tool edge is also orthogonal decomposed along the x-directions and y-directions, and the vector coordinates (\( dx, dy \)) are obtained:

\[ dx = R \cos \left[ \theta + \omega_s (t - t_{sc}) \right] - R \cos \left[ \theta \right]; \]
\[ dy = R \sin \left[ \theta + \omega_s (t - t_{sc}) \right] - R \sin \left[ \theta \right]; \]  \hspace{1cm} (9)

When the component value of the vibration displacement in the direction of the cutting displacement is equal to the displacement produced by the cutting speed, the tool will contact the workpiece again. There is a relationship between the values as follows:

\[ (\Delta y - dy)/(\Delta x - dx) = - (dx/dy); \]  \hspace{1cm} (10)

The value of \( t \) solved by simultaneous Eq. (8) to Eq. (10) is the re-contact time \( t_{rc} \).

### 2.3 Analysis of tool-workpiece contact rate

According to the definition of tool-workpiece contact rate, it is the proportion of the contact time between the tool and the workpiece in a single vibration cycle:

\[ r = t_c / T = (t_{sc} - t_{rc}) / T \]  \hspace{1cm} (11)

Combined with the solution of separation time and re-contact time, the contact rate model of EUVAM is established. Due to the complexity of the formula involved in the process, the changing trend of contact rate with various parameters can only be determined by numerical solution. Through the mathematical software calculation and drawing the contact rate change trend surface diagram under the action of the vibration frequency, amplitude, cutting speed, amplitude ratio, and time-varying tooth position angle, as shown in Fig. 2 (a)–(c).

It can be seen from the figure that with the decrease of cutting speed, the increase of vibration frequency or amplitude, the corresponding contact rate will become smaller, which is consistent with the common-sense cognition, and also the same as the conclusions drawn in other studies. When these three factors change separately, the trend of contact rate with tooth position angle is similar. On the contrary, it is obvious from Fig. 2 (d) that under different amplitude ratio conditions, with the change of time-varying tooth position angle, the change trend of contact rate will be obviously different. The further analysis is shown in Fig. 2 (e).

By changing the amplitude \( a \) to control the change of amplitude ratio \( a/b \), it can be seen that when \( a/b = 0 \), there is only one dimension of ultrasonic vibration at this time, so the obtained contact rate curve is the one-dimensional ultrasonic milling contact rate curve. When \( a/b = 1 \), the vibration is a standard circle vibration, and the contact rate curve is a horizontal straight line. This is because the vibration of any tooth position angle is equal in both tangential and radial directions, so the change of contact rate is no longer related to the tooth position angle. When \( a/b > 1 \), the contact rate curve is a
Fig. 2 Curve chart of contact rate change trend

a. $V_c=600\text{mm/s}, a=0.015\text{mm}, b=0.010\text{mm}$

b. $V_c=600\text{mm/s}, f=20000\text{Hz}, a=1.5$

c. $f=20000\text{Hz}, a=0.015\text{mm}, b=0.010\text{mm}$

d. $V_c=600\text{mm/s}, f=20000\text{Hz}, a=0.015\text{mm}$

e. Variation trend of contact rate under different amplitude ratio

f. Comparison of contact rate between 1D-UVAM and EUVAM

Fig. 2 Curve chart of contact rate change trend
concave curve and the contact rate increases with the increase of tooth position angle, which is due to the decrease of tangential amplitude at the position with larger tooth angle. It can be concluded that EUVAM can improve the phenomenon that the contact rate increases rapidly with the change of tooth position angle in 1D-UVAM.

Figure 2 (f) shows the difference of contact rate between one-dimensional ultrasonic vibration and elliptical ultrasonic vibration under similar vibration parameters and cutting speed. It can be seen from the simulation results that when the tooth position angle is near 0, the contact rate of 1D-UVAM and EUVAM is very close. However, with the decrease of the time-varying tooth position angle, the trend difference between them becomes larger and larger. For 1D-UVAM, when the variable tooth position angle is near $-\pi/2$, because the vibration direction is perpendicular to the tangential direction, the tangential separation will not occur, so the contact rate must be 1. For EUVAM, the tangential separation still occurs because the vibration direction also changes periodically.

### 3 Force model of ultrasonic milling

In the field of milling research, the part of milling force is undoubtedly the most important. As the purpose and development of kinematics research, on the one hand, it can be directly used as the evaluation standard in the machining process, on the other hand, it has a significant impact on the surface processing quality and other processing results. It has been a consensus that UVAM has the advantage of reducing the milling force. However, most of the research focuses on the more superficial content, such as the characteristics and performance of ultrasonic milling force, but lack the mechanism analysis of the causes of its change. In this paper, by calculating the undeformed cutting thickness and combining with the previous contact rate model, the ultrasonic milling force model is established to explore the specific mechanism of milling force reduction.

#### 3.1 Undeformed cutting thickness

In common milling, the difference of adjacent cutter tooth path can directly represent the chip thickness, but in UVAM, the situation is more complicated because of the tool edge will generate whirl path under the ultrasonic vibration. The thereafter vibration period will interfere with the surface machined in the previous vibration period, and produce a new machined surface. Therefore, it is necessary to establish a new model for undeformed cutting thickness in EUVAM. In order to simplify the calculation process, the ultrasonic vibration is transformed to the milling cutter equivalently.

As shown in Fig. 3 (a), the position of the tool edge in the cutting state at a certain time is $A$, and the coordinates of $A$ can be expressed as a univariate parametric equation with respect to $t$ by Eq. (12):

$$
x = R\cos(2\pi nt - 2\pi/z) + \cos(2\pi ft) + V/t;
$$

$$
y = R\sin(2\pi nt - 2\pi/z) + b\sin(2\pi ft);
$$

Then the parameter equation of the last cutter tooth is:

$$
x = R\cos(2\pi nt) + \cos(2\pi ft) + V/t;
$$

$$
y = R\sin(2\pi nt) + b\sin(2\pi ft);
$$

The coordinates of the milling tool center point $O$ at this time can be expressed as:

$$
O_x = \cos(2\pi ft) + V/t;
$$

$$
O_y = b\sin(2\pi ft);
$$

Thus, the equation of $OA$ is determined:

$$
y-b\sin(2\pi ft) = \tan(2\pi nt)[x-Vt\cos(2\pi ft)]
$$

Assuming that point $B$ is the intersection of straight-line $OA$ and machined surface, the coordinate of point $B$ is brought into Eq. (15):

$$
R\sin(2\pi nt_b + 2\pi/z) + b\sin(2\pi ft_b) - b\sin(2\pi ft) = \tan(2\pi nt)
$$

$$
[R\cos(2\pi nt_b + 2\pi/z) + \cos(2\pi ft_b) + V/t - V/nt - \cos(2\pi ft)]
$$

Through Eq. (16), the corresponding $t_b$ can be solved at a given time $t$, so as to determine the coordinates of point $B$. The distance between $AB$ is the instantaneous chip thickness. However, in the actual solution process, it is found that tool edge will generate whirl path under the ultrasonic vibration, so there will be multiple solutions to the equation. As shown in Fig. 3 (a), the intersection of line $AB$ and blue curve is not unique. Therefore, it is necessary to choose the solution according to the actual situation. The choice condition is to judge the distance value between $AB$ corresponding to several solved $t_b$. $t_b$ corresponding to the minimum $AB$ value is taken as the correct solution, and the actual cutting thickness can be calculated through the judgment process.

For a cutter tooth from cut-in to cut-out, the corresponding cutting thickness change is shown in Fig. 3 (b). In the whole cutting area, the phenomenon of cycle lag caused by ultrasonic vibration leads to the cutting thickness greater than the feed per tooth in some areas, and the cutting thickness will drop to zero in each vibration cycle. The curve shown in Fig. 3 (c) is obtained by enlarging the content in (b) locally. It can be seen that the change of time-varying tooth position angle causes the
change of cutting thickness curve shape in different vibration periods. Select three more special angles to zoom in, it can be seen in Fig. 3 (d), $\theta=0$ and the cutting thickness is pulse shape. In Fig. 3 (e) $\theta=\pi/4$, the pulse shape has disappeared, and the peak value on the right side is decreasing, forming an approximate triangle shape. In this case, the cutting thickness increases rapidly and then decreases slowly. In Fig. 3 (f) $\theta=\pi/2$, the tooth position angle is close to the cut-out angle, and the cutting thickness should be close to 0 for common milling. However, due to the radial vibration in EUVAM, there will still be a small cutting thickness. The curve is wedge-shaped, and there is a large proportion in a single vibration cycle that the corresponding cutting thickness is zero.

3.2 Simulation results analysis of ultrasonic milling force model

For common milling, the most commonly used milling force model is mechanistic model, namely instant rigidity force model, which discretizes the cutting edge along the axis. In this model, the cutting edge is discretized along the axial direction, and the cutting force elements of the $k$-th cutting edge element on the $j$-th cutter tooth in the tangential, radial and axial directions are expressed as:

\[
\begin{align*}
    dF_{tj} &= g(\theta_j) \left[ K_w h(\theta_j) \, dz + K_v \, ds \right] \\
    dF_{rj} &= g(\theta_j) \left[ K_w h(\theta_j) \, dz + K_v \, ds \right] \\
    dF_{aj} &= g(\theta_j) \left[ K_w h(\theta_j) \, dz + K_v \, ds \right]
\end{align*}
\] (17)

Whether the cutting edge participates in milling depends on many factors, such as tool diameter, number of cutting edges, radial cutting depth and so on. In this model, whether the time varying tooth position angle is between the cut-in angle and the cut-out angle is used to judge whether the cutting edge participates in milling:

\[
g(\theta_j) = \begin{cases} 
1 & \theta_u \leq \theta_j \leq \theta_e \\
0 & \text{others}
\end{cases}
\] (18)

The undeformed cutting thickness can be expressed as:

\[
h(\theta_j) = f_s \sin(\theta_j)
\] (19)

Combined with the previous research on EUVAM kinematics, the model is supplemented to complete the establishment of elliptical ultrasonic milling force model. The
complementary process mainly occurs in the change of $g(\theta, z)$ in Eq. (17) caused by tangential separation and the change of cutting thickness $h(\theta, z)$ caused by radial separation. The simulation results are obtained by mathematical software, as shown in Fig. 4.

It can be seen that compared with common milling force, it has similar overall fluctuation shape in three directions in a single cycle. The simulation parameters include tool parameters, diameter $D=6$ mm, number of teeth $N=2$, helix angle $\beta=\pi/6$. Interdental angle $\varphi_p=\pi$. The process parameters, rotational speed $n=20$ r/s, feed rate $f_z=0.02$ mm/z, cutting thickness $a_p=0.2$ mm, cut-in angle of groove milling $\varphi_{st}=0$, cut-out angle $\varphi_{ex}=\pi$; The vibration parameters, ultrasonic frequency $f=19300$ Hz, ultrasonic amplitude $a=b=6$ μm.

The ultrasonic milling force in X and Y directions is mainly analyzed, and some special angles are selected for magnification, as shown in the left and right sides of Fig. 4. It can be seen that under different angles, different pulse shapes will be formed, which is due to the influence of feed rate per tooth on cycle lag, which further affects the change of cutting thickness; Another point is that the separation time is different at different angles, which is consistent with the relevant research on contact rate.

In Fig. 5, the simulation results of ultrasonic milling force in X direction are compared with common milling force, and it is obvious that the overall trend of the two is similar. After local amplification, it can be seen that in each vibration cycle, the ultrasonic milling force is in pulse shape, which is slightly higher than the common milling force in the peak value. However, in terms of the average value in a single cycle, the ultrasonic milling force is significantly lower than the common milling force. This is also the principle that ultrasonic milling can reduce the milling force, which ensures that there is enough milling force to complete milling in the process of machining. Through the continuous separation-contact process, intermittent machining is realized, which can reduce the cutting heat and improve the processing environment.

4 Experimental setup and procedure

4.1 Design and calibration of elliptical ultrasonic vibration device

The hinge structure is designed by ANSYS to couple the two-dimensional ultrasonic vibration. The vibration modes with phase difference in the X and Y directions are determined near the frequency of 20 kHz. This platform is used for coupling ultrasonic vibration from two directions, and then the workpiece is driven to carry out elliptical ultrasonic vibration.

The vibration performance of the elliptical ultrasonic vibration platform is tested. The layout of the test site is shown in Fig. 6 (b). The model of the laser displacement sensor used in the test is Virgins Technology M70LL/2, which consists of a laser probe, a high-frequency acquisition card, an analog to digital converter, a data cable and the corresponding data.
processing software (Multi-instrument). The range of the sensor is 2 mm and the resolution is 1 μm. The sampling bandwidth is up to 10 MHz, which can meet the test requirements. In the test process, the sampling frequency is set to 500 KHz to meet the requirements of the ultrasonic vibration displacement signal with the frequency of 20 kHz.

Firstly, aiming at the calibration of the corresponding relationship between power and amplitude under different phase angles, three phase angles including 90°, 120°, and 180° are selected as the test targets, and 4-5 power percentages are selected for each phase angle. In the process of testing, each group of sampling for each parameter is one second. In this one second, four points are randomly selected, five cycles are selected near each point, and the average value of these twenty cycles is taken as the amplitude corresponding to the parameter.

The obtained amplitude is linearly fitted with the corresponding transducer power, and the results are shown in Fig. 6 (c). It can be seen that under three kinds of phase angles, the coefficient of determinability $R^2$ is close to 1, which can show that it has a good degree of linear fitting. The amplitude can be solved by the corresponding three formulas in the figure.

The next step is to test the vibration trajectory of the elliptical ultrasonic vibration platform. Ideally, two laser displacement probes should be used for dual channel test to detect the two planes AB, so as to determine the real phase angle between the actual vibration displacement generated by the two transducers. However, limited by the experimental conditions, only a single laser displacement probe can be used for the test, so the indirect test method is adopted, that is, to infer the actual vibration trajectory of the vibration platform by testing the vibration of plane C. It can be seen from Fig. 6 (d) that when the amplitudes on the AB planes are the same, the amplitudes on the C plane will change greatly under different phase angles. Based on this principle, the vibration track is tested.

Three groups of parameters including phase angle 90° power 80%, phase angle 120° power 60% and phase angle 180° power 60% are selected to measure the three planes of ABC, and the specific measurement results are shown in Table 1.

It can be seen from the data in the table that the deviation values of the test results of the two planes AB are within 10%, and the deviation of the average amplitude under the same power is small, so it can be judged that the stability of the vibration device is good. The results of the last three groups of data measured on the C plane show that for the specific shape of the vibration trajectory, there is a certain degree of error between the actual situation and the ideal situation, but the amplitude value on the C plane is significantly different from that on the AB two planes, so it can be concluded that the vibration platform does produce an elliptical trajectory.

### 4.2 Experimental equipment and parameters

The elliptical UVAM device and dynamometer are installed on the end mill machining center, and the processing experimental platform is built, as shown in Fig. 7. The experimental platform consists of four parts: ultrasonic vibration system, machining system, vibration measurement system and milling force measurement system. The ultrasonic vibration system and vibration measurement system have been stated above. The machining system consists of Demage vertical machining center (DMC 635V) and carbide end mill, the machine parameters are shown in Table 2.

The cemented carbide milling cutter is produced by Chengdu Chengliang Tool Group Co., Ltd. The TiAlSiN coating on the end mill cutter is prepared by adding appropriate amount of Si element on the basis of the commonly used TiAlN coating, which has stronger oxidation resistance and thermal stability, and is more suitable for processing titanium.
Fig. 6 Elliptical ultrasonic vibration device and its calibration test
alloy under dry cutting conditions; The milling force measurement system is composed of three-direction force measurement platform (Kistler 9257b), charge amplifier (5070), data acquisition card and data processing software.

Table 1  Vibration track test data

| Plane | Vibration parameters | Average amplitude | Calibration amplitude | Amplitude ratio | Ideal ratio | Deviation |
|-------|----------------------|-------------------|-----------------------|----------------|-------------|-----------|
| A     | 90°-80%              | 6.2 μm            | 6.1 μm                | 1.01           | 1           | 1.1%      |
|       | 120°-60%             | 6.5 μm            | 6.7 μm                | 0.97           | 1           | 3.0%      |
|       | 180°-60%             | 9.8 μm            | 9.8 μm                | 1.00           | 1           | 0         |
| B     | 90°-80%              | 6.3 μm            | 6.1 μm                | 1.04           | 1           | 3.8%      |
|       | 120°-60%             | 6.3 μm            | 6.7 μm                | 0.94           | 1           | 6.0%      |
|       | 180°-60%             | 10.3 μm           | 9.8 μm                | 1.05           | 1           | 5.0%      |
| C     | 90°-80%              | 5.2 μm            | 6.1 μm                | 0.85           | 1           | 15.3%     |
|       | 120°-60%             | 3.2 μm            | 6.7 μm                | 0.48           | 0.58        | 17.6%     |
|       | 180°-60%             | 1.8 μm            | 9.8 μm                | 0.18           | 0           | -         |

Fig. 7  Ultrasonic milling force experiment platform
Dynaware) on PC. It can measure a group of milling forces in three-dimensional orthogonal direction in real time. The sampling frequency is 7000 Hz and the threshold is less than 0.01 N.

The comparative experimental schemes of different spindle speed, cutting depth and feed rate with and without ultrasonic are designed respectively. Before the beginning of the

| Machine parameters        | numerical value |
|---------------------------|-----------------|
| Spindle speed range / r·min⁻¹ | 20-8000         |
| Movement accuracy / mm    | 0.001           |
| Maximum feed speed / m·min⁻¹ | 24             |
| Fast moving speed / m·min⁻¹ | 30             |
| Positioning accuracy / mm | 0.008           |

Table 2 Parameters of DMC 635 V machine tool

Fig. 8 Comparison of simulation and experimental value of milling force
processing experiment, the oxidation layer on the surface of titanium alloy can be removed by using the end mill with a diameter of 10 mm, and the more accurate z-height and the smoother processing surface can be obtained. Each group of experimental conditions was processed three times and the milling force signal was recorded. After completing a group of experiment, replace the same tool to avoid experimental errors caused by tool wear.

5 Discussion and results

5.1 Verification of force model in ultrasonic milling

The ultrasonic milling force model established in the previous paper needs to be verified by the experimental data. Before that, the milling force experimental data of common milling are used to compare with the model to determine the accuracy of the mechanistic model. The experimental data in the stable cutting area is selected as the basis for evaluating the milling force model, and the peak and valley value and the time to reach the peak and valley in a single cutting cycle are selected as the indexes for evaluating the trend of waveform change.

Figure 8 (a) and (b) shows the comparison between the simulation value and the experimental value of common milling force in the milling force model used. Due to the factors not considered by the milling force model such as chatter in the cutting process, the overall trend of the two models is similar, but the simulation value is a smooth curve and the original experimental value fluctuates a lot. It can be seen that both the overall trend and the specific peak value are fairly close. In the experimental data acquired under the same parameter, 20 cutting cycles are randomly selected and their peak and trough values are compared with the average time to reach the peak and trough as the experimental values.

The comparison of experimental data and simulation values is shown in Table 3. The comparison data shows the good similarity between the model and the reality from the numerical level. However, some simulation errors are large, and there is a stage at the end of the cutting cycle which is obviously inconsistent with the actual value, which shows that the simulation method has some limitations.

Figure 8 (c) and (d) shows the comparison between the simulated values of ultrasonic milling forces in X and Y directions and the actual measured values obtained from experiments. In general, the simulation results of the two directions are similar to the experimental values to a certain extent. In the experimental data obtained under the same parameters, 20 cutting cycles are randomly selected, and the average value of the peak and valley value and the time to reach the peak and valley is taken as the experimental value for comparison. The corresponding data are shown in Table 4.

If only this part of data is used as the evaluation standard, the simulation model has a certain accuracy, but it is not

| Wave crest of X-direction | Simulation value | Experimental value | Error |
|--------------------------|------------------|--------------------|-------|
| Wave trough of X-direction | -11.72 N | -13.76 N | 14.82% |
| Time to crest of X-direction | 15.31 ms | 18.28 ms | 16.25% |
| Time to trough of X-direction | 42.97 ms | 42.13 ms | 2.00% |
| Wave crest of Y-direction | 7.629 N | 10.58 N | 27.89% |
| Wave trough of Y-direction | -39.15 N | -39.29 N | 0.36% |
| Time to crest of Y-direction | 0.814 ms | 1.143 ms | 28.78% |
| Time to trough of Y-direction | 29.27 ms | 31.99 ms | 8.50% |

Table 4 Comparison of experimental and simulation values of ultrasonic milling force model

| Wave crest of X-direction | Simulation value | Experimental value | Error |
|--------------------------|------------------|--------------------|-------|
| Wave trough of X-direction | -23.44 N | -27.26 N | 14.01% |
| Time to crest of X-direction | 14.00 ms | 15.14 ms | 7.53% |
| Time to trough of X-direction | 44.75 ms | 45.13 ms | 0.84% |
| Wave crest of Y-direction | 12.04 N | 19.56 N | 38.44% |
| Wave trough of Y-direction | -54.80 N | -54.75 N | 0.01% |
| Time to crest of Y-direction | 0.65 ms | 0.57 ms | 13.86% |
| Time to trough of Y-direction | 29.43 ms | 29.42 ms | 0.03% |
difficult to see from the figure that there are differences in many areas. The obvious difference is that under ideal conditions, the ultrasonic milling force will form a pulse shape from zero to peak and then to zero.

Considering that the sampling frequency of the experimental data is limited to 7000hz, the sampling frequency of the simulation value is also reduced to 7000hz, which cannot more accurately reflect the real change of the milling force.

Fig. 9  Variation trend of average milling force
in ultrasonic milling, but can be compared with the milling force signal collected by the dynamometer at the same frequency level. As shown in Fig. 8 (e) and (f), the waveform changes of the simulation value and the experimental value have higher similarity, and in some ultrasonic vibration cycles, the milling force does not drop to zero. There is still a certain error between the simulation value and the actual value, and the serious area is marked with a red box.

Analysis of the causes of error: it is speculated that in the actual processing situation, due to a series of reasons such as tool or workpiece chatter, untimely chip removal, tangential friction between tool and workpiece, it is difficult to form the pulse waveform that can return to zero as shown in the simulation.

5.2 Analysis on variation trend of ultrasonic milling force

After confirming that the simulation model has certain accuracy, this paper focuses on analyzing the influence of four parameters including cutting speed, amplitude, feed and cutting depth on the common milling force and the ultrasonic milling force and the corresponding change trend. Unlike the peak point coordinates selected in the previous section to evaluate the accuracy of the model, in order to better reflect the separation characteristics of ultrasonic cutting, the average milling force needs to be selected as the criterion to measure the impact of milling force.

For the average value of common milling force, with the increase or decrease of feed per tooth, the average milling force will change linearly. The corresponding data are counted and plotted as a histogram, as shown in Fig. 9 (a) and (b). The linear change of average milling force in XY direction is affected by the feed rate of each tooth. With the increase of feed rate, the reduction percentage of milling force caused by ultrasonic vibration increases, but with the further increase of feed rate, the increment of this percentage begins to decrease, and finally it will stabilize at a specific value. This is the same as the change trend of tangential contact rate studied in the previous article, this means that, for the average milling force, the main reason for the reduction of ultrasonic vibration is tangential separation.

When the cutting depth changes, the average milling force in X or Y direction also changes and presents a linear trend as shown in Fig. 9 (c) and (d). For different cutting depth, the reduction percentage of ultrasonic vibration on milling force does not change greatly.

In the common milling force model, the cutting speed are not involved, but in the actual processing experiment, it can be seen that the milling force shows a downward trend with the increase of cutting speed. For ultrasonic milling, when the vibration parameters remain unchanged, with the increase of cutting speed, the time proportion of tangential contact will increase (means the contact rate increases) in a vibration cycle, and the tangential separation will not occur when the cutting speed exceeds a certain value. In this case, the ultrasonic effect only changes the distribution of the cutting thickness, but it does not affect the average milling force in essence. The specific form is that the average milling force in the feed direction (X-direction) increases, while the average milling force in the vertical feed direction (Y-direction) decreases, as shown in Fig. 9 (e) and (f).

When the applied amplitude changes, the average milling force will also change. Figure 9 (g) shows the average milling force in X-direction and Y-direction when the amplitude is 0 (normal milling), 3, 6, and 9 μm respectively. The more special is that when the amplitude is 3 μm, the average milling force in X-direction is greater than that of common milling, because the tangential separation conditions are not satisfied when the amplitude is 3 μm, so the tangential separation does not occur, but only the change of cutting thickness in each vibration period is caused by the existence of frequency ratio and feed quantity. The milling force in X-direction becomes larger and the milling force decreases in Y-direction. For amplitude of 6 μm and 9 μm, because the tangential separation condition is satisfied, the contact time decreases with the amplitude increasing in each vibration period, which leads to the decrease of average milling force.

6 Conclusion

In this paper, the contact rate model of EUVAM which can be applied to groove milling is established, and on this basis, the ultrasonic milling force model is established combined with the cutting thickness of UVAM. Designing an experimental device to verify the model, and the influence of different parameters on the average ultrasonic milling force is analyzed. In actual production, the research conclusion can guide the selection of process and vibration parameters, so as to effectively cause the required separate milling. The specific conclusions are as follows:

1. In this paper, a new model is proposed to ensure that the established tool workpiece-contact rate model can be applied to EUVAM and groove milling. Through the analysis of the model, with the decrease of cutting speed, the increase of vibration frequency or amplitude, the corresponding contact rate will become smaller. When amplitude ratio \( a / b = 0 \), the contact rate curve obtained is the one-dimensional ultrasonic milling contact rate curve. When \( a / b = 1 \), the vibration is the standard circle vibration, and the change of contact rate is no longer related to the tooth position angle. It can be concluded that EUVAM can improve the phenomenon that the contact rate increases rapidly with the change of tooth position angle in 1D-UVAM.
Based on the common milling force model, the window function \( g(\theta) \) of cutting area is modified by tangential separation calculation, the cutting thickness function \( h(\theta) \) is modified by calculating the undeformed cutting thickness. And the elliptical ultrasonic milling force model is established. Through the model, it can be seen that the milling force in a single vibration cycle will form different pulse shapes under the influence of the separation and the change of cutting thickness. Comparing the simulation results of ultrasonic milling force and common milling force, it can be seen that the overall trend of the two forces is similar. For each vibration cycle, the ultrasonic milling force is pulse like, and the peak value is slightly higher than the common milling force, but the average value in a single cycle is significantly lower than the common milling force.

The elliptical ultrasonic vibration device is calibrated by laser displacement sensor, and its amplitude has a significant positive linear relationship with the device power. Indirect measurement of elliptical vibration trajectory shows that although there are some errors, the device can meet the vibration requirements.

According to the experimental data, the average coordinate value of the peak point is selected to verify the ultrasonic milling force model. The error can be controlled within 15%, and the matching degree is good. Taking the average milling force as the evaluation standard, the influence of cutting speed, amplitude, feed rate and cutting depth on the ability of ultrasonic vibration to reduce milling force was discussed. With the increase of cutting speed, the contact rate of ultrasonic milling increases significantly, and the milling force reduction ability decreases; The cutting depth has the least influence on the reduction ability of milling force; With the increase of feed rate, milling force reduction ability is enhanced.

Limited by the author’s level, experimental conditions, research time and other factors, this paper can only complete part of the research content. Here we share some existing research ideas or follow-up research that can be carried out on the basis of this article: For the established ultrasonic milling force model, only the change of instantaneous cutting thickness is considered. The subsequent research can classify and refine the phenomena such as friction, chatter and ultrasonic impact in the actual cutting process, and conduct quantitative research, so as to establish a more practical ultrasonic milling force model: In addition, the rigidity of the designed EVAM platform is weak, which is easy to produce chatter and affect the machining effect in the actual machining experiment. On this basis, the universal support can be added to increase the stiffness.

Nomenclature

- \( V_c \), cutting velocity
- \( \theta \), time-varying tooth position angle
- \( a \), amplitude in feed direction
- \( f \), vibration frequency
- \( V_f \), feed velocity
- \( t_s \), separation time
- \( b \), amplitude perpendicular to feed direction
- \( \theta_0 \), initial tooth position angle
- \( \omega_0 \), tool angular velocity
- \( t_{rc} \), re-contact time
- \( z \), the number of cutter teeth
- \( t_m \), cutting time corresponding to the previous cutter tooth
- \( f_r \), Feed per tooth
- \( \theta_{ex} \), Angle of start cutting
- \( \theta_{ex} \), Angle of exit cutting
- \( \theta_{j,rc} \), Angle of the \( j \)-th cutting edge element on the \( j \)-th cutter tooth

Author contribution

Zongyuan Li and Lida Zhu conceived the idea. Zongyuan Li and Zhichao Yang provided modeling and calculation method. Zongyuan Li drafted the manuscript, and Zongyuan Li, Zhu Lida, Zhihao Yang, Jian Ma, Wenbin Cao interpreted, discussed and edited the manuscript. Zongyuan Li and Zhichao Yang finalized the manuscript, including preparing the detailed response letter. Lida Zhu supervised the work.

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Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Declarations

Ethics approval

Not applicable.

Consent to participate

Not applicable.

Consent for publication

Manuscript is approved by all authors for publication. I would like to declare on behalf of my coauthors that the work described was original research that has not been published previously, and not under consideration for publication elsewhere, in whole or in part.

Competing interests

The authors declare no competing interests.

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