Experimental and theoretical study of compressive mechanical behavior of red sandstone after heating-cooling treatment

Dejian Li¹,², Xiaolin Liu¹,²*, Yiming Shao¹,², Chao Han¹,²

¹State Key Laboratory for Geomechanics and Deep Underground Engineering, China University of Mining and Technology, Beijing 100083, China
²School of Mechanics and Civil Engineering, China University of Mining and Technology (Beijing), Beijing 100083, China

Abstract: In order to investigate compressive mechanical behaviors of rock materials after different heating-cooling treatments, in this paper, a series of uniaxial compressive experiments are carried out on red sandstone samples after various heating temperature (from 25°C to 1000°C) and water cooling treatments (10°C) to obtain evolution laws of mechanical property. The evolution laws of peak strength, elastic modulus, primary wave velocity and micro-structure are analyzed in details. And for better reflecting compressive stress-strain behaviors of red sandstone after heating-cooling treatments, based on Caputo variable-order fractional calculus, considering strain correlation and constant strain loading rate, we propose a novel variable-order fractional constitutive model to describe stress-strain behaviors of red sandstone samples after heating-cooling treatments. The validation of proposed model is well verified and a comparative study between proposed variable-order fractional constitutive model and constant-order fractional constitutive model is performed to highlight the advantage of proposed model. The evolutions of mechanical characteristics are revealed by presented varying-order function related to strain and the influence of fitting parameters on stress-strain behaviors are also discussed for deeply comprehending compressive mechanical mechanism of red sandstone after heating-cooling treatments.

Keywords: Red sandstone; Heating-cooling treatment; Compressive stress-strain behavior; Variable-order fractional constitutive model; Evolution of mechanical property

1 Introduction

With development of green energy resource, geothermal energy has been recommended as an alternative clean energy with high reliability, low cost and environmentally friendliness that comparing to traditional energy (Lund et al. 2005; Zaigham and Nayyar 2010). Geothermal energy is generally stored in underground space with depth of 4-6 km and distributed in rock mass with high temperature (Bai et al. 2012; Feng et al. 2012; Richards et al. 1994). During the extraction of geothermal energy, heat-carrying fluid is a significant operation and the cold water is input in rock stratum and high temperature rock mass will encounter and be cooled rapidly, where hot dry rock within deep rock stratum is forced to cool and hot energy is released and applied (Chandrasek 2010; Collin and Rowcliffe 2002). These high temperature water cooling actions will induce serious influence on physical and mechanical property of rock mass (e.g. weak of strength and instability of rock) (Peng et al. 2016; Zhang et al. 2014; Zhang et al. 2014). Therefore it is necessary and valuable to explore various mechanical property of rock subjected to high temperature and water cooling treatment.

For investigating the various mechanic al characteristics of rock after high temperature or heating cooling treatment, much effort have been attached and numerous achievements have been obtained (Yang et al. 2020; Chen and Wang 1980; Carlos et al. 2011; Liu et al. 2020). Shao et al.
studied the effect of cooling rate on mechanical property of heated Strathbogie granite with different grain sizes (Shao et al. 2014). Zhang et al. explored the influence of mineral components within rock material on physical and mechanical property of rock (Zhang et al. 2016). The investigation of primary wave velocity of Yantai granite after thermal treatment were performed and Zhu et al. found the increasing of temperature will cause attenuation of primary wave velocity (Zhu et al. 2017). Hu et al. conducted a series of uniaxial and tri-axial tests on sandstone after high temperature treatment and its rheological property were also studied and modeled (Hu et al. 2019). The coupling effect of high-temperature and water-cooling on the rutting resistance of rock asphalt mixture is investigated and a Bayesian approach is constructed to model the dynamic stability and predict resistance of rock asphalt mixture (Ren et al. 2020). From these current researches, it can be demonstrated the mechanical property of rock material is deeply affected by different high temperature treatments and high temperature treatment will weaken strength, elastic modulus and primary wave velocity of rock material (Zhu et al. 2020). However, there are few studies related to rock after heating-cooling treatment and the corresponding researches of mechanical behavior also have been few focused. Heating-cooling treatment has been applied on rock materials to carry out uniaxial and tri-axial experiments and thermal mechanical parameters of treated rock were obtained (Zhang et al. 2017). And cyclic loading tests of rock after various heating-cooling treatments were also performed to achieve mechanical characteristics and deformation laws (Zhu et al. 2019).

For the current study of mechanical behavior of rock material after heating-cooling treatment, it should be noticed that very few researches were devoted to clarify and describe compressive mechanical behavior of rock materials after heating-cooling treatments. Thus, how to well model and depict stress-strain relationship of rock after heating-cooling treatment under compressive experiments is a main purpose in this study. The description of stress-strain behavior of material has employed many mathematic methods (Kong et al. 2018; Tang et al. 2000; Liu et al. 2020). A mechanical model based on Weibull distribution was proposed by K. Weddfelt to study the bearing capacity and crack propagation of hard rock (Weddfelt et al. 2017). Fu et al. introduced Cauchy rotation equation to model axial compressive behaviors of layered rock (Fu et al. 2018). But it is well-known that the mechanical property of rock material is varying with time when these were subjected to different heating-cooling treatments. For describing these varying mechanical behaviors, variable-order fractional calculus is introduced and applied in depiction of stress-strain relationship due to its strong time-memory characteristics. And the application of variable-order fractional calculus in modeling stress-strain relationship has appeared in study of polymer materials (Meng et al. 2016; Meng et al. 2019). Meng et al. proposed a variable-order fractional model to describe strain softening and hardening behavior of polymer material and a linear varying-order function was also presented (Meng et al. 2019). The tensile and shear behaviors of sintered nano-silve paste were reflected by variable-order fractional model that proposed by Cai et al (Cai et al. 2020). Hence, based on the advantages of variable-order fractional calculus in modeling stress-strain behaviors of materials, in this study, based on variable-order fractional theory and strain correlation of deformation, a novel variable-order fractional constitutive model will be proposed to describe varying stress-strain relationship of rock after different heating-cooling treatments.

And considering to above statements, the outline of this paper is illustrated as follows. Section 2 introduces a series of compressive experiments that carried out on red sandstone samples after
various heating-cooling treatments and the variations of elastic modulus, peak strength and primary wave velocity were also analyzed in details. In section 3, a novel variable-order fractional constitutive model related to strain is proposed and its applicability and validation in experimental stress-strain data of red sandstone samples after heating-cooling treatments will be verified. And in section 4 the sensitivity of fitting parameters is discussed and the evolution mechanisms of mechanical property of treated samples are revealed. Finally, several conclusions will be drawn.

2 Uniaxial compressive experiments

2.1 Experimental samples, program and apparatus

The red sandstone experimental samples are selected from Liuyang mountain area of Hunan province, China. Cylindrical samples were drilled from full red sandstone rock mass without obvious cracks and fracture on surface, whose size are 100 mm in height and 50 mm in diameter. In accordance with the standard requirements of International Society of Rock Mechanics [31], the parallelism and surface fatness are controlled within ±0.05 mm and ±0.02 mm, respectively. As displayed in Fig. 1, for ensuring the uniformity of initial samples, a series of primary wave (P-wave) velocity tests were performed on initial red sandstone samples to select 30 experimental samples with consistent mechanical property. Then the selected initial red sandstone samples are heated to 200, 300, 400, 500, 600, 700, 800, 900 and 1000 °C with a rising rate of 5°C/min by industrial muffle furnace and then the target temperature will be kept for 4h. After continuous heating treatments on samples, the heated red sandstone samples will be cooled by water without air in glass container with a volume of 50L for 2h until arriving at set temperature of 10°C. Different target heating temperature symbols different cooling rate and by monitoring cooling rate of each heated samples, the average cooling rate approaches to 20°C/min, which is greater than previous results in Chaki’s researches (Franklin et al. 1979). For each group of red sandstone samples after heating-cooling treatment, we have prepared three samples under same heating-cooling treatment to insure the reliability of experimental results. The used experimental apparatus is uniaxial-triaxial compressive experimental system in State Key Laboratory for Geomechanics and Deep Underground Engineering and the all testing procedures are controlled automatically by PC software. During the uniaxial compressive tests, the loading style is set as strain loading with constant strain rate of 0.003 mm/s until failing sample.

![Fig. 1 Experimental initial red sandstone samples and red sandstone samples after heating-cooling treatment](image-url)
As shown in Fig. 1, we can see with an increase in temperature under same cooling conditions, there are gradually much white fine particles on the surface of red sandstone samples, possibly changing of mineral composition induced by heating-cooling treatment. And the color of each treated sample exhibits the transformation of dark red, red and white. In order to explore the influence of different heating-cooling treatments on microstructure evolutions of red sandstone samples, a series of scanning electron microscopy (SEM) tests were conducted on each experimental sample and for simplifying analysis of effect of different heating-cooling treatments on sample, the red sandstone samples that subjected to 25, 200-10, 400-10, 600-10, 800-10 and 1000-10℃ are selected as analyzed examples. It can be observed in Fig. 2(a) that there is a little number of initial cracks within the sample at room temperature with 25℃, which indicates initial red sandstone sample has almost complete microstructure. And when the sample is heated to 200℃ and cooled to target temperature with 10℃, there are small number of micro-cracks and particles generated on the surface of sample where in the weak junction of aperture and it symbols the effect of heating-cooling treatment has appeared. With an increase in temperature of 400 and 600℃, not only there are a large quantity of micro-cracks generating, but also long cracks within apertures begin to connect and penetrate each other. And when the heated temperature arrives at 800 and 1000℃, due to the softening effect of montmorillonite in water resulted by high temperature water cooling, the relatively complete structure of initial sample is divided into small particle with clear cracks by the trans-granular cracks, which will induce the weaken of strength of red sandstone sample.
For exploring the influence of different heating-cooling treatments on axial stress-strain behaviors of red sandstone samples, the self-developed uniaxial and tri-axial experimental system by SKLGDUE was employed and its vertical loading pressure can arrive at 2000kN. In order to eliminate testing errors of each treated sample, three prepared samples under same treated conditions were tested and the average value of mechanical parameters of three prepared samples is calculated as analyzed value. As shown in Fig. 3, it presents general stress-strain mechanical behaviors of red sandstone samples under 10 kinds of heating-cooling treatment. At the beginning of initial heating-cooling treatment, the evolutions of stress-strain curves exhibit relatively consistent tendency. But with rising of heated temperature, it can be seen the development of compaction stage of sample needs more time and when it enters the status of heated temperature with 700, 800, 900 and 1000°C, the peak strength will appear a sudden drop, which can be accounted for that high temperature water cooling reduces the components of quartz and clay minerals within red sandstone samples and the great ductility resulted by bonding force inside sample rising down.

And Fig. 4 presents variations between peak strength of treated red sandstone sample and
different heating-cooling treatments. It is demonstrated that the peak strength of sample will be upgraded by heating-cooling treatment and then it will decrease with an increase in heated temperature, which is same as other previous study (Zhang et al. 2014; Shao et al. 2014). That may be interpreted that during the initial heating-cooling process, the corresponding heating-cooling treatment will reduce viscous substance and eliminate relative sliding inside rock material. And with processing of heating-cooling treatment on sample, the effect of temperature impaction that induced by heating-cooling will cause much developments of initial cracks within rock and the rising down of peak strength will appear. The maximum peak strength is 91.430 MPa that corresponding to heating temperature of 500℃ and cooling temperature of 10℃. From this peak point, peak strength will decrease sharply until arriving at 40.569 MPa, which symbols for this specific red sandstone, heating-cooling treatment of 500-10℃ is a critical temperature condition and peak strength of red sandstone can be enhanced or weakened by heating-cooling treatment.

2.3 Effect of heating-cooling treatment on elastic modulus and primary wave velocity

Initial loading tangent modulus (called as $E_0$) is a significant parameter in analysis of evolution of mechanical property of red sandstone after heating-cooling treatment, which is the initial slope of stress-strain curve after compaction stage under compressive experiments. As shown in Fig. 5,
the evolutions of average tangent modulus, i.e., elastic modulus of red sandstone samples that
subjected to different heating-cooling treatments (25, from 200-10 to 1000-10°C) are illustrated
and $E_0$ increases with temperature until arriving at 300°C and then it decreases with an constant
rate, whose changes correspond to evolutions of compaction stages of each treated samples.
Meanwhile, it is well-known that the P-wave velocity is closely related to elastic modulus (Zhang
et al. 2017) and we can see in Fig. 6 that the evolutions of P-wave velocity under different
heating-cooling treatments are well agreement with variations of elastic modulus.

![Fig. 6 Evolution of P-wave velocity of red sandstone after different heating-cooling treatments](image)

### 3 Modelling stress-strain relationship of red sandstone after heating-cooling treatment

#### 3.1 Variable-order fractional stress-strain model

In this section, a well-known viscoelastic constitutive model is introduced, which was proposed
by Smit and Vries (Smit and Vries 1970)

$$\sigma(t) = E \tau^\alpha D^\alpha \varepsilon(t)$$  \hspace{1cm} (1)

where $\sigma(t)$, $\varepsilon(t)$, $E$ and $\tau$ represent stress, strain, elastic modulus and relaxation time,
respectively, $D^\alpha$ is constant-order fractional calculus. And when $\alpha = 0$, Eq. (1) denotes
stress-strain relationship of pure elastic solid body. When $\alpha = 1$, Eq. (1) characterizes pure
viscosity fluid body. If $0 < \alpha < 1$, Eq. (1) can be used to describe mechanical behaviors of
viscoelastic materials. However, during the continuous deformation of material, the mechanical
property within material is varying with time or space. Corresponding to these variations,
variable-order fractional calculus is presented to reflect changing mechanical behaviors.

In recent years, variable-order fractional calculus has obtained much achievement and there are
many kinds of variable-order fractional calculus with different definitions (Ross and Samko 1995;
Mainardi 2010). In this study, for matching continuous viscoelastic deformation between 0 and 1,
the variable-order fractional calculus that proposed by Coimbra (Coimbra 2003) is introduced,
whose order $\alpha$ is assumed as $\alpha(t)$. And considering to time yields, the actual expression for $0 < \alpha(t) < 1$ is defined

$$D^{\alpha(t)} f(t) = \frac{1}{\Gamma[1 - \alpha(t)]} \int_0^t (t - \theta)^{-\alpha(t)} D^1 f(\theta) d\theta + \frac{[f(0^+) - f(0^-)] e^{-\alpha(t)}}{\Gamma[1 - \alpha(t)]}$$  \hspace{1cm} (2)

where $\alpha(t)$ is varying-order function related to time, stress, strain, relaxation time and so on.

Among, the definition of Euler gamma function is expressed (Coimbra 2003)

$$\Gamma(t) = \int_0^t e^{-\theta} \theta^{t-1} d\theta$$  \hspace{1cm} (3)
By combining Eq. (2) and Eq. (1), Eq. (1) can be rewritten as Eq. (4).

\[ \sigma(t) = E \tau^{(t)} D^{(t)} \varepsilon(t) \]  

(4)

And then the variable-order fractional expression of strain \( D^{(t)} \varepsilon(t) \) within Eq. (4) can be obtained

\[ D^{(t)} \varepsilon(t) = \frac{1}{\Gamma[1 - \alpha(t)]} \int_{0^+}^{t} (t - \theta)^{-\alpha(t)} D^1 \varepsilon(\theta) d\theta + \frac{[\varepsilon(0^+) - \varepsilon(0^-)] t^{-\alpha(t)}}{\Gamma[1 - \alpha(t)]} \]  

(5)

It can be demonstrated that during the uniaxial compression experiments, when vertical strain loading function is set as \( \varepsilon(t) = at \), the Eq. (5) can be formulated

\[ D^{(t)} \varepsilon(t) = \frac{a}{\Gamma[1 - \alpha(t)]} \int_{0^+}^{t} (t - \theta)^{-\alpha(t)} d\theta = \frac{at^{1-\alpha(t)}}{\Gamma[2 - \alpha(t)]} \]  

(6)

where \( a \) represents strain rate. In this study, \( a \) is vertical loading rate, i.e., \( a = 0.003 \text{ mm/s} \).

Then the Eq. (6) is substituted into Eq. (4) and the stress-strain constitutive model can be derived as follows:

\[ \sigma(t) = E (at)^{\alpha(t)} \frac{\varepsilon(t)^{1-\alpha(t)}}{\Gamma[2 - \alpha(t)]} \]  

(7)

### 3.2 Determination of varying-order function

In order to better employ proposed variable-order fractional constitutive model to depict stress-strain relationship of rock material under different experimental conditions in next section, how to determine a reasonable varying-order function within proposed model is significant. It is well-known that the total deformation process of rock material can be regarded as an evolution of viscoelastic plasticity. As mentioned in previous researches (Koeller 1984; Zhou et al. 2011), fractional order \( \alpha \) that varying from 0 to 1 can be used to reflect viscoelastic behaviors of material, which characterize the transformation between ideal elastic body and viscous fluid body.

And for the description of viscoplasticity of material, as said in theory of shear thinning (Yin et al. 2012; Meng et al. 2019), fractional order \( \alpha \) that exceeding to 1 can be used to describe plastic behavior of material. Based on above stated, in this study, the total compressive deformation process of rock material is divided as two parts, i.e., pre-peak stage and peak post stage, that is shown in Fig. 7.

![Fig. 7 Segment of total compressive deformation process of rock material](image)

Fig. 7 Segment of total compressive deformation process of rock material

For exhibiting the evolution of mechanical property during compressive deformation, corresponding to presented pre-peak stage and peak post stage, a piecewise varying-order function
with exponential form is determined, which is related to time-varying strain, that is expressed as Eq. (8). And in terms of determination of varying-order function, it can be interpreted that with an consumption in full viscosity of material, the ability of resisting to deformation of material is gradually weaken, this attenuation that from 1 to 0 can be characterized by classical exponential function, i.e., \( \alpha(\varepsilon) = e^{-\varepsilon}/\varepsilon_0 \). Then considering to peak post stage of compressive deformation, corresponding to fractional order \( \alpha \) exceeding to 1, a varying-order function with same exponential form is assumed, i.e., \( \alpha(\varepsilon) = e^{-\varepsilon/\varepsilon_1} \). Finally, in conjunction of constitutive model (Eq. (7)) and presented varying-order function (Eq. (8)), a novel variable-order fractional constitutive model related to strain is formulated as Eq. (9). And in next section, its applicability and validation will be verified by stress-strain experimental data of red sandstone after heating-cooling treatment.

\[
\alpha(\varepsilon) = \begin{cases} 
e^{-\varepsilon/\varepsilon_0}, & 0 \leq \varepsilon(t) \leq \varepsilon_p \varepsilon(t), & \varepsilon(t) > \varepsilon_p \\
\end{cases} \tag{8}
\]

where \( \alpha(\varepsilon) \) is varying-order function related to strain, \( \varepsilon_p \) is peak strain, \( \varepsilon_0 \) and \( \varepsilon_1 \) represent controlling strain parameters within pre-peak stage and peak post stage, respectively.

\[
\sigma(t) = E(\alpha T)^{\alpha(\varepsilon)} \varepsilon(t)^{1-a(\varepsilon)} \frac{\varepsilon_0}{[2-a(\varepsilon)]} \tag{9}
\]

### 3.3 Model validation and comparison

![Fig. 8 Comparisons among experimental stress-strain curve, predicted curves of proposed variable-order fractional model and constant-order fractional model. And variations between and varying-order function and strain. a) Comparisons and variations of red sandstone under normal temperature with 25°C; b) Comparisons and variations of red sandstone after heating 200°C and cooling 10°C treatment.](image)

In order to verify the applicability of proposed variable-order fractional constitutive model, in this section, based on obtained stress-strain experimental data of red sandstone after heating-cooling treatment, a series of model validations are performed and the related analysis of evolution of varying-order function are also given. Fig. 8 illustrates comparisons among experimental stress-strain curves of red sandstone samples with 25°C and 200-10°C, predicted curve by proposed variable-order fractional model and predicted curve by constant-order fractional model. It should be noted due to the consistent well agreements of fitting results for experimental data, the experimental samples with 25°C and 200-10°C are selected as analyzed.
examples. It can be obtained in Fig. 8(a) that the experimental stress-strain data is well correspondence with proposed variable-order fractional model and the constant-order fractional model is inconsistent with experimental stress-strain data. There are obvious differences between proposed variable-order fractional model and constant-order fractional model that displayed in Fig. 8(b).

And what need to be highlighted is evolution of varying-order function and its corresponding interpretation. We can see with growth of strain, when time-varying strain is less than peak strain $\varepsilon_p$, varying-order function will decay from 1 to 0, which can be interpreted the full viscosity of material, that characterized by varying-order function equaling to 1, will be consumed to resist deformation and the changes of mechanical property can be exhibited by presented varying-order function related to strain, these mentioned are similar to previous study (Cai et al. 2020; Liu and Li 2020). Meanwhile, when concentrating to peak post stage, by referring from other researches (Liu and Li 2020; Zhang et al. 2015), for better describing plastic evolution and strain softening behavior, a varying-order function that greater than 1 with exponential form is established. And the presented varying-order function within peak post stage varies from 1 to 2, which is same as previous researches of soft rock and polymer materials (Liu and Li 2020; Zhang et al. 2015). Last but not least, the evolution of mechanical property of red sandstone after heating-cooling treatment can be well characterized by proposed variable-order fractional constitutive model and the presented varying-order function is valuable in description of dynamic mechanical property. Finally, Fig. 9 illustrates other comparisons of red sandstone samples after eight kinds of heating-cooling treatments and similar conclusions can be achieved from above mentioned. And the related fitting parameters of proposed variable-order fractional constitutive model based on experimental data are given in Tab. 1.
Fig. 9 Comparisons among experimental stress-strain curve, predicted curves of proposed variable-order fractional model and constant-order fractional model. And variations between and varying-order function and strain. a) Comparisons and variations of red sandstone after heating and cooling treatment of 300-10℃; b) That of red sandstone after 400-10℃; c) That of red sandstone after 500-10℃; d) That of red sandstone after 600-10℃; e) That of red sandstone after 700-10℃; f) That of red sandstone after 800-10℃; g) That of red sandstone after 900-10℃; h) That of red sandstone after 1000-10℃.

Table. 1 Fitting parameters of proposed variable-order fractional constitutive model

| Style      | $E_0$/GPa | $\tau_0$ | $\varepsilon_0$ | $E_1$/GPa | $\tau_1 \times 10^3$ | $\varepsilon_1$ |
|------------|-----------|-----------|------------------|-----------|-----------------------|-----------------|
| 25℃        | 127.8     | 0.0184    | 0.0130           | 1.3287    | 5.9106                |                 |
| 200-10℃    | 100.4     | 45.65     | 0.1777           | 3.3329    | 3.8112                | 1.2886          |
| 300-10℃    | 170.6     | 70.15     | 0.0344           | 18.587    | 0.2235                | 1.9343          |
| 400-10℃    | 80.01     | 267.9     | 1.4380           | 3.6191    | 8.0798                | 4.7618          |
| 500-10℃    | 65.21     | 289.0     | 1.7000           | 3.2471    | 1.0259                | 2.6035          |
| 600-10℃    | 72.85     | 3.771     | 0.5493           | 3.6239    | 5.0129                | 3.1405          |
| 700-10℃    | 84.54     | 39.99     | 0.9649           | 5.3354    | 4.0884                | 3.0245          |
| 800-10℃    | 95.64     | 118.3     | 1.3110           | 0.1934    | 2.3616                | 3.0710          |
| 900-10℃    | 28.46     | 993.3     | 1.4940           | 3.6775    | 3.1849                | 3.5050          |
| 1000-10℃   | 28.13     | 1897      | 16.010           | 0.0742    | 3.6676                | 4.2385          |

3.4 Analysis of sensitivity of parameters
From the above comparisons between experimental stress-strain data and proposed variable-order fractional model, it can be observed the proposed constitutive model is well agreement with experimental data. Corresponding to pre-peak stage and peak post stage, there are six divided fitting parameters of proposed variable-order fractional constitutive model with piecewise varying-order function, i.e., $E_0$, $\tau_0$, $\varepsilon_0$, $E_1$, $\tau_1$ and $\varepsilon_1$. For better clarifying the influence of fitting parameters on stress-strain behaviors, a series of analysis of sensitivity of fitting parameters were performed on selected sample of 200-10℃. It is illustrated in Fig. 10 that the effect of elastic modulus $E_0$ and $E_1$ on stress-strain behavior of red sandstone sample. For the pre-peak stage in Fig. 10(a), the increasing of $E_0$ will upgrade its peak strength and elastic deformation and the variations of $E_0$ has no effect on initial compaction stage. Likewise, in Fig. 10(b), the alterations of $E_1$ only change stress value and have no influence on tendency of stress-strain curve of peak post stage.

And considering to effect of relaxation time $\tau_0$ and $\tau_1$ on stress-strain curves, Fig. 11(a) presents with an increase in relaxation time $\tau_0$, the influence of $\tau_0$ on peak strength is little, but it reduces the length of compaction stage. And it is similar as phenomenon in Fig. 10(b), Fig. 11(b) shows the rising of relaxation time $\tau_1$ only upgrade the position of stress-strain curve of peak post stage.
post stage and it has no obvious effect on evolution of stress-strain curve. Nevertheless, when concentrating on controlling strain parameter $\varepsilon_0$ and $\varepsilon_1$ within pre-peak stage and peak post stage, in Fig. 12(a), the influence of controlling strain $\varepsilon_0$ on stress-strain behavior is same as displayed in Fig. 10(a) and Fig. 11(a). It is worth noting that the variations of $\varepsilon_1$ has prominent effect on stress-strain curve of peak post stage. When $\varepsilon_1$ rises down, the peak strength is still invariant, but decreasing rate of stress rises up gradually. And it is noticeable that the decreasing of $\varepsilon_1$ will keep last strength of peak post stage constant and reduce the peak strength of treated sample. The decreasing rate of stress will rise down gradually with an increase in controlling strain $\varepsilon_1$. In short, based on above discussions on sensitivity of parameters, it can be concluded that elastic modulus and relaxation time have obvious effect on peak strength and controlling strain parameter will induce peak strength and evolution characteristics of stress-strain curve within peak post stage, which will provide references for interpreting deformation mechanism of total stress-strain curve under compressive experiments.

![Fig. 12 Effect of strain controlling parameter $\varepsilon_0$ and $\varepsilon_1$ on stress-strain relationships of pre-peak stage and peak post stage](image)

4 Discussions

The main mineral composition of red sandstone includes clastic and clay minerals, e.g., quartz, feldspar, montmorillonite and illite etc. When red sandstone is subjected to water, the clay minerals within red sandstone are easy to disintegrate and soften quickly, which is the main factor affecting mechanical property of red sandstone. Different high temperature treatments will cause variations of thermal expansion and contraction coefficient of mineral particles within rock itself, which will lead to thermal stress due to disharmony of thermal deformation. When thermal stress resulted by high temperature heating exceeding to tensile yield strength of crystal particles, numerous micro-cracks will be developed and the internal structure of rock mass will be destroyed (Zhu et al. 2020). Subsequently, thermal rupture will be induced and affect mechanical property. During the processing of rising temperature, the interlayer water, crystal water and structural water within rock mass will be taken off and even decomposed and then a specific mineral composition with little water or no water is formed, which will weaken physical and mechanical property of rock (Zhang et al. 2015). Then when the rock after high temperature treatment is subjected to water for cooling rapidly, thermal shock that resulted by rapid cooling will cause temperature gradient inside rock mass, which is greater than that leaded by supplying steady heated flow. And the micro-cracks surrounding crystal particles will extend and penetrate and then run through the
whole rock sample by rapid water cooling impaction and at last, it can be concluded the surface of rock is intact and a large number of cracks and fissures have been produced inside rock mass. In short, the changes of physical and chemical, e.g., microstructure, mineral composition and growth of cracks, after heating-cooling treatment is the main reason for weakness and damage of mechanical property of rock mass.

5 Conclusions

For studying compressive mechanical property of red sandstone after heating-cooling treatments including ten kinds of high temperature, a series of compressive experiments were carried out and corresponding elastic modulus, peak strength and primary wave velocity were also analyzed. And in order to better describe compressive stress-strain behavior of red sandstone after different heating-cooling treatments, a variable-order fractional constitutive model was proposed and its applicability and validation have been verified by experimental data. Several detailed conclusion are expressed as follow.

(1) A series of compressive experiments were conducted on red sandstone samples after heating-cooling treatments to investigate evolution of mechanical property. Peak strength of treated red sandstone samples exhibit increasing and then decreasing with heating temperature and it will arrive at maximum peak strength with heating temperature of 500°C. The evolution laws of elastic modulus are same as that of p-wave velocity. Both experience short increase and then decrease with an increase in heating temperature.

(2) The compressive stress-strain behaviors exhibit various distributions and especially in compaction stage, high temperature water cooling will induce the growth of compaction stage of treated samples. For better depicting stress-strain behaviors, based on variable-order fractional calculus, considering strain correlation and constant loading strain rate, a variable-order fractional constitutive model is proposed with clear physical meaning and expressions. The applicability and validation of proposed variable-order fractional constitutive model have been verified by obtain experimental stress-strain data.

(3) By comparing constant-order fractional constitutive model, the advantage of proposed variable-order fractional model is highlighted and it is well agreement with experimental data. Then the varying-order function related to strain is presented and the variation of varying-order function is used to exhibit the evolution of mechanical property. Finally, the sensitivity of fitting parameters is analyzed and the effect of elastic modulus, relaxation time and controlling strain on stress-strain behaviors is illustrated. The damage mechanism of heating-cooling treatment on red sandstone is discussed from the micro-structure in details.

Acknowledgment

The work introduced in this paper was supported by the National Key R&D Program of China (2016YFC0600901), National Natural Science Foundation of China (41572334, 11572344), and Fundamental Research Funds for the Central Universities (2010YL14).

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Reference

Bai M, Reinicke KM, Teodoriu C et al (2012) Investigation on water–rock interaction under geothermal hot dry rock conditions with a novel testing method, J Petrol Sci Eng 90-91:26-30.

Cai W, Wang P, Fan J (2020) A variable-order fractional model of tensile and shear behaviors for
sintered nano-silver paste used in high power electronics, Mech Mate 145:103391.

Carlos MR, Frazier P, Tighe M et al (2011) Assessing ground cover at patch and hillslope scale in semi-arid woody vegetation and pasture using fused Quickbird data, Int J Appl Earth Obs Geoinf 14(1):674-682.

Chaki S, Takarli M, Agbodjan WP (2015) Influence of thermal damage on physical properties of a granite rock: porosity, permeability and ultrasonic wave evolutions, Constr Build Mater 22(7):1456–1461

Chandrasekharam D, Chandrasekar V (2010) Geothermal Energy Resources, India: Country Update, In: Proceedings of the world geothermal congress. Bali. Indonesia. 4:25-29.

Chen Y, Wang CY (1980) Thermally induced acoustic emission in Westerly granite, Geophys. Res. Lett. 7(12):1089-1092.

Coimbra CF (2003) Mechanics with variable order differential operators, Ann Phys Berlin 12: 692-703.

Collin M, Rowcliffe D (2002) The morphology of thermal cracks in brittle materials, J Eur Ceram Soc 22(4):35-45.

Feng Z, Zhao Y, Zhou A, et al (2012) Development program of hot dry rock geothermal resource in the Yangbajing Basin of China, Renew Energy 39:490-495.

Franklin JA, Vogler UW, Szlavin J (1979) Suggested methods for determining water content, porosity, density, absorption and related properties and swelling and slake-durability index properties: part 1: suggested methods for determining water content, porosity, density, absorption and related properties, Int J Rock Mech Min Sci 3:143-151

Fu H, Zhang J, Huang Z, et al (2018) A statistical model for predicting the triaxial compressive strength of transversely isotropic rocks subjected to freeze-thaw cycling, Cold Reg Sci Tech 145:237-248.

Hu B, Yang SQ, Tian WL (2019) Creep-permeability behavior of sandstone considering thermal-damage, Geomech Eng 1(18):71-83.

Koeller RC (1984) Applications of Fractional Calculus to the Theory of Viscoelasticity, J Appl Mech 51:299-307.

Kong R, Feng XT, Zhang X, et al (2018) Study on crack initiation and damage stress in sandstone under true triaxial compression, Int J Rock Mech Min Sci 106:117-123.

Liu R, Jing H, Li X, et al (2020) An experimental study on fractal pore size distribution and hydro-mechanical properties of granites after high temperature treatment, Fractal

Liu XL, Li DJ (2020) A link between a variable-order fractional Zener model and non-Newtonian time-varying viscosity for viscoelastic material: relaxation time, Acta Mech 1-13.

Liu XL, Li DJ, Han C (2020) Nonlinear damage creep model based on fractional theory for rock materials, Mech Time-Depend Mate 10.

Lund JW, Freeston DH, Boyd TL (2005) Direct application of geothermal energy: 2005 Worldwide review, Geothermics 34:691-727.

Mainardi F (2010) Fractional Calculus and Waves in Linear Viscoelasticity: An Introduction to Mathematical Models, World Scientific Chapter 3.

Meng R, Yin D, Drapaca CS (2019) Variable-order fractional description of compression deformation of amorphous glassy polymers, Comput Mech 13:1-9.

Meng R, Yin D, Zhou C, et al (2016) Fractional description of time-dependent mechanical property evolution in materials with strain softening behavior, Appl Math Model 40:398-406.
Meng RF, Yin DS, Lu S, et al (2019) Variable-order fractional constitutive model for the time-dependent mechanical behavior of polymers across the glass transition, Europ Phys J Plus 134:376-384.

Peng J, Rong G, Cai M, et al (2016) Physical and mechanical behaviors of a thermal-damaged coarse marble under uniaxial compression, Eng Geol 200(18):88-93.

Ren J, Xue B, Zhang L, et al (2020) Characterization and prediction of rutting resistance of rock asphalt mixture under the coupling effect of water and high-temperature, Construct Build Mate 254(10):119316.

Richards HG, Parker RH, Green ASP, et al (1994) The performance and characteristics of the experimental hot dry rock geothermal reservoir at Rosemanowes, Cornwall (1985–1988), Geothermics 23(2):73-109.

Ross B, Samko S (1995) Fractional Integration Operator of Variable Order in the Holder Spaces. Int J Math Math Sci 18:777-788.

Shao S, Wasantha PLP, Ranjith PG, et al (2014) Effect of cooling rate on the mechanical behavior of heated Strathbogie granite with different grain sizes, Int J of Rock Mech Min Sci 70:381-387.

Smit W, Vries HD (1970) Rheological models containing fractional derivatives, Rheol Acta 9:525-534.

Tang CA, Liu H, LEE PKK, et al (2000) Numerical studies of the influence of microstructure on rock failure in uniaxial compression-Part I: effect of heterogeneity, Int J Rock Mech Min Sci 37:555-569.

Weddfelt K, Saadati M, Larsson PL (2017) On the load capacity and fracture mechanism of hard rocks at indentation loading, Int J Rock Mech Min Sci 100(1):170-176.

Yang R, Hong C, Liu W, et al (2020) Non-contaminating cryogenic fluid access to high-temperature resources: Liquid nitrogen fracturing in a lab-scale Enhanced Geothermal System-ScienceDirect, Renew Energy. 165: 125-138.

Yin D, Zhang W, Cheng C, et al (2012) Fractional time dependent Bingham model for muddy clay, J Non-Newton Fluid Mech (187-188):32-35.

Zaigham NA, Nayyar ZA (2010) Renewable hot dry rock geothermal energy source and its potential in Pakistan, Renew Sustain Energy Rev 14:1124-1129.

Zhang F, Zhao J, Hu D, et al (2017) Laboratory Investigation on Physical and Mechanical Properties of Granite After Heating and Water-Cooling Treatment, Rock Mech Rock Eng

Zhang F, Zhao J, Hu D, et al (2017) Laboratory Investigation on Physical and Mechanical Properties of Granite After Heating and Water-Cooling Treatment, Rock Mech Rock Eng

Zhang L, Mao X, Liu R, et al (2014) The Mechanical Properties of Mudstone at High Temperatures: an Experimental Study, Rock Mech Rock Eng 47:1479-1484.

Zhang W, Sun Q, Hao S, et al (2016) Experimental study on the variation of physical and mechanical properties of rock after high temperature treatment, Appl Therm Eng 98:1297-1304.

Zhang WQ, Qian HT, Sun Q, et al (2015) Experimental study of the effect of high temperature on primary wave velocity and microstructure of limestone, Environ Earth Sci 74: 5739-5748.

Zhou HW, Wang CP, Han BB, et al (2011) A creep constitutive model for salt rock based on fractional derivatives, Int J Rock Mech Min Sci 48:116-121.

Zhu S, Zhang W, Sun Q, et al (2017) Thermally induced variation of primary wave velocity in granite from Yantai: Experimental and modeling results, Int J Therm Sci 114: 320-326.

Zhu Z, Tian H, Chen J, et al (2019) Experimental investigation of thermal cycling effect on
physical and mechanical properties of heated granite after water cooling, Bull Eng Geo Environ 6:1-9.

Zhu ZN, Tian H, Jiang G, et al (2020) Experimental investigation on physical and mechanical properties of thermal cycling granite by water cooling, Acta Geotech 15(5).