A dark energy model interacting with dark matter and unparticle

Songbai Chen\textsuperscript{1,2} and Jiliang Jing\textsuperscript{1,2}

\textsuperscript{1} Institute of Physics and Department of Physics, Hunan Normal University, Changsha, Hunan 410081, People’s Republic of China
\textsuperscript{2} Key Laboratory of Low Dimensional Quantum Structures and Quantum Control of Ministry of Education, Hunan Normal University, Changsha, Hunan 410081, People’s Republic of China

E-mail: csb3752@163.com and jijing@hunnu.edu.cn

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Abstract
We study dynamical behaviors of dark energy models interacting with dark matter and unparticle in the standard flat Friedmann–Robertson–Walker cosmology. We considered four different interacting models and examined the stability of the critical points. We find that there exist late-time scaling attractors corresponding to an accelerating universe, and the alleviation of the coincidence problem depends on the choice of parameters in the models.

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1. Introduction

Many observations confirm that we live in an accelerating universe. Within the framework of Einstein gravity, this acceleration can be explained by the mysterious energy component called dark energy (DE) with negative pressure, which occupies almost 70\% of the content of the universe at present [1–4]. The simplest explanation for dark energy is the cosmological constant, which is a term that can be added to Einstein’s equations. This term acts like a perfect fluid with an equation of state (EoS) $\omega_x = -1$, and the energy density is associated with the quantum vacuum. Although this interpretation is consistent with observational data, at the fundamental level it fails to be convincing. The vacuum energy density is far below the value predicted by any sensible quantum field theory, and it suffers the coincidence problem, namely, why the DE and the dark matter (DM) are comparable in size exactly right now. To overcome the coincidence problem, some sophisticated dynamical scalar field DE models, such as quintessence, k-essence and phantom field, have been put forth to replace the cosmological constant [5]. However, these scalar field DE models still cannot resolve the coincidence problem.
The interacting DE model has been regarded as a possible way to alleviate this problem [6]. In these models it is assumed that there exists a nonzero interaction between DE and DM in the universe that gauges DE transfers to DM which allows us to create an equilibrium balance in the evolution of the universe, so that the density of DE keeps the same order as that of DM at late times. Although lacking a fundamental explanation for the source of the interaction, the interacting models are manifested as the useful and robust models at the order of one standard deviation by some data sets from observational cosmology including CMB shift parameter, BAO, age parameter and supernova observations and so on [6–8]. Therefore, the interacting models have attracted a great deal of interest [9–25]. Recently, some attempts have been proposed to describe the interaction between DE and DM from a fundamental field theory point of view [26, 27].

The investigations above have been focused only on the interaction between DE and DM. However, it is physically reasonable and even expected from a theoretical point of view, that DM as well as DE can interact with other dark components of the universe [28, 29]. Recent investigations show that unparticles can be treated theoretically as another important dark components in the universe. The concept of unparticle is introduced first by Georgi [30], which is based on the hypothesis that there could be an exact scale invariant hidden sector resisted at a high-energy scale (for a recent review of unparticles, see [33, 34]). Although the fundamental energy scale of such a sector is far beyond the reach of today’s or near future accelerators, it is possible that this new sector affects the low-energy phenomenology. These effects are described as unparticle in the effective low-energy field theory because the behaviors of these new degrees of freedom are quite different from those of the ordinary particles. For example, their scaling dimension does not have to be an integer or half an integer. Recently, a lot of work [31, 32] have been focused on the new collider signals for unparticle physics. One of the interesting features of unparticle is that it does not have a definite mass and instead has a continuous spectral density as a consequence of scale invariance [30]

\[
\rho(P^2) = A_{du} \theta(P^0) \theta(P^2) (P^2)^{d_u - 2},
\]

(1)

where \(P\) is the 4-momentum, \(A_{du}\) is the normalization factor and \(d_u\) is the scaling dimension. The theoretical bounds of the scaling dimension \(d_u\) are \(1 \leq d_u \leq 2\) (for boson unparticle) or \(3/2 \leq d_u \leq 5/2\) (for fermion unparticle) [34]. The pressure and energy densities of the thermal boson unparticle are given by [35]

\[
p_u = g_s T^4 \left( \frac{T}{\Lambda_{1u}} \right)^{2(2d_u - 1) \frac{C(d_u)}{4\pi^2}},
\]

\[
\rho_u = (2d_u + 1) g_s T^4 \left( \frac{T}{\Lambda_{1u}} \right)^{2(2d_u - 1) \frac{C(d_u)}{4\pi^2}},
\]

(2)

where \(C(d_u) = B(3/2, d_u) \Gamma(2d_u + 2) \zeta(2d_u + 2)\), while \(B, \Gamma, \zeta\) are the Beta, Gamma and Zeta functions, respectively. Thus, the EoS of the boson unparticle reads [35]

\[
\omega_u = \frac{1}{2d_u + 1}.
\]

(3)

For the fermion unparticle we find that the EoS has the same form as that of the boson one. Obviously, the EoS of unparticle \(\omega_u\) is positive which is different from that of DE and DM. Since the unparticle interacts weakly with standard model particles, it can be regarded as a new form of a dark component. Recent investigations show that the unparticles play an important role in the early universe [36] and black hole physics [37]. Therefore, it is natural to ask whether there exist some new properties in the late-time evolution of the universe if the unparticle takes part in the interaction with DE and DM? In this paper we choose four
different coupling terms and study the dynamical behaviors of the interacting DE models with DM and unparticle by the phase-space analysis method to discuss further the stability of the critical points and their cosmological implications.

The paper is organized as follows: in section 2, we construct a cosmological scenario in which DE interacts with DM and unparticle, and then we present the formalism for its transformation into an autonomous dynamical system which is suitable for a phase-space stability analysis. In section 3, we consider some special coupling forms and perform the phase-space analysis of the corresponding interacting DE model with DM and unparticle, and then discuss their cosmological implications. Our conclusions and discussions will be presented in the last section.

2. An interacting dark energy model with dark matter and unparticle

In the Einstein theory, a flat Friedmann–Robertson–Walker (FRW) universe is described by the standard Friedmann equation and Raychaudhuri field equation

\[ H^2 = \frac{\kappa}{3} \rho, \]
\[ \dot{H} = -\frac{\kappa}{2} (\rho + p). \]

Here \( H \) is the Hubble parameter and \( \kappa \) is the constant \( 8\pi G \). The total energy density \( \rho = \rho_m + \rho_x + \rho_u \), where \( \rho_m, \rho_x \) and \( \rho_u \) correspond to the energy densities of DM, DE and unparticle, respectively. For simplicity here we have neglected the radiation and baryons since we concentrate on the late-time accelerating universe.

The interaction among DE, DM and unparticle can be described in the background by the balance equations

\[ \dot{\rho}_x + 3H(1 + \omega_x)\rho_x = \Gamma_1, \]
\[ \dot{\rho}_m + 3H\rho_m = \Gamma_2, \]
\[ \dot{\rho}_u + 3H(1 + \omega_u)\rho_u = \Gamma_3. \]

Here the terms \( \Gamma_1, \Gamma_2 \) and \( \Gamma_3 \) describe the coupling among DE, DM and unparticles. The total conservation equation demands that

\[ \Gamma_1 + \Gamma_2 + \Gamma_3 = 0. \]

To analyze the evolution of the dynamical system, we introduce the dimensionless variables

\[ x \equiv \frac{\kappa \rho_x}{3H^2}, \quad y \equiv \frac{\kappa \rho_m}{3H^2}, \quad z \equiv \frac{\kappa \rho_u}{3H^2}, \quad \frac{d}{dN} = \frac{1}{H} \frac{d}{dt}, \]

where \( N \equiv \ln a \) is the number of e-folding to represent the cosmological time. Using the above definitions, the Hubble equations can be rewritten as

\[ x + y + z = 1, \]
\[ \frac{\dot{H}}{H^2} = -\frac{3}{2} \left[ 1 + \frac{\omega_x x + \omega_u z}{x + y + z} \right] = -\frac{3}{2} (1 + \omega_x x + \omega_u z). \]

The effective total EOS \( \omega_{\text{tot}} \) is given by

\[ \omega_{\text{tot}} = \frac{\omega_x \rho_x + \omega_u \rho_u}{\rho_x + \rho_m + \rho_u} = \frac{\omega_x x + \omega_u z}{x + y + z} = \omega_x x + \omega_u z. \]
Once the concrete forms of the coupling terms $\Gamma_1$, $\Gamma_2$ and $\Gamma_3$ are given, the equations of motion (4), (5) and (6) can be transformed to an autonomous system containing the variables $x$ and $y$ and their derivatives with respect to $N = \ln a$. This autonomous system generally has the form: $X' = f(X)$, where $X$ is the column vector constituted by the auxiliary variables, $f(X)$ is the corresponding column vector of the autonomous equations, and the prime denotes derivative with respect to $N = \ln a$. Solving the equations $X' = 0$, we can obtain the critical points $X_c$. Then, in order to describe the stability properties of these critical points, we can expand the equations $X' = f(X)$ around $X_c$ by setting $X = X_c + \delta U$ with $\delta U$ being the perturbations of the variables considered as a column vector. Thus, for each critical point we can expand the equations for the perturbations up to the first order as $\delta U' = \Xi \cdot \delta U$, where the matrix $\Xi$ contains the coefficients of the perturbation equations. Through the analysis of the eigenvalues of $\Xi$, we can describe the stability of each critical point. In general, for an arbitrary coupling term $\Gamma_i$ it is difficult to obtain the analytical forms of the critical points. Here we only consider some specific forms of $\Gamma_i$ and examine the stability of the interacting DE models with DM and unparticle in the next section.

3. Analysis of stability in the phase space

In this section, we will consider some concrete coupling forms $\Gamma_i$ and analyze the stability of the corresponding interacting DE models with DM and unparticle, and then discuss their cosmological implications.

3.1. Interacting model I

In general, there is as yet no basis in fundamental theory for a specific coupling in the dark sectors because the nature of dark sectors remains unknown. Thus, all coupling models discussed at the present moment are necessarily phenomenological [6]. There are two criteria to determine whether some models can be more physical justification than the others. One is to confront observations. The other is to examine whether the coupling can lead to accelerated scaling attractor solutions [15], which is a decisive way to achieve similar energy densities in dark sectors and alleviate the coincidence problem. Motivated by analogy with dissipation of cosmological fluids, the coupling terms $\Gamma_i$ which are proportioned to the densities $\rho_x$ and $\rho_m$ have been studied in the context of quintessence [6] and phantom [13, 16, 17, 22] models. In this section, we first consider the case in which all coupling terms $\Gamma_i$ are proportioned to the density of DE $\rho_x$. Here we choose $\Gamma_2 = \Gamma_3$ so that we can obtain the analytical expression for the critical point, which is very convenient for us to study the dynamics of the model in the following calculations. The concrete expressions of $\Gamma_i$ are

$$
\Gamma_1 = -6bH\rho_x, \quad \Gamma_2 = \Gamma_3 = 3bH\rho_x,
$$

where $b$ is a positive coupling constant. It means that in this model only DE can convert to DM and unparticle. Using dimensionless variables, the dynamical equations of the system can be expressed as

$$
x' = -6bx - 3\omega_x x + 3x(\omega_x x + \omega_u z),
y' = 3bx + 3y(\omega_x x + \omega_u z),
z' = 3bx - 3\omega_u z + 3z(\omega_x x + \omega_u z).
$$

Solving the equations $x' = 0$, $y' = 0$ and $z' = 0$, we obtain the critical points $(x_c, y_c, z_c)$:
b< ωx

The points $A_1$ and $B_1$ imply that our universe is dominated by DM and unparticle, respectively. The condition for the existence of the point $C_1$ is $0 < b < -\frac{\omega_x}{2}$. The critical point $C_1$ denotes that the DE, DM and unparticle can be coexisted in the late times of the universe.

After some operations, we can obtain the $3 \times 3$ matrix $\Xi$ of the linearized perturbation equations

\[
\Xi = \begin{pmatrix}
\omega_x(6\omega_x - 3) + 3\omega_uy_z - 6b & 0 & 3\omega_ax_c \\
3(\omega_x\omega_z + b) & 3(\omega_x + \omega_uy_z) & 3\omega_uy_c \\
3(\omega_z + b) & 0 & 3\omega_x + \omega_uy(z_x - 3)
\end{pmatrix}.
\]

The eigenvalues of the coefficient matrix $\Xi$ encode the behavior of the dynamical system near the critical points. If the real parts of all eigenvalues of the matrix $\Xi$ are negative, then the critical point is a stable point; otherwise it is unstable. Through some careful calculations, we obtain the eigenvalues of the coefficient matrix for these critical points:

- Point $A_1$ : $\lambda_1 = 0$, $\lambda_2 = -3\omega_uy$, $\lambda_3 = -3(2b + \omega_x)$,
- Point $B_1$ : $\lambda_1 = 3\omega_uy$, $\lambda_2 = 3\omega_uy$, $\lambda_3 = -3(2b + \omega_x - \omega_uy)$,
- Point $C_1$ : $\lambda_1 = 3(2b + \omega_x)$, $\lambda_2 = 3(2b + \omega_x)$, $\lambda_3 = 3(2b + \omega_x - \omega_uy)$.

For point $A_1$, the eigenvalue $\lambda_3$ is non-negative, which indicates that $A_1$ is not a stable point. For point $B_1$, we also find that both eigenvalues $\lambda_1$ and $\lambda_2$ are positive because the EoS of the unparticle $\omega_uy > 0$. Therefore the point $B_1$ is not the stable point. For point $C_1$, when $0 < b < -\frac{\omega_x}{2}$, all eigenvalues $\lambda_1$, $\lambda_2$ and $\lambda_3$ are negative, which means that $C_1$ is a stable point as shown in figure 1. The stable region of $C_1$ is not affected by the EoS of the unparticle, but the position of $C_1$ in the phase space is decided together by $\omega_x$, $\omega_uy$ and the coupling constant $b$. From equation (11), we also learn that the effective total EoS at point $C_1$ is $\omega_{tot} = 2b + \omega_x$, which is independent of $\omega_uy$. When $b \rightarrow -\omega_x/2$, we find that $\omega_{tot}$ tends to zero. It is consistent with the result shown in figure 2. As the coupling is strong enough the universe will be dominated by DM. Moreover, for the critical point $C_1$, we find that in its stable region none of the coordinates $x$, $y_c$ and $z_c$ in the phase space vanishes, which means that the coincidence problem can be alleviated in the universe described by the critical point $C_1$. Since the effective total EoS at point $C_1$ is $\omega_{tot} = 2b + \omega_x$, we else obtain $\ddot{a} \propto (3\omega_x + 6b + 1)(\omega_{tot})^{-2}$ and $\rho \propto a^{-3(\omega_x + 2b + 1)}$. This means that point $C_1$ is an accelerated scaling solution as $b < -\omega_x/2 - \frac{3}{5}$ and there is singularity in the finite future as $b < \frac{\omega_x}{2} - \frac{1}{2}$.

3.2. Interacting model II

In the previous discussions, we only studied the case DE converts to DM and unparticle and did not consider the exchange of energy between DM and unparticle. In this section, we will consider another special case in which besides DE transfer to DM, DM can also be converted to unparticle. Similarly, in order to obtain the analytical forms for the critical points, we assume that the coupling terms $G_i$ have the forms

\[
\Gamma_1 = -3bH\rho_x, \quad \Gamma_2 = 3bH(\rho_x - \rho_m), \quad \Gamma_3 = 3bH\rho_m,
\]

respectively.
The dynamics of the system can be described by
\begin{align}
x' &= -3bx - 3\omega_x x + 3x(\omega_x x + \omega_u z), \\
y' &= 3b(x - y) + 3y(\omega_x x + \omega_u z), \\
z' &= 3by - 3\omega_u z + 3z(\omega_x x + \omega_u z),
\end{align}

Figure 1. The phase diagram of interacting dark energy with DM and unparticle through the coupling terms (12). The point $C_1$ is the critical point. Here we choose the values $\omega_x = -1.2$, $\omega_u = 0.28$ and $b = 0.5$ in the stable region $b < -\omega_x/2$.

Figure 2. Variety of $x_c$, $y_c$ and $z_c$ with $b$ at the critical point $C_1$ for fixed $\omega_x = -2$ and $\omega_u = 0.3$. The coupling constant $b$ is located in the stable region $0 < b < -\omega_x/2$. 
and the critical points \((x_c, y_c, z_c)\) are
\[
\begin{align*}
\text{Point } A_2 : & \ (0, 0, 1), \\
\text{Point } B_2 : & \ (0, 1 - \frac{b}{\omega_x}, \frac{b}{\omega_x}), \\
\text{Point } C_2 : & \ \left(\frac{\omega_x(b + \omega_x - \omega_u)}{\omega_x^2 + \omega_u(b - \omega_x)}, -\frac{b(b + \omega_x - \omega_u)}{\omega_x^2 + \omega_u(b - \omega_x)}, \frac{b^2}{\omega_x^2 + \omega_u(b - \omega_x)}\right).
\end{align*}
\]

(19)

The condition that the point \(C_2\) exists is \(0 < b < \omega_u - \omega_x\). Similarly, the \(3 \times 3\) matrix \(\mathcal{Z}\) of the linearized perturbation equations reads
\[
\mathcal{Z} = \begin{bmatrix}
-3(\omega_x + b - 2\omega_x x_c - \omega_u z_c) & 3\omega_u x_c & 3\omega_u y_c \\
3\omega_x (y_c + b) & 3(\omega_x x_c + \omega_u z_c) - 3b & 3\omega_u y_c \\
3\omega_x z_c & 3b & 3\omega_x x_c + \omega_u(6z_c - 3)
\end{bmatrix}.
\]

(20)

The eigenvalues of the coefficient matrix for these critical points are
\[
\begin{align*}
\text{Point } A_2 : & \ \lambda_1 = 3\omega_u, \quad \lambda_2 = 3(\omega_u - b) , \quad \lambda_3 = -3(b + \omega_x - \omega_u), \\
\text{Point } B_2 : & \ \lambda_1 = 3b, \quad \lambda_2 = -3\omega_x, \quad \lambda_3 = 3(b - \omega_u), \\
\text{Point } C_2 : & \ \lambda_1 = 3\omega_x, \quad \lambda_2 = 3(\omega_x + b), \quad \lambda_3 = 3(b + \omega_x - \omega_u).
\end{align*}
\]

(21)

For point \(A_2\), since \(\omega_u > 0\), the eigenvalue \(\lambda_1\) is always positive, which indicates that \(A_2\) is an unstable point. Moreover, the EoS of DE \(\omega_x < 0\) and the coupling constant \(b > 0\) means that both eigenvalues \(\lambda_1\) and \(\lambda_2\) of the point \(B_2\) are positive. Thus the point \(B_2\) is an unstable point. For point \(C_2\), when \(-\omega_x < b < \omega_u - \omega_x\), the sign of \(\lambda_2\) is always opposite to the signs of \(\lambda_1\) and \(\lambda_2\), which leads \(C_2\) to a saddle point. However, when \(0 < b < -\omega_x\), we find that all eigenvalues \((\lambda_1, \lambda_2\) and \(\lambda_3\) are negative, which indicates that \(C_2\) is a stable point as shown in figure 3. Meanwhile, we also find none of the coordinate components of \(C_2\) in the phase space disappears, which means that in this case there exist three components (DE, DM and unparticle) in the late-time universe and the coincidence problem can be alleviated. From equation (11), we learn that the effective total EoS at point \(C_2\) is \(\omega_{\text{tot}} = b + \omega_x\). Therefore we obtain \(\ddot{a} \propto -(1 + 3b + 3\omega_x)H^2 + \frac{1}{\rho_{\text{tot}}}\) and \(\rho \propto a^{-3(1+b+\omega_x)}\). This means that point \(C_2\) is an accelerated scaling solution as \(b < -\omega_x - 1/3\) and there exists singularity in the finite future as \(b < -\omega_x - 1\). For fixed \(\omega_x\), the effective total EoS increases with \(b\). Figure 4 shows that in the universe described by \(C_2\) the density of the unparticle increases and DE decreases with the coupling constant \(b\), while the density of DM first increases and then decreases. This means that the universe will be dominated by the unparticle if the coupling constant \(b\) is large enough.

### 3.3. Interacting model III

In [21], we have investigated the dynamics of the interacting DE models in which the coupling terms \(\Gamma_i\), contain the product of the densities of DE and DM and find that this new type of dark sector coupling leads to the more interesting accelerated scaling solutions and presents us more complicated features in the dynamical phase space. Therefore, in the following sections, we will consider the cases in which the coupling terms \(\Gamma_i\) contain the product of the density of DE, DM and the unparticle and to see whether it presents some new properties or not in the evolution of the universe with the unparticle component. For mathematical simplicity, we first assume that the interaction among DE, DM and unparticle has the forms
\[
\Gamma_1 = -6b\kappa H^{-1}\rho_x\rho_u, \quad \Gamma_2 = \Gamma_3 = 3b\kappa H^{-1}\rho_x\rho_u.
\]

(22)

As in model (12), these coupling terms also denote that DE can be transferred to DM and the unparticle. The dynamical equations of the system can be written as
Figure 3. The phase diagram of interacting dark energy with DM and unparticle through the coupling terms (17). The point $C_2$ is the critical point. Here we choose the values $\omega_x = -1.2$, $\omega_u = 0.28$ and $b = 0.5$ in the stable region $b < -\omega_x$.

Figure 4. Variety of $x_c$, $y_c$ and $z_c$ with $b$ at the critical point $C_2$ for fixed $\omega_x = -1.8$ and $\omega_u = 0.2$.

\[
\begin{align*}
    x' &= -6bxz - 3\omega_x x + 3x(\omega_x x + \omega_u z), \\
    y' &= 3bxz + 3y(\omega_x x + \omega_u z), \\
    z' &= 3bxz - 3\omega_u z + 3z(\omega_x x + \omega_u z),
\end{align*}
\]
and the critical points \((x_c, y_c, z_c)\) are

- Point \(A_1\) : \((1, 0, 0)\),
- Point \(B_3\) : \((0, 0, 1)\),
- Point \(C_3\) : \[
\begin{pmatrix}
\omega_x (2b + \omega_x - \omega_u) & (\omega_x - \omega_u)^2 + 3b(\omega_x - \omega_u) + 2b^2 & -b(b + \omega_x - \omega_u) \\
\omega_y (2b + 2\omega_y - \omega_u) & b(2b + 2\omega_y - \omega_u) & -b(2b + 2\omega_y - \omega_u)
\end{pmatrix}
\]

(24)

The condition that the point \(C_3\) exists is \(b > \omega_u - \omega_x\). Repeating the previous operations, we can obtain a 3 × 3 matrix \(\Xi\) of the linearized perturbation equations,

\[
\Xi = \begin{bmatrix}
\omega_x (6\omega_x - 3) + 3\omega_u z_c - 6b & 0 & 3\omega_u x_c - 6b \\
3(\omega_x y_c + b z_c) & 3(\omega_x x_c + \omega_u z_c) & 3(\omega_u x_c + b x_c) \\
3(\omega_x + b) z_c & 0 & 3\omega_x x_c + \omega_u (6z_c - 3) + 3b
\end{bmatrix}
\]

(25)

and the eigenvalues of the coefficient matrix for these critical points

- Point \(A_3\) : \(\lambda_1 = 3\omega_x\), \(\lambda_2 = 3\omega_y\), \(\lambda_3 = 3(b + \omega_x - \omega_u)\),
- Point \(B_3\) : \(\lambda_1 = 3\omega_u\), \(\lambda_2 = 3\omega_y\), \(\lambda_3 = -3(b + \omega_x - \omega_u)\),
- Point \(C_3\) : \(\lambda_1 = \frac{3\omega_u \omega_x}{2b + 2\omega_x - \omega_u}\), \(\lambda_{2,3} = \frac{3(b\omega_u \omega_x \pm \sqrt{(b\omega_u \omega_x)^2 + 4b^2(\omega_x - \omega_u)^2(2\omega_x - \omega_u) + 4(2\omega_x - \omega_u)^2(2\omega_x - \omega_u) + b^2(2\omega_x - \omega_u)^2)}}{2b(2b + 2\omega_x - \omega_u)}\)

(26)

Through similar analysis, we find that \(A_3\) is a stable point when \(b < \omega_u - \omega_x\) and is a saddle point when \(b > \omega_u - \omega_x\). The point \(B_3\) is an unstable point because \(\omega_u > 0\), which leads to the signs of the eigenvalues \(\lambda_{1,2}\) that are always positive. For the point \(C_3\), it is a stable point when \(b > \omega_x - \omega_u\). We can obtain the total effective EoS \(\omega_{\text{tot}} = \frac{4\omega_x \omega_u}{2b + 2\omega_x - \omega_u}\). It depends not only on the EoS of DE and unparticle, but also on the coupling constant \(b\). From the total effective EoS \(\omega_{\text{tot}}\), it is easy to obtain that point \(C_3\) is an accelerated scaling solution as \(\omega_u - \omega_x < b < (\omega_u - 2\omega_x - 3\omega_x \omega_u)/2\). While for \(b > (\omega_u - 2\omega_x - 3\omega_x \omega_u)/2\), we find that \(\omega_{\text{tot}} > -1/3\), which means that it is a decelerated scaling solution. This can be explained by the fact that for the larger \(b\) more DE transfers to DM and unparticle in the universe. While, as \(b < \omega_x - \omega_u\), the point \(C_3\) is an unstable point. Moreover, in this model, when \(b\) is small coupling the universe will enter the era dominated by DE because \(A_3\) is a stable point (as shown in the left figure in figure 5). When \(b\) is larger the universe will enter a stable stage described by point \(C_3\) (as shown in the right figure in figure 5), which contains the DE, DM and unparticle. Thus, when \(b > \omega_u - \omega_x\) this model can resolve the coincidence problem. Moreover, from figure 6, we also find that with the increase of \(b\) the density of DM increases in the universe, but both DE and unparticle decrease. This implies that the universe will be dominated by DM when the interaction is very strong. Figure 7 also shows that with the increase of the coupling constant \(b\) the EoS \(\omega_{\text{tot}}\) increases so that the universe can make a transition from an accelerating expansion phase to a decelerating one. As \(b\) tends to infinity the EoS \(\omega_{\text{tot}}\) approaches zero, which is consistent with that of DM.

3.4. Interacting model IV

In model (III), we consider only the case the coupling terms do not contain the density of DM. In the following model, we will consider that the interaction includes \(\rho_m\) and suppose that \(\Gamma_i\) have the forms

\[
\Gamma_1 = -3b\kappa H^{-1}\rho_x \rho_u, \quad \Gamma_2 = 3b\kappa H^{-1}(\rho_x \rho_u - \rho_m \rho_u), \quad \Gamma_3 = 3b\kappa H^{-1}\rho_m \rho_u
\]

(27)
Figure 5. The phase diagram of the interacting dark energy with DM and unparticle through the coupling terms (22). In the left figure, we fix the values \((\omega_x = -1.2, \omega_u = 0.28\) and \(b = 0.5\)) which meets \(b < \omega_u - \omega_x\) and the critical point is \(A_3\). In the right, we choose the values \((\omega_x = -1.2, \omega_u = 0.28\) and \(b = 1.8\)) which satisfies \(b > \omega_u - \omega_x\) and the critical point is \(C_3\).

Figure 6. A variety of \(x_c, y_c,\) and \(z_c\) with \(b\) at the critical point \(C_3\) for fixed \(\omega_x = -1.2\) and \(\omega_u = 0.3\). Here the coupling constant \(b\) is located in the region \(b > \omega_u - \omega_x\).

Although the coupling forms are more complicated than model (22), it also presents us the analytical expressions for the critical points which bring simplicity in the later calculation. This type of \(\Gamma_1\) also denotes that DE can be transferred to DM and at the same time DM can also be converted to unparticle. The dynamical equations of the system are given by

\[
\begin{align*}
x' &= -3b x z - 3\omega_x x + 3x(\omega_x x + \omega_u z), \\
y' &= 3b(x z - yz) + 3y(\omega_x x + \omega_u z), \\
z' &= 3b y z - 3\omega_u z + 3z(\omega_x x + \omega_u z),
\end{align*}
\]  

(28)
and the critical points \((x_c, y_c, z_c)\) can be expressed as

- Point \(A_4\) : \((1, 0, 0)\),
- Point \(B_4\) : \((0, 0, 1)\),
- Point \(C_4\) : 
  \[
  \frac{(b + \omega_x - \omega_u)}{b}, \frac{(\omega_u - \omega_x)(b + \omega_x - \omega_u)}{b(b - \omega_u)}, -\frac{\omega_x(x_c)}{b(b - \omega_u)}.
  \]

The critical point \(C_4\) exists only when \(b > \omega_u - \omega_x\). Similarly, we can obtain the \(3 \times 3\) matrix \(\Xi\):

\[
\Xi = \begin{bmatrix}
  \omega_x(6x_c - 3) + 3\omega_uz_c - 3b & 0 & 3(\omega_u - b)x_c \\
  3(\omega_ux_c + b_zc) & 3(\omega_ux_c + \omega_uz_c - b_zc) & 3(\omega_uy_c + bx_c - by_c) \\
  3\omega_ux_c & 3b_zc & 3\omega_ux_c + \omega_u(6z_c - 3) + 3by_c
\end{bmatrix}.
\]

The eigenvalues of the coefficient matrix for these critical points are

- Point \(A_4\) : \(\lambda_1 = 3\omega_x, \quad \lambda_2 = 3\omega_x, \quad \lambda_3 = 3(\omega_x - \omega_u)\),
- Point \(B_4\) : \(\lambda_1 = 3\omega_u, \quad \lambda_2 = 3(\omega_u - b), \quad \lambda_3 = -3(b + \omega_x - \omega_u)\),
- Point \(C_4\) : 
  \[
  \lambda_1 = \frac{3\omega_u(b + \omega_x - 2\omega_u)}{b - \omega_u}, \quad 
  \lambda_{2,3} = \frac{3(b\omega_x \pm \sqrt{b\omega_x[4(\omega_x - \omega_u)^2 + b(5\omega_x - 4\omega_u)]})}{2b}.
  \]

Obviously, all eigenvalues \((\lambda_1, \lambda_2, \lambda_3)\) for the point \(A_4\) are negative because the EoS of DE \(\omega_x < 0\), which indicates that in this case point \(A_4\) is always a stable point. \(B_4\) is an unstable point since \(\lambda_1\) is positive. From the previous discussion we find that point \(C_4\) exists only when \(b > \omega_u - \omega_x\), which leads to \(\lambda_2 > 0\) and \(\lambda_3 < 0\). This means that point \(C_4\) is a saddle point. Since point \(A_4\) describes a universe filled with only DE, the interaction (27) cannot resolve the coincidence problem.
4. Conclusions and discussions

In this paper we have studied the dynamical behaviors when DE is coupling to DM and unparticle in the standard flat FRW cosmology. We considered four different interacting models and examined the stability of critical points. In all the examined models we found that there exists a stable late-time scaling solution which corresponds to an accelerating universe. This feature was expected since DE cosmology has been constructed to always satisfy this condition. Moreover, for all the studied cases, we also find the accelerating stable solutions if we choose the appropriate parameters in the models. Except the fourth model, the stable solutions in other models admit that DE coexists with DM and unparticle in the universe. Our result also implies that in these models the coincidence problem can be alleviated only when the coupling is not strong enough.

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