Analysis on load distribution of beam-arch combination system bridge

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Abstract. Many prestressed concrete beam-arch combination system bridges have been built in China, but the research on the mechanical characteristics of beam-arch combination system bridges is insufficient. By adopting the structural mechanics and taking the three-span continuous beam-arch combination bridge as the research object, a theoretical formula for calculating the load distribution of the beam, arch and boom of the beam-arch combination system is put forward. By comparing the theoretical calculation with the finite element calculation, the theoretical calculation formula is verified, and the factors affecting the load sharing ratio and the bending moment distribution ratio are analysed. The results show that the load sharing ratio is less affected by the beam-arch bending stiffness ratio, and more affected by beam-to-boom equivalent stiffness ratio. The load sharing ratio increases with the increase of rise-span ratio and the decrease of side-to-span ratio. The beam-arch bending moment distribution ratio is related to the beam-arch bending stiffness ratio, the side-to-span ratio and the rise-span ratio, and beam-to-boom equivalent stiffness ratio.

1. Introduction
In recent years, beam-arch combination bridges have been widely used in bridge construction due to appealing design and reasonable mechanical distribution. Under load, arch bears the pressure, and beam bears the thrust generated by the arch foot, which not only solves the problem of excessive thrust of the traditional arch bridge, but also improves the stress state of the beam bridge. In order to better understand the mechanical characteristics of beam-arch composite bridge, scholars at home and abroad have carried out a series of studies on it. Zhu, W.G. and Liu, Z.Y. studied the effects of the rise-span ratio and beam-arch bending stiffness ratio on internal forces under dead load, live load, and temperature changes and obtained the change rule[1-2]. Yi, Y.K. studied the mechanism of beam-arch synergistic deformation, explored the influencing factors of the arch-beam load sharing ratio and arch-beam deflection ratio, and redefined the boundary values of rigid girder flexible arch, flexible beam rigid arch and rigid beam rigid arch[3]. Li, Y. analyzed the role of beam-arch bending stiffness ratio in the distribution of axial force and bending moment[4]. Cai, J.B, Chen, H.L. and Hu, M. studied the factors that affect the load sharing ratio of the arch[5]. Zhu, Y. deduced the formulas of beam-arch bending moment ratio of the flexible arch rigid beam combination system, and analyzed the causes for the error of formulas [6]. Teng, L.P. analyzed the structural stress changes of simply supported beam-arch composite system bridge under different rise-span ratios, beam-arch bending stiffness ratios and initial tension of booms. It was also proposed that there exists an optimal boom stiffness for the internal force distribution[7].

With the rapid development of finite element calculation, the design of beam-arch composite bridge is more convenient, but it is difficult for finite element calculation to clearly show the
relationship between structural mechanical characteristics and design parameters. Calculating the load sharing situation of the beam-arch combination system bridge through the structural mechanics will provide the designer with foundation in the preliminary design stage, reduce the number of trial calculations and improve design efficiency.

2. Formula Derivation
By taking three-span continuous beam-arch composite system bridge as the research object, the mechanical characteristics under even load are studied based on the principles of structural mechanics and material mechanics.

2.1. Basic Assumption
In order to simplify the calculation, the basic assumptions in the process of calculating the tension of the boom are as follows:

(1) The relative rotation angle and displacement of the beam-arch joint are not considered.
(2) The beam and arch are all members with equal section and the arch axis is a parabola.
(3) The booms are equally spaced, bear the same force, and comply with the assumption of equivalent tension.

According to the basic assumption (1), the three-span continuous beam-arch combination bridge can be replaced by the corresponding three-span continuous beam and fixed arch. According to the basic assumption (2), the equation of arch axis is $y = \frac{4fc^2}{L^2}$, and the calculation process can be greatly simplified. According to the basic assumption (3), the beam bears a load of $(1-t)q$ and the arch bears a load of $tq$ under even load $q$.

2.2. Solution Procedure
Take the following three-span continuous beam-arch combination bridge as an example to calculate the load sharing ratio in this paper. As shown in figure 1, the side span is $L_1$; the middle span is $L$; the boom spacing is $a$; and the rise is $f$. The structure can be simplified as shown in figure 2 under the above basic assumptions.

![Figure 1. Figure of a beam-arch combination bridge.](image)

![Figure 2. Simplified structure.](image)

The beam-arch combination bridge deforms under the action of even load. According to the deformation coordination conditions, equation (1) is obtained. In equation (1), $\delta_1, \delta_2$ and $\delta_3$ represent mid-span deflection of beam, arch and boom respectively, which can be calculated by the simplified structure.

$$\delta_1 = \delta_2 + \delta_3 \quad (1)$$

The three-span continuous beam is statically indeterminate structure. Take the basic structure as shown in figure 3. $X_1$ is solved by canonical equation of force method, and then $\delta$ can be obtained.
according to structural mechanics and material mechanics. The expression of $\delta_1$ is shown in equation (2). For the coherence of the article, each symbolic expression is listed in the appendix.

$$\delta_1 = \frac{5(1-\eta)qL^3}{384EI_a} \left(1-\eta \right)$$

Figure 3. Basic structure of the three-span continuous beam.

The fixed arch is also statically indeterminate structure. Take the basic structure as shown in Figure 4. $X_1$ and $X_2$ are solved by elastic centre method. The expression of $\delta_2$ can also be obtained after calculation, as shown in equation (3).

$$\delta_2 = \frac{RqL}{8E_bI_b}$$

Figure 4. Basic structure of the fixed arch.

As a tension member, the deformation of boom is shown in formula (4) under even load.

$$\delta_3 = \frac{fqa}{E_cA_c}$$

$\delta_1$, $\delta_2$, and $\delta_3$ are all expressions with $t$, and the value of $t$ can be obtained by simultaneous equations (1)-equation (4), as shown in equation (5).

$$t = \frac{5-12/(2\eta+3)}{5-12/(2\eta+3)+48RK_{wa}+96\mu K_{wa}}$$

The load sharing ratio is an important index of beam-arch composite system bridge. The expression is shown in equation (6).

$$\lambda = \frac{(1-\eta)q}{tq} = \frac{1}{t} - 1$$

3. Formula Verification

In order to verify the accuracy of equation (6), a continuous beam-arch composite bridge is taken as an example to calculate the finite element and equation (6) respectively. It is assumed that the span arrangement of the bridge is (75+125+75) m, the arch axis is a quadratic parabola, and the rise-span ratio is 1/5. The beam and arch are made of C50 concrete with an elastic modulus of $3.45 \times 10^4$MPa. The suspender adopts parallel high-strength wire rope with elastic modulus of $1.95 \times 10^5$MPa and spacing of 7.5m. There are 15 booms in total. The middle span of the bridge bears an even load of 10kN/m. Equation (6) and finite element are used to calculate the load sharing ratio of bridges with three different beam-arch bending stiffness ratios respectively. The calculation results of the two are shown in table 1.
Table 1. Comparison of finite element and equation calculation results

| Beam-arch stiffness ratio | finite element calculation | equation calculation |
|--------------------------|-----------------------------|----------------------|
|                         | 1/80 1 80                   | 1/80 1 80            |
| Mid-span deflection of beam /(mm) | 0.223 0.837 0.511 0.195 0.751 0.462 | 0.120 0.536 0.203 0.091 0.419 0.168 |
| Mid-span deflection of arch /(mm) | 0.950 0.950 0.320 1.000 0.990 0.380 |

It can be seen from table 1 that the results of beam and arch deflection calculated by finite element method are greater than those calculated by formula, which has nothing to do with the beam-arch bending stiffness ratio. The load sharing ratio is exactly the opposite. Moreover, compared with the finite element calculation, the error of the beam deflection and load sharing ratio calculated by the equation is smaller, while the error of the arch deflection is larger.

4. Analysis of load sharing ratio and bending moment sharing ratio

It can be seen from equation (5) that the load sharing ratio is affected by the beam-arch bending stiffness ratio, the rise-span ratio, the side-to-span ratio and beam-to-boom equivalent stiffness ratio. Taking real bridge built in China as an example, the influence of above factors on the load sharing ratio and bending moment ratio is discussed. The main parameters are shown in table 2.

Table 2. Main parameter table

| span/(m) | rise-span ratio | Beam-arch bending stiffness ratio | Beam-boom equivalent stiffness ratio |
|---------|----------------|---------------------------------|-------------------------------------|
| 52+89+52 | 0.2            | 2.42                            | 0.0685                              |

4.1. Beam-arch bending stiffness ratio

The load sharing ratio and the moment distribution ratio are calculated by changing the bending stiffness ratio of the bridge. The beam-arch bending stiffness ratio is changed by changing the arch bending stiffness in this paper. In order to intuitively express the influence of beam-arch bending stiffness ratio on the load distribution of the bridge, the calculation results are divided by the moment distribution ratio and the load distribution ratio of the bridge, as shown in figure 5.

As we can see from figure 5, with the increase of bending moment ratio, the load sharing ratio changes little because the value of R is too small. When the value of R is increased, the load sharing ratio and bending moment sharing ratio change obviously with the increase of beam-arch bending stiffness ratio. When \( K_{ab} < 1 \), load sharing ratio is basically unchanged, which is due to the influence of the value of \( RK_{ab} \). When \( K_{ab} \geq 1 \), the load sharing ratio decreases at the beginning and then keeps stable with the increase of \( K_{ab} \). The moment sharing ratio decreases with the increase of the bending moment ratio.
4.2. Rise-span ratio
Taking rise-span ratio as the variable, calculate load sharing ratio and bending moment sharing ratio respectively. The results are shown in figure 6.

The rise-span ratio affects the load sharing ratio by affecting the deformation of the fixed arch. It can be seen from figure 6 that the load sharing ratio and the bending moment distribution ratio gradually increase, and the increase gradually becomes larger as the rise-span ratio increases. Besides, moment sharing ratio is more affected by the rise-span ratio. This may be due to the fact that with the rise-span ratio increasing, the moment produced by the external force is balanced through the moment formed by the pressure and the arch foot thrust, so that the bending moment of the arch decreases and the moment sharing ratio increases significantly.

4.3. Side-to-span ratio
Taking side-to-span ratios as the variable, calculate load sharing ratio and bending moment sharing ratio respectively. The results are shown in figure 7.

The side-to-span ratio mainly affects the load sharing ratio by affecting the deflection of the main beam. It can be seen from figure 7 that with the increase of the side-to-span ratio, the load sharing ratio and the bending moment sharing ratio gradually decrease, but the change in the bending moment sharing ratio is smaller. Since there are many influencing factors that determine the side-to-span ratio, such as navigation requirements, geological conditions, economical rationality and so on, the load sharing ratio can be adjusted by changing the side-to-span ratio within the limited conditions.

4.4. Beam-to-boom equivalent stiffness ratio
The beam-to-boom equivalent stiffness ratio is changed by changing the boom stiffness in this paper. Assuming that the boom stiffness is the times of that of the real bridge, the influence of the beam-to-boom equivalent stiffness ratio is analysed. The results are shown in figure 8.

It can be seen from figure 8 that the load sharing ratio decreases as the boom stiffness increases. When the boom stiffness is small, it decreases greatly, and the bending moment distribution ratio tends to be stable when the boom stiffness is 1.5 times the design stiffness. When the boom is flexible, the load sharing ratio is relatively large. When the boom is rigid, the load is mainly borne by the arch, but the beam still bear part of the load.

5. Conclusion
In this paper, the structural mechanics and material mechanics are used to analyze the mechanical characteristics of beam-arch combination bridges. Taking a three-span continuous beam-arch composite bridge under even load as an example, theoretical calculation and finite element calculation are carried out respectively, and the change rule of the influence of design parameters on the structure is obtained. The results show that:
(1) The influence of the beam-arch bending moment ratio on the load sharing ratio is related to the value of R. When the value of R is small, the load sharing ratio keeps unchanged as the beam-arch bending stiffness ratio increases. When the R value is large, the load sharing ratio changes little when \( K_{ab} < 1 \); the load sharing ratio decreases with the increase of beam-arch bending stiffness ratio, and then tends to be stable when \( K_{ab} > 1 \). Even if the bending stiffness of the arch is relatively large, the arch will still bear part of the load.

(2) With the increase of the side-to-span ratio and the boom stiffness, the load sharing ratio will decrease. The side-to-span ratio has a smaller effect on the load sharing ratio, and the slope is relatively even; the stiffness of the boom has a greater influence on the load sharing ratio. As the stiffness of the boom increases, the change is greater early and gradually tends to be stable.

(3) The load sharing ratio and bending moment distribution ratio increase with the increase of the rise-span ratio. The slope of curves increases gradually and bending moment ratio is more affected by the rise-span ratio.

6. Appendices

\[ \eta = \frac{L_2}{L} \]  
\[ R = \frac{(f - 6y_i)[(\omega + 1) \mu^2 k / f + 2\omega \mu^2 / L]}{f^2(8\mu^2 + 3\omega \mu^2 + 2) - 6y_i f(2\mu^2 - \omega) + 4f(\omega + 1) \mu^2 k} \]  
\[ \mu = \frac{4f}{L} \]  
\[ \omega = \frac{\mu \sqrt{1 + \mu^2}}{\mu \sqrt{1 + \mu^2}} \]  
\[ k = \frac{I_2}{A_b} \]  
\[ y_i = \frac{(2\mu^2 - \omega)f}{4\mu^2(\omega + 2)} \]  
\[ K_{ab} = \frac{E_a I_a}{E_b I_b} \]  
\[ K_w = \frac{E_w I_w a}{E_L I_L} \]

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