Research And Analysis Of Three Degree Of Freedom Drivetrain Model For Wind Turbine

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Abstract. According to the characteristics of the drivetrain shaft system of large-scale wind turbines, a three-degree-of-freedom wind turbine drivetrain dynamic model is established using Lagrangian equations and dynamic equations, and the drivetrain torsion mathematical model and multi-mass simplified model are extended to Three-degree-of-freedom multi-body dynamics mathematical model. Using this mathematical model can effectively carry out theoretical analysis on the vibration model of wind power generators, and provide a theoretical reference for the design and simulation of the drivetrain. Taking this three-degree-of-freedom mathematical model as a prototype, a three-degree-of-freedom(3-DOF) rigid-flexible coupling multi-body dynamics simulation model is established in Simpack. Perform frequency domain analysis in the dynamic equilibrium state of the drivetrain model to obtain frequency characteristics such as the natural frequency and energy distribution of the drivetrain. According to the frequency characteristics, the potential resonance frequency is identified, and the dynamic response characteristics of the transmission chain are analyzed to determine the dangerous resonance frequency of the dynamic model.

1. Introduction

Wind turbines operate under complex wind conditions, with a wide range of wind speeds and a large bandwidth of rotation frequency of the wind wheel, which will cause resonance of the components and corresponding excitations of the wind turbine. Therefore, how to avoid resonance between components and the corresponding excitation in the design stage is a problem that must be studied in depth.

The purpose of analyzing the vibration of the Drivetrain system is to obtain the inherent characteristics of the system, thereby avoiding resonance and harmful vibration modes of the structure, and providing the necessary basis for the response analysis [1, 2]. Haider Al-Hamadani [3] et al. found that increasing the complexity of the model can predict more complex vibration shapes. The increase in gear meshing stiffness will result in a wider frequency range, but it has no effect on the lowest natural frequency of the fan drivetrain. At this stage, many documents use simple 2-mass models, more detailed 3-mass models, and even more complex 11-mass models for theoretical simulation modeling calculations, but the degrees of freedom of these simplified models are one-dimensional torsion [4,6]. Zhu [7] and Du Jing [8] used the multi-body dynamics simulation software Simpack as the simulation platform to establish a multi-flexible six-degree-of-freedom dynamics simulation model for the Drivetrain of a large wind turbine to determine the dangerous resonance point [9].

The model in this paper extends the simplified model of the concentrated mass block to the topology model of the complete drivetrain. According to the structural characteristics of the complete machine of the wind turbine drivetrain, a three-degree-of-freedom dynamics mathematical model of the drivetrain is established. According to the mathematical model, a 3-DOF rigid-flexible coupling multi-body
dynamics simulation model is established in Simpack and analyzes the frequency domain response and time domain response, calculates the natural frequency of the Drivetrain and screens out the dangerous resonance frequency.

2. Mathematical model of multibody dynamics

2.1. Drivetrain dynamic model

This article takes a 5MW horizontal axis wind turbine as the modeling object. Its structure diagram is shown in figure 1. The transmission system includes three speed stages, one stage of planetary gear and two stage of parallel shafts. The total transmission ratio of the drivetrain is 106:1.

![Figure 1. Dynamic Model of Gear Transmission System](image)

The planetary gear train of the drivetrain dynamic system is composed of a sun gear, three planetary gears and an inner gear ring. Let the direction from the sun gear to the planet gear be the positive direction of the meshing line. Assuming that the displacement of the sun gear is $y_s$ and $z_s$, the displacement of each planetary gears are $y_{pi}$ and $z_{pi}$, the displacement of the ring gear is $y_r$ and $z_r$, and the displacement of the planet carrier is $y_c$ and $z_c$, the structure diagram of the gear train is shown in Figure 2. Project the displacement of the sun gear and planetary gears to the meshing line direction [10] to obtain the relative displacement $\delta_{spi}$ of the sun gear the planetary gears along the meshing direction, the relative displacement $\delta_{rpi}$ of the ring gear and the planetary gears along the meshing direction, and the relative displacement $\delta_{cpi}$ of the carrier and planetary gear along the tangent direction of the tie rod are as follows:

\[
\begin{align*}
\delta_{spi} &= (y_{pi} - y_s)\sin\psi_{spi} + (z_{pi} - z_s)\cos\psi_{spi} + u_s + u_n - u_c \cos \alpha_s \\
\delta_{rpi} &= (y_{pi} - y_r)\sin\psi_{rpi} + (z_{pi} - z_r)\cos\psi_{rpi} + u_r - u_n - u_c \cos \alpha_r \\
\delta_{cpi} &= (y_{pi} - y_c)\sin\psi_{cpi} + (z_{pi} - z_c)\cos\psi_{cpi} + u_c
\end{align*}
\] (1)
Where, $\alpha_s$ is the meshing angle between the sun gear and the planet gears, $\alpha_r$ is the meshing angle between the ring gear and the planet gears.

Figure 2. Relative displacement analysis of planetary gear

The high-speed stage of the drivetrain dynamic system is composed of two-stage parallel gear. In the state of motion, due to the vibration of the system, the rotation centers of the driving gear and driven gear will shift, the rotation center is offset to $O_{G1}(y_{G1}, z_{G1}), O_{G3}(y_{G3}, z_{G3})$. Meanwhile, the center of mass of the gear equivalent mass will shift to $C_{G1}(y_{G01}, z_{G01}), C_{G3}(y_{G03}, z_{G03})$, the offset is $\rho_i = C_iO_i (i = G1, G3)$.

Figure 3. Analysis of Relative Displacement of Parallel Gears

When the dynamic equation of the model starts to respond, $O_{G1}C_{G1}$ is parallel to the $y_{G1}$ axis and $O_{G3}C_{G3}$ is parallel to the $y_{G3}$ axis, as shown in Figure 3. The mathematical relationship between the coordinates of the gear centroid of $C_{G1}$ and $C_{G3}$ and the coordinates of the gear rotation center $O_{G1}$ and $O_{G3}$ is as equation (2).
\[ \begin{align*}
\begin{cases}
y_{G01} &= y_{G1} + \rho_{G1} \cos \theta_{G1} \\
z_{G01} &= z_{G1} - \rho_{G1} \sin \theta_{G1} \\
y_{G03} &= y_{G3} - \rho_{G3} \cos \theta_{G3} \\
z_{G03} &= z_{G3} - \rho_{G3} \sin \theta_{G3}
\end{cases}
\end{align*} \] (2)

\( y_{G01}, z_{G01}, y_{G03}, z_{G03} \) are the displacement of driving gear and driven gear in the \( y \) and \( z \) directions in the rotation plane. In the same way, the displacement of the driving gear \( G_2 \) and the driven gear \( G_4 \) in the \( y \) and \( z \) directions in the rotation plane can be calculated:

\[ \begin{align*}
\begin{cases}
y_{G02} &= y_{G2} + \rho_{G2} \cos \theta_{G2} \\
z_{G02} &= z_{G2} + \rho_{G2} \sin \theta_{G2} \\
y_{G04} &= y_{G4} + \rho_{G4} \cos \theta_{G4} \\
z_{G04} &= z_{G4} + \rho_{G4} \sin \theta_{G4}
\end{cases}
\end{align*} \] (3)

Write the displacement of the parallel gear pair in equations (2) and (3) in the rotation plane in matrix form:

\[ \begin{align*}
\begin{bmatrix}
y_{G0} \\
z_{G0}
\end{bmatrix} &= \begin{bmatrix}
A_y & \lambda_y \\
A_z & \lambda_z
\end{bmatrix} \begin{bmatrix}
y_{G} \\
z_{G}
\end{bmatrix} \tag{4}
\end{align*} \]

Where: \( Y_{G0} = \begin{bmatrix} y_{G01} & y_{G02} & y_{G03} & y_{G04} \end{bmatrix}^T \) is the displacement vector of the gear center in the direction \( y \) of its rotation plane, \( Z_{G0} = \begin{bmatrix} z_{G01} & z_{G02} & z_{G03} & z_{G04} \end{bmatrix}^T \) is the displacement vector of the gear center in the direction \( z \) of its rotation plane, \( A_y, A_z \) are 4th-order matrices with diagonal elements of 1, \( \lambda_y = \text{diag}[1 \ 1 \ -1 \ 1] \) \( \lambda_z = \text{diag}[-1 \ 1 \ -1 \ 1] \) \( p_{G} = [\rho_{G1} \ \rho_{G2} \ \rho_{G3} \ \rho_{G4}]^T \).

According to Figure 4, using the linear relationship of displacement, the relationship between the bearing displacement and the gear displacement can be obtained [3]:

\[ Z = L\zeta_z \] (5)
\[ Y = L\zeta_y \] (6)

Where,

\[ Z = \begin{bmatrix}
z_{b1} & z_{b2} & z_{b6} & z_{b7} & z_{b8} & z_{b9} & z_{b10} & z_{b11} & z_{b12} & z_{b13}
\end{bmatrix}^T, \quad Y = \begin{bmatrix}
y_{b1} & y_{b2} & y_{b6} & y_{b7} & y_{b8} & y_{b9} & y_{b10} & y_{b11} & y_{b12} & y_{b13}
\end{bmatrix}^T, \quad \zeta_z = \begin{bmatrix}
\zeta_{br} & \zeta_a & \zeta_s & \zeta_{br} & \zeta_{b6} & \zeta_{b7} & \zeta_{b8} & \zeta_{b9} & \zeta_{b10} & \zeta_{b11} & \zeta_{b12} & \zeta_{b13}
\end{bmatrix}^T, \quad \zeta_y = \begin{bmatrix}
\zeta_c & \zeta_{br} & \zeta_a & \zeta_s & \zeta_{br} & \zeta_{b6} & \zeta_{b7} & \zeta_{b8} & \zeta_{b9} & \zeta_{b10} & \zeta_{b11} & \zeta_{b12} & \zeta_{b13}
\end{bmatrix}^T.
\]

\( L \) is the relation matrix of the distance between bearing and gear in Figure 4.

![Figure 4](Image)

Figure 4. Z-direction displacement relation of each axis. (a) LSS,(b) shaft2,(c) shaft3,(d) HSS1,(e) HSS2
2.2. Dynamic mathematical model of Drivetrain

In this paper, the dynamic mathematical model of the wind turbine drivetrain adopts the Lagrangian mechanics analysis method. The Lagrange equation establishes the relationship between the kinetic energy, potential energy and work of the system from the energy point of view, and its analysis steps are standardized and unified. In addition, the Lagrangian equation analysis method can avoid the constraint reaction force of each motion pair, reduce the unknowns of the equation, and reduce the amount of calculation. Therefore, Lagrange's equation is a universal method for studying constraint dynamics. The basic Lagrangian equation is as follows (7):

$$\frac{d}{dt} \left( \frac{\partial E_k}{\partial \dot{q}_i} \right) - \frac{\partial E_k}{\partial q_i} + \frac{\partial E_p}{\partial \dot{q}_i} = Q_i$$

(7)

Where, $E_k$ is Kinetic energy of the system, $E_p$ is Potential energy of the system, $q_i$ is Generalized Coordinate System Vector, $Q_i$ is Generalized force array.

1) $E_k$ Kinetic energy of the system

$$E_k = E_{k1} + E_{k2}$$

(8)

Where, $E_{k1}$, $E_{k2}$ are the kinetic energy of the main and driven wheel systems respectively

$$E_{k1} = \frac{1}{2} [m_{cw} (\dot{y}_{cw} + \dot{z}_{cw}) + I_{cw} \dot{\theta}_{cw}] + \frac{1}{2} [m_{s1} (\dot{y}_{s1} + \dot{z}_{s1}) + I_{s1} \dot{\theta}_{s1}] + \frac{1}{2} [m_{s2} (\dot{y}_{s2} + \dot{z}_{s2}) + I_{s2} \dot{\theta}_{s2}] + \frac{1}{2} [m_{s3} (\dot{y}_{s3} + \dot{z}_{s3}) + I_{s3} \dot{\theta}_{s3}]$$

$$E_{k2} = \frac{1}{2} [m_{cw} (\dot{y}_{cw} + \dot{z}_{cw}) + I_{cw} \dot{\theta}_{cw}] + \frac{1}{2} [m_{s1} (\dot{y}_{s1} + \dot{z}_{s1}) + I_{s1} \dot{\theta}_{s1}] + \frac{1}{2} [m_{s2} (\dot{y}_{s2} + \dot{z}_{s2}) + I_{s2} \dot{\theta}_{s2}] + \frac{1}{2} [m_{s3} (\dot{y}_{s3} + \dot{z}_{s3}) + I_{s3} \dot{\theta}_{s3}]$$

(9)

2) $E_p$ Potential energy of the system

$$E_p = E_{p1} + E_{p2}$$

(10)

Where, $E_{p1}$, $E_{p2}$ are the potential energy of the main and driven wheel system respectively:

$$E_{p1} = \frac{1}{2} k_{s1} (y_{s1}^2 + z_{s1}^2) + \frac{1}{2} k_{s2} (y_{s2}^2 + z_{s2}^2) + \frac{1}{2} \sum_{i=1}^{3} k_{r1} (\delta_{s1i}^2 + \delta_{s2i}^2) + \frac{1}{2} \sum_{i=1}^{3} k_{s1} (y_{s1i}^2 + z_{s1i}^2) + \frac{1}{2} \sum_{i=1}^{3} k_{s2} (y_{s2i}^2 + z_{s2i}^2)$$

$$E_{p2} = \frac{1}{2} k_{s1} (y_{s1}^2 + z_{s1}^2) + \frac{1}{2} k_{s2} (y_{s2}^2 + z_{s2}^2) + \frac{1}{2} \sum_{i=1}^{3} k_{r1} (\delta_{s1i}^2 + \delta_{s2i}^2) + \frac{1}{2} \sum_{i=1}^{3} k_{s1} (y_{s1i}^2 + z_{s1i}^2) + \frac{1}{2} \sum_{i=1}^{3} k_{s2} (y_{s2i}^2 + z_{s2i}^2)$$

(11)

3) $Q_i$ Generalized force array

In this paper, the influence of gravity is not considered, so $Q_i$ is the external force array of the system, which is the aerodynamic load acting on the wind wheel and the electromagnetic torque acting on the generator

$$Q_i = [0 \quad 0 \quad F_C \quad \cdots \quad 0 \quad 0 \quad F_{GEN}]^T$$

(12)

4) $q_i$ matrix of generalized coordinate system $q_i$:

$$q_i = [y_i \quad z_i \quad \theta_i]^T$$

(13)

Where:

- $i = \text{Rotor, } c, \text{ cp1, cp2, cp3, } s, \text{ G1, G2, G3, G4, GEN}$

Reorganize the above formulas to obtain the dynamic vibration control equation in matrix form:
Where, \( M \) is system quality matrix, \( k \) is system stiffness matrix, \( Q \) is array of the system of the external forces.

3. Modeling and analysis of Multi-body dynamic drivetrain

This paper uses the multi-body dynamics simulation platform Simpack as the simulation tool to establish the multi-body dynamics model of the drivetrain and perform simulation.

3.1. Dynamic simulation model of Drivetrain

According to the requirements of GL 2012, He et. al. [11] established a multi-body flexible wind turbine drivetrain analysis model with six degrees of freedom. In this paper, taking 5MW wind turbine as the prototype, a three degree of freedom rigid flexible coupling multi-body dynamic drivetrain model was established in SIMPACK. According to the connection state of the model, the topological diagram of the drivetrain shown in Figure 5 is drawn, and the connection relations and degrees of freedom of each component are defined. All the data in the model are measured by 5MW wind turbine, including mass, moment of inertia, stiffness and other parameters. This paper does not consider the influence of damping on the model calculation results. There are many flexible bodies in the topological graph of drivetrain. In this paper, the finite element model reduction is carried out by Craig–Bampton modal reduction method [12].

3.2. Model excitation frequency calculation

Wind turbines operate under complex wind speed conditions. The operating conditions of the wind turbine can be divided into three operating conditions: cut-in, rated and cut-out. Calculate the excitation frequency of the drivetrain system under the three working conditions, including the rotation frequency of the rotating shaft and the meshing frequency of the gear meshing. The calculated excitation frequencies for each working condition are shown in Table 1. Taking into account the wind shear-tower shadow effect, the excitation frequency of the low-speed shaft should take into account 6 times its rotation frequency.

| source | Excitation frequency/Hz |
|--------|------------------------|
| lss_1p | 0.101 0.189 0.215 |
| lss_2p | 0.203 0.378 0.431 |
| lss_3p | 0.304 0.567 0.646 |
| lss_6p | 0.608 1.134 1.292 |
| ims1_1p | 0.394 0.735 0.838 |
| ims1_2p | 0.788 1.470 1.675 |
| ims1_3p | 1.183 2.205 2.513 |

Table 1. Excitation frequency under different working conditions
3.3. Dynamic system vibration analysis

The wind turbine drive train model is a complex dynamics model. Before using Simpack for natural frequency analysis, Dynamic balance should be down to eliminate the gaps in the model firstly so that the calculated natural frequency is closer to the actual situation. After modal analysis in Simapck, the multi-order modal frequencies and the modal energy distribution under the corresponding natural frequencies are obtained. The above modal frequencies are screened according to certain rules, only the frequencies below the highest excitation frequency are selected for analysis [13], and the final natural frequencies are shown in Table 2.

| Order 1 | 2   | 3   | 4   | 5   | 6   |
|---------|-----|-----|-----|-----|-----|
| Frequency/Hz | 1.6 | 13.3 | 19.9 | 32.2 | 53.1 |

| Order 1 | 2   | 3   | 4   | 5   |
|---------|-----|-----|-----|-----|
| Frequency/Hz | 6 | 7 | 8 | 9 | 10 |
| Order 1 | 2   | 3   | 4   | 5   |
| Frequency/Hz | 53.1 | 70.4 | 139.2 | 187.6 | 295.7 |

| Order 1 | 2   | 3   | 4   |
|---------|-----|-----|-----|
| Frequency/Hz | 369.1 | 549.3 | 563.1 |
| Order 1 | 2   | 3   | 4   |
| Frequency/Hz | 807.6 | 839.9 | 919.9 |
| Order 1 | 2   | 3   |
| Frequency/Hz | 1201.3 | 1280.6 |

Combining the natural frequency and excitation frequency of the system, the Campbell diagram can be obtained, as shown in Figure 6.

Figure 6. Campbell diagram of drivetrain system (10-100Hz)
Among them, the oblique solid line represents the excitation frequency of the drivetrain system, the horizontal dotted line represents the natural frequency of the drivetrain system, and the vertical dotted line represents the hub speed under three working conditions of cut in, rated and cut out respectively. The intersection point of the oblique solid line and the straight dotted line represents the possible resonance point of the system. It can be observed from the figure 6 that the potential resonance point exists in each natural frequency. However, according to the selection principle, only when the speed level and excitation frequency of parts with energy proportion greater than 20% in energy distribution are in the same speed level, the intersection point can be considered as potential resonance point. The energy distribution diagram of the second-order natural frequency as shown in figure 7, where the energy of the gen_stand account for more than 20%, and the speed class is high-speed, and in figure 6 the excitation frequency that intersects with the second-order natural frequency includes the high-speed axis st3_p_shaft, which is composed of high speed shaft. Therefore, the modal vibration frequency and the excitation frequency are at the same speed level, that is, resonance may occur. Such resonance points are called potential resonance points.

![Energy Diagram](image)

Figure 7. Energy distribution diagram of 2nd order natural frequency

According to this method, the intersection frequencies in the Campbell diagram of figure 5 are screened one by one, and multiple potential resonance points can be obtained, as shown in Table 3.

| Order | Natural frequency /Hz | Source of excitation frequency                  |
|-------|------------------------|-------------------------------------------------|
| 2     | 13.3                   | st3_w_shaft/st3_p_shaft                        |
| 3     | 19.9                   | generator_rotor/generator_frame                |
| 4     | 32.2                   | generator_frame                                |
| 6     | 53.1                   | generator_stand                                |
| 9     | 187.6                  | st2_carrier                                    |
| 13    | 563.1                  | coupling                                       |
| 14    | 728.6                  | coupling                                       |
| 18    | 919.9                  | st3_p_shaft                                    |

Input the load of the corresponding working condition, and add the offset of the fixed load, so that the total load is input to ensure that the running speed of the whole machine can exactly cover the speed of the entire cut-in and cut-out working conditions. At the speed point corresponding to the resonance frequency point, analyze the frequency domain response of the acceleration of each excitation source around the time domain. At the 6th order natural frequency, the corresponding excitation source is the generator_stand, and the corresponding hub speed is 8.25rpm.
Therefore, at this speed, the acceleration of the generator_stand is performed with fast Fourier transform, and the frequency domain response shown in figure 8 is obtained. It can be observed from the figure 8 that under the 6th natural frequency of 53.1hz, the acceleration frequency domain response of the generator_stand appears peak value, which means that the generator_stand may appear resonance in the process of operation, which should be improved in the design of this part.

4. Conclusions
The dynamic characteristics of drivetrain of large-scale wind turbine is a factor to be fully considered in the research of wind turbine characteristics. In this paper, the torsional mathematical model and mass simplified model of drivetrain are extended to the three degree of freedom multi-body dynamic mathematical model through Lagrange equation, which provides a theoretical basis for the study of vibration of wind turbine. According to the three degree of freedom mathematical model, a three degree of freedom rigid flexible coupling multi-body dynamic simulation model is established in SIMPACK and dynamic characteristics of the simulation model are analyzed. The time and frequency domain analysis of the model shows that the model will produce dangerous resonance point at the 6th natural frequency of 53.1hz, and the response part is the generator_stand, which should be paid attention to.

Acknowledgments
The paper is sponsored by Natural Science Foundation of China (approval No.:51975066). The Authors acknowledge assistance or encouragement from colleagues and valuable comments from reviewers.

Reference
[1] Cui,Y.X. (2017) Mechanical system dynamics. Science Publishing House, Beijing.
[2] Ding,P. (2017) Research On Active Control of Torsional Vibration In Wind Turbine Drive Train. North China Electric Power University (Beijing), Beijing.
[3] Al-Hamadani H, Long H, Cartmell M. (2016) Effects of model complexity on torsional dynamic responses of NREL 750 kw wind turbine drivetrain. In: International Conference on Power Transmissions Lept. Chongqing. pp.205-212.
[4] Li,D.D., Chen,C. A Study On Dynamic Model Of Wind Turbine Generator Sets. Proceedings of the CSEE,2005(03) :117-121.
[5] Zhang,J.H., Gao,Y., Dai,C.L. et al. (2019) Torsional vibration Characteristics analysis Of Wind Turbine Drive Train. Acta Energiae Solaris Sinica, 40(05):1448-1455.
[6] E. B. Muhando, T. Senjyu, A. Uehara, T. Funabashi and C. Kim, (2009)LQG Design for Megawatt-Class WECS With DFIG Based on Functional Models' Fidelity Prerequisites. In: IEEE Transactions on Energy Conversion, vol. Japan. pp.893-904.
[7] Zhu, C., Chen, S., Song, C. et al. (2015) Dynamic analysis of a megawatt wind turbine drive train. J Mech Sci Technol. ,29, 1913–1919
[8] Du,J., Qin,Y., Li,C.W. (2014) Dynamics Modeling And Simulation Analysis Of Wind Turbine Drive Train. Acta Energiae Solaris Sinica, 35(010):1950-1957.
[9] Germanischer Lloyd. (2010) Guideline for the certification of wind turbines edition 2010. Hamburg: Germanischer Lloyd.

[10] Zhang, C. (2008) Machinery Dynamics. Beijing: Higher Education Press, Beijing.

[11] He, Y. L., Huang, W., Li, C. W. et al. (2014) Flexible Multibody Dynamics Modeling and Simulation Analysis of Large-scale Wind Turbine Drivetrain. Journal of Mechanical Engineering, 50(01):61-69.

[12] CRAIG R R. (1985) A Review Of Time-Domain And Frequency Domain Component Mode Synthesis Method. Combined Experimental/Analytical Modeling of Dynamic Structural Systems, 12(3): 1-30.

[13] Hou, H. B. (2012) Analysis and Research on the Dynamic Performance of MW Grade Wind Turbine’s Simulation. Chongqing University, Chongqing.