Regge trajectories in light and heavy mesons: the pattern of appearances and possible dynamical explanations

S. S. Afonin

Saint Petersburg State University, 7/9 Universitetskaya nab., St.Petersburg, 199034, Russia
E-mail: s.afonin@spbu.ru

Abstract

I briefly review the Regge approach to the hadron spectrum and advocate a dynamical emergence of principal quantum number in the known spectrum of light non-strange mesons. Further it is shown how the linear radial trajectories with universal slope can be extended to heavy quarkonia and a qualitative string interpretation is given. After that I discuss a recently proposed non-string model leading to a natural appearance of linear Regge trajectories and explaining many mass relations.

1 Regge trajectories and degeneracy in light meson spectrum

The Regge phenomenology emerged from the dual amplitudes \cite{1} and gave rise to various hadron string models. This approach continues to play an important role in the study of hadron spectroscopy. The dual amplitudes and some related string approaches predict the following behavior of meson masses,

\[ M_n^2 = a(J + bn + c), \quad J, n = 0, 1, 2, \ldots \]  

(1)

where \( J \) is the spin, \( n \) enumerates the radially excited states, \( a \) represents a universal slope and \( b, c \) are some constants. The relation (1) reproduces the classical Regge behavior \( M^2 \sim J \) observed in the light baryons and mesons. Also this relation predicts the equidistant daughter Regge trajectories. The number \( n \) enumerates the states on these ”radial” Regge trajectories. The available experimental data on light non-strange mesons seem to confirm the linear Regge behavior (1) \cite{1,2,14}.

A popular assumption about the structure of mesons is the hypothesis that the mesons represent a gluon string with a quark/antiquark at the ends.

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A careful analysis of light non-strange meson spectrum showed, however, that instead of spin $J$ in Eq. (1) one should use the angular momentum $L$ of the quark-antiquark pair [8–10],

$$M^2 \sim a(L + bn). \tag{2}$$

In addition, the data suggest $b = 1$ that is a remarkable property leading to numerous Hydrogen-like degeneracies [15–18].

## 2 Universal description

Various attempts to generalize the relation (1) to the sector of heavy quarks resulted in the emergence of strong non-linearities with respect to $J$ and $n$. Let us look, however, at the experimental data. A relatively rich set of data on radial recurrences in this sector exists only for the unflavored vector heavy quarkonia. Using the relevant data from the Particle Data [19], see Refs. [20, 21], one can see again the emergence of linear behavior as a function of consecutive number $n$. Aside from the ground state, the masses approximately lie on the linear radial trajectories

$$M^2_n = a(n + b), \tag{3}$$

where we re-denoted the vector intercept $b = 1 + c$. In general, the slope and intercept in (3) depend strongly on the quark flavor [20, 21].

The universal linearity suggests that some universal gluodynamics lies behind the observed behavior. We can propose a simple string scheme explaining this universality. The central idea is that at the conditions when a quark-antiquark pair form a resonance in the QCD vacuum, the pair may be viewed as a virtually static system. The binding is provided by the exchange of some particle (one might speculate that this particle is a pomeron). And one must quantize the motion of this particle (not the radial motion of quarks as in the standard hadron string approaches!). The total energy (mass) of the system is

$$M = m_1 + m_2 + p + \sigma r. \tag{4}$$

Here $m_1$ and $m_2$ are the masses of quark and antiquark separated by the distance $r$, $p$ is the momentum of exchanged particle, and $\sigma$ represents the standard string tension. Let us apply the semiclassical quantization to the momentum $p$

$$\int_0^l p dr = \pi(n + b), \quad n = 0, 1, 2, \ldots. \tag{5}$$
Here \( l \) is the maximal quark separation and the constant \( b \) depends on the boundary conditions. Substituting \( p \) from (4) to (5) and making use of the definition \( \sigma = \frac{m}{r} \) we obtain the linear radial trajectory

\[
(M_n - m_1 - m_2)^2 = 2\pi \sigma(n + b).
\] (6)

In our unflavored case \( m_1 = m_2 \equiv m \) and the relation (6) can be simplified to

\[
(M_n - 2m)^2 = a(n + b),
\] (7)

which is our generalization of the linear spectrum (3) to non-zero quark masses. In this relation, the universal gluodynamics (the slope \( a \)) and dependence on quantum numbers (the dimensionless intercept \( b \)) are clearly separated from the contribution of quark masses. For this reason, the parameters \( a \) and \( b \) in the relation (7) should be flavor-independent. The analysis of Refs. [20,21] indeed confirmed this expectation. A successful check of relation (6) in the case heavy-light systems was performed in Ref. [22].

3 A possible non-string mechanism for linear Regge trajectories

Usually the observation of linear trajectories is interpreted as an evidence for string picture of mesons. In spite of many attempts, however, a satisfactory quantized hadron string has not been constructed. Among typical flaws of this approach one can mention the absence of spontaneous chiral symmetry breaking, rapidly growing (with mass) size of meson excitations, and totally unclear role of higher Fock components in the hadron wavefunction.

A novel realization of quark model concept has been recently proposed in Ref. [23]. This realization leads to a natural (and alternative to hadron strings) explanation of Regge recurrences and is potentially free of typical shortcomings inherent to string, potential and some other approaches. Stated more strictly, the given approach represents a phenomenological mass counting scheme in which the relativistic invariance, renormalization invariance, chiral symmetry breaking, higher Fock components, and linear Regge behavior are qualitatively built-in. The approach allows to classify the light mesons without use of any angular momentum associated with hadron constituents. The states \( \pi_1 \) (which are exotic for the standard quark model) emerge in a natural way while the other exotic quantum numbers remain forbidden. The constructed mass counting scheme permits to obtain hadron masses from very simple relations with a typical accuracy comparable to numerical calculations in complicated dynamical models.
The underlying motivation is as follows. The hadron masses must be renorminvariant. On the other hand, the light non-strange mesons can be viewed, in one way or another, as some quantum excitations of pion. Basing on the Gell-Mann–Oakes–Renner relation for the pion mass and the known formula for the stress-energy tensor in QCD it is motivated that the masses of these mesons can be represented as

$$m_h^2 = \Lambda(E_h + 2m_q) = \Lambda E_h + m_{\pi}^2.$$  \hspace{1cm} (8)

The energy parameter $E_h$ seems to be related to the renorminvariant gluon condensate, $E_h \sim \alpha_s \langle G_{\mu\nu}^2 \rangle / \langle \bar{q}q \rangle$ [23]. A phenomenological mass counting scheme based on the relation (8) and a demonstration how it describes the meson spectroscopy is scrutinized in Ref. [23].

It is also suggested that the parameter $E_h$ can be interpreted as an effective energy of some constituents different from the current quarks. The problem is to propose a model for these constituents which should represent some excitations inside the pion. Three basic excitations are postulated. The first one appears when one of quarks absorbs a gluon of certain energy $E_h = E_\rho$. The spin of excited quark changes its direction to the opposite one, converting the original spin-singlet $q\bar{q}$-pair ($\pi$-meson) to the spin-triplet one ($\rho$-meson). The second kind of excitation emerges due to formation of spin-singlet $q\bar{q}$-pair with effective mass $E_h = E_0$ (the lower index stays for total spin of $S$-wave $q\bar{q}$-pair). The third basic excitation is the formation of spin-triplet $q\bar{q}$-pair with effective mass $E_h = E_1$. The formation of these constituent pairs are likely a QCD analogue of formation of para- and ortho-positronia from photons. By assumption, any excitation inside pion leading to an observable resonance can be represented as a combination of these basic excitations so that the total effective energy $E_h$ in (8) is just a sum (appropriate number of times) of $E_\rho$, $E_0$, and $E_1$. Simultaneously this will dictate the quantum numbers of constructed resonance. In a sense, the introduction of these constituent pairs reflects the excitations of higher Fock components in the pion wavefunction. These pairs in excited hadrons, in some sense, bear a superficial resemblance to neutrons and protons in atomic nuclei. The given approach represents thus a new realization of the quark model concept, a realization in which highly excited states appear due to multiquark components in hadron wavefunctions.

The inclusion of strange quarks into this approach is straightforward. It would be interesting to extend the underlying ideas to light baryons and to hadrons with heavier quarks.
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