Asset volatility forecasting:  
The optimal decay parameter in the EWMA model*  

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Abstract  
The exponentially weighted moving average (EMWA) could be labeled as a competitive volatility estimator, where its main strength relies on computation simplicity, especially in a multi-asset scenario, due to dependency only on the decay parameter, $\lambda$. But, what is the best election for $\lambda$ in the EMWA volatility model? Through a large time-series data set of historical returns of the top US large-cap companies; we test empirically the forecasting performance of the EWMA approach, under different time horizons and varying the decay parameter. Using a rolling window scheme, the out-of-sample performance of the variance-covariance matrix is computed following two approaches. First, if we look for a fixed decay parameter for the full sample, the results are in agreement with the RiskMetrics suggestion for 1-month forecasting. In addition, we provide the full-sample optimal decay parameter for the weekly and bi-weekly forecasting horizon cases, confirming two facts: i) the optimal value is as a function of the forecasting horizon, and ii) for lower forecasting horizons the short-term memory gains importance. In a second way, we also evaluate the forecasting performance of EWMA, but this time using the optimal time-varying decay parameter which minimizes the in-sample variance-covariance estimator, arriving at better accuracy than the use of a fixed-full-sample optimal parameter.  

Keywords: Volatility Forecasting, Exponentially weighted moving average, EWMA, backtesting.  
JEL Classification: C5.

1 Introduction  
Volatility is a key parameter in financial problems as derivative pricing, portfolio allocation, and value-at-risk. Thus, optimal (and sometimes efficient) forecasting plays a fundamental role in ex-ante valuation. Despite there are several sophisticated methods to deal with the volatility estimation, as the ARCH-GARCH family (Bollerslev, 2010) or stochastic volatility (SV) models (Shephard and Andersen, 2009); these models are complex and computationally expensive, especially in a multi-asset framework. Then, and depending on their purposes, some practitioners prefer to use costless implementations. One of those is the exponentially weighted moving average (EWMA) model, capable to address two well-known “stylized facts” as heteroskedasticity, and volatility clustering. This model assigns different weights to the past information, where more recent lags receive more importance than old observations. The weight assignment decays exponentially at a rate $\lambda$.  

One of the greater advantages of the EWMA over the standard GARCH models is related to computation simplicity. While in the former, the variance is easily and quickly updated; in the latter, a new likelihood maximization should be performed for every run. This point is particularly problematic in the multivariate context, where MGARCH models become very time-costly, especially when the number of assets rises. Also, EWMA captures the autocorrelation of the squared returns better than standard GARCH approaches (Bee, 2012).

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variance nor mean-reversion; so it could be not suitable for long-term forecasting. Moreover, the decay parameter should be adjusted in function of the time horizon.

Ding and Meade (2010) point that EWMA exhibits a greater forecasting accuracy to real data compared to GARCH and SV models. González-Rivera et al. (2004) found EWMA brings good results in the option pricing context. In addition, and for portfolio purposes, Zakamulin (2015) reveals that EWMA performance is very similar to the DCC-GARCH, in both covariance-matrix forecasting error and portfolio tracking error.

The EWMA method was popularized by RiskMetrics (1996), who recommends a smoothing factor equal to 0.97 for a 1-month forecast. These values are obtained minimizing the average squared errors for a large number of time-series (detailed procedure on section 5.3.2 of the Technical Document). However, Bollen (2015) and González-Rivera et al. (2007) tested empirically the optimal $\lambda$ under several criteria, found that the RiskMetrics suggested values are overestimated and far from optimal.

The goal of this paper is to deal with a common issue that should be faced by the practitioners in their implementation: what should be the optimal value for the decay parameter in the EMWA model? This question is answered taking into account different stock returns time-series and forecasting horizons: weekly, bi-weekly and monthly. For this, we use large historical data (daily adjusted-closing prices for the 27 years period 1994-2018) of the top blue-chip companies in the US stock market (DJIA components), and we evaluate the volatility estimation in a multi-asset context (variance-covariance matrix) of the EWMA approach under different specifications, measuring its out-of-sample performance, in a rolling-window scheme. The use of daily data obeys two reasons. First, for forecasting horizons greater or equal than a week, as the selected ones, the daily frequency offers similar results to high-frequency for volatility prediction (Lýócsa et al., 2021). And second, the high-frequency data is less accessible, and computationally (and economically) costly, which is in the opposite direction to this work aims. We address and compare two different ways to obtain the optimal smoothing parameter: i) the use of a full-sample optimal $\lambda$ (RiskMetrics methodology), and ii) the optimal in-sample time-varying $\lambda$, which maximizes the accuracy of the covariance matrix previous to the forecasting. We found a better predictive accuracy for the second way for the selected forecasting horizons.

The outline of this document is the following. First, a brief review of the realized volatility and covariances, and their estimation by the EMWA method, is developed. After that, in section 3 the methodology of the paper (data, procedure, and error measure), is described. The results are displayed and analyzed in section 4 and finally, the main conclusions are summarized.

2 Definitions

2.1 Realized Volatility and Covariances

First, the continuously compounded return (log-return) of the $i-$asset is defined as:

$$ r_{i,t} = \ln \left( \frac{P_{i,t}}{P_{i,t-1}} \right) $$

where $P_{i,t}$ is the price of the asset $i$ at time $t$.

An unbiased estimator for the ex-post true volatility is the realized volatility. At the day $t$, the realized volatility the $T$ trading days period (from $t-T+1$ to $T$), is computed as the square root of the $T-$most recent squared daily returns (Andersen et al., 1999; Bollen, 2015):

$$ \sigma_{i,t,T} = \sigma_{i,[t-T+1,t]} = \sqrt{T-1 \sum_{k=0}^{T-1} r_{i,t-k}^2} $$

Later, the entries of the covariance matrix $C_{tt}$ depends on both the volatilities and correlations for the recent $T-$days:
\[(C_{iT})_{ij} = \sigma_{ij,tT} = \sigma_{i,tT} \sigma_{j,tT} \rho_{ij,tT}\]

being \(\rho_{ij,tT}\) the correlation coefficient among the returns \(i\) and \(j\), between the days \(t - T + 1\) and \(t\). Equivalently, we have:

\[\sigma_{ij,tT} = \sum_{k=0}^{T-1} r_{i,t-k} r_{j,t-k}\]

### 2.2 EWMA volatility

The equally weighted moving average model, estimates the one-day-ahead daily conditional variance as:

\[
\hat{\sigma}^2_{i,t+1} = (1 - \lambda) \sum_{n=0}^{\infty} \lambda^n r^2_{i,t-n} \tag{1}
\]

with \(0 < \lambda < 1\).

If \(\lambda\) moves away from 1, the EWMA assigns higher weights to the recent than the past observations. Then, the quality of the results depends on the election of the parameter \(\lambda\). A value greater (lower) than the optimal goes to an under-reaction (over-reaction) to the new information input.

Equivalently, Eq. (1) could be expressed by the following recurrence relation:

\[
\hat{\sigma}^2_{i,t+1} = (1 - \lambda) r^2_{i,t} + \lambda \hat{\sigma}^2_{i,t} \tag{2}
\]

The first term of the RHS of (2) updates the variance due to the new information, while the second one represents the persistence effect. It’s easily to show that new iterations in (2) conditional to the information up to time \(t\), won’t update the variance result. Thus, for any \(k \in \mathbb{N}^+\):

\[
\hat{\sigma}^2_{i,t+k+1|t} = \hat{\sigma}^2_{i,t+1} \tag{3}
\]

Then, Eq. (3) implies that the volatility estimation should be scaled to the length of the time horizon; i.e., for the \(T\)-day period:

\[
\hat{\sigma}^2_{i,T} = T \cdot \hat{\sigma}^2_{i,t+1} \tag{4}
\]

Eq. (4) could be considered as a particular case of the of Engle and Bollerslev (1986) IGARCH(1,1), which at the same time is a restricted case of the Bollerslev (1986) standard GARCH(1,1) such that the coefficients of both lagged squared-returns and lagged variances sum one (i.e., a unit-root GARCH). This restrictions drive to an infinite unconditional variance and persistence in the variance shocks.

A big feature of the EWMA is its straightforward extension to the multivariate case, where the co-variance forecasting among the assets \(i\) and \(j\) is given by:
\[
\sigma_{ij,t+1} = (1 - \lambda) \sum_{n=0}^{\infty} \lambda^n r_{i,t-n} r_{j,t-n} \tag{4}
\]
\[
= (1 - \lambda) r_{i,t-n} r_{j,t-n} + \lambda \sigma_{ij,t-n} \tag{5}
\]

It should be marked that the simplicity of Eq. (5) is due to the use of the same \( \lambda \) for all the time-series. In fact, it also guarantees a positive semi-definite variance-covariance matrix. Besides, the multivariate EWMA could be read as a special case of the Engle and Kroner (1995) multivariate GARCH.

We can note from eq. (1) that the EWMA use all the past information (summation up to infinity). However, since the importance of the past values decays exponentially tending to zero, we can select an effective number of historical data (cut-off) according to a desired level of tolerance RiskMetrics (1996):

\[
N = \frac{\ln (\Upsilon_L)}{\ln (\lambda)} \tag{6}
\]

where \( N \) is the cutoff point and \( \Upsilon_L \) the tolerance level. Thus, Eq. (6) means that for a given \( \lambda \) the information not considered (above the \( N \)-lag) would have contributed with a total weight of \( \Upsilon_L \).

If we truncate the summation up to \( N \) terms, the weights don’t sum the unity, Eqs. (4) and (5) should be corrected as:

\[
\hat{\sigma}_{ij,t+1} = (1 - \lambda) (1 - \lambda^N) \sum_{n=0}^{N-1} \lambda^n r_{i,t-n} r_{j,t-n} \tag{7}
\]
\[
\hat{\sigma}_{ij,t+1} = (1 - \lambda) r_{i,t} r_{j,t} + \lambda \sigma_{ij,t} (1 - \lambda^N) \tag{8}
\]

Note that in the limit \( \lambda \to 1 \), the above equations converge to the raw-sample variance and covariances using \( N \) lags.

3 Methodology

3.1 Data

We use the continuously compounded returns of the daily adjusted-closing prices of 22 stocks listed continuously in the Dow Jones Industrial Average (DJIA) index from 2000 up to 2020 (see Table 1). We consider the historical records from January 3, 1994 to December 31, 2020 (6799 values for each asset) and we examine the forecasting for each day in the period 2000-2020. This time-span arises crisis, bull markets, and standard periods. The database is freely available at Yahoo! Finance (www.finance.yahoo.com). The data prior to 2000 will be used for calibration purposes and for exploratory analysis.

3.2 Exploratory analysis (1994-1999)

Table 2 shows some common statistical tests over the log-return series for each one of the considered assets, inside the six-year period 1994-1999. The \( W \)-values of the Shapiro-Wilk test exhibits non-normality in the return time-series. In terms of ARCH disturbances, Engle’s Lagrange multiplier test considering 20 lags, indicates the presence of heteroskedasticity. Besides, the Augmented Dickey-Fuller (ADF) unit-root test reveals stationarity for all the variables.

\(^1\)PFE, RTX and XOM were excluded from the index in the middle of 2020, but we also considered them in the analysis.
### Table 1: List of the used stocks

| Company Name | Symbol |
|--------------|--------|
| 3M           | MMM    |
| American Express | AXP   |
| Caterpillar Inc. | CAT   |
| ExxonMobil    | XOM    |
| Intel         | INTC   |
| JP Morgan Chase | JPM  |
| Merck & Co.   | MRK    |
| Pfizer        | PFE    |
| The Coca-Cola Co. | KO   |
| The Walt Disney Co. | DIS  |
| Verizon       | VZ     |
| Alcoa Inc.    | AA     |
| Boeing        | BA     |
| DowDuPont     | DD     |
| IBM           | IBM    |
| Johnson & Johnson | JNJ  |
| McDonald’s    | MCD    |
| Microsoft     | MSFT   |
| Procter & Gamble | PG   |
| The Home Depot | HD    |
| Raytheon Tech. Corp | RTX  |
| Walmart       | WMT    |

*a Formerly E.I. Du Pont de Nemours & Co.
*b Formerly United Tech. Corp.

### Table 2: Statistics for daily returns from 2Jan1994 to 31dec1999.

| Asset | SW | ARCH(20) | ADF |
|-------|----|----------|-----|
| AA    | 0.97* | 123.3* | -34.1* |
| BA    | 0.91* | 46.4** | -35.6* |
| DD    | 0.98* | 138.6* | -32.3* |
| HD    | 0.98* | 143.4* | -36.7* |
| INTC  | 0.99* | 33.7*** | -37.1* |
| JPM   | 0.97* | 202.8* | -32.6* |
| MCD   | 0.97* | 135.3* | -35.1* |
| MRK   | 0.98* | 39.1*** | -36.2* |
| PFE   | 0.99* | 114.0* | -33.8* |
| RTX   | 0.98* | 162.9* | -32.7* |
| VMT   | 0.99* | 91.8* | -37.0* |
| AXP   | 0.97* | 311.37* | -35.1* |
| CAT   | 0.97* | 46.1** | -34.9* |
| DIS   | 0.97* | 167.4* | -37.1* |
| IBM   | 0.94* | 13.4** | -35.6* |
| JNJ   | 0.99* | 65.0* | -34.5* |
| KO    | 0.97* | 136.0* | -34.6* |
| MMM   | 0.97* | 56.1* | -35.5* |
| MSFT  | 0.99* | 65.0* | -38.1* |
| PG    | 0.99* | 181.1* | -36.6* |
| VZ    | 0.98* | 126.5* | -40.1* |
| XOM   | 0.99* | 102.4* | -36.3* |

*a Significant at p<0.0001, ** Significant at p<0.005, *** Significant at p<0.05

*No ARCH effects are found using 20 lags. However, if considering only one lag, the null hypothesis is rejected at the 10% significance level. This fact is confirmed by the Ljung–Box \( Q^2(20) \) test to detect autocorrelation over the squared returns; where \( Q^2(20) \) provides p<0.0001.
3.3 Procedure
First, we select as forecast horizons: 1 week (5 days), 2 weeks (10 days), and 1 month (21 days). Besides, we consider 99 different values for \( \lambda \), from 0.01 to 0.99. We predict the 21 years period from 2000-2020.

Then, for a given \( \lambda \), the first \( T \)-covariance-matrix estimator will be given (through the EWMA approach) for the first weekday of 2000 year (Jan 3) using the information available \( T \) days back. The forecasted values will be compared with the \( T \)-realized-covariances on that day. Later, the procedure continue using a rolling-window framework up to the last considered day (December 31, 2020). Thus, we forecast 253 covariances, for each value of \( \lambda \), and for each working day in the 21 years interval 2000-2020 (\( \sim 1.32 \times 10^8 \) computations).

3.4 Error measure
As pointed by [Patton (2011)](#), one of the robust error measures consistent with a noise volatility proxy -as the sum of squared returns- is the mean squared error (MSE). Since the original RiskMetrics methodology uses the MSE as the main source, the results will be presented considering that loss function.

Let \( \hat{C}_{tT}(\lambda) = [\hat{\sigma}_{ij,tT}(\lambda)] \) the \( T \)-period forecasted variance-covariance matrix at time \( t \) for a given \( \lambda \), and \( C_{tT} = [\sigma_{ij,tT}] \) the realized one. The square forecasted error for the period \( T_k \) is given by [Zakamulin (2015)](#):

\[
SE_{tT}(\lambda) = \sum_{i=1}^{#A} \sum_{j=1}^{#A} (\hat{\sigma}_{ij,tT}(\lambda) - \sigma_{ij,tT})^2
\]

where \( #A = 22 \) is the number of assets considered. Then, the MSE is computed averaging among all the periods (windows):

\[
MSE_{T}(\lambda) = \frac{1}{#W} \sum_{k=1}^{#W} SE_{kT}(\lambda)
\]

being \( #W \) the number of rolling windows (5284).

The optimal \( \lambda \) at time \( t \), is who provides the minimum squared error in each date:

\[
\lambda^*_t = \arg \min [SE_{T}(\lambda)]
\]

On the other hand, the optimal smoothing parameter for the full-sample minimizes the \( MSE_T \):

\[
\lambda^*_T = \arg \min [MSE_T(\lambda)]
\]

3.5 Predictive accuracy
In order to make a pair-wise comparison of two sets of predicted values, we employ the widely-used [Diebold and Mariano (1995)](#) test who reveals statistically significant differences in the forecasting accuracy.

A brief description of the test is addressed in the following. Let \( d_t = SE_{a_t} - SE_{b_t} \) the loss differential, at time \( t = \{1, 2, \ldots, N\} \), between the squared errors of the approaches a and b. The Diebold-Mariano (DB) test identify if the predicted values have the same accuracy or not. Then, the null hypothesis correspond to zero expected value in the loss differential for all \( t \); i.e., \( H_0: \mathbb{E}(d_t) = 0 \). In consequence, the alternative hypothesis considers different levels of accuracy in the forecasting. Defining \( D \) and \( \bar{d} \) as

\[\text{Considering only the upper triangular sections of } \hat{C}_{tT} \text{ and } C_{tT}, \text{ the double accounting of the errors is avoided.}\]
Figure 1: Squared loss function for monthly forecasting

(a) Optimal $\lambda$ over time

(b) MSE using a fix $\lambda$

Figure 2: MSE for one-week and two-week forecasting using a fix $\lambda$

(a) One-week

(b) Two-week

The auto-covariance and sample mean of $d_t$, respectively; the t-statistics is given by $DM = \bar{d} / \sqrt{D/N}$. This test assumes normal distribution under the null hypothesis, $DM \sim N(0,1)$. So, the null hypothesis is rejected if $|DM| > z_{\alpha/2}$ being $z_{\alpha/2}$ the upper z-value of the standard normal distribution for the half of the desired level of tolerance $\alpha$.

4 Results

We have computed the $T$-variance-covariance estimator by EWMA, varying the decay parameter $\lambda = \{0.01, 0.02, \ldots, 0.99\}$ and forecasting horizon $T = \{5, 10, 21\}$, for each business day from Jan/03/2000 to Dec/31/2000 using the data available up to $T$ days earlier.

For the 1-month forecasting horizon, we found an optimal full-sample decay parameter equals 0.98 (see Fig. 1b) which corresponds to a MSE equal to 0.01486. The recommended value given by RiskMetrics, equal to 0.97, ranks in second place with a very little increase in the MSE value (0.01489). As could be anticipated by this minimal difference, the Diebold and Mariano test couldn’t reject the null hypothesis of equal accuracy ($p$-value >0.9) for the EWMA model using these two smoothing parameters.

Figures 2a and 2b display the results for 5 and 10 days as forecasting horizons finding optimal decays equal to $\lambda_5^* = 0.92$ and $\lambda_{10}^* = 0.95$. It confirms that the optimal decay parameter is a function of the forecasting horizon. Besides, the decrease in $\lambda_T^*$ when $T$ decline, implies a diminution in the persistence in favor of short-memory.

On a second approach, in addition to the full-sample optimal ($\lambda_T^*$) we also evaluated the use of the
Table 3: MSE and $t$-Statistics of Diebold-Mariano (DM) joint test for $\lambda_t = \lambda_T^*$ and $\lambda_t = \lambda_{(t-1)}^*$. 

|       | $T=5$         | $T=10$        | $T=21$        |       |
|-------|---------------|---------------|---------------|-------|
| MSE   | 0.001355      | 0.001160      | 0.004159      | 0.003056 |
| DM    | 3.214**       | 5.895*        | 5.174*        |       |

* Significant at $p<0.001$, ** Significant at $p<0.005$

one-time lagged optimal decay parameter; i.e., $\lambda_{(t-1)}^*$; to compute the covariance-matrix at time $t$ taking into account the realizations at $(t-1)$. For illustrative purposes, Fig. 1A plots the daily optimal decay parameter for the monthly forecast horizon. Table 3 shows, for the three selected forecasting horizons, the MSE under the two used approaches; i.e., the full-sample optimal smoothing parameter and the time-varying one that minimizes the SE at the immediately previous time. We can observe that there is an improvement in the MSE loss function if we consider $\lambda_{(t-1)}^*$ instead of the constant-valued $\lambda_T^*$. To analyze if the results are statistically significant or not, the $t$-statistics of the pairwise Diebold and Mariano are reported in the table (positive sign indicates a better accuracy of the in-sample time-varying optimal decay parameter), finding a superior forecasting performance when we use $\lambda_{(t-1)}^*$ in place of $\lambda_T^*$.

5 Summary

In this paper, the EWMA volatility model is addressed by studying the optimal decay parameter under different time horizons: 1 week, 2 weeks, and 1 year. Using the historical price database of the top 22 blue-chip companies in the US stock market in the period 2000-2020, we evaluate the forecasting performance on that time-span through the squared error loss function of the variance-covariance matrix.

If we look for a full-sample optimal smoothing parameter, the results are in agreement with the original RiskMetrics suggestion for the 1-month forecasting, finding an optimal value equal to 0.98 (RiskMetrics suggest 0.97, which in our analysis ranked second with a very minimal difference in the MSE). However, for daily forecast, our findings report that the recent lags should receive more weight ($\lambda_1^* = 0.89$) than the RiskMetrics recommendation ($\lambda_1^* = 0.94$); Nevertheless, despite the lower MSE of the former, there are no statistical differences in terms of forecasting accuracy. We also obtain the optimal full-sample decay parameter considering 1-week and 2-week as forecasting horizons: $\lambda_5^* = 0.92$ and $\lambda_{10}^* = 0.95$, respectively.

Unlike other research outputs where the estimation of $\lambda_T^*$ relies on the single-variance analysis of one or more time series, here the full variance-covariance matrix is estimated considering 22 time-series (i.e., 253 entries per day) and 21 years of data.

Moreover, we also tested the use of the optimal value of $\lambda$ at time $t-1$ (namely $\lambda_{(t-1)}^*$) to predict the covariance by EWMA for the time $t$. We found an improvement in the MSE using this way instead of a fixed $\lambda$, with the exception of the 1-day forecasting. On one hand, this improvement in the MSE attempts to the simplicity of the EWMA approach because at every time we need to run the EWMA model under a different set of $\lambda$ values and select the optimal in each computation. On the other hand, this approach takes much less time than other multivariate models (for example M-GARCH) and could be developed without high computational skills (for instance, in an Excel spreadsheet).

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