An alternate method for finding an optimal solution to Mixed Type Transportation Problem under a Fuzzy Environment

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Abstract. In this paper, we develop a new algorithm for the initial fuzzy basic feasible solution to a transportation problem with imprecise parameters. All the decision variables are assumed to be triangular fuzzy numbers or trapezoidal fuzzy numbers or real numbers. By applying the efficient ranking function and arithmetic operations the solution is obtained without changing it into its crisp equivalent form. Using the parametric form of fuzzy numbers such as left fuzziness index, right fuzziness index, modal value and by using a proposed algorithm we obtained the initial basic fuzzy feasible solution to the problem. We further discuss the optimality by applying the modified distribution method. A numerical example is solved to show the effectiveness of the proposed algorithm.

1. Introduction

The linear programming model has been utilized in a wide range of fields such as inventory management, scheduling, production, transportation and distribution, finance, agriculture, etc. The class of transportation problems is one of the relevant and well-structured types of linear programming problems. Transportation Problem (TP) is a logistical problem for enterprises, particularly for transport and manufacturing companies. The transportation problem deals with delivering a homogeneous product from suppliers to targets. The purpose of the transportation problem is to decide the delivery strategy that reduces overall delivery cost that satisfying demand and supply points. The parameters of the transportation problem are the unit cost, which is the cost of shipping an item from a certain point of supply to a certain point of demand, the quantities available at the points of supply and the quantities needed at the points of demand.

In the actual situation, the parameters of a transportation problem may not be defined; it may be inexact due to some unexpected circumstances. Modeling of real-world mathematical optimization issues is practically impossible due to the existence of vague and incomplete information. Therefore, as the constraints and priorities are not clearly defined, the field of optimization faces the challenge of quantifying unknown data in a meaningful way. The definition of Fuzzy Sets could be used as an efficient decision-making technique to tackle this ambiguity. In 1965, Zadeh [10] introduced a fuzzy set as a sub-discipline of mathematics to capture the instability in real systems.
In the field of optimization, the implementation of fuzzy set theory develops rapidly after the primary effort made by Bellman and Zadeh [2]. Fuzzy transportation problem (FTP) has received a great deal of researchers’ attention as it is relevant to real-life problems with ambiguous phenomena. Chanas and Kuchta [3] suggested the theory of the optimal solution to the problem of transportation including fuzzy parameters and a methodology that determines this solution.

This imprecision may result from the lack of precise details or, possibly, from the flexibility of the company to plan its capabilities. Therefore, to solve transport problems effectively, conventional traditional methods should not be used. A Fuzzy Transportation Problem (FTP) is a problem where the demand, cost and supply for transport are fuzzy quantities. The use of FTP is therefore more appropriate for modelling and solving the problems of the real world. In a practical sense and a real-life problem, we meet uncertainty in some situations but not in each. The transportation problem where the parameters are either real numbers or fuzzy numbers is called a mixed type FTP.

To obtain the initial basic feasible solution for problem of fuzzy transportation containing both real numbers and fuzzy numbers, Nizam Uddin Ahmed et.al [7] used Robust's ranking feature and fuzzy zero-point method. Nirbhay Mathur [6] presented an innovative method for optimizing transportation problems through generalized trapezoidal numbers in a fuzzy environment. Senthil Kumar [8] used the PSK method to determine the solution to the Type 3 fuzzy transportation problem. Kumar[4] studied three different types in the Pythagorean fuzzy environment.

In this paper, we proposed a process for finding an optimal solution to a mixed type of fuzzy transport problem without being transferring it to a classical one. To develop the proposed method, we have used the proposed fuzzy ranking method and the fuzzy arithmetic operations introduced by Ming Ma et al. Also, we discuss the numerical example of a mixed type fuzzy transportation problem and its comparative study to illustrate the process developed in this chapter. The rest of the paper is organized as follows: In section 2 we define the fundamental concepts of fuzzy number, rank, and their arithmetic operations. Section 3 gives the formulation of a mixed type fuzzy transportation problem. In section 4, we present a method for the solution of a mixed type fuzzy transportation problem whose parameters are either fuzzy numbers or real numbers. In section 5, a numerical example is provided to show the efficiency of the presented method.

2. Preliminaries

Definition 2.1:

A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set $\tilde{A}$ of the real line $\mathbb{R}$ such that 
$\mu_\tilde{A}(x): \mathbb{R} \rightarrow [0,1]$, $\forall \ x \in \mathbb{R}$ where $\mu_\tilde{A}(x)$ is called the membership function of the fuzzy set and it is piece-wise continuous.

Definition 2.2:

A fuzzy number $\tilde{A}$ is a Trapezoidal Fuzzy Number denoted by $\tilde{A} = (a_1, a_2, a_3, a_4)$ and its membership function $\mu_\tilde{A}(x)$ is given below:
A trapezoidal fuzzy number \( \tilde{A} = (a_1, a_2, a_3, a_4) \) is reduced to a triangular fuzzy number when \( a_2 = a_3. \) A fuzzy number \( \tilde{A} \) is said to be a trivial fuzzy number (crisp number) if its membership function is given by

\[
\mu_{\tilde{a}}(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & \text{if } a_1 \leq x \leq a_2 \\
1, & \text{if } a_2 \leq x \leq a_3 \\
\frac{a_4-x}{a_4-a_3}, & \text{if } a_3 \leq x \leq a_4 \\
0, & \text{otherwise}
\end{cases}
\]

2.1. Parametric representation of Fuzzy Numbers

A fuzzy number \( \tilde{A} \in F(R) \) can be defined as an ordered pair of functions through the \( r \)-cut approach as to \( \tilde{A} = (\bar{a}(r), \bar{a}(r)) \) where \( r \in [0,1] \). We use \( F(R) \) to denote the set of all fuzzy numbers defined on \( R \).

The \( r \)-cut form is known as

I) \( \bar{a}(r) \) is a bounded monotonic increasing left continuous function over \([0, 1]\).

II) \( \bar{a}(r) \) is a bounded monotonic decreasing left continuous function over \([0, 1]\).

III) \( \bar{a}(r) \leq \bar{a}(r) \), where \( 0 \leq r \leq 1 \).

Moreover, any arbitrary fuzzy number \( \tilde{A} \in F(R) \) with parametric form \( \tilde{A} = (\bar{a}(r), \bar{a}(r)) \) can also be written as \( \tilde{A} = (\bar{a}_0, \bar{a}_r, \bar{a}_r^*) \) where \( \bar{a}_0 = (\bar{a} - \bar{a}), \bar{a}_r^* = (\bar{a} - \bar{a}_0) \) are said to be left fuzziness index function and the right fuzziness index function respectively. For an arbitrary fuzzy number, \( \tilde{A} = (\bar{a}, \bar{a}^*) \) the number \( \bar{a}_0 = \left( \frac{\bar{a}(1) + \bar{a}_r(1)}{2} \right) \) is known as location index number of \( \tilde{A} \).

2.2. Ranking of Triangular Fuzzy Numbers

Many different methods were suggested for the ranking of fuzzy numbers within the literature. Abbasbany and Hajjari [1] have suggested a different ranking method dependent on left and right spreads at a certain \( r \) level of fuzzy numbers. For any arbitrary fuzzy number \( \tilde{A} \in F(R) \) with parametric form \( \tilde{A} = (\bar{a}(r), \bar{a}(r)) \), we define the magnitude of the fuzzy number \( \tilde{A} \) by

\[
\text{Mag}(\tilde{A}) = \frac{1}{2} \int_0^1 ((\bar{a}(r) + \bar{a}(r) + \bar{a}_0(r))f(r)dr)
\]

where the function \( f(r) \) is nonnegative and increasing on \([0, 1]\) with \( f(0) = 0, f(1) = 1 \) and \( \int_0^1 f(r)dr = \frac{1}{2} \). According to the situation, the function \( f(r) \) can be assigned by the decision-maker.
2.3. Arithmetic Operations on Fuzzy Numbers

Ming Ma et al. [5] have developed new fuzzy arithmetic based on location index and fuzziness index functions. For any arbitrary fuzzy number, \( \tilde{A} = (a_0, a_+, a^-) \) and \( \tilde{B} = (b_0, b_+, b^-) \) the arithmetic operations on them are defined as \( \tilde{A} + \tilde{B} = (a_0 + b_0, a_+ + b_+, a^- + b^-) \). The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is the least upper bound in the lattice \( L \). We define \( a_+ b = \max\{a, b\} \) and \( a_\wedge b = \min\{a, b\} \), for all \( a, b \in L \).

In particular, for any two fuzzy numbers \( \tilde{A} = (a_0, a_+, a^-) \) and \( \tilde{B} = (b_0, b_+, b^-) \), we define

(I) Addition: \( \tilde{A} + \tilde{B} = (a_0 + b_0, \max\{a_+, b_+\}, \max\{a^-, b^-\}) \)

(II) Subtraction: \( \tilde{A} - \tilde{B} = (a_0 - b_0, \max\{a_+, b_+\}, \max\{a^-, b^-\}) \)

(III) Multiplication: \( \tilde{A} \times \tilde{B} = (a_0 \times b_0, \max\{a_+, b_+\}, \max\{a^-, b^-\}) \)

(IV) Division: \( \tilde{A} ÷ \tilde{B} = (a_0 ÷ b_0, \max\{a_+, b_+\}, \max\{a^-, b^-\}) \) provided \( b_0 \neq 0 \).

3. Mixed Type Fuzzy Transportation Problem

The fuzzy transportation problem can be expressed mathematically as follows:

Minimize \[ z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij} \]

Subject to \[ \sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i \quad (i = 1, 2, ..., m) \]

\[ \sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j \quad (j = 1, 2, ..., n) \]

and \( \tilde{x}_{ij} \geq 0 \),

where \( \tilde{a}_i \) represent the amount of supply at source \( i \) and \( \tilde{b}_j \) represent the amount of demand and \( j \). \( \tilde{c}_{ij} \) denotes the unit transportation cost from source \( i \) to destination \( j \) and \( \tilde{x}_{ij} \) represents the number of units to be shifted from source \( i \) to destination \( j \). Here \( \tilde{x}_{ij} \) are either fuzzy or real numbers.

4. Algorithm for Approximation Method

(i) Transform the mixed type fuzzy transportation problem into a fully fuzzy transportation problem by applying the concept of trivial Trapezoidal fuzzy number.

(ii) Represent each fuzzy number in parametric form

(iii) Look for the lowest cost for each row using the proposed ranking function and deduct it from all costs for the respective row. Do the same column-wise, and in each row and column, there will be at least one zero.

(iv) Find the differences in each row and column between the smallest cost and the next smallest cost, and pick the row or column with the largest difference. Break the tie arbitrarily if there is a tie.

(v) Choose a cell with zero cost in the chosen column (or) row and allocate the maximum feasible amount to that cell. Adjust the demand and supply. Repeat the steps (iii) to (v) unless all the criteria of demands and supplies are satisfied.

(vi) Finally, we get the required optimal solution to the problem.
5. Numerical Example
Consider a mixed type fuzzy transportation problem discussed by Senthil Kumar[8]. Here the cost coefficient, supply, and demand are either fuzzy numbers are real numbers.

| Table 1. Mixed Type Fuzzy Transportation Problem |
|-----------------|-----------------|-----------------|
| 6               | (1,2,3)         | (5,7,9,11)      |
| (1,2,3)         | (4,8,16,20)     | 10              |
| (1,3,5,7)       | 3               | (6,8,10)        |
| (11,22,30,41)   | (5,8,11)        | 12              |

| Table 2. Initial Basic Feasible Solution of Example 2. |
|-----------------|-----------------|-----------------|
| (6,0-0r,0-0r)   | (2,1-r,1-r)     | (8,3-2r,3-2r)   |
| (2,1-r,1-r)     | (12,8-4r,8-4r)  | (10,0-0r,0-0r)  |
| (20,10-6r,10-6r)| (8,3-3r,3-3r)   | (8,2-2r,2-2r)   |
| (4,3-2r,3-2r)   | (8,3-3r,3-3r)   | (10,15-11r,15-11r) |
| (6,15-11r,15-11r)| (8,3-3r,3-3r)  | (10,15-11r,15-11r) |

Hence the fuzzy initial basic feasible solution in the form of fuzziness index and location index is given by

\[ \tilde{x}_{12} = (8,3-3r,3-3r), \tilde{x}_{13} = (2,3-3r,3-3r), \tilde{x}_{21} = (20,10-6r,10-6r), \]
\[ \tilde{x}_{31} = (6,15-11r,15-11r), \tilde{x}_{33} = (10,15-11r,15-11r) \]

It can be checked that the current initial fuzzy basic feasible solution is optimum.

The fuzzy optimal transportation cost is \[ \min \tilde{z} = (161+11r,176,191-11r) \] where \( 0 \leq r \leq 1 \). Therefore the fuzzy optimal transportation cost in terms of \( (a_1,a_2,a_3,a_4) \) is given by \[ \min \tilde{z} = (161,172,172,191) \].

| Table 3. Comparative study of example |
|-----------------|-----------------|-----------------|
| Fuzzy optimal solution of the proposed method | Fuzzy optimal solution of Senthil Kumar |
| \[ \min \tilde{z} \] | (161,172,180,191) | (104,168,184,248) |
| For \( r = 1 \) | 176 | 176 |

6. Conclusion
We have suggested the approximation method in this paper to determine the optimal solution to the problem of mixed form fuzzy transport without converting to a crisp equivalent problem. To demonstrate the efficiency of the proposed process, a numerical illustration is given. From the example, we see that the optimal solution in the general form is given by \( (161,172,180191) \) whose crisp value is 176. In addition, compared to the existing methods, the fuzzy optimal solution determined by the proposed methods has minimized spreads which helps the decision-maker more.
7. References

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