A finite differences approach for fluid-structure interaction in a hydrostatic compliant thrust bearing

A A Marinescu\textsuperscript{1} and T Cicone\textsuperscript{1}

\textsuperscript{1}Machine Elements and Tribology Department, University POLITEHNICA of Bucharest, Bucharest, Romania

Email: traian.cicone@upb.ro

Abstract. It is well known since the pioneering work of Dowson and Castelli that load capacity increases in compliant (elastic pad) hydrostatic bearings, compared to their rigid counterparts. In a recently published paper, a new design solution of a flexible pad hydrostatic bearing was proposed and analysed theoretically. A simplified analytical model was proposed for the fluid-structure interaction. The model was based on two important simplifications: linear pressure distribution on the bearing land, used to calculate the deformation of the pad, and parabolic variation of film thickness in the Reynolds equation. The analysis was made for isothermal conditions and constant pressure in the recess. This new approach relaxes the two simplifications by considering a linearly elastic model for the flexible pad and including its deformation in the Reynolds equation. A finite differences scheme is proposed for both the fluid flow and elastic deformation, considering the flexible plate simply supported. The consistency and convergence of the iterative solution are analysed, and comparisons with previous analytical solutions are shown. These results show the limits of the accuracy of the simplified analytical model and allow the selection of the design parameters of a test rig.

1. Introduction

It has been observed for decades that the presence of compliant elements in bearings leads in some circumstances to a higher load capacity\cite{1,4}. One method of achieving compliance is through the means of the material used (i.e. a highly deformable material, characterized by a low modulus of elasticity). The interest for such a constructive solution has been cultivated for almost half a century, since the first applications involving rubber\cite{1,4} as their compliant medium. However, an important pitfall was not foreseen for this solution, that is, the decreasing elasticity and crumbling – a process informally known as the “aging” of rubber.

Another possible way of achieving compliance is by structural means – namely, the constructive solution of the otherwise rigid bearing – by involving a flexible element, whose deformation is of an order of magnitude comparable to that of the film thickness. Several solutions involving structural compliance, which used gases\cite{5} or liquids\cite{6} as their lubrication medium, have emerged prior to the present one, but these solutions have only focused on particular design solutions. Both papers\cite{5,6} included numerical solutions of the fluid – structure interaction, which revealed an improved stiffness, but the results lacked generality.

In a recent paper\cite{2}, the authors have analysed a new possible design solution employing the latter compliance method, where a thin, elastic plate replaces the otherwise rigid upper surface of the bearing. Since the deformation was not produced by external forces, being instead dictated by an
internal mechanism – the pressure, an increase in the difficulty of the problem-solving process occurs, which leads to the necessity of considering a coupled FSI approach.

The paper proposed a simplified analytical solution for the coupled fluid film – elastic deformation mechanism, considering instead an uncoupled, “quasi-FSI” solution, based on an approximation of the elastic deformation of the flexible pad. The elastic deformation was assumed parabolical, with its maximum at the central axis of the bearing, which allowed an analytical solution for the Reynolds equation. A second simplifying assumption, that is, linear pressure distribution on the land, led to a closed form solution for the maximum elastic deformation of the plate. The solution allowed a parametric analysis, which revealed an improved load capacity and an increased rate of flow in contrast with an identical, rigid bearing.

An improved elastic model where the maximum deformation of the plate was computed using FE by assuming a quasi-parabolical variation of pressure distribution on the land was also considered. The comparison between the two models has shown a slightly overestimated load capacity in the case of the analytical solution, especially for highly elastic plates. However, neither of the two models were realistic, due to the fact that they were uncoupled.

The present paper aims to present a different approach, which relaxes both simplifications – linear pressure distribution on the bearing land and parabolic variation of film thickness, respectively – by solving iteratively the coupled system of equations. A finite differences scheme is proposed for both the fluid flow and elastic deformation. The consistency and convergence of the iterative solution are analysed, and comparisons with the quasi-FSI solution are shown, thus revealing the accuracy limits of the simplified analytical model.

**Nomenclature**

\[ a, b, c, d, e, f, g \] - constants [-]

\[ D = \frac{Et^3}{12(1 - \nu^2)} \] - plate elastic constant [Pa·m³]

\[ E \] - Young modulus of elasticity [Pa]

\[ F \] - load capacity [N]

\[ \bar{F} = \frac{F}{F_r} \] - dimensionless load capacity [-]

\[ \bar{F} = \frac{F}{F_r} \] - relative load carrying capacity [-]

\[ h \] - film thickness [m]

\[ \bar{h} = \frac{h}{h_0} \] - dimensionless film thickness [-]

\[ K = \frac{p_r r^2}{E t^4} \] - complex elastic parameter [-]

\[ N \] - number of mesh intervals

\[ p \] - pressure [Pa]

\[ \bar{p} = \frac{p}{p_i} \] - dimensionless pressure [-]

\[ r \] - radius [m]

\[ \bar{r} = \frac{r}{r_i} \] - dimensionless radius [-]

\[ R = \frac{r_o}{r_i} \] - relative recess radius [-]

\[ \bar{R} = \frac{r_o}{r_i} \] - dimensionless radius [-]

\[ t \] - plate thickness [m]

\[ \bar{t} = \frac{t}{h_0} \] - dimensionless plate thickness [-]

\[ y \] - plate deformation [m]

\[ \bar{y} = \frac{y}{t} \] - dimensionless plate deformation [-]

**Greek**

\[ \epsilon \] - root mean square error [%]

\[ \eta \] - viscosity [Pa·s]

\[ \nu \] - Poisson ratio [-]

\[ \sigma \] - stress [Pa]

**Subscripts/ Superscripts**

\[ * \] - analytic

\[ i \] - inner

\[ o \] - outer

\[ r \] - rigid

\[ s \] - supply

**Abbreviations**

BC - boundary condition(s)

FDM - finite differences method

FEM - finite element method

FSI - fluid-structure interaction

HS - hydrostatic

SOR - successive over-relaxation
2. The model
The analysis is conducted upon a HS thrust bearing with a centrally-placed recess (figure 1), having the following geometrical configuration: the lower, rigid circular pad of the bearing is provided with a recess (radius \( r_s \)), connected to the supply hole, through which the lubricant is pumped. The upper bearing pad, provided with a cavity of small depth with respect to its own radius, is covered by a thin, metallic membrane, simply supported on its contour.

For the current approach, the operating conditions are assumed isothermal and the influence of the restrictor in the supply is neglected. According to this hypothesis, the supplied pressure \( (p_s) \) is the same as the pressure in the recess, which in turn, is proportional to the force acting upon the bearing \( (F) \).

Since the computation of the bearing performance characteristics – with its main focus on the analysis of the lift force – requires an FSI approach, it is necessary to solve concomitantly the fluid flow and elastic deformation equations.

![Schematic representation of the model](image)

**Figure 1.** Schematic representation of the model

2.1. The elastic model
The circular, flexible plate is considered perfectly flat prior to its deformation, and having a uniform thickness, which does not change when the plate is being deformed. The thickness of the upper bearing pad ridge modifies the effective diameter of the elastic pad; however, its influence is neglected in this analysis. Other assumptions taken into account are the homogeneity and isotropy of the material of the plate, and the concentricity and parallelism of the two bearing pads. Thus, axi-symmetry prevails.

Since the maximum deformation of the plate is considered small relatively to its thickness \( (\bar{y} < 0.5) \), one can use a linearly elastic model, which is equivalent to resorting to the Kirchhoff – Love theory of small deformations[8], which neglection the effect of the shear forces. In this case, the plate is only subjected to bending. According to the classical theory of elasticity for plates[3][7], the model of thin plate (also known as “membrane model”) which neglection the effect of transverse shear is best applied when the relative thickness \( \frac{t}{r_0} > 0.04 \). This limit is slightly exceeded for the practical design proposed herein and Reisnner – Mindlin[7] plate theory would give more realistic theoretical predictions. However, it was demonstrated[3] that the thin plate assumption can be extended up to \( \frac{t}{r_0} < 0.2 \), neglecting the increase in deflection due to transverse shear, which is less than 5.7%.

The present approach neglects any variation of stresses and deformations across the thickness of the plate and is considered acceptable as long as the thickness of the plate is much smaller than its radius, condition which is satisfied by the design solution involved in the current analysis.
The plate is loaded with uniformly distributed pressure in circumferential direction, which leads to an axi-symmetric problem. The fibre deformation is described by the following differential equation of the fourth degree, also known as the “elastica equation”:

$$\frac{d^4 y}{d r^4} + 2 \frac{d^3 y}{r d r^3} - \frac{1}{r^2} \frac{d^2 y}{d r^2} + \frac{1}{r^3} \frac{d y}{d r} = \frac{p(r)}{D}$$  

where $D = \frac{E t^3}{12(1-\nu^2)}$ is the elastic constant of the plate, and $p(r)$ is the pressure distributed axi-symmetrically.

In dimensionless form, equation (1) becomes:

$$\frac{d^4 \hat{y}}{d \hat{r}^4} + 2 \frac{d^3 \hat{y}}{\hat{r} d \hat{r}^3} - \frac{1}{\hat{r}^2} \frac{d^2 \hat{y}}{d \hat{r}^2} + \frac{1}{\hat{r}^3} \frac{d \hat{y}}{d \hat{r}} = 12K(1 - \nu^2)\hat{p}(\hat{r})$$  

where $K = \frac{p r^2}{E t^4}$ is the elastic complex parameter.

**Figure 2.** Schematic representation of the simply supported plate under distributed load

The subsequent analysis focuses on the case of a simply supported plate (figure 2), considering the following boundary conditions in dimensionless form:

(BC$_1$) Null deformation on the contour of the plate:

$$\hat{y} = 0 \text{ at } \hat{r} = 1$$  

(BC$_2$) Null radial bending moment on the contour of the plate:

$$\left( \frac{d^2 \hat{y}}{d \hat{r}^2} + \frac{\nu}{\hat{r}} \frac{d \hat{y}}{d \hat{r}} \right) = 0 \text{ at } \hat{r} = 1$$  

(BC$_3$) Shear force on the contour of the plate equal to the reaction of the axial load on the plate; in the case of the plate loaded with radially variable pressure, one obtains:

$$\frac{d}{d \hat{r}} \left( \frac{d^2 \hat{y}}{d \hat{r}^2} + \frac{1}{\hat{r}} \frac{d \hat{y}}{d \hat{r}} \right) = 12K(1 - \nu^2) \int_0^1 \hat{p}(\hat{r}) d\hat{r}$$  

(BC$_4$) Maximum deformation (null rotation angle) at the centre of the plate:

$$\frac{d \hat{y}}{d \hat{r}} = 0 \text{ at } \hat{r} = 0$$
2.2. The fluid flow model

Classical assumptions of the hydrostatic lubrication theory are used for the fluid flow analysis: the flow, which involves a Newtonian and incompressible lubricant is laminar and isoviscous. The deformation of the plate has a very small variation with respect to the depth of the recess, and consequently, the pressure inside the recess remains constant even after deformation. The pressure variation across the film thickness, as well as the inertial, gravitational and external forces, are neglected.

Therefore, the axi-symmetric form of the isoviscous Reynolds equation is:

\[ \frac{d}{dr} \left( r \frac{dh}{dr} \frac{dp}{dr} \right) = 0 \]

where the film thickness \( h(r) \) includes the elastic deformation of the plate under the pressure load, denoted by \( y(r) \):

\[ h(r) = h_0 + y(r) \]

The numerical solution of the Reynolds equation is obtained for its dimensionless form:

\[ \frac{d}{d\tilde{r}} \left( \tilde{r} \tilde{h}^3 \frac{d\tilde{p}}{d\tilde{r}} \right) = 0 \]

where

\[ \tilde{h}(\tilde{r}) = 1 + \tilde{y}(\tilde{r})\tilde{\ell} \]

For the current bearing geometry, the dimensionless pressure boundary conditions are the following:

\( \text{(BC}_1) \) Constant pressure in the bearing recess:

\[ \tilde{p} = 1 \text{ for } 0 < \tilde{r} < R \]

\( \text{(BC}_2) \) Atmospheric pressure at the outer bearing radius:

\[ \tilde{p} = 0 \text{ at } \tilde{r} = 1 \]

Because the elastic deformation of the plate is available numerically, the Reynolds equation can be solved only numerically.

3. The numerical solution

A nodal unidimensional mesh consisting of \( N+1 \) equidistant points distributed along the bearing radius is defined. The first node is situated at the edge of the plate \( (\tilde{r}_1 = 1) \), while the last one is positioned at the axis of symmetry \( (\tilde{r}_{N+1} = 0) \). The step of the nodal mesh is constant and equal to \( \Delta \tilde{r} = \frac{1}{N} \). The mesh step is selected in a manner that allows a node to be placed exactly on edge of the bearing land. The same mesh is used for both the fluid flow and the elastic model.

In order to solve the equation of deformation, its dimensionless form (2) and the associated boundary conditions (3)…..(6) are formulated using finite differences[7]. Despite possessing the disadvantage of giving inaccurate higher order derivatives for the approximate solution, the FDM allows the two models (elastic and fluid flow) to be easily connected. Even though FDM is inappropriate for complex geometries and curved boundaries, these restrictions do not apply for the current chosen geometry. Craig[3] showed that the differences between the FDM and FEM are under 1% for a rectangular plate loaded with constant pressure distributed on a mesh of 12 x 12 points for a relative thickness ratio of the plate \( \frac{t}{h_0} < 0.2 \).

A second order precision scheme for the fourth-degree differential equation would imply an exaggerated complexity for the algebraic equations used to obtain a superior precision, which is inconvenient from a practical point of view. Therefore, for the finite difference form of the elasticity
equation, first order central finite differences are used. These ensure an accuracy proportional to the squared value of the mesh step. For BCs 2 and 3 forward differences are used, whereas BC 4 is written using backward differences.

By rewriting equation (2) in a finite differences form, and after the rearrangement of terms, one arrives to an algebraic equation from which \( \tilde{y}_{j+1} \) is written as a function of the values of four neighbouring points.

\[
\tilde{y}_{j+1} = \frac{a \cdot \tilde{y}_{j+2} + c \cdot \tilde{y}_j - d \cdot \tilde{y}_{j-1} + e \cdot \tilde{y}_{j-2} - 6K(1 - \nu^2)\Delta \tilde{r}^4 \tilde{p}_j}{b}
\]  

(13)

where

\[
a = \tilde{r}^3 + \tilde{r}^2 \Delta \tilde{r}
\]

\[
b = 4\tilde{r}^3 + 2\tilde{r}^2 \Delta \tilde{r} + \tilde{r} \Delta \tilde{r}^2 - \frac{\Delta \tilde{r}^3}{2}
\]

\[
c = 6\tilde{r}^3 + 2\tilde{r} \Delta \tilde{r}^2
\]

\[
d = 4\tilde{r}^3 - 2\tilde{r}^2 \Delta \tilde{r} + \tilde{r} \Delta \tilde{r}^2 + \frac{\Delta \tilde{r}^3}{2}
\]

\[
e = \tilde{r}^3 - \tilde{r}^2 \Delta \tilde{r}
\]

Similarly, the boundary conditions (3) to (6) become:

\[
\tilde{y}_1 = 0
\]  

(14)

\[
\tilde{y}_2 = \frac{\tilde{y}_3}{2 - \nu \Delta \tilde{r}}
\]  

(15)

\[
\tilde{y}_3 = \frac{1}{6} \tilde{y}_4 + \frac{3}{2} \frac{2\Delta \tilde{r} - \Delta \tilde{r}^2}{\Delta \tilde{r} - 3} - 12K\Delta \tilde{r}^3(1 - \nu^2) \int_0^1 \tilde{p}(\tilde{r}) d\tilde{r}
\]  

(16)

\[
\tilde{y}_{N+1} = \tilde{y}_N
\]  

(17)

Equation (13), including boundary conditions (14) to (17), yields to a linear system of \( N+I \) equations with \( N+I \) unknowns (i.e. deformations for each node) which can be solved by various classical methods. Assuming the existence of boundary conditions at both edges of the integration domain, the substitution method was considered a suitable choice. Even though it might imply a slower solving time, the substitution method is more robust and leads to results regardless of the input values.

A Fortran code run in double precision was implemented for this method. The deformations obtained in dimensionless form depend only on two parameters: the complex elastic parameter \( K \), which varies over a very narrow range and the Poisson coefficient \( \nu \), which is considered constant in this study: \( \nu=0.31 \).

An FD scheme for 4th order differential equations requires an accuracy analysis. The FD formulation given by equations (13) to (17) was first applied to a simple bending plate problem: constant pressure on the entire plate has been selected because it is closer to the bending case of the envisaged EHS bearing and an analytical solution[8] is available. Increasingly finer discretisations, obtained by dividing the initial size of the integration interval by 2 were considered. The present analysis uses discretisations with 250, 500, 1000, 2000 and 4000 intervals. The overall analysis of the accuracy of each discretization was performed by comparing the root mean square error:
\[ \varepsilon_N = \left( \frac{1}{N} \sum_{j=1}^{N+1} (y_j - y_j^*)^2 \right)^{\frac{1}{2}} \]

where \( y_j \) represents the deformation in node \( j \) obtained numerically and \( y_j^* \) is the deformation predicted analytically[8]:

\[ y_j^* = \frac{3}{16} K (1 - \nu^2) (1 - \bar{r}^2) \left( \frac{5 + \nu}{1 - \nu} - \bar{r}^2 \right) \]

The accuracy of the numerical method was analysed for two cases of practical interest \((K = 0.1 \) and \( K = 0.7 \)). To have a clear image on the precision order, it is useful to analyse the error variation with the integration step in a logarithmic scale representation, as shown in figure 3:

![Figure 3. Root mean square error](image)

Figure 3 suggests a high accuracy for the chosen method, which can be explained by the use of central finite differences. From the equation used to approximate the evolution of the error with the number of intervals, it follows that the rate of convergence is of the second order. The straight lines show that the numerical method is consistent and convergent. The higher the value of the parameter \( K \), the higher were the errors. Based on these results, we chose to perform the following analyses for \( N=1000 \) nodes, which seemed to be a good compromise between accuracy and computing time.

For the discretized Reynolds equation, the pressures are located in each node, while the film thickness is computed in mid-intervals, between nodes. Thus, the Reynolds equation (9) is rewritten in a finite differences form using a first order precision scheme. Therefore:

\[ \bar{p}_j = \frac{f \cdot \bar{p}_{j+1} + g \cdot \bar{p}_{j-1}}{f + g} \]

where

\[ f = \frac{(\bar{h}_j + \bar{h}_{j+1})^3}{8} (\bar{r}_j - \Delta \bar{r}) \]

\[ g = \frac{(\bar{h}_j + \bar{h}_{j-1})^3}{8} (\bar{r}_j + \Delta \bar{r}) \]
One should note that the Reynolds equation is integrated only on the land of the bearing. Consequently, equation (20) written for each node on the land \((j=2...N_L-1)\) leads to a linear system of \(N_L\) equations, solved using the Gauss – Seidel SOR algorithm with an overrelaxation factor of 1.95. This numerical method has been implemented in a Fortran code run in double precision, which allows the prediction of the load capacity for given film thickness variation and recess size. The iterative procedure stops once the difference between two successive values of the load capacity is less than a chosen tolerance (i.e. \(10^{-6}\)). The method has proven a very quick convergence.

Finally, an iterative procedure which couples the two codes has been implemented (figure 4). The calculation starts with an initial guess value implemented for the parabolic pressure distribution on the land, which allows the calculation of the elastic deformation followed by the solution of the Reynolds equation, which yields a new pressure distribution. The iterations stop when relative differences between two consecutive values of pressure, calculated in each nodal point, do not exceed the imposed tolerance of \(10^{-6}\). The procedure is rapidly convergent with no more than some tens of iterations and yields to a dimensionless load capacity for given operating and constructive data, which are included in the dimensionless parameters \(R, K, \bar{t}, \nu\).

![Figure 4. FSI solving algorithm](image)

4. Results and discussion
The numerical applications require first an analysis of the reasonable intervals for the three dimensionless parameters \(R, K, \bar{t}\). The relative recess radius \(R\) has typical values around 0.5; as a consequence, two more neighbouring values have been selected for comparisons: 0.3 and 0.7, respectively. One can remark that \(K\) and \(\bar{t}\) are not independent parameters, as both depend on the plate thickness.

A numerical evaluation for the ranges of values for the dimensionless parameters \(K\) and \(\bar{t}\) has been made, based on the following ranges of operating and constructive parameters: \(E = 2\cdot 10^{11} [\text{Pa}], r_o = 20..200[\text{mm}], p_s = 1..10[\text{bar}], t = 0.5..5[\text{mm}]\).

From the extended limitation for the moderately thin plate model (see chapter 2.1), it results that \(\frac{\bar{t}}{r_o} < 0.2\). On the other hand, a highly elastic plate is limited by its maximum stress in the centre, which must be lower than the yield stress. Therefore, in the case of a constant pressure \(p_s\) acting upon the plate, considering the maximum nominal stress:
\[ \sigma_{\text{max}} \big| r=0 = \frac{3}{8} (3 + \nu) p_s \left( \frac{r_o}{t} \right)^2 < R_{p0.2} \]  

one obtains a lower bound for the relative thickness \( \frac{t}{r_o} \geq 1.24 \frac{p_s}{R_{p0.2}} \). Considering the range of values for the yield stress \( R_{p0.2} = 250..800 [\text{MPa}] \) and the previously mentioned supply pressure interval, it results that \( 0.014 < \frac{t}{r_o} < 0.2 \) for thin plates assumptions.

Further consideration is given to the convergent gap on the bearing land. From the theory of HD lubrication for an inclined plane configuration, reasonable bounds of the film thickness ratio are \( 1.1 < \frac{h_{\text{max}}}{h_o} = \frac{h_o + y_{\text{max}}}{h_o} < 8 \). For higher values, the rate of flow becomes prohibitive. Thus, the inequality is further applied for our case:

\[ 0.1 < \frac{y_{\text{max}}}{h_o} = \tilde{y}_{\text{max}} \cdot \tilde{t} < 7 \]  

The same simple uniform pressure bending plate model allows a rough estimation of the maximum deformation:

\[ \tilde{y}_{\text{max}} = \frac{3}{16} K (1 - \nu^2) \frac{5 + \nu}{1 + \nu} = 0.69 K \]  

Hence, for linearly elastic model, \( \tilde{y}_{\text{max}} = 0.69 K < 0.5 \) (see chapter 2.1), which gives further \( K < 0.73 \). However, this is a conservative limitation, based on an overestimated pressure loading. A more precise limit, which counts for the recess size, can be calculated if a simple model of combined pressure distribution (linearly variable on the land and constant on the recess) is used. The formula, which can be found in \([2]\), gives the maximum limit of \( K \) within a very narrow interval for typical values of the recess relative radius \( R \), for a relative deformation less than 0.5, as shown in figure 5. Therefore, since \( K \) and \( \tilde{t} \) are coupled, the limit of their product can be obtained combining equations (22) and (23), thus arriving to \( K \tilde{t} < 10 \).

![Figure 5. Maximum values of the plate elastic constant for various recess sizes](image)

The numerical analysis, done in terms of force related to the same force obtained for rigid case, is focused on the comparison between the results of the actual iterative solution and those predicted with the simplified analytical model based on an uncoupled system of equations \([2]\).
First, a qualitative comparison of pressure distributions predicted analytically and numerically is depicted in figure 6, for two different elastic plates defined by the $K\bar{t}$ factor and two extreme values for the recess radii ($R=0.3$ and $R=0.7$). One can note that the pressure distributions for the two models – analytical and numerical – are relatively close, with differences attenuated as $R$ increases or $K\bar{t}$ decreases. It can be also remarked that the difference between linear pressure variation on the land and the real (quasi-parabolic) one is more pronounced for low recess radii. This affects the deformation calculation.

**Figure 6.** Pressure distribution on the land – analytical vs. numerical

**Figure 7.** Relative load carrying capacity
Subsequently, a parametric analysis was performed in terms of the ratio between the dimensionless lift force and its rigid counterpart with the parameters $K$ and $\bar{t}$. The parameter $K$ varied on a limited interval, from 0.1 to 1, for several values of the relative recess radius $R$ and thickness ratio $\bar{t}$, while $\nu$ was kept constant at 0.31. The boundaries of these parameters for practical cases are shown with a dotted line in figure 7, while taking into account the limits imposed by figure 5, and the condition $K\bar{t}<10$.

One can observe that the load capacity increases with the increase of the complex elastic parameter $K$, reaching for small recess radii almost twice the value corresponding to the same rigid bearing. The effect is more pronounced for higher values of $\bar{t}$.

The following analysis focuses on the relative differences in terms of load capacity between the analytical[2] and numerical solutions, calculated as $\frac{F-F^*}{F}$. Figure 8 shows the differences between the two models – numerical and analytical – calculated for a range of values varying from 0 to 10 for $K\bar{t}$, and three representative values of $R$ – 0.3, 0.5 and 0.7, respectively. It is not surprising that the differences for each $R$ value are practically the same, whatever value of the product $K\bar{t}$, a behaviour which was presented in detail at the beginning of this chapter.

From figure 8, one can notice that, for values of the relative recess radius higher than 0.5, the analytical model overestimates the load capacity with errors that slightly exceed 1% for the extreme case of $R=0.7$. For $R=0.53$ – an optimal value for rigid bearings – the analytical solution is very accurate.

The accuracy diminishes gradually for bearings having smaller relative recess radii, and the differences between the numerical and analytical models increase up to 6% when $R=0.3$ and $K\bar{t}$ is close to its maximum limit. In the case of small recesses, the analytical model underestimates the load capacity – an effect which was expected due to the fact that one simplification (i.e. linear pressure distribution for plate deformation), affects a greater land zone. This is put in evidence better by pressure distributions shown in figure 6.

It is also of interest to remark that the relative differences are very small if $K\bar{t}<1$, regardless of the recess size.

![Figure 8. Load capacity – relative differences between analytical and numerical models](image-url)
5. Conclusions
The accuracy of a simplified analytical model (previously published) for an original EHS bearing was evaluated with numerical solutions of coupled equations of elasticity and fluid flow. The results strengthen the main advantage of the EHS bearing: improved load capacity.

The numerical approach includes an original numerical solution for the axi-symmetric elastic plate based on finite differences, whose accuracy was analysed in detail.

The comparison, which covers the usual practical range for the variable parameters, show reasonably good approximation of the simplified model. The approximation is better for low and medium elasticity of the compliant member.

The comparison shows that for typical values of the recess radius \((R=0.5...0.7)\), the analytical model has an excellent accuracy. It can be also concluded that the analytical model can be used with confidence for the design of experimental models and prediction of their behaviour.

The numerical model and its corresponding written code are a basis of future development of refined theoretical analyses which will include the effects of restrictors or non-linear elastic effects.

6. References
[1] Castelli V, Rightmire G K and Fuller D D 1967 On the analytical and experimental investigation of a hydrostatic, axisymmetric compliant-surface thrust bearing ASME Journal of lubrication technology 89(4) pp 510-519
[2] Cicone T, Marinescu A A and Sorohan Şt 2020 A simple analytical model for an elastohydrostatic thrust bearing IOP Conference Series: Materials Science and Engineering 724
[3] Craig R J Finite difference solutions of Reissner’s plate equations 1987 ASCE Journal of Engineering Mechanics 113(1) pp 31-48
[4] Dowson D and Taylor C M Elastohydrostatic lubrication of circular plate thrust bearings 1967 ASME Journal of lubrication technology 89(3) pp 237-242
[5] Hao D, Xiao-Long Z and Juan-An Z 2015 Static characteristic analysis and experimental research of aerostatic thrust bearing with annular elastic uniform pressure plate Advances in mechanical engineering 7(3) pp 1-13
[6] Hayashi K and Hirasata K 1981Theoretical Investigation on the back-pressured elastohydrostatic thrust bearing Journal of JSLE International Edition 26(4) pp 277-283
[7] Szilard R 2004 Theories and applications of plate analysis: Classical, numerical and engineering methods (New Jersey: John Wiley & Sons, Inc.)
[8] Young W and Budynas R 2011 Roark’s formulas for stress and strain (McGraw-Hill Companies)