Penetration and transformations of vortices in bulk current-carrying superconductors

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Abstract

The equations of viscous evolution of 3D arbitrarily shaped vortices in an isotropic type II superconductor and necessary boundary conditions are formulated in the frame of London approximation. The theory is applied to analyse characteristic scenaria of vortex penetration into current-carrying thick plate or bulk samples with another geometry. It is shown that regarding of the surface transport current value a vortex penetrates either as "flexible stick" or as "elastic thread". The latter regime is accompanied by giant stretching of the vortex core along the current-induced surface magnetic field. This geometrical transformation leads to decrease of viscous friction and large increase of the vortex drift velocity as compared with the stick-like regime. As a result, the vortex first winds round the sample cross-section and forms a ring-like curve, and only later begins to move deep into the sample interior. The analytical estimates of the vortex shape and stretching and its velocity are obtained.

I. INTRODUCTION

Not long ago a variety of complex magnetic structures formed by many strongly curved and entangled vortices was discovered in bulk superconductors [1]. The origin of these structures can not be explained if treat the motion of vortices like that of stick-like objects. It is necessary to consider the evolution of three-dimensional magnetic flux lines with potentially arbitrary shape.

For the first, it is useful to investigate the motion and shaping of a single vortex but interacting with a surface supercurrent which represents either transport current or Meissner current induced by external field. As far as we know, even this simple task previously was not under careful consideration.

Of course, a vortex never lives alone, without interactions with other vortices, and no stable many-vortex structure could exist without mutual repulsion of vortices. However, one can suppose that the scenario of magnetic flux penetration into a current-carrying bulk superconductor should be dominated only by vortex interaction with the surface current distribution, i.e. eventually by geometry of superconducting sample, not by the...
vortex-vortex interaction. The latter can not seriously affect this scenario, merely because it itself is unable to ensure a deep penetration at all.

To prove this statement, let us imagine the steady flow of vortices which arise at the flat current-carrying boundary of half-infinite superconductor and then move deep into the sample due to their repulsion. Clearly, because of viscous character of vortex motion, such the flow needs in nonzero gradient of concentration of vortices. As a consequence, both the concentration and the local drift velocity of vortices must be decreasing functions of the depth. Hence, their product is not constant, that is the magnetic flux conservation can not be satisfied. This discrepancy means that no steady flow could be supported by the inter-vortex forces only. In particular, it is impossible to realize the stationary lasting penetration of vortices from infinite flat boundary parallel to external magnetic field.

Therefore, the only force what can push a vortex through the sample interior is nothing but self-action of vortex caused by its distorsion. But in order to involve this force into the evolution, the vortex must feel the shape of the sample boundary.

Hence, the true picture looks as follows. After nucleation in a surface layer with thickness of order of London penetration depth $\lambda$, a vortex firstly expands over the sample boundary remaining in this layer. At this stage only the end fragments of vortex are factually moving. The ends slide along the boundary, and the resulting shape of vortex core reflects that of boundary. This process lasts until the curvature of main middle part of the core becomes sufficiently strong in order to cause the deepening of vortex as a whole.

In view of these reasonings, the geometry of steady transport of magnetic flux into a bulk superconductor looks rather insensible to inter-vortex interactions and thus can be testified in terms of a single vortex, at least if not consider details of vortex nucleation and processes like annihilation and reconnections of vortices [1] which take place deep inside the sample.

For example, many aspects of resistivity in supercurrent-carrying wires can be described as evolution of ring-like vortices as if thats instantly arise near the boundary, then contract independently one on another and finally self-annihilate [2]. However, more correct consideration should include the first stage when vortex transforms from small nucleus into a ring. We shall see that in fact this stage may result also in a non-ring penetration geometry, and more detailed theory can predict what the scenario realizes under given transport current value and sample dimensions.

Though a lot of works were published previously touching upon a role of vortex distorsions, for instance, under a pinning by randomly distributed centers [3], always some preliminary restrictions of the vortex geometry were attracted. In the present work the general equations of evolution of arbitrarily curved vortex lines in isotropic superconductors are formulated and analysed. We shall especially discuss the true formulation of boundary conditions for these equations.

It will be shown that in a sample whose dimensions noticably exceed $\lambda$ the vortex can penetrate either as flexible stick or as elastic (similarly to a rubber thread attracted by its ends through water). of surface supercurrent. The latter case occurs only if surface supercurrent exceeds $H_{c1}/4\pi$ (in CGS units) and is characterized by giant stretching of the vortex core along the sample boundary in the direction parallel to drift of the ends. The stretching is accompanied by decrease of both the vortex energy and viscous
dissipation per unit drift velocity, and results in strong increase of the vortex drift velocity under given transport current. In the framework of this scenario, the vortex core firstly transforms into a ring-like curve winding round the wire cross-section (or into a spiral, if there is an external magnetic field parallel to current), and only later the vortex begins to cut the wire and enter deep into its interior. This general picture is in agreement with the known simplific model of magnetic flux penetration into round wires. Additionally, our approach allows to scope very different stages of vortex evolution in unified manner and obtain quantitative estimates for each stage.

II. LONDON APPROXIMATION

We shall confine ourselves by the London approximation. Of course, it would have no sense if one could not apply it to actually moving vortices. But in any case the requirement must be satisfied that characteristic velocity $u_0$ of viscous vortex motion influenced by magnetic fields comparable with low critical magnetic field $H_{c1}$, must be significantly smaller than the speed of electromagnetic waves.

The velocity scale $u_0$ can be naturally estimated as

$$u_0 \equiv \mu \varepsilon / \lambda$$

where

$$\varepsilon = \Phi_0 H_{c1} / 4\pi$$

is the self-energy per unit length of long straight-line vortex, $\varepsilon / \lambda$ is the characteristic scale of Lorentz force also related to unit length, and $\mu$ is mobility of the vortex core. Below it will be seen that so defined $u_0$ really serves as the velocity unit. If combine this definition and the known relations [4]

$$\frac{c^2}{\Phi_0 \mu} \sim \sigma_n H_{c2} , \quad \sigma_n \sim \frac{h}{\Delta (\frac{c}{\lambda})^2}$$

with standard notations, $\sigma_n$ being the normal conductivity and $\Delta \sim 2k_B T_c$ being the order parameter, one obtains

$$u_0 \sim \frac{\lambda k_B T_c H_{c1}}{2\pi \hbar H_{c2}}$$

As a typical example, at $T_c \sim 100 K$, $\lambda \sim 3 \cdot 10^{-5} cm$ and $H_{c2}/H_{c1} \sim 500$, one gets the estimate $u_0 \sim 10^5 cm/s$. This value looks small enough to allow for applicability of quazi-static London approximation. In fact, such an approach was used in large number of works on motion of separate vortices as well as vortex lattices. The obvious exception is very dense lattice, with small inter-vortex distances of order of coherence length. But our present subjects of interest are far from such complications.
III. EVOLUTION EQUATIONS

In the framework of London approximation, the free energy $E$ of vortex, placed into a given surroundings, is completely determined by the shape of its core, $R(p) = \{X(p), Y(p), Z(p)\}$, with $X, Y, Z$ being coordinates of the core points and $p$ being a scalar parameter. In accordance with the principles of mechanics and nonequilibrium thermodynamics, the simplest equation of a massless viscous evolution of the core line looks as

$$\mu^{-1} \partial R/\partial t = f(R)$$

with Lorentz force on the right-hand side and friction force on the left, both being related to unit core length.

By its sense, the parameter $\mu^{-1}$ is the effective drag coefficient which is determined by all the dissipative energy losses conjugated with the core motion. Generally, there are at least two sorts of dissipative processes accompanying the motion (see, for example, the review [4]), namely, relaxation of the order parameter and normal currents induced by time-dependent own magnetic field of the vortex. A concrete expression for $\mu$ can be derived from more detailed theory, for instance, from the Ginzburg-Landau functional approach, under its reduction to London approximation [4]. The reduction is possible because normal self-current of moving vortex and corresponding dissipation are located mainly in a close vicinity of the core line, at distance comparable with coherence length. After the transition to London’s description, the effect of normal currents becomes hidden in $\mu$, but these currents give no contribution to the Lorentz force [4]. Therefore, the reduction results in the identity whose meaning is balance of friction force and Lorentz force, as it is stated by the Eq.1, with $f(R)$ being determined only by supercurrents.

To write $f(R)$, one has not to evaluate the supercurrent distribution. Instead, as in general in mechanics and statistical thermodynamics, $f(R)$ can be expressed by means of $E$’s variation under a small displacement of a local core fragment, that is as the functional derivative $\delta E/\delta R(p)$. However, the latter itself is not invariant with respect to arbitrary (non-degenerated) transformations of the parametrization $R(p)$ and to physical dimensionality of $p$. In case of isotropic media, the only true invariant expression for the Lorentz force is

$$f = -\frac{dp}{dL} \frac{\delta E}{\delta R(p)} = - \left| \frac{\partial R}{\partial p} \right|^{-1} \frac{\delta E}{\delta R(p)} = \frac{\Phi_0}{c} [J \times N]$$

Here the vector $\partial R/\partial p \equiv R'$ is locally parallel to the core, $dL = |R'| dp$ is the differential of the core length, $N \equiv R' |R'|^{-1} = \partial R/\partial L$, and $J$ is the density of full effective supercurrent which streams around core and pushes a given core fragment.

The energy $E = E\{R(p)\}$ includes self-interaction of vortex and its interaction with surroundings, in particular, with other vortices. Correspondingly, in general $J$ consists of external currents and self-current of vortex determined by its distortion. The Eqs.1 and 2 could be directly extended to a number of interacting vortices. Besides, in principle, one may add into $E$ also interactions with pinning potentials. However, below we are interested only in motion of separate vortex in absence of pinning.
The parameter $p$ enumerates strictly the core points. But in practice it is preferable to use another kind of parametrization, concretely, to introduce the parameter $q$ which enumerates some suitable continuum of surfaces $Q(r) = q$, $r = \{x, y, z\}$, each possessing only one intersection with core line. The connection between $p$ and new parameter $q$ is implied by the obvious relation $Q(R(p(q, t), t)) = q$, and simple algebraic manipulations lead to the modified form of the evolution equations,

$$
\frac{\partial R}{\partial t} = \mu \left[ 1 - \frac{\partial R}{\partial q} \otimes \frac{\partial Q(R)}{\partial R} \right] f(R), \quad f(R) = -\left| \frac{\partial R}{\partial q} \right|^{-1} \frac{\delta E}{\delta R(q)}
$$

where the symbol $\otimes$ denotes the tensor product of two vectors.

These equations describe how the intersection points marked by $q$ move along the corresponding surfaces $Q(r) = q$. Clearly, this is factually two-dimensional motion. This feature becomes quite obvious if it is possible to identify $q$ as one of cartesian coordinates, that is to use parallel planes as the marking surfaces. For instance, if that are XY-planes, $q = Z$ and $Q(r) = z$, then the Eqs.3 reduces to the equation

$$
\frac{\partial}{\partial t} \begin{pmatrix} X \\ Y \end{pmatrix} = -\frac{\mu}{\sqrt{1 + X'^2 + Y'^2}} \begin{pmatrix} 1 + X'^2 & X'Y' \\ Y'X' & 1 + Y'^2 \end{pmatrix} \begin{pmatrix} \delta E/\delta X(Z) \\ \delta E/\delta Y(Z) \end{pmatrix}
$$

with shortened notations $X' \equiv \partial X/\partial Z$, $Y' \equiv \partial Y/\partial Z$. The Eq.4 describes the time evolution of $X$ and $Y$ coordinates of the core points marked with their $Z$-coordinate.

**IV. BOUNDARY CONDITIONS AND LOCAL APPROXIMATION**

In absence of pinning and more vortices, the vortex energy $E = E_s + E_i$ consists of two parts: the energy $E_i$ of the vortex interaction with transport or Meissner supercurrent and the self-energy $E_s$. Therefore, the current in the Eq.2 also can be devided into two parts, $J = J_s + J_i$.

Formally, $E_s$ is a complicated spatially non-local functional [5] depending on both the core configuration $R(p)$ and the shape of sample. Among other factors, $E_s$ includes the vortex interaction with the sample boundary what can be interpreted as attraction of the end fragments of the core to their mirror images placed outside superconductor.

But, if the curvature radius of the core everywhere is not too small as compared with $\lambda$, and besides, if the core nowhere is too close to itself, then the so-called local approximation is possible,

$$
E_s \approx \varepsilon L = \varepsilon \int |dR(p)|
$$

where $L$ is the core length. This well known approximation was argued and used as long ago as in 1968 by Galaiko [6], and later by many other authors (in particular, in [2-5]).

Our own computer simulations showed that the relative error of evaluation of self-action force by means of local approximation does not exceed a few percents even if the curvature radius is as small as $0.1\lambda$.

In the local approximation the Eqs.1 and 2 take the form
\[
\frac{\partial R}{\partial t} = \mu \frac{\Phi_0}{c} [(J_s + J_i) \times N], \quad J_s = \frac{cH_{c1}}{4\pi} \left[ N \times \frac{\partial^2 R}{\partial \rho^2} \right] \left| \frac{\partial R}{\partial \rho} \right|^{-2}
\] (6)

Here \( J_s \) is the self-current what flows through the very core. Its absolute value is inversely proportional to the local curvature radius of core.

However, the local approximation needs to be accompanied by correct boundary conditions. The true conditions should take into account the vortex interaction with superconductor boundary. There are two ways to show that this interaction results in the orthogonality of the end fragments of core to the boundary. Thought these conditions are known at least since [6], sometimes that's are neglected, so it is desirable to present more argumentation.

First, let us note that the force vector \( f(R) \) is always perpendicular to the local core direction. Indeed, any variation \( \delta R \) parallel to this core direction, \( \delta R \parallel \partial R/\partial \rho \), merely is identical to a change of parametrization, without factual change of the shape, so it has no physical meaning and should result in \( \delta E = 0 \) (therefore the last expression in (2) always is consistent with previous ones). The same is seen from (6). As a consequence, any core point displaces perpendicularly to the core, in particular, the end points do which are placed just on the boundary. Hence, we must conclude that the end fragments always are oriented to be orthogonal with respect to the boundary.

Secondly, the non-orthogonality would mean that the contour formed by core and its mirror image is broken at the end point, i.e. has infinitely small curvature radius here. From the point of view of exact \( E_s \) [5], if such a sharp ”knee” occurred it would cause infinitely strong Lorentz self-action force and consequently would be immediately straightened thus restoring the orthogonality.

But, we must to underline that the principal conclusions to be deduced do not refer to the local approximation and can be derived from general non-local Eqs.1 and 2 only.

**V. STICK-ELASTIC VORTEX TRANSFORMATION IN CURRENT-CARRYING PLATE**

To avoid a complicated mathematics, we confine ourselves by simplifying superconductor geometry. Consider the vortex evolution in an infinitely wide plate, \(-D < Z < D\), without pinning but in presence of transport surface supercurrent uniformly distributed over the boundary planes and obeying the London equation. If this current flows along Y-axis then

\[
J_x = J_z = 0, \quad J_y = \frac{c}{4\pi \lambda} H_{c1} j(Z), \quad j(Z) \equiv h \frac{\cosh(Z/\lambda)}{\cosh(D/\lambda)}
\]

with \( h \) being the dimensionless measure of current density.

Let initially the vortex pierces the plate in Z-direction being described with \( R = \{X(Z,0) = 0,0,Z\} \). It has similar orientation soon after nucleation near the edge of a real finite plate. Then, due to obvious spatial symmetry, the vortex will remain inside the XZ-plane \( Y = 0 \) and keep only one intersection with any of XY-planes. In this situation the Eq.4 can be applied and, besides, reduced to only equation for X-coordinate, \( X = X(Z,t) \), as a function of time and Z-coordinate:
\[
\frac{\partial X}{\partial t} = -\mu \sqrt{1 + X'^2} \frac{\delta E}{\delta X(Z)}
\]  

(7)

The energy can be expressed as

\[
E = E_s + E_i = E_s - \frac{\varepsilon}{\chi} \int X(Z, t) j(Z) dZ
\]

(8)

where the integral represents the energy \( E_i \) of vortex interaction with transport current (this expression differs only by some constant from the general \( E_i \) representation [5]). In the local approximation (5), the Eq.7 looks as strongly nonlinear diffusion-type equation

\[
\frac{\partial X}{\partial t} = u_0 \left[ \frac{\lambda}{1 + X'^2} + \sqrt{1 + X'^2} j(Z) \right]
\]

(9)

with notation \( X'' \equiv \frac{\partial^2 X}{\partial Z^2} \) and characteristic velocity \( u_0 \) introduced in Sec.2.

Here the left side is responsible for the friction, and two terms on the right-hand side represent the self-action force and transport current-induced Lorenz force, respectively. Clearly, because of the latter force both the vortex ends will forwardly move in one and the same direction (to opposite edge of the plate), while the middle of vortex will be more or less backward, and the larger is transport current the longer should be the distance \( \Delta X \equiv X(\pm D, t) - X(0, t) \) (below termed vortex stretching).

Some predictions of further vortex behaviour can be deduced merely from the energy expression (8). Just after start the middle is still in rest. As the Eq.8 shows, in thick plate \( (D >> \lambda) \) the unit displacement of every end leads to the \( E_i \)'s decrease by \( h \varepsilon \). At the same time, the corresponding lengthening of each of two symmetrical core branches results in the \( E_s \)'s increase by \( \varepsilon \) per unit length. Consequently, if \( h > 1 \) then the total energy decreases, and the vortex stretching along the drift direction becomes profitable. The lengthening process should last until the curvature of the most backward central part of the core becomes so large that the self-action force makes this part moving as quickly as the ends do.

Thus, at \( h > 1 \) the vortex gets over the friction like elastic in water. Oppositely, at \( h < 1 \) the stretching is energetically unprofitable, and the vortex should move as deformed flexible stick. The transition from this stick-like behaviour to elastic-like one, when transport current increases from \( h < 1 \) to \( h > 1 \), is the example of so-called "nonequilibrium phase transitions".

Let us consider the steady drift of vortex as a whole, without change of shape. The corresponding solution on the Eqs. 7 or 9 can be written as \( X(Z, t) = ut + X(Z) \). The stationary shape \( X(Z) \) and the drift velocity \( u = u(h, D) \) should be obtained from (7) or (9) with the help of above discussed orthogonality boundary conditions

\[
\frac{dX}{dZ} (\pm D) = 0
\]

Besides, due to the mirror symmetry, the condition \( X'(\pm 0) = 0 \) should be satisfied.

In this steady nonequilibrium state the self-energy \( E_s \) is constant, therefore, the work \( M_j \) produced by transport current per unit time,
\[ M_j = -\frac{dE_i}{dt} = \frac{u\Phi_0}{c} \int_{-D}^{D} J_y(Z) dZ = 2uh\varepsilon \tanh(D/\lambda) \]

coincides with the energy dissipation per unit time \( M_d \). In accordance with (1) and (2),

\[ W_d = \frac{1}{\mu} \int \left| \frac{\partial R}{\partial t} \right|^2 dL = \frac{1}{\mu} \int \left( \frac{u}{Q} \right)^2 Q dZ = 2\delta u^2/\mu \]

where the notations

\[ Q \equiv \sqrt{1 + X'^2} = \frac{dL}{dZ}, \quad \delta \equiv \int_0^D \frac{dZ}{Q} \]

are introduced. We took into account that actual displacement of the core always is locally perpendicular to its orientation. Only such displacements are physically meaningful and really cause the friction. Therefore, the drift velocity and the local core velocity are connected by the relation

\[ \left| \frac{\partial R}{\partial t} \right| = \frac{u}{Q} \]

Evidently, the factor \( Q \) determines at one and the same time local orientation of the core and degree of its stretching.

Hence, the equality \( M_d = M_j \) yields

\[ U \equiv \frac{u}{u_0} = \frac{h\lambda}{\delta} \tanh(D/\lambda) \approx \frac{h\lambda}{\delta} \quad (10) \]

In view of above reasonings, at \( h < 1 \) the vortex stretching is weak, therefore, \( X'^2 \) is comparable with unit, \( Q \sim 1, \Delta X \sim D \) and \( \delta \sim D \). Then the Eq.10 shows that in this stick-like regime \( U \sim h\lambda/\delta \approx h\lambda/D << h \), i.e. the drift velocity is inversely proportional to the plate thickness. This is quite natural, because the surface current-induced Lorentz force acts only on the ends, while the friction almost equally acts on any core fragment.

In general, the parameter \( \delta \) serves as the effective plate half-thickness. Obviously, always \( \delta < D \). In the stretched elastic-like regime in thick plate anywhere at \( D - |Z| >> \lambda \) the inequalities \( |X'| >> 1 \) and \( Q >> 1 \) take place. Hence, \( \delta << D \) and what is essential it becomes almost insensitive to thickness. As a consequence, both the drift velocity and mobility \( u/h \) strongly increase as compared with stick-like regime and both become independent on thickness (below we shall see that \( \delta \sim \lambda \) and \( U \sim h \), i.e. \( U \) becomes approximately \( D/\lambda \) times larger). According to the \( M_d \)'s expression, the matter is that thought the energy dissipation \( Q \) times increases due to the core lengthening this effect is overpowered by its \( Q^2 \) decrease because of \( Q \) times decrease of the factual core velocity \( \left| \frac{\partial R}{\partial t} \right| \). As the result, the vortex stretching leads to smaller friction and smaller entropy production, under fixed vortex velocity, and to larger velocity under fixed transport current. The picture looks as if most part of core slides along itself, but this process does not mean a real motion of core and so does not cause a friction and dissipation.
VI. DRIFT OF THE VORTEX ENDS

To be convinced in what was said, let us consider vortex shape in the stretched elastic-like regime. Because at $\Delta X >> D$ most part of the core inevitably has a small curvature, it can be considered with neglecting the self-action. Then any of the Eqs.7 and 9 reduces to

$$U \approx \sqrt{1 + X'^2 j(Z)}$$  \hspace{1cm} (11)

Here from the characteristic exponential asymptotics does follow,

$$X(Z) \approx -\frac{\lambda U}{h} \left[ \exp \left( \frac{D - Z}{\lambda} \right) - 1 \right]$$  \hspace{1cm} (12)

Here $Z > 0$, $X(-Z) = X(Z)$, and for definitness the position $X = 0$ is prescribed to the end fragments. It is easy to verify that corresponding self-action contribution in the Eq.9 indeed is negligibly small as compared with the current-induced force.

We can get a rough estimate of the stretching if put on $Z = 0$ in (12) and take into account that $U > h\lambda/D$. Then the Eq.12 yields

$$\frac{\Delta X}{\lambda} \approx \frac{U}{h} \exp \left( \frac{D}{\lambda} \right) > \frac{\lambda}{D} \exp \left( \frac{D}{\lambda} \right)$$

Hence, $\Delta X/\lambda$ possesses exponentially strong dependence on $D/\lambda$, and it can be giantly large if $D$ exceeds $\lambda$ by an order of value or more.

In view of this circumstance, the ratio $\Delta X/W$ with $W$ being the width of a real finite plate, becomes of principal importance. Clearly, if $\Delta X >> W$ then the steady drift of the vortex as a whole is impossible: the ends of vortex will achieve the opposite edge before the displacement of its backward central part will be comparable with $W$ (all the more, before the velocity of this part becomes equal to that of the ends).

Consider the drift of the ends in such a non-stationary situation. Because the vortex lengthening is profitable, this drift can do independently on the motion of deepened backward part, as if thickness was infinitely large ($D/\lambda \rightarrow \infty$). To estimate the drift velocity, let us multiply the Eq.7 or 9 (with $\partial X/\partial t \Rightarrow u$) by $Q^{-1}$ and integrate over variable $z = D - Z$ from zero to infinity, with the condition $X'(z \rightarrow \infty) = \infty$ which evidently corresponds to infinitely far backward center. Then both the Eqs.7 and 9 result in

$$U = \frac{\lambda}{\delta}(h - 1), \quad \delta = \int_0^\infty \frac{dz}{Q}$$

To evaluate this integral, note that in accordance with the orthogonality boundary conditions the shape of the end fragments of the core is parabolic, for instance, at upper end $X(Z) = X(D) - (Z - D)^2/2\rho$, with $\rho$ being the curvature radius at the end point. It follows from the Eq.9 that $\lambda/\rho = h - U$. In this parabolic region the integration diverges but becomes cut after transition to exponential asymptotics (12). The estimate of the integral leads to approximate equation

$$U \approx h(h - 1)/\{h - 1 + \ln[2z_0(h - U)/\lambda]\}$$  \hspace{1cm} (13)
where $z_0$ is the depth of the crossover point, $z_0 \sim 4\lambda$. The Eq.12 helps to estimate the end drift velocity in thick plate. Obviously, it turns into zero at $h \to 0$, in agreement with $D \to \infty$ limit of the estimate for stick-like regime. It can be shown that velocity of the steady drift of the vortex as a whole is only slightly smaller differing by a multiplier of order of unit.

VII. GIANT VORTEX STRETCHING

The exponentially large vortex stretching is the most significant possibility of vortex evolution in thick plate, as well as in bulk samples in general. Therefore it would be useful to more correctly justify the above simplistic estimate of $\Delta X$.

Note that $\Delta X > L/2 - D$. Divide both sides of (9) by $j(Z)$ and integrate from zero to $D$. This results in

$$L/2 = B - A, \quad B \equiv U \int \frac{dZ}{j(Z)}, \quad A \equiv \int \arctan(X') \left| \frac{d\lambda}{dZ} j(Z) \right| dZ$$

It is easy to notice that

$$A < A_0 \equiv \frac{\pi\lambda}{2} \left[ \frac{1}{j(0)} - \frac{1}{j(D)} \right]$$

so $\Delta X > B - A_0 - D$. The calculation of integral $B$ gives

$$B = \frac{2U\lambda}{h} \cosh(D/\lambda) \{ \arctan[\exp(D/\lambda)] - \frac{\pi}{4} \}$$

Then, after simplifications possible due to $D >> \lambda$, one finally obtains

$$\Delta X/\lambda > \frac{\pi(U - k)}{4h} \exp(D/\lambda)$$

with $k < 1$. Because $u$ is monotonously growing function of $h$, the coefficient in front of exponent is positive if $h$ exceeds some level larger than unit, for example, if $h > 2$. Hence, at least at $h > 2$ the vortex stretching is exponentially strong.

This estimate is obtained in the framework of local approximation. More correct estimate should give a lesser value, because of self-attraction of the core in middle part of the plate where two symmetrical exponential tails described by (12) meet one another and form an arc. Such a non-local effect is most essential just under the specific plate geometry. However, the non-local correction can not change the shape of the front vortex part where the non-local interaction is weak as compared with other forces. It is not hard to show that the latter requirement is satisfied if $|Z| > Z_0$, where $Z_0$ is the solution on equation

$$h \exp[-(D - |Z|)/\lambda] \approx \sqrt{\lambda/2\pi |Z|} \exp(-2 |Z|/\lambda)$$

If take into account that the more is $h$ the less is $Z_0$, then this equation yields $Z_0 < D/3$. Hence, at $|Z| > D/3$ the mutual attraction of two core branches can be neglected, and
the asymptotics (12) remains valid. This means that the maximally possible effect of non-locality is the replacing $D$ in the exponent by $\alpha D$ with $\alpha > 2/3$. Consequently, the lower bound for the stretching with confidence can be estimated as

$$\Delta X/\lambda > \frac{\lambda}{D} \exp(2D/3\lambda)$$

Thus, even in the worst case the non-local effects do not abolish the exponential character of stretching.

For example, if $\lambda \approx 3 \cdot 10^{-5} \text{cm}$ and $h$ equals to a few units, then even at $D \sim 20\lambda < 10^{-3} \text{cm}$ one gets $\Delta X > m\lambda \exp(2D/3\lambda)$, with $m \sim 1$, i.e. $\Delta X > 1 \text{cm}$ what exceeds a width of any realistic sample. Thus at first the vortex should form a ring whose shape approximately copies that of the sample cross-section. During this process the velocity of backward deepened core part is primarily determined by its distortion which is created in the beginning of stretching and thus has curvature radius of order of $D$. Hence, this velocity is of order of $u_0\lambda/D$, and at the moment when the ends will meet one another the displacement of most backward point will be yet as small as $\sim \lambda W/Dh << W$.

VIII. DISCUSSION AND RESUME

It seems clear that both the above conclusions can be extended to bulk current-carrying superconductors with another geometry, for instance, to round wires, if treat $2D$ and $W$ as minimal and maximal diameters of cross-section of the wire, respectively.

Due to possibility of giant deformation and stretching of vortices, the thermodynamically nonequilibrium process of vortex penetration can promote formation of complicated many-vortex dynamical configurations which seem rather strange and unprofitable from the point of view of equilibrium thermodynamics. The presence of an external magnetic field parallel to transport current should lead to formation of spiral-like configuration instead of ring-like one and thus especially ensure the entangling of vortices. The Eq.4 enables us to describe this scenario in details, if choose Z-axis to be directed along the wire. Besides, the presence of weak pinning should amplifier the stretching of vortex and additionally complicate its shaping, because the motion of deepened part of vortex is characterized by relatively small forces of order of $\epsilon\lambda/D$ (much smaller than forces $\sim \epsilon$ what act on the end fragments) and so may be easily held back by pinning centers.

We would like to underline the role of orthogonality boundary conditions. In the work [3] the equation was under use similar to our Eq.9, but boundary conditions was formulated in terms of the tension of core line. One can see from [3] that such conditions make it impossible to consider the case of high surface transport current $> H_{c1}$ corresponding to the elastic-like regime.

To resume, we formulated the invariant equations of viscous motion of arbitrarily shaped 3D vortex lines, and applied them to careful analysis of the scenario of vortex penetration into a thick superconducting sample. As it was argued, the vortex-vortex interaction does not significantly affect the penetration process. But, of course, a full description of resistive state leads to more complicated tasks about vortex-vortex interactions deep inside the sample.
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REFERENCES
1. M.V.Indenbom, C.J.van der Beek, V.Berseth, W.Benoit, G.D’Anna, A.Erb, E.Walker and R.Flukiger, Nature, 1997, Feb.20 .
2. Yu.A.Genenko, Phys.Rev., B 49, 1994, 6950.
3. Chao Tang, Shechao Feng and L.Golubovich, Phys.Rev.Lett., 472, 1994, 1264.
4. L.P.Gorkov and N.B.Kopnin, Uspekhi fizicheskikh nauk, 116, 1975, 411 (transl. in English in Sov.Phys.-Usp., 1975).
5. Yu.E.Kuzovlev, Physica, C 292, 1997, 117.
6. V.P.Galaiko, Zh.Teor.Eksp.Fiz., 50 , 1966, 1322.