Using ray-traversal for 3D particle matching in the context of particle tracking velocimetry

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An innovative method based on the traversal of rays, originating from detected particles, through a three-dimensional grid of voxels is presented. The methodology has as main advantage that the outcome of the method is independent of the order of the input: the order of the cameras and the order of the rays presented as input to the algorithm does not influence the outcome. The algorithm finds matches in decreasing value of match quality, ensuring that globally best matches are matched before bad matches. The algorithm is found to scale efficiently with the number of cameras and particles.

I. INTRODUCTION

The process of calculating the 3D position based on the views of multiple cameras is traditionally called stereomatching, and is based on the biological process of stereopsis. Though most animals have binocular vision, experiments in fluid dynamics have been using more than two cameras to improve depth perception and to provide extra robustness and accuracy by data-redundancy, in particular in the context of Lagrangian particle tracking techniques$^{[1–4]}$. The angles between the cameras should, however, be optimized. A small angle between two cameras causes large errors in the estimation of the depth. It is fairly common to put cameras at relative angles of 90 degrees, such that one of the coordinates is redundant, which makes matching easier. It is, though, not strictly necessary; it can be any angle. When more than two cameras are used, in order to minimize the error in all directions, it is generally a good strategy to globally maximise the relative angles between all the cameras.

This manuscript is on the matching algorithm of light rays from a set of several cameras used for 3D particle tracking methods. In this context, the 3D position of a particle is retrieved from the intersection of rays of light (epipolar lines) coming from each of the cameras. These rays can be obtained e.g. from Tsai’s pinhole model$^{[5]}$ based on the optics of the camera and the objectives or more elaborate models$^{[6]}$. Like ray-tracing in computer graphics, we will consider the rays to origin from each camera, towards the detected particle—in the opposite direction of light being scattered by a particle that is captured by a camera. For a given particle and a given camera, this ray $r$ has an origin $p$ (let’s say at the position of the particle image on the camera sensor) and a direction $v$. The complexity of the problem arises when many particles are to be tracked simultaneously with several cameras. In this situation, a bundle of epipolar lines (one line for each particle) emerges from each cameras. In order to determine the actual 3D position of all the particles, one needs to find the inter-bundle lines that cross (or nearly cross) with each other. To get a feeling of the difficulty of the problem, we show an example set of rays from experiments having slightly over 1600 rays in total, emerging from 4 cameras at different view angles, see Fig. 1. The problem of 3D matching can then be stated as follows: given $c$ cameras, each with $m_i$ rays, find sets of rays $r_{i,j}$ that minimize the distance from a point to rays coming from different cameras. Here the $i$ index is the index of the camera and $j$ the index of the ray for that camera.

FIG. 1: Example of rays for the setup shown in Fig. 8a. Gray scale bar has a length of 50 mm. A total of $\sim 1600$ rays intersect a measurement volume. Rays originating from the same camera have the same color. Data taken from Ref. [7].
For a valid match, the rays from each set of rays should be from distinct cameras. A simple approach would be to consider all possible sets and pick out the best matches. However this would lead to \(m^c\) combinations to be tested. For a typical case of 400 particles, tracked with 4 cameras, this would lead to a large number of candidate matches (\(400^4 = 2.56 \times 10^{10}\)). This is computationally prohibitive as too much time would be spent to go through all combinations looking for crossing or nearly crossing lines. Various strategies can be thought of that will eliminate the majority of these candidate matches. Classical alternative strategies are generally based on projections of epipolar lines between pairs of cameras \([1, 3, 8]\), in order to reduce the dimension of space in which crossings are to be found. The most efficient schemes can reduce the computational need from \(O(m^c)\) to \(O(cm \log m)\). However, such strategies usually operate by successive stereo-matching searches of correspondences between pairs of cameras. When more than two camera are used, this requires then either to consider one of the cameras as a reference for the pairs (with time complexity \(O(cm \log m)\), what may lead to ambiguities (as due to imperfection of the optical models, the matches found may depend on the choice of the reference camera) or to consider all possible pairs of cameras (with time complexity \(O(c^2 m \log m)\) and apply sophisticated combinatorial algorithms to perform the required consistency checks between all the pairwise stereo-matches to avoid ambiguities.

Here we propose a new strategy, based on ray traversal across 3D voxels, which efficiently allows to perform the stereo-matching with an arbitrary number of cameras, by combining all the cameras simultaneously, without requiring pair-wise operations and consistency post-checks. Note that we will focus exclusively on the matching part of particles tracking velocimetry (PTV) method, and not on the tracking part; i.e. the following of matched particles over time. Beyond PTV, the present method could also be used for the 3D reconstruction of an object by matching image keypoints from multiple images from different angles.

II. METHOD

In this article we will focus on the traversal of rays through a 3D array of voxels (in analogy to pixels, volume elements) with constant spacing in each direction, but it can be generalized to voxels with varying widths, and even further to an octree where a (cubic) space is recursively subdivided into 8 sub-cubes in order to locally refine the 3D volume. For simplicity and didactic purposes, we will consider a rather simple scenario where there are only 3 cameras with a total of 7 rays, and where the situation can be visualized in 2D such that it is comprehensible, see fig. 2.

The first step of the algorithm is to traverse the rays through the voxels, this can be done very fast and is linear with the number of voxels in each direction, see e.g. Ref. [9]. During this process we will maintain a list of all the voxels traversed, denoted by the two indices \(x\) and \(y\) (and \(z\) in real experiments), along with the ray who traversed it, see step 1 of Table I. Furthermore, we also add all the neighbours of the visited voxels (here we use a \(\ell_1\) norm of 1 giving 6 neighbours: left, right, above, below, front, back). Such that these rays allow for some ‘play’ during the matching. For our near-2D toy-problem shown in Fig. 2 we have 319 voxels that are visited. Our first element in the list is \(x = 4, y = 11\), and \(Ray = 1.2\) meaning that ray 2 from camera 1 has visited the voxel with horizontal index 4, and vertical index 11.

The second step is to gather, for each voxel, what rays have traversed through that voxel, see step 2 of Table I. One can see that a lot of voxels will have only a single
TABLE I: In step 1 we maintain a list of all the traversed voxels (denoted by the $x$ and $y$ indices) and the ray identifier in the camera-ray format. In step 2 the list of step 1 is grouped by the voxel indices $x$ and $y$, each voxel which is traversed by multiple rays will show multiple rays in the ray column. In step 3, we discard each voxel which is only traversed once because we need at least 2 rays (from different cameras) in order to get a match. In case there are more cameras we can require e.g. at least rays from 3 different cameras. We also discard the voxel indices, as they are no longer needed. In step 4 we remove any duplicates from list the list of step 3. For each list of rays, we expand it in to all subsets i.e. 1.1, 2.1, 3.1, 3.3 is expanded to two tuplets: 1.1, 2.1, 3.1 and 1.1, 2.1, 3.3, after this ‘expansion’ we remove again all the duplicates, the result is shown in the list of step 5. In step 6 we calculate the point which gives the position of the best match for each tuplet of rays, and the square root of the mean square distance is given (the matching error). This result is then sorted by number of rays (descending), and then by the error (ascending). Note that in real measurements the voxel indices are in 3D: $x$, $y$, and $z$. The last step is to pick matches from the top working down, while making sure each ray is only matched once.

| Step 1 | Step 2 | Step 3 | Step 4 | Step 5 | Step 6 |
|--------|--------|--------|--------|--------|--------|
| $x$ $y$ Ray | $x$ $y$ Rays | Rays | Candidates | Candidates | Best fit | $\sqrt{d^2/n}$ |
| 4 11 1.2 | 5 12 1.2 | 1.1, 2.1, 3.1, 3.3 | 1.1, 2.1, 3.1, 3.3 | 1.1, 2.1, 3.1 | 21.7, 0.6, 0.5 | 0.43 |
| 5 11 1.2 | 14 4 1.1, 2.1, 3.1, 3.3 | 1.2, 2.1 | 1.2, 2.1 | 1.2, 2.2 | 21.5, 0.3, 0.5 | 0.44 |
| 5 12 1.2 | 11 1.2, 2.1 | 1.2, 2.1 | 1.2, 2.2 | 1.2, 2.1 | 21.9, 0.0, 0.5 | 0.65 |
| 6 12 1.2 | 6 12 1.2 | 1.2, 2.2 | 1.2, 2.2, 3.2 | 1.2, 2.2 | 21.6, 0.7, 0.8 | 0.06 |
| 7 12 1.2 | 5 11 1.2, 2.1 | 1.2, 2.1, 3.2 | 1.2, 2.1 | 1.2, 2.1, 3.3 | 15.1, 4.7, 0.5 | 0.50 |
| 7 13 1.2 | 5 11 1.2 | 1.1, 2.1, 3.1 | 1.1, 2.1 | 1.2, 2.2, 3.2 | 15.1, 4.7, 0.5 | 0.50 |
| 8 13 1.2 | 5 11 1.2, 2.2 | 1.2, 2.1, 3.1 | 1.1, 2.2 | 1.1, 2.2 | 22.0, 0.4, 0.5 | 0.50 |
| 8 14 1.2 | 13 16 1.2, 2.2 | 1.2, 2.1, 3.2 | 1.1, 2.2 | 1.1, 2.2 | 30.0, 1.1, 0.5 | 0.50 |
| 9 14 1.2 | 8 13 1.2 | 1.2, 2.1 | 1.1, 3.3 | 1.2, 2.1 | 31.7, -0.1, 1.0 | 0.53 |
| 10 14 1.2 | 8 14 1.2 | 1.2 | 2.1, 3.1 | 1.1, 3.3 | 21.3, -8.1, 1.0 | 0.53 |
| 10 15 1.2 | 3 11 1.2, 2.1, 3.2 | 1.2, 2.2, 3.2 | 1.2, 2.1 | 1.1, 2.2 | 21.6, 0.7, 0.8 | 0.06 |
| ... | ... | ... | ... | ... | ... | ... |
| 319 | 261 | 46 | 10 | 9 | 5 | 3 |

We can again walk through the sorted list and only keep an element if the one before was not the same, this can be done in linear time $O(n)$. Note that during step 2 we sorted the rays for each entry, this makes this sorting and deleting duplicates much easier. We are left with 10 entries in our list.

In the fifth step we extract all possible candidate from each of the set of rays. In general this means that the rays are grouped by camera, and then the Cartesian product is applied to get all the tuples of possible candidates. To exemplify, say that we have an entry: 1.1, 1.2, 2.1, 3.1, 3.2 this would give 2 possibilities for camera 1, 1 possibility for camera 2, and 2 possibilities for camera 3 or a total of 4 combinations of candidates ($\{1.1, 1.2, 1.3, 3.1\}$, $\{1.1, 1.2, 1.3, 2.3\}$, $\{1.2, 1.2, 1.3, 3.1\}$, and $\{1.2, 2.1, 1.3, 2.3\}$). For our toy-problem we can see that the first entry of step 4 is expanded in to 2 possible candidate matches (the first two entries of step 5).

In the sixth and penultimate step we calculate the point for which the sum of the square distances is minimum for each candidate set of rays. This can be efficiently solved using a set of linear equations (3 of them to be precise, one for each coordinate) that can be solved using standard matrix algebra, see Fig. 2 for an illustration. For this point we calculate the root mean square value of the distances from this point to the rays (matching error); a smaller value means the rays cross more closely; a better and more probable match. Finally these candidates are sorted, first by number of rays in descending order (matches with more rays are more reliable than matches with less rays), and then by matching error in ascending order (smaller error is better). See the list in step 6 of Table I.

The last step would be to walk through the list of candidates, starting from the top, and picking each of them for which the rays are not matched before. A naïve ap-
approach would be to remove all future entries which has one of the rays of the accepted candidate, however this would lead to $O(n^2)$ time scaling which is unfavourable. We can do this more efficiently by keeping a hash map (hash table) which records the number of times a ray is used. This allows for fast insertion, looking up, and modification of the number of times a ray is used. This leads to an algorithm that scales with time as $O(n)$ on average, and with a worst-case of $O(n \log n)$.

As one can see, most of the steps have $O(n \log n)$ time complexity or better, where $n$ is the number of traversed voxels. We can safely assume that the number of traversed voxels scales linearly with the number of rays (and independent of the number of cameras). We do, however, have to consider the size of the voxel. The voxels should be equal or larger than the maximum distance we allow as ‘error’ for our matching. If we allow for a maximum distance of (say) 1 mm, we need to make sure that all the voxels within a neighbourhood of 1 mm from the ray are also traversed, this is done by also traversing the neighbours, see the light-shaded voxels in fig. 2. It ensures us that matches with errors twice this 1 mm will not occur, as two (or more) rays that are more than twice the distance away from each other will not traverse the same cells. It is therefore not a good idea to introduce a smaller voxel size than this distance in combination with a larger neighborhood of say 2 in $\ell_1$ distance in order to fulfil the requirement of maximum distance. This would just lead to more duplicate entries in the lists of steps 2 through 4. For speed reasons we do have to make a compromise in the choice of our voxel spacing. A very fine spacing will give much better pruning in possible candidates (the list of step 5) but memory usage and time consumption will be high in the first steps as more cells are traversed; the $n$ in the aforementioned time complexity measures will increase inversely with the voxel size. However, if we make the voxel very large, say, in the extreme case, 1 voxel that fills the entire volume, then all the rays will traverse this single voxel, so the list of step 1, 2, 3, and 4 will be of length $\sum_i m_i$, however the length of step 5 will be $\prod_i m_i$, and will scale as $m^c$, where $m$ is the number of particles and $c$ the number of cameras; this is our naive approach of trying out all combination of rays. And optimum is to be found with a voxel size (and maximum distance) for which the entries in steps 1 through 4 are manageable, while the number of candidates in step 5 is also kept manageable.

III. TIMING

In order to find the optimum number of divisions, we will consider an artificial experiment with 256 randomly placed particles in a cubic volume with sides of length 1. We will add 4 virtual cameras, and position them around the volume in a configuration. For now, we will consider a perfect arrangement of pinhole cameras such that the virtual rays are perfectly going through the virtual particles. For simplicity we will keep the voxels cubic, and vary the voxel size in all three direction simultaneously and time the duration of the execution, see Fig. 4. During step 3 we have only kept voxels which are visited by at least 3 cameras.

![FIG. 4: Time consumption as a function of the number of divisions of the voxel grid for 256 particles and 4 cameras. The fastest execution was found for 85 divisions (shown in red), balancing the cost of traversing many voxels (large number of divisions) and having many candidates (small number of divisions). Error bars are based on repeated timings.](image)
ically for a real measurement by e.g. a golden-section search, before processing the entire recording. We can repeat this procedure for different number of particles, see Fig. 5. One can see that the same balance can be found for all datasets of different number of particles. As expected, the optimal number of divisions \((d)\) increases with the number of particles \((m)\). We find that \(d \propto m^{0.465 \pm 0.006}\) for a 95% confidence level. The number of cells traversed, for optimal timing (and therefore divisions), scales therefore as \(n \propto m^{1.465}\).

In Fig. 5 the optimal computing time as a function of number of particles is shown. It is found that the time \(t\) scales as \(t \propto m^{1.44 \pm 0.01}\) for a 95% confidence level. Which is close to the predicted \(m^{1.465}\) scaling that was predicted above. The scaling of \(m^{1.44}\) means that doubling the number of particles results in only \(2.7 \times\) more processing power.

Next, the performance of the matching algorithm will be tested by randomly disturbing the particles in 3D for each camera. This causes the rays not to perfectly intersect, in order to mimic optical imperfections, which are unavoidable in a real PTV experiments. We artificially perturb the synthetic particles with a distance \(\delta\) normalized by the average normal distance between adjacent particles \(\langle d_{\text{closest}} \rangle\). 4 cameras are used in a tetrahedral configuration. We compute the average distance between every particle and its closest neighbour as seen by the camera which is denoted by \(\langle d_{\text{closest}} \rangle\). When \(\delta / \langle d_{\text{closest}} \rangle \geq 0.5\) the particles can be disturbed so much that they can start ‘touching’. In Fig. 6 we show the matching statistics for 50 frames of 256 random particles, perturbed for a variety of disturbances \(\delta\). It can be seen that if the particles are mismatched 20% of the mean inter-particle distance more than 90% is still correctly matched. Note that this highly depends on the arrangement of the cameras, the shape of the measurement volume, and if the particles exhibit clustering.

V. MEMORY

We now focus on the memory usage. We set up a synthetic case of 256 particles seen by 4 cameras in a tetrahedral configuration. We vary the number of divisions over a wide range and monitor the time and memory used by the program, more specifically it is set-up to monitor the so-called maximum resident set size, see Fig. 7. We observe that both the memory consumption and time consumption show an optimum. However, these are slightly
shifted, which is expected as storing the traversed voxels and the candidates has a different memory ratio than the time-ratios for the time taken for traversing through the voxels and going through the candidate matches—the algorithms used have different memory and time dependencies. For the user it is best to select the number of divisions in the gray area, optimizing for memory (around 55 divisions) or for speed (around 85 divisions). These numbers will of course depend on the exact implementation and the implementation language (C++11), the compiler (GNU++11), the operating system (macOS Catalina), the hardware, the number of particles (256) and their arrangement, and the number of cameras (4) and their arrangement, where the values in parentheses are the parameters used for this publication.

VI. EXAMPLES

The above matching algorithm has already been successfully applied to a variety of geometries with different number of cameras. We note that the tracking of the particles was done separately with standard particle tracking algorithms [10, 11]. We will go over several of the use-cases:

The code, as described above, has been used in the Twente Water Tunnel facility [12], which is a vertically oriented water tunnel that is 8 m tall, which has an active grid to create near homogeneous isotropic stationary turbulence. It was used to track millimetric bubbles in 3D using 4 high-speed cameras, see Ref. [7] and Fig. 8a for an overview of the measurement section and the camera arrangement. An example trajectory, colored by its horizontal speed, of one of the bubbles is shown in Fig. 8b.

Another use-case is for the measurements of a turbulent jet. Here 3 high-speed cameras were used to track neutrally-buoyant tracer particles of 250 μm in size, see Fig. 8d. Lastly, the algorithm is also already used in Rayleigh-Bénard convection experiments to track tracer particles and in the Lagrangian Exploration Module [14] at the École normale supérieure in Lyon to track neutrally-buoyant particles for which publications are in progress.

VII. CONCLUSION

In conclusion, we have explained a new algorithm for finding matches of rays, the results of which can be used for Particle Tracking Velocimetry (PTV) or 3D reconstruction. The main advantage over other algorithms is that the order of cameras and the order of the rays (detected particles in 2D) do not influence the outcome. Another advantage is that it will find the globally best match, rather than a greedy algorithm (locally optimal choice). Moreover, the algorithm shows good scaling behavior in terms of time and memory as the number of cameras and particles increase. We have shown several use-cases with different geometries and with different number of high-speed cameras, however this algorithm is not limited to those geometries, as it could also be used to track particles (bubbles, droplets) in geometries such as pipes, von Kármán flow, oscillating grid turbulence setups, or the V-ONSET [15].
FIG. 8: (a) Measurement section of the Twente Water Tunnel with a cross section of 45 cm × 45 cm. Four Photron 1024PCI cameras view a common volume in the center of the tunnel. (b) Trajectory of a bubble (colored by its horizontal speed) tracked during 595 frames (corresponding to a duration of 2.38 s) at Re, = 230. Figure adapted with permission from Mathai et al. [7]. Copyright 2018 by the American Physical Society. (c) Trajectories of 3 mm density-matched spheres (color by its speed |\vec{V}|) in counter-rotating Taylor Couette turbulence with \( f_i = -0.4 f_o \) and Re = \((2\pi f_i r_i - 2\pi f_o r_o)(r_o - r_i)/\nu = 8 \times 10^4\). Data is obtained by combining 8 high-speed cameras. Inner cylinder has a diameter of 150 mm. (d) Visualization of a 1000 tracks of a turbulent jet with particles obtained from tracer particles detected by 3 cameras. The box has sides of 50 mm.

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