Why the astrophysical black hole candidates are not rotating black holes

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It is believed that the basic component of the central engine of quasars, microquasars, and energetic Gamma Ray Bursts are the rotating or the Kerr Black Holes (BH)[1]. But by using a generic property[2-4] of the metric components of a stationary axisymmetric rotating metric in its standard form, namely, $g_{\phi\phi} = \sin^2 \theta g_{\theta\theta}$, where $\phi$ is the azimuth angle and $\theta$ is the polar angle measured from the axis of symmetry, we have found the unexpected and surprising result that (i) in order to have a mass of a Kerr BH $m \geq 0$, it is necessary that its rotation parameter $a = 0$ and if one insists for an $a \geq 0$, one must have $m \leq 0$! Thus if the suspected Black Hole candidates with $m > 0$ are really rotating they cannot be BHs at all which is in agreement with some detailed analysis of recent observations[5-8]. However, if it is assumed that such objects are strictly non-rotating, they could be non-rotating Schwarzschild BHs ($a = 0$) with $m \geq 0$ if we ignore the physical difficulties associated with the existence of such objects. This result calls for new theoretical efforts to understand a vast range of astrophysical phenomenon. If one derives the Kerr Metric in a straightforward manner by using the Backlund transformation, it is seen that $a = m \sin \phi$. This relationship confirms that $a = m = 0$ for Kerr BHs.

To describe the axially symmetric stationary Kerr spacetime or any other axially symmetric stationary spacetime, it is convenient to consider $x^0 = t$, the time coordinate, and $x^1 = \phi$, the azimuth angle. By definition, for such a spacetime, the metric coefficients are independent of $t$ and $\phi$:

$$g_{ik} = g_{ik}(x^2, x^3)$$

(1)

Further for a spacetime rotating with increasing $\phi$, it is also required that the spacetime is invariant under simultaneous inversion of of $t$ and $\phi$, i.e, under the transformation $t \rightarrow -t$ and $\phi \rightarrow -\phi$[2-4]. This demands that

$$g_{r2} = g_{r3} = g_{\phi2} = g_{\phi3} = 0$$

(2)

This brings the metric in the following form:

$$ds^2 = g_{tt} dt^2 + g_{\phi\phi} d\phi^2 + g_{22} (dx^2)^2 + g_{33} (dx^3)^2 + 2g_{\phi t} d\phi dt + 2g_{23} dx^2 dx^3$$

(3)

Further, this form of a metric remains unchanged under a coordinate transformation of the form[2-4]

$$x^2 = x^2(x^{2'}, x^{3'}), \quad x^3 = x^3(x^{2'}, x^{3'})$$

(4)
The above constraints, in turn, imply additional constraints on the metric, and in particular, if we choose spherical polar coordinates with \( x^2 = r \) and \( x^3 = \theta \), then we should have (see Eqs. (3.3) and (3.30) in Ref.[2] and Eq. (74) in Ref. [3])

\[
g_{23} = g_{r\theta} = 0
\]  

(5)

and

\[
g_{\phi\phi} = g_{\theta\theta} \sin^2 \theta
\]  

(6)

so that eventually the metric is of the form

\[
ds^2 = g_{\phi\phi} d\phi^2 + g_{rr} dr^2 + g_{\theta\theta} d\theta^2 + 2 g_{\phi t} d\phi dt + g_{tt} dt^2
\]  

(7)

It can be seen that Eq.(6) incorporates the physical condition of asymptotic freedom of the metric[2]

“that the star and its manifold are axially symmetric; i.e. that there exists a Killing-vector field, \( \xi_\phi \), with closed orbits, which, at radial infinity, is spacelike and is orthogonal to \( \xi_t \) and has length \( r \sin \theta \).”

However, note that while \( g_{\phi\phi}, g_{tt} \) and \( g_{\phi t} \) are invariant under arbitrary transformation of the form (4), \( g_{\theta\theta} \) in Eq.(6) corresponds to a unique choice of \( \theta \)[2] Suppose we subject the metric(7) to the following coordinate transformation (S. Antoci, private communication):

\[
r \to r' = r; \quad t \to t' = t; \quad \phi \to \phi' = \phi; \quad \theta \to \theta' = f(\theta)
\]  

(8)

where \( f(\theta) \) is a monotonic function of \( \theta \). It can be seen that while, Eq.(5) remains unaltered under this transformation, Eq.(6) would not be valid unless one fixes \( f(\theta) = \theta \) uniquely. Physically, this unique choice of \( \theta \) corresponds to measuring it from the axis of symmetry and ensuring that \( g_{\phi\phi} = 0 \) along the same and the azimuthal plane lies at \( \theta = 90^\circ \). Whenever we adopt this convention of defining \( \theta \) uniquely, Eq.(6) would be applicable. On the other hand, if we do not fix \( \theta \) uniquely, as mentioned above, i.e, even if we allow the limited freedom of \( \theta \to \theta' = \theta + \theta_0 \), Eq.(6) cannot be invoked.

This form of the metric, known as the Standard Form, simplifies the Einstein equations considerably and is widely used for studies of rotating compact objects[9], stationary axially symmetric rotating wormholes[10], and above all, the rotating black holes[1].

It may be emphasized that the general form of stationary axisymmetric metric (in the Standard Form) widely used in the literature is[2,3,9,10]:

\[
ds^2 = e^\lambda dr^2 - e^\nu dt^2 + e^\mu r^2 [d\theta^2 + \sin^2 \theta (d\phi - \omega dt)^2]
\]  

(9)

where \( \lambda = \lambda(r, \theta), \nu = \nu(r, \theta), \mu = \mu(r, \theta) \). And \( \omega = \omega(r, \theta) \) is associated with the Relativistic Frame Dragging effect.

Note that Eq.(6) is already incorporated in the foregoing form, i.e, the physical conditions of asymptotic flatness and measurement of \( \theta \) w.r.t. the axis of symmetry are already imposed on the metric.
The Kerr metric, in the so-called Boyer and Lindquist[1] coordinate has also this
**Standard Form:**

\[
\begin{align*}
    ds^2 &= \left(r^2 + a^2 + \frac{2mr}{\rho^2}a^2\sin^2\theta\right)\sin^2\theta d\phi^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2d\theta^2 \\
    &\quad + \frac{4amr}{\rho^2}\sin^2\theta d\phi dt - \left(1 - \frac{2mr}{\rho^2}\right)dt^2
\end{align*}
\]  

(10)

Here \(m\) is the mass of the Kerr BH, \(a\) is the angular momentum per unit mass and

\[
\rho^2 = r^2 + a^2\cos^2\theta; \quad \Delta = r^2 - 2mr + a^2
\]  

(11)

It is easily seen that Eq.(9) is in the form of Eq.(7) where all cross terms except the one containing \(d\phi dt\) are 0 and where

\[
g_{\phi\phi} = [r^2 + a^2 + \frac{2mr}{\rho^2}a^2\sin^2\theta]\sin^2\theta
\]  

(12)

and

\[
g_{\theta\theta} = \rho^2
\]  

(13)

Metric.(10) too uses \(x^2 = r\) and \(x^3 = \theta\), is also asymptotically flat and measures \(\theta\) uniquely w.r.t. the axis of symmetry.

For instance suppose we subject Eq.(12) to the coordinate transformation (8). While the LHS \(g_{\phi\phi} \rightarrow g'_{\phi\phi} = g_{\phi\phi}\) would remain unchanged under this transformation, the RHS will not do so unless \(\theta' = f(\theta) = \theta\) is already fixed uniquely. And unless we measure \(\theta\) in this unique fashion, \(g_{\phi\phi}\) may not vanish along the axis of symmetry. It is also known that when one uses the Boyer-Lindquist coordinate, the azimuthal plane lies at \(\theta = 90^\circ\) which is also the case with the original Kerr coordinates[11] (but the original Kerr metric is not in the standard form and Eq.6) cannot be invoked there). Thus Eq.(6) should be applicable to the Boyer-Lindquist metric but not to the original Kerr metric[11] (see a cross-check to this effect at the end of the paper).

Then combining Eqs. (6), (12) and (13), we obtain

\[
[r^2 + a^2 + \frac{2mr}{\rho^2}a^2\sin^2\theta]\sin^2\theta = \rho^2\sin^2\theta
\]  

(14)

In the foregoing Eq., by first cancelling \(\sin^2\theta\) from both sides, then using Eq.(11) on the R.H.S., then transposing and finally using the identity \(\sin^2\theta + \cos^2\theta = 1\), it follows that

\[
a^2\sin^2\theta(1 + 2mr/\rho^2) = 0
\]  

(15)

The Eq.(15) tells that either

\[
a = 0; \quad m \geq 0
\]  

(16)

or,

\[
a \geq 0; \quad 2mr = -\rho^2; \quad m \leq 0, \quad if \ r \geq 0
\]  

(17)

These foregoing conditions show that there cannot be any rotating BH with \(m \geq 0\) and if it is assumed that the compact object is strictly non-rotating, i.e, \(a = 0\), only
then it is possible to have \( m \geq 0 \). Hence if the astrophysical BH candidates having \( m > 0 \) would at all be BHs, must be strict Schwarzschild ones. Further, probable occurrence of Kerr BHs with negative masses, as apparently allowed by Eq.(17) can actually be ruled out on the following grounds: Combining Eqs. (11) and (17), one would have

\[
\begin{align*}
  r^2 + 2mr + a^2 \cos^2 \theta &= 0 \\
  r &= -m \pm \sqrt{m^2 - a^2 \cos^2 \theta}
\end{align*}
\]  

This is the equation of a 2-D surface and thus if there would be a negative mass Kerr BH, the associated 4-D spacetime would collapse to a 3-D spacetime. Hence negative masses (i.e, Eq.[17]) can be rejected in the present context even without invoking any “Positive Energy Theorem”.

Therefore, we find that although the Kerr solution appears very appealing, in a strict sense, it is not so because it cannot describe the gravitational field of any spinning object of finite mass, not even the supposed BHs.

Newman and Janis[12] derived the Kerr metric by starting from the Schwarzschild form in a method which is “curious” as admitted by themselves. Recall that the radial variable appearing in the Schwarzschild metric \( R \) is very much a real quantity and is in fact a scalar too. Newman and Janis first effected a coordinate transformation of the form:

\[
R \rightarrow r \theta' = \theta_{\text{Schwarzschild}}
\]

where \( r \) is allowed to be a complex variable. After this, they introduced another transformation of the form

\[
r' = r + ia \cos \theta;
\]

where \( r' \) is seen to be the radial coordinate of the Kerr metric. In effect, Newman and Janis pretended as if \( r' \) were a real variable. But a careful consideration would convince that no physically allowed coordinate transformation can transform a purely real variable into a complex one and thus \( r \) must be a real variable under physically admissible transformations. Consequently \( r' \) can truly be a real variable iff \( a = 0 \) which is our Eq. (16). In fact Newman and Janis admitted that

“there is no simple, clear reason for the series of operations performed on the tetrad to yield a new (different from Schwarzschild) solution.”

On the other hand, Chandrasekhar[13] derived the Kerr metric by starting with a general axisymmetric stationary metric of the form (7) (but in cylindrical coordinates), and one may wonder why such a general approach too should eventually lead to apparent incongruities like Eqs. (16-17). While deriving the Kerr metric, Chandrasekhar not only used the justified condition that for an empty spacetime, energy momentum = 0, but he also assumed beforehand that the resultant vacuum spacetime must contain an Event Horizon (EH), mathematically, a “null surface” spanned by two Killing vectors corresponding to \( \phi \) and \( t \) symmetries. This is in contrast to the derivation of the vacuum Schwarzschild metric where one does not, beforehand, force the existence of any EH and where the EH arises on its own. This suggests that while a spherically symmetric vacuum spacetime does allow the existence of
an EH for \( m \geq 0 \) (as long as we ignore associated physical problems) a stationary axisymmetric vacuum spacetime does not do so even when we consider only mathematical symmetry arguments (as long as we insist for \( m \geq 0 \)). If there would be a Kerr BH with \( m > 0 \), as noted by Carter[14], there would be very severe violation of causality for the internal solutions because of the occurrence of non-removable closed timelike curves in regions of finite positive \( r \). Consequently, he noted that there would be a “breakdown of general relativity” and “the whole theory might have to be abandoned”. But we see here that there is no “breakdown of general relativity” and the apparent “breakdown” was either due to pretentious mathematical “trics” or because of undue assumptions. So what need to be “abandoned” are such aspects rather than “the whole theory”.

Our result is probably in conformity with Mach’s principle in that for a purely empty spacetime, rotation cannot be meaningfully defined. It is also in definite agreement with the recent detection of ultra-relativistic flow with bulk Lorentz factor \((\geq 10)\) from the Cir X-1, an object without an EH but with strong intrinsic magnetic field[14]. The latter aspect is again in conformity with interpretation of recent observations of black hole candidates[5-8]: the so-called Black Hole Candidates do possess intrinsic magnetic fields which the astrophysical BHs cannot and hence the BH candidates are not BHs. In practice, none of the astrophysical BH candidates or any other compact object is expected to be strictly non-rotating and even if we ignore the evidence for intrinsic magnetic moment in the BHCs, they can hardly be BHs in view of Eq.(16). On the other hand, they must be ultra compact objects with physical surface and without any EH.

To summerize, we find that though the Kerr metric has fascinated both relativists and astrophysicists for 40 years, as long as we are interested in exact spacetime of finite mass spinning BHs or compact objects, Kerr metric is only of academic interest. However, for compact objects spinning slowly, the far off gravitational field might (or might not) be approximated by Kerr metric[2, 16]. Note that despite attempts made for almost 40 years, nobody has been able to match the external/internal spacetime of any known physical body (other than a Kerr BH) possessing pressure, temperature and positive mass-energy density by the Kerr metric[2,16].

This is not to tell that there cannot be spinning objects with valid physical properties; on the other hand, it only to remind that we have yet not been able to derive the exact form of the metric associated with any spinning object (except Kerr BH). And it is a long overdue task for the relativists to derive the exact metric for a spinning physical object (other than Kerr BH). It is also not difficult to see why the Kerr metric is valid only for Kerr BHs:

A spinning BH has no mass moment of order higher than \( l = 1 \)[2,16]. On the other hand, even if a physical body with finite mass and extent is perfectly spherical to start with, it would develop deviation from spherical symmetry once it starts spinning. And a spinning physical fluid with asphericity is likely to develop moments higher than the \( l = 1 \)[2,16].m For instance, one can see the expression for quadrupole \((l = 2)\) deformity formula for an originally spherical spinning fluid in Eq(3.60) of ref[2].

Our result that even a spinning BH cannot have finite mass can be cross checked in the following manner. The Boyer-Lindquist metric(10) can be most directly derived by using the Backlund transformation[17]. When one does so, the
following relationship between \( a \) and \( m \) emerges automatically:

\[
a = m \sin \phi
\]  

(22)

Since \( a \) and \( m \) are constants, but \( \phi \) is a variable and \( \sin \phi \neq 0 \) (in general), the foregoing equation can be satisfied iff

\[
a = m = 0
\]  

(23)

Thus, unequivocally, \( a = 0 \) for Kerr BHs. And this result also confirms that the Boyer-Lindquist metric (10) incorporates the physical condition \( f(\theta) = \theta \) as much as Eqs.(6) and (9) do. If one would go through any article on BHs where Boyer-Lindquist coordinates are used one would find that \( \theta \) is uniquely measured from the axis of symmetry and the azimuthal plane is marked by \( \theta = 90^\circ \), the prerequisite for Eq.(6). And since all astronomical objects and physical objects (except few elementary particles) including the BHCs have \( m > 0 \), they are not Kerr BHs. Thus neither the Quasars, nor the micro-quasars, nor the Gamma Ray Bursts nor anything else is powered by spinning Kerr BHs contrary to the present astrophysical paradigm.

However, it is possible that all such astrophysical objects are powered by spinning BH Candidates which have physical surface and intrinsic magnetic fields but no EH. In other words, all such powerful astrophysical central engines might be powered, along with likely accretion power, by spinning objects which are somewhat akin to (relativistic) pulsars. Such spinning objects with physical surface and intrinsic magnetic fields, rather than BHs from which nothing can escape (atleast classically), are most suitable for understanding the origin of powerful collimated jets and radiation. Such BH candidates, unlike cold Neutron Stars, will be “hot”, i.e, trapped radiation pressure will play an important role in supporting them. The conventional mass upper limit of “cold” objects will be irrelevant for them.

Also, unlike strictly Neutron Stars, in strict hydrostatic equilibrium, these objects may be collapsing at an incredible slow rate and thus generate radiation pressure/heat at their core by virtue of virial theorem (negative specific heat). If any reader is desires to see Ref.[17] but may not have easy access to it may request the author for a photocopy of the same.

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