Two-field Models of Dark Energy with Equation of State Across -1

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In this paper, we study the possibility of building two-field models of dark energy with equation of state across -1. Specifically we will consider two classes of models: one consists of two scalar fields (Quintessence + Phantom) and another includes one scalar (Phantom) and one spinor field (Neutrino). Our studies indicate to some extent that two-field models give rise to a simple realization of the dynamical dark energy model with the equation of state across $w = -1$.

I. INTRODUCTION

Recent observational data\textsuperscript{1, 2, 3} strongly indicate that the Universe is spatially flat and accelerating at the present time. In the frame of Friedmann-Robertson-Walker (FRW) cosmology, the acceleration may be attributed to some mysterious source called dark energy. The simplest candidate for dark energy seems to be a small positive cosmological constant, but it suffers from difficulties associated with the fine tuning and coincidence problem. An alternative is a dynamical scalar field, such as Quintessence\textsuperscript{4} or Phantom\textsuperscript{5}. The Phantom field violates the energy conditions, which leads to many interesting cosmological phenomena\textsuperscript{6, 7, 8}.

Despite the current theoretical ambiguity for the nature of dark energy, the prosperous observational data (e.g. supernova, CMB and large scale structure data and so on ) have opened a robust window for testing the recent and even early behavior of dark energy using some parameterizations for its equation of state. The recent fits to current supernova(SN) Ia data, CMB and LSS\textsuperscript{7} find that even though the behavior of dark energy is to great extent in consistency with a cosmological constant, an evolving dark energy with the equation of state $w$ larger than -1 in the recent past but less than -1 today is weakly favored. If such a result holds on with the accumulation of observational data, this would be a great challenge to the current cosmology.

The evolving dark energy with an equation of state $w$ crossing -1 during its evolution was firstly advocated and named as Quintom in Ref.\textsuperscript{8}. The Quintom models of dark energy are different from the Quintessence or Phantom in the determination of the evolution and fate of the universe. Generically speaking, the Phantom model has to be more fine tuned in the early epochs to serve as dark energy today, since its energy density increases with expansion of the universe. Meanwhile the Quintom model can also preserve the tracking behavior of Quintessence, where less fine tuning is needed. The Quintom model with an oscillating equation of state considered in\textsuperscript{8} can lead to the oscillations of the Hubble constant and a recurring universe, which in some sense unifies the early (Phantom) inflation\textsuperscript{10} and current acceleration of the universe. This oscillating Quintom would not lead to a big crunch nor big rip. The scale factor keeps increasing from one period to another and leads naturally to a highly flat universe. Since in this model the universe recurs itself and we are only staying among one of the epochs, the coincidence problem in some sense is reconciled. Ref.\textsuperscript{11} has considered a variation of this oscillating Quintom model and studied its constraints from SN, CMB and LSS.

There have been some efforts made on the model building of the Quintom dark energy based on field theory. First of all, a single scalar field with the canonical kinetic term (Quintessence) or a negative kinetic term (Phantom) will not be able to give rise to the equation of state across -1. The similar conclusion has also been obtained for the k-essence models\textsuperscript{12}. Beyond the single scalar field theory, Refs.\textsuperscript{8} and\textsuperscript{13} have proposed an explicit model of Quintom with two scalar fields, one being the Quintessence and another being the Phantom field. This type of model can easily lead to a scenario where at early time the Quintessence dominates with $w > -1$ and lately the Phantom dominates with $w < -1$, satisfying current observations. Some recent relevant studies are given in\textsuperscript{14}.

The Quintom model considered in refs.\textsuperscript{8} and\textsuperscript{13} does not include the interaction between the two scalars and is not the most general one. In this paper, we revisit this type of two-field Quintom model by introducing a interaction term. We will study in detail the cosmological evolution of this model. In ref.\textsuperscript{16} it has been shown that in the two-field Quintom model the scaling solution dominated by the Phantom field will be a late-time attractor in the absence of the interactions. In this paper by a explicit calculation we will show the interactions do not affect the Phantom-domination attractor behavior. In addition our studies also show that this class of models will provide some interesting possibilities of the evolution of the equation of state which have not been considered in refs.\textsuperscript{8} and\textsuperscript{13}. Furthermore we will propose in this paper a two-field Quintom model with a scalar and a fermion which specifically we take to be the neutrinos and study its evolution of the equation of state. This paper is organized as follows: in section II we study in detail the model with one Quintessence and one Phantom; in section III we present our model of Quintom with a Phantom and neutrinos; section IV is our conclusion and discussions.
II. QUINATOM MODEL WITH PHANTOM FIELD AND QUINTESSENCE FIELD

In this section, we study a Quintom model with two scalar fields, in which one is the Phantom field and another is the Quintessence field. The Lagrangian for such a coupled system is given by

\[ \mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_\sigma + V_{\text{int}}, \]  

(1)

where

\[ \mathcal{L}_\phi = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi), \]  

(2)

with \( V(\phi) \) being the potential for the Phantom field and

\[ \mathcal{L}_\sigma = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - V(\sigma) \]  

(3)

with \( V(\sigma) \) the potential for the Quintessence field, and \( V_{\text{int}} \) denotes the interaction between the Phantom and the Quintessence fields. The equation of state of this system is

\[ w = -\frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\sigma}^2 - V(\phi) - V(\sigma) - V_{\text{int}}, \]  

(4)

The evolution equations of the fields and the fluid for a spatially flat FRW model are

\[ \ddot{\phi} + 3H \dot{\phi} - \frac{dV(\phi)}{d\phi} dV_{\text{int}} = 0, \]  

(5)

\[ \ddot{\sigma} + 3H \dot{\sigma} + \frac{dV(\sigma)}{d\sigma} dV_{\text{int}} = 0, \]  

(6)

\[ \dot{\rho}_\gamma + 3H (\rho_\gamma + P_\gamma) = 0, \]  

(7)

where \( \rho_\gamma \) is the density of fluid with a barotropic equation of state \( P_\gamma = (\gamma - 1)\rho_\gamma \) where \( \gamma \) is a constant in the range of \( 0 < \gamma \leq 2 \).

Now we study the cosmological evolution of the model with lagrangian (1). The authors of ref. 13 have considered a model where \( V(\phi) = V_\phi 0 e^{-\lambda_1 \phi}, V(\sigma) = V_\sigma e^{-\lambda_2 \sigma} \) \( \mu \) and show that the late time behavior of this model is a Phantom-domination attractor. Here we introduce an interaction term \( V_{\text{int}} = \lambda (V(\phi)V(\sigma))^{1/2} \) and study its effect on the cosmological evolution. We follow closely the conventional phase-plane analysis for the spatially flat FRW models in [13] and the detailed studies on models with multi coupled Quintessence fields in Ref. [10]. Defining firstly the variables

\[ x_\phi = \frac{\phi'}{\sqrt{6}}, \ y_\phi = \sqrt{\frac{V(\phi)}{3H}} - x_\sigma = \frac{\sigma'}{\sqrt{6}}, \ y_\sigma = \frac{\sqrt{V(\sigma)}}{\sqrt{3H}}, \ z = \frac{\sqrt{\rho}}{\sqrt{3H}}, \]  

(8)

the evolution Eqs. (5-7) can be written to an autonomous system

\begin{align*}
    x'_\phi &= -3x_\phi + \frac{\gamma z^2}{2} - \frac{\sqrt{6}}{2} \lambda_1 y_\phi^2 - \frac{\sqrt{6}}{4} \lambda_1 \lambda y_\phi y_\sigma, \\
    y'_\phi &= 3y_\phi - x_\phi + \frac{\gamma z^2}{2} - \frac{\sqrt{6}}{2} \lambda_1 x_\phi y_\phi, \\
    x'_\sigma &= -3x_\sigma + \frac{\gamma z^2}{2} + \frac{\sqrt{6}}{2} \lambda_2 y_\sigma^2 + \frac{\sqrt{6}}{4} \lambda_2 \lambda y_\phi y_\sigma, \\
    y'_\sigma &= 3y_\sigma - x_\sigma + \frac{\gamma z^2}{2} - \frac{\sqrt{6}}{2} \lambda_2 x_\sigma y_\sigma, \\
    z' &= 3z - x_\phi^2 + x_\sigma^2 - 3z^2, \quad \gamma z, \quad (9-13)
\end{align*}

where a prime denotes a derivative with respect to the logarithm of the scale factor. The critical points correspond to the fixed points where \( x_\phi = 0, x_\sigma = 0, y_\phi = 0, y_\sigma = 0, z' = 0 \), which have been calculated and given in Table I.

FIG. 1: Plot of the equation of state \( w \) as a function of the scale factor \( \ln a \).

Eqs. (9-13) can be reduced to four independent equations. To study the stability of the critical points, we substitute the linear perturbations about the critical points into these independent equations and keep terms to the first-order in the perturbations. The four perturbation equations give rise to four eigenvalues. The stability requires the real part of all eigenvalues be negative (see Table II for the eigenvalues of perturbation equations and the stability of critical points).

From Table I and II, one can see that even if there exists the interaction between the two fields, the Phantom-dominated solution is still a late-time stable attractor. If this coupled system is initially dominated by the Quintessence field, it will eventually evolve into the Phantom-domination phase. Thus an equation of state across \(-1\) will be inevitable. This provides a natural
realization of the Quintom scenario. As an illustration we in Fig. 1 plot the evolution of the equation of state \( w \) as a function of the scale factor where in the numerical calculation we have taken \( V_{\phi_1} = V_{\phi_2} = 0.35 \times 10^{-46} (GeV)^4 \), \( \lambda_1 = \lambda_2 = \lambda = 1 \).

Before concluding this section we point out some interesting behavior of the equation of the state of the two-field models. In Fig. 2 and Fig. 3, by choosing potentials and specific values of the model parameters we show that the equation of state could be oscillated. And the oscillations mainly occur in Fig. 2 in the late time when redshift \( z < 0 \), while Fig. 3 give rise to one example more interesting, in which the oscillations across \( -1 \) occur in the near past, which might lead to some effects testable observationally.

![FIG. 2: The state equation \( w \) as a function of scale factor \( \ln a \) for the potential \( V = \lambda_1 \cos (\xi \phi_1) + \lambda_2 \cos (\alpha \phi_2) + \lambda \phi_1^2 \phi_2^2 \) where \( \xi = 0.5, \alpha = 1, \lambda_1 = \lambda_2 = 2.47 \times 10^{-46} GeV^{-4} \), the four lines from the top to the bottom at \( \ln a = 0 \) correspond to \( \lambda = (0.2, 0.3, 0.4, 0.5) \times 10^{-120} \) respectively.]

![FIG. 3: The state equation \( w \) as a function of scale factor \( \ln a \) for the potential \( V = \frac{\lambda_1}{\lambda_2} m_1^2 \phi_1^2 + \frac{\lambda_2}{\lambda_1} m_2^2 \phi_2^2 + \lambda \phi_1^2 \phi_2^2 \), where \( \frac{\lambda_1}{\lambda_2} m_1^2 = 5.9 \times 10^{-84} GeV^2 \), \( \frac{\lambda_2}{\lambda_1} m_2^2 = 1.19 \times 10^{-82} GeV^2 \), and \( \lambda = 50 \times 10^{-120} \).]

### III. QUINTOM MODEL WITH PHANTOM FIELD AND NEUTRINO

The two-field models of Quintom dark energy considered above consist of two scalar fields. In the following we study a model where the Quintessence in (11) is replaced by the neutrinos. In this model the neutrino will become a part of the dark energy. There have been a lot of studies recently in the literature on the coupled system of neutrinos interacting with the dark energy scalars [17, 18, 19, 20, 21, 22, 23, 24, 25], however in all of these studies the scalar fields have canonical kinetic terms. In the following we will show that a system with a Phantom and neutrinos can naturally give rise to a realization of Quintom models. Furthermore since the neutrinos are the particles existing in the standard model of the elementary particle physics this model introduces less
degree of freedom in comparison to the Quintom model considered above with two scalar fields.

The Lagrangian for such a coupled system is given by

\[ \mathcal{L} = \mathcal{L}_\nu + \mathcal{L}_\phi + M(\phi)\bar{\nu}\nu, \]

(14)

where

\[ \mathcal{L}_\phi = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi), \]

(15)

with \( V(\phi) \) the potential for Phantom field and

\[ \mathcal{L}_\nu = \bar{\nu} \iota \nu, \]

(16)

which is the kinetic term of the neutrino. In \( M(\phi)\bar{\nu}\nu \) characterizes the interaction between the Phantom and the neutrinos which for instance in Ref.\(^2\) is given by

\[ \mathcal{L}_{\text{int}} = e^{-\beta \frac{\phi^2}{M_{\text{pl}}}^2} \mathcal{L}_{\text{L}} HH + h.c., \]

(17)

where \( \beta \) is the coefficient characterizing the strength of the Phantom interaction with the neutrinos, \( f \) is the scale of new physics beyond the Standard Model which generates the \( B - L \) violation, \( \mathcal{L}_{\text{L}} \), \( H \) are the left-handed lepton and Higgs doublets respectively. When the Higgs field gets a vacuum expectation value \( < H > \) \( \sim \nu \), the left-handed neutrino receives a Phantom field dependent Majorana mass \( m_\nu \sim e^{-\beta \frac{\phi^2}{M_{\text{pl}}}^2} \nu^2 \).

In general, with different \( V(\phi) \) and \( M(\phi) \) in (14) and (15) this two-field Quintom model of dark energy behaves differently. Firstly we will analyze the general feature of the cosmological evolution for this class of models without specifying the explicit form of \( V(\phi) \) and \( M(\phi) \). Due to the existence of the interaction term \( M(\phi) \), the evolution of the Phantom field is determined by the effective potential which is the combination of the potential \( V(\phi) \) and \( E(\phi) \)

\[ \bar{V}(\phi) = n M(\phi)(\frac{M(\phi)}{E}), \]

(18)

with \( n \) and \( E \) being the number density and energy of the neutrinos respectively and \( \bar{\phi} \) indicating the thermal average. For relativistic neutrinos, the term \( \bar{V}(\phi) \) is greatly suppressed and the neutrinos and dark energy decouple. For non-relativistic neutrinos, the effective potential of the system is given by \( V_{\text{eff}}(\phi) = V(\phi) + n M(\phi) \). Consequently for the equation of motion of the scalar field \( \phi \) it is

\[ \ddot{\phi} + 3H\dot{\phi} - \frac{dV}{d\phi} - \frac{d\bar{V}}{d\phi} = 0. \]

(19)

The equation of state for such a coupled system is

\[ w = \frac{-\frac{1}{2} \dot{\phi}^2 - V(\phi)}{-\frac{1}{2} \dot{\phi}^2 + V(\phi) + n M(\phi)}, \]

(20)

which can be rewritten as

\[ w = \frac{\Omega_\phi}{\Omega_\phi + \Omega_\nu} w_\phi + \frac{\Omega_\nu}{\Omega_\phi + \Omega_\nu} w_\nu. \]

(21)

From the equation above one can see that during the radiation dominant period since \( \Omega_\nu \) is much larger than \( \Omega_\phi \), \( w \) of the coupled system will be around \( 1/3 \) and in the matter dominant regime \( w \) will be close to 0. However in the late time when the Phantom energy dominant over neutrinos \( \Omega_\phi \gg \Omega_\nu \), \( w \) gradually evolves into the value smaller than -1. To illustrate this behavior in Fig. 4 we plot the evolution of the equation of state \( w \) of a model where \( V = V_0 e^{-\lambda \frac{\phi^2}{M_{\text{pl}}}} \) and \( M = M e^{-\gamma \frac{\phi^2}{M_{\text{pl}}}} \). In the numerical calculation we have taken that \( \lambda = 5.5 \) and \( \gamma = 2.5 \). One can see from this figure that \( w \) changes from above -1 to below -1 as the redshift decreases.

\[ \text{FIG. 4: Plot of the } w \text{ of the system as a function of the scale factor } \ln a \text{ for } V = V_0 e^{-5.5 \frac{\phi^2}{M_{\text{pl}}}}, \text{ and } M = M e^{-2.5 \frac{\phi^2}{M_{\text{pl}}}}. \]

IV. CONCLUSION

In this paper we have studied theoretically the possibility of building the two-field dark energy model with an equation of state across -1. In general within the framework of the general relativity and the field theory, it is difficult to realize the Quintom with a single scalar field model. Thus two fields are required. In models with two scalar fields there will be a lot of possibilities for the model building by choosing the potentials, interactions and the model parameters. As shown in Figs. 2 and 3

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\(^1\) The studies on the interacting Phantom dark energy with dark matter are given in \(^2\)
the physics associated with this type of models and their implications in cosmology are quite rich. In the model with Phantom and neutrinos, as we emphasized in the paper since the neutrinos are the particles existing in the standard model of the elementary particle physics, compared with the model with two scalar fields this model introduces less degree of freedom. Furthermore the neutrino masses vary during the evolution of the Phantom field, which makes this model more interesting. Before concluding we should also point out that there exist possibilities of building models of dark energy with equation of state across -1 beyond the field theory with minimal couplings to the gravity and the four dimension[27].

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