Wind Speed using Improved Mixture Weibull Distribution

D. Indhumathy, C.V. Sesdaiah, B.Vennila

Abstract: Wind is random in nature both in space and in time. Several technologies are used in wind resource assessment (WRA). The appropriate probability distribution used to calculate the available wind speed at that particular location and the estimation of parameters is the essential part in installing wind farms. The improved mixture Weibull distribution $\text{M\tilde{W}bl}(2,3)$ is proposed model which is the mixture of two and three parameter Weibull distribution with parameters including scale, shape, location and weight component. The basic properties of the proposed model and estimation of parameters using various methods are discussed.

Index Terms: Improved Mixture Weibull Distribution, Maximum likelihood method, Parameter Estimation, Wind speed.

I. INTRODUCTION

In this paper we discuss about the application of advanced Weibull distribution models in wind energy. Enormous number of authors accept Weibull distribution as the best fit in wind resource assessment[1]. A few mixture distributions are available in literature but we have taken only the mixture of Weibull distribution because of flexibility[2].

Several authors have used Two and three parameter Weibull distribution for wind sites having single peak distribution. But this does not suit all the locations with varying wind speed data. In this case we have to implement the mixture Bimodal Weibull distribution [3]. A few areas experience calm wind speed for several days of the whole year which is not included in the estimation of average wind speed often which leads to some drawback in the computation part. So we have considered calm wind speed also in improved mixture Weibull distribution $\text{M\tilde{W}bl}(2,3)$ [4]. In section two of this paper the improved mixture Weibull distribution is defined. In the next section we have the description of the sites. In the fourth chapter the distribution of wind speed is given. In fifth chapter we have the computation of the parameter using MLE method continued with wind power equation. The objective of the study is to develop a new mathematical formulation for computing wind speed.

II. WEIBULL DISTRIBUTION

A. Two Parameter Weibull Distribution

The probability density function of the two parameter Weibull distribution is given by

$$f(v; \alpha, \beta) = \frac{\alpha}{\beta} \left(\frac{v}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{v}{\beta}\right)^\alpha\right] \quad \text{for } v > 0$$  (1)

$\alpha$ and $\beta$ are the shape and scale parameters respectively. The unit of wind speed is $(m/s)$ [5].

The cumulative distribution function (cdf) [6] is defined by

$$F(v) = 1 - \exp\left[-\left(\frac{v - \gamma}{\beta}\right)^\alpha\right]$$  (2)

B. Three Parameter Weibull Distribution

The PDF of the three parameter Weibull distribution is given by

$$f(v; \alpha, \beta, \gamma) = \frac{\alpha}{\beta} \left(\frac{v - \gamma}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{v - \gamma}{\beta}\right)^\alpha\right] \quad \text{for } v > \gamma$$  (3)

where $\gamma$ is the location parameter.

The associated cumulative distribution function is

$$F(v) = 1 - \exp\left[-\left(\frac{v - \gamma}{\beta}\right)^\alpha\right]$$  (4)

C. Improved Mixture Weibull Distribution

The proposed Improved mixture Weibull distribution $\text{M\tilde{W}bl}(2,3)$ is a mixture of two and three parameter Weibull distributions with a weight component [7]. A random variable $V$ that is denoted as $\alpha$ with mixing parameter $\omega$ is said to have an improved mixture Weibull distribution $\text{M\tilde{W}bl}(2,3)$ with probability density function

$$f_{(2,3)}(v; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega, \gamma_0) = \omega f(v; \alpha_1, \beta_1) + (1-\omega) f(v; \alpha_2, \beta_2, \gamma_0)$$

$$= \omega \left[ \frac{\alpha_1}{\beta_1} \left(\frac{v}{\beta_1}\right)^{\alpha_1-1} \exp\left[-\left(\frac{v}{\beta_1}\right)^\alpha_1\right] \right]$$

$$+ (1-\omega) \left[ \frac{\alpha_2}{\beta_2} \left(\frac{v - \gamma_0}{\beta_2}\right)^{\alpha_2-1} \exp\left[-\left(\frac{v - \gamma_0}{\beta_2}\right)^\alpha_2\right] \right]$$  (5)
Cumulative Distribution function:
The cumulative distribution function of $\text{MWbI}(2, 3)$ is given by
\[
FF_{(2,3)}(v; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega, \gamma_0) = P(V \leq v) = \omega F(v; \alpha_1, \beta_1)(1 - \omega)F(v; \alpha_2, \beta_2, \gamma_0)
\]
\[
= \omega \left[ 1 - \exp \left( -\left( \frac{v}{\beta_1} \right)^{\alpha_1} \right) \right] + (1 - \omega) \left[ 1 - \exp \left( -\left( \frac{v - \gamma_0}{\beta_2} \right)^{\alpha_2} \right) \right]
\]

III. WIND RESOURCE ASSESSMENT
Wind resource assessment at two sites are specifically considered for the study because of the variation in distribution of the wind.

A Site Specification
The site is located in Tamil Nadu with Latitude $78^\circ 7' 46''$ N and Longitude $10^\circ 79' 04''$ E.

B Wind Source Data
The wind speed data are observed using an anemometer at 10m height above the sea level. The collected data are saved in the data logger for further process.

C Site Specification - Sulur
The latitude $11^\circ 06' 65''$ N and longitude $77^\circ 13' 12''$ E are the geo coordinates of Sulur. The measured wind speed at a height of 10 m is observed and the average wind speed for a particular day or month is calculated. The sample wind speed or an average wind speed calculated month wise are given in the graph.

D Wind Source Data
A twelve month time series data (considering hourly averages over a 24 hr period is given) in TABLE -1.

| Hr | Dir/Speed (km/hr) | Hr | Dir/Speed (km/hr) |
|----|------------------|----|------------------|
| 1  | EAST 11.1        | 13 | ESE 11.1         |
| 2  | ENE 20.4         | 14 | ENE 14.8         |
| 3  | ENE 11.1         | 15 | ESE 11.1         |
| 4  | ENE 7.4          | 16 | ENE 18.5         |
| 5  | NE C             | 17 | ENE 9.3          |
| 6  | NE C             | 18 | ENE 11.1         |
| 7  | NE 14.8          | 19 | ENE 9.3          |
| 8  | NE 11.1          | 20 | EAST C           |
| 9  | NE 9.3           | 21 | EAST 9.3         |
| 10 | NNE 9.3          | 22 | EAST 7.4         |
| 11 | NNE 11.1         | 23 | ESE 11.1         |
| 12 | ENE 9.3          | 24 | ESE 7.4          |

IV. WIND SPEED FREQUENCY
The observed wind speed plotted in a graph shows single peak frequency distribution [8].
The Weibull using

\[ P = \frac{1}{2} \rho A |v|^3 \]  

where \( v \) is the wind speed in \((m/s)\) and \( A \) is the rotor area of the wind turbine blades.

Moreover the power produced by the turbine depends on the power coefficient \( C_p \), [11] and \( \eta \in (0,1) \) as efficiency constant [12].

The power output of the turbine can be written as:

\[ P = \frac{1}{2} C_p \eta A |v|^3 \]  

A. Proposition

The cumulative distribution function of power is defined by

\[ FF_{p,2,3}(v) = \omega \left( 1 - \exp \left[ - \frac{1}{\beta_2^{\alpha_2}} \left( \frac{2v}{\rho A} \right)^{\alpha_2} \right] \right) \]

with sufficient conditions:

\[ \frac{\sum_{i=1}^{n} v_{i}^{\alpha_1} \log \left( v_{i} \right)}{n} = \frac{1}{\alpha_1} \sum_{i=1}^{n} \log \left( v_{i} \right) \]  

\[ \frac{\sum_{i=1}^{n} (v_{i} - \gamma_0)^{\alpha_2} \log \left( v_{i} - \gamma_0 \right)}{n} = \frac{1}{\alpha_2} \sum_{i=1}^{n} \log \left( v_{i} - \gamma_0 \right) \]  

\[ \sum_{i=1}^{n} v_{i}^{\alpha_1} \log \left( v_{i} \right) = \sum_{i=1}^{n} v_{i}^{\alpha_1} \log \left( v_{i} \right) - \frac{1}{\alpha_1} \frac{n}{n} \]

\[ \sum_{i=1}^{n} (v_{i} - \gamma_0)^{\alpha_2} \log \left( v_{i} - \gamma_0 \right) = \frac{1}{\alpha_2} \frac{n}{n} \]

Proof: 

\[ FF_{p,2,3}(v) = P(P \leq v) = P \left( V \leq \frac{2v}{\rho A} \right) \]

\[ = \omega \left( 1 - \exp \left[ - \frac{1}{\beta_2^{\alpha_2}} \left( \frac{2v}{\rho A} \right)^{\alpha_2} \right] \right) + (1-\omega) \left( 1 - \exp \left[ - \frac{1}{\beta_2^{\alpha_2}} \left( \frac{2v}{\rho A} \right)^{\alpha_2} \right] \right) \]  

Differentiating both sides with respect to \( v \), we get the pdf

\[ f_{p,2,3}(v) = \omega \left[ \frac{1}{\beta_2^{\alpha_2}} \left( \frac{2v}{\rho A} \right)^{\alpha_2 - 1} \right] \frac{1}{\beta_2^{\alpha_2}} \frac{2}{\rho A} \]

\[ + (1-\omega) \left[ \frac{1}{\beta_2^{\alpha_2}} \left( \frac{2v}{\rho A} \right)^{\alpha_2 - 1} \right] \frac{1}{\beta_2^{\alpha_2}} \frac{2}{\rho A} \]
Thus the PDF is obtained.

**B. Power Distribution**

The wind speed that flows through the turbine producing maximum power between the cut-in and rated wind speed, having maximum efficiency between the rated and cut-off wind speed [13] is given by

\[ V_{\text{turbine}} = \begin{cases} 0 & V < v_{\text{cut-in}} \\ v_{\text{cut-in}} & V \leq v_{\text{rated}} \\ v_{\text{rated}} & V < v_{\text{cut-off}} \\ 0 & V \geq v_{\text{cut-off}} \end{cases} \]  

(17)

Where \( v_{\text{cut-in}} \) is the wind speed at which the turbine starts operating and \( v_{\text{cut-off}} \) is the highest wind speed at which the turbine shuts down. And these units are fixed by the manufacturer [14].

And

\[ P_{\text{turbine}} = \begin{cases} 0 & V < v_{\text{cut-in}} \\ \frac{1}{2} \rho C_D \eta A V^2 & v_{\text{cut-in}} \leq V \leq v_{\text{rated}} \\ \frac{1}{2} \rho C_D \eta A v_{\text{rated}}^2 & v_{\text{rated}} < V < v_{\text{cut-off}} \\ 0 & V \geq v_{\text{cut-off}} \end{cases} \]  

(18)

**VII. RESULTS**

The PDF drawn from the estimated parameters clearly shows that only improved Weibull parameters computed using Maximum likelihood method gives a best fit. \( \gamma_0 \) is the minimum wind speed experienced at that location.

**VIII. CONCLUSION**

An improved mixture Weibull distribution function plays a vital role in the assessment of wind resource and for the installation of wind turbines. A new mathematical formulation has been derived to estimate the wind experienced by the turbine. The improved mixture Weibull distribution is different from the other mixture Weibull distribution because of the location parameter. This method can be used in specific location where the site experiences wind speed which represents a bimodal frequency peak.

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