New Electroweak Tests for the Topflavour Model $^a$

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We explore phenomenologies of the topflavour model for the LEP experiment at $m_Z$ scale. Implications of the model on the $Z$ peak data are studied in terms of the precision variables $\epsilon_i$'s.

1 Introduction

In this work we consider the model with additional SU(2) acting only on the third generation, which has been suggested by several authors$^1$, $^2$ as a possible solution of the $R_b$ problem. The third generation undergoes different flavour dynamics from the first and second generations and we expect that this type of model would help us to explain the discrepancy of $R_b$ of which measurement is about 1.8 (LEP) and 1.5 (LEP+SLC) standard deviations higher than the SM prediction. As an analogy to the topcolor model$^3$, this model is called topflavour model$^2$.

In order to parametrize new physics effects on the observables from the LEP experiments, we calculate the precision variables $\epsilon_i$'s introduced by Altarelli et al.$^4$, $^5$ with the new LEP data reported by the LEP Electroweak Working Group$^6$ in the framework of the topflavour model. Because there is no direct evidence for new physics beyond the Standard Model (SM) at LEP until now, the new physics contribution, if it exists, are thought to be comparable with the radiative correction effects of the SM at most. Hence it will be interesting to study the new physics effects in terms of the precision variables. Among four epsilon variables, $\epsilon_b$ has been of particular interest because it encodes the corrections to the $Z \rightarrow b\bar{b}$ vertex and we focus on $\epsilon_b$.

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2 The Epsilon Variables and the Standard Model Predictions

The precision variables $\epsilon_1$, $\epsilon_2$, $\epsilon_3$ and $\epsilon_b$ are defined from the basic observables, the mass ratio of $W$ and $Z$ bosons $m_W/m_Z$, the leptonic width $\Gamma_l$, the forward–backward asymmetry for charged leptons $A_{FB}$ and the $b$–quark width $\Gamma_b$, which are all defined at the $Z$–peak. In terms of the epsilon variables, we have the virtue that the most interesting physical results are already obtained at a completely model independent manner without assumptions like the dominance of vacuum polarisation diagrams. The $m_t$–dependence for all observables via loops come out through the $\epsilon_i$’s.

The $\epsilon_{1,2,3}$ variables are defined by the linear combinations of the correction terms $\Delta \rho$, $\Delta k$ and $\Delta r_W$ in the eqs. (9) of ref. [5], which are extracted from the vector and axial–vector couplings for charged leptons, $g_{lv}$, $g_{la}$ and the mass ratio of $W$ and $Z$ bosons, $m_W/m_Z$. The vector and axial–vector couplings are obtained from the observables $\Gamma_l$ and $A_{FB}$. The formular for $\epsilon_b$ is rather complicated. It is defined by the equations:

$$
\frac{g_{bA}}{g_{bV}} = \frac{1 - \frac{4}{3}(1 + \Delta k)\epsilon_0^2 + \epsilon_b}{1 + \epsilon_b}.
$$

(1)

We obtain the relation between $\epsilon_b$ and $\Gamma_b$ by insertion of $g_{bV}$ and $g_{bA}$ into the formular of $\Gamma_b$. The SM predictions for $\epsilon_i$’s are given by

$$
\epsilon_1 = 5.8 \times 10^{-3}, \quad \epsilon_2 = -7.4 \times 10^{-3}, \quad \epsilon_3 = 5.0 \times 10^{-3}, \quad \epsilon_b = -5.3 \times 10^{-3},
$$

(2)

with the Higgs boson mass $m_H = 100$ GeV.

The epsilon variables are obtained using the recent LEP data from ref. [6]:

$$
\epsilon_1 = (2.9 \sim 5.3) \times 10^{-3}, \quad \epsilon_2 = (-6.6 \sim -3.6) \times 10^{-3},
$$

$$
\epsilon_3 = (1.0 \sim 4.5) \times 10^{-3}, \quad \epsilon_b = (-4.5 \sim -0.8) \times 10^{-3}.
$$

(3)

Note that the lepton universality is assumed for the values of $\Gamma_l$ and $A_{FB}$.

3 The Topflavour Model

We study the topflavour model with the extended electroweak gauge group $SU(2)_l \times SU(2)_h \times U(1)_Y$ where the first and second generations couple to $SU(2)_l$ and the third generation couples to $SU(2)_h$. The left–handed quarks and leptons in the first and second generations transform as $(2,1,1/3)$, $(2,1,-1)$ under $SU(2)_l \times SU(2)_h \times U(1)_Y$, and those in the third generation as $(1,2,1/3)$, $(1,2,-1)$ while right–handed quarks and leptons transform as $(1,1,2Q)$ where $Q = T_3u + T_3d + Y$ is the electric charge of a fermion.
The covariant derivative is given by $D^\mu = \partial^\mu + ig_l T^a_l W_{la}^\mu + ig_h T^a_h W_{ha}^\mu + ig' B^\mu$, where $T^a_l$ and $T^a_h$ denote the $SU(2)_{l,h}$ generators and $Y$ is the $U(1)$ hypercharge generator. Corresponding gauge bosons are $W^\mu_{la}, W^\mu_{ha}$ and $B^\mu$ with the coupling constants $g_l, g_h$ and $g'$ respectively. The gauge couplings may be written as $g_l = \frac{e}{\sin \theta \cos \phi}, \quad g_h = \frac{e}{\sin \theta \sin \phi}, \quad g' = \frac{e}{\cos \theta}$ in terms of the weak mixing angle $\theta$ and the new mixing angle $\phi$ between $SU(2)_l$ and $SU(2)_h$.

The symmetry breaking is accomplished by the vacuum expectation values (VEV) of two scalar fields $\Sigma$ and $\Phi$: $\langle \Phi \rangle = (0, v/\sqrt{2})^*$, $\langle \Sigma \rangle = u I$ where $I$ is $2 \times 2$ identity matrix. The scalar field $\Sigma$ transforms as $(2,2,0)$ under $SU(2)_l \times SU(2)_h \times U(1)$ and we choose that $\Phi$ transforms as $(2,1,1)$ corresponding to the SM Higgs field. In the first stage, the scalar field $\Sigma$ gets the vacuum expectation value and breaks $SU(2)_l \times SU(2)_h \times U(1)$ down to $SU(2)_{l+h} \times U(1)_Y$ at the scale $\sim u$. The remaining symmetry is broken down to $U(1)_{em}$ by the the VEV of $\Phi$ at the electroweak scale. Since the third generation fermions do not couple to Higgs fields with this particle contents, they should get masses via higher dimensional operators. We do not study the mass generation problem of this model in details here. We demand that both $SU(2)$ interactions are perturbative so that the value of the mixing angle $\sin \phi$ is constrained $g^2(l,h)/4\pi < 1$, which results in $0.03 < \sin^2 \phi < 0.96$. We also assume that the first symmetry breaking scale is much higher than the electroweak scale, $v^2/u^2 \equiv \lambda \ll 1$.

The basic observables in the topflavour model depend upon the model parameters $\lambda$ and $\sin^2 \phi$ as well as the Higgs mass $m_\mu$. With the correction terms we obtain the epsilon variables in the topflavour model:

$$
\begin{align*}
\epsilon_1 &= \epsilon_1^{SM} - 2\lambda \sin^4 \phi \\
\epsilon_2 &= \epsilon_2^{SM} + \lambda \sin^4 \phi \left( \frac{s_0^2}{c_0^2 - s_0} - 2 \right) \\
\epsilon_3 &= \epsilon_3^{SM} - \lambda \sin^4 \phi \\
\epsilon_b &= \epsilon_b^{SM} - \frac{1}{2} \lambda \frac{\sin^2 \phi}{g_{lA}^{SM}} (1 + \epsilon_b^{SM} \sin^2 \phi) 
\end{align*}
$$

We used the ZFITTER for numerical calculations of the epsilon variables in this model and we use 175 GeV as input value of $m_t$, which is reported by the CDF and D0.

In Figs. 1 and 2, the experimental ellipses are shown in the $\epsilon_1 - \epsilon_b$ and $\epsilon_3 - \epsilon_b$ planes respectively with the results of the SM and the topflavour model. We express the results with variations of $\sin^2 \phi$ and $m_\mu$, where

$$
m^2_{Z'} = \frac{m^2_{Z'}}{\lambda \sin^2 \phi \cos^2 \phi} \left[ \cos^2 \theta_{SM} + \lambda \sin^4 \phi \left( \frac{c_0^2 s_0^2}{c_0^2 - s_0^2} + 2 s_0^2 \right) \right],
$$

We used the ZFITTER for numerical calculations of the epsilon variables in this model and we use 175 GeV as input value of $m_t$, which is reported by the CDF and D0.
with the Weinberg angle of the SM, $\theta_{SM}$. The ellipses are shifted along the minus direction of $\epsilon_b$ compared with those from 1996 data\cite{11} and thus come closer to the SM prediction. We find that the SM predictions still lie outside the 90% C.L. ellipses in both planes. These deviations are caused by the $Z\bar{b}b$ coupling and related to the $R_b$ discrepancy. The lower limit of the mass of heavy gauge boson $Z'$ is about 1.2 TeV (1.1 TeV) at 90 (95) % C.L. which agrees with the result of ref. [1].

4 Concluding Remarks

In this work, we explore the phenomenologies of the topflavour model, which are extension of the SM with the additional $SU(2)$ symmetry. The third gen-

![Figure 1: Plots of the model predictions in units of $10^{-3}$ with varying the model parameter $\sin^2\phi$, $m_{Z'}$, and the Higgs boson mass $m_H$ in $\epsilon_1-\epsilon_b$ plane. The experimental ellipses at 1-$\sigma$, 90% C.L. and 95% C.L. are given. Points are labelled from $\sin^2\phi = 0.1 \sim 0.9$ from right to left for each value of $m_{Z'}$.](image)

![Figure 2: Plots of the model predictions in units of $10^{-3}$ with varying the model parameter $\sin^2\phi$, $m_{Z'}$, and the Higgs boson mass $m_H$ in $\epsilon_3-\epsilon_b$ plane.](image)
eration is special in this model and it is also expected to explain the hierarchy of the fermion mass spectrum as well as the $R_b$ discrepancy observed in LEP experiment. The experimental values of $\epsilon$ variables are obtained from the recent LEP results reported by the LEP Electroweak Working Group. At present the SM predictions is still out of 90% C. L. ellipses mainly caused by the deviations of $\epsilon_b$ originated by the $Zb\bar{b}$ vertex. Since the topflavour model provides the different flavour dynamics on the third generations, the value of $\epsilon_b$ can shift to the experimental value.

In conclusion, we investigated the phenomenological implications of the topflavour model with the LEP data at $Z$–peak. We found that the best fitted mass of $Z'$ boson to the LEP data is about 2 TeV and with this value, the signature of $Z'$ boson is expected to be found via the excess in the $t\bar{t}$ pair production at the LHC or at the NLC, as is discussed in ref. [1].

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