Gauge–invariant nonlocal quark condensates in QCD: a new interpretation of the lattice results

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We study the asymptotic short–distance behaviour as well as the asymptotic large–distance behaviour of the gauge–invariant quark–antiquark nonlocal condensates in QCD. A comparison of some analytical results with the available lattice data is performed.

1. INTRODUCTION

In a recent paper \cite{1} we have presented a lattice determination of the quark–antiquark nonlocal condensates, defined as:

\[ C_i(x) = -\sum_{f=1}^{4} \langle \text{Tr}[\bar{q}_a^f(0) (\Gamma^i)_{ab} S(0,x) q_b^f(x)] \rangle, \] (1)

where \( S(0,x) \) is the Schwinger line needed to make \( C_i(x) \) gauge–invariant and \( \Gamma^i \) are the sixteen independent matrices of the Clifford’s algebra acting on the Dirac indices \( a, b \). The trace in (1) is taken with respect to the colour indices.

Making use of T,P invariance one can prove that all the correlators (1) vanish, except those with \( \Gamma^i = 1 \) (“scalar” nonlocal condensate) and with \( \Gamma^i = \gamma^\mu_E \) and \( \mu \) in the direction of \( x \) (“longitudinal–vector” nonlocal condensate):

\[ C_0(|x|) = -\sum_{f=1}^{4} \langle \text{Tr}[\bar{q}_a^f(0) S(0,x) q_b^f(x)] \rangle ; \]
\[ C_v(|x|) = -\frac{x^\mu}{|x|} \sum_{f=1}^{4} \langle \text{Tr}[\bar{q}_a^f(0) (\gamma^\mu_E)_{ab} S q_b^f(x)] \rangle. \] (2)

These two quantities play a relevant role in many applications of QCD sum rules, especially for studying the meson form factors and the meson wave functions \cite{2,3}. The nonlocal quark condensates have been also determined within the single instanton approximation of the instanton liquid model \cite{4}.

The lattice computations of Ref. \cite{1} have been performed both in the \textit{quenched} approximation and in full QCD using four degenerate flavours of \textit{staggered} fermions [whence the sum over the flavour index \( f \) in (1)] and the \( SU(3) \) Wilson action for the pure–gauge sector.

In full QCD the nonlocal condensates have been measured on a \( 16^3 \times 24 \) lattice at \( \beta = 5.35 \) and two different values of the quark mass: \( a \cdot m_q = 0.01 \) and \( a \cdot m_q = 0.02 \) (\( a \) being the lattice spacing). For the \textit{quenched} case the measurements have been performed on a \( 16^4 \) lattice at \( \beta = 6.00 \), using valence quark masses \( a \cdot m_q = 0.01, 0.05, 0.10 \), and at \( \beta = 5.91 \) with a quark mass \( a \cdot m_q = 0.02 \). Further details, as well as a remark about the reliability of the results obtained for the longitudinal–vector nonlocal condensate, can be found in \cite{1}.

In what follows we shall concentrate on the scalar nonlocal condensate \( C_0(x) \). In Ref. \cite{1} a best fit to the data with the following function has been tried:

\[ C_0(x) = A_0 \exp(-\mu_0 x) + \frac{B_0}{x^2}. \] (3)

The form of the perturbative–like term \( B_0/x^2 \) is that obtained in the leading order in perturbation theory, in the chiral limit \( m_q \to 0 \).

The quantity of greatest physical interest which can be extracted from our lattice determinations is the correlation length \( \lambda_0 \equiv 1/\mu_0 \) of the scalar quark–antiquark nonlocal condensate. At the lightest quark mass \( a \cdot m_q = 0.01 \) we have obtained the value \( \lambda_0 \simeq 0.63 \text{ fm} \) \cite{1}.

Here we study the asymptotic short–distance behaviour as well as the asymptotic large–
distance behaviour of the gauge–invariant quark–antiquark nonlocal condensates in QCD. The large–distance behaviour is derived by making use of a relation of these correlators to the two–point functions for the scalar and pseudoscalar \( \bar{q}q \) meson operators in the limit of the heavy–quark mass \( M_Q \to \infty \). A comparison of some analytical results with the available lattice data will be performed.

2. SHORT–DISTANCE BEHAVIOUR

The short–distance behaviour of the correlators is described by an “operator product expansion” (O.P.E.), having the form:

\[
C_0(x) = \frac{\bar{A}_0}{x^2} - \langle : \bar{q} q : \rangle + \ldots
\]

(4)

The vacuum expectation values of the local operators, such as \( \langle : \bar{q} q : \rangle \), appear as expansion coefficients of the nonlocal condensate \( C_0(x) \) in a Taylor series in the variable \( x^2 \). In the literature \[7\] the nonperturbative part of the scalar nonlocal condensate is often parametrized with a Gaussian function:

\[
C_0^{(n.p.)}(x) = \bar{A}_0 \exp \left( -\frac{1}{8} \mu_q^2 x^2 \right).
\]

(5)

The parameter \( \mu_q^2 \) characterizes the nonlocality of the quark condensate and it is given by \[8\]:

\[
\mu_q^2 = \frac{\langle : \bar{q} D^2 q : \rangle}{\langle : \bar{q} q : \rangle},
\]

(6)

where \( D_\mu = \partial_\mu + igA_\mu^a T^a \) is the covariant derivative.

From the QCD sum–rules phenomenology one finds the following estimate for \( \mu_q^2 \) \[8\]:

\[
\mu_q^2 = 0.50(5) \text{ GeV}^2.
\]

(8)

Therefore we have also tried a best fit to the data of the scalar nonlocal condensate with the function

\[
C_0(x) = \bar{A}_0 \exp \left( -\frac{1}{8} \mu_q^2 x^2 \right) + \frac{\bar{B}_0}{x^2}.
\]

(9)

It comes out that the data are well fitted by this function [even if the \( \chi^2/N_{d.o.f.} \) is slightly larger than in the exponential case, Eq. (3)]. The following value for \( \mu_q^2 \) is obtained from the best fit to the full–QCD data at \( \beta = 5.35 \) and \( a \cdot m_q = 0.01 \):

\[
\mu_q^2 = 0.46(5) \text{ GeV}^2.
\]

This value is in good agreement with the QCD sum–rule estimate (8).

3. LARGE–DISTANCE BEHAVIOUR

In this section we study the asymptotic large–distance behaviour of the quark correlators. The starting point is the fermion propagator in an external gluon field \( A_\mu = A_\mu^a T^a \), in the infinite mass limit \( M_Q \to \infty \) (static fermion limit) \[9\]:

\[
\langle Q_{\alpha, \beta}(z) \bar{Q}_{\alpha', \beta'}(z') \rangle = \delta^{(3)}(z - z') \left[ P \exp \left( ig \int_{z_0}^{z} dz_0 A_\mu(z, t) \right) \right]_{\alpha' \beta'}
\]

\[
\times \left[ \theta(z_0 - z_0')(P_+)_a b e^{-iM_Q(z_0 - z_0')} + \theta(z_0' - z_0)(P_-)_a b e^{iM_Q(z_0 - z_0')} \right],
\]

(11)

where \( \alpha, \beta = 1, \ldots, N_c \) are colour indices, \( a, b \) are Dirac indices and

\[
P_\pm = \frac{1 + \gamma^0}{2} \quad ; \quad P_- = \frac{1 - \gamma^0}{2}.
\]

Let us consider now the following mesonic correlators:

\[
M_S^{(Q)}(x^0) = \sum_{f=1}^{4} \int d^3 z' \langle O_{S}^{\dagger}(x^0, z') O_{S}^{f}(0) \rangle ;
\]

\[
M_{PS}^{(Q)}(x^0) = \sum_{f=1}^{4} \int d^3 z' \langle O_{PS}^{\dagger}(x^0, z') O_{PS}^{f}(0) \rangle,
\]

(13)

where \( x^0 > 0 \) and the mesonic operators \( O_{S}^{f} \) and \( O_{PS}^{f} \) are so defined:

\[
O_{S}^{f} \equiv \bar{q} f^0 \gamma Q \quad ; \quad O_{PS}^{f} \equiv \bar{q} f^0 \gamma^5 Q.
\]

(14)

The spin and colour indices are contracted. In other words, \( O_{S}^{f} \) is the time component of the vector current \( J_\mu^v = \bar{q} \gamma^\mu Q \), while \( O_{PS}^{f} \) is the time component of the axial–vector current \( J_\mu^a = \bar{q} \gamma^\mu Q \).
\[ q^I \gamma^\mu \gamma^5 Q \] we call \( O^I_5 \) “scalar” meson operator and \( O^I_{PS} \) “pseudoscalar” meson operator. Using Eq. (11) and performing the analytic continuation from Minkowskian to Euclidean space \((x^0 \to -ix_4, \text{with } x^0, x_4 > 0)\), one easily finds the following relations:

\[
M^{(Q)}_{E,PS}(x_4) - M^{(Q)}_{E,S}(x_4) = e^{-M_0 x_4} C_0(x_4) ; \\
M^{(Q)}_{E,PS}(x_4) + M^{(Q)}_{E,S}(x_4) = e^{-M_0 x_4} C_4(x_4) .
\]

(15)

\( C_0(x_4) \) and \( C_4(x_4) \) are precisely the quark correlators measured on the lattice, defined by Eqs. (2). As a candidate for the heavy quark \( Q \), we can take the \( b \) (bottom) quark, for which we know the pseudoscalar mesons of the type \( I(J^P) = \frac{1}{2}(-) \), with mass \( M_B \approx 5.3 \text{ GeV} \). We now consider the Eqs. (15) in the asymptotic limit \( x_4 \to \infty \). If we make the assumption (supported by the experimental evidence) that the lightest \( B \)-mesons are the pseudoscalar ones, we find that:

\[
C_{0,4}(x_4) \sim A_\infty e^{-\mu_\infty x_4} ,
\]

(16)

where

\[
A_\infty = 2F^2_B M_B ; \quad \mu_\infty = M_B - M_b .
\]

(17)

\( M_b \) is the mass of the \( b \) quark. \( F_B \) is the \( B \)-meson decay constant, defined in the usual way:

\[
\langle B(\vec{p}) | J_{S}^{I}(x) | 0 \rangle = -i F_B \mu e^{i p x} .
\]

(18)

We have indicated with \( | B(\vec{p}) \rangle \) the state of a \( B \)-meson with spatial momentum \( \vec{p} \). This state is normalized in the following way:

\[
\langle B(\vec{p}) | B(\vec{q}) \rangle = 2p^0_B (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{q}) ,
\]

(19)

where \( p^0_B = \sqrt{p^2 + M_B^2} \) is the energy.

Using the above-mentioned value of \( M_B \) and the rough estimate \( M_b \approx 5 \text{ GeV} \) for the mass of the \( b \) quark, we derive:

\[
\mu_\infty = M_B - M_b \approx 300 \text{ MeV} .
\]

(20)

That is, after conversion to length units, \( \mu_\infty \equiv 1/\mu_\infty \approx 0.66 \text{ fm} \). This value is in good agreement with the value \( \lambda_0 \approx 0.63 \text{ fm} \) that we have obtained at the lightest quark mass \( a \cdot m_q = 0.01 \) from a best fit with the function (3). An approach which uses the QCD sum–rules techniques gives the value \( \mu_\infty \approx 290 \div 360 \text{ MeV} \), which is in good agreement with the estimate (20) and with the above-mentioned lattice result.

4. Discussion and Conclusions

We have reconsidered the available lattice data for the gauge–invariant quark–antiquark nonlocal condensates. A gaussian–type parametrization, inspired by the O.P.E. at short distances, fits well the data and gives results in agreement with the QCD sum–rules phenomenology. The original exponential–type parametrization fits also very well the data, in agreement with the expected large–distance behaviour of the correlators. The natural question which arises is: are the available lattice data for the correlators in the short–distance or in the large–distance regime? It seems that they are just in an intermediate range. Further study is required in order to investigate more accurately both the short–distance and the large–distance asymptotic behaviour.

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