Comprehensive Analysis of the Tidal Effect in Gravitational Waves and Implication for Cosmology

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Abstract

Detection of gravitational waves (GWs) produced by coalescence of compact binaries provides a novel way to measure the luminosity distance of GW events. Combining their redshift, they can act as standard sirens to constrain cosmological parameters. For various GW detector networks in second-generation (2G), 2.5G, and 3G, we comprehensively analyze the method to constrain the equation-of-state (EOS) of binary neutron stars (BNSs) and extract their redshifts through the imprints of tidal effects in GW waveforms. We find for these events that the observations of electromagnetic counterparts in the low-redshift range $z < 0.1$ are important for constraining the tidal effects. Considering 17 different EOSs of NSs or quark stars, we find GW observations have strong capability to determine the EOS. Applying the events as standard sirens, and considering the constraints of NS’s EOS derived from low-redshift observations as prior, we can constrain the dark energy EOS parameters $w_0$ and $w_a$. In the 3G era, the potential constraints are $\Delta w_0 \in (0.0006, 0.004)$ and $\Delta w_a \in (0.004, 0.02)$, which are 1–3 orders smaller than those from traditional methods, including Type Ia supernovae (SNe Ia) and baryon acoustic oscillations. The constraints are also made 1 order smaller than the method of GW standard siren by fixing the redshifts through short–hard $\gamma$-ray bursts, due to more available GW events in this method. Therefore, GW standard sirens, based on the tidal effect measurement, provide a realizable and much more powerful tool in cosmology.

Unified Astronomy Thesaurus concepts: Gravitational waves (678); Gravitational wave detectors (676); Neutron stars (1108); Dark energy (351)

1. Introduction

The discovery of the first gravitational-wave (GW) event GW150914 (Abbott et al. 2016a, 2016b, 2016c, 2016d, 2016e, 2016f, 2016g), produced by the coalescence of two massive black holes (BHs), marks the new era of GW astronomy. Slightly later, the first binary neutron star (BNS) coalescence GW event, GW170817, together with its electromagnetic (EM) counterparts, opens the window of multi-messenger GW astronomy (Abbott et al. 2017a). As the primary targets of ground-based GW detectors, such as LIGO and Virgo, the GW events of BNSs and/or neutron star–black holes (NSBHs) provide a new approach to explore the internal structure of NSs (Abbott et al. 2017a, 2020). In addition, these events can be treated as the standard sirens to measure various cosmological parameters (Schutz 1986), in particular the Hubble constant (Abbott et al. 2017b) and the equation-of-state (EOS) of dark energy.

The internal structure of NSs remains a challenge for current astrophysics. The ultra-high-density state inside NS is an unknown field with new scientific potential. This extreme condition is inaccessible for terrestrial collider experiments and is theoretically challenging. In the EM observations of astrophysics, the constraints on the EOS of an NS mainly come from the measurement of the radius and mass of NSs. Several NSs with a mass greater than $2M_\odot$ have been observed (Antoniadis et al. 2013; Fonsela et al. 2016; Arzoumanian et al. 2018; Cromartie et al. 2020; https://stellarcollapse.org/nmasses), which set the maximum of allowable NS mass larger than $2M_\odot$ and can exclude some EOS models. However, the constraint from the mass observations is not a strong one. Lindblom proposed that the EOS of NSs can be further constrained by observing the radius-mass relation (Lindblom 1992). Related studies have been carried out and provided some constraints on the EOS (Miller et al. 2019; Riley et al. 2019). However, the uncertainties are still large in deriving constraints from current radius measurements. With the increasing sensitivities of GW detectors, and the detection of GW170817 and GW190425, the GW signal together with its EM counterpart provides a new method to explore the EOS of NSs. In the final stage of the inspiral BNS or NSBH, the tidal deformation that contains imprints of the internal structure of an NS can affect the orbital motion as well as the waveform of the emitted GWs (Vines et al. 2011; Bini et al. 2012; Vines & Flanagan 2013; Abdulalhin et al. 2018; Landry 2018; Banihashemi & Vines 2020). Thus, the EOS of an NS can be determined by analyzing the GW signals from these coalescences (Read et al. 2009; Hinderer et al. 2010). GW170817 set a constraint on tidal deformability $\lambda$, whose relation with gravitational mass alone, $\lambda(m)$, depends on the EOS (Lovel 1911; Hinderer 2008; Hinderer et al. 2010), with the observation of a GW for the first time (Abbott et al. 2017a), and excluded some of the EOS models (Abbott et al. 2018a.

In order to obtain a better measurement of the EOS of NSs, it is necessary to combine the information from an ensemble of BNSs. The first idea of combining multiple sources with the Fisher matrix method was reported in Markakis et al. (2012). Del Pozzo et al. (2013) proposed the first fully Bayesian study that combined information to constrain the EOS by using the tidal effects. Their results showed that when taking the NS mass as $1.4M_\odot$, tens of GW observations can constrain $\lambda$ to an accuracy of $\sim$10%. Lackey & Wade (2015) showed that in the
range of 1–2$M_{\odot}$, $\lambda$ can be measured to an accuracy of 10%--50%, and the overwhelming majority of this information comes from the loudest ~5 GW events. In order to reduce the computational cost of the study, these authors adopted a Fisher matrix approach. Agathos et al. (2015) improved this analysis by including the effects of NS spin in the GW waveforms and obtained the consistent results. Vivanco et al. (2019) carried out Bayesian inference and optimized the previous data processing method. They found that LIGO and Virgo are likely to place similar constraints on the EOSs of NSs (Lackey & Wade 2015) by combining the first 40 BNS detections (Vivanco et al. 2019).

In all these works, the authors only investigated the potential constraint of the EOS of NSs by focusing on the 2G GW detectors, including LIGO (Aasi et al. 2015), Virgo (Acernese et al. 2014), the Kamioka Gravitational wave detector (KAGRA; Aso et al. 2013; Abbott et al. 2018b), and LIGO India (Iyer et al. 2011). More importantly, these works did not consider the important role of EM counterparts in the analysis. From the observations of GW170817, BNS mergers are known to have the EM counterparts (Abbott et al. 2017c) called “kilonova” or “macronova,” which have nearly isotropic optical emission (Li & Paczyński 1998). For the events at the low-redshift range, i.e., $z \lesssim 0.1$, they can be detected by various optical telescopes including the Zwicky Transient Factory (ZTF), Large Synoptic Survey Telescope (LSST; Cowperthwaite et al. 2019), Wide Field Survey Telescope (WFST), and so on. Once the EM counterparts are detected, the position and redshift of these GW events can be fixed by their host galaxies, and the degeneracy between these parameters and the EOS parameters of an NS can be broken. For this reason, with the help of EM-counterpart observations for the low-redshift GW events, the constraint of an NS’s EOS is expected to be significantly improved. In this work, we consider a sample of representative EOS models (Özel & Freire 2016; Zhu et al. 2018; Zhou et al. 2018; Xia et al. 2019) under the 2 $M_{\odot}$ constraint. We linearly expand the relationship of NS $\lambda$--$m$ at the average mass of NSs $1.35M_{\odot}$ as $\lambda = Bm + C$, which is similar to the treatment in Del Pozzo et al. (2013). Based on the latest BNS merger rate (Abbott et al. 2019), we simulated a bunch of BNS sources. Using the networks of t2G, 2.5G LIGO A+ (Barsotti et al. 2018), the 3G Einstein telescope (ET; Punturo et al. 2010; https://www.et-gw.eu/), and the Cosmic Explorer (CE; Abbott et al. 2017d; Dwyer et al. 2015) GW detectors, we assume that every detected GW signal at a low redshift of $z < 0.1$ or $z < 0.05$ has a detectable EM counterpart, which gives the precise redshift and location of the GW source. We adopt the Fisher matrix approach to calculate the covariance matrix of $B$ and $C$, which quantifies the capabilities of GW detectors for the determination of an NS’s EOS. To highlight the importance of detectable EM counterparts, we also compare the results with those in the case without EM counterparts.

As an important issue in GW astronomy, the GW events can be treated as the standard sirens to probe the expansion history of the universe, since the observations on the GW waveform can measure the luminosity distance $d_L$ of the events, without having to rely on a cosmic distance ladder (Schutz 1986). For the standard sirens in the low-redshift range—for instance, in 2G and/or 2.5G era—one can measure the Hubble constant $H_0$ (Abbott et al. 2017b) and hope to solve the H$_0$-tension problem in the near future (Chen et al. 2018). While, in the 3G era, a large number of high-redshift events will be detected, and the standard sirens will provide a novel way to detect the cosmic dark energy (Sathyaprakash et al. 2010; Zhao et al. 2011). The key task of this issue is to determine the redshift of these GW events. The most popular way is from the observations of their EM counterparts and/or host galaxies (Sathyaprakash & Schutz 2009; Sathyaprakash et al. 2010; Zhao et al. 2011; Abbott et al. 2017b; Cai & Yang 2017; Yan et al. 2020; Yu et al. 2020a; Tamanini et al. 2016). As mentioned above, for the low-redshift BNS mergers, the kilonovae as the primary EM counterparts could emit nearly isotropic EM radiations (Li & Paczyński 1998), which are expected to be detected by various optical telescopes. However, for the high-redshift events, the expected EM counterparts are the short–hard $\gamma$-ray bursts (shGRBs) and the afterglows (Nakar 2007). This method of redshift measurement has three defects. First, the optical radiation of afterglow is faint, and only the nearby sources can be detected. Second, the radiation of shGRBs is believed to be emitted in a narrow cone more or less perpendicular to the plane of the inspiral. Therefore, these EM counterparts are expected to be detected only if they are nearly face-on. Third, in this method, the inclination angle $i$ of the events should be well determined by EM observations, which is needed to break the degeneracy between $i$ and $d_L$ (Gupta et al. 2019; Zhao & Santos 2019). For these reasons, only a small fraction of the total number of BNSs are expected to be observed with shGRBs, and the application of GW sirens in cosmology is limited (Sathyaprakash et al. 2010; Zhao et al. 2011; Cai & Yang 2017; Zhao & Wen 2018).

The alternative method to measure the redshift of GW events is by counting the tidal effects in GW waveform, which was first proposed in Messenger & Read (2012). The basic idea is that, the contributions of tidal effects in the GW phase evolution break a degeneracy between the mass parameters and redshift, thereby allowing the simultaneous measurement of both the effective distance and the redshift for individual sources. Since this method depends only on the GW observations, all the detectable GW events can be used as standard sirens. Thus, in particular in the 3G era, the number of useful events will be dramatically large. For the first time, Del Pozzo et al. (2017) proposed to use the Bayesian method to investigate the measurement errors of cosmic parameters with the GW events detected by ET. However, they directly assumed that the EOS of an NS is known and did not discuss how to determine it from the observations. In this work, with the EOS model determined at low redshift as a prior, in light of the tidal effect in GW, we calculate the errors of redshift $\Delta z$ and luminous distance $\Delta d_L$ of GW sources with the GW observation from the high-redshift range of $0.1 < z < 2$ observed by the assumed 3G detector networks. Then, with the help of the property of the Fisher matrix, we convert $\Delta z$ and $\Delta d_L$ into the errors of parameters of dark energy. For comparison, we also calculate the measurement errors of the dark energy parameters without EM counterparts observed at low redshift.

This paper is organized as follows. In Section 2, we review the mathematical preliminaries of GW waveforms from BNS coalescence and the response of GW detectors to these signals. In Section 3, we briefly introduce the EOS models adopted in this paper. In Section 4, we describe our method to calculate the Fisher matrix for a set of parameters required to fully describe a GW signal from BNS coalescence. In Section 5, we show the results of distinguishing different EOS models by 2G, 2.5G, and 3G detector networks and compare the results with
Table 1

| Detector                  | Latitude (°) | Longitude (°) | Orientation Angle (°) | Radius (km) |
|---------------------------|--------------|---------------|-----------------------|-------------|
| LIGO Livingston           | 30°56        | -90°77        | 243°                  | 1.44        |
| LIGO Hanford              | 46°45        | 119°51        | 171°                  | 1.44        |
| Virgo                     | 43°33        | 10°5          | 116°                  | 1.44        |
| KAGRA                     | 36°25        | 137°18        | 0°                    | 1.44        |
| LIGO India                | 19°09        | 74°05         | 0°                    | 1.44        |
| Einstein Telescope Europe | 43°54        | 10°42         | 19°48                 | 1.44        |
| Cosmic Explorer in USA    | 30°54        | -90°53        | 162°15                | 1.44        |
| Assumed detector in Australia | -31°51   | 115°74        | 0°                    | 1.44        |

Note. Orientation is the smallest angle made by any of the arms and the local north direction (Chu et al. 2012; Vitale & Evans 2017).

and without the presence of detectable EM counterparts. In Section 6, we discuss the determination of the parameters for dark energy by using 3G detectors. At the end, in Section 7, we summarize and discuss our main results of this work.

Throughout this paper, we choose the units in which $G = c = 1$, where $G$ is the Newtonian gravitational constant and $c$ is the speed of light in a vacuum.

2. GW Waveforms and the Detector Response

In this section, we briefly review the detection of GWs by a ground-based network, which includes $N_d$ GW detectors. The sizes of these detectors are all much smaller than the wavelength of GWs. We use the vector $r_I$ with $I = 1, 2, ..., N_d$ to denote the locations of the detectors, which are given by

$$r_I = R_e (\sin \varphi_I \cos \alpha_I, \sin \varphi_I \sin \alpha_I, \cos \varphi_I),$$

in the celestial coordinate system. Here $\varphi_I$ is the latitude of the detector, $\alpha_I \equiv \lambda_I + \Omega \cdot t$, where $\lambda_I$ is the east longitude of the detector, $\Omega$ is the rotational angular velocity of the Earth, and $R_e$ is the radius of the Earth. In this paper, we take the zero Greenwich sidereal time at $t = 0$.

For a transverse-traceless GW signal detected by a single detector labeled by $I$, the response is a linear combination of the plus mode $h_+$ and cross modes $h_\times$ as the following:

$$d_I(t_0 + \tau_I + t) = F_I^+ h_+(t) + F_I^\times h_\times(t), \quad 0 < t < T,$$

where $t_0$ is the arrival time of the wave at the coordinate origin and $\tau_I = n \cdot \tau_I(t)$ is the time required for the wave to travel from the origin to reach the $I$th detector at time $t$, with $t \in [0, T]$ being the time label of the wave, $T$ being the signal duration, and $n$ is the propagation direction of a GW. In the above, the antenna pattern functions $F_I^+$ and $F_I^\times$ (Jaranowski et al. 1998; Maggiore 2008) depend on the location of the source ($\theta_s, \phi_s$), the polarization angle $\psi_s$, the latitude $\varphi_s$, and the longitude $\gamma$ of the detector on the Earth, the orientation angle $\gamma$ of the detector’s arms, which is measured counterclockwise from east of the Earth to the bisector of the interferometer arms and the angle between the interferometer arms $\zeta$. In Table 1, we list the parameters for LIGO, Virgo, KAGRA, and the LIGO India, as well as the potential ET in Europe, the CE experiment in the U.S., and the assumed detector in Australia, respectively.

![Figure 1](https://www.et-gw.eu/). The design amplitude spectral density of ET-D, CE, and LIGO A+; and LIGO, Virgo, and KAGRA noise levels (Dwyer et al. 2015; Abbott et al. 2016b, 2018b, 2017d; Barsotti et al. 2018, https://www.et-gw.eu/). We take the noise level of the assumed detector in Australia to be the same as CE in the USA and take LIGO India to be the same as LIGO-H.

![Figure 1](https://www.et-gw.eu/)

Figure 1. The design amplitude spectral density of ET-D, CE, and LIGO A+; and LIGO, Virgo, and KAGRA noise levels (Dwyer et al. 2015; Abbott et al. 2016b, 2018b, 2017d; Barsotti et al. 2018, https://www.et-gw.eu/). The noises in 3G detectors are observed to be lower than the 2G detectors by about one order of magnitude, and the 2.5G detector lies in the middle. The change in orbital frequency over a single GW cycle is negligible during the inspiral phase of the binary merger. Therefore, under the stationary phase approximation (SPA; Zhang et al. 2017a, 2017b; Zhao & Wen 2018), the Fourier transform of the time series data from the $I$th GW detector can be obtained as follows:

$$d_I(f) = \int_0^T d_I(t) e^{2\pi i f t} dt.$$  

We define a whitened data set in the frequency domain in terms of the one-side noise spectral density $S_f$ as the following (Wen & Chen 2010):

$$\hat{d}_I(f) \equiv S_f^{-1/2}(f) d_I(f).$$

For a detector network, this can be rewritten as (Wen & Chen 2010)

$$\hat{d}(f) = e^{-i\Phi} \hat{A} h(f),$$

where $\Phi$ is the $N_d \times N_d$ diagonal matrix with $\Phi_{II} = 2\pi f|V_I|(n \cdot \tau_I(f))$, and

$$\hat{A} h(f) = \begin{bmatrix} F_1^+ h_+(f) + F_2^+ h_+(f) + F_3^+ h_+(f) \sqrt{S_1(f)} \\ F_2^+ h_+(f) + F_3^+ h_+(f) \sqrt{S_2(f)} \\ \vdots \end{bmatrix}.$$  

Note that $F_i^+$, $F_i^\times$, and $\Phi_{II}$ are all functions with respect to frequency in general and are taken as the values at $f = f_0 = (5/256) M_\odot^{-5/3} (\pi f)^{-8/3}$ (Maggiore 2008) under
SPA, where \( t_c \) is the binary merger time, \( M_c \equiv M_1 t_c^{3/5} \) is the chirp mass of binary system with component masses \( m_1 \) and \( m_2 \), and \( M \equiv (1 + z)(m_1 + m_2) \), \( \eta \equiv m_1 m_2 / (m_1 + m_2)^2 \). We also note that the masses in this paper are defined as the physical (intrinsic) mass, and the observed mass \( m_{\text{obs}} \) is related to the physical mass through

\[
m_{\text{obs}} = (1 + z)m, \tag{7}
\]

where \( z \) is the redshift of the GW source.

In general, in order to estimate parameters, one needs to use the spinning inspiral-merge-ringdown merger waveform template of the compact binaries. For simplicity, similar to previous works (Sathyaprakash et al. 2010; Zhao et al. 2011; Taylor & Gair 2012; Zhao & Wen 2018), we only consider the waveforms in the inspiralling stage and adopt the restricted post-Newtonian (PN) approximation of the waveform for the non-spinning systems (Cutler et al. 1993; Blanchet & Iyer 2005; Sathyaprakash & Schutz 2009), which is expected to have no significant differences in our result with the full waveforms.

The SPA Fourier transform of the GW waveform from a coalescing BNS is given by (Sathyaprakash & Schutz 2009)

\[
F^i_{\ell m}(\phi_f) = A_{\ell m} f^{-\gamma/6} \exp[i(2\pi f t_c - \pi/4 + 2\psi(f/2) + \Phi_{\text{tidal}}(f)) - \varphi_{f,(2,0)}],
\tag{8}
\]

with the Fourier amplitude \( A_{\ell m} \) given by

\[
A_{\ell m} = \frac{1}{d_L} \sqrt{(F^i_{\ell m}(1 + \cos^2 \iota))^2 + (2F^i_{\ell mm}(1 - \cos^2 \iota))^2} \sqrt{5\pi/96} \pi^{-\gamma/6} M_{\text{chirp}}^{7/6},
\tag{9}
\]

where \( d_L \) is the luminosity distance of the GW source, and \( \iota \) is the inclination angle between the binary’s orbital angular momentum and the line of sight. In the above, the phase \( \psi \) is contributed by the point-particle approximation of the NSs, \( \varphi \) is related to the detection capability of a GW detector, and \( \Phi_{\text{tidal}} \) is the contribution of the finite-size deformation effects of the BNS. Under the 3.5 PN approximation for the phase, \( \psi \) and \( \varphi_{f,(2,0)} \) are given by (Blanchet & Iyer 2005; Sathyaprakash & Schutz 2009)

\[
\psi = -\psi_c + \frac{3}{256\eta} \sum_{i=0}^{7} \psi_i (2\pi Mf)^{i/3}, \tag{10}
\]

\[
\varphi_{f,(2,0)} = \tan^{-1}\left(-\frac{2 \cos \iota F^x_{f,0}}{(1 + \cos^2 \iota) F^x_{f,0}}\right), \tag{11}
\]

and \( \Phi_{\text{tidal}} \) is given by (Messenger & Read 2012)

\[
\Phi_{\text{tidal}}(f) = \frac{3 \lambda}{128\pi M^5} \left[ -\frac{24}{\lambda_a} \left( 1 + \frac{11\eta}{\lambda_a} \right) (\pi Mf)^5/3 
- \frac{5}{28 \lambda_a} (3179 - 919 \lambda_a - 2286 \lambda_a^2 + 260 \lambda_a^3) (\pi Mf)^7/3 \right], \tag{12}
\]

where the sum is over the components of the binary, \( \lambda_a \equiv m_a / (m_1 + m_2) \) and \( \lambda \equiv \lambda(m_a) \). The parameter \( \lambda \) characterizes changes of the quadruple of an NS, given an external gravitational field, and is comparable in magnitude with the 3PN and 3.5PN phasing terms for NSs (Messenger & Read 2012). The \( \lambda \)-\( m \) relation depends on the EOS models of NSs, thus different EOS models will lead to different functions of \( \lambda(m) \). Since the NS masses are approximately taken to be a Gaussian distribution with a mean of \( 1.35 M_\odot \) and a standard deviation of \( 0.15 M_\odot \) (Thorsett & Chakrabarty 1999; Stairs 2004; Del Pozzo et al. 2017), to parameterize EOS models, we express the \( \lambda \)-\( m \) relation as a linear function within 1\( \sigma \) range of the NS mass distribution as the following:

\[
\lambda = B m + C, \tag{13}
\]

with \( B \) and \( C \) as two tidal effect parameters, which are expected to be determined through GW detection. Each set of values of \( B \) and \( C \) represents one EOS model of an NS. The EOS can be determined by fixing \( B \) and \( C \) through observations.

3. EOS Models and Tidal Deformability

Since the astrophysical observations indicate the existence of NSs with masses larger than \( \sim 2 M_\odot \) (Demorest et al. 2010; Antoniadis et al. 2013; Fonseca et al. 2016; Arzoumanian et al. 2018; Cromartie et al. 2020), in this paper we consider a sample of candidate NS EOSs (Ozel & Freire 2016; Zhu et al. 2018; Zhou et al. 2018; Xia et al. 2019) with varying stiffness under the \( 2M_\odot \) constraint. The \( m \)-\( r \) relations are plotted in Figure 2 (a), and the corresponding \( \lambda \)-\( m \) relations are shown in Figure 2 (b). Note that we supplement four EOSs of quark stars, as illustrated by the dashed lines in Figure 2, which are self-bound compact stars discussed in the literature. The corresponding fitted values of \( B \) and \( C \) are listed in Table 2, together with the maximum masses and the radii of \( 1.35 M_\odot \) stars. We mention here that ms1 and ms1b are incompatible, to the 90% credibility interval, with the tidal deformability constraint from GW170817 (Abbott et al. 2017a), while H4 is marginally consistent. Among NS EOS models, ap4, ms1, ms1b, and wff2 are outside the boundaries, to the 68.3% credibility interval, of the mass and radius constraints obtained for PSR J0030+0451 (Miller et al. 2019; Riley et al. 2019). We make the reasonable assumption that all NSs are described by the same EOS (Forbes et al. 2019), i.e., all NSs have the same values of \( B \) and \( C \). By the numerical simulation of the GW from BNSs with observable electromagnetic counterparts at low redshift, we study the discrimination of these models by networks of GW detectors.

For most BNS mergers at high redshift, the electromagnetic counterparts are not expected to be observed. However, from Equation (12), we found that when considering the tidal effects, \( z \) and \( m \) do not bind with each other, contrary to the point-particle approximation of Equation (10). This property enables us to estimate redshifts by only using information coming only from the GW observations without electromagnetic counterparts, which will be discussed in later sections when determining the parameters of dark energy.

4. Fisher Matrix and Analysis Method

The match-filter method is commonly used to estimate the parameters from detected waveforms against certain theoretical templates. For an unbiased estimation (the ensemble average of which is the true value), according to the Cramer–Rao bound (Cramér 1946), the lower bound for the covariance matrix of estimated parameters is the inverse of the Fisher matrix (Kay 1993) when considering statistical errors, which is method-independent. Thus, the errors of the estimated parameters \( \mu \) can be calculated by using Fisher matrix \( F_{ij} \) through the expression \( (\delta \mu_{\mu}^2 (\delta \mu_{\mu}^2) = (F^{-1})_{\mu \mu} \) in many fields (Martinez et al. 2009).
network including $N_d$ independent GW detectors is given by (Finn 1992; Finn & Chernoff 1993)

$$F_{ij} = \frac{\partial \mathbf{d}}{\partial \mathbf{d}}.$$

where $\partial \mathbf{d} = \partial \mathbf{d}/\partial \mathbf{p}_i$ and $\langle \partial 0 \rangle = 2 \int_{\delta_{\text{min}}}^{\delta_{\text{max}}} \{a(f) b^*(f) + a^*(f) b(f)\} df$. For a given BNS system, $\mathbf{d}(f)$ in Equation (5) depends on 12 system parameters ($M_c$, $\eta$, $t$, $\psi_c$, $\theta_c$, $\phi$, $\psi_s$, $\psi_{\text{sys}}$, $\delta t$, $z$, $B$, and $C$), where $\psi_c$ is defined in Equation (10), $B$ and $C$ are defined in Equation (12), and the other parameters are all defined previously. By taking the noise in detectors as stationary and Gaussian, the optimal squared signal-to-noise ratio ($S/N$) is given by (Finn 1992; Finn & Chernoff 1993)

$$\rho^2 = \langle \partial \mathbf{d}/\partial \mathbf{d} \rangle.$$

Once the total Fisher matrix $F_{ij}$ is calculated, an estimate of the rms error $\Delta \mathbf{p}_i$ in measuring the parameter $\mathbf{p}_i$ can then be calculated,

$$\Delta \mathbf{p}_i = (F^{-1})_{ii}/2.$$

The correlation coefficient between two parameters will be $r_{ij} = (F^{-1})_{ij}/(F^{-1})_{ii}(F^{-1})_{jj}/2$. Note that $r_{ij} = 0$ indicates the independency of $\mathbf{p}_i$ and $\mathbf{p}_j$, and $|r_{ij}| = 1$ indicates the complete correlation of $\mathbf{p}_i$ and $\mathbf{p}_j$, i.e., $\mathbf{p}_i$ and $\mathbf{p}_j$ are degenerate in the data analysis, and the Fisher matrix is not invertible.

For practical computation, the analytical expression of the solution for $\partial \mathbf{d}/\partial \mathbf{p}_i$ is usually not available, and one needs to use numerical methods. We adopt an approximation $\partial \mathbf{d} = \partial \mathbf{d} = (\mathbf{d}(f; p_i + \delta p_i) - \mathbf{d}(f; p_i))/\delta p_i$, which was used by Zhao & Wen (2018), and calculate the Fisher matrix numerically. Note that the Fisher matrix yields only the lower
Abbott et al. samples as shown in panel densities of the detectors detectors are listed in Table1, the design amplitude spectral m we will generate the random BNS samples simulatively with According to the detection rates in Abbott et al. simulations. from the corresponding results derived from Monte Carlo limit of the covariance matrix, which might be slightly different from the corresponding results derived from Monte Carlo simulations.

5. Determination of the EOS of the NSs

To extract EOS information from the GW detection, great detector sensitivity is required. In this paper, we investigate four types of detector networks, including 2G, 2.5G, and 3G detectors, for GW detection, and compare their abilities for the EOS determination.

1. LHV: one LIGO Livingston detector, one LIGO Hanford detector, and one Virgo detector.
2. LHVIK: LHV and additionally one LIGO India detector and one KAGRA.
3. 3LA+: three assumed LIGO A+ detectors locate in the sites of LHV, respectively.
4. ET2CE: one ET-D detector in Europe and two CE detectors in the USA and in Australia, respectively. The optimal site localizations for this kind of network with planning 3G detectors have been studied in Raffai et al. (2013).

The coordinates, orientations, and open angles of the detectors are listed in Table 1, the design amplitude spectral densities of the detectors’ noise are shown in Figure 1. According to the detection rates in Abbott et al. (2019, 2020), we will generate the random BNS samples simulatively with \( m_1 = m_2 = 1.35M_\odot \) at low redshift \( z < 0.1 \) to investigate the resolution of \( B \) and \( C \) for the tidal effects in Equation (13).

5.1. The LHV Network

We consider the case with LHV network to determine the EOS model in low redshift \( z < 0.1 \). Given a merger rate of 250–2810 Gpc\(^{-3}\) yr\(^{-1}\) at 90% confidence level (CL; Abbott et al. 2019), and assuming that BNSs are uniformly distributed in comoving volume, there will be \( 2.5 \times 10^2 \sim 2.8 \times 10^3 \) BNS merger events for a three year observation. In this paper, \( S/N > 10 \) is taken as the criterion for detectable GWs. As an illustration, we perform a numerical simulation to generate GW samples from a BNS merger. In Figure 3, the \( S/N \) distribution of the generated samples is presented. There are observed to be 30 detectable samples among the total 250 merger events and 384 detectable samples among the total 2800 merger events. For every detectable GW, we assume that the corresponding electromagnetic counterparts are observable at this low-redshift region, which provides the precise redshift \( z \) and location \((\theta_\ast, \phi_\ast)\) of the source. The remaining nine parameters \((M_\ast, \eta, t_\ast, \psi_\ast, i_\ast, \psi_\ast, d_L, B, and C)\) need to be determined by GW observation. To explore the detection uncertainties of \( B \) and \( C \), one needs to marginalize the remaining seven parameters, which is complicated with a Bayesian approach (Del Pozzo et al. 2017). We adopt an easy-to-handle process by using a Fisher matrix (Coe 2009; Amendola & Sellentin 2016). For a detectable GW sample \( k \), we calculate the nine parameter Fisher matrix and inverse it to get the nine parameter covariance matrix. The removal of the rows and columns of the seven parameters except \( B \) and \( C \) is equivalent to the marginalization of these seven parameters, and inverting the remaining \( 2 \times 2 \) submatrix yields the Fisher matrix \((F_{ij})_k \) of \( B \) and \( C \). The total Fisher matrix is obtained by summing the Fisher matrices of \((B, C)\) over all detectable GW samples, \( F_{ij} = \sum_k(F_{ij})_k \). Then, it is straightforward to get the covariance matrix of \( B \) and \( C \) as the inverse of \( F_{ij} \), with which we can plot the error ellipse of \( B \) and \( C \) as illustrated in Figure 4 at a 2\( \sigma \) CL. If two error ellipses overlap, the two corresponding models of the EOS cannot be distinguished from each other. We find that when taking 250 samples of merger events, under a 2\( \sigma \) CL, some models can be distinguished from the others, such as ms1, H4, ms1b, alf2, bsk21, qmf60, MIT2, and MIT2cfI. The remaining models cannot be distinguished due to the low sensitivity of LHV. When sampling 2800 merger events, all models can be distinguished from each other.

In the above, we only consider the BNS coalescences with the same individual NS masses as 1.35\( M_\odot \). In reality, the actual individual masses are more likely to be different. In order to investigate the effect of different NS masses on constraining...
the EOS parameters, we set all the merging BNS masses as the medium numbers of the NS masses derived from GW190425, which is $m_1 = 1.75 M_\odot$ and $m_2 = 1.57 M_\odot$ (Abbott et al. 2020), and repeat the same simulation process as before. We find that the error ellipses of $B$ and $C$ are similar to Figure 4, which leads to the same conclusion for distinguishing EOS models. So different NS masses do not significantly affect the results. However, since the premised $\lambda-m$ relation in Equation (13) is only valid within the $1\sigma$ range of the NS mass distribution adopted in this paper, the differentiation of EOS models may be modified according to the true merging NS mass distribution, which currently remains unknown unfortunately.
models are distinguishable. These samples are generated at the corresponding S/N distributions are illustrated in Figure 5. In panel (a), the error ellipses of qmf40, eng, ap4, and wff2 are observed to have certain overlaps with other models, so that they are indistinguishable with this GW observation. In panel (b), there is no overlap between any two ellipses, thus in this case, all the EOS models are distinguishable.

5.2. The LHVIK Network

Next, we consider the LHVIK network. The S/N distribution is shown in Figure 5. When a total of 250 samples are taken, we can see that there 66 detectable samples, which is more than that in LHV case. The corresponding error ellipses of B and C for different EOS models are shown in Figure 6(a) under a 2σ CL. Compared with Figure 4(a) of LHV, LHVIK can distinguish five additional EOS models, namely, sly, mpa1, ap3, pQCD800, and sqm3. When a total of 2800 samples are taken, there are 748 detectable samples, and the error ellipses are presented in Figure 6(b). We find that all the EOS models are distinguishable. The error ellipses by LHVIK are seen to be slightly smaller than those by LHV. This is because the S/N by LHVIK is greater than by LHV and the number of detectable samples by LHVIK is also larger.

5.3. The 3LA+ Network

Similar to the previous process, Figure 7 plots the S/N distribution of 3LA+ in the redshift range of z < 0.1. Among a total of 250 samples, 152 samples are detectable, and the 2σ error ellipses of B and C are plotted in Figure 8(a). Among a total of 2800 samples, 603 samples can be detected, and the 6σ error ellipses are plotted in Figure 8(b). From the two figures, all the EOS models are found to be distinguished by 3LA+ given any merger rate. Thus, the detection capability of 3LA+ is stronger than both LHV and LHVIK, and the proportion of samples that can be detected by 3LA+ is higher.

5.4. The ET2CE Network

Similarly, the S/N distribution by the ET2CE network is plotted in Figure 9 in the redshift z < 0.1. Under criterion of S/N > 10, all GW samples are detectable in the given merger rates. When the merge events are taken as the lower limit of 250, the 25σ error ellipses of B and C are plotted in Figure 10(a). When the merger events are taken to be its upper limit of 2800 samples, the 80σ error ellipses are in Figure 10(b). All the EOS models observed to be distinguished for every possible merger rate. This improvement is due to the great sensitivity of the ET2CE network.

In order to compare the detection capabilities of different detector networks, at z < 0.1 with a total of 250 samples, we take the average of the 2σ errors of B and C given by each network over all the EOS models. The result is shown in Figure 11(a). The number of the detectable samples at z < 0.1 With the threshold of S/N > 10 for each network is also plotted in Figure 11(b). The errors of B and C detected by the 2.5G or 3G detector network are found to be smaller than the 2G ones. This is due to the greater sensitivity of the 2.5G and 3G detectors, which leads to more detectable samples as shown in Figure 11(b). And a network with more detectors such as LHVIK can increase the detectable sample size and reduce the errors of B and C but not very significantly.

5.5. Detectable EM Counterpart in z < 0.05

The second BNS merger GW event GW190425 is noticeable at z ∼ 0.03, and no EM counterpart is reported (Abbott et al. 2020). Thus, as a more conservative consideration, we assume
that in the future, the detectable EM counterpart of the GWs generated by the BNS merger is within the redshift range of \( z < 0.05 \). According to Abbott et al. (2019), in a three year observation, the number of BNS merger events will be 32–370. We repeat a simulation similar to the previous subsections to estimate the error ellipses of the tidal parameters of each EOS model. The S/N distributions by the LHV, LHVIK, 3LA+, and ET2CE networks are plotted in Figures 12, 13, 14, and 15, respectively. Comparing Figures 12 and 3, by LHV, the rate of the detectable samples in \( z < 0.05 \) and \( z < 0.1 \) is seen to be quite similar. Comparing Figures 13 and 5, by LHVIK, the number of the detectable samples in \( z < 0.05 \) is seen to be
greater than that in \( z < 0.1 \); the same is true for 3LA+ and ET2CE. This is because the detection capability of LHV is weaker than LHVIK, 3LA+, or ET2CE, so the main part of the detectable objects by LHV comes from low redshift. In particular, the number of the detectable samples by LHV is almost unchanged when the redshift is reduced to a lower level.

Using these samples, the error ellipses of \( B \) and \( C \) for different EOS models by using LHV, LHVIK, 3LA+, and ET2CE are presented in Figures 16, 17, 18, and 19, respectively. Comparing Figures 16 and 4, the error ellipses obtained in \( z < 0.05 \) and \( z < 0.1 \) are observed to be similar in size, and the distinguishable EOS models for the two redshift ranges are the same.

### Figure 9
The S/N distribution of the samples generated in the redshift range of \( z < 0.1 \) detected by ET2CE. There are a total of 250 samples in panel (a), and a total of 2800 samples in panel (b). With the threshold of S/N > 10, all the GW samples are found to be detectable with ET2CE.

### Figure 10
The error ellipses of \( B \) and \( C \) for different EOS models under the detection of ET2CE. The EM counterparts are assumed to be detectable for every GW sample. Panel (a) is plotted with the total 250 samples, and the error ellipses in panel (a) are at 25\( \sigma \) CL; panel (b) is plotted with the total 2800 samples, and the error ellipses in panel (b) are at 80\( \sigma \) CL. These samples are generated at \( z < 0.1 \), and the corresponding S/N distributions are illustrated in Figure 9. The EOS models are found to be distinguishable by ET2CE and the CL is higher than the networks of 2G and 2.5G detectors. These improvements are due to the great sensitivity of the 3G detectors.
same. The error ellipses in Figure 17 by LHVIK are smaller than in Figure 6, but not significantly, which leads to the same distinguishable EOS models in these two figures. Due to the great sensitivity of ET2CE, in $z < 0.05$, all the EOS models are distinguishable by ET2CE at the 20$\sigma$ level with a total of 32 samples, and at the 50$\sigma$ level with a total of 370 samples. The result for 3LA+ is similar to ET2CE in which when using the network of 3LA+, all the EOS models are distinguishable at the 2$\sigma$ level with a total of 32 samples as well as at the 4$\sigma$ level with a total of 370 samples. Therefore, we can conclude that if the redshift range of the detectable EM counterparts is reduced to a smaller but reasonable value, the outcome of the EOS determination still stands.

5.6. No EM Counterpart

For comparison, we also consider the case where there is no detectable EM counterpart for each GW signal in a low-redshift range. Under this assumption, the position parameters ($\theta_s$, $\phi_s$) and redshift $z$ of the source remain unknown. Subsequently, for each GW event, we need to calculate the Fisher matrix of all 12 parameters, ($M_c$, $\eta$, $I_c$, $\psi_c$, $\iota$, $\theta_s$, $\phi_s$, $\psi_p$, $d_L$, $z$, $B$, and $C$), and then marginalize it into a Fisher matrix of two tidal effect
parameters \((B, C)\) as the processing adopted in Section 5.1. Then, we sum the Fisher matrices of \((B, C)\) over all detectable GW samples to obtain a total Fisher matrix whose inverse is the covariance matrix of \((B, C)\).

In this subsection, we still consider the redshift range of \(z < 0.1\) and the number of BNS merger events for three year observation ranges from 250 to 2800. With these setups, we repeat a simulation similar to Sections 5.1, 5.2, 5.3, and 5.4. The distributions for \(S/N\) by using LHV, LHVIK, 3LA+, and ET2CE are given in Figures 3, 5, 7, and 9, respectively. The corresponding error ellipses of \(B\) and \(C\) by using LHV, LHVIK, 3LA+, and ET2CE are plotted in Figures 20, 21, 22, and 23, respectively. If there is no electromagnetic counterpart, in the case of a lower merge rate with 250 samples, when using LHV, all the EOS models are indistinguishable at the 1\(\sigma\) CL; but when using LHVIK, MIT2cfl can be distinguished from other models. When taking a higher merge rate with 2800 samples, the EOS models ms1, H4, ms1b, qmf40, and qmf60 can be distinguished by LHV or LHVIK at the 1\(\sigma\) CL, but other models are indistinguishable. By 3LA+, at the 1\(\sigma\) CL, with a total of 250 samples, the EOS models ms1, H4, and MIT2cfl can be distinguished, but the others are indistinguishable. With a total of 2800 samples, except for mpa1 and ap3, which have similar prediction of radii and \(\lambda - m\) slopes for NSs, but other EOS models can be distinguished. By ET2CE, all the EOS models are distinguishable, which is the same as the conclusion.

![Figure 13](image-url)

**Figure 13.** The S/N distribution of the samples generated in the redshift range of \(z < 0.05\) detected by LHVIK. With the threshold of \(S/N > 10\), there are 25 detectable samples in the total 32 samples as shown in panel (a); and there are 279 detectable samples in the total 370 samples as shown in panel (b).

![Figure 14](image-url)

**Figure 14.** In the redshift range of \(z < 0.05\) and detected by 3LA+, the S/N distribution of the generated samples is illustrated. With the threshold of \(S/N > 10\), there are 30 detectable samples in the total 32 samples as shown in panel (a); and there are 341 detectable samples in the total 370 samples as shown in panel (b).
Comparing these figures with Figures 4, 6, 8, and 10, the errors of $B$ and $C$ are observed to be much larger than those with a detectable EM counterpart. Thus, EM counterparts are important for the determination of EOS, especially for the networks of 2G and 2.5G detectors.

### 6. Determination of Dark Energy

Dark energy with positive density but negative pressure is believed to be an impetus of the accelerated expansion of the universe (Albrecht et al. 2006; Amendola & Tsujikawa 2010), and a large number of dark energy models have been proposed.
Figure 17. The error ellipses of $B$ and $C$ at the 2σ CL for different EOS models under the detection of LHVIK. Panel (a) is plotted with a total of 32 samples and panel (b) is plotted with a total of 370 samples. These samples are generated at $z < 0.05$, and the corresponding S/N distributions are shown in Figure 13. We have assumed the EM counterparts for every GW sample are detectable. The overlaps between the ellipses as well as the distinguishable EOS models are observed to be the same as in Figure 6.

Figure 18. The error ellipses of $B$ and $C$ for different EOS models under the detection of 3LA+. Panel (a) is plotted with a total of 32 samples at the 2σ CL and (b) is plotted with a total of 370 samples at the 4σ CL. These samples are generated at $z < 0.05$, and the corresponding S/N distributions are shown in Figure 14. We have assumed the EM counterparts for every GW sample are detectable. All the EOS models are observed to be distinguishable, but the CL in panel (b) of this figure is lower than that in Figure 8 (b).
Figure 19. The error ellipses of $B$ and $C$ for different EOS models under the detection of ET2CE. Panel (a) is plotted with a total of 32 samples at the $20\sigma$ CL and (b) is plotted with a total of 370 samples at the $50\sigma$ CL. These samples are generated at $z < 0.05$, and the corresponding S/N distributions are shown in Figure 15. We have assumed the EM counterparts for every GW sample are detectable. All the EOS models are observed to be distinguishable, but the CLs are lower than that in Figure 10.

Figure 20. The error ellipses of $B$ and $C$ for different EOS models at $1\sigma$ CL under the detection of LHV. Panel (a) is plotted with a total of 250 samples and panel (b) is plotted with a total of 2800 samples, which are generated at $z < 0.1$ and correspond to the lowest and highest merger rates of a BNS (Abbott et al. 2019). The corresponding S/N distributions are the same as that in Figure 3. We have assumed the EM counterparts cannot be detected in these two figures. These EOS models are observed to be indistinguishable in panel (a), and there only the models of ms1, H4, ms1b, qmf40, and qmf60 can be distinguished in panel (b).
Figure 21. The error ellipses of B and C for different EOS models at the 1σ CL under the detection of LHVIK. Panel (a) is plotted with a total of 250 samples and panel (b) is plotted with a total of 2800 samples. The samples are generated at \( z < 0.1 \), and the corresponding S/N distributions are the same as that in Figure 5. We have assumed the EM counterparts cannot be detected in these two figures. MIT2cf is observed to be distinguishable in panel (a), and though the error ellipses in panel (b) are smaller than that in Figure 20 (b), the distinguishable models in panel (b) are the same as Figure 20(b).

Figure 22. The error ellipses of B and C for different EOS models at the 1σ CL under the detection of 3LA+. Panel (a) is plotted with a total of 250 samples and panel (b) is plotted with a total of 2800 samples. The samples are generated at \( z < 0.1 \), and the corresponding S/N distributions are the same as that in Figure 7. We have assumed the EM counterparts cannot be detected in these two figures. ms1, H4, and MIT2cf are observed to be distinguished in panel (a), and except for mpa1 and ap3, the other EOS models can be distinguished in panel (b).
in the literature (Frieman et al. 2008). In order to differentiate these different models, it is essential to measure the EOS of dark energy. Currently, the main methods include observations of Type Ia supernova (SN Ia), CMB, a large-scale structure, weak gravitational lensing, and so on (Albrecht et al. 2006; Amendola & Tsujikawa 2010), which are based on the observations of various electromagnetic waves. Future GW events of the compact binary coalescences can act as the standard sirens to explore the physics of dark energy. This issue has been widely discussed in the previous works (Sathyaprakash et al. 2010; Zhao et al. 2011; Cai & Yang 2017; Zhao & Wen 2018), under the assumption that for each detectable GW event in the BNS merger, an EM counterpart can be found, thereby the position and the redshift of the source are fixed. However, the detection ability of this method is limited by the number of available sources, since for most high-redshift (e.g., \( z > 0.1 \)) events, it is unlikely that the EM counterparts will be observed. In this section, we analyze the GW waveforms containing the tidal effects, from which both the redshift and luminosity distance can be obtained. Furthermore, the evolution of dark matter can be studied. For the EOS model of a BNS, we will adopt the \( B \) and \( C \) obtained from the GW and the corresponding EM-counterpart observations at the low-redshift as a prior condition and add it to the process of analyzing the dark energy EOS with only GW observation in the high-redshift range.

In order to quantify the EOS of dark energy, in this work, we adopt the widely used Chevallier–Linder–Polarski parameterization (Chevallier & Polarski 2001; Linder 2003; Albrecht et al. 2006), in which a parameter, \( w(z) \), is introduced as the ratio of the pressure \( \rho_{\text{de}} \) and energy density \( \rho_{\text{de}} \) of dark energy as

\[
w(z) = \frac{\rho_{\text{de}}}{\rho_{\text{de}}} = w_0 + w_a \frac{z}{1 + z},
\]

where \( w_0 \) and \( w_a \) are two constants. \( w_0 \) represents the present-day value of EOS and \( w_a \) represents its evolution with redshift. In the \( \Lambda \)CDM model, a cosmological constant, \( \Lambda \), corresponds to \( w_0 = -1 \) and \( w_a = 0 \). In a general Friedmann–Lemaître–Robertson–Walker (FLRW) universe, the luminosity distance of astrophysical sources is a function of redshift \( z \) as (Weinberg 2008)

\[
d_L(z) = (1 + z) \left\{ \begin{array}{ll}
|k|^{-1/2} \sin \left[ |k|^{1/2} \int_0^z \frac{dz'}{H(z')} \right] & (\Omega_k < 0) \\
\int_0^z \frac{dz'}{H(z')} & (\Omega_k = 0), \\
|k|^{-1/2} \sinh \left[ |k|^{1/2} \int_0^z \frac{dz'}{H(z')} \right] & (\Omega_k > 0) 
\end{array} \right.
\]

where \( \Omega_k \) is the contribution of spatial curvature density, and the Hubble parameter \( H(z) \) is

\[
H(z) = H_0[\Omega_m(1 + z)^3 + \Omega_k(1 + z)^2 + (1 - \Omega_m)E(z)]^{1/2},
\]

Figure 23. The error ellipses of \( B \) and \( C \) for different EOS models under the detection of ET2CE. Panel (a) is plotted at the 4\( \sigma \) CL with a total of 250 samples and panel (b) is plotted at 15\( \sigma \) CL with a total of 2800 samples. The samples are generated at \( z < 0.1 \), and the corresponding S/N distributions are the same as that in Figure 9. We have assumed the EM counterparts cannot be detected in these two figures. All the EOS models are observed to be distinguishable. However, the CLs are smaller than those in Figure 10.
with

\[ E(z) = (1 + z)^{3(1+w_0+w_a)}e^{-3w_a(z/1+z)}. \]  

There are five cosmological parameters to be determined in the above \( d_L - z \) relation, i.e., \((H_0, \Omega_m, \Omega_z, w_0, w_a)\). However, as Zhao et al. (2011) have shown, the background parameters \((H_0, \Omega_m, \text{and } \Omega_z)\) and the dark energy parameters \((w_0, w_a)\) have strong degeneracy, thus one cannot constrain the full parameters from GW standard sirens alone. The SN Ia and BAO methods for dark energy detection also suffer from the same problem. A general way to break this degeneracy is to use the Fisher matrix for the constraints (Shoemaker et al. 2010; Zhao et al. 2011). \( \Delta w_0 \) is induced from the redshift error \( \Delta z \) as

\[ \Delta d_L = \frac{\partial d_L(z)}{\partial z} \Delta z. \]  

In Equations (21) and (22), \( \Delta d_L \) and \( \Delta z \) are the 1σ errors. To get \( \Delta d_L \) and \( \Delta z \), we first calculate a 12 × 12 Fisher matrix from the \( k \)th GW sample of the full 12 parameters \((M_c, \eta, \tau_c, \psi_c, \tau, \phi, \psi, d_L, z, B, \text{and } C)\) in the redshift range of \( 0.1 < z < 2 \). Then, we add the Fisher matrix of \( B \) and \( C \) obtained from the low-redshift GW and corresponding EM counterparts to the \( B \)- and \( C \)-related elements in this 12 × 12 Fisher matrix as a prior (Coe 2009). \( \Delta d_L \) and \( \Delta z \) are the square root of the \( (d_L, d_L) \)- and \( (z, z) \)-diagonal elements of the inverse of this new 12 × 12 matrix.

There is also another quantity, the figure of merit (FoM; Albrecht et al. 2006), indicating the goodness of constraints from observational data sets, which is proportional to the inverse area of the error ellipse in the \( w_0-w_a \) plane as the following:

\[ \text{FoM} = [\text{Det } C(w_0, w_a)]^{-1/2}, \]  

where \( C(w_0, w_a) \) is the covariance matrix of \( w_0 \) and \( w_a \). A larger FoMs means stronger constraint on the parameters, since it corresponds to a smaller error ellipse.

By adopting the merger rates in Abbott et al. (2019), there will be \( 2.1 \times 10^5 \sim 7.2 \times 10^6 \) merger events for a three year observation. And when the number of events is large, the sum of events in Equation (21) can be replaced by the following integral (Zhao et al. 2011):

\[ F_{ij} = N \times \int_0^{\infty} \frac{\partial \ln d_L(z)}{\partial p_i} \frac{\partial \ln d_L(z)}{\partial p_j} f(z) dz \times \left\{ \frac{1}{(\Delta d_L/d_L(k))^2 + (\Delta d_L/d_L(k))^2 + \sigma_i^2} \right\} dz, \]  

where \( f(z) \) is the number distribution of the GW sources over redshift \( z \), and \( (\cdot) \) is the average over the angles \((\theta_c, \phi_c, \tau, \text{and } \psi_c)\). Thus, the 1σ errors of \( \Delta w_0 \) and \( \Delta w_a \), and FoM are related to a large number \( N \) of the total events as the following:

\[ \Delta w_0 = \frac{A_{w_0}}{\sqrt{N}}, \quad \Delta w_a = \frac{A_{w_a}}{\sqrt{N}}, \quad \text{FoM} = A_{\text{FoM}} N, \]  

where the values of the coefficients \( A_{w_0}, A_{w_a} \) and \( A_{\text{FoM}} \) depend on the type of detector network. Equation (25) makes it easy to convert the estimation errors for one large number of events to another.

Since in the 3G era only a larger number of high-redshift BNSs are expected to be detected by GW observations, in this work, we will focus on the 3G detector network for the determination of dark energy. We perform a simulation to estimate the errors of \( w_0 \) and \( w_a \) with the ET2CE network. Following Section 5.4, we also take the assumption that the EM counterparts are detectable only in \( z < 0.1 \). Thus, at the lowest merger rate, we generate \( 2.1 \times 10^5 \) GW samples within the redshift range of \( 0.1 < z < 2 \) numerically, and its S/N distribution is shown in Figure 24(a), where the number of detectable sample is \( 1.6 \times 10^5 \), accounting for \( \sim 76\% \) of the total samples. At the highest merger rate, the S/N distribution with a total of \( 7.2 \times 10^6 \) GW samples is shown in Figure 24(a), where the number of detectable sample is \( 5.5 \times 10^6 \), accounting for \( \sim 76\% \) of the total samples too. Note that Del Pozzo et al. (2017) generated 1000 GW samples and plotted the S/N distribution detected by ET, which showed a \( \sim 5\% \) detectable sample proportion under S/N > 10. The difference between these two S/N distributions is due to ET2CE’s better detection capability. Taking the Fisher matrix of \( B \) and \( C \) in Section 5.4 as a prior, with the total \( 2.1 \times 10^5 \) GW samples, we calculate the Fisher matrix of Equation (21) for fiducial values of \( w_0 = -1 \) and \( w_a = 0 \), and yield \( \Delta w_0, \Delta w_a \) at the 1σ CL for different EOS models, which are listed in Table 3. At the highest merger rate with \( 7.2 \times 10^6 \) events, the corresponding \( \Delta w_0, \Delta w_a \), and FoM are listed in Table 4. Consistent with the claim in Del Pozzo et al. (2017), we find that all the EOSs of NSs yield very similar results. For the case with a pessimistic estimation of the event rate, GW standard sirens in this method can follow the constraints of \( \Delta w_0 \sim 0.004, \Delta w_a \sim 0.02 \), and FOM \( \sim 4 \times 10^4 \). While in the case with optimistic estimation of event rate, the constraints are improved to \( \Delta w_0 \sim 0.0006, \Delta w_a \sim 0.004 \), and FOM \( \sim 2 \times 10^5 \).

As a continuation of the discussion in Section 5.5, we also assume that the detectable EM counterparts are only distributed within \( z < 0.05 \). Repeating the similar numerical simulation process as previous under this assumption, we find that the errors of \( w_0 \) and \( w_a \), and the FoM values are a total of the same as in Tables 3 and 4. This result is conceivable from the results in Section 5.5, which shows that there is no significant difference between these two redshift ranges in the determination of the EOS model.
To investigate the importance of the observation of EM counterparts, we consider a situation that there are no detectable EM counterparts at low redshift. Similar numerical simulation process as previous yields the values of $\Delta w_0$, $\Delta w_a$ and FoM for each EOS model, which are listed in Tables 5 and 6. Comparing with Tables 5 and 3, with no EM counterparts, the errors of $\Delta w_0$ and $\Delta w_a$ are observed to become...
significantly larger. Therefore, detectable EM counterparts at low redshift can reduce the errors of \( w_0 \) and \( \omega_a \) to a certain extent.

It is important to compare our results with those in previous works. Table 3 in Del Pozzo et al. (2017) listed the errors of \( w_0 \) and \( \omega_a \), which are determined by a single ET through a Bayesian approach. The 1\( \sigma \) errors of \( w_0 \) and \( \omega_a \) for 2.1 \( \times \) 10^{5} GW events can be derived from Table 3 of Del Pozzo et al. (2017) as \( \Delta w_0 = 0.028 \) and \( \Delta \omega_a = 0.031 \), which are larger than our results in Table 3. This is mainly due to the better sensitivity of the ET2CE network than a single ET. More importantly, Del Pozzo et al. (2017) assumed the EOSs of NSs are known in advance. In this work, we consider a more realistic case, in which we assume the EOSs of NSs will be determined by the BNS events at low redshifts with both GW and EM observations. We find that, in the 3G era, this approach is nearly equivalent to the case with known NS’s EOS, and the assumption in Del Pozzo et al. (2017) is justified in our analysis.

In previous works, the authors investigate the potential constraint of dark energy by GW standard sirens based on the assumption that the high-redshift BNS events have detectable shGRB counterparts. Due to the difficulty of EM observations, the available event number is always to be \( \sim 1000 \) in the 3G detection era (Sathyaprakash et al. 2010; Zhao et al. 2011; Cai & Yang 2017; Zhao & Wen 2018), which is justified in our recent simulations (J. M. Yu et al. 2020b, in preparation). In this approach, the potential constraints of dark energy parameters by various 3G detector networks have been explicitly studied in our previous work (Zhao & Wen 2018). For instance, the detector network, including three CE-like detectors, is expected to give the constraints of \( \Delta w_0 \sim 0.03 \) and \( \Delta \omega_a \sim 0.2 \), if assuming 1000 face-on BNSs at \( z < 2 \) have the detected EM counterparts. In comparison with these results, we find that GW standard sirens with tidal effect have a much stronger detection ability for dark energy, which might guide the research direction in this issue.

The most recent determination by Planck+SN1a+BAO of the parameters \( w_0 \) and \( \omega_a \) reports \( \Delta w_0 \sim 0.077 \) and \( \Delta \omega_a \sim 0.3 \) (Aghanim et al. 2018), which are much larger than our results. In the next generation of SN Ia and BAO observations, with the CMB priors, the potential constraints on the dark energy parameters are around \( \Delta w_0 \sim 0.3 \) and \( \Delta \omega_a \sim 0.1 \) (Bock et al. 2006). Therefore, in comparison with the tradition EM methods, we find that in the 3G era with GW standard sirens, the constraints on dark energy parameters would be able to improve by 1–3 orders in magnitude. This is mainly attributed to much larger event numbers of GW sources. So, we conclude that in the near future, the GW standard sirens provide a much more powerful tool for constraining the cosmological parameters.

### 7. Conclusions

In this paper, we study the potential detection of the NS’s EOS by using the 2G, 2.5G, and 3G detectors to detect the GW with corresponding EM in the low redshift under the framework of a Fisher matrix analysis. We also study the potential of
using 3G detector network with a GW from a BNS merger as a cosmological probe to determine the dark energy parameters. Specifically, we fit the relation between the tidal deformation $\lambda$ and the NS mass $m$ around $m = 1.35 M_\odot$ by a linear approximation of $\lambda = B m + C$ and choose the parameters $B$ and $C$ to represent different EOS models. At low redshift, the detectable EM counterparts can give the accurate redshift and position of the GW sources. With this, we estimate the errors of $B$ and $C$ by using the tidal effect in GW observation and propose to use the overlap of two error ellipses of $B$ and $C$ to study the ability of distinguishing different EOS models, which is different from other papers in the literature that only set constraints on $\lambda$ (Hinderer et al. 2010; Abbott et al. 2017a, 2020). Using the EOS model determined by the low redshift as a prior, the GW events of BNS mergers from high redshift can be used as a standard siren to determine its luminosity distance and redshift, which can be used to constrain the dark energy parameters.

According to the BNS merger rate given by Abbott et al. (2019), for a three year duration of observation, we simulate and generate 250 and 2800 events within the low redshift $z < 0.1$, respectively. With the tidal effect in GW from BNS merger, under the Fisher matrix analysis, we compare the capability of distinguishing EOS models by different 2G, 2.5G, and 3G detectors. The main result of the study is that, at the 2$\sigma$ CL, if EM counterparts are detectable at low redshift, when the merge rate of a BNS is taken as the upper limit of the theoretical value, the 2G detector networks can distinguish all the 17 EOS models adopted in this paper. While, if the merge rate is taken as its lower limit, most EOS models can be distinguished by the 2G detector networks, but there are still some indistinguishable EOS models. For any merger rate in the theoretical range, the 2.5G and 3G detector networks can distinguish all these EOS models due to their great sensitivity. We also check that if the detectable EMs lie in the redshift range of $z < 0.05$, the above results still holds true. To emphasize the importance of detectable EM, we have taken the assumption that there is no EM detectable at the low redshift as shown in Section 5.6. The result shows that in the given range of merger rates, none of the 17 EOS models can be distinguished by the 2G detector networks, some of the EOS models can be distinguished by the 2.5G detector networks, and only the 3G detector networks will be able to distinguish all 17 EOS models. Thus, EM counterparts are important for the determination of EOS, especially for the networks of 2G and 2.5G detectors.

Notice that the above conclusions are based on the assumption that all merging NSs have the same mass as $1.35 M_\odot$. The actual merging NS masses are more likely to be different. As discussed at the end of Section 5.1, different NS masses will not significantly affect the differentiation of the EOS models under the approximated NS mass distribution adopted in this paper. However, the true mass distribution remains unknown. Therefore, to distinguish EOS models with more realistic mass distributions requires a more careful analysis in the future.

With the EOS model determined in low redshift by ET2CE as a prior, and considering tidal effect, we calculate the errors of redshift and luminosity distance of GW sources by only using the GW observation from the high-redshift range of $0.1 < z < 2$ by ET2CE. Then, we convert these errors into the errors of parameters of dark matter $w_0$ and $w_a$ according to Equation (21). The results are listed in Tables 3 and 4 for the lowest and highest merger rate, respectively. We have shown that our results are consistent with Zhao & Wen (2018) and Del Pozzo et al. (2017), and the future GW observation will give better constraints on $w_0$ and $w_a$ than the CMB observation (Aghanim et al. 2018). We also calculate the errors of $w_0$ and $w_a$ with no EM counterparts at low redshift, which are listed in Tables 5 and 6. Without EM counterparts, the errors of $w_0$ and $w_a$ are seen to become significantly larger.

In our analysis, for simplicity, we neglect the NS spins, which are believed to be small in BNS systems (O’Shaughnessy & Belczynski 2008). In addition, we only consider the post-Newtonian expansion at the inspiral stage of a BNS merger, which has been proved to be in good agreement with observations (Abbott et al. 2017a, 2020). We have also made a strong assumption in our analysis that the EOS of all NSs is the same, which allows us to refer to the EOS parameters obtained from the low redshift to the high redshift for constraining parameters of dark energy. But in reality, NSs may have different EOSs depending on the internal material composition. For example, the EOS of a quark star and an NS are likely to be different (Arroyo-Chávez et al. 2020; Weber et al. 2012). We leave this problem to our future research.

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