The quantum Hall effect (QHE) was discovered in Si metal-oxide-semiconductor field-effect transistors (MOSFETs) more than 25 years ago. This system is still under study, mainly because of the problem of the metal-insulator transition in two-dimensional electron systems (2DES) (see, for example, Ref. and references therein). The main interest of researches of the QHE is focused on much less disordered 2DES based on Si/SiGe and GaAs/AlGaAs structures with high electron mobility. We are aware of only a few publications where the Hall resistivity $\rho_{xy}$ was measured in Si-MOSFET in a narrow interval of magnetic fields $B$ and gate voltages $V_g$. In the present work, we report the results of transport measurements in two Si-MOSFET samples in the QHE regime in a wide interval of $B$ (up to 14 T) and electron densities $n$ (up to $1.5 \cdot 10^{16}$ m$^{-2}$), controlled by $V_g$.

Measurements of $n$ and electron mobility $\mu$ at $T = 0.3$ K in sample #1 yield a linear dependence $n(V_g) = 1.41 \cdot 10^{15} (V_g - 0.4 \text{ V})$ m$^{-2}$ within the interval of $V_g$ from 5.5 to 11 V with $\mu \approx 1.0 \div 1.5$ m$^2$/V·s. Sample #2 was measured in the interval $V_g$ between 8 and 11 V, and $n(V_g)$ was approximated by a different linear dependence $n(V_g) = 1.25 \cdot 10^{15} (V_g + 0.6 \text{ V})$ m$^{-2}$, with very similar values for $n$ in the measured interval of $V_g$. The mobility in sample #2 was also within the above interval, changing from $\mu = 1.46$ m$^2$/V·s at $V_g = 8$ V to 1.27 m$^2$/V·s at $V_g = 11$ V. The sample resistance was measured using a standard lock-in technique with the measuring current 20 nA at a frequency of 10.6 Hz.

Figure 1 shows $\rho_{xy}$ of sample #1 as a function of perpendicular magnetic field $B$ at different $V_g$. One can see two features. The first one is the “overshoot” effect which is observed at almost every plateau, being especially large at filling factor $\nu = 3$ (Fig. 2). In incremental magnetic fields, when $\nu$ approaches the integer 3, $\rho_{xy}$ overshoots the normal plateau value $1/(3(h/e)^2)$ by 8.6 kOhm. However, as $B$ increases further, $\rho_{xy}$ drops to its normal value. The overshoot effect has been previously observed in GaAs/AlGaAs and Si/SiGe heterostructures (see, for example, Ref. and references therein).

The second feature is new. It consists of a “dip” of $\rho_{xy}$ from the plateau at $\nu = 4$ (6.45 kOhm) to the plateau at $\nu = 5$ (5.16 kOhm) at magnetic fields when the filling factor approaches $\nu = 5$. This effect can be seen more clearly in Fig. 2, where the dimensionless Hall resistivity (in units of $h/e^2$) is plotted as a function of the filling factor $\nu = nh/eB$.

Figure 3 shows that the longitudinal resistivity $\rho_{xx}$ also exhibits a “dip” at $\nu = 5$ and a less pronounced “dip” at $\nu = 7$.

In the present work, the “dip” effect was invariably observed in all experiments, in both samples, for different voltage probes and reversed directions of the current and magnetic field. Moreover, Figs. 1 and 2 show the
FIG. 2: Dependence of the dimensionless resistivity $\rho_{xy} / (h/e^2)$ on the filling factor $\nu = nh/eB$ for different gate voltages $V_g$. The insert shows the enhanced view of the “dip” effect.

development of the "dip" with variation of the gate voltage for the same sample and probes. These facts give us confidence that the observed "dip" is not connected with heterogeneity of the 2DES and possible admixture of $R_{xx}$ into $R_{xy}$. Let us discuss the origin of the "dip" effect.

For Si-based 2DES, like Si/SiGe and Si-MOSFETs, the energy spectrum in a magnetic field is

$$E_n = \hbar \omega_c (N + \frac{1}{2}) + \frac{\Delta E_s}{2} + \frac{\Delta E_v}{2}$$

where $N = 0, 1, \ldots$ is the Landau level (LL) number, $\omega_c = eB / m_e$ is the cyclotron frequency, $\Delta E_s = g^* \mu_B B$ is the Zeeman splitting, $g^*$ is the effective Landé factor, $\mu_B = e\hbar / 2m_e$ is the Bohr magneton, $B = (B_\perp^2 + B_\parallel^2)^{1/2}$ is the total magnetic field, $\Delta E_v [K] = \Delta_0^v + 0.6B_\perp [T]$ is the valley splitting energy, $\Delta_0^v$ is assumed[28] to be 2.4 K or 0.9 K. In accordance with this scheme, odd filling factor corresponds to the Fermi level position $\varepsilon_F$ midway between two adjacent valley-split LL (see insert in Fig. 4). If one takes into account the disorder in real samples, each LL is broadened into a Landau band (LB), with the width determined by the scale of the disorder energy $W$.

In Fig. 4, the dependences of $\rho_{xy}(\nu)$ for $n$-Si/SiGe[6,7] and for $n$-Si-MOSFET with almost the same electron concentration $n$ are shown together for comparison. In Si/SiGe, all plateaus are clearly observed, including plateaus at odd filling factor. In Si-MOSFET, plateaus at odd filling factors with $\nu > 3$ are not observed. This can be explained by the increase of disorder in Si-MOSFET, because the interface between Si and SiO$_2$ is much less perfect than the interface between Si and SiGe. Increase of disorder results in the broadening of LB in Si-MOSFET. If the width of LB is of order of the valley-splitting energy $\Delta E_v$, the density-of-states $N(\varepsilon)$ does not have a deep minimum between valley-split adjacent Landau bands, the Fermi level does not linger between them, and the corresponding plateau is missed.

However, when $\nu$ approaches the integer $\nu = 5$, the value of $\rho_{xy}$ starts to fall toward the missed plateau and finally reaches this plateau with increasing $n$ (see insert in Fig. 2). This effect can be explained by the temporary enhancement of the valley splitting. It was shown[9,10] that the occupied Landau levels undergo a self-energy shift roughly proportional to their occupation and inversely proportional to the screening of the system. Therefore, the enhancement oscillates as a function of the electron occupation of LB and has the maximum value when the filling factor approaches an integer and the Fermi level lies midway between the adjacent valley-split LB (see Fig. 5). We believe that this exchange enhancement of the valley splitting is responsible for the “dip” effect at $\nu = 5$.

The model of spin-split exchange enhancement was used to explain the enhanced $g$-factor in GaAs/AlGaAs[11] and the bistable switching between quantum Hall conduction and dissipative conduction near $\nu = 1$ in a quantum Hall system in GaInAs quantum well[12]. Exchange

FIG. 3: Longitudinal resistivity in dimensionless units $\rho_{xx} / (h/2e^2)$ as a function of filling factor $\nu = nh/eB$ for different $V_g$.

FIG. 4: Comparison of QHE in Si/SiGe ($n = 8.94 \cdot 10^{15}$ m$^{-2}$, solid line) and in Si-MOSFET with similar electron concentration ($n = 8.80 \cdot 10^{15}$ m$^{-2}$, dashed line).
enhancement was also used to explain the overshoot effect at \( \nu = 3 \) in Si/SiGe heterostructure. However, overshoot is observed at the low magnetic-field edge of the \( \rho_{xy} \) plateau when \( \nu \) is far from an integer. Furthermore, after overshoot, \( \rho_{xy} \) remains at its “normal” plateau value, while the enhanced splitting due to exchange interaction oscillates and has maximum at the integer \( \nu \). Therefore, the manifestation of the exchange enhanced splitting is expected in the close vicinity of integer \( \nu \) in the form of a “dip”, which is observed in our experiment for the first time (Fig. 2).

We would like to mention that non-monotonic behavior of the Hall resistance with several reentrances of the plateau values \( 1/4(h/e^2) \) and \( 1/3(h/e^2) \) was observed in a modulation-doped GaAs quantum well at very low temperatures (15 mK) in Ref. and explained by the existence of collective insulating states in the \( N = 1 \) Landau level. Most likely, physics behind this phenomenon and our “dip” is entirely different, although the two effects look similar.

One can see also from Fig. 2 that in Si-MOSFET, the integer values of \( \nu \) do not correspond to the midpoint of the plateaus, in contrast with more perfect Si/SiGe (Fig. 4). This can be considered as evidence of asymmetry of LB in Si-MOSFET, when delocalized states are displaced from the center of LB (Fig. 5). As a result of this asymmetry and considerable overlap of the adjacent valley-split LB, the Fermi level \( \varepsilon_F \) is situated in the interval of localized states corresponding to the plateau with \( \rho_{xy} = 1/4 \) (in units of \( h/e^2 \)) even at \( \nu \gtrsim 5 \), Fig. 5(a). However, as \( \varepsilon_F \) approaches exactly \( \nu = 5 \), the valley splitting increases leading to LB separation, Fig. 5(b). The localized states, which correspond to the plateau 1/5, show up, and \( \rho_{xy} \) dives to 1/5. When \( \nu \) is further decreased, the exchange interaction-induced enhancement of the valley splitting disappears and \( \varepsilon_F \) finds itself again in the interval of localized states which corresponds to the plateau \( \rho_{xy} = 1/4 \), Fig. 5(c).

Asymmetry in the position of the delocalized states could be a consequence of the asymmetry of large potential fluctuations caused by the fact that an excess of the local electron concentration above the average value \( \langle n \rangle \) is, in principle, unlimited, while the local deficit of electron density is limited by the value of \( \langle n \rangle \) itself. To satisfy neutrality, the area occupied by the negatively charged fluctuations is less than the area of the positively charged fluctuations. Correspondingly, the integral number of localized states above the percolation level is less than the integral number of states below this level which explain the asymmetry of the position of delocalized states in disorder-broadened LB. Computer simulation also shows that the increase of disorder leads to the displacement of delocalized states from the central part of LB. In more perfect systems, potential fluctuations are small, and asymmetry is negligible, which explain why in Si/SiGe, the integer values of \( \nu \) correspond approximately to the middle point of each plateau.

The question arises why the “dip” effect is clearly observed in Si-MOSFET at high electron densities in relatively strong magnetic fields but barely observed at low electron densities (Fig. 1); why it does not exist in Si/SiGe? We believe that the necessary condition for observation of the “dip” effect is the equality of the splitting energy \( \Delta E \) and the width of the adjacent LB. The last parameter could be estimated roughly as the average energy of disorder \( W \). In more perfect systems, \( W \ll \Delta E \) and the Fermi level is fixed between the narrow adjacent LB even without enhanced splitting. As a result, the odd plateaus are clearly observed in Si/SiGe. In the opposite case, \( W \gg \Delta E \) (Si-MOSFET with low electron density), the adjacent strongly broadened valley-split LB remain unresolved even in the case of enhanced splitting. The valley-splitting is small in weak magnetic fields, which explains why the “dip” effect is barely observed at \( \nu = 7 \).

What follows from these considerations is that in our case, \( W \approx \Delta E_\nu \). One can roughly estimate the average energy of disorder as \( W \approx h/\tau \), where \( \tau \) is the time between elastic collisions which determines the mobility \( \mu = e\tau/m^* \). In our sample, \( \mu = 1.0 \pm 1.5 \) m²/V · s. Using \( m^* = 0.2m_0 \) for strained Si layers, we obtain \( W \approx 7 \div 10 \) K. Figure 1 shows that the “dip” effect is clearly observed at \( B \approx 10 \div 12 \) T. In these fields, \( \Delta E_\nu \approx 8 \div 10 \) K, which is indeed equal to the above estimate of \( W \) and confirms our model.

It was also predicted that the exchange enhancement drops drastically around a certain temperature due to a two-fold positive feedback mechanism: with increasing temperature the difference in occupation numbers of the two valleys decreases and simultaneously the screening increases.

This effect was observed in our experiment. Figure 6 shows the temperature dependence of the “dip” at \( \nu = 5 \) measured at \( V_g = 11 \) V and \( B = 12.5 \) T plotted in the Arrhenius scale. The maximum amplitude of the “dip” is the difference between two plateaus at 1/4 and 1/5 in units of \( h/e^2 \) and is obtained at low temperatures. With increasing \( T \), the amplitude of “dip” decreases first.
weakly but at $T > 1$ K the amplitude drops drastically with the energy of activation $T_0 = 11 \pm 3$ K, which is indeed approximately equal to the valley splitting $\Delta E_v$ at $B = 12.5$ T.

In summary, we have observed a new “dip” effect in the Hall resistivity of Si-MOSFET measured in the quantum Hall effect regime. This effect can be considered as a manifestation of the oscillating enhancement of the valley splitting due to the exchange interactions. Observation of the “dip” effect is preferable if the width of the adjacent LB is approximately equal to the initial valley-split energy.

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