Single Heavy Flavour Baryons using Coulomb plus Power law interquark Potential
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Abstract

Properties of single heavy flavor baryons in a non relativistic potential model with colour coulomb plus power law confinement potential have been studied. The ground state masses of single heavy baryons and the mass difference between the \((J^P = \frac{3}{2}^+\) and \(J^P = \frac{1}{2}^+\)) states are computed using a spin dependent two body potential. Using the spin-flavour structure of the constituting quarks and by defining an effective confined mass of the constituent quarks within the baryons, the magnetic moments are computed. The masses and magnetic moments of the single heavy baryons are found to be in accordance with the existing experimental values and with other theoretical predictions. It is found that an additional attractive interaction of the order of \(-200\) MeV is required for the antisymmetric states of \(\Lambda_Q\) (\(Q \in c, b\)). It is also found that the spin hyperfine interaction parameters play decisive role in hadron spectroscopy.

1 Introduction

Confirmation of the existence of the charmed baryons at Fermilab[1] arose an increasing interest on heavy-baryon spectroscopy. It is striking that baryons containing one or two heavy charm or beauty flavour could play an important role in our understanding of QCD at the hadronic scale [2]. The copious production of heavy quarks at LEP, Fermilab Tevatron, CERN and B factories, opened up rich spectroscopic study of heavy hadrons. Many theoretical models [3, 4, 5, 6, 7, 8, 9, 10, 11, 12] have also predicted the heavy baryon mass spectrum. The non-relativistic quark model (NRQM) [13] has been able to explain very nicely the mass spectrum of light baryons. Though the experimental and theoretical data on the properties of heavy flavour mesons are available plenty in literature, the masses of most of the heavy baryons have not been measured yet experimentally [14]. Thus the recent predictions about the heavy baryon mass spectrum have become a subject of renewed interest due to the experimental facilities at Belle, BABAR, DELPHI, CLEO, CDF etc [15, 16, 17, 18, 19, 20]. These experimental groups have been successful in discovering heavy baryonic states along with other heavy flavour mesonic states and it is expected that more heavy flavour baryon states will be detected in near future. Most of the new states are within the
heavy flavour sector with one or more heavy flavour content and some of them are far from most of the theoretical predictions. Though there are consensus among the theoretical predictions on the ground state masses [6, 7], there are little agreement among the model predictions of the properties like spin-hyperfine splitting among the $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ baryons, the form factors [6], magnetic moments etc [9]. All these reasons make the study of the heavy flavour spectroscopy extremely rich and interesting. The study of heavy baryons further provide excellent laboratory to understand the dynamics of light quarks in the vicinity of heavy flavour quarks in bound states.

For the present study of the properties of the low-lying baryonic states with a heavy quark, we consider coulomb plus power form as the interquark interaction potential [9, 21, 22]. The spin hyperfine interactions similar to the one employed in [2] have been used with mass dependance of the constituting quarks in the present study. The magnetic moments of the baryons are computed based on the non-relativistic quark model using spin-flavour wave functions of the constituting quarks [9].

The paper is organized as follows. In sect.II of this paper, the basic methodology adopted for the present study of computing the binding energy of the constituting quarks within the baryon containing single heavy quarks is described. In sect.III, we present the calculation of magnetic moments of the baryons. In sect.IV, we present our results and discuss the important features and conclusions of the present study.

## 2 Methodology and Binding energy of the Single Heavy Flavour Baryons

We start with the color singlet Hamiltonian of the system as

$$H = -\sum_{i=1}^{3} \frac{\nabla^2_i}{2m_i} + \sum_{i<j} V_{ij}$$

Where, the interquark potential

$$V_{ij} = -\frac{2\alpha_s}{3} \frac{1}{x_{ij}} + \beta x_{ij}^\nu + V_{\text{spin}}(ij);$$

$$x_{ij} = |\vec{x}_i - \vec{x}_j|$$

Following ref [8], the single particle position co-ordinates are replaced by the CM co-ordinates plus the interquark distances of $q_1$, $q_2$ and $q_3 = Q$, as

$$\vec{X} = \frac{1}{\sum m_i} \sum_{i=1}^{3} m_i \vec{x}_i;$$
Table 1: The Model Parameters with the Variational Parameter $\lambda$

| System        | $\nu$ | $\lambda$ in units of Bohr radius | $\beta$ in units of (Bohr Energy)$^{\nu+1}$ |
|---------------|-------|-----------------------------------|-----------------------------------------------|
| Charm Baryons | 0     | 1.31                              | 1.018                                         |
|               | 1     | 3.07                              | 1.262                                         |
|               | 2     | 3.64                              | 1.830                                         |
| Beauty Baryons| 0     | 1.31                              | 4.782                                         |
|               | 1     | 5.57                              | 10.000                                        |
|               | 2     | 6.72                              | 26.530                                        |

$m_u = m_d = 338, m_s = 420, m_c = 1380, m_b = 4275$ (in MeV)

Table 2: Hyperfine Parameters for the symmetric spin-flavour combinations of single heavy baryons

| System | $\nu = 0$ | $\nu = 1$ | $\nu = 2$ |
|--------|-----------|-----------|-----------|
| cqq    | 13        | 1         | 0.625     |
| cqs    | 39        | 3         | 1.875     |
| bqq    | 78        | 1         | 0.625     |
| bqs    | 468       | 6         | 3.750     |

$$\vec{r}_1 = \vec{x}_1 - \vec{x}_3;$$
$$\vec{r}_2 = \vec{x}_2 - \vec{x}_3$$

and

$$r_{12} = |\vec{r}_2 - \vec{r}_1| = |\vec{x}_2 - \vec{x}_1|;$$

$$m_{i3} = \frac{m_i m_3}{m_i + m_3}; \quad m_3 = m_Q$$

Accordingly, the Hamiltonian can be separated into the translationally invariant CM part and the part governing the relative motion given by, $h$ as

$$h = \frac{\nabla^2 r_{13}}{2 m_{13}} - \frac{\nabla^2 r_{23}}{2 m_{23}} - \frac{\nabla r_{1} \cdot \nabla r_{2}}{m_3} + V(r_{1}, r_{2})$$

Here, $m_1, m_2, m_3$ are the constituent quark masses and all the independent orientations of co-ordinates $\vec{r}_1$ and $\vec{r}_2$ are assumed. Under the spherically symmetric approximation, the potential is then written in terms of the magnitudes of the co-ordinates $\vec{r}_1$ and $\vec{r}_2$. 
and $r_2$ as

$$V(r_1, r_2) = -\frac{2\alpha_s}{3} \left( \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_{12}} \right) + \beta(r_{12}^\nu + r_{12}^\nu + r_{12}^\nu) + V_{\text{spin}}$$  \hspace{1cm} (8)

Here, $\alpha_s$ is the running strong coupling constant and $V_{\text{spin}}$ is the spin dependent part of the three body system. We choose the spatial part of the trial wave function as

$$\Psi(r_1, r_2) = f(r_1)f(r_2)$$  \hspace{1cm} (9)

with a normalization condition given by

$$\int |\Psi(r_1, r_2)|^2 d^3r_1 d^3r_2 = 1.$$

Here, $f(r) = \frac{\lambda^{3/2}}{\sqrt{\pi}} e^{-\lambda r}$ and $\lambda$ is a common variational parameter corresponds to both $f(r_1)$ and $f(r_2)$.

We find the expression for the ground state energy $E$ without the spin dependent part of the potential given by Eqn 8, as

$$E = -\lambda^2 + 2\lambda(\lambda - 1) - \frac{5\lambda}{8} + 8\beta\lambda^32^{-3-\nu}\lambda^{-3-\nu}\Gamma(\nu + 3)$$

$$+ \frac{8\beta2^{-\nu}\Gamma(\nu + 6)G_\nu \lambda^{-\nu}}{\nu + 2}$$  \hspace{1cm} (10)

Where,

$$G_\nu = \frac{1}{128}[1 - (-1)^{2\nu + 6} \ _2F_1(1; \nu + 6; 3; 2)] - \frac{1}{192}[1 + (-1)^{2\nu + 6} \ _2F_1(1; \nu + 6; 4; 2)]$$

and $\ _2F_1$ is the Hypergeometric function. We minimize the energy expression given by Eqn 10 to find the variational parameter $\lambda$.

In the single heavy quark baryonic system, the heavy quark acts as a static colour source, with two light quarks revolving around. For simplicity, we will take the baryonic units, in which all the length and energy scales are measured in the unit of the Bohr radius $(m_{\text{red}}2\alpha_s/3)^{-1}$ and Bohr energy $m_{\text{red}}(2\alpha_s/3)^2$ as in [8]. Where $m_{\text{red}}$ is computed with reference to the combination of the lightest flavour with the heavy flavour at the center, as $m_{\text{LQ}} = \frac{m_l m_Q}{m_l + m_Q}$, $m_l$ is mass of the lightest flavour quark. For example, in the case of $qqc$ and $qsc$, we consider $m_{\text{red}} = \frac{m_l m_Q}{m_l + m_Q}$, while in case of the $ssc$, $m_{\text{red}} = \frac{m_s m_Q}{m_s + m_Q}$. The running strong coupling constant is computed using the relation

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \frac{33 - 2n_f}{12\pi} \alpha_s(\mu_0)\ln(\frac{\mu}{\mu_0})}$$  \hspace{1cm} (11)

For the present study, we considered $\alpha_s(\mu_0 = 1\text{GeV}) \approx 0.7$. Though it is an ad-hoc choice, the same value has been employed in our earlier studies on light-heavy mesons [23]. The energy eigen value given by Eqn 10 is computed for the choices of the potential
The spin average mass of baryonic system (without spin contribution) is then obtained as

\[ M_{Qqq} = \Sigma m_i + E \]  

(12)

The quark masses and the potential parameters of the model as listed in Table 1 are obtained so as to get the ground state spin average masses of qqc (2486 MeV), qsc (2568 MeV), qsb (5820 MeV), qsb (5902 MeV) and ssc (6176 MeV) for each potential index \( \nu = 0, 1 \) and 2.

One of the most tricky and important components of hadron spectroscopy is to predict correctly the spin hyperfine split among the \( J^P = \frac{3}{2}^+ \) and \( J^P = \frac{1}{2}^+ \) baryons. There exist many attempts starting from the standard OGE potential corresponds to a free gluon exchange to confined gluon exchange \([24, 25, 26]\). The free gluon exchange interaction between the point like quarks leads to the delta function behavior, while the effect of confinement of the gluons as well as the finite size of the quarks warrant smoothing of the delta function. One of the most suitable form is provided by \([2]\)

\[ V_{spin} = -\frac{A}{4} \alpha_s \sum_{i<j} \frac{e^{-r_{ij}/r_0}}{r_{ij} r_0} \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{6 m_i m_j} \]  

(13)

Here, the parameter \( A \) and the regularization parameter \( r_0 \) are considered as the hyperfine parameters of the model. It is closely similar to the form given by \([2, 9]\) except the way we treat the parameter \( r_0 \). Here we treat \( r_0 \) as a hyperfine parameter related to gluon dynamics, and hence independent of the masses of the interacting quarks as treated by \([2]\). As no well established procedure to evaluate \( r_0 \) from colour gluon dynamics is known, we obtain the optimum values of these hyperfine parameters graphically by studying the behavior of the hyperfine split of \( \Sigma_Q^* - \Sigma_Q, \Xi_Q^* - \Xi_Q \) and \( \Omega_Q^* - \Omega_Q \) for both the charm \((Q = c)\) and beauty \((Q = b)\) baryons at different \( r_0 \) values. The behavior is shown in Fig 1. It is seen that below the range of \( r_0 < 10^{-4} \text{ MeV}^{-1} \) as seen in Fig 1, the split gets saturated while for \( r_0 \geq 0.01 \), the corresponding hyperfine mass difference approaches to zero. Thus we fix our regularization parameter well within the saturation region of \( r_0 \) value to about \( 10^{-6} \text{ MeV}^{-1} \). The other hyperfine parameter, \( A \) of Eqn [13] for different choices of the power index \( \nu \) and for the different quark combinations \((Qqq)\) are listed in Table [2]. The computed masses of \( J^P = \frac{1}{2}^+ \) and \( J^P = \frac{3}{2}^+ \) of the symmetric compositions of Qqq states are listed in Table [3] along with other contemporary model predictions and with known experimental values.

In the case of spin antisymmetric state of \( \Lambda_Q \) baryons, similar study on the masses of \( \Lambda_c \) and \( \Lambda_b \) with \( r_0 \) is shown in Fig 2 and 3 respectively. A similar saturation property of the \( \Lambda_Q \) masses with \( r_0 \) below \( 10^{-4} \text{MeV}^{-1} \) is observed. However, the saturated
masses with respect to $r_0$ of $\Lambda_b (5820 \text{ MeV})$ and $\Lambda_c (2486 \text{ MeV})$ as seen from Fig 2 and 3 are about 200 MeV more than their experimental masses of 5624 MeV and 2286 MeV respectively. It suggests then the requirement of an attractive interaction of about 200 MeV magnitude to the antisymmetric state irrespective of its heavy quark content and potential index. One can add such attractive part to the Hamiltonian of the $\Lambda_Q$ state or can absorb it in the regularization parameter of the spin-hyperfine interaction.

Hence, we attain $r_0$ values away from the mass saturation region nearer to the pole, as shown in Fig 2 and 3. Accordingly, without changing $A$ parameters for $\Lambda_Q (Q \in c, b)$, we get the regularization parameter $r_0$ corresponding to the experimental masses of $\Lambda_c$ and $\Lambda_b$. The details are summarized in Table 4. Our results are also compared with other model predictions as well as with experimental values. The mass difference between $\Sigma_Q^* - \Lambda_Q$, $\Xi_Q^* - \Xi_Q$ and $\Sigma_Q^* - \Sigma_Q$ are also listed in Table 5 along with the experimental values as well as with other model predictions.

3 Magnetic Moments of the Single Heavy Flavour Baryons

For the computation of the magnetic moments, we consider the mass of bound quarks inside the baryons as its effective mass taking into account of its binding interactions with other two quarks described by the Hamiltonian given in Eqn 1. The effective mass
Table 3: Single heavy baryon masses of symmetric spin-flavour combinations (masses are in MeV)

| Baryon | Quark Content | J P | Others | J P | Others |
|--------|---------------|-----|--------|-----|--------|
| \(\Sigma_c\) | qqc | 0 | 2444 | 2453 | 2507 | – |
| | | 1 | 2445 | 2451 | 2507 | 2516 |
| | | 2 | 2443 | 2460±80 | 2508 | 2440±70 |
| | | | | | 2454 | 2518 |
| | | | | | 2448 | 2505 |
| \(\Xi_c\) | qsc | 0 | 2455 | 2466 | 2625 | – |
| | | 1 | 2464 | 2485 | 2637 | 2672 |
| | | 2 | 2452 | 2468 | 2627 | 2650 |
| | | | | | 2473 | 2680 |
| | | | | | 2496 | 2633 |
| | | | | | 2468 | 2646 |
| | | | | | 2481 | 2654 |
| \(\Omega_c\) | ssc | 0 | 2674 | 2698 | 2758 | – |
| | | 1 | 2674 | 2696 | 2758 | 2757 |
| | | 2 | 2673 | 2710 | 2759 | 2770 |
| | | | | | 2678 | 2752 |
| | | | | | 2701 | 2759 |
| | | | | | 2698 | 2768 |
| \(\Sigma_b\) | qqb | 0 | 5806 | 5820 | 5827 | – |
| | | 1 | 5807 | 5801 | 5827 | 5823 |
| | | 2 | 5805 | 5806±70 | 5828 | 5780±70 |
| | | | | | 5805 | 5834 |
| | | | | | 5808 | 5829 |
| | | | | | 5789 | 5844 |
| \(\Xi_b\) | qsb | 0 | 5826 | 5624 | 5940 | – |
| | | 1 | 5827 | 5872 | 5939 | 5936 |
| | | 2 | 5820 | 5820 | 5943 | 5980 |
| | | | | | 5847 | 5959 |
| | | | | | 5825 | 5967 |
| | | | | | 5805 | 5963 |
| \(\Omega_b\) | ssb | 0 | 6156 | 6040 | 6187 | – |
| | | 1 | 6156 | 6005 | 6186 | 6065 |
| | | 2 | 6154 | 6060 | 6187 | 6090 |
| | | | | | 6040 | 6060 |
| | | | | | 6037 | 6090 |
| | | | | | 6065 | 6088 |
Figure 2: Dependance of $r_0$ on the masses of $\Lambda_c$.

Figure 3: Dependance of $r_0$ on the masses of $\Lambda_b$. 
Table 4: The Hyperfine Parameters for $\Lambda_Q$ Baryons

| Baryon ($\Lambda_Q$) | $\nu$ | $A$  | $r_0$ in MeV$^{-1}$ |
|----------------------|-------|------|---------------------|
| $\Lambda_c(2286)$    | 0     | 13   | $3.266 \times 10^{-3}$ |
|                      | 1     | 1    | $1.395 \times 10^{-3}$ |
|                      | 2     | 0.625 | $1.173 \times 10^{-3}$ |
| $\Lambda_b(5624)$   | 0     | 78   | $4.860 \times 10^{-3}$ |
|                      | 1     | 1    | $1.144 \times 10^{-3}$ |
|                      | 2     | 0.625 | $0.940 \times 10^{-3}$ |

Table 5: $J^P = \frac{3}{2}^+$ and $\frac{1}{2}^+$ ground state Mass difference at different Potential index, $\nu$ (Masses in MeV)

| Mass difference | $\nu = 0$ | $\nu = 1$ | $\nu = 2$ | Expt. | [2] | [9] |
|-----------------|-----------|-----------|-----------|-------|-----|-----|
| $\Sigma_c^{*+} - \Lambda_c^+$ | 221       | 221       | 222       | 232   | 233 | 235 |
| $\Sigma_c^{*+} - \Sigma_c^+$   | 63        | 62        | 65        | 64    | 57  | 65  |
| $\Sigma_b^{*0} - \Lambda_b^0$  | 203       | 203       | 204       | 205   | 220 | 205 |
| $\Sigma_b^{*0} - \Sigma_b^0$   | 21        | 20        | 23        | 21    | 55  | 23  |
| $\Xi_c^{*0} - \Xi_c^0$         | 170       | 173       | 175       | 178   | 137 | 187 |
| $\Xi_b^{*0} - \Xi_b^0$         | 114       | 112       | 123       | $-$   | 178 | 64  |

for each of the constituting quark $m_i^{eff}$ can be defined as [9]

$$m_i^{eff} = m_i \left(1 + \frac{\langle H \rangle}{\sum_i m_i}\right) \quad (14)$$

such that the corresponding mass of the baryon is given by

$$M_B = \sum_i m_i + \langle H \rangle = \sum_i m_i^{eff} \quad (15)$$

Here, $\langle H \rangle$ includes the spin hyperfine interaction also. For example, the effective mass of the $u$ and $d$ quark from the chosen values of the mass parameter will be different when it is in udc combinations or in udb combinations as $\sum_i m_i^{udc} \neq \sum_i m_i^{udb}$. Now, the magnetic moment of baryons are obtained in terms of the bound quarks as

$$\mu_B = \sum_i \langle \phi_{sf} | \mu_i \overline{\sigma}_i | \phi_{sf} \rangle \quad (16)$$
Table 6: Magnetic moments of single heavy charm and beauty baryons in terms of Nuclear magneton $\mu_N$ (* indicates $J^P = \frac{3}{2}^+$ state.)

| Baryon | $\nu = 0$ | $\nu = 1$ | $\nu = 2$ | 9  | 10  | RQM [5] | NRQM [5] |
|--------|-----------|-----------|-----------|----|-----|---------|---------|
| $\Sigma^{++}_c$ | 1.9487 | 1.9479 | 1.9494 | 2.2720 | 2.5320 | 1.7600 | 1.8600 |
| $\Sigma^{*++}_c$ | 3.4071 | 3.4071 | 3.4058 | 3.8420 | -- | -- | -- |
| $\Sigma^{+}_c$ | 0.3918 | 0.3917 | 0.3920 | 0.5000 | 0.5480 | 0.3600 | 0.3700 |
| $\Sigma^{*+}_c$ | 1.1306 | 1.1306 | 1.1301 | 1.2520 | -- | -- | -- |
| $\Xi^{+}_c$ | 0.5105 | 0.5087 | 0.5112 | 0.7090 | 0.2110 | 0.4100 | 0.3700 |
| $\Xi^{*+}_c$ | 1.2700 | 1.2642 | 1.2690 | 1.5130 | -- | -- | -- |
| $\Omega^{0}_c$ | -0.9497 | -0.9497 | -0.9501 | -0.9580 | -0.8350 | -0.8500 | -0.8500 |
| $\Omega^{*0}_c$ | -0.8339 | -0.8339 | -0.8336 | -0.8650 | -- | -- | -- |
| $\Sigma^{0}_c$ | -1.1650 | -1.1645 | -1.1660 | -1.0120 | -1.4350 | -1.0400 | -1.1100 |
| $\Sigma^{*0}_c$ | -1.1460 | -1.1460 | -1.1455 | -0.8480 | -- | -- | -- |
| $\Xi^{0}_c$ | -1.1011 | -1.0971 | -1.1025 | -0.9640 | 0.3600 | -0.9500 | -0.9800 |
| $\Xi^{*0}_c$ | -0.9910 | -0.9865 | -0.9902 | -0.6880 | -- | -- | -- |
| $\Sigma^{+}_b$ | 2.1249 | 2.1246 | 2.1253 | 2.2260 | 2.6690 | 2.0700 | 2.0100 |
| $\Sigma^{*+}_b$ | 3.0827 | 3.0827 | 3.0821 | 3.2390 | -- | -- | -- |
| $\Sigma^{0}_b$ | 0.54683 | 0.54673 | 0.54692 | 0.5910 | 0.6820 | 0.5300 | 0.5200 |
| $\Sigma^{*0}_b$ | 0.72404 | 0.72404 | 0.72392 | 0.7910 | -- | -- | -- |
| $\Omega^{0}_b$ | -0.8047 | -0.8047 | -0.8050 | -0.9580 | -0.7030 | -0.8200 | -0.7100 |
| $\Omega^{*0}_b$ | -1.2918 | -1.2920 | -1.2918 | -1.1990 | -- | -- | -- |
| $\Sigma^{0}_b$ | -1.0313 | -1.0311 | -1.0315 | -1.0450 | -1.3050 | -1.0100 | -0.9700 |
| $\Sigma^{*0}_b$ | -1.6346 | -1.6346 | -1.6343 | -1.6550 | -- | -- | -- |
| $\Xi^{0}_b$ | 0.6580 | 0.6579 | 0.6587 | 0.7650 | 0.6600 | 0.6500 | 0.6500 |
| $\Xi^{*0}_b$ | 0.8751 | 0.8753 | 0.8747 | 1.0410 | -- | -- | -- |
| $\Xi^{0}_b$ | -0.9407 | -0.9406 | -0.9417 | -0.9010 | -0.0550 | -0.9100 | -0.8400 |
| $\Xi^{*0}_b$ | -1.4770 | -1.4773 | -1.4762 | -1.0950 | -- | -- | -- |
| $\Lambda^{+}_c$ | 0.4077 | 0.4077 | 0.4077 | 0.3840 | 0.3410 | 0.3800 | 0.3700 |
| $\Lambda^{0}_b$ | -0.0640 | -0.0640 | -0.0640 | -0.0640 | -0.0600 | -0.0690 | -0.0600 |
Table 7: Baryon-meson mass Inequalities for $J^P = \frac{3}{2}^+$ single heavy Baryons. (masses are in MeV)

| Baryon-meson Inequalities | $\nu$ | Baryon mass in MeV | R.H.S of the inequality relation |
|---------------------------|-------|--------------------|---------------------------------|
| $m_{\Sigma^{++}}(qqc) \geq \frac{1}{2}(m_\rho + 2m_{D^*})$ | 0     | 2507               | $\geq$ 2394                     |
|                           | 1     | 2507               | $\geq$                          |
|                           | 2     | 2508               | $\geq$                          |
| $m_{\Xi}(qsc) \geq \frac{1}{2}(m_{K^*} + m_{D^*} + m_{D_{s^*}})$ | 0     | 2625               | $\geq$ 2507                     |
|                           | 1     | 2637               | $\geq$                          |
|                           | 2     | 2627               | $\geq$                          |
| $m_{\Omega}(ssc) \geq \frac{1}{2}(m_\phi + 2m_{D^*})$ | 0     | 2758               | $\geq$ 2518                     |
|                           | 1     | 2758               | $\geq$                          |
|                           | 2     | 2759               | $\geq$                          |
| $m_{\Sigma^b}(qqb) \geq \frac{1}{2}(m_\rho + 2m_{B^*})$ | 0     | 5827               | $\geq$ 5710                     |
|                           | 1     | 5827               | $\geq$                          |
|                           | 2     | 5828               | $\geq$                          |
| $m_{\Xi^b}(qsb) \geq \frac{1}{2}(m_{K^*} + m_{B^*} + m_{B_{s^*}})$ | 0     | 5940               | $\geq$ 5793                     |
|                           | 1     | 5939               | $\geq$                          |
|                           | 2     | 5943               | $\geq$                          |
| $m_{\Omega^b}(ssb) \geq \frac{1}{2}(m_\phi + 2m_{B_{s^*}})$ | 0     | 6187               | $\geq$ 5879                     |
|                           | 1     | 6186               | $\geq$                          |
|                           | 2     | 6187               | $\geq$                          |
Table 8: Baryon-meson mass Inequalities for $J^P = \frac{1}{2}^+$ single heavy Baryons. (masses are in MeV)

| Baryon-meson Inequalities | $\nu$ | Mass in MeV | R.H.S of the inequ. relation |
|---------------------------|-------|-------------|------------------------------|
| $m_{\Sigma_c}(qqc) \geq \frac{1}{4}(2m_{\rho} + 3m_D + m_{D^*})$ | 0     | 2444        | $\geq$ 2288                  |
|                           | 1     | 2445        | $\geq$                       |
|                           | 2     | 2443        | $\geq$                       |
| $m_{\Xi_c}(qsc) \geq \frac{1}{4}[(m_K + m_{K^*}) + (m_{D_1} + m_D) + (m_{D^*} + m_{D_1^*})]$ | 0     | 2455        | $\geq$ 2336                  |
|                           | 1     | 2464        | $\geq$                       |
|                           | 2     | 2452        | $\geq$                       |
| $m_{\Omega_c}(ssc) \geq \frac{1}{4}(2m_{\phi} + 3m_{D_s} + m_{D_s^*})$ | 0     | 2674        | $\geq$ 2514                  |
|                           | 1     | 2674        | $\geq$                       |
|                           | 2     | 2673        | $\geq$                       |
| $m_{\Sigma_b}(gqb) \geq \frac{1}{4}(2m_{\rho} + 3m_B + m_{B^*})$ | 0     | 5806        | $\geq$ 5675                  |
|                           | 1     | 5807        | $\geq$                       |
|                           | 2     | 5805        | $\geq$                       |
| $m_{\Xi_b}(qsb) \geq \frac{1}{4}[(m_K + m_{K^*}) + (m_{B_1} + m_B) + (m_{B^*_1} + m_{B_1^*})]$ | 0     | 5826        | $\geq$ 5682                  |
|                           | 1     | 5827        | $\geq$                       |
|                           | 2     | 5820        | $\geq$                       |
| $m_{\Omega_b}(ssb) \geq \frac{1}{2}(m_{\phi} + 2m_{B_s^*})$ | 0     | 6156        | $\geq$ 5880                  |
|                           | 1     | 6156        | $\geq$                       |
|                           | 2     | 6154        | $\geq$                       |
| $m_{\Lambda_c}(qqc) \geq \frac{1}{4}(2m_{\pi} + 3m_D + m_{D^*})$ | 0     | 2286        | $\geq$ 1973                  |
|                           | 1     | 2286        | $\geq$                       |
|                           | 2     | 2286        | $\geq$                       |
| $m_{\Lambda_b}(gqb) \geq \frac{1}{4}(2m_{\pi} + 3m_{B} + m_{B^*})$ | 0     | 5624        | $\geq$ 5359                  |
|                           | 1     | 5624        | $\geq$                       |
|                           | 2     | 5624        | $\geq$                       |
where

\[ \mu_i = \frac{e_i}{2m_i} \tag{17} \]

Here, \( e_i \) and \( \sigma_i \) represents the charge and the spin of the quark constituting the baryonic state. We have employed the spin flavour wavefunction \( |\phi_{sf}\rangle \) of the symmetric and antisymmetric states of the baryons as used in [9]. Using the spin flavour wave functions corresponds to \( J^P = \frac{1}{2}^+ \) and \( J^P = \frac{3}{2}^+ \), we compute the magnetic moments of the baryons containing a single charm or beauty quark. Our results are listed in Table 5 for the choices of \( \nu = 0,1 \) and 2. Other theoretical model predictions of the magnetic moments are also listed for comparison.

### 4 Results and Discussions

We have employed a simple nonrelativistic approach with coulomb plus power potential to study the masses and magnetic moments of the single heavy flavour baryons. The model parameters for each choices of the potential index, \( \nu \) are listed in Table 1 along with the corresponding variational parameter \( \lambda \) of the trial wavefunction. The model parameters are obtained to get the ground state spin average masses of the Qqq systems.

The spin hyperfine interaction has been taken into account according to [2] where the interaction is expressed in terms of two hyperfine parameters, \( A \) and a regularization parameter \( r_0 \). It is interesting to see that the hyperfine mass splitting of the symmetric spin combinations of the two body interaction gets saturated with \( r_0 \) within a range, \( r_0 < 10^{-4} \text{ MeV}^{-1} \). Similar saturation property for the masses of \( \Lambda_Q (Q \in b, c) \) is also seen with respect \( r_0 \). The spin hyperfine parameter \( A \), is now fixed to yield the experimental mass difference of \( \Sigma^*_Q - \Sigma_Q \) well within the saturated range of \( r_0 = 10^{-6} \text{ MeV}^{-1} \). The resulting hyperfine parameter, \( A \) for different choices of \( \nu \) are shown in Table 2 for different quark compositions. It can be seen that the parameter \( A \) vary from \( \nu = 0 \) to 2 according to a fixed ratio given by \( 13 : 1 : \frac{5}{8} \) for the cqq system and as \( 78 : 1 : \frac{5}{8} \) in the case of bqq systems. For the \( \Xi_Q \) states, it is found that the ratio triples in the case of \( \Xi_c \) as \( \nu \) varies from 0 to 2 and becomes six times in the case of \( \Xi_b \). It probably corresponds to a statistical factor related to the combination of the light quark composition \( 3C_2 \) with charm (cqs) and \( 4C_2 \) in the case of (bqs) systems.

In Table 5 we provide the \( \Sigma^*_Q - \Lambda_Q, \Xi^*_Q - \Xi_Q \) and \( \Sigma^*_Q - \Sigma_Q \) mass differences obtained from the present study for each case of the model potential, \( \nu = 0 \) to 2. The masses of \( J^P = \frac{1}{2}^+ \) and \( J^P = \frac{3}{2}^+ \) baryons as listed in Table 6 are in agreement with the well known baryon-meson mass inequalities [31]. The inequality relations are shown
in Table 7 and 8 for the $J^P = \frac{3}{2}^+$ and $\frac{1}{2}^+$ states respectively. The masses of single heavy baryons studied here are found to be well above the equality limit satisfying the inequality relations.

In conclusion, our results of single heavy flavour baryons are found to be in accordance with predictions of more realistic computations like the lattice results [27] and Faddeev approach [2] as well as with other model predictions. The behavior of hyperfine mass splitting studied here with respect to the regularization parameter, $r_0$ and the resultant saturation property of the mass splitting as seen in Fig 1 and consequently for the masses of $\Lambda_Q (Q \in b,c)$ seen in Fig 2 and 3 in the range of $r_0 \leq 10^{-4} \text{ MeV}^{-1}$ is an important feature that might help us to understand the nature of gluon exchange interactions between the pair of quarks inside the Baryon. For $\Lambda_Q$ Baryons, the extra 200 MeV, at the saturation range of $r_0 \leq 10^{-4} \text{ MeV}^{-1}$, above their experimental masses indicates the requirement of an attractive interaction potential for the description of $\Lambda_Q$ Baryons which is independent of the potential model index as well as the heavy flavour content. Detail analysis of these issues related to the single heavy flavour baryons will be more meaningful when we acquire more experimental information about these states and their orbital excitations. Some of these baryons and orbital excitation are expected to be observed at future experiments.

It is important to note that the predictions of the magnetic moment of single heavy Baryons studied here are with no additional parameters. Our results on magnetic moments are compared with one of our recent predictions using hypercentral potential [9] and predictions based on Faddeev formalism for the baryons [10] as well as with relativistic (RQM) and the nonrelativistic quark model (NRQM) predictions of [5] in Table 6. The special feature of the present study to compute the magnetic moments of single heavy flavour baryons is the consideration of the effective interactions of the bound state quarks by defining an effective bound state mass to the quarks within the baryon, which vary according to different interquark potential as well as with quark compositions.

Experimental measurements of the heavy flavour baryon magnetic moments are sparse and only few experimental groups (BTeV and SELEX Collaborations) are expected to do measurements in near future.

We conclude that the interquark potential interactions and the particular spin hyperfine structure assumed in the present study play significant role in the description of heavy flavour baryonic properties in particular for their spin hyperfine splitting and magnetic moments. We look forward future observations of more experimental bary-
onic states in the single heavy flavour sector at different heavy flavour high luminosity experiments.

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