Casimir pressure on a thin metal slab

M S Tomaš and Z Lenac

1 Rudjer Bošković Institute, P. O. Box 180, 10002 Zagreb, Croatia
2 Department of Physics, University of Rijeka, 51000 Rijeka, Croatia

E-mail: tomas@thphys.irb.hr

Abstract. We consider the vacuum-field pressure on boundaries of a metal slab in the middle of a cavity with perfectly reflecting mirrors adopting the plasma model for the metal and paying special attention to the surface plasmon polariton contribution to the pressure. We demonstrate that, with increasing cavity length, the pressure on a thin \((d \ll \lambda_P)\) slab in this system decreases from the Casimir pressure \(F_C = -\pi^2 \bar{h} c / 240 d^4\) at zero slab-mirror distances to the non-retarded force per unit area \(F_{nr} = 1.19 (d/\lambda_P) F_C\) in the case of an isolated slab. In the first case the pressure is entirely due to the photonic modes propagating through the metal whereas in the second case it is entirely due to the (non-retarded) surface plasmon modes supported by the free-standing thin slab. In either case the pressure decreases with the slab thickness. These considerations indicate that the vacuum-field pressure on a thin metal layer (and its modal structure) can be in a symmetric cavity significantly influenced when changing the cavity length.

1. Introduction

The Casimir effect most commonly refers to the existence of forces between neutral macroscopic bodies due to the change of the zero-point energy of the electromagnetic field in confined space [1, 2, 3, 4]. Soon upon its prediction for two perfectly conducting plates in vacuum [5], the theory of the Casimir effect was extended to more realistic systems consisting of two dielectrics separated by a vacuum gap [6] and by a gap filled by a medium [7, 8, 9] and, more recently, to systems involving more layers [10] including general dielectric and magnetodielectric multilayers [11, 12, 13, 14]. Evidently, when the gap between two stacks of layers is filled by a medium, as in multilayers, the vacuum-field force (per unit area) on the stacks can also be regarded as the pressure on the medium between them. Moreover, since the vacuum-field fluctuations are always present, the pressure on the medium persists even in the absence of other layers, that is, even in the case of a free standing single material slab. Consequently, as pointed out recently [15], in addition to the traditional Casimir force due to the presence of other layers, every material layer in a multilayered systems experiences a vacuum-field pressure on its boundaries. Changes caused by this pressure (e.g. change of the layer thickness) can be taken as an alternative signature of the Casimir effect and it is therefore of fundamental interest to explore it in more details. From the practical point of view, however, particularly interesting systems in this respect are those involving metallic plates as metal components are often met in micromechanical (MEMS) and nanomechanical (NEMS) devices [16, 17].

Vacuum-field pressure on a metal slab has already been addressed (to some extent) by Dzyaloshinskii et al. [7] when discussing the Casimir force between two dielectric media separated by a metal layer. The same system has also recently been considered by Imry [18] who pointed out strong dependence of the zero-point radiation pressure on a metal film on properties of
the surrounding media. Very recently, Benassi and Calandra [15] used the Lifshitz formula to calculate the pressure on a metal slab and explore its dependence on the properties of the slab as well as on the distance and properties of a nearby (metallic) substrate. By combining a Lifshitz-like formula and the mode summation method, in our previous work [19] we have explored the effect of surrounding media as well as of a cavity on the pressure on a metal slab paying special attention to the contribution of the surface polariton (SP) modes to the pressure. In the present work, we consider in more details the effect of a cavity on the pressure on surfaces of a metal slab in its center and derive several new results concerning this pressure. Since the ordinary Casimir force on the slab vanishes in symmetric configurations, consideration of this system is a very convenient way to explore the pressure on the slab surfaces. As in aforementioned works, we adopt free-electron (plasma) model to describe the metal and, for simplicity, assume perfectly reflecting cavity mirrors. Accordingly, our aim here is to establish trends of the pressure with the system parameters and calculate its limiting values rather than to discuss it for a realistic system.

The paper is organized as follows. In Section II, we briefly adapt the theory of the ordinary Casimir force on a slab in a cavity [11, 14, 20] to include also the vacuum-field pressure on the slab surfaces and derive a Lifshitz-like formula for this pressure (see also [15]). In Sections III-V, we use this formula to discuss the pressure on surfaces of a free standing metal slab, of a metal layer between perfect mirrors and of a metal slab in an ideal cavity, respectively. Our conclusions are summarized in Section VI.

2. Preliminaries

Consider a dielectric slab inserted in a planar cavity, as depicted in Fig. 1. The total force (per unit area) acting on the slab consists of the pressure \( F = f_s \) on the slab boundaries and the slab-mirror interaction force \( F' = f_2 - f_1 \) [15], where according to the theory of the Casimir force in multilayers [11]

\[
f_j = -\frac{\hbar}{2\pi^2} \int_0^\infty d\xi \int_0^\infty dkk\kappa_j \sum_{q=p,s} \frac{1 - D_{qj}(i\xi, k)}{D_{qj}(i\xi, k)}, \quad D_{qj}(i\xi, k) = 1 - r_j^q r_j^q e^{-2\kappa_j d_j}. \tag{1}
\]

Here \( \kappa_j(i\xi, k) = \sqrt{\varepsilon_j(i\xi)k^2/c^2 + k^2} \) is the perpendicular wave vector at the imaginary frequency in the \( j \)th layer and \( r_j^q(i\xi, k) \) are the reflection coefficients of the right and left stack of layers bounding the layer.

![Figure 1. System considered schematically.](image-url)
Considering first the pressure on the slab, we have

\[ F = -\frac{\hbar}{2\pi^2} \int_0^{\infty} d\xi \int_0^{\infty} dk k \kappa_s \sum_{q=p,s} r^q_{s-} r^q_{s+} e^{-2\kappa_d d_s} \left( 1 - r^q_{s-} r^q_{s+} e^{-2\kappa_d d_s} \right), \tag{2} \]

where

\[ r^q_{s-}(i\xi, k) = \frac{-\rho^q + R^q_{1(2)} e^{-2\kappa_d d_s}}{1 - \rho^q R^q_{1(2)} e^{-2\kappa_d d_s}} \tag{3} \]

are reflection coefficients for the waves reflected within the slab. Here \( \kappa(i\xi, k) \equiv \kappa_1 = \kappa_2 = \sqrt{\xi^2/k^2 + k^2} \) is the perpendicular wave vector in the cavity,

\[ \rho^P(i\xi, k) = \frac{\varepsilon_s k - \kappa_s}{\varepsilon_s k + \kappa_s}, \quad \rho^S(i\xi, k) = \frac{\kappa - \kappa_s}{\kappa + \kappa_s}, \tag{4} \]

are the vacuum-slab reflection coefficients and \( R^q_{1(2)}(i\xi, k) \) are those of the mirrors. According to Eqs. (2) and (3), \( f_s \) can be rewritten as

\[ F = -\frac{\hbar}{2\pi^2} \int_0^{\infty} d\xi \int_0^{\infty} dk k \kappa_s \sum_{q=p,s} \frac{\rho^q - R^q_{1} e^{-2\kappa_d d_s} (\rho^q - R^q_{2} e^{-2\kappa_d d_s}) e^{-2\kappa_s d_s}}{D_{qs}}, \tag{5} \]

where

\[ \tilde{D}_{qs}(i\xi, k) = 1 - \rho^q e^{-2\kappa_s d_s} \rho^q (1 - e^{-2\kappa_s d_s}) (R^q_{1} e^{-2\kappa_d d_s} + R^q_{2} e^{-2\kappa_d d_s}) + (\rho^q e^{-2\kappa_s d_s} R^q_{1} R^q_{2} e^{-2\kappa_d (d_1 + d_2)}). \tag{6} \]

The traditional force per unit area on the slab due to the presence of the mirrors \( F' = f_2 - f_1 \) is found similarly using [11]

\[ r^q_{1-2+}(i\xi, k) = R^q_{1(2)}, \quad r^q_{1+(2-)}(i\xi, k) = r^q + \frac{\rho^q e^{-2\kappa_d d_s} (R^q_{1} e^{-2\kappa_d d_s} - R^q_{2} e^{-2\kappa_d d_s})}{1 - \rho^q R^q_{2(1)} e^{-2\kappa_d d_s}}, \tag{7} \]

where \( r^q(i\xi, k) \) and \( t^q(i\xi, k) \) are the Fresnel coefficients for the (whole) slab. We find that \( F' \) is given by Eq. (5) provided that the denominators in that equation are replaced by [11]

\[ \rho^q(1 - e^{-2\kappa_s d_s}) (R^q_{2} e^{-2\kappa_d d_s} - R^q_{1} e^{-2\kappa_d d_s}) \tag{8} \]

and that \( \kappa_s \to \kappa \) in front of the sum. Clearly, in contrast to \( F \), the pressure \( F' \) vanishes when the slab is in the center (\( d_1 = d_2 \)) of a symmetric \((R^q_{1} = R^q_{2})\) cavity.

Frequencies \( \omega_n^R(k) \) of SP and other bound modes supported by the system are given as solutions of

\[ D_{qs}(\omega, k) = 1 - r^q_{s-} r^q_{s+} e^{-2\alpha_s d_s} = 0, \quad \alpha_s(\omega, k) \equiv \kappa_s(-i\omega, k) = \sqrt{k^2 - \varepsilon_s \omega^2/c^2}, \tag{9} \]

or, equivalently, \( \tilde{D}_{qs}(\omega, k) = 0 \) in the corresponding part of the \((\omega, k)\)-plane. Separate contribution of each SP mode to the Casimir force can therefore be obtained by using the real frequency counterpart of Eq. (5) and calculating the corresponding residuum. Alternatively, since this equation is (for a lossless system) compatible with the standard definition of the Casimir energy (with respect to the slab)

\[ E_s = A \int \frac{d^2 k}{(2\pi)^2} \sum_{q=p,s} \sum_{n} \frac{\hbar}{2} [\omega_n^R(k, d_s) - \omega_n^R(k, d_s \to \infty)], \tag{10} \]
we may calculate the surface contribution to the pressure using directly
\[
F_S = -\frac{1}{A} \frac{\partial E^{\text{SP}}_s}{\partial d_s} = -\frac{\hbar}{4\pi} \int_0^\infty \frac{dk}{\kappa_s} \sum_\sigma \frac{\partial \omega_\sigma(k)}{\partial d_s}, \tag{11}
\]
where \(\sigma\) enumerates SP modes. In the following, we use this method to perform a modal analysis of the pressure on a thin metal slab in a symmetric cavity. Clearly, in such a configuration, frequencies of SP (and other) modes are found as solutions of
\[
r^q_s(\omega, k)e^{-\alpha_s d_s} = \pm 1, \quad r^q_s(\omega, k) = \frac{-\rho^s + R^q_i e^{-2\alpha_s d_s}}{1 - \rho^s R^q_i e^{-2\alpha_s d_s}}, \quad \alpha(\omega, k) = \sqrt{k^2 - \omega^2/c^2}, \tag{12}
\]
and the modes are further characterized by an index \(\nu = \pm\) describing their symmetry with respect to the central plane of the system.

3. Free-standing metal slab
Let us start with the case of a free-standing metallic slab. Adopting the electron plasma model, the slab is described by the dielectric function
\[
\varepsilon_s(i\xi) = 1 + \frac{\omega_P^2}{\xi^2}, \tag{13}
\]
where \(\omega_P\) is the metallic plasma frequency. Letting \(R^q_i(2) = 0\) Eq. (5), the pressure on the slab is then given by
\[
F = -\frac{\hbar}{2\pi^2} \int_0^\infty d\xi \int_0^\infty d\kappa_s \kappa_s \sum_{q=p,s} \frac{[\rho^q(i\xi, k)]^2 e^{-2\alpha_s d_s}}{1 - [\rho^q(i\xi, k)]^2 e^{-2\alpha_s d_s}}, \tag{14}
\]
with \(\rho^q(i\xi, k)\) given by Eqs. (4) and (13). Note that this formula differs from the corresponding formula for the standard Casimir force (per unit area) between two metal half-spaces only in the (explicit) presence of \(\kappa_s\) instead of \(\kappa\).

For a thin, \(d_s \ll c/\omega_P\), slab the main contribution to \(F\) comes from large wave vectors. Accordingly, we may let \(\kappa_s \simeq \kappa \simeq k\) and consequently \(\rho^s \simeq \rho^s_{nr} = (\varepsilon_s - 1)/(\varepsilon_s - 1)\) and \(\rho^s \simeq \rho^s_{nr} = 0\) in Eq. (14). Thus, the pressure on the slab is in this nonretarded (quasistatic) approximation equal to the pressure on two identical semi-infinite metals separated by a thin vacuum gap. Introducing \(x = \xi/\omega_P\) and \(t = 2kd\) as the integration variables, in this way we obtain
\[
F_{nr} = -\frac{\hbar \omega_P}{16\pi^2 d^3} \int_0^\infty dx \int_0^\infty dt t^2 \frac{e^{-t}}{(2x^2 + 1)^2 - e^{-t}} = -0.00781 \frac{\hbar \omega_P}{d^3}. \tag{15}
\]
It is easy to see that \(F_{nr}\) is entirely due to the two surface plasmon modes supported by the slab (or two semi-infinite free-electron metals) [18, 24, 25, 26, 27]. Indeed, from the nonretarded limit of Eq. (12)
\[
\frac{\varepsilon_s(\omega)}{\varepsilon_s(\omega)} - 1 e^{-kd_s} = \mp 1
\]
we find familiar frequencies of surface plasmons
\[
\omega_\pm(k) = \frac{\omega_P}{\sqrt{2}} \sqrt{1 \pm e^{-kd_s}}, \tag{16}
\]
so that the integrand in Eq. (3) becomes:
\[
-k \frac{\partial \omega_\pm(k)}{\partial d_s} = \pm \frac{\omega_P}{2\sqrt{2}} \frac{k^2 e^{-kd_s}}{\sqrt{1 \pm e^{-kd_s}}}.\]
Using this in Eq. (3), the total (nonretarded) contribution to the pressure of the two plasmon modes is easily calculated to be equal to $F_{nr}$ given above as a result of compensation between the pressing ($F_{SP}^- = 7.83F_{nr}$) and relaxing ($F_{SP}^+ = -6.83F_{nr}$) contributions from the $\omega_-$ and $\omega_+$ mode, respectively.

To estimate the pressure on a thick slab, we proceed in the standard way [6, 7] and introduce in Eq. (14) the variable $p$ instead of $k$ by letting $\kappa_s = \sqrt{\varepsilon_s(\xi^2)}\xi p/c$. With $x = \xi/\omega_P$, this gives

$$F \simeq \frac{hc k_P^3}{2\pi^2} \int_0^\infty dx (1 + x^2)^{3/2} \int_1^\infty dpp^2 \sum_{q=p,s} [\rho^q(i\xi\omega_P,p)]^2 e^{-2p\sqrt{1 + x^2}k_P d_s},$$

(17)

where $k_P = \omega_P/\lambda_P = 2\pi/\lambda_P$ and

$$\rho^p(i\xi\omega_P,p) = \frac{(1 + x^2)s - x^2p}{(1 + x^2)s + x^2p}, \quad \rho^s(i\xi\omega_P,p) = \frac{s - p}{s + p}, \quad s = \sqrt{p^2 - 1 + \frac{x^2}{1 + x^2}}.$$ (18)

Since for $k_P d_s \gg 1$ the main contribution to the integral comes from small-$x$ region, we can let $x = 0$ everywhere except in the exponents where we use $\sqrt{1 + x^2} \approx 1 + x^2/2$. Retaining only the leading terms, in this way we obtain

$$F \simeq \frac{hc k_P^3}{4\pi \sqrt{\pi k_P d_s}} \int_1^\infty dpp^3/2 e^{-2p\sqrt{1 + x^2}k_P d_s} \left[ 1 + \frac{(\sqrt{p^2 - 1} - p)^2}{\sqrt{p^2 - 1} + p^2} \right]$$

$$\simeq \frac{hc k_P^3}{4} \frac{e^{-2k_P d_s}}{(\pi k_P d_s)^{3/2}}, \quad k_P d_s \gg 1,$$ (19)

where the final result follows upon a partial integration. Accordingly, as already noted by Dzyaloshinskii et al [7], in the plasma model for the metal the pressure on the slab surfaces exponentially decreases with $d_s$.

**Figure 2.** Vacuum-field pressure on a free standing metal slab relative to its nonretarded value as a function of the slab thickness. Lower line gives the surface polariton contribution to the total vacuum-field pressure.

**Figure 3.** Surface polariton contribution (dashed line) to the total vacuum-field pressure (full line) on a free standing metal slab. Separate contributions $F_{SP}^\pm$ of the two surface polariton modes are presented by dotted lines.
pressure on a free standing slab relative to its nonretarded value \( F_{ur} \) [Eq. (15)] and the surface polariton contribution to it \( F_S \), respectively. As seen, owing to the field retardation, the true Casimir pressure \( F \) deviates significantly from \( F_{ur} \) for slab thicknesses \( d_s / \lambda_P > 0.01 \) and can be in this region rather well approximated by the contribution of two SP modes supported by the slab \( F_S \). Of course, at even larger slab thicknesses \( (d_s / \lambda_P > 1) \), \( F_S \) vanishes owing to the decoupling of SP modes at two slab surfaces. The same happens to \( F \) since, as shown above, it attenuates exponentially for large thicknesses of the slab.

4. Metal slab between perfect mirrors

Next we consider the other limiting case namely that of a metal layer sandwiched between perfect mirrors. Such a situation is described by Eqs. (2) and (3) when letting \( R_i^q = \delta_{qp} - \delta_{qs} \) and \( d_i = 0 \). From Eq. (3) it then follows that also \( r_{s\pm}^q = \delta_{qp} - \delta_{qs} \) and we have

\[
F = -\frac{\hbar}{\pi^2} \int_0^\infty d\xi \int_0^\infty dk k \kappa_s \frac{e^{-2\kappa_s d_s}}{1 - e^{-2\kappa_s d_s}}. \tag{20}
\]

This formula can be significantly simplified by exploiting the fact that the integral over \( k \) vanishes when \( \xi \to \infty \). Following Schaden et al. [21], we partially integrate over \( \xi \) and simultaneously introduce \( \kappa_s \) as the integration variable in the integral over \( k \). This gives

\[
F = \frac{\hbar e^{\kappa_p}}{2\pi^2} \int_0^\infty d\xi \xi^2 \frac{d}{d\xi} \frac{\kappa_s}{\sqrt{\varepsilon_s(i\xi)} / c} \frac{e^{-2\kappa_s d_s}}{1 - e^{-2\kappa_s d_s}} \sum_q e^{-2\kappa_s d_s} \frac{1}{\varepsilon_s(i\xi)}
\]

where, in the second step, we have noted that for the free-electron dielectric function \( (d/d\xi) \sqrt{\varepsilon_s(i\xi)} = 1/\sqrt{\varepsilon_s(i\xi)} \). Finally, making the substitution \( y = \sqrt{\varepsilon_s(i\xi)} \xi / \omega_P \) and expanding the integrand, we obtain for the pressure on the slab surfaces

\[
F = -\frac{\hbar c k_p}{\pi^2} \sum_{n=1}^{\infty} \frac{d^2}{da_n^2} \frac{K_1(a_n)}{a_n}, \quad a_n = 2nkpd_s, \tag{22}
\]

where

\[
K_1(a) = a \int_1^\infty dy \sqrt{y^2 - 1} e^{-ay} = \int_1^\infty dy \frac{y}{\sqrt{y^2 - 1}} e^{-ay} \tag{23}
\]

is recognized as modified Bessel function of the second kind [22, 23].

To find pressure on a thin slab we use

\[
\frac{d^2}{da^2} \frac{K_1(a)}{a} = \frac{6}{a^2} - \frac{1}{2a^2} + O(a^0), \quad a \ll 1.
\]

This gives

\[
f_s = F_C \left[ 1 - \frac{5}{\pi^2} (k_p d_s)^2 + O((k_p d_s)^4) \right], \quad F_C = -\frac{\pi^2 \hbar c}{240 d_s^3}, \tag{24}
\]

so that the pressure on a thin metal layer is, to the leading order, given by famous Casimir result. Evidently, it is due to the modes propagating through the layer. Indeed, as follows from Eq. (12), this system supports only modes with frequencies \( \alpha_s = -i\pi / d_s \)

\[
\omega_n^2(k) = c \sqrt{k_P^2 + k^2 + n^2 \pi^2 / d_s^2},
\]
where \( n \) is an integer. For \( k_P \ll d_s^{-1} \), these mode frequencies lead to the same Casimir energy as in the original Casimir configuration. The pressure on a thick slab can be obtained using

\[
\frac{d^2}{da^2} K_1(a) = e^{-a} \sqrt{\frac{\pi}{2a^3}} \left[ 1 + \frac{27}{8a} + O(a^{-2}) \right], \quad a \gg 1
\]

and is, to the leading order, again given by Eq. (19). These considerations are illustrated by the black full line in Fig. 4 giving the pressure \( F \) (relative to the Casimir pressure \( F_C \)) on a free-electron metal layer between ideal mirrors as a function of \( k_P d_s \).

5. Metal slab in an ideal cavity

The pressure on a metal slab in the center of a cavity with perfectly reflecting mirrors is given by Eqs. (2) and (3) [or Eqs. (5)]] with \( R_q = \delta_{qp} - \delta_{qs} \) and \( d_1 = d_2 = d \) and can only be calculated numerically. Its behaviour with the slab thickness and the mirror-slab distance is illustrated in Figs. 4 and 5.

As seen, cavity strongly affects the pressure on a thin \( (k_P d_s < 0.1) \) slab; one observes its strong drop from \( F_C \) for non zero mirror-slab distances. As shown above, with increasing \( d \) the pressure on a thin slab decreases to \( F_{nr} = 0.19k_P d_s F_C \). We also recall that, whereas for \( d = 0 \) the pressure is due to modes propagating through the metal, at large mirror-slab separations it is due solely to surface plasmon (evanescent) modes. This can be understood when realizing that nearby mirrors effectively damp surface modes since their field cannot accommodate to the perfect mirror boundary condition. Indeed, according to Eq. (12) (with \( q = p \)) the dispersion relations of SP modes in the present configuration can be rewritten as

\[
\varepsilon(\omega) = \frac{\alpha_s \tanh^{\pm 1} \alpha_s d_s}{\alpha \tanh \alpha d}.
\]

Accordingly, when \( d \to 0 \) frequencies of SP modes tend to zero. Thus when changing the cavity length, in addition to changing its magnitude, one effectively modify the mode spectrum of the pressure on the slab.
As expected, the effect of the cavity on the pressure on a thick \((kpd > 1)\) slab is less pronounced and diminish with the slab thickness owing to the attenuation of the vacuum-field fluctuations with \(d_s\). We also observe from the red and black dashed curve in Fig. 4 that for systems with mirror-slab distances \(kpd \geq 1\) the cavity effect on the pressure is very small and the pressure on the slab differs very little (on this scale) from the pressure on a free standing slab. Thus, the effect of the cavity on the pressure on a slab in its center is largest for systems with the mirror-slab distances \(kpd < 1\), as illustrated in Fig. 5. Since \(\Delta p\) for (noble) metals is of the order of \(10^2\)nm, this corresponds to the mirror-slab distances less than (say) 20nm.

6. Summary
In this work we have considered and performed a modal analysis of the vacuum-field pressure on surfaces of a metal slab in the center of a cavity adopting the plasma model for the metal and assuming ideally reflecting cavity mirrors. We have confirmed analytically previous results for the pressure on a thin and a thick free-standing slab and derived an exact formula for the pressure on a metallic layer between perfect mirrors. According to these results, the pressure on a thin slab decreases with increasing slab-mirror distances from the well-known Casimir pressure \(F_C\) to the quasistatic pressure \(F_{nr} = 1.19(d/\lambda_P)^2 F_C\). In the first case it is entirely due to the photonic modes propagating through the metal and in the second case it is entirely due to the surface plasmon modes of the free-standing slab. We recall that the pressure on metallic plates in the standard Casimir configuration behaves with decreasing their separation precisely in the same way: at large separations between the plates it is equal to \(F_C\) and is due to the photonic modes \([5, 6]\) whereas at small separations between the plates it is equal to \(F_{nr}\) and is due to the surface plasmon modes supported by the plates \([24, 25, 26, 27]\). This similarity of the pressures in two configurations is lost in the case of a thick slab as the pressure on its surfaces exponentially decreases for large slab thicknesses. These considerations demonstrate that a cavity may influence the vacuum-field pressure on a thin metal slab (and its modal structure) to a considerable extent.

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