The Strongly Coupled Fourth Family and a First-Order Electroweak Phase Transition. I

Quark Sector

Yoshio KIKUKAWA,¹,∗) Masaya KOHDA²,∗∗) and Junichiro YASUDA³,∗∗∗)

1Institute of Physics, University of Tokyo, Tokyo 153-8092, Japan
2Department of Physics, Saitama University, Saitama 338-8570, Japan
3Center for the Studies of Higher Education, Nagoya University, Nagoya 464-8601, Japan

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In models of dynamical electroweak symmetry breaking due to strongly coupled fourth-family quarks and leptons, their low-energy effective descriptions may involve multiple composite Higgs fields, leading to a possibility that the electroweak phase transition at finite temperature is first-order due to the Coleman-Weinberg mechanism. We examine the behavior of the electroweak phase transition on the basis of the effective renormalizable Yukawa theory, which consists of the fourth-family quarks and two $SU(2)$-doublet Higgs fields corresponding to the bilinear operators of the fourth-family quarks with/without imposing the compositeness condition. The strength of the first-order phase transition is estimated using the finite-temperature effective potential at one loop with ring improvement. In the Yukawa theory without the compositeness condition, it is found that there is a parameter region where the first-order phase transition is sufficiently strong for the electroweak baryogenesis with the experimentally acceptable Higgs boson and fourth-family quark masses. On the other hand, when the compositeness condition is imposed, the phase transition turns out to be weakly first-order, or possibly second-order, although the result is rather sensitive to the details of the compositeness condition. By combining with the result of the Yukawa theory without the compositeness condition, it is argued that with the fourth-family quark masses in the range of 330–480 GeV, corresponding to the compositeness scale in the range of 1.0–2.3 TeV, the four-fermion interaction among the fourth-family quarks does not lead to the strongly first-order electroweak phase transition.

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§1. Introduction

The standard model (SM) can in principle fulfill all three Sakharov conditions¹ for generating a baryon asymmetry in the universe.²–⁴) The model fails, however, for two reasons, to explain the value of the asymmetry required for the primordial nucleosynthesis,⁵) or the value measured through the cosmic microwave background.⁶) The first reason is that the $CP$ violation from the Kobayashi-Maskawa mechanism,⁷) which nicely explains $CP$ violation in K- and B-systems, is highly suppressed.⁸)–¹³) The second reason is that the electroweak phase transition (EWPT) is not strongly first-order. The experimental lower bound on the Higgs mass, $m_h > 114$ GeV,¹⁴) im-

*1 E-mail: kikukawa@hep1.c.u-tokyo.ac.jp
**1 E-mail: mkohda@krishna.th.phy.saitama-u.ac.jp
***1 E-mail: yasuda@cshe.nagoya-u.ac.jp
plies that there is no EWPT in SM. Consequently, sphaleron-induced (B+L)-violating interactions are not sufficiently suppressed in the broken phase and wash out the baryon asymmetry. Therefore, if the physics at the electroweak scale is to explain the baryon asymmetry in the universe, a better understanding of the structure of the Higgs sector and the source of CP violation would be required.

The fourth family is still a viable phenomenological possibility beyond SM. The constraint from the invisible Z width is insignificant for the fourth-family neutrino being heavier than $m_Z/2$. Although the electroweak precision data give stringent constraint on the fourth family, it is known that the data do not exclude their existence. It was shown that there still remains a parameter region being consistent with all current experimental bounds.

The existence of the fourth-family quarks accommodates the extra mixings and CP-violating phases within the Cabibbo-Kobayashi-Maskawa scheme. It has been argued that the observed anomaly in B-CP asymmetries may be explained by the effect of the fourth-family quarks. Recently, it has been pointed out that the CP violation from these new phases could be large enough to explain the baryon asymmetry in the universe on the basis of the dimensional analysis using the Jarlskog invariants extended to four families.

The question is then whether EWPT can be strongly first-order with the fourth family: the mere addition of the fourth family to SM is of no help in this respect, as long as the standard Higgs sector with the single SU(2) doublet is considered. (See, for example, a recent study by Fok and Kribs.) Carena et al. first discussed the possibility of a first-order EWPT due to new heavy fermions coupled strongly to Higgs bosons. They found that some heavy and strongly interacting bosonic fields are required both to stabilize the effective potential against the large effect of the heavy fermions and to cause a first-order EWPT. This led the authors to consider a supersymmetric model. EWPT in the supersymmetric model with the fourth family has recently been examined in Ref. 34). (See Ref. 36) for earlier works.)

If the masses of the fourth-family quarks and leptons are quite large and are comparable to the unitarity bounds, the fourth family must couple strongly to the Higgs sector. In this case, the masses of the fourth-family quarks and leptons (or their vacuum condensates) may be regarded as the order parameters of the electroweak symmetry breaking (EWSB). The effective description of the fluctuations of the order parameters may involve multiple Higgs scalar fields. This leads to the possibility that EWPT would be first-order due to the Coleman-Weinberg mechanism (the fluctuation-induced first-order phase transition).

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** The model of dynamical electroweak symmetry breaking due to top quark condensate has been proposed by Nambu and Miransky-Tanabashi-Yamawaki and examined in detail through renormalization group methods by Marciano and Bardeen-Hill-Lindner.

*** The original Higgs sector of SM, if the electroweak interactions are turned off, is nothing but the $O(4)$ linear sigma model, and its finite-temperature phase transition is second-order, which is governed by the Wilson-Fisher IR-stable fixed point. It is the effect of the gauge interaction, which makes the fixed point IR-unstable and causes a first-order phase transition. Once the Higgs sector
The goal of this paper is to explore the above possibility of a first-order EWPT due to the heavy fourth family. We start from a model of dynamical electroweak symmetry breaking due to the effective four-fermion interactions of the fourth-family quarks and leptons at the scale $\Lambda_{4f}$ around a few TeV. We adopt the four-fermion interactions considered by Holdom. This four-fermion theory may be rewritten into a Yukawa theory by introducing auxiliary scalar fields that correspond to the bilinear operators of the fourth-family quarks and leptons. These scalar fields consist of three $SU(2)$ doublets and one $SU(2)$ triplet. (It is assumed that the right-handed neutrino is extra heavy, acquiring its mass at the flavor scale around 1000 TeV.) The renormalization group evolution from the scale $\Lambda_{4f}$ down to the electroweak scale $v (= 246 \text{ GeV})$ may generate operators such as the kinetic and interaction terms of the scalar fields and other higher dimensional operators. We then extend this model by including the kinetic, cubic, and quartic terms of the scalar fields so that it becomes renormalizable, neglecting the effect of the higher dimensional operators. It is this effective renormalizable model for which we examine EWPT through the finite-temperature effective potential at one loop with the ring improvement. Strictly speaking, in our case, the renormalization group equations must be subject to the compositeness condition as a boundary condition at the scale $\mu = \Lambda_{4f}$. Accordingly, the values of the renormalized couplings at the lower scale $\mu = v$ are restricted in a certain region of the parameter space. In our analysis, however, we will first explore the parameter space of the renormalizable theory without the constraint due to the compositeness condition, in order to locate the parameter region where a strongly first-order EWPT is realized. We then examine the possible overlap of these two regions.

In formulating the effective Yukawa theory, one encounters the vacuum instability and the singularity of the Landau pole due to the large Yukawa couplings of the fourth-family quarks and leptons. We then need to introduce a cutoff $\Lambda$ before the problems occur, by imposing a certain stability condition for the scalar quartic couplings, as well as upper bounds for the scalar and Yukawa couplings. We should require that the nonrenormalizable operators are sufficiently suppressed, $m^2/\Lambda^2 \ll 1$, as well as the perturbativity conditions for both the scalar and Yukawa couplings, simply because our analysis at finite temperature is basically perturbative. This restricts the range of the masses of the fourth-family fermions, which we can explore: $m_{q'} \lesssim 480 \text{ GeV}$ for the quark masses “on shell”. Although it is lower than the typical value in the models of dynamical electroweak symmetry breaking owing to the extended, the number of scalar fields is increased and there appear additional quartic couplings among them. The fixed points of the multiple quartic coupling constants may be IR-unstable and one can expect a first-order phase transition even in the pure scalar sector. A related approach is to consider the single Higgs doublet model (the $O(4)$ model) with the dimension-six or higher operators. The quartic coupling may then be assumed negative so that the model is out of the domain of the Wilson-Fisher fixed point. Such higher dimensional operators may be induced by the effect of heavy particles coupled to the Higgs doublet, or more generally, by the effect of a certain dynamical system behind the Higgs sector. The EWPT has been examined in various dynamical models of the Higgs sector: walking technicolor theories in Refs. using low-energy effective sigma models, pseudo Goldstone Higgs boson models or little Higgs models in Refs. (64) and 68), models of the gauge-Higgs unification in Refs. 69–71).
strongly coupled fourth family, the upper value reaches approximately 500 GeV and we consider that it is possible to obtain some useful information about the nature of the EWPT even for those dynamical models.

Since the critical behavior of the first-order phase transition at finite temperature that we are concerned with, is not universal in general, the result of our analysis would depend on our choice of low-energy effective theory, where a certain truncation of fields and operators has to be carried out. Therefore, our analysis, must be semi-quantitative (or qualitative), showing a possibility to realize the strongly first-order EWPT required for the electroweak baryogenesis.

In this paper, we concentrate on the effect of the heavy quarks and consider two $SU(2)$ doublets out of four scalar fields. The analysis of the effect of the heavy charged lepton and neutrino will be reported in a subsequent paper. To be consistent with the electroweak precision date, we simply assume that the masses of the fourth-family quarks are degenerate. The bosonic sector of our model then reduces to the two Higgs doublet model (2HDM), with the global $SU(2)_R$ symmetry. This $SU(2)_R$ symmetry may be broken due to the effects of the large Yukawa coupling for the top quark and the $SU(2) \times U(1)$ gauge interaction. We will discuss the possibly large effect of the top quark in the last section, but will leave the detailed analysis of the effect for future study. We neglect the $SU(2) \times U(1)$ gauge interaction, because we do not expect a large effect of the electroweak interaction to the dynamics of the first-order phase transition in this model. We also neglect the effect of the quasi infrared fixed point due to the $SU(3) \times SU(2) \times U(1)$ gauge interaction.

This paper is organized as follows. In §2, we formulate the effective Yukawa theory and introduce the cutoff scale $\Lambda$ by considering the vacuum instability and the triviality bounds. We then specify the compositeness condition for our model. In §3, we derive the finite-temperature effective potential at one loop with the ring improvement. In §4, on the basis of the numerical analysis of the effective potential, we examine the strength of the first-order phase transition in the Yukawa theory with/without the compositeness condition. Section 5 is devoted to conclusions and discussion.

§2. Fourth family and electroweak symmetry breaking

2.1. Fourth family and four-fermion interactions

We assume the existence of the fourth-family quarks and leptons, which we denote by $q' = (t', b')^T$, $\ell' = (\nu'_{\tau L}, \tau'_{L})^T$, $\tau'_R$. The right-handed neutrino $\nu'_{\tau R}$ is assumed to acquire its mass at the flavor scale around 1000 TeV and to be absent below the flavor scale. To be consistent with the electroweak precision data, the masses of the fourth-family quarks should be almost degenerate with a small mass splitting. For simplicity, we assume $m_{t'} = m_{b'}$.

*) If $SU(3) \times SU(2) \times U(1)$ gauge interaction is included, there appears an effective infrared fixed point in the renormalization group equation of the Yukawa coupling Eq. (2.14). But its value is about $y_c \simeq 1.6$, which is less than the values of $y$ considered in our analysis. Thus, we neglect the effect of the infrared fixed point.
Following Holdom,\textsuperscript{72) we introduce the four-fermion interactions of the fourth-family fermions as follows:

\[
\mathcal{L}_{4f} = G_{q'}(q_{Lij}^iq_{Rj}^q)(q_{Rj}^iq_{Lij}) + G_{\tau'}(\bar{\ell}_{Li}^i\tau_{Ri})(\tau_{Ri}^i\ell_{Li}) - G_{\nu'^i}(\ell_{Li}^iC^i\ell_{Li}^i)(\bar{\nu}_{Li}C\bar{\nu}_{Li}^T),
\]

(2.1)

where \( C \) is the charge conjugation matrix, and color indexes are contracted within a bracket. The scale of these interactions is assumed to be \( \Lambda_{4f} \): \( G_{q'}, G_{\tau'}, G_{\nu'^i} \approx 1/\Lambda_{4f}^2 \). The interaction term among the quarks has \( SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A \) symmetry, where \( U(1)_V \) corresponds to the baryon number. On the other hand, the interaction terms among the leptons have \( SU(2)_L \times U(1)_V \times U(1)_R \) symmetry, which includes the vectorlike \( U(1) \) symmetry corresponding to the lepton number. The above four-fermion interactions, therefore, have the extra symmetries compared with the SM Higgs sector, which has \( O(4) \) symmetry. We then assume the existence of subleading multifermion operators, which are suppressed compared with Eq. (2.1), so that the extra symmetries are broken explicitly and, hence, the possible pseudo Nambu-Goldstone (NG) bosons acquire nonzero masses.

The four-fermion interactions may be rewritten into the form of Yukawa interactions by introducing the auxiliary scalar fields \( \Phi \), \( H_{\tau'} \), and \( \chi^a \) \((a = 1, 2, 3)\), which correspond to the bilinear operators of the fourth-family quarks and leptons as follows:

\[
\Phi_{ij} \sim \bar{q}_{Rj}^iq_{Li}, \quad H_{\tau'} \sim \bar{\tau}_{Ri}^i\ell_{Li}, \quad \chi^a \sim \bar{\nu}_{Li}^T\tau^aC\bar{\nu}_{Li}^T.
\]

(2.2)

Then one obtains

\[
\mathcal{L}'_{4f} = -m_{\Phi_0}^2 \text{tr}(\Phi^\dagger\Phi) - m_{H_{\tau'}}^2 H_{\tau'}^\dagger H_{\tau'} - m_{\chi}^2 \chi^a\chi^a + \mathcal{L}_Y,
\]

(2.3)

where \( \mathcal{L}_Y \) is the Yukawa interaction term given by

\[
\mathcal{L}_Y = -y_{q'0}(\bar{q}_{Lj}\Phi q_{Ri}^i + \text{c.c.}) - y_{\tau'0}(\bar{\tau}_{Ri}^iH_{\tau'}^\dagger + \text{c.c.}) - f(\ell_{Li}^iC^i\tau^a\chi^a\ell_{Li}^i + \text{c.c.}).
\]

(2.4)

2.2. Effective renormalizable theory

Through the renormalization group evolution from the scale \( \mu = \Lambda_{4f} \) down to the electroweak scale \( \mu = v \) (=246 GeV), the kinetic and interaction terms of the scalar fields and other higher dimensional operators may be generated. We then extend this model by including the kinetic, cubic, and quartic terms of the scalar fields so that it becomes renormalizable, neglecting the effect of the higher dimensional operators. The effective renormalizable theory is then given by the following Lagrangian:

\[
\mathcal{L} = \mathcal{L}_k + \mathcal{L}_Y - V,
\]

(2.5)

where \( \mathcal{L}_k \) consists of the kinetic terms for fourth-family fermions and scalar bosons and \( V \) is the scalar potential. The explicit form of \( V \) is given in Appendix A.\textsuperscript{*})

\textsuperscript{*} V includes a scalar cubic term \( H_{\tau'}^\dagger C^i\chi^aH_{\tau'} \), which may enhance the strength of a first-order phase transition at high temperature.
Strictly speaking, the renormalization group equations are subject to the compositeness condition as a boundary condition at the scale $\mu = \Lambda_{4f}^{44}$. Accordingly, the values of the renormalized couplings at the lower scale $\mu = v$ are restricted in a certain region of the parameter space. In the following analysis, however, we will first explore the parameter space of the renormalizable theory without the constraint due to the compositeness condition, in order to locate the parameter region where a strongly first-order EWPT is realized. We then examine the possible overlap of these two regions.

In this paper (I), we concentrate on the effect of the fourth-family quarks and consider two $SU(2)$ doublets out of four scalar fields. We also neglect, in this paper, the $SU(3) \times SU(2) \times U(1)$ gauge interaction and consider the global symmetry limit, simply because we do not expect a large effect of the color and the electroweak interactions on the dynamics of the first-order phase transition in this model. Then the Lagrangian Eq. (2.5) reduces to

$$\mathcal{L} = \bar{q}' i\partial q' - y(q_L' \Phi q_R' + \text{c.c.}) + \text{tr}(\partial_{\mu} \Phi^\dagger \partial^\mu \Phi) - m_\Phi^2 \text{tr} \Phi^\dagger \Phi$$

$$- \frac{\lambda_1}{2}(\text{tr} \Phi^\dagger \Phi)^2 - \frac{\lambda_2}{2} \text{tr}(\Phi^\dagger \Phi)^2 + c(\text{det} \Phi + \text{c.c.}).$$

(2.6)

We include the last term that breaks the $U(1)_A$ symmetry and induces the mass of the pseudo NG boson. Then, the symmetry of the theory is the chiral symmetry $SU(2)_L \times SU(2)_R$ plus the $U(1)_V$ symmetry corresponding to the baryon number. We do not include the terms that consist of $\epsilon \Phi^* \epsilon$ other than in the determinant term.

2.3. Electroweak symmetry breaking

We assume that the chiral symmetry $SU(2)_L \times SU(2)_R$ breaks down to the diagonal subgroup $SU(2)_V$ by the vacuum expectation value (VEV) of $\Phi(x)$:

$$\langle \Phi(x) \rangle = \frac{\phi}{\sqrt{2N_f}} \begin{pmatrix} 1 \\ \sqrt{2N_f} \end{pmatrix},$$

(2.7)

where $N_f(=2)$ is the number of fourth-family quark flavors, $I$ is the $N_f \times N_f$ unit matrix, and $\phi \geq 0$. At tree-level, VEV is determined using the effective potential:

$$V_0(\phi) = \frac{1}{2}(m_\Phi^2 - c)\phi^2 + \frac{1}{8} \left( \lambda_1 + \frac{\lambda_2}{N_f} \right) \phi^4.$$  

(2.8)

For $(m_\Phi^2 - c) < 0$, VEV is given by

$$\phi_0 = \sqrt{\frac{-2(m_\Phi^2 - c)}{\lambda_1 + \lambda_2/N_f}}.$$  

(2.9)

For the effective potential to be stable in this channel, the following conditions must be satisfied:

$$\lambda_1 + \lambda_2/N_f \geq 0, \quad \lambda_2 \geq 0.$$  

(2.10)

Around VEV, we may parametrize the fluctuation of $\Phi(x)$ as follows:

$$\Phi(x) = \frac{\phi + h + i\eta}{\sqrt{2N_f}} I + \sum_{\alpha=1}^{3} (\xi^{\alpha} + i\pi^{\alpha}) \sigma^{\alpha} \frac{\sigma^\dagger}{2},$$

(2.11)
The bosonic sector of this model is just the 2HDM. \( h \) is the singlet of \( SU(2)_V \) and corresponds to the SM Higgs boson. The adjoint \( \pi^\alpha \) are the NG bosons of the breaking of \( SU(2)_L \times SU(2)_R \), while the singlet \( \eta \) is the pseudoscalar Higgs boson and is also the pseudo NG boson associated with the breaking of the \( U(1)_A \) symmetry. The adjoint \( \xi^\alpha \) consist of the extra neutral Higgs boson and the charged Higgs bosons. Three NG bosons \( \pi^\alpha \) are eaten by \( W \) and \( Z \) bosons when the electroweak interactions are introduced. As for the fourth-family quarks, the experimental lower bound from the direct search \( m_{q'} \gtrsim 256 \text{ GeV} \) implies \( y \gtrsim 2.1 \) at tree level by taking \( \phi_0 = v \) (= 246 GeV).

2.4. Cutoff scale of the effective theory

The applicability of the effective theory defined by the Lagrangian Eq. (2.6) would break down at some energy scale and one needs to introduce a cutoff \( \Lambda \). The running coupling constants in this model, \( \widetilde{\lambda}_1(\mu), \widetilde{\lambda}_2(\mu), \) and \( \tilde{y}(\mu) \), obey the following renormalization group equations at one loop:

\[
\mu \frac{\partial}{\partial \mu} \tilde{\lambda}_1 = \frac{1}{8\pi^2} \left[ (N_f^2 + 4) \tilde{\lambda}_1^2 + 4N_f \tilde{\lambda}_1 \tilde{\lambda}_2 + 3\tilde{\lambda}_2^2 + 2N_c y^2 \tilde{\lambda}_1 \right],
\]

\[
\mu \frac{\partial}{\partial \mu} \tilde{\lambda}_2 = \frac{1}{8\pi^2} \left[ 6\tilde{\lambda}_1 \tilde{\lambda}_2 + 2N_f \tilde{\lambda}_2^2 + 2N_c y^2 \tilde{\lambda}_2 - 2N_c \tilde{y}^4 \right],
\]

\[
\mu \frac{\partial}{\partial \mu} \tilde{y} = \frac{1}{16\pi^2} \left[ (N_f + N_c) \tilde{y}^3 \right],
\]

with the initial conditions \( \tilde{\lambda}_1(v) = \lambda_1, \tilde{\lambda}_2(v) = \lambda_2, \) and \( \tilde{y}(v) = y \) given at the electroweak scale. As one can see in Fig. 1, the Yukawa coupling, which is large at the
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Fig. 1. (color online) Renormalization group flows of the Yukawa coupling for various initial values at the electroweak scale. The values of $y$ are 2.0, 2.5, and 3.0 from bottom to top. The dashed line (gray) indicates the value $y = 2.1$, which corresponds to the experimental lower bound from the direct search of the fourth-family quarks, $m_{q'} \gtrsim 256$ GeV. The dot-dashed line (blue) indicates the upper limit of the perturbativity condition, which is adopted in our analysis as a criterion for the Landau pole.

The ultraviolet behaviors of the scalar quartic couplings then take two types of pattern depending on the relative size of the scalar quartic couplings and Yukawa coupling at the electroweak scale: (i) the scalar quartic couplings $\bar{\lambda}_1(\mu) + \bar{\lambda}_2(\mu)/N_f$ and/or $\bar{\lambda}_2(\mu)$ are driven to negative at some energy scale, implying that the electroweak vacuum is unstable; (ii) the scalar quartic couplings encounter the Landau pole at some energy scale. In both cases, one should introduce a cutoff before these problems occur.

We estimate the cutoff $\Lambda$, for given initial values of the couplings at the electroweak scale, as the scale at which one of the following conditions is first met:

$$\bar{\lambda}_1(\Lambda) + \bar{\lambda}_2(\Lambda)/N_f = 0, \quad \bar{\lambda}_2(\Lambda) = 0,$$  

which correspond to case (i) (vacuum instability) and

$$\bar{\lambda}_1(\Lambda) + \bar{\lambda}_2(\Lambda)/N_f = \frac{16\pi^2}{N_f}, \quad \bar{\lambda}_2(\Lambda) = \frac{16\pi^2}{N_f}, \quad \bar{y}(\Lambda)^2 = \frac{16\pi^2}{N_c},$$  

which correspond to case (ii) (Landau pole). Here, we adopt the upper limits of the perturbativity bounds,

$$\bar{\lambda}_1(\Lambda) + \bar{\lambda}_2(\Lambda)/N_f \leq \frac{16\pi^2}{N_f}, \quad \bar{\lambda}_2(\Lambda) \leq \frac{16\pi^2}{N_f}, \quad \bar{y}(\Lambda)^2 \leq \frac{16\pi^2}{N_c},$$  

as a criterion for the Landau pole.

In Fig. 2, we show the contours of the estimated cutoff $\Lambda$ for $y = 2.0$ in $\lambda_1 - \lambda_2/N_f$ plane. We see that in the most regions, the cutoff scale is around 1 TeV or
Fig. 2. An estimate of the cutoff scale $\Lambda$ for $y = 2.0$ in $\lambda_1 - \lambda_2/N_f$ plane. The dashed, dot-dashed, solid, and dotted contours correspond to $\Lambda = 5.0, 1.5, 1.0, \text{and } 0.5$ TeV, respectively. In the shaded region, the effective potential at tree level is unstable and we consider the region where $\lambda_1 + \lambda_2/N_f \geq 0$ only.

Fig. 3. An estimate of the cutoff scale $\Lambda$ for $y = 2.5$ in $\lambda_1 - \lambda_2/N_f$ plane. The dot-dashed, solid, and dotted contours correspond to $\Lambda = 1.5, 1.0, \text{and } 0.5$ TeV, respectively.

lower. For the fixed $\lambda_1$, $\Lambda$ tends to increase with $\lambda_2/N_f$ for small $\lambda_2/N_f$ and tends to decrease with $\lambda_2/N_f$ for large $\lambda_2/N_f$. The former (latter) behavior is due to the fact that $\Lambda$ is determined via the vacuum instability (the Landau pole) conditions in that region. Figure 3 is a similar plot for $y = 2.5$. We see that for the larger value of the Yukawa coupling, relatively larger values of $\lambda_2/N_f$ are required to fulfill the vacuum stability condition. In Fig. 4, we show the contours of the estimated cutoff $\Lambda$ for $\lambda_1 + \lambda_2/N_f = 0.05$ in $y - \lambda_2/N_f$ plane.

In order to ensure the applicability of the effective renormalizable theory, the
cutoff $\Lambda$ should be large enough compared with other mass scales in the theory: $\Lambda \gg m_i(\phi_0), \phi_0$. In the following analysis of the first-order EWPT, it turns out that the largest mass scale is given by $m_\xi(\phi_0)$ around 400–700 GeV. Then, we require $\Lambda \geq 1$ TeV and exclude the region of the parameter space where this condition is not fulfilled. This requirement leads to the constraint on the quark masses, $m_{q'} \lesssim 370$ GeV at tree level, corresponding to the Yukawa coupling $y \leq 3.0$.

2.5. Compositeness condition

Just below the scale of the four-fermion interaction $\mu \lesssim \Lambda_{4f}$, the four-fermion theory Eq. (2.3) with only quark fields $q'$,

$$\mathcal{L}_{4f}' = \bar{q}' i \not\!D q' - y_{q'0}(\Phi_L \Phi q'_R + \text{c.c.}) - m_{\Phi_0}^2 \text{tr}(\Phi^\dagger \Phi),$$

is renormalized to the Yukawa theory Eq. (2.6), where the renormalized couplings are given by

$$\bar{\lambda}_1(\mu) = 0, \quad \bar{\lambda}_2(\mu) = \frac{32\pi^2}{N_c} \frac{1}{\ln(\Lambda_{4f}^2/\mu^2)}, \quad \bar{y}(\mu)^2 = \frac{16\pi^2}{N_c} \frac{1}{\ln(\Lambda_{4f}^2/\mu^2)}.$$  \hfill (2.19)

(See Appendix B for detail.) In the limit $\mu \to \Lambda_{4f}$, one finds

$$\bar{\lambda}_1(\mu) \to 0, \quad \bar{\lambda}_2(\mu) \to \infty, \quad \bar{y}(\mu)^2 \to \infty,$$

$$\bar{\lambda}_2(\mu)/\bar{y}(\mu)^2 \to 2.$$  \hfill (2.20)

This provides the compositeness condition in terms of the renormalized couplings as the boundary condition of the renormalization group equations at $\mu = \Lambda_{4f}$.\footnote{44}
Fig. 5. (color online) The electroweak-scale values of the running coupling constants, which are subject to the compositeness conditions at various $\Lambda_{4f}$, are shown in $\lambda_1-\lambda_2/N_f$ plane. The blue, red, and green data sets (on the dashed, dotted, and dot-dashed curve, respectively) correspond to criteria A, B, and C, respectively. The blue filled circles (on the dashed curve) indicate the values $\lambda_1, \lambda_2/N_f$ for $\Lambda_{4f}$ ($y$) equal to 0.5 (4.0), 1.0 (3.0), 1.5 (2.7), and 5.0 TeV (2.2) from top left to bottom right, respectively.

In our effective theory formulated as above, however, the compositeness condition should be modified. One cannot impose the above condition Eq. (2.20) literally because the values of the couplings $\bar{\lambda}_2(\mu)$ and $\bar{y}(\mu)$ must exceed the perturbativity bounds. However, the divergence of $\bar{\lambda}_2(\mu)$ and $\bar{y}(\mu)$ is due to the Landau pole. Then, it seems reasonable in our case to substitute the upper limit of the perturbativity bounds for the compositeness condition:

$$\bar{\lambda}_1(\Lambda_{4f}) = 0, \quad \bar{\lambda}_2(\Lambda_{4f}) = \frac{16\pi^2}{N_f}, \quad \bar{y}(\Lambda_{4f})^2 = \frac{16\pi^2}{N_c},$$

$$\frac{\bar{\lambda}_2(\Lambda_{4f})}{\bar{y}(\Lambda_{4f})^2} = \frac{N_c}{N_f}. \quad \text{[Criterion A]}$$

In Fig. 5, we plot the electroweak-scale values $\lambda_1, \lambda_2$ of the running coupling constants, which are subject to the compositeness conditions Eq. (2.21) at various scales. The blue filled circles (on the dashed curve) correspond to the values of $\Lambda_{4f}$ ($y$) = 0.5 (4.0), 1.0 (3.0), 1.5 (2.7), and 5.0 TeV (2.2) from top left to bottom right, respectively. The case with $\Lambda_{4f}$ = 1.0 TeV comes close to the stability boundary at the electroweak scale, taking the value $\lambda_1 + \lambda_2/N_f = 0.05$. Then, in Fig. 4, the values $y, \lambda_2$ of this case are indicated by the blue filled circle (indicated by “A”), which is located at the “cusp” on the boundary of the allowed region with $\Lambda \geq 1.0$ TeV.

In fact, the electroweak-scale values of the couplings are rather sensitive to the choice of their values at the compositeness scale $\Lambda_{4f}$. To see this, let us refer to the above criterion for the compositeness condition as “A”, and introduce slightly modified criteria “B” and “C” as follows:

$$\bar{\lambda}_1(\Lambda_{4f}) = 0, \quad \bar{\lambda}_2(\Lambda_{4f}) = \frac{8\pi^2}{N_f}, \quad \bar{y}(\Lambda_{4f})^2 = \frac{8\pi^2}{N_c},$$
\[
\lambda_2(A_{4f})/\bar{y}(A_{4f})^2 = N_c/N_f, \quad \text{[Criterion B]} (2.22)
\]

and
\[
\bar{\lambda}_1(A_{4f}) = 0, \quad \bar{\lambda}_2(A_{4f}) = \frac{4\pi^2}{N_f}, \quad \bar{y}(A_{4f})^2 = \frac{4\pi^2}{N_c}, \quad \bar{\lambda}_2(A_{4f})/\bar{y}(A_{4f})^2 = N_c/N_f, \quad \text{[Criterion C]} (2.23)
\]

In Fig. 5, the red and green filled circles (on the dotted and dot-dashed curve, respectively) show the electroweak-scale values $\lambda_1$, $\lambda_2$ of the running coupling constants subject to the compositeness conditions Eqs. (2.22) and (2.23), respectively. We will discuss this point further in §4 in relation to the analysis of EWPT.

§3. Effective potential

3.1. Zero-temperature effective potential

At zero temperature, the one-loop effective potential is given by
\[
V^{(0)}(\phi) = V_0(\phi) + V_1^{(0)}(\phi), \quad (3.1)
\]

where $V_0$ is the tree-level effective potential, $V_1^{(0)}$ is the one-loop contribution at zero temperature.

$V_0$, the tree-level effective potential, is given by
\[
V_0(\phi) = \frac{1}{2} (m_{\phi}^2 - c) \phi^2 + \frac{1}{8} \left( \lambda_1 + \frac{\lambda_2}{N_f} \right) \phi^4. \quad (3.2)
\]

$V_1^{(0)}$, the one-loop contribution at zero temperature, is given by
\[
V_1^{(0)}(\phi) = \frac{1}{64\pi^2} \sum_{i=h,\xi,\eta,\pi,q'} n_i m_i^4(\phi) \left[ \ln \frac{m_i^2(\phi)}{\mu^2} - \frac{3}{2} \right] + \frac{1}{2} A \phi^2. \quad (3.3)
\]

$m_i(\phi)$ and $n_i$ are the effective masses depending on $\phi$ and the number of degrees of freedom, respectively, which are given in Table I. In the calculation of the loop integral in $V_1^{(0)}$, we have taken the limit $\Lambda \to \infty$ and have used the $\overline{\text{MS}}$ scheme with a slight modification to renormalize the ultraviolet divergences. The first term is the one-loop contribution in ordinary $\overline{\text{MS}}$ scheme with the renormalization scale $\mu$. The modification is the existence of the second term, which is added to preserve the tree-level VEV $\phi_0 = \sqrt{-2(m_{\phi}^2 - c)/(\lambda_1 + \lambda_2/N_f)}$ and, then, we set $\phi_0 = v (=246 \text{ GeV})$. The parameter $A$ is determined through $0 = \partial V_1^{(0)}(\phi)/\partial \phi$ at $\phi = v$ and is given by
\[
A = -\frac{1}{16\pi^2} \sum_{i=h,\xi,\eta,\pi,q'} n_i a_i m_i^2(v) \left( \ln \frac{m_i^2(v)}{\mu^2} - 1 \right). \quad (3.4)
\]
At one loop, the Higgs boson mass $m_h$ is shifted from the tree-level value $(m_h)_{\text{tree}} = \sqrt{\lambda_1 + \lambda_2/N_f v}$. In this paper, we adopt the following definition for the Higgs boson mass $m_h$ at one loop:

$$m_h^2 \equiv \left( \lambda_1 + \frac{\lambda_2}{N_f} \right) v^2 + \frac{v^2}{8\pi^2} \sum_{i=h,\xi,q'} n_i a_i^2 \ln \frac{m_i^2(v)}{\mu^2}. \quad (3.5)$$

This is the curvature of the effective potential at $\phi = v : V(0)''(\phi = v)$ with neglecting the contributions from the light or massless scalar bosons $\eta$ and $\pi$s.

As for the mass of the extra scalar bosons $\xi$, $\eta$ and the fourth-family quarks $q'$, we adopt the formula at the tree level:

$$m_{\xi}^2 \equiv 2c + \frac{\lambda_2}{N_f} v^2, \quad m_{\eta}^2 \equiv 2c, \quad m_{q'}^2 \equiv \frac{y^2}{2N_f} v^2. \quad (3.6)$$

In the following analysis, we use these definitions for the masses.

### 3.2. Finite-temperature effective potential

The one-loop contribution at finite temperature, $V_1(T)$, is given by

$$V_1(T)(\phi, T) = \frac{T^4}{2\pi^2} \left( \sum_{i=h,\xi,q} n_i J_B[m_i^2(\phi)/T^2] + n_{q'} J_F[m_{q'}^2(\phi)/T^2] \right), \quad (3.7)$$

where $J_B$ and $J_F$ are defined by

$$J_B(a) = \int_0^\infty dx \, x^2 \ln \left( 1 - e^{-\sqrt{x^2+a}} \right), \quad J_F(a) = \int_0^\infty dx \, x^2 \ln \left( 1 + e^{-\sqrt{x^2+a}} \right). \quad (3.8)$$

In the following analysis of EWPT, we carry out a numerical integral for $J_B$ and $J_F$ without high-temperature expansion.

In the ordinary perturbation theory at finite temperature, the perturbative expansion breaks down near the critical temperature due to the existence of the higher-loop IR divergent diagrams in the massless limit. To improve the reliability of the

---

*The right-hand side of this equation is written in terms of the renormalized parameters. By adding the counterterms, the quadratically divergent contributions to the Higgs boson mass are fine-tuned away. To get a sense of fine tuning, we compare the quadratically divergent contributions $\sum_{i=h,\xi,q} n_i J_B[m_i^2(\phi)/T^2]$ with the one-loop Higgs boson mass squared $m_h^2$ defined here. In most parts of the interesting parameter region where $y \gtrsim 2.1$, the cutoff scale is around 1 TeV or lower (see §2.4) and fine tuning is not required to obtain a Higgs boson mass of a few hundred GeV order (see Fig. 9). However, in the parameter region where the cutoff scale is comparable to 5 TeV or higher, the degree of fine tuning is sizable (about one part in one hundred). For instance, at point “A” (“B”) in Fig. 4, $A \simeq 1$ TeV (5 TeV) and the size of the quadratically divergent contributions is $\sum_{i=h,\xi,q} n_i J_B[m_i^2(\phi)/T^2] \simeq -(450 \text{ GeV})^2 [-70(200 \text{ GeV})^2]$, whereas $m_h^2 \simeq (450 \text{ GeV})^2 [(200 \text{ GeV})^2]$.**

**) The contributions of $\eta$ and $\pi$s are divergent and may be regarded as artifacts due to the fact that $V^{(0)''}(\phi = v)$ corresponds to the off-shell Higgs boson mass at the zero momentum $p^2 = 0$, while the on-shell (physical) mass should be finite. One expects that it is valid to neglect the contributions from $\eta$ and $\pi$s as long as $\lambda_1 + \lambda_2/N_f$ is much smaller than $\lambda_2/N_f$ or $y$. This argument is in line with Ref. 75).
perturbative expansion, we include the contributions from the ring diagrams, which are the most dominant IR contributions at each order of the perturbative expansion.\textsuperscript{76)–82)}

One can include the contribution of ring diagrams, $V_{\text{ring}}(\phi, T)$, by replacing $m_i^2(\phi)$ ($i = h, \xi, \eta, \pi$) in $V_1^{(0)}$ and $V_1^{(T)}$ with the effective $T$-dependent masses $M_i^2(\phi, T) \equiv m_i^2(\phi) + \Pi_\phi$, where $\Pi_\phi$ is the self-energy of the scalar bosons in the IR limit where the Matsubara frequency and the momentum of the external fields become zero and in the leading order of $m_i(\phi)/T$ and is given by

$$\Pi_\phi = \frac{1}{12} \left[ (N_f^2 + 1) \lambda_1 + 2N_f \lambda_2 + N_c y^2 \right] T^2,$$

at one-loop order.

After all, the one-loop ring-improved effective potential is given by

$$V(\phi, T) = V_0(\phi) + V_1^{(0)}(\phi) + V_1^{(T)}(\phi, T) + V_{\text{ring}}(\phi, T)$$

$$= V_0(\phi) + \frac{1}{2} A \phi^2$$

$$+ \sum_{i = h, \xi, \eta, \pi} n_i \left[ \frac{1}{64\pi^2} M_i^4(\phi, T) \left( \ln \frac{M_i^2(\phi, T)}{\mu^2} - \frac{3}{2} \right) + \frac{T^4}{2\pi^2} J_B[M_i^2(\phi, T)\beta^2] \right]$$

$$+ n_q' \left[ \frac{1}{64\pi^2} m_{q'}^4(\phi) \left( \ln \frac{m_{q'}^2(\phi)}{\mu^2} - \frac{3}{2} \right) + \frac{T^4}{2\pi^2} J_F[m_{q'}^2(\phi)\beta^2] \right].$$

In the following, we study the finite-temperature EWPT by numerically evaluating this effective potential.

\section*{§4. Numerical analysis of electroweak phase transition}

EWPT should be strongly first-order in order to avoid the washout of the generated baryon asymmetry in the broken phase. How strongly first-order the phase transition must be depends on the energy of the sphaleron solution\textsuperscript{100), 101)} in the model considered. As far as the classical (static) solution of the equation of motions is concerned, one may neglect the effect of the fourth-family quarks even when they

\textsuperscript{*)} The ring-improved perturbation theory is valid when the nonring diagrams are suppressed with respect to the ring diagrams. We infer the expansion parameter in the ring-improved perturbation theory by the power counting argument as in the $\lambda \phi^4$ theory.\textsuperscript{76)} By inspecting the higher order diagrams for the scalar field self-energies, the expansion parameters are expected to be $n_i \max(\lambda_1 + \lambda_2 N_f, \lambda_2 N_f) T_4/[4\pi M_i(\phi, T)] (i = h, \xi, \eta, \pi)$, where $M_i(\phi, T)$ and $n_i$ are the effective $T$-dependent masses of the scalar bosons and corresponding numbers of degrees of freedom. This is maximized for $i = \pi$ and $\phi = 0$, then, $(N_f^2 - 1) \max(\lambda_1 + \lambda_2 N_f, \lambda_2 N_f) T_4/[4\pi M_\pi(0, T_c)] \ll 1$ should be held for the ring-improved perturbation theory to be valid at $T = T_c$. In the analysis of EWPT, we have observed that for $y \gtrsim 2.1$, the above expansion parameter is smaller than 1 but is not so small (greater than 0.5).
are heavy. Then, one may use the condition
\[ \frac{\phi_c}{T_c} \gtrsim 1, \]  
as the criterion for a strongly first-order EWPT, as discussed in our previous work.\(^5\)

In the following analysis, assuming a first-order phase transition, we solve the conditions
\[ \left. \frac{\partial V(\phi, T_c)}{\partial \phi} \right|_{\phi = \phi_c} = 0, \quad V(\phi_c, T_c) = V(0, T_c), \]  
numerically for various parameters, \(m_h, m_\xi, m_{q'}, \) and \(m_\eta\) (see Eqs. (3.5) and (3.6) for definitions). Then we evaluate \(\frac{\phi_c}{T_c}\) in order to estimate the strength of the first-order phase transition.

We first explore the parameter space of the renormalizable theory without the constraint due to the compositeness condition, in order to locate the parameter region where a strongly first-order EWPT is realized. We consider only the region where the stability condition at tree level, \(\lambda_1 + \lambda_2/N_f \geq 0,\) is satisfied; otherwise, the mass parameter squares in the one-loop effective potential become negative. We also consider only the region where the perturbation theory is reliable:
\[ \lambda_1 + \lambda_2/N_f \ll \left(\frac{4\pi}{N_f}\right)^2, \quad \frac{\lambda_2}{N_f} \ll \left(\frac{4\pi}{N_f}\right)^2, \quad y^2 \ll \left(\frac{4\pi}{N_f}\right)^2. \]  

To fulfill the requirement \(\Lambda \geq 1\) TeV, we also concentrate on a region where \(y \leq 3.0\). For an estimated \(\Lambda\), we require that \(\phi = v\) is the global minimum of the one-loop zero-temperature effective potential \(V(0)(\phi)\) for \(0 < \phi < \Lambda\). The renormalization scale of the effective potential is set to the electroweak scale, \(\mu = v\). We next examine the possible overlap of the parameter region of the strongly first-order EWPT and the region subject to the compositeness condition.

4.1. Yukawa theory without compositeness condition

We first discuss the numerical results for the Yukawa theory without the constraint due to the compositeness condition. In this analysis, we neglect the effect of the \(U(1)_A\) symmetry breaking term, setting \(c = 0\).

In Fig. 6, we show the contours of \(\phi_c/T_c\) in the \(\lambda_1 - \lambda_2/N_f\) plane for \(y = 2.0\) in the allowed region with \(\Lambda \geq 1\) TeV. (The contours with \(1\) TeV \(\geq \Lambda \geq 0.5\) TeV are also shown for reference.) We clearly see that the region of the strongly first-order transition lies above the stability boundary \(\lambda_1 + \lambda_2/N_f = 0\). When \(\lambda_1 + \lambda_2/N_f\) gets larger, \(\phi_c/T_c\) tends to become smaller. For a fixed small \(\lambda_1 + \lambda_2/N_f\), there is a tendency that \(\phi_c/T_c\) decreases as \(\lambda_2/N_f\) increases.

Figure 7 is a similar contour diagram for \(y = 2.5\). We note that the region with \(\phi_c/T_c > 1\) disappears in this case. The substantial region close to the stability boundary is excluded by the condition \(\Lambda \geq 1\) TeV. This is because one encounters

\(^1\) The large Yukawa couplings of the fourth-family quarks and leptons may affect the baryon number diffusion rate, which is usually computed at one loop including the effect of the fluctuations around the sphaleron solution.\(^102\)–\(^108\)
Fig. 6. Contour plots for various $\phi_c/T_c$ in $\lambda_1-\lambda_2/N_f$ plane at $y = 2.0$, $c = 0$. The dashed lines are the contours of $\phi_c/T_c = 1.0$, 0.8, 0.6, and 0.4 from bottom left to top right as indicated. The solid lines indicate the boundary of the allowed region with $\Lambda \geq 1$ TeV. (The region with $1$ TeV $> \Lambda \geq 0.5$ TeV is also shown.) In the shaded region, the effective potential at tree level is unstable. The black filled circle indicates the values of the couplings $\lambda_1, \lambda_2/N_f$ when the compositeness condition is satisfied at $\Lambda = 9$ TeV so that $y = 2.0$.

Fig. 7. Contour plots for various $\phi_c/T_c$ in $\lambda_1-\lambda_2/N_f$ plane at $y = 2.5$, $c = 0$. The dashed lines are the contours of $\phi_c/T_c = 0.8$, 0.6, and 0.4 from bottom left to top right as indicated. The black filled circle indicates the values of the couplings $\lambda_1, \lambda_2/N_f$ when the compositeness condition is satisfied at $\Lambda = 2.3$ TeV so that $y = 2.5$.

the instability at a lower scale for the larger Yukawa coupling. However, even when the allowed region is extended to $\Lambda \geq 0.5$ TeV, the transition is weakly first-order with $\phi_c/T_c < 1$. For larger values of $y$, as far as we explored, the transition becomes more weakly first-order, or possibly second-order.
In Fig. 8, we show the contours of $\phi_c/T_c$ in $y-\lambda_2/N_f$ plane for $\lambda_1 + \lambda_2/N_f = 0.05$, $c = 0$. We see clearly that a strongly first-order phase transition is realized even for $y > 2.1$, which corresponds to the experimental lower bound from the direct search of the fourth-family quarks $m_{q'} \gtrsim 256$ GeV, in the range $3 \lesssim \lambda_2/N_f \lesssim 4$ (corresponding to $430$ GeV $\lesssim m_\xi \lesssim 500$ GeV). For larger values of $y$, however, the stability of the electroweak vacuum requires larger values of $\lambda_2/N_f$ and then the strength of the first-order phase transition becomes weaker, or possibly second-order.

In Fig. 10, $\phi_c/T_c$ is plotted as a function of $m_h$ for several values of $m_\xi$ with $m_{q'} = 260$ GeV fixed. We can see that with $m_\xi$ fixed, $\phi_c/T_c$ decreases as $m_h$ increases,
Fig. 10. $\phi_c/T_c$ as a function of $m_h$ for $m_{q'} = 260$ GeV and $m_\xi = 400, 450, 500, \text{ and } 550$ GeV from bottom left to top right.

Fig. 11. $\phi_c/T_c$ as a function of $m_h$ for $m_\xi = 450$ GeV and $m_{q'} = 200, 260, 280, \text{ and } 300$ GeV from top to bottom.

whereas, with $m_h$ fixed, $\phi_c/T_c$ increases as $m_\xi$ increases. In the whole range, $m_h$ exceeds the experimental bound, $m_h > 114$ GeV. In Fig. 11, on the other hand, $\phi_c/T_c$ is plotted as a function of $m_h$ for several values of $m_{q'}$ with $m_\xi = 450$ GeV fixed. In this case, we note that with $m_h$ fixed, $\phi_c/T_c$ decrease as $m_{q'}$ increases.

4.2. Effect of explicit symmetry breaking term

We next examine the effect of the explicit $U(1)_A$ symmetry breaking term by taking a value of $m_\eta$ (or $c$) nonzero. The effect can be read from Fig. 12. For a fixed value of $m_h$, $\phi_c/T_c$ decreases as $m_\eta$ increases. Thus, we see that the explicit symmetry breaking term reduces the strength of the first-order phase transition. The experimental bound on the mass of the pseudo NG boson depends on how $\eta$ couples to the other particles, which is not specified in our model. If we adopt the bound for the pseudoscalar Higgs boson in supersymmetric models (with $\tan\beta > 0.4$), the allowed value is $m_\eta > 93.4$ GeV.\(^{109}\) For such value of $m_\eta$, we see that the region of $\phi_c/T_c \geq 1$ becomes quite narrow.\(^{\ast}\)

Typical values of $\phi_c$ and $T_c$ are shown separately in Fig. 13 as a function of $m_h$

\(^{\ast}\) We note that the tree-level formula is used for $m_\eta$ in this analysis and an inclusion of the one-loop correction to $m_\eta$ would affect the extent of the region with $\phi_c/T_c \geq 1$. 

\(m_c=260\text{GeV}, m_\eta=0\)

\(m_c=450\text{GeV}, m_\eta=0\)
Fig. 12. $\phi_c/T_c$ as a function of $m_h$ for $m_\xi = 450$ GeV, $m_{q'} = 260$ GeV and $m_\eta = 0, 50, 100,$ and 150 GeV from top to bottom.

Fig. 13. Plots of $\phi_c$ (black) and $T_c$ (gray) as a function of $m_h$ with $m_{q'} = 260$ GeV, $m_\xi = 450$ GeV, and $m_\eta = 100$ GeV fixed.

for $m_{q'} = 260$ GeV, $m_\xi = 450$ GeV, and $m_\eta = 100$ GeV fixed.

4.3. Compositeness condition

Finally, we discuss the numerical results for the Yukawa theory with the compositeness condition imposed. Let us recall the plots in Figs. 4 and 5 of the electroweak-scale values of the running coupling constants, which are subject to the compositeness conditions Eq. (2.21) (Criterion A) at various scales: the values $\lambda_1, \lambda_2$ for $\Lambda_{4f} = 1.0$ TeV come close to the stability boundary at the electroweak scale, taking the value $\lambda_1 + \lambda_2/N_f = 0.05$, and it is located at the “cusp” on the boundary of the allowed region with $\Lambda \geq 1$ TeV. In this case, corresponding to the blue filled circle (indicated by “A”) in Fig. 8, although we could not find the solution of Eq. (4.2), we have checked that the phase transition is weakly first-order with $\phi_c/T_c < 0.001$, or possibly second-order.

The above conclusion deserves discussion. If one adopts the other criterion for the compositeness condition, the phase transition can be first-order. In the case of criterion B, for example, the electroweak-scale values of the running coupling constants, which are subject to the compositeness condition at $\Lambda_{4f} = 3.7$ TeV come close to the stability boundary at the electroweak scale, taking the value $\lambda_1 + \lambda_2/N_f =$
In Fig. 8, we show the electroweak-scale values $y, \lambda_2$ of this case with the red filled circle (indicated by “B”). One immediately sees that the phase transition is rather strongly first-order with $\phi_c/T_c \simeq 1$. Thus, the critical behavior of the phase transition is rather sensitive to the choice of values of the coupling constants at the compositeness scale $\Lambda_{4f}$. However, by combining the above results with those of the Yukawa theory without the compositeness condition, in particular, with the result for $y = 2.5$ shown in Fig. 7, it seems fair to say that the four-fermion interaction of the fourth-family quarks, which causes EWSB at zero temperature and produces the mass of the heavy quarks larger than $m_{q'} \simeq 310$ GeV ($y \gtrsim 2.5, \Lambda_{4f} \lesssim 2.3$ TeV), does not lead to the strongly first-order EWPT at finite temperature.\(^\ast\) We note that a region with $\Lambda_{4f} < 1$ TeV ($y > 3.0, m_{q'} > 370$ GeV) is beyond the scope of our analysis using the effective renormalizable theory.

\section{Conclusions and discussion}

In this paper, we have discussed the finite-temperature electroweak phase transition in the model where the electroweak symmetry is dynamically broken due to the four-fermion interaction of the fourth-family quarks. Based on the effective renormalizable Yukawa theory with/without compositeness condition, we have estimated the strength of the first-order phase transition $\phi_c/T_c$, using the finite temperature effective potential at one loop with the ring improvement. In the Yukawa theory without compositeness condition, the phase transition can be strongly first-order with $\phi_c/T_c \gtrsim 1$ for the experimentally acceptable Higgs boson and fourth-family quarks masses in the range $256$ GeV $\lesssim m_{q'} \lesssim 290$ GeV when $\lambda_1 + \lambda_2/N_f = 0.05$. On the other hand, once the compositeness condition is imposed as the boundary condition for the running coupling constants, the values of the couplings at the electroweak scale are restricted in a certain region of the parameter space. There, the phase transition turns out to be weakly first-order, or possibly second-order. The above result depends on how to specify the compositeness condition. In fact, we observed that the values of the running couplings at $\mu = v$ are rather sensitive to the choice of the boundary condition at $\mu = \Lambda_{4f}$, and if one takes the smaller values as the boundary condition at $\mu = \Lambda_{4f}$ (for example, Criterion B), the phase transition can be strongly first-order with $\phi_c/T_c \sim O(1)$. In spite of this ambiguity, combining with the results of the Yukawa theory without the compositeness condition, it seems plausible that for $3.0 \geq y \gtrsim 2.5$ (corresponding to $1$ TeV $\lesssim \Lambda_{4f} \lesssim 2.3$ TeV and $480$ GeV $\gtrsim m_{q', \text{phys}} \gtrsim 330$ GeV “on shell”\(^\ast\ast\)), the four-fermion interaction of the fourth-family quarks, which causes EWSB at zero temperature, does not lead to the strongly first-order EWPT at finite temperature. We note that a region with $y > 3.0$ is beyond the scope of our analysis using the effective renormalizable theory.

\(^\ast\) There remains a possibility that the effect of the electroweak interactions might enhance slightly the strength of the first-order phase transition.

\(^\ast\ast\) For $y = 3.0$ ($m_{q'} = 370$ GeV), the fourth-family quark mass defined by Eq. (3.6) receives a rather large correction due to the running of the Yukawa coupling. The “on-shell” mass, defined by $m_{q', \text{phys}} = \bar{y}(m_{q', \text{phys}})/\sqrt{2N_f}$, is $m_{q', \text{phys}} \simeq 480$ GeV for $y = 3.0$ and is $m_{q', \text{phys}} \simeq 330$ GeV for $y = 2.5$ (see Fig. 14).
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Fig. 14. (color online) Plots of the running mass of the fourth-family quarks, $\bar{m}_{q'}(\mu) = \bar{y}(\mu)v/\sqrt{2N_f}$, as a function of the renormalization scale $\mu$. The value of “on-shell” mass is defined by $m_{q',\text{phys}} = \bar{y}(m_{q',\text{phys}})v/\sqrt{2N_f}$. The dotted line (red) indicates $\bar{m}_{q'}(\mu) = \mu$.

In our analysis, we have assumed that the masses of the fourth-family quarks are degenerate and imposed the $SU(2)_R$ symmetry, neglecting the effects of the large Yukawa coupling for the top quark and the $SU(2) \times U(1)$ gauge interaction. If one allows the breaking of the $SU(2)_R$ symmetry, the Lagrangian Eq. (2.6) should be extended to that of the more generic 2HDM, and VEV of $\Phi(x)$ assumes the form:
$$\langle \Phi \rangle = \text{diag}(v_u, v_d)/\sqrt{2N_f}.$$ Even in such a case, one may expect that the behavior of EWPT does not change qualitatively as long as the fourth-family quarks are relatively heavier than the top quark. However, the large top Yukawa coupling may make the vacuum instability more severe. Then, this would narrow the allowed parameter region (with $\Lambda \geq 1$ TeV) where our effective theory may be applicable.

Since the critical behavior of the first-order phase transition at finite temperature that we are concerned with, is not universal in general, the result of our analysis would depend on our choice of low-energy effective theory, where a certain truncation of fields and operators has to be carried out.\(^{110}\) Our analysis, therefore, must be semiquantitative (or qualitative), showing a possibility to realize the strongly first-order EWPT required for the electroweak baryogenesis.

Fortunately, it is now possible to formulate the model considered in this paper nonperturbatively on the lattice, preserving $SU(2) \times U(1)$ chiral gauge symmetry exactly.\(^{111} - 117\) A direct numerical analysis of EWPT by Monte Carlo simulations with the state-of-art technique\(^{118} - 126\) may shed light on the issues discussed above. Even in the global symmetry limit, where $SU(2) \times U(1)$ gauge interactions are turned off, there are several questions worth studying: one may study the phase structure and the critical behavior of chiral symmetry restoration at finite temperature in relation to the triviality bounds for the masses of the Higgs bosons and the fourth-family quarks and leptons.\(^{127} - 129\)

In this paper, we have focused on the effect of the fourth-family quarks. The analysis of the effect of the fourth-family charged lepton and neutrino will be reported in a subsequent paper.\(^{97}\)
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Appendix A

Scalar Potential

In this appendix, we give the scalar potential in the full effective Yukawa theory, which includes both fourth-family quarks and leptons as well as all scalar fields $\Phi$, $H_{\tau'}$, and $\chi^a$ ($a = 1, 2, 3$).

The most general renormalizable scalar potential, which is consistent with symmetry (including the general $U(1)_A$ breaking terms) is given by

$$V(\Phi, H_{\tau'}, \chi) = V_2 + V_3 + V_4,$$

where

$$V_2 = m_\Phi^2 \text{tr}(\Phi^\dagger \Phi) + m_{H_{\tau'}}^2 H_{\tau'}^\dagger H_{\tau'} + m_{\chi^a}^2 \chi^a \chi^a - c(\text{det} \Phi + \text{c.c.}),$$

$$V_3 = a H_{\tau'}^\dagger \epsilon \chi^a \chi^a + \text{c.c.},$$

and

$$V_4 = \sum_{i,j,k,l} \left[ \rho_{ijkl} \text{tr}(\phi_i^\dagger \phi_j) \text{tr}(\phi_k^\dagger \phi_l) + \rho'_{ijkl} \text{tr}(\phi_i^\dagger \phi_j \phi_k^\dagger \phi_l) \right] + \alpha (H_{\tau'}^\dagger H_{\tau'})^2 + \beta_1 (\chi^a \chi^a)^2$$

$$+ \beta_2 (\chi^a \chi^a)(\chi^{bs} \chi^{bs}) + \sum_{i,j} \left[ \gamma_{ij} \text{tr}(\phi_i^\dagger \phi_j) H_{\tau'}^\dagger H_{\tau'} + \gamma'_{ij} H_{\tau'}^\dagger \phi_i \phi_j \phi_i \phi_j H_{\tau'} \right]$$

$$+ \sum_{i,j} \gamma''_{ij} \text{tr}(\phi_i^\dagger \phi_j) \chi^a \chi^a + \kappa H_{\tau'}^\dagger H_{\tau'} \chi^a \chi^a,$$

where we have used the following notation:

$$\phi_1 \equiv \Phi, \ \phi_2 \equiv \epsilon \Phi^* \epsilon,$$

and $i, j, k, l = 1, 2$.

Appendix B

Renormalization Group Evolution of the Four-Fermion Theory

Just below the scale of the four-fermion interaction $\mu \lesssim A_{4f}$, the four-fermion theory equivalent, Eq. (2.3),

$$L_{4f}' = \bar{q}^i i \not{\partial} q^i - y_{q0}(\bar{q}^i L \Phi q^i_R + \text{c.c.}) - m_{q0}^2 \text{tr}(\Phi^\dagger \Phi)$$

is renormalized as

$$L_0 = \bar{q}^i i \not{\partial} q^i - y_{q0}(\bar{q}^i L \Phi q^i_R + \text{c.c.}) + Z_\Phi \text{tr}(\partial_\mu \Phi^\dagger \partial^\mu \Phi) - m_\Phi^2 \text{tr} \Phi^\dagger \Phi$$

$$- \frac{\lambda_1}{2} (\text{tr} \Phi^\dagger \Phi)^2 - \frac{\lambda_2}{2} (\text{tr} \Phi^\dagger \Phi)^2 + c(\text{det} \Phi + \text{c.c.}),$$

(B.2)
in terms of the bare field variables, where

\[ Z_\Phi = \frac{N_c}{16\pi^2} g_0^2 \ln(A_{4f}^2/\mu^2), \quad m_{\Phi 0}^2 = m_{\Phi 0}^2 - \frac{2N_c}{16\pi^2} g_0^2 \ln(A_{4f}^2 - \mu^2), \]

\[ \lambda_1 = 0, \quad \lambda_2 = \frac{2N_c}{16\pi^2} g_0^4 \ln(A_{4f}^2/\mu^2), \quad c = 0. \quad (B.3) \]

Then, the renormalized couplings read

\[ \bar{\lambda}_1(\mu) = 0, \quad \bar{\lambda}_2(\mu) = \frac{32\pi^2}{N_c} \frac{1}{\ln(A_{4f}^2/\mu^2)}, \quad \bar{y}(\mu)^2 = \frac{16\pi^2}{N_c} \frac{1}{\ln(A_{4f}^2/\mu^2)}. \quad (B.4) \]

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