Mesoscopic fluctuations of tunneling through double quantum dots

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We study fluctuations of conductance of two connected in series dots in the Coulomb blockade regime. The pattern of the fluctuations turns out to be extremely sensitive to a magnetic field. These conductance fluctuations also provide information about the mesoscopic fluctuations of charge of a partially-opened single quantum dot, which are hard to measure directly.

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Charge of a mesoscopic conductor attached to leads by high-resistance junctions is quantized. This quantization (Coulomb blockade) is progressively smeared with the reduction of the junction resistance to the level $\sim h/e^2$. Most directly, the smearing can be studied by a measurement of charge or differential capacitance of a quantum dot connected to a particle reservoir by a tunable junction of conductance $G_i$. This proved to be a hard experiment however, performed only very recently.

Spatial quantization of the electron motion in a dot leads to mesoscopic fluctuations of its transport and thermodynamic characteristics in the Coulomb blockade regime. Fluctuations of the conductance in a single dot in both peaks and valleys of the Coulomb blockade are of the order of the average conductance, and were studied experimentally in Refs. [4,5]. Fluctuations of the ground state energy of a partially opened dot result in a random contribution to its charge and differential capacitance. However these fluctuations are small [4], therefore hard to observe directly. The accuracy of the experiments [1], where the average differential capacitance of a quantum dot was studied, is insufficient for quantitative study of the fluctuations.

In this paper we demonstrate that a number of features of the mesoscopic fluctuations in the Coulomb blockade regime is much easier to observe in a double-dot geometry, shown in Fig. 1. The dots are statistically identical, with average level spacing $\Delta$, and charging energy $E_C$. The temperature $T$ is much smaller than $\Delta$ but exceeds the level widths associated with electron escape to the leads. Conductances of the three junctions satisfy relation $e^2/h \gtrsim G_i \gg G_L, G_R$. The dependence of the conductance $G$ of the system on $V_{g1}$ and $V_{g2}$ in this case reveals sharp twin peaks [3].

We calculate the variance of the peak splitting and show that it gives a direct measure of the random component of the ground state energy and charge of a partially opened dot. The positions of the peak doublets with respect to each other also fluctuate. In the absence of the fluctuations, the peaks would form a regular pattern (see Fig. 2). Random displacements of peak doublets manifest themselves most prominently in strong fluctuations of the conductance measured along the line $V_{g1} = V_{g2} \equiv V_g$ on the plane $(V_{g1}, V_{g2})$. These fluctuations are unusually sensitive to a magnetic field. The corresponding correlation field is parametrically smaller than the field leading to crossover from orthogonal to unitary ensemble.

Qualitative consideration and the main results — The dependence of conductance $G$ of the double-dot system on the gate voltages $(V_{g1}, V_{g2})$ forms a three-dimensional landscape. The ridges on this landscape, shown by solid lines on the map Fig. 2, correspond to the activation-less hopping of an electron on and off the double dot. When $G_i \rightarrow 0$, the ridges form a rectangular pattern (dashed lines on Fig. 2). The conductance has peaks when both dots are at resonance (crossings of the dashed lines). At these special values of the gate voltages, the four states with $n$ or $n+1$ electrons in the left dot and $m$ or $m+1$ electrons in the right dot have the same energy. Finite inter-dot tunneling ($G_i \neq 0$) removes this degeneracy. The two states with the total number of electrons $n + m + 1$ hybridize, lowering the energy of the ground state by $U$, with $\langle U \rangle = (\ln 2/\pi^2)(hG_i/e^2)E_C$ [7]. This results in anticrossing of the conductance ridges and formation of twin peak structures shown in Fig. 2. The peak splitting (see Fig. 2) in dimensionless units $N = C_g V_g / e$ is

$$N_s = \frac{2CU}{e^2}, \quad \text{with} \quad \langle N_s \rangle = \frac{\ln 2 \hbar G_i}{\pi^2 e^2}. \quad (1)$$

FIG. 1. Schematic view of the double-dot system. The dots are formed by applying a negative voltage to the gates. The conductance $G_L$ and $G_R$ of the channels connecting the double-dot to the electron reservoirs is controlled by $V_L$ and $V_R$; voltage $V_i$ controls the conductance $G_i$ of the interdot channel. Voltages $V_{g1}$ and $V_{g2}$ are used to change the number of electrons in the dots.
We will show that typically the displacement $\mathcal{E}$ obtained in nature, peak doublets form a square lattice, and the line doublets with respect to each other. In an ideal structure, peak splitting, measured along this line, $\mathcal{G}(\mathcal{N}) \equiv \mathcal{G}(\mathcal{N}, \mathcal{N})$. (Note the difference in definitions of $\mathcal{G}_{max}$ and the height of a peak in $\mathcal{G}(\mathcal{N}_1, \mathcal{N}_2)$).

The displacement $\mathcal{E}$ is a sum of two independent terms. One of them, $\mathcal{E}_1 \propto \delta U$, stems from the fluctuations in the interdot interaction. The second one, $\mathcal{E}_2 \sim \Delta$, is due to the randomness of one-electron spectra in the dots $[8]$. The peak width in the direction perpendicular to the line $\mathcal{N}_1 = \mathcal{N}_2$ is determined by the hybridization between the discrete levels mediating the tunneling through the double-dot, and is of the order of $\sqrt{\hbar \mathcal{G}_i/ e^2 \Delta}$, which is much smaller than $\sqrt{\langle \mathcal{E}^2 \rangle + \langle \mathcal{E}^2 \rangle}$. The ratio of the two latter parameters yields the probability for the line $\mathcal{N}_1 = \mathcal{N}_2$ to “meet” a peak, and thus describes the fluctuations of $\mathcal{G}_{max}$:

$$\frac{\langle \mathcal{G}^2_{max} \rangle}{\langle \mathcal{G}_{max} \rangle^2} \sim \frac{\sqrt{\langle \mathcal{E}^2 \rangle}}{\hbar \mathcal{G}_i/ e^2 \Delta} \gg 1.$$ 

The exact result obtained in Quantitative discussion differs from the above estimate only by numerical factors:

$$\frac{\langle \mathcal{G}^2_{max} \rangle}{\langle \mathcal{G}_{max} \rangle^2} \sim A \sqrt{0.25 \frac{h \mathcal{G}_i \mathcal{E}_C}{2e^2 \Delta} + \frac{2e^2}{h \mathcal{G}_i} \frac{E_C}{\Delta}}. \quad (3)$$

Here $A \approx 0.45$ and $A \approx 0.23$ for the orthogonal and unitary ensembles respectively. We see that the variance of $\mathcal{G}_{max}$ is parametrically larger than $\langle \mathcal{G}_{max} \rangle$, in all the considered domain $\mathcal{G}_i \lesssim e^2/\hbar$.

The characteristic magnetic field altering the pattern of fluctuations in $\mathcal{G}(\mathcal{N})$ also can be obtained from the above qualitative consideration. The applied magnetic field shifts the two discrete levels mediating the conductance $\mathcal{G}_{max}$. A shift exceeding the peak width leads to a considerable suppression of a maximum conductance $\mathcal{G}_{max}$. Such a shift can be produced by a small variation of the magnetic field,

$$B_c = \frac{\Phi_0}{S} \left[ \frac{h \mathcal{G}_i}{4\pi^4 e^2} \frac{\Delta}{4\pi E_T} \right]^{1/2}. \quad (4)$$

Here $S$ and $E_T$ are the area and the Thouless energy $\mathcal{E}_T$ of a single dot, $\Phi_0$ is the flux quantum. Field $B_c$ is much smaller than the characteristic magnetic field $B_\Delta = (\Phi_0/S)\sqrt{\Delta/4\pi E_T}$ changing the electron states in a single dot $[11]$. Therefore $B_c$ does not lead to the crossover from the orthogonal to unitary ensemble.

Quantitative description — To calculate the conductance of the double-dot system, we use the Hamiltonian in the following form

$$\hat{H} = \hat{H}_d + \hat{H}_C + \hat{H}_{lead} + \hat{H}_t,$$ 

$$\hat{H}_d = \sum_k \xi_k a_k^\dagger a_k + \sum_l \xi_l a_l^\dagger a_l.$$ 

![Figure 2](image-url)
\[ \hat{H}_l = \sum_{kp} \left[ t_{kp} a_k^p c_p + h.c. \right] + \sum_{lq} \left[ t_{lq} a_l^q c_q + h.c. \right] + \sum_{kl} \left[ t_{kl} a_k a_l^\dagger + h.c. \right], \]

\[ \hat{H}_C = E_C \left[ (\hat{n}_1 - N_1)(\hat{n}_1 + 1) + (\hat{n}_2 - N_2)(\hat{n}_2 + 1) \right]. \]

The subscripts \( k \) and \( l \), and \( p \) and \( q \) denote the states in the left and right dot, and left and right lead respectively; \( \xi_{k,l} \) are the one-electron energy spectra of the dots, \( n_{L,R} \) are the numbers of electrons in the left and right dot. The dimensionless gate voltages are given by \( \xi_{k,l} \) are the numbers of electrons in the left and right dot. The dimensionless gate voltages are \( \xi_{k,l} = C_g V_{gs}/e \), and \( E_C = e^2/2C \) is the charging energy of each dot; \( C_g \) and \( C \) are the capacitance to the gate and the total capacitance of a single dot. The height of the conductance maximum is then given by

\[ \hat{\tilde{G}}_\text{max} = \frac{2e^2}{h} \frac{\Gamma_k \Gamma_l}{\Gamma_k + \Gamma_l + \left( \Gamma_k - \Gamma_l \right) \Delta} \left\{ \frac{2\sqrt{2} |t_{kl}|}{\sqrt{E^2 + 2|t_{kl}|^2}} \right\} \left[ \frac{\pi}{2T} \right], \]

with \( k = k_0, l = l_0 \). Here \( \Gamma_k \propto |\psi_k(r_L)|^2 \) and \( \Gamma_l \propto |\psi_l(r_R)|^2 \) are the widths of the states \( k \) and \( l \) with respect to the electron tunneling into the adjacent lead; \( r_L, r_R \) are the points of the dot-lead contacts. At \( \Delta > \Delta \) there can be no inter-dot resonance on the line \( \xi_1 = \xi_2 \), and the conductance inevitably involves cotunneling through one of the dots. At low temperature \( T < \sqrt{E_C \Delta} \) only the elastic cotunneling is possible. The resulting conductance \( \hat{\tilde{G}}_\text{max} \sim (\Delta/E)G_I G_{L,R} \) yields parametrically small contributions to the ensemble averages. To calculate \( \langle \tilde{G}_\text{max} \rangle \) and \( \langle \tilde{G}_\text{max}^2 \rangle \), one needs to use \( \text{Eq. (12)} \) and average over all the random quantities entering in it. The statistical properties of \( \psi_{k,l} \) are well-known, and now we discuss those of \( \Delta \).

One source of fluctuations of \( \Delta \) is the randomness of the interdot tunneling matrix elements, which leads to random shifts of energies of electron states, see Eqs. \( \text{[3], [4]} \). This contribution yields:

\[ \langle \delta U^2 \rangle = \frac{2}{\beta} \left[ 1 - \left\{ \frac{\ln 2}{\pi^2} \right\}^2 \right] \left( \frac{hG_I}{2e^2} \right)^2 E_C \Delta, \]

which leads to Eq. \( \text{[3]} \). The corrections \( \text{[10]} \) to the ground state energy are given by a sum over the one-electron levels with energies \( \lesssim E_C \). Since there are many statistically independent terms in this sum, the distribution of the fluctuations \( \delta U \) is Gaussian.

Now we will consider the fluctuations of conductance \( \tilde{G} = G(N_1, N_2) \). The height of its maxima is a random quantity, depending not only on the one-electron wave functions, but also on the random displacement \( \mathcal{E} \) of a peak from the line \( N_1 = N_2 \). If \( \mathcal{E} < \Delta \), the electron transport occurs by resonant tunneling via two states, \( k_0, l_0 \). The height of the conductance maximum is then given by

\[ \langle \tilde{G}_\text{max} \rangle = \frac{2e^2}{h} \frac{\Gamma_k \Gamma_l}{\Gamma_k + \Gamma_l + \left( \Gamma_k - \Gamma_l \right) \Delta} \left[ \frac{2\sqrt{2} |t_{kl}|}{\sqrt{E^2 + 2|t_{kl}|^2}} \right] \left[ \frac{\pi}{2T} \right], \]

with \( k = k_0, l = l_0 \). Here \( \Gamma_k \propto |\psi_k(r_L)|^2 \) and \( \Gamma_l \propto |\psi_l(r_R)|^2 \) are the widths of the states \( k \) and \( l \) with respect to the electron tunneling into the adjacent lead; \( r_L, r_R \) are the points of the dot-lead contacts. At \( \Delta > \Delta \) there can be no inter-dot resonance on the line \( \xi_1 = \xi_2 \), and the conductance inevitably involves cotunneling through one of the dots. At low temperature \( T < \sqrt{E_C \Delta} \) only the elastic cotunneling is possible. The resulting conductance \( \hat{\tilde{G}}_\text{max} \sim (\Delta/E)G_I G_{L,R} \) yields parametrically small contributions to the ensemble averages. To calculate \( \langle \tilde{G}_\text{max} \rangle \) and \( \langle \tilde{G}_\text{max}^2 \rangle \), one needs to use \( \text{Eq. (12)} \) and average over all the random quantities entering in it. The statistical properties of \( \psi_{k,l} \) are well-known, and now we discuss those of \( \mathcal{E} \).

One source of fluctuations of \( \mathcal{E} \) is the randomness of the interdot tunneling matrix elements, which leads to random shifts of energies of electron states, see Eqs. \( \text{[3], [4]} \). This contribution yields:

\[ \langle \mathcal{E}^2 \rangle = \langle \delta U^2 \rangle / 2. \]
maxima along the line $(N_1^{(0)} + N, N_2^{(0)} + N)$. The number of maxima $N$ should be large enough to exceed the “correlation length” the ground state energy. In a partially open quantum dot, the characteristic correlation length of the fluctuations of the ground state energy at different gate voltages is $\sim N_{corr} = E/\Delta$. Thus, one must average over $N \approx N_{corr}$ peaks which gives

$$\langle \varepsilon_2^2 \rangle = \frac{4\Delta^2}{\beta \pi^2} \ln \left( \frac{EC}{\Delta} \right), \quad (14)$$

The distribution of $\varepsilon_2$ is Gaussian, except for the tails that are unimportant for our purposes.

The distribution of the energy mismatch $\varepsilon$ is Gaussian; its variance is $\langle \varepsilon^2 \rangle = (\varepsilon_1^2) + (\varepsilon_2^2)$, since the two contributions are independent. The value of $\varepsilon$ is determined by a large number of one-electron states. Therefore one can neglect the correlations between $\varepsilon$ and the one-electron wave functions of two states $k_0$ and $l_0$, involved in Eq. (2). Averaging of Eq. (12) yields:

$$\langle \tilde{G}^{\max} \rangle = A_1 G_0 \sqrt{\frac{\hbar G_1}{2e^2} \frac{\Delta}{\sqrt{\langle \varepsilon^2 \rangle}} \frac{\Delta}{4T}} , \quad (15)$$

$$\langle \tilde{G}^{\max}_2 \rangle = A_2 G_0^2 \sqrt{\frac{\hbar G_1}{2e^2} \frac{\Delta}{\sqrt{\langle \varepsilon^2 \rangle}} \left( \frac{\Delta}{4T} \right)^2} . \quad (16)$$

The conductances of the dot-lead contacts for simplicity here are taken equal: $G_1 = G_R = G_L$. The numerical factors are ensemble-dependent; $A_1 = 1/\sqrt{2\pi^2}$ (GOE) or $A_1 = \pi^2/64\sqrt{2}$ (GUE), and

$$A_2 = \frac{3\pi - 8}{\sqrt{16\pi^2}} \text{ (GOE) or } A_2 = \left( \frac{32 - 9\pi}{1024} \right) \text{ (GUE)} .$$

Substitution of $\langle \varepsilon^2 \rangle$ [Eqs. (13), (14)] into Eqs. (15), (16) gives Eq. (3).

The fluctuations of $G_{\max}$ are very sensitive to the magnetic field because the leading contribution to $\langle \tilde{G}^{\max}_2 \rangle$ comes from the peaks, whose narrow resonant region is traversed by the line $\varepsilon = \varepsilon_1$. A variation of the magnetic field by $B_c$ [Eq. (3)] would shift these peaks away from the line by $\sim \sqrt{\hbar G_1/e^2\Delta}$. This leads to a significant decrease of the height of the corresponding maximum in $G(N)$. The one-electron wave functions are not changed by such a small variation of the magnetic field. Therefore the correlation function of the peak conductance at different magnetic fields, $\langle \tilde{G}^{\max}(B_1)\tilde{G}^{\max}(B_2) \rangle$, is controlled by the parametric correlations of $\varepsilon_1$ with itself at two different fields. If $|B_1 - B_2| \ll B_\Delta \equiv (\Phi_0/S)/\sqrt{\Delta/4\pi e^2}$, the distribution function of the field-induced variation $\varepsilon(B_1 - B_2)$ of the energy $\varepsilon$ is

$$P(\varepsilon) = \frac{1}{\sqrt{2\pi \alpha}} \exp \left[ -\frac{\varepsilon^2}{2\alpha^2} \right] , \quad (17)$$

$$\alpha = \left( \frac{\Delta}{\sqrt{2}} \right) (|B_1 - B_2|/B_\Delta) .$$

Using Eqs. (12)-(14), (17), we obtain the correlation function $\langle \tilde{G}^{\max}(B_1)\tilde{G}^{\max}(B_2) \rangle$. Its full-width-half-maximum is $\approx 6B_c$, and the asymptote at $|B_1 - B_2| > B_c$ is

$$\frac{\langle \tilde{G}^{\max}(B_1)\tilde{G}^{\max}(B_2) \rangle}{\langle \tilde{G}^{\max}_2(B_1) \rangle} \approx \frac{1}{16\sqrt{2}A_2} \frac{B_c}{|B_1 - B_2|} . \quad (18)$$

The full expression for $\langle \tilde{G}^{\max}(B_1)\tilde{G}^{\max}(B_2) \rangle$ is too cumbersome to present here.

**Conclusion** — In this paper mesoscopic fluctuations of Coulomb blockade in a double-dot structure are studied. Electron tunneling between the dots leads to the formation of peak doublets in the dependence of conductance on gate voltages $V_{g1}, V_{g2}$. We find the variance of spacings within the doublets, see Eq. (3). This variance provides information on the mesoscopic fluctuations of the ground state energy of a partially opened quantum dot, which is hard to measure directly. The pattern of the conductance as a function of $V_{g1}, V_{g2}$ is affected by an unusually small magnetic field given by Eq. (3).

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