Innovative Approach of EOQ Structure for Decaying Items with Time Sensitive Demand, Cash-Discount, Shortages and Permissible Delay in Payments

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Abstract
Growing business process and rising aggressive conditions are encouraged to use the inventory control scheme and components in an ideal way. Cash discount and permissible delay are beneficial for vendor and buyer both. This study considers an EOQ model through demand rate depends on the time. A lower or higher time leads to lower or higher demand after feedback vice versa. In this paper deterioration, cash- discount, shortages and permissible delay are also considered. Mathematical models are discussed under four different states of affair. Solution method is given for finding the finest answer. The main aim is to maximize total profit. Numerical examples are provided for all four dissimilar situations. Optimal values with strictures are calculated to analyze the sensitivity investigation of optimal strategy concerning the parameters of the system. It is revealed that the total income is concave by means of cycle time.

Keywords Cash-discount; Inventory; Deterioration · Shortages · Demand · Trade credit

Mathematics Subject Classification 90B05

Introduction

The production of commodities is the first stage of manufacture. Three stage system, a manufacturer, seller and market are considered in the process. The retailer’s term represents for part of the system works as a bridge between market and manufacturer. EOQ models begin by taking into account that the demand rate was stable along with cycle time. This assumption was a serious restriction because in real life, demand of a commodity can depend on such multiple issue at the time, stock – level, quantity, selling price, lead time, advertisings, rebate, exponential etc.

Silver and Peterson [1] pointed out that the demand rate of some goods may be influenced by the stock stage. Indeed, huge heaps of supplies exhibited in a superstore sometimes guides
buyers to purchase more. There are two common rules for the inventory manager always keep high stock-level to make best use of profit of the inventory administration and depleting the order a new order begins. It might be profitable to raise the order level in each cycle and request a fresh order previous two stock runs out. Baker and Urban [2] designed an EOQ model with a store-linked, where the aim was profit maximization. Tripathi [3] analyzed an EOQ structure of items whose demand is a decreasing association of trade cost. Alfares [4] published the inventory model containing stock-echelon sensitive demand and a storage time-connected, carrying cost taking two times-linked holding cost functions. Tripathi and Mishra [5] described a production inventory model for time-associated demand. Time induced demand EOQ systems are established by Dave [6] and Maiti et al. [7]. Yang [8] described a structure under store-linked demand rate and stock-connected carrying cost under shortages. Several research papers published in this direction by Chang et al. [9], Soni and Shah [10], Tripathi [11], Teng et al. [12], Hsieh and Dye [13], Wee and Wang [14], Zhou et al. [15], Shah et al. [16] etc.

Deterioration of items is a major problem in the universe. Generally almost all items deteriorate with time. Deterioration means freshness decay of commodities. A normal common man declines his activity after passing the time and at last comes to the end. Tripathi and Pandey [17] studied an EOQ model to find the optimal total cost for price sensitive demand with Weibull distribution. Geetha and Kumar [18] addressed a model in which inventory cost will be lowered, if the seller can effectively reduce deterioration by improving the storage facility. Jaggi et al. [19] offered a model, including selling price dependent demand under deterioration. Mishra and Talati [20] proposed a single set-up multiple deliveries for fading stuff with fixed life time. Jani et al. [21] have studied an inventory policy for the item which has expiry date with two levels of trade credit depending on the quantity of order. It is considered that a supplier is ready to give a mutually agreed credit period to retailer only if the order quantity purchased by retailer is more than the predetermined quantity of ordered. Some researches in this in this direction are Chaudhury et al. [22], Duan et al. [23], Pervin et al. [24], Teng and Chang [25], Dye [26], Ghiami and Williams [27], Shah et al. [28] and others.

In the world economics, inflation and time value of capital cannot be unnoticed because of uncertainty of demand, weather, climate change abruptly, storm lock down (for example COVID-19), labor strike, flood etc. Buzacott [29] presented first an EOQ system by assuming inflationary effect on costs. Yang et al. [30] designed a variety of EOQ models with time-unstable demand pattern under inflation. Sarkar and Moon [31] explored a manufacture EOQ model for random demand through cause of inflation. Chaudhari et al. [32] designed a single retailer and single product which deteriorates continuously for time dependent deteriorating item with seasonal demand, quadratic demand is debated here which is suitable for the items whose demand with starting of the season increases initially and after end of the season, it starts decreases. Reduction of deterioration is reduced by preservation technology. Some researchers like Sarkar et al. [33], Yang [34], Dey et al. [35], Tripathi and Chaudhary [36] consequences special type of structures under inflation and time-reduction.

Trade credits and shortages both play an important role in any type of business. Tripathi [37] published at an EOQ system for spoilage products with curvilinear time-linked demand, shortages under traffic credits. Pal and Chandra [38] established a sporadic review EOQ model under traffic credits and price discounts. Chen et al. [39] extended an EOQ system to include particulars that (i) advertising cost is considerably superior to unit acquiring cost and (ii) interest rate charged by the trader is not essentially advanced than the seller’s savings return rate under permitted delay. Jiangtao et al. [40] addressed a multi-commodity system for unpreserved substances where demand rates of goods are stored-linked two-echelon
traffic credits. Chern and Teng [41] designed an EOQ system for trader for finding his/her best possible replenishment cycle time, included the fact that (i) failing foodstuffs decline constantly and having utmost life time and (ii) a seller frequently proposes an allowed delay in payments to draw additional purchasers.

The remainder of the work designed as follows. In the subsequent Sect. 2, assumptions and notation are mentioned. Mathematical formulation is argued in Sect. 3. The most beneficial explanation is renowned in part 4. After that, numerical examples are offered of all four cases to display hypothetical fallouts. Sensitivity exploration by means of distinct parameters is conversed in segment 6. We present conclusion and future research in the last.

**Notations and Assumptions**

**Notations**

\( K \) cost of ordering (in $)

\( c, p, h, s \) unit purchasing, selling, carrying and shortage cost/item

\( Q_1 \) and \( Q_2 \) highest inventory level and maximum shortage quantity

\( Q \) Lot – size

\( D(t) \) demand rate

\( I_p \) and \( I_e \) unit interest paid and earned/$

\( Q(t) \) level of inventory at moment ‘t’

\( \phi \) deterioration rate.

\( r \) cash reduction rate, ‘\( r \)’ lies between zero and one

\( t_1 \) time to end up inventory

\( M_1 \) and \( M_2 \) stage of cash reduction and allowable delay (\( M_2 > M_1 \))

\( T \) cycle time.

\( C_H, C_D \) and \( C_S \) carrying, deterioration and shortages cost

\( S_R \) sales profits

\( IP_1 \) and \( IP_3 \) interest payable (in $) (cases 1 and 3)

\( IE_i \) interest earned (in $), \( i = 1 - 4 \)

\( T_i^* \) optimal \( T \)

\( P_i (T) \) total profit/yr (in $)

**Assumptions**

1. Demand rate is time-sensitive, i.e. \( D(t) = \alpha + \beta t \), \( \alpha \) is positive, \( 0 \leq \beta \leq 1 \), \( \beta \) is not zero.
2. Shortages are permitted.
3. Vendor suggests cash discount, if payment is ready in \( t_1 \), or as a well full expense is charged. Inside credit period \( M_2 \).
4. Replenishment arises immediately at endless pace.
5. Deterioration rate is steady, \( 0 \leq \phi < 1 \).

**Mathematical Models**

Reduction of originality is a shapeless and natural phenomenon for items with passing time. Some preservation technologies can maintain freshness for some time, but they cannot con-
continue for a long time. The level of inventory \( Q(t) \) for some moment ‘\( t \)’ is ruled by subsequent equations are:

\[
\frac{dQ(t)}{dt} = \begin{cases} 
- \phi Q(t) - D(t), & 0 \leq t \leq t_1 \\
- D(t), & t_1 \leq t \leq T
\end{cases}
\]  

(1)

Solution of (1) and (2) with \( Q(t_1) = 0 \) is:

\[
Q(t) = \begin{cases} 
\frac{1}{\phi} \left\{ (\alpha - \beta) \left( \frac{e^{\phi t_1} - 1}{\phi} - \phi_t - t \right) \right\} \\
-(t - t_1) \left\{ \alpha + \frac{\beta (t_1 + t)}{2} \right\}
\end{cases}
\]

(2)

\[
Q = Q_1 + Q_2
\]

(3)

where \( Q_1 \) and \( Q_2 \) is obtained by substituting \( t = 0 \) and \( t = T \) in (4) and (5) respectively?

\[
CH = \frac{h}{\phi} \left\{ \left( \alpha - \beta \right) \left( \frac{e^{\phi t_1} - 1 - \phi t_1}{\phi} \right) + \beta t_1 \left( \frac{e^{\phi t_1} - 1 - \phi t_1}{2} \right) \right\}
\]

(4)

\[
CS = s \int_{t_1}^{T} -Q(t)dt = \frac{s(T - t_1)^2}{6} \left\{ \frac{3\alpha + \beta (T + 2t_1)}{2} \right\}
\]

(5)

Since seller offers permitted delay of cash reduction and payment. As a result, two situations may occur (i) payment is pleased at \( M_1 \) with reduction and (ii) payment is paid at \( M_1 \), lacking the cash diminish.

Case 1: \( M_1 \leq t_1 \leq T \)

In nearby learning, \( T \geq M_1 \). Since supplier presents allowed delay of cash concession, interest paid is nil. Figures of all mentions cases are as follows:

Therefore,

\[
IP_1 = \frac{e^{t_p}}{\phi} \left( \alpha - \beta \right) \left( \frac{e^{\phi (t_1 - M_1)} - 1 - \phi (t_1 - M_1)}{\phi} \right) + \beta \left( \frac{t_1 (e^{\phi (t_1 - M_1)} - 1)}{\phi} - \frac{(t_1^2 - M_1^2)}{2} \right)
\]

(6)

\[
IE_1 = p I_c \int_{0}^{M_1} (\alpha + \beta t).t.dt = p I_c M_1^2 \left( \frac{\alpha}{2} + \frac{\beta M_1}{3} \right)
\]

(7)

And,

\[
SR = p \int_{0}^{T} (\alpha + \beta t)dt = p T \left( \alpha + \frac{\beta T}{2} \right)
\]

(8)

The trader gets a cash allowance from supplier, due to payment is remunerated at \( M_1 \). Thus

\[
CD = rp \left\{ \frac{1}{\phi} \left\{ (\alpha - \beta) (e^{\phi t_1} - 1) + \beta t_1 e^{\phi t} \right\} + \frac{(T - t_1)(2\alpha + \beta (t_1 + T))}{2} \right\}
\]

(9)

Thus,

\[
P_1(T) = \{SR - (A + CH + CS + IP_1 - IE_1 - CD)\}/T
\]

(10)

Case 2: \( M_1 > T \)

In this situation, \( M_1 > T \), therefore \( IC_2 = 0 \) and

\[
IE_2 = p I_c t_1 \left\{ \alpha \left( M_1 - \frac{t_1}{2} \right) + \frac{\beta t_1}{2} \left( M_1 - \frac{t_1}{3} \right) \right\}
\]

(11)
Thus,

\[ P_2(T) = \left\{ S_R - (A + C_H + C_S - IE_2 - C_D) \right\}/T \]  \hspace{1cm} (12)

**Payment is Compensated at Credit Phase** \( M_2 \)

Case 3: \( M_2 < T \)

In such case, \( T > M_2 \), the seller has no cash price cut, thus

\[
I P_3 = c I_p \int_{M_2}^{t_1} Q(t) dt
\]

\[
IE_3 = p I_e \int_0^{M_2} (\alpha + \beta t) dt = p I_e M_2^2 \left( \frac{\alpha}{2} + \frac{\beta M_2}{3} \right)
\]  \hspace{1cm} (14)

As a result

\[ P_3(T) = \left\{ S_R - (A + C_H + C_S + IP_3 - IE_3) \right\}/T \]  \hspace{1cm} (15)

Case 4: \( M_2 \geq T \)

In this situation, \( T \leq M_2 \), the vendor has no cash price cut & \( IP_4 = 0 \), thus

\[
IE_4 = p I_e t_1 \left\{ \alpha \left( M_2 - \frac{t_1}{2} \right) + \frac{\beta t_1}{2} \left( M_2 - \frac{t_1}{3} \right) \right\}
\]

\[ P_4(T) = \left\{ S_R - (A + C_H + C_S - IE_4) \right\}/T \]  \hspace{1cm} (17)

For small decline rate, we can presume (Figs. 1, 2, 3 and 4).

\[ e^{\phi T} \approx 1 + \phi T + \frac{\phi^2 T^2}{2} \text{ etc, } \phi T < 1 \]  \hspace{1cm} (18)

Hence, the entirety income of case 1–4 is falling too:

\[
P_1 = \frac{(1 + r)p(2\alpha + \beta T)}{2} - \frac{K}{T} - \frac{h t_1^2 [\alpha(3 + \phi t_1) + t_1(2 + \theta T_1)\beta]}{6T} - \frac{c I_p(t_1 - M_1)^2}{6T}
\]

\[
[\alpha(3 + (t_1 - M_1)\phi) + \beta(M_1 + 2t_1 + \phi(t_1 - M_1)\phi)] + \frac{p I_e M_1^2(3\alpha + 2\beta M_1) + r p \phi t_1^2(\alpha + \beta t_1)}{6T}
\]

\[
P_2 = p(1 + r) \left( \alpha + \frac{\beta T}{2} \right) - \frac{K}{T} - \frac{h t_1^2 [a(3 + \phi t_1) + t_1(2 + \phi t_1)\beta]}{2T} - \frac{s(T - t_1)^2[3\alpha + (T + 2t_1)\beta]}{6T}
\]

\[
+ \frac{p I_e t_1}{6T} \left[3(2M_1 - t_1) + \beta t_1(3M_1 - t_1)\right] + \frac{r(p(2\alpha + \beta T))}{2T} + \frac{r p t_1^2(\alpha + \beta t_1)}{2T} \]

\[
P_3 = p \left( \alpha + \frac{\beta T}{2} \right) - \frac{K}{T} - \frac{h t_1^2 [a(3 + \phi t_1) + \beta(2 + \phi t_1)t_1]}{6T} - \frac{s(T - t_1)^2[3\alpha + \beta(2t_1 + T)]}{6T}
\]

\[
- \frac{c I_p(t_1 - M_2)^2[\alpha(3 + \phi(t_1 - M_2)) + \beta(M_2 + 2t_1 + \phi(t_1 - M_2)\phi)]}{6T} + \frac{p I_e M_2^2(\alpha + \beta M_2)}{T} \left( \frac{\alpha}{2} + \frac{\beta M_2}{3} \right)
\]  \hspace{1cm} (21)
\[
\frac{p l t_1}{6T} \{3\alpha(2M_2 - t_1) + \beta(3M_2 - t_1)t_1\}
\]

Since \( t_1 < T \), taking, \( t_1 = \gamma T \), \( \gamma \) is stable (0 < \( \gamma \) < 1). Equations (19) – (22) become:

\[
P_1(T) = \frac{p(1 + r)(2\alpha + \beta T)}{2} - \frac{K}{T} - \frac{h\gamma^2\{\alpha(3 + \gamma \phi T) + \beta(2 + \phi \gamma T)\gamma T\}T}{6} - \frac{s(1 - \gamma)^2\{3\alpha + (1 + 2\gamma)\beta T\} T}{6} - \frac{c I_p(T \gamma - M_1)^2[\alpha(3 + \phi(\gamma T - M_1)) + \beta(M_1 + 2T \gamma + \gamma \phi(T \gamma - M_1)]T}{3T}
\]
\[ P_2(T) = \frac{(1 + r)p(2\alpha + \beta T) + r\gamma^2(\alpha + \beta \gamma T)T}{6T} - \frac{K}{T} - \frac{h\gamma^2[\alpha(3 + \phi_T) + \beta \gamma (2 + \gamma \phi_T)T]}{6} 
- \frac{s(1 - \gamma)^2[\alpha + \beta T(1 + 2\gamma)]T}{6} \]

\[ P_3(T) = p\left(\alpha + \frac{\beta T}{2}\right) - \frac{K}{T} - \frac{h\gamma^2[\alpha(3 + \phi_T) + \beta (2 + \gamma \phi_T)\gamma_T]T}{6} 
- \frac{s(1 - \gamma)^2[\alpha + \beta (1 + 2\gamma)T]}{6} \]
Necessary and sufficient circumstances for maximization are:

\[ dP_i \frac{dP_i}{dT} = 0, \quad \text{and} \quad \frac{d^2 P_i}{dT^2} < 0. \quad \text{for} \quad i = 1, 2, 3, 4. \]

Putting first derivative of (23) – (26) w.r.t. \( T \), to zero, we find

\[ 3\beta (1 + r) p T^2 + 6K - h\gamma^2 T^2 \{ \alpha (3 + 2\gamma T + \beta (4 + 3\gamma T) T) - s (1 - \gamma)^2 \} \{ 3\alpha + 2(1 + 2\gamma) \beta T \} T^2 \]

\[ -c I_p \left\{ (\gamma T^2 - M_1^2) \left[ \alpha (3 + \phi (\gamma T - M_1)) + \beta (\gamma T - M_1) \right] \right\} \]

\[ \frac{d^2 P_i}{dT^2} = \frac{2K}{T^3} - \frac{h\gamma^3 (\alpha \phi + \beta (2 + 3\gamma T))}{3} - \frac{s \beta (1 - \gamma)^2 (2\gamma + 1)}{3} - \frac{c I_p (\gamma^2 T^2 - M_1^2) [\alpha \phi + \beta (2 + \phi (2\gamma T - M_1))]}{3} \]

\[ \frac{c I_p M_2^2}{T^3} \left[ \alpha \left\{ 1 + \phi (\gamma T - M_1) \right\} + \beta (\alpha \phi + \beta (2 + \phi (2\gamma T - M_1))) \right] + \frac{c I_p \lambda^2 \beta \phi}{T^3} + r \beta P_i \gamma^3 \]

\[ \frac{d^2 P_2}{dT^2} = -\frac{2K}{T^3} + \frac{h\gamma^3 (\alpha \phi + \beta (2 + 3\gamma T))}{3} + \frac{s \beta (1 - \gamma)^2 (1 + 2\gamma)}{3} + \frac{\beta P_i \gamma^3}{3} - r \beta P_i \gamma^3 < 0 \]
\[
\frac{d^2 P_i}{dT^2} = -\frac{2K}{T^3} - \frac{h \gamma^3 [\alpha \phi + \beta (2 + 3 \phi \gamma T)]}{3} - \frac{s \beta (1 - \gamma)^2 (1 + 2 \gamma)}{3} - \frac{c I_p (\gamma^2 T^2 - M_i^2)}{T^3} \left[ \alpha \left\{ 1 + \frac{(\gamma T - M_i) \phi}{3} \right\} + \left\{ M_i + 2 \gamma T + \gamma \phi T (\gamma T - M_i) \right\} \right] < 0 \quad (33)
\]

\[
\frac{d^2 P_4}{dT^2} = -\frac{2K}{T^2} + \frac{2 \gamma^2 \beta \phi p l e M_i^2}{T^3} \left( \frac{\alpha}{2} + \frac{\beta M_i}{3} \right) < 0 \quad (34)
\]

Since \( \frac{d^2 P_i}{dT^2} < 0 \), \( i = 1 - 4 \). \( P_i \) is maximum at \( T_i^* \). We have also shown by graphs in numerical examples.

**Algorithm**

In this section, we provide a solution procedure and flow diagram for finding an optimal resolution.

*Step 1* locate \( T_i^* \) by resolve (27)–(30), \( i = 1 - 4 \).

*Step 2* if \( T_1^* \geq M_1 \), come across \( P_1^* \) by (23).

*Step 3* if \( T_2^* < M_1 \), discover \( P_2^* \) by (24).
Step 4 if $T_{3}^{*} \geq M_{2}$, find $P_{3}^{*}$ by (25).

Step 5 if $T_{4}^{*} < M_{2}$, locate $P_{4}^{*}$ by (26).

Step 6 Obtain most favorable income $P_{i} = \max P_{i}$.

Step 7 end.

Numerical Examples

Examples are supplied to make obvious conclusions of structure discussed in each case:

Example 1 ($M_{1} \leq T$).

Bearing in mind subsequent constrains in proper component:

$\alpha = 1.5 \times 10^{3}$, $\beta = 150$, $\phi = 1/100$, $s = 100$, $K = 1.0 \times 10^{3}$, $p = 250$, $h = 10$, $I_{e} = 13/100$, $I_{p} = 3/20$, $M_{1} = 1/5$ yr, $c = 100$, $r = 1/50$ & $\gamma = 3/5$. Putting these in (27), and resolving for $T$, we find, $T_{1}^{*} = 0.65989$ yr, that validate case 1, corresponding $Q^{*} = 1023.7$ and $P_{1}^{*} = $ 2973.7.

Example 2 ($M_{1} > T$).

Considering following strictures in their suitable units:

$\alpha = 1.5 \times 10^{3}$, $\beta = 150$, $\phi = 1/100$, $s = 500$, $h = 10$, $K = 100$, $p = 300$, $I_{e} = 13/100$, $I_{p} = 3/20$, $M_{1} = 1/5$ yr, $c = 50$, $r = 1/50$ & $\gamma = 3/5$. Replacement of those in (28) and solving for $T$, we gain, $T_{2}^{*} = 0.1157$ yr, which proves case 2, accordingly $Q^{*} = 174.57$ & $P_{2}^{*} = $ 794.37.
Table 1 Deviation of $T$, $Q$ and $P_i$ by $s$, $K$, $c$, $p$, $h$, $\phi$, $\alpha$ and $\beta$

| $s$ | $T^*$ | $Q^*$ | $P_i^*$ | $K$ | $T^*$ | $Q^*$ | $P_i^*$ |
|-----|-------|-------|--------|-----|-------|-------|--------|
| 105 | 0.55973 | 863.97 | 2448.7 | 110 | 0.69855 | 1085.8 | 2882.8 |
| 110 | 0.48786 | 750.29 | 2020.7 | 120 | 0.73295 | 1141.2 | 2793.4 |
| 115 | 0.43481 | 666.92 | 1717.6 | 130 | 0.76408 | 1237.8 | 2619.0 |
| 120 | 0.39437 | 603.65 | 1347.1 | 140 | 0.79263 | 1280.8 | 2533.9 |

| $c$ | $T^*$ | $Q^*$ | $P_i^*$ | $p$ | $T^*$ | $Q^*$ | $P_i^*$ |
|-----|-------|-------|--------|-----|-------|-------|--------|
| 105 | 0.63346 | 981.41 | 2903.7 | 260 | 0.80909 | 1264.6 | 3860.7 |
| 110 | 0.61039 | 944.57 | 2842.4 | 270 | 1.01837 | 1608.3 | 4993.1 |
| 115 | 0.59018 | 912.36 | 2788.4 | 280 | 1.27802 | 2044.3 | 6417.3 |
| 120 | 0.57241 | 884.10 | 2740.8 | 290 | 1.56867 | 2544.8 | 8148.3 |
| 125 | 0.55671 | 859.18 | 2698.5 | 300 | 1.87570 | 3088.0 | 9976.4 |

| $h$ | $T^*$ | $Q^*$ | $P_i^*$ | $\phi$ | $T^*$ | $Q^*$ | $P_i^*$ |
|-----|-------|-------|--------|--------|-------|-------|--------|
| 12   | 0.57088 | 881.67 | 2505.4 | 0.02   | 0.65874 | 1023.2 | 2969.2 |
| 14   | 0.50394 | 775.66 | 2113.6 | 0.03   | 0.65771 | 1022.7 | 2964.8 |
| 16   | 0.45278 | 695.11 | 1777.9 | 0.04   | 0.65664 | 1022.1 | 2960.4 |
| 18   | 0.41281 | 632.46 | 1483.8 | 0.05   | 0.65559 | 1021.6 | 2956.0 |
| 20   | 0.38084 | 582.53 | 1221.2 | 0.06   | 0.65454 | 1021.2 | 2951.7 |
| 1250 | 1.8827  | 2533.8 | 8242.99| 200    | 2.24652 | 3890.5 | 13,933.5 |

| $\alpha$ | $T^*$ | $Q^*$ | $P_i^*$ | $\beta$ | $T^*$ | $Q^*$ | $P_i^*$ |
|-----------|-------|-------|--------|---------|-------|-------|--------|
| 1000      | 2.9219 | 3581.6 | 14,854.0| 160  | 0.92394 | 1456.7 | 4323.9 |
| 1050      | 2.6585 | 3337.9 | 12,968.5| 170  | 1.26014 | 2029.8 | 6151.1 |
| 1100      | 2.3968 | 3080.9 | 11,238.0| 180  | 1.60881 | 2654.0 | 8405.0 |
| 1200      | 2.1378 | 2812.2 | 9662.94 | 190  | 1.94021 | 3279.6 | 11,017.0|
| 1250      | 1.8827 | 2533.8 | 8242.99 | 200  | 2.24652 | 3890.5 | 13,933.5|

Example 3 ($M_2 < T$).

Let us choose following constraints in proper entities:

$\alpha = 1.1 \times 10^3$, $\beta = 150$, $\phi = 1/100$, $s = 100$, $K = 1.0 \times 10^3$, $p = 250$, $h = 10$, $I_e = 13/100$, $I_p = 3/20$, $M_1 = 1/4$ yr, $c = 100$, $r = 1.50$, and $M_2 = 140$ days. On putting these (29) and solving for $T$, we find, $T_3^* = 0.42588$ yr, which confirms case 3, related $Q^* = 652.92$ & $P_3^* = \$3082.5$.

Example 4 ($M_2 \geq T$).

Following constraints are taken in suitable units:

$\alpha = 1.5 \times 10^3$, $\beta = 150$, $\phi = 1/100$, $s = 100$, $K = 1.0 \times 10^3$, $p = 500$, $h = 10$, $I_e = 13/100$, $I_p = 3/20$, $c = 50$, $r = 1/100$, and $M_2 = 1/4$ yr. On substituting those in (30), and solving for $T$, we obtain, $T_4^* = 0.19933$ years, that verifies case 4, resultant $Q^* = 338.0$ & $P_4^* = \$2328.2$.

Using above algorithm Case 3 gives the optimal (maximum) solution (Figs. 5, 6, 7 and 8).
Sensitivity Analysis

Case 1

Considering identical data as in Ex. 1, sensitivity study is conversed. Fallouts are reviewed in Table 1.

Case 2

Using the same figures as in Ex. 2, sensitivity scrutiny is conversed in Table 2.

Case 3

With parallel information as in Ex 3, sensitivity inquiry is discussed below:

Case 4

By means of alike data as design in Ex. 4, sensitivity inspection is as follows:

Following judgment can be finished from Table 1:

1. Enlarge of $s$, $K$, $c$ and it will cause a drop of $P_1$.
2. Elevate of $p'$ and $\phi$ will lead augment in $P_1$.

Following submission can be equipped from Table 2:

1. Lift of $s$, $K$ and $h$ will cause weakening in $P_2$. 

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### Table 3

| s     | T*     | Q*     | P3*     | K     | T*     | Q*     | P3*     |
|-------|--------|--------|---------|-------|--------|--------|---------|
| 75    | 1.67627| 2733.5 | 6987.5  | 1100  | 0.49783| 766.03 | 2866.4  |
| 80    | 1.31673| 2110.2 | 5093.2  | 1200  | 0.55300| 853.29 | 2676.3  |
| 85    | 1.00272| 1582.4 | 4340.7  | 1300  | 0.59869| 925.92 | 2502.7  |
| 90    | 0.74353| 1158.3 | 3786.5  | 1400  | 0.63815| 988.91 | 2341.1  |
| 95    | 0.55171| 857.21 | 3383.2  | 1500  | 0.67315| 1045.0 | 2186.6  |

### Dissimilarity of \( T, Q \) and \( P_i \) by \( s, K, c, h, \alpha, \beta \) and \( \phi \)

2. Raise of \( p \) consequences enhances \( P_2 \).

Following proposition can be finished from Table 3.

1. Boost of \( s, K, \) and \( h \) will direct diminish \( P_3 \).
2. Augment of will direct decline in \( P_3 \).
3. Amplification of \( p \) causes moves up in \( P_3 \).

Deductions made from Table 4 are as follows:

1. Boost of \( s \) and \( p \) will express make bigger in \( P_4 \).
2. Improve of \( A \) and \( h \) lead, turn down in \( P_4 \).

### Managerial Insights

These above deviations have the following managerial implications.

- Higher values of \( s, c, h, \phi \) and \( \alpha \) implies lower values of cycle time, order quantity and total profit for case I and III.
Table 4 Discrepancy of $T$, $Q$ & $P_i$ by $s$, $K$, $c$, $p$ and $h$

| $s$ | $T$  | $Q^*$ | $P_4^*$ | $K$  | $T^*$ | $Q^*$ | $P_4^*$ |
|-----|------|-------|---------|------|-------|-------|---------|
| 460 | 0.21621 | 327.94 | 2178.7  | 950  | 0.21711 | 329.32 | 2657.3  |
| 470 | 0.21024 | 318.80 | 3026.5  | 960  | 0.21824 | 331.06 | 2590.9  |
| 480 | 0.20475 | 310.39 | 1872.6  | 970  | 0.21837 | 332.80 | 2524.7  |
| 490 | 0.19968 | 302.62 | 1872.6  | 980  | 0.22050 | 334.52 | 2454.9  |
| 500 | 0.19498 | 285.42 | 1615.5  | 990  | 0.22162 | 336.24 | 2393.4  |

| $p$ | $T^*$ | $Q^*$ | $P_4^*$ | $h$  | $T^*$ | $Q^*$ | $P_4^*$ |
|-----|------|-------|---------|------|-------|-------|---------|
| 450 | 0.20427 | 306.45 | 700.10  | 12   | 0.21976 | 333.39 | 2261.4  |
| 460 | 0.20758 | 314.72 | 1022.0  | 14   | 0.21691 | 329.02 | 2194.0  |
| 470 | 0.21107 | 320.06 | 1361.2  | 16   | 0.21417 | 324.81 | 2126.1  |
| 480 | 0.21474 | 325.70 | 1687.2  | 18   | 0.21153 | 320.80 | 2052.7  |
| 490 | 0.21862 | 331.65 | 2009.7  | 20   | 0.20899 | 316.87 | 1989.0  |

| $\alpha$ | $T^*$ | $Q^*$ | $P_3^*$ | $\beta$ | $T^*$ | $Q^*$ | $P_3^*$ |
|-----------|-------|-------|--------|---------|-------|-------|--------|
| 1600      | 0.20466 | 330.71 | 2584.1 | 100     | 0.17645 | 266.32 | 727.25 |
| 1700      | 0.19034 | 326.41 | 2865.0 | 110     | 0.18345 | 277.12 | 1047.4 |
| 1800      | 0.17865 | 324.06 | 3171.4 | 120     | 0.19134 | 289.31 | 1369.4 |
| 1900      | 0.16886 | 323.07 | 3501.8 | 130     | 0.20032 | 303.20 | 1691.9 |
| 2000      | 0.16051 | 232.05 | 3854.8 | 140     | 0.21065 | 319.21 | 2012.7 |

- Higher values of $p$ and $\beta$ implies higher values of cycle time, order quantity and total profit for case I and III.
- Higher values of $K$ implies higher values of cycle time and order quantity while lower values of total profit.
- Higher values of $s$, $h$, and $\alpha$ implies lower values of cycle time, order quantity and lower values total profit for case II and IV.
- Higher values of $K$, $p$, and $\beta$ implies higher values of cycle time, order quantity and lower values total profit for case II and IV.

**Conclusion**

In this study, we have deliberated EOQ models under trade credits permit for four unlike conditions. We have attempted to locate characteristic of cash decline into the conventional model with permitted delay. Numerical examples are completed on credible attempt. Optimal explanation is acquired for finding optimal variables. Solution process is communicated to find most advantageous solution. Sensitivity reading of the clarification for dissimilar constraints has been conferring. This research is obliging for returning products since demand for continuing foodstuffs is usually time allied. It is seen that disparity in shortage, ordering, procure, carrying and selling costs, lead to momentous possessions on finest $P_i$, $i = 1$–$4$.

Entire income is around stable with adjust in weakening rate. Outcomes came into view in sensitivity analysis is conflicting, like expand in cost fallouts reject of earnings whereas intensify in selling price argued lift in income.

A variety of likely extensions of the model that can be presented as like: (i) variable decay and Weibull deterioration (ii) to assume a variable carrying cost (iii) to comprise fall in the
purchasing cost/unit (iv) to study the case of inflation and shipment charges and (v) to study stock-sensitive demand.

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