On the role of gluons and the quark sea in the proton spin

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The real, interacting elementary particle always consists of a 'bare' particle and a cloud of virtual particles mediating a self-interaction and/or the binding inside a composite object. In this note we discuss the question of spin content of the virtual cloud in two different cases: electron and quark.

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1. INTRODUCTION

This report is an addendum to our recent study [1] related to the spin of composite particle. In general, description of real interacting particles can be related to their 'bare' or 'dressed' form. In our present discussion we address the general questions:

a) How much do the virtual particles surrounding bare particle contribute to the spin of corresponding real, dressed particle?

b) How much do the virtual particles mediating binding of the constituents of a composite particle contribute to its spin?

First, we will illustrate the problem with the case of electron. In quantum mechanic the total angular momentum (AM) of any particle including composite ones is given by the sum of the orbital AM (OAM) and spin, \( J = L + S \).

The corresponding quantum numbers are discrete sets of integers or half-integers and in the relativistic case only total AM conserves, so only \( J \) and \( J_z \) can be the good quantum numbers. The electron, as a Dirac particle, in its rest frame has AM defined by its spin, \( s = 1/2 \). This value is the same for the dressed electron (as proved experimentally) and for the bare one (as follows from the QED Lagrangian). The dressed electron is a bare electron surrounded by the virtual cloud of \( \gamma \) and \( e^-e^+ \) pairs, as symbolically sketched in Fig. 1 for different scales represented by the parameter \( Q^2 \). So the renormalization as a continuous change of the scale should not change the AMs represented by the discrete numbers, \( J^e(Q^2) = s = 1/2 \). But what about the projections \( J^e_z(Q^2) + J^\gamma_z(Q^2) = \pm 1/2 \)? Can the contribution of virtual cloud \( J^\gamma_z(Q^2) \) differ from zero and how much? Apparently, we are not able to make a complete calculation of \( J^\gamma_z \) in the framework of the perturbative QED, however we can make at least its semiclassical calculation as follows.

The electromagnetic field, or its \( \gamma \)-quanta, are according to Maxwell equations created by the electric current. We consider the current generated by the electron states represented by the spinor spherical harmonics

\[
|j, j_z\rangle = \Phi_{j_\mu j_z} (r) = \frac{1}{\sqrt{2\epsilon}} \begin{pmatrix}
\sqrt{\epsilon + mR_{k\ell} \Omega_{j_\nu j_z} (\omega)} \\
-\sqrt{\epsilon - mR_{k\lambda} \Omega_{j_\nu j_z} (\omega)}
\end{pmatrix},
\]

FIG. 1: Scale dependent image of a real particle, see text.
A few examples of deal with their coordinate representation. The corresponding current read
momentum (AM) and have been discussed before \[1\] in momentum representation, while in the present report we

| \(j, j_z\) | \(\rho_{j, j_z}(\omega)\) |
|---|---|
| \(\frac{1}{2}, \frac{1}{2}\) | \(\frac{1}{4} - \frac{3}{4} \cos \theta\) |
| \(\frac{1}{2}, -\frac{1}{2}\) | \(\frac{1}{4} + \frac{3}{4} \cos \theta\) |
| \(\frac{1}{2}, 0\) | \(\frac{5}{4} - \frac{3}{4} \cos \theta\) |
| \(1, 1\) | \(45 \cos 2\theta + 15 \cos 4\theta\) |
| \(1, 0\) | \(45 \cos 2\theta - 45 \cos 4\theta\) |
| \(1, -1\) | \(27 - 12 \cos 2\theta - 45 \cos 4\theta\) |
| \(5, 0\) | \(64\) |
| \(5, 1\) | \(64\) |
| \(5, 2\) | \(64\) |

TABLE I: The examples of the angular distributions \(\rho_{j, j_z}\). The common factor \(1/4\pi\) is omitted. For opposite \(j_z\) it holds: \(\rho_{j, -j_z} = \rho_{j, j_z}\).

where \(\omega\) represents the polar and azimuthal angles \((\theta, \varphi)\) of the space coordinates \(r\) with respect to the axis of quantization \(z\), \(l_p = j \pm \frac{1}{2}\), \(\lambda_p = 2j - l_p\) (\(l_p\) defines the parity), energy \(\epsilon = \sqrt{k^2 + m^2}\) and

\[
\Omega_{j, j_z} (\omega) = \left( \begin{array}{c}
\frac{j + j_z + 1}{2j} Y_{j, j_z+1/2} (\omega) \\
\frac{j - j_z + 1}{2j} Y_{j, j_z+1/2} (\omega)
\end{array} \right), \quad \lambda_p = j + \frac{1}{2}
\]

The functions \(Y_{j, j_z}\) are usual spherical harmonics. The radial functions in the case of free electron read:

\[
R_{kl}(r) = \frac{2\pi k}{r} J_{l+1/2}(kr), \quad \int r^2 R_{kl} R_{k'l'} dr = 2\pi \delta(k - k'),
\]

where \(k = |k|\) and \(J_{l'}(z)\) are Bessel functions of the first kind, otherwise (e.g. for electron in the hydrogen atom) the radial functions will be different due to an external field. However it is important that the information about AM of the electron is completely absorbed in the angular terms. The states \(|j, j_z\rangle\) are eigenstates of the total angular momentum (AM) and have been discussed before \([1]\) in momentum representation, while in the present report we deal with their coordinate representation. The corresponding current read

\[
I_\mu = \langle 0, I \rangle = \Phi_{j, j_z}^\dagger (r) \gamma^\mu \gamma_\mu \Phi_{j, j_z} (r)
\]

and one can check that

\[
I_0 = \hbar I \rho_{j, j_z} (\cos \theta), \quad I = \hbar I I \rho_{j, j_z} (\cos \theta) r,
\]

where

\[
\hbar I = \frac{1}{2} \left( \left(1 + \frac{m}{\epsilon}\right) R_{k l p}^2 + \left(1 - \frac{m}{\epsilon}\right) R_{k l p}^2 \right), \quad \hbar II = -\frac{k}{\epsilon r} R_{k l p} R_{k l p}.
\]

A few examples of \(\rho_{j, j_z}\) are given in Table [1] where one can observe the following. The stationary current \(I_\mu\) depends only on \(j\) and \([j_z]\), therefore it does not involve any information on the direction of electron polarization. So, there is no reason to expect any correlation between electron polarization and polarization of the electromagnetic field generated by this current, or equivalently polarization of the statistical set of emitted and reabsorbed \(\gamma\). In other words the average polarization of virtual cloud of \(\gamma\) and consequently also e⁻e⁺ pairs should be zero. The AM of the electromagnetic field is given by the relation

\[
J^\gamma = \int r \times (E \times H) d^3r,
\]

where \(E, H\) are the corresponding intensities of electric and magnetic field. Due to the symmetry of current \([5]\) that generates these fields, the corresponding AM satisfies

\[
J^\gamma = 0,
\]
the proof is given in Appendix. This result follows only from the angular terms in the wave function \( |1\rangle \) and does not depend on the radial ones. The result represents a mean value, which is not influenced by the fluctuations generated by single \( \gamma \). So, this calculation suggests AM of the cloud of virtual \( \gamma \) is zero despite the fact that AM of its source, the electron in a state \( |1\rangle \), is not zero. The same holds for any combination of the states \( |1\rangle \).

In fact one can expect the similar results also for the virtual photons mediating the binding of electrons in an atom, we take the hydrogen for an illustration. While the free electron emits and reabsors virtual photons by itself, the electron bounded in hydrogen atom in addition exchanges (emits and absorbs) virtual photons with the proton. As we have already noted, different radial terms in the electron wave function do not influence relation (8). Since the AM of the electromagnetic field generated by the proton is zero as well, the total AM of hydrogen will be given only by AM's of the electron and proton, without contribution of the electromagnetic field generated by both the particles.

Perhaps similar arguments can be valid also for the nucleons bounded in a nucleus. This would suggest the virtual particles mediating the binding of nucleons also do not contribute to the resulting spin of nucleus, which is always integer or half-integer.

The situation with quarks inside a nucleon is more complicated. The quark at different scales is sketched in Fig. 1 (in which the bare electron surrounded by virtual cloud of \( \gamma \) and \( e^-e^+ \) pairs is now replaced by the bare quark with a cloud of virtual \( g \) and \( q \bar{q} \) pairs). The terminology is as follows:

i) The bare quark can be identified with the current quark, which can be described by the distribution functions \( q_a(x) \) defined in the quark-parton model. They are related to the sets of quarks and antiquarks in the figure for \( Q^2 \rightarrow \infty \).

ii) The constituent quark can be identified with the dressed quark at a low \( Q^2 \) scale.

iii) The valence quark can be identified with the set of quarks, from which the cloud of virtual \( q \bar{q} \) pairs and gluons is separated off. In the figure the valence quarks are represented by the central spots. Strictly speaking, depending on the scale, valence and sea quarks may not be clearly distinguishable. In a short time interval \( \Delta \tau \), a quark from the virtual \( q \bar{q} \) pair is indistinguishable from the source, valence quark, see Fig. 2. However, the usual definition in terms of the quark-parton model distributions

\[
q_{\text{val}}^a(x,Q^2) = q^a(x,Q^2) - \bar{q}^a(x,Q^2)
\]

is unambiguous. For quarks the parameter \( Q^2 \) represents the renormalization scale, but also the DIS parameter \( -Q^2 = \text{photon four-momentum square} \) or equivalently a scale of the space-time domain inside which the photon absorption takes place \( |1\rangle \).

Now, we can put the question b) for the quarks bounded inside the proton: How much the field of virtual gluons generated by the valence quarks and the sea of virtual \( q \bar{q} \) pairs created by the gluons contribute to the proton spin? This question is being studied in the experiments, which measure contribution of the gluons and sea quarks to the proton spin. Available data from the experiments COMPASS \( |2\rangle \) and HERMES \( |3\rangle \) suggest rather small gluon contribution, in fact the data are consistent with zero within statistical errors. The very recent results of the RHIC experiments \( |4\rangle |5\rangle \) suggest a positive gluon contribution which, however still cannot fully compensate for a small quark contribution to the proton spin.

These results can be interpreted in the framework of covariant approach presented in \( |1\rangle \). In the paper we studied the relativistic interplay between the quark spins and orbital AM's, which collectively contribute to the proton spin. The simplest scenario assuming

a) the quarks are in the state \( j = 1/2 \), see Eq. 113\( |1\rangle \),

b) mass of quarks can be neglected, \( \langle m/c \rangle \rightarrow 0 \),

c) there is no gluon contribution, i.e. proton spin \( J = 1/2 \) is generated only by the AM of quarks, see Eq. 109\( |1\rangle \)
gave a prediction for the contribution of the quark spins in DIS region,

\[
\Delta \Sigma = \frac{1}{3}.
\]
while the “missing” part of the proton spin is fully compensated by the quark OAM. This prediction fits the data [7–9] surprisingly well.

However, in a more general case, if only a) can be assumed, then AM of the quarks consists of the spin and orbital parts

\[ J_q = \frac{1}{2} \Delta \Sigma + \langle L_z \rangle, \]

where

\[ \Delta \Sigma = \frac{1}{3} (1 + 2\bar{\mu}), \quad \langle L_z \rangle = \frac{1 - \bar{\mu}}{3}, \quad \bar{\mu} = \left\langle \frac{m}{c} \right\rangle > 0. \]

Further, if the proton spin consists of the quark and gluon contributions,

\[ \frac{1}{2} = J_q + J_g, \quad J_q = \frac{1}{2} \kappa, \quad J_g = \frac{1}{2} (1 - \kappa), \]

then the value \( \Delta \Sigma \) from Eq. (12) is rescaled correspondingly

\[ \Delta \Sigma = \frac{\kappa}{3} \left( 1 + 2\bar{\mu} \right). \]

On the other hand, Eq. (13) gives

\[ \kappa = 1 - 2J_g, \]

so we have

\[ \Delta \Sigma = \frac{1}{3} \left( 1 - 2J_g \right) (1 + 2\bar{\mu}). \]

This relation means that the quark spin content is correlated with two parameters, gluon contribution \( J_g \) and the quark effective mass ratio \( \bar{\mu} \). This dependence is demonstrated in Fig. 3. One can observe:

1) \( \Delta \Sigma < 1/3 \) gives \( J_g > 0 \),
2) \( \Delta \Sigma = 1/3 \) gives \( J_g = 0 \) for \( \bar{\mu} \to 0 \), and \( J_g > 0 \) for \( \bar{\mu} > 0 \),
3) for \( \Delta \Sigma > 1/3 \) the sign of \( J_g \) depends on \( \bar{\mu} \), for \( \bar{\mu} \to 0 \) we have \( J_g < 0 \).

In this way the COMPASS and HERMES data [7–9] giving \( \Delta \Sigma \approx 1/3 \), are compatible also with a positive gluon contribution \( J_g \) suggested by the recent data on RHIC. Note that positive \( J_g \) correlates with a non-zero quark effective mass ratio \( \bar{\mu} \). The reported \( J_g \) still admit a significant role of the quark OAM.

The most general scenario, in which some admixture of the quark states with \( j = 3/2, 5/2, ... \), or \( l_p > 0 \) is present, gives (cf. Tab. II[II])

\[ \Delta \Sigma = a + b\bar{\mu}. \]

Since the probability of this admixture is rather small, we will ignore it here.
Finally, the spin contribution of the sea quarks is known to be small or compatible with zero \[9\]. This confirms the expectation that the sea quark contribution correlates with the gluon contribution.

To conclude, interpretation of the available sets of experimental data in framework of the covariant approach suggests an important role of the quark OAM for the creation of the proton spin on the scale \(Q^2\) defined by the data. The positive gluon contribution to the proton spin does not contradict the covariant approach. Moreover, the values \(J^g\) and \(\Delta \Sigma\) obtained experimentally allow us, in principle, to estimate the quark mass parameter \(\langle m/\epsilon \rangle\). However, the precise data on \(J^g\) are still missing and so the existing experimental data do not disprove the hypothesis \(J^g \approx 0\) based on the analogy with AM of virtual photons given by Eq. \[5\].

**Appendix A: Proof of the relation \[8\]**

The current \[5\] generate electric and magnetic field

\[
E(r) = \int I_0(r') \frac{r - r'}{|r - r'|^{3/2}} d^3r',
\]

\[
H(r) = \int I(r') \times \frac{r - r'}{|r - r'|^{3/2}} d^3r'.
\]

If we define

\[
W_X^r(r) = \int \frac{h_X(r') p_{j,j'} (\cos \theta')}{|r - r'|^{3/2}} d^3r', \quad S(r) = \int \frac{h_1(r') p_{j,j'} (\cos \theta')}{|r - r'|^{3/2}} d^3r',
\]

where \(X = I, II\), then with the use of \[5\], \[6\] we get:

\[
E(r) = -W^I(r) + S(r) r, \quad H(r) = W^{II}(r) \times r.
\]

In terms of spherical coordinates

\[
r_1 = r \sin \theta \cos \varphi, \quad r_2 = r \sin \theta \sin \varphi, \quad r_3 = r \cos \theta
\]

we have

\[
S(r) = \int \frac{h_1(r') p_{j,j'} (\cos \theta')}{(r^2 + r'^2 - 2rr' \sin \theta \sin \theta' \cos (\varphi - \varphi') + \cos \theta \cos \theta')^{3/2}} r'^2 \sin \theta' d\varphi' d\theta' dr',
\]

\[
W_X^r(r) = \int \frac{h_X(r') p_{j,j'} (\cos \theta') r'}{(r^2 + r'^2 - 2rr' \sin \theta \sin \theta' \cos (\varphi - \varphi') + \cos \theta \cos \theta')^{3/2}} r'^2 \sin \theta' d\varphi' d\theta' dr'.
\]

Obviously \(S(r)\) does not depend on \(\varphi\) so we have

\[
S(r) = S(r, \theta).
\]

In the second integral, after substitution \(\psi = \varphi' - \varphi\) we replace correspondingly in \(r'\):

\[
x' = r' \sin \theta' \cos \varphi' \rightarrow r' \sin \theta' (\cos \psi \cos \varphi - \sin \psi \sin \varphi),
\]

\[
y' = r' \sin \theta' \sin \varphi' \rightarrow r' \sin \theta' (\cos \psi \sin \varphi + \sin \psi \cos \varphi),
\]

and instead of \[A7\] we obtain

\[
W_1^X(r) = \int \frac{h_X(r') p_{j,j'} (\cos \theta') r' \sin \theta' (\cos \psi \cos \varphi - \sin \psi \sin \varphi)}{(r^2 + r'^2 - 2rr' \sin \theta \sin \theta' \cos \psi + \cos \theta \cos \theta')^{3/2}} r'^2 \sin \theta' d\psi d\theta' dr',
\]

\[
W_2^X(r) = \int \frac{h_X(r') p_{j,j'} (\cos \theta') r' \sin \theta' (\cos \psi \sin \varphi + \sin \psi \cos \varphi)}{(r^2 + r'^2 - 2rr' \sin \theta \sin \theta' \cos \psi + \cos \theta \cos \theta')^{3/2}} r'^2 \sin \theta' d\psi d\theta' dr',
\]

\[
W_3^X(r) = \int \frac{h_X(r') p_{j,j'} (\cos \theta') r' \cos \theta}{(r^2 + r'^2 - 2rr' \sin \theta \sin \theta' \cos \psi + \cos \theta \cos \theta')^{3/2}} r'^2 \sin \theta' d\psi d\theta' dr'.
\]
Where in general
\[
\int_{-\pi}^{\pi} f_{\text{even}}(\psi) \sin \psi \, d\psi = 0
\]  
(A14)

where \( f_{\text{even}}(\psi) = f_{\text{even}}(-\psi) \), then the second term in (A11), (A12) vanishes and the expressions are simplified
\[
W_1^X(r, \theta) = W^X(r, \theta) r_1, \quad W_2^X(r, \theta) = W^X(r, \theta) r_2, \quad W_3^X(r, \theta) = W^X(r, \theta),
\]  
(A15)

where
\[
W^X(r, \theta) = \frac{1}{r \sin \theta} \int \frac{h_X(r') \rho_{j,j} (\cos \theta') r' \sin \theta' \cos \psi}{(r^2 + r'^2 - 2rr' \sin \theta \sin \theta' \cos \psi + 2 \cos \theta \cos \theta')^{3/2}} \, r^2 \sin \theta' \, \psi \, d\theta' \, dr',
\]  
(A16)

\[
W_3^X(r, \theta) = \int \frac{h_X(r') \rho_{j,j} (\cos \theta') r' \sin \theta' \cos \psi}{(r^2 + r'^2 - 2rr' \sin \theta \sin \theta' \cos \psi + 2 \cos \theta \cos \theta')^{3/2}} \, r^2 \sin \theta' \, \psi \, d\theta' \, dr'.
\]  
(A17)

After inserting from (A4) into (7) we integrate AM density
\[
\mathbf{j}^\gamma = r \times \left( (-W^I(r) + S(r)r) \times (W^{II}(r) \times r) \right),
\]  
(A18)

which with the use of (A8), (A15) gives
\[
\mathbf{j}^\gamma = \left\{ \begin{array}{ll}
(W^I W^{II} (r_1^2 r_2 r_3 + r_1 r_2^2 r_3) + W^{II} W^I r_2^2 r_3^2 - W^{II} W^I (r_1^2 r_2 + r_1 r_2^2) - W^{II} W^I r_3^2) ,

(-W^{II} W^I (r_1^2 r_2 r_3 + r_1 r_2^2 r_3) - W^{II} W^I r_2^2 r_3^2 + W^{II} W^I (r_1^2 r_2 + r_1 r_2^2) + W^{II} W^I r_3^2) ,

0
\end{array} \right\}
\]  
(A19)

These terms depend on \( \varphi \) only via coordinates \( r_1 \) and \( r_2 \) (A5). Since each term involves just one odd power of \( r_1 \sim \cos \varphi \) or \( r_2 \sim \sin \varphi \), the integral satisfies (8).

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