Determination of the ultimate load for centrally compressed concrete filled steel tubular columns based on the deformation theory of plasticity

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Abstract. The article presents a methodology for determining the stress-strain state and breaking load for concrete filled steel tubular columns in a physically non-linear setting. The calculation is performed step by step, for each step in the load, the problem is reduced to a system of three linear algebraic equations. The results are compared with known analytical dependences, as well as with the results of calculations in a three-dimensional formulation using the finite element method.

Introduction
Compared to traditional reinforced concrete elements, concrete filled steel tubular (CFST) columns have a number of significant advantages, which include savings on formwork during the construction of buildings, an increase in the bearing capacity of a concrete core due to work in conditions of comprehensive compression, a decrease in shrinkage deformations due to the lack of moisture exchange between concrete and the external environment, the possibility of using high-strength concretes by increasing the limit of elastic work of structures and the plastic nature of destruction, higher fire resistance compared to metal structures, increased torsional rigidity, etc [1-7].

At the same time, CFST columns have disadvantages, one of which is the lack of a generally accepted method for calculating their bearing capacity taking into account the effect of lateral compression of concrete.

The existing calculation methods, for example [8-11], are mainly based on an empirical approach that does not reflect the physics of the processes of change in the stress-strain state of CFST elements, which results in a limited scope and impossibility of spreading to all the variety of existing steel grades and concrete compositions.

The aim of this work is to develop a method for calculating CFST columns based on the most general assumptions of the mechanics of a deformable solid.

Methods
The element of the considered structure is shown in figure 1. When deriving the resolving equations, we will take into account the presence of forced deformations \( \varepsilon_{br}^*, \varepsilon_{bd}^*, \varepsilon_{bc}^* \) in concrete, which may include creep deformations, shrinkage, expansion deformations. The total deformations of concrete will be written as:
\[ \varepsilon_{b\theta} = \varepsilon_{br} = \frac{1}{E_b} \left( \sigma_{b\theta} - \nu_b \left( \sigma_{br} + \sigma_{bz} \right) \right) + \varepsilon_{b\theta}^*; \]

\[ \varepsilon_{bz} = \frac{1}{E_b} \left( \sigma_{bz} - \nu_b \left( \sigma_{br} + \sigma_{b\theta} \right) \right) + \varepsilon_{bz}^*. \]

The modulus of elasticity of concrete \( E_b \) and Poisson's ratio \( \nu_b \) in equations (1) may depend on the magnitude of the stresses.

Radial and hoop stresses in concrete are equal to the contact pressure between the steel shell and concrete with a "−" sign:

\[ \sigma_{br} = \sigma_{b\theta} = -p. \] (2)

Taking into account (2), expressions (1) take the form:

\[ \varepsilon_{b\theta} = \frac{1}{E_b} \left( -p \left(1 - \nu_b \right) - \nu_b \sigma_{bz} \right) + \varepsilon_{b\theta}^*; \]

\[ \varepsilon_{bz} = \frac{1}{E_b} \left( \sigma_{bz} + 2 \nu_b p \right) + \varepsilon_{bz}^*. \] (3)

Hoop stresses in the steel shell are determined from the Laplace equation:

\[ \sigma_{s\theta} = \frac{pD}{2\delta}, \] (4)

where \( D \) is the shell diameter, \( \delta \) is the shell thickness.

The deformations of the steel shell are calculated by the formulas:
When concrete and steel work together, the equality of their deformations on $z$ and $\theta$ should be ensured:

$$
\varepsilon_{bc} = \varepsilon_{sc} \rightarrow \frac{1}{E_s} \left( \sigma_{bc} + 2\nu_b \sigma_s \right) + \varepsilon_{sc}^* = \frac{1}{E_s} \left( \sigma_{sc} - \nu_s \frac{pD}{2\delta} \right);
$$

$$
\varepsilon_{b\theta} = \varepsilon_{s\theta} \rightarrow \frac{1}{E_b} \left( 1 - \nu_b \right) \sigma_{bc} - \nu_s \sigma_{sc} = E_b \varepsilon_{bc}^*;
$$

The relationship between compressive force $F$ and stresses in concrete is represented as:

$$
\sigma_s A_s + \sigma_{bc} A_b = F,
$$

where $A_s = \pi D \delta$ - steel shell cross-sectional area.

After transformations, the problem is reduced to a system of three equations for the unknowns $p, \sigma_{bc}, \sigma_{sc}$:

$$
p \left( \frac{D}{2\delta} + \alpha (1 - \nu_b) \right) + \alpha \nu_b \sigma_{bc} - \nu_s \sigma_{sc} = E_b \varepsilon_{bc}^*;
$$

$$
p \left( 2\alpha \nu_b + \frac{\nu_s D}{2\delta} \right) + \alpha \sigma_{bc} - \sigma_{sc} = -E_b \varepsilon_{bc}^*;
$$

$$
\sigma_{sc} \pi D \delta + \sigma_{bc} \frac{\pi D^2}{4} = -F,
$$

where $\alpha = E_s / E_b$.

As equations establishing the relationship between stresses and instantaneous deformations, we use the dependences of the deformation theory of plasticity of concrete by G.A. Geniev [12,13]. In cylindrical coordinates, these equations take the form:

$$
\varepsilon_{b\theta} = \frac{1}{E_h (\Gamma)} \left( \sigma_{b\theta} - \nu_b \left( \sigma_{br} + \sigma_{bc} \right) \right) + \varepsilon_d^*;
$$

$$
\varepsilon_{bc} = \frac{1}{E_h (\Gamma)} \left( \sigma_{bc} - \nu_b \left( \sigma_{br} + \sigma_{bc} \right) \right) + \varepsilon_d^*;
$$

where $\varepsilon_d = -g_0 \Gamma^2 / 3$ - dilatation deformations, $g_0$ - dilatation modulus,

$$
\Gamma = \frac{2}{3} \sqrt{\left( \varepsilon_1 - \varepsilon_2 \right)^2 + \left( \varepsilon_2 - \varepsilon_3 \right)^2 + \left( \varepsilon_1 - \varepsilon_3 \right)^2} - \text{the intensity of shear strain}.
$$

Dilatation deformations can be considered as a special case of forced deformations $\varepsilon_{b\theta}^*$ and $\varepsilon_{bc}^*$.

The dilatation modulus $g_0$ in Geniev's theory is defined as:

$$
g_0 = -\frac{\theta_c}{\Gamma_c^2},
$$

where $\theta_c = -1 \cdot 10^{-4}$ is the ultimate volumetric deformation of concrete in pure shear, $\Gamma_c$ is the limiting intensity of shear strains in pure shear, calculated as:

$$
\Gamma_c = \frac{2T_c}{G_0}.
$$
In formula (11) \( T_c = \sqrt{\frac{R_b R_{bt}}{3}} \) is the limiting intensity of tangential stresses, \( R_b \) and \( R_{bt} \) - respectively compressive and tensile strength of concrete, \( G_0 = E_0 / (2(1 + \nu_b)) \) - initial concrete shear modulus, \( E_0 \) - initial elastic modulus of concrete.

The secant module \( E_b(\Gamma) \) is determined by the formula:

\[
E_b(\Gamma) = E_0 \left(1 - \frac{\Gamma}{2\Gamma_s}\right),
\]

where \( \Gamma_s \) is the limiting intensity of shear deformations, depending on the nature of the stress state:

\[
\Gamma_s = \Gamma_s k(\lambda, \delta),
\]

\[
k(\lambda, \delta) = \frac{\lambda(1 + \delta)}{2} + \sqrt{\frac{\lambda^2(1 + \delta)^2}{4} + (1 + \delta)}.
\]

The parameter \( \delta \) in (13) is defined as:

\[
\delta = e \left(\frac{S}{T}\right)^3,
\]

where \( T = \frac{1}{\sqrt{6}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} \) - shear stress intensity, \( e = (R_b R_{bt}) / (3T_c^2) - 1, \)

\[
S = \sqrt{3} \left[\frac{1}{2}(\sigma_1 - \sigma)(\sigma_2 - \sigma)(\sigma_3 - \sigma)\right]^{\frac{1}{3}}, \sigma = (\sigma_1 + \sigma_2 + \sigma_3) / 3 - mean stress.
\]

The parameter \( \lambda \) in (13) is calculated by the formula:

\[
\lambda = \frac{f \sigma}{T},
\]

where \( f = \frac{3T_c (R_b - R_{bt})}{R_b R_{bt}} \).

The calculation is performed by a stepwise method, the load is increased in small portions \( \Delta F \). At each step, the stress increments \( \Delta \sigma, \Delta \sigma_{sz}, \) and \( \Delta \sigma_{sz} \) are determined from the solution of system (8) by substituting \( \Delta F \) instead of \( F \). As the modulus of elasticity of concrete in (8), the tangent modulus is substituted, determined by the formula:

\[
E_{bt, tangent}(\Gamma) = E_0 \left(1 - \frac{\Gamma}{2\Gamma_s}\right).
\]

The dilatation deformation increment is defined as:

\[
\Delta \varepsilon_d = \varepsilon_d(\Gamma + \Delta \Gamma) - \varepsilon_d(\Gamma) = -\frac{g_0}{3} \left(\Gamma^2 + 2\Gamma \Delta \Gamma + \Delta \Gamma^2\right) - \frac{8g_0}{3} \Gamma^2 = -\frac{8g_0}{3} \left(2\Gamma \Delta \Gamma + \Delta \Gamma^2\right).
\]

The value of \( \Delta \Gamma^2 \) in comparison with the first term in the parenthesis can be neglected, since it is a quantity of a higher order of smallness, and then formula (17) takes the form:

\[
\Delta \varepsilon_d = -\frac{2g_0 \Gamma \Delta \Gamma}{3}.
\]

The material of the cage is assumed to be ideal elastoplastic; the Huber-Mises-Hencky plasticity condition is used:

\[
\frac{1}{\sqrt{2}} \sqrt{\sigma_{sz}^2 - \sigma_{sc} \sigma_{sc} \sigma_{sd} + \sigma_{sd}^2} = \sigma_y,
\]

where \( \sigma_y \) is the yield strength of steel.

**Results and Discussion**
To test the constructed deformation model, we used the results presented in the article [8]. In this publication, 4 analytical methods for calculating centrally compressed concrete filled steel tubular columns are considered:

1. The unified method proposed by Chinese scientists Min Yu, Xiaoxiong Zha, Jianqiao Ye, Yuting Li [9], according to which the bearing capacity $N_0$ of centrally compressed CFST columns is determined by the formula:
   \[ N_0 = (1 + \eta)\left( f_y A_y + f_c A_{ck}\right), \]
   (20)
   where $\eta$ is a coefficient depending on the shape of the concrete column (for round elements $\eta = 0$), $f_y$ and $f_c$ are the ultimate compressive strength of steel and concrete, $A_y$ and $A_{ck}$ are the cross-sectional areas of steel and concrete.

2. Method of the authors Min Yu, Xiaoxiong Zha, Jianqiao Ye, Chunyan She [10] for strength and stability analysis of the CFST columns with circular cross-section. Calculations for strength analysis are performed in the following sequence:
   a) The yield strength of the CFST structure is calculated by the formula:
   \[ f_{sc} = 1 + \frac{\Omega \varepsilon_{sc}}{2\Omega + 0.05\varepsilon_{sc} + \varepsilon_{sc} \left(0.2 \frac{f_{ck}}{f_y} + 0.05\right)} \]
   (21)
   where $\Omega = A_c / (A_c + A_h)$ is the ratio of the concrete core area to the total area of the element. In the case of a continuous section, $\Omega = 1$, and for a hollow element: $\Omega = 1 - \psi$, where $\psi$ is the void coefficient of the cross section; $\varepsilon_{sc} = A_y f_y / (A_y f_{ck})$ is the ratio between the bearing capacity of steel and the bearing capacity of concrete, $f_{ck}$ and $f_y$ are the prismatic strength of concrete and the yield strength of steel, respectively.
   b) The bearing capacity of the CFST element is calculated using the formula:
   \[ N_0 = f_{sc} A_{sc}, \]
   (22)
   where $A_{sc}$ is the cross-sectional area of the element.

3. L.I. Storozhenko method [11] for calculating the ultimate axial load on CFST elements with a circular cross section, included in the current standards of Ukraine. The calculation is performed in the following sequence:
   a) The concrete strength is calculated using the formula:
   \[ R^* = 0.65 B \left(1 + 16.1 \mu_{pb} \beta\right), \]
   (23)
   where $\mu_{pb} = \left(D / (D - 2t)\right)^2 - 1$ - reinforcement coefficient, which acts as the main parameter that takes into account the relationship between the pipe thickness and its outer diameter.
   b) the bearing capacity of the CFST element is calculated by the formula:
   \[ N_{stab} = \gamma_{bs} \left( R^*_b A_b + \gamma_{s2} R_{ys} A_{ys} \right). \]
   (24)
   Coefficients $\gamma_{bs}$ and $\gamma_{s2}$ in formula (24) take into account the joint work of the concrete core and the steel shell.

4. Method presented in Eurocode 4 (EN 1994) [14].

To compare the listed methods, the calculation of a CFST element is performed with the following initial data: $R_0 = 11.5$ MPa, $R_{bs} = 0.9$ MPa, $E_0 = 2.75 \cdot 10^5$ MPa, $\alpha_z = 235$ MPa, $E_s = 2.10^5$ MPa, $D = 200$ mm, $d = 3$ mm, element length $l = 500$ mm. Figure 2 shows a graph of changes in axial deformation $\varepsilon_z$ depending on the magnitude of the compressive force, obtained by our method. The breaking load was 860 kN. Also in this figure, the dashed line shows a curve constructed without taking into account the lateral compression of concrete. In this case, the breaking load was found to be 732 kN. The increase in strength due to the appearance of a three dimensional stress state in concrete was 17.5%. The dash-dotted line in figure 2 corresponds to the case when the steel shell accepts only circumferential stresses, while the ultimate load was 777 kN. The results obtained indicate the need to ensure the joint work of the concrete and the steel tube in the circumferential and axial directions. 
Figure 2. Dependence of axial deformation on compressive load: solid line - when concrete and steel work together in the circumferential and axial directions, dashed line - when working together only in the axial direction, dash-dot line - when working together only in the circumferential direction.

The values of the ultimate load obtained on the basis of four methods, as well as their deviations from the author's solution are given in table 1. The best agreement of the results with the author's solution is observed for the method of L.I. Storozhenko.

Table 1. Comparison of ultimate load values obtained by different methods

| Method | 1   | 2   | 3   | 4   | Authors |
|--------|-----|-----|-----|-----|---------|
| Ultimate load, kN | 1128 | 980 | 815 | 1045 | 860     |
| Deviation from the author's solution, % | 31.2 | 14  | 5.23 | 21.5 | -       |

Also, this problem was simulated in a three-dimensional formulation, taking into account physical nonlinearity in the LIRA-SAPR software package. The concrete was represented by volumetric prismatic finite elements, and the steel tube was represented by rectangular finite elements of the shell. Due to symmetry, a quarter of the structure was considered. The design scheme is shown in figure 3. At the top end, the displacements in z direction were combined. For concrete and steel, an exponential law of deformation was set. The breaking load as a result of the calculation was 840 kN, which differs from our solution by only 2.3%.
Summary
The constructed model of the CFST columns deformation under central compression is based on the general equations of the mechanics of a deformable solid and, in contrast to the existing approaches, does not contain empirical coefficients.

The reliability of the results is confirmed by comparison with the analytical dependences presented in the works of other authors, as well as by calculations in the LIRA-SAPR software package in a three-dimensional setting. On the basis of the performed calculation, it is shown that it is necessary to ensure the joint work of concrete and steel both in the axial and in the circumferential direction.

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