QCD TESTS USING $b\bar{b}g$ EVENTS
AND A NEW MEASUREMENT OF THE B-HADRON
ENERGY DISTRIBUTION*

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Abstract

We present new studies of 3-jet final states from hadronic $Z^0$ decays recorded by
the SLD experiment, in which jets are identified as quark, antiquark or gluon. Our gluon
energy spectrum, measured over the full kinematic range, is consistent with the predictions
of QCD, and we derive a limit on an anomalous chromomagnetic $b\bar{b}g$ coupling. We
measure the parity violation in $Z^0$ decays into $b\bar{b}g$ to be consistent with the predictions
of electroweak theory and QCD, and perform new tests of T- and CP-conservation at
the $b\bar{b}g$ vertex. We also present a new technique for reconstructing the energy of a $B$
hadron using the set of charged tracks attached to a secondary vertex. The $B$ hadron
energy spectrum is measured over the full kinematic range, allowing improved tests of
predictions for the shape of the spectrum. The average scaled energy is measured to be
$<x_B> = 0.719 \pm 0.005 (\text{stat.})$ (Preliminary).

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1. Introduction

Experimental studies of the structure of events containing three hadronic jets in $e^+e^-$ annihilation have been limited by difficulties in identifying which jet is due to the quark, which to the antiquark and which to the gluon. Since the gluon is expected to be the lowest-energy jet in most events, the predictions of QCD have been tested using energy and angle distributions of energy-ordered jets, and this is sufficient to confirm the $q\bar{q}g$ origin of such events and to determine the gluon spin [1]. Tagging of the origin of any two of the three jets in such events would allow more complete and stringent tests of QCD predictions.

Here we present a study [2] of 3-jet final states in which two of the jets have been tagged as $b$ or $\bar{b}$ jets using the long lifetime of the $B$-hadrons in the jets and the precision vertexing system of the SLD. The remaining jet is tagged as the gluon jet, and its energy spectrum is studied over its full kinematic range. Adding a tag of the charge of one of the $b/\bar{b}$ jets, and exploiting the high electron beam polarization of the SLC, we measure two angular asymmetries sensitive to parity violation in the $Z^0$ decay, and also construct new tests of T- and CP-conservation at the $bbg$ vertex. The study of $b$-flavor events is especially useful as input to measurements of electroweak parameters such as $R_b$ and $A_b$, and as a probe of new physics, which is expected in many cases to couple more strongly to heavier quarks. A test of the flavor-independence of the strong coupling using 3-jet final states was presented separately [3] at this conference.

Experimental studies of the fragmentation of bottom quarks into $B$ hadrons have been limited by the efficiency for reconstructing the energies of individual $B$ hadrons with good resolution, especially for low energy $B$ hadrons. Improved measurements would allow stringent tests of QCD predictions for the fragmentation function as well as provide crucial input to fragmentation models used for a number of important electroweak and heavy flavor measurements.

Here we present a study [4] of the $B$-hadron energy distribution using a novel kinematic technique to estimate the energy $E_B$ of each individual $B$ hadron. $B$ hadrons are again tagged using their long lifetime, by reconstructing secondary vertices. The tracks in each vertex are used, along with the vertex flight direction, to compute kinematic quantities that are then used to select a final sample and estimate each $E_B$. This technique has high efficiency and good energy resolution for all $E_B$, and the resulting measurement covers the full kinematic range.

2. The Gluon Energy Spectrum

Well contained hadronic events [5] in which exactly 3 jets are found using the JADE algorithm at $y_{cut} = 0.02$ are selected. The jet energies are calculated from the angles between them [5] and the jets are energy ordered such that $E_1 > E_2 > E_3$. In each jet we count the number $n_{sig}$ of ‘significant’ tracks, i.e. those with normalized transverse impact...
Figure 1: The measured scaled gluon energy distribution (dots). The predictions of first and second order QCD and of a parton shower calculation are also shown as the dotted, dashed and solid lines, respectively.

parameter with respect to the primary interaction point (IP) $d/\sigma_d > 3$. We require exactly two of the three jets to have $n_{sig} > 1$, and the remaining jet is tagged as the gluon jet. This yields 1533 events with an estimated purity of correctly tagged gluon jets of 91%. In 2.5% (12.5%) of these events, jet 1(2), the (second) highest energy jet, is tagged as the gluon jet, giving coverage over the full kinematic range.

The background from non-$b\bar{b}g$ events and events with an incorrect gluon tag is subtracted, and the resulting distribution of scaled gluon energy $z = 2E_g/\sqrt{s}$ is corrected for the effects of selection efficiency and resolution. The fully corrected spectrum is shown in fig. and shows the expected falling behaviour with increasing $z$. The distribution is cut off at low $z$ by the finite $y_{cut}$ value used for jet finding. Also shown are the predictions of first and second order QCD. Both reproduce the general behaviour, but fail to describe the
data in detail. The prediction of the JETSET [6] parton shower simulation is also shown and reproduces the data. Our data thus confirm the predictions of QCD, although higher order effects are clearly important in the intermediate gluon energy range, $0.2 < z < 0.4$.

The gluon energy spectrum is particularly sensitive to the presence of an anomalous chromomagnetic term in the QCD Lagrangian. A fit of the theoretical prediction [7] including an anomalous term parametrized by a relative coupling $\kappa$, yields a value of $\kappa = -0.03 \pm 0.06$ (Preliminary), consistent with zero, and corresponding to limits on such contributions to the $b\bar{b}g$ coupling of $-0.15 < \kappa < 0.09$ at the 95% confidence level.

3. Parity Violation in 3-jet $Z^0$ Decays

We now consider two angles, the polar angle of the quark with respect to the electron beam direction $\theta_q$, and the angle between the quark-gluon and quark-electron beam planes $\chi = \cos^{-1}(\hat{p}_q \times \hat{p}_g) \cdot (\hat{p}_q \times \hat{p}_e)$. The cosine $x$ of each of these angles is expected to be distributed as $1 + x^2 + 2A_P A_Z x$, where the $Z^0$ polarization $A_Z = (P_e - A_e)/(1 - P_e A_e)$ depends on the electron beam polarization $P_e$, and each $A_P$ is predicted by QCD.

Three-jet events (Durham algorithm, $y_{\text{cut}} = 0.005$) are selected and energy ordered,
and a topological vertex finder is applied to the tracks in each jet. The 3420 events containing any vertex with invariant mass above 1.5 GeV/c^2 are kept; these have an estimated b\(\bar{b}\)g purity of 87%. We calculate the momentum-weighted charge of each jet \(j\), 

\[ Q_j = \sum_i q_i |\vec{p}_i \cdot \hat{p}_j|^{0.5}, \]

using the charge \(q_i\) and momentum \(\vec{p}_i\) of each track \(i\) in the jet. In this case we assume that the highest-energy jet is not the gluon, and tag it as the \(b\) (\(\bar{b}\)) jet if \(Q = Q_1 - Q_2 - Q_3\) is negative (positive). We then define the \(b\)-quark polar angle 

\[ \cos \theta_b = -\text{sign}(Q)(\hat{p}_e \cdot \hat{p}_1). \]

The left-right-forward-backward asymmetry \(\tilde{A}_{LRFB}\) in this polar angle is shown as a function of \(|\cos \theta_b|\) in fig. 3. A clear asymmetry is seen, which increases with \(|\cos \theta_b|\) in the expected way. A fit yields an asymmetry parameter of \(A_P = 0.89 \pm 0.06 \pm 0.07\) (Preliminary), where the first error is statistical and the second systematic, consistent with the QCD prediction of \(A_P = 0.93 A_b = 0.87\).

We then tag one of the two lower energy jets as the gluon jet, using the impact parameters of their tracks. If jet 2 has \(n_{\text{sig}} = 0\) and jet 3 has \(n_{\text{sig}} > 0\), then jet 2 is tagged as the gluon jet; otherwise jet 3 is tagged as the gluon jet. In each event we construct the angle \(\chi\), and \(A_{FB}^{\chi}\) is shown as a function of \(\chi\) in fig. 3. Here we expect only a small deviation from zero as indicated by the dashed line on fig. 3. Our measurement is consistent with the prediction, as well as with zero. A fit (solid line) yields \(A_{\chi} = -0.015 \pm 0.045 \pm 0.001\) (Preliminary), to be compared with an expectation of \(-0.060\).
4. Symmetry Tests in 3-jet $Z^0$ Decays

Using these fully tagged events, we can construct observables that are formally odd under time and/or CP reversal. The energy-ordered triple product $\cos \omega^+ = \vec{\sigma}_Z \cdot (\hat{p}_1 \times \hat{p}_2)$, where $\vec{\sigma}_Z$ is the $Z^0$ polarization vector, is $T_N$-odd and CP-even. Since the true time reversed experiment is not performed, this quantity could have a non-zero $\tilde{A}_{FB}$, and we have previously set a limit [5] using events of all flavors. A calculation [9] including Standard Model final state interactions predicts that $\tilde{A}_{FB}^+$ is largest for $b\bar{b}g$ events, but is only $\sim 10^{-5}$. The fully flavor-ordered triple product $\cos \omega^- = \vec{\sigma}_Z \cdot (\hat{p}_q \times \hat{p}_{\bar{q}})$ is both $T_N$-odd and CP-odd.

Our measured $\tilde{A}_{FB}^+$ and $\tilde{A}_{FB}^-$ are shown in fig. 3. They are consistent with zero at all $|\cos \omega|$. Fits (solid lines in fig. 3) to the data yield the asymmetry parameters consistent with zero, and we set limits on any $T_N$- and CP-violating asymmetries of $-0.039 < A_T^+ < 0.035$ and $-0.086 < A_T^- < 0.040$, respectively.

5. The $B$-Hadron Energy Spectrum

For the $B$ energy measurement, each selected hadronic event is divided into two hemispheres by the plane perpendicular to the thrust axis. The topological vertex finder is applied to the set of tracks in each hemisphere, and any vertex with invariant mass greater than 2 GeV/$c^2$ is retained as a candidate $B$ hadron vertex. The flight direction of the $B$ hadron is taken to be along the line joining the IP and the vertex position.

The four-vector sum of the tracks in the vertex (assigned the charged pion mass) is calculated, and its momentum component transverse to the flight direction is equated with the transverse component of the “missing” momentum from the $B$ hadron decay. At this point two quantities are still needed in order to determine the energy of the $B$ hadron, the missing mass and the missing momentum along the flight direction. Assuming a mass $M_B$ for the $B$ hadron eliminates one of these unknowns, and also allows an upper limit to be calculated on the missing mass:

$$M_{0\text{max}}^2 = M_B^2 - 2M_B\sqrt{M_{\text{chg}}^2 + P_t^2 + M_{\text{chg}}^2},$$ \hspace{1cm} (1)

where $M_{\text{chg}}$ and $P_t$ are the invariant mass and momentum transverse to the flight direction, respectively, of the set of tracks in the vertex. Using $M_B = 5.28$ GeV/$c^2$, equating $M_0^2$ with $M_{0\text{max}}^2$ and solving for the $B$ hadron energy is found to provide a good estimate of the true $B$ hadron energy for the mixture of $B$ hadron species produced in our simulation of $Z^0$ decays. As expected, the simulated resolution on this estimate is best for vertices with low values of $M_{0\text{max}}^2$, approaching 6% as $M_{0\text{max}}^2 \rightarrow 0$. It does not degrade very rapidly with increasing $M_{0\text{max}}^2$ due to the strong tendency for the true missing mass in hadronic $B$ decays to cluster near the maximum value.

A cut is placed on $M_{0\text{max}}^2$ that depends on the measured energy in such a way that the efficiency for selecting $B$ vertices, estimated from our simulation, is roughly independent
Figure 4: Left-right-forward-backward asymmetries of the a) energy- and b) flavor-ordered triple product. The solid (dashed) line represents a fit to the data (95% confidence limits).
of energy; it is 4% on average and is above 3% for $E_B > 8$ GeV. A sample of 1938 vertices is selected, with an estimated $B$ hadron purity of 99.5%. The energy resolution is estimated to be 10% on average, roughly independent of $E_B$. The measured energy is then divided by the beam energy, and the distribution of $x_B = E_B/E_{beam}$ is shown in fig. 3. It can be seen that the measurement covers the entire kinematic range from the $B$ hadron mass ($x_B \approx 0.12$) to the beam energy.

Also shown in fig. 3 is the prediction of our simulation, generated using the JETSET program with the Peterson fragmentation option and $\epsilon_b = 0.006$. The predicted distribution peaks at a value of $E_B$ consistent with the peak in the data, but the width is significantly larger than that of the data.

The correction of the measured distribution to obtain the true $x_B$ distribution depends on the form assumed for the true distribution, due to the rapid variation of the distribution on the scale of the bin size. We have tested several functional forms for the true $x_B$ distribution by weighting simulated $B$ hadrons at the generator level to reproduce the function for a given set of parameter values. The weighted detector-level distribution is compared with that measured in the data, and a $\chi^2$ is minimized to find the best parameter values. Most of the test functions we tried are not able to describe the data adequately for any values of their parameters. Three functions, the Peterson function and two generalizations thereof, are able to describe the data; at their respective fitted parameter values they are very similar to each other for $x_B < 0.7$, but show substantial variation for $x_B > 0.8$, indicating that there remains nonnegligible model-dependence in the correction. From these three fitted functions we extract a measurement of the mean value,

$$<x_B> = 0.719 \pm 0.005 \text{ (stat.)} \pm 0.001 \text{ (shape)} \quad \text{(Preliminary)}.$$

The experimental systematic error and the correction procedure are under study.

6. Conclusions

In summary, we use the excellent vertexing capability of the SLD and the high electron beam polarization of the SLC to make several new tests of QCD, using 3-jet final states in which jets are tagged as quark, antiquark or gluon jets. The gluon energy spectrum is measured over its full kinematic range; we confirm the prediction of QCD and set limits on anomalous chromomagnetic couplings. The parity violation in $Z^0$ decays to $b\bar{b}g$ is found to be consistent with the predictions of electroweak theory plus QCD, and new tests of T- and CP-conservation in strong interactions are performed.

In addition, we have developed a new technique for measuring the energies of individual $B$ hadrons using information only from the charged tracks attached to a secondary vertex. The method has high efficiency and good energy resolution that are roughly independent of the true energy. The $B$-hadron energy distribution has been measured over the full kinematic range, providing a basis for improved tests of QCD predictions and better input into fragmentation models.
Figure 5: Uncorrected distribution of measured $B$ hadron energies (dots). The histogram is the prediction of our simulation.
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