Abstract. There is strong evidence that new physical degrees of freedom and new phenomena exist and may be revealed in future collider experiments. The best hints of what this new physics might be are provided by electroweak symmetry breaking. I briefly review certain theories for physics beyond the standard model, including the top-quark seesaw model and universal extra dimensions. A common feature of these models is the presence of vector-like quarks at the TeV scale. Then I discuss the role of a linear $e^+e^-$ collider in disentangling this new physics.

THE CASE FOR NEW PHYSICS

The observed fundamental particles, namely the $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge bosons, the longitudinal degrees of freedom of the $W^\pm$ and $Z^0$, and three generations of quarks and leptons, may explain in principle all observed physical phenomena. However, there is strong evidence for the existence of new phenomena at higher energy scales than the ones probed so far. The most robust argument is provided by the perturbative violation of unitarity in the $WW$ scattering, at a scale of order 1 TeV [1]. Therefore, either the $W$ and $Z$ have strongly coupled self-interactions at the TeV scale, or new fundamental degrees of freedom exist. The scale of these new phenomena is within the reach of future collider experiments, and exploring them is at the heart of high-energy physics.

The standard model accommodates rather well all available data, especially when the Higgs boson is light [2]. More generally, any model with a decoupling limit [3] given by the standard model is viable, at least in that limit. These are models in

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which all the particles beyond the standard model may be given large electroweak-symmetric masses. I will refer to them as “decoupling models”. In practice, the decoupling may be only partial, so that new particles with electroweak-symmetric masses of order 1 TeV give rise at the electroweak scale to higher-dimensional operators which may change the fit of the standard model to the data. In particular, the Higgs boson mass may be larger in this case, even close to the triviality bound [4]. In what follows I will concentrate on decoupling models, but one should keep in mind that it is not currently possible to rule out all models without a Higgs boson, because the physical quantities relevant for comparing with the data typically cannot be computed when the interactions are non-perturbative.

Even if a standard-model-like Higgs boson will be discovered, and maybe for a while no experimental data will hint at other new physics, there are robust theoretical reasons to expect physics beyond the standard model. First, the Higgs self-coupling increases with the energy and the theory breaks down at some scale. Second, the $U(1)_Y$ gauge coupling is also ill-behaved at high energy. Third, the standard model does not include gravity, while the measured gravitational effects are produced by matter composed of standard model particles. Additional reasons to expect new physics are given by the large number of parameters in the standard model, the large hierarchies between their values, and the rather strange field content of the standard model (for example, the Higgs doublet is the only scalar field, and does not appear to be theoretically motivated.)

It would be highly desirable that a theory which includes both the standard model and gravity explains why the electroweak interactions are so much stronger than the gravitational interactions. Currently, the only possible explanations for this hierarchy between the electroweak and Planck scales require new particles with mass of order 1 TeV. The decoupling models may be classified according to the solution for the hierarchy problem into supersymmetric extensions of the standard model, theories with a composite Higgs doublet, and theories with extra dimensions. Although convenient, this classification is not clear-cut given that there are supersymmetric models in extra dimensions [5,6], models of extra dimensions that generate composite Higgs doublets [7–9], as well as supersymmetric models of Higgs compositeness [10].

The supersymmetric extensions of the standard model and theories with extra dimensions accessible only to gravity have been covered in other talks at this con-
ference [11], so here I review only models with composite Higgs doublets and/or standard model fields in extra dimensions. I also discuss their implications for experiments at a linear $e^+e^-$ collider.

**TOP QUARK SEESAW-THEORY**

The quarks and leptons are chiral, in the sense that their left- and right-handed components have different $SU(2)_W \times U(1)_Y$ charges. New chiral fermions would have large contributions to the electroweak observables and the current data place strong constraints on their number, charges and mass splittings. On the other hand, vector-like fermions, defined as having the same gauge charges for the left- and right-handed components, are much less constrained. This can be seen from the fact that their masses preserve the electroweak symmetry, so that in the limit where their mass $m_v$ is much above the electroweak scale $v \approx 246$ GeV, their contribution to the electroweak observables is suppressed by $v^2/m_v^2$ [12].

If a vector-like fermion has the same $SU(3)_C \times SU(2)_W \times U(1)_Y$ charges as a standard model fermion, then it may couple to the Higgs doublet. In the case where this coupling is large, or the number of vector-like fermions is large, the properties of the Higgs field change rapidly at scales above the fermion masses. To see this, consider a vector-like quark $\chi$ that has the same charges as the right-handed top quark $t_R$. Then the most general renormalizable coupling (up to an $U(2)$ global transformation acting on $t_R, \chi_R$) of $\chi$ to the Higgs doublet $H$ may be written as

$$L = - (\mathbf{q}_L^3 \cdot \bar{\chi}_L) \begin{pmatrix} 0 & \xi \chi H \\ m_{\chi t} & m_{\chi \chi} \end{pmatrix} \begin{pmatrix} t_R \\ \chi_R \end{pmatrix} + \text{h.c.} \quad (1)$$

Here $q_L^3$ is the top-bottom left-handed doublet, $\xi_{\chi}$ is a Yukawa coupling constant, while $m_{\chi t}$ and $m_{\chi \chi}$ are electroweak symmetric masses. The eigenvalues of the mass matrix obtained by replacing the Higgs doublet with its VEV represent the top quark mass, $m_t$, and the mass $m_{\chi}$ of the new quark:

$$m_{t,\chi}^2 = \frac{1}{2} \left( m_{\chi \chi}^2 + m_{\chi t}^2 + \xi_{\chi}^2 v^2 \right) \left[ 1 \mp \sqrt{1 - 2 \left( \frac{\xi_{\chi} v m_{\chi t}}{m_{\chi \chi}^2 + m_{\chi t}^2 + \xi_{\chi}^2 v^2/2} \right)^2} \right] \quad (2)$$

If the Yukawa coupling is large at the scale $m_{\chi}$, then its scale dependence is very strong and it blows up at a scale $\Lambda_c$ not far above $m_{\chi}$. The dominant contribution to this effect is the Higgs wave function renormalization shown in Fig. 1. In fact this
would be the only leading contribution if the number of colors \( N_c \) were very large (rather than \( N_c = 3 \)), which may be seen as an indication that this phenomenon persists at strong coupling.

![Diagram](image)

**FIGURE 1.** Vector-like quark contribution to the Higgs self-energy.

The Yukawa coupling that seemingly blows up is a signal for new physics at the scale \( \Lambda_c \). The simplest interpretation is that the scalar \( H \) ceases to be a physical degree of freedom above \( \Lambda_c \). A confirmation for this interpretation is given by viewing the one-loop graph in Fig. 1 as a correction to the Higgs kinetic term rather than a change in the Yukawa coupling. Normalizing the coefficient of the Higgs kinetic term to unity at the electroweak scale, one finds that it vanishes at the scale \( \Lambda_c \) because the correction is negative and sufficiently large at that scale. Therefore, the Higgs doublet is no longer a propagating field at higher scales. It is then natural to assume that \( H \) is a composite field, with some constituents bound by a strongly-coupled interaction. Furthermore, the large Yukawa coupling reveals the constituents: \( \chi_R \) and \( q_L^3 \). This implies that the strongly-coupled interaction should not confine, otherwise the top-bottom left-handed doublet would not be present in the effective low-energy theory. \( \Lambda_c \) is called the compositeness scale, and may be in the TeV range if the Yukawa coupling is sufficiently large at the electroweak scale (\( \xi_\chi \sim 3 - 4 \) based on the large-\( N_c \) computation.) The exchange of the non-propagating \( H \) field at the compositeness scale induces a four-quark contact term of the form \( (\bar{\tau}_L\chi_R)(\bar{\chi}_Rq_L^3)/\Lambda_c^2 \) with a large dimensionless coupling. This interaction represents the dominant effect of the strongly-coupled interaction responsible for binding the Higgs doublet, and is indeed non-confining, being weak at long-distance.

The model of Higgs compositeness described so far is called the top-quark seesaw theory [13,14], because in the limit \( \xi_\chi v, m_{\chi t} \ll m_{\chi\chi} \) the top mass is suppressed.
compared with the electroweak asymmetric fermion mass, $\xi\chi v/\sqrt{2}$, by a seesaw mechanism.

The Higgs self-coupling, $(-\lambda_h/2)(H^+H)^2$, is also affected by the large Yukawa coupling. The one-loop RGE for $\lambda_h$ is given by

$$\frac{d\lambda_h}{d\ln \mu} = \frac{3}{4\pi^2} \left( \lambda_h \xi^2 - \xi^4 + \lambda_h^2 \right).$$

The Higgs boson mass is determined by the self-coupling, $M_h = \sqrt{\lambda_h v}$. The case where the Higgs boson is light, $\lambda_h \ll \xi^2$, implies that $\lambda_h$ decreases at higher scales. This situation would be hard to reconcile with the fact that the Higgs self-coupling is given by the strongly-coupled interaction between the Higgs constituents. One may infer that $\lambda_h$ is rather large and positive at the electroweak scale, so that the Higgs boson is heavy, with a mass $M_h$ estimated to be of order 500 GeV [4,15]. This Higgs mass prediction has clearly large theoretical uncertainties due to the non-perturbative interactions involved in compositeness, but the large Higgs self-coupling appears to be a rather generic consequence of the strongly-coupled binding interaction. It is possible though that there exist several composite Higgs fields [14], in which case the mixing among the various CP-even Higgs bosons may allow the lightest one to have standard-model-like couplings and a mass close to the current LEP II bound [17].

Whether the Higgs boson is as light as $\sim 115$ GeV, or as heavy as 500 GeV, the top-quark seesaw theory remains viable: in the former case it would be in the decoupling limit $m_\chi \gg v$; in the latter case it would require the vector-like quark to have a mass closer to the TeV scale, such that its isospin-violating effects render the electroweak observables in agreement with the current data [4,16].

When the effective theory below the compositeness scale includes an extended Higgs sector, it is possible that a light Higgs boson, with nearly standard couplings to fermions and gauge bosons, has completely non-standard decay modes. This happens whenever a $CP$-odd scalar has a mass less than half the Higgs mass and the coupling of the Higgs to a pair of $CP$-odd scalars is not suppressed. The Higgs boson decays into a pair of $CP$-odd scalars, each of them subsequently decaying into a pair of standard model particles, with model dependent branching ratios [18]. A linear $e^+e^-$ collider may prove very useful in disentangling the composite nature of the Higgs boson with non-standard decays, by measuring its width and branching ratios.
The heavy quark constituent of the Higgs has a mass $m_\chi$ of a few TeV. Nevertheless, a linear $e^+e^-$ collider operating at $\sqrt{s} = 500$ GeV, or above, may already see the effects of the mixing between the top and $\chi$ [19]. For this it will be necessary to determine the $Z\bar{t}t$ coupling at the few percent level.

The interaction responsible for binding the Higgs field, and approximated with a four-quark interaction at the $\Lambda_c$ scale, is provided by a spontaneously broken gauge symmetry, such as topcolor [20], or some flavor or family symmetry [21]. Such interactions are non-confining, and also asymptotically free, allowing for a solution to the hierarchy problem. Above the compositeness scale there must be some additional physics that leads to the spontaneous breaking of the gauge symmetry responsible for binding the Higgs. This may involve new gauge dynamics [16], or fundamental scalars and supersymmetry. A linear $e^+e^-$ collider could study these interesting strongly-interacting particles only if it operates at energies above a TeV. Evidently, the CLIC design, with the acceleration provided by a high-intensity beam that allows a center of mass energy of a few TeV, is highly desirable in this context.

Other models of Higgs compositeness have been proposed recently [22–24], and each of them has interesting phenomenological consequence with implications for a linear $e^+e^-$ collider.

**STANDARD MODEL IN EXTRA DIMENSIONS**

Extra spatial dimensions accessible to standard model particles are constrained by the Tevatron and LEP data to be smaller than of order $(1 \text{ TeV})^{-1}$. The existence of TeV-size extra dimensions is a logical possibility, motivated by various theoretical considerations, such as the generation of hierarchical quark and lepton masses, or the potential for gauge coupling unification at a scale in the TeV range [6].

**Kaluza-Klein resonances**

The immediate consequence of this scenario is the existence of towers of Kaluza-Klein (KK) modes for the particles that propagate in the TeV-size extra dimensions. For example, the gluons would have spin-1 color-octet excitations. Their masses are given by $\sqrt{j/R}$ where $R$ is the radius of the extra dimensions and $j$ is an integer that labels the KK level. The number of states on each level is a function of the number of extra dimensions. For example, with one extra dimension, the only
occupied levels have $j = k^2$ where $k$ is an integer. Both the density of occupied levels and the average number of states ($D_n$) on a level increase with the number of extra dimensions.

The $W, Z$ and photon would have color-singlet spin-1 excitations with a mass spectrum similar to the KK gluons, slightly perturbed due to the electroweak symmetry breaking. In the often considered case where the quarks and leptons are localized on a three-dimensional domain wall (a 3-brane), the KK excitations of the gauge bosons have the same couplings up to a factor of order one as the corresponding standard model states. Therefore, the $Z$ and photon KK states may be produced in the $s$ channel in $e^+e^-$ collisions. At the same time, the KK excitations of the electroweak gauge bosons contribute at tree-level to the electroweak observables, and are constrained to lie above $\sim 4$ TeV in the absence of other compensating effects. If some standard model fermions propagate in extra dimensions, for each of these chiral quarks and leptons there is a tower of vector-like fermions with mass separations of order $1/R$. A linear $e^+e^-$ collider with $\sqrt{s} = 0.5$–1 TeV is unlikely to produce directly any of these KK excitations, but is very sensitive to their presence via virtual effects. With $\sqrt{s}$ in the TeV range, however, a linear $e^+e^-$ collider may produce a series of KK resonances, which not only would establish the existence of extra dimensions but also would determine the number of extra dimensions and their structure.

A qualitatively distinct case is that all standard model particles propagate in extra dimensions. These are called universal extra dimensions. The KK number is then conserved at each vertex, so that the KK excitations may be produced only in groups of two or more. Hence, the direct bounds from the Tevatron and LEP are significantly lower. Moreover, the KK states do not contribute at tree level to the electroweak observables. The current mass bounds on the first KK states are as low as 300 GeV for $\delta = 1$ universal extra dimension, and in the 400 – 800 GeV range for $\delta = 2$ [25]. These loose bounds make the universal extra dimensions particularly interesting for collider experiments. At $\sqrt{s} > 600$ GeV, a linear $e^+e^-$ collider could already pair-produce KK leptons, quarks and gauge bosons. One possibility is that the KK states decay outside the detector, so that the signal would be pairs of highly ionizing tracks. An alternative is that some interactions that do not conserve momentum in the extra dimensions allow the KK states to decay promptly into pairs of standard model particles.
Composite Higgs from universal extra dimensions

Apart from these phenomenological implications, phenomena due to the extra dimensions shed a new light on the origin of electroweak symmetry breaking. An interesting example is provided by the universal extra dimensions. The KK excitations of the standard model gauge bosons and fermions give rise to a scalar bound state with the quantum numbers of the standard model Higgs doublet [9]. The Higgs boson appears as a composite scalar with a combination of KK modes of the top-quark playing the role of constituents. This can be easily understood given that the KK excitations of the gluons and electroweak gauge bosons induce a strong attractive interaction between the left- and right-handed top-quark fields. It is remarkable that out of the many possible bound states involving the quarks and leptons, the most deeply-bound state has the quantum numbers of the Higgs doublet. This state acquires a vacuum expectation value and, thus, breaks the electroweak symmetry. Furthermore, this composite Higgs doublet has a large Yukawa coupling to the top-quark. Hence, electroweak symmetry breaking and the large top mass are direct consequences of the experimentally determined gauge charges of the quarks and leptons.

At a scale in the TeV range, called for convenience the string scale, $\Lambda_s$, the only degrees of freedom are the $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge bosons and the three generations of quarks and leptons, all of them propagating in a higher-dimensional spacetime. (At even higher energy scales, the fundamental degrees of freedom of a theory incorporating quantum gravity are expected to become relevant, such as the winding modes of string theory.) Below the scale $\Lambda_s$, fermion–anti-fermion pairs bind via the $SU(3)_C \times SU(2)_W \times U(1)_Y$ interactions. Then, below the scale $1/R < \Lambda_s$ that sets the size of the extra dimensions, the physics is described by an effective four-dimensional theory. This effective theory is the usual standard model, with the possible addition of a few other scalars, such as the heavy states of a two-Higgs-doublet model.

This simple model is consistent with all the experimental data, and is also predictive. The top mass is predicted with an uncertainty of about 20% and is consistent with the experimental value. This and, more importantly, the prediction of the Higgs quantum numbers are unmatched by the standard model or its supersymmetric extensions. However, like the standard model or the MSSM, this model only
accommodates without predictions the light quark and lepton masses.

The Higgs mass also is determined theoretically, by solving the RGE for \( x_H \equiv \lambda_h / \lambda_t^2 \), where \( \lambda_h \) is the Higgs boson self-coupling and \( \lambda_t \) is the top Yukawa coupling (\( \lambda_t \approx 1 \) at the electroweak scale). Ignoring the electroweak gauge couplings, the one-loop RGE for \( x_H \) is given in the case of two universal extra dimensions by

\[
\frac{d \ln x_H}{d \ln \mu} = \frac{\lambda_t^2(\mu) N_{KK}(\mu)}{(4\pi)^2} \left[ 12 x_H + 9 - \frac{24}{x_H} + \frac{64 g_3^2(\mu)}{3 \lambda_t^2(\mu)} \right].
\]

Due to the strong scale-dependence of the number of KK modes lighter than the scale \( \mu \), \( N_{KK}(\mu) \), the solution of this RGE lies in a narrow interval, being rather insensitive to the value of \( x_H \) at \( \Lambda_s \). The Higgs boson mass is predicted in turn to lie between the \( WW \) threshold and the electroweak scale [9]. The main theoretical uncertainty in this computation comes from the possible existence of other composite scalars, such as the heavy states of a two-Higgs-doublet sector. The mixing between the CP-even states could in principle reduce the mass of the lightest Higgs boson.

Assuming that the effects of heavy scalars may be ignored, the Higgs mass prediction implies that the Higgs boson decays mostly to \( WW \) and \( ZZ \). The capability of a linear \( e^+e^- \) collider with \( \sqrt{s} = 500 \) GeV in this case has been analyzed in Ref. [26]. More striking than the Higgs physics would be the non-standard phenomena discussed above, associated with the KK modes. A linear \( e^+e^- \) collider operating at a center of mass energy of a few TeV would probe the new physics at the electroweak, compactification, and string scales.

REFERENCES

1. B. W. Lee, C. Quigg and H. B. Thacker, “The Strength Of Weak Interactions At Very High-Energies And The Higgs Boson Mass,” Phys. Rev. Lett. 38, 883 (1977).
2. P. Langacker, “Physics implications of precision electroweak experiments,” hep-ph/0102085;
   J. Erler, “Fundamental parameters from precision tests,” hep-ph/0102143.
3. T. Appelquist and J. Carazzone, “Infrared Singularities And Massive Fields,” Phys. Rev. D 11, 2856 (1975).
4. R. S. Chivukula, C. Holbling and N. Evans, “Limits on a composite Higgs boson,” Phys. Rev. Lett. 85, 511 (2000) [hep-ph/0002022].
5. I. Antoniadis, “A Possible New Dimension At A Few TeV,” Phys. Lett. B246, 377 (1990).
6. K.R. Dienes, E. Dudas and T. Gherghetta, “Extra spacetime dimensions and unification,” Phys. Lett. B436, 55 (1998) [hep-ph/9803466].

7. B. A. Dobrescu, “Electroweak symmetry breaking as a consequence of compact dimensions,” Phys. Lett. B461, 99 (1999) [hep-ph/9812349].

8. H. Cheng, B. A. Dobrescu and C. T. Hill, “Electroweak symmetry breaking and extra dimensions,” Nucl. Phys. B589, 249 (2000) [hep-ph/9912343].

9. N. Arkani-Hamed, H. Cheng, B. A. Dobrescu and L. J. Hall, “Self-breaking of the standard model gauge symmetry,” Phys. Rev. D 62, 096006 (2000) [hep-ph/0006238].

10. M. A. Luty, J. Terning and A. K. Grant, “Electroweak symmetry breaking by strong supersymmetric dynamics at the TeV scale,” hep-ph/0006224.

11. R. M. Godbole and N. Arkani-Hamed, plenary talks at the Linear Collider Workshop 2000, http://www-lc.fnal.gov/lcw2000/.

12. M. B. Popovic and E. H. Simmons, “Weak-singlet fermions: Models and constraints,” Phys. Rev. D 62, 035002 (2000) [hep-ph/0001302].

13. B. A. Dobrescu and C. T. Hill, “Electroweak symmetry breaking via top condensation seesaw,” Phys. Rev. Lett. 81, 2634 (1998) [hep-ph/9712319].

14. R. S. Chivukula, B. A. Dobrescu, H. Georgi and C. T. Hill, “Top quark seesaw theory of electroweak symmetry breaking,” Phys. Rev. D 59, 075003 (1999) [hep-ph/9809470].

15. R. S. Chivukula, private communication.

16. H. Collins, A. K. Grant and H. Georgi, “The phenomenology of a top quark seesaw model,” Phys. Rev. D 61, 055002 (2000) [hep-ph/9908330].

17. B. A. Dobrescu, “Minimal composite Higgs model with light bosons,” Phys. Rev. D 63, 015004 (2001) [hep-ph/9908391].

18. B. A. Dobrescu, G. Landsberg and K. T. Matchev, “Higgs boson decays to CP-odd scalars at the Tevatron and beyond,” hep-ph/0005308.

19. M. B. Popovic, “Third generation seesaw mixing with new vector-like weak-doublet quarks,” hep-ph/0101123.

20. C. T. Hill, “Topcolor: Top quark condensation in a gauge extension of the standard model,” Phys. Lett. B266, 419 (1991).

21. G. Burdman and N. Evans, “Flavour universal dynamical electroweak symmetry breaking,” Phys. Rev. D 59, 115005 (1999) [hep-ph/9811357].

22. H. Georgi and A. K. Grant, “A topcolor jungle gym,” Phys. Rev. D 63, 015001 (2001) [hep-ph/0006050].

23. H. He, T. Tait and C. P. Yuan, “New topflavor models with seesaw mechanism,” Phys. Rev. D 62, 011702 (2000) [hep-ph/9911266].

24. A. Aranda and C. D. Carone, “Bounds on bosonic topcolor,” Phys. Lett. B488, 351 (2000) [hep-ph/0007020].

25. T. Appelquist, H. Cheng and B. A. Dobrescu, “Bounds on universal extra dimensions,” hep-ph/0012100.

26. P. F. Derwent, et al, “Linear Collider Physics”, report FERMILAB-FN-701, January 2001.