Jet quenching in thin plasmas

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We investigate the energy loss of quarks and gluons produced in hard processes resulting from final state rescatterings in a finite quark-gluon plasma. The angular distribution of the soft gluon bremsstrahlung induced by \( n_s = 1 \) rescatterings in the plasma is computed in the Gyulassy-Wang model. Special focus is on how the interference between the initial hard radiation amplitude, the multiple induced Gunion-Bertsch radiation amplitudes, and gluon rescattering amplitudes modifies the classical parton cascade results.

1. INTRODUCTION

At RHIC energies (\( \sqrt{s} \sim 200 \text{ AGeV} \)) a new observable was predicted in nuclear collisions: jet quenching \([1]-[3]\). Collisional energy loss of a hard jet propagating through quark-gluon plasma has been estimated to be modest \([1,2]\): \( dE_{\text{coll}}/dx \ll 1 \text{ GeV/fm} \), however the induced radiative energy loss estimated \([3,4]\) from the Gunion-Bertsch amplitude \([5]\) is expected to be significantly larger, \( dE_{\text{rad}}/dx > \text{few GeV/fm} \). In Ref. \([6]\) a model for non-abelian energy loss was developed and the analog of the QED class of radiation diagrams was summed. This leads to a constant \( dE/dx \sim 1 \text{ GeV/fm} \) independent of the path length. However, in BDMPS \([8]\) it was shown that gluon rescattering diagrams in the medium considerably increased the energy loss. For an incident very high energy jet penetrating a plasma of thickness, \( L \), in which the mean free path, \( \lambda = 1/(\sigma \rho) \), and the color electric fields are screened on a scale \( \mu \), the analytic BDMPS prediction \([8]\) is \( dE/dx \propto \alpha_s \mu^2 L/\lambda \). An alternative path integral approach led to similar results \([9]\). The transverse momentum dependence of the energy loss in QED was calculated recently \([10]\).

Unfortunately, the analytic approximations are not applicable to “thin” plasmas of only a few mean free path thickness, and more importantly they do not apply at all to the problem of computing the angular distribution. The main complication is that the angular distributions require a complete treatment of all gluon and jet final state interactions. In a direct brute force pQCD approach, in the thin plasma limit where the main simplicity arises from the fact that for a few rescatterings, \( n_s \leq 3 \), the number of amplitudes is however still sufficiently small to allow direct computation of all interference terms. Here we display the \( n_s = 1 \) case, a complete investigation of \( n_s \leq 3 \) rescattering can be found in Ref. \([11]\).

The single rescattering case, \( n_s = 1 \), was first considered by Gunion and Bertsch \([3]\) (GB). They estimated the hadron rapidity distribution due to gluon bremsstrahlung associated with a single elastic valence quark scattering in the Low-Nussinov model. The
most important difference compared to the problem addressed in this paper is that GB computed the non-abelian analog of the Bethe-Heitler formula while we are interested in the analog of radiative energy loss in sudden processes analogous to $\beta$-decay. The GB problem therefore is to compute the soft radiation associated with a single scattering of an incident on-shell quark prepared in the remote past, i.e., $t_0 = -\infty$, relative to the collision time, $t_1$. In our case, the radiation associated with a hard jet processes in nuclear reactions must take into account the fact that the jet parton rescatters within a short time, $t_1 - t_0 \sim R_A$, after it is produced. The sudden appearance of the jet color dipole moment within a time interval $\sim 1/E$ results in a broad angular distribution of gluons even with no final state scattering. The induced radiation caused by rescattering necessarily interferes strongly with this hard radiation.

To model the scattering in a thin plasma, we employ the same model of the plasma as considered in Gyulassy–Wang (GW) \cite{GW}. The scattering centers are approximated by static color-screened potentials with Fourier components $V(\vec{q}) = 4\pi\alpha_s/(\vec{q}^2 + \mu^2)$ and color screening mass, $\mu = 4\pi\alpha_s T^2$.

### 2. Angular Distribution with One Rescattering

The amplitude for the production of a hard jet with momentum $P_0$ localized initially near $x_0^n = (t_0, \vec{0})$ we denote by $M_J(J(P)e^{iPx_0})$. The hard vertex is localized within a distance $\sim 1/P_0$, and the amplitude, $J(P)$, is assumed to vary slowly with $P$ on the infrared screening scale $\mu$. This jet radiates a gluon with momentum $\vec{k}$, which can scatter on $n_s = 1$ scattering center.

![Fig. 1](image)

**Fig. 1.** Three contributions to the soft gluon radiation amplitude $M_J \otimes M_1$.

The conditional double inclusive jet and gluon probability distribution is the following:

$$d^6D_{\text{rad}}^{(n_s=1)}(k,p) = \rho_{\text{rad}}^{(n_s=1)}(k,p) \frac{d^3\vec{k}}{(2\pi)^32\omega} \frac{d^3\vec{p}}{(2\pi)^32p_0} = \langle |M_J \otimes M_{n_s=1}|^2 \rangle \frac{d^3\vec{k}}{(2\pi)^32\omega} \frac{d^3\vec{p}}{(2\pi)^32p_0}, \quad (1)$$

which involves averaging over all initial target colors and summing over all final colors and the locations $x_i$ of the target centers. After averaging one obtains:

$$\rho_{\text{rad}}^{(1)}(k,p) \approx |J(p+k)|^2 \int \frac{d^2\vec{q}_1}{(2\pi)^2} \frac{d^2\vec{q}_1'}{(2\pi)^2} V(\vec{q}_1)V^*(\vec{q}_1') \frac{C_2(i)}{D_A} T(\vec{q}_1 - \vec{q}_1') \left\langle \sum_{i,j=1}^3 M_{1,i}(k,p;\vec{q}_1\perp)M_{1,j}^\dagger(k,p;\vec{q}_1'\perp) \right\rangle_t. \quad (2)$$

The main source of non-diagonal dependence arises through the interference terms in $M_1M_1^\dagger$ and the $T(\vec{q}_1\perp - \vec{q}_1'\perp)$ transverse form factor.
Following Ref. [8], the three radiative amplitudes $M_{1,i}$ of Fig.1. are given by

\[
M_{1,1} = 2ig_s \frac{\bar{\epsilon}_1 \cdot \vec{k}_1}{k_1^2} (e^{i\ell_1 \cdot \vec{k}_1^2/2\omega_1} - e^{i\ell_0 \cdot \vec{k}_1^2/2\omega_0}) a_1 c, \quad M_{1,2} = 2ig_s \frac{\bar{\epsilon}_1 \cdot \vec{k}_1}{k_1^2} (-e^{i\ell_1 \cdot \vec{k}_1^2/2\omega_1}) c a_1, \\
M_{1,3} = 2ig_s \frac{\bar{\epsilon}_1 \cdot (\vec{k} - \vec{q})}{(k - q_1)^2} e^{i\ell_1 \cdot \vec{k}_1^2/2\omega_1} \times (e^{i\ell_1 \cdot (k - q_1)^2/2\omega_1} - e^{i\ell_0 \cdot (k - q_1)^2/2\omega_1}) [c, a_1]. \quad (3)
\]

The momentum distribution of gluon radiation can be derived from the conditional probability distribution of radiation $\rho_{rad}^{(1)}(p, k)$, normalized by the elastic scattering $\rho_{el}^{(1)}(p)$ \[11\]. Before averaging on scattering centers one can obtain from eq. (2):

\[
\frac{dN_{g}^{(1)}}{dy d^2k_\perp} = \frac{\rho_{rad}^{(1)}(p, k)}{\rho_{el}^{(1)}(p)} \Rightarrow C_R \frac{\alpha_s}{\pi^2} \left\{ \tilde{H}^2 + R (\tilde{B}_1^2 + t(\tilde{b}_1) \tilde{C}_1^2) - R \left( \tilde{H} \cdot \tilde{B}_1 \cos(t_{10} \omega_0) \right) - R t(\tilde{b}_1/2) \left( \tilde{H} \cdot \tilde{C}_1 \cos(t_{10} \omega_0) - 2\tilde{C}_1 \cdot \tilde{B}_1 \cos(t_{10} \omega_0) \right) \right\}. \quad (4)
\]

Here we rearranged the radiation amplitudes and introduced the hard term $\tilde{H} = \vec{k}_1/k_1^2$, the cascading term $\tilde{C}_1 = (\vec{k} - \vec{q}_1)/(k - q_1)^2$, and the GB term $\tilde{B}_1 = \vec{H} - \tilde{C}_1$. The interference terms contain phase factors composed from transverse energy transfer terms $\omega_0 \equiv k_\perp^2/2\omega$, $\omega_1 \equiv (k - q_1)^2/2\omega$, $\omega_{10} \equiv \omega_1 - \omega_0$ and time difference term $t_{10} \equiv t_1 - t_0$. These interference terms are responsible for “quantum cascading”: without them one can recover the classical cascading limit in eq. (4); with them a complex behavior of “quantum cascading” dominated by these interference terms appears. The transverse form factor $t(\tilde{b}) = T(\tilde{b})/T(0)$ connects to the target size and it regulates automatically the gluon cascading on the finite size target object.

In the limit $t_1 \rightarrow +\infty$ the hard radiation formula for the no-scattering case, $\tilde{H}^2$ is recovered. The limit $t_0 \rightarrow -\infty$ is corresponding to the isolated bremsstrahlung amplitude and it leads to the characteristic GB radiation spectrum, $\tilde{H}^2 + R \tilde{B}_1^2$ \[11\].

At finite $t_1$ and $t_0 = 0$ the numerical evaluation of the eq. (2) leads to the angular distribution of the radiated gluon. Fig. 2. displays our numerical results, normalizing the one gluon radiation one scattering distribution, $dN_g^{(1)}/dy d^2k_\perp$, by the one gluon radiation no scattering distribution, $dN_g^{(0)}/dy d^2k_\perp$. Fig.2a, displays the contribution of the time-independent, classical cascading terms and their sum. Fig.2b, shows the strong influence of the interference terms. Especially at small $k_\perp$ the hard radiation limit is recovered through the suppression of the GB contribution. This can be proved exactly in eq.(3) by means of $k_\perp \rightarrow 0$. At large $k_\perp$ kinematical cuts appear from the appropriate jet energy, however the enhancement connecting the presence of the $\tilde{C}_1$ cascading term can be seen clearly.

In the case of more scattering ($n_s > 1$) the existence of $\tilde{C}_i$ cascading terms leads to approximate exponentiation of the relative gluon radiation distribution. Naive interpretation of this result would suggest that the energy loss exponentiates with the length. However, in that case multiple emission processes have to be also considered and this necessitates a renormalization of the wavefunction as discussed in detail in Ref. [11]. With that renormalization one finds approximately constant $dE/dx$ for $n_s \leq 3$ up to $L \sim 3$ fm.
Fig. 2. (a) The normalized time-independent envelopes and separate contributions to the conditional probability distribution of gluons associated with a single rescattering $n_s = 1$ of quark jet (ECM = 100GeV) is shown as a function of the logarithmic “angle”, $\log_{10} k_\perp^2/\mu^2$ with $\mu = 0.5$ GeV. These curves correspond to the incoherent parton cascade limit. (b) For finite $t_{10} = 1$ fm/c case destructive interference limits the corrections to the hard self-quench distribution to higher angles as shown for different gluon energies.

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