Electricity-Aware Bid Format for Heat Commitment and Dispatch

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Coordination between heat and electricity markets is essential to achieve a cost-effective and efficient operation of the overall energy system. In the current sequential market practice, the heat market is cleared before the electricity market, and has no insight into the impacts of heat dispatch on the electricity market. While preserving this sequential practice, this paper introduces an “electricity-aware” bid format for the coordination of heat and electricity systems. This novel market mechanism defines heat bids conditionally on day-ahead electricity prices. Prior to clearing heat and electricity markets, the proposed bid selection mechanism selects the “valid” bids which minimize the operating cost of the heat system, while anticipating heat and electricity market clearing. This mechanism is modeled as a trilevel optimization problem, which we recast as a mixed-integer linear program using a lexicographic function. We use a realistic case study based on the Danish electricity and heat system, and show that the proposed bid selection mechanism yields a 4.5\% reduction in total operating cost of heat and electricity systems compared to the existing market-clearing procedure, while reducing the financial losses of combined heat and power plants and heat pumps due to invalid bids by up to 20.3 million euros.

Key words: Electricity-aware bid format, Market mechanism, Unit commitment, Multi-carrier energy systems, trilevel optimization.

1. Introduction
Exploiting potential synergies between electricity and other energy systems has been identified as a key solution to achieve a flexible and sustainable energy future (Lund 2007, Meibom et al. 2013, Pinson et al. 2017). In particular, an coordinated electricity and heat system operation is central in
a sustainable energy system due to strong technical and economic interdependencies between these two systems (Heat Road Map Europe 2018, Lund et al. 2010, 2014). In several countries, energy markets operate heat and electricity systems in the day-ahead stage sequentially and independently. For instance, in Denmark, the day-ahead heat market is cleared before the electricity market (Pinson et al. 2017). The main shortcoming of such a sequential framework is that the myopic dispatch of assets at the interface of electricity and heat systems, including Combined Heat and Power (CHP) units and large-scale Heat Pumps (HPs), may limit their operational flexibility in the electricity market (Chen et al. 2015), i.e., their ability to adjust their schedule to facilitate integration of additional renewable energy production (Zhao et al. 2016). Importantly, the potential lack of flexibility may hinder the increasing penetration of renewable energy sources. Additionally, Virasjoki et al. (2018) and Mitridati et al. (2020) show that in Nordic countries, the myopic dispatch of CHPs and HPs in the heat market may impact day-ahead electricity prices, which, in return, impact the production costs and profitability of CHPs and HPs.

This challenge raises an important research question: How to improve the dispatch of CHPs and HPs in the heat market in order to achieve a cost-effective and efficient operation of the overall energy system while preserving the current sequential energy market-clearing framework?

This question has been extensively addressed in the literature by proposing a drastic change in the current market design, i.e., moving towards fully integrated approaches based on electricity and heat co-optimization. Chen et al. (2015) propose a combined economic dispatch for heat and electricity systems, which increases the flexibility of CHPs at the interface between both systems. This study illustrates the benefits of electricity and heat co-optimization to reduce the operating cost of the overall energy system and to increase the penetration of renewable energy in the day-ahead dispatch. However, a co-optimization approach can only realistically be considered as an ideal benchmark, due to i) the disruptive regulatory changes it requires in the way the current independent and sequential heat and electricity markets operate, ii) the vast difference in geographical areas covered by both systems, and iii) the different (and sometimes conflicting) objectives of each market and system operator (Pinson et al. 2017). Therefore, based on extensive talks with regulators and heat market operators, we are convinced that a fully integrated market is not a feasible and realistic framework for sector coordination.

By opposition with these fully integrated operational approaches, Mitridati et al. (2020) introduce a novel formulation of the heat market-clearing problem based on a hierarchical optimization model. This approach claims to provide a soft coordination between heat and electricity systems while respecting the sequential clearing of their respective markets. This coordination is achieved by allowing the heat market clearing to model the heat production costs of CHPs and HPs as a function of day-ahead electricity prices in the upper-level problem, while anticipating the impact of
heat dispatch on the electricity market clearing in the lower level problem. Although the resulting hierarchical heat market-clearing procedure proposed in Mitridati et al. (2020) respects the sequential heat and electricity market-clearing framework, it results in increased computational complexity for the heat market operator, and pricing issues arising from non-convexities. Therefore, in practice, this approach still requires important regulatory changes.

Furthermore, the market-clearing procedure proposed in Mitridati et al. (2020) does not account for the non-convex temperature dynamics in district heating networks and the complex techno-economic characteristics of CHPs and HPs, such as their start-up and no-load costs as well as minimum on/off times. To address these issues, Li et al. (2016), Zheng et al. (2018), Mitridati and Taylor (2018) and Dai et al. (2018) develop optimization models accounting for the dynamics of heat transfer in the district heating network, making it possible to utilize the capability of storing energy in heat pipelines. These studies illustrate the benefits of the accurate representation of techno-economic characteristics of the district heating network to increase flexibility of the overall energy system at the day-ahead stage. In addition, as CHPs and HPs are required to produce the energy traded in the day-ahead heat market regardless of electricity market outcomes, an accurate representation of their techno-economic characteristics is essential to achieve an efficient and secure operation of the heat system. However, adding such complexities requires solving a heat unit commitment model, and brings pricing challenges due to introducing 0/1 binary variables (Ruiz et al. 2012, O’Neill et al. 2005).

To tackle the above challenges, we draw inspiration from the notion of bid-validity constraints introduced in Byeon and Van Hentenryck (2020), that addresses the coordination problem of electricity and natural gas (but not heat) systems. This work introduces a hierarchical mechanism to commit electricity producers and select their valid bids, while anticipating the impact on the electricity and natural gas day-ahead markets.

This paper makes two contributions to the heat and electricity market mechanism design. The first contribution is to introduce an electricity-aware bid format in the day-ahead heat market based on the concept of bid-validity conditions. This format enables CHPs and HPs to design bids for the heat market, which are conditioned on day-ahead electricity prices. Contrary to traditional bid formats, e.g., price-quantity bids, this novel bid format implicitly takes into account the complex techno-economic characteristics of CHPs and HPs at the interface between heat and electricity markets, and ensure cost recovery in the heat market for all market participants. The second contribution is to develop an electricity-aware bid selection mechanism which commits heat producers and selects their valid bids, i.e., those bids that ensure cost recovery of heat producers prior to their participation in heat and electricity markets. This mechanism takes into account the sequential clearing of heat and then electricity markets, as it is prevalent in some countries such as Denmark. The hierarchical
framework of the proposed mechanism allows anticipating the electricity market prices as a function of the electricity-aware heat bids. Furthermore, this mechanism accounts for the flexibility of the district heating network by optimizing time delays in heat transfer. Note that in this approach, after the heat bids have been selected, heat and electricity markets are still cleared sequentially by independent market operators, and their market-clearing formulation remains unchanged. One major advantage of this approach compared to a fully integrated one is that the heat and electricity market operators remain independent entities, optimizing their own, and sometimes conflicting, objectives, and rely on information exchange between them to achieve coordination. In particular, the heat bid-selection problem aims at minimizing solely the heat system operating costs, rather than the overall system operating costs. In the case of a fully integrated model, while the overall operating costs may be decreased, the operating costs of one individual sector may be drastically increased, as has previously been observed in the literature (Mitridati and Pinson 2016, Mitridati et al. 2020).

The main methodological contribution of this paper pertains to computational scalability: It shows that the proposed electricity-aware bid selection mechanism can be modeled as a trilevel optimization problem, which can be reformulated as a tractable single-level mixed-integer linear program (MILP) using a lexicographic function and the strong duality of the middle- and the lower-level problems.

Finally, the last contribution of this paper concerns a thorough numerical evaluation of the proposed mechanism on two substantial case studies. This evaluation compares the proposed electricity-aware bid selection mechanism to the existing decoupled heat and electricity market mechanism, and an ideal fully integrated market mechanism. The first illustrative case study shows that the proposed electricity-aware mechanism can achieve 77.6% of the so-called value of coordination achieved by the fully integrated mechanism, while maintaining a sequential heat and electricity market framework. The second case study, based on the realistic electricity and heat systems in Denmark, provides additional insights on how the proposed mechanism is able to ensure cost recovery for CHPs and HPs in the heat market, while reducing the operating cost of the overall energy system by 4.5% and wind curtailment by 0.4% compared to the decoupled mechanism.

The remainder of this paper is organized as follows. Section 2 provides the required preliminaries and describes the techno-economic interdependencies between heat and electricity systems and the current status-quo in the markets that operate them. Section 3 introduces the proposed electricity-aware heat bid format and the bid selection mechanism. In addition, the resulting heat and electricity market-clearing procedure with the proposed bid selection mechanism as well as their market properties are explained. Section 4 details the mathematical formulation of the proposed electricity-aware heat bid selection mechanism. Section 5 presents numerical results. Finally, Section
concludes and discusses potential extensions of this work. The extended formulations are provided in the online appendix.

2. Preliminaries

By convention, any set indexed by an electricity or heat market zone \( z \in \mathcal{Z}_E \) or \( n \in \mathcal{Z}_H \) represents the subset located/connected to this zone. Furthermore, all variables and vectors of variables in the heat unit commitment, and day-ahead heat and electricity market-clearing problems are represented in bold. A full list of notation is provided below.

Nomenclature

A. Sets

\( \mathcal{B}_E \) Set of bids for each day-ahead electricity market participant and time step

\( \mathcal{B}_H \) Set of bids for each day-ahead heat market participant and time step

\( \mathcal{I}_{\text{CHP}} \) Set of CHPs in the heat system

\( \mathcal{I}_E \) Set of electricity market participants

\( \mathcal{I}_{\text{HP}} \) Set of HPs in the heat system

\( \mathcal{I}_H \) Set of heat market participants

\( \mathcal{T} \) Set of time steps in day-ahead heat and electricity markets (24 hourly steps)

\( \mathcal{Z}_E \) Set of electricity market zones

\( \mathcal{Z}_H \) Set of heat market zones

\( \Phi_{\text{r}} \) Set of counts of time periods with distinct start-up costs of generation unit \( j \)

\( \Phi_{\text{u,init}} \) Set of initial (fixed) up- or down-time periods of generation unit \( j \)

B. Parameters

\( \overline{ATC}_{z\tilde{z}t} \) Upper bound of the available transfer capacity (ATC) between electricity market zones \( z \) and \( \tilde{z} \) at time step \( t \) (Wh)

\( \overline{f}_{nmt} \) Maximum heat flow between market zones \( n \) and \( m \) at time step \( t \) (Wh)

\( F_j \) Maximum fuel consumption of CHP \( j \) (Wh)

\( \overline{s}_{j\tilde{b}t} \) Quantity of electricity bid \( \tilde{b} \) of power plant \( j \) at time step \( t \) (Wh)

\( \overline{s}_{j\tilde{b}t} \) Quantity of heat bid \( b \) of heat market participant \( j \) at time step \( t \) (Wh)

\( \rho_{\text{r}}^{E_j} \) Electricity efficiency ratio of CHP \( j \) (-)

\( \rho_{\text{r}}^{H_j} \) Heat efficiency ratio of CHP \( j \) (-)

\( COP_j \) Coefficient of performance of HP \( j \) (-)

\( \Phi_{\text{d}}^j \) Minimum down-time of generation unit \( j \) at time step \( t \) (h)

\( \Phi_{\text{u}}^j \) Minimum up-time of generation unit \( j \) at time step \( t \) (h)

\( \overline{ATC}_{z\tilde{z}t} \) Lower bound of the ATC between electricity market zones \( z \) and \( \tilde{z} \) at time step \( t \) (Wh)
\( F_j \) Minimum fuel consumption of CHP \( j \) (Wh)
\( S^E_{jt} \) Self-committed minimum production of electricity market participant \( j \) at time step \( t \) (Wh)
\( S^H_j \) Minimum heat production of heat market participant \( j \) (Wh)
\( c^E_{jbt} \) Price of electricity bid \( \hat{b} \) of power plant \( j \) at time step \( t \) (€/Wh)
\( c^H_{jbt} \) Price of heat bid \( b \) of heat market participant \( j \) at time step \( t \) (€/Wh)
\( K_{nmtk} \) Thermal loss coefficient flow traveling between market zones \( n \) and \( m \), from time \( t \) to \( t + k \)
\( L^E_{zt} \) Electricity load in zone \( z \) at time step \( t \) (Wh)
\( L^H_{zt} \) Heat load at market zone \( z \) and time \( t \) (Wh)
\( r_j \) Minimum power-to-heat ratio of CHP \( j \) (-)
\( u^\text{init}_{jt} \) Initial commitment status of generation unit \( j \) at time \( t \in \Phi_j^{\text{init}} \)

C. Electricity-aware bid selection variables (\( \Omega^E \))
\( \overrightarrow{U}_{nmt} \) Direction of heat flow between market zones \( n \) and \( m \) at time step \( t \) (-)
\( \Phi_{nmt} \) Time delay of water injected between market zones \( n \) and \( m \) at time step \( t \) (h)
\( r_j^\Phi \) Start-up cost of generation unit \( j \) at time step \( t \) (€)
\( u_{nmtk}^\Phi \) Binary variable representing the time delay between market zones \( n \) and \( m \) at time step \( t \)
\( u^\text{bid}_{jbt} \) Binary variable representing whether bid \( b \) of generation unit \( j \) at time step \( t \) is valid
\( u^\text{commit}_{jbt} \) Binary variable representing the commitment status of generation unit \( j \) at time step \( t \)
\( u^\text{shut}_{jt} \) Binary variable representing whether generation unit \( j \) at time step \( t \) is shut down
\( u^\text{start}_{jt} \) Binary variable representing whether generation unit \( j \) at time step \( t \) is started up

D. Day-ahead heat market variables (\( \Omega^H \))
\( \overline{P}_{jt} \) Maximum electricity production of CHP \( j \) at time step \( t \) (Wh)
\( P^E_{jt} \) Self-committed minimum electricity production of CHP \( j \) at time step \( t \) (Wh)
\( f^\text{in}_{nmt} \) Inlet heat flow between market zones \( n \) and \( m \) at time step \( t \) (Wh)
\( f^\text{out}_{nmt} \) Outlet heat flow between market zones \( n \) and \( m \) at time step \( t \) (Wh)
\( L^\text{HP}_{jt} \) Electricity consumption of HP \( j \) at time step \( t \) (Wh)
\( Q^b_{jbt} \) Dispatch of heat bid \( b \) of generation unit \( j \) at time step \( t \) (Wh)
\( Q^b_{jt} \) Total heat production of generation unit \( j \) at time step \( t \) (Wh)

E. Day-ahead electricity market variables (\( \Omega^E \))
\( \lambda^E_{zt} \) Electricity market price at market zone \( z \) at time step \( t \) (€/Wh)
\( P^\text{bid}_{jbt} \) Dispatch of electricity bid \( \hat{b} \) of generation unit \( j \) at time step \( t \) (Wh)
\( P^E_{jt} \) Total electricity production of generation unit \( j \) at time step \( t \) (Wh)

2.1. Existing day-ahead heat and electricity markets
In Nordic countries such as Denmark, heat and electricity systems are operated by competitive auction-based markets which interface the physical and economic aspects of each system. Energy
markets such as day-ahead heat and electricity markets operate on the principles of energy exchanges, in which market participants submit bids which implicitly embed their techno-economic characteristics. Contrary to energy markets operating on the principles of energy pools such as the North American markets, unit commitment costs and constraints are not accounted for explicitly by the market operator. Furthermore, physical flows and congestion in each market zone are managed a posteriori by the transmission systems operators (Meeus et al. 2005). Therefore, uniform prices are obtained as the marginal prices in each market zone. In European countries, interconnected electricity market zones are coupled based on Available Transfer Capacities (ATC) (ETSO 2000, Tosatto and Chatzivasileiadis 2020), which represent inter-zonal power transfer bounds. Similarly, heat transfer bounds between market zones of the transmission network are enforced in the day-ahead heat market clearing. Due to the thermal losses and longer time delays in transmission, heat networks cover a smaller area than electricity networks In Nordic countries, each heat network typically covers a few geographically close urban areas. These heat networks are isolated from one another and operated by independent heat market operators. Each isolated heat network typically encompasses a single heat market zone.

Additionally, day-ahead heat and electricity markets are cleared sequentially and independently, with the heat market being cleared before the electricity market. In the day-ahead heat market, each market participant $j \in I^H$ submits bids $b \in B^H$ for each hour of the following day $t \in T$, in the form of independent price-quantity pairs $(c_{jbt}^H, s_{jbt}^H)$. These bids are dispatched by the heat market operator based on a merit-order and least-cost principle. Once the heat market has been cleared, each electricity market participant $j \in I^E$ submits bids $\tilde{b} \in B^E$ in the day-ahead electricity market for each hour of the following day, in the form of independent price-quantity pairs $(c_{j\tilde{bt}}^E, s_{j\tilde{bt}}^E)$. Furthermore, market participants may choose to self-commit a certain minimum electricity production $s_{jlt}^E$, which will be dispatched regardless of the equilibrium price in the market. These bids are dispatched by the electricity market operator based on a merit-order and least-cost principle. Note that heat and electricity market operators are not necessarily the same entities. Therefore, co-optimizing heat and electricity systems would require major institutional and regulatory changes.

### 2.2. Physical and economic interfaces between heat and electricity systems

CHPs ($j \in I^{CHP}$) and large-scale HPs ($j \in I^{CHP}$) participate in both day-ahead heat and electricity markets. The techno-economic characteristics of these units, linking their heat and electricity outputs, create implicit interdependencies in the operation of both systems. The heat $Q_{jlt}$ and electricity $P_{jlt}$ production of CHPs are constrained by their joint Feasible Operating Region (FOR) (Lahdelma and Hakonen 2003). The majority of CHPs are extraction units that can produce heat
and electricity at different ratios. For each time period $t \in \mathcal{T}$ considered, their FOR can be modeled by a set of linear equations, such that

$$P_{jt} \geq r_j Q_{jt}, \forall j \in \mathcal{CHP}, t \in \mathcal{T} \tag{1a}$$

$$E_j u^0_{jt} \leq \rho_j^E P_{jt} + \rho_j^H Q_{jt} \leq E_j u^0_{jt}, \forall j \in \mathcal{CHP}, t \in \mathcal{T}, \tag{1b}$$

where (1a) represents their minimum heat-to-power ratio $r_j$, and (1b) represents their commitment status $u^0_{jt}$, and the upper $E_j$ and lower $E_j$ bounds on their fuel consumption, expressed as a linear function of their heat and electricity production with the heat and power efficiency coefficients, $\rho_j^H$ and $\rho_j^E$, respectively.

Similarly, the heat production $Q_{jt}$ of HPs is proportional to their electricity consumption $L_{jt}^{HP}$, with a heat-to-power ratio, called the Coefficient of Performance (COP), such that

$$Q_{jt} = \text{COP}_j L_{jt}^{HP} \forall j \in \mathcal{HP}, t \in \mathcal{T}. \tag{2}$$

The COP of HPs varies based on their input and output temperatures. However, it can be accurately estimated for the following day. Therefore, for the purpose of this study, it is considered as a fixed and known parameter of the system at each time period.

In addition to the aforementioned physical interdependencies, the variable production costs of CHPs and HPs are intrinsically linked to their heat and electricity outputs. As HPs produce heat from electricity purchased in the electricity market, their variable heat production cost $\Theta^H_{jt}$ is linearly dependent on their electricity consumption $L_{jt}^{HP}$ at each market-clearing time period $t \in \mathcal{T}$, such that

$$\Theta^H_{jt} = \text{COP}_j \lambda_{zt}^E L_{jt}^{HP}, \forall j \in \mathcal{HP}_z, t \in \mathcal{T}, \tag{3}$$

where $\lambda_{zt}^E$ represents the day-ahead electricity prices in market zone $z \in \mathcal{Z}^E$ where HP $j \in \mathcal{HP}_z$ is located.

Similarly, as CHPs simultaneously produce heat and electricity, their variable production cost can be represented by a linear function of their heat and electricity production. Due to the strong linkage between the heat and electricity production of CHPs, the cost allocation between heat and electricity production is not straightforward. A common approach used in Nordic countries, and followed in the literature (Pinson et al. 2017, Mitridati and Pinson 2016, Mitridati et al. 2020), defines the variable heat production cost of CHPs as their total variable production cost $\Gamma_{jt}$ minus revenues from electricity sales, such that

$$\Gamma^H_{jt} = \Gamma_{jt} - P_{jt} \lambda_{zt}^E, \forall z \in \mathcal{Z}^E, j \in \mathcal{CHP}_z, t \in \mathcal{T} \tag{4}.$$  

Furthermore, the total variable production cost of CHPs can typically be approximated by a linear function of their fuel consumption, such that $\Gamma_{jt} = c_j (\rho^E P_{jt} + \rho^H Q_{jt})$. Note that this linear variable cost assumption can be relaxed to assume convex quadratic or convex piece-wise linear costs, without loss of generality.
2.3. Interdependencies in a market environment

In the current sequential market framework described in Section 2.1, CHPs participate in both heat and electricity markets, creating implicit interdependencies between these independently-operated markets. In addition, in Nordic countries, a growing number of large-scale HPs are participating in wholesale heat and electricity markets. In order to achieve an efficient dispatch for the whole system while ensuring their profitability, the bids of CHPs and HPs in heat and electricity markets must implicitly reflect their techno-economic characteristics. In particular, the bids of CHPs and HPs in the heat market must reflect their marginal heat production cost. Due to the linkage between their heat and electricity outputs and costs, CHPs and HPs must anticipate their day-ahead electricity dispatch and day-ahead electricity prices in order to accurately compute their marginal heat production cost. As the COP of HPs is considered fixed and known over each market-clearing time period, the marginal heat production cost $\dot{\Gamma}_{jt}^H$ of HPs is an affine function of the foreseen day-ahead electricity prices, such that

$$\dot{\Gamma}_{jt}^H = \frac{\partial \Gamma_{jt}^H}{\partial Q_{jt}} = \frac{\lambda_{zt}^E}{\text{COP}_{jt}}, \forall z \in Z^E, j \in I^\text{HP}, t \in T. \quad (5)$$

Unlike HPs, the heat-to-power ratio $r_{jt} = \frac{P_{jt}}{Q_{jt}}$ of CHPs is variable, and therefore, their marginal heat production cost must be computed at the optimal heat-to-power ratio given the heat production and day-ahead electricity prices. For low day-ahead electricity prices the marginal heat production cost of CHPs represents the incremental heat production cost at the minimum heat-to-power ratio, i.e., $\dot{\Gamma}_{jt}^H = (c_j\left(r_{jt}^H + r_{jt}^E\right) - r_{jt}\lambda_{zt}^E)$, and for high day-ahead electricity prices it represents the opportunity cost of producing an extra unit of heat at the maximum heat-to-power ratio, i.e.,

$$\dot{\Gamma}_{jt}^H = \left(\lambda_{zt}^E \frac{\rho_{jt}^H}{\rho_{jt}^E}\right).$$

Therefore, the marginal heat production cost of CHPs can be expressed as a convex piece-wise linear function\(^1\) of electricity prices, such that

$$\dot{\Gamma}_{jt}^H = \max\left\{\lambda_{zt}^E \frac{\rho_{jt}^H}{\rho_{jt}^E}, c_j\left(r_{jt}^H + r_{jt}^E\right) - r_{jt}\lambda_{zt}^E\right\}, \forall z \in Z^E, j \in I^\text{CHP}, t \in T. \quad (6)$$

As day-ahead electricity prices are unknown prior to the heat market clearing, CHPs and HPs must anticipate these prices in order to compute their heat bids. In practice, they use a deterministic electricity price forecast to approximate their marginal heat production costs. This cost allocation method allows CHPs and HPs to implicitly model the linkage between their heat and electricity outputs and costs. However, the resulting bids may differ from the marginal heat production costs due to forecast errors on day-ahead electricity markets, or due to the exercise of market power. This makes market monitoring challenging in the current heat market framework. Therefore, making the

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\(^1\) Convex piece-wise linear functions are defined as the upper envelop of the lines forming their linear pieces.
bidding process in the heat market more transparent is essential. Furthermore, the heat market clearing is myopic to the impacts of the heat dispatch of CHPs and HPs on the electricity market clearing and, in turn, to the impact of day-ahead electricity prices on the marginal heat production cost and profitability of CHPs and HPs in the heat market (Virasjoki et al. 2018).

Once their heat dispatch for each hour of the following day is fixed, CHPs and HPs can participate in the day-ahead electricity market (Nordpool 2019). Due to the strong linkage between their heat and electricity outputs and costs, the bids of CHPs and HPs in the electricity market are dependent on their day-ahead heat dispatch and commitment. As CHPs and HPs are required to provide the heat dispatched in the day-ahead heat market, they must self-commit their minimum electricity production \( P_{jt} \) and inflexible electricity consumption \( L_{HP}^{jt} \) in the electricity market, such that

\[
L_{HP}^{jt} = \frac{Q_{jt}}{\text{COP}_{jt}}, \forall j \in \mathcal{I}^{HP}, t \in \mathcal{T} \tag{7}
\]

\[
P_{jt} \geq \max \left( \frac{F_j - \rho_{H}^{E} Q_{jt}}{\rho_{j}^{E}} u_{jt}^{0}, r_{j} Q_{jt}, \forall j \in \mathcal{I}^{CHP}, t \in \mathcal{T} \right). \tag{8}
\]

Furthermore, CHPs must reduce their maximum offered electricity production \( \overline{P}_{jt} \) to ensure their electricity dispatch is feasible with respect to their heat dispatch and commitment, such that

\[
P_{jt} \leq \overline{P}_{jt} \leq \frac{F_j - \rho_{H}^{E} Q_{jt}}{\rho_{j}^{E}} u_{jt}^{0}, \forall j \in \mathcal{I}^{CHP}, t \in \mathcal{T}. \tag{9}
\]

In this context, (7)-(9) show that the heat-driven dispatch of CHPs and HPs limits their flexibility in the day-ahead electricity market, which may limit the integration of renewable energy sources and impact day-ahead electricity prices. In return, (5) and (6) show that these day-ahead electricity prices may impact the merit order in the heat market and the profitability of CHPs and HPs. Hence, one major challenge for CHPs and HPs in the current market framework is to devise heat bids which ensure cost recovery. Yet, in the current sequential and independent market framework, the heat market clearing is myopic to these interdependencies. In particular, the current simple price-quantity bid format is ill suited to model these complex interdependencies.

Additionally, while geographically-isolated heat networks are operated by independent markets, they are connected to the same electricity network. In practice multiple independent heat markets may interface with the same electricity market. Therefore, the merit order in each independent heat market is impacted by the same electricity market prices, and by extension, by the heat dispatch of CHPs and HPs in other heat markets. As a result, the clearing of these independent day-ahead heat markets should be coordinated.
3. Towards an electricity-aware heat system

As discussed in Section 2, the current sequential heat and electricity market-clearing framework does not effectively take into account the interdependencies between energy systems. As a result, the heat market clearing is myopic to the impact of the heat dispatch on the electricity system. While various studies have showed the benefits of simultaneously clearing heat and electricity markets, this integrated approach requires major regulatory changes, and therefore it is not realistically applicable. In order to address the aforementioned challenges, we focus on designing a novel day-ahead heat market mechanism which respects the current sequential market framework. Our proposed approach introduces a novel electricity-aware heat bid format which (i) accurately embeds the techno-economic characteristics of the units at the interface between heat and electricity systems such as CHPs and HPs, and (ii) ensures cost recovery in the heat market for all market participants. Furthermore, in order to optimally select the submitted electricity-aware heat bids, we introduce an electricity-aware heat bid selection mechanism, which (i) centrally coordinates the participation of CHPs and HPs on independent and geographically-isolated heat markets, and (ii) anticipates and improves the sequential clearing of heat and electricity markets through the proposed electricity-aware heat bids.

Note that in this approach, after the heat bids have been selected, heat and electricity markets are still cleared sequentially by independent market operators, and their market-clearing formulation remains unchanged.

3.1. A novel electricity-aware heat bid format: A bid-validity approach

As discussed above, the current price-quantity bid format in the day-ahead heat market is unable to account for the complex interdependencies between heat and electricity markets. In particular, this bid format cannot guarantee cost recovery for CHPs and HPs. Therefore, our proposed approach is to introduce a new bid format which ensures cost recovery for all heat market participants, i.e.,

\[(c^H_{jbt} - M_j) u^\text{bid}_{jbt} \geq \tilde{\Gamma}^H_{jt} - M_j, \; \forall j \in I^H, t \in T, b \in B^H,\]

where \(M_j\) is a big-enough constant, which provides an upper bound on the marginal heat production costs of CHPs and HPs. Decision variables \(u^\text{bid}_{jbt} \in \{0, 1\}\) ensure that \(u^\text{bid}_{jbt} = 1\) if and only if bid \(b\) is selected in the heat market. Therefore, the cost-recovery condition (10) guarantees that a necessary condition for a bid \(b\) to be selected is that its bidding price \(c^H_{jbt}\) is greater than or equal to the marginal heat production cost of this unit, i.e., \(\tilde{\Gamma}^H_{jt}\). Note that, for \(u^\text{bid}_{jbt} = 0\), (10) simply enforces the upper bound, i.e., \(M_j \geq \tilde{\Gamma}^H_{jt}\). In practice, this condition ensures cost recovery of CHPs and HPs in the heat market, conditionally on future electricity prices.

In order to enforce this condition, we introduce a novel electricity-aware bid format in the day-ahead heat market, in the form of simple price-quantity bids \(\{c^H_{jbt}, s^H_{jbt}\}\), conditioned on day-ahead
electricity prices. As the marginal heat production cost of CHPs and HPs is a function of electricity prices, their electricity-aware bids define a range of electricity prices over which cost recovery is guaranteed. This new bid format is defined formally below.

**Definition 1 (Electricity-aware heat bid format).** For each heat market participant \( j \in \mathcal{I}_H \) and time step \( t \in \mathcal{T} \), an electricity-aware heat bid \( b \in \mathcal{B}_H \) is defined as a price-quantity pair \( \{c_{jbt}, s_{jbt}\} \) associated with a range \( \{\lambda_{jbt}^E, \lambda_{jbt}^E\} \) of electricity prices over which this bid is considered valid.

This electricity-aware bid format is implemented by the heat system operator through enforcing the following linear *bid-validity conditions* for each bid:

\[
\begin{align*}
\lambda_{zt}^E - \lambda_{jbt}^E &\leq M_j (1 - u_{jbt}^{\text{bid}}), \quad \forall z \in \mathcal{Z}_E, j \in \mathcal{I}_H, t \in \mathcal{T}, b \in \mathcal{B}_H \quad (11a) \\
\lambda_{jbt}^E - \lambda_{zt}^E &\leq M_j (1 - u_{jbt}^{\text{bid}}), \quad \forall z \in \mathcal{Z}_E, j \in \mathcal{I}_H, t \in \mathcal{T}, b \in \mathcal{B}_H \quad (11b)
\end{align*}
\]

These conditions enforce that a bid can only be selected, i.e., \( u_{jbt}^{\text{bid}} = 1 \), if electricity prices are within the bounds \( \{\lambda_{jbt}^E, \lambda_{jbt}^E\} \). These electricity price bounds are directly computed by market participants based on their marginal heat production costs, such that the cost-recovery condition (10) is guaranteed within this range of prices. In particular, for heat market participants whose marginal heat production costs can be expressed as an affine or convex piece-wise linear functions of the day-ahead electricity prices such as CHPs and HPs, the detailed expression of these electricity price bounds is provided in the online appendix.

### 3.2. A hierarchical approach for electricity-aware bid selection

We introduce an electricity-aware bid selection mechanism for the proposed electricity-aware bid format. This mechanism aims at selecting the *valid* bids of heat market participants which minimize the operating cost of the overall heat system while anticipating their impact on the participation of CHPs and HPs in the sequential heat and electricity markets. Once the valid bids have been selected, CHPs and HPs participate in the sequential heat and electricity markets by solely submitting this set of valid bids, in the form of independent price-quantity pairs. This sequence of bid selection and market clearing is highlighted in Figure 1. As isolated heat networks may interface with the same electricity market zones, this electricity-aware bid selection mechanism must coordinate the bids of CHPs and HPs across multiple heat market zones. Therefore, the proposed mechanism is solved centrally, as illustrated in Figure 1. As a result, this electricity-aware mechanism coordinates the participation of these units in both heat and electricity markets, while the heat and electricity market-clearing mechanisms remain unchanged, as illustrated in Figure 1.

The proposed mechanism is built upon a Stackelberg game with a single leader and multiple followers. The leader is the heat unit commitment and bid selection problem that determines the
Sequential bid selection mechanism and clearing of heat and electricity markets

on/off commitment decisions in the heat sector, while anticipating the reaction of the followers, i.e., the sequential heat and electricity market-clearing problems. The action of the leader constrains the reaction of the follower, and, in return, the reaction of the followers impacts the objective function of the leader. In our case, the commitment decisions of heat producers affect the sequential heat and electricity market-clearing problems. In return, the heat dispatch of CHPs and HPs impacts the operating cost of the heat system in the heat unit commitment problem. Additionally, day-ahead electricity prices affect the marginal heat production costs and the profitability of CHPs and HPs. This is accounted for in the heat bid selection mechanism in the form of bid-validity conditions (10), which enforce cost recovery for all market participants.

This mechanism relies on the exchange of information on the electricity market supply and demand curves. This information is solely exchanged between the electricity market operator and the heat system operator. Market participants in both sectors do not have access to it prior to the market clearings. Therefore, this proposal does not raise privacy issues for electricity market participants, or provide any additional strategic advantage to heat market participants. Furthermore, this framework is in line with recent regulatory changes, encouraging sector coordination and information exchange between market and system operators. For instance, the joint Federal Energy Regulatory Commission (FERC) and North American Electric Reliability Corporation (NERC) report on the 2011 polar vortex put an emphasis on increased information sharing between energy sectors to improve systems reliability and prevent extreme events FERC and NERC (Aug. 2011,N).

Assuming the availability of perfect information on the input data within the heat and electricity market-clearing problem, our proposed model results in a hierarchical trilevel optimization problem, whose structure is illustrated in Figure 1. The upper-level problem represents the heat unit commitment problem, which is constrained by the lower-level optimization problem. The lower-level problem will be modeled as a bilevel optimization model, representing the sequential heat and electricity market-clearing problem. The next subsection further explains it.
3.2.1. A closer look at the lower-level problem: As illustrated by Figure 2, the lower-level problem in the proposed electricity-aware bid selection mechanism represents the sequential clearing of heat and electricity markets, in which the heat market is cleared first and its outcomes impact the bids in the electricity market. Different approaches have been studied in the literature to model such sequential and interdependent market-clearing problems (Pineda and Morales 2016). Traditionally, these markets have been modeled as an equilibrium problem, where the electricity market-clearing problem takes as input the optimal solutions of the heat market-clearing problem, as illustrated in Figure 2(a). It is worth noting that this formulation of the lower-level problem is not a bilevel optimization problem. Rather, both the heat and electricity market-clearing problems are lower-level problems for the upper-level problem in Figure 1. However, the dual formulation of this sequential market-clearing model contains non-convex bilinear terms, making the solution of the whole electricity-aware bid selection mechanism computationally challenging.

![Figure 2](image-url) The structure of the sequential heat and electricity market-clearing model as (a) an equilibrium problem, (b) a bilevel problem proposed by Pineda and Morales (2016), and (c) a bilevel problem proposed by this paper. For clarity, only one heat and electricity market zone is represented.

In order to tackle these challenges, Pineda and Morales (2016) propose an interesting alternative that models these sequential market-clearing problem in the lower-level problem as a bilevel optimization problem, whose leader is the electricity market-clearing problem, and whose follower is the heat market-clearing problem. This alternative is illustrated in Figure 2(b). With this setup, the electricity market-clearing problem becomes the middle-level problem of the electricity-aware bid selection mechanism, while the heat market-clearing problem becomes the lower-level problem. Despite its computational tractability, this alternative has its own shortcoming. These equilibrium (Figure 2(a)) and bilevel (Figure 2(b)) formulations are not equivalent. In the bilevel formulation proposed in Pineda and Morales (2016), the electricity market in the middle level is able to anticipate the outcomes of the heat market in the lower level, and, in case of multiple solutions to the
lower-level problem, to choose the optimistic solution which minimizes its own objective. Due to the existing sequential order of these market-clearing problems, this formulation is not realistic.

We propose a third alternative, which is also a bilevel optimization model. However, the middle-level problem represents the heat market clearing, which is constrained by the electricity market clearing in the lower level, as illustrated in Figure 2(c). Therefore, the electricity market in the lower level is constrained by the solutions of the heat market in the middle level.

**Proposition 1.** The proposed bilevel formulation of the sequential heat and electricity market-clearing problem, in which the middle-level problem represents the heat market clearing and the lower-level problem represents the electricity market clearing, as illustrated in Figure 2(c), is equivalent to the equilibrium formulation of these sequential market-clearing problems, as illustrated in Figure 2(a).

A proof of Proposition 1 is provided in the online appendix. Furthermore, the main advantage of the proposed bilevel formulation is its computational tractability, as it will be discussed later in Section 4. Note that in the proposed model, the middle-level problem does not anticipate the solutions of the lower-level problem, as its constraints and objective function are not dependent on the variables of the lower-level problem.

As a result, the proposed electricity-aware bid selection mechanism is formulated as a trilevel optimization problem, in which the middle-level problem represents the heat market clearing, and the lower-level problem represents the electricity market clearing.

### 3.3. Heat and electricity market properties

After the bid selection step, the sequential clearing of heat and electricity markets remains unchanged, and therefore the economic properties of the current sequential heat and electricity markets are still valid. In particular, day-ahead heat and electricity markets with price-quantity bids and uniform pricing are *budget balanced* (Kazempour et al. 2018). Furthermore, as the bid selection mechanism may reject certain bids, the revenue of certain heat market participants may be reduced. However, as previously discussed, solely bids that do not guarantee cost recovery are discarded. Therefore, after the bid selection step, the heat and electricity markets guarantee *cost recovery* for all market participants with respect to the variable production costs announced. These two important properties guarantee that the proposed electricity-aware approach can be implemented in the current sequential market framework without major changes.

However, *incentive compatibility* can be guaranteed for these market mechanisms only under the assumption of perfect competition (Wilson 1977, Hobbs et al. 2004). Otherwise, individual market participants may manipulate market outcomes by submitting strategic bids. Roberts and Postlewaite (1976) show that incentive-compatibility can be achieved in the limit, as the number of agents
tends to infinity. In fact, in the existing sequential and decoupled market framework, CHPs have an incentive to bid strategically due to high opportunity costs between heat and electricity markets. In particular, they can implicitly subsidize their heat or electricity production by freely allocating their production costs across the different sectors, in order to gain a strategic advantage in one or the other market. This market advantage can induce shifts in heat production from heat only generators (e.g. biomass) to CHPs, and in electricity production from thermal generators (e.g. gas turbines) and renewable sources to CHPs. This market power issue is discussed in detail in (Pinson et al. 2017, Mitridati et al. 2020, Virasjoki et al. 2018). As the proposed bid-selection mechanism discards the bids of CHPs or heat pumps that are not realistic, in the sense that they do not properly allocate their heat and electricity costs, it is not expected to provide any additional market advantage to CHPs compared to the existing market framework. On the other hand, while a fully integrated market framework may mitigate certain strategic behaviors related to the sequential nature of heat and electricity markets and high opportunities costs, due to the previously mentioned regulatory and organizational challenges, such a market mechanism is not realistically applicable.

Furthermore, heat and electricity markets with uniform pricing may not be efficient. Tang and Jain (2013) show that a Nash equilibrium may not exist in the general case. However, these markets are efficient under the assumption of a perfect competition.

Note that, similarly to all new market framework proposals, our mechanism may result in new strategic behaviors from market participants that will need to be studied rigorously. Addressing the issue of market power and strategic behaviors in heat and electricity markets and their impact on social welfare is out of the scope of this paper.

4. Formulation of the proposed trilevel optimization problem
The proposed electricity-aware heat bid format provides CHPs and HPs with greater flexibility to take into account their techno-economic characteristics and the linkage between their heat and electricity outputs in the current sequential market design. In addition, this electricity-aware approach can be implemented in the current sequential market framework without major regulatory and organizational changes by introducing a bid selection mechanism prior to the clearing of sequential heat and electricity markets, as described in this section.

4.1. Upper level: Electricity-aware bid selection
The proposed electricity-aware bid selection problem in the upper-level problem seeks to minimize the operating cost of the overall heat system for each hour of the following day \( t \in T \), while anticipating the impact of its decisions on the heat market-clearing problem in the middle level and the electricity market-clearing problem in the lower level. The set of decision variables \( \Omega_{EA} \) of this problem includes the commitment state \( u_{jt}^0 \), validity \( u_{jbt}^{\text{bid}} \) of all heat bids \( b \in B^H \), start-up \( u_{jt}^{\uparrow} \)
and shut-down $v^\uparrow_{jt}$ states, and start-up costs $r_{jt}$ of each heat producer $j \in \mathcal{I}^H$, flow direction $\overrightarrow{u}_{nmt}$ and time delay $\Phi_{nmt}$ between market zones $n \in \mathcal{Z}^H, m \in \mathcal{Z}_n^H$ of the district heating network. The upper-level problem receives feedback on the outcomes of the heat and electricity market-clearing problems, including the heat dispatch $Q_{jbt}^H$ of all bids and the electricity prices $\lambda_{st}^E$ in each market zone $z \in \mathcal{Z}^E$ that are needed to ensure the validity of all bids. Therefore, the electricity-aware bid selection problem writes as

$$\min_{\Omega_{\text{EA}j}(Q_{jbt}, \lambda_{st}^E)} \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{I}^H} \left( c^0_{jt} u^0_{jt} + r_{jt} + \sum_{k=1}^{g_{jt}} c^H_{jbt} Q_{jbt} \right) \quad (12a)$$

Subject to

$$r_{jt} \geq c^\downarrow_{jt} \left( u^0_{jt} - \sum_{k=t-h}^{t} u^0_{j,k} \right), \forall j \in \mathcal{I}^H, t \in \mathcal{T} \setminus \Phi_{j}^{u,\text{init}}, h \in \Phi_{j^t} \quad (12b)$$

$$r_{jt} \geq 0, \forall j \in \mathcal{I}^H, t \in \mathcal{T} \quad (12c)$$

$$u^0_{jt} = u^{\text{init}}_{jt}, \forall j \in \mathcal{I}^H, t \in \Phi_{j}^{u,\text{init}} \quad (12d)$$

$$\begin{array}{l}
\sum_{k=t-\Phi_{jt}^\downarrow}^{t} v^\downarrow_{j,k} \leq u^0_{jt}, \forall j \in \mathcal{I}^H, t \in \mathcal{T} \setminus \Phi_{j}^{u,\text{init}} \\
\sum_{k=t-\Phi_{jt}^\uparrow+1}^{t-\Phi_{jt}^\uparrow} u^0_{jt} = u^{0}_{j(t-\Phi_{jt}^\uparrow+1)}, \forall j \in \mathcal{I}^H, t \in \mathcal{T} \setminus \Phi_{j}^{u,\text{init}} \\
v^\uparrow_{jt} - v^\downarrow_{jt} = u^0_{jt} - u^{0}_{j(t-1)}, \forall j \in \mathcal{I}^H, t \in \mathcal{T} \setminus \Phi_{j}^{u,\text{init}} \\
u^{\text{bid}}_{jt} \leq u^{\text{bid}}_{jt}, \forall j \in \mathcal{I}^H, t \in \mathcal{T}, b \in \mathcal{B}^H, \tilde{b} \in \mathcal{B}^H, \text{ s.t. } b < \tilde{b} \quad (12g)$$

$$\Phi_{nmt} = \sum_{k=0}^{F} k u^\Phi_{nmtk}, \forall n \in \mathcal{Z}^H, m \in \mathcal{Z}_n^H, t \in \mathcal{T} \quad (12i)$$

$$\sum_{k=0}^{F} u^\Phi_{nmtk} = 1, \forall n \in \mathcal{Z}^H, m \in \mathcal{Z}_n^H, t \in \mathcal{T} \quad (12j)$$

$$\overrightarrow{u}_{nmt} + \overrightarrow{u}_{mtn} = 1, \forall n \in \mathcal{Z}^H, m \in \mathcal{Z}_n^H, t \in \mathcal{T} \quad (12k)$$

$$\overrightarrow{v}_{jt}, v^\uparrow_{jt}, u^0_{jt}, u^{\text{bid}}_{jbt} \in \{0, 1\}, \forall j \in \mathcal{I}^H, b \in \mathcal{B}^H, t \in \mathcal{T} \quad (12l)$$

$$\overrightarrow{u}_{nmt}, u^\Phi_{nmtk} \in \{0, 1\}, \forall n \in \mathcal{Z}^H, m \in \mathcal{Z}_n^H, k \in \{0, ..., \Phi\}, t \in \mathcal{T} \quad (12m)$$

Bid-validity conditions (11) \quad (12n)$$

The upper-level objective function (12a) minimizes the heat system operating cost, which includes start-up costs $r_{jt}$, no-load costs $c^0_{jt}$, and the cost of dispatching heat bids, i.e., $c^H_{jbt}$. Constraints (12b) and (12c) model the start-up cost depending on the time the generation units have been offline. The expression $\left( u^0_{jt} - \sum_{k=t-h}^{t} u^0_{j,k} \right)$ in (12b) is equal to 1 when unit $j$ becomes online after it has been turned off for $h$ time periods. Constraint (12d) fixes the initial minimum up- and down-time of the
generation units. Constraints (12e) and (12f) enforce the minimum up- and down-time, respectively. In particular, (12e) states that a generation unit has to remain on if it has been turned on less than $\Phi_j^\uparrow$ time steps ago, i.e., if $v_{jk}^\uparrow = 1$ for $k > t - \Phi_j^\uparrow$. Similarly, (12f) states that a generation unit cannot be turned on, i.e., $v_{jk}^\uparrow = 0$ for $k > t - \Phi_j^\downarrow$, if it has been turned on less than $\Phi_j^\downarrow$ time steps ago. Constraint (12g) states the relationship between the binary variables for the on/off, start-up, and shut-down statuses of each unit. Constraint (12h) ensures that a bid $\tilde{b}$ can be selected only if the previous bids $b < \tilde{b}$ have been selected. Constraints (12i) and (12j) define the discrete time delays $\Phi_{nmt}$ between the market zones of the heat network, such that $\Phi_{nmt} = k$ if and only if the binary variable $u_{nmtk} = 1$. Constraint (12k) states the relationship between the direction of the heat flows between two connected market zones of the district heating network, where $\overrightarrow{u}_{nmt} = 1$ if and only if the flow entering the pipeline at time step $t$ is directed from market zone $z$ to market zone $\tilde{z}$. Furthermore, (12n) represents the bid-validity conditions for all heat market participants whose marginal heat production costs are expressed as convex piece-wise linear functions of day-ahead electricity prices, as linear constraints. This ensures that a bid can be selected only if its price is greater than or equal to the marginal heat production cost of the corresponding unit. In particular, the expression of these piece-wise linear marginal heat production costs for CHPs and HPs is provided in the online appendix. Finally, (12o) represents the feedback from the primal and dual solutions of the middle- and lower-level problems.

4.2. Middle level: heat markets clearing

For given values of the selected bids, heat flow directions and heat time delays determined in the upper-level problem, the middle-level problem clears the day-ahead heat markets across all isolated heat networks. As the heat networks are isolated, the heat market-clearing problems in the respective market zones are independent. Therefore, multiple independent market-clearing problems can be modeled as a single optimization problem in the middle-level without the loss of generality. The set of middle-level variables $\Omega^H$ includes the dispatch of all submitted bids $Q_{jbt}$, as well as inlet $f_{in}^\text{out}$ and outlet $f_{out}^\text{out}$ heat flows between market zones of the district heating network. In addition, the heat market computes the adjusted minimum and maximum electricity outputs $P_{jH}$, and the electricity consumption $L_{HP}$ of HPs. The middle-level problem is

$$\min_{\Omega^H, \{\lambda^E_{zt}\}} \sum_{t \in T} \sum_{j \in I_H} \sum_{b \in B_H} c_{jbt} Q_{jbt}$$

s.t.

$$\sum_{j \in I_H} Q_{jt} + \sum_{z \in Z_H} (f_{out}^\text{out} - f_{in}^\text{in}) = L_{zt}^H, \forall z \in Z_H, t \in T$$

$$Q_{jt} = \sum_{b \in B_H} Q_{jbt}, \forall j \in I_H, t \in T$$

$$s_j^H u_{0j}^t \leq Q_{jt}, \forall j \in I_H, t \in T$$

(13a) (13b) (13c) (13d)
\[
\sum_{j \in J} u_{jbt} \leq Q_{jbt} \leq \sum_{j \in J} u_{jbt}, \forall j \in J^H, t \in T, b \in B^H, \tilde{b} \in B^H, \text{ s.t. } b < \tilde{b}
\]

(13e)

\[
\sum_{k=0}^{K_{jtk}} f_{nmt}^{in} = f_{nmt}^{out} \forall n \in Z^H, m \in Z^H_n, t \in T
\]

(13f)

\[
0 \leq f_{nmt}^{in} \leq f_{nmt}^{out}, \forall n \in Z^H, m \in Z^H_n, t \in T
\]

(13g)

\[
\{\lambda_{zt}^E\} \in \text{dual solution of (14)}.
\]

(13i)

The middle-level objective function (13a) minimizes the operating cost of the heat system for given valid bids submitted by the heat market participants. Constraint (13b) enforces the heat balance in each market zone of the district heating network. Constraint (13c) defines the total heat production of each market participant as the sum of its bids dispatched. Constraint (13d) provides a lower bound on the heat production, for the commitment decision \(u_{0jt}\) fixed in the upper-level problem (12). Constraint (13e) enforces the heat production bounds for all heat bids \(b\), based on the validity \(u_{jbt}\) of the bids selected in the upper-level problem (12). This constraint imposes that if one of the following bids \(\tilde{b} > b\) is selected, bid \(b\) should fully be dispatched. Constraint (13f) defines the outlet heat flow in each pipeline as a function of the inlet heat flow and the discrete time delay variables \(u_{nmtk}^E\) fixed in the upper-level problem (12). Constraint (13g) restricts the heat flow entering each pipeline based on the direction of the flow \(\tilde{u}_{nmt}\) fixed in the upper-level problem (12). Furthermore, (13h) computes the electricity consumption of HPs, and the adjusted electricity production bounds of CHPs based on their commitment and heat dispatch. These constraints model the interdependencies between the heat and electricity outputs of CHPs and HPs, and ensure that their bids in the electricity market are feasible with respect to their heat dispatch and commitment. Note that the bilinear terms \(Q_{jbt}u_{0jt}\) in (13h) are a product of continuous and binary variables, and therefore, they can be exactly linearized using McCormick envelopes. Finally, the electricity market clearing is represented as the lower-level problem in (13i). Although the solutions of the lower-level problem do not appear in the objective function or constraints of the middle-level problem, they provide feedback on the electricity prices \(\lambda_{zt}^E\) to the upper-level problem (12).

4.3. Lower level: electricity market clearing

The lower-level problem represents the day-ahead electricity market clearing, for given values of the adjusted electricity bids of CHPs and HPs computed in the middle-level problem (13). The set of electricity market variables \(\Omega^E\) includes the dispatch \(P_{jbt}\) of all electricity bids \(\tilde{b} \in B^E\), and the electricity transfers \(f_{z\tilde{z}t}^E\) between market zones \(z \in Z^E\) and \(\tilde{z} \in Z_{\tilde{z}}^E\). The lower-level problem reads as

\[
\min_{\Omega^E} \sum_{t \in T} \sum_{j \in J^E} \sum_{b \in B^E} c_{jbt}^E P_{jbt}
\]

(14a)
The lower-level objective function (14a) minimizes the operating cost of the electricity system. Constraint (14b) imposes the power balance in each electricity market zone, for the inflexible electricity consumption of HPs $L_{jt}^{HP}$ fixed in the middle-level problem (13). Constraint (14c) sets the total electricity production of each market participant as the sum of its bids dispatched. Constraint (14d) provides bounds on the electricity production of each bid for CHPs and other electricity market participants. Constraints (14e) and (14f) set the minimum and maximum self-committed electricity production of CHPs and other electricity market participants, respectively. Constraint (14g) limits the power exchanged between the electricity market zones, whereas (14h) links the directed power exchanged between these market zones. Finally, the electricity zonal prices $\lambda_{zt}^E$ are defined as the dual variables of the balance equations (14b).

### 4.4. Reformulation as a single-level optimization problem

The electricity-aware bid selection problem introduced above can be formulated in a compact way as the following linear trilevel optimization problem:

$$
\begin{align*}
\min_{\substack{z \in \{0,1\}^N \cap \mathbb{R}^E, x^H \geq 0, y^E}} & \quad c^T z + c^H x^H \\
\text{s.t.} & \quad z \in Z^{UC} \\
& \quad A^{bid} z + B^{bid} y^E \geq b^{bid} \\
& \quad x^H, y^E \in \text{ solutions of } \min_{x^H, y^E \geq 0} c^H x^H \\
& \quad \text{s.t. } A^H x^H + B^H z \geq b^H \\
& \quad y^E \in \text{ dual sol. of } \min_{x^E \geq 0} c^E^T x^E \\
& \quad \text{s.t. } A^E x^E + B^E x^H \geq b^E,
\end{align*}
$$

where $z$ represents the vector of primal variables of the upper-level problem (12), $x^H$ represents the vector of primal variables of the middle-level problem (13), and $(x^E, y^E)$ represent the vectors...
of primal and dual variables of the lower-level problem (14). The expression of the feasible space \( \mathcal{UC} \), the vectors \((c^0, b^{bid}, c^H, b^H, c^E, b^E)\) and matrices \((A^{bid}, B^{bid}, A^H, B^H, A^E, B^E)\) of parameters can be derives from the detailed formulations of problems (12) to (14).

In this section, we explain how the proposed electricity-aware bid selection mechanism, formulated as trilevel optimization problem (15), can be reformulated as a single-level MILP. To this purpose, we need to reformulate the middle- and lower-level problems as a linear program. Notice, that the objective function (15d) and constraints (15e) of the middle-level problem do not depend on the lower-level primal and dual variables, \(x^E\) and \(y^E\). This implies that the solutions of the middle-level problem are not affected by the solutions of the lower-level problem. This allows solving bilevel problem (15d)-(15g) in two steps: (i) solve the middle-level problem and obtain the optimal solutions \(x^H\), (ii) solve the lower-level problem with \(x^H\) fixed to the values \(x^H^*\) and obtain the optimal solutions \(y^E^*\). We approximate the middle- and lower-level problems by a single-level linear program using a lexicographic function (Byeon and Van Hentenryck 2020). The electricity-aware bid selection problem can then be reformulated as a single-level MILP, exploiting the strong duality of such an approximation of the middle- and lower-level problems.

**Proposition 2.** The trilevel optimization problem (15) can be asymptotically approximated by the following single-level MILP:

\[
\begin{align*}
\min_{x \in \{0,1\}^N, z \geq 0, x^E \geq 0, y^H, y^E} & \quad \gamma c^0^T z + \gamma c^H^T x^H + (1 - \gamma) c^E^T x^E \\
\text{s.t.} & \\
& z \in \mathcal{UC} \\
& A^{bid} z + \frac{1}{(1 - \gamma)} B^{bid} y^E \geq b^{bid} \\
& A^H x^H + B^H z \geq b^H \\
& A^E x^E + B^E x^H \geq b^E \\
& y^H^T A^H + y^E^T B^E \leq \gamma c^H^T \\
& y^E^T A^E \leq (1 - \gamma) c^E^T \\
& y^H^T (b^H - B^H z) + y^E^T b^E \geq \gamma c^H^T x^H + (1 - \gamma) c^E^T x^E.
\end{align*}
\]

When the penalty factor \(\gamma\) tends to 1, the solutions of (16) converge to the solutions of (15).

The proof of Proposition 2 is provided in the online appendix. Note that the bilinear terms in (16h) can be linearized using an exact McCormick relaxation (McCormick 1976).

5. **Numerical analyses with two case studies**

This section illustrates the benefits of the proposed electricity-aware bid selection mechanism in terms of renewable energy penetration, cost-effectiveness, and profitability of CHPs and HPs,
through two case studies. The first case study focuses on quantifying the cost benefits achieved by the proposed electricity-aware mechanism compared to a fully integrated one. The second case study provides further insights on the operation of the proposed mechanism.

In both case studies, each mechanism studied is solved daily over the optimization horizon. In order to ensure a fair comparison between the electricity-aware and decoupled mechanisms, the hourly heat bids of CHPs and HPs are computed each day as their heat marginal costs using the day-ahead electricity price forecast, as discussed in Section 2. These forecast prices are derived by jointly clearing the heat and electricity markets. Finally, the penalty factor $\gamma$ in (16) is fixed to 0.99. A sensitivity analysis reveals that solutions are stable around this value, and therefore, are assumed to have converged.

Details on these case studies’ setup and all relevant data are provided in the online appendices (Mitridati et al. 2021a,b).

5.1. Case study 1: modified IEEE 24-node electricity system

This modified version of the IEEE 24-node electricity system consists of twelve thermal power plants, six wind farms, two extraction CHPs, and two HPs. Each node represents an electricity market zone. The heat system consists of two isolated district heating networks, each comprising one CHP, one waste incinerator heat-only (HO) unit, one heat-only peak boiler, and one large-scale HP each. This case study compares the proposed electricity-aware mechanism over 2 consecutive months to:

i) the sequential and decoupled unit commitment and market mechanism presented in Section 2. This mechanism represents a base-case against which we test the benefits of the proposed mechanism.

ii) the fully integrated unit commitment and market mechanism. By jointly and simultaneously optimizing the heat and electricity systems, this mechanism represents an ideal benchmark, which provides the basis to quantify the value of coordination. We use this metric to quantify the costs benefits of the proposed electricity-aware mechanism.

Results - Value of coordination: As summarized in Table 1, the integrated market mechanism reduces the overall system cost by 11.3% compared to the decoupled mechanism. The absolute

| Decoupled | Electricity-aware | Integrated |
|-----------|-------------------|------------|
| Overall system cost ($10^3\text{€}$) | 11.75 | 10.72 | 10.42 |
| Heat system cost ($10^3\text{€}$) | 2.66 | 2.96 | 10.23 |
| Electricity system cost ($10^3\text{€}$) | 9.09 | 7.76 | 0.19 |
| Value of coordination achieved (%) | 0 | 77.6 | 100 |

Table 1 Case study 1: Overall, heat and electricity system costs, in $10^3\text{€}$, and value of coordination achieved, in % of total value of coordination.
Table 2  Case study 2: Heat, electricity and overall energy systems commitment and dispatch costs, in 10^6 €, and renewable energy utilization in the electricity system, as a percentage of the available production.

|                        | Decoupled | Electricity-aware | Difference |
|------------------------|-----------|-------------------|------------|
| Overall system cost*   | 1,967     | 1,881             | −4.4%      |
| Heat system cost       | 1,360     | 1,287             | −5.4%      |
| Electricity system cost| 608       | 594               | −2.2%      |
| Renewable utilization  | 95.9      | 96.3              | +0.4%      |

cost difference, i.e., 1,330€, between the decoupled and integrated market mechanisms represents the value of coordination between heat and electricity markets (Mitridati and Taylor 2018). However, as previously discussed, this integrated mechanism is not a realistic alternative to a sequential heat and electricity market framework. Indeed, it can be observed that the heat system operating cost is drastically increased with the integrated mechanism compared to the decoupled and hierarchical mechanisms, which supports the findings by Mitridati and Pinson (2016), Mitridati et al. (2020). Meanwhile, the proposed electricity-aware mechanism achieves 77.6% of this so-called value of coordination, while maintaining the sequential and independent structure of the markets.

The proposed electricity-aware heat dispatch achieves these benefits by better anticipating the impact of CHPs and HPs on the electricity market, and exploiting the operational flexibility at the interface between heat and electricity systems, as will further be discussed in Case study 2.

5.2. Case study 2: Danish energy system

Case study setup: The Danish electricity system is divided into two market zones, DK1 and DK2, connected via an interconnection. Additionally, we consider three disconnected district heating networks, which geographically cover, respectively, the greater Copenhagen area, Aarhus area, and the multicity TVIS area (Fredericia, Middelfart, Kolding and Vejle). These district heating networks comprise 11 CHPs, 6 incinerators (IS), i.e., CHPs with fixed heat-electricity ratio, 6 HPs, 20 HO units and peak boilers, and 3 heat storage tanks (HS). This realistic case study provides additional insights on the differences in dispatch and commitment between the proposed electricity-aware mechanism and the existing decoupled one over 1 year.

Results - Dispatch and profits of heat and electricity generators: The simulation of the sequential operation of the heat and electricity systems over 1 year with both mechanisms shows that the proposed electricity-aware bid selection mechanism is able to efficiently anticipate the impact of the commitment decisions of CHPs and HPs on the electricity market. As summarized in Table 2, the proposed mechanism achieves lower heat, electricity and overall system costs compared to the decoupled approach. This is achieved by switching off certain CHPs during extended periods of low day-ahead electricity prices and when their bids are invalid. For instance, as illustrated in Figure 3(a) the bids of CHP6 are rejected by the electricity-aware bid selection mechanism because
they are invalid, and CHP5 is used to partially cover this heat production, despite offering more expensive bids. While, for a given hour, this rejected bid may be replaced with more expensive bids, over multiple days, this electricity-aware approach yields a more efficient and less expensive commitment and dispatch in the heat system.

Additionally, as the decoupled mechanism fails to anticipate the impact of day-ahead electricity prices on the heat production costs of CHPs and HPs, in the hours where invalid bids are selected, these units suffer large financial losses, as summarized in Table 3 for each unit. As illustrated in Figure 3(b) for CHP6, in the days where the electricity-aware bid selection mechanism rejects invalid bids, the decoupled unit commitment incurs large financial losses by selecting these invalid bids. These losses are more significant during the winter time due to the higher production level of CHP6 during these days.

6. Conclusion

This paper proposes a novel electricity-aware bid format and bid selection mechanism, which are modeled using a trilevel optimization problem. This mechanism improves the coordination between heat and electricity markets by allowing CHPs and HPs to offer electricity-aware heat bids
which are conditioned on day-ahead electricity prices, while respecting the current sequential heat and electricity market clearing procedure. These electricity-aware bids are modeled using linear bid-validity conditions. As a result, a tractable MILP reformulation for the proposed trilevel optimization problem is developed. Finally, the value of improving the coordination between heat and electricity systems is illustrated in two case studies. The first illustrative case study shows that the proposed electricity-aware mechanism can achieve 77.6% of the value of coordination achieved by the fully integrated mechanism, while maintaining a sequential heat and electricity market framework. The second case study, based on the realistic electricity and heat systems in Denmark, shows that the proposed mechanism is able to ensure cost recovery for CHPs and HPs in the heat market, while reducing the operating cost of the overall energy system by 4.5% and wind curtailment by 0.4% compared to a decoupled mechanism. These benefits are achieved by anticipating the impact of CHPs and HPs on electricity markets. This work allows us to harness and remunerate the flexibility of CHPs and HPs at the interface between heat and electricity systems, and to achieve an efficient and cost-effective operation of the overall energy system.

This study opens up various opportunities for future work. Firstly, the proposed bid selection mechanism coordinates the participation of CHPs and HPs across multiple isolated heat networks...
and market zones in a centralized way. This requires a central heat system operator to collect information from independent heat market operators, which may be challenging in practice. As a potential alternative, decomposition techniques relying on consensus-based distributed algorithms could be investigated to facilitate the application of the proposed approach in the current energy system. Recent advances in the literature have introduced performance guarantees on the application of such algorithms to large-scale MILPs (Vujanic et al. 2016, Dvorkin et al. 2018, Falsone et al. 2019). Furthermore, this work does not take into account additional energy products, such as natural gas, and how their day-ahead prices may impact the validity of heat bids. As the day-ahead gas market is cleared after the heat and electricity markets, this limitation may be accounted for by developing an electricity- and gas-aware heat bid selection mechanism, modeled as trilevel optimization problem (Byeon and Van Hentenryck 2020).

Secondly, as previously discussed, the proposed bid selection mechanism does not guarantee efficiency and incentive compatibility, due to the unchanged design of the sequential heat and electricity markets. In order to quantify the loss of efficiency in heat and electricity markets resulting from the exercise of market power, further agent-based analysis should be conducted. Providing a rigorous analysis of potential strategic behaviors, exercise of market power, and opportunity costs across both heat and electricity markets would provide useful insights for further market designs. Furthermore, market power may be reduced by designing new market-clearing mechanisms for the day-ahead heat and electricity markets (Koçyiğit et al. 2020, Karaca and Kamgarpour 2019), which would require major regulatory and organizational changes.

Thirdly, the proposed model assumes perfect information on the bids of the market participants and wind power availability in the electricity market clearing. However, such information may not be communicated by the independent market operator, due to privacy concerns. Therefore, imperfect information on the parameters of the lower-level problem may be assumed using a scenario-based stochastic programming framework, or a robust counterpart of the middle- and lower-level problems. In addition, in order to mitigate the inefficiencies related to the lack of information exchange, a privacy-preserving extension of the proposed model can be developed, based on the Privacy-Preserving Stackelberg Mechanism (PPSM) introduced in Fioretto and Van Hentenryck (2018). The PPSM allows the follower in a Stackelberg game, e.g., the electricity market operator, to share differentially-private information, e.g. bids, with the leader, e.g. the heat system operator, while ensuring near-optimal coordination of the sectors.

Finally, while this work focuses on the coordination of different energy markets in the day-ahead stage, accounting for uncertainty of energy delivery in real time is essential to ensure reliability of the overall energy system. Following the work on policy-based reserves proposed by Warrington et al. (2013) and Ratha et al. (2020), the proposed unit commitment and bid selection mechanism could be extended to co-optimize energy and operating reserves.
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Author Biographies

Lesia Mitridati obtained her Ph.D. in Electrical Engineering from the Technical University of Denmark (DTU), and is currently a postdoctoral researcher in the H. Milton Steward School of Industrial and Systems Engineering at Georgia Tech. Her research goal is to efficiently address the emerging challenges of the energy transition by bridging the gap between the electricity systems community and other disciplines, such as optimization, machine learning, and game theory.

Pascal Van Hentenryck is the A. Russell Chandler III Chair and Professor, and the associate chair of innovation and entrepreneurship in the H. Milton Steward School of Industrial and Systems Engineering at Georgia Tech. His research focuses on artificial intelligence and operations research for engineering applications: He explores methodologies that includes large-scale optimization and machine learning and applies them in challenging applications in energy, mobility, privacy, and resilience.

Jalal Kazempour is an Associate Professor at the Energy Analytics & Markets (ELMA) group at the Technical University of Denmark (DTU). He received the Ph.D. degree in electrical engineering from the University of Castilla-La Mancha, Ciudad Real, Spain, in 2013. He was a postdoctoral fellow with the Johns Hopkins University, Baltimore, MD, USA, in 2014, and DTU from 2015 to 2016. He is interested in applications of advanced data-driven optimization, game-theoretic, and learning techniques to answer economic questions about power and energy systems.

The work originated when the energy systems groups from DTU and Georgia Tech met at the wonderful summer school in electricity systems organized by DTU. The two teams realized that they were working on sequential energy markets of different types. The present paper represents the cross-fertilization of ideas from electricity and gas networks in the United States and district heating networks in Denmark studied in the demonstration project "EnergyLab Nordhavn" in Copenhagen, Denmark.

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Appendix A: Computation of electricity-aware heat bids for CHPs and HPs

Let us first consider a heat market participants whose marginal heat production cost can be expressed as affine functions of the day-ahead electricity prices, i.e., such that
\[ \hat{\lambda}_{jt}^H = a_{jt} \lambda_{zt}^E + b_{jt}, \forall j \in T_z, t \in T, \]  
with \( a_{jt} \in \mathbb{R}, b_{jt} \in \mathbb{R} \) the fixed affine parameters. As a result, the cost-recovery conditions (10) for each electricity-aware heat bid \( b \) with price \( c^{H}_{jkt} \) rewrite as
\[ (c^{H}_{jkt} - M_j) u^{bid}_{jkt} \geq a_{jt} \lambda_{zt}^E + b_{jt} - M_j, \forall j \in T_z, t \in T, b \in B^H. \]  

Furthermore, we assume that in each electricity market zone \( z \in \mathcal{Z}^E \), the set of day-ahead electricity prices is bounded, i.e., \( \lambda_{zt}^E \leq \Lambda_{zt}^E \leq \bar{\lambda}_{zt}^E \). This assumption is without loss of generality, as the bidding prices in electricity market are typically bounded, and bounds on day-ahead electricity prices can be derived from these bounds and the value of lost loads. Therefore, the bounds on electricity prices for each electricity-aware heat bid \( b \), is computed as
\[ \lambda_{zt}^E = \begin{cases} \frac{c^{H}_{jkt} - b_{jt}}{a_{jt}}, & \text{if } a_{jt} < 0, \forall j \in T_z^H, t \in T, b \in B^H, \\ \lambda_{zt}^E, & \text{else} \end{cases} \]  
(19a)

\[ \bar{\lambda}_{zt}^E = \begin{cases} \frac{c^{H}_{jkt} - b_{jt}}{a_{jt}}, & \text{if } a_{jt} > 0, \forall j \in T_z^H, t \in T, b \in B^H, \\ \bar{\lambda}_{zt}^E, & \text{else} \end{cases} \]  
(19b)

This general expression of the electricity price bounds in electricity-aware bids can directly be applied to HPs, using the expression of their convex piece-wise linear marginal heat production costs provided in (5).

Let us now generalize to heat market participants whose marginal heat production cost can be expressed as a convex piece-wise linear functions of the day-ahead electricity prices, i.e., such that
\[ \hat{\Gamma}_{jt}^H = \max_{k=1,...,K} a_{jkt} \{ \lambda_{zkt}^E + b_{jkt} \}, \forall j \in T_z^H, t \in T, \]  
(20)

with \( k \in \{1,...,K\} \) the number of affine pieces, and \( a_{jkt}, b_{jkt} \in \mathbb{R} \) their fixed affine parameters. In this expression, \( \Gamma_{jt}^H \) is defined as the (convex) upper envelope of the lines forming the affine pieces. As a result, the cost-recovery conditions (10) for each electricity-aware heat bid \( b \) with price \( c^{H}_{jkt} \) rewrite as
\[ (c^{H}_{jkt} - M_j) u^{bid}_{jkt} \geq a_{jkt} \lambda_{zkt}^E + b_{jkt} - M_j, \forall j \in T_z^H, k \in \{1,...,K\}, t \in T, b \in B^H. \]  
(21)

Therefore, the bounds on electricity prices for each electricity-aware heat bid \( b \), is computed as
\[ \lambda_{zkt}^E = \begin{cases} \max \left\{ \frac{c^{H}_{jkt} - b_{jkt}}{a_{jkt}}, \forall k \in \{1,...,K\} \mid a_{jkt} < 0 \right\}, & \text{if } \exists a_{jkt} < 0, \forall j \in T_z^H, t \in T, b \in B^H, \\ \lambda_{zt}^E, & \text{else} \end{cases} \]  
(22a)

\[ \bar{\lambda}_{zkt}^E = \begin{cases} \min \left\{ \frac{c^{H}_{jkt} - b_{jkt}}{a_{jkt}}, \forall k \in \{1,...,K\} \mid a_{jkt} > 0 \right\}, & \text{if } \exists a_{jkt} > 0, \forall j \in T_z^H, t \in T, b \in B^H, \\ \bar{\lambda}_{zt}^E, & \text{else} \end{cases} \]  
(22b)

This general expression of the electricity price bounds in electricity-aware bids can directly be applied to CHPs, using the expression of their convex piece-wise linear marginal heat production costs provided in (6).
Appendix B: Proofs of propositions

B.1. Proof of Proposition 1

For a given value of the upper-level variables \( z^* \), the equilibrium formulation of the sequential heat and electricity market-clearing problems can be expressed in a compact form as

\[
\begin{align*}
\mathbf{x}^H & \in \text{primal solution of } \min_{x^H \geq 0} c^H \mathbf{x}^H \quad \text{(23a)} \\
\text{s.t. } & A^H \mathbf{x}^H + B^H z^* \geq b^H \quad \text{(23b)} \\
\mathbf{y}^E & \in \text{dual solution of } \min_{x^E \geq 0} c^E \mathbf{x}^E \quad \text{(23c)} \\
\text{s.t. } & A^E \mathbf{x}^E + B^E \mathbf{x}^H \geq b^E \quad \text{(23d)}
\end{align*}
\]

where (23a)-(23b) represent the heat market-clearing problem, and (23c)-(23d) represent the electricity market-clearing problem. Therefore \( \{x^H^*, y^E^*\} \) are optimal solutions to (23) if and only if \( x^H^* \) is an optimal solution to the heat market-clearing problem (23a)-(23b), and \( y^E^* \) is an optimal solution to the electricity market-clearing problem (23c)-(23d) with the variables \( x^H \) fixed to the values \( x^H^* \).

Similarly, for a given value of the upper-level variables \( z^* \), \( \{x^H^*, y^E^*\} \) are solutions to the proposed bilevel formulation of the sequential heat and electricity market-clearing problems (15d)-(15g), if and only if \( x^H^* \) is an optimal solution to the heat market-clearing problem (15d)-(15e), and \( y^E^* \) is an optimal solution to the electricity market-clearing problem (15f)-(15g) with the variables \( x^H \) fixed to the values \( x^H^* \).

Therefore, any solutions to the equilibrium problem (23) is equivalent to the bilevel formulation (15d)-(15g), and these two formulations are equivalent.

B.2. Proof of Proposition 2

Firstly, by strong duality of the linear lower-level problem (15f)-(15g), the middle- and lower-level problems (15d)-(15g) are equivalent to

\[
\begin{align*}
\mathbf{x}^H, \mathbf{y}^E \in & \text{arg min } x^E \geq 0 c^H \mathbf{x}^H \quad \text{(24a)} \\
\text{s.t. } & A^H \mathbf{x}^H + B^H z^* \geq b^H \quad \text{(24b)} \\
\mathbf{y}^E \in & \text{arg min } x^E \geq 0 c^E \mathbf{x}^E \quad \text{(24c)} \\
\text{s.t. } & A^E \mathbf{x}^E + B^E \mathbf{x}^H \geq b^E \quad \text{(24d)} \\
& y^E^T A^E \leq c^E \quad \text{(24e)} \\
& y^E^T (B^E - B^E \mathbf{x}^H) \geq c^E \mathbf{x}^E \quad \text{(24f)}
\end{align*}
\]

Constraint (24e) represents the dual constraints of the lower-level problem (15f)-(15g), i.e., dual feasibility. Constraint (24f) enforces equality of the primal and dual objective values of the lower-level problem (15f)-(15g) at optimality (strong duality).

Furthermore, objective function (24a) and constraints (24b) of the middle-level problem do not depend on the lower-level variables \( x^E \) and \( y^E \). Therefore, the solutions of middle-level optimization problem are not affected by the solutions of the lower-level problem. Problem (24) can thus be solved in two steps: (i) solve the heat market-clearing problem (24a)-(24b) and obtain the optimal solutions \( x^H^* \), (ii) solve the electricity
market-clearing problem (24c)-(24f) with the variable $x^H$ fixed to $x^H^*$ and obtain the optimal solutions $y^E^*$. Therefore, the middle-level problem (24) can be reformulated using a lexicographic function as

\[
\begin{align*}
    \mathbf{x}^H, \mathbf{y}^E & \in \arg \min_{x^H, x^E, y^E \geq 0} < c^H \mathbf{x}^H, c^E \mathbf{x}^E > \\
    \text{s.t.} \quad & A^H \mathbf{x}^H + B^H z \geq b^H \\
    & A^E \mathbf{x}^E + B^E \mathbf{x}^H \geq b^E \\
    & y^E^T A^E \leq c^E \\
    & y^E^T (B^E - B^E \mathbf{x}^H^*) \geq c^E^T \mathbf{x}^E.
\end{align*}
\]

(25a)

Any optimal solution $x^H^*, x^E^*, y^E^*$ of problem (25) satisfies the following properties:

\[
\begin{align*}
    x^H^* & \in \arg \min_{x^H} c^H^T \mathbf{x}^H \\
    \text{s.t.} \quad & A^H \mathbf{x}^H + B^H z \geq b^H ,
\end{align*}
\]

(26a)

and

\[
\begin{align*}
    x^E^*, y^E^* & \in \arg \min_{x^E, y^E \geq 0} c^E^T \mathbf{x}^E \\
    \text{s.t.} \quad & A^E \mathbf{x}^E + B^E \mathbf{x}^H^* \geq b^E \\
    & y^E^T A^E \leq c^E \\
    & y^E^T (B^E - B^E \mathbf{x}^H^*) \geq c^E^T \mathbf{x}^E.
\end{align*}
\]

(27a)

In addition, by strong duality of problem (27), any feasible solution $\hat{x}^E, \hat{y}^E$ is optimal. Therefore, $\hat{x}^E$ is an optimal solution of the primal formulation of the lower-level problem (15f)-(15g), such that

\[
\begin{align*}
    \hat{x}^E & \in \arg \min_{x^E \geq 0} c^E^T \mathbf{x}^E \\
    \text{s.t.} \quad & A^E \mathbf{x}^E + B^E \mathbf{x}^H^* \geq b^E ,
\end{align*}
\]

(28a)

and $\hat{y}^E$ is an optimal solution of the dual formulation of the lower-level problem (15f)-(15g), such that

\[
\begin{align*}
    \hat{y}^E & \in \arg \max_{y^E \geq 0} y^E^T (B^E - B^E \mathbf{x}^H^*) \\
    \text{s.t.} \quad & y^E^T A^E \leq c^E 
\end{align*}
\]

(29a)

As problem (28) is a relaxation of problem (27) and the set of solutions $\hat{x}^E, \hat{y}^E$ is feasible to problem (27), then problem (25) can be approximated by the following linear program, with $\gamma \in [0,1]$:

\[
\begin{align*}
    \min_{x^H, x^E \geq 0} \gamma c^H^T \mathbf{x}^H + (1 - \gamma) c^E^T \mathbf{x}^E \quad & (30a) \\
    \text{s.t.} \quad & A^H \mathbf{x}^H + B^H z \geq b^H \\
    & A^E \mathbf{x}^E + B^E \mathbf{x}^H \geq b^E 
\end{align*}
\]

(30b)

(30c)
where $y^E$ is obtained as the dual variable associated with constraint (30c) (Byeon and Van Hentenryck 2020). As a result, problem (15) can be approximated by the following linear bilevel optimization problem:

$$\begin{align*}
\min_{x \in \{0,1\}^N, z \geq 0} & \quad \gamma c^{0^T} z + \gamma c^{H^T} x^H + (1 - \gamma) c^{E^T} x^E \\
\text{s.t.} & \quad z \in \mathbb{Z}^{UC} \\
& \quad A^{\text{bid}} z + B^{\text{bid}} y^E \geq b^{\text{bid}} \\
& \quad A^H x^H + B^H z \geq b^H \\
& \quad A^E x^E + B^E x^H \geq b^E \\
& \quad g^{H^T} A^H + \frac{1 - \gamma}{\gamma} g^{E^T} B^E \leq c^H \\
& \quad g^{E^T} A^E \leq c^E \\
& \quad g^{H^T} (b^H - B^H z) - c^H x^H \geq \frac{1 - \gamma}{\gamma} \left( c^{E^T} x^E - g^{E^T} (b^E - B^E x^H) \right).
\end{align*}$$

(32a) (32b) (32c) (32d) (32e) (32f) (32g) (32h)

Besides, by strong duality of the lower-level problem (31d), problem (31) is equivalent to problem (16).

It remains to show that problem (16) is an asymptotic approximation to problem (15), i.e., as $\gamma \to 1$ the solutions to problem (16) become optimal solutions to problem (15). By introducing the auxiliary variables $\tilde{y}^H = \frac{y^H}{\gamma}$, and $\tilde{y}^E = \frac{y^E}{1 - \gamma}$, problem (16) is equivalent to

$$\begin{align*}
\min_{x \in \{0,1\}^N, z \geq 0} & \quad \gamma c^{0^T} z + \gamma c^{H^T} x^H + (1 - \gamma) c^{E^T} x^E \\
\text{s.t.} & \quad z \in \mathbb{Z}^{UC} \\
& \quad A^{\text{bid}} z + B^{\text{bid}} \tilde{y}^E \geq b^{\text{bid}} \\
& \quad A^H x^H + B^H z \geq b^H \\
& \quad A^E x^E + B^E x^H \geq b^E \\
& \quad \tilde{y}^{H^T} A^H + \frac{1 - \gamma}{\gamma} \tilde{y}^{E^T} B^E \leq c^H \\
& \quad \tilde{y}^{E^T} A^E \leq c^E \\
& \quad \tilde{y}^{H^T} (b^H - B^H z) - c^H x^H \geq \frac{1 - \gamma}{\gamma} \left( c^{E^T} x^E - \tilde{y}^{E^T} (b^E - B^E x^H) \right).
\end{align*}$$

(33a) (33b)

Let us denote $P(z^*)$ and $\tilde{P}(z^*)$ the optimal objective value of problems (25) and (32), respectively, with the value of the unit commitment variable $z$ fixed to $z^*$. Let $\{x^{H^*}, x^{E^*}, y^{H^*}, y^{E^*}\}$ be the optimal solutions to $\tilde{P}(z^*)$. As $\gamma \to 1$, (32f) and (32h) become

$$\tilde{y}^{H^T} A^H \leq c^H \quad \quad \tilde{y}^{H^T} (b^H - B^H z) - c^H x^H \geq \frac{1 - \gamma}{\gamma} \left( c^{E^T} x^E - \tilde{y}^{E^T} (b^E - B^E x^{H^*}) \right).$$

(34)
It follows from (32d) that, for any gamma $\gamma \in [0, 1]$

$$\tilde{y}^E \top (b^E - B^E x^H) \geq c^E \top x^E.$$  \hspace{1cm} (35)

Constraints (32e) and (32g) guarantee that $x^E^*$ and $y^E^*$ are feasible to problem (27) with $x^H$ fixed to $x^H^*$. Additionally, (35) guarantees that $x^E^*$ and $y^E^*$, together, satisfy the strong duality equation of problem (27) with $x^H$ fixed to $x^H^*$. Therefore, $x^E^*$ and $y^E^*$ are the primal and dual optimal solutions to problem (27) with $x^H$ fixed to $x^H^*$ for any $\gamma \in [0, 1]$.

In summary, $x^H^*$ is a feasible solution to $P(z^*)$, which converges towards the optimal solution when $\gamma \to 1$, and $y^E^*$ is the optimal solution of the lower-level problem with respect to $x^H^*$ for any $\gamma \in [0, 1]$. Hence, problem (16) always provides a feasible solution to problem (12), which converges towards the optimal solution when $\gamma \to 1$.

**Appendix C: Case studies setup**

**C.1. Case study 1: modified IEEE 24-node electricity system**

In the associated paper, a 24-bus power system and two isolated 3-node district heating systems, respectively connected to nodes $(n_8, n_9, n_{10})$ and $(n_{15}, n_{18}, n_{19})$ of the power system are considered, as illustrated in Figure 4.

A modified version of the 24-bus IEEE Reliability Test System composes the integrated energy system. It consists of 12 thermal power plants, 6 wind farms, 2 extraction CHPs, and 2 HPs. Data for power generation, costs, loads and transmission for the 24-bus IEEE Reliability Test System is derived from Ordoudis et al. (2016). Additionally, 6 wind farms of 250MW each are installed at buses $n_3, n_5, n_7, n_{16}, n_{21}, n_{23}$. Additionally, the transmission capacity of the lines connecting the node pairs $(n_{15}, n_{21}), (n_{14}, n_{16})$ and $(n_{13}, n_{23})$ is reduced to 400MW, 250MW and 250MW, respectively, in order to introduce congestion in the transmission network.

Additionally, two isolated 3-node DHNs connected to the power system are considered. Each DHN comprises one CHP, one waste incinerator heat-only unit, one heat-only peak boiler, and 1 large-scale HP. The techno-economic characteristics of these units and DHNs are derived from Zugno et al. (2016), Li et al. (2016), Mitridati and Taylor (2018) and representative of the greater Copenhagen area. Heat generation data is summarized in Table 4. District heating transmission networks data can be derived from Mitridati and Taylor (2018).

The heat loads in both networks, and electricity loads are derived from Madsen (2015) and Tosatto and Chatzivasileiadis (2020), Energinet.dk (2020) for two consecutive months and representative of heat load profiles in the greater Copenhagen area. Additionally, spatially and temporally correlated profiles of wind power generation for two consecutive months at six locations are derived from Tosatto and Chatzivasileiadis (2020), Energinet.dk (2020). These time series and all relevant case study data are available in the online appendix (Mitridati et al. 2021a).
Figure 4  Case study 1: Modified IEEE 24-node electricity system with 6 wind farms (the bottom system) connected to two isolated 3-node district heating systems (the two systems on the top of the figure)

Table 4  Heat generation data in isolated DHNs 1 and 2.

| Type | CHP   | HO₁  | HO₂  | HP₁  | CHP   | HO₃  | HO₄  | HP₂  |
|------|-------|------|------|------|-------|------|------|------|
|  CHP |  2.4  |  -   |  -   |  -   |  2.4  |  -   |  -   |  -   |
|  HO₁ |  0.25 |  -   |  -   |  -   |  0.25 |  -   |  -   |  -   |
|  HO₂ |  0.6  |  -   |  -   |  0.6 |  -    |  -   |  -   |  -   |
|  HP₁ |  100  |  100 |  100 |  10  |  200  |  200 |  200 |  20  |
|  CHP |  0    |  0   |  0   |  0   |  0    |  0   |  0   |  0   |
|  HO₂ |  0    |  0   |  0   |  0   |  0    |  0   |  0   |  0   |
|  HP₂ |  25   |  50  |  50  |  50  |  50   |  50  |  50  |  50  |
|  COP |  -    |  -   |  2.5 |  -   |  -    |  -   |  -   |  2.5 |
|  Φ↑  |  2    |  2   |  1   |  1   |  2    |  2   |  1   |  1   |
|  Φ↓  |  2    |  2   |  1   |  1   |  2    |  2   |  1   |  1   |
|  |  2    |  2   |  1   |  1   |  2    |  2   |  1   |  1   |
|  c   |  10.5 |  13.5|  30  |  0   |  10.5 |  13.5|  30  |  0   |
|  c₂  |  100  |  100 |  25  |  -   |  100  |  100 |  25  |  -   |
|  c₃  |  5    |  5   |  2   |  0   |  5    |  5   |  2   |  0   |
|  c₆  |  100  |  250 |  100 |  100 |  100  |  250 |  100 |  100 |

C.2. Case study 2: Danish energy system

As illustrated in Figure 5, the Danish electricity system is divided into two market zones, *DK1* and *DK2*, connected via an interconnection. Generation costs and parameters, ATCs, as well as electricity loads, wind
and solar power generation for one year are available on the website of Danish electricity system operator, Energinet.dk (Energinet.dk 2020) as well as Tosatto and Chatzivasileiadis (2020).

Additionally, we consider three disconnected district heating networks, which geographically cover, respectively, the greater Copenhagen area, Aarhus area, and the multicity TVIS area (Fredericia, Middelfart, Kolding and Veje). These district heating networks comprise 11 CHPs, 6 incinerators (IS), i.e., CHPs with fixed heat-electricity ratio, 6 HPs, 20 HO units and peak boilers, and 3 heat storage tanks (HS). Heat load profiles for 1 year, generation costs, and technical parameters for these district heating networks are derived from Madsen (2015), Mitridati and Taylor (2018) and the website of Danish electricity system operator, Energinet.dk (Energinet.dk 2020). These time series and all relevant case study data are available in the online appendix (Mitridati et al. 2021b).