The Application of Bayesian Technique for Particle Identification

Ding Tian

Teaching and Research Section of Physics, School of Materials Science and Technology, China University of Geosciences (Beijing), Beijing 100083, P.R. China

The PID problem in high energy physics experiments is analysed with Bayesian technique. The corresponding applicable method is presented.

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I. INTRODUCTION

Particle identification (PID) is important in high energy physics experiments, and it mainly refers to charged particles. Different techniques are used to study this problem. In this paper, the problem is analysed with Bayes’ theorem of probability theory. It is well known that the best classification methods are based on Bayesian techniques if all the probability distributions are known. However, from a literature survey, it appears that how to use Bayesian technique in PID problem has not yet been thoroughly investigated.

Different detectors use different variables to do PID, such as TOF $t$ from TOF detector, $dE/dx$ from wire chamber, deposited energy $E$ from shower counter, Cherenkov radiation emission angle $\theta$ from RICH counter, the deposited energy $W$ or transition radiation (TR) photon hits $N$ from TR detector, etc. For different particles with same momentum, the random variables $(t, dE/dx, E, \theta, W, N \text{ etc})$ may have different distributions which can be used for PID, therefore in this paper we call the random variables PID variables. Sometimes more than one PID variables which have different character can be obtained from one detector, such as in shower counter, both the deposited energy $E$ of a shower and one or two variables which describe the shape of the shower can be used for electron/hadron separation.

For an unknown charged particle, its momentum is usually known (e.g., given by drift chamber). Therefore, all the calculations of probabilities in this paper are under the condition that the particle’s momentum vector is known and indicated with $p$, $\theta$ and $\phi$ which are magnitude, polar and azimuthal angles of the particle’s momentum vector respectively.

The paper is organized as follows: In section two, the PID problem is analysed with Bayesian technique when there is only one PID variable (use TOF $t$ as example) is obtained for an unknown charged particle. In section three, similar analysis is done when two and more PID variables (use TOF $t$ and the deposited energy $E$ in shower counter as example) are available. Section four is the conclusions.

*Electronic address: tianding@cugb.edu.cn

1 In this paper, random variable and its value are denoted with same symbol.
II. CASE FOR ONE PID VARIABLE

For a fixed momentum denoted with the parameters $p$, $\theta$ and $\phi$, $P(i)(i = 1, 2, 3, 4, 5)$ are used to represent the appearing probabilities of particle $e^+, \mu^+, \pi^+, K^+, p^+$ (or $e^-, \mu^-, \pi^-, K^-, p^-$) \(^2\) respectively, because five kinds of particles can have the same parameters’ values $p$, $\theta$, $\phi$. Here and below, $i$ and $j$ are used to represent one particle in $e^+, \mu^+, \pi^+, K^+, p^+$ or $e^-, \mu^-, \pi^-, K^-, p^-$. And because only five kinds of particle can be the unknown particle, the appearing probabilities should be normalized to unit for the fixed momentum:

$$\sum_{i=1}^{5} P(i) = 1. \quad (1)$$

if some kind particle does not appear, the corresponding $P(i) = 0$; and if the number of charged particle kinds is larger than five (e.g., cosmic rays or particles from nuclear reaction), the sum terms will exceed five.

When there is only one PID variable (e.g. TOF $t$), what we know is the TOF $t$ of the unknown charged particle and the conditional probability $P(t|i)$ which is the probability of TOF $t$, given that the unknown charged particle is $i$. \(^3\) From the point of view of probability theory, only the probability that the unknown charged particle is $i$ can be determined. Then the PID problem can be written as follows:

Given the momentum of the unknown charged particle and $P(t|i)$, calculate $P(i|t)$, where $P(i|t)$ is the conditional probability that the unknown charged particle is $i$, given that the TOF of the unknown charged particle is $t$. In the light of the definition of conditional probability and Bayes’ theorem, we have

$$P(i|t) = \frac{P(t|i)P(i)}{P(t)} = \frac{P(t|i)P(i)}{\sum_{j=1}^{5} P(t|j)P(j)}$$

$$= \frac{\int_{t_0}^{t_0} f_i(t)dt \cdot P(i)}{\sum_{j=1}^{5} \int_{t_0}^{t_0} f_j(t)dt \cdot P(j)} = \frac{f_i(t)P(i)}{\sum_{j=1}^{5} f_j(t)P(j)} \quad (2)$$

where $P(j)$ is the appearing probability of the charged particle $j$, $P(t)$ is the probability that TOF $t$ occurs, and $f_j(t)$ is the probability density function (p.d.f.) of variable $t$ for the charged particle $j$. The denominator in equation \(^2\) is the normalizing constant which only makes $P(i|t)$ have the probability meaning. The probability $P(i|t)$ is proportional to $f_i(t)P(i)$ in which $f_i(t)$ is determined by the detector, while $P(i)$ has no concern with any detector. The p.d.f. for TOF $t$ is usually a Gaussian distribution, i.e.

$$f_i(t) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{(t - t_{i0})^2}{2\sigma_i^2}\right] \quad (3)$$

where $\sigma_i$ is the resolution of TOF for the charged particle $i$, $t_{i0}$ is the expected value of TOF for the charged particle $i$. The general result for above pattern recognition can be easily found \(^4\).

According to the physical meaning of $P(i|t)$, after five values $P(i|t)$ have been calculated, the reasonable hypothesis for the unknown charged particle is $i$ which makes $P(i|t)$ the largest in the five values. For any other PID variable $X$, if all p.d.f.s of TOF $t$ in equation \(^2\) are replaced with the corresponding p.d.f.s of variable $X$, equation \(^2\) can be

\(^2\) The unknown particle’s charge is known.

\(^3\) Because TOF $t$ has continuous distribution, the values of $P(t|i)$ are all infinitesimals.
used for PID variable $X$. But the PID variable $X$ may not have a Gaussian distribution for every $i$ as TOF $t$ has in equation(3). For example, the deposited energy $E$ of a fixed momentum electron in EM shower counter has Gaussian distribution, while for $\pi,K$, the deposited energy usually has not.

In equation(2), the p.d.f.s $f_i(t)$ (or $P(t|i)$) can be obtained from calibration of the detector. Thus the appearing probability(or prior probability)$P(i)$ is the only unknown quantity. And it is $P(i)$ that makes PID problem complicatedly because $P(i)$ varies with studied final states. Here, we give some remarks on $P(i)$.

1. $P(i)$ is the appearing probability of the charged particle $i$ for studied final state and the momentum vector($p$, $\theta$, $\phi$). This means different final states have different $P(i)$, while different cuts(e.g. charged track number) in analysis result in different final states. For example, if all events of $J/\psi$ decay are considered, we get a set of $P(i)$ for the momentum vector ($p$, $\theta$, $\phi$); for the same momentum vector ($p$, $\theta$, $\phi$), if only those four-charged-tracks events from $J/\psi$ decay are considered, we will obtain another set of $P(i)$. But why do we need a second set of $P(i)$? In fact, the second set of $P(i)$ can be used to enhance the efficiency of PID when we select the events which only have four charged hadron tracks. In the four-charged-tracks events of $J/\psi$ decay, the appearing probabilities of leptons ($e,\mu$) are by far less than that of hadrons($\pi,K,p$). If the second set of $P(i)$ is used to select events, the affection of leptons will be reduced greatly. Therefore, analyses which use corresponding $P(i)$ will have better event selecting. After a series of cuts are used to obtain $P(i)$, the correct use of $P(i)$ is that the cuts used in the event selecting should not be looser than those cuts used in obtaining $P(i)$, because the $P(i)$ can not be used to select the events which do not belong to the corresponding final state. Once $P(i)$ has been figured out, it is not necessary to to change it when a new analysis is performed so long as the conditions which determine $P(i)$ do not change.

2. $P(i)$ can be obtained from M.C. process. But a more reliable way of obtaining $P(i)$ is recurrence approach in real data. M.C. results or theoretical values(if any) can be used as initial values.

3. If the difference between $P(i)$ is not large, PID will mainly rely on the inherent PID capability of detector, i.e., p.d.f.s of PID variable(e.g., $f_i(t)$ in equation(2)). And if the difference between $\sigma_i$ is neglected, we derive the conventional PID method(for TOF detector) which is only the contribution of exponential part(the weight of the unknown charged particle to be particle $i$) in equation(3):

$$W_i = \exp\left[ -\frac{(t - t_{0i})^2}{2\sigma_i^2} \right]$$

(4)

However, the difference between $P(i)$ can not be neglected at will. For example, in the final states of $J/\psi$ decay, difference between the appearing probabilities of $\pi^\pm$, $K^\pm$ varies with momentum from several to ten times. So it is valuable and more accurate to consider the effect of $P(i)$ when large difference between $P(i)$ exists. For example, if the weights of an unknown particle to $\pi$, $K$ are equal, i.e., $W_3 = W_4$, one may have no idea of what the unknown particle is. But according to equation(2), the probability which the unknown particle is $\pi$ is several to ten times larger than the probability which the unknown particle is $K$. Furthermore, if $W_3 < W_4$, the particle will be identified to be $K$, but $P(3|t) > P(4|t)$ may occur because $P(3) > P(4)$, this suggests that the unknown particle is more likely to $\pi$. Finally, if one does not use $P(i)$, one may have set all $P(i)$ a same value(equals 0.2) which is groundless.
4. PID problem will become troublesome if \( P(i) \) depends on three parameters \((p, \theta, \phi)\). To reduce the number of the parameters is favourite. For final states come from the colliders which have equal energy particle and anti-particle colliding, it is not difficult to find that \( P(i) \) is independent of polar angle \( \phi \) because of axis symmetry, and because there are all kinds of channels in one final state(e.g. \( J/\psi \) decay or four-charged-tracks final state in \( J/\psi \) decay), \( P(i) \) may be independent of azimuthal angle \( \theta \). Thus, for the final states from most colliders, if the cuts of obtaining \( P(i) \) are loose enough, \( P(i) \) may only depend on one parameter \( p \), the magnitude of momentum vector. In applications, the particle’s possible momentum region can be divided into many small regions (e.g. 50MeV/c or less for a region’s width). For every region, we have five values \( P(i) \). Then, for an unknown charged particle, \( P(i|t) \) in which we are interested can be calculated.

Obviously the above procedure has no difficulty of correlations between particles mentioned in reference[6].

III. CASE FOR TWO AND MORE PID VARIABLES

When there are two PID variables(e.g. TOF \( t \) and the deposited energy \( E \) in EM shower counter) for one unknown charged particle, then the PID problem can be written as follows:

Given the momentum of the unknown charged particle, \( P(t|i) \) and \( P(E|i) \), calculate \( P(i|t,E) \), where \( E \) is the measured value of the deposited energy in shower counter, \( P(E|i) \) is the conditional probability that the deposited energy is \( E \) given that the unknown charged particle is \( i \), and \( P(i|t,E) \) is the conditional probability that the unknown charged particle is \( i \), given that TOF \( t \) and the deposited energy \( E \) occur simultaneously. By virtue of the definition of conditional probability, we have again

\[
P(i|t,E) = \frac{P(i,t,E)}{P(t,E)} = \frac{P(t,E|i)P(i)}{P(t,E)}
\]  

(5)

where \( P(i,t,E) \) is simultaneous occurrence probability of \( i,t \) and \( E \); \( P(t,E) \) is the probability that TOF \( t \) and the deposited energy \( E \) occur simultaneously; \( P(t,E|i) \) is the conditional probability that TOF \( t \) and the deposited energy \( E \) occur simultaneously given that the unknown charged particle is \( i \). Because measurements of TOF \( t \) and the deposited energy \( E \) are independent, we have

\[
P(t,E|i) = P(t|i)P(E|i)
\]  

(6)

Here, it should be noted that the situation of variable \( E \) is not the same as that of TOF \( t \), the probability that \( E = 0 \) may not be infinitesimal because of finite sensitivity of the detector, i.e. the distribution of \( E \) is not a pure continuous distribution, but a mixed one:

\[
P(E|i) = \begin{cases} 
P(E = 0|i) & \text{if } E = 0; \\
[1 - P(E = 0|i)]g_i(E)dE & \text{if } E > 0
\end{cases}
\]  

(7)

where \( g_i(E) \) is the p.d.f. of variable \( E \) for the charged particle \( i \) when the deposited energy \( E > 0 \). If \( E = 0 \) for the unknown charged particle, then

\[
P(i|t,E = 0) = \frac{P(t,E = 0|i)P(i)}{P(t,E = 0)} = \frac{P(t,E = 0|i)P(i)}{\sum_{j=1}^{5} P(t,E = 0|j)P(j)}
\]
\[
\frac{f_i(t)P(E = 0|i)P(i)}{\sum_{j=1}^{5} f_j(t)P(E = 0|j)P(j)}
\] (8)

If \( E > 0 \) for the unknown charged particle, then

\[
P(i|t, E > 0) = \frac{P(t, E > 0|i)P(i)}{P(t, E > 0)} = \frac{P(t, E > 0|i)P(i)}{\sum_{j=1}^{5} P(t, E > 0|j)P(j)}
\]

\[
= \frac{f_i(t)P(E > 0|i)P(i)}{\sum_{j=1}^{5} f_j(t)P(E > 0|j)P(j)}
\]

\[
= \frac{f_i(t)[1 - P(E = 0|i)]g_i(E)P(i)}{\sum_{j=1}^{5} f_j(t)[1 - P(E = 0|j)]g(j(E)P(j)}
\] (9)

Similarly, the reasonable hypothesis for the unknown charged particle is \( i \) which makes \( P(i|t, E) \) the largest in the five values.

Obviously, it is not difficult to generalize above calculation to the case of many independent PID variables. Two PID variables from two different detectors are usually independent. Furthermore, the method can be used all the same when a PID variable has discrete distribution (e.g. \( \mu \) detector hits probability), and using it is straightforward in this case.

If two PID variables \( X \) and \( Y \) are correlative, the conditional probability

\[
P(X,Y|i) = f_i(X,Y)dXdY
\] (10)

where \( f_i(X,Y) \) is the joint p.d.f. of PID variables \( X \) and \( Y \) for particle \( i \). Similarly, we have

\[
P(i|X,Y) = \frac{f_i(X,Y)P(i)}{\sum_{j=1}^{5} f_j(X,Y)P(j)}
\] (11)

Since the joint p.d.f. \( f_i(X,Y) \) is difficult to obtain, the above equation(11) is not very useful.

**IV. CONCLUSIONS**

By employing Bayes’ theorem of probability theory, we have clarified the usage of all types of PID information. The corresponding applicable method to PID problem is also proposed. The method has some attracting properties. First, the final results (e.g., \( P(i|t) \), \( P(i|t,E) \)) are probabilities which have definite physical meaning. Second, when one PID variable has no-Gaussian distribution (e.g. Landau distribution of \( dE/dx \)), this method can be used as well. Finally, the conventional PID method can be derived from it after some approximation.

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