Forced Oscillations in Fluid Tori and Quasi–Periodic Oscillations

WILLIAM H. LEE
Instituto de Astronomía, UNAM, Apdo. Postal 70–264, Cd. Universitaria, México D.F. 04510

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Abstract. The kilo-Hertz Quasi–Periodic Oscillations in X–ray binaries could originate within the accretion flow, and be a signature of non–linear fluid oscillations and mode coupling in strong gravity. The possibility to decipher these systems will impact our knowledge of fundamental parameters such as the neutron star mass, radius, and spin. Thus they offer the possibility to constrain the nuclear equation of state and the rotation parameter of stellar–mass black holes. We review the general properties of these oscillations from a hydrodynamical point of view, when the accretion flow is subject to external perturbations and summarize recent results.

Key words: accretion disks — gravitation — hydrodynamics — stars: neutron — X–rays: binaries

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1. Introduction

The modulations of the X–ray flux in Low–Mass X–ray Binaries (LMXBs) and their observed range in frequencies, from \(\approx 300\) Hz to \(\approx 1200\) Hz (see van der Klis 2000; McClintock & Remillard 2006; van der Klis, these proceedings, for reviews) strongly suggest that dynamical time scales in the inner regions of the accretion disks occurring in these systems are being probed (i.e., at orbits only a few gravitational radii in circumference).

In Black Hole X–ray Binaries (BHXRBs), the high frequency Quasi–Periodic Oscillations (QPOs) in the X–ray flux seem to be fairly stable, exhibiting nearly constant frequencies. Additionally, in the four galactic microquasars known to show more than one peak, their frequencies are in a 3:2 ratio, strongly suggesting that dynamical time scales in the inner regions of the accretion disks occurring in these systems are being probed (i.e., at orbits only a few gravitational radii in circumference).

In Neutron Star (NS) systems, the twin peaks observed in the frequency spectrum drift over a considerable interval (a few hundred Hz, as mentioned above), yet are linearly correlated throughout this range. In many sources the peak separation, while not constant, is consistent with being half the burst oscillation frequency, and in other sources with the burst frequency directly. In the observed instances where burst oscillations, the spin frequency of the pulsar, and twin QPO peaks in the kHz range have been observed in the same source (SAX J1808.4-3658), the burst oscillation is the spin frequency, and is twice the peak separation, i.e., \(\nu_{\text{burst}} = \nu_s = 2\Delta\nu(\text{QPO})\) (Wijnands et al. 2003). In XTEJ1807–294, the peak separation is consistent with the spin frequency (Linares et al. 2005), and there generally appears to be a trend whereby the “slow” neutron stars have twin peaks with \(\Delta\nu \approx \nu_s\) and the “fast” ones show \(\Delta\nu \approx \nu_s\) (with the transition at about 300 Hz).

We have previously suggested (Kluźniak et al. 2004; Lee, Abramowicz & Kluźniak 2004) that the peculiar behavior of SAX J1808 can be understood in terms of a nonlinear response of the accretion disk, when subject to an external perturbation at a fixed frequency. While it is currently not possible to study the detailed structure and dynamical behavior of the accretion disk and its modes of oscillations in full detail, general physical principles can still be applied, and yield suggestive results. We have proposed that within the accretion disk, as in other nonlinear systems, a subharmonic response will appear in the presence of a perturbation, as will higher harmonics of the perturbation itself. Specifically, a second QPO frequency is to appear when two possible modes of oscillation are separated by half the perturbing frequency, and thus couple.

This presentation is devoted to a numerical study of the non–linear oscillations in such systems, by considering bound fluid tori representing a portion of the accretion flow around the compact object (in particular thin tori). We believe strong gravity plays an important role in their behavior, and consider its effects in a simplified approach. This is a purely hydrodynamical study, and so the potentially important effects of the magnetic field have not been considered for now.
2. Hydrostatic equilibrium for fluid tori

Astrophysical fluid bodies often appear in two topologically different varieties: spheres and tori. In the former, support against gravitational collapse is mostly provided by pressure from within (e.g., by photons, degenerate or ideal gas particles, neutrinos), with a possible centrifugal contribution, while in the latter, it comes mostly from rotation, with pressure providing corrections from simple test particle motion. Each of these terms: hydrodynamical and mechanical in nature, play an important role in the behavior of the system under external perturbations, and on the propagation of waves within such systems. As we will consider here the behavior and possible observational consequences of accretion disks in the particular context of LMXBs, we will focus mainly on toroidal configurations. Nevertheless, knowledge gathered from quasi–spherical systems is relevant, and indeed quite useful for interpreting these systems.

2.1. Gravitational potentials

At this point we may consider various forms for the gravitational potential of the central mass, \( M \). In the Newtonian regime, obviously \( \Phi = \Phi_{\text{N}} = -GM/r \). Since we are interested in applications to systems containing compact objects such as Neutron Stars and Black Holes, it is useful to consider pseudo–Newtonian potentials of the kind proposed originally by Paczyński and Wiita (1980), namely \( \Phi_{\text{PW}} = GM/(r - r_g) \), where \( r_g = 2GM/c^2 \) is the gravitational, or Schwarszchild, radius of the central mass. The important feature of this class of potentials is that they reproduce the existence of a marginally stable orbit, and that of capture orbits for finite values of the orbital angular momentum, \( \ell \), both unique features of General Relativity. In addition to the original form, we have also used extensively a new pseudo-potential, \( \Phi_{\text{KL}} = GM[1 - \exp(r_{ms}/r)]/r_{ms} \), where \( r_{ms} = 3r_g \) (Kluźniak & Lee 2002). The main advantage of this expression is related to the epicyclic frequencies for test particle motion.

In Newtonian gravity, a test particle in circular motion with frequency \( \nu_z \) around a point mass, \( M \), will perform small epicyclic motions when perturbed. These can be radial or vertical, but will always occur at frequencies \( \nu_r = \nu_z = \nu_\phi = \Omega/2\pi \). This is just a re-statement of the well-known fact that the Newtonian orbits of a point mass are closed curves. In addition, no matter what the value of angular momentum a particle has, \( \ell \), one can always place it at a certain radius, \( r_{\text{circ}} = \ell^2 / GM \) such that it will be in circular orbit.

In strong gravity this is no longer the case, as the central well in the potential is so powerful that two qualitatively new effects appear. First, capture orbits exist even at finite \( \ell \). Second, in the most simple case of the Schwarszchild metric (static, spherically symmetric geometry), the three–fold degeneracy between orbital and epicyclic frequencies is broken, such that \( \nu_r < \nu_z = \nu_\phi \), and \( \nu_r^2 < 0 \) for \( r < r_{ms} \). Radial oscillations are thus unstable inside \( r_{ms} \), and no circular orbit is possible in that region.

![Fig. 1. Effective potential wells for the Newtonian (N), Paczyński–Wiita (PW) and Kluzniak–Lee (KL) formulae given in the text, in units of \( c^2/2 \). The value of angular momentum is given in units of \( r_g c \). Capture orbits are not possible in Newtonian physics, but they appear qualitatively in the pseudo–Newtonian potentials, as in General Relativity. The local minimum in each curve gives the radius of the circular orbit possible at the corresponding value of \( \ell_0 \).](image)

The particular form of \( \Phi_{\text{KL}} \) is such that the ratio of radial to vertical epicyclic frequencies as a function of radius, \( \nu_r/\nu_z \), is exactly equal to the value computed in full General Relativity in the Schwarszchild metric. Specifically, we have

\[
\nu_r^2 = \frac{1}{4\pi^2} \frac{GM}{r^3} \exp(r_{ms}/r), \tag{1}
\]

and

\[
\nu_z^2 = \frac{1}{4\pi^3} \left(1 - \frac{r_{ms}}{r}\right) \frac{GM}{r^4} \exp(r_{ms}/r). \tag{2}
\]

Figure 1 shows the effective potential well as a function of radius for the three forms mentioned here and the same value of the orbital angular momentum.

2.2. Fluid equilibrium configurations

The fluid constituting the torus is located in the vicinity of the mass \( M \), which produces the potential \( \Phi \). For simplicity we shall neglect the contribution of the mass density of the fluid, \( \rho \), to the potential, and assume azimuthal symmetry, so that the problem can be studied in cylindrical coordinates \((r, z)\). The gravitational pull from the mass \( M \) is countered by the fluid pressure gradients and the centrifugal force, such that

\[
\frac{1}{\rho} \nabla P = - \nabla \Phi_{\text{eff}},
\]

where

\[
\Phi_{\text{eff}} = \Phi + \int \frac{\ell(r')^2}{2r'^3} dr'.
\]
3 NUMERICAL METHOD AND INITIAL CONDITIONS

3.1 Numerical Method

We have used the SPH (Smooth Particle Hydrodynamics) numerical method to perform our simulations (Monaghan 1992). This is a Lagrangian formulation, which interpolates a given function $A(r)$, with a set of given quantities $A(r')$ through a kernel function $W(r,h)$, using the convolution integral:

$$A(r) = \int A(r') W(r-r',h) \, dr.$$  \hfill (6)

We have implemented the prescriptions of Monaghan & Lattanzio (1985) for azimuthal symmetry, using the kernel:

$$W(r,h) = \frac{\sigma}{h^3} \begin{cases} 
1 - 3 \left( \frac{r}{h} \right)^2 + 3 \left( \frac{r}{h} \right)^3 / 4 & \text{if } 0 \leq \frac{r}{h} \leq 1; \\
\left( 2 - \frac{r}{h} \right)^3 / 4 & \text{if } 1 \leq \frac{r}{h} \leq 2; \\
0 & \text{if } 2 \leq \frac{r}{h}.
\end{cases} \hfill (7)$$

Here $h$ represents the smoothing length, and is comparable to the typical separation between fluid elements (it essentially gives the spatial resolution of the calculation), and $r$ is the radial coordinate. In two dimensions, $\nu = 2$ and $\sigma = 10/7\pi$. The gas is modeled as an inviscid fluid, and so the Navier–Stokes equations take the form:

$$\frac{d\rho}{dt} = 0,$$

$$\frac{dP}{dt} = -\frac{1}{\rho} \frac{\partial P}{\partial r} - \frac{GM_{BH}r}{(R-r_g)^2} + r\Omega^2 + \left( \frac{dv_r}{dt} \right)_{art},$$  \hfill (8)

$$\frac{d\rho z}{dt} = -\frac{1}{\rho} \frac{\partial \rho z}{\partial z} - \frac{GM_{BH}z}{(R-r_g)^2} + \left( \frac{dv_z}{dt} \right)_{art},$$  \hfill (9)

where $R = \sqrt{r^2 + z^2}$ is the distance to the central mass $M$. The sub–index $art$ indicates the artificial viscosity terms,
which is used to account for the presence of shocks and to avoid interpenetration of the fluid elements.

The energy equation is:

\[
\frac{du}{dt} = -\left(\frac{P}{\rho}\right) \nabla \cdot \mathbf{v} + \left(\frac{\rho}{2} \frac{ds}{dt}\right)_{\text{art}}
\]

where \(u\) is the internal energy per unit mass. No external (i.e., radiative) losses are assumed, and thus the only change in \(u\) arises from work done. When discretized over a finite number of fluid elements, often called particles in SPH, the convolution integral becomes a sum over all elements.

We have used the prescription given by Balsara (1995) for the artificial viscosity, which reduces the artificial shearing stress. The explicit forms for the acceleration and the energy dissipation due to artificial viscosity for the \(i\)th SPH fluid element are:

\[
\left(\frac{du}{dt}\right)_{i,\text{art}} = -\sum_{j \neq i} m_j \Pi_{ij} \nabla_i \mathbf{W}_{ij},
\]

and

\[
\left(\frac{ds}{dt}\right)_{i,\text{art}} = \frac{1}{2} \sum_{j \neq i} m_j \Pi_{ij} (\mathbf{v}_i - \mathbf{v}_j) \cdot \nabla_i \mathbf{W}_{ij}.
\]

where \(\Pi\) is defined by (see, e.g., Lee and Ramirez-Ruiz 2002)

\[
\Pi_{ij} = \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2}\right) = (-\alpha_0 \mu_{ij} + \beta_0 \mu_{ij}^2),
\]

\[
\mu_{ij} = \begin{cases} \frac{\rho_i (\mathbf{v}_i - \mathbf{v}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j)}{2 \alpha_{ij} + \eta^2} & \text{if } |\mathbf{v}_{ij} \cdot \mathbf{r}_{ij}| < 0; \\ 0 & \text{if } |\mathbf{v}_{ij} \cdot \mathbf{r}_{ij}| \geq 0; \end{cases}
\]

The function \(f_i\) is defined by:

\[
f_i = \frac{|\nabla \cdot \mathbf{v}|_i}{|\nabla \cdot \mathbf{v}|_i + |\nabla \times \mathbf{v}|_i + \eta' c_i / h_i},
\]

and \(\eta = 10^{-2}\). The sound speed and smoothing length of each element are denoted by \(c_i\) and \(h_i\) respectively, and the factor \(\eta' \approx 10^{-4}\) in the denominator prevents numerical divergences. \(\alpha_0 = \beta_0 = \gamma/2\) are constants of order unity and \(\gamma\) is the polytropic index from the equation of state. This form of the artificial viscosity supresses the shearing stresses when the compression in the fluid is low and the vorticity is high, \(|\nabla \cdot \mathbf{v}| \ll |\nabla \times \mathbf{v}|\), but remains in effect if the compression dominates in the flow \(|\nabla \cdot \mathbf{v}| \gg |\nabla \times \mathbf{v}|\).

### 3.2. Initial Conditions and Applied Perturbations

To construct a particular fluid torus, one needs only to specify the equation of state, the distribution and absolute value of angular momentum, and the degree to which the potential well is filled (through the constant \(\Phi_0\)). We restrict ourselves here to systems in which \(\ell(r) = \ell_0 = \text{cst}\). Thus the actual value of angular momentum fixes the position of the center of the torus, \(r_0\), as defined in \(\text{cst}\). Numerically, this is done by means of a Monte Carlo technique, by distributing fluid elements over the prescribed volume according to the analytical density profile, and subsequently relaxing them with artificial damping included in the equations of motion. The rotation law is strictly enforced during this procedure, and the internal energy is fixed by the adiabatic relation so that one has complete control over the initial condition. We have verified that our initial conditions are indeed in equilibrium by evolving them explicitly in time without any applied perturbations. No global evolution is observed during these computations.

We have applied two fundamentally different types of perturbations to initial models: impulsive and periodic. In the first, an additional velocity field is imposed on the initial condition at \(t = 0\), and the system is evolved without additional perturbations for many (typically one hundred) dynamical times. In the second, an additional, periodic term is added to the equations of motion to produce an acceleration. This can be applied only in the radial direction, or vertical, or both.

Finally, one can hold \(\ell_0\) constant during a calculation, or vary it slowly (i.e., over many dynamical times). In the first case, the torus will remain essentially in the same radial region, oscillating due to the applied perturbation. In the second, in addition to these oscillations it will either move inward or outward, depending on whether \(\ell_0\) decreases or increases. We have considered this possibility in view of the fact that gas within an accretion disk will, in reality, drift radially as angular momentum is transported by viscous stresses.

The temporal profile of the applied forcing can be varied. We have used single–frequency sinusoids as well as narrow pulse–like functions with repetition time \(T_s = 1/\nu_s\). This can be thought of as the rotation period of the central mass, which affects the accretion disk through some unspecified mechanism (e.g., the pulsar magnetic field, asymmetries in the quadrupole moment or other effects).

### 4. Forced Oscillations

We will hereafter restrict the presentation to the case of slender tori, where their radial and vertical extent, \(L\), is small compared to the radial position of their center, i.e., \(L \ll r_0\). The thin annulus can then be viewed as a small section of the entire accretion structure surrounding the compact object. The main difficulty with this picture, and in relating it to real systems, lies in the fact that the artificial zero–pressure boundaries obviously make wave propagation into the exterior of this torus impossible. Mode leaking is certainly an important issue (Frager 2005), but for now we address only closed systems. The dynamical response of thick tori under global impulsive perturbations has been addressed in a series of papers by Rezzolla and collaborators (Zanotti et al. 2003; Rezzolla et al. 2003a,b; Montero et al. 2004; Zanotti et al. 2005), while global and localized periodic perturbations have been considered by Rubio–Herrera & Lee (2005a,b).

As a result of the induced forcing, the torus experiences small–amplitude oscillations, which can be measured in various ways. One of these is to simply consider the position of its center, \((r_0, z_0)\), defined, as above, as the location of maximum density, as a function of time. The corresponding time series are complicated, as they prominently display the perturbing frequency \(\nu_s\) and the local epicyclic frequency for small radial and vertical oscillations, \(\nu_r\) and \(\nu_z\) respectively. There are however, combination frequencies and cross–over
Fig. 3. Fourier transforms of the radial (solid) and vertical (dashed) oscillations of the center of a slender torus with constant specific angular momentum when perturbed periodically at frequency $\nu_s = 400$ Hz in a pseudo–Newtonian potential. The local values of the radial and vertical epicyclic frequencies are $\nu_r = 500$ Hz and $\nu_z = 700$ Hz, respectively. Even though the perturbation is purely radial, vertical oscillations are excited because of pressure coupling. The power was re–scaled along the vertical axis for illustrative purposes. In reality the power in vertical motions is much smaller than in the radial ones.

phenomena, so that the whole behavior is best analyzed by performing Fourier transforms of $r_0(t)$ and $z_0(t)$.

### 4.1 Radial and Vertical Mode Coupling

For calculations where the angular momentum does not vary in time, since it is also constant in space, there are no mechanisms for its transport, and the perturbation does not produce any torques for its transport, and the perturbation does not produce any torques. The fluid must remain close to its equilibrium radius $r_0$. Figure 5 shows the Fourier transform of $r_0$ and $z_0$ for such a calculation, assuming the potential $\Phi_{KL}$, a central mass $M = 1.4 M_\odot$, $r_0 = 6.1 r_g$ and a purely radial, sinusoidal perturbation at frequency $\nu_s = 400$ Hz. The corresponding values of the local radial and vertical epicyclic frequencies are $\nu_r = 500$ Hz and $\nu_z = 700$ Hz. The power spectrum of radial motions clearly shows the perturbation frequency, $\nu_s$, and the radial epicyclic frequency, $\nu_r$. Likewise, the power spectrum of vertical motions exhibits the vertical epicyclic frequency $\nu_z$. In this particular case, the value of $r_0$ and $\nu_s$ is such that the difference between the two epicyclic frequencies is equal to half the spin frequency, i.e., $\nu_z - \nu_r = \nu_s/2$. There is thus clear evidence for mode coupling, through pressure, since the perturbation was initially only applied in the radial direction. Beats between the various frequencies are also present. In Figure 4 the power in vertical motions shows peaks at $\nu_z - \nu_s$, $\nu_r$, and $\nu_s + \nu_r$. The very mode responsible for the coupling between the radial and vertical oscillations is weakly visible at $\nu_z - \nu_r = 200$ Hz.

A calculation with varying angular momentum (see §3.2) where the torus drifts inward is even more revealing. Starting at $r_0 = 6.7 r_g$, and terminating at $r_0 = 5.35 r_g$ over two hundred initial orbital periods, the radial and vertical oscillations of the center of the torus occur at a range of frequencies, covering values between the extremes in epicyclic frequencies $\nu_r$ and $\nu_z$. This choice of parameters implies that, with $\nu_s = 400$ Hz, the calculation begins with $\nu_s - \nu_r < \nu_s/2$ and ends with $\nu_z - \nu_r > \nu_s/2$.

A power spectrum of the radial oscillations of $r_0$ during the a complete simulation using a pulse–like perturbation shows several features (see Figure 4). The perturbation at $\nu_s$ is clearly visible, as are its three harmonics (it is not a pure sine wave, see Figure 3 in Lee, Abramowicz & Kluźniak 2004). A broad, flat–topped peak is also apparent, covering the range $440\text{Hz} < \nu_r < 580\text{Hz}$. These are simply the local values of $\nu_r$ at the inner and outer radii, respectively. Harmonics of the radial epicyclic frequency, as well as a beat with $\nu_s$ are also visible, at greatly reduced power. The corresponding power spectrum for the oscillations of $z_0$ shows a similar broad peak, for $590\text{Hz} < \nu_z < 880\text{Hz}$, as expected (see Figure 4).

To explore the behavior of the oscillations in more detail, we have split the time series of $r_0(t)$ and $z_0(t)$ into eight equal segments, each thus covering one eighth of the simulation. During this time, the torus is in a narrow radial window, and we may say that its position is nearly constant (except for the oscillations induced by the external forcing). We then extract the individual Fourier transforms for each segment. The results for the radial oscillations are shown in Figure 5 for the first and last time segments (they are all qualitatively similar). As expected, each one shows the perturbation and its harmonics, as well as a narrow peak centered at the value of $\nu_r$ corresponding to the average position of the circle of maximum pressure during the time interval. The power spectrum of the corresponding vertical oscillations is shown in Figure 5. There is a strong peak at $\nu_z$, the amplitude of which varies greatly as the torus drifts radially. In Figure 5 we show the power in this peak as a function of the average position of the center of the torus, $r_0$. Two facts are immediately clear. First, the intensity of the coupling between radial and vertical modes is a function of the external perturbation. Second, this is a non–linear coupling, because the interaction is through a sub–harmonic of the perturbation, $\nu_s/2$, and not with the perturbation itself or its higher harmonics. This fact points clearly to rich non–linear behavior in these systems, and, we believe, has been directly seen in accretion disks in LMXBs (Kluźniak et al. 2004; Lee, Abramowicz & Kluźniak 2004), in particular in SAXJ1808.4–3658 (Wijnands et al. 2003).

If the coupling between modes is due to some interaction with the external perturbation, and this excites a resonance, one would naively expect the consequences to display resonance–like behavior. In particular, the resonant window should be narrow, and if the corresponding condition is not
met (e.g., above $\nu_z - \nu_r = \nu_s/2$), then no coupling should occur. However, this is not what we observe in the calculation we have just described. As the torus drifts inward, the power in vertical motions rises sharply as the resonance is approached, but remains high even after it has been passed. The corresponding range in frequencies for $\nu_z$ spans more than 100 Hz. This appears to indicate that once excited, the modes have locked (Kluźniak et al. 2004) and continue to be excited for some time. The curve in Figure 4 is reminiscent in shape of the amplitudes of meridional oscillations for slightly perturbed orbits in nearly geodesic motion, as computed by Abramowicz et al. (2003, see their Figure 2). In that case a generic coupling between radial and polar oscillators was assumed. In Neutron Star sources, the frequencies of the kHz QPOs drift over large ranges, sometimes hundreds of Hz. Mode locking following resonant excitation could in principle be responsible for this behavior.

### 4.2. Additional Vertical Modes

Inspection of Figures 3 and 5 reveals that the power spectrum of vertical oscillations has a secondary peak at frequency $\nu_z > \nu_r$. The power in this oscillation is variable as the torus drifts radially, but it appears to be continuously excited. In some instances it seems to increase in power when $\nu_z - \nu_r \approx \nu_s/2$, but its presence is not limited to the corresponding radial window.

Thus it would seem again, in agreement with our previous results, that a non–linear coupling between the radial and vertical modes transfers energy to the latter. Further, a second oscillation at frequency $\nu_2 \approx \nu_z + \nu_s/2$ is excited within the same vertical motions, with power comparable to that at frequency $\nu_z$. Previously, when we examined peaks in the power spectrum at two different frequencies, they occurred in different variables (e.g., radial vs. vertical) and thus it was impossible to directly compare their amplitudes (as the strength of the real coupling is unknown, and the applied perturbation is arbitrary). Here we now show for the first time evidence for similar amplitudes in different modes of the same general motions.

### 5. Conclusions and Directions for Future Work

The oscillations of fluid confined by gravity and/or centrifugal forces can lead to rich behavior under a variety of astrophysical scenarios (we have focused on QPOs in X–ray binaries, the observations of solar oscillations provide another, much closer example). Under quite general considerations we have shown that simple, acoustic modes which have their origin in free particle oscillations are modified by the hydrodynamics and can couple to one another in a non–linear fashion. The locking of excited modes could explain the fact that the observed QPO frequencies drift over considerable ranges while still arising from resonant interactions. We believe that their signature is present in the observations of accretion flows in strong gravitational fields, and will allow for the measurement of important parameters related to the central, compact object.

Clearly, meaningful constraints will be derived only through much more detailed work. For example, it is not clear how such oscillations will actually modify the X–ray lightcurve (see, e.g., Schnittman 2005 for work along these lines). In addition, the strong gravitational field needs to be modeled in full General Relativity, with consideration of the self–gravity of the disk and detailed thermodynamics and radiation transport.

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Fig. 4. Fourier transforms of the radial (top) and vertical (bottom) oscillations of the center of a radially drifting slender torus when perturbed periodically at frequency $\nu_s = 400$ Hz in a pseudo–Newtonian potential. The outer (initial) and inner (final) central radii are $r_0 = 6.7r_g$ and $r_0 = 5.35r_g$ respectively.

Fig. 5. Fourier transforms of the radial (top) and vertical (bottom) oscillations of the center of a radially drifting slender torus when perturbed periodically at frequency $\nu_s = 400$ Hz in a pseudo–Newtonian potential. Only the first and last segments of the calculation (1/8 of the total) are shown.
Fig. 6. Power in vertical motions at the local value of $\nu_z$ as a function of the average central position of the torus, $r_0$, for the calculation with a radially drifting (inwards) perturbed torus. The calculation was divided into eight segments of equal duration. During each segment the torus drifted by $0.16r_g$ in the radial direction (shown by the horizontal bars). The strongest response occurs at $r_0 \approx 6.15r_g$, where $\nu_z = \nu_c = \nu_s/2$. Note how the curve is not symmetrical with respect to the point of maximum power. See the text for details.