Microwave-induced “zero-resistance” states and second-harmonic generation in an ultraclean two-dimensional electron gas

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Microwave-induced resistance oscillations and “zero-resistance” states were discovered in ultraclean two-dimensional electron systems in 2001–2003 and have attracted great interest from researchers. A comprehensive theory of these phenomena was developed in 2011: It was shown that all experimentally observed dependencies can be naturally explained by the influence of the ponderomotive forces which arise in the near-contact regions of two-dimensional electron gas under the action of microwaves. Now we show that the same near-contact physical processes should lead to another nonlinear electrodynamic phenomenon – the second-harmonic generation. We calculate the frequency, magnetic field, mobility, and power dependencies of the second-harmonic intensity and show that it can be as large as \( \sim 0.2 \text{ mW/cm}^2 \) under realistic experimental conditions.
The microwave-induced resistance oscillations (MIRO) and “zero-resistance” states (MIZRS) is a very nice physical phenomenon which was discovered in very-high-electron-mobility two-dimensional (2D) electron systems in GaAs-AlGaAs quantum wells in 2001—2003 [1,2]. The magnetoconductance $\Delta\sigma_{xx} = U_{xx}/I$ of a 2D electron gas, measured between the side contacts to a Hall-bar sample, see Figure 1, demonstrates very large oscillations under the action of microwaves. These oscillations are governed by the ratio $\omega/\omega_c$, where $\omega$ and $\omega_c = eB/mc$ are the microwave and cyclotron frequencies, respectively, $B$ is the external magnetic field, perpendicular to the 2D electron-gas plane, $e$ and $m$ are the charge and effective mass of the 2D electron, and $c$ is the velocity of light. If the microwave power is sufficiently large, the measured magnetoconductance $\Delta\sigma_{xx}$ increases by a factor of $3 - 7$ at $\omega_c \gtrsim \omega$, and decreases down to zero at $\omega_c \lesssim \omega$, remaining unchanged at the exact coincidence of the cyclotron and microwave frequencies. The effect is observed not only around $\omega_c = \omega$, but also around harmonics $\omega_n = \omega/k$, with $k$ up to $\sim 10$. It is seen in classical magnetic fields ($\hbar \omega_c \ll \epsilon_F$, where $\epsilon_F$ is the Fermi energy of the 2D electron gas), at low temperatures $T \lesssim 1$ K, and demonstrates an activation dependence on $T$ and microwave power, $\ln(\sigma^{min}_{xx}) \propto - P/T$, Ref. [4], where $\sigma^{min}_{xx}$ is the magnetoconductance at the principal minimum of $\sigma_{xx}$. In contrast to the absorption of microwave radiation, the MIRO/ZRS effects are insensitive to the circular polarization sense [5], and observed only in samples with an extremely high electron mobility $\mu \gtrsim 1.5 \times 10^7$ $\text{cm}^2/\text{Vs}$. The scattering parameter $\omega\tau$ in the MIRO/ZRS experiments exceeds several hundred; e.g., in [2], $\omega\tau \approx 360$, where $\tau = m\mu/e$ is the scattering time extracted from mobility. In such a collisionless plasma, electrons freely move around the cyclotron orbits, and their scattering is not important. The cyclotron radius $R_c$ of electrons is about $0.4 - 2.2 \mu m$ in [2] and up to $\sim 20 \mu m$ in [4]. A typical microwave power density is about $1 \text{mW/cm}^2$; the corresponding microwave electric field in the incident wave is $E_0 \approx 0.6$ $\text{V/cm}$.

The entire set of all experimentally observed dependencies can be explained as follows [6]. As seen from Figure 1, the shape of the investigated sample reminds a spider with a small semiconductor “body” and several thin long metallic “legs” – contact wires. In conventional magnetoconductance measurements (without microwaves) the contact wires do not play any active role. Under the microwave irradiation, however, they serve as antennas, focusing the microwave radiation in small near-contact areas, like lightning rods. The near-contact ac electric field $E_c$ exceeds the incident-wave field $E_0$ by two-three orders of magnitude, $E_c \gg E_0$, and is strongly inhomogeneous on the cyclotron radius scale. In addition, this field is linearly polarized, independent of the polarization of the incident wave, since near metallic surfaces the field is always normal to the metal boundary. This strong and strongly inhomogeneous electric field $E(r,t)$ acts on the near-contact electrons by a time-independent second-order ponderomotive force $F_{pm}(r) \propto \nabla (E^2(r,t))_t$, where $\langle \ldots \rangle_t$ means averaging over time. The force $F_{pm}(r)$ is proportional to the imaginary part of the nonlocal conductivity of the 2D electron gas $\sigma''(\omega,\omega_c)$, and, therefore, changes its direction depending on the sign of $\omega - k\omega_c$: it repels electrons from the contacts at $\omega_c \lesssim \omega/k$ and attracts them to the contacts at $\omega_c \gtrsim \omega/k$, thus forming depletion or accumulation regions in the near-contact areas. Since the near-contact ac electric field is strongly inhomogeneous at the $R_c$-scale, the effect is seen not only around the fundamental $\omega \approx \omega_c$, but also

![Figure 1](image-url)

FIG. 1. The geometry of a standard quantum Hall sample in which the MIRO/ZRS effects have been observed. The sample is placed in a perpendicular magnetic field $B$ and is irradiated by microwaves. Shaded areas show contacts.
around higher harmonics $\omega \simeq k\omega_c$. In the depletion regime, $\omega_c \lesssim \omega/k$, the ponderomotive potential may become larger than the Fermi energy near the contacts. In this case the strong ponderomotive force isolates the bulk of the 2D gas from the contacts, thus leading to the vanishing voltage $U_{xx}$ (which is no longer proportional to the bulk magnetoresistance $R_{xx}$), and to the apparent “zero-resistance” states. The near-contact density of the 2D electrons is then proportional to the Boltzmann factor $\sim \exp(-P/T)$ thus explaining the activation dependence of the resistance minima, $\ln R_{min} \propto -P/T$, Ref. [4]. The strength of the ponderomotive force substantially depends on the sharpness of the cyclotron resonances at $\omega \simeq k\omega_c$, thus requiring large values of the scattering parameter $\omega\tau \gg 1$. This explains why the MIRO/ZRS phenomena have been observed only in samples with an extremely high electron mobility.

It is physically clear, that a strongly inhomogeneous ac electric field, needed for observation of the discussed effects, arises not only near the contacts, really touching the 2D gas, but also near the edges of any metallic elements placed in the vicinity of it. This can be a metallic grating lying on top of the quantum-well system, or another similar structure. For example, the “contactless” MIRO measurements have been performed in a GaAs quantum well covered by a coplanar waveguide [7]. The observed oscillations were weaker than in the original MIZRS papers [2, 3], since the distance between the sharp metallic edges of the waveguide and the 2D gas was larger than between the real contacts and the 2D layer.

The outlined theory [6] correctly describes the microwave frequency, polarization, power, magnetic field, mobility, and temperature dependencies of the MIRO/ZRS effects (an overview of alternative theories, which attempted to explain the MIRO/ZRS effects by the influence of microwaves on the scattering of electrons in the bulk of the 2D gas can be found in [8]).

The metallic wires (contacts), attached to the 2D layer, thus distort transport measurements of bulk properties of the system irradiated by microwaves. However, they can be very useful for detecting other, nonlinear optical, phenomena. In this paper, we show that the concentration of microwave power near metallic edges should lead to a second harmonic generation in the 2D electron plasma. This effect, directly related to the MIRO/ZRS phenomena, can be used for creating microwave and terahertz sources of radiation and is therefore of great importance.

Consider the (almost) collisionless motion of a 2D electron in the close vicinity of a metallic contact, Figure 2(a). We assume that the 2D gas and the contact layer lie in the plane $z = 0$, and the 2D gas (the contact) occupies the half-plane $x > 0$ $(x < 0)$. Further, we assume that the external permanent magnetic field points in $z$-direction, $B = (0, 0, B)$, and the microwave electric field near the contact,

$$E_x(x, t) = E_x(x) \cos \omega t,$$

is strongly inhomogeneous. In zeroth order in $E_x$ an electron rotates around the cyclotron orbits

$$(x^{(0)}(t) \quad y^{(0)}(t)) = (x_0 + R_c \sin(\omega_c t + \phi) \quad y_0 - R_c \cos(\omega_c t + \phi)),$$
where \( x_0, y_0, \) and \( \phi \) are determined by initial conditions. Since the microwave field is inhomogeneous on the cyclotron radius scale, the electron experiences different forces in different parts of its trajectory. In the first order in \( E_x \), the force

\[
F_x^{(1)}(t) = -eE_x[x^{(0)}(t)] \cos\omega t = -eE_x[x_0 + R_c \sin[\omega t + \phi]] \cos\omega t
\]

contains an infinite number of harmonics with the frequencies \( \pm \omega + k\omega_c \),

\[
F_x^{(1)}(t) = -\frac{e}{2} \sum_{k=-\infty}^{\infty} \epsilon_k(x_0)e^{ik(\phi - \pi/2)} \left( e^{i(k\omega_c + \omega)t} + e^{i(k\omega_c - \omega)t} \right),
\]

where the factors \( \epsilon_k(x_0) \) are determined by the inverse Fourier transform of the field, s. Ref. [6],

\[
\epsilon_k(x_0) = \epsilon_{-k}(x_0) = \frac{1}{\pi} \int_0^\pi E_x(x_0 + R_c \cos x) \cos k\xi d\xi.
\]

Solving equations of the electron motion,

\[
\dot{r} = v, \quad m\ddot{v} = -\frac{e}{c}v \times B - \gamma mv + F_x(t)e_x,
\]

with the force \( F_x(t) = F_x^{(1)}(t) \) from [10] (here \( \gamma = 1/\tau \)), we get the first-order correction to the coordinate,

\[
x^{(1)}(t) = \frac{e}{2m} \sum_{k=-\infty}^{\infty} \epsilon_k(x_0)e^{ik(\phi - \pi/2)} \left( \frac{e^{i(k\omega_c + \omega)t}}{(k\omega_c + \omega - i\gamma)^2 - \omega_c^2} + \frac{e^{i(k\omega_c - \omega)t}}{(k\omega_c - \omega - i\gamma)^2 - \omega_c^2} \right).
\]

In the second order in \( E_x \), the force

\[
F_x^{(2)}(t) = -e\cos\omega t \frac{\partial E_x[x^{(0)}(t)]}{\partial x} x^{(1)}(t),
\]

acting on the electron, is then

\[
F_x^{(2)}(t) = -\frac{e^2}{4m} \sum_{k,k'=-\infty}^{\infty} \frac{\partial^2 \epsilon_k'(x_0)}{\partial x_0^2} \epsilon_k(x_0)e^{i(k+k')(\phi - \pi/2)}
\]

\[
\times \left( \frac{e^{i[(k+k')\omega_c + 2\omega_c]t} + e^{i[(k+k')\omega_c - 2\omega_c]t}}{(k\omega_c + \omega - i\gamma)^2 - \omega_c^2} + \frac{e^{i[(k+k')\omega_c + 2\omega_c]t} + e^{i[(k+k')\omega_c - 2\omega_c]t}}{(k\omega_c - \omega - i\gamma)^2 - \omega_c^2} \right).
\]

It depends on the initial conditions of the electron motion, \( x_0 \) and \( \phi \). Averaging Eq. [10] over \( \phi \), we get

\[
\langle F_x^{(2)} \rangle_{\phi} = -\frac{e^2}{16m\omega_c} (1 + e^{i2\omega t}) \frac{\partial}{\partial x_0} \sum_{k=-\infty}^{\infty} \frac{\epsilon_{k+1}^2(x_0) - \epsilon_k^2(x_0)}{k\omega_c + \omega - i\gamma} + c.c.,
\]

where c.c. means the complex conjugate.

The force [10] is proportional to the factor \( (1 + e^{i2\omega t}) \) and thus contains two effects. The time-independent term is the ponderomotive force considered in Ref. [6]. The corresponding ponderomotive potential,

\[
U_{pm}(x_0) = \frac{e^2}{8m\omega_c} \sum_{k=1}^{\infty} \left( \epsilon_{k+1}^2(x_0) - \epsilon_k^2(x_0) \right) \left( \frac{\omega - k\omega_c}{(\omega - k\omega_c)^2 + \gamma^2} + \frac{\omega + k\omega_c}{(\omega + k\omega_c)^2 + \gamma^2} \right),
\]

depends on the distance \( x_0 \) of an electron from the contact. Since \( E_x(x) \) and, hence, \( \epsilon_k(x_0) \), dramatically grow approaching the contact, electrons closest to the contact (at \( x_0 \approx R_c \)) experience the largest force. The near-contact change of the density, which determines the visible change of the measured voltage \( U_{xx} \), is then determined by the density factor [8],

\[
\mathcal{N} = \frac{n_s(R_c)}{n_s^0} = \frac{T}{E_F} \left( \frac{E_F - U_{pm}(R_c)}{T} \right),
\]
which is defined as the ratio of the near-contact density, \( n_s(R_c) \), to the density of electrons far from the contacts, in the bulk of the 2D layer, \( n_s^0 \). The function

\[
F(z) = \int_0^\infty \frac{dx}{1 + \exp(x - z)}
\]  

(13)
in \([12]\) is the Fermi integral. This formula was obtained in \([8]\) and describes experimentally observed resistance oscillations and zero-resistance states under the influence of microwaves, for details see \([8]\).

The second contribution to the force \([10]\) oscillates in time with the frequency \( 2\omega \). Substituting it in the equation of motion \([9]\) we calculate the second-order velocity

\[
\begin{pmatrix}
v_x^{(2)}(t) \\
v_y^{(2)}(t)
\end{pmatrix}
= \frac{1}{2m} \frac{\partial U_{pm}(x_0)}{\partial x_0} \begin{pmatrix}
-\sin(2\omega t) \left( \frac{2\omega - \omega_0}{(2\omega - \omega_0)^2 + \gamma^2} + \frac{2\omega + \omega_0}{(2\omega + \omega_0)^2 + \gamma^2} \right) \\
\cos(2\omega t) \left( \frac{2\omega - \omega_0}{(2\omega - \omega_0)^2 + \gamma^2} - \frac{2\omega + \omega_0}{(2\omega + \omega_0)^2 + \gamma^2} \right)
\end{pmatrix}.
\]  

(14)

One sees that electrons oscillate with the double microwave frequency \( 2\omega \), both in the \( x \)- and \( y \)-directions, with the oscillation amplitude depending on their distance from the contact \( x_0 \). Averaging (14) over \( x_0 \) according to the rule

\[
\langle v^{(2)}_a(t) \rangle \equiv \langle v^{(2)}_a(x_0, t) \rangle_{x_0} = \frac{1}{W} \int_{R_c}^W v^{(2)}_a(x_0, t) dx_0,
\]  

(15)

where we take the cyclotron radius \( R_c \) for the lower, and the width of the sample \( W (W \gg R_c) \) for the upper integration limit, and multiplying the result by \(-\epsilon \tilde{n}_s \), where \( \tilde{n}_s \) is the effective density of electrons oscillating with the double frequency, we get the ac current, \( j_a^{(2)} = -\epsilon \tilde{n}_s \langle v^{(2)}_a(t) \rangle \), at the frequency \( 2\omega \),

\[
\begin{pmatrix}
j_x^{(2)} \\
j_y^{(2)}
\end{pmatrix}
= \frac{\epsilon \tilde{n}_s U_{pm}(R_c)}{2mW} \begin{pmatrix}
-\sin(2\omega t) \left( \frac{2\omega - \omega_0}{(2\omega - \omega_0)^2 + \gamma^2} + \frac{2\omega + \omega_0}{(2\omega + \omega_0)^2 + \gamma^2} \right) \\
\cos(2\omega t) \left( \frac{2\omega - \omega_0}{(2\omega - \omega_0)^2 + \gamma^2} - \frac{2\omega + \omega_0}{(2\omega + \omega_0)^2 + \gamma^2} \right)
\end{pmatrix}.
\]  

(16)

Here we have ignored the value of the ponderomotive potential far from the contact, \( U_{pm}(W) \), since it rapidly decreases when \( x_0 \gg R_c \).

Equations (10) is the main result of this work. The second-harmonic current has both \( x \)- and \( y \)-polarization. Its \( \omega \)- and \( B \)-dependence is determined by the resonance factors in big parenthesis and by the ponderomotive potential \( U_{pm}(R_c) \), Eq. (11). The amplitude of \( j^{(2)} \) is proportional to the effective density of electrons participating in the \( 2\omega \)-oscillations, \( \tilde{n}_s \), and inversely proportional to the sample width \( W \). The density \( \tilde{n}_s \) is small as compared to the density of electrons in the bulk of the sample, since only the electrons in the very vicinity of the metallic contacts feel the second-harmonic force and oscillate with the frequency \( 2\omega \). The width of the sample designed for detecting the second-harmonic radiation should therefore be small (not larger than a few cyclotron radii). Important is, that the sample should be asymmetric (i.e. no contacts on one side of the sample); otherwise, radiation from left and right contacts cancels each other. The radiating elements shown in Figure 2(b) can be arranged in an array. The electric field of the radiated wave is then \( E_{x,y}^{(2)} \approx 2\pi j_{x,y}^{(2)}/c \), and the intensity of radiation polarized perpendicularly (\( \perp \)) and parallel (\( \parallel \)) to the boundary 2D gas – contact is

\[
J_{\perp}^{(2)} = \frac{\pi}{c} (J_x^{(2)})^2, \quad J_{\parallel}^{(2)} = \frac{\pi}{c} (j_y^{(2)})^2.
\]  

(17)

In order to analyze the \( B \)-dependencies of the obtained results we have to specify the shape of the function \( E_x(x) \). Assuming that the field \( E_x(x) \) varies in space (at \( x > 0 \)) as

\[
E_x(x) = E_0 + (E_c - E_0) e^{-x/L_c}, \quad x > 0,
\]  

(18)

where \( E_c \gg E_0 \), and \( L_c \) is a field penetration length, we get for the function \( \epsilon_k(x_0) \):

\[
\epsilon_k(x_0) = E_0 \delta_{k0} + (E_c - E_0) e^{-x_0/L_c} (-1)^k I_k(R_c/L_c).
\]  

(19)

Substituting (19) into Eqs. (11), (12), (16), and (17), we get the results shown in Figures 3-5. Figure 3 shows the ponderomotive potential \( U_{xx} \) and the density factor \( \tilde{n}_s \), which determines the observed potential difference \( U_{xx} \) between the contacts, cf. \([8]\). The calculated curves are in very good agreement with the experimentally observed resistance oscillations. The amplitude of the second-harmonic current \( j^{(2)} \), Figure 5 exhibits the same oscillations.
as the ponderomotive potential, Figures 3(a), and in addition, a resonance near $\omega_c = 2\omega$. The intensity of radiation polarized perpendicular to the contact boundary is close to $(\omega_c \approx 2\omega)$ or larger than $(\omega_c \lesssim \omega)$ that of the wave polarized parallel to it, Figure 5.

It is important to emphasize, that the current $j^{(2)}$ has a much weaker temperature dependence than the MIRO/ZRS effect. While in the MIZRS regime the measured $R_{xx}$ has the activation temperature dependence, and the effect disappears when the temperature grows from $\sim 1$ K up to $\sim 4$ K, the $T$-dependence of the second harmonics current is contained only in the mobility. One can therefore expect that the harmonic-generation effect will be observable not only at liquid helium but also at higher temperatures.

Let us estimate the expected power of the second-harmonic radiation. As seen from [11] and Figure 4, the maximum of the second-harmonic current is achieved at $\omega_c = 2\omega \approx \pm \gamma$, so that $(j^{(2)})_{\text{max}} \approx \tilde{n}_s U_{pm}(R_c)/4\hbar W \gamma$. Since in the MIZRS regime the absolute value of the ponderomotive potential, $U_{pm}(R_c)$, exceeds the Fermi energy, we get the estimate

$$ (j^{(2)})_{\text{max}} \gtrsim \frac{\tilde{n}_s E_F}{4mW \gamma}, \quad \text{and} \quad E^{(2)} \gtrsim \frac{\pi e\tilde{n}_s E_F}{2meW \gamma} = \frac{\pi e\tilde{n}_s v_F v_F \tau}{4c} W. \quad (20) $$

Assuming that the effective density of near-contact electrons, $\tilde{n}_s$, oscillating with the frequency $2\omega$, is two orders of magnitude smaller than the bulk electron density, $n_s \approx n_s/100 \approx 3 \times 10^9$ cm$^{-2}$, and that the mean-free-path $v_F \tau$ can be comparable with the width of the sample $W$, we obtain $E^{(2)} \approx 0.25$ V/cm and $J^{(2)} \approx 0.2$ mW/cm$^2$. The estimated power density of the emitted second harmonic is comparable with the power of the incident wave, which shows that the predicted effect is not small and should be observable in the ultra-clean two-dimensional electron systems which demonstrate the MIRO/ZRS effect.

The predicted effect of the harmonic generation may shed light on the nature of the recently observed photoresistance spike at the magnetic field corresponding to the condition $\omega \approx 2\omega_c$, Refs. [4, 12]. Due to the frequency doubling this spike may be a consequence of the conventional cyclotron resonance of the second harmonic.

Plasma phenomena in metals and semiconductors have been studied since 1950-ies [13], but a much larger influence of the charge-carrier scattering, as compared to gaseous plasmas, did not allow one to observe in solids all the variety of interesting plasma phenomena, including nonlinear ones. Due to a great improvement of quality of semiconductor quantum-well samples, with the electron mobility exceeding $10^7$ cm$^2$/Vs, a practically collisionless solid-state plasma has become available. The MIRO/ZRS effect [1, 5], as well as the second harmonic generation predicted here, open up new perspectives in studying nonlinear electrodynamic phenomena in such two-dimensional solid-state plasma.

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FIG. 3. (a) The ponderomotive potential \( U_{\text{pm}}, \text{arb. units} \) and (b) the density factor \( f \) as a function of \( \omega_c/\omega \). The parameters used: \( \omega_\tau = 72, E_c/E_0 = 20, v_F/\omega L_c = 2 \), and \( T/E_F = 0.02 \).

FIG. 4. The amplitude of the second-harmonic current \( j \) as a function of \( \omega_c/\omega \). The parameters used: \( \omega_\tau = 72, E_c/E_0 = 20 \), and \( v_F/\omega L_c = 2 \).
FIG. 5. The squared amplitudes of the second-harmonic current (proportional to the intensities \[^{[17]}\]) around (a) \(\omega_0 \simeq \omega\) and (b) \(\omega_0 \simeq 2\omega\). The parameters used: \(\omega \tau = 72\), \(E_c/E_0 = 20\), and \(v_F/\omega L_c = 2\).