Nucleon Axial Form Factor from Domain Wall on HISQ

Aaron S. Meyer

Lawrence Livermore National Laboratory

August 1, 2023

This work is supported by Lawrence Livermore National Security, LLC under Contract No. DE-AC52-07NA27344 with the U.S. Department of Energy.
Outline

▶ Neutrino Oscillation
▶ Quasielastic Scattering
▶ LQCD Fit Setup
▶ Fit Stability
▶ Axial Form Factor
▶ Future Prospects

Special thanks: Daniel Xing, Jinchen He

Note: all references in online slides are hyperlinked
Neutrino Oscillation
Neutrino Physics Goals

Flagship long baseline experiments to measure neutrino oscillation

DUNE: USA, HyperK: Japan

Seek to answer fundamental questions about neutrinos:

- mass ordering ($\Delta m_{32}^2 > 0$?)
- octant ($\sin^2 \theta_{23} = 0.5$?)
- CP violation ($\delta_{CP} =$?)
- PMNS unitarity?
- 3 $\nu$ flavors?
- precision constraints

Measurements of solar, supernova $\nu$

Data collection starts 2028–2029 $\implies$ need support from theory!
Neutrino Oscillation and Quasielastic

Compute nucleon amplitudes, ingredients for nuclear models

Quasielastic is lowest $E_\nu$, simplest $\Rightarrow$ most important

Question:
How well do we know nucleon quasielastic cross section from elementary target sources?

▶ Hydrogen/Deuterium scattering
▶ Lattice QCD

[Rev.Mod.Phys. 84]

$\nu_\mu$ flux [arb.unit]

HyperK [1805.04163[physics.ins-det]]

DUNE [1512.06148[physics.ins-det]]

$\nu_\mu$ $\rightarrow$ $\mu^-$ $\nu_\mu$
Quasielastic Form Factors

Quasielastic (QE) scattering assumes quasi-free nucleon inside nucleus

\[ \mathcal{M}_{\text{nucleon}} = \langle \ell | \mathcal{J}^\mu | \nu_\ell \rangle \langle N' | \mathcal{J}_\mu | N \rangle \]

\[ = u(p') \left[ \gamma_\mu F_1(q^2) + \frac{i}{2M_N} \sigma_{\mu\nu} q^\nu F_2(q^2) + \gamma_\mu \gamma_5 F_A(q^2) + \frac{1}{2M_N} q_\mu \gamma_5 F_P(q^2) \right] u(p) \]

- \( F_1, F_2 \): constrained by eN scattering
- \( F_P \): subleading in cross section, \( \propto F_A \) from pion pole dominance constraint

Axial form factor \( F_A \) is leading contribution to nucleon cross section uncertainty

Induced pseudoscalar form factor \( F_P \) can be determined independently
Deuterium Constraints on $F_A$

- Outdated bubble chamber experiments:
  - Total $O(10^3) \nu\mu$ QE events
  - Digitized event distributions only
  - Unknown corrections to data
  - Deficient deuterium correction

- Dipole overconstrained by data
  underestimated uncertainty $\times O(10)$

- Prediction discrepancies could be from nucleon and/or nuclear origins

Coming soon:
MINER$\nu$A $\bar{\nu}_\mu p \rightarrow \mu^+ n$ dataset
& updated form factor fits
See [Nature 614 (2023)]
Matrix Elements from LQCD
Fit Setup

\[ \mathcal{R}_{A_z}(t, \tau, q) = \frac{C_{A_z}^{3pt}(t, \tau, q)}{\sqrt{C^{2pt}(t-\tau, 0)C^{2pt}(\tau, q)}} \sqrt{\frac{C^{2pt}(\tau, 0)}{C^{2pt}(t, 0)}} \frac{C^{2pt}(t-\tau, q)}{C^{2pt}(t, q)} \]

\[ \lim_{t-\tau, \tau \to \infty} \frac{1}{\sqrt{2E_q(E_q + M)}} \left[ -\frac{q_z^2}{2M} \hat{F}_P(Q^2) + (E_q + M)\hat{F}_A(Q^2) \right] \]

\[ Q^2 = |q|^2 - (E_q - M)^2 \]

\[ \mathcal{A}_z \text{ with } q_z = 0 \implies \mathcal{R}_{A_z}(t, \tau, q) \to \sqrt{\frac{E_q + M}{2E_q}} \hat{g}_A(Q^2) \]

\[ \implies \text{No induced pseudoscalar} \]

\[ \implies \text{Simplified analysis of } \hat{F}_A(Q^2) = \hat{g}_A(Q^2) \]

\[ \implies 3\text{-state Bayesian fits to excited states} \]

\[ \implies \text{a12m130 ensemble only: } a \approx 0.12 \text{ fm, } M_\pi \approx 130 \text{ MeV, } M_\pi L \approx 3.8 \]
Correlation Function Ratio

- Horizontal: source-insertion time, centered about midpoint
- Vertical: correlator ratio $\sim$ axial matrix element
- Color: source-sink separation time; $t_{\text{sep}}/a \in \{3,\ldots,12\}$
- Colored bands: fit range
- Gray band: $\hat{g}_A$ posterior value

PRELIMINARY

source side ($p \neq 0$)
sink side ($p = 0$)
$\hat{g}_A(Q^2)$ Correlators

$RA_3(t_{ins},t_{sep})[qL/2\pi= (4,2,0)]$

$RA_3(t_{ins},t_{sep})[qL/2\pi= (3,2,0)]$

$RA_3(t_{ins},t_{sep})[qL/2\pi= (0,0,0)]$

$RA_3(t_{ins},t_{sep})[qL/2\pi= (4,3,0)]$

$RA_3(t_{ins},t_{sep})[qL/2\pi= (2,2,0)]$

$RA_3(t_{ins},t_{sep})[qL/2\pi= (1,0,0)]$
Stability – Maximum Momentum

Correlated difference with nominal fit

Systematic drift of $\hat{g}_A$ as more data added to fit

$\left(\frac{qL}{2\pi}\right)^2 = 50$ fit: 516 parameters, 1732 timeslices, 1000 samples

> 1200 eigenvalues modified by SVD cut

$\Rightarrow$ poorly conditioned covariance matrix?
Stability – Maximum Momentum

Remove subset of momenta \(\Rightarrow\) fewer data
Symptoms improve... reduce degrees of freedom further?
Fit **pairs of momenta** \( (q = 0 \text{ and one } q \neq 0) \)
Final step: drop excited state parameters,
   perform **weighted average** over \( q = 0 \) parameters,
   \( q \neq 0 \) allowed to float due to correlations but not refit

Pair fit: 60 parameters, 212 timeslices
Averaging fit, \( (qL/2\pi)^2 = 50 \): 88 parameters
Axial Form Factor Fit

Trend of high-$Q^2$ enhancement seen in other LQCD results
2–4% LQCD uncertainty vs 10% uncertainty on $D_2$ result

TODO list:

$qL/2\pi = (1, 0, 0)$ matrix element larger than expectation
Deep dive into excited states systematics, prior dependence
More momenta, $q_z \neq 0$, full set of ensembles
Free Nucleon Cross Section

LQCD prefers 30–40% enhancement of $\nu_\mu$ CCQE cross section

recent Monte Carlo tunes require 20% enhancement of QE

[Phys.Rev.D 105 (2022)] [2206.11050 [hep-ph]]

QE enhancements produce 10-20% event rate enhancement, $E_\nu$-dependent

cross section changes at ND $\neq$ effective cross section changes at FD: insufficient CCQE model freedom $\rightarrow$ bias in FD prediction
Concluding Remarks
Outlook

- Nucleon form factor uncertainty significantly underestimated in neutrino cross sections
- LQCD is a proxy for missing experimental data, potential for big impact in neutrino oscillation
- Fits to LQCD data limited by number of samples $\implies$ need to work around poorly conditioned covariance
- Excited state contamination is a significant systematics in LQCD

Thank you for your attention!
Backup
Form Factor Parameterizations

Most common in experimental literature: dipole ansatz —

\[ F_A(Q^2) = g_A \left( 1 + \frac{Q^2}{m_A^2} \right)^{-2} \]

▶ Overconstrained by both experimental and LQCD data (revisit later)
▶ Inconsistent with QCD, requirements from unitarity bounds
▶ Motivated by \( Q^2 \to \infty \) limit, data restricted to low \( Q^2 \)

Model independent alternative: \( z \) expansion [Phys.Rev.D 84 (2011)] —

\[ F_A(z) = \sum_{k=0}^{\infty} a_k z^k \quad z(Q^2; t_0, t_{\text{cut}}) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}} - t_0}} \quad t_{\text{cut}} \leq (3M_\pi)^2 \]

▶ Rapidly converging expansion
▶ Controlled procedure for introducing new parameters
Axial Radius \( r_A^2 \)

Radius related to slope: \( r_A^2 = -\frac{6}{g_A} \frac{dF_A}{dQ^2} \bigg|_{Q^2=0} \)

Good agreement with \( r_A^2 \) from experiment, poor agreement with large \( Q^2 \)

Fixing radius to agree at large \( Q^2 \) would bring radius down to \( r_A^2 \sim 0.25 \text{ fm}^2 \)

\[ \Rightarrow \text{ Incompatible with dipole ansatz} \]
Large model uncertainty, not included in world averages

- Valid only in $M_\pi \to 0$, $q \to 0$ limits
- Expansion to $O(M_\pi^2, Q^2)$:
  - restricted $Q^2$ validity
  - lacks shape freedom in $Q^2$
- Predates Heavy Baryon $\chi$PT, no systematic power counting

Modern experiments do not report $F_A(Q^2) \implies$ averages out of date
Possible argument for comparing to $r_A^2$ from low $Q^2$; high $Q^2$ untrustworthy
Effort needed to update prediction from photo/electro pion production