RELATIVISTIC LIDOV-KOZAI RESONANCE IN BINARIES

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Abstract. We consider the secular dynamics of a binary and a planet in terms of non-restricted, hierarchical three-body problem, including the general relativity corrections to the Newtonian gravity. We determine regions in the parameter space where the relativistic corrections may be important for the long-term dynamics. We try to constrain the inclinations of putative Jovian planets in recently announced binary systems of HD 4113 and HD 156846.

1 Introduction

In the recent sample of detected extrasolar planetary systems, some planets exhibit large eccentricities. It may be explained by the Lidov-Kozai resonance (LKR) acting in binary stellar systems (e.g., Innanen et al. 1997, Takeda and Rasio 2005, Verrier and Evans 2008). If the inner planetary orbit is inclined to the orbital plane of the binary, the exchange of the angular momentum between orbits may force large amplitude eccentricity oscillations of the planetary orbit, and simultaneously its argument of pericenter $\omega_1$ librates around $\pm \pi/2$. However, the LKR may be suppressed by the general relativity (GR) correction to the Newtonian gravity (NG) through changing frequencies of pericenters. Here, we focus on the non-restricted problem and relatively compact systems, and the dynamical effects of including the GR interactions in the model of motion.

2 The secular dynamics of the hierarchical triple system

We consider the hierarchical triple system. The Hamiltonian written with respect to canonical Poincaré variables (e.g., Laskar and Robutel 1995), $\mathcal{H} = \mathcal{H}_{\text{kepl}} + \mathcal{H}_{\text{pert}}$, where

\begin{equation}
\mathcal{H}_{\text{kepl}} = \sum_{i=1}^{2} \left( \frac{p_{i}^2}{2\beta_i} - \frac{\mu^*_i \beta_i}{r_i} \right), \quad \mathcal{H}_{\text{pert}} = \left( - \frac{k^2 m_1 m_2}{\Delta} + \frac{p_1 \cdot p_2}{m_0} \right) + \mathcal{H}_{\text{GR}},
\end{equation}

(2.1)

describes perturbed Keplerian motions of the inner binary (the central mass $m_0$ and $m_1$), and the outer binary ($m_0$ and more distant point-mass $m_2$), $\mu^*_i = k^2 (m_0 + m_i)$, where $k$ is

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the Gauss gravitational constant, $\beta_i = (1/m_i + 1/m_0)^{-1}$ are the reduced masses, $r_{1,2}$, are the radius vectors of $m_{1,2}$ relative to $m_0$, $p_{1,2}$ stand for their conjugate momenta relative to the barycenter, and $A = \|r_1 - r_2\|$. $H_{GR}$ stands for GR correction to the Newtonian potential of $m_0$ and $m_1$ (see, e.g., [Richardson and Kelly 1983]). We assume that the ratio of semi-major axes $\alpha = a_1/a_2 < 0.2$, and $H_{pert} \ll H_{kept}$. It means that both $m_{1,2}$ are small (planetary regime) or one of $m_{1,2} \sim m_0$ is relatively large, and one of these bodies is enough distant from $m_0$ (binary regime).

We expand $H_{NG}$ with respect to $\alpha$ and the Hamiltonian is averaged out with respect to the mean longitudes (Migaszewski and Goździewski 2008a), that leads to the secular term $H_{sec} = \langle H_{NG} \rangle + \langle H_{GR} \rangle$, where

$$
\langle H_{NG} \rangle = -\frac{k^2 m_1 m_2}{a_2} \left[ 1 + \sqrt{1 - e_2^2} \sum_{l=2}^{\infty} X^l \mathcal{R}(e_1, e_2, \omega_1, \omega_2, l) \right], \quad X = \alpha/(1 - e_2^2), \quad (2.2)
$$

$I$ stands for the mutual inclination, $\omega_{1,2}$ are the pericenter arguments, and perturbing terms $\mathcal{R}$ are derived in (Migaszewski & Goździewski, in preparation). The averaged GR term is $\langle H_{GR} \rangle = -3\beta_1 \mu_2^2 e^{-2} a_1^{-2} (1 - e_1^2)^{-1/2}$, where $c$ is the velocity of light. The expansion in Eq. (2.2) generalizes the octupole theory (e.g., [Ford et al. 2000]) and the coplanar model (Migaszewski and Goździewski 2008a). After the Jacobi’s elimination of nodes ($\Delta \Omega = \pm \pi$), we eliminate one degree of freedom thanks to the integral of the total angular momentum, $C$. Then $H_{sec} \equiv H_{sec}(G_1, G_2, \omega_1, \omega_2)$ parameterized by $C = |C|$ (or the Angular Momentum Deficit AMD $\equiv L_1 + L_2 - C$, where $L_{1,2}$, $G_{1,2}$ are the Delaunay actions) is reduced to two degrees of freedom. For $\alpha = 0.1$, the relative errors of $H_{sec}$ approximated by the 10-th order expansion do not exceed $10^{-8}$ in the relative magnitude (see Fig. [1]).

To study $H_{sec}$, we apply the representative plane of initial conditions, $\Sigma$, introduced in (Michchenko and Malhotra 2004) (Michchenko et al. 2006) which crosses all phase-space trajectories. Due to symmetries of $H_{sec}$ with respect to the apsidal and nodal lines:

$$
\frac{\partial H_{sec}}{\partial \omega_1} = \frac{\partial H_{sec}}{\partial \omega_2} = 0, \quad (\omega_1, \omega_2) \in \{ (0, 0), (0, \pm \pi), (\pm \pi/2, \pm \pi/2), (\pm \pi/2, \mp \pi/2) \}, \quad (2.3)
$$

and these conditions define the $\Sigma$-plane, $\Sigma = \{ e_1 \cos \Delta \mathcal{O}, e_2 \cos 2\omega_1 \}$, $\Delta \mathcal{O} \equiv \mathcal{O}_1 - \mathcal{O}_2$, $(e_1, e_2) \in [0, 1]$, see the left-hand panel of Fig. [2] for an illustration. Restricting $(\omega_1, \omega_2)$ to the above set, we also define $\Sigma_\mathcal{C} = \{ e_1 \sin \omega_1, e_2 \sin \omega_2 \}$, $\Sigma_\mathcal{E} = \{ e_1 \cos \omega_1, e_2 \cos \omega_2 \}$ revealing levels of $H_{sec}$ without discontinuities (Libert and Henrard 2007).

### 3 Stationary solutions and the Lidov-Kozai resonance

The equilibria of $H_{sec}$ provide much information on the structure of the phase space. In the $\Sigma$-planes, these equilibria appear as quasi-elliptic or quasi-hyperbolic (saddle) points of the levels of $H_{sec}$, according with the equations of motion:

$$
\frac{\partial H_{sec}}{\partial G_1} = 0, \quad \frac{\partial H_{sec}}{\partial G_2} = 0, \quad \text{or} \quad \frac{\partial H_{sec}}{\partial e_1} = 0, \quad \frac{\partial H_{sec}}{\partial e_2} = 0. \quad (3.1)
$$

The stability and bifurcations of equilibria in the full and in the restricted three-body problem were studied in many works (see, e.g., [Kozai 1962], [Krasinsky 1972], [Krasinsky 1974].
Fig. 1. A test of the relative accuracy of the 10-th order expansion of $H_{\text{sec}}$ and levels of $H_{\text{sec}}$ in the $\Sigma_3$-plane (see the text for details) for different values of AMD compared with the semi-analytical (exact) averaging (see Michtchenko and Malhotra 2004 or Migaszewski and Goździewski 2008b). Differences between the theories are expressed in terms of the relative log-scale.

Lidov and Ziglin 1974, Féjoz 2002, Michtchenko et al. 2006, Libert and Henrard 2007, Migaszewski and Goździewski 2008b) regarding the NG model. Here, we investigate more closely the equilibrium at the origin ($e_1 = e_2 = 0$), which is well known since Poincaré, in the presence of the GR interactions. According with the terminology of Krassinsky 1974 that is the trivial space solution of the 3rd kind ($e = 0, I \neq 0$), see Fig. 1a. The zero-eccentricity equilibrium (ZEE) is related to the maximum of $H_{\text{sec}}$ and is Lyapunov stable. For a given value of $C$, the mutual inclination of circular orbits, $i_0$, is also a maximal mutual inclination if $I_{1,2} < \pi/2$. Moreover, for some smaller $C$ (larger AMD), the origin may change its stability due to bifurcations illustrated in the $\Sigma_3$-plane (Figs. 1b,c). For instance, Fig. 1b illustrates a saddle accompanied by two elliptic points. Close to these points, the phase-space trajectories exhibit librations of $\omega_{1,2}$ around $\pm \pi/2$. This structure (see also Fig. 2) is associated with the LKR; the elliptic points may be called nontrivial, negative solutions of the 3rd kind, ($e \neq 0, I \neq 0$), as in Krassinsky 1974). They appear when $C < i_{\text{crit}}$, or, equivalently, when $i_0 > i_{\text{crit}}$ for initially circular orbits. For more details see e.g., Libert and Henrard 2007, Migaszewski and Goździewski 2008b).

Here, we restrict our calculations to $i_{\text{crit}} < \pi/2$ (the case of direct orbits), hence we do not follow the second bifurcation (Fig. 1b) appearing for $i_{\text{crit}} \sim \pi/2$, $e_1 \sim 1$. We compute $i_{\text{crit}}$ causing the stability change of ZEE in the NG model for mass ratio $\mu = m_1/m_2 \in [10^{-3}, 10^3]$, and $\alpha \in [10^{-3}, 0.2]$ (see the right-hand panel of Fig. 2). Two kinds of LK bifurcation may appear (Krassinsky 1972): at $i_{\text{crit}} \sim 40^\circ$ (the inner LKR, amplifying $e_1$) and for $i_{\text{crit}} \sim 64^\circ$ (we call it the outer LKR; e.g., in a case of a circumbinary planet). Moreover, in the planetary regime of $m_{1,2}$, $i_{\text{crit}}$ depends only on $\alpha$ and $\mu$, and not on individual semi-major axes nor masses.

4 Effects of the General Relativity correction

After introducing the $H_{\text{GR}}$ correction to $H_{\text{sec}}$, the structure of the phase space changes qualitatively (Fig. 3). We choose the same $i_0$ (a function of constant $L_{1,2}$ and $C$) for fixed
The representative plane of initial conditions, $\Sigma$. The critical inclination $i_{\text{crit}}$ in the $(\mu, \alpha)$-plane, the NG model. See the text for more details.

The $\Sigma$-plane for $m_0 = 1 M_\odot$, $a_1 = 0.5 \text{au}$, $\alpha = 0.01$, $\mu = 0.01$, and $i_0 = 60^\circ$. The left-hand panel is for the NG model, next panels are for the GR model, and $m_1 = \{1, 0.1\} m_1$, respectively.

$\alpha = 0.01$ and $\mu = 0.01$. For the NG model, a clear LKR structure appears (Fig. 3b). However, in terms of the GR model, the saddle structure may shrink (Fig. 3c), and finally, for small enough masses, it disappears (Fig. 3d). That effect may be characterized globally through the critical inclination $i_{\text{crit}} = i_{\text{crit}}(\mu, \alpha)$ for varying $m_1, a_1$ (Fig. 4). The structure of the $(\mu, \alpha)$-plane in terms of the GR model is very different from the NG case (Fig. 2). We may see three distinct regions related to the inner LKR ($i_{\text{crit}} \sim 40^\circ$), and to the outer LKR ($i_{\text{crit}} \sim 64^\circ$), smoothly passing into a new region emerging in the bottom-left corner, which is colored in yellow, where $i_{\text{crit}} \rightarrow \pi/2$, and the LKR may be totally suppressed.

5 An application to the HD 4113 and HD 156846 planetary systems

We apply the results to test a hypothesis that highly eccentric orbits of recently detected Jovian planets in HD 4113 and HD 156846 planetary systems (Tamuz et al. 2008) can be explained by the LKR resonance (in the sense considered here, i.e., of the direct orbits) forced by more distant and unseen (likely massive) objects. Indeed, the radial velocity (RV) of HD 4113 exhibits annual trend $RV_t \sim 28 \text{ ms}^{-1}$ that implies the minimal mass
Fig. 4. The critical inclination \(i_{\text{crit}}\) in the \((\mu, \alpha)\)-plane, in terms of the relativistic model. Orbital parameters of the innermost body are labeled in the respective plots, \(m_0 = 1m_\odot\).

of putative distant companion \(\sim 10m_1\) and \(a_2 \sim 10\text{au}\). A simulation of \(i_{\text{crit}}\) is illustrated in Fig. [5a]. Here, we assume that the orbit of HD 4113b is edge-on. The red lines in the \((\mu, \alpha)\)-plane mark raw limits of the orbital parameters of a putative distant object. It must be also massive enough to induce the observed RV drift, hence the skew line is for the RV, estimated under an assumption that its orbit is circular. Moreover, to force \(\max e_1 \sim 0.9\) (to generate large enough “loop” in the \(\Sigma\)-plane, see Fig. [5]), appropriately large \(i_0 < \pi/2\), \((i_0 > i_{\text{crit}})\), is required. Figs. [5b,c] illustrate \(\min i_0(\max e)\), i.e., the minimal inclination \(i_0\) for which \(e_1\) may reach given \(\max e\) (here, \(\max e = 0.9\). Figure [5b] is for the NG model and Fig. [5c] is for the GR model, respectively. In both cases, \(\min i_0(0.9) \sim 70^\circ\) and the putative body may be responsible for the detected large eccentricity of HD 4113b.

The Jovian planet HD 156846b belongs to a wide binary with \(a > 250\text{au}\) and \(m_B \sim 0.56m_\odot\). We found that the putative system is located in such a \((\mu, \alpha)\)-region, in which the LKR can be suppressed at all because \(i_{\text{crit}} \sim \pi/2\) (Fig. [5]). Moreover, \(\min i_0(0.85) \sim 66^\circ\) for the NG model (Fig. [5]), while \(\min i_0(0.85) > \pi/2\) for the GR model (Fig. [5]), also the structure of the \((\mu, \alpha)\)-plane is qualitatively different in these two cases. Hence, in the GR model, the eccentricity cannot be explained by the LKR (in the sense considered here).

6 Conclusions

Recently (Migaszewski and Goździewski 2009), we found that apparently subtle GR correction to the Newtonian model of coplanar planetary system may lead to significant, qualitative changes of the secular dynamics. In the present work, we try to extend such a quasi-global study to non-coplanar model, applying the averaging and the concept of representative plane of initial conditions. The results indicate that the 3D dynamics are also very different in the both 3D models. We continue the work on this problem.

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Fig. 5. The critical inclination $i_{\text{crit}}$ (the left-hand column), $i_{\text{max}}(\mu e)$ for the NG-model (the middle column), and $i_{\text{min}}(\mu e)$ for the GR-model (the right-hand column). The top row is for the HD 4113 system (max $e = 0.9$), the bottom row is for the HD 156846 system (max $e = 0.85$).

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