Mathematical characterization of fluid compressibility impact on fluid drive’s output parameters

A S Lunev, D A Sokolov, M V Litvinchuk, S P Dunaeva and I V Andreychikov
Siberian Federal University, 79, Svobodny Avenu, Krasnoyarsk, 660041, Russia

E-mail: Allynev@mail.ru

Abstract. All hydraulic drive fluids may contain undissolved gas to varying degrees. The urgency of an issue is induced by the fact that the high compressibility of liquid and gas mixture can affect the characteristics of fluid drives. The article concerns mathematical model of fluid drive which allows assessing the degree of influence of working fluid gas content on the character of parameter changes.

1. Mathematical model

Figure 1 shows a diagram of a stand for testing a mathematical model of a hydraulic drive with an open liquid circulation system. The diagram shows a regulated pump 2, an unregulated hydraulic motor 3 with reverse rotation of the shaft, safety valves 11 that protect the hydraulic lines from high pressures above the permissible ones (each of them may be delivery). A feed system consisting of an auxiliary pump, an overflow valve and of two check valves that protect the hydraulic lines from lower pressures (in order to avoid cavitation in the pump).

In the mathematical characterization of transients occurring in a hydraulic drive, we assume the following assumptions: the fluid flow is considered one-dimensional, the fluid temperature is constant, wave processes can be ignored, hydraulic coefficients of friction and local resistances can be determined by formulas for stationary processes, the pipeline walls are considered to be absolutely rigid.

We assume that the pump shaft is connected to the drive motor shaft by means of a coupling. In this case, the angular velocities of the engine shaft \( \omega_e \) and the pump shaft \( \omega_p \) are equal

\[
\omega_e = \omega_p, \tag{1}
\]

Motion equation of pump shaft

\[
M_e - M_p = J_{br.p} \frac{d\omega_p}{dt}, \tag{2}
\]

where \( M_e \) – engine torque to pump shaft; \( M_p \) – pump side resistance during pump shaft rotation; \( J_{br.p} \) – total moment of inertia brought to the pump shaft; \( \omega_p \) – pump shaft velocity.

The relationship of engine torque to speed can be found in the directories. Approximate engine torque to speed, let’s describe

\[
M_e(\omega_p) = M_{e,\text{max}} - K_e \omega_p, \tag{3}
\]

where \( M_{e,\text{max}} \) – maximum torque value of the engine to the pump shaft; \( K_e \) – engine speed coefficient.
Figure 1. Scheme of the bench. A hydraulic actuator with an open fluid circulation system: 1 - engine; 2 – pump; 3 – motor; 4 – tank; 5 – pressure sensors; 6 – flow meter; 7 – adjustable throttle; 8 – coupling; 9 – thermometer; 10 – ultrasonic sensors of fixation; 11 – valve.

The formula below determinates the moment resistance \( M_p \)

\[
M_p = \frac{V_p(p_2 - p_1)}{2 \pi \eta_{h.p}},
\]

(4)

where \( V_p \) – the working volume of the pump; \( p_2 \) – pump outlet pressure; \( p_1 \) – pump inlet pressure; \( \eta_{h.p} \) – hydro-mechanical efficiency of the pump.

Total moment of inertia brought to the pump shaft \( J_{e.p} \) can be defined as:

\[
J_{br.p} = J_p + J_e,
\]

(5)

where \( J_p \) – pump rotor moment; \( J_e \) – moment of inertia of the engine rotor brought to the pump shaft.

Hydromotor shaft movement equation

\[
M_m - M_r = J_{br.m} \frac{d\omega_m}{dt},
\]

(6)

where \( M_m \) – torque at the shaft of the hydraulic motor; \( M_r \) – moment of resistance brought to the hydraulic motor shaft; \( J_{br.m} \) – cumulative moment of inertia brought to the hydromotor shaft; \( \omega_m \) – angular velocity of the shaft of the hydraulic motor.

Fluid-power motor shaft’s spin moment is determined below \( M_m \).
\[ M_m = \frac{V_m(p_3-p_4)\eta_{h.m}}{2\pi}, \]  

(7)

where \( V_m \) – the working volume of the hydraulic motor; \( p_3 \) – hydromotor inlet pressure; \( p_4 \) – hydromotor outlet pressure; \( \eta_{h.m} \) – hydro-mechanical efficiency of a hydraulic motor.

The resistance moment \( M_r \) can be defined by the formula

\[ M_r(t) = M_{r0}\sin(\omega t), \]  

(8)

where \( M_{r0} \) – a constant of the moment of resistance brought to the shaft of the hydraulic motor; \( t \) – time; \( \omega \) – circular frequency.

The moment of inertia \( J_{br.m} \) can be determined by the formula

\[ J_{br.m} = J_m + J_c, \]  

(9)

where \( J_m \) – moment of inertia of the rotor of the hydraulic motor; \( J_c \) – cumulative moment of inertia of the parts and assemblies propelled by the hydraulic motor shaft.

The volumetric modules of the liquid-air \( B_{m,1}, B_{m,2}, B_{m,3} \) and \( B_{m,4} \) mixture are determined by the ratios

\[ B_{m,1} = \frac{(1-\alpha_{g1}) A_{1}(B_{1,0}+A_{P0})/(B_{1,0}+A_{P1})+\alpha_{g1}(p_0/p_2)^{1/\gamma}}{(1-\alpha_{g1}) A_{1}(B_{1,0}+A_{P0})/(B_{1,0}+A_{P1})+\alpha_{g1}(p_0/p_2)^{1/\gamma}}, \]  

(10)

\[ B_{m,2} = \frac{(1-\alpha_{g2}) A_{2}(B_{1,0}+A_{P0})/(B_{1,0}+A_{P2})+\alpha_{g2}(p_0/p_2)^{1/\gamma}}{(1-\alpha_{g2}) A_{2}(B_{1,0}+A_{P0})/(B_{1,0}+A_{P2})+\alpha_{g2}(p_0/p_2)^{1/\gamma}}, \]  

(11)

\[ B_{m,3} = \frac{(1-\alpha_{g3}) A_{3}(B_{1,0}+A_{P0})/(B_{1,0}+A_{P3})+\alpha_{g3}(p_0/p_2)^{1/\gamma}}{(1-\alpha_{g3}) A_{3}(B_{1,0}+A_{P0})/(B_{1,0}+A_{P3})+\alpha_{g3}(p_0/p_2)^{1/\gamma}}, \]  

(12)

\[ B_{m,4} = \frac{(1-\alpha_{g4}) A_{4}(B_{1,0}+A_{P0})/(B_{1,0}+A_{P4})+\alpha_{g4}(p_0/p_2)^{1/\gamma}}{(1-\alpha_{g4}) A_{4}(B_{1,0}+A_{P0})/(B_{1,0}+A_{P4})+\alpha_{g4}(p_0/p_2)^{1/\gamma}}, \]  

(13)

where \( \alpha_{g1}, \alpha_{g2}, \alpha_{g3} \) and \( \alpha_{g4} \) – volume content of gas in the liquid at the pump inlet, at the pump outlet, at the hydraulic motor inlet and at the hydraulic motor outlet, respectively; \( B_{1,0} \) – bulk modulus of the liquid at atmospheric pressure \( p_0 \); \( A \) – coefficient depending on the type of liquid and temperature; \( n \) – polytropic coefficient.

Leakage flow of the pump and the hydraulic motor can be found from the formula

\[ Q_{lf.p} = Q_p \left(1 - \eta_{v.p}\right) \frac{p_2}{p_{pn.p}}, \]  

(14)

\[ Q_{lf.m} = Q_m \left(1 - \eta_{v.m}\right) \frac{p_3}{p_{pn.m}}, \]  

(15)

where \( \eta_{v.p} \) – pump volumetric efficiency; \( p_{pn.p} \) – rated pump pressure; \( \eta_{v.m} \) – hydraulic motor volumetric efficiency; \( p_{pn.m} \) – rated hydraulic motor pressure.

Bypass valve flow can be determined by the formula

\[ Q_{v} = \mu_{v} \pi d_{v} h_{v} \sin \beta \sqrt{\frac{2(p_{v1}-p_{v2})}{\rho}}, \]  

(16)

where \( \mu_{v} \) – valve flow coefficient; \( d_{v} \) – diameter of the valve supply channel; \( h_{v} \) – opening of the valve throttle gap; \( \beta \) – cone-generating angle of the valve throttling gap; \( p_{v1} \) – the pressure at the valve inlet, can be assumed to be equal to the pump outlet pressure \( p_2 \), it can be considered equal to the pump pressure.
pressure, since the length of the pipeline connecting the valve to the pump is small and its hydraulic resistance can be ignored; \( \rho_2 \) – the pressure at the valve outlet, can be assumed to be equal to the pressure on the liquid open surface in the tank \( p_0 \), since the length of the pipeline connecting the valve to the tank is small and its hydraulic resistance can be ignored; \( \rho \) – fluid density.

Opening the valve throttle gap \( h \), can be found from the valve equilibrium equation

\[
S_v(p_{v1} - p_{v2})\psi_v - F_{sp} - c_{sp}h_v = 0,
\]

where \( S_v \) – cross-sectional area of the valve supply channel; \( \psi_v \) – trial valve coefficient; \( F_{sp} \) – spring pretension force; \( c_{sp} \) – spring constant.

The relationship between the pressure \( p_0 \) on the liquid open surface in the tank and the pump inlet pressure \( p_1 \) for the turbulent flow regime can be found from equation

\[
p_0 = \rho g z_1 + p_1 + \frac{\rho \nu_1^2}{2} + p_{l1} + \rho l_1 \frac{d\nu_1}{dt},
\]

where \( p_0 \) – the pressure on the liquid open surface in the tank; \( \rho \) – fluid density; \( g \) – acceleration of gravity; \( z_1 \) – the height of the pump inlet pipe center, measured from the liquid open surface in the tank (changing the position of the liquid open surface in the tank is ignored); \( p_{l1} \) – pressure losses in the suction line due to friction and form loss; \( l_1 \) – length of the suction line.

The relationship between the pump outlet pressure \( p_2 \) and the hydraulic motor inlet pressure \( p_3 \) for a turbulent flow regime can be found from the equation

\[
\rho g z_2 + p_2 = \rho g z_3 + p_3 + p_{l2} + \rho l_2 \frac{d\nu_2}{dt},
\]

where \( z_2 \) – the height of the pump outlet pipe center, measured from the liquid open surface in the tank; \( z_3 \) – the height of the hydraulic motor inlet pipe center, measured from the liquid open surface in the tank; \( p_{l2} \) – pressure losses in the pump line due to friction and form loss; \( l_2 \) – length of the pump line.

The system of equations (1)-(19) represents a mathematical model of a fluid drive. For computer calculations, it is more convenient to reduce the mathematical model to the Cauchy form

\[
\frac{d\omega_p}{dt} = \frac{1}{f_{br,p}} \left( M_{e,max} - K_{e}\omega_p - \frac{V_p(p_2-p_1)}{2\pi} \right),
\]

\[
\frac{dp_2}{dt} = \frac{B_{m,2}}{0.5(V_p+V_2)} \left[ \frac{V_p\omega_p}{2\pi} - S_2v_2 - Q_p(1-\eta_{v,p}) \right] p_2 - Q_v,
\]

\[
\frac{dp_1}{dt} = \frac{B_{m,1}}{(V_1+0.5v_p)} \left[ S_1 v_1 - \frac{V_p\omega_p}{2\pi} \right],
\]

\[
\frac{d\nu_2}{dt} = \frac{1}{\rho l_2} [p_2 - p_3 + \rho g(z_2-z_3) - p_{l2}],
\]

\[
\frac{d\nu_1}{dt} = \frac{1}{\rho l_1} [p_0 - p_{l1} - \rho g z_1 - \frac{\nu_1^2}{2}],
\]

\[
\frac{d\omega_m}{dt} = \frac{1}{f_{br,m}} \left[ \frac{V_m(\nu_3-p_4)\eta_{m}}{2\pi} \right] p_3 - M_T,
\]

\[
\frac{dp_3}{dt} = \frac{B_{m,3}}{0.5(V_2+V_m)} \left( S_2v_2 - \frac{V_m\omega_m}{2\pi} - Q_m \right) \left( 1 - \frac{\eta_{v,m}}{\eta_{m}} \right) \frac{p_3}{p_{pn,m}},
\]

\[
\frac{dp_4}{dt} = \frac{B_{m,4}}{(0.5 V_m+V_3)} \left( \frac{V_m\omega_m}{2\pi} - S_3v_3 \right),
\]

\[
\frac{d\nu_3}{dt} = \frac{1}{\rho l_3} \left( \rho g z_3 + \frac{\nu_3^2}{2} + p_4 - p_0 - p_{l3} \right).
\]

Figure 2 shows the results of calculations of a fluid drive mathematical model with an open liquid circulation.
Figure 2. Dependence of the pump outlet pressure on time for two values of the undissolved gas content in the liquid.

2. Conclusion
It follows from the graphs that an increase in the amount of undissolved gas content in the working fluid affects the character of changes in the fluid drive parameters. When the undissolved gas content increases from 0 to 19%, a local maximum appears on the graph of the pump outlet pressure.

References
[1] Lunev A S, Nikitin A A, Kaizer Y F, Lysyannikov A V, Sokolov D A and Obvintseva V Y 2019 Comparative analysis of the dependence of the bulk elastic modulus of the liquid on pressure and gas factor J. Phys.: Conf. Ser. 1399 055083
[2] Lunev A S, Nikitin A A, Ionova V A, Novik A V and Kramarenko V A 2020 Apparatus for controlling the amount of undissolved gas in the hydraulic fluid J. Phys.: Conf. Ser. 1515 052012
[3] Danilov Y A, Kirillovskii Y L and Kolpakov Y G 1990 Equipment of Fluid Power Drive: Work Processes and Characteristic (Moscow: Machinery Construction)
[4] Gorbeshko M V 1997 Development of Mathematical Models for the Hydraulic Machinery of Systems Controlling the Moving Components of Water Development Works Hydrotechnical construction 31(12) 745-50
[5] Nikitin A A and Mandrakov E A 2014 Influence of undissolved gas in the hydraulic liquid on the dynamics of logger’s hydraulic drive Proc. of Tomsk Polytechnic University. Math. and mech. Phys. 325(2) 65-71
[6] Rabie M G 2009 Fluid Power Engineering (New York City: McGraw Hill Professional)