Torque Calculation of Five-Phase Synchronous Reluctance Motors With Shifted-Asymmetrical-Salient-Poles Under Saturation Condition

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Abstract—In traditional analytical method (AM), the magnetic saturation is always ignored to simplify the calculation process. However, synchronous reluctance motors (SynRMs) often operate around saturation point to achieve higher torque density. Therefore, a new AM is proposed, in which the saturation of stator iron has been considered. The key of the proposed method includes a saturation factor, and an iterative method is adopted to compute the saturation factor in the SynRM by increasing the air-gap length. Especially, the proposed AM can be applied to a SynRM even with shifted-asymmetrical-salient-poles. In the process of AM, the expression of stator magnetomotive force (MMF) is built firstly. Additionally, the air-gap density including slotting effect and salient-poles is calculated. Then, the rotor MMF under saturation of the stator iron is obtained. Therefore, the precision of the instantaneous torque can be improved significantly. Eventually, by the verification of finite elements method (FEM) and experiments, the torque performance of SynRMs with shifted asymmetrical rotor can be predicted accurately by the proposed AM.

Index Terms—Synchronous reluctance motors, torque, shifted asymmetrical salient pole, analytical method, saturation.

I. INTRODUCTION

With the continuous increasing of the price of permanent magnets, the machines with less or none of rare-earth materials have received widespread attention. Among them, synchronous reluctance motors (SynRMs) have been widely used due to their low cost, wide flux-weakening capability, high fault tolerant capability and acceptable torque density [1], [2]. However, the fatal flaw of SynRMs is high torque ripple. Since the average torque of SynRMs increases by making the operating point to be saturated, the torque ripple becomes worse. Hence, the reduction of torque ripple for SynRMs is imperative.

For the sake of reducing the torque ripple of SynRMs, many methods have been presented. Some papers focus on optimized control [3], [4], while the majorities are absorbed in the design of parameters of SynRMs or the structure of SynRMs [5], [6]. On the basis of multi-objective genetic optimization and finite elements method (FEM), the optimal design of SynRMs was proposed in [6]. However, it cannot reveal the principle of torque ripple reduction well, and the computation cost of FEM is very high. Hence, analytical methods (AMs) are proposed to reveal the mechanism of torque and the reduction of torque ripple [7] because AMs are beneficial to build the relationship between the parameters and the torque performance. Various AMs are put forward to evaluate torque and torque ripple, such as the Maxwell Stress Tensor [8]-[10], the winding function theory [11], [12] and the Lorentz force law [13]-[15]. First of all, subdomain field model [8] and magnetic vector potential method [9] found on the Maxwell Stress Tensor were aimed to the calculation of the air-gap flux density. But, the two methods need heavy calculation, and they are mostly applied to permanent-magnet motors. Then, a semi-numerical method [10] was proposed to calculate the harmonic torque components in SynRMs, but the radial and tangential air-gap flux density are obtained by FEM rather than AM. Meanwhile, on the basis of the winding function theory, the inductance matrix of the motor is obtained. After that, the instantaneous torque was calculated by the energy formula [11]. However, in this method, the magnetic permeability of stator and rotor is assumed as infinity. Therefore, when the motor is saturated, the results of the method are much larger than the actual values. Besides, an analytical method found on the Lorentz force law was proposed to take the saturation of stator and rotor into account [13]. Through the method, the air-gap flux density can be computed correctly, then high-precision torque, torque ripple and the loss are obtained. However, this method is only applied to the SynRM with symmetrical rotor.

This paper proposes an AM of torque calculation for SynRMs with shifted-asymmetrical-salient-poles based on the Lorentz force law, in which the magnetic saturation and stator slotting effect are considered. For computing the torque, the
expressions of stator MMF, the air-gap flux density and rotor MMF are listed. Meanwhile, the saturation and the slotting effect are considered, which improve the accuracy of calculated torque. Besides, the results of AM are validated through FEM and experiment. In part II, the topologies and features of SynRMs will be shown. In part III, stator MMF, the process of saturation of stator, air-gap flux density and rotor MMF will be calculated by AM, and the derivation of torque and torque ripple will be given. Besides, values of AM and FEM will be compared to verify the correctness of AM in part IV. Moreover, the result of the experiment will be shown in part V. Eventually, in part VI, the conclusion will be drawn.

II. TOPOLOGIES AND FEATURES

The topologies of studied SynRM with shifted asymmetrical rotor have been proposed in [11], and the two-dimensional model of the studied SynRM is shown in Fig. 1(a). The number of stator slots is 40, rotor poles is 8, and the winding is single layer, integral slot and distributed. The SynRM with shifted-asymmetrical salient-poles can be received in three steps:

1) First, a SynRM with symmetrical salient-poles is built.
2) Second, the asymmetrical rotor is derived in the rational combination of unequal pole to reduce the second order torque ripple.
3) Eventually, the asymmetrical rotor structure is selected as whole, and the shifting angle is 4.5 mechanical degrees, which aims to cut down the first order torque ripple.

The symmetrical, asymmetrical and shifted asymmetrical rotor structures are listed in Fig. 1(b)-(d). The major parameters are given in Ref. 11.

III. AM FOR SYNRM S

In this part, the method for calculating torque is proposed based on the Lorentz force law. First, the stator MMF is obtained by the winding function \( N(\theta) \) and the current excitation. Then, the air-gap flux density is computed through considering the stator slotting effect and the structure of the rotor. Besides, an iterative method is put forward to solve the saturation of the stator iron and compute the correct saturation factor. Furthermore, the rotor MMF is calculated by the stator MMF and the air-gap flux density which considered the stator slotting effect and the saturation of stator iron. Finally, the torque is calculated according to the harmonics of the stator MMF and the rotor MMF.

A. Stator MMF

![Fig. 2. (a) The principle of the winding function. (b) Turn function of phase E.](image)

The principle of winding function is given in Fig. 2(a), and the corresponding winding function waveform is given in Fig. 2(b). The amplitude \( N \) is the half number of turns per slot. Besides, Fourier series of the winding function \( N(\theta) \) are computed as follows:

\[
N(\theta) = \sum_{h=1,1,5} N_h \cos(h \theta)
\]

where \( h \) is the harmonic order, \( N_h \) is the \( h \)th-order stator winding harmonic, and \( \theta \) represents the angular position (elec. deg.) under the stator reference frame, which is based on the phase E-axis. In Fig. 2(b), the winding function of phase E is symmetrical, and the winding function of phase A, B, C, D are the same as the phase E, and just displacement is \( 2\pi/5 \) elec. deg. respectively.

The current of the phase E is computed as follows:

\[
I_E(\alpha) = I_m \cos(\alpha - \delta)
\]

where \( I_m \) is the value of peak current, \( \alpha t \) is the instantaneous position of rotor, and \( \delta \) is the current angle which is measured from the \( d \)-axis.

According to (1) and (2), the stator MMF is computed as:

\[
F_s = \sum_{k=A,B,C,D,E} N_k(\theta) I_k
= \sum_{h=1,1,5} \frac{5N_i}{2} \cos(h \theta \pm \alpha \mp \delta)
= \sum_{h=1,1,5} F_{sh} \cos(h \theta \pm \alpha \mp \delta)
\]

where \( F_{sh} \) represents the \( h \)th-order of stator MMF, while the value is simplified as:

\[
F_{sh} = \begin{cases} 
\frac{5N_i}{2} & h = 10m \pm 1 (m = 1,2,3,\ldots) \\
0 & h \neq 10m \pm 1 (m = 1,2,3,\ldots)
\end{cases}
\]
B. Air-gap Flux Density

The air-gap flux density considering the structure of stator and rotor is calculated as follows:

\[ B_g = \frac{F}{g} \]  

(5)

\[ B_g' = B_g \Lambda \]  

(6)

where \( \mu_0 \) represents the permeability of a vacuum, \( g \) is the length of the air-gap, \( \Lambda \) is the ratio of the air-gap length \( g \) and the equivalent air-gap length \( l'(\theta) \), which is based on the structures of the stator and rotor. Then, the expression of \( \Lambda \) is as follow:

\[ \Lambda(\theta) = \frac{g}{l'(\theta)} = \frac{g}{g + l_1(\theta) + l_2(\theta - \gamma)} \]  

(7)

where \( l_1(\theta) \) is the function of length of flux lines in stator slots in Fig. 3(a). Meanwhile, in Fig. 3(b), the function of \( l_2(\theta - \gamma) \) is similar to \( l_1(\theta) \). However, \( l_2(\theta - \gamma) \) is obtained by the length of the flux lines in the rotor. Since the flux lines in stator and rotor pass through the smallest reluctance path, and then, the formula of \( l_1(\theta) \) is:

\[ l_1(\theta) = \begin{cases} 
0, & 0 \leq r_1 \theta \leq k_1 \\
\frac{\pi}{2}(r_1 \theta - k_1), & k_1 \leq r_1 \theta \leq k_3 \\
\frac{\pi}{2}(r_1 \theta - k_1) + (r_1 \theta - k_1 - h_s) \varepsilon, k_3 \leq r_1 \theta \leq k_3 \\
l_1(2k_3 - r_1 \theta), & k_3 \leq r_1 \theta \leq \frac{2\pi r_1}{N_s} 
\end{cases} \]  

(8)

\[ \varepsilon = \frac{\pi}{2} - \arctan \left( \frac{h_s}{(h_s - h_r) / 2} \right) \]  

(9)

where \( r_1 \) represents the inner radius of stator, \( N_s \) represents the number of the stator slots, the \( b_0, b_1, h_s, h_r \) and \( \varepsilon \) are the parameters of slot in Fig. 3(b). Similar to \( l_1(\theta) \), \( l_2(\theta - \gamma) \) is:

\[ l_2(\theta - \gamma) = \begin{cases} 
0, & 0 \leq \theta - \gamma \leq m_1 \\
\frac{\pi}{2} r_2 (\theta - \gamma - m_1), & m_1 \leq \theta - \gamma \leq m_2 \\
h_m, & m_2 \leq \theta - \gamma \leq m_3 \\
\frac{g}{2}(2m_3 - (\theta - \gamma)), m_3 \leq \theta - \gamma \leq \frac{\pi}{p} 
\end{cases} \]  

(10)

where \( r_2 \) is the rotor outer radius, \( m_2, m_4 \) are the locations where the length of arc is equal to the depth of the rotor slot.

Fig. 4 describes the structure of symmetrical rotor under one pole-pair and its corresponding function diagram of \( l_2(\theta - \gamma) \). The width of the rotor teeth is \( 2\varepsilon_1 \). Then, the relative air-gap permeance function can be obtained by (7) and (9), which can be shown in Fig. 6. Fig. 7 describes the asymmetrical rotor structure under one pole-pair, in which one of the rotor teeth is \( 2\varepsilon_1 \) and another is \( 2\varepsilon_2 \). The corresponding \( l_2(\theta - \gamma) \) is given in Fig. 8. Similar to the symmetrical and asymmetrical rotor structure, the shifted asymmetrical model under two pole-pair is shown in Fig. 9, and the corresponding \( l_2(\theta - \gamma) \) is given in Fig. 10. With regard to the asymmetrical and shifted asymmetrical, the relative air-gap permeance function is similar to that under the symmetrical rotor structure.
C. Magnetic Saturation

Magnetic saturation is ignored in most AMs, which lead to higher air-gap flux density when the electrical loading increases. Fig. 11 shows the flux densities of three rotor structures under rated current. It can be observed that the saturation of stator is serious, while the saturation of rotor can be ignored. Besides, if the saturation of rotor cannot neglect, the same iterative method can be applied to the rotor. Hence, only the saturation of stator iron needs to be considered, and the same iterative method can be applied to the rotor. Hence, only the saturation factor ($K_{sat}$) is employed in the proposed AM, which can be seemed as increasing the appropriate length of the air-gap.

In stator iron, an iteration approach is applied to calculate the $K_{sat}$. When computing the saturation of stator iron needs to be considered, and the same iterative method can be applied to the rotor. Hence, only the saturation factor ($K_{sat}$) is employed in the proposed AM, which can be seemed as increasing the appropriate length of the air-gap.

The magnetic flux which flows through each stator tooth is calculated through integrating the air-gap flux density $B'_y$, the formula is as follows:

$$\Phi_y = \int_{\theta_y}^{\theta_y+2\pi} B'_y(\theta) r_y L_{sl} d\theta, \quad (11)$$

where $\theta_y$ is the angle of the first stator slot center under mechanical period, $\omega_{slot}$ is the slot mechanical angle.

Afterwards, the flux density of each tooth is as follows:

$$B_y = \Phi_y / (\omega h_t) \quad (12)$$

The value of $H_i$ can be obtained by the B-H curve, in which the $B_y$ is received by (12). Therefore, the magnetic voltage drop $U_{yi}$ in $i$th is computed as:

$$U_{yi} = H_i h_t \quad (13)$$

In Fig. 12, it is obviously that the flux $\Phi_{y1}$ which flows through the yoke between the first and second teeth is the same flux of the first tooth $\Phi_{t1}$.

$$\Phi_{y1} = \Phi_{t1} \quad (14)$$

Besides, the fluxes through the yoke between other teeth can be obtained as:

$$\Phi_{yi} = \Phi_{ti} / (L_{sl} h_t) \quad (15)$$

The flux density in the part of stator yoke is computed as:

$$B_{yi} = \Phi_{yi} / (L_{sl} h_t) \quad (16)$$

The method to obtain $H_{yi}$ is the same as $H_{ti}$, and magnetic voltage drop $H_{yi}$ in $i$th yoke is computed as:

$$U_{yi} = H_{yi} h_t \quad (17)$$

where $l_y$ is the flux path length computed as:

$$l_y = \pi (2 r_s - h_y) / Q_y \quad (18)$$

Through summing the $U_{yi}$ in the yoke of stator, the magnetic voltage drop in all flux paths $U_{pathyi}$ is obtained. Meanwhile, the calculation of $\Phi_{y1}$ should be consistent with the first tooth of each pole.

According to $U_{yi}$ and $U_{pathyi}$, the saturation factor of each stator tooth $K_{sat}$ is an intermediate quantity in the iteration approach, which is received by:

$$K_{sat} = 1 + \frac{U_{yi} + U_{pathyi}}{H_{yi} h_t} \quad (19)$$

where $H_{yi}$ is the field intensity in front of each stator tooth.

$$K_{sat-var} = K_{sat-new} \cdot K_{sat-i} \quad (20)$$

$$K_{sat-pre} = K_{sat-i} \quad (21)$$

$$K_{sat-i} = K_{sat-pre} \cdot K_{sat-step} \quad (22)$$

$$B_{sat-var} = B_{sat-i} \quad (23)$$

At the first of the flowchart, the initial value of $K_{sat}$ and the intermediate value $K_{sat-pre}$ are set as 1, the step $K_{sat-step}$ is set as 1.001. It means that the air-gap length is uncharged and the saturation isn’t considered. Then, compute the $K_{sat-new}$ in (19) and the ratio $K_{sat-var}$ in (20). In the rest part of the iteration, for example, when compute the saturation of the first stator tooth, judge the ratio $K_{sat-var}$ is greater than 1 or not. If $K_{sat-var}$ is greater than 1, then calculate the expressions (21) and (22). It makes the old $K_{sat}(i)$ multiply a step $K_{sat-step}$, which intends to reduce the ratio $K_{sat-var}$. Next, judge the ratio $K_{sat-var}$ again, if the ratio $K_{sat-var}$ is less than 1, give the computed value of $K_{sat-pre}$ to $K_{sat}(i)$ and the $K_{sat}(i)$ is the value of saturation of the first stator tooth. At last, the air-gap flux density considered the saturation of stator iron is calculated in (23). The approach of iteration is shown in Fig. 13. And the $K_{sat-var}$ of the second tooth under 11A is shown in Fig. 14.
D. Rotor MMF

The rotor MMF is obtained by the stator MMF, and the air-gap flux density which takes the stator slotting effect and the structure of rotor into account. Since the saturation of stator iron has been considered, which makes the result of rotor MMF more precise.

Fig. 15 explains a sketch under one rotor pole, while the rotor slot can be seen as a flux barrier and the corresponding magnetic network can be described in Fig. 16. In the magnetic network, the rotor MMF is computed as:

$$ F_r = r \frac{1}{L} \int_{\frac{\pi}{2\theta_s}}^{\frac{\pi}{2\theta_s} + \theta_r} U_r - \frac{F_r}{g} d\theta $$

(24)

where $\theta_h$ is half angle of the flux-barrier, $\theta_m$ is related to the rotor position, $\theta_s$ is the angular coordinate in the stator stationary reference frame. $t$ and $l$ are the height and length of the flux barrier. $r$ is the inner stator radius. (In this part, the flux-barrier is equivalent to a rectangular.)

Then, the distributions of equivalent rotor MMF schematic diagrams of three models are given in Figs. 17-19. $F_{rm1}$, $F_{rm2}$ and $F_{rm3}$ are amplitudes of rotor MMF.

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**Fig. 12.** The distribution of flux lines in stator.

**Fig. 13.** The flowchart of calculation the saturation factors in stator.

**Fig. 14.** The convergence of $K_{sat\_st}$.

**Fig. 15.** Planar geometry of one rotor pole.

**Fig. 16.** Magnetic network of the one pole of the symmetrical motor.

**Fig. 17.** Equivalent rotor MMF schematic diagram of symmetrical model under one pole-pair.

**Fig. 18.** Equivalent rotor MMF schematic diagram of asymmetrical model under one pole-pair.
The torque is computed based on the Lorentz force law, and the torque formula obtained by the harmonics of stator MMF and rotor MMF can be expressed as:

\[ T = \frac{\mu_0}{g} p r_g L \pi \sum_{h=1,2,3...} \left\{ h F_{s h} F_{rh} \sin \left( (h \pm 1) \alpha \delta \right) \right\} \]  \hspace{1cm} (25)

where \( p \) represents the number of pole-pairs, \( r_g \) represents the air-gap radius, \( L \) represents the stack length, and \( F_{s h} \) represents the \( h \)-th-order harmonic of rotor MMF. The formulae of the average torque and torque ripple of SynRMs are shown as follows:

\[ T_{avg} = \frac{\mu_0}{g} p r_g L \pi F_{s 1} F_{r 1} \sin (\delta) \]
\[ T_{ripple} = -\frac{\mu_0}{g} p r_g L \pi \sum_{n=1,2,3...} \left\{ F_{s n} F_{r n} \sin \left( (h \pm 1) \alpha \delta \right) \right\} \]  \hspace{1cm} (26)

where \( F_{s 1}, F_{r 1} \) represent the fundamental harmonic of stator MMF and rotor MMF.

IV. VERIFICATION BY FEM

In this part, the air-gap flux density and instantaneous torque waveforms obtained by the AM and FEM are compared, which is aimed at verifying the correctness of the proposed AM.

A. Air-gap Flux Density

Figs. 20-22 illustrates the air-gap flux density and key harmonics of the air-gap flux density obtained by FEM and AM under symmetrical model, asymmetrical model and shifted asymmetrical model. In Fig. 20, the fundamental harmonics of air-gap flux density under FEM and AM are 0.676 and 0.681 respectively, and the error of the third harmonic order between FEM and AM is 0.04. Meanwhile the main fundamental components in the SynRM with asymmetrical rotor under FEM and AM are 0.68 and 0.67 as shown in Fig. 21. Additionally, the first harmonic order of the shifted asymmetrical rotor is 0.50 and 0.46 under FEM and AM, the error of the first harmonic order between FEM and AM is 0.04 in Fig. 22. Although the waveforms of air-gap flux density computed by FEM shows some differences compared with the results obtained by AM, the errors of the key harmonics between FEM and AM are small.

In Fig. 23, the air-gap flux density of the asymmetrical SRM under different AMs is compared. Among them, AM means that the stator slotting effect and saturation of stator iron are considered. Nevertheless, in the AM1, the stator slotting effect and saturation of stator iron are not considered, AM2 just considers the stator slotting effect and the saturation of stator iron is ignored [11]. In Fig. 23(a), the matching of two methods is very weak, which is caused by the stator slotting effect. Besides, Fig. 23(b) illustrates that the ignorance of the saturation of stator iron makes the peak values higher than that of AM. Hence, ignoring stator slotting effect and saturation of stator iron results in the inaccurate calculation of the air-gap flux density. Further, it affects the accuracy of instantaneous torque and causes the larger errors of torque harmonics.

B. Torque

Figs. 24-26 show the instantaneous torque waveforms and
key harmonic of instantaneous torque at \( I_{\text{max}} = 11 \) A and \( \delta = 45^\circ \) of the three models. The average torque of the SynRMs calculated by FEM is 5.75 Nm, 5.77 Nm and 5.48 Nm, respectively. Meanwhile, the values received by AM are 5.53 Nm, 5.60 Nm and 5.42 Nm, respectively. Besides, the error of fundamental harmonic is 0.46, 0.05 and 0.04, respectively. Moreover, the torque ripple in three SynRMs are 91.60\%, 44.4\%, and 10.94\%, respectively. Hence, the proposed AM offers high calculation accuracy, and the torque ripple of the SynRM with shifted asymmetrical rotor is decreased greatly.

Fig. 24. Torque performance of symmetrical SynRM. (a) Instantaneous torque waveform. (b) Key harmonic of the instantaneous torque.

Fig. 25. Torque performance of asymmetrical SynRM. (a) Instantaneous torque waveform. (b) Key harmonic of the instantaneous torque.

Fig. 26. Torque performance of shifted asymmetrical SynRM. (a) Instantaneous torque waveform. (b) Key harmonic of the instantaneous torque.

Fig. 27 reveals the torque-angle function of SynRM with shifted asymmetrical rotor at 11 A. It is obviously that the torque-angle performance obtained by AM is consistent with that of FEM.

V. EXPERIMENTAL VERIFICATION

The SynRM with shifted asymmetrical rotor is set up and measured to verify the correctness of the proposed AM. Fig. 28 reveals the prototype of the machine.

For the sake of measuring torque ripple correctly, the test speed demands to be selected cautiously. In this platform, the cut-off frequency of the torque sensor is 1 kHz. When the speed is 30 r/min, the frequency of one electrical cycle is 2 Hz. At the same time, the first-order and second-order of torque ripple are 10 and 20. Therefore, the maximum frequency of one cycle of the torque ripple is 40 Hz, and it is much smaller than the cut-off frequency. Thus, the instantaneous torque and current at different current angles can be measured under 11 A at 30 r/min. Fig. 29 reports the measured torque performance under 11 A at 30 r/min. Then, the torque comparison among experiment, FEM and AM at 11 A is shown in Fig. 30. It can be observed that the average torque is about 5.26 Nm and the peak-peak torque is 0.65 Nm under experiment, and that values are 5.28 Nm and 0.6 Nm under FEM, while the values are 5.29 Nm and 0.62 Nm under AM. In conclusion, the proposed AM offers high predictable precision. Compared with Fig. 24-26, the measured result become lower because of the island (as shown in Fig. 15) of outside rotor is not considered in Section IV.

In Fig. 31(a), the torque-angle performance computed by AM under 6 A is consistent with that of the FEM and experiment. With the increase of saturation under 11 A, there are some differences between the AM and FEM. Nevertheless, compared with the AM without considering saturation [11], the predicted accuracy is improved significantly. Meanwhile, the torque ripple obtained by FEM, AM and experiments is shown in Fig. 31(b). It is obviously that the AM result agrees with the FEM and experimental ones while the current angle is between 30 to 50. Hence, the proposed AM is effective to calculate the torque performance of SynRMs with shifted-asymmetrical-salient-poles.

Fig. 28. Proto type machine and experiment platform. (a) Prototype of SynRM with shifted asymmetrical rotor. (b) Experimental platform.

Fig. 29. Measured steady torque performance under 11 A at 30 r/min (1 Nm/div, 5A/div).

Fig. 30. Torque comparison among AM, FEM and experiment under 11 A.
VI. CONCLUSION

In this paper, a new torque analytical method according to the Lorenz force law has been presented for SynRMs with shifted-asymmetrical-salient-roples, in which the saturation of the stator iron is taken into account. In the prediction of torque, the theoretical stator MMF is generated by the winding function theory, and the rotor MMF is generated by air-gap flux density and the stator MMF. In the process of the calculation of these MMFs, the saturation of the stator iron and the stator slotting effect have been included. Then, in comparison with AM, FEM and the experiments, the errors of them are within acceptable range, which verified the significance of the proposed AM. Additionally, the AM also can be applied to a conventional SynRM.

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