Multiplicity distributions associated to subthreshold events in heavy-ion collisions

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Abstract

Subthreshold events (pion production, for instance) at energies $E < m_{\pi}$ are treated as rare events. The associated multiplicity distribution $P_c(n)$ and the unconstrained distribution $P(n)$, when there is no rare event-trigger, are related by the model independent relation

$$P_c(n) = \frac{n^2}{\langle n^2 \rangle} P(n).$$

This relation, and in particular its improved version inspired in clustering of nucleons, is in fair agreement with data. Moreover, it allows to extract from the multiplicity data information on the number of nucleons involved in meson production processes, occurring at energies below the production threshold in free nucleon collisions.
Production of particles, pions or other hadrons, energetic photons or fast protons, in nucleus-nucleus collisions at energies per nucleon well below the free nucleon-nucleon threshold for such production \cite{1} gives clear evidence for nucleonic correlations in nuclear matter. These correlations may simply reflect the fermionic nature of the nucleons or they may be seen as true collective effects, like clustering of nucleons (see \cite{2} for a review).

By clustering processes we mean here pion production mechanisms involving sub-systems with a number of nucleons between 2 and the mass number $A$, the simplest example being the interaction of a nucleon with a deuteron. In fact, schematically we can represent pion production in the $pd \rightarrow \pi^0pd$ reaction via resonance formation as in the diagram of Fig.1a. The threshold for the mentioned reaction is lower than the free nucleon threshold (Fig.1b). In contrast to this diagram, the one of Fig.1a is kinematically allowed at subthreshold production energies, because the two nucleons of the deuteron are correlated. Basic diagrams \cite{3-6}, like the one in Fig.1b, require a medium to be non vanishing, for energies below the production energy threshold.

Higher mass resonances can be obtained if larger clusters supply the required energy. Our emphasis here is on clustering rather than on the specific mechanism of pion production (like higher-mass resonances \cite{7,8}).

In order to see, in a simplified manner, what the problem is, let us write the laboratory energy $E_L$ of nucleus $A$ as

$$E_L = A \langle E \rangle_N,$$ \hfill (1)

where $\langle E \rangle_N$ is the average nucleon energy. If one assumes that a collision results from the superposition of nucleon-nucleon collisions and ignores Fermi momentum and binding energy, the threshold kinetic energy per nucleon to produce, say, a pion is,

$$E_{th} \equiv \langle E \rangle_N - \frac{m_N}{2m_N} = 2m_N \left[ (1 + \frac{m_\pi}{2m_N})^2 - 1 \right] \approx 2m_\pi \approx 280 \text{ MeV}. \hfill (2)$$

This is the free nucleon threshold. In nuclear matter one may have nucleons with energy above $\langle E \rangle_N$ and the threshold becomes lower than (2).

The lowering of the threshold (2) can be very easily visualized if one allows for clustering of nucleons. If $\alpha \leq A$ nucleons of nucleus $A$ — with mass number $A$— collide with one nucleon of nucleus $B$ — with mass number $B$ — (or vice-versa) the threshold energy per nucleon becomes,

$$E_{th} \approx \frac{1 + \frac{\alpha}{\alpha} m_\pi}{\alpha} \geq m_\pi \approx 140 \text{ MeV} . \hfill (3)$$

This is, of course, the threshold for free nucleon-nucleus collisions.

As experimental production of pions occurs even at energies bellow $m_\pi$, this requires clustering from both nuclei, $\alpha \leq A$ and $\beta \leq B$,

$$E_{th} \approx \frac{\alpha + \beta}{\alpha \beta} m_\pi , \hfill (4)$$

with the absolute threshold, naturally occurring for $\alpha = A$ and $\beta = B$,

$$E_{th} \approx \frac{A + B}{AB} m_\pi . \hfill (5)$$

In the case of $^{12}C-^{12}C$ interactions, for instance, this threshold corresponds to $\approx 23$ MeV. The binding energies will affect this value within less than a keV, provided they
are considered simultaneously in the final and initial states. This lower bound cannot be obtained in a simple way from Fermi-motion based arguments.

Independently of the underlying model for the nucleus-nucleus collisions, the point we would like to make is that these sub NN threshold events are rare, in the sense that their probability of occurrence is very small. While total inelastic cross-sections are of the order of several millibarn, the cross-sections we are talking about here are of the order of the microbarn or nanobarn. In other words, by imposing kinematic restrictions through lowering the energy available in the system, one moves to the tails of the fermionic distributions or requires simultaneous clustering, and the events become rare, the probability of occurrence being very small.

It is interesting to note that as one unconstrains the kinematics, i.e., increases the energy, the π production cross-section increases very rapidly reaching the millibarn values for \( E_{th} > 2m_\pi \) (see Fig.2)). That is the region of (free) nucleon-nucleon interactions where the Glauber approach becomes valid.

We shall next make a short discussion on rare events (see references [9]), adapted to the present situation, where we are interested in clustering processes in both colliding nuclei. If in a multi nucleon-nucleon collision process, \( \nu \) elementary collisions occur, and \( \tau_c \) is the probability of clustering nucleons, and \( N(\nu) \) is the number of events with \( \nu \) elementary collisions, we can write the identity

\[
N(\nu) = \left[ \sum_{k=0}^{\nu} \binom{\nu}{k} \tau_c^k (1-\tau_c)^{\nu-k} \right] \left[ \sum_{k'=0}^{\nu} \binom{\nu}{k'} \tau_c^{k'} (1-\tau_c)^{\nu-k'} \right] N(\nu) .
\]  

(6)

The meaning of (6) can be easily grasped by expanding in powers of \( \tau_c \) to obtain,

\[
N(\nu) = \left[ (1-\tau_c\nu)^2 + 2(1-\tau_c\nu)(\tau_c\nu) + (\tau_c\nu)^2 + \ldots \right] N(\nu) .
\]  

(7)

In this sum the first term means the normal contribution (Glauber-type, i.e. no clustering in both nuclei, giving \( E_{th} \sim 2m_\pi \)), the second term means cluster in one of the nuclei (reducing \( E_{th} \) to \( E_{th} \sim m_\pi \)) and the third term, \( N_c(\nu) = \tau_c^2 \nu^2 N(\nu) \), represents what interests us, namely nucleon clustering in both nuclei (allowing \( E_{th} < m_\pi \)). The subsequent contributions, corresponding to multiple clustering are negligible. In a sense, the events described by (8) are doubly-rare events.

If one makes the reasonable assumption that the number \( \nu \) of collisions is a measure of the number \( n \) of produced particles, from (8) we obtain the \( \tau_c \) independent, universal relation

\[
P_c(n) = \frac{n^2}{\langle n^2 \rangle} P(n) ,
\]  

(9)

where \( P(n) \) is the unconstrained particle multiplicity distribution, \( \langle \ldots \rangle \) denotes the average value with respect to the \( P(n) \) distribution and \( P_c(n) \) is the multiplicity distribution associated to the rare event (π emission below the proton-nucleus threshold).

Note that

\[
\Sigma_{n=0} P_c(n) = \Sigma_{n=0} P(n) = 1 ,
\]  

(10)
and that [9]

\[ \langle n \rangle_c = \frac{C_3}{C_2} \langle n \rangle , \tag{11} \]

where \( C_q \equiv \langle n^q \rangle / \langle n \rangle^q \). Equation (11) implies that

\[ \langle n \rangle_c > \langle n \rangle . \tag{12} \]

It is also clear that

\[ P_c(n) < P(n) \iff n^2 < \langle n^2 \rangle , \]
\[ P_c(n) > P(n) \iff n^2 > \langle n^2 \rangle , \tag{13} \]

i.e., the distributions must cross at some point \( n \), corresponding to \( n^2 = \langle n^2 \rangle \).

In Fig. 3 we test Eq.(9) for the particular case of the data of ref. [10] for \(^{36}\)Ar on \(^{27}\)Al collisions. The data \( P(n) \) [10] on the inclusive charged particles distribution (nucleons, etc...) is shown by the open circles. The data \( P_c(n) \) [10] on the equivalent distribution when a sub-threshold pion, at \( E = 95 \) MeV/Nucleon, is detected is shown by the open squares. Qualitatively, equation (9), implying equations (11) and (12), satisfies the data: the average multiplicity, with a trigger on \( \pi \), is larger, \( \langle n \rangle_c \simeq 9 \) while \( \langle n \rangle \simeq 4.4 \) [11], the two distributions cross at \( n^2 = \langle n^2 \rangle \simeq 49 > \langle n \rangle^2 \simeq 19.4 \) (the inequality \( \langle n^2 \rangle > \langle n \rangle^2 \) has to be satisfied), the distribution is independent of the rare event trigger [12].

For the description of \( P(n) \) (full line) the generalized gamma function was used

\[
P(n) = \frac{1}{\langle n \rangle} \frac{\mu}{\Gamma(k)} \left[ \frac{\Gamma(k + 1/\mu)}{\Gamma(k)} \right]^{k\mu} \left( \frac{n}{\langle n \rangle} \right)^{k\mu - 1} \times \exp \left[ - \left( \frac{\Gamma(k + 1/\mu)}{\Gamma(k)} \frac{n}{\langle n \rangle} \right)^\mu \right] \tag{14} \]

with parameters \( k = 0.77, \mu = 2.0, \langle n \rangle = 4.4 \). To describe \( P_c(n) \) equation (9) was used. The results (short-dashed line) show that the agreement of (9) with the rare event triggered distribution is not quantitative enough, namely to allow a correct description of the crossing point with the inclusive distribution curve.

To improve this situation, one should notice that in the case of the production of a pion, subsequent absorption may affect the relation (9). The multiplicity associated to a fast photon, for instance, would be better to test (9) as the photon is less affected by absorption.

On the other hand, recently, in the context of nucleus-nucleus collisions at high energy it was argued that when clustering effects occur the multiplicity has an additional increase because the clusters themselves produce more particles [13]. In other words, (9) has to be modified to become

\[
P_c(n + \delta) = \frac{n^2}{\langle n^2 \rangle} P(n) , \tag{15} \]

where \( \delta \) traces the average number \( \alpha (> 1) \) of nucleons in the clusters, accordingly to

\[ \delta \simeq 2\alpha - 2 . \tag{16} \]

For \( \alpha = 1, \delta = 0 \) as it should (no clustering) and (15) reduces to (9).
Using (4), with \( \alpha \simeq \beta \), and requiring

\[
E_{th} = \frac{2}{\alpha} m_\pi > 95 \text{ MeV/Nucleon}
\]  

(17)

one estimates

\[
\alpha \leq 3
\]

(18)

and, from (16),

\[
\delta \leq 4
\]

(19)

In Fig. 3 we also included the description of the pion-triggered data \( P_c(n) \) by means of equation (15) with \( \delta = 3 \) (long-dashed line). In comparison with the \( P_c(n) \) results given by (9) (short-dashed line), improvement is achieved with the curve corresponding to (15). Indeed the shift by \( \delta \) originated by cluster formation describes better the cross-over point of the constrained and un-constrained multiplicities. Moreover, the value of the parameter \( \delta \), which is found to be consistent with the distribution data, is also consistent with (18) and (19), originated only from the observed sub-threshold energy. Thus, Eq. (19) constitutes an important bound constraining the overall behavior of the data. In conclusion, clustering arguments relate directly the observed sub-threshold production energies with the behavior of the distribution data.

The physical picture is that at sub-threshold energies the corresponding wavelength scale of the process is large enough for clusters of nucleons in the nucleus to recoil as a whole. At these energies correlations due to nucleon-nucleon interactions may dominate Pauli blocking or Fermi-motion correlations. An absolute threshold may be calculated corresponding to a recoil of the complete nucleus as a whole (equation (5)). The deviation of the physical threshold from the absolute one is due to clustering formation involving only part of the constituent nucleons. The clustering also affects the distribution data for particle production events triggered by rare events such as pion production.

Therefore, rare-event triggered measurements give direct information on the number of nucleons involved in particle production at threshold. Accordingly, few-nucleon production reactions, near threshold, such as \( pd \to \pi^+ \alpha \) and \( pd \to \eta \alpha \) for example, which may be exactly calculable at present, are worth being investigated experimentally at the existing strong focusing synchrotons facilities (COSY, CELSIUS), since they may confirm the proposed relation between threshold energies and number of nucleons involved in the production mechanisms. We note here also that the failure already found in reference [14] to describe by means of the impulse approximation the \( ^3He(\pi, \eta)^3H \) reaction below the free production threshold for forward scattering, is already an evidence for a cluster-enhanced rare event.

As we mentioned before, it is not our purpose to test models of interacting nuclei but to test the validity of (9) when the kinematic constraints are such that occurrence of event c (pion, fast proton, . . .) is rare. However, Eq.(15), which in any case is approximate, as \( \alpha \) and \( \beta \) may fluctuate, only makes sense in a model with interaction via clusters. This is similar to what is proposed in [13] for cumulative effects at very high energies. Models with independent \( NN \) collisions (\( \alpha = 1 \)) in medium (based on Fermi-motion distributions arguments alone) do not give support to the arguments leading to (15). So, although relation (9) is universal, (15) depends upon assumptions concerning the type of interactions involved and thus can distinguish between models.
The fact that particles can be produced below threshold via a clusterization mechanism raises still another important question, namely, whether particle production normally forbidden by Zweig’s rule could occur in an environment where (microscopic) clusterization takes place. In particular, this effect may have important consequences for $\phi$ meson production in heavy ion collisions at SPS and LHC energies. It remains also to be clarified whether this effect has any consequences for $J/\psi$ production at collider energies.

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Figure Captions

Figure 1 - Pion production mechanisms mediated by resonance formation in the medium (a) and in free nucleon scattering (b).

Figure 2 - Experimental cross-section for $^{12}C-^{12}C$ from the fourth reference in [1]. The full line is only to guide the eye.

Figure 3 - Data for $^{36}Ar$ on $^{27}Al$ collisions taken from reference [10]. The full curve is the fit to $P(n)$, the short-dashed line is $P_c(n)$ given by equation 9, the long-dashed line is $P_c(n)$ given by equation 15.
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$\sigma_{\text{inclusive } \pi^0} (\mu b)$

$E_{\text{Lab}}/A \text{ (MeV)}$

$C+C \rightarrow \pi^0 + X$

Absolute threshold
