Correlation Effects in the Uncertainty Estimation of Two-Pressure Humidity Generators

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Abstract. This paper presents an analysis of the correlation effect in the uncertainty estimation of a system for humidity generation based on the two pressures principle. In order to compare the validity of the calculations, two methods for evaluating uncertainty, including the correlation between the input variables, were used. The first one was the classical approximation based on the applicability of the central limit theorem, while the second consisted of the propagation of the input distributions using Monte Carlo simulations. The obtained results show that the usual assumption of negligible correlations between the input variables is valid for uncertainty levels in calibrations of hygrometers.

1. Introduction

Humidity is an important parameter in industrial processes. As a variable that affects the characteristic of the final product, it is a variable that must be controlled. In order to provide traceability to measurements, a calibration process that includes a stable source of humidity is required. Humidity generators based on the two-pressure principle are one of the most used devices in this field.

The two-pressure method consists of the saturation of an air-water vapor mixture and its posterior isothermal expansion in a measurement chamber[1]. Under these conditions, the relative humidity can be approximated as the ratio of the pressure in the chamber and the saturator pressure. A system that operates on this technology constitutes the national standard of the NMI Colombia.

One feature of this humidity generator is that for values above 344.7 kPa, the pressure in the saturator and the pressure in the chamber are monitored by the same sensor. It is well known that a usual source of correlation is the use of the same instrument to measure two quantities. The classical method to evaluate the effect of the correlation between the input variables has been established in the Guide to the Expression of Uncertainty in Measurement, JCGM 100:2008 (GUM) [2]. The application of the GUM method strongly depends on the fulfillment of the conditions of the central limit theorem. To overcome limitations or prove the validity of the GUM approach, an alternative method based on the propagation of the input distribution using the Monte Carlo method has been proposed[3]. The objective of this paper is to evaluate the contribution of the correlation effect on the humidity standard of NMI Colombia using the GUM method and to test the results against the Monte Carlo method.
2. Mathematical Model

The definition of relative humidity, considering the non-ideal behavior of the gases in a two-pressure humidity generation, can be determined using equation (1).

\[
HR = \left( \frac{f_e(T_e, P_e) \cdot e_n(T_e) \cdot P_e}{f_e(T_s, P_s) \cdot e_n(T_s) \cdot P_s} \right) \cdot 100
\]  

(1)

Where \( f_e \) is the enhancement factor, \( e_n \) is the vapor pressure, \( P_s \) is the pressure in the chamber, \( P_e \) is the pressure in the saturator, \( T_s \) is the temperature in the saturator and \( T_e \) is the pressure in the chamber. Using the GUM framework [2], uncertainty according to equation (1) can be expressed as

\[
u(HR)_{\text{uncor}} = \left( \frac{\partial HR}{\partial P_e} \right)^2 u^2(\delta(P_{e\text{op}})) + \left( \frac{\partial HR}{\partial P_s} \right)^2 u^2(\delta(P_{s\text{op}})) + \left( \frac{\partial HR}{\partial e_n} \right)^2 u^2(\delta(e_n(T))) + \left( \frac{\partial HR}{\partial f_e} \right)^2 u^2(\delta(f_e(T))) + \left( \frac{\partial HR}{\partial T_e} \right)^2 u^2(\delta(T_e)) \]  

(2)

The terms for the correlation are presented in equation (3) and must be added to equation (2) to calculate the total combined uncertainty.

\[
u(HR)_{\text{total}} = \left( \frac{\partial HR}{\partial P_e} \right)^2 u^2(\delta(P_{e\text{op}})) + \left( \frac{\partial HR}{\partial P_s} \right)^2 u^2(\delta(P_{s\text{op}})) + \left( \frac{\partial HR}{\partial e_n} \right)^2 u^2(\delta(e_n(T))) + \left( \frac{\partial HR}{\partial f_e} \right)^2 u^2(\delta(f_e(T))) + \left( \frac{\partial HR}{\partial T_e} \right)^2 u^2(\delta(T_e)) + \sum_{i=1}^{n} \left( \frac{\partial HR}{\partial T_i} \right)^2 u^2(\delta(T_i)) \]  

(3)

An alternative method to propagate the uncertainty is the Monte Carlo method as described in Supplement 1 to the Guide to the Expression of Uncertainty in Measurement, JCGM 101:2008 [3]. This methodology is useful when the conditions for the GUM approach are not fulfilled or when its evaluation is not practical. Examples of the first situation are presented when the dominant contribution is a non-Gaussian distribution, when the linearization of the model provides an inadequate representation and, in general, when the conditions of applicability of the central limit theorem do not hold. Basically, the Monte Carlo method propagates probability distributions assigned to input quantities through a numerical simulation of the mathematical model to evaluate a probability distribution for the output value.

3. Measurements

The pressure in chamber \( P_e \) and the pressure in saturator \( P_s \) were measured with a piezoresistive transducer. The temperature in the chamber and saturator was measured with 10 kΩ thermistors. Vapor pressure was estimated using the international equations for vapor pressure proposed by Saul and Wagner[4], using coefficients according to ITS90 [5]. The enhancement factor is calculated through the Greenspan equation [6] using the coefficients adjusted by Hardy [7]. The saturator’s efficiency is determined through the factory specifications for this equipment model. The uncertainty in the determination of the enhancement factor to the Greenspan equation is taken by adjusting the data by Hyland [8], as shown by Garcia Skabar [9].
The atmospheric pressure is about 750 hPa ± 5 hPa. Therefore, the correlation effect is presented for humidity above 20 %HR. The measurements were taken between 20 %HR and 85 %HR in a period of eight months in the Humidity Laboratory of the NMI Colombia. In order to assure the quality of the measurements, a chilled mirror hygrometer and a capacitive hygrometer were used as check standards of the process for each test. Three resistance thermometers were attached in the chamber to check uniformity. An example of the uncertainty budget is presented in Table 1 for three levels of humidity.

The correlation effect computed in equation (3) is calculated through the correlation coefficients between \( P_s, P_c, T_s \) and \( T_c \) as defined in equation (5)

\[
    r(y_i, y_m) = \frac{u(y_i, y_m)}{u(y_i)u(y_m)} \tag{5}
\]

Where \( u(y_i, y_m) \) represents the covariance between \( y_i \) and \( y_m \).

### 4. Results

Table 1 shows an example of the uncertainty calculation for the two-pressure humidity generator.

| Uncertainty component | 30 %HR       | 50 %HR       | 80 %HR       |
|-----------------------|--------------|--------------|--------------|
|                       | Estimated u, %HR | Estimated u, %HR | Estimated u, %HR |
| \( P_s \)            | 251969 Pa     | 1.63E-03     | 150015 Pa    | 2.41E-03     | 93504 Pa    | 2.60E-03     |
| \( P_c \)            | 75206 Pa      | 9.57E-05     | 75165 Pa     | 1.44E-03     | 75165 Pa    | 1.36E-03     |
| \( T_s \)            | 20.00 °C      | 1.26E-03     | 20.00 °C     | 2.63E-03     | 20.00 °C    | 3.04E-03     |
| \( T_c \)            | 20.02 °C      | 1.07E-03     | 20.08 °C     | 9.26E-04     | 20.09 °C    | 1.55E-03     |
| Saturator efficiency  | 0 %HR         | 4.28E-02     | 0 %HR        | 7.14E-02     | 0 %HR       | 1.14E-01     |
| Vapor pressure in the saturator | 2339 Pa | 8.65E-04 | 2339 Pa | 1.44E-03 | 2339 Pa | 2.31E-03 |
| Enhancement factor in the saturator | 1.008 --- | 4.60E-03 | 1.005 --- | 7.68E-03 | 1.004 --- | 1.23E-02 |
| Vapor pressure in the chamber | 2342 Pa | 8.65E-04 | 2350 Pa | 1.44E-03 | 2351 Pa | 2.31E-03 |
| Enhancement factor in the chamber | 1.003 --- | 6.05E-04 | 1.003 --- | 0.00101 | 1.003 --- | 1.62E-03 |

An example of the output distributions using the Monte Carlo method for the example in Table 1 are presented in Figure 1, both including and not including correlation. For each distribution 1E6 samples were used.

The output distributions using the Monte Carlo method are very close to normal distributions. The difference with the interval calculated with the GUM method for a coverage probability of 95% is less than 0.4% for all cases, indicating that the use of the GUM approach is valid.

![Output distributions of the example](image-url)
Figure 2 shows the effects of computing the correlation effect in the combined uncertainty of all the tests performed using equations (2) and (4)

![Figure 2. Combined uncertainty for correlation effect](image)

For the entire interval evaluated the effect of the correlation is negative, which means that ignoring the correlation produces an uncertainty overestimation. The magnitude of this effect increases with the relative humidity. However, the maximum difference is 0.005 %HR which for 85 %HR is about 2%.

5. Conclusions
The effect of correlation in the standard humidity generator of the NMI Colombia was evaluated. The comparison between the methods evaluated shows that the classical GUM approach is adequate for estimating the measurement uncertainty, including the correlation effect. In regard to the correlation effect itself, the contribution to the total uncertainty is negative, increasing with relative humidity and with its magnitude between 20 %HR and 85 %HR, it is less than 0.005 %HR. According to the GUM recommendations the uncertainty reported should be “safe” or “conservative.” Therefore, ignoring the slight effect of the correlation between the input variables is a valid assumption for the uncertainties reported in the calibration services at NMI Colombia.

6. References
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