Study on Structural Reliability of Prestressed Reinforced Concrete Beam Bridges Based on Full Probability Analysis

Yubao Liu¹*, Hanxiong Liu² and Shouyi Sun³

¹CCCC First Harbor Engineering Co. Ltd. Tianjin, China
²CCCC Second Highway Consultants Co., Ltd. Wuhan, Hubei, China
³School of transportation and logistics engineering, Wuhan University of technology, Wuhan, Hubei, China

Email: 2321547575@qq.com

Abstract. A three dimensional(3D) finite element model is established for a prestressed concrete girder bridge, list the limit state equation and variable probability distribution model, obtain the failure probability and reliability index of the limit state equation based on the Monte-Carlo method, investigate the reliability index’s sensitivity to random variables for limit state equations of bridge bearing capacity. The specific method of the research on the structure reliability index calculation is to select a suitable way that can fit the research’s object among the center point method, the checking-point method, the Monte-Carlo method, the importance sampling method, and so on. Authors use finite element software ANSYS to build model and perform 3D force analysis. According to the results, authors can determine the bridges’ failure modes under the state of 3D stress. Then, authors can list the possible limit state equations related to Stiffness, strength, function, durability, and some character else. After determining the bridge’s structural limit state equation and the probability distribution model of the variables, determine the distribution function of each impact factor and its characteristic parameters through the site survey data and other related data surveys. Then, use the Monte-Carlo method for the calculation of bridge reliability index, obtain the failure probability and reliability index of each limit state equation, analyze the sensitivity of each variable to the reliability index under the ultimate state of the bridge's bearing capacity. At last, authors give construction quality control plans and suggestions according to the data above.

Keywords. Bridge engineering; Full probability analysis; Structural reliability; Prestressed Bridges; Reinforced concrete Bridges

1. Introduction

According to the statistics, as of the end of 2006, there were 6,282 dangerous highway bridges in the fifth category of health in our country. Among of them, After the pre-stressed concrete bridge is built and put into use for a period of time, the bridges’ structural materials will have the problem of natural aging, coupled with the increasing vehicle load and improper use and maintenance, the bridges will have various structural damages and defects. These will bring difficulties to the corresponding maintenance and management of the bridge. Therefore, how to evaluate the reliability of bridge operations and carry out corresponding maintenance and management have become a topic of concern to the domestic and foreign bridge workers.

The reliability analysis example of PC beam bridge shows: evaluate the safety status of the reinforced concrete bridges in service scientifically, determine the reliability of bridges from different aspects and provide a practical basis for highway and bridge operation management unit will help
Construction Department take reasonable reinforcement methods for the bridge to restore and improve the bridge's bearing capacity.

Until 2012, 43.88 billion yuan was invested in the renovation of dangerous bridges across the country and 21,610 dangerous bridges with a total of 1.741 million meters were retrofit. To replace these seriously diseased old bridges will cost a lot of money and the traffic will be closed. This situation is not allowed by our national conditions and financial resources. Although the development of the reliability theory of prestressed concrete beam bridge structure has a considerable scale at present, there is still quite a large gap in the application of reliability analysis theory to the engineering practice. Therefore, the relevant research is needed to continue.

At present, in the relevant specifications, for prestressed concrete structural members designed by limit state control. The target and reliable indicators of its crack resistance (crack width), deflection and stress are not clear yet. Although the limits of the above indicators are given in the design specifications, their reliability levels have not been determined. Therefore, it’s our duty to establish a reliability analysis model of prestressed concrete bridge structure considered the ultimate state of bearing capacity and the limit state of normal use. Apart from this, understand the reliability level of various indicators of prestressed concrete bridge members designed in accordance with the country’s standards and analyze the influencing parameters are also necessary.

The research objects of this paper include the basic principles and calculation methods of the reliability of prestressed concrete structures, as well as the probability distribution types of the commonly used variables. By using the finite element software ANSYS to establish a model, authors completely analyze Reinforced concrete beam bridge and give the construction quality control plan and suggestions to the relevant departments.

2. Structural reliability

The probability of the structure or the structural member complete a predetermined function is called the reliability probability, also can be called reliability $P_r$. The probability that the structure cannot complete its intended function is called the failure probability. It also can be expressed as $P_f$. And assuming that the basic random variable in the structure is $x_1, x_2, ..., x_n$, the corresponding probability density function is $f_x(x_1, x_2, ..., x_n)$, the functional function represented by these random variables is $Z = g_x(x_1, x_2, ..., x_n)$. Then the failure probability of the structure can be expressed as

$$P_f = P(Z < 0) = \int_{Z<0} f_x(x_1, x_2, ..., x_n)dx_1dx_2...dx_n$$  \hspace{1cm} (1)

Assuming that the random variable of resistance of the structure is $R$ and the random variable of the load is $S$. The corresponding probability density functions and probability distribution functions are $f_R(r)$ and $f_S(s)$. The probability distribution functions are $F_R(r)$ and $F_S(s)$. $R$ and $S$ are independent of each other. Then the probability of structural failure is:

$$P_f = \int_{R<S} f(r,s)drds$$  \hspace{1cm} (2)

If $R$ and $S$ are normal distribution, their mean and standard deviation are $\mu_R$, $\mu_S$ and $\sigma_R$, $\sigma_S$. Then $Z$ is also a normal random variable. Then its mean is $\mu_Z = \mu_R - \mu_s$ and its standard deviation is

$$\sigma_Z = \sqrt{\sigma^2_R + \sigma^2_S}$$  \hspace{1cm} (3)

And $\beta = \frac{\mu_Z}{\sigma_Z}$ is the reliable index.

3. Failure Mode and Limit State Equation

The limit state is a sign which can distinguish the success and the failure of the structural working state. In the course of use, if the entire structure or part of the structure exceeds a specific state and cannot meet a certain functional requirement specified by the design, this specific state is called the limit state of the function. And the limit state can be divided into two types: the bearing capacity limit state and the
normal service limit state.

3.1. Load capacity limit state.

The ultimate state of bearing capacity refers to the structure or the structural member reaching the maximum bearing capacity, and then there will be fatigue, overturning, instability, drift, continuous collapse and other failures or deformation that is not suitable for continued bearing. The design expression for the ultimate state of the bearing capacity is as follows:

$$\gamma_0 S \leq R$$

$$R = R(f_c, f_s, \alpha_k ...)$$ (4)

In the formula, $\gamma_0$ is the important factor of the structure. $S$ is the design value of the effect combination under the limit state of the bearing capacity. $R$ is the design value of the resistance of the structural member. $\gamma_{RD}$ is the uncertainty coefficient of the structural member resistance model. $f_c$, $f_s$ is the design value of concrete and steel bar strength. $\alpha_k$ is the standard value of the geometric parameter.

3.1.1. Variable load effect control combination.

$$S = \gamma_G S_{GK} + \gamma_{Q1} S_{Q1K} + \sum_{i=2}^{n} \Phi_{ci} \gamma_{Qi} S_{QiK}$$ (5)

3.1.2. Combination of permanent load effect control.

$$S = \gamma_G S_{GK} + \sum_{i=1}^{n} \Phi_{ci} \gamma_{Qi} S_{QiK}$$ (6)

In the formula, $\gamma_G$ is the partial coefficient of permanent load, with a numerical value of 1.2. $\gamma_{Q1}$ and $\gamma_Qi$ is the partial coefficient of the first and i-th variable load, with a numerical value of 1.4. $S_{GK}$ is the load effect value calculated according to the permanent load standard value $G_K$. $S_{QiK}$ is the load effect value calculated according to the variable load standard value $Q_iK$. $\Phi_{ci}$ is the wind load with a value of 0.6, other coefficients are with the value of 0.7. $n$ is the variable number of loads participating in the combination.

3.2. Normal use limit state.

The normal use limit state refers to a certain limit corresponding to the normal service and durability performance of the structure or structural member. The design expression for the serviceable limit state is: $S_d < C$

In the formula, $S$ is the design value of load effects such as deformation, cracks, and so on. $C$ is the limit for structural members to reach the deformation, stress, crack width and natural frequency specified in the normal use requirements.

Taking $S_d$ under the standard combination, the structure function function is:

$$Z = R - S$$

$$= C - S_d$$

$$= C - (S_{GK} + S_{Q1K} + \sum_{i=2}^{n} \Phi_{ci} S_{QiK})$$ (7)

Taking $S_d$ under the permanent combination, the structure-function is:

$$Z = R - S$$

$$= C - S_d$$

$$= C - (S_{GK} + S_{Q1K} + \sum_{i=2}^{n} \Phi_{ci} S_{QiK})$$ (8)

4. Reliability analysis and calculation

4.1. The flexural limit state equation

$$Z = R - S$$
The shear limit state equation

\[
Z = R - S
\]

\[
= V_u - \gamma_0 V_d
\]

\[
= \alpha_1 \alpha_2 \alpha_3 (0.45 \times 10^{-3}) bh_0 \sqrt{(2 + 0.6p) f_{cu,k} f_{sv} f_{sd}} + (0.75 \times 10^{-3}) f_{sd} \sum A_{sb} \sin \theta_s - \gamma
\]

\[
= \alpha_1 \alpha_2 \alpha_3 (0.45 \times 10^{-3}) bh_0 \sqrt{(2 + 0.6 \times 100) \frac{(A_p + A_{pb} + A_{a})}{bh_0} f_{cu,k} f_{sv}}
\]

\[
+(0.75 \times 10^{-3}) f_{sd} \sum A_{sb} \sin \theta_s - \gamma_0 V_d
\]

Crack resistance limit state equation

\[
Z = |W_{fk}| - W_{f_k}
\]

\[
= C_1 C_2 C_3 \times \frac{M_S + N_{P0}(Z - e_p)}{(A_p + A_S)Z} \times \frac{30 + d}{0.28 + 10 \frac{A_S + A_p}{bh_0 + (b' - b)h_f}}
\]

\[
= |W_{fk}| - C_1 C_2 C_3 \times \frac{M_S + (\sigma_{con} - \sigma_i)A_p - \sigma_{con}A_S(Z - e_p)}{(A_p + A_S)Z} \times \frac{30 + d}{0.28 + 10 \frac{A_S + A_p}{bh_0 + (b' - b)h_f}}
\]

Deflection limit state equation

\[
Z = R - S
\]

\[
= \frac{l}{600} - (f_{st} - f_{gl})
\]

\[
= \frac{l}{600} - \eta \left( f_s - f_g \right)
\]

\[
= \frac{l}{600} - \eta_\theta \times \frac{\varepsilon_t^2}{48 \times 0.95 \varepsilon_c} (M_S - M_G)
\]

Reliability analysis results.
Authors use a calculation program compiled by MATLAB to calculate the reliability of the deflection
according to the Monte Carlo method, combine the existing literature and the statistical parameters in data statistics according to the limit state equation of the deflection of the prestressed concrete bridge structure defined above. Then, authors can reveal and analyze the reliability level of prestressed concrete simply supported beams designed according to the current specifications. Furthermore, authors use Matlab to check the calculation, get the corresponding reliability index and compare it with the target reliability index required by the relevant specifications at last.

The following contents are part of the code of the flexural reliability program and the target reliability index during the bridge design period which is shown in the Table1:

\[
\begin{align*}
\text{muX} &= [1312.16; 4587; 25.81; 2403.7739; 202.4263; 202.3524; 1822.6324] \\
\text{cvX} &= [0.01; 0.008; 0.0344; 0.0029; 0.0004; 0.0056; 0.0021] \\
\text{sigmaX} &= \text{cvX} \times \text{muX}
\end{align*}
\]

\[
\text{nS}=5e6; \quad \text{ig}=\text{ones(nS}:1)
\]

\[
\text{n}=\text{length(muX)}
\]

\[
\text{for k}=1; \text{n},x(:,k)=\text{normrnd(muX(k),sigmaX(k),nS,1);} \quad \text{end}
\]

\[
\text{t}=(x(:,1).*x(:,2))/(x(:,3).*x(:,4));
\]

\[
\text{if t}<x(:,6)
\]

\[
\text{g}=x(:,1).*x(:,2)./x(:,7)-x(:,1)*x(:,2)./(2.*x(:,3).*x(:,4))... -100043.34e6
\]

\[
\text{else}
\]

\[
\text{g}=x(:,3).*x(:,4)-x(:,5).*x(:,6).*x(:,7)-x(:,6)./2+... x(:,7).*x(:,1).*x(:,2)-x(:,3).*x(:,4)-x(:,5).*x(:,6)... -x(:,1).*x(:,2)-x(:,3).*x(:,4)+... x(:,5).*x(:,6).*x(:,7).*x(:,3).*x(:,4)-10043.34e6;end}
\]

\[
\text{nF}=\text{sum(ig(g<0))};
\]

**Table 1.** Target reliability index during the bridge design period

| Combination of action and effect | 1st level | 2nd level | 3rd level |
|---------------------------------|-----------|-----------|-----------|
|                                 | ductile damage | brittle damage | ductile damage | brittle damage | ductile damage | brittle damage |
| Main combination                | 4.7       | 5.2       | 4.2       | 4.7          | 3.7          | 4.2          |
| extra combination               | 4.2       | 3.7       | 3.7       | 4.2          | 3.2          | 3.7          |

5. **Random Variable Sensitivity Analysis**

Sensitivity analysis is while a random variable changes within a certain range, authors can study the impact of changes in these variables on the model results. And the main purpose of sensitivity analysis is to obtain the sensitivity coefficient of each variable by analyzing the relevant variables of the model. As examples, the sensitivity of the reliability index to the average value of \( f_p \) and the sensitivity of the reliability index to the coefficient of variation of \( f_p \) are shown in the Figure1 and the Figure2.

In the actual work, researchers can ignore random variables with small sensitivity coefficients based on our work experience. By treating some random variables as constants, the complexity of the model can be greatly reduced and the workload of data analysis can be reduced. We need to focus on random variables with large sensitivity coefficients, and strictly control them during the design, so as to ensure that the structure is safe enough. Under such measures, the work efficiency is improved without affecting the calculation accuracy.
By changing the mean value and the coefficient of variation \( \delta \) of the resistance reliability random variables, sensitivity analysis is performed on the seven random variables in the expression. Finally, a summary of the results can be got as shown in the Table2 and the Table3:

| Random Variable | Mean Sensitivity |
|-----------------|------------------|
| \( f_p \)       | 6                |
| \( A_p \)       | 6                |
| \( f_{cd} \)    | 37.5             |
| \( b' \)        | 37.5             |
| \( b \)         | -                |
| \( h' \)        | -                |
| \( h_0 \)       | 6                |

Table 2. Random Variable Mean Sensitivity

| Mean Sensitivity | \( f_p \) | \( A_p \) | \( f_{cd} \) | \( b' \) | \( b \) | \( h' \) | \( h_0 \) |
|------------------|-----------|-----------|--------------|--------|--------|--------|--------|
| Average percentage increase(%) | 6 | 6 | 37.5 | 37.5 | - | - | 6 |
| Reliability index \( \beta \) increase percentage(%) | 2800 | 6207 | 21 | 18 | - | - | 1991 |
Table 3. Random Variable Coefficient of Variation Sensitivity

|                   | $f_p$ | $A_P$ | $f_{cd}$ | $b'_f$ | $b$ | $b'_f$ | $h_0$ |
|-------------------|-------|-------|----------|--------|-----|--------|-------|
| Percentage increase in the coefficient of variation(%) | 56    | 57    | 60       | 55     | -   | -      | 57    |
| Reliability index $\beta$ reduce percentage(%)       | 34    | 22    | 4        | 4      | -   | -      | 10    |

According to the table above, some constructive suggestions can be given as follows:

- 1) Strictly control the performance of prestressed steel bar materials (tensile strength, area) during design and construction
- 2) PC beam web width $b$ and roof thickness $h'_f$ can be treated as constants in resistance analysis, and the requirements can be appropriately relaxed during construction acceptance.

6. Conclusion

This subject is adopted for the first time that the total probability method is used to analyze the reliability of the prestressed reinforced concrete beam bridge structure with multiple failure modes. Authors combine the existing structural finite element analysis software and Monte Carlo numerical simulation program to solve the establishment of the relevant model of the random process of each variable and the solution of the equation.

In actual engineering applications, on the basis of reliability basic theory and method research foundation, authors study the failure modes and limit states of the prestressed concrete beam bridges and analyze the sensitivity of each random variable to the reliability index under the ultimate state of the bridge's bearing capacity. Finally, authors can use these as the basis to obtain cost-effective construction quality control plans and suggestions which meet the requirements.

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