THE MACHO PROJECT LARGE MAGELLANIC CLOUD VARIABLE STAR INVENTORY. XI. FREQUENCY ANALYSIS OF THE FUNDAMENTAL-MODE RR LYRAE STARS

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ABSTRACT

We have frequency-analyzed 6391 variables classified earlier as fundamental-mode RR Lyrae (RR0) stars in the MACHO database on the Large Magellanic Cloud (LMC). The overwhelming majority (i.e., 96%) of these variables have been proved to be indeed RR0 stars, whereas the remaining ones have fallen into one of the following categories: single- and double-mode Cepheids, binaries, first-overtone and double-mode RR Lyrae stars, and nonclassified variables. Special attention has been paid to the properties of the amplitude- and phase-modulated RR0 stars (the Blazhko stars). We found altogether 731 Blazhko variables showing either a doublet or an equidistant triplet pattern at the main pulsation component in their frequency spectra. This sample overwhelmingly exceeds the number of Blazhko stars known in all other systems combined. The incidence rate of the Blazhko variables among the RR0 stars in the LMC is 11.9%, which is 3 times higher than their rate among the first-overtone RR Lyrae stars. No difference is found in the average brightness between the single-mode and Blazhko variables. However, the latter ones show a somewhat lower degree of skewness in their average light curves and a concomitant lower total amplitude in their modulation-free light curves. From the frequency spectra we found that variables with larger modulation amplitudes at the higher frequency side of the main pulsation component are 3 times more common than the ones showing the opposite amplitude pattern. A search for a modulation component with the Blazhko period in the average brightness of the individual variables showed the existence of such a modulation with an overall amplitude of ≈0.006 mag. On the other hand, a similar search for quadruple modulation patterns around the main pulsation component has failed to clearly detect such components at the ≈0.004 mag level. This means that the amplitudes of the quadruple components (if they exist) should be, on average, at least 10 times smaller than those of the triplet components. This finding and the existence of Blazhko variables with highly asymmetric modulation amplitudes not only question the validity of the magnetic oblique rotator model but also put stringent constraints on models based on mode-coupling theories.

Subject headings: globular clusters: general — stars: horizontal-branch — stars: oscillations — stars: variables: other (RR Lyrae)

On-line material: color figures

1. INTRODUCTION

The mystery of the physical cause of amplitude modulation in certain RR Lyrae stars, popularly known as the “Blazhko (BL) Effect,” remains despite almost a century of study. In a previous paper in this series (Alcock et al. 2000, hereafter A00) we discussed the results of the frequency analysis of a sample of 1350 Large Magellanic Cloud (LMC) variables, formerly classified as first-overtone RR Lyrae (RR1) stars. In addition to the discovery of more than...
100 double-mode variables, we found 52 stars that displayed long-period amplitude and phase modulations. Until very recently (Olech, Kaluzny, & Thompson 2001 and references therein; Cseresnjes 2001; Moskalik & Poretti 2003; Soszyński et al. 2003), BL-type variability was established as a well-observed phenomenon only among fundamental-mode variables. The study of A00 gave the first reliable statistics on the incidence rate of this behavior among RR1 stars. A rate of 4% was found, which is surprisingly low compared to the most often cited rate of 20%-30% for the fundamental-mode (RR0) stars (Szejdl 1988). The main purpose of the present study is to verify whether this large difference does indeed exist between these two groups of stars. The large and homogeneous sample of the MACHO database stars used in this study yield much more significant results for the global statistical properties of this population than any other previous investigations, which were typically based on very limited and inhomogeneous samples of objects from various stellar systems. (Exceptions to this characterization are the surveys of Soszyński et al. 2003 on the 7612 RR Lyrae stars of the LMC from the OGLE-II database, Moskalik & Poretti 2003 for the 215 variables of the Galactic bulge from the OGLE-I database, and Cseresnjes 2001 for the 3700 variables of the Fornax dwarf galaxy.)

Apart from the task of identification of the variable type for RR Lyrae in the LMC MACHO database, we also study the global properties of the BL stars and compare them with their singly periodic counterparts. A key discriminant between rival explanations for the BL behavior is found in the predicted frequency and amplitude pattern distributions of the amplitude spectra of these stars. The existence (or nonexistence) of triplet or quintuplet structures and the degree of asymmetry of the corresponding modulation amplitudes strongly restrict the range of acceptable models. The present large data set enables us to put significant constraints on the possible ranges of the various modulation components. A brief summary of the results obtained by the analysis of a fraction of the present data set has already been published by Welch et al. (2002).

2. THE DATA, THE METHOD OF ANALYSIS, AND THE VARIABLE CLASSIFICATION

The data set analyzed in this paper comprises 6391 light curves selected from the MACHO database as variables satisfying certain conditions on brightness, color, and period, corresponding to the expected properties of RR0 stars in the LMC.19 Because of overlapping fields, 364 light curves turned out to represent variables already appearing in the database with other identification numbers. Since these double (or multiple) identifications decrease the number of stars in each variable type by only a few percent and therefore do not influence the incidence rates in a significant way, all of our subsequent results and statistics refer to the full sample of 6391 identifications. The list of stars with multiple identifications is presented at the end of this section.

This paper analyzes RR Lyrae stars identified in 30 LMC MACHO project fields (1, 2, 3, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 22, 23, 24, 47, 53, 55, 57, 76, 77, 78, 79, 80, 81, and 82). These fields include 54% of the observed stars in all fields because they cover the densely populated bar of the LMC (see the home page of the MACHO project).20 The data utilized span a period of 2700 days (7.4 yr), which is 1 yr longer than the time baseline of the RR1 stars analyzed in A00. The magnitudes have been transformed to the Johnson V system according to the method described by Alcock et al. (1999). As a result of the longer time baseline, the average number of observation epochs per star is also greater, often in the range 700–1000. The sampling rate and data distribution are the same as in the case of the RR1 stars. In particular, there is only a 1 day quasi-periodicity in the sampling but no trace of 1 yr aliasing. The accuracy of the individual data points is also the same as for the RR1 data, but, because of the larger amplitudes, the overall signal-to-noise ratio (S/N = A/σ, where A is the total [peak-to-peak] amplitude and σ is the standard deviation of the residuals) is larger (∝ S/N) than for the RR1 stars (∝ 2.6, although this figure refers to the data in the “r” color, which further lowers the S/N; see A00).

Apart from spectrum averaging (see § 5), the method of analysis in this survey is essentially the same as that in A00. We performed a standard frequency analysis based on a Fourier approach and a “prewhitening” technique (see, e.g., Deeming 1975, with the fast transformation method recommended by Kurtz 1985). Before execution of the Fourier transformation, outlying points have been omitted by checking the distribution of the magnitudes and leaving out data points separated from a compact set that is thought to represent the light curve plus random noise. The relative number of omitted points in this way varied between 1% and 4%, and most often these omitted points were obtained in very poor seeing. The classification of the variables was performed in three major steps:

1. Frequency analysis with prewhitening in the [0.6] day−1 band, visual inspection of the frequency spectra and folded light curves, and preliminary selection of variable types.
2. Reanalysis of the preselected multiperiodic variables by three successive prewhitenings and a closer examination of the frequency spectra.
3. Visual inspection of the light curves of all variables to check the above classifications.

Except for the first prewhitening cycle, when the main pulsation component was filtered out from the time series, we used a single-component least-squares fit for whitening. Because of the high nonlinearity, for the main pulsation component we utilized all harmonics up to the third one.

It is important to emphasize that the classification is based on the observed frequency pattern in a wide frequency band. Therefore, the significance of a given pattern is determined through a comparison of a large number of other

19 The selection of the RR Lyrae stars for this paper was made many years before the completion of the MACHO project. In the time since the submission of this paper, we have become aware that there are no RR Lyrae variables in the eastern half of field 5 and an underabundance in a small number of other locations. This was apparently due to an access problem of the program used to phase the photometry. Our ability to test this hypothesis end-to-end has been hampered by the unavailability of scripts present on Mount Stromlo Observatory computers, which were lost during the devastating fire of 2003 January 18. We have since extracted a more complete list of RR Lyrae stars from the full 8 yr database, and we will make this list available to interested investigators. It is our strong expectation that the frequency of BL behavior reported in this paper will not be significantly altered by a similar analysis of the additional candidates in the 8 yr list.

20 See http://www.macho.mcmaster.ca.
features that might make that pattern less significant than if we had made the comparison in a smaller frequency band (the chance of detecting higher, noise-induced peaks increases with the increase of the frequency band). For example, if we choose a narrow band around the main pulsation component and search for an equidistant triplet in the spectrum, a considerable fraction of the BL stars that were classified after the inspection of the wide frequency band as BL stars with doublet patterns (BL1 stars) will prove to have triplet patterns and will be classified as BL2 stars (see § 4). Although some arguments can be given for the preference of a classification based on pattern search in a narrow frequency band, we follow a more conservative approach, and our classification (if not stated otherwise) refers to the results obtained from the above wide frequency band.

Variable types and their respective totals obtained in the course of the above analysis are listed in Table 1. The notation is the same as in A00, except for the BL stars, in which we use similar symbols for both variables with apparently single modulation components (BL1 stars) and those with two symmetrically spaced modulation components straddling the main pulsation component (BL2 stars). In A00 we assigned the symbol “BL” only to the BL2 stars, in order to maintain a coherent nomenclature with the handful of variables that have been analyzed in the Galactic field. These few variables all display triplets in their frequency spectra. However, the analysis of our larger sample strongly suggests that the two types of modulation overlap and that they are indicative of the same underlying phenomenon, which we denote by “BL.”

As discussed earlier, our statistics have not been corrected for multiple identifications because of field overlap. This results in incidence rates that differ from the true rates by, at most, 0.1%. One reason that we did not make this correction is that, in critical cases, classification of the same variable star may depend on the particular time series we analyzed. For example, the same star might appear in one field with many observed epochs and in the other with a much smaller number. If a modulation component has an amplitude near the noise level, it could potentially be classified as both a BL and a single-mode variable (for example, a variable star identified as both MACHO ID 17.3194.3230 and 10.3194.499). A similar ambiguity might arise in the classification of BL1 and BL2 stars if the S/N is low (e.g., MACHO ID 6.6811.969 and 13.6811.4172). We found five variables that have been classified as both BL1- and BL2-types. Among the 400 BL1 stars, there are 13 duplicates with the same BL1 classification. Similarly, we found eight duplicate identifications among the 331 BL2 stars that were classified identically in both instances. These duplications suggest that our classification is repeatable and reliable.

The notation in Table 1 incorporates the notation of Udalski et al. (1999) for the radial pulsation mode, for which the symbols FU, FO, and SO denote the fundamental and first- and second-overtone modes, respectively. The RR0-νM variables are similar to the BL stars in that their amplitude spectra exhibit close secondary components. However, their frequency patterns are more complicated and show no obvious sign of dominant, equidistant frequency components (around the main pulsation component). For period change (PC) variables, the structure remaining around the main pulsation component cannot be resolved, and the few successive prewhitenings do not eliminate these features from the amplitude spectrum. Although the failure of the prewhitening technique indicates a long-term variation in the signal frequency (or even in the amplitude), we must note that it is also possible (at least in some cases) that we are witnessing BL modulations with very long periods. This contention is supported by the fact that the present sample contains BL stars with modulation periods of several years. With these long BL periods, the modulation components are quite close to each other but are still well resolved as a result of the long temporal baseline of the observations. Several BL stars additionally display a pulsation component character. In such cases the variables have been classified as BL-type, despite the additional pulsation component behavior.

A significant fraction (5%) of the stars in this sample display frequency components at integer per day. These are labeled RR0-D variables (such stars also were found in A00). Clearly this behavior indicates some instrumental or reduction artifact. At present, the source of this is unknown.

### Table 1

| Type     | Short Description                              | Number | Percent | σ |
|----------|------------------------------------------------|--------|---------|---|
| RR0-S    | Singly periodic FU RR Lyrae                    | 4882   | 79.3    | 0.5|
| RR0-BL1  | RR0 with one close frequency component         | 400    | 6.5     | 0.3|
| RR0-BL2  | RR0 with two close symmetric frequency components | 331    | 5.4     | 0.3|
| RR0-νM   | RR0 with several close components              | 20     | 0.3     | 0.1|
| RR0-PC   | RR0 with period change                         | 177    | 2.9     | 0.2|
| RR0-D    | RR0 and integer per day frequencies            | 308    | 5.0     | 0.3|
| RR0-MI   | RR0 with some miscellany                       | 40     | 0.6     | 0.1|
| RR1      | Singly periodic FO RR Lyrae                   | 36     | ...     | ...|
| RR01     | FU/FO RR Lyrae                                 | 6      | ...     | ...|
| NC       | Nonclassified variables                        | 32     | ...     | ...|
| SOCEP    | Singly periodic overtone Cepheid               | 60     | ...     | ...|
| FU/FO    | FU/FO double-mode Cepheid                      | 2      | ...     | ...|
| FO/SO    | FO/SO double-mode Cepheid                      | 58     | ...     | ...|
| BI       | Eclipsing binary                               | 39     | ...     | ...|

Notes.—The symbol σ denotes the standard deviation of the population ratio, assuming Poisson distribution (see eq. [1]). In computing population ratios, only fundamental-mode RR Lyrae stars have been considered.
The difference between the RR0-MI and NC variables is that in the former class (most probably) all variables are RR0 stars, but they exhibit extra peaks in their frequency spectra, which are not easy to explain within any simple framework (e.g., multimode pulsation). In the case of NC stars, classification is not possible because of either the limited data set or the “strange-looking” light curve (e.g., driftlike mean-light variation). While additional interesting physical behavior may be present in these stars, we expect (but have not established) that extrinsic effects are responsible for many such classifications.

Many new LMC Cepheid variables have already been discovered by both the MACHO and OGLE projects. After cross-identification of the apparent Cepheid variables found in our initial sample with both databases, we find that 67% of the FO/SO Cepheids reported in this paper are new discoveries.

Statistical errors (standard deviations $\sigma$) of the incidence rates are calculated from the assumption that the population follows a Poisson distribution [i.e., $\sigma_i = (N_i)^{1/2}$, where $N_i$ is the number of objects in the $i$th population]. This yields the following expression for the standard deviation of the population ratio $N_{i}/N$

$$\sigma_{N_{i}/N} = \frac{1}{\sqrt{N}} \sqrt{\frac{N_i}{N} (1 - \frac{N_i}{N})}.$$  

BL stars constitute a significant fraction of the RR0 population in the LMC, according to Table 1. Therefore, for the first time, BL stars can be used for reliable statistical studies. Primary periods (those that appear with the highest amplitude in the frequency spectra of the original data) are listed in Table 3. Fourier decompositions of all BL stars are given in Table 4. Denoting the RR0 and BL (or modulation) frequencies by $f_0$ and $f_m$, respectively, a Fourier sum with frequencies $f_1 = f_0 - f_m$, $f_2 = f_0$, $f_3 = f_0 + f_m$, $f_4 = 2f_0 - f_m$, $f_5 = 2f_0$, $f_6 = 2f_0 + f_m$, $f_7 = 3f_0$, $f_8 = 4f_0$, $f_9 = 5f_0$, and $f_{10} = 6f_0$ has been fitted to each light curve, and outliers were successively omitted until no data points deviated from the fitted curve by more than $3 \sigma$ (i.e., 3 times the standard deviation of the residuals). The principle effect of these omissions is to remove photometry acquired in conditions of very poor seeing. Amplitudes and phases refer to the following type of decomposition:

$$V(t) = \mathcal{V} + a_1 \sin[2\pi(t - t_0) + \varphi_1] + \ldots .$$

Here $t$ is given in HJD$-2400000.0$, and the epoch $t_0$ is set equal to zero for all variables.

| Table 2 |
| --- |
| **Star-by-Start Comments on the Frequency Analysis** |

| MACHO ID | Type | Remarks |
| --- | --- | --- |
| 1.3441.1031 | . | . |
| 1.3442.1051 | . | . |
| 1.3442.1107 | . | . |
| 1.3442.1243 | BL2 | ? Long mod. period, Df=0.00063 (1/T=0.00037) |
| 1.3442.503 | . | . |
| 1.3443.1313 | . | . |
| 1.3443.1335 | . | . |
| 1.3444.1187 | RR1 & PC | . |
| 1.3444.56 | SOCEP | . |
| 1.3446.1877 | . | . |

Notes.—Table 2 is published in its entirety in the electronic edition of the *Astrophysical Journal*. A portion is shown here for guidance regarding its form and content. Table 2 is available in its entirety via the link to the machine-readable version above. For the definition of the variable types, please see the machine-readable version of this table.

Fig. 1.—Field-by-field distribution of the number of all fundamental-mode RR Lyrae stars (bottom, with the field number shown on the top of the columns) and the ratio of the BL stars to the total number of the fundamental-mode RR Lyrae stars (top). [See the electronic edition of the *Journal for a color version of this figure.*]
TABLE 3

COORDINATES AND MAIN PERIODS OF THE VARIABLES

| MACHO ID     | \(a(J2000.0)\) | \(\delta(J2000.0)\) | Period (days) |
|--------------|-----------------|---------------------|---------------|
| 1.3443.1335  | 05 01 26.0      | -69 15 38           | 0.4509916     |
| 1.3444.1187  | 05 01 55.4      | -69 11 31           | 0.4239698     |
| 1.3444.56    | 05 01 49.9      | -69 12 46           | 0.8057676     |
| 1.3446.1877  | 05 01 42.7      | -69 02 06           | 0.5997655     |
| 1.3446.1886  | 05 01 35.1      | -69 02 06           | 0.6061866     |
| 1.3446.2004  | 05 01 37.9      | -69 03 26           | 0.4876528     |
| 1.3449.1327  | 05 01 42.3      | -68 52 06           | 0.4839085     |

Notes.—Units of right ascension are hours, minutes, and seconds, and units of declination are degrees, arcminutes, and arcseconds. Table 3 is published in its entirety in the electronic edition of the Astrophysical Journal. A portion is shown here for guidance regarding its form and content. Table 3 is available in its entirety via the link to the machine-readable version above.

3. COMPARISON OF THE GLOBAL PROPERTIES OF THE RR0 AND BL STARS

The purpose of this section is to compare the overall photometric parameters of singly periodic and BL variables. We include all BL stars because a preliminary investigation did not find any differences between the two subgroups with respect to the relations to be discussed in this section.

First, we examine the average magnitudes. These values are computed from a Fourier fit described in the previous section. We used 10th-order fits for both the BL and singly periodic variables. For the BL stars the fits include the basic pulsation component and its harmonics only up to order 6 because of the addition of the modulation components at \(f_0\) and \(2f_0\). We found that this fitting procedure gives reliable values for the low-order modulation and Fourier components. The average magnitudes are plotted in Figure 2. In order to avoid unnecessary crowding, we took a subsample of 1000 variables from MACHO fields 1, 10, 11, 12, 13, and 14 to represent the RR0-S stars.

First of all, note that both groups of stars cover the same ranges of parameters in this diagram. Solely on the basis of their absolute magnitudes, BL variables are indistinguishable from singly periodic RR0 stars. Indeed, as shown in Table 6, the average of the mean magnitudes of both classes agree within their statistical errors. Both classes of stars display large scatter in \(\overline{V}\). This is due to various effects, including crowding, inhomogeneous reddening within the LMC, differences in the absolute magnitudes, finite geometrical extension of the LMC, and, of course, measurement/calibration errors. Despite the large scatter, some trend in the average magnitudes is observable. This trend is in qualitative agreement with the well-known relation between period and luminosity observed in other stellar clusters (e.g., Kovacs & Walker 2001) and also expected from stellar evolution theory, which predicts brighter horizontal branches at lower metallicities (i.e., at longer periods; see Castellani, Chieffi, & Pulone 1991).

TABLE 4

FOURIER DECOMPOSITIONS OF THE BL VARIABLES

| MACHO ID     | \(f_0\) | \(f_m\) | \(N\) | \(\overline{V}\) | \(\sigma\) | \(a_1\) | \(a_2\) | \(a_3\) | \(a_4\) | \(a_5\) | \(a_6\) | \(a_7\) | \(a_8\) | \(a_9\) | \(a_{10}\) | \(\varphi_1\) | \(\varphi_2\) | \(\varphi_3\) | \(\varphi_4\) | \(\varphi_5\) | \(\varphi_6\) | \(\varphi_7\) | \(\varphi_8\) | \(\varphi_9\) | \(\varphi_{10}\) |
|--------------|--------|--------|------|---------------|-------|------|------|------|------|------|------|------|------|------|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1.3446.2004  | 2.0564 | 0.0118 | 762  | 19.1238       | 0.0922 | 0.0234 | 0.2982 | 0.0461 | 0.0246 | 0.1305 | 0.0557 | 0.0808 |
| 1.3570.930   | 1.7092 | 0.0088 | 304  | 19.0981       | 0.0750 | 0.0138 | 0.3227 | 0.0641 | 0.0284 | 0.1305 | 0.0557 | 0.1033 |

Notes.—Table 4 is published in its entirety in the electronic edition of the Astrophysical Journal. A portion is shown here for guidance regarding its form and content. Table 4 is available in its entirety via the link to the machine-readable version above.
In the LMC is confined to a relatively narrow interval, as
implies that the metallicity of the majority of the RR0 stars
of the single-mode RR0 stars has recently been studied by
decompositions of a few BL stars. (Lack of similarity
suggested by these authors on the basis of the temporal Fourier
applied to the average light curve—than was originally sug-
if phase of the Fourier component at frequency
Table 6). The less significant difference between the phases
suggests that the
amplitudes than their singly periodic counterparts (see
Table 6). The less significant difference between the phases
suggests that the \( \langle P_0, \phi_{31} \rangle \rightarrow [\text{Fe/H}] \) relation derived by
Jurcsik & Kovács (1996) may have a wider validity—when
applied to the average light curve—than was originally sug-
gested by these authors on the basis of the temporal Fourier
decompositions of a few BL stars. (Lack of similarity
between the temporal light curves of the BL stars and those
of the single-mode RR0 stars has recently been studied by
Jurcsik, Benkő, & Szeidl 2002.) It is also worth noting that
the definite correlation between the period and the phases
implies that the metallicity of the majority of the RR0 stars
in the LMC is confined to a relatively narrow interval, as

Next we turn to the comparison of the Fourier param-
eters related to the main pulsation component \( f_0 \) and its
harmonics. Denoting by \( A_i \) the amplitude and by \( \phi_i \) the
phase of the Fourier component at frequency \( f_0 \) and
employing the standard, epoch-independent phase differ-
ce \( \phi_0 = \phi_1 - i \phi_1 \), Figures 3 and 4 show the variations of
these quantities as functions of the period. It is seen that
with increasing order, the Fourier amplitudes of the BL
stars become significantly lower than those of the RR0-S
stars (at the same period). A similar but much weaker effect
also exists among the phases, as can be inferred from the
computed average values and from their errors (see Table
6). This result implies that the modulation-free light curves
of the BL stars show less skewness and have lower total
amplitudes than their singly periodic counterparts (see
Table 6). The less significant difference between the phases
suggests that the \( \langle P_0, \phi_{31} \rangle \rightarrow [\text{Fe/H}] \) relation derived by
Jurcsik & Kovács (1996) may have a wider validity—when
applied to the average light curve—than was originally sug-
gested by these authors on the basis of the temporal Fourier
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between the temporal light curves of the BL stars and those
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\text{Notes.—Units of right ascension are hours, minutes,}
\text{and seconds, and units of declination are degrees, arcminutes,}
\text{and arcseconds. Table 5 is published in its entirety in the electronic}
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\text{entirety via the link to the machine-readable version above.}
\]

\[
\text{Fig. 3.—Comparison of the Fourier amplitudes of the}
\text{\( f_0 \), 2\( f_0 \), and 3\( f_0 \) components of the singly periodic}
\text{RR0 stars (open circles) and the BL stars (filled circles). [See the}
\text{electronic edition of the Journal for a color version of this figure.]}
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\[
\text{4. MODULATION FREQUENCY AND}
\text{AMPLITUDE DISTRIBUTIONS}
\]

In this section we will discuss both the frequency with
which modulation timescales appear in the BL population
and the incidence with which the stronger modulation com-
ponent is at higher or lower frequency than the primary
pulsation peak. For this purpose, we define the sign of the
modulation frequency \( f_m \) (\( |f_m| \equiv |f_0 - f_\lambda| \)) to be positive if
\( A_+ > A_- \) and negative if the reverse is true (here \( A_+ \) and \( A_- \)
are, respectively, the amplitudes of the modulation compo-
nents on the higher and lower frequency sides of the main
pulsation component). With the above definition we can
include all BL stars and study whether there is a preference
for a dominant frequency/amplitude pattern. Figure 5
shows the resulting distribution function. It is clearly seen
that there is a strong preference for frequency patterns in
which the modulation amplitude on the higher frequency
side is larger than its lower frequency counterpart. We find
that 74% of all BL stars have positive \( f_m \). Similar ratios are
observed when the two types of BL stars are studied sepa-
rately: \( f_m > 0 \) for 79% of the BL1 stars and 68% of the BL2
stars. BL variables with very long modulation periods and
those with periods as short as 50 days occur with about the

\[
\text{TABLE 5}
\text{ VARIABLES WITH MULTIPLE IDENTIFICATION NUMBERS}
\]

| MACHO ID | Period (days) | \( \alpha \) (J2000.0) | \( \delta \) (J2000.0) |
|----------|---------------|---------------------|---------------------|
| 18.3446.4944........ | 0.6061888 | 05 01 35.0 | -69 02 06 |
| 1.3446.1886........ | 0.6061866 | 05 01 35.1 | -69 02 06 |
| 1.3446.2004........ | 0.4876528 | 05 01 37.9 | -69 03 26 |
| 18.3446.4648........ | 0.4876523 | 05 01 37.8 | -69 03 26 |
| 18.3449.4465........ | 0.4939059 | 05 01 21.3 | -68 51 30 |
| 1.3449.1459........ | 0.4939031 | 05 01 21.3 | -68 51 31 |

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\text{Notes.—Units of right ascension are hours, minutes,}
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\text{FIG. 3.—Comparison of the Fourier amplitudes of the}
\text{\( f_0 \), 2\( f_0 \), and 3\( f_0 \) components of the singly periodic}
\text{RR0 stars (open circles) and the BL stars (filled circles). [See the}
\text{electronic edition of the Journal for a color version of this figure.]}
\]

\[
\text{TABLE 6}
\text{ AVERAGE FOURIER PARAMETERS OF THE SINGLY PERIODIC}
\text{RR0 AND BL STARS}
\]

| Parameter | Average (RR0) | Average (BL) |
|-----------|---------------|--------------|
| \( A_0 \) | 19.340 ± 0.008 | 19.351 ± 0.010 |
| \( A_1 \) | 0.293 ± 0.003 | 0.295 ± 0.003 |
| \( A_2 \) | 0.136 ± 0.001 | 0.123 ± 0.001 |
| \( A_3 \) | 0.094 ± 0.001 | 0.073 ± 0.001 |
| \( A_{\text{tot}} \) | 0.843 ± 0.008 | 0.755 ± 0.007 |
| \( \phi_{31} \) | 2.389 ± 0.005 | 2.365 ± 0.008 |
| \( \phi_{11} \) | 5.076 ± 0.011 | 4.922 ± 0.014 |
| \( \phi_{13} \) | 1.571 ± 0.019 | 1.301 ± 0.025 |

\[
\text{Notes.—The symbol \( A_{\text{tot}} \) denotes the total (peak-}
\text{to-peak) amplitude without modulation. Errors show the}
\text{standard deviations of the means as estimated from}
\text{the scatter of the observed values. A 4 \( \sigma \) cut is applied in}
\text{the computation of the \( A_0 \) averages.}
\]
same probability. For shorter modulation periods, the number of BL stars sharply decreases and reaches very small incidence rates for modulation periods shorter than about 25 days. Nevertheless, one can still find a few stars with very short Blazhko periods: 9.5360.804 and 6.6212.1121 have $f_m = 0.050$ and 0.122 days$^{-1}$, respectively, with the latter value being the only one in the whole sample exceeding 0.1 day$^{-1}$. We also note that a similarly short modulation period was observed in AH Cam by Smith et al. (1994).

It is also a matter of interest whether the small deviations from a strictly equidistant frequency spacing in the case of the BL2 stars are statistically significant. We recall that in the course of a similar analysis of the RR1 stars it was shown that the observed deviations are not significant (see A00). Following the same notation as in A00, we compute the quantity $\delta f \equiv f_+ + f_- - 2f_0$ to characterize the degree of deviation from equidistant spacing. We show the distribution function obtained from the analysis of the BL2 stars in Figure 6. We select the triplets from the frequency spectra through the following automated procedure:

1. Compute successively prewhitened amplitude spectra in the $0.1$ day$^{-1}$ neighborhood of the main pulsation component. Five prewhitening cycles are computed and the highest peak obtained after each cycle is stored.

2. The strongest secondary component $f$ is chosen from the stored peak frequencies by requiring $|f - f_0| > c_1/T$, where $c_1 = 1.5$ and $T$ is the total time span of the respective data set. This condition produces a distinction between PC and long-period BL stars.

3. A third peak is selected from frequencies appearing on the side opposite to the main component where the strongest secondary component was found. The variable is classified as a BL2 type if $|\delta f| < c_2/T$, where $c_2 = 3.0$. This condition is used to select triplets with “approximately” equidistant frequency spacings.

In nine cases out of the 331 BL2 variables, this automatic selection method could not find a triplet, usually because the large PC coexisting with the triplet did not allow the process to reach the level of the Blazhko components within the five prewhitening cycles. A high noise level was also a factor in some cases. Therefore, Figure 6 shows the results for the remaining 322 stars. (It is noted that five variables from this sample yielded considerably smaller $f_m$ values than the ones listed in Table 4. Again, this difference is accounted for in the analysis.)
for by the large PC in these stars. In order to maintain a close compatibility with the subsequent statistical tests, we keep these different modulation frequencies in the investigation of the distribution of the nonequidistant spacings.)

We see that, with few exceptions, all frequency deviations are within $\pm 0.0002$ day$^{-1}$. This is about one-half of the characteristic line width (i.e., $1/T \approx 0.00037$ day$^{-1}$). In order to assess if this degree of unequal spacing can still be explained as the result of a noise-induced frequency shift, we perform the following test. Artificial time series are generated, with equidistant triplets given by the Fourier decompositions (see §2) of the 331 BL2 stars. Then, a Gaussian white noise is added to these data, with the standard deviation of the fit of the original data. In the next step, these test data are processed in the same way as the real observations. Finally, we use the peak frequencies obtained from the analysis to compute $\delta f$ and plot the corresponding histogram. The result is shown in Figure 7. Comparison with Figure 6 reveals that the two distributions are very similar to each other, although the test data show a smaller dispersion and no outliers above $|\delta f| > 0.0002$ day$^{-1}$. The derived standard deviations are consistent with this conclusion. After applying a $3\sigma$ criterion for the omission of the outliers, we find $\sigma_{\delta f}^{\text{obs}} = 6.3 \times 10^{-5}$ day$^{-1}$ and $\sigma_{\delta f}^{\text{test}} = 4.9 \times 10^{-5}$ day$^{-1}$. By performing the above test for 10 different realizations of the noise for each time series, we get $\sigma_{\delta f}^{\text{test}} = (4.59 \pm 0.26) \times 10^{-5}$ day$^{-1}$, where the error represents the standard deviation of the 10 $\sigma_{\delta f}^{\text{test}}$ values obtained with the various realizations. We conclude that the difference between the observed and test values is statistically significant. Therefore, the observed small deviations from equidistant spacing cannot be accounted for by purely noise-induced frequency shifts.

In checking the notes (see Table 2) related to the nine variables mentioned before for not passing the automatic selection criteria and those 18 stars that did not pass the $3\sigma$ criterion with respect to the symmetric spacing, we find the following. Except for two cases, the light curves of the variables have some peculiarity—often of a PC nature—but we also find remarks on high noise level and small second-order modulation component. In one case there is no comment, and in a single case the observed frequency is declared “nice.”

Our conclusion from all these tests is that the cause of nonequidistant frequency spacing has various origins, including cases in which neither the noise level nor peculiar additional features are suspected to be responsible for the deviation. The cause of the nonequidistant spacing should be studied in detail on a star-by-star basis by considering various effects in addition to the noise-induced frequency shift, as discussed above.

In some models a resonance between the fundamental and Blazhko periods $P_{\text{BL}} = 1/f_m$ could be a matter of interest. Therefore, we compute the fractional part of $P_{\text{BL}}/P_0$ and plot its distribution function. As can be verified from Figure 8, there are no resonances between the two periods, and the distribution is essentially flat for all fractional values. The fluctuations are of statistical origin, because the relative standard deviation in an average bin is 23% (assuming a Poisson distribution).

We now investigate the distribution of the modulation amplitudes. There are two important properties to address: (1) range of the relative (pulsation vs. modulation) strength of the modulation amplitudes and (2) degree of asymmetry of the left- ($A_-$) and right- ($A_+$) modulation components. We employ the Fourier decompositions for both the BL1 and BL2 stars to get the observed values of $A_1$, $A_-$, and $A_+$. Since noise always introduces some nonzero excess power at all frequencies, a distortion of the distributions at low-modulation amplitudes is expected.

We plot the distribution of the relative modulation strength in Figure 9. In order to minimize the effect of noise mentioned above, we take the largest modulation component in each case. The distribution peaks sharply at $\approx 0.15$ and decays with a long tail after $\approx 0.2$. It is important to observe that there is a wide range of possible modulation strengths from very weak to very strong, with the largest having $A_+ / A_1 > 0.4$. Considering extreme modulation strengths, we call attention to the specific cases of variables
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The asymmetry in the modulation amplitudes is a significant problem for both currently available models (Shibahashi 2000, Nowakowski & Dziembowski 2001). To quantify the degree of asymmetry, we introduce the following parameter

\[ Q = \frac{A_+ - A_-}{A_+ + A_-}. \]

We recall that the amplitudes are computed from Fourier fits that include both the \( A_- \) and \( A_+ \) components: \( Q \) is found to cover almost all values, partly as a result of noise and partly because of the presence of “hidden” BL2 variables among the BL1 stars. In fact, if we perform an automated search for symmetric frequency patterns in the way mentioned at the beginning of this section, we find that 160 out of the 400 BL1 stars could be classified as BL2-type.

Figure 10 displays the distribution function of all BL stars. As we have already seen in Figure 5, there is a strong dominance of variables with \( A_+ > A_- \). The distribution is very broad, but the most probable cases are those with \( Q \approx 0.1 \). Of course, when the above distribution is plotted only for the BL1 variables, we find a deficit at small \( Q \) values, and the distribution is double-peaked at larger/smaller \( Q \) values. The BL2 variables are mostly confined to the \( Q = (0.2, 0.4) \) interval, but some extreme asymmetric cases can also be found. For example, variable 77.7793.1432 shows a large positive asymmetry with \( Q = 0.823 \).

5. A SEARCH FOR ADDITIONAL MODULATION COMPONENTS BY SPECTRUM-AVERAGING

Our standard method of analysis described in § 2 did not find any definite signature of two interesting features (both of these are expected within the frameworks of either the oblique rotator or of the resonant mode-coupling models). The first feature we expect is due to the long-term periodic variation of the average brightness. The presence of this component is expected from nonlinear mode coupling. The other possible feature is an equidistant quintuplet structure around \( f_0 \). This property is predicted by the oblique rotator model and is also expected when mode-coupling occurs. Since such features are not visible in the observed (individual) frequency spectra, we employ a method of spectrum averaging in order to amplify such small signals (if they are present).

Our procedure is the following. The huge differences in the lengths of the Blazhko periods require us to normalize the spectra in both the amplitude and frequency domains before averaging. Therefore, after prewhitening by the “trivial” components (i.e., \( f_0 - f_m, f_0, f_0 + f_m \), etc.) and taking the Fourier transform of the residuals, each spectrum is divided by its variance for more effective noise filtering of the different quality spectra. (We always use amplitude spectra—in both the computation of the individual and average spectra.) Then all peaks are identified and shifted by the amount necessary to position the modulation components in the same place in each case. This unified frequency band is divided into equidistant bins, and the maximum amplitude values of the peaks are found for each bin. These values are used in the binned-and-normalized version of original frequency spectrum. Finally, all these spectra are added together, and the average is plotted for visual inspection. To demonstrate the effectiveness of the method, Figure 11 reveals the result obtained by the analysis of a small sample of BL2 variables.

We see that the S/N of the average spectrum has dramatically improved compared to the majority of the individual spectra. The prewhitening by the \( f_0 \) component (and its harmonics) results in a dip at \( \Delta f / f_0 \), as we see in the following sections, dips related to prewhitening will show up more obviously when using a large set of variables. Occasionally, a rather large separation is observable between the peaks in the individual normalized spectra (e.g., variable 5.4288.4400). This effect is due to the long modulation periods of those stars, which results in a greater stretch in the frequency domain. Therefore, because of the finite...
Fig. 11.—Example of spectrum averaging. Individual normalized spectra are shown in each panel, except for the top right-hand panel, where the resulting average spectrum is displayed. Each spectrum has been computed after prewhitening by $f_0$ and its harmonics. Only the peak amplitudes are displayed. The symbol $\Delta f$ denotes $f - f_0$, where $f$ is the test frequency. All variables are classified as BL2 stars.
temporal baseline of the observations, we observe fewer peaks in the very narrow frequency band corresponding to the long modulation period. PC behavior is also observable in some cases (e.g., 5.4285.302). The variable 5.4891.855 represents one of the handful cases in which a quadruple structure is observed but with unequal frequency spacings.

5.1. Average Light Level

In order to reach the highest possible S/N, we employ all BL stars in the computation of an average spectrum. Individual frequency spectra are computed in the [0, 0.2] day\(^{-1}\) band. By using 200 bins in all cases, each spectrum is transformed in the frequency domain of \(0 < \Delta f / f_m < 3\), where \(\Delta f\) denotes the test frequency \(f\). The result is somewhat sensitive to the choice of the number of bins. If the bins are too wide (compared to the line width), then the hidden signal will be less effectively recovered because the neighboring smaller peaks (which come from the noise) will lower the amount of power collected in the bin of interest. On the other hand, bins that are too narrow will result in a division of signal power into several adjacent bins, as a result of the noise-induced frequency shift of the peak associated with the signal. With the above choice of bin number we cover approximately the characteristic line width at the moderately short Blazhko period of \(\approx 50\) days. This bin width is also appropriate for possible frequency shifts because of noise (see Figs. 6 and 7). In order to avoid too large a deformation in the plot due to the large dip at zero frequency (each time series has zero average), the final spectrum is shifted downward by an appropriate amount and then stretched in the vertical axis to cover the \([0, 1]\) interval by the \([\text{min}, \text{max}]\) range of the spectrum.

Figure 12 shows the resulting amplitude spectrum. We observe a definite peak at \(\Delta f / f_{\text{mod}} = 1.0\). Some earlier studies on Galactic Blazhko variables also indicated the presence of this component (Borkowski 1980; Kovács 1995; Nagy 1998). However, the data used in those studies suffered from very strong aliasing by the main periods of the variables caused by the observational technique employed when those variables were observed. As a result, the possibility of power leakage from the \(f_0 \pm f_m\) components to frequencies differing by \(f_0\) from these modulation components is expected.

Since the observed amplitude is influenced by the size of bins and noise level, the simplest way to get an estimate on the amplitude and significance of the peak is to add a sinusoidal tracer of a given frequency to the signal. This has been done, and the result shown in Figure 13 is obtained. By comparing the two peaks we estimate the overall amplitude of the observed modulation in the average light level of about 0.006 mag.

A further characteristic feature observable in the spectrum is the slow rise of the signal power from \(\Delta f / f_{\text{mod}} \approx 1.0\) to zero. This mild trend is most probably due to long-term instrumental or photometric calibration effects.

5.2. Quintuplet Structure

Side peaks at \(f = f_0 \pm 2f_m\), have been searched for in the same manner as the \(f_m\) component. Here the number of bins used was 400 because of the larger bandwidth required by this analysis. The average spectrum of all the BL stars is shown in Figure 14. In addition to some excess power very close to the prewhitened \(f_0\) component and to \(f_0 \pm f_m\), we
also suspect that there is a peak at \( f_0 - 2f_m \), very close to the noise level. The excess power near \( f_0 \) (and probably also at \( f_0 \pm f_m \)) is associated with the observed PC character of some variables. On average, this is a small effect, but in a few individual cases this may be comparable to the Blazhko effect (e.g., 6.6570.771 and 78.5615.905).

In order to estimate an upper limit for possible quadruplet components, we injected tracer signals into the individual time series in the same fashion as in our analysis for the \( f_m \) component. The effect of an 0.007 mag amplitude tracer is shown in Figure 15. We see that this type of component would have been easily detected from the present database. A test with an even lower amplitude of 0.005 mag shows that this would be also detectable, albeit with a lower significance. Therefore, we can safely conclude that if equidistant quadruplet structures exist in the BL stars, they would have amplitudes (on average) less than 0.004 mag.

6. CONCLUSIONS

We have studied the global statistical properties of amplitude- and phase-varying fundamental-mode RR Lyrae (BL) stars by performing a frequency analysis of 6391 variables from the MACHO LMC database. We found 731 BL stars, which is by far the largest population known in a single stellar system. Because of the large sample and the long time baseline of the MACHO project observations, it was possible to obtain statistically significant results, describing certain overall properties of the BL phenomenon. The main results are the following:

1. Fundamental-mode (RR0) BL stars constitute 11.9% of the presently analyzed LMC RR0 population (6158 stars). This incidence rate is 3 times larger than that of their first-overtone (RR1) counterparts (see Alcock et al. 2000).

2. A continuous transition seems to exist between variables showing an equidistant triplet around (and including) their main pulsation component in their frequency spectra and those displaying only a close doublet (BL2 and BL1 types, respectively). This suggests that strongly asymmetric modulation amplitudes are not rare among the BL2 stars, and their BL1 counterparts represent only extreme cases in which the missing component has such a low amplitude that it becomes buried by the observational noise.

3. Seventy-four percent of the BL stars have larger modulation amplitudes on the higher frequency side of the main pulsation component.

4. Statistical tests indicate that some fraction of the BL2 stars may have nonequidistant frequency spacing. There is also a small subgroup of 20 variables (RR0−νM class) that display several close peaks at the main pulsation component but without apparent regularity in their frequency spacing.

5. There is a nearly uniform distribution of BL modulation periods (\( P_{BL} \)) from \( \approx 50 \) to several hundred or even thousand days. In comparison, variables with \( P_{BL} \) < 30 days are rare.

6. Modulation amplitude (\( A_+ \) and \( A_- \)) components around the main pulsation component (\( A_1 \)) in the frequency spectra are usually in the range \( 0.1 < A_+ / A_1 < 0.3 \), but there are also more extreme values.

7. There is a variation with the overall amplitude of \( \approx 0.006 \) mag with a period of \( P_{BL} \) in the average brightness of the BL stars.

8. No clear trace of a quintuplet structure is found around the main pulsation component. The overall upper limit of the amplitude of such a component is estimated to be less than 0.004 mag.

9. No difference is found in the long-term average brightness between the singly periodic and BL RR0 stars. However, there are small, but systematic, differences in the Fourier decompositions of the modulation-free light curves of the BL stars and those of the singly periodic stars. The differences yield average BL light curves that have smaller amplitudes and lower degree of skewness than their singly periodic counterparts.

10. The pulsation component variables (stars with unresolved power at the main pulsation component) show an incidence rate of 2.9%, which is much lower than the rate of 10.6% of the RR1-PC stars.

Most of the above conclusions are very similar to those given by Moskalik & Poretti (2003) for the Galactic bulge RR Lyrae stars. The main difference is that they find a much higher incidence rate for RR0-BL stars. Their rate is 23%, which is in the often-quoted range for the BL stars (Szeidl 1988) but a factor of 2 higher than what we obtained for our LMC sample. Since the 215 RR0 stars analyzed by Moskalik & Poretti (2003) can be considered a statistically significant sample, we tend to agree with them that metallicity is the most probable agent for causing the difference in the incidence rates.

Very recently, Soszyński et al. (2003) published a similar analysis on 7612 RR Lyrae stars in the LMC from the OGLE-II database. Although a detailed comparison of our study with theirs is beyond the scope of the present paper, we note the following. Their incidence rates for the BL stars are 15% and 6% for the RR0 and RR1 stars, respectively. Both of these rates are somewhat larger than ours. It is unclear at this moment what the reason is for this slight discrepancy. It may come from either the higher S/N obtained from the use of Differential Image Analysis, or the higher density of variables in the LMC bar of the OGLE sample, or the details in the search for the modulation components (for example, a search in a narrower frequency band around the main pulsation component may lead to higher detection rates; see § 2). Further comparative studies are needed to answer these questions.
Unfortunately, neither the oblique magnetic rotator (Shibahashi 2000) nor the rotating resonant pulsator models (Van Hoolst, Dziembowski, & Kawaler 1998; Dziembowski & Cassisi 1999; Nowakowski & Dziembowski 2001) are capable of explaining the observed properties of the BL population. One major problem for these models is the production of strongly asymmetric modulation components. No hint of this behavior is found in the oblique magnetic rotator model, whereas the resonant one is able to explain only mild asymmetry. Although no current observational effort is known to be attempting to detect magnetic fields in RR Lyrae stars, the last result in this field by Chadid (2001) does not support the presence of strong magnetic field in RR Lyrae itself. The situation is also complicated concerning the direct detection of nonradial pulsation through a surface velocity field (Kolenberg 2002).

We conclude that the underlying reason for BL behavior is still not understood. The large sample reported here will allow investigators to select subsamples with common features for further study and will hopefully lead to a theoretical understanding of this rather obvious observational phenomenon. Precise spectroscopic observations to disentangle the large radial pulsation velocity field and the small nonradial one, together with a definite measurement of an upper limit on the magnetic field, are among the most important observational tasks for the near future.

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