Digital model of the oil spill process on the Earth’s surface

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Abstract. The Fourth Industrial Revolution currently taking place has an impact on human interaction with the environment, which must move to a new level, ensuring the harmonization of the needs of mankind. Cyber-physical systems can play an important role in the environmental sphere. The possibility of creating a digital model of the process of oil spills with their penetration into the ground and the adjacent water body, which causes significant damage to the environment and the economy, especially in the northern regions, is being considered. An approach to solving this problem is proposed, based on the representation of the soil contamination area in the form of a set of flat layers, each of which is calculated by the method of movable grids.

1. Introduction
The ongoing Fourth Industrial Revolution (Industry 4.0) being created at the same time, imply the massive introduction of cyber physical systems (CPS) in the production and service of human needs, including everyday life, labor, security, and leisure. It is expected that the changes caused by this revolution will cover the most diverse aspects of life: the labor market, living environment, political systems, technological order, human identity, and others. Human interaction with the environment will not be left aside, which should move to a new level, ensuring the harmonization of humanity’s needs.

Cyber-physical systems can play an important role in the environmental sphere. Is of interest to create digital models of one class of wild objects, which are called dynamic processes on the Earth’s surface [1]. In the general case, such a process can be called any phenomenon (natural or anthropogenic), which can be represented in the form of a moving region, contour or traveling wave propagating over the surface of the Earth. Examples of such processes are desertification, floods, mudflows, landslides, natural fires, oil spills on the surface of the earth and water bodies [2-9]. These objects have a different physical nature, but they have many common features. Due to these features, their dynamics and control can be described using a single mathematical apparatus. Such a device, in turn, allows you to create digital data models of objects, which are an indispensable part of all cyber-physical systems. In this paper, we consider the possibility of creating a digital model of one of the types of dynamic processes - the process of oil spill and their penetration into the ground and the adjacent reservoir. The processes of spills of polluting liquids on the surface of the Earth and water bodies occur quite often and harm the environment. The relevance of creating such models became obvious after an environmental disaster, a federal emergency that occurred on May 29, 2020, when the diesel fuel tank was depressurized at Power station-3 in Kayerkan (Norilsk region) [9]. This is one of the largest oil spills in the history of the Arctic zone, posing a threat to the ecosystem of the Arctic Ocean.
Therefore, the creation of digital models of such processes can contribute to the creation of decision support systems for the design of storage facilities for hazardous liquids and their operation. Present work proposes approach to solving this problem, based on the representation of the soil contamination area in the form of a set of flat layers, each of which is calculated by the method movable nets.

2. Model of the process of spreading liquid that has got into the ground

We will consider the process of liquid propagation in the soil in a three-dimensional region $D$, each point of which is determined by the vector $A = (x, y, z) \in D$, where the coordinates $x, y$ are the projection of the point onto the horizontal plane, $z$ is the depth of the point location. The polluting liquid trapped in the ground spreads in accordance with the law of liquid filtration in a porous medium. In accordance with Darcy's law \( \| \cdot \| \), the velocity vector filtration rate is determined by the equation

$$w = -k_\varphi \frac{dh}{dt} = -k_\varphi grad \ p \ [m/s],$$

(1)

where $k_\varphi$ – filtration coefficient $k_\varphi = \frac{\rho g k_\mu}{\mu} \ [m/s]$,

$\rho$ – density of the liquid [kg/m$^3$],

$g$ – acceleration due to gravity [kg/m$^2$],

$\mu$ – coefficient of dynamic viscosity [kg/m s],

$k$ – coefficient of permeability of the medium [m$^2$],

$grad \ p = p_0 = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$ – vector of the slope of the terrain [m/m].

The process contour (its outer boundary) at each moment of time is considered as a continuous differentiable surface, the equation of which is given in an implicit form \( \| \cdot \| \):

$$\phi(x, y, z, t) = 0, \ (x, y, z) \in D.$$  

(2)

At each point of the process boundary, the continuity condition must be satisfied, which is determined by the ratio $\frac{d\phi}{dt} = 0$. Since $x=x(t), y=y(t), z=z(t)$, the total derivative $\frac{d\phi}{dt} = 0$ can be represented in the following form

$$\frac{d\phi}{dt} = \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial x} \frac{dx}{dt} + \frac{\partial \phi}{\partial y} \frac{dy}{dt} + \frac{\partial \phi}{\partial z} \frac{dz}{dt} = 0.$$  

(3)

Let’s use the matrix representation of the written expression and denote:

$$v = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{bmatrix}$$ – velocity vector,  

$grad\phi = \begin{bmatrix} \frac{\partial \phi}{\partial x} \\ \frac{\partial \phi}{\partial y} \\ \frac{\partial \phi}{\partial z} \end{bmatrix}$ – process gradient,

whence the Hamilton-Jacobi equation follows \( \| \cdot \| \):

$$\frac{d\phi}{dt} + v^T grad\phi = 0,$$

(4)

where the T sign stands for transposition.

The expression $H = v^T grad\phi$ is called the Hamilton function of the given process.

You can enter the normal speed of the border movement

$$v_n = \frac{\left( grad\phi \right)^T}{\left| grad\phi \right|} \cdot v$$

(5)

where $\left( grad\phi \right)^T = n$ is the unit normal vector to the boundary of the process.

Considering (3), the equation of motion of the boundary takes the form
\[
\frac{d\varphi}{dt} = v_n |\nabla \varphi|, \quad (6)
\]
with initial conditions:
\[
t = 0 \quad \varphi(x, y, z, 0) = \varphi_0(x, y, z). \quad (7)
\]
Here \(\varphi_0(x, y, z) = 0\) is the equation of the outer boundary of the region \(D(0)\) at the initial moment of time.

In the model under consideration, the normal velocity vector \(v_n\) is the defining parameter of the process. This vector at each point of the boundary depends on the filtration velocity vector \(w\) and the direction of the normal to the process boundary - the vector \(n\). This dependency can be represented as follows:

\[
v_n = w \cdot \chi(\alpha). \quad (8)
\]

The function \(\chi(\alpha)\) is called the indicatrix of the normal speed of the propagation process (figurotrix). It shows how the speed of propagation of the process changes depending on the direction of the normal \(n\) [15]. In formula (8) \(\alpha\) is the angle between the direction of the normal to the process boundary and the direction of the vector \(p_0\), or \(\alpha = \arccos (w^*n)\), where the * sign denotes the scalar product of vectors. Figurotrix has the following properties:

1. \(0 \geq \chi(\alpha) \geq 1\).
2. \(\chi(0) = 1\) – the highest propagation speed of the process occurs when the direction of the normal \(n\) coincides with the direction of the filtration velocity vector \(w\).
3. \(\chi(\alpha) = \chi(-\alpha), \quad 0 \geq \alpha \geq \pi\) – the figurotrix is symmetric with respect to the sign of the angle \(\alpha\).

3. Digital terrain model

To solve practical problems of modeling the process under consideration, a digital terrain model should be prepared using a GIS system, where this process takes place, which was designated as area \(D\). The basis of the terrain model is an electronic map of the area of interest to us with the topology of layers and soil characteristics plotted on it to the required depth. It is also necessary to select homogeneous soil areas and for them determine the filtration coefficients \(k_\varphi\), the slope vectors of the terrain \(p_0\) and figurotrix \(\chi(\alpha)\).

The characteristics of the spilled liquid should be determined, the initial contour of the spill and its debit indicated. A detailed analysis of the process of filtration of oil products through the soil is contained in works [3,4]. In this work, an approximate estimate of the filtration coefficient \(k_\varphi\) is used based on the tabular data below (table 1), which may be sufficient for solving engineering problems (http://sdspmk.ru/dvizeniepw).

| Rock characteristics                                      | Filtration coefficient \(k_\varphi\) m/day |
|-----------------------------------------------------------|------------------------------------------|
| Very well permeable pebbles with coarse sand              | 100–1000 and more                        |
| Well-permeable pebbles and gravel with fine sand, coarse sand, clean medium-grained sand | 100–10 |
| Permeable pebbles and gravels with fine sand, medium and fine sands | 10–1 |
| Low-permeability fine-grained sands, sandy loam           | 1–0.1                                    |
| Low-permeability loams                                   | 0.1–0.001                                |
| Clays, marls, monolithic rocks                           | less than 0.001                          |

Table 1. Filtration coefficient for some rock characteristics
4. Algorithm for modeling the process of liquid propagation in soil

The algorithm is based on the use of the Darcy equation and the solution of the Hamilton-Jacobi equation by the method of moving grids [5,6] considering the three-dimensional space.

It is assumed that a polluting liquid with a volume of $V_0$ has spilled on the surface of the soil, which penetrates the soil and spreads in it.

At the same time, the hypothesis is accepted that the propagation process can be represented as a set of flat horizontal layers $C_k$, $k = 0, 1, ..., N$, located one below the other at different depths.

The propagation process is calculated in discrete time $t = 0, 1, ..., N$ with a step $\Delta t$, while the $C_k$ layers are displayed on the terrain map.

The speed of fluid movement in the soil is determined by Darcy’s law, considering the slope of the terrain, determined by the vector $\mathbf{p}_0$, and the characteristics of the soil. The figurotrix of the propagation process $\chi(\alpha)$ is determined by the slope of the terrain, the characteristics of the soil and the intensity of the process.

At the zero step at $t = 0$, the initial contour of the liquid spill $C_0$ is placed on the soil surface. Further, using the method of movable grids, the process contours are calculated for subsequent layers.

When passing from layer $C_k$ to the next layer $C_{k+1}$ at time $t + 1$, for all nodes of the process contour $C_k$, the normal vectors $p_i$ are determined as for the two-dimensional problem, and all points of the layer $C_{k+1})$. The new layer $C_{k+1}$ is "lowered" to the required depth. The distance between adjacent layers in depth is equal to $\Delta z = w \frac{z}{\rho} \Delta t$, so the layer $C_{k+1}$ turns out to be at the depth $z_{k+1} = w \frac{z}{\rho} \Delta t (k+1)$.

For each layer, its area $S_k$ and the volume of the liquid entering the layer $V_k = S_k \Delta z$ are calculated, as well as its total volume:

$$V_{ok} = \sum_{i=1}^{k} V_i. \quad (9)$$

The modeling process stops at the $N$ step either when the specified limiting calculation depth $z_{\text{max}}$ is reached, or when the total volume of the liquid that has fallen into the ground becomes equal to the volume of the source $V_0 : V_{0N} \geq V_0$.

If the process of spreading a liquid meets an aquifer or an open reservoir, the rate of spread of the pollution spot is determined by the laws of hydrodynamics. This issue is not considered in this paper.

5. An example of modeling the process of liquid propagation in soil

Below is a simple example of calculating the process of spreading oil products in the ground.

Figure 1 shows a sketch of the situation. The terrain model $D$ contains three homogeneous ground areas, as well as one water object (river). The soil on the plots consists of gravel of various compositions and slopes towards the river.

Initial data for simulation:

- liquid - diesel fuel, density at $20^{\circ}C$ – 800 kg/m$^3$
- kinematic viscosity at $20^{\circ}C$ - 0.08 cm$^2$/s,
- soil - homogeneous area: gravel with fine sand, filtration coefficient for different areas is $k_\varphi = 10$ m/day, $k_\varphi = 15$ m/day and $k_\varphi = 20$ m/day,
- the value of the slope of the terrain $|p_0| = 0.1$ m/m,
- filtration rate for different areas $w = k_\varphi |p_0| = 1$ respectively is 1, 1.5, and 2 m/day,
- figurotrix $\chi(\varphi) = (1 - e)/(1 - e \cos \varphi)$ – ellipse with eccentricity $e = 0.5$,
- time step $\Delta t = 1$ day,
- distance between adjacent layers $\Delta z = 0.1$ m,
- maximum number of layers $N = 4$.
Figure 1. Schematic diagram of the situation with an oil product spill and its penetration into the ground. It is shown a source of liquid $C_0$ and projections on the horizontal plane of the 3 contours of layers $C_k$ located one below the other at different depths.

On diagram: (a) – initial spill on surface of ground $C_0$; (b) – the spill on second day $C_2$ on depts 0.2 m; (c) - the spill on third day $C_3$ on depts 0.3 m, the spilled fuel reaches the water; (d) – the spill on fourth day $C_4$ on depts 0.4 m.

The area of the layers and their volume in ground obtained by numerical simulation are shown in table 2.

| t, days | Liquid spill | Spill depths $z_t$, m | Spill area, m² | Spill volume, m³ |
|---------|--------------|------------------------|----------------|------------------|
| 0       | $C_0$        | 0                      | 267            | 26.7             |
| 1       | $C_1$        | 0.1                    | 823            | 82.3             |
| 2       | $C_2$        | 0.2                    | 2702           | 270.2            |
| 3       | $C_3$        | 0.3                    | 3470           | 347.0            |
| 4       | $C_4$        | 0.4                    | 6371           | 637.1            |
|         | Total volume, m³ |                |                | 1383.3           |

6. Conclusion
The article proposes a new approach to solving the urgent problem of creating a digital model of the process of spilling a polluting liquid on the Earth's surface. A mathematical model of the process has been developed and an algorithm for solving this problem, based on the representation of the soil contamination area has been proposed, in the form of a set of flat layers, each of which is calculated by the method of moving meshes. With the further development of this direction, there is a lot of work to be done on the creation on the basis of GIS technologies of three-dimensional digital terrain models for
various types of soils, as well as databases containing the characteristics of soils and pollutants. These models should also include a description of the spread of pollution in aquifers and on the surface of water bodies. In addition, it is necessary to improve the mathematical and software tools to predict the process of spillage and filtration of polluting liquids in complex soils, including aquifers.

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