Dynamic Effects of Persistent Shocks

Mario Alloza*    Jesús Gonzalo*    Carlos Sanz*
Bank of Spain    Universidad Carlos III de Madrid    Bank of Spain

June 26, 2020

Abstract

We provide evidence that many narrative shocks used by prominent literature are persistent. We show that the two leading methods to estimate impulse responses to an independently identified shock (local projections and distributed lag models) treat persistence differently, hence identifying different objects. We propose corrections to re-establish the equivalence between local projections and distributed lag models, providing applied researchers with methods and guidance to estimate their desired object of interest. We apply these methods to well-known empirical work and find that how persistence is treated has a sizable impact on the estimates of dynamic effects.

Keywords: impulse response function, local projection, shock, fiscal policy, monetary policy.

JEL classification: C32, E32, E52, E62.

*We thank Fabio Canova, Jesús Fernández-Villaverde, Alessandro Galesi, Gergely Gánics, Juan F. Jimeno, Mikkel Plagborg-Møller, Juan Rubio-Ramírez, Enrique Sentana, and seminar participants at the I Workshop of the Spanish Macroeconomics Network (Universidad Pública de Navarra), Bank of Spain, CFE 2018 (University of Pisa), II Workshop in Structural VAR models (Queen Mary University of London), VII Workshop on Empirical Macroeconomics (Ghent University), 2019 American Meeting of the Econometric Society (University of Washington), and 2019 edition of the Padova Macro Talks for insightful comments. Alloza: m.alloza@bde.es. Gonzalo: jesus.gonzalo@uc3m.es. Sanz: carlossanz@bde.es.
1 Introduction

Estimating the impact of economic shocks is a crucial aspect of macroeconomics. To identify economically meaningful shocks, the literature has traditionally relied on systems of equations coupled with restrictions implied by economic theory. Recently, researchers are increasingly using narrative identification, e.g., looking at written official documentation or newspapers and exploiting arguably exogenous variation in these series.\(^1\) While its focus on identifying exogenous variation is appealing, the lack of restrictions in narrative methods yields objects with less standard time series properties.

In this paper, we analyze how the presence of persistence in narrative shocks affects the identification and estimation of their dynamic effects, providing empirical researchers with methods and guidance to deal with this issue.\(^2\)

We begin by showing that many narrative shocks used by prominent literature are serially correlated. In particular, we systematically test for serial correlation in eight shocks used in leading economics journals. We find evidence of serial correlation in seven of them. The presence of persistence in the shock does not necessarily preclude these variables from being categorized as “shocks” following standard definitions of aggregate shocks. More concretely, according to Ramey (2016), a shock should represent unanticipated movements. What this condition implies is that shocks are unforecastable, i.e., they are forecast errors. In particular, when the forecasting loss function is not quadratic, for instance, the check function, the forecasting errors may not be a martingale difference sequence (m.d.s) and therefore could be serially correlated. However, serial correlation poses additional challenges for the identification of the macroeconomic experiment of interest.

When estimating the dynamic response of some variable to a serially correlated shock, some part of this persistence may be passed on to the impulse response function (IRF).

\(^1\)See Romer and Romer (2004), Romer and Romer (2010), or Ramey and Zubairy (2018) for prominent examples of narrative identification.

\(^2\)Throughout the paper we use the term persistence as a phenomenon captured or reflected by serial correlation, a testable condition. We use both terms interchangeably.
Hence, a researcher may want to identify two objects of interest: the response as if the shock were uncorrelated, i.e., to a counter-factual serially uncorrelated shock ($R(h)^*$), or the response to the shock as it is, i.e., including the effect of persistence in the IRF ($R(h)$). Deciding for one or the other depends on what specific question the researcher is trying to address. On the one hand, $R(h)^*$ allows to compare effects with those obtained from a theoretical or empirical model, and facilitates comparisons across different types of shocks (e.g., monetary versus fiscal shocks) or across countries. On the other hand, $R(h)$ is more appropriate if the researcher is interested in evaluating the most likely dynamic response of a variable to a shock based on historical data. Regardless of which object is preferred by the researcher, the difference between $R(h)$ and $R(h)^*$ is informative about how much of the dynamic transmission of a shock is due to the presence of persistence.

We consider the two most popular methods to estimate impulse responses when a shock has already been identified (e.g., using narrative methods). These are local projections (LPs) (Jordà (2005)) and distributed lag models (DLMs).\footnote{By DLMs we refer to single-equation regressions of an outcome variable against the contemporaneous value and lags of the shock with or without an autoregressive component. These methods are also known as truncated moving average regressions. These specifications are frequent in the applied literature—see, e.g., Romer and Romer (2004), Cerra and Saxena (2008), Romer and Romer (2010), Alesina et al. (2015), Arezki et al. (2017), and Coibion et al. (2018).} We show that, if there is no serial correlation, the two methods identify the same object. However, we demonstrate that this equivalence breaks down in the presence of serial correlation. In this case, LPs identify $R(h)$ while DLMs regressions identify $R(h)^*$. The intuition is that LPs compute the response at horizon $h$ by regressing the outcome variable in $t+h$ against the shock in time $t$. Since the standard setting does not account for how the shock evolves between $t$ and $t+h$, the responses include two components: an economic effect (the economic impact of the shock on the endogenous variables) and an effect that exclusively depends on the degree of serial correlation of the shock. By contrast, DLMs implicitly account for the evolution of the shock, hence identifying the effect as if the shock were not persistent.

While this result might seem discouraging, we then show that it is possible to adjust
both estimating methods to obtain the desired object of interest. Consider a researcher who wants to use LPs and is interested in identifying $R(h)^*$. As mentioned, if she runs standard LPs with a persistent shock, she will identify $R(h)$ instead. Perhaps surprisingly, the most obvious solution of including lags of the shock will not address this issue. However, we show that, by including leads of the shock, she will recover $R(h)^*$. Likewise, we show how standard DLMs can be adapted so that they identify $R(h)$.

To illustrate how our methods work, we consider an actual empirical application, which also serves to assess the quantitative relevance of persistence in a real case by comparing estimates of $R(h)$ and $R(h)^*$. In particular, we consider Ramey and Zubairy (2018)'s LPs estimation of the dynamic effects to a shock constructed from news about future changes in defense spending. We find that, after two years, the responses that exclude the effect of persistence in the shock are about 40% lower than the original Ramey and Zubairy (2018)'s estimates. The effect of serial correlation also seems to have an effect on the short-run response of fiscal multipliers during recessions. In the appendix, we consider additional applications, based on Guajardo et al. (2014), Romer and Romer (2004), Gertler and Karadi (2015), and Romer and Romer (2010). Overall, we find that how persistence is treated can have a sizable impact on the estimated effects.

The results of this paper generalize in at least three important aspects. First, the results of the (lack of) equivalence between LPs and DLMs when the shock is persistent carry over to multivariate settings popularly used in the empirical literature. Building on a result by Plagborg-Møller and Wolf (2019), we show that the dynamic response from a VAR with the shock embedded as an endogenous variable is equivalent to that of a VAR with the shock included as an exogenous variable only when that shock has no serial correlation.\footnote{This result arises because a VAR with a shock as an exogenous variable (often known as VAR-X) can be seen as multivariate generalization of a DLM (see Mertens and Ravn (2012) or Favero and Giavazzi (2012) for examples of VAR-X specifications). Furthermore, Plagborg-Møller and Wolf (2019) show that, under some assumptions, LPs are equivalent to a VAR when the shock is included as an endogenous variable (as in Bloom (2009) or Ramey (2011)).} We believe this result has relevant practical implications for applied macroeconomic researchers. Second,
we also show that our results generalize to specific contexts where a researcher employs an instrument in a LP setting (also known as LP-IV). Lastly, a researcher interested in using LPs to uncover the dynamic relations of two variables may be interested in including leads of a third variable to construct counterfactual responses as if the behavior of that third variable had remained constant over the response horizon. This can be seen as the LP counterpart of constructing counterfactual responses in a VAR that allow to separate a direct effect of a regressor on a dependent variable from other indirect effects. This procedure has been frequently used in the empirical VAR literature.\footnote{Our paper makes four contributions to the literature. First, we formally and systematically test for the presence of serial correlation in shocks used by previous work. Although the issue of persistence in shocks has been noted before,\footnote{Ramey (2016) finds that the time aggregation required to convert the shock in Gertler and Karadi (2015) to monthly frequency, inserts serial correlation. Miranda-Agrippino and Ricco (2018) corroborate this finding, by regressing the shock on four lags and testing their joint significance. They also find that other measures of monetary shocks such as Romer and Romer (2004) exhibit serial correlation.} we believe we are the first to formally and systematically test for serial correlation in prominent narratively-identified shocks.}

Our second contribution is to show that, while both LPs and DLMs identify the same object if the shock is serially uncorrelated, this equivalence breaks down in the presence of persistence. Plagborg-Møller and Wolf (2019) prove that LPs and VAR methods identify the same impulse responses when both methods have an unrestricted lag structure. This result formalizes some of the examples provided in Ramey (2016), which implies that different identification schemes in a VAR setting can be implemented in a LP context. Our result builds on a different premise: we consider the cases where the shock has already been identified using narrative measures and the researcher wants to use LPs or DLMs to estimate dynamic effects.

Our third contribution is to provide methods to re-establish the LP-DLM equivalence when there is persistence, providing applied researchers with a menu of options to identify their desired object of interest. In this regard, our method of adding leads to LPs is related to the tradition in factor analysis by Geweke and Singleton (1981) and on the DOLS estimation

\footnote{See, for example, Bernanke et al. (1997), Sims and Zha (2006), or Bachmann and Sims (2012). In recent research, Cloyne et al. (2020) propose an alternative method based on a Blinder-Oaxaca-type decomposition.}
of cointegration vectors (Stock and Watson (1993)). Dufour and Renault (1998) introduce leads in some of their IRFs to study causality at different horizons. Faust and Wright (2011) find that including ex-post forecast errors results in an accuracy improvement when forecasting excess bond and equity returns. More recently, Teulings and Zubanov (2014) find that estimating dynamic effects of a dummy variable (e.g., banking crisis) in a panel data context with fixed effects and LPs suffers from a negative small-sample bias, since the estimation of the fixed effect picks up the value of future realization of the dummy variable. The authors show that this bias is attenuated either by increasing the sample size or by including future realizations of the dummy variable over the response horizon.\footnote{By contrast, the difference between LPs and DLMs that we identify is not due to a bias in the estimates, but instead to differences in identification due to the persistence of the shock. Since our problem still persists asymptotically, increasing the sample does not reduce the LP-DLM difference. Additionally, this difference is not necessarily negative, but will depend on the nature of the data generating process that drives the persistence.}

Finally, we speak to some recent and well-known empirical work on the effects of monetary and fiscal policy (Ramey and Zubairy (2018) Guajardo et al. (2014), Romer and Romer (2004), and Gertler and Karadi (2015)). Our contribution is to apply our methods to these works and re-assess their empirical evidence. We do not claim that any of these papers is “wrong”. Rather, what our results indicate is that the correct interpretation of their results depends on the desired object of interest and the employed estimating method.

The rest of the paper proceeds as follows. Section 2 provides evidence of serial correlation in shocks used by previous work. Section 3 describes that LPs and DLMs treat persistence differently, and proposes a solution to re-establish the equivalence between them. It also provides simulations to help understand the results. Section 4 discusses the previous findings and the options available to applied researchers working with a persistent shock. Section 5 lays out an application. Section 6 concludes. The online appendix contains proofs of the theoretical results and further material, including the generalization of the results to VAR and IV settings, additional robustness exercises, and other empirical applications.
2 Evidence and implications of serial correlation in shocks

When shocks are identified from within an empirical model, the researcher imposes a set of restrictions to recover shocks that can be economically meaningful. Typically, this implies that the resulting shocks are well-behaved and display some statistical features that might be seen as desirable—in particular, no persistence. Alternatively, shocks may be identified without the use of a model, for example, by using narrative methods. This alternative identification relies on the existence of historical sources, such as official documentation, periodicals, etc., from which a shock variable is constructed. In this section, we provide evidence that it is common that shocks identified this way are persistent. We then take stock on this finding in light of Ramey (2016)’s canonical definition of a shock.

We study eight aggregate shocks used by prominent literature on monetary and fiscal policy. Some of these shocks are identified using narrative methods, while some employ alternative strategies such as timing restrictions using high-frequency methods.\textsuperscript{8}

To test for the presence of persistence we use a \textit{portmanteau}-type test following Box and Pierce (1970).\textsuperscript{9} The null hypothesis is that the data are not serially correlated. We test for the presence of autocorrelation in 40 periods, although results are robust to different horizons (see Table D.1).

\textsuperscript{8} In particular, Romer and Romer (2010) and Cloyne (2013) construct measures of exogenous tax changes for the US and the UK, respectively. The authors classify legislated tax measures according to the motivation, as reflected in official documentation, and consider those tax changes that are the result of causes non-related to the state of the economy. In a similar vein, Ramey and Zubairy (2018) construct a measure of government spending shocks by looking at the announcements of future changes in defense spending. Guajardo et al. (2014) construct a series of fiscal consolidations in OECD countries motivated by a desire to reduce the deficit (as opposed to motivated by current or prospective economic conditions). Romer and Romer (2004) and Cloyne and Hürten (2016) identify exogenous changes in monetary policy by looking at the minutes and discussion of the monetary policy committees of the Federal Reserve and Bank of England, respectively (they also orthogonalize the resulting series using forecastable information available at that time). Alternatively, Gertler and Karadi (2015) identify a proxy of monetary policy shocks using high frequency surprises around policy announcements. Lastly, Arezki et al. (2017) construct a measure of news shocks based on the date and size of worldwide giant oil discoveries. While some of these papers employ auxiliary regressions to isolate forecastable information, all have in common that the shocks have not been exclusively identified from a time series model.

\textsuperscript{9} We implement the small sample correction following Ljung and Box (1978). For the cases of Arezki et al. (2017) and Guajardo et al. (2014), which refer to panel data, we test serial correlation using a generalized version of the autocorrelation test proposed by Arellano and Bond (1991) that specifies the null hypothesis of no autocorrelation at a given lag order.
The results from these tests are displayed in Table 1. Out of the eight considered shocks, six show very large test statistics that result in rejections of the hypothesis of serial uncorrelation for any level of significance. One of them (Romer and Romer (2004)) displays some degree of serial correlation which leads to failure to reject the null hypothesis only for significance levels above 5%.\(^{10}\) As further evidence of the presence of serial correlation in the above series, Figure D1 plots the associated correlograms. Romer and Romer (2010) constitutes the only considered shock for which we fail to detect the presence of persistence.\(^{11}\)

According to the canonical definition (Ramey (2016)), empirical shocks should (i) be exogenous to current and lagged endogenous variables, (ii) be uncorrelated to other exogenous shocks, and (iii) represent unanticipated movements (or news about future shocks). While one might think that the presence of persistence violates the third condition, this is not necessarily the case. When the forecasting loss function is the quadratic one, it is well known that the forecasting errors must be a m.d.s with respect to some information set and therefore uncorrelated.\(^{12}\) This is the case when the shocks come directly from a conditional expectation model, like a VAR model. When the forecasting loss function is not quadratic, for instance, the check function (popular in quantile regressions), the forecasting errors are not a m.d.s and therefore they could be serially correlated. They still are forecasting errors (satisfy (iii)) but are serially correlated.

This indicates that serially-correlated shocks can still be labeled “shocks” according to the previous definition. However, even if a researcher always operates under the quadratic

\(^{10}\)The hypothesis of serial uncorrelation is rejected for significance levels below 5\% when considering fewer lags in the test or when considering a longer series (with updated data) from Coibion (2012). The presence of some degree of autocorrelation is shown in Panel E of Figure D1.

\(^{11}\)Persistence may have different origins. In some instances, it arises because of the method used to convert a nominal series into real terms. For example, Cloyne (2013) and Arezki et al. (2017) divide their series by lagged GDP, while Ramey and Zubairy (2018) use the GDP deflator and a measure of trend GDP. In other instances, the serial correlation arises because of the mapping between different time frequencies. This is usually the case with the identification of monetary policy shocks, such as Romer and Romer (2004), Gertler and Karadi (2015), or Cloyne and Hürtgen (2016), where daily monetary changes are converted into monthly series. Finally, there are other shocks that are more likely to appear together, because of their multi-period nature (for example, episodes of fiscal consolidations, as identified by Guajardo et al. (2014), tend to be spread over the course a few years) or because they cluster around events like wars (as in Ramey and Zubairy (2018)).

\(^{12}\)See Granger and Machina (2006) and Lee (2008) for a description and analysis of loss functions.
Table 1: Persistence in macroeconomic shocks

| paper                     | type of shock                    | Box-Pierce (40) test | p-value |
|---------------------------|----------------------------------|----------------------|---------|
| Arezki et al. (2017)      | news about oil discoveries       | 177.903              | 0.000   |
| Cloyne (2013)             | tax (UK)                         | 98.751               | 0.000   |
| Cloyne and Hürtgen (2016) | monetary policy (UK)             | 84.422               | 0.000   |
| Gertler and Karadi (2015) | monetary policy (US)             | 124.568              | 0.000   |
| Guajardo et al. (2014)    | fiscal consolidations            | 185.810              | 0.000   |
| Ramey and Zubairy (2018)  | government spending              | 182.950              | 0.000   |
| Romer and Romer (2004)    | monetary policy (US)             | 53.758               | 0.072   |
| Romer and Romer (2010)    | tax (US)                         | 19.023               | 0.998   |

The third column implements the Box and Pierce (1970) test of serial correlation using the small sample correction following Ljung and Box (1978). The null hypothesis of this test assumes that the data are not serially correlated within 40 periods. For Arezki et al. (2017) and Guajardo et al. (2014), which refer to panel data, we use a generalized version of the autocorrelation test proposed by Arellano and Bond (1991). The serial correlation test yields p-values smaller than 0.05 when testing the shocks of Romer and Romer (2004) with fewer lags or when using the updated data from Coibion (2012) (p-value drops to 0.0041). Ramey and Zubairy (2018) use extended data from Ramey (2011).
loss function and considers that serially-correlated shocks should not be called “shocks”, in the rest of the paper we show that such shocks can still provide valuable information for empirical analysis.

3 Theoretical framework

We consider the following VAR as the data generating process:

\[
y_t = \sum_{\ell=1}^{\infty} A_{\ell} y_{t-\ell} + \sum_{q=0}^{\infty} \delta_q x_{t-q} + u_t
\]
\[
x_t = \sum_{r=1}^{\infty} \gamma_r x_{t-r} + \varepsilon_t,
\]

where \( y_t \) is a vector of endogenous time series, \( x_t \) is a strictly exogenous variable such that \( \mathbb{E}(u_t | y_{t-s}, x_{t-p}) \) for all \( s > 0, p \leq 0 \), and \( u_t \) and \( \varepsilon_t \) are a vector and a scalar i.i.d. variables, with mean and variance given by \( u_t \sim (0, \Sigma_u^2) \) and \( \varepsilon_t \sim (0, \sigma_\varepsilon^2) \), respectively. Following the evidence discussed in the previous section, \( x_t \) is considered to be a shock identified using narrative methods and is allowed to be persistent.

This general framework encompasses several empirical specification often found in the literature. For example, when ignoring the second equation, system (1) becomes a VAR with an exogenous variable (or VAR-X).\(^{13}\) Additionally, when \( y_t \) is a scalar and \( A_{\ell} = 0 \) for all \( \ell \), system (1) becomes a DLM.\(^{14}\) Alternatively, when \( x_t \) is instead included in the vector of endogenous variables \( y_t \), system (1) becomes a standard VAR.\(^{15}\) We explore the implications of this last representation in Appendix B.1.

Without loss of generality, we consider a simpler version of system (1) with \( A_{\ell} = 0 \) for all \( \ell \),

\(^{13}\)See, for example, Mertens and Ravn (2012) or Favero and Giavazzi (2012), which assume \( \ell \) and \( q \) are finite numbers.
\(^{14}\)As in Romer and Romer (2004) or Romer and Romer (2010).
\(^{15}\)As in Bloom (2009) or Ramey (2011).
\( \delta_q = 0 \ \forall q > 0 \) and \( \gamma_r = 0 \ \forall r > 1 \):

\[
\begin{align*}
y_t &= \delta x_t + u_t \\
x_t &= \gamma x_{t-1} + \varepsilon_t,
\end{align*}
\] (2)

where \( y_t \) is now the economic outcome variable for interest (for example, GDP), \( x_t \) is an economic shock (e.g., a fiscal or monetary policy shock) which is strictly exogenous \( \mathbb{E}(u_t|x_{t-p}) \) \( \forall p \geq 0 \), and \( u_t \) and \( \varepsilon_t \) are i.i.d variables with mean and variance given by \( u_t \sim (0, \sigma_u^2) \) and \( \varepsilon_t \sim (0, \sigma^2_{\varepsilon}) \), respectively. \( \delta \) measures the contemporaneous impact of variable \( x_t \) on \( y_t \) and is the main parameter of interest.

The data generating process described by system (2) is intentionally simple to illustrate how the dynamic relationship between the dependent variable \( y_t \) and the shock \( x_t \) depends on the persistence of the latter. Importantly, the obtained results also arise in more complex settings when we incorporate more general characteristics as in system (1).\(^{16}\)

We are interested in recovering the response of our variable of interest \( y_t \) when a shock \( x_t \) hits the system in period \( t \). We consider two different IRFs. The first one, denoted by \( \mathcal{R}(h) \) for period \( h \), is:

\[
\mathcal{R}(h) = \mathbb{E} [y_{t+h} | x_t = 1, \Omega_{t-1}] - \mathbb{E} [y_{t+h} | x_t = 0, \Omega_{t-1}],
\] (3)

where \( \Omega_{t-1} \) represents all the history of previous realizations of \( \varepsilon_t \) and \( x_t \) up to period \( t-1 \). Importantly, note that the above definition does not condition for future realizations of \( x_t \). Hence, if \( \gamma \neq 0 \), an initial unit impulse in \( x_t \) does not imply that \( x_{t+j} = 0 \).\(^{17}\) In other words, equation (3) describes dynamic responses that include the possible persistence of the

\(^{16}\)For example, in Subsection 3.3, we consider models that also include persistence in the dependent variable and lagged effects of the shock. Appendix B.2 proposes a DGP that calls for the use of instruments in LP regressions. Appendix B.3 provides an alternative specification where the degree of serial correlation in the shock \( x_t \) is taken from the actual data, instead of following an autoregressive process.

\(^{17}\)This impulse response is equivalent to \( \mathcal{R}(h) = \mathbb{E} [y_{t+h} | \varepsilon_{t+1} = 1, \varepsilon_{t+1} = 0, ..., \varepsilon_{t+h} = 0, \Omega_{t-1}] - \mathbb{E} [y_{t+h} | \varepsilon_{t} = 0, \varepsilon_{t+1} = 0, ..., \varepsilon_{t+h} = 0, \Omega_{t-1}] \). See, for example, Koop et al. (1996).
shock $x_t$. For example:

\[
R(0) = \frac{\partial y_t}{\partial x_t} = \delta \\
R(1) = \frac{\partial y_{t+1}}{\partial x_t} = \delta \gamma \\
R(2) = \frac{\partial y_{t+2}}{\partial x_t} = \delta \gamma^2 \\
\ldots
\]

However, the researcher might also be interested in the response to the shock as if the shock had no persistence. We call this second IRF $R(h)^\ast$ and define it as:

\[
R(h)^\ast = \mathbb{E}[y_{t+h}|x_t = 1, x_{t+1}, \ldots, x_{t+h}, \Omega_{t-1}] - \mathbb{E}[y_{t+h}|x_t = 0, x_{t+1}, \ldots, x_{t+h}, \Omega_{t-1}].
\]

Contrary to $R(h)$, $R(h)^\ast$ explicitly controls for future realizations of $x_t$ so that it describes dynamic responses that do not incorporate the effect of persistence (regardless of the value of $\gamma$), i.e., the responses are observationally equivalent to those that would arise from a data generating process with $\gamma = 0$:\footnote{The definition of $R(h)^\ast$ is not new. When $x_t$ is the shock variable of interest, this impulse response is referred to as the “traditional impulse response function” by Koop et al. (1996): $R(h)^\ast = \mathbb{E}[y_{t+h}|x_t = 1, x_{t+1} = 0, \ldots, x_{t+h} = 0, \Omega_{t-1}] - \mathbb{E}[y_{t+h}|x_t = 0, x_{t+1} = 0, \ldots, x_{t+h} = 0, \Omega_{t-1}]$. It provides an answer to the question “what is the effect of a shock of size 1 hitting the system at time $t$ on the state of the system at time $t + h$ given that no other shocks hit the system?”.
}

\[
R(0)^\ast = \frac{\partial y_t}{\partial x_t} = \delta \\
R(1)^\ast = \frac{\partial y_{t+1}}{\partial x_t}
|_{x_{t+1} = 0} = 0 \\
R(2)^\ast = \frac{\partial y_{t+2}}{\partial x_t}
|_{x_{t+1}, x_{t+2} = 0} = 0 \\
\ldots
\]

Note that, if $\gamma = 0$ (the shock is not persistent), then $R(h) = R(h)^\ast$ $\forall$ $h$. By contrast, if $\gamma \neq 0$, then $R(h) \neq R(h)^\ast$ $\forall$ $h > 0$. 


3.1 Differences between DLMs and LPs under persistence

We now consider the two most frequently used methods to estimate impulse responses when a shock is independently identified, DLMs and LPs, and compare the objects that they identify when the shock is persistent. We first consider the case of DLMs. The use of these models is widespread in applied macroeconomics.\footnote{See, for example, Romer and Romer (2004), Cerra and Saxena (2008), Romer and Romer (2010), Alesina et al. (2015), Arezki et al. (2017), Coibion et al. (2018) for interesting applications based on DLM methods, or Baek and Lee (2020) for a discussion of their properties. As mentioned in the introduction, these methods are also a special case of more general specifications such as VARs with exogenous variables (or VAR-X). We develop this point further in Appendix B.1, when generalizing some of the results of the paper.}

In the case of system (2), note that we can recover the response function $\mathcal{R}(h)^{DLM}$ using the following regression:\footnote{This regression should include as many lags as the response horizon $h = 0, 1, \ldots, H$.}

$$y_t = \theta_0 x_t + \theta_1 x_{t-1} + \theta_2 x_{t-2} + \theta_3 x_{t-3} + \theta_4 x_{t-4} + \ldots + e_t,$$

(5)

and it follows that $\mathcal{R}(h)^{DLM} = \frac{\partial y_{t+h}}{\partial x_t} = \theta_h \forall h$.

The second main method to compute impulse responses is LPs, proposed by Jordà (2005). LPs are more robust to certain sources of misspecification and for this reason, their use has increased in recent times (see Ramey (2016) for examples). LPs compute impulse responses by estimating an equation for each response horizon $h = 0, 1, \ldots, H$:

$$y_{t+h} = \delta_h x_t + \xi_{t+h},$$

(6)

where the sequence of coefficients $\{\delta_h\}_{h=0}^{H}$ determines the response of the variable of interest $\mathcal{R}(h)^{LP} = \delta_h$ for each horizon $h$.\footnote{Unrelated to our case at hand, note that the structure of the LPs induce serial correlation in the residuals $\xi_{t+h}$. This is usually corrected by computing autocorrelation-robust standard errors (Jordà (2005)). See Olea and Plagborg-Møller (2020) for a recent contribution on inference in LPs.}

We now consider under which conditions both methods identify the same objects.

Proposition 1. Given the data generating process described by system (2), if the shock $x_t$ is serially uncorrelated, then the response functions identified by DLMs and LPs are equal for all response horizons, that is:

\[\mathcal{R}(h)^{DLM} = \mathcal{R}(h)^{LP} = \delta_h \forall h.\]
If $\gamma = 0$, then $R(h)^{DLM} = R(h)^{LP} = R(h)^* = R(h) \forall h$.

If the shock is serially correlated, then the response functions identified by DLMs and LPs are different for all $h > 0$:

If $\gamma \neq 0$ and $h = 0$, then $R(h)^{DLM} = R(h)^{LP} = R(h)^* = R(h)$.

If $\gamma \neq 0$ and $h \geq 1$, then $R(h)^{DLM} = R(h)^* \neq R(h)^{LP} = R(h)$.

**Proof.** See Appendix A.1. \hfill \Box

Following the above proposition, when $\gamma \neq 0$, LPs recover a dynamic response that includes three dynamic effects: (i) the effect that $x_t$ has directly on $y_{t+h}$ (due to a lagged impact of the shock), (ii) the effect that $x_t$ has through the persistence of $y_t$, and (iii) the effect that $x_t$ has on $y_{t+h}$ through $x_{t+h}$ (since $\text{cov}(x_t, x_{t+h}) \neq 0$ when $\gamma \neq 0$). The first two effects are independent of $\gamma$ and are shut down in our simple specification of system (2) (we will incorporate them in our simulation exercises in the next subsection). The last effect (the *persistence* effect of $x_t$) drives the difference between $R(h)^{DLM}$ and $R(h)^{LP}$. In particular, $R(h)^{LP} = R(h) = \delta \gamma^h$, while $R(h)^{DLM} = R(h)^* = 0$ for all $h \geq 1$.

To understand why LPs, unlike DLMs, incorporate this third effect, consider the LPs when $h = 1$:

$$y_{t+1} = \delta_1 x_t + \xi_{t+1},$$

(7)

where $\delta_1 = R(1)^{LP}$. The direct effect of $x_t$ on $y_{t+1}$ is 0. If $x_t$ had no persistence, then $\delta_1$ would be 0. However, when $\gamma \neq 0$, we can use system (2) to express $y_{t+1}$ as a function of $x_t$:

$$y_{t+1} = \delta x_{t+1} + u_{t+1}$$

$$= \delta (\gamma x_t + \varepsilon_{t+1}) + u_{t+1}$$

$$= \delta \gamma x_t + u^*_{t+1},$$

where $u^*_{t+1} = \delta \varepsilon_{t+1} + u_{t+1}$. This shows that the coefficient $\delta_1$ in equation (7) will also recover the persistence effect of $x_t$: $\delta_1 = \delta \gamma$. The intuition is that between period $t$ and period $t + 1$,
$x_t$ affects $x_{t+1}$ when $\gamma \neq 0$. Since $x_{t+1}$ is not a regressor in equation (7), then this effect is absorbed by $\delta_1$.

When impulse responses are identified using DLMs, the treatment of the persistence of $x_t$ is different. Consider a version of equation (5) expressed in terms of $t+1$:

$$y_{t+1} = \theta_0 x_{t+1} + \theta_1 x_t + \theta_2 x_{t-1} + \theta_3 x_{t-2} + \theta_4 x_{t-3} + \ldots + \epsilon_{t+1}.$$ (8)

As noted earlier, the sequence of coefficients $\theta_h$ determines the response function. Consider the response when $h = 1$, i.e., $\mathcal{R}(1)^{DLM} = \theta_1$. Note that, while we know from system (2) that $\frac{\partial y_{t+1}}{\partial x_t} = \delta \gamma$, the coefficient recovered by $\theta_1$ is indeed $\frac{\partial y_{t+1}}{\partial x_t} \bigg|_{x_{t+1}} = 0$. That is, since the DLM controls for $x_{t+1}$, the persistence effect of $x_t$ is accounted for.

In other words, DLMs identify:

$$\mathcal{R}(h)^{DLM} = \mathbb{E} [y_{t+h}|x_t = 1, \Omega_{t-1}, x_{t+h-1}, \ldots, x_{t+1}] - \mathbb{E} [y_{t+h}|x_t = 0, \Omega_{t-1}, x_{t+h-1}, \ldots, x_{t+1}],$$

while LPs identify:

$$\mathcal{R}(h)^{LP} = \mathbb{E} [y_{t+h}|x_t = 1, \Omega_{t-1}] - \mathbb{E} [y_{t+h}|x_t = 0, \Omega_{t-1}].$$

Note that the difference between $\mathcal{R}^{LP}$ and $\mathcal{R}^{DLM}$ is positive (negative) when $\gamma > 0$ ($\gamma < 0$). In empirical applications, $\gamma$ may be positive or negative.

### 3.2 Reestablishing the equivalence between DLMs and LPs

In this subsection we lay out two methods that can render the responses from DLMs and LPs identical, even under the presence of persistence.

---

22This omitted variables problem is also briefly mentioned in Alesina et al. (2015) in the particular context of fiscal consolidation plans.

23For example, $\gamma$ seems to be positive in Ramey and Zubairy (2018), and negative in Romer and Romer (2004).
3.2.1 Adapting LPs to exclude the effect of serial correlation

A researcher may be interested in recovering responses as if the shock were serially uncorrelated ($\mathcal{R}(h)^*$). (We discuss in Section 4 when the object of interest may be $\mathcal{R}(h)^*$, or $\mathcal{R}(h)$ instead.) However, we have shown that $\mathcal{R}^{LP}(h) \neq \mathcal{R}(h)^*$ if $\gamma \neq 0$ and $h \geq 1$.

Two apparent methods to avoid LPs picking up the effect of persistence in $x_t$ are: (i) to include lags in the regression (6), or (ii) to replace $x_t$ with the error term that purges out the persistence:

$$\varepsilon_t = x_t - \gamma x_{t-1}. \quad (9)$$

However, neither of these methods yields $\mathcal{R}^*(h)$. The reason is that replacing $x_t$ with $\varepsilon_t$ does not include any further information between $t$ and $t + h$, so the responses of the dependent variable will still be affected by $x_{t+h}$. This point is further developed in Appendix B.4.

A third potential method to exclude the effect of persistence would be recasting system (2) as a VAR that includes the shock as an endogenous variable. However, since in this case LPs and a VAR would identify the same impulse responses (see Plagborg-Møller and Wolf (2019)) the VAR responses would also include an effect due to the persistence of the shock—we explore this in more detail in Appendix B.1.

Instead, we propose a method based on the inclusion of leads of the persistent shock variable. In particular, given that the DGP of system (2) poses an AR(1) for $x_t$, one should regress:

$$y_{t+h} = \delta_{h,0} x_t + \delta_{h,1} x_{t+1} + \xi_{t+h}, \quad (10)$$

where $\delta_{h,0}$ is the $h$-horizon response identified by LPs that include leads of the shock $x_t$, which we denote as $\mathcal{R}^F(h)$. In more general processes, in which the autocorrelation of the shock may be of an order larger than one, the optimal choice of leads can be derived adapting the procedure from Choi and Kurozumi (2012).24 The most conservative procedure would be to include $h$ leads of the shock in each period $h$. This is the choice implemented in Section 5,

---

24See also Lee (2020) for lag order selection in LPs.
when considering empirical applications.

**Proposition 2.** Given the data generating process described by system (2), the response function identified by modified LPs to a shock \( x_t \) as described in equation (10) is equal to the response as if the shock had no persistence (and to the response obtained from DLMs as in equation (5)), that is:

\[ R(h)^F = R(h)^* = R(h)^{DLM} \forall \gamma \text{ and } h. \]

*Proof.* See Appendix A.2.

Intuitively, leads of \( x_t \) in equation (10) act as controls for the persistence of the shock throughout the response horizon, so that the parameter \( \delta_{h,0} \) reflects the dynamic response to a counterfactual serially-uncorrelated shock, that is, controlling for the effect due to \( \frac{\partial x_{t+1}}{\partial x_t} \neq 0 \) built in system (2) when \( \gamma \neq 0 \).

### 3.2.2 Adapting DLMs to include the effect of persistence

As noted earlier, \( R(h)^{DLM} = R(h)^* \) regardless of the value of \( \gamma \). However, in some instances, the researcher may be interested in the response that includes the effect of persistence (\( R(h) \)). In this subsection, we show how to adapt DLMs to recover these responses. Intuitively, the idea is to compute the impulse responses in system (2) with respect to \( \varepsilon_t \) instead of \( x_t \).

Consider a recursive substitution of \( x_t \) in system (2):

\[ y_t = \delta \gamma^t x_0 + \delta \sum_{i=0}^{t} \gamma^i \varepsilon_{t-i} + u_t. \]  

(11)

The responses of \( y_t \) to \( \varepsilon_t \), which we denote by \( R(h)^{DLM-per} \), can be obtained from the coefficients \( \tilde{\theta}_h \) in:

\[ y_t = \tilde{\theta}_0 \varepsilon_t + \tilde{\theta}_1 \varepsilon_{t-1} + \tilde{\theta}_2 \varepsilon_{t-2} + \tilde{\theta}_3 \varepsilon_{t-3} + \tilde{\theta}_4 \varepsilon_{t-4} + \ldots + \varepsilon_t. \]  

(12)
Proposition 3. Given the data generating process described by system (2), the response function identified by DLMs of \( y_t \) to the innovation \( \varepsilon_t \) as described in equation (12) is equivalent to the response that includes the effects of persistence (and to the response obtained from LPs as in equation (6)):

\[
R(h)^{DLM-per} = R(h) = R(h)^{LP} \forall \gamma \text{ and } h.
\]

Proof. See Appendix A.3. \qed

Proposition 3 establishes a direct equivalence between the coefficients obtained from equation (12) and those obtained from LPs in equation (6): \( \tilde{\theta}_h = \delta_h \forall h \). The former are also related to the coefficients estimated from the DLM in terms of \( x_t \), as in equation (5):

\[
\theta_0 = \tilde{\theta}_0 = \delta, \quad \theta_1 = \tilde{\theta}_1 - \gamma \tilde{\theta}_0, \ldots, \quad \theta_h = \tilde{\theta}_h - \gamma \tilde{\theta}_{h-1}.
\]

Intuitively, the response of \( y_{t+1} \) to \( x_t \) has an overall effect of \( \delta_1 = \tilde{\theta}_1 \), which includes (i) the direct effect of \( x_t \) on \( y_{t+1} \) (0, in our simple case) and (ii) the effect on \( y_{t+1} \) that is due to the persistence in \( x_t \) (given by \( \gamma \delta \)). The standard DLM estimation from equation (5), since it accounts for the evolution of \( x_t \) over the response horizon, is implicitly subtracting the part of the response that is given by the persistence of \( x_t \) from the overall effect.

3.3 Examples

In this subsection, we perform stochastic simulations of the asymptotic behavior of the impulse response functions using both LPs and DLMs. Our goal is twofold. First, to evaluate quantitatively the conclusions reached in the previous subsection using a plausible calibration of the parameters that determine the model. Second, to consider a slightly more complex (and realistic) version of the data generating process that includes richer features frequently present in real empirical applications. In particular, we consider the following process:

\[
\begin{align*}
y_t &= \rho y_{t-1} + B_0 x_t + B_1 x_{t-1} + u_t \\
x_t &= \gamma x_{t-1} + \varepsilon_t,
\end{align*}
\]

(13)
where \( E(\varepsilon_{t-s}u_{t-r}) = 0 \ \forall s, r \geq 0 \), and \( u_t \) and \( \varepsilon_t \) follow \( \mathcal{N}(0,1) \) distributions. We set \( B_0 = 1.5 \), \( B_1 = 1 \) and \( \rho = 0.9 \).

Compared to system (2), the new DGP described in system (13) includes persistence in the outcome variable through \( \rho \), and allows the shock \( x_t \) to have lagged effects on \( y_t \) through \( B_1 \).

We simulate system (13) for 100 million periods and recover the dynamic responses of \( y_t \) to the shock \( x_t \) using LPs:

\[
y_{t+h} = \rho y_{t-1} + \beta_{h,0} x_t + \beta_{h,1} x_{t-1} + \beta_{h,f} x_{t+1} + \xi_{t+h}.
\]  

We consider three cases: (i) no persistence (\( \gamma = 0 \)), without including leads in the estimation (i.e., setting \( \beta_{h,f} = 0 \)); (ii) some persistence (\( \gamma = 0.2 \)) and still \( \beta_{h,f} = 0 \); (iii) some persistence (\( \gamma = 0.2 \)) and including a lead of the explanatory variable (i.e., allowing \( \beta_{h,f} \neq 0 \)).

Note that equation (14) must include a lag of shock \( x_t \) to capture the effect of \( B_1 \) in system (13). However, this does not control for the potential persistence of shock \( x_t \), as will be apparent in the simulations.

Figure 1 shows the results of our simulations. In case (i) (dark-blue solid line), the response has a contemporaneous effect of \( \hat{\beta}_{1,0} = 1.5 \) and peaks at the following period due to the fact that both \( \rho \) and \( B_1 \) have positive values. Using the language of the previous section, the impulse response function estimated by LPs with no persistence is asymptotically equivalent to the one obtained directly from equation (14), that is, \( \hat{\mathcal{R}}(h)^{LP} \rightarrow \mathcal{R}(h)^\star \).

In case (ii) (red solid line), the introduction of persistence in the shock \( x_t \) results in a larger effect on \( y_t \) on all horizons after impact. This has potentially important implications: if a macroeconomist is interested in the effects of a serially-uncorrelated shock (as in most

\footnote{We introduce this extra lag of the shock to make explicit the distinction between the effect due to the persistence of the shock and the effect of lagged values of the shock on current outcomes.}

\footnote{The choice of \( \gamma = 0.2 \) is based on an empirical application that we will present in Section 5. Of course, larger values of \( \rho \) would yield higher biases due to the persistence of the process.}
This figure shows the response of a simulated outcome variable to a shock with different degrees of persistence, using LPs. The dark blue line shows the results of estimating equation (14) assuming $\gamma = 0$ in equation (13). The red line shows the same estimation when $\gamma = 0.2$. The dashed grey line shows the response after including leads of the shock as in equation (14) and still assuming $\gamma = 0.2$.

In general equilibrium models, but naively estimates equation (14), implicitly setting $\beta_{h,f} = 0$, then the dynamic response is upwardly biased due to the persistence of the shock, i.e., $\hat{R}(h)^{LP} > R(h)^*$ for $h > 0$. Given the assumptions on the autocorrelation of the process $x_t$, the bias is particularly large in the short and medium run. Higher values of the persistence parameters $\gamma$ and $\rho$ would increase the difference between both responses (blue and red lines in Figure 1).

In case (iii) (dashed grey line in Figure 1), we see that the inclusion of leads of $x_t$ renders the response of the outcome variable to a persistent shock identical to the one obtained when considering a shock without persistence, i.e., $\hat{R}(h)^F \rightarrow R(h)^*$. In Appendix B.3 we provide an alternative simulation where the shock $x_t$ in (13) is not assumed to follow an AR(1) process but it is instead taken from actual data.

Next, we use these simulations to show that the computation of impulse responses using DLMs always yields the same estimates regardless of the persistence in $x_t$, that is,
First, note that, since $\rho < 1$, system (13) can be inverted and re-written as:

$$y_t = (1 - \rho L)^{-1} (B_0 + B_1 L) x_t + (1 - \rho L)^{-1} u_t,$$

(15)

where $L$ represents the lag operator.

Given the independence of $u_t$ and $x_t$, the representation from equation (15) suggests that the dynamic responses of $y_t$ from $x_t$ can be obtained from the coefficients $\vartheta_h$ in the following regression:

$$y_t = \vartheta_0 x_t + \vartheta_1 x_{t-1} + \vartheta_2 x_{t-2} + \vartheta_3 x_{t-3} + \ldots + \vartheta_H x_{t-H} + \xi_t,$$

(16)

where $H$ is the response horizon.$^{27}$

We estimate equation (16) for three different cases: (i) assuming that $\gamma = 0$ in the data generating process described in system (13), (ii) assuming that $\gamma = 0.2$ and (iii) replacing $x_t$ with $\hat{\epsilon}_t$ in equation (16) (i.e., following equation (12)).

The results are shown in Figure 2. Cases (i) and (ii) are displayed in blue and dashed grey lines, respectively. As argued earlier, since equation (16) controls for all potential dynamic effects of $x_t$, including its persistence, the coefficients $\vartheta_h$ reflect the responses to a shock as if the variable $x_t$ showed no persistence, regardless of the value of $\gamma$. Hence, we have that $\hat{R}^{DLM}(h) \rightarrow R^*(h)$ for any $\gamma$. Note that these impulse response functions are the same as those obtained with LPs ($\hat{R}(h)^{LP}$) when $\gamma = 0$, or when we include leads in the LPs ($\hat{R}(h)^F$).

Case (iii) is shown in the red line in Figure 2. As argued in the previous subsection, when computing the impulse response with respect to $\epsilon_t$, we are allowing the DLMs to pick up the effect that is due to the persistence in $x_t$. In other words, since we do not implicitly control for the leads of $x_t$ but for those of $\epsilon_t$ in the DLM, we are not taking into account the persistence of $x_t$. In this case, the responses are equal to those obtained from LPs when

$^{27}$Baek and Lee (2020) show that for autoregressive distributed lag models, setting the lag order to $H$ is a necessary condition to achieve consistency.
This figure shows the response of a simulated outcome variable to a shock with different degrees of persistence, using DLMs. The dark blue line shows the results of estimating equation (16) assuming $\gamma = 0$ in system (13). The dashed grey line shows the same estimation when $\gamma = 0.2$. The red line shows the response when substituting $x_t$ in equation (16) by $\hat{\varepsilon}_t$, an OLS estimate of $\varepsilon_t$ (see equation (9)), where serial correlation has been removed.
Table 2: Adapting LPs and DLMs when shocks are persistent

| Object of interest / Method | LPs | DLMs |
|----------------------------|-----|------|
| Response as if no persistence \((\mathcal{R}(h)^*)\) | include leads | no action needed |
| Response with persistence \((\mathcal{R}(h))\) | no action needed | replace \(x_t\) with \(\varepsilon_t\) |

\(\gamma \neq 0: \hat{\mathcal{R}}(h)^{DLM-per} = \hat{\mathcal{R}}(h)^{LP} \rightarrow \mathcal{R}(h)\).

4 Discussion: A guide to practitioners

In the presence of a persistent shock, a researcher needs to determine what object to identify. Table 2 summarizes the adjustments required in LPs and DLMs depending on the choice of the object of interest.

The researcher faces two options: to identify the response as if the shock were uncorrelated \((\mathcal{R}(h)^*)\) or the response that includes the effect of persistence \((\mathcal{R}(h))\). There are arguments in favor of both. Ultimately, deciding for one or the other may depend on what specific question the researcher is trying to address.

Since \(\mathcal{R}(h)^*\) can be understood as the IRF resulting from a standardized shock (so that it becomes serially uncorrelated), it should be the desired object when the researcher wants to establish comparisons across dynamic responses. There are at least three instances when \(\mathcal{R}(h)^*\) can facilitate comparisons. First, a shock identified from within a model (say, a structural VAR) or the innovation to a stochastic process in a DSGE model are, by construction, a m.d.s. (they are non-persistent). Given the absence of serial correlation, the thought experiment carried out in such cases is equivalent to constructing and IRF such as the shock takes the value of 1 on impact and 0 afterwards. Contrary to VAR-identified shocks or innovations in a DSGE model, narratively-identified shocks may display serial correlation. If this is the case, \(\mathcal{R}(h)\) (resulting, for example, from standard LP) will identify a different object, since the effect of serial correlation is included in the IRFs. In this instance, \(\mathcal{R}(h)^*\) will provide
the same macroeconomic experiment as, for example, a DSGE model.\textsuperscript{28}

Second, $R(h)^*$ can also be an object of interest when the researcher wants to compare the effects of different shocks, e.g., whether fiscal or monetary policy is more effective in stimulating output. For example, it may be the case that fiscal shocks tend to show more persistence or that a given identification procedure tends to generate shocks with different degree of serial correlation. If the effect of persistence amounts to a non-negligible amount of the dynamic response, this could wrongly lead to the conclusion that one shock is more effective than the other when the true underlying cause is that the DGP of both shocks is different. Since $R(h)^*$ effectively standardizes the dynamic responses to shocks with different data generating processes, this would facilitate such comparison.

Third, in a similar vein, $R(h)^*$ can be useful when the researcher wants to compare the effects of the same shock using data from different countries. This is because $R(h)^*$ provides a standardization of the data generating processes of the shocks, which may be heterogeneous across countries.\textsuperscript{29}

On the other hand, $R(h)$ should be the object of interest when the researcher is interested in estimating the \textit{most likely dynamic response} of a variable to a shock according to the historical data. This argument is similar to the one posed by Fisher and Peters (2010) and Ramey and Zubairy (2018) to support the use of the cumulative multiplier (the ratio of the integral of the output response to that of the government spending response) to evaluate the effectiveness of fiscal policies. If we consider the effects of a monetary policy shock that cuts the policy rate by one percentage point, it is important to note that, if that shock displays persistence, then the total monetary policy action (the evolution of the nominal

\textsuperscript{28} As mentioned earlier, when $x_t$ is the shock of interest, $R(h)^*$ is defined as the “traditional impulse response” in Koop et al. (1996).

\textsuperscript{29} Consider the following example: we want to compare the effects of fiscal policy in the US (using a news variable) and in another country (where we have availability of an alternative news variables). Consider the case that the news variables have different amounts of serial correlation and we obtain estimates of the government spending multipliers in both countries. Could we conclude that government spending is more effective in one country versus the other? Potentially, both policies could be equally effective but their sources of identification (news variables) may have different DGPs (one with more serial correlation than other), what leads to different multipliers.
interest following the initial tightening) may be different to what would occur if the shock were non-persistent.

Importantly, and regardless of the experiment that one wants to run, looking at the difference between $R(h)$ and $R(h)^*$ is informative by itself, as it speaks about how much of the dynamic response is due to the implied DGP of the shock variable. Put differently, it informs the researcher of a propagation mechanism: $R(h)$ includes the propagation through the persistence of $x_t$ while $R(h)^*$ does not.

Further to this, the methods that underlie the construction of $R(h)^*$ when using local projections can be exported to more general uses. Hence the inclusion of leads of different variables can help in decomposing an IRF in different channels of propagation (where serial correlation is just one of them). This avenue could be particularly informative in highlighting what economics models can bring the dynamic responses closer to the data.

5 Application

In this section we use the empirical work of Ramey and Zubairy (2018) to show the quantitative relevance of serial correlation in an actual example. We do so by computing two types of IRFs, $R(h)$ and $R(h)^*$, as described above.\(^{30}\)

Ramey and Zubairy (2018), building on previous work by Ramey (2011) and Owyang et al. (2013), produce a series of announces about future defense spending between 1890q1-2014q1, scaled by previous quarter trend real GDP.\(^{31}\) This series, plotted in panel D of Figure D2, has a positive autocorrelation of 18.4% (47.0% in the subsample after WWII).\(^{32}\)

---

\(^{30}\)In the appendix, we consider additional applications, based on Guajardo et al. (2014), Romer and Romer (2004), Gertler and Karadi (2015), and Romer and Romer (2010).

\(^{31}\)Ramey and Zubairy (2018) estimate trend GDP as sixth degree polynomial for the logarithm of GDP and multiplier by the GDP deflator. In fact, it is the use of the GDP deflator and trend GDP as a way to scale the shocks what seems to induce the persistence. The persistence is also present when the shock is scaled by previous-quarter GDP, as in Owyang et al. (2013).

\(^{32}\)This positive autocorrelation is significant at a confidence level of 90% when considering standard errors that are robust to the presence of heteroskedasticity and persistence (with more than one lag) for the whole sample. For the subsample starting after WWII, the autocorrelation is significant at any level.
Ramey and Zubairy (2018) use LPs to estimate the response of output and government spending to a shock in future defense spending. We follow their same approach and sample and estimate the following equations for output \( y_t \) and government spending \( g_t \):

\[
\begin{align*}
y_{t,h} &= \beta_y^{shock} + \sum_{j=1}^{P} \rho_{j,h} z_{t-j} + \sum_{f=1}^{h} \gamma_{f,h}^{shock} + \xi_t \\
g_{t,h} &= \beta_g^{shock} + \sum_{j=1}^{P} \rho_{j,h} z_{t-j} + \sum_{f=1}^{h} \gamma_{f,h}^{shock} + \varepsilon_t,
\end{align*}
\]

where \( z_t \) includes \( P \) lags of \( y_t, g_t \) and \( shock_t \). Note that, following the discussion in previous sections, we include \( h \) leads of the variable \( shock_t \). In particular, for each horizon \( h \) we include \( h \) leads.

To replicate Ramey and Zubairy (2018)'s estimates, we set \( \gamma_{f,h} = 0, \forall f, h \). The black, solid line in Figure 3 represents the estimated responses of output (left panel) and government spending (right panel) to the shock.\(^{33}\) As noted in Section 3, these dynamic responses are the equivalent to the \( R(h) \) as defined in equation (3) (with the only difference being that \( \Omega_{t-1} \) includes now the past history of \( z_t \)). The results closely resemble those in Ramey and Zubairy (2018) (Figure 5 of their paper).\(^{34}\)

Next, we allow \( \gamma_{f,h} \neq 0 \). As discussed in Section 3, this amounts to estimating \( R(h)^* \) as defined in equation (4). In the red lines in Figure 3, we observe that the dynamic responses change considerably when the leads are included. For example, after two years, output and government spending are 40% lower than in Ramey and Zubairy (2018)'s estimates. The large observed difference between \( R(h) \) and \( R(h)^* \) suggests that the persistence of the news variable plays a non-negligible role in explaining the dynamic transmission of the fiscal shock to output and government spending.

Whether to include leads or not also has implications for inference. The 95\% confidence

---

\(^{33}\)Figure D3 also replicates the original 95\% confidence intervals computed using the Newey-West correction.\(^{34}\)We drop the last \( h \) observations of the sample, so that the specifications with and without leads can be fully comparable. This does not have any discernible effect when replicating the original results from Ramey and Zubairy (2018).
Figure 3: Output and government spending responses, with and without leads

Black lines show the results of estimating the system (17) without including any lead (as in Ramey and Zubairy (2018)). Red solid lines represent the results of estimations when including $h$ leads of the Ramey and Zubairy (2018) news variable (with 95% confidence intervals).

Intervals when leads are included (shown in dashed lines in Figure 3) are substantially narrower than when they are not (grey areas in Figure D3). The latter are around 50% broader after two years, and more than twice as big after three years.

The dynamic responses of output and government spending are informative about the expected path of these variables after a shock. To obtain a measure of the efficiency of fiscal policy (i.e., the increase of output per each dollar increase in government spending), Ramey and Zubairy (2018) use the cumulative multiplier, computed as:\footnote{Ramey and Zubairy (2018) show that the cumulative multiplier can be obtained in one step yielding identical results to those obtained combining equations (17) and (18).}

$$M_{t,h} = \frac{\sum_{i=1}^{h} \beta_h^y}{\sum_{i=1}^{h} \beta_h^g}$$

(18)

We find that this statistic is not substantially affected by persistence of the shock (Figure D4). Given that both output and government spending react similarly when including...
leads of the shock, taking the ratio of the two variables attenuates the differences between both specifications.\footnote{Even though the multiplier does not change much when accounting for persistence, the fact that the expected responses of output and government spending do change substantially is very relevant from a policymaker point of view. For example, a higher response of government spending can affect other important variables such as public debt or future changes in tax liabilities.}

**Non-linear effects.** We now investigate whether the effect of persistence in the shock can affect the responses in a non-linear setting, i.e., if government spending multipliers are different in expansions and recessions.\footnote{See Ramey (2019) for a recent summary of this debate. For example, an influential study by Auerbach and Gorodnichenko (2012) finds that government spending multipliers are higher during recessions using a non-linear VAR. Altoza (2018) highlights the role of the information used to define a period of recession, and finds that output responds negatively to government spending shocks in a post-WWII sample under different identification and estimation approaches.} For this, we follow Ramey and Zubairy (2018) and estimate a series of non-linear LPs:

\[
x_{t+h} = S_{t-1} \left[ \alpha_{A,h} + \sum_{j=1}^{P} \rho_{A,j,h} z_{t-j} + \beta_{A,h} \text{shock}_t + \sum_{f=1}^{h} \delta_{A,f,h} \text{shock}_{t+f} \right] + \\
(1 - S_{t-1}) \left[ \alpha_{B,h} + \sum_{j=1}^{P} \rho_{A,j,h} z_{t-j} + \beta_{B,h} \text{shock}_t + \sum_{f=1}^{h} \delta_{B,f,h} \text{shock}_{t+f} \right] + \xi_{t+h}, \tag{19}
\]

where \(x_t\) is either output or government spending and \(S_t\) is a binary variable indicating the state of the economy. When \(S_t = 1\), the economy is booming and, when \(S_t = 0\), the economy is in recession, which is defined as when the unemployment rate is above the threshold of 6.5. In this setting, all the variables (and the constant), are allowed to have differential effects during expansions and recessions.

We first replicate the non-linear responses of output and government spending during booms and recessions obtained by Ramey and Zubairy (2018). Hence, we estimate equation (19) setting \(\delta_{A,f,h} = \delta_{B,f,h} = 0 \ \forall f, h\), which identifies \(\mathcal{R}(h)\). Our results, shown in Figure 4 in black lines, resemble very closely those from the authors. Next, we repeat the experiment accounting for potential persistence, that is, including leads of the shock (iden-
Black lines show the results from system of equations (19) without including any lead (as in Ramey and Zubairy (2018)). Grey areas represent 68 and 95% Newey-West confidence intervals for these estimates. Red solid lines represent the results of estimations when including $h$ leads of the Ramey’s news variable. Red dashed lines represent the 95% Newey-West confidence intervals for these estimates.

The results are shown in red lines in Figure 4. While relatively similar in the case of expansions, the responses are quantitatively different during recessions. The estimates that include leads lie outside of the 95% confidence bands during much of the response horizon. The results suggest that ignoring the effect of persistence could yield responses during recessions that, after 2–3 years, are twice as large as the responses that account for the effect of persistence. Or, in other words, persistence in the shock is responsible for up to 50% of the dynamic transmission of the shock during recessions.

In Figure 5, we show how these responses map into estimates of non-linear fiscal multipliers. In the case of expansions, the results do not change much depending on whether the persistence is accounted for (red solid line, $R(h)^*$) or not (black solid line, $R(h)$). In either case, they resemble those in Ramey and Zubairy (2018) (see Figure 6 of their paper). In recessions, however, the results change substantially depending on whether the persistence is
The black solid and dashed lines show the cumulative multiplier during periods of expansion and recession, respectively, without including any lead (as in Ramey and Zubairy (2018)). The red solid and dashed lines show the cumulative multiplier during periods of expansion and recession, respectively, when including leads of the shock.

controlled for or not. If it is not (black solid line), the multiplier has a negative value upon impact and substantially falls in the following quarter to a value of -2. It becomes positive before the end of the first year, and fully converges to the value of the multiplier during expansions after six quarters. If the persistence is excluded from the dynamic responses (red dashed line), the cumulative multiplier is -1 (instead of -2) and becomes positive after the first year. Furthermore, the multiplier during recessions remains lower than the multiplier during expansions for a much longer period. When the persistence is not accounted for, this convergence is achieved after 6 quarters, as mentioned above. However, when including leads of the shock, this convergence is not fully reached during our considered response horizon. These results suggest that during the short and medium-run the government spending multiplier could be lower during recessions than during expansions, and part of this difference may be attributable to the presence of persistence in the shock.

One of the main advantages of LPs is that they allow to accommodate non-linear settings,
as those in equation (19). This is particularly useful since, contrary to threshold VARs, LPs do not impose any restriction on the evolution of state $S_t$ (while non-linear VARs that interact the shock with a state dummy do assume that $S_t$ remains fixed during the response horizon). The framework explained in the previous section allows to consider additional macroeconomic experiments that can help understand how restrictive this condition is. In particular, by including leads of the state $S_t$ in equation (19) we are identifying the counterfactual response to a fiscal shock when the underlying state of the economy is not allowed to change (as in threshold VARs). We perform this experiment and report the multipliers during booms in recessions in green lines in Figure D5. We observe that, when the state is not allowed to change, the multiplier during recessions is slightly higher in the short run, but essentially unchanged at medium and longer horizons. This exercise allows us to illustrate how the use of leads of variables in conjunction with LPs can help understand interesting counterfactual exercises and shed light on the dynamic transmission of shocks.

6 Conclusions

We have shown that persistence results in the estimation of different responses when using LPs versus traditional methods based on DLMs. For a researcher interested in the response as if the shock were not persistent, DLMs yield the desired object, but LPs need to be adapted. The opposite is true if the object of interest is the response to the shock “as it is”. Regardless of which is the thought experiment that the researcher seeks to carry out, the difference between both types of responses is informative about how much of the dynamic transmission of a shock is due to the presence of persistence.

The use of leads can be generalized to other interesting contexts, as it allows to shut down channels of transmission. For example, one may be interested in the effects of monetary policy shocks on output due to a particular instrument while holding other variables (e.g., changes to fiscal policy) constant. In the context of LPs, leads of a selected variable (e.g., tax
changes) will deliver responses holding that variable constant. This methodology allows to separate the direct (due to the impact through the regressor of interest) and indirect effects (due to other variables in the regression). This has often been used in the context of VARs, by imposing restrictions on the coefficients of selected impulse responses. The inclusion of leads achieves a similar goal in LPs, hence allowing to construct interesting macroeconomic experiments. We leave these questions for future research.

References

Alesina, A., Barbiero, O., Favero, C., Giavazzi, F., and Paradisi, M. (2017). The effects of fiscal consolidations: Theory and evidence. NBER Working Paper, 23385.

Alesina, A., Favero, C., and Giavazzi, F. (2015). The output effect of fiscal consolidation plans. Journal of International Economics, 96:S19–S42.

Alloza, M. (2018). Is Fiscal Policy More Effective in Uncertain Times or During Recessions? Banco de España Working Paper, 1730.

Alloza, M. and Sanz, C. (2020). Jobs Multipliers: Evidence from a Large Fiscal Stimulus in Spain. Scandinavian Journal of Economics, forthcoming.

Arellano, M. and Bond, S. (1991). Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations. Review of Economic Studies, 58(2):277–297.

Arezki, R., Ramey, V. A., and Sheng, L. (2017). News shocks in open economies: Evidence from giant oil discoveries. Quarterly Journal of Economics, 132(1):103–155.

Auerbach, A. and Gorodnichenko, Y. (2012). Measuring the Output Responses to Fiscal Policy. American Economic Journal: Economic Policy, 4(2):1–27.
Bachmann, R. and Sims, E. R. (2012). Confidence and the transmission of government spending shocks. *Journal of Monetary Economics*, 59(3):235 – 249.

Baek, C. and Lee, B. (2020). A Guide to Single Equation Regressions for Impulse Response Estimations. *Working Paper, University of California, Berkeley*.

Barnichon, R. and Matthes, C. (2017). Understanding the Size of the Government Spending Multiplier: It’s in the Sign. *Working Paper, Federal Reserve Bank of San Francisco*.

Barnichon, R. and Mesters, G. (2020). Identifying modern macro equations with old shocks. *Quarterly Journal of Economics*, forthcoming.

Bernanke, B. S., Gertler, M., Watson, M., Sims, C. A., and Friedman, B. M. (1997). Systematic monetary policy and the effects of oil price shocks. *Brookings Papers on Economic Activity*, 1997(1):91–157.

Bloom, N. (2009). The Impact of Uncertainty Shocks. *Econometrica*, 77(3):623–685.

Box, G. E. and Pierce, D. A. (1970). Distribution of residual autocorrelations in autoregressive-integrated moving average time series models. *Journal of the American Statistical Association*, 65(332):1509–1526.

Cerra, V. and Saxena, S. C. (2008). Growth Dynamics: The Myth of Economic Recovery. *American Economic Review*, 98(1):439–57.

Choi, I. and Kurozumi, E. (2012). Model selection criteria for the leads-and-lags cointegrating regression. *Journal of Econometrics*, 169(2):224–238.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (1999). Monetary policy shocks: What have we learned and to what end? *Handbook of Macroeconomics*, 1:65–148.

Cloyne, J. (2013). Discretionary Tax Changes and the Macroeconomy: New Narrative Evidence from the United Kingdom. *American Economic Review*, 103(4):1507–28.
Cloyne, J. and Hürtgen, P. (2016). The Macroeconomic Effects of Monetary Policy: A New Measure for the United Kingdom. *American Economic Journal: Macroeconomics*, 8(4):75–102.

Cloyne, J. S., Jordà, Ò., and Taylor, A. M. (2020). Decomposing the Fiscal Multiplier. *NBER Working Paper*, 26939.

Coibion, O. (2012). Are the Effects of Monetary Policy Shocks Big or Small? *American Economic Journal: Macroeconomics*, 4(2):1–32.

Coibion, O., Gorodnichenko, Y., and Ulate, M. (2018). The Cyclical Sensitivity in Estimates of Potential Output. *Brookings Papers on Economic Activity*, page 343.

Devries, P., Guajardo, J., Leigh, D., and Pescatori (2011). A New Action-Based Dataset of Fiscal Consolidation. *IMF Working Paper*, 11/128.

Dufour, J.-M. and Renault, E. (1998). Short Run and Long Run Causality in Time Series: Theory. *Econometrica*, 66(5):1099–1125.

Faust, J. and Wright, J. H. (2011). Efficient prediction of excess returns. *Review of Economics and Statistics*, 93(2):647–659.

Favero, C. and Giavazzi, F. (2012). Measuring Tax Multipliers: The Narrative Method in Fiscal VARs. *American Economic Journal: Economic Policy*, 4(2):69–94.

Fisher, J. D. and Peters, R. (2010). Using Stock Returns to Identify Government Spending Shocks. *Economic Journal*, 120(544):414–436.

Gertler, M. and Karadi, P. (2015). Monetary Policy Surprises, Credit Costs, and Economic Activity. *American Economic Journal: Macroeconomics*, 7(1):44–76.

Geweke, J. F. and Singleton, K. J. (1981). Maximum likelihood” confirmatory” factor analysis of economic time series. *International Economic Review*, pages 37–54.
Gilchrist, S. and Zakrajek, E. (2012). Credit Spreads and Business Cycle Fluctuations. *American Economic Review*, 102(4):1692–1720.

Goujard, A. (2017). Cross-Country Spillovers from Fiscal Consolidations. *Fiscal Studies*, 38(2):219–267.

Granger, C. W. and Machina, M. J. (2006). Chapter 2 Forecasting and Decision Theory. In Elliott, G., Granger, C., and Timmermann, A., editors, *Handbook of Economic Forecasting*, volume 1, pages 81 – 98. Elsevier.

Guajardo, J., Leigh, D., and Pescatori, A. (2014). Expansionary austerity? International evidence. *Journal of the European Economic Association*, 12(4):949–968.

Jordà, Ó. (2005). Estimation and inference of impulse responses by local projections. *American Economic Review*, 95(1):161–182.

Koop, G., Pesaran, M., and Potter, S. M. (1996). Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics*, 74(1):119 – 147.

Lee, B. (2020). Lag Order Selection in Local Projections. *Working Paper, Hong Kong University of Science and Technology.*

Lee, T.-H. (2008). Loss functions in time series forecasting. *International Encyclopedia of the Social Sciences*, pages 495–502.

Ljung, G. M. and Box, G. E. (1978). On a measure of lack of fit in time series models. *Biometrika*, 65(2):297–303.

Mertens, K. and Ravn, M. O. (2012). Empirical evidence on the aggregate effects of anticipated and unanticipated US tax policy shocks. *American Economic Journal: Economic Policy*, 4(2):145–81.

Miranda-Agrippino, S. and Ricco, G. (2018). The transmission of monetary policy shocks. *CEPR Discussion Paper No. DP13396.*
Olea, J. L. M. and Plagborg-Møller, M. (2020). Local Projection Inference is Simpler and More Robust Than You Think. *Working Paper, Columbia University.*

Owyang, M. T., Ramey, V. A., and Zubairy, S. (2013). Are government spending multipliers greater during periods of slack? Evidence from twentieth-century historical data. *American Economic Review, 103*(3):129–34.

Plagborg-Møller, M. and Wolf, C. K. (2019). Local Projections and VARs Estimate the Same Impulse Responses. *Working Paper, Princeton University.*

Ramey, V. (2011). Identifying Government Spending Shocks: It’s all in the Timing. *Quarterly Journal of Economics, 126*(1):1.

Ramey, V. A. (2016). Macroeconomic shocks and their propagation. In *Handbook of Macroeconomics*, volume 2, pages 71–162. Elsevier.

Ramey, V. A. (2019). Ten Years after the Financial Crisis: What Have We Learned from the Renaissance in Fiscal Research? *Journal of Economic Perspectives, 33*(2):89–114.

Ramey, V. A. and Zubairy, S. (2018). Government spending multipliers in good times and in bad: Evidence from US historical data. *Journal of Political Economy, 126*(2):850–901.

Romer, C. D. and Romer, D. H. (2004). A New Measure of Monetary Shocks: Derivation and Implications. *American Economic Review, 94*(4):1055–1084.

Romer, C. D. and Romer, D. H. (2010). The macroeconomic effects of tax changes: estimates based on a new measure of fiscal shocks. *American Economic Review, 100*(3):763–801.

Sims, C. A. and Zha, T. (2006). Does monetary policy generate recessions? *Macroeconomic Dynamics, 10*(2):231–272.

Stock, J. and Watson, M. (1993). A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems. *Econometrica, 61*(4):783–820.
Stock, J. H. and Watson, M. W. (2018). Identification and Estimation of Dynamic Causal Effects in Macroeconomics Using External Instruments. *Economic Journal*, 128(610):917–948.

Teulings, C. N. and Zubanov, N. (2014). Is economic recovery a myth? Robust estimation of impulse responses. *Journal of Applied Econometrics*, 29(3):497–514.
Online Appendices

A Proofs

A.1 Proof of Proposition 1

Consider equation (6) (rewritten here for convenience):

\[ y_{t+h} = \delta_h x_t + \xi_{t+h}, \quad (A.1) \]

where \( \delta_h = \mathcal{R}(h)^{LP} \) represents the impact of variable \( x_t \) on \( y_{t+h} \) (the response function).

Since \( \delta_h \) is the linear projection coefficient of equation (A.1):

\[ \delta_h = \frac{\text{cov}(y_{t+h}, x_t)}{\text{var}(x_t)}. \quad (A.2) \]

The dynamic effect of \( x_t \) on \( y_{t+h} \) can also be obtained from DLMs as in equation (5):

\[ y_t = \theta_0 x_t + \theta_1 x_{t-1} + \theta_2 x_{t-2} + \theta_3 x_{t-3} + \theta_4 x_{t-4} \ldots + u_t. \]

Since this expression holds \( \forall t \), it can be written as:

\[ y_{t+h} = \theta_0 x_{t+h} + \theta_1 x_{t+h-1} + \theta_2 x_{t+h-2} + \theta_3 x_{t+h-3} + \ldots + \theta_h x_t + u_t, \]

where the coefficient \( \theta_h = \mathcal{R}(h)^{DLM} \) represents the impulse response in period \( h \), obtained from:

\[ \theta_h = \frac{\text{cov}(y_{t+h}, x_t)}{\text{var}(x_t)} \bigg|_{x_{t+1}, \ldots, x_{t+h}}. \quad (A.3) \]

When \( \gamma = 0 \) in the process described by system (2), we have that \( x_t = \varepsilon_t \sim \text{white noise}(\mu_\varepsilon, \sigma_\varepsilon^2) \), so:

\[ \theta_h = \frac{\text{cov}(y_{t+h}, x_t)}{\text{var}(x_t)} \bigg|_{x_{t+1}, \ldots, x_{t+h}} = \frac{\text{cov}(y_{t+h}, x_t)}{\text{var}(x_t)}. \]
In this case, $\delta_h = \theta_h$ and LPs and DLMs yield the same responses: $\mathcal{R}(h)^{LP} = \mathcal{R}(h)^{DLM}$ $\forall h$. Note that $\mathcal{R}(h)^{DLM} = \mathcal{R}(h)^*$ $\forall h, \gamma$ since (under linearity):

$$
\mathcal{R}(h)^* = \frac{\partial y_{t+h}}{\partial x_t} \bigg|_{x_{t+1},\ldots,x_{t+h}} = \frac{\text{cov}(y_{t+h}, x_t)}{\text{var}(x_t)} \bigg|_{x_{t+1},\ldots,x_{t+h}}.
$$

When $\gamma \neq 0$, equation (A.2) becomes (using the equations in system (2)):

$$
\delta_h = \frac{\text{cov}(y_{t+h}, x_t)}{\text{var}(x_t)} = \frac{\text{cov}(\delta x_{t+h} + u_{t+h}, x_t)}{\text{var}(x_t)} = \delta \frac{\text{cov}(x_{t+h}, x_t)}{\text{var}(x_t)} = \delta \gamma^h, \quad (A.4)
$$

using the expression:

$$
x_{t+h} = \gamma^h x_t + \sum_{j=0}^{h-1} \gamma^j \varepsilon_{t+h-j}. \quad (A.5)
$$

However, the dynamic response obtained from DLMs is:

$$
\theta_h = \frac{\text{cov}(y_{t+h}, x_t)}{\text{var}(x_t)} \bigg|_{x_{t+1},\ldots,x_{t+h}} = \delta \frac{\text{cov}(x_{t+h}, x_t)}{\text{var}(x_t)} \bigg|_{x_{t+1},\ldots,x_{t+h}}.
$$

When $h = 0$, the above expression becomes $\theta_0 = \delta \frac{\text{cov}(x_t, x_t)}{\text{var}(x_t)} = \delta$. For $h > 0$, we have that $\theta_h = \delta \frac{\text{cov}(x_{t+h}, x_t)}{\text{var}(x_t)} \bigg|_{x_{t+1},\ldots,x_{t+h}} = 0$. This shows that when $\gamma \neq 0$, we have that $\mathcal{R}(h)^{LP} = \mathcal{R}(h)^{DLM}$ if and only if $h = 0$. ■

### A.2 Proof of Proposition 2

Consider equation (10) (rewritten here for convenience):

$$
y_{t+h} = \delta_{h,0} x_t + \delta_{h,1} x_{t+1} + \xi_{t+h}, \quad (A.6)
$$

where $\delta_{h,0} = \mathcal{R}(h)^F$ represents the impact of the shock $x_t$ on $y_{t+h}$ when including leads of the former. Since $\delta_{h,0}$ is the linear projection coefficient of equation (A.6), then:

$$
\delta_{h,0} = \delta \frac{\text{cov}(x_{t+h}, x_t)}{\text{var}(x_t)} \bigg|_{x_{t+1},\ldots,x_{t+h}} = \delta \frac{\text{cov}(x_{t+h}, x_t)}{\text{var}(x_t)} \bigg|_{x_{t+1}}. \quad (A.7)
$$
Note that the data generating process in system (2) considers that $x_t$ is an AR(1) so it can be represented in terms of $x_{t+1}$ (see equation (A.5)).

Then, we have that DLMs and LPs with leads recover the same object:

$$\theta_h = \frac{cov(y_{t+h}, x_t)}{var(x_t)} \bigg|_{x_{t+1}, \ldots, x_{t+h}} = \delta \frac{cov(x_{t+h}, x_t)}{var(x_t)} \bigg|_{x_{t+1}} = \delta_{h,0}. $$

To see this, note that in period $h = 0$ we have that:

$$\theta_0 = \frac{cov(y_t, x_t)}{var(x_t)} \bigg|_{x_{t+1}, \ldots, x_{t+h}} = \frac{cov(y_t, x_t)}{var(x_t)} = \delta = \delta_{0,0}. $$

In periods $h > 0$, we can rewrite equation (A.7) as:

$$\delta_{h,0} = \delta \frac{cov(x_{t+h}, x_t)}{var(x_t)} \bigg|_{x_{t+1}} = \delta \gamma^{h-1} \frac{cov(x_{t+1}, x_t)}{var(x_t)} \bigg|_{x_{t+1}} = 0. $$

Similarly, equation (A.3) becomes:

$$\theta_h = \frac{cov(y_{t+h}, x_t)}{var(x_t)} \bigg|_{x_{t+1}, \ldots, x_{t+h}} = \delta \gamma^{h-1} \frac{cov(x_{t+1}, x_t)}{var(x_t)} \bigg|_{x_{t+1}} = 0. $$

So we have $\mathcal{R}(h)^F = \mathcal{R}(h)^{DLM} \forall h, \gamma$. And we know that $\mathcal{R}(h)^{DLM} = \mathcal{R}(h)^* $, from the section above.

**A.3 Proof of Proposition 3**

Consider a version of equation (12) rewritten here for convenience:

$$y_t = \tilde{\theta}_0 \varepsilon_t + \tilde{\theta}_1 \varepsilon_{t-1} + \tilde{\theta}_2 \varepsilon_{t-2} + \tilde{\theta}_3 \varepsilon_{t-3} + \tilde{\theta}_4 \varepsilon_{t-4} \ldots + u_t, \quad (A.10)$$

where $\tilde{\theta}_0 = \mathcal{R}(h)^{DLM-per}$ represents the impact of variable $\varepsilon_t$ on $y_{t+h}$. Note that $\varepsilon_t$ is not observable but can be obtained if we know the data generating process described in

1Note also that the results easily generalize to cases when $x_t$ is an autoregressive process of higher order.
system (2). Since equation (A.10) represents the linear projection of \(y_t\) on \(\varepsilon_t\) and its lags, with \(\varepsilon_t \sim iid(\mu_\varepsilon, \sigma_\varepsilon^2)\), we have:

\[
\tilde{\theta}_h = \frac{\text{cov}(y_{t+h}, \varepsilon_t)}{\text{var}(\varepsilon_t)} \bigg|_{\varepsilon_{t+1}, \ldots, \varepsilon_{t+h}} = \frac{\text{cov}(y_{t+h}, \varepsilon_t)}{\text{var}(\varepsilon_t)} = \delta \frac{\text{cov}(x_{t+h}, \varepsilon_t)}{\text{var}(\varepsilon_t)}. \tag{A.11}
\]

This expression is equivalent to equation (A.2) which implies that \(\tilde{\theta}_h = \delta_h\) and \(R(h)^{DLM-per} = R(h)^{LP}\). To see this, substitute for \(x_{t+h}\) in equation (A.11) using expression (A.5) and system (2):

\[
\tilde{\theta}_h = \delta \gamma^h \frac{\text{cov}(x_t, \sum_{j=0}^{h-1} \gamma^j \varepsilon_{t+h-j}, \varepsilon_t)}{\text{var}(\varepsilon_t)} = \delta \gamma^h \frac{\text{cov}(x_t, \varepsilon_t)}{\text{var}(\varepsilon_t)} = \delta \gamma^h. \tag{A.12}
\]

Note that the above expression yields the same result as equation (A.4), which shows that \(\tilde{\theta}_h = \delta_h \ \forall \ h\).  ■
B Additional results

B.1 Including the shocks as endogenous variables in a VAR

A researcher may consider including a shock with persistence as an endogenous variable in a VAR. Does this approach eliminate the effect of the persistence of the shock on the impulse responses? A VAR, since it explicitly models the persistence of the shock, includes this effect in the estimated impulse responses and, hence, yields the same dynamic effects as LPs (contrary to what is obtained when including the shock as a distributed lag structure within a VAR).\(^1\)

To see this in an intuitive way, consider the data generating process given by system (13) and rewritten here for convenience (with a slightly different notation):

\[
\begin{align*}
y_t &= \rho y_{t-1} + \delta_0 x_t + \delta_1 x_{t-1} + \varepsilon_t^y \\
x_t &= \gamma x_{t-1} + \varepsilon_t^x.
\end{align*}
\]

This process can be recast as a structural VAR of the form \(A_0 Y_t = B^* Y_{t-1} + \varepsilon_t\), with:

\[
\begin{bmatrix}
1 & 0 \\
-\delta_0 & 1
\end{bmatrix}
\begin{bmatrix}
x_t \\
y_t
\end{bmatrix}
= \begin{bmatrix}
\gamma & 0 \\
\delta_1 & \rho
\end{bmatrix}
\begin{bmatrix}
x_{t-1} \\
y_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_t^x \\
\varepsilon_t^y
\end{bmatrix}.
\]

An econometrician would estimate the following reduced-form VAR:

\[
Y_t = B Y_{t-1} + u_t,
\]

where \(B = A_0^{-1} B^*\); and \(u_t = A_0^{-1} \varepsilon_t\) are reduced-form residuals. Since the data generating process given by equation (B.1) already incorporates restrictions on the contemporaneous

\(^1\)Bloom (2009), Romer and Romer (2010), and Ramey (2011) are examples of studies that include shocks as endogenous variables in a VAR. These specifications are also known as hybrid VARs (see Coibion (2012)). Plagborg-Møller and Wolf (2019) formally show that VARs and LPs identify the same impulse responses. Here we illustrate that when one of the endogenous variables in the VAR is a persistent shock, this effect will be carried over to the dynamic responses.
behavior of the variables, a researcher may identify the structural impulse responses by computing the Choleski decomposition (when $x_t$ is ordered first) of the variance-covariance matrix of reduced-form residuals $u_t$.

However, note that, when $\gamma \neq 0$, the response of $y_t$ to $\varepsilon_t^x$ will include an effect due to the persistence of the shock $x_t$. Intuitively, consider the case of $\rho = \delta_1 = 0$. In this scenario, a researcher may be interesting in recovering a one-off shock to $x_t$. However, the response of $y_t$ will be given by $R(h)^{VAR} = \delta_0 \sum_{k=0}^{\infty} \gamma^k \varepsilon_{t-k}$, that is, the one-off shock will still have effects along the response horizon because of the persistence of $x_t$ (when $\gamma \neq 0$).

To see this point in a more general way, consider the same calibration as used in the main text (i.e., $\rho = 0.9, \delta_0 = 1.5, \delta_1 = 1$, and either $\gamma = 0$ or $\gamma = 0.2$). We compute the impulse responses of variables $y_t$ and $x_t$ (the measured shock) to $\varepsilon_t^x$, shown in Figure B1. We also estimate the same impulse-response functions for $y_t$ using LPs, as in Section 3.3.

The results illustrate that, regardless of the value $\gamma$, a VAR that considers $x_t$ as endogenous variable and LPs estimate the same impulse responses. When there is some persistence
in the shock $x_t$, both the VAR (that considers $x_t$ as an endogenous variable) and LPs include a dynamic effect due to the persistence of $x_t$.

Importantly, when $x_t$ displays persistence, the response function estimated by a VAR will vary depending on whether $x_t$ is included as an endogenous variable (as shown before) or an exogenous one (e.g., with a distributed lag or moving average structure, as in Mertens and Ravn (2012) or Favero and Giavazzi (2012)). To show this point, Figure B2 displays the response of $y_t$ when (i) $x_t$ is included as an endogenous variable in the VAR, or (ii) when estimating a regression of $y_t$ on $x_t$ and a lag of both variables. As it can be seen, considering $x_t$ as an exogenous regressor always delivers the same dynamic responses as if $x_t$ were serially uncorrelated, regardless of the actual value of $\gamma$.

The discussion above highlights that the result regarding the equivalence of LPs and DLMs under no serial correlation of the shock can be generalized to multivariate settings of VARs with the shock included as an endogenous variable (in the case of LPs) or as an exogenous variable (in the case of DLMs).
Figure B2: Responses when the shock is included as an endogenous or exogenous variable in a VAR

The figure shows the VAR responses of $y_t$ to $e_t^x$ estimated from (B.3) under different assumptions of the persistence parameter $\gamma$ and two different specifications: solid lines display the responses when the shock is included as an endogenous variable in the VAR and dashed line shows the responses when the same shock is included as an exogenous variable in the VAR.
B.2 Local projections with instrumental variables

Recently, there has been an increased attention to the use of external sources of variation as instruments in LPs. In this section, we investigate how persistence may affect the estimation of dynamic effects when using instrumental variables in local projections (LP-IV).

Stock and Watson (2018) provide the conditions under which a researcher can exploit external variation to estimate impulse response functions. A valid instrument $z_t$ should be both relevant and contemporaneously exogenous, that is, $z_t$ should not be correlated to any shock in the system except with the one that the researcher is interested in. Lastly, Stock and Watson (2018) impose a restriction called lead/lag exogeneity, which implies that the instrument should not be correlated with any lead or lag of any of the shocks in the system.

Consider the following data generating process:

\[
y_t = \beta g_t + u_t \\
u_t = m_t + a_t \\
g_t = \lambda x_t + (1 - \lambda) m_t \\
z_t = x_t + \nu_t \\
x_t = \gamma x_{t-1} + \varepsilon_t,
\]

where $a_t$, $\nu_t$, $m_t$ and $\varepsilon_t$ follow independent $\mathcal{N}(0, 1)$ distributions. A researcher may be interested in estimating the dynamic effects of variable $g_t$ on $y_t$ (e.g., the effects of government spending on output). However, $g_t$ is endogenous due to the presence of an omitted variable $m_t$. The researcher may have the availability of an instrument $z_t$, which is contemporaneously exogenous by construction and relevant when $\lambda \neq 0$. This instrument, since it depends

\footnote{See Ramey and Zubairy (2018) for an example. Related to this, Ramey (2016) discusses the distinction between shock, innovation, and instrument. Barnichon and Mesters (2020) show how independently identified shocks can be used as instruments to estimate the coefficients of structural forward looking macroeconomic equations.}
directly on the shock $x_t$, displays persistence when $\gamma \neq 0$. When there is persistence in
the instrument (and the shock), the lead/lag exogeneity condition mentioned above is not
satisfied. To illustrate this point, we simulate system (B.4) setting $\beta = 2$ (and different
values of $\lambda$ and $\gamma$) for 100 million periods, and estimate the dynamic effects of $g_t$ on $y_t$ using
LP.

We first consider the case of $\gamma = 0$ and $\lambda = 1$, that is, there is no problem of endogeneity
or persistence. The estimated effect of $g_t$ on $y_t$ recovered by LPs is represented by a solid
blue line in Panel A of Figure B3. As expected, the contemporaneous impact of government
spending on output is equal to 2. When considering $\lambda = 0.5$ (but still no persistence, i.e.,
$\gamma = 0$), LPs that employ OLS will deliver biased estimates of the contemporaneous effect of
$g_t$ (solid red line). The difference between the red and the blue lines in the first period is a
measure of the endogeneity bias. The problem of endogeneity can be addressed by using $z_t$
as an instrument for $g_t$ to recover the exogenous variation in government spending (given by
$x_t$). This result (still considering $\gamma = 0$) is represented by the dashed grey line in Panel A
of Figure B3, which shows how the use of LP-IV can overcome the presence of endogeneity,
delivering a response function identical to the benchmark case without omitted variables
bias.

Next, we repeat the previous exercise but now we allow for persistence in the instrument
(due to persistence of the shock); in particular, we set $\gamma = 0.2$. The results are shown in Panel
B of Figure B3 (we still represent, in solid blue line, the benchmark case of $\gamma = \lambda = 1$ for
reference). When there is endogeneity and persistence, LPs estimates of the dynamic effects
of $g_t$ are affected by both an endogeneity bias on impact, and by the effect of persistence in
the instrument during the rest of the response horizon (as shown in the previous section).
This result is displayed by the solid red line in Panel B of Figure B3, which is different
from zero after impact. Now consider estimating the dynamic effects using LP-IV with an
instrument $z_t$ (that displays persistence). The results (dashed green line) show that the use
of the instrument addresses the problem of endogeneity (on impact, the effect from the LP-IV
This figure shows the response of a simulated outcome variable to a shock using local projections with instruments, with an underlying DGP given by system (B.4) and calibrated for different degrees of persistence in the shock ($\gamma = 0$ in panel A and $\gamma = 0.2$ in Panel B). In both panels, red lines refer to estimation using LPs estimated using OLS and green dashed lines refer to LPs estimated using instrumental variables, when the DGP generates endogeneity. For reference, the blue solid lines (in both panels) display responses when the DGP does not generate persistence or endogeneity. In Panel B, the grey pointed line displays responses estimated using instrumental variables in LPs and including leads of the shock.

estimates is able to recover the true effect of $\beta = 2$). However, the dynamic effect from the rest of the response horizon still reflects the presence of persistence.

As discussed above, persistence in the instrument violates the lead/lag exogeneity condition. Stock and Watson (2018) state that, in general, this condition could be satisfied by the inclusion of further controls in the LP-IV regression. If the source of persistence is strictly restricted to the instrument, Stock and Watson (2018) show that the lead/lag exogeneity condition could be reestablished by including lags of the instrument. However, in cases like system (B.4), where the instrument inherits its persistence from the shock, lags of the instrument will not satisfy the lead-lag exogeneity condition. We build on the intuition from Stock and Watson (2018) and adapt it to the problem of persistence by including leads of the instrument in the set of exogenous variables in the LP-IV estimates. The results, shown in dashed grey lines in Panel B of Figure B3, corroborate this intuition: despite the presence of both endogeneity and persistence, enhancing the LP-IV estimates with leads of the shock
allows to recover the dynamic effects as if the instrument were not serially correlated, i.e., \( R(h)^* \).

In sum, the presence of persistence can potentially violate the lead-lag exogeneity assumption, invalidating inference under LP-IV. The solution to reestablish this condition will depend on the source of persistence in the model. When the instrument inherits its persistence from the shock, our proposed solution builds on the general intuition from Stock and Watson (2018), showing that the inclusion of leads of the instrument can deliver valid inference under LP-IV.

### B.3 Alternative simulation: using the persistence from an actual shock

In this subsection we compute the impulse response of a simulated variable \( y_t \) to a shock \( x_t \) with the following DGP:

\[
y_t = \rho y_{t-1} + B_0 x_t + B_1 x_{t-1} + u_t,
\]

where \( x_t \) is the actual government spending shock from Ramey and Zubairy (2018) as shown in Panel D of Figure D2. \( u_t \) is a random variable following \( u_t \sim N(0, 1) \). We set \( \rho = 0.9 \), \( B_0 = 1.5 \), and \( B_0 = 1 \).

Equation (B.5) is simulated for 497 periods (the length of Ramey and Zubairy (2018)’s shock), and we then compute the relevant IRFs. We repeat this process 10,000 times, and compute the average impulse responses across all repetitions. The results are shown in Figure B4.

When computing the dynamic response with standard LPs (i.e., without including any lead), the estimates diverge from the expected response when the shock has no persistence (distance between red and dark blue lines in Figure B4). Adding one lead improves the estimates, bringing the impulse-response into line with the theoretical response in the first period (green line). The accuracy of the impulse-response converges to the theoretical response when
This figure shows the response of a simulated outcome variable to the government spending shock from Ramey and Zubairy (2018). The dark blue line is the theoretical impulse-response to a shock that shows no persistence. The red line shows the LPs estimation of the impulse-response to the Ramey and Zubairy (2018) without including any lead. Green line repeats the same estimation adding one lead. Dashed grey line shows the response when including 20 leads.

more leads are included. When we include as many leads as periods in the response horizon (20), the dynamic response estimated from LPs using the actual shock (with persistence) is equivalent to the response to a non-serially correlated shock (dashed grey line).

B.4 Responses in LPs using variables adjusted for serial correlation

An apparent potential alternative to the use of leads proposed in the main text might be to adjust the shock $x_t$ so that it does not display persistence (e.g., by regressing $x_t$ on its own lags and using the resulting residual). Once the persistence is removed, one may expect the dynamic responses not to include the effect due to the persistence of the shock. However, this is not the case in a LPs setting, as we show next.

Consider the case where we obtain a variable adjusted for serial correlation: $\varepsilon_t = x_t -$
\( \gamma x_{t-1} \), as shown in equation (9). Then, \( \varepsilon_t \) can be used as substitute of the original shock \( x_t \).

Assuming \( B_1 = 0 \) in system (13) (for simplicity) consider the following series of LPs:

\[
y_{t+h} = \rho_h y_{t-1} + \lambda_h \varepsilon_t + \xi_{t+h}. \tag{B.6}
\]

To obtain the dynamic responses of \( y_t \) to the shock \( \varepsilon_t \) (adjusted for persistence), we rewrite the first equation in system (13) as a function of \( \varepsilon_t \) and compute the relevant partial derivatives. For the cases of \( h = 0 \) and \( h = 1 \) these are:

\[
\begin{align*}
\lambda_0 &= \frac{\partial y_{t+1}}{\partial \varepsilon_t} = B_0 \\
\lambda_1 &= \frac{\partial y_{t+1}}{\partial \varepsilon_t} = \rho \frac{\partial y_{t}}{\partial \varepsilon_t} + B_0 \frac{\partial x_{t+1}}{\partial \varepsilon_t} = \rho B_0 + B_0 \gamma = B_0 (\gamma + \rho). \tag{B.7}
\end{align*}
\]

That is, even after correcting for the persistence in shock \( x_t \), conventional LPs yield responses \( R(h) \), i.e., still containing the effect of persistence of the shock.

While this result may seem counter-intuitive, it arises from the fact that LPs do not have an explicit dynamic structure as a DLM. Hence, removing the persistence from \( x_t \) does not eliminate its effect on \( y_{t+1}, y_{t+2} \), etc.

To empirically show this point, we simulate series of \( y_t \) and \( x_t \) following system (13) and the calibration used in Section 3.3 (we now allow \( B_1 \neq 0 \)). We then obtain the residuals \( \hat{\varepsilon}_t \) as an estimate of \( \varepsilon_t \) described above and estimate the following equation:

\[
y_{t+h} = \rho y_{t-1} + \lambda_{h,0} \hat{\varepsilon}_t + \lambda_{h,1} \hat{\varepsilon}_{t-1} + \xi_{t+h}. \tag{B.8}
\]

Results are shown in Figure B5. The simulations corroborate the above results and we find that the use of a variable adjusted for serial correlation as \( \hat{\varepsilon}_t \) in equation (B.6) fails to retrieve an impulse response as the one obtained when \( \gamma = 0 \) in equation (13).
This figure shows the response of a simulated outcome variable to a shock with different degrees of persistence. The dark blue line shows the results of estimating equation (B.8) assuming $\gamma = 0$ in equation (13). The red line shows the same estimation when $\gamma = 0.2$. The dashed grey line shows the response when including a predicted regressor where persistence has been removed as explanatory variable (as in equation (B.6)).
C Additional empirical applications

C.1 Guajardo et al. (2014)

In this subsection we explore the relevance of our results in the context of episodes of fiscal consolidation, as produced in Guajardo et al. (2014). The authors employ a panel of OECD economies to analyze the response of economic activity to discretionary changes in fiscal policy motivated by a desire to reduce the budget deficit and not correlated with the short-term economic outlook.\(^1\) As mentioned in Table 1, this measure of fiscal changes exhibits some degree of persistence.\(^2\)

To explore the effects of persistence in this context, we compute the responses estimating a series of LPs:\(^3\)

\[
y_{i,t+h} = \mu_{h,i} + \lambda_{h,t} + \beta_{h,0}\text{shock}_{i,t} + \sum_{f=1}^{h} \beta_{h,f}\text{shock}_{i,t+f} + \beta_{h,s}X_{i,t} + \xi_{i,t+h},
\]

where \(y_{i,t}\) is a measure of economic activity (either private consumption or real GDP), \(\mu_{h,i}\) and \(\lambda_{h,t}\) represent country and time fixed effects, respectively, and \(X_{it}\) is a vector of variables that includes a lag of the shock, output, and private consumption, and a deterministic trend. In our setting, responses to the fiscal shocks are given by the estimates of coefficients \(\beta_{h,0}\) for different horizons \(h\).

We first estimate equation (C.1) by setting \(\beta_{h,f} = 0\ \forall h, f\). The results, shown in black solid lines in Figure C1 qualitatively replicate the benchmark results of Guajardo et al. (2014), with a fiscal consolidation shock significantly reducing output during the first 6

\(^{1}\)A detailed description of these shocks can be found in Devries et al. (2011).

\(^{2}\)Regressions of the fiscal consolidations measure (expressed as % of GDP) on its own lags and including time and country fixed effects reveal persistence in the previous two or three years (depending on the number of lags included). Intuitively, some degree of persistence is expected in these series since they often involved multi-year plans, as noted in Alesina et al. (2015) and Alesina et al. (2017).

\(^{3}\)Note that Guajardo et al. (2014) do not construct responses using LPs and hence their computed responses do not show the effect of persistence, as noted in the previous section. There are, however, a number of studies that employ their fiscal consolidations dataset with LPs (see, for example, Barnichon and Matthes (2017) or Goujard (2017)).
Black lines show the results from equation (C.1) with output as dependent variable and setting $\beta_{h,f} = 0 \forall h, f$, i.e., without including any leads of the shock. Grey areas represent 90% Newey-West confidence intervals for these estimates (as in Guajardo et al. (2014)). Red solid lines represent the results of estimations when allowing $\beta_{h,f} \neq 0$ and including $h$ leads of the consolidations variable.

Next, we estimate equation (C.1) but allow $\beta_{h,f} \neq 0$ (red lines in Figure C1). Three points are worth noting regarding these results. First, when accounting for the effects of persistence, the point estimates are smaller in absolute value. On average, the new responses are 35% lower during the first six years after the shock. Two years after a fiscal consolidation, output is almost 60% smaller when accounting for persistence (-0.2 vs -0.5).

Second, when including leads of the shock, the estimates are more precise, which translates into smaller confidence intervals (set at 90% as in the original paper of Guajardo et al. (2014)). During the first six years, these intervals are about 20% smaller on average in the specifications that include leads of the shock.

---

4Guajardo et al. (2014) focus on the dynamic effects of output and private consumption during 6 years after the shock. We also compute results for private consumption, shown in Figure D6 in Appendix D. As in the original paper, we also find a significant reduction in this variable during the first 6 years after a consolidation shock.
Third, these narrower intervals now include zero for most of the response horizon. Ignoring the persistence of the shock would lead to the conclusion that the output contraction after a fiscal consolidation is significant throughout the six years after the shock. However, when accounting for persistence, the effect of the shock is significant only during the first year after the shock, while it seems less plausible to conclude that the effect is statistically different from zero during the rest of the response horizon.

This exercise suggests that the policy implications from fiscal consolidations may be different when estimating $R(h)$ vs. $R(h)^*$. 

C.2 Romer and Romer (2010)

What happens when including leads of non-persistent shocks? In this section we conduct a placebo test based on Romer and Romer (2010), who investigate the output effects of legislated tax changes. Romer and Romer (2010) identify exogenous changes in tax revenues by classifying fiscal reforms according to their motivation (i.e., whether or not they are the response to changing macroeconomic conditions). As discussed in Section 2, it is the only shock considered here for which we unambiguously fail to reject the null hypothesis of no persistence. Hence, the inclusion of leads of the shock should not have a discernible impact on the estimation of dynamic responses. Beyond corroborating the previous statement, this subsection shows that the unnecessary inclusion of leads does not negatively affect inference in this application.

We estimate the response of output to exogenous tax changes following Romer and Romer (2010). We adapt the original estimation from the authors to the LPs setting:

\[
\frac{y_{t+h} - y_{t-1}}{y_{t-1}} = \beta_{0} + \sum_{f=1}^{h} \beta_{f} + \xi_{t+h}. \tag{C.2}
\]

In our first exercise, we set $\beta_{h,f} = 0 \forall h, f$ in equation (C.2) to replicate the results from Romer and Romer (2010). The results are shown Figure C2 (black lines). The response of

\footnote{Adding controls such as lags of output or the own shock do not affect the obtained results shown next.}
Black solid line shows the responses to a tax shock estimated from equation (C.2) with $\beta_{h,f} = 0$, i.e., without including any lead. Grey areas represent 68 and 95% Newey-West confidence intervals for these estimates. Red solid line shows the responses to a tax shock estimated from equation (C.2) with $\beta_{h,f} \neq 0$ and including $h$ leads of the shock. Red dashed lines represent 95% Newey-West confidence intervals for these estimates.

output is similar to that in Romer and Romer (2010): it falls persistently after a tax hike of 1% of GDP, with a peak effect reached in the 10th quarter.\(^6\)

Next, we allow for $\beta_{h,f} \neq 0$. The results, shown in Figure C2 (red lines), suggest that the inclusion of leads does not significantly affect the results. The point estimations with and without leads of the shock overlap each other for most of the response horizon and only diverge slightly during the quarters 8 to 11th.

While, given the results of Table 1 we should not expect a change in the point estimates (which we have corroborated) the same cannot be say about issues regarding inference. However, Figure C2 shows that confidence bands are not distinguishable between both specifications during the first seven quarters and differ only slightly afterwards.

\(^6\)The difference with the original estimations from Romer and Romer (2010) are only quantitative: the peak tax multiplier is about 3 in the 10th quarter. Our estimations suggest a peak multiplier of 2.25 also reached in the same quarter.
In sum, this placebo exercise is reassuring in that the inclusion of leads only matters when the explanatory variable displays some persistence. These results suggest that including leads in LPs is a conservative way to address the effects of persistence when there is a suspicion that the shock is persistent and the researcher wants to identify $\mathcal{R}(h)^\ast$.\(^7\)

### C.3 Romer and Romer (2004)

We now consider the measure of monetary policy shocks produced by Romer and Romer (2004). The authors identify exogenous monetary policy changes following a three-step procedure. First, they follow narrative methods to identify the Federal Reserve’s intentions for the federal funds rate around FOMC meetings. Second, they regress the resulting measure on the Federal Reserve’s internal forecasts (Greenbook) to account for all relevant information used by the Fed. Lastly, the series is aggregated from FOMC frequency to monthly frequency.

As shown in Table 1, the resulting measure displays some degree of persistence.\(^8\) Interestingly, the correlogram of the series seems to show a pattern consistent with negative persistence (see Panel E in Figure D1). This implies that standard LPs that do not account for persistence in the shock will identify $\mathcal{R}(h)$.

Romer and Romer (2004) estimate the response of output to the monetary policy shock using a lag-distributed regression of the log of industrial output and the measure of monetary policy shocks. Here, we adapt the estimation to a LPs setting by following the exact data and specification from Ramey (2016) (adapted in turn from Coibion (2012)) for the original sample of 1969m3-1996m12):

\[
y_{t+h} = \beta_{h,0}shock_t + \theta_h(L)x_t + \beta_{h,f} \sum_{f=1}^{h} shock_{t+f} + \xi_{t+h},
\]

where $y_t$ is either the federal funds rates, the log of industrial production, the log of consumer price index, the unemployment rate, or the log of a commodity price index, and $shock_t$ is

\(^7\)See Alloza and Sanz (2020) for another example that adds leads to LPs using a non-persistent shock. Similarly to the evidence provided in this section, they also show that adding leads does not affect inference.

\(^8\)The degree of persistence is higher when using the updated series produced by Coibion (2012).
the original Romer and Romer (2004) measure of monetary policy shocks. The regressions include a set of controls \( x_t \), with two lags and the contemporaneous values of all dependent variables, and two lags of the shock. By including the contemporaneous values of the the dependent variables, we are implementing the recursiveness assumption often used in VARs to identify monetary policy shocks.\(^9\)

The results of estimating equation (C.3) when we set \( \beta_{h,f} = 0 \) \( \forall h, f \) are shown in solid black lines (with 90% confidence bands) in Figure C3. Since we employ the same data and specification, they replicate the results from Ramey (2016) (Figure 2B). Ramey (2016) argues that the responses using LPs show more plausible dynamics than those obtained from a standard VAR (a persistent fall in industrial output and a rise in unemployment that slowly converge to 0). The drop in output after a monetary shock is broadly consistent with the original results from Romer and Romer (2004) but there are, however, two important differences. First, the trough in the response of output is reached after two years. In the estimates of Figure C3 and Ramey (2016), the trough is reached after a first year and lasts for about twelve months with a slight rebound in between. Second, although both results refer to the same impulse (a realization of the policy measure of one percentage point), the magnitude of the output fall in the original estimates of Romer and Romer (2004) is substantially bigger than when using LPs (-4.3 vs -1.7).

Next, we investigate whether accounting for the persistence in the shock has an effect on the dynamic responses. We re-estimate equation (C.3), but allowing \( \beta_{h,f} \neq 0 \). The results are shown in red lines in Figure C3. We observe that the dynamics of output are closer to the original estimates of Romer and Romer (2004): a continuous drop in output that reaches the trough after the second year. Furthermore, the magnitude of the fall is now substantially higher (-3.1) and closer to the results from Romer and Romer (2004). Another noticeable difference is that the effects on unemployment and the initial positive reaction on prices (the

---

\(^9\)This assumption implies that the monetary shock does not affect macroeconomic variables (such as output, prices, employment...) contemporaneously, and monetary variables (e.g., money stock, reserves...) do not affect the federal funds rates within a month. See Christiano et al. (1999) for further details. Later on, we show estimates that relax this assumption (Figure C4).
Figure C3: Responses to monetary policy shock from Romer and Romer (2004), with and without leads

Black solid lines refer to a benchmark specification that preserves the recursive assumption and does not include leads of the shock. Grey areas show 90% Newey-West confidence intervals. Red solid lines include $h$ leads of the monetary shocks.
so-called price puzzle) are now larger. All in all, the results from Figure C3 suggest that accounting for the persistence of the monetary policy shock can lead to larger estimates of the dynamic responses.

Ramey (2016) also investigates the role of the recursiveness assumption in the dynamic responses (the inclusion of contemporaneous values for some variables in the LPs estimation to replicate the identification in a VAR). She finds that relaxing this assumption results in weird dynamics of unemployment in the short run. We replicate these results by dropping the contemporaneous values in $x_t$ in equation (C.3) and setting $\beta_{h,f} = 0$. We indeed find that unemployment rate significantly drops in the first months after a monetary policy contraction (black solid lines in Figure C4). We investigate whether these strange dynamics may be the result of the persistence in the monetary policy shock. We estimate again equation (C.3) relaxing both the recursiveness assumption and allowing $\beta_{h,f} \neq 0$. The results (red solid lines in Figure C4) are very similar to those from Figure C3. Interestingly, unemployment responds positively to the monetary policy contraction.
Figure C4: Responses to monetary policy shock from Romer and Romer (2004) with no recursive assumption, with and without leads

Black solid lines refer to a benchmark specification that relaxes the recursive assumption and does not include leads of the shock. Grey areas show 90% Newey-West confidence intervals. Red solid lines include $h$ leads of the monetary shocks.

C.4 Gertler and Karadi (2015)

In this section we explore another application of the effects of monetary policy shocks based on Gertler and Karadi (2015). The authors identify exogenous changes in monetary policy by looking at variations in the 3-month-ahead futures of the federal funds within a 30-minute window of a FOMC announcement.\footnote{This scheme is often denoted as High-Frequency Identification (HFI). See Ramey (2016) for a comparison with other identification procedures.} By relying on this identification scheme, rather than on standard timing assumptions (e.g., Christiano et al. (1999)), the authors are able to explore the effects on measures of financial market frictions or other variables that are often assumed to be contemporaneously invariant to a monetary policy shock.

As shown in Table 1, this measure of monetary policy shock displays some persistence. This was first noted by Ramey (2016), who highlights that the procedure followed by Gertler and Karadi (2015) to convert FOMC shocks (expressed at FOMC frequency) to monthly frequency in-
troduces this serial correlation.\textsuperscript{11}

Gertler and Karadi (2015) embedded the identified monetary policy shocks in a VAR, using the measure of monetary policy surprises as an instrument of the residuals in the VAR. Here we explore what consequences the persistence of the shock might have if the researcher were to use standard LPs (and estimate $R(h)$).

To do so, we implement the following specification, suggested by Ramey (2016):

$$y_{t+h} = \beta_{h,0}\text{shock}_t + \theta_h(L)x_t + \beta_{h,f}\sum_{f=1}^{\min\{h,12\}} \text{shock}_{t+f} + \xi_{t+h}, \quad (C.4)$$

where $y_t$ is either the 1-year government bond rate, the log of industrial production, the excess bond premium spread from Gilchrist and Zakrajek (2012), or the log of consumer price index, and $\text{shock}_t$ is the measure of monetary policy shocks from Gertler and Karadi (2015). The regressions also include a set of controls $x_t$, with two lags and the contemporaneous values of all dependent variables, and two lags of the shock.\textsuperscript{12} Following Ramey (2016), we estimate equation (C.4) for a sample of 1991m1-2012m6.\textsuperscript{13} Given this reduced sample, we limit the number of leads introduced in the estimation to a maximum of 12 (i.e., we use $h$ leads for $h < 12$ and 12 leads for longer horizons).\textsuperscript{14}

The results of these estimations are shown in Figure C5 for two cases: setting $\beta_{h,f} = 0 \forall h, f$ (black solid lines with 68 and 90\% confidence intervals) and allowing $\beta_{h,f} \neq 0$ (red

\textsuperscript{11}In particular, Gertler and Karadi (2015) cumulate the surprises on any FOMC days during the last 31 days, effectively introducing a first-order moving-average structure. This is a variation of the procedure followed by Romer and Romer (2004) and that also results in a measure of monetary policy shocks that displays persistence.

\textsuperscript{12}Note that while the inclusion of lags of the shocks are meant to account for persistence in the shock, our analysis from Section 3 shows that they are not effective for this role. In our results, the inclusion of lags of the shock did not have any noticeable effect. We keep them here in order to replicate the results from Ramey (2016).

\textsuperscript{13}Gertler and Karadi (2015) proceed in two steps: they first estimate the dynamic coefficients and residuals from a VAR during the period 1979-2012. Then they estimate the contemporaneous effects of monetary policy using both the residuals from the previous step and the monetary policy instrument in a proxy VAR during 1991-2012.

\textsuperscript{14}We also preserve the sample at the end of the period until 2012m06 (which will be otherwise reduced when including leads) by considering values of the leads of the shock equal to 0 for the last 12 periods. Although this is not important for our results, it allows us to compare our estimates to those from Ramey (2016).
Benchmark estimations that do not account for persistence reproduce the results from Ramey (2016) (Figure 3B). She notes that LPs using directly the Gertler and Karadi (2015) instrument as an explanatory variable give rise to puzzling results, namely: a sluggish response of the policy rate, output increases after the monetary expansion, and the credit spread and prices do not show significant dynamics during most of the response horizon. Ramey (2016) suggests that the persistence exhibited by Gertler and Karadi (2015) and potential predictability of the series could be sources of concern. When we incorporate leads of the shock in the estimation of equation (C.4) (red lines in Figure C5) we see that both the response of the government bond rate and industrial production seem to be overestimated when persistence is not accounted for (by contrast, the results do not change much for the excess bond premium and prices).  

15 Alternatively, Ramey (2016) concludes that these differences may be due to the fact that the reduced-form parameters (used to construct the impulse responses) are estimated for a longer sample (1970-2012 instead of 1991-2012) or to potential misspecification of the original VAR estimates due to the rising importance of forward guidance, which may lead to a problem of non-fundamentality in the VAR.
Figure C5: Responses to monetary policy shock from Gertler and Karadi (2015), with and without leads.

Black solid lines refer to a benchmark specification that does not include leads of the shock. Grey areas show 68 and 90% Newey-West confidence intervals. Red solid lines include $h$ leads of the monetary shocks up to horizon $h = 12$, after then, the number of leads is kept to 12.
### D Additional Tables and Figures

Table D.1: Robustness: different lag structures for tests

|                         | 5 lags       | 10 lags      | 20 lags      | 40 lags      | 60 lags      |
|-------------------------|--------------|--------------|--------------|--------------|--------------|
| Arezki et al. (2017)    | 175.944      | 175.953      | 176.049      | 177.903      | 177.907      |
|                         | (0.000)      | (0.000)      | (0.000)      | (0.000)      | (0.000)      |
| Cloyne (2013)           | 11.365       | 21.521       | 40.041       | 98.751       | 120.270      |
|                         | (0.045)      | (0.018)      | (0.005)      | (0.000)      | (0.000)      |
| Cloyne and Hürtgen (2016)| 17.723       | 20.771       | 47.357       | 84.422       | 103.001      |
|                         | (0.003)      | (0.023)      | (0.001)      | (0.000)      | (0.001)      |
| Gertler and Karadi (2015)| 53.802       | 84.284       | 106.133      | 124.568      | 131.030      |
|                         | (0.000)      | (0.000)      | (0.000)      | (0.000)      | (0.000)      |
| Guajardo et al. (2014)  | 160.740      | 173.315      | 182.866      | 185.810      | 185.810      |
|                         | (0.000)      | (0.000)      | (0.000)      | (0.000)      | (0.000)      |
| Ramey and Zubairy (2018) | 79.298       | 89.916       | 104.414      | 182.950      | 190.974      |
|                         | (0.000)      | (0.000)      | (0.000)      | (0.000)      | (0.000)      |
| Romer and Romer (2004)  | 15.536       | 23.965       | 43.824       | 53.758       | 64.576       |
|                         | (0.008)      | (0.008)      | (0.002)      | (0.072)      | (0.320)      |
| Romer and Romer (2010)  | 1.578        | 3.080        | 6.562        | 19.023       | 24.783       |
|                         | (0.904)      | (0.980)      | (0.998)      | (0.998)      | (1.000)      |

The columns report the values of a Box and Pierce (1970) test (with Ljung and Box (1978) correction) including different lags. P-values are shown in brackets. In Arezki et al. (2017) and Guajardo et al. (2014) is tested using a generalized version of the autocorrelation test proposed by Arellano and Bond (1991) that specifies the null hypothesis of no autocorrelation at a given lag order.
Figure D1: Autocorrelograms

Panel A: Cloyne (2013)  
Panel B: Cloyne and Hürtgen (2016)  
Panel C: Gertler and Karadi (2015)  
Panel D: Ramey and Zubairy (2018)  
Panel E: Romer and Romer (2004)  
Panel F: Romer and Romer (2010)

95% confidence intervals computed using Bartlett’s formula for MA(q) processes.
Figure D2: Time series of shocks

Panel A: Cloyne (2013)

Panel B: Cloyne and Hurtgen (2016)

Panel C: Gertler and Karadi (2015)

Panel D: Ramey and Zubairy (2018)

Panel E: Romer and Romer (2004)

Panel F: Romer and Romer (2010)
Figure D3: Output and government spending responses, with and without leads

Black lines show the results of estimating the system (17) without including any lead (as in Ramey and Zubairy (2018)). Grey areas represent 68 and 95% Newey-West confidence intervals for these estimates. Red solid lines represent the results of estimations when including \( h \) leads of the Ramey and Zubairy (2018) news variable (with 95% Newey-West confidence intervals).
Figure D4: Government spending multiplier, with and without leads

Black lines show the cumulative multiplier without including any lead. Red solid lines represent the estimates of the cumulative multiplier when including a number of leads of the Ramey and Zubairy (2018) news variable that increase with the response horizon.
Figure D5: Government spending multiplier during expansions and recessions, with and without leads

The black solid and dashed lines show the cumulative multiplier during periods of expansion and recession, respectively, without including any lead (as in Ramey and Zubairy (2018)). The red solid and dashed lines show the cumulative multiplier during periods of expansion and recession, respectively, when including leads of the shocks and the state. Green solid and dashed lines refer to estimates of the expansion and recession multipliers, respectively, when including leads of the shock and the regime.
Figure D6: Private consumption response to a fiscal consolidation shock, with and without leads

Black lines show the results from equation (C.1) with private consumption as dependent variable and setting $\beta_{h,f} = 0$, i.e., without including any lead of the shock. Grey areas represent 90% Newey-West confidence intervals for these estimates (save interval as reported in Guajardo et al. (2014)). Red solid lines represent the results of estimations when allowing $\beta_{h,f} \neq 0$ and including $h$ leads of the consolidations variable.