On Galilean and Lorentz invariance in pilot-wave dynamics

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It is argued that the natural kinematics of the pilot-wave theory is Aristotelian. Despite appearances, Galilean invariance is not a fundamental symmetry of the low-energy theory. Instead, it is a fictitious symmetry that has been artificially imposed. It is concluded that the search for a Lorentz-invariant extension is physically misguided.

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1 Introduction

The de Broglie–Bohm pilot-wave formulation of quantum theory [1–4] is known to be mathematically equivalent to standard quantum theory (at least insofar as the latter is applicable). But with regard to the physical interpretation of the pilot-wave theory itself, there is as yet no clear consensus.

The dynamics of the theory may be written in a first-order, velocity-based form [1,5]. For the case of a nonrelativistic system of \( N \) particles with masses \( m_i \) and three-vector positions \( X_i(t) \) \( (i = 1, 2, ..., N) \), the motion of the particles is determined by the de Broglie guidance equation

\[
m_i \frac{dX_i}{dt} = \nabla_i S(X, t) .
\]

Here, \( S \) is the phase of the wavefunction \( \Psi \) (defined on \( 3N \)-dimensional configuration space), which satisfies the Schrödinger equation (units \( \hbar = 1 \))

\[
\frac{\partial \Psi}{\partial t} = \sum_{i=1}^{N} -\frac{1}{2m_i} \nabla_i^2 \Psi + V \Psi .
\]

In principle, these equations determine the motion of the particles, given their initial positions and the initial wavefunction. In practice, the initial positions are not known. If, for an ensemble of (approximately isolated) systems with the same wavefunction \( \Psi \), the initial distribution of configurations is given by \( P = |\Psi|^2 \), the statistical predictions of quantum theory will be accounted for.

It is widely believed that Galilean invariance is a fundamental symmetry of the above theory. Mathematically, if the coordinates are subjected to a Galilean transformation

\[
x_i' = x_i - vt, \quad t' = t
\]

(where, like \( v \), each \( x_i \) is a three-vector), the accompanying transformation [3]

\[
\Psi' = \Psi \exp \left[ i \left( \frac{1}{2} \sum_i m_i v^2 t - \sum_i m_i v \cdot x_i \right) \right]
\]

of the wavefunction leads to the Galilean invariance of (1) and (2), where

\[
S' = S + \frac{1}{2} \sum_i m_i v^2 t - \sum_i m_i v \cdot x_i , \quad \nabla_i S' = \nabla_i S - m_i v .
\]

It then seems natural to search for a relativistic extension of the theory, that should be fundamentally Lorentz invariant. This leads to serious difficulties for the many-body case, as was first noted by de Broglie [1]. Some critics might regard these difficulties as evidence against the credibility of the whole theory.

In this note we argue that, if the pilot-wave theory is correctly interpreted, Galilean invariance is not a fundamental symmetry of the above low-energy theory. The search for a Lorentz-invariant extension then seems misguided. In
our view, the difficulties encountered in such a search are no reflection on the plausibility of the pilot-wave theory. Rather, they show that the theory is not being interpreted correctly.

2 Background

The above dynamics was published by de Broglie in 1928 [1]. It was a natural generalisation of his earlier theory of particle motion, in which velocities are determined by ‘phase waves’ [11]. For de Broglie, the guidance equation (1) was an expression of the identity of the principles of Maupertuis and Fermat, and was central to his view of the dynamics. (From 1925 to 1927 de Broglie tried to derive (1) from an underlying ‘double solution’ theory. But then he saw that it could simply be postulated [12], leading to what he called the ‘pilot-wave theory’ based on (1) and (2).)

Bohm revived the theory in 1952, but in a second-order form based on acceleration [2]. In Bohm’s mechanics, (1) is not regarded as an equation of motion, but as a restriction imposed on the initial momenta [2, p. 170]. This boundary condition happens to be preserved by the time evolution, which is thought to be generated by a pseudo-Newtonian equation for acceleration, containing a ‘quantum potential’. It was suggested by Bohm that the restriction (1) could be relaxed, leading to corrections to quantum theory. In contrast, in de Broglie’s (mathematically equivalent) dynamics of 1928, the motion is regarded as determined by (1) alone, as de Broglie himself made very clear [1]. And there is no question of relaxing (1), for it is the expression of de Broglie’s guiding idea – that the principles of Maupertuis and Fermat are the same.

Bohm showed that the theory – which de Broglie had applied to interference and scattering – could account for the general quantum theory of measurement. Recently, de Broglie’s original first-order dynamics was revived by Bell [5].

The first-order approach, in which (1) is regarded as the basic law of motion for the system, was adopted by the author [13,14] and by Dürr et al. [15,16]. There are some differences, however, which are crucial for the issue of Galilean and Lorentz invariance.

In the author’s view, the pilot wave Ψ should be interpreted as a new causal agent, more abstract than forces or ordinary fields [14]. This causal agent is grounded in configuration space – which is not surprising in a fundamentally ‘holistic’, nonlocal theory. Heuristically, however, its action in three-space may be visualised in terms of ‘Aristotelian forces’. The ‘Aristotelian force’ $f_i = \nabla_i S$

\[^2\text{In 1927 de Broglie proposed the theory for a many-body system with a wavefunction in configuration space, contrary to many erroneous claims, made in the literature, that he only considered the one-body theory with a wavefunction in three-space. (The historical study by Cushing [6] is also mistaken on this point, as Cushing has recently recognised [7,8,] See Ref. [1], pp. 118, 119, where de Broglie generalises the one-particle pilot-wave theory to a many-body system. A full analysis and outline of Ref. [1] is given elsewhere [9,10].}\]

\[^3\text{Some authors have erroneously referred to de Broglie’s dynamics, defined by (1) and (2), as ‘Bohmian mechanics’. This misnomer not only ignores de Broglie’s priority; it also misrepresents the views of Bohm.}\]
on the right-hand-side of (1) is analogous to the Newtonian force $F_i = -\nabla_i V$ (where $V$ is the potential energy) that appears on the right-hand-side of the classical law of motion

$$m_i \frac{d^2 X_i}{dt^2} = -\nabla_i V(X, t).$$

(5)

According to (5), the ratio of Newtonian force to mass gives the acceleration. While according to (1), the ratio of ‘Aristotelian force’ to mass gives the velocity.

Clearly, it is not logically necessary that causal thinking be based on ‘Newtonian forces’ proportional to the acceleration. Generically, ‘force’ simply means ‘agency causing motion (of a specific type)’. And the causal agency acting on a particle might, in reality, be an ‘Aristotelian force’ proportional to the velocity (as was in fact widely believed before Galileo). In our view the physical, causal explanation for why, say, a particle slows down as it approaches a certain point is simply that the ‘Aristotelian potential’ $S$ has a turning point there.

In Ref. [14], it was noted that Aristotelian causes are physically incompatible with the relativity of motion, as remarked by Russell [17]. (This is partly why they were abandoned in the 17th century.) If one invokes such causes, there should be a natural state of absolute rest. Thus, in Ref. [14], only time-independent transformations of three-space coordinates were considered, with respect to which $\Psi$ is a scalar.

In contrast, Dürr et al. [15,16] do not explicitly invoke such causal agents, and regard Galilean invariance as a fundamental symmetry of the theory (so that, as in classical mechanics, uniform motion is relative). Thus $\Psi$ is subject to the transformation law (3). Indeed, Dürr et al. appeal to Galilean invariance in order to motivate the choice (1) of velocity field.

This appeal to Galilean invariance has recently been questioned by Brown et al. [18] who conclude that, indeed, in the context of the pilot-wave theory, the Aristotelian forces introduced by the author seem more natural than the usual Newtonian ones. However, the issue of the status of Galilean invariance was unfortunately not resolved. In particular, it seemed doubtful that a natural state of rest could be reconciled with the Galilean invariance of (1) and (2). Brown et al. remark that there appears to be no frame-independent definition of Aristotelian forces (since these forces will themselves transform under a Galilean transformation).

We shall now attempt to clarify this potentially very confusing situation.

3 The role of kinematics and dynamics

In present physical theories at least, the burden of describing Nature is shared between kinematics and dynamics. The former defines the structure of space-time; the latter accounts for motion within this structure (in so far as this motion deviates from the ‘natural’, force-free state).

Now the division of labour between kinematics and dynamics cannot be defined uniquely. For instance, it is usual in classical general relativity to assume a curved spacetime. But one may equally think in terms of flat spacetime, on
which a ‘metric field’ distorts rods and clocks so as to give the appearance of a curved geometry. In the former approach, a wealth of phenomena are accounted for by the kinematics of spacetime itself. In the latter, these same phenomena are accounted for by dynamical influences on flat spacetime. And it is impossible to say which picture is ‘true’.

Nevertheless, to say that classical spacetime is flat is usually regarded as a mistake, for the following reason: The whole purpose of kinematics is to embrace universal features of the dynamics. Any effects that are found to be independent of the particular material bodies involved are best assumed to be part of the kinematics.

This point has recently been made particularly clearly by Sonego [19]. The ‘zeroth law of mechanics’ – that the behaviour of free bodies is independent of their mass and composition – plays a key role in defining the geometry of spacetime.⁴

Such arguments are, however, usually presented in ‘operational’ terms: Further, a body is in general assumed to be ‘free’ when it is far away from other bodies of a specified type (charged bodies in general relativity, massive bodies in Newtonian theory). Thus it is implicitly assumed that: (i) Forces have their origin in other bodies; (ii) The effect of these forces diminishes with distance.

This approach would not be appropriate in the pilot-wave theory. For we are, at least at present, unable to perform operations with subquantum trajectories. Further, the origin of forces in the pilot-wave theory is not other bodies, but the wavefunction itself.⁵ And, of course, these forces do not necessarily diminish with distance.

But it is still possible to define a natural kinematics in the pilot-wave theory. For while operational arguments sometimes act as guides in the construction of theories, if one is given a theory, the essential point – that kinematics should embrace universal features of the dynamics – may be implemented without recourse to strictly operational arguments. For the theory itself tells us what ‘universal’ features there are in Nature (assuming the theory to be true). And once we have identified these, the most convenient definition of kinematics becomes clear. In other words, if we are given a theory, a natural definition of kinematics is singled out by the theory itself.

Note that, as Poincaré [21] in particular was well aware, it is possible in principle, in virtually any theory, to adopt virtually any spacetime structure, provided one adds appropriate compensating dynamical factors. So the issue is not whether a certain kinematics is possible, but whether it is the most suitable.

⁴On this basis Sonego argues that, because the spreading of free wavepackets is mass-dependent, quantum theory alone does not clearly define a spacetime structure. Some sort of underlying dynamics is necessary.

⁵In (1), the Aristotelian force acting on a particle depends on the positions of the other particles. But it does not make sense to regard these other particles as the origin of this force because, unlike in classical mechanics, the force is not generated by a fixed (Ψ-independent) function of relative particle positions. In this sense the pilot-wave theory is distinctly non-mechanical, as pointed out (in terms of the quantum potential) by Bohm et al. [20].
4 Aristotelian kinematics and Aristotelian spacetime

We now show that the Aristotelian dynamics of the pilot-wave theory naturally selects an Aristotelian kinematics and an Aristotelian spacetime.

In order to define a kinematics, one must first of all find a natural definition of a ‘free’ system. Otherwise, it will be impossible to find any ‘universal’ properties of motion at all.

The procedure is a familiar one. In Newtonian mechanics, for instance, forces are identified as the cause of motion. A system is considered to be ‘free’ when the right-hand-side of (5) vanishes. One then has a set of ‘natural motions’

\[ X_i(t) = v_i t + X_i(0) , \]

where \( X_i(0) \) and \( v_i \) are arbitrary constants. These trajectories are completely independent of the masses of the particles. It is therefore expedient to regard them as features of spacetime itself (that is, as geodesics associated with an appropriate affine structure). In this way, one arrives at the kinematics of Galilean spacetime. Uniform motion is the natural state, and Galilean invariance is a fundamental symmetry.

Similarly, in general relativity one defines a ‘free’ body to be one upon which no nongravitational forces act. The resulting trajectories are independent of the body’s mass and composition. So again, it makes sense to regard the trajectories as properties of spacetime itself. Hence the picture of geodesics in curved spacetime.

What is the natural definition of a ‘free’ system in the pilot-wave theory? If one writes the dynamics in the first-order form given by (1) and (2), a system can only be regarded as ‘free’ if the ‘Aristotelian force’, appearing on the right-hand-side of (1), vanishes. One then has a set of ‘natural motions’

\[ X_i(t) = X_i(0) , \]

where \( X_i(0) \) are arbitrary constants. Again, these (rather trivial) trajectories are completely independent of the masses of the particles. It is both natural and expedient to regard them as features of spacetime itself. Thus one arrives at Aristotelian spacetime, in which the natural state of motion is rest.

Formally speaking, Aristotelian spacetime may be characterised as a product \( E \times E^3 \) of a Euclidean time line with Euclidean three-space [22]. The natural invariance group corresponds to time-independent transformations on three-space. In particular, the physics is invariant under translations and rotations in \( E^3 \).

With respect to this natural group, the guiding field \( \Psi \) is, as we have mentioned, a scalar. This ensures that (1) is invariant. (If \( V \) depends only on the distances between particles, (2) will of course also be invariant.)

Note that Aristotelian spacetime \( E \times E^3 \) is a perfectly abstract, geometrical structure. In principle, no coordinates are required for its description.
5 Galilean invariance: a fictitious symmetry

At this point, it might be objected that the equations (1) and (2) are nevertheless Galilean invariant. But this mathematical symmetry is misleading.

Consider, by way of analogy, the case of classical mechanics. And consider a transformation of coordinates

\[ x'_i = x_i - \frac{1}{2} a t'^2, \quad t' = t, \]

to a new frame with an *acceleration* \( a \). If one defines a transformation

\[
V'(x) = V(x) - \frac{1}{2} \sum_i m_i a^2 t'^2 + \sum_i m_i a \cdot x_i,
\]

\[
-\nabla'_i V'(x) = -\nabla_i V(x) - m_i a
\]

of the Newtonian potential energy and force, the classical dynamical equations (5) become invariant. They have the same form in the accelerated frame. The usual view, of course, is that in (6) one has introduced ‘fictitious inertial forces’ in the accelerated frame, in order to make Newton’s laws *appear* invariant. By *imposing* an appropriate ‘transformation law’ on the Newtonian force, one can (artificially) make classical mechanics invariant under transformations to accelerated frames of reference.

Physically, and mathematically, the transformations (4) and (6) are very similar. In (6) the Newtonian force \(-m_i a\) acting on the \( i \)th particle (in the accelerating frame) is proportional to the mass; its effect is therefore independent of the mass. It affects all bodies equally. This universality betrays it as a fictitious entity that ought to be eliminated by an appropriate choice of kinematics. Similarly, in (4) the Aristotelian force \(-m_i v\) acting on the \( i \)th particle (in the uniformly moving frame) is proportional to the mass, has a universal effect, and should be eliminated by redefining the kinematics.

Thus the supposed ‘Galilean invariance’ of the pilot-wave theory is, in our view, a first-order analogue of the above fictitious invariance of (second-order) classical mechanics. Just as the true, physical invariance group of classical mechanics leaves acceleration and (Newtonian) force invariant, so the true, physical invariance group of pilot-wave dynamics leaves velocity and (Aristotelian) force invariant. (This resolves the difficulty raised by Brown et al.)

Dürre et al. [15,16] have proposed what is, in effect, a mixture of first-order (Aristotelian) dynamics with second-order (Galilean) kinematics. We assert on the basis of the above reasoning that such a mixture is physically incongruous. An Aristotelian dynamics requires an Aristotelian kinematics.

\[\text{Note that, for a symmetry to be included in the physical kinematics, it is not enough that it be a mere mathematical symmetry. For example, it has been known since 1910 that Maxwell’s equations are invariant under a 15-parameter Lie group whose coordinate transformations include not only Lorentz transformations but also a scale transformation and a class of nonlinear transformations (where a subset of the latter corresponds, in the nonrelativistic limit, to a transformation to an accelerated frame). See, for example, Ref. [23].}\]
6 ‘Detection’ of the natural state

In the pilot-wave theory of particles, at the fundamental level of an individual system, the effects of absolute velocity (of the reference frame) may be compensated for by transforming the wavefunction via (3).

As a result, the natural state of zero velocity cannot be detected. This may seem peculiar, in view of our assertion that this state exists.

But the situation is similar in classical mechanics. There, strictly speaking, the natural state of zero acceleration cannot be detected – without implicit assumptions about the origin of forces. For by transforming the Newtonian force according to (6), the effects of absolute acceleration may be cancelled (that is, the effect of an acceleration of the reference frame on the observed laws of physics may be compensated for).

Let us be clear about this. Classically, the transformation (6) introduces what are usually called ‘fictitious inertial forces’. They are regarded as fictitious because they appear to have no source. But strictly speaking, the real existence of such forces cannot be ruled out. They might be generated, for instance, by acceleration with respect to distant matter. Perhaps they are real after all. Indeed, even without distant matter, it might simply be that classical forces really do transform in this way – just as a magnetic field may be generated by the Lorentz transformation of an electric field.

Strictly speaking, then, the natural state of unaccelerated motion cannot be detected in classical mechanics – unless one assumes that real forces have their origin in nearby bodies. With this assumption (or consensus) as to what real forces are, a particle far away from other bodies provides a standard of unaccelerated motion.

There can be no comparable assumption in the pilot-wave theory, where motion is generated by the wavefunction. One would need a consensus on what the wavefunction of the universe really is (up to a constant phase), in order to detect the true frame.

None of this affects our argument, however.

First of all, in both theories, a natural kinematics is in any case singled out by the dynamics as above, simply by identifying ‘forces’ with the entities appearing on the right-hand-side of the equations of motion. No further consensus on the nature of these forces is needed to single out the natural kinematics, even if practical detection of the natural state should turn out to be problematic without such a consensus.

Secondly, things are much more clear cut in field theory. There, the theory not only picks out a natural Aristotelian kinematics (where the natural state is a static field configuration). The natural state of rest is also singled out by the detailed behaviour of the field. The (static) vacuum field is not Lorentz invariant [20]. And if fundamental nonlocality is assumed to define an absolute simultaneity, the speed of approximately classical electromagnetic waves (measured by absolutely synchronised clocks) will be isotropic only if the clocks are at absolute rest [9,14,24]. (If $P \neq |\Psi|^2$, there will be observable instantaneous signals [13]. These may be used to synchronise distant clocks.)
Even in the case of field theory, no doubt, one could construct other, more contrived theories with a different (possibly Lorentz-invariant) natural state. For one may always introduce compensating factors that make unnatural motions appear natural, as seen above in the case of classical mechanics, where the transformation (6) makes accelerated motion appear force-free. In practice, however, such compensating factors are easily recognised to be rather contrived.

In conclusion, then, so-called ‘operational detection’ of natural motion never really occurs, in any theory, without implicit, simplifying assumptions. In the end, it is simply the case that the theory itself singles out a natural, convenient kinematics. And this is as true in the pilot-wave theory as in any other.

7 Origin of the velocity field

It might be thought [15] that Galilean invariance plays an important role, ensuring that the coefficients \( m_i \) in (1) are equal to those in (2). But the structure of the Schrödinger equation itself, in one frame of reference, singles out the natural velocity field \( \nabla_i S / m_i \) — just as the structure of general relativity singles out a natural set of trajectories (the geodesics). There is no need to explain why particles follow precisely the natural trajectories singled out by the theory. (In contrast, in Newtonian physics it is cogent to ask why inertial and gravitational mass are equal, because there is nothing in the structure of the theory that prefers this.)

For the Schrödinger equation (2) implies the continuity equation

\[
\frac{\partial |\Psi|^2}{\partial t} + \sum_{i=1}^{N} \nabla_i \cdot \left( |\Psi|^2 \frac{\nabla_i S}{m_i} \right) = 0
\]

for the naturally-occurring quantity \( |\Psi|^2 \). Any function proportional to \( |\Psi|^2 \) is preserved by the velocity field \( \nabla_i S / m_i \), if and only if the coefficients \( m_i \) are equal to those in (2). Thus, given that the velocity field is proportional to \( \nabla_i S \), the structure of the Schrödinger equation singles out the natural coefficients.

As already noted, the issue is not whether Lorentz invariance is possible, but whether it is suitable (as is always the case with kinematics). Despite the difficulties arising from nonlocality in Minkowski spacetime, one might be able to construct a high-energy theory with a fictitious Lorentz invariance, analogous to the fictitious Galilean invariance of the low-energy theory. As we have seen, it would be an error to regard this symmetry as part of the kinematics. (On the other hand, should attempts to impose Lorentz invariance fail, it would be a mistake to be alarmed by this. For such attempts have no real motivation, since even the low-energy theory is not really fundamentally invariant under boosts.)

The situation is no better in special or general relativity. The former may be given a Lorentz interpretation, with an absolute rest frame; while the latter may, as noted, be cast in terms of a flat spacetime background. Only theoretical expediency singles out the usual kinematics.

The same cannot be said for the equilibrium distribution \( P = |\Psi|^2 \), even though \( |\Psi|^2 \) is the natural measure. For (1) and (2) are fundamental laws, while ensemble distributions are contingent. The distribution \( P = |\Psi|^2 \) should be given a dynamical explanation along the lines of classical statistical mechanics [19,13,14,25].
There is no need to appeal to Galilean invariance to fix these coefficients, as done by Dürr et al. [15].

Note that we are appealing here, not to the empirical conservation of the equilibrium ensemble distribution \( P = |\Psi|^2 \), but to mathematical properties of the Schrödinger equation for an individual system. For if \( \Psi \) is associated with an individual system, so is the quantity \( |\Psi|^2 \). (Since probabilities are not fundamental in the pilot-wave theory, the question of the equality of masses in (1) and (2) must be addressed at the level of individual dynamics. And at that level, the mathematical structure of the Schrödinger equation does pick out a natural velocity field \( \nabla_i S/m_i \).)

8 Galilean invariance and the quantum potential

To convincingly maintain Galilean invariance as part of the fundamental symmetry group, one could write the pilot-wave theory in Bohm’s second-order, pseudo-Newtonian form based on the quantum potential. For then, the natural definition of a ‘free’ system would be analogous to that in classical mechanics: the total Newtonian force acting on the system (including that generated by the quantum potential) must vanish. One then obtains the classical set of natural motions, with arbitrary uniform velocity, leading to a natural Galilean kinematics. Thus Holland [3] is consistent when he asserts that Galilean invariance is a fundamental symmetry, for he bases the dynamics on the quantum potential.

But then things become rather inelegant, and also difficult. The quantum potential itself is inelegant. The Galilean transformation (3) of the wavefunction is mathematically peculiar, having no simple geometrical interpretation. And a Galilean-invariant theory invites attempts at a Lorentz-invariant extension, leading to enormous complications.

In contrast, the guidance equation (1) is simple and elegant. So is Aristotelian spacetime \( E \times E^3 \). The wavefunction is simply a scalar. And there is no need to think of Lorentz invariance as anything other than a phenomenological feature of the equilibrium distribution \( P = |\Psi|^2 \) – as becomes clear in field theory [4,9,14,20,24].

9 Remarks

The definition of spacetime structure, and the choice of a true invariance group, is by no means a merely semantic issue. It has serious ramifications both for our understanding of the physics and for the subsequent development of the theory. Field theory may in fact be developed with advantage on Aristotelian spacetime. Electrodynamics is greatly simplified, since the time-component of the vector potential (which leads to ‘ghosts’ in standard QED) makes no appearance at all [14,26]. Similarly, non-Abelian gauge theories such as QCD
have a natural ghost-free formulation, equivalent to standard field theory written in the temporal gauge [9].

It has been proved [27] that ‘York time’ [28] provides a unique slicing of curved, classical spacetime, for a closed universe satisfying reasonable conditions. York time may then be regarded as an absolute cosmic time [29,30]. It is natural to identify York time with our Aristotelian time, and the associated curved spacelike slices with curved Aristotelian three-space [9,26]. One may then identify gravitation as a curvature of Aristotelian spacetime $E \times E^3$, rather than as a curvature of Minkowski spacetime $M_4$. Since the former has a definite foliation built into it, the notorious ‘problem of time’ in quantum gravity is eliminated from the outset. (Technical problems remain, however.)

An Aristotelian kinematics, with an absolute slicing of spacetime, is in fact naturally singled out by the pilot-wave dynamics of gravity. For the momentum canonically conjugate to the three-metric is given essentially by the extrinsic curvature tensor – which tells how the three-geometry is embedded in spacetime. Just as the momentum of a particle is determined by its position and wavefunction in the low-energy theory, so the embedding of a spacelike slice in spacetime should be determined by its three-geometry and wavefunction.

10 Conclusions

The second-order structure of classical mechanics defines a physical, Galilean spacetime with a natural state of zero acceleration. Similarly, the first-order structure of pilot-wave dynamics defines an Aristotelian spacetime, with a natural state of zero velocity.

To impose Galilean invariance on the pilot-wave theory is like imposing, on Newtonian mechanics, an invariance under transformations to uniformly accelerated frames. The Galilean transformation (3) of $\Psi$ amounts to the introduction of fictitious inertial (Aristotelian) forces.

Despite appearances, then, Galilean invariance is not a fundamental symmetry of the low-energy pilot-wave theory. There is then no reason to impose Lorentz invariance in the high-energy domain.

Some authors, following Bell, have portrayed the current situation as a sort of two-horse race for fundamental Lorentz invariance, the contestants being the pilot-wave and dynamical-reduction theories. From the above perspective, this is quite misguided. For in the pilot-wave theory, uniform motion is not relative – so the ‘problem’ of finding a Lorentz-invariant extension simply does not arise. Whether the theory of dynamical reduction is also able to circumvent this problem remains to be seen. (Perhaps the reduction mechanism could be shown to single out a natural state of rest.)

$^{10}$It is therefore wrong to assume, as many authors do, a guidance equation for the three-metric with arbitrary lapse and shift functions (that is, with arbitrary spacetime slicing). For a full discussion see Ref. [9].
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