On the history of fourth order metric theories of gravitation

By RAINER SCHIMMING (Greifswald) and HANS-JÜRGEN SCHMIDT (Potsdam)

Present address for correspondence:
http://www.physik.fu-berlin.de/~hjschmi, e-mail: hjschmi@rz.uni-potsdam.de

H.-J. Schmidt, Institut für Mathematik, Universität Potsdam
Am Neuen Palais 10, D-14469 Potsdam, Germany

Abstract

We present the history of fourth order metric theories of gravitation from its beginning in 1918 until 1988.

1 Introduction

From the advent of the general relativity theory (GRT) in 1915 by Albert Einstein (1879-1955) until today numerous geometrized theories of gravitation have been proposed.

Here, we shall review the history of a class of theories which is conceptually rather close to GRT:

- The gravitational field is described by a space-time metric only.

- The field equation follows from a Hamiltonian principle. The Lagrangian \( L \) is a quadratic scalar in die Riemannian curvature of the metric. (Note that \( L \) in GRT is linear in the curvature, i.e. proportional to the scalar curvature \( R \).)

- The constants appearing in this ansatz are adjusted such that the theory is compatible with experimentally established facts. Hence, the Lagrange

---

\(^1\)Reprint of the original paper which appeared in NTM-Schriftenr. Gesch. Naturw., Tech., Med. (Leipzig) 27 (1990) 1, pages 41-48; ISSN 0036-6978. The only difference in comparison with the original is that those text-parts which had been given only in German language, are now (in brackets just after the German text) translated into English.
function reads \(^2\)
\[
L = aR^2 + bR_{ij}R^{ij} + kR + \Lambda
\]  
with constants \(a\), \(b\), \(k\), \(\Lambda\) where \(a\) and \(b\) do not vanish simultaneously. The variational derivative of \(R_{ijkl}R^{ijkl}\) with respect to the metric can be linearly expressed by the variational derivatives of \(R_{ij}R^{ij}\) and of \(R^2\) \[1\]. Thus we may omit \(R_{ijkl}R^{ijkl}\) in (1) without loss of generality. The theory is scale-invariant if and only if \(\Lambda \cdot k = 0\). It is even conformally invariant if and only if \(\Lambda = k = 0\) and \(3a + b = 0\). The field equation following from \(L\) eq. (1) is of fourth order, i.e. it contains derivatives up to the fourth order of the components of the metric with respect to the space-time coordinates. (Note that Einstein’s equation of GRT is of second order.)

The fourth order metric theories of gravitation are a very natural modification of the GRT. Historically, they have been introduced as a specialization of Hermann Weyl’s (1885-1955) nonintegrable relativity theory from 1918 \[2\]. Later on, just the fourth order theories became interesting and more and more physical motivations supported them: The fourth order terms can prevent the big bang singularity of GRT; the gravitational potential of a point mass is bounded in the linearized case; the inflationary cosmological model is a natural outcome of this theory. But all the arguments from classical physics were not so convincing as those from quantum physics: the quantization of matter fields with unquantized gravity background leads to a gravitational Lagrangian of the above form \[3\]. Moreover, fourth order theories turned out to be renormalizable at the one-loop quantum level \[4\], but at the price of losing the unitarity of the S-matrix. (Note that Einstein’s equation is not renormalizable.) These circumstances caused a boom of fourth order gravity (classical as well as quantum) in the seventies. We will stop our record of the history before this boom. We restrict ourselves to the purely metrical theories (i.e., the affinity is always presumed to be Levi-Civita) and want only to mention here that fourth order field equations following from a variational principle can be formulated in scalar-tensor theories, theories with

\(^2\)We apply the usual notations of tensor calculus and differential geometry. Particularly: \(R_{ijkl}\) = components of the curvature tensor of a Riemannian metric, \(R_{ij}\) = components of the Ricci tensor = \(R^k_{\;ikj}\), \(R =\) scalar curvature = \(R^k_{\;k}\), \(C_{ijkl}\) = components of the conformal curvature tensor.
independent affinity, and other theories alternative to GRT as well.

2 Papers inspired by Weyl’s theory

In 1918, soon after Albert Einstein’s proclamation of the GRT, Hermann Weyl proposed a new kind of geometry and a unified theory of gravitation and electromagnetism based on it. He dwelled on the matter in a series of papers [2, 5-10] until it became superseded by the modern gauge field interpretation of electromagnetism [11 - 13]. Note that the gauge concept together with the words “Eichung” (gauge) and “Eichinvarianz” (gauge invariance) came into use in theoretical physics through Weyl’s ansatz. For a broader discussion and evaluation we refer to [14]. A. Einstein [15] pointed out that the nonintegrability of the lengths of vectors under Weyl-like parallel propagation contradicts to physical experience. His argument has been refuted not earlier than in 1973: Paul Adrien Maurice Dirac (1902-1984) discusses the possibility of a varying gravitational constant. He writes:

“Such a variation would force one to modify Einstein’s theory of gravitation. It is proposed that the modification should consist in the revival of Weyl’s geometry, in which lengths are nonintegrable when carried around closed loops, the lack of integrability being connected with the electromagnetic field”. [16, p. 403]

H. Weyl’s aesthetically very appealing modification of GRT unfortunately does not directly describe the real dynamics of fields and particles; however it deeply influenced the “dynamics of theories”. By this we mean that various fundamental ideas have been formed or promoted by Weyl’s papers:

- the search for alternatives to the GRT based on geometrization;
- the unification of the interactions or forces of nature, beginning with gravity and electromagnetism;
- field theories based on the geometry of an affine connection;
- conformal geometry and conformally invariant field theories;
- the gauge field idea, and
Here we are interested just in the last item. Weyl required the Lagrangian to be a polynomial function of the curvature and to be conformally invariant. He states:

“Dies hat zur Folge, dass unsere Theorie wohl auf die Maxwellschen elektromagnetischen nicht aber auf die Einsteinschen Gravitationsgleichungen führt; an ihre Stelle treten Differentialgleichungen 4. Ordnung.” [2, S. 477] (This has the consequence that our theory, though it leads to Maxwell’s equations of electromagnetism, fails to lead to Einstein’s gravitational equations; they are replaced by differential equations of fourth order.)

The ambiguity in the concrete choice of $L$ appeared as a difficulty which is opposed to the spirit of unification: any linear combination of $R^2$ and $R_{ij} R^{ij}$ would do. The variation of $R_{ij} R^{ij}$ or of $R_{ijkl} R^{ijkl}$ with respect to the vector field yields Maxwell-like equations, while for the choice of $R^2$ an electromagnetic Lagrangian $F_{ij} F^{ij}$ together with a coupling constant $\alpha$ has to be added by hand: $L = R^2 + \alpha F_{ij} F^{ij}$ [6, 2]. Weyl himself favoured different Lagrangians in different papers. Moreover, he took trouble to produce results compatible with Einstein’s GRT. For this aim he destroyed the conformal invariance by a special gauge. Ernst Reichenbächer criticizes:

“Um so auffallender ist es, dass Weyl in dem von ihm durchgerechneten Beispiel für die Wirkungsfunktion durch Festlegung der Eichung vor der Variation den Grundsatz der Eichinvarianz durchbricht.” [17, S. 157]. (It is even more conspicuous, that Weyl in his chosen calculated example of an action has broken the axiom of gauge invariance before performing the variation.)

A more detailed analysis of the theory was necessary then. Roland Weitzenböck [18] produced and studied all scalar invariants of the curvature in Weyl’s geometry. Wolfgang Pauli jun. (1900-1958) [19, 20] and a little later Ferencz Jüttner (geb.(born) 1878) [21] calculated the spherically symmetric static gravitational field for variants of Weyl’s theory. Pauli [20, S. 748] comes to an important conclusion:

“Hiernach ist klar, dass aus Beobachtungen der Merkurperihelbewegung und der Strahlenablenkung, die mit Einsteins Feldgleichungen im Einklang sind, niemals ein Argument gegen Weyls Theorie entnommen werden kann,
wenigstens solange die letztere eine der drei Invarianten \( R_{ij}, R_{ijkl} \) als Weltfunktion zugrunde legt.” (From this it becomes obvious, that from observations of the Mercury perihelion change and from the light ray deviation, which are in agreement with Einstein’s field equations, one can never deduce an argument against Weyl’s theory, at least, as long as one restricts to action functions combined from the three invariants \( R_{ij}, R_{ijkl} \).)

In other words, fourth order gravitational field equations following from (1) are not falsifiable by experimental physics! Pauli [20, 22] and other authors did not even consider the vector field (i.e. assumed it to be equal to zero) in Weyl’s theory, thus making it unaffected by the criticism of nonintegrability [15]. Rudolf Bach [23] realized the possibility to keep the conformal invariance in a purely metrical theory: a Lagrangian

\[
L = C_{ijkl} C^{ijkl}
\]

or equivalently,

\[
L = R^2 - 3 R_{ij} R^{ij}
\]

yields a conformally invariant field equation for the metric, later on called “Bach’s equation”. In a similar spirit and in the same year 1921, Albert Einstein [24] proposed a conformally invariant theory. His expressions suffer from being non-rational in the metric. This theory is sometimes cited but has never been studied in details.

Reichenbächer [25, 17] proposed a variant of Weyl’s theory based on a non-rational Lagrangian resembling nonlinear Born - Infeld electrodynamics.

In [26], also \( L = R^2 \) is used to get a field-theoretical model for the electron, but the fourth order terms are lost by an error in the calculations.

Cornel Lanczos (1893-1974) [27] tried a programme of “Electromagnetismus als natürliche Eigenschaft der Riemannschen Geometrie” (Electromagnetism as natural property of Riemannian geometry). He also assumed the vector field in Weyl’s theory to be zero, but reintroduced it then in an alternative way as a set of Lagrangian multipliers. Unfortunately, Lanczos was, working with hyperbolic differential equations, misled by a formal analogy with elliptic differential equations. He varied the speculations with Lagrangian multipliers in a series of papers [28-34]. To take it positive, many useful
mathematical formulas for fourth order theories resulted from Lanczos’ work. Particularly, the paper [1] became a “citation classic”.

In the twenties, the programme of classical field theory with its two cornerstones geometrization and unification lost some of its attractiveness in virtue of the quickly progressing quantum theory, cf. [35]. Moreover, there were the refutation of Weyl’s theory and objections to fourth order equations. Lanczos expressed them as follows:

“Der Grund, weshalb diese Untersuchungen nicht weiter gediehen sind und zu keinem Fortschritt führten, lag an zwei Momenten. Einerseits war es entmutigend, dass man zumindest drei anscheinend gleichwertige Invarianten zur Verfügung hatte: $R^2$, $R_{\alpha\beta} R^{\alpha\beta}$, $R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$, ohne ein plausibles Auswahlprinzip zwischen ihnen zu besitzen. Andererseits erscheinen diese Gleichungen, solange man ihre innere Struktur nicht verstehe, als Differentialgleichungen viert er Ordnung für die die $g_{ik}$ von einer Kompliziertheit sind, die für jede weitere Schlussfolgerung ungeeignet ist.” [27, p.75] (The reason, why these investigations did not give rise to further results, is twofold. On the one hand, it was discouraging, that one had at least three seemingly similar invariants: $R^2$, $R_{\alpha\beta} R^{\alpha\beta}$, $R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$, without possessing any plausible principle of choice among them. On the other hand, these equations appear, as long as one does not understand their inner structure, as differential equations of fourth order for the $g_{ik}$ to be of such a complexity which makes them unsuited for drawing any further consequences from them.)

Similarly, Bergmann argues in his text-book [36] that, first, fourth order equations admit too many solutions and, second, their Lagrangian is rather ambiguous.

This situation explains why only few papers on fourth order gravitation appeared in the period from the thirties to the sixties and why these did not follow the actual trends at their time. H. A. Buchdahl dealt with the subject in the period 1948-1980. In his papers he covered the following problems:

- Invariant-theoretical considerations continuing those of Weitzenböck and Lanczos [37];

- General expressions for the variational derivatives of Lagrangians built from the curvature and, possibly, its derivatives are obtained [38-42].
Einstein spaces are solutions of a rather general class of fourth order equations [43, 44];

- Static gravitational fields in fourth order theories [45];

- Cosmological solutions in theories where the Lagrangian is a function of the scalar curvature [46, 47];

- Conformal gravity [48];

- Reinterpretation of some fourth order equations in five dimensions [49].

Sir Arthur Stanley Eddington (1882-1944) in 1921, see [50], and Erwin Schrödinger (1887-1961) in 1948, see [51], also discussed gravitational field equations of fourth order to get field theoretical particle models, i.e., they tried to realize Einstein’s particle programme.

3 A new view

Fourth order metric theories of gravitation have been discussed from 1918 up to now. One original motivation was the scale invariance of the action, a property which does not hold in GRT. Another motivation was the search for a unification of gravity with electromagnetism, which is only partially achieved with the Einstein-Maxwell system. There was no experimental fact contradicting GRT which could give motives for replacing it by a more complicated theory.

But a lot of problems appeared:

1. The Lagrangian became ambiguous in sharp contrast to the required unification.

2. The higher order of the field equation brought

2.1. mathematical problems in the search for solutions and

2.2. physical problems for the interpretation of the additional degrees of freedom.
3. The well-founded Newtonian theory of gravitation did not result as the weak-field limit of scale invariant fourth order gravity.

The third problem was the last of these to be realized but the first to be solved, both in 1947: One has to break the scale invariance of the theory by adding the Einstein-Hilbert action to the purely quadratic Lagrangian. Then, up to an exponentially small term, the correct Newtonian limit appears [52].

The original scale invariant theory then, again emerges as the high-energy limit of that sum. The items 1., 2. and the absence of experimental facts contradicting GRT seemed to restrain the research on these theories already in the twenties. Only in 1966 a renewed interest in these theories arose in connection with a semiclassical description of quantum gravity [53-55]. The coefficient of the quadratic term became calculable by a renormalization procedure, thus solving problem 1, at least concerning the vacuum equation. Further, the fact that fourth order gravity is one-loop renormalizable in contrast to GRT; a fact which was realized in 1977, [4] initiated a boom of research. It is interesting to observe that it is just the scale invariance of the curvature squared terms – the original motivation – which is the reason for the renormalizability. Also the latest fundamental theory – the super-string theory – gives in the field theoretical limit (besides other terms) just a curvature-squared contribution to the action [56, 57]. The use of modern mathematics and computers has led to a lot of results to clarify the structure of the space of solutions thus solving problem 2.1. in the eighties. The more profound problem 2.2 has now three kinds of answers:

a) In spite of the higher order of the differential equation, a prescribed matter distribution plus the $O(1/r)$-behaviour of the gravitational potential suffice – such as it takes place in Newtonian theory – to determine the gravitational potential for isolated bodies in a unique way for the weak-field slow-motion limit, [52, 53]

b) the observation that the additional degrees of freedom are just the phases of damped oscillations which become undetectably small during the cosmic evolution, and, by the way, can solve the missing mass problem and prevent the singularity problem of GRT [58], and
c) it is supposed that there exist massive gravitons besides the usual massless gravitons known from GRT, but they are very weakly coupled [59].

The last point to be mentioned is the experimental testability: In the recent three years many efforts have been made to increase the accuracy in determining the constants $G$, $\alpha$ and $l$ if the gravitational potential is assumed (also in other theories than fourth order gravity) to be

$$Gmr^{-1}(1 + \alpha e^{-r/l}).$$

The term proportional to $\alpha$, the “fifth force”, can be interpreted as the fourth order correction to GRT. Up to now, it has not been possible to exclude $\alpha = 0$ by experiments [60-62].

Finally, let us say: Fourth order gravity theories will remain an essential link between GRT and quantum gravity for a long time.

References

[1] Lanczos, K.: A remarkable property of the Riemann - Christoffel tensor in four dimensions. Annals of Math. 39 (1938) 842-850.

[2] Weyl, H.: Gravitation und Elektrizität. Sitzungsber. Preuss. Akad. d. Wiss. Teil 1 (1918) 465-480.

[3] Utiyama, R., B. de Witt: Renormalization of a classical gravitational field interacting with quantized matter fields, J. Math. Phys. 3 (1962) 608.

[4] Stelle, K.: Renormalization of higher-derivative quantum gravity. Phys. Rev. D 16 (1977) 953-969.

[5] Weyl, H.: Reine Infinitesimalgeometrie. Mathemat. Zeitschr. 2 (1918) 384-411.

[6] Weyl, H.: Eine neue Erweiterung der Relativitätstheorie. Ann. d. Phys. Leipz. (4) 59 (1919) 101-133.

[7] Weyl, H.: Elektrizität und Gravitation. Physik. Zeitschr. 21 (1920) 649-650.

[8] Weyl, H.: Über die physikalischen Grundlagen der erweiterten Relativitätstheorie. Physik. Zeitschr. 22 (1921) 473-480.

[9] Weyl, H.: Electricity and Gravitation. Nature 106 (1921) 800-802.

[10] Weyl, H.: Raum, Zeit, Materie. 5. Auflage, Berlin: Springer-Verl. 1923.
[11] Schrödinger, E.: Über eine bemerkenswerte Eigenschaft der Quantenbahnen eines einzelnen Elektrons. Zeitschr. f. Physik 12 (1923) 13-23.

[12] Weyl, H.: Elektron und Gravitation I. Zeitschr. f. Physik 56 (1929) 330-352.

[13] London, F.: Quantenmechanische Deutung der Theorie von Weyl, Zeitschr. f. Physik 42 (1927) 375-389.

[14] Vizgin, V. P.: Einstein, Hilbert, Weyl: Genesis des Programms der einheitlichen geometrischen Feldtheorien. NTM-Schriftenr. Leipzig 21 (1984) 23-33.

[15] Einstein, A.: Nachtrag zu [2]; P. 478.

[16] Dirac, P. A. M.: Long range forces and broken symmetries. Proc. R. Soc. Lond. A 333 (1973) 403-418.

[17] Reichenbächer, E.: Die Eichinvarianz des Wirkungsintegrals und die Gestalt der Feldgleichungen in der Weylschen Theorie. Z. f. Physik 22 (1924) 157-169.

[18] Weitzenböck, R.: Über die Wirkungsfunktion in der Weylschen Physik 1, 2, 3. Sitzungsber. Akad. d. Wiss. Wien, Math.-naturwiss. Kl. Abt. Iia, 129 (1920) 683-696; 697-703; 130 (1921) 15-23.

[19] Pauli, W.: Zur Theorie der Gravitation und der Elektrizität von Hermann Weyl. Physik. Zeitschr. 20 (1919) 457-467.

[20] Pauli, W.: Merkurperihelbewegung und Strahlenablenkung in Weyls Gravitationstheorie. Berichte d. Deutschen Phys. Ges. 21 (1919) 742-750.

[21] Jüttner, F.: Beiträge zur Theorie der Materie. Math. Annalen 87 (1922) 270-306.

[22] Pauli, W.: Relativitätstheorie, Enc. math. Wiss. Bd. 5, Teil 2, S. 543-775. Leipzig: Teubner Verl. 1922.

[23] Bach, R.: Zur Weylschen Relativitätstheorie und der Weylschen Erweiterung des Krümmungsbegriffs. Math. Zeitschr. 9 (1921) 110-135.

[24] Einstein, A.: Eine naheliegende Ergänzung des Fundamentes der allgemeinen Relativitätstheorie. Sitzungsbericht. Preuss. Akad. d. Wiss. Teil 1, (1921) 261-264.

[25] Reichenbächer, E.: Eine neue Erklärung des Elektromagnetismus, Z. f. Physik 13 (1923) 221-240.

[26] Kakinuma, U.: On the structure of an electron, Part I, II. Proc.
[27] Lanczos, C.: Elektromagnetismus als natürliche Eigenschaft der Riemannschen Geometrie. Zeitschr. f. Physik 73 (1932) 147-168.

[28] Lanczos, C.: Zum Auftreten des Vektorpotentials in der Riemannschen Geometrie. Zeitschr. f. Physik 75 (1932) 63-77.

[29] Lanczos, C.: Electricity as a natural property of Riemannian geometry. Phys. Rev. 39 (1932) 716-736.

[30] Lanczos, C.: Ein neuer Aufbau der Weltgeometrie. Zeitschr. f. Physik 96 (1935) 76-106.

[31] Lanczos, C.: Matter waves and Electricity. Phys. Rev. 61 (1942) 713-720.

[32] Lanczos, C.: Lagrangian multipliers and Riemannian spaces. Rev. Mod. Phys. 21 (1949) 497-502.

[33] Lanczos, C.: Electricity and General Relativity. Rev. Mod. Phys. 29 (1957) 337-350.

[34] Lanczos, C.: Quadratic action principle of Relativity. J. Math. Phys. 10 (1969) 1057-1065.

[35] Vizgin V. P.: Hermann Weyl, die Göttinger Tradition der mathematischen Physik und einheitliche Feldtheorien. Wiss. Zeitschr. d. E.-M.-Arndt-Univ. Greifswald, Math.-naturwiss. Reihe 33 (1984) 57-60.

[36] Bergmann, P. G.: Introduction to the theory of relativity. New York: Prentice Hall 1942.

[37] Buchdahl, H.: On functionally constant invariants of the Riemann tensor. Proc. Cambr. Philos. Soc. 68 (1970) 179-185.

[38] Buchdahl, H.: Über die Variationsableitung von Fundamentalinvarianten beliebig hoher Ordnung. Acta Mathematica 85 (1951) 63-72.

[39] Buchdahl, H.: On the Hamilton derivatives arising from a class of gauge-invariant action principles in a $W_n$. J. Lond. Math. Soc. 26 (1951) 139-149.

[40] Buchdahl, H.: An identity between the Hamiltonian derivatives of certain fundamental invariants in a $W_4$. J. Lond. Math. Soc. 26 (1951) 150-152.

[41] Buchdahl, H.: On the gravitational field equations arising from the square of the Gaussian curvature. Nuovo Cim. 23 (1962) 141-156.
[42] Buchdahl, H.: The Hamiltonian derivatives of a class of fundamental invariants. Quart. J. Math. Oxford 19 (1948) 150.

[43] Buchdahl, H.: A special class of solutions of the equations of the gravitational field arising from certain gauge-invariant action principles. Proc. Nat. Acad. Sci. USA 34 (1948) 66-68.

[44] Buchdahl, H.: Reciprocal static metrics and non-linear Lagrangians. Tensor 21 (1970) 340-344.

[45] Buchdahl, H.: Quadratic Lagrangians and static gravitational fields. Proc. Cambr. Philos. Soc. 74 (1973) 145-148.

[46] Buchdahl, H.: Non-linear lagrangians and cosmological theory. Monthly Not. R. Astron. Soc. 150 (1970) 1-8.

[47] Buchdahl, H.: The field equations generated by the square of the scalar curvature: solutions of the Kasner type. J. Phys. A 11 (1978) 871-876.

[48] Buchdahl, H.: On a set of conform-invariant equations of the gravitational field. Proc. Edinburgh Math. Soc. 10 (1953) 16-20.

[49] Buchdahl, H.: Remark on the equation $\delta R^2/\delta g^{ij} = 0$. Intern. J. Theor. Phys. 17 (1978) 149-151.

[50] Eddington, A. S.: Relativitätstheorie in mathematischer Behandlung. Berlin: Springer-Verl. 1925.

[51] Schrödinger, E.: Space-time structure. Cambridge: University Press 1950.

[52] Gregory, C.: Non-linear invariants and the problem of motion. Phys. Rev. 72 (1947) 72-75.

[53] Pechlaner, E., R. Sexl: On quadratic Lagrangians in General Relativity. Commun. Math. Phys. 2 (1966) 165-173.

[54] Sacharov, A. D.: Vakuumnye kvantovye fluktuacii v iskrivlennom prostranstve i teoria gravitacii. Dokl. Akad. Nauk SSSR 177 (1967) 70-71.

[55] Treder, H.-J.: Zur unitarisierten Gravitationstheorie mit lang- und kurzreichweitigen Termen. Ann. Phys. Leipz. 32 (1975) 383-400.

[56] Ivanov, B.: Cosmological solution with string correction. Phys. Lett. B 198 (1987) 438.

[57] Hochberg, D., T. Shimada: Ambiguity in determining the effective action for string-corrected Einstein gravity. Progr. Theor. Phys. 78 (1987)
680.

[58] Müller, V., H.-J. Schmidt: On Bianchi type I vacuum solutions in $R + R^2$ theories of gravitation. I. The isotropic case. Gen. Relat. Grav. 17 (1985) 769-781.

[59] Stelle, K.: Classical gravity with higher derivatives. Gen. Relativ. Grav. 9 (1978) 353-371.

[60] Mio, N.: Experimental test of the law of gravitation at small distances. Phys. Rev. D 36 (1987) 2321.

[61] Stacey, F., G. Tuck, G. Moore: Quantum Gravity: Observational constraints on a pair of Yukawa terms. Phys. Rev. D 36 (1987) 2374.

[62] Ander, M., M. Nieto: Possible resolution of the Brookhaven and Eötvös experiments. Phys. Rev. Lett. 60 (1988) 1225.

Acknowledgement: We thank Dr. sc. U. Kasper for critically reading the manuscript.

Anschriften der Verfasser: (Addresses of the authors in the year 1990:)

Dr. sc. mit. R. Schimming
Fachbereich Mathematik
Ernst-Moritz-Arndt-Universität
F.-L.-Jahn-Str. 15a
Greifswald
DDR - 2200

Dr. sc. nat. H.-J. Schmidt
Zentralinstitut für Astrophysik
Akademie der Wissenschaften der DDR
R.-Luxemburg-Str. 17a
Potsdam
DDR - 1591