Prediction of high-temperature flow stress of HMn64–8–5–1.5 manganese brass alloy based on modified Zerilli-Armstrong, Arrhenius and GWO-BPNN model

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Abstract

An accurate constitutive model is essential for designing the process of hot precision forging and numerical simulation. Based on the isothermal compression tests of as-extruded HMn64–8–5–15 manganese brass alloy at the deformation temperature of 873–1073 K and strain rate of 0.01–10 s⁻¹, the effect of the friction and deformation temperature rise on the flow stress during the hot compression process was analyzed, and the flow stress curves were corrected. Three constitutive models based on the modified Zerilli-Armstrong, Arrhenius, and a back-propagation neural network (BPNN) optimized by the grey wolf optimization (GWO) algorithm (GWO-BPNN) models were established to describe the high-temperature flow stress of this alloy. Meanwhile, the prediction ability of the three models was evaluated by the calculated values of mean absolute percentage error (MAPE) and root mean square error (RMSE). The values of MAPE for the modified Zerilli-Armstrong, Arrhenius, and GWO-BPNN models were computed to be, 3139 %, 2448 % and 1265 %, and the values of RMSE were calculated to be 1804, 1482 and 0467 MPa, respectively. The GWO-BPNN model was with the greatest prediction ability for the flow stress among these models. The GWO algorithm was introduced to optimize the initial weights and thresholds of the BPNN model, and it has good prediction accuracy and better stability. It can better describe the high-temperature flow behavior of HMn64–8–5–15 alloy.

1. Introduction

The high-strength and wear-resistant Cu-Zn-Mn brass alloys are widely used in the manufacture of automobile synchronizer gear rings due to their good thermoplasticity and excellent wear resistance [1, 2]. With the rapid development of the automobile industry, the gear ring material was required to have higher strength and wear resistance. In recent years, the properties of HMn64–8–5–15 manganese-brass alloy obtained by optimizing the content of each element in the alloy are close to the advanced foreign level. Therefore, the construction of an accurate constitutive model of HMn64–8–5–15 alloy is of great significance to the formulation of the hot precision forging process of the gear ring.

At present, the methods for constitutive model construction were divided into three categories: the phenomenological type, the physical type and the machine learning type [3]. The Arrhenius model was one of the most widely used phenomenological constitutive models. Furthermore, some researchers have proposed a variety of correction methods for the original Arrhenius model due to its poor prediction accuracy, such as strain compensation [4–7], multi-condition of deformation to find material parameters [8–10], optimal algorithm to optimize material parameters [11], etc. The Zerilli-Armstrong model was a fundamental physical constitutive...
model based on the theory of thermally activated dislocation motion. Because the original model ignored the influence of the effects between the deformation conditions on the hardening and softening terms, its prediction accuracy was poor, which lead to various correction methods of the Zerilli-Armstrong model\[12–14\]. In recent years, with the rapid development of artificial intelligence, researchers gradually focused on the simulation, extension and expansion of stress-strain curves through machine learning algorithms. For its great algorithm to solve the problem of nonlinear regression and excellent prediction accuracy of material rheological behavior, machine learning provided a new idea and method for the establishment of material constitutive models. The current machine learning models for flow stress prediction include artificial neural network\[15–19\], support vector machine regression\[20, 21\] and K-nearest neighbor regression\[22\], etc.

As a result, this material is essential for gear rings and very little research concentrates on it. Therefore, the extruded HMn64–8–5–15 manganese brass alloy was employed as the research object in this paper and the hot compression tests were conducted to explore and predict its flow stress. The modified Zerilli-Armstrong model, Arrhenius model and GWO-BPNN model were introduced to construct their constitutive models based on the compression tests. And these three models were compared with each other and the prediction accuracy was analyzed. Then the most accurate constitutive model could provide theoretical guidance for the subsequent hot precision forging process designing and numerical simulation.

2. Experimental and results

2.1. Experiment

The material used in this experiment was extruded HMn64–8–5–15 manganese brass, provided by Sichuan Luzhou Changjiang Machinery Co, Ltd. The chemical composition is shown in Table 1. The phase analysis of the alloy was carried out on a DX-2700B x-ray diffractometer. Figure 1 shows the XRD pattern of the alloy. The main phases are: CuZn (β phase) + Mn₅Si₃ (hard phase). To observe the microstructure of this material, a sample was ground and polished, then it was etched in a corrosive solution configured by 20 ml nitric acid + 30 ml of glacial acetic acid + 50 ml of anhydrous ethanol. Figure 2 shows the metallographic structure of the manganese brass alloy, in which the yellow matrix is β-phase and irregular polygonal distribution is the hard phase Mn₅Si₃.

The experimental material was processed into some cylindrical standard samples with a diameter of 8 mm and a height of 12 mm using a wire cutting machine, and the roughness of the upper and lower end surfaces of the samples was less than 16 μm. Then the isothermal and constant strain rate thermal compression experiments were carried out on a Gleeble 3500 thermal simulator with strain rates of 0.01, 0.1, 1, and 10 s⁻¹ and deformation

| Cu | Mn | Al | Si | Fe | Pb | Zn |
|----|----|----|----|----|----|----|
| 64 | 8  | 5  | 15 | 08 | 04 | Bal |

Figure 1. XRD pattern.

Table 1. The chemical composition of as-extruded HMn64–8–5–15 (mass fraction, %).

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temperatures ranging from 873 to 1073 K at intervals of 50 K. The total true strain of each sample was 0.9. To reduce the influence of friction on the experimental results, graphite was added as lubrication and the experiments should be in a unidirectional compression state as much as possible.

2.2. Results and friction correction of flow stress
The friction of the compression tests played an important role in the measured flow stress. With the gradual increase of the friction between the end face of the sample and the flat anvil mold, the ‘bulging’ of the sample occurred after the compression test, as shown in figure 3. The appearance of the drum-shaped phenomenon was inevitable. At the same time, the data value collected by the thermal compression experiment was higher than the theoretical value. Therefore, it is necessary to evaluate the influence caused by the ‘bulging belly’ to decide whether friction correction is required.

At present, Roebuck [23] proposed a criterion to evaluate the influence of the friction through a barreling coefficient, which is expressed as follows:

$$B = \frac{H_r R_m^2}{H_o R_0^2}$$

Where $B$ is the barreling coefficient, $H_r$ and $R_m$ are the height and the maximum radius of the deformed sample; $R_0$ and $H_0$ are the initial radius and the height of the sample, respectively.

When $B < 11$, the influence on the measured data was slight and the friction correction was not needed. When $B > 11$, the measuring flow stress must be corrected. By calculating the $B$ value of these 20 group compression tests, the results were displayed in table 2. It could be concluded that all the compression tests
should be corrected because the barreling coefficient $B > 11$. Therefore, the flow stress was corrected according to the widely used method proposed by Gholamzadeh [24]:

$$\sigma = \frac{\sigma_0}{1 + \left(\frac{2}{3} \sqrt{3}\right) m(R_0/H_0) e^{3b/2}}$$

Where $\sigma_0$ and $\sigma$ are the flow stresses before and after the friction correction, respectively; $\varepsilon$ is the true strain; $m$ is a friction coefficient calculated by formulas below:

$$m = \frac{R_i b}{3\sqrt{3}}$$  \hspace{1cm} (3)

$$b = 4R_m - R_i \frac{H_i}{R_1 - H_0 + H_i}$$  \hspace{1cm} (4)

$$R_i = \sqrt{\frac{H_0 R_m^2 - 2R_m^2}{H_i}}$$  \hspace{1cm} (5)

Where $b$ is the drum parameter; $R_i$ is the average radius of the corresponding deformed sample, defined as $R_i = R_0 \sqrt{H_0/H_i}$; $R_1$ is the top end surface radius of the deformed sample.

Not only the friction plays an important in the compression tests, but also the deformation temperature affects the flow stress. The temperature distribution of the sample is not uniform during the hot compression process, the deformation of the core area of the sample is larger than that of the other area, and the heat caused by the deformation will lead to a higher temperature of the core of the sample than the set temperature.

Simultaneously, it is considered that the temperature collection of the compression experiment is obtained by the thermocouple welded on the outer contour surface, the collected temperature is the lower surface layer, and the temperature difference between the core and the surface of the sample makes the experimental temperature deviate from the set temperature and directly affects the experimental flow stress. Therefore, the temperature correction was conducted in this paper according to the method proposed by Laasraoui et al [25].

The temperature rise caused by deformation heat was calculated by the formula (6):

$$\Delta T = \frac{\eta \beta}{\rho c} \int_0^\varepsilon \sigma_0 d\varepsilon$$

Where $\rho$ and $c$ are the density of the sample (g cm$^{-3}$) and the specific heat (J g$^{-1}$ K$^{-1}$), respectively, and $\rho = 850$ g cm$^{-3}$, $c = 0.3777$ J g$^{-1}$ K$^{-1}$ according to reference [26]; $\eta$ is the thermal conversion rate, and the value is 0.95 generally in a thermal compression experiment; $\beta$ is the adiabatic factor According to the work of Mataya et al [27], $\beta$ was related to the strain rate as follows:

$$\beta = \begin{cases} 
0 & \varepsilon \leq 0.001 \text{s}^{-1} \\
0.316 \log_{10} \varepsilon + 0.95 & 0.001 \text{s}^{-1} < \varepsilon < 1 \text{s}^{-1} \\
0.95 & \varepsilon \geq 1 \text{s}^{-1}
\end{cases}$$  \hspace{1cm} (7)

Therefore, the flow stress change $\Delta \sigma$ caused by the temperature rise $\Delta T$ is calculated by the formula (8):

$$\Delta \sigma = \left( \frac{\partial \sigma}{\partial (1/T_n)} \right)_{\varepsilon, h} \left( \frac{1}{T_n + \Delta T} - \frac{1}{T_n} \right)$$  \hspace{1cm} (8)

Where $T_n$ is the deformation temperature (K) set in the compression experiments.

Figure 4 shows the stress-strain curves of HMn64--8--5--15 alloy before and after the friction and temperature correction above. The comparison shows that the changing trend of the rheological curves before and after the correction is similar to the original experimental curves. The influence of friction on the flow stress is more significant than that of the temperature rise effect, and with the increase of the strain, the influence of friction

| Strain rates /s$^{-1}$ | Deformation temperature /K |
|------------------------|---------------------------|
|                        | 873  | 923  | 973  | 1023 | 1073 |
| 0.01                   | 1.128| 1.115| 1.106| 1.147| 1.142|
| 0.01                   | 1.111| 1.168| 1.111| 1.133| 1.133|
| 1.00                   | 1.133| 1.157| 1.101| 1.144| 1.132|
| 10.00                  | 1.217| 1.234| 1.159| 1.147| 1.133|
and temperature rise effect on the flow stress becomes more and more significant and it is shown that the difference value between the corrected value and the experimental value gradually increases.

3. Development of constitutive model

In order to evaluate the accuracy and predictive ability of the constructed models before and after improvement, the correlation coefficient (R), the mean absolute percentage error (MAPE) and the root mean square error (RMSE) are introduced and expressed as shown in equations (9)–(11). The correlation coefficient (R) ranging from −1 to 1 expresses the degree of linear correlation between the two variables. The value of R close to 1, 0 and −1 represents a positive correlation, no relationship and negative correlation between the predicted value and the experimental value, respectively. The mean absolute percentage error (MAPE) can accurately estimate the prediction ability of models. The root mean square error (RMSE) evaluates the deviations from the experimental value and the predicted value. The lower mean absolute percentage error (MAPE) and the root mean square error (RMSE) indicate good prediction performance of the models.

\[
R = \frac{\sum_{i=1}^{N} (E_i - \bar{E})(P_i - \bar{P})}{\sqrt{\sum_{i=1}^{N} (E_i - \bar{E})^2 \sum_{i=1}^{N} (P_i - \bar{P})^2}} \tag{9}
\]

\[
MAPE = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{E_i - P_i}{E_i} \right| \times 100\% \tag{10}
\]

\[
RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (E_i - P_i)^2} \tag{11}
\]

Where \(E_i\) is the real experimental value; \(P_i\) is the predicted value from the models; \(\bar{E}\) and \(\bar{P}\) are the mean value of the real experimental value and the predicted value, respectively; \(N\) is the number of samples.

3.1. Modified Zerilli-Armstrong model

At present, the Zerilli-Armstrong model proposed by Samantaray D [28] is widely used as a correction method, but the prediction accuracy is not very well. So Jin Chaoyang et al [29] proposed a modified Zerilli-Armstrong model (M-ZA) based on the strain compensation. The model is expressed as follows:

![Corrected and uncorrected flow true strain-stress curves of HMn64–8–5–1.5 alloy at the strain rate of: (a) \(\dot{\varepsilon} = 0.01\ s^{-1}\), (b) \(\dot{\varepsilon} = 0.1\ s^{-1}\), (c) \(\dot{\varepsilon} = 1\ s^{-1}\), (d) \(\dot{\varepsilon} = 10\ s^{-1}\).](image-url)
Where $\dot{\varepsilon}^* = \dot{\varepsilon}/\dot{\varepsilon}_{\text{ref}}$ is the dimensionless strain rate; $\dot{\varepsilon}_{\text{ref}}$ is the reference strain rate, s$^{-1}$; $T^*$ = $T - T_{\text{ref}}$; $T_{\text{ref}}$ is the reference temperature, K; $I_1$, $S_1$, $D_5$ and $D_6$ are material parameters.

The solution process of the M-ZA model is shown in figure 5. A reference condition at the temperature of 873 K and strain rate of 0.01 s$^{-1}$ was selected to calculate the material constant in the M-ZA model. When the strain rate was equal to the reference strain rate, equation (12) was transformed into equation (13). Under different strains, $I_1$ and $S_1$ can be determined by linear fitting, this also identifies that the value of $I_1$ and $S_1$ needs to consider the influence of the strain. Similarly, under different strains and deformation temperatures, made $S_2$ equal to $(D_5 + D_6 T^*)$, then $S_2$ can be determined by linear fitting. Under different strains, $D_5$ and $D_6$ can be determined by $S_2 - T^*$ linear fitting. $D_5$ and $D_6$ also changed with the changing strain. According to the solution process above, the material parameter values in the M-ZA model were all obtained.

$$\ln \sigma = I_1 + S_1 T^*$$

However, whether it is the original Zerilli-Armstrong model or the modified Zerilli-Armstrong model proposed by Samantaray D and Jin Chaoyang, a reference deformation condition was needed specifically. It also means that the model was solved at a certain reference deformation temperature and strain rate, and the reference condition was not considered as a reason for the effect of prediction accuracy of the predicted model. Therefore, some different reference conditions were applied to calculate the model constants, and the prediction accuracy was shown in table 3. It can be known from the table that the prediction accuracy of the ZA model modified by Samantaray D is lower than that of the M-ZA model modified by Jin Chaoyang, and the prediction accuracy of the two models are both affected by the reference deformation condition. At the same time, the selection of the strain rate for the reference condition has a more significant impact on the prediction accuracy of the predicted model compared with the deformation temperature. To sum up, 873 K & 001 s$^{-1}$ was selected as the reference condition, and the M-ZA model of HMn64–8–5–15 alloy was constructed. The formula for M-ZA is shown below:
Table 3. Prediction accuracy of M-ZA model under different reference conditions.

| Type      | Deformation temperature /K | Strain rates /s⁻¹ | 001 | 01 | 10 |
|-----------|-----------------------------|-------------------|-----|---|----|
|           | R   | RMSE /MPa | MAPE /% | R   | RMSE /MPa | MAPE /% | R   | RMSE /MPa | MAPE /% | R   | RMSE /MPa | MAPE /% |
| Samantaray D | 873 | 0.987    | 4.692 | 5.691 | 0.991 | 3.683 | 5.246 | 0.992 | 3.760 | 5.983 | 0.993 | 3.349 | 6.902 |
|           | 923 | 0.988    | 4.639 | 5.605 | 0.991 | 3.595 | 5.036 | 0.993 | 3.589 | 5.832 | 0.993 | 3.160 | 6.720 |
|           | 973 | 0.988    | 4.589 | 5.527 | 0.991 | 3.528 | 4.879 | 0.993 | 3.456 | 5.748 | 0.994 | 3.004 | 6.574 |
|           | 1023 | 0.988   | 4.542 | 5.465 | 0.992 | 3.481 | 4.782 | 0.994 | 3.364 | 5.741 | 0.994 | 2.887 | 6.498 |
|           | 1073 | 0.989   | 4.499 | 5.420 | 0.992 | 3.456 | 4.730 | 0.994 | 3.315 | 5.777 | 0.995 | 2.812 | 6.458 |
| JIN       | 873 | 0.998    | 1.804 | 3.139 | 0.998 | 2.128 | 3.321 | 0.996 | 2.701 | 4.877 | 0.997 | 2.016 | 3.581 |
|           | 923 | 0.998    | 1.804 | 3.139 | 0.998 | 2.128 | 3.321 | 0.996 | 2.701 | 4.877 | 0.997 | 2.016 | 3.581 |
|           | 973 | 0.998    | 1.804 | 3.139 | 0.998 | 2.128 | 3.321 | 0.996 | 2.701 | 4.877 | 0.997 | 2.016 | 3.581 |
|           | 1023 | 0.998   | 1.804 | 3.139 | 0.998 | 2.128 | 3.321 | 0.996 | 2.701 | 4.877 | 0.997 | 2.016 | 3.581 |
|           | 1073 | 0.998   | 1.804 | 3.139 | 0.998 | 2.128 | 3.321 | 0.996 | 2.701 | 4.877 | 0.997 | 2.016 | 3.581 |
\[
\sigma = \exp \left[ I_1 + S_1 T^* + (D_5 + D_6 T^*) \ln \dot{\varepsilon} \right]
\]
\[
I_1 = \ln \left( 19.659 - 3.94e + 147.79{e}^2 - 800.29{e}^3 
+ 1.661 \times 10^3{e}^4 - 1.536 \times 10^5{e}^5 + 537.002{e}^6 \right)
\]
\[
S_1 = \begin{cases} 
-0.005 & -0.0018\varepsilon - 0.0019\varepsilon^2 + 0.0349\varepsilon^3 - 0.0585\varepsilon^4 \\
0.0221\varepsilon^5 + 0.0065\varepsilon^6 & 
\end{cases}
\]
\[
D_5 = 0.2844 + 0.003\varepsilon - 1.7809\varepsilon^2 + 8.9893\varepsilon^3 - 18.2884\varepsilon^4 
+ 16.7791\varepsilon^5 - 5.846\varepsilon^6 
\]
\[
D_6 = -2.472 \times 10^{-8} + 0.0029\varepsilon - 0.0182\varepsilon^2 + 0.0612\varepsilon^3 - 0.1173\varepsilon^4 
+ 0.1168\varepsilon^5 - 0.0455\varepsilon^6 
\]

(14)

3.2. Modified Arrhenius model

A modified Arrhenius (M-Arrhenius) model proposed by Mohamadizadeh A [8] with consideration of deformation conditions is defined as follows:

\[
\dot{\varepsilon} = A(\varepsilon, \dot{\varepsilon}, T) \left[ \sinh \left( \alpha(\varepsilon, T) \sigma \right) \right] \exp \left( -\frac{Q(\varepsilon, \dot{\varepsilon}, T)}{RT} \right) 
\]

Where \( Q \) is the deformation activation energy (kJ·mol\(^{-1}\)), which is related to the material; \( R \) is the molar gas constant (8.314 J·mol\(^{-1}\)·K\(^{-1}\)); \( n \) is the stress index; \( A \) and \( \alpha \) are parameters related to the material.

The construction diagram of the M-Arrhenius model is shown in figure 6. Firstly, at 5 deformation temperatures, 17 sets of stress-strain data with strains between 0.05 and 0.85 at an interval of 0.05 were collected. Then, the \( \beta \) and \( n_1 \) values can be determined by the \( \ln \dot{\varepsilon} - \sigma \) and \( \ln \dot{\varepsilon} - \ln \sigma \) linear fitting curves under each deformation temperature and strain, respectively. The constant \( \alpha \) was defined as \( \alpha = \beta n_1 \). The values of \( \alpha \) under different strains and deformation temperatures were obtained based on the values of \( \beta \) and \( n_1 \). And the \( \alpha (\varepsilon, T) \) considered the deformation temperature and strain was determined by nonlinear fitting. According to the data calculated above, the values of \( n \) under different deformation temperatures and strains were obtained by \( \ln \dot{\varepsilon} - \ln [\sinh (\alpha(\varepsilon, T) \sigma)] \) linear fitting. Consequently, \( n(\varepsilon, T) \) which considered the deformation temperature and strain was also determined by nonlinear fitting. Thirdly, at 4 strain rates, the values of \( S \) under different strain rates and strains were determined by \( \ln [\sinh (\alpha(\varepsilon, T) \sigma)] - 1/T \) linear fitting, and \( S(\varepsilon, \dot{\varepsilon}) \) considered the strain rate and strain was obtained by nonlinear fitting. Fourth, the value of \( Q \) is defined as \( Q(\varepsilon, \dot{\varepsilon}, T) = Rn(\varepsilon, T)S(\varepsilon, \dot{\varepsilon}) \), and the \( Q \) was then calculated. Finally, the intercept \( F \) was obtained through \( \ln \dot{\varepsilon} - \ln [\sinh (\alpha(\varepsilon, T) \sigma)] \) linear regression at different strains and deformation temperatures. And \( F(\varepsilon, T) \) considered the deformation temperature and the strain effect was obtained by nonlinear fitting. Simultaneously, the \( \ln A(\varepsilon, \dot{\varepsilon}, T) = F(\varepsilon, T) + \frac{m(\varepsilon, T)S(\varepsilon, \dot{\varepsilon})}{T} \) was determined indirectly. According to the calculation above, all the material parameter values of the M-Arrhenius model were determined.

The prediction accuracy is shown in table 4 which the modified Arrhenius model (L-Arrhenius) of LIN [4] and the M-Arrhenius model with consideration of deformation conditions were compared. The prediction accuracy of the M-Arrhenius model is significantly better than that of the L-Arrhenius model, which also means the modified model was remarkably improved based on the original model. The formula of the M-Arrhenius model for H15 alloy is shown in equation (16).

\[
\dot{\varepsilon} = A(\varepsilon, \dot{\varepsilon}, T) \left[ \sinh \left( \alpha(\varepsilon, T) \sigma \right) \right] \exp \left( -\frac{Q(\varepsilon, \dot{\varepsilon}, T)}{RT} \right) 
\]

\[
A(\varepsilon, T) = -63.767 - 0.288\varepsilon + 0.203T^2 - 2.129 \times 10^{-4}T^2 + 7.331 \times T^3 
+ 1 - 11.312N + 33.712\varepsilon^2 - 28.259\varepsilon^3 - 0.0893\varepsilon^4 + 6.283 \times 10^{-3}\varepsilon^2 
- 2888.654 + 82.307\varepsilon + 9.369T - 0.01T^2 + 3.445 \times 10^{-6}T^3 
+ 1 + 43.379\varepsilon - 21.314\varepsilon^2 + 10.749\varepsilon^3 - 0.0328T - 8.066 \times 10^{-5}T^2 
+ 6.211 - 3.917\varepsilon - 0.355\varepsilon^2 + 0.244\varepsilon^3 - 0.028\varepsilon^4 
+ 1 - 0.21\varepsilon - 0.794\varepsilon^2 + 0.661\varepsilon^3 - 0.118\varepsilon^4 + 0.076\varepsilon^5 
+ \frac{900.916 - 5.818\varepsilon - 2.802T - 0.00283T^2 - 9.075 \times 10^{-7}T^3}{1 - 0.97\varepsilon + 0.535\varepsilon^2 - 0.232\varepsilon^3 - 0.00634T + 6.817 \times 10^{-6}T^2} 
\]

(16)

3.3. GWO-BPNN model

Error Back Propagation Neural Network (BPNN) is a method to realize machine learning and it is also one of the most widely used neural network models [17]. At present, there are three layers (input layer, hidden layer and
output layer) of the BPNN model used for material rheological behavior prediction. The three parameters of the model which are the number of neural nodes in the hidden layer, the initial weights and thresholds of the neural network affect the prediction accuracy. Therefore, the grey wolf optimization (GWO) algorithm was introduced to optimize the initial weights and thresholds of the BPNN model based on the original BPNN model.

The diagram of the GWO-BPNN model construction is displayed in figure 7.

**Figure 6.** Diagram of the construction procedure for M-arrhenius model.

**Table 4.** Comparison of prediction accuracy between L-Arrhenius and M-Arrhenius models.

| Type       | R  | RMSE (MPa) | MAPE (%) |
|------------|----|------------|----------|
| L-Arrhenius| 0.978 | 6253       | 12112    |
| M-Arrhenius| 0.998 | 1482       | 2448     |

Step 1 – Data processing. The input variables were the deformation conditions of the isothermal compression experiments at the deformation temperature ranging from 873 K to 1073 K with an interval of 50 K, the strain rate of 0.01 – 10 s⁻¹, and strain from 0.05 to 0.85 with an interval of 0.05. The responding output was true stress. There were a total of 340 groups of data. Then 324 groups were selected randomly as the training groups and the other 16 groups were selected as the test groups. At the same time, in order to eliminate the influence of the dimension between different indicators on the model results, the data must be normalized. The data were normalized to [−1, 1] by equation (17).
where $x_i$ is the normalized data; $x'_i$ is the $i$th original data; $x_{\text{max}}$ and $x_{\text{min}}$ are the maximum and minimum values of the corresponding input and output data, respectively.

After completing the data processing, a 3-layer BPNN network structure was selected. There were 3 nodes in the input layer (deformation temperature, strain rate and strain), 1 node in the output layer (stress), and a hidden layer. It must be noticed that insufficient neural nodes of the hidden layer may lead to underfitting and redundant neural nodes may cause overfitting. Therefore, the number of hidden layer neural nodes was set to be 3–15 to train the neural network in this paper. Then the mean squared error (MSE) between the predicted value of the training group and the actual value was introduced as the filter index (equation (18)). The test results showed that the model prediction accuracy was the best when there were 12 neural nodes of the hidden layer. The other settings of this model were the tanh and the purlin for the activation functions of the hidden layer and the output layer, respectively. The training algorithm was the trainlm The maximum number of iterations was equal to 500 The learning rate was equal to 0.1 and the target error was equal to $1 \times 10^{-5}$.

Where $\tilde{y}_i$ is the predicted output of the neural network; $y_i$ is the expected output value of the neural network; num is the number of the samples in the training group.

Step 2 – Optimizing the initial weights and thresholds of the BPNN model using the GWO algorithm. The population of the GWO algorithm was initialized at first. The number of wolves was set to be 10, the maximum number of iterations was 500, the dimension was $61 \times 12 + 12 \times 1 + 12 + 1$, and the boundary was also determined. Secondly, the BPNN model was trained with the initial weights and thresholds contained in each grey wolf within the boundary range. Then the fitness function was constructed according to the mean square error of the predicted value of the test group and the actual value. By calculating the fitness value, 3 grey wolf and their positions were determined. At the same time, the number of iterations was evaluated to know whether the maximum number of iterations has been reached during this calculation process. If the maximum number of iterations was not reached, updated the parameters in the GWO algorithm, and recalculate the fitness value of each grey wolf. Until the maximum number of iterations was achieved, then output the initial weights and thresholds of the optimal grey wolf.
Step 3 - Build a BPNN model based on the optimal initial weights and thresholds obtained by the GWO algorithm. Train the neural network using the training groups. When the error met the requirements, the trained BPNN model was obtained. Then input the data from the test group into the trained BPNN model, and the stress values predicted by the model were obtained.

A representative group of the BPNN model was selected for comparison after 10 independent runs due to the randomness of the BPNN model. Figure 8(a) is the iterative curve of the initial weights and thresholds of the BPNN model optimized by the GWO algorithm. The fitness value was stable at 0.00989 after 302 iterations. Therefore, the optimal solution of the fitness value was obtained, which also meant that the optimal initial weights and thresholds of the BPNN model were both determined. The BPNN model was trained using the GWO algorithm optimization results, and the prediction accuracy of the model was estimated with the test group. Figure 8(b) showed the comparison of the prediction accuracy of these two models. Figure 8(c) showed the comparison results of the prediction value errors of these two models to distinctly explain the prediction difference between the two models. The prediction value error of the GWO-BPNN model was better than the BPNN model. Figure 8(d) showed the prediction results of the model running 10 times independently. The prediction fluctuation of the GWO-BPNN model was significantly smaller than the BPNN model, which indicated that the reliability and stability of the GWO-BPNN model were better than those of the BPNN model.

3.4. Comparison of three constitutive models

The prediction accuracy of the M-ZA, M-Arrhenius and GWO-BPNN models constructed in Sections 3.1–3.3 was compared. As shown in figure 9, the three models are all fluctuated and distributed on a 45° straight line, and the RMSE and MAPE of the GWO-BPNN model are 0.467 MPa and 12.65 %, respectively. The prediction accuracy of GWO-BPNN is better than the M-ZA and M-Arrhenius models. Figure 10 shows the comparison of prediction accuracy of the three models under different deformation conditions. The prediction accuracy of these three models is great at the strain ranging from 0.2 to 0.6, but does not perform well at the small strain and large strain range. However, the GWO-BPNN model under all the corresponding strains was still better than the other two models. At the same time, it can be known that the prediction accuracy of the GWO-BPNN model is less affected by the deformation temperature and strain rate compared with the other two models, and it shows higher prediction accuracy.

The three constructed constitutive models were compared and analyzed. The prediction accuracy of the M-ZA model constructed by the correction method proposed by Jin Chaoyang was improved significantly.
compared with the original and the commonly used Samantaray D correction model. But the number of material parameters to be determined was increased to 28, and this method was also influenced by the selection of the reference deformation condition. The prediction accuracy of the M-Arrhenius model is better than the M-ZA model, but the consideration of deformation conditions in calculating the material parameters of the M-Arrhenius model increases the complexity of the calculation and uncertainty of the parameters. Especially, the calculation method of nonlinear surfaces to fit the relationship between material parameters and different deformation conditions is difficult for complex data, and the fitting effect also directly affects the prediction accuracy of the model. For the GWO-BPNN model proposed in this paper, the prediction accuracy of the model was improved by finding the optimal number of neural nodes in the hidden layer. At the same time, the GWO optimization algorithm was introduced to optimize the initial weights and thresholds to avoid the problem of
poor learning ability and prediction ability caused by parameter setting dependence of the BPNN model. Therefore, the GWO-BPNN model has better prediction accuracy and stability, and can better describe the high-temperature rheological behavior of HMn64–8–5–15 alloy.

4. Conclusion

The compression tests of HMn64–8–5–15 magnesium brass alloy were conducted on an isothermal simulator and the true stress-strain curves were obtained. Based on the experimental data, three constitutive models were constructed to describe the flow behavior. Some conclusions are as follows:

(1) The flow stress of HMn64–8–5–15 manganese brass alloy was corrected based on the friction theory and temperature rise theory caused by deformation. The correction results show that the changing trend of the flow stress-strain curves before and after the correction is similar, and the effect of friction on flow stress is more significant than that of temperature rise.

(2) The M-ZA constitutive model, M-Arrhenius constitutive model and GWO-BPNN constitutive model of HMn64–8–5–15 alloy were constructed. By comparing the prediction accuracy of the three models, the \( RMSE \) and \( MAPE \) of the GWO-BPNN model were 0.467 MPa and 12.65\%, respectively. Its prediction accuracy is better than that of the M-ZA and M-Arrhenius models.

(3) The three models were compared and analyzed: the M-ZA model has many material parameters to be determined, and the prediction accuracy is affected by the selection of reference deformation conditions. The consideration of deformation conditions in calculating the material parameters of the M-Arrhenius model increases the complexity of the calculation and uncertainty of the parameters, and the fitting effect also directly affects the prediction accuracy of the model. The GWO-BPNN model introduced the GWO algorithm to optimize the initial weights and thresholds of BPNN, which results in better prediction accuracy and better stability. In the end, GWO-BPNN can better describe the high-temperature rheological behavior of HMn64–8–5–15 alloy.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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