Towards Massive, Ultra-Reliable, and Low-Latency Wireless: The Art of Sending Short Packets

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Abstract—Most of the recent advances in the design of high-speed wireless systems are based on information-theoretic principles that demonstrate how to efficiently transmit long data packets. However, the upcoming 5G wireless systems will need to support novel traffic types that use short packets. For example, short packets represent the most common form of traffic generated by sensors and other devices involved in machine-to-machine communications. Furthermore, there are emerging applications in which small packets are expected to carry critical information that should be received with low latency and ultra-high reliability.

In current systems involving long packets, the metadata (control information) is typically of negligible size compared to the actual information payload, and it is often transmitted using heuristic methods without affecting the overall system performance. When the packets are short, however, metadata may be of the same size as the payload, and the conventional methods to transmit it may be highly suboptimal.

In this article, we review recent advances in information theory that provide the theoretical principles governing the transmission of short packets. We then apply these principles to three exemplary scenarios (the two-way channel, the downlink broadcast channel, and the uplink random access channel), thereby illustrating how the tradeoff brought by short-packet transmission affects the design of metadata, and—more generally—of wireless protocols.

I. INTRODUCTION

The fifth generation (5G) of wireless systems is expected to turn wireless connectivity into a true commodity “...for anything that may benefit from being connected...” [1], ranging from tiny static sensors to vehicles and drones. Hence, the focus in 5G will not be only on higher data rates, but also on the introduction of new wireless modes, such as ultra-reliable communication (URC) and massive machine-to-machine communications (MM2M) [2]–[4]. URC refers to communication services where data packets are exchanged at moderately low throughput (e.g., 50 Mbit/s) but with stringent requirements in terms of reliability (e.g., 99.999%) and latency (e.g., 4 ms). Example of URC include reliable cloud connectivity, critical connections for industrial automation, and reliable wireless coordination among vehicles [4]. With MM2M one refers to the scenario where a massive number of devices (e.g., 10,000) needs to be supported within a given area. This is relevant for large-scale distributed cyber-physical systems (e.g., smart grid) or industrial control. Also in this case, the data packets are short (and often contain correlated measurements) and reliability must be high to cope with critical events.

The central challenge with these two new wireless modes, i.e., URC and MM2M, is the capability to support short-packet transmission. This requires a fundamentally different design approach than the one used in current high-data-rate systems, such as 4G LTE and WiFi. High data rate over those links is achieved by sending sufficiently large data packets. Hence, the transmission design can rely on efficient coding methods. In other words, for large data packets the law of large numbers does not deteriorate the efficiency of the overall transmission, see Fig. 1(a). However, the data sent by one out of a massive set of sensors, or the data sent among wirelessly coordinated vehicles is small in size and comparable to the metadata, see Fig. 1(b). This has two fundamental implications: (1) the overall data packet is short and asymptotic information theory is not applicable; (2) the size of the metadata is comparable to the size of the data and inefficient encoding of metadata significantly affects the overall efficiency of the transmission.

During the last few years, significant progress has been made within the information theory community to address the problem of transmitting short packets. Particularly for point-to-point scenarios, information theorists have gained some understanding of the theoretical principles governing short-packet transmission and possess metrics that allow them to assess their performance.

Fig. 1. Data (D) and metadata (M) in a packet; (a) long data packets used in current wireless systems; (b) short data packets needed to support novel 5G applications, such as URC and MM2M.
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that are needed for the correct functioning of the wireless proto-

cols. Such bits, which will be referred throughout as metadata—

in contrast to the actual data to be transmitted—including control information, such as packet initiation and termination, logical addresses, synchronization and security information, etc.

As illustrated in Fig. 2, a packet consists of $k$ payload bits, which are made up of $k_i$ information bits (information payload) and $k_o$ additional bits, containing metadata from the MAC layer and higher layers. The payload bits are typically encoded into a block of $n$ data symbol (complex numbers) to increase reliability in packet transmission. Finally, $n_o$ additional symbols are added to enable packet detection, efficient synchronization (in time and frequency), or estimation of channel state information (CSI), which is needed by the receiver to compensate for the distortion of the transmitted signal introduced by the wireless channel. With a slight abuse of notation, we shall refer to the additional $k_o$ bits and $n_o$ symbols as metadata.

The ratio $R = k_i/n$, i.e., the number of information bits per complex symbol (or, equivalently, the number of transmitted payload bits per second per unit bandwidth) represents the net transmission rate and is a measure of the spectral efficiency of a communication system. In some wireless standard (such as LTE) specific physical/logical channels are reserved to carry exclusively metadata (control channels). This effectively lowers further the net transmission rate $R$.

In most current wireless systems, we have that $k_i \gg k_o$ and that $n_e \gg n_o$, so the net transmission rate $R$ is roughly $k/n_e$. Consequently, the performance of such systems essentially depends on the efficiency of the channel encoder. In other words, an efficient transmission of the data payload suffices to provide high spectral efficiency. Furthermore, $k_i$ (and hence also $n_e$) is typically large. Consequently, information-theoretic metrics such as capacity [5] and capacity-versus-outage [6] are accurate, in spite of being defined for asymptotically large packet sizes. Specifically, the capacity $C$—defined as the largest rate $k/n_e$ for which the packet error probability can be made arbitrarily small

by choosing $n_e$ sufficiently large—is the relevant performance metric for channels that exhibit an ergodic behavior over the duration of each transmitted codeword, such as the additive white Gaussian noise (AWGN) channel and the block-fading channel. The capacity-versus-outage $C_\epsilon$ (also known as outage or $\epsilon$-capacity)—defined as the largest rate $k/n_e$ for which a packet error probability less than $\epsilon$ can be achieved by choosing $n_e$ sufficiently large—is more relevant for nonergodic channels, such as the quasi-static fading channel, i.e., a fading channel in which the fading coefficient stays constant over the packet duration [7, p. 2631], [8, Sec. 5.4.1]. Both quantities require that the codeword length $n_e$ (i.e., the packet size) and, hence, also the size of the data payload $k$ be large.

When the packets are short, the situation changes drastically. On the one hand, new information-theoretic performance metrics other than capacity or capacity-versus-outage are needed to capture the tension between reliability and throughput, as well as the cost incurred in exploiting time-frequency and spatial resources (PHY overhead). On the other hand, when the packets are short, the MAC overhead is significant and needs to be designed optimally, perhaps together with the data. We shall address the former issue in Section III and the latter issue in Section IV.

II. ANATOMY OF A PACKET

Practically, all of today’s wireless systems transmit data in packets. Each transmitted packet over the air carries not only the information bits intended to the receiver but also additional bits that are needed for the correct functioning of the wireless protocols. Such bits, which will be referred throughout as metadata—in contrast to the actual data to be transmitted—include control information, such as packet initiation and termination, logical addresses, synchronization and security information, etc.

Fig. 2. Structure of a packet.

In contrast, so far information theorists have mostly viewed the design of metadata as something outside their competence area. Consequently, the transmission of metadata has been largely left to heuristic approaches.

In this article, we present the theoretical principles that govern the transmission of short packets and present metrics that allow us to assess their performance. We then highlight the challenges that need to be addressed to optimally design URC and MM2M applications by means of three examples that illustrate how the tradeoffs brought by short-packet transmission affect protocol design.

III. RE Thinking PHY Performance Metrics

A. Backing off from the Infinite Blocklength Asymptotics

In this section, we account for the metadata symbols required for the estimation of CSI, but ignore other issues such as packet detection or synchronization. As is common in information theory, we view the blocks channel encoder and PHY overhead in Fig. 2 as one encoder block and consider the transmission of metadata symbols for channel estimation (such as pilot symbols) as a possible encoding strategy.

Let $R^*(n, \epsilon)$ be the largest rate $k/n$ for which there exists a coding scheme (i.e., an encoder/decoder pair and a set of length-$n$ codewords) whose packet error probability does not exceed $\epsilon$.

The capacity-versus-outage $C_\epsilon$ can be obtained from $R^*(n, \epsilon)$ via

$$C_\epsilon = \lim_{n \to \infty} R^*(n, \epsilon).$$

The Shannon capacity $C$ can be obtained from (1) by letting $\epsilon$ tend to 0:

$$C = \lim_{\epsilon \to 0} C_\epsilon.$$

While Shannon capacity or capacity-versus-outage are reasonable performance metrics for current wireless systems, where the packet size is typically large, assessing the performance of short-packet communications requires a more refined analysis of $R^*(n, \epsilon)$. Unfortunately, the exact value of $R^*(n, \epsilon)$ is unknown even for channel models such as the binary symmetric channel that are much simpler to analyze than the one encountered in wireless communications. Indeed, determining $R^*(n, \epsilon)$ is an NP-hard problem [9], and its complexity is conjectured to be doubly exponential in the packet length $n$.

Nevertheless, during the last few years, significant progress has been made within the information theory community to address the problem of quantifying $R^*(n, \epsilon)$, and, hence, solve
the long-standing problem of accounting for delay constraints in a satisfactory way. Building upon Dobrushin’s and Strassen’s previous asymptotic results, Polyanskiy, Poor, and Verdú [10] recently provided a unified approach to obtain tight bounds on $R^*(n, \epsilon)$. They showed that for various channels with positive Shannon capacity $C$, the maximal coding rate $R^*(n, \epsilon)$ can be expressed as

$$R^*(n, \epsilon) = C - \frac{V}{n} Q^{-1}(\epsilon) + O\left(\frac{\log n}{n}\right)$$

(3)

where $O(\log n/n)$ summarizes remainder terms of order $\log n/n$. Here, $Q^{-1}(\cdot)$ denotes the inverse of the Gaussian $Q$ function and $V$ is the so-called channel dispersion [10, Def. 1]. The approximation (3) implies that to sustain the desired error probability $\epsilon$ for a given packet size $n$, one incurs a penalty on the rate (compared to the channel capacity) that is proportional to $1/\sqrt{n}$.

**B. AWGN Channel**

For the case of a real AWGN channel with signal-to-noise ratio (SNR) $\rho$, the capacity and the channel dispersion are given by

$$C(\rho) = \frac{1}{2} \log(1 + \rho)$$

(4)

$$V(\rho) = \frac{\rho (2 + \rho)}{2 (1 + \rho)^2} (\log e)^2.$$  

(5)

It has been observed that a good approximation for $R^*(n, \epsilon)$ can be obtained by replacing the remainder terms on the RHS of (3) by $(\log n)/(2n)$ [10]. The resulting approximation, which is commonly referred to as normal approximation, is plotted in Fig. 3, together with nonasymptotic achievability and converse bounds on $R^*(n, \epsilon)$ (see [10] for details).

As shown in the figure, the achievability and converse bounds provide an accurate characterization of $R^*(n, \epsilon)$, which lies in the shaded region. According to the bounds, to operate at 70% of capacity with a packet error rate of $10^{-3}$, i.e., at 0.35 bits/channel use, it is sufficient to use codes whose blocklength is between 220 and 276 channel uses. For the parameters considered in the figure, the normal approximation is indistinguishable from the achievability bound. We also see that capacity is an inaccurate performance metric for packet sizes that are as short as the ones considered in the figure.

**C. Fading Channels**

We shall next discuss how to extend the results reported in Section III-B for the AWGN case to fading channels. Throughout, we shall focus on the block-fading model [12], depicted in Fig. 4, according to which the fading coefficient stays constant for $n_c$ channel uses and changes then independently. In general, $n_c$ can be interpreted as the number of “time-frequency slots” over which the channel does not change. Within the $k$th coherence interval, the input-output relation of the block-fading channel with $m_t$ transmit and $m_r$ receive antennas is given by

$$Y_k = X_k H_k + W_k.$$  

(6)

Here, $X_k \in \mathbb{C}^{m_t \times n}$ and $Y_k \in \mathbb{C}^{m_r \times n}$ are the transmitted and received matrices, respectively; $H_k \in \mathbb{C}^{m_t \times m_r}$ denotes the fading matrix; $W_k \in \mathbb{C}^{n \times m_r}$ denotes the additive noise, which is assumed to have independent and identically distributed (i.i.d.), zero-mean, unit-variance, complex Gaussian entries. For the sake of simplicity, we assume Rayleigh fading, i.e., we assume that the fading matrix $H_k$ has i.i.d., zero-mean, unit-variance, complex Gaussian entries. However, this assumption is not essential. In fact, most results presented in this paper were either originally derived for more general fading distributions or can be generalized with little effort. For convenience, we shall assume that each codeword spans $\ell$ coherence intervals, i.e., $n = \ell n_c$.

1) Capacity-versus-outage at finite blocklength: We shall first discuss the case where the channel remains constant over the packet duration, i.e., $\ell = 1$. In this case, the fading channel is said to be quasi static, to reflect that the fading matrix is random but stays constant during the packet transmission. It is well-known that a quasi-static fading channel can be in outage, in which case the packet error probability is bounded away from zero, even if the packet size tends to infinity [6], [7]. Consequently, for such channels the capacity is zero and a more relevant performance metric is the capacity-versus-outage $C_\epsilon$, defined as the largest rate for which the probability of an outage is less than or equal to $\epsilon$.

Capacity-versus-outage is often regarded as a performance metric for delay-constrained communication over slowly-
varying fading channels (see, e.g., [15]). In fact, the assumption that the fading matrix stays constant during the packet transmission seems plausible only if the packet size is small. Nevertheless, the definition of capacity-versus-outage requires that the blocklength tends to infinity. For example, for a single-antenna system, the outage probability as a function of the rate $R$ is given by [16], [15], [13]

$$P_{\text{out}}(R) = \mathbb{P} \left[ \log \left( 1 + |H|^2 \rho \right) < R \right]$$

and the capacity-versus-outage $C_\epsilon$ is the supremum of all rates $R$ satisfying $P_{\text{out}}(R) \leq \epsilon$. The rationale behind this result is that, for every realization of the fading coefficient $H = h$, the quasi-static fading channel can be viewed as an AWGN channel with channel gain $|h|^2$, for which communication with arbitrarily small packet error probability is feasible if, and only if, $R < \log(1 + |h|^2 \rho)$, provided that the blocklength is sufficiently large. However, it is prima facie unclear whether the quantity $\log(1 + |h|^2 \rho)$ is meaningful when the packet size is small.

To better understand the relevance of capacity-versus-outage for delay-constrained communication, a more refined analysis of $R^*(n, \epsilon)$ was presented in [17]. It was shown that [17, Ths. 3 and 9]

$$R^*(n, \epsilon) = C_\epsilon + O \left( \frac{\log n}{n} \right)$$

irrespective of the number of transmit and receive antennas, and irrespectively of whether CSI is available to transmitter, receiver, or both. Comparing (8) with (3), we observe that for the quasi-static fading case the channel dispersion is zero, i.e., the $1/\sqrt{n}$ rate penalty is absent. This suggests that $R^*(n, \epsilon)$ converges quickly to $C_\epsilon$ as $n$ tends to infinity, thereby indicating that capacity-versus-outage is indeed a meaningful performance metric for delay-constrained communication over slowly-varying fading channels. Numerical examples that support this claim can be found in [17, Sec. VI]. Thus, (8) provides mathematical support to the observation reported by several researchers in the past that the outage probability describes accurately the performance over quasi-static fading channels of actual codes (see [15] and references therein). The intuition behind this result is that the dominant error event over quasi-static fading channels is that the channel is in a “deep fade”. Since coding is not helpful against deep fades in the quasi-static fading scenario, it follows that $R^*(n, \epsilon)$ is close $C_\epsilon$ already for small blocklengths.

As we shall show in the next section, when the number of independent time-frequency branches spanned by each codeword increases, the outage capacity becomes rapidly an inaccurate performance metric, because it does not capture the channel-estimation overhead.

2) Tradeoff between diversity, multiplexing, and channel estimation: When communicating over multiple-input multiple-output fading channels, a common question is whether the spatial degrees of freedom offered by the antennas should be used to lower the packet error probability for a given data rate (through the exploitation of spatial diversity) or to increase the data rate for a given packet error probability (through the exploitation of spatial multiplexing). These two effects cannot be harvested concurrently, but there exists a fundamental tradeoff between diversity and multiplexing. This tradeoff admits a particularly simple characterization in the high-SNR regime [18].

It has been recently demonstrated that for data packets of 1000 channel uses or more and for moderately low packet-error probabilities (around $10^{-2}$), one should typically operate at maximum multiplexing [19]. In this regime, which is relevant for current cellular systems, diversity-exploiting techniques are detrimental both for high- and for low-mobility users. For high-mobility users (where $n_c$ is significantly smaller than the packet size $n$), abundant time and frequency selectivity is available, so diversity-exploiting techniques are essentially superfluous. For low-mobility users (where $n_c$ is typically large), the fading coefficients can be learnt at the transmitter and outage events can be avoided altogether by rate adaption.

However, when the packet size becomes small and/or smaller packet-error probabilities are required, these conclusions may cease to be valid. For example, for packet lengths of, say, 100 channel uses (which is roughly equal to a LTE resource block) and packet-error probability of $10^{-5}$ or lower, spatial diversity may be more beneficial than spatial multiplexing. Furthermore, when the coherence interval $n_c$ is small, the cost of estimating the fading coefficients may be significant and must therefore be taken into consideration.

Studies based on capacity or capacity-versus-outage are inherently incapable of illuminating the entire diversity-multiplexing-channel estimation tradeoff. Indeed, the capacity of the block-fading channel characterizes the largest rate that is achievable if the number of time-frequency diversity branches $\ell$ grows to infinity while the coherence interval $n_c$ is held fixed. Consequently, it reflects the cost of estimating the fading coefficients but hides away the effects of spatial diversity, since an infinite diversity gain can already be achieved via time-frequency diversity. Conversely, the definition of capacity-versus-outage is based on the assumption that the coherence interval $n_c$ grows to infinity while the number of diversity-branches $\ell$ is held fixed. It therefore captures the effects of spatial diversity but hides away the cost of estimating the fading coefficient, since for an infinite coherence interval $n_c$ the channel can be estimated perfectly without a rate penalty.

To investigate the entire diversity-multiplexing-channel estimation tradeoff for small packet lengths, bounds on $R^*(n, \epsilon)$ were presented in [20]–[22]. Here, we provide an example, taken from [22], which illustrates the benefit of a nonasymptotic analysis of the diversity-multiplexing-channel estimation tradeoff. Specifically, we consider a scenario based on the 3GPP LTE standard [19] where the packet size is $n = 168$ symbols (which corresponds to 14 OFDM symbols, each consisting of 12 tones). We set the SNR to 6 dB and the packet error rate to $10^{-5}$, which corresponds to a URC scenario, and compute the bounds on the maximum coding rate obtained in [22] as a function of the coherence time $n_c$ or, equivalently, the number of diversity branches $\ell$ (recall that $n = \ell n_c$) for a $2 \times 2$ MIMO system.

The bounds are depicted in Fig. 5. We see from the figure that, given $n$ and $\epsilon$, the rate $R^*(n, \epsilon)$ is not monotonic in the coherence interval $n_c$, but there exists a value $n_c^*$ (in this case 14) that maximizes the rate. This accentuates the fundamental tension between time-frequency diversity (which decreases with $n_c$) and the ability of estimating the fading coefficient (which increases with $n_c$).

We further observe that both outage and ergodic capacity
(computed for the so called noncoherent scenario where CSI is not available at the receiver—see [23] for a recent review) fail to capture this tension, although their intersection predicts surprisingly well the rate-maximizing coherence interval. We also note that when the coherence interval is smaller than 8 channel uses, one of the two transmit antennas should actually be switched off, because the cost of estimating the fading coefficients overcomes the benefit of using two antennas at the transmitter.

In Fig. 5, we also depict bounds on the maximum coding rate obtainable using an Alamouti inner code [24], a configuration in which the transmit antennas are used to provide exclusively transmit diversity. Since the gap between the rate achievable using Alamouti and the maximum coding rate converse is small, we conclude that for the scenario considered in Fig. 5, the available transmit antennas should be used to provide diversity and not multiplexing.

D. $R^*(n, \epsilon)$ versus Error Exponents

In addition to refined analyses of $R^*(n, \epsilon)$, error exponents are often used as performance metrics to assess the tension between reliability and throughput for small packet lengths. Their definition is based on the observation that for any rate $R$ below capacity, the packet-error probability decays exponentially in the blocklength $n$. Specifically, the error exponent $E(R)$ corresponding to the rate $R < C$ is defined by

$$ \epsilon = e^{-n[E(R)+o(1)]} $$

where $o(1)$ summarizes remainder terms that vanish as $n$ tends to infinity. While the maximal rate $R^*(n, \epsilon)$ as a function of $n$ and $\epsilon$ and the minimum packet-error probability $\epsilon^*(R, n)$ as a function of $R$ and $n$ are equivalent characterizations of the triple $(R, n, \epsilon)$, channel dispersion and error exponents (which describe $R^*(n, \epsilon)$ and $\epsilon^*(R, n)$) asymptotically as $n$ tends to infinity) do not necessarily characterize the same regime. Indeed, the asymptotic expansion (3) corresponds to the region where $\epsilon$ is bounded away from zero and $C - R$ is very small (since $R^*(n, \epsilon)$ converges to $C$ as $n$ tends to infinity). In contrast, (9) corresponds to the region where $C - R$ is bounded away from zero and $\epsilon$ is very small (since $\epsilon$ decays exponentially in $n$). For wireless communications, where a small but bounded packet-error probability can be tolerated, an asymptotic expansion of $R^*(n, \epsilon)$ such as (3) seems more meaningful.

E. Further Works

The work by Polyanskiy, Poor, and Verdú [10] has triggered a renewed interest in the problem of finite blocklength information theory. This is currently a very active research area. Here, we provide a (necessarily not exhaustive) list of related works dealing with wireless communications at finite blocklength.

When CSI is available at the receiver, the dispersion of fading channels was obtained in [26]–[28] for specific scenarios. Upper and lower bounds on the second-order coding rate of quasi-static multiple-input multiple-output (MIMO) Rayleigh-fading channels have been reported in [29] for the asymptotically ergodic setup when the number of antennas grows linearly with the blocklength. The channel dispersion of single-antenna, quasi-static fading channels with perfect CSI at both the transmitter and receiver and a long-term power constraint has been given in [30], [31].

For discrete-memoryless channels, feedback combined with variable-length coding has been shown to dramatically improve the speed at which the maximum coding rate approaches capacity [32]. Such improvements can be achieved by letting the receiver feed-back a single bit to inform the transmitter that decoding has been successful (stop feedback, also known as decision feedback). One can relax the assumption that decoding is attempted after each symbol, with marginal performance losses [33].

Coding schemes approaching the performance predicted by finite-blocklength bounds have been also proposed. In [34], list decoding of polar codes is shown (through numerical simulations) to operate close to the maximum coding rate (see Fig. 6). The finite-blocklength gap to capacity exhibited by polar codes has been characterized up to second order (in terms of the so-called scaling exponent) in [35]–[37]. A comparison between the finite-blocklength performance of convolutional codes (both with Viterbi and with sequential decoding) and LDPC codes is provided in [38]. Bounds and exact characterizations on the
error-vs-delay tradeoff for codes of very small cardinality have been recently provided in [39].

For the case of channels with feedback, designs based on tail-biting convolutional codes combined with the reliability-output Viterbi algorithm have been proposed in [40]. For short blocklengths, these schemes operate above the achievability bound provided in [32].

Finally, second-order characterizations of the coding rates for some problems in network information theory have recently been obtained. A comprehensive review is provided in [41].

**F. spectre: short-packet communication toolbox**

To optimally design communication protocols for short-packet transmission, one needs to rely on accurate physical layer performance metrics. spectre—short-packet communication toolbox [42] is a collection of numerical routines for the evaluation of achievability and converse bounds on the maximum coding rate for popular channel models, including the AWGN channel, the quasi-static fading channel, and the Rayleigh block-fading channel. This toolbox can be freely accessed online and is under development. All the numerical simulations reported in this paper can be reproduced using spectre routines.

**IV. COMMUNICATION PROTOCOLS FOR SHORT PACKETS**

In simple terms, a communication protocol is a distributed algorithm that determines the actions of the actors involved in the communication process. Protocol information, also referred to as metadata or control information, can be understood as a source code [43] that ensures correct operation of the protocols and describes, e.g., the current protocol state, the packet length, or the addresses of the involved actors.

Only few results are available on the information-theoretic design of communication protocols, e.g., [44]–[46], and most of them deal with the (source coding) problem of how to encode the network/link state that needs to be communicated as a protocol information. The problem of how to transmit the protocol-related metadata has been largely left to heuristic approaches, such as the use of repetition coding. Broadly speaking, whereas information theorists busy themselves with developing capacity-approaching schemes for the reliable transmission of the information payload, they often see the design of metadata as something outside their competence area, or as stated in [32]: “…control information is not under the purview of the physical layer…” Such a line of thinking is fully justifiable when the ratio between the data and metadata is the one depicted in Fig. 1(a), where the metadata occupy a small fraction of the overall packet length. However, for applications where the data is comparable in size to the metadata—see Fig. 1(b)—this approach seems questionable.

In the following, we shall argue that a thorough understanding of how the maximum transmission rate \( R^*(n, \epsilon) \) depends on the packet length \( n \) and on the packet error probability \( \epsilon \) is also beneficial for protocol design. As mentioned above, only few results are available on the information-theoretic design of protocols, and there is virtually no work that considers protocol design for short-packet transmission. This section is therefore based on three simple examples that illustrate how the tradeoffs brought by short-packet transmissions affect protocol design. We believe that these examples unveil a number of interesting tradeoffs worth exploring and we hope that they may motivate the research community to pursue a better theoretical understanding of protocol design.

For simplicity, we assume throughout this section an AWGN channel with SNR \( \rho = 10 \) and we approximate \( R^*(n, \epsilon) \) as

\[
R^*(n, \epsilon) \approx C - \sqrt{\frac{V}{n}} Q^{-1}(\epsilon) + \frac{1}{2n} \log n \tag{10}
\]

where \( C \) and \( V \) are given in (4) and (5), respectively.\(^2\) It is expected that the tradeoffs encountered for the AWGN channel will also be relevant for fading channels. Solving (10) for \( \epsilon \) yields the following approximation of the packet error probability as a function of the packet length \( n \) and the number of information bits \( k = Rn \)

\[
\epsilon^*(k, n) \approx Q\left(\frac{nC - k + 0.5 \log n}{\sqrt{nV}}\right) \tag{11}
\]

which we shall use throughout this section.\(^3\)

**A. Reliable Communication Between Two Nodes**

Consider a two-way communication protocol where the nodes acknowledge the correct reception of a data packet transmitting an ACK. The correct transmission of a data packet from, say, node 1 to node 2 would result in the following protocol exchange sequence:

1) The packet from node 1 is correctly received by node 2. We shall denote the probability of this event by \( 1 - \epsilon_1 \);
2) Node 2 sends an ACK to node 1. We shall denote the probability that an ACK is received correctly by \( 1 - \epsilon_2 \).

As noted in [47], if we communicate over a noisy channel and we are restricted to use a finite number of channel uses, then no protocol will be able to achieve perfectly reliable communication. Indeed, it is possible that either a packet is received incorrectly (an event which has probability \( \epsilon_1 \)) or that the ACK is received incorrectly (which happens with probability \( \epsilon_2 \)). By (11), decoding errors are particularly relevant if the packet size is small, in which case \( \epsilon_1 \) and \( \epsilon_2 \) are large. Thus, the often-made assumptions of perfect error detection or perfect ACK-transmission (so-called “1-bit feedback”) are particularly misleading if the considered packet length is small.

Let us consider the following example. Let each node have a 6-byte address and assume that node 1 has 12 data bytes to

\(^2\)Recall that, as mentioned in Section III, replacing the remainder terms in (3) by \( \frac{1}{2n} \log n \) yields a good approximation for \( R^*(n, \epsilon) \).

\(^3\)In the remainder of the paper, we assume that all \( \log \) functions are in base 2.
send. Assume that the packet sent by node 1 contains the source address, the destination address, one bit for flow control and the data bytes. Hence, node 1 transmits \(k_{s,1} = 96\) data bits and \(k_{o,1} = 97\) metadata bits, resulting in \(k_1 = k_{s,1} + k_{o,1} = 193\) bits. The ACK packet sent by node 2 consists of the source address and the destination address and one ACK bit. For the ACK packet, this yields \(k_{s,2} = 0\) data bits, \(k_{o,2} = 97\) metadata bits, so \(k_2 = k_{s,2} + k_{o,2} = 97\) bits. Let \(n\) be the total number of channel uses available to send the data and the ACK. Then we need to find the optimal number of channel uses \(n_1\) by node 1 and \(n_2 = n-n_1\) by node 2 such that the reliability of the transmission, given by \((1-e^\epsilon(k_1, n_1))(1-e^\epsilon(k_2, n_2))\), is maximized. These values can be found numerically using the approximation (11). For example, the minimum value of \(n\) that offers reliability of transmission

\[(1-e^\epsilon(k_1, n_1))(1-e^\epsilon(k_2, n_2)) > 0.999\]

is \(n = 203\), out of which \(n_1 = 132\) channel uses are used to send the data packet and \(n_2 = 71\) channel uses are used to send the ACK. As another example, fix \(n = 250\) as the maximal allowed number of channel uses. The numerical optimization that yields the largest reliability \((1-e^\epsilon(k_1, n_1))(1-e^\epsilon(k_2, n_2))\) gives \(n_1 = 158\) and \(n_2 = 92\). The resulting reliability is almost 1 and the resulting throughput is \((1-e^\epsilon(k_1, n_1))(1-e^\epsilon(k_2, n_2))k_{s,1}/n = 0.384\) bits/channel use.

In many cases, it is not practical to have variable values for \(n_1\) and \(n_2\). In this case, a fixed time division duplex (TDD) structure in which \(n_1 = n_2\) is preferred. In such a structure, there is no need of explicit ACK packets, since the acknowledgement is typically piggybacked on a data packet. In order to align this scenario with the last example, we assume that \(n_1 = n_2 = 125\), such that the acknowledgment for the packet arrives within \(n = 250\) channel uses from the start of the data transmission. A packet sent by nodes 1 and 2 contains 194 bits, of which 96 are data bits, 96 are bits for addresses, 1 bit is for flow control, and 1 bit for the acknowledgment. Evaluating (11) for these parameters gives \(e^\epsilon(k_1, n_1) = e^\epsilon(k_2, n_2) = 0.0118\). Observe that the reliability is markedly decreased, although the throughput is almost doubled to 0.759 bits/channel use.

These simple examples show that adjusting the packet length and the coding rate has the potential to yield high reliability. Note, however, that flexibility in the packet length necessarily implies that the receiver needs to acquire information about it. This means that the protocol needs to reserve some bits within each packet for the metadata that describes the packet length. Our simple calculations have not accounted for this overhead.

The use of a predefined slot length yields a robust system design, since no additional error is caused by the exchange of length-related metadata. This indicates that, in designing protocols that support ultra-high reliability, a holistic approach is required that includes all elements of the protocol/metadata that are commonly assumed to be perfectly received.

B. Downlink Multi-User Communication

We now turn to an example in which a base station (BS) transmits in the downlink to \(M\) devices. The BS needs to unicast \(D\) bits to each device. Hence, it sends in total \(MD\) bits. As a reference, we consider a protocol where the BS serves the users in a time division multiple access (TDMA) manner: each device receives its \(D\) bits in a dedicated time slot that consists of \(n\) channel uses. Thus, the TDMA frame consists of \(M\) slots with a total of \(MN\) channel uses. In order to avoid transmission of metadata, we assume that the system operates in a circuit-switched TDMA manner: (a) all devices and the BS are perfectly synchronized to a common clock; (b) each device knows the slot in which it will receive its data. The performance of this idealized scheme can be considered as an upper bound on the performance of practical systems, such as GSM, as it assumes that there is a genie that helps the devices to remain synchronized.

The approximation on \(e^\epsilon(k, n)\) in (11) suggests that, for short packet sizes, it may be more efficient to encode a larger amount of data than the one intended to each device. Thus, instead of using TDMA, the BS may concatenate all the data packets for the individual devices. In this way, the BS constructs a single data packet of \(MD\) bits that should be broadcasted by using \(MN\) channel uses. Each receiving device then decodes the whole data packet and extracts the bits it is interested in from the decoded \(MD\) bits.

As a concrete example, assume that the BS wishes to transmit \(D = 192\) bits to each device and that there are \(M = 10\) devices. Furthermore, assume that \(n = 125\). We consider for simplicity one-shot communication. Accounting for retransmissions would require a more elaborate discussion.

In the reference scheme, the probability of error experienced by each device is 0.007. If concatenation is used, however, the probability of error drops to about 10\(^{-12}\), which puts the transmission scheme in a different reliability class, while preserving the same overall delay. The price paid is the fact that each device needs to decode more data than in the reference scheme. Also privacy and security considerations may make this second solution undesirable.

Note that if one ignores the dependency of the packet error probability \(e^\epsilon\) on the packet size \(n\), one would conclude that the circuit-switched TDMA protocol is the most efficient, since all channel uses can be devoted to the transmission of payload bits. In contrast, by taking the dependence of \(e^\epsilon\) on \(n\) into account, we see that an unconventional protocol that concatenates the data intended to different devices outperforms the traditional TDMA protocol by orders of magnitude in terms of reliability.

C. Uplink Multi-User Communication

Our last example is related to a scenario in which \(M\) devices run a random access protocol in order to transmit to a common receiver BS. Specifically, there are \(M\) users, each sending \(D\) bits to the BS. Each packet should be delivered within a time that corresponds to \(n\) channel uses. These \(n\) channel uses are divided into \(K\) equally-sized slots of \(n_K = n/K\) channel uses. The devices apply a simple framed ALOHA protocol: each device picks randomly one of the \(K\) slots in the frame and send its packet. If two or more users pick the same slot, then a collision occurs and none of the packets is received correctly (see [48] for a more elaborate example). If only one device picks a particular slot (singleton slot), then the error probability is calculated using (11) for \(D\) bits and \(n_K\) channel uses.

Note that the source/destination addresses are necessary in order to uniquely identify the link to which the ACK belongs.
We are interested in the following question: given $M$, $D$, and $n$, how should we choose the slot size $K$ in order to maximize the packet transmission reliability experienced by each individual device? This problem entails a tradeoff between the probability of collision and the number of channel uses available for each packet, which by (11) affects the achievable packet error probability in a singleton slot. Indeed, if $K$ increases, then the probability of a collision decreases, while the packet error probability for a singleton slot increases. Conversely, if $K$ decreases, then the probability of collision increases, while the packet error probability for a singleton slot decreases. The probability of successful transmission is given by

$$P_S = \frac{M}{K} \left( 1 - \frac{1}{K} \right)^{M-1} \left( 1 - \epsilon^* (D, n_K) \right) \quad (12)$$

where $(M/K) (1-1/K)^{M-1}$ is the probability of not experiencing collision and $\epsilon^* (D, n_K)$ is the probability of error for a packet of $D$ bits sent over $n_K$ channel uses, which can be approximated by (11).

As a concrete example, let us take $D = 192$ bits, $M = 10$ devices, and $n = 800$ channel uses. The number of slots that maximizes (12) is $K = 6$. In contrast, the classic framed-ALOHA analysis, which assumes that packets are decoded correctly if no collisions occur (i.e., $\epsilon^* = 0$ in (12)), yields $K = M = 10$.5

V. Conclusions

Motivated by the advent of wireless applications such as massive machine-to-machine and ultra-reliable communications, we have provided a review of recent advances in the theory of short-packet communications and demonstrated through three examples how this theory can help designing novel efficient communication protocols that are suited to short-packet transmissions. The key insight is that—when short-packet are transmitted—it is crucial to take into account the communication resources that are invested in the transmission of metadata. This unveils tradeoffs that are not well understood yet and that deserve further research, both on the theoretical and on the applied side.

Acknowledgment

We would like to thank Yury Polyanskiy for letting us reproduce Fig. 3 and Fig. 6 and Erik G. Ström for fruitful discussions.

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