Charged Dilatonic Black Holes and their Transport Properties

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We briefly explain the consistency conditions imposed on the effective holographic theories, which are parameterized by two real exponents $(\gamma, \delta)$ that control the IR dynamics. The general scaling of DC resistivity with temperature at low temperature and AC conductivity at low frequency across the whole $(\gamma, \delta)$ plane are explained. There is a codimension-one region where the DC resistivity is linear in the temperature. For massive carriers, it is shown that when the scalar operator is not the dilaton, the DC resistivity scales as the heat capacity (and entropy) for $(2 + 1)$-dimensional systems. Regions are identified where the theory at finite density is a Mott-like insulator. This contribution is based on [C. Charmousis, \textit{et al.}, JHEP 1011, 151 (2010)]\textsuperscript{1} with emphasis on the transport properties of charged dilatonic black holes with potential.

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1 Introduction

In recent years, AdS/CFT correspondence opens up new ways to investigate low temperature dynamics of strongly correlated electron systems \textsuperscript{2}. The physics of condensed matter system (CM) is very diverse, thus we need to find various theoretical models and to classify them in terms of its universality classes at low temperature. Classification criteria may include the appropriate phases via their thermodynamic properties including phase transitions and their transport coefficients, which is directly connected to what is measured by experiments.

One of the first and comprehensively investigated example of such classes is charged Reissner Nordström (RN) black holes of Einstein-Maxwell gravity with a cosmological constant. Such solutions proved to be the simplest laboratory for the study of phase transitions at finite density \textsuperscript{3}, fermionic quasi-particles with non-Fermi liquid behavior \textsuperscript{4}, as well as an emerging scaling symmetry at zero temperature \textsuperscript{5}, and superfluid (superconductivity) phase transitions \textsuperscript{6}. While this model acquired interesting and important developments, it has a large entropy at extremality signaling instability. Anticipating CM application, it is desirable to find many more models, which have qualitatively distinct features with each other.

A family of charged black hole solutions with one parameter to the low energy string theory with dilaton has interesting and several distinctive thermodynamic properties \textsuperscript{7}: The Maxwell field $(F)$ in the solution has a generalized coupling with the dilaton $(\phi)$ as $e^{-2\alpha \phi} F^2$, where $\alpha$ is a parameter. When $\alpha = 0$, it is nothing but the RN black hole, which has vanishing extremal temperature $T = 0$ and finite entropy $S = S_0$ at extremality. The extremal behavior of the solution changes according to the parameter $\alpha$: The extremal entropy vanishes for non-zero $\alpha$, while the extremal temperature changes as $T = 0$ for $0 < \alpha < 1$, $T = T_0$ for $\alpha = 1$ and $T = \infty$ for $\alpha > 1$. Thus it is natural to expect that we can generate several qualitatively distinct classes with a dilaton potential and a coupling constant to the Maxwell field with a scalar dilation.

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2 Effective Holographic Approach

We would like to model strongly-coupled holographic systems at finite charge density by including a leading relevant scalar operator, which is generically uncharged and drives the renormalization group flow of the system from UV to IR. The corresponding holographic theory is an Einstein-Maxwell-Dilaton system with a scalar potential. In the case of zero charge density such theories have been analyzed (see e.g. [3]) and generalized to the finite charge density case (see e.g. [9]). This is a phenomenological approach based on the concept of Effective Holographic Theory (EHT), in analogy with Effective Field Theories which is low energy approximations to QFT. It allows a parametrization of large classes of IR dynamics and a survey of important observables, while it is not obvious if concrete EHTs can be embedded in well-defined string theories, or if classical singular solutions are acceptable as saddle points of the dual CFTs [8][10][11].

Our strategy is to select a minimal set of operators (or dual string fields) that are expected to dominate the low energy dynamics and then parametrize their EHT in terms of a general two-derivative action. This set is composed of the bulk metric, \( g_{\mu \nu} \), controlling the energy distribution, bulk gauge field, \( A_\mu \), associated with a conserved U(1) current \( J_\mu \), and a scalar leading-relevant operator, \( \phi \), controlling the coupling constant and its running in the IR. For a CM system, \( \phi \) may represent the strong interactions of the ion lattice or the effect of spins on the charge carriers. If the charge carriers are dilute, we expect that the dynamics can be treated in the probe approximation, while the full set of equations must be solved at sufficiently large doping.

At the two-derivative level, the EHT in the Maxwell case is described in the Einstein frame by the action

\[
S = M^{p-1} \int d^{p+1} x \sqrt{-g} \left[ R - \frac{1}{2} (\partial \phi)^2 + V(\phi) \right] - M^{p-1} \int d^{p+1} x \sqrt{-g} \frac{Z(\phi)}{4} F_{\mu\nu} F^{\mu\nu}
\]

Indeed, the most general, \( p + 1 \) dimensional, two-derivative action containing a metric, a vector and a single scalar can be brought to this form via field redefinitions. On the other hand, the linear Maxwell action, \( S_{Max} \), must be replaced by the DBI action \( S_{DBI} = -M^{p-1} \int d^{p+1} x e^{2k\phi} Z(\phi) \sqrt{-\det(g + e^{-k\phi} F^2)} \), in the fundamental charge case. Here \( g = e^{-k\phi} F^2 \) is the Einstein frame metric.\( Z \) has been normalized such that the DBI action reduces to the Maxwell one for \( F^2 \ll 1 \). The parameter \( k \) encodes the dilatonic nature of the scalar field \( \phi \). If \( \phi \) is the dilaton, \( k = -\sqrt{-\frac{1}{2}} \). Even though the discussions can be generalized to other dimensions straightforwardly, we would like to concentrate on the 4 dimensional case, targeting \((2+1)-dimensional CM phenomena.

While, in holographic theories, it is standard to assume that there is a non-trivial fixed point in the UV to ensure a globally controllable behavior with an appropriate UV boundary, this assumption is not necessary if our only interest is to understand the infrared (IR) dynamics.\( ^1 \) For example, the non-trivial dynamical condition, which defines the “vacuum” and expectation values, is imposed in the IR typically by a regularity condition in the far IR. For this purpose and also motivated by controlled examples, we model \( V(\phi) = V_0 e^{-\delta \phi} \) as generic scalar operators have non-trivial n-point functions in all realistic examples. We further demand that \( e^{-\delta \phi} \to 0 \) in the UV to guarantee that simple modifications of the potential like \( V \to \frac{V_0 e^{-\delta \phi}}{p+1} + V_0 e^{-\delta \phi} \) will lead to asymptotically AdS solutions without modification of the IR behavior. We also model \( Z(\phi) \sim e^{\gamma \phi} \) motivated from both gauge supergravity actions stemming from string theory as shown in the introduction.

With this EHT setup, we generically encounter naked singularities deep in IR. Thus we investigate the consistency condition of our EHT model in IR with the following listed criteria and constrained the parameter space \( (\gamma, \delta) \), which is depicted in Fig[1]:

- Gubser’s Criteria : A naked singularity is unphysical if the scalar potential is not bounded below when evaluated in the solution [10]. In holography, well-defined and controllable perturbations of \( N=4 \) sYM lead to mild naked singularities in the bulk, which are generically resolved by lifting to

\[1 \] In the holographic approach, This direction is explicitly pursued recently in [12].

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higher dimensions or eventually by the inclusion of the stringy states. Thus not all naked singularities are unacceptable.

- Well-defined spectrum: A naked singularity describes unreliable holographic physics if the second-order equations describing the spectrum of small fluctuations around the solution are not well-defined Sturm-Liouville problems. We change our low energy fluctuation equations into Schrödinger form and require our solutions to have only one normalizable mode in the deep IR for spin 1 and spin 2 spectra. This also allows us to classify the nature of low energy spectrum, which is discussed when we discuss the transport properties.

- Thermodynamic Stability: We accept the AdS completable solutions in the thermodynamically unstable case as well as thermodynamically stable solutions. Please consult the details of the thermodynamic properties of the various solutions in the companion contribution of these proceedings.

### 3 Transport Properties

Conductivities can be calculated by linear response theory using a Kubo formula from the two-point function of the fluctuations of the gauge field. We will consider long wavelengths compared to their frequency, which is a good approximation for optical CM experimental measurements. Following [17][18], we calculate the zero-temperature asymptotics of the AC conductivity \( \sigma(\omega) \sim \omega^{2|\nu|-1} \) by transforming our fluctuation equations into a Schrödinger equation with potential \( V(z) = \frac{\nu^2}{z^2} \) in the near horizon region. While the DC conductivity is the zero frequency limit of the AC conductivity, this limit may be ill-defined [13]. A direct calculation can be done by turning on a constant electric field, calculating the induced current and thus obtaining the ratio using the probe DBI action which describes massless, self-interacting but non-backreacting charge carriers. We also summarize the low energy spectrum of our models, which was analyzed fully in [1].

We will in turn consider four different classes of solutions: uncharged, near extremal, exact solution with \( \gamma \delta = 1 \) and \( \gamma = \delta \). The thermodynamic properties and phase structure of these solutions are summarized in [15]. The exact solutions were found earlier in [16].

- Uncharged solution: The solution depends on one parameter \( \delta \) and it has an acceptable singularity when \( 0 \leq |\delta| < 1 \) according to the criteria in the previous section. We analyze the spectrum of current excitations with probe Maxwell term with a parameter \( \gamma' \) on the uncharged background. When \( \frac{\gamma}{\delta} > \frac{3}{2} \) or \( \frac{\gamma}{\delta} < -\frac{1}{2} \) and the UV dimension of the scalar \( \Delta < 1 \), the Schrödinger potential diverges both in the UV and the IR. Thus the spectrum is discrete and gaped, which resembles to an insulator. Otherwise it is a conductor. When \( \frac{1}{2} < \frac{\gamma}{\delta} < \frac{3}{2} \), the spectral problem is unacceptable and our EHT becomes unreliable.

When \( |\delta| < 1 \), AC conductivity is given by \( \sigma \sim \omega^{1-\frac{2}{3}} \) with \( c = \frac{(\gamma\delta+1-\delta^2)\gamma\delta}{(1-\delta^2)^2} \). This reveals interesting scaling behavior

\[
\sigma \sim \omega^{-\frac{2}{3}} \quad \text{when} \quad \gamma = \frac{\delta^2 - 1}{3\delta} \quad \text{or} \quad \frac{2(\delta^2 - 1)}{3\delta},
\]
which seems to fit to an available experimental data \cite{19}.

At the lowest densities the DC conductivity shows \( \rho = 1/\sigma \sim T^{\frac{2(\gamma - 1)}{\gamma - 2}} \). In particular, in this dilute regime it is independent of the dilatonic nature of the scalar field, which can be parametrized by a parameter \( k \). Thus the conductivity can be linear in temperature \( T \) for

\[
\rho = \frac{1}{\sigma} \sim T \quad \text{when} \quad \gamma = \delta = \frac{1}{2},
\]

which is universal for unconventional superconductor at the optimal doping \cite{20}. This DC resistivity has an interesting relation with entropy and heat capacity in temperature \( T \rho \sim S \sim C_V \sim T^\gamma \). As the zero charge density system never exhibits an entropy linear in temperature, we conclude that when linear resistivity appears, it is due to the presence of charge carriers. For higher charge density, \( \rho \sim T^{2\delta + 2} \langle J' \rangle \), where \( \langle J' \rangle \) is charge density. In this case, the resistivity can never be linear.

- Near extremal solution: We view this solution as near extremal limit of full general solutions of the intermediate range \( 1 < \gamma \leq 2 \) near the horizon at extremality, and hence the system is insulating, while it becomes conducting away of states at extremality.

- Exact solution with \( \gamma \delta = 1 \): For \( 0 \leq \delta^2 < 2/\sqrt{3} \), the AC conductivity at extremality scales as \( \sigma(\omega) \approx \omega^n \), with \( n = \frac{(\delta^2 - 1)(\sqrt{3} - 2\delta)}{\sqrt{3}(\delta^2 - 1)} - 1 \). The exponent is always larger than \( 5/3 \) in this region, and diverges at \( \delta^2 = 1 + 2/\sqrt{3} \). The spectrum is continuous and has no mass gap in this region and thus behaves as a conductor. For \( 1 + 2/\sqrt{3} < \delta^2 < (5 + \sqrt{33})/4 \), the Schrödinger potential diverges near the horizon at extremality, and hence the system is insulating, while it becomes conducting away from extremality (see Fig. 3). For \( (1 + 2/\sqrt{3})/4 < \delta^2 < 3 \), the system is conducting with a continuum of states at extremality.

The DC conductivity for massive charge carriers is depicted in Fig. 4. In the lower range \( 0 \leq \delta^2 \leq 1 \), the DC resistivity vanishes at zero temperature and then rises in the black hole phase. In the intermediate range \( 1 < \delta^2 < 1 + \frac{2}{\sqrt{3}} \), the resistivity drops steeply with temperature in the unstable large black hole branch and rises slowly with temperature in the thermodynamically dominant small black hole branch. In this latter branch, the low-temperature resistivity is linear for \( \delta^2 = 1 + 2/\sqrt{5} \).

The validity of linear regime is determined by the IR dynamical scale, \( \ell \). Finally, in the upper branch...
1 + 2/\sqrt{3} < \delta^2 < 3$, the resistivity is decreasing with the temperature. In the special case $\delta^2 = 1$, the resistivity is finite at zero temperature, then increases, and finally diverges at a higher temperature signaling potentially a critical behavior.

- Exact solution with $\gamma = \delta$: As explained in [15], the upper range of $\gamma \delta = 1$ case has no equivalent in $\gamma = \delta$. The zero temperature AC conductivity for all such black holes behaves as $\omega^2$ independent of $\delta$. In both ranges the resistivity is non-zero at zero temperature and has a regular low-temperature expansion in integer powers of the temperature starting with a linear term. Transport properties of $\gamma = \delta$ case are similar to those of $\gamma \delta = 1$ case in an appropriate range.

4 Outlook

After [1], there are several attempts to investigate the transport properties of the dilatonic background. The near extremal solutions is also considered in [22]. For the important case with black branes carrying both electric and magnetic charges in Einstein-Maxwell theory coupled to a dilaton-axion in asymptotically anti de Sitter space is analyzed in [23].

It is worthwhile to mention recent attempts, using holographic finite density systems, to describe the benchmark conductivity features of unconventional superconductors: a linear temperature dependence of resistivity, $\rho \sim T$, at the optimal doping, its crossover to $T^2$ in the over-doped region, and a quadratic temperature dependence of inverse Hall angle, $\cot \Theta_H = \frac{\sigma_{xy}}{\sigma_{xx}} \sim T^2$. The linear temperature dependence of the Ohmic resistivity was found in spaces with AdS or Lifshitz scaling [1, 22, 24, 25], as well as Schrödinger space with special conditions [26, 27]. A linear resistivity and a crossover to quadratic was found in a larger class of scaling geometries in [1]. The Hall angle behavior was recently discussed using Lifshitz type metric with broken rotational symmetry [28]. Finally these all three benchmark behaviors including a crossover were reproduced in a single holographic system in [29].

We discuss a set of effective holographic theories parametrized by two exponents $(\gamma, \delta)$ associated with dilatonic coupling to the gauge fields and potential. The parameter space is severely constrained by the nature of the IR singularity. Imposing constraints for the singularity to be a good kind removes part of the $(\gamma, \delta)$ plane as discussed. The transport properties of the allowed parameter space are studied and characterized by their scaling behavior. They suggest that there are EHTs that display the hallmark behavior of general strange metals, namely linear resistivity. Moreover, the DC resistivity is correlated to the entropy and the specific heat.

There are however more features of the physics of the EHTs that need to be analyzed so that their suitability as theories of strange metal behavior can be assessed. An important general ingredient are the zero and low temperature spectra of fluctuations, that should be derived in order to completely characterize the low energy degrees of freedom, as well as the energy-energy and charge-charge correlators. In particular the interplay between insulating versus conducting behavior must be further analyzed. It is a generic property of the holographic systems described here, in the $(2+1)$–dimensional case, to favor conducting
behavior. Indeed, with the exception of strongly relevant dynamics, the systems at small charge density seem to be conductors at any small but non-zero temperature. There is generically a continuous phase transition to the extremal solution which is insulating.

The phase structure also seems to be interestingly varying with the IR “strength” of the scalar operator captured by the exponent $\delta$. There are indications from our analysis that the appearance of discrete spectra at low temperature is “delayed” by the finite charge density. At zero charge density, such spectra appear when $\delta^2 > 1$ while at finite density in the $\gamma = \delta$ case they never appear, and for $\gamma = \delta = 1$ their appearance is delayed until $\delta^2 = 1 + \gamma$. This structure needs verification from a detailed analysis of low energy spectra.

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