A linear streaming algorithm for community detection in very large networks

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ABSTRACT
In this paper, we introduce a novel community detection algorithm in graphs, called SCoDA (Streaming Community Detection Algorithm), based on an edge streaming setting. This algorithm has an extremely low memory footprint and a lightning-fast execution time as it only stores two integers per node and processes each edge strictly once. The approach is based on the following simple observation: if we pick an edge uniformly at random in the network, this edge is more likely to connect two nodes of the same community than two nodes of distinct communities. We exploit this idea to build communities by local changes at each edge arrival. Using theoretical arguments, we relate the ability of SCoDA to detect communities to usual quality metrics of these communities like the conductance. Experimental results performed on massive real-life networks ranging from one million to more than one billion edges show that SCoDA runs more than ten times faster than existing algorithms and leads to similar or better detection scores on the largest graphs.

KEYWORDS
Community Detection; Graph Streaming; Network Analysis

1 INTRODUCTION
1.1 Motivations
Networks arise in a wide range of fields from biology [27] to social media [23] or web analysis [12][29]. In most of these networks, we observe groups of nodes that are densely connected between each other and sparsely connected to the rest of the graph. One of the most fundamental problems in the study of such networks consists in identifying these dense clusters of nodes. This problem is commonly referred to as community detection.

A major challenge for community detection algorithms is their ability to process very large networks that are commonly observed in numerous fields. For instance, social networks have typically millions of nodes and billions of edges (e.g. Friendster [23]). Many algorithms have been proposed during the last ten years, using various techniques ranging from combinatorial optimization to spectral analysis [17]. Most of them fail to scale to such large real-life networks [31].

1.2 Contributions
In this paper, we introduce a novel approach to detect communities in very large graphs. This approach is based on edge streams where network edges are streamed in a random order. The algorithm processes each edge strictly once. Moreover, the algorithm only stores two integers for each node: its current community index and the number of adjacent edges that have already been processed. Hence, the time complexity of the algorithm is linear in the number of edges and its space complexity is linear in the number of nodes. In the experimental evaluation of the algorithm we show that this streaming algorithm, called SCoDA (Streaming Community Detection Algorithm), is able to handle massive graphs [40] with low execution time and memory consumption.

1.3 Related work
A number of algorithms have been developed for detecting communities in networks or graphs [13]. Many rely on the optimization of some objective function that measures the quality of the detected communities. The most popular quality metric is the modularity [25], which is based on the comparison between the number of edges that are observed in each cluster and the number of edges that would be observed if the edges were randomly distributed. Other metrics have been used with success, like the conductance, the out-degree fraction and the clustering coefficient [40]. Another popular class of algorithms uses random walks [30][37]. These methods are based on the fact that random walks tend to get "trapped" in the dense zones of the graph. These techniques have proved to be efficient but are often time-consuming and fail to scale to large...
In this section, we define SCoDA, a streaming algorithm for community detection in graphs. We use randomized insert-only edge streams and define a minimal sketch by storing only two integers per node.

2.2 Motivation

Although there is no universal definition of what a community is, most existing algorithms rely on the principle that nodes tend to be more connected within a community than across communities. Hence, if we pick uniformly at random an edge \( e \) in \( E \), this edge is more likely to link nodes of the same community (i.e., \( e \) is an \textit{intra-community} edge), than nodes from distinct communities (i.e., \( e \) is an \textit{inter-community} edge). Equivalently, if the edges of \( E \) are processed in a random order, we expect many intra-community edges to arrive before the inter-community edges.

More formally, let \( C \subset V \) be a community that we want to detect. If the edges of \( E \) are randomly drawn without replacement, we can consider the event where the first \( k \) edges drawn in \( e(C) \) are \textit{intra-community} edges, i.e. in \( e(C, C) \):

\[
\text{Intra}_k(C) = \text{the first } k \text{ edges that are drawn from } e(C) \text{ are in } e(C, C)
\]

The probability of this event is:

\[
\mathbb{P} \left[ \text{Intra}_k(C) \right] = \prod_{l=0}^{k-1} \frac{|e(C, C)| - l}{|e(C)| - l} \approx \prod_{l=0}^{k-1} \left( 1 - \phi_l(C) \right),
\]

where

\[
\phi_l(C) = \frac{|e(C, C)|}{|e(C, C)| + |e(C, C)| - l},
\]

for all \( l = 0, \ldots, k-1 \). Observe that the definition of \( \phi_0(C) \) is very close to that of the conductance \( \psi(C) \) of \( C \),

\[
\psi(C) = \frac{|e(C, C)|}{2|e(C, C)| + |e(C, C)|}.
\]

In particular, \( \phi_0(C) = \frac{2\psi(C)}{1 + \psi(C)} \approx 2\psi(C) \) for small values of the conductance. We refer to \( \phi_l(C) \) as the \textit{pseudo-conductance} in the rest of the paper. It is well known that good communities are subsets of \( V \) with low conductance [33]. We then expect \( \phi_l(C) \) to be low for small values of \( l \) if \( C \) is a good community and the probability of picking an \textit{inter-community} edge within the first \( k \) edges picked at random in \( e(C) \) to be low for small values of \( k \).

2.3 A streaming approach

This observation is used to design an algorithm that streams the edges of the network in a random order. For each arriving edge \((u, v)\), the algorithm places \( u \) and \( v \) in the same community if the edge arrives early (intra-community edge) and splits the nodes in distinct communities otherwise (inter-community edge). In this formulation, the notion of an early edge is of course critical. In the proposed algorithm, we consider that an edge \((u, v)\) arrives early if the current degrees of nodes \( u \) and \( v \), accounting for previously arrived edges only, is low.

More formally, the first step of the algorithm consists in shuffling the list of edges \( E \), i.e., in generating a random permutation of the list of edges. The algorithm then considers edges in this particular order, say \( e_1, e_2, \ldots, e_m \). Each node is initially in its own community. For each new edge \( e_j = (u, v) \), the algorithm performs one of the following actions:

- \( u \) joins the community of \( v \);
- \( v \) joins the community of \( u \);
- no action.

The choice of the action depends on the updated degrees \( d(u) \) and \( d(v) \) of nodes \( u \) and \( v \), i.e., the degree computed using the edges \( e_1, \ldots, e_j \). If \( d(u) \) or \( d(v) \) is greater than a given threshold \( D \), then we do nothing; otherwise, the node with the lowest degree joins the community of the other node.
2.4 Algorithm

The algorithm SCoDA is defined in Algorithm 1. It takes the list of edges of the graph and one integer parameter $D \geq 1$. The algorithm builds two arrays $d$ and $c$ of size $n$. At the end of the algorithm, $d_i$ is the degree of node $i$, and $c_i$ the community of node $i$. When the algorithm starts, each node has degree zero and is in its own community ($d_i = 0$ and $c_i = i$ for all $i$). Then, the list of edges is shuffled and the main loop iterates over the edges in this random order. For each new edge $e_j = (u, v)$, the degrees of $u$ and $v$ are updated. Then, if these degrees are both lower than the threshold parameter $D$, the node with the lower degree joins the community of the other node. Otherwise, the communities remain unchanged.

![Algorithm 1 SCoDA](image)

**Algorithm 1 SCoDA**

**Require:** List of edges $E$ between nodes $\{1, ..., n\}$ and parameter $D \geq 1$

1. For all $i = 1, ..., n$, $d_i \leftarrow 0$ and $c_i \leftarrow i$
2. Shuffle the list of edges $E$
3. **for** $j = 1, ..., |E|$ **do**
4. \[(u, v) \leftarrow j^{th} \text{ edge of } E\]
5. $d_u \leftarrow d_u + 1$ and $d_v \leftarrow d_v + 1$
6. **if** $d_u \leq D$ and $d_v \leq D$ **then**
7. **if** $d_u \leq d_v$ **then** $c_u \leftarrow c_v$
8. **else** $c_u \leftarrow c_v$
9. **end if**
10. **end if**
11. **end for**
12. **return** $(c_i)_{i=1,...,n}$

Observe that, in case of equality $d_u = d_v \leq D$, $v$ joins the community of $u$. Of course, this choice is arbitrary and can be made random (e.g., $u$ joins the community of $v$ with probability $1/2$ and $v$ joins the community of $u$ with probability $1/2$). Equivalently, the random shuffling of the list of edges may include for each edge $e = (u, v)$ a random choice between $(u, v)$ and $(v, u)$.

An example of execution of the algorithm on a toy network of 13 nodes and 25 edges is shown in Figure 1. Observe that this execution of SCoDA is able to perfectly recover the two underlying communities. This depends on the random shuffling of the edges, however, and another instance may give a different output. In the next section, we analyse the ability of SCoDA to detect communities in real-life graphs, as well as the variance of the results provided by different executions of the algorithm.

Note that when an edge $(u, v)$ arrives, only the community memberships of $u$ and $v$ are modified. In particular there is no propagation of a community change to the neighbors of $u$ or $v$. In Figure 1, we see for instance that, when edge 20 is streamed, only node 7 is transferred to community $C_9$. A consequence of this absence of propagation is that SCoDA is embarrassingly parallel: the execution of the algorithm can be split into tasks, each processing a subset of the edges, with $d$ and $c$ stored in a shared memory.

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*The choice of these subsets does not matter since edges are considered in random order.*

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**Figure 1:** Example of SCoDA execution on a small network: The order of arrival of the edges is indicated as a label on each edge. The table lists the communities with more than two nodes for each step of the algorithm. It shows the execution of SCoDA for $D = 4$. 
We also remark that the algorithm easily extends to weighted graph. Indeed, we can consider that each edge is drawn with a probability proportional to its weight instead of considering uniform probabilities.

2.5 Complexity

The algorithm contains three parts: the initialization of the vectors \(d\) and \(c\), which is linear in \(n\), the shuffling of the list of edges, which is linear in \(m\) with the Fisher-Yates algorithm [11], and the main loop which is also linear in \(m\). Thus, the time complexity of the algorithm is linear in \(m\) (assuming that \(m\) is larger than \(n\), which is the case in practice).

Concerning the space complexity, we only use two arrays of integers of size \(n\), \(d\) and \(c\). Note that the algorithm does not need to store the list of edges in memory, but can simply read it in a random order, which is the main benefit of the streaming approach. Hence, the space complexity of the algorithm is \(2n \cdot \text{sizeOf(int)} = O(n)\).

2.6 Degree threshold

In the rest of the paper, the only parameter of SCoDA, \(D\), is set to the mode of the degree distribution of the network, i.e., the degree that appears most often in the graph, excluding the leaf nodes. Hence we take \(D = d_{\text{mode}}\) with

\[
d_{\text{mode}} = \arg \max_{d > 1} |\{u \in V : d(u) = d\}|.
\]

(1)

This choice is justified in Section 5.

Note that the computation of \(d_{\text{mode}}\) for a given graph is linear in the number of edges \(m\) and can be done in a streaming way like SCoDA (but before the execution of SCoDA). Indeed, it is sufficient to know the degree of each node; computing \(d_{\text{mode}}\) then requires \(n\) comparisons.

3 EXPERIMENTAL RESULTS

3.1 Datasets

We use real-life networks provided by the Stanford Social Network Analysis Project (SNAP [40]) for the experimental evaluation of SCoDA. These datasets include ground-truth community memberships that we use to measure the quality of the detection. We consider datasets of different natures:

- **Social networks**: The YouTube, LiveJournal, Orkut and Friendster datasets correspond to social networks [2][23] where nodes represent users and edges connect users who have a friendship relation. In all these networks, users can create groups that are used as ground-truth communities in the dataset definitions.

- **Co-purchasing network**: The Amazon dataset corresponds to a product co-purchasing network [21]. The nodes of the graph represent Amazon products and the edges correspond to frequently co-purchased products. The ground-truth communities are defined as the product categories.

- **Co-citation network**: The DBLP dataset corresponds to a scientific collaboration network [2]. The nodes of the graph represent the authors and the edges the co-authorship relations. The scientific conferences are used as ground-truth communities.

The size of these networks ranges from approximately one million edges to more than one billion edges. It enables us to test the ability of SCoDA to scale to very large networks. The characteristics of these datasets can be found in Table 1.

| Dataset | \(|V|\)  | \(|E|\)  | # communities |
|---------|--------|--------|--------------|
| Amazon  | 334,863| 925,872| 311,782      |
| DBLP    | 317,080| 1,049,866 | 1,449,666  |
| YouTube | 1,134,890| 2,987,624 | 8,455,253   |
| LiveJournal | 3,997,962| 3,468,189 | 137,177     |
| Orkut   | 3,072,441| 117,185,083| 49,732      |
| Friendster | 65,608,366| 1,806,067,135| 2,547       |

Table 1: SNAP datasets used for the benchmark on real networks

3.2 Benchmark algorithms

For assessing the performance of SCoDA we use a wide range of state-of-the-art algorithms that are based on various approaches:

- **SCD** (S) partitions the graph by maximizing the WCC, which is a community quality metric based on triangle counting [31].

- **Louvain** (L) is based on the optimization of the well-known modularity metric [5].

- **Infomap** (I) splits the network into modules by compressing the information flow generated by random walks [32].

- **Walktrap** (W) uses random walks to estimate the similarity between nodes, which is then used to cluster the network [30].

- **OSLOM** (O) partitions the network by locally optimizing a fitness function which measures the statistical significance of a community [20].

3.3 Performance metrics

We use two metrics for the performance evaluation of the selected algorithms. The first is the average \(F1\)-score [39][31]. Given an estimate \(\hat{C}\) of a true community \(C\), the precision and recall of this estimation, that respectively penalize false positive and false negative, are defined as:

\[
\text{Precision}(\hat{C}, C) = \frac{|\hat{C} \cap C|}{\lvert \hat{C} \rvert}, \quad \text{Recall}(\hat{C}, C) = \frac{|\hat{C} \cap C|}{\lvert C \rvert}.
\]

The \(F1\)-Score of the estimation \(\hat{C}\) of \(C\) is then defined as the harmonic mean of precision and recall:

\[
F1(\hat{C}, C) = \frac{2 \cdot \text{Precision}(\hat{C}, C) \cdot \text{Recall}(\hat{C}, C)}{\text{Precision}(\hat{C}, C) + \text{Recall}(\hat{C}, C)}.
\]

Now consider some partition of the graph into \(K\) communities, \(C = \{C_1, \ldots, C_K\}\). The \(F1\)-Score of the partition \(\hat{C} = \{\hat{C}_1, \ldots, \hat{C}_L\}\) with respect to \(C\) is defined by:

\[
\text{F1}(\hat{C}, C) = \frac{1}{K} \sum_{k=1}^{K} \max_{\ell \leq L} \text{F1}(\hat{C}_\ell, C_k).
\]
Finally, the average F1-Score between the set of detected communities \( \hat{C} \) and the set of ground-truth communities \( C \) is:

\[
\text{F1}(\hat{C}, C) = \frac{\text{F1}(\hat{C}, C) + \text{F1}(C, \hat{C})}{2}
\]

The second metric we use is the Normalized Mutual Information (NMI), which is based on the mutual entropy between indicator functions for the communities [18].

### 3.4 Benchmark setup

The experiments were performed on EC2 instances provided by Amazon Web Services of type m4.4xlarge with 64 GB of RAM, 100 GB of disk space, 16 virtual CPU with Intel Xeon Broadwell or Haswell and Ubuntu Linux 14.04 LTS.

SCoDA is implemented in C++ and the source code can be found on GitHub\(^3\). For the other algorithms, we used the C++ implementations provided by the authors, that can be found on their respective websites. Finally, all the scoring functions were implemented in C++. We used the implementation provided by the authors of [18] for the NMI and the implementation provided by the authors of SCD [31] for the F1-Score.

### 3.5 Benchmark results

**Execution time.** We compare the execution times of the different algorithms on SNAP networks in Table 2. The entries that are not reported in the table corresponds to algorithms that returned execution errors or algorithms with execution times exceeding 6 hours. In our experiments, only SCD, except from SCoDA, was able to run on all datasets. The fastest algorithms in our benchmarks are SCD and Louvain and we observe that they run more than ten times slower than our streaming algorithm. More precisely, SCoDA runs in less than 50ms on the Amazon and DBLP networks, which contain millions of edges, and in 5 minutes on the largest network, Friendster, that has more than one billion edges. In comparison, it takes seconds for SCD and Louvain to detect communities on the smallest networks, and several hours to run on Friendster. Figure 2 shows the execution times of all the algorithms with respect to the number of edges in the network. We remark that there is more than one order of magnitude between SCoDA and the other algorithms.

In order to compare the execution time of SCoDA with a minimal algorithm that only reads the list of edges without doing any additional operation, we measured the run time of the Unix command `cat` on the largest dataset, Friendster. `cat` reads the edge file sequentially and writes each line corresponding to an edge to standard output. In our experiments, the command `cat` takes 152 seconds to read the list of edges of the Friendster dataset, whereas SCoDA processes this network in 314 seconds. That is to say, reading the edge stream is only twice faster than the execution of SCoDA.

**Memory consumption.** We measured the memory consumption of SCoDA and compared it to the memory that is needed to store the list of the edges for each network, which is a lower bound of the memory consumption of the other algorithms. We use 64-bit integers to store the node indices. The memory needed to represent the list of edges is 14.8 MB for the smallest network, Amazon, and

### 3.6 Variance of the algorithm

SCoDA is not a deterministic algorithm as it depends on a random permutation of the list of the edges. We study its variance over multiple runs by computing the standard deviation of the F1-Score.

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\(^3\)https://github.com/ahollocou/scoda
and of the number of communities over 1000 independent runs. The results are collected in Table 5. We see that the standard deviation is 100 times lower than the average value of these metrics. Note that these experiments were only run on the smaller datasets because the run times on bigger networks were prohibitive.

4 THEORETICAL ANALYSIS

In this section, we provide a theoretical analysis of SCoDA explaining its good performance. We consider successively the precision and recall of the algorithm.

4.1 Notation

Observe that, as far as the analysis is concerned, we can assume that the algorithm performs at each step a random sampling without replacement of the edges, instead of the initial random shuffling. When an edge \( e = (u, v) \) is drawn, there are two cases depending on the degrees \( d_u \) and \( d_v \).

- If \( d_u \leq D \) and \( d_v \leq D \) then \( u \) joins the community of \( v \) or conversely, \( v \) joins the community of \( u \). In this case we say that \( e \) is a transfer edge and we use \( t(e) \) to denote this event.
- Otherwise, \( u \) and \( v \) remain in their communities. We say that \( e \) is a blank edge.

### 4.2 Precision

In this part of the analysis, we are interested in the false positives detected by SCoDA.

Let \( C \subset V \) be the community that we want to detect. Let \( S \) be a community returned by SCoDA such that \( S \cap C \neq \emptyset \). Note that there is necessarily such a \( S \) because the algorithm performs a partition of the set of nodes \( V \). We say that node \( u \in V \) is a false positive in \( S \) with respect to community \( C \) if \( u \in S \setminus C \). Note that if SCoDA returns a false positive, then one of the edges between \( C \) and \( \overline{C} = V \setminus C \) is necessarily a transfer edge. Here, we study the quantity of such edges that can lead to false positives, that we call false-positive edges. We use \( FPE(C) \) to denote their number:

\[
FPE(C) = |\{e \in e(C, \overline{C}) : t(e)\}|
\]

Let \((u, v)\) be an edge of \( e(C, \overline{C}) \). In what follows, we consider by convention that, for such edges, \( u \in C \) and \( v \in \overline{C} \). We observe that if the first \( D \) edges that are drawn in \( e((u)) \) (i.e., edges with an end equal to \( u \)) are in \( e(C, C) \) (intra-community edges), then \((u, v)\) cannot be a transfer edge because \( d_u > D \) when \((u, v)\) is streamed. The probability for the \( k^{th} \) edge in \( e(u) \) to be in \( e(C, C) \) knowing that the previous edges were in \( e(C, C) \) is:

\[
1 - \frac{d^-(u)}{d(u) - k}
\]

Therefore, the expected value of \( FPE(C) \) satisfies:

\[
\mathbb{E}[FPE(C)] = \sum_{e \in e(C, \overline{C})} \mathbb{P}[t(e)] \\
\leq \sum_{(u, v) \in e(C, \overline{C})} 1 - \prod_{k=0}^{D-1} \left(1 - \frac{d^-(u)}{d(u) - k}\right)
\]

The quantity \( d^-(u)/d(u) \) is known as the Out Degree Fraction (ODF) of node \( u \) for the community \( C \). We use ODF\((u, C)\) to denote this quantity. Observe that a good community corresponds to low values of ODF\((u, C)\) for \( u \in C \).

The ratio of the expected number of false positive edges to the total number of edges in the community satisfies:

\[
\frac{\mathbb{E}[FPE(C)]}{|e(C)|} \leq \phi_0(C) \left[1 - \frac{\sum_{(u, v) \in e(C, \overline{C})} \prod_{k=0}^{D-1} \left(1 - \frac{\text{ODF}(u, C)}{|e(C)|} \right) k}{|e(C)|}\right],
\]

where \( \phi_0(C) \) is the pseudo-conductance defined in Section 2. Note that the term \( \prod_{k=0}^{D-1} \left(1 - \frac{\text{ODF}(u, C)}{|e(C)|} \right) k \) is null if \( D > d_C(u) \). We remark that the lower the pseudo-conductance \( \phi_0(C) \) and the Out Degree Fraction ODF\((u, C)\) for \( u \in C \), the less false-positive edges are observed. For good communities \( C \), these quantities are typically small, leading to a good precision.

### 4.3 Recall

Now we analyze the performance of SCoDA in terms of recall, i.e., its ability to recover all the nodes of a given community \( C \).

**Intuition.** We would like to have a low probability of splitting a community \( C \) into several sub-communities. We remark that SCoDA may split a community \( C \) into two sub-communities \( C_1 \) and \( C_2 \) if the edges \((u, v) \in e(C_1, C_2) (u \in C_1 \text{ and } v \in C_2) \) satisfy...
$d_{C_1}(u) > D$ or $d_{C_2}(v) > D$. In this case, when $(u, v) \in e(C_1, C_2)$ is processed, we can potentially have $d_u > D$ or $d_v > D$.

Now if the community $C$ is homogeneous, we have typically $d_{C_1}(u) \approx \bar{d}|C_1|$ and $d_{C_2}(u) \approx \bar{d}|C_2|$, where $\bar{d}$ corresponds to the average intra-community degree in $C$. Therefore, our community $C$ is potentially split into $C_1$ and $C_2$ if:

$$\frac{|C_1|}{|C|} > D \text{ or } \frac{|C_2|}{|C|} > D$$

In particular, if the parameter $D$ is close or larger than $\bar{d}$, then these inequalities are not satisfied and the probability of splitting $C$ into sub-clusters is low.

**Analysis on random graphs.** The previous argument is not rigorous and only provides an insight into the behavior of SCoDA. In order to study the ability of SCoDA to recover an entire community, we study its results on a random graph model. We represent a community as a small, homogeneous and well-connected random graph. For such a random graph, we expect SCoDA to return the entire graph as a community. Hence, we measure the average value of the relative size of the largest community returned by the algorithm:

$$\max_{C \in C} \frac{|C|}{n}$$

We choose the simplest random graph model, the Erdős–Rényi graph [15] (other experiments, not reported here, showed the same type of results on the configuration model [24][26]). Recall that this model has two parameters $n$ and $p$: $n$ is the number of nodes, and each edge $(u, v)$ is included in the graph with probability $p$ independently of every other edge. Although this model is inappropriate for modeling entire real-life networks, it is reasonable for representing small communities.

We experimentally generate graphs using this model for different values of $n$ and $p$. We plot the average value of the relative size of the largest community returned by SCoDA. For each value of $(n, p)$, 1000 graphs were generated and SCoDA was run once on each graph. The degree threshold $D$ was set to $d_{\text{med}}$ as in previous experiments.

We see that the relative size of the largest community is close to 1 when the parameter $p$ tends to 1. The case $p = 1$ corresponds to the situation where the graph is complete and where we want to recover the densest possible community, the clique. We see that SCoDA is able to recover almost the entire community in this situation. Besides, as soon as $p > 0.5$, SCoDA recovers more than 75% of the nodes in its largest community.

5 **SETTING THE DEGREE THRESHOLD**

In this section, we give a deeper insight into the choice of the only parameter of SCoDA, the degree threshold $D$.

5.1 **Some strategies**

Consider the dynamic graph $H$ having the same nodes as $G$ and whose edges are those successively considered by SCoDA. Note that $D$ corresponds to the maximum updated degree in this graph until a change in the communities occurs. Indeed, when an edge $(u, v)$ is streamed, there is a change in the communities if and only if both $d_u$ and $d_v$ are lower than $D$.

![Figure 3: Average relative size of the largest community returned by SCoDA on an Erdős-Rényi random graph of parameters $(n, p)$](image)

As observed before, with high probability, intra-community edges are streamed before inter-community edges. If $D$ is too low, SCoDA might not take some intra-community edges into account and can potentially split communities into sub-clusters. Remark that, if $D = 1$, then the algorithm only outputs communities with at most two nodes. On the contrary, if $D$ is too high, community transfers due to inter-community edges will occur and deteriorate the quality of the detected communities.

Hence, $D$ is intuitively related to the degree distribution of the graph $G(V, E)$. We can consider several options for the choice of this parameter:

- **Average degree**: $D = d_{\text{avg}}$, the average value of the degree $d(u)$ over the network;
- **Median degree**: $D = d_{\text{med}}$, the median value of the degree $d(u)$ over the network;
- **Mode of the degree distribution**: $D = d_{\text{mode}}$, the most common degree in the network excluding leaf nodes, as defined by (1).

Recall that we have chosen $D = d_{\text{mode}}$ until now. We justify this choice below both experimentally on SNAP datasets and theoretically using the previous analysis.

5.2 **Experimental analysis**

We evaluate the performance of SCoDA as a function of $D$ on real-life networks. For this purpose, we perform experiments on the SNAP datasets introduced above and we use the Average F1 Score to measure the quality of the detected communities. In order to evaluate the accuracy of the different choices of $D$, we consider the relative quality ratio $Q$ between the F1-score obtained for a given $D$, noted $\text{F1}(D)$, and the maximum of this score observed for any
value of $D$:

$$Q(D) = \frac{F_1(D)}{\max_{D'} F_1(D')}$$

In Figure 4, we plot the Average F1-Score of SCoDA with respect to $D$. Note that, the singletons returned by SCoDA were excluded from the computation of the F1-Score in order to boost the execution time of this scoring metric for different values of $D$, which explains why the scores slightly differ from the ones listed in Table 3. As expected, we observe that the F1-Score first increases with $D$ until it reaches a maximum value, and then decreases as $D$ continues to increase.

In Figure 5, we plot the ratio $Q(D)$ defined above for the three choices for $D$ listed above (average, median and mode). We see that $Q(D) > 0.9$ for $D = d_{\text{mode}}$ whereas $Q(D)$ shows poor values for the other choices of $D$ on the social networks datasets LiveJournal, Orkut and Friendster. This justifies experimentally our previous choice.

Table 6 collects different statistics on the degree distribution of the datasets, including the values of $d_{\text{avg}}$, $d_{\text{med}}$ and $d_{\text{mode}}$. We remark that for YouTube, LiveJournal, Orkut and Friendster the density ($\frac{m}{n(n-1)}$) is lower and the maximum degree $d_{\text{max}}$ is higher than for the DBLP and Amazon datasets. The different nature of the networks could explain the differences in these statistics and in the behavior of SCoDA. On the one hand, we have social networks where the performance of SCoDA decreases drastically when $D$ increases, and, on the other hand, we have a co-citation and a co-purchasing networks for which the F1-Score decreases much more slowly.

Note that one could ask why we do not simply use a fixed value for $D$ (e.g., $D = 2$) but experiments on the random graphs defined in §4.2 shows that it deteriorates the recall of the algorithm.

5.3 Theoretical insights

In Section 4, we observed that the ODFs of nodes at the boundary of community $C$ need to be higher than $D$ in order to obtain few false positive edges. Besides, we have seen that $D$ must be close to the intra-community degree in $C$ in order to decrease the likelihood for $C$ to be split into sub-clusters by SCoDA. These arguments suggest that the parameter $D$ should be close to the typical value of intra-community degrees in the network. Since most nodes have few inter-community links, this is well approximated by the most probable degree in the graph, that is the mode of the degree distribution as chosen in our experiments. Observe that the average and the median are not typical values of the degree of a node, which may explain the worse performance of SCoDA in these cases.

6 CONCLUSION AND FUTURE WORK

We introduced a novel approach for community detection based on a random stream of edges. This approach is based on simple properties of such edge streams, that are closely related to important concepts in network analysis such as conductance and out-degree fraction. We designed an algorithm, named SCoDA, that stores only two integers for each node and runs linearly in the number of edges. In our experiments, SCoDA runs more than 10 times faster than
state-of-the-art algorithms such as Louvain and SCD and shows better detection scores on the largest networks. Thus SCoDA would be useful in many applications where massive graphs arise. For example, the web graph contains around $10^{10}$ nodes which is much more than in the Friendster dataset.

While we evaluated the performance of the algorithm on static graphs only, it would be interesting for future work to measure the ability of SCoDA to handle evolving networks by conducting benchmarks on dynamic datasets [28] with existing approaches [14][9]. Note that modifications to the algorithm design could be made to handle events such as edge deletions.

Another interesting research direction would be to exploit the fact that, between two runs of SCoDA, the transfer-edges and the blank-edges can change. For each edge of the network, we could count how many times it corresponds to a transfer-edge over several runs and use this result to refine the community detection with, for instance, a boosting aggregating approach [6].

Furthermore, we remark that the condition $d_{old} \leq D$ and $d_{new} \leq D$ plays an important role in the definition of the algorithm. Future works could explore different ways to define when an edge arrives early or late. For instance, the general condition $f(d_{old}, d_{new}) \leq D$, could lead to different results for certain choices of $f$.

Finally, SCoDA only returns disjoint communities, whereas, in many real networks, overlaps between communities can be observed [18]. An important research direction would consist in adapting SCoDA to overlapping community detection and compare it to existing approaches [38][39].

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