1. Introduction

With the advent and development of nanostructure technology, a variety of nanostructures with complex geometries were successfully fabricated, such as corrugated semiconductor films [1–6], rolled-up nanotubes [7–12], Möbius stripes [13–15], peanut-shaped C₆₀ polymers [16–20]. These successes in experiment found the basis of the emerging nanoelectronics. In the two-dimensional (2D) reduced systems, the dynamics of a confined electron is affected by the surface curvature. As an important consequence, the geometric potential induced by the curvature appears in the effective surface quantum equation [21–24] by the thin-layer quantization procedure [25–28]. In the procedure, a squeezing potential is introduced to accomplish the reduction of the number of the spatial variables of the curved systems. Physically, the squeezing process is probably severe and breaks with the natural limits set by the uncertainty principle. Even so, the present quantization scheme can be safely applied to study the motion of confined electrons when the quantum excitation energies in the normal direction are raised far beyond those in the tangential direction. Actually, the thin-layer quantization method has successfully been employed to calculate the band-structure of real systems [29–31], determine the localized surface states in geometrically deformed quantum systems [32–38], and study the transport properties of electrons confined in systems with complex geometries [39–42]. Furthermore, the experimental evidences for the geometrical effects of the curved surface have been presented, such as the realization of an optical analog of the curvature-induced geometric potential [43], the observation of the influence of geometry on the proximity effect [44] and the observation of Riemannian geometrical effects on electronic states [19]. In other words, the geometrical deformation can be concluded as the presence of geometric potential in the dimensionally reduced quantum equation.

In the same vein, the advent and development of nanostructures have clearly led to a frontier field in 2D semiconductor research [45–50]. It is now possible to design and fabricate materials with prescribed electronic and photonic properties by artificial band-gap engineering. In order to satisfy the requirement of the development of nanoelectronics, theoretical physicists have tried to study the effects of the geometrical deformation on the tunneling rate [51], the electrical resistivity [52, 53] and the persistent current [54, 55]. Moreover, on the basis of the geometric potential, quantum-electromechanical circuits [56–58] and thin film transistors [59] have been proposed.
In fact, over the years, the electronic properties of the periodically curved surface have been a subject of active studies, both theoretical and experimental, due to the demand in understanding the physics involved and its great application potential. The ability to confine electrons nearly to 2D regions on nanostructures has given a new impetus toward understanding the physics of reduced dimensionality systems. An important property of nanostructures is how their geometries affect their behaviors. Usually, one thinks of geometrical effects being connected primarily with the physical size and barrier configuration that define the confining region of the nanostructure.

In the present study, we consider a model [51] (shown in figure 1) which contains two barriers and a well, the well is fabricated as a periodically corrugated thin layer. In the model, $R_1$ denotes the free electron beam source, $R_2$ is a barrier with width $d = 1 \, \text{Å}$, $R_3$ is a corrugated thin layer with width $L = 1000 \, \text{Å}$, $R_4$ is the other barrier with width $d = 1 \, \text{Å}$, and $R_5$ denotes a drain, from left to right. The double barriers’ resonant tunneling structure is a typical microstructure that has been the focus of many investigations [60–65]. In the present model, the corrugations are employed to provide transmission gaps and resonant tunneling domains, the double barriers and the boundaries between adjacent regions, in which the electron has different effective masses, are adopted to generate the tunneling peaks and valleys in the tunneling domains. The theoretical basis is that when the spatial dimension is reduced to a scale being comparable with the de Broglie wavelength of the electron in the vicinity of the Fermi energy in the model, the wave nature of the electron is expected to play an increasingly important role in transport properties.

This paper is organized as follows. In section 2, we briefly review the effective quantum equation for the electron confined in a periodically corrugated layer by the thin-layer quantization scheme, and analyze the curvature-induced geometric potential. In section 3, we obtain a new geometric potential by the extended thin-layer quantization scheme, and also analyze the thickness-modified geometric potential. In section 4, we investigate the effects of the corrugations, double barriers and boundaries on transmission probability. Finally in section 5 the conclusions are given.

2. Quantum dynamics of a particle confined on a periodically corrugated surface

A 2D curved surface $S$ embedded in the usual three-dimensional (3D) space can be parametrized by $\mathbf{r} = \mathbf{r}(q_1, q_2)$, where $q_1$ and $q_2$ are the curvilinear coordinate variables over $S$. With respect to $q_1$ and $q_2$, two unit basis vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ over $S$ are defined by $\mathbf{e}_1 = \left. \frac{\partial \mathbf{r}}{\partial q_1} \right|_{q_2}$ and $\mathbf{e}_2 = \left. \frac{\partial \mathbf{r}}{\partial q_2} \right|_{q_1}$, respectively. By introducing a curvilinear coordinate variable $q_3$ along the direction normal to $S$, a 3D subspace $V_N$ consisting of points near to $S$ and on $S$ can be described by $\mathbf{R}(q_1, q_2, q_3) = \mathbf{r}(q_1, q_2) + q_3 \mathbf{e}_3(q_1, q_2)$, where $\mathbf{e}_3(q_1, q_2)$ is the unit basis vector perpendicular to $S$, with the definition $\mathbf{e}_3 = \left( \mathbf{e}_1 \times \mathbf{e}_2 \right)$. In $V_N$, the covariant components of the metric tensor are defined by $g_{ij} = \left. \frac{\partial \mathbf{r}}{\partial q_i} \cdot \frac{\partial \mathbf{r}}{\partial q_j} \right|_{q_2}$, $i, j = 1, 2, 3$. On $S$, the covariant components of the reduced metric tensor are determined by $g_{ab} = \left. \frac{\partial \mathbf{r}}{\partial q_a} \cdot \frac{\partial \mathbf{r}}{\partial q_b} \right|_{q_2}$ ($a, b = 1, 2$). The relationships between $G_{ab}$ and $g_{ab}$ are

$$G_{ab} = g_{ab} + (\alpha g + g^{T} \alpha T)_{ab}q_3 + (\alpha g \alpha^{T})_{ab}(q_3)^2,$$

and $G_{aa} = G_{aa} = 0$, $G_{ai} = 1$, where $T$ denotes the matrix transpose, $\alpha$ is the Weingarten curvature matrix

$$\alpha = \frac{1}{8} \begin{pmatrix} g_{23} h_{21} - g_{21} h_{23} & g_{23} h_{21} - g_{21} h_{23} & g_{23} h_{21} - g_{21} h_{23} \\ g_{21} h_{23} - g_{23} h_{21} & g_{21} h_{23} - g_{23} h_{21} & g_{21} h_{23} - g_{23} h_{21} \\ g_{23} h_{21} - g_{21} h_{23} & g_{23} h_{21} - g_{21} h_{23} & g_{23} h_{21} - g_{21} h_{23} \end{pmatrix},$$

wherein $h_{ab}$ are the coefficients of the second fundamental form, $h_{ab} = \mathbf{e}_n \cdot \frac{\partial \mathbf{R}}{\partial q_a \partial q_b}$. By means of $\alpha$, the mean curvature $M$ is $M = \frac{1}{2} \text{Tr}(\alpha)$ and the Gaussian curvature $K = \det(\alpha)$, and then the relation between $G = \det(G_{ab})$ and $g = \det(g_{ab})$ is $G = f^2 g$ with $f = 1 + 2Mg_1 + K(q_3)^2$.

Based on the above mathematical formula, we can confine a free electron on a curved surface by the thin-layer quantization scheme [25, 26], the effective surface Schrödinger equation is obtained as

$$-\frac{\hbar^2}{2m} \frac{\partial}{\sqrt{g}} \big( \sqrt{g} g^{ab} \partial_b \chi_i \big) + V_g \chi_i = E \chi_i,$$

where $V_g$ is the geometric potential

$$V_g = -\frac{\hbar^2}{2m} (M^2 - K).$$

Practically, nanocorrugated thin-films are often found in nanomaterial experiments. For the sake of simplicity, a curved surface $S$ shown in figure 2 is considered. It is corrugated...
along the direction of $x$ with period $2\pi/\gamma$, amplitude $a$, but is flat along that of $y$. In the Monge form, $S$ can be described as

$$
\mathbf{r} = \mathbf{e}_x x + \mathbf{e}_y y + a \cos(\gamma x). \tag{5}
$$

According to the procedure mentioned above, from equation (5) we obtain three unit basis vectors and the three corresponding derivative elements

$$
\begin{align*}
\mathbf{e}_x &= w \mathbf{e}_x + w a \gamma \sin(\gamma x) \mathbf{e}_y, & d\xi &= \frac{1}{w} dx, \\
\mathbf{e}_y &= \mathbf{e}_y, & d\eta &= dy, \\
\mathbf{e}_z &= -w a \gamma \sin(\gamma x) \mathbf{e}_x + w \mathbf{e}_y, & d\zeta &= d\eta,
\end{align*}
$$

respectively, with

$$
w = \frac{1}{\sqrt{1 + a^2 \gamma^2 \sin^2(\gamma x)}}. \tag{6}
$$

Subsequently, we obtain $(G_{ab})$ and $G$

$$
(G_{ab}) = \begin{pmatrix}
(1 + q_\eta^2) & 0 \\
0 & 1
\end{pmatrix}, \quad G = (1 + q_\eta^2)^2, \tag{7}
$$

the Weigarten curvature matrix $\alpha$

Figure 3. (a) Surface of $V_g$ with $n = 3$, as a function of $\gamma x$ and $a$. (b) Slices of the geometric potential $V_g$ at $a = 5$ nm, 10 nm, 20 nm, 30 nm, as a function of $\gamma x$. (c) Slices of the geometric potential $V_g$ at $\gamma x = \pi, 1.1 \pi, 1.2 \pi, 1.3 \pi$, as a function of $a$.

$$
\alpha = \begin{pmatrix}
\kappa & 0 \\
0 & 0
\end{pmatrix}. \tag{8}
$$

Figure 4. Schematic of a periodically curved layer with a uniform thickness $h$. The standard surface of the layer is described by $z = a \cos(\gamma x)$. Here $a$ and $2\pi/\gamma$ are the amplitude and period length of corrugations, respectively. $(\xi, \eta)$ are the two curvilinear coordinates over the layer.

$$
\begin{pmatrix}
\kappa & 0 \\
0 & g
\end{pmatrix} \quad g = 1, \tag{10}
$$

where $k = w^3 a \gamma^2 \cos(\gamma x)$. In the above calculated process, the factor $f$, the mean curvature $M$ and the Gaussian curvature $K$ can be given as

$$
M = \frac{\kappa}{\sqrt{\kappa^2 + g^2}}, \quad K = \frac{\kappa}{\sqrt{\kappa^2 + g^2}}. \tag{11}
$$
Consequently, the expected Schrödinger equation (3) is

\[ \frac{\hbar^2}{2m^*} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \chi_z = E_z \chi_z, \]

where \( m^* \) denotes the effective mass of the electron, and \( V_g \) is the geometric potential

\[ V_g = -\frac{\hbar^2}{8m^*} \frac{[\alpha \gamma^2 \cos(\gamma x)]^2}{[1 + \alpha^2 \gamma^2 \sin^2(\gamma x)]^{3/2}}. \]

These results are the same as those given by Ono and Shima [52]. The quantum motion in the normal direction is neglected, because the introduced squeezing potential raises the quantum excitation energies in normal direction far beyond those in tangential direction [25, 26].

As the central result of the thin-layer quantization scheme, the geometric potential appears in the expectant quantum equation. With \( n = 3 \) and \( m^* = 0.067m_0 \) (0.067 is the effective mass of the electron in the GaAs substrate) the geometric potential \( V_g(x) \) shown in figure 3 is a function of \( \gamma x \) and \( a \), where \( n \) denotes the period number of the corrugations in \( R_3 \), \( m_0 \) is the mass of a free electron. As observed in figure 3(a), the \( x \) dependence of \( V_g(x) \) deviates considerably from a cosinusoidal surface, whereas the surface corrugation is exactly cosinusoidal. It is shown in figure 3(b) that the downward peaks are formed at \( \gamma \pi = \pm \pi, 3\pi, 5\pi, \ldots \), here the height of the surface \( S \) is either maximum \( (z = a) \) or minimum \( (z = -a) \). The amplitude of the peaks grows sharply with increasing \( a \) that is highlighted in figures 3(b) and (c). As a consequence, we can provide a list of attractive potential wells by introducing corrugations, tune their depths by \( a \), and design their number by \( n \).

3. The modification of the layer thickness to the geometric potential

Really, the corrugated layer has a certain thickness. The thickness effects on the effective quantum equation (12) can be investigated by the extended thin-layer quantization scheme [28]. In the present model the thickness effects on the kinetic
term can be neglected, because the thickness lengthens the displacement of the electron across the layer very slightly, whereas a thickness-modified geometric potential $V'_g$ can be given as

$$V'_g = V_g + \frac{\hbar^2}{4m'} \left[ a^2 \gamma^2 \cos^2(\gamma x) \right]^\frac{3}{2} h,$$

(14)

where $h$ denotes the layer thickness sketched in figure 4 and $V_g$ is the geometric potential (13). The second term on the right-hand side of equation (14) is attributed to the layer thickness. It needs to be noted that $h$ must be less than the minimum curvature radius on $S$. For an arbitrary point on $S$, there is only one principle curvature $k = w^2a^2\gamma^2\cos(\gamma x)$, the corresponding curvature radius is

$$\rho = \frac{[1 + a^2\gamma^2 \sin^2(\gamma x)]^3}{a^2 \gamma^2 \cos(\gamma x)}.$$ 

(15)

It is obvious that the minimum curvature radius is $\rho_{\text{min}} = 1/(a^2\gamma^2)$ at $\gamma x = n\pi$ ($n = 0, \pm 1, \pm 2, \cdots$). When $h = \rho_{\text{min}}$, at new points (they have a uniform distance $\rho_{\text{min}}$ to $S$) $\rho = 0$, the corresponding mean curvature $M$ becomes $\infty$. In order to avoid the trouble, we primitively define $h$ ranging between 0 and 1/(2a$^2\gamma^2$).

As an important result for the extended thin-layer quantization scheme [28], the modification of the layer thickness is included in $V'_g(x)$, as a function of $\gamma x$ and $h$, described in figure 5 with $n = 3$ and $a = 12$ nm. As mentioned above, the downward peaks are still formed at $\gamma x = \pm n\pi (l = 0, 1, 2, \cdots)$. However, there is a change shown in figure 5(a), the downward peaks at $\gamma x = \pm (2m + 1)\pi (m = 0, 1, 2, \cdots)$ grow with increasing $h$, but those at $\gamma x = \pm 2m\pi$ dwarf, which is highlighted in figures 5(b)–(d).

The scales of $V_g(x)$ and $V'_g(x)$ are substantially influenced by $n$ described in figures 6(a) and (b), the larger the period number $n$, the deeper the peaks at $\gamma x = l\pi (l = 0, 1, 2, \cdots)$. Obviously, as shown in figures 6(c) and (d), $h$ evidently deepens the peaks at $\gamma x = (2l + 1)\pi$, but remarkably shallows those at $\gamma x = 2l\pi$. In other words, we can adjust the discrepancy between adjacent wells by depositing the layer thickness $h$ finely.

---

**Figure 6.** (a) The downward peaks of $V_g$ at $\gamma x = \pi$ versus $n$ for $a = 4$ nm, 8 nm, 12 nm. (b) The downward peaks of $V'_g$ at $\gamma x = 2\pi$ and $h = 0$ nm versus $n$ for $a = 4$ nm, 8 nm, 12 nm. (c) The downward peaks of $V'_g$ at $\gamma x = \pi$ and $h = 1$ nm versus $n$ for $a = 4$ nm, 8 nm, 12 nm. (d) The downward peaks of $V'_g$ at $\gamma x = 2\pi$ and $h = 1$ nm versus $n$ for $a = 4$ nm, 8 nm, 12 nm.
Figure 7. Transmission probability versus incident energy $E$ for the model with $V(x) = 20$ meV in $R_2$ and $R_4$, $V(x) = 0$ meV otherwise, and $m^*$ for (a) $m^* = m_0$ and (b) $m^* = 0.067m_0$.

Figure 8. Transmission probability versus incident energy $E$ for the model with the effective mass $m^* = 0.067m_0$ in $R_3$, $m^* = m_0$ otherwise. No barriers are plotted as the solid blue curve; with barriers are displayed as the dotted red curve.
4. Transmission probability in the periodically corrugated thin layer

4.1. Transmission probability

According to equation (12), with the limit \( h \to 0 \) the quantum equation for an electron in the model depicted in figure 1 can be

\[
\frac{\hbar^2}{2m^*} \frac{d}{dx} \left[ w \frac{d}{dx} \psi(x) \right] + V(x) \psi(x) = E \psi(x),
\]

(16)

where \( m^* = 0.067m_0 \) (the effective mass of the electron in the GaAs substrate) if \( x \in R_3 \) and \( m^* = m_0 \) otherwise, \( w = \sqrt{1 + a^2 \gamma^2 \sin^2(\gamma x)} \) if \( x \in R_3 \) and \( w = 1 \) otherwise, \( \psi \) is a wave function, \( E \) is the energy with respect to \( \psi \) and \( V(x) \) is

\[
V(x) = \begin{cases} 
0 \text{ meV}, & x \in R_1, \\
20 \text{ meV}, & x \in R_2, \\
V_3(x), & x \in R_3, \\
20 \text{ meV}, & x \in R_4, \\
0 \text{ meV}, & x \in R_5,
\end{cases}
\]

(17)

wherein \( V_3(x) \) is the geometric potential (13) and meV denotes milli electron volts. When the effects of the layer thickness are considered, \( V_3(x) \) should be replaced by \( V_3'(x) \) in equation (14).

With the help of the transfer matrix technique [66], instead of dealing with continuous variations of \( V(x) \) in \( R_3 \), we split \( R_3 \) up into segments, in each segment \( V(x) \) can be regarded as a constant. And then let us assume \( R_3 \) will be a sequence \( N_3 \) small segments, \( R_2 \) one segment \( (N_2 = 1) \) and \( R_4 \) one segment \( (N_4 = 1) \). It is straightforward to obtain that the total number of segments is \( N = N_2 + N_3 + N_4 \), and that of boundaries is \( N + 1 \). For an arbitrary segment, the \( j \)th region, in which the wave function \( \psi_j(x) \) can be given by

\[
\psi_j(x) = A_j \exp[i k_j \xi_j(x)] + B_j \exp[-i k_j \xi_j(x)],
\]

(18)

where

\[
k_j = \frac{\sqrt{[2m^*(E - U_j)]}}{\hbar}.
\]

(19)

wherein \( \hbar \) is the reduced Planck constant, \( m^*_j \) is the effective mass for the electron at the middle point in the \( j \)th region, \( U_j \) is a constant, \( U_j = V(x_j^j) \). For the wave function (18), it is particular that the curvilinear coordinate variable \( \xi(x) \) as a function of \( x \) has the derivative form \( d\xi = \sqrt{1 + a^2 \gamma^2 \sin^2(\gamma x)} dx \) if \( x \in R_3 \) and \( d\xi = dx \) otherwise.
On account of the continuities of $\psi_j(x)$ and $\psi'_j(x)/m^*_j$ at each boundary, we can determine $A_j$ and $B_j$ in equation (18) by the following multiplication:

$$
\begin{bmatrix}
A_j \\
B_j
\end{bmatrix}
= \prod_{l=0}^{j-1} M_l
\begin{bmatrix}
A_0 \\
B_0
\end{bmatrix}
$$
(20)

where

$$
M_l = \frac{1}{2}
\begin{bmatrix}
(1 + s_l) \exp[-i(k_{l+1} - k_l)x_l] & (1 - s_l) \exp[-i(k_{l+1} + k_l)x_l] \\
(1 - s_l) \exp[i(k_{l+1} + k_l)x_l] & (1 + s_l) \exp[i(k_{l+1} - k_l)x_l]
\end{bmatrix}
$$
(21)

with

$$
s_l = \frac{m^*_{l+1}}{m^*_l} \frac{k_l}{k_{l+1}}.
$$
(22)

As $j = N$, the equation (20) becomes

$$
\begin{bmatrix}
A_N \\
B_N
\end{bmatrix}
= M
\begin{bmatrix}
A_0 \\
B_0
\end{bmatrix}
$$
(23)

where

$$
M = \begin{pmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{pmatrix}
= \prod_{l=1}^{N} M_l
$$
(24)

Without any loss of generality, we assume that in the model under study $R_1$ is an electron source consisting of free electrons, and $R_5$ is a drain. Using the plane wave approximation, the wave function in $R_1$ can be

$$
\psi_0(x) = \exp(ik_0x_0) + B_0 \exp(-ik_0x_0),
$$
(25)

and in $R_3$

$$
\psi_N(x) = A_N \exp(ik_Nx_N),
$$
(26)

where $B_0$ and $A_N$ are the coefficients of reflection and transmission, respectively. In the case of $A_0 = 1$, $B_N = 0$, $m^*_0 = m^*_N = m_0$ and $k_0 = k_{N+1}$, we can obtain the transmission amplitude $A_N$ and the transmission probability $T$ as

$$
A_N = \frac{1}{M_{22}},
$$
(27)

and

$$
T = \frac{1}{(M_{22})^2},
$$
(28)

respectively.
4.2. Numerical results and analysis

In this section, we will investigate how the corrugations affect the transmission probability by the transfer matrix method. Before beginning the investigation, we briefly analyze the effects of other ingredients in the model on the transmission probability. The effective masses of electron in the model are closely related to the resonant tunneling peaks as shown in figure 7. When the effective mass is less, the number of the peaks is less, their amplitudes smaller. These results are highlighted in the insets in figure 7.

In the case of the model with \( m' = 0.067m_0 \) in \( R_3 \) and \( m' = m_0 \) otherwise, the two boundaries between \( R_2 \) and \( R_3 \), \( R_3 \) and \( R_4 \) are essential to influence the transmission probability. In response to the boundaries, the amplitudes of the tunneling peaks considerably grow shown in figure 8. In other words, these peaks are mostly provided by the continuities of \( \psi(x)/m' \), the contributions of the double barriers could be neglected. It is worth noticing that the peaks almost reach 1 and the valleys are about 0.235. Physically, the peaks occur when the length of \( R_3 \) strictly equals the integer multiple times of a half wave length of the electron, the bottom of the valleys is eventually determined by the ratio of the effective masses in \( R_3 \) and \( R_2 \) (or \( R_4 \)).

In a special case, the model without the double barriers, with \( m' = 0.067m_0 \) in all regions, \( V(x) = V(x) \) in \( R_3 \) and \( V(x) = 0 \) meV otherwise, transmission gaps [67–71] and resonant tunneling domains [72–76] appear in the present system shown in figure 9. The presence of the transmission gaps is the most fascinating finding in this study. It is readily proved that the widths of the transmission gaps grow with increasing the corrugation amplitude \( a \). The cause is that the larger the distances between adjacent wells in the geometric potential \( V_g(x) \), the less the communication or coupling between adjacent wells [77]. As periodical corrugations presented in \( R_3 \), the factor \( w \) in equation (16) is a function of \( x \) as \( w = 1/\sqrt{1 + a^2\gamma^2 \sin^2(\gamma x)} \), in \( R_3 \), \( x_j \) must be replaced by \( \xi_j \)

\[
\xi_j = \int_0^{\gamma_j} \sqrt{1 + a^2\gamma^2 \sin^2(\gamma x)} \, dx.
\]  

(29)

According to this integral, the distance between adjacent wells naturally grows with increasing the corrugation amplitude. Another aspect of the transmission probability is that the transmission gaps become wider when \( n \) is larger with a fixed \( a \), the tunneling domains become narrower correspondingly. The reason is that the greater the number of wells, the stronger their reflections. The resonant tunneling domains are formed essentially by the coupling between adjacent wells, with respect to resonant energy domains [77, 78]. As an application potential, the transmission gaps mean that the electron is mostly reflected, but the tunneling domains allow the electron to pass readily.

In the considered case, the model with \( m' = 0.067m_0 \) in \( R_3 \) fabricated by the GaAs substrate and \( m' = m_0 \) otherwise, with \( V(x) \) in equation (17), the transmission probability as a function of \( E \) and \( a \) is described in figure 10 for (a) \( n = 1.5 \), (b) \( n = 2 \), (c) \( n = 2.5 \) and (d) \( n = 3 \). In striking contrast to figure 9, figure 10 shows that tunneling peaks and valleys evidently occur in the tunneling domains. Significantly, the...
transmission gaps are still kept. That is to say that the boundaries and the double barriers cannot considerably influence the transmission gaps, but they construct the tunneling peaks and valleys in the tunneling domains, especially when the amplitude $a$ is small. This result can be directly manifested by the fact that the bottom of the valleys is still a certain constant at $a = 0$, which agrees well with that described in figure 8. Additionally, it is worthwhile to note that the number of tunneling peaks in each of the tunneling domains is equivalent to that of the wells in the geometric potential.

In order to highlight the influences of the double barriers and the layer thickness $h$ to the transmission probability, in the case of $n = 3$ and $a = 14\text{nm}$, the transmission probability as a function of $E$ is plotted in figure 11. Strikingly, it shows that the transmission gaps are mostly provided by the corrugations. The peaks and valleys in the tunneling domains are mainly formed by the boundaries between adjacent regions in which the electron has different masses. And the boundaries can make the transmission gaps flatter. Trivially, the double barriers and the layer thickness affect the valleys at a small scale.

5. Conclusions

A particular component of the considered model is the presence of a periodically corrugated thin layer. The corrugated deformation contributes the geometric potential $V_g$, a sequence of attractive potential wells, to the electron across the corrugated layer. The number of wells is determined by the period number $n$ of the corrugations as $2n$. The depth of the wells grows with increasing the amplitude $a$ of the corrugations. Approximately, the geometric potential can be roughly equivalent to a sequence of square potential wells. The square wells can be structured by introducing periodically magnetic fields. In terms of the magnetic field, the filter designed for the electron with a certain energy can be named as a curvature-tunable filter [79]. By means of the surface curvature, the filter structured for the electron with a certain energy can be named as a curvature-tunable filter. When the layer thickness $h$ is considered, the potential $V_g$ should be replaced by $V'_g$, which includes the modification given by the layer thickness $h$. For the sake of application, the difference between adjacent wells can be controlled by depositing the layer thickness.

The most fascinating finding in the present study is the presence of the transmission gaps and resonant tunneling domains resulting from the periodic corrugations. In the gaps the electron is mostly reflected, in the tunneling domains the electron readily passes. Additionally, in the resonant tunneling domains the resonant splitting peaks and valleys are essentially attributed to the boundaries between adjacent regions in which the electron has different effective masses, and is slightly influenced by the layer thickness $h$. These results can provide a new route for experimenters to fabricate nanoelectronic device. As a potential application, we can control the widths of the transmission gaps by the amplitude $a$ and period number $n$ of the corrugations, design the resonant splitting peaks and valleys by depositing different materials in adjacent regions with certain thickness. Experimentally, the nanocorrugated thin films can be obtained by the detachment method [1, 2]. The corrugated nanofilms fabricated from narrow-gap and gapless semiconductors can be the most promising objects for experiment. In real physical experiments, Coulomb electron interaction, spin-orbital coupling, screening effects and atomic structure need to be considered for two-dimensional curved systems. These interesting questions need to be studied further.

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