Enhanced Tensor-Force Contribution in Collision Dynamics

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The tensor and spin-orbit forces contribute essentially to the formation of the spin mean field, and give rise to the same dynamical effect, namely spin polarization. In this paper, based on time-dependent density functional calculations, we show that the tensor force, which usually acts like a small correction to the spin-orbit force, becomes more important in heavy-ion reactions and the effect increases with the mass of the system.

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INTRODUCTION

The tensor force, which is necessary to explain the properties of the deuteron, attracts special attention recently, because it has turned out to play an essential role in the existence limit of exotic nuclei, as well as the nuclear shell structure far from the $\beta$-stability line (for example, see \textsuperscript{[1]}\textsuperscript{3}). An important feature is that the spatial average of the tensor force is exactly to zero, so that its effect is spatially localized. On the other hand, the spin-orbit force, which is necessary to explain the large spin polarizations of scattered nucleons, plays a crucial role in the nuclear shell structure. The origin of the tensor force can be found in the one-pion exchange potential, and that of the spin-orbit force in the relativistic aspects of quantum dynamics.

Thus the tensor and spin-orbit forces are quite different in their origins, while resulting in the same dynamical effect, namely, spin polarization. Spin polarization, which arises mostly from the spin-orbit force, spontaneously takes place in the early stage of heavy-ion reactions, and affects the equilibration process to a large extent. As long as the microscopic time-dependent Skyrme energy density functional (Skyrme-EDF) calculations are concerned, the appearance of spontaneous spin polarization even in central collisions between $\beta$-stable nuclei was shown, and its origin was clarified to be the time-odd part of the spin-orbit force \textsuperscript{[9]}. Therefore, the enhancement or reduction of spin polarization gives an ideal framework to pin down the properties of the tensor force in collision situations.

In this paper, the role of the tensor force in heavy-ion reactions is investigated based on time-dependent density functional calculations with explicitly implemented tensor force, where the time-odd part of the spin-orbit force is also fully taken into account. Special attention is paid to the effect of the tensor force on time evolution. As a result, some information on the importance of the contribution from the tensor force in heavy-ion reactions is presented.

THEORETICAL FRAMEWORK

Mean field due to spin-orbit and tensor forces

The contribution of the tensor force, whose role was underestimated and mostly neglected for a long time, was substantially studied in the context of Skyrme-EDF only recently \textsuperscript{[4]}-\textsuperscript{[8]}. Here we begin with the functional form of the tensor and spin-orbit forces in Skyrme-EDF. Let $\rho$, $\sigma$, and $J$ represent the number density, spin density, and spin-orbit density, respectively. The contribution of the tensor and spin-orbit forces to the energy density functional has the form

$$W_q(r) \cdot (-i)(\nabla \times \sigma)$$

(1)

where $q = n, p$ ($n$ and $p$ stand for neutron and proton, respectively). $W_q(r)$, which is called the form factor of the spin mean field, is decomposed into the contributions from spin-orbit and tensor forces.

$$W_q(r) = W^{LS}_q(r) + W^T_q(r),$$

(2)

where $W^{LS}_q(r)$ and $W^T_q(r)$ denote the form factors of spin-orbit and tensor mean fields, respectively. The contribution of the spin-orbit force to the functional \textsuperscript{[10]} is represented by

$$W^{LS}_q(r) = \frac{1}{2} W_0 (\nabla \rho(r) + \nabla \rho_q(r)) + \frac{1}{8} (t_1 - t_2) J_q(r).$$

(3)

Note that the second term on the right-hand side, whose contribution in collision situations was discussed in \textsuperscript{[11]}, has never been taken into account for some modern Skyrme parameterizations such as SLy4d and SKM*, because it makes fitting spin-orbit splittings more difficult. Although there are several versions of the tensor force, we are concerned with the natural tensor force only. Its contribution to the energy functional is represented by

$$W^T_q(r) = \alpha J_q(r) + \beta J'_{q'}(r)$$

(3)

with $q' = n, p$ satisfying $q \neq q'$, according to Stancu-Brink-Flocard \textsuperscript{[12]}, where the unique contribution of the tensor force can be found in $J'_{q'}(r)$ to $W'_q(r)$. The full
introduction of the tensor force requires to refit additionally the corresponding central-force parameters. Although the full introduction brings about largely different and complicated contributions depending on the choice of force parameter sets, it has been shown to mostly result in weakening the natural contribution. Several versions of the tensor force are compared in Ref. [14]. Here we restrict discussion to the tensor force as defined by Eq. (3), because the aim is not a discussion of the existence limit of exotic nuclei, but rather the general features of the tensor contribution in reactions. It is readily seen that the effect of the tensor force corresponds to a quantitative modification of that due to the spin-orbit force. Accordingly, the contribution of the tensor force should be discussed in association with the spin-orbit force.

### Tensor-force contribution in collision situations

A framework for measuring the effects of the tensor force is presented with a focus on collision dynamics. Concerning the spin polarization, it is reasonable to begin with a discussion of spin-orbit coupling. It is defined by the scalar triple product

\[
L \cdot S = - \frac{\hbar}{2} \left( r \times p \right) \cdot \left( \sigma + \sigma' \right)
\]

or

\[
= - ih \left( r \times \left( p + \left( \sigma + \sigma' \right) \right) \right),
\]

where \( \sigma \) and \( \sigma' \) denote the spins of the two nucleons. In collision situations \( r \times p \) is related to the impact parameter. Comparing Eqs. (1) and (3), \( W_q(r) \) in Eq. (1) plays the role of the vector \( r \) in Eq. (4), where the momentum \( p \) is replaced approximately by \( \nabla \) in the Skyrme-EDF.

In order to evaluate the contribution of the tensor force to spontaneous spin polarization, we introduce a proper theoretical setting of heavy-ion collisions. Our starting point is that the tensor and the spin-orbit forces are localized effects, which are not easy to compare in collision dynamics, if there is some similarity in their localized patterns. Let the reaction plane be \((x, z)\) with the initial collision direction \( z \), and the direction perpendicular to the reaction plane be \( y \). For simplicity, the spin direction of the initial state is assumed to be parallel to the \( y \)-axis. In this setting, because only the \( z \)-component of \( \mathbf{p} \) and the \( y \)-component of \( \mathbf{\sigma} \) are non-zero, we have

\[
L \cdot S = - i \hbar x \left( p_y (\sigma + \sigma')_z - p_z (\sigma + \sigma')_y \right)
\]

or

\[
= i \hbar x p_z (\sigma + \sigma')_y.
\]

We see that only the \( x \)-component of the vector \( r \), and thus the \( x \)-component of \( W_q(r) \) play a role. In this setting, the role of the tensor force in the spin polarization can be evaluated by the corresponding \( x \)-component of \( W^T_q(r) \). Accordingly, the tensor and spin-orbit forces can be compared, if there is a certain similarity between the \( x \)-components of \( W^T_q(r) \) and \( W^L_q(r) \) (otherwise attraction or repulsion happen irregularly from place to place). Note that their similarity, which will be shown to be true, is not trivial. In the following the \( x \)-components of \( W^T_q(r) \) and \( W_q^{L_S}(r) \) are simply represented by \( W^T_q(r) \) and \( W_q^{L_S}(r) \), if there is no ambiguity.

### ROLE OF THE TENSOR FORCE

#### Spontaneous spin polarization

A systematic three-dimensional time-dependent density functional calculation is carried out in a spatial box \( 48 \times 48 \times 48 \text{ fm}^3 \) with a spatial grid spacing of 0.8 fm, in which the Skyrme-force parameter set SV-tls [14] is used for the tensor part, and SkM* and SLy4d [16] for the remainder including the spin-orbit force: \( \alpha = 71.102 \text{ [MeV fm}^{-5} \text{]} \) and \( \beta = 35.142 \text{ [MeV fm}^{-5} \text{]} \). The parameter set SV-tls was lately introduced in the context of the refit tensor force; it is one of the most reliable parameter sets in terms of reproducing the contribution of the form factor \( W^T_q(r) \) of the tensor mean field. The relative velocity in the collisions is set to 10 % of the speed of light, and the initial distance of the colliding nuclei to 20.0 fm; their initial positions are \((0,0,10)\) and \((0,0,-10)\). In order to pay special attention to the mass-dependent general
features, we consider central collisions between identical N=Z nuclei: $^{16}\text{O} + ^{16}\text{O}$, $^{40}\text{Ca} + ^{40}\text{Ca}$ and $^{56}\text{Ni} + ^{56}\text{Ni}$. The contributions from $J_q$ and $J_{q'}$ in Eq. (3) are not so different for collisions between N=Z nuclei, therefore the parameter dependence mostly arises from the sum of $\alpha$ and $\beta$. Some features of the tensor force acting on bound nuclei were studied in [8].

Figure 1 shows the time evolution of $^{40}\text{Ca} + ^{40}\text{Ca}$ resulting in fusion, where the terms associated with the tensor force (SV-tls) are explicitly included. The same calculation without the spin-orbit force does not achieve fusion. Omitting the tensor force while including the spin-orbit force shows no notable difference to the density evolution with all force terms included. This suggests that large dissipation arises from the spin-orbit force, while the tensor-force contribution is definitely small. The composite nucleus evolves with a continuing oscillation; the two nuclei get into contact around time $= 4.2 \times 10^{-22} \text{ s}$, and the first full-overlap is achieved at $5.6 \times 10^{-22} \text{ s}$.

Let us consider the y-projection of spin for each single nucleon. The spin distribution of the colliding nuclei is calculated by their superposition:

$$P(t, r) = \rho(t, r)_{\uparrow} - \rho(t, r)_{\downarrow},$$

where $\rho(t, r)_{\uparrow}$ and $\rho(t, r)_{\downarrow}$ denote the densities of spin-up and spin-down components, respectively. In this definition, the density plays the role of weight. The value of $P(t, x)$ is positive if the spin-up component is more abundant, zero for saturated spins, and negative otherwise. As is seen from the presence of $(\sigma + \sigma')_q$ in Eq. (5), the problem of comparing the different role of tensor and spin-orbit forces becomes meaningless if spontaneous spin polarization is absent. Spin polarization appears for all the reactions and all the force parameter sets used; e.g., in Fig. 2 the presence of spin polarization is shown for $^{40}\text{Ca} + ^{40}\text{Ca}$. As a result, the concept of examining the role of the tensor force in the presence of spin polarization is valid and will be carried out in the following.

Figure 2 shows that strong spin polarization is located on the edge of the density distribution. The localized pattern of the spin structure is complicated, leading to a complicated localization of attraction and repulsion due to the tensor force. The spin distribution is point-symmetric with respect to the origin, which reflects the symmetry of the central collision. Note that the spatial average of spin polarization for the spin-saturated system is equal to zero.

**Comparison between tensor and spin-orbit forces**

Let us begin with the effect of the tensor force in a compound nucleus formed briefly after the full-overlap situation (time $= 6.0 \times 10^{-22} \text{ s}$). In case of $^{40}\text{Ca} + ^{40}\text{Ca}$, Fig. 3 compares the x-components of $W_q(r)$ for the tensor and spin-orbit forces. Both distributions are antisymmetric with respect to the z-axis, and have similar distributions but different signs and amplitudes. It is clearly seen that the tensor-force contribution is op-
The contribution of the tensor force is quite small before contact time, achieves local maximum at times $6.75 \times 10^{-22}$ s and $9.00 \times 10^{-22}$ s, and relaxes afterwards.

Several points should be remarked here. First, the tensor force contribution is enhanced in collision situations, being up to 10 times larger than before the contact time. Second, the opposite sign and the smallness of the tensor compared to the spin-orbit contributions are apparent during the heavy-ion collision but not before contact. The opposite sign means that the contribution of the tensor force continues to weaken the spin polarization during the reaction. Third, the similarity between protons and neutrons is confirmed throughout the reaction.

The tensor-force contribution is compared for different force parameter sets in Table I, where the enhancement is calculated by the ratio

$$\frac{W_p^T}{W_p^{LS}}(t = 6.5 \times 10^{-22}s),$$

where $W_p^T/W_p^{LS}(t)$ is calculated as shown in Eq. (6). This table shows that the enhancement is true independent of the choice of force parameter sets, and no significant difference exists between protons and neutrons.

### Mass dependence

As already mentioned, the opposite sign and the smallness of the tensor compared to the spin-orbit force contribution is valid independent of the mass. Note that a calculation using SkM* + SV-tls showed the same features, hinting that this is probably not strongly force-dependent.

Figure 6 shows the mass dependence of the ratio of the tensor to spin-orbit contributions for $^{40}$Ca + $^{40}$Ca. Values of Eq. (6) are calculated for different force parameter sets and isospins.

| Parameter set          | Protons ($q = p$) | Neutrons ($q = n$) |
|------------------------|-------------------|--------------------|
| SkM* + SV-tls          | 6.94              | 6.23               |
| SLy4d + SV-tls         | 6.34              | 7.11               |

This table shows that the enhancement is true independent of the choice of force parameter sets, and no significant difference exists between protons and neutrons.

The isoscalar dipole mode shown in Fig. 5 suggests that the isoscalar dipole contribution is not strongly force-dependent, and shows a relatively large tensor contribution close to the first maximum, so that it is legitimate to compare the magnitude for the three cases. While the values are not exactly the same for the two parameter sets, they show the same trend; the tensor force contribution becomes larger for reactions involving a heavier nucleus.

The relaxation of the tensor contribution is not strongly correlated with that of the isoscalar dipole oscillation (density oscillation towards the fused system). The contribution of the tensor force is quite small before contact time ($4.2 \times 10^{-22}$ s), increases after the contact time, achieves local maximum at times $6.75 \times 10^{-22}$ s and $9.00 \times 10^{-22}$ s, and relaxes afterwards.

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where $W_p^T/W_p^{LS}(t)$ is calculated as shown in Eq. (6). This table shows that the enhancement is true independent of the choice of force parameter sets, and no significant difference exists between protons and neutrons.
FIG. 5: (color online) The time evolution of the ratio of contributions from tensor force to those of the spin-orbit force is shown for protons and neutrons, respectively (upper panel), where the calculated points (at multiples $0.75 \times 10^{-22}$ s) are connected by 3rd-order spline functions. This reaction corresponds to the case shown in Fig 1. For reference, the time evolution of the isoscalar dipole (is-dipole) mode is shown by a dotted line (upper panel). The corresponding reaction-plane snapshots of the $x$-components of $W_q^T(r)$ and $W_{LS}^q(r)$ are shown in a square ($30 \times 15$ fm$^2$) (lower panel), where the maximum amplitude $A$ of the function is shown in the lower right hand side of each plot.

TABLE II: Mass dependence of the growth of spin polarization (for an explanation see text). For both parameter sets, values are normalized by the values obtained for $^{16}\text{O} + ^{16}\text{O}$.

| Parameter set         | $^{16}\text{O} + ^{16}\text{O}$ | $^{40}\text{Ca} + ^{40}\text{Ca}$ | $^{56}\text{Ni} + ^{56}\text{Ni}$ |
|-----------------------|-----------------|-----------------|-----------------|
| SkM* + SV-tls         | 1.000           | 0.396           | 0.260           |
| SLy4d + SV-tls        | 1.000           | 0.571           | 0.085           |

(SkM* + SV-tls). This is not a negligible effect considering the remarkable spin-orbit splitting in the ground states of heavy nuclei. This should have a certain impact on superheavy synthesis; the tensor force is suggested to play a considerable role in whether a heavy composite nucleus is formed successfully or not. On the other hand, the spin polarization becomes smaller for reactions involving heavier nuclei. The statistical ratio of spin polarization

$$\frac{\max_r (\rho_1(t,r) - \rho_2(t,r))}{\sum_r (\rho_1(t,r) + \rho_2(t,r))}$$

between time $= 6.0 \times 10^{-22}$ s and $1.5 \times 10^{-22}$ s, which corresponds to the amplitude of spin polarization due to the collision, is summarized in Table 11. Thus the tensor-force contribution tends to survive for the heavier cases, while the spin-orbit force contribution decreases sharply with mass. Note that there is no serious discrepancy between neutrons and protons visible in Fig. 6.

Finally, the validity of the obtained results is also confirmed by additionally examining an old tensor force parameter set proposed by Stancu-Sprung [12, 17] ($\alpha = 154.390$ [MeV fm$^{-5}$] and $\beta = 139.910$ [MeV fm$^{-5}$]). The major difference is that its amplitude is actually smaller than the spin-orbit force contribution, but reaches as much as 50% of the spin-orbit contribution in $^{56}\text{Ni} + ^{56}\text{Ni}$. The difference between the two parameters can be related to the largeness of the $\alpha$ and $\beta$ values proposed in the Refs. [12, 17] compared to Ref. [14].

CONCLUSION

Based on time-dependent density functional calculations with explicitly implemented tensor force, the role of the tensor force has been studied in the context of collision dynamics. It is remarkable that the contribution from the tensor force is enhanced in collision situations. Its contribution is mass-dependent and has considerable influence on reactions involving a heavier nucleus. As long as heavy-ion reactions between $N = Z$ identical nuclei are concerned, the opposite sign and the smallness of the tensor force contribution compared to that of the spin-orbit force has been confirmed independent of mass. In particular, the opposite sign means that the spin polarization, thus the large dissipation due to the spin-orbit force, is reduced by the tensor force. We conclude that the tensor-force contribution is rather important in heavy-ion reactions with respect to the magnitude of dissipation. The results presented in this paper give a solid starting point for future researches clarifying the role of the tensor force in heavy-ion reactions involving exotic nuclei, where the drastically different contribution...
FIG. 6: (color online) The ratios between tensor and spin-orbit force contributions for protons (left panel; $q = p$) and neutrons (right panel; $q = n$) as functions of the mass of the composite nucleus. The values at time $= 6.0 \times 10^{-22}$ s are chosen to calculate the ratio.

from $J_q$ and $J_q'$ in Eq. (3) might play a significant role.

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