Limits on non-minimal Lorentz violating parameters through FCNC and LFV processes

Y. M. P. Gomes and J. A. Helayel-Neto

Centro Brasileiro de Pesquisas Físicas (CBPF), Rua Dr Xavier Sigaud 150, Urca, Rio de Janeiro, Brazil, CEP 22290-180

In this work we analyse a non-minimal Lorentz-violating extension of the electroweak theory in the fermionic sector. Firstly we analyse the relation between the CKM rotation in the quark sector and possible contributions of this new coupling to flavour changing neutral currents (FCNC) processes. In sequel we look for non-diagonal terms through possible leptonic flavour violation (LFV) decays. Strong bounds are presented to the Lorentz violating parameters of both the quark and the leptonic sectors.

I. INTRODUCTION

Despite its great success, the Standard Model of Particle Physics should not be the final description of nature and it has been shown that in some of its extensions, string theory, for example, is possible that Lorentz symmetry is violated \cite{1, 2}. Observation of any, albeit small, sign of Lorentz symmetry violation (LSV) would represent a major paradigm shift and would require re-examination of the very basis of modern physics, i.e., relativity theory and quantum field theory \cite{1–4}.

A possible realization of LSV is achieved with Lagrangian model where a spin field acquires a non-zero vacuum expectation value - see for example, Ref. \cite{1}. Given this approach, one can introduce non-dynamic tensors \cite{5} and explore various differences between couplings for the SM gauge and matter sectors \cite{1–6}. For a review of theory and experimental tests of CPT and Lorentz invariance, see references \cite{4–11}.

In the present study we investigate the case proposed in \cite{12} of a non-minimally coupling with a constant 4-vector background with a specific SM sector, the fermion - electroweak Bosons interacting sector. More specifically, we follow two main paths which are; possible contributions for flavour Changing Neutral Currents (FCNC) decays and contributions for Lepton flavour Violation (LFV) decays \cite{13, 14}. Studies involving mesons and LSV can be seen in \cite{15, 16}. In the first path we use the strong bounds in the FCNC mesons decays given by \(K^0 \rightarrow \mu^+\mu^-\), \(B^0 \rightarrow \mu^+\mu^-\) and \(B_s^0 \rightarrow \mu^+\mu^-\) to find constrains involving the LSV parameters. Going further, we analyse the leptonic sector through the LFV decays \(\mu \rightarrow e + \gamma\), \(\tau \rightarrow \mu + \gamma\) and \(\tau \rightarrow e + \gamma\). In the final comments we discuss our results.

II. NON-MINIMAL COUPLING IN THE ELECTROWEAK SECTOR

Based on the non-minimal coupling used in \cite{12} one can extend the idea of a non-minimal coupling in the \(SU(2) \times U(1)\) sector of the Standard Model. Starting from the implementation of the following covariant derivative proposed we have

\[
(D_\mu')_{AB} = \left(\partial_\mu + igYB_\mu + ig'W^I_\mu\sigma^I / 2\right)\delta_{AB} - i\xi^\nu_{AB}F_{\mu\nu} - i\rho^\nu_{AB}\sigma^IF^I_{\mu\nu},
\]

where \(Y = 2(Q - T^3), g \) and \(g' \) are the \(U(1)_Y\) and \(SU(2)_L\) coupling constants, respectively, \(B_\mu \) and \(W^I_\mu \) are the \(U(1)_Y\) and \(SU(2)_L\) Gauge bosons, the indexes \(A,B\) refers to Standard Model fermionic families, \(F_{\mu\nu} = \partial_\mu B_\nu\),

\[
F^I_{\mu\nu}\sigma^I = \left(\partial_{[\mu}W^3_{\nu]} + gW^{3+}_{[\mu}W^-_{\nu]} - \partial_{[\mu}W^3_{\nu]} + gW^3_{[\mu}W^+_{\nu]}\right),
\]

\*Electronic address: ymuller@cbpf.br
†Electronic address: helayel@cbpf.br
\( \sigma^I \) refers to Pauli’s matrices. Our analysis will begin in the quarks sector, where we expect to analyse the relationship between the Lorentz violation and the CP violation that is represented by the CKM Matrix. In the sequence we will analyse the leptonic sector, where the absence of Right neutrinos and the universality of weak interactions in this sector demand a thorough analysis.

### A. The Quark sector and FCNC:

Taking into account the flavour structure, the new covariant derivative will act in the quarks sector as follows:

\[
\mathcal{L} = (\bar{Q}_L)(i\gamma^\mu D_\mu')_{AB}(Q_L)_B + (\bar{u}_R)(i\gamma^\mu D_\mu')_{AB}(u_R)_B + (\bar{d}_R)(i\gamma^\mu D_\mu')_{AB}(d_R)_B ,
\]

where \( A, B \) refers to the respective quark family (i.e., flavour). Here we use \((Q_L)_A = (u_{L,A} \ d_{L,A})^T\) doublet under \(SU(2)_L\) and \((u_R)_A, (d_R)_A\) singlets under \(SU(2)_L\). The Lagrangian above brings us the following new interaction terms in the Left sector:

\[
\mathcal{L}_{LSV}^{Left} = \xi^\mu_{AB}(\bar{Q}_L)A\gamma^\nu(Q_L)_B F_{\mu\nu} + \rho^\mu_{AB}(\bar{Q}_L)A \gamma^\nu \sigma^I(Q_L)_B F^I_{\mu\nu} ,
\]

Since this coupling acts in the interaction sector of the fermionic sector with the electroweak bosonic fields, there will be no changes in the fermion masses neither in the gauge boson mass. As we know, the relationship between the Bosons \( B, W^3 \) and \( A, Z \) is given from the mass matrix diagonalization after the spontaneous symmetry breaking of the Higgs potential, and from this diagonalization appears the very known relationship:

\[
A = \cos \theta_W B + \sin \theta_W W^3 \quad , \quad Z = - \sin \theta_W B + \cos \theta_W W^3 .
\]

Rewriting the Lagrangian \((3)\) in terms of the photon and Z-boson fields \( A, Z \), we reach:

\[
\mathcal{L}_{LSV}^{Left} = \bar{u}_{L,A}(\Delta_{AB} + \tilde{\Delta}_{AB})u_{L,B} + \bar{d}_{L,A}(\tilde{\Delta}_{AB} - \Delta_{AB})d_{L,B} + \bar{u}_{L,A}(\tilde{\Delta}_{AB}^{\dagger})d_{L,B} + h.c. ,
\]

where

\[
\Delta_{AB} = \xi^\mu_{AB} \gamma^\nu (\cos \theta_W \partial_\mu A_\nu - \sin \theta_W \partial_\mu Z_\nu) ,
\]

\[
\tilde{\Delta}_{AB} = \rho^\mu_{AB} \gamma^\nu (\sin \theta_W \partial_\mu A_\nu + \cos \theta_W \partial_\mu Z_\nu + ig' W^+_{\mu} W^-_{\nu}) ,
\]

and

\[
\Delta_{AB}^{\dagger} = \rho^\mu_{AB} \gamma^\nu (\partial_\mu W^+_{\nu} + ig' \sin \theta_W A_\mu W^+_{\nu} + ig' \cos \theta_W Z_\mu W^+_{\nu}) .
\]

Similarly, the couplings in the Right sector gives us the following interaction Lagrangian:

\[
\mathcal{L}_{LSV}^{Right} = \xi^\mu_{AB} \bar{u}_{R,A} \gamma^\nu u_{R,B} F_{\mu\nu} + \xi^\mu_{AB} \bar{d}_{R,A} \gamma^\nu d_{R,B} F_{\mu\nu} .
\]

The equation above is justified by the fact that that the Right sector is singlet under \(SU(2)_L\) group transformations. Rewriting the above equation on the same \( A, Z \) basis we have:

\[
\mathcal{L}_{LSV}^{Right} = \bar{u}_{R,A} \Delta_{AB} u_{R,B} + \bar{d}_{R,A} \Delta_{AB} d_{R,B} .
\]

Finally, introducing the CKM matrix we have that with the standard parametrization choice, \( d_A = V_{AB}d_B \) where \( d_A \) represent the physical quarks. The CKM matrix expansion in terms of standard parameters \( \lambda, \rho \in \eta \) \((\lambda \approx |V_{us}| \approx 0.23, \ s_{12} = \lambda, \ s_{23} = A \lambda^2, \ s_{13} e^{-i\delta} = A \lambda^3 (\rho - i \eta)\) with \( A \approx 0.83, \ \rho \approx 0.12 \) and \( \eta \approx 0.35 \), gives the following CKM matrix approximation \cite{CKM}:

\[
V_{CKM} \approx \begin{pmatrix}
1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8} & \lambda & A \lambda^3 (\rho - i \eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8} (4A^2 + 1) \lambda^4 & A \lambda^2 \\
A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 + \frac{1}{2} A \lambda^4 (1 - 2(\rho - i \eta)) & 1 - \frac{\lambda^2 \lambda^4}{2}
\end{pmatrix} .
\]
So we can rewrite the new Lagrangian and interaction with the Left and Right sectors. \( \mathcal{L}_{LSV}^{total} = \mathcal{L}_{LSV}^{Left} + \mathcal{L}_{LSV}^{Right} \) and it is given by:

\[
\mathcal{L}_{LSV}^{total} = \tilde{U}_A (\Delta_{AB} + \bar{\Delta}_{AB} P_L) U_B + \bar{D}_A \mathcal{V} A C (\Delta_{CD} - \bar{\Delta}_{CD} P_L) V^\dagger_D E + \bar{U}_i \Delta_{AB}^+ V^\dagger_{BC} P_L D_C + h.c. \quad .
\] (12)

Here \( U_A, D_A \) are Dirac spinors \( U_A = (u_{L,A} \ u_{R,A})^T, D_A = (d_{L,A}^T \ d_{R,A}^T) \) and \( P_L = \frac{1 - \gamma_5}{2} \). The Interaction Lagrangian can be written explicitly as follows:

\[
\mathcal{L}_{LSV}^{total} = \tilde{U}_A (\epsilon_1^\mu \gamma^\nu + \epsilon_2^\mu \gamma^\nu \gamma_5)_{AB} U_B A_{\mu \nu} + \tilde{D}_A (\epsilon_3^\mu \gamma^\nu + \epsilon_4^\mu \gamma^\nu \gamma_5)_{AB} D_B A_{\mu \nu} +
\]

\[
+ \bar{U}_A (\epsilon_5^\mu \gamma^\nu + \epsilon_6^\mu \gamma^\nu \gamma_5)_{AB} U_B Z_{\mu \nu} + \bar{D}_A (\epsilon_7^\mu \gamma^\nu + \epsilon_8^\mu \gamma^\nu \gamma_5)_{AB} D_B Z_{\mu \nu} +
\]

\[
+ \frac{\gamma}{2} (\epsilon_{9}^\mu)_{AB} \bar{U}_A \gamma^\nu (1 - \gamma_5) D_B \left( \partial_{[\mu} W_{\nu]} \right) + \frac{i}{2} g (\epsilon_{10}^\mu)_{AB} \bar{U}_A \gamma^\nu (1 - \gamma_5) U_B - (\hat{\rho}^\mu)_{AB} \bar{D}_A \gamma^\nu (1 - \gamma_5) D_B \right] + h. c. \quad ,
\] (13)

where \( A_{\mu \nu} = \partial_{[\mu} A_{\nu]} \), \( Z_{\mu \nu} = \partial_{[\mu} Z_{\nu]} \), \( \epsilon_{AB} = (V n V^\dagger)_{AB} \) for any matrix \( n_{AB} \), \( \epsilon_1^\mu = \frac{1}{2} c_W \xi^\mu + \frac{1}{2} s_W \rho_\mu \), \( \epsilon_2^\mu = \frac{1}{2} c_W \xi^\mu - \frac{1}{2} s_W \rho_\mu \), \( \epsilon_3^\mu = \frac{1}{2} c_W \xi^\mu \), \( \epsilon_4^\mu = \frac{1}{2} c_W \xi^\mu \), \( \epsilon_5^\mu = -\frac{1}{2} s_W \rho_\mu \), \( \epsilon_6^\mu = \frac{1}{2} c_W \xi^\mu \), \( \epsilon_7^\mu = -\frac{1}{2} s_W \rho_\mu \), \( \epsilon_8^\mu = \frac{1}{2} c_W \xi^\mu \), \( \epsilon_9^\mu = \frac{1}{2} c_W \xi^\mu \), \( \epsilon_{10}^\mu = \frac{1}{2} c_W \xi^\mu \), \( c_W = \cos \theta_W \), \( c_\rho = \rho \), \( c_\delta = \delta \), and we omit flavour indexes for simplicity. We shall pay attention to \( c_\delta \) since it was the main parameter in which contributes to the process we are interested.

From the equation [14], we can see that the CKM Matrix will not only appear in the charged currents as in the Standard Model, but now we will also get Lorentz violation parameters which can be complex due to the CP violation parametrized by the \( \delta \) complex phase.

We assume Lorentz violation vectors such that families do not mix, but show dependence on the family. In other words, let’s say \( \xi_{AB}^\mu \) is given by:

\[
\xi_{AB}^\mu = \begin{pmatrix} \xi_{11}^\mu & 0 & 0 \\ 0 & \xi_{22}^\mu & 0 \\ 0 & 0 & \xi_{33}^\mu \end{pmatrix} .
\] (14)

where \( \xi_{11}^\mu \neq \xi_{22}^\mu \neq \xi_{33}^\mu \), in principle. We assume that \( \rho \) shares the same characteristics. From this structure, after the rotation induced by the CKM matrix we obtain:

\[
\xi_{11} = (V \xi V^\dagger)_{11} \approx \xi_{11} + (\xi_{22} - \xi_{11}) \lambda^2 + O(\lambda^3) \quad ,
\] (15)

\[
\xi_{12} \approx (-\lambda + \frac{\lambda^3}{2})(\xi_{11} - \xi_{22}) + O(\lambda^5) \quad ,
\] (16)

\[
\xi_{13} \approx (\xi_{11} - \xi_{22})(\lambda^3) - (\xi_{11} - \xi_{22})(\lambda^3) + O(\lambda^5) \quad ,
\] (17)

\[
\xi_{22} \approx \xi_{22} + (\xi_{11} - \xi_{22}) \lambda^2 + (\xi_{33} - \xi_{22}) A^2 \lambda^4 + O(\lambda^5) \quad ,
\] (18)

\[
\xi_{23} \approx (\xi_{33} - \xi_{22}) A^2 \lambda^4 + \xi_{11} A^2 \lambda^4 (-i \eta + \rho - 1) + \xi_{22} A^2 \lambda^4 (-i \eta - \rho + 1) + O(\lambda^5) \quad ,
\] (19)

\[
\xi_{33} \approx \xi_{33} + (\xi_{22} - \xi_{33}) A^2 \lambda^4 + O(\lambda^5) \quad ,
\] (20)

and \( \xi_{ij} = \xi_{ij}^* \) for \( i \neq j \). Note that the complex characteristic resides in the non-diagonal components of Lorentz violation parameters, but depends on the difference in magnitudes expressed in our assumption [14]. In the low energy limit, that is, in the limit where the massive gauge bosons decay fast enough, the \( Z \) decay generates the following new effective interaction between neutral currents in the \( D \)-quark sector:

\[
\mathcal{H} = \frac{\alpha^2}{\sqrt{2}} \mathcal{V} F^0 \times \bar{D}_A \gamma^\mu q^\dagger (c_7 \gamma^\nu + c_8 \gamma^\nu \gamma_5)_{AB} D_B \quad ,
\]
where \( J_\mu^0 = \sum_f \bar{f} \gamma_\mu (v_f - a_f \gamma_5) f \) in which couples to \( Z \) in the SM, \( \frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W} \), \( v_f = T^f_3 - 2 Q_f s^2_W \), \( a_f = T^f_3 \) and \( q = p_f - p_f = p_D - p_D \). Interestingly, while \( c_5 \) and \( c_6 \) remain diagonal in flavour space, \( c_7 \) and \( c_8 \) contain contributions from the CKM matrix, so they will not be diagonal, assuming Eq. (14). The \( c_3 \) and \( c_4 \) parameters also share this property. Importantly, the terms \( c_7 \) and \( c_8 \) generate the so-called Flavour Changing Neutral Current (FCNC) in a tree-level process. In SM these processes are forbidden in a tree level process, so it is an important result to be discussed. In addition, it is also important to point out that \( c_3 \) and \( c_4 \) can generate flavour exchange through photon coupling, a phenomenon that does not occur in the SM.

Analysing in detail the interaction that brings us the possibility of FCNC processes given by the equation (21), after some algebraic manipulations we can rewrite it as follows:

\[
\mathcal{H}_{FCNC} = \frac{c^2_W G_F}{\sqrt{2}} \left[ \bar{D}_A \gamma^\mu D_B \times \sum_f \bar{f} M_{\mu AB}^\nu (v_f - a_f \gamma_5) f + \right. \\
+ \bar{D}_A \gamma^\mu \gamma_5 \gamma_5 D_B \times \sum_f \bar{f} N_{\mu AB}^\nu (v_f - a_f \gamma_5) f \right],
\]

where \( M_{\mu AB}^\nu = (\delta^\mu_q c_7 - q_\mu c_6^\nu)_{AB} \) and \( N_{\mu AB}^\nu = (\delta^\mu_q c_8 - q_\mu c_6^\nu)_{AB} \). Now we can calculate the Decay Rate of some possible processes in the non-diagonal sector in the flavour space.

Neutral Kaon Decay: If there is a contribution to the neutral current with FCNC then the neutral K mesons (\( s \bar{d} \) and \( d \bar{s} \) ) could easily decay into a pair of muons, as is shown in Fig. [1].

![Figure 1: Hypothetical decay of the neutral Kaons K⁰ and K̄⁰ in a FCNC process.](image)

The standard method to work in meson decays is given by the following parametrization \( \langle 0 | \bar{\psi} \gamma_\mu \gamma_5 | K^0(q) \rangle = i F_{K^0} q_\mu \) [12]. The vector current will be responsible for the vector mesons (e.g., \( K^* \)), and a similar analysis can be done. Thus, the scattering matrix of the Kaon decay is given by:

\[
\mathcal{M} = \frac{c^2_W G_F}{\sqrt{2}} (N_{12})^{\mu \nu} \times \langle 0 | \bar{\psi} \gamma_\mu \gamma_5 b | K^0(q) \rangle \times \bar{\mu}(p) \gamma_\nu (v_f - a_f \gamma_5) \mu(q - p),
\]

where \( (N)_{12}^{\mu \nu} = (N_\mu \nu q_\mu (c_8)_12 - q_\mu (c_8)_12) \). Therefore, summing over spins of the square of the scattering matrix we reach:

\[
\langle |\mathcal{M}|^2 \rangle = \sum |\mathcal{M}|^2 = \frac{c^4_W G^2_F}{2} F^2_{K^0} q_\mu q_\nu N_{12}^{\mu \nu} (N_1^\nu \gamma_5)^{\mu \lambda} \sum J_\lambda J^\dagger_\lambda,
\]

where \( J_\lambda = \bar{\mu}(p) \gamma_\lambda (v_\mu - a_\mu \gamma_5) \mu(q - p) \). After simplifications and using the rest frame of the neutral kaon where \( q = (M_{K^0}, 0) \) we reach:

\[
\langle |\mathcal{M}|^2 \rangle \approx \frac{c^4_W G^2_F}{2} F^2_{K^0} q_\mu q_\nu M_{K^0}^6 \left| \bar{c}^\dagger \gamma_5 - 4 \left( \frac{\bar{c} \cdot \bar{p}}{M_{K^0}} \right)^2 \right|^2,
\]

where we ignore terms proportional to \( v_\mu = 1 - 4 s^2_W \ll 1 \) and \( c = c_{8,12} = \frac{1}{4} c_W \hat{p}_{12} \). By the use of the golden rule for two body decay we have:

\[
\Gamma = \frac{1}{2(4\pi)^2 M_{K^0}} \int d^3 \bar{p} \frac{\langle |\mathcal{M}|^2 \rangle}{E_{\mu} E_{\mu'}} \delta(M_{K^0} - E_{\mu} - E_{\mu'})
\]

(24)
where \( p \) and \( p' \) are the 4-momentum of the final states. Thus, the integrals are given as follows:

\[
\Gamma = \frac{c_W^4 G_F^2 F_{K^0}^2 a_\mu^2 M_{K^0}^5}{24\pi} \int \frac{PdP}{2\sqrt{P^2 + y^2 M_{K^0}^2}} \delta(P - \kappa) \int d\Omega \left( |\vec{c}|^2 - 4 \left( \frac{\vec{c} \cdot \vec{p}}{M_{K^0}} \right)^2 \right),
\]

where \( P = |\vec{p}|, \kappa = \frac{M_{K^0}}{2}\sqrt{(1 - 4y^2)} \) and \( y = \frac{m_c}{M_{K^0}} \). Defining \( \vec{c} = |\vec{c}|(\sin \theta_c \cos \phi_c, \sin \theta_c \sin \phi_c, \cos \theta_c) \) with \( \phi_c, \theta_c \) generic angles, the angular integral can be calculated and it are given by:

\[
\int d\Omega \left( |\vec{c}|^2 - 4 \left( \frac{\vec{c} \cdot \vec{p}}{M_{K^0}} \right)^2 \right) = \frac{4}{3\pi} |\vec{c}|^2 \left( 3 - \frac{4P^2}{M_{K^0}^2} \right),
\]

Therefore:

\[
\Gamma(K^0 \rightarrow \mu^+ \mu^-) = \frac{c_W^4 G_F^2 F_{K^0}^2 a_\mu^2 M_{K^0}^5}{12\pi} |\vec{c}|^2 \int \frac{PdP}{2\sqrt{P^2 + y^2 M_{K^0}^2}} \delta(P - \kappa) \left( 3 - \frac{4P^2}{M_{K^0}^2} \right) = \frac{c_W^4 G_F^2 F_{K^0}^2 a_\mu^2 M_{K^0}^5}{12\pi} |\vec{c}|^2 \sqrt{(1 - 4y^2)} (1 + 2y^2) .
\]

So, applying the value of the constants we found the decay rate as follows:

\[
\Gamma(K^0 \rightarrow \mu^- \mu^+) = 4.3 \times 10^{-7} |\vec{c}_{12}|^2 \text{MeV}^{-3} ,
\]

where \( a_\mu \approx 0.5, G_F = 1.16 \times 10^{-11} \text{MeV}^{-2}, M_{K^0} = 497.61 \text{MeV} \) and \( F_{K^0} = 164 \text{MeV} \). According to [18], in the Standard Model neutral Kaons with long half-lives \( K^0_L \) have a Decay Rate given by:

\[
\Gamma(K^0_L) = \frac{1}{\tau_{K^0_L}} \approx 1.3 \times 10^{-14} \text{MeV} ,
\]

such way that the Branching Ratio \( BR(K^0_L \rightarrow \mu^+ \mu^-) \), experimentally limited to a value less than \( 6.8 \times 10^{-9} \) [18], will be given by :

\[
BR(K^0_L \rightarrow \mu^+ \mu^-) = \frac{2\Gamma(K^0 \rightarrow \mu^+ \mu^-)}{\Gamma(K^0_L)} = 6.7 \times 10^7 \left( \frac{|\vec{c}|}{\text{MeV}^{-1}} \right)^2 .
\]

Therefore, the contribution to this decay arise from a possible Lorentz violation must be smaller or in the order of contribution from the Standard Model. From this statement we find the following bound for the spacial components of the background vectors:

\[
|\vec{p}_{22} - \vec{p}_{11}| \approx \frac{4}{c_W \lambda} |\vec{c}_{12}| < 5.6 \times 10^{-10} \text{MeV}^{-1} .
\]

Similarly we can calculate the Branching Ratio for the \( B_0 \) (\( d\bar{b} \)) meson:

\[
BR(B_0 \rightarrow \mu^+ \mu^-) = 1.7 \times 10^8 \left( \frac{|\vec{c}_{13}|}{\text{MeV}^{-1}} \right)^2 ,
\]

and using the latest data from [18] we reach the following bound:

\[
|\vec{p}_{11} - \vec{p}_{22} - (\rho - i\eta)(\vec{p}_{11} - \vec{p}_{33})| \approx \frac{4|\vec{c}_{13}|}{\lambda \lambda^3 c_W} < 4.2 \times 10^{-8} \text{MeV}^{-1} .
\]
Finally, for the $B_s^0$ meson ($s\bar{b}$) we find:
\[
BR(B_s^0 \to \mu^+\mu^-) = 2.7 \times 10^8 \left( \frac{|\tilde{c}_{23}|}{MeV^{-1}} \right)^2 ,
\]
and therefore we have:
\[
|\tilde{\rho}_{33} - \tilde{\rho}_{22}| \approx \frac{4|\tilde{c}_{23}|}{A\lambda^2 c_W} < 1.4 \times 10^{-8} MeV^{-1} .
\]

In summary, the bounds obtained are organized in the table II. In the next section we will analyse the leptonic sector.

### B. The Leptonic sector and LFV:

Analysing now the lepton sector we shall recall that there is no CKM mechanism in this sector, so we shall be presenting an analysis based on an assumption of non-diagonal LSV parameters. As we will see, this non-diagonal terms gives us stronger bounds than the diagonal ones showed in the quark sector. Shallowly speaking, we have that after modification of the covariant derivative the Lagrangian corresponding to the leptonic sector can be written as follows [12, 19]:
\[
\mathcal{L}_\ell = \sum_{A,B} i(\bar{\ell}_L)_A \gamma^\mu (D^\mu_\mu)_A (\ell_L)_B + i(\bar{\ell}_R)_A \gamma^\mu (D^\mu_\mu)_A (\ell_R)_B ,
\]
here $(L_L)_A = ((\nu_L)_A, (\ell_L)_A)^T$ and $(\ell_R)_B$ are $SU(2)_L$ doublet and singlet respectively, and mass terms from Yukawa interactions are omitted as LSV terms do not influence them. Lagrangian (36) can be split into two components $\mathcal{L}_\ell = \mathcal{L}_{\ell,SM} + \mathcal{L}_{LSV}$ and the last component can be written as follows
\[
\mathcal{L}_{LSV} = \frac{1}{2} \epsilon^\mu_{AB} [(\bar{L}_L)_A \gamma^\nu (L_L)_B + (\bar{\ell}_R)_A \gamma^\nu (\ell_R)_B] F_{\mu\nu} + \frac{1}{2} \rho^\mu_{AB} (\bar{L}_L)_A \gamma^\nu F^{\mu\nu}_{L\mu} \sigma^I (L_L)_B .
\]

Writing the $B_\mu$ and $W_\mu^3$ fields at the base of the $Z$ and the photon fields the above equation will bring us the following Lagrangian interaction for the Left sector:
\[
\mathcal{L}_{LSV}^{Left} = (v_1)_A (\bar{\ell}_L)_A \gamma^\nu (\ell_L)_B \partial_\mu A_\mu + (v_2)_A (\bar{\nu}_L)_A \gamma^\nu (\nu_L)_B \partial_\mu A_\mu + (v_3)_A (\bar{\ell}_L)_A \gamma^\nu (\ell_L)_B \partial_\mu Z_\mu + (v_4)_A (\bar{\nu}_L)_A \gamma^\nu (\nu_L)_B \partial_\mu Z_\mu + \frac{ig}{2\sin\theta_W} (v_1 - v_2)_A W_\mu^\mu W_\mu^\nu (\bar{\ell}_L)_A \gamma^\nu (\ell_L)_B + (\bar{\nu}_L)_A \gamma^\nu (\nu_L)_B + \frac{(v_1 - v_2)_A (\ell_L)_A \gamma^\nu (\nu_L)_B \left( \partial_\mu W_\mu^\nu + i e A_\mu W_\mu^\nu + i e c W_\mu^{\nu} \right) + h.c}.
\]

where we define for convenience $v_{1\mu} = c_W \xi_\mu + s_W \rho_\mu , v_{2\mu} = c_W \xi_\mu - s_W \rho_\mu , v_{3\mu} = -s_W \xi_\mu + c_W \rho_\mu , v_{4\mu} = -s_W \xi_\mu - c_W \rho_\mu$. The Lorentz violation implemented into the Right leptonic sector will be given by
\[
\mathcal{L}_{LSV}^{Right} = \xi_{AB} (\bar{\ell}_R)_A \gamma^\nu (\ell_R)_B F_{\mu\nu} .
\]

Rewriting the above equation on the $\{A_\mu, Z_\mu\}$ basis, we reach:
\[
\mathcal{L}_{LSV}^{Right} = \cos \theta_W \xi_{AB} (\bar{\ell}_R)_A \gamma^\nu (\ell_R)_B \partial_\mu A_\mu - \sin \theta_W \xi_{AB} (\bar{\ell}_R)_A \gamma^\nu (\ell_R)_B \partial_\mu Z_\mu .
\]

Let’s now return to the full LSV Lagrangian. Using the definitions $(\ell_L)_A = P_L \ell_A = \frac{1 + \Delta}{2} \ell_A$ and $(\ell_R)_A = P_R \ell_A = \frac{1 - \Delta}{2} \ell_A$, $(\nu_L)_A = \nu_A$, with $\ell_A$ and $\nu_A$ Dirac spinors, we are able to rewrite the
Lagrangian as follows:

\[ \mathcal{L}_{LSV} = \bar{\ell}_A (c_1^\mu \gamma^\nu + c_2^\mu \gamma^{\nu \gamma_5})_{AB} \ell_B \partial_{[\mu} A_{\nu]} + \frac{1}{2} (\nu_2)_{AB}^\mu \nu_B \partial_{[\mu} A_{\nu]} + \]
\[ + \frac{1}{2} \rho_{AB}^\mu \bar{\ell}_A \gamma^\nu \nu_B \left( \partial_{[\mu} W_{\nu]} - i e A_{[\mu} W_{\nu]} + i e \cot \theta_W Z_{[\mu} W_{\nu]} \right) + \]
\[ + \frac{i}{2} g \rho_{AB}^\mu W_{\mu}^+ W_{\nu}^+ \left[ \bar{\ell}_A \gamma^\nu \gamma^5 \ell_B - \bar{\nu}_A \gamma^\nu \gamma^5 \nu_B \right] + h.c. , \]  

(40)

where \( c_{1\mu} = \frac{1}{2} (c_W \xi_\mu + \frac{1}{2} s_W \rho_\mu) \), \( c_{2\mu} = -\frac{1}{2} s_W \rho_\mu \), \( c_{3\mu} = -\frac{1}{2} (s_W \xi_\mu + \frac{1}{2} c_W \rho_\mu) \) e \( c_{4\mu} = \frac{1}{2} c_W \rho_\mu \) and the flavour indexes are omitted for simplicity. The above Lagrangian is a generalization of the model seen in Ref. [12] [19], since in this work the flavour structure are take into account.

| Interaction | Vertex |
|-------------|--------|
| \( \gamma \ell_A \ell_B \) | \( q_\nu (c_1^{[\mu} \gamma^{\nu]} + c_2^{[\mu} \gamma^{\nu]} \gamma_5)_{AB} \) |
| \( \gamma \nu_A \nu_B \) | \( \frac{1}{2} (\nu_2)_{AB}^{[\mu} \gamma^{\nu]} \frac{(1-\gamma_5)}{2} q_\nu \) |
| \( Z^0 \ell_A \ell_B \) | \( q_\nu (c_3^{[\mu} \gamma^{\nu]} + c_4^{[\mu} \gamma^{\nu]} \gamma_5)_{AB} \) |
| \( Z^0 \nu_A \nu_B \) | \( \frac{1}{2} (\nu_3)_{AB}^{[\mu} \gamma^{\nu]} \frac{(1-\gamma_5)}{2} q_\nu \) |
| \( W^- \ell_A \nu_B \) | \( \frac{i}{\sqrt{2}} (\rho)_{AB}^{[\mu} \gamma^{\nu]} \frac{(1-\gamma_5)}{2} q_\nu \) |
| \( W^- \gamma \ell_A \nu_B \) | \( \frac{i e}{\sqrt{2}} (\rho)_{AB}^{[\mu} \gamma^{\nu]} \frac{(1-\gamma_5)}{2} \) |
| \( W^- Z^0 \ell_A \nu_B \) | \( \frac{i e}{\sqrt{2}} \cot \theta_W (\rho)_{AB}^{[\mu} \gamma^{\nu]} \frac{(1-\gamma_5)}{2} \) |
| \( W^+ W^- \ell_A \ell_B \) | \( \frac{i q_\nu}{\sqrt{2}} (\rho)_{AB}^{[\nu]} \frac{(1-\gamma_5)}{2} \) |
| \( W^+ W^- \nu_A \nu_B \) | \( -\frac{i q_\nu}{\sqrt{2}} (\rho)_{AB}^{[\nu]} \frac{(1-\gamma_5)}{2} \) |

Table I: Vertex factors obtained from Eq.(40). Here \( q^\mu \) represents the \( A \), \( W \) or \( Z \) 4-momentum.

As we can see a novel coupling between the photon and the neutral current is generated. Going further, we can also see that a coupling between the electromagnetic current and the \( Z^0 \) boson also arises. A possible decay from Lorentz violation will be the so-called neutrino-free muon decay, \( \mu \to e + \gamma \). This process is prohibited in the Standard Model, so that experimentally there are strong limits to this decay, with Branching Ratio \( BR(\mu \to e + \gamma) < 4.2 \times 10^{-13} \text{ (90\% C.L.)} \) [14]. In the same way as the lepton tau decays we have \( BR(\tau \to e\gamma) < 3.3 \times 10^{-8} \text{ e } BR(\tau \to \mu\gamma) < 4.4 \times 10^{-8} \), both with 90\% of confidence level [14].

From a momentum conservation perspective, the \( \ell_A \to \ell_B + \gamma \) decay can occur as long as the mass of the \( \ell_A \) lepton is greater than the \( \ell_B \) mass. However in the Standard Model this decay is prohibited, so we can use this decay to find bounds for the Lorentz violation parameters.

Directly, the scattering matrix which describes the decay of the Feynman diagram shown in Fig. 2 is given by:

\[ \langle |M|^2 \rangle = \sum_{s,s',k} |M|^2 = -Tr \left[ (\gamma - m_A)(\Gamma_{AB}^\mu)(\gamma - \bar{k} - m_B)\Gamma_{AB}^\nu \right] \sum_k \epsilon_{\mu,k} \epsilon_{\nu,k}^* , \]  

(41)

where \( \Gamma_{AB}^\mu = q_\nu (c_1^{[\mu} \gamma^{\nu]} + c_2^{[\mu} \gamma^{\nu]} \gamma_5)_{AB} \), \( m_A > m_B \) and are no sum over \( A,B \). Using the identity \( \sum_k \epsilon_{\mu,k} \epsilon_{\nu,k}^* = \eta_{\mu\nu} - \frac{2q_\nu}{q^2} \) and calculating the traces over gamma matrices we get the following expression:

\[ \langle |M|^2 \rangle = -12 m_A^2 - E_q (c_{20}^2 m_A (1 - y^2) + E_q (y - 3) y) + c_{10}^2 m_A^2 (1 - y^2) + 2 E_q (y - 3) y) + c_{11}^2 \cdot q m_A (1 - y^2) + E_q (y (3 + y)) + \]
\[ + c_{22}^2 \cdot q m_A (1 - y^2) + 2 E_q (y - 3) y) + c_{11}^2 \cdot (c_i \cdot q)^2 (3 + y) \]  

(42)
where $y = m_B/m_A$ and we hide the flavour indexes $c^0_{1,AB}$ and $c^0_{2,AB}$ for simplicity. With that, we can calculate the decay rate of this process and, using the rest frame of the lepton $\ell_A$, is given by:

$$
\Gamma(\ell_A \rightarrow \ell_B + \gamma) = \frac{1}{2(4\pi)^2 m_A} \int \frac{d^3q}{E_q E_k} \delta(m_A - E_q - E_{k_B}) = \frac{1}{2(4\pi)^2 m_A} \int dE_q E_q^2 \frac{1}{E_q E_{k_B}} \delta(m_A - E_q - E_{k_B}) \int d\Omega(|M|^2),
$$

(43)

where

$$
\int d\Omega(|M|^2) \approx -\frac{4}{3} E_q \pi \left(3(c_2^0)^2 + 3(c_1^0)^2\right) m_A + O(y).
$$

So:

$$
\Gamma(\ell_A \rightarrow \ell_B + \gamma) = \left(\frac{E_q}{8\pi m_A E_{k_B}}\right) \int \frac{d\Omega}{4\pi} \left(|M|^2\right)\bigg|_{E_q=m_A-E_{k_B}} = \frac{3}{4\pi} \left(\frac{(c_{1,AB}^0)^2 + (c_{2,AB}^0)^2}{m_A^3}\right) m_A + O(y).
$$

(44)

Using the most recent measurements, we have that the mean life of the muon is given by $\tau_\mu = 3.3 \times 10^{10} \text{MeV}^{-1}$, and for the tau-lepton $\tau_\tau = 4.4 \times 10^8 \text{MeV}^{-1}$ [18]. Thus we have the Branching ratio for the muon decay is given by [18]:

$$
BR(\mu \rightarrow e + \gamma) = \frac{\Gamma(\mu \rightarrow e + \gamma)}{\tau_\mu^{-1}} < 4.2 \times 10^{-13}.
$$

(45)

Then we reach the following bound:

$$
|\Delta_{12}| < 2.11 \times 10^{-17} \text{MeV}^{-1},
$$

(46)

where $\Delta_{12}^2 = (c_{1,12}^0)^2 + (c_{2,12}^0)^2$. Similarly from the Branching Ratio $BR(\tau \rightarrow \mu + \gamma) < 4.4 \times 10^{-8}$ [18] we get the bound as follows:

$$
|\Delta_{23}| < 2.7 \times 10^{-13} \text{MeV}^{-1},
$$

(47)

where $\Delta_{23}^2 = (c_{1,23}^0)^2 + (c_{2,23}^0)^2$. Finally, in the process $\tau \rightarrow e + \gamma$, we have the Branching ratio upper bound given by $BR(\tau \rightarrow e + \gamma) < 3.3 \times 10^{-8}$ [18]. Thus we get the third limit which will be given by:

$$
|\Delta_{13}| < 2.4 \times 10^{-13} \text{MeV}^{-1},
$$

(48)

where $\Delta_{13}^2 = (c_{1,13}^0)^2 + (c_{2,13}^0)^2$. Rewriting the limits obtained in terms of the initial 4-vectors we have $\Delta_{AB}^2 = \frac{1}{4}(1+\cos 2\theta_W)(\xi_{0,AB}^0)^2 + \frac{1}{4}c_W s_W \rho_{AB}^0 c_{0,AB} + \frac{1}{16}(1-\cos 2\theta_W)(\rho_{1,AB}^0)^2$, with no sum over flavour indexes. The bounds are grouped in table [III] and the region plots are shown in Fig. [3]
The decays $\mu \to e + \gamma$ and $\tau \to \mu + \gamma$ are drastically suppressed in the SM; this is very important to distinguish between the neutrino flavours. Now, with LSV, these decays, though very tiny, can occur and they signal a very meaningful aspect of LSV in particle physics.

III. FINAL COMMENTS

In this work we analyse the non-diagonal sectors (in the flavour space) of the proposed model [12] in the quark and leptonic sectors. In the quark sector we find that the CKM rotation could generate FCNC processes even if the Lorentz violation parameters were diagonal in flavour space. We realize that, through the FCNC processes, only the spatial components or the 4-vector parameters $\xi$ and $\rho$ contribute to the FCNC decays and we found limits between $10^{-8} - 10^{-10}$ MeV$^{-1}$ by use of the experimental FCNC bounds.

As can be seen in [19], we need to use the Sun Centered Frame (SCF) in order to took a better inertial frame than the earth frame. So, in the SCF the module of any vector $\vec{V} = (V_x, V_y, V_z)$ in the SCF are given by:

$$|\vec{V}_{SCF}| = \sqrt{\left(\frac{1}{2} + \sin^2 \chi\right)V_x^2 + \left(\frac{1}{2} + \cos^2 \chi\right)V_y^2 + V_z^2 - 2V_xV_y \cos \chi \sin \chi},$$

where $\chi$ is the colatitude of the laboratory and $\vec{V}_{SCF} = (V_X, V_Y, V_Z)$. In the case of the LHC collaboration we have $\chi \approx 44^\circ$.

| Decay                        | Bound (MeV$^{-1}$) |
|------------------------------|--------------------|
| $K_L^0 \to \mu^+ + \mu^-$    | $\sqrt{0.98 V_{12,X}^2 + 1.02 V_{12,Y}^2 + V_{12,Z}^2 - V_{12,X}V_{12,Y}} < 5.6 \times 10^{-10}$ |
| $B^0 \to \mu^+ + \mu^-$      | $|\vec{c}^0_{SCF}| < 4.2 \times 10^{-8}$ |
| $B_s^0 \to \mu^+ + \mu^-$    | $\sqrt{0.98 V_{23,X}^2 + 1.02 V_{23,Y}^2 + V_{23,Z}^2 - V_{23,X}V_{23,Y}} < 1.4 \times 10^{-8}$ |

Table II: Bounds for Lorentz violation parameters from experimental limits of FCNC processes.

where $\vec{V}_{AB} = (\bar{\rho}_{AA} - \bar{\rho}_{BB})_{SCF}$, for $A, B = 1, 2, 3$,

$$|\vec{c}^0_{SCF}|^2 = 0.98 (W_X^2 - 0.12 V_{13,X}^2) + 1.02 (W_Y^2 - 0.12 V_{13,Y}^2) + (W_Z^2 - 0.12 V_{13,Z}^2) +$$

$$-(W_XW_Y - 0.12 V_{13,Y}V_{13,X}),$$

with $\vec{W} = \bar{V}_{12} - \rho \bar{V}_{13} = \bar{\rho}_{11} - \bar{\rho}_{22} - \rho (\bar{\rho}_{11} - \bar{\rho}_{33})$, $\rho \approx 0.12$ and $\eta \approx 0.35$ [17]. As can be seen, the limits are not simple, specially the limit from the $B^0$ decay. Besides the complexity of the bound obtained from the $B^0$ decay, the bound reached from the $K^0$ and the $B_s^0$ decays can be visualized as a hyperboloid with width approximately below $10^{-10} MeV^{-1}$ and $10^{-8} MeV^{-1}$, respectively.

In the leptonic sector the bound are functions of the temporal components $\rho_{AB}^0$. Where analysed in the SCF perspective we have $\rho_{AB}^0 \approx \rho_{AB}^0$, where $\rho_{AB}^0$ is the temporal component of $\rho_{AB,SCF}$. Therefore, we can affirm that we stronger bounds ($|10^{-12} - 10^{-16}|$ MeV$^{-1}$) are found for the non-diagonal components of the LSV parameters by use of the lepton flavour violation branching ratios. Is important to highlight that our bounds in the leptonic sector are, comparing with our weakest bound, five times more accurate than the bound reach in Ref. [19] (where $\rho_{SCF}^0 < 8 \times 10^{-7} MeV^{-1}$).
Table III: Limits for Lorentz violation parameters from the experimental limits of the leptonic LFV sector.

| Decay | Bound (MeV$^{-1}$) |
|-------|-------------------|
| $\mu \rightarrow e + \gamma$ | $|\Delta_{12}| = \sqrt{0.19 (\xi_{12}^T)^2 + 0.10 \xi_{12}^T \rho_{12}^T + 0.03 (\rho_{12}^T)^2} < 2.1 \times 10^{-17}$ |
| $\tau \rightarrow \mu + \gamma$ | $|\Delta_{23}| = \sqrt{0.19 (\xi_{23}^T)^2 + 0.10 \xi_{23}^T \rho_{23}^T + 0.03 (\rho_{23}^T)^2} < 2.7 \times 10^{-13}$ |
| $\tau \rightarrow e + \gamma$ | $|\Delta_{13}| = \sqrt{0.19 (\xi_{13}^T)^2 + 0.10 \xi_{13}^T \rho_{13}^T + 0.03 (\rho_{13}^T)^2} < 2.4 \times 10^{-13}$ |

Figure 3: Plot of the allowed regions in $\xi_{AB}^T \times \rho_{AB}^T$ parameter space. Here $A, B = 1, 2$ refers to $\mu \rightarrow e + \gamma$ decay, $A, B = 2, 3$ refers to $\tau \rightarrow \mu + \gamma$ decay and $A, B = 1, 3$ refers to $\tau \rightarrow e + \gamma$ decay. We use $\theta_W = \arccos(80/91)$.

Acknowledgments

This work was funded by the Brazilian National Council for Scientific and Technological Development (CNPq).

[1] V Alan Kostelecký and Stuart Samuel. Spontaneous breaking of lorentz symmetry in string theory. Physical Review D, 39(2):683, 1989.
[2] Irina Mocioiu, Maxim Pospelov, and Radu Roiban. Breaking cpt by mixed noncommutativity. Physical Review D, 65(10):107702, 2002.
[3] Stefano Liberati. Tests of lorentz invariance: a 2013 update. Classical and Quantum Gravity, 30(13):133001, 2013.
[4] David Mattingly. Modern tests of lorentz invariance. Living Reviews in relativity, 8(1):5, 2005.
[5] C Adam and Frans R Klinkhamer. Causality and cpt violation from an abelian chern–simons-like term. *Nuclear Physics B*, 607(1-2):247–267, 2001.

[6] Rodolfo Casana, Manoel M Ferreira Jr, and Carlos EH Santos. Classical solutions for the carroll-field-jackiw-proca electrodynamics. *Physical Review D*, 78(2):025030, 2008.

[7] AP Baeta Scarpelli, Humberto Belich, JL Boldo, and JA Helayel-Neto. Aspects of causality and unitarity and comments on vortexlike configurations in an abelian model with a lorentz-breaking term. *Physical Review D*, 67(8):085021, 2003.

[8] Don Colladay and V Alan Kostelecký. Lorentz-violating extension of the standard model. *Physical Review D*, 58(11):116002, 1998.

[9] V Alan Kostelecký and Matthew Mewes. Signals for lorentz violation in electrodynamics. *Physical Review D*, 66(5):056005, 2002.

[10] Yunhua Ding and V Alan Kostelecký. Lorentz-violating spinor electrodynamics and penning traps. *Physical Review D*, 94(5):056008, 2016.

[11] V Alan Kostelecký and Zonghao Li. Gauge field theories with lorentz-violating operators of arbitrary dimension. *Physical Review D*, 99(5):056016, 2019.

[12] Victor E Mouchrek-Santos and Manoel M Ferreira Jr. Repercussions of dimension five nonminimal couplings in the electroweak model. In *Journal of Physics: Conference Series*, volume 952, page 012019. IOP Publishing, 2018.

[13] Anesesh V Manohar and Mark B Wise. *Heavy quark physics*, volume 10. Cambridge university press, 2000.

[14] E Ripiccini, MEG Collaboration, et al. New result from the meg experiment at psi and the meg upgrade. *Nuclear and Particle Physics Proceedings*, 260:147–150, 2015.

[15] Benjamín R Edwards and V Alan Kostelecký. Searching for cpt violation with neutral-meson oscillations. *Physics Letters B*, 795:620–626, 2019.

[16] Roel Aaij, C Abellán Beteta, B Adeva, M Adinolfi, Z Ajaltouni, S Akar, J Albrecht, F Alessio, M Alexander, S Ali, et al. Search for violations of lorentz invariance and c p t symmetry in b (s) 0 mixing. *Physical review letters*, 116(24):241601, 2016.

[17] Gerhard Buchalla and Andrzej J Buras. Qcd corrections to rare k-and b-decays for arbitrary top quark mass. *Nuclear Physics B*, 400(1-3):225–239, 1993.

[18] Masaharu Tanabashi, K Hagiwara, K Hikasa, K Nakamura, Y Sumino, F Takahashi, J Tanaka, K Agashe, G Aielli, C Amsler, et al. Review of particle physics. *Physical Review D*, 98(3):030001, 2018.

[19] YMP Gomes, PC Malta, and MJ Neves. Testing lorentz-symmetry violation via electroweak decays. *arXiv preprint arXiv:1909.10398*, 2019.