Order Tracking Based on Robust Peak Search Instantaneous Frequency Estimation

Y Gao¹, Y Guo¹, Y L Chi¹ and S R Qin²

¹ Faculty of Mechanical and Electrical Engineering, Kunming University of Science and Technology, Kunming, 650093, China
² Test Center of Mechanical Engineering College, Chongqing University, 400030, Chongqing, China

E-mail: km_gy@yahoo.com.cn

Abstract. Order tracking plays an important role in non-stationary vibration analysis of rotating machinery, especially to run-up or coast down. An instantaneous frequency estimation (IFE) based order tracking of rotating machinery is introduced. In which, a peak search algorithms of spectrogram of time-frequency analysis is employed to obtain IFE of vibrations. An improvement to peak search is proposed, which can avoid strong non-order components or noises disturbing to the peak search work. Compared with traditional methods of order tracking, IFE based order tracking is simplified in application and only software depended. Testing testify the validity of the method. This method is an effective supplement to traditional methods, and the application in condition monitoring and diagnosis of rotating machinery is imaginable.

1. Introduction

Order analysis performs the conversion of non-stationary signals in time domain to stationary ones in angle domain by constant angle increment sampling about the reference shaft, so it is more effective to show the speed-dependent vibration. The key to order analysis is the constant angle increment sampling about the reference shaft or so called equiangular sampling. The accurate timing marks of order sampling and the reference speed or frequency at the timing marks are required for order analysis; this procedure to obtain the timing marks is called order tracking.

Now, the hardware based order tracking and the computed order tracking (COT) [1][2] are two popular order tracking methods. Special hardware equipments are employed to realize the order tracking in hardware based order tracking. Keyphasor device and interpolation algorithm just do the same work in COT. COT is more convenient and lower cost in practice than hardware based order tracking, and put into engineering use successfully. However, the keyphasor device is still required by COT for speed tracking, this making the order analysis still complex. In the situation where the keyphasor equipment’s installation is inconvenient, both methods mentioned above are invalid.

A new method of order tracking of rotating machinery based on instantaneous frequency estimation (IFE) is proposed [3], however, the peak search work can be destroyed by strong noises or other non-order components in the older version of IFE based order tracking. A new version of IFE based order tracking is introduced in this paper. A robust peak search method is employed, which makes the peak
search little affected by noises or other non-order components, and a good prospect of application in
case monitoring and diagnosis of rotating machinery is imaginable.

2. Instantaneous Frequency Estimation

2.1. Instantaneous Frequency

Instantaneous frequency (IF) is one of the most intuitive concepts, but is also an argued topic. Some
deep and detailed discussion about it can be found in literatures [4]-[6]. Here is just a simple
instruction on IF.

A ‘standard’ definition of IF was first given by Gabor in 1946 and Ville who formally introduced
the definition two years later. The IF of a real signal is the derivative of the phase of the associated
‘analytic signal’ \( z(t) \) [4]:

\[
f_i(t) = \frac{1}{2\pi} \frac{d}{dt} \arg(z(t))
\]

(1)

Where the subscript, \( i \), represents instantaneous.

Another interpretation of the IF comes from the time–frequency distributions (TFD) point of view,
where the IF of a signal at the time \( t_0 \) is defined as the weighted average of the frequencies which exist
in the signal at the time \( t_0 \) [6][7] or the first moment of TFD:

\[
f_i(t) = \frac{\int_{-\infty}^{\infty} f \hat{P}_i(t, f) df}{\int_{-\infty}^{\infty} \hat{P}_i(t, f) df}
\]

(2)

Where \( \hat{P}_i(t, f) \) is the estimation of TFD.

2.2. Instantaneous Frequency Estimation

Recent years, many research results on instantaneous frequency estimation (IFE) can be found in
documents [4]-[7]. However, most of them are just valid for single-component signals, only TFD
based IFE is able to apply to multi-component signals under some conditions [5][6]. This advantage
makes the TFD based IFE fit more the practice of engineering.

In general, there are two ways to get IFE from TFD: 1. Calculate directly by equation (2); 2. Search
the peaks of TFD [5]. The latter is more intuitive, simple, and also effective, so it is adopted in this
paper. Another reason is that it is more successful in estimating the IFE of multi-component signals.

Many kinds of TFD can be used for the peak searching based IFE [5]. Here the most
understandable TFD, Short-Time Fourier Transform (STFT) was employed in our research.

2.3. IFE Based on The Peak of STFT

The STFT of a signal \( x(\tau) \) is defined by:

\[
\text{STFT}_x(\tau, f) = \int_{-\infty}^{\infty} [x(\tau) \gamma^* (\tau - t)] e^{-j2\pi ft} dt
\]

(3)

Where the asterisk stand for complex conjugate; \( \gamma(\tau) \) is a short time window, viz. a Fourier transform
of the product of signal \( x(\tau) \) and a ‘analysis window’ \( \gamma(t) \) which is centered at \( t \). Multiply a signal by
a short time window \( \gamma(t) \) means to take a slice of the signal at time \( \tau = t \), So \( \text{STFT}_x(\tau, f) \) can be
considered as the “local spectrum” of signal \( x(\tau) \) at ‘analysis time’ \( t \) [8]. The spectrogram of STFT is
the energy of signal concentrate at time \( t \) and frequency \( f \), and is defined as the square of the absolute
value of STFT \( (t, f) \)

\[
\text{SPEC}(t, f) = |\text{STFT}(t, f)|^2
\]

(4)
The corresponding discrete representations of equation (3) and equation (4) are given by

\[ STFT(n, k) = \sum_{i=0}^{N-1} x(i) \cdot e^{-j \frac{2\pi}{N} (i - n)} \]  

(5)

\[ SPEC(n, k) = |STFT(n, k)|^2 \]  

(6)

Where \( N \) is the length of FFT; \( n, k \) are the “nodes” in the time frequency grid of the spectrum respectively. In practice, Fast Fourier Transform (FFT) is taken in the calculation of equation (5).

IFE based the peak of STFT’s will perform well if the IF of the signal changed slowly[5], and STFT is the simplest, the fastest calculation, with clear physical concept in all TFDs and it is also robust to noise. The drawback of STFT is the poor time-frequency resolution when compared with other TFDs, such as Wigner-Ville Distribution (WVD). So, if a more accurate IFE is required, other high resolution TFDs can be employed with peak search.

What we need is only the IFE corresponding to the speed of reference shaft. So, here we declare that if no special statement, the IFE is the correspondence of reference shaft’s speed in this paper.

### 3. IFE Based Order Tracking

As what mentioned above, The key to order tracking is to obtain the \( n_i(t) \), the instantaneous speed of the reference shaft, which represent the function relation about speed \( n \) and \( t \). The suffix \( i \) stands for instantaneous. It induces us recalled a basic notion, \( f_i(t) \), instantaneous frequency, which is often used in time-frequency analysis. Obviously, the speed of reference shaft is corresponding to its instantaneous frequency, given by

\[ n_i(t) = 60 \times f_i(t) \]  

(7)

The scheme of IFE base order tracking see reference [3].

#### 3.1. Robust Peak Search

When the spectrogram is attained, the order tracking can be done through peak search based IFE.

The main steps of peak search are described as follows:

- Pick some points along a prominent order component in the spectrogram, and use cubic spline data interpolation to obtain initial discrete search starting points \( SPEC(n, k_n), n \in (0, N-1) \), in which every frequency bin \( k_n \) is employed as a peak search starting point at time slot \( n \).
- By peak search, to get the discrete instantaneous frequency \( f_i(n, k) \) of the reference shaft speed.

The algorithm of peak search is given by:

\[ f_i(n, k_i) = \text{arg max} \{SPEC(n, k)\}, \quad k_n - p \leq k \leq k_n + p, \quad n \in (0, N - 1) \]  

(8)

Where the notion, arg max, represents the argument (here is \( k \)) of the maximum; \( n, k \) are the time slots and frequency bins on the time-frequency plane respectively; \( p \) is the search range. When the peak search is finished, \( (n, k_i) \), the coordinates of peaks just represent the IFE, \( k_i \), at time \( n \).

The main improvement in the new version of IFE based order tracking is at the first step, which makes the peak search little affected by noise or other non-order components. In section 4, a test example shows how to do these steps in an actual test.

#### 3.2. Curve Fitting Based on LSM

The discrete IFE data from peak search are used to make curve fitting based on Least Squares Method (LSM) in a narrow range. If \( (t_k, f_k), k=1, 2, \ldots, K \), are the discrete coordinates of IFE on time-frequency grids, and a simple quadric function is taken as a curve fitting equation for an example, namely

\[ f_i(t) = at^2 + bt + c \]  

(9)
The sum of $\varepsilon$, Square Error, is

$$F(a, b, c) = \sum_{k=1}^{K} \varepsilon_k^2 = \sum_{k=1}^{K} \left[f_i(t_k) - f_k\right]^2$$  \hspace{1cm} (10)

Let

$$\frac{\partial F}{\partial a} = 0 ; \quad \frac{\partial F}{\partial b} = 0 ; \quad \frac{\partial F}{\partial c} = 0$$  \hspace{1cm} (11)

The coefficients $a, b, c$ in equation (9) can be gotten. Where $K$ is the number of fitting data, calculated by algorithms. If the max order analysis range is 20 and trig speed of order analysis is $n$, corresponding instantaneous is $f_i(t)$, an equation is given by

$$\int_{t_k}^{t_{k+1}} f_i(t) \times dt = \text{FFT length}$$  \hspace{1cm} (12)

And the corresponding digital representation is

$$\sum_{k=1}^{K} f_{ik} \cdot \Delta T = \text{FFT length}$$  \hspace{1cm} (13)

Where $\Delta T = n\Delta t$ ($\Delta t = 1 / f_c$); $m$ depends on the slipping points number of analysis window of STFT per time; $K$ is the data points number under test speed $n$ required by curve fitting.

### 3.3. Keyphasor Timing Mark Based on IFE

The speed plus as keyphasor signal or timing mark is required by traditional order tracking methods to realize equiangular sampling or re-sampling in angle domical. There is no such explicit keyphasor timing marks can be used in the IFE based order tracking, but the LSM based curve fitting equation about $f_i(t)$, equation (9), in a narrow range can provide the keyphasor timing marks needed by order tracking. Supposing $T_n$, sampling time, $f_i(t)$, instantaneous frequency, an equation is given by

$$2\pi \int_{0}^{T_n} f_i(t) dt = n\Delta \theta \quad n = 1, 2, \cdots, \text{FFT length}$$  \hspace{1cm} (14)

Where $n$ is index of sampling series; $\Delta \theta = 2\pi / \text{order}_{\text{max}}$ (the max order range), is constant angular increment.

In a state of the run-up or coast down, if $f_i(t)$ satisfies the smooth and continuous condition, the approximation with polynomial or spline function is feasible in a narrow range. So, from equation (9) and equation (14), an equation is given by

$$\frac{aT_n^3}{3} + \frac{bT_n^2}{2} + cT_n = \frac{n}{2\text{order}_{\text{max}}} + \frac{aT_0^3}{3} + \frac{bT_0^2}{2} + cT_0$$  \hspace{1cm} (15)

Solve equation (15) and attain the valid value of $T_n$ (in practice the numerical value of $T_n$ is able to be attained, $T_n \ , n=1, 2, 3, \cdots, N$), it is just the keyphasor timing marks. Where $N$ is the FFT length of order analysis.

### 3.4. Sampling in Angle Domain

As mentioned above, the equiangular sampling about the reference shaft is required by order analysis. To traditional order analysis, it is performed by keyphasor pulses, which provided by hardware (e.g. tachometer). For our method the keyphasor timing marks or resampling times $T_n \ , n=1, 2, 3, \cdots N$ (FFT length), equation (15), is used to do the angle interpolation to original high rate sampling data $x(T_n)$. In test example of this paper, good results were attained only by the simple linear interpolation.

At last, the FFT about $x(T_n) \ (n=1, 2, 3, \cdots, N)$, is performed to get the order spectrum, such as order amplitude spectrum, power spectrum, and so on. If an analyzer input a start-speed and compute
different order spectrum under different speed with a speed increment, the order waterfall can be attained in the same way.

3.5. The Applying Condition
Under test speed, if a prominent order component of signal can be separated from other order components on time-frequency plane, a good result can be attained by the IFE based order tracking.

4. Test
An actual test to a rotor was done to demonstrate the validity of IFE based order tracking. Some information about the test is given: The rotating speed of the rotor is about 6800rpm; the test was done during coast down with sampling rate 40,000 Hz; the length of sampling data was 1258×1024 points. The B&K accelerometer type 4370 and B&K charge amplifier type 2635 are used in this test to obtain the signal of acceleration of a bearing support of the rotor. To reveal the anti-noise ability of the peak search method, a strong artificial noise is added to the original acceleration signal with constant frequency 150 Hz and amplitude 0.1 scaled by acceleration of gravity. The result is shown as figure 1. The horizontal axis represents time \( t \) in seconds while vertical axis denotes frequency \( f \) scaled by Hz. The order components of signal were clear exposed.

![Figure 1. Spectrogram under down-sampling rate 16.](image1.png)

By obtaining a more accurate IFE of rotating speed, a down-sampling rate 64 is applied, and the corresponding spectrograms are shown as figure 2 and figure 3. The horizontal axis represents time \( t \) in seconds while vertical axis denotes frequency \( f \) scaled by Hz. According to the order tracking steps mentioned in section 3, the first step is to pick some points along the second order component in the spectrogram, and use cubic spline data interpolation to obtain initial discrete search starting points, which are the dark ‘line’, discrete points, along the second order

![Figure 2. Cubic spline data interpolation result under down-sampling rate 64.](image2.png)

![Figure 3. Peak search result under down-sampling rate 64.](image3.png)
component are shown in figure 2. The second step is doing the peak search, and the white points shown in figure 3 are the search result, whose time and frequency coordinates indicate the IFE of the second order component.

The order spectrum analysis result was shown as figure 4. The horizontal axis represents orders, $l\omega$, while vertical axis denotes the amplitude of power scaled by the square of acceleration of gravity. Note the first order component is weaker than other high order components. Compared with figure 5, which was the order spectrum of original signal, it is able to find the spectrum line locates between order 1 and order 2 was caused by the strong artificial noise, see figure 4.

![Figure 4. Order spectrum at 6000rpm under strong artificial noise.](image)

![Figure 5. Order spectrum at 6000rpm with original signal.](image)

5. Conclusion
Simulations and actual tests indicate if the applying condition is satisfied, the IFE based order tracking is feasible in engineering applications, and a good resolution of order analysis can be attained by it.

This method is an effective supplement to traditional methods on order tracking and is more attractive on the condition where the keyphasor equipment’s installation is inconvenient. A good prospect of application in engineering is imaginable.

Acknowledgements
The project supported by Natural Science Foundation of Yunan, China (No. 2004E0011Q).

References
[1] Fyfe K R and Munck E D S 1997 Analysis of computed order tracking Mechanical Systems and Signal Processing 11 187-205
[2] Bossley K M, Mckendrick R J, Harris C J and Mercer C 1999 Hybrid computed order tracking Mechanical Systems and Signal Processing 13 627-641
[3] Guo Y, Qin S and Liang Y 2003 Order tracking of rotating machinery based on instantaneous frequency estimation Chinese Journal of Mechanical Engineering 39 32-36
[4] Boashash B 1992 Interpreting and estimating the instantaneous frequency of a signal—Part 1: Fundamentals Proc. IEEE 80 520-538
[5] Boashash B 1992 Interpreting and estimating the instantaneous frequency of a signal—Part 2: Algorithms and Applications Proc. IEEE 80 540-568
[6] Cohen L 1995 Time–Frequency Analysis (New Jersey: Prentice Hall)
[7] Emresoy M K and El-Jaroudi A 1998 Iterative instantaneous frequency estimation and adaptive matched spectrogram Signal Processing 64 157-165
[8] Zhang X D and Bao Z 1999 Non-stationary Signal Analysis & Proceeding (Beijing: Defense Industry)