Requirement of the on/off ratio of superconducting heat switch used for the continuous stage of cADR

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Abstract. As an important component of the adiabatic demagnetization refrigerator (ADR), the superconducting heat switch uses the different thermal conductivities of a metal in the normal state and the superconducting state to turn the heat switch on/off for the continuous stage. Switch ratio is an important parameter to evaluate the performance of a heat switch, which is related to many factors. Peter Kittel calculated the different switch ratio requirements in the single-stage and multi-stage ADR. Based on that, this paper focuses on the relationship between the required switch ratio and operating parameters of the ADR system, especially the ratio of the heat leak to the effective cooling power.

1. Introduction

Adiabatic demagnetization refrigeration (ADR) is an important refrigeration technology in space exploration and other scientific research. It has the advantages of high refrigeration efficiency, gravity-independence, etc. To reach lower temperatures efficiently, researchers connected several single-stage ADRs in series to form a multi-stage ADR, which can be classified into single-shot ADR and continuous ADR (cADR)[1-3]. As an important component of ADR, the heat switch plays an important role in controlling the thermodynamic cycle of the system and completing the continuous operation of cADR. According to their different working principles, the heat switch can be classified into gas gap heat switch, superconducting heat switch, magneto-resistive heat switch, and mechanical heat switch, etc. The gas gap heat switch uses the adsorption/desorption of gas, for instance $^3$He, to turn the heat switch on/off. It has a simple structure and high reliability, but the disadvantage is that when the temperature is lower than 0.2 K, the gas gap heat switch cannot continue to work due to the extremely low saturated vapor pressure[4, 5]. The mechanical heat switch works by making and breaking a mechanical high conductance contact. It can be completely disconnected in the off state, but the disadvantage is that the structure is complex and the durability is not good[6]. The magneto-resistive heat switch uses the magneto-resistive effect of a metal to work. It has a wide working temperature range and a large switch ratio, but the disadvantage is that it needs a considerably large magnetic field [7, 8]; The superconducting heat switch uses the different thermal conductivities of superconducting metals in the normal state and superconducting state. Due to its low working temperature, simple configuration, and large switch ratio, it becomes the preferred choice for temperatures far below 1 K [9]. This paper focuses on the superconducting heat switch.
Superconductivity was discovered in 1922 by Kamerlingh Onnes while studying the resistivity of pure metals[10]. Usually, in a normal metal, heat can be carried by electrons and phonons, and the electronic thermal conductivity is considerably larger than the phonon thermal conductivity, \( k_{\text{normal}} \propto T^\eta (\eta \approx 1) \). When the temperature is lower than the superconducting temperature, some of the electrons are paired and become so-called Cooper pairs. They all sit in the low energy state of zero entropy, and cannot leave this ground state to carry heat, so only the remaining unpaired electrons and phonons can carry heat. Therefore, the thermal conductivity of the superconductor approaches the thermal conductivity of an insulator, \( k_{\text{super}} \propto T^\eta (\eta \approx 3) \). When the magnetic field applied on the superconductor is larger than the critical magnetic field at this corresponding temperature, the magnetic field will destroy Cooper pairs, and the electrons will return to the free state to participate in the thermal conduction, and the thermal conductivity will rise sharply[2]. This is the working principle of the superconducting heat switch. Usually, the switch ratio \( (\gamma(T) = k_u/k_s \propto T^2) \) is used to evaluate the performance of the superconducting heat switch[1].

In 1949, Heer and Daunt first used the superconducting metal of tin and thallium to make heat switches. At \( T = 0.65 \) K, the switch ratio of tin and thallium is 40 and 60 respectively[11]. In 1961, Reese and William obtained the switch ratio of 457\( T^2 \) for a lead heat switch in the temperature range of 0.1 K-0.5 K[12]. In 1964, Peshkov obtained the switch ratio of 510\( T^2 \), 620\( T^2 \) for single crystal tin and etched foil tin, respectively[9]. In 1978, Mueller successfully made a gold-plated aluminum superconducting heat switch. The switch ratio of the heat switch is up to 1600\( T^2 \), which is the highest switch ratio reported ever[13]. In 2000, Ho and Hallock made an indium superconducting heat switch using 5N 0.13 mm thick indium tape, which has a switch ratio of 657\( T^2 \) at 0.1-0.4 K[14].

To guide the design of a superconducting heat switch, in 2000, Kittel studied the relationship between the required switch ratio and operating parameters in different stages[15]. The main findings are:

For the single-stage, the requirement of switch ratio is:

\[
\gamma_1(T_h) > \frac{1}{m+1} \frac{T_h^2}{\Delta T_c} \frac{1}{\tau}
\]  

(1)

For the intermediate stage, the requirement of switch ratio is:

\[
\gamma_2(T_i) > \frac{1}{m+1} \left( \frac{T_h}{T_i} \right)^{m+1} \frac{T_i^2}{\Delta T_c} \frac{1}{\tau}
\]

(2)

For the continuous stage, the requirement of switch ratio is:

\[
\gamma_3(T_c) > \frac{1}{m+1} \left( \frac{T_h}{T_c} \right)^{m+1} \frac{T_c}{\Delta} \frac{1}{\tau}
\]

(3)

Where \( T_h \) is the high temperature of the second stage, \( T_c \) is the temperature of the continuous stage, \( T_i \) is the high temperature of the third stage, \( \tau \) is time ratio, \( \Delta \) is the temperature difference, \( m \) is the power of temperature, \( \gamma(T_h) \), \( \gamma(T_i) \) and \( \gamma(T_c) \) are the switch ratio at temperature \( T_h \), \( T_i \) and \( T_c \) respectively. Assuming that the temperature span ratio and time ratio \( \tau \) are the same in different stages, the conclusion is that the single-stage ADR has the least stringent requirement for the switch ratio \( \gamma_1 \), the intermediate stage has the most difficult switch ratio requirements \( \gamma_3 \), the isothermal stage switch ratio requirements \( \gamma_3 \) are in between the former two.

\[
\gamma_1 < \gamma_3 < \gamma_2
\]

(4)

However, in practical applications, the superconducting heat switch will not be used in single-stage and intermediate-stage, because the gas gap heat switch will be a better choice in a relatively high temperature range. Furthermore, it is also necessary to refine the analysis, especially considering the ratio of the heat leak (heat flow through the heat switch during isothermal demagnetization) to the effective cooling power (applied heat load).
In the following, section 2 establishes the relationship between the required switch ratio and operating parameters through physical model. Section 3 gives a few sample calculations to show how the results can be used to guide the design of a superconducting heat switch. Finally, a conclusion is drawn.

2. Physical Model

Typical ADR consists of four processes: adiabatic magnetization, isothermal magnetization, adiabatic demagnetization, and isothermal demagnetization. For a cADR, the continuous stage (M1) is directly connected to the applied heat load and is magnetized or demagnetized periodically, to keep a constant temperature. The remaining stages act to periodically cascade heat from the continuous stage up to the heat sink [16]. When the continuous stage (M1) is in the state of isothermal demagnetization, the superconducting heat switch turns off to provide an adequate adiabatic environment. The temperature of the continuous stage (M1) \( T_1 \) is \( T_C \), while the temperature of the second stage (M2) \( T_2 \) is \( T_H \). The effective cooling power (applied heat load) is \( Q_L \), and the heat flow through the superconducting heat switch is \( Q_{OFF} \) (heat leak). When the continuous stage is in the state of isothermal magnetization, the superconducting heat switch superconducting heat switch turns on to ensure the cooling power generated by the second stage can be efficiently transferred to the continuous stage in a small temperature difference. The temperature of the continuous stage (M1) \( T_1 \) is \( T_C \), while the temperature of the second stage (M2) \( T_2 \) is \( T_C - \Delta \) (\( \Delta \) is the temperature difference). The effective cooling power is still \( Q_L \) and the heat flow through the superconducting heat switch is \( Q_{ON} \). As shown in figure 1.

![Figure 1](image)

**Figure 1.** Heat switch used for the continuous stage during isothermal demagnetization (Upper) and magnetization stage (Lower).

For the continuous stage containing salt pill 1, according to the second law of thermodynamics, the entropy change rate of the system is equal to the sum of the entropy production rate caused by irreversible loss and the entropy flow rate caused by heat and mass transfer. Since there is no mass transfer and the irreversible loss is small enough to be ignored, the entropy change rate \((d\hat{S}_{\text{mag}})\) of the salt pill is equal to the entropy flow rate \((\delta\hat{S}^q)\) caused by heat transfer through thermal conduction:

\[
d\hat{S}_{\text{mag}} = \delta\hat{S}^q
\]

(5)

For the isothermal magnetization state and the isothermal demagnetization state, the entropy change rate of the salt pill \( \Delta S_{\text{mag}} \), \( \Delta S_{\text{demag}} \) is:
\[ \Delta S_{\text{mag}} = \frac{Q_{\text{ON}}}{T_c} - \frac{Q_L}{T_c} \]  
\[ \Delta S_{\text{demag}} = \frac{Q_{\text{OFF}}}{T_c} + \frac{Q_L}{T_c} \]  

(6)  
(7)  

The entropy change of isothermal magnetization \( \Delta S_{\text{mag}} \) should be equal to that of isothermal demagnetization \( \Delta S_{\text{demag}} \) in one cycle:

\[ \Delta S_{\text{mag}} t_{\text{ON}} = \Delta S_{\text{demag}} t_{\text{OFF}} \]  

(8)  

Where \( t_{\text{ON}} \) is the time of the isothermal magnetization and \( t_{\text{OFF}} \) is the time of the isothermal demagnetization. Defining the time ratio of \( t_{\text{ON}} \) to \( t_{\text{OFF}} \) as \( \tau \):

\[ \tau = \frac{t_{\text{ON}}}{t_{\text{OFF}}} \]  

(9)  

For the single-shot ADR, the time ratio \( \tau \) is always far larger than 1. For the cADR, the value of time ratio \( \tau \) is not so strictly defined. Here a typical value \( \tau = 1 \) is used for the analysis. Combining the equations (6) ~ (9) leads to:

\[ \frac{Q_{\text{ON}}}{T_c} - \frac{Q_L}{T_c} \tau = 1 \]  
\[ \frac{Q_{\text{OFF}}}{T_c} + \frac{Q_L}{T_c} \]  

(10)  

This equation can be re-written in a simplified form:

\[ \frac{Q_{\text{ON}}}{Q_L} \cdot \frac{Q_L}{Q_{\text{OFF}}} \tau = 1 \]  

(11)  

Generally, the cooling power produced by the salt pill should be utilized as much as possible, that is, the heat leak \( Q_{\text{OFF}} \) of the heat switch should be far less than the effective cooling power \( Q_L \). To better show the relationship between the two, the parameter \( \chi \) (the ratio of the heat leak to the effective cooling power) is defined:

\[ \chi = \frac{Q_{\text{OFF}}}{Q_L} \]  

(12)  

For an efficient thermodynamic system, there should be \( \chi << 1 \). Combining the equation (11) and equation (12), the relationship between \( Q_{\text{ON}} \) and \( Q_{\text{OFF}} \) becomes:

\[ \frac{Q_{\text{ON}}}{Q_{\text{OFF}}} = \frac{1 + \chi + \tau}{\chi \tau} \]  

(13)  

For a superconducting heat switch, the thermal conductivity in the normal state and superconducting state are shown in the following equations:

\[ k_{\text{ON}} = \alpha T^n (n \approx 1) \]  
\[ k_{\text{OFF}} = \beta T^m (m \approx 3) \]  

(14)  
(15)  

Defining the ratio of switch ratio as \( \gamma(T) \):

\[ \gamma(T) = \frac{k_{\text{ON}}}{k_{\text{OFF}}} = \frac{\alpha}{\beta} T^{n-m} \]  

(16)  

The ratio of heat flow through the heat switch during isothermal magnetization \( (Q_{\text{ON}}) \) and isothermal demagnetization stage \( Q_{\text{OFF}} \) is:
Combining the equations (15) and (16), and assuming that $T_{Cm}^m << T_{Hm}$ and $\Delta << T_{Cm}$, and then the ratio of $Q_{ON}$ to $Q_{OFF}$ can be written in the following equation:

$$\frac{Q_{ON}}{Q_{OFF}} = (m+1)\gamma(T_{C})\alpha(T_{H}) \frac{T_{C}}{T_{Cm}} \left(\frac{T_{C}}{T_{H}}\right)^{m+1}$$

(18)

Combining the equations (12) and (17), the relationship between the required switch ratio and operating parameters becomes:

$$\gamma(T_{C}) = \frac{\chi + 1 + \tau}{\chi \tau} \frac{T_{C}}{T_{Cm}} \left(\frac{T_{H}}{T_{C}}\right)^{m+1}$$

(19)

3. Analysis

Based on the established physical model, the relationship between the required switch ratio of a heat switch and operating parameters, especially the ratio of the heat leak to the effective cooling power $\chi$ can be obtained. It is helpful to select the appropriate working temperature range for different types of heat switches while ensuring the ratio of the heat leak to the cooling power $\chi$ can meet the requirement.

To better show how the results can be used to guide the design of ADR, a few samples were analyzed. The lead heat switch with a switch ratio of 45$T^2$, the indium heat switch with a switch ratio of 65$T^2$, the aluminum heat switch with a switch ratio of 1600$T^2$ and the tin (single crystal) heat switch with a switch ratio of 510$T^2$ are used as examples for calculation. For the cADR, a typical value $T_{C} = 50 \text{ mK}$, $\Delta = 5 \text{ mK}$, $\tau = 1$, and $m = 3$ are brought into the equation (19). The relationship between $\chi$ and $T_{H}$ is shown in figure 2.

![Figure 2. Relationship between $\chi$ (the ratio of the heat leak to the effective cooling power) and $T_{H}$ (the high temperature of M2) for different heat switches ($T_{C} = 50 \text{ mK}$, $\Delta = 5 \text{ mK}$, $m=3$, $\tau=1$)](image)

$\chi$ vs $T_{H}$ plot for different heat switches.
It can be found from figure 2 that when $T_H$ increases, $\chi$ will increase accordingly. Taking the aluminum heat switch with a switch ratio of 1600$T^2$ as an example. When the $T_H$ is 0.2 K, the ratio of the heat leak to the cooling power $\chi$ is 0.002, and when the $T_H$ increases to 0.4K, the ratio of the heat leak to the cooling power $\chi$ increases to 0.03. Obviously, reducing $T_H$ can effectively reduce the ratio of the heat leak to the cooling power, but for the ADR system, reducing $T_H$ will increase the magnetic field strength required for the latter stage and increase the complexity of system design and operation. Therefore, a reasonable strategy is to increase the $T_H$ to reduce the complexity of the system while ensuring that the $\chi$ can meet the requirement. For example, assuming the $\chi$ should be less than 0.1, taking the other parameters mentioned above into equation (19), the maximum permissive $T_H$ that can meet the operating requirement for different heat switches are listed in table 1.

**Table 1.** The maximum $T_H$ for different heat switches ($T_c = 50$ mK, $\Delta = 5$ mK, $m=3$, $\tau=1$, $\chi=0.1$)

| Heat switch         | $\chi(T)$     | Maximum permissive $T_H$ (K) |
|---------------------|---------------|------------------------------|
| Al                  | 1600$T^2$[13] | 0.52                         |
| Sn (single crystal) | 510$T^2$[9]   | 0.40                         |
| In                  | 65$T^2$[14]   | 0.24                         |
| Pb                  | 45$T^2$[12]   | 0.22                         |

4. Conclusion

Based on the physical model, this paper establishes the relationship between the required switch ratio $\chi(T)$ and operating parameters of the lowest temperature stage in a cADR. Specially, the ratio of heat leak to the effective cooling power $\chi (Q_{off}/Q_{on})$ is introduced, which is one of the important parameters used to evaluate the thermodynamic efficiency. Typical operating parameters are used for detailed calculation, which shows the maximum permissive $T_H$ of different heat switches.

Besides the parameter $\chi$, there are other interesting parameters worth considering. Such as the impact of the fluctuation of $m$, which represents the thermal conductivity of the superconductor, on the whole result. These conclusions are useful for choosing the superconducting heat switch and designing the ADR system.

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