Degenerate and Other Neutrino Mass Scenarios and Dark Matter

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Abstract

I discuss in this talk mainly three topics related with dark matter motivated neutrino mass spectrum and a generic issue of mass pattern, the normal versus the inverted mass hierarchies. In the first part, by describing failure of a nontrivial potential counter example, I argue that the standard $3\nu$ mixing scheme with the solar and the atmospheric $\Delta m^2$'s is robust. In the second part, I discuss the almost degenerate neutrino (ADN) scenario as the unique possibility of accommodating dark matter mass neutrinos into the $3\nu$ scheme. I review a cosmological bound and then reanalyze the constraints imposed on the ADN scenario with the new data of double beta decay experiment. In the last part, I discuss the $3\nu$ flavor transformation in supernova (SN) and point out the possibility that neutrinos from SN may distinguish the normal versus inverted hierarchies of neutrino masses. By analyzing the neutrino data from SN1987A, I argue that the inverted mass hierarchy is disfavored by the data.

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I. INTRODUCTION

The hot and cold dark matter cosmology [1] has been served as one of the rival models which solves the problem of structure formation in the universe in a consistent way with observation of fluctuation of cosmoc microwave background radiation and galaxy correlations [2]. While the necessity of the hot component of the dark matter becomes less prominent in the light of recent observations [3,4] the problem still remains as to what extent the possible dark matter neutrinos can have a ”market share” in the cosmos. Moreover, the possibility that neutrinos have masses of a few eV range, if realized in nature, should shed light on underlying physics of neutrino mass, the unique hint to date for physics beyond the standard model of elementary particles.

In this talk, I examine the question of neutrinos with masses of eV scale which is suitable for cosmological hot dark matter. I assume the three-flavor mixing scheme of leptons in the standard model. It is important to emphasize that two robust evidences in favor of neutrino oscillations, the atmospheric [5] and the solar neutrino [6] anomalies perfectly fit into the three-flavor mixing scheme. While the LSND experiments [7] suggests that the scheme is too tight, I would like to wait for the confirmation by independent experiments before deciding to go beyond the three-flavor framework. I will use, throughout this article, the standard Particle Data Group notation for the elements of lepton flavor mixing matrix, the Maki-Nakagawa-Sakata (MNS) matrix [8].

I will convey you, in this talk, mainly the following three messages:

(i) The standard three-neutrino mixing scheme, in which two $\Delta m^2$ are assigned to the atmospheric and the solar neutrino oscillations, is robust. I will describe an attempt at challenging to this conventional wisdom but I will badly fail.

(ii) The above fact seems to indicate that if the neutrinos have dark matter mass of a few eV they must be almost degenerate in masses. I discuss the powerful constraints imposed on such almost degenerate neutrino (ADN) scenario by cosmological observations and by the neutrinoless double $\beta$ decay experiments.

(iii) I then move on to the question of mass pattern of neutrinos. I will address one of the key questions among the remaining problems in the three-neutrino mixing scheme, namely
the sign of \( \Delta m_{13}^2 \equiv m_3^2 - m_1^2 \). I point out that observation of neutrinos from supernova will tell us about it \[9\].

II. ACCOMMODATING DARK MATTER MASS NEUTRINOS IN THE THREE-FLAVOR MIXING SCHEME?

Let me start by introducing the first problem mentioned in (i). Since this is a dark matter conference, it is natural to raise the following question; is it possible to embed \( \Delta m_{DM}^2 \) into the three-neutrino mixing scheme? Well, the answer appears to be trivially No! as far as one wants to keep \( \Delta m_{\text{atm}}^2 \) and \( \Delta m_{\odot}^2 \) which exhaust the available two \( \Delta m^2 \) in 3 \( \nu \) scheme. The only way out of the dilemma is to give up one of the two \( \Delta m^2 \) in favor of \( \Delta m_{DM}^2 \), but in such a way that the resuling scheme is still capable of explaining the atmospheric and the solar neutrino observations. Of course, it would not be possible to account for all the aspects of the data and probably we must live with e.g., the energy-independent deficit by a factor of \(~ 2\) in solar neutrinos \[10\], probably at the price of sacrificing one of the 4 solar neutrino experiments. But, it is still highly nontrivial to prove or refute the possibility that such scenario exists in a consistent manner with all the other constraints. So, let me try.

In the following, I examine scenarios of 3 \( \nu \) mixing in which the first \( \Delta m^2 \) is assigned to \( \Delta m_{DM}^2 \) and the second to \( \Delta m_{\text{atm}}^2 \) or \( \Delta m_{\odot}^2 \). One has to recognize, first of all, that such scenario is strongly constrained by the reactor and the accelerator experiments. As noticed in Refs. \[11,12\] there are only three tiny regions on parameter space spanned by \( s_{13}^2 \) and \( s_{23}^2 \), as schematically indicated in Fig. 1:

(a) \( c_{13}^2 \sim \epsilon \) and \( s_{23}^2 \) is arbitrary
(b) \( s_{13}^2 \sim \epsilon \) and \( s_{23}^2 \sim \delta \)
(c) \( s_{13}^2 \sim \epsilon \) and \( c_{23}^2 \sim \delta \)

where \( \epsilon \) and \( \delta \) are of the order of a few \( \times 10^{-2} \) for a value of \( \Delta m_{DM}^2 \) which is appropriate for hot dark matter. If you want to know more precise shape of the allowed region for a given value of \( \Delta m_{DM}^2 \), see Ref. \[12\]. Therefore, the two mixing angles \( s_{13}^2 \) and \( s_{23}^2 \) are essentially determined depending upon your choice of the regions in (a) - (c).

Next I must decide which option I take; assignment of remaining \( \Delta m^2 \) to either \( \Delta m_{\text{atm}}^2 \)
or $\Delta m^2_{\odot}$. Once I decide the option the scenario is completely determined up to the arbitrary parameter $\theta_{12}$. Then, by adjusting $\theta_{12}$ I try to explain all the data of the solar and the atmospheric neutrino observations under the constraints of the terrestrial experiments.

I do not try to describe the details of the actual process of the analysis but summarize the results in Table 1. In the Table the symbols "N" and "I" refer to the normal and the inverted mass hierarchies respectively, which imply mass patterns, N (normal): $m_3 \gg m_1 \sim m_2$ and I (inverted): $m_1 \sim m_2 \gg m_3$. The regions (a)-(c) (in order from above) are indicated symbolically in Table 1.

Table I: Grading Hierarchical Mass Dark Matter Neutrino Scenarios.

| Mixing Angles | Normal vs. Inverted | $\nu_\odot$ | $\nu_{\text{atm}}$ | $\beta\beta$ | CHOOZ |
|---------------|---------------------|-------------|-------------------|--------------|-------|
|               |                     | $\sim \frac{1}{2}$ shape | $\sim 60\% \nu_\mu \rightarrow \nu_\tau$ |            |       |
| $\Delta m^2 = \Delta m^2_{\odot}$ |                      | $\bigcirc$ $\bigcirc$ $\times$ $\times$ $\times$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ |
|               |                     | $\bigcirc$ $\bigcirc$ $\times$ $\times$ $\times$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ |
|               |                     | $\bigcirc$ $\bigcirc$ $\times$ $\times$ $\times$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ |
| $\Delta m^2 = \Delta m^2_{\text{atm}}$ |                      | $\bigcirc$ $\bigcirc$ $\times$ $\times$ $\bigcirc$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ |
|               |                     | $\bigcirc$ $\bigcirc$ $\times$ $\times$ $\bigcirc$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ |
|               |                     | $\bigcirc$ $\bigcirc$ $\times$ $\times$ $\bigcirc$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ | $\bigcirc$ $\bigcirc$ |

As you see in Table 1 there is no satisfactory case. In Table 1 "if vac" implies that if
the vacuum solar neutrino solution of non-just-so type, i.e., energy independent reduction of about factor of 2, turns out to be the case. Explanation of average reduction of solar and atmospheric muon neutrino rate is achieved in the mass pattern (b) of $\Delta m_{atm}^2$ case, but it fails at the CHOOZ [13] as well as at the Superkamiokande experiments [5] which jointly provide evidence for dominance of $\nu_\mu \rightarrow \nu_\tau$ channel.

III. ALMOST DEGENERATE NEUTRINOS

The fact that we badly fail in our attempt at incorporating $\Delta m_{DM}^2$ into 3 $\nu$ scheme strongly suggests that the standard three-flavor mixing scheme is robust. Then, the only way to accommodate the dark matter mass neutrinos is to assume that three $\nu$ states are almost degenerate with masses of a few eV range [14], the almost degenerate neutrino (ADN) scenario [15].

The ADN scenario is the most natural possibility if all of the three mixing angles are large. At the moment, we do know that $\theta_{23}$ is large and almost maximal, while one of the other angles $\theta_{13}$ is small [13]. We still do not know if $\theta_{12}$ is large or small.

The ADN scenario can be constrained by cosmological observations as well as by laboratory experiments. The latter includes the direct mass measurement using $\beta$ decay end point spectrum, and the neutrinoless double $\beta$ decay experiments.

Since a few eV mass neutrinos play important role in cosmology it is conceivable that it is constrained by cosmological observations. This is a natural place where one can place a bound on neutrino masses because the streaming motion of light neutrinos washes out seeds for structure formations at small scales. In fact, it is argued by Fukugita, Liu, and Sugiyama [16] that it is the case; they used matching condition of fluctuation powers at COBE and clusters scales as the most sensitive probe and obtained $m_\nu \lesssim 0.6 - 1.8$ eV depending upon $\Omega_{matter} = 0.3 - 0.4$ at the Hubble parameter 80 in units of 100 km s$^{-1}$ Mpc$^{-1}$. This type of treatment should be valid for dark matter massive neutrinos with more generic mass spectrum, with which the similar bound presumably results. The bound of course applies both to Dirac and Majorana neutrinos.

Let us move on to the laboratory bound. I discuss here only the double $\beta$ decay bound
because it achieves the greatest sensitivity among the laboratory experiments. Of course, the double \( \beta \) bound only applies to Majorana neutrinos. There is a simple reason why the experiment gives rise to nontrivial constraints on neutrino mixing parameters. The dark matter motivated ADN scenario requires the neutrino mass of the order of \( \sim \) a few eV. On the other hand, the sensitivity of the double \( \beta \) decay experiments went down to less than 0.5 eV. It means that an efficient cancellation must take place among contributions from three mass eigenstates, which implies a tight constraint on mixing angles \[17\].

Observable in the neutrinoless double \( \beta \) decay experiments can be written as

\[
\langle m_{\nu e} \rangle = \left| \sum_i U_{ei}^2 m_i \right|
\]

where \( U_{ei} \) denotes the elements of the MNS matrix \[8\]. Under the ADN approximation, \( |m_i - m_j| \ll m_i \simeq m \) (i=1-3) and by using the standard parametrization by Particle Data Group, it can be written as

\[
\langle m_{\nu e} \rangle = m \left| c_{12}^2 c_{13}^2 e^{-i(\beta+\gamma)} + s_{12}^2 c_{13}^2 e^{i(\beta-\gamma)} + s_{13}^2 e^{2i(\gamma-\delta)} \right|,
\]

where \( \beta \) and \( \gamma \) are the extra CP-violating phases characteristic to Majorana neutrinos. Since \( r \equiv \langle m_{\nu e} \rangle/m \lesssim 0.3 \) there must be cancellation between three angle factors in (2). The resulting constraint has first been examined in Ref. \[17\] in a manner completely independent of unknown Majorana phases and it was shown that the ADN scenario is inconsistent with the SMA MSW solar neutrino solution. (See also Ref. \[18\] for an update.) Later, the similar analyses have been repeated or extended by a number of authors who exploit newer (more stringent) constraint, or cover a wider class of solar neutrino solutions and/or more general mass patterns \[18,19\].

I take the chance of presentation at Dark2000 to update our analysis done in Ref. \[17\]. It is timely to do reanalysis now because the most stringent bound on \( \langle m_{\nu e} \rangle \) provided by the Heidelberg-Moscow Group has been changed to

\[
\langle m_{\nu e} \rangle < 0.35\text{eV},
\]

as announced at this conference \[20\], which is relaxed by a factor of \( \sim 2 \) compared with the previous one. Therefore, constraints derived in some of the earlier analyses can be artificial.
We obtain the parameter region allowed by the bound (3) with use of the degenerate neutrino mass $m = 2.3$ eV, the lowest mass in Pogosyan-Starobinsky analysis (21). Notice that it is the case of mildest constraint. In Fig. 2 we draw a hexagon-shaped region allowed by the double $\beta$ decay experiment (assuming no constraint on unknown Majorana phases) together with parameter region allowed for the MSW solution of solar neutrino problem on the solar triangle plot introduced by the Bari group (22). For the solar neutrino allowed region we also use the updated results by the group (23).

We observe:

(1) All the MSW solutions of the solar neutrino problem is excluded at $\sim 1$ $\sigma$ level in the ADN scenario, except for the LOW solution; its allowed region bridges between the LMA region and the bottom of the hexagon.

(2) The LMA MSW solution barely survives only when we take into account of the factor of 2 uncertainty in estimation of the nuclear matrix elements.

(3) The vacuum solution is obviously consistent with the double $\beta$ bound because it spans a wide region at the lowest quarter of the double $\beta$ hexagon due to the nearly maximal $\theta_{12}$; wide region because of the freedom from the CHOOZ bound. See, e.g., Ref. (18). This statement presumably generalizes to the "quasi-vacuum" solution (24).

Now let us turn the argument around. Namely, I try to derive the upper bound on degenerate neutrino mass which is consistent with the LMA MSW solution, the best favored solution by the data at this moment. By looking into Fig. 3 one can safely argue that $r \equiv \langle m_{\nu e} \rangle / m > 0.24$ in order to have overlapping region between the allowed regions by the double $\beta$ and the solar neutrino data. It implies the upper bound

$$m < 1.5 eV. \quad (4)$$

in the ADN scenario. If a factor of $\sim 2$ uncertainty of the nuclear matrix elements is taken into account, the bound would become loosen by the same factor.

It is interesting to note that the cosmology argument by Fukugita et al. (16) and our double $\beta$ bound give numerically similar upper bounds while the physics and the underlying assumptions involved differ completely. I also want to stress that, given the present accuracy of the experimental bound (of the order of $\langle m_{\nu e} \rangle \lesssim$ a few $\times$ 0.1 eV) the estimation of mass
bound with the ADN assumption is not so bad as an order of magnitude estimation even for hierarchical spectrum. I should also emphasize that this will no longer be true when the bound goes down to \( \langle m_{\nu e} \rangle \lesssim 0.01 \text{ eV} \) because it is below \( \sqrt{\Delta m_{\text{atm}}^2} \) and starts to distinguish the various mass patterns.

**IV. NORMAL VS. INVERTED MASS HIERARCHIES BY SUPERNOVA NEUTRINOS**

Now we address the last point (iii) mentioned at the beginning of this article, namely the issue of normal vs. inverted hierarchies of neutrino masses. I guess that it is one of the most important questions in the 3 \( \nu \) mixing scheme which presumably will provide the key to understand the underlying physics of neutrino mass spectrum. Furthermore, it is the crucial question for the neutrinoless double \( \beta \) decay experimentalists. The required sensitivities for detecting positive signal are \( \langle m_{\nu e} \rangle \sim 0.001 \text{ eV} \) and \( \langle m_{\nu e} \rangle \simeq 0.04 - 0.07 \text{ eV} \) for the normal and the inverted mass hierarchies, respectively [25,26].

Now I want to point out that observation of neutrino events from supernova (SN) provides us with a mean for discriminating the normal vs. the inverted mass hierarchies. Furthermore, I argue that the inverted hierarchy of neutrino mass is strongly disfavored by the neutrino data from SN1987A [27] unless the mixing angle \( \theta_{13} \) is very small, that is, unless \( s_{13}^2 \lesssim a \text{ few } \times 10^{-4} \). Of course, this conclusion must be checked against the direct determination of the sign of \( \Delta m_{13}^2 \) which may be done in future long-baseline accelerator experiments [28–30]. However, the result we have obtained appears to be the unique hint which is available before such experiments are actually done.

Toward the goal of showing that the inverted mass hierarchy is disfavored, I must first explain some key features of neutrino flavor conversion in SN. For more detailed explanation see our recent paper [9].

We start by summarizing the common knowledges on neutrinos from supernova [31] and their properties inside neutrinosphere [32,33].

1. Consideration of energetics of SN collapse indicates that almost all (\( \sim 99\% \)) of the gravitational binding energy of neutron star is radiated away via neutrino emission. The
total energy is estimated to be several $\times 10^{53}$ erg, and it is expected that the equipartition of energy into three flavors in a good approximation \[32,33\].

(2) It is discussed that the shape of the energy spectra of various flavor neutrinos can be described by a ”pinched” Fermi-Dirac distribution \[34\]. The pinched form may be parametrized by introducing an effective ”chemical potential”.

(3) There is no physical distinction between $\nu_\mu$ and $\nu_\tau$ and their antiparticles in neutrinosphere. It is because $\nu_\mu$ and $\bar{\nu}_\mu$ are not energetic enough to produce muons by the charged current interactions, and the neutral current cross sections of $\nu$ and $\bar{\nu}$ are similar in magnitude. Therefore, we collectively denote them as ”heavy neutrinos” hereafter.\[\]

(4) The location of neutrinosphere of heavy neutrinos, $\nu_\mu$ and $\nu_\tau$, is believed to be in deeper place than $\bar{\nu}_e$ and $\nu_e$ in SN. It is due to the fact that the heavy neutrinos have weaker interactions with surrounding matter; they interact with matter only via the weak neutral current, whereas, $\bar{\nu}_e$ and $\nu_e$ do have additional charged current interactions. Hence, the heavy neutrinos have to have deeper neutrinosphere because their trapping requires higher matter density compared to those required for $\bar{\nu}_e$ and $\nu_e$.

This last feature is of crucial importance for our business. It implies that the heavy neutrinos are more energetic when they are radiated off at neutrinosphere because the temperature is higher in denser region. It may be characterized by the temperature ratios of $\nu_e$ and $\bar{\nu}_e$ to $\nu_{\text{heavy}}$

$$\tau \equiv \frac{T_{\nu_{\text{h}}}}{T_{\bar{\nu}_{e}}} \simeq \frac{T_{\nu_{\text{h}}}}{T_{\bar{\nu}_{e}}} \simeq 1.4 - 2.0$$

according to the simulation of supernova dynamics which is carried out in Ref. \[32-34\]. We ignore in the present treatment the temperature difference between $\bar{\nu}_{\text{h}}$ and $\nu_{\text{h}}$.

We now turn to the the neutrino flavor conversion in supernova (SN), the core matter in our discussion in this part of my presentation. In fact, it has a number of characteristic features which makes SN unique among other astrophysical and terrestrial sources.

(i) Because of extremely high matter density inside neutrinosphere all the neutrinos with

*The terminology implicitly assumes that the normal mass hierarchy is the case. Nevertheless, we will use it even when we discuss the inverted mass hierarchy.
cosmologically interesting mass range, \( m_\nu \lesssim 100 \text{ eV} \), are affected by the MSW effect \cite{37}. (Earlier references on the MSW effect in supernova include Ref. \cite{38}.) Consequently, the three neutrino and three antineutrino eigenstates have two level crossings, first at higher (H) density and the second at lower (L) density, inside SN as schematically indicated in Fig. 4.

(ii) The key question in the neutrino flavor conversion in SN is whether the H level crossing is adiabatic or not. If it is adiabatic, then the physical properties of neutrino conversion is simply \( \nu_e - \nu_{\text{heavy}} \) exchange in the normal mass hierarchy. It should be emphasized that this feature holds irrespective of the possible complexity of the solar neutrino conversion which governs the L resonance. These are nothing but the key features that have been pointed out in our earlier paper, Ref. \cite{39}, and was called as "\( \nu_e - \nu_\tau \) exchange".

(iii) The second important question is if the neutrino mass spectrum adopts the normal or inverted mass hierarchies. If the mass hierarchies is of normal (inverted) type, the H level crossing is in the neutrino (antineutrino) channel.

The last two remarks are crucial in our business. It will allow us to determine which mass hierarchy is realized by analyzing neutrino data from SN without knowing the parameters in the solar neutrino solution. Notice that this statement is valid not only for the MSW but also for the vacuum solar neutrino solutions.

One can elaborate (ii) by treating the neutrino evolution equation in high density matter of SN envelope, as explained in Ref. \cite{9}. See also Ref. \cite{40} for a recent comprehensive treatment of neutrino flavor conversion in SN in the framework of three-flavor mixing.

The adiabaticity of the H resonance is guaranteed if the following adiabaticity parameter \( \gamma \) is much larger than unity at the resonance point:

\[
\gamma \equiv \frac{\Delta m^2 \sin^2 2\theta}{2E} \left| \frac{d \ln N_e}{dr} \right|^{-1} \left| _{\text{res}} \right.
= \left( \frac{\Delta m^2}{2E} \right)^{1-1/n} \frac{\sin^2 2\theta (\cos 2\theta)^{1+1/n}}{n} \frac{\rho_0 Y_e}{m_p} \left[ \sqrt{2} G_F \rho_0 Y_e \right]^{1/n},
\]

Here, we assumed that the density profile of the relevant region of the star can be described as \( \rho(r) = \rho_0 (r/r_\odot)^{-n} \) to obtain the second line in the above equation, where \( r_\odot = 6.96 \times 10^{10} \) cm denotes the solar radius. With the choice \( n = 3 \) and \( \rho_0 \simeq 0.1 \text{ g/cc} \) \cite{11}, we get,

\[
\gamma \simeq 0.63 \times \left[ \frac{\sin^2 \theta_{13}}{10^{-4}} \right] \left[ \frac{\Delta m^2}{10^{-3} \text{eV}^2} \right]^{2/3} \left[ \frac{E}{20 \text{ MeV}} \right]^{-2/3},
\]
for the small value of $\theta_{13}$. Since the conversion probability $P$ is approximately given by 

$$P = \exp\left[-\frac{\pi}{2} \gamma\right], \quad s_{13}^2 \gtrsim \text{a few} \times 10^{-4}$$

assures adiabaticity in a good accuracy.

Now we notice that the basic elements for the argument toward disfavoring inverted mass hierarchy is actually very simple. Because of (iii), the resonance is in the antineutrino channel if the inverted mass hierarchy is the case as illustrated in Fig. 4b. It means that, if the H resonance is adiabatic, all the $\bar{\nu}_e$'s at neutrinosphere are converted into heavy antineutrino states, and vice versa. It is also known [40] that if H resonance is adiabatic, final $\bar{\nu}_e$ spectrum at the detector is not affected by the earth matter effect.

Since the $\bar{\nu}_e$-induced charged current reaction is dominant in water Cherenkov detector, one can severely constrain the scenario of inverted mass hierarchy by utilizing this feature of neutrino flavor transformation in SN. When the next supernova event comes it can be used to make clear judgement on whether the inverted mass hierarchy is realized in nature, a completely independent information from those that will be obtained by the long-baseline neutrino oscillation experiments.

We show in the rest of my talk that by analyzing the neutrino data from SN1987A one can obtain a rather strong feeling against the inverted hierarchy of neutrino masses. In the following analysis, we assume that $s_{13}$ is not very small, $s_{13}^2 \gtrsim \text{a few} \times 10^{-4}$, to guarantee the adiabaticity of the H resonance.

In fact, very similar analyses have been done by several authors [42,43]. Our work, in comparison with theirs, may be characterized in the following way; We formulate the problem in a proper setting of the three-flavor mixing scheme of neutrinos, which is essential for the SN neutrinos. With this setting one can clearly identify the cases that the conclusion reached in the previous analyses does and does not apply. To our understanding disfavoring

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$^1$In fact the reason is very simple; let us first note that $\bar{\nu}_3$ state which carry the original $\bar{\nu}_e$ spectrum oscillate very little into $\bar{\nu}_e$ in the earth because $|\Delta m_{13}^2|/E$ is much larger than the earth matter potential and also because $\theta_{13}$ is small [13]. Therefore, the oscillation in the earth takes place essentially only between $\bar{\nu}_1$ and $\bar{\nu}_2$, decoupling the $\bar{\nu}_3$ state. It would lead to regeneration of $\bar{\nu}_e$ but it would not give any significant effect for the $\bar{\nu}_e$ component at the detector because both $\bar{\nu}_1$ and $\bar{\nu}_2$ carry original energy spectrum of heavy flavors at the neutrinosphere.
the inverted mass hierarchy is the most solid statement one can draw from the analysis of SN1987A data, assuming that $\theta_{13}$ is not extremely small.

We follow Jegerlehner, Neubig and Raffelt [43] who employed the method of maximum likelihood. We define the Likelihood function as follows [43]:

$$L = C \exp \left(-\int_0^\infty n(E)dE\right) \prod_{i=1}^{N_{\text{obs}}} n(E_i),$$

where $N_{\text{obs}}$ is the total number of experimentally observed events and the $C$ is some constant which is irrelevant for our purpose of parameter estimation and the determination of confidence regions. Here, $n(E)$ is the expected positron energy spectrum at Kamiokande or IMB detector which is computed taking into account the detector efficiency as well as energy resolution in the same way as in Ref. [43]. For a combined analysis of the Kamiokande and IMB detectors, the likelihood function is defined as the product of the likelihood function for each detector.

We draw in Fig. 5 equal likelihood contours as a function of the heavy to light temperature ratio $\tau$ on the space spanned by $\bar{\nu}_e$ temperature and total neutrino luminosity by giving the neutrino events from SN1987A observed by Kamiokande and IMB detectors [27]. In addition to it we introduce an extra parameter $\eta$ defined by $L_{\nu_s} = L_{\bar{\nu}_s} = \eta L_{\nu_e} = \eta L_{\bar{\nu}_e}$ which describe the departure from equipartition of energies to three neutrino species and examine the sensitivity of our conclusion against the change in the SN neutrino spectrum. For simplicity, as in Ref. [43], we set the “effective” chemical potential equal to zero in the neutrino distribution functions because we believe that our results would not depend much even if we introduce some non-zero chemical potential.

At $\tau = 1$, that is at equal $\bar{\nu}_e$ and $\nu_e$ temperatures, the 95 % likelihood contour marginally overlaps with the theoretical expectation [34] represented by the shadowed box in Fig. 5. When the temperature ratio $\tau$ is varied from unity to 2 the likelihood contour moves to the left, indicating less and less consistency, as $\tau$ increases, between the standard theoretical expectation and the observed feature of the neutrino events after the MSW effect in SN is taken into account. This is simply because the observed energy spectrum of $\bar{\nu}_e$ must be interpreted as that of the original one of $\bar{\nu}_{\text{heavy}}$, in the presence of the MSW effect in the anti-neutrino channel, which implies that the original $\bar{\nu}_e$ temperature must be lower by a factor $\tau$ than the observed one, leading to stronger inconsistency at larger $\tau$. 
The solid lines in Fig. 5 are for the case of equipartition of energy into three flavors, \( \eta = 1 \), whereas the dotted and the dashed lines are for \( \eta = 0.7 \) and 1.3, respectively. We observe that our result is very insensitive against the change in \( \eta \).

We conclude that if the temperature ratio \( \tau \) is in the range 1.4-2.0 as the SN simulations indicate, the inverted hierarchy of neutrino masses is disfavored by the neutrino data of SN1987A unless the H resonance is nonadiabatic [9].

A summary of the features of neutrino events that we expect in the three-flavor mixing scheme of neutrinos are given in Ref. [9]. Recently some related works have appeared on the web [44–47].

In summary, I addressed three topics in my talk;

(i) failure of three-flavor hierarchical-mass dark matter neutrino hypothesis,
(ii) almost degenerate neutrino (ADN) scenario as the unique possibility of accommodating dark matter mass neutrinos and the constrains imposed on it,
(iii) likely possibility that supernova neutrinos may distinguish the normal versus inverted hierarchies of neutrino masses.

To conclude, I would like to say that the dark matter neutrino hypothesis has had profound implications to stimulate many inspirations in neutrino physics, despite the fact that the chance for it being the reality became less likely now. Yet, the question of neutrino mass of a few eV range, which is still compatible with data, should be settled both theoretically and experimentally.

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FIG. 1. The three shaded regions allowed by all the terrestrial experiments are drawn on parameter space spanned by $s_{23}^2$ and $s_{13}^2$ in dark matter $\nu$ embedded three neutrino mixing scheme. Two of the three regions allow large deficit of solar and atmospheric neutrinos, as indicated in the figure.
FIG. 2. Plotted is the allowed region with 90% CL of the double $\beta$ constraint $\langle m_{\nu e} \rangle < 0.35$ eV for neutrino masses $m=2.3$ eV. The strips with $(++-)$ etc. indicate regions with CP conservation with the CP parities indicated. Also plotted as a darker shaded area is the allowed region with 90% CL for the three-flavor MSW solution of the solar neutrino problem obtained by Fogli et al. The SMA MSW solutions, which are drawn almost on the axis of $s_{12}^2 = 0$ in the plot, as well as LMA solution at its CHOOZ allowed parameters are not compatible with the double $\beta$ decay constraint at 90% CL.
FIG. 3. The same as in Fig. 2 but with \( r \equiv \langle m_{\nu e} \rangle/m < 0.24 \).
FIG. 4. The schematic level crossing diagram for the case of (a) normal and (b) inverted mass hierarchies considered in this work. The circles with the symbol H and L correspond to resonance which occur at higher and lower density, respectively.
FIG. 5. Contours of constant likelihood which correspond to 95.4 % confidence regions for the inverted mass hierarchy under the assumption of adiabatic H resonance. From left to right, \( \tau = T_{\bar{\nu}_e}/T_{\nu_e} = T_{\bar{\nu}_x}/T_{\nu_e} = 2, 1.8, 1.6, 1.4, 1.2 \) and 1.0 where \( x = \mu, \tau \). Best-fit points for \( T_{\bar{\nu}_e} \) and \( E_b \) are also shown by the open circles. The parameter \( \eta \) parametrizes the departure from the equipartition of energy, \( L_{\nu_x} = L_{\bar{\nu}_x} = \eta L_{\nu_e} = \eta L_{\bar{\nu}_e} \) (\( x = \mu, \tau \)), and the dotted lines (with best fit indicated by open squares) and the dashed lines (with best fit indicated by stars) are for the cases \( \eta = 0.7 \) and 1.3, respectively. Theoretical predictions from supernova models are indicated by the shadowed box.