An Improved Crack Initiation Stress Criterion for Brittle Rocks under Confining Stress

Lai Wei1,2,*, Quansheng Liu1 and Xuewei Liu1

1State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Wuhan 430071, China
2University of Chinese Academy of Science, Beijing 100049, China

*Corresponding author e-mail: weilaiwhrsm@foxmail.com

Abstract. The stress level associated with crack initiation is an important damage threshold in rock failure process, which can be applied in the prediction of brittle failure in tunnels. In the present study, the uniaxial and triaxial compression tests are conducted on two sets of hard sandstones, while the dilation strains and AE counts are employed to characterize the crack accumulation. The crack initiation stress is then determined through the inflection point on strain and AE curves. Furthermore, two crack initiation modes: the splitting mode for low confinements and the sliding mode for high confinements are presented by analysing the fracturing mechanism. Based on the derivation of maximum tensile stress around the rock defects, an improved crack initiation criterion is proposed to predict the \( \sigma_{ci} \) of confined rocks. In the new criterion, the frictional components on crack surface are considered by introducing the parameter \( m_{ci} \), which can be applied for various types of rocks as well. At last, the triaxial compression test data of different rock types are reviewed to verify the reliability and applicability of the new criterion. The results indicate that the new criterion can predict the crack initiation stress of confined rock with high accuracy.

1. Introduction

Underground excavation in the highly stressed rock mass often encounters brittle failure, resulted from the initiation and accumulation of crack damage. These micro cracks are readily initiated under the induced local tensile stress, owing to the inhomogeneous nature of rock. In addition, stress concentration would also arise with the propagation of micro cracks, leading to progressive strength degradation and sudden localized failure of the rock eventually. Therefore, the stress levels associated with crack initiation and interaction are of much importance in predicting the brittle failure of rock.

The crack initiation stress \( \sigma_{ci} \) and the crack damage stress \( \sigma_{cd} \) are the critical characteristic stress thresholds in the failure process. While crack initiation represents the onset of fracturing in micro level, crack damage represents the start of crack coalesce and dilatation deformation (volumetric strain). These two stress thresholds can be identified respectively according to the cracking state within the rock mass, and they can divide the failure process as well, distinguishing the material property of rock between different failure stages.

The stress thresholds of \( \sigma_{ci} \) and \( \sigma_{cd} \) have been extensively studied by many researchers. For the most rock types under uniaxial compression, the \( \sigma_{ci} \) is about 0.3-0.6 times of its UCS [1], while the \( \sigma_{cd} \)
is about 0.7-0.8 times of its UCS [2]. So that the $\sigma_{ci}$ is commonly expressed as: $\sigma_{ci} = K \times UCS$, with the parameter $K$ representing the CI stress level under the uniaxial compression. In the laboratory tests, the crack initiation and crack damage can be identified through different methods, relying on the measured strains and AE techniques [2-4]. Moreover, the predictions of $\sigma_{ci}$ and $\sigma_{cd}$ had been applied at the URL test tunnel in Canada to better quantify rock damage [5]. They can also be regarded as the lower bound and upper bound of the wall yield strength respectively in tunnel designing [6]. In addition, $\sigma_{ci}$ also had a good performance in interpreting the extent of EDZ when combined with micro-seismic monitor data [7].

There are many factors that can influence the characteristic stress thresholds, like rock types, mineral composition, particle size and structural types. Martin had proposed the linear crack initiation criterion for the Lac du bonnet granite under low confining stress ($0-5$ MPa): $\sigma_1 - \sigma_3 = (0.4\pm0.05) \sigma_c$ [5]. But attempts to define the crack initiation criterion for higher confining stress and for various rock types have not met with much success.

The hard sandstone, as a typical brittle rock, can produce the tensile cracks readily under compressive loading, resulting in a great tendency for the brittle failure. In the present study, two sets of sandstone specimens are tested under uniaxial and triaxial compression, while the $\sigma_{ci}$ and $\sigma_{cd}$ are identified through the strain analysis and AE techniques. Furthermore, an improved crack initiation criterion is proposed in the basis of the Griffith strength criterion, which can be applied for various types of rock. The friction parameter $m_{ci}$ is introduced in the new criterion to characterize the friction resistance on the crack surface for high confining conditions. The reliability and applicability of the new criterion are verified by the test results in present study and the test data of other rock types.

2. Rock specimens and test procedures

The laboratory tests were performed on two sets of hard sandstones, Chongqing green sandstone and Hubei red sandstone (Figure 1). The green sandstones are classified as lithic arenites with silica cement. Its rock density is 2.304 g/cm$^3$, and its average P-wave velocity is 4872 m/s. The red sandstones are classified as Feldspathic arenites with hematite cement. Its rock density is 2.235 g/cm$^3$, and its average P-wave velocity is 4649 m/s. All the sandstones were processed into cylindrical specimens with the diameter of 50mm and the height of 100mm according to the ISRM Suggested Method [8].

![Figure 1. Rock test specimens: (a) Chongqing green sandstone; (b) Hubei red sandstone](image-url)

Uniaxial and triaxial compression tests were carried out on a rock mechanics servo-controlled testing system RMT-150C (developed by Institute of Rock and Soil Mechanics, Chinese Academy of Sciences). The specimens were first subjected to the uniaxial compression in displacement control
mode at a loading rate of 0.005 mm/s. And the axial and lateral strains of rock specimens were recorded in real time. Meanwhile the AE sensors (Nano30) and the AE monitoring system (PCI-2) were installed to record the AE signals during the loading. The AE trigger threshold was set as 40 dB. Subsequently, triaxial compression tests were carried out. For green sandstone specimens, the confining stress were set as 6.0, 12.0, 18.0 and 24.0 MPa. While for red sandstone specimens, the confining stress were set as 4.0, 8.0, 12.0, 16.0 and 20.0 MPa. The axial and lateral strains of rock specimens were also recorded in the triaxial compression tests.

Figure 2. AE hit of sandstone specimens under uniaxial compression: (a) green sandstone GS-1; (b) red sandstone RS-1
3. Detection of stress thresholds for sandstone

3.1. AE and strain characteristic

The dilatancy of the rock specimens under compression is mainly controlled by its cracking state. Hence the strain response can be used to characterize the accumulated micro cracks in rock specimens. Meanwhile, the acoustic emission signal is generated upon the crack initiation as well, so the AE techniques can also be employed to represent the crack accumulation. Figure 2 shows the axial stress-strain curve with associated AE hits for green sandstone specimen (GS-1) and red sandstone specimen (RS-2). It is noted that the failure process of rock specimens can be divided by 4 inflection points [1]: (1) crack closure, (2) crack initiation, (3) crack damage, (4) peak.

![Figure 3](image-url)
As shown in Figure 2, the stress-strain responses exhibit a concave part in the initial stage, resulting from the closure and compaction of the pre-existing cracks. Meanwhile the AE signals are also recorded during the crack closure stage. The extent and amplitude of this nonlinear part is dependent on the pre-existing crack density, therefore the porosity of red sandstone would assume to be much higher. The crack closure ($\sigma_{cc}$) is then reached at the end of the nonlinear part, and the elastic deformation stage ($\sigma_{cc}$ to $\sigma_{ci}$) starts subsequently. The associated AE activities also drop to a relatively low level, entering the quiet period (AE hits less than 1/10 of its peak).

The micro cracks would initiate when the axial stress reaches $\sigma_{ci}$, and these cracks accumulate stably with the increasing load ($\sigma_{ci}$ to $\sigma_{cd}$). As a result, AE signals are detected from the background noisy again, although still in a relatively low level. However, there is a more obvious impact on the strain response of rock specimen due to the crack initiation. As shown in Figure 3a, the volumetric strain curve of uniaxial compressed specimen departs form the linearity at $\sigma_{ci}$. While for triaxial compressed specimen ($\sigma_3=24$ MPa), the onset of dilational trends can still be identified at $\sigma_{ci}$ in Figure 3b. These newly initiated cracks are mainly tensile cracks, which would extend in axial direction (parallel to principal compressive stress) [9]. So upon the crack initiation, while the axial strain keeps linear, distinctive lateral dilation deformation occurs. Hence the $\sigma_{ci}$ can also be regarded as the elastic limit of rock specimen.

When the axial stress increases to $\sigma_{cd}$, the crack density in rock specimen would reach a critical level, leading to the unstable propagation and interaction of micro cracks. As a result, the local shear band forms with the crack coalescence, and the rate of lateral deformation increases due to the violent destruction of micro structure. The volumetric strain curve reaches its reversal point at $\sigma_{cd}$, transforming from contraction to dilation. Meanwhile, the associated AE hits also increases dramatically at $\sigma_{cd}$, as shown in Figure 2. After this, the axial strain also departs from linearity due to the sliding of oblique shear band. And the macro rupture plane forms eventually as the shear bands coalesce.

![Figure 4. Lateral strain and lateral strain stiffness of red sandstone specimen (RS-3)](image)

The deformation parameters of sandstone specimens can also be used to reflect the cracking state. Figure 4 shows the lateral strain and lateral stiffness of red sandstone specimen (RS-3). The lateral strain respond exhibits a strong nonlinear behavior, while the segmented feature can still be detected. This lateral dilatation is controlled by the crack accumulation, and it is made up by the crack openings in the initiation stage and the crack slippages in the damage stage. As for the lateral stiffness, it
decreases rapidly with the crack initiation, from initially 300 GPa to about 20 GPa in the crack damage stage. It is noted that the lateral dilatation deformation takes place mainly in crack damage stage, while the total amount of lateral deformation in crack damage stage is about 5 times compared to the crack initiation stage.

Figure 5. Ratio of lateral strain to axial strain and AE counts of red sandstone specimen (RS-3)

Figure 5 shows the ratio of lateral strain to axial strain (apparent Poisson’s ratio $\nu$) and the associated cumulative AE counts of red sandstone specimen (RS-3). The apparent Poisson’s ratio is 0.18 when the axial stress reaches the $\sigma_{ci}$. And as the axial stress increasing to the $\sigma_{cd}$, it rises up to 0.5, which corresponds to the reversal point of volumetric strain. Meanwhile, the associated AE counts also accumulate with the increasing load, which exhibits a good correlation with the apparent Poisson’s ratio. These two parameters represent the damage caused by the crack evolution. It is also indicated that the deformation parameters of rock specimen are directly controlled by its cracking state.

3.2. Identification of the $\sigma_{ci}$ and $\sigma_{cd}$

Based on the test results, AE techniques and strain responses can well characterize the accumulation of micro cracks in rock specimen. Both of them can be used to identify the characteristic stress level for rock specimen.
Figure 6 shows the relationship between axial stress and cumulative AE counts of sandstone specimen. From this curve, the total amount of newly initiated cracks can be reflected intuitively. It is noted that the cumulative AE counts curve shows an obvious segmented feature during the failure process, while each inflection stands for the onset of a new stage, corresponding to the characteristic stresses. Firstly, the crack closure ($\sigma_{cc}$) can be determined from the end of initial convex part on the cumulative AE counts curve. And as for the identification of crack initiation ($\sigma_{ci}$), a linear reference line can be employed to determine the departure point, which represents the end of the elastic deformation stage. Figure 6a shows the cumulative AE counts curve of green sandstone specimen (GS-1), its $\sigma_{ci}$ is determined accordingly as 26.0 MPa. Furthermore for the crack damage ($\sigma_{cd}$), the AE rate increases dramatically due to the crack interaction. The intersection of two reference lines on the curve can be identified as the $\sigma_{cd}$ [6], so that the $\sigma_{cd}$ of GS-1 is about 42 MPa. While Figure 6b shows
the cumulative AE counts curve of red sandstone specimen (RS-2), which has the consistent developing pattern. Thus its $\sigma_{ci}$ and $\sigma_{cd}$ are obtained as 16.3 MPa and 29 Mpa respectively.

In addition, the characteristic stresses can also be identified through the strain analyses. For the crack damage stress, the $\sigma_{cd}$ is directly obtained by the reversal point on volumetric strain curve. While for the crack initiation stress, the lateral strain respond method [4] is introduced in the present study. Firstly, the linear strain reference line is added to the lateral strain curve from zero to $\sigma_{cd}$ in Figure 7a. And by finding the maximum change of $\Delta LSR$ between the loading and reference line (Figure 7b), the onset of lateral dilation can be determined accurately, thus the $\sigma_{ci}$ is identified. So the $\sigma_{ci}$ of GS-6 is obtained as 61.37 MPa in Figure 7.

![Figure 7](image)

**Figure 7.** Identification of crack initiation stress by LSR method: (a) illustration of the LSR; (b) LSR result of green sandstone specimen GS-6

According to the methods above, the characteristic stresses of all the rock specimens are summarized in Table 1 and Table 2. It is noted that the results obtained from AE techniques and strain analyses are approximately equal, which mutually examines these two methods. For green sandstone
specimens, the average $\sigma_{ci}$ is about 0.42 times of its UCS, while the average $\sigma_{cd}$ is about 0.75 times of its UCS. For red sandstone specimens, the average $\sigma_{ci}$ is about 0.48 times of its UCS, while the average $\sigma_{cd}$ is about 0.78 times of its UCS.

| Sample no. | $\sigma_{ci}$ (MPa) | $\sigma_{cd}$ (MPa) | $\sigma_c$ (MPa) |
|------------|---------------------|---------------------|-----------------|
| AE LSR     | 26.0                | 26.65               | 43.25           |
| GS-1       | 28.0                | 29.39               | 47.51           |
| GS-2       | 29.5                | 28.01               | 53.48           |
| GS-3       | 19.5                | 21.21               | 35.74           |
| RS-1       | 16.3                | 17.94               | 31.97           |
| RS-2       | 22.0                | 21.42               | 32.88           |
| RS-3       | 24.0                | 21.21               | 35.74           |

Table 1. Results of uniaxial tests on sandstones

| Sample no. | $\sigma_3$ (MPa) | $\sigma_{ci}$ (MPa) | $\sigma_{cd}$ (MPa) | $\sigma_c$ (MPa) |
|------------|------------------|---------------------|---------------------|-----------------|
| GS-4       | 6                | 44.99               | 84.38               | 108.98          |
| GS-5       | 12               | 52.64               | 99.68               | 135.57          |
| GS-6       | 18               | 61.37               | 120.03              | 151.64          |
| GS-7       | 24               | 74.95               | 131.95              | 161.01          |
| RS-4       | 4                | 30.56               | 47.20               | 63.59           |
| RS-5       | 8                | 43.40               | 65.87               | 88.91           |
| RS-6       | 12               | 52.88               | 84.69               | 110.26          |
| RS-7       | 16               | 53.14               | 87.53               | 109.18          |
| RS-8       | 20               | 56.17               | 90.52               | 113.15          |

Table 2. Results of triaxial tests on sandstones

4. Crack initiation criterion

As the lower bound of in situ rock strength, the $\sigma_{ci}$ is a more important threshold in rock engineering. In this section, the crack initiation mechanisms are studied, and an improved crack initiation criterion is proposed.

4.1. Modes for crack initiation

Most micro cracks in rock originate from the grain boundaries, which acts as the tensile stress raiser. For different confining conditions, the cracks are generated in two different patterns: the splitting mode and the sliding mode.

Under low confining stress, the crack initiation would develop in the splitting mode. In this condition, the pre-existing cracks remain opened during the failure process, which promotes higher local tensile stress around its perimeter. Splitting failure in micro level would take place when the induced tensile stress exceeds the cohesive strength between rock minerals. Without sufficient confinements, these cracks would extend rapidly along the major principal stress direction, resulting in the macro splitting rupture eventually. As shown in Figure 8, the brittle spalling failure of wall rock should belong to such splitting failure mode. In the tunnel excavation, stress conditions of wall rock would degrade to uniaxial compression with the release of radial confinements. While the tangential stress of wall rock increases due to the stress redistribution. When the tangential stress reaches the $\sigma_{ci}$, tensile cracks would initiate sub-parallel to the free surface of tunnel. And these cracks are allowed to extend and dilate as the splitting plane in the absence of supporting, hence separating a thin layer from the wall rock. With the further crack extension, the rock layer would become much more slender and buckle under compression, leading to the spalling failure. Therefore in the tunnel excavation, the crack initiation thresholds can be a useful index to predict the spalling failure of wall rock.
Figure 8. Rock spalling [10] and its formation mechanism

In terms of high confining conditions, the pre-existing cracks are closed and tensile stress regions shrank. Very few splitting cracks can generate, thus the crack initiation would develop in the sliding mode. The frictional resistance has to be overcome firstly to produce the relative movements along flaw surfaces before crack initiation [11]. After the slippage, local tensile stress would increase at the flaw tips, and the tensile cracks (wing cracks) would initiate along the maximum tensile stress direction when $\sigma_c$ is reached. But in this condition, the newly initiated cracks are limited on extending due to the confining stress. They would then curve into s-shape (secondary cracks) on account of the shear movements. With the increasing compression, the s-shape cracks gradually extend and coalesce to form the macro shear band eventually. The sliding mode for crack initiation is similar to the mix I-II mode in fracture mechanics, as the tensile loading superposed onto the shear mode [12]. Local tensile stress arises at the flaw tips after slippage, leading to the crack deflection away from plane geometry (shear movements).

So in the basis of the confining conditions, crack initiation modes can be divided into the splitting mode for low confining stress and the sliding mode for high confining stress. For both modes, the tensile fracturing is the beginning of damage process, but their fracturing mechanisms are slightly different. Under low confining stress, crack initiation occurs when cohesive strength ($c$) is exceeded. While under high confining stress, the frictional components ($\phi$) are mobilized after the crack closure. Frictional resistance need to be overcome before the crack initiation, thus raising the crack initiation threshold.

4.2. Theoretical interpretation for crack initiation

Through the analyses in previous section, the concentration of local tensile stress is supposed to be the main drive of the crack initiation. Different from the classical strength theory, Griffith [13] took the material defects into consideration in the analysis of crack growth under biaxial compression. In his theory, crack would start to extend when the maximum local tensile stress on its perimeter reaches a certain critical value. Erdogan and Sih [14] also proposed a similar strength theory, in which cracks would extend when the stress intensity factor $K$ reaches the fracture toughness $K_{IC}$ in the direction of the maximum tensile stress. There is an implicit assumption in the Griffith theory on the unstable propagation of cracks, so that the materials are supposed to rupture once the crack initiated. But it is noted that the assumption is invalid for the rock, thus the Griffith theory is not appropriate to deal with the eventual shear failure in compression [15]. However, it can still be applied with the initiation of tensile micro cracks.

For opened cracks, the defects in brittle rock from which tensile crack originates can be regarded as a flat elliptical opening, while the frictional resistance is neglected. And the tangential stress components along the inclined crack boundary can be calculated [16]. Differentiate the tangential stress with respect to the eccentric angle and equate it to zero, then the maximum stress on crack surface are obtained:
\[ \xi \cdot \sigma_{\theta} = -\frac{(\sigma_t - \sigma_3)^2}{4(\sigma_1 + \sigma_3)} \]  

(1)

Where \( \sigma_{\theta} \) is the maximum tensile stress on the crack boundary, \( \xi \) is the axial ratio of the elliptical crack. The tensile cracks initiate when \( \xi \sigma_{\theta} \) reaches a certain value.

From equation (1), it is noted that the \( \sigma_{\theta} \) is driven by the principal stress difference \( \sigma_1 - \sigma_3 \). So the cohesion strength can be exceed even in the compressive stress field, as \( \sigma_1 - \sigma_3 \) increased. This is consistent with the empirical linear crack initiation criterion: \( \sigma_1 - \sigma_3 = (0.4 \pm 0.05) \sigma_c \) [5]. Since the stress difference has a similar form with the deviatoric stress, it has been assumed that the crack initiation is originated from the shear failure. But as a matter of fact, the shear failure is prohibited during the initiation stage for intact rocks, owing to the kinematic freedom restrictions [6]. As the tensile strength of rock is much weaker, tensile cracking dominates the crack initiation process.

Furthermore, considering the crack subjected to axial tension, thus \( \xi \sigma_{\theta} = -2 \sigma_t \). Substituting it into equation (1) gives the Griffith strength criterion:

\[ (\sigma_1 - \sigma_3)^2 = 8 \sigma_t \cdot (\sigma_1 + \sigma_3) \]  

(2)

So for the unconfined rock (\( \sigma_3 = 0 \), splitting mode), the \( \sigma_{ci} = 8 \sigma_t \) in the Griffith strength criterion. Although underestimated, it can be a rough estimate of the \( \sigma_{ci} \) in uniaxial compression.

While in terms of the closed cracks, the frictional components should be considered. The frictional resistance \( \tau_n \) acted on crack surface can be assumed as: \( \tau_n = \mu \sigma_n \), where \( \mu \) is the coefficient of friction. Thus the maximum local tensile stress transforms into:

\[ \xi \cdot \sigma_{\theta} = -\frac{\sqrt{1 + \mu^2} - \mu}{2} (\sigma_1 - \sigma_3) + \mu \sigma_3 \]  

(3)

Here, cracks are assumed to be closed completely under the confining stress. So the tangential stress decreases to \( (\tau - \mu \sigma_n) \). While the normal stress is entirely transmitted across the crack surface, producing no effects on the crack initiation. Thereby in equation (3), cracks are generated as mode II fractures, in which the \( \sigma_{ci} \) threshold increases significantly. The slope gets steeper as the frictional property enhanced. Substituting the internal frictional angle of sandstone specimens into equation (3), while it is found that the predicted \( \sigma_{ci} \) is higher than the test results.

Equation (1) and (3) are the theoretical simplification for the crack initiation in splitting mode and sliding mode, which can be seen as the upper and lower limits respectively. While in fact, the real microstructure of pre-existing flaws is so complicated that the completely open or closure cannot be accomplished. For opened cracks, the frictional resistance acted on the surface cannot be neglected completely. And for closed cracks, the normal stress is not transmitted and counteracted across the surface entirely. Thus the actual crack initiation threshold should fall in between the equation (1) and equation (3).

4.3. Improved criterion for crack initiation

In the light of the analyses above, a modification to the Griffith’s theory is proposed, aiming at the crack initiation in sliding mode. A resistance term \( (\mu \sigma_3 \sigma_t) \) is added to represent the frictional components of the closed cracks, thus the equation (1) transforms into:

\[ \xi \cdot \sigma_{\theta} = -\frac{(\sigma_1 - \sigma_3)^2 - \mu \sigma_3 \sigma_t}{4(\sigma_1 + \sigma_3)} \]  

(4)
Compared to equation (1), the principal stress difference \( \sigma_1-\sigma_3 \) needs to further increase to overcome the resistance component for crack initiation. Also considering the axial tension condition: \( \xi \sigma_0 = -2\sigma_t \), and substituting it into equation (4) then gives the explicit form:

\[
\sigma_1 - \sigma_3 = \sqrt{(16 + \mu) \cdot \sigma_t \sigma_3 + 16\sigma_t^2 + 4\sigma_t}
\]  

(5)

Furthermore, considering the crack initiation under uniaxial compression where \( \sigma_i = \sigma_{ci} \), \( \sigma_t = 0 \), and substituting it into equation (5) gives the C/T ratio: \( \sigma_{ci} = 8\sigma_t \). Cai had proposed that the suggested C/T ratio: \( \sigma_{ci}/\sigma_t = 8 \) in Griffith theory, which is smaller for common rocks, is more suitable to characterize the fracturing in micro level [17]. So it should be replaced as \( \sigma_{ci} = K\sigma_t = 8\sigma_t \) here to deal with the crack initiation. Substituting it into equation (5) then gives:

\[
\sigma_1 - \sigma_3 = K\sigma_c \cdot \sqrt{m_{ci} \cdot \frac{\sigma_t}{\sigma_c} + \frac{1}{4} + \frac{1}{2}K\sigma_c}
\]  

(6)

In equation (6), the coefficient of \( \sigma_3 \) is the frictional parameter \( m_{ci} \), whose magnitude is in direct proportion to the coefficient of friction \( \mu \), representing the frictional resistance acted on the crack surface. In the present study, the \( m_{ci} \) is determined as the product of \( m_c \) and \( K \):

\[
m_{ci} = K \cdot m_c
\]  

(7)

Where \( m_i \) is the ‘slope’ parameter of the Hoek-Brown criterion and \( K \) is the CI stress level in uniaxial compression.

Based on the derivation in fracture mechanics, Zuo pointed out that the \( m_i \) has a real physical meaning related to the frictional property of rock [18]. And it can be expressed as: \( m_i = (\mu/\kappa) \cdot (\sigma_c/\sigma_t) \), where the coefficient \( \kappa \) is a constant \( (\kappa = \sqrt{3/2}) \) in maximum-stress criterion, \( \kappa = 1 \) in maximum energy release-rate criterion. In addition, the C/T ratio can be replaced with the uniaxial CI stress level \( K \) as: \( \sigma_{ci}/\sigma_t = K\sigma_t = 8/K \) for crack initiation as mentioned above. Cai also pointed out that the \( K \) should be a function of C/T ratio and grain size \( a \): \( K \sim (\sigma_c/\sigma_t)^{-1} \cdot a^{-1/3} \) [19]. Hence the frictional parameter \( m_{ci} \), as the product of \( m_i \) and \( K \), is directly proportional to coefficient of friction \( \mu \) and grain size \( a \) of rock.

According to the estimates of H-B parameter concluded by Hoek and Brown [20], the reliable value of \( m_i \) for various types of rock can be determined specifically. And the \( K \) can be obtained from the uniaxial compression tests. By multiplying, the frictional parameter \( m_{ci} \) is determined. Substituting the \( m_{ci} \) into equation (6) gives the improved Griffith crack initiation criterion, thus the \( \sigma_{ci} \) for confined rock can be predicted.

5. Verification and discussion

In this section, the improved Griffith crack initiation criterion is examined with the test results of sandstone specimens in present study and other types of triaxial compressed rocks from literature.

Firstly, the proposed crack initiation criterion is verified on Chongqing green sandstone, the \( \sigma_{ci} \) is identified through LSR method, and the confining stress ranges from 0-24 MPa. According to the estimates of \( m_i \) concluded by Hoek and Brown [20], \( m_i = 7 \pm 2 \) for the fine grain sandstone. While based on the peak strengths, the value of \( m_i \) is determined to be 9. And the uniaxial CI stress level \( K = 0.42 \). Hence, the frictional parameter \( m_{ci} = K \cdot m_i = 3.78 \). Substituting it into equation (6) gives the prediction of the \( \sigma_{ci} \), as shown in Figure 9a. In addition, the linear crack initiation criterion \( (\sigma_{ci}/\sigma_3 + UCS = m \cdot UCS/\sigma_3) \cdot a \) [21] are also draw for comparison.

For Hubei red sandstone, the confining stress ranges from 0-20 MPa, while the value of \( m_i \) is determined to be 9 according to the recommendation, and the uniaxial CI stress level \( K = 0.47 \). Hence,
the frictional parameter $m_{c_i} = K_{mi} = 4.23$. Also substituting it into equation (6) gives the prediction of the $\sigma_{ci}$ as shown in Figure 9b.

From the results, the improved Griffith crack initiation criterion predicts the $\sigma_{ci}$ thresholds accurately under different confining stresses. While the linear criterion, mainly focused on low confining conditions, underestimates the $\sigma_{ci}$ distinctly for high confining conditions. It is noted that the $\sigma_{ci}$ of the two sets sandstone are underestimated in linear criterion for over 30%. As for the H-B spalling criterion, which replaces the m and s with crack initiation parameters, it gives a higher prediction for the $\sigma_{ci}$. The crack initiation originates from the tensile fracturing, which may not be suitable to be represented by shearing criterion.
In addition, the triaxial compression test results of other rock types are also used to verify the proposed crack initiation criterion as shown in Figure 9c-g, including: Darley Dale sandstone [22]; Lac du Bonnet granite [23]; Dagang mountain granite [24]; Beishan granodiorite [25] and Jinping marble [26]. The values of the parameter $m_i$ vary with the rock types, they are determined appropriately according to the value recommendation [20], which contributes to the accurate prediction of the $\sigma_{ci}$. The $m_i$ and the associated initiation parameters for each rock specimen are shown in Figure 9 as well. It is noted that the predicted $\sigma_{ci}$ thresholds fit the test results very well under different confining stresses. Thus the verifications cover igneous rocks, sedimentary rocks and metamorphic rocks, which prove the accuracy and the applicability of the improved Griffith crack initiation criterion.

Moreover, the linear criterion ($\sigma_1 - \sigma_3 = K\sigma_c$) is found to underestimate the $\sigma_{ci}$ of different rock types for 20-40% generally. In this criterion, the coefficient of $\sigma_3$ equals to 1, i.e., the slope $a = (1+\sin\phi)/(1-\sin\phi) = 1$, which suggests that the rocks are frictionless during the initiation stage. Although all the primarily cracks initiated in rocks originate from the tensile fracturing basically, but due to the crack closure under the confining stress, the initiation mechanism alters from splitting mode to sliding mode. So the linear criterion may be suitable for the unconfined rocks, in which the crack initiation develops in the splitting mode. While for the confined rocks, it is necessary to employ the proposed criterion to consider the frictional components, which gives an accurate prediction of the $\sigma_{ci}$ thresholds. While H-B spalling criterion ($\sigma_i = \sigma_3 + \text{UCS} (m_i \text{UCS}/\sigma_3) a$) [21] is found to overestimate the $\sigma_{ci}$ of different rock types, especially for sedimentary rocks and metamorphic rocks. This criterion may be more suitable for the granite rocks, which have a higher CI stress level.

6. Conclusion
The brittle failure of rock mass is closely related to the crack accumulation, while this failure process can be divided into several stages according to crack initiation $\sigma_{ci}$ and crack damage $\sigma_{cd}$. In the present study, the failure process of the two sets brittle sandstone specimens are studied in uniaxial and triaxial compression tests. Through the strain analyses and the AE techniques, the crack accumulations within the rock specimens are characterized. In addition, the crack initiation stress $\sigma_{ci}$ and the crack damage stress $\sigma_{cd}$ are identified. For different confining conditions, the cracks are found to initiate in two different patterns: the splitting mode and the sliding mode. While all the micro cracks originate from the tensile fracturing, the frictional resistance needs to be overcome to produce the slippage before the crack initiation in confining conditions, thus raising the $\sigma_{ci}$ threshold.
For sliding mode, the improved crack initiation criterion is proposed based on the Griffith theory. By introducing the frictional parameter $m_{ci}$, which is the product of $m_i$ and $K$, the frictional component acted on crack surface is considered. According to the value recommendation of $m_i$, the parameter $m_{ci}$ can be determined to represent the frictional property of various types of rock. Test results from seven groups of rock specimens under triaxial compression, including granite, sandstone, marble, are used to verify the proposed crack initiation criterion. All the predicted $\sigma_{ci}$ under different confining stress are in good agreement with the test data. Thus the proposed improved crack initiation criterion proves its reliability and applicability in predicting the crack initiation stress of confined rocks.

Acknowledgments
This work was supported by the National Basic Research Program of China (973 Program) (Grant Nos: 2014CB046904, 2015CB058102) and the National Nature Science Foundation of China (Grant No: 41602324).

References
[1] Brace, W. F, Paulding, B. W, & Scholz, C. H. (1966). Dilatancy in the fracture of crystalline rocks. Journal of Geophysical Research, 71 (16), 3939 - 3953, DOI 10.1029/JZ071i016p03939.
[2] Martin, C. D., & Chandler, N. A. (1994.). The progressive fracture of Lac du Bonnet granite. Int. J. Rock. Mech. Min. Sci. Geomech. Abstr. (Vol. 31, No. 6, pp. 643-659), DOI 10.1016/0148 - 9062 (94) 90005 - 1.
[3] Eberhardt, E., Stead, D., Stimpson, B., & Read, R. S. (1998). Identifying crack initiation and propagation thresholds in brittle rock. Canadian Geotechnical Journal, 35 (2), 222 - 233, DOI 10.1139/97 - 091.
[4] Nicksiar, M., & Martin, C. D. (2012). Evaluation of methods for determining crack initiation in compression tests on low-porosity rocks. Rock. Mech. Rock. Eng., 45 (4), 607 - 617, DOI 10.1007/s00603 - 012 - 0221 - 6.
[5] Martin, C. D. (1997). Seventeenth Canadian geotechnical colloquium: the effect of cohesion loss and stress path on brittle rock strength. Can. Geotech. J., 34 (5), 698 - 725, DOI 10.1139/t97 - 097 - 030.
[6] Diedrichs, M. S., Kaiser, P. K., & Eberhardt, E. (2004). Damage initiation and propagation in hard rock during tunnelling and the influence of near-face stress rotation. Int. J. Rock. Mech. Min. Sci., 41(5), 785-812, DOI 10.1016/j.ijrmms.2004.02.003.
[7] Cai, M., & Kaiser, P. K. (2005). Assessment of excavation damaged zone using a micromechanics model. Tunnelling and underground space technology, 20 (4), 301 - 310, DOI 10.1016/j.tust.2004.12.002.
[8] Fairhurst, C. E., & Hudson, J. A. (1999). Draft isrm suggested method for the complete stress-strain curve for intact rock in uniaxial compression. International Journal of Rock Mechanics & Mining Science & Geomechanics Abstracts, 36 (3), 281 - 289.
[9] Castro, L. A., Grabinsky, M. W., & McCreath, D. R. (1997). Damage initiation through extension fracturing in a moderately jointed brittle rock mass. Int. J. Rock. Mech. Min. Sci., 34 (3-4), 110-e1, DOI 10.1016/S1365 - 1609 (97) 00053 - 1.
[10] Andersson, J. C., & Martin, C. D. (2009). The Åspö pillar stability experiment: part I - experiment design. International Journal of Rock Mechanics & Mining Sciences, 46 (5), 865 - 878, DOI 10.1016/j.ijrmms.2009.02.010.
[11] Zhang, P., Li, N., Li, X. B., & Nordlund, E. (2009). Compressive failure model for brittle rocks by shear faulting and its evolution of strength components. Int. J. Rock. Mech. Min. Sci., 46 (5), 830 - 841, DOI 10.1016/j.ijrmms.2009.01.002.
[12] Lawn, B. (1993). Fracture of brittle solids. Cambridge university press.
[13] Griffith AA. Theory of rupture. In: Proceedings of the 1st international Congress of applied mechanics. Delft: Tech. Boekhandel en Drukkerij J Walter Jr; 1924. pp. 55e63.
[14] Erdogan, F., & Sih, G. C. (1963). On the crack extension in plates under plane loading and transverse shear. Journal of basic engineering, 85 (4), 519-525, DOI 10.1115/1.3656897.

[15] Hoek, E., & Martin, C. D. (2014). Fracture initiation and propagation in intact rock—a review. Journal of Rock Mechanics and Geotechnical Engineering, 6 (4), 287 - 300, DOI 10.1016/j.jrmge.2014.06.001.

[16] Hoek, E., & Bieniawski, Z. T. (1984). Brittle fracture propagation in rock under compression. Int. J. of Fracture, 26 (4), 276 - 294, DOI 10.1016/BF00962960.

[17] Cai, M. (2010). Practical estimates of tensile strength and Hoek–Brown strength parameter mi of brittle rocks. Rock. Mech. Rock. Eng., 43 (2), 167 - 184, DOI 10.1007/s00603-009-0053-1.

[18] Zuo, J. P., Li, H. T., Xie, H. P., Ju, Y., & Peng, S. P. (2008). A nonlinear strength criterion for rock-like materials based on fracture mechanics. Int. J. Rock. Mech. Min. Sci., 45(4), 594-599, DOI 10.1016/j.ijrmms.2007.05.010.

[19] Cai, M., Kaiser, P. K., Tasaka, Y., Maejima, T., Morioka, H., & Minami, M. (2004). Generalized crack initiation and crack damage stress thresholds of brittle rock masses near underground excavations. International Journal of Rock Mechanics & Mining Sciences, 41(5), 833 - 847, DOI 10.1016/j.ijrmms.2004.02.001.

[20] Hoek, E., & Brown, E. T. (1997). Practical estimates of rock mass strength. Int. J. Rock. Mech. Min. Sci., 34 (8), 1165-1186, DOI 10.1016/S1365 - 1609 (97) 80069 - X.

[21] Nicksiar, M., & Martin, C. D. (2013). Crack initiation stress in low porosity crystalline and sedimentary rocks. Eng. Geol., 154, 64 - 76, DOI 10.1016/j.engeo.2012.12.007.

[22] Pestman, B. J., & Van Munster, J. G. (1996, September). An acoustic emission study of damage development and stress-memory effects in sandstone. Int. J. Rock. Mech. Min. Sci. Geomech. Abstr. (Vol. 33, No. 6, pp. 585-593), DOI 10.1016/0148 - 9062 (96) 00011 - 3.

[23] Lau JSO, Gorski B. Uniaxial and triaxial compression tests on URL rock samples from boreholes 207 – 045 - GC3 and 209-069-PH3. CANMET Divisional Report MRL 92-025 (TR); 1992. p. 46.

[24] Liu, Q., Hu Y. H., Liu B. (2009). Progressive damage constitutive models of granite based on experimental results [J]. Rock and Soil Mechanics, 30 (2), DOI 10.16285/j.rsm.2009.02.044. (in Chinese).

[25] Zhao, X. G., Cai, M., Wang, J., & Ma, L. K. (2013). Damage stress and acoustic emission characteristics of the Beishan granite. Int. J. Rock. Mech. Min. Sci., 64, 258 - 269, DOI 10.1016/j.ijrmms.2013.09.003.

[26] HUANG Shu-ling. Study on mechanical model of brittle rock under high stress condition and its engineering applications. PhD Thesis, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, 2008. (in Chinese).