Calculation of higher-twist evolution kernels for polarized deep inelastic scattering

D. Müller

CERN, Geneva, Switzerland

Abstract

Based on the non-local light-ray operator technique, we develop an algorithm for the computational calculation of evolution kernels for higher-twist operators in leading order of the perturbation theory. We compute the evolution kernel for the twist-3 operators in the flavour singlet channel. Our result confirms the local anomalous dimensions computed by Bukhvostov, Kuraev, and Lipatov as well as the non-local evolution kernels of Balitzky and Braun.

I. INTRODUCTION

Recently, in deep inelastic scattering (DIS) the first moments of the transverse polarized structure function $g_2(x_{Bj}, Q^2)$ have been measured [1]. Among the leading twist-2 part $g_2(x_{Bj}, Q^2)$ contains a non-power suppressed twist-3 part, which it is intended to measure

*Permanent address: Institut für Theoretische Physik, Universität Leipzig, 04109 Leipzig, Germany
at HERMES and the SMC. Although at present the statistic is too low to extract the evolution of the twist-2 part of $g_2(x_{Bj}, Q^2)$ with respect to the momentum transfer $Q^2$, it may be important for future high-precision measurements to know the theoretical prediction also for the twist-3 part. This higher-twist contribution is in fact expressed in terms of three particle operators sandwiched between the polarized nucleon state; therefore, it possesses no simple parton interpretation. Furthermore, also for (higher-twist) operators with fixed spin there remains a mixing problem due to the renormalization. Consequently, the evolution equation is no longer of the type of a Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation.

Introducing so-called quasi-partonic operators, Bukhvostov, Kuraev and Lipatov derived evolution equations for higher-twist correlation functions and computed the twist-3 evolution kernels and the anomalous dimensions for both the non-singlet and the singlet channels \[2,3\]. Their non-singlet result was confirmed in the axial gauge as well as in the covariant gauge by different authors using quite different techniques. The results in \[4,5\] coincide with the original result; however, in \[6\] the transposed local anomalous dimension matrix was obtained. Recently, two more independent calculations \[7,8\] support the Bukhvostov, Kuraev and Lipatov result.

It should be mentioned that special emphasis was given in \[7\] (see also \[9\]) to the equation of motion (EOM) operators. As is expected from the renormalization properties of gauge-invariant operators \[10\], the result obtained in the off-shell scheme coincides with the previous ones in which the mixing with the EOM operators was neglected from the beginning of the calculation. In the minimal subtraction scheme this result is obvious at leading order. However, beyond the leading order it is not excluded that a non-proper regularization of infrared singularities spoils the general renormalization principles.

Here we apply an appropriate technique, which is based on the non-local operator product expansion introduced by Anikin and Zavialov \[11\] and the renormalization group equation for light-ray operators \[12\]. For several years Geyer, Robaschik and others applied this approach to the calculation of leading twist non-forward evolution equations \[13\] relevant
for deeply inelastic Compton scattering and, more recently, for the forward twist-3 non-singlet channel \[8\]. A similar approach was independently developed by Balitzky and Braun \[5\] and among other studies, applied to the one-loop calculation of the non-forward twist-3 singlet channel.

Although the singlet result \[5\] coincides with the local anomalous dimension matrix given in \[3\] (up to obvious misprints in both papers) it is desired to have a third independent calculation. On the other hand it is also theoretically interesting to study the evolution of power-suppressed contributions. Calculating the first few moments of twist-4 operators, a first step in this direction was done in \[14\]. It should be stressed that also one-loop order calculations are cumbersome and will be more difficult for increasing twist. Thus, it would be desired to have an algorithm that allows the use of computer power. Such an algorithm will be given in this paper for light-ray operators.

II. HIGHER-TWIST CONTRIBUTIONS TO TRANSVERSE POLARIZED DIS

Applying the non-local operator product expansion \[12,15,5\] the structure functions for DIS are factorized in coefficient functions and forward matrix elements of light-ray operators. The well-known result for the longitudinal structure function \(g_1\) reads in leading order

\[
g_1(x_{Bj}, Q^2) = \frac{1}{2} \sum_{q=u,d,\ldots} e_q^2 \Delta q_q(x_{Bj}, Q^2). \tag{2.1}
\]

The quark distribution functions (containing quark and antiquark) are defined as matrix elements of leading twist-2 operators:

\[
\Delta q_q(x, Q^2) = \int \frac{d\kappa}{2\pi(xS)} \langle P, S | O_q^{tw2}(0, \kappa) + (\kappa \to -\kappa) | P, S \rangle |_{\mu^2=Q^2} e^{i\kappa \cdot (\vec{x}P)} , \tag{2.2}
\]

where the renormalization point \(\mu\) of the operator is set equal to the momentum transfer \(Q\).

The twist-2 operators are defined as

\[
O_q^{tw2}(\kappa_1, \kappa_2) = \bar{\psi}_q(\kappa_1 \vec{x}) \vec{\gamma}_5 \psi_q(\kappa_2 \vec{x}), \tag{2.3}
\]
where $\tilde{x}$ is a light-like vector. Here and in the following we apply for simplicity the light-cone gauge, i.e. $\tilde{x}A = 0$. A gauge invariant definition in a general gauge is obtained by including a path-ordered link factor.

The operator product analysis to leading order suggests that the structure function $g_2$ can be written in the form of a generalized Wandzura-Wilczek relation [16]:

$$g_2(x_{Bj}, Q^2) = -\bar{g}_2(x_{Bj}, Q^2) + \int_{x_{Bj}}^{1} \frac{dy}{y} \bar{g}_2(y, Q^2),$$

so that the Burkhardt-Cottingham sum rule [17], i.e. $\int_0^1 dx g_2(x, Q^2) = 0$, is obviously fulfilled. Here $\bar{g}_2(x, Q^2) = g_1(x, Q^2) + \tilde{g}_2(x, Q^2)$ is decomposed in the twist-2 part given by $g_1$ and a remaining, not-power suppressed, twist-3 part $\tilde{g}_2$ [18]. The twist-3 contribution

$$\tilde{g}_2(x, Q^2) = \frac{1}{2} \sum_{q=u,d,..} e_q^2 \Delta\tilde{q}_q(x, Q^2)$$

(2.5)

can now be formally expressed as

$$\Delta\tilde{q}_q(x, Q^2) = \frac{1}{x} \int \frac{d\kappa}{2\pi(\tilde{x}P)} S^\rho \left< P, S \left| \tilde{O}_q^{tw3}(0, \kappa) - (\kappa \to -\kappa) \right| P, S \right> |_{\mu^2=Q^2} e^{ix\kappa(\tilde{x}P)},$$

(2.6)

where $S^\rho$ is the polarization vector and the twist-3 light-ray operator is defined as

$$\tilde{O}_q^{tw3}(\kappa_1, \kappa_2) = \int_0^1 du \tilde{\psi}_q(\kappa_1 \tilde{x}) \gamma_5 D^\sigma(u, \kappa_1 \tilde{x}, \kappa_2 \tilde{x}) \psi_q(\kappa_2 \tilde{x}),$$

(2.7)

with

$$D^\rho(u, \kappa_1 \tilde{x}, \kappa_2 \tilde{x}) = \partial_\kappa^\rho + igA^\rho([\kappa_1 \bar{u} + \kappa_2 u] \tilde{x}), \quad \bar{u} = 1 - u.$$  

(2.8)

Unlike $\Delta q_q(x, Q^2)$ this new twist-3 function $\Delta\tilde{q}_q(x, Q^2)$ has no simple parton interpretation. In fact, using the equation of motion $(i \not{D} - m)\psi = 0$ it turns out that the twist-3 operator (for more details see [8]) can be decomposed in a basis of three-particle operators containing also the gluon field and in a mass-dependent two-particle operator. Introducing an appropriate definition of a three-particle correlation function the net contribution to $\tilde{g}_2$ in leading order can be written as

$$x\bar{g}_2(x, Q^2) = \frac{1}{2} \sum_{q=u,d,..} e_q^2 \left[ \frac{1}{x} m_q(x, Q^2) + \frac{d}{dx} \int_0^1 du u \Delta\tilde{q}_q(x, u, Q^2) \right].$$

(2.9)

The three-particle correlation function
\[ \Delta \bar{q}(x, u, Q^2) = \int \frac{d\kappa}{4\pi} \frac{S_\rho}{(\bar{x} P)^2} \langle P, S | Y^\rho_-(-\kappa u, \kappa \bar{u}) + (\kappa \to -\kappa) | P, S \rangle_{\mu^2=Q^2} e^{i \kappa x (\bar{x} P)} \]  

(2.10)

is even under charge conjugation and depends on the variable \( u \), which gives the relative position of the gluon field on the light cone and on the variable \( x \). For \( 0 \leq u \leq 1 \) the gluon field lies between the two quark fields. Because of the support property \( |x| \leq \text{Max}(1, |2u-1|) \), the variable \( x \) is then restricted to \( |x| \leq 1 \) and can be interpreted as an effective momentum fraction. The introduced ligth-ray operator \( \bar{Y}_q^\rho(\kappa_1, \kappa_2) = \bar{S}_q^\rho(\kappa_1, 0, \kappa_2) + S_q^\rho(\kappa_2, 0, \kappa_1) \) is the non-local generalization of the so-called Shuryak-Vainshtein operators [19]:

\[ \pm \bar{S}_q^\rho(\kappa_1, \tau, \kappa_2) = i g \bar{\psi}_q(\kappa_1 \bar{x}) \not{\!} F^{\alpha \rho}(\tau \bar{x}) \pm \gamma^5 F^{\alpha \rho}(\tau \bar{x}) \bar{x}_\alpha \psi_q(\kappa_2 \bar{x}), \]  

(2.11)

where \( \not{\!} F^{\alpha \beta} = \frac{1}{2} \epsilon^{\alpha \beta \mu \nu} F_{\mu \nu} \) is the dual field strength tensor. Furthermore, we introduced a two-particle distribution function

\[ m_q(x, Q^2) = \int \frac{d\kappa}{2\pi} \frac{S_\rho}{(\bar{x} P)^2} \langle P, S | M^\rho_q(0, \kappa) + (\kappa \to -\kappa) | P, S \rangle_{\mu^2=Q^2} e^{i \kappa x (\bar{x} P)}, \]  

(2.12)

where the operator

\[ M^\rho_q(\kappa_1, \kappa_2) = m_q \bar{\psi}_q(\kappa_1 \bar{x}) \sigma^{\alpha \rho} \bar{x}_\alpha \gamma^5(\bar{x} D)(\kappa_2 \bar{x}) \psi_q(\kappa_2 \bar{x}), \quad \sigma_{\alpha \beta} = \frac{i}{2} [\gamma_\alpha, \gamma_\beta] \]  

(2.13)

is proportional to the current mass \( m_q \).

Using the definition (2.10) the evolution equation for the twist-3 correlation function can be obtained in a straightforward manner from the renormalization group equation (RGE) of the non-local ligth-ray operators. From the non-singlet result given in [5,8] one obtains an extended DGLAP equation [18]:

\[ Q^2 \frac{d}{dQ^2} \bar{q}^{\text{NS}}(y, u, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int \frac{dz}{z} \int dv \left\{ \bar{P}_{qq}^{\text{NS}}(z, u, v) \Delta \bar{q}^{\text{NS}} \left( \frac{y}{z}, v, Q^2 \right) + \delta(v - u - \bar{u}z) \frac{z}{|v|} \bar{P}_{qm}(z) m^{\text{NS}} \left( \frac{y}{z}, Q^2 \right) \right\} \]  

\[ Q^2 \frac{d}{dQ^2} m^{\text{NS}}(y, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int \frac{dz}{z} \bar{P}_{mm}(z) m^{\text{NS}} \left( \frac{y}{z}, Q^2 \right). \]  

(2.14)

Here, the integration region is determined by both the support of \( \Delta \bar{q}^{\text{NS}} \) and by the kernel
where 

\[ A_{\tilde{n}} \text{ for } \tilde{n} \text{ definite spin } \leq 0 \text{ for the remaining mixing problem there is no known analytical solution.} \]

In two limits for \( u \):

- Note that, due to the evolution, the variable \( u \) will be mixed with the antisymmetric gluon operator: 

\[ \Theta_1(z, u, v) = \theta(z)\theta(u - vz)\theta(\bar{u} - z\bar{v}), \quad \Theta_2(z, u, v) = \theta(-\bar{u}\bar{v}z)\theta(\{1 - vz\}\bar{u}\theta(\{z - u\}\bar{u}), \]

\[ \Theta_3(z, u, v) = \theta(\bar{u}\bar{v}z)\theta(\{vz - u\}\bar{u}, \]

\[ K(z, u, v) = z + \left\{ \frac{u^2}{v(v - u)}\delta(u - vz) + \left( \frac{u \to \bar{u}}{v \to \bar{v}} \right) \right\}, \quad L(z, u, v) = -\text{sign}(\bar{u})\frac{\bar{v}u^2}{\bar{u}^2}\delta(u - z), \]

\[ M(z, u, v) = \frac{2z(1 - zv)}{\bar{u}^2}, \quad N(z, u, v) = \frac{\text{sign}(\bar{u})\bar{v}}{\bar{u}(v - u)}\left\{ \frac{\bar{v}}{\bar{u}}\delta(1 - z) + \frac{u^2}{v}\delta(u - vz) \right\}. \]

Note that, due to the evolution, the variable \( u \) can no longer be restricted to the region \( 0 \leq u \leq 1 \). The moments with respect to \( y \) correspond to matrix elements of operators with definite spin \( n \), so that their evolution equation will be diagonal with respect to \( n \). However, for the remaining mixing problem there is no known analytical solution. In two limits for \( x \to 1 \) and (only in the non-singlet channel) also in the large \( N_c \) limit, the evolution equation for \( \tilde{g}_2(x, Q^2) \) reduces to an equation of the DGLAP type [20].

In the singlet channel, there appears a mixing problem. The \( C \)-even quark operator

\[ Y^\rho(\kappa_1, \kappa_2) = \sum_{q=u,d,...} Y^\rho_q(\kappa_1, \kappa_2) = \sum_{q=u,d,...} \bar{S}^\rho_q(\kappa_1, 0, \kappa_2) + \bar{S}^\rho_q(\kappa_2, 0, \kappa_1) \]

will be mixed with the antisymmetric gluon operator:

\[ C^\rho(\kappa_1, \kappa_2) = g f^{abc} \tilde{x}_a \bar{x}_b \bar{x}_c \tilde{F}_{\alpha\beta}(\kappa_1 \bar{x}) F^\mu_{\beta\gamma}(0) F^\gamma_{\gamma\mu}(\kappa_2 \bar{x}), \quad C^\rho(\kappa_2, \kappa_1) = -C^\rho(\kappa_1, \kappa_2). \]

### III. Calculation of Twist-3 Evolution Kernels

The relevant Feynman diagrams for the calculation of the evolution kernel to leading order are shown in Fig. 1. In addition to the one-particle irreducible diagrams, the application
of the equation of motion can be represented by reducible diagrams [3]. In the following we describe shortly the computational calculation of the twist-3 evolution kernels in the light-cone gauge. The applied method can be easily modified for a covariant gauge and can be extended to twist higher than 3.

Because of the light-cone gauge all operator vertices

\[ O_v(\kappa_1, \kappa_2) = o_v e^{-i\kappa_1(\bar{x}p_1) - i\kappa_2(\bar{x}p_2)} \quad \text{with} \quad v = \{^+S,^-S, G\} \]  

have only a pure exponential \( \kappa \)-dependence. Here \( o_v \) includes the Lorentz, flavour, and colour structure of the operator. However, the gluon propagator contains a spurious pole:

\[ -ig^{\mu\nu} - \frac{(k^\mu x^\nu + k^\nu x^\mu)}{k^2 + i\epsilon}, \]  

which will be regularized by the Leibbrandt-Mandelstam prescription [21]:

\[ \frac{1}{k \bar{x}} = \frac{k \bar{x}^*}{(k \bar{x})(k \bar{x}^*) + i\epsilon} \]  

with the dual light-like vector \( \bar{x}^* \), i.e. \( \bar{x}^2 = \bar{x}^*2 = 0 \) and \( \bar{x} \bar{x}^* \neq 0 \).

The Dirac algebra can be performed with one of the usual high-energy programs (such as FORM, FeynCalc or Tracer). The integration over the loop momentum and the final simplification of the result were done with a rule-based program written in Mathematica. After cancellation of common scalar products between numerator and denominator, as well as the decomposition of \( 1/\bar{x}k \ldots 1/\bar{x}(k+p_1) \), each diagram is expressed in terms of the following dimensional regularized tensor integrals:

\[ I_{\mu_1 \ldots \mu_j}(a, b, c, \alpha; \kappa) = \int \frac{d^{4-2\epsilon}k}{(2\pi)^{4-2\epsilon}} \frac{k^\mu_1 \ldots k^\mu_j}{(k^2)^a((k + p_1)^2)^b((k + p_2)^2)^c} e^{-i\kappa(\bar{x}k)}, \]  

With the help of the +-prescription

\[ x_+^{\alpha} f(x) = \frac{f(x) - \sum_{i=0}^{\alpha-1} x^i f^{(i)}(0)}{x^\alpha}, \quad f^{(i)}(0) = \frac{d^i}{dx^i} f(x)|_{x=0}, \]  

the integral \( I_{\mu_1 \ldots \mu_j}(a, b, c, \alpha; \kappa) \) can be decomposed in

\[ I_{\mu_1 \ldots \mu_j}(a, b, c, \alpha; \kappa) = I_+^{\mu_1 \ldots \mu_j}(a, b, c, \alpha; \kappa) + \sum_{\beta=0}^{\alpha-1} (-i\kappa)^\beta I_{\mu_1 \ldots \mu_j}(a, b, c, \alpha - \beta; \kappa = 0), \]  

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where \( I^\mu_1\cdots\mu_j(a,b,c,\alpha;\kappa) \) contains no more spurious poles. The remaining integrals with \( \kappa = 0 \) give only a trivial contribution to the evolution kernel. Such integrals are common in light-cone gauge and can be found in [22]. Since the integrand of \( I^\mu_1\cdots\mu_j(a,b,c,\alpha;\kappa) \) is analytic in \( \tilde{x}k \) and \( \tilde{x}^2 \) vanishes, the momentum integration results in

\[
I^\mu_1\cdots\mu_j(a,b,c,\alpha;\kappa) = \frac{i(1-\epsilon)}{(4\pi)^{2-\epsilon}} \Gamma(a+b+c-j-2+\epsilon) \int_0^1 dy \int_0^y dz (1-y-z)^{a-1} \times \frac{\partial}{\partial b_{\mu_1}} \cdots \frac{\partial}{\partial b_{\mu_j}} \left\{ b^2 - D(y,z) \right\}^{-(a+b+c-j-2+\epsilon)} (\tilde{x}b)^{\alpha} e^{i\kappa(\tilde{x}b)} |_{\mu = B^\mu(y,z)},
\]

where \( B^\mu(y,z) = yp_1^\mu + zp_2^\mu \), \( D(y,z) = yp_1^2 + zp_2^2 \). (3.7)

Performing the derivation with respect to \( b_{\mu_j} \) and taking into account only the ultra-violet (UV) divergent part provide the result for the corresponding Feynman diagram. Unfortunately, the output is cumbersome and must be simplified. In a first step all exponentials appearing in the result of a given Feynman diagram will be transformed in a unique form in such a way that the measure of the parameter integrals remains unchanged, for instance:

\[
e^{-i[\kappa_1(1-y)+\kappa_2y]\tilde{x}p_1+\kappa_2(z+y)+\kappa_1(1-y-z)]\tilde{x}p_2} \to \tilde{x}p_1 + \tilde{x}p_2 \quad e^{-i[\kappa_1(1-y)+\kappa_2y]\tilde{x}p_1+\kappa_2(1-z)+\kappa_1z]\tilde{x}p_2}. \quad (3.8)
\]

Finally, such rational functions in the external momenta as

\[
\int_0^1 dy \int_0^y dz \delta(1-y-z) \frac{\tilde{x}p_1 + \tilde{x}p_2}{y\tilde{x}p_1 - z\tilde{x}p_2} e^{-i[\kappa_1y+\kappa_2y]\tilde{x}p_1+\kappa_2\tilde{z}+\kappa_1z]\tilde{x}p_2} \quad (3.9)
\]

have to be transformed into momenta-independent ones. Applying partial integration, this expression can be written in the desired form

\[
\int_0^1 dy \int_0^y dz \left[ \delta(z) \frac{1}{y_+] - \delta(y) \frac{1}{z_+] \right] e^{-i[\kappa_1y+\kappa_2y]\tilde{x}p_1+\kappa_2\tilde{z}+\kappa_1z]\tilde{x}p_2}. \quad (3.10)
\]

Summing up the contribution of the diagrams and taking into account the renormalization of the quark and gluon fields, the evolution kernels are given by the \( \frac{1}{\epsilon} \) pole [23]:

\[
\mu^2 \frac{d}{d\mu^2} Y(\kappa_1,\kappa_2) = \frac{\alpha_s}{2\pi} \int_0^1 dy \int_0^y dz \left\{ \left( C_F - \frac{C_A}{2} \right) \left[ y\delta(z)Y(-\kappa_1y,\kappa_2-\kappa_1y) - 2zY(\kappa_1 - \kappa_2\tilde{z},-\kappa_2y) + [K(y,z)]_+Y(\kappa_1y + \kappa_2y,\kappa_2\tilde{z} + \kappa_1z) \right] + \right.
\]

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\[ \frac{C_A}{2} \left[ (2\bar{z} + [N(y, z)]_+ - \frac{7}{2} \delta(y)\delta(z)) Y(\kappa_1 - \kappa_2 z, \kappa_2 y) + [N(y, z)]_+ Y(\kappa_1 y, \kappa_2 - \kappa_1 z) \right] - 2N_f y\bar{y} \delta(z) Y(\kappa_1 y + \kappa_2 \bar{y}, \kappa_1 y + \kappa_2 \bar{y}) + C_F y^2 \delta(z) [M(\kappa_1 - \kappa_2 y) + M(\kappa_2 y - \kappa_1)] - \\
N_f(\kappa_1 - \kappa_2) \left[ (1 - y - z + 4yz) G(\kappa_1 \bar{y} + \kappa_2 y, \kappa_1 z + \kappa_2 \bar{z}) + (1 - y - z) \times \right. \\
\left. \{ G(-\kappa_1 - \kappa_2)(\bar{y} - z), -\kappa_1 \bar{y} - \kappa_2 y) - G(-\kappa_1 z - \kappa_2 \bar{z}, \kappa_1 - \kappa_2)((\bar{y} - z)) \} \right], \tag{3.11} \]

\[ \mu^2 \frac{d}{d\mu^2} G(\kappa_1, \kappa_2) = \frac{\alpha_s}{2\pi} C_A \int_0^1 dy \int_0^\bar{y} dz \left\{ (1 - y - z + 3yz) [G(\kappa_1 \bar{y} + \kappa_2 y, \kappa_2 \bar{z} + \kappa_1 z) + \right. \\
G(\kappa_1 y - \kappa_2, \kappa_1 \bar{z} - \kappa_2)] + (1 - y - z) [G(\kappa_1(y - z), \kappa_2 - \kappa_1 y) - G(\kappa_2 - \kappa_1 \bar{z}, -\kappa_1(\bar{y} - z)) + \right. \\
\left. \left( [Q(y, z)]_+ - \frac{8C_A + N_f}{9C_A} \delta(y)\delta(z) \right) \right\} [G(\kappa_1 \bar{y} + \kappa_2 y, \kappa_2 \bar{z} + \kappa_1 z)/2 + G(\kappa_1(\bar{y} - z), \kappa_2 - \kappa_1 z)] - \\
\frac{1}{16} [2 - \delta(y)\delta(z)][Y(\kappa_1 y - \kappa_2, \kappa_1 \bar{z} - \kappa_2) - Y(\kappa_1 \bar{z} - \kappa_2, \kappa_1 y - \kappa_2)]/\kappa_1 - \\
\frac{1}{16} [2 - 12yz \delta(1 - y - z) + \delta(y)\delta(z)] [2Y(\kappa_1 \bar{y} + \kappa_2 y, \kappa_2 \bar{z} + \kappa_1 z)/(\kappa_1 - \kappa_2) - \\
\{ Y(\kappa_1 y - \kappa_2, \kappa_1 \bar{z} - \kappa_2) + Y(\kappa_1 \bar{z} - \kappa_2, \kappa_1 y - \kappa_2) \} /\kappa_1 - (\kappa_1 \leftrightarrow \kappa_2) \right\], \tag{3.12} \]

\[ \mu^2 \frac{d}{d\mu^2} M^\rho(\kappa) = \frac{\alpha_s}{2\pi} C_A \int_0^1 dy \frac{1}{(1 - y)_+} \left\{ y^2 M^\rho(\kappa y) \right\}, \tag{3.13} \]

where we used the following plus-prescriptions:

\[ [A(y, z)]_+ = A(y, z) - \delta(y)\delta(z) \int_0^1 dy' \int_0^\bar{y} dz' A(y', z'), \quad \text{for } A = \{ K, Q \}, \]

\[ [N(y, z)]_+ = N(y, z) - \delta(\bar{y})\delta(z) \int_0^1 dy' \int_0^\bar{y} dz' N(y', z'), \quad N(y, z) = \delta(\bar{y} - z) \frac{y^2}{\bar{y}} + \delta(z) \frac{y}{\bar{y}}, \]

\[ K(y, z) = 1 + \delta(z) \frac{\bar{y}}{y} + \delta(y) \frac{\bar{z}}{z}, \quad Q(y, z) = \frac{1}{2} \left( \delta(z) \frac{\bar{y}^2}{y} + \delta(y) \frac{\bar{z}^2}{z} \right). \]

Taking into account the different operator definitions, our massless result coincides (up to a missing factor \( N_f \)) in the quark-gluon kernel and an obvious misprint in the gluon-quark sector) with the evolution kernels calculated by Balitzky and Braun [3], restricted to the forward case. The Taylor expansion with respect to \( \kappa_1, \kappa_2 \) provides the local operators and their anomalous dimension matrix. If we set the missing upper sum limit in Eq. (55) of [3] to \( n \), we agree (up to a constant normalization factor in the definition of the gluon operator) for the massless case with the general analytical expression for the anomalous dimension matrix given by Bukhvostov, Kuraev, and Lipatov. Our mass dependent terms coincide with their non-singlet result.
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Figure 1: The relevant Feynman diagrams for the calculation of twist-3 singlet evolution kernels in the light-cone gauge for: a) the mass operator $M^\rho$, b) the mixing of the quark operator $\pm S^\rho$ with the gluon operator $G^\rho$, c) the quark operator itself and its mixing with the mass operator, d) the mixing of the gluon operator with the quark operator, and e) the gluon operator itself.