Dilemmatic situations for learning mathematics-related beliefs

T Y E Siswono, A W Kohar, S Hartono, A H Rosyidi, P Wijayanti and R Ekawati
Universitas Negeri Surabaya, Kampus Unesa Ketintang, Surabaya, Indonesia.

E-mail: tatagsiswono@unesa.ac.id

Abstract. This study aims to investigate how dilemmatic situations were used as context in a set of instructional materials for learning teachers' mathematics-related beliefs. The instructional materials were developed through formative evaluation steps: self-evaluation, expert review, one-to-one, small group, and field test. The formative evaluation step involved three experts and 31 pre-service teachers in examining the materials' validity, practicality, and potential effects. The dilemmatic situations included in the instructional materials provoke pre-service teachers to select one of the three types of teaching sequences corresponding to three hierarchical beliefs about the nature of mathematics (instrumentalist, Platonist, and problem-solving) in accordance with beliefs about teaching and learning. Results showed that the instructional materials were valid and practical, and have a potential effect on improving pre-service teachers' mathematics-related beliefs. In particular, the dilemmatic situations could encourage pre-service teachers to consider which beliefs they should follow regarding problem-solving teaching. On the one hand, the pre-service teachers agree to the problem-solving view, on the other hand, they are confronted with the immediate situations found in their instructional practice which makes them hesitant in having such view. However, this instructional practice lead teachers show a good understanding of the borderline between those three beliefs. Besides, the instructional practice is indicated to potentially effect on changing the pre-service teachers' mathematics-related belief to be more constructivist.

1. Introduction

Mathematics-related beliefs are known to play a great role in teaching and learning mathematics. This is because the teacher mathematics-related beliefs influence what teachers will teach, how they teach, and what they learn in the classroom [1]. In addition, Ernest [2] asserts that beliefs also affect student learning outcomes. However, some reports show that teachers, as well as pre-service teachers, are reported to have traditional beliefs about mathematics. For instance, Siswono et al. [3] reported that there were no participants in their case study having a problem-solving view. Instead, the participants considered mathematics as a discipline with a static body of knowledge with regards to Platonist view [2], or science which merely study about number and formula with regards to Instrumentalist view [2]. Therefore, change of beliefs from traditional to constructivist are considered very important for changes in teaching practice.

Considerable concern has been given to the issue of changing teacher beliefs as one of the current reform of topics in mathematics teacher education. Research provides attention on improving teacher beliefs concerning nature of mathematics [4], mathematical and pedagogical view [5], students' problem-solving [6], and students' engagement in mathematics instruction [7]. A variety of programme supporting the successfulness of reaching such attention has also been examined. Guberman & Leikin [5], for example, explored the development of teacher beliefs through a programme using systematic
utilize of a multiple-solution task. Blömeke et al. [8], on the other hand, found that teachers' beliefs develop when teachers are provided with a strong climate of trust from the school where they work. Within a professional learning applying principles: fidelity, engagement, and change. Lomas & Clarke [4] reported the improvement of teachers' beliefs about mathematics and their teaching and learning. All such effort was carried out to confront teachers’ initial beliefs that make them learn mathematics-related beliefs. This is because an individual learns when he/she attempts acquiring new or modifying existing knowledge, behaviors, skills, values/beliefs, or preferences [9].

Learning mathematics-related beliefs through confronting situations can also be undertaken by providing dilemmatic situations within an instructional practice. A dilemma is defined as situations in which two values, commitments, or obligations conflict and there seems to be an incorrect thing to do [10]. The conflicts cause multiple, equally possible alternatives, each of which has advantages and disadvantages [11]. By employing dilemmatic situations as contexts of learning, the authors argue that teachers would learn new schemes of knowledge and beliefs which may confront their initial beliefs to be maintained or even changed.

One aspect of dilemmas experienced by teachers is pedagogical beliefs [12]. The dilemmas occur when teachers are confronted with at least two distinct pedagogical views: traditional and constructivist view. In the traditional view, the teacher is described as the transmitter of knowledge and the director of learning, whereas the students are expected to be passive recipients of knowledge, whose responsibility is to follow the director’s lead [13]. In the constructivist view, a teacher is described as a facilitator of student learning, in which students are actively engaged in constructing knowledge through collaborative learning [13]. When choosing one of those two views, teachers are faced with some variables that need to be considered occurring in their daily teaching such as immediate classroom situations (students' abilities, attitudes, and behavior; the mathematics topic at hand; time constraints), social norms (demand of curriculum; learning resources), or their previous school experience.

This study makes an effort to develop a set of instructional materials (lesson plan and worksheet) for a teacher development programme to encourage pre-service teachers to learn mathematics-related beliefs through dilemmatic situations. Therefore, the authors developed a set of instructional materials which support pre-service teachers’ understanding of mathematics-related beliefs as well as improve their mathematics-related beliefs to be more constructivist. In this study, the mathematics-related beliefs learned by the pre-service teachers are those proposed by Ernest [2] and then summarised by Beswick [14]. Ernest describes three types of epistemological beliefs, namely Platonist views, instrumentalist views, and problem-solving views. In the Platonist view, mathematics is seen as a static yet integrated knowledge, whose structures and interconnections between various topics are essentially important. The instrumentalist view states that mathematics is a collection of facts, rules, and skills that are useful and essentially unrelated. Meanwhile, the problem-solving view sees mathematics as a dynamic and creative human discovery. Beswick's framework [14] present a theoretical consideration which put beliefs about mathematics [2] with their corresponding beliefs about mathematics teaching: content-focused with an emphasis on performance, content-focused with an emphasis on understanding, learner-focused [15] and learning: skill mastery, passive reception of knowledge, active construction of understanding, and autonomous exploration of learner's interests [2].

Thus, the research questions of this present study are: 1) how to develop valid and practical instructional materials supporting pre-service teachers’ understanding of mathematics-related beliefs? 2) how are dilemmatic situations employed as context for learning mathematics-related beliefs? and 3) what is the potential effect of the developed instructional materials?

2. Research method
This is a design research with the type of development study design employing formative evaluation [16]. The stages of the research are preliminary stage and prototyping stage (formative evaluation) which includes self-evaluation, expert reviews and one-to-one, small group, and field test [17, 18].
Tessmer [17] defines formative evaluation as a judgment of the strengths and weaknesses of the instruction in its development stages for the purposes of revising the instruction to improve its effectiveness and practicality. The components involved in this development process were the designer of the instructional materials (in this case we as the authors), the experts, and the learners (in this case the pre-service teachers). The subject of this research is 31 pre-service teachers from primary school at Surabaya city and experts in mathematics education. In detail, the participants involved are five preservice teachers (one-to-one), three experts (expert review), six pre-service teachers (small group), and 20 pre-service teachers (field test). The participants of field test consist of 13 female and seven male pre-service teachers studying at the master program of primary education at Universitas Negeri Surabaya; all of them have not joined any mathematics-related course during their master program. The stages of one-to-one, expert review, small group, and field test are described to answer the first research question by examining the validity and practicality of the instructional materials.

To answer the second research question, in particular, a mathematics-related beliefs task was developed. The task encourages pre-service teachers to select one of the three teaching sequences which may arouse teachers' dilemmas on the teaching of area measurement which portrays the three philosophical beliefs summarised by Beswick [14]. Figure 1 presents one of the three options used in field test which have been revised through the development process.

Some participants auditioned to become a singer in a queue area outside a television station building. All the participants are standing in line. With the queue area shown in the right picture, determine the best estimate of the number of participants of the audition.

Option 1 (Instrumentalist)
1. Providing an explanation on how to find the area of combined geometrical figures by firstly giving some simple examples;
2. Asking students to practice how to find the area of the combined geometrical figure based on the teacher's method;
3. Introducing the application of the area of the combined geometrical figure in real life, such as finding the number of tiles required in a floor with a particular area;
4. Ensuring that students understand how to solve such 'Tiling' problem so that it can be applied in solving the 'Queue of Singer Audition' problem;
5. Starting to present the 'Queue of Singer Audition' problem which is considered as a more difficult problem than the 'Tiling' problem;
6. Telling that there is important missing information that needed to be found, i.e., the space that a participant might occupy while standing in the queue area;
7. Directing students to a specific strategy, i.e., dividing the area occupied by participants with the size of space of a person while standing;
8. By teachers' guidance, students finally obtain the solution to the problem; and
9. Correcting students' answers based on the procedures and the final solution they performed.

Figure 1. Mathematics-related beliefs task.

At field test, the pre-service teachers followed two learning activities: 1) mathematics problem-solving knowledge for teaching developed from the Chapman's framework (see more detail at [19]), in which this topic also become another concern of supporting teachers' improvement on beliefs, and 2) mathematics-related beliefs. The dilemmatic situations were provided in the latter activities. Therefore, this does not aim to describe the first learning activities. The pre-service teachers were asked to select individually one of the options at the beginning of the lesson, give their arguments, discuss with other colleagues, confirm the results of their discussion with theoretical perspectives guided by the lecturer, and reflect on whether they change their choices at the end of the lesson.
The third research question, on the other hand, was answered by describing the change of beliefs related to teaching problem-solving task after joining the learning activities. The change was indicated by the number of choices for each of the options: Instrumentalist, Platonist, and Problem-solving selected by pre-service teachers in the field test, which was counted at the beginning of the learning activities. At the end of the activities, they were asked to consider their choices about whether they would keep or change their types of teaching sequences. Thus, the author counts the number of the pre-service teachers' choices at the end of learning activities.

3. Results and discussion

3.1. Development process of the instructional materials
At the prototyping stage, the instructional materials were developed within self-evaluation, expert review, one-to-one, small group, and field test. At the self-evaluation stage, the authors re-examined the initial prototype design while preparing the research instrument related to the development process. The study was conducted by checking the suitability of the lesson plan and learning activities in the worksheet regarding its content, constructs, and language. The results from the self-evaluation were then tested in one-to-one and expert review simultaneously. One-to-one was conducted to see the practicality of the developed materials. Information obtained from this activity is the students' comments about the clarity of the teaching materials intent and the model of pre-service teachers' responses to the instructional materials. Meanwhile, three experts in expert review validated the instructional materials concerning content, constructs, and languages.

The results of one-to-one indicates some pre-service teachers’ comments to the quality of the instructional materials which relates to (1) unclear images, (2) some less familiar terms, (3) too long sentences, (4) grammatical errors, and (5) the emergence of alternative responses to problems examined in the worksheet. All these comments were then used as a basis to revise the instructional materials to meet practicality criteria. Based on the expert review, the researcher revised the instructional materials regarding some issues, i.e., correction of the typos, the correct use of capital letters, the use of more effective sentences, and the use of more legible drawings. Furthermore, the revision also concerned to the addition of questions about the selection of teaching sequences based on the type of beliefs on teaching mathematics, the refinement of mathematics-related task regarding the sequences of teaching problem-solving task that would be discussed by pre-service teachers which looks not represent the Instrumentalist, Platonist, and problem-solving view. The revised instructional materials were then used in a small group and field test. In these two stages, the learning activities designed in the instructional materials were firstly trialled and evaluated in actual classroom teaching setting although with the small number of participants.

3.2. Dilemmatic situations: instrumentalist, Platonist, or problem-solving?
Based on the results of the analysis of the subject response, four subjects chose the Instrumental option, seven subjects chose the Platonic option, and nine subjects chose the problem-solving option before the discussion took place. Figure 2 shows examples of the subject's response to the choice of the learning sequence.

Figure 2a shows the reason for the subject choosing option 1. In the subject's opinion, option 1 is the best choice because students need to learn from something simpler to something more complex gradually. Meanwhile, Figure 2b indicates the reason for the subject choosing option 2. According to him, option 2 will provide space for students to be more independent in solving mathematical problems because teachers only guide as necessary at each stage of Polya. Figure 2c shows the subject's lack of confidence in choosing option 1 and 2, due to too much teacher intervention in helping the student to solve the problem, and the timeliness of the initial problem, which should be suitable only for students who like challenges, not students that passive in learning.
Translation:

Option 1.
In this teaching sequence, students learn how to construct initial knowledge from simple things so that it can be easily understood, and then, they learn a more complex knowledge gradually before learning to solve a complex problem. Thus, it will make students solve such a problem.

b) Option 2. Platonist view

Translation:

Option 2. This option follows Polya's model of problem-solving because to solve a problem, Polya's stages should be used. Besides, teachers guide their students at each stage so that the students will be independent in solving a problem-solving task.

c) Option 3. Problem-solving view

Translation:

Option 3, because option 1 indicates that the teacher provide too much initial knowledge or topics being taught and students tend to be more passive. Option 2 indicates that students are directly provided with a problem, in which not all students have sufficient initial knowledge to understand the problem. This option will be effective for those who are active and challenged. In option 3, the teacher not only keeps guiding students but also providing opportunities to be actively engaged in solving a problem.

Figure 2. Pre-service teachers' responses to mathematics-related beliefs task.

Pre-service teachers encountered conflicting ideologies regarding choosing the best instructional sequences provided at the field test. Following the findings of Ding & Wang [20] regarding decision making patterns of teachers when facing dilemmatic situations, the authors categorized pre-service teachers' response within the lesson which used the developed instructional materials as follows.

3.2.1 Maintain personal beliefs and practices
In this pattern, participants who reported that they maintained their own beliefs and practices mainly justified their decisions by comparing the opposing beliefs and practices represented by other two options to highlight the beliefs on evidence found in his field teaching such as their immediate classroom situation, past school experiences. The following excerpt indicates their view.

PsT1 : I will keep my beliefs that option 1 (the instrumentalist view) is the best although many other of my friends choose another one. I believe that students need to understand a certain mathematical concept first. I have tried to give such kind of problem to my students in the previous lesson, and they got stuck to understand the problem. My teachers in the past often give my friends and I many exercises after we understand the particular basic rules of mathematics. That makes us more flexible to solve any exercises about mathematics problem. Furthermore, I think my students will be more interested in learning concepts before a real-world problem like this.
3.2.2 Change and adapt to the opposing beliefs and practices
The second pattern was to change and adapt to the opposing beliefs and practices. Participants who changed their beliefs justified the changes by viewing the potential benefits of the opposing beliefs emerging during class discussion. This gives rise to reconceptualize the opposing beliefs by modifying the situations provided in the teaching sequences with the context where they teach. This causes them to reconceptualize the opposing beliefs by adjusting the situations provided in the teaching sequences with the context where they teach. The following excerpt indicates this pattern.

PsT2: Yes, you [another PST] are correct. When guiding students to find the strategies, the teacher in option 1 (Instrumentalist) seems to give too much direction, like when he exemplifies some methods of finding the area. This will reduce students' opportunity to present their own strategies. Previously I select this options [option 1] since it best represents my daily teaching, but when you say it should better to give this problem at the beginning of a lesson, instead of at the end of a lesson, I start thinking to find a more suitable problem which fits students' prior knowledge. So, I agree that it is not about when we teach problem-solving task, but it is about how we select and modify an appropriate problem, and I think this problem is ok for my students.

Lecturer: Why did you change your options into option 2 (problem-solving) instead of option 3 (Platonist)?

PsT2: Basically, I agree with the teacher in option 3 because he involves students to find the solution actively. However, the worksheet completed with a set of questions as guidance will not make students find their own methods. Thus, I change my option to option 2 which does not provide such kind of guidance.

3.2.3 Avoid taking action
In this pattern, participants show a decision-making pattern by choosing not to confront their initial beliefs or opposing beliefs. This is indicated by their view about a trust given to the curriculum designer, in this case, is the government. In other words, regardless of whether their initial beliefs or the opposing beliefs are correct, they will rely on the mandate requested by the current curriculum. That being said, these prospective teachers experienced dilemmatic tensions because of their identities as teachers as curriculum implementer. This, as Ding & Wang [20] said, is called a self-identification which became the way teachers justified their decisions to avoid taking action in response to the dilemmas. The following excerpt exemplifies one of pre-service teachers' pattern of decision making.

PsT3: I previously select option 1, and now it is still option 1. This is not because I have a good experience in handling such kind of learning activities during my daily teaching. But, it is about how to complete all the targeted topics in the curriculum in the expected time. That is why I can only do that. However, if you give me extra time for teaching, I will choose option 2. That is good. Teachers in this case are a good facilitator.

3.3 Potential effect of the instructional materials on improving pre-service teachers' mathematics-related beliefs
Results of field test show that there were changes in the selection of belief types on the selection of learning sequences. Of the 20 participants, 12 participants changed the choice, and eight participants remain on the choice. From the 12 participants, four participants changed the choice from the instrumental option to the problem-solving option, 5 participants changed the choice from the Platonic option to the problem-solving option, and interestingly, there were 3 participants who changed the choice from problem-solving to platonic option. The latter result seems surprising since it means the beliefs of those three subjects become more traditional after joining the course. When analyzing their written work about the reasons why they changed their belief, we found that the reasons are around their students' mathematical ability background in which students at their class seems difficult to solve a real-life problem which is introduced at the beginning of the lesson. Meanwhile, from the 8 participants with unchanged options, there are six that remain on the problem-solving option, whereas
only 2 participants remain on the Platonist option. In general, these results indicate that after following the course, the majority of subjects (15 out of 20 subjects) choose a learning sequence that describes the problem-solving. These results indicate that there is an influence of learning using the instructional materials developed in this study on the change of belief type choices. In other words, these teaching materials have a potential effect on the change in teachers' beliefs in mathematics and learning. Nevertheless, these results seem to need further clarified through the practice of learning which the subject will do while teaching in the classroom. This can be clarified with the findings of Siswono et al. [21] demonstrating that teacher's teaching practices of their selected participants, even with the problem-solving approach, have not fully met the requirements for problem-solving learning. However, this seems to be incompatible with the studies of Siswono, Kohar, Kurniasari, and Astuti [22] who reported that high school teachers in their studies tend to be convinced to apply problem-solving ideas as dynamic approaches while teaching math.

Accordingly, the gap between teacher beliefs and teacher teaching practices needs to be the focus of further concern, especially by professional developers of teachers to develop their knowledge and skills in problem-solving teaching practice. Minimizing such a gap have been actually carried out by a professional teacher programme designer such as by Siswono et al. [23,24]. Derived from the findings that there is a relationship between teachers' beliefs and knowledge regarding problem-solving [25], two interrelated programs have been designed with different concern, i.e. programs for improving teachers' mathematics problem solving knowledge for teaching [23], and program for improving teachers' knowledge in posing context-based problem solving task [24]. These two programs employ problem-solving and problem-posing approach as learning tools for teachers participant at those programs. Since the results of these programs were experimentally able to improve teachers' problem-solving-related knowledge, future programs may be designed by considering problem-solving and problem-posing approach as tools for improving teacher beliefs, knowledge, and practice more comprehensively. For instances, the programs may consider to include real-world situation in mathematics learning [26], support teachers to create context-based task emphasizing on the authenticity of context [27] as well as solve authentic context-based problem, instead of only camouflage context-based problem [28], and improve mathematics content knowledge which often leads teachers to have instrumentalist beliefs about mathematics [29].

4. Conclusion

The dilemmatic situations were used as context for learning mathematics-related beliefs. This context was refined along with the development process of the instructional materials within a developmental research following formative evaluation stages of Tessmer [17]. The dilemmatic situations could encourage pre-service teachers to consider which values they should follow since such values conflict, and there seems to be no right thing to do. On the one hand, the pre-service teachers agree to the problem-solving view, on the other hand, they are confronted with the immediate situations found in their instructional practice which makes them hesitant in having the problem-solving view. In addition, the instructional materials were proved to potentially effect on improving pre-service teachers' mathematics-related beliefs and attract pre-service teachers' interest in learning mathematics-related beliefs through dilemmatic situations.

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