Asymmetric Inert Scalar Dark Matter

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In the quite minimal inert scalar doublet dark matter framework, we analyze what would be the effect of a $B - L$ asymmetry that could have been produced in the Universe thermal bath at high temperature. We show that, unless the $\lambda_5$ scalar interaction is tiny, this asymmetry is automatically reprocessed in part into a DM asymmetry that can easily dominate the DM relic density today. This scenario requires the inert DM mass scale to lie in the few-TeV range. Two types of relic density suppressions render this scenario viable: thermalization, from the same $\lambda_5$ interaction, of the asymmetries at temperature below the dark matter particle threshold, and DM particle-antiparticle oscillations.

\section{Introduction}

The similarity of baryonic and dark matter (DM) abundances, determined by the observation of the Cosmic Microwave Background (CMB) anisotropies, $\Omega_{DM}/\Omega_B = 5.4 \pm 0.1$ \cite{Ade:2013zuv}, has motivated a long series of scenarios where both abundances have a related or even very same origin. Since the baryon asymmetry is to a very good approximation totally asymmetric – no primordial population of antibaryons has been observed in the Universe – a common origin of both abundances suggests that the DM abundance today would be associated to the generation of a DM particle-antiparticle asymmetry (see e.g. the reviews of Refs. \cite{Chun:2008pi,Bezrukov:2012sa,Gerb Nicolini:2014mya}). In the following, we will show that this is naturally realized in the particulate Standard Model (SM) a single scalar doublet, $H_2$, odd under a $Z_2$ symmetry. The most general scalar potential is in this case

$$V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + \lambda_1 |H_1|^4 + \lambda_2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_1|^4 |H_2|^2 + \frac{\lambda_5}{2} (H_1^* H_2)^2 + h.c.,$$

where the SM and the inert scalar doublets can be written as

$$H_1 = \left( \frac{\phi^+}{\sqrt{2}} + \phi^0 \right) \quad \text{and} \quad H_2 = \left( \frac{\eta^+}{\eta^0} \right),$$

with $\eta^0 \equiv (H^0 + i A^0)/\sqrt{2}$. In the scalar potential, $m_2^2$ is assumed to be positive to insure that $H_2$ doesn’t acquire a vev, so that it’s lightest (neutral) component is stable. Before electroweak symmetry breaking (EWSB), all $H_2$ components have mass $m_{H_2} = m_2$, whereas after EWSB ($v = 246$ GeV), they get split in mass

$$m_{H^0}^2 = m_2^2 + \lambda_{H^0} v^2, \quad m_{A^0}^2 = m_2^2 + \lambda_{A^0} v^2, \quad m_{\eta^0}^2 = m_2^2 + \lambda_{H^0} v^2,$$

with $\lambda_{H^0} = \lambda_5/2$ and $\lambda_{H^0, A^0} = (\lambda_3 + \lambda_4 + \lambda_5)/2$. In the following, we will assume, without loss of generality, that $\lambda_5$ is negative, so that $H^0$ is the DM particle. Various well-known constraints hold on the parameters of the theory. Tree level vacuum stability requires $\lambda_{1,2} > 0$, $\lambda_{H^0, A^0, H^c} > -\sqrt{\lambda_1 \lambda_2} \approx -0.36/\sqrt{2}$. EW precision test observables require $\Delta \Gamma = (m_{\eta^0} - m_{H^0}) (m_{\eta^0} - m_{H^0}) / 12 \pi^2 \alpha^2 \lesssim 10^{-1}$. Z decay width constraint at LEP requires $m_{\eta^0} + m_{H^0} > m_Z$ and $m_{\eta^0} > m_Z/2$. Direct detection constraint importantly requires that the $Z$ exchange diagram is kinematically forbidden, i.e. $m_{A^0} - m_{H^0} \gtrsim \mu_{\eta^0 \beta_{DM}}/2$, where $\beta_{DM}$ is the DM halo velocity with respect to the earth, and $\mu_{\eta^0} = m_{H^0} m_{\eta^0}/(m_{H^0} + m_{\eta^0})$ is the reduced mass of the system for the nuclear $\lambda_0$ used by the experiment. For $m_{H^0} \gtrsim m_{\eta^0}$ and Xenon nucleus, using an average velocity of $\sim 270$ km/s, this constraint can be rephrased as $m_{\eta^0} - m_{H^0} \gtrsim 50$ keV. Taking into account the velocity distribution around this central value, and the recoil energy sensitivity of the experiments, the condition becomes $m_{\eta^0} - m_{H^0} \gtrsim 180$ keV, although a more robust constraint is $m_{\eta^0} - m_{H^0} \gtrsim 100$ keV \cite{Olive:2016xmw}, or equivalently

$$|\lambda_5| \gtrsim 3.3 \cdot 10^{-6} \left( \frac{m_{H^0}}{\text{TeV}} \right) \left( \frac{m_{\eta^0} - m_{H^0}}{100 \text{keV}} \right).$$

It is well known that the IDM can account for the observed DM relic abundance via the usual freeze-out mechanism, and be in agreement with direct detection constraints, for DM masses in the ranges $\sim [50, 80]$ GeV and above $\sim 540$ GeV, up to the $\sim 40-50$ TeV unitarity bound \cite{Hambye:2011ky,Chung:2011th,Buttazzo:2015coa,Buttazzo:2012vq}. However, as we will show, in an all parameter space region the DM abundance is instead dominated by asymmetric production, and such a possibility can perfectly lead to the observed abundance.

\section{Asymmetric Production of the DM Relic Density}

Let us make two simple starting assumptions. First, let us assume that the symmetric component of the relic density left after freeze-out is smaller than the observed value. Fast SM gauge scatterings automatically care for that for $m_{DM}$ within the $\sim 120 - 540$ GeV range, whereas for other values of $m_{DM}$ large enough $\lambda_{3,4}$ interactions can take care of that \cite{Hambye:2011ky}. This implies a symmetric annihilation cross section larger than the
usual thermal freeze-out value $\sim 1$ pb, which means a freeze-out temperature $T_{fo}$ smaller than the usual $T_{fo} \sim m_{DM}/25$ value. Second, let us assume that a $B-L$ asymmetry has been generated at a temperature $T_{eq}$ above $m_{H_2}$ and above the EWPT temperature $T_{EW}$ (which we take as the temperature where the vacuum expectation value of the SM scalar field becomes sizable, that is $T_{EW} \approx 165$ GeV from Ref. [13]). We do not care about the way this $B-L$ asymmetry could have been generated. It could be due for example to the straightforward leptogenesis mechanism. Note that, as well-known, if a $B-L$ asymmetry is generated at high temperature, a $H_1-H_2^*$ asymmetry will also be created automatically at high temperature from thermal equilibrium SM interactions [14].

If a $B-L$ (and thus $H_1$) asymmetry is created at high temperature, an inert doublet $H_2-H_2^*$ asymmetry is to be expected too. The scalar potential of Eq. (1) contains the $\lambda_5$ interaction which uniquely does not conserve the number of $H_2$ minus the number of $H_2^*$ (as well as the number of $H_1$ minus the number of $H_1^*$). This interaction is in thermal equilibrium at $T \sim m_{H_2}$ if at this temperature the corresponding $\Gamma_5$ scattering rate, given in the Appendix, is larger than the Hubble rate, which gives the condition

$$|\lambda_5| \gtrsim 2 \cdot 10^{-6} \cdot (m_{H_2}/\text{TeV})^{1/2}. \quad (5)$$

Given the lower bound of Eq. (4), it turns out that this relation must anyway hold for TeV masses. For instance, Eq. (4) with a 100 keV (180 keV) mass splitting implies Eq. (5) for $m_{H_2} \gtrsim 400$ GeV (100 GeV). Therefore, one expects the $\lambda_5$ interaction to equilibrate the $H_2$ and $H_1$ (and $B-L$) asymmetries.\(^1\) In particular even if, as we assume here, no $H_2$ asymmetry is created at high energies, such an asymmetry will be created anyway as soon as the $B-L$ asymmetry is created. In other words the inert DM model contains an interaction which basically implies that “Higgsogenesis”\(^{16}\) production of a DM asymmetry is at work.\(^2\) Note that the scenario could work also the other way around, i.e. a primordial DM asymmetry could be at the origin of baryogenesis via the same $\lambda_5$ equilibrium interaction, a possibility we will not consider here (for a scenario of this kind see Ref. [17]).

In the following, we will consider in details and chronologically what happens when the temperature of the Universe cools down from $T \gg m_{H_2}$ to today $T \ll T_{EW}$, crossing $m_{H_2} > T_{\lambda_5} > T_{fo} > T_{EW}$, with $T_{\lambda_5}$ the temperature where the scattering induced by the $\lambda_5$ interaction decouples and $T_{fo}$ the freeze-out temperature at which the total annihilation cross section decouples. Given that the inert doublet components undergo gauge interactions, $T_{\lambda_5}$ is sizably larger than $T_{fo}$, unless $\lambda_5$ is of order one, which as we will see is not a viable option for the case we are interested in (where the DM asymmetry is responsible for most of the relic density). Similarly, as we will see, $T_{fo} \gtrsim T_{EW}$, i.e. few-TeV DM, is also generically necessary in order to have a viable scenario (as in the scenario of Ref. [15]), but some violation of this inequality is possible. A sketch of the scenario, applied to our framework, is shown in Fig. 1.

\section{A. $T \gtrsim m_{H_2}$}

At temperature above $T_{EW}$, all 4 inert doublet components have a common mass $m_{H_2} = m_2$. If Eq. (5) is satisfied, the chemical potential of both scalar doublets are equal, $\mu_{H_2} = \mu_{H_1}$. Together with the usual SM chemical equilibrium relations (from thermal equilibrium SM processes\([14]\)), the $\mu_{H_2} = \mu_{H_1}$ relation (from e.g. $\eta^-\eta^0 \leftrightarrow \text{SM}$ processes), and the hypercharge relation

$$\sum_i (\Delta Q_i + 4\Delta u_i - 2\Delta d_i - \Delta l_i - 2\Delta e_i) + \Delta H_1 + \Delta H_2 = 0, \quad (6)$$

it simply gives

$$\Delta H_2 = \Delta H_1 = -\frac{4}{23} \Delta B-L. \quad (7)$$

From now on, we define for each species $X$ the asymmetry $\Delta X \equiv X_+ - X_-$ and the total density $\Sigma X \equiv X_+ + X_-$, where $X_+ \equiv n_X/s$ is the particle number density-to-entropy ratio of $X$. Since we are dealing with asymmetries, we also define the number of degrees of freedom by summing the number of particles (or antiparticles but not both), i.e. $g_X = 1$ for a $SU(2)_L$ singlet, and $g_X = 2$ for a doublet.

As well-known, for similar $B-L$ and DM asymmetries, the DM relic density constraint requires $m_{DM}$ to have a mass of few GeV (more exactly, from Eq. (7) and taking into account the $Y_{B-L}$ to $Y_R$ ratio which holds in this case, Eq. (22) below, one would need $m_{DM} \approx 10$ GeV). As this possibility is excluded by collider constraints, this implies that a subsequent suppression of the DM asymmetry by a factor of $\sim (10 \text{ GeV}/m_{DM})$ must necessarily occur. Two different types of suppressions can naturally take place. A first one is a Boltzmann suppression from asymmetry violating scatterings, used in several other DM models, see e.g. Refs. [16][13]. In our scenario, it can arise from the $\lambda_5$ interaction within the period $m_{H_2} > T > T_{\lambda_5}$. The other possible suppression can arise later when $T \lesssim T_{EW}$ from the combined effect of DM oscillations and symmetric annihilations.

\section{B. $m_{H_2} \gtrsim T \gtrsim T_{\lambda_5}$}

Once the temperature drops below $m_{H_2}$, if the $\lambda_5$ interaction goes on to be in thermal equilibrium, the $H_2$ asymmetry gets Boltzmann suppressed, simply from the fact that in this case the $H_2H_2 \rightarrow H_1H_1$ process is still fully open, whereas the

\footnotesize\begin{itemize}
\item \textsuperscript{1} Actually in the few-TeV asymmetric inert DM scenario considered in Ref. [13], it is assumed instead that the $\lambda_5$ interaction could have never been in thermal equilibrium. In this case, the DM asymmetry would have been created explicitly at high energies, basically independently of the $B-L$ asymmetry.
\item \textsuperscript{2} In Ref. [16], a $X_1$ fermion singlet DM framework is considered with an extra $X_2$ fermion doublet and an $X$-symmetry. An $X_2$ asymmetry is created from a $X$-symmetry violating $\lambda_2H_1^*$ non-renormalizable interaction, which is afterwards reprocessed into a $X_1$ asymmetry through $X_2$ decays.
\end{itemize}
$H_1H_1 \rightarrow H_2H_2$ is doubly Boltzmann suppressed. In this way, the $\lambda_5$ interaction leaves intact the sum of the asymmetries of $H_1$ and $H_2$ but not each asymmetry individually. This can be directly seen from the Boltzmann equation of $\Delta H_2$, valid for $T \geq T_{EW}$,

$$
\frac{d\Delta H_2}{dz} = -\frac{4}{m_{H_2}^2} \frac{\Delta_{H_2}}{y_{H_2}} \frac{\Delta_{H_2}}{y_{H_1}^2} \gamma_{\lambda} \, ,
$$

(8)

where $z = m_{H_2}/T$, $H(z)$ is the Hubble rate and $\gamma_{\lambda}$ (z) is the reaction density of the $\lambda_5$ scatterings, given in Appendix. Thus, since $T$ drops below $m_{H_2}$, the $H_2H_2 \rightarrow H_1H_1$ term is enhanced with respect to the inverse term by the factor $y_{H_2}^{-1}$, as the Hubble rate $H$ and Boltzmann suppression, unlike $y_{H_1}$. This Boltzmann suppression of the asymmetry lasts until the $\lambda_5$ induced scatterings decouple, at $T = T_{\lambda_5}$, when $\Gamma_{\lambda_5} \simeq H$. Quantitatively, this can be accounted by the usual k-factor which gives the asymmetry as a function of the temperature

$$
\Delta H_2(z) = \frac{k(z)}{2} \Delta H_1 \, ,
$$

(9)

Down to $T_{\lambda_5}$, the chemical potential relation $\mu_{H_2} = \mu_{H_1}$ still holds, and we get

$$
\Delta H_2(z) = k(z) \Delta H_1 \, .
$$

(10)

This is nothing but the solution which makes the r.h.s. of Eq. (2) to vanish. Together with the other chemical potential relations above, the asymmetry reads at $T = T_{\lambda_5}$ (similarly to the fermion doublet case of Ref. [16])

$$
\Delta H_2(z_{\lambda_5}) = -\frac{16k(z_{\lambda_5})}{158 + 13k(z_{\lambda_5})} \Delta_{B-L} \, .
$$

(12)

For practical reasons, it is convenient to define

$$
\Delta_{H_2}^{\lambda_5} = |\Delta H_2(z_{\lambda_5})| \, .
$$

(13)

Let us note that since $z_{\lambda_5} \gg 1$, the $k$-factor can be approximated by

$$
k(z_{\lambda_5}) \simeq 12 \left(\frac{z_{\lambda_5}}{2\pi}\right)^{3/2} e^{-z_{\lambda_5}} \, .
$$

(14)

Clearly, the $\lambda_5$ coupling must not be too large in order to avoid a too strong exponential suppression of the $H_2$ asymmetry.

The value of $z_{\lambda_5}$ is approximately given by the condition that the $\Gamma_{\lambda_5} = n_{H_2}^0 \langle \sigma_{\lambda_5} v \rangle$ rate (given in the Appendix) is equal to the Hubble rate $H$. The corresponding value of $z_{\lambda_5}$ can be found approximately by solving the equation

$$
z_{\lambda_5} \simeq \ln \left[ \frac{0.0038 \cdot m_{Pl}^2 g_{*} T_{EW}^2 m_{H_2}^2 \langle \sigma_{\lambda_5} v \rangle}{\sqrt{g_{*} z_{\lambda_5}}} \right] \, ,
$$

(15)

with $g_{*} = 106.75$ the number of relativistic degrees of freedom at this temperature, and $m_{Pl}$ the Planck scale.

C. $T_{\lambda_5} > T > T_{EW}$

During this period, the $\Delta H_2$ asymmetry stays constant unlike the total abundance $\Sigma_{H_2}$, whose Boltzmann equation for this period reads

$$
\frac{d\Sigma_{H_2}}{dz} = -\frac{\langle \sigma_{eff} v \rangle s}{z H} \left[ \frac{\Delta_{H_2}^{\lambda_5} - \Delta_{H_2}^{\lambda_5} \gamma_{\lambda}^2}{2} \right] \, ,
$$

(16)

where $\langle \sigma_{eff} v \rangle$ is the effective thermal cross section of the $H_2H_2 \leftrightarrow SM SM$ annihilations, given in the Appendix. With a constant $\Delta_{H_2}^{\lambda_5}$, as it is the case during this period, the solution of Eq. (16) at freeze-out is to a good approximation given by

$$
\Sigma_{H_2}(z_{fo}) \simeq \left[ \Delta_{H_2}^{\lambda_5} + \Delta_{H_2}^{\lambda_5} \gamma_{\lambda}^2 \right]^{1/2} \, ,
$$

(17)

which is nothing but the expression which makes the r.h.s. of Eq. (16) to vanish. Here, by $z_{fo}$ we mean the usual freeze-out value given by the equation

$$
z_{fo} \simeq \ln \frac{0.0038 \cdot m_{Pl}^2 g_{*} T_{EW}^2 m_{H_2}^2 \langle \sigma_{eff} v \rangle}{\sqrt{g_{*} z_{fo}}} \, .
$$

(18)

If the annihilations are fast enough to leave at $T_{fo}$, a symmetric component smaller than the asymmetric one (which is typically satisfied for $\langle \sigma_{eff} v \rangle \gtrsim 1$ pb), the following relation holds,

$$
\Sigma_{H_2}(z_{fo}) \simeq \Delta_{H_2}^{\lambda_5} \gg \Sigma_{H_2}^{\eta_{L}}(z_{fo}) \, .
$$

Given the sign of the baryon asymmetry, this means at $T_{fo}$, $\Sigma_{H_2} \sim -\Delta_{H_2} \sim -\Delta_{H_2}^{\lambda_5} \gg \Sigma_{H_2}^{\eta_{L}} \sim \Sigma_{H_2}^{\eta_{L}} \sim \Sigma_{H_2}^{\eta_{L}} \sim \Sigma_{H_2}^{\eta_{L}} \sim \Sigma_{H_2}^{\eta_{L}}$.

D. $T_{fo} > T > T_{EW}$

Nothing is expected to happen during this period. The $H_2$ total density left at $T_{fo}$ is left intact until $T_{EW}$, temperature at which the total density and asymmetry are given by (for a dominant asymmetric component)

$$
\Sigma_{H_2}(z_{EW}) \simeq \Sigma_{H_2}(z_{fo}) \simeq \Delta_{H_2}^{\lambda_5} \text{ and } |\Delta_{H_2}(z_{EW})| = |\Delta_{H_2}^{\lambda_5}| \, .
$$

(19)

E. $T < T_{EW}$

Next, once the temperature drops below $T_{EW}$, two new effects arise: generation of mass splittings and fast inert particle-antiparticle oscillations $\eta^0 \leftrightarrow \eta^{0*}$. The effect of the mass splittings generated between the $H^0, A^0$ and $\eta^+$ components by the SM scalar vev, Eq. (3), is of moderate importance. Assuming, as said above, that the $H^0$ component is the lightest one (i.e. $\lambda_5 < 0$), they imply that the other components will ultimately decay to $H^0$. But these decays conserve the number of inert scalar particles. They just convert the $H_2$ asymmetry created before EWSB (with mass $m_{H_2}$) into a DM relic density of selfconjugated DM particles $H^0$ (with mass $m_{H^0} = m_{DM}$, different from $m_{H_2}$, unless $\lambda_{H^0}$ vanishes). More important is the potential effect of the much faster inert particle-antiparticle oscillations $\eta^0 \leftrightarrow \eta^{0*}$ caused by the
that means the DM density is equal to the asymmetry left stored in the day will be simply equal to the number of inert scalar particles states \[19\]. In this case, the number of will occur in this case. Let us consider both possible cases anyway reduce the DM abundance as no inverse processes will quickly give a number density of each population much larger than their thermal equilibrium values, roughly \(n_{\eta^0} \sim n_{\eta^0} \sim |\Delta n_{\eta^0}|/2 \gg n_{\eta^0}^e\), so that \(|\Delta n_{\eta^0}(\sigma_{\nu})| > H\) can hold even if \(n_{\eta^0}^e(\sigma_{\nu}) < H\). If these annihilations occur, they will anyway reduce the DM abundance as no inverse processes will occur in this case. Let us consider both possible cases separately.

1. \(T < T_{EW}\): No symmetric annihilations after EWSB

If no symmetric annihilations arise after EWSB, oscillations have simply no effect. They quickly reconvert a pure \(\eta^0\) population, or a pure \(\eta'^0\) population, into an oscillating mixed \(\eta^0 - \eta'^0\) population, but they do not change the number of inert states \[19\]. In this case, the number of \(H^0\) DM particles left today will be simply equal to the number of inert scalar particles stored in the \(H_2\) asymmetry before EWSB, i.e. \(Y^d_{DM} \simeq \lambda^5\), that means the DM density is equal to the asymmetry left after \(\lambda^5\) interaction’s decoupling. From Eqs. (12) and \(19\), this gives

\[
Y^d_{DM} = \frac{16k(\zeta_{\lambda^5})}{158 + 13k_{H_2}(\zeta_{\lambda^5})} \Delta_{B-L},
\]

with \(k(\zeta_{\lambda^5})\) given by Eq. (14). Only the relation between the value of \(Y^d_{B}\) today and \(\Delta_{B-L}\) changes after EWSB, as a result of the fact that below \(T_{EW}\) the conservation of electric charge holds rather than conservation of \(Y\) and \(T_3\). We get

\[
Y^d_{B} \simeq \frac{12}{37} \Delta_{B-L}.
\]

As a result, the DM density reads

\[
Y^d_{DM} = \frac{148k(\zeta_{\lambda^5})}{474 + 39k(\zeta_{\lambda^5})} Y^d_{B},
\]

and the actual DM to baryon density ratio is given by

\[
\frac{\Omega_{DM}}{\Omega_B} = \frac{Y^d_{DM}}{Y^d_{B}} \left( \frac{m_{H^0}}{1 \text{ GeV}} \right) = \frac{148k(\zeta_{\lambda^5})}{474 + 39k(\zeta_{\lambda^5})} \left( \frac{m_{H^0}}{1 \text{ GeV}} \right).
\]

This is the final result if no symmetric annihilations occur after EWSB. Below, we will see for which values of the parameters this holds. It is worth to note that in this case, by plugging the relic density constraint in the \(\lambda^5\) interaction decoupling condition, \(\Gamma_{\lambda^5}(\zeta_{\lambda^5}) \simeq H(\zeta_{\lambda^5})\), with \(\Gamma_{\lambda^5}\) as given in the Appendix, we find the following equality, \(m_{H^0}/\lambda^5 \sim (6 \times 10^4 \text{ TeV}) \cdot (10/\zeta_{\lambda^5})^{1/2}\). This equality requires a value of \(m_{H^0}/\lambda^5\) which is in agreement with the direct detection constraint of Eq. \(4\). This explains why, in the numerical results obtained below, we will find viable values of the parameters within this regime. However, it must be stressed that it is not mandatory to avoid symmetric annihilations after EWSB. On the contrary, if the \(\lambda^5\) interaction above does not provide enough suppression, these scattering processes could easily provide it, without the need of any special tuning. This is what we will now quantify.

2. \(T < T_{EW}\): Symmetric annihilations after EWSB

The neutral states \(\eta^{0(+)}\) do oscillate but the charged ones \(\eta^\pm\) do not. Let’s first see what happen’s to the neutral density, like if there were no charged states. The effect of the fast \(\eta^0\) oscillations together with the symmetric annihilations can be accounted for, by using the Boltzmann equations that have been derived in the general context of an oscillating DM particle in Refs. \[19\] \[20\].

\[
\frac{d\Sigma_{\eta^0}}{dz} = -\frac{(\sigma_{0V}) s}{2zH} \left( \delta^2_{\eta^0} - \Delta^2_{\eta^0} - \delta^2_{\eta^0} - \delta^2_{\eta'^0} \right),
\]

\[
\frac{d\Delta_{\eta^0}}{dz} = 2i\Gamma_{\eta^0} z H \xi_{\eta^0},
\]

\[
\frac{d\Sigma_{\eta'^0}}{dz} = 2i\Gamma_{\eta'^0} z H \xi_{\eta'^0} - (\sigma_{0V}) s \xi_{\eta^0} \Sigma_0,
\]

where for any \(T \leq T_{EW}\) we define \(z \equiv m_{H^0}/T\), with \(\langle \sigma_{0V}\rangle\) the thermally averaged \(\eta'^0 - \nu\) annihilation cross section, and \(\xi_{\eta^0}\) a quantity that accounts for the coherence between the \(\eta^0\) and \(\eta'^0\) components (see \[19\] for further details). The resolution of these equations leads to a monotonically decreasing \(\Sigma_{\eta^0}(z)\) function and to oscillating functions \(\Delta_{\eta^0}(z) \propto \cos[f(z)]\) and \(\xi_{\eta^0}(z) \propto \sin[f(z)]\) whose amplitudes also decrease monotonically. For fast oscillations, and neglecting the \(\Sigma_{\eta'^0}\) term in Eq. (25), the set of Boltzmann equations can be simplified and solved analytically, at an approximate level, as explained in the Appendix. The solution it gives for \(\Sigma_{\eta^0}\) is \[3\]

\[
\Sigma_{\eta^0}(z \geq z_{EW}) = \frac{\Sigma_{\eta^0}(z_{EW})}{1 + \frac{1}{2}\frac{(\sigma_{0V}) s}{H(z)} \left( \frac{z}{z_{EW}} - 1 \right)} \xi_{\eta^0}(z_{EW}).
\]

\[3\] This result is approximately the same than the one obtained in \[19\] for much smaller \(\delta m\) values – see Eqs (25) and (33) therein – but in which \(x_{\eta^0, \eta'^0}\) (which depends on \(\delta m\)) is now simply replaced by \(z_{EW}\).
where \( z_{EW} = m_{Pl}/T_{EW} \) and where we fixed the initial abundance and asymmetry to be equal to \( \Sigma_{0}^{0}(z_{EW}) = \Delta_{H_{2}}^{2}/2 \). The asymmetry \( \Delta_{0}^{0} \) and \( \Sigma_{0}^{0} \) are, in turn, fast oscillatory functions which are equal to zero on average.

The result of Eq. (28) can also be qualitatively understood in the following way. Once \( T \leq T_{EW} \), the fast oscillations reprocess quasi instantaneously the \( \eta^{0} \) asymmetry in oscillatory abundances for \( \eta^{0} \) and \( \eta^{0*} \). On average, just after EWSB, we have therefore \( n_{\eta^{0}} \simeq n_{\eta^{0*}} \simeq |\Delta_{0}^{0}| \frac{z}{z_{EW}}/2 \). Since \( T_{EW} > T_{EW} \), when two conjugate particles annihilate to SM particles, the reduction of inert doublet state it implies will not be compensated by any inverse processes. As a result, the Boltzmann equation for \( \Sigma_{0}^{0} \) one gets along this way is simply given by

\[
\frac{d\Sigma_{0}^{0}}{dz} = -\frac{(\sigma_{0} v z)}{2zH} \Sigma_{0}^{2},
\]

whose resolution leads to nothing else than Eq. (28).

The next step is to include the contribution of the charged states. Since these states do not oscillate, one could naively expect that the charged asymmetry is essentially left intact until the charged states decay to \( H^{0} \) states. This doesn’t work this way. To see that precisely, one should in principle solve the corresponding set of five coupled Boltzmann equations, for \( \Sigma_{0}^{0} \), \( \Delta_{0}^{0} \), \( \Sigma_{1}^{+} \), \( \Delta_{1}^{+} \), but we don’t need to go that far. It turns out in practice that, just before EWSB, there are essentially only \( \eta^{-} \) and \( \eta^{0*} \) states as considered here, as soon as oscillations start they put the neutral state asymmetry to zero (on average), and processes which can transfer a charged asymmetry into a neutral one will very quickly put the charged asymmetry to zero too. This will be done in particular by \( \eta^{0} \rightarrow \eta^{+} \rightarrow \eta^{0*} \) inelastic scatterings and \( \eta^{+} \rightarrow \eta^{0} \) decays. The decrease of the charged component asymmetry due to these processes is exponential (\( \Delta_{1}^{+} \propto e^{-z} \)), as can be seen from the corresponding term in the Boltzmann equation, \( sH_{2}/dz \Delta_{1}^{+} \propto -\Delta_{1}^{+} \eta^{+} \eta^{-} + \eta^{0*} + ... \). This “re-equilibration” of the asymmetries by these processes, which follows their “un-equilibration” by the oscillations when these oscillations start, occurs much faster than the process of suppression of \( \Sigma_{0}^{0} \) in Eq. (51). As a result, in the same way as for the neutral states, one can adopt the simple assumption that as soon as oscillations start, the particle and antiparticle densities for charged states are equilibrated, \( Y_{\eta^{0*}} = Y_{\eta^{0}} = Y_{\eta^{-}} = Y_{\eta^{+}} \). At this point, the annihilation processes such as \( \eta^{+} \eta^{-} \rightarrow SMSM, \eta^{0} \eta^{0*} \rightarrow SMSM \) and \( \eta^{-} \eta^{0} \rightarrow SMSM \) can start again, in the same way as the \( \eta^{0} \eta^{0*} \rightarrow SMSM \) ones. The whole effect can be approximatively accounted by the simple Boltzmann equation

\[
\frac{d\Sigma_{0}^{0}}{dz} = -\frac{(\sigma_{e f f} v z)}{zH} \Sigma_{0}^{2},
\]

Similarly to what has been obtained in Eq. (28), the resolution of Eq. (30) integrated from \( T_{EW} \) until now and using the initial condition in (19), leads to

\[
\Sigma_{H_{2}}(z \geq z_{EW}) = \frac{\Delta_{H_{2}}^{5}}{1 + \frac{(\sigma_{e f f} v z)}{m_{\lambda}^{5}} (\frac{z}{z_{EW}} - 1) \Delta_{H_{2}}^{2}}.
\]

This equation holds for the case where the total number density just before EWSB is given by the asymmetry. If there is also a non-negligible part which is left from the symmetric freeze-out, one must simply replace the asymmetry at \( z_{EW} \), \( \Delta_{H_{2}}^{5} \), by the total number density at the same temperature, \( \Sigma_{H_{2}}(z_{EW}) \), since this is the number which determines the number of symmetric annihilations which will occur after EWSB,

\[
\Sigma_{H_{2}}(z \geq z_{EW}) = \frac{\Sigma_{H_{2}}(z_{EW})}{1 + \frac{(\sigma_{e f f} v z)}{m_{\lambda}^{5}} (\frac{z}{z_{EW}} - 1) \Delta_{H_{2}}^{2}}.
\]

F. Final DM relic density from an initial \( H_{2} \) symmetry

We summarize in Fig. 1 the evolution of the asymmetry \( |\Delta_{H_{2}}| \) and total density \( \Sigma_{H_{2}} \). We remind the main steps:

1. \( T \gtrsim m_{H_{2}} \). The \( H_{2} \) asymmetry, proportional to the \( B - L \) asymmetry, is generated through the \( \lambda_{\lambda} \) interactions:

\[
\Delta_{H_{2}}(z \lesssim 1) = -\frac{1}{23} \Delta_{B - L}.
\]

2. \( m_{H_{2}} \gtrsim T \gtrsim T_{\lambda} \). The asymmetry undergoes a Boltzmann suppression until the \( \lambda_{\lambda} \) interaction decouples,

\[
\Delta_{H_{2}}(z_{\lambda}) = \frac{16k (z_{\lambda} / 158 + 13k (z_{\lambda}))}{\Delta_{B - L}}.
\]

---

4 The reason why the two results coincide is in fact more subtle. Since the \( \eta^{0*} \) oscillatory behavior is given by

\[
Y_{\eta^{0*}} = \frac{1}{2} f^{i}(z) (1 \pm \cos g(z)),
\]

the Boltzmann equation in this naive approach should read

\[
\frac{d\Sigma_{0}^{0}}{dz} = -\frac{(\sigma_{0} v z)}{2zH} \Sigma_{0}^{2} \sin^{2} g(z).
\]

Averaging this expression, we find Eq. (29) up to an extra factor 1/2. An extra factor 2 must nevertheless be added to take into account the contribution of the coherence \( \Sigma_{0}^{0} \) part, giving back Eq. (28).

5 If there is no asymmetry and if the freeze-out has occurred prior to EWSB, one recognizes in Eq. (32) the usual asymptotic freeze-out behavior, i.e. the freeze-out is not instantaneous, but reaches asymptotically its final value as given in this equation. In practice, as well known, the effect is negligible in this case, i.e. the denominator is equal to unity to a good approximation. Here, instead, the denominator at \( z_{fa} \) can be much larger due to the asymmetry.
Since ultimately no asymmetry survives, the relation between the baryon and the $B - L$ asymmetry is still given by Eq. (22), and the final DM to baryon density ratio is given by

$$\frac{\Omega_{DM}}{\Omega_B} = \frac{\sum H_2(z_{EW})}{1 + \kappa \cdot \sum H_2(z_{EW})} \cdot \frac{1}{y_{DM}^{today}} \cdot \frac{m_{H_0}}{1 \text{ GeV}},$$

(36)

or equivalently, if the asymmetric component dominates, using Eqs. (12) and (19),

$$\frac{\Omega_{DM}}{\Omega_B} = \frac{148 \kappa \cdot \sum H_2(z_{EW})}{474 + \left(39 + 148 \kappa \cdot y_{DM}^{today}\right) \cdot k(z_{\lambda_5})} \cdot \frac{m_{H_0}}{1 \text{ GeV}},$$

(37)

A number of comments can be done regarding these results:

- Eqs. (34) and (36) show that beside the $\lambda_5$ interaction induced “k-factor” suppression in $\Delta H_2$, see Eq. (12), oscillations drive a $1 + \kappa \cdot \sum H_2(z_{EW})$ factor suppression. This “k-factor” suppression can be sizable as soon as $\kappa \cdot \sum H_2(z_{EW}) \gtrsim 1$.

- As Eq. (32) shows, this suppression is neither instantaneous nor exponential. It goes as the inverse of $z/z_{EW} - 1$ until it reaches an asymptotic value. In this sense, imposing that the cross section satisfies the unitarity bound, it is naturally limited but still can be responsible for most of the $\sim (10 \text{ GeV}/m_{DM})$ suppression needed, see below.

- The appearance of the $\kappa \cdot \sum H_2(z_{EW})$ factor is not surprising. The condition $\kappa \cdot \sum H_2(z_{EW}) < 1$ is nothing but the condition $(n_{H_2} + n_{\tilde{H}_2})/\langle \sigma v \rangle < H$ at $T = T_{EW}$.

- Interestingly, for large values of $\kappa \cdot \sum H_2(z_{EW})$, the $y_{DM}^{today}$ relic density obtained doesn’t depend anymore on the asymmetry left at $T_{EW}$, even if this asymmetry is the source of the final DM abundance. In this case, we simply get

$$y_{DM}^{today} = \frac{1}{\kappa},$$

and the ratio reads

$$\frac{\Omega_{DM}}{\Omega_B} \simeq 0.15 \cdot z_{EW} \cdot \left(\frac{1 \text{ pb}}{\langle \sigma v \rangle}\right).$$

(38)

This means, as we could have anticipated, that for large cross section the asymmetry left is independent of the initial asymmetry, provided this initial asymmetry is large enough. In other terms, if the $\kappa$-factor suppression is small, both baryon and DM asymmetries are directly connected. If instead it is large, they are not related anymore in a so direct way, since in this case the final relic density depends only on the annihilation cross section.\footnote{But still, even in this case, they remain similar as the $\kappa$ factor is bounded from above by unitarity considerations on the total cross section.}
Note interestingly that Eq. (35) is nothing but the result of the standard freeze-out scenario, but with the important difference that in the standard case, $\zeta_{\text{EW}}$ in Eq. (38) must be replaced by $\zeta_{\text{fo}}$.

III. RESULTS AND DISCUSSION

The final result of Eq. (36) depends on three parameters: $m_{\rho_0}, \Sigma_{H_2}(z_{\text{EW}})$ and the total cross section $\langle \sigma_{\text{eff}}/v \rangle$ via $\kappa$ in Eq. (35). This means that for given values of the input parameters $m_{\rho_0}$ and $\langle \sigma_{\text{eff}}/v \rangle$, there is only one value of $\Sigma_{H_2}(z_{\text{EW}})$ which gives the observed value of $\Omega_{DM}/\Omega_B$, as given by the PLANCK best fits, $\Omega_{DM} h^2 = 0.120$ and $\Omega_B h^2 = 0.022$ [11]. Since $\Sigma_{H_2}(z_{\text{EW}})$ depends only on these two input parameters and on $\Delta_{H_2}^{\lambda_3}$, this means also that there is only one value of $\Delta_{H_2}^{\lambda_3}$ which gives the correct relic density for fixed values of the two input parameters. We show in Fig. 2 this value of $\Delta_{H_2}^{\lambda_3}$ as a function of $m_{\rho_0}$ for different values of the cross section. By comparing this value of $\Delta_{H_2}^{\lambda_3}$ to the value this asymmetry would have if there were no “k-factor” suppression – given by the $H_2$ upper horizontal line – one can read off what is the value of this $\lambda_3$ induced “k-factor” suppression, Eq. (12) as compared to Eq. (7).

As said above, to dominate the final relic density, the asymmetry cannot be suppressed by more than a factor $m_{\text{DM}}/10$ TeV. Figure 2 also shows the corresponding values of the $\kappa \cdot \Sigma_{H_2}(z_{\text{EW}})$ factor which lead to the other suppression, i.e. the $1/(1 + \kappa \cdot \Sigma_{H_2}(z_{\text{EW}}))$ factor in Eq. (36). It also shows for which values of the various parameters the asymmetry produced before the EW transition is responsible for 50% of the final DM relic density (black line). Above (below) this line the relic density is dominantly of asymmetric (symmetric) origin. Similarly, the dotted upper (lower) black line gives the values of the parameters above (below) which the asymmetry is responsible for more (less) than 90% (10%) of the final relic density. For masses which give a freeze-out below $T_{\text{EW}}$, the $\kappa \cdot \Sigma_{H_2}(z_{\text{EW}})$ factor becomes exponentially large because in this case $\Sigma_{H_2}(z_{\text{EW}})$ is still exponentially larger than its value at freeze-out. Thus, the proportion of $\Sigma_{H_2}(z_{\text{EW}})$ which is due to $\Delta_{H_2}^{\lambda_3}$ is therefore exponentially suppressed. This explains why the black lines quickly go up for $m_{\text{DM}}$ below 4 – 5 TeV. Note nevertheless that this suppression, even if exponential, is far from instantaneous. As a result we find that, still, the asymmetry can dominate the relic density for a mass equal to 3.7 TeV which is substantially lower than the 4.7 TeV value which gives $T_{\text{fo}} = T_{\text{EW}}$. A comment which must be made at this point concerns the fact that we have considered the electroweak phase transition as if it was an instantaneous process, i.e. as a step function at the temperature $T_{\text{EW}} \sim 165$ GeV – from Ref. [13] (see also Ref. [21]) – which as said above is the temperature where the vacuum expectation value of the SM scalar field becomes sizable (i.e. where oscillations are about to start to reprocess the asymmetry). As the electroweek transition is a crossover, it is clearly an approximation which could be refined. A change of $T_{\text{EW}}$ by a given factor would shift all $m_{\rho_0}$ values in Fig. 2 by about the same factor.

As expected from the discussion above, Fig. 2 also shows that, for large value of $\kappa \cdot \Sigma_{H_2}(z_{\text{EW}})$, the observed relic density doesn't depend anymore on the value of $\Delta_{H_2}^{\lambda_3}$, provided this later quantity is above a certain value.

Note that the r.h.s. green curve of Fig 2 is obtained by imposing that all quartic couplings are perturbative, $\lambda_{3,4} < 4\pi$. This line shows that a dominant asymmetric component requires that $m_{\text{DM}} \lesssim 25$ TeV (whereas the same condition gives $m_{\text{DM}} \lesssim 30$ TeV for the standard freeze-out scenario and for a small value of the $\lambda_5$ coupling). Such a bound also implies an upper bound on the $\langle \sigma_{\text{eff}}/v \rangle$ cross section of about 2.5 pb, that is to say a value about 4 times larger than the $\sim 0.7$ pb value one needs at these energies along the standard freeze-out scenario. Imposing instead that $\lambda_{3,4} < \sqrt{4\pi}$ one gets $m_{\text{DM}} \lesssim 8$ TeV and $\langle \sigma_{\text{eff}}/v \rangle \lesssim 1.1$ pb (dashed green light).

The minimum value of the $\lambda_3^2 + \lambda_4^2$ coupling combination (which enters in $\langle \sigma_{\text{eff}}/v \rangle$) that this scenario requires is $\sim 2$, corresponding to $m_{\text{DM}} \sim 4$ TeV and a cross section of $\sim 0.5$ pb. This is smaller than the usual $\sim 0.7$ pb because the associated asymmetry $\Delta_{H_2}^{\lambda_3} \sim |\Delta_{H_2}|$ also participates to the depletion of the total density. No need to say that with such large values of these quartic couplings, Landau poles are to

\[7\] To get this 3.7 TeV value we simply applied Eq. (31) neglecting the fact that in this case the $\Sigma_{H_2}^{\phi_1}$ inverse scattering term must be taken into account in the Boltzmann equations (as in Eq. (25)). The incorporation of this term would slightly lower further this minimum value of $m_{\text{DM}}$.\]
be typically expected far below the Planck scale. Although the energy scale at which we get a Landau pole depends on the value of other couplings such as $\lambda_2$, if there is no cancellations between the contributions of various couplings in the beta functions, a value of $\lambda_{3,4} \sim 1.5$ gives a Landau pole at $\sim 10^{5}-10^6$ GeV. This means that new physics is to be expected in this case below this value. The scale of $B-L$ asymmetry production has not to be necessarily below this scale. All what matters for the value of $\Omega_{DM}/\Omega_B$ is the value of the $B-L$ asymmetry at $T \sim m_{H^0}$.

In Fig. 3 as a function of the same two input parameters $m_{H^0}$ and $(\sigma_{eff}v)$, we show the value of $\lambda_5$ which leads to the $\Delta_{H^0}^\lambda$ value needed in Fig. 2. The corresponding value of the $m_{A_0}^2 - m_{H_0}^2$ mass splitting is also given on Fig. 3. This figure shows that the scenario works for $\lambda_5 \in \{10^{-3},10^{-2}\}$, which corresponds to approximately $m_{A_0} - m_{H_0} \in \{10^{-5},10^{-3}\}$ GeV, i.e. a mass splitting much smaller than the DM mass. The dashed red area is excluded by direct detection experiment, taking in Eq. 4 a mass splitting equal to 100 keV. Taking instead 180 keV one get the dashed red line.

The direct detection constraints in Fig. 3 exclude a part of the parameter region displayed in Fig. 2. This is shown in Fig. 4. In most of the parameter space allowed in this figure, both the “k” and “K” are active. It is not possible to get the observed value of $\Omega_{DM}/\Omega_B$ by invoking only the $\kappa$ suppression, but still it can be responsible for most of the suppression needed. It is possible to have suppression only from the $\lambda_5$ interaction but only within a relatively narrow region. Fig. 4 also shows that, including the direct detection mass splitting constraint, the minimum value of $m_{H^0}$ that allows a dominant asymmetric production of the DM relic density is not anymore 3.7 TeV but 4 TeV.

**IV. ADDITIONAL DIRECT DETECTION CONSTRAINTS**

Beyond the direct detection bound of Eq. (4), which is required to avoid a too fast Z mediated interaction between $H^0$ and a nucleon $N$, such an interaction can also occur through tree-level SM scalar exchange and loop-level gauge boson exchange. The cross section induced by these 2 contributions is given by \[ \sigma_{IDM}^L = f_N^2 \frac{m_N^4}{m_h^4} \left[ \frac{\lambda_{H^0}}{\pi m_{H^0}} + \frac{9\pi\alpha_0}{64 m_W^2} \right] \left[ 1 + \frac{m^2_h}{m_W^2} \right]^2 \].

The electroweak part is subleading if $\lambda_{H^0}$ is of order unity or higher. This must be confronted to the current best constraint on such a cross section, from the LUX experiment [23], which for $m_{H^0} > 1$ TeV is approximately given by \[ \sigma_{LUX}^L \lesssim 1.2 \times 10^{-11} \left( \frac{m_{H^0}}{1 \text{ GeV}} \right) \text{ pb}. \] Taking the nucleon form factor $f_N \approx 0.3$, $m_N$ as the proton mass, $m_h \sim 125$ GeV, and $\alpha_0 = \sqrt{2}G_F m_W^2 / \pi$, this gives the bound \[ \lambda_{H^0} \lesssim 0.5 \left( \frac{m_{H^0}}{1 \text{ TeV}} \right)^{3/2}. \]

This bound on $\lambda_{H^0} \sim (\lambda_3 + \lambda_4)/2$ doesn’t put any one-to-one constraint on $(\sigma_{eff}v)$, see Eq. (47) in the Appendix. However, interestingly, if there is no cancellation between $\lambda_3$ and $\lambda_4$ in $\lambda_{H^0}$, so for instance if both couplings have same sign or if $|\lambda_3| \gg |\lambda_4|$ or $|\lambda_4| \gg |\lambda_3|$, the value of the cross section we need in Fig. 2 is a value of $\lambda_{H_0}$ which is of the order of the bound of Eq. (41). For instance for $|\lambda_3| \gg |\lambda_4|$ or $|\lambda_4| \gg |\lambda_3|$ and $m_{DM} = 5$ TeV, in order that the asymmetry dominates the DM density, the range of cross section values we need in Fig. 2 gives $1.4 \lesssim \lambda_{H_0} \lesssim 1.6$, to be compared with $\lambda_{H_0} \lesssim 4.6$ from Eq. (41). For $m_{DM} = 10$ TeV, we get instead in the same way $3.2 \lesssim \lambda_{H_0} \lesssim 4.6$, to be compared to the direct interaction constraint.
detection constraint which in this case doesn’t give any relevant bound, as it gives a non-perturbative value $\lambda_{H_0} \lesssim 15.8$.

The scenario is therefore expected to be testable by the next generation of direct detection experiments. Even if a cancellation occurs so that $\lambda_{H_0}$ is small, still the electroweak part turns out to be large enough to allow possibilities of direct detection signals in the future, for $m_{DM} \sim$ few TeV. At 10 TeV, it is 10 times weaker than the expected Xenon 1T sensitivity [9].

Finally note also that, since in this model the DM is ultimately not asymmetric (even if it has been created asymmetrically), the model avoids the usual strong constraints which hold on an asymmetric scalar component, related to the absence in this case of annihilation following DM accumulation in stars, and associated Bose-Einstein condensation effects [24, 25]. The inert DM component does annihilate in stars through $H^0H^0 \rightarrow SMSM$ processes, and thus avoid these constraints.

V. SUMMARY

In summary, we have shown that if DM is made of the lightest neutral component of an inert scalar doublet, its relic density could be dominantly due to an asymmetric production. This is based on the remarkable facts that: a) unless the $\lambda_5$ interaction is tiny (a possibility basically excluded by direct detection constraints), this interaction will automatically creates a DM asymmetry as soon as a $B-L$ asymmetry is produced, b) the same interaction leads to the neutral component mass splitting which is necessary to avoid a too large Z-exchange contribution to the direct detection cross section, c) the inert DM model has also other renormalizable scalar interactions $\propto \lambda_{3,4}$ which can suppress easily the symmetric component, and d) this DM asymmetry, which is left intact at least until a temperature equal to the mass of the scalar doublet, is not necessarily erased too strongly afterwards. There are two types of suppressions which could occur, that we have studied in details: i) from the very same $\lambda_5$ interaction, the “$k$-factor” suppression, and ii) from the combined effect of DM oscillation and DM symmetric annihilation, the “$\kappa$-factor” suppression. This leads to a scenario which chronologically occurs as represented in Fig. 1. We showed that in the few-TeV range, there is an all region of parameter space where the DM asymmetry survives enough to dominate the final DM relic density. Fig. 4 summarizes well our results by showing, as a function of the input parameters of the model, $m_{DM}$ and the total ($\sigma_{eff}v$) annihilation cross section, what are the effect of both types of suppressions. In most of the allowed parameter space they are both active, and both effects must be taken into account. The scenario requires that the freeze-out temperature of the symmetric component occurs before or slightly after the electroweak transition.

As most asymmetric DM scenarios, the framework we consider does not explain why the baryon and DM abundances are so similar. Our scenario trades this abundance coincidence for a coincidence between the mass of the proton, the mass of the dark matter particle, the value of the $\lambda_5$ equilibrating coupling, and the value of the annihilation cross section. Even if both abundances have same origin, these 4 parameters must “cooperate” to lead to a DM abundance so close to the baryon one. For instance, the scenario requires that the $\lambda_5$ interaction lies within $\sim 10^{-5}$ to $\sim 10^{-3}$. A larger value of the $\lambda_5$ coupling would give a much smaller DM asymmetric component (in which case only the symmetric contribution could account for the observed relic density). Also, to avoid a $\kappa$ suppression, and have a sufficiently suppressed symmetric component, the value of the cross section must lie within a rather narrow range. For larger values of the cross section, even if the abundance is of asymmetric origin, one gets back a result which scales as the inverse of the cross section. Requiring perturbative couplings, this leads to an abundance which cannot differ by more than $\sim 2$ orders of magnitude from the value one would obtain without $\kappa$ suppression, but still this means that this effect can change the abundance ratio by such an amount.

Rather than providing a real explanation for the abundance coincidence, this scenario shows that, by adding nothing but a DM scalar doublet to the SM, the origin of the DM relic density could be of asymmetric origin and due to the generation of a $B-L$ asymmetry at high temperature. This framework constitutes, to our knowledge and in various ways, the most minimal scenario of asymmetric production of the DM relic density that has been proposed. One remark one could do nevertheless about the simplicity of this scenario, is that it requires that at least one scalar coupling has a relatively large value. This basically implies new physics at an energy scale far below the Planck scale to avoid that this quartic coupling develops a Landau pole at a higher energy.

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Note added: few days ago, the possibility of equilibration of a $H_1$ asymmetry with a DM asymmetry has also been considered in Ref. [26], in the framework of models with a $SU(2)_L$ DM multiplet. Both approaches are the same, save for the fact that different minimality criteria have been considered. If one adopts, as in Ref. [25], the minimality criteria that the annihilation cross section is dominated by the known SM gauge interactions, the scalar scenario we consider above doesn’t work (since for a scalar doublet with mass above $\sim 540$ GeV this leads to a too large symmetric component [9, 11]). Ref. [26] also assumes, as another minimality criteria, that there are no annihilations taking place after EWSB. As said above, for the specific scalar doublet case, we find that this is possible (i.e. to have no sizable $\kappa$ suppression effect) for a small part of the otherwise allowed parameter space, see Fig. 4. We thank the authors of Ref. [26] for discussions.
Appendix

Rates and cross sections

In Eq. (8), the reaction density of the $\lambda_5$ scatterings for the $\eta^+$ (and similarly for $\eta^0$) is given by

$$\gamma_{\lambda_5} = \gamma^\phi_{\eta^+} + \gamma^0_{\eta^+},$$

(42)

where

$$\gamma^\phi_{\eta^+} = \frac{m_{H^2}}{64\pi z} \int_0^\infty dx \sqrt{x} K_1(z\sqrt{x}) \hat{\sigma}(ab \to cd).$$

(43)

with $\hat{\sigma}(ab \to cd)$ the reduced cross section. These are given by

$$\hat{\sigma}(\phi^+\phi^+ \to \eta^+\eta^+) = 2\hat{\sigma}(\phi^0\phi^+ \to \eta^0\eta^+) = \frac{\lambda^2}{4\pi} \sqrt{1 - \frac{4}{x}}.$$

(44)

In the non-relativistic limit, the corresponding rate is given by

$$\Gamma_{\lambda_5} \equiv \frac{\gamma_{\lambda_5}}{\eta^+} = n_{\eta^+} \langle \sigma_{\lambda_5} v \rangle,$$

(45)

where

$$\langle \sigma_{\lambda_5} v \rangle = \frac{3\lambda^2}{32 \pi m^2_{H^2}}.$$

(46)

In Eq. (16) and (30), the effective cross section of the $H_2H_2 \to SM SM$ annihilations is given by [9]

$$\langle \sigma_{c_{fj}} v \rangle = \sum_{i,j} \langle \sigma_{ij} v \rangle \frac{y^e_{ij} y^e_{ij}}{y^e_{H_2} y^e_{H_2} \sqrt{\sum_{ij} y^e_{ij} y^e_{ij}}} \frac{3}{8} (\lambda_3^2 + \lambda_3^2 + \lambda_3^2).$$

(47)

with $g$ the weak coupling constant. We neglected the $\lambda_5$ contribution, and the corrections due to the contributions proportional to $\langle v^2 \rangle$.

Analytical resolution of the Boltzmann equations

The Boltzmann equations given in Eqs. (25)-(27) do not in general have a simple analytical solution. However, in the case of very fast oscillations, like it is the case here, a good approximation consists in symmetrizing the equations for $\Delta_{\eta^0}$ and $\Xi_{\eta^0}$, i.e. replacing Eqs. (26)-(27) by

$$\frac{d\Delta_{\eta^0}}{dz} = \frac{\Gamma_{\text{osc}}}{zH} \Delta_{\eta^0} - \frac{1}{2} \langle \sigma_{\eta^0} v \rangle \Delta_{\eta^0} \Xi_{\eta^0},$$

(48)

$$\frac{d\Xi_{\eta^0}}{dz} = \frac{\Gamma_{\text{osc}}}{zH} \Xi_{\eta^0} - \frac{1}{2} \langle \sigma_{\eta^0} v \rangle \Delta_{\eta^0} \Xi_{\eta^0}.$$

(49)

In this approximation, the solutions for $\Delta_{\eta^0}$ and $\Xi_{\eta^0}$ are of the form

$$\Delta_{\eta^0}(z) = f(z) \cos[g(z)], \quad \Xi_{\eta^0}(z) = i f(z) \sin[g(z)].$$

(50)

Furthermore, since we are interested in oscillations happening after the freeze-out, we can neglect $\Sigma_{\eta^0}^{\eta^0}$ in Eq. (25). With these approximations, integrating from $z_{EW}$ to $z$ with the initial conditions $\Delta_{\eta^0}(z_{EW}) = \Sigma_{\eta^0}(z_{EW})$ and $\Xi_{\eta^0}(z_{EW}) = 0$, the analytical solutions of the Boltzmann equations Eqs. (25)-(29) are given by Eq. (50) and

$$\Sigma_{\eta^0}(z) = \sqrt{\Delta_{\eta^0}(z) - \Xi_{\eta^0}(z)^2} = f(z),$$

(51)

with

$$f(z) = \frac{\Sigma_{\eta^0}(z_{EW})}{1 + \frac{1}{2} \langle \sigma_{\eta^0} v \rangle (z_{EW}) \left( \frac{z}{z_{EW}} - 1 \right) \Sigma_{\eta^0}(z_{EW})},$$

(52)

$$g(z) = \frac{\Gamma_{\text{osc}}}{H(z)} \left( \frac{z^2}{z_{EW}} - 1 \right).$$

(53)

The abundance $\Sigma_{\eta^0}$ decreases therefore monotonically until it reaches an asymptotical value given by

$$\Sigma_{\eta^0}(z \gg z_{EW}) = \frac{\Sigma_{\eta^0}(z_{EW})}{1 + \frac{1}{2} \langle \sigma_{\eta^0} v \rangle (z_{EW}) \left( \frac{z}{z_{EW}} - 1 \right) \Sigma_{\eta^0}(z_{EW})}.$$

(54)

Let's note that despite appearances, the denominator doesn't depend on $z$, since $z^2/(z_{EW}) = 12\sqrt{3} M_P T_{EW}/5\pi^2$.

[1] P. A. R. Ade et al. [Planck Collaboration], arXiv:1502.01589 [astro-ph.CO].
[2] H. Davoudiasl and R. N. Mohapatra, New J. Phys. 14 (2012) 095011 [arXiv:1203.1247 [hep-ph]].
[3] K. Petraki and R. R. Volkas, Int. J. Mod. Phys. A 28 (2013) 1330028 [arXiv:1305.4939 [hep-ph]].
[4] K. M. Zurek, Phys. Rept. 537 (2014) 91 [arXiv:1308.0338 [hep-ph]].
[5] S. M. Boucenna and S. Morisi, Front. Phys. 1 (2014) 33 [arXiv:1310.1904 [hep-ph]].
[6] R. Barbieri, L. J. Hall and V. S. Rychkov, Phys. Rev. D 74 (2006) 015007 [hep-ph/0603188].
[7] L. Lopez Honorez, E. Nezri, J. F. Oliver and M. H. G. Tytgat, JCAP 0702 (2007) 028 [hep-ph/0612275].
[8] E. Ma, Phys. Rev. D 73 (2006) 077301 [hep-ph/0601225].
[9] T. Hambye, F.-S. Ling, L. Lopez Honorez and J. Rocher, JHEP 0907 (2009) 090 [Erratum-ibid. 1005 (2010) 066] arXiv:0903.4040 [hep-ph].
10. N. Nagata and S. Shirai, arXiv:1411.0752 [hep-ph].
11. M. Cirelli, N. Fornengo and A. Strumia, Nucl. Phys. B 753 (2006) 178 [hep-ph/0512090].
12. N. Fonseca, R. Z. Funchal, A. Lessa and L. Lopez-Honorez, arXiv:1501.05957 [hep-ph].
13. M. D’Onofrio, K. Rummukainen and A. Tranberg, Phys. Rev. Lett. 113 (2014) 14, 141602 [arXiv:1404.3565 [hep-ph]].
14. J. A. Harvey and M. S. Turner, Phys. Rev. D 42 (1990) 3344.
15. C. Arina and N. Sahu, Nucl. Phys. B 854 (2012) 666 [arXiv:1108.3967 [hep-ph]].
16. G. Servant and S. Tulin, Phys. Rev. Lett. 111 (2013) 15, 151601 [arXiv:1304.3464 [hep-ph]].
17. S. Davidson, R. Gonzalez Felipe, H. Serdio and J. P. Silva, JHEP 1311 (2013) 100 [arXiv:1307.6218 [hep-ph]].
18. E. Nardi, F. Sannino and A. Strumia, JCAP 0901 (2009) 043 [arXiv:0811.4153 [hep-ph]].
19. M. Cirelli, P. Panci, G. Servant and G. Zaharijas, JCAP 1203 (2012) 015 [arXiv:1110.3809 [hep-ph]].
20. S. Tulin, H. B. Yu and K. M. Zurek, JCAP 1205 (2012) 013 [arXiv:1202.0283 [hep-ph]].
21. Y. Burnier, M. Laine and M. Shaposhnikov, JCAP 0602 (2006) 007 [hep-ph/0511246].
22. M. Klasen, C. E. Yaguna and J. D. Ruiz-Alvarez, Phys. Rev. D 87 (2013) 075025 [arXiv:1302.1657 [hep-ph]].
23. D. S. Akerib et al. [LUX Collaboration], Phys. Rev. Lett. 112 (2014) 9, 091303 [arXiv:1310.8214 [astro-ph.CO]].
24. S. D. McDermott, H. B. Yu and K. M. Zurek, Phys. Rev. D 85 (2012) 023519 [arXiv:1103.5472 [hep-ph]].
25. C. Kouvaris and P. Tinyakov, Phys. Rev. Lett. 107 (2011) 091301 [arXiv:1104.0382 [astro-ph.CO]].
26. S. M. Boucenna, M. B. Krauss and E. Nardi, arXiv:1503.01119 [hep-ph].