The neutral Higgs boson masses of MSSM at one-loop level in explicit CP violation scenario

S.W. Ham\textsuperscript{(1)}, S.K. Oh\textsuperscript{(1,2)}, E.J. Yoo\textsuperscript{(2)}, C.M. Kim\textsuperscript{(2)}, D. Son\textsuperscript{(1)}

\textsuperscript{(1)} Center for High Energy Physics, Kyungpook National University
Daegu 702-701, Korea

\textsuperscript{(2)} Department of Physics, Konkuk University, Seoul 143-701, Korea

Abstract

The neutral Higgs sector of the minimal supersymmetric standard model (MSSM) in explicit CP violation scenario is investigated at the one-loop level. Within the context of the effective potential formalism, the masses of the neutral Higgs bosons are calculated at the one-loop level by taking into account the contributions of the following loops of ordinary particles and superpartners: top quark, the scalar top quarks, bottom quark, the scalar bottom quarks, tau lepton, the scalar tau leptons, $W$ boson, the charged Higgs boson, the charginos, $Z$ boson, the scalar and pseudoscalar Higgs bosons, and the neutralinos. Our calculation is an improvement in the sense that both the terms which are quartic in the electroweak coupling constants into account, and the pseudoscalar Higgs loop contribution are explicitly included.
1. Introduction

For new physics beyond the standard model (SM) [1], the enlargement of the Higgs sector is considered as one of indispensable ingredients, especially for supersymmetric models. Among various supersymmetric extensions of the SM, the minimal supersymmetric standard model (MSSM) is the simplest one which has just two Higgs doublets in its Higgs sector [2]. In principle, in the models with multiple Higgs doublets, the violation of CP symmetry may be accomplished either spontaneously or explicitly by the mixing between the scalar and pseudoscalar Higgs parts [3, 4, 5]. In practice, however, it is known that at the tree level the Higgs potential of the MSSM conserves CP symmetry, because the complex phase can always be eliminated by rotating the Higgs fields. Thus, neither explicit nor spontaneous CP violation can happen in the MSSM at the tree level. Even at the one-loop level the scenario of spontaneous CP violation is excluded, because the radiatively corrected Higgs potential of the MSSM leads to a very light Higgs boson which is unacceptable by the CERN $e^+e^-$ LEP2 data.

The remaining possibility at the one-loop level for the MSSM is then the explicit CP violation scenario. A number of investigations have been devoted to examine the explicit CP violation in the MSSM at one-loop level [6]. In those investigations it is observed that the tree-level situation is considerably modified when radiative contributions are included. In particular, the one-loop MSSM Higgs potential with explicit CP violation is found to allow the lightest neutral Higgs boson to possess a mass above the experimental lower bound from the LEP2 data. Therefore, the general consensus is that the MSSM at the one-loop level can accommodate explicit CP violation.

Recently, Ibrahim and Nath [7] have investigated explicit CP violation scenario, paying their attention to the phenomenological implications of the non-trivial CP phase on the chargino sector. They also have computed the scalar-pseudoscalar mixings arising from the neutralino sector, and compared with those from the chargino sector [8]. Their calculations are quite exhaustive since almost all the relevant loops are included: top quark, the scalar top quark, bottom quark, the scalar bottom quark, tau lepton, the scalar tau lepton, $W$ boson, the charged Higgs boson, the charginos, $Z$ boson, the neutral Higgs bosons, and the neutralinos. They have reported that in the MSSM with explicit CP violation the neutralino exchange corrections to the mixings of the CP-even sector and the CP-odd sector are comparable to the chargino exchange corrections [8].

We would also investigate, in more detail, the Higgs sector of the MSSM with explicit CP violation at the one-loop level. In our investigation, we take explicitly the terms which are quartic in the electroweak coupling constants into account. We regard it consistent to consider the quartic terms in the one-loop effective potential contributed by the scalar top quark, the scalar bottom quark, the scalar tau lepton, the chargino, and the neutralinos, because the Higgs self contributions are essentially induced by them.

Moreover, in our investigation, the pseudoscalar Higgs loop contribution is included, when the one-loop contributions to the neutral Higgs boson masses are evaluated, as well as all the loops of the relevant particles and superpartners. The inclusion of the pseudoscalar Higgs contribution is not only reasonable but also necessary, because there is a non-trivial correlation between the ordinary particles and the corresponding superpartners in the one-loop effective potential. In the neutral sector of the MSSM, there are four neutralinos as superpartners, and as ordinary particles there are $Z$ boson and three neutral Higgs bosons, one of them being the pseudoscalar Higgs boson. Thus, the radiative contribution of the pseudoscalar Higgs loop should simultaneously be included, whenever the loops of $Z$ boson and the two scalar Higgs bosons are
taken into account, corresponding to the loops of four neutralinos. But for the pseudoscalar Higgs contribution, the mass matrix for the neutral Higgs bosons at the one-loop level might be inconsistent and would yield incomplete masses for them and mixing angles among them. In order to obtain reliable results for the Higgs productions and their decays, it is quite necessary to include the pseudoscalar Higgs contribution.

In this paper, we evaluate the masses of the neutral Higgs bosons in the MSSM with explicit CP violation at the one-loop level. We take into account the loops of the pseudoscalar Higgs boson as well as all the loops that have been considered in the investigations of Ref. [8], namely, the loops of top quark, the scalar top quarks, bottom quark, the scalar bottom quarks, tau lepton, the scalar tau leptons, $W$ boson, the charged Higgs boson, the charginos, $Z$ boson, the scalar Higgs bosons, and the neutralinos. We examine the effect of the contributions of the quartic terms and the pseudoscalar Higgs loops. We find that the mass matrix of the neutral Higgs bosons in the MSSM at the one-loop level is evidently affected by the inclusion of the quartic terms and the pseudoscalar Higgs contribution.

2. The MSSM Higgs sector

A starting point to evaluate the neutral Higgs boson masses in the MSSM with explicit CP violation may be the tree-level Higgs potential, which is given in terms of two Higgs doublets $H_i$ ($i = 1, 2$) by

$$V^0 = \frac{g_1^2}{8} (H_1^\dagger \sigma H_1 + H_2^\dagger \sigma H_2)^2 + \frac{g_2^2}{8} (|H_2|^2 - |H_1|^2)^2 + m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m_3^2 (H_1^T \epsilon H_2 + \text{H.c.}) ,$$

where $\epsilon$ is an antisymmetric $2 \times 2$ matrix with $\epsilon_{12} = 1$, $\sigma$ denotes the three Pauli matrices, $g_1$ and $g_2$ are the U(1) and SU(2) gauge coupling constants, respectively, and $m_i^2$ ($i = 1, 2, 3$) are the soft SUSY breaking masses. Two soft SUSY breaking masses $m_i^2$ ($i = 1, 2$) may be assumed to be real, without loss of generality. We assume $m_3^2 = |m_3^2| e^{i\phi_3}$ to be complex.

Transforming to a unitary gauge, one may express two Higgs doublets in the mass eigenstates as

$$H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 + h_1 + i \sin \beta h_3 \\ \sin \beta C^+ \end{pmatrix},
H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos \beta C^+ + v_2 + h_2 + i \cos \beta h_3 \end{pmatrix} e^{i\phi_0} ,$$

where $h_1$, $h_2$, $h_3$ are three neutral Higgs fields, $C^+$ is the charged Higgs field, and $\phi_0$ is the relative phase between the two Higgs doublets. In the presence of the explicit CP violation, $h_1$, $h_2$, $h_3$ are mixed states of the CP parity. With respect to these three neutral Higgs fields, we have the corresponding minimum conditions as

$$m_1^2 = m_3^2 \tan \beta \cos(\phi_3 + \phi_0) - \frac{m_2^2}{2} \cos 2\beta ,
\quad m_2^2 = m_3^2 \cot \beta \cos(\phi_3 + \phi_0) + \frac{m_2^2}{2} \cos 2\beta ,
0 = m_3^2 \sin(\phi_3 + \phi_0) ,$$

respectively.
At the tree level, the phases $\phi_3$ and $\phi_0$ can be set to zero. Thus, it is well observed that there is no CP phase in the above tree-level Higgs potential. In order to accommodate the explicit CP violation scenario, the coefficients in the soft SUSY breaking terms are assumed to be complex in general. To be specific, we assume that the following quantities may be complex: The Higgs mixing parameters with mass dimension $\mu$, the U(1) gaugino mass $M_1$, the SU(2) gaugino mass $M_2$, and the trilinear soft SUSY breaking masses $A_t$, $A_b$, and $A_\tau$. The complex phases in these quantities are responsible for the CP violation. On the other hand, we take the soft SUSY breaking masses $m_Q$, $m_T$, $m_B$, $m_L$, and $m_E$ be real.

Now, as the electroweak symmetry is broken spontaneously, the neutral Higgs fields develop non-trivial vacuum expectation values (VEVs). The ratio of the two VEVs are defined as $\sqrt{\frac{v_1}{v_2}}$. In terms of $v_1$ and $v_2$, the fermion masses are given as $m_{t_r} = (h_t v_2)^2/2$ for top quark, $m_{b_r} = (h_b v_1)^2/2$ for bottom quark, $m_{t_r} = (h_t v_1)^2/2$ and the gauge boson masses as $m_{Z} = (g_2 v)^2/4$ and $m_{W} = (g_1^2 + g_2^2)v^2/4$ with $v^2 = v_1^2 + v_2^2$.

We calculate the radiative corrections to the tree-level Higgs sector. We employ the effective potential method [9] to estimate radiative corrections to the tree-level Higgs sector. The Higgs potential at the one-loop level is

$$V = V^0 + V^1,$$

where $V^1$ represents the radiative corrections due to various loops, and is calculated by the effective potential method. It may conveniently be decomposed as

$$V^1 = V^t + V^b + V^\tau + V^h + V^{\chi^0}, \quad \text{(4)}$$

where

$$V^t = \sum_{i=1}^{2} \frac{3M_i^4}{32\pi^2} \left( \log \frac{M_i^2}{\Lambda^2} - \frac{3}{2} \right) - \frac{3M_4^4}{16\pi^2} \left( \log \frac{M_4^2}{\Lambda^2} - \frac{3}{2} \right),$$

for the one-loop contributions of top quark and the scalar top quarks,

$$V^b = \sum_{i=1}^{2} \frac{3M_i^4}{32\pi^2} \left( \log \frac{M_i^2}{\Lambda^2} - \frac{3}{2} \right) - \frac{3M_4^4}{16\pi^2} \left( \log \frac{M_4^2}{\Lambda^2} - \frac{3}{2} \right),$$

for those of bottom quark and the scalar bottom quarks,

$$V^\tau = \sum_{i=1}^{2} \frac{M_i^2}{32\pi^2} \left( \log \frac{M_i^2}{\Lambda^2} - \frac{3}{2} \right) - \frac{M_4^2}{16\pi^2} \left( \log \frac{M_4^2}{\Lambda^2} - \frac{3}{2} \right),$$

for those of tau lepton and the scalar tau leptons,

$$V^h = \frac{3M_W^4}{32\pi^2} \left( \log \frac{M_W^2}{\Lambda^2} - \frac{3}{2} \right) + \frac{3M_4^4}{32\pi^2} \left( \log \frac{M_4^2}{\Lambda^2} - \frac{3}{2} \right) - \sum_{i=1}^{2} \frac{M_{\chi_i}^4}{16\pi^2} \left( \log \frac{M_{\chi_i}^2}{\Lambda^2} - \frac{3}{2} \right),$$

for those of $W$ boson, the charged Higgs boson and the charginos,

$$V^{\chi^0} = -\sum_{i=1}^{4} \frac{M_{\chi_i}^4}{32\pi^2} \left( \log \frac{M_{\chi_i}^2}{\Lambda^2} - \frac{3}{2} \right),$$

for those of $Z$ boson and both scalar and pseudoscalar neutral Higgs bosons, and finally,
for those of the neutralinos.

In the above expressions, $\Lambda$ is the renormalization scale in the modified minimal subtraction ($\overline{MS}$) scheme and $\mathcal{M}$ stands for the field-dependent mass matrix of the relevant ordinary particles and their superpartners. Note that the coefficient of $V^\tau$ differs from those of $V^t$ and $V^b$ as the color factor is absent in it. One may group the above components into two parts: $V^t$, $V^b$, $V^\tau$, and $V^\tilde{\chi}$ into the charged part of $V^1$, whereas $V^h$ and $V^\tilde{\chi}^0$ into the neutral part. Note that the loops of all the relevant particles and superpartners are included: top quark, bottom quark, tau lepton, $W$ boson, the charged Higgs boson, $Z$ boson, the neutral scalar and pseudoscalar Higgs bosons, and then the scalar top quarks, the scalar bottom quarks, the scalar tau leptons, the charginos, and the neutralinos.

In order to calculate the radiative corrections, one has to know the tree-level masses of particles and superpartners. The scalar top quarks, the scalar bottom quarks, the scalar tau leptons, and the charginos obtain their tree-level masses as follows:

$$m^2_{\tilde{\chi}_{1,2}} = m^2_t + \frac{1}{2}(m^2_Q + m^2_T) + \frac{m^2_Z}{4} \cos 2\beta \pm \left\{ \frac{1}{2}(m^2_Q - m^2_T) \right\}^{1/2},$$

$$m^2_{\tilde{h}_{1,2}} = m^2_b + \frac{1}{2}(m^2_Q + m^2_B) - \frac{m^2_Z}{4} \cos 2\beta \pm \left\{ \frac{1}{2}(m^2_Q - m^2_B) \right\}^{1/2},$$

$$m^2_{\tilde{\tau}_{1,2}} = m^2_{\tau} + \frac{1}{2}(m^2_L + m^2_E) - \frac{m^2_Z}{4} \cos 2\beta \pm \left\{ \frac{1}{2}(m^2_L - m^2_E) \right\}^{1/2},$$

$$m^2_{\tilde{\chi}_1} = \frac{1}{2}(M^2_2 + \mu^2) + m^2_W \pm \left\{ \frac{1}{2}(M^2_2 - \mu^2) - m^2_W \cos 2\beta \right\}^{1/2},$$

$$+ 2m^2_W \cos^2 \beta (M^2_2 + \mu^2 \tan^2 \beta + 2M_2 \mu \tan \beta \cos \phi_c) \right\}^{1/2},$$

where $D$-terms are included. Notice that four non-trivial CP phases appear in the above expressions. They are defined as follows: $\phi_t$ is the relative phase of $A_t$ and $\mu$, $\phi_b$ is that of $A_b$ and $\mu$, $\phi_\tau$ is that of $A_\tau$ and $\mu$, and $\phi_c$ is that of $M_2$ and $\mu$.

The tree-level neutralino mass matrix is given as

$$\mathcal{M}_{\tilde{\chi}^0} = \begin{pmatrix}
M_1 e^{i\phi_1} & 0 & -\frac{g_1}{\sqrt{2}} H^0_1 & \frac{g_2}{\sqrt{2}} H^0_2 \\
0 & M_2 & \frac{g_2}{\sqrt{2}} H^0_1 & -\frac{g_2}{\sqrt{2}} H^0_2 \\
-\frac{g_1}{\sqrt{2}} H^0_1 & \frac{g_2}{\sqrt{2}} H^0_1 & 0 & -\mu e^{i\phi_2} \\
\frac{g_1}{\sqrt{2}} H^0_2 & -\frac{g_2}{\sqrt{2}} H^0_2 & -\mu e^{i\phi_2} & 0
\end{pmatrix},$$

where additional complex phases appear: $\phi_1$ is the relative phase between $M_1$ and $M_2$ and $\phi_2$ is the relative phase between $M_2$ and $\mu$. However, $\phi_2$ is identical to $\phi_c$. Thus, the neutralino
mass matrix introduces only one more phase. Consequently, we have in general five non-trivial CP phases: \( \phi_t, \phi_b, \phi_\tau, \phi_c = \phi_2 \) and \( \phi_1 \). The above neutralino mass matrix is complex and symmetric, but not Hermitian. By diagonalizing the Hermitian matrix \( M_{\tilde{\chi}_i}^\dagger M_{\tilde{\chi}_j} \) through a similarity transformation, the tree-level neutralino masses are calculated. They are denoted as \( m_{\tilde{\chi}_i}^2 (i = 1, 2, 3, 4) \), and sorted such that \( m_{\tilde{\chi}_1} < m_{\tilde{\chi}_2} \) for \( i < j \).

For the Higgs sector, the tree-level mass of the pseudoscalar Higgs boson is obtained as

\[
m_{A^0}^2 = \frac{2m_3^2 \cos(\phi_3 + \phi_0)}{\sin 2\beta} .
\]

(7)

Here we retain \( \phi_3 \) and \( \phi_0 \) because they become non-zero at the one loop level even though they are zero at the tree level. The tree-level masses of the charged Higgs boson and the remaining neutral Higgs bosons are given as

\[
m_{c^+}^2 = m_{W}^2 + m_A^2 , \quad m_{\tilde{\chi}_i^0, H^0}^2 = \frac{1}{2} \left[ m_Z^2 + m_{A^0}^2 \mp \sqrt{\left( m_Z^2 + m_{A^0}^2 \right)^2 - 4m_Z^2 m_{A^0}^2 \cos^2 2\beta} \right] ,
\]

(8)

from the tree-level Higgs potential of the MSSM. Within the context of perturbation theory, we eliminate both \( m_2^2 \) and \( m_3^2 \) in the one-loop functions of the Higgs bosons by using the tree-level minimum equations.

At the one loop level, there is one nontrivial minimum condition with respect to \( h_3 \) field. The CP-odd tadpole minimum equation is obtained as

\[
0 = m_3^2 \sin(\phi_3 + \phi_0) + \frac{3m_2^2 \mu A_t \sin \phi_t}{16\pi^2 v^2 \sin^2 \beta} f_1(m_{t_1}^2, m_{t_2}^2) + \frac{3m_2^2 \mu A_b \sin \phi_b}{16\pi^2 v^2 \cos^2 \beta} f_1(m_{b_1}^2, m_{b_2}^2) + \frac{m_2^2 \mu A_\tau \sin \phi_\tau}{16\pi^2 v^2 \cos^2 \beta} f_1(m_{\tau_1}^2, m_{\tau_2}^2) - \frac{m_2^2 \mu M_2 \sin \phi_c}{4\pi^2 v^2} f_1(m_{\tilde{\chi}_1}^2, m_{\tilde{\chi}_2}^2) + \sum_{k=1}^4 \frac{m_{\tilde{\chi}_k}^2}{4\pi^2 v^2} \left\{ \log \left( \frac{m_{\tilde{\chi}_k}^2}{\Lambda^2} \right) - 1 \right\} \frac{E_3}{\prod_{a \neq k}^4 (m_{\tilde{\chi}_a}^2 - m_{\tilde{\chi}_k}^2)} ,
\]

(9)

where

\[
E_3 = -(m_{\tilde{\chi}_0}^2 - M_1^2)(m_{\tilde{\chi}_0}^2 - m_{\tilde{\chi}_k}^2)M_2\mu m_W^2 \sin \phi_2
- (m_{\tilde{\chi}_k}^2 - M_2^2)(m_{\tilde{\chi}_0}^2 - m_{\tilde{\chi}_k}^2)M_1\mu (m_Z^2 - m_W^2) \sin(\phi_1 + \phi_2) .
\]

(10)

The dimensionless function \( f_1 \) is defined as

\[
f_1(m_x^2, m_y^2) = \frac{2}{(m_x^2 - m_y^2)} \left\{ m_x^2 \log \frac{m_x^2}{\Lambda^2} - m_y^2 \log \frac{m_y^2}{\Lambda^2} \right\} - 2 .
\]

The six terms on the right-hand side of Eq. (9) come respectively from the tree-level Higgs potential, and the one-loop contributions of the scalar top quark, the scalar bottom quark, the scalar tau lepton, the chargino, and the neutralinos.

By differentiating the Higgs potential at the one-loop level with respect to the three neutral Higgs fields, a \( 3 \times 3 \) symmetric mass matrix \( M_{ij} \) is obtained in the \( (h_1, h_2, h_3) \)-basis. It may be decomposed as

\[
M_{ij} = M_{ij}^0 + M_{ij}^1
\]
where \( M^0_{ij} \) is obtained from \( V^0 \), and \( M^1_{ij} \) from \( V^1 \). In particular, \( M^1_{ij} \) may further be decomposed as

\[
M^1_{ij} = M^t_{ij} + M^b_{ij} + M^\tau_{ij} + M^X_{ij} + M^{h0}_{ij} + M^{\chi0}_{ij}
\]

where, as the superscripts suggest, \( M^t_{ij} \) represents the radiative contributions from \( V^t \). Likewise, \( M^b_{ij}, M^\tau_{ij}, M^X_{ij}, M^h_{ij} \), and \( M^{h0}_{ij}, M^{\chi0}_{ij} \) represent respectively the radiative contributions from \( V^b, V^\tau, V^X, V^h \), and \( V^{h0} \).

The matrix elements of \( M_{ij} \) in the \((h_1, h_2, h_3)\)-basis are easily calculated as

\[
\begin{align*}
M_{11} &= m_Z^2 \cos^2 \beta + m_A^2 \sin^2 \beta + M_{11}^1, \\
M_{22} &= m_Z^2 \sin^2 \beta + m_A^2 \cos^2 \beta + M_{22}^1, \\
M_{33} &= \tilde{m}_A = M_{33}^1, \\
M_{12} &= -(m_Z^2 + m_A^2) \cos \beta \sin \beta + M_{12}^1, \\
M_{13} &= M_{13}^1, \\
M_{23} &= M_{23}^1.
\end{align*}
\]

(11)

where the mass parameter \( \tilde{m}_A \) is defined as

\[
\tilde{m}_A = \frac{2}{\sin 2\beta} \left[ m_3^2 \cos(\phi_3 + \phi_0) + \frac{3m_t^2 \mu A_1 \cos \phi_t}{16\pi^2 v^2 \sin^2 \beta} f_1(m_{t^1}^2, m_{t^2}^2) \\
+ \frac{3m_b^2 \mu A_b \cos \phi_b}{16\pi^2 v^2 \cos^2 \beta} f_1(m_{b^1}^2, m_{b^2}^2) + \frac{m_Z^2 \mu A_\tau \cos \phi_\tau}{16\pi^2 v^2 \cos^2 \beta} f_1(m_{\tau^1}^2, m_{\tau^2}^2) \\
+ \frac{m_W^2 \mu M_2 \cos \phi_c}{4\pi^2 v^2} f_1(m_{\chi^1}^2, m_{\chi^2}^2) \\
+ \frac{4}{4\pi^2 v^2} \left( \log \left( \frac{m_{\chi^0}^2 L^2}{\Lambda^4} \right) - 1 \right) \prod_{a \neq k} \left( \frac{m_{a^0}^2}{m_{a^0}^2 - m_{\chi^0}^2} \right) \right],
\]

(12)

with

\[
E_{33} = (m_{\chi^0}^2 - M_t^1)(m_{\chi^0}^2 - m_b^2)M_{22}m_{W^2} \cos \phi_2 + (m_{\chi^0}^2 - M_{\tau^2}^1)(m_{\chi^0}^2 - m_{W^2}^2)M_{11} \mu (m_Z^2 - m_W^2) \cos(\phi_1 + \phi_2).
\]

(13)

In Eq. (11), the matrix elements \( M_{3i} \) \((i = 1, 2)\) represent the mixing between the scalar and the pseudoscalar components. Thus, \( M_{3i} = M_{3i}^1 \) \((i = 1, 2)\) indicate that the mixing occurs only at the one loop level. There is no mixing at the tree level.

For \( M_{ij}^1 \), we first calculate \( M_{ij}^h \) from \( V^h \). The result is given as follows:

\[
\begin{align*}
M_{11}^h &= -\frac{v^2 \sin^2 \beta \Delta_{h^1}^2}{32\pi^2} \left( f_2(m_{h^1}^2, m_{h^2}^2) - \frac{m_Z^2}{32\pi^2 v^2} (m_{A^0}^2 \sin^2 \beta - 4m_Z^2 \cos^2 \beta) f_1(m_{h^1}^2, m_{h^2}^2) \right) \\
&+ \frac{m_b^2 \cos^2 \beta}{32\pi^2 v^2} \log \left( \frac{m_{h^2}^2}{m_{h^1}^2} \right) + \frac{m_T^2 \cos^2 \beta \Delta_{h^1} \log(m_{h^1}^2/m_{h^2}^2)}{16\pi^2 (m_{h^2}^2 - m_{h^1}^2)} \\
&+ \frac{m_W^2 \cos^2 \beta \cos^2 2\beta}{8\pi^2 v^2} \log \left( \frac{m_{W^2}^2}{m_{A^0}^2} \right) + \frac{m_Z^2 \cos^2 \beta}{8\pi^2 v^2} \log \left( \frac{m_Z^2}{m_{W^2}^2} \right),
\end{align*}
\]

\[
\begin{align*}
M_{22}^h &= -\frac{v^2 \sin^2 \beta \Delta_{h^2}^2}{32\pi^2} \left( f_2(m_{h^2}^2, m_{h^1}^2) - \frac{m_Z^2}{32\pi^2 v^2} (m_{A^0}^2 - 4m_Z^2) f_1(m_{h^2}^2, m_{h^1}^2) \right) \\
&- \frac{m_b^2 \cos^2 \beta}{32\pi^2 v^2} \log \left( \frac{m_{h^1}^2}{m_{h^2}^2} \right) + \frac{m_T^2 \cos^2 \beta \Delta_{h^2} \log(m_{h^2}^2/m_{h^1}^2)}{16\pi^2 (m_{h^1}^2 - m_{h^2}^2)} \\
&+ \frac{m_W^2 \cos^2 \beta \cos^2 2\beta}{8\pi^2 v^2} \log \left( \frac{m_{W^2}^2}{m_{A^0}^2} \right) + \frac{m_Z^2 \cos^2 \beta}{8\pi^2 v^2} \log \left( \frac{m_Z^2}{m_{W^2}^2} \right),
\end{align*}
\]
In the above expressions, \( m \) the radiative corrections due to superpartners, via the CP phases in soft SUSY breaking terms. Eq. (14). Therefore, the explicit CP violation scenario in the Higgs sector of the MSSM may be

\[
\begin{align*}
M_{33}^h &= -\frac{m_Z^2}{32\pi^2 v^2} \left\{ m_h^2 \left( \frac{m_h^2}{\Lambda^2} - 1 \right) + m_H^2 \left( \frac{m_H^2}{\Lambda^2} - 1 \right) \right\} \\
M_{12}^h &= -\frac{v^2 \sin 2\beta \Delta_{h_1} \Delta_{h_2}}{64\pi^2} f_2(m_h^2, m_H^2) + m_Z^2 \sin 2\beta \frac{\sin 2\beta}{32\pi^2 v^2} (m_{A_0}^2 - 2m_Z^2) f_1(m_h^2, m_H^2) \\
M_{13}^h &= 0, \\
M_{23}^h &= 0,
\end{align*}
\]

(14)

where

\[
\begin{align*}
f_2(m_x^2, m_y^2) &= \frac{m_x^2 + m_y^2}{m_x^2 - m_y^2} \log \frac{m_y^2}{m_x^2} - 2, \\
\Delta_{h_1} &= \frac{2m_Z^2}{v^2} (m_Z^2 - m_{A_0}^2) \cos^2 \beta + \frac{4m_Z^2 m_{A_0}^2}{v^2} \sin^2 \beta, \\
\Delta_{h_2} &= \frac{2m_Z^2}{v^2} (m_Z^2 - m_{A_0}^2) \sin^2 \beta + \frac{4m_Z^2 m_{A_0}^2}{v^2} \cos^2 \beta.
\end{align*}
\]

In the above expressions, \( m_{h^0} \), and \( m_{H^0} \), and \( m_{A^0} \) are the tree-level masses of the scalar and the pseudoscalar Higgs bosons. Since there is no CP phase in \( V^h \), we have \( M_{13}^h = M_{23}^h = 0 \) in Eq. (14). Therefore, the explicit CP violation scenario in the Higgs sector of the MSSM may be regarded as the radiative CP violation, in the sense that the CP violation occurs only through the radiative corrections due to superpartners, via the CP phases in soft SUSY breaking terms.

Next, we calculate \( M_{ij}^\chi \), the contributions of radiative corrections due to the four neutralinos. We obtain

\[
M_{ij}^\chi = -\sum_{k=1}^4 \frac{m_{\chi_k^0}^2}{16\pi^2} \left( \log \frac{m_{\chi_k^0}}{\Lambda^2} - 1 \right) \frac{\partial^2 m_{\chi_k^0}^2}{\partial h_i \partial h_j} - \sum_{k=1}^4 \frac{1}{16\pi^2} \log \frac{m_{\chi_k^0}}{\Lambda^2} \left( \frac{\partial m_{\chi_k^0}^2}{\partial h_i} \right) \left( \frac{\partial m_{\chi_k^0}^2}{\partial h_j} \right),
\]

(16)

where the first-order derivative \( \frac{\partial m_{\chi_k^0}^2}{\partial h_i} \) is given explicitly by

\[
\frac{\partial m_{\chi_k^0}^2}{\partial h_i} = -\frac{A_i m_{\chi_k^0}^6 + B_i m_{\chi_k^0}^4 + C_i m_{\chi_k^0}^2 + D_i}{\prod_{a \neq k} (m_{\chi_a^0}^2 - m_{\chi_k^0}^2)},
\]

8
and the second-order derivative $\partial^2 m_{\chi_k^0}^2 / \partial h_i \partial h_j$ by

$$
\frac{\partial^2 m_{\chi_k^0}^2}{\partial h_i \partial h_j} = - \frac{A_{ij} m_{\chi_k^0}^6 + B_{ij} m_{\chi_k^0}^4 + C_{ij} m_{\chi_k^0}^2 + D_{ij}}{4 \prod_{a \neq k} (m_{\chi_k^0}^2 - m_{\chi_a^0}^2)} + \sum_{a \neq k} \frac{1}{(m_{\chi_k^0}^2 - m_{\chi_a^0}^2)} \left( \frac{\partial m_{\chi_k^0}^2}{\partial h_i} \frac{\partial m_{\chi_a^0}^2}{\partial h_j} + \frac{\partial m_{\chi_a^0}^2}{\partial h_i} \frac{\partial m_{\chi_k^0}^2}{\partial h_j} \right) \right). \tag{17}
$$

The formulas for coefficients $A_i, B_i, C_i$, and $D_i$ ($i = 1, 2, 3$) are given in Appendix A, and those for $A_{ij}, B_{ij}, C_{ij}$, and $D_{ij}$, $(i, j = 1, 2, 3)$ are given in Appendix B. Actually, $A_{ij} = 0$ for all $i, j$ because four elements of the neutralino mass matrix are zero.

We also carry out calculations for the remaining components of the one-loop mass matrices, namely, $M_{ij}^t, M_{ij}^b, M_{ij}^\tau$, and $M_{ij}^3$, and the results are given in Appendices C, D, E, and F, respectively.

In the scenario of explicit CP violation, among the elements of the mass matrix for the neutral Higgs bosons, $M_{i3} = M_{i3}^t = M_{i3}^b = M_{i3}^\tau + M_{i3}^h + M_{i3}^\tau + M_{i3}^0$ ($i = 1, 2$) are responsible for the scalar-pseudoscalar mixing and eventually account for the CP violation. It is worthwhile to notice that our calculations show the general dependence of $M_{i3}$ on the various CP phases: $M_{i3}^t (i = 1, 2)$ are proportional to $\sin \phi_t$, $M_{i3}^h (i = 1, 2)$ to $\sin \phi_b$, $M_{i3}^\tau (i = 1, 2)$ to $\sin \phi_\tau$, $M_{i3}^\tau (i = 1, 2)$ to $\sin \phi_\tau$. Meanwhile, $M_{i3}^b (i = 1, 2)$ are zero, as noticed above. As for $M_{i3}^0 (i = 1, 2)$, they have complicated expressions, but one can easily see that every term is proportional to $\sin(\phi_1 + \phi_2)$ or $\sin \phi_3$ since $B_{3i}, C_{3i}, D_{3i}$ are proportional to them whereas $B_{3i}, C_{3i}, D_{3i}$ ($i = 1, 2, 3$) are zero. Consequently, if any of the CP phases be not zero, they might give rise to the CP violation via the scalar-pseudoscalar mixing at the one-loop level.

3. Numerical analysis

We investigate the effects of CP phases in the one-loop corrected MSSM Higgs sector. As we have emphasized, if there exist some CP phases in the Higgs potential at the one-loop level, they would produce the scalar-pseudoscalar mixing through non-zero $M_{13}$ and $M_{23}$ and thus generate explicit CP violation in the neutral Higgs sector. In the MSSM Higgs sector, we show that there are in general five independent CP phases. They are contained in the masses of the scalar top quarks, the scalar bottom quarks, the scalar tau leptons, the charginos, and the neutralinos.

The mass matrix for the neutral Higgs bosons is derived by differentiating twice the Higgs potential with respect to the three Higgs fields. It is expressed as a $3 \times 3$ matrix. By calculating its three eigenvalues, one can obtain the three masses of the neutral Higgs bosons, analytically in principle. However, we would not write down the full expressions for the neutral Higgs boson masses, since they are very complicated functions of many parameters coming from the soft SUSY breaking in the MSSM. Rather, we would like to analyze numerically the effect of the CP phases on the mass matrix of the neutral Higgs boson by focusing our attention to $M_{i3} (i = 1, 2)$.

For the numerical calculations, we need concrete numbers for the parameter values. We set $m_t = 175.0$ GeV, $m_b = 4.0$ GeV, $m_\tau = 1.7$ GeV, $m_W = 80.4$ GeV, $m_Z = 91.1$ GeV, and $\sin^2 \theta_W = 0.231$. Then the remaining relevant free parameters are $\Lambda, \tan \beta, \mu, m_Q, m_T, m_B, m_L, m_E, A_t, A_b, A_\tau, M_1, M_2, \phi_t, \phi_b, \phi_\tau, \phi_\phi(\phi_2)$, and $\phi_1$. Since for a moderate values of the tan $\beta$, the contribution for the scalar bottom sector as well as the scalar tau lepton one is relatively small, we set $m_Q = m_L, m_T = m_B = m_E, A_t = A_b = A_\tau$. At the electroweak scale one can take
the relation of $M_1 = 5 \tan^2 \theta_W M_2 / 3$ between U(1) and SU(2) gaugino masses. Thus, we have $\Lambda$, $\tan \beta$, $m_Q$, $m_T$, $A_t$, $M_2$, and five CP phases as free parameters. Note that we will choose different values for the soft SUSY breaking masses of SU(2) doublet and singlet scalar fermions such that $m_Q = m_L$ is different from $m_T = m_S = m_E$ in order to consider large mass splitting between the left- and right- handed scalar fermions.

Now, as a typical illustration, we set $\phi_t = \phi_b = \phi_c = \phi_v(\phi_2) = \phi_1 = \pi/3$. We fix the renormalization scale $\Lambda$ in the effective potential at 300 GeV. We set the remaining free parameters as $m_A = 300$ GeV, $m_Q = 800$ GeV, $m_T = M_2 = 400$ GeV, and $A_t = 200$ GeV. With these input values, we calculate the elements of the neutral Higgs boson mass matrix for some values of $\tan \beta$ and $\mu$.

Our results are shown in Tables 1-8. These Tables are divided into two categories: in Tables 1-4 the contributions from the quartic terms of $O(g_i^4)$ ($i = 1, 2$) and the pseudoscalar Higgs contribution are excluded whereas they are included in Tables 5-8, in order to exhibit the effect of these contributions. Tables 1, 2, 5 and 6 displays numerical results for small $\tan \beta = 5$ while Tables 3, 4, 7 and 8 for relatively large $\tan \beta = 30$. For Tables 1, 3, 5 and 7 we set $\mu = -400$ GeV while for Tables 2, 4, 6 and 8 we set $\mu = 400$ GeV. The entries of Tables are the elements of the mass matrix of the neutral Higgs bosons in the $(h_1, h_2, h_3)$-basis, where all the contributions are fully listed. The unit is (GeV)$^2$. Note that, as the CP phases induce the CP violation through the scalar-pseudoscalar mixing, the columns for $M_{13}$ and $M_{23}$ are filled with generally non-zero numbers. The number in the last row in each column is the sum of the numbers in the preceding rows in the same column.

In all of the Tables 1-8, one can easily notice that, among the contributions to $M_{13}$ and $M_{23}$, the tree-level ones and the Higgs ones vanish: $M_{13}^0 = M_{23}^0 = 0$ and $M_{13}^h = M_{23}^h = 0$. This reflects the fact that there is no CP phase in the tree-level Higgs sector of the MSSM. All the other elements are not zero, accounting for the scalar-pseudoscalar mixing, because the CP phases are present there. Actually, one can trace the effects of the CP phases on various contributions to $M_{13}$ and $M_{23}$. If $\phi_t = 0$, one would have $M_{13}^t = M_{23}^t = 0$. Further, one would have $M_{13}^b = M_{23}^b = 0$ for $\phi_b = 0$. Likewise, one would have $M_{13}^c = M_{23}^c = 0$ for $\phi_c = 0$, $M_{13}^\tau = M_{23}^\tau = 0$ for $\phi_\tau(\phi_2) = 0$, and $M_{13}^{\chi_0} = M_{23}^{\chi_0} = 0$ for $\phi_2 = \phi_1 = 0$. Among the non-zero contributions to $M_{13}$ and $M_{23}$, we find that $M_{13}^{\chi_0}$ of Table with $\mu = -400$ GeV contribute about $25 \sim 30\%$ to the scalar-pseudoscalar mixing. Thus, Tables 1, 3, 5, and 7 indicate that the radiative corrections loops contribute roughly $25\%$ to the CP mixing between $h_1$ and $h_3$ components.

Now, let us compare Table 1 with Table 5, Table 2 with Table 6, and so on to examine the effects of the contributions from the quartic terms of $O(g_i^4)$ ($i = 1, 2$) and the contribution by the pseudoscalar Higgs boson $A^0$. It is quite evident that there are non-negligible differences between them. The effects can most clearly be seen in $M_{33}^h$. In Tables 1-4, we see that $M_{33}^h$ is identically zero. This is expected since neither $A^0$ nor quartic term contributions are included in Tables 1-4. On the other hand, Tables 5-8 show $M_{33}^h$ is definitely far from zero, indicating that both of them contribute in Tables 5-8. For the particular parameter values, $M_{33}^h$ is calculated as $55.9, 58.0, 30.2,$ and $39.2$ (GeV)$^2$ in Tables 5, 6, 7, and 8, respectively.

The effects of the contributions from the quartic terms and $A^0$ are also seen in other entries, too. By comparing Table 1 with Table 5, for example, one can see that those contributions alter the values of $M_{11}^h$, $M_{22}^h$, and $M_{12}^h$. The value of $M_{33}^h$, as well as those of $M_{11}^h$, $M_{22}^h$, and $M_{12}^h$, is essential to evaluate the neutral Higgs boson masses from their mass matrix and the mixing among them, at the one-loop level. Consequently, these contributions should be included,
as they play roles definitely in the matrix elements $M^h_{ij}$ at the one-loop level. Without these contributions, the neutral Higgs boson masses at the one-loop level would be changed, if the changes be small. We further note that $M^h_{33}$ is in particular also affected by the inclusion of these contributions.

Now, to be concrete, we evaluate the neutral Higgs boson masses. From Table 5, we obtain $m_{h_i} (i = 1, 2, 3) = 109.8, 300.1, 302.6$ GeV and we obtain $108.6, 300.1, 302.8$ GeV from Table 6. Since Tables 5 and 6 differ in the value of $\mu$, the dependence on $\mu$ is relatively insignificant, for $\tan \beta = 5$. For relatively large $\tan \beta = 30$, the masses of the three neutral Higgs bosons are obtained as $m_{h_i} (i = 1, 2, 3) = 115.2, 300.0, 300.1$ GeV for Table 7 ($\mu = -400$ GeV) and 115.0, 300.0, 300.2 GeV for Table 8 ($\mu = 400$ GeV). We see that the mass of the lightest neutral Higgs boson becomes slightly larger for large $\tan \beta$. All of Tables 5-8 produce the neutral Higgs boson masses without contradicting LEP2 data.

We have some comments on the radiative corrections due to the loops of $Z$ boson, the neutral Higgs bosons, and the neutralinos. One can observe in Table 5-8 that the sum of $M_{12}^h$ and $M_{12}^0$ are comparatively large. This implies that the contributions of $Z$ boson, the neutral Higgs bosons (both scalar and pseudoscalar), and neutralinos play important role in the radiative corrections to the mixing between $h_1$ and $h_2$ components at the one-loop level. Especially, this mixing occurs most largely in Table 6 as $(M_{12}^h + M_{12}^0) = -78.1 \, (\text{GeV})^2$.

The contributions of the neutralino loops depend crucially on the CP phase $\phi_1$. In other words, the CP phase $\phi_1$ occurs only in the expressions for the neutralino contributions. Now, in order to examine in more detail the dependence of the contributions of the neutralino loops on $\phi_1$, we plot $M_{13}^h, M_{13}^0, M_{23}^h, M_{23}^0, M_{13}^\tilde{\chi}, M_{23}^\tilde{\chi}$ at the one-loop level. Fig. 1 shows that the contributions of the neutralino loops are smaller than those of the scalar fermions for some values of $\phi_1$. For both $M_{13}^h$ and $M_{23}^h$, the contributions of the neutralino loops are larger than those of the scalar fermions for some values of $\phi_1$. For both $M_{13}^h$ and $M_{23}^h$, Fig. 1 shows that the contributions of the neutralino loops are smaller than the chargino ones for the whole range of $0 < \phi_1 < \pi$.

4. Conclusions

In the MSSM, the tree-level neutral Higgs sector may be divided into the scalar part and pseudoscalar part, and there is no mixing between them. Any phase that can cause the scalar-pseudoscalar mixing, hence the CP violation, can be absorbed away at the tree level. At the one-loop level, where explicit CP violation is viable by introducing several CP phases in the effective Higgs potential, the scalar-pseudoscalar mixing occurs in general. The scalar-pseudoscalar mixing is manifested by the non-vanishing $M_{13}$ and $M_{23}$ matrix elements of the neutral Higgs bosons. Evidently, these off-diagonal elements affect the masses of the neutral Higgs bosons when the mass matrix is diagonalized.

The mass matrix of the neutral Higgs boson in the MSSM is evaluated at the one-loop level with explicit CP violation. In explicit CP violation scenario, five non-trivial CP phases are introduced, from the soft SUSY breaking terms of the MSSM Lagrangian, in the masses of the scalar top quarks, the scalar bottom quarks, the scalar tau leptons, the charginos, and the neutralinos. These phases penetrate into the mass matrix of the neutral Higgs bosons. All the contributions of relevant loops are taken into account: The loops of the pseudoscalar Higgs boson as well as all the loops of top quark, the scalar top quarks, bottom quark, the scalar bottom
quarks, tau lepton, the scalar tau leptons, \( W \) boson, the charged Higgs boson, the charginos, \( Z \) boson, the scalar Higgs bosons, and the neutralinos.

Especially, we consider the contributions of the terms quartic in the electroweak coupling and the pseudoscalar Higgs loop contribution in the one-loop effective potential to the neutral Higgs boson masses at the MSSM with five explicit CP phases. It is found that the contributions of quartic terms and \( A^0 \) are definitely non-zero to the \((3,3)\)-element of the mass matrix of the neutral Higgs bosons. The CP mixing between \( h_1 \) and \( h_3 \) components can be induced about 25\% of the considered total contribution by the neutralino contributions. The contributions of \( Z \) boson, the neutral Higgs bosons (both scalar and pseudoscalar), and the neutralinos contribute largely to the mixing between \( h_1 \) and \( h_2 \) components above other ones.

Acknowledgments

This work was supported by Korea Research Foundation Grant (2001-050-D00005).
Appendix A

The coefficients that appear in the first derivatives of the neutralino masses with respect to the neutral Higgs fields in the radiatively corrected mass matrix for the neutral Higgs bosons are given as follows:

\[
A_1 = - \frac{4m_W^2 \cos \beta}{v},
\]
\[
B_1 = 4M_1^2 m_W^2 \cos \beta + 4M_1^2 (m_Z^2 - m_W^2) \cos \beta + 4M_2^2 (m_Z^2 + \mu^2) \cos \beta - 4M_2 m_W \mu \sin \beta \cos \phi_2 + \frac{4M_1 \mu (m_W^2 - m_Z^2) \sin \beta \cos (\phi_1 + \phi_2)}{v},
\]
\[
C_1 = - \frac{4m_Z^2 \mu^2 \sin \beta \sin 2\beta}{v} - \frac{4M_1^2 m_W^2 (m_Z^2 + \mu^2) \cos \beta}{v} + 4M_1 \mu (m_Z^2 - m_W^2) (m_Z^2 + \mu^2) \sin \beta \cos (\phi_1 + \phi_2)v + 4M_1 \mu (m_Z^2 - m_W^2) (m_Z^2 + \mu^2) \sin \beta \cos (\phi_1 + \phi_2),
\]
\[
D_1 = \frac{4M_1^2 m_W^4 \mu^2 \sin \beta \sin 2\beta}{v} + 4M_2^2 \mu^2 (m_Z^2 - m_W^2)^2 \sin \beta \sin 2\beta + 8M_1 M_2 m_W^2 (m_Z^2 - m_W^2) \cos \beta \cos \phi_1 v + 4M_2^2 m_W^2 \mu^3 \sin \beta \cos \phi_1 + \frac{4M_1 \mu (m_Z^2 - m_W^2) (m_Z^2 + \mu^2) \sin \beta \cos (\phi_1 + \phi_2)}{v},
\]

and

\[
A_2 = A_1 (\cos \beta \leftrightarrow \sin \beta),
\]
\[
B_2 = B_1 (\cos \beta \leftrightarrow \sin \beta),
\]
\[
C_2 = C_1 (\cos \beta \leftrightarrow \sin \beta),
\]
\[
D_2 = D_1 (\cos \beta \leftrightarrow \sin \beta).
\]  

and

\[
A_3 = 0,
\]
\[
B_3 = -4M_2 m_W \mu \sin \phi_2 + \frac{4M_1 \mu (m_W^2 - m_Z^2) \sin (\phi_1 + \phi_2)}{v},
\]
\[
C_3 = \frac{4M_2 m_W \mu (M_1^2 + \mu^2) \sin \phi_2}{v} + \frac{4M_1 \mu (m_W^2 - m_Z^2) (M_2^2 + \mu^2) \sin (\phi_1 + \phi_2)}{v},
\]
\[
D_3 = -4M_2^2 M_1 m_W^3 \mu^3 \sin \phi_2 \frac{v}{v} + \frac{4M_1 M_2^2 \mu^3 (m_W^2 - m_Z^2) \sin (\phi_1 + \phi_2)}{v}.
\]  

Appendix B

The coefficients that appear in the second derivatives of the neutralino masses with respect to the neutral Higgs fields in the radiatively corrected mass matrix for the neutral Higgs bosons are given as follows:

\[
A_{11} = 0,
\]
\[ B_{11} = \frac{8m_2^2 \cos^2 \beta}{v^2}, \]
\[ C_{11} = -\frac{8M_t^2m_W^2 \cos^2 \beta}{v^2} - \frac{8M_Z^2(m_2^2 - m_W^2)^2 \cos^2 \beta}{v^2} + \frac{16M_1M_2m_W^2(m_2^2 - m_W^2) \cos^2 \beta \cos \phi_1}{v^2}, \]
\[ D_{11} = 0, \] (21)

and
\[ A_{22} = A_{11}(\cos \beta \leftrightarrow \sin \beta) = 0, \]
\[ B_{22} = B_{11}(\cos \beta \leftrightarrow \sin \beta), \]
\[ C_{22} = C_{11}(\cos \beta \leftrightarrow \sin \beta), \]
\[ D_{22} = D_{11}(\cos \beta \leftrightarrow \sin \beta) = 0, \] (22)

and
\[ A_{12} = 0, \]
\[ B_{12} = \frac{4m_2^2 \sin 2\beta}{v^2}, \]
\[ C_{12} = -\frac{4M_t^2m_W^4 \sin 2\beta}{v^2} - \frac{8m_2^4 \mu^2 \sin 2\beta}{v^2} - \frac{4M_Z^2(m_2^2 - m_W^2)^2 \sin 2\beta}{v^2} + \frac{8M_1M_2m_W^2(m_2^2 - m_W^2) \sin 2\beta \cos \phi_1}{v^2}, \]
\[ D_{12} = \frac{8M_2^2m_W^2 \mu^2 \sin 2\beta}{v^2} + \frac{8M_2^2 \mu^2(m_2^2 - m_W^2)^2 \sin 2\beta}{v^2} + \frac{16M_1M_2m_W^2 \mu^2(m_2^2 - m_W^2) \sin 2\beta \cos \phi_1}{v^2}, \] (23)

and \( A_{13} = B_{13} = C_{13} = D_{13} = 0 \) for \( l = 1, 2, 3. \)

Appendix C

The elements for the mass matrix of the neutral Higgs bosons due to the radiative contributions of the top quark and scalar top quarks are

\[ M_{11}^i = -\frac{3}{4\pi^2 v^2} \left\{ \frac{m_i^2 \Delta_i}{\sin \beta} + \frac{\cos \beta \Delta_i}{2} \right\}^2 f_2(m_{t_1}^2, m_{t_2}^2) \left( \frac{m_i^2}{m_{t_2}^2 - m_{t_1}^2} \right)^2 + \frac{3m_2^2 \cos^2 \beta}{64\pi^2 v^2} \log \left( \frac{m_i^2 m_{t_2}^2}{\Lambda^4} \right) \]
\[ + \frac{3\cos^2 \beta}{16\pi^2 v^2} \left( \frac{4m_W^2}{3} - \frac{5m_Z^2}{6} \right)^2 f_1(m_{t_1}^2, m_{t_2}^2) \]
\[ + \frac{3m_2^2 \cos \beta}{8\pi^2 v^2} \left\{ \frac{m_i^2 \mu \Delta_i}{\sin \beta} + \frac{\cos \beta \Delta_i}{2} \right\} \frac{\log(m_{i_2}^2/m_{i_1}^2)}{(m_{i_2}^2 - m_{i_1}^2)}, \]

\[ M_{22}^i = -\frac{3}{4\pi^2 v^2} \left\{ \frac{m_i^2 \Delta_i}{\sin \beta} - \frac{\sin \beta \Delta_i}{2} \right\}^2 f_2(m_{t_1}^2, m_{t_2}^2) \left( \frac{m_i^2}{m_{t_2}^2 - m_{t_1}^2} \right)^2 - \frac{3m_2^4}{2\pi^2 v^2 \sin^2 \beta} \log \left( \frac{m_i^2}{\Lambda^2} \right) \]
\[ + \frac{3\sin^2 \beta}{16\pi^2 v^2} \left( \frac{4m_W^2}{3} - \frac{5m_Z^2}{6} \right)^2 f_1(m_{t_1}^2, m_{t_2}^2) \]
\[ + \frac{3\sin \beta}{8\pi^2 v^2} \left( \frac{4m_i^2}{\sin^2 \beta} - m_Z^2 \right) \left\{ \frac{m_i^2 \Delta_i}{\sin \beta} - \frac{\sin \beta \Delta_i}{2} \right\} \frac{\log(m_{i_2}^2/m_{i_1}^2)}{(m_{i_2}^2 - m_{i_1}^2)}. \]
of the bottom quark and scalar bottom quarks are

The elements for the mass matrix of the neutral Higgs bosons due to the radiative contributions

Appendix D

with

\[ \Delta_{\tilde{t}_1} = \mu \cot \beta - A_t \cos \phi_t, \]
\[ \Delta_{\tilde{t}_2} = A_t - \mu \cot \beta \cos \phi_t, \]
\[ \Delta_{\tilde{t}} = \left( \frac{4}{3} m_W^2 - \frac{5}{6} m_Z^2 \right) \left\{ m_Q^2 - m_t^2 + \left( \frac{4}{3} m_W^2 - \frac{5}{6} m_Z^2 \right) \cos 2\beta \right\}. \]
\[ M_{22}^b = \frac{3}{16\pi^2 v^2} \left( \frac{2m_b^2}{\cos \beta} - \frac{m_Z^2 \cos \beta}{2} \right)^2 \log \left( \frac{m_{b_1}^2 m_{b_2}^2}{\Lambda^4} \right), \]

\[ M_{33}^b = -\frac{3m_b^2 A_b^2 \sin^2 \phi_b}{4\pi^2 v^2 \cos^4 \beta} \left( \frac{m_{b_1}^2}{m_{b_1}^2 + \frac{m_{b_2}^2}{2}} \right) \left( \frac{m_{b_1}^2}{m_{b_1}^2 - \frac{m_{b_2}^2}{2}} \right), \]

\[ M_{12}^b = \left( \frac{m_b^2 A_b \Delta_{b_1}}{\cos \beta} \right) \left( \frac{m_b^2 A_b \Delta_{b_1}}{\cos \beta} - \frac{m_{b_2}^2 \Delta_{b_2}}{2} \right) \left( \frac{m_{b_2}^2 \Delta_{b_2}}{m_{b_1}^2 - m_{b_2}^2} \right)^2 \log \left( \frac{m_{b_2}^2}{m_{b_1}^2} \right) \frac{m_{b_2}^2}{m_{b_1}^2 - m_{b_2}^2}, \]

\[ M_{13}^b = -\frac{3m_b^2 A_b \sin \phi_b}{4\pi^2 v^2 \cos^2 \beta} \left( \frac{m_b^2 A_b \Delta_{b_1}}{\cos \beta} \right) \left( \frac{m_b^2 A_b \Delta_{b_1}}{\cos \beta} - \frac{m_{b_2}^2 \Delta_{b_2}}{2} \right) \left( \frac{m_{b_2}^2 \Delta_{b_2}}{m_{b_1}^2 - m_{b_2}^2} \right)^2 \log \left( \frac{m_{b_2}^2}{m_{b_1}^2} \right) \frac{m_{b_2}^2}{m_{b_1}^2 - m_{b_2}^2}, \]

\[ M_{23}^b = -\frac{3m_b^2 m_Z^2 \mu A_b \tan \beta \sin \phi_b}{16\pi^2 v^2 \cos \beta} \left( \frac{m_b^2 m_Z^2 \mu \Delta_{b_1}}{\cos \beta} \right) \left( \frac{m_b^2 m_Z^2 \mu \Delta_{b_1}}{\cos \beta} - \frac{m_{b_2}^2 \Delta_{b_2}}{2} \right) \left( \frac{m_{b_2}^2 \Delta_{b_2}}{m_{b_1}^2 - m_{b_2}^2} \right)^2 \log \left( \frac{m_{b_2}^2}{m_{b_1}^2} \right) \frac{m_{b_2}^2}{m_{b_1}^2 - m_{b_2}^2}, \]

with

\[ \Delta_{b_1} = A_b - \mu \tan \beta \cos \phi_b, \]

\[ \Delta_{b_2} = \mu \tan \beta - A_b \cos \phi_b, \]

\[ \Delta_b = \left( 1 - \frac{m_{b_1}^2}{3m_W^2} - \frac{m_{b_2}^2}{3m_W^2} \right) \left( m_{b_1}^2 - m_{b_2}^2 \right) \left( \frac{m_{b_1}^2}{6} - \frac{m_{b_2}^2}{3} \right) \cos 2\beta \].

Appendix E

The elements for the mass matrix of the neutral Higgs bosons due to the radiative contributions
of the tau lepton and scalar tau leptons are

\[
M_{11}^\tau = - \frac{1}{4\pi^2 v^2} \left\{ \frac{m_\tau^2 A_\tau \Delta \tau_1}{\cos \beta} - \frac{\cos \beta \Delta \tau_2}{2} \right\}^2 \left( \frac{f_2(m_{\tau_1}^2, m_{\tau_2}^2)}{(m_{\tau_2}^2 - m_{\tau_1}^2)^2} - \frac{m_\tau^4}{2\pi^2 v^2 \cos^2 \beta} \log \left( \frac{m_\tau^2}{\Lambda^2} \right) \right.
+ \frac{\cos^2 \beta}{16\pi^2 v^2} \left( \frac{3m_Z^2}{4} - m_W^2 \right)^2 f_1(m_{\tau_1}^2, m_{\tau_2}^2)
+ \frac{\cos \beta}{8\pi^2 v^2} \left( \frac{4m_\tau^2}{\cos \beta} - m_Z^2 \right) \left\{ \frac{m_\tau^2 A_\tau \Delta \tau_1}{\cos \beta} + \frac{\cos \beta \Delta \tau_2}{2} \right\} \log \left( \frac{m_{\tau_2}^2 / m_{\tau_1}^2}{(m_{\tau_2}^2 - m_{\tau_1}^2)} \right)
+ \frac{1}{16\pi^2 v^2} \left( \frac{2m_\tau^2}{\cos \beta} - \frac{m_Z^2}{2} \right)^2 \log \left( \frac{m_\tau^2 m_{\tau_2}^2}{\Lambda^4} \right),
\]

\[
M_{22}^\tau = - \frac{1}{4\pi^2 v^2} \left\{ \frac{m_\tau^2 \mu \Delta \tau_2}{\cos \beta} - \frac{\sin \beta \Delta \tau_2}{2} \right\}^2 \left( \frac{f_2(m_{\tau_1}^2, m_{\tau_2}^2)}{(m_{\tau_2}^2 - m_{\tau_1}^2)^2} + \frac{m_\tau^4 \sin^2 \beta}{64\pi^2 v^2} \log \left( \frac{m_\tau^2 m_{\tau_2}^2}{\Lambda^4} \right) \right.
+ \frac{\sin^2 \beta}{16\pi^2 v^2} \left( \frac{3m_Z^2}{4} - m_W^2 \right)^2 f_1(m_{\tau_1}^2, m_{\tau_2}^2)
+ \frac{m_\tau^2 \sin \beta}{8\pi^2 v^2} \left( \frac{4m_\tau^2}{\cos \beta} - m_Z^2 \right) \left\{ \frac{m_\tau^2 \mu \Delta \tau_2}{\cos \beta} - \frac{\sin \beta \Delta \tau_2}{2} \right\} \log \left( \frac{m_{\tau_2}^2 / m_{\tau_1}^2}{(m_{\tau_2}^2 - m_{\tau_1}^2)} \right)
+ \frac{1}{4\pi^2 v^2} \left( \frac{m_\tau^2 \mu A_\tau \Delta \tau_2}{\cos \beta} + \frac{\cos \beta \Delta \tau_2}{2} \right) \left\{ \frac{m_\tau^2 \mu A_\tau \Delta \tau_2}{\cos \beta} - \frac{\sin \beta \Delta \tau_2}{2} \right\} \log \left( \frac{m_{\tau_2}^2 / m_{\tau_1}^2}{(m_{\tau_2}^2 - m_{\tau_1}^2)} \right),
\]

\[
M_{33}^\tau = - \frac{m_\tau^2 \mu A_\tau^2 \sin^2 \phi_\tau f_2(m_{\tau_1}^2, m_{\tau_2}^2)}{4\pi^2 v^2 \cos^4 \beta \left( m_{\tau_2}^2 - m_{\tau_1}^2 \right)^2},
\]

\[
M_{12}^\tau = - \frac{1}{4\pi^2 v^2} \left\{ \frac{m_\tau^2 A_\tau \Delta \tau_2}{\cos \beta} + \frac{\cos \beta \Delta \tau_2}{2} \right\} \left\{ \frac{m_\tau^2 \mu A_\tau \Delta \tau_2}{\cos \beta} + \frac{\sin \beta \Delta \tau_2}{2} \right\} \left( \frac{f_2(m_{\tau_1}^2, m_{\tau_2}^2)}{(m_{\tau_2}^2 - m_{\tau_1}^2)^2} - \frac{m_\tau^4}{32\pi^2 v^2 \cos^2 \beta} \log \left( \frac{m_\tau^2}{\Lambda^2} \right) \right.
+ \frac{\cos \beta}{16\pi^2 v^2} \left( \frac{4m_\tau^2}{\cos \beta} - m_Z^2 \right) \left\{ \frac{m_\tau^2 \mu A_\tau \Delta \tau_2}{\cos \beta} + \frac{\sin \beta \Delta \tau_2}{2} \right\} \log \left( \frac{m_{\tau_2}^2 / m_{\tau_1}^2}{(m_{\tau_2}^2 - m_{\tau_1}^2)} \right)
+ \frac{1}{32\pi^2 v^2} \left( \frac{3m_Z^2}{4} - m_W^2 \right)^2 f_1(m_{\tau_1}^2, m_{\tau_2}^2)
+ \frac{m_\tau^2 \sin 2 \beta}{128\pi^2 v^2} \left( \frac{4m_\tau^2}{\cos \beta} - m_Z^2 \right) \log \left( \frac{m_{\tau_2}^2 m_{\tau_2}^2}{\Lambda^4} \right),
\]

\[
M_{13}^\tau = - \frac{m_\tau^2 \mu A_\tau \sin \phi_\tau}{4\pi^2 v^2 \cos^2 \beta} \left\{ \frac{m_\tau^2 A_\tau \Delta \tau_1}{\cos \beta} + \frac{\cos \beta \Delta \tau_2}{2} \right\} \left( \frac{f_2(m_{\tau_1}^2, m_{\tau_2}^2)}{(m_{\tau_2}^2 - m_{\tau_1}^2)^2} - \frac{m_\tau^4}{16\pi^2 v^2 \cos \beta} \log \left( \frac{m_\tau^2}{\Lambda^2} \right) \right.
+ \frac{m_\tau^2 \mu A_\tau \sin \phi_\tau}{16\pi^2 v^2 \cos \beta} \left( \frac{4m_\tau^2}{\cos \beta} - m_Z^2 \right) \log \left( \frac{m_{\tau_2}^2 / m_{\tau_1}^2}{(m_{\tau_2}^2 - m_{\tau_1}^2)} \right)
+ \frac{1}{16\pi^2 v^2 \cos \beta} \left( \frac{m_\tau^2 A_\tau \Delta \tau_1}{\cos \beta} - \frac{\cos \beta \Delta \tau_2}{2} \right) \left( \frac{f_2(m_{\tau_1}^2, m_{\tau_2}^2)}{(m_{\tau_2}^2 - m_{\tau_1}^2)^2} - \frac{m_\tau^4}{16\pi^2 v^2 \cos \beta} \log \left( \frac{m_\tau^2}{\Lambda^2} \right) \right)
+ \frac{m_\tau^2 m_\tau^2 \mu A_\tau \tan \beta \sin \phi_\tau}{16\pi^2 v^2 \cos \beta} \log \left( \frac{m_{\tau_2}^2 / m_{\tau_1}^2}{(m_{\tau_2}^2 - m_{\tau_1}^2)} \right),
\]

\[
M_{23}^\tau = - \frac{m_\tau^2 \mu A_\tau \sin \phi_\tau}{4\pi^2 v^2 \cos^2 \beta} \left\{ \frac{m_\tau^2 \mu A_\tau \Delta \tau_2}{\cos \beta} - \frac{\sin \beta \Delta \tau_2}{2} \right\} \left( \frac{f_2(m_{\tau_1}^2, m_{\tau_2}^2)}{(m_{\tau_2}^2 - m_{\tau_1}^2)^2} - \frac{m_\tau^4}{4\pi^2 v^2 \cos \beta} \log \left( \frac{m_\tau^2}{\Lambda^2} \right) \right.
+ \frac{m_\tau^2 \mu A_\tau \sin \phi_\tau}{16\pi^2 v^2 \cos \beta} \left( \frac{4m_\tau^2}{\cos \beta} - m_Z^2 \right) \log \left( \frac{m_{\tau_2}^2 / m_{\tau_1}^2}{(m_{\tau_2}^2 - m_{\tau_1}^2)} \right)
+ \frac{m_\tau^2 m_\tau^2 \mu A_\tau \tan \beta \sin \phi_\tau}{16\pi^2 v^2 \cos \beta} \log \left( \frac{m_{\tau_2}^2 / m_{\tau_1}^2}{(m_{\tau_2}^2 - m_{\tau_1}^2)} \right),
\]

with

\[
\Delta \tau_1 = A_\tau - \mu \tan \beta \cos \phi_\tau,
\]

\[
\Delta \tau_2 = \mu \tan \beta - A_\tau \cos \phi_\tau.
\]
\[ \Delta_\tilde{\tau} = \left( \frac{3}{4} m_Z^2 - m_W^2 \right) \left( m_L^2 - m_E^2 + \frac{3}{4} m_Z^2 - m_W^2 \right) \cos 2\beta \].

(29)

Appendix F

The elements for the mass matrix of the neutral Higgs bosons due to the radiative contributions of the $W$ boson, charged Higgs boson, and charginos are

\[
M_{11}^\tilde{\tau} = \frac{\cos^2 \beta}{8\pi^2 v^2} (4m_W^2 M_2 \Delta \tilde{\chi}_1 - \Delta \tilde{\chi}) \left( \frac{2f_2(m_{\tilde{\chi}_1}^2, m_{\tilde{\chi}_2}^2)}{(m_{\tilde{\chi}_2}^2 - m_{\tilde{\chi}_1}^2)^2} + \frac{m_W^2 \cos^2 \beta}{4\pi^2 v^2} \log \left( \frac{m_W^2 m_{\tilde{\chi}_1}^2}{m_{\tilde{\chi}_2}^2 m_{\tilde{\chi}_1}^2} \right) \right)
\]

\[- \frac{m_W^2 \cos^2 \beta (m_W^2 M_2 \Delta \tilde{\chi}_1 - \Delta \tilde{\chi}) \log \left( \frac{m_{\tilde{\chi}_2}^2}{m_{\tilde{\chi}_1}^2} \right) - \frac{m_W^2 \cos^2 \beta}{2\pi^2 v^2} f_1(m_{\tilde{\chi}_1}^2, m_{\tilde{\chi}_2}^2) \],

\[
M_{22}^\tilde{\tau} = \frac{\sin^2 \beta}{8\pi^2 v^2} \left( \frac{(4m_W^2 \mu \cot \beta \Delta \tilde{\chi}_2 + \Delta \tilde{\chi})^2 f_2(m_{\tilde{\chi}_1}^2, m_{\tilde{\chi}_2}^2)}{(m_{\tilde{\chi}_2}^2 - m_{\tilde{\chi}_1}^2)^2} + \frac{m_W^2 \sin^2 \beta}{4\pi^2 v^2} \log \left( \frac{m_W^2 m_{\tilde{\chi}_2}^2}{m_{\tilde{\chi}_1}^2 m_{\tilde{\chi}_2}^2} \right) \right)
\]

\[- \frac{m_W^2 \sin^2 \beta (4m_W^2 \mu \cot \beta \Delta \tilde{\chi}_2 + \Delta \tilde{\chi}) \log \left( \frac{m_{\tilde{\chi}_2}^2}{m_{\tilde{\chi}_1}^2} \right) - \frac{m_W^2 \sin^2 \beta}{2\pi^2 v^2} f_1(m_{\tilde{\chi}_1}^2, m_{\tilde{\chi}_2}^2) \],

\[
M_{33}^\tilde{\tau} = \frac{2(m_W^2 M_2 \sin \phi_c)^2 f_2(m_{\tilde{\chi}_1}^2, m_{\tilde{\chi}_2}^2)}{\pi^2 v^2} \left( \frac{m_{\tilde{\chi}_2}^2}{m_{\tilde{\chi}_1}^2} - m_{\tilde{\chi}_1}^2 \right)^2 ,
\]

\[
M_{12}^\tilde{\tau} = \frac{\sin 2\beta}{16\pi^2 v^2} \left( \frac{(4m_W^2 M_2 \Delta \tilde{\chi}_1 - \Delta \tilde{\chi})(4m_W^2 \mu \cot \beta \Delta \tilde{\chi}_2 + \Delta \tilde{\chi}) \left( \frac{f_2(m_{\tilde{\chi}_1}^2, m_{\tilde{\chi}_2}^2)}{(m_{\tilde{\chi}_2}^2 - m_{\tilde{\chi}_1}^2)^2} \right)}{(m_{\tilde{\chi}_2}^2 - m_{\tilde{\chi}_1}^2)^2} \right)
\]

\[- \frac{m_W^2 \cos \beta (m_2 \sin \beta \Delta \tilde{\chi}_1 + \mu \cos \beta \Delta \tilde{\chi}_2) \log \left( \frac{m_{\tilde{\chi}_2}^2}{m_{\tilde{\chi}_1}^2} \right) \right)
\]

\[- \frac{f_2(m_{\tilde{\chi}_1}^2, m_{\tilde{\chi}_2}^2)}{(m_{\tilde{\chi}_2}^2 - m_{\tilde{\chi}_1}^2)^2} \log \left( \frac{m_{\tilde{\chi}_2}^2}{m_{\tilde{\chi}_1}^2} \right) + \frac{m_W^2 \sin 2\beta}{4\pi^2 v^2} f_1(m_{\tilde{\chi}_1}^2, m_{\tilde{\chi}_2}^2) ,
\]

\[
M_{13}^\tilde{\tau} = \frac{m_W^2 M_2 \mu \cos \beta \sin \phi_c \left( \frac{(4m_W^2 M_2 \Delta \tilde{\chi}_1 - \Delta \tilde{\chi}) f_2(m_{\tilde{\chi}_1}^2, m_{\tilde{\chi}_2}^2)}{(m_{\tilde{\chi}_2}^2 - m_{\tilde{\chi}_1}^2)^2} \right)}{2\pi^2 v^2}
\]

\[- \frac{m_W^2 M_2 \mu \cos \beta \sin \phi_c \log \left( \frac{m_{\tilde{\chi}_2}^2}{m_{\tilde{\chi}_1}^2} \right) ,
\]

\[
M_{23}^\tilde{\tau} = \frac{m_W^2 M_2 \mu \sin \beta \sin \phi_c \left( \frac{(4m_W^2 \mu \cot \beta \Delta \tilde{\chi}_2 + \Delta \tilde{\chi}) f_2(m_{\tilde{\chi}_1}^2, m_{\tilde{\chi}_2}^2)}{(m_{\tilde{\chi}_2}^2 - m_{\tilde{\chi}_1}^2)^2} \right)}{2\pi^2 v^2}
\]

\[- \frac{m_W^2 M_2 \mu \sin \beta \sin \phi_c \log \left( \frac{m_{\tilde{\chi}_2}^2}{m_{\tilde{\chi}_1}^2} \right) .
\]

(30)

with

\[
\Delta_{\tilde{\chi}_1} = M_2 + \mu \tan \beta \cos \phi_c ,
\]

\[
\Delta_{\tilde{\chi}_2} = M_2 \cos \phi_c + \mu \tan \beta ,
\]

\[
\Delta_\tilde{\chi} = 2m_W^2 (M_2^2 - \mu^2 - 2m_W^2 \cos 2\beta) .
\]

(31)
Reference

[1] S. Glashow, Nucl. Phys. 22, 579 (1961); S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in Proc. 8th Nobel Symposium, 367, edited by N. Svartholm (Almquist and Wiksell, Stockholm, 1968).

[2] H.P. Nilles, Phys. Rep. 110, 1 (1984); J.F. Gunion and H.E. Haber, Nucl. Phys. B 272, 1 (1986); J.F. Gunion, H.E. Haber, G.L. Kane, and S. Dawson, The Higgs Hunters’ Guide (Addison-Wesley Pub. Co., Redwood City, CA, USA, 1990).

[3] S. Weinberg, Phys. Rev. Lett. 37, 657 (1976); M. Masip and A. Rasin, Phys. Rev. D 52, 3768 (1995); M. Masip and A. Rasin, Nucl. Phys. B 460, 449 (1996); M. Masip and A. Rasin, Phys. Rev. D 58, 035007 (1998).

[4] A. Pomarol, Phys. Lett. B 287, 331 (1992); N. Maekawa, Phys. Lett. B 282, 387 (1992); N. Haba, Phys. Lett. B 398, 305 (1997); O. Lebedev, Eur. Phys. J. C 4, 363 (1998).

[5] A. Pilaftsis, Phys. Lett. B 435, 88 (1998); A. Pilaftsis, Phys. Rev. D 58, 096010 (1998). D.A. Demir, Phys. Rev. D 60, 055006 (1999).

[6] M. Carena, J. Ellis, A. Pilaftsis, and C.E.M. Wagner, Nucl. Phys. B 586, 92 (2000); S.Y. Choi, M. Drees, and J.S. Lee, Phys. Lett. B 481, 57 (2000); G.L. Kane and L.T. Wang, Phys. Lett. B 488, 383 (2000). M. Carena, J. Ellis, A. Pilaftsis, and C.E.M. Wagner, Phys. Lett. B 495, 155 (2000); S.W. Ham, S.K. Oh, E.J. Yoo, and H.K. Lee, J. Phys. G 27, 1 (2001); A.G. Akeroyd and A. Arhrib, Phys. Rev. D 64 095018 (2001); S.Y. Choi, K. Hagiwara, and J.S. Lee, Phys. Rev. D 64, 032004 (2001); Phys. Lett. B 529, 212 (2002).

[7] T. Ibrahim and P. Nath, Phys. Rev. D 63, 035009 (2001).

[8] T. Ibrahim and P. Nath, Phys. Rev. D 66, 015005 (2002).

[9] S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).
Figure Captions

Fig. 1: The plot of the (1, 3)- and (2, 3)-elements of the mass matrix of the neutral Higgs bosons at the one-loop level with the contribution of terms of $O(g_i^4)$ ($i = 1, 2$), $M_{13}^0$ (dot-dashed curve), $M_{23}^0$ (dashed curve), $M_{13}$ (dotted curve) and $M_{23}$ (solid curve), as a function of $\phi_1$ for $\phi_t = \phi_b = \phi_\tau = \phi_\chi(\phi_2) = \pi/3$, $\Lambda = 300$ GeV, and $\bar{m}_A = 300$ GeV, $m_Q = 800$ GeV, $m_T = 400$ GeV, $A_t = 200$ GeV, and $M_2 = 400$ GeV. We set $\tan \beta = 5$ and $\mu = -400$ GeV. These values for the parameters are the same as Table 5, except that $\phi_1$ is taken as a variable.
Table 1: The elements of the symmetric mass matrix of the neutral Higgs bosons in the $(h_1, h_2, h_3)$-basis, at the one-loop level with CP violation for $\phi_t = \phi_b = \phi_\tau = \phi_c(\phi_2) = \phi_1 = \pi/3$. Here, the contributions of terms of $O(g_4^i)$ $(i = 1, 2)$ and $A^0$ are neglected. The unit is (GeV)$^2$. The values of the relevant parameters are $\Lambda = 300$ GeV, $m_A = 300$ GeV, $m_Q = 800$ GeV, $m_T = 400$ GeV, $A_t = 200$ GeV, and $M_2 = 400$ GeV. We set $\tan\beta = 5$ and $\mu = -400$ GeV. The number in the first row in each column is the tree-level value. The number in the last row in each column is the sum of all numbers in the preceding rows, representing the value at the one-loop level. The numbers in each column in between the two rows represent the various loop contributions as decomposed in the one-loop Higgs potential.

| $(i, j)$ | (1, 1)    | (2, 2)    | (3, 3)    | (1, 2)    | (1, 3)    | (2, 3)    |
|---------|-----------|-----------|-----------|-----------|-----------|-----------|
| $M^0_{ij}$ | 86856.9  | 11424.0   | 90000.0   | -18900.1 | 0.0       | 0.0       |
| $M^t_{ij}$ | -7.2     | 6362.6    | -6.9      | 229.3    | 7.0       | -225.0    |
| $M^b_{ij}$ | 0.1      | -0.008    | -0.001    | 0.01     | -0.007    | 0.003     |
| $M^\tau_{ij}$ | 0.002    | -0.0001   | -0.0001   | 0.0002   | -0.00009  | 0.00004   |
| $M^\chi_{ij}$ | 3.0      | -268.3    | 16.5      | -31.5    | 17.1      | 40.7      |
| $M^h_{ij}$ | -3.7     | -150.4    | 0.0       | -12.3    | 0.0       | 0.0       |
| $M^\tilde{\chi}_{ij}$ | 0.1      | -146.1    | 12.4      | -17.6    | 10.2      | 31.3      |
| $M_{ij}$ | 86849.3  | 17221.8   | 90022.0   | -18732.3 | 34.4      | -152.9    |

Table 2: The same as Table 1, except for $\mu = 400$ GeV.

| $(i, j)$ | (1, 1)    | (2, 2)    | (3, 3)    | (1, 2)    | (1, 3)    | (2, 3)    |
|---------|-----------|-----------|-----------|-----------|-----------|-----------|
| $M^0_{ij}$ | 86856.9  | 11424.0   | 90000.0   | -18900.1 | 0.0       | 0.0       |
| $M^t_{ij}$ | -0.08    | 6267.7    | -6.9      | -25.5    | 0.7       | 226.0     |
| $M^b_{ij}$ | 0.1      | -0.006    | -0.001    | 0.01     | 0.009     | -0.003    |
| $M^\tau_{ij}$ | 0.002    | -0.00008  | -0.00001  | 0.0002   | 0.0001    | -0.00003  |
| $M^\chi_{ij}$ | -14.6    | -288.7    | 16.8      | 77.9     | 1.7       | -37.1     |
| $M^h_{ij}$ | -3.6     | -147.2    | 0.0       | -13.2    | 0.0       | 0.0       |
| $M^\tilde{\chi}_{ij}$ | -9.3     | -155.1    | 12.7      | -42.1    | -1.9      | -29.9     |
| $M_{ij}$ | 86829.3  | 17100.5   | 90022.6   | -19059.0 | 0.6       | 158.9     |
Table 3: The same as Table 1, except for $\tan \beta = 30$.

| $(i, j)$ | (1, 1) | (2, 2) | (3, 3) | (1, 2) | (1, 3) | (2, 3) |
|----------|--------|--------|--------|--------|--------|--------|
| $M^0_{ij}$ | 89909.3 | 8371.6 | 90000.0 | −3272.3 | 0.0 | 0.0 |
| $M^t_{ij}$ | −2.7 | 6088.2 | −6.4 | 139.1 | 4.1 | −212.8 |
| $M^b_{ij}$ | 7.2 | −9.3 | −1.7 | 1.7 | −0.7 | 4.0 |
| $M^\tau_{ij}$ | 0.1 | −0.1 | −0.02 | 0.02 | −0.009 | 0.05 |
| $M^{\tilde{\chi}}_{ij}$ | 6.7 | −279.2 | 16.7 | 13.7 | 10.9 | 40.0 |
| $M^{H}_{ij}$ | 0.1 | −170.2 | 0.0 | −1.6 | 0.0 | 0.0 |
| $M^{\tilde{\chi}}_{0ij}$ | 2.1 | −155.7 | 12.5 | 7.1 | 5.3 | 31.3 |
| $M_{ij}$ | 89922.7 | 13845.2 | 90021.1 | −3112.2 | 19.7 | −137.3 |

Table 4: The same as Table 2, except for $\tan \beta = 30$.

| $(i, j)$ | (1, 1) | (2, 2) | (3, 3) | (1, 2) | (1, 3) | (2, 3) |
|----------|--------|--------|--------|--------|--------|--------|
| $M^0_{ij}$ | 89909.3 | 8371.6 | 90000.0 | −3272.3 | 0.0 | 0.0 |
| $M^t_{ij}$ | −1.6 | 6073.0 | −6.4 | −106.5 | 3.2 | 212.9 |
| $M^b_{ij}$ | 3.2 | −9.0 | −1.7 | 6.2 | 2.7 | −3.9 |
| $M^\tau_{ij}$ | 0.05 | −0.1 | −0.02 | 0.08 | 0.03 | −0.05 |
| $M^{\tilde{\chi}}_{ij}$ | 3.7 | −309.5 | 16.7 | −33.0 | 8.3 | −39.4 |
| $M^{H}_{ij}$ | −0.1 | −136.4 | 0.0 | −3.0 | 0.0 | 0.0 |
| $M^{\tilde{\chi}}_{0ij}$ | 0.5 | −157.3 | 12.6 | −17.4 | 3.2 | −31.1 |
| $M_{ij}$ | 89915.1 | 13832.2 | 90021.2 | −3426.1 | 17.5 | 138.4 |
Table 5: The elements of the symmetric mass matrix of the neutral Higgs bosons in the $(h_1, h_2, h_3)$-basis, at the one-loop level with the contributions of terms of $O(g_i^4)$ ($i = 1, 2$) and $A^0$. The setting of the parameters is the same as Table 1.

| $i,j$ | (1, 1) | (2, 2) | (3, 3) | (1, 2) | (1, 3) | (2, 3) |
|-------|--------|--------|--------|--------|--------|--------|
| $M^0_{ij}$ | 86856.9 | 11424.0 | 90000.0 | −18900.1 | 0.0 | 0.0 |
| $M^t_{ij}$ | −0.7 | 5832.3 | −7.0 | 265.7 | 4.3 | −212.3 |
| $M^b_{ij}$ | 1.0 | 30.6 | −0.001 | −5.1 | 0.02 | −0.1 |
| $M^\tau_{ij}$ | 0.1 | 4.7 | −0.00001 | −0.8 | 0.002 | −0.01 |
| $M^\tilde{\chi}_i^0_{ij}$ | 1.7 | −311.8 | 16.6 | −23.8 | 17.3 | 40.0 |
| $M^\tilde{\chi}_i^0_{ij}$ | 53.1 | −148.6 | 55.9 | −31.5 | 0.0 | 0.0 |
| $M^\chi_i^0_{ij}$ | 2.8 | −82.3 | 12.4 | −15.5 | 10.5 | 31.8 |
| $M_{ij}$ | 86915.1 | 16749.0 | 90069.7 | −18711.4 | 32.2 | −140.6 |

Table 6: The same as Table 5, except for $\mu = 400$ GeV.

| $i,j$ | (1, 1) | (2, 2) | (3, 3) | (1, 2) | (1, 3) | (2, 3) |
|-------|--------|--------|--------|--------|--------|--------|
| $M^0_{ij}$ | 86856.9 | 11424.0 | 90000.0 | −18900.1 | 0.0 | 0.0 |
| $M^t_{ij}$ | 0.1 | 5744.0 | −6.9 | 24.7 | 3.5 | 213.3 |
| $M^b_{ij}$ | 1.0 | 30.5 | −0.001 | −5.3 | −0.02 | 0.1 |
| $M^\tau_{ij}$ | 0.1 | 4.7 | −0.00001 | −0.9 | −0.002 | 0.01 |
| $M^\tilde{\chi}_i^0_{ij}$ | −16.2 | −331.7 | 16.9 | −69.6 | 1.6 | −36.4 |
| $M^\tilde{\chi}_i^0_{ij}$ | 53.1 | −146.1 | 58.0 | −31.8 | 0.0 | 0.0 |
| $M^\chi_i^0_{ij}$ | −6.7 | −95.5 | 13.0 | −46.3 | 0.5 | −30.1 |
| $M_{ij}$ | 86889.3 | 16630.0 | 90072.5 | −19029.5 | 5.6 | 146.9 |
Table 7: The same as Table 5, except for $\tan \beta = 30$.

| $(i,j)$ | (1, 1) | (2, 2) | (3, 3) | (1, 2) | (1, 3) | (2, 3) |
|---------|--------|--------|--------|--------|--------|--------|
| $M^0_{ij}$ | 89909.3 | 8371.6 | 90000.0 | −3272.3 | 0.0 | 0.0 |
| $M^t_{ij}$ | -2.1 | 5561.2 | -6.4 | 139.2 | 3.7 | -200.4 |
| $M^b_{ij}$ | 6.7 | 51.3 | -1.7 | 8.4 | -0.5 | -2.4 |
| $M^\tau_{ij}$ | 0.07 | 7.0 | -0.02 | 0.3 | 0.006 | -0.4 |
| $M^\tilde{\chi}_{ij}$ | 6.7 | -324.4 | 16.7 | 14.7 | 10.9 | 39.2 |
| $M^h_{ij}$ | 40.0 | -168.3 | 30.2 | -4.0 | 0.0 | 0.0 |
| $M^\tilde{\chi}_0_{ij}$ | 5.0 | -91.3 | 12.8 | 12.8 | 8.1 | 32.1 |
| $M_{ij}$ | 89965.7 | 13407.2 | 90043.2 | -3100.9 | 22.4 | -131.8 |

Table 8: The same as Table 6, except for $\tan \beta = 30$.

| $(i,j)$ | (1, 1) | (2, 2) | (3, 3) | (1, 2) | (1, 3) | (2, 3) |
|---------|--------|--------|--------|--------|--------|--------|
| $M^0_{ij}$ | 89909.3 | 8371.6 | 90000.0 | −3272.3 | 0.0 | 0.0 |
| $M^t_{ij}$ | -2.0 | 5547.0 | -6.4 | -92.1 | 3.6 | 200.5 |
| $M^b_{ij}$ | 3.2 | 51.1 | -1.7 | 5.5 | 2.4 | 2.4 |
| $M^\tau_{ij}$ | 0.06 | 6.9 | -0.02 | -0.1 | 0.01 | 0.4 |
| $M^\tilde{\chi}_{ij}$ | 3.6 | -354.7 | 16.8 | -31.2 | 8.3 | -38.6 |
| $M^h_{ij}$ | 38.9 | -140.8 | 39.2 | -3.8 | 0.0 | 0.0 |
| $M^\tilde{\chi}_0_{ij}$ | 2.5 | -93.6 | 12.9 | -22.7 | 6.0 | -31.7 |
| $M_{ij}$ | 89955.7 | 13387.7 | 90052.3 | -3417.0 | 20.5 | 133.0 |
Fig. 1: The plot of the (1, 3)- and (2, 3)-elements of the mass matrix of the neutral Higgs bosons at the one-loop level with the contribution of the quartic terms of $O(g_i^4)$ ($i = 1, 2$), $M_{13}^{\phi_1}$ (dot-dashed curve), $M_{23}^{\phi_1}$ (dashed curve), $M_{13}$ (dotted curve) and $M_{23}$ (solid curve), as a function of $\phi_1$ for $\phi_t = \phi_b = \phi_c = \phi_2 = \pi/3$, $\Lambda = 300$ GeV, and $\bar{m}_A = 300$ GeV, $m_Q = 800$ GeV, $m_T = 400$ GeV, $A_t = 200$ GeV, and $M_2 = 400$ GeV. We set $\tan \beta = 5$ and $\mu = -400$ GeV. These values for the parameters are the same as Table 5, except that $\phi_1$ is taken as a variable.