Limitations on ACI Code Minimum Thickness Requirements for Flat Slab

Bilal Ismaeel Abd Al-Zahra 1, Maitham Alwash 1, Ameer Baiee 1, Ali A. Shubbar 2, 3*

1 Lecturer in Civil Engineering, University of Babylon, Hilla, Babil, Iraq.
2 Research Assistant in Civil Engineering, Liverpool John Moores University, Liverpool, United Kingdom.
3 Researcher in Department of Building and Construction Technical Engineering, College of Technical Engineering, The Islamic University, 54001, Najaf, Iraq.

Received 03 August 2021; Revised 25 September 2021; Accepted 06 October 2021; Published 01 November 2021

Abstract

Reinforced concrete two-way flat slabs are considered one of the most used systems in the construction of commercial buildings due to the ease of construction and suitability for electrical and mechanical paths. Long-term deflection is an essential parameter in controlling the behavior of this slab system, especially with long spans. Therefore, this study is devoted to investigating the validation of the ACI 318-19 Code long-term deflection limitations of a wide range of span lengths of two-way flat slabs with and without drop panels. The first part of the study includes nonlinear finite element analysis of 63 flat slabs without drops and 63 flat slabs with drops using the SAFE commercial software. The investigated parameters consist of the span length (4, 5, 6, 7, 8, 9, and 10 m), compressive strength of concrete (21, 35, and 49 MPa), the magnitude of live load (1.5, 3, and 4.5 kN/m²), and the drop thickness (0.25t slab, 0.5t slab, and 0.75t slab). In addition, the maximum crack width at the top and bottom are determined and compared with the limitations of the ACI 224R-08. The second part of this research proposes modifications to the minimum slab thickness that satisfy the permissible deflection. It was found, for flat slabs without drops, the increase in concrete compressive strength from 21MPa to 49MPa decreases the average long-term deflection by (56, 53, 50, 44, 39, 33 and 31%) for spans (4, 5, 6, 7, 8, 9, and 10 m) respectively. In flat slab with drop panel, it was found that varying drop panel thickness t2 from 0.25t slab to 0.75t slab decreases the average long-term deflection by (45, 41, 39, 35, 31, 28 and 25%) for span lengths (4, 5, 6, 7, 8, 9 and 10 m) respectively. Limitations of the minimum thickness of flat slab were proposed to vary from Ln/30 to Ln/19.9 for a flat slab without a drop panel and from Ln/33 to Ln/21.2 for a flat slab with drop panel. These limitations demonstrated high consistency with the results of Scanlon and Lee’s unified equation for determining the minimum thickness of slab with and without drop panels.

Keywords: Long-term Deflection; Allowable Deflection; Flat Slab; Drop Panel Thickness; Concrete Compressive Strength; Crack width; Span Length.

1. Introduction

A flat plate slab (or known also as a flat slab without a drop panel) is a two-way reinforced concrete slab that transfers loads directly to the supporting columns without the aid of beams or drop panels or capitals. In case of the presence of column capitals, drop panels, or both the slab is called a flat slab. The flat plate, that is common in

*Corresponding author: a.a.shubbar@ljmu.ac.uk
http://dx.doi.org/10.28991/cej-2021-03091769
© 2021 by the authors. Licensee C.E.J, Tehran, Iran. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC-BY) license (http://creativecommons.org/licenses/by/4.0/).
residential building, has several advantages such as cost savings due to low story height and simple/quick construction and formwork, and flat ceiling that has high fire resistance (few sharps corners for concrete spalling) and less obstruction to light diffusion. The flat slab is satisfactory for long spans and heavy loads, in particular, the flat slab is economical for parking, warehouses, and industrial buildings [1-3].

The deflection is a crucial issue in the design of flat slabs with or without drop panels. Most Standards like ACI 318-19 [4], CSA A23.3-04 [5], AS 3600 [6] and Euro code 2 [7] propose two alternative ways for the control of deflection. The first approach is to calculate the deflection and to compare the calculated deflection with the allowable limits. The second approach controls indirectly the deflection by limiting minimum slab thickness or maximum span/depth ratio.

The flexural stiffness EI (E: the concrete modulus of elasticity and I: the moment of inertia) of a flexural member is an essential variable in the calculation of deflection. For the reinforced concrete members, the amount of section cracking affects significantly the moment of inertia and consequently, this effect must be considered in the analysis of deflection [8]. Generally, there are two different methods for considering the cracking effect: the effective moment of inertia method [9] and the mean curvature method [10]. Furthermore, creep and shrinkage have important effects on the long-term deflection, and therefore literature provides several ways for considering this effect, the most famous one is the ACI 318 method. The analysis for deflection can be done by using a range of refined methods [11], like a non-linear analysis or finite element analysis [12]. Recent work has used the Artificial Neural Network approach [13] for the prediction of deflection. However, the approaches for calculating the deflection in flat slabs are complicated and involving several approximations due to complex behavior at the service load stage (cracking, time-dependent effect, tension stiffening). Therefore, the direct calculation of deflection for the typical situations is impractical and engineers prefer to control the deflection using the minimum slab thickness or maximum span/depth ratio approach.

The minimum slab thickness or maximum span/depth ratio approach is the focus of many researches for decades. Several studies [14-18] have proposed different expressions for the maximum allowable span/depth ratio for slabs (including flat plate and flat slabs) considering the effects of different factors such as sustained load, aspect ratio, reinforcement ratio, support condition, concrete modulus of elasticity, target maximum permissible incremental deflection and long-term deflection effects.

Vollum and Hossain [19] have studied the span/depth rules given in Euro code 2 and they have found that the deflections calculated in flat slabs dimensioned with span/depth rules of Euro code 2 can be excessive in external and corner panels since the rules fail to allow for the effect of cracking during construction. Lee and Scanlon [20] have compared the minimum slab (one-way and two-way) provisions of various Standards (ACI 318-08, Euro code 2, BS 8110-1:1997, and AS 3600-2001 and the unified equation proposed by Scanlon and Lee [15]) by performing a parametric study to evaluate the effects of several relevant design parameters. The results show that ACI 318 conditions need a revision to cover the range of the affected design parameters. Furthermore, applicability limitations require to be added to ACI provisions, especially for flat slab provisions which seem to be sufficient for the limit of L/240 (for typical loading and spans) but insufficient in many cases for the limit of L/480. Bertero [21] has investigated the effectiveness of ACI 318 provisions for minimum thickness of two-way slabs for controlling the deflection to be within the allowable limits. This study evaluates (from a statistical viewpoint) the calculated deflections for two-way slabs having minimum thickness specified according to the ACI 318-14 requirements and as a result, it provides recommendations for upcoming ACI code revision. Hasan and Taha [22] have investigated the effects of several parameters (aspect ratio, live load, concrete strength) on the long-term deflection of flat plate slabs without edge beams (corner panels). They have highlighted the effect of not account for the aspect ratio in five Standards (ACI 318-14, CSA A23.3-14, AS 3600, BS8110, Euro code 2) provisions for the minimum slab thickness. Moreover, the applicability of the ACI 318-14 requirements for the thickness of flat plate slab without edge beam appeared to be sufficient to satisfy the permissible deflection limits L/360 and L/240 for typical spans and concrete strength while they were insufficient in many cases for the limit of L/480. Sanabra-Loewe et al. [23] have assessed the ACI 318 code and Eurocode 2 methods for the minimum slenderness ratio of R.C. slabs. The evaluated factors were: load, span, and permissible deflection. The results highlight the shortcoming of the Eurocode 2 and ACI 318 code provisions. Al-Nu'man & Abdullah [24] have developed a simulation model that considering the materials and loads uncertainties and along with the sensitivity analysis of results. The results indicate that the ACI 318-14 minimum thickness requirements are adequate for 4m and 6m span or less for flat plate and flat slab respectively. Depending on the characteristic strength of concrete, the redistribution factor, and the total steel ratio, Santos and Henriques [25] have proposed new span/depth limits satisfying both deflection and ductility requirements. However, these limits are restricted to the cases of beams and one-way slabs.

From the above review of literature, it is clear that there is a common consensus that the minimum thickness provisions required by ACI 318 code for flat slabs cannot ensure the deflection to comply with the maximum permissible limits for all flat slabs. Therefore, the objective of the present paper is to study the domain of applicability of ACI minimum thickness provisions for flat slab for controlling the long-term deflection and to provide the
community of engineers the limitations for these provisions. The present paper addresses this issue by selecting the slab thickness according to the ACI 318-19 provisions, then, calculating the deflections using the Nonlinear Finite element Analysis for 126 case studies of flat slabs (with and without drop panels) for a range of span lengths and practical selected values of several influencing parameters (live loads, materials strengths, and drop panel thickness) and comparing the computed deflections with the ACI 318-19 permissible values (L/240, L/480).

2. Nonlinear Finite Element Analysis

The methodology of the present study is devoted to calculate the long-term deflection of flat slabs with thicknesses that determined according to ACI 318-19 Code minimum thickness requirements and to compare the calculated deflections with ACI 318-19 Code permissible limits. To achieve this goal, a nonlinear Finite Element Analysis was performed to investigate the long-term deflection in flat slabs. The SAFE software was considered here for this purpose. The long-term deflection was calculated according to the procedure illustrated in [26]. This procedure includes the calculation of deflection for three cases:

- Case 1: the immediate deflection due to short-term loads: DL + SDL + LL,
- Case 2: the immediate deflection due to sustained loads: DL + SDL + ΨLLL,
- Case 3: the long-term deflection due to sustained loads: DL + SDL + ΨL LL.

Where DL, SDL and LL represent the slab self-weight, superimposed dead load and live load applied on the slab respectively. ΨL is the percentage of live load considered to be sustained.

Using SAFE software analysis options, the nonlinear (cracked) analysis was performed for cases 1 and 2, instead, for case 3 the nonlinear (long-term cracked i.e. with creep and shrinkage effects) analysis was carried out.

The value of long-term deflection was determined as a linear combination of case 3 + case 1 - case 2, where the difference between case 1 and case 2 represents the incremental deflection (without creep and shrinkage) due to non-sustained loading on a cracked structure.

Two layouts of the flat slabs were considered for analysis in the present study. both cases consist of three equal spans in each direction without edge beams, however, the first one is without drop panels (i.e. flat plate), see Figure 1, and the second layout with drop panels as shown in Figure 2. The drop panel dimensions were selected to comply with ACI 318-19 requirements for the drop panel as detailed in Figure 3.

The ACI code provisions for the minimum thickness of flat slab take into account only two effects: span length and yield strength of steel fy. However, this paper considers the effects of several factors on the long-term deflection and as a result on the minimum thickness requirements, these are: span length L, concrete compressive strength fc′, service live load, and drop panel thickness t2. The range of values for each one of these factors was selected to be consistent with that used in the real practice and with available ACI 318-19 provisions. The selected values were: span length L (4, 5, 6, 7, 8, 9, 10) m, concrete compressive strength fc (21, 35, 49) MPa, service live load (1.5, 3, 4.5) kN/m2, and drop panel thickness t2 (0.25t slab, 0.5t slab, 0.75t slab). On the contrary, the other parameters were considered fixed through the analysis and their specified values were:

- Steel reinforcement properties: yield strength fy = 420 MPa (Grade 60), Modulus of elasticity Es = 200 GPa,
- Modulus of elasticity of concrete Ec = 4700√fc′.
- Superimposed dead load = 2 kN/m2,
- Dimension of squared columns supporting flat slabs with span length 4, 5, 6, 7, 8, 9 and 10 m are 300, 300, 350, 400, 450, 500 and 550 mm respectively,
- The percentage of live load that considered to be sustained ΨL = 25%.
- The time-dependent factor or creep coefficient = 2, i.e. for sustained load duration five years or more as specified in ACI 318-19.
Consequently, in total 126 case studies of flat slabs were analyzed to study the effects of factors considered in this paper. These case studies were divided equally into two main groups. The first one includes 63 case studies of the flat slab without drop panels and the second one comprises 63 case studies of flat slabs with drop panels. The two groups were similar in the range of values for span length and live load (values stated above), however, the concrete compressive strength was varied in the first one and had a fixed value $f'_c=21$ MPa in second group. Furthermore, the range of values for drop panel thickness (given above) was considered in the second group only.

3. Results and Discussion

Using the nonlinear Finite Element Analysis, the long-term deflection was investigated at different points of 126 case studies of flat slabs. Figures 4 and 5 show the resulting long-term deflection for two extreme case studies of the flat slab without drop panel having the same concrete strength ($f'_c=21$MPa) and live load ($LL=4.5$ kN/m$^2$) but with different values for span length ($L=4m$ for Figure 4 and $L=10m$ for Figure 5). Figures 6 and 7 illustrate the long-term deflection for another two case studies similar to that shown in Figures 4 and 5 respectively but for the flat slab with a drop ($t_2=0.25t_{slab}$). From these four figures, it is clear that the maximum long-term deflection occurs at corner panels and nearly at the midpoint of the diagonal line between the corner and interior columns. The same finding was drawn from all other cases and therefore the long-term deflection given in the next sections will be at the midpoint of the diagonal line between the corner and interior columns for the corner panels.

Due to the large campaign of case studies considered in the present paper, it is convenient to discuss the results into two subsections, firstly for the cases of flat slabs without drop panel and secondly for the cases of flat slabs with drop panel.
3.1. Two-way Flat Slab without Drop Panels

Analysis results of maximum long-term deflection for the 63 case studies of the flat slab without drop panel are given in Table 1 and shown graphically in Figures 8, 9 and 10. As shown, the results were obtained from analyzing flat slabs having span lengths varied from 4 to 10 m, and for three values of concrete compressive strengths (21, 35, 49) MPa and three values of live loads (1.5, 3, 4.5) kN/m². The resulting maximum long-term deflections were compared with the ACI 318-19 allowable deflection limits: L/480 (roof or floor construction supporting or attached to non-structural elements likely to be damaged by large deflections) and L/240 (roof or floor construction supporting or attached to non-structural elements not likely to be damaged by large deflections). Although the slab thickness was dimensioned according to ACI 318-19 minimum thickness requirements (Ln/30) for all cases, the calculated maximum long-term deflection exceeds one or both allowable limits in many cases. As an example, for the cases with LL=1.5 kN/m², the calculated deflections exceed the limit of L/240 when the span length is larger than 4, 6, 8 m for f_c values of 21, 35, 49 MPa respectively. Furthermore, Figures 8, 9 and 10 show a nearly linear increase in maximum long-term deflection as the span length changes from 4 to 10 m, but with a slope that becomes steeper for weaker concrete strength. In other words, improving the concrete compressive strength from 21 to 49 MPa reduces the maximum long-term deflection by an average of (56, 53, 50, 44, 39, 33 and 31%) for spans (4, 5, 6, 7, 8, 9 and 10 m) respectively. These percentages indicate that the efficiency of using stronger concrete (f_c=49 MPa) is the highest when the slab span length is 4 m. Regarding the effect of live loads, as expected, changing the live load from 1.5 to 4.5
kN/m² leads to more deflection, however, this effect is more pronounced for a small span length of 4 m and is diminished gradually for a larger span length. This behavior can be explained by referring to any short-term deflection elastic equation (for example with $384EI$) where the span length $L$ has power 4 while the loads $w$ has power 1 and consequently the effect of the increase in span length is dominated.

Table 1 also compares the maximum cracks width at the top and bottom faces of the slab with the ACI 224R-08 [27] allowable limit of 0.3 mm that corresponds to the exposure condition: humidity, moist air and soil. From these analysis results, there is a clear trend of increasing the crack width with the increase in span length and as a result exceeding the allowable limits 0.3 mm for span length more than 7 m.

Table 1. Analysis results for flat slab without drop panel with different values of spans length, concrete compressive strength and live loads

Two-way flat slab without drop panels, $f_c = 420$ MPa

| Span (L), m | $L_a/30$ mm | $f'_c = 21$MPa | $f'_c = 35$MPa | $f'_c = 49$MPa | Allowable deflections (mm) | Allowable crack width mm |
|-----------|-------------|----------------|----------------|----------------|--------------------------|-------------------------|
| 4         | 125         | 12.6           | 0.12           | 0.15           | 8.1          | 0.17                     | 0.12                     | 6.1                      | 0.16                     | 0.12                     | 8.3                      | 16.6                      | 0.30                     |
| 5         | 160         | 22.9           | 0.18           | 0.17           | 12.7         | 0.19                     | 0.17                     | 10.5                     | 0.20                     | 0.17                     | 10.4                     | 20.8                      | 0.30                     |
| 6         | 190         | 34.8           | 0.22           | 0.21           | 19.5         | 0.23                     | 0.21                     | 16.3                     | 0.24                     | 0.22                     | 12.5                     | 25.0                      | 0.30                     |
| 7         | 220         | 47.2           | 0.25           | 0.25           | 30.7         | 0.26                     | 0.26                     | 22.2                     | 0.26                     | 0.26                     | 14.5                     | 29.1                      | 0.30                     |
| 8         | 255         | 55.7           | 0.29           | 0.32           | 44.8         | 0.30                     | 0.32                     | 30.1                     | 0.31                     | 0.33                     | 16.6                     | 33.3                      | 0.30                     |
| 9         | 285         | 66.6           | 0.32           | 0.38           | 53.2         | 0.32                     | 0.38                     | 41.2         | 0.31                     | 0.37                     | 18.7                     | 37.5                      | 0.30                     |
| 10        | 315         | 79.1           | 0.31           | 0.38           | 67.0         | 0.32                     | 0.38                     | 53.0         | 0.31                     | 0.37                     | 20.8                     | 41.6                      | 0.30                     |

$LL = 1.5$ kN/m²

| Span (L), m | $L_a/30$ mm | $f'_c = 21$MPa | $f'_c = 35$MPa | $f'_c = 49$MPa | Allowable deflections (mm) | Allowable crack width mm |
|-----------|-------------|----------------|----------------|----------------|--------------------------|-------------------------|
| 4         | 125         | 18.8           | 0.15           | 0.14           | 10.8         | 0.15                     | 0.13                     | 7.7                      | 0.15                     | 0.13                     | 8.3                      | 16.6                      | 0.30                     |
| 5         | 160         | 31.6           | 0.18           | 0.19           | 19.7         | 0.18                     | 0.18                     | 12.4                     | 0.19                     | 0.18                     | 10.4                     | 20.8                      | 0.30                     |
| 6         | 190         | 40.4           | 0.21           | 0.23           | 28.3         | 0.22                     | 0.22                     | 17.7                     | 0.22                     | 0.22                     | 12.5                     | 30.0                      | 0.30                     |
| 7         | 220         | 52.1           | 0.24           | 0.27           | 40.0         | 0.23                     | 0.26                     | 29.0                     | 0.24                     | 0.26                     | 14.5                     | 29.1                      | 0.30                     |
| 8         | 255         | 60.0           | 0.30           | 0.33           | 45.9         | 0.29                     | 0.32                     | 37.4         | 0.29                     | 0.33                     | 16.6                     | 33.3                      | 0.30                     |
| 9         | 285         | 70.2           | 0.30           | 0.35           | 57.8         | 0.30                     | 0.35                     | 48.3         | 0.30                     | 0.34                     | 18.7                     | 37.5                      | 0.30                     |
| 10        | 315         | 82.2           | 0.32           | 0.39           | 70.7         | 0.31                     | 0.39                     | 55.4         | 0.31                     | 0.39                     | 20.8                     | 41.6                      | 0.30                     |

$LL = 3$ kN/m²

| Span (L), m | $L_a/30$ mm | $f'_c = 21$MPa | $f'_c = 35$MPa | $f'_c = 49$MPa | Allowable deflections (mm) | Allowable crack width mm |
|-----------|-------------|----------------|----------------|----------------|--------------------------|-------------------------|
| 4         | 125         | 22.5           | 0.14           | 0.14           | 15.0         | 0.16                     | 0.14                     | 9.5                      | 0.15                     | 0.14                     | 8.3                      | 16.6                      | 0.30                     |
| 5         | 160         | 33.9           | 0.17           | 0.18           | 23.8         | 0.18                     | 0.18                     | 18.1                     | 0.18                     | 0.18                     | 10.4                     | 20.8                      | 0.30                     |
| 6         | 190         | 44.9           | 0.21           | 0.22           | 33.1         | 0.21                     | 0.22                     | 26.4                     | 0.21                     | 0.22                     | 12.5                     | 25.0                      | 0.30                     |
| 7         | 220         | 54.3           | 0.24           | 0.26           | 43.1         | 0.24                     | 0.26                     | 35.5                     | 0.24                     | 0.26                     | 14.5                     | 29.1                      | 0.30                     |
| 8         | 255         | 62.1           | 0.29           | 0.31           | 51.6         | 0.26                     | 0.32                     | 41.2         | 0.29                     | 0.32                     | 16.6                     | 33.3                      | 0.30                     |
| 9         | 285         | 73.3           | 0.31           | 0.34           | 63.7         | 0.30                     | 0.34                     | 50.3         | 0.30                     | 0.35                     | 18.7                     | 37.5                      | 0.30                     |
| 10        | 315         | 85.2           | 0.35           | 0.35           | 74.3         | 0.35                     | 0.35                     | 60.7         | 0.35                     | 0.36                     | 20.8                     | 41.6                      | 0.30                     |

$LL = 4.5$ kN/m²
3.2. Two-way Flat Slab with Drop Panels

From the analysis of 63 case studies of the flat slab with drop panel, Table 2 provides the resulting maximum long-term deflections, also, Figures 11, 12, and 13 show these results graphically. The variables in this analysis were the span lengths (varied from 4 to 10 m), live loads (1.5, 3, 4.5 kN/m²) and drop panel thickness $t_2$ (0.25$t_{slab}$, 0.5$t_{slab}$, 0.75$t_{slab}$). Since the effect of concrete compressive strength became clear from the above analysis of the flat slabs without drop panel, a fixed value of $f_c'$ = 21MPa was considered here for the analysis of flat slabs with drop panel. The resulting maximum long-term deflections were compared with the ACI 318-19 allowable deflection limits L/480 and L/240. In spite of the slab thickness was selected to comply with ACI 318-19 minimum thickness requirements (Ln/33) for all cases, the computed maximum long-term deflection exceeds one or both allowable limits in many cases. As an example, for the cases with LL=1.5 kN/m², the calculated deflections exceed the limit of L/240 when the span length is larger than 4, 5, 7 m for drop panel thickness $t_2$ of 0.25$t_{slab}$, 0.5$t_{slab}$, 0.75$t_{slab}$ respectively. Moreover, Figures 11, 12 and 13 show a nearly linear relation between the resulting maximum long-term deflection and the span length. However, the slopes of these relations reduce as the drop panel becomes thicker. In other words, varying drop panel thickness $t_2$ from 0.25$t_{slab}$ to 0.75$t_{slab}$ decreases the average long-term deflection by (45, 41, 39, 35, 31, 28 and
25%) for span lengths (4, 5, 6, 7, 8, 9 and 10 m) respectively. These percentages show that the positive effect of drop panel thickness is important for small spans and it becomes less significant for larger spans. Concerning the live load effect, a similar finding to that drawn above for flat slab without drop was found here i.e. increasing the live load leads to larger long-term deflection but this effect becomes less important with the increase in span lengths.

In addition to the maximum long-term deflection, Table 2 shows the resulting maximum cracks width at the top and bottom faces of slab. These results exhibit a logical increase in the width of the cracks as the span length varies from 4 to 10 m. The comparison of the resulting maximum cracks width with the ACI 224R-08 [27] allowable limit of 0.3 mm (that corresponds to the exposure condition: humidity, moist air and soil) indicates that the crack width fails to comply with the allowable limit (0.3 mm) when the span length is more than 7 m.

Table 2. Analysis results for flat slab with drop panel with different values of spans length, drop panel thickness and live loads

| Span (L) | t_{slab} | t_{L} | Maximum crack width | t_{L} | Maximum crack width | t_{L} | Maximum crack width |
|----------|----------|-------|----------------------|-------|----------------------|-------|----------------------|
| (m)     | (mm)     | (mm)  | mm  | mm  | mm  | mm  | mm  | (mm)     | (mm)     | (mm)     |
| 4       | 115      | 29    | 12.8 | 0.18 | 0.11 | 58  | 9.1  | 0.21 | 0.10 | 87  | 7.2  | 0.24 | 0.10 | 8.3  | 16.6 | 0.30 |
| 5       | 145      | 37    | 24.9 | 0.21 | 0.16 | 73  | 18.8 | 0.23 | 0.15 | 109 | 13.1 | 0.28 | 0.15 | 10.4 | 20.8 | 0.30 |
| 6       | 175      | 44    | 34.8 | 0.24 | 0.20 | 88  | 25.1 | 0.28 | 0.19 | 132 | 20.5 | 0.31 | 0.19 | 12.5 | 25.0 | 0.30 |
| 7       | 200      | 50    | 46.1 | 0.26 | 0.23 | 100 | 37.1 | 0.29 | 0.22 | 150 | 26.7 | 0.31 | 0.22 | 14.5 | 29.1 | 0.30 |
| 8       | 230      | 58    | 55.4 | 0.29 | 0.27 | 115 | 43.9 | 0.31 | 0.27 | 173 | 35.3 | 0.32 | 0.26 | 16.6 | 33.3 | 0.30 |
| 9       | 260      | 65    | 65.7 | 0.31 | 0.33 | 130 | 53.7 | 0.31 | 0.33 | 195 | 45.7 | 0.32 | 0.29 | 18.7 | 37.5 | 0.30 |
| 10      | 290      | 73    | 78.2 | 0.32 | 0.36 | 145 | 63.8 | 0.34 | 0.38 | 218 | 55.9 | 0.34 | 0.37 | 20.8 | 41.6 | 0.30 |

| Span (L) | t_{slab} | t_{L} | Maximum crack width | t_{L} | Maximum crack width | t_{L} | Maximum crack width |
|----------|----------|-------|----------------------|-------|----------------------|-------|----------------------|
| (m)     | (mm)     | (mm)  | mm  | mm  | mm  | mm  | mm  | (mm)     | (mm)     | (mm)     |
| 4       | 115      | 29    | 18.5 | 0.15 | 0.12 | 58  | 13.5 | 0.18 | 0.12 | 87  | 9.8  | 0.21 | 0.11 | 8.3  | 16.6 | 0.30 |
| 5       | 145      | 37    | 29.7 | 0.19 | 0.16 | 73  | 22.2 | 0.23 | 0.16 | 109 | 18.3 | 0.24 | 0.16 | 10.4 | 20.8 | 0.30 |
| 6       | 175      | 44    | 40.1 | 0.23 | 0.20 | 88  | 29.1 | 0.27 | 0.20 | 132 | 23.5 | 0.28 | 0.20 | 12.5 | 25.0 | 0.30 |
| 7       | 200      | 50    | 50.7 | 0.26 | 0.23 | 100 | 42.3 | 0.27 | 0.23 | 150 | 32.6 | 0.30 | 0.23 | 14.5 | 29.1 | 0.30 |
| 8       | 230      | 58    | 60.1 | 0.26 | 0.27 | 115 | 50.4 | 0.28 | 0.27 | 173 | 42.1 | 0.31 | 0.27 | 16.6 | 33.3 | 0.30 |
| 9       | 260      | 65    | 70.2 | 0.32 | 0.30 | 130 | 58.5 | 0.30 | 0.32 | 195 | 50.6 | 0.31 | 0.32 | 18.7 | 37.5 | 0.30 |
| 10      | 290      | 73    | 80.7 | 0.32 | 0.36 | 145 | 69.1 | 0.33 | 0.35 | 218 | 60.9 | 0.34 | 0.34 | 20.8 | 41.6 | 0.30 |

| Span (L) | t_{slab} | t_{L} | Maximum crack width | t_{L} | Maximum crack width | t_{L} | Maximum crack width |
|----------|----------|-------|----------------------|-------|----------------------|-------|----------------------|
| (m)     | (mm)     | (mm)  | mm  | mm  | mm  | mm  | mm  | (mm)     | (mm)     | (mm)     |
| 4       | 115      | 29    | 23.1 | 0.15 | 0.13 | 58  | 17.2 | 0.18 | 0.12 | 87  | 12.8 | 0.21 | 0.12 | 8.3  | 16.6 | 0.30 |
| 5       | 145      | 37    | 35.7 | 0.19 | 0.17 | 73  | 26.2 | 0.22 | 0.16 | 109 | 22.4 | 0.23 | 0.16 | 10.4 | 20.8 | 0.30 |
| 6       | 175      | 44    | 43.2 | 0.23 | 0.20 | 88  | 35.1 | 0.26 | 0.20 | 132 | 27.6 | 0.25 | 0.20 | 12.5 | 25.0 | 0.30 |
| 7       | 200      | 50    | 54.3 | 0.26 | 0.23 | 100 | 46.2 | 0.25 | 0.23 | 150 | 39.3 | 0.27 | 0.23 | 14.5 | 29.1 | 0.30 |
| 8       | 230      | 58    | 64.1 | 0.25 | 0.27 | 115 | 54.5 | 0.26 | 0.27 | 173 | 47.6 | 0.35 | 0.27 | 16.6 | 33.3 | 0.30 |
| 9       | 260      | 65    | 73.5 | 0.27 | 0.31 | 130 | 62.8 | 0.29 | 0.31 | 195 | 55.2 | 0.36 | 0.31 | 18.7 | 37.5 | 0.30 |
| 10      | 290      | 73    | 84.5 | 0.30 | 0.35 | 145 | 75.1 | 0.32 | 0.33 | 218 | 65.3 | 0.36 | 0.31 | 20.8 | 41.6 | 0.30 |
4. Proposed Minimum Thickness of Flat Slab

4.1. Modifications of the ACI-318 Code Limitations

Based on the above discussion of the results obtained in the present study, it is clear that for the control of deflection the use of a single formula for the minimum thickness for all flat slabs without a drop (Ln/30) or with drop (Ln/33) as specified by ACI code (for $f_y = 420$ MPa, exterior panel without edge beam) is a serious issue. The main shortcoming of the ACI formulas is its restriction to a single variable (span length) and the ignoring of other influencing factors like concrete compressive strength, applied live loads, and drop panel thickness.

Consequently, the 126 case studies considered here were re-analyzed using the nonlinear finite element analysis in order to specify, for each case, the appropriate minimum thickness that can ensure the complying of long-term deflection with the allowable limit of $L/240$. For this purpose, the re-analysis was performed with a gradual increase in the slab thickness (increments of 5 mm) for each case and then the maximum long-term deflection was investigated and compared the limit $L/240$. According to ACI 318-19 code, in any case, the flat slab thickness should be at least 125 mm for slab without a drop and 100 mm for slab with a drop, therefore these values were considered as the starting values for the slab thickness in the analysis.

Table 3 gives the analysis results for the 63 cases of the flat slab without a drop. It shows, for each case study, the resulting appropriate minimum slab thickness and the corresponding maximum calculated long-term deflection. Based
on these results, a new proposed formula for minimum slab thickness that corresponds to each case study was proposed and provided in Table 3. As shown, these formulas vary from Ln/30 to Ln/19.9 which is a wide range as compared with the single formula provided by ACI code (Ln/30).

Regarding the re-analysis of the 63 cases of the flat slab with a drop, the analysis results were given in Table 4. These results include, for each case study, the investigated appropriate minimum slab thickness, the maximum computed long-term deflection and as a result the proposed new formula for the minimum slab thickness. As shown, the proposed formulas for the cases of slab with drop panel have a range from Ln/33 to Ln/21.2 which provides evidence that the single ACI code formula (Ln/33) cannot be satisfactory for all cases.

Table 3. Proposed minimum thickness of flat slab without drop panels that satisfies the ACI limit L/240

| Span (L.L) m | f'_c = 21MPa | f'_c = 35MPa | f'_c = 49MPa | Allowable deflections mm |
|-------------|--------------|--------------|--------------|-------------------------|
| L.L = 1.5 kN/m² | f'_c = 21MPa | f'_c = 35MPa | f'_c = 49MPa | Allowable deflections mm |
| L.L = 3 kN/m² | f'_c = 21MPa | f'_c = 35MPa | f'_c = 49MPa | Allowable deflections mm |
| L.L = 4.5 kN/m² | f'_c = 21MPa | f'_c = 35MPa | f'_c = 49MPa | Allowable deflections mm |
The aspect ratio for edge supported slab, and the modulus of elasticity of concrete. The general form of the proposed geometrical and material characteristics of the flat slab is as follows:

\[
\frac{l_n}{h} = \beta \left[ \frac{\Delta_{inc}}{L_{allow}} \right] \frac{0.0167 \times K_{DP} \times E_c \times b}{K \times K_{SZ} \times K_{AR} \times (AW_{Slab} + \Delta_{def})} \right]^{1/3}
\]

(1)
where: \(L_n\): is the clear span in mm; \(h\): is the minimum thickness in mm; \(\beta\): edge support coefficient (for slab without edge support equals to 1.0 and for edge supported equals to long span / short span); \((\Delta_{in}/L)_{allow}\): is the targeted incremental deflection which equals to 1/480 for flat slab; \(K_{D,P}\): drop panel coefficient which equals to 1.0 for slabs without drop and 1.35 for slabs with drop panels; \(E_C\): is the modulus of elasticity of concrete; \(b\): is the strip width which equals to 1000mm; \(K\): is the coefficient of end support condition which equals to 1.4 for both ends continuous, 2.0 for one end continuous and 5.0 for both ends continuous; \(K_{SS}\): is the coefficient column supported condition of two way slabs which equal to 1.35 for column supported and 1.0 for other cases; \(K_{AR}\): is the edge supported condition which equals to 0.2 + 0.4 \(\beta\) for edge supported slabs and 1.0 for other cases; \(\lambda\): is the time-dependent factor of sustained loads according to ACI 318-14 Code. \(W_C\): is the sustained load in kN/m² which equals to the self-weight plus superimposed dead load plus 0.25 of the live load; and \(W_L(\text{add})\): is the additional live load in kN/m² which equals to 0.75 of the live load.

For comparison reasons, Equation 1 is implemented on the investigated cases of slabs with and without drop panels. The results were listed in Tables 5 and 6. In general, high consistence was found between the results of the Scanlon and Lee equation and the proposed limitations, especially for slabs without drop panels. Higher thickness was recorded by using the equations of Scanlon and Lee than the proposed limitations and the ACI 318 Code limitations. All the output of the equation and the proposed limitations were satisfied the required allowable deflection that indicated by the ACI-318 Code. That demonstrated the efficiency of the proposed limitations by means of agree with the results of the equation and at the same time satisfying the allowable deflection requirements. Moreover, the proposed limitations considered effect of thickness of the drop panels which is neglected in the Scanlon and Lee equation.

Table 5. Minimum thickness of flat slab without drop panels based on the proposed limitations, ACI-318 Code limitations and Scanlon and Lee equation

| Span (L) m | \(f'_{c} = 21\)MPa | \(f'_{c} = 35\)MPa | \(f'_{c} = 49\)MPa |
|-----------|-----------------|-----------------|-----------------|
|           | Proposed t mm   | ACI-318 t mm   | Scanlon and Lee eq. t mm | Proposed t mm   | ACI-318 t mm   | Scanlon and Lee eq. t mm | Proposed t mm   | ACI-318 t mm   | Scanlon and Lee eq. t mm |
| 4         | 125             | 125             | 130.0           | 125             | 125             | 118             | 125             | 125             | 110             |
| 5         | 160             | 160             | 174.0           | 160             | 160             | 157             | 160             | 160             | 147             |
| 6         | 220             | 190             | 222.0           | 190             | 190             | 200             | 190             | 190             | 187             |
| 7         | 260             | 220             | 271.0           | 225             | 220             | 245             | 220             | 220             | 228             |
| 8         | 310             | 255             | 325.0           | 275             | 255             | 292             | 260             | 255             | 273             |
| 9         | 360             | 285             | 381.0           | 330             | 285             | 342             | 300             | 285             | 319             |
| 10        | 425             | 315             | 440.0           | 385             | 315             | 394             | 355             | 315             | 368             |

| Span (L) m | \(f'_{c} = 21\)MPa | \(f'_{c} = 35\)MPa | \(f'_{c} = 49\)MPa |
|-----------|-----------------|-----------------|-----------------|
|           | Proposed t mm   | ACI-318 t mm   | Scanlon and Lee eq. t mm | Proposed t mm   | ACI-318 t mm   | Scanlon and Lee eq. t mm | Proposed t mm   | ACI-318 t mm   | Scanlon and Lee eq. t mm |
| 4         | 125             | 125             | 138.0           | 125             | 125             | 125             | 125             | 125             | 117             |
| 5         | 175             | 160             | 183.0           | 160             | 160             | 165             | 160             | 160             | 155             |
| 6         | 220             | 190             | 231.0           | 200             | 190             | 209             | 190             | 190             | 195             |
| 7         | 280             | 220             | 282.0           | 245             | 220             | 255             | 230             | 220             | 238             |
| 8         | 330             | 255             | 336.0           | 290             | 255             | 303             | 270             | 255             | 283             |
| 9         | 380             | 285             | 393.0           | 340             | 285             | 354             | 320             | 285             | 331             |
| 10        | 445             | 315             | 452.0           | 400             | 315             | 407             | 370             | 315             | 380             |

| Span (L) m | \(f'_{c} = 21\)MPa | \(f'_{c} = 35\)MPa | \(f'_{c} = 49\)MPa |
|-----------|-----------------|-----------------|-----------------|
|           | Proposed t mm   | ACI-318 t mm   | Scanlon and Lee eq. t mm | Proposed t mm   | ACI-318 t mm   | Scanlon and Lee eq. t mm | Proposed t mm   | ACI-318 t mm   | Scanlon and Lee eq. t mm |
| 4         | 135             | 125             | 144.0           | 125             | 125             | 131             | 125             | 125             | 123             |
| 5         | 185             | 160             | 191.0           | 170             | 160             | 173             | 160             | 160             | 162             |
| 6         | 235             | 190             | 240.0           | 210             | 190             | 217             | 200             | 190             | 204             |
| 7         | 290             | 220             | 292.0           | 260             | 220             | 264             | 245             | 220             | 247             |
| 8         | 340             | 255             | 347.0           | 305             | 255             | 314             | 290             | 255             | 293             |
| 9         | 400             | 285             | 405.0           | 355             | 285             | 365             | 335             | 285             | 341             |
| 10        | 460             | 315             | 465.0           | 410             | 315             | 419             | 390             | 315             | 391             |
Table 6. Minimum thickness of flat slab without drop panels based on the proposed limitations, ACI-318 Code limitations and Scanlon and Lee equation

| Span (L) m | $t_{2=0.25\text{t}_{\text{slab}}}$ | $t_{2=0.5\text{t}_{\text{slab}}}$ | $t_{2=0.75\text{t}_{\text{slab}}}$ |
|-----------|----------------------------------|----------------------------------|----------------------------------|
|           | Proposed t mm | ACI-318 t mm | Scanlon and Lee eq. t mm | Proposed t mm | ACI-318 t mm | Scanlon and Lee eq. t mm | Proposed t mm | ACI-318 t mm | Scanlon and Lee eq. t mm |
| 4         | 115            | 115            | 116.0             | 115            | 115            | 116.0             | 115            | 115            | 116.0             |
| 5         | 150            | 145            | 154.0             | 145            | 145            | 154.0             | 145            | 145            | 154.0             |
| 6         | 200            | 175            | 196.0             | 185            | 175            | 196.0             | 175            | 175            | 196.0             |
| 7         | 235            | 200            | 240.0             | 220            | 200            | 240.0             | 200            | 200            | 240.0             |
| 8         | 290            | 230            | 287.0             | 260            | 230            | 287.0             | 240            | 240            | 287.0             |
| 9         | 340            | 260            | 366.0             | 305            | 260            | 366.0             | 280            | 280            | 366.0             |
| 10        | 405            | 290            | 387.0             | 370            | 290            | 387.0             | 340            | 340            | 387.0             |

| Span (L) m | $t_{2=0.25\text{t}_{\text{slab}}}$ | $t_{2=0.5\text{t}_{\text{slab}}}$ | $t_{2=0.75\text{t}_{\text{slab}}}$ |
|-----------|----------------------------------|----------------------------------|----------------------------------|
|           | Proposed t mm | ACI-318 t mm | Scanlon and Lee eq. t mm | Proposed t mm | ACI-318 t mm | Scanlon and Lee eq. t mm | Proposed t mm | ACI-318 t mm | Scanlon and Lee eq. t mm |
| 4         | 120            | 115            | 123.0             | 115            | 115            | 123.0             | 115            | 115            | 123.0             |
| 5         | 165            | 145            | 163.0             | 145            | 145            | 163.0             | 145            | 145            | 163.0             |
| 6         | 205            | 175            | 205.0             | 185            | 175            | 205.0             | 175            | 175            | 205.0             |
| 7         | 245            | 200            | 250.0             | 225            | 200            | 250.0             | 210            | 210            | 250.0             |
| 8         | 305            | 230            | 298.0             | 270            | 230            | 298.0             | 250            | 250            | 298.0             |
| 9         | 360            | 260            | 348.0             | 325            | 260            | 348.0             | 295            | 295            | 348.0             |
| 10        | 425            | 290            | 400.0             | 380            | 290            | 400.0             | 350            | 350            | 400.0             |

| Span (L) m | $t_{2=0.25\text{t}_{\text{slab}}}$ | $t_{2=0.5\text{t}_{\text{slab}}}$ | $t_{2=0.75\text{t}_{\text{slab}}}$ |
|-----------|----------------------------------|----------------------------------|----------------------------------|
|           | Proposed t mm | ACI-318 t mm | Scanlon and Lee eq. t mm | Proposed t mm | ACI-318 t mm | Scanlon and Lee eq. t mm | Proposed t mm | ACI-318 t mm | Scanlon and Lee eq. t mm |
| 4         | 130            | 115            | 129.0             | 120            | 115            | 129.0             | 115            | 115            | 129.0             |
| 5         | 175            | 145            | 170.0             | 165            | 145            | 170.0             | 145            | 145            | 170.0             |
| 6         | 215            | 175            | 214.0             | 200            | 175            | 214.0             | 180            | 175            | 214.0             |
| 7         | 260            | 200            | 260.0             | 240            | 200            | 260.0             | 220            | 220            | 260.0             |
| 8         | 315            | 230            | 308.0             | 285            | 230            | 308.0             | 265            | 265            | 308.0             |
| 9         | 375            | 260            | 359.0             | 340            | 260            | 359.0             | 305            | 305            | 359.0             |
| 10        | 445            | 290            | 411.0             | 410            | 290            | 411.0             | 370            | 370            | 411.0             |

5. Conclusions

The nonlinear Finite Element Analysis was used in order to study the effectiveness of using ACI minimum thickness provisions for flat slab for the controlling of long-term deflection. The analysis involved 126 case studies that considered the effects of several influencing parameters: slab span length, concrete compressive strength, the applied live load, and the thickness of the drop panel. From the analysis results, the main findings can be summarized as follow:

- ACI 318-19 minimum thickness provisions required for the control of deflection in flat slab (with or without drop) cannot be satisfactory (i.e. to comply with the ACI allowable limits L/480 and L/240) for all cases because they consider the effects of only the span length and yield strength of steel and ignoring the effects of the other influencing factors like the concrete compressive strength, live load, and the drop panel thickness. Therefore, these ACI code provisions have a serious problem and need a real revision;

- The effect of using high concrete compressive strength on reducing the long-term deflection was found to be significant especially for small spans. It was observed that the increase in concrete compressive strength from 21MPa to 49MPa decreases the average long-term deflection by (56%, 53%, 50%, 44%, 39%, 33% and 31%) for spans (4, 5, 6, 7, 8, 9 and 10 m) respectively;
In flat slab with drop panel, the use of thicker drop panel has an important positive effect on the reduction of long-term deflection especially for small spans. It was found that varying drop panel thickness $t_2$ from $0.25t_{shlab}$ to $0.75t_{shlab}$ decreases the average long-term deflection by (45, 41, 39, 35, 31, 28 and 25%) for span lengths (4, 5, 6, 7, 8, 9 and 10 m) respectively;

Concerning the live load effect, it was observed that increasing the live load leads to larger long-term deflection but this effect becomes less important with the increase in span lengths;

Formulas for calculating the minimum thickness of flat slab were proposed to vary from $L_n/30$ to $L_n/19.9$ for flat slab without drop panel and from $L_n/33$ to $L_n/21.2$ for flat slab with drop panel;

High constancy was observed between the results of Scanlon and Lee equation and the proposed limitations of the minimum thickness of slabs with and without drop panels.

6. Declarations

6.1. Author Contributions

Conceptualization, B.I.A., M.A., A.B. and A.A.S.; methodology, B.I.A., M.A., A.B. and A.A.S.; software, B.I.A.; validation, M.A., A.B. and A.A.S.; formal analysis, B.I.A.; investigation, B.I.A., M.A., A.B. and A.A.S.; data curation, A.B. and A.A.S.; writing—original draft preparation, B.I.A. and M.A.; writing—review and editing, A.B. and A.A.S.; visualization, B.I.A., M.A., A.B. and A.A.S.; supervision, B.I.A., M.A., A.B. and A.A.S.; project administration, B.I.A., M.A., A.B. and A.A.S. All authors have read and agreed to the published version of the manuscript.

6.2. Data Availability Statement

The data presented in this study are available on request from the corresponding author.

6.3. Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

6.4. Conflicts of Interest

The authors declare no conflict of interest.

7. References

[1] Fanella, David. “Reinforced Concrete Structures: Analysis and Design.” McGraw-Hill Companies, (2016).
[2] McCormac, Jack C., and Russell H. Brown. “Design of Reinforced Concrete.” John Wiley & Sons, (2015).
[3] Darwin, David, Charles William Dolan, and Arthur H. Nilson. Design of concrete structures. Vol. 2. New York, NY, USA:: McGraw-Hill Education, (2016).
[4] ACI Committe 318, “Building Code Requirements for Structural Concrete and Commentary (ACI 318-19),” Farmington Hill, MI, (2019).
[5] Canadian Standards Association, “Design of Concrete Structures for Buildings (CSA A23.3-04),” Rexdale, (2004).
[6] Standards Association of Australia. AS 3600: Concrete Structures.” North Sydney 2009 (2009): 1–108.
[7] European Committee for Standardization CEN, “Eurocode 2: Design of concrete structures -Part 1-1: General rules and rules for buildings (EN 1992-1-1:2004),” Brussels, (2004).
[8] Setareh, Mehdi, and Robert Darvas. “Concrete Structures” (2017). doi:10.1007/978-3-319-24115-9.
[9] Ashraf, Syed Mehdi. Practical Design of Reinforced Concrete Buildings. Practical Design of Reinforced Concrete Buildings. CRC Press, (2017). doi:10.1201/b22298.
[10] Gilbert, R. I. “The Serviceability Limit States in Reinforced Concrete Design.” Procedia Engineering 14 (2011): 385–95. doi:10.1016/j.proeng.2011.07.048.
[11] Pack, Lonnie. Australian Guidebook for Structural Engineers. Australian Guidebook for Structural Engineers. CRC Press, (2017). doi:10.4324/9781315197326.
[12] Tošić, Nikola, Nenad Pecić, Mauro Poliotti, Antonio Mari, Lluis Torres, and Jelena Dragaš. “Extension of the \( \zeta \) -Method for Calculating Deflections of Two-Way Slabs Based on Linear Elastic Finite Element Analysis.” Structural Concrete 22, no. 3 (2021): 1652–70. doi:10.1002/suco.202000558.
[13] Hossain, Tahsin Reza, Salah Uddin Ahmed, and Mohammed Saiful Alam Siddiquee. “Prediction of Short- and Long-Term Deflections of Reinforced Concrete Flat Plates Using Artificial Neural Network.” Journal of Civil Engineering (IEB) 47, no. 2 (2019): 167–77.
[14] Gilbert, R. I. “Deflection Control of Slabs Using Allowable Span To Depth Ratios.” Journal of the American Concrete Institute 82, no. 1 (1985): 67–72. doi:10.14359/10316.

[15] Scanlon, Andrew, and Young Hak Lee. “Unified Span-to-Depth Ratio Equation for Nonprestressed Concrete Beams and Slabs.” ACI Structural Journal 103, no. 1 (2006): 142–48. doi:10.14359/15095.

[16] Caldentey, Alejandro Pérez, Javier Mendoza Cembranos, and Hugo Corres Peiretti. “Slenderness Limits for Deflection Control: A New Formulation for Flexural Reinforced Concrete Elements.” Structural Concrete 18, no. 1 (2017): 118–27. doi:10.1002/suco.201600062.

[17] K. A. Ahmat, "Probabilistic Assessment of ACI 318 Minimum Thickness Requirements for Two-way Slabs", MSc Thesis, University of Sharjah, Sharjah - United Arab Emirates, (2017).

[18] Fahmi, Mereen H., and Ayad Z. Saber. “Modified Minimum Depth-Span Ratio of Beams and Slabs.” International Journal of Emerging Trends in Engineering Research 8, no. 9 (2020): 5571–80. doi:10.30534/ijeter/2020/107892020.

[19] Vollum, R. L., and T. R. Hossain. “Are Existing Span-to-Depth Rules Conservative for Flat Slabs?” Magazine of Concrete Research 54, no. 6 (2002): 411–21. doi:10.1680/macr.2002.54.6.411.

[20] Lee, Young Hak, and Andrew Scanion. “Comparison of One- And Two-Way Slab Minimum Thickness Provisions in Building Codes and Standards.” ACI Structural Journal 107, no. 2 (2010): 157–63. doi:10.14359/51663531.

[21] Bertero, Raul, and Agustin Bertero. “Statistical Evaluation of Minimum Thickness Provisions for Slab Deflection Control.” ACI Structural Journal 116, no. 5 (2019): 301–2. doi:10.14359/51720209.

[22] Hasan, S A, and B O Taha. “Aspect Ratio Consideration in Flat Plate Concrete Slab Deflection.” Zanco Journal of Pure and Applied Sciences 32, no. 5 (2020). doi:10.21271/zjpas.32.5.6.

[23] Sanabra-Loewe, Marc, Joaquín Capellà-Llovera, Sandra Ramírez-Anaya, and Ester Pujadas-Gispert. “A Path to More Versatile Code Provisions for Slab Deflection Control.” Proceedings of the Institution of Civil Engineers - Structures and Buildings (April 15, 2021): 1–15. doi:10.1680/jstbu.20.00205.

[24] Al-Nu’man, B S, and C S Abdullah. “Investigation of a Developed Deflection Control Model of Reinforced Concrete Two Way Slab Systems.” Eurasian Journal of Science and Engineering 4, no. 1 (2018): 18–31. doi:10.23918/eajse.v4i1s18.

[25] Santos, José, and António Abel Henriques. “Span-to-Depth Ratio Limits for RC Continuous Beams and Slabs Based on MC2010 and EC2 Ductility and Deflection Requirements.” Engineering Structures 228 (2021): 111565. doi:10.1016/j.engstruct.2020.111565.

[26] Portland Cement Association. Notes on ACI 318-11. Illinois: Portland Cement Association, (2013).

[27] “ACI Committe 224, “Control of Cracking in Concrete Structures (ACI 224R-08).” Farmington Hill, MI, (2008).