Cosmic homogeneity: a spectroscopic and model-independent measurement

R. S. Gonçalves1⋆, G. C. Carvalho1,2, C. A. P. Bengaly Jr.1,3, J. C. Carvalho1, A. Bernui1, J. S. Alcaniz1,4, and R. Maartens3,5,
1Observatório Nacional, 20921-400, Rio de Janeiro - RJ, Brasil
2Departamento de Astronomia, Universidade de São Paulo, 05508-090 São Paulo - SP, Brasil
3Department of Physics & Astronomy, University of the Western Cape, Cape Town 7535, South Africa
4Physics Department, McGill University, Montreal QC, H3A 2T8, Canada
5Institute of Cosmology & Gravitation, University of Portsmouth, Portsmouth PO1 3FX, United Kingdom

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ABSTRACT
Cosmology relies on the Cosmological Principle, i.e., the hypothesis that the Universe is homogeneous and isotropic on large scales. This implies in particular that the counts of galaxies should approach a homogeneous scaling with volume at sufficiently large scales. Testing homogeneity is crucial to obtain a correct interpretation of the physical assumptions underlying the current cosmic acceleration and structure formation of the Universe. In this Letter, we use the Baryon Oscillation Spectroscopic Survey to make the first spectroscopic and model-independent measurements of the angular homogeneity scale $\theta_h$. Applying four statistical estimators, we show that the angular distribution of galaxies in the range $0.46 < z < 0.62$ is consistent with homogeneity at large scales, and that $\theta_h$ varies with redshift, indicating a smoother Universe in the past. These results are in agreement with the foundations of the standard cosmological paradigm.

Key words: Cosmology: observations – (cosmology:) large-scale structure of Universe

1 INTRODUCTION
The Cosmological Principle constitutes one of the most fundamental pillars of modern cosmology. In past decades, it has been indirectly established as a plausible physical assumption, given the observational success of the standard ΛCDM cosmology, which assumes large-scale homogeneity and isotropy, with structure formation described via perturbations. Although isotropy has been directly tested (Blake & Wall 2002; Bernui et al. 2013; Tiwari & Nusser 2016; Bengaly 2016; Schwarz et al. 2016; Bengaly et al. 2017; Bernal et al. 2017; Javanmardi & Kroupa 2017), homogeneity is much harder to probe by observations (see, e.g., Clarkson & Maartens 2010; Maartens 2011; Clarkson 2012).

As is well known, the smaller the scale we observe, the clumpier the universe appears. However, non-uniformities such as groups and clusters of galaxies, voids, walls, and filaments, are expected in a Friedmann-Lemaître-Robertson-Walker (FLRW) universe according to cosmological simulations. In such a background, a transition scale is also expected, above which the patterns composed by these structures become smoother, eventually becoming indistinguishable from a random distribution of sources. This homogeneity scale $r_h$ has been identified and estimated at $70 - 150\,\text{Mpc}/h$, using data from several galaxy and quasar surveys (Hogg et al. 2005; Scrimgeour et al. 2012; Nadathur 2013; Alonso et al. 2015; Pandey & Sarkar 2015; Laurent et al. 2016; Sarkar & Pandey 2016; Ntelis 2017), although other authors have claimed no evidence for it (Sylos Labini et al. 1998; Sylos Labini 2011; Park et al. 2017). In the context of the ΛCDM paradigm, an upper limit for the homogeneity scale was estimated by Yadav et al. (2010) to be $r_h \sim 260\,\text{Mpc}/h$.

Tests of homogeneity of the matter distribution by counting sources in spheres or spherical caps are not direct tests of geometric homogeneity, i.e. of the Cosmological Principle. Source counts on spatial hypersurfaces inside the past lightcone cannot be accessed by this method, since the counts are restricted to the intersection of the past lightcone with the spatial hypersurfaces. Instead, source counts provide consistency tests: if the count data show that the matter distribution does not approach homogeneity on large scales, then this can falsify the Cosmological Principle. Alternatively, if observations confirm an approach to count ho-
2 R. S. Gonçalves et al.

mogeneity, then this strengthens the evidence for geometric homogeneity – but cannot prove it. A test of homogeneity of the galaxy distribution that does probe inside the past light-cone has been developed by [Heavens et al. (2011), Hoyle et al. (2012)] – but this test is unable to determine a homogeneity scale.

When a length scale $r_0$ is used to probe homogeneity, a further assumption is made – a fiducial FLRW model is assumed a priori, in order to convert redshifts and angles to distances. In order to circumvent this model dependence, one can use an angular homogeneity scale $\theta_h$ (Alonso et al. 2014). It was shown by [Alonso et al. (2015)] that the $\theta_h$ determined from the 2MASS photometric catalogue is consistent with ACDM-based mock samples within 90% confidence level.

In this Letter, we make tomographic measurements of $\theta_h$ in the Luminous Red Galaxies (LRG) sample from the Baryon Oscillation Spectroscopic Survey (BOSS), data release DR12. Because DR12 is a dense, deep galaxy catalogue covering roughly 25% of the sky, it provides an excellent probe of the large-scale galaxy distribution, allowing us to make robust measurements in six very thin ($\Delta z = 0.01$), separated redshift shells in the interval $0.46 < z < 0.62$. This also avoids the additional correlations that would arise due to projection effects [Sarkar & Pandey 2016 Carvalho et al. 2016 2017]. To our knowledge, this is the first time that the characteristic homogeneity scale is obtained with a spectroscopic and model-independent measurement, at intermediate redshifts. In addition, we are able to determine the redshift evolution of $\theta_h$. We ensure further robustness by using four different estimators, which produce results that are compatible with each other and with the predictions of standard cosmology, without assuming any cosmological model a priori.

2 ANALYSIS

2.1 Observational data

The total effective area covered by BOSS DR12 is 9.329 deg$^2$, with completeness parameter $c > 0.7$. As in previous BOSS data releases, DR12 is divided into two target samples: LOWZ (galaxies up to $z \approx 0.4$) and CMASS (massive galaxies with $0.4 < z < 0.7$). They cover different regions in the sky, named north and south galactic cap. Here we are interested in exploring the homogeneity transition at redshifts $z > 0.46$, and we use only the north galactic cap of the CMASS LRG sample.

We divide the DR12 CMASS sample into six thin redshift bins of width $\Delta z = 0.01$, between $0.46 < z < 0.62$. As observed in Table 1, the number of galaxies in each bin is $N_{\text{galaxies}} \geq 18,800$, thus providing good statistical performance for the analysis. Moreover, we choose non-contiguous bins to suppress correlations between neighbouring bins.

2.2 Methodology

For a homogeneous angular distribution, the number counts in spherical caps of angular radius $\theta$ are given by

$$\bar{N}(\theta) = \bar{n} A(\theta), \quad A(\theta) = 2\pi(1 - \cos \theta),$$

where $\bar{n}$ is the angular number density and $A$ is the solid

| $z$     | redshift bins | $N_{\text{galaxies}}$ |
|---------|--------------|-----------------------|
| 0.465   | 0.46 - 0.47  | 22551                 |
| 0.495   | 0.49 - 0.50  | 31763                 |
| 0.525   | 0.52 - 0.53  | 32794                 |
| 0.555   | 0.55 - 0.56  | 29486                 |
| 0.585   | 0.58 - 0.59  | 23997                 |
| 0.615   | 0.61 - 0.62  | 18800                 |

Table 1. The six redshift bins used in the analysis and their properties: mean redshift, bin width, and number of galaxies.

angle of the cap. If the observed number is $N$, we define the scaled number count $N = N/N$, which is obtained in four different ways as presented below. The correlation dimension is

$$D_2(\theta) = \frac{d \ln N}{d \ln \theta} = \frac{d \ln N}{d \ln \theta} + \frac{\theta \sin \theta}{1 - \cos \theta},$$

where the second equality follows from Eq. 1. The homogeneous limit is

$$D_2(\theta) = \frac{\theta \sin \theta}{1 - \cos \theta} \approx 2,$$

where the approximation is accurate to sub-percent level for $\theta \leq 0.34$ rad, i.e., $\sim 20^\circ$.

Estimators for $N$ are defined below, based on their counterparts for $r_0$ [Alonso et al. 2015 Laurent et al. 2016 Ntelis 2017]. In order to estimate the observational results we need to compare the observational data, previously described, with mock catalogues. In our analysis we use twenty random catalogues, generated by a Poisson distribution with the same geometry and completeness as the SDSS-DR12.

2.2.1 Average

This is the most common approach in the literature [Alonso et al. 2014 Ntelis 2017]. We define a cap in the sky of a given angular separation $\theta$ around one galaxy, counting how many galaxies are inside this region. We repeat the process considering each galaxy as the centre (‘cen’) of a cap for different angular separation values, and for each redshift bin, thus obtaining a number count average in each case. The same process is replicated for the random catalogue, and we define the estimator as the ratio of the averages:

$$N(\theta)_{\text{Ave}} = \frac{\sum_i N_{\text{cen}}^{obs} / M_{\text{cen}}^{obs}}{\sum_i N_{\text{ran}}^{cen} / M_{\text{cen}}^{cen}},$$

where the total number of galaxies used as centres of caps are equal in both catalogs, $M_{\text{cen}}^{obs} = M_{\text{cen}}^{cen}$. Then we calculate $D_2(\theta)_{\text{Ave}}$ via Eq. 2. Finally, we repeat the previous steps for twenty random catalogues, obtaining a mean value and a standard deviation for $D_2(\theta)_{\text{Ave}}$.

2.2.2 Centre

First we calculate the ratio of the observed and random counts-in-caps centred on the first galaxy, using the equiva-

https://data.sdss.org/sas/dr12/boss/iss/

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lent position in the random catalogue. Then we repeat the process for each centre in both catalogues, obtaining
\[ N(<\theta)_{\text{Con}} = \frac{1}{M_{\text{ran}}^{\text{cen}}} \sum N_{\text{ran}}^{\text{obs}} / N_{\text{ran}}^{\text{cen}}. \]  
(5)

We calculate \( D_2(\theta)_{\text{Con}} \) via Eq. (2), and then repeat the previous steps for twenty random datasets in order to calculate its mean and standard deviation.

2.2.3 Peebles-Hauser (PH)

We follow the Peebles-Hauser estimator (Peebles & Hauser 2000) estimator, but instead of using the number of galaxies, we estimate the scaled counts-in-caps by the number of pairs within a given angular separation in the catalog. We define \( DD(\theta) \) as the number of pairs of galaxies (for a given \( \theta \)) normalized to the total number of pairs, \( M_{\text{ran}}^{\text{obs}}(M_{\text{ran}}^{\text{obs}} - 1)/2 \). We define \( RR(\theta) \) equivalently for the random catalogue. Then
\[ N(<\theta)_{\text{PH}} = \frac{\sum_{\phi=0}^\theta DD(\phi)}{\sum_{\phi=0}^\theta RR(\phi)}, \]  
(6)

and \( D_2(\theta)_{\text{PH}} \) follows from Eq. (2). As above, this procedure is repeated for the other random catalogues, from which we obtain the mean and standard deviation for \( D_2(\theta)_{\text{PH}} \).

2.2.4 Landy-Szalay (LS)

We use an estimator based on the Landy-Szalay correlation function (Landy & Szalay 1993). In addition to the previous definition, we define \( DR(\theta) \) as the number of pairs of galaxies between the observational and random catalogues, for a given \( \theta \), normalized by \( M_{\text{ran}}^{\text{obs}} M_{\text{ran}}^{\text{obs}} \). Following a similar routine to the PH estimator, we obtain
\[ N(<\theta)_{\text{LS}} = 1 + \frac{\sum_{\phi=0}^\theta [DD(\phi) - 2DR(\phi) + RR(\phi)]}{\sum_{\phi=0}^\theta RR(\phi)}. \]  
(7)

We again calculate \( D_2(\theta)_{\text{LS}} \) via Eq. (2), and after repeating this step for the other random data, we obtain the mean and standard deviation for \( D_2(\theta)_{\text{LS}} \).

2.2.5 Estimation of \( \theta_h \)

In order to estimate the homogeneity scale, \( \theta_h \), for each one of the previous methods, we perform the following approach: we make a model-independent polynomial fit for each \( D_2 \) set, in each redshift slice (exemplified for one redshift slice in Fig. 1). Following previous analyses (Alonso et al. 2014, 2015; Ntelis 2017), we identify the scale of transition as the angle at which the fits of our estimator are within one per cent of the homogeneous limit \( \theta_h \), given by Eq. (3). Although arbitrary, the 1%-criterion is widely used in the literature, and is justified given the sample noise. Given the values of \( \theta_h \), we perform a bootstrap analysis (Efron & Gons 1983) on these values with 1000 realisations and we obtain the mean and error with 68% c.l. for the \( \theta_h \) (Table 2).

3 RESULTS

Figure 1 presents the fits of the correlation dimension for the four estimators, showing the crossing of the homogeneity threshold. We illustrate only the redshift slice \( 0.49 < z < 0.50 \), since the results for the other slices are very similar. The corresponding numerical results for \( \theta_h \) and their
and calculate the parameters $\alpha$ and $\beta$ for each estimator. The results are shown in Table 3. One can see the four estimators produce similar $\theta_h$ values. Additionally, there is a clear correlation between $\theta_h$ and $z$: for lower $z$, the transition angular scale increases, as illustrated in Fig. 2. This is the expected behaviour, since matter perturbations grow stronger in later epochs, so that the Universe should appear clumpier as the redshift decreases. To better visualize this correlation, we perform a linear fit,

$$\theta_h(z) = \alpha + \beta z,$$

and calculate the parameters $\alpha$ and $\beta$ for each estimator. The results are shown in Table 3. One can see the four estimators produce similar $\theta_h$ values.

In order to compare our results with previous model-dependent analyses, we convert the $\theta_h$ measurements in Table 3 into the corresponding physical distance, as illustrated in Fig. 2. This is the expected behaviour, since matter perturbations grow stronger in later epochs, so the Universe should appear clumpier as the redshift decreases. To better visualize this correlation, we perform a linear fit,

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