Observed quantization of anyonic heat flow

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The quantum of thermal conductance of ballistic (collisionless) one-dimensional channels is a unique fundamental constant1. Although the quantization of the electrical conductance of one-dimensional ballistic conductors has long been experimentally established2, demonstrating the quantization of thermal conductance has been challenging as it necessitated an accurate measurement of very small temperature increase. It has been accomplished for weakly interacting systems of phonons3–5, photons6 and electronic Fermi liquidsd–g; however, it should theoretically also hold in strongly interacting systems, such as those in which the fractional quantum Hall effect is observed. This effect describes the fractionalization of electrons into anyons and chargeless quasiparticles, which in some cases can be Majorana fermions2. Because the bulk is incompressible in the fractional quantum Hall regime, it is not expected to contribute substantially to the thermal conductance, which is instead determined by chiral, one-dimensional edge modes. The thermal conductance thus reflects the topological properties of the fractional quantum Hall electronic system, to which measurements of the electrical conductance give no access9–12. Here we report measurements of thermal conductance in particle-like (Laughlin–Jain series) states and the more complex (and less studied) hole-like states in a high-mobility two-dimensional electron gas in GaAs–AlGaAs heterostructures. Hole-like states, which have fractional Landau-level fillings of 1/2 to 1, support downstream charged modes as well as upstream neutral modes13, and are expected to have a thermal conductance that is determined by the net chirality of all of their downstream and upstream edge modes. Our results establish the universality of the quantization of thermal conductance for fractionally charged and neutral modes. Measurements of anyonic heat flow provide access to information that is not easily accessible from measurements of conductance.

The fractional quantum Hall state, first observed in 1982, still provides a spectrum of challenges. The universal quantized electrical Hall conductance $G_{1T}$, with current $I$ flowing in downstream, chiral, one-dimensional edge modes, is directly related to the filling $v$ of Landau levels in the bulk: $G_{1T} = dI/dV = vG_0$, with $V$ the voltage, $G_0 = e^2/h$ the quantum of electrical conductance, $e$ the elementary charge and $h$ the Planck constant. The nature and number of the edge modes are not dictated by topological considerations and may take different values for different fractional quantum Hall states at the same Landau-level filling10,11. Consequently, the electrical conductance reflects the number and conductance of downstream charged chiral modes, but is independent of the total number of modes, their chirality and their character.

The thermal conductance $g_0 = dJ_0/dT$, with $J_0$ the heat current and $T$ the temperature, was studied for a single heat-conducting channel first by John Pendry in 198312, who set an upper bound on the thermal conductance of $g_0 = k_0 T$, with $k_0 = \pi^2 k_B^2/(3h)$ and $k_B$ the Boltzmann constant. For edge states of Abelian quantum Hall states, both integer and fractional, Kane and Fisher13 showed that the thermal conductance is closely related to the net number of edge modes: $g_0 = N_{\text{net}} k_0 T$, with $N_{\text{net}} = N_d - N_u$ defined as the difference between the numbers of downstream and upstream edge modes10.

Here we extend previous studies of thermal transport13–16 to a strongly interacting system provided by the fractional quantum Hall effect2. Our aim is to test the notion that the quantum of thermal conductance is universal and therefore independent of the charge of the quasiparticles. Specifically, we first establish our method by studying non-interacting electrons ($v = 1$ and $v = 2$). We then extend the study to the particle-like $v = 1/3$ Laughlin state, with electrical conductance $G_{1T} = (1/3)G_0$ and quasiparticle charge $e^* = e/3$, which is expected to have one unit of the thermal quantum conductance ($g_0 = 1/k_0 T$, without the factor of $1/3$). Finally, we study the intriguing and complex hole-like states $v = 2/3$, 3/5 and 4/7, in which case the fractional quantum Hall liquid also supports upstream chiral edge modes13; for example, the standard picture of the $v = 2/3$ state is a downstream charge mode with electrical conductance $G_{1T} = (2/3)G_0$ and an upstream chargeless (neutral) mode8,10,13–16. In general, these topological modes might be augmented by extra pairs of counter-propagating modes if the confining potential is soft and if edge reconstruction is established16. For $v = 2/3$, because $N_{\text{net}} = 0$ the heat flow must be fully diffusive and so the net thermal conductance should approach zero for increasing system size15. For the other hole-like states

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Figure 1 | Configuration of the device. The device is fabricated in a high-mobility two-dimensional electron gas embedded in a GaAs–AlGaAs heterostructure. An SEM micrograph of the four-armed device, which has a small floating ohmic contact at its centre (green; the depleting groves underneath are not visible) and a quantum point contact (QPC) in each arm (an air bridge shorts the two sides of the split-gate of the QPC). The source (S), drain (D1 and D3) and ground (G) contacts are drawn not to scale. In this example, $v = 2$ and QPC2 and QPC4 are fully pinched ($M = N = 2$), while QPC1 and QPC3 transmit only the outmost edge mode and fully reflect the inner mode ($t_0$ is the transmission coefficient of QPC). The source current $(I_s, \text{red})$ impinges on QPC1, which transmits a current $I_0$ (here $I_{in} = I_2/2$), which is absorbed in the floating contact. Edge modes (green) at potential $V_{\text{DC}}$ and temperature $T_{\text{m}}$ move from the floating contact into the four arms (in arms 2 and 4 they are fully reflected). Cold edge modes, at temperature $T_{\text{b}}$ (blue) arrive from the grounded contacts. A resonant (LC) circuit at each drain, with centre frequency $f_0 = 695 \text{kHz}$ and bandwidth $\Delta f = 10–30 \text{kHz}$, depending on $v$, filters the signal, which is amplified by the voltage pre-amplifier (cooled to 4.2 K) and then by a room-temperature amplifier (total gain is about 1,000). The amplified signal is measured by a spectrum analyser with similar $f_0$ and $\Delta f$. 

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studied, \( v = 3/5 \) and \( v = 4/7 \), the net heat current is expected to propagate in the upstream direction\(^{10,12}\).

Our experimental set-up is based on the core idea put forward in ref. 8, wherein the thermal conductance in the integer quantum Hall regime was measured. However, here we exploit a more flexible implementation: a scanning electron microscopy (SEM) image of the ‘heart’ of the four-armed structure is shown in Fig. 1 (for a full description of the device, see Extended Data Fig. 1). A DC input power is provided by a source voltage \( V_s \) and driving current \( I_s = G_j V_s \), with propagating power \( P_0 = 0.5 I_s V_s \). The current impinging on a floating reservoir, \( I_m = I_0 \beta \), where \( I_0 = V_{QPC}/v \) is the transmission coefficient of quantum point contact 1 (QPC1), \( V_{QPC} \) is its Landau-level filling and \( v \) is the filling of the system as a whole. The outgoing current from the floating reservoir splits into \( M \) arms (\( M = 2, 3 \) or 4, as determined by the QPCs in the arms; equal splitting of the current into the open arms is demonstrated in Extended Data Fig. 2), carrying power \( P_{out} = P_m/N \), where \( N \) is the number of outgoing current-carrying modes, with the dissipated power in the reservoir given by \( \Delta P = P_m - P_{out} = 0.5I_m V_s (1 - N^{-1}) \). The reservoir reaches a new thermal equilibrium at temperature \( T_m \), at which point the dissipated power is equal to the sum of the power carried by phonons (to the bulk) and that of the chiral edge modes: \( \Delta P = \Delta P_{ph} + \Delta P_\beta \). If universality is established, then \( \Delta P_\beta = 0.5Nc_\beta(T_m^2 - T_0^2) \) for the electron base temperature; the heat carried by the phonons is expected to obey \( \Delta P_{ph} = \beta(T_m^3 - T_0^3) \), where \( \beta \) is a constant that depends on the size of the heated reservoir\(^{17}\).

The heat carried by the phonons is negligible in comparison to the heat carried by the phonons (to the bulk) and that of the chiral edge modes. We deduce the thermal conductance with a reasonable accuracy (as shown in Extended Data Fig. 2c, inset). We also find the phonon coefficient \( \beta \) for the floating reservoir, \( \beta \approx 3 - 5 \) nW K\(^{-1}\). Similar measurements were also performed for \( v = 1 \), for which there are fewer configurations (because there is one charge mode in each arm). Here, the average thermal conductance per one-dimensional mode was found to be \( g_0 = (0.98 \pm 0.03) \kappa_0 T \) (where here and henceforth \( T = (T_0 + T_m)/2 \) and the specified error is the standard deviation), in an excellent agreement with the expected quantization (Fig. 2c, inset). We also find the phonon coefficient \( \beta \) for the floating reservoir, \( \beta \approx 3 - 5 \) nW K\(^{-1}\). Similar measurements were also performed for \( v = 1 \), for which there are fewer configurations (because there is one charge mode in each arm). Here, the average thermal conductance per one-dimensional mode was found to be \( g_0 = (0.98 \pm 0.03) \kappa_0 T \) (see Extended Data Fig. 3). Note that the error specified for \( g_0 \) was calculated from only the randomly scattered data; systematic errors (if they exist) were neglected (see Methods).

We now describe measurements for the most prominent fractional state, \( v = 1/3 \). Because this state is the first filled Landau level of composite fermions (\( v_{CF} = 1 \)), with electronic filling \( v = v_{CF}/(2v_{CF} + 1) \) (or otherwise must be subtracted); (iv) the outgoing current must split equally between all of the conducting arms; (v) full charge and temperature equilibration was assumed to take place in the floating contact (charging of the floating contact is negligible due to the very short charging (RC) time)\(^2\); and (vi) heat flow via phonons is independent of the number of conducting arms, and depends only on the temperature \( T_m \). We address issues concerning propagation length and energy flow through the bulk later.

First we elaborate on measurements at \( v = 2 \), at which two edge modes leave the source and QPC1 determines the number of modes that impinge on the floating contact. The excess thermal noise \( S_p \) was measured as a function of \( I_0 \) for a few different filling factors of the QPCs in the arms, \( V_{QPC} \) (Fig. 2a). The temperature \( T_m \), determined from the thermal noise (see Methods), is plotted as a function of the dissipated power \( \Delta P \) in Fig. 2b (in this case \( T_0 = 30 \) mK). Because the temperature of the electrons was relatively high in these measurements, the phonon contribution was subtracted (as demonstrated in Fig. 2c): \( \delta P = \Delta P(N_i, T_m) - \Delta P(N_j, T_m) \), with \( N_i \) and \( N_j \) the total number of outgoing modes from the central floating contact for different QPC settings. The normalized coefficient of the dissipated power, defined as \( \lambda = \delta P/(\kappa_0/2) \), is plotted for six different configurations of \( N_i = N_j \) in Fig. 2c. The average thermal conductance of a single electronic mode was found to be \( g_0 = (0.98 \pm 0.03) \kappa_0 T \) (see Extended Data Fig. 3). Note that the error specified for \( g_0 \) was calculated from only the randomly scattered data; systematic errors (if they exist) were neglected (see Methods).

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(ref. 18), it harbours a single chiral downstream edge mode. (This is expected when the edge potential is sharp; for a soft edge potential, edge reconstruction may add pairs of counter-propagating modes, but these should not affect the thermal conductance\(^{16,19}\).) Consequently, the thermal conductance of this state is expected to be \( g_Q = 1 \kappa_0 T \) — the same as that of the \( v = 1 \) state. We studied this state in the lower-density sample (see Methods), which has a larger floating contact (the measured reflection coefficient from that contact was about 2%). With a lower electron temperature, \( T_0 \approx 10 \) mK, the power dissipation was kept small, thus avoiding a large increase in \( T_m \) and enabling us to neglect the contribution of the phonons altogether. Source noise (on top of the direct current), multiplied by a factor of \( N^{-2} = N^{-2} \) (here and henceforth, because each arm has one charge mode, \( N = M \)), was subtracted from the measured thermal noise. Because both the electrical conductance \( (G_0) \) and temperature range \( (T_m) \) were small, the excess thermal noise was small, thereby necessitating a long averaging time.

The dissipated power as a function of \( T_m^2 \) is plotted in Fig. 3a for two configurations, \( N = 4 \) and \( N = 2 \), along with the theoretically expected values. A linear fit (dashed line) to the data gives \( g_Q = (1.00 \pm 0.045) \kappa_0 T \), once again in excellent agreement with expectations.

The hole-like states in the first Landau level, with fractional fillings \( 1/2 < v < 1 \), belong to the same composite-fermion hierarchy as the particle-like states. (However, the residual magnetic field that remains after flux attachment of two flux quanta is opposite in direction to the original field and so edge modes propagate in both chiralities.) The composite-fermion filling in these states is related to the actual electron fractional filling via \( v = v_{CF}/(v_{CF} - 1) \). We studied the states \( v = 2/3, v = 3/5 \) and \( v = 4/7 \), which have theoretically predicted thermal conductances of \( g_Q = 0 \), \( g_Q = -1 \kappa_0 T \) and \( g_Q = -2 \kappa_0 T \), respectively. Astonishingly, for \( v = 3/5 \) and \( 4/7 \), heat is expected to propagate in the opposite direction to that of the charge current (hence the negative signs)\(^{10,12}\), however, the actual sign of the net chirality could not be determined with our device configuration.

In our experiment, excited downstream modes leave the floating contact, but retain an electrochemical potential and temperature \( T_m \) equal to those of the floating contact, and propagate along one edge of each arm, while excited upstream neutral modes, also heated to \( T_m \), emanate from the opposite side of the contact and propagate along the other edge of the arm. Charge and neutral modes arrive back from the grounded contacts (which are at \( T_0 \); blue lines in Fig. 1), intermix and equilibrate with the hotter downstream modes, thereby returning energy to the floating contact. The initial ballistic propagation of downstream modes undergoes an inter-edge scattering at long enough propagation length, and heat dissipation via the edge modes is restricted.

To understand the difference between the quantization of the thermal conductance at \( v = 2/3 \) and at other filling factors, we consider a simplified model of thermal transport on an edge of a quantum Hall state with one downstream mode \( (N_1 = 1) \) and \( N \) upstream modes, with \( N_1 = 1 \) (see below) at \( v = 2/3 \), \( N_3 = 2 \) at \( v = 3/5 \) and \( N_3 = 3 \) at \( v = 4/7 \). Using conservation of energy and adopting Newton’s law of cooling for thermal relaxation between counter-propagating modes on the same edge, we obtain a linear system of differential equations that describes the heat flow in this system (see equations (4)–(7) in Methods). We find that for \( N_0 > N_1 \) the temperature of the modes converges to that of the upstream modes. The temperature difference between the downstream mode and the upstream modes vanishes as a decaying exponential function of the distance from the floating contact (with a relaxation length \( \xi \)). When the size of the system is much larger than the relaxation length \( \xi \), the heat is carried along one edge of each arm and the quantization should be precise. In contrast, for \( N_0 < N_1 \) (as is the case for \( v = 2/3 \), the temperatures of the modes depend only linearly on the distance from the contacts, and the temperature difference between the downstream and upstream modes remains constant along the edge for a certain length of each arm. Consequently, the thermal conductance approaches zero only linearly in \( 1/L: g_Q \rightarrow 2 \kappa_0 T/(1 + L/\xi) \). Furthermore, because this is true for both edges, heat flows equally along both edges of each arm.

We measured the thermal noise in these three hole-like states. The total dissipated power (including phonons) for \( N = 4 \) is plotted as function of \( T_m^2 \) in Fig. 4a, along with the theoretically expected values. The normalized power \( \lambda \), for \( \Delta N = N_1 - N_j = 4 - 2 = 2 \), is plotted as function of \( T_m^2 \) in Fig. 4b. The state with \( v_{CF} = 2 \) or \( v = 2/3 \) is expected to support an equal number of downstream and upstream modes\(^{9,10}\). Our current understanding is that there are two downstream modes, each with conductance \( g_0/3 \), and two upstream modes, which are neutral\(^{19,21}\). The partitioned charge in a partly pinched QPC (for example, for \( v_{CF} = 1/3 \)) has previously been measured via shot noise\(^{21}\) and has been verified to be \( v^* = 2e/3 \) (see Extended Data Fig. 4 for the thermal noise and Extended Data Fig. 5 for the shot noise).

The total out-flowing electronic power \( \Delta P \) for \( M = N = 4 \) is plotted as function of \( T_m^2 \) in Fig. 4a \( (v = 2/3 \text{, green data}) \); the data deviates from the expected zero thermal conductance (solid line). In Fig. 4b we plot \( \lambda \), for \( \Delta N = N_1 - N_j = 4 - 2 = 2 \), as a function of \( T_m^2 \) \( (v = 2/3 \text{, green data}) \). In both measurements, we find a thermal conductance of \( g_Q = (0.33 \pm 0.02) \kappa_0 T \). This value decreased to \( g_Q \approx 0.25 \kappa_0 T \) when the electron temperature was increased to \( T_0 \approx 30 \) mK. With the two edges of each arm being symmetric (a downstream hot mode and an upstream cold mode), the thermal conductance in each edge is
Figure 4 | Measurements of the fractional quantum Hall effect at hole-like states. a, Total power $\Delta P$ carried out from the floating contact by the fractional edge modes for $N = 4$ in the three hole-like states as function of $T_m^2$ ($v = 2/3$, green; $v = 3/5$, blue; $v = 4/7$, red). The corresponding expected theoretical results, $\Delta P = 0.5 \times 4N_{\text{net}}^2 \gamma^2 \Delta T / m$, with $N_{\text{net}}^2 \gamma$ the expected thermal conductance ($N_{\text{net}} = 0$ for $v = 2/3$, $N_{\text{net}} = -1$ for $v = 3/5$ and $N_{\text{net}} = -2$ for $v = 4/7$), are plotted as solid lines; note that, in our device, we see only the absolute value of the thermal conductance. The root-mean-square of the data points deviates from the expected values by 12% for $v = 3/5$ and by 8.5% for $v = 4/7$; deviation in the case of $v = 2/3$ is large (see text). $T_b = 10$ mK. b, A plot of $\lambda / \Delta N = 8P / (K_2/2) / \Delta N$, with $\delta P = \Delta P(N = 4, T_m) - \Delta P(N = 2, T_m)$, so any phonon contribution is subtracted, as function of $T_m^2$. The slopes of the linear fits (dashed lines) agree quite well with the predictions for the thermal conductance, except for the $v = 2/3$ state. Arrows show models of the minimal edge channels for each filling factor: solid arrows depict downstream charged modes and dashed arrows depict upstream neutral modes.

g_0 \approx (0.17–0.13) \gamma^2 \Delta T$, which is not a large deviation from the expected thermal conductance of zero.

Taking an average arm length of $L \approx 150 \mu$m, we find that $\xi \approx 30 \mu$m for $T_b = 10$ mK and $\xi \approx 20 \mu$m for $T_b = 30$ mK. These findings are consistent with the increased rate with which energy is exchanged as the base temperature is increased. Interestingly, further increases in the rate of energy exchange lead to a quantization of thermal conductance that improves with increasing temperature. However, non-zero heat flow through the bulk cannot be excluded in this case, owing to the low thermal conductivity of the edges.$^{16}$

The state with $v = 3$ or $v = 3/5$ is expected to have a single upstream neutral mode ($N_+ = 1$, $N_- = 0$), which carries heat in the limit of long propagation length. Although the total number of modes that are supported by the state is unknown, by gradually pinching a QPC we find two clear lower plateaus at $v_{\text{QPC}} = 2/5$ and $v_{\text{QPC}} = 1/3$, suggesting that at least two underlying modes (see Extended Data Fig. 5). Shot noise of the partitioned current in one of these plateaus revealed a quasiparticle charge of $e^* = 3e/5$ (see Extended Data Fig. 5), accompanied by upstream neutral modes.$^{13}$

We plot the total power dissipation $\Delta P$ for $N = 4$ in Fig. 4a, and the normalized coefficient $\lambda$ in Fig. 4b ($v = 3/5$, blue data). We find an average thermal conductance $g_Q = (1.04 \pm 0.03) \gamma^2 \Delta T$, which agrees very well with our expectations. Increasing the electron temperature to $T_b = 30$ mK did not alter the result, presumably owing to the short equilibration length $\xi$.

In the state with $v = 3$ or $v = 4/7$, $N_+ = N_- = -2$ and the equilibration length is expected to be even shorter (see Methods). Following a similar procedure as described above, we observed three plateaus in a gradually pinched QPC, at $v_{\text{QPC}} = 3/7$, $v_{\text{QPC}} = 2/5$ and $v_{\text{QPC}} = 1/3$, suggesting a larger number of underlying modes (see Extended Data Fig. 5). The partitioned quasiparticle charge (measured in any of these plateaus) was found to be $e^* = 4e/7$. We found a nicely quantized thermal conductance of $g_Q = (2.04 \pm 0.05) \gamma^2 \Delta T$ (Fig. 4a, b), reconfirming, again, that the equilibration length is much shorter than the system size ($\xi \ll L$).

To summarize, the extraordinarily precise quantization of the electrical Hall conductance is a good example of a topological phenomenon in physics. Yet, the conductance is just one signature of the topological order, and different orders may exhibit the same conductance. The measurement of topological properties that distinguish between such orders is a key challenge. We address this challenge here by successfully measuring the thermal Hall conductance experimentally. Beyond the obvious difficulties associated with the measurement of thermal Hall conductance, our measurements indicate that the topological protection of this quantization is weaker than that of the electrical Hall conductance; for example, the thermal conductance might depend on the propagation length and on possible heat flow through the bulk.

We verified the value of the quantum thermal conductance of six different quantum Hall states: two integer states, a particle-like fractional state and three hole-like states. Our results are consistent with the fundamental theory that predicts that fractional modes and chargeless modes carry the same heat as the non-interacting electronic ones (irrespective of their velocities and of the interaction with other chiral modes or the bulk). This type of experiment can be extended to other poorly understood quantum Hall states; for example, there are important questions about the second Landau level, which might host a family of non-Abelian states (such as $v = 5/2$ and $v = 12/5$). Some of the proposed topological orders may serve as a platform for universal (topologically based) quantum computations. Thus far, these states have remained enigmatic, with no definitive proof of their nature. Measurements of their thermal conductance would provide compelling evidence of their topological order.

Online Content Methods, along with any additional Extended Data display items and Source Data, are available in the online version of the paper; references unique to these sections appear only in the online paper.

Received 31 October 2016; accepted 3 March 2017.

Published online 17 April 2017.

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Acknowledgements We acknowledge the help and advice of R. Sabo, I. Gurman, N. Ofek, H.-K. Choi and D. Mahalu. M.H. acknowledges the European Research Council under the European Community’s Seventh Framework Programme, grant agreement number 339070, the partial support of the Minerva Foundation, grant number 711752, and, together with V.U., the German Israeli Foundation (GIF), grant number I-1241-303.10/2014, and the Israel Science Foundation (ISF). A.S. and Y.O. acknowledge the European Research Council under the European Community’s Seventh Framework Programme (FP7/2007-2013)/ERC, grant agreement number 339070, and the Israel Science Foundation, ISF agreement number 13335/16. Y.O. acknowledges CRC 183 of the DFG. D.E.F. acknowledges support by the NSF under grant number DMR-1205715.

Author Contributions M.B., A.R. and M.H. designed the experiment. M.B. and A.R. fabricated the devices with input from M.H. A.R. participated in initial measurements. M.B. and M.H. performed the measurements, did the analysis and guided the experimental work. Y.O., D.E.F. and A.S. worked on the theoretical aspects. V.U. grew the heterostructures in which the two-dimensional electron gas (2DEG) is embedded. All authors contributed to writing the manuscript.

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Reviewer Information Nature thanks G. Gervais and the other anonymous reviewer(s) for their contribution to the peer review of this work.
Device fabrication. Two different heterojunctions, each hosting a 2DEG, were used: the high-density 2DEG, used in the integer regime, with electron density $n_e = 1.1 \times 10^{11} \text{cm}^{-2}$, and a 4.2-K dark mobility of $\mu = 4 \times 10^6 \text{cm}^2 \text{V}^{-1} \text{s}^{-1}$, a 2DEG depth below the surface of 113 nm, a spacer to donors of 80 nm and a quantum-well width of 30 nm; and the low-density 2DEG, used in the fractional regime, with $n_e = 0.88 \times 10^{11} \text{cm}^{-2}$, a 4.2-K dark mobility of $\mu = 4.6 \times 10^5 \text{cm}^2 \text{V}^{-1} \text{s}^{-1}$, a 2DEG depth below the surface of 128 nm, a spacer to donors of 95 nm and a quantum-well width of 30 nm. Because the electron density tends to decrease near the edge of the contacts, a larger floating contact was fabricated in the low-density 2DEG to minimize reflection from the contact. Etched groves under the floating contact (not visible in the SEM micrograph) made sure that the impinging current enters the bulk of the metal ohmic contact before splitting to the different arms.

Evaluation of QPCs and contacts. All of the QPCs were biased at a gate voltage of +0.3 V during the cooling process. This process makes the QPCs more stable and tends to sharpen the confining potential. Yet, in the low-density 2DEG and at composite fractions, such as $v = 2/5$ and $v = 4/5$, the inner edge mode was found to be partly reflected. Even for a zero-bias-cooling process, the QPCs always reflected a sizable part of the impinging current.

Source noise. Some ohmic contacts produced noise at high magnetic field (fillings $v < 1/2$), which could be related to a lower density near their periphery. The source noise could be as high as $1.5 \times 10^{−20} \text{A} \cdot \text{Hz}^{−1}$ at the highest source current. In such a case (for $v = 1/3$), the source noise was divided by 4, 9, or 16, for $N = 2, 3, 4$, respectively, and subtracted from the measured thermal noise.

Small floating contact. Reflection was measured by comparing the fully reflected current from a pinched QPC (current impinging from the source) and the reflected current for $N = 2$ (or larger). The reflection was always less than 2% in all of the measured fractional states. Assuming this reflection led to shot noise, its magnitude would have been much smaller than the measured thermal noise.

Branching of current to arms. Because the small floating contact might not have the same contact resistance in each of the four arms, equal branching of the impinging current into each arm must be verified. Hence, the transmitted current(s) into the arms of the amplifier(s) was (were) measured when the QPCs in the other arms were either open or fully pinched (see Extended Data Fig. 2).

Calibration of the gain. Knowing the gain of the amplification chain is crucial for determining the electron temperature $T_e$. Two calibration regions, each one in a different arm, were each composed of an additional QPC and two contacts (see Extended Data Fig. 1). Two methods were used to calculate the gain: (i) verification of the well-known quasiparticle charge at a known temperature; and (ii) comparison of the gains in different Hall states. The gain (of the amplification chain, composed of two amplifiers and a spectrum analyser) was calibrated using shot-noise measurements. The temperature is first determined, independent of the gain, by a linear extrapolation of the noise–current curve to zero noise. The intersection point is $v_{th} = 2k_BT_e$ with $v_{th} = ln(h/e\tau)$ the Hall voltage (see Extended Data Fig. 3b).

$\Delta P = \frac{1}{2} G \frac{\partial V_{QPC}}{\partial I_{QPC}} - \frac{\partial V_{QPC}}{\partial I_{QPC}}$.

The temperature $T_m$ is related to the thermal noise via $S_n = 2G^2 k_B(T_m - T_0)$, with

$\frac{1}{G} = \frac{1}{G_{\text{amp}}} + \frac{1}{\sum_{n=1}^{N} G_{i}}$, $G_{i}$

where $G_{i}$ is the conductance of the ith arm.

Thermal conductance in hole-like states. (See Extended Data Fig. 6.) Here, we discuss heat transport from the floating ohmic contact at the position $x_p$ at temperature $T_m$ and from the grounded contact at position $x_0$ at a bath temperature $T_0$. In the Abelian phases of the fractional quantum Hall regime, we expect to have $n_q$ modes emanating from the floating ohmic contact (and moving downstream) and $n_q$ modes emanating from the bath (grounded contacts, moving upstream). In the upper edge, $n_q$ and $n_q$ interchange —$n_q$ modes emanate from the floating ohmic contact and $n_q$ from the bath. For example, for $v = 1$ and $v = 1/3$, $n_q = 1$ and $n_q = 0$; for $v = 2/3$, $n_q = 1$; and for $v = 3/5$, $n_q = 1$ and $n_q = 2$. Henceforth, we discuss the heat flow only at the lower edge of the arms, and add the contribution from the upper edge of the arms at the end of the calculation. To analyse the system, we assume that there is a negligible heat flow through the bulk between the lower and the upper edges. This assumption might not be always valid.

The counter-propagating modes along one edge emanate from two different contact boundaries and are therefore generally at different temperatures. We expect that energy will flow from the hot to the cold modes, owing to their interaction, and denote the energy current between them by $j_{x}(x)$. Because thermal energy is conserved, we have the following relation between the downstream heat current $j_{x}(x)$, the upstream heat current $j_{x}(x)$ and $j_{x}(x)$:

Using the one-dimensional relation between the heat current and temperature, $J = KT^2$, with

$K = \frac{1}{2} \frac{\kappa_{th}^2}{3h}$

we obtain the differential relations

$\frac{d I_{x}^2(x)}{dx} = j_{x}(x)$

$\frac{d I_{x}^2(x)}{dx} = j_{x}(x)$

Here $j_{x}(x)$ is the heat current density between the downstream and upstream modes (heat exchanged divided by $k_B$). In practice, the physical behaviour of $j_{x}(x)$ can be complex because a few upstream and downstream modes with different energy relaxation rates are in action and the temperature imbalance between them may depend on the position $x$. However, in a simplified model, we group together all downstream modes and all upstream modes, and adopt Newton’s law of cooling, which suggests that

$\frac{d}{dx} = - \frac{[2D_{x}(x)^2 - T_{x}(x)^2]}{\zeta}$

This relation is expected to hold for a small temperature difference, and the relaxation length $\zeta$ is expected to depend on the average temperature. A solution of equations (1) and (2) (which we describe below), together with the boundary conditions for the lower edge,

$T_{x}(x_0) = T_m$

provides a full phenomenological description of the heat flow in the system (with $n_q$ and $n_q$ exchanged for the upper edge).
By integrating equations (1) and (2) with respect to \(x\), with the boundary conditions in equation (3), we obtain the temperature of the downstream mode at the lower edge:

\[
T_d^{\text{lower}}(x)^2 = \frac{T_m^2 n_d e^{x/L(\xi_m)} + (T_m^2 - T_u^2) n_u e^{x/L(\xi_u)} - T_0^2 n_d e^{x/L(\xi_u)}}{n_u e^{x/L(\xi_u)} - n_d e^{x/L(\xi_u)}}
\]

with \(n = n_u n_d/(n_u - n_d)\). The temperature of the upstream mode at the lower edge is

\[
T_u^{\text{lower}}(x)^2 = \frac{T_m^2 n_d e^{x/L(\xi_m)} + (T_m^2 - T_u^2) n_u e^{x/L(\xi_u)} - T_0^2 n_d e^{x/L(\xi_u)}}{n_u e^{x/L(\xi_u)} - n_d e^{x/L(\xi_u)}}
\]

To determine the temperature profile of the modes in the upper edge, we interchange \(n_u\) and \(n_d\). The normalized two-terminal thermal conductance coefficient (which is unity in case of \(v = 1\)) is positive by definition and is composed of two parts, corresponding to the heat flow along the upper and lower edges. The contribution of the lower edge is

\[
K_{\text{lower}} = \frac{K_{\text{lower}}(x)^2 - n_u e^{x/L(\xi_u)} - (n_u - n_d) T_0^2}{n_u e^{x/L(\xi_u)} - n_d e^{x/L(\xi_u)}}
\]

where \(L = x_0 - x_m\). To obtain \(K_{\text{upper}}\) we interchange \(n_u\) and \(n_d\) (also in the expression for \(\xi\)), which yields

\[
K_{\text{upper}} = \frac{n_u (n_u - n_d)}{n_d e^{x/L(\xi_u)} - n_u e^{x/L(\xi_u)}}
\]

In the experiment, heat is flowing out of the floating contact from both of its sides, the lower and the upper edges of each arm, so that we actually measure the sum of them:

\[
K = K_{\text{lower}} + K_{\text{upper}} = \frac{2 n_d}{n_d e^{x/L(\xi_u)} - n_u e^{x/L(\xi_u)}} - 1
\]

Using the general expressions given in equations (4), (5) and (7)–(9), we reach several conclusions about the structure of the temperature profile and the thermal conductance in the fractional Hall states.

For \(\xi \gg L\), the upstream and downstream modes do not fully equilibrate and two modes (the charge mode at the lower edge and the neutral mode at the upper edge) conduct heat. When \(\xi \ll L\), the system does not conduct electronic heat. The interpolation between the two extremes is algebraic.

In our experiment, \(K\) was measured for two bath temperatures: for \(T_0 = 10\) mK we measure a normalized thermal conductance \(K = 0.33\) (and not zero); and for \(T_0 = 30\) mK we find \(K \approx 0.25\). Counter-intuitively, the quantization increases as the temperature increases. This is explained by the improved equilibration at high temperatures. Taking and average arm length of \(L = 150\) \(\mu\)m, we find \(\xi \approx 30\) \(\mu\)m for \(T_0 = 10\) mK and \(\xi \approx 20\) \(\mu\)m for \(T_0 = 30\) mK. Note that, because the sample has several arms of different length \(L\), the estimates of the relaxation lengths are approximate.

For \(v = 3/5\) and \(v = 4/7\). For the case \(v = 3/5\), \(n_u = 2\) and \(n_d = 1\). According to equations (4) and (5), the temperatures of the downstream and upstream modes equilibrate after a length \(\xi_c\), with \(\xi_c = n_u n_d/(n_u - n_d) = 2\).

On the lower edge, the temperature of the single downstream mode (which emanates from the hot floating contact at temperature \(T_{0h}\) is reduced exponentially the temperature of the two upstream modes (which emanate from the cold bath at temperature \(T_0c\)). On the upper edge, the situation is reversed: the temperature of all of the modes in the upper edge is equal to \(T_{0c}\) at \(x_m\) and starts to decrease only at a distance \(\xi_c\) away from the cold bath. There, the temperature of the downstream mode approaches \(T_{0c}\) and that of the two upstream modes is reduced from \(T_{0c}\) to a lower value.

In the limit \(L \gg \xi\), we find normalized values \(K_{\text{lower}} = 0\) and \(K_{\text{upper}} = 1\), with small exponential corrections. The heat is conducted by the modes on the upper edge and the measurements show quantization of the thermal conductance with high accuracy. Taking \(L/(2\xi_c) = 3\), similar to the low-temperature value for \(v = 2/3\), we obtain a correction of about 0.05, which is similar to our experimental accuracy.

An interesting experimental fact. The hole-like states appeared to be ‘experimentally friendly’ in these experiments. Their conductance plateaus in the transmission of the QPCs lend themselves to a constant conductance in the nonlinear regime (independent of the current in the measurement regime). In particular, the dip in the density of the electrons, that may take place near the periphery of the contacts, does not seem to lead to any reflection and excess noise (as we find in the integer regime or in the particle-like states). We attribute this effect to an increase in the composite-fermion filling as the density decreases (as the filling approach \(v = 1/2\)), enabling a smooth traversal of the edge modes of lower composite-fermion filling fractions.\(^{10}\)

Data availability. The datasets generated and analysed during this study are available from the corresponding author on reasonable request.

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Extended Data Figure 1 | Detailed schematic of the structure.
The SEM micrograph shows details away from the floating ohmic contact. In particular, note the two calibration regions, each hosting an added source contact (CS1 or CS3), a ground contact (CG1 or CG3) and a QPC (CQ1 or CQ3, which is fully pinched when thermal noise is measured).

When QPC1 is fully pinched and CQ1 is partly pinched, shot noise is measured by the amplifier at D1 (in the same arm), with respect to ground. There are also voltage probes (VP) in each arm for low-frequency lock-in measurements. Q1–Q4 denote QPC1–QPC4.
Extended Data Figure 2 | Current splitting among multiple arms.

a, $\nu = 1/3$. Current is sourced from S and measured by the amplifier at D1 (see notations in Fig. 1). The blue curve is the reflection coefficient $r_{\text{rel}}$ into D1 (a function of the pinching of QPC1) when all of the QPCs in the other arms are open. The other lines correspond to when the QPC in each of the arms pinches separately, while QPC1 is fully open. We see that each arm consumes the same current. $V_{\text{QPC}}$ represents the pinching of the relevant QPC. b, $\nu = 3/5$. A similar experiment as for a, but in this case the current is sourced from S and measured at D3 as each of the other QPCs pinch. The red and blue lines are the transmissions of QPC1 and QPC3 while both QPC2 and QPC4 were completely pinched; hence, the relative transmission starts from $t_{\text{rel}} = 0.5$ and goes down to zero. The other two lines are the transmissions of QPC2 while all QPCs except QPC4 were open, and of QPC4 while all QPCs except QPC2 were open. In these cases, the relative transmission starts from 0.33 and reaches 0.5 when QPC2 or QPC4 was completely pinched.
Extended Data Figure 3 | Thermal noise and analysis at \( \nu = 1 \).

**a.** Thermal noise \( S_{th} \) as a function of \( I_S = I_{in} \) for different numbers of arms \( (N) \). **b.** The normalized subtracted power dissipation \( \lambda \) (see main text) as function of \( T_m^2 \) \( (T_m < 40 \, \text{mK}) \). Subtracting the contributions for different \( N \) cancels out the phonon contribution; the notation in the legend, for example, \( 'N = 4 - N = 2' \), indicates the power dissipation for four arms, less that for two arms. The fit lines lead to an average thermal conductance per channel of \( g_Q = (0.9 \pm 0.1) \kappa_0 T \).
Extended Data Figure 4 | Raw data of thermal noise $S_{kh}$ and $T_m$ for hole-like states. a, Thermal noise $S_{kh}$ for two different arm configurations and $v = 2/3$, as a function of source current $I_s$. b, Deduced $T_m$ as function of the dissipated power $\Delta P$ in the floating ohmic contact for the two arm configurations and $v = 3/5$ or $v = 4/7$ ($T_0 = 10$ mK).
Extended Data Figure 5 | Gain and temperature calibration via shot noise. a, Transmission $t$ of a QPC for $\nu = 2/3$ (blue, top), $\nu = 3/5$ (red, middle) and $\nu = 4/7$ (green, bottom). Plateaus are visible for the different chiral charge modes at the edge. b, Shot noise $S_t$ (green circles) measured for $\nu = 2/3$ at the $t = 0.5$ plateau to calibrate the gain. The black solid line shows a fit with the equation $2e^* I_s(t(1-t)\coth(e^* V_S/(2k_B T_0) - 2k_B T_0/(e^* V_S)))$, with $e^* = 2e/3$ and $T_0 = 10$ mK. Linear extrapolation of this fit to zero noise from $V_S = (e^*/h) I_s = 2k_B T_0$ (dashed magenta line) provides an exact measure of $T_0$ that is independent of the gain. c, Shot noise $S_t$ (red circles) measured for $\nu = 3/5$ at the $t = 0.55$ plateau, with the blue solid line corresponding to a fit using the expression in b, but with $e^* = 3e/5$ and $T = 11$ mK. d, Resonant response of the LCR$_0$ circuit (connected to the drain in parallel with the actual device) at different filling factors. Gains at different filling factors are compared by the ratio of the areas in the appropriate bandwidths. At smaller filling factors, the bandwidth is smaller (largest 30 kHz, smallest 10 kHz).
Extended Data Figure 6 | Schematics aiding the calculation of the equilibration length in Methods (and the main text). The schematic shows a single arm with $n_d$ downstream modes (in the chiral direction; solid arrows) and $n_u$ upstream modes (in the achiral direction; dashed arrows). The arrows marked as $J_t$ represent energy exchange in the equilibration process. The relevant length of each arm is $L = x_0 - x_m$. 