Non-Gaussian continuous-variable entanglement and steering

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Two Kerr-squeezed optical beams can be combined in a beam splitter to produce non-Gaussian continuous-variable entangled states. We characterize the non-Gaussian nature of the output by calculating the third-order cumulant of quadrature variables and predict the level of entanglement that could be generated by evaluating the Duan-Simon and Reid Einstein-Podolsky-Rosen criteria. These states have the advantage over Gaussian states and non-Gaussian measurement schemes in that the well known, efficient, and proven technology of homodyne detection may be used for their characterization. A physical demonstration maintaining the important features of the model could be realized using optical fibers, beam splitters, and homodyne detection.

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I. INTRODUCTION

Continuous-variable (CV) systems provide flexible and powerful means for implementing quantum-information schemes [1], in large part, because there are mature and precise techniques for measuring the quadratures of light, most of which are familiar from classical communications technologies. Despite the need to deal with transmission losses, recent work has demonstrated useful distances comparable to those achieved with discrete-variable systems [2]. Furthermore, research and development has progressed to the stage where CV quantum key distribution systems have advantages over discrete-variable methods [3–5].

The remaining stumbling block for the wider use of CV systems is that the most-readily available CV systems and the most-developed detection techniques produce only Gaussian statistics. This limitation rules out tasks, such as entanglement distillation [6], quantum error correction [7], and quantum computation. One way of introducing non-Gaussian statistics is through nonlinear measurements [8], but this approach negates one of the main advantages of CV systems, namely, the highly developed technology that is available for performing Gaussian homodyne measurements.

In this paper, we proceed along an alternative approach, namely, to use CV sources that produce non-Gaussian outputs. The importance of this area of research was shown recently by Ohliger et al. [9], who demonstrated there are serious limitations to the use of Gaussian states for quantum-information tasks which may be avoided by developing useful and relatively simple non-Gaussian sources.

Non-Gaussian light can be produced by means of a $\chi^{(3)}$ nonlinear medium, such as a single-mode optical fiber. For an intense pulse of light, the Kerr effect distorts the initially symmetric noise distribution of a coherent state, leading to a quadrature-squeezed state [10,11]. Departures from the classic squeezed ellipse are increasingly evident for longer interaction times and stronger Kerr effects, leading, for example, to the crescent-shaped phase-space distributions [12,13] and non-positive Wigner functions [14] associated with non-Gaussian behavior.

Pairs of Kerr-squeezed pulses can be combined on a beam splitter to produce CV entangled states involving a very large number of photons. The type of state produced and the level of nonlinearity required to produce it are in contrast to those of the “quantum scissors” method of engineering photon-number entanglement by means of parametric interactions and giant Kerr nonlinearities [15,16]. Despite the successful demonstration of the Kerr-squeezing approach [17], the non-Gaussian character of these entangled states has not, to our knowledge, been explicitly demonstrated.

In this paper, we use a single-mode anharmonic oscillator [18] to determine the non-Gaussian entanglement that can, in principle, be achieved with Kerr-squeezed states. We characterize the non-Gaussian statistics through higher-order cumulants and gauge the level of entanglement by calculating the Duan-Simon and Einstein-Podolsky-Rosen (EPR) correlations.

II. TESTING FOR NON-GAUSSIAN STATISTICS

A Gaussian state can be most-simply defined as a state with a Gaussian Wigner function, i.e., a state whose marginal distributions are Gaussian. For a CV state, the departures from non-Gaussian behavior can, thus, be characterized by the skewness of the distributions of its quadrature moments as revealed in nonzero higher-order cumulants [19]. Other proposed approaches have used Hilbert-Schmidt distances [20] and quantum relative entropy [21], the latter having been successfully implemented experimentally [22]. One advantage of using skewness as a non-Gaussian measure is that the relevant cumulants follow directly from quadrature measurements and do not require tomographic reconstruction of the entire quantum state.

We define the generalized quadrature $\hat{X}(\theta)$ at angle $\theta$ as

$$\hat{X}(\theta) = \hat{a} e^{-i\theta} + \hat{a}^\dagger e^{i\theta},$$

so that the canonical $X$ quadrature is found at $\theta = 0$ with conjugate $\hat{Y} = \hat{X}(\pi/2)$.

For a Gaussian distribution, all cumulants higher than second order vanish, and therefore, we can test for non-Gaussian statistics by the presence of a nonzero third-order cumulant,

$$\kappa_3(\theta) = \left< \hat{X}^3(\theta) \right> + 2\left< \hat{X}(\theta) \right>^3 - 3\left< \hat{X}(\theta) \right> \left< \hat{X}^2(\theta) \right>.$$  

Although $\kappa_3 \neq 0$ is a sufficient condition for non-Gaussian statistics, it is not a necessary one. In particular, $\kappa_3$ will vanish for a symmetric distribution in phase space. In the presence...
of such symmetry, the fourth-order moment $\kappa_4$ provides the lowest-order test for non-Gaussian behavior,
\[
\kappa_4(\theta) = \langle \hat{X}^4(\theta) \rangle + 2 \langle \hat{X}(\theta) \rangle^4 - 3 \langle \hat{X}^2(\theta) \rangle^2 - \langle \hat{X}(\theta) \rangle \kappa_3(\theta).
\]
(3)

The fourth-order cumulant can be used to infer the negativity of the Wigner function [23], which is considered to be a direct measure of the nonclassicality of a state. It also allows comparison to the nonclassical states that have been experimentally demonstrated to be non-Gaussian, such as the number state [8] and the photon-subtracted squeezed vacuum [24,25]. For both of these states, $\kappa_4$ scales quadratically with number. For the number state, for example,
\[
\kappa_4 = -6n(n+1).
\]
(4)

In the analysis below, we will determine the regimes in which the Kerr-squeezed state is skewed to a similar level.

III. NON-GAUSSIAN STATISTICS IN THE KERR-SQUEEZED STATE

The Hamiltonian for the single-mode model, ignoring any effects due to loss and excess noise, is
\[
\mathcal{H} = \hbar \chi (a^\dagger a)^2,
\]
(5)
where $\chi$ represents the third-order nonlinearity of the medium and $\hat{a}$ is the bosonic annihilation operator for the electromagnetic-field mode.

For an input Glauber-Sudarshan coherent state,
\[
|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle,
\]
(6)
where $|n\rangle$ represents a Fock state of fixed number, we may find analytical expressions for all the operator moments necessary to calculate the first four cumulants.

The Heisenberg equation of motion for $\hat{a}$ can formally be solved to give
\[
\dot{a}(t) = e^{-i\chi t(2\hat{a}^\dagger + 1)} \hat{a}(0),
\]
(7)
whose expectation value in a coherent state is
\[
\langle \hat{a}(t) \rangle = |\alpha| e^{-i \chi t \cos 2\chi t} e^{i \chi t \sin 2\chi t}.
\]
(8)

Defining $\hat{a}_\theta \equiv \hat{a} e^{-i\theta}$, we can write the mean of a quadrature moment as
\[
\langle \hat{X}(\theta, t) \rangle = \langle \hat{a}_\theta(t) + \hat{a}_\theta^\dagger(t) \rangle
\]
(9)
for which we already have a solution. The second moment is
\[
\langle \hat{X}^2(\theta, t) \rangle = (2\hat{a}_\theta^2 + \hat{a}_\theta^4 + \hat{a}_\theta^2) + \langle \hat{X}(\theta, t) \rangle^2,
\]
(10)
where we have dropped the time argument on the right-hand side for simplicity. The third- and fourth-order moments are
\[
\langle \hat{X}^3(\theta) \rangle = (\hat{a}_\theta^3 + 3 \hat{a}_\theta^2 \hat{a}_\theta + 3 \hat{a}_\theta \hat{a}_\theta^2 + \hat{a}_\theta^3 + 3 \hat{a}_\theta^4 + 3 \hat{a}_\theta),
\]
\[
\langle \hat{X}^4(\theta) \rangle = (\hat{a}_\theta^4 + 4 \hat{a}_\theta^3 \hat{a}_\theta + 6 \hat{a}_\theta^2 \hat{a}_\theta^2 + 4 \hat{a}_\theta^3 \hat{a}_\theta + 4 \hat{a}_\theta^4 + 4 \hat{a}_\theta + 6 \hat{a}_\theta^2 + 12 \hat{a}_\theta \hat{a}_\theta + 6 \hat{a}_\theta^3 + 3).
\]
(11)

To analytically calculate these moments, we use the following expectation values and their complex conjugates:
\[
\langle \hat{a}_\theta^2(t) \rangle = \alpha^2 e^{-2i\chi t} e^{-4\chi t} e^{i|\alpha|^2(\cos 4\chi t - i \sin 4\chi t)},
\]
\[
\langle \hat{a}_\theta^3(t) \rangle = \alpha^3 e^{-3i\chi t} e^{-6\chi t} e^{i|\alpha|^2(\cos 6\chi t - i \sin 6\chi t)},
\]
\[
\langle \hat{a}_\theta^4(t) \rangle = \alpha^4 e^{-4i\chi t} e^{-16\chi t} e^{i|\alpha|^2(\cos 8\chi t - i \sin 8\chi t)}.
\]
(12)

These equations reveal several kinds of contributions to the dynamics with different time scales. The sine and cosine terms in the exponents can each be expanded, and for sufficiently small interaction time $\chi t$, we can keep the first two terms in each, i.e., up to fourth order in time. We are left with a number of different contributions to the exponent.

First, there is the nonlinear phase factor proportional to $N\chi t$, where $N = |\alpha|^2$. This mean-field frequency shift can be removed by a switch to a rotating frame, i.e., setting $\theta = \theta_0 + 2N\chi t$.

Second, the real exponent proportional to $N\chi t^2$ is responsible for squeezing, although in order to obtain quantum squeezing, i.e., below the coherent-state level, we also require the zero-point phase factors.

Finally, there are the third- and fourth-order terms $N\chi^3 t^3$ and $N\chi^4 t^4$, which for large $N$, give the leading-order contribution to the third- and fourth-order cumulants and, hence, are responsible for most of the skewness we see in the quadrature statistics.

In typical Kerr-squeezing experiments, the number of photons is large $N \gg 1$ in order to compensate for a weak nonlinearity $\chi \ll 1$. In this limit, we can derive a simple expression for the third-order cumulant of the $Y$ quadrature rotating at the mean-field frequency, which is where skewness is most evident,
\[
\kappa_3 \left( \frac{\chi}{2} \right) \approx -256 \frac{1}{\sqrt{N}} (\chi Nt)^3.
\]
(13)

The validity of this expression is demonstrated in Fig. 1, which plots the third-order cumulant for various photon numbers. The exact results for $N > 10^6$ are indistinguishable on this time scale from the simple cubic growth described by Eq. (13). Time is, here, scaled by $N\chi$ in order to compare results that give the same Kerr effect. On this scale, the third-order cumulant decreases in proportion to the square root of the number of photons. Note however, that the absolute size of the third-order cumulant increases with particle number at a rate faster than $\langle \hat{X} \rangle^3$.

The fourth-order cumulant $\kappa_4$ is plotted in Fig. 2. Again, for large photon numbers, the cumulant approaches a limiting scaling behavior,
\[
\kappa_4 \propto \frac{1}{N} (\chi Nt)^4,
\]
(15)
which gives the same relative growth of
\[
\frac{\kappa_4}{\langle \hat{X} \rangle^4} \sim N,
\]
(16)
for large $N$. The validity of this expression is demonstrated in Fig. 1, which plots the third-order cumulant for various photon numbers. The exact results for $N > 10^6$ are indistinguishable on this time scale from the simple cubic growth described by Eq. (13). Time is, here, scaled by $N\chi$ in order to compare results that give the same Kerr effect. On this scale, the third-order cumulant decreases in proportion to the square root of the number of photons. Note however, that the absolute size of the third-order cumulant increases with particle number at a rate faster than $\langle \hat{X} \rangle^3$.
although if the time is adjusted as a function of \( N \) to keep the Kerr-squeezing constant, the fourth-order cumulant decreases in proportion to the particle number.

Figures 3 and 4 show the cumulants as a function of \( N \) for a fixed value of \( \chi N t = 25 \). One can clearly see two different regimes of behavior with the crossover between the two occurring just above \( N \sim 10^4 \). For \( \kappa_4 \), the number-state result is also plotted for comparison. One can see that \( \kappa_4/(Nt)^3 \) scales as described above for large \( N \) but, for small \( N \), is limited to values of the order of corresponding number-state results (increasing with \( N \) quadratically). This result suggests that a Kerr-squeezed state can be as non-Gaussian by this measure as the number state for sufficiently long interaction times.

### IV. QUADRATURE VARIANCES, ENTANGLEMENT, AND EINSTEIN-PODOLSKY-ROSEN CORRELATIONS

Although entangled states have already been predicted to be produced by the intracavity nonlinear coupler [26,27], the linearization process used to obtain the spectra in those cases forces Gaussian statistics on the outputs. Here, we wish to produce entangled states that maintain non-Gaussian statistics, so we will proceed by mixing the outputs of two Kerr oscillators on a beam splitter [28] as experimentally demonstrated by Ref. [17]. We will now show the quadrature variances as we need squeezed states in order to obtain entangled modes in the outputs. We note that, although this could be performed by mixing one squeezed mode with vacuum, better results in terms of the degree of violation of the relevant inequalities are obtained by mixing two squeezed

![Diagram](image-url)

**FIG. 1.** (Color online) The third-order cumulant \( \kappa_3 \) of \( \hat{Y} = \hat{X}(\pi/2) \) in a rotating frame as a function of time for various photon numbers ranging from 100 to 10^6 as labeled. In this and subsequent plots, time is scaled by the inverse of the mean-field interaction strength and, hence, is a dimensionless quantity; \( \kappa_3 \) is scaled by \( 1/\sqrt{N} \). The dashed lines give the exact results [Eqs. (13)], and the solid line gives the approximate result [Eq. (13)], which is accurate for large \( N \) or small mean-field interaction time \( N\chi t \). The inset shows \( \kappa_3 \) for \( N = 1000 \) in more detail.

![Diagram](image-url)

**FIG. 2.** (Color online) The fourth-order cumulant \( \kappa_4 \) of \( \hat{Y} = \hat{X}(\pi/2) \) in a rotating frame as a function of time for various photon numbers ranging from 100 to 10^6 as labeled. Time is scaled by the mean-field interaction strength, and \( \kappa_3 \) is scaled by \( 1/N \). For large \( N \) or small mean-field interaction time \( N\chi t \), the results approach the same limiting curve \( \propto t^4 \). The inset shows \( \kappa_4 \) for \( N = 1000 \) in more detail, both for the dashed line: Kerr-squeezed state and for the dotted line: number state.

The Heisenberg uncertainty principle demands that

\[
V(\hat{X}(\theta))V(\hat{X}(\theta + \pi/2)) \geq 1,
\]

so that any quadrature with variance below 1 is squeezed. Figure 5 shows the variances in the canonical \( \hat{X} = \hat{X}(0) \) and \( \hat{Y} = \hat{X}(\pi/2) \) quadratures with time, again, scaled by \( N\chi t \) so that, apart from small-number effects, the same level of squeezing is obtained for different photon numbers. As for the cumulants, the results for \( N > 10^6 \) cannot be distinguished on this time scale. In fact, above \( N = 1000 \), the small-number

![Diagram](image-url)

**FIG. 3.** (Color online) Third-order cumulant \( \kappa_3(\pi/2) \) of the Kerr-squeezed state at fixed \( \chi N t = 25 \) as a function of \( N \). The behavior at \( N \gtrsim 10^5 \) reveals the scaling behavior described by Eq. (13); the behavior at \( N \lesssim 10^4 \) is due to the saturation effect seen in the inset of Fig. 1.
effects only appear as small different amounts of squeezing, so for the remainder of the paper, we quote results for \( N = 1000 \).

Considering a beam splitter with reflectivity \( \eta \) and labeling the inputs by \( \hat{a}_1 \) and \( \hat{a}_2 \) and the outputs by \( \hat{b}_1 \) and \( \hat{b}_2 \), we find

\[
\begin{align*}
\hat{b}_1 &= \sqrt{\eta} \hat{a}_1 + i \sqrt{1 - \eta} \hat{a}_2, \\
\hat{b}_2 &= i \sqrt{1 - \eta} \hat{a}_1 + \sqrt{\eta} \hat{a}_2.
\end{align*}
\] (18)

For notational convenience, we will now make the simplification \( \hat{X}_{bj} \rightarrow \hat{X}_j \) and, similarly, for \( \hat{Y}_{bj} \). This allows us to define the variances of the beam-splitter outputs as

\[
\begin{align*}
V(\hat{X}_1) &= \eta V(\hat{X}_{a_1}) + (1 - \eta)V(\hat{Y}_{a_2}), \\
V(\hat{X}_2) &= (1 - \eta)V(\hat{Y}_{a_1}) + \eta V(\hat{X}_{a_2}), \\
V(\hat{Y}_1) &= \eta V(\hat{X}_{a_1}) + (1 - \eta)V(\hat{X}_{a_2}), \\
V(\hat{Y}_2) &= (1 - \eta)V(\hat{Y}_{a_1}) + \eta V(\hat{Y}_{a_2}).
\end{align*}
\] (19)

Along with the covariances,

\[
\begin{align*}
V(\hat{X}_1, \hat{X}_2) &= -\sqrt{\eta(1 - \eta)}[V(\hat{X}_{a_1}, \hat{Y}_{a_1}) + V(\hat{X}_{a_2}, \hat{Y}_{a_2})], \\
V(\hat{Y}_1, \hat{Y}_2) &= \sqrt{\eta(1 - \eta)}[V(\hat{X}_{a_1}, \hat{Y}_{a_1}) + V(\hat{X}_{a_2}, \hat{Y}_{a_2})],
\end{align*}
\] (20)

we now have all the expressions needed to calculate the quantities necessary to check for violation of the continuous-variable Duan-Simon inequality [29,30]. For the purposes of this article, we define this as

\[
V(\hat{X}_1 \pm \hat{X}_2) + V(\hat{Y}_1 \mp \hat{Y}_2) \geq 4,
\] (21)

with any violation of this inequality being sufficient to demonstrate the presence of entanglement for a non-Gaussian state. The result for this correlation with \( \eta = 0.5 \) is shown in Fig. 6. The (red) dotted line gives the maximum violation, optimized over quadrature angle \( \theta \). Clearly, the outputs from two Kerr oscillators mixed on a beam splitter can give a continuous-variable non-Gaussian entangled bipartite resource.

As shown by Wiseman et al. [31] and Cavalcanti et al. [32], the inseparability of the system density matrix describes a set of states which includes, within it, subsets which are more deeply nonclassical than evidenced by entanglement alone, such as those which demonstrate the EPR paradox [33]. For our purposes here, we will use the inequality developed by Reid [34], written as

\[
V_{\text{inf}}(\hat{X}_{aj})V_{\text{inf}}(\hat{Y}_{bj}) \geq 1,
\] (22)

where \( j = 1, 2 \) and

\[
\begin{align*}
V_{\text{inf}}(\hat{X}_{aj}) &= V(\hat{X}_{aj}) - \frac{[V(\hat{X}_{aj}, \hat{X}_{aj})]^2}{V(\hat{X}_{aj})}, \\
V_{\text{inf}}(\hat{Y}_{bj}) &= V(\hat{Y}_{bj}) - \frac{[V(\hat{Y}_{bj}, \hat{Y}_{bj})]^2}{V(\hat{Y}_{bj})}.
\end{align*}
\] (23)

From the expressions given above for the Duan-Simon criterion, it can be seen that all the moments necessary to calculate
FIG. 7. (Color online) Reid EPR correlation [left-hand side of Eq. (22)] after mixing on a 50:50 beam splitter as a function of interaction time $\chi t$ for $N = 1000$. A value below 1 signifies a demonstration of the EPR paradox. The dashed lower line is optimized for the quadrature angle, whereas, the upper solid line is for the canonical quadratures.

These expressions are available analytically. As shown in Fig. 7, the two modes after the beam splitter exhibit a strong violation of the Reid inequality.

Finally, we consider the skewness of the final entangled state. For a 50:50 beam splitter, the third- and fourth-order cumulants of the $X$ quadrature in output port 1 can be shown to be as follows:

$$\kappa_3(X_1) = \frac{1}{\sqrt{8}}[\kappa_3(X_{a1}) - \kappa_3(Y_{a2})],$$

$$\kappa_4(X_1) = \frac{1}{4}[\kappa_4(X_{a1}) + \kappa_4(Y_{a2})] + 4\langle X_1 \rangle \kappa_3(X_1),$$

which confirms that skewed inputs to a beam splitter lead to skewed outputs with cumulants generally on the same order of magnitude.

V. CONCLUSIONS

To summarize, we have employed a simple model of the $\chi^{(3)}$ nonlinear process to demonstrate that violations of the Duan-Simon and Reid entanglement criteria occur at the same time as significant departures from Gaussian behavior. For sufficiently long interaction time $N \chi t$, the nonlinear interaction will skew the distribution of the quadrature variables, leading to large third- and fourth-order cumulants.

Moreover, such non-Gaussian entanglement occurs in regimes accessible to optical-fiber experiments [35]. However, for accurate quantitative predictions, one would need to go beyond the single-mode model to include the effects of pulse dynamics and extra noise sources, using the simulations methods, for example, that were used in Ref. [36].

As with any coherent scheme employing the $\chi^{(3)}$ nonlinearity of optical fibers, the intrinsic weakness of the nonlinearity itself can be a limiting factor. Besides using large photon numbers, this factor may be overcome by use of electromagnetically induced transparency to produce giant cross-Kerr nonlinear phase shifts and, hence, highly non-Gaussian quantum states [37]. On the other hand, the presence of large numbers of photons makes optical fiber a very bright source of non-Gaussian entanglement, which may well be a practical advantage over number-state schemes.

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