Second order interference of chaotic light reflected from random medium

A. Yu. Zyuzin
A.F. Ioffe Physico-Technical Institute of Russian Academy of Sciences, 194021 St. Petersburg, Russia

We consider the reflection from a random medium of light with short coherence length. We found that the second order correlation function of light can have a peak in a direction where the reflection angle is equal to angle of incidence. This occurs when the size of the region, from which light is collected, is larger than the coherence length.

I. INTRODUCTION

Considerable theoretical and experimental interest have recently arisen in the field of strong scattering of quantum states of light.

It was theoretically proposed \cite{1} that the entanglement of light, i.e. the quantum nature of light, can be probed in the multi-photon scattering experiments \cite{2, 3}. Of basic interest is a question of optical noise propagation in random medium \cite{4, 5}, and photon counting statistics of multiple scattered light \cite{6}

Interestingly, the reflection of light from multiple scattering medium on average is an angle independent and has a weak localization peak due to the constructive interference of light in the backscattering direction. It is a precursor manifestation of Anderson localization

Pioneering work on weak localization of photon noise have been reported recently \cite{7}. It was found weak localization narrow peak in backscattering of photon noise.

We note, however, that the experiment was limited by a large light coherence length

In this paper we consider a situation of arbitrary relation between the coherence length and system size. We find that when the coherence length is smaller than the system size, probability of two photon absorption develops a peak at reflection angle equal to incidence angle. Together with the standard backscattering peak resulting from the weak localization of light, the obtained peak constitutes a new characteristic of a light scattering from random medium

II. DEFINITIONS

We consider the light incident at direction \( \mathbf{n} \) on the surface of disordered medium and reflected after multiple scattering on disorder in direction \( \mathbf{m} \). Diffusion transport of light is characterized by the mean free path \( l \), which is much smaller than the size of the medium. The light reflected from the area of size \( S \) in the direction \( \mathbf{m} \) is collected by the detector, as it is shown in figure 1.

We assume that light is chaotic and characterized by the coherence length comparable with the linear size of the area. The stationary Gaussian density operator \( \rho \) of incident light is defined by correlation function:

\[
\text{Sp}[\rho c_{\alpha_1}^+(t_1)c_{\alpha_2}^+(t_2)c_{\beta_1}(t_2)c_{\beta_2}(t_1)] = \nu(\omega_1)\nu(\omega_2)(\delta_{\alpha_1,\beta_2}\delta_{\alpha_2,\beta_1} + \delta_{\alpha_1,\beta_1}\delta_{\alpha_2,\beta_2}) \exp(i\omega_1t_1t_2)),
\]

where \( c_\alpha \) and \( c_\alpha^\dagger \) are creation and annihilation operators of photons in state \( \alpha \equiv k, s \), where \( k \sim \mathbf{n} \) and \( s = 1, 2 \) are respectively the photon momentum and the polarization with complex polarization vector \( \mathbf{e}(k, s) \).

The difference between frequencies of photon in states \( \alpha_1 \) and \( \alpha_2 \) is defined by \( \omega_{12} = \omega_1 - \omega_2 = \epsilon(k_1 - k_2) \).

In order to study the crossover from large to short coherence length of light compare to linear size of the area we assume that the spectral function \( \nu(\omega) \) is Gaussian:

\[
\nu(\omega) = N_0 \exp[-\sigma(\omega - \Omega_0)^2].
\]

The coherence length of the chaotic light, which is characterized by the this Gaussian spectral function is defined as:

\[
L_c = c\sqrt{\sigma}.
\]

The probability of absorption of photons at points \( x_1 = (r_1, t_1) \) and \( x_2 = (r_2, t_2) \) of detector is defined by the second order correlation function:

\[
G^{(2)} = \text{Sp}[\rho A_i^+(x_1)A^k(x_2)A^k(x_2)A_i^+(x_1)],
\]

where \( A_+ \) and \( A_- \) are positive and negative frequency parts of the vector fields, with \( i, k = (x, y, z) \).

Since we are considering diffusive scattering medium from which the light is reflected we will calculate the disorder averaged value of the probability of absorption \( \langle G^{(2)} \rangle \).

We will calculate the absorption probability normalized to the time independent intensity of light: \( \langle G^{(1)} \rangle = \langle \text{Sp}[\rho A_-A_+(x)] \rangle \). As a function of direction of reflection \( \mathbf{m} \) intensity of light contains narrow peak in the backscattering direction which gives small correction to
functions and the dashed lines represent scattering. The solid lines denote the light Green’s function. The solid lines denote the light Green’s function. We therefore choose it as a normalization constant in the definition (1).

\[
\langle G^{(1)} \rangle \text{ averaged over direction of } m \text{ defined as } \langle G^{(1)} \rangle.
\]

We therefore choose it as a normalization constant in the probability of absorption:

\[
g^{(2)}(n, m, t_{12}) \equiv \langle G^{(2)} \rangle / \langle G^{(1)} \rangle^2
\]

Fourier transformation of \(g^{(2)}(n, m, t_{12})\) over \(t_{12}\) might be represented as

\[
g^{(2)}(n, m, \Omega) = 2\pi g_0^{(2)}(n, m)\delta(\Omega) + g_1^{(2)}(n, m, \Omega) \quad (6)
\]

First and second terms here correspond to that in the definition [11].

III. CALCULATION OF \(g^{(2)}(n, m, \Omega)\)

Diagrams describing Cooperon, Diffuson, and mixed Cooperon-Diffuson contributions are shown in Fig. 2. In the study of the crossover from large to small coherence length compared to the system size we restrict ourselves to the case of scalar waves.

A. Diffusion ladders

We use standard impurity technique while calculating correlation functions. In case of multiple scattering diffusion ladder appears, which at \(\omega l/c << 1\) \(P(r, r', \omega)\) satisfies equation

\[
-(D\nabla^2 - i\omega)P(r, r', \omega) = \delta(r - r')
\]

Here \(D = cl/3\) is light diffusion coefficient, \(c\) anf \(l\) are light velocity and mean free path, correspondingly. Consider the random medium occupying the half-space \(z > 0\). Then the boundary condition for the ladder is given by \(P(r, r', \omega) = 0\) at \(z, z' = 0\). Performing the Fourier transformation over coordinates \(x, y\) we obtain:

\[
P(z, z', Q, \omega) = \frac{\sinh [q \min(z, z')] - q \max(z, z')] \exp[-q \max(z, z')]}

where \(q^2 = Q^2 - i\omega\)

When considering the scattering at large angles we must distinguish between Cooperon and Diffuson propagators in the integral with four Green’s functions [12, 13].

\[
P_c(\omega, Q) \equiv \int_0^{\infty} dz dz' P(z, z', \omega, Q) e^{-\frac{(z + z')}{2D} \frac{\mu_n \mu_m}{\mu_n + \mu_m}}
\]

and

\[
P_d(\omega, Q) \equiv \int_0^{\infty} dz dz' P(z, z', \omega, Q) e^{-\frac{(z + z')}{2D} \frac{\mu_n \mu_m}{\mu_n + \mu_m}},
\]

where \(\mu_n\) and \(\mu_m\) are the normal projections of directions \(n\) and \(m\) of incident and of scattered waves, respectively.

Integrating over \(z\) and \(z'\) in [9] and [10] in the limit \(|q|l << 1\), we obtain an expression for the Cooperon propagator:

\[
P_c(\omega, Q) = \frac{l^3}{2D} \frac{(2\mu_n \mu_m)^3}{\mu_n + \mu_m^2} \left(1 - 4ql \frac{\mu_n \mu_m}{\mu_n + \mu_m} \right)
\]

and for the Diffuson propagator:

\[
P_d(\omega, Q) = \frac{l^3}{2D} \frac{(\mu_n \mu_m)^2}{\mu_n + \mu_m} \left[1 - (\mu_n + \mu_m)ql \right].
\]

B. Cooperon contributions

Cooperon contributions are shown in fig. 2 a, b. They contribute to \(g_1^{(2)}\) when diffusion ladders couple states \(k_i\) and \(k_j\) with \(i = j\), and \(k_i\) and \(k_p\) with \(l = p\) so vertex couples states with \(i \neq l\). Cooperon propagators in this case do not depend on the frequency of light.

Let us consider first diagram, shown in Fig.2a. Phase factors of light incident in the direction \(n\) and reflected back in the direction \(m\) in expression

\[
e^{-ik_1(r-r')}(n+m) P_c(0, r-r') e^{-ik_2(R-R')(n+m)} P_c(0, R-R')
\]

must be integrated over the surface of the medium.

Integrating over the surface of the medium with coordinates \((r, r', R, R')\) we obtain:

\[
\int \frac{d^2 Q}{(2\pi)^2} P_c(0, Q) |F(Q + k_1(n + m))|^2
\]

\[
\times \int \frac{d^2 Q'}{(2\pi)^2} P_c(0, Q') |F(Q' + k_2(n + m))|^2.
\]
where
\[ F(Q) = \frac{1}{S} \int_S d^2r \exp(iQr) \] (15)
is form factor of the surface.

At \(|k_i(n + m)|\) larger than the inverse of linear dimension of area we can calculate [13] as:
\[ P_c(0, k_1(n + m))P_c(0, k_2(n + m))S^2. \] (16)

The contribution from the second diagram shown in Fig. 2b, after integration of phase factors can be written as:
\[ \int \frac{d^2Q}{(2\pi)^2} P_c(\omega_{12}, Q)F(Q + k_1n + k_2m)F(Q + k_2n + k_1m)^2 \] (17)
Form-factors vary with momentum much faster than \(P_c(\omega_{12}, Q)\), therefore the integral in [17] can be calculated as:
\[ |P_c(\omega_{12}, k_0(n + m))|^2 |F(k_{12}(n - m))|^2 \] (18)
where Cooperon propagators depend on the frequency \(\omega_{12} = c(k_1 - k_2) = ck_{12}\). We note that the contribution from the second diagram strongly depends on the ratio between the coherence length of light and the size of the surface from which the radiation is collected.

At \(\sqrt{S}/L_c < 1\) and \(|k_{12}|L_c \leq 1\) the form-factor can be approximated as \(|F(k_{12}(n - m))| \approx |F(0)| = 1\). As a result, the second contribution [18] has the same properties as the first one [16].

In the opposite case when \(\sqrt{S}/L_c > 1\) radiation from \(P_c(\omega_{12}, Q)\) scatters in direction \((n - m)\parallel = 0\).

C. Diffusion contributions

Diffusion contributions are given by two diagrams, shown on Fig 2 c, d.

Similarly to the calculation of the Cooperon we integrate the phase factors over the surface and obtain for the first diagram:
\[ P_d(0, 0)P_d(0, 0)S^2 \] (19)
Note that phase factors do not give rise to the frequency and angle dependence in the first diagram. Second diagram can be calculated as:
\[ |P_d(\omega_{12}, k_{12}n)F(k_{12}(n - m))|^2 \] (20)
Here \(k_{12} = \omega_{12}/c\). Here we neglect the momentum dependence of the diffusion ladder as \(P_d(\omega_{12}, k_{12}n) \approx P_d(\omega_{12}, 0)\) in the limit \(\omega_{12}/c < 1\).

Again, if detector collects the radiation from the area \(S < L_c^2\) then the form factor becomes \(F(k_{12}(n - m)) = 1\), and (20) does not depend on the scattering angle. Contrary, if detector collects the radiation from large area \(S > L_c^2\) then the contribution [20] of the second diagram is maximal in the direction \(m\parallel = n\parallel\) which corresponds to the case when the angle of reflection equals the angle of incidence.

D. mixed cooper-diffusion contributions

Diagrams that describe these contributions are shown in Fig. 2e,f. After the integration over the surface of the medium the contribution of mixed diagrams results in:
\[ \int \frac{d^2Q}{(2\pi)^2} [P_c(0, Q)F^2(Q + k_1(m + n) + (k_1 \rightarrow k_2)] P_d(0, 0)S. \] (21)
We then perform the integration over the momentum and obtain:
\[ 2P_c(0, k_0(m + n))P_d(0, 0)S^2. \] (22)

The diagram shown in Fig. 2f after the integration of phase factors over the surface of the medium yields:
\[ 2 \text{Re} \int \frac{d^2Q}{(2\pi)^2} P_c(\omega_{12}, Q)F(Q + k_2m + k_1n) \]
\[ \times F(Q + k_1m + k_2n)P_d(-\omega_{12}, k_1n)F(k_{12}(n - m)). \]
Again, the integration over momentum gives:
\[ 2\text{Re}P_c(\omega_{12}, k_0(n + m))P_d(-\omega_{12}, 0)|F(k_{12}(n - m))|^2. \] (24)

E. sum of all contributions

At \(l \ll L_c\) and \(l \ll \sqrt{S}\) we can neglect the momentum dependence of \(P_d\). Thus in the limits \(L_c, \sqrt{S} \gg k_0|n\parallel + m\parallel\) we collect all Cooperon [16], [18], diffusion contributions [19] [20], mixed contributions [22] [24] and obtain:
\[ \Sigma(\omega_1, \omega_2) \equiv [P_c(0, k_0(n + m)) + P_d(0, 0)]^2 + \]
\[ + |P_c(\omega_{12}, k_0(n + m)) + P_d(\omega_{12}, 0)|^2 |F(k_{12}(n - m))|^2. \] (25)

Diagrams that determine contribution \(g_0^{(2)}\) can be obtained from that, shown in Fig. 1, by interchanging Green’s functions in such a way that there is no change of state of light at the vertex, i.e. state \(i = p\) couples to \(j = l\). It can be shown that the sum of such diagrams is equal to [26].

F. integrating over frequencies

To obtain \(g_1^{(2)}\) we must integrate [25] over frequencies as
\[ \int d\omega_1 d\omega_2 \delta(\omega_1 - \omega_{12}) \Sigma(\omega_1, \omega_2) \] (26)
\[ \approx \sqrt{\frac{\pi}{2\sigma}} N_0^2 \Sigma(\omega_0 + \Omega/2, \omega_0 - \Omega/2) \exp(-\sigma \Omega^2/2), \]
Here we assume that \( \Sigma(\omega_1, \omega_2) \) is slow function of \( (\omega_1 + \omega_2) \).

Following integration of \( g_1^{(2)} \) over \( \Omega \) gives \( g_0^{(2)} \).

IV. RESULTS

Calculation shows that \( g^{(2)}(\textbf{n}, \textbf{m}, \Omega) \) as a function of angles depends on the ratio of coherence length of light \( L_c \) to the size of the area from which light is collected.

We note that when the system size is larger than the coherence length, \( g^{(2)}(\textbf{n}, \textbf{m}, \Omega) \) contains weak localization peak in the backscattering direction, defined by \( \textbf{n} + \textbf{m} = 0 \). In addition to this peak, \( g^{(2)}(\textbf{n}, \textbf{m}, \Omega) \) has a peak in the forward-scattering direction at which the angle of reflection equals the angle of incidence, defined by \( \textbf{n}_|| - \textbf{m}_|| = 0 \), where \( \textbf{n}_|| \) and \( \textbf{m}_|| \) are the components of direction of light parallel to the surface.

We now summarize our results for the second order correlation function \( g^{(2)}(\textbf{n}, \textbf{m}, \Omega) \) derived in the limit of strong disorder \( l/c \ll \sqrt{\sigma} \), \( \Omega l/c \ll 1 \), \( L < L_c k_0 \), and in the case of scalar waves.

Normalization factor in the definition of \( g^{(2)}(\textbf{n}, \textbf{m}, \Omega) \) is \( \pi |N_0P_2(0,0)|^2/\sigma \).

Near the backscattering direction \( k_0|\textbf{n} + \textbf{m}| < 1 \) and \( \mu_n = \mu_m \) therefore we obtain:

\[
g_0^{(2)}(\textbf{n}, \textbf{m}) = 4 \left[ 1 - 2k_0\mu_m|\textbf{n}_|| + \textbf{m}_|| \right] (1 + \Phi(2\textbf{n})) \tag{27}
\]

and

\[
g_1^{(2)}(\textbf{n}, \textbf{m}, \Omega) = \sqrt{2\pi\sigma}(1 + \Gamma(2\textbf{n}))e^{-\sigma\Omega^2/24}
\times (1 - 2k_0\mu_m|\textbf{n}_|| + \textbf{m}_||). \tag{28}
\]

Second term in \( (27) \) and \( (28) \) corresponds to the peak in the backscattering direction: \( \textbf{n} = -\textbf{m} \).

Reflection in forward scattering direction defined by \( \textbf{n}_|| - \textbf{m}_|| = 0 \) is described by:

\[
g_0^{(2)}(\textbf{n}, \textbf{m}) = 1 + \Phi(\textbf{n} - \textbf{m}) \tag{29}
\]

and

\[
g_1^{(2)}(\textbf{n}, \textbf{m}, \Omega) = \sqrt{2\pi\sigma}(1 + \Gamma(\textbf{n} - \textbf{m}))e^{-\sigma\Omega^2/2}. \tag{30}
\]

Functions \( \Phi(p) \) and \( \Gamma(p) \) are determined by the form-factor \( \Sigma(p) \) of the surface, from which the light is collected, as

\[
\Gamma(p) = |F(\Omega p/c)|^2 = \frac{1}{\mathcal{S}} \int_{\mathcal{S}} d^2r e^{\imath \textbf{p} \cdot \textbf{r}/c^2}, \tag{31a}
\]

\[
\Phi(p) = \sqrt{\frac{\sigma}{2\pi}} \int_{-\infty}^{\infty} d\omega |F(\omega p/c)|^2 e^{-\sigma\omega^2/2}. \tag{31b}
\]

We note that \( \Phi(0) = 1 \) and \( \Gamma(p) = \frac{\sqrt{\sigma}}{|p|} \sqrt{\sigma/\mathcal{S}} \approx 1 \). By definition, functions \( \Phi(p) \) and \( \Gamma(p) \) depend only on the components of the vector \( \textbf{p} \) parallel to the surface.

V. CONCLUSION

Let us now comment on how the divergence of the incident beam limits the proposed interference effect. In the case of two incident beams characterized by \( k_1, \textbf{n}_1 \) and \( k_2, \textbf{n}_2 \) the form-factor is given by \( F(k_{12}(\textbf{m} - (\textbf{n}_1 + \textbf{n}_2)/2) - (k_1 + k_2)(\textbf{n}_1 - \textbf{n}_2)/2) \). If \( k_0 \gg |k_{12}| \) the most important limitation is associated with the second term of the argument of the form-factor. Therefore, we need condition \( k_0|\textbf{n}_1 - \textbf{n}_2| \sqrt{\sigma} < 1 \) to be satisfied for observing the peak in absorption probability.

Note that recent experiments Ref. [11] were performed in the regime of large coherence length compared to the size of the system.

To conclude, we calculate the probability of absorption of two photons reflected from the random medium as a function of the reflection angle. We show that this result depends on the ratio between the size of the medium and coherence length of light. We predict a peak in absorption probability when the angle of reflection equals the angle of incidence if coherence length is smaller than the system size.

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