Nonlinear gravitational-wave memory from cusps and kinks on cosmic strings

Alex Jenkins

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with Mairi Sakellariadou
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GWxBSM
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KING'S
College
LONDON
1. GW memory

2. Cosmic strings

3. Memory from strings
GW memory (linear)

- permanent GW strain offset
- well-known since 1970s
- typical of unbound systems
  (gravitational scattering, supernovae, ...)

SÁNDOR J. KOVÁCS, JR.
W. K. Kellogg Radiation Laboratory, California Institute of Technology
AND
KIP S. THORNE
Center for Radiophysics and Space Research, Cornell University; and
W. K. Kellogg Radiation Laboratory, California Institute of Technology
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THE GENERATION OF GRAVITATIONAL WAVES.
IV. BREMSSTRAHLUNG*††

GW memory from cosmic strings
GW memory (nonlinear)

- discovered by Christodoulou '91
- nonlinear GR effect
- intuition (Thorne '92): linear memory of unbound gravitons
Why is nonlinear memory interesting?

- prediction of **nonlinear, dynamical** GR
- **observable** with GW observatories
- deep connections (Strominger et al):
  - global structure of spacetime (BMS)
  - IR quantum gravity (soft theorems)
Existing results

Focus is usually on BBHs
- Memory from SMBBH mergers is a promising target for PTAs (e.g. Aggarwal et al, '20 ApJ)
- Should be detectable by LIGO/Virgo after $O(2000)$ BBHs (Hübner et al, '20 PRD)

Other sources?
- Scattering, SNe, GRBs, ... are good sources of linear memory
- Aurrekoetxea et al, '20 CQG studied linear + nonlinear memory from collapsing cosmic string loops
1. GW memory

2. Cosmic strings

3. Memory from strings
What are cosmic strings?

- 1-dimensional topological defects
- generic in many theories beyond SM

- $\sim 1$ long string per horizon,
  chops off many loops
- loops decay through GWs

Ringeval, Sakellariadou, & Bouchet '07 JCAP, arXiv:astro-ph/0511646
Key assumptions

1. **Nambu-Goto approximation:**
   - string width $\ll$ loop size
   - no non-gravitational long-range interactions
   - single parameter: string tension $G\mu$

2. **Linearised gravity:**
   - loop evolves on a flat background,
   - generates weak GWs suppressed by $G\mu \ll 1$
GW bursts from loops

three main mechanisms for loop GW emission:

\[ \tilde{h}(f) \sim f^{-4/3} \]
beamed

cusp

\[ \tilde{h}(f) \sim f^{-5/3} \]
beamed

kink

\[ \tilde{h}(f) \sim f^{-2} \]
isotropic

kink-kink collision

illustrations from Long, Hyde, & Vachaspati '14 JCAP, arXiv:1405.7679
1. GW memory

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3. Memory from strings
Why memory + cosmic strings?

1. memory literature has focused on BBHs
2. strings are a promising GW source for LVK/LISA/PTAs
3. strong high-frequency emission
   (memory pushes things to lower frequencies)
4. memory could “leak” out of GW beam
Leading-order cusp memory

- Primary emission
- First-order memory
- Higher-order memory

- Memory “leaks” out of beam
- Same $\sim f^{-4/3}$ as original waveform

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alexander.jenkins@kcl.ac.uk

GW memory from cosmic strings

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Higher-order memory

- the memory GWs are themselves sources of memory ("memory of the memory")
- need to iterate to all orders to get complete memory signal
- higher-order contributions diverge for large enough loops
- is the weak-field description of cusps unphysical?
Backreaction?

- gravitational backreaction could smooth out the cusp?
- see e.g. Chernoff et al, ’18 PRD or Blanco-Pillado et al, ’19 PRD
- typical timescale too long, $\tau \sim \mathcal{O}(1/G\mu)$

(figure from Quashnock & Spergel, ’90 PRD)
One possible cure: PBHs

- predicted for the same range of loop lengths!
- horizon forms, “traps” GWs
- higher-order memory converges

ACJ & Sakellariadou, arXiv:2006.16249

alexander.jenkins@kcl.ac.uk

GW memory from cosmic strings

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Memory from kinks

- memory suppressed at high frequencies due to interference
- higher-order memory always converges
- PBHs not predicted for kinks—makes sense that the memory is well-behaved here
Observable implications?

- key GW observables:
  1. individual bursts
  2. combined stochastic background

- assuming PBHs, memory is unobservable

- other scenarios could be more interesting?
Wrapping up

- nonlinear GW memory is interesting and worth studying beyond BBHs
- memory from cusps *diverges* using the standard waveform
- need some (unknown) strong-gravity physics to fix this—PBH formation is one possible resolution
- assuming PBH formation, observational prospects are poor...
  ...but nature might surprise us

*thanks for listening!*
Backup Slides
How do we observe GW memory?

given linear GW signal \( h^{(0)} \), can compute memory signal

\[
h^{(1)}(t, r) = \frac{1}{2r} \int_{-\infty}^{t} dt' \int_{r'} |r \dot{h}^{(0)}(t, r')|^2
\]

1. “late-time memory”

\[
\Delta h^{(1)} \equiv \lim_{t \to \infty} h^{(1)}(t)
\]

unobservable with ground-based interferometers

2. frequency domain waveform

\[
\tilde{h}^{(1)}(f) = -\int_{\mathbb{R}} dt \int_{r'} \frac{ie^{-2\pi ift}}{4\pi fr} |r h^{(0)}|^2
\]

this is our best bet
Higher-order memory

• the memory GWs can themselves act as a source “memory of the memory”

\[
\tilde{h}^{(n)}(f) = -\frac{4G}{fr} \int_{\mathbb{R}} dt \int_{\hat{r}'} ie^{-2\pi ift} \frac{d^3 E_{gw}^{(n-1)}}{dt \, d^2 \hat{r}'}
\]

• for cusps, this introduces factors of \( \ell / \ell_* \), where

\[
\ell_* \sim \delta / (G\mu)^3 \sim 90 \text{ m} \times \left( \frac{G\mu}{10^{-11}} \right)^{-7/2}
\]
Finite-width regularisation

- original waveform assumes string has zero width (Nambu Goto approximation)
- UV divergence due to $\dot{h}^{(0)} \to \infty$
- “hidden” by narrow beam, but leaks out due to GR nonlinearity
- natural solution: high-frequency cutoff at string width scale
  \[
  \delta \sim \ell_{\text{Pl}} / \sqrt{G \mu}
  \]
  \[
  f < \frac{1}{\delta} \sim 10^{38} \text{ Hz} \times \left( \frac{G \mu}{10^{-11}} \right)^{1/2}
  \]
Understanding the divergence

- toy model suggests a necessary (but not sufficient) condition:

\[ \max_t |\dot{r}| \gg 1, \quad \max_t |\dot{E}_{gw}| \gg \frac{1}{G} = \frac{m_{pl}}{t_{pl}} \]

- for BBHs,

\[ \max_t |\dot{r}| \sim \frac{m_1 m_2}{(m_1 + m_2)^2} \lesssim 1 \]

(which makes sense)