Discrete symmetries and QM studies with entangled neutral kaons at KLOE-2

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Abstract. The long history of kaon physics results produced by KLOE is being continued at the upgraded KLOE-2 detector. Entangled neutral kaon pairs produced at DAΦNE are a unique tool to probe discrete symmetries and QM basic principles at the utmost precision. The status of the newest ongoing analyses using the most refined analysis tools will be presented and discussed: (i) search for decoherence and CPT violation effects in the $\phi \to K_S K_L \to \pi^+\pi^-\pi^+\pi^-$ decay, (ii) test of CP and CPT symmetries in $K_S$ semileptonic decays, (iii) test of time reversal and CPT in transitions in $\phi \to K_S K_L \to \pi e\nu, 3\pi^0, 2\pi$ decays, (iv) study of the $K_S \to \pi^+\pi^-\pi^0$ decay.

1. Introduction
The test of fundamental discrete symmetries with the neutral K meson system has a long and rich history. The properties of this system are directly related to the CP, T and CPT symmetries and provide the potential of designing new tests and to search for violation effects. Such potential can be further enhanced by using the quantum entanglement of neutral kaons produced in certain decays, thus opening a whole new field of quantum interferometry studies. The KLOE experiment, being the only facility which can study entangled neutral kaon pairs, is one of the most important contributors to the state of knowledge of kaon physics and related discrete symmetries violation. Event though a lot of focus in the field of discrete symmetry studies in the recent years has been shifted towards heavier neutral meson systems such as B and D, the KLOE experiment strives to prove that the potential of neutral kaon physics is far from being fully exploited.

The KLOE (K LOng Experiment) detector located at Laboratori Nazionali di Frascati, Italy operates at the DAΦNE electron-positron collider, which provides $e^+e^-$ collisions with center-of-mass energy set at the $\phi$ resonance mass peak. The decays of $\phi$ into charged and neutral kaon pairs recorded by KLOE provide an opportunity to extensively study kaon physics, inevitably related to fundamental discrete symmetries. A unique property of KLOE and DAΦNE is the production of neutral kaon pairs in a quantum-entangled state, which opens a way to probe discrete symmetries by means of quantum interferometry.

Operation of the KLOE detector yielded a dataset with an integrated luminosity of about 2.5 fb$^{-1}$, corresponding to $10^{10}$ of produced $\phi$ mesons. This data have been used to test discrete symmetries in the neutral kaon systems using diverse processes and techniques bringing multiple results which are reported on in this work. Moreover, improvement of some of the results...
is possible by ongoing refined data analyses whose progress is presented. Finally, the recent upgrade of the detector and the new measurements presently performed by the KLOE-2 setup show perspectives to improve almost all of the KLOE results related to discrete symmetries. The expected improvements to be brought by KLOE-2 are also discussed in this work.

2. Search for CPT violation effects with the $\phi \rightarrow K_S K_L \rightarrow \pi^+\pi^-\pi^+\pi^-$ process

One of the possible indirect probes of the CPT symmetry violation is a search for Lorentz noninvariance. The Standard Model Extension (SME) and the Greenberg Anti-CPT theorem [1, 2] state that CPT violation should appear together with Lorentz invariance breaking which may be manifested as a direction-dependent modulation of the usual $\delta_K$ parameter in neutral $K$ meson system. This modulation can be parametrized as:

$$\delta_K \simeq i \sin \phi_{SW} e^{i\phi_{SW}} \gamma_K (\Delta a_0 - \beta^*_K \Delta a)/\Delta m,$$

where $\phi_{SW}$ is the superweak phase, $\gamma_K$ and $\beta^*_K$ are the Lorentz factor and velocity of a kaon in the laboratory frame and $a_\mu$ denote coefficients of the SME lagrangian part. It is thus possible to extract $a_\mu$ from a measurement of the $\delta_K$ dependence on the kaon momentum direction.

At KLOE, the pairs of neutral $K$ mesons are produced in an entangled state for which the double decay rate for the kaons decaying into final states $f_1$ and $f_2$ in their proper times differing by $\Delta \tau$ reads:

$$I_{f_1f_2}(\Delta \tau) = C_{12} e^{-\Gamma|\Delta \tau|} \left[ |\eta_1|^2 e^{2i\phi_{SW}/2} e^{-\Delta \phi} + |\eta_2|^2 e^{-2i\phi_{SW}/2} e^{\Delta \phi} - 2\Re \left( \eta_1 \eta_2^* e^{-i\Delta m \Delta \tau} \right) \right],$$

where $\Delta \Gamma = \Gamma_S - \Gamma_L$, $\Gamma = (\Gamma_S + \Gamma_L)/2$ and

$$\eta_i = \frac{\langle f_i|T|K_L\rangle}{\langle f_i|T|K_S\rangle}, \quad i = 1, 2.$$ (3)

In fact, if two identical final states are chosen e.g. as $\pi^+\pi^-$, the aforementioned CPT violation effects can lead to a discrepancy between the $\eta_1$ and $\eta_2$ parameters due to different momenta of the two kaons. Their momentum dependence would appear in the following form:

$$\eta_1 \simeq \epsilon_K - \delta_K (\vec{p}_1),$$

$$\eta_2 \simeq \epsilon_K - \delta_K (\vec{p}_2),$$ (5)

where $\epsilon_K$ is the usual parameter of the neutral kaon system related to T and CP violation and $\delta_K$ depends on the kaon momentum. Therefore, a measurement of the double decay rate for the $\phi \rightarrow K_S K_L \rightarrow \pi^+\pi^-\pi^+\pi^-$ as a function of kaons momenta configuration in a given reference frame allows for extraction of the direction-dependent modulation of $\delta_K$ and, as seen from equation 1, for a measurement of the $\Delta a_\mu$ parameters of the quark sector of the Standard Model Extension.

As the frame of fixed stars (sidereal frame) is natural choice of a reference frame to observe the modulation, the set of $\phi \rightarrow K_S K_L \rightarrow \pi^+\pi^-\pi^+\pi^-$ events recorded by KLOE was divided into two subsamples of kaon emission angle in the laboratory ($\vec{p}_1 \cdot \vec{p}_\phi \lesssim 0$ where $p_1$ is the momentum of the kaon with a larger $z$ momentum component of the kaons pair) and four bins of sidereal time corresponding to varying location of the laboratory w.r.t. the sidereal frame due to Earth rotation. The double decay rates $I_{\pi^+\pi^-\pi^+\pi^-}(\Delta \tau)$ for thus obtained 8 subsamples were simultaneously fit in order to extract the $\Delta a_\mu$ parameters. The $I_{\pi^+\pi^-\pi^+\pi^-}(\Delta \tau)$ distributions and the fit results are shown in figure 1.

Table 1 presents values $\Delta a_\mu$ SME parameters measured by KLOE. These results, reaching the sensitivity at the level of $10^{-18}$ GeV expected for the kaon sector, presently are the most precise...
measurement for the quark sector of the SME. It is worth noting that these results are dominated by statistical uncertainty, leaving room for improvement by the KLOE-2 detector which aims to collect a data sample of at least $5 \text{ fb}^{-1}$. On top of the larger integrated luminosity, KLOE-2 is also expected to reduce the systematic error thanks to the improvement of the tracking and vertexing resolutions provided by the new inner tracking detector [4].

Table 1. Values of the SME lagrangian parameters measured by the KLOE experiment [3].

| Parameter | KLOE measurement |
|-----------|------------------|
| $\Delta a_0$ | $(-6.0 \pm 7.7_{\text{stat}} \pm 3.1_{\text{syst}}) \times 10^{-18}$ GeV |
| $\Delta a_X$ | $(0.9 \pm 1.5_{\text{stat}} \pm 0.6_{\text{syst}}) \times 10^{-18}$ GeV |
| $\Delta a_Y$ | $(-2.0 \pm 1.5_{\text{stat}} \pm 0.5_{\text{syst}}) \times 10^{-18}$ GeV |
| $\Delta a_Z$ | $(3.1 \pm 1.7_{\text{stat}} \pm 0.6_{\text{syst}}) \times 10^{-18}$ GeV |

3. Search for quantum decoherence with $\phi \rightarrow K_S K_L \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

The process involving decays of both entangled kaons into a final state with two charged pions can also be studied to test the basic principles of quantum mechanics by searching for possible decoherence of the kaons state. According to quantum mechanics, the entangled state of the two kaons produced in a $\phi$ meson decay prohibits their simultaneous decay into an identical final state. This effect is caused by the completely destructive interference between states with identical kaon decay times, which results in a dip in the double decay intensity $I_{\pi^+ \pi^- \pi^+ \pi^-}(\Delta \tau)$ for $\Delta \tau \approx 0$ (visible e.g. in the distributions shown in figure 1). However, according to the Furry hypothesis of “spontaneous factorization” [5], a decoherence effect might take place immediately after the $\phi \rightarrow K^0 \bar{K}^0$ decay, leading to factorization of the kaons into a non-entangled state. This hypothetical effect violating the quantum mechanics principles can be parametrized by an $\zeta$.
parameter accounting for weakening of the interference as follows:

\[
I(\pi^+\pi^-, \pi^+\pi^-, \Delta t) = \frac{N}{2} \left[ |(\pi^+\pi^-, \pi^+\pi^-|K^0\bar{K}^0(\Delta t))|^2 + |(\pi^+\pi^-, \pi^+\pi^-|K^0\bar{K}^0(\Delta t))|^2 - (1 - \zeta_{0\bar{0}}) \cdot 2\Re (\langle \pi^+\pi^-, \pi^+\pi^-|K^0\bar{K}^0(\Delta t)\rangle\langle \pi^+\pi^-, \pi^+\pi^-|\bar{K}^0K^0(\Delta t)\rangle^*) \right].
\]

(6)

The \(\zeta_{0\bar{0}}\) is the decoherence parameter defined in the \(\{K^0, \bar{K}^0\}\) basis and a similar \(\zeta_{SL}\) parameter may be defined in the \(\{K_S, K_L\}\) basis.

At KLOE, a sample of 1.7 fb\(^{-1}\) was used to select \(e^+e^- \rightarrow \phi \rightarrow K_SK_L \rightarrow \pi^+\pi^-\pi^+\pi^-\) events and to determine the \(I_{\pi^+\pi^-, \pi^+\pi^-}(|\Delta \tau|)\) double decay rate as a function of the time difference between kaon decays. The residual background originated mostly from non-resonant \(e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\). The resulting \(I_{\pi^+\pi^-, \pi^+\pi^-}(|\Delta \tau|)\) spectrum shown in figure 2 was fitted with the intensity function given by equation 6, including the effects of resolution and of background contamination, which account for the reduction of visible destructive interference pattern in the region around \(|\Delta \tau| \approx 0\). With the fit (also shown in figure 2) the following results on the decoherence parameters were obtained [6]:

\[
\begin{align*}
\zeta_{0\bar{0}} &= (1.4 \pm 0.9_{\text{stat}} \pm 3.8_{\text{syst}}) \times 10^{-7}, \\
\zeta_{SL} &= (0.3 \pm 1.8_{\text{stat}} \pm 0.6_{\text{syst}}) \times 10^{-2}.
\end{align*}
\]

(7) (8)

Figure 2. The double decay rate \(I_{\pi^+\pi^-, \pi^+\pi^-}(|\Delta \tau|)\) for \(e^+e^- \rightarrow \phi \rightarrow K_SK_L \rightarrow \pi^+\pi^-\pi^+\pi^-\) (black points) and the result of a fit including effects of resolution and background contamination (solid histogram). Red hatched area denotes uncertainty related to the efficiency correction. Figure adapted from [6].

Figure 3. Preliminary double decay rate for \(\phi \rightarrow K_SK_L \rightarrow \pi^+\pi^-\pi^+\pi^-\) obtained from KLOE data with refined event selection (black circles). Hollow red circles denote the fit result.

The above results are compatible with the predictions of quantum mechanics, showing no indications of decoherence of the kaons’ states. The achieved sensitivity on the \(\zeta_{0\bar{0}}\) parameter
at the level of $10^{-6}$ is especially good due to the CP suppression in the used decay channel and improves by several orders of magnitude the previous measurement with neutral kaons [7] and a result obtained with B mesons [8]. However, the sensitivity reached by KLOE in this measurement is still limited by the presence of residual background in the interference region (see figure 2). Recently, improved techniques of selection of the $\phi \to K_S K_L \to \pi^+ \pi^- \pi^+ \pi^-$ events have been devised and applied to KLOE data, yielding a cleaner sample with the same dataset of 1.7 fb$^{-1}$ used in the previous study. The new event selection criteria are similar to those used in the search for CPT and Lorentz invariance violation [3] described in section 2. At present, a re-analysis of KLOE data is in progress and a preliminary $I_{\pi^+\pi^-\pi^+\pi^-}(\Delta \tau)$ distribution is shown in figure 3 where a more pronounced region of destructive interference ($\Delta \tau \approx 0$) is clearly visible (compare to figure 2). This improved analysis is expected to bring a substantial reduction of the uncertainties on $\zeta_{00}$ and $\zeta_S$. Moreover, a further improvement of sensitivity on the decoherence parameters is expected from the KLOE-2 experiment, both due to the larger statistical sample and to the use of the inner tracker detector [4], which will improve the tracking performances for $\pi^+\pi^-$ events thus allowing to reduce by a factor of 3 the resolution spread in the $\Delta \tau \approx 0$ region of the double kaon decay rate $I_{\pi^+\pi^-\pi^+\pi^-}$.

4. Tests of CP and CPT in $K_S$ semileptonic decays

The possible charge asymmetry in semileptonic decays of neutral K mesons, i.e. asymmetry between the $K_{S,L} \to \pi^+ e^- \bar{\nu}_e$ and $K_{S,L} \to \pi^- e^+ \nu_e$ decays, is closely related to fundamental discrete symmetries as its value may be expressed in terms of the following CP and CPT violating parameters:

$$A_{S,L} = \frac{\Gamma(K_{S,L} \to \pi^- e^+ \nu_e) - \Gamma(K_{S,L} \to \pi^+ e^- \bar{\nu}_e)}{\Gamma(K_{S,L} \to \pi^- e^+ \nu_e) + \Gamma(K_{S,L} \to \pi^+ e^- \bar{\nu}_e)} = 2 \left[ Re(\epsilon_K) \pm Re(\delta_K) - Re(y) \pm Re(x_-) \right].$$  (9)

In the above expression, $\epsilon_K$ and $\delta_K$ are the usual parameters for the kaon system corresponding respectively to CP and CPT violation whereas $y$ describes CPT violation in $K^0(\bar{K}^0) \to \pi e\nu$ decays and $x_-$ is a function of semileptonic kaon decay amplitudes which vanishes if the $\Delta S = \Delta Q$ rule holds. Therefore, if CPT invariance is assumed, the semileptonic asymmetries for $K_L$ and $K_S$ should both reduce to the same value, related to CP violation and expected to be at the level of $10^{-3}$:

$$A_S = A_L = 2 Re(\epsilon_K) \approx 3 \cdot 10^{-3}. \quad (10)$$

Consequently, measurements of the semileptonic asymmetries in the decays of both short and long-lived neutral kaons not only provide a probe of CP violation but also allow for a test of the CPT symmetry through a comparison between $A_S$ and $A_L$

The most precise results available to date comprise a measurement of $A_L = (3.332 \pm 0.058_{stat} \pm 0.047_{syst}) \times 10^{-3}$ by the KTeV experiment [9] and a determination of $A_S$ performed by KLOE using a data sample of 0.41 fb$^{-1}$ with the following result [10]:

$$A_S = (1.5 \pm 9.6_{stat} \pm 2.9_{syst}) \times 10^{-3}. \quad (11)$$

Due to a significant difference of semileptonic decay rates for $K_S$ and $K_L$, the uncertainty on the $A_S$ asymmetry is the limiting factor of a possible CPT test and its reduction is crucial. The uncertainty on KLOE result is dominated by the statistical error which can be improved by using a larger data sample available from KLOE. Therefore presently the whole KLOE dataset of 1.7 fb$^{-1}$ is subject to an improved analysis aiming at a new determination of the semileptonic asymmetry of the $K_S$.

In this analysis, $K_S$ decays at KLOE are tagged by an interaction of $K_L$ in the electromagnetic calorimeter and, subsequently, decays involving charged pion and electron/positron tracks are
Figure 4. Distribution of the difference between missing energy and missing momentum for selected events. Shaded area denotes the signal region where semileptonic $K_S$ decays with electron and positron are counted. Data is presented in black histogram, total MC is the dotted line and colored histograms show MC-based shapes for major background components used for background normalization and subtraction.

identified based on the particles’ time of flight from the decay vertex to the calorimeter. In order to count the events with $K_S \rightarrow \pi^+ e^- \bar{\nu}_e$ and $K_S \rightarrow \pi^- e^+ \nu_e$, the spectrum of the difference between the missing energy and the missing momentum of the $K_S$ decay $\Delta E(\pi, e)$ is built up, assuming that the two reconstructed tracks are an electron and a pion: as shown in figure 4 the semileptonic decays with an undetected neutrino result in a pronounced peak around zero. A certain amount of background mostly from $K_S \rightarrow \pi^+ \pi^- (\gamma) (\rightarrow \pi \mu \nu)$ is allowed in the final $\Delta E(\pi, e)$ distribution as MC-simulated shapes for the major background components are normalized to the data and subtracted from the events counted in the signal region (see figure 4). More details of this ongoing analysis can be found in reference [11]. As the analyzed data sample is 4 times larger, an twofold reduction of the statistical error is expected. Moreover, the KLOE-2 experiment arises prospects of measuring $A_S$ with the uncertainty at the level of $3 \times 10^{-3}$.

5. Test of time-reversal and CPT symmetries in neutral kaon transitions with $\phi \rightarrow K_SK_L \rightarrow \pi e\nu, 3\pi^0, 2\pi$ decays

Neutral meson pairs in a quantum entangled state provide a unique opportunity to test fundamental symmetries by means of a comparison between a certain transition of the neutral meson state and a process obtained through conjugation of the transition e.g. with the T or CPT operator. The symmetry under reversal in time (T) is a notable case where the processes compared in a direct test are obtained simply by an exchange of initial and final states. Such a test is feasible with the system of neutral K mesons by observing transitions between kaon states in the strangeness basis $\{K^0, \bar{K}^0\}$ and eigenstates of the CP operator corresponding to eigenvalues of +1 and −1, i.e. $\{K_+ \approx K_S, K_- \approx K_L\}$ [12].

In this test, the final state of a certain kaon transition is identified in the $\{K^0, \bar{K}^0\}$ basis by observation of its decay through a semileptonic channel with an electron or positron, where the charge of the lepton identifies the strangeness of the decaying kaon according to the $\Delta S = \Delta Q$ rule. Similarly, decays into a hadronic final state with $\pi^+ \pi^- (\text{CP}=+1)$ or $3\pi^0 (\text{CP}=-1)$ define the kaon state in the CP basis $\{K_+, K_-\}$. The identification of the initial state of a kaon transition must be performed before the decay occurs, and it is only possible by exploiting the quantum entanglement of neutral kaon pairs produced in a $\phi$ decay. An observation of the earlier between the two kaon decays provides information about the state of its partner particle being in an orthogonal state at the same moment of time.

Out of four possible transitions which may be compared with their T-conjugates, two can be

While $K_L$ and $K_S$ are not exact CP eigenstates, it was shown that kaons mixing due to CP violation has a negligible effect on the T symmetry test [12].
used at KLOE-2 to determine with high statistics the asymmetry ratios dependent on the time difference $\Delta t$ between the two kaon decays (refer to [12] for a detailed study). The probabilities entering these ratios are estimated by double decay rates $I(f_1, f_2; \Delta t)$ corresponding to processes characterized by a pair of kaon decays into specific final states $f_1$ and $f_2$ at times separated by $\Delta t$:

$$R_2(\Delta t) = \frac{P[K^0(0) \to K^- \Delta t]}{P[K^0(0) \to K^- \Delta t]} \sim \frac{I(\pi^+ e^- \nu_\beta, 3\pi^0; \Delta t)}{I(\pi^+, \pi^- e^+ \nu; \Delta t)}$$  \hspace{1cm} (12)

$$R_4(\Delta t) = \frac{P[\bar{K}^0(0) \to K^+ \Delta t]}{P[\bar{K}^0(0) \to K^+ \Delta t]} \sim \frac{I(\pi^- e^+ \nu_\beta, 3\pi^0; \Delta t)}{I(\pi^-, \pi^+ e^+ \nu; \Delta t)}$$  \hspace{1cm} (13)

It was shown that in the asymptotic region of large time differences ($\Delta t \gg \tau_S$ where $\tau_S$ is the lifetime of $K_S$) the deviation of $R_2(\Delta t)$ and $R_4(\Delta t)$ from unity should be proportional to the real part of $\epsilon_K$ which is a $T$ violating parameter of the neutral kaon system [12].

It is worth noting that although several experiments exist with access to entangled $B$ mesons and such time-reversal symmetry test has been implemented at the BaBar experiment [13], KLOE-2 is the only existing experimental setup with entangled kaons, and represents the unique possibility to perform $T$ symmetry tests in the neutral kaon system. With its goal integrated luminosity, a statistically significant test of time reversal symmetry is expected with a sensitivity at the level of $10^{-3}$.

Moreover, the aforementioned concept of direct discrete symmetry test in transitions can be extended to the CPT symmetry by an appropriate choice of the compared processes. Thus, a similar set of time-dependent ratios sensitive to CPT violation is measurable at KLOE-2 [14]:

$$R_{2,\text{CPT}}(\Delta t) = \frac{P[K^0(0) \to K^- \Delta t]}{P[K^0(0) \to K^- \Delta t]}$$  \hspace{1cm} (14)

$$R_{4,\text{CPT}}(\Delta t) = \frac{P[\bar{K}^0(0) \to K^+ \Delta t]}{P[\bar{K}^0(0) \to K^+ \Delta t]}$$  \hspace{1cm} (15)

Additionally, in case of the CPT test, the above ratios may be combined into a double ratio which is a very robust CPT-violation observable in the $\Delta t \gg \tau_S$ limit:

$$R_{2,\text{CPT}}^{\text{exp}}(\Delta t \gg \tau_S) = 1 - 8 \Re \delta_K - 8 \Re \epsilon_\pi.$$  \hspace{1cm} (16)

Any deviation from unity in this observable is produced by a genuine CPT noninvariance effect without the need for $\Delta S = \Delta Q$ rule assumption and with a negligible impact of $CP$ violation [14]. The value of this CPT-violation observable has never been measured before. Therefore, determination of $R_{2,\text{CPT}}(\Delta t)$ and $R_{4,\text{CPT}}(\Delta t)$ establishes a test of CPT symmetry complementary to the one described in section 4 and is one of the goals of the KLOE-2 experiment along with the time-reversal symmetry test.

The experimental realization of both aforementioned tests requires selection and reconstruction of two classes of events: $K_S K_L \to \pi^+ \pi^- \pi^0 \nu$ and $K_S K_L \to \pi^0 \nu_\beta$. Methods to select the signal and to reconstruct the kaon decay times with sufficient resolution are already being devised using both KLOE dataset and KLOE-2 data collected so far. Among the kaon decays of interest, $K_L \to 3\pi^0 \to 6\gamma$ involving no charged particles is of special importance as its reconstruction must be based solely on information from the electromagnetic calorimeter. A dedicated reconstruction algorithm has been prepared in order to allow for analytical calculation of the $K_L$ decay location and time. It is based on the trilateration technique also used in GPS positioning and was tested to provide a resolution of $K_L$ decay time of about $2 \tau_S$ [15]. A preliminary spectrum of the double decay rate $I(\pi^0 \nu_\beta, 3\pi^0; \Delta t)$ at its present stage of event selection and obtained with the dedicated $K_L \to 3\pi^0 \to 6\gamma$ reconstruction is shown in figure 5. The major residual background component is presently due to $K_S \to \pi^+ \pi^-$ and methods to purify the sample are being elaborated.
Figure 5. Raw distribution of double decay rate $I(\pi e\nu, 3\pi^0; \Delta t)$ for $K_S K_L \rightarrow \pi e\nu \, 3\pi^0$ events as a function of time difference between kaon decays. The spectrum was obtained with KLOE data (black points) and corresponding MC simulations (histograms). The major residual background component is presently due to $K_S \rightarrow \pi^+ \pi^- \pi^0$.

6. Search for CPT violation with rare $K_S$ decays

Another possible approach to study the mechanism of CP violation is to search for CP-forbidden rare decays of neutral K mesons, e.g. decay of $K_S$ into three pions. The $K_S \rightarrow 3\pi^0$ decay is a pure CP-violating process whereas $K_S \rightarrow \pi^+ \pi^- \pi^0$ violates CP for $I=1$ or $3$ in the final state.

CP violation in the system of neutral kaons is usually expressed in terms of the following amplitude ratios (for $K_S$ decays):

\[
\eta_{+0} = \frac{\langle \pi^+\pi^-\pi^0 | H | K_S \rangle}{\langle \pi^+\pi^-\pi^0 | H | K_L \rangle} = \epsilon_K + \epsilon'_{+0},
\]

\[
\eta_{000} = \frac{\langle \pi^0\pi^0\pi^0 | H | K_S \rangle}{\langle \pi^0\pi^0\pi^0 | H | K_L \rangle} = \epsilon_K + \epsilon'_{000},
\]

where $\epsilon_K$ is the usual parameter of CP violation in mixing and $\epsilon'_{+0}$ and $\epsilon'_{000}$ describe CP violation in decay.

Although the $K_S \rightarrow 3\pi^0$ decay has never been observed to date, the KLOE experiment has set the most precise upper limit on its branching ratio using a dataset of $1.7 \text{ fb}^{-1}$. The presence of short lived neutral kaons was tagged by detecting the associated $K_L$ interaction in the calorimeter, similarly to the case of the charge asymmetry measurement described in section 4. The 6 photon candidates from $K_S \rightarrow 3\pi^0$ are then reconstructed based on calorimeter information and a kinematic fit is applied. Finally, a discriminant analysis based on the kinematical difference between the signal and the $K_S \rightarrow 2\pi^0$ overwhelming background is performed. For further details of the analysis the reader is referred to [16].

The upper limit obtained by KLOE [16] amounts to:

\[
\text{BR}(K_S \rightarrow 3\pi^0) < 2.6 \times 10^{-8},
\]

which yields the limit $|\eta_{000}| < 0.0088$ at 90% confidence level. Further improvement is expected from the KLOE-2 experiment which is going to collect at least 5 fb$^{-1}$ of data allowing to push its sensitivity to $\text{BR}(K_S \rightarrow \pi^0\pi^0\pi^0)$ below $10^{-8}$.

The same KLOE dataset used to study $K_S \rightarrow 3\pi^0$ is presently being explored in a direct search for $K_S$ decays into the $\pi^+\pi^-\pi^0$ final state. Current knowledge of the branching ratio for this process is an average of three indirect measurements and amounts to $\text{BR}(K_S \rightarrow \pi^+\pi^-\pi^0) = 3.5^{+1.1}_{-0.9} \times 10^{-7}$ [17] with uncertainty at the level of 30%. The ongoing analysis of KLOE data shows prospects to improve this result by a direct measurement with relative uncertainty below 20%.

7. Summary and perspectives

The KLOE experiment in the past years already produced important contributions to a variety of discrete symmetry studies, such as the search for Lorentz invariance and, consequently,
CPT symmetry tests, CP and CPT testing with charge asymmetry in semileptonic kaon decays, searches for deviation from quantum mechanics principles manifested by decoherence of entangled kaons’ state and searches for rare kaon decays. Despite that, the data collected by KLOE still did not reveal all its secrets and currently two enhanced re-analyses as well as a new search for rare \( K_S \rightarrow \pi^+\pi^-\pi^0 \) are in progress to further improve the sensitivity to discrete symmetries’ violation.

At the same time, the KLOE-2 detector is half way through its new data-taking campaign with novel subdetectors and larger integrated luminosity. These two factors are expected to allow for an improvement of all of the KLOE measurements reported in this work.

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