NEUTRINO MASS AND AN EVER
EXPANDING UNIVERSE
(AN IRREVERENT PERSPECTIVE)

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Abstract

There have been two significant recent findings. One is a cul-
mination of the Superkamiokande experiments, which demon-
strate Neutrino oscillation and therefore a non-zero mass. The other is the
finding that the universe will continue to expand without declaration
based on distant supernovae observations. At the very least these two
findings call for a review of the existing and generally accepted the-
ories. In this talk it is pointed out how such a Neutrino Mass can
in fact be deduced from a theoretical model, as also the eternal ex-
pansion feature of the universe, and how both these findings do not
contradict each other.

1 Introduction and Review

We start with a brief background on the neutrino\[1\]–\[7\]. The neutrino was
proposed in 1929 by W. Pauli. In his words, ”Dear Radioactive Ladies and
Gentlemen,..... as a desperate remedy to save the principle of energy conser-
vation in $\beta$ decay, ..... I propose the idea of a neutral particle of spin half.”

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A few years later Fermi introduced the four Fermi Hamiltonian for $\beta$ decay using the Neutrino and so the theory of weak interactions was born. Finally in 1956 Reines and Cowan discovered the Neutrino. The Neutrino turned out to be a massless, chargeless spin half particle with handedness. There are an estimated $10^{90}$ neutrinos in the universe. From the beginning it has been an enigmatic particle. In the words of Pauli, again, nearly thirty years after he had first postulated it “this particle neutrino, of the existence of which I am not innocent, still persecutes me.” The generally accepted standard model retains these features of the neutrino, except that there are three families, the electron, the muon and the tau neutrinos. Because of their vanishing mass, mixings and magnetic moment also vanish. However higher order weak interactions endow it with a charge radius. Infact the electromagnetic processes appear only in the elastic scattering of neutrinos with electrons or quarks. Though the standard model predicts zero neutrino mass, all that we can say from experiment is that there are the following upper bounds for the neutrino mass:

$$\text{Mass of } \nu_e < 12eV$$
$$\text{Mass of } \nu_\mu < 170keV$$
$$\text{Mass of } \nu_\tau < 24MeV$$

This could provide a motivation for a study of physics beyond a standard model. Further there is no apriori theoretical reason why the right handed neutrino field should not exist, unlike in the case of the masslessness of the photon. In other words there are only left handed neutrinos in the standard model, just to conform to observation. On the other hand, many unification schemes do predict a neutrino mass. Though the idea of a neutrino mass and neutrino oscillation goes a long way back to authors like Markov, Pontecarvo and others, several puzzling questions have persisted. These include, the question of the smallness of the neutrino mass and the fact that the observed number of solar neutrinos is less than half the expected number. This latter problem would be solved if the neutrinos are a superposition of different mass Eigen states, leading to neutrino oscillation and therefore mass. As suitable mass for the neutrino would also solve the problem of dark matter in the galaxies, and it could also resolve the problem of the missing mass
of the universe. This is because standard big bang cosmology predicts the existence of relic background neutrinos. If these neutrinos had a mass of about $10eV$, the universe would be closed.

Though neutrinos are considered not to have any electric charge, they could have a magnetic moment induced by quantum loops in electro weak theories. In fact a magnetic moment $\sim 10^{-10}\mu_B$ would also solve the solar neutrino puzzle. This is because the neutrinos can undergo a spin flip in the sun’s magnetic field, and go over to righthanded neutrinos, which because of their extremely weak interactions cannot be detected. However, this value appears to be very high and theory predicts a value $<\sim 10^{-19}\mu_B$.

Interestingly the Kamiokande experiments based on observations of the supernova SN 1987 A indicate that if the neutrino has a charge at all, this will be less than or equal to $10^{-17}e$, rather than the earlier limit, greater than or equal to $10^{-13}e$.

It may also be mentioned that recent versions of the neutrino as a Dirac particle suggest a mass given by,

$$m_\nu \approx 10^{-7}m_l,$$

where $m_l$ is the relevant lepton.

Finally we sum up the experimental evidence which suggested a non zero neutrino mass before the recent SuperKamiokande experiments:

i) Solar neutrino deficit.

ii) The deficit of muon neutrinos relative to electron neutrinos produced in the atoms.

iii) The neutrino oscillation observed at the Los Alamos Liquid Scintillation Neutrino Detector (LSND).

iv) The Russian Tritium experiment.

v) The astronomical Tritium experiment.

The latest experiments suggest a neutrino mass $\sim 10^{-8}$ electron mass.

## 2 Kerr-Newman Formulation and Consequences

We now consider the neutrino in a slightly different context. According to a recent model, elementary particles, typically leptons, can be treated as, what may be called Quantum Mechanical Black Holes (QMBH) [8]-[12], which share certain features of Black Holes and also certain Quantum Mechanical
characteristics. Essentially they are bounded by the Compton wavelength within which non local or negative energy phenomena occur, these manifesting themselves as the Zitterbewegung of the electron. These Quantum Mechanical Black Holes are created out of the background Zero Point Field and this leads to a consistent cosmology, wherein using \( N \), the number of particles in the universe as the only large scale parameter, one could deduce from the theory, Hubble’s law, the Hubble constant, the radius, mass, and age of the universe and features like the hitherto inexplicable relation between the pion mass and the Hubble constant \( [8] \). The model also predicts an ever expanding universe, as recent observations do confirm.

Within this framework, it was pointed out that the neutrino would be a mass less and charge less version of the electron and it was deduced that it would be lefthanded, because one would everywhere encounter the pseudo spinorial (“negative energy”) components of the Dirac spinor, by virtue of the fact that its Compton wavelength is infinite (in practise very large). Based on these considerations we will now argue that the neutrino would exhibit an anomalous Bosonic behaviour which could provide a clue to the neutrino mass.

3 The anomalous neutrino

As detailed in Ref.\([9]\) the Fermionic behaviour is due to the non local or Zitterbewegung effects within the Compton wavelength effectively showing up as the well known negative energy components of the Dirac spinor which dominate within while positive energy components predominate outside leading to a doubly connected space or equivalently the spinorial or Fermionic behaviour. In the absence of the Compton wavelength boundary, that is when we encounter only positive energy or only negative energy solutions, the particle would not exhibit the double valued spinorial or Fermionic behaviour: It would have an anomalous anyonic behaviour.

Indeed, the three dimensionality of space arises from the spinorial behaviour outside the Compton wavelength\([13]\). At the Compton wavelength, this disappears and we should encounter lower dimensions. As is well known\([14]\) the low dimensional Dirac equation has like the neutrino, only two components corresponding to only one sign of the energy, displays handedness and has no invariant mass.
Of course the above model strictly speaking is for the case of an isolated non interacting particle. As neutrinos interact through the weak or gravitational forces, both of which are weak, the conclusion would still be approximately valid particularly for neutrinos which are not in bound states.

We will now justify the above conclusion from three other standpoints: Let us first examine why Fermi-Dirac statistics is required in the Quantum Field Theoretic treatment of a Fermion satisfying the Dirac equation. The Dirac spinor has four components and there are four independent solutions corresponding to positive and negative energies and spin up and down. It is well known that in general the wave function expansion of the Fermion should include solutions of both signs of energy:

\[ \psi(\vec{x}, t) = N \int d^3p \sum_{\pm s} [b(p, s)u(p, s) \exp(-ip^\mu x_\mu/\hbar) + d^*(p, s)v(p, s) \exp(+ip^\mu x_\mu/\hbar)] \] (1)

where \( N \) is a normalization constant for ensuring unit probability.

In Quantum Field Theory, the coefficients become creation and annihilation operators while \( bb^+ \) and \( dd^+ \) become the particle number operators with eigenvalues 1 or 0 only. The Hamiltonian is now given by:

\[ H = \sum_{\pm s} \int d^3p E_p [b^+(p, s)b(p, s) - d(p, s)d^+(p, s)] \] (2)

As can be seen from (2), the Hamiltonian is not positive definite and it is this circumstance which necessitates the Fermi-Dirac statistics. In the absence of Fermi-Dirac statistics, the negative energy states are not saturated in the Hole Theory sense so that the ground state would have arbitrarily large negative energy, which is unacceptable. However Fermi-Dirac statistics and the anti commutators implied by it prevent this from happening.

From the above, it follows that as only one sign of energy is encountered for the \( \nu \), we need not take recourse to Fermi-Dirac statistics.

We will now show from an alternative viewpoint also that for the neutrino, the positive and negative solutions are delinked so that we do not need the negative solutions in (1) or (2) and there is no need to invoke Fermi-Dirac statistics.
The neutrino is described by the two component Weyl equation\cite{17}:

\[ i\hbar \frac{\partial \psi}{\partial t} = ihc\vec{\sigma} \cdot \Delta \psi(x) \]  

\hspace{1cm} (3)

It is well known that this is equivalent to a mass less Dirac particle satisfying the following constraint (ref.\cite{17}):

\[ \Gamma_5 \psi = -\psi \]  

\hspace{1cm} (4)

We now observe that in the case of a massive Dirac particle, if we work only with positive solutions for example, the current or expectation value of the velocity operator \( c\vec{\alpha} \) is given by (ref.\cite{15}),

\[ J^+ = \langle c\alpha \rangle = \langle \frac{c^2 \vec{p}}{E} \rangle_+ = \langle v_{gp} \rangle_+ \]  

\hspace{1cm} (5)

in an obvious notation.

(5) leads to a contradiction: On the one hand the eigen values of \( c\vec{\alpha} \) are \( \pm c \).

On the other hand we require, \( \langle v_{gp} \rangle < 1 \).

To put it simply, working only with positive solutions, the Dirac particle should have the velocity \( c \) and so zero mass. This contradiction is resolved by including the negative solutions also in the description of the particle also. This in fact is the starting point for (1) above.

In the case of mass less neutrinos however, there is no contradiction because they do indeed move with the velocity of light. So we need not consider the negative energy solutions and need work only with the positive solutions.

There is another way to see this. Firstly, as in the case of massive Dirac particles, let us consider the packet (1) with both positive and negative solutions for the neutrino. Taking the \( z \) axis along the \( \vec{p} \) direction for simplicity, the acceptable positive and negative Dirac spinors subject to condition (4) are

\[ u = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \quad v = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} \]

The expression for the current is now given by,

\[ J^z = \int d^3p \sum_{s} [\langle |b(p,s)|^2 + |d(p,s)|^2 \rangle \frac{P^z c^2}{E} \]

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Using the expressions for $u$ and $v$ it can easily be seen that in (6) the cross (or Zitterbewegung) term disappears. Thus the positive and negative solutions stand delinked in contrast to the case of massive particles, and we need work only with positive solutions (or only with negative solutions) in (1).

Finally this can also be seen in yet another way. As is known (ref.[17]), we can apply a Foldy-Wouthuysen transformation to the mass less Dirac equation to eliminate the "odd" operators which mix the components of the spinors representing the positive and negative solutions.

The result is the Hamiltonian,

$$H' = \Gamma^0 pc$$

(7)

Infact in (7) the positive and negative solutions stand delinked. In the case of massive particles however, we would have obtained instead,

$$H' = \Gamma^0 \sqrt{p^2 c^2 + m_o c^4}$$

(8)

and as is well known, it is the square root operator on the right which gives rise to the "odd" operators, the negative solutions and the Dirac spinors. Infact this is the problem of linearizing the relativistic Hamiltonian and is the starting point for the Dirac equation.

Thus in the case of mass less Dirac particles, we need work only with solutions of one sign in (1) and (2). The equation (2) now becomes,

$$H = \sum_{\pm s} \int d^3 p E_p [b^+(p, s)b(p, s)]$$

(9)

As can be seen from (9) there is no need to invoke Fermi-Dirac statistics now. The occupation number $bb^+$ can now be arbitrary because the question of a ground state with arbitrarily large energy of opposite sign does not arise. That is, the neutrinos obey anomalous statistics.

In a rough way, this could have been anticipated. This is because the Hamiltonian for a mass less particle, be it a Boson or a Fermion, is given by

$$H = pc$$
Substitution of the usual operators for $H$ and $p$ yields an equation in which the wave function $\psi$ is a scalar corresponding to a Bosonic particle.

4 The Spin Statistics Theory

According to the spin-statistics connection, microscopic causality is incompatible with quantization of Bosonic fields using anti-commutators and Fermi fields using commutators ([16]). But it can be shown that this does not apply when the mass of the Fermion vanishes.

In the case of Fermionic fields, the contradiction with microscopic causality arises because the symmetric propagator, the Lorentz invariant function,

$$\Delta_1(x - x') \equiv \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_k} [e^{-ik(x-x')} + e^{ik(x-x')}],$$

does not vanish for space like intervals $(x - x')^2 < 0$, where the vacuum expectation value of the commutator is given by the spectral representation,

$$S_1(x-x') \equiv i <[\psi_\alpha(x), \psi_\beta(x')]|0>= - \int dM^2 [\rho_1(M^2)\Delta_x + \rho_2(M^2)]_{\alpha\beta} \Delta_1(x-x').$$

Outside the light cone, $r > |t|$, where $r \equiv |\vec{x} - \vec{x}'|$ and $t \equiv |x_o - x'_o|$, $\Delta_1$ is given by,

$$\Delta_1(x' - x) = -\frac{1}{2\pi^2 r} \frac{\partial}{\partial r} K_o(m\sqrt{r^2 - t^2}),$$

where the modified Bessel function of the second kind, $K_o$ is given by,

$$K_o(mx) = \int_0^\infty \frac{\cos(xy)}{\sqrt{m^2 + y^2}} dy = \frac{1}{2} \int_{-\infty}^\infty \frac{\cos(xy)}{\sqrt{m^2 + y^2}} dy$$

(cf.[18]). In our case, $x \equiv \sqrt{r^2 - t^2}$, and we have,

$$\Delta_1(x - x') = const \frac{1}{x} \int_{-\infty}^\infty \frac{y \sin xy}{\sqrt{m^2 + y^2}} dy$$

As we are considering massless neutrinos, going to the limit as $m \to 0$, we get, $|Lt_{m \to 0} \Delta_1(x - x')| = |(const.) Lt_{m \to 0} \frac{1}{x} \int_{-\infty}^\infty \sin xy dy| < \frac{\theta(1)}{x}$. That is, as the Compton wavelength for the neutrion is infinite (or very large), so is $|x|$. 

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and we have $|\Delta_1| << 1$. So the invariant $\Delta_1$ function nearly vanishes everywhere except on the light cone $x = 0$, which is exactly what is required. So, the spin-statistics theorem or microscopic causality is not violated for the mass less neutrinos when commutators are used.

5 Neutrino Mass

The fact that the ideally, massless, spin half neutrino obeys anomalous statistics could have interesting implications. For, given an equilibrium collection of neutrinos, we should have if we use the Bose-Einstein statistics\[19]\).

\[ PV = \frac{1}{3} U, \quad (10) \]

instead of the usual

\[ PV = \frac{2}{3} U, \quad (11) \]

where $P, V$ and $U$ denote the pressure, volume and energy of the collection. We also have, $PV \alpha NkT$, $N$ and $T$ denoting the number of particles and temperature respectively.

On the other hand for a fixed temperature and number of neutrinos, comparison of (10) and (11) shows that the effective energy $U'$ of the neutrinos would be twice the expected energy $U$. That is in effect the neutrino acquires a rest mass $m$. It can easily be shown from the above that,

\[ \frac{m e^2}{k} \approx \sqrt{3} T \quad (12) \]

That is for cold background neutrinos $m$ is about a thousandth of an $eV$ at the present background temperature of about $2^o K$:

\[ 10^{-9} m_e \leq m \leq 10^{-8} m_e \quad (13) \]

This can be confirmed, alternatively, as follows. As pointed out by Hayakawa, the balance of the gravitational force and the Fermi energy of these cold background neutrinos, gives[20],

\[ \frac{G N m^2}{R} = \frac{N^{2/3} \hbar^2}{m R^2}, \quad (14) \]
where $N$ is the number of neutrinos. Further as in the Kerr-Newman Black Hole formulation equating (14) with the energy of the neutrino, $mc^2$ we immediately deduce

$$m \approx 10^{-8}m_e$$

which agrees with (12) and (13). It also follows that $N \sim 10^{90}$, which is correct. Moreover equating this energy of the quantum mechanical black hole to $kT$, we get (cf. also (12))

$$T \sim 1^oK,$$

which is the correct cosmic background temperature. Alternatively, using (12) and (13) we get from (14), a background radiation of a few millimeters wavelength, as required. So we obtain not only the mass and the number of the neutrinos, but also the correct cosmic background temperature, at one stroke.

6 Discussion

1) Hayakawa (cf.ref.[20]) in effect assumes the above neutrino mass and equating the energy of oscillation of the background neutrino gas which is in equilibrium under the Fermi gas pressure and gravitational attraction, with the energy due to the weak Fermi interaction deduces the correct value of the Fermi coupling constant $G_F$.

On the other hand, if the weak interaction is mediated by an intermediate particle of mass $M$ and Compton wavelength $L$, we will get exactly as in the model described in Section 2 for the electrons[21] (cf.ref.[8] also), from the fluctuation of particle number $N$, on using (13),

$$g^2\sqrt{NL^2} \approx mc^2 \sim 10^{-14},$$

From (15), on using the value of $N$, we get,

$$g^2L^2 \sim 10^{-59}$$

This agrees with experiment and the theory of massless particles the neutrino specifically acquiring mass due to interaction[22], using the usual value of
\[ M \sim 100\text{Gev}.. \]

Alternatively, from (14), we get the correct value of \( g^2/m_W^2 \).

Thus a complete characterization of the weak interaction is possible.

2) The present value of the neutrino mass, as given by equation (13), for example falls well short of the 10eV required to close the universe. Thus there is no contradiction with the latest observations which indicate that the universe is expanding for ever [23].

3) Interestingly, if we use Bose-Einstein statistics, and equation (10) for solar neutrinos, rather than for the background neutrinos as we have done, their number would need to be halved, as in the solar neutrino puzzle.

4) The preceding considerations do not contradict Hayakawa’s earlier work, as seen above. Indeed, the fact that the background neutrino temperature equals the Fermi temperature would also explain the Bosonization effect. One way to see this is as follows:

For a collection of Fermions, we know that the Fermi energy is given by [19],

\[
\epsilon_F = \frac{p_F^2}{2m} = \left(\frac{\hbar^2}{2m}\right)\left(\frac{6\pi^2}{v}\right)^{2/3} 
\]  

(16)

where \( v^{1/3} \) is the interparticle distance. On the other hand, in a different context, for phonons, the maximum frequency is given by, (cf.ref.[19]).

\[
\omega_m = c\left(\frac{6\pi^2}{v}\right)^{1/3} 
\]  

(17)

This occurs for the phononic wavelength \( \lambda_m \approx \text{inter-atomic distance between the atoms, } v^{1/3}, \) being, again, the mean distance between the phonons. \( \text{'}c\text{'} \) in (17) is the velocity of the wave, the velocity of sound in this case. The wavelength \( \lambda_m \) is given by,

\[
\lambda_m = \frac{2\pi c}{\omega_m} 
\]

We can now define the momentum \( p_m \) via the de Broglie relation,

\[
\lambda_m = \frac{\hbar}{p_m} 
\]

which gives,

\[
p_m = \frac{\hbar}{c}\omega_m, 
\]  

(18)
We can next get the maximum energy corresponding to the maximum frequency $\omega_m$ given by (17), which as is known is,

$$\epsilon_m = \frac{p_m^2}{2m} = \frac{\hbar^2}{2m} \left( \frac{6\pi^2}{v} \right)^{2/3} \tag{19}$$

Comparing (16) and (19), we can see that $\epsilon_m$ and $p_m$ exactly correspond to $\epsilon_F$ and $p_F$.

The Fermi energy in (16) is obtained as is known by counting all single particle energy levels below the Fermi energy $\epsilon_F$ using Fermi-Dirac statistics, while the maximum energy in (19) is obtained by counting all energy levels below the maximum value, but by using Bose-Einstein statistics (cf.ref.[19]).

We can see why inspite of this, the same result is obtained in both cases. In the case of the Fermi energy, all the lowest energy levels below $\epsilon_F$ are occupied with the Fermionic occupation number $< n_p > = 1$, $p < p_F$. Then, the number of levels in a small volume about $p$ is $d^3p$. This is exactly so for the Bosonic levels also. With the correspondence given in (18), the number of states in both cases coincide and it is not surprising that (16) and (19) are the same. In effect, Fermions below the Fermi energy have a strong resemblance to phonons. This is reminiscent of Fermi-Bose transmutation [24].

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