New light mediators for the $R_K$ and $R_{K^*}$ puzzles

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The measurements of $R_K$ and $R_{K^*}$ provide hints for the violation of lepton universality. However, it is generally difficult to explain the $R_{K^*}$ measurement in the low $q^2$ range, $0.045 \leq q^2 \leq 1.1 \text{ GeV}^2$. Light mediators offer a solution by making the Wilson coefficients $q^2$ dependent. We check if new lepton nonuniversal interactions mediated by a scalar ($S$) or vector particle ($Z'$) of mass between $10-200 \text{ MeV}$ can reproduce the data. We find that a 25 MeV $Z'$ with a $q^2$-dependent $b \to s$ coupling and that couples to the electron but not the muon can explain all three anomalies in conjunction with other measurements. A similar 25 MeV $S$ provides a good fit to all relevant data except $R_{K^*}$ in the low $q^2$ bin. A 25 MeV $Z'$ with a $q^2$-dependent $b \to s$ coupling and that couples to the muon but not the electron provides a good fit to the combination of the $R_K$ and $R_{K^*}$ data, but does not fit $R_{K^*}$ in the low $q^2$ bin well.

I. INTRODUCTION

The search for new physics in $B$ decays is an ongoing endeavor. Recently, anomalies in semileptonic $B$ decays have received a lot of attention. These anomalies are found in the charged current $b \to c \tau \bar{\nu}_\tau$ and neutral current $b \to s \ell^+ \ell^-$ transitions. Here we focus on the neutral current anomalies though the anomalies might be related [1]. Other anomalies appear in $B \to K^* \mu^+ \mu^-$ where the LHCb [2,3] and Belle [4] Collaborations find deviations from the Standard model (SM) predictions, particularly in the angular observable $P^\phi_{5,8}$ [5]. The ATLAS [6] and CMS [7] Collaborations have also made measurements of the $B \to K^* \mu^+ \mu^-$ angular distribution with results consistent with LHCb. Further, the LHCb has made measurements of the branching ratios and angular distributions in $B^0_s \to \phi \mu^+ \mu^-$ [8,9] which are at variance with SM predictions based on lattice QCD [10,11] and QCD sum rules [12].

The measurements discussed above are subject to unknown hadronic uncertainties [13] making it necessary to construct clean observables to test for new physics (NP). One such observable is $R_K \equiv \mathcal{B}(B^+ \to K^+ \mu^+ \mu^-)/\mathcal{B}(B^+ \to K^+ \ell^+ \ell^-)$ [14,15], which has been measured by LHCb [16]:

$$R^K_{\rm exp} = 0.745^{+0.099}_{-0.074} \, \text{(stat)} \pm 0.036 \, \text{(syst)}, \quad 1 \leq q^2 \leq 6.0 \text{ GeV}^2.$$  \hspace{1cm} (1)

This differs from the SM prediction, $R^K_{\text{SM}} = 1 \pm 0.01$ [17] by $2.6\sigma$. Note, the observable $R_K$ is a measure of lepton flavor universality and requires different new physics for the muons versus the electrons, while it is possible to explain the anomalies in the angular observables in $b \to s \mu^+ \mu^-$ in terms of lepton flavor universal new physics [18].

Recently, the LHCb Collaboration reported the measurement of the ratio $R_{K^*} \equiv \mathcal{B}(B^0 \to K^{*0} \mu^+ \mu^-)/\mathcal{B}(B^0 \to K^{*0} \ell^+ \ell^-)$ in two different ranges of the dilepton invariant mass-squared $q^2$ [19]:

$$R^{\exp}_{K^*} = \begin{cases} 0.660^{+0.110}_{-0.070} \, \text{(stat)} \pm 0.024 \, \text{(syst)}, & 0.045 \leq q^2 \leq 1.1 \text{ GeV}^2, \quad \text{(low } q^2) \\ 0.685^{+0.069}_{-0.066} \, \text{(stat)} \pm 0.047 \, \text{(syst)}, & 1.1 \leq q^2 \leq 6.0 \text{ GeV}^2, \quad \text{(central } q^2). \end{cases}$$  \hspace{1cm} (2)

These differ from the SM predictions by $2.2-2.4\sigma$ (low $q^2$) and $2.4-2.5\sigma$ (central $q^2$), which further strengthens the hint of lepton non-universality observed in $R_K$.

Lepton universality violating new physics may occur in $b \to s \mu^+ \mu^-$ and/or $b \to s \ell^+ \ell^-$ transitions. The fact that the measurement of $\mathcal{B}(B^+ \to K^+ e^+ e^-)$ is found to be consistent with the prediction of the SM may lead one to conclude that NP is more likely to be in $b \to s \mu^+ \mu^-$. However, the branching ratios suffer from hadronic uncertainties [20] unlike the ratios $R_K$ and $R_{K^*}$, and so new physics in $b \to s \mu^+ \mu^-$ and/or in $b \to s \ell^+ \ell^-$ is still allowed.

Since the announcement of the $R_{K^*}$ result, a number of papers have analyzed the new measurements, mostly in terms of new physics with heavy mediators [21,22]. The general conclusion is that there is a significant disagreement with the SM, possibly as large as $\sim 6\sigma$, and that theoretical hadronic uncertainties [23,24] are insufficient to understand the data. However, with heavy new physics it is difficult to understand the $R_K$-measurement in the very low $q^2$ bin.
0.045 \leq q^2 \leq 1.1 \text{ GeV}^2$, although the predictions are consistent with measurements within 1.5\sigma. A resolution to this problem may be possible if the new physics is light.

In models with light mediators \cite{30,32,39,40}, the new physics cannot be integrated out, resulting in a $q^2$ dependence of the Wilson coefficients (WCs). If the light mediator mass is between $m_B$ and twice the lepton mass, and the mediator width is narrow, then it is observable as a resonance in the dilepton invariant mass. To avoid constraints from the search for such states, one generally takes the mediator mass to be $m_B$ or less than $2m_\ell$. In this paper we study a light scalar mediator denoted by $S$ and a light vector mediator denoted by $Z'$.

II. LIGHT SCALAR

We start our discussion with a light scalar $S$ with mass in the $10 - 200 \text{ MeV}$ range. For this scenario, we assume the following flavor-changing $bsS$ vertex,

$$F(q^2) \bar{s}_b \left[ g_{bs}^S P_L + g_{bs}^{S'} P_R \right] b_s S,$$

where $F(q^2)$ is a form factor. The matrix elements for the processes $b \to s\ell^+\ell^-$ and the mass difference in $B_s$ mixing are

$$M_{b \to s\ell^+\ell^-} = \frac{F(q^2)}{q^2 - M_S^2} \left[ \bar{s}(g_{bs}^S P_L + g_{bs}^{S'} P_R)b \right] \left[ \bar{\ell}(g_{\ell\ell}^F P_L + g_{\ell\ell}^{F'} P_R)\ell \right],$$

$$\Delta M_{B_s}^{NP} = \frac{(F(q^2))^2}{2q^2 - 2M_S^2} f_B^2 m_{B_s} \left[ -\frac{5}{12} \left( (g_{bs}^S)^2 + (g_{bs}^{S'})^2 \right) + 2g_{bs}^S g_{bs}^{S'} \frac{7}{12} \right],$$

where we have used Ref. \cite{42} for $B_{s0}^0 - B_{s0}$ mixing. The mass difference in the SM for the $B_s$ system is \cite{43}

$$\Delta M_{B_s}^{SM} = (17.4 \pm 2.6) \text{ ps}^{-1},$$

which is consistent with experimental measurement \cite{44},

$$\Delta M_{B_s} = (17.757 \pm 0.021) \text{ ps}^{-1}.$$  

We will choose the new physics contribution, $\Delta M_{B_s}^{NP}$, to be as large as the uncertainty in the SM prediction.

We now consider $b \to s\ell^+\ell^-$ transitions. For light scalars coupling to muons, $R_K$ and $R_{K^*}$ are generally increased from their SM values in contradiction with experiment. Moreover, the measured $B_s \to \mu^+\mu^-$ rate also puts strong constraints on new scalar couplings to muons.

We therefore suppose the scalar couples mainly to electrons in which case the matrix element for $b \to se^+e^-$ from Eq. (2) is

$$M_{b \to se^+e^-}^{S,S'} = \frac{g_S}{q^2 - M_S^2} F(q^2) \left[ g_{bs}^S (sP_L b) + g_{bs}^{S'} (sP_R b) \right] \left( \bar{e}e \right) + \frac{g_{ee}^{S'}}{q^2 - M_S^2} F(q^2) \left[ g_{bs}^S (sP_L b) + g_{bs}^{S'} (sP_R b) \right] \left( \bar{e}e \right),$$

where $g_{ee}^{S'} = (g_{ee}^{S'} + g_{ee}^{S'})/2$ and $g_{ee}^{S'} = (g_{ee}^{S'} - g_{ee}^{S'})/2$. In the following discussion, we chose different structures for the form factor $F(q^2)$.

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1 In our effective theory approach, the structure in Eq. (3) is of the general form consistent with the assumed symmetries. As an illustration of how a flavor changing vertex with a $q^2$-dependent form factor may occur, consider the following Lagrangian at the $b$-quark mass scale in the gauge basis:

$$\mathcal{L} = \frac{g}{\Lambda^2} \bar{b} \chi \chi + \bar{g} \chi \chi S,$$

where $\chi$ is a hidden sector fermion (which may serve as a dark matter candidate) of mass $m_\chi \lesssim m_b$, and we have suppressed all Lorentz structures in the Lagrangian. (In the context of Section III, for a light vector mediator $Z'$, one may consider a similar Lagrangian of the form, $g_\chi \bar{\chi} \gamma^\mu Z' \chi$. The first term in the Lagrangian represents an effective coupling between the $b$ and $\chi$ fields that might arise via the exchange of a heavy mediator of mass $\Lambda \gg m_b$, which has been integrated out of the theory at the $m_b$ scale. Although there is no direct coupling between $b$ and $S$ (or $Z'$), a $bbS$ (or $bbZ'$) vertex with a $q^2$-dependent coupling will be generated by a $\chi$ loop. Transferring the $b$ quark from the gauge to the mass basis then generates a $bbS$ (or $bbZ'$) coupling. In the case of the scalar mediator the form factor contains terms of the form, $m_\chi^2$ and $q^2/\Lambda^2$. For the latter term to dominate, $q^2 \gg m_\chi^2$, which implies that $m_\chi \lesssim 30 \text{ MeV}$ for the $q^2$ values of interest. For the $Z'$ case, the leading term in the form factor goes as $q^2$ due to the conserved vector current. We note that the situation is similar to the SM case where $\chi$ is replaced by the charm quark and $S$ (or $Z'$) by the photon. In this case the first term in the Lagrangian, of the form $\frac{g}{M_W} \bar{b} \gamma^\mu \chi \chi$ is just one of the terms in the SM effective Lagrangian after integrating out the $W$ boson. The charm quark then induces an effective $\bar{b} s \gamma^*$ vertex which yields $\bar{b} s \ell^+\ell^-$ via $\gamma^* \to \ell^+\ell^-$.}
TABLE I. The fit results and the predictions for $R_K$ and $R_{K^*}$ at the best fit point for three scenarios of a light mediator with a mass of 25 MeV.

| Case | Experimental results | $R_{K^*}[0.045-1.1]$ | $R_{K^*}[1.1-6.0]$ | $R_{K^*}[1.0-6.0]$ | Pull |
|------|----------------------|------------------------|---------------------|---------------------|------|
|      |                      | 0.66 ± 0.09            | 0.69 ± 0.10         | 0.73 ± 0.09         |      |
| Standard model predictions | 0.93                 | 0.99                  | 1.0                 |                    |      |

(i) Light scalar with electron coupling

$F(q^2) \equiv 1, \ g_{bs} = 2.0 \times 10^{-4}$

- $g_{bs} g_{bs}^\prime = (12.6 \pm 2.2) \times 10^{-9}$
  - $a_{bs} \neq 0$
  - $g_{bs} g_{bs}^\prime = (4.0 \pm 1.6) \times 10^{-9}$
  - $a_{bs} = 0$
  - $g_{bs} g_{bs}^\prime = (2.7 \pm 2.6) \times 10^{-9}$
  - $g_{bs}^\prime g_{bs} = (15.5 \pm 2.6) \times 10^{-8}$

- $a_{bs} \neq 0$
  - $g_{bs} g_{bs}^\prime = (6.5 \pm 3.5) \times 10^{-10}$
  - $a_{bs} = 0$
  - $g_{bs} g_{bs}^\prime = (5.7 \pm 2.3) \times 10^{-10}$
  - $g_{bs}^\prime g_{bs} = (0.2 \pm 0.1) \times 10^{-11}$

- $a_{bs} \neq 0$
  - $g_{bs} g_{bs}^\prime = (3.9 \pm 1.8) \times 10^{-10}$
  - $g_{bs}^\prime g_{bs} = (4.4 \pm 4.2) \times 10^{-11}$

(ii) Light vector with muon coupling

$F(q^2) \equiv 1, \ g_{bs} = 8.0 \times 10^{-4}$

- $g_{bs} g_{bs} = (2.3 \pm 2.0) \times 10^{-10}$
  - $a_{bs} \neq 0$
  - $g_{bs} = 0, \ g_{bs}^\prime = g_{bs}^\prime$
  - $g_{bs} g_{bs} = (1.3 \pm 2.2) \times 10^{-10}$
  - $g_{bs}^\prime g_{bs} = (0.1 \pm 0.1) \times 10^{-11}$
  - $g_{bs}^\prime g_{bs} = (0.0 \pm 1.5) \times 10^{-11}$

(iii) Light vector with electron coupling

$F(q^2) \equiv 1, \ g_{bs} = 2.5 \times 10^{-4}$

- $g_{bs} g_{bs} = (6.0 \pm 1.0) \times 10^{-10}$
  - $a_{bs} \neq 0$
  - $g_{bs} = 0, \ g_{bs}^\prime = g_{bs}^\prime$
  - $g_{bs} g_{bs} = (0.8 \pm 5.0) \times 10^{-10}$
  - $g_{bs}^\prime g_{bs} = (1.4 \pm 1.0) \times 10^{-11}$

- $a_{bs} \neq 0$
  - $g_{bs} = 0, \ g_{bs}^\prime \neq g_{bs}^\prime$
  - $g_{bs} g_{bs} = (4.3 \pm 4.2) \times 10^{-10}$

- $a_{bs} = 0$
  - $g_{bs} = 0, \ g_{bs}^\prime \neq g_{bs}^\prime$
  - $g_{bs}^\prime g_{bs} = (0.4 \pm 1.4) \times 10^{-8}$

A. $F(q^2) \equiv 1$

First, we consider the situation in which the $bsS$ vertex is generated either at tree level or at loop level with internal particles with masses much greater than the $b$ quark mass. Then, the form factor $F(q^2) \equiv 1$, and to avoid a pole contribution to the measurements of $B(B^0 \to K^\ast \pi^\pm \pi^\mp)$ in the dilepton invariant mass range, $m_{ee} = [30 - 1000]$ MeV [45], we choose $M_S = 25$ MeV.

Note that the BaBar [46] and Belle [47,48] measurements require $m_{ee}$ to be larger than 30 MeV [49] and 140 MeV, respectively. We fix $g_{bs} = 2.0 \times 10^{-4}$, which is the largest value allowed by the anomalous magnetic moment of the electron [50] for $M_S = 25$ MeV at the 2σ CL. Then we perform a $\chi^2$-fit to the theoretically clean observables $R_K$ and $R_{K^*}$, and the new physics contribution to the $B_s$ mass difference, $\Delta M_{NP} = 0.2 \pm 0.6$ ps$^{-1}$. In Ref. [51] the lepton flavor dependent angular observables $Q_{L,5}$ were measured but since the errors in the measurements are large we do not use them in our fit. We use flavio [52] to calculate the theoretical values of the observables $O_{th}$. We then compute

$$\chi^2(g_{bs}, g_{bs}^\prime) = \sum_{R_K, R_{K^*}, \Delta M_{NP}} (O_{th}(g_{bs}, g_{bs}^\prime) - O_{exp})^T C^{-1} (O_{th}(g_{bs}, g_{bs}^\prime) - O_{exp}),$$

where $O_{exp}$ are the experimental measurements of the observables, and the total covariance matrix $C$ is the sum of theoretical and experimental covariance matrices. The SM gives a very poor fit to the $R_K$ and $R_{K^*}$ measurements with

$$\chi^2_{SM}/\text{dof} = 25.5/3.$$  

The best fit values of the couplings $g_{bs}$ and $g_{bs}^\prime$ along with predictions at the best fit point, for $M_S = 25$ MeV and $g_{ee} = 2.0 \times 10^{-4}$, are provided in Table I. As a good fit is obtained in this case, we check if these values are consistent with the various measured branching ratios in $b \to s e^+ e^-$ modes. If $S$ can decay to $e^+ e^-$ with a branching ratio $\sim 1$ then the decays $B \to K(e^+ e^-)$ will be dominated by the two-body decays, $B \to K(e^+ e^-)$, with $S$ decaying to $e^+ e^-$. 

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TABLE II. Constraints from $\mathcal{B}(B^0 \to K^0 e^+ e^-)$ and $\mathcal{B}(B^0 \to K^{*0} e^+ e^-)$. See the text for details.

| $S$, $a_{bs}$ ≠ 0 | $|g_{bs}^S + g_{bS}^S| \lesssim 9.9 \times 10^{-7}$ | $|g_{bs}^S - g_{bs}^S| \lesssim 9.0 \times 10^{-7}$ | $|g_{bs}^S|, |g_{bS}^S| \lesssim 9.5 \times 10^{-7}$ |
|-------------------|----------------------------------|----------------------------------|----------------------------------|
| $S$, $a_{bs} = 0$| $|g_{bs}^S + g_{bS}^S| \lesssim 4.4 \times 10^{-2}$ \left(\frac{25 \text{ MeV}}{M_Z^*}\right)$ | $|g_{bs}^S - g_{bs}^S| \lesssim 4.0 \times 10^{-2}$ \left(\frac{25 \text{ MeV}}{M_Z^*}\right)$ | $|g_{bs}^S|, |g_{bS}^S| \lesssim 4.2 \times 10^{-2}$ \left(\frac{25 \text{ MeV}}{M_Z^*}\right)$ |
| $Z'$, $a_{bs} ≠ 0$| $|g_{bs}^S + g_{bS}^S| \lesssim 5.8 \times 10^{-9}$ \left(\frac{M_{Z'}}{25 \text{ MeV}}\right)$ | $|g_{bs}^S - g_{bs}^S| \lesssim 5.4 \times 10^{-9}$ \left(\frac{M_{Z'}}{25 \text{ MeV}}\right)$ | $|g_{bs}^S|, |g_{bS}^S| \lesssim 5.6 \times 10^{-9}$ \left(\frac{M_{Z'}}{25 \text{ MeV}}\right)$ |
| $Z'$, $a_{bs} = 0$| $|g_{bs}^S + g_{bS}^S| \lesssim 2.6 \times 10^{-4}$ \left(\frac{25 \text{ MeV}}{M_{Z'}}\right)$ | $|g_{bs}^S - g_{bs}^S| \lesssim 2.4 \times 10^{-4}$ \left(\frac{25 \text{ MeV}}{M_{Z'}}\right)$ | $|g_{bs}^S|, |g_{bS}^S| \lesssim 2.5 \times 10^{-4}$ \left(\frac{25 \text{ MeV}}{M_{Z'}}\right)$ |

For the two body $B \to KS$ decay, the branching ratio is

$$\mathcal{B}(B \to KS) = \frac{(g_{bs}^S + g_{bS}^S)^2 |\bar{p}_K| (m_B^2 - m_K^2)^2 f_0^2 (m_s^2 / m_B^2) \tau_B}{32 \pi m_s^2 m_B^2} ,$$  \hspace{1cm} (11)

where the form factor $f_0(\tau)$ can be found in Ref. 53.

For the two body $B \to K^*S$ decay, the branching ratio is

$$\mathcal{B}(B \to K^*S) = \frac{(g_{bs}^S - g_{bS}^S)^2 |\bar{p}_K| |A_0| (m_s^2 / m_B^2) \tau_B}{8 \pi m_s^2} ,$$

where $\tau_B$ is the lifetime of $B$ meson, $|\bar{p}_K| = \lambda^{1/2}(m_B^2, m_K^2, m_s^2)/2m_B$, and the form factor $A_0$ is taken from Ref. 54.

To bound the NP coupling constants $g_{bs}^S$ and $g_{bS}^S$, we require the $B \to K^{(*)}S$ branching ratio to be less than 1%. This choice is consistent with uncertainties in the calculation of the $B$ meson width 53. For $M_S$ between 10 – 200 MeV, $\mathcal{B}(B^0 \to K^{(*)0} e^+ e^-)$ and $\mathcal{B}(B^0 \to K^{(*)0} e^+ e^-)$ impose the constraints shown in Table II. The best-fit values of the coupling given in Table II are in contradiction with these constraints. Hence, a light scalar with form factor $F(q^2) \equiv 1$ is ruled out.

B. $F(q^2) \neq 1$

Now we consider a $q^2$-dependent form factor $F(q^2) \neq 1$ which may be loop induced. For momentum transfer $q^2 \ll m_B^2$, $F(q^2)$ can be expanded as

$$F(q^2) = a_{bs} + b_{bs} \frac{q^2}{m_B^2} + \ldots ,$$  \hspace{1cm} (13)

where $m_B$ is the $B$-meson mass. We do not include the $B_s$ mass difference and $\mathcal{B}(B_s \to e^+ e^-)$ as constraints since $F(q^2)$ is unknown for $q^2 \sim m_B^2$. We assume that $S$ does not couple to neutrinos so that $B \to K^{*0} \nu \bar{\nu}$ 54, 57 does not constrain $a_{bs}$. Redefining $a_{bs}g_{bS}^S$ as $g_{bs}^S$, and $a_{bs}g_{bS}^S$ as $g_{bS}^S$, we perform a $\chi^2$-fit to the theoretically clean observables $R_K$ and $R_{K^*}$. The best fit values of the couplings and the predictions for $R_K$ and $R_{K^*}$ are shown in Table II. Taking into account the constraints on $g_{bs}^S$ and $g_{bS}^S$ from Table II along with the constraints on $g_{e\gamma}$ from the anomalous magnetic moment of the electron, we see that the best fit values $O(10^{-8})$ cannot be achieved in this case.

To avoid the strong constraints from the two-body decays we set $a_{bs} = 0$ in Eq. (13) (thereby also evading the $B \to K^{(*)0} \nu \bar{\nu}$ constraint if the mediator couples to neutrinos 54), and absorbing the factor $b_{bs}$ to redefine $g_{bs}^S$ and $g_{bS}^S$, the matrix element for $b \to se^+ e^-$ is given by

$$M_{b \to se^+ e^-} = \frac{q^2}{m_B^2} \frac{g_{e\gamma}^S}{q^2 - M_S^2} \left[ g_{bs}(sP_L b) + g_{bs}(sP_R b) \right] (ee) + \frac{q^2}{m_B^2} \frac{g_{e\gamma}^S}{q^2 - M_S^2} \left[ g_{bs}(sP_L b) + g_{bs}(sP_R b) \right] (e\gamma e) .$$  \hspace{1cm} (14)

With the form factor $q^2/M_B^2$, requiring $\mathcal{B}(B^0 \to K^{(*)0} e^+ e^-)$ and $\mathcal{B}(B^0 \to K^{(*)0} e^+ e^-)$ to be less than 1% gives the constraints on $g_{bs}^S$ and $g_{bS}^S$ in Table II. The best-fit values can be found in Table II. A reasonable fit is obtained in this case with a pull of 4.4. We see that $R_K$ and $R_{K^*}$ values in the central $q^2$ bin can be reasonably accommodated, while the effect on $R_{K^*}$ in the low $q^2$ bin is small in this case. We also evaluated the branching ratios for various $b \to se^+ e^-$ observables; see Table II. Our prediction for $\mathcal{B}(B \to K e^+ e^-)$ [1.0–6.0] is somewhat in tension with the
The prediction for the inclusive mode where we have used Ref. [42] for $T_{AB}$ Table III. The experimental results for various physics cases that fit the $R_K$ and $R_K^*$ data and satisfy the $B(B \to K^{(*)}e^+e^-)$ constraints. The light mediator mass is 25 MeV, $F(q^2) \neq 1$ and $a_{bs} = 0$.

| Experimental results | $R_K [0.045 \pm 0.0]$ | $B(B \to Ke^+e^-) [1.0 \pm 0.0]$ | $B(B \to Xe^+e^-) [1.0 \pm 0.0]$ | $B(B^0 \to K^{(*)}e^+e^-) [0.03 \pm 0.1$ |
|----------------------|------------------------|-------------------------------|-------------------------------|----------------------------------|
| Standard model predictions | 0.98 | 1.69 x 10^{-7} | 1.74 x 10^{-6} | 2.6 x 10^{-7} |
| Light scalar $g_{bs}^2 g_{e}^2 = 2.7 \times 10^{-8}$, $g_{bs}^2 g_{e}^4 = -15.5 \times 10^{-8}$ | 0.93 | 2.5 x 10^{-7} | 2.3 x 10^{-6} | 2.6 x 10^{-7} |
| Light vector $g_{bs} g_{e} = -3.9 \times 10^{-8}$, $g_{bs}^2 g_{e} = 1.4 \times 10^{-8}$ | 0.73 | 2.4 x 10^{-7} | 2.6 x 10^{-6} | 2.8 x 10^{-7} |
| Light vector, $g_{bs} = 0$ | 0.66 | 2.7 x 10^{-7} | 2.5 x 10^{-6} | 2.7 x 10^{-7} |
| Light vector, $g_{bs} = 0$, $g_{bs}^2 g_{e} = 2.0 \times 10^{-8}$ | 1.04 | 2.4 x 10^{-7} | 2.5 x 10^{-6} | 2.8 x 10^{-7} |

Experimental result. Allowing for a 10% uncertainty in the theoretical prediction [59], the discrepancy is about 2.3σ. The prediction for the inclusive mode $B(B \to Xe^+e^-) [1.0 \pm 0.0]\). which suffers from less hadronic uncertainties, is consistent with measurement.

Finally, we considered the case with a pseudoscalar coupling of the electron and find similar results to that of the scalar coupling.

### III. Light $Z'$

A $Z'$ with mass less than $2m_\mu$ was recently proposed in Ref. [39] to simultaneously explain the measurements of $R_K$ and the anomalous magnetic moment of the muon, with implications for nonstandard neutrino interactions. Such a $Z'$ may potentially explain $R_K^*$ in the low $q^2$ bin [31]. A $Z'$ with a mass in the few GeV range was discussed recently [30, 32] but the $q^2$ dependence of the WC is not strong enough to explain the $R_K^*$ at low $q^2$ [32]. Here we focus on an MeV $Z'$.

We assume the flavor-changing $bsZ'$ vertex to have the form,

$$F(q^2) \bar{s}\gamma^\mu [g_{bs} P_L + g_{bs}' P_R] b Z'_\mu.$$  \hfill (15)

The matrix elements for $b \to s\ell^+\ell^-$ and the mass difference in $B_s$ mixing are

$$M_{b \to s\ell^+\ell^-} = \frac{F(q^2)}{q^2 - M_{Z'}^2} [s\gamma^\mu (g_{bs} P_L + g_{bs}' P_R) b] (\bar{\ell}\gamma^\mu (g_{L}^\ell P_L + g_{R}^\ell P_R) \ell)$$

$$+ \frac{F(q^2)}{q^2 - M_{Z'}^2} m_{b} m_{\ell} (g_{R}^\ell - g_{L}^\ell) \bar{s} (g_{bs} P_R + g_{bs}' P_L) b (\bar{\ell}\gamma^\mu \ell),$$

$$\Delta M_{B_s}^{NP} = \frac{(F(q^2))^2}{2q^2 - 2M_{Z'}^2} \frac{3}{2} f_{B_s}^2 m_{B_s} \left[ \left( g_{bs}^2 + g_{bs}'^2 \right) \left( 1 - \frac{5}{6} \frac{m_b^2}{M_{Z'}^2} \right) - 2g_{bs} g_{bs}' \left( \frac{5}{6} - \frac{m_b^2}{M_{Z'}^2} \right) \right],$$ \hfill (16)

where we have used Ref. [32] for $B_s^0-\bar{B}_s^0$ mixing. Also, we define $g_{\ell\ell} \equiv (g_{L}^\ell + g_{R}^\ell)/2$ and $g_{\ell\ell}' \equiv (g_{R}^\ell - g_{L}^\ell)/2$ for convenience.

#### A. $Z'$ with muon coupling

We begin with the case where the $Z'$ couples to muons and not to the electrons.

1. $F(q^2) \equiv 1$

We first assume that $F(q^2) \equiv 1$ and consider the case $g_{L}^{\mu\mu} = g_{R}^{\mu\mu} = g_{\mu\mu}$, so the leptonic term is a purely vector current. We perform a fit to the $R_K$ and $R_K^*$ data, and the new physics contribution to the $B_s$ mass difference. We
choose $M_{Z'} = 25$ MeV and fix $g_{ee} = 8.0 \times 10^{-4}$, which is the 2$\sigma$ upper bound from the anomalous magnetic moment of the muon. The fit results are shown in Table I. We see that the overall improvement over the SM is insignificant because $g_{bs}^b$ and $g_{bs}^{b'}$ are suppressed by $B_s$ mixing.

2. $F(q^2) \neq 1$

Now we consider $F(q^2) \neq 1$ and assume an expansion as in Eq. \ref{eq:fit_expansion}. Keeping only the leading $a_{bs}$ term, we perform a fit to the observables $R_K$ and $R_{K^*}$ for $M_S = 25$ MeV. We do not employ the new physics contribution to the $B_s$ mass difference as a constraint since $F(q^2)$ is unknown for $q^2 \sim m_B^2$. The fit results are shown in Table I. The overall improvement over the SM is poor, with a pull of 2.4. Clearly, a light $Z'$ with pure vector coupling to the muon is unable to explain the $R_{K[1.0-6.0]}$, $R_{K^*[0.045-1.1]}$ and $R_{K^*[1.1-6.0]}$ anomalies simultaneously. However, on removing $R_{K^*[0.045-1.1]}$ from the fit, one can easily accommodate the measured values of $R_{K[1.0-6.0]}$ and $R_{K^*[1.1-6.0]}$, and a pull of around 4.0 is obtained.

We next consider the case with $a_{bs} = 0$ and the $Z'$ also has nonzero axial vector coupling with the muons, i.e., $g_{L}^{\mu} \neq g_{R}^{\mu}$. To keep the number of new couplings unchanged, we take either $g_{bs}^{\ell} = 0$ or $g_{bs}^{b'} = 0$. This case also does not give a good fit to the data; see Table I.

As can be seen from Table I overall two of the scenarios with $a_{bs} = 0$ provide good fits except to the $R_{K^*}$ measurement in the low $q^2$ bin. Moreover, a $Z'$ with purely vector muon coupling is easily compatible with other $B \rightarrow s\ell^+\ell^-$ observables \cite{32}.

B. $Z'$ with electron coupling

We now consider the case where the $Z'$ couples to electrons and not to muons.

1. $F(q^2) \equiv 1$

We first assume that $F(q^2) \equiv 1$ and we start by considering the case $g_{ee}^e = g_{ee}^{\ell} = g_{ee}$ so the leptonic term is a purely vector current. We perform a fit to the $R_K$ and $R_{K^*}$ data, and the new physics contribution to the $B_s$ mass difference. We fix $g_{ee} = 2.5 \times 10^{-4}$, which is within the 90% CL upper limit from NA48/2 \cite{60}. The fit results are shown in Table I. The fit to $R_K$ and $R_{K^*}$ is close to the SM predictions because of $B_s$ mixing.

2. $F(q^2) \neq 1$

Now we consider $F(q^2) \neq 1$. We fit to the observables $R_K$ and $R_{K^*}$ only since $F(q^2)$ is unknown for $q^2 \sim m_B^2$. The best fit results are shown in Table I. While a good fit to $R_K$ and $R_{K^*}$ is obtained, we need to check if these couplings are consistent with other measurements. As in the scalar case there is a two-body contribution to $\mathcal{B}(B \rightarrow K^{(*)} e^+ e^-)$ from $B \rightarrow K^{(*)} Z'$ and $Z'$ decaying to $e^+ e^-$ with a branching ratio $\sim 1$.

The branching ratio for $B \rightarrow K Z'$ is \cite{61,62}.

$$\mathcal{B}(B \rightarrow K Z') = \frac{|g_{bs} + g_{bs}^{b'}|^2 m_B^2 \beta_{BKZ'}^2}{64\pi M_{Z'}^2 \Gamma_B} \left[ f^{BK}(M_{Z'}^2) \right]^2,$$ \hspace{1cm} (17)

where $\beta_{XYZ} = \lambda^{1/2}(1, M_X^2/M_Y^2, M_Z^2/M_X^2)$ and $f^{BK}$ is a form factor. For $B \rightarrow K^* Z'$ the branching ratio is given by,

$$\mathcal{B}(B \rightarrow K^* Z') = \frac{\beta_{BK^*Z}}{16\pi m_B \Gamma_B} \left( |H_0|^2 + |H_+|^2 + |H_-|^2 \right),$$ \hspace{1cm} (18)

where the helicity amplitudes are defined as,

$$H_0 = (g_{bs} - g_{bs}^{b'}) \left[ -\frac{1}{2} (m_B + M_{K^*}) \xi A_1(M_{Z'}^2) + \frac{M_{K^*} M_{Z'}}{m_B + M_{K^*}} \sqrt{\xi^2 - 1} A_2(M_{Z'}^2) \right],$$ \hspace{1cm} (19)

and

$$H_\pm = \frac{1}{2} (g_{bs} - g_{bs}^{b'}) \left[ (m_B + M_{K^*}) A_1(M_{Z'}^2) \right] \pm (g_{bs} + g_{bs}^{b'}) \frac{M_{K^*} M_{Z'}}{m_B + M_{K^*}} \sqrt{\xi^2 - 1} V(M_{Z'}^2).$$ \hspace{1cm} (20)
we get the constraints shown in Table II. The best fit satisfies all constraints on $g_{ee}$.

We next consider the case when $Z'$ also has nonzero axial vector coupling with the electrons, i.e., $g_{L}^{ee} \neq g_{R}^{ee}$. The best-fit results are shown in Table I. While a good fit to $R_K$ and $R_{K^{*}}$ is obtained, the best-fit values do not satisfy the two-body constraints of Table II along with the constraint on $g_{ee}$.

Now, to avoid the two-body constraint, like in the scalar case, we set $a_{bs} = 0$ in Eq. (13). In this case, assuming $g_{L}^{ee} = g_{R}^{ee} = g_{ee}$, i.e., pure vector coupling to the electron, and for $M_{Z'} = 25$ MeV, we fit the product $g_{ee} a_{bs}$ and $g_{ee} g_{bs}^L$ to the $R_K$ and $R_{K^{*}}$ data. The results are summarized in Table II. Clearly, at the best fit point the predictions for $R_K$ and $R_{K^{*}}$ are within the 1σ range of the measurements. Requiring $B(B^0 \to K^0 e^+ e^-) < 1\%$ and $B(B^0 \to K^{*0} e^+ e^-) < 1\%$, we get the constraints shown in Table II. The best fit satisfies all constraints on $g_{bs}$, $g_{bs}^L$ and $g_{ee}$. From Table II we see that $R_K$ and $R_{K^{*}}$ values in all measured $q^2$ bins can be reasonably accommodated. We also checked that the predictions for the branching ratios to electron modes are consistent with the various observables; see Table III.

Our prediction for $B(B \to K e^+ e^-)_{[1.0-6.0]}$ is somewhat higher than the measurement and this tension could become significant with a reduction in the theoretical and experimental uncertainties. The prediction for the inclusive mode $B(B \to X_s e^+ e^-)_{[1.0-6.0]}$, which suffers from less hadronic uncertainties, is consistent with measurement.

Next we consider the case when $Z'$ also has nonzero axial vector coupling with the electrons, i.e., $g_{L}^{ee} \neq g_{R}^{ee}$. Again, we either set $g_{bs} = 0$ or $g_{bs} = 0$. The best-fit values shown in Table II satisfy the constraints on the NP couplings, and the $R_K$ and $R_{K^{*}}$ values in all measured $q^2$ bins can be reasonably accommodated. The corresponding branching ratios with electron modes are provided in Table III.

IV. SUMMARY

In this work we have addressed the recent measurement of $R_{K^{*}}$, with particular attention to the low $q^2$ bin, $0.045 \leq q^2 \leq 1.1$ GeV$^2$. This measurement has been difficult to explain with new physics above the GeV scale. For mediators in the $10 - 200$ MeV mass range, we find:

1. A (pseudo)scalar that only couples to muons cannot explain the $R_K$ and $R_{K^{*}}$ measurements as the predicted values are larger than in the SM, in conflict with experiment. An S coupling to only electrons can reproduce the $R_K_{[1.0-6.0]}$, $R_{K^{*}}_{[0.045-1.1]}$ and $R_{K^{*}}_{[1.1-6.0]}$ data, but the desired values of the couplings are not consistent with the measurements of the branching ratios $B(B \to K^{(*)} e^+ e^-)$. A $q^2$-dependent flavor changing $b \to s$ coupling to the scalar can produce compatibility with $B(B \to K^{(*)} e^+ e^-)$ and gives a good fit to $R_K$ and $R_{K^{*}}$ in the central $q^2$ bin, but the deviation of $R_{K^{*}}$ from the SM in the low $q^2$ bin is small.

2. A $Z'$ with general vector and axial vector couplings to the muon and a $q^2$-dependent $b \to s$ coupling provides a good fit to the combination of the three $R_K$ and $R_{K^{*}}$ measurements, but does not fit $R_{K^{*}}_{[0.045-1.1]}$ well.

3. A $Z'$ with general vector and axial vector couplings to the electron can explain $R_K$ and $R_{K^{*}}$ data in all measured bins but the desired values of the couplings are not consistent with the measurements of $B(B \to K^{(*)} e^+ e^-)$. However, a $q^2$-dependent flavor changing $b \to s$ coupling to the vector is compatible with $B(B \to K^{(*)} e^+ e^-)$ and gives good fits to $R_K$ and $R_{K^{*}}$; of the cases we considered, the case with purely vector electron coupling provides the best agreement with the data with a pull of 4.8.

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