Density and correlations of topological objects near the transition temperature in lattice gluodynamics

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(Dated: August 26, 2019)

Topological lumps are known to be present in gluonic fields of SU(3) gluodynamics. Near the transition temperature they were classified either as constituents of nondissociated (ant)calorons, or as constituents of (anti)dyon pairs, or as isolated (anti)dyons. In this paper we study the density and correlation functions of these objects at temperature $T/T_c = 0.96$.

PACS numbers: 11.15.Ha, 12.38.Gc, 12.38.Aw
Keywords: Lattice gauge theory, overlap Dirac operator, caloron, dyon

I. INTRODUCTION

In Ref. [1] we have investigated topological objects formed in gluonic fields of SU(3) gluodynamics near the transition temperature. It was done with the help of low lying modes of the overlap Dirac operator. These modes allow to construct the topological charge density corresponding to the three constituents (dyons) of a caloron with nontrivial holonomy [2–4] when three types of fermionic temporal boundary conditions are used.

The dyons are playing the decisive role in a recent model of the QCD vacuum proposed by Shuryak and collaborators [5–9]. The density of dyons and their interaction determined in lattice simulations are important inputs for this model. This motivated us to return to investigation of SU(3) pure gluodynamics. We will present results obtained just below the confinement-deconfinement phase transition at $T/T_c = 0.96$. In particular, we present a numerical value for the dyon number density and compare it with the model prediction [7]. Results for various dyon correlation functions are presented for the first time.

We study the density and interaction of dyons using the same method to construct the topological charge density as in Ref. [1]. The new element introduced into this method is the new criterion to determine the number of low lying modes of the overlap Dirac operator that should be used for the construction of the UV-filtered fermionic topological charge density.

In Section II the details of the lattice ensemble created at a temperature near the deconfining transition are described. In Section III we sketch the fermionic construction of the topological charge densities as applied for three types of fermionic temporal boundary conditions. A cluster analysis of the resulting topological charge densities provides us with a possibility to localize dyons of different types. The results for dyon densities and dyon correlation functions are presented and discussed. Finally, we present our conclusions in Section IV.

II. SETUP OF THE INVESTIGATION

The SU(3) gauge field configurations for this investigation have been generated on a lattice of size $24^3 \times 6$ by sampling the pure SU(3) gauge theory using the Lüscher-Weisz action [10].

In addition to the plaquette term (pl), the Lüscher-Weisz action includes a sum over all $2 \times 1$ rectangles (rt) and a sum over all parallelograms (pg), i.e. all possible closed loops of length 6 along the edges of all 3-cubes

$$S[U] = \beta \left( \sum_{pl} \frac{1}{3} \text{Re Tr}[1 - U_{pl}] + c_1 \sum_{rt} \frac{1}{3} \text{Re Tr}[1 - U_{rt}] + c_2 \sum_{pg} \frac{1}{3} \text{Re Tr}[1 - U_{pg}] \right),$$

where $\beta$ is the principal inverse coupling parameter, while the coefficients $c_1$ and $c_2$ are computed using

arXiv:1908.08709v1 [hep-lat] 23 Aug 2019
In terms of the average plaquette coupling constant $\beta$ for a given $\beta$, results of one-loop perturbation theory and tadpole improvement [11,13]:

$$c_1 = -\frac{1}{20u_0^3}[1 + 0.4805\alpha], \quad c_2 = -\frac{1}{u_0^3}0.03325\alpha.$$  \hspace{1cm} (2)

For a given $\beta$, the tadpole factor $u_0$ and the lattice coupling constant $\alpha$ are self-consistently determined in terms of the average plaquette

$$u_0 = \left(\frac{1}{3}\text{Re Tr} U_{pl}\right)^{1/4}, \quad \alpha = -\frac{\ln \left(\frac{1}{3}\text{Re Tr} U_{pl}\right)}{3.06839}$$  \hspace{1cm} (3)

in the course of a series of iterations.

The ensemble of 100 configurations for the overlap analysis has been generated at $\beta = 8.20$. According to previous work [14], this ensemble corresponds to a temperature of $T = 287$ MeV [18] close to the phase transition temperature $T_c = 300$ MeV [15].

### III. TOPOLOGICAL CLUSTERING

We have analyzed the configurations of the ensemble by identifying and investigating $N \leq 30$ zero and near-zero eigenmodes of the overlap Dirac operator. The spectral analysis has been performed for three types of temporal boundary conditions (b.c.) applied to the fermion field $\psi$:

$$\psi(1/T) = \exp(i\phi)\psi(0)$$  \hspace{1cm} (4)

with

$$\phi = \begin{cases} 
\phi_1 & \equiv -\pi/3, \\
\phi_2 & \equiv +\pi/3, \\
\phi_3 & \equiv \pi.
\end{cases}$$  \hspace{1cm} (5)

For these three types of b.c.'s the fermionic zero mode is maximally localized at one of the three constituent dyons in the case of a single-caloron solution with maximally nontrivial holonomy. For each of these b.c.'s we have determined the topological index and have checked that it was independent of the choice of $\phi$. The obtained spectra are also independent of b.c.'s and have a nonzero spectral density around zero value (signalling spontaneous violation of chiral symmetry).

In order to proceed further, we have reconstructed from the zero and non-zero modes the profiles of the UV-filtered topological charge density corresponding to the chosen fermionic boundary condition according to its spectral representation (for details see [16,17])

$$q_{i,N}(x) = -\sum_{j=1}^{N} \left(1 - \frac{\lambda_{i,j}}{2}\right) \psi_{i,j}^\dag(x)\gamma_5\psi_{i,j}(x),$$  \hspace{1cm} (6)

where $j$ enumerates the eigenvalues $\lambda_{i,j}$ equal and closest to zero. These precise eigenvalues $\lambda_{i,j}$, as well as the corresponding modes $\psi_{i,j}(x)$, are characterized by the $i$-th boundary condition. Correspondingly, the UV-filtered topological density $q_{i,N}(x)$ depends on the boundary condition, too.

We have applied a cluster analysis with a variable lower cut-off $|q_{i,N}(x)| > q_{\text{cut}} > 0$ to these density functions. The cut-off $q_{\text{cut}}$ has been chosen such as to resolve a maximal number of internally connected (while mutually separated clusters). It has been independently adapted for each configuration. The purpose of the cluster analysis was to discover extended objects that we are going to consider as dyon candidates (of the respective type).

We have found the following average numbers of clusters per configuration (comprising all 100 configurations) corresponding to the three boundary conditions $i = 1, 2, 3$:

$$N_{i,30} = 27.0(4), \quad N_{2,30} = 27.0(4), \quad N_{3,30} = 27.0(4)$$

which completely agree within errors (the latter given in parentheses). Therefore in the confining (center symmetric) phase the abundance of all three types of clusters is equal, and the clusters can be interpreted as dyons with maximally nontrivial holonomy. For further comparison we made the same calculations with $N = 10$ or $N = 20$ low lying modes taken into account for the reconstruction of the topological density

$$N_{1,10} = 14.6(3), \quad N_{2,10} = 14.2(3), \quad N_{3,10} = 14.1(3)$$

$$N_{1,20} = 21.0(3), \quad N_{2,20} = 21.2(4), \quad N_{3,20} = 21.0(4)$$

While the mutual equality of multiplicity remains, the total number of clusters changes monotonically with the number of analyzing modes.

Taking the lattice spacing value $a = 0.115$ fm from Ref. [14] we obtain the physical three-dimensional dyon cluster density $\rho = (N_{1,30} + N_{2,30} + N_{3,30})/(24a)^3 = 3.9(6)$ fm$^{-3}$.

Next we have checked whether clusters of different types are correlated among themselves. We found (in average per configuration)

$$N_{d,30} = 40(1), \quad 2N_{dd,30} = 24.3(7), \quad 3N_{ddd,30} = 16.7(7),$$

where $N_{d,30}$ is the number of isolated clusters, $N_{dd,30}$ is the number of pairs of connected clusters and $N_{ddd,30}$ is the number of triplets of connected clusters (full calorons). The clusters of different type were counted as connected in pairs or triplets if the distance between them was less than two lattice spacings. Interpreted in terms of calorons of non-trivial holonomy this means that we see full caloron-like clusters consisting of three constituents on one hand and also completely dissolved caloron constituents on the other hand.
Again for further comparison we made the same calculations with $N = 10, 20$ low lying modes taken into account

$$N_{d,10} = 22(1),\ 2N_{dd,10} = 11.6(4),\ 3N_{ddd,10} = 9.1(5)$$

$$N_{d,20} = 32.4(9),\ 2N_{dd,20} = 17.4(6),\ 3N_{ddd,20} = 13.3(7)$$

We computed the topological susceptibility $\chi = < Q^2 > / V_4$ (with $Q$ being the topological charge of configuration and $V_4$ being its 4d volume) and found $\chi = (187 \pm 6 \text{ MeV})^4$. This result is in very good agreement with that of Ref. [14] obtained at same temperature and for same action as in our work. Our modelling of the topological susceptibility by ensemble of dyons, dyon pairs and full calorons gives rise to

$$\chi_{\text{model}} = Q^2_{d} n_{d,N} + Q^2_{dd} n_{dd,N} + Q^2_{ddd} n_{ddd,N}$$

equal to $(169 \pm 2 \text{ MeV})^4, (187 \pm 2 \text{ MeV})^4$ and $(201 \pm 2 \text{ MeV})^4$ for $N = 10, 20, 30$, respectively. Here $n_{d,N} = N_{d,N}/V_4$, $n_{dd,N} = N_{dd,N}/V_4$, $n_{ddd,N} = N_{ddd,N}/V_4$ are the densities of isolated dyons, dyon pairs and full calorons respectively and $Q_d = \pm 1/3$, $Q_{dd} = \pm 2/3$, $Q_{ddd} = \pm 1$ are their modelled topological charges. As one can see the best agreement of our modelling with the result of Ref. [14] is obtained for $N = 20$ modes of the overlap operator.

We take this agreement as a criterium for choosing the number of modes $N$ and we use further exclusively $N = 20$ modes of the overlap operator in calculations of correlation functions. The three-dimensional dyon cluster density in this case is equal to $\rho = 3.03 \text{ fm}^{-3}$. Note, that respective dimensionless value $\rho/T^3 = 0.98$ should be compared with the value 0.74 obtained in Ref. [2] for the density of dyons in case of the SU(2) dyon model at a temperature close to $T_c$.

For each configuration we calculated the distances between topological lump objects of different types (dyons/antidyons of three types): two dyons (or antidyons) of the same type $(d_i, d_j)$, dyon and antidyon of the same type $(d_i, \bar{d}_j)$, two dyons (or antidyons) of different type $(d_i, d_j)$, dyon and antidyon of different type $(d_i, \bar{d}_j)$. The numbers of these pairs in bins of distances over the range from zero distance to the maximal distance $\sqrt{(24/2)^2 + (24/2)^2 + (24/2)^2} = 20$ lattice units. Normalized (to the total density of dyons and antidyons of all types squared) the correlators of dyons and antidyons densities

$$<d_i d_j(x)> = \sum_i <d_i(x)d_j(0) + \bar{d}_i(x)d_j(0)> / (\sum_i <d_i + d_j>)^2$$

$$<d_i \bar{d}_j(x)> = \sum_i <d_i(x)\bar{d}_j(0) + \bar{d}_i(x)d_j(0)> / (\sum_i <d_i + d_j>)^2$$

are shown in Fig. 1 as histograms with bins of one lattice spacing size. The errors are shown in the centers of the corresponding bins.

In Fig. 1 the first two bins are not shown. Respective correlator values are equal to 51.6 and 4.47 for first and second bins, correspondingly. We see that two dyons (or antidyons) of different type are attracting (positively correlated) at small distances. At this point we are in agreement with model results of Ref.[7]. As a result of this attraction the half of dyons and antidyons are combined in dyon pairs and dyon triplets (full calorons). We see also some attraction at small distance for two dyons (or antidyons) of the same type (which cannot form dyon pairs and full calorons) (see Fig. 1a) This is in contrast to the repulsion postulated in Ref.[7]. As for dyon and antidyon interaction we observe a repulsion at small distances of strength independent of the types of dyons and antidyons (see Fig. 1a and Fig. 1b). At larger distances in all four cases we do not see any interaction (non-trivial correlation).

We note that our results are qualitatively the same for $N = 10$ and 30.

IV. CONCLUSIONS

In SU(3) lattice gauge theory, using a small number of modes of the overlap Dirac operator with eigenvalues closest to zero, we have investigated clusters formed by the UV-filtered fermionic topological charge density. The topological charge density has been computed for three different types of temporal boundary conditions applied to the overlap Dirac operator. Assuming that these clusters correspond to dyons, we have obtained their frequency of occurrence and demonstrated the tendency to combine into triplets (calorons) or to form pairs of dyons apart from remaining isolated dyons.

We accomplished a first lattice computation of the dyon correlation functions defined in eqs. (7,10). We found at small distances attraction for two dyons (or antidyons) and repulsion for dyon and antidyon. The attraction for two dyons (or antidyons) of different type is larger than the attraction for two dyons (or antidyons) of same type. Repulsion for dyon and antidyon does not depend on types of dyons and antidyons. At larger distances in all cases we do not see any correlations.
FIG. 1: Correlators (normalized to the total density of dyons and antidyons of all types squared) of dyon and antidyon densities are shown as histograms over distance with bins of one lattice spacing. The errors are shown in the centers of corresponding bins.

Acknowledgments

V.G.B. and B.V.M. are supported by the RFBR grant 18-02-40130.

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