J/ψ decay into ϕ(ω) and vector-vector molecular states

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Abstract Based on the picture that the f0(1370), f0(1710), f2(1270), f2(1525), K∗0(1430) resonances are dynamically generated from the vector-vector interaction, we study the decays J/ψ → ϕ(ω)f0(1370)[f0(1710)], J/ψ → ϕ(ω)f2(1270)[f2(1525)], and J/ψ → K∗0K∗0 (1430) and make predictions for seven independent ratios that can be done among them. The starting mechanism is that the J/ψ decays into three vectors, followed by the final state interaction of a pair of them. The weights of the different three vector primary channels are obtained from the basic assumption that the J/ψ (c̅c) is an SU(3) singlet. By means of only one free parameter we predict four ratios in fair agreement with experiment, make two extra predictions for rates yet unmeasured, and disagree on one data for which only upper bounds are reported. Further measurements are most welcome to complete the information required for these ratios which test the nature of these resonances as dynamically generated.

1 Introduction

The undeniable success of the quark model to put in order the bulk of hadronic states [1–5] has not precluded that some states show a richer structure than the standard q̅q meson and qqq baryon composition. Many different structures have been proposed to understand different hadronic states, as tetraquarks, pentaquarks, hybrids, molecules (see recent reviews on these topics [6–14]). The molecular picture to describe many hadronic states [9] received an undeniable boost, with a broad consensus reached that the recently observed pentaquark states [15] are of molecular nature [15–26,26–30], as originally predicted in Ref. [31]. It is fair to recall that there are also many such cases of molecular nature in the light sector. The case of light scalar mesons, f0(500), f0(980), a0(980), as dynamically generated resonances from the pseudoscalar-pseudoscalar interaction [32–35] has received much attention (see reviews [36, 37]). So is the case of axial vector mesons, generated from the interaction of pseudoscalar and vector mesons [38,39], or particular baryons like the two Λ(1405), N*(1535), · · · , generated from the meson-baryon interaction [40–48].

With so much work done in these sectors, it is surprising that the interaction of vector mesons among themselves has received comparatively much less attention. The reason probably is that the chiral Lagrangians [49,50] do not deal with the interaction of vector mesons among themselves and the works reported before rely upon the chiral Lagrangians using a unitary extension that matches them at low energies. The link of the former works to the vector-vector interaction can be traced to the work of Ref. [51], where it was shown that the chiral Lagrangians up to O(p4), and implementing vector meson dominance, can be equally obtained using the local hidden gauge approach [52–55] through the exchange of vector mesons. The local hidden gauge approach, dealing explicitly with vector mesons, provides the tools for the vector-vector interaction through a contact term and the exchange of vector mesons. This formalism was used in [56,57] to study first the ρρ interaction [56] and then extended to SU(3) [57]. As a consequence of that, vector-vector bound states that decay into lighter mesons were found in these works, and the f0(1370), f2(1270), f0(1710), f2(1525), K∗0(1430) resonances, among others, were generated within this approach. The consistency of this picture

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with many reactions where these resonances are produced has been tested, for instance, in $B^0$ and $\bar{B}^0$ decays [58], photoproduction of the $f_2(1270)$, $f_2^*(1525)$ [59, 60], the coupling of $f_0(1370)$ and $f_2(1270)$ to $\gamma\gamma$ [61], the $J/\psi$ decay into $\phi(\omega)$ and some of these resonances, together with decay to $K^{*0}$ and the $K_2^{*0}(1430)$ [62], and the production of these resonances in $\psi(nS)$ and $\Upsilon(nS)$ decays [63, 64].

In the present work we want to retake the issue of the $J/\psi$ decay [62]. In that work we introduced some Lagrangians, involving transition of $J/\psi$ to three vectors by analogy to a similar work in $J/\psi \to \phi(\omega)PP$, with $P$ a pseudoscalar meson, where the PP mesons interact producing the $f_0(500)$ and $f_0(980)$ resonances [65–67]. In Refs. [65] and [66] two different formalisms, although equivalent, were used to study the process. In between, different methods, based on the $c\bar{c}$ character as a singlet of $SU(3)$, have been developed, which are conceptually easier. Indeed, in Ref. [68], where the $J/\psi \to h_1(1380)$ BESIII reaction has been studied [69], the basic ingredient is the $J/\psi \to VVP$ ($V$ for vector) transition, followed by $VP$ interaction that, according to [38, 39] generates the axial vector mesons. The $J/\psi$ is assumed to be an SU(3) singlet and then two structures are used $(VV\bar{P})$ and $(V\bar{V}\langle P\rangle)$, with $V$ and $P$ the SU(3) matrices of vectors and pseudoscalars in SU(3), and $\langle P\rangle$ standing for the trace of these matrices. The formalism of Ref. [68] is shown to be equivalent to those of Refs. [65, 66], yet technically easier, and more intuitive, with the dominant $(VV\bar{P})$ contribution found to be equivalent to the main term in Refs. [65] and [66].

The dominance of the trace of three mesons has also been established in the $\chi_{c1}$ decay into $\eta\pi^+\pi^-$ [70], where the $(PP\bar{P})$ structure for primary production of three pseudoscalars in the $\chi_{c1}$ decay is followed by the interaction of $P\bar{P}$ pairs to produce the $a_0(980)$ and the $f_0(500)$ resonances [71]. Similarly, the $(VV\bar{P})$ structure has also been tested as the dominant one in the study of the $\chi_{cJ}$ decay into $\phi h_1(1380)$ [72], where $\phi$ is a spectator and the other $VP$ pair interacts to produce the $h_1(1380)$ [73].

In the present work we want to study the $J/\psi$ decay into $\phi(\omega)$ and vector resonances that come from the vector-vector interaction. We then assume that the $J/\psi(c\bar{c})$ is an SU(3) singlet and take the $(VV\bar{V})$ primary production, followed by the interaction of the $VV$ pairs. We shall also consider a possible small contribution of the $(VV\langle V\rangle)$ structure, which has some similarity to the $(VV\langle P\rangle)$ structure of [68], but cannot be taken from there since this corresponds to different structures. We shall discuss this issue in detail to clarify what the differences are.

The issue of the vector-vector interactions has been vindicated recently after the debate created by the works [74, 75] questioning the $f_2(1270)$ as a $\rho\rho$ molecular state [56, 57]. However, the conclusion of Refs. [74, 75] was reached after some approximations were taken which are not valid in the special case of the $f_2(1270)$. Ref. [74] used the on-shell factorization of the $\rho$-exchange term that produced a singularity in the potential and an imaginary part at energies below $\sqrt{3m_\rho}$, above the mass of the $f_2(1270)$. The deficiencies of this method, putting on-shell ($p^2 = m_\rho^2$) the $\rho$-meson in the loops and in the external $\rho$-mesons of the molecular states, were discussed in Ref. [76]. A new method was used in Ref. [75] based on dispension relations to overcome the problem of factorization. Yet, the approximations done in Ref. [75] also ran into problems. By making more subtractions in the dispersion integral, it was shown in Ref. [77], that the method used in Ref. [75] did not converge for energies far from the $\rho\rho$ threshold below $\sqrt{3m_\rho}$, and hence, it could not be used to make predictions at $1270$ MeV, where the $f_2(1270)$ resonance appears. Apart from that, the on-shell factorized potential of Ref. [74] was used also in Ref. [75] as input in the dispersion relation, what caused problems around the singularity at $E = \sqrt{3m_\rho}$ [77]. The discussions, nevertheless, had their fruit, since in Ref. [76] a new method that overcomes the on-shell factorization was used, improving on the early results of Ref. [56]. The method of Ref. [76] did an exact evaluation of the diagrams involved in the $\rho\rho$ interaction at one loop level, that contain, two, three and four internal propagators, showing that there are no singularities, nor imaginary part below threshold, and then an effective potential was used such that, $V_{eff}G_{\rho\rho}V_{eff}$, with $G_{\rho\rho}$ the ordinary two $\rho$ loop function of the Bethe Salpeter equation, gave the same result as the exact one loop result. Then, this effective potential was used as kernel in the Bethe Salpeter equation, $T = [1 − V_{eff}G]^{-1}V_{eff}$, which produced poles that would be associated to the physical states. With a fine tuning of the regulator, a cut-off in the integral of the $G$ function, to obtain a pole for $J = 2$ at 1270 MeV, an interesting result emerged: the coupling of the state to $\rho\rho$ differed in less than 10% from the one obtained in Ref. [56], and in a much smaller amount for the $f_0(1370)$. This remarkable agreement, in spite of the very different methods used, was attributed in Ref. [76] to the memory of the system on the Weinberg compositeness condition [78], which links the coupling to the binding for small energies and the fact that, when one takes into account the mass distribution of the $\rho$ mesons, the $f_2(1270)$ is not as bound as it looks. With this reassurance, we shall continue to use the couplings and $G$ functions of Refs. [56, 57] which one needed as input for the calculations that we perform here, but we shall also take into account the difference between the couplings in Refs. [56] and [76] to estimate uncertainties from the method used.

Apart from the new perspective and formalism discussed above, there is one more reason to retake this problem since new data are available. Then we shall evaluate the rates for $J/\psi \to \phi(\omega)f_0(1370)[f_0(1710)]$, $J/\psi \to \phi(\omega)f_2(1270)[f_2^*(1525)]$, and $J/\psi \to K^{*0}\bar{K}_2^{*0}(1430)$ and from there we shall evaluate ratios that we will compare with experiment.
This paper is organized as follows. In Sect. 2, we explain how the $J/\psi$ first decays into three vector mesons and then a pair of them rescatter to generate dynamically the $f_0(1370)$, $f_0(1710)$, $f_2(1270)$, $f_2(1525)$, and $K^{*0}(1430)$. We then construct seven ratios to eliminate unknown couplings and fix one remaining relative weight by fitting to four experimental known ratios and make predictions for three remaining ones in Sect. 3, followed by a short summary and outlook in Sect. 4.

2 Formalism

We study first the $J/\psi \to VVV$ transition for which we will have the operator $\mathcal{C}(VVV)$ mixed with $\mathcal{C}\beta(VVV)\{V\}$, being $\mathcal{C}$ a normalization constant, $\beta$, a parameter to fit to the data, and $V$ the matrix containing the nonet of vector mesons,

$$V_\mu = \left( \begin{array}{ccc} \frac{1}{\sqrt{2}} & 0 & 1 \\ \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{array} \right).$$

These two kind of operators are studied separately in the subsections below.

2.1 Terms coming from $VVV$

Then, we can construct the trace, $\langle VVV \rangle$, and look at the terms which accompany the $\omega$, $\phi$ and $K^{*0}$ mesons.

We find,

1. for $\omega$ meson

$$\frac{3}{\sqrt{2}} \left\{ 3 \rho^0 \rho^0 + 3 \rho^+ \rho^- + 3 \rho^- \rho^+ + \omega \omega + 3 K^{*+} K^{*-} + 3 K^{*0} K^{*0} \right\},$$

2. for $\phi$ meson

$$\phi \left\{ 3 K^{*+} K^{*-} + 3 K^{*0} K^{*0} + \phi \phi \right\},$$

3. for $K^{*0}$ meson

$$K^{*0} \left\{ 3 \rho^+ K^{*-} - \frac{3}{\sqrt{2}} \rho^0 K^{*0} + \frac{3}{\sqrt{2}} \omega K^{*0} + 3 \phi K^{*0} \right\}.$$

It is convenient to write these in terms of isospin states in order to use the information of Ref. [57]. We have the isospin multiplets $(-\rho^+, \rho^0, \rho^-)$, $(K^{*+}, K^{*0})$, $(K^{*0}, -K^{*-})$ and then

$$|\rho \rho, I = 0 \rangle = -\frac{1}{\sqrt{6}} |\rho^0 \rho^0 + \rho^+ \rho^- + \rho^- \rho^+ \rangle;$$

$$|\omega \omega, I = 0 \rangle = \frac{1}{\sqrt{2}} |\omega \omega \rangle;$$

$$|\phi \phi, I = 0 \rangle = \frac{1}{\sqrt{2}} |\phi \phi \rangle,$$

where we introduced the extra $1/\sqrt{2}$ factor of the unitary normalization for identical particles,

$$|K^+ K^-, I = 0 \rangle = -\frac{1}{\sqrt{2}} |K^{*+} K^{*-} + K^{*0} K^{*0} \rangle,$$

$$|\rho K^*, I = 1/2, I_3 = 1/2 \rangle = \frac{\sqrt{2}}{3} |\rho^+ K^{*-} \rangle - \frac{1}{\sqrt{3}} |\rho^0 K^{*0} \rangle.$$

From Eqs. (2–4) and (5–7) we find the weights for $|\rho \omega, I = 0 \rangle$, $|K^+ K^-, I = 0 \rangle$, $\omega \omega$ and $\phi \phi$ production in isospin basis.

We substitute the combinations of the pair of mesons in Eqs. (2–4) in terms of the isospin states in Eqs. (5–7) and the coefficients in front of these states provide the weights, $h_i$, given by

$$h_{\omega \rho \rho} = -\frac{3 \sqrt{3}}{2}; \quad h_{\omega K^+ K^-} = -3; \quad h_{\omega \omega \omega} = 3;$$

$$h_{\omega \rho \phi} = 0; \quad h_{\phi \rho \rho} = 0; \quad h_{\phi K^+ K^-} = -3 \sqrt{2};$$

$$h_{\phi \phi \phi} = 3 \sqrt{2}; \quad h_{\rho \omega \omega} = 0; \quad h_{K^{*0} \rho K^*} = 3 \sqrt{2};$$

$$h_{K^{*0} \omega K^*} = \frac{3}{\sqrt{2}}; \quad h_{K^{*0} \phi K^*} = 3,$$

where the symmetry factor $n!$ for production of $n$ identical particles has been taken into account.

2.2 Terms with $\langle VVV \rangle$

Alternatively, we can see what weight we get with the $\langle VVV \rangle$ structure. In this case we find that

$$\langle VVV \rangle \equiv \left\langle \sqrt{2} \omega + \phi \right\rangle \left\{ \rho^0 \rho^0 + \rho^+ \rho^- + \rho^- \rho^+ + \omega \omega + 2K^{*+} K^{*-} + 2K^{*0} K^{*0} + \phi \phi \right\},$$

and, similarly as done in the former subsection, the weights in isospin basis are now

$$h'_{\omega \rho \rho} = -\frac{3 \sqrt{3}}{2}; \quad h'_{\omega K^+ K^-} = -4; \quad h'_{\omega \omega \omega} = 6; \quad h'_{\omega \rho \phi} = 2;$$

$$h'_{\phi \rho \rho} = -\frac{3 \sqrt{2}}{2}; \quad h'_{\phi K^+ K^-} = -2 \sqrt{2}; \quad h'_{\phi \phi \phi} = 3 \sqrt{2};$$

$$h'_{\rho \omega \omega} = \frac{3 \sqrt{2}}{2}; \quad h'_{K^{*0} \rho K^*} = 0; \quad h'_{K^{*0} \omega K^*} = 2 \sqrt{2}; \quad h'_{K^{*0} \phi K^*} = 2.$$

2.3 Production of resonances

The production of the resonances proceeds as described graphically in Fig. 1.

Analytically we can translate the mechanism of Fig. 1 into the transition matrix

$$t_j = C \sum_i h_i G_i (M_{R_j}) g_{R_j, i},$$

where $G_i$ is the normalizing constant for the $i$-th pair of mesons.
where \( h_i \) are the weights calculated before for the different intermediate channels, \( G_i \) is the loop function of the two intermediate mesons and \( g_{R,i} \) are the couplings of the resonance \( g_i \) to these channels \( i \). This information we obtain from Ref. [57] and is shown in Table 1, together with the same uncertainties in Table 2 that were evaluated in [62].

If we consider the \( \langle VV \rangle \langle V \rangle \) structure, then we replace
\[
h_i \to h_i + \beta h_i^\prime. \tag{12}
\]
We have ignored the spin structure with the four vectors that we have in the production vertex. One can easily implement it using the spin operators of Refs. [56,57]. However, it is unnecessary since we only calculate ratios between spin \( J = 2 \) states and \( J = 0 \) states independently, and factors coming from the spin structure cancel in the ratios.

2.4 Comparison with the formalism of a former approach

In the introduction we have stated that the formalisms of Refs. [65] and [66] are different but equivalent, and also equivalent to the formalism of Ref. [68]. However, the latter, which is used as a guideline for the formalism used here, is not equivalent to the one used in Ref. [62]. In Ref. [65] the \( J/\psi \to \phi(\omega)PP \) reaction was studied and two mechanisms were considered which are depicted in Fig. 2a, b. Analytically this results into a structure of the \( \phi PP \) production of the type,
\[
\tilde{g}\psi_{\mu}\phi^{\mu}S, \tag{13}
\]
being \( \psi_{\mu}, \phi^{\mu} \) the \( J/\psi \) and \( \phi(\omega) \) vector fields, and \( S \) a scalar source which is written as
\[
S = \bar{s}s + \lambda_{\phi}\bar{n}n; \quad \bar{n}n = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d). \tag{14}
\]
The source \( S \) is then written in terms of meson-meson pairs, \( PP \), upon hadronization, containing \( \pi\pi, K\bar{K}, \pi\eta, \eta\eta \). The \( PP \) pairs are then allowed to undergo final state interaction producing resonances.

The approach of Ref. [66] is technically different. A structure like the one of Eq. (13) is also taken, \( \psi_{\mu}V^{\mu}PP' \), with \( P, P' \) being the two meson fields. Then all the meson-meson pairs as members of an SU(3) octet, are written in terms of \( \bar{S}_1, \bar{S}_8, \bar{S}_8', \bar{S}_8'' \), the singlet (\( \bar{S}_1 \)), a symmetric-octet (\( \bar{S}_8 \)) and an antisymmetric octet (\( \bar{S}_8' \)) representations of \( P \otimes P' \). The same is done for the \( V^{\mu}(\phi \otimes \omega) \), writing \( \phi \) and \( \omega \) in terms of a singlet and an octet, and finally \( J/\psi \) is assumed to be an SU(3) singlet. Then all matrix elements can be written in terms of two amplitudes, \( t_1, t_5 \). The two degrees of freedom in this formalism can be written in terms of those of Ref. [65], \( \tilde{g} \), \( \lambda, \phi \), and one finds the relations,
\[
t_1 = 4\sqrt{\frac{7}{3}}\tilde{g}v \quad t_5 = -\sqrt{\frac{10}{3}}\tilde{g}, \tag{15}
\]
with
\[
v = \frac{\sqrt{2} + 2\lambda\phi}{\sqrt{2} - \lambda\phi}. \tag{16}
\]
The parameter \( v \) in Ref. [65] is conveniently chosen and when \( \lambda\phi = 0 (v = 1) \) there is exact OZI fulfillment. The approach of Ref. [68] is also technically different. In the latter one also produces primarily \( J/\psi \to VPP \) but there V and one \( P \) undergo final state interaction to produce an axial vector state, \( h_1(1380), h_1(1170) \) and \( b_1(1235) \). Besides that, one also assumes that \( J/\psi \) is an SU(3) singlet and the Lagrangian for \( J/\psi \to VPP' \) is taken as
\[
\mathcal{L}_{J/\psi, VPP} = A_1 J/\psi(\langle V^{\mu} PP > + \beta < PP > < V^{\mu} \rangle), \tag{17}
\]
where \( V^{\mu} \) is the matrix of the vectors in the present work, Eq. (1), and \( P \) the equivalent matrix of the pseudoscalar mesons (Eq. (2) of Ref. [68]). With these two degrees of freedom it was also possible to show the equivalence to the formalisms of Refs. [65] and [66] with the relations,
\[
A_1 \equiv -\tilde{g} \quad \beta \equiv \frac{v - 1}{3}. \tag{18}
\]
We can see that the case with \( v = 1 \) (OZI complying case) is obtained with \( \beta = 0 \) in the formalism, and in [68] it was found that the \( < V^{\mu} PP > \) term in Eq. (17) was the dominant one in order to reproduce the experimental data.

With this perspective it is possible to see the differences between the approach of [62] to study the reactions addressed here and the present approach. In Ref. [62] the formalism of Ref. [66] was followed, substituting \( PP \) by \( VV \) with the same SU(3) structure and isolating a third vector \( V^{\mu} \) as done in [66]. By doing that one relies upon the success of the formalism of Ref. [66] for \( VPP \) production, but one is
somewhat breaking the symmetry for the production of three vectors. Because of that and the easy formulation in terms of SU(3) invariants used in [68], we follow this latter formalism and take the transition Lagrangian as,

$$\mathcal{L}_{J/\psi,3V} = C (<VVV> + \beta <VV>V) ,$$

where all vectors are now considered on the same footing. The formalism of Ref. [62] and the present one give rise to different weights. Indeed, the weights obtained in Ref. [62] for \(J/\psi \to \phi (\omega)VV'\) are given in Table 3 and for \(J/\psi \to K^{*0}VV'\) in Table 4.

We can see that for \(v = 1\) the weights \(W(\phi)\) for \(\phi \rho \rho\) and \(\phi \omega \omega\) vanish in Table 3. Comparing with the weights in Eqs. (7) and (9) we see that one needs \(\beta = 0\) in our formalism for this case, and we find the coefficients obtained in both formalisms equivalent up to a global factor. However, if \(v \neq 1\)

Table 1 Couplings and values of the loop functions for the different channels at the resonance energy for the \(f_0(1370)\) and \(f_0(1710)\) states

| Channel | \(\rho\) (MeV) | \(K^*K^*\) | \(\omega\omega\) | \(\phi\phi\) |
|---------|----------------|-------------|----------------|-------------|
| \(f_0(1370)\) | \(g_i\) | 7914 \(-i1048\) | 1210 \(-i415\) | \(-39 + i31\) | 12 \(+i24\) |
| Error \(g_i\) (%) | 4 | 3 | 22 | 22 |
| \(G_i(\times10^{-3})\) | \(-7.69 + i1.72\) | \(-4.13 + i0.26\) | \(-8.97 + i0.87\) | \(-0.63 + i0.14\) |
| Error \(G_i\) (%) | 10 | 29 | 42 | 220 |
| \(f_0(1710)\) | \(g_i\) (MeV) | \(-1030 + i1087\) | 7127 \(+i94\) | \(-1764 + i109\) | \(-2494 + i205\) |
| Error \(g_i\) (%) | 12 | 6 | 2 | 2 |
| \(G_i(\times10^{-3})\) | \(-9.7 + i6.18\) | \(-7.68 + i0.58\) | \(-10.85 + i8.19\) | \(-2.16 + i0.13\) |
| Error \(G_i\) (%) | 6 | 17 | 19 | 141 |

Table 2 Couplings and values of the loop functions for the different channels at the resonance energy for the \(f_2(1270)\), \(f_2'(1525)\) and \(\tilde{K}^{*0}(1430)\) states

| Channel | \(\rho\) (MeV) | \(K^*\tilde{K}^*\) | \(\omega\omega\) | \(\phi\phi\) | \(\rho\tilde{K}^*\) | \(\omega\tilde{K}^{*0}\) | \(\phi\tilde{K}^{*0}\) |
|---------|----------------|-------------|----------------|-------------|----------------|----------------|-------------|
| \(f_2(1270)\) | \(g_i\) (MeV) | 10551 | 4771 | \(-503\) | \(-771\) | 0 | 0 | 0 |
| Error \(g_i\) (%) | 4 | 3 | 22 | 22 | 0 | 0 | 0 |
| \(G_i(\times10^{-3})\) | \(-4.74\) | \(-3.00\) | \(-4.97\) | 0.475 | 0 | 0 | 0 |
| Error \(G_i\) (%) | 10 | 29 | 42 | 220 | 0 | 0 | 0 |
| \(f_2'(1525)\) | \(g_i\) (MeV) | \(-2611\) | 9692 | \(-2707\) | \(-4611\) | 0 | 0 | 0 |
| Error \(g_i\) (%) | 12 | 6 | 2 | 2 | 0 | 0 | 0 |
| \(G_i(\times10^{-3})\) | \(-8.67\) | \(-4.98\) | \(-9.63\) | \(-0.710\) | 0 | 0 | 0 |
| Error \(G_i\) (%) | 6 | 17 | 19 | 141 | 0 | 0 | 0 |
| \(\tilde{K}^{*0}(1430)\) | \(g_i\) (MeV) | 0 | 0 | 0 | 0 | 10613 | 2273 | \(-2906\) |
| Error \(g_i\) (%) | 0 | 0 | 0 | 0 | 3 | 5 | 5 |
| \(G_i(\times10^{-3})\) | 0 | 0 | 0 | 0 | \(-6.41\) | \(-5.94\) | \(-2.70\) |
| Error \(G_i\) (%) | 0 | 0 | 0 | 0 | 12 | 19 | 43 |

Fig. 2 Diagrams a and b for the decay of the \(J/\psi\) into the \(\phi\) and \(\pi\pi\), where \(\pi\) refers to the light \(u, d\) quarks. The gray blob in a depicts the final state interactions in the coupled \(\pi\pi - K\bar{K}\) system.
Table 3 Weights for the processes $J/\psi \to \phi(\omega) V' V'$ in $I = 0$

| ch. | $\rho \rho$ | $K^* K^*$ | $\omega \omega$ | $\phi \phi$ |
|-----|-------------|------------|---------------|-------------|
| $W^{(\phi)}$ | $-\frac{1}{\sqrt{3}}(v - 1)$ | $-\frac{\sqrt{3}}{2}(2v + 1)$ | $\frac{1}{\sqrt{3}}(v - 1)$ | $\frac{1}{\sqrt{3}}(v + 2)$ |
| $W^{(\omega)}$ | $-\frac{1}{\sqrt{3}}(2v + 1)$ | $-\frac{1}{\sqrt{3}}(4v - 1)$ | $\frac{1}{\sqrt{3}}(2v + 1)$ | $\frac{1}{\sqrt{3}}(v - 1)$ |

Table 4 Weights for the process $J/\psi \to K^{*0} V' V'$ in isospin $I = 1/2, I_1 = 1/2$

| Ch. | $\rho K^*$ | $K^* \omega$ | $K^* \phi$ |
|-----|-------------|--------------|-------------|
| $W^{(K^{*0})}$ | $\sqrt{2}$ | $\frac{1}{\sqrt{2}}$ | 1 |

we can compare the ratio of $W^{(\phi)}_{\rho \omega} / W^{(\phi)}_{\rho \rho} = -1/\sqrt{3}$, while in the present formalism one gets the ratio $-\beta \sqrt{2} / \beta \sqrt{3/2} = -2/\sqrt{3}$. This is sufficient to see that the formalisms are not equivalent, but given the fact that the dominant term comes from $<VVV>$ ($\beta = 0$), where the formalisms are equivalent, the results that we obtain here, although different, should resemble those obtained in Ref. [62]. In addition, we have more ratios to compare now, and the experience and success of the works of Refs. [68, 70–73] using the new formulation in terms of SU(3) invariants gives us confidence to better rely on these new results.

3 Results and discussions

For the evaluation of the $J/\psi$ decay into $\omega(\phi)$ and a molecular VV state, the standard formula for the width is given by

$$\Gamma = \frac{1}{8\pi} \frac{1}{M^2_{J/\psi}} |t|^2 q,$$

where $q = \lambda^{1/2}(M^2_{J/\psi}, M^2_R, M^2_V)/2M_{J/\psi}$, where $M_R$ stands for the mass of the VV resonance, and $M_V$ refers to the mass of the vector meson, $\omega, \phi$ or $K^{*0}$, involved in the $J/\psi$ decay. For the evaluation of $\Gamma$ and $q$ in Eq. (20), we take the values of the masses of the resonances given in Ref. [57], although, if the nominal masses of the resonances according to the PDG [79] are used instead, numerical results barely change. The ratios,

$$R_1 \equiv \frac{\Gamma_{J/\psi \to \phi f_2(1270)}}{\Gamma_{J/\psi \to \phi f_2'(1270)}} \quad R_2 \equiv \frac{\Gamma_{J/\psi \to \omega f_2(1270)}}{\Gamma_{J/\psi \to \omega f_2'(1270)}}$$

are estimated as it was done in Ref. [62], but with the new weights given in Eqs. (8) and (10), by using Eqs. (11), (12) and (20). In addition, we evaluate here three more ratios:

$$R_5 \equiv \frac{\Gamma_{J/\psi \to \omega f_2(1370)}}{\Gamma_{J/\psi \to \omega f_2(1710)}} \quad R_6 \equiv \frac{\Gamma_{J/\psi \to \omega f_2'(1370)}}{\Gamma_{J/\psi \to \omega f_2'(1710)}} \quad R_7 \equiv \frac{\Gamma_{J/\psi \to \omega f_2(1710)}}{\Gamma_{J/\psi \to \omega f_2'(1710)}} \quad (22)$$

First of all, the parameter $\beta$ in Eq. (12) is fitted to get the experimental values of the ratios of Eqs. (21) and (22). These values are gathered in the last column of Table 5. For comparison, we also show the results obtained in Ref. [62]. In addition, we have more ratios to compare now, and the experience and success of the works of Refs. [68, 70–73] using the new formulation in terms of SU(3) invariants gives us confidence to better rely on these new results.

1 We obtain $\chi^2 = 5.99$ inside the 90% CI [0.6, 6.3]. Thus, the $\chi^2/v = 1.99$. If we include the ratio $R_1$ in the fit, then, the fit does not pass the Pearson test, the total $\chi^2$ increases around three units, $\approx 9.4$, and $\chi^2/v \approx 2.35$. However, the value of $\beta$ obtained barely changes, we still obtain $\beta \approx 0.320$ (before we obtained $\beta = 0.323$).
third column of this table. In both methods, (I) and (II), the 95% CI obtained are comparable. For the sample size used here, the 0.5 quantiles obtained are almost identical to the values of the ratios for $\beta = 0.32$ in both methods, indicating that the sample is large enough in the present problem, and we omit those in the table.

The experimental values fall inside the 95% CI for the ratios for $i = 1, 3, 4, 7$, and also these come out similar to those of Table 7 in Ref. [62], showing a fair agreement between theory and experiment, and also with the theory used in Ref. [62]. For the ratio $R_2$, we obtain a value below the experimental lower limit. It is unclear whether this discrepancy is a genuine failure of the theory or it may be attributed to the unprecise measurement of the partial decay rate $\Gamma(J/\psi \to \omega f_2'(1525))$. Precise measurements of the rate in the future, hopefully providing a partial decay width rather than an upper bound, will be helpful to elucidate this question.

The results obtained here for the ratios $i = 1, 3, 4$ support the molecular $VV$ nature of the tensor resonances, $f_2'(1270), f_2'(1525)$ and $K_2^*(1430)$, investigated in Refs. [56, 57]. The new ratio $R_7$ studied here in perfect agreement with experiment, also supporting the vector-vector nature of the scalar $f_0(1710)$.

It is interesting to look at the dependence of the ratios with the parameter $\beta$. For that, we generate a new random sample with the same size $n$ for the ratios $R_i's$ as in method (I), but with different values of $\beta$. Then, confidence intervals of 68%, 95% probability are evaluated. These are shown in Fig. 3, with darker and lighter error bands respectively. The experimental result is shown as a gray band, while the brown vertical line indicates the result for $\beta = 0.32$ for comparison.

Although, in principle, different values of $\beta$ rather than that obtained in the fit, 0.32, are possible, since the ratios obtained in the second and third columns are close or inside the 95% CI given in Table 5, the experimental value of $R_3$ does not favor $\beta = 1$, as shown in Fig. 3, where the theory for smaller values of $\beta$ than 0.5 is preferred. The study done here favors an admixture of both terms, $\langle VVV \rangle$ and $\langle VV \rangle \langle V \rangle$.

As shown in Ref. [76], if the one loop calculation is performed exactly for the $\rho\rho$ interaction, the coupling of the $f_2(1270)$ to $\rho\rho$ was slightly larger by 10% than in Ref. [56]. In order to look at this effect, we redid the calculation increasing the uncertainty of the coupling to 10% and repeated the evaluation of uncertainties of the ratios $R_i, i = 1, 4$, with a sample of same size ($n = 30$). The average difference between the 0.5 quantiles in the new calculation compared with the previous one is less than 1%, while for the extremes of the CI given in Table 5, these differences are less than 7%.

In any case, the results shown here for the four ratios $i = 1, 3, 4, 7$ provide further support to the molecular interpretation of the $f_2(1270), f_2'(1525), K_2^*(1430)$ and $f_0(1710)$ of Refs. [56, 57]. The predictions for the ratios $i = 5, 6$ involving the scalar resonances $f_0(1370)$ and $f_0(1710)$ are also given in Table 5, where $R_6$ turns out to be very small, indicating that the $J/\psi \to \phi f_0(1370)$ decay is suppressed. This can be easily understood in terms of the dominant term in Eq. (2), since the pair of vectors that create the $f_0(1370)$ are the $K^* K^*$ and $\phi \phi$, but not $\rho \rho$ which is the main building block of the $f_0(1370)$ [56, 57].

### 4 Summary and outlook

We have correlated different decays of the $J/\psi$ meson, like $J/\psi \to \phi f_0(1370)[f_0(1710)], J/\psi \to \phi(\omega) f_2(1270)[f_2'(1525)],$ and $J/\psi \to K^* K^* \phi\phi(1430)$. These decay processes have in common the production of a vector meson together with some $f_0, f_2, K_2$ resonances, which in previous works were shown to have properties consistent with the vector-vector molecular hypothesis. The assumptions made are very basic, starting from the $J/\psi$, being a $c\bar{c}$ state, as a singlet of $SU(3)$, in the same way as a $s\bar{s}$ state is a singlet of isospin $SU(2)$. The nature of these resonances as dynamically generated implies that the production process begins by the $J/\psi$ decaying into three vectors, followed by the interaction of a $VV$ pair that leads to the formation of these resonances. Based on information gathered from related reactions, we find that the two basic $SU(3)$ invariant structures that can be made from three vector $SU(3)$ matrices, and are relevant to these reactions, are $\langle VVV \rangle$ and $\langle VV \rangle \langle V \rangle$, with the first structure being dominant. We have also taken advantage to show the differences between this approach and the former one used in Ref. [62].

We determine seven independent ratios that can be made with these decay rates and fit the strength of the $\langle VV \rangle \langle V \rangle$ structure versus the $\langle VVV \rangle$ one. We find indeed a relative strength of this term of about 0.3. By means of this only parameter we can obtain good results compared to experiment for four ratios. Two extra ratios have no data to compare, and hence are genuine predictions of the theory, and one ratio disagrees with the data. It would be better to wait for precisemeasurements of that width, for which there is only an upper bound at present, to draw conclusions. While certainly future measurements of the magnitudes involved in this ratio are most welcome, the overall fair agreement found with most data, with rates that are quite different to each other, speaks much in favor of the picture where these resonances are dynamically generated. The measurement of the rates needed to determine the two predictions for which there is no data at present would provide a further test for this idea and should be most welcome.

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Fig. 3 Dependence of the ratios $R_i$'s with $\beta$. Darker and lighter bands represent 68% and 95% CI. Gray bands represent the experimental result, while the brown vertical line indicates the result for $\beta = 0.32$. 
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References

1. S. Godfrey, N. Isgur, Phys. Rev. D 32, 189 (1985)
2. N. Isgur, G. Karl, Phys. Rev. D 18, 4187 (1978)
3. S. Capstick, N. Isgur, Phys. Rev. D 34, 2809 (1986)
4. S. Capstick, W. Roberts, Prog. Part. Nucl. Phys. 45, S241 (2000)
5. J. Vijande, F. Fernandez, A. Valcarce, J. Phys. G 31, 481 (2005)
6. H.X. Chen, W. Chen, X. Liu, S.L. Zhu, Phys. Rept. 639, 1 (2016)
7. R.F. Lebed, R.E. Mitchell, E.S. Swanson, Prog. Part. Nucl. Phys. 93, 143 (2017)
8. A. Esposito, A. Fillion, A.D. Polosa, Phys. Rept. 668, 1 (2017)
9. F.K. Guo, C. Hanhart, U.G. Meissner, Q. Wang, Q. Zhao, B.S. Zou, Rev. Mod. Phys. 90, 015004 (2018)
10. A. Ali, J.S. Lange, S. Stone, Prog. Part. Nucl. Phys. 97, 123 (2017)
11. S.L. Olsen, T. Skwarnicki, D. Zieminska, Rev. Mod. Phys. 90, 015003 (2018)
12. M. Karlner, J.L. Rosner, T. Skwarnicki, Ann. Rev. Nucl. Part. Sci. 68, 17 (2018)
13. Y.R. Liu, H.X. Chen, W. Chen, X. Liu, S.L. Zhu, Prog. Part. Nucl. Phys. 107, 237 (2019)
14. N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C.P. Shen, C.E. Thomas, A. Vairo, C.Z. Yuan, arXiv:1907.07583 [hep-ex]
15. R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 122 (2019) no.22, 222001
16. H.X. Chen, W. Chen, S.L. Zhu, Phys. Rev. D 100(5), 051501 (2019)
17. M. Z. Liu, Y. W. Pan, F. Z. Peng, M. Sanchez Sanchez, L. S. Geng, A. Hosaka and M. Pavon Valderrama, Phys. Rev. Lett. 122(24), 242001 (2019). https://doi.org/10.1103/PhysRevLett.122.242001
18. J. He, Eur. Phys. J. C 79(5), 393 (2019). https://doi.org/10.1140/epjc/s10052-019-6906-1
19. R. Chen, Z.F. Sun, X. Liu, S.L. Zhu, Phys. Rev. D 100, 011502 (2019)
20. C.W. Xiao, J. Nieves, O. Eset, Phys. Rev. D 100, 014021 (2019)
21. J.R. Zhang, Eur. Phys. J. C 79(12), 1001 (2019)
22. L. Meng, B. Wang, G.J. Wang, S.L. Zhu, Phys. Rev. D 100, 014031 (2019)
23. M.B. Voloshin, Phys. Rev. D 100, 034020 (2019)
24. Y. Yamaguchi, H. Garci'a-Tequecoatl, A. Giachino, A. Hosaka, E. Santopinto, S. Takeuchi and M. Takizawa, arXiv:1907.04684 [hep-ph]
25. Z.G. Wang, X. Wang, arXiv:1907.04582 [hep-ph]
26. M. Pavon Valderrama, Phys. Rev. D 100, no. 9, 094028 (2019)
27. Y.J. Xu, C.Y. Cui, Y.L. Liu, M.Q. Huang,, arXiv:1907.05097 [hep-ph]
28. T.J. Burns, E.S. Swanson., arXiv:1908.03528 [hep-ph]
29. Y.H. Lin, B.S. Zou, Phys. Rev. D 100(5), 056005 (2019)
30. Y. Yamaguchi, A. Hosaka, S. Takeuchi, M. Takizawa, arXiv:1908.08790 [hep-ph]
31. J.J. Wu, R. Molina, E. Oset, B.S. Zou, Phys. Rev. Lett. 105, 232001 (2010)
32. J.A. Oller, E. Oset, Nucl. Phys. A 620, 438 (1997)
33. N. Kaiser, Eur. Phys. J. A 3, 307 (1998)
34. M.P. Locher, V.E. Markshin, H.Q. Zheng, Eur. Phys. J. C 4, 317 (1998)
35. J. Nieves, E. Ruiz Arriola, Nucl. Phys. A 679, 57–117 (2000). https://doi.org/10.1016/S0375-9474(00)00321-3
36. J.A. Oller, E. Oset, A. Ramos, Prog. Part. Nucl. Phys. 45, 157 (2000)
37. E. Oset et al., Int. J. Mod. Phys. E 25, 1630001 (2016)
38. M.F.M. Lutz, E.E. Kolomeitsev, Nucl. Phys. A 730, 392 (2004)
39. L. Roca, E. Oset, J. Singh, Phys. Rev. D 72, 014002 (2005)
40. N. Kaiser, P.B. Siegel, W. Weise, Nucl. Phys. A 594, 325 (1995)
41. E. Oset, A. Ramos, Nucl. Phys. A 635, 99 (1998)
42. J.A. Oller, U.G. Meissner, Phys. Lett. B 500, 263 (2001)
43. M.F.M. Lutz, E.E. Kolomeitsev, Nucl. Phys. A 700, 193 (2002)
44. T. Inoue, E. Oset and M.J. Vicente Vacas, Phys. Rev. C 65 (2002) 035204
45. C. Garcia-Recio, J. Nieves, E. Ruiz Arriola and M.J. Vicente Vacas, Phys. Rev. D 67 (2003) 076009
46. D. Jido, J.A. Oller, E. Oset, A. Ramos, U.G. Meissner, Nucl. Phys. A 725, 181 (2003)
47. T. Hyodo, D. Jido, Prog. Part. Nucl. Phys. 67, 55 (2012)
48. Y. Kamiya, K. Miyahara, S. Ohnishi, Y. Ikeda, T. Hyodo, E. Oset, W. Weise, Nucl. Phys. A 954, 41 (2016)
49. S. Weinberg, Phys. Rev. 166, 1568 (1968)
50. J. Gasser, H. Leutwyler, Ann. Phys. 158, 142 (1984)
51. G. Ecker, J. Gasser, H. Leutwyler, A. Pich, E. de Rafael, Phys. Lett. B 223, 425 (1989)
52. M. Bando, T. Kugo, K. Yamawaki, Phys. Rept. 164, 217 (1988)
53. M. Harada, K. Yamawaki, Phys. Rept. 381, 1 (2003)
54. U.G. Meissner, Phys. Rept. 161, 213 (1988)
55. H. Nagahiro, L. Roca, A. Hosaka, E. Oset, Phys. Rev. D 79, 014015 (2009)
56. R. Molina, D. Nicmorus, E. Oset, Phys. Rev. D 78, 114018 (2008)
57. L.S. Geng, E. Oset, Phys. Rev. D 79, 074009 (2009)
58. J.J. Xie, E. Oset, Phys. Rev. D 90, 094006 (2014)
59. J.J. Xie, E. Oset, Eur. Phys. J. A 51, 111 (2015)
60. J.J. Xie, E. Oset, L.S. Geng, Phys. Rev. C 93, 025202 (2016)
61. H. Nagahiro, J. Yamagata-Sekihara, E. Oset, S. Hirenzaki, R. Molina, Phys. Rev. D 79, 114023 (2009)
62. A. Martinez Torres, L. S. Geng, L. R. Dai, B. X. Sun, E. Oset and B. S. Zou, Phys. Lett. B 680 (2009) 310
63. L.R. Dai, E. Oset, Eur. Phys. J. A 49, 130 (2013)
64. L.R. Dai, J.J. Xie, E. Oset, Phys. Rev. D 91, 094013 (2015)
65. U.G. Meissner, J.A. Oller, Nucl. Phys. A 679, 671 (2001)
66. L. Roca, J.E. Palomar, E. Oset, H.C. Chiang, Nucl. Phys. A 744, 127 (2004)
67. T.A. Lahde, U.G. Meissner, Phys. Rev. D 74, 034021 (2006)
68. W.H. Liang, S. Sakai, E. Oset, Phys. Rev. D 99, 094020 (2019)
69. M. Ablikim et al., BESIII Collaboration. Phys. Rev. D 98, 072005 (2018)
70. M. Ablikim et al., BESIII Collaboration. Phys. Rev. D 95, 032002 (2017)
71. W.H. Liang, J.J. Xie, E. Oset, Eur. Phys. J. C 76, 700 (2016)
72. M. Ablikim et al., BESIII Collaboration. Phys. Rev. D 91, 112008 (2015)
73. S.J. Jiang, S. Sakai, W.H. Liang, E. Oset, Phys. Lett. B 797, 134831 (2019)
74. D. Gülmez, U.G. Meissner, J.A. Oller, Eur. Phys. J. C 77, 460 (2017)
75. M.L. Du, D. Gülmez, F.K. Guo, U.G. Meissner, Q. Wang, Eur. Phys. J. C 78, 988 (2018)
76. L.S. Geng, R. Molina, E. Oset, Chin. Phys. C 41, 124101 (2017)
77. L. S. Geng, R. Molina and E. Oset, PTEP 2019, no. 10, 103B05 (2019)
78. S. Weinberg, Phys. Rev. 137, B672 (1965)
79. M. Tanabashi et al., Particle data group. Phys. Rev. D 98, 030001 (2018)