Self-stabilization of high frequency oscillations in semiconductor superlattices by time-delay autosynchronization

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We present a novel scheme to stabilize high-frequency domain oscillations in semiconductor superlattices by a time–delayed feedback loop. Applying concepts from chaos control theory we propose to control the spatio-temporal dynamics of fronts of accumulation and depletion layers which are generated at the emitter and may collide and annihilate during their transit, and thereby suppress chaos. The proposed method only requires the feedback of internal global electrical variables, viz current and voltage, which makes the practical implementation very easy.

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Semiconductor superlattices have been demonstrated to give rise to self-sustained current oscillations ranging from several hundred MHz to 150 GHz at room temperature. Various mechanisms with or without the involvement of propagating field domains have been discussed. In any case, a superlattice constitutes a highly nonlinear system, and instabilities are likely to occur. Indeed, chaotic scenarios have been found experimentally and described theoretically in periodically driven as well as in undriven systems. For a reliable operation of a superlattice as an ultra-high frequency oscillator such unpredictable and irregular conditions should be avoided. In principle, synchronization of oscillations in a superlattice by an external signal could be exploited to achieve a desired periodic behavior. However, in reality, the control of the forcing frequency in the ultrahigh range presents substantial technical problems.

Here, we propose a simple self-stabilizing scheme that is especially suitable for semiconductor devices like superlattices. It uses a profound concept of chaos control from nonlinear dynamics and chaos theory. Within this approach, an intrinsically unstable time-periodic motion is stabilized using a simple feedback loop with a time delay. This type of control needs only small control forces initially, and they vanish once control has been achieved. A sound advantage is that the oscillation mode to be stabilized need not be known beforehand, in contrast to other chaos control schemes. Rather, a simple delay line leads to autosynchronization of the system. Methods of nonlinear control theory have been usefully applied to real world problems in various areas of physics, chemistry and biology, but no use has been made of this in the field of semiconductor self-oscillators.

Control methods can be either local or global. Local methods require our ability to measure, and apply forcing directly to, the spatially resolved state variables of the system under study. However, in nanotechnology such variables, being e.g. electron densities in some quantum wells, are not easily accessible, and thus local methods cannot be applied. Unlike those, global methods require access only to some macroscopic variable(s) characterizing some integral output of the system. Such output can generally be reliably measured, and thus global methods seem to be the only option for the control of devices like superlattices. However, as we will show below, they are not straightforwardly applicable to nano-systems whose structure is spatially discrete. In this Letter we present a general approach to self-stabilization of irregular oscillations in semiconductor devices based upon essentially discrete quantum structures.

We consider a model for nonlinear electronic transport in semiconductor superlattices that yields complex and chaotic dynamic behavior under fixed time-independent external voltage in a regime where self-sustained dipole waves are spontaneously generated at the emitter. Those dipole waves are associated with traveling field domains, and consist of electron accumulation and depletion fronts that in general travel at different velocities and may merge and annihilate. Such moving fronts are widespread in nonlinear, spatially extended systems, and similar chaotic front patterns occur in many other systems, e.g., spatially continuous models describing bulk impurity impact ionization breakdown in semiconductors or globally coupled heterogeneous catalytic reactions. Thus the time-delay autosynchronization method proposed in this work could be readily applied to stabilize similar space-time patterns in a variety of systems.

Our model of a superlattice is based on sequential tunneling of electrons. In the framework of this model the quantum wells are assumed to be only weakly coupled, and electrons are localized at these wells. The tunneling rate to the next well is lower than the typical relaxation rate between the different energy levels.

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within one well. The electrons within one well are then in quasi-equilibrium and transport through the barrier is incoherent. The resulting tunneling current density $J_{m\rightarrow m+1}(F_m, n_m, n_{m+1})$ from well $m$ to well $m + 1$ depends only on the electric field $F_m$ between both wells and the electron densities $n_m$ and $n_{m+1}$ in the respective wells (in units of cm$^{-2}$). A typical dependence of $J_{m\rightarrow m+1}$ on the electric field between two consecutive wells is N-shaped and exhibits a pronounced regime of negative differential conductivity.

The rate of variation of electron density in well $m$ is governed by the continuity equation

$$\frac{dn_m}{dt} = J_{m-1\rightarrow m} - J_{m\rightarrow m+1} \quad \text{for } m = 1, \ldots, N$$

and Gauss’s law

$$\epsilon_r \epsilon_0 (F_m - F_{m-1}) = \epsilon (n_m - N_D) \quad \text{for } m = 1, \ldots, N,$$

where $N$ is the number of wells in the superlattice, $\epsilon_r$ and $\epsilon_0$ are the relative and absolute permittivities, $\epsilon < 0$ is the electron charge, $N_D$ is the donor density, and $F_0$ and $F_N$ are the fields at the emitter and collector barrier, respectively. The total applied voltage $U$ between emitter and collector imposes a global constraint $U = -\sum_{m=0}^{N} F_m d$, where $d$ is the superlattice period. This, together with (2), allows us to eliminate the field variables $F_m(n_1, \ldots, n_N, U)$ from the dynamic equations.

At the contacts Ohmic boundary conditions $J_{0\rightarrow 1} = \sigma F_0$, $J_{N-1\rightarrow N} = \sigma F_N n_N / N_D$ are chosen, where $\sigma$ is the Ohmic contact conductivity, and the factor $n_N / N_D$ is introduced in order to avoid negative electron densities at the collector. The value of $\sigma$ essentially determines the oscillation mode.

If the contact conductivity $\sigma$ is chosen appropriately, electron accumulation and depletion fronts are generated at the emitter. Those fronts form a traveling high field domain, with leading electron depletion front and trailing accumulation front. This leads to self-generated current oscillations. A fixed voltage $U$ imposes a constraint on the lengths of the high-field domains and thus on the front velocities. If $N_a$ accumulation fronts and $N_d$ depletion fronts are present, the respective front velocities $v_a$ and $v_d$ must obey $v_a/v_d = N_a/N_d$. If the accumulation and depletion fronts have different velocities, they may collide in pairs and annihilate. At certain combinations of contact conductivity $\sigma$ and voltage $U$, chaotic motion arises, when the annihilation of fronts of opposite polarity occurs at irregular positions within the superlattice. The inset of Fig. 1 shows the plane of $\sigma$ and $U$, where regions with distinct regimes are marked by different shading. Black regions are those where chaotic behavior has been found. As a computationally convenient criterion for chaos we have used the rapid decay of the autocorrelation function estimated from $n_{200}(t)$. Chaotic regimes are found at low contact conductivity and low voltages, where dipole oscillations with leading accumulation and trailing depletion fronts occur, and at higher contact conductivity and higher voltage, where the role of accumulation and depletion fronts is interchanged. In Fig. 4, a one-parameter bifurcation diagram is given, obtained by plotting the time differences $\Delta t$ between two consecutive maxima of the electron density in well $m$.

We shall now introduce a time delayed feedback loop to control the chaotic front motion and stabilize a periodic oscillation mode which is inherent in the chaotic attractor. In the extended time–delay autosynchronization scheme first suggested by Socolar et al [34], multiple time delays are used to improve the control performance. Analytical insight into those schemes has been gained only recently in [35, 36, 37], and various ways of coupling of the control force, including local and global schemes, have been compared in [38, 39, 40]. Whereas local coupling schemes usually lead to efficient control in a large control domain, they are not easily implemented in real systems since local, spatially resolved measurements are necessary. Therefore, here we propose a much

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**FIG. 1:** One-parameter bifurcation diagram: Time differences between consecutive maxima of the electron density in well $n = 20$ vs voltage at $\sigma = 0.5 \Omega^{-1} m^{-1}$. Time series of length 600 ns have been used for each value of the voltage. The inset shows a two-parameter bifurcation diagram: black squares denote chaotic oscillations, light shading indicates periodic oscillations, and the white region shows the absence of oscillations. Simulation of an $N = 100$ superlattice with $Al_{0.5}Ga_{0.7}As$ barriers of width $b = 5nm$ and GaAs quantum wells of width $w = 8nm$, doping density $N_D = 1.0 \times 10^{15} cm^{-2}$ and scattering induced broadening $\Gamma = 8meV$ at $T = 20K$. 

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**Equations:**

1. $\frac{dn_m}{dt} = J_{m-1\rightarrow m} - J_{m\rightarrow m+1}$ for $m = 1, \ldots, N$
2. $\epsilon_r \epsilon_0 (F_m - F_{m-1}) = \epsilon (n_m - N_D)$ for $m = 1, \ldots, N$.
out by using e.g. the following low–pass filter:

$$J(t) = \alpha \int_0^t J(t') e^{-\alpha(t-t')} dt', \quad (3)$$

with a cut-off frequency $\alpha$.

The multiple time delays of the resulting signal $\overline{J}$ (Fig. 2 black curve) are then used to modulate the volt-

FIG. 2: Control of chaotic front dynamics by extended time-delay autosynchronization. a) Space-time plot of the uncontrolled charge density, and current density $J$ vs. time. b) Same with global voltage control with exponentially weighted current density (denoted by the black curve). Parameters as in Fig. 4: $U = 1.15 V$, $\tau = 2.29$ ns, $K = 3 \times 10^{-6}$, $R = 0.2$, $\alpha = 10^6 s^{-1}$. Light and dark regions denote electron accumulation and depletion fronts in the space-time plots of the charge densities, respectively.

simpler global scheme. In our problem, as a global output signal that is coupled back in the feedback loop, it is natural to use the total current density $J$ defined as follows: $J = \frac{1}{N} \sum_{m=0}^{N} J_{m \rightarrow m+1}$ [31]. For the uncontrolled chaotic oscillations, $J$ is given in Fig. 2b, by grey, showing irregular spikes at those times when two fronts annihilate. Note that the grey current time trace is modulated by fast small-amplitude oscillations (due to well-to-well hopping of depletion and accumulation fronts in our discrete model) which are not resolved in the plot. However, as the variable $J$ is fed back to the system for the purposes of control, these high–frequency oscillations render the control loop unstable. They need to be filtered out by using e.g. the following low–pass filter:

$$\overline{J}(t) = \alpha \int_0^t J(t') e^{-\alpha(t-t')} dt', \quad (3)$$

with a cut-off frequency $\alpha$.

The multiple time delays of the resulting signal $\overline{J}$ (Fig. 2 black curve) are then used to modulate the volt-

FIG. 3: Control domain for global voltage control with exponentially weighted current density. Full circles denote successful control, small dots denote no control. Parameters as in Fig. 2

age $U$ across the superlattice:

$$u = U_0 + U_c(t)$$

$$U_c(t) = -K (\overline{J}(t) - \overline{J}(t - \tau)) + R U_c(t - \tau)$$

$$= -K \sum_{\nu = 0}^{\infty} R^\nu (\overline{J}(t - \nu \tau) - \overline{J}(t - (\nu + 1) \tau))$$

where $U_0$ is a time–independent external bias, and $U_c$ is the control voltage. $K$ is the amplitude of the control force, $\tau$ is the delay time, and $R$ is a memory parameter. Such a global control scheme is easy to implement experimentally. It is non-invasive in the sense that the control force vanishes when the target state of period $\tau$ has been reached. This target state is an unstable periodic orbit of the uncontrolled system. The period $\tau$ can be determined by observing the resonance-like behavior of the mean control force versus $\tau$. The result of the control is shown in Fig. 2b. The front dynamics exhibits annihilation of front pairs at fixed positions within the superlattice, and stable periodic oscillations of the current are obtained. In Fig. 4 the control domain is depicted in the parameter plane of $R$ and $K$. A typical horn-like control domain similar to the ones known from other coupling schemes [31] is found. Typically, the left-hand control boundary corresponds to a period-doubling bifurcation, while the right-hand boundary is associated with a Hopf bifurcation. These findings show that our control scheme is robust.

To conclude, we have demonstrated that time-delay autosynchronization represents a convenient and simple scheme for the self-stabilization of high-frequency current oscillations due to moving domains in superlattices. This approach lacks the drawback of synchronization by an external ultrahigh-frequency forcing, since it requires nothing but delaying of the global electrical system output by the specified time lag. The proposed low-pass filte-
ing of the output signal presents a solution of the problem one necessarily encounters when trying to control a nano-system with a crucially discrete quantum structure leading to superimposed fast well-to-well hopping oscillations in our case.

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