INDIRECT $CP$ VIOLATION IN AN ELECTROWEAK $SU(2)_L \times U(1)$ GAUGE THEORY OF CHIRAL MESONS.

B. Machet

Laboratoire de Physique Théorique et Hautes Energies, Universités Pierre et Marie Curie (Paris 6) et Denis Diderot (Paris 7); Unité associée au CNRS UMR 7589.

Abstract: Indirect $CP$ violation is analyzed in the framework of the electroweak gauge theory of $J = 0$ mesons proposed in [1] in which they transform like composite fermion-antifermion operators by the chiral $U(N)_L \times U(N)_R$ group and by the $SU(2)_L \times U(1)$ gauge group of the Glashow-Salam-Weinberg model [2]. It is shown that, in this model where, in particular, mass terms are introduced for the mesons themselves, and unlike what happens in the standard model for fermions [3]:
- electroweak mass eigenstates can differ from $CP$ eigenstates even in the case of two generations;
- the existence of a complex entry in the mixing matrix for the constituent fermions is no longer a sufficient condition for indirect $CP$ violation to occur at the mesonic level.

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1 Introduction; theoretical setting.

The interpretation of mesons as fermion-antifermion composites is widely accepted. However, a field theory in which the fields in the Lagrangian (quarks) are not the particles (asymptotic states) steps on the unsolved problem of confinement. To bypass this difficulty, I proposed in a gauge theory in which $J = 0$ mesons are both the fields and the particles, but in which they transform, by the relevant symmetry groups, like $\bar{q}_i q_j$ or $\bar{q}_i \gamma_5 q_j$ operators. It incorporates in particular the chiral properties of the quarks. Requiring that, in the quest for unification, the gauge group of symmetry be the same as for leptons, it is chosen to be the $SU(2)_L \times U(1)_g$ gauge group of the Glashow-Salam-Weinberg model and acts on the “constituent” fermions accordingly; so, the symmetry properties of the mesons also reflect the underlying electroweak symmetry of the standard model of quarks. The latter are however no longer considered as dynamical objects (they do not appear in the Lagrangian).

Let

$$\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ d \\ s \\ \vdots \end{pmatrix}$$

(1)

be a $N$-vector of fermions lying in the fundamental representation of $U(N)$. There are $N/2$ families of fermions ($N$ is restricted to be even).

Any meson of the type $\bar{q}_i q_j$ or $\bar{q}_i \gamma_5 q_j$ is represented by $\Psi \bar{M} \Psi$ or $\Psi \gamma_5 \bar{M} \Psi$, where $\bar{M}$ is an $N \times N$ matrix with a single nonvanishing entry, equal to 1, at the crossing of the $i$th line and $j$th column; such matrices define the “flavour” or “strong” eigenstates (the flavour group of symmetry is the diagonal subgroup of the chiral group, and supposed unbroken by strong interactions). Any $N \times N$ matrix $\bar{M}$ represents, up to the parity quantum number, a $J = 0$ meson which is a linear combination of the previous eigenstates; its behaviour, when acted upon by a symmetry group is determined by the laws of transformations of the fermions. The $\bar{M}$’s are the dynamical meson fields of the model.

The $\gamma_5$ matrix plays an essential role in the transformation of the composite pseudoscalar operators by the chiral group; it in particular introduces, in addition to commutators, anticommutators of $\bar{M}$ with the generators of $U(N)_L \times U(N)_R$, which are $N \times N$ matrices, too. The action of the chiral group on the mesons consequently involves the associative character of the $U(N)$ algebra.

Since we want to drop any reference to fermions, hence to $\gamma_5$, we swap the latter for a doubling in the space of $\bar{M}$ matrices and we distinguish $\bar{M} \bar{P}_{even}$ and $\bar{M} \bar{P}_{odd}$ mesons, corresponding to the combinations $\Psi \bar{M} \Psi + \Psi \gamma_5 \bar{M} \Psi$ and $\Psi \bar{M} \Psi - \Psi \gamma_5 \bar{M} \Psi$, which are respectively even or odd by the action of the parity changing operator $\bar{P}$, and in terms of which (see 1) the laws of transformation by the chiral group $U(N)_L \times U(N)_R$ are specially simple. The $2N^2$ independent $\bar{M}$ matrices so obtained match the total number of $J = 0$ scalar and pseudoscalar mesons built with $N$ flavours of fermions.

Care has also to be taken of the role of $\gamma_5$ as far as the transformation by charge conjugation is concerned (see below).

The electroweak gauge group naturally appears as a subgroup of the chiral group. The three $SU(2)_L$
generators $T_L^a$ are also now $N \times N$ matrices (see \[1\] and the introduction of section 3 below)

$$T_L^3 = \frac{1}{2} \left( \begin{array}{cc} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{array} \right), \quad T_L^+ = \left( \begin{array}{cc} 0 & K \\ 0 & 0 \end{array} \right), \quad T_L^- = \left( \begin{array}{cc} 0 & 0 \\ K^\dagger & 0 \end{array} \right),$$

(2)

which act trivially on the $N$-vector $\Psi_L = \left( (1 - \gamma_5)/2 \right) \Psi$. $\mathbb{I}$ and $K$ are respectively the $N/2 \times N/2$ identity matrix and the most general unitary mixing matrix, which can in particular be of the Cabibbo-Kobayashi-Maskawa (CKM) type \[3\]. Consequences of the relationship that results between the electroweak and chiral breaking have been emphasized in \[6\].

The $U(1)$ generator satisfies the Gell-Mann-Nishijima relation \[4\] (written in its “chiral” form)

$$(Y_L, Y_R) = (Q_L, Q_R) - (T_L^3, 0),$$

(3)

and that the electric charge be represented by the customary (diagonal) operator

$$Q_L = Q_R = Q = \left( \begin{array}{cc} 2/3 & 0 \\ 0 & -1/3 \end{array} \right),$$

(4)

yields back the usual expressions for the “left” and “right” hypercharges

$$Y_L = \frac{1}{6} \mathbb{I}, \quad Y_R = Q_R.$$  

(5)

$Q$ turns out to be the “third” generator of the custodial $SU(2)_V$ symmetry investigated in \[1, 6\].

The orientation of the electroweak subgroup inside the chiral group is controlled by a unitary matrix, $(R, R)$, acting diagonally:

$$R = \left( \begin{array}{cc} \mathbb{I} & 0 \\ 0 & K \end{array} \right);$$

(6)

indeed, the electroweak group defined by eq. (2) is the one with generators

$$R^\dagger t_L^a R;$$

(7)

with

$$t_L^3 = \frac{1}{2} \left( \begin{array}{cc} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{array} \right), \quad t_L^+ = \left( \begin{array}{cc} 0 & \mathbb{I} \\ 0 & 0 \end{array} \right), \quad t_L^- = \left( \begin{array}{cc} 0 & 0 \\ \mathbb{I} & 0 \end{array} \right).$$

(8)

In practice, this rotation only acts on the $t^\pm$ generators (we require $t^- = (t^+)^\dagger$, such that the unit matrices in eqs. (8) have the same dimension).

The $2N^2$ electroweak eigenstates can be classified into $N^2/2$ quadruplets, split into two sets, respectively “even” and “odd” by the parity changing operator $P$. All of them can be written in the form \[1\]

$$\Phi(D) = (M^0, M^3, M^+, M^-)(D)$$

$$= \left[ \frac{1}{\sqrt{2}} \left( \begin{array}{cc} D & 0 \\ 0 & K^\dagger DK \end{array} \right), \quad i \frac{1}{\sqrt{2}} \left( \begin{array}{cc} D & 0 \\ 0 & -K^\dagger DK \end{array} \right), \quad i \left( \begin{array}{cc} 0 & DK \\ 0 & 0 \end{array} \right), \quad i \left( \begin{array}{cc} 0 & 0 \\ K^\dagger D & 0 \end{array} \right) \right]$$

(9)
where $\mathbb{D}$ is a real $N/2 \times N/2$ matrix.

That the entries $M^+$ and $M^-$ are, up to a sign, hermitian conjugate requires that the $\mathbb{D}$’s are restricted to symmetric or antisymmetric matrices. Because of the presence of an “$i$” for the for $M_3^{\pm}$ and not for $M_0^0$, the quadruplets $(\mathbb{D})$ always mix entries which have different behaviour by hermitian conjugation: they are consequently not hermitian representations.

Each of them is the sum of two doublets of $SU(2)_L$, and also the sum of one singlet plus one triplet of the custodial diagonal $SU(2)_V$ $(\mathbb{1}, \mathbb{3})$. The $P$-even and $P$-odd quadruplets do not transform in the same way by $SU(2)_L$ (the Latin indices $i, j, k$ below run from 1 to 3); for $P$-even quadruplets, one has

$$T_{LL}^i \cdot M_{P\text{even}}^j = -\frac{i}{2} \left( \epsilon_{ijk} M_{P\text{even}}^k + \delta_{ij} M_0^0 \right),$$

$$T_{LL}^i \cdot M_0^0 = \frac{i}{2} M_{P\text{even}}^i;$$

while $P$-odd quadruplets transform according to

$$T_{LL}^i \cdot M_{P\text{odd}}^j = -\frac{i}{2} \left( \epsilon_{ijk} M_{P\text{odd}}^k - \delta_{ij} M_0^0 \right),$$

$$T_{LL}^i \cdot M_0^0 = -\frac{i}{2} M_{P\text{odd}}^i,$$

and only representations transforming alike, $P$-even or $P$-odd, can be linearly mixed. The (diagonal) charge operator acts indifferently on both types of representations by:

$$Q \cdot M^i = -i \epsilon_{ij3} M^j,$$

$$Q \cdot M_0^0 = 0.$$ 

(12)

The misalignment of “strong” and electroweak eigenstates, resulting from the one of the electroweak group inside the chiral group, is conspicuous from the presence of the mixing matrix $K$ in $(\mathbb{3})$.

Adding or subtracting eqs. $(\mathbb{1})$ and $(\mathbb{2})$, and defining scalar ($S$) and pseudoscalar ($P$) fields by

$$(M_{P\text{even}} + M_{P\text{odd}}) = S,$$

and

$$(M_{P\text{even}} - M_{P\text{odd}}) = P,$$

yields two new types of stable quadruplets which include objects of different but definite parities, corresponding to $CP$ eigenstates, depending whether $\mathbb{D}$ is a symmetric or antisymmetric matrix

$$\varphi = (S^0, \bar{P}),$$

and

$$\chi = (P^0, \bar{S});$$

(16)

they transform in the same way by the gauge group, according to eq. $(\mathbb{2})$, and thus can be linearly mixed. As they span the whole space of $J = 0$ mesons too, this last property makes them specially convenient to consider.

Taking the hermitian conjugate of any representation $\Phi$ swaps the relative sign between $M_0^0$ and $\bar{M}$; as a consequence, $\Phi^\dagger_{P\text{even}}$ transforms by $SU(2)_L$ as would formally do a $P$-odd representation, and vice-versa; on the other hand, the quadruplets $(\mathbb{3})$ are also representations of $SU(2)_R$, the action of which
is obtained by swapping eqs. (10) and (11); so, the hermitian conjugate of a given representation of SU(2)_L is a representation of SU(2)_R with the same law of transformation, and vice-versa. The same result holds for any (complex) linear representation of quadruplets transforming alike by the gauge group.

The conjugate of a pseudoscalar \( \bar{q}_i \gamma_5 q_j \) composite operator being \(-\bar{q}_j \gamma_5 q_i\) (the “-” sign comes from the anticommutation of \(\gamma_5\) with \(\gamma^0\)), one must distinguish between the matrix conjugate \(M^\dagger\) and the charge conjugate \(M\) of any \(M\) meson: the latter, corresponding to charge conjugation, gets an extra “-” sign with respect to the former when pseudoscalars are concerned; one has the following relation between the charge and hermitian conjugates of parity eigenstates:

\[
\mathcal{S} = \mathcal{S}^\dagger, \quad \mathcal{P} = -\mathcal{P}^\dagger, \tag{17}
\]

yielding in particular:
- for symmetric \(D\)’s:

\[
(S^0, \bar{P})_{sym} = (S^0, \bar{P})_{sym}, \quad (P^0, \bar{S})_{sym} = -(P^0, \bar{S})_{sym}. \tag{18}
\]

- for antisymmetric \(D\)’s:

\[
(S^0, \bar{P})_{antisym} = -(S^0, \bar{P})_{antisym}, \quad (P^0, \bar{S})_{antisym} = (P^0, \bar{S})_{antisym}. \tag{19}
\]

Only for \((S^0, \bar{P})\) and \((P^0, \bar{S})\) quadruplets is there any link between matrix and charge conjugation. For \(\Phi_{Peven}\) and \(\Phi_{Podd}\) quadruplets, which mix scalars and pseudoscalars, one has instead the relations:

\[
\Phi_{Peven} = (\Phi_{Podd})^\dagger, \quad \Phi_{Podd} = (\Phi_{Peven})^\dagger. \tag{20}
\]

The link between the dynamical (matricial) fields and the usually defined \(J = 0\) “strong” mesonic eigenstates is the following: consider for example the case \(N = 4\), for which the matrix \(K\) shrinks back to the Cabibbo matrix; the pseudoscalar \(\pi^+\) meson, which is a flavour or “strong eigenstate, is represented in our notation, up to a normalization factor, by the matrix-valued field

\[
\Pi^+ = i \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \tag{21}
\]

sandwiched between two 4-vectors \(\Psi\) of quarks \([\underline{\underline{4}}]\), it gives, indeed (restoring the \(\gamma_5\)), the wave function

\[
\Psi \gamma_5 \Pi^+ \Psi = i \bar{u} \gamma_5 d, \tag{22}
\]

of the \((+1)\) charged pion \([\underline{4}]\).

The normalization of the state, determined from its leptonic decays (see \([\underline{[4]}, \underline{4}, \underline{[4]}, \underline{4}]\)), is \(a = 2f/\langle H\rangle\): the relation between the \(\Pi^+\) matrix and the field (of dimension \(|mass|\)) of the observed \(\pi^+\) particle is \(\Pi^+ = a\pi^+\). \(f\) is the leptonic decay constant of the mesons (considered to be the same for all of them) and \(H = S^0(D_1)\) is the Higgs boson (see the remark at the end of Appendix A).
One likewise identifies the other pseudoscalar mesons, $K^+, D^+, D_s^+$, such that $\mathbb{P}^+(D_1)$ (see section 2 and Appendix A)

$$
\mathbb{P}^+(D_1) = i \begin{pmatrix}
  c_\theta & s_\theta \\
  -s_\theta & c_\theta
\end{pmatrix},
$$

(23)

one of the three Goldstone bosons of the broken electroweak symmetry, is the linear combination of pseudoscalar mesons

$$
\mathbb{P}^+(D_1) = \frac{2f}{\langle H \rangle} \left( c_\theta (\pi^+ + D_s^+) + s_\theta (K^+ - D^+) \right).
$$

(24)

2 The quadratic invariants.

In order to construct a $SU(2)_L \times U(1)$ Lagrangian, one needs invariant quadratic polynomials in the $J = 0$ mesonic fields.

To every quadruplet $(M^0, \vec{M})$ is associated such an invariant:

$$
\mathcal{I} = (M^0, \vec{M}) \otimes (M^0, \vec{M}) = M^0 \otimes M^0 + \vec{M} \otimes \vec{M};
$$

(25)

the “$\otimes$” product is a tensor product, not the usual multiplication of matrices and means the product of fields as functions of space-time; \( \vec{M} \otimes \vec{M} \) stands for \( \sum_{i=1,2,3} M_i^0 \otimes M_i^0 \).

For the relevant cases \( N = 2, 4, 6 \), there exists a set of \( D \) matrices (see appendix A) such that the algebraic sum of invariants specified below, extended over all representations defined by (15,16,9)

$$
\begin{align*}
\frac{1}{2} \left( \sum_{\text{symmetric } D's} - \sum_{\text{antisym } D's} \right) \left( (S^0, \vec{P})(D) \otimes (S^0, \vec{P})(D) - (P^0, \vec{S})(D) \otimes (P^0, \vec{S})(D) \right) \\
= \frac{1}{2} \sum_{\text{all } D's} \left( (S^0, \vec{P})(D) \otimes (S^0, \vec{P})(D) + (P^0, \vec{S})(D) \otimes (P^0, \vec{S})(D) \right) \\
= \frac{1}{4} \sum_{\text{all } D's} \left( \Phi^+_\text{P_{odd}}(D) \otimes \Phi^+_\text{P_{even}}(D) + \Phi^-_\text{P_{odd}}(D) \otimes \Phi^-_\text{P_{even}}(D) \right) \\
= \frac{1}{4} \sum_{\text{all } D's} \left( \Phi^+_\text{P_{even}}(D) \otimes \Phi^+_\text{P_{even}}(D) + \Phi^-_\text{P_{odd}}(D) \otimes \Phi^-_\text{P_{odd}}(D) \right)
\end{align*}
$$

(26)

is diagonal both in the electroweak basis and in the basis of strong eigenstates: in the last one, all terms are normalized alike to (+1). Two “−” signs occur in the first line of eq. (26):  
- the first between the $(P^0, \vec{S})$ and $(S^0, \vec{P})$ quadruplets, because, as seen on eq. (9), the $P^0$ entry of the former has no “$i$” factor, while the $\vec{P}$’s of the latter do have one; as we define all pseudoscalars with an “$i$” (see eqs. (21,23,24)), a $(\pm i)$ relative factor has to be introduced between the two types of representations, yielding a “−” sign in eq. (26); 
- the second for the representations corresponding to antisymmetric \( D \) matrices, which have an opposite behaviour by matrix conjugation as compared to the ones with symmetric \( D \)’s.

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\( \text{Eq. (26) specifies eq. (25) of [1], in which the “−” signs were not explicitly written.} \)
The kinetic part of the $SU(2)_L \times U(1)$ Lagrangian for $J = 0$ mesons is built from the combination (26) of invariants above, now used for the covariant derivatives of the fields with respect to the gauge group; it is thus also diagonal in both the strong and electroweak basis.

Other invariants can be built which are tensor products of two representations transforming alike by the gauge group: two $P$-odd or two $P$-even, two $(S^0,\vec{P})'$ s, two $(P^0,\vec{S})'$ s, or one $(S^0,\vec{P})$ and one $(P^0,\vec{S})$; such is for example

$$I_{12} = (S^0,\vec{P})(D_1) \otimes (P^0,\vec{S})(D_2) = S^0(D_1) \otimes P^0(D_2) + \vec{P}(D_1) \otimes \vec{S}(D_2).$$

(27)

According to the remark made in the previous section, all the above expressions are also invariant by the action of $SU(2)_R$.

The quadratic $SU(2)_L$ invariants (23) are not a priori self conjugate expressions and have consequently no definite property by hermitian conjugation.

However, unitarity compels the electroweak Lagrangian of $J = 0$ mesons to be hermitian.

Any invariant quadratic expression constructed from the quadruplets (15,16) is hermitian (a "i" factor has eventually to be introduced), like in particular the expression (26) that yields the kinetic terms when used for the covariant derivatives of the fields; each entry of theirs has indeed a well defined behaviour by hermitian conjugation.

In general, the invariant $\Omega \otimes \Omega$ attached to a given quadruplet $\Omega$ is hermitian if and only if the condition $\bar{\Omega} = \pm \Omega$ is satisfied; from eqs. (18,19), it can only be a representation of the type (15) or (16).

For a general quadruplet (8), for example $\Phi_{P_{\text{even}}}$, the invariant $\Phi_{P_{\text{even}}} \otimes \Phi_{P_{\text{even}}}$ is not hermitian, its charge conjugate being, by eq. (20)

$$\bar{\Phi}_{P_{\text{even}}} \otimes \Phi_{P_{\text{even}}} = \Phi_{P_{\text{odd}}} \otimes \Phi_{P_{\text{odd}}} = \pm \Phi_{P_{\text{even}}} \otimes \Phi_{P_{\text{even}}},$$

(28)

Instead, for the representations (3), the quadratic expressions

$$\bar{\Phi}_{P_{\text{even}}} \otimes \Phi_{P_{\text{even}}} = \Phi_{P_{\text{odd}}} \otimes \Phi_{P_{\text{even}},}$$

$$\Phi_{P_{\text{odd}}} \otimes \Phi_{P_{\text{odd}}} = \Phi_{P_{\text{even}}} \otimes \Phi_{P_{\text{odd}}},$$

(29)

are hermitian and each one connects two quadruplets transforming alike by the gauge group ($\Phi_{P_{\text{even}}}$ and $\Phi_{P_{\text{odd}}}$, $\Phi_{P_{\text{odd}}}$ and $\Phi_{P_{\text{even}}}$) (see section 1).

If the $\Phi$'s in (23) correspond to matrices $D$ with definite symmetry properties, the above hermitian invariants do not depend whether $\Phi$ is $P_{\text{odd}}$ or $P_{\text{even}}$, but only whether $D$ is symmetric or skew-symmetric (this determines the signs in the second lines of the two equalities in eq. (30) below, though the third lines are formally identical). One has

$$T_{\Phi}^{\text{sym}} = \Phi_{P_{\text{even}}} (D_{\text{sym}}) \otimes \Phi_{P_{\text{even}}} (D_{\text{sym}}) = \Phi_{P_{\text{odd}}} (D_{\text{sym}}) \otimes \Phi_{P_{\text{odd}}} (D_{\text{sym}})$$

$$= (S^0,\vec{P})(D_{\text{sym}})^\otimes 2 - (P^0,\vec{S})(D_{\text{sym}})^\otimes 2$$

$$= (S^0,\vec{P})(D_{\text{sym}}) \otimes (S^0,\vec{P})(D_{\text{sym}}) + (P^0,\vec{S})(D_{\text{sym}}) \otimes (P^0,\vec{S})(D_{\text{sym}}),$$

$$T_{\Phi}^{\text{antisym}} = \Phi_{P_{\text{even}}} (D_{\text{antisym}}) \otimes \Phi_{P_{\text{even}}} (D_{\text{antisym}}) = \Phi_{P_{\text{odd}}} (D_{\text{antisym}}) \otimes \Phi_{P_{\text{odd}}} (D_{\text{antisym}})$$

$$= - (S^0,\vec{P})(D_{\text{antisym}})^\otimes 2 + (P^0,\vec{S})(D_{\text{antisym}})^\otimes 2$$

$$= (S^0,\vec{P})(D_{\text{antisym}}) \otimes (S^0,\vec{P})(D_{\text{antisym}}) + (P^0,\vec{S})(D_{\text{antisym}}) \otimes (P^0,\vec{S})(D_{\text{antisym}});$$

(30)
they correspond, for a given \( \mathbb{D} \) matrix, to degenerate \((S^0, \tilde{P})(\mathbb{D})\) and \((P^0, \tilde{S})(\mathbb{D})\) quadruplets.

The expressions above can be used to build the mass terms of the Lagrangian.

There are \textit{a priori} as many \((N^2/2)\) independent mass scales as there are independent representations (quadruplets). Preserving the electroweak and custodial symmetries, they share with the leptonic case the same arbitrariness; the hierarchy of mesonic masses has here the same status as, e.g., the one between the muon and the electron.

The breaking of \([SU(2)_L \times U(1)]\) to \([U(1)_{em}]\) is compatible with preserving the custodial \([SU(2)_V]\) symmetry \([1]\); the number of allowed mass doubles up to \(N^2\) and a splitting can occur inside each quadruplet between the triplet and the singlet of \(SU(2)_V\) (this allows in particular a scalar-pseudoscalar splitting inside each quadruplet \((\ref{15})\) or \((\ref{16})\)). This is to be compared with the standard model in which the custodial symmetry is broken by any mass splitting inside a doublet of \(SU(2)_L\) \([11]\). As the electroweak group is a subgroup of the chiral group, chiral and electroweak symmetry breaking are connected. In particular, introducing a vacuum expectation value for the Higgs boson is equivalent (see Appendix A) to allowing for diagonal \(\langle q_i q_i \rangle\) condensates.

The traditional picture \([12]\) associates the \(N^2\) pseudoscalar mesons (flavour or “strong” eigenstates) with the \(N^2\) (pseudo)-Goldstones generated by the breaking of the chiral group \([U(N)_L \times U(N)_R]\) into its diagonal flavour subgroups. They become (perturbatively) massive by electroweak radiative corrections \([11]\) and the (non-perturbative) bulk of their masses is parameterized by introducing (soft) mass terms for the quarks in the QCD Lagrangian \([5]\). This point of view is now modified since:

- as observed experimentally, none of the \(2N^2 J = 0\) “strong” mesons corresponds to a Goldstone particle: in addition to electroweak radiative corrections, they can be given arbitrary masses, the number of which depends on the symmetries that are preserved (chiral, or electroweak, or just the custodial symmetry);
- scalar and pseudoscalar electroweak mass eigenstates are on the same footing, but can be splitted while preserving the custodial symmetry;
- there exist only three genuine Goldstones bosons \(\tilde{P}(\mathbb{D}_I)\) which arise when the electroweak symmetry is spontaneously broken down to \(U(1)_{em}\), or, equivalently, chiral \(SU(2)_L \times SU(2)_R\) broken into \(SU(2)_V\); they are linear combinations of flavour eigenstates (pseudoscalar mesons) (see eq. \((\ref{24})\)).

The mass splittings, which can be arbitrarily large, have a purely electroweak origin since, from eq. \((\ref{26})\), equal electroweak mass terms also correspond to equal mass terms for strong eigenstates \([1]\); however, their hierarchy obviously lies outside the realm of perturbation theory.

We define electroweak mass eigenstates as states which diagonalize both the kinetic terms and the mass terms of the electroweak Lagrangian.

### 3 Indirect CP violation.

If the stakes are high for the observation of “direct” \(CP\) violation \([13, 14]\), and, in particular, for discovering whether the so-called \(\epsilon'\) parameter is vanishing or not \([15]\), all phenomena of \(CP\) violation \([16, 17]\) are, up to now, compatible with the so-called “indirect” violation \([13, 14]\), which means that the electroweak mass eigenstates are not \(CP\) eigenstates.

The only known mechanism to trigger it in the electroweak standard model for fermions is through a complex mixing matrix for quarks \([3, \text{\footnote{and other invariant tensor products mixing quadruplets with different symmetry properties like } \Phi_{P_{even}(\mathbb{D}_{asym})} \otimes \Phi_{P_{even}(\mathbb{D}_{sym})} + h.c., \Phi_{P_{even}(\mathbb{D}_{asym})} \otimes \Phi_{P_{even}(\mathbb{D}_{sym})} + h.c., \ldots}}\) for a number of generations at least equal to three, in the process of diagonalization of the most general mass matrix for quarks, one phase cannot be reabsorbed in their wave functions. Hence, the combined experimental evidence for indirect \(CP\) violation and
the preeminence of the Glashow-Salam-Weinberg model as “the” theoretical tool to interpret the data led, before its discovery \[19, 20\], to a strong prejudice in favour of the existence of a third generation of fermions.

Despite our conservative attitude, \(CP\) violation nevertheless appears now from a different perspective, which shows in particular how sensitive can be the interpretation of experimental data to the theoretical filter used for their analysis. The mechanism which was sufficient to trigger it at the quark level, and which used to be extrapolated at the level of the asymptotic states despite our ignorance of the dynamics of confinement, now fails when the mesons themselves are chosen to be the dynamical fields of the theory, though the chiral and electroweak symmetry properties of their building blocks have been preserved. As mass terms are now introduced for the mesons themselves and no longer for quarks, that all phases in the mixing matrix can be or not reabsorbed has lost its previous relevance.

I show below that, in the \(SU(2)_L \times U(1)\) electroweak gauge theory for mesons (and leptons – but in this sector it is the genuine Glashow-Salam-Weinberg model –) proposed above, the role of the mixing matrix fades away as far as indirect \(CP\) violation is concerned. It only now determines the orientation of the electroweak group inside the chiral group; that it is the same matrix as the one controlling, in the standard electroweak model, the misalignment between leptons and quarks, allows the identification (2) of \(SU(2)_L \times U(1)\) as a precise subgroup of the chiral group. The identification that we make here of the building blocks of the \(\mathcal{M}\) mesons with the quarks of the standard model could eventually be loosened into a less conservative approach and more exotic enlargements of the latter.

Another mixing matrix now triggers indirect \(CP\) violation, the one which diagonalizes the kinetic terms with a new set of states different from the “strong” or “flavour” eigenstates (15, 16) occurring in eq. (26); that it includes at least one complex entry is the necessary condition for the existence of electroweak mass eigenstates for mesons which are not \(CP\) eigenstates.

### 3.1 Electroweak mass eigenstates can be \(CP\) eigenstates in the presence of a complex CKM matrix.

The demonstration of this first result is straightforward.

In the following Lagrangian for \(J = 0\) mesons, which is hermitian and \(SU(2)_L \times U(1)\) invariant, and where the sum is extended to the representations defined by eqs. (13,16) and all \(\mathcal{D}\) matrices defined in Appendix A: (\(D_{\mu}\) is the covariant derivative with respect to \(SU(2)_L \times U(1)\))

\[
\mathcal{L} = \frac{1}{2} \sum_{\text{symmetric } \mathcal{D}'\text{s}} \left( D_{\mu}(S^0, \tilde{P})(\mathcal{D}) \otimes D^\mu(S^0, \tilde{P})(\mathcal{D}) - m^2_D(S^0, \tilde{P})(\mathcal{D}) \otimes (S^0, \tilde{P})(\mathcal{D}) \right) \\
- \left( D_{\mu}(P^0, \tilde{S})(\mathcal{D}) \otimes D^\mu(P^0, \tilde{S})(\mathcal{D}) - \tilde{m}^2_D(P^0, \tilde{S})(\mathcal{D}) \otimes (P^0, \tilde{S})(\mathcal{D}) \right) \\
- \left( D_{\mu}(\bar{S}^0, \tilde{P})(\mathcal{D}) \otimes D^\mu(\bar{S}^0, \tilde{P})(\mathcal{D}) - \bar{m}^2_D(\bar{S}^0, \tilde{P})(\mathcal{D}) \otimes (\bar{S}^0, \tilde{P})(\mathcal{D}) \right) \\
- \left( D_{\mu}(\bar{P}^0, \tilde{S})(\mathcal{D}) \otimes D^\mu(\bar{P}^0, \tilde{S})(\mathcal{D}) - \bar{m}^2_D(\bar{P}^0, \tilde{S})(\mathcal{D}) \otimes (\bar{P}^0, \tilde{S})(\mathcal{D}) \right),
\]

\[
= \frac{1}{2} \sum_{\text{all } \mathcal{D}'\text{s}} \left( D_{\mu}(S^0, \tilde{P})(\mathcal{D}) \otimes D^\mu(S^0, \tilde{P})(\mathcal{D}) - m^2_D(S^0, \tilde{P})(\mathcal{D}) \otimes (S^0, \tilde{P})(\mathcal{D}) \right) \\
+ \left( D_{\mu}(P^0, \tilde{S})(\mathcal{D}) \otimes D^\mu(P^0, \tilde{S})(\mathcal{D}) - \tilde{m}^2_D(P^0, \tilde{S})(\mathcal{D}) \otimes (P^0, \tilde{S})(\mathcal{D}) \right) \\
+ \left( D_{\mu}(\bar{S}^0, \tilde{P})(\mathcal{D}) \otimes D^\mu(\bar{S}^0, \tilde{P})(\mathcal{D}) - \bar{m}^2_D(\bar{S}^0, \tilde{P})(\mathcal{D}) \otimes (\bar{S}^0, \tilde{P})(\mathcal{D}) \right) \\
+ \left( D_{\mu}(\bar{P}^0, \tilde{S})(\mathcal{D}) \otimes D^\mu(\bar{P}^0, \tilde{S})(\mathcal{D}) - \bar{m}^2_D(\bar{P}^0, \tilde{S})(\mathcal{D}) \otimes (\bar{P}^0, \tilde{S})(\mathcal{D}) \right),
\]

(31)

the mass eigenstates, being the \(S^0, \tilde{P}, P^0, \tilde{S}\) mesons, are also \(CP\) eigenstates. The complex matrix \(\mathcal{K}\) is entirely absorbed in their definition, and no complex coupling constant appears in the Lagrangian. It is of course straightforward to build hermitian \(SU(2)_L \times U(1)\) invariant quartic terms.
Conclusion: The existence of a complex phase in the mixing matrix for quarks is not a sufficient condition for electroweak mass eigenstates of $J = 0$ mesons transforming like $\bar{q}_i q_j$ or $\bar{q}_i \gamma_5 q_j$ to differ from CP eigenstates.

3.2 Indirect CP violation can occur at the mesonic level with only two generations of fermions.

Whatever be the number of generations, to every $\Phi_{P_{even}}$ or $\Phi_{P_{odd}}$ quadruplet can be attached (see section 3) an $SU(2)_L \times U(1)$ invariant and hermitian mass term $\bar{\Phi} \Phi$; hence it is trivial to have electroweak mass eigenstates which are not CP eigenstates because they are not $P$ eigenstates. For example, any set of linear combinations of the $\Phi_{P_{even}}$ (see Appendix A) that also diagonalize the kinetic terms can be mass eigenstates corresponding to $(S + P)$ combinations.

However, it seems not to be questioned at present that observed electroweak mass eigenstates have a well defined parity, and that, consequently, indirect CP violation can only spring from $C$ violation.

I show how, in the proposed model, electroweak eigenstates of $J = 0$ mesons can be $P$ but not CP eigenstates with only two generations of fermions.

The simple mechanism lies in that the kinetic terms of the Lagrangian can also be diagonalized with eigenvectors which are complex linear combinations $\xi$ of the quadruplets of parity eigenstates $(S^0, \vec{P})$ (the same can be done with $(P^0, \vec{S})$ quadruplets), for which hermitian and $SU(2)_L \times U(1)$ invariant mass terms $\bar{\xi} \xi$ can straightforwardly be written.

Let us work in the basis made with the quadruplets $\varphi$ and $\chi$ given by eqs. (15,16), which we split into $\varphi_{sym}$, $\varphi_{antisym}$, $\chi_{sym}$, $\chi_{antisym}$ according to the symmetry property of the matrix $\mathbb{D}$. We cast them into a vector $\Upsilon$ with dimension $2N^2$, written in an abbreviated notation

$$\Upsilon = \begin{pmatrix} \varphi_{sym}(\mathbb{D}) \\ \varphi_{antisym}(\mathbb{D}) \\ \chi_{sym}(\mathbb{D}) \\ \chi_{antisym}(\mathbb{D}) \end{pmatrix}.$$ \hspace{1cm} (32)

For example, for two generations ($N = 4$), $\varphi_{sym}(\mathbb{D})$ stands above for the three independent $(S^0, \vec{P})(\mathbb{D})$ quadruplets corresponding to the three $2 \times 2$ symmetric $\mathbb{D}_1, \mathbb{D}_2, \mathbb{D}_3$ matrices (see Appendix A), $\varphi_{antisym}(\mathbb{D})$ for the unique $(S^0, \vec{P})(\mathbb{D}_4)$ quadruplet corresponding to the unique antisymmetric $\mathbb{D}_4$ matrix etc.

The kinetic terms of the Lagrangian write

$$\mathcal{L}_{kin} = \frac{1}{2} \partial_{\mu} \Upsilon^\dagger [\text{I}] \partial^{\mu} \Upsilon + \cdots$$ \hspace{1cm} (33)

where $[\text{I}]$ is the $2N^2 \times 2N^2$ unit matrix.

Let us make a change of basis described by the $2N^2 \times 2N^2$ matrix $U$

$$\Upsilon = U \Xi,$$

such that $U^\dagger U$ is diagonal.

$\Xi$ is a vector of dimension $2N^2$

$$\Xi = \begin{pmatrix} [\xi] \\ [\omega] \end{pmatrix},$$ \hspace{1cm} (35)
in which the first \( N^2 \) entries, generically called \([\xi]\) are linear combinations of the \( \varphi(D) \)’s, and the last \( N^2 \), \([\omega]\) are linear combinations of the \( \chi(D) \)’s.

We furthermore request that \( \mathbb{U} \) does not mix states with different parities: it writes

\[
\mathbb{U} = \begin{pmatrix}
A & 0 \\
0 & B
\end{pmatrix}
\]  

(36)

where both \( A \) and \( B \) are \( N^2 \times N^2 \) matrices.

For the sake of simplicity, and without losing any generality, let us suppose that \( B \) is the unit matrix, such that the new eigenvectors diagonalizing the kinetic terms only differ from the original ones in the subspace of \((S^0, \vec{P})\) quadruplets.

The hermitian mass term that can be constructed for the \( \xi \)’s,

\[
\mathcal{L}_m^\xi \propto \sum_{i=1}^{N^2} \mu_i^2 \bar{\xi}_i \otimes \xi_i
\]

(37)

is \( SU(2)_L \times U(1) \) invariant since it only involves invariant tensor products of pairs or \((S^0, \vec{P})\) quadruplets transforming alike by the gauge group (see section 1).

• If \( A \) is purely real, the mass eigenstates split into \( CP \) eigenstates: consider indeed a real linear combination \( \xi \) among those which diagonalize the kinetic terms

\[
\xi = \sum_{i=1}^{N_{sym}} a_i \varphi^i_{sym} + \sum_{i=1}^{N_{antisym}} b_i \varphi^i_{antisym}
\]

(38)

with \( a_i, b_i \) real, and \( N_{sym} + N_{antisym} = N^2 \). The corresponding hermitian mass term for \( \xi \) is proportional to

\[
\bar{\xi} \xi = \left( \sum_{i=1}^{N_{sym}} a_i \varphi^i_{sym} - \sum_{i=1}^{N_{antisym}} b_i \varphi^i_{antisym} \right) \otimes \left( \sum_{i=1}^{N_{sym}} a_i \varphi^i_{sym} + \sum_{i=1}^{N_{antisym}} b_i \varphi^i_{antisym} \right)
\]

\[
\left( \sum_{i=1}^{N_{sym}} a_i \varphi^i_{sym} \right) \otimes \left( \sum_{i=1}^{N_{antisym}} b_i \varphi^i_{antisym} \right)
\]

\[
\left( \sum_{i=1}^{N_{sym}} b_i \varphi^i_{antisym} \right) \otimes \left( \sum_{i=1}^{N_{antisym}} a_i \varphi^i_{sym} \right)
\]

(39)

(where we have used eqs. (18,19) for the last equality) such that the electroweak mass eigenstate \( \xi \) splits into two degenerate \( CP \) eigenstates \( \sum_{i=1}^{N_{sym}} a_i \varphi^i_{sym} \) and \( \sum_{i=1}^{N_{antisym}} b_i \varphi^i_{antisym} \).

• If there is to be any indirect \( CP \) violation, it can thus only occur through a complex mixing matrix between mesons. That it can indeed happen is easily demonstrated on a simple example.

Consider in eq. (36), still for \( N = 4 \), a matrix \( A \)

\[
A = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & a & b & 0 \\
0 & c & d & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

(40)

It couples here the two symmetric quadruplets \( \varphi_2 = (S^0, \vec{P})(D_2) \) and \( \varphi_3 = (S^0, \vec{P})(D_3) \) but the demonstration can be made with any matrix \( A \).
That the kinetic term can be diagonalized with $\xi_2$ and $\xi_3$ as well as with $\varphi_2$ and $\varphi_3$ requires $V^\dagger V$ to be diagonal, $V$ being the $2 \times 2$ complex matrix

$$ V = \begin{pmatrix} a & b \\ c & d \end{pmatrix}; \quad (41) $$

this gives the conditions

$$ \bar{a}b + \bar{c}d = 0 = ab + cd. \quad (42) $$

Writing

$$ \Delta = ad - bc = -\frac{b}{c} (|a|^2 + |c|^2), \quad (43) $$

where we have used (42) for the last equality, the new eigenstates are

$$ \xi_2 = \frac{1}{\Delta} (d\varphi_2 - b\varphi_3) = -\frac{1}{\Delta} \frac{b}{c} (\bar{a}\varphi_2 + \bar{c}\varphi_3) $$
$$ \xi_3 = \frac{1}{\Delta} (-c\varphi_2 + a\varphi_3), \quad (44) $$

for which one can introduce the hermitian mass terms with real coefficients

$$ \mu_2^2 \xi_2 \xi_2 + \mu_3^2 \xi_3 \xi_3 = $$

$$ \frac{\mu_2^2}{|\Delta|^2} (|d|^2 \varphi_2^2 + |b|^2 \varphi_3^2 - (\bar{b}d + \bar{d}b)\varphi_2 \otimes \varphi_3) + \frac{\mu_3^2}{|\Delta|^2} (|c|^2 \varphi_3^2 + |a|^2 \varphi_2^2 - (\bar{a}c + \bar{c}a)\varphi_2 \otimes \varphi_3); \quad (45) $$

they are again $SU(2)_L \times U(1)$ invariant because they involve tensor products of quadruplets $\varphi_2$ and $\varphi_3$ behaving alike by the gauge group.

Take for example $a$ and $b$ real, $c$ complex; the condition (43) entails that $d = -(\bar{a}/\bar{c})b = -abc/|c|^2$ is also complex.

The hermitian and $SU(2)_L \times U(1)$ invariant Lagrangian

$$ L = \frac{1}{2} (\partial_\mu \varphi_2 \otimes \partial^\mu \varphi_2 + \partial_\mu \varphi_3 \otimes \partial^\mu \varphi_3) + \cdots $$

$$ -\frac{1}{2|\Delta|^2} (\mu_2^2 |d|^2 + \mu_3^2 |c|^2) \varphi_2^2 - (\mu_2^2 |b|^2 + \mu_3^2 |a|^2) \varphi_3^2 - (\mu_2^2 (\bar{b}d + \bar{d}b) + \mu_3^2 (\bar{b}d + \bar{d}b)) \varphi_2 \otimes \varphi_3) + \cdots $$

$$ \equiv \frac{1}{2} (|a|^2 + |c|^2) \left( \partial_\mu \xi_2 \otimes \partial^\mu \xi_2 + \frac{|b|^2}{|c|^2} \partial_\mu \xi_3 \otimes \partial^\mu \xi_3 \right) + \cdots $$

$$ -\frac{1}{2} (\mu_2^2 \xi_2 \otimes \xi_2 + \mu_3^2 \xi_3 \otimes \xi_3) + \cdots \quad (46) $$

admits $\xi_2$ and $\xi_3$ as electroweak mass eigenstates; they are $P$ eigenstates, but not $C$ eigenstates because they are not $C$ eigenstates.

Conclusion: indirect $CP$ violation can occur with two generations only for $J = 0$ mesons transforming like $\bar{q}_i q_j$ or $\bar{q}_i \gamma_5 q_j$ composite operators.

Remark: one needs more than one generation to be able to combine several quadruplets with the same definite parity quantum numbers.
4 Conclusion: outlook and perspectives.

Renormalizable extensions of the electroweak standard model usually enlarge its gauge group of symmetry [21], eventually incorporate a “flavour” or “horizontal” symmetry [22], or supersymmetry [23], often increase the number of particles, but seldom question the parallel between quarks and leptons. Among other reasons, this attitude finds its justifiication in the subtle mechanism of cancelation of anomalies [24] between the two types of fields [25]. The price to pay lies in ad-hoc procedures to circumvent the problem of confinement [26].

Other extensions, based on effective Lagrangian [27], partially break the parallel mentioned above, incorporate constraints imposed by chiral dynamics [12] and some of the features of Quantum Chromodynamics [5], but abandon renormalizability.

Some, still non-renormalizable, exploit the analogy between the high mass limit of the Higgs boson [28] and $\sigma$-models [29] to give predictions for a strongly interacting scalar sector [30].

Models with dynamical symmetry breaking are often plagued, too, with non-renormalizability [31], or with the difficult issue of flavour changing neutral currents [32].

We followed here a different approach: preserving renormalizability and limiting the spectrum to the one of observed particles, we chose to break the parallel between quarks and leptons and to promote the symmetry which exists between true asymptotic states, mesons (bosons) and leptons (fermions).

The misalignment between the electroweak and the chiral groups of symmetry reflects into the one between electroweak and flavour (or “strong”) eigenstates. The first type of eigenstates being, unlike the second, $a$ priori linear combinations of states of different parities and behaving differently by charge conjugation, indirect CP violation is naturally expected to occur. We have shown that this is what happens, independently of the number of generations (if greater than one), through a complex mixing matrix which now occurs at the mesonic level.

The quadruplet $(S^0, \vec{P})(D^1)$, isomorphic to the complex scalar doublet of the Glashow-Salam-Weinberg model, includes the Higgs boson and the three Goldstones of the broken electroweak symmetry. The Goldstone triplet being directly related to observed pseudoscalar mesons (and not to “technimesons”), those turn out to be connected to the Higgs boson by the electroweak group of symmetry. The latter is consequently expected to play a role in electroweak decays of pseudoscalar mesons. This is the subject of a forthcoming work [33].

The problem of the cancelation of anomalies now requires that the leptonic sector be by itself anomaly-free, which happens, for example, if the observed $V - A$ couplings are effective vertices of a more fundamental purely vectorial theory (this has been investigated in [34]).

The question of higher spins [35] for the mesons needs investigation, and also the sector of baryons. About the last point, it is tempting to advocate for the existence of a dual sector [36] since charge quantization is effective as soon as the custodial symmetry is preserved [1, 6] and dyon-like solutions have been exhibited in a similar model [37]. Then the baryons could be thought of as extended objects (solitons) [38] which would be the strongly interacting fields of the dual sector. This is currently under investigation.
Appendix

A Diagonalizing eq. (26) in the basis of strong eigenstates: a choice of \( D \) matrices.

The property is most simply verified for the “non-rotated” \( SU(2)_L \times U(1) \) group and representations corresponding to eq. (8) and setting \( K = I \) in (9).

A.1 \( N = 2 \) (1 generation).

Trivial case: \( D \) is a number.

A.2 \( N = 4 \) (2 generations).

The four \( 2 \times 2 \) \( D \) matrices (3 symmetric and 1 antisymmetric) can be taken as

\[
D_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad D_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad D_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad D_4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (47)
\]

A.3 \( N = 6 \) (3 generations).

The nine \( 3 \times 3 \) \( D \) matrices (6 symmetric and 3 antisymmetric), can be taken as

\[
D_1 = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]

\[
D_2 = \frac{2}{\sqrt{3}} \begin{pmatrix} \sin \alpha & 0 & 0 \\ 0 & \sin(\alpha \pm \frac{2\pi}{3}) & 0 \\ 0 & 0 & \sin(\alpha \mp \frac{2\pi}{3}) \end{pmatrix},
D_3 = \frac{2}{\sqrt{3}} \begin{pmatrix} \cos \alpha & 0 & 0 \\ 0 & \cos(\alpha \pm \frac{2\pi}{3}) & 0 \\ 0 & 0 & \cos(\alpha \mp \frac{2\pi}{3}) \end{pmatrix},
\]

\[
D_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad D_5 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix},
\]

\[
D_6 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad D_7 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad D_8 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad D_9 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}. \quad (48)
\]

where \( \alpha \) is an arbitrary phase.
Remark: as $\mathbb{D}_1$ is the only matrix with a non vanishing trace, $\mathbb{S}^0(\mathbb{D}_1)$ is the only neutral scalar matrix with the same property; we take it as the Higgs boson.

Considering that it is the only scalar with a non-vanishing vacuum expectation value prevents the occurrence of a hierarchy problem \[39\].

This last property is tantamount, in the “quark language”, to taking the same value for all condensates $\langle \bar{q}_i q_i \rangle, i = 1 \cdots N$, in agreement with the flavour independence of “strong interactions” between fermions, supposedly at the origin of this phenomenon in the traditional framework.

As the spectrum of mesons is, in the present model, disconnected from a hierarchy between quark condensates (see section 2), it is not affected by our choice of a single Higgs boson.
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