Design and Synchronization of Chaotic System using Threshold Controller Coupling

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Abstract—A new autonomous chaotic system is designed from the Lorenz family of systems and studied its dynamics both theoretically and numerically. Then two such systems are synchronized using a threshold controller as the coupling element by which complete and asynchronous is achieved. The new system is investigated using MATLAB Simulink and Multisim. The results show that based on the control parameter value the system switches from complete to asynchronous. Keywords—chaotic system, synchronization, threshold controller

1. INTRODUCTION

The study of chaos has grown rapidly and it has a tremendous effect in the field of science and engineering. Chaotic behaviour was first observed by Vander Pol while he was doing an experiment on neon lamp oscillator. A chaotic model was first designed by Lorenz in 1963. The most recent development in non-linear dynamics is synchronization. The synchronization in a coupled chaotic system was first reported by Fujiiyama and Yamada [1] and later by Pecora and Carroll [2]. Since then there has been a great interest in understanding the mechanism of synchronization in a various chaotic system. Synchronization is a non-linear study that occurs between two or more chaotic system to adjust for the common behaviour. In recent years many different types of synchronization and coupling methods have been reported e.g. complete synchronization [3], generalized synchronization [4], phase synchronization [5], lag synchronization [6], anti-synchronization [7], projective synchronization [8], etc. In 2004, Yu and Zhang [9] have presented synchronization of two coupled chaotic systems with bidirectional linear error feedback. Projective synchronization (PS) was introduced by Mainieri and Rehacek in 1999 [10]. In this paper, a new chaotic system is designed and synchronization is done using a threshold controller.

Section II introduces the dynamical and numerical simulation of the new chaotic system. In Section III analog circuit design of the proposed chaotic system using Multisim is discussed. Section IV shows the synchronization of the systems using a threshold controller. Finally, the results are discussed and concluded in the last section and its applicability in secure communication.

2. NUMERICAL ANALYSIS

A set of three first order non-linear state equations of the proposed chaotic system is given below:

\[ \dot{x} = ay - bx \]
\[ \dot{y} = x(c - z) - y \]
\[ \dot{z} = dx - z \]

The system has four real parameters a, b, c, and d. The chaotic system has been model in MATLAB Simulink in Fig. 1. The system is studied by varying parametric values and phase portrait has been obtained for the parameter values a=0.06, b=50, c=26.5 and d=50 with initial condition...
\((\dot{X}, \dot{Y}, \dot{Z})=(0.01,0.01,0.01)\). Theoretical calculation gives that the system has three equilibrium points E1, E2 and E3:

E1(0,0,0), E2(\(\pm \sqrt{\frac{c-1}{d}}, \pm \sqrt{\frac{c-1}{d}}, c-1\)) and E3(\(-\sqrt{\frac{c-1}{d}}, -\sqrt{\frac{c-1}{d}}, c-1\))

The Jacobian matrix of the given chaotic system is:

\[
J = \begin{vmatrix}
-\alpha b & \alpha b & 0 \\
\alpha c - z & -1 - \alpha x \\
\alpha d y & \alpha d x & -1
\end{vmatrix}
\] (2)

Case1: when E2(\(\pm \sqrt{\frac{c-1}{d}}, \pm \sqrt{\frac{c-1}{d}}, c-1\)) the Jacobian Matrix becomes:

\[
J(E2) = \begin{vmatrix}
-\alpha b & \alpha b & 0 \\
\alpha c - (c - 1) & -1 - \sqrt{\frac{c-1}{d}} \\
\alpha d & \alpha d & -1
\end{vmatrix}
\] (3)

Therefore, the characteristics equation of the equilibrium E2 is:

\[
\det(J(E2) - \lambda I) = 0
\] (4)

\[
\det\begin{pmatrix}
-3 & 3 & 0 \\
1 & -1 & -7 \\
4 & 4 & -1
\end{pmatrix} - \lambda \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} = 0
\] (5)

\[
\det\begin{pmatrix}
-3 - \lambda & 3 & 0 \\
1 & -1 - \lambda & -7 \\
4 & 4 & -1 - \lambda
\end{pmatrix} = 0
\] (6)

By solving the above determinant, we get:

\[
\lambda^2 + 5\lambda + 60 = 0
\] (7)

Solving eq. (7) we get the Eigen values:

\(\lambda_1 = -3, \lambda_2 = -5 - i\frac{\sqrt{215}}{2}\) and \(\lambda_3 = -5 + i\frac{\sqrt{215}}{2}\) which is

\((\lambda_1, \lambda_2, \lambda_3) = (-3, -5 + 7.3314i, -5 - 7.3314i)\) (8)

The equilibrium points are hyperbolic in nature and the trajectories diverge from the equilibrium point called a saddle which is stable.

Case2: when E3(\(-\sqrt{\frac{c-1}{d}}, -\sqrt{\frac{c-1}{d}}, c-1\)) then the Jacobian Matrix becomes:

\[
J(E2) = \begin{vmatrix}
-\alpha b & \alpha b & 0 \\
\alpha c - (c - 1) & -1 - \sqrt{\frac{c-1}{d}} \\
-\alpha d & \alpha d & -1
\end{vmatrix}
\] (9)

Therefore, the characteristics equation of the equilibrium E2 is:

\[
\det(J(E3) - \lambda I) = 0
\] (10)
\[ \det \begin{pmatrix} -3 & 3 & 0 \\ 1 & -1 & 7 \\ -4 & -4 & -1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 0 \] (11)

\[ \det \begin{pmatrix} -3 - \lambda & 3 & 0 \\ 1 & -1 - \lambda & 7 \\ -4 & -4 & -1 - \lambda \end{pmatrix} = 0 \] (12)

By solving the above determinant, we get:
\[ \lambda^2 + 5\lambda - 108 = 0 \] (13)

Solving eq. (13) we get the Eigen values:
\[ \lambda_1 = -3, \lambda_2 = -5 + i \sqrt{407 \over 2}, \text{ and } \lambda_3 = -5 - i \sqrt{407 \over 2} \text{ which is} \]
\[ (\lambda_1, \lambda_2, \lambda_3) = (-3, -5 + 10.087i, -5 - 10.087i) \] (14)

The equilibrium points are hyperbolic in nature and the trajectories diverge from the equilibrium point called a saddle which is stable.

Using the MATLAB Simulink, the phase portrait of the model is achieved in Fig.2, Fig.3 and Fig.4.

**Fig.1:** MATLAB Simulink model of a chaotic system.

**Fig. 2:** x-y phase portrait
Fig. 3: y-z phase portrait

Fig. 4: x-z phase portrait

Fig. 5: 3-D attractor of the new chaotic system
From the Fig.6 it is evident that the waveform is irregular and random in nature. This confirms the presence of chaotic behaviour in the model.

A bifurcation diagram helps to visualize the dynamics as a function of a bifurcation parameter in a system [11]. It is a standard way to verify the period doubling bifurcation and route towards chaos.

Bifurcation diagram for the chaotic system has been implemented using MATLAB. In fig7. all the control parameter for b, c and d have been fixed and by only varying the value of a from 0.1 to 0.38 we can observe period doubling bifurcation. As we further increase the value of a beyond 0.35, we can see its route towards chaos.

Lyapunov exponent is defined as the average exponential rates of divergence or convergence of a nearby orbits in phase space. Any system with at least one positive Lyapunov exponent is defined to
be chaotic. From the fig8. it is observed that the first Lyapunov exponent is zero in the chaotic region, when it is negative the system is periodic and when it is beyond 3.5 it is positive which means the system behaves chaotically in this region.

![Fig.8: Lyapunov exponent of the system](image)

### 3. ANALOG CIRCUIT DESIGN OF CHAOTIC SYSTEM IN MULTISIM

The proposed new system is also implemented using Multisim and the circuit diagram is given in Fig.9.

![Fig.9: Circuit diagram of the new system](image)

The given circuit is implemented in multisim using electronic components such as resistors, capacitors, operational amplifiers, and multipliers. There are 3 capacitors, 17 resistors, 8 operational amplifier, and 3 multipliers are used in the circuit. Op-amp LM741CN, AD633JN multipliers with resistors
R2=R3=R6=R7=R8=R10=R12=R14=R16=R17=100kΩ, R1=R11=R15=10kΩ, R4=R5=33.3kΩ, R9=3.77kΩ, R13=200kΩ, C1=C2=C3=10nF, \( V_N = -12\)V and \( V_P = 12\)V. Inputs to the multiplier AD633JN are -12V and +12V. The waveform obtained from the oscilloscope is shown in Fig.10, Fig.11 and Fig 12. Whereas the chaotic attractor in xyz plane is shown in Fig. 13, Fig.14 and Fig15.

Fig.10: x waveform of the system.

Fig.11: y waveform of the system
Fig. 12: z waveform of the system

Fig. 13: x-y phase portrait
4. SYNCHRONIZATION OF THE CHAOTIC SYSTEM USING THRESHOLD CONTROLLER

The given system is synchronized using a control parameter where the identical drive system drives the response system under a suitable threshold control parameter $G(y)$ which is a non-linear function in the system and is given by:

$$G(y) = \begin{cases} y^* & y > y^* \\ y & -y^* \leq y \leq y^* \\ -y^* & y < -y^* \end{cases}$$
The threshold controller works when the observed value is greater than the threshold value then it is reset to a positive threshold i.e. \( y > y^* \) and when the observed value is less than the threshold value then it is reset to negative threshold i.e. \( y < -y^* \). While varying the value of the control parameter the system goes from complete synchronization to unsynchronized state.

The drive and the response of the proposed chaotic system is given below:

**Drive:**

\[
\begin{align*}
\dot{x} &= ab(y - x) \\
\dot{y} &= x(c - z) - y + G(\dot{y}) \\
\dot{z} &= dxy - z
\end{align*}
\]  

**Response:**

\[
\begin{align*}
\dot{x}' &= ab(y - x) \\
\dot{y}' &= x(c - z) - y + \varepsilon_1*\varepsilon_2*G(\dot{y} - \dot{y}') \\
\dot{z}' &= dxy - z
\end{align*}
\]  

MATLAB Simulink block for the given drive and response system is given in fig16. The threshold value for the drive and response system is fixed at \( y^* = 3.4 \). When the value of \( \varepsilon_1 = 1.4 \) and \( \varepsilon_2 = 2.08 \) the system exhibits complete synchronization. From the fig.17 it is clearly seen that the signal \( x \) and \( x_1 \) are completely synchronized and the synchronization between them is shown in the fig18.

![MATLAB Simulink model of synchronization using threshold controller](image-url)
Fig. 17: Complete synchronization of the signal x and x1 at ε1=1.4 and ε2=2.08

When the value of ε1=0.68 and ε2=2.08 the system is unsynchronized. Fig.19 shows that the signal x and x1 are not synchronized and the original signal cannot be obtained. In fig20. It shows the phase of synchronization between x and x1 and the error between the transmitted x and received x1 is very high which can be seen in fig21.

So, by varying the value of control parameter one can achieve the original signal at the receiver side.
Fig. 19: Unsynchronized of the signal x and x1 at $\varepsilon_1=0.68$ and $\varepsilon_2=2.08$

Fig. 20: Synchronization between signal x and x1

Fig. 21: Error between the signal x and x1
5. CONCLUSION

The theoretical and numerical analysis of the proposed system is done in MATLAB and in Multisim. The proposed system is synchronized using threshold controller and by varying the different values of the control parameter the system exhibits complete synchronization which can be used for secure communication.

6. REFERENCES

[1]. T Yamada and H Fujisaka, Prog. Theor. Phys. 70, 1240 (1983)
[2]. L M Pecora and T L Carroll, Phys. Rev. Lett. 64, 821 (1990)
[3]. Thomas L. Carroll and Louis M. Pecora, "Synchronizing chaotic circuits", IEEE Trans. Circuits Systts., vol. 38, pp. 453-456, Apr. 1991.
[4]. Louis M. Pecora and Thomas L. Carroll, "Driving systems with chaotic signals", Phys. Rev. A, vol. 44, pp. 2374-2383, Aug. 1991.
[5]. Kevin M. Cuomo and Alan V. Oppenheim, "Circuit Implementation of Synchronized Chaos with Applications to Communications", Phys. Rev. Lett., vol. 71, pp. 65-68, Jul. 1993.
[6]. A Tarai(Poria), S Poria and P Chatterjee, Chaos, Solitons and Fractals 40, 885 (2009)
[7]. Lakshmanan M, Rajasekar S. Nonlinear dynamics: integrability, chaos and pattern formation. Berlin: Springer-Verlag: 2003.
[8]. Stankovski T, McClintock PVE, Stefanovska A. Coupling functions enable secure communications. Phys Rev X. 2014;4(1):011026 (1–9).
[9]. Ren H-P, Baptista MS, Grebogi C. Wireless communication with chaos. Phys Rev Lett. 2013;110(18):184101.
[10]. Suresh, R., Srinivasan, K., Senthilkumar., V, Murali, K., Lakshmanan, M., Kurths, J.:Dynamic Environment coupling induced synchronized states in coupled time-delayed electronic circuits. Int. J. Bifurcat. Chaos 24 (2014)
[11]. Oliva RA, Strogatz SH. Dynamics of a large array of globally coupled lasers with distributed frequencies. Int J Bifurc Chaos. 2001;11(9):2359–2374.
[12]. Hirosawa K, Kittaka S, Oishi Y, et al. Phase locking in a Nd:YVO4 waveguide laser array using Talbot cavity. Opt Express. 2013;21(21):24952–24961.
[13]. Eckhardt B, Ott E, Strogatz SH, et al. Modeling walker synchronization on the millennium bridge. Phys Rev E. 2007;75(2):021110 (1–10).
[14]. Strogatz SH, Abrams DM, McRobie A, et al. Crowd synchrony on the Millennium Bridge. Nature. 2005 Nov;438:43–44.
[15]. Belykh I, Jeter R, Belykh V. Foot force models of crowd dynamics on a wobbly bridge; 2016, arXiv:1610.05366v1. p. 1–15.
[16]. Kocarev L, Parlitz U. General approach for chaotic synchronization with applications to communication. Phys Rev Lett. 1995;74(25):5028–5031.
[17]. Stankovski T, McClintock PVE, Stefanovska A. Coupling functions enable secure communications. Phys Rev X. 2014;4(1):011026 (1–9).
[18]. Ren H-P, Baptista MS, Grebogi C. Wireless communication with chaos. Phys Rev Lett. 2013;110(18):184101.
[19]. Parlitz U. Estimating model parameters from time series by autosynchronization. Phys Rev Lett. 1996;76(8):1232–1235.
[20]. Yu D, Parlitz U. Estimating parameters by autosynchronization with dynamics restrictions. Phys Rev E. 2008;77(6):066221.
[21]. NHK K. ISMAIL*, “Estimation Of Reliability Of D Flip-Flops Using Mc Analysis”, Journal of VLSI Circuits And Systems 1 (01), 10-12,2019.
[22]. Sulyukova,”Analysis of Low power and reliable XOR-XNOR circuit for high Speed Applications”,Journal of VLSI Circuits And Systems 1 (01), 23-26,2019.