Unitary Query for the $M \times L \times N$ MIMO Backscatter RFID Channel

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Abstract — A MIMO backscatter RFID system consists of three operational ends: the query end (with $M$ reader transmitting antennas), the tag end (with $L$ tag antennas) and the receiving end (with $N$ reader receiving antennas). Such an $M \times L \times N$ setting in RFID can bring spatial diversity and has been studied for STC at the tag end. Current understanding of the query end is that it is only an energy provider for the tag and query signal designs cannot improve the performance. However, we propose a novel unitary query scheme, which creates time diversity within channel coherent time and can yield significant performance improvements. To overcome the difficulty of evaluating the performance when the unitary query is employed at the query end and STC is employed at the tag end, we derive a new measure based on the ranks of certain carefully constructed matrices. The measure implies that the unitary query has superior performance. Simulations show that the unitary query can bring $5-10$ dB gain in mid SNR regimes. In addition, the unitary query also can improve the performance of single-antenna tags significantly, allowing employing low complex and small-size single-antenna tags for high performance. This improvement is unachievable for single-antenna tags when the conventional uniform query is employed.

Index Terms—RFID, backscatter channel, MIMO, query method, space-time coding

I. INTRODUCTION

Radio-frequency identification (RFID) is a wireless communication technology that allows an object to be identified automatically and does not require LOS transmission [1]. It is one important infrastructure of the internet of things, and adds significant values in many applications, such as inventory systems, product tracking, access control, libraries, museums, sports and social networks. An RFID system includes three major components: RFID readers (also known as interrogators), RFID tags (also known as labels), and RFID software or RFID middleware [2]. An RFID tag is a small electronic device that has a unique ID. It transmits data over the air in response to interrogation by an RFID reader. Depending on power supplying methods, the RFID tags can be categorized into passive, active, and semi-active tags. An active tag utilizes its internal battery to continuously power its RF communication circuitry, while a passive RFID tag has no internal power supply and relies on RF energy transferred from the reader to the tag. A semi-passive tag is powered by both its internal battery and RF energy from the reader.

Most RFID tags deployed are based on backscatter modulation, which does not require the modulated signal to be amplified and retransmitted, and thus the RF tags can be made extraordinary small and inexpensive. By the principal of backscatter modulation, the RF tag simply scatters a portion of the incident continuous wave signals from the reader transmitter back to the reader receiver using load modulation [3]. Such signals sent from the reader transmitters are known as query signals. The backscatter RFID can operate at ultra-high frequency (UHF) at 860 – 960 MHz, 2.45 GHz and 5.8 GHz with the operating range of the order of 10 meters. Measurements in [3] and [4] showed that the backscatter RFID channel can be modeled as a two-way channel with a forward sub-channel and a backscattering sub-channel, and both sub-channels can be modeled as certain fading, depending on the radio propagation environment. This two-way channel fades deeper than the conventional one-way channel and degrades the data transmission reliability and reading range, which are two important performance metrics in RFID systems.

Many efforts have been made on improving the performance of the backscatter RFID [4]–[25]. Among those efforts, using multiple antennas for both tags and readers appears to be one practical and promising way. Such multiple-input multiple-output (MIMO) systems had a great success in conventional wireless communications [26]–[30] and were also investigated and found promising in RFID [4]–[10]. A general MIMO backscatter RFID channel has $M$ query antennas on the reader, $L$ tag antennas on the tag and $N$ receiving antennas on the reader, as shown in Fig. 1. This MIMO setting can create spatial diversity and thus can improve the bit error rate (BER) performance and reading range of backscatter RFID. In [5], simulations showed that with the MIMO setting, the range of backscatter RFID can be extended by a factor of four or more in the pure diversity configuration and that capacity can be increased by a factor of ten or more in the spatial multiplexing configuration. In [6], it was shown that backscatter diversity can mitigate the fading by changing the shape of the fading distribution which, along with the increased RF tag scattering aperture, can result in a 10 dB gain at a BER of $10^{-4}$ and thus can lead to increased backscatter radio communication reliability and range (e.g., up to a 78 percent range increase), which is consistent with a later result in [8]. Except diversity gain, in [10] it was shown that additional antenna gains can be realized to mitigate or overcome extra path loss by using multiple antennas for narrowband signals centered at 5.8 GHz. The radio measurements of backscatter RFID with MIMO settings have also been investigated: in [4] the measurement was conducted at 5.8 GHz, and experiment showed that diversity gains are available for multiple-antenna RF tags and the results matched well with the gains predicted using the analytic
fading distributions derived in [6]. In [9], a method for the
determination of the channel coefficients between all antennas
was presented. Another interesting research was conducted in
[7], where researchers described a developed analog frontend
for an RFID rapid prototyping system which allows for various
real-time experiments to investigate MIMO techniques.

A. Related Work

The spatial diversity brought by MIMO settings for the
backscatter RFID has been analytically studied recently. With
the quasi-static fading assumption, it was shown that for the
$M \times L \times N$ backscatter RFID channel, the diversity
order achieves $\min(N, L)$ for the uncoded case [12], and the
diversity order achieves $L$ for the orthogonal space-time coded
case [13], [14]. Moreover, the diversity order cannot be greater
than $L$ [14]. All the above studies that use MIMO settings to
exploit the diversity gain for the backscatter RFID are based
on the uniform query, for which the query antennas send the
same signal over all symbol times. Since [5], [6], where the
$M \times L \times N$ backscatter RFID channel was formulated, there is
no other query signaling methods have been considered. This
is because the previous understanding is that, since spatial
diversity can only be obtained by duplicating the information
and transmitting it over multiple branches, while the query
end is not the information source, designs of the query signal
cannot bring spatial diversity in quasi-static channels. In this
paper, however, we show that in quasi-static channels, the
query signals can create time diversity via multiple query
antennas and thus improve the performance for the backscatter
RFID significantly. Our result does not follow the achievable
diversity order of the $M \times L \times N$ channel reported in previous
findings for the uniform query. We also analytically study the
performance of the proposed unitary query. Due to the specific
signaling and fading structure of the backscatter RFID channel,
the pairwise error probability (PEP) and even the diversity
order are not trackable for the unitary query, we thus derive a new measure which can compare the PEP
performance of the unitary query with that of the uniform
query. The derived measure in this paper can be used as criteria for designing the MIMO backscatter RFID
system.

• We present analysis to show that the proposed unitary
query has a very practical meaning: for conventional
uniform query, to improve the performance significantly,
equipping multiple antennas on the tag is a must. By
contrast, for the proposed unitary query, to improve the
performance significantly, equipping multiple antennas on
the tag is not a must. The proposed unitary can transfer
the complexity requirements from the tag to the reader,
and allows single-antenna tag to have high performance.

This paper is organized as follows: We give a brief intro-
duction of the MIMO backscatter RFID channel in Section II
We propose the unitary query in Section III and derive a new
measure for the performance of the unitary query. In Section IV
we study a few examples and conduct the corresponding
simulations. Finally we summarize our work in Section V.

Notations: In this paper, $Q(\cdot)$ means the $Q$ function; $P(\cdot)$,
$E_X(\cdot)$, $X|Y$, $\|\cdot\|_F$, $\text{rank}(\cdot)$, $\|\cdot\|$, $(\cdot)^T$, and $(\cdot)^H$ denote
the probability of an event, the expectation over the density of
$X$, the conditional random variable of $X$ given $Y$, the Frobenius
norm of a matrix, the rank of a matrix, the magnitude of a
complex number, the transpose, and the conjugate transpose,
respectively; $X \sim Y$ means that $X$ is identically distributed
with $Y$.

II. THE $M \times L \times N$ MIMO BACKSCATTER RFID
CHANNEL

The backscatter RFID has three operational ends: the reader
query end (i.e., the set of reader transmitting antennas), the tag
end (i.e., the set of tag antennas), and the reader receiver end
(i.e., the set of reader receiving antennas). These three ends
can be mathematically modeled by an $M \times L \times N$ dyadic
backscatter channel which consists of $M$ reader transmitter
antennas, $L$ RF tag antennas, and $N$ reader receiver antennas
[5], [6], [8], [13], [14], as shown in Fig. 1. In a quasi-static
wireless channel, this MIMO structure can be summarized by
using the following matrices: More specifically,

$$Q = \begin{pmatrix}
q_{1,1} & \cdots & q_{1,M} \\
\vdots & \ddots & \vdots \\
q_{L,1} & \cdots & q_{L,M}
\end{pmatrix} \quad (1)$$

is the query matrix (with size $T \times M$), representing the
query signals sent from the $M$ reader query (transmitting)
antennas to the tag over $T$ time slots (i.e. $T$ symbol times);

$$H = \begin{pmatrix}
h_{1,1} & \cdots & h_{1,L} \\
\vdots & \ddots & \vdots \\
h_{M,1} & \cdots & h_{M,L}
\end{pmatrix} \quad (2)$$
is the channel gain matrix (with size \( M \times L \)) from the reader transmitter to the tag, representing the forward sub-channels;

\[
C = \begin{pmatrix}
c_{1,1} & \cdots & c_{1,L} \\
\vdots & \ddots & \vdots \\
c_{T,1} & \cdots & c_{T,L}
\end{pmatrix}
\]  

(3)

is the coding matrix (with size \( T \times L \)), where the tag transmits space-time coded or uncoded symbols from its \( L \) antennas over \( T \) time slots; and

\[
G = \begin{pmatrix}
g_{1,1} & \cdots & g_{1,N} \\
\vdots & \ddots & \vdots \\
g_{L,1} & \cdots & g_{L,N}
\end{pmatrix},
\]  

(4)

is the channel gain matrix (with size \( L \times N \)) from the tag to the reader receiver, representing the backscattering sub-channels. Finally the received signals at \( N \) reader receiving antennas over \( T \) time slots are represented by matrix \( R \) with size \( T \times N \):

\[
R = ((QH) \circ C)G + W
\]  

(5)

where \( \circ \) is the Hadamard product, and the matrix \( W \) is with the same size as that of \( R \), representing the noise at the \( N \) reader receiving antennas over \( T \) time slots. Typically, both \( H \) and \( G \) are modeled as full rank matrices with i.i.d complex Gaussian entries, and \( W \) is AWGN.

The signal-channel structure of the \( M \times L \times N \) RFID channel is radically different from conventional wireless channels, and can be characterized as a \textit{query-fading-coding-fading} structure. Compared with the conventional one-way wireless channel, this signal-channel structure not only has one more layer of fading \( H \) but also one more signaling mechanism represented by the query matrix \( Q \). In addition, the backscatter principle makes the received signals not a simple series of linear transformations of transmitted signals and channel gains, but actually there involves a non-linear structure in the backscatter RFID channel, which is the result from the Hadamard product in (5). Because it has such a special signaling-channel structure, the backscatter RFID channel behaves completely different from that of the one-way channel [13], [14]. It is worth mentioning here that the keyhole channel also has two layers of fading, however, the keyhole channel and the backscatter RFID channel are essentially different. The keyhole channel is still a one-way channel, as the signals sent out will not be reflected back. In addition, the keyhole channel has only two operational ends (the transmitter and the receiver), while the backscatter channel has three operational ends and the information to be transmitted is at the middle end (the tag end). The essential differences of the two channels have been discussed in [13], [14], especially there is a detailed discussion in [14]. In general, the \( M \times L \times N \) backscatter RFID channel is more complicated than the keyhole channel.

III. UNITARY QUERY FOR BACKSCATTER RFID

Recall that in the backscatter RFID channel, there are three operational ends: the query end, the tag end, and the receiving end. In the previous literature, the understanding of the query end was that the design of query signals can not improve the BER performance. This is based on the following explanation:

\[
Q_{\text{uniform}} = \frac{1}{\sqrt{M}} \begin{pmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{pmatrix}.
\]  

(6)

The uniform query is used as the query method for all previous studies of the \( M \times N \times L \) backscatter channel, and no further investigation has been made on the query signal design since the \( M \times N \times L \) backscatter channel has been formulated in [5], [6]. The reason that no other query signal design method has been considered probably from the understanding that the

![Fig. 1. The \( M \times L \times N \) backscatter RFID channel. The channel consists of three operational ends: the query end (with \( M \) query antennas), the tag end (with \( L \) tag antennas) and the receiving end (with \( N \) receiving antennas). The query antennas transmit unmodulated (query) signals to the RF tag and the RF tag scatters a modulated signal back to the reader.](image)
spatial diversity from the transmitter can only be made when transmitting duplicated information from different antennas, and since the query signals do not carry information, the spatial diversity can only be made from the tag antennas and the reader receiving antennas.

However, in general, query signals can be designed to follow any arbitrary \( Q \). In this paper, we propose the so-called unitary query, which satisfies the unitary condition:

\[
Q_{\text{unitary}} Q_{\text{unitary}}^H = I. \tag{7}
\]

Note that to satisfy the unitary condition we must have \( T = M \), while, as long as there are at least \( M \) symbol times during the transmission period, we can always cast the query signals into blocks each of which has \( T = M \) symbol times, and obtain the unitary query. Since the above query matrix is unitary and the entries of \( H \) are i.i.d complex Gaussian, we have

\[
Q_{\text{unitary}} H \sim X = \begin{pmatrix} x_{1,1} & \cdots & x_{1,L} \\ \vdots & \ddots & \vdots \\ x_{T,1} & \cdots & x_{T,L} \end{pmatrix}. \tag{8}
\]

The resulting matrix \( X \) (with size \( T \times L \)) has i.i.d complex Gaussian entries \( x_{t,l} \)'s, so the unitary query actually transforms the forward channel \( H \), which is invariant over the \( T \) time slots, into a channel \( X \) which varies over the \( T \) time slots. We will show later that this variation over the \( T \) time slots is the fundamental reason that the unitary query can bring additional time diversity and significant performance improvement for some STCs in the backscatter RFID channel. When compared with that of the uniform query

\[
Q_{\text{uniform}} H \sim Y = \begin{pmatrix} y_1 & \cdots & y_L \\ \vdots & \ddots & \vdots \\ y_1 & \cdots & y_L \end{pmatrix}. \tag{9}
\]

where \( y_l \)'s are i.i.d complex Gaussian. Clearly the resulting matrix \( Y \) (also with size \( T \times L \)) has identical rows. Thus the uniform query transforms the full rank matrix \( H \) into a rank-one matrix, while the unitary query transforms \( H \) into another full rank matrix. \( X \) varies both temporally and spatially, while \( Y \) only varies spatially. In the following sub-section, we give a brief interpretation of the diversity of the proposed unitary query.

A. Interpretation for Unitary Query: Time Diversity within Coherent Interval

In the quasi-static channel, where the channel is highly correlated across consecutive symbols, no time diversity can be provided within one coherent time interval for the one-way channel, and time diversity can only be provided by interleaving symbols in different coherent time intervals. This also applies to the backscatter RFID channel when the conventional uniform query is employed. The unitary query, however, utilizes the multiple query antennas, to create time diversity within channel coherent time. Fig. 2 shows that, with the conventional uniform query, the backscatter RFID channel still behaves like a quasi-static channel: the channel changes every \( T \) symbol times; by contrast, when the unitary query is employed, the channel changes every 1 symbol time.

An alternative interpretation based on geometry is shown in Fig. 3. We consider the codewords \( (c_{1,1}, c_{2,1}, \cdots, c_{T,1}) \), which can be viewed as a point in a \( T \)-dimensional space. We can see that when the uniform query is applied, possible locations of the point \( (c_{1,1}, c_{2,1}, \cdots, c_{T,1}) \) can only be mapped to the points on a straight line, which is only 1-dimensional. However, when the unitary query is applied, possible locations of the point \( (c_{1,1}, c_{2,1}, \cdots, c_{T,1}) \) can be mapped to any points in the entire \( T \)-dimensional space. From Fig. 3 it is clear that this kind of full-dimensional spreading out of possible locations of the codewords by the unitary query may yield significant performance improvements.

B. New Performance Measure for the \( M \times N \times L \) Channel

Now we need to study the performance of the unitary query. In previous literature, the performance when the \( M \times N \times L \) channel employs the uniform query was investigated in [6].
it was shown that, the analysis is very difficult even with the conventional uniform query. With the proposed unitary query being employed, the analysis will be more difficult, the diversity order is not trackable. We thus derive a new measure other than the conventional diversity order for the performance analysis. The new measure is based on the ranks of certain carefully constructed random matrices.

When the $M \times N \times L$ channel employs the unitary query at the query end, and employs space-time coding at the tag end, it has an equivalent channel model as

$$R = (X \circ C)G + W,$$

where $X$ is given in [3]. When the $M \times N \times L$ channel applies uniform query at the query end, and space-time coding at the tag end, it has an equivalent channel model as

$$R = (Y \circ C)G + W,$$

where where $Y$ is given in [9]. Now we define the codewords difference matrix for codewords matrices $C$ and $C'$ as

$$\Delta = C - C' = \left( \begin{array}{ccc} \delta_{1,1} & \cdots & \delta_{1,T} \\ \vdots & \ddots & \vdots \\ \delta_{L,1} & \cdots & \delta_{L,T} \end{array} \right).$$

The PEP is the probability that the receiver decide erroneously in favor of the codewords matrix $C'$ a when the $C$ is actually transmitted, for unitary query, the PEP is can be evaluated as

$$\text{PEP}_X(\gamma) = \mathbb{E}_{H,G} \left( Q \left( \sqrt{\gamma Z_X} / 2 \right) \right),$$

where

$$Z_X = \sqrt{(X \circ C)G - (X \circ C')G} \|_F^2 = \sqrt{\| (X \circ \Delta)G \|_F^2},$$

is the random variable which represents the squared distance between the codewords matrix $C$ and $C'$ when unitary query is employed and the tag uses space-time coding. Similarly, for the uniform query, the PEP is given by

$$\text{PEP}_Y(\gamma) = \mathbb{E}_{H,G} \left( Q \left( \sqrt{\gamma Z_Y} / 2 \right) \right),$$

where

$$Z_Y = \sqrt{(Y \circ C)G - (Y \circ C')G} \|_F^2 = \sqrt{\| (Y \circ \Delta)G \|_F^2},$$

is the random variable which represents the squared distance between the codewords matrices $C$ and $C'$ when uniform query is employed and the tag uses space-time coding. $\gamma$ in the above equations is the averaged signal-to-noise ratio (SNR).

Quite different from that of the one-way channel, directly evaluating the PEPs in (13) and (15) is not feasible because the distributions of $Z_X$ and $Z_Y$ are not trackable when general space-time code is considered at the tag end. Even for the case when the uniform query is employed (corresponds to the distribution of $Z_Y$), the asymptotic PEP can only be obtained for two special coding cases: the orthogonal space-time code [13], [14] and the uncoded case [12]. When the proposed unitary query is employed (corresponds to the distribution of $Z_X$), evaluating the PEP will be even harder. In this paper, we reconsider the evaluation of PEP and provide a new measure for the PEP performance for the $M \times L \times N$ channel, to overcome the above difficulties. This new measure can provide a deep understanding of the performance of the channel, and can be used to compare the performances between the unitary query and the uniform query. Instead of considering the squared codewords distance as a whole, we treat it in a time fashion. When the unitary query is employed, at time $t$, the squared codewords distance is given by

$$Z_X^t = \sqrt{(x_{t,1}, \cdots, x_{t,L}) \circ (\delta_{1,t}, \cdots, \delta_{L,t})}G \|_F^2 = \sqrt{\| (x_{t,1}, \cdots, x_{t,L}) \Delta_t G \|_F^2},$$

where $\Delta_t$ is defined as

$$\Delta_t \triangleq \left( \begin{array}{ccc} \delta_{1,t} \\ \vdots \\ \delta_{L,t} \end{array} \right),$$

then over the $T$ time slots we have

$$Z_X = \sum_{t=1}^{T} \sqrt{(x_{t,1}, \cdots, x_{t,L}) \Delta_t G} \|_F^2 = \sum_{t=1}^{T} \sqrt{(x_{t,1}, \cdots, x_{t,L})E_t \|_F^2},$$

where $E_t$ is defined as

$$E_t \triangleq \Delta_t G.$$

We will see later that the ranks of the carefully constructed random matrices $E_t$’s determine the performance for the unitary query.

When the uniform query is employed, the squared codewords distance at time $t$ is given by

$$Z_Y^t = \sqrt{(y_1, \cdots, y_L) \circ (\delta_{1,t}, \cdots, \delta_{L,t})}G \|_F^2 = \sqrt{\| (y_1, \cdots, y_L) \Delta_t G \|_F^2},$$

and over the $T$ time slots we have

$$Z_Y = \sum_{t=1}^{T} \sqrt{(y_1, \cdots, y_L)E_t \|_F^2} = \sum_{t=1}^{T} \sqrt{(y_1, \cdots, y_L)(E_1, \cdots, E_T) \|_F^2}.$$
for $n = 1, \cdots, N$. Also, we will see later that the rank of the carefully constructed random matrix
\begin{equation}
D \triangleq (D_1, \cdots, D_N) \tag{26}
\end{equation}
determines the performance for the uniform query.

Now we give the following two Lemmas about the ranks of the random matrices $E_i$’s and the rank of the matrix $D$.

**Lemma 1.** For the matrices $E_i$’s defined in (20), we have $\text{rank}(E_i) = \min(N, L_i^*)$ with probability (w.p.) 1 for all $t \in \{1, \cdots, T\}$, where $L_i^*$ is the number of non-zero elements of the $t$-th column of the codewords difference matrix $\Delta$.

**Proof of Lemma 1.** See the appendix.

**Lemma 2.** For the matrix $D$ defined in (26), we have $\text{rank}(D) = \min(N \times \text{rank}(\Delta), L)$ with probability 1, where $L$ is the number of non-zero columns of the codewords difference matrix $\Delta$.

**Proof of Lemma 2.** See the appendix.

With understanding the ranks of the matrices $E_i$’s and the rank of the matrix $D$, we introduce the following theorem of the new measure for the unitary query and the uniform query.

**Theorem 1.** In asymptotic high SNR regimes, the PEP performances of space-time codes with the unitary query and the uniform query in the $M \times N \times L$ backscatter RFID channel given in (5) can be measured by
\begin{equation}
R_{\text{unitary}} = \sum_{t=1}^{T} \min(N, L_i^*), \tag{27}
\end{equation}
and
\begin{equation}
R_{\text{uniform}} = \min(N \times \text{rank}(\Delta), L), \tag{28}
\end{equation}
respectively, where $L_i^*$ is the number of non-zero elements of the $t$-th column of the codewords difference matrix $\Delta$. In other words, if
\begin{equation}
R_{\text{unitary}} > R_{\text{uniform}}, \tag{29}
\end{equation}
we have
\begin{equation}
\lim_{\gamma \to \infty} \frac{\text{PEP}_{Z_X}(\bar{\gamma})}{\text{PEP}_{Z_Y}(\bar{\gamma})} \to 0; \tag{30}
\end{equation}
if
\begin{equation}
R_{\text{unitary}} < R_{\text{uniform}}, \tag{31}
\end{equation}
we have
\begin{equation}
\lim_{\gamma \to \infty} \frac{\text{PEP}_{Z_Y}(\bar{\gamma})}{\text{PEP}_{Z_X}(\bar{\gamma})} \to 0; \tag{32}
\end{equation}
and if
\begin{equation}
R_{\text{unitary}} = R_{\text{uniform}}, \tag{33}
\end{equation}
we have
\begin{equation}
\lim_{\gamma \to \infty} \frac{\text{PEP}_{Z_X}(\bar{\gamma})}{\text{PEP}_{Z_Y}(\bar{\gamma})} = \epsilon > 0; \tag{34}
\end{equation}
where $\epsilon$ is some positive constant.

**Proof of Theorem 1.** See the appendix.

The new measure in Theorem 1 can be used to compare the PEP performances of the unitary query and the uniform query in large scale (i.e., if one measure is large than the other, its performance will be much better than that of the other). Therefore in some sense the new measure is similar to the diversity order, but not exactly the same. We give a brief discussion of three possible cases.

**Case 1:** $R_{\text{unitary}} > R_{\text{uniform}}$

In this case, the performance of the unitary query will be much better than that of the uniform query. Most well designed space-time codes fall into this case, and can drive the full potential of the $M \times L \times N$ backscatter RFID channel.

**Case 2:** $R_{\text{unitary}} < R_{\text{uniform}}$

In this case, the performance of the uniform query will be much better than that of the uniform query. However, only rare space-time codes fall into this case. Such space-time codes cannot drive the full potential of the $M \times L \times N$ backscatter RFID channel and thus are not preferred in the $M \times L \times N$ channel.

**Case 3:** $R_{\text{unitary}} = R_{\text{uniform}}$

In this case, the performance of the unitary query will be similar as that of the uniform query, while the unitary query will still outperform the uniform query, though the improvement will not be significant.

Note that in the above three cases, Case 1 can achieve the full potential of the $M \times L \times N$ channel, and is usually preferred.

**IV. PERFORMANCE EVALUATIONS**

In this section, we give a few examples and provide corresponding simulation results for the proposed unitary query and the conventional uniform query. We will see by how much the unitary query can improve the performance and how the unitary query transfers the complexity requirement from the tag end to the reader end for high performance systems. In the following simulations, we use the same channel model as in previous real measurements (3) (4) and analytical studies (6), (12)–(14) of the $M \times N \times L$ backscatter RFID channel. More specifically, the entries of both $H$ in (2) and those of $G$ in (4) follow i.i.d complex Gaussian distribution, with zero mean and unit variance, and the fading is quasi-static. Given a codewords difference matrix $\Delta$, the new performance measure ($R_{\text{uniform}}$) for the conventional uniform query given in Theorem 1 is based on the rank of the random matrix $D$ defined in (26), and the new performance measure ($R_{\text{unitary}}$) for the proposed unitary query is based on the ranks of the random matrices $E_i$’s defined in (20).

**A. Tag with Multiple Antennas**

When the uniform query is employed, the limit of the performance is given by
\begin{equation}
R_{\text{uniform}} = \min(N \times \text{rank}(\Delta), L) \leq L, \tag{35}
\end{equation}
which means no matter how many antennas are equipped in the channel and whatever the space-time code is, the performance has a bottleneck determined by $L$. However, the unitary query
can break through this bottleneck and bring a significant improvement:

\[ R_{\text{unitary}} = \sum_{t=1}^{T} \min(N, L_t^*) \leq T L. \]  

(36)

With some space-time codes, the above measure \( R_{\text{unitary}} \) can achieve \( TL \). We give the following example to illustrate this and show how much gain the unitary query can bring.

**Example 1** Consider the \( 2 \times 2 \) backscatter RFID channel, i.e.

\[ H = \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix}, \quad G = \begin{pmatrix} g_{1,1} & g_{1,2} \\ g_{2,1} & g_{2,2} \end{pmatrix}, \]

(37)

where the entries of \( H \) and \( G \) are i.i.d. complex Gaussian with zero mean and unit variance, and the following codewords difference matrix resulted from the space-time code employed at the tag end:

\[ \Delta = \begin{pmatrix} 1 & -2 \\ 1.5 & 2.5 \end{pmatrix}. \]

(38)

In this case \( M = 2, L = 2, N = 2, T = 2, \) \( \text{rank}(\Delta) = 2, \) and \( L_1^* = L_2^* = 2. \) Based on Theorem 1 when the unitary query is employed we have

\[ R_{\text{unitary}} = \sum_{t=1}^{T} \min(N, L_t^*) = \min(2, 2) + \min(2, 2) = 4, \]

(39)

and when the uniform query is employed we have

\[ R_{\text{uniform}} = \min(N \times \text{rank}(\Delta), L) = \min(2 \times 2, 2) = 2. \]

(40)

Therefore the performance of the unitary query is expected to be much better than that of the uniform query. Simulations confirm this as we can see in Fig. 4 there is a significant gain by employing the unitary query for the \( 2 \times 2 \times 2 \) backscatter channel. We observe a 5 to 7 dB gain in the SNR regimes of 10 to 15 dB when the system employs unitary query, and the gain increases as the SNR increases. This gain brought by the unitary query can be considered as the time diversity gain that has been illustrated in Section III-A.

**Example 2** We consider a case that the tag end employs the same space-time code as that in Example 1 but a different antenna setting: a \( 2 \times 2 \times 1 \) backscatter RFID channel, i.e.

\[ H = \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix}, \quad G = \begin{pmatrix} g_{1,1} \\ g_{2,1} \end{pmatrix}. \]

(41)

In this case \( M = 2, L = 2, N = 1, T = 2, \) \( \text{rank}(\Delta) = 2, \) and \( L_1^* = L_2^* = 2. \) and the measures are given by

\[ R_{\text{unitary}} = \sum_{t=1}^{T} \min(N, L_t^*) = \min(1, 2) + \min(1, 2) = 2, \]

(42)

\[ R_{\text{uniform}} = \min(N \times \text{rank}(\Delta), L) = \min(1 \times 2, 2) = 2. \]

(43)

Since \( R_{\text{unitary}} = R_{\text{uniform}}, \) by Theorem 1 the unitary query still outperforms the uniform query but the improvement is not significant, as shown in Fig. 5. In this case, with the given code difference matrix in (35), the \( 2 \times 2 \times 1 \) channel achieves the full potential for the uniform query but does not achieve the full potential for the unitary query, that is reason why the unitary query outperforms the uniform query but the gain is not significant.

**B. Tag with Single Antenna**

In practice, since equipping multiple antennas on the tag increases the complexity and even the size of the tag, single-antenna tags are always preferred. However, with the conventional uniform query, the performance of the single-antenna tag \( (L = 1) \) is quite limited. As we can see that when \( L = 1, \) the performance measure for the conventional query uniform is

\[ R_{\text{uniform}} = \min(N \times \text{rank}(\Delta), 1) = 1. \]

(44)

It means that, when the conventional uniform query is employed, significant performance improvement can never be made for single-antenna tags. However, for the unitary query, the measure is given by

\[ R_{\text{unitary}} = \sum_{t=1}^{T} \min(N, L_t^*) \leq \sum_{t=1}^{T} \min(N, 1) = T. \]

(45)

Clearly when \( L_t^* = L = 1 \) for all \( t, \) \( R_{\text{unitary}} \) achieves \( T. \) Therefore, with the unitary query, carefully choosing coding scheme can lead to significant improvements for single-antenna tags. We use the following example to illustrate this.

**Example 3** We consider the BPSK with repetition code of order of 2, and the \( 2 \times 1 \times 2 \) backscatter RFID channel, i.e.

\[ \Delta = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \]

(46)

and

\[ H = \begin{pmatrix} h_{1,1} \\ h_{2,1} \end{pmatrix}, \quad G = \begin{pmatrix} g_{1,1} & g_{1,2} \end{pmatrix}. \]

(47)

In this case \( M = 2, L = 1, N = 2, T = 2, \) \( \text{rank}(\Delta) = 1, \) and \( L_1^* = L_2^* = 1. \) We have

\[ R_{\text{unitary}} = \sum_{t=1}^{T} \min(N, L_t^*) = \min(2, 1) + \min(2, 1) = 2, \]

(48)

and

\[ R_{\text{uniform}} = \min(N \times \text{rank}(\Delta), L) = \min(2 \times 1, 1) = 1. \]

(49)

Thus we expect that the unitary query can bring significant gain. The simulation results of this example are shown in Fig.
The conventional uniform query
The proposed unitary query

Fig. 4. Example 1: Performance comparisons between the unitary query and the uniform query. In this example, the space-time code that has a code difference matrix defined in (38) is employed at the tag end. We can see that when the proposed unitary query is employed at the reader query end, the performance is much better than the case when the conventional unitary is employed. In the $2 \times 2 \times 2$ backscatter RFID channel, the unitary query can bring a significant gain: 5-7 dB improvement in the 10-20 dB SNR regime, this improvement can be even larger than 10 dB in higher SNR regimes. The simulations in this example agrees the performance measure in Theorem 1.

Fig. 5. Example 2: Performance comparisons between the unitary query and the uniform query. In this example, the space-time code which has a code difference matrix defined in (38) is employed at the tag end. In the $2 \times 2 \times 1$ backscatter RFID channel, the unitary query outperforms the uniform query while the gain is not significant in this case. The simulations in this example agrees the performance measure in Theorem 1.

Fig. 6. Example 3: The unitary query can also significantly improve the performance of single-antenna tag. About 10 dB gain brought by the unitary query is observed in the $2 \times 1 \times 2$ backscatter RFID channel in the mid SNR regimes when the tag employs repetition code. With the conventional uniform query, this level of improvement is only achievable when multiple antennas are equipped on the tag.

V. Conclusion

In this paper, we proposed the unitary query at the reader query end in the $M \times L \times N$ MIMO backscatter RFID channel. We showed that even in the quasi-static fading, the unitary query can provide time diversity via multiple reader query antennas and thus can improve the performance of the RFID channel significantly. Due to the difficulty of evaluating the PEP and the diversity order directly, we derived a new performance measure based on the ranks of certain carefully constructed matrices. Simulations showed that the proposed unitary query can improve the performance by 5 to 10 dB in mid-range SNR regimes, and the gain increases as the SNR increases. The unitary query can also improve the performance for the case of having single-antenna tag significantly, making it possible to employ inexpensive, small and low complex tags for high performance. In other words, for high performance RFID systems, the proposed unitary query can transfer the complexity requirement from the tag end to the reader end.

VI. Appendix

Proof of Lemma 2

Let $g_1, \cdots, g_N$ denote the columns of $G$. We consider a set of scalars $\{a_1, \cdots, a_N\}$ where $a_n \in \mathbb{C}$, for any linear combination of the set of vectors, $\{g_1, \cdots, g_N\}$,

$$b = \sum_{n=1}^{L} a_n g_n$$

is a zero-mean complex Gaussian random vector with covariance matrix $\sum_{n=1}^{L} \|a_n\|^2 I$, therefore

$$\mathbb{P}(b = 0) = 0.$$
When $N \leq L$, (51) implies that
\[ \mathbb{P}(\text{rank}(G) < N) = 0, \]  
(52)
or
\[ \mathbb{P}(\text{rank}(G) = N) = 1. \]  
(53)
When $N > L$, by performing a linear combination of the rows of $G$ and following a procedure similar to the case that $N \leq L$, we can obtain
\[ \mathbb{P}(\text{rank}(G) = L) = 1. \]  
(54)
Hence the matrix $G$ is of full rank with probability 1, i.e.
\[ \mathbb{P}(\text{rank}(G) = \min(N, L)) = 1. \]  
(55)
Now notice that $\Delta_i$ is diagonal, therefore $E_i = \Delta_i G$ has $L_i^*$ non-zero rows. Because $G$ is full rank w.p. 1, we have
\[ \text{rank}(E_i) = \min(L_i^*, N) \]  
(56)
w.p. 1.

**Proof of Lemma 2.** Following similar steps to prove that $G$ is of full rank w.p. 1, we can show that
\[ \mathbb{P}(\text{rank}(G_n) = L) = 1, \]  
(57)
i.e., $G_n$ is also of full rank w.p. 1. Since
\[ D_n = \Delta G_n, \]  
(58)
we have
\[ \mathbb{P}(\text{rank}(\Delta G_n) = \text{rank}(\Delta)) = 1, \]  
(59)
i.e. the rank of $D_n$ is the same as the rank of $\Delta$ w.p. 1.

Now let us consider the following two cases:

**Case 1:** $N \times \text{rank}(\Delta) \leq L$

By Eqn. (59), clearly the columns of each of $D_n$’s span a subspace of dimension $\text{rank}(\Delta)$ in $\mathbb{C}^L$ w.p. 1. Now consider a set of scalars $a_{i,j}$’s, where $i \in \{1, \cdots, N\}$, and $j \in \{1, \cdots, T\}$. If for $i \in \{2, \cdots, N\}$ and $j \in \{1, \cdots, T\}$, $a_{i,j}$’s are not all zero, it is not hard to verify that
\[ \mathbb{P} \left( \sum_{j=1}^{T} a_{1,j} D_{1,j} = \sum_{i=2}^{N} \sum_{j=1}^{T} a_{i,j} D_{i,j} \right) = 0. \]  
(60)
This implies that the rows of all $D_n$’s span a subspace of dimension $N \times \text{rank}(\Delta)$ in $\mathbb{C}^L$ w.p. 1, i.e. the rank of the block matrix $D$ is $N \times \text{rank}(\Delta)$ w.p. 1 in this case.

**Case 2:** $N \times \text{rank}(\Delta) > L$

Following the similar procedure as in Case 1, we can find that the dimension of the subspace spanned by the columns of all $D_n$’s is $L$, i.e. the rank of the block matrix $D$ is $L$ w.p. 1 in this case.

With the results from Cases 1 and 2, we have Lemma 2 hold.

**Proof of Theorem 1.** We consider singular value decompositions of $E_i$’s and $D$, i.e.,
\[ E_i = U_i \Lambda_i V_i, \]  
(61)
and
\[ D = U^* \Lambda^* V^*. \]  
(62)
Note that, for the unitary query, for a realization of $G$ the squared distance between codewords can be given as
\[ Z_X|G = \sum_{t=1}^{T} \|x_{t,1}, \cdots, x_{t,L}\|_F^2 \]  
\[ = \sum_{t=1}^{T} \|x_{t,1}, \cdots, x_{t,L}\|_F^2 \]  
\[ \approx \sum_{t=1}^{T} \|x_{t,1}, \cdots, x_{t,L}\|_F^2 \]  
\[ = \sum_{i=1}^{\text{rank}(E_i)} \lambda_i \|x_{t,i}\|^2, \]  
(63)
where $\lambda_i$’s ($i = 1, \cdots, \text{rank}(E_i)$) are the non-zero eigenvalues of $E_i$. Given a realization of $G$, the conditional PEP on $G$ is given by
\[ \text{PEP}_{Z_X|G}(\bar{\gamma}) = \mathbb{E}_{Z_X|G} \left( Q \left( \frac{\sum_{i=1}^{\text{rank}(E_i)} \lambda_i \|x_{t,i}\|^2/2}{\bar{\gamma}} \right) \right) \]  
\[ = \prod_{t=1}^{T} \frac{1}{1 + \lambda_i \bar{\gamma}/4} \]  
(64)
Therefore the PEP for the uniform query can be obtained as
\[ \text{PEP}_{Z_X|G}(\bar{\gamma}) = \mathbb{E}_G \left( \text{PEP}_{Z_X|G}(\bar{\gamma}) \right) \]  
\[ = \mathbb{E}_G \left( \prod_{t=1}^{T} \frac{1}{1 + \lambda_i \bar{\gamma}/4} \right) \]  
\[ = \mathbb{E}_G \left( \prod_{t=1}^{\text{rank}(D)} \frac{1}{1 + \lambda_i \bar{\gamma}/4} \right) \]  
(65)
The last step of the above derivation is obtained by using the result from Lemma 1 and the fact that $0 < \frac{1}{1 + \lambda_i \bar{\gamma}/4} < \infty$.

Similarly, for the uniform query, for a realization of $G$, the squared distance between codewords can be given by
\[ Z_Y|G = \|y_1, \cdots, y_L\|_F^2 \]  
\[ = \|y_1, \cdots, y_L\|_U^* \Lambda^* V^* \]  
\[ \approx \|y_1, \cdots, y_L\|_F^2 \]  
\[ = \sum_{i=1}^{\text{rank}(D)} \lambda_i^* \|y_{1,i}\|^2, \]  
(66)
where $\lambda_i^*$’s are the eigenvalues of $D$. For a realization of $G$, the conditional PEP is given by
\[ \text{PEP}_{Z_Y|G}(\bar{\gamma}) = \mathbb{E}_{Z_Y|G} \left( Q \left( \frac{\sum_{i=1}^{\text{rank}(D)} \lambda_i^* \|y_{1,i}\|^2/2}{\bar{\gamma}} \right) \right) \]  
\[ = \prod_{i=1}^{\text{rank}(D)} \frac{1}{1 + \lambda_i^* \bar{\gamma}/4} \]  
(67)
Therefore the PEP for the uniform query is given by

$$\text{PEP}_G(\gamma) = E_G \left( \prod_{i=1}^{\min(N \times \text{rank}(D), L)} \frac{1}{1 + \lambda_i^* \gamma/4} \right)$$

$$= E_G \left( \prod_{i=1}^{T} \prod_{t=1}^{\min(N, L_i)} \frac{1}{1 + \lambda_i^* \gamma/4} \right). \quad (68)$$

The last step of the above derivation is obtained by using the result from Lemma 2 and the fact that \(0 < \frac{1}{1 + \lambda_i^* \gamma/4} < \infty\). With the assumption that

$$E_G \left( \prod_{i=1}^{T} \prod_{t=1}^{\min(N, L_i)} \frac{1}{1 + \lambda_i \gamma} \right) < \infty,$$ \quad (69)

$$E_G \left( \prod_{i=1}^{T} \prod_{t=1}^{\min(N, L_i)} \frac{1}{1 + \lambda_i \gamma} \right) < \infty,$$ \quad (70)

and applying the Dominated Convergence Theorem (DCT), we have

$$\lim_{\gamma \to \infty} \left( \gamma \text{P_{unitary}} \times \text{PEP}_{Z_X}(\gamma) \right) = E_G \left( \prod_{i=1}^{T} \prod_{t=1}^{\min(N, L_i)} \frac{1}{1 + \lambda_i \gamma} \right),$$ \quad (71)

and

$$\lim_{\gamma \to \infty} \left( \gamma \text{P_{unitary}} \times \text{PEP}_{Z_Y}(\gamma) \right) = E_G \left( \prod_{i=1}^{T} \prod_{t=1}^{\min(N, L_i)} \frac{1}{1 + \lambda_i \gamma} \right).$$ \quad (72)

**Case 1:** \(\text{P_{unitary}} > \text{P_{uniform}}

In this case,

$$\lim_{\gamma \to \infty} \frac{\text{PEP}_{Z_X}(\gamma)}{\text{PEP}_{Z_Y}(\gamma)} = \lim_{\gamma \to \infty} \frac{\gamma \text{P_{unitary}} \times E_G \left( \prod_{i=1}^{T} \prod_{t=1}^{\min(N, L_i)} \frac{1}{1 + \lambda_i \gamma} \right)}{\gamma \text{P_{unitary}} \times E_G \left( \prod_{i=1}^{T} \prod_{t=1}^{\min(N, L_i)} \frac{1}{1 + \lambda_i \gamma} \right)} = 0.$$ \quad (73)

**Case 2:** \(\text{P_{unitary}} < \text{P_{uniform}}

In this case,

$$\lim_{\gamma \to \infty} \frac{\text{PEP}_{Z_X}(\gamma)}{\text{PEP}_{Z_Y}(\gamma)} = \lim_{\gamma \to \infty} \frac{\gamma \text{P_{unitary}} \times E_G \left( \prod_{i=1}^{T} \prod_{t=1}^{\min(N, L_i)} \frac{1}{1 + \lambda_i \gamma} \right)}{\gamma \text{P_{unitary}} \times E_G \left( \prod_{i=1}^{T} \prod_{t=1}^{\min(N, L_i)} \frac{1}{1 + \lambda_i \gamma} \right)} = 0.$$ \quad (74)

**Case 3:** \(\text{P_{unitary}} = \text{P_{uniform}}

In this case, we have

$$\lim_{\gamma \to \infty} \frac{\text{PEP}_{Z_X}(\gamma)}{\text{PEP}_{Z_Y}(\gamma)} = \lim_{\gamma \to \infty} \frac{\gamma \text{P_{unitary}} \times E_G \left( \prod_{i=1}^{T} \prod_{t=1}^{\min(N, L_i)} \frac{1}{1 + \lambda_i \gamma} \right)}{\gamma \text{P_{unitary}} \times E_G \left( \prod_{i=1}^{T} \prod_{t=1}^{\min(N, L_i)} \frac{1}{1 + \lambda_i \gamma} \right)} = \frac{E_G \left( \prod_{i=1}^{T} \prod_{t=1}^{\min(N, L_i)} \frac{1}{1 + \lambda_i \gamma} \right)}{E_G \left( \prod_{i=1}^{T} \prod_{t=1}^{\min(N, L_i)} \frac{1}{1 + \lambda_i \gamma} \right)} = c.$$ \quad (75)
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