Unified model of primordial black holes and dark matter formation

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Abstract. We propose a unified model of primordial black holes and soliton dark matter formation. Dynamic of spherically symmetric clumps of scalar field is considered in Newtonian approximation. The formation of hidden mass of the Universe is discussed. Numerical solution of the system of interacting scalar and gravitational fields is used to obtain the mass of a clumps.

1. Introduction

There are a lot of different ways to form a primordial black holes in early Universe. Some of them based on a phase transition phenomenon occurred during the process of inflation. For instance [1–3] there was elaborated a mechanism of primordial black holes (PBH) formation with wide range of masses. That kind of a mechanism could be suited to match hidden mass of the Universe very well.

Apparently the existence of complex scalar field implies the possibility of scalar clumps formation. Scalar clump is a self-gravitating (quasi)stable soliton configuration of the field. It’s existence was predicted by Wheeler and then studied very thoroughly in a number of papers [4–6].

Here we build a toy model and numerically estimate the number of PBHs and scalar clumps with different mass. In frames of that model a lot of bubbles with different value of vacua appears. Some of them could evolve into PBHs, but other stay in form of stable scalar field configuration, such as scalar clumps. Both entities could be used to explain hidden mass of the Universe.

2. Black holes dark matter

Let’s start with a mechanism of primordial black holes (PBH) formation based on first order phase transition during inflationary stage of the Universe [7], [8]. Consider complex scalar field \( \psi = \chi \exp(i\theta) \) during the inflation process. This field being additional to inflaton is described by the potential:

\[
V(\chi, \theta) = \lambda(\chi^2 - \frac{f^2}{2})^2 + \Lambda^4(1 - \cos(\theta))
\]
The process of quantum fluctuations of the field during its classical motion would lead to closed domain walls formation. In the case when domain wall collapses down to its Schwarzschild radius then the black hole forms.

The process of quantum fluctuations at inflation is described in detail in [9–12]. An amplitude of quantum fluctuations obeys the Gauss distribution with the average value \( \chi(N) \) equals to the classical component of the field at the e-fold number \( N \)

\[
dP(\chi_{\text{crit}}) = \frac{1}{\sqrt{2\pi(N_u - N)}} \exp \left( -\frac{(\chi_{\text{crit}} - \chi(N))^2}{2\delta\chi^2(N_u - N)} \right).
\]  

Here \( \delta\chi = H/(2\pi) \) is the average value of the amplitude of the field fluctuation, \( H \) - is a Hubble parameter on inflationary stage and \( N_u = 60 \) is a duration of the inflationary period.

The example of PBHs spectrum obtained numerically is presented on the figure 1. Since the mechanism of PBH formation implies that energy domain wall concentrates inside the Schwarzschild radius \( r_{\text{Sch}} \) PBHs it should be \( r_{\text{Sch}} > h \), where \( h \) is the width of the wall. If \( h > r_{\text{Sch}} \) then PBHs can’t form. Scalar field configuration formed in that case may be stable or quasi-stable and decay with time. In general one should deal with self-gravitating complex scalar field. Let us simplify the task and consider non-relativistic complex scalar field in frames of Newtonian approximation instead of General Relativity. The system of interacting gravitational and complex scalar field would lead to non-singular scalar configurations, scalar clumps or so-called boson stars [4–6].

Take Einstein action in the form

\[
S = S_g + S_m = \frac{M_{\text{pl}}^2}{2} \int R \sqrt{-g} d^4x + \int (g_{\mu\nu}\partial_{\mu}\psi^*\partial_{\nu}\psi - U(\psi)) \sqrt{-g} d^4x.
\]

here we take into account only quadratic term in potential \( U(\psi) \simeq m^2\psi^*\psi = 2\sqrt{\lambda}f\psi^*\psi \), and metric tensor:

\[
ds^2 = g_{\mu\nu}dx^\mu dx^\nu = (1 + 2V)dt^2 - (1 - 2V)(dx^2 + dy^2 + dz^2),
\]
here $V$ - is a gravitational potential, $\psi = \phi(x, t)e^{-imt}$ - is a field representation. Following the standard formalism we solve the system of two Poisson equation under assumption of spherical symmetry.

$$E\phi = \frac{1}{2r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \phi}{\partial r} \right) - V\phi$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial V}{\partial r} \right) = \phi \phi^*$$

We use dimensionless variables $m_{x_{\text{phys}}}, m_{t_{\text{phys}}} \rightarrow (x, t)$ and $\sqrt{4\pi G} \phi_{\text{phys}} \rightarrow (\phi)$ and consider clump in the stationary state.

Scalar clump mass is given by:

$$M = M_{\text{pl}}^2 \frac{r_c}{m} \int_0^{r_c} \phi^2 4\pi r^2 \, dr = \frac{M_{\text{pl}}^2 I}{m},$$

where $m$ - is a mass of field quanta, $r_c$ - is a characteristic size of a clump, i.e. the distance where gathered main amount of field energy. In general the mass of a clumps depends on the mass of scalar field quanta. Note, that in case of PBH we calculate its mass as energy of domain wall that being collected inside the Schwarzschild radius, but in case of clumps this is not true. Only a part of energy that being delivered by domain wall constitutes the mass of scalar clump. The rest of energy is reemitted in space. So the mass distribution of scalar clumps presented on figure 1 should be distorted.

3. Conclusion

Mass of the clump for the parameters we use is about 10 g, but their number is enormous. Such entities along with black holes could easily pretend to be a dark matter, contributing to full energy density of the Universe. Notice that only scalar field that has been recently discovered is the Higgs field, and there is no evidence of scalar clumps existence. So they remain purely theoretical entities by now along with primordial black holes. A mechanism of primordial black hole formation is developed in a number of papers. In turn the model of associated production of scalar clumps and black holes needs to be refined. Since the formation of scalar clump is possible only under a certain parameters and initial value of field, it seems unlikely that all domain walls, that failed to become PBHs, can form scalar clumps. One more thing about scalar clump formation in frames of Newtonian regime. Mass of the clump is given by the formula $M = \frac{M_{\text{pl}}^2 I}{m}$, and Schwarzschild radius is $r_{\text{Sch}} = \frac{2M}{M_{\text{pl}}^2} = \frac{2I}{m}$. On the other hand the physical size of the clump $r_{\text{phys}} = \frac{r_c}{m}$. If $r_{\text{Sch}} > r_{\text{phys}}$ or equivalently $2I > r_c$, then we have a contradiction with Newtonian approximation. To sum up the mechanism of numerous scalar clumps production needs further development.

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