Multiple-qubit Rydberg Quantum Logic Gate via dressed-state-based shortcut to adiabaticity

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We present a scheme to realize multiple-qubit quantum state transfer and quantum logic gate by combining the advantages of Vitanov-style pulses and dressed-state-based shortcut to adiabaticity (STA) in Rydberg atoms. The robustness of the scheme to spontaneous emission can be achieved by reducing the population of Rydberg excited states through the STA technology. Meanwhile, the timing control errors can also be minimized through using the well-designed pulses. Moreover, it can be smoothly turned on or off with high fidelity and faster than traditional shortcut to adiabaticity methods by applied dressed-state method. By using Rydberg antiblockade (RAB) effect, we show that multiple-qubit Toffoli gate can be constructed and give the general conditions of the parameters. As an example, we use the three-qubit case as to make numerical simulations, which show that the average fidelity can be higher than 99.0%.

I. INTRODUCTION

Quantum computation shows its remarkable properties in processing information and solving physical problems. Quantum algorithms designed for quantum computer are predicted to overwhelm some traditional algorithm. It can finish many difficult tasks like factoring large integers1 and searching unsorted databases2 that will cost traditional computation hundreds of years. Moreover, as people research forward, they found that apart from its strong application, information in qubits and its further topics about It from Qubit, help us get deep insights in fundamental physics. Topics including holographic duality3, quantum gravity4, quantum chaos and dynamics5, make quantum information science a very deep subject.

As for application aspect, building reliable quantum gates is of vital requirement for quantum information processing, since quantum algorithm is operated via quantum logic gates. In order to construct a quantum logic gate, a large variety of platforms and systems have been studied extensively, including Cavity Quantum Electrodynamics(CQED)6, trapped ions7,8, optical lattices9,10, quantum dot system11,12, diamond-based system13, etc. Recent years, because of its long lifetime, clean platform with small decoherence rate and robust against spontaneous emission, Rydberg atoms was regarded as an ideal candidate for realizing quantum information processing.

Among Rydberg atoms14, there exists a phenomenon named Rydberg blockade which was induced by the large electric dipole among ground states and high-excited states. In blockade radius range, no more than one atom could not be excited to Rydberg states. In contrast to the blockade regime, another interesting phenomenon was predicted by Ates et al.15, when operating two-step excitation scheme of creating an ultracold lattice gas. When the shifted energy compensated by the two-photon detuning, the so-called Rydberg antiblockade (RAB) regime will be constructed. Meanwhile, corresponding experimental realization was presented by Anthor et al.16. Controlling the detuning between the laser field and atomic transition could compensate the energy shift caused by Rydberg-Rydberg interaction(RRI) which motivate us to build RAB based quantum logic gates both theoretically17–21 and experimentally22–31. They can be divided as following categories. For the phase used to construct gate, there are dynamical gates18,32–48 and geometric gate49–57. According to the method for constructing interaction among atoms, they could be divided as blockade32–37,45–57, antiblockade17,39,58, dipole-dipole resonant interaction41 or Förster resonance42,43. Moreover, as the idea for multibit $C_N$ $NOT$ gate was presented59, heated research based on many-body cases was triggered60–71.

Apart from feasible platform, quantum information processing(QIP) requires short operating time and high fidelity, with the admissible error of gate operations being below $10^{-4}$ for a reliable quantum processor. The most famous one is stimulated Raman adiabatic passage(STIRAP) but the adiabatic conditions limited the operating time. Hence, quite a few of methods were presented to construct shortcuts72,73, for example, transitionless tracking algorithm74–78, Lewis-Riesenfeld invariants theory79,80, dressed state81,82, universal SU(2) transformation83 and so on. Although these methods speed up the evolution process, there still exist some experimental difficulties. When designing laser
pulses of schemes, we may face problems that perfect operation need an infinite energy gap or laser shape is not smooth enough to be turn on or off. Fortunately, Baksic et al. presented dressed-state protocol for solving the problems[81]. Consequently, many ideas based on this method was extended successfully[84–92]. Dressed-state approach, as a kind of shortcut to adiabatic scheme, can shorten the operating time but still remains the outstanding properties of adiabatic. Namely, by choosing dressed-state basis and control Hamiltonian $H'_c$ properly, the unwanted off-diagonal terms in Hamiltonian of the whole system can be canceled dramatically. However, though the methods stated above are good to show their advantages, some defects are still remained respectively. Because dressed-state approach, comparing to STIRAP, has the defect of suffering from spontaneous emission especially in the intermediate state.

Thus, here we present a combination of protocols that allows the advantages of each single method to cancel the defects of each other. For instance, in Rydberg system, the spontaneous emission is extremely small[93] and can promote the effectiveness of dressed-state approach. And hence it becomes a more promising scheme to prepare C-NOT gate with the advantages of long lifetime, fast speed, high fidelity and robust against pulsing area as well as timing error. Moreover, we also visualized the process of the dressed-state method that could help people understand, utilize and optimize this idea sufficiently.

The rest parts of this paper are organized as follow: First, in part II.1 we will give the basic model and effective model. Then, in part II.2 we will discuss the specific physical process of the gate in our system. After that, in part III we generalize our method to multiple qubits cases[59]. The numerical simulation will also be given in IV. Finally, V is the conclusions part.

II. TWO-QUBIT CASE

II.1. Basic physical model

The configuration of two-qubit quantum state transfer and quantum logic gate are shown in FIG.1(a). The physical process of the gate in our system. After that, in part III we generalize our method to multiple qubits cases[59]. The numerical simulation will also be given in IV. Finally, V is the conclusions part.

\[
\hat{H} = \hat{H}_1 + \hat{H}_2 + \hat{R},
\]

where

\[
\hat{R} = V |rr\rangle \langle rr|,
\]

\[
\hat{H}_1 = \Omega_c \cos \Delta t |1\rangle \langle 1| + H.c.,
\]

\[
\hat{H}_{2, step1} = (\Omega_1 |r\rangle \langle r| + \Omega_m |m\rangle \langle m| + H.c.) \cos \Delta t.
\]

\[
\hat{H}_{2, step2} = (\Omega_0 |r\rangle \langle r| + \Omega_1 |r\rangle \langle r| + H.c.) \cos \Delta t.
\]

\[
\hat{H}_{2, step3} = (\Omega_m |r\rangle \langle r| + \Omega_0 |r\rangle \langle r| + H.c.) \cos \Delta t.
\]

$V$ is the Rydberg interaction strength, $\Omega_0, \Omega_1, \Omega_m$ and $\Omega_c$ are the Rabi frequency of laser shown in FIG.1(a). $\Delta$ represents the detuning of laser and atom transition frequency. Our approach can be divided into three periods, while all of them are similar to each other: i) Add $\Omega_1$ and $\Omega_m$ to $|1\rangle$ and $|m\rangle$ respectively, let the population on $|1\rangle$ migrate to $|m\rangle$; ii) Add $\Omega_0$ and $\Omega_1$ to $|0\rangle$ and $|1\rangle$, make the population of $|0\rangle$ shift to $|1\rangle$; iii) Add $\Omega_m$ and $\Omega_0$ to $|m\rangle$ and $|0\rangle$ e, make the population of $|m\rangle$ transfer to $|0\rangle$.

As a matter of fact, for each step, the pulse scheme we used are same. All of the pulses are Vitanov-style pulses (Eq.(17,18)) after being dressed, which will be defined later in the following content.

Though exciting the control atom to Rydberg state, we can turn on or off this process which corresponds to a C-NOT gate operation.

FIG. 1. (a)Level structure of control and target Rydberg atoms driven by lasers to realize two-qubit quantum state transfer and quantum controlled-NOT gate. Control atom consists of two stable ground states $|0\rangle, |1\rangle$ and one Rydberg state $|r\rangle$ while the target atom has an additional ground state $|m\rangle$. $V$ is the Rydberg-Rydberg interaction strength between two atoms. (b) The effective model of each step. $|In\rangle$ (the state that population need to be transferred) together with $|Out\rangle$ (the state that will accepts population transferred from $|In\rangle$) show the direction of population transfer in each step. The $\Delta_{eff}, \Omega'_p$ and $\Omega'_s$ are effective detuning and pulses after calculated in Eq.(4) respectively.

In large detuning condition $\Delta \gg \Omega_{c,p,s}$ and Rydberg blockade condition $V \gg \Omega_{c,p,s}$, the dynamics of system can be described by effective Hamiltonian

\[
H_{eff} = \left( \Omega'_p |In\rangle \langle r| + \Omega'_s |Out\rangle \langle rr| + H.c. \right) + \Delta_{eff} |rr\rangle \langle rr|,
\]

where

\[
\Omega'_{p,s} = \frac{\Omega_{c,p,s}}{2\Delta}, \quad \Delta_{eff} = V - 2\Delta + \frac{\Omega^2 + \Omega'^2}{3\Delta}.
\]
Ω = control Hamiltonian method to process quantum information, the additional problem. Dressed-state method is to choose a new set of basis and \( \psi_{\text{new}} \) are spin-1 matrices in adiabatic basis. They obey the commutation relation \( [\hat{\psi}_{\text{new}}, \hat{\psi}^\dagger_{\text{new}}] \) represents the Hamiltonian under adiabatic basis 

\[
\langle \psi_{\text{in}} | H_{\text{new}} | \psi_{\text{out}} \rangle = \Omega(t) \hat{M}_x + \hat{\theta}(t) \hat{M}_y, \tag{7}
\]

where

\[
\hat{M}_x = |\varphi_+\rangle \langle \varphi_+| - |\varphi_-\rangle \langle \varphi_-|, \quad \hat{M}_y = i (|\varphi_-\rangle \langle \varphi_+| + |\varphi_+\rangle \langle \varphi_-|)/\sqrt{2} + \text{H.c.,}
\]

are spin-1 matrices in adiabatic basis. They obey the commutation relation \( [\hat{M}_i, \hat{M}_j] = i \varepsilon^{ijk} \hat{M}_k \) where \( \varepsilon \) is the anti-symmetric tensor. \( |\varphi_\pm\rangle \) and \( |\varphi_0\rangle \) are "Bight" states and "Dark" state in this three level system. At which, \( \Omega = \sqrt{(\Omega_\rho')^2 + (\Omega_\rho')^2} \) and \( \theta = - \arctan (\Omega_\rho'/\Omega_\rho) \). \( \hat{U}_{\text{ad}} \) is the unitary transformation from the original site of states \( \{|\text{In}\rangle, |\text{Out}\rangle, |rr\rangle\} \) to adiabatic states \( \{|\varphi_\pm\rangle, |\varphi_0\rangle\} \):

\[
\hat{U}_{\text{ad}}(t) = \begin{pmatrix}
\sin(\theta(t))/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} & -\cos(\theta(t))/\sqrt{2} \\
\cos(\theta(t)) & 0 & 0 & \sin(\theta(t)) \\
\sin(\theta(t))/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & -\cos(\theta(t))/\sqrt{2}
\end{pmatrix} \tag{9}
\]

Furthermore, adiabatic basis can be changed into a set of dressed-state basis via unitary operator \( V \) by using Eular angle \( \mu, \xi \) and \( \eta \):

\[
|\varphi_D(t)\rangle = V(t) |\varphi_{\text{ad}}(t)\rangle = \exp \left[ i\eta(t)\hat{M}_x \right] \exp \left[ i\mu(t)\hat{M}_y \right] \exp \left[ i\xi(t)\hat{M}_z \right] |\varphi_{\text{ad}}(t)\rangle. \tag{10}
\]

For the additional control field, which can cancel the off-diagonal term, defined as

\[
\hat{H}_c(t) = \hat{U}_{\text{ad}}^\dagger(t) \left( g_x(t)\hat{M}_x + g_z(t)\hat{M}_z \right) \hat{U}_{\text{ad}}(t). \tag{11}
\]

that do not directly couple \(|\text{In}\rangle\) and \(|\text{Out}\rangle\), \( \eta(t) \) only change the phase of off-diagonal element \( \text{see Appendix A} \) and since we set \( \eta(t) = 0 \) for simple analysis. In order to eliminate the off-diagonal elements, control field \( g_x \) and \( g_z \) should satisfy:

\[
g_x(t) = \frac{\mu}{\cos\xi} - \hat{\theta} \tan\xi, \quad g_z(t) = -\Omega + \hat{\xi} + \frac{\mu \sin\xi \hat{\theta}}{\sin\mu \cos\xi}. \tag{12}
\]

Eventually, the ultimate Hamiltonian in dressed-state basis comes into

\[
\hat{H}_{\text{new}} = \hat{\theta} + \hat{\xi} + \frac{\mu \sin\xi \hat{\theta}}{\sin\mu \cos\xi} \hat{M}_z, \tag{13}
\]

which shows there exists a dressed-dark state. The evolution operator can be written as

\[
\hat{U} = U_{\text{ad}}(t_f)\hat{V}^\dagger(t_f) \exp \left(-i \int_{t_i}^{t_f} d\tau \hat{H}_{\text{new}} \right) \hat{V}(t_i)\hat{U}_{\text{ad}}(t_i). \tag{14}
\]

Supposing the initial (final) state \(|\text{In}\rangle\) \(|\text{Out}\rangle\) corresponding eigen state \(|\varphi(t_i)\rangle\) \(|\varphi(t_f)\rangle\) of system, we can select the evolution path along with the dressed-dark state \(|\varphi_0\rangle\) and then the evolution process can be accelerated. The population of \(|r\rangle\) can be deduced in Eq.(13), or directly through the geometric relation via the projection of vector on B axis in FIG.2 (b), by following simple relation:

\[
|\langle \psi(t)|rr\rangle|^2 = \sin^2\mu(t) \cos^2\xi(t). \tag{15}
\]

One can also simply conduct that the population on state \(|10\rangle\) and \(|11\rangle\) are:

\[
|\langle \psi(t)|1\rangle|^2 = [\cos(\theta(t)) \cos(\mu(t)) + \sin(\theta(t)) \sin(\mu(t)) \sin(\xi(t))]^2, \quad |\langle \psi(t)|1\rangle|^2 = [\sin(\theta(t)) \cos(\mu(t)) - \cos(\theta(t)) \sin(\mu(t)) \sin(\xi(t))]^2 \tag{16}
\]

For prototype pulses shapes’ selection, we use Vitanov-style pulses to keep the process could be smoothly turn on and off:

\[
\Omega_p(t) = -\Omega(t) \sin\theta(t), \quad \Omega_s(t) = \Omega(t) \cos\theta(t). \tag{17}
\]

\[
\theta(t) = \frac{\pi}{2} \frac{1}{1 + \exp \left( \frac{t-t_0}{\tau} \right)}. \tag{18}
\]
parameters should be replaced by \(\tau\) and after modifying and the dynamical evolution of two-atom system is \(\hat{H} = \hat{H}_{\text{atom}} + \hat{H}_{\text{f}}\). More specifically, in three qubits case, the Hamiltonian could be written as follow:

\[
H_{\text{atom}}(t) = (\Omega_c |r\rangle_1 \langle 1| + \Omega_r |r\rangle_2 \langle 1| + \Omega_p |r\rangle_3 \langle 0| + \Omega_r |r\rangle_3 \langle 1| + \text{H.c.}) \cos(\Delta t),
\]

III. MULTIPLE QUBITS CASE VIA MANY-BODY ANTI-BLOCKADE

III.1. Three-qubit case

For three-atom case, which is a special case of n-bit case (FIG.3 (a, Left)) [58, 95], the interaction between atoms and the energy level diagram are plotted in FIG.3 (a, Right) and (b) respectively. The Hamiltonian of the n-atom system is \(\hat{H} = \hat{H}_{\text{atom}} + \hat{H}_{\text{f}}\). More specifically, in three qubits case, the Hamiltonian could be written as follow:
and interaction terms:

\[
H_I = V_{12} \langle \uparrow \uparrow \rangle \langle \uparrow \uparrow \rangle \otimes I + V_{23} I \otimes \langle \uparrow \uparrow \rangle \langle \uparrow \uparrow \rangle \\
+ V_{13} \langle \uparrow \rangle \langle \uparrow \rangle \otimes I \otimes \langle \uparrow \rangle \langle \uparrow \rangle
\]

(22)

where \( I \) refer to the identity operator. Similarly, we can calculate the effective Hamiltonian in this system:

\[
H_{\text{eff}} = \Omega_{\text{eff}}^{(p)} \langle \uparrow \uparrow \rangle \langle \uparrow \uparrow \rangle \langle \uparrow \rangle \langle \uparrow \rangle + \Delta_{\text{eff}} \langle \uparrow \uparrow \rangle \langle \uparrow \rangle \langle \uparrow \rangle \langle \uparrow \rangle \\
+ \Delta_{\text{eff}} \langle \uparrow \rangle \langle \uparrow \rangle \langle \uparrow \rangle \langle \uparrow \rangle + \text{H.c.}
\]

(23)

where

\[
\Omega_{\text{eff}}^{(p)} = \frac{30\Omega_0}{\Delta} \\
\Delta_{\text{eff}} = V_{12} + V_{23} + V_{13} - 3\Delta + \frac{\Omega_0^2 + (\Omega_0^2 + \Omega_2^2)}{3\Delta}.
\]

(24)

To make this simplify valid, the large detuning condition \( \Delta \gg \max\{\Omega, \Omega_p, \Omega_4\} \) must be satisfied. From above, we can find that the interaction term \( \langle \uparrow \rangle \langle \uparrow \rangle \langle \uparrow \rangle \langle \uparrow \rangle \) only depends on the sum of \( V_{12}, V_{13}, V_{23} \), which means that this gate can be trapped in any desired shape. Here we use the same dressed-state method in II.1 for this effective three level model. The long lifetime of Rydberg state will suppress the occurrence of error. By choosing appropriate parameters to satisfy RAB condition

\[
(9 - 2\alpha^2)\Delta^2 - 3(V_{12} + V_{23} + V_{13})\Delta - 3(\Omega_0^2 + \Omega_2^2) = 0, \quad (25)
\]

and large detuning condition, we may observe the population transfer by using auxiliary basis method was shown in FIG.4 (a, Top). Meanwhile, the average fidelity when we implement dressed-state method was shown in FIG.4 (a, Bottom). When \( \tau \) chosen as a large number, the out come corresponds to a adiabatic case FIG.4 (a, Middle).

III.2. \textbf{N-qubit case}

Moreover, to make our methods more malleable to be applied by multiple qubits. Here we post the general form of Hamiltonian in N-body case.

\[
H_{\text{eff}} = \frac{A_n^c \Omega_0^{n-1} \Omega_p^{n-1}}{2^n \Delta^{n-1}} \langle \uparrow \rangle \langle \uparrow \rangle \langle \uparrow \rangle \langle \uparrow \rangle \langle \downarrow \ldots \rangle \langle \downarrow \rangle + \frac{A_n^c \Omega_0^{n-1} \Omega_p^{n-1}}{2^n \Delta^{n-1}} \langle \uparrow \rangle \langle \uparrow \rangle \langle \uparrow \rangle \langle \uparrow \rangle \langle \downarrow \ldots \rangle \langle \downarrow \rangle + \text{H.c.} + \Delta_{\text{eff}} \langle \uparrow \rangle \langle \uparrow \rangle \langle \uparrow \rangle \langle \uparrow \rangle \\
+ \text{H.c.} + \Delta_{\text{eff}} \langle \uparrow \rangle \langle \uparrow \rangle \langle \uparrow \rangle \langle \uparrow \rangle \
\]

(26)

where \( \hat{\rho} \) is the density matrix of the quantum system and \( \mathcal{D} \) is the dissipaters superoperator.

\[
\mathcal{D}[\hat{\Gamma}]\rho = \sum_{i=1}^{N-1} \sum_{j \in \{0, 1\}} \left[ \hat{L}_{ij} \hat{\rho} \hat{L}_{ij}^\dagger - \frac{1}{2} \left\{ \hat{L}_{ij}^\dagger \hat{L}_{ij}, \hat{\rho} \right\} \right] \\
+ \sum_{k \in \{0, 1, 2\}} \left[ \hat{L}_{NK} \hat{\rho} \hat{L}_{NK}^\dagger - \frac{1}{2} \left\{ \hat{L}_{NK}^\dagger \hat{L}_{NK}, \hat{\rho} \right\} \right],
\]

(28)

where \( \hat{\Gamma} \) are the quantum jump operators with \( \Gamma_{ij} = \sqrt{\gamma_{ij}} \langle \downarrow \rangle \langle \downarrow \rangle \), \( \Gamma_{NK} = \sqrt{\gamma_{NK}} \langle \downarrow \rangle \langle \downarrow \rangle \) and \( \gamma_{NK} \) represents N-atom system. For simplicity, we assume \( \gamma_{ij} = \gamma / 2 \).
\[ \gamma_Nk = \gamma/3 \] and take the spontaneous-emission rate \( \gamma \) of Rydberg state \( |r\rangle \) as \( \gamma \simeq 2\pi \times 1 \text{kHz} \) which is proper for Rydberg atoms\[96\]. For the two-qubit C-NOT gate, FIG. 2 (d, Top) shows the population transfer when with decay. And the FIG. 2 (d, Bottom) corresponds to for Rydberg atoms\[96\]. For the two-qubit C-NOT gate, the parameters of Rydberg state \( |r\rangle \) and \( |\tilde{r}\rangle \) are chosen as \( |r\rangle = \cos \theta/2 |110\rangle + \sin \theta/2 |111\rangle \). The numerical simulation results show that the average fidelities of the three-qubit controlled-NOT gate in FIG.4(b) is 99.37%.

V. CONCLUSIONS

In conclusion, our scheme to construct C-NOT gate is presented based on dressed-state method in Rydberg system using Vitanov-style pulses. We gave both equivalent model and initial model as numerical simulation. The result shows that they match well under large detuning condition, robust against parameter fluctuations and decay than other adiabatic case. We also generalize our strategy to multi-qubit cases, by choosing appropriate parameters, n-bit Toffoli gate could be achieved.

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Appendix A: Elimination of off-diagonal elements

Hamiltonian \( \hat{H}_{\text{new}} \) under dressed-state basis

\[
\hat{H}_{\text{new}} = \left( g_x \sin \mu \sin \xi - \eta - \hat{\theta} \cos \xi \sin \mu + (\Omega + g_x - \hat{\xi}) \cos \mu \right) M_z \\
+ \frac{\omega_1}{2\sqrt{\gamma}} \left( -i g_x \cos \mu \sin \xi - g_x \cos \xi + i \sin \mu \left( g_x - \hat{\xi} + \Omega \right) + \hat{\theta} (-\sin \xi + i \cos \mu \cos \xi + \hat{\mu} \right) \left| \tilde{\varphi}_+ \rangle \langle \tilde{\varphi}_0 \right| \\
+ \frac{\omega_2}{2\sqrt{\gamma}} \left( -e^{2i\xi}(\cos \mu - \Omega)(g_x - i\hat{\theta}) + (\cos \mu + \Omega)(g_x + i\hat{\theta}) + 2ie^{i\xi}(\sin \mu(g_x - \hat{\xi} + \Omega) + i\hat{\mu}) \right) \left| \tilde{\varphi}_- \rangle \langle \tilde{\varphi}_0 \right| + \text{H.c.}
\]

(A1)

and by solving equations which eliminate all off-diagonal elements, we get

\[
g_x = \frac{\hat{\mu}}{\cos \xi} - \hat{\theta} \tan \xi \\
g_x = -\Omega + \hat{\xi} + \frac{\hat{\mu} \sin \xi - \hat{\theta}}{\tan \mu \cos \xi}
\]

(A2)

1. Effective Hamiltonian

Appendix B: Effective Hamiltonian

1. two-atom case

In this article, the way to calculate effective Hamiltonian is based on James’s method\[97\]. By rotating to the interaction picture, the Hamiltonian contains three kinds of terms with different frequencies, i.e., \( \omega_1 = \Delta \), \( \omega_2 = \Delta + V \) and \( \omega_3 = V - \Delta \). In our system, the interaction Hamiltonian could have the form:

\[
H_I = \sum_{n=1}^{\infty} h_n \exp(-i\omega_n t) + h_n^\dagger \exp(i\omega_n t)
\]

(B1)

where

\[
h_1 = \frac{\Omega_x}{\sqrt{2}} \left| 10 \right\rangle \left\langle r0 \right| + \frac{\Omega_x}{\sqrt{2}} \left| 11 \right\rangle \left\langle r1 \right| \\
+ \frac{\Omega_x}{\sqrt{2}} \left| 01 \right\rangle \left\langle 0r \right| + \frac{\Omega_x}{\sqrt{2}} \left| 11 \right\rangle \left\langle 1r \right| \\
+ \frac{\Omega_x}{\sqrt{2}} \left| 00 \right\rangle \left\langle 0r \right| + \frac{\Omega_x}{\sqrt{2}} \left| 10 \right\rangle \left\langle 1r \right| + \text{H.c.}
\]

\[
h_2 = \frac{\Omega_y}{\sqrt{2}} \left| 1r \right\rangle \left\langle rr \right| + \frac{\Omega_y}{\sqrt{2}} \left| r1 \right\rangle \left\langle rr \right| + \frac{\Omega_y}{\sqrt{2}} \left| r0 \right\rangle \left\langle rr \right| + \text{H.c.}
\]

(B2)
where $|0\rangle\langle 1\rangle$ represents $|In\rangle\langle Out\rangle$, the effective Hamiltonian can be written as

$$H_{\text{eff}}(t) = \sum_{m,n=1}^{N} \frac{1}{2} \left( \frac{1}{\omega_m} + \frac{1}{\omega_n} \right) |h_m^\dagger, h_n\rangle \exp \left( i \left[ \omega_m - \omega_n, t \right] \right)$$ (B3)

and by discarding all high frequency terms. Near the conditional anti-blockade condition $\Delta \approx 2\Delta$, the effective Hamiltonian projected into subspace \{10\}, {11}, {rr}\} can be written as

$$H_{\text{eff}} = \Omega'_p |11\rangle \langle rr| + \Omega'_s |10\rangle \langle rr| + \Delta_{\text{eff}} |rr\rangle \langle rr| \quad (B4)$$

where

$$\Omega'_p = \frac{\Omega \Omega_p}{4} \left( \frac{1}{V - \Delta} \right), \quad \Omega'_s = \frac{\Omega \Omega_s}{4} \left( \frac{1}{V - \Delta} + \frac{1}{\Delta} \right)$$

$$\Delta_{\text{eff}} = V - 2\Delta + \frac{\Omega^2_p + \Omega^2_s + \Omega^2_p}{4} \left( \frac{1}{V - \Delta} + \frac{1}{\Delta} \right) \quad (B5)$$

We suppose $V = 2(1 + \beta)\Delta$ where $\beta \ll 1$, then we get

$$\Omega'_p = \frac{\Omega \Omega_p}{2\Delta} (1 - \beta), \quad \Omega'_s = \frac{\Omega \Omega_s}{2\Delta} (1 - \beta)$$

$$\Delta_{\text{eff}} = 2\beta\Delta + \frac{\Omega^2_p + \Omega^2_s + \Omega^2_s}{3\Delta} \left( 1 - \frac{5\beta}{3} \right) \quad (B6)$$

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