Abstract. Convective heat transfer between a vegetal structure and its surrounding medium remains poorly described. However, for some applications, such as forest fire propagation studies, convective heat transfer is one of the main factors responsible for vertical fire transitions, from ground level to the tree crowns. These fires are the most dangerous because their rates of spread can reach high speeds, around one meter per second. An accurate characterization of this transfer is therefore important for fire propagation modelling. This study presents an attempt to formulate a theoretical modelling of the convective heat transfer coefficient for vegetal structures generated using an Iterated Function Systems (IFS). This model depends on the IFS parameters. The results obtained using this approach were compared with previously computed numerical results in order to evaluate their accuracy. The maximal discrepancies were found to be around 12% which proves the efficiency of the present model.

1. Introduction
It is well known that the main thermal process involved in forest fire propagation is generally the radiative heat transfer. This transfer occurs between the flame and the vegetal structures. However, when the fire propagation is driven by wind, the convective heat transfer can become as important as the radiative transfer [1]. The convective heat transfer not only increases the fire rate of spread in improving the combustion, but it is also one of the factors responsible for vertical fire transitions, from ground level (bushfires for instance) to the tree crowns. These crown fires are the most dangerous because their rates of spread can reach high speeds, around one meter per second.

In order to predict these dangerous phenomena, an accurate characterization of this transfer is therefore important for fire propagation modelling at the macroscopic scale (at the tree scale). However, at the mesoscopic scale (at the branch scale), the heat transfer between vegetal sets and the surrounding medium (air) in the context of forest fires has not yet been fully investigated and understood in existing studies.

This paper is devoted to the characterization of the convective heat transfer between a vegetal set (a tree for example) and the surrounding medium. The aim is to define macroscopic laws to predict this heat transfer. To develop a macroscopic model it is necessary to upscale a model at mesoscopic scale using any homogenisation method [2]. Using an average method similar to Marle [3], Séro-Guillaume and Margerit [4] have obtained a physical model which is used in the
The present work. This set of equations considers two temperatures and can be written as:

\[
\Phi \rho_f C_{pf} \left( \frac{\partial T_f}{\partial t} + \mathbf{U}_f \cdot \nabla T_f \right) - \nabla \cdot \left( \lambda_f \nabla T_f - \mathbf{Q}_f \right) = Q_{fs} + R_f \tag{1}
\]

\[
(1 - \Phi) \rho_s C_{ps} \frac{\partial T_s}{\partial t} - \nabla \cdot \left( \lambda_s \nabla T_s - Q_s^\prime \right) = -Q_{fs} + R_s \tag{2}
\]

In these relations, \( T_f \) and \( T_s \) are the averaged temperatures of the gas and of the solid phases respectively, \( \mathbf{U}_f \) is the averaged velocity of the gas phase, \( \lambda_f \) and \( \lambda_s \) are the heat conductivities of the two equivalent phases, \( \mathbf{Q}_f^\prime \) and \( Q_s^\prime \) are the radiative fluxes and \( R_f \) and \( R_s \) are the heat sources due to chemical reactions, \( \Phi \) is the porosity. \( Q_{fs} \) represents the heat exchange from fluid phase to solid phase. As it has been demonstrated in [5], this term can be expressed as:

\[
Q_{fs} = \chi (T_s - T_f) \tag{3}
\]

The aim of this study is to define and identify the heat transfer coefficient \( \chi \) for vegetal structures such as bushes, trees or forests by numerical and theoretical means. This heat exchange coefficient is defined as a variable by volume unit. This coefficient can be simply expressed as a variable by surface unit and denoted \( h \). The relation between \( \chi \) and \( h \) is given by,

\[
\chi = h \sigma \tag{4}
\]

where \( \sigma \) is the specific area of the medium considered (ratio of the total solid surface to the equivalent medium volume).

The present paper is organised as follows: Section 2 is dedicated to the geometrical reconstruction of vegetation by Iterated Function System tool. Section 3 deals with the numerical model developed in this work to evaluate the convection coefficient \( \chi \). Section 4 is devoted to a theoretical attempt to express the \( \chi \) coefficient as a function of macroscopic dimensionless numbers. Section 5 presents the numerical results and proposes a new correlation to estimate the convection coefficient for vegetal structures. The last section will draw conclusions form the present work and suggests potential research directions for future studies.

2. Vegetal reconstruction
In this study, only the ligneous part of trees is considered. The leaves are not taken into account. As demonstrated by Mandelbrot [6], fractal geometry, in particular Iterated Function Systems (IFS) can be used to model any kind of geometrical structures. In this work, IFS are employed...
to represent vegetal structures. A deterministic IFS algorithm is a set of $N_t$ transformations $\{\omega_1, ..., \omega_{N_t}\}$. An IFS process considers an initial geometrical set $A_0$ in $\mathbb{R}^3$ and the sequence:

$$A_{j+1} = \bigcup_{i=1}^{N_t} \omega_i(A_j)$$ (5)

The sequence converges toward a fractal set. $A_j$ is the set of elements generated at the $j^{th}$ generation. In practical terms, iterations are stopped at an order $n$, called the IFS order. For the $N_t$ transformations, affine transformations are chosen. They are obtained by a matrix multiplication of rotation matrices (regarding the Euler’s angles $\phi$, $\theta$ and $\varphi$) and a contraction coefficient $s$, as proposed in [7] and developed in [8]. For all the structures in this work, the parameters were set as follows: $s = 0.6$ and $\theta = 0$. Transformations $\omega_i$ were considered with $\phi_i = (i - 1) \frac{\pi}{2}$. Generating the sequence given by Eq. (5), tree-like structures were obtained at different orders. In setting $N_t = 4$, an example is depicted in Fig. 1 for three different iteration orders. The vegetal structures are then represented by a framework made up of cylinders with different diameters and different lengths.

The transformations considered are useful for the evaluation of morphological features such as the length $L_j$ or the diameter $D_j$ of the elements generated at order $j$ with the ones at the preceding order $j - 1$ through the contraction coefficient $s$. In considering $S_j$ the surface of every elements generated at order $j$, $S_0$ the surface of the initial element and $S_t$ the total surface of the structure, the relations are:

$$D_j = sD_{j-1} = s^jD_0$$ (6)

$$S_j = (N_t s^2)^j S_{j-1} = (N_t s^2)^j S_0$$ (7)

$$S_t = \sum_{j=0}^{n} S_j = S_0 \sum_{j=0}^{n} (N_t s^2)^j = S_0 \frac{1 - (N_t s^2)^{n+1}}{1 - N_t s^2}$$ (8)

As the elements (the branches) considered in the present study are cylindrical, $S_0$ is given by:

$$S_0 = \pi D_0 L_0$$ (9)

These discrete vegetal structures are both used for the theoretical formulation and the numerical estimation of the convection coefficient $\chi$.

### 3. Heat transfer modelling: Numerical model

For this study, a low Mach number flow is considered because the pressure is near constant for these flows. The numerical model used for this study is based on a $k - \epsilon$, $k_\theta - \epsilon_\theta$ turbulence model, according to Deng et al. [9]. The justification concerning the use of this model is due to the turbulence which is strongly anisotropic. As a consequence the assumption of a constant turbulent Prandtl number could not be made for these configurations. This model is not recalled here for the sake of completeness. The set of equations used for this work is detailed in [5].

Concerning the numerical algorithm, a QUICK scheme is used for the convective fluxes and a SIMPLEC algorithm is considered to treat the pressure-velocity coupling. The flow and the temperature fields are computed using FLUENT 6.3. The use of a CFD model such as FLUENT is justified by the easiness in adding user-defined scalars and functions, which allow the user to control the way in which the solver is applied to the system being modelled. For this study, the mesh generation, the $k_\theta-\epsilon_\theta$ model and the post-processing results are computed with user-defined functions.
The validation of new functions introduced in FLUENT is improved by the computational simulation [10] on the downstream flow of a backward-facing step for which the results have been compared with the works of Vogel et al. [11].

Concerning the boundary conditions, the ground and the vegetation are considered to be solid walls ($U = k = 0$, $\epsilon = \nu \frac{\partial^2 k}{\partial y^2}$, $k = 0$, $\epsilon = \frac{\partial u \partial \theta}{\partial y \partial y}$). The full resolution of the viscous sublayer allows the use of such boundary conditions. The boundary layer and the fully turbulent region were then matched using the Chen and Patel two-layer technique which is explained below. For the temperature, the vegetation is considered to be hot $T = 600$ K and the inlet air is fresh $T = 300$ K. These thermal boundary conditions are relevant for a heat transfer quantification in steady state. The ground was considered to be an adiabatic wall.

The plane $YZ$ at $X = 0$ is the inlet. In order to compute realistic situations, the velocity follows a logarithmic profile corresponding to an atmospheric wind profile [12]. For the present study, the inlet velocity at $z = 1$ m was in the range from 0 m/s to 5 m/s. The plane $XZ$ at $Y = 0$ was a symmetry plane whereas all the other planes were considered to be pressure outlets.

In order to resolve the near-wall part of the flow the two-layer technique of Chen and Patel [13] were used. Roughly speaking the domain is divided into two parts: (I) a region which is affected by the viscosity and (II) a fully turbulent zone. These parts are defined from a wall distance based on Reynolds number $Re_y = \frac{\rho u \sqrt{k}}{\mu}$, $y$ being the distance between cell adjacent to the wall and the nearest wall. If $Re_y > 200$ the region is considered as fully turbulent, elsewhere the flow is considered as viscosity affected. In this region an enhanced turbulent viscosity was calculated [14]. Then the outer flow and the inner flow were matched [15] for the treatment of the thermal part.

The computational domain is a parallelepipedic 3-D box. The location of the vegetal structure inside this box is shown in Figure 2. The computational domain is large enough to capture the recirculating loop behind the tree. The typical size of the computational domain is about 30 m x 20 m x 20 m. To reduce calculation costs, a symmetry plane according to the flow direction was used to reduce the cell numbers. Commonly, the total number of cells reached 3 000 000 elements which limited the number of studied cases despite of the use of a cluster of 6 workstations.

Once the results numerically obtained, the heat exchange between the solid and air phases is evaluated on the tree surface ($S_{tree}$) by:

$$Q_{fs} = -Q_{sf} = -\frac{1}{V_{\text{env}}} \int_{S_{\text{tree}}} \lambda_f \frac{\partial T}{\partial n} dS$$

where $n$ is the surface normal to the tree. According to Eq. (3), the heat transfer coefficient $\chi$ was therefore estimated by,

$$\chi = \frac{1}{V_{\text{env}}} \int_{S_{\text{tree}}} \frac{\lambda_f \frac{\partial T}{\partial n} dS}{T_{\text{mean}} - T_{\text{tree}}}$$

The envelope volume $V_{\text{env}}$ represents a convex envelope containing both the vegetal structure and air. An example of envelope volume and numerical details concerning this approach are given in [5]. The mean enthalpy and momentum are estimated over the envelope volume which contained the fluid ($V_{\text{env}}$) by,

$$C_{pT}T_{\text{mean}} = \frac{1}{V_{\text{env}}} \int_{V_{\text{env}}} C_p T dV$$
\[ \begin{align*}
\rho V_{\text{intrinsic}} &= \frac{1}{V_{\text{env}}} \int_{V_{\text{env}}} \rho f U dV \\
V_{\text{intrinsic}} &= \| V_{\text{intrinsic}} \| 
\end{align*} \]  \hspace{1cm} (13)

\[ \lambda_f \text{ is then estimated at } T_{\text{mean}}. \]

\[ \begin{align*}
\frac{Nu_{D_i}}{\lambda_f} &= C_i Re_{D_i}^{m_i} Pr^{1/3} \\
Re_{D_i} &= \frac{\rho f V_{\text{intrinsic}} D_i}{\mu_f} = \frac{Re_{D_0} s^i}{Pr} = \frac{Re_H D_0}{H s^i} \\
V_{\text{intrinsic}} \text{ is the intrinsic mean velocity of the fluid phase inside the crown and } H \text{ the crown height.} \end{align*} \]  \hspace{1cm} (15)

\[ \chi = \frac{1}{V_{\text{hull}}} \sum_{j=0}^{n} h_j S_j 
\]  \hspace{1cm} (17)
\begin{table}[h]
\begin{center}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Structure number & Order & Rotation angles & \(\sigma\) [m\(^{-1}\)] & \(H\) [m] \\
\hline
\# 1 & 4 & 0 & \(\pi/4\) & 23.6 & 0.76 \\
\# 2 & 5 & 0 & \(\pi/5\) & 33.3 & 0.76 \\
\# 3 & 5 & 0 & \(\pi/4\) & 41.9 & 0.74 \\
\# 4 & 6 & 0 & \(\pi/4\) & 44.0 & 0.85 \\
\hline
\end{tabular}
\end{center}
\caption{Properties of the vegetal structures used in this study}
\end{table}

\(V_{\text{hull}}\) is the volume of the convex hull surrounding the crown [5]. Using Eq. (15) to estimate \(h_i\), Eq. (17) becomes:

\[ \chi = \frac{\lambda_i P_r^{1/3} \pi L_0}{V_{\text{hull}}} \sum_{j=0}^{n} C Re_H^m D_0^m H^m s^{(1+m)} N_t^j \]  

(18)

Due to the tensorial order of the conductivity (second order), the Stanton number is preferred to the Nusselt number. In consequence, the Stanton number is expressed by:

\[ St = \frac{\chi H}{\rho_l C_p V_{\text{intrinsic}}} = Pr^{-2/3} C \left( \sigma D_0^{n-1} H^{2-m} \right) \frac{(1 - s^2 N_t) \left(1 - (s^{1+m} N_t)^{n+1}\right)}{(1 - (s^2 N_t)^{n+1}) (1 - (s^{1+m} N_t))} Re_H^{m-1} \]  

(19)

Eq. (19) demonstrates that the heat transfer between air and the vegetal structure can be modelled by a power law depending on the Reynolds number. Moreover, the exponent of this law seems to be constant and the prefactor coefficient could depend on the vegetal structure through the IFS parameters. \(C\) and \(m\) are estimated on the basis of \(Re_H\) according to the Chilton Colburn correlation.

5. Numerical results

This study is carried out on several academic trees with different geometrical properties. All the parameters needed to generate the vegetal structures are given in Table 1. The specific surface area and the crown heights are mentioned in this Table too.

The inlet velocity is in the range between 0 to 5 m/s. Consequently, the Reynolds numbers \(Re_H\) vary from 15,000 - 85,000. This broad range justifies the assumption done that the flow is fully turbulent. Considering these Reynolds numbers, the Chilton-Colburn parameters \(C\) and \(m\) can be set at nominal values, 0.683 and 0.466 respectively corresponding to a Reynolds number in the range between 40 to 40 000.

A first comparison is carried out on the results obtained with each model (numerical and theoretical model). Figure 3(a) shows the Stanton numbers as a function of Reynolds numbers for the 4 vegetal structures. The numerical results are depicted with symbols whereas the estimated results obtained with the analytical model are plotted in solid lines. Even if the results obtained with the structure #1 are very close, the maximal and the mean discrepancies are estimated at 70\% and 40\% respectively. This point demonstrates that the interactions between branches and the fluid flow are very important and that the assumptions used for the Chilton Colburn correlation are not well satisfied.

Clearly the analytical model cannot predict accurately the heat transfer coefficient. However, the slope given by each numerical results seems to be identical whatever the vegetal structures. It justifies that the \(m\) parameter can be considered as a constant in the theoretical model.
Nevertheless, the $C$ parameter seems to be structure dependent. The second attempt is then carried out in order to identify the $C$ and $m$ parameters thanks to the numerical results. For this identification, a genetic algorithm has been used, similar to the one developed by Duan et al. [16]. The results have been summed up in Table 2. The results obtained with this second model demonstrates that the maximal and mean discrepancies are lower than 12% and 3% respectively which proves its efficiency. However, the main drawback of this model is that one parameter ($C$) needs to be adjusted regarding the vegetal structure.

The third attempt consists in discarding $C$ in formulating a new expression for the theoretical model. Table 2 summarizes the product $C\sigma$ for each structure. It should be noted that this product is obviously constant whatever the studied vegetal structure. This observation is important, because the structure dependence introduced by $C$ can be substituted by another constant for which the value is set at 100. Consequently, the theoretical model proposed in this work can be reformulated as:

$$St = Pr^{-2/3} \left( \frac{100D_0^{m-1}H^{2-m}}{\eta_s} \right) \left( \frac{1 - s^2N_t}{1 - (s^{1+m}N_t)^{n+1}} \right) \left( \frac{1 - (s^2N_t)^{n+1}}{1 - (s^{1+m}N_t)} \right) Re_H^{m-1}$$

(20)
with $m = 0.226$. Figure 3(b) presents the results obtained with this new relation. The maximal and the mean discrepancies observed for each tree structure presented in Table 2 are lower than 12% and 6% respectively. These observations demonstrate the relevance of this new model.

6. Conclusions
The present study provides an analytical expression to quantify the heat transfer coefficient of a vegetal structure surrounded by ambient air. The vegetal structure has been generated thanks to an Iterated Function System tool. The developed model only depends on the IFS parameters used. The results obtained here exhibit a 12% maximal discrepancy proving the efficiency and the relevance of the present convective coefficient formulation. This coefficient can be now introduced inside a forest fire propagation model to take into account the heat transfer by convection between a vegetal set and surrounding air.

The next stage of this work will focus on the development of the same methodology in order to characterize the extinction coefficient of vegetal structures to improve radiative transfer modelling.

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