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SIMULATION STUDY OF ENERGY RESOLUTION OF THE ELECTROMAGNETIC SHASHLYK CALORIMETER FOR DIFFERENT OF LAYERS AND ABSORBER COMBINATIONS

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The response simulation of an ideal KOPIO-type electromagnetic sampling calorimeter was carried out in the energy range of 50 MeV – 16 GeV using Geant4-10.6.0 toolkit. In this work, we obtained energy resolution parameters for prototypes of Shashlyk calorimeter modules (ECAL SPD) of the NICA collider SPD setup for different thicknesses of a lead absorber with different numbers of layers. The NICA scientific experiment provides a unique opportunity to study parton distributions and correlations in hadron structure when working with high-intensity polarized relativistic ion beams. The ECAL electromagnetic calorimeter is one of the key detectors of the SPD device. There are some preliminary requirements for an electromagnetic calorimeter, in particular, for energy resolution in the energy range from 50 MeV to 16 GeV. It has been shown in detail that a more accurate obtaining of stochastic as well as permanent coefficients acting as parameters of the energy resolution parameterization formula is possible when longitudinal energy leakages from the calorimeter tower are taken into account. Such leakages are always present even in small amounts. Thus, the energy resolution parameterization of an ideal sampling calorimeter with a good $\chi^2 / \text{ndf}$ value is fitted with function of the type:

$$\sigma_E = \frac{a}{\sqrt{E}} \oplus b \oplus ( p_1 \cdot \ln E + p_2 \cdot \ln^2 E + p_3 \cdot \ln^3 E ),$$

where the logarithm $\ln E$ means $\ln \left( \frac{E}{E_c} \right)$, where $E_c$ is the effective critical energy. Based on the results of detailed modeling, the dependence of these parameters on the number of calorimeter plates and absorber thicknesses was found. The approach is based on careful selection and analysis of the energy spectra obtained by modeling according to the $\chi^2$-square criterion and an adequate choice of the approximation functions of the energy resolution. The methods proposed in this paper can be easily extended to other combinations of absorber-scintillator thicknesses.

KEY WORDS: sampling calorimeter, Shashlyk detector, electromagnetic calorimeter, Geant4 simulation

The NICA SPD experiment [1] provides a unique opportunity to study parton distributions and correlations in the structure of hadrons when working in beams of high-intensity polarized relativistic ions. One of the key detectors of the SPD project is the $4\pi$ electromagnetic calorimeter (ECAL), which consists of the central barrel part and two end-cap. The total weight of the electromagnetic calorimeter is about 120 tons. is that The energy resolution in the 50 MeV–16 GeV energy range should be at the level $5\% / \sqrt{E(\text{GeV})}$. It is the main preliminary requirement for the ECAL SPD. As known [2], the electromagnetic calorimeter energy resolution depends on several factors [2], which can be varied and evaluated by the Monte Carlo method with the aim of further optimization:

- sampling fluctuation;
- longitudinal and lateral energy leakage due to limited module size;
- the effect of heterogeneity of the calorimeter module medium due the design features: holes for fibers, steel studs for coupler module, fibers etc;
- fiber attenuation length;
- inhomogeneous light collection in scintillator layer;
- inaccuracies in the dimensions of calorimeter parts due the manufacturing technological features in mass production;
- photo statistics;
- electronic noise.

The total energy resolution of the ideal electromagnetic calorimeter is usually parameterized by the (in quadrature) sum of the stochastic term $a$, due to fluctuations of the Poisson type, and constant term $b$, associated with systematic effects:

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b$$

(1)

The parameter $a = \sqrt{a_1^2 + a_2^2}$ has statistical nature and represents a combination fluctuation sampling effects ($a_1$) and photo statistics fluctuations ($a_2$). In this paper we considered for the ideal calorimeter when fluctuations $a_2 = 0$. © O.P. Gavrishchuk, V.E. Kovtun, T.V. Malykhina, 2020
The parameter $b$ is associated with heterogeneous effects in medium of ECAL module which begins to dominate at high energies. Therefore, at high energies, the energy resolution is determined only by the magnitude of the constant term. The term $b$ must be zero if we take into account only sampling of fluctuations. This is clearly insufficient and uninformative to study the features of even an ideal electromagnetic calorimeter.

The purpose of this work is the correct determination the parameter $b$ for all energy range for Shashlyk ideal calorimeter. The good agreement on the $\chi^2$ criterion will be after added (in quadrature) the energy leakage terms. In this case, the correct definition of the constant term is obtained for the prototypes of ECAL SPD modules. The dependence of the $a$, $b$ terms from various combinations of absorber thicknesses $Pb$ and number of plates $L$ was obtained from the Monte-Carlo calorimeter simulation.

**ECAL KOPIO MODULE CHARACTERISTICS**

Proposed for as a prototype ECAL SPD [1] Shashlyk calorimeter based on the experience of various developments of the installation KOPIO [3] calorimeter module. This calorimeter is a lead-scintillator-type sandwich structure with a wavelength shifter (WLS) fiber. At first module KOPIO was designed and manufactured in Institute for Nuclear Research (Moscow) in 1991 for KOPIO (E865, BNL) [3, 4] experiment.

The module had experimental energy resolution $8\%\sqrt{E(GeV)}$. The calorimeter worked for a long time in the experiment and showed good stability of the parameters. Good physical characteristics with low cost and understandable design made it attractive for many other experiments. In the future, similar calorimeters were used in PHENIX (RHIC, BNL) [5], HERA-B (DESY) [6] and other installations.

Subsequently for the purposes of the KOPIO experiment resolution improvement was required to $3\%\sqrt{E(GeV)}$. This requirement led to the creation and development of new prototypes, testing in particle beams, the detailed Monte Carlo simulation. The new module prototype [7] in the $3 \times 3$ assembly tested on positrons and pions beam AGS BNL accelerator in the range $0.5 - 2 GeV/c$.

Monte Carlo simulations the optimal energy resolutions of the prototype KOPIO module and results of testing in particle beams are shown in Figure 1 [7, 8].

![Figure 1](image_url)

Figure 1. Test results KOPIO module prototypes: in particle beams (a) and computer simulation (b) by Geant3

An improved module prototype was developed and studied in detail in [8]. The final calorimeter energy resolution was obtained in the new design with best optical fibers and scintillator on the photon beam in the range of 50 - 1000 MeV.

$$\frac{\sigma_E}{E} = (2.74 \pm 0.05)\% \oplus (1.96 \pm 0.1)\%$$

(2)

Relatively large constant term ($\sim 2\%$) authors explained of the small radiation length equal to $15.9X_0$, when $X_0$ – effective radiation length. Further it was supposed to improve the energy resolution by increasing the length of the module.

In works on improving the electromagnetic calorimeter parameters, the authors [7, 8], drew attention to the dependence of the constant term on energy. Using Geant3 computer simulation of the module parameters with 6000
layers showed the “ideal” resolution of the calorimeter (see Fig. 1.), which depended only on the calorimeter sampling structure. The final module design had next parameters: $D_{Pb} = 0.275 \text{ mm}$, $D_{Sc} = 1.5 \text{ mm}$, $L = 300$. Thus, it was not possible to improve the resolution due to the improvement of the calorimeter structure and the choice of fibers and scintillator with better parameters. In reality, the energy resolution was obtained $\sim 5\%/\sqrt{E(\text{GeV})}$ on the beam.

In our opinion, formula (1), which was used by the developers of the KOPIO calorimeter, is not quite accurate, because the constant term, as we will show below, depends on energy. Formula (1) gives a good approximation of the energy resolution for small energy leakage from the module. Therefore, in real designs of calorimeters, the analysis of experimental data was not very accurate, and the desired resolution was not achieved. We illustrate (Fig. 2) the incorrectness of formula (1) in the following example. Take a very large number of layers $L = 3000$. Then, in this case, formula (1) gives good agreement with the simulation results ($\chi^2/\text{ndf} = 1.2$). In the practical case of a small number of layers with $L = 140$ we have: $\chi^2/\text{ndf} = 42.7$ and the application of formula (1) is incorrect. For comparison, the approximation of the same data is given by our refined formula (4).

For the purposes of solving physical problems at the SPD, it is desirable that ECAL have a high resolution in a wider energy range of $0.05 - 10 \text{ GeV}$ than at the KOPIO. The addition of terms to formula (1) that describe explicitly longitudinal energy leakages allows making correct parameter estimates also in a wide energy range. The energy dependence associated with energy leakage is transferred to these additional terms and the constant term is really independent of energy. The chi-square test also performs well over a wide range of energies.

**SIMULATION OF THE ELECTROMAGNETIC SAMPLING CALORIMETER**

Computer simulation of the deposit energy in the Shashlyk calorimeter modules was performed using the Monte Carlo method using C++ and the Geant4 class library \[9\] version 10.6 (December 6, 2019) in the Linux Ubuntu operating system 18.04. The ROOT6 framework \[10\] was used to process large amounts of modeling data.

For computer simulation of absorbed energy in sandwich-calorimeter modules, the most suitable model of physical processes is QGSP_BERT \[11, 12\] for primary electrons with energies in the range from $50 \text{ MeV}$ to $20 \text{ GeV}$. The Monte Carlo counting speed and the accuracy of the results are significantly affected by simulation parameters that require a certain setting. One of them in the PhysicsList from Geant4 class library is cutting energy $E_{\text{cut}}$.

If the particle energy value is lower than the threshold energy value, then the particle is considered stopped and is not monitored programatically for most processes. In the PhysicsList Geant4 class, the threshold value is specified in units of length, and is converted into values in energy units for each particle and for each material included in the simulated instrument and described in the DetectorConstruction class.

The dependence of the simulated energy resolution of the Shashlyk calorimeter from $E_{\text{cut}}$ parameter (length units) is shown in Fig. 3. The following module design was used: $D_{Pb} = 0.4 \text{ mm}$, $D_{Sc} = 1.5 \text{ mm}$, $L = 3000$.

Thus, when the parameter $E_{\text{cut}}$ changing from $10 \mu\text{m}$ to $1000 \mu\text{m}$ calorimeter response varies within $1\%$. We used the value $E_{\text{cut}} = 100 \mu\text{m}$ in the simulation.

In this work, the so-called ideal calorimeter is studied in detail. That is, only sampling and longitudinal leaks are taken into account. Transverse leakage were excluded by the appropriate choice of module dimensions ($YZ = 300 \text{ mm}$). The response of the calorimeter during the passage of an electron along the $X$ axis is proportional the total energy loss in all $L$ layers of the scintillator.
Figure 3. The calorimeter response depending on the Geant4 cut parameter. The beam energy $E_0=1000$ MeV.
-10000 µm, ▲-1000 µm, • -100 µm, ◊-10 µm.

For each energy point (beam energy $E_0$), the same number of events $N=10^4$ was simulated.

OBTAINING PARAMETERS $\bar{E}_S, \sigma_S$ FROM ENERGY SPECTRUM

In the first step calorimeter response was simulated for a fixed incoming beam energy $E_0(i)[i=1-60]$. For the beam energy points $i$ unevenly distributed throughout the energy range from 50 MeV to 16 GeV. From the energy spectrum of the response calorimeter module by the ROOT the average energy $\bar{E}_S(i)$ and standard deviation $\sigma_S(i)$ values was obtained (index S means that the values refer to the scintillator). This procedure was equally performed for a different number of layers pairs: $L=30, 100, 120, 140, 160, 180, 200, 220, 240, 300, 500, 600, 3000$. A similar calculation of the calorimeter was made and a new set of pairs of points was obtained $\bar{E}_S(i, L)$ and $\sigma_S(i, L)$. These values used to obtain a detailed dependence on the calorimeter energy resolution $\sigma_S/E$ from the beam energy $E_0(i)$.

Usually the calorimeter response in high-energy physics is approximated using the CBF function, first used Crystal Ball Collaboration [13, 14]. It has the following disadvantages: poor convergence and dependence on the obtained parameters from statistics. Therefore, the approximation was carried out by the same type function (3) proposed in [15], but with better convergence in the entire energy range and the independence of the obtained values from statistics and a smaller number of parameters.

$$f(E; \bar{E}_S, \sigma_S, kL_S, kH_S) = \begin{cases} \frac{kL^2}{e^2} e^{\frac{k}{kL} \frac{(E-\bar{E}_S)}{\sigma_S}}, & \text{for } \frac{E-\bar{E}_S}{\sigma_S} \leq -kL_S \\ \frac{1}{2} (\frac{E-\bar{E}_S}{\sigma_S})^2, & \text{for } -kL_S < \frac{E-\bar{E}_S}{\sigma_S} \leq kH_S \\ \frac{kH^2}{e^2} e^{\frac{k}{kH} \frac{(E-\bar{E}_S)}{\sigma_S}}, & \text{for } kH_S < \frac{E-\bar{E}_S}{\sigma_S} \end{cases}$$

(3)

The parameter values in function (3) have a clear meaning. Parameters $kL_S$ and $kH_S$ show the boundary values at which the “cross linking” of the Gaussian peak with exponential “tails” occurs. Data sampling $\bar{E}_S$ and $\sigma_S$ produced
in the range $\chi^2/\text{NDF} [0.8 \div 1.5]$. Those energy points for which the value $\chi^2/\text{ndf}$ went beyond the specified range, did not participate in further consideration.

Figure 4. Examples of approximating the energy response of a calorimeter by function (3)

The approximation is performed on channels with filled bins in the all channel range from 0.0 to 16.0 GeV. Examples of good approximation results are shown in Fig. 4. In the entire range of energies of primary particles function (3) gives good convergence for $\chi^2$. Sometimes the peak parameters are not determined (nan) during automated processing of a large number of spectra by ROOT. Such energy points do not take part in further analysis, but there are no more 10%.

After simulation calorimeter responses the values $E_s$ and $\sigma_s$ are founded and energy resolution curves of an ideal calorimeter are built.

**DETERMINATION ENERGY RESOLUTION COEFFICIENTS FOR THE APPROXIMATION FUNCTION**

For small number of plates the approximation function of the energy resolution of an ideal calorimeter has poor convergence according to the criterion $\chi^2$, especially for modules with few plates. Also the poor convergence may occur in the following cases: a small number significant energy points $E_i$, a narrow energy range. Accordingly, the constant term will also have different values depending on the energy range and the number of energy points $E_i$.

We take into account longitudinal leakage in the energy resolution by adding additional terms to the quadratic sum of formula (1). The form these terms is proposed for leakage in [16].

The final formula that we will use to approximate the energy resolution of the calorimeter will take the form:

$$
\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} + b \oplus \left( p_1 \cdot \ln^1 E + p_2 \cdot \ln^2 E + p_3 \cdot \ln^3 E \right)
$$  

(4)

The designation $\ln E$ implies that the quantity $\ln \frac{E}{E_s}$ - dimensionless, when $E_s$ – effective critical energy.

Table 1 shows the values $\chi^2/\text{ndf}$ for approximated functions of energy resolution according to formula (4) with longitudinal energy leakage from module. Families of energy resolution dependency curves $\frac{\sigma_E}{E}$ from beam energy $E_0$ shown in Fig. 5 – 7 for different absorber thicknesses $D_{pb}$. Only the selected energy points according to the criterion $\chi^2$ were used.

Data analysis (Table) shows that formula (4) works well in a wide range of layers numbers. In those rare cases, when $\chi^2/\text{ndf}$ is relatively large, the situation can be improved by increasing the number of simulated energy points $E_s(i)$ and narrowing the energy range. Very big $\chi^2/\text{ndf} = 17.5$ for $L=100$ means that necessary next logarithmic terms of a higher order to add in the formula (4). This indicates a very large energy leakage from the calorimeter module. From a physical point of view, we can assume that the calorimeter at such small radiation lengths ceases to work as a calorimeter. Most of the energy of an electromagnetic shower goes away in the form of leaks.

From a consideration on Fig. 5 families of curves, it can be seen that approximation (4) also well describes the energy resolution data for different $D_{pb}$ and $L$. It is important to note that in this approach, the constant term $b$ will really not depend from energy of an ideal calorimeter.
Table.

| $L$  | $D_{pb} = 0.3$ mm | $D_{pb} = 0.4$ mm | $D_{pb} = 0.5$ mm |
|------|------------------|------------------|------------------|
| 100  | 17.5             | 1.37             | 2.20             |
| 120  | 4.00             | 2.01             | 1.14             |
| 140  | 2.33             | 1.54             | 0.89             |
| 160  | 1.29             | 0.53             | 2.07             |
| 180  | 1.71             | 1.23             | 1.61             |
| 200  | 1.66             | 0.85             | 2.65             |
| 240  | 1.18             | 1.64             | 2.32             |
| 300  | 1.69             | 1.27             | 1.23             |
| 500  | 2.09             | 1.94             | 2.55             |
| 600  | 2.04             | 2.18             | 0.98             |
| 3000 | 1.96             | 1.05             | 1.32             |

Figure 5. Energy resolution of Shashlyk calorimeter depending on the layers number at different absorber thicknesses $D_{pb}$. 

GETTING DEPENDENCE OF ECAL PARAMETERS FROM THE NUMBER
OF PLATES AND THEIR THICKNESS

From the families of energy resolution curves, we can obtain (Fig. 6) the dependences of the stochastic and constant terms from the number of plates and different absorber thicknesses. The family of curves of the stochastic term $a$ was approximated by a linear function. The family of curves of the constant term $b$ was approximated by a power function.

Figure 6. The dependences of the stochastic coefficient $a$ (on left) and constant coefficient $b$ (on right) ideal calorimeter on the number of plates at different absorber thicknesses.

The curves designations: ▲ - $D_{pb} = 0.3$ mm; ■ - $D_{pb} = 0.4$ mm; ● - $D_{pb} = 0.5$ mm.

With a small number of plates, their deviation from the linear function can be explained by the small number of energy points in the region 50 MeV – ~1 GeV after selecting pairs of points $E_S(i, L)$ and $\sigma_S(i, L)$ according to selection criteria $\chi^2 / ndf$ and the inapplicability of the formula with such a small amount of substance.
Based on the results presented in Fig. 6, let’s consider an improved prototype ECAL KOPIO [8] with module parameters: $D_{ps} = 0.275 \text{ mm}$, $D_s = 1.5 \text{ mm}$, $L = 300$. We got the limiting estimates: $a = 2.5 \cdot \sqrt{\text{GeV}}\%$, $b = 0.75\%$ from the figures for an ideal KOPIO calorimeter. Thus, a resolution of 3% at 1 GeV cannot be reached. Our approach shows that longitudinal leakage with such a large number of plates do not lead to a decrease the constant term, and the magnitude $b \approx 2\%$ refers to other reasons indicated above (in particular, the dependence on energy).

Similar estimates can be made for the module ECAL SPD [1]: $D_{ps} = 0.3 \text{ mm}$, $D_s = 1.5 \text{ mm}$, $L = 220$. We have: $a = 2.5 \cdot \sqrt{\text{GeV}}\%$, $b = 1.75\%$ and the limit resolution at 1 GeV is value about 4.25%.

**CONCLUSIONS**

This paper shows that it is possible to obtain an ideal sampling calorimeter with a constant term which independent of energy by a computer simulation. The physical basis for this fact is the explicit accounting of calorimeter leakage. It was also shown that poor convergence $\chi^2$ can be associated with an insufficient number of energy points and/or a narrow region of approximation of the energy resolution.

The basis of the used approach is a rigorous selection of electromagnetic showers obtained by the Monte Carlo method in a narrow range of $\chi^2 / \text{ndf}$ values. The approximation function (3) was used for these purposes, which describes the asymmetric shape of the spectrum peak with account for leakage much better than the CBF function. The curve of energy resolution depends on energy is built on the basis of a set of selected pairs of points $E_s$ and $\sigma_s$. The correct values of the stochastic and constant terms are obtained from approximation this curve by the new formula (4) with a good convergence. Families of curves are obtained for stochastic and constant terms for lead absorber thicknesses $D_{ps} = 0.3, 0.4, 0.5 \text{ mm}$. It is possible to make estimates of $a, b$ parameters for almost any sampling and the number of calorimeter module layers based on the analysis of these curves. ECAL SPD calorimeter parameters are obtained in comparison with its prototype KOPIO of the Shashlyk type calorimeter.

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Моделювання енергетичного розрішення електромагнітного Shashlyk-калориметра при різних комбінаціях кількості пластин і товщин поглиніта

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Моделювання откликів ідеального електромагнітного сьмплінг-калориметра типу КОРИО проведено в діапазоні енергій 50 МэВ – 16 ГэВ з використанням бібліотеки класов Geant4 версії 10.6.0. В роботі побудовано параметри енергетичного розрішення для прототипів модулів Shashlyk-калориметра (ECAL SPD) установки SPD колайдера NICA [1] для різних товщин свічцового аборберу при різних кількостях пластин. Науковий експеримент NICA надає унікальну можливість для вивчення пардонних розподілень і кореляцій у структурі адронів при роботі з різноманітними енергіями.

Одним із ключових детекторів установки SPD є електромагнітний калориметр ECAL. Суттєвою є наявність енергорозрішення в діапазоні енергій від 50 МэВ до 16 ГэВ. В роботі показано, що більш точне отримання стохастичного, а особливо, константного коефіцієнта у якості параметрів формули параметризації енергетичної розподільної здатності можливо при врахуванні продольних утіків енергії з калориметра, які завжди присутні навіть у малих кількостях. Таким чином, формула для аппроксимації енергетичної розподільної здатності ідеального сьмплінг-калориметра з прийнятним значенням $\chi^2 / ndf$ набирає вигляду: 

$$\sigma_E = \frac{a}{\sqrt{E}} \oplus b \oplus \left( p_1 \cdot \ln^2 E + p_2 \cdot \ln E + p_3 \cdot \ln^2 E \right),$$

де під знаком логарифма $\ln E$ мається на увазі величина $\ln cE$, де $cE$ - ефективна критична енергія.

За результатами детального моделювання знайдена залежність цих параметрів від числа пластин калориметру і товщини аборберу. Підхід заснований на ретельному відборі і аналізі отриманих при моделюванні спектрів енергії згідно з критерієм $\chi$-квадрат та адекватному виборі функції аппроксимації енергетичної розподільної здатності. Метод легко може бути розширений на інші комбінації товщин аборбер-сцинтиллятор.

КЛЮЧЕВІ СЛОВА: сьмплінг-калориметр, Shashlyk-детектор, електромагнітний калориметр, Geant4 моделювання