Lorentz Violation on The Primordial Baryogenesis.

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ABSTRACT

Recently many studies have considered the possibility of a Lorentz Invariance Violation (LIV), and explored its consequences in a wide range of experiments. If this is true, a LIV could explain some mysteries in Cosmology. In this paper specifically, we will analyze the effects on The Primordial Baryogenesis because it is one of the more important and mysterious phenomena of the Big-Bang, that happened at very high energies, so we have a real chance to obtain an important effect. We will see that this effect could exist, depending directly on the temperature, that is very high at this time in the history of the Universe. So, it is possible to use this result as a test for a LIV and explore the possibility that the boson that started the baryogenesis explains, in part, the dark matter. We will obtain estimates about the beginning time of the baryogenesis and the boson mass too, that come directly from the LIV.

INTRODUCTION

In 1928 Paul Dirac, in his famous equation, predicted the existence of anti-particles. Now this has been experimentally checked: to each particle one anti-particle is associated, where both have the same mass, half life and opposite charge. It is natural to think that they must exist in the same number, but the reality clearly shows the opposite; it shows an asymmetry baryon-antibaryon. The production of both and this asymmetry is named Primordial Baryogenesis.

What Baryogenesis’s theories have in common are the $B$, $C$ and $CP$ violation, maintaining $CPT$ symmetry, and Departure from Thermal Equilibrium. One of the favorite theories to explain the Baryogenesis consists of introducing a new particle named $X$ Boson that is of the same form as $Z$ and $W$ bosons, and should be related to a new force that is necessary for the Grand Unification Theory.

We will study this phenomenon using statistical mechanics and cosmology, but we will introduce a Lorentz Invariance Violation (LIV) too, that is translated in a small modification of the usual energy dispersion relation.

In the Section 1 we will show the usual procedure to represent the Baryogenesis in the most simple model, explaining in detail the conditions previously mentioned. In the Section 2 we will show the effects when the LIV is included through Threshold Energy conditions and we will incorporate them to the development of the previous section. Finally we will expose the results and some predictions obtained by the LIV. An extra effect by the LIV is a possible breaking of $CPT$ symmetry, but we will forget it.

\footnote{These conditions were proposed for first time by Sakharov.}
because these effects turned out to be very little.

The LIV that, apparently, is the most important at high energies, and that we will use here, has the form \[ E^2 = v_{max}^2 p^2 + m^2 c^4 \] (1)

where \( v_{max} = c(1 - \alpha) \) is the maximum particle velocity. \( \alpha \) is very little such that it will be marked to high \( p \). Some estimates of this factor \( \alpha \), given by theoretical analysis of the physics of each particle and experimental limit of some parameters, say that is approximately \( 10^{-22} \) or \( 10^{-23} \). This value will always be little but it can be different for distinct particles \[7\ \[11\].

1 BARYOGENESIS.

For Baryogenesis to be possible, we need a Boson \( X \) that produces baryons and anti-baryons when decaying. Three conditions have to be also carried out that will allow the present baryonic asymmetry \[1\ \[2\]:

a) Baryon number violation. This mean that, while the Universe expansion is happening, the factor:

\[ B = \frac{n_b - n_{\bar{b}}}{s} \] (2)

where \( n_b, n_{\bar{b}} \) and \( s \) are the baryon and anti-baryon numbers and the entropy per comobile volume, must change. If \( B \) violation had not existed, the asymmetry would be only given due to the initial condition because \( B = cte \). So, if we consider a \( X = \bar{X} \) model, there must been a reaction of the kind:

\[ X \leftrightarrow b + \bar{b} \]
\[ X \leftrightarrow \bar{b} + b \] (3)

b) C and CP Violation, with CPT symmetry. Of this form baryons and anti-baryons have the same physics (same mass, Lorentz violation, etc) and that the boson decay rate can be different (in a small factor) to the inverse boson decay rate. If these violations did not exist, baryons and anti-baryons will be created in the same number, without preference for the matter above the anti-matter. We can interpret the CP violation, in term of the amplitude of probability, as:

\[ | M(X \to b + b) |^2 - | M(b + b \to X) |^2 = \epsilon_1 \]
\[ | M(\bar{b} + \bar{b} \to X) |^2 - | M(X \to \bar{b} + \bar{b}) |^2 = \epsilon_2 \] (4)

where \( 1 \gg \epsilon_i > 0 \) allow a little inclination in favor of the matter. As we also want to keep CPT, we must have:

\[ | M(X \to b + b) |^2 = | M(\bar{b} + \bar{b} \to X) |^2 \]
\[ | M(X \to \bar{b} + \bar{b}) |^2 = | M(b + b \to X) |^2 \] (5)

So, we can deduce the most simple model:
\[
| M(X \rightarrow b + b) |^2 = | M(\bar{b} + b \rightarrow X) |^2 = \frac{1}{2} (1 + \epsilon) | M_0 |^2
\]

\[
| M(X \rightarrow \bar{b} + \bar{b}) |^2 = | M(b + b \rightarrow X) |^2 = \frac{1}{2} (1 - \epsilon) | M_0 |^2
\]  

(6)

Where \(| M_0 |^2 | is constant.

c) Departure from Thermal Equilibrium. If equilibrium exists, we permanently have that \( n_b = n_\bar{b} \), and the matter and anti-matter amount would always be the same. A Departure from Equilibrium process is described by the Boltzmann equation [2]:

\[
\hat{L}[f] = \hat{C}[f]
\]

(7)

Where \( \hat{L} \) is the Liouville operator, \( \hat{C} \) is the Collision operator and \( f \) is the distribution function. The Liouville operator corresponds to:

\[
\hat{L} = p^\mu \frac{\partial}{\partial x^\mu} - \Gamma^\mu_{\nu \rho} p^\nu p^\rho \frac{\partial}{\partial p^\mu}
\]

(8)

and the Collision operator must consider all possible combinations of the process. Evaluating:

\[
p^\mu \frac{\partial f}{\partial x^\mu} - \Gamma^\mu_{\nu \rho} p^\nu p^\rho \frac{\partial f}{\partial p^\mu} = \hat{C}[f]
\]

(9)

Using the FRW metric and considering that \( p^\mu = [E, v_{\text{max}} \vec{p}] \) and \( x^\mu = [v_{\text{max}} \vec{x}, t] \), is reduced to:

\[
E v_{\text{max}} \frac{\partial f}{\partial t} + \frac{\dot{R}}{R} v_{\text{max}} |\vec{p}|^2 \frac{\partial f}{\partial E} = \hat{C}[f]
\]

(10)

Applying \( \frac{g_{\nu \mu}}{(2\pi \hbar)^3} \int \frac{d^3 p}{E} \), it is obtained:

\[
\frac{g}{(2\pi \hbar)^3} \int \frac{\partial f}{\partial t} d^3 p - H(t) \frac{g_{\nu \mu}}{(2\pi \hbar)^3} \int p^2 \frac{\partial f}{\partial E} d^3 p = \frac{g_{\nu \mu}}{(2\pi \hbar)^3} \int \hat{C}[f] \frac{d^3 p}{E}
\]

(11)

Where \( H(t) = \frac{\dot{R}(t)}{R(t)} \) is the Hubble constant. Integrating by parts the second term on the left and using the definition of particles number density \( n(t) = \frac{g}{(2\pi \hbar)^3} \int f d^3 p \), we obtain:

\[
\frac{\partial n(t)}{\partial t} + 3H(t)n(t) = \frac{g_{\nu \mu}}{(2\pi \hbar)^3} \int \hat{C}[f] \frac{d^3 p}{E}
\]

(12)

Where the term of the right represents the departure from equilibrium. If it is declared null, it is obtained \( n(t) = n_0 R^{-3}(t) \) that is the case of an evolution in equilibrium.
2 THRESHOLD ENERGY AND COLLISION FACTOR.

As in this paper we want to see Lorentz Violation effect, we can have energy levels where the reaction is not produced. To obtain this energy level, we will use the Threshold Energy \[9\] \[10\] \[12\], that consists on the following:

The total energy of a system of many particles is:

\[
E = \sum_{i} E_i (|p_i|) + \xi_j \left( p_i^0 - \sum_{i} p_i^j \right)
\] (13)

where \( \xi_j \) are Lagrange multipliers that impose the conservation of momentum, \( i \) level to every particle and \( j \) represent the vectorial components. Deriving with respect to \( p_i^j \), to minimize \( E \), we obtain:

\[
\frac{\partial E_i}{\partial p_i^j} = v_i^j = \xi_j
\] (14)

So, \( E \) is minimum when the velocity is the same for all particles. We will name this energy \( E_{min} = \sum_i E_i \) where all particles \( i \) have the same velocity. Now, if we have two particles that collide to produce others, we can write the energy as:

\[
E = E_1 (\vec{p} - |p_2|\hat{n}) + E_2 (|p_2|) + \chi (\hat{n}^2 - 1)
\] (15)

where \( \hat{n} \) is normalized vector of \( \vec{p}_2 \), \( \vec{p} = \vec{p}_1 + \vec{p}_2 \) is the total momentum and \( \chi \) is other Lagrange multipliers that impose the normalization of \( \hat{n} \). Maximizing \( E \) with respect to \( \hat{n} \), we obtain:

\[
\hat{n} = \frac{\vec{v}_1 |p_2|}{2\chi}
\] (16)

Since \( |\hat{n}| = 1 \), we have that \( \chi = \pm \frac{\vec{v}_1 |p_2|}{2\chi} \), so:

\[
\hat{n} = \pm \frac{\vec{v}_1}{v_1}
\] (17)

Evaluating in \( E \), we can see that the maximization occurs when \( \hat{n} = -\frac{\vec{v}_1}{v_1} \), so when a frontal collision happen. This case is represented for \( E_{max} = E_a + E_b \) where \( a \) and \( b \) are particles that collide face to face.

Finally, the threshold condition is given by:

\[
E_{max} \geq E_{min}
\] (18)

Since \( a \) and \( b \) go from one to the other and the particles \( i \) go in the same direction, we have:

\[
E_a + E_b \geq \sum_i E_i \quad p_a - p_b = \sum_i p_i
\] (19)
\subsection{Reactions Allowed Zones}

\subsubsection{\( X \longrightarrow b_1 + b_2 \)}

\( b_i \) can be a baryon or anti-baryon. Using (19):

\begin{align}
E_X &\geq E_{b_1} + E_{b_2} = 2E_b \\
p_X &= p_{b_1} + p_{b_2} = 2p_b
\end{align}

as they are the same kind particle and have the same velocity, we have that \( p_{b_1} = p_{b_2} = p_b \) and \( E_{b_1} = E_{b_2} = E_b \). As a dispersion relation is carried out of the form \( E^2 = v^2_{\text{max}}p^2 + m^2 c^4 \), we have:

\begin{align}
(v_b^2 - v_X^2)p_X^2 &\leq (m_X^2 - 4m_b^2)c^4 \quad (21)
\end{align}

If \( v_b > v_X \):

\begin{align}
p_X &\leq \sqrt{\frac{m_X^2 - 4m_b^2}{v_b^2 - v_X^2}}c \\
p_b &\leq \sqrt{\frac{m_X^2 - 4m_b^2}{4v_b^2 - v_X^2}}c
\end{align}

(22)

giving an important superior restriction to momentum. If we use \( m_X \gg m_b \) and \( v_b^2 - v_X^2 = (v_b + v_X)(v_b - v_X) \approx 2v_b^2 \partial \alpha \cos \partial \alpha = \alpha_X - \alpha_b \), it is reduced to:

\begin{align}
p_X &\leq \frac{m_X c}{\sqrt{2\partial \alpha}} \\
p_b &\leq \frac{m_X c}{2\sqrt{2\partial \alpha}}
\end{align}

(23)

If \( v_b \leq v_X \), we do not have a bound, since \( p_X^2, p_b^2 \geq 0 \).

\subsubsection{\( b_1 + b_2 \longrightarrow X \)}

Doing the same development using (19), we obtain:

\begin{align}
E_{b_1} + E_{b_2} &\geq E_X \\
p_{b_1} - p_{b_2} &= p_X
\end{align}

(24)

\begin{align}
4v_b^2p_{b_1}^2 - 4v_X^2p_Xp_{b_1} + p_X^2(v_b^2 - v_X^2) - m_X^2 c^4 &\geq 0
\end{align}

(25)

Where the approximation \( m_X \gg m_b \) was used. The solution, in \( p_{b_1} \), is a parabola with a minimum. So the zeros will give us the bounds of the reaction. If we define \( f(p_{b_1}) = ap_{b_1}^2 + bp_{b_1} + c \), we will see that \( a = 4v_b^2 \), \( b = -4v_b^2p_X \) and \( c = p_X^2(v_b^2 - v_X^2) - m_X^2 c^4 \) where the zeros are given by:

\begin{align}
p_{b_1,0} &= \frac{-b \pm \Delta}{2a} \\
\Delta &= b^2 - 4ac
\end{align}

(26)

If \( \Delta^2 < 0 \), zeros do not exist and \( f(p_b) \geq 0 \) is always carried out without bound. On the other hand, if \( \Delta^2 \geq 0 \), a zone exists where the reaction is prohibited. Evaluating, it is seen that \( \Delta^2 = 16v_b^2E_X^2 > 0 \), that is, a bound exists. The zeros are:

\begin{align}
p_{b_1,0} &= \frac{v_b p_X \pm E_X}{2v_b}
\end{align}

(27)

So, the bounds for this reaction are:
For the first bounds:

\[ p_{b_1} \geq \frac{v_B p_X + \sqrt{v_B^2 p_X^2 + m_X^4 c^4}}{2v_B} \quad \text{and} \quad p_{b_2} = p_{b_1} - p_X \leq \frac{-v_B p_X + \sqrt{v_B^2 p_X^2 + m_X^4 c^4}}{2v_B} \]  

If \( p_X \geq 0 \), clearly always \( p_{b_1} \geq 0 \), but \( p_{b_2} \geq 0 \) will only be if \( (v_B^2 - v_X^2)p_X^2 \leq m_X^2 c^4 \). If \( p_X \leq 0 \), \( p_{b_2} \geq 0 \) and \( p_{b_1} \geq 0 \) only if \( (v_B^2 - v_X^2)p_X^2 \leq m_X^2 c^4 \).

For the second bound:

\[ p_{b_1} \leq \frac{v_B p_X - \sqrt{v_B^2 p_X^2 + m_X^4 c^4}}{2v_B} \quad \text{and} \quad p_{b_2} = p_{b_1} - p_X \leq \frac{-v_B p_X - \sqrt{v_B^2 p_X^2 + m_X^4 c^4}}{2v_B} \]  

In this case, if \( p_X \geq 0 \), always \( p_{b_2} \leq 0 \), but \( p_{b_1} \leq 0 \) will only be if \( (v_B^2 - v_X^2)p_X^2 \leq m_X^2 c^4 \). The same happens if \( p_X < 0 \). So, the condition to \( \text{sign}(p_{b_1}) = \text{sign}(p_{b_2}) \) is that:

\[ (v_B^2 - v_X^2)p_X^2 \leq m_X^2 c^4 \]  

That corresponds to the decay condition that in \( X \rightarrow b_1 + b_2 \). Analyzing, we can simplify, considering \( |p_X| = |p_{b_{\text{max}}}| - |p_{b_{\text{min}}}| \), in:

\[ |p_{b_{\text{max}}}| \geq \frac{v_B |p_X| + E_X}{2v_B} \quad |p_{b_{\text{min}}}| \geq \frac{-v_B |p_X| + E_X}{2v_B} \]  

but \( p_{b_1} \) can be in any of both regions. So, the bound can be simplified to:

\[ p_{b_1} \geq \frac{-v_B |p_X| + E_X}{2v_B} \]  

That, together to (31), represent the allowed zone for the reaction. Now that we have the thresholds, we can do our calculations.

### 2.2 Collision Factor and Departure from Equilibrium Condition

The Collision Factor is:

\[ \frac{g v_{\text{max}}}{(2\pi)^3} \int C[f] \frac{d^3p}{E} = - \int (2\pi \hbar)^4 \delta^4(p_X - p_{b_1} - p_{b_2}) \sum Y_{X,b_1,b_2} \ d\Pi_1 d\Pi_2 d\Pi_X \]  

\[ Y_{X,b_1,b_2} = f_X \left( \frac{M(X \rightarrow b_1 + b_2)}{M(X \rightarrow b_2 + b_1)} \right)^2 + \frac{M(X \rightarrow b_1 + \bar{b}_2)}{M(X \rightarrow \bar{b}_1 + b_2)} \right)^2 \]  

\[ - f_{b_1} f_{b_2} \left( M(b_1 + b_2 \rightarrow X) \right)^2 - f_{b_1} f_{b_2} \left( M(\bar{b}_1 + \bar{b}_2 \rightarrow X) \right)^2 \]

Where \( f_X \), \( f_{b_1} \) and \( f_{b_2} \) are Boson, baryons and anti-baryons distribution functions respectively, and \( d\Pi_i = \frac{g_i v_i}{(2\pi \hbar)^3} d^3p_i \) and \( d\Pi_X = \frac{g_X v_X}{(2\pi \hbar)^3} d^3p_X \). The amplitudes are given by the model previously mentioned but are cancelled out of zone of calculated thresholds. This expression follows from assuming that the distribution of boson, baryons and anti-baryons approach a Boltzmann distribution of the kind:
\[
\begin{align*}
    f_X &= e^{-\frac{E_X - \mu_X}{k_B T}} \\
    f_{b_1} &= e^{-\frac{E_{b_1} - \mu}{k_B T}} \\
    f_{b_2} &= e^{-\frac{E_{b_2} + \mu}{k_B T}} \\
    f_{\bar{b}_i} &= e^{-\frac{E_{\bar{b}_i} + \mu}{k_B T}} \\
\end{align*}
\]

Where \( \mu_X, \mu \) are the boson and baryon chemical potential. We use this to simplify the expression of the collision factor. This is acceptable in the high temperature approximation, that we are using throughout this work.

We must mention that the boson is not decoupled, since if this is not like that, the effect on the baryonic asymmetry would be little for being practically Non-Relativistic (\( T \lesssim m_X \)). This is that bosons, baryons and anti-baryons are still in chemical equilibrium with the thermal bath. So we have that \( \mu_{b_i} = -\mu_{\bar{b}_i} = \mu \).

Analyzing the product \( f_{b_1} f_{b_2} \), we can see that:

\[
\begin{align*}
    f_{b_1} f_{b_2} &= e^{-\frac{E_{b_1} + E_{b_2}}{k_B T}} e^{\frac{2\mu}{k_B T}} \\
    &= e^{-\frac{E_{b_1}}{k_B T}} e^{\frac{2\mu}{k_B T}} \\
    &= f_X e^{\frac{2\mu}{k_B T}} \\
\end{align*}
\]

Where \( f_X^q \) is the chemical equilibrium boson distribution (\( \mu_X = 0 \)). The same way for the product \( f_{\bar{b}_1} f_{\bar{b}_2} \), we have:

\[
\begin{align*}
    f_{\bar{b}_1} f_{\bar{b}_2} &= f_X^eq e^{-\frac{2\mu}{k_B T}} \\
\end{align*}
\]

With these relations and using our probability of amplitudes, we obtain:

\[
\frac{g v_{\text{max}}}{(2\pi \hbar)^3} \int C[f] \frac{d^3 p}{E} = - \int (2\pi \hbar)^4 | M_0 |^2 \delta^4(p_X - p_{b_1} - p_{b_2}) \times \left[ f_X - f_X^q \right] d\Pi_1 d\Pi_2 d\Pi_X
\]

Where we have considered that \( \mu \ll k_B T \) and we keep up to first order in \( \mu \) and \( \epsilon \). As the term \( f_X \) comes from the boson decay and \( f_X^q \) of the inverse decay, they have different integration ranges. For this, we must separate them in the integral, so:

\[
\frac{\partial n(t)}{\partial t} + 3H(t)n(t) = \frac{g v_{\text{max}}^u v_X}{4v_b(2\pi \hbar)^3} \left| M_0 \right|^2 \left( I^{N_{\text{eq}}}_{b_1, b_2, X} - I^{eq}_{b_1, b_2, X} \right)
\]

Where \( I^{eq}_{b_1, b_2, X} \) and \( I^{N_{\text{eq}}}_{b_1, b_2, X} \) contain the distribution with and without equilibrium respectively and \( p_i = [E_i; v_{\text{max}}, i\hat{p}_i] \). We will resolve them in general form and then we will distinguish them. For this, we use the relation [13]:

\[
\frac{v_{b_2}^3 d^3 p_{b_2}}{2E_{b_2}} = d^3 p_{b_2} \delta \left( p_{b_2}^2 - m_{b_2}^2 c^4 \right) \Theta(E_{b_2})
\]
Then we integrate in $d^4p_{b_2}$ using $\delta^4(p_X - p_{b_1} - p_{b_2})$, that is simply replacing $p_{b_2} = p_X - p_{b_1}$, having:

$$I^a_{b_1,b_2,X} = \int \frac{f_X^a \delta \left((p_X - p_{b_1})^2 - m_{b_2}^2 c^4\right)}{P_{b_1} E_X} |p_{b_1}|^2 \sin(\theta_1) dp_{b_1} d\theta_1 d\phi_1 d^4p_X \quad (41)$$

Now we use the second delta to integrate in $\phi_1$. For this, we have the identity:

$$\delta(F(\phi_1)) = \sum_{\phi_{1,i}} \frac{1}{|F'(\phi_{1,i})|} \delta(\phi_1 - \phi_{1,i}) \quad (42)$$

With:

$$F(\phi_1) = p_X^2 + P_{b_1}^2 - 2p_X p_{b_1} - m_{b_1}^2 c^4$$
$$\quad = m_{b_1}^2 c^4 - 2E_X E_{b_1} + 2v_b v_X |\vec{p}_{b_1}| |\vec{p}_X| (\cos(\theta_X) \cos(\theta_1) + \sin(\theta_X) \sin(\theta_1) \cos(\phi_1))$$
$$\quad = -2v_b v_X |\vec{p}_{b_1}| |\vec{p}_X| \sin(\theta_X) \sin(\theta_1) \sin(\phi_1) \quad (43)$$

We can see that two values of $\phi_{1,i}$ exist; The first between 0 and $\pi$, where $\sin(\phi_{1,i}) > 0$, and other between $\pi$ and $2\pi$, where $\sin(\phi_{1,i}) < 0$ and equal in module to the previous one. So, we evaluate in $|F'(\phi_{1,i})|$ with one of them and multiply by 2, obtaining:

$$\delta(F(\phi_1)) = \frac{2}{|F'(\phi_{1,i})|} \delta(\phi_1 - \phi_{1,i}) \Theta(F^{2}(\phi_{1,i}))$$

$$I^a_{b_1,b_2,X} = 2 \int \frac{f_X^a \Theta(E_X - E_{b_1}) \Theta(F^{2}(\phi_{1,i})) |p_{b_1}|^2 |\vec{p}_X|^2 \sin(\theta_1) \sin(\theta_X) dp_{b_1} d\theta_1 dP_X d\phi_X \quad (44)$$

Where $\Theta(F^{2}(\phi_{1,i}))$ appears to assure that $\cos^2(\phi_1) \leq 1$. Besides, the value of $\phi_{1,1}$ carries out:

$$2 (E_X E_{b_1} - v_b v_X |\vec{p}_{b_1}| |\vec{p}_X| \cos(\theta_X) \cos(\theta_1) + \sin(\theta_X) \sin(\theta_1) \cos(\phi_{1,1})) = m_{b_1}^2 c^4 \quad (45)$$

Therefore:

$$|F'(\phi_{1,1})| = \sqrt{a \cos^2(\theta_1) + b \cos(\theta_1) + c} \quad (46)$$

With:

$$a = -(2v_b v_X |\vec{p}_{b_1}| |\vec{p}_X|)^2$$
$$b = -4v_b v_X |\vec{p}_{b_1}| |\vec{p}_X| \cos(\theta_X)(-2E_X E_{b_1} + m_{b_1}^2 c^4)$$
$$c = (2v_b v_X |\vec{p}_{b_1}| |\vec{p}_X| \sin(\theta_X))^2 - m_{b_1}^2 c^8 - 4E_X^2 E_{b_1}^2 + 4E_X E_{b_1} m_{b_1}^2 c^4 \quad (47)$$

So, the integral in $\theta_1$ is reduced to:

$$\int_{-1}^{1} \frac{\Theta(ax^2 + bx + c)}{\sqrt{ax^2 + bx + c}} dx \quad (48)$$

With $x = \cos(\theta_1)$. The parabola $ax^2 + bx + c$ has a maximum ($a < 0$) and the zeros are within $-1$ and 1. This means that the integral interval can be extended to $[-\infty, \infty]$ without affecting anything thanks to the Heaviside’s $\theta$. Using the relation:
we obtain:

\[
I_{b_1,b_2,X}^{\nu} = \frac{\pi}{v_0^2 v_X} \int f_X^2 \frac{\Theta(E_X - E_{b_1}) \Theta(b^2 - 4ac)}{E_X p_X} dE_{b_1} d^3 p_X
\]

(50)

Where we have used that \( E_{b_1} = v_0 p_{b_1} \). Analyzing the second Heaviside’s, we can see that its argument is positive if:

\[
\frac{E_X - v_X p_X}{2} \leq E_{b_1} \leq \frac{E_X + v_X p_X}{2}
\]

(51)

Now we must distinguish the following processes.

2.2.1 \( X \) Decay

\( a = N_{eq} \) From (23), we have the bound:

\[
E_{b_1} \leq \frac{m_X c v_b}{2\sqrt{2\alpha}} \quad \vee \quad p_X \leq \frac{m_X c}{\sqrt{2\alpha}} \quad \text{If: } v_b > v_X
\]

(52)

\[
E_{b_1} < \infty \quad \vee \quad p_X < \infty \quad \text{If: } v_b \leq v_X
\]

(53)

But we also must carry out the limits imposed by the Heaviside in the integral. With a bit of analysis, we can see that the bound given by the Threshold Energy is always greater than the inferior Heaviside limit. Therefore, the integration limits are:

\[
\frac{E_X - v_X p_X}{2} \leq E_{b_1} \leq \frac{E_X + v_X p_X}{2}
\]

(54)

If: \((v_b \leq v_X) \vee [(v_b > v_X) \text{ and } \left(p_X \leq \frac{m_X c}{2\sqrt{2\alpha}}\right)]\)

\[
\frac{E_X - v_X p_X}{2} \leq E_{b_1} \leq \frac{m_X c v_b}{2\sqrt{2\alpha}}
\]

(55)

If: \( (v_b > v_X) \text{ and } \left(\frac{m_X c}{2\sqrt{2\alpha}} < p_X < \frac{m_X c}{\sqrt{2\alpha}}\right) \)

Where we have used that \( \partial \alpha \ll 1 \). Therefore, evaluating in (50) when \( v_b \leq v_X \), we obtain:

\[
I_{b_1,b_2,X}^{N_{eq}}(v_b \leq v_X) = \frac{\pi}{v_0^2 v_X} \int f_X^2 \frac{f_X}{E_X} d^3 p_X \quad \text{With: } 0 \leq p_X \leq \infty
\]

(56)

These is almost no difference to the case without Lorentz Violation. But, if \( v_b > v_X \) we have:

\[
I_{b_1,b_2,X}^{N_{eq}}(v_b > v_X) = \frac{\pi}{v_0^2 v_X} \times
\]

\[
\left[v_X \int_A \frac{f_X}{E_X} d^3 p_X + y v_b \int_B \frac{f_X}{E_X p_X} d^3 p_X - \frac{1}{2} \int_B \frac{f_X}{p_X} d^3 p_X + \frac{v_X}{2} \int_B \frac{f_X}{E_X} d^3 p_X \right]
\]

(57)
with \( A \to (p_X \leq y) \) and \( B \to (y \leq p_X \leq 2y) \) where \( y = \frac{m_X c}{2\sqrt{2}\alpha} \). We can see that many extra factors due to the Lorentz violation and his prohibited energy zones appear. If we call \( C \to (p_X \geq 2y) \) and consider that in the \( B \) and \( C \) region have that \( E_X = v_X p_X \), we obtain:

\[
I_{b_1, b_2, X}^{N_q}(v_b \geq v_X) = \frac{\pi}{v_b^3} \times \\
n\left[ \int \frac{f_X}{E_X} d^3p_X + \frac{4\pi v_b}{v_X} \int_B f_X dE_X - \frac{4\pi}{v_X} \int_{B+C} E_X f_X dE_X \right]
\]  

Where the integration zone in the first integral extend to all momenta.

### 2.2.2 Inverse \( X \) Decay

\( (a = eq) \) In this case, the bound is given by (31) and (33), so:

\[
E_{b_1} \geq \frac{-v_b p_X + E_X}{2}
\]  

with:

\[
p_X \leq \frac{m_X c}{\sqrt{2}\alpha} \quad \text{If: } v_b > v_X
\]
\[
p_X \leq \infty \quad \text{If: } v_b \leq v_X
\]

So, the \( E_{b_1} \) limits, considering the Heaviside of the integral, are:

\[
\frac{E_X - v_X p_X}{2} \leq E_{b_1} \leq \frac{E_X + v_X p_X}{2} \quad \text{If: } v_b > v_X
\]
\[
\frac{E_X - v_b p_X}{2} \leq E_{b_1} \leq \frac{E_X + v_b p_X}{2} \quad \text{If: } v_b \leq v_X
\]

Therefore, if \( v_b > v_X \), we have:

\[
I_{b_1, b_2, X}^{eq}(v_b > v_X) = \frac{\pi}{v_b} \int_{A+B} \frac{f_X^{eq}}{E_X} d^3p_X
\]
\[
I_{b_1, b_2, X}^{eq}(v_b > v_X) = \frac{\pi}{v_b} \left[ \int \frac{f_X^{eq}}{E_X} d^3p_X - \frac{4\pi}{v_X} \int_{B+C} f_X^{eq} E_X dE_X \right]
\]

Where we have used that in the region \( C \) the boson is ultra relativistic. And if \( v_b \leq v_X \):

\[
I_{b_1, b_2, X}^{eq}(v_b \leq v_X) = \frac{\pi(v_b + v_X)}{2v_b^3 v_X} \int \frac{f_X^{eq}}{E_X} d^3p_X
\]
\[
I_{b_1, b_2, X}^{eq}(v_b \leq v_X) = \frac{\pi}{v_b^3} \int \frac{f_X^{eq}}{E_X} d^3p_X
\]

Where we use that \( \frac{v_X - v_b}{v_X} \ll 1 \).
2.3 Differential Equation Solution and Analysis

The distribution functions inside and outside of equilibrium are related by $f_X = f_X^e \frac{e^{\frac{1}{2}\frac{\mu X}{\pi T}}}{M_0}$. Now we will evaluate in (59).

a) If $v_b \leq v_X$, we use (50) and (64):

$$\frac{\partial n(t)}{\partial t} + 3H(t)n(t) = \frac{g^2 g_X v_X \pi}{4v_X^3 (2\pi \hbar)^3} | M_0 |^2 \left( \frac{e^{rac{\mu X}{kT}}}{e^{rac{\mu X}{kT}}} - 1 \right) \int \frac{f_X^e}{E_X} d^3 p_X$$

(65)

b) If $v_b > v_X$, we use (58) and (63):

$$\frac{\partial n(t)}{\partial t} + 3H(t)n(t) = \frac{g^2 g_X v_X \pi}{4v_X^3 (2\pi \hbar)^3} | M_0 |^2 \times$$

$$\left[ \left( \frac{e^{rac{\mu X}{kT}}}{e^{rac{\mu X}{kT}}} - 1 \right) \int \frac{f_X^e}{E_X} d^3 p_X + \frac{4\pi}{v_X} \int_{B+C} f_X^e dE_X \right]$$

(66)

As the unique part that depends on the time in $f_X^e$ is the temperature $T$, if we derive it, we have

$$\frac{\partial f_X^e}{\partial t} = -\frac{\beta}{v_X} E_X f_X^e$$

with $\beta = \frac{1}{v_X}$. So, deriving the differential equation and remembering that $n_X^e = \frac{g_X}{(2\pi \hbar)^3} \int f_X^e d^3 p_X$, with $n_X = e^{\beta \mu X} n_X^e$, we obtain:

a) If $v_b \leq v_X$:

$$\ddot{n}(t) + 3 \left[ \dot{H}(t)n(t) + H(t)\dot{n}(t) \right] = M(t) [n_X^e(t) - n_X(t)] + \mu_X \frac{\partial}{\partial t} e^{\beta \mu X} - 1 [\dot{n}(t) + 3H(t)n(t)]$$

(67)

b) If $v_b > v_X$:

$$\ddot{n}(t) + 3 \left[ \dot{H}(t)n(t) + H(t)\dot{n}(t) \right] = M(t) \left[ n_X^e(t) - n_X(t) + \frac{4\pi g_X}{v_X^3 (2\pi \hbar)^3} \frac{\partial J}{\partial t} \right]$$

$$+ \mu_X \frac{\partial}{\partial t} e^{\beta \mu X} - 1 [\dot{n}(t) + 3H(t)n(t)]$$

(68)

With $M(t) = \frac{g_X}{16\pi v_X \hbar} | M_0 |^2 \frac{\partial}{\partial t}$ and

$$J = e^{\beta \mu X} \left( y_b \int_B f_X^e dE_X - \int_{B+C} E_X f_X^e dE_X \right) + \int_C E_X f_X^e dE_X$$

(69)

It is the factor that represents the Lorentz violation effect. Integrating:

$$J = \frac{1}{B^2} e^{-\beta y_b} \left[ e^{-\beta y_b} (2\beta y_b + 1) - e^{\beta \mu X} (\beta y_b e^{-\beta y_b} + 1) \right]$$

(70)

By the high temperature, we know that $\beta y_b \sim 1$, moreover if the reactions that produce the Baryogenesis are sufficiently fast, we have that $\beta \mu_X \ll 1$. So:
\[ J \sim \frac{1}{\beta^2} = (k_B T)^2 \]
\[ \frac{\partial J}{\partial \beta} \sim -\frac{1}{\beta^3} = -(k_B T)^3 \]  
\( \text{(71)} \)

As \( \frac{\partial H}{\partial t} = \beta H(t) \), so \( M(t) = \frac{\hat{\beta}^2 v_X}{16\pi^2 \hbar c^2} | M_0 |^2 \beta H(t) \geq 0 \). This means that:
\[ F(\bar{n}_X, \bar{n}_X, n_X, \mu_X) \propto -H(t)T^2 \propto -T^4 \]  
\( \text{(72)} \)

where \( F \) is the usual differential equation that represents Baryogenesis without Lorentz violation (or \( v_b \leq v_X \)). As the Baryogenesis temperature is very high (Grand Unification Level), the Lorentz violation effect, when the Baryogenesis starts, is very important; whenever \( v_b > v_X \). The effects of this factor on the solution will be seen in a subsequent work. So far, the important result is that it is possible to find a trace of a possible Lorentz violation in the Baryogenesis.

Remembering the bound found with the Threshold Energy for the boson decay, if \( v_b > v_X \):
\[ p_X \leq \frac{m_X c}{\sqrt{2\partial \alpha}} \]  
\( \text{(73)} \)

we can find a limit to the temperature when these reactions start. For this, we are looking for the temperature to fullfill that:
\[ \langle p_X \rangle = \frac{m_X c}{\sqrt{2\partial \alpha}} \]  
\( \text{(74)} \)

For this, we need the relation between average momentum and temperature. Using a Fermi statistic and \( E_X = v_X p_X \), we obtain:
\[ \langle p_X \rangle = \frac{k_B T \pi^4}{30c\zeta(3)} \]  
\( \text{(75)} \)

So, the temperature at the beginning of the Baryogenesis is:
\[ k_B T_B = \frac{30\zeta(3)m_X c^2}{\pi^4\sqrt{2\partial \alpha}} \approx 0.3702 \times 10^{11} \]  
\( \text{(76)} \)

Where we used \( \partial \alpha = 5 \times 10^{-23} \) [11]. As the energies are in the Grand Unification level, it is required that \( k_B T_B \) or \( m_X c^2 \gtrsim 10^{16} \) [GeV]. If we impose this limit to \( m_X \), we obtain a temperature:
\[ k_B T_B \gtrsim 0.3702 \times 10^{27} \text{[GeV]} \]  
\( \text{(77)} \)

That matches a too much early era in the universe (Planck era). On the other hand, if we impose the limit to \( T_B \):

\[ k_B T_B \gtrsim 0.3702 \times 10^{27} \text{[GeV]} \]  
\( \text{(78)} \)
\[ m_X c^2 \gtrsim 2.7012 \times 10^5 \text{[GeV]} \]

So, in spite of having an extremely high mass \((m_X \gg m_b)\), these values are far below of the Grand Unification level (Desert). So, it is possible that the \(X\) Boson would be observed in the LHC because the maximum energies are \(\sqrt{s} = 14\) [TeV] in proton-proton collisions \([14]\).

**CONCLUSION.**

As we are at a high energy level, in the Grand Unification scale, we have a greater possibility to find a LIV effect. In this work, we saw that this effect really exists, but it becomes more important if the baryon and boson maximum velocities are related by \(v_b > v_X\). Owing to the fact that we do not know the \(X\) boson properties and difficulty to estimate the \(\alpha\) parameter, it is difficult to know if we are really in this case. But if this is the case, we would have an important trace of a LIV. Additionally, it is possible that the modification to the differential equation of the \(X\) boson decay will give some information about the probability amplitudes. It could be interesting if a LIV could be interpreted as a parameter of the type of \(\epsilon\) in \([9]\). If this is possible, the LIV could explain the preference for the matter over the anti-matter.

We estimated a condition for the moment when the Baryogenesis begun, given by the LIV. This condition tells us that \(k_B T_B = 0.262 \times 10^{11} m_X c^2\). Then the majority of bosons start to decay. The condition give us a estimation of the mass or temperature if we have other method to know one of them. Anyway, it is a condition that only appears if we impose the LIV.

Other point to analyze, that we leave proposed, is the possibility that the bosons are part of the dark matter. The inferior limit found to the boson mass is very high yet, but it is very close to the energy limit obtained by the LHC, so the \(X\) boson could be produced. We must say that this mass limit is obtained only by the LIV too.

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