Statistical properties magnetar bursts and FRB 121102

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ABSTRACT

In this paper, we present statistics of soft gamma repeater (SGR) bursts from SGR J1550-5418, SGR 1806-20 and SGR 1900+14 by adding new bursts from Kırmızıbayrak et al. (2017) detected with the Rossi X-ray Timing Explorer (RXTE). We find that the fluence distributions of magnetar bursts are well described by power-law functions with indices 1.84, 1.68, and 1.65 for SGR J1550-5418, SGR 1806-20 and SGR 1900+14, respectively. The duration distributions of magnetar bursts also show power-law forms. Meanwhile, the waiting time distribution can be described by a non-stationary Poisson process with an exponentially growing occurrence rate. These distributive features indicate that magnetar bursts can be regarded as a self-organizing critical process. We also compare these distributions with the repeating FRB 121102. The statistical properties of repeating FRB 121102 are similar with magentar bursts, combing with the large required magnetic filed ($B \geq 10^{14} G$) of neutron star for FRB 121102, which indicates that the central engine of FRB 121102 may be a magnetar.

Key words: stars: magnetars - radio continuum: transients

1 INTRODUCTION

Soft gamma repeaters (SGRs) are characterized by short ($\sim 0.1 s$) recurrent emission of gamma-rays and X-rays at irregular intervals (Kouveliotou 1995; Turolla et al. 2015; Kaspi & Beloborodov 2017). Different from normal gamma-ray bursts, the repeated bursts come from the same object and the photons are less energetic in soft gamma-ray and hard X-ray band. It has long been accepted that SGRs come from the dissipation of magnetic energy of an extremely magnetized neutron star (Thompson & Duncan 1995; Woods & Thompson 2006; Mereghetti 2008). The ultra-strong magnetic fields rotating with the central engine cause great stress to build up through the crust. When the neutron star crust could no longer support the stress, it fractures and produces the so-called ‘starquakes’. Finally, the ejecting high-energy particles are captured by the near-surface magnetic field, causing emissions of soft gamma rays and X-rays. Thompson et al. (2002) proposed that the magnetospheres of neutron stars are globally twisted. Lyutikov (2003) suggested that bursts may be produced by magnetic reconnection in the magnetosphere of magnetars. Recent studies have found that the distribution of SGR burst energy can be well fitted with a power-law function (Göğüş et al. 1999, 2000, 2001; Nakagawa et al. 2007; Israel et al. 2008; Lin et al. 2011; van der Horst et al. 2012; Prieskorn & Kaaret 2012). This power-law distribution of energy release has been seen in many natural systems featuring nonlinear energy dissipation and is commonly regarded as a symbol of self-organized critical (SOC) systems (Bak et al. 1987; Aschwanden 2011). First introduced by Bak, Tang, and Wiesenfeld in 1987 (Bak et al. 1987), the concept of SOC has been applied to a wide variety of astrophysics (Lu & Hamilton 1991; Göğüş et al. 1999; Aschwanden 2011; Wang & Dai 2013; Wang et al. 2015; Li et al. 2015; Harko et al. 2015; Yi et al. 2016; Wang, Wang & Dai 2017; Wang & Yu 2017; Yan et al. 2018; Zhang et al. 2019). SOC refers to a critical state with instability threshold in a nonlinear energy dissipation system. The great success of Lu & Hamilton (1991) in explaining solar flares by an SOC system made SOC a widely popular concept. Earthquakes, sharing many statistical properties with SGR bursts, have been described by a SOC system (Bak & Tang 1989; Olami et al. 1992). Inspired by the earthquakes models, a SOC interpretation of SGR bursts was derived (Cheng et al. 1996).

Many researches on the statistical characteristics of SGRs have been carried out, most of which focus on the distribution of burst energy (Cheng et al. 1996; Göğüş et al. 1999, 2000; Prieskorn & Kaaret 2012). The energy or total counts of SGR bursts are proved to have power-law-like size distributions. The frequency distributions of duration and waiting time are also important features of SOC systems (Aschwanden 2011). However, they have not been investigated for SGRs. Recently, Kirmizibayrak et al. (2017) presented broadband (2-250 keV) time-averaged spectral analysis of a large sample of bursts from SGR J1550-5418, SGR...
1900+14, and SGR 1806-20 detected with the Rossi X-ray Timing Explorer (RXTE). Our analysis is based on these public data.

Fast radio bursts (FRBs) are milliseconds mysterious radio transients with anomalously high dispersion measure ( Lorimer et al. 2007; Thornton et al. 2013; Petroff et al. 2019). FRBs are believed to occur at cosmological distances, which is supported by the direct localization of FRB 121102 (Chatterjee et al. 2017), FRB 180924 (Bannister et al. 2019), and FRB 190523 (Ravi et al. 2019). Ten FRBs have been found to be repeating (Spitler et al. 2016; CHIME/FRB Collaboration 2019a,b). The central engine of FRBs is still a mystery. Many theoretical models invoke magnetars (Platts et al. 2018; Cordes & Chatterjee 2019, and references therein). FRB 121102 has an extremely high Faraday rotation measure, indicating strongly magneto-ionic surrounding medium (Michilli et al. 2018), combing a coincidental continuum radio source (Chatterjee et al. 2017; Marcote et al. 2017), has inspired a model involving a young magnetar in an expanding supernova remnant (Metzger et al. 2017; Cao et al. 2017; Metzger et al. 2019).

In this paper, we give statistical analysis of over 1,500 bursts from SGR J1550−5418, SGR 1806-20 and SGR 1900+14 observed by the RXTE between 1996 and 2011. The cumulative distributions of total counts, burst duration and waiting time are shown in section 2. The distributions of total counts and burst duration show a threshold power-law form, while the waiting time distribution can be well described by a non-stationary Poisson process with an exponentially growing occurrence rate. The best-fitting results for each distribution can be found in section 3. We compare the distributions between magnetar bursts and FRB 121102 in section 4. The conclusions are given in section 5.

## 2 DATA AND METHODS

According to the updated catalog offered by the McGill Pulsar Group, a total of 15 SGRs (11 confirmed, 4 candidates) have been found (Olausen & Kaspi 2014). The most well-studied sources include SGR J1550−5418, SGR 1806−20 and SGR 1900+14. Kouveliotou (1995) have built an on-line database of magnetar bursts observed by the Rossi X-ray Timing Explorer (RXTE) which can be found at http://magnetars.sabanciuniv.edu. In this paper, we consider 1,535 bursts from SGR J1550−5418, SGR 1806−20 and SGR 1900+14 detected in 15 years. A short description of the database entries is given in Table 1. We focus on the distributions of fluence, duration and waiting time of magnetar bursts. The value of fluence and duration can be directly obtained from the on-line database. Power-law distribution of duration is also an important prediction of SOC theory (Bak et al. 1987; Aschwanden 2011). However, this prediction has not been explored for magnetar bursts. The frequency distribution of duration is studied in this paper. We have measured the waiting times between successive bursts through \( \delta t = t_{start,i+1} − t_{start,i} \), where \( t_{start,i} \) and \( t_{start,i+1} \) are the start times for the \( i \) th and \( i+1 \) th bursts, respectively. However, in order to avoid the effects of Earth occultation and data gaps, only the continuous observation data is used.

Generally, a nonlinear process can be simply expressed by an exponential growth phase with saturation after a random time interval, which has been applied to many different scientific areas (Willis & Yule 1922; Rosner & Vaiana 1978; Newman 2005; Pinto et al. 2012). The size distribution \( N(x) \) of exponentially growing avalanches can be written as

\[
N(x) = n_0(x_0 + x)^{-\alpha - 1} dx.
\]

(1)

For \( x_1 \leq x \leq x_2 \), the normalization constant \( n_0 \) is expressed as

\[
N_0 = n_{ev}(1 - \alpha n_00(x_0 + x)^{-1} - (x_0 + x_1)^{-1} - 1).
\]

(2)

Since a constant \( x_0 \) has been added to the ideal power-law distribution function, such a distribution can be called as thresholded power-law size distribution, or differential occurrence frequency distribution (Bour & Davy 1997; Aschwanden 2015). Hereafter, we simply refer it as the differential distribution.

When the data sample is too small, a cumulative distribution function is more often used in fitting. The cumulative size distribution is defined as the integral number of events above a given value \( x \). If \( x_1 \) and \( x_2 \) represent the minimum and maximum value of the size distribution, and \( n_{ev} \) refers to the total number of events, the thresholded cumulative size distribution can be expressed as

\[
N_{cum}(x) dx = \int_n x \, n_0(x_0 + x)^{-\alpha - 1} dx + 1 + (n_{ev} - 1) \left( \frac{(x_0 + x)^{-\alpha - 1} - (x_0 + x_1)^{-\alpha - 1}}{(x_0 + x)^{-\alpha} - (x_0 + x_1)^{-\alpha}} \right).
\]

(3)

It can be seen that \( x_0 \) and \( \alpha \) are the only two free variables in the cumulative distribution.

The parameter \( x_0 \) can largely improve the fitting result at the lower bound of a power-law-like distribution, and has its own physical meanings. It could be attributed to the instability threshold of the system, as well as the sub-sampling below the detection threshold (Aschwanden 2015). These two causes are physically different, but can be mathematically treated in the same way.

For each source, we derived both differential and cumulative distributions of total counts, burst duration and burst waiting time. The dataset of each distribution is a series of event sizes with a total number of \( n_{ev} \). To begin with, the data is uniformly binned on a logarithmic scale between the minimum and maximum sampled data \( (x_1 \) and \( x_2 \)). Empirically, for differential distributions, we set the number of bins to be \( \log(x_2/x_1) \times 10 \), while for cumulative distributions, the number is \( \log(x_2/x_1) \times 5 \) (Aschwanden 2015). The threshold \( x_0 \) is determined from the bin with the maximum number of events, ensuring that an ideal power-law fit can be applied to the data range \( [x_0, x_2] \). The expected uncertainty of the differential distribution can be written as

\[
\sigma_{diff,i} = \sqrt{N_{bin,i}/Ax_i}.
\]

(4)

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Table 1.

| Source Name | Observation Period | Burst Number |
|-------------|--------------------|--------------|
| SGR J1550-5418 | Oct 2000-Apr 2010 | 179         |
| SGR 1806-20 | Nov 1996-Jun 2011 | 924         |
| SGR 1900+14 | Jun 1998-Aug 2006 | 432         |

List of the database entries.
While for the cumulative distribution, the uncertainty is

\[ \sigma_{\text{cum},i} = \sqrt{N_{\text{bin},i}/\Delta t_i}. \]  

(5)

Each data bin as well as the uncertainties are demonstrated in Figures 1, 2 and 3. The threshold \( \delta \) is marked with a red dashed line.

For the distributions of total counts and burst duration, the dataset can be well described by Equation (1). Since the total number of events \( n_{\text{ev}} \) and the constant \( \delta \) have already been identified, the normalization constant \( N_0 \) can be expressed as a function of \( \alpha_\delta \). Thus, the power-law slope \( \alpha_\delta \) is the only free variable to optimize, which can be obtained by Bayesian statistical analysis.

For the distributions of burst waiting time, the data reflects a superposition of multiple exponential distributions with different time scales, which could be attributed to a non-stationary Poisson process (Wheatland et al. 1998; Aschwanden 2011). Such waiting time distributions indicate random processes with time-dependent event rates, and can also be characterized with Bayesian statistics (Wheatland et al. 1998; Wheatland & Litvinenko 2002). The probability function can be approximately expressed as a subdivision into discrete time intervals, within which the occurrence rate is constant (Equation 6). Thus, the non-stationary process is regarded as a superposition of several stationary processes with occurrence rates \( \lambda_1, \lambda_2, \ldots, \lambda_n \). The waiting time distribution is

\[ P(\Delta t) = \begin{cases} 
\lambda_1 e^{-\lambda_1 \Delta t} & \text{for } t_1 \leq t \leq t_2 \\
\lambda_2 e^{-\lambda_2 \Delta t} & \text{for } t_2 \leq t \leq t_3 \\
\vdots & \text{for } t_n \leq t \leq t_{n+1} \\
\lambda_n e^{-\lambda_n \Delta t} & \text{for } t_n \leq t \leq t_{n+1+1} 
\end{cases} \]  

(6)

The dataset shows power-law-like distributions with slopes close to 2 for large waiting times. We suppose that the variability of burst rate shows spikes like \( \delta \)-functions (Aschwanden & McTiernan 2010). The occurrence rate \( \lambda \) is exponentially growing (equation 7) and fulfills the normalization requirement

\[ \int_0^\infty \lambda f(\lambda) d\lambda = \lambda_0. \]

(7)

The waiting time distribution in a given time interval can be written as

\[ P(\Delta t) = \frac{\lambda_0}{(1 + \lambda_0 \Delta t)^2}. \]

(8)

where the mean burst rate \( \lambda_0 \) is the only variable to optimize.

3 FITTING RESULTS

We use the open source probabilistic programming framework PyMC3 to perform the fitting. PyMC3 uses Theano to compute gradients via automatic differentiation and allows model specification directly in Python code (Salvatier et al. 2016). For the distributions of total counts and burst duration, the power-law slope \( \alpha_\delta \) is created as a stochastic random variable with normal prior distribution and the step method is run for 5000 iterations. The mean value and standard deviation of the collected samples in the returned trace object are regarded as the best-fit slope and its uncertainty for each distribution.

For cumulative distributions, the best-fit power-law slopes of total counts are \( 1.840 \pm 0.033, 1.682 \pm 0.008 \), and \( 1.654 \pm 0.014 \) for SGR J1550-5418, SGR 1806-20 and SGR 1900+14, respectively. These value are consistent with those derived by Gögüş et al. (1999, 2000) and Prieskorn & Kaaret (2012). SOC theory not only predicts the power-law distribution of energy, but also predicts power-law distribution of duration (Aschwanden 2011). For duration distributions, the power-law slopes are \( 1.698 \pm 0.034, 1.723 \pm 0.008 \), and \( 1.821 \pm 0.016 \) for SGR J1550-5418, SGR 1806-20 and SGR 1900+14, respectively.

The waiting time distribution of these three datasets can be described by a Poisson processes with mean burst rate \( \lambda \). Similarly, the best-fit \( \lambda \) and its uncertainty for each waiting time distribution is obtained by PyMC3. Because observations cover a long time, the burst rate varies with times, which can be treated as a non-stationary Poisson process.

A compilation of the best-fitting parameters for all the distributions is listed in Table 2. Also, the fits of cumulative size distributions are shown in Figures 1, 2 and 3 for SGR J1550-5418, SGR 1806-20 and SGR 1900+14, respectively. Figure 4 shows the histograms of the burst waiting times. We have fit the waiting time distributions to a log-Gaussian function. The peaks are 208 s (with \( \sigma \sim 4.27 \)), 478 s (\( \sigma \sim 6.31 \)) and 115 s (with \( \sigma \sim 5.75 \)) for SGR J1550-5418, SGR 1806-20 and SGR 1900+14, respectively. Gögüş et al. (1999) found that the waiting times of SGR 1900+14 can be fitted with log-Gaussian distribution, but with a low peak about 498 s. For waiting times of SGR 1806-20, Gögüş et al. (2000) derived a log-Gaussian distribution with a peak at 103 s from observation and numerical simulations. Considering the errors of fitting, our results for SGR 1806-20 and SGR 1900+14 are larger than those of Gögüş et al. (1999) and Gögüş et al. (2000). The main reason is that the difference of burst identification methods. In Gögüş et al. (1999) and Gögüş et al. (2000), the bursts are selected using a phase-folding technique (Woods et al. 1999). The bursts used in this paper are analyzed using Bayesian blocks algorithm provided in Scargle et al. (2013) and Lin et al. (2013). For SGR 1900+14, 837 bursts are derived by Gögüş et al. (1999), compared to 432 bursts with Bayesian blocks algorithm in a long observation period (Kurniazbayrak et al. 2017). Therefore, the peak of waiting time distributions is larger for the bursts in Kurniazbayrak et al. (2017). Interestingly, Hurley et al. (1994) found that the waiting times of SGR 1806-20 can be fitted with log-Gaussian distribution with a peak about 1.63 \times 10^6 s. This result may be affected by data gap. Overall, our result is between the value of Gögüş et al. (1999) and Hurley et al. (1994).

4 COMPARISON WITH FAST RADIO BURSTS

In this section, we compare the distributions between SGRs and fast radio bursts (FRBs). By now, dozens of FRBs have been discovered, which are listed in FRB Catalogue (http://frbcat.org/) (Petroff et al. 2016). Eleven FRBs are repeating (Spitler et al. 2016; CHIME/FRB Collaboration 2019a,b). There are some phenomenological similarities be-
between SGRs and FRBs, including repeatability, timescales and the duty factor of pulses (Kulkarni et al. 2014; Katz 2016; Wang & Yu 2017).

The high brightness temperatures \( T_b \geq 10^{37} \) K of FRBs require a coherent emission process (Katz 2016; Lyutikov 2019). The two most common mechanisms are coherent curvature radiation produced near the surface of the neutron star (Lu & Kumar 2018; Yang & Zhang 2018) and the synchrotron maser process (Lyubarsky 2014; Metzger et al. 2019). For the synchrotron maser process, the emission is from an ultra-relativistic shock moving towards observer, which propagates into medium of moderately high magnetization, \( \sigma > 10^3 \) (Lyubarsky 2014; Beloborodov 2017). In the magnetar scenario, these shocks result from the sudden release of energy during the earliest stages of a flare. Some theoretical models for FRBs basing on magnetars have been proposed (Popov & Postnov 2013; Lyubarsky 2014; Beloborodov 2017, 2019; Metzger et al. 2019), which can explain most observational properties of FRB 121102. For example, the persistent radio nebula associated with FRB 121102 can be produced by ion ejecta from the magnetar flares (Beloborodov 2017). Meanwhile, both energy and particle content of the nebula are consistent with this scenario, calibrated by observations of ejecta from SGR 1806-20. Margalit & Metzger (2018) proposed that electron-ion nebula may explain the rotation measure observed in FRB 121102.

We use the largest sample of FRB 121102, which is observed by GBT at 4-8 GHz (Zhang et al. 2018). This sample contains 21 pulses reported in Gajjar et al. (2018) and 72 pulses identified by machine learning. These pulses were observed within a 6 hours observation. They share the same observation conditions and were observed by the same telescope. Therefore, we can put them together to analysis and ignore complex selection effects. Power-law distributions of energy \( \alpha_E = 1.63 \pm 0.06 \) and distributions \( \alpha_T = 1.57 \pm 0.13 \) for these 93 FRB 121102 bursts are shown in Figure 5. Gourdji et al. (2019) discovered a low-energy sample with 41 bursts for FRB 121102 and found \( \alpha_E \approx 1.7 \) if all bursts are included (see their Figure 5). However, if the low-energy bursts are discarded, a steeper \( \alpha_E \approx 2.8 \) is found. Wang & Zhang (2019) also found that six samples of FRB 121102 bursts observed by different telescopes at different frequencies show a universal energy distribution with \( \alpha_E \) around 1.7. Meanwhile, similar power-law index of energy distribution for non-repeating FRBs is also found (Lu & Piro 2019; Zhang & Wang 2019).

The waiting time distribution of FRB 121102 also can be described by a non-stationary Poisson process with mean burst rates \( \lambda_0 = 1.23 \pm 0.08 \times 10^{-5} \text{ ms}^{-1} \) (Zhang et al. 2018) found that the rate of detection is not stationary and the distribution of waiting time cannot be well fitted using Poissonian distribution for the same sample. For a small sample of waiting times of FRB 121102, Oppermann et al. (2018) modeled the distribution of waiting times as Weibull distribution, which can describe non-Poissonian distributions with clustering. It must be noted that the non-stationary Poissonian distribution used in this paper is similar to the Weibull distribution. Because the rate of bursts in a non-stationary Poisson process also varies with time (Wheatland et al. 1998). The mean burst rate is \( 1.23 \pm 0.08 \times 10^{-5} \text{ ms}^{-1} \). Zhang et al. (2018) found that the rate of detection is not stationary and the distribution of waiting time cannot be well fitted using Poissonian distribution for the same sample. For a small sample of waiting times of FRB 121102, Oppermann et al. (2018) modeled the distribution of waiting times as Weibull distribution, which can describe non-Poissonian distributions with clustering. It must be noted that the non-stationary Poissonian distribution used in this paper is similar to the Weibull distribution. Because the rate of bursts in a non-stationary Poisson process also varies with time (Wheatland et al. 1998). The mean burst rate is \( 1.23 \pm 0.08 \times 10^{-5} \text{ ms}^{-1} \). Using the same data, Zhang et al. (2018) found the rate is \( 0.05 \text{ s}^{-1} \) for Poissonian distribution. Using a sparse waiting time sample, Oppermann et al. (2018) derived a mean repetition rate of \( 5.73 \pm 3.0 \text{ day}^{-1} \). The large discrepancy between the two rates is that the waiting times used in this paper are derived from 93 bursts in 5 hr observation, comparing to 17 bursts in about 74 hr observation in Oppermann et al. (2018).

We also show the fitting results of energy, duration

| Burst Statistics | SGR 1550-5418 | Cumulative Distribution |
|------------------|---------------|--------------------------|
| Total counts \( \alpha_E \) | 1.667 ± 0.078 | 1.840 ± 0.033 |
| Burst Duration \( \alpha_T \) | 1.503 ± 0.008 | 1.698 ± 0.034 |
| Waiting Time \( \lambda_0 \) | 1.702 ± 0.080 s\(^{-1}\) | / |

| Burst Statistics | SGR 1806-20 | Cumulative Distribution |
|------------------|---------------|--------------------------|
| Total counts \( \alpha_E \) | 1.741 ± 0.067 | 1.682 ± 0.008 |
| Burst Duration \( \alpha_T \) | 1.430 ± 0.005 | 1.723 ± 0.008 |
| Waiting Time \( \lambda_0 \) | 0.697 ± 0.039 s\(^{-1}\) | / |

| Burst Statistics | SGR 1900+14 | Cumulative Distribution |
|------------------|---------------|--------------------------|
| Total counts \( \alpha_E \) | 1.672 ± 0.048 | 1.654 ± 0.014 |
| Burst Duration \( \alpha_T \) | 1.505 ± 0.006 | 1.821 ± 0.016 |
| Waiting Time \( \lambda_0 \) | 1.311 ± 0.049 s\(^{-1}\) | / |

| Burst Statistics | FRB 121102 | Cumulative Distribution |
|------------------|---------------|--------------------------|
| Energy \( \alpha_E \) | - | 1.63 ± 0.06 |
| Duration \( \alpha_T \) | - | 1.57 ± 0.13 |
| Waiting Time \( \lambda_0 \) | \( 1.23 \pm 0.08 \times 10^{-5} \text{ ms}^{-1} \) | / |

Table 2. The best-fit parameters of cumulative distributions and differential distributions for SGR J1550-5418, SGR 1806-20, SGR 1900+14 and FRB 121102.
and waiting time for FRB 121102 in Table 2. Therefore, similar distributions between FRB 121102 and magnetar bursts are found. Meanwhile, Metzger et al. (2019) discussed that FRBs could arise from synchrotron maser emission at ultra-relativistic magnetized shocks, such as produced by flare ejecta from young magnetars. This model can explain the observational properties of FRBs, including burst duration, high intrinsic linear polarization, an spectral energy distribution with complex frequency structure, the downward evolution of frequency structure in FRB 121102 and 180814.J0422+73, and time-varying dispersion measure. More interestingly, their model has a testable prediction. According to their analysis (equation (40) of Metzger et al. (2019)), the luminosity of FRBs is
\[ L_{\text{FRB}} \approx 3 \times 10^{32} f_3 E_{39} \gamma^{-1} \text{erg s}^{-1}, \]
where \( f \) is the radiative efficiency, and \( E_b \) is the energy of magnetar burst. From this equation, we can see that the luminosity of FRB is linearly proportional to the energy of magnetar burst. Our results support that the central engine of FRB 121102 is a magnetar. More recently, a similar model basing on magnetar has been proposed (Beloborodov 2019), which can also explain most features of FRBs.

The millisecond duration of FRBs requires a compact object origin, i.e., neutron stars. From the energy budget, the repeating FRB 121102 must be powered by the magnetic energy (Lyutikov et al. 2017), which is similar to but more extreme than giant flares produced by Galactic magnetars. The internal magnetic energy of neutron star is
\[ E_B = 4\pi R_{\text{NS}}^3 B^2 / 2 \pi = 3 \times 10^{45} \text{erg} B_{14}^2, \]
where \( B \) is the magnetic field and \( R_{\text{NS}} = 12 \) km is the radius of the neutron star. The maximum number of bursts produced by a given magnetar is
\[ N_{\text{FRB}} = \frac{E_b}{E_{\text{FRB}}} \approx 0.03 f_b^{-1} \left( \frac{f_6}{10^{-8}} \right)^2 \left( \frac{B}{10^{16} \text{G}} \right)^2 \left( \frac{E_{\text{FRB}}}{10^{39} \text{erg}} \right)^{-1}, \]
where \( f_b \) is the fraction of the flare energy placed into FRB emission, \( E_b \) is the beaming factor, and \( E_{\text{FRB}} \) is the typical energy of FRB 121102 (Wang & Zhang 2019). The FRB efficiency is fixed to a value of \( f_6 = 10^{-6} \) (Lyubarsky 2014). From observation, FRB 121102 has been active at least 7 years with more than 200 bursts. From equation (11), the neutron star magnetic field has to be larger than a threshold value:
\[ B \geq 8 \times 10^{15} G f_b \left( \frac{f_6}{10^{-8}} \right)^{1/2} \left( \frac{E_{\text{FRB}}}{10^{39} \text{erg}} \right)^{1/2}. \]

Therefore, for a large range of \( f_b \geq 1.5 \times 10^{-4} \), the central neutron star must be a magnetar (\( B \geq 10^{14} \text{G} \)). Although the value of \( f_b \) is uncertain, it cannot be too small, which causes that the beaming-correct FRB rate (proportional to \( f_b^{-1} \)) would greatly exceed the rate of known objects (Zhang 2016), such as core-collapse supernova (Dahlen et al. 2004), binary neutron star merger (Abbott et al. 2017), long gamma-ray bursts (Yu et al. 2015) and short gamma-ray bursts (Zhang & Wang 2018).

It is important to search radio bursts associated with high-energy bursts of magnetars. Interestingly, XTE J1810-197 is the first ever magnetar emitting transient radio burst. More recently, the second radio outburst that has been observed from this magnetar (Maan et al. 2019). The bursts show a characteristic intrinsic width of the order of 0.5–0.7 ms. It is also found that the bursts exhibit possible similar spectral structures to that of FRB 121102 (Maan et al. 2019). The magnetar J1810-197 is only the third object after repeating FRBs and the Crab pulsar which is found to exhibit frequency structures, may provide a link between the underlying emission mechanisms for magnetars and repeating FRBs. However, higher time-resolution of observation is required.

5 CONCLUSIONS

In this paper, we find power-law distributions of total counts and durations for bursts from three magnetars. Power-law energy distributions have also been found for solar flares with \( \alpha_E = 1.53 – 1.73 \) (Crosby et al. 1993; Lu et al. 1993; Aschwanden 2011), for flares from gamma-ray bursts with \( \alpha_E = 1.06 \) (Wang & Dai 2013; Yi et al. 2016, 2017), for flares from black holes with \( \alpha_E = 1.6 – 2.1 \) (Wang et al. 2015; Yan et al. 2018). Power-law duration distributions also found in above systems. This is a typical behavior for SOC systems. The concept of SOC (Bak et al. 1987) states that subsystems self-organize due to some driving force to a critical state at which a slight perturbation can cause a chain reaction of any size within the system. Magnetar power-law energy and duration distributions, along with a non-stationary waiting time distribution, support that systems responsible for magnetar bursts are in a SOC state. For magnetars, the critical systems are neutron star crusts strained by evolving magnetic stresses (Thompson & Duncan 1995). We also found similar energy, duration and waiting time distributions between magnetars and FRB 121102, together with some theoretical models of FRBs basing on magnetar, which indicate that the central engine of FRB 121102 is a magnetar. In future, much more repeating FRBs will be discovered. The connection between repeating FRBs and magnetars can be tested.

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Statistical properties of SGRs and FRBs
Figure 1. Distributions of total counts, durations and waiting times and best-fitting results (red lines) for SGR J1550-5418.

Figure 2. Distributions of total counts, durations and waiting times and best-fitting results (red lines) for SGR 1806-20.
Figure 3. Distributions of total counts, durations and waiting times and best-fitting results (red lines) for SGR 1900+14.

Figure 4. Frequency distribution histograms of waiting time for SGRs. The best-fit log-Gaussian distribution curves are marked as red dashed lines.
Figure 5. Frequency distributions of energy, duration and waiting time for FRB 121102. The best-fit results (red lines) are also shown.