Local Cosmology

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Abstract

We define the concept of a Maximally symmetric osculating space-
time at any event of any given Robertson-Walker model. We use this
definition in two circumstances: i) to approximate any given cosmo-
logical model by a simpler one sharing the same observational param-
eters, i.e, the speed of light, the Hubble constant and the deceleration
parameter at the time of tangency, and ii) to shed some light on the
problem of considering an eventual influence of the overall behaviour
of the Universe on localized systems at smaller scales, or viceversa.

1 Introduction

Expressions such as Local cosmology, or Local cosmology of isolated sys-
tems sound as a contradiction of terms because by cosmology we mean the
physics of that single isolated system that we call our Universe. What our
title reminds is that between the scale of the Universe as a whole, which is
certainly a more complex subject that any present or future model will be
able to contemplate, and that of much smaller structures, there is room to
be interested in those scales of time and space which without spanning the
whole universe include a significant fraction of it.

In Sect. 2 we propose to approximate a complex cosmological model by a
simpler one in the neighbourhood of any event at which the three fundamen-
tal observational cosmological parameters are known. This approach mimics
an elementary geometrical construction. Consider a curve in a plane whose
Cartesian graph is a function $F(T)$. The parallel to the $T$ axis through a
point with coordinates $(T, F(T))$ is a very rough approximation to the curve.
If we want a better approximation we can draw the tangent to the curve at the point. We still can do better by drawing the osculating circle at the point. We will see that this process suggests to us to approximate a general Robertson-Walker model by either one of five possible Robertson-Walker geometries with constant space-time curvature.

In Sect. 3 we compare the Robertson-Walker-like description of models with constant space-time curvature to their static one, emphasizing the problem of identifying the meaning of the radial coordinate in each case. We conclude that the two descriptions do not correspond to different frames of reference but to different operational meanings of the radial coordinate.

In Sect. 4 we choose an appropriate expression for the energy-momentum source of a Robertson-Walker geometry to be able to consistently derive the evolution through time $T$ of space and space-time curvatures of the corresponding osculating model.

In Sect. 5 we consider the problem of describing the perturbation of a cosmological model by a single inhomogeneity, that we assume to be spherically symmetric. Or equivalently the perturbation of a spherically symmetric compact source that feels that spatial infinity is not flat because it is modified by a Robertson-Walker behaviour. This problem has been already considered in several papers ([1]-[4]). We take here a simplifying view restricting ourselves to consider the problem in the context of the dual description of Kottler’s solution as a static or time dependent metric.

Our concluding remarks include some hints about how to deal with eventual discrepancies between theory and observations that could be relevant to the subject of this paper.

### 2 Local osculating cosmological models

Let us consider a Robertson-Walker Model that we shall write as:

$$dS^2 = -dT^2 + \frac{1}{c^2(T)} \left( \frac{dR^2}{1 - kR^2} + R^2 d\Omega^2 \right), \quad d\Omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2$$

where $c(T)$ has dimensions of velocity and $k$ is the space curvature of the model. We make thus explicit an obvious possible interpretation, [5], of Robertson-Walker cosmological models as describing a continuous sequence, labelled by a speed parameter constant $\frac{\ddagger}{\ddagger} c(T)$, of static space-times where any physical process whose evolution time-scale is fast compared to the time-scale evolution of the effective speed of light $c(T)$ can be described.

\footnote{The word constant is here a language licence as one says the Hubble constant}
From a geometrical point of view the approximation that this interpretation suggests amounts to approach a graph \( c(T) \) by a parallel to the time axis taken at the time when the process is considered. In this paper we want to improve on this idea of approximating a cosmological model by a simpler one that in some precise sense is locally equivalent to it in some interval of \( T \).

Let us consider the line-element \([120x701]\) at the time \( T_{ref} \) when we start observing the local properties of the space-time around our location. We shall assume that we have reset the clock we use to:

\[
T_{ref} = 0, \tag{2}
\]

so that if we measure the effective speed of light at this time we obtain some value \( c_0 \) so that:

\[
c(0) = c_0. \tag{3}
\]

What is usually called the scale factor is then the function:

\[
F(T) = \frac{c_0}{c(T)} \tag{4}
\]

and from our point of view it is in fact a refractive index for which the speed of reference is \( c_0 \). We assume also that both the Hubble constant and the deceleration parameter are known at \( T = 0 \):

\[
H_0 = -\frac{\dot{c}_0}{c_0}, \quad q_0 = -2 + \frac{c_0 \ddot{c}_0}{\dot{c}^2_0} \tag{5}
\]

We shall say that a Robertson-Walker model with line-element:

\[
dS^2 = -dT^2 + \frac{1}{\bar{c}^2(T)} \left( \frac{dR^2}{1 - k R^2} + R^2 d\Omega^2 \right) \tag{6}
\]

is maximally symmetric and osculating to the model defined by the line-element \([120x701]\) at \( T = 0 \) iff:

i) it has constant space-time curvature, i.e. the function \( \bar{c}(T) \) is a solution of the following two equations:

\[
\ddot{\bar{c}}^2 + \bar{k} \bar{c}^4 = 1/3 \bar{\Lambda} \bar{c}^2, \tag{7}
\]

\[
2\bar{c}^2 - \bar{c} \dddot{\bar{c}} = 1/3 \bar{\Lambda} \bar{c}^2. \tag{8}
\]

where \( \bar{k} \) and \( \bar{\Lambda} \) are respectively the space and the space-time curvature constants.
ii) the two functions \( c(T) \) and \( \bar{c}(T) \) have in common their value and the values of their first and second derivatives at \( T = 0 \).

\[
\bar{c}_0 = c_0, \quad \dot{\bar{c}}_0 = \dot{c}_0, \quad \ddot{\bar{c}}_0 = \ddot{c}_0 \tag{9}
\]

ensuring thus that a line element and its osculating one have the same observable values of \( c_0, H_0 \) and \( q_0 \).

From (3), (5) and the preceding conditions i) and ii) it follows that:

\[
\bar{\Lambda} = -3q_0H_0^2, \quad \bar{k} = -\frac{H_0^2}{c_0^2}(q_0 + 1) \tag{10}
\]

These formulas allow to distinguish five types of osculating maximally symmetric space-times to a given Robertson-Walker model, depending on the value of \( q_0 \):

1.- If \( q_0 = 0 \) the osculating space-time is Milne’s model. In this case one has:

\[
\bar{\Lambda} = 0, \quad \bar{k} = -\frac{H_0^2}{c_0^2} < 0 \tag{11}
\]

and the function \( \bar{c}(T) \) is:

\[
\bar{c}(T) := \frac{1}{p(T + A)}, \quad A = \frac{1}{c_0p} \tag{12}
\]

where here and below the constant \( A \) has been chosen such that \( c(0) = c_0 \) and \( p \) such that \( p = \sqrt{|k|} \).

2.- If \( q_0 = -1 \) the osculating space-time is the proper de Sitter model \((dS_0)\). In this case one has:

\[
\bar{\Lambda} = 3H_0^2 > 0, \quad \bar{k} = 0 \tag{13}
\]

and the function \( \bar{c}(T) \) is:

\[
\bar{c}(T) = c_0 \exp(\lambda T), \tag{14}
\]

where here and below \( \lambda = \sqrt{1/|\bar{\Lambda}/3|} \). These two models are not generic in the sense that unless one demands explicitly the corresponding condition for the line-element \( (\Pi) \) one can not expect to have either \( k = 0 \) or \( \Lambda = 0 \) exactly.

The generic models are the following:

3.- if \( q_0 < -1 \) the osculating space-time is a positive space-curvature de Sitter model \((dS_+\)). In this case one has:
\[ \vec{k} > 0, \quad \vec{\Lambda} > 0 \]  
and the function \( \vec{c}(T) \) is:

\[ \vec{c}(T) = \frac{\lambda}{p} \text{sech}(\lambda(T + A)), \quad A = \frac{1}{\lambda} \text{arcsech} \left( \frac{pc_0}{\lambda} \right) \]  
\( 16 \)

4.- if \(-1 < q_0 < 0\) the osculating space-time is a negative space-curvature de Sitter model (\(dS_-\)). In this case one has:

\[ \vec{k} < 0, \quad \vec{\Lambda} > 0 \]  
and the function \( \vec{c}(T) \) is:

\[ \vec{c}(T) = \frac{\lambda}{p} \text{csch}(\lambda(T + A)), \quad A = \frac{1}{\lambda} \text{arccsch} \left( \frac{pc_0}{\lambda} \right) \]  
\( 18 \)

5.- if \( q_0 > 0 \) the osculating space-time is known as anti de Sitter model (AdS). In this case one has:

\[ \vec{k} < 0, \quad \vec{\Lambda} < 0 \]  
and the function \( \vec{c}(T) \) is:

\[ \vec{c}(T) = \frac{\lambda}{p} \text{sec}(\lambda(T + A)), \quad A = \frac{1}{\lambda} \text{arcsec} \left( \frac{pc_0}{\lambda} \right) \]  
\( 20 \)

3 Interpretation of the static description

As it is well-known the line-elements corresponding to these five models admit local time-like integrable Killing fields and therefore they can be brought with an appropriate coordinate transformation to the static form:

\[ ds^2 = -(1 - \frac{\vec{\Lambda}}{3c_0^2} r^2) dt^2 + \frac{1}{c_0^2} (1 - \frac{\vec{\Lambda}}{3c_0^2} r^2)^{-1} dr^2 + \frac{1}{c_0^2} r^2 d\Omega^2 \]  
\( 21 \)

We give below the corresponding coordinate transformations from this static form to the line-elements corresponding to the five functions \( \vec{c}(T) \) listed in the preceding section. The \( r \) coordinate transformation is in every case:

\[ r = \frac{Rc_0}{\vec{c}(T)} \]  
\( 22 \)

while the corresponding time transformations are:

1.- For Milne’s space-time:
\[ t = \sqrt{1 + p^2 R^2 (T + A)} \]  

(23)

2.- For the proper de Sitter’s \(dS_0\) space-time:

\[ t = T - \frac{1}{2\lambda} \ln \left( 1 - \frac{\lambda^2 R^2}{c(T)^2} \right) \]  

(24)

3.- For de Sitter’s \(dS_+\) model:

\[
\begin{align*}
  t &= \frac{1}{\lambda} \text{arctanh} \left( \frac{(1 + pR) \tanh \left( \frac{1}{2} \lambda (T + A) \right)}{\sqrt{1 - p^2 R^2}} \right) \\
  &\quad + \frac{1}{\lambda} \text{arctanh} \left( \frac{(1 - pR) \tanh \left( \frac{1}{2} \lambda (T + A) \right)}{\sqrt{1 - p^2 R^2}} \right)
\end{align*}
\]  

(25, 26)

4.- For de Sitter’s \(dS_-\) model:

\[
\begin{align*}
  t &= \frac{1}{\lambda} \text{arctanh} \left( \frac{pR + \tanh \left( \frac{1}{2} \lambda (T + A) \right)}{\sqrt{1 + p^2 R^2}} \right) \\
  &\quad + \frac{1}{\lambda} \text{arctanh} \left( \frac{-pR + \tanh \left( \frac{1}{2} \lambda (T + A) \right)}{\sqrt{1 + p^2 R^2}} \right)
\end{align*}
\]  

(27, 28)

5.- And finally for the anti de Sitter model:

\[
\begin{align*}
  t &= \frac{1}{\lambda} \text{arctan} \left( \sqrt{1 + p^2 R^2} \tan \left( \frac{1}{2} \lambda (T + A) \right) + pR \right) \\
  &\quad + \frac{1}{\lambda} \text{arctan} \left( \sqrt{1 + p^2 R^2} \tan \left( \frac{1}{2} \lambda (T + A) \right) - pR \right)
\end{align*}
\]  

(29, 30)

The dual description of any model with constant space-time curvature either in a static Killing frame of reference or in a Robertson-Walker time-dependent conformal Killing frame of reference raises a problem of physical interpretation that we want to illustrate with the consideration of a particularly simple scenario. Let us consider the static description of the Anti de Sitter model. An elementary calculation shows that a free test particle can describe circles of any radius \(r\) around any point with angular velocity:

\[ \frac{d\phi}{dt} = \lambda \]  

(31)

in the plane \(\theta = \pi/2\). On the other hand these circular orbits will be described from the Robertson-Walker point of view as spiralling orbits such that:
\[ R = \frac{\lambda r}{p} \sec(\lambda(T + A)) \]  

(32)

The mathematics is transparent. With different variables we have different laws of evolution, but we physicists have to understand better the problem. We have to know the meaning of \( r, t, R \) and \( T \) and the meaning of the coordinate transformations (22)-(29) to reach some understanding of the reality of the physical process we are describing.

Let us assume that a test body, located at a point with radial coordinate \( r \), in the space-time and the frame of reference described by (21) has initial velocity zero at some instant \( t \). An elementary calculation shows that at the same instant it has a radial acceleration:

\[ \frac{d^2 r}{dt^2} = \frac{1}{3} \tilde{\Lambda}(1 - \frac{1}{3}c^2_0)\tilde{r}^2 r \quad \left( \frac{dr}{dt} = 0 \right) \]  

(33)

This shows that even when \( r \) is infinitesimal it can not be considered to be a distance that can be measured with a stretched thread because the origin \( r = 0 \) being arbitrary such an interpretation is in contradiction with the homogeneity of the space-time described by (21). On the contrary if we consider the same solution in the frame of reference described by (6) we obtain:

\[ \frac{d^2 R}{dT^2} = 0 \quad \frac{dR}{dT} = 0 \]  

(34)

which is consistent with the interpretation of small values of \( R \) as distances measured with an stretched thread. The meaning of \( r \) that follows from (22) is that small infinitesimal values of this variable can be interpreted as Radar distances.

The interpretation of the variables \( t \) and \( T \) offers no problem because they are both adapted synchronizations derived from proper time along the same world-line \( r = R = 0 \).

4 To follow through: an example

Let us consider a particular Robertson-Walker model with line-element (6). We derive below convenient expressions of \( \tilde{\Lambda}(T) \) and \( \tilde{k}(T) \) corresponding to the osculating model at each time \( T \) as explicit functions of \( c(T), \rho(T) \) and \( P(T) \). The latter being respectively the mass density function of the model and the pressure function. To do that we shall use the field equations in the following form:
\[ S_{\alpha\beta} + \Lambda g_{\alpha\beta} = 8\pi GT_{\alpha\beta} \]  

(35)

where \( S_{\alpha\beta} \) is the Einstein tensor of the line-element (6), \( \Lambda \) is the proper global cosmological constant of the model, with dimensions Time\(^{-2}\) and \( T_{\alpha\beta} \) is:

\[
T_{\alpha\beta} = \rho(T)u_{\alpha}u_{\beta} + \frac{P(T)}{c(T)^2}(g_{\alpha\beta} + u_{\alpha}u_{\beta})
\]

(36)

with \( u_0 = -1 \) and \( u_i = 0 \). Eqs. (35) are then the usual equations with a small but important difference: the function \( c(T) \) is a replacement for a constant \( c_0 \) considered to be a universal constant. Notice that this writing of the field equations does not require any Einstein's gravitational constant different from Newton's constant.

The explicit Einstein's equations (35) reduce to the following two:

\[
3\frac{k c(T)^4 + \dot{c}(T)^2}{c(T)^2} = 8\pi G\rho(T) + \Lambda
\]

(37)

\[
\frac{k c(T)^4 + 5\dot{c}(T)^2 - 2c(T)\ddot{c}(T)}{c(T)^2} = -8\pi G \frac{P(T)}{c(T)^2} + \Lambda
\]

(38)

Using the definitions (5) for an arbitrary time:

\[
H(T) = -\frac{\dot{c}(T)}{c(T)}, \quad q(T) = -2 + \frac{c(T)\ddot{c}(T)}{\dot{c}(T)^2}
\]

(39)

we get solving the two algebraic equations with unknowns \( H(T)^2 \) and \( q(T) \):

\[
H^2 = -kc(T)^2 + \frac{1}{3}(8\pi G\rho(T) + \Lambda)
\]

(40)

\[
q = \frac{4\pi G(3P(T) + \rho(T)c(T)^2) - \Lambda c(T)^2}{c(T)^2(-3kc(T)^2 + 8\pi G\rho(T) + \Lambda)}
\]

(41)

Using then the definitions (10) with \( c(T) \), \( H(T)^2 \) and \( q(T) \) instead of \( c_0 \), \( H_0^2 \) and \( q_0 \), we obtain finally:

\[
\bar{\Lambda}(T) = \frac{-4\pi G(3P(T) + \rho(T)c(T)^2) + \Lambda c(T)^2}{c(T)^2}
\]

(42)

\[
\bar{k}(T) = \frac{k c(T)^4 - 4\pi G(\rho(T)c(T)^2 + P(T))}{c(T)^4}
\]

(43)

In [5] we considered a model characterized by the appealing equation of state:
\[ P(T) = \frac{1}{3} \rho(T)c(T)^2 \] (44)

Using the field equations (35), (36) leads to a particularly simple density dependence on \( c(t) \):

\[ \rho(T) = Bc(T) \] (45)

where \( B \) is a constant. The corresponding functions \( \bar{\Lambda}(T) \) and \( \bar{k}(T) \) being:

\[ \bar{\Lambda}(T) = \Lambda - 8\pi G \rho(T), \quad \bar{k}(T) = k - \frac{16\pi G}{3c(T)^2} \rho(T) \] (46)

5 Local cosmology of isolated systems

Let us consider the Kottler solution [6]:

\[ ds^2 = -(1 - \frac{2GM}{c_0^2 r} - \frac{\Lambda}{3c_0^2 r^2})dt^2 + \]
\[ + \frac{1}{c_0^2} (1 - \frac{2GM}{c_0^2 r} - \frac{\Lambda}{3c_0^2 r^2})^{-1} dr^2 + \frac{1}{c_0^2} r^2 d\Omega^2 \] (47)

which is a solution of Einstein’s vacuum field equations including a cosmological constant \( \Lambda \). If \( \Lambda = 0 \) it becomes the proper Schwarzschild solution; and if \( M = 0 \) then it reduces to the static descriptions of the space-times of constant curvature we have considered in the preceding sections.

We consider the Kottler solution to be a convenient starting point to present our ideas about what could be called the Local cosmology of isolated systems. Roughly speaking this means assigning reality, up to some approximation, to the asymptotically Robertson-Walker description of the Kottler solution obtained by the coordinate transformations (22)-(29).

Our approximation will consider only the linear approximation with respect to \( M \) and neglect every power of \( \lambda r/c_0 \), \( \lambda R/c_0 \) or \( \lambda/(c_0 \rho) \) greater than 2. We shall assume also that \( GM/c_0^2 << R \) and use it consistently with the preceding conditions neglecting, for instance, terms as \( (\lambda^2/c_0^2)(GM/c_0^2)R \).

The final result of the calculation can be written as the line-element:

\[ dS^2 = - \left( 1 - \frac{2GM\bar{c}(T)}{c_0^2 R} \right) dT^2 + \]
\[ + \frac{1}{\bar{c}(T)^2} \left( \frac{1}{1 - kR^2} + \frac{2GM\bar{c}(T)}{c_0^2 R} \right) dR^2 + \frac{1}{\bar{c}(T)^2} R^2 d\Omega^2 \] (48)
where \( \bar{c}(T) \) is each of the five functions listed in Sect 2. To reach this final result has required an additional linear time re-synchronization:

\[
T \rightarrow T + GMQ(T)R/c_0^3
\]

(49)
to make zero the coefficient \( dTdR \). The corresponding function \( Q(T) \) is for each case:

1. \(- Q(T) = 4p \) \hspace{1cm} (50)
2. \(- Q(T) = 4 \exp(\lambda T)\lambda/c_0 \) \hspace{1cm} (51)
3. \(- Q(T) = 2p \cosh(\lambda(T + A)) \) \hspace{1cm} (52)
4. \(- Q(T) = 2p \sinh(\lambda(T + A)) \) \hspace{1cm} (53)
5. \(- Q(T) = -4p \sin(\lambda(T + A)) \) \hspace{1cm} (54)

The Lagrangian derived from (48) at the Newtonian approximation is:

\[
L = \frac{1}{2} F(T)^2 (\ddot{R}^2 + R^2 (\dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2)) + \frac{GM}{R} F(T)^{-1}
\]

(55)

Notice that besides the variation of \( G \) given by:

\[
\tilde{G} = GF(T)^{-1}
\]

(56)

the kinetic term gets also a time-dependent factor. And while the Hamiltonian is obviously not conserved, because of the spherical symmetry, the angular momentum:

\[
P = F(T)^2 R^2 \sin^2 \theta \dot{\phi}
\]

(57)
is conserved, which means that the classical expression \( R^2 \sin^2 \theta \dot{\phi} \) it is not.

On the other hand the zero order approximation of the line-element [18] is the line-element [6].

Our conclusion at this point is therefore that in the framework that we have been proposing

i) the equations of motion of a test body in the field of a spherically symmetric source at the lowest approximation are the equations derived from [55]:

\[
\ddot{R} + 2H \dot{R} = - \frac{GM}{R^2} F(T)^{-3} + \frac{P^2}{R^3} F(T)^{-4}
\]

(58)

\[
R^2 \ddot{\phi} + 2R \dot{\phi} (\ddot{R} + HR) = 0.
\]

(59)
assuming that the trajectory lies on the plane $\theta = \pi/2$.

ii) The Maxwell equations have to be written taking into account that the space-time metric is \( g \) and the transit time of light between two points of space has to be calculated taking into account that light propagates along a medium with time-dependent index of refraction $n = \bar{F}(T)$ as explained at the beginning of Sect. 2.

We believe that two points of view allow to implement this second condition:

a.- The simplest one is to accept the minimally coupled vacuum Maxwell equations. In this case light will propagate along null geodesics of \( \bar{g} \) and transit times should be calculated accordingly.

b.- The second possibility consists in sticking closer to the interpretation of $\bar{F}(T)$, as suggested before for $\bar{F}(T)$, i.e.:

$$ \bar{F}(T) = \frac{c_0}{\bar{c}(T)} $$

assuming accordingly that the space-time trajectories of light are the null geodesics of the space-time metric with coefficients \( \bar{\gamma} \):

$$ \bar{\gamma}_{\alpha\beta} = g_{\alpha\beta} + (1 - \bar{F}^{-2})u_\alpha u_\beta, \quad u_\alpha = -\delta_{0\alpha} $$

where $g_{\alpha\beta}$ are the metric coefficients of \( \gamma \). Or, explicitly:

$$ dS^2 = -\frac{1}{\bar{F}(T)^2}dT^2 + \frac{\bar{F}(T)^2}{c_0^2} \left( \frac{dR^2}{1 - kR^2} + R^2(d\theta^2 + \sin(\theta)^2d\phi^2) \right) $$

### 6 Concluding remarks

Let us assume for simplicity that using Newtonian theory, i.e. the physics derived from \( \gamma \) with $\bar{c}(T) = c_0$, and after taking into account every otherwise controlled perturbation, we reach the conclusion that a discrepancy remains between the theory and observations\(^a\). It follows from what we said in the previous section that we could try to explain the discrepancy keeping in mind one or both of these two remarks:

i) Take into account that the variable $R$ is derived from telescopic observations while radar distances, say $R_{\text{rad}}$, measure round-trip travel-times of light. To the second order approximation the relationship between these two quantities is:

\(^a\)Two hypothetical discrepancies have been suggested recently [8]-[10]
\[ R_{\text{rad.}} = R + \frac{f}{2c_0}H_0R^2 \]  \hspace{1cm} (63)

with \( f = 1 \) if we choose option a.- to describe the propagation of light, and \( f = 2 \) if we choose option b.-.

ii) Consider the possibility of using (48) and (6) with free parameters \( \bar{\Lambda} \) and \( \bar{k} \) and use consistently the mechanics and the optics derived for them. Of course a single successful explanation of a discrepancy using such freedom will not be reliable until the same framework, with the same values of \( \bar{\Lambda} \) and \( \bar{k} \), can be applied consistently to other aspects of the physics of the same system.

The overall image that we feel legitimate to draw from this paper is that together with the usual approach to Cosmology, that relies on models that are supposed to describe the history of the Universe from its origin to its end, it might be also worthwhile to describe it piecewise considering different scales and different moments of its history. Possible interesting space scales could start with the Earth-Moon and solar systems scales and beyond these reach those of larger scales.

We want to suggest that each scale of this hierarchy of structures could have its own Cosmology, so to speak, and have their own local cosmological parameters \( c_0, H_0 \) and \( q_0 \). How much of it would be just a reflection of the global behaviour of the Universe and how much would be their own intrinsic properties remains an intriguing question. We can even imagine that the global behaviour of the Universe is the result of averaging effects and causes with roots at much smaller scales. From this point of view the ideas and elementary results presented in this paper could be considered as being more than mere approximations. They could be seen as a more realistic way of trying to understand the overall behaviour of our Universe.

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