Effects of flow condition and chute geometry on the shockwaves formed on chute spillway

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ABSTRACT

Shock waves have undesirable effects, such as excessive dynamic pressure on spillway walls and the extension of flow beyond spillway conduits. To eliminate these detrimental impacts, designers have attempted to detect the characteristics of these waves. Flow interaction with chute piers generates triple waves in the middle walls and sidewalls of spillway conduits. The present study quantitatively investigated the characteristics of these waves with respect to variations in the bottom inclination angle (θ), wall contraction (ψ), and Fr numbers (Fr₀). The results indicated that with the increment of θ, Fr₀, (ψ, Fr₀), and (θ, Fr₀), the height (Hₘ) and distance (lₘ) of the first wave (w₁) increased, which can be helpful for flow aeration. Furthermore, owing to a boost in θ, Fr₀, (ψ, Fr₀), and (θ, Fr₀), the height of the second wave (w₂) was decreased. Therefore, the amount of dynamic pressure on the spillway walls was reduced. Moreover, the distance of w₂ decreased with a rise in θ, ψ and increased with the increment of Fr₀ and (ψ, Fr₀). As for w₃, raising ψ and (ψ, Fr₀) elevated the height of this wave and declined its distance. An increase in the height of w₃ boosted the flow turbulence and aeration.

Key words: chute spillways, Flow-3D, numerical model, shockwave characteristics

HIGHLIGHT

- The characteristics of the shockwaves were investigated with respect to various bottom inclination angles, variations in wall contraction, and different Fr numbers.

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NOTATIONS

$Fr_0$ Froude number

$g$ gravitational acceleration (m/s²)

$H$ shockwave height (m)

$H_{max}$ Maximum shockwave height (m)

$H_{avg}$ average shockwave height in transverse direction of $H_{max}$ (m)

$\theta$ bottom inclination angle (degree)

$\varphi$ wall contraction (degree)

$l_s$ distance between location of $H_{max}$ and tip of chute pier (m)

$a_1$ First shock angle (degree)

$a_2$ Second shock angle (degree)

$\rho$ water density (kg/m³)

$\mu$ dynamic viscosity of water (Pa·s)

$\sigma$ surface tension (N/m)

$V$ average flow velocity in the spillway (m/s)

$b_p$ pier width (m)

$b_s$ pier space (m)

$w_1$ First shockwave

$w_2$ Second shockwave

$w_3$ Third shockwave

↑ increase

↓ decrease

→ move toward downstream

← move toward upstream
1. INTRODUCTION

Spillways, as principal sections of dams, play a critical role in receiving water and delivering it to the downstream so as to prevent dam overtopping during floods. In spillways, due to high flow velocity and geometrical changes, some undesirable phenomena, such as local vortices, cavitation, and shock waves, may occur during operations. Although numerous studies have been conducted to determine flow characteristics in dam structures, limited information is currently available about shock waves (Hunt et al. 2008; Woolbright et al. 2008; Bai et al. 2016; Jahani et al. 2018; Juan César et al. 2018).

Shock waves are usually formed in chute piers and inclined chutes (Reinauer & Hager 1994, 1997; Wu et al. 2006; Wu & Yan 2008; Abdo et al. 2018). Piers are used to install gates, divide the longitudinal section of spillways, and enhance aeration; therefore, using the piers is inevitable. Moreover, due to a decline in chute width due to bridges, a transition from side-channel intakes to tunnel spillways, and a decrease in excavation cost, designers make use of inclined chutes (Reisi et al. 2015; Salazar et al. 2019).

According to the literature, three kinds of shock waves are produced in chute spillways (Figure 1). The first wave ($w_1$), called rooster tail waves, is generated just after piers (Wu et al. 2005; Chen et al. 2013). The first wave moves in the transverse direction and collides with sidewalls, which leads to the formation of the second wave ($w_2$). Finally, owing to the development of the second wave and channel convergence, the third wave ($w_3$) is created at the mid-latitude of the channel axis (Biabani et al. 2019). The formation of these waves brings about unbalanced hydraulic conditions and imposes considerable dynamic pressure on the sidewalls of the structure. Therefore, it is necessary to explore the features of these waves in order to achieve more practical and efficient designs for dam structures.

Jan et al. (2009) studied the characteristics of $w_3$ in inclined chutes considering the effects of sidewall contraction ($27.45^\circ < \varphi < 40.17^\circ$), the bottom inclination angle ($6.22^\circ < \theta < 25.38^\circ$), and the Fr number ($1.04 < Fr_0 < 3.51$). They offered empirical dimensionless relations to predict maximum shock wave height ($H_m$), the distance of the maximum shock wave ($l_s$), and the shock angle ($\alpha$). The results of their study showed that under the same inflow conditions, with an increase in sidewall convergence, the values of $\alpha$ and $H_m$ were raised while the value of $l_s$ decreased. Pagliara & Kurdistani (2011) observed that increasing $Fr_0$ led to a boost in the height of rooster tail waves and air concentration. Hassanzadeh et al. (2019) investigated the characteristics of rooster tail waves in horseshoe spillways and concluded that the height of these waves had a linear relationship with the flow velocity, and length had a reverse relationship with the spillway length. In their study, Xue et al. (2018) pointed out that the height of rooster tail waves was augmented by increasing spillway slope. Nikpura et al. (2018) showed that decreasing the length of contraction walls in open channels raised the flow velocity and the shock wave height. Mousavimehr et al. (2021) investigated the behavior of shock waves along chute spillways and found that with an increase in the Froude number, the wavelength at walls was augmented while the wave height was shortened. Furthermore, the maximum end sections of the waves were transmitted to the downstream area.

![Figure 1](image-url) | Sketch of hydraulic shockwaves in a chute spillway.
In plenty of studies, the K-ε (RNG) turbulence model and the VOF method have been used to investigate the characteristics of flow on hydraulic structures. For instance, Aminoroayaie Yamini et al. (2021) evaluated the hydrodynamic performance of flow and cavitation indices in the bottom outlet of Sardab Dam. Stamou et al. (2008) examined the characteristics of supercritical flows in gradual open channel expansions. Ebrahimnejzad er al. (2020) simulated flow on the spillway of Gavoshan Dam to determine the effect of the bucket edge angle on hydraulic flow characteristics. Jahani et al. (2018) assessed the impact of the geometry of guide walls and piers on flow patterns at the spillway entrance of Jareh Dam. Bayon et al. (2018) studied flow behaviors in the non-aerated region of stepped spillways. The cavitation index was calculated for different values of bed roughness in chute spillways by Samadi-Boroujeni et al. (2019). Also, in his studies, Yakhot et al. (1992) observed that in comparison with the k-ε model, the Renormalized Group Equations had higher accuracy in measuring flow behaviors which has stronger shear regions, and lower turbulence intensity.

As obvious, previous studies have focused on certain aspects of Dam, which restricts their practical implications. In this vein, there is a lack of knowledge about the features of shock waves in chute spillways. Therefore, it is required to identify the characteristics of these waves under hydraulic and geometric variations. In the present study, the characteristics of shock waves, such as \( H/H_{aw} \) (at the transverse direction of the \( H_m \) location), \( l_i \) (the distance between the location of \( H_m \) and the tip of the chute pier), and \( \alpha \) (the shock angle), were investigated with respect to various bottom inclination angles (\( 4^\circ < \theta < 12^\circ \)), variations in wall contraction (\( 3^\circ < \varphi < 7^\circ \)), and different \( Fr \) numbers (2.18 < \( Fr_0 \) < 5.85). Then, the simultaneous effects of (\( \varphi, Fr_0 \)) and (\( \theta, Fr_0 \)) were studied in 76 scenarios (see Table 1). Ultimately, the obtained results were compared with the findings of previous research.

2. BASIC EQUATIONS OF FLOW FIELD

There are two basic equations of fluid motion: the continuity and momentum equations expressed as Equations (1) and (2) for incompressible and turbulent flow with constant viscosity and density (Ferziger & Peric 2012).

\[
\frac{\partial U_i}{\partial x_i} = 0 \tag{1}
\]

\[
\frac{\partial U_i}{\partial t} + U_i \frac{\partial U_i}{\partial x_i} = - \frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial U_i}{\partial x_j} - \frac{\nu'}{2} \frac{\partial U_j}{\partial x_i} \right) \tag{2}
\]

where \( x_i \), \( t \), and \( \rho \) represent the Cartesian system, time, and fluid density, respectively. \( P, U_i, \frac{\nu'}{2}, \frac{\partial U_j}{\partial x_i} \) denote the mean pressure, velocity, and fractional area of fluid (VOF) method and a maker function \( F \) (x, y, z, t). This function reflects the VOF #1 per unit volume and satisfies the following equations:

\[
\frac{\partial F}{\partial t} + \frac{1}{V_F} \left[ \frac{\partial}{\partial x} (FA_x u) + R \frac{\partial}{\partial y} (FA_y v) \right] + \frac{1}{V_F} \left[ \frac{\partial}{\partial z} (FA_z w) + \frac{\xi}{\nu A_x} \frac{FA_x u}{x} \vphantom{\frac{\xi}{\nu A_x}} \right] = F_{DIF} + F_{SOR} \tag{3}
\]

\[
FDIF = \frac{1}{V_F} \left[ \frac{\partial}{\partial x} \left( v \nu A_x \frac{\partial F}{\partial x} + R \frac{\partial}{\partial y} \left( \nu \nu A_y \frac{\partial F}{\partial y} \right) \vphantom{\frac{\partial}{\partial x}} \right) \right] + \frac{1}{V_F} \left[ \frac{\partial}{\partial z} \left( v \nu A_z \frac{\partial F}{\partial z} \vphantom{\frac{\partial}{\partial z}} \right) + \frac{\xi}{\nu A_x} \frac{FA_x F}{X} \right] \tag{4}
\]

where \( V_F \) is the volume fraction of flow, \( u, v, w \). \( A_x, A_y, \) and \( A_z \) signify the velocity and fractional area of flow components along the x, y, and z axes, respectively. When Cartesian coordinates are used, \( R \) is set to 1 and \( \xi \) to 0. The term \( F_{SOR} \)

Table 1 | Variations of Fr number (\( Fr_0 \)), bottom inclination angle (\( \theta \)), and wall contraction (\( \varphi \)) used in the cases of simulations

| \( Fr_0 \) | \( \theta \) | \( \varphi \) |
|---|---|---|
| 2.18 | 3 | 4 |
| 2.58 | 4 | 6 |
| 2.74 | 5 | 8 |
| 2.95 | 6 | 10 |
| 3.20 | 7 | 12 |
corresponds to $R_{\text{SOR}}$ (the density source) in Equation (3). $F_{\text{SOR}}$ refers to the time rate of change in fluid volume fraction $\#1$ associated with the mass source for fluid $\#1$.

$F$ is construed based on the type of problem. For a single fluid, $F$ represents the volume fraction occupied by the fluid. Thus, fluid exists where $F = 1$. Void regions refer to the locations where there is no fluid mass ($F = 0$). Thus, a uniform pressure was assigned to void regions. Physically, they refer to the areas that are filled with vapor or gas and have a negligible density with respect to fluid density (Hirt & Nichols 1981; Zhang et al. 2011).

### 3. GENERAL CHARACTERISTICS OF NAZLOOCHAY DAM AND ITS FACILITIES

The embankment of Nazloochay Dam with a clay core is located in the Northwest of Urmia Province, Iran. This dam controls flows in the Nazloo River. This dam is designed to effectively manage and use water and soil resources in the region, prevent downstream floods, supply municipal water, and irrigate farmlands. A chute spillway, constructed on the right side of the dam, consists of an entrance channel with a pier, spillway crest, chute section, and stilling basin. Figure 2 illustrates the plan and longitudinal sections of the spillway. As seen in Figure 2, the 2-m pier on the central axes of the entrance channel divides the longitudinal section of the crest into two spaces, each with a length of 21 m. The upstream and downstream faces of the spillway crest are designed based on the recommended international standards. These upstream and downstream faces had circle radii of 4.75 and 1.5 m, and a parabolic shape with Equation $Y = 0.1015x^{1.836}$, respectively. The chute section is comprised of two parts. One of them has a bottom slope of 4° and a convergent-sidewall degree of 3° and the other one has a bottom slope of 32° and a consistent-width value of 50 m. This spillway was designed for a maximum discharge rate of 2,270 m$^3$s$^{-1}$ (The final report of Nazloochay dam’s spillway hydraulic model 2008).

### 4. NUMERICAL MODEL

In this research, the Flow-3D software was used to develop a three-dimensional (3D) numerical model for the spillway in Nazloochay Dam. The 3D geometry of this spillway was initially drawn in actual size by the Auto Cad3D software. Then, it was exported to the Flow-3D in the STL format. The boundary conditions of the numerical model are reported in Table 2.
The fluid was considered incompressible with a single-phase flow. To benefit from the features of the Renormalization Group, it was substituted for constant factors. In this regard, the turbulent model of K-ε (RNG) was used. The VOF method was also employed to calculate the free surface profile for the flow downstream of the service gate.

The accuracy of the computational meshes was increased by considering the interior part of the spillway as a meshing component and the volume flow rate boundary condition as model input. Due to the longitudinal symmetry of the conduit, half of the flow was simulated so as to boost the number of simulated cells with small sizes. Therefore, the flow simulation errors decreased in the regions under study (Table 3).

5. PHYSICAL MODEL OF SPILLWAY AND VALIDATION OF NUMERICAL MODEL

The physical model of the spillway comprises Plexiglas sheets and wood in the 1:40 scale. In order to measure the flow depth along the spillway channel, a point gauge, a ruler, and scale side walls were utilized. A pitot tube and a current meter were

| Table 2 | Boundary condition of the numerical model |
|---------|-------------------------------------------|
| Item    | Definition                                |
| Model input | Volume flow rate                         |
| Model output | Outflow                                 |
| Walls   | Wall                                      |
| Border between the blocks | Symmetry                                |

| Table 3 | Sensitivity of numerical model under various boundary conditions and simulated areas |
|---------|-------------------------------------------------------------------------------------|
| Row | Boundary condition of model input | Simulated area | Cell dimensions | The computational time (s) | R² | Depth | Velocity |
|      |                                  |                | Upstream of the crest | Downstream of the crest | Total cells | | Q830 | Q1200 | Q2270 | Q830 | Q1200 | Q2270 |
| 1    | Volume flow rate                  | Whole spillway duct | 0.4 | 0.3 | 9,633,362 | 25 | 0.829 | 0.818 | 0.845 | 0.819 | 0.828 | 0.835 |
| 2    | Volume flow rate                  | Interior part of the spillway duct | 0.4 | 0.3 | 7,076,576 | 25 | 0.889 | 0.879 | 0.890 | 0.879 | 0.889 | 0.899 |
| 3    | Volume flow rate                  | Half of the interior part of spillway | 0.4 | 0.3 | 3,816,681 | 25 | 0.889 | 0.879 | 0.890 | 0.879 | 0.889 | 0.899 |
| 4    | Volume flow rate                  | Half of the interior part of spillway | 0.3 | 0.25 | 9,245,654 | 25 | 0.989 | 0.985 | 0.976 | 0.990 | 0.961 | 0.974 |
| 5    | Volume flow rate                  | Half of the interior part of spillway | 0.3 | 0.2 | 12,251,520 | 25 | 0.989 | 0.985 | 0.976 | 0.990 | 0.961 | 0.974 |
| 6    | Flow velocity                     | Half of the interior part of spillway | 0.3 | 0.2 | 12,251,520 | 25 | 0.943 | 0.942 | 0.934 | 0.921 | 0.932 | 0.912 |
| 7    | Flow pressure                     | Half of the interior part of spillway | 0.3 | 0.2 | 12,251,520 | 25 | 0.954 | 0.957 | 0.962 | 0.971 | 0.968 | 0.975 |
also used to measure the flow velocity. A rectangular channel was also placed in the downstream of the physical model to compute the flow discharge rate (Figure 3). To calculate the flow pressure, 214 piezometers were used on the sidewalls and at the bottom of the physical model. The flow depth and velocity were measured using various service flow rates. In the physical model, some measurement errors were observed in model construction, velocity and pressure measurements, and the reading of water level and flow depth.

The numerical simulations were performed based on the prototype spillway for the total flow rate of 830 m$^3$/s (less than the average flow rate), 1,200 m$^3$/s (the average flow rate), and 2,270 m$^3$/s (the maximum flow rate). The performance of the numerical model was validated using the flow depth and velocity on the central axes of the spillway in the physical model (Figures 4(a)–4(c) and 5(a)–5(c)). There was a discrepancy between the numerical and experimental values of flow depth and velocity at the stilling basin section where the flow encountered turbulence and hydraulic jumps. However, the estimated errors were less than 4%, which was acceptable in engineering practice. Therefore, the numerical model was verified.

6. DIMENSIONAL ANALYSIS OF SHOCK WAVE HEIGHT ($H$)

Due to the steep slope of the second part of the chute section ($32^\circ$), the shock waves dissipated in this part. Consequently, the upstream of the above-mentioned section was simulated to explore the characteristics of these waves. As depicted in Figure 1, the amount of $H$ was determined by the geometric and hydraulic parameters, including water density $\rho$ (kg/m$^3$), the dynamic viscosity of water $\mu$ (N s/m$^2$), the average flow velocity in the spillway $V$ (m/s), surface tension $\sigma$ (N/m), the acceleration of gravity $g$ (m/s$^2$), the average flow depth $h_{av}$ (m), the bottom inclination angle $\theta = \arctan (h/l)$, wall contraction $\varphi = \arctan (h/l)$, pier width $b_p$ (m), and pier space $b_a$ (m). The general role of these variables in the amount of $H$ is defined via the following equation:

$$H = f (\rho, \mu, g, h_{av}, V, \theta, \varphi, b_p, b_a)$$  

(5)

Given that the Reynolds number and flow depth were bigger than $10^5$ and 5 mm, respectively, $\mu$ and $\sigma$ were ignored. Also, geometrical parameters $b_p$ and $b_a$ were not included. Parameters $\rho$, $g$, and $h$ signify three independent basic dimensionless quantities. Considering the dimensions of these parameters, the following non-dimensional equation is formed using the $\pi$ theorem:

$$\frac{H}{h_{av}} = f (\rho, g, V, \theta, \varphi)$$  

(6)

Equation (6) is rewritten as follows:

$$\frac{H}{h_{av}} = f (\theta, \varphi, Fr_0)$$  

(7)

As seen in Equation (7), the amount of $H/h_{av}$ is a function of the bottom inclination angle ($\theta$), wall contraction ($\varphi$), and the Fr number ($Fr_0$).

7. RESULTS AND DISCUSSION

7.1. CHARACTERISTICS OF SHOCK WAVES

7.1.1. Impact of bottom inclination angle ($\theta$)

This part presents different characteristics of the triple waves ($w_1$, $w_2$, $w_3$), such as $H/H_{av}$, $l_s$, and $\alpha$, with regard to the variations in the bottom inclination angle ($4^\circ < \theta < 12^\circ$). It is worth noting that the Fr number ($Fr_0 = 2.18$ ($Q = 1,200$ m$^3$/s)) and wall contraction ($\varphi = 3^\circ$) were constant in all simulations. Furthermore, $H_{av}$ was the average flow height in the transverse direction of the $H_m$ location. Figure 6 depicts the transverse and longitudinal profiles of $w_1$ and $w_2$. As clear, by increasing $\theta$ from $4^\circ$ to $12^\circ$, the value of $H/H_{av}$ increased in $w_1$. Additionally, this wave was generated at greater distances. To put it in other terms, the amount of $H/H_{av}$ was elevated from 1.120 to 1.32, and that of $l_s$ was raised from 9.2 to 15.5 m. Hassanzadeh et al. (2019) found that the height and length of $w_1$ had a linear relationship with the flow velocity. Therefore, with a boost in $\theta$, the flow velocity
Figure 3 | Physical model of Nazloochay spillway and laboratory measuring instruments.
Figure 4 (a–c) Numerical and experimental results of flow depth along central axes of spillway at flow rates of (a) 830, (b) 1,200, and (c) 2,270 m$^3$/s.
Figure 5 | (a-c) Numerical and experimental results of flow velocity along central axes of spillway at flow rates of (a) 830, (b) 1,200, and (c) 2,270 m$^3$/s.
rose, which brought about an increase in the height of \( w_1 \). As the result of Pagliara & Kurdistani (2011) also indicated, the higher the height of \( w_1 \), the more the amount of air concentration. Hence, it can be declared that increasing \( \theta \) elevates pressure along the conduit of chute spillways.

**Figure 6** | Transverse and longitudinal profiles of \( w_1 \) and \( w_2 \) with respect to variations in bottom inclination angle (\( \theta \)).

**Figure 7** | Sketch of shock angle (\( \alpha \)) with respect to variations in bottom inclination angle (\( \theta \)).
Concerning the characteristics of \(w_2\), the amounts of \(H = H_{av}\) and \(l_s\) decreased by increasing the angle. The amount of \(H = H_{av}\) declined from 1.614 in \(\theta = 4^\circ\) to 1.162 in \(\theta = 12^\circ\). Furthermore, \(H_m\) was pushed 4.5 m toward the upstream. It shows that increasing \(\theta\) makes \(w_2\) longer and thinner.

Based on the obtained results, it can be asserted that with a boost in \(\theta\), the amount of \(l_s\) in \(w_1\) increases, and the amount of \(l_s\) in \(w_2\) is reduced, which leads to the increment of \(\alpha_1\). Figure 7 exhibits the sketch of \(\alpha_1\) in various degrees of the angle (\(\theta\)). The amount of \(\alpha_1\) was augmented from 10.06° in \(\theta = 4^\circ\) to 11.8° in \(\theta = 12^\circ\).

As a result of increasing \(\theta\), the height of \(w_2\) declined and the flow entered the second part of the chute. Consequently, the height of \(w_3\) plummeted and dissipated. Therefore, the characteristics of this wave were not measured.

7.1.2. Impact of wall contraction (\(\varphi\))

The behavior of the shock waves was assessed with respect to variations in \(\varphi\). It is worth noting that the bottom inclination angle (\(\theta = 4^\circ\)) and the \(Fr\) number (\(Fr = 2.18\)) remained constant. Owning to the relevance of \(\tan \varphi \leq (2Fr)^{-1}\), the maximum amount of contraction angle can increase to 12.91° in \(Fr = 2.18\). Thus, in this study, \(\varphi\) was set in the range of 3° to 7°.

The results revealed that regarding \(w_1\), the increment of \(\varphi\) had no significant effect on the values of \(H = H_{av}\) and \(l_s\). The contraction of the walls started at the pier, which largely influenced the flow in the downstream. As a result, the amount of \(H = H_{av}\)
varied from 1.197 to 1.22 and that of $l_s$ ranged from 9 to 11 m. Boosting $\varphi$ not only had a remarkable impact on $w_1$ but also substantially affected $w_2$ and $w_3$. As illustrated in Figure 8, the amount of $H/H_{aw}$ increased from 1.56 to 1.72 in $w_2$ and from 1.339 to 1.68 in $w_3$. The amount of $l_s$ decreased by 16 m in $w_2$ and by 35 m in $w_3$. Consequently, both $w_2$ and $w_3$ moved toward the upstream. Because of the wall contraction, the shock waves had already collided with the sidewalls. All of the aforementioned results are consistent with the findings of Biabani et al. (2019) who observed that by increasing $\varphi$ from 3° to 7°, the amount of $H/H_{aw}$ in $w_3$ increased from 1.9 to 2.2. In a similar vein, the amounts of $w_1$ characteristics were varied with variations in $\varphi$.

Concerning $\alpha_1$ and $\alpha_2$, Figure 9(a) and 9(b) show that with an increase in $\varphi$, the degree of $\alpha_1$ was raised from 10.09° to 15.63°. This can be attributed to the formation of $w_2$ at lower distances and a drop in the amount of $l_s$. In addition, $\alpha_2$ had an upward trend and increased from 13.61° to 26.77°. Both $w_2$ and $w_3$ moved toward the upstream; however, variations in $w_3$ were larger.

7.1.3. Impact of Fr number

According to Reinauer & Hager (1994), Fr numbers greater than 5 have no significant effect on the characteristics of shock waves, meaning that, similar shock wave specifications would be obtained for $Fr_0 > 5$. In the current study, since the Fr numbers were less than 5 (2.18 < $Fr_0$ < 3.85), the amounts of bottom inclination angle ($\theta = 4°$) and wall contraction ($\varphi = 3°$) remained stable.

The transverse and longitudinal profiles of $w_1$ and $w_2$ are displayed in Figure 10. The amounts of $H/H_{aw}$ and $l_s$ rose by increasing the Fr number in $w_1$. To be more exact, the amount of $H/H_{aw}$ was elevated from 1.197 to 1.52 and the amount of $l_s$ was raised from 9.2 to 13.80 m. As a result of increasing the Fr number, the velocity of the flows colliding with each other increased after the chute pier, which led to the extension of the height of $w_1$.

Considering $w_2$, increasing the Fr number pushed this wave toward the downstream and declined the amount of $H/H_{aw}$. As presented in Figure 10 the amount of $H/H_{aw}$ lessened from 1.614 in $Fr = 2.18$ to 1.21 in $Fr = 3.85$. Consequently, $w_2$ moved 6.7 m toward the downstream. As a result of a boost in the Fr number, the flow velocity increased, which prevents the

Figure 9 | Sketch of shock angle ($\alpha_1$ and $\alpha_2$) with respect to variations in wall contraction ($\varphi$).
first shock wave from developing or colliding with sidewalls at the upstream parts. In a similar vein, Mousavimehr et al. (2021) found that boosting the Fr number led to an increase in the wavelength at the walls and a reduction in the wave height. Furthermore, the maximum end sections of the waves were transmitted to the downstream.

Given that with a rise in the Fr number, $w_2$ exhibited a greater movement than $w_1$, $\alpha_1$ declined from 10.05° in Fr = 2.18 to 9.905° in Fr = 3.85 in $w_2$ (Figure 11).

When the flow entered the second part of the chute, the height of $w_2$ decreased, the characteristics of $w_3$ were not examined.

### 7.1.4. Simultaneous impact of $\psi$ and Fr$^0$

In this study, the influence of the prominent parameters ($H/H_{aw}$, $l_x$, $\alpha_1$, and $\alpha_2$) on the features of shock waves was evaluated individually. However, exploring the simultaneous effects of these parameters can also provide invaluable information on the most influential factors and substantially assist designers in constructing more efficient structures.

In this regard, as pointed out in Sections 7.1.2 and 7.1.3, the contraction angle of the walls ranged from 3° to 7° (3° < $\psi$ < 7°), and the Fr number was set between 2.18 and 5.85 (2.18 < Fr$^0$ < 3.85). The relevance of $\tan \psi \leq (2Fr)^{-1}$ confirmed the maximum amount of $\psi$ (7°) in Fr = 3.85.

Figure 12 showed that as for $w_1$, the increment of the Fr number increased the value of $H_m/H_{aw}$ from 0.070 to 0.105 and the value of $l_x$ from 6.6 to 8 m. On the other hand, with a boost in $\psi$, the amount of $H_m/H_{aw}$ varied between −0.020 and +0.011 and the amount of $l_x$ ranged between −0.10 and +1.30. Hence, compared to $\psi$, the Fr number had a more remarkable impact
on the formation of \( w_1 \). As a result, it can be claimed that by simultaneously increasing \( \varphi \) and \( Fr_0 \) in the mentioned ranges, this wave moves toward the downstream and the amount of \( H_m = H_{av} \) is elevated.

With regard to \( w_2 \), it was found that the amount of \( H_m = H_{av} \) was determined by the \( Fr \) number. A closer analysis revealed that the increment of \( \varphi \) increased the amount of this parameter at most to 0.156 while an increase in the \( Fr \) number lessened it by more than 0.35. Nonetheless, the location of \( H_m \) was determined by \( w \). Therefore, this wave moved toward the upstream. In other words, although a boost in the \( Fr \) number pushed this wave toward the downstream, in all the states, \( \varphi \) pushed this wave more toward the upstream (Figure 12).

The variations of the \( H_m = H_{av} \) and \( l_s \) values in \( w_3 \) were the same as those in \( w_2 \), meaning that, the amounts of the \( H_m = H_{av} \) and \( l_s \) decreased in \( w_3 \) as well (Figure 12). It is worth mentioning that this wave in \( \varphi = 3^\circ \) and \( Fr = 2.18 \) entered the second part of the chute and dissipated.

7.1.5. Simultaneous impact of \( \theta \) and \( Fr_0 \)

The results associated with the simultaneous influence of \( \theta \) \( (4^\circ \leq \theta \leq 12^\circ) \) and \( Fr_0 \) \( (2.18 < Fr_0 < 3.85) \) on the shock wave characteristics demonstrated that by increasing both parameters, the amounts of \( H_m / H_{av} \) and \( l_s \) were boosted in \( w_1 \). As seen in Figure 13, the amounts of \( H_m / H_{av} \) and \( l_s \) were raised at least by 0.078 and 2.80 m, respectively, with an increase in the \( Fr \) number. Moreover, the \( H_m / H_{av} \) and \( l_s \) amounts increased by 0.068 and 3.00 m with a boost in \( \theta \).

Concerning \( w_2 \), with a rise in each parameter, the amount of \( H_m / H_{av} \) decreased by at least 0.2. The location of \( H_m \) was variable and did not follow any special pattern. In other words, this wave sometimes moved toward the upstream and sometimes toward the downstream (Figure 13). Also, owing to a reduction in the height of \( w_2 \), \( w_3 \) was not formed.

8. CONCLUSION

In the present study, the characteristics of shock waves (\( H/H_{av} \), \( l_s \), \( \alpha_1 \), and \( \alpha_2 \)) were investigated in chute spillways with respect to variations in the bottom inclination angle \( (4^\circ < \theta < 12^\circ) \), wall contraction \( (3^\circ < \varphi < 7^\circ) \), and \( Fr \) numbers \( (2.18 < Fr_0 < 3.85) \). The simultaneous effects of \( (\varphi, Fr_0) \) and \( (\theta, Fr_0) \) on aforementioned parameters were also studied. The numerical model was simulated in the Flow-3D software and verified through the experimental results of the chute spillway in Nazloochay Dam. The results can substantially contribute to designing more efficient chute spillways with a pier.

Based on the results, the following conclusions are drawn:

- The amounts of \( H/H_{av} \) and \( l_s \) in \( w_1 \) increased with the increment of \( \theta \), \( Fr_0 \), \( (\varphi, Fr_0) \), and \( (\theta, Fr_0) \). The major reason lies in the fact that elevated \( \theta \) and \( Fr_0 \) boost flow velocity, which induces the collision of water flows after the pier and pushes them toward the downstream. Furthermore, increasing the height of \( w_1 \) raises the amount of air concentration. Nonetheless, an increase in \( \varphi \) did not have any significant impact on the above-mentioned shock wave characteristics.

Figure 11 | Sketch of shock angle \( (\alpha_1) \) with respect to variations in \( Fr \) number \( (Fr_0) \).
In $w_2$, the amount of $H_m/H_{aw}$ decreases by increasing $\theta$, $Fr_0$, ($\varphi$, $Fr_0$), and ($\theta$, $Fr_0$). The increment of $\theta$ and $Fr_0$ enhances flow velocity, which makes $w_2$ shorter and longer. However, the amount of the mentioned parameter is augmented with a boost in $\varphi$.

In $w_2$, $l_i$ declines by increasing $\theta$ and $\varphi$ and is raised with the increment of $Fr_0$ and ($\varphi$, $Fr_0$). An increase in the wall contraction angle brings about the collision of shock waves with sidewalls in the upstream parts. Furthermore, with a boost in $Fr_0$, flow velocity increases, which leads to the development of the mentioned wave and its collision with sidewalls at longer distances.
The amount of \( l_s \) reduces in \( w_3 \) owing to an increase in \( \theta \) and \( (\theta, Fr_0) \). The amount of \( H / H_{av} \) is also raised due to a boost in \( \varphi \) and it declined in \( (\varphi, Fr_0) \). This wave dissipates as a result of a drop in the height of \( w_2 \) and enters the second part of the chute owing to an increase in \( \theta, Fr_0 \), and \( (\theta, Fr_0) \).

**Figure 13** | Variations of \( H / H_{av} \) and \( l_s \) in \( w_1 \) and \( w_2 \) under the simultaneous influence of \( (\theta, Fr_0) \).

**Table 4** | Summary of the impact of bottom inclination angle \( (\theta) \), wall contraction \( (\varphi) \), \( Fr \) number \( (Fr_0) \), \( (\varphi, Fr_0) \), and \( (\theta, Fr_0) \) on shock wave characteristics

| \( w_1 \) | \( w_2 \) | \( w_3 \) |
|---------|---------|---------|
| \( H / H_{av} \) | \( l_s \) | \( H / H_{av} \) | \( l_s \) | \( H / H_{av} \) | \( l_s \) |
| \( \theta \) | ↑ | → | ↓ | ← | Not formed | ↑ | → | Not formed |
| \( \varphi \) | ↑ | → | ↓ | ← | Not formed | ↑ | ← | Not formed |
| \( Fr_0 \) | ↑ | → | ↓ | ← | Not formed | ↓ | ← | Not formed |
| \( (\varphi, Fr_0) \) | ↑ | → | ↓ | ← | Not formed | ↓ | ← | Not formed |
| \( (\theta, Fr_0) \) | ↑ | → | ↓ | ← | Not formed | ↓ | ← | Not formed |

- The amount of \( l_s \) reduces in \( w_3 \) owing to an increase in \( \varphi \) and \( (\varphi, Fr_0) \). The amount of \( H / H_{av} \) is also raised due to a boost in \( \varphi \) and it declined in \( (\varphi, Fr_0) \). This wave dissipates as a result of a drop in the height of \( w_2 \) and enters the second part of the chute owing to an increase in \( \theta, Fr_0 \), and \( (\theta, Fr_0) \).
The degree of first shock angle ($\alpha_1$) is augmented with the increment of $\theta$ and $\varphi$ while it lessens with an increase in $Fr_0$. Furthermore, due to the dissipation of $w_1$ induced by the increment of $\theta$ and $Fr_0$, $\alpha_2$ was calculated just with respect to $\varphi$ variations, and the results confirmed the increment of this angle (Table 4).

DATA AVAILABILITY STATEMENT
All relevant data are included in the paper or its Supplementary Information.

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