Theory of CP Violation

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CP violation in $K$ and $B$ decays is reviewed in the Standard Model (SM) and beyond the SM. In $K$ decays, one is seeking first evidence for CP violation in direct $K \to \pi \pi$ decays. This would not give a precise quantitative test for the present explanation of CP violation in terms of a phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Such tests are provided by a variety of CP asymmetries in neutral and charged $B$ decays. Certain features, characterizing CP violation beyond the standard model, are outlined in the $B$ meson system.

1. THE CKM MATRIX

In the Standard Model (SM), CP violation is due to a nonzero complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V$, describing the interaction of the three families of quarks with the charged gauge boson. This unitary matrix can be approximated by the following two useful forms:\textsuperscript{[1]}

$$V \approx \begin{pmatrix} 1 & \frac{1}{2} s_{12} & s_{13} e^{-i \gamma} \\ -s_{12} & 1 - \frac{1}{2} s_{12}^2 & s_{23} \\ s_{12} s_{23} - s_{13} e^{i \gamma} & -s_{23} & 1 \end{pmatrix} \text{(1)}$$

$$\approx \begin{pmatrix} 1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (\rho - i \eta) \\ -\lambda & 1 - \frac{1}{2} \lambda^2 & A \lambda^2 \\ A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1 \end{pmatrix}$$

The measured values of the three intergenerational mixing angles, $\theta_{ij}$, and the phase $\gamma$ are given by:\textsuperscript{[2]}

$$s_{12} \equiv \sin \theta_{12} \approx |V_{us}| = 0.220 \pm 0.002 ,$$
$$s_{23} \equiv \sin \theta_{23} \approx |V_{cb}| = 0.039 \pm 0.003 ,$$
$$s_{13} \equiv \sin \theta_{13} \equiv |V_{ub}| = 0.0031 \pm 0.0008 ,$$
$$35^\circ \leq \gamma \equiv \text{Arg}(V^{*}_{ub}) \leq 145^\circ . \text{(2)}$$

The only evidence for a nonzero value of $\gamma$ comes from CP violation in the $K^0 - \bar{K}^0$ system.

Unitarity of $V$ implies quite a few triangle relations. The $db$ triangle,

$$V_{ud} V_{ub}^\ast + V_{cd} V_{cb}^\ast + V_{td} V_{tb}^\ast = 0 , \text{ (3)}$$

which has large angles, is shown in the latest Review of Particle Physics \textsuperscript{[2]}. The phase $\beta = \text{Arg}(V^{*}_{td})$ is determined to lie within the limits

$$10^\circ \leq \beta \leq 35^\circ , \text{ (4)}$$

while $\alpha \equiv \pi/2 - \beta - \gamma$ has the present bounds

$$20^\circ \leq \alpha \leq 120^\circ . \text{ (5)}$$

In addition to the separate constraints on $\alpha$, $\beta$ and $\gamma$, pairs of these angles are correlated. Due to the rather limited range of $\beta$, the angles $\alpha$ and $\gamma$ are almost linearly correlated through $\alpha + \gamma = \pi - \beta$ \textsuperscript{[2]}. A special correlation exists also between small values of $\sin 2\beta$ and large values of $\sin 2\alpha$ \textsuperscript{[2]}

In contrast to the $B^0$ unitarity triangle which is expected to have three large angles, the neutral $K$ meson triangle consisting of the elements $V_{qd} V_{qs}^\ast (q = u, c, t)$ has two long sides (length $\sim \lambda$) and one extremely short side (length $\sim \mathcal{O}(\lambda^5)$). This explains why CP asymmetries in $K$ decays, which are related to the tiny angle of this triangle ($\mathcal{O}(\lambda^4)$), are of order $10^{-3}$.

The only present information about a phase in the CKM matrix comes from the measured value of the CP impurity $K^0 - \bar{K}^0$ mixing parameter $\epsilon_K$. Although this single measurement can be accommodated in the CKM theory, it does not test
2. CP VIOLATION IN THE K MESON SYSTEM

2.1. CP Violation in $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ Mixing

The flavor states $P^0$ and $\bar{P}^0$ ($P$ can be either a $K$ or a $B$ pseudoscalar meson) mix through the weak interactions to form the "Light" and "Heavy" mass-eigenstates $P_L$ and $P_H$:

$$|P_L\rangle = p|P^0\rangle + q|\bar{P}^0\rangle, \quad |P_H\rangle = p|P^0\rangle - q|\bar{P}^0\rangle. \quad (6)$$

These states have masses $m_{L,H}$ and widths $\Gamma_{L,H}$. The Hamiltonian eigenvalue equation (using CPT)

$$\begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix} \begin{pmatrix} p \\ \pm q \end{pmatrix} = (m_{L,H} - \frac{i}{2}\Gamma_{L,H}) \begin{pmatrix} p \\ \pm q \end{pmatrix} \quad (7)$$

has the following solution for the mixing parameter $q/p \equiv (1 - \epsilon)/(1 + \epsilon)$:

$$\frac{q}{p} = \sqrt{\frac{M_{12}^2 - \frac{i}{2}\Gamma_{12}^2}{M_{12}^2 - \frac{i}{2}\Gamma_{12}^2}} = -\frac{2(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)}{\Delta m - \frac{i}{2}\Delta \Gamma}, \quad (9)$$

where $\Delta m \equiv m_H - m_L$, $\Delta\Gamma \equiv \Gamma_H - \Gamma_L$. $M_{12}$ and $\Gamma_{12}$ describe respectively transitions from $P^0$ to $\bar{P}^0$ via virtual states and contributions from decay channels which are common to $P^0$ and $\bar{P}^0$.

The CP impurity parameter $\epsilon$ gives the mass-eigenstates in terms of states with well-defined CP

$$|P_L\rangle = \frac{1}{\sqrt{1 + |\epsilon|^2}}(|P^0\rangle + \epsilon|P^0\rangle), \quad (10)$$

$$|P_H\rangle = \frac{1}{\sqrt{1 + |\epsilon|^2}}(|P^0\rangle + \bar{\epsilon}|P^0\rangle). \quad (11)$$

$q/p$ has a phase freedom under redefinition of the phases of the flavor states $P^0$, $\bar{P}^0$. Thus the phase of $q/p$ can be rotated away and $|q/p| = 1$ means CP conservation in $P^0 - \bar{P}^0$ mixing. The deviation of $|q/p|$ from one measures CP violation in the mixing: $1 - |q/p| \approx 2\text{Re}\epsilon$.

It is clear from eq.(11) that CP violation in neutral meson mixing is expected to be small under two different circumstances:

$$\text{Arg}M_{12} \approx \text{Arg}(-\Gamma_{12}) \quad (K \text{ meson}), \quad (12)$$
The first case applies to the neutral $K$ meson system and the second one - to $B$ mesons. The different circumstances allude to the reason for the small and theoretically uncertain CP violation in $K$ decays in contrast to the large and theoretically clean CP violation in $B$ decays. In $K$ decays $\Gamma_{12}$ is dominated by the $2\pi$ channel, the amplitude of which involves (in the CKM phase convention) a very small phase which is even smaller than the small phase of $M_{12}$. The calculation of both phases involve hadronic uncertainties. On the other hand, the second condition, which applies to the neutral $B$ meson system, says nothing about phases of decay amplitudes which can be, and in fact are, large. The phase of $q/p$, which can be approximated by the phase of $M'_{12}$, appears in the relation between the expected large CP asymmetries and pure CKM parameters.

In the neutral $K$ system, $M_{12}$ obtains a small imaginary contribution from $t$ and $c$ quarks in the box-diagrams, and $\Gamma_{12}$ has a much smaller imaginary part from $K \to 2\pi$ (see following subsection).

$$2|M_{12}| = |\Delta m_K| = m_L - m_S,$$

$$2|\Gamma_{12}| = -|\Delta \Gamma_K| = |\Gamma_S - \Gamma_L|,$$

where we used the conventional notations for the long- and short-lived kaons. Hence

$$|\tilde{c}| \approx \frac{i \text{Im} M_{12}}{\Delta m_K - \frac{i}{2} \Delta \Gamma_K} = \frac{\text{Im} M_{12}}{\sqrt{2} \Delta m_K} e^{i\phi_K},$$

and one finds

$$|\tilde{c}| \approx B_K \text{const.} f(m_t, m_c, \eta_q, S_{ij})(S_{12}S_{23}S_{13})\sin\gamma,$$

where $\phi_K \equiv -2\Delta m_K/\Delta \Gamma_K$, $\phi_K = (43.6 \pm 0.2)^0$. The coefficient $\text{const.} f$ includes factors such as $\pi^2, G_F^2, m_W^2, \Delta m_K/m_K, f_K$, and $c$ and $t$ quark masses, mixing angles and QCD corrections $\eta_q$, and is of order 100. It multiplies the intrinsic CP violating factor $S_{12}S_{23}S_{13}\sin\gamma$, which is of order $10^{-5}$. Thus, a value $|\tilde{c}| \sim O(10^{-3})$ is expected to originate naturally from the CKM matrix. Theoretical and experimental errors in some of the above parameters and in the hadronic matrix element of the box diagram, $B_K = 0.8 \pm 0.2$, imply that the prediction for $|\tilde{c}|$ involves a substantial uncertainty, which leads to the large range of the presently allowed phase $\gamma$ in Eq.(2).

### 2.2. Direct CP Violation in $K \to 2\pi$

The weak amplitudes of neutral $K$ mesons to charged and to neutral two pion states can be decomposed into amplitudes of final states with isospin $I = 0, 2$. One then defines

$$\eta_+ \equiv \frac{\langle \pi^+ \pi^- | H_W | K_L \rangle}{\langle \pi^+ \pi^- | H_W | K_S \rangle}, \quad \eta_0 \equiv \frac{\langle \pi^0 \pi^0 | H_W | K_L \rangle}{\langle \pi^0 \pi^0 | H_W | K_S \rangle},$$

and one finds

$$\eta_+ = \epsilon + \epsilon', \quad \eta_0 = \epsilon - 2\epsilon',$$

$$\epsilon = \tilde{c} + \text{Im} \phi_0,$$

$$\epsilon' = \frac{w}{\sqrt{2}} (\tan \phi_2 - \tan \phi_0) e^{i(\delta_2 - \delta_0 + \tilde{s})},$$

where $\delta_1$ is the elastic phase shift for $\pi\pi$ scattering at the kaon mass in an isospin $I$ channel. $A_I$ involves a weak CKM phase $\phi_I$, which changes sign under charge-conjugation, $A_I = |A_I|e^{i\phi_I}$.

A calculation of $\epsilon'/\epsilon$ requires knowing the phases $\phi_0$, $\phi_2$. These can be estimated in the Standard Model using the tree and penguin diagrams. Whereas the tree operator has real contributions to both $A_0$ and $A_2$, the penguin operator comes with a complex CKM phase and contributes only to $A_0$. Thus, one finds $\phi_2 = 0$ and $\phi_0$ can be estimated to be given by

$$\tan \phi_0 \sim \frac{\text{Im}(V_{ud}V_{us}^*)}{V_{ud}V_{us}^*} \langle P \rangle \sim \text{a few} \times 10^{-4} \langle P \rangle.$$
3. METHODS OF MEASURING CKM PHASES IN $B$ DECAYS

3.1. Decays to CP-eigenstates

The most frequently discussed method of measuring weak phases is based on neutral $B$ decays to final states $f$ which are common to $B^0$ and $\bar{B}^0$. CP violation is induced by $B^0 - \bar{B}^0$ mixing through the interference of the two amplitudes $B^0 \to f$ and $B^0 \to \bar{B}^0 \to f$. When $f$ is a CP-eigenstate, and when a single weak amplitude (or rather a single weak phase) dominates the decay process, the time-dependent asymmetry

$$A(t) \equiv \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)} \quad (21)$$

obtains the simple form

$$A(t) = \xi \sin(2(\phi_M + \phi_f) - \Delta m t) \quad (22)$$

$\xi$ is the CP eigenvalue of $f$, $2\phi_M$ is the phase of $B^0 - \bar{B}^0$ mixing, $|\phi_M| = 0$ for $B^0$, $B_s^0$, respectively, $\phi_f$ is the weak phase of the $B^0 \to f$ amplitude, and $\Delta m$ is the neutral $B$ mass-difference.

The two very familiar examples are:

(i) $B^0 \to \psi K_S$, where $\xi = -1$, $\phi_f = \text{Arg}(V_{cb}^* V_{cs}) = 0$,

$$A(t) = -\sin 2\beta \sin(\Delta m t) \quad (23)$$

and

(ii) $B^0 \to \pi^+ \pi^-$, where $\xi = 1$, $|\phi_f| = |\text{Arg}(V_{ub}^* V_{ud})| = \gamma$,

$$A(t) = -\sin 2\alpha \sin(\Delta m t) \quad (24)$$

Thus, the two asymmetries measure the angles $\beta$ and $\alpha$.

3.2. Decays to other States

A similar method can also be applied to measure weak phases when $f$ is a common decay mode of $B^0$ and $\bar{B}^0$, but not necessarily a CP eigenstate. In this case one measures four different time-dependent decay rates, $\Gamma_f(t)$, $\bar{\Gamma}_f(t)$, $\Gamma_\bar{f}(t)$, $\bar{\Gamma}_{\bar{f}}(t)$, corresponding to initial $B^0$ and $\bar{B}^0$ decaying to $f$ and its charge-conjugate $\bar{f}$. The four rates depend on four unknown quantities, $|A|$, $|\bar{A}|$, $\sin(\Delta\delta_f - \Delta\phi_f - 2\phi_M)$, $\sin(\Delta\delta_f + \Delta\phi_f + 2\phi_M)$, $(A$ and $\bar{A}$ are the decay amplitudes of $B^0$ and $\bar{B}^0$ to $f$, $\Delta\delta_f$ and $\Delta\phi_f$ are the the strong and weak phase-differences between these amplitudes). Thus, the four rate measurements allow a determination of the weak CKM phase $\Delta\phi_f + 2\phi_M$. This method can be applied to measure $\alpha$ in $B^0 \to p^+ \pi^-$, and to measure $\gamma$ in $B^0_s \to D^+_s K^-$. Other ways of measuring $\gamma$ in $B^0_s$ decays were discussed in Ref. [1].

3.3. Penguin Pollution

All this assumes that a single weak phase dominates the decay $B^0(\bar{B}^0) \to f$. As a matter of fact, in a variety of decay processes, such as in $B^0 \to \pi^+ \pi^-$, there exists a second amplitude due to a “penguin” diagram in addition to the usual “tree” diagram [12]. As a result, CP is also violated in the direct decay of a $B^0$, and one faces a problem of separating the two types of asymmetries. This can only be partially achieved through the more general time-dependence

$$A(t) = \frac{(1 - |A|^2)c(t) - 2\text{Im}(e^{-2\im\phi_M A/A}s(t))}{1 + |A|^2} \quad (25)$$

where $c(t) \equiv \cos(\Delta m t)$, $s(t) \equiv \sin(\Delta m t)$. The $\cos(\Delta m t)$ term implies direct CP violation, and the coefficient of $\sin(\Delta m t)$ obtains a correction from the penguin amplitude. The two terms have a different dependence on $\Delta\delta$, the final-state phase-difference between the tree and penguin amplitudes. The coefficient of $\cos(\Delta m t)$ is proportional to $\sin(\Delta\delta)$, whereas the correction to the coefficient of $\sin(\Delta m t)$ is proportional to $\cos(\Delta\delta)$. Thus, if $\Delta\delta$ were small, this correction might be large in spite of the fact that the $\cos(\Delta m t)$ term were too small to be observed.

3.4. Resolving Penguin Pollution by Isospin

The above “penguin pollution” may lead to dangerously large effects in $B^0(t) \to \pi^+ \pi^-$ decay, which would avoid a clean determination of $\alpha$ [13]. One way of removing this effect is by measuring also the (time-integrated) rates of $B^0 \to \pi^0 \pi^0$, $B^+ \to \pi^+ \pi^0$ and their charge-conjugates [14]. One uses the different isospin properties of the penguin ($\Delta I = 1/2$) and tree ($\Delta I = 1/2, 3/2$)
operators and the well-defined weak phase of the tree operator. This enables one to determine the correction to $\sin 2\alpha$ in the coefficient of $\sin(\Delta m t)$. Electroweak penguin contributions could, in principle, spoil this method, since unlike the QCD penguins they are not pure $\Delta I = 1/2$. These effects are, however, very small and consequently lead to a tiny uncertainty in determining $\alpha$. The difficult part of this method may perhaps be the decay rate measurement into two neutral pions. It is of major importance to settle experimentally the question of color-suppression of this mode. Other methods of resolving the “penguin pollution” in $B^0 \to \pi^+\pi^-$, which do not rely on decays to neutral pions, will be described in Sec. 4.

3.5. Measuring $\gamma$ in $B^z \to D K^\pm$

In $B^z \to D K^\pm$, where $D$ may be either a flavor state $(D^0, \bar{D}^0)$ or a CP-eigenstate $(D^0, \bar{D}^0)$, one can measure separately the magnitudes of two interfering amplitudes leading to direct CP violation. This enables a measurement of $\gamma$, the relative weak phase between these two amplitudes. This method is based on a simple quantum mechanical relation among the amplitudes of three different processes,

$$\sqrt{2}A(B^+ \to D^0 K^+) = A(B^+ \to \bar{D}^0 K^+) + A(B^+ \to D^0 K^+) \ .$$

(25)

The CKM factors of the two terms on the right-hand-side, $V_{ub}V_{cs}$ and $V_{cb}V_{us}$, involve the weak phases $\gamma$ and zero, respectively. A similar triangle relation can be written for the charge-conjugate processes. Measurement of the rates of these six processes, two pairs of which are equal, enables a determination of $\gamma$. The present upper limit on the branching ratio of $B^+ \to \bar{D}^0 K^+$ is already very close to the value expected in the SM. The major difficulty of this method may be in measuring $B^+ \to D^0 K^+$ which, following the example of the suppressed $B^0 \to \bar{D}^0 \pi^0$ rate, is expected to be color-suppressed. For further details and a feasibility study see Ref. [17].

If indeed $B(B^+ \to D^0 K^+)$ is found to be suppressed to a level of $10^{-6}$, then one of the sides of the triangle Eq.(25) would be much smaller than the other two, which would create a serious difficulty in observing an asymmetry. The other consequence of such suppression would be a difficulty in determining the flavor of $D^0$ through its hadronic decays, which interfere with Cabibbo-suppressed decays of $\bar{D}^0$ from $B^+ \to \bar{D}^0 K^+$. This problem will be addressed below. The easy way out would be to compare other two processes of this kind, again induced by $V_{ub}V_{cs}$ on-the-one-hand and $V_{cb}V_{us}$ on-the-other-hand, which are equally suppressed. Although this does not improve statistics, the resulting CP asymmetries are expected to be larger. Two variants, based on this simple idea, use the following processes:

- $B^0 \to D^0(\bar{D}^0)K^+\pi^0$, where the flavor of $K^+\pi^0$ is determined through $K^+\pi^0 \to K^+\pi^-$. Both decays to $D^0$ and $\bar{D}^0$ are color-suppressed.

- $B^+ \to D^0(\bar{D}^0)K^+$, where $D^0$ and $\bar{D}^0$ are identified by their respective Cabibbo-allowed and Cabibbo-suppressed decays to $K^-\pi^+$. In this case the two interfering amplitudes forming a triangle with their sum may be of comparable magnitudes, one being color-suppressed and the other being doubly-Cabibbo-suppressed.

4. Methods based on flavor SU(3)

4.1. $\alpha, \beta$ and $\gamma$ from $B$ decays to two light pseudoscalars

Now we turn to other methods of determining CKM phases, which are more involved both theoretically and experimentally. Here one is using a larger variety of two body $B$ decays, including $B^0 \to K^+\pi^-$, which was recently reported to have a somewhat larger rate than $B^0 \to \pi^+\pi^-$. The precision of these methods must be studied carefully. One may use approximate flavor SU(3) symmetry of strong interactions, including first order SU(3) breaking, to relate all two body processes of the type $B \to \pi\pi, B \to \pi K$ and $B \to K\bar{K}$. Since SU(3) is expected to be broken by effects of order 20%, such as in $f_K/f_\pi$, one must introduce SU(3) breaking terms in such
an analysis. A great deal of effort was made recently along this direction \[23, 27\]. In the present section we will discuss two applications of this analysis to a determination of weak phases. Early applications of SU(3) to two-body B decays can be found in Ref. \[28\].

The weak Hamiltonian operators associated with the transitions \( \bar{b} \to \bar{u}qq \) and \( \bar{b} \to \bar{f} \) (\( q = d \) or \( s \)) transform as a \( 3^* \), \( 6 \) and \( 15^* \) of SU(3). The \( B \) mesons are in a triplet, and the symmetric product of two final state pseudoscalar octets in an S-wave contains a singlet, an octet and a 27-plet. Thus, these processes are given in terms of five SU(3) amplitudes: \( \langle 1 \mid 3^* \mid 3 \rangle \), \( \langle 8 \mid 3^* \mid 3 \rangle \), \( \langle 8 \mid 6 \mid 3 \rangle \), \( \langle 8 \mid 15^* \rangle \), \( \langle 27 \mid 15^* \rangle \).

An equivalent and considerably more convenient representation of these amplitudes is given in terms of an overcomplete set of six quark diagrams occurring in five different combinations. These diagrams are denoted by \( T \) (tree), \( C \) (color-suppressed), \( P \) (QCD-penguin), \( E \) (exchange), \( A \) (annihilation) and \( PA \) (penguin annihilation). The last three amplitudes, in which the spectator quark enters into the decay Hamiltonian, are expected to be suppressed by \( f_B/m_B \) \( (f_B \approx 180 \) MeV) and may be neglected to a good approximation.

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The presence of higher-order electroweak penguin contributions introduces no new SU(3) amplitudes, and in terms of quark graphs merely leads to a substitution \[16\].

\[
T \to t \equiv T + P_{EW}^C, \quad C \to c \equiv C + P_{EW},
\]

\[
P \to p \equiv P - \frac{1}{3} P_{EW}^C,
\]

where \( P_{EW} \) and \( P_{EW}^C \) are color-favored and color-suppressed electroweak penguin amplitudes. \( \Delta S = 0 \) amplitudes are denoted by unprimed quantities and \( |\Delta S| = 1 \) processes by primed quantities. Corresponding ratios are given by ratios of CKM factors

\[
\frac{T'}{T} = \frac{C'}{C} = \frac{V_{us}}{V_{ud}}, \quad \frac{P'}{P} = \frac{P_{EW}'}{P_{EW}} = \frac{V_{ts}}{V_{td}}.
\]

(27)

\( t \)-dominance was assumed in the ratio \( P'/P \). The effect of \( u \) and \( c \) quarks in penguin amplitudes can sometimes be important \[28\].

The expressions of all thirteen two body decays to two light pseudoscalars in the SU(3) limit are given in Tables 1 and 2 of \[10\]. The vanishing of three other amplitudes, associated with \( B_d^0 \to K^+K^- \), \( B_s^0 \to \pi^+\pi^- \), \( B_s^0 \to \pi^0\pi^0 \), follows from the assumption of negligible exchange \( (E) \) amplitudes. This can be used to test our assumption which neglects final state rescattering effects. If rescattering is important, then the rates of the above processes could be considerably larger than estimated using naive factorization. This possibility will be discussed in Section 5.

First-order SU(3) breaking corrections can be introduced in a most general manner through parameters describing mass insertions in the above quark diagrams \[31\]. The interpretation of these corrections in terms of ratios of decay constants and form factors is model-dependent. There is, however, one case in which such interpretation is quite reliable. Consider the tree amplitudes \( T \) and \( T' \). In \( T \) the \( W \) turns into a \( ud \) pair, whereas in \( T' \) it turns into \( u\bar{u} \). One may assume factorization for \( T \) and \( T' \), which is supported by data on \( B \to D\pi \) \[31\], and is justified for \( B \to \pi\pi \) and \( B \to \pi K \) by the high momentum with which the two color-singlet mesons separate from one another. Thus,

\[
\frac{T'}{T} = \frac{V_{us} f_K}{V_{ud} f_{\pi}}.
\]

(28)

Similar assumptions for \( C'/C \) and \( P'/P \) cannot be justified.

Tables 1 and 2 of \[10\] and Eq.(28) can be used to separate the penguin term from the tree amplitude in \( B_d^0 \to \pi^+\pi^- \), and thereby determine simultaneously all the three angles of the unitarity triangle. In one of the schemes \[22\] one uses only \( B \) decays to final states with kaons and charged pions, \( B_d^0 \to \pi^+\pi^- \), \( B_d^0 \to \pi^-K^+ \), \( B^+ \to \pi^+K^0 \) and the corresponding charge-conjugated processes. Measurement of these six processes enables a determination of both \( \alpha \) and \( \gamma \), with some remaining discrete ambiguity associated with the size of final-state phases. A sample corresponding to about 100 \( B_d^0 \to \pi^+\pi^- \) events, 100 \( B_d^0 \to \pi^+K^0 \) events, and a somewhat smaller number of detected \( B^\pm \to \pi^\pm K_S \) events, attainable in future \( e^+e^- \) factories, is sufficient for reducing
the presently allowed region in the \((\alpha, \gamma)\) plane by a considerable amount. The reader is referred to Ref.32 for more details. A few alternative ways to learn the penguin effects in \(B_d^0 \to \pi^+\pi^-\) were suggested in Ref.33.

### 4.2. Use of the Recently Observed \(B \to \eta'K\)

The use of \(\eta\) and \(\eta'\) allows a determination of \(\gamma\) from decays involving charged \(B\) decays alone \(^{[54]}\). When considering final states involving \(\eta\) and \(\eta'\) one encounters an additional penguin diagram (a so-called “vacuum cleaner” diagram), contributing to decays involving one or two flavor SU(3) singlet pseudoscalar mesons \(^{[34]}\). This amplitude \((P_1)\) appears in a fixed combination with a higher-order electroweak penguin contribution in the form \(p_1 = P_1 - (1/3)P_{\text{EW}}\). The importance of this diagram was demonstrated very recently by the large branching ratio \(B(B^+ \to \eta'K^+) = (7.8^{+2.7}_{-2.2} \pm 1.0) \times 10^{-5}\) reported at this conference \(^{[36]}\).

Writing the physical states in terms of the SU(3) singlet and octet states, \(\eta = \eta_8 \cos \theta - \eta_1 \sin \theta\), \(\eta' = \eta_8 \sin \theta + \eta_1 \cos \theta\) (where \(\sin \theta \approx 1/3\)), one finds the following expressions for the four possible \(\Delta S = 1\) amplitudes of charged \(B\) decays to two charmless pseudoscalars:

\[
A(B^+ \to \pi^+ K^0) = p',
\]

\[
A(B^+ \to \pi^0 K^+) = \frac{1}{\sqrt{2}} (-p' - t' - c'),
\]

\[
A(B^+ \to \eta K^+) = \frac{1}{\sqrt{3}} (t' - c' - p'_1),
\]

\[
A(B^+ \to \eta' K^+) = \frac{1}{\sqrt{6}} (3p' + t' + c' + 4p'_1).
\]

These amplitudes satisfy a quadrangle relation

\[
\sqrt{6} A(B^+ \to \pi^+ K^0) + \sqrt{3} A(B^+ \to \pi^0 K^+) = 2\sqrt{2} A(B^+ \to \eta K^+) + A(B^+ \to \eta' K^+).
\]

A similar quadrangle relation is obeyed by the charge-conjugate amplitudes, and the relative orientation of the two quadrangles holds information about weak phases. However, it is clear that each of the two quadrangles cannot be determined from its four sides given by the measured amplitudes. A closer look at the expressions of the amplitudes shows that the two quadrangles share a common base, \(A(B^+ \to \pi^+ K^0) = A(B^- \to \pi^- K^0)\), and the two sides opposite to the base (involving \(\eta\)) intersect at a point lying 3/4 of the distance from one vertex to the other. This fixes the shapes of the quadrangles up to discrete ambiguities. Finally, the phase \(\gamma\) can be determined by relating these amplitudes to that of \(B^+ \to \pi^+\pi^0\)

\[
|A(B^+ \to \pi^0 K^+) - A(B^- \to \pi^0 K^-)| = 2 \frac{V_{us}}{V_{td}} \frac{f_K}{f_{\pi}} |A(B^+ \to \pi^+\pi^0)| \sin \gamma.
\]

This method becomes particularly appealing due to the recent CLEO measurement of an anomalously large branching ratio of \(B^+ \to \eta' K^+\) \(^{[50]}\).

### 5. LARGE FINAL STATE PHASES IN \(B\) DECAYS

In order to have large asymmetries in charged \(B\) decays one requires an interference between two amplitudes of comparable magnitude, involving both a large weak CKM phase-difference and a large final state interaction phase-difference. So far, there exists no experimental evidence for final state phases in \(B\) decays, and it has been often assumed that such phases are likely to be small in decays of a heavy \(B\) meson to two light high momentum particles. Evidence for strong phases, related to final states with well-defined isospin and angular momentum, can be obtained from \(B \to D\pi\) decays. The amplitudes into \(D^-\pi^+\), \(D^0\pi^0\), \(D^{*0}\pi^+\) obey a triangle relation, from which the phase-difference between the \(I = 1/2\) and \(I = 3/2\) amplitudes may be determined. The present branching ratios of these decays already imply an upper limit \(^{[37]}\), \(\delta_{1/2} - \delta_{3/2} < 35^\circ\). Improved measurements of these branching ratios may lead to first evidence for strong phases or to more stringent bounds. An important question is, therefore, where would one expect final state interaction phases to be large? In the present section we will demonstrate two cases in which large phases may be anticipated.
5.1. Interference between Resonance and Background

Consider the decay $B^+ \rightarrow \chi_{c0}\pi^+$, $\chi_{c0} \rightarrow \pi^+\pi^-$, where one is looking for a final state with three pions, two of which have an invariant mass around $m(\chi_{c0}) = 3415$ MeV [38]. The width of this $J^P = 0^+ \sigma \pi$ state, $\Gamma(\chi_{c0}) = 14 \pm 5$ MeV, is sufficiently large to provide a large, and probably maximal, CP conserving phase. The decay amplitude into three pions, where two pions are at the resonance, consists of two terms with different CKM phases (we neglect a small penguin term):

- $R$: a resonating amplitude, consisting of a product of the weak decay amplitude of $B^+ \rightarrow \chi_{c0}\pi^+$ involving a real CKM factor $V^*_{cb}V_{cd}$ ($a_w = \text{real}$), the strong decay amplitude of $\chi_{c0} \rightarrow \pi^+\pi^-$ ($a_s = \text{real}$), and a Breit-Wigner term for the intermediate $\chi_{c0}$.

- $D$: a direct decay amplitude of $B^+ \rightarrow \pi^+\pi^-\pi^+$ involving a CKM factor $V^*_{ub}V_{ud}$ with phase $\gamma$, which we write as $(d/m_B)\exp(i\gamma)$ ($d = \text{real}$):

$$ R = a_w a_s \frac{\sqrt{m_{\pi^+}\Gamma}}{s - m^2 + im_{\pi^-}}, D = \frac{d}{m_B} \exp(i\gamma). $$

The total amplitude is $R + D$.

The $B^+ - B^-$ decay rate asymmetry, integrated symmetrically around the resonance, is given by

$$ Asym. \approx -2 \frac{d}{a_w a_s} \frac{\sqrt{m_{\pi^+}\Gamma}}{m_B} \sin\gamma. \quad (30) $$

The strong phase difference between the resonating and direct amplitudes is approximately $\pi/2$. Phases other than due to the resonance width were neglected. Reasonable estimates of the amplitudes $d$ and $a_w a_s$ show that the coefficient of $\sin\gamma$ in the asymmetry is of order one [38]. That is, a large CP asymmetry is expected in this channel, requiring for its observation $10^8$ to $10^9 B$ mesons.

5.2. Rescattering in Quark Annihilation Processes

A large number of $B$ meson decays may proceed only through participation of the spectator quark, whether through amplitudes proportional to $f_B/m_B$ or via rescattering from other less-suppressed amplitudes. A recent analysis of this class of processes was carried out [38], assuming that rescattering from a dominant process leads to suppression by only factor $\lambda \sim 0.2$ compared to $f_B/m_B \approx \lambda^2$. Such an assumption can be justified, for instance, by a Regge-based analysis [40]. The consequences of this assumption are twofold:

- An expected hierarchy of amplitudes in the absence of rescattering will be violated by rescattering corrections, leading to much larger rates. As an example, the branching ratio of $B^0 \rightarrow K^+D_s^-$ can be enhanced by rescattering through a $\pi^+D^-$ intermediate state from about $10^{-6}$ to somewhat less than $10^{-4}$.

- Such violations could point the way toward channels in which final-state interactions could be important. Cases in which final state phases lead to large CP asymmetries are those to which both tree and penguin amplitudes contribute. Two examples are $B^0 \rightarrow D^0\bar{D}^0(D_s^+D_s^-)$ and $B_s \rightarrow \pi^+\pi^-(\pi^0\pi^0)$.

6. CP VIOLATION BEYOND THE STANDARD MODEL

6.1. Modifying the Unitarity Triangle

The above discussion assumes that the only source of CP violation is the phase of the CKM matrix. Models beyond the SM involve other phases, and consequently measurements of CP asymmetries may violate SM constraints on the three angles of the unitarity triangle [1]. Furthermore, even in the absence of new CP violating phases, these angles may be affected by new contributions to the sides of the triangles. The three sides, $V_{cd}V^*_{cb}$, $V_{ud}V^*_{ub}$ and $V_{td}V^*_{tb}$, are measured in $b \rightarrow c\nu$, $b \rightarrow u\bar{c}\nu$ and in $B^0 - \bar{B}^0$ mixing, respectively. A variety of models beyond the SM provide new contributions to $B^0 - \bar{B}^0$ and $B_s - \bar{B}_s$ mixing, but only very rarely [12] do such models involve new amplitudes which can compete with the $W$-mediated tree-level $b$ decays. Therefore, whereas two of the sides of the unitarity triangle are usually stable under new physics effects, the side involving $V_{td}V^*_{tb}$ can be modified by such
effects. In certain models, such as a four generation model and models involving $Z$-mediated flavor-changing neutral currents (to be discussed below), the unitarity triangle turns into a quadrangle.

In the phase convention of Eq. (1) the three angles $\alpha$, $\beta$, $\gamma$ are defined as $\gamma \equiv \text{Arg}(V_{ub}V_{us}^\ast)$, $\beta \equiv \text{Arg}(V_{td}V_{ts}^\ast)$, $\alpha \equiv \pi - \beta - \gamma$. Assuming that new physics affects only $B^0 - \bar{B}^0$ and $B_s - \bar{B}_s$ mixing, one can make the following simple observations about CP asymmetries beyond the SM:

- The asymmetry in $B_d^0 \to \psi K_S$ measures the phase of $B^0 - \bar{B}^0$ mixing and is defined as $2\beta'$, which in general can be different from $2\beta$.

- The asymmetry in $B_d^0 \to \pi^+\pi^-$ measures the phase of $B^0 - \bar{B}^0$ mixing plus twice the phase of $V_{ub}$, and is given by $2\beta' + 2\gamma \equiv 2\pi - 2\alpha'$, where $\alpha' \neq \alpha$.

- The time-dependent rates of $B_s/\bar{B}_s \to D_+^0 K^-$ determine a phase $\gamma'$ given by the phase of $B_s - \bar{B}_s$ mixing plus the phase of $V_{ub}$; in this case $\gamma' \neq \gamma$.

- The processes $B^\pm \to D^0 K^\pm$, $B^\pm \to \bar{D}^0 K^{\mp}$, $B^\pm \to D^{0(\pm)} K^\pm$ measure the phase of $V_{ub}$ given by $\gamma$.

Measuring a nonzero value for the phase $\gamma$ through the last method would be evidence for CP violation in direct decay, thus ruling out superweak-type models $[43]$. Such a measurement will obey the triangle relation $\alpha' + \beta' + \gamma = \pi$ with the phases of $B_d^0 \to \psi K_S$ and $B_d^0 \to \pi^+\pi^-$, irrespective of contributions from new physics to $B^0 - \bar{B}^0$ mixing. On the other hand, the phase $\gamma'$ measured by the third method violates this relation. This demonstrates the importance of measuring phases in a variety of independent ways.

Another way of detecting new physics effects is by determining the phase of the $B_s - \bar{B}_s$ mixing amplitude which is extremely small in the SM, corresponding to an angle of the almost flat $sb$ unitarity triangle $[44]$. This can be achieved through CP asymmetry measurements in decays such as $B_s \to \psi\phi$ governed by the quark process $b \to c\tau s$.

Let us note in passing that in certain models, such as multi-Higgs doublet models with natural flavor conservation (to be discussed below), in spite of new contributions to $B^0 - \bar{B}^0$ and $B_s - \bar{B}_s$ mixing, the phases measured in $B_d^0 \to \psi K_S$ and in $B_s/\bar{B}_s \to D_+^0 K^-$ are unaffected, $\beta' = \beta$, $\gamma' = \gamma$. Nevertheless, the values measured for these phases may be inconsistent with the CP conserving measurements of the sides of the unitarity triangle.

6.2. CP Asymmetries vs. Penguin Decays

Models in which CP asymmetries in $B$ decays are affected by new contributions to $B^0 - \bar{B}^0$ mixing will usually also have new amplitudes contributing to rare flavor-changing $B$ decays, such as $b \to sX$ and $b \to dX$. We refer to such processes, involving a photon, a pair of leptons or hadrons in the final state, as “penguin” decays.

In the SM both $B^0 - \bar{B}^0$ mixing and penguin decays are governed by the CKM parameters $V_{ts}$ and $V_{td}$. Unitarity of the CKM matrix implies $|V_{ts}/V_{cb}| \approx 1$, and $|V_{td}/V_{cb}| < 0.33$, and $B^0 - \bar{B}^0$ mixing only imposes the second constraint slightly due to large hadronic uncertainties, $0.15 < |V_{td}/V_{cb}| < 0.33$.

The addition of contributions from new physics to $B^0 - \bar{B}^0$ mixing relaxes the above constraints in a model-dependent manner. The new contributions depend on new couplings and new mass scales which appear in the models. These parameters also determine the rate of penguin decays. A recent comprehensive model-by-model study $[13]$, updating previous work, showed that the values of the new physics parameters, which yield significant effects in $B^0 - \bar{B}^0$ mixing, will also lead in a variety of models to large deviations from the SM predictions for certain penguin decays. Here we wish to briefly summarize the results of this analysis:

- **Four generations**: The magnitude and phase of $B^0 - \bar{B}^0$ mixing can be substantially changed due to new box-diagram contributions involving internal $t'$ quarks. For
such a region in parameter space, one expects an order-of-magnitude enhancement (compared to the SM prediction) in the branching ratio of $B^0 \rightarrow l^+l^-$ and $B^+ \rightarrow \phi \pi^+$.

- Z-mediated flavor-changing neutral currents: The magnitude and phase of $B^0 - \overline{B}^0$ mixing can be altered by a tree-level $Z$-exchange. If this effect is large, then the branching ratios of the penguin processes $b \rightarrow sl^+l^-$, $B_s \rightarrow l^+l^-$, $B_s \rightarrow \phi \pi^0$ ($b \rightarrow dl^+l^-$, $B^0 \rightarrow l^+l^-$, $B^+ \rightarrow \phi \pi^+$) can be enhanced by as much as one (two) orders-of-magnitude.

- Multi-Higgs doublet models with natural flavor conservation: New box-diagram contributions to $B^0 - \overline{B}^0$ mixing with internal charged Higgs bosons affect the magnitude of the mixing amplitude but not its phase (measured, for instance, in $B^0 \rightarrow \psi K_S$). When this effect is large, the branching ratios of $B^0, B_s \rightarrow l^+l^-$ are expected to be larger than in the SM by up to an order of magnitude.

- Multi-Higgs doublet models with flavor-changing neutral scalars: Both the magnitude and phase of $B^0 - \overline{B}^0$ mixing can be changed due to a tree-level exchange of a neutral scalar. In this case one expects no significant effects in penguin decays.

- Left-right symmetric models: Unless one fine-tunes the right-handed quark mixing matrix, there are no significant new contributions in $B^0 - \overline{B}^0$ mixing and in penguin $B$ decays.

- Minimal supersymmetric models: There are a few new contributions to $B^0 - \overline{B}^0$ mixing, all involving the same phase as in the SM. Branching ratios of penguin decays are not changed significantly. However, certain energy asymmetries, such as the $l^+l^-$ energy asymmetry in $b \rightarrow sl^+l^-$ can be largely affected.

- Non-minimal supersymmetric models: In non-minimal SUSY models with quark-squark alignment, the SUSY contributions to $B^0 - \overline{B}^0$ mixing and to penguin decays are generally small. In other models, in which all SUSY parameters are kept free, large contributions with new phases can appear in $B^0 - \overline{B}^0$ mixing and can affect considerably SM predictions for penguin decays. However, due to the many parameters involved, such schemes have little predictivity.

We see that measurements of CP asymmetries and rare penguin decays give complementary information and, when combined, can distinguish among the different models. In models of the first, second and fourth types one has $\beta' \neq \beta$, $\gamma' \neq \gamma$. One expects different measurements of $\gamma$ in $B^\pm \rightarrow DK^\pm$ and in $B_s/\overline{B}_s \rightarrow D^\pm K^\mp$, and a nonzero CP asymmetry in $B_s \rightarrow \psi \phi$. These three models can then be distinguished by their different predictions for branching ratios of penguin decays. On the other hand, both in the third and sixth models one expects $\beta' = \beta$, $\gamma' = \gamma$. In order to distinguish between these two models, one would have to rely on detailed dilepton energy distributions in $b \rightarrow sl^+l^-$. 6.3. Large CP Asymmetries in Radiative Neutral B Decays

Certain CP asymmetries, such as in $B_s \rightarrow J/\psi \phi$, are expected to be extremely small in the SM, and are therefore very sensitive to sources of CP violation beyond the SM. This is a typical case, in which large effects of new physics in CP asymmetries originate in additional sizable contributions to $B_q - \overline{B}_q$ ($q = d, s$) mixing. Much smaller effects, which are harder to measure and have considerable theoretical uncertainties, can occur as new contributions to $B$ decay amplitudes. There is one class of processes, namely radiative $B^0$ and $B_s$ decays, in which large mixing-induced asymmetries are due to new contributions to the decay amplitude.

Consider decays of the type $B^0, B_s \rightarrow M^0 \gamma$, where $M^0$ is any hadronic self-conjugate state $M^0 = \rho^0, \omega, \phi, K^{*0}$ (where $K^{*0} \rightarrow K_S \pi^0$), etc.
As in $B^0 \to J/\psi K_S$ and $B_s \to J/\psi\phi$, the asymmetries in $B \to M^0\gamma$ are due to the interference between mixing and decay. We neglect direct CP violation which is expected to be small [53]. In the Standard Model, the photon in $b \to q\gamma$ is dominantly left-handed; only a fraction $m_d/m_b$ of the amplitude corresponds to a right-handed photon, where the quark masses are current masses. The final $M^0\gamma$ states are not pure CP-eigenstates; they consist to a good approximation (neglecting the ratio $m_q/m_b$) of equal admixtures of states with positive and negative CP-eigenvalues. Thus, due to an almost complete cancellation between contributions from positive and negative CP-eigenstates, the asymmetries in $b \to q\gamma$ are very small, given by $m_d/m_b$. A few examples of time-dependent asymmetries $A(t)$ expected in the SM are [47]:

\[
\begin{align*}
B^0 &\to K^{*0}\gamma : (2m_s/m_b)\sin(2\beta)\sin(\Delta mt) \\
B^0 &\to \rho^0\gamma : 0 \\
B_s &\to K^{*0}\gamma : (2m_d/m_b)\sin(2\beta)\sin(\Delta mt) \\
B_s &\to \phi\gamma : 0
\end{align*}
\]

where $K^{*0}$ is observed through $K^{*0} \to K_S\pi^0$.

Much larger CP asymmetries can occur in extensions of the Standard Model such as the $SU(2)_L \times SU(2)_R \times U(1)$ left-right symmetric model [3], $SU(2) \times U(1)$ models with exotic fermions (mirror or vector-doublet quarks) [50], and nonminimal supersymmetric models [51]. As an example, consider the left-right symmetric model, in which mixing of the weak-eigenstates $W_L, W_R$ into the mass-eigenstates $W_1, W_2$ is given by the matrix

\[
\begin{pmatrix}
\cos \zeta & e^{-i\omega} \sin \zeta \\
-\sin \zeta & e^{-i\omega} \cos \zeta
\end{pmatrix}.
\]

The process $b \to q\gamma$ obtains in addition to the SM penguin amplitude with $W$ (and $t$) exchange, two penguin-type contributions, from $W_L - W_R$ mixing and from charged scalar exchange. These terms can be sizable in spite of present severe constraints on the parameters of the model, the $W_R$ and charged Higgs masses and the mixing angle $\zeta$. The measured branching ratio of $b \to s\gamma$ [52] implies certain constraints on these parameters. However, even if experiments were to agree precisely with the SM prediction, the asymmetry could be very large. For this case, we list the largest possible asymmetries in the above-mentioned processes, obtained when $\zeta$ takes its present experimental upper limit $\zeta = 0.003$ [17].

\[
\begin{align*}
B^0 &\to K^{*0}\gamma : \mp 0.67 \cos(2\beta)\sin(\Delta mt) \\
B^0 &\to \rho^0\gamma : \mp 0.67 \sin(\Delta mt) \\
B_s &\to K^{*0}\gamma : \mp 0.67 \cos(2\beta)\sin(\Delta mt) \\
B_s &\to \phi\gamma : \mp 0.67 \sin(\Delta mt)
\end{align*}
\]

That is, whereas in the SM all asymmetries are at most a few percent, they can be larger than 50% in the left-right symmetric model. Observing asymmetries at this level would be a clear signal of physics beyond the SM.

7. CONCLUSION: FUTURE OUTLOOK

- Future $K$ decay experiments can potentially measure a nonzero value for $\epsilon'/\epsilon$ at a level of $10^{-3}$, thus confirming the expected phenomenon of direct CP violation in $K$ decays. However, due to theoretical uncertainties, this cannot provide a precise test of the CKM mechanism.

- Observation of CP asymmetries in $B^0, B^+, B_s$ decays would provide first evidence for CP violation outside the neutral kaon system.

- Determination of $\alpha, \beta, \gamma$ from these asymmetries and from $B$ decay rates is complementary to information about the unitarity triangle from CP conserving measurements and from CP violation in the neutral $K$ meson system. Such measurements will test the CKM origin of CP violation.

- Detection of deviations from Standard Model asymmetry predictions, combined with information about rates of rare penguin $B$ decays, could provide clues towards a more complete theory.

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