Full-shape cosmology analysis of SDSS-III BOSS galaxy power spectrum using emulator-based halo model: a 5% determination of $\sigma_8$

Yosuke Kobayashi$^{1,2,*}$, Takahiro Nishimichi$^{3,2}$, Masahiro Takada$^2$,†, and Hironao Miyatake$^{4,2}$

1 Department of Astronomy/Steward Observatory, University of Arizona, 933 North Cherry Avenue, Tucson, AZ 85721-0065, USA
2 Kavli Institute for the Physics and Mathematics of the Universe (WPI), The University of Tokyo Institutes for Advanced Study (UTIAS), The University of Tokyo, Chiba 277-8588, Japan
3 Center for Gravitational Physics, Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
4 Kobayashi-Maskawa Institute for the Origin of Particles and the Universe (KMI), Nagoya University, Nagoya, 464-8602, Japan

We present the results obtained from the full-shape cosmology analysis of the redshift-space power spectra for 4 galaxy samples of the SDSS-III BOSS DR12 galaxy catalog over 0.2 < $z$ < 0.75. For the theoretical model, we use an emulator that was built from an ensemble of N-body simulations, which enables fast and accurate computation of the redshift-space power spectrum of “halos”. Combining with the halo occupation distribution to model the galaxy-halo connection, we can compute the redshift-space power spectrum of BOSS-like galaxies in less than a CPU second, for an input model under flat $\Lambda$CDM cosmology. In our cosmology inference, we use the monopole, quadrupole and hexadecapole moments of the redshift-space power spectrum and include 7 nuisance parameters, with broad priors, to model uncertainties in the galaxy-halo connection for each galaxy sample, but do not use any information on the abundance of galaxies. We demonstrate a validation of our analysis pipeline using the mock catalogs of BOSS-like galaxies, generated using different recipes of the galaxy-halo connection and including the assembly bias effect. Assuming weak priors on cosmological parameters, except for the BBN prior on $\Omega_b h^2$ and the CMB prior on $n_s$, we show that our model well reproduces the BOSS power spectra. Including the power spectrum information up to $k_{\text{max}} = 0.25 \, \text{h Mpc}^{-1}$, we find $\Omega_m = 0.301^{+0.012}_{-0.011}$, $H_0 = 68.2 \pm 1.4 \, \text{km s}^{-1}\text{Mpc}^{-1}$, and $\sigma_8 = 0.786^{+0.036}_{-0.037}$ for the mode and 68% credible interval, after marginalization over galaxy-halo connection parameters. We find little improvement in the cosmological parameters beyond a maximum wavelength $k_{\text{max}} \simeq 0.2 \, \text{h Mpc}^{-1}$ due to the shot noise domination and marginalization of the galaxy-halo connection parameters. Our results are consistent with the Planck CMB results within 1$\sigma$ statistical uncertainties.

I. INTRODUCTION

The three-dimensional distribution of galaxies, measured from wide-area spectroscopic surveys of galaxies, is a powerful probe of cosmology, e.g., for constraining cosmological parameters such as parameters characterizing the nature of dark energy and for testing gravity theory on cosmological scales [1–7]. To attain the fundamental cosmology, there are various exiting, ongoing and planned galaxy redshift surveys: the SDSS-III Baryon Oscillation Spectroscopic Survey (BOSS; [8]), the SDSS-IV extended Baryon Oscillation Spectroscopic Survey (eBOSS; [9]), the Subaru Prime Focus Spectrograph (PFS; [10]), the Dark Energy Spectroscopic Instrument (DESI; [11]), the ESA Euclid satellite mission [12], and the NASA Nancy Grace Roman Space Telescope [13].

The galaxy distribution observed by spectroscopic surveys is modulated by the Doppler effect due to the line-of-sight peculiar velocities of galaxies, and exhibits characteristic anisotropies, called the redshift-space distortion (RSD) [14–16]. The RSD effect is useful to improve cosmological constraints by breaking degeneracies between the cosmological parameters and uncertainties in galaxy bias relative to the underlying matter distribution [17]. In addition, since the RSD effect is a gravitational effect, it can be used, if precisely measured, to probe the strength of gravitational field in large-scale structure, which can be in turn used to test gravity theory on cosmological scales [18].

In order to exploit the full information from galaxy redshift surveys, we need a sufficiently accurate theoretical template that enables a high-fidelity comparison with the measured clustering statistics of galaxies to obtain a robust estimation of cosmological parameters. The linear theory of cosmological fluctuations, which has been in a remarkable success in CMB analyses, ceases to be accurate at $k \gtrsim 0.1 \, \text{h Mpc}^{-1}$ due to nonlinear effects of structure formation [17]. The standard approach to tackle this difficulty has been analytic prescriptions based on the perturbation theory (PT) of large-scale structure [19, 20]. This approach describes the distribution of galaxies in terms of a series expansion of both the matter density and velocity fields with a set of free coefficients/terms including bias parameters, under the single-stream approximation [21, 22]. A further refined model consistently separating short-scale physics including the galaxy bias from large-scale dynamics of interest, so-called Effective Field Theory of Large-Scale Structure (EFTofLSS),

* yosukekobayashi@email.arizona.edu
† masahiro.takada@ipmu.jp
has also been developed [23]. These models have been applied to actual datasets to obtain cosmological constraints [5, 24–29]. While these PT-based templates give useful predictions at linear and quasi-nonlinear scales up to $k \sim 0.2 \, h\,\text{Mpc}^{-1}$, an application of these models to even smaller scales is still disturbed by even higher-order contributions of both the density and velocity fields as well as non-perturbative effects arising from the dynamics beyond shell crossing, *i.e.*, formation of galaxies (or dark matter halos) [e.g., 30–36]. Consequently, the cosmology analysis of the galaxy power spectrum has been typically limited to the wavenumber $k \lesssim 0.15 – 0.2 \, h\,\text{Mpc}^{-1}$, depending on the redshift and the accuracy of the model required to meet the statistical precision of data [25, 26, 37]. In other words, the clustering information on the higher-$k$ scales does not seem practical for cosmology in this method, because the information is used to basically constrain higher-order bias parameters and other nuisance parameters that need to be introduced for the theoretical consistency of models.

As an alternative approach, in this paper we use a simulation-based theoretical template, the *emulator*, which enables fast and accurate computation of the redshift-space power spectrum of “halos” in the flat \( \Lambda \)CDM framework, developed in our previous paper [38]. Dark matter halos are locations where galaxies likely form. It is relatively straightforward to accurately simulate the formation and evolution processes of halos using \( N \)-body simulations and then have an accurate prediction of their clustering properties including the redshift-space power spectrum [20, 39]. Kobayashi et al. [38] developed an emulator by training a feed-forward neural network with a dataset of the redshift-space power spectra of halos measured from halo catalogs in an ensemble set of \( N \)-body simulations for 80 models within flat \(  \omega \)CDM framework in the DARK QUEST campaign [40]. The emulator was validated using the test dataset consisting of 20 cosmological models that are not in training, and it was shown that the power spectra are sufficiently accurate up to $k = 0.6 \, h\,\text{Mpc}^{-1}$. The emulator includes all the non-perturbative, nonlinear effects relevant to the formation and evolution of halos: nonlinear clustering, nonlinear RSD, nonlinear bias of halos, and halo exclusion effect. By combining with a halo occupation distribution (HOD) model [41–46], the emulator enables to compute the redshift-space power spectrum of galaxies in less than a CPU second on a typical recent laptop computer. Thus, the emulator allows for cosmological parameter inference of the galaxy power spectrum in a multidimensional parameter space. Such an emulator-based method is equivalent to estimating cosmological parameters from comparison of the observed power spectrum with mock spectra from simulated galaxy catalogs generated from costly high-resolution \( N \)-body simulations with varying cosmological models.

Hence, the purpose of this paper is to perform a cosmology analysis of the redshift-space galaxy power spectrum measured from the public BOSS DR12 large-scale structure catalog over $0.2 < z < 0.75$ [47], using the aforementioned emulator. In doing this, we analyze the “full-shape” information in the monopole, quadrupole and hexadecapole moments of the redshift-space power spectra, beyond the traditional approach to extract only geometrical information through the features of baryon acoustic oscillations (BAO) and the anisotropy originating from RSD. We first show a series of validation checks of our method to ensure unbiased cosmological inference beyond the accuracy assessment of the emulator at the halo power spectrum level already presented in our previous paper. We apply the full analysis pipeline to mock signals of the galaxy power spectrum measured from the galaxy mock catalogs and then check whether our method can recover the cosmological parameters to within the statistical errors. For this validation, we use the mock catalogs generated using different recipes of galaxy-halo connection from our fiducial HOD model [also see 37, for similar analyses but with different observables] and also use mock catalogs including the assembly bias effect that is one of the most dangerous, physical systematic effects in the halo model approach. We then apply the analysis method to the BOSS galaxy power spectra under the flat \( \Lambda \)CDM cosmology. In doing this, we employ weak priors on the cosmological parameters, except for the BBN prior on $\Omega_b h^2$ [48] and the CMB prior on $n_s$ [49], employ very broad priors for the galaxy-halo connection parameters, and then estimate the cosmological parameters after marginalization over uncertainties in the nuisance parameters. We also compare our cosmological constraints with the recent PT-based results [50, 51] and the Planck 2018 cosmological results [49].

The structure of this paper is as follows. In Sec. II we will describe the power spectrum data and covariance matrix we use in this work. In Sec. III we will describe details of the emulator-based halo model as the theoretical template and then the parameters and priors used in the cosmology analysis. In Sec. IV we show the main results of this paper: the cosmological parameters for the flat \( \Lambda \)CDM model. In Sec. V we give discussion on possible residual systematic effects in the data and the theoretical template. Sec. VI is devoted to the conclusion.

II. DATA

For the galaxy power spectrum data and the covariance matrix, we use the updated measurement of the BOSS DR12 power spectrum recently provided in Ref. [52], which are publicly available from https://fbeutler.github.io/hub/deconv_paper.html. We use four data chunks, named “NGC z1”, “SGC z1”, “NGC z3”, and “SGC z3”, which mean the measurements for different galaxy samples in two non-overlapping redshift bins (“z1” and “z3”) both for the Northern and Southern Galactic caps (“NGC” and “SGC”). “z1” (“z3”), which we hereafter call “low-z” (“high-z”), corresponds to the range $0.2 < z < 0.5$ ($0.5 < z < 0.75$) with its effective
redshift $z_{\text{eff}} = 0.38$ (0.61). We compute the theoretical model of the power spectrum for each redshift bin at its effective redshift.

The estimator adopted in Ref. [52] gives the power-spectrum multipoles convolved with the survey window function, which can be expressed by a matrix-vector multiplication. Therefore, we construct a consistent model prediction of the window-convolved power spectrum as

$$P_{\text{conv}} = W M P_{\text{true, flat-sky}}^\text{true, flat-sky}$$

where $P_{\text{conv}}$ is the vector of the convolved power spectrum multipoles, $P_{\text{true, flat-sky}} = \{P_0, P_2, P_4\}$ is that of the true (unconvolved) power spectrum multipoles computed in the global plane-parallel approximation, where the monopole, quadrupole, and hexadecapole moments are concatenated. $M$ is the matrix that induces the odd multipoles (dipole and octopole) arising from the wide-angle effect. $W$ is the matrix that consists of Fourier-space multipole moments of the survey window function (see Ref. [52] for details). Due to the correction for the wide-angle effect through the matrix $M$, the dipole and octopole moments (the real parts, since they are purely imaginary) are induced by the leakage from the standard even multipole moments, but we do not use these odd multipoles in this work. We compute the power spectrum monopole, quadrupole, and hexadecapole using the emulator-based theoretical model we describe in the next section and substitute them to $P_{\text{true, flat-sky}}$ in Eq. (1).

The matrices, $W$ and $M$, are also made publicly available on the same website.

In the measurement of the convolved power spectrum multipoles, Ref. [52] assumes the flat-geometry ΛCDM model with $\Omega_m = 0.31$, which is the same as the model assumed in the original measurement of the BOSS DR12 public spectra for their cosmology analysis [26]. They measure the power spectrum in a cubic box by a fast Fourier transform (FFT) with the side length $L = 3500$ $h^{-1}$ Mpc and Nyquist wavenumber $k_{\text{Ny}} = 0.628$ $h$ Mpc$^{-1}$. Each Fourier mode is divided into wavenumber bins in the range $k \in (0, 0.4)$ $h$ Mpc$^{-1}$ with width $\Delta k = 0.01$ $h$ Mpc$^{-1}$.

As for the power spectrum covariance matrix, we use the matrix provided by Ref. [52], which was estimated from 2048 realizations of the MultiDark-Patchy (hereafter Patchy) mock catalogs [53, 54]. These mock catalogs were generated using an approximate $N$-body solver which combines the Lagrangian perturbation theory, a small-scale halo collapse model, and a semi-analytical galaxy biasing scheme, augmented by calibration to a reference large-volume $N$-body simulation sample selected from the BigMultiDark simulations [55]. To correct the biased estimate of the inverse covariance, we multiply the so-called Hartlap factor [56] to the inverse of the estimated covariance:

$$f_{\text{Hartlap}} = \frac{N_f - n_{\text{bin}} - 2}{N_f - 1},$$

where $n_{\text{bin}}$ is the number of bins we use in the parameter inference. For instance, in the case that we use the monopole, quadrupole and hexadecapole moments up to $k_{\text{max}} = 0.25$ $h$ Mpc$^{-1}$, the number of bins is $n_{\text{bin}} = 25 \times 3 = 75$ for each galaxy sample, yielding $f_{\text{Hartlap}} = 0.9629$.

### III. THEORETICAL MODEL AND PARAMETERS

#### A. Theoretical model

To investigate the cosmological parameter constraint from the BOSS galaxy power spectrum, we use the theoretical template computed using the emulator for the redshift-space power spectrum of halos combined with the HOD model, developed in our previous work [38]. We below give a brief description of our theoretical template, and see Ref. [38] [also 17] for further details.

We employ the five-parameter HOD model in Ref. [45], which splits the galaxies into central and satellite galaxies. The mean halo occupation number of central and satellite galaxies within host halos with mass $M$ are given as

$$\langle N_c(M) \rangle = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\log M - \log M_{\text{min}}}{\sigma_{\log M}} \right) \right],$$

and

$$\langle N_s(M) \rangle = \langle N_c(M) \rangle \lambda_s(M) = \langle N_c(M) \rangle \left( \frac{M - \kappa M_{\text{min}}}{M_1} \right)^{\alpha_{\text{ext}}},$$

respectively, where erf$(x)$ is the error function and the logarithms in Eq. (3) are base 10. Note that, in our model we adopt $M = M_{200} = (4\pi/3)200\rho_0 R_{200}^3$ as the halo mass definition, where $\rho_0$ is the mean comoving mass density in the universe and $R_{200}$ is the spherical comoving radius within which the mean mass density is 200 times $\rho_0$. Here $\{\log M_{\text{min}}, \sigma_{\log M}, \log M_1, \alpha_{\text{sat}}, \kappa, \alpha_{\text{ext}}\}$ are model parameters. The probability distribution of galaxies given the mean number is assumed to be Bernoulli for centrals and Poisson for satellites given that the halo of interest has a central halo. This HOD model is also used in the cosmology analysis of the galaxy-galaxy weak lensing and projected galaxy correlation function measured from the Hyper Suprime-Cam Year1 catalog and the BOSS DR11 catalog [57].

Using the above HOD model, our full model of the redshift-space galaxy power spectrum is given by the sum of the one- and two-halo terms,

$$P_{gg}(k) = P_{1h}(k) + P_{2h}(k),$$

where

$$P_{1h}(k) = \frac{1}{n_{\text{eff}}^2} \int dM \frac{dn}{dM}(M) \langle N_c(M) \rangle \times \left[ 2\lambda_s(M) \tilde{H}(k; M) + \lambda_s(M)^2 \tilde{H}(k; M)^2 \right],$$

and

$$P_{2h}(k) = \frac{1}{n_{\text{eff}}^2} \int dM \frac{dn}{dM}(M) \langle N_c(M) \rangle \times \left[ 2\lambda_s(M) \tilde{H}(k; M) + \lambda_s(M)^2 \tilde{H}(k; M)^2 \right].$$
As described above, we use our emulator [38] to compute the redshift-space halo power spectrum $P_{hh}(k; M_1, M_2)$ in the two-halo term (Eq. 7) for an input model in the flat ΛCDM cosmology. The emulator outputs $P_{hh}(k; M_1, M_2)$ given as a function of the two-dimensional wavevector $k$, rather than the multipole moments such as $P_{hh,ℓ}(k; M_1, M_2)$. This modeling allows us to straightforwardly include the Alcock-Paczyński effect (see the next subsection), even in the presence of the FoG effect. We also use the publicly available Dark Emulator (https://dark-emulator.readthedocs.io/en/latest/) [40] to compute the halo mass function, and the Colossus code (http://www.benediktiemer.com/code/colossus/) [63] to compute the halo concentration-mass relation. In the theoretical model the redshift-space halo power spectrum in the two-halo term, $P_{hh}(k; M_1, M_2)$, carries cosmological information, and other functions such as the HOD and the distribution of galaxies in their host halos are all nuisance and are needed to account for uncertainties in the galaxy-halo connection.

B. Alcock-Paczyński effect

Since we need to assume a reference cosmology ($Ω_m = 0.31$) in the measurement of the power spectrum, we incorporate the Alcock-Paczyński (AP) effect [64, 65] into the theoretical model. Due to the difference between the true and reference cosmologies, the Fourier-space wavenumbers are transformed as

$$k^\text{ref}_\bot = α_\bot k_\bot \equiv \frac{D_\Lambda(z)}{D_\text{ref}_\Lambda(z)} k_\bot,$$

$$k^\text{ref}_\| = α_\| k_\| \equiv \frac{H^\text{ref}(z)}{H(z)} k_\|,$$  

(12)

where $D_\Lambda(z)$ and $H(z)$ are the angular diameter distance and Hubble parameter at the effective redshift $z$. Quantities with superscript “ref” denote those for the reference cosmology. By this coordinate transformation, the power spectrum multipole is transformed as

$$P^{\ell}_{gg,\ell}(k^\text{ref}) = \frac{2ℓ + 1}{2α_\bot^2} \int_{-1}^{1} dμ^\text{ref} P_{gg}(k, μ) \mathcal{L}_\ell(μ^\text{ref}).$$  

(13)

where $k$ and $μ$ in the argument of $P_{gg,\ell}$ are given in terms of $k^\text{ref}$ and $μ^\text{ref}$; $k = k(k^\text{ref}, μ^\text{ref})$ and $μ = μ(μ^\text{ref})$, given as

$$k(k^\text{ref}, μ^\text{ref}) = k^\text{ref} \frac{1}{α_\bot} \left[ 1 + (μ^\text{ref})^2 \left( \frac{α_\bot^2}{α_\|} - 1 \right) \right]^{1/2} ,$$

(14)

$$μ(μ^\text{ref}) = μ^\text{ref} \frac{α_\bot}{α_\|} \left[ 1 + (μ^\text{ref})^2 \left( \frac{α_\bot^2}{α_\|} - 1 \right) \right]^{-1/2} .$$

(15)

The velocity dispersion, $σ_v^2(M)$, is given by

$$σ_v^2(M) = \frac{1}{a^2 H^2} \frac{GM}{2 a R_{200}(M)},$$

(11)

where $G$ is the gravitational constant, $a$ is the scale factor, $H$ is the Hubble parameter at scale factor $a$, and $R_{200}(M)$ is the halo radius $R_{200}$ for halos with mass $M$. The factor $1/(aH)^2$ is to convert the velocity to the redshift-space displacement. The above velocity distribution models the fingers-of-God (FoG) effect due to virial motions of satellite galaxies in their host halos. In order to further include uncertainties in the FoG effect, we will introduce a nuisance parameter to model the uncertainty, $σ_v(M) → c_{vel} σ_v(M)$, and treat $c_{vel}$ as a model parameter in the cosmology analysis.
TABLE I. Model parameters and priors used in our cosmology analysis in flat ΛCDM cosmology. \(\mathcal{N}(\mu, \sigma)\) denotes the Gaussian distribution with mean \(\mu\) and standard deviation \(\sigma\). \(U(a, b)\) denotes the uniform distribution between the minimum value \(a\) and the maximum value \(b\). The Gaussian priors on \(\omega_b\) and \(n_s\) are based on the BBN and Planck CMB constraints, respectively (see text for details). The flat priors on other cosmological parameters are set to be within the parameter ranges on which the emulator is supported.

| Parameter | Prior |
|-----------|-------|
| **Cosmological parameters** | |
| \(\omega_b\) | \(\mathcal{N}(0.02268, 0.00038)\) |
| \(\omega_c\) | \(U(0.10782, 0.13178)\) |
| \(\Omega_{\Lambda}\) | \(U(0.54752, 0.82128)\) |
| \(\ln(10^{10}A_s)\) | \(U(2.4752, 3.7128)\) |
| \(n_s\) | \(\mathcal{N}(0.9649, 0.0042)\) |
| **HOD parameters** | |
| \(\log M_{\text{min}}\) | \(U(12.0, 15.0)\) |
| \(\sigma^2_{\log M}\) | \(U(0.0001, 2.0)\) |
| \(\log M_1\) | \(U(12.0, 16.0)\) |
| \(\sigma_{\text{sat}}\) | \(U(0.01, 5.0)\) |
| \(\kappa\) | \(U(0.01, 5.0)\) |
| **Other nuisance parameters** | |
| \(c_{\text{vel}}\) | \(U(0.01, 10.0)\) |
| \(P_{\text{shot}}\) | \(U(-10^4, 10^4) h^{-2} \text{Mpc}^3\) |
| **Derived parameters** | |
| \(\Omega_{\text{m}}\) | – |
| \(H_0\) | – |
| \(\sigma_8\) | – |

Since the reference cosmology is generally different from the underlying true cosmology, the AP effect induces an additional anisotropy in the measured power spectrum. In other words, the measured anisotropy enables us to infer the true cosmology through the geometrical information.

### C. Model parameters and priors

In this subsection, we describe the model parameters that we infer, as well as their prior settings. The default setting is summarized in Table I.

#### 1. Cosmological parameters

Since we want to infer the cosmological parameters within the flat-geometry ΛCDM framework, we sample all of the five parameters:

\[
P_{\text{cosmo}} = \{\omega_b, \omega_c, \Omega_{\Lambda}, \ln(10^{10} A_s), n_s\},
\]

where \(\omega_b = \Omega_b \langle h^2 \rangle\) and \(\omega_c = \Omega_c \langle h^2 \rangle\) are the physical energy density parameters of baryon and cold dark matter, \(\Omega_{\Lambda}\) is the energy density parameter of the cosmological constant, and \(A_s\) and \(n_s\) are the amplitude (at the pivot scale \(k_{\text{pivot}} = 0.05 \text{Mpc}^{-1}\)) and the spectral tilt of the power spectrum of primordial curvature perturbations. For \(\omega_b\) and \(n_s\), we impose priors that are inferred from other cosmological probes. More specifically, we adopt a Gaussian prior on \(\omega_b\) from the primordial deuterium and helium abundance data compared with the standard Big Bang nucleosynthesis (BBN) model [48, 66, 67]. On the other hand, we impose a Gaussian prior on \(n_s\) given in Table 1 of Ref. [49] for the Planck 2018 “TT, TE, EE+lowE+lensing”. We treat \(\Omega_m(=1-\Omega_{\Lambda})\), the total matter energy density, \(H_0 = 100 h \text{ km s}^{-1} \text{Mpc}^{-1}\), the Hubble constant, and \(\sigma_8\), the standard deviation of linear matter perturbations at \(z = 0\) averaged within a sphere with comoving radius \(8 h^{-1} \text{Mpc}\), as the derived parameters. We fix the density parameter of neutrinos to \(\omega_\nu = 0.00064\), corresponding to 0.06 eV for the total mass of the three mass eigenstates, which is used when computing the linear matter power spectrum used to set up the initial conditions of cosmological simulations that are used for the DARK EMULATOR development [40].

#### 2. Nuisance parameters

As we described in Sec. III A, we employ five parameters to specify the HOD model for each galaxy sample. In addition, we include two additional nuisance parameters.

- \(c_{\text{vel}}\) — The multiplicative coefficient on the velocity dispersion of galaxies relative to the halo center. It regulates the uncertainty on the strength of the FoG effect (see around Eq. 10).
- \(P_{\text{shot}}\) — The residual shot noise contribution apart from the simple Poisson shot noise. We add \(P_{\text{shot}}\) to the galaxy power spectrum \(P_{\gg}(k, \mu)\), and hence it is relevant only to the monopole moment.

We employ a flat prior over the range \([-10^4, 10^4] (h^{-1} \text{Mpc})^3\) for the latter. The mean number density of galaxies for each sample is a factor of a few times \(10^{-4} (h^{-1} \text{Mpc})^3\), so the prior range is sufficiently wide. Thus, we have seven nuisance parameters on the galaxy-halo connection:

\[
P_{\text{galaxy}} = \{\log M_{\text{min}}, \sigma^2_{\log M}, \log M_1, \sigma_{\text{sat}}, \kappa, c_{\text{vel}}, P_{\text{shot}}\},
\]

for each of the 4 galaxy samples. Hence, the total number of parameters is \(5 + 7 \times 4 = 33\) within the flat ΛCDM framework. As can be found from Table I, we employ a broad prior range for each of the galaxy-halo connection parameters. For example, the range of \(M_{\text{min}}\), which is one of the parameters that are sensitive to the linear bias of the galaxy sample, corresponds to halos that have \(b_1 \approx [1.2, 10.2]\) at \(z = 0.5\) for the Planck cosmology.
approach can be considered conservative in parameter inference.

\section{Parameter inference}

We employ the Bayesian inference to derive the parameter posterior distribution:

$$p_{\text{post}}(p|D) \propto \mathcal{L}(D|p)p_{\text{prior}}(p),$$  \hspace{1cm} (18)

where \(p_{\text{prior}}\) and \(p_{\text{post}}\) are the prior and posterior distributions of model parameters, and \(\mathcal{L}(D|p)\) is the likelihood function of the observational data \(D\) given parameters \(p\). Using the power spectrum data vector and the covariance matrix, we compute the log-likelihood function:

$$\ln \mathcal{L}(D|p) = -\frac{1}{2} \sum_{\text{samp}} \sum_{\ell,\ell'} \sum_{i,j} \left[ P^D_\ell(k_i) - P_\ell(k_i; p) \right] \times \text{Cov}^{-1} \left[ P_\ell(k_i), P_\ell(k_j) \right] \left[ P^D_\ell(k_j) - P_\ell(k_j; p) \right],$$  \hspace{1cm} (19)

where we assume the Gaussian likelihood and omit the normalization factor. \(P^D_\ell(k_i)\) denotes the data of the \(\ell\)-th multipole moment of the power spectrum in the \(i\)-th wavenumber bin, \(P_\ell(k_i; p)\) is its theoretical model prediction, and \(p\) is the model parameters. We include the power spectrum information over \(k_{\text{min}} \leq k \leq k_{\text{max}}\), and we employ \(k_{\text{min}} = 0.005 \, h \, \text{Mpc}^{-1}\) and \(k_{\text{max}} = 0.25 \, h \, \text{Mpc}^{-1}\) as our fiducial choices, respectively. We will below give a validation of the choice of \(k_{\text{max}} = 0.25 \, h \, \text{Mpc}^{-1}\) and discuss how different choices of \(k_{\text{min}}\) or \(k_{\text{max}}\) change the cosmological results. The summation \(\sum_{\text{samp}}\) denotes the summation over galaxy samples when the power spectrum information for different galaxy samples are combined.

For the parameter sampling, we employ the Markov-chain Monte Carlo (MCMC) sampling of the standard Metropolis algorithm [68], or the nested sampling algorithm MultiNest [69] implemented in the public Python package PyMultiNest [70]. In the MCMC sampling, we monitor the convergence for the cosmological parameters by using a method in Ref. [71], which is an improved variant of the Gelman-Rubin diagnostic \([72, 73]\). More specifically, we apply the rank-normalization [71] to the MCMC chains and measure the Gelman-Rubin statistic \(\hat{R}\) of the chains split in half (so-called the split-\(\hat{R}\) [74]), after discarding 1000 points at the beginning of each chain as the burn-in phase. We run the MCMCs until the criteria \(\hat{R} < 1.05\) for the cosmological parameters are met. We use GetDist [75] package to draw triangle plots of parameter posteriors.

We should note that, throughout this paper, we do not include the abundance of galaxies (the mean number density) nor the BAO information after reconstruction \([\text{e.g., see 76, for such a study}]\) in the data vector in the parameter inference.

\section{RESULTS}

We show the main results of our cosmology analysis in this section. Throughout this paper, we mainly focus on the constraints on three cosmological parameters \(\Omega_m\), \(H_0\), and \(\sigma_8\), which are well constrained by the redshift-space galaxy power spectrum, in flat \(\Lambda\text{CDM}\) model.

\subsection{Validation tests}

Before showing the main results, we first present validation tests of our emulator based method. To test the validity and usefulness of the emulator-based method, we performed various cosmology challenges: we apply our cosmology analysis pipeline to simulated mock signals of the redshift-space galaxy power spectrum to address whether the pipeline can recover the underlying true cosmological parameters used in the simulations. Please see Ref. [37] for details of the procedures and aims. Y. K., who is one of the authors of this paper and led the actual cosmology inference analysis of the BOSS data, applied the pipeline to the mock signals for the BOSS-like galaxies used in Ref. [37]. The mock galaxy catalogs were generated using a different recipe for the galaxy-halo connection based on subhalos, so it is not entirely clear whether our HOD method can recover the underlying cosmological parameters. For instance, the spatial and velocity structures of satellite galaxies in host halos are generally different from those in our fiducial halo model. In this test, Y. K. was not informed of the cosmological parameters, and the validation test was done effectively in a blind manner. He submitted the results to T. N., who is a co-author of this paper and is the main organizer and the maintainer of the challenge program, and then the ground truth cosmological parameters were revealed with the mutual agreement not to update the analysis anymore. The submitted results are recorded and presented on the challenge webpage (\url{https://www2.yukawa.kyoto-u.ac.jp/~takahiro.nishimichi/data/PTchallenge/}).

In Figure 1 we show the results of our validation tests. The left panel shows the results using the simulated signals that are generated from the mock catalogs of the above cosmology challenges of Ref. [37]. To mimic the cosmology analysis of the BOSS power spectra, we use the mock catalogs at \(z = 0.38\) and \(0.61\) to simulate the redshift-space spectra for the 4 subsamples, the NGC/SGC in the low-\(z\) and high-\(z\) bins, and use the same covariance matrix as that used in the following cosmology analysis of BOSS spectra (see Sec. II). These mock catalogs are based on the realizations of \(N\)-body simulations for the total volume of \(566 \, (h^{-1}\text{Gpc})^3\) [37], which is about a hundred times that of the BOSS DR12 galaxy sample. The large volume of the simulations allows us to sufficiently reduce the sample variance errors in the simulated power spectra. Therefore, we can conduct a fairly stringent test of the systematic error due to an imper-
fect modeling of the power spectrum. The figure shows that our analysis method recovers the true cosmological parameters to within the statistical errors of the BOSS galaxy spectra. We also stress that our emulator-based method passes the validation test even if including the power spectrum information up to $k_{\text{max}} = 0.3 \, h \, \text{Mpc}^{-1}$, where the perturbation theory breaks down. Here the constraints on $\Omega_m$ and $H_0$ are mainly from the BAO features and partly from the power spectrum shape via the AP effect. On the other hand, the constraint on $\sigma_8$ is from the power spectrum amplitude, after the degeneracies with galaxy bias uncertainty (uncertainties in the galaxy-halo connection in our model) are lifted by measurements of the RSD effect as we will discuss later (Sec. V F). However, including the information beyond $k \simeq 0.2 \, h \, \text{Mpc}^{-1}$ gives little improvement in the cosmological parameters, due to the shot noise domination and the degeneracies with the galaxy-halo connection parameters. On the other hand, we find that the constraints on HOD parameters are improved by including the higher-$k$ information.

As another sanity check, we also perform the validation tests using the mock galaxy catalogs generated in Refs. [17, 40]. They are based on the simulations for the Planck cosmology using the same HOD prescription as that in our analysis. Similarly to the above test, we used the outputs of N-body simulations at $z = 0.251$ and 0.617, adopted the same HOD model as that in Ref. [77] to populate galaxies into halos of each realization, and then simulated the mock galaxy spectra for the 4 galaxy samples. We used the simulations of total volume $128 \, (h^{-1} \text{Gpc})^3$ to generate the simulated data vector [see 17, for details of the simulations], and performed the cosmology analysis using the same covariance matrix of the BOSS spectra. The right panel of Figure 1 shows that our analysis method recovers the cosmological parameters to within the statistical errors, for all the $k_{\text{max}}$ values. One might notice a slight bias in each cosmological parameter, even if both the theoretical template and the mock catalogs employ the same form of HOD model. We ascribe the parameter shift to the projection effect of the mock galaxy catalogs used in the Cosmology Challenges in Ref. [37] (see text for details). Since we would like to blind the true values of the cosmological parameters used in the mock catalogs, i.e., those for the Planck cosmology.
ters. On the other hand, the credible intervals of the cosmological parameters are not much improved by including the information from $k_{\text{max}} = 0.25$ to $0.30\,h\,\text{Mpc}^{-1}$, as can be found from Figure 1. Hence, from these results, we conclude that our method can robustly recover the cosmological parameters to within the statistical errors of the BOSS power spectra, after marginalization over the galaxy-halo connection parameters. Here note that the cosmological information is extracted from the redshift-space power spectrum of halos predicted by the emulator in our method. In this paper, we employed $k_{\text{max}} = 0.25\,h\,\text{Mpc}^{-1}$ as our fiducial choice of the maximum wavenumber.

We will also later show the validation test of our analysis method using the mock catalogs including the assembly bias effect, which is one of the most dangerous, physical systematic effects in the halo model approach.

B. Results: $\Lambda$CDM cosmology

Figure 2 shows the main results of this paper, the cosmological parameters obtained from the actual BOSS power spectra. The figure shows the projected posterior distributions of $\Omega_m$, $H_0$ and $\sigma_8$ for the flat $\Lambda$CDM model, after marginalizing over the other parameters such as the galaxy-halo connection parameters. These constraints include the BAO and full shape information of the redshift-space power spectrum for the 4 samples of the low-z/high-z NGC and SGC samples, up to $k_{\text{max}} = 0.25\,h\,\text{Mpc}^{-1}$ where the nonlinear effects such as nonlinear bias, nonlinear clustering and nonlinear RSD are properly included. Our method achieves precise measurements of the cosmological parameters:

$$\begin{align*}
\Omega_m &= 0.301_{+0.012}^{-0.011} \\
H_0 &= 68.2 \pm 1.4 \\
\sigma_8 &= 0.786_{+0.036}^{-0.037},
\end{align*}$$

where we report the mode and the 68% credible interval of the marginalized posterior distribution for each parameter, respectively. Note that $H_0$ is in units of $\text{km}\,\text{s}^{-1}\text{Mpc}^{-1}$.

Table III compares our result with those from the recent similar full-shape analyses of the power spectrum multipoles using the PT-based theory models: the EFTofLSS model [50] and the model based on Lagrangian perturbation theory (LPT) [51]. They used the same power spectrum signals and window function of BOSS DR12 provided by Ref. [52], i.e., the same dataset as that we use. It is interesting to note that our results are consistent with these two analyses to within the statistical errors, even though our halo model-based method and the PT-based ones are constructed based on totally different frameworks. The size of the error bar in each parameter is comparable with those of the their results. This might be counter-intuitive, because one might think that our halo model based method is a more restrictive model than PT-based models, so expect our method gives tighter constraints on the cosmological parameters. Probably this is due to the wide priors of the galaxy-halo connection parameters in our analysis, and we will later discuss a possible room of improvements in the cosmological parameters within our method.

We also note other works [27, 28, 79–81] that recently performed the full-shape cosmology analysis of the BOSS power spectrum. However, these works used the different dataset and/or analysis method from those in this paper, so it is not easy to make an apple-to-apple comparison with our result. We here give a note for completeness of our discussion, stressing that the full-shape cosmology analysis is becoming possible to obtain cosmological constraints for a given cosmological framework such as the flat $\Lambda$CDM model.

Our results can also be compared with the Planck 2018 CMB constraints: we overplot the Planck 2018 cosmological constraints from the baseline likelihood of “TT, TE, EE+lowE”; the MCMC chains of Planck 2018 cosmology analysis are downloaded from the Planck Legacy Archive (http://pla.esac.esa.int/pla/#cosmology), where the neutrino mass is fixed to 0.06 eV as we did in our analysis. Our results are in good agreement with the Planck results for all of the three cosmological parameters. In Table II, we also show the parameter value at the maximum a posteriori (MAP), the median, and the mode with 68% credible interval (CI) for the 1d posterior distribution of each parameter. For comparison with the weak lensing survey results [e.g. 57, 82], we also give the results for $S_8$, the parameter on which the weak lensing surveys can give most stringent constraint. In addition, we show the results for $f(z)\sigma_8(z)$, which is the parameter often used to characterize the constraint mainly from the RSD measurement on linear scales. Note that, since we assume the flat $\Lambda$CDM cosmology, the linear growth rate $f(z)$ is determined solely by $\Omega_m$ through

$$f(z) = \frac{3}{2} \Omega_m(z) + \left[ \int_0^1 \frac{dx}{\Omega_m(z)x^{-1} + (1 - \Omega_m(z))x^{2}} \right]^{-1},$$

where $\Omega_m(z) = \Omega_m(1 + z)^3[H_0/H(z)]^2$ is the time-dependent matter density parameter. It is in contrast to traditional RSD analyses such as Ref. [26], where one performs model independent linear-growth measurements, marginalizing over possible nonlinear corrections and focusing only on the amplitude of the apparent anisotropies arising from the linear RSD effect. Therefore, as we have already quoted earlier in this subsection, we can determine fundamental cosmological parameters, breaking the degeneracy between $\sigma_8$ and $f$ under the assumed cosmological model. The table gives the constraints for $f(z)\sigma_8(z)$ that are obtained from the MCMC analyses using either of the low-z NGC+SGC sample or high-z NGC+SGC sample at the effective redshift $z_{\text{eff}} = 0.38$ or 0.61, respectively (see below). Our constraint is from the combined information of the BAO, the AP effect, the
FIG. 2. The posterior distributions of $\Omega_m$, $H_0$ and $\sigma_8$, obtained from the cosmology inference including the full shape of the monopole, quadrupole and hexadecapole moments of the BOSS DR12 galaxy power spectra up to $k_{\text{max}} = 0.25 \, h \, \text{Mpc}^{-1}$, for the flat $\Lambda$CDM model. For the theoretical templates we use the emulator-based halo model, and the posteriors include marginalization over uncertainties in other cosmological parameters and the nuisance parameters including the galaxy-halo connection parameters. For comparison, the red contours show the results from the Planck 2018 “TT, TE, EE+lowE” analysis.

| Parameter | MAP Median | 68% CI | Planck 68% CI |
|-----------|------------|--------|---------------|
| $\ln(10^{10} A_s)$ | 2.93 | 3.01 $^{+0.080}_{-0.087}$ | 3.045 $^{+0.011}_{-0.012}$ |
| $\Omega_m$ | 0.300 | 0.302 $^{+0.009}_{-0.012}$ | 0.3166 $^{+0.0084}_{-0.0073}$ |
| $H_0$ [km s$^{-1}$ Mpc$^{-1}$] | 68.3 | 68.2 $^{+1.4}_{-1.4}$ | 67.27 $^{+0.60}_{-0.51}$ |
| $\sigma_8$ | 0.754 | 0.786 $^{+0.036}_{-0.037}$ | 0.8120 $^{+0.0073}_{-0.0072}$ |
| $S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$ | 0.754 | 0.787 $^{+0.034}_{-0.035}$ | 0.834 $^{+0.016}_{-0.015}$ |
| $f \sigma_8(z_{\text{eff}} = 0.38)$ | 0.474 | 0.471 $^{+0.028}_{-0.029}$ | 0.4771 $^{+0.0066}_{-0.0062}$ |
| $f \sigma_8(z_{\text{eff}} = 0.61)$ | 0.434 | 0.434 $^{+0.028}_{-0.026}$ | 0.4696 $^{+0.0053}_{-0.0051}$ |

TABLE II. The results of the cosmological parameter inference of the BOSS power spectra for $\Lambda$CDM model as in Figure 2. Note that we include the BBN prior on $\omega_b$ and the Planck CMB prior on the spectral tilt $n_s$ (see Table I). For each parameter, we show the parameter value at the maximum a posteriori (MAP), the median, and the mode with 68% credible interval (CI) for the 1d posterior distribution of each parameter. The parameters other than $\ln(10^{10} A_s)$ are derived parameters (see Table I). For comparison with constraints from other experiments, we also show the constraints on $S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$ and $f(z)\sigma_8(z)$, where $f$ is the linear growth rate $f \equiv d \ln D/d \ln a$. For $f \sigma_8$, we show the results obtained from the cosmology analysis using the subsample of low-$z$ NGC+SGC or high-$z$ NGC+SGC sample at the effective redshift $z_{\text{eff}} = 0.38$ and 0.61, respectively (see the right panel of Figure 5). For comparison, we also show the 68% credible interval from the Planck 2018 “TT, TE, EE+lowE” analysis, taken from Table 2 of Ref. [49].
FIG. 3. The comparison of monopole (blue), quadrupole (red), and hexadecapole (green) moments between the data (symbols with error bars) and the model predictions (solid lines) at MAP of the MCMC chains in our cosmology analysis shown in Figure 2. The error bars are computed from the diagonal elements of the covariance matrix, described in Section II.

FIG. 4. The median and 68% percentile interval of the HOD functions for each of the BOSS galaxy samples (low-z/high-z NGC/SGC samples), obtained from the MCMC chains in Figure 2. The solid and dashed lines are the medians of central+satellite and central-only HOD functions, respectively.

| Parameter | This work | EFTofLSS model [50] | LPT model [51] |
|-----------|-----------|---------------------|----------------|
| \( \Omega_m \) | \( 0.301^{+0.012}_{-0.011} \) | \( 0.312^{+0.011}_{-0.012} \) | \( 0.305 \pm 0.01 \) |
| \( H_0 \) [\( \text{km s}^{-1} \text{Mpc}^{-1} \)] | \( 68.2 \pm 1.4 \) | \( 68.5^{+1.1}_{-1.3} \) | \( 68.5 \pm 1.1 \) |
| \( \sigma_8 \) | \( 0.786^{+0.036}_{-0.037} \) | \( 0.737^{+0.040}_{-0.044} \) | \( 0.738 \pm 0.048 \) |

TABLE III. Comparison of the parameter bounds from recent “full-shape” analyses of the BOSS DR12 power spectrum (68% CI). For the analysis of the EFTofLSS model, we quote the results using the window-convolved power spectrum (top row of Table IV of Ref. [50]). All the results shown here are obtained from exactly the same power spectrum data provided by Ref. [52].
RSD effect and the amplitude and shape information of the power spectrum under the flat $\Lambda$CDM framework.

Figure 3 shows that our model at MAP well reproduces all the measured multipoles of redshift-space power spectra. The reduced chi-square value at the MAP is $\chi^2$/d.o.f. $\simeq 1.05$ for 267 (= 300 – 33) degrees of freedom (the p-value $p \simeq 0.28$), implying that the MAP model gives an acceptable fit to the data. Taking a closer look at the figure, our model has a slightly weaker BAO feature than data. This tendency reflects the fact that our power spectrum emulator is based on the training data which have a larger $k$-bin width ($\Delta k = 0.02\ h\ Mpc^{-1}$) than the data ($\Delta k = 0.01\ h\ Mpc^{-1}$), to suppress the statistical scatters on the training data (also see Appendix A of Ref. [38]). Since the BAO feature leads to a tight constraint on $\Omega_m$ through the AP effect, the sharpening of the BAO feature in the emulator could improve the cosmological constraints.

Figure 4 shows the mean HOD functions obtained from the MCMC chains. It can be found that the galaxy population inferred from the BOSS DR12 galaxy power spectra is almost confined to halos with masses, $M \gtrsim 10^{12} h^{-1} M_{\odot}$. This figure also shows that there is no remarkable difference in the HOD among the 4 galaxy samples. Our results are qualitatively consistent with the HODs estimated in the previous works [83–85]. We again stress that the HOD constraints are obtained from the fitting of the emulator-based halo model to the redshift-space power spectrum, without employing any strong priors on the HOD parameters nor using the abundance information (the mean number density of galaxies). Hence, the posterior distributions of HOD are purely from the redshift-space clustering information, while the previous works took into account different clustering information such as the galaxy-galaxy weak lensing and/or the projected correlation function [e.g. 57, for such study].

V. DISCUSSION

In this section, we discuss the robustness of our cosmology results: we study how different analysis methods and datasets change the inferred cosmological parameters.

A. Variations in the cosmological parameters for different galaxy subsamples

In Figure 5 we study how the cosmological parameters are changed when using different subsets of the data vector: the left panel shows the results for 4 individual galaxy samples, and the right panel shows the combined results for each of the two redshift bins, where the two galactic hemispheres, NGC and SGC, are combined. Shifts in each parameter display a similar trend to those shown in Figure 2 of Ref. [27], even though we use a totally different theoretical template, i.e., the halo model based method, compared to the EFTofLSS in their paper. Hence, we think that the parameter shifts are likely due to the sample variances in each subsample. For completeness of our discussion, in Appendix B we show the 2d posterior distributions for the full parameters for each of the 4 individual galaxy samples (see Figure 15). The low-$z$ SGC sample displays a sizable difference in some parameters compared to the other samples, but the difference is still within the statistical errors. Hence, we cannot give any definite conclusion as to that sample could have a potential observational systematics compared to the others.

B. The impact of hexadecapole moments

Figure 6 shows that the inclusion of the hexadecapole moment of the redshift-space power spectrum yields only a subtle improvement in the cosmological parameters, because the hexadecapole moments have lower signal-to-noise ratios than the monopole and quadrupole moments as shown in Figure 3. However, we note that the hexadecapole indeed improves some of the HOD parameters, which is consistent with the finding in Ref. [86].

C. The cosmological information in the different range of $k$

One advantage of our approach, e.g., compared to the PT-based method such as the EFTofLSS, is that ours enables to compare the model predictions with the measurements up to higher $k_{\text{max}}$. As described in our previous paper [38], our emulator is designed to give an accurate prediction of the redshift-space power spectrum up to $k = 0.6\ h\ Mpc^{-1}$. However the power spectrum measurements at $k > 0.2\ h\ Mpc^{-1}$ are in the shot noise dominated regime, and therefore it is not clear whether the cosmological parameters are improved even if we include the data points at higher wavenumbers. In Figure 7 we study how the cosmological parameters are changed for different choices of $k_{\text{max}}$: we consider $k_{\text{max}} = 0.2$, 0.25 or 0.3 $h\ Mpc^{-1}$, respectively. The figure shows that the size of the credible intervals is not largely changed for the different $k_{\text{max}}$ values, confirming the shot noise domination in the power spectrum measurements at $k > 0.2\ h\ Mpc^{-1}$. We confirmed that the statistical errors of the galaxy-halo connection parameters such as the residual shot noise and the HOD parameters are indeed improved by including the information on the higher $k$. A closer look reveals that the results for $k_{\text{max}} = 0.2$ and 0.25 $h\ Mpc^{-1}$ are consistent with each other. However, the result for $k_{\text{max}} = 0.3\ h\ Mpc^{-1}$ shows a sizable shift in $\sigma_8$. For the validation tests using the noiseless mock signals in Figure 1, we did not find this level of shift in the cosmological parameters.

As a further test of this shift, we use 50 realizations of noisy signals that are generated by adding random noise realizations drawn from the BOSS covariance ma-
FIG. 5. The posterior distributions for cosmological parameters, obtained from the different galaxy subsamples. The left panel shows the results for 4 individual galaxy samples: low-z NGC, low-z SGC, high-z NGC, and high-z SGC. The right panel shows the results for each of low-z or high-z NGC+SGC samples. In the right panel, we overplot the gray contours to show the distribution for the full sample for comparison, which is the same as the blue contours in Figure 2.

| Parameter                   | low-z NGC MAP Median | low-z NGC 68% CI | low-z SGC MAP Median | low-z SGC 68% CI |
|-----------------------------|----------------------|------------------|----------------------|------------------|
| $\ln(10^{19}A_s)$          | 3.05                 | 3.10             | 3.17                 | 3.14             |
| $\Omega_m$                 | 0.311               | 0.318            | 0.261               | 0.259            |
| $H_0$ [km s$^{-1}$ Mpc$^{-1}$] | 66.1               | 66.5             | 71.1                 | 73.0             |
| $\sigma_8$                 | 0.782               | 0.817            | 0.811                | 0.842            |
| $S_b \equiv \sigma_b(\Omega_m/0.3)^{0.5}$ | 0.796              | 0.841            | 0.756                | 0.783            |
| $f_{\sigma_b(z_{\text{eff}} = 0.38)}$ | 0.457              | 0.481            | 0.451                | 0.467            |

| Parameter                   | high-z NGC MAP Median | high-z NGC 68% CI | high-z SGC MAP Median | high-z SGC 68% CI |
|-----------------------------|----------------------|------------------|----------------------|------------------|
| $\ln(10^{19}A_s)$          | 2.65                 | 2.76             | 3.02                 | 3.11             |
| $\Omega_m$                 | 0.298               | 0.299            | 0.305                | 0.310            |
| $H_0$ [km s$^{-1}$ Mpc$^{-1}$] | 70.3               | 70.1             | 67.6                 | 67.6             |
| $\sigma_8$                 | 0.685               | 0.717            | 0.783                | 0.824            |
| $S_b \equiv \sigma_b(\Omega_m/0.3)^{0.5}$ | 0.683              | 0.711            | 0.789                | 0.838            |
| $f_{\sigma_b(z_{\text{eff}} = 0.61)}$ | 0.393              | 0.410            | 0.450                | 0.470            |

TABLE IV. Similar to Table II, but this table shows the results obtained from the cosmology analysis using each of the 4 galaxy samples.

trix to the noiseless mock signals (the mock signals in the right panel of Figure 1). Then we applied the same cosmology analysis pipeline to each of the mock signals to estimate the cosmological parameters. Figure 8 shows the distributions of shifts in the cosmological parameters at $k_{\text{max}} = 0.25$ or $0.30$ h Mpc$^{-1}$ with respect to those at $k_{\text{max}} = 0.20$ h Mpc$^{-1}$, for the low-z and high-z NGC samples which give the dominant contributions to the cosmological constraints of our full analysis. The figure shows that there is a reasonable chance to have the parameter shifts for $k_{\text{max}} = 0.25$ h Mpc$^{-1}$ seen from the actual BOSS data. However, the shifts in some parameters for $k_{\text{max}} = 0.3$ h Mpc$^{-1}$ are at the tail of the distribution of the mock results, indicating a possible hint in the systematic effects at $k \gtrsim 0.25$ h Mpc$^{-1}$, e.g., a limitation of the halo model approach at such high $k$ scales or a residual
FIG. 6. The posterior distributions of cosmological parameters in our fiducial cosmology analysis for the full galaxy sample, obtained if or not we include the hexadecapole moments of the redshift-space power spectrum in the parameter inference. The red contours are the same as those in Figure 2.

FIG. 7. Comparison of the posterior distributions obtained when including the redshift-space power spectrum information up to a different $k_{\text{max}}$ in the analysis. We show the cases of $k_{\text{max}} = 0.2$ (blue), 0.25 (red), and 0.3 (green) $h$ Mpc$^{-1}$ and also show the results of the Planck 2018 CDM “TT, TE, EE+lowE” (gray). The cases of $k_{\text{max}} = 0.25 h$ Mpc$^{-1}$ and the Planck are identical to those in Figure 2.

FIG. 8. The distribution of scatters in each cosmological parameter, obtained from the cosmology analysis of noisy mock power spectra for the low-z and high-z NGC samples, using the different $k_{\text{max}}$ cuts; the x-axis shows a shift in the mode value in the 1d posterior of each parameter at $k_{\text{max}} = 0.25$ or $0.3 h$ Mpc$^{-1}$, compared to that at $k_{\text{max}} = 0.2 h$ Mpc$^{-1}$. We used the 50 realizations of the noisy mock spectra, and the histogram in each panel displays the distribution of parameter shifts. The vertical red and green lines denote the shift found from the cosmology analyses of the real BOSS data, shown in Figure 7. The dashed curves denote the Gaussian distributions specified by the mean and variance of the parameter shifts among the 50 realizations.

systematic error in the power spectrum data. Hence our fiducial choice of $k_{\text{max}} = 0.25 h$ Mpc$^{-1}$ seems reasonable against possible systematic effects.

In Appendix C we also study possible effects of the fiber collision and the minimum wavenumber $k_{\text{min}}$ on the cosmological results. Here $k_{\text{min}}$ is the minimum wavenumber in that we include the power spectrum information over $k_{\text{min}} \leq k \leq k_{\text{max}}$ in the cosmology analysis. A brief summary is that these effects do not appear to cause any major systematic effect in our cosmological results.

D. A further test of emulation accuracy

Now we turn to discussion on a possible uncertainty in the model predictions. As discussed in Refs. [40] and [38], our emulator for the redshift-space halo power spectrum is calibrated using a dataset of $N$-body simulations for the 101 flat wCDM models, where we used 15 realizations for the fiducial Planck cosmology and one realization for each of 100 models (more exactly among these we used the data for 80 models as training data). Here the 100 models are sampled using the optimal maximum-distance sliced Latin hypercube design in the 6-dimensional parameter space of wCDM cosmology [see 40, for details]. One might think that our emulator has a better accuracy
around the Planck cosmology, which is different from the MAP model or the model preferred by our cosmology analysis of the BOSS power spectra. To test this possible uncertainty, we ran a new set of N-body simulations for a model at the MAP cosmology in Table II: the model with $\Omega_m = 0.300, H_0 = 68.3\,\text{km}\,\text{s}^{-1}\text{Mpc}^{-1}$, $\sigma_8 = 0.754$, and other cosmological parameters are set to the values at MAP. More exactly, we ran each N-body simulation with box side length of $2.5\,\text{h}^{-1}\text{Gpc}$ and $3000^3$ particles, and use 5 realizations. The total volume is about $78\,(\text{h}^{-1}\text{Gpc})^3$, much larger than the BOSS volume ($\sim 5.7\,(\text{h}^{-1}\text{Gpc})^3$), so we can sufficiently reduce the sample variance effect in the simulated power spectra. Then we populate galaxies into halos using the same recipe of the galaxy-halo connection used in the Cosmology Challenge paper [37], which is different from our fiducial HOD method. Using the same covariance matrix as we used in the actual cosmology analysis of BOSS data, we perform the same cosmology analysis to the simulated power spectra. This is very similar to the validation test of our method using the simulated power spectrum used in the Cosmology Challenge [37], but this new test gives a validation of our method at the MAP model. Also note that the new set of N-body simulations employs the fixed neutrino mass of 0.06 eV as in the simulations used for the emulator development, while the simulations in the Cosmology Challenges assume zero neutrino mass. Hence, this test also gives a confirmation on the subtle effect of the non-zero neutrino mass, which mainly affects the transfer function of matter density fluctuations. Figure 9 shows that our method recovers each cosmological parameter accurately.

E. Assembly bias

Another concern of the halo model approach is the impact of the “assembly bias” effect: although the simple halo model assumes that the clustering amplitudes of halos (and galaxies) are determined by halo masses, they might depend on a secondary parameter, depending on the assembly history of halos/galaxies [87, 88]. Even in the presence of the assembly bias, the redshift-space distortion effect due to the peculiar velocities are unlikely to be affected by the assembly bias, because the peculiar velocities arise directly from the gravitational field [17]. To test a possible effect of the assembly bias on cosmology analysis, we use the mock catalogs of galaxies including the assembly bias effect in Ref. [17], where galaxies are populated preferentially into halos that have lower concentrations as a proxy of the assembly history [see 17, for details]. The mock catalogs including the assembly bias effect are generated using the same N-body simulation as those used in the right panel of Figure 1; therefore the total volume is about $128\,(\text{h}^{-1}\text{Gpc})^3$. The mock galaxies have about 30% higher amplitudes in the real-space correlation function at large scales, compared to that of the mock galaxies without the assembly bias effect, which otherwise have the same HOD (see Figure 12).

We generated the mock signals for each of the BOSS-like galaxy samples, and applied the same pipeline of cosmology analysis to the mock signals. Note that the assembly bias has not been detected with high significance from the BOSS galaxies [e.g. 89].

Figure 10 shows the cosmological parameters obtained from the mock catalogs including the assembly bias effect. It can be found that the assembly bias does not cause a significant bias in the inferred cosmological parameters, $\Omega_m, H_0$ and $\sigma_8$. This is consistent with the Fisher forecast in the previous work [17], confirming that the BAO and RSD information are not affected by the assembly bias effect, even after marginalization over the galaxy-halo connection parameters. In other words, the cosmology analysis using the redshift-space power spectrum does not rely on the halo mass estimate. This result is contrasted with that in Ref. [77]: they found that the assembly bias causes a significant bias in the cosmological parameters, especially $\sigma_8$ and $\Omega_m$, if a hypothetical joint-probe cosmology analysis using the galaxy-galaxy weak lensing and the projected correlation function is applied to the mock signals including the assembly bias effect. In such a method, the galaxy-galaxy weak lensing plays an important role to constrain the average mass of host halos, which in turn helps determine the galaxy bias uncertainties to obtain the clustering ampli-
FIG. 10. The posterior distributions of $\Omega_m$, $H_0$, and $\sigma_8$, obtained when we apply our analysis pipeline to the mock power spectrum signals that are generated from the mock galaxy catalogs including the assembly bias effect (see text for the details). The horizontal and vertical dashed lines show the input parameter values. The mock galaxy catalogs have the same HOD shape as that of the mock catalogs in the right panel of Figure 1, but were generated by populating mock galaxies preferentially into halos that have lower concentrations, in each halo mass bin.

Finally we comment on the impact of the galaxy-halo connection parameters on the cosmological parameters. We have so far employed broad priors of these nuisance parameters. Other observables such as galaxy-galaxy weak lensing [85, 91] can be used to infer the HOD parameters. Throughout this paper, we have employed a fairly broad prior range for each of 7 galaxy-halo connection parameters. In this subsection, we study how the cosmological parameter estimation can be improved if we have some knowledge on the HOD parameters.

As a working example, in Figure 13 we show the posterior distributions of the cosmological parameters obtained from the BOSS data by fixing the galaxy-halo connection parameters to their values at MAP in our fiducial analysis (the results for Figure 2). Therefore, this can be regarded as the best-case scenario where the galaxy-halo connection of the galaxy sample is perfectly known. The figure shows that the cosmological parameters are significantly improved. It is interesting to observe that the level of improvement of the parameter constraint varies with the parameters. We can see that $\Omega_m$ and $H_0$, which are mostly determined by the geometrical information through the BAO feature and also partly from the spectral shape information, are not significantly improved as compared to $\sigma_8$, which is an amplitude-related parameter. This is telling that the overall amplitude of the cosmological fluctuations is the hardest to interpret from observations due to the strong degeneracy with the galaxy bias uncertainty or the uncertainty in galaxy-halo connection.

VI. CONCLUSION

In this paper, we used an emulator of halo clustering statistics to estimate the cosmological parameters from the full-shape analysis of the redshift-space power spectra measured from the BOSS DR12 galaxy catalog over $0.2 < z < 0.75$. Combining with the HOD model, the emulator allows us to compute the redshift-space power spectra of galaxies for a given cosmological model within the flat $\Lambda$CDM cosmology, in less than a CPU second. It enables the parameter inference of the BOSS spectra in a multi-dimensional parameter space (33 parameters in this study). We showed that the emulator model well reproduces the monopole, quadrupole and hexadecapole moments of the redshift-space power spectra simultaneously for all of the 4 galaxy subsamples. Our method yields stringent constraints on the cosmological parameters, $\Omega_m$, $H_0$ and $\sigma_8$, even after marginalization over uncertainties in the nuisance parameters including the galaxy-halo connection parameters: more precisely, $\Omega_m = 0.301^{+0.012}_{-0.011}$, $H_0 = 68.2 \pm 1.4 \, \text{km s}^{-1}\text{Mpc}^{-1}$, and $\sigma_8 = 0.786^{+0.037}_{-0.036}$, for the mode and the 68% credible interval of the 1d posterior distribution (Figure 2 and
Table II). The cosmological parameters we obtained are consistent with those from the independent studies for the same BOSS spectra using PT-based models as the theoretical template [50, 51], even though the theoretical templates are totally different. This shows the robustness of the redshift-space power spectrum method for estimating the cosmological parameters: the BAO, AP effects and RSD information can be robustly extracted as long as the uncertainties in the nonlinear effects and galaxy properties are marginalized over. The statistical precision for each of the main parameters, $\Omega_m$, $H_0$ and $\sigma_8$ is also comparable with that from the PT-based methods. One might think that the halo model could allow us to use the redshift-space power spectra down to a larger $k$ and therefore lead to a more stringent constraint on the parameters, compared to the PT-based methods that treat all the galaxy bias parameters as nuisance parameters. However, this is not the case. We think there are a few reasons for this result. First, we employed quite broad priors for each of the HOD parameters and did not include any information on the abundance of galaxies. Hence our analysis might be consid-
Planck are also in good agreement with the template information at fixed $k_{min}$. Second, the power spectrum is sensitive. The galaxy-galaxy weak lensing arises mainly from Fourier modes in the three-dimensional space perpendicular to the line-of-sight direction, and it was shown that it carries almost independent information from the redshift-space power spectrum that arises from Fourier modes in the three-dimensional space [94]. Hence, combining the redshift-space power spectrum with the galaxy-galaxy weak lensing helps disentangle the degeneracy between the galaxy bias uncertainty (the galaxy-halo connection) and the cosmological parameters, leading to an improved estimation of the cosmological parameters (see Figure 13 for a possible improvement in the best-case scenario). To do this, our emulator based method easily enables to jointly combine the two observables in the same halo model framework for parameter inference. This is definitely an interesting direction, and will be our future work.

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There is a promising route to improving our cosmological constraints. It is a joint-probes cosmology: although we use the redshift-space power spectrum as the data vector in this paper, there is another observable available in the BOSS footprint. The promising one is the galaxy-galaxy weak lensing, which can be obtained by cross-correlating the positions of BOSS galaxies with shapes of background galaxies, where the background galaxies can be taken from an imaging survey overlapping with some portion of the BOSS footprint. For this, the Subaru HSC and some parts of the DES and KiDS surveys have an overlapping region with the BOSS footprints, and the galaxy-galaxy weak lensing can be measured from the joint analysis of BOSS and these imaging surveys. Since the galaxy-galaxy weak lensing can measure the average mass distribution around the BOSS galaxies, it helps observationally disentangle the galaxy bias uncertainty or yields stringent constraints on the HOD parameters in the halo model picture. In fact, Ref. [57] combined the galaxy-galaxy weak lensing, measured from the HSC data of small area coverage 140 sq. degrees, with the projected correlation function of BOSS galaxies in the cosmology analysis and then obtained the accuracy of $\sigma(S_8) \approx 0.05$, compared to our constraint of $\sigma(S_8) \approx 0.04$ although $S_8$ is not a best parameter to which the redshift-space power spectrum is sensitive. The galaxy-galaxy weak lensing arises mainly from Fourier modes in the two-dimensional space perpendicular to the line-of-sight direction, and it was shown that it carries almost independent information from the redshift-space power spectrum that arises from Fourier modes in the three-dimensional space [94]. Hence, combining the redshift-space power spectrum with the galaxy-galaxy weak lensing helps disentangle the degeneracy between the galaxy bias uncertainty (the galaxy-halo connection) and the cosmological parameters, leading to an improved estimation of the cosmological parameters (see Figure 13 for a possible improvement in the best-case scenario). To do this, our emulator based method easily enables to jointly combine the two observables in the same halo model framework for parameter inference. This is definitely an interesting direction, and will be our future work.
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Appendix A: Posterior distributions of the galaxy-halo connection parameters in the validation test

Figure 14 show the posterior distributions of the galaxy-halo connection parameters, obtained from the cosmology analysis of the mock data vector that are generated from the mock catalogs of BOSS-like galaxies. The mock galaxy catalog is the same as that used in the right panel of Figure 1. The figure shows that some of the galaxy-halo connection parameters are not necessarily well recovered, although the cosmological parameters are fairly well recovered as shown in Figure 1. The size of the posterior contours for the galaxy-halo connection parameters indeed shrinks with including the power spectrum information on the higher $k$.

Appendix B: Posterior distributions in full parameter space

For comprehensiveness of our discussion, in Figure 15 we show the posterior distributions in each 2d subspace of the full parameters for the $\Lambda$CDM model for each galaxy sample, based on our baseline analysis setup (Table 1).

Appendix C: Tests of the fiber collision effect and the minimum wavenumber $k_{\text{min}}$

Next we investigate the impact of the fiber collision on the cosmological parameter inference from the redshift-space galaxy power spectrum. The fiber collision, which occurs due to the inability of the two adjacent optical fibers to be closer than some finite separation angle, is a potential systematic effect on the cosmological parameter inference. The likelihood of the fiber collision depends on the number density of target galaxies on the celestial sphere, and it leads to an anisotropic effect in the measured power spectrum. Ref. [95] shows that the correction to the fiber collision effect on the galaxy power spectrum depends on the true power spectrum itself, and suggests the way for the correction. In this work, instead of implementing the correction in the model prediction, we examine the extent to which the fiber collisions affect the cosmology inference.

Figure 16 shows a comparison of the parameter inference between the cases of with/without the fiber collisions. To investigate the influence of the fiber collisions on the cosmological parameter inference, we use the PATCHY mocks which have the fiber collision weights. The fiber collision weights in the PATCHY mocks are assigned following Ref. [54], which reflects the nearest neighbor (NN) method. In this figure we see that there are only marginal differences in the parameter posteriors between the case with the fiber collisions corrected by the NN weights (empty, dashed line contours) and that of the true power spectrum with no fiber collisions (filled, solid line contours), for both the low-$z$ and high-$z$ NGCs, up to $k_{\text{max}} = 0.3\ h\ Mpc^{-1}$.

Figure 17 shows another test based on mock power spectra. We quote the shifts of the power spectrum due to the fiber collisions [96] in the NSeries mocks shown in the left panel of Figure 3 of Ref. [95], and add them to our fiducial mock signals after spline interpolation to reduce the noise contributions. This mock fiber collision effect causes almost no change to the posterior distribution of the cosmological parameters, even in the case of $k_{\text{max}} = 0.3\ h\ Mpc^{-1}$. From these studies, we conclude that the fiber collision effect has no significant impact on the cosmology analysis. Therefore, we conclude that the systematic shift in the $\sigma_8$ posterior in our real data analysis when we increase $k_{\text{max}}$ (shown in Figure 7) is not due to the fiber collisions.

We mention the influence of the minimum wavenumber $k_{\text{min}}$ of the power spectrum signals we use in the cosmological inference. Figure 18 shows the parameter inference results for different values of $k_{\text{min}} = 0.005, 0.02,$ and $0.05\ h\ Mpc^{-1}$ while we keep $k_{\text{max}} = 0.25\ h\ Mpc^{-1}$. Here $k_{\text{min}} = 0.005\ h\ Mpc^{-1}$ is our default choice throughout this paper. It shows that the choice of $k_{\text{min}}$ has no significant effect on the cosmological parameter inference.
FIG. 14. The posterior distributions of the galaxy-halo connection parameters, for the low-z NGC sample as an example, obtained from the combined analysis using the mock catalogs that are generated using the same HOD model as in our theoretical template (the right panel of Figure 1). The dashed lines in each plot denote the input value of each parameter that is used when generating the mock galaxy catalogs.

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FIG. 15. The posterior distributions in each 2d subspace of the full parameters for each of the 4 galaxy samples, for the cosmology analysis in Figure 2: the low-z NGC, low-z SGC, high-z NGC and high-z SGC samples, respectively. The posterior distributions are from the joint parameter inference of these parameters (33 parameters in total as given in Table 1): 5 cosmological parameters (while we impose tight Gaussian priors to $\omega_b$ and $n_s$) and each sample is characterized by 7 nuisance parameters (5 HOD parameters, the virial velocity parameter, and the residual shot noise parameter).

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Ref. [95] (see their Figure 3) to the power spectrum monopole and quadrupole of the fiducial mock signals.

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