Dissipation-assisted prethermalization in long-range interacting atomic ensembles

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We theoretically characterize the semiclassical dynamics of an ensemble of atoms after a sudden quench across a driven-dissipative second-order phase transition. The atoms are driven by a laser and interact via conservative and dissipative long-range forces mediated by the photons of a single-mode cavity. These forces can cool the motion and, above a threshold value of the laser intensity, induce spatial ordering. We show that the relaxation dynamics following the quench exhibits a long prethermalizing behaviour which is first dominated by coherent long-range forces, and then by their interplay with dissipation. Remarkably, dissipation-assisted prethermalization is orders of magnitude longer than prethermalization due to the coherent dynamics. We show that it is associated with the creation of momentum-position correlations, which remain nonzero for even longer times than mean-field predicts. This implies that cavity cooling of an atomic ensemble into the self-organized phase can require longer time scales than the typical experimental duration. In general, these results demonstrate that noise and dissipation can substantially slow down the onset of thermalization in long-range interacting many-body systems.

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The quest for a systematic understanding of non-equilibrium phenomena is an open problem in theoretical physics for its importance in the description of dynamics from the microscopic up to astrophysical scales [1–3]. Aspects of these dynamics are studied in the relaxation of systems undergoing temporal changes (quenches) of the control field across a critical point [4–6]. Quenches across a non-equilibrium phase transition provide further insight into the interplay between noise and external drives on criticality and thermalization [7, 8]. In this context photonic systems play a prominent role, thanks to their versatility [9–15].

Polarizable particles in a high-finesse cavity, like in the setup illustrated in Fig. 1(a), offer a unique system to study relaxation in long-range interacting systems. Here, multiple photon scattering mediates particle-particle interactions whose range scales with the system size in a single-mode cavity [15–18]. In this limit, atomic ensembles in cavities are expected to share several features with other long-range interacting systems such as gravitational clusters and non-neutral plasmas in two or more dimensions [3, 10, 19]. The equilibrium thermodynamics of these systems can exhibit ensemble inequivalence [3, 20], while quasi-stationary states (QSS) typically characterise the out-of-equilibrium dynamics [3, 21, 22]. QSS are metastable states in which the system is expected to remain trapped in the thermodynamic limit, they are Vlasov-stable solutions and thus depend on the initial state. So far, however, evidence of QSS has been elusive. It has been conjectured that noise and dissipation can set a time scale that limits the QSS lifetime [23, 24], and possibly gives rise to dynamical phase transitions [25]. In Ref. [28] it was shown that, in presence of dissipation due to viscous damping or local inelastic collisions, the relaxation dynamics of long-range interacting systems can be cast in terms of so-called scaling QSS, which are solutions of the kinetic mean-field equation and asymptotically tend to a unique QSS [28]. Accordingly, one would expect to observe QSS in cavity systems [19]. In Ref. [16], however, we found no evidence of the typical superlinear dependence on $N$ of the QSS time scale [3], which we attributed to the effect of noise and dissipative processes. Nonetheless, the dissipative dynamics is here due to retardation effects in the coupling between the atoms and a global variable, the cavity field, and can also establish long-range correlations [29, 30] whose influence on the relaxation dynamics is still unexplored.

![FIG. 1. (Color online) (a) Atoms interact with the standing-wave mode of a cavity and are transversally driven by a laser. The laser amplitude ($\Omega$) and/or frequency ($\Delta_c$) are suddenly quenched across the threshold, above which the atoms organize in regular spatial patterns at steady state. The coherent scattering amplitude per atom, $S$, is tuned by the laser, $S \propto \Omega$, the resonator dissipates photons at rate $\kappa$. (b) Phase diagram of the second-order self-organization transition as a function of $\bar{n}$ (proportional to $S^2$) and $\Delta_c/\kappa$ (that determines the asymptotic temperature). The black line separates the homogeneous phase (with order parameter $\Theta = 0$) from the self-organized one (with $\Theta \rightarrow \pm 1$). The red dashed lines A and B illustrate the initial and final values of the sudden quenches we analyse.]
In this work we characterise the interplay between dissipative and conservative long-range forces in the semiclassical dynamics of \( N \) polarizable particles (atoms) confined within a high-finesse single-mode cavity and transversally driven by a laser [10, 31, 33] (see Fig. 1(a)). The particles motion is along the cavity axis (\( x \)-axis), and the dynamics results from their opto-mechanical coupling with the cavity mode at wave number \( k \) and spatial mode function \( \cos(kx) \). We focus on the regime where the laser frequency \( \omega_L \) is smaller than the cavity frequency \( \omega_c \), such that \( \Delta_c = \omega_L - \omega_c < 0 \). Here, the dynamics is characterised by a thermal stationary state, which can exhibit a second-order driven-dissipative phase transition (spatial selforganization) as a function of the laser intensity and of \( \Delta_c \). This transition is due to the interplay between the dispersive and the dissipative forces: The dispersive forces tend to order the particles positions, and the intracavity photon number is maximum. The dissipative forces, instead, are due to retardation effects in the dynamics of atoms and field: For \( \Delta_c < 0 \) they cool the atoms into a thermal state whose effective temperature \( T_{eff} \) is determined by \( \Delta_c \) and the cavity loss rate \( \kappa \): \( k_B T_{eff} = h(\Delta_c^2 + \kappa^2)/(\Delta_c) \), with \( k_B \) Boltzmann constant. \( T_{eff} \) determines the threshold \( S_c \) of the coherent laser scattering amplitude \( S \) per atom at which spatial self-organization occurs, such that \( \sqrt{N}S_c = 2k_BT_{eff}/\hbar \) and separates the regime where the spatial distribution is uniform and \( \Theta \approx 0 \) from the symmetry broken phase in which the atoms form Bragg gratings, as shown in Fig. 1(b).

We analyse the semiclassical dynamics of the atoms after a quench across the transition using a Fokker-Planck equation (FPE) for the phase space distribution \( f(x_1, \ldots, x_N; p_1, \ldots, p_N; t) \) at time \( t \) and as a function of the positions \( x_j \) and the momenta \( p_j \). The FPE is valid when the cavity linewidth \( \kappa \) exceeds the recoil frequency \( \omega_r = h\kappa^2/(2m) \) and the width of the momentum distribution \( \Delta p \) is larger than the photon linear momentum \( \hbar k \). It reads [31, 39]

\[
\partial_t f = \{H, f\} + \hat{n}\mathcal{L}_\beta f + O(U_0),
\]

where Hamiltonian \( H = \sum_{j=1}^{N} p_j^2/(2m) + h\Delta_n\hat{n}N\Theta^2 \) determines the coherent dynamics and is a realization of the anisotropic Hamiltonian Mean Field model (HMF) [10, 21, 40]. The dimensionless parameter \( \hat{n} = NS^2/(\kappa^2 + \Delta^2) \) scales the depth of the conservative potential. It also scales the dissipator \( \mathcal{L}_\beta \), describing the effective long-ranged friction and diffusion [30, 31]:

\[
\mathcal{L}_\beta f = \frac{N}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \sin(kx_i)\partial_{p_j} \sin(kx_j) \left(p_j + \frac{m}{\beta} \partial_{p_j}\right) f.
\]

with \( \Gamma = 2\omega_r h\kappa\beta \) and \( \beta = (k_B T_{eff})^{-1} \). For \( \Delta_c < 0 \) the incoherent dynamics drives the system into the stationary state \( f_S(\beta, \hat{n}) = f_0 \exp(-\beta H) \), where \( f_0 \) warrants normalization. This state is well defined in the thermodynamic limit we choose, according to which as \( N \) is increased, the quantity \( NS^2 \) (and thus \( \hat{n} \)) is kept constant. This choice warrants that the Hamiltonian satisfies Kac's scaling [3].

The relaxation dynamics following a sudden quench at \( t = 0 \) is numerically evaluated by means of stochastic differential equations (SDE). Averages are taken over several trajectories, sampling the dynamics of \( N \) atoms according to the given initial distribution [30, 41]. Before the quench is performed \( (t < 0) \), we assume that the system has reached the equilibrium solution \( f_S(\beta, \hat{n}) \) of the FPE at a given value of \( \hat{n} = \hat{n}_i \) and \( \Delta_c \). At \( t = 0 \) the value of \( \hat{n} \) is quenched from \( \hat{n}_i < \hat{n}_c \), deep in the disordered phase, to \( \hat{n}_f > \hat{n}_c \), well inside the ordered phase. This corresponds to the horizontal path A of Fig. 1(b), keeping \( \Delta_c \), and hence the asymptotic temperature, constant. We evolve the initial state setting \( \hat{n} = \hat{n}_f \) in Eq. 1. In what follows we focus on quenches from the disordered to the ordered phase along path A, nevertheless the essential features of the dynamics we will report on characterize also the quenches in the opposite direction as well as along paths of type B, which connects points with different asymptotic temperatures (see Supplemental Material, SM, [12]).

The time evolution of the modulus of the order parameter \( |\langle \Theta \rangle| \) is displayed in Fig. 2(a) for different values of \( \hat{n}_f \): \( |\langle \Theta \rangle| \) tends towards an asymptotic value, that is closer to unity the larger is \( \hat{n}_f \). Before reaching the steady state the dynamics go through different stages, which we classify as: (i) a fast relaxation towards an intermediate value of the magnetization with time scale \( t \lesssim 10^2\kappa^{-1} \); this time scale decreases with \( \hat{n}_f \). (ii) A transient regime where \( |\langle \Theta \rangle| \) seems to grow logarithmically with time. (iii) Finally, the dissipation becomes dominant and brings the system to the asymptotic value, which is exponentially approached over time scales of the order of \( 10^9\kappa^{-1} \). These time scales are illustrated in Fig. 2(a) and here reported for \( N = 50 \) particles but generally depend on \( N \), as we discuss later on.

We first observe that, being \( \Delta_c \) negative, the growth of \( |\langle \Theta \rangle| \), Fig. 2(a), corresponds to a monotonic decrease of the potential energy, \( V = h\Delta_c\hat{n}N\Theta^2 \). In the fast relaxation stage (i), this decrease is well-fitted by an exponential, and is associated with a corresponding decrease of the relative fluctuations (see inset), indicating that the cavity field exponentially grows and creates a mechanical potential, which increasingly localizes the atoms at its minima. The exponential potential depth growth is due to this nonlinearity: the more the atoms become localized in the Bragg grating the larger is the scattering amplitude, and thus the potential depth. The increasing localization correspondingly augments the ki-
the time evolution of the relative localization

Here, $\Gamma$ which has the same physical origin as the long-range

the initial Gaussian ("thermal") state, for

the inset of Fig. 2(b): $K$ exponentially deviates from the value of the initial Gaussian ("thermal") state, for which $K_{\text{gaussian}} = 3$. We have verified that this dynamics is well described by a Vlasov equation for the single-particle distribution $f_{x}(x,p;t)$, which we derive assuming $f(x_{1},\ldots,x_{N};p_{1},\ldots,p_{N};t) = \prod_{j=1}^{N}f_{x}(x_{j},p_{j};t)$, integrating out the $N-1$ variables from Eq. (1) for the ini-
the observable for QSS [28], which we here define as:

$$\phi_{11} = \frac{\langle |\sin(kx)p| \rangle}{\langle |\sin(kx)| \rangle \langle |p| \rangle} - 1. \quad (3)$$

When \(\phi_{11} \neq 0\), the distribution is not factorizable into a kinetic and a potential term. Figure 3(b) displays the time evolution of \(\phi_{11}\) for the Hamiltonian, mean-field, and full dynamics. In stages (i) and (ii) the three models predict approximately the same behaviour. Instead, in stage (iii), \(\phi_{11}\) evolves differently: For both MF and full FPE it exhibits a minimum, however reached at different times, which seems to possess the features of a scaling QSS, namely, a sequence of QSS with identical correlations [28]. Its nature could be understood in terms of the onset of collective oscillations which are (almost) decoupled from noise and dissipation. Analogous behaviours have been reported for the case of atomic arrays in a cavity [29] [41]. Since the trajectories of \(\phi_{11}\) are different for the three types of simulations, the corresponding QSS are expected to not be the same. In particular, the discrepancy between full FPE and MF in stage (iii) remains of the same order when scaling up the system, while instead Hamiltonian prethermalization tends towards the corresponding mean-field prediction. Figure 4 displays the relaxation time scales for the MF and the full FPE: the two curves suggest a linear increase with \(N\) for both cases, nevertheless they run parallel thus showing that the discrepancy is a scalable effect. We deduce that this discrepancy is due to the momentum-position correlations due to noise, which are otherwise discarded in the MF treatment.

This prethermalization is not related to the critical slowing down observed in Ref. [15], but is due to the creation of correlations between momentum and position, and is reminiscent of kinetic-stop dynamics [40]. It implies that cavity-cooling of a large sample of atoms into the self-organized phase, corresponding to a sudden quench along path B, can be very slow and thus inefficient (see also Ref. [34]). Our analysis sets the stage for the development of a kinetic equation that is valid in the full quantum regime [47,51].

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FIG. 4. Relaxation after a sudden quench along path A for the parameters of Fig. 3 ($\bar{n}_a = 2\bar{n}_c$) and for different $N$. Inset: Time evolution of $\langle \Theta^2 \rangle$ for $N = 50$ (gray) and $N = 200$ (black). The dashed lines are the prediction of the MF model (see SM, 32). The horizontal dashed line indicates where $\langle \Theta^2 \rangle$ has reached 60% of its stationary value and identifies the corresponding time $\kappa t^*_N$. Onset: time $\kappa t^*_N$ (in units of $\kappa^{-1}$) as a function of $N$ for the full FPE (squares) and MF (stars). The lines are the corresponding linear fits in the log-log plot.

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Mean-Field equation and Vlasov limit

The mean-field equation is derived from Eq. (1) in the main text by assuming \( f(x_1, \ldots, x_N; p_1, \ldots, p_N; t) = \prod_{j=1}^{N} f_1(x_j, p_j; t) \), and integrating out the \( N-1 \) variables. It reads

\[
\partial_t f_1 = \{ H_{MF}[f_1], f_1 \} + \bar{n} \mathcal{L}_{\beta, MF} f_1
\]

(S1)

where the mean-field Hamiltonian \( H_{MF}[f_1] \) is given by

\[
H_{MF}[f_1] = \frac{p^2}{2m} + \frac{2\hbar \Delta_c \bar{n}}{N} \left( \frac{1}{2} \cos(kx) + (N-1) \langle \cos(kx') \rangle f_1 - \frac{\hbar k \beta \kappa}{2m \Delta_c} (N-1) \langle p' \sin(kx') \rangle f_1 \right) \cos(kx),
\]

with \( \langle A(x', p') \rangle f_1 = \int_0^\infty \int_{-\infty}^\infty dp' \mathcal{A}(x', p') f_1(x', p') \) and \( \mathcal{A} \) a phase space function. Furthermore the mean-field dissipator \( \mathcal{L}_{\beta, MF} \) is defined as

\[
\mathcal{L}_{\beta, MF} f_1 = \frac{\Gamma}{N} \sin^2(kx) \partial_p \left( p + \frac{m}{\beta} \partial_p \right) f_1.
\]

The dissipator \( \mathcal{L}_{\beta, MF} \) is responsible for the relaxation of the system to the thermal stationary state with temperature \( \beta^{-1} = k_B T_{\text{ee}} = \hbar (\Delta_c^2 + \kappa^2)/(-4 \Delta_c) \) \([S1][S3]\). Since the dissipator decreases with \( N^{-1} \) (for increasing \( N \)), the mean-field predicts a relaxation timescale that extends linearly with \( N \). Although the relaxation for the full FPE, Eq. (1) in the main text, is orders of magnitudes slower (see Fig. 4 in the paper), the corresponding growth of the timescale with \( N \) is almost indistinguishable from a linear one.

In order to make some statements for the short time dynamics we derive the Vlasov equation. This equation is derived from Eq. (S1) after performing the limit \( N \to \infty \) with \( NS^2 = \text{const.} \) and reads

\[
\partial_t f_1 + \frac{p}{m} \partial_x f_1 - \partial_x V[f_1] \partial_p f_1 = 0,
\]

(S2)

where the Vlasov potential \( V[f_1] \) is

\[
V[f_1] = 2\hbar \Delta_c \bar{n} \left( \langle \cos(kx') \rangle f_1 - \frac{\hbar k \beta \kappa}{2m \Delta_c} \langle p' \sin(kx') \rangle f_1 \right) \cos(kx).
\]

The stability analysis of Eq. (S2) shows that a spatially homogeneous distribution is unstable against small fluctuations \( \delta f \) when \( \bar{n}_f > \bar{n}_c \). The fluctuations exhibit exponential growth at rate \( \gamma \), which monotonously increases with \( \bar{n}_f \) and is a solution of the equation

\[
[1 - 2\kappa \gamma/(\Delta_c^2 + \kappa^2)] F(\gamma) \bar{n}_f / \bar{n}_c = 1,
\]

(S3)

with \( F(\gamma) = 1 - \sqrt{\pi} b \exp(b^2) \text{erfc}(b), b^2 = h \gamma^2 \beta/(4 \omega_c) \) and \( \text{erfc} \) is the complementary error function. The solution (red line in Fig. 3(a) in the main text for a fixed value of \( \bar{n}_f \)) well fits the numerical result in the fast relaxation regime. Thus, this initial behaviour is analogous to the violent relaxation observed in the HMF and has mainly Hamiltonian origin.

Quenches along path B

Figure S1 displays a sudden quench in the detuning \( \Delta_c \) while keeping \( \Omega_c \) hence the laser amplitude, constant. This quench corresponds to path B of Fig. 1(b) in the paper and alters both \( \bar{n} \) and \( \beta \) in Eq. (1), namely both the asymptotic order and temperature. We consider quenches from the disordered (with \( \Delta_c = -\kappa \)) to the ordered phase (with \( \Delta_c = -4 \kappa \)), and vice versa, assuming that the initial state is the asymptotic state of the parameter choice before the quench. Also in this case the three regimes can be identified. Remarkably, for the quench from the ordered to the disordered phase, the system remains for long times trapped in an ordered pattern. The pattern stays stable due to
the long-range forces. This transient is further accompanied by a momentum distribution that is narrower than the initial and the asymptotic value, as visible in the inset of subplot (b). On the other hand, the momentum distribution is markedly non-Gaussian, as visible in (c). This behaviour shows that, even if the final value of the parameter $\bar{n}$ is well below threshold and the asymptotic number of intracavity photons $\langle \hat{a}^\dagger \hat{a} \rangle \approx N \bar{n} (\Theta^2)$ is small, yet there is a metastable regime in which the number of intracavity photons is significantly larger, due to the metastable atomic patterns which support superradiant scattering of photons into the resonator until they decay.

![FIG. S1. Numerical simulation of the dynamics for $N = 50$ atoms following a sudden quench along path B, where $\Delta_c$ is varied but the laser intensity is kept fixed. The black line corresponds to the results for the evolution when the value of the detuning is suddenly quenched from $\Delta_c = -4\kappa$ with $\bar{n}_i \approx 0.44 \bar{n}_c$ to $\Delta_c = -\kappa$ with $\bar{n}_f = 2 \bar{n}_c$. The grey line displays the case where initial and final points are swapped. (a) and (b): Time evolution of $\langle |\Theta| \rangle$ and $\Delta p$ (inset) as a function of time (in units of $\kappa^{-1}$). Subplot (c) displays the behaviour of the kurtosis $K$.](image)

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