An Analysis of the $^{12}\text{C} + ^{12}\text{C}$ Reaction Using a New Type of Coupling Potential

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Abstract

A new approach has been used to explain the experimental data for the $^{12}\text{C} + ^{12}\text{C}$ system over a wide energy range in the laboratory system from 32.0 MeV to 126.7 MeV. This new coupled-channels based approach involves replacing the usual first derivative coupling potential by a new, second-derivative coupling potential. This paper first shows and discusses the limitation of the standard coupled-channels theory in the case where one of the nuclei in the reaction is strongly deformed. Then, this new approach is shown to improve consistently the agreement with the experimental data: the elastic scattering, single-2$^+$ and mutual-2$^+$ excitation inelastic scattering data as well as their 90$^\circ$ elastic and inelastic excitation functions with little energy-dependent potentials. This new approach makes major improvement on all the previous coupled-channels calculations for this system.
Keywords: optical model, coupled-channels calculations, DWBA, elastic and inelastic scattering, dynamical polarization potential (DPP), $^{12}$C+$^{12}$C reaction.

I. INTRODUCTION

Forty years ago, it was observed that the elastic cross-section of the $^{12}$C+$^{12}$C system varies rapidly with bombarding energy. This structure in the excitation functions, which could also be observed in other systems such as $^{12}$C+$^{16}$O and $^{16}$O+$^{16}$O, has remained a subject attracting continuous interest from both theoretical and experimental points of views. Consequently, a large body of data over a wide energy range has been accumulated for the $^{12}$C+$^{12}$C system from the systematic studies of this reaction [1–4].

However, there has been no global model that describes consistently the available elastic and inelastic scattering data over a wide energy range and this reaction presents a challenge to the many different theoretical models. Some of the problems can be summarised as: (1) no consistent description of the elastic scattering, single-$2^+$ and mutual-$2^+$ excitation inelastic scattering data as well as their $90^\circ$ excitation function; (2) the out of phase problem between the theoretical predictions and the experimental data for these states; (3) no simultaneous description of the individual angular distributions and resonances; (4) the magnitude of the mutual-$2^+$ excitation inelastic scattering data is unaccounted for.

The elastic scattering data of this system has been studied systematically and progress has been made using the optical model (see the review by Brandan and Satchler [5]). However, the inelastic scattering has received little attention and there is no systematic study over a wide energy range and the above-mentioned problems could not be explained using the standard coupled-channels models (see for example [3,6–11]).

Stokstad et al. [3] were the first to study the elastic and single-$2^+$ excitation inelastic scattering data using the DWBA and the coupled-channels methods from $E_{Lab}=74.2$ MeV to 126.7 MeV. They obtained reasonable agreement with the elastic scattering data. However, they could not reproduce the correct oscillatory structure for the single-$2^+$ excitation inelastic scattering.
scattering data and the magnitude of the data could not be accounted for correctly. They did not study the mutual-2\(^+\) excitation inelastic scattering data in their calculations. No theoretical calculations has predicted the magnitude of this data correctly over a wide energy range.

Wolf, Satchler and others [6] studied this system at three different energies. They used a double-folding potential and an angular-momentum dependent imaginary potential in their coupled-channels calculations. They could not reproduce the experimental data measured at \(E_{Lab}=74.2, 93.8,\) and \(126.7\) MeV. In particular, the theoretical predictions for the mutual-2\(^+\) excitation inelastic scattering data were very small by factors of 3 to 10 with respect to the experimental data. The results of the single-2\(^+\) excitation inelastic scattering calculations were also very oscillatory in comparison with the experimental one. We encountered the same problems in our standard coupled-channels calculations. Varying the parameters and changing the shape of the real and imaginary potentials do not provide a solution, as discussed in section [11].

Fry et al [7,8] also worked on this reaction to obtain the integrated cross-section (also known as Cormier’s resonances) for the single-2\(^+\) and mutual-2\(^+\) excitation channels using the coupled-channels method. They made use of a double-folding potential like the one of Stokstad et al [3] and an angular momentum-dependent imaginary potential. However, this method totally failed and no improvement of the densities in the double-folding potential would solve the magnitude problem of the mutual-2\(^+\) excitation inelastic scattering data. The same problems are observed in other authors’ works such as Sakuragi [9] and Ito [10].

Another interesting analysis was made by Ordoñez et al [12]. They showed the necessity of using a real potential that has a minimum in the surface region. They reported a detailed phase-shift analysis of the \(^{12}\text{C}+^{12}\text{C}\) elastic scattering data in the range of \(11.0 \leq E_{Lab} \leq 66.0\) MeV. This analysis revealed a striking sequence of gross structure resonances that appeared to form a rotational band from \(l=0\) to at least \(16\hbar\). These resonances were simulated by shape-resonances in a real potential with a secondary minimum at large radii related to the shape-isomeric doorway states in \(^{24}\text{Mg}\).
The interesting feature of their work is the double-peaked nature of the real potential. It is clear that this potential does explain the resonance data, which other models have failed to reproduce within such a large energy range. As will be argued in section IV, there is a resemblance between this potential and our total nuclear potential ($V_{\text{real}}+V_{\text{coupling}}$).

Ordoñez et al could not justify this double-peaked potential, other than by asserting it was required to fit the experimental data. This paper and a forthcoming paper [13] shall argue that this deepening at the surface is due to the strongly deformed structure of the $^{12}$C and may indicate a super-deformed state of the compound nucleus, $^{24}$Mg. It is also clear that Ordoñez et al took into account the coupling effects in their optical model calculations by introducing such a deepening at the surface without running coupled-channels calculations.

The literature clearly shows that the standard coupled-channels approach can fit neither any of the individual angular distributions nor the 90° elastic scattering excitation function simultaneously. For the resonance calculations, the situation is the same. That is, even if one fitted the Cormier’s resonances observed for the single-2$^+$ and mutual-2$^+$ excitation channels, it would be, at the same time, impossible to fit the 90° elastic scattering excitation function. Clearly, the $^{12}$C+$^{12}$C system has numerous problems to which no consistent global solution has been provided yet.

The overview of previous works indicates that the central potentials are actually quite reasonable since they have given the resonances at the correct energies and with sensible widths. Within the optical model calculations, they have also given very good agreements for the elastic scattering angular distributions or the 90° elastic scattering excitation functions independently. However, the calculations for the mutual-2$^+$ excitation inelastic scattering data is in general under-predicted by a large factor and the oscillatory structure of the data can not be reproduced correctly. They have remained unsolved so far.

Therefore, our aim of analysing the $^{12}$C+$^{12}$C system is to search for a global solution for some of these problems with little energy-dependent potentials within the coupled-channels formalism from 32.0 MeV to 126.7 MeV in the laboratory system.

In the next section, we introduce the model potentials used to analyse the experimental
data and the results of these analyses are shown in section III, where we also make a discussion of the limitations of the standard coupled-channels method and highlight the problems. Section IV is devoted to the analyses of the experimental data using our new coupling potential and the results are shown in section V. Finally, the section VI gives a summary and a discussion of the new and standard coupled-channels calculation.

II. THE STANDARD COUPLED-CHANNELS CALCULATIONS

A recent critical review by Kondō et al [14] found that a potential with a real depth of $\sim 300$ MeV was able to account for the $90^\circ$ elastic excitation function at low energies ($E_{Lab} \leq 75.0$ MeV). The real potentials proposed in this paper are tested and their parameters have to be readjusted due to the coupling effects in the coupled-channels calculations.

In our coupled-channels calculations, the interaction between the $^{12}$C nuclei is described by a deformed optical potential. The real potential has the square of a Woods-Saxon shape:

$$V_N(r) = \frac{-V_N}{(1 + \exp(r - R_N)/a_N)^2}$$

and the parameters, as shown in table I, are fixed to reproduce the $90^\circ$ elastic scattering excitation function. The Coulomb potential is assumed to be that of a uniformly charged nucleus with a radius of 5.5 fm.

The imaginary potential has the standard Woods-Saxon volume shape:

$$W(r) = -\frac{W}{1 + \exp((r - R_W)/a_W)}$$

and its depth increases quadratically with energy as:

$$W = -2.69 + 0.145E_{Lab} + 0.00091E_{Lab}^2$$

The parameters of the radius and diffuseness are shown in table I.

Since the $^{12}$C nucleus is strongly deformed, its collective excitation has been treated in the framework of the coupled-channels formalism. The $^{12}$C nucleus has a static quadrupole
deformation, which is taken into account by deforming the real optical potential with a
Taylor expansion about \( R = R_0 \) in the usual way \[15\]:

\[
U(r - R) = U(r - R_0) - \delta R \frac{\partial}{\partial R_0} U(r - R_0) + \frac{1}{2!} (\delta R)^2 \frac{\partial^2}{\partial R_0^2} U(r - R_0) - \ldots
\]

(4)

For the projectile \( P \) and the target \( T \)

\[
\delta R_P = R_P \beta_2 Y_{20}(\theta, \phi)
\]

\[
\delta R_T = R_T \beta_2 Y_{20}(\theta, \phi).
\]

(5)

\( R_P \) and \( R_T \) are the radii of the projectile and target. The form factors \[15\] are

\[
F_P(r) = R_P \left[ \frac{\partial}{\partial R_0} U(r, R_0) \right], \quad F_T(r) = R_T \left[ \frac{\partial}{\partial R_0} U(r, R_0) \right]
\]

(6)

\[
H_{P,T}(r) = \frac{1}{(4\pi)^{1/2} R_P R_T} \frac{\partial^2 U(r, R_0)}{\partial R_0^2}
\]

(7)

\( F_P(r) \) and \( F_T(r) \) in equation (6) are the first-order form factors that account for the
excitations of the projectile and target nuclei, while \( H_{P,T}(r) \) in equation (7) is the second
order form factor that accounts for their mutual excitation.

In equation (5), \( \beta_2 = -0.6 \) is the deformation parameter of the \(^{12}\text{C} \) nucleus. This empirical
value is derived from its known \( \text{B(E2)} \) value. The value of \( \text{B(E2)} \) is \( 42 \text{ e}^2\text{fm}^4 \) \[16\]. (A more
recent measurement gives an average value of \( 39 \pm 4 \text{ e}^2\text{fm}^4 \) \[17\]).

In the standard coupled-channels calculations of inelastic scattering involving mutual
excitation of the two nuclei, the codes CHUCK \[18\] and FRESCO \[19\] are used in such
a way that the two nuclei are excited sequentially. However, we think it essential that
\textit{simultaneous} mutual excitation of the two nuclei be included in the calculations. To do
so, we use the mutual-2\(^+\) excitation inelastic scattering data that are available. We modify
the code CHUCK to take into account the \textit{simultaneous} mutual excitation process \[20\].
It is observed that the simultaneous mutual excitation of the two nuclei does affect the
calculations, in particular in the resonance region where the calculations are very sensitive
to the small variations of the potential parameters. This is demonstrated in figure \[1\] at
$E_{\text{Lab}} = 93.8$ MeV since we have available experimental data for all the states considered in this paper.

III. RESULTS

The result of the 90° elastic scattering excitation function obtained using the parameters of table I is shown in figure 2. The theoretical predictions and the experimental data are in very good agreement, but, as Kondo et al found, this potential family does not fit the individual elastic scattering and inelastic scattering data as well as their excitation functions simultaneously.

We have attempted to obtain reasonable fits to the individual angular distributions by changing the parameters of the real potential, shown in table I, but without success. Some authors [9,21] also found a potential family that reproduces the individual angular distributions, but does not fit the 90° elastic scattering excitation function.

To overcome this difficulty, we searched for a new potential family by readjusting the parameters of the real potential and letting the imaginary potential change freely. The parameters are shown in table II. Except in the resonance regions, we obtained satisfactory agreement for the elastic scattering data as shown in figures 3 and 4 with dashed lines. However, the theoretical predictions of the magnitudes and the phase of the oscillations are not in good agreement with the experimental data for the single-$2^+$ state, as shown in figure 5 with dashed lines. The out-of-phase and magnitude problems between the theoretical calculations and the experimental data are clearly seen at many energies. These results for the elastic and single-$2^+$ excitation inelastic scattering are almost identical to those obtained by Stokstad et al [3]. For the mutual-$2^+$ excitation inelastic scattering data, as shown in figure 6 with dashed lines, there is no agreement and the theoretical predictions of the magnitude of mutual-$2^+$ excitation inelastic scattering data are much smaller than the experimental one; they are under-predicted by a factor of 3 to 10. Nevertheless, our results for the mutual-$2^+$ excitation inelastic scattering data are in conformity with the findings
of the references [3, 4, 11], a problem mentioned by many authors in a recent international conference on clustering (ICC ‘99) [10, 22, 23]. In order to make a comparison with the new calculations, presented in the next section, some of the results for the single-2\(^+\) and mutual-2\(^+\) states are shown in figure 3 and 6.

We had anticipated that the inclusion of the simultaneous mutual excitation of two nuclei could solve the magnitude problem of the mutual-2\(^+\) excitation data. However, although this effect has improved the details of the fits to the experimental data, it failed to provide a solution. The magnitude of the mutual-2\(^+\) excited state cross-section is still one of the major outstanding problems of this reaction.

In the past, the magnitude problem for the single-2\(^+\) excitation inelastic scattering calculations was solved for different reactions by changing the empirical \(\beta\) value [24, 25]. Thus, the same solution was expected to apply to the \(^{12}\text{C} + ^{12}\text{C}\) system for the single-2\(^+\) and mutual-2\(^+\) excitations inelastic scattering calculations. For this purpose, we increased the \(\beta\) value to -1.2, which is twice the actual value and has no physical justification. However, although the agreement between theoretical predictions and the experimental data for the magnitudes of the single-2\(^+\) and mutual-2\(^+\) excitations inelastic scattering data is improved, the theoretical predictions for the elastic scattering data are very poor; the same holds for the 90° elastic scattering excitation functions.

Within the coupled-channels formalism, the reason for this failure may be understood if the effect of changing the real potential on the inelastic scattering cross-section is considered. The method of obtaining the coupling potential that describes the inelastic scattering has been based on perturbation theory. Since the coupling potential is connected to the real term by a Taylor expansion around the surface of the nucleus, changing the real potential has a substantial effect on the elastic scattering data, but not on the inelastic scattering one. Therefore, according to this standard procedure, the coupling potential has the same energy-dependence as the central term. Actually, Smithson et al [26] analysed the inelastic scattering data for the \(^{16}\text{O} + ^{208}\text{Pb}\) system and asserted that the standard deformation procedure is inadequate for the description of the inelastic scattering data. They also concluded that
there is no reason for the coupling potential to have the same energy dependence as the central potential.

**IV. NEW COUPLING POTENTIAL**

If we consider two $^{12}$C nuclei approaching each other, the double-folding model will generate an *oblate* potential which is correct at large distance. When these two nuclei come close enough, they create the compound nucleus $^{24}$Mg which is a *prolate* nucleus in its ground state, whereas the folding model yields an oblate (attractive) potential in this case. How well the double-folding model describes a prolate nucleus with an oblate potential is unclear and this may be the reason why the earlier calculations using a double-folding model in the coupled-channels method were unable to provide a consistent solution to the problems of this reaction.

The limitations of the standard coupled-channels method, on the one hand, and the *oblate* character of the $^{12}$C and the *prolate* character of the compound nucleus $^{24}$Mg, on the other hand, have motivated us to use a second-derivative coupling potential. In order to describe the above-mentioned configuration, the coupling potential must be *oblate* (attractive) when two $^{12}$C nuclei are at large distances and must be *prolate* (repulsive) when they are at short distances. The standard and the new coupling potential are shown in figure 7.

One possible interpretation of such a second-derivative coupling potential can be made if we express the total potential as a function of the radii for different orientations of the two colliding $^{12}$C nuclei. If $\theta_{P,T}$ are the angles between the symmetry axes and the axis joining the centers of the projectile and target, then the total potential, as an approximation, can be expressed in the following way:

$$V(r) = V_N + \beta_2 R \frac{dV_C}{dR} (Y_{20}(\theta_P, \phi_P) + Y_{20}(\theta_T, \phi_T)) + \beta_2^2 R^2 \frac{d^2V_C}{dR^2} (Y_{20}(\theta_P, \phi_P) + Y_{20}(\theta_T, \phi_T)) \quad (8)$$

where $V_N$ is the nuclear potential and $V_C$ is the new second-derivative coupling potential. The final term is due to the mutual excitation.
The result for the $^{12}$C+$^{12}$C system is shown in figure 8. A second local minimum is observed in the interaction potential for certain orientations. This feature, included only in an ad hoc way in the work of Ordoñez et al [12], has not been taken into account in the standard coupled-channels calculations. To investigate this minimum, we looked at the total inverted potential, i.e. the dynamical polarization potential (DPP) plus the bare potential, obtained by the inversion of the S-Matrix [13]. Our analysis suggests that the new coupling potential points to the presence of the super-deformed configurations in the compound nucleus $^{24}$Mg, as it has been speculated [27,28].

The real and imaginary potentials in these new calculations have the same shapes as in previous calculations (see equation 1 and 2) and the parameters of the potentials are displayed in table III. We have analysed the experimental in the same energy range.

V. RESULTS

The results of the analyses using the new coupling potential are displayed in figures 3 and 4 for the ground state, in figure 5 for the first excited state (single-$2^+$) and in figure 6 for the mutual excited state (mutual-$2^+$).

The agreement is very good for the elastic scattering, single-$2^+$ and mutual-$2^+$ excitation inelastic scattering data over the whole energy range studied. The theoretical predictions of the magnitudes and the phase of the oscillations for the single-$2^+$ and mutual-$2^+$ excitations inelastic scattering data, which have been the major outstanding problems of this reaction, are in a very good agreement with the empirical values. This new coupling potential has made a substantial improvement at all the energies considered.

The $90^\circ$ elastic scattering excitation function is also analysed and the result is shown in figure 9. The agreement with the experimental data is excellent over the whole energy range.

Table III indicates that the parameters are almost constant (1 to 3 % changes) away from the resonance regions. However, at certain energies in the energy range $E_{Lab} \sim 90$ to
110.0 MeV, the parameters fluctuate. We interpreted the fluctuations at small energies in table III as the effect of the resonances observed by Cormier et al [29,30], Chappell et al [31–33] and Fulton et al [34]. The changes of the potential parameters in the energy range $E_{\text{Lab}} \sim 90$ to 110.0 MeV might be related to resonances, which have not yet been observed in the $^{12}\text{C}^{+}\text{^{12}C}$ system. Within such an interpretative scheme, one may infer that these resonances might be associated with the single and mutual-4$^+$ excited states of $^{12}\text{C}$, states which are strongly coupled to the ground state.

These predictions motivated us to run an experiment in this energy range. The initial analyses of the experimental data indicate that the variation of these parameters are not actually random since structures relating to the 4$^+$ state of the $^{12}\text{C}$ are seen in this energy range. The detailed analyses and the full results will be given in the forthcoming paper [35].

This new, second-order coupling potential, has also been applied successfully to the $^{16}\text{O}^{+}\text{^{28}Si}$ and $^{12}\text{C}^{+}\text{^{24}Mg}$ systems [20,36]. This model has explained the experimental data successfully.

VI. SUMMARY AND CONCLUSION

We considered the elastic and inelastic scattering of the $^{12}\text{C}^{+}\text{^{12}C}$ system from 32.0 MeV to 126.7 MeV in the laboratory system. Although this reaction has been one of the most extensively studied reaction over the last forty years, there has been no global model that explains consistently the measured experimental data over a wide energy range. In the introduction, we presented the problems that this reaction manifests. We attempted to find a consistent solution to these problems. However, within the standard coupled-channels method, we failed, as others did, to describe certain aspects of the data, in particular, the single-2$^+$ and mutual-2$^+$ excitation inelastic scattering data although the optical model and coupled-channels models explain perfectly some aspects of the elastic scattering data.

As discussed in section III, the standard coupled-channels method entails that the coupling potential has the same energy-dependence as the central term. However, our analysis
reveals that the coupling potential has a vital importance in explaining the experimental
data for the reactions that involve at least one strongly deformed nucleus and that there
is no reason for the coupling potential to have the same energy dependence as the central
potential. This may explain the failure of the standard coupled-channels calculations.

The comparison of the results obtained using the standard and new coupled-channels cal-
culations indicates that a global solution to the problems of the scattering observables of this
reaction over a wide energy range (32.0 MeV to 126.7 MeV) with little energy-dependence
on the potentials has been provided by this new coupling potential. The significance of the
new approach should be underlined because it does not only fit the present experimental
data, but it also leads to other novel and testable predictions. To our knowledge, this has not
been yet achieved over such a wide energy range. Studies using this new coupling potential
may also lead to new insights into the formalism and a new interpretation of such reactions.

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TABLES

**TABLE I.** The parameters of the potentials required to fit the $90^\circ$ elastic excitation function, displayed in figure 2.

| $V_N$ (MeV) | $R_N$ (fm) | $a_N$ (fm) | $W$ (MeV) | $R_W$ (fm) | $a_W$ (fm) |
|------------|-----------|-----------|-----------|-----------|-----------|
| 345.0      | 3.62      | 1.60      | eq. (3)   | 5.50      | 0.51      |

**TABLE II.** The numerical values of the potentials used in the standard coupled-channels calculations. $V_N$, $r_N$ and $a_N$ stand for the depth, radius and diffuseness of the real potential respectively and $W$, $r_W$ and $a_W$ stand for the depth, radius and diffuseness of the imaginary potential respectively.

| $E_{Lab}$ (MeV) | $V_N$ (MeV) | $r_N$ (fm) | $a_N$ (fm) | $W$ (MeV) | $r_W$ (fm) | $a_W$ (fm) |
|-----------------|-------------|-----------|-----------|-----------|-----------|-----------|
| 32.0            | 290.0       | 0.80      | 1.30      | 3.0       | 1.20      | 0.51      |
| 40.0            | 290.0       | 0.79      | 1.28      | 3.6       | 1.20      | 0.51      |
| 45.0            | 290.0       | 0.80      | 1.15      | 3.8       | 1.20      | 0.51      |
| 49.0            | 290.0       | 0.79      | 1.23      | 4.2       | 1.20      | 0.51      |
| 50.0            | 290.0       | 0.80      | 1.21      | 4.5       | 1.20      | 0.51      |
| 55.0            | 290.0       | 0.80      | 1.15      | 5.0       | 1.20      | 0.51      |
| 57.75           | 290.0       | 0.81      | 1.35      | 6.3       | 1.20      | 0.51      |
| 60.0            | 290.0       | 0.80      | 1.30      | 6.6       | 1.20      | 0.51      |
| 65.0            | 290.0       | 0.79      | 1.43      | 7.0       | 1.20      | 0.51      |
| 70.7            | 290.0       | 0.81      | 1.20      | 8.5       | 1.20      | 0.51      |
| 78.8            | 290.0       | 0.81      | 1.30      | 9.5       | 1.20      | 0.51      |
| 93.8            | 290.0       | 0.82      | 1.35      | 12.0      | 1.20      | 0.51      |
| 98.2            | 290.0       | 0.81      | 1.30      | 12.5      | 1.20      | 0.51      |
| 102.1           | 290.0       | 0.81      | 1.30      | 14.0      | 1.20      | 0.51      |
| 105.0           | 290.0       | 0.81      | 1.30      | 14.4      | 1.20      | 0.51      |
| 112.0           | 290.0       | 0.80      | 1.30      | 13.0      | 1.20      | 0.51      |
| 117.1           | 290.0       | 0.80      | 1.35      | 14.0      | 1.20      | 0.51      |
| 121.6           | 290.0       | 0.80      | 1.35      | 14.1      | 1.20      | 0.51      |
| 126.7           | 290.0       | 0.81      | 1.30      | 14.2      | 1.20      | 0.51      |
TABLE III. The numerical values of the potentials used in the new coupled-channels calculations. $W$ denotes the imaginary potential. $V_N$, $r_N$ and $a_N$ stand for the depth, radius and diffuseness of the real potential respectively and $r_C$ and $a_C$ stand for the radius and diffuseness of the coupling potential respectively ($V_C=210.0$ MeV).

| $E_{Lab}$ (MeV) | $V_N$ (MeV) | $r_N$ (fm) | $a_N$ (fm) | $W$ (MeV) | $r_C$ (fm) | $a_C$ (fm) |
|-----------------|-------------|------------|------------|-----------|------------|------------|
| 32.0            | 290.0       | 0.804      | 1.19       | 2.21      | 0.69       | 0.70       |
| 40.0            | 288.0       | 0.806      | 1.28       | 2.40      | 0.69       | 0.70       |
| 45.0            | 290.0       | 0.809      | 1.28       | 2.97      | 0.69       | 0.70       |
| 49.0            | 290.0       | 0.810      | 1.28       | 3.07      | 0.69       | 0.70       |
| 50.0            | 290.0       | 0.813      | 1.24       | 3.07      | 0.69       | 0.70       |
| 55.0            | 290.0       | 0.813      | 1.26       | 3.17      | 0.69       | 0.70       |
| 57.75           | 290.0       | 0.813      | 1.26       | 3.17      | 0.69       | 0.70       |
| 60.0            | 290.0       | 0.813      | 1.28       | 3.37      | 0.69       | 0.70       |
| 65.0            | 290.0       | 0.811      | 1.28       | 3.57      | 0.69       | 0.70       |
| 70.7            | 289.0       | 0.799      | 1.29       | 3.71      | 0.69       | 0.70       |
| 78.8            | 287.0       | 0.785      | 1.28       | 5.50      | 0.68       | 0.70       |
| 93.8            | 292.0       | 0.790      | 1.34       | 11.9      | 0.67       | 0.67       |
| 98.2            | 289.0       | 0.785      | 1.27       | 11.5      | 0.66       | 0.65       |
| 102.1           | 289.0       | 0.810      | 1.33       | 11.5      | 0.65       | 0.63       |
| 105.0           | 289.0       | 0.810      | 1.37       | 11.5      | 0.66       | 0.66       |
| 112.0           | 287.0       | 0.800      | 1.28       | 13.8      | 0.68       | 0.67       |
| 117.1           | 290.0       | 0.810      | 1.32       | 14.7      | 0.69       | 0.68       |
| 121.6           | 290.0       | 0.810      | 1.33       | 15.3      | 0.68       | 0.67       |
| 126.7           | 288.0       | 0.795      | 1.30       | 17.3      | 0.66       | 0.67       |
FIG. 1. A comparison of the results of the simultaneous mutual excitation (the dashed line) and the sequential one (the solid line) for the elastic, single-$2^+$ and mutual-$2^+$ excitations at $E_{Lab}=93.8$ MeV.

FIG. 2. The comparison of the experimental data and the results of the standard coupled-channels calculation for the $90^\circ$ elastic scattering excitation function.
FIG. 3. Ground state results: The dashed lines show the predictions of the standard coupled-channels calculations (see table II for the parameters) while the solid lines show the results of the new coupled-channels calculations, obtained using new coupling potential with the empirical $\beta$ value ($\beta_C^2 = \beta_N^2 = -0.6$) (see table III for the parameters).
FIG. 4. Ground state results: The dashed lines show the predictions of the standard coupled-channels calculations (see table I for the parameters) while the solid lines show the results of the new coupled-channels calculations, obtained using new coupling potential with the empirical $\beta$ value ($\beta_2^C=\beta_2^N=-0.6$) (see table II for the parameters) (continued from figure 3).
FIG. 5. Single-2\(^+\) state results: The dashed lines show the predictions of the standard coupled-channels calculations (see table II for the parameters) while the solid lines show the results of the new coupled-channels calculations, obtained using new coupling potential with the empirical $\beta$ value ($\beta_2^C=\beta_2^N=-0.6$) (see table III for the parameters).
FIG. 6. Mutual-2\(^+\) state results: The dashed lines show the predictions of the standard coupled-channels calculations (see table I for the parameters) while the solid lines show the results of the new coupled-channels calculations, obtained using new coupling potential with the empirical \(\beta\) value \((\beta_2^C = \beta_2^N = -0.6)\) (see table II for the parameters).
FIG. 7. The comparison of the standard coupling potential (1) with $\beta=-0.6$, (2) with $\beta=-1.2$ and our new coupling potential for $E_{\text{Lab}}=32.0$ MeV. The parameters of the latter are shown in table III.

FIG. 8. The orientation potentials of two nuclei at different angles including the hexadecupole deformation of $^{12}$C.
FIG. 9. 90° elastic scattering excitation function, obtained using new coupling potential with the empirical $\beta$ value ($\beta^C_2 = \beta^N_2 = -0.6$).