Abstract: We present $T$-dual pairs of tachyon free (stable) AdS as well as dS solutions in the context of type II orientifold compactifications. In our construction, the type IIA setting is a purely geometric flux model which, in addition, includes the usual NS-NS three-form flux $H_3$ and the RR $p$-form fluxes $F_p$ for $p \in \{0, 2, 4, 6\}$, while the $T$-dual type IIB model includes (non-)geometric fluxes along with the usual $S$-dual pair of the three-form $(F_3, H_3)$ fluxes. In both the models there is one combination of the RR axions which remains flat, and the dS solution is realized through the $D$-term effects induced by (non-)geometric fluxes, without which one can only have AdS solutions. Using flux scaling arguments we also discuss how to engineer a parametric control in these models, in the sense of realizing the respective AdS/dS solutions in large volume, weak string-coupling as well as large complex-structure regime. We also discuss open possibilities (e.g. about the unknown status on having the complete set of Bianchi identities) under which such tachyon free dS solutions may or may not be viable.

Keywords: T-duality, Non-geometric Flux, de-Sitter Vacua, Swampland Conjecture
1 Introduction

Realizing de-Sitter solutions and possible obstructions on the way of doing it have been always in the center of attraction since decades. In this regard, the historicity of efforts on finding de-Sitter vacua in string theory (inspired) setups can be classified into the following three categories:

- Existence: Starting from a simple model building with minimal ingredients at hand, there have been several de-Sitter no-go scenarios proposed from time to time. For example, several no-go theorems forbidding de-Sitter (dS) and inflationary realizations have been proposed in a series of works [1–23], mostly in the context of type IIA orientifold compactifications. In fact these no-go results have played a central role
in the recent revival of the swampland conjectures [24, 25]. However, the good thing about any no-go result is the fact that it happens to hold in a framework with a given set of assumptions, and for a model builder aiming to realize de-Sitter solution, the very first task is to explore the loopholes or the limits under which the no-go necessities can be evaded, e.g. see [26] for a recent charting of the (anti-) de Sitter vacua from the perspective of ten-dimensional supergravities. In contrary to the (minimal) de-Sitter no-go scenarios, in the meantime there have been several proposals for realizing (stable) de-Sitter vacua [27–46]; see [47, 48] also for the F-theoretic initiatives taken in this regard.

- Stability: After evading some no-go results by adding more ingredients, the immediate question is about the stability of the subsequent de-Sitter solution, especially to ensure if those solutions are true minima or tachyonic in nature. This is a quite crucial condition to check as there have been plethora of de-Sitter solutions constructed by simple attempts of evading the no-go results, many of which eventually turned out to be tachyonic in nature [3–5, 10, 17, 49–51]. In fact the existence of such tachyonic de-Sitter solutions demanded the need of refinements in the original swampland conjectures [24, 25] leading to numerous amount of work with similar interests including the Quintessence alternative [52–74] and the challenges it faces regarding the discrepancy in Hubble constant [75, 76].

- Viability: This step is usually combined with what we referred to ‘Stability’ in the second step. However, given the fact that this appears to cover a wider range of open possibilities, it may be worth considering it separately. Given that very little is known about the corrections which induce the scalar potential pieces to perform moduli stabilization, it is an important question to ask if the de-Sitter vacua which pass the tests in the first two steps are genuine or not. For example, the questions regarding scale separation and field excursions in moduli space [77–93] without breaking effective field theory (EFT) assumptions, tadpole conjecture [94–96] and ways to avoid it [97] may be considered in this class. In fact, it happens very often that the scalar potential corrections are known only in pieces and are sometimes discovered/challenged with new updates, and in this regard, a perfect check about viability may be considered as the toughest task for string phenomenologists!

In the current work our aim is to analyse the vacua arising from a $T$-dual pair of type II models along the aforesaid three steps. For that purpose we begin with considering type IIA orientifold setup which includes geometric flux along with the standard three-form NS-NS ($H_3$) flux and the RR $p$-form $F_p$ fluxes for $p \in \{0, 2, 4, 6\}$. In our approach we first investigate the de-Sitter scenarios which can be obtained by evading the standard no-go results, e.g. those obtained from the moduli dynamics restricted to the volume/dilaton plane [4–6]. In this process we make some specific choice of fluxes such that

- The only geometric flux which arise in the scalar potential corresponds to the $D$-term effects which are positive semi-definite. All the geometric flux contributions arising
from the $F$-terms effects are set to zero, which facilitates in satisfying NS-NS Bianchi identities without losing any $D$-term fluxes.

- We consider the only some suitable RR flux components to be non-zero while setting many of those to zero. This facilitates in making the tadpole terms independent of one flux which subsequently does not get bounded by $O6$-charge through $D6$ tadpole conditions, and turns out to be useful in realizing larger volume and weaker string coupling values after the minimization process. However, recalling the fact that Romans mass term along with geometric flux is needed to evade the previously known no-go results, we always keep non-zero Romans mass along with some geometric flux components to be non-zero.

The $T$-dual completions of the four-dimensional type II effective theories by including the (non-)geometric fluxes have been initiated in the toroidal context [8, 98–102], and a couple of interesting efforts have been initiated in establishing a concrete connection between the (non-)geometric ingredients of the two theories in [103, 104], and a full mapping with the $T$-duality at the level of NS-NS non-geometric flux components and the two scalar potentials, has been presented in [105]. In [21], it was shown that a geometric type IIA setup corresponds to a non-geometric type IIB under a set of $T$-dual transformations relating the fluxes and moduli of the two theories (e.g. see [105]). In fact many of the (geometric) type IIA de-Sitter no-go scenarios have been $T$-dualized to type IIB case with non-geometric fluxes [21], which has refined the regime of de-Sitter search in various possible ways. By using $T$-dual transformations we present the corresponding type IIB model forming what we call a $T$-dual pair of type II models which help us realizing AdS/dS vacua according to the criteria of existence and stability as we have discussed.

However, regarding the viability there remains a subtle and open issue. It is naively assumed that a consistent incorporation of the various (non-)geometric fluxes enriches the compactification backgrounds which creates better possibilities for model building, however one does not known how many and which type of fluxes can be simultaneously turned-on, respecting the various Bianchi identities and the tadpole cancellation conditions. In this regard, there have been two formulations for computing the Bianchi identities; the standard one mostly applicable to toroidal orientifolds involves fluxes with non-cohomology indices [106–108], while in the cohomology formulation fluxes are represented using the non-trivial cohomology indices [103, 104, 107]. However, a mismatch between the two sets of Bianchi identities of these two formulations have been observed in [107, 109] and studied in some good detail in [110, 111]. The additional unknown identities in the cohomology formulation, if they exist, might be relevant for our de-Sitter solution along with the recent studies performed in [112–117].

This article is organised as follows. Section 2 presents a brief review of the relevant ingredients about the generic non-geometric type IIA and type IIB models, collecting the explicit $T$-duality transformations among the various moduli and fluxes on the two sides. In section 3 we present a useful formulation of the generic geometric type IIA scalar potential, and discuss possible scenarios in which the standard/known de-Sitter no-go results can be evaded. Section 4 presents two simple constructions aiming to evade the known dS no-go
scenarios corresponding to the type IIA setting, and its $T$-dual counterpart as a type IIB model. Subsequently, we present a detailed analysis of the $T$-dual pairs of AdS as well as dS solutions in section 5. Finally we summarise our results in section 6 and provide the additional useful pieces of information in appendix A.

2 Moduli, fluxes and the $T$-duality

The four-dimensional effective potentials arising from type II flux compactifications and their applications towards moduli stabilization have been extensively studied in series of works [27, 118–127]. In particular, the study of nongeometric flux compactifications and their four-dimensional scalar potentials have led to a continuous progress in various phenomenological aspects such as towards moduli stabilization, in constructing de-Sitter vacua and also in realizing the minimal aspects of inflationary cosmology [98, 99, 107, 108, 112–116, 128–137]. In the context of Type II supergravity theories, such (non-)geometric fluxes can generically induce tree level contributions to the scalar potential for all the moduli and hence can subsequently help in dynamically stabilizing them through the lowest order effects. Moreover, the common presence of the nongeometric fluxes in Double Field Theory (DFT), superstring flux-compactifications, and the gauged supergravities has helped in understanding a variety of interconnecting aspects in these three formulations along with opening new windows for exploring some phenomenological aspects as well [98, 101, 102, 106, 109, 130, 138–153]. Moreover, the nongeometric flux compactification scenarios also present some interesting utilizations of the symplectic geometries [154, 155] to formulate the effective scalar potentials; e.g. see [156–158], which generalize the work of [118, 119] by including the nongeometric fluxes. The ten-dimensional origin of the four-dimensional nongeometric scalar potentials have been explored via an iterative series of works in the supergravities [147–150, 152, 153, 155, 159–163] and through some robust realization of the Double Field Theory (DFT) reduction on Calabi Yau threefolds [164, 165].

Moreover, a concrete connection among the type II effective potentials derived from DFT reductions and those of the symplectic approach has been established in [156, 157].

In this section we recollect the relevant ingredients for type IIA and type IIB orientifold models, and more details can be directly refereed to [105]. The moduli dynamics in the low energy four dimensional effective supergravity theories are governed by the so-called $F$- and $D$-term contributions which are encoded in the Kähler potential, the flux superpotential and the gauge kinetic functions. We present these generic ingredients for the $T$-dual completed version of the type IIA and type IIB setups.

2.1 Type IIA orientifold model

The Kähler potential in type IIA model is given as below

$$K_{\text{IIA}} = - \ln \left( \frac{4}{3} \kappa_{\alpha\beta\gamma} t^\alpha t^\beta t^\gamma + 2 \, p_0 \right) - 4 \ln (z^0)^{-1} - 2 \ln \left( \frac{1}{6} k_{\lambda\mu\rho} z^\lambda z^\mu z^\rho + \frac{\bar{p}_0}{4} \right) , \quad (2.1)$$
defined in the following manner:

\[ T^a = b^a - i t^a, \]
\[ N^0 = \xi^0 + i (z^0)^{-1}, \]
\[ N^k = \xi^k + i (z^0)^{-1} z^k, \]
\[ U_\lambda = \xi_\lambda - i (z^0)^{-1} \left( \frac{1}{2} k_{\lambda \rho \kappa} z^\rho z^\kappa - \frac{1}{2} \hat{k}_{\lambda \kappa m} z^k z^m - \bar{p}_\lambda z^k - \bar{p}_\lambda \right), \]

Here \( t^a \) are two-cycle volume moduli and \( b^a \)'s denote the axionic moduli arising from the NS-NS two form potential \( B_2 \), while \( \xi^0, \xi^k \) and \( \xi_\lambda \) are components of the RR three-form potential \( C_3 \) defined as \( C_3 = \xi^k \alpha_k - \xi_\lambda \beta^\lambda \) where \( \hat{k} = \{0, k\} \). Moreover, the quantities \( \{z^k, z^\rho\} \) collectively denote the dilaton and the complex structure moduli. In addition, we use the following topological quantities [35, 166],

\[ p_{ab} = \frac{1}{2} \int_{CY} \hat{D}_a \wedge \hat{D}_b \wedge \hat{D}_b \quad (\text{mod} \ Z), \]
\[ p_a = \frac{1}{24} \int_{CY} c_2(CY) \wedge \hat{D}_a, \quad p_0 = - \frac{\zeta(3) \chi(CY)}{8 \pi^3}, \]

where \( \hat{D}_a, c_2(CY) \) and \( \chi(CY) \) respectively denote the dual to the divisor class, the second Chern class and the Euler characteristic of the compactifying CY threefold. We denote these topological quantities on the mirror CY threefold with a tilde with an appropriate change of indices, e.g. \( \tilde{p}_{k\lambda}, \tilde{p}_\lambda \) and \( \tilde{p}_0 = -p_0 \) due to a sign flip in the Euler characteristic of the mirror as compared to the compactifying CY threefold.

For the later purpose, we also define \( V = \frac{1}{6} \kappa_{abc} t^a t^b t^c \) as the overall volume of the compactifying (CY) threefold and similarly an analogous quantity for the complex structure side defined as \( U = \frac{1}{6} k_{\lambda \rho \kappa} z^\lambda z^\rho z^\kappa \), and subsequently the Kähler potential can also be written as below,

\[ K_{\text{IIA}} = - \ln (8 V + 2 p_0) - 4 \ln (z^0)^{-1} - 2 \ln \left( U + \bar{p}_0 \right). \]

Now, the type IIA flux superpotential can be generically given in terms of the fluxes and the chiral variables introduced in Eq. (2.2) in the following form,

\[ \sqrt{2} W_{\text{IIA}} = \left[ \bar{c}_0 + T^a \bar{c}_a + \frac{1}{2} \kappa_{abc} T^a T^b m^c + \frac{1}{6} \kappa_{abc} T^a T^b T^c m^0 - i p_0 m^0 \right] \]
\[ - N^0 \left[ \Pi_0 + T^a \bar{w}_{a0} + \frac{1}{2} \kappa_{abc} T^b T^c Q^a_{0} + \frac{1}{6} \kappa_{abc} T^a T^b T^c R_0 - i p_0 R_0 \right] \]
\[ - N^k \left[ \Pi_k + T^a \bar{w}_{ak} + \frac{1}{2} \kappa_{abc} T^b T^c Q^a_{k} + \frac{1}{6} \kappa_{abc} T^a T^b T^c R_k - i p_0 R_k \right] \]
\[ - U_\lambda \left[ \Pi^\lambda + T^a \bar{w}_{a\lambda} + \frac{1}{2} \kappa_{abc} T^b T^c Q^a_{\lambda} + \frac{1}{6} \kappa_{abc} T^a T^b T^c R^{\lambda} - i p_0 R^{\lambda} \right], \]
where we have introduced a shifted version of the flux parameters to absorb the effects from \( p_{ab}, p_a \) in the following manner,

\[
\begin{align*}
\bar{e}_0 &= e_0 - p_a m^a, \\
\bar{e}_a &= e_a - p_{ab} m^b + p_a m^0, \\
\bar{H}_0 &= H_0 - p_a Q^a_0, \\
\bar{H}_a &= H_a - p_{ab} Q^b_k + p_a R_k, \\
\bar{H}^\lambda &= H^\lambda - p_{ab} Q^{a\lambda}, \\
\bar{w}_a &= w_a - p_{ab} Q^b + p_a R_0,
\end{align*}
\]

where \( \{e_0, e_a, m^a, m^0\} \) corresponds to the RR \( p \)-form fluxes \( \{F_0, F_4, F_2, F_0\} \) while the remaining four types of fluxes, namely \( \{H, w, Q, R\} \) with appropriate components, arise from the NS-NS three-form flux \( H_3 \), geometric flux, non-geometric flux. In addition, the various topological quantities used in shifted fluxes in Eq. (2.6) are defined as Eq. (2.3).

For type IIA model, the explicit expressions of the \( D \)-terms can be given as below,

\[
\begin{align*}
D_\alpha &= \left(\frac{z(0)}{z}\right)^{-1} e^{K_0} \left[ \left( \mathcal{U} + \bar{p}_0 - \frac{1}{2} \hat{K}_{\lambda \alpha} z^\lambda z^\alpha \right) \hat{w}_\alpha^0 + \hat{K}_{\lambda \alpha} z^\lambda z^\alpha \hat{w}_\alpha + z^\lambda \hat{w}_\alpha \right], \\
D^\alpha &= - \left(\frac{z(0)}{z}\right)^{-1} e^{K_0} \left[ \left( \mathcal{U} + \bar{p}_0 - \frac{1}{2} \hat{K}_{\lambda \alpha} z^\lambda z^\alpha \right) \hat{Q}^\alpha_0 + \hat{K}_{\lambda \alpha} z^\lambda z^\alpha \hat{Q}^\alpha_0 + z^\lambda \hat{Q}^\alpha_0 \right].
\end{align*}
\]

Here \( e^{K_0} = (z(0))^2 / (\mathcal{U} + \bar{p}_0) \), and the electric part of the gauge kinetic couplings given as

\[
\text{Re}(f^{\text{ele}}_{g})_{\alpha \beta} = - \frac{1}{2} \hat{K}_{\alpha \beta} t^a
\]

while the magnetic part is defined as the dual of the electric part. This leads to the following \( D \)-term contributions to the four-dimensional scalar potential,

\[
V_{IIA}^D = \frac{1}{2} D_\alpha \left[ \text{Re}(f^{\text{ele}}_{g})_{\alpha \beta} \right]^{-1} D_\beta + \frac{1}{2} D^\alpha \left[ \text{Re}(f^{\text{mag}}_{g})_{\alpha \beta} \right]^{-1} D^\beta.
\]

Also note that \( \text{Re}(f^{\text{ele}}_{g}) > 0, \text{Re}(f^{\text{ele}}_{g}) > 0 \) as these are related to moduli space metrics which is positive definite, and can be shown to be \( V_{IIA}^D \geq 0 \).

### 2.2 Type IIB orientifold model

The chiral variables governing the moduli dynamics in the 4D low energy effective type IIB supergravity model are given as,

\[
\begin{align*}
U^i &= \bar{u}^i - i \, v^i, \\
S &= c_0 + i \, s, \\
G^a &= (c^a + c_0 \, b^a) + i \, s \, b^a, \\
T_\alpha &= \hat{c}_\alpha - i \, s \left[ \frac{1}{2} \ell_{\alpha \beta \gamma} t^\beta t^\gamma - \frac{1}{2} \ell_{\alpha ab} b^a b^b - p_{\alpha a} b^a - p_\alpha \right],
\end{align*}
\]

where \( t^a \) denotes the two-cycle volumes moduli and \( \{b^a, c^a\} \) are the so-called odd axions pairs arising from the two form potentials \( (B_2, C_2) \), while \( c_0 \) is the universal axion and \( s = e^{-\phi} \) denoted the dilaton \( \phi \). In addition, \( v^i \) denotes the complex structure axions and \( u^i \) denotes the complex structure saxionic moduli. Also we have introduced \( \hat{c}_\alpha \) as an
axionic combination given as \( \hat{\epsilon}_\alpha = c_\alpha + \hat{\epsilon}_{\alpha ab} b^a b^b \), where \{\( \hat{\epsilon}_{\alpha \beta \gamma}, \hat{\epsilon}_{\alpha ab} \)\} denote the even/odd sector triple intersection numbers, and \( c_\alpha \) arises from the reduction of the RR four-form (C4) potential. Now, for the type IIB model, the (T-dual) Kähler potential is given as

\[
K_{\text{IIB}} = -\ln \left( \frac{4}{3} l_{ijk} u^i u^j u^k + 2 \tilde{p}_0 \right) - 4 \ln s - 2 \ln \left( \frac{1}{6} \hat{\epsilon}_{\alpha \beta \gamma} t^\alpha t^\beta t^\gamma + \frac{p_0}{4} \right),
\]

which is a real function of the complexified moduli \( S, U^i, G^a \) and \( T_\alpha \). Also, by introducing the compact notations, this can also be given as

\[
K_{\text{IIB}} = -\ln (8 U + 2 \tilde{p}_0) - 4 \ln s - 2 \ln \left( \mathcal{V} + \frac{p_0}{4} \right),
\]

where the topological quantities \( p_0 \) for the compactifying CY and \( \tilde{p}_0 \) for the mirror CY are given by

\[
p_0 = -\frac{\zeta_3(CY)}{8 \pi^4} = -\tilde{p}_0.
\]

Further, similar to the type IIA model, the generalized flux superpotential for the type IIB side can be given in terms of the fluxes and the chiral variables (2.9) in the following form,

\[
\sqrt{2} W_{\text{IIB}} = \left[ F_0 + U^i \overline{F}_i + \frac{1}{2} l_{ijk} U^i U^j F^k - \frac{1}{6} l_{ijk} U^i U^j U^k F^0 - i \tilde{p}_0 F^0 \right] (2.12)
\]

\[
- S \left[ \overline{H}_0 + U^i \overline{H}_i + \frac{1}{2} l_{ijk} U^i U^j H^k - \frac{1}{6} l_{ijk} U^i U^j U^k H^0 - i \tilde{p}_0 H^0 \right]
\]

\[
- G^a \left[ \overline{\omega}_{a0} + U^i \overline{\omega}_{ai} + \frac{1}{2} l_{ijk} U^i U^j \omega_{ak}^i - \frac{1}{6} l_{ijk} U^i U^j U^k \omega_{ak}^i - i \tilde{p}_0 \omega_{ak}^i \right]
\]

\[
- T_\alpha \left[ \overline{Q}^0_\alpha + U^i \overline{Q}_i ^\alpha + \frac{1}{2} l_{ijk} U^i U^j \hat{Q}^\alpha k - \frac{1}{6} l_{ijk} U^i U^j U^k \hat{Q}^\alpha k - i \tilde{p}_0 \hat{Q}^\alpha k \right],
\]

where because of the \( \alpha' \)-corrections on the mirror side, the complex structure sector is modified such that to induce rational shifts in the usual flux components given as,

\[
\overline{F}_0 = F_0 - \tilde{p}_i F^i, \quad \overline{F}_i = F_i - \tilde{p}_j F^j - \tilde{p}_k F^k, \quad (2.13)
\]

\[
\overline{H}_0 = H_0 - \tilde{p}_i H^i, \quad \overline{H}_i = H_i - \tilde{p}_j H^j - \tilde{p}_k H^k, \quad \overline{\omega}_{a0} = \omega_{a0} - \tilde{p}_i \omega_{ai}^i, \quad \overline{\omega}_{ai} = \omega_{ai} - \tilde{p}_j \omega_{aj}^j - \tilde{p}_k \omega_{ak}^k, \quad \overline{Q}^\alpha_0 = \hat{Q}^\alpha_0 - \tilde{p}_i \hat{Q}^{\alpha i}, \quad \overline{Q}^\alpha_i = \hat{Q}^\alpha_i - \tilde{p}_j \hat{Q}^{\alpha j} - \tilde{p}_k \hat{Q}^{\alpha k},
\]

where \{\( F_0, F_i, F^i, F^0 \)\} corresponds to the RR three-form flux \( F_3 \) while the remaining four types of fluxes, namely \{\( H, \omega, Q, R \)\} with appropriate indices/components, arise from the NS-NS three-form flux \( H_3 \), geometric flux (\( \omega \)), and the non-geometric (\( Q, R \)) fluxes. In addition, the various topological quantities used in shifted fluxes in Eq. (2.13) are defined as Eq. (2.3). Finally, the D-terms for the type IIB model are given as

\[
D_K = -\frac{s e^{\frac{\kappa(Q)}{2}}}{2} \left[ R_K (\mathcal{V} + p_0 - \frac{1}{2} \hat{\epsilon}_{\alpha ab} t^\alpha b^a b^b) + Q^\alpha K \hat{\epsilon}_{\alpha ac} b^c - t^\alpha \hat{\omega}_{\alpha K} \right],
\]

(2.14)
\[ D^K = \frac{s e^{\frac{K(Q)}{2}}}{2} \left[ R^K (\mathcal{V} + p_0 - \frac{1}{2} \hat{\ell}_{aabb} t^a b^b) + Q^a B^K \hat{\ell}_{aabc} t^a - t^a \hat{\omega}_a^K \right], \]

which subsequently leads to the following contributions in the 4D scalar potential \cite{107},

\[ V^{D}_{1IB} = \frac{1}{2} D_J \left( \text{Re}(f_{JK}) \right)^{-1} D_K + \frac{1}{2} D^J \left( \text{Re}(f^{JK}) \right)^{-1} D^K, \quad (2.15) \]

where the electric/magnetic gauge kinetic couplings are given as \( \text{Re}(f_{JK}) = -\frac{1}{2} \hat{l}_{iJK} u^i = \frac{1}{2} l_{iJK} \), and \( \text{Re}(f^{JK}) = -\frac{1}{2} l^{iJK} \) leading to a positive semi-definite scalar potential \( V^{D}_{1IB} \geq 0 \).

### 3 Type IIA model with geometric flux

#### 3.1 Reformulating the scalar potential

The geometric type IIA scalar potential (A.1) can be also collected as under,

\[ V \equiv (V_R + V_{NS} + V_{loc}) = (V_{f_6} + V_{f_4} + V_{f_2} + V_{f_0}) + (V_h + V_\omega) + V_{loc}, \quad (3.1) \]

where

\begin{align*}
V_{f_6} &= \frac{e^{4D}}{4V} f_6^2, \\
V_{f_4} &= \frac{e^{4D}}{4} f_a \tilde{g}^{ab} f_b, \\
V_{f_2} &= \frac{e^{4D}}{4} t^a \tilde{g}_{ab} f^b, \\
V_{f_0} &= \frac{e^{4D}}{4} \mathcal{V} f_0^2, \\
V_h &= \frac{e^{2D}}{4V} \left[ \frac{\hat{h}_0}{\mathcal{U}} + \tilde{g}^{ij} h_{i0} h_{j0} + \tilde{g}_{\lambda \rho} h_{\lambda 0} h_{\rho 0} \right], \\
V_\omega &= \frac{e^{2D}}{4V} \left[ t^a t^b \left( \frac{h_a h_b}{\mathcal{U}} + \tilde{g}^{ij} h_{ai} h_{bj} + \tilde{g}_{\lambda \rho} h_{a \lambda} h_{b \rho} \right) \right. \\
&\quad + \frac{1}{\mathcal{U}} \left( h_a - \frac{k_\lambda}{2} h_{a \lambda} \right) \left( \mathcal{V} \tilde{g}^{ab} - t^a t^b \right) \left( h_b - \frac{k_\rho}{2} h_{b \rho} \right) \left. \right] + \frac{1}{\mathcal{U}} \left( \mathcal{U} \hat{h}_a^0 + z^a \hat{h}_a \right) \mathcal{V} \left( \tilde{v}_{a 0}^a - \frac{1}{2} \left( \mathcal{U} \hat{h}_b^0 + z^b \hat{h}_b \right) \right), \\
V_{loc} &= \frac{e^{3D}}{2 \sqrt{\mathcal{U}}} \left[ \left( f_0 h_0 - f^a h_a \right) - \left( f^0 h_0^a - f^a h_0 \right) \left( \frac{k_\lambda}{2} \right) \right].
\end{align*}

where the various non-zero “axionic flux orbits” are given in eqn. (A.2). This scalar potential can be studied for the searching the stable vacua through minimization of the moduli/axions, however one can consider the two-field volume/dilaton analysis to rule out certain scenarios \cite{2, 5}. In a more general analysis, one can also include the complex structure moduli in order to further check the de-Sitter solutions allowed by the volume/dilaton analysis.

For that purpose, we further introduce two new moduli, namely \( \rho \) and \( \sigma \) via a redefinition in the overall volume (\( \mathcal{V} \)) of the Calabi Yau threefold and its mirror volume \( \mathcal{U} \) by
considering the two-cycle volume moduli $t^a$ and $z^l$ as

$$t^a = \rho \gamma^a, \quad \Rightarrow \quad \nu = \rho^3, \quad \kappa_{abc} \gamma^a \gamma^b \gamma^c = 6,$$  \hspace{1cm} (3.3)

$$z^l = \sigma \theta^l, \quad \Rightarrow \quad \mathcal{U} = \sigma^3, \quad k_{\rho \gamma} \theta^\rho \theta^\gamma \theta^\delta = 6,$$

where $\gamma^a$'s denote the angular Kähler moduli while $\theta^l$'s corresponds to the angular Kähler moduli on the mirror Calabi Yau threefold. Now we can extract the volume factor $\rho$ from the Kähler moduli space metric and its inverse in the following way,

$$\tilde{g}_{ab} = \frac{\kappa_a \kappa_b - 4 \mathcal{V} \kappa_{ab}}{4 \mathcal{V}} = \rho \tilde{g}_{ab}, \quad \tilde{g}^{ab} = \frac{2 \epsilon^a \epsilon^b - 4 \mathcal{V} \kappa^{ab}}{4 \mathcal{V}} = \frac{1}{\rho} \tilde{g}^{ab},$$  \hspace{1cm} (3.4)

$$\tilde{g}_{\lambda \rho} = \frac{k_\lambda k_\rho - 4 \mathcal{U} k_{\lambda \rho}}{4 \mathcal{U}} = \sigma \tilde{g}_{\lambda \rho}, \quad \tilde{g}^{\lambda \rho} = \frac{2 x^\lambda x^\rho - 4 \mathcal{U} k^{\lambda \rho}}{4 \mathcal{U}} = \frac{1}{\sigma} \tilde{g}^{\lambda \rho},$$

$$\tilde{g}_{ij} = - \hat{k}_{ij} = \sigma \hat{g}_{ij}, \quad \tilde{g}^{ij} = - \hat{k}^{ij} = \frac{1}{\sigma} \hat{g}^{ij}.$$

Here the matrix $\tilde{g}_{ab}$ and its inverse $\tilde{g}^{ab}$ do not depend on $\rho$ modulus. Similarly, the matrices $\tilde{g}_{\lambda \rho}, \tilde{g}^{\lambda \rho}, \tilde{g}_{ij}$ and $\tilde{g}^{ij}$ do not depend on the analogous complex structure modulus $\sigma$. Using these new redefinitions the explicit dependence of the $\rho$ and $\sigma$ moduli can be extracted out from the generic type IIA scalar potential pieces given in eqn. (3.2), which can be rewritten in the following pieces,

$$V_{f_0} = e^{4D} \rho^3 A_1, \quad V_{f_2} = e^{4D} \rho A_2, \quad V_{f_4} = \frac{e^{4D}}{\rho} A_3, \quad V_{f_6} = \frac{e^{4D}}{\rho^3} A_4,$$  \hspace{1cm} (3.5)

$$V_h = \frac{e^{2D}}{\rho^3 \sigma^3} \left( A_5 + \sigma^2 A_6 + \sigma^4 A_7 \right),$$

$$V_\omega = \frac{e^{2D}}{\rho \sigma^3} \left( A_8 + \sigma^2 A_9 + \sigma^4 A_{10} + \sigma^6 A_{11} \right),$$

$$V_{loc} = \frac{e^{3D}}{\sigma^2} \left( A_{12} + A_{13} \sigma^2 \right),$$

where $A_i$'s are some functions of moduli other than volume modulus $\rho$, the complex structure modulus $\sigma$ and the 4-dimensional dilaton $D$. For completeness, the explicit expressions of $A_i$'s are given below,

$$A_1 = \frac{1}{4} (t^0)^2, \quad A_2 = \frac{1}{4} t^a \tilde{g}_{ab} t^b, \quad A_3 = \frac{1}{4} \tilde{L}_a \tilde{g}^{ab} \tilde{L}_b, \quad A_4 = \frac{1}{4} \tilde{f}_0^2,$$  \hspace{1cm} (3.6)

$$A_5 = \frac{1}{4} h_{00}^2, \quad A_6 = \frac{1}{4} h_{00} \tilde{g}^{ij} h_{ij}, \quad A_7 = \frac{1}{4} h^\lambda \tilde{L}_\lambda h^\rho_\rho, \quad A_8 = \frac{1}{4} h_a \tilde{g}^{ab} h_b,$$

$$A_9 = \frac{1}{4} \left[ (\gamma^a \gamma^b \tilde{g}^{ij} h_{ai} h_{bj}) + (\theta^\lambda \tilde{h}_{\alpha \lambda}) (\tilde{k}_{\alpha \beta} \gamma^\alpha)^{-1} (\theta^\rho \tilde{h}_{\beta \rho}) 
- \frac{1}{2} (\tilde{g}^{ab} - \gamma^a \gamma^b) \left( k_\lambda \rho \gamma \theta^\rho \theta^\gamma h_\lambda h_b + k_\rho \rho \gamma \theta^\rho \theta^\gamma h_\rho h^\rho \right) \right],$$

$$A_{10} = \frac{1}{4} \left[ (\gamma^a \gamma^b \tilde{g}_{\lambda \rho} h_\alpha h^\rho) + (\tilde{h}_\alpha (\theta^\rho \tilde{h}_{\beta \rho}) + (\theta^\lambda \tilde{h}_{\alpha \lambda}) \tilde{h}_\beta^\rho) (\tilde{k}_{\alpha \beta} \gamma^\alpha)^{-1} 
- \frac{1}{2} (\tilde{g}^{ab} - \gamma^a \gamma^b) \left( k_{\alpha \beta} \gamma \theta^\rho \theta^\gamma h_\rho h^\rho \right) \right].$$
\[ + \frac{1}{4} \left( \bar{g}^{ab} - \gamma^{a} \gamma^{b} \right) (h_{a} \lambda) (h_{b} \rho) (k_{\lambda \rho \gamma} \theta^{\rho \theta \gamma}) \],

\[ A_{11} = \frac{1}{4} \left( \hat{h}_{a}^{0} \right) (k_{\alpha \beta} \gamma^{\alpha})^{-1} (\hat{h}_{\beta}^{0}), \]

\[ A_{12} = \frac{1}{2} \left( f^{0} h_{b} - f^{a} h_{a} \right), \quad A_{13} = -\frac{1}{4} (k_{\lambda \rho \gamma} \theta^{\rho \theta \gamma}) \left( f^{0} h_{a}^{\lambda} - f^{a} h_{a}^{\lambda} \right). \]

This shows that all the \( A_{i} \)'s except \( A_{9}, A_{10}, A_{12} \) and \( A_{13} \) are positive semi-definite. Let us make an observation that the scalar potential pieces in Eq. (3.5) have complicated coefficients in the volume/dilaton plane and hence de-Sitter no-go results arising from some simple scale separation analysis (e.g. see [92, 93]), may be evaded for the most generic constructions.

### 3.2 Some scenarios evading the known dS no-go results

There have been several no-go results against the de-Sitter realization in type IIA model, especially arising from the volume/dilaton analysis [2, 5]. One possible method to evade such no-go results is to include the geometric flux along with a non-zero Romans mass term. Several attempts for de-Sitter realization have been made exploiting this observations [5], however best to our knowledge there is no proposed model which realizes non-tachyonic de-Sitter solution using integer valued fluxes. Using the derivatives for generic scalar potential as given in Eq. (3.1) along with (3.5) w.r.t. the volume modulus \( \rho \) and the dilaton \( D \), we find the following extremization conditions,

\[ \frac{\partial V}{\partial D} = 0 = 4 V_{f_{6}} + 4 V_{f_{4}} + 4 V_{f_{2}} + 4 V_{f_{0}} + 2 V_{h} + 2 V_{\omega} + 3 V_{\text{loc}} = 0, \]

\[ \rho \frac{\partial V}{\partial \rho} = 0 = 3 V_{f_{6}} + V_{f_{4}} - V_{f_{2}} - 3 V_{f_{0}} + 3 V_{h} + V_{\omega} = 0, \]

(3.7)

Moreover, the volume-dilaton sector of the Hessian matrix can be now given as under,

\[ M_{11} = 20 V_{f_{6}} + 20 V_{f_{4}} + 20 V_{f_{2}} + 20 V_{f_{0}} + 6 V_{h} + 6 V_{\omega} + 12 V_{\text{loc}}, \]

\[ M_{12} = 12 V_{f_{6}} + 4 V_{f_{4}} - 4 V_{f_{2}} - 12 V_{f_{0}} + 6 V_{h} + 2 V_{\omega} = M_{21}, \]

\[ M_{22} = 9 V_{f_{6}} + V_{f_{4}} + V_{f_{2}} + 9 V_{f_{0}} + 9 V_{h} + V_{\omega}. \]

(3.8)

In a nutshell, in order to investigate the possibility of de-Sitter realization, we need to check if there is a simultaneous solution to the following set of conditions (e.g. see [13]),

\[ 4 V_{f_{6}} + 4 V_{f_{4}} + 4 V_{f_{2}} + 4 V_{f_{0}} + 2 V_{h} + 2 V_{\omega} + 3 V_{\text{loc}} = 0, \]

\[ 3 V_{f_{6}} + V_{f_{4}} - V_{f_{2}} - 3 V_{f_{0}} + 3 V_{h} + V_{\omega} = 0, \]

\[ V_{f_{6}} + V_{f_{4}} + V_{f_{2}} + V_{f_{0}} + V_{h} + V_{\omega} + V_{\text{loc}} > 0, \]

\[ Tr[M] > 0, \quad \frac{(Tr[M])^{2}}{4} \geq Det[M] > 0. \]

(3.9)

Generically, there can be many solutions in support of finding the de-Sitter solution, however note that this analysis considers only the volume-dilaton sector and does not include all moduli, and hence that can have possibility to further rule out possible solution allowed
at this stage. However, in case there are some no-go against finding de-Sitter in this sector itself, then there is no need to include all the moduli together, and the result remains conclusive.

Scenario 1:

\[ V_{f_2} = 0, \quad V_{f_6} = 0, \quad V_{f_4} > 0, \quad 10V_{f_0} + 2V_{f_4} + 3V_{loc} = 4V_h, \quad 3V_{f_0} = V_{f_4} + 3V_h + V_\omega, \]
\[ V_h \geq 0, \quad 24V_{f_4} + 9V_h + \sqrt{516V_{f_4}^2 + 972V_{f_4}V_h + 441V_h^2} > 30V_{f_0}, \quad V_{f_0} > V_{f_4} + V_h. \quad (3.10) \]

Scenario 2:

\[ V_{f_2} = 0, \quad V_{f_4} = 0, \quad 24V_{f_0} + 9V_{loc} + 2V_\omega = 6V_h, \quad (3.11) \]
\[ V_h \geq 0, \quad V_\omega > 0, \quad 3V_{f_0} < 3V_h + 2V_\omega, \quad 3V_{f_0} = 3V_{f_6} + 3V_h + V_\omega, \]
\[ 27V_h + 19V_\omega + \sqrt{2025V_h^2 + 1458V_hV_\omega + 265V_\omega^2} < 72V_{f_0}. \]

Scenario 3:

\[ V_{f_6} = 0, \quad V_{f_4} = 0, \quad \frac{7V_\omega}{11} < V_{f_2} < V_\omega, \quad V_h > \frac{4V_{f_2}^2 - 9V_\omega V_{f_2} + 5V_\omega^2}{33V_{f_2} - 21V_\omega}, \quad (3.12) \]
\[ V_\omega > 0, \quad V_{loc} = -\frac{8V_{f_2} + 18V_h + 10V_\omega}{9}, \quad V_{f_0} = \frac{3V_h + V_\omega - V_{f_2}}{3}. \]

In the next section, we will present some concrete models where we will present benchmark examples with specific choice of fluxes such that all these necessary conditions can be met along with realizing the basic EFT requirements such as large volume, large complex structure and weak coupling values at the minimum.

3.3 Investigating the possible flux scalings

In this subsection we will explore the possible flux scaling which could be introduced in order to facilitate the moduli stabilization in some generic fashion.

Scenario 1:

As mentioned in Eq. (3.10), demanding \( V_{f_2} = 0 \) and \( V_{f_6} = 0 \) simultaneously we get,
\[ e_0 = \frac{m^a e_a}{m_0} - \frac{1}{3} \kappa_{abc} \frac{m^a m^b m^c}{m_0^2} + \xi^0 H_0 + \xi^k H_k + \xi^\lambda H^\lambda \]
\[ - \frac{m^a}{m_0} \left( \xi^0 w_{a0} + \xi^k w_{ak} + \xi^\lambda w_{a\lambda} \right), \quad b^a = -\frac{m^a}{m_0}. \]

This shows that \((h_{\lambda}^{\lambda} + 1)\) number of axions are fixed. Noting the fact there one can always make a rotation of the flux orbits [23] such that all the axionic dependences are absorbed in the new flux combinations like those clubbed in Eq. \((A.2)\). With this observation one can satisfy both the demands \( V_{f_2} = 0 \) and \( V_{f_6} = 0 \) along with satisfying \( m^a = 0 \) and \( e_0 = 0 \).
subject to imposing the following conditions on the axions,
\[ b^a = 0, \quad \xi^0 H_0 + \xi^k H_k + \xi^\lambda H^\lambda = 0, \]  
(3.14)

which results in a stronger constraint on the axions. Moreover, let us consider the flux scalings relevant for the saxionic moduli by looking into the scalar potential pieces given in Eq. (3.5). Assuming that \( V_f^2 = 0 = V_f^4 \) and all remaining terms to be non-zero and comparable to one another at the extremum, we anticipate the following flux scaling for the corresponding moduli VEVs,
\[ \rho \sim \sqrt{\frac{A_3}{A_1}} \sim \sqrt{\frac{A_5}{A_8}}, \quad e^D \sim \frac{1}{\rho^3 \sigma^{3/2}} \sqrt{\frac{A_5}{A_1}} \sim \frac{1}{\sigma^{3/2}} \frac{A_8}{A_3}, \]
(3.15)

Scenario 2:

As mentioned in Eq. (3.11), demanding \( V_f^2 = 0 \) and \( V_f^4 = 0 \) simultaneously we get,
\[ e_a = \frac{1}{2} \kappa_{abc} b^b m^c + \xi^0 w_{a0} + \xi^k w_{ak} + \xi^\lambda w_{a^\lambda}, \quad b^a = -\frac{m^a}{m_0}. \]
(3.16)

In fact one can satisfy \( V_f^2 = 0 \) and \( V_f^4 = 0 \) along with satisfying \( m^a = 0 \) and \( e_a = 0 \) subject to imposing the following conditions on the axions,
\[ b^a = 0, \quad \xi^0 w_{a0} + \xi^k w_{ak} + \xi^\lambda w_{a^\lambda} = 0, \]  
(3.17)

which results in a stronger constraint on the axions. Similar to the previous scenario, the flux scalings relevant for the saxionic moduli can be estimated by looking into the scalar potential pieces given in Eq. (3.5). Assuming that \( V_f^2 = 0 = V_f^4 \) and all remaining terms to be non-zero and comparable to one another at the extremum, we anticipate the following flux scaling for the corresponding moduli VEVs,
\[ \rho \sim \left( \frac{A_4}{A_1} \right)^{1/6} \sim \sqrt{\frac{A_5}{A_8}}, \quad e^D \sim \frac{1}{\rho^3 \sigma^{3/2}} \sqrt{\frac{A_5}{A_1}}, \]
(3.18)

Scenario 3:

As mentioned in Eq. (3.12), demanding \( V_f^6 = 0 \) and \( V_f^4 = 0 \) simultaneously we get,
\[ \epsilon_a + \kappa_{abc} b^b m^c + \frac{1}{2} \kappa_{abc} b^b b^c m_0 = \xi^0 w_{a0} + \xi^k w_{ak} + \xi^\lambda w_{a^\lambda}, \]
(3.19)
\[ \epsilon_0 = \frac{1}{2} \kappa_{abc} b^a b^b m^c + \frac{1}{3} \kappa_{abc} b^a b^b b^c m_0 + \xi^0 H_0 + \xi^k H_k + \xi^\lambda H^\lambda. \]
In fact one can satisfy $V_{f_6} = 0$ and $V_{f_4} = 0$ along with satisfying $e_0 = 0$ and $e_a = 0$ subject to imposing the following conditions on the axions,

$$
\xi^a_0 w_a^b + \xi^k w_{ak} + \xi_\lambda w_\lambda^b = \kappa_{abc} b^b m^c + \frac{1}{2} \kappa_{abc} b^c b^c m_0,
$$

$$
\xi^0 H_0 + \xi^k H_k + \xi_\lambda H_\lambda = -\frac{1}{2} \kappa_{abc} b^a b^b m^c - \frac{1}{3} \kappa_{abc} b^a b^b b^c m_0.
$$

Similar to the previous scenario, the flux scalings relevant for the saxionic moduli can be estimated by looking into the scalar potential pieces given in Eq. (3.5). Assuming that $V_{f_4} = 0 = V_{f_6}$ and all remaining terms to be non-zero and comparable to one another at the extremum, we anticipate the following flux scaling for the corresponding moduli VEVs,

$$
\rho \sim \sqrt{\frac{A_2}{A_1}}, \quad e^D \sim \frac{1}{\rho^3 \sigma^{3/2}} \sqrt{\frac{A_5}{A_1}},
$$

$$
\sigma^2 \sim \frac{A_5}{A_6} \sim \frac{A_6}{A_7} \sim \frac{A_8}{A_9} \sim \frac{A_9}{A_{10}} \sim \frac{A_{10}}{A_{11}} \sim \frac{A_{12}}{A_{13}}.
$$

We will discussed these flux scalings in more explicit settings in the upcoming section.

Finally, let us not forget that the choice of fluxes is restricted by the following NS-NS Bianchi identities [107],

$$
H^\lambda \hat{w}_{a\lambda} = H_k \hat{w}_{ak}, \quad w_a^\lambda \hat{w}_{a\lambda} = w_{ak} \hat{w}^k_{a},
$$

which shows that one can nullify the second identities by imposing that all the $F$-term geometric fluxes (which appear without a hat) identically vanish, i.e. $w_{ak} = 0 = w_a^\lambda$. Here we note that given the positive semi-definite nature of the D-term scalar potential, one would not want to set $D$-term geometric fluxes to zero as those may help in inducing the possible uplifting to de-Sitter solutions. Subsequently the only geometric flux which remain non-trivial arise from the $D$-term effects. However, such fluxes (denoted as $\hat{w}_{a\lambda}$ and $\hat{w}^k_{a}$) will also need to satisfy the first constraint in Eq. (3.22) and hence are not fully arbitrary.

4 Constructing a pair of simple $T$-dual type II models

In [21] it has been shown that the type IIA geometric flux models and a certain class of the type IIB non-geometric models (with special solutions of Bianchi identities) are $T$-dual to each other in the sense that there are certain transformations on the moduli/fluxes which exchange the scalar potential on the one theory with the same on the other. Now, we consider a pair of $T$-dual type IIA/IIB constructions which will be used to investigate the possibility of realizing $T$-dual pairs of AdS, Minkowskian and dS solutions.
4.1 Model A: type IIA with geometric flux

In order to make some more simplification for the explicitness of the computations, let us consider the following Hodge numbers for the compactifying threefold,

\[ h_{-1}^{1,1} = 1, \quad h_{+1}^{1,1} = 1, \quad h_2^{1,1} = 1. \] (4.1)

Given the fact that splitting of \( \{ \hat{k}, \lambda \} \equiv \{ 0, k, \lambda \} \) indices is such that \( k + \lambda = h_2^{1,1} \), and so for our particular model, we take \( k = 0 \) and \( \lambda = 1 \). This means that the complex variables \( N^k \) will be absent along with the flux components involving the \( k \)-indices. Using Eq. (2.1) and the triple intersection numbers as \( \kappa_{111} = 6 \) and \( k_{111} = 6 \), the simplified Kähler potential can be given as

\[ K_{\text{IIA}} = -3 \ln \left( i (T^1 - T^1) \right) - \ln \left( -i (N^0 - \overline{N^0}) \right) - 3 \ln \left( i (U_1 - \overline{U}_1) \right) + \text{const.}, \] (4.2)

which can be used to compute the \( F \)-term scalar potential by directly using the flux superpotential. However we already have a compact version of the scalar potential, from which one can read-off the various pieces of the simplified scalar potential. For that purpose, first we read-off the non-zero axionic flux orbits which are expressed as under,

\[
\begin{align*}
f_0 &= e_0 + b^1 e_1 + \frac{1}{2} \kappa_{111} (b^1)^2 m^1 + \frac{1}{6} \kappa_{111} (b^1)^3 m_0 \\
&\quad - \xi^0 (H_0 + b^1 w_{10}) - \xi_1 (H^1 + b^1 w_{11}), \\
f_1 &= e_1 + \kappa_{111} b^1 m^1 + \frac{1}{2} \kappa_{111} b^1 m_0 - \xi^0 w_{10} - \xi_1 w_{11}, \\
&\quad - \xi^0 (H_0 + b^1 w_{10}), \quad \kappa_1 = (H^1 + b^1 w_{11}), \\
h_0 &= (H_0 + b^1 w_{10}), \quad h_1 = (H^1 + b^1 w_{11}), \\
h_1 &= w_{10}, \quad h_1 = w_{11}, \quad h_1 = h_{11} = w_{11}, \quad h_1 = h_{0} = w_{0}.
\end{align*}
\] (4.3)

Moreover the moduli space metrics in the complex structure and Kähler moduli sector and the same are given as under,

\[
\begin{align*}
\tilde{G}_{ab} : \quad &\tilde{G}_{11} = \frac{1}{2} \kappa_{111} t^1, \quad G^{11} = \frac{2}{\kappa_{111} t^1}, \quad \mathcal{V} = \frac{1}{6} \kappa_{111} (t^1)^3, \\
&\tilde{G}_{\lambda\rho} : \quad \tilde{G}_{11} = \frac{1}{2} \kappa_{111} z^1, \quad G^{11} = \frac{2}{\kappa_{111} z^1}, \quad \mathcal{U} = \frac{1}{6} \kappa_{111} (z^1)^3.
\end{align*}
\] (4.4)

Now in order to normalize \( \mathcal{V} = (t^1)^3 \) and \( \mathcal{U} = (z^1)^3 \) we choose the intersection numbers to be \( \kappa_{111} = 6 \) and \( k_{111} = 6 \). Also let us denote \( t^1 = \rho \) and \( z^1 = \sigma \) to avoid too many indices. The scalar potential can be given as under,

\[
\begin{align*}
V_{\text{IIA}} = \frac{e^{4D}}{4 \rho^3} \left[ (f_0)^2 + \frac{\rho^2}{3} (f_1)^2 + 3 \rho^4 (f^1)^2 + \rho^6 (f^0)^2 \right] \\
&\quad + \frac{e^{2D}}{4 \rho^3 \sigma^3} \left[ h_0^2 + 3 \sigma^4 (h_1^0)^2 + \frac{\rho^2}{3} (h_1)^2 - 3 \rho^2 \sigma^4 (h_1^0)^2 \right]
\end{align*}
\]

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\[ +4 \rho^2 \sigma^2 h_1 h_1^1 + \frac{\rho^2 \sigma^2}{\kappa_{111}} \left( \sigma^2 \hat{h}_1^0 + \hat{h}_1 \right)^2 \]  
\[ + \frac{\epsilon^{3D}}{2 \sigma^{3/2}} \left[ (f_0^0 h_0 - f_1^1 h_1) - (f_0^0 h_1^0 - f_1^1 h_1^1) \right] 3 \sigma^2 \].

Also note that the scalar potential given in eqn. (4.5) shows that apart from the local/tadpole piece given in the last line, all pieces are positive semidefinite except for the piece involving the flux \( h_1^1 \equiv w_1^1 \), and therefore for the purpose of hunting for the de-Sitter solutions, it would be a good idea to set this flux to zero. However this will have impact on the two Bianchi identities given in eqn. (3.22) which translates into the following two constraints,

\[ H^1 \hat{w}_{11} = H_0 \hat{w}_1^0, \quad w_1^1 \hat{w}_{11} = w_{10} \hat{w}_1^0 \]  
(4.6)

This shows that setting \( w_1^1 = 0 \) will demand either to set \( w_{10} = 0 \) or \( \hat{w}_1^0 = 0 \). However, given that we do not want to switch-off any of the \( D \)-term pieces which could be useful to make the dS uplift, we take \( w_{10} = 0 \), which means a further simplification of the scalar potential (4.5) through the subsequent flux orbits set to zero via \( h_1^1 = h_1 = 0 \). In addition, as we know that \( F_0 \) with the presence of any one of the \( F_2, F_4 \) or \( F_6 \) flux being non-zero can evade the simplest version of the de-Sitter no-go conditions obtained in the volume-dilaton analysis [13], let us make the flux choice for the RR flux such that 

\[ e_0 = 0, \quad m^1 = 0 \].

(4.7)

Subsequently, the type IIA scalar potential takes the following form,

\[ V_{\text{IIA}} = V_R + V_{NS} + V_{loc}, \]  
(4.8)

where

\[ V_R = \frac{e^{4D}}{4 \rho^3} \left[ (f_0)^2 + \frac{\rho^2}{3} (f_1)^2 + 3 \rho^4 (f_1^1)^2 + \rho^6 (f_0^0)^2 \right] \]  
(4.9)

\[ V_{NS} = \frac{e^{2D}}{4 \rho^3 \sigma^3} \left[ h_0^2 + 3 \sigma^4 (h_1^0)^2 + \frac{\rho^2 \sigma^2}{\kappa_{111}} \left( \sigma^2 \hat{h}_1^0 + \hat{h}_1 \right)^2 \right] \]

\[ V_{loc} = \frac{e^{3D}}{2 \sigma^{3/2}} \left[ f_0^0 (h_0 - 3 \sigma^2 h_1^0) \right]. \]

The simplified flux orbits are given as,

\[ f_0 = b^1 e_1 + \frac{1}{6} \kappa_{111} (b^1)^3 m_0 + \xi^0 H_0 - \xi_1 H^1, \]  
(4.10)

\[ f_1 = e_1 + \frac{1}{2} \kappa_{111} b^1 b^1 m_0, \quad f_1^1 = b^1 m_0, \quad f_0 = m_0, \]

\[ h_0 = H_0, \quad h_1^0 = H^1, \quad \hat{h}_{11} = \hat{w}_{11}, \quad \hat{h}_1^0 = \hat{w}_1^0. \]
4.2 Model B: type IIB with (non-)geometric flux

In this section, we consider the $T$-dual type IIB construction for the previous type IIA geometric flux setting. This model is based on the compactification with an orientifolded (CY) threefold having the following hodge numbers,

$$h^{1,1}_+ = 1, \quad h^{1,1}_- = 0, \quad h^{2,1}_+ = 1, \quad h^{2,1}_- = 1.$$

The above hodge numbers confirm that the underlying type IIB setup has one $T$ modulus, one $U$ modulus along with the axion-dilaton $S$ whereas there are no odd-moduli counted by $a \in \{1, 2, \ldots, h^{1,1}_-\}$. With this information in hand we conclude that there would be 16 flux components in total (4 for RR-fluxes, 4 for each of the $\{H_3, Q\}$ fluxes while 2 for each of the $\{\omega, R\}$ fluxes), which would be allowed by the orientifold projection, and have to be further constrained by the Bianchi identities and tadpole cancellation conditions. After reading-off the flux orbits, the only thing we need to know for writing down the scalar potential is the moduli space metrics in the complex structure and Kähler moduli sector and the same are given as under,

$$G_{ij}: \quad G_{11} = \frac{1}{2} l_{111} u^1, \quad G^{11} = \frac{2}{l_{111} u^1}, \quad U = \frac{1}{6} l_{111} (u^1)^3, \quad \mathcal{V} = \frac{1}{6} l_{111} (t^1)^3. \quad (4.12)$$

$$G_{\alpha\beta}: \quad G_{11} = \frac{1}{2} \ell_{111} t^1, \quad G^{11} = \frac{2}{\ell_{111} t^1}, \quad \mathcal{V} = \frac{1}{6} \ell_{111} (t^1)^3.$$

Similar to the previous case, we again take the triple intersection numbers as $l_{111} = 6$ and $\ell_{111} = 6$ so that to normalize the volume as $\mathcal{V} = (t^1)^3 \equiv \rho^3$ and $\mathcal{U} = (u^1)^3 \equiv \sigma^3$. Subsequently we can directly write down the full scalar potential in the following manner,

$$V_{\text{IIB}} = \frac{e^{4\phi}}{4 \rho^6 \sigma^3} \left[ (f_0)^2 + 3 \sigma^4 (f_1)^2 + \frac{\sigma^2}{3} (f_1)^2 + \sigma^6 (f_0)^2 \right]$$

$$+ \frac{e^{2\phi}}{4 \rho^6 \sigma^3} \left[ h_0^2 + 3 \rho^4 (h_0^1)^2 + \frac{\sigma^2}{3} (h_1)^2 - 3 \sigma^2 \rho^4 (h_1^1)^2 \right]$$

$$+ 4 \sigma^2 \rho^2 h_1 h_1^1 + \frac{\sigma^2}{\ell_{111}} \left( \rho^3 \hat{h}_0^1 + \rho \hat{h}_{11} \right)^2$$

$$+ \frac{e^{3\phi}}{2 \rho^6} \left[ (f_0 h_0 - f_1 h_1) - 3 \left( f_0 h_0^1 - f_1 h_1^1 \right) \rho^2 \right],$$

where the simplified version of the non-trivial axionic flux orbits are given as below,

$$f_0 = F_0 + v^1 F_1 + 3 (v^1)^2 F^1 - (v^1)^3 F_0^0 - c_1 \hat{Q}_0^1 - c_0 h_0,$$

$$f_1 = F_1 + 6 v^1 F_1 - 3 (v^1)^2 F^0 - c_1 \hat{Q}_1^1 - c_0 h_1,$$

$$f^1 = F^1 - v^1 F^0,$$

$$f^0 = - F^0,$$

$$h_0 = H_0 + v^1 H_1, \quad h_1 = H_1,$$

$$h_0 = H_0 + v^1 H_1, \quad h_1 = H_1,$$
\[ h^{10} = \hat{Q}^{10} + v^i \hat{Q}^{11}, \quad h^{11} = \hat{Q}^{11}, \]
\[ \hat{h}_{11} = \hat{\omega}_{11}, \quad \hat{h}^{10} = - R_1. \]

However, there are some additional Bianchi identities arising from the collection in eqn. (A.4) which simplify into the following two constraints,
\[ H_0 R_1 + \hat{Q}^{10} \hat{\omega}_{11} = 0, \quad H_1 R_1 + \hat{Q}^{11} \hat{\omega}_{11} = 0. \] (4.15)

The second constraint can be trivially satisfied by further setting two of the 6 NS-NS fluxes to zero via \( H_1 = 0 = \hat{Q}^{11} \) which is equivalent to setting \( h_1 = 0 = h^{11} \) in the scalar potential (4.13). Thus we are left with only four NS-NS fluxes to be non-zero and satisfying the first constraint of (4.15). In addition, let us also set two of the RR flux components to zero, namely \( F_0 = 0 = F^1 \). Subsequently, the scalar potential in eqn. (4.13) takes the following simpler form,
\[ V_{\text{IIB}} = V_{\text{NS}} + V_R + V_{\text{loc}} \] (4.16)
where
\[ V_R = \frac{e^{4\phi}}{4 \rho^6 \sigma^3} \left[ (f_0)^2 + 3 \sigma^4 (f^1)^2 + \frac{\sigma^2}{3} (f_1)^2 + \sigma^6 (f^0)^2 \right], \] (4.17)
\[ V_{\text{NS}} = \frac{e^{2\phi}}{4 \rho^6 \sigma^3} \left[ \hat{h}_0^2 + 3 \rho^4 (h^{10})^2 + \frac{\sigma^2}{\ell_{111}} \left( \hat{h}_{11} + \rho^2 \hat{h}^{10} \right)^2 \right], \]
\[ V_{\text{loc}} = \frac{e^{3\phi}}{2 \rho^3} \left[ f^0 h_0 - 3 \rho^2 f^0 h^{10} \right], \]
and the axionic fluxes are given as,
\[ f_0 = v^1 F_1 - (v^1)^3 F^0 - c_1 \hat{Q}^{10} - c_0 H_0, \] (4.18)
\[ f_1 = F_1 - 3 (v^1)^2 F^0, \quad f^1 = - v^1 F^0, \quad f^0 = - F^0, \]
\[ h_0 = H_0, \quad h^{10} = \hat{Q}^{10}, \quad \hat{h}_{11} = \hat{\omega}_{11}, \quad \hat{h}^{10} = - R_1. \]

**Summary**

To summarize the story so far, we have establishing the explicit one-to-one correspondence between the two T-dual models via the scalar potentials given in eqn. (4.9) for type IIA model and in eqn. (4.17) for type IIB, which are further supplemented by their respective axionic flux polynomials as given in eqn. (4.10) and eqn. (4.18). The relevant fluxes and moduli/axions in the two T-dual pair of models are presented in Table 1.
Table 1: The various $T$-dual pairs of non-zero fluxes and moduli in type IIA/IIB models are presented in this table. In addition, the other fluxes are set as: $e_0 = 0 = m_1$ in type IIA and $F_0 = 0 = F_1$ in the type IIB model. Also, the flux $\hat{w}_{11}$ can be eliminated by using the Bianchi identity $H_1 \hat{w}_{11} = H_0 \hat{w}_{10}$ in type IIA, and similarly the non-geometric $R$-flux in type IIB model can be eliminated by using the constraint: $H_0 R_1 + \hat{Q}_{10} \hat{\omega}_{11} = 0$.

Subsequently, the moduli stabilization in two model goes exactly the same way, though one has to be careful if the VEVs are within the physical domain of validity of the given model. For example, it is worth to mention that some solutions which are allowed at the type IIB side may fall (though not necessarily) in the unphysical regime in the $T$-dual type IIA side. In order to trust the EFT description, the 10D dilaton determining the string coupling in the type IIA model should follow the constraint below,

$$e^{-\varphi} = \frac{e^{-D}}{\sqrt{V}} \gg 1.$$  \hspace{1cm} (4.19)

Therefore, after making all those simplifications, we end up effectively having five non-zero independent flux parameters which determine the dynamics of the various moduli and the axions.

5 Analyzing the $T$-dual pair of flux vacua

Given that we have completely identified the two $T$-dual scalar potentials in terms of the transformation of various moduli/fluxes and saxions/axions, here we will focus on type IIA model only and the type IIB computations turns out to be completely analogues after considering the identifications using Table 1. However we will present the numerical results for both the models.

Axion stabilization

Noting the fact that axions do not appear in the $D$-term scalar potential due to our simplistic construction, a common $F$-term axion minimization can be presented with/without the inclusion of $D$-terms. Using the scalar potential in Eqs. (4.8)-(4.9), the extremization
of axions $\xi^0$, $\xi_1$ and $b^1$ results in the following conditions,

$$\begin{align*}
\partial_{\xi^0} V_{\text{IIA}} &= \frac{e^{AD}}{2V} f_0 h_0 = 0, \\
\partial_{\xi_1} V_{\text{IIA}} &= \frac{e^{AD}}{2V} f_0 h^1_0 = 0, \\
\partial_{b^1} V_{\text{IIA}} &= \frac{e^{AD}}{2V} (f_0 f_1 + f_1 G^{11} \kappa_{111} f^1 + f^1 G_{11} f^0) = 0.
\end{align*}$$

(5.1)

Given that we do not want to lose any NS-NS flux now, the above three conditions can be satisfied by setting

$$f_0 = f^1 = 0,$$

(5.2)

where we keep the Romans mass parameter $f^0 = m_0 \neq 0$ as that is necessary to avoid the well-known de-Sitter no-go theorems for geometric type IIA scenarios [3, 5]. This choice of RR-flux helps in setting axions $b^1$ to zero and hence simplifying the axionic flux orbits. Subsequently, two (out of three) axionic moduli are stabilized as under,

$$\langle b^1 \rangle = 0, \quad \langle \xi^0 \rangle H_0 + \langle \xi_1 \rangle H^1 = 0.$$

(5.3)

Now, after the axion minimization, the scalar potential takes a simpler form,

$$V_{\text{IIA}}^{\text{axion}}(D, \rho, \sigma) = \frac{e^{2D}}{4 \rho^3 \sigma^3} \left[ H_0^2 + 3 \sigma^4 (H^1)^2 + \frac{\rho^2 \sigma^2 \hat{w}_{11}^2}{\hat{\kappa}_{111}} \left( 1 + \frac{\sigma^2 H^1}{H_0} \right)^2 \right] + \frac{e^{3D}}{2 \rho^3 / 2} \left[ m_0 (H_0 - 3 \sigma^2 H^1) \right],$$

(5.4)

Using the scalar potential in Eq. (4.8) along with the various pieces given in Eq. (4.9), we can explicitly determine the coefficients $A_i$’s as defined in Eq. (3.6) and subsequently use them for making flux scaling estimated for the three scenarios which evades the dS no-go results of the volume/dilaton analysis. These are given as below,

$$\begin{align*}
A_1 &= \frac{1}{4} m_0^2, \quad A_2 = \frac{3}{4} (f^1)^2, \quad A_3 = \frac{1}{12} f_1^2, \quad A_4 = \frac{1}{4} f^0_0, \\
A_5 &= \frac{1}{4} H_0^2, \quad A_6 = 0, \quad A_7 = \frac{3}{4} (H^1)^2, \quad A_8 = 0, \\
A_9 &= \frac{1}{4 \hat{\kappa}_{111}} \hat{w}_{11}^2, \quad A_{10} = \frac{1}{2 \hat{\kappa}_{111}} \hat{w}_{11} \hat{w}^0_{11}, \quad A_{11} = \frac{1}{4 \hat{\kappa}_{111}} (\hat{w}_{11}^0)^2, \\
A_{12} &= \frac{1}{2} m_0 H_0, \quad A_{13} = -\frac{3}{2} m_0 H^1.
\end{align*}$$

(5.5)

We note that the process of axion minimization has also set $A_2 = 0 = A_4$ and $A_3 = \frac{1}{12} e_1^2$. Now one can determine the flux scaling corresponding to the simplified scalar potential in
Eq. (5.4) which turns out to be given as below,

\[ e^{-D} \sim \frac{(e_1)^2}{(m_0)^{\frac{1}{2}} (H_0)^{\frac{3}{4}}} \left( \frac{H_1}{H_0} \right)^{\frac{1}{4}}, \quad \rho \sim \sqrt{\frac{e_1}{m_0}} \left( \frac{H_1}{H_0} \right)^{\frac{1}{4}}, \quad \sigma \sim \sqrt{\frac{H_0}{H_1}}, \]  

(5.6)

which shows that the string coupling \( g_s = e^{\langle \phi \rangle} \) obtained from the VEV of the ten dimensional dilaton \( \langle \phi \rangle \) should scale as follows,

\[ e^{-\langle \phi \rangle} = e^{-D} \sqrt{V} \sim \frac{(e_1)^2 (m_0)^{\frac{1}{2}}}{(H_0)^{\frac{3}{4}}} \left( \frac{H_1}{H_0} \right)^{\frac{1}{4}}, \]  

(5.7)

Note that similar flux scalings for saxion VEVs have been observed for the rigid toroidal orientifold of \( T^6/(\mathbb{Z}_3 \times \mathbb{Z}_3) \) in [167]. We will examine these naive expectations based on scaling arguments through explicitly solving the extremization conditions.

**Saxion stabilization**

Using the scalar potential pieces collected in Eqs. (4.8)-(4.9), the dilaton derivative is given as below,

\[ \partial_D V_{\text{IIA}} = 0, \quad \langle 2 V_{\text{NS}} \rangle + 4 \langle V_R \rangle + 3 \langle V_{\text{loc}} \rangle = 0, \]  

(5.8)

which implies that fixing the dilaton to its VEV results in the following simplification in the scalar potential at the extremum,

\[ \langle V_{\text{IIA}} \rangle_{\partial_D V_{\text{IIA}} = 0} = \frac{\langle V_{\text{NS}} \rangle}{3} - \frac{\langle V_R \rangle}{3} \]  

(5.9)

Subsequently, we are only left with fixing the complex structure and volume moduli, namely \( \sigma \) and \( \rho \) respectively. These saxionic moduli are to be stabilized in the following effective potential,

\[ V_{\text{IIA}} = \frac{e^{2\langle D \rangle}}{12 \rho^3 \sigma^4} \left[ H_0^2 + 3 \sigma^4 (H_1)^2 + \frac{\rho^2 \sigma^2 \bar{w}_{11}}{\hat{k}_{111}} \left( 1 + \frac{\sigma^2 (H_1)}{H_0} \right)^2 \right] \]  

\[ - \frac{e^{4\langle D \rangle}}{12 \rho} \left[ \frac{1}{3} (e_1)^2 + \rho^4 (m_0)^2 \right], \]  

(5.10)

which does not involve the \( V_{\text{loc}} \) terms at all. Also, the positive semi-definite nature of the \( D \)-term effects creates hope for possible uplifting or towards the existence of a new tachyon free (stable) de-Sitter vacua.

### 5.1 AdS vacua: without \( D \)-terms

Now, let us have the exact conditions for moduli stabilization in the absence of \( D \)-term effects, with the aim to engineer the fluxes so that we could look for the possibility of de-Sitter solutions by such effect as a second step.

One can observe that in the absence of the \( D \)-term contribution, i.e. for \( \bar{w}_{11} = 0 = \bar{w}_{1}^{0} \), the saxionic extremization conditions can be satisfied for the following two sets of AdS
solutions:

\[ \text{AdS1 : } e^{-\langle D \rangle} = -\frac{2 m_0}{5 H_0} \langle \rho \rangle^3 \langle \sigma \rangle^\frac{3}{2}, \quad \langle \sigma \rangle^2 = -\frac{H_0}{H_1}, \quad \langle \rho \rangle^2 = \pm \frac{5 e_1}{9 m_0}, \quad (5.11) \]

\[ \langle V_{\text{IIA}} \rangle = -\frac{3 e^{4(D)} \langle \rho \rangle^3 m_0^2}{25} = -\frac{3 e^{2(D)} H_0^2}{4 \langle \rho \rangle^3 (\sigma)^3}; \]

\[ \text{AdS2 : } e^{-\langle D \rangle} = -\frac{4 m_0}{5 H_0} \langle \rho \rangle^3 \langle \sigma \rangle^\frac{3}{2}, \quad \langle \sigma \rangle^2 = -\frac{H_0}{4 H^1}, \quad \langle \rho \rangle^2 = \pm \frac{5 e_1}{3 \sqrt{6} m_0}, \quad (5.12) \]

\[ \langle V_{\text{IIA}} \rangle = -\frac{2 e^{4(D)} \langle \rho \rangle^3 m_0^2}{25} = -\frac{e^{2(D)} H_0^2}{8 \langle \rho \rangle^3 (\sigma)^3}. \]

The Hessian analysis further shows that \textbf{AdS1} correspond to a tachyonic solution while \textbf{AdS2} is a minimum. Given that saxionic VEVs has to be positive, we need to choose \( m_0 \) and \( H_0 \) of opposite sign, and subsequently there can be two possibilities for choosing the sign of the fluxes,

\[ (i). \quad m_0 > 0, \quad e_1 < 0, \quad H_0 < 0, \quad H^1 > 0, \quad (5.13) \]

\[ (ii). \quad m_0 < 0, \quad e_1 > 0, \quad H_0 > 0, \quad H^1 < 0. \]

Note that the tadpole contributions in both the cases are \( N_{\text{flux}} < 0 \) which is consistent with the standard literature of type IIA models without geometric flux (e.g. see [167, 168]). Without loss of any generality, let us take the first choice. Subsequently, the two AdS solutions can be equivalently expressed entirely in terms of fluxes as below,

\[ \text{AdS1 : } \langle D \rangle = -\frac{2 \sqrt{5} (-e_1)^\frac{3}{2}}{27 \sqrt{m_0} (-H_0)^\frac{1}{2} (H^1)^\frac{1}{4}}, \quad \langle \rho \rangle = \frac{\sqrt{5}}{3} \sqrt{-\frac{e_1}{m_0}}, \quad \langle \sigma \rangle = \sqrt{-\frac{H_0}{H^1}}, \quad (5.14) \]

\[ e^{-\langle \varphi \rangle} = \frac{2 (-e_1)^\frac{1}{4} m_0^{\frac{1}{2}}}{3^\frac{3}{2} 5^\frac{1}{4} (-H_0)^\frac{1}{2} (H^1)^\frac{1}{4}}, \quad \langle V_{\text{IIA}} \rangle = -\frac{59049 (m_0)^{\frac{3}{2}} (-H_0) (H^1)^3}{400 \sqrt{5} (-e_1)^{\frac{3}{2}}}; \]

\[ \text{AdS2 : } \langle D \rangle = \frac{\sqrt{5} (-e_1)^\frac{3}{2}}{9 \times 6^\frac{1}{4} \sqrt{m_0} (-H_0)^{\frac{1}{4}} (H^1)^\frac{1}{4}}, \quad \langle \rho \rangle = \frac{\sqrt{5}}{2^\frac{1}{4} 3^\frac{3}{4}} \sqrt{-\frac{e_1}{m_0}}, \quad \langle \sigma \rangle = \frac{1}{2} \sqrt{-\frac{H_0}{H^1}}, \quad (5.14) \]

\[ e^{-\langle \varphi \rangle} = \frac{2^\frac{3}{4} (-e_1)^\frac{1}{4} m_0^{\frac{1}{2}}}{3^\frac{3}{4} 5^\frac{1}{4} (-H_0)^{\frac{1}{4}} (H^1)^\frac{1}{4}}, \quad \langle V_{\text{IIA}} \rangle = -\frac{1458 \times 2^\frac{1}{4} 3^\frac{3}{4} m_0^\frac{3}{2} (-H_0) (H^1)^3}{25 \sqrt{5} (-e_1)^{\frac{3}{2}}}. \]

These analytic results are extremely useful in the many ways. First let us observe that the exact AdS solutions in Eq. (5.14) confirm the previous estimates about the flux scalings for the saxionic VEVs as given in Eqs. (5.6)-(5.7). Also such flux scaling of the saxionic VEVs can help in determining the flux regions where the solutions can be physically acceptable and trustworthy. In this regard, we make the following points:

- We observe that choosing suitably large values for the \( |e_1| \) flux can be used to realize larger VEVs for volume modulus \( \rho \) along with weaker values for the string coupling \( g_s = e^{\langle \varphi \rangle} \). Given that there are no non-geometric fluxes present in the model, the \( e_1 \) flux is not restricted by the tadpole condition as well.
• Also, we observe that of the $m_0$ flux should be minimum for realizing large VEVs for volume modulus $\rho$ as well as the having weak coupling, and therefore we prefer to set $m_0 = 1$ in our numerical analysis.

• It is obvious that the $H^1$ flux should be taken to minimum values in order to have larger VEV for the complex structure modulus $\sigma$ along with weak string coupling, and therefore we prefer to set $H^1 = 1$ in our numerical analysis.

• Although larger values for $H_0$ could also help for large complex structure modulus ($\sigma$) realization, however this can happen only at the cost of enhancing the string coupling by a reduction of the dilaton factor $e^{-\phi}$. Thus we observe that large complex structure and weak coupling requirements are apparently contradictory to each other with respect to the choice of the $H_0$ flux, and one has to find a viable balance to keep both in the physically valid regime.

• However, choosing larger values for $|e_1|$ flux can help in having weak-coupling and large volume realizations, and also leaves a scope for large complex structure realization by keeping the factor $|e_1|/|H_0|$ large while having $|H_0|$ large as well.

• With these scaling arguments, we find that the moduli dynamics can be mainly controlled by two fluxes $|e_1|$ and $|H_0|$. However, demanding $|e_1|/H_0$ and $H_0$ to take large values has to be balanced by the need of large tadpole charge compensation, and so one would not prefer to take too large values for $|H_0|$ and should be satisfied with those which are just enough to ensure $\langle \sigma \rangle > 1$.

Given that the type IIB story can be read-off from this type IIA analysis using Table 1, we do not repeat the whole analytics about moduli stabilization again. However, we present the numerical samplings for the completion of “T-dual pairs” of the AdS/dS solutions.

Numerical samplings:

For some numerical estimates from our $T$-dual type IIA/IIB analysis, now we present a couple of concrete flux samplings along with the other moduli stabilization details. The $T$-dual pair of solutions to the type IIA/IIB models are given in Tables 2, 3, 4, and 5.

**Table 2:** Numerical samplings for type IIA models resulting in tachyonic and stable AdS solutions where we have set the other flux parameters as: $e_0 = 0$, $m^1 = 0$, $m_0 = 1$, $H^1 = 1$ and $\hat{w}_{11} = 0$, which leads to (two of the three) axions being stabilized as $\langle b^1 \rangle = 0$ and $\langle \xi^0 \rangle H_0 + \langle \xi^1 \rangle H^1 = 0$.

| IIA | $e_1$ | $H_0$ | $\langle \rho \rangle$ | $\langle \sigma \rangle$ | $\langle e^{-D} \rangle$ | $\langle V \rangle$ | $\langle e^{-\phi} \rangle$ | AdS Vacua type |
|-----|-------|-------|-----------------|----------------|-----------------|----------------|-----------------|----------------|
| S1  | -50   | -5    | 5.27046         | 2.23607        | 39.1619         | 146.402        | 3.23661         | Tachyonic      |
|     |       |       | 5.83273         | 1.11803        | 37.5333         | 198.433        | 2.66446         | Stable         |
Table 2: Numerical samplings for type IIA models resulting in tachyonic and stable AdS solutions

where we have set the other flux parameters as: \( e_0 = 0, m_1 = 0, m_0 = 1, H^1 = 1 \) and \( \hat{w}_{11} = 0 \), which leads to (two of the three) axions being stabilized as \( \langle b^1 \rangle = 0 \) and \( \langle \xi^0 \rangle H^0 + \langle \xi^1 \rangle H^1 = 0 \).

| IIA | \( e_1 \) | \( H_0 \) | \( \langle \rho \rangle \) | \( \langle \sigma \rangle \) | \( \langle e^{-D} \rangle \) | \( \langle V \rangle \) | \( \langle e^{-\varphi} \rangle \) | AdS Vacua type |
|-----|--------|------|-----------------|----------|----------------|-------|----------------|--------|
| S2  | -50    | -10  | 5.27046         | 3.16228  | 32.9311        | 146.402 | 2.72166        | Tachyonic |
|     |        |      | 5.83273         | 1.58114  | 31.5616        | 198.433 | 2.24054        | Stable   |
| S3  | -75    | -5   | 6.45497         | 2.23607  | 71.945         | 268.957 | 4.38691        | Tachyonic |
|     |        |      | 7.1436          | 1.11803  | 68.9531        | 364.545 | 3.61142        | Stable   |
| S4  | -75    | -10  | 6.45497         | 3.16228  | 60.4983        | 268.957 | 3.68894        | Tachyonic |
|     |        |      | 7.1436          | 1.58114  | 57.9825        | 364.545 | 3.03683        | Stable   |
| S5  | -100   | -5   | 7.45356         | 2.23607  | 110.767        | 414.087 | 5.44331        | Tachyonic |
|     |        |      | 8.24872         | 1.11803  | 106.16         | 561.254 | 4.48108        | Stable   |
| S6  | -100   | -10  | 7.45356         | 3.16228  | 93.1432        | 414.087 | 4.57726        | Tachyonic |
|     |        |      | 8.24872         | 1.58114  | 89.2698        | 561.254 | 3.76812        | Stable   |
| S7  | -50    | -50  | 5.27046         | 7.07107  | 22.0224        | 146.402 | 1.82008        | Tachyonic |
|     |        |      | 5.83273         | 3.53553  | 21.1065        | 198.433 | 1.49834        | Stable   |
| S8  | -100   | -50  | 7.45356         | 7.07107  | 62.2886        | 414.087 | 3.061          | Tachyonic |
|     |        |      | 8.24872         | 3.53553  | 59.6983        | 561.254 | 2.51989        | Stable   |
| S9  | -100   | -100 | 7.45356         | 10       | 52.3783        | 414.087 | 2.57398        | Tachyonic |
|     |        |      | 8.24872         | 5        | 50.2001        | 561.254 | 2.11897        | Stable   |

Table 3: Numerical samplings for type IIB models which are T-dual to the type IIA models presented in Table 2. For type IIB case, we have set: \( F_0 = 0, F^1 = 0, F^0 = -1 \) and \( Q^1_0 = 1 \) which leads to (two of the three) axions being stabilized as \( \langle v^1 \rangle = 0 \) and \( \langle c^0 \rangle H_0 + \langle c^1 \rangle Q^1_0 = 0 \).

| IIB | \( F_1 \) | \( H_0 \) | \( \langle \sigma \rangle \) | \( \langle \rho \rangle \) | \( \langle e^{-\phi} \rangle \) | \( \langle V \rangle \) | \( \langle V_E \rangle \) | AdS Vacua type |
|-----|--------|------|-----------------|----------|----------------|-------|----------------|--------|
| S1’ | -50    | -5   | 5.27046         | 2.23607  | 11.7121        | 11.1803 | 448.136        | Tachyonic |
|     |        |      | 5.83273         | 1.11803  | 31.7493        | 1.39754 | 250.016        | Stable   |
**Table 3:** Numerical samplings for type IIB models which are $T$-dual to the type IIA models presented in Table 2. For type IIB case, we have set: $F_0 = 0$, $F^1 = 0$, $F^0 = -1$ and $\tilde{Q}_{10} = 1$ which leads to (two of the three) axions being stabilized as $\langle \nu \rangle = 0$ and $\langle \nu_0 \rangle H_0 + \langle \nu_1 \rangle \tilde{Q}_{10} = 0$.

| IIB   | $F_1$ | $H_0$ | $\langle \sigma \rangle$ | $\langle \rho \rangle$ | $\langle e^{-\phi} \rangle$ | $\langle V \rangle$ | $\langle V_E \rangle$ | AdS Vacua type |
|-------|-------|-------|-----------------------------|-------------------------|---------------------------|-------------------|---------------------|----------------|
| $S2^\prime$ | -50   | -10   | 5.27046                     | 3.16228                 | 5.85607                   | 31.6228         | 448.136             | Tachyonic        |
|       |       |       | 5.83273                     | 1.58114                 | 15.8747                   | 3.95285          | 250.016             | Stable           |
| $S3^\prime$ | -75   | -5    | 6.45497                     | 2.23607                 | 21.5166                   | 11.1803         | 1115.87             | Tachyonic        |
|       |       |       | 7.1436                      | 1.11803                 | 58.3273                   | 1.39754          | 622.547             | Stable           |
| $S4^\prime$ | -75   | -10   | 6.45497                     | 3.16228                 | 10.7583                   | 31.6228         | 1115.87             | Tachyonic        |
|       |       |       | 7.1436                      | 1.58114                 | 29.1636                   | 3.95285          | 622.547             | Stable           |
| $S5^\prime$ | -100  | -5    | 7.45356                     | 2.23607                 | 33.1269                   | 11.1803         | 2131.7              | Tachyonic        |
|       |       |       | 8.24872                     | 1.11803                 | 89.8007                   | 1.39754          | 1189.28             | Stable           |
| $S6^\prime$ | -100  | -10   | 7.45356                     | 3.16228                 | 16.5635                   | 31.6228         | 2131.7              | Tachyonic        |
|       |       |       | 8.24872                     | 1.58114                 | 44.9003                   | 3.95285          | 1189.28             | Stable           |
| $S7^\prime$ | -50   | -50   | 5.27046                     | 7.07107                 | 1.17121                   | 353.553         | 448.136             | Tachyonic        |
|       |       |       | 5.83273                     | 3.53553                 | 3.17493                   | 44.1942         | 250.016             | Stable           |
| $S8^\prime$ | -100  | -50   | 7.45356                     | 7.07107                 | 3.31269                   | 353.553         | 2131.7              | Tachyonic        |
|       |       |       | 8.24872                     | 3.53553                 | 8.98007                   | 44.1942         | 1189.28             | Stable           |
| $S9^\prime$ | -100  | -100  | 7.45356                     | 10.                     | 1.65635                   | 1000.           | 2131.7              | Tachyonic        |
|       |       |       | 8.24872                     | 5.                      | 4.49003                   | 125.            | 1189.28             | Stable           |

**Table 4:** Hessian Eigenvalues and the scalar potential VEVs corresponding to the AdS solutions for the type IIA flux samplings presented in Table 2. This shows that the first AdS solution is tachyonic while the second AdS solution is tachyon free, but having a flat axionic combination.

| IIA   | $\langle V_0 \rangle \cdot 10^6$ | Eigenvalues of $\langle V_{ij} \rangle \cdot 10^6$ |
|-------|---------------------------------|---------------------------------------------------|
| $S1$  | -7.46917                        | $\{8.88289, 3.00399, -1.49383, 0.0619177, -0.00844809, 0\}$ |
Table 4: Hessian Eigenvalues and the scalar potential VEVs corresponding to the AdS solutions for the type IIA flux samplings presented in Table 2. This shows that the first AdS solution is tachyonic while the second AdS solution is tachyon free, but having a flat axionic combination.

| IIA | $\langle V_0 \rangle \cdot 10^6$ | Eigenvalues of $\langle V_{ij} \rangle \cdot 10^6$ |
|-----|-------------------------------|------------------------------------------------|
|     | $-7.99900$                   | $\{50.7392, 3.29533, 1.05382, 0.0647094, 0.000892266, 0\}$ |
| S2  | $-14.9383$                   | $\{17.7737, 6.27177, -1.49383, 0.175051, -0.0628746, 0\}$ |
|     | $-15.9980$                   | $\{62.2931, 6.77616, 1.72381, 0.182272, 0.0674246, 0\}$ |
| S3  | $-1.20465$                   | $\{0.954538, 0.32024, -0.240931, 0.00296067, -0.000407189, 0\}$ |
|     | $-1.29011$                   | $\{7.57355, 0.352403, 0.12203, 0.0309812, 0.0000428714, 0\}$ |
| S4  | $-2.40931$                   | $\{1.90945, 0.653175, -0.240931, 0.00837239, -0.00310205, 0\}$ |
|     | $-2.58021$                   | $\{8.796, 0.713663, 0.210689, 0.00875237, 0.00328944, 0\}$ |
| S5  | $-0.330094$                  | $\{0.196128, -0.0660188, 0.0656144, 0.000342326, -0.0000472137, 0\}$ |
|     | $-0.353509$                  | $\{1.99277, 0.0722849, 0.0261409, 0.000358322, 4.9654 \times 10^{-6}, 0\}$ |
| S6  | $-0.660188$                  | $\{0.3923, 0.1327, -0.0660188, 0.000968137, -0.000362747, 0\}$ |
|     | $-0.707019$                  | $\{2.24158, 0.145594, 0.0465027, 0.00101296, 0.0000383062, 0\}$ |
| S7  | $-74.6917$                   | $\{89.0392, 69.4097, -3.51704, 1.95339, -1.49383, 0\}$ |
|     | $-79.99$                     | $\{158.674, 63.944, 3.77999, 1.82423, 0.442318, 0\}$ |
| S8  | $-3.30094$                   | $\{1.96242, 0.887765, -0.0660188, -0.0335668, 0.010819, 0\}$ |
|     | $-3.53509$                   | $\{4.32243, 0.892626, 0.121494, 0.0112401, 0.00386789, 0\}$ |
| S9  | $-6.60188$                   | $\{3.92622, 3.067, -0.155412, -0.0660188, 0.03059, 0\}$ |
|     | $-7.07019$                   | $\{6.99006, 2.82553, 0.152914, 0.0312391, 0.0195449, 0\}$ |

Table 5: Hessian Eigenvalues and the scalar potential VEVs corresponding to the AdS solutions for the T-dual type IIB flux samplings presented in Table 3. This shows that the first AdS solution is tachyonic while the second AdS solution is tachyon free, but having a flat axionic combination.

| IIB | $\langle V_0 \rangle \cdot 10^6$ | Eigenvalues of $\langle V_{ij} \rangle \cdot 10^6$ |
|-----|-------------------------------|------------------------------------------------|
| S1' | $-7.46917$                   | $\{50.0532, 7.44615, 3.00399, -0.024647, -0.00844809, 0\}$ |
Table 5: Hessian Eigenvalues and the scalar potential VEVs corresponding to the AdS solutions for the $T$-dual type IIB flux samplings presented in Table 3. This shows that the first AdS solution is tachyonic while the second AdS solution is tachyon free, but having a flat axionic combination.

| IIB | $\langle V_0 \rangle \cdot 10^6$ | Eigenvalues of $\langle V_{ij} \rangle \cdot 10^6$ |
|-----|---------------------------------|-----------------------------------------------|
| S2’ | -7.999                          | $\{173.081, 11.9961, 3.29533, 0.00232891, 0.000892266, 0\}$ |
|     | -14.9383                        | $\{57.6423, 14.4722, 6.27177, -0.176186, -0.0628746, 0\}$ |
|     | -15.998                         | $\{174.097, 24.0616, 6.77616, 0.0184691, 0.00674246, 0\}$ |
| S3’ | -1.20465                        | $\{7.89435, 0.80887, 0.32024, -0.00119216, -0.000407189, 0\}$ |
|     | -1.29011                        | $\{27.8915, 1.28928, 0.352403, 0.00011436, 0.0000428714, 0\}$ |
| S4’ | -2.40931                        | $\{8.34774, 1.58473, 0.653175, -0.000920718, -0.00310205, 0\}$ |
|     | -2.58021                        | $\{27.9512, 2.58154, 0.713663, 0.000888553, 0.000328944, 0\}$ |
| S5’ | -0.330094                       | $\{2.14617, 0.167052, 0.0656144, -0.000138224, -0.0000472137, 0\}$ |
|     | -0.353509                       | $\{7.64046, 0.264933, 0.0722849, 0.0000128873, 4.965 \times 10^{-6}, 0\}$ |
| S6’ | -0.660188                       | $\{2.21206, 0.329076, 0.1327, -0.00108925, -0.000362747, 0\}$ |
|     | -0.707019                       | $\{7.64915, 0.530156, 0.145594, 0.000102924, 0.0000383062, 0\}$ |
| S7’ | -74.6917                        | $\{860.701, 75.8463, 69.4097, -3.51704, -1.40715, 0\}$ |
|     | -79.99                          | $\{308.868, 117.167, 63.944, 1.33617, 0.442318, 0\}$ |
| S8’ | -3.30094                        | $\{6.91363, 1.6158, 0.887765, -0.0443618, -0.0335668, 0\}$ |
|     | -3.53509                        | $\{8.35218, 2.69498, 0.892626, 0.0115894, 0.00386789, 0\}$ |
| S9’ | -6.60188                        | $\{38.038, 3.35196, 3.067, -0.155412, -0.0621878, 0\}$ |
|     | -7.07019                        | $\{13.6502, 5.17812, 2.82553, 0.0590508, 0.0195449, 0\}$ |

Let us recall here that so far we have been working with the string-frame variables, including the overall volume $\mathcal{V} = \rho^3$ and this volume can be given in the Einstein frame by the relation $\mathcal{V}_E = g_s^{-\frac{3}{2}} \mathcal{V}$. Thus we find that depending on the string coupling values, the Einstein-frame volume which can be responsible for supergravity EFT computations is quite better than the string-frame values! So the $T$-dual pairs of these AdS minimum can be reasonably stable against the sub-leading corrections. This way one can fix all the moduli in this model, except an axionic combination, namely the one in $(\xi^0, \xi_1)$ plane for type IIA and $(c_0, c_1)$ plane for type IIB case, which however still remain flat. We observe that the VEVs
of \( \rho \) and \( \sigma \) moduli are exchanged for the type IIA and type IIB models, while the dilaton VEVs are exchanged according to the relation \((e^D)_{\text{IIA}} \leftrightarrow (e^\phi \rho^{-\frac{1}{2}})_{\text{IIB}}\).

### 5.2 dS vacua: with D-terms

In this section we investigate the effects of including D-term contributions which are generically positive semi-definite in nature, and hence one may expect to realize stable de-Sitter solution. Just to have some naive estimates (which may or may not be true as we will explore later on), momentarily if we simply assume that the stabilized values of the moduli/axions do not change significantly, then the D-term contributions to the scalar potential at the previous AdS minimum may be given as,

\[
\langle V_D \rangle = \frac{2187 \times 3^{\frac{1}{2}} (m_0)^{\frac{3}{2}} (H^1)^2 \hat{w}_{11}}{80 \sqrt{5} \times 2^{\frac{1}{2}} \hat{\kappa}_{111} (-e_1)^{\frac{3}{2}}},
\]

(5.15)

where we have used the saxion/axion VEVs corresponding to the AdS2 solution in (5.14) which is a tachyon free minimum in the absence of D-terms. This naive estimate shows that a priory there appears to be a chance of uplifting the AdS to some dS solution under the assumption that the previously stabilized values remain (almost) fixed at their respective minimum. For that we need,

\[
\langle V_{\text{IIA}} \rangle = -\frac{1458 \times 2^{\frac{1}{2}} \times 3^{\frac{3}{2}} (m_0)^{\frac{3}{2}} (-H_0) (H^1)^3}{25 \sqrt{5} (-e_1)^{\frac{3}{2}}}
\]

\[
+ \frac{2187 \times 3^{\frac{1}{2}} (m_0)^{\frac{3}{2}} (H^1)^2 \hat{w}_{11}^2}{80 \sqrt{5} \times 2^{\frac{1}{2}} \hat{\kappa}_{111} (-e_1)^{\frac{3}{2}}} \geq 0,
\]

(5.16)

which can be satisfied if,

\[
\hat{w}_{11}^2 \geq \frac{32 \sqrt{2}}{5 \sqrt{3}} \times \frac{(m_0) (-H_0) (H^1)}{(-e_1) \hat{\kappa}_{111}}.
\]

(5.17)

Just to have some rough estimate, for the flux choices taken in the samples S1-S9, the requirement in Eq. (5.17) simplifies into the following form,

\[
\hat{w}_{11}^2 \geq \lambda \hat{\kappa}_{111}, \quad \text{where} \quad \lambda = \left\{ 0.522558, 1.04512, 0.348372, 0.696744, 0.261279, 0.261279, 0.522558 \right\}.
\]

(5.18)

So there appears to be a hope of uplifting the previous AdS solution to some de-Sitter for some choice of geometric flux and the triple intersection number. However, recall again that in arriving at this naive estimate, we have assumed that the previous VEVs of saxions are not significantly changed after including the D-term, which may not turn out to be correct, given that it depends on all the three saxions \( \varphi, \rho \) and \( \sigma \) and every piece in the scalar potential is on the same footing, in the sense of having no hierarchy to begin with.
Now we turn to explicitly solving the extremization conditions to investigate about the possibility dS solutions. Using the flux scaling arguments suggested in Eq. (5.6) and Eq. (5.7) along with their verification for the absence of geometric flux scenarios in Eq. (5.14) we take the following ansatz to begin with,

$$\langle b^1 \rangle = 0, \quad \langle \xi_1 \rangle = -\frac{H_0}{H^1} \langle \xi_0 \rangle, \quad e^{-\langle D \rangle} = \frac{\alpha_1 (-e_1)^{\frac{3}{2}}}{\sqrt{m_0} (-H_0)^{\frac{3}{4}} (H^1)^{\frac{1}{4}}},$$  \hspace{1cm} (5.19)

$$\langle \rho \rangle = \alpha_2 \sqrt{-\frac{e_1}{m_0}}, \quad \langle \sigma \rangle = \alpha_3 \sqrt{-\frac{H_0}{H^1}}.$$ 

In addition, we introduce a new parameter $\alpha_4$ defined through the following flux ratio which is induced only through the $D$-term effects,

$$\alpha_4 = \frac{e_1 \hat{\omega}_1^2}{\kappa_{111} m_0 H_0 H^1}.$$  \hspace{1cm} (5.20)

By considering these ansatz, similar to the previously realized AdS solutions which correspond to the flux choice considered as \{ $m_0 > 0, e_1 < 0, H_0 < 0, H^1 > 0$ \}, we find that it does not solve the extremization conditions for \{ $\alpha_1 > 0, \alpha_2 > 0, \alpha_3 > 0, \alpha_4 \geq 0$ \}. Therefore we conclude that the previous AdS solutions cannot be lifted to Minkowskian or dS solution by adding the $D$-terms of the type we considered.

However we find that there can be new Minkowskian/dS solutions (which does not arise as an uplifted version of the previous AdS solutions) for a different type of flux choice given as \{ $m_0 > 0, e_1 < 0, H_0 > 0, H^1 > 0$ \} and for exploring that possibility we consider the following ansatz,

$$\langle b^1 \rangle = 0, \quad \langle \xi_1 \rangle = -\frac{H_0}{H^1} \langle \xi_0 \rangle, \quad \langle \tau \rangle = e^{-\langle D \rangle} = \frac{\alpha_1 (-e_1)^{\frac{3}{2}}}{\sqrt{m_0} (H_0)^{\frac{3}{4}} (H^1)^{\frac{1}{4}}} > 0,$$  \hspace{1cm} (5.21)

$$\langle \rho \rangle = \alpha_2 \sqrt{-\frac{e_1}{m_0}} > 0, \quad \langle \sigma \rangle = \alpha_3 \sqrt{\frac{H_0}{H^1}} > 0, \quad \alpha_4 = \frac{(-e_1) \hat{\omega}_1^2}{\kappa_{111} (m_0) (H_0) (H^1)}, \quad \forall \alpha_i > 0.$$ 

To elaborate more on it we use Eq. (5.21) to get the following expressions for the three saxion extremization conditions,

$$\partial_\tau V = -\frac{(m_0)^3 (H_0)^{\frac{5}{2}} (H^1)^{\frac{15}{8}}}{6 (-e_1) \alpha_1^5 \alpha_1^5 \alpha_2^3 \alpha_3^3} \left[ 9 \alpha_1 \alpha_3^{3/2} + (1 - 3 \alpha_3^2) \alpha_2^3 + (2 \alpha_4^2 + 1) \alpha_3^3 \alpha_2^2 \right]$$  \hspace{1cm} (5.22)

$$+ 3 \alpha_1^2 (\alpha_2^2 \alpha_4 \alpha_3^{2} + (2 \alpha_2 \alpha_4^2 + 3) \alpha_3^4 + \alpha_2 \alpha_4 \alpha_3^{2} + 1) \right],$$

$$\partial_\sigma V = -\frac{(m_0)^{\frac{5}{2}} (H_0)^{\frac{7}{2}} (H^1)^{\frac{1}{4}}}{4 (-e_1) \alpha_1^5 \alpha_1^5 \alpha_2^3 \alpha_3^3} \left[ (\alpha_1 + 1) \left( \alpha_1 (3 \alpha_2^2 \alpha_4 \alpha_3^{2} + (3 - \alpha_2 \alpha_4) \alpha_3^{2} \alpha_2^{2}) - 3 \alpha_2 \alpha_3^{3/2} \right) \right],$$

$$\partial_\rho V = -\frac{(m_0)^3 (H_0) (H^1)^{\frac{1}{2}}}{12 (-e_1) \alpha_1^5 \alpha_1^5 \alpha_2^3 \alpha_3^3} \left[ \alpha_2^2 (1 - 9 \alpha_2^2) \alpha_3^{3} + 3 \alpha_1 \alpha_2^2 \alpha_4 \alpha_3^{2} + (2 \alpha_4^2 + 9) \alpha_3^{4} + \alpha_2^2 \alpha_4 \alpha_3^{2} + 3 \right],$$

while the scalar potential defined through Eqs. (4.8)-(4.10) or equivalently its axion mini-
mized version given in Eq. (5.4) takes the following form,

\[ V = \frac{\left( m_0 \right)^2 (H_0)^3}{12 \left( -e_1 \right)^2 \alpha_1^2 \alpha_3^3 \alpha_3^3} \left[ 6 \alpha_2 \alpha_3^{3/2} \left( 1 - 3 \alpha_3^2 \right) \alpha_2^3 + \left( 3 \alpha_2^4 + 1 \right) \alpha_3^2 \alpha_2^3 \right. \\
\left. + 3 \alpha_2 \left( \alpha_2^2 \alpha_3^6 + 2 \alpha_2 \alpha_3^2 + 3 \right) \alpha_3^2 + \alpha_2^3 \alpha_3^2 + 1 \right], \tag{5.23} \]

Subsequently one can get a Minkowskian or dS solution by solving the extremization conditions, in addition to consistently demanding the following constraint,

\[ \alpha_4 \geq -\frac{6 \alpha_2 \alpha_3^{3/2} \left( 1 - 3 \alpha_3^2 \right) \alpha_2^3 + \left( 3 \alpha_2^4 + 1 \right) \alpha_3^2 \alpha_2^3 + \alpha_1^2 \left( 9 \alpha_3^4 + 3 \right)}{3 \alpha_2^2 \alpha_3^2 \alpha_3^2 \left( \alpha_3^2 + 1 \right)^2}, \tag{5.24} \]

where equality corresponds to the Minkowskian solution. Moreover, there is a unique numerical solution for the Minkowskian case which corresponds to solving four polynomial equations in four unknowns \( \alpha_i \)'s which takes the following solution,

\[ \alpha_1 = 0.386877, \quad \alpha_2 = 1.03182, \quad \alpha_3 = 1.84403, \quad \alpha_4 = 0.424169. \tag{5.25} \]

Note that these \( \alpha_i \)'s can generically be some irrational numbers and a true Minkowskian solution will demand them to be take those precise values. However for numerical estimates we have presented rounded off figures for these \( \alpha_i \) parameters.

**Numerical samplings for Minkowskian solutions:**

For a set of flux choice with non-zero \( D \)-term flux \( \hat{\omega}_{11} \), the results for various VEVs of the moduli/axions along with their Hessian Eigenvalues for type IIA are mentioned in Table 6 and Table 7. In addition, the corresponding \( T \)-dual type IIB information is collected in Table 8 and Table 9 respectively.

| HA | \( e_1 \) | \( H_0 \) | \( \langle \rho \rangle \) | \( \langle \sigma \rangle \) | \( \langle e^{-D} \rangle \) | \( \langle \mathcal{V} \rangle \) | \( \langle e^{-\mathcal{V}} \rangle \) |
|----|----|----|----|----|----|----|----|
| S1 | -50 | 5 | 7.29606 | 4.12337 | 91.4714 | 388.388 | 4.64144 |
| S2 | -50 | 10 | 7.29606 | 5.83133 | 76.9179 | 388.388 | 3.90297 |
| S3 | -75 | 5 | 8.93581 | 4.12337 | 168.044 | 713.514 | 6.29102 |
| S4 | -75 | 10 | 8.93581 | 5.83133 | 141.307 | 713.514 | 5.29009 |
| S5 | -100 | 5 | 10.3182 | 4.12337 | 258.72 | 1098.53 | 7.80593 |
| S6 | -100 | 10 | 10.3182 | 5.83133 | 217.557 | 1098.53 | 6.56398 |
| S7 | -50 | 50 | 7.29606 | 13.0392 | 51.4381 | 388.388 | 2.61007 |
| S8 | -100 | 50 | 10.3182 | 13.0392 | 145.489 | 1098.53 | 4.3896 |
| S9 | -100 | 100 | 10.3182 | 18.4403 | 122.341 | 1098.53 | 3.6912 |

**Table 6:** Numerical samplings for type IIA geometric flux model corresponding to the Minkowskian solution, i.e. \( \langle V_0 \rangle = 0 \). Here, we have set the other flux parameters as: \( e_0 = 0, m^1 = 0, m_0 = 1, H^1 = 1, \) which leads to axions being stabilized as: \( \langle b^{1} \rangle = 0 \) and \( \langle \xi^{0} \rangle H_0 + \langle \xi^{1} \rangle H^1 = 0. \)
Table 7: Hessian Eigenvalues for the Minkowskian solution, i.e. $\langle V_0 \rangle = 0$ corresponding to the flux samplings of Table 6. This shows that there are no tachyons present, though there is an axionic combination still remaining flat.

| IIA | Eigenvalues of $\langle V_{ij} \rangle$.10^6 |
|-----|-----------------------------------------------|
| S1  | {0.531391, 0.23274, 0.104622, 0.000267061, 0.000988211, 0} |
| S2  | {0.918269, 0.269479, 0.210475, 0.00206273, 0.000279392, 0} |
| S3  | {0.0697052, 0.0307571, 0.0112366, 0.0000127765, 4.724.10^{-6}, 0} |
| S4  | {0.105215, 0.0407598, 0.0225316, 0.0000990067, 0.000133599, 0} |
| S5  | {0.0175195, 0.00689047, 0.00230835, 1.478.10^{-6}, 5.462.10^{-7}, 0} |
| S6  | {0.0234751, 0.0102855, 0.00462343, 0.000114627, 1.545.10^{-6}, 0} |
| S7  | {4.29194, 1.29677, 0.289438, 0.207258, 0.00311117, 0} |
| S8  | {0.09601, 0.0242602, 0.0125822, 0.00135235, 0.0000172598, 0} |
| S9  | {0.189403, 0.0573058, 0.0127681, 0.0091575, 0.000048772, 0} |

Table 8: Numerical samplings corresponding to the Minkowskian solution, i.e. $\langle V_0 \rangle = 0$, for the $T$-dual type IIB models presented in Table 6. Other flux parameters are set as: $F_0 = 0, F_1 = 0, F_0 = -1$ and $\hat{Q}^0 = 1$ which leads to axions being stabilized as: $\langle v^1 \rangle = 0$ and $\langle a_0 \rangle H_0 + \langle c_1 \rangle \hat{Q}^0 = 0$.

| IIB | $F_1$ | $H_0$ | $\langle \sigma \rangle$ | $\langle \rho \rangle$ | $\langle e^{-\phi} \rangle$ | $\langle V \rangle$ | $\langle V_E \rangle$ |
|-----|-------|-------|-----------------|-----------------|-----------------|-----------------|-----------------|
| S1' | -50   | 5     | 7.29606         | 4.12337         | 10.9246         | 70.1064         | 2531.44         |
| S2' | -50   | 10    | 7.29606         | 5.83133         | 5.46231         | 198.291         | 2531.44         |
| S3' | -75   | 5     | 8.93581         | 4.12337         | 20.0698         | 70.1064         | 6303.36         |
| S4' | -75   | 10    | 8.93581         | 5.83133         | 10.0349         | 198.291         | 6303.36         |
| S5' | -100  | 5     | 10.3182         | 4.12337         | 30.8995         | 70.1064         | 12041.6         |
| S6' | -100  | 10    | 10.3182         | 5.83133         | 15.4497         | 198.291         | 12041.6         |
| S7' | -50   | 50    | 7.29606         | 13.0392         | 1.09246         | 2216.96         | 2531.44         |
| S8' | -100  | 50    | 10.3182         | 13.0392         | 3.08995         | 2216.96         | 12041.6         |
| S9' | -100  | 100   | 10.3182         | 18.4403         | 1.54497         | 6270.51         | 12041.6         |

Table 9: Hessian Eigenvalues for the Minkowskian solution, i.e. $\langle V_0 \rangle = 0$ corresponding to the flux samplings of Table 8. This shows that there are no tachyons present, though there is an axionic combination still remaining flat.

| II B | Eigenvalues of $\langle V_{ij} \rangle$.10^6 |
|-----|-----------------------------------------------|
| S1' | {1.54193, 0.300266, 0.178242, 0.0031758, 0.0004048, 0} |
| S2' | {2.03816, 0.600957, 0.295961, 0.0227268, 0.00314276, 0} |
| S3' | {0.231281, 0.0322809, 0.0203511, 0.000150858, 0.000193471, 0} |
| S4' | {0.270132, 0.064582, 0.0360282, 0.0016734, 0.000150265, 0} |
| S5' | {0.0613253, 0.00663379, 0.00430614, 0.0000175041, 2.2366.10^{-6}, 0} |
| S6' | {0.0681444, 0.0132699, 0.00787725, 0.000173739, 0} |
| S7' | {34.1741, 13.08255, 1.19388, 0.379295, 0.210008, 0} |
| S8' | {0.284565, 0.0667359, 0.0245968, 0.0060399, 0.00213864, 0} |
| S9' | {1.5103, 0.136229, 0.0527624, 0.0167577, 0.00928112, 0} |
Numerical samplings for de-Sitter solutions:

We have learnt from the numerical analysis done so far that for slightly larger values of the uplifting parameter $\alpha_4$ as compared to the Minkowskian value mentioned in Eq. (5.25), one can realize tachyon free dS solutions. For illustration purpose we take the flux sampling $S8$ and show the details on dS uplifting starting from AdS solutions via crossing the Minkowskian value, by simply varying the $\alpha_4$ parameter. This is presented in Table 10.

| Model | $\alpha_4$ | $\langle V \rangle \times 10^3$ | $\langle \rho \rangle$ | $\langle \sigma \rangle$ | $\langle e^{-D} \rangle$ | $\langle V \rangle$ | $\langle e^{-\phi} \rangle$ |
|-------|------------|-------------------------------|-------------------|-------------------|---------------------|-----------------|---------------------|
| S8    | 0.40       | -25.2996                      | 9.97367           | 13.6722           | 128.538             | 992.12          | 4.08083             |
| S8    | 0.41       | -13.6743                      | 10.0964           | 13.421            | 134.502             | 1029.2          | 4.19255             |
| S8    | 0.42       | -3.68518                      | 10.2449           | 13.1564           | 141.826             | 1075.28         | 4.32508             |
| S8    | 0.425      | 0.026406                      | 10.3188           | 13.0384           | 145.518             | 1098.71         | 4.39011             |
| S8    | 0.43       | 0.700693                      | 10.3339           | 13.0152           | 146.278             | 1103.55         | 4.40335             |
| S8    | 0.43       | 4.67438                       | 10.4382           | 12.8638           | 151.559             | 1137.29         | 4.49412             |
| S8    | 0.44       | 11.2997                       | 10.7427           | 12.4923           | 167.416             | 1239.78         | 4.75471             |

Table 10: The numerical samplings for AdS and dS (via crossing the Minkowskian) solutions for a range of values for the $\alpha_4$ parameter. The flux parameters for model S8 are: $e_1 = -100$, $e_0 = 0$, $m_1 = 0$, $m_0 = 1$, $H_0 = 50$, $H^1 = 1$. The plots showing the uplift are presented in Figure 1.

Figure 1: One dimensional slices of scalar potential showing AdS to dS uplifting by $\alpha_4$ parameter.
5.3 Comments on viability of the dS vacua

In this section we present some comments about the viability of the tachyon free dS solutions we have obtained, and try to explore under which conditions their stability/existence could be questioned, though we do not anticipate such possibilities to be generic and always creating issues for the class of AdS/dS solutions we have presented.

On the integrality of the fluxes

Let us recall that our AdS solutions are realized with integer valued fluxes while in order to get the Minkowskian/dS solution, we need to find fluxes such that one satisfies the following condition according to the $\alpha^4$ parameter defined in Eq. (5.20),

$$\alpha^4 = \frac{(-e_1) \hat{w}_{11}^2}{\hat{\kappa}_{111} m_0 H_0 H^1} \geq 0.424169...$$

Note that the equality corresponds to the Minkowskian solution for which $\alpha^4$ needs to satisfy Eq. (5.24) and this generically leads to an irrational value of $\alpha^4$. Therefore, for integral values of fluxes and the triple intersection number $\hat{\kappa}_{111}$, a truly Minkowskian solution cannot be achieved. However for de-Sitter solutions one would need to take slightly larger values for $\alpha^4$ as compared to the bound given in Eq. (5.26). For a chosen value of the $\alpha^4$ parameter, we tabulate the positive real values for $\{\alpha_1, \alpha_2, \alpha_3\}$ obtained as the possible solutions to the extremization conditions which are presented in Table 11.

In Table 11 we have also introduced a new parameter $\gamma_0$ which determines the sign of the VEV of the scalar potential at a given extremum following from Eq. (5.23). This parameter $\gamma_0$ is subsequently defined as below:

$$\langle V \rangle = \gamma_0 \frac{(m_0)^{\frac{5}{2}} (H_0) (H^1)^3}{(-e_1)^{\frac{9}{2}}},$$

where

$$\gamma_0 = \frac{1}{12 \alpha_1^2 \alpha_2^2 \alpha_3^2} \left[ 6\alpha_1 \alpha_3^{3/2} (1 - 3\alpha_3^2) \alpha_2^3 + (3\alpha_2^4 + 1) \alpha_3^3 \alpha_2^2 
+ 3\alpha_1^2 (\alpha_2^2 \alpha_4 \alpha_6^6 + (2\alpha_4 \alpha_2^2 + 3) \alpha_4^4 + \alpha_2^2 \alpha_4 \alpha_3^2 + 1) \right].$$

Using the set of values for $\alpha_i$ parameters from Table 11 one can easily determine the VEVs of saxions for a chosen flux values using Eq. (5.21).

As mentioned in Table 11, a detailed numerical analysis shows that for $\alpha_4 \geq 0.45$, the three extremization conditions in Eq. (5.22) do not result in positive real solutions for the set of parameters $\{\alpha_1, \alpha_2, \alpha_3\}$ which are used for determining the VEVs of their respective saxions. Given that one needs to satisfy the bound in Eq. (5.26) this shows that there is only a narrow width for $\alpha_4$ parameter which is available for having the dS solutions. This
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
$\alpha_4$ & $\alpha_1$ & $\alpha_2$ & $\alpha_3$ & $\gamma_0$ & Vacua type \\
\hline
0.40 & 0.341801 & 0.997367 & 1.93354 & -0.505992 & Stable AdS \\
0.40 & 0.865109 & 1.31971 & 1.58217 & 0.124422 & Tachyonic dS \\
0.41 & 0.35766 & 1.00964 & 1.89801 & -0.273486 & Stable AdS \\
0.41 & 0.804922 & 1.28963 & 1.59365 & 0.145916 & Tachyonic dS \\
0.42 & 0.377135 & 1.02449 & 1.8606 & -0.0737035 & Stable AdS \\
0.42 & 0.743727 & 1.2576 & 1.60788 & 0.171904 & Tachyonic dS \\
0.4242 & 0.386954 & 1.03188 & 1.8439 & 0.000528121 & Stable dS \\
0.4242 & 0.717112 & 1.24318 & 1.61508 & 0.184542 & Tachyonic dS \\
0.425 & 0.388974 & 1.03339 & 1.84063 & 0.0140139 & Stable dS \\
0.425 & 0.711939 & 1.24034 & 1.61656 & 0.187088 & Tachyonic dS \\
0.43 & 0.403017 & 1.04382 & 1.81921 & 0.0934877 & Stable dS \\
0.43 & 0.678523 & 1.2217 & 1.62688 & 0.204158 & Tachyonic dS \\
0.44 & 0.445184 & 1.07427 & 1.76667 & 0.225995 & Stable dS \\
0.44 & 0.599241 & 1.17522 & 1.65781 & 0.2465 & Tachyonic dS \\
0.45 & … & … & … & … & … \\
\hline
\end{tabular}
\caption{For a chosen value of the (uplifting) parameter $\alpha_4$, the set of positive real solutions for the other three parameters $\{\alpha_1, \alpha_2, \alpha_3\}$ are obtained by solving the three extremization conditions in Eq. (5.22). The parameter determining the sign of potential at a given extremum is defined in Eq. (5.28). The Minkowskian solution defined by Eq. (5.25) lies in the middle of the table.}
\end{table}

Table 11: For a chosen value of the (uplifting) parameter $\alpha_4$, the set of positive real solutions for the other three parameters $\{\alpha_1, \alpha_2, \alpha_3\}$ are obtained by solving the three extremization conditions in Eq. (5.22). The parameter determining the sign of potential at a given extremum is defined in Eq. (5.28). The Minkowskian solution defined by Eq. (5.25) lies in the middle of the table.

can be estimated as:

$$0.42417 \leq \alpha_4 = \frac{(-e_1) \hat{w}_{11}^2}{\hat{\kappa}_{111} m_0 H_0 H^1} \leq 0.44. \quad (5.29)$$

Note that each of the quantities defining the parameter $\alpha_4$ as seen above, are either fluxes or triple intersection numbers and hence should usually take integral values. Moreover as we have already argued to set $m_0 = 1$ and $H^1 = 1$ for having large VEVs for volume modulus and complex structure modulus respectively (as seen from Eq. (5.23)), we have the following condition on the remaining quantities:

$$0.42417 \leq \alpha_4 = \frac{(-e_1) \hat{w}_{11}^2}{\hat{\kappa}_{111} H_0} \leq 0.44. \quad (5.30)$$
Note that increasing $|e_1|$ to have larger volume will demand either choosing quite (unnaturally) large value of intersection number $\hat{k}_{111}$ or a large value of $H_0$ flux in order to stay with the required range of the uplifting parameter $\alpha_4$. However, let us not forget that $H_0$ flux enters in the tadpole relation and hence can be bounded, unlike the $|e_1|$ flux. So one may not have much freedom to enlarge $H_0$ flux to a high value, though one cannot deny to have this possibility in generic CY orientifold models.

Now this suggests that there is a need for a delicate choice of fluxes as $|e_1|$ large is needed for large volume VEV and a couple of such flux samplings with $\alpha_4 = 0.44$, covering a bit the extreme possibilities can be taken as below,

$$S_{10}: \quad e_1 = -44, \quad \hat{w}_{11} = 1, \quad \hat{k}_{111} = 20, \quad H_0 = 5,$$

$$S_{11}: \quad e_1 = -44, \quad \hat{w}_{11} = 1, \quad \hat{k}_{111} = 10, \quad H_0 = 10. \quad (5.31)$$

| IIA  | $\alpha_4$ | $\langle V \rangle \times 10^8$ | $\langle \rho \rangle$ | $\langle \sigma \rangle$ | $\langle e^{-D} \rangle$ | $\langle V \rangle$ | $\langle e^{-\phi} \rangle$ | Vacua type |
|------|------------|-------------------------------|----------------|----------------|----------------|----------------|----------------|------------|
| S10  | 0.44       | 4.54497                       | 7.12593        | 3.9504         | 86.8912        | 361.847        | 4.56786        | Stable dS  |
| S10  | 0.44       | 4.95734                       | 7.79555        | 3.7097         | 116.96         | 473.741        | 5.37363        | Tachyonic dS|
| S11  | 0.44       | 9.08994                       | 7.12593        | 5.58671        | 73.0665        | 361.847        | 3.8411         | Stable dS  |
| S11  | 0.44       | 9.91469                       | 7.79555        | 5.24245        | 98.3513        | 473.741        | 4.51866        | Tachyonic dS|

Table 12: Numerical samplings for the two new benchmark model defined in Eq. (5.31) which give stable and tachyonic dS solutions for integral valued fluxes and the triple intersection numbers.

Scale separation arguments

Let us note that our flux choice is such that one of the RR fluxes, namely $|e_1|$, which governs the VEV of the overall volume modulus, does not appear in the tadpole condition at all, and hence it is not restricted by any upper bound. Subsequently, it is possible to realize quite large values for the VEV of the overall volume of the compactifying sixfold, i.e. $\langle V \rangle \gg 1$ along with weak string coupling $\langle g_s \rangle \ll 1$ as can be seen from the benchmark models presented in Table 10 and Table 12. Therefore, it is expected that the string scale ($M_s$) and the Kaluza-Klein states ($M_{KK}$) will be separated from the AdS/dS scale for appropriate choice of fluxes.

Obstructions from the Bianchi identities

For type IIA orientifold case, it has been found [107, 111] that the two known formulations of Bianchi identities do not result in equivalent sets of constraints on the fluxes, and there are always some “missing identities” which can have impact on the vacua realized within the so-called symplectic or cohomology formulation of (non-)geometric fluxes. So it might be possible that in such a simple setting which allows only a couple of moduli, it could get hard to consistently turn-on the all the needed (non-)geometric fluxes.

For our type IIA model we have six non-zero fluxes in the low energy dynamics, namely $\{e_1, m_0, H_0, H^1, \hat{w}_{11}, \hat{w}_{1}^{(0)}\}$ and these are to be used for stabilizing six scalars fields. To be
specific, there are three saxions \( \{D, \sigma, \rho\} \) and three axions \( \{\xi_0, \xi_1, b^1\} \). However, as we switch-off the \( F \)-term geometric fluxes in order to avoid a possibly negative contribution to the scalar potential, and allow their presence only through the positive semi-definite \( D \)-term effects, the cohomology form of the Bianchi identities results in just one constraint. This also reduces the number of independent fluxes to five, which can be attributed to the root cause of having one axionic combination still flat in the end. So the observation that there is at least one axion combination unfixed due to BIs, one cannot afford to make any other fluxes to zero, in case some additional constraints on the remaining fluxes arise in an explicit model through the so-called missing Bianchi identities. Similar observations have been made for the type IIB models as well [110], which in the case of rigid compactification has resulted in a (partial) restoration of the no-scale structure even in the presence of non-geometric fluxes [22]. To investigate the issue of missing Bianchi identities is beyond the scope of the current plan as the main task and goal in the current work has been limited to follow a balanced approach in the search of finding stable de-Sitter solutions for integer fluxes, and if such a candidate model is found, then to enumerate the possible loopholes for future refinements!

6 Summary and conclusions

Using the dictionary of [105], in this work we have presented some simple and explicit \( T \)-dual pairs of type II models which have been subsequently used for looking at the possibilities of realizing tachyon free (stable) AdS/dS solutions. On type IIA side we include the so-called geometric flux along with the usual NS-NS three-form flux \( H_3 \) and the RR \( p \)-form fluxes \( F_p \) for \( p \in \{0, 2, 4, 6\} \). This geometric type IIA model corresponds to a non-geometric type IIB setting related by a set of \( T \)-duality transformations as studied in [21], which makes it possible to exchange the information from one model to the other. For this reason we have focused on type IIA geometric flux model in detail and have presented the numerics about the \( T \)-dual pair of AdS/dS vacua for both the models.

First we have presented the generic type IIA scalar potential in a suitable formulation as given in Eqns. (3.5)-(3.6) which has been used for exploring the possible scenario that could evade the well known de-Sitter no-go theorems of geometric type IIA setup. We have engineered the flux choice such that:

• Given that the de Sitter no-go following from the volume/dilaton analysis can be evaded by simultaneously including the Romans mass term and geometric fluxes [2, 5]. Therefore, we consider Romans mass term \( m_0 \) and (some of) the geometric flux to be always non-zero. To be specific, we make only those geometric fluxes non-zero which appears in the \( D \)-term effects and such that to make only a positive semi-definite contribute to the scalar potential.

• As said above, all the known NS-NS Bianchi identities (of the cohomology formulation) are satisfied without nullifying any of the \( D \)-term fluxes which could be useful for uplifting purpose, given their positive semi-definite nature.
Some of the RR fluxes which couple to $H_3$ flux and the geometric flux in the tadpole relation are set to zero. To be specific, these are the ones following from the two-form potential ($F_2$) and six-form potential ($F_6$) denoted as: $m^a = 0$ and $e_0 = 0$.

There is one flux $|e_1|$ which does not receive an upper bound from the tadpole relation as it can couple only to non-geometric $Q_f$-flux, and hence can facilitate the realization of large volume, large complex structure and weak string coupling VEVs for the AdS as well as dS solutions we have.

There is one combination of axions which remains flat, and it may be attributed to making too restrictive choice of fluxes for various aforesaid reasons.

We have correlated the type IIA solutions with their $T$-dual type IIB cousins using Table 1, and have argued about their viability under the so-called missing Bianchi identities [107, 111] which still remains an open issue to explore, and can lead to extra constraints which may or may not rule out the $T$-dual pair of de Sitter solutions we have realized. However, settling the issue of missing Bianchi identities is beyond the scope of the current plan and we leave that for a future work.

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A Two equivalent scalar potentials

In this section we establish an equivalence between two classes of scalar potentials corresponding to type IIA and type IIB models.

A.1 Type IIA scalar potential with geometric flux

In the absence of any non-geometric fluxes, the type IIA scalar potential can be given as [22, 105],

$$V_{\text{IIA}} = \frac{e^{4D}}{4V} \left[ f_0^2 + \mathcal{V}_f + \mathcal{G}_{ab} f_a f_b + \mathcal{V} \mathcal{G}_{ab} f^a f^b + \mathcal{V}^2 (f^0)^2 \right] + \frac{e^{2D}}{4V} \left[ \frac{h_0^2}{U} + \tilde{G}^{ij} h_{i0} h_{j0} + \tilde{G}_{\lambda\rho} h^\lambda_0 h^\rho_0 \right]$$

$$+ \frac{e^{2D}}{4V} \left[ \mathcal{V} + \tilde{G}^{ij} h_{ai} h_{bj} + \tilde{G}_{\lambda\rho} h^\lambda_0 h^\rho_0 \right] + \frac{1}{U} \left( h_a - \frac{k_\lambda}{2} h^\lambda_0 \right) \left( V \tilde{G}^{ab} - t^a t^b \right)$$

$$\times \left( h_b - \frac{k_\rho}{2} h^\rho_0 \right) + \frac{1}{U} \left( \mathcal{V} \tilde{h}_a^0 + z^\lambda \tilde{h}_a^\lambda \right) \mathcal{V}^{-1} \left[ \mathcal{V} \tilde{h}_b^0 + z^\rho \tilde{h}_b^\rho \right] , \quad (A.1)$$

$$+ \frac{e^{3D}}{2 \sqrt{U}} \left[ (f^0 h_0 - f^a h_a) - \left( f^0 h^\lambda_0 - f^a h^\lambda_a \right) \frac{k_\lambda}{2} \right] .$$
where the various non-zero “axionic flux orbits” can be written in the following form,

\[
f_0 = e_0 + b^a e_a + \frac{1}{2} \kappa_{abc} b^a b^b m^c + \frac{1}{6} \kappa_{abc} b^a b^b b^c m_0 - \xi^0 (H_0 + b^a w_{a0}) - \xi^k (H_k + b^a w_{ak}) - \xi^\lambda (H^\lambda + b^a w_{a^\lambda}),
\]

\[
f_a = e_a + \kappa_{abc} b^b m^c + \frac{1}{2} \kappa_{abc} b^b b^c m_0 - \xi^0 w_{a0} - \xi^k w_{ak} - \xi^\lambda w_{a^\lambda},
\]

\[
f^a = m^a + m_0 b^a,
\]

\[
h_0 = (H_0 + b^a w_{a0}) + z^k (H_k + b^a w_{ak}) + \frac{1}{2} \hat{k}_{\lambda mn} z^m z^n (H^\lambda + b^a w_{a^\lambda}),
\]

\[
h_{k0} = (H_k + b^a w_{ak}) + \hat{k}_{\lambda kn} z^n (H^\lambda + b^a w_{a^\lambda}),
\]

\[
h_a = w_{a0} + z^k w_{ak} + \frac{1}{2} \hat{k}_{\lambda mn} z^n w_{a^\lambda},
\]

\[
h_{a^\lambda} = w_{a^\lambda} + \hat{k}_{\lambda kn} z^n w_{a_k^\lambda} + \frac{1}{2} \hat{k}_{\lambda mn} z^n w_{a^\lambda}.
\]

Let us also note that the fluxes allowed by the orientifold projection will be further constrained by the following Bianchi identities,

\[
H^\lambda \hat{w}_{a^\lambda} = H_k \hat{w}_{a_k^\lambda} + \hat{k}_{\lambda mn} z^n \hat{w}_{a_k^\lambda} + \frac{1}{2} \hat{k}_{\lambda mn} z^n \hat{w}_{a_0^\lambda},
\]

\[
\hat{h}_{a^\lambda} = \hat{w}_{a^\lambda} + \hat{k}_{\lambda kn} z^n \hat{w}_{a_k^\lambda} + \frac{1}{2} \hat{k}_{\lambda mn} z^n \hat{w}_{a_0^\lambda}.
\]

We consider the type IIA setup with geometric flux such that fluxes with \(k\)-indices are absent. This is equivalent to not having any odd-moduli \(G^a\) in the dual type IIB theory.

### A.2 T-dual Type IIB scalar potential with ‘special’ non-geometric fluxes

In order to arrive at a type IIB model which is \(T\)-dual of the type IIA geometric flux scenario, it was shown it [21] that one has to consider the so-called ‘special solution’ of Bianchi identities, which basically corresponds to switching-off the fluxes with upper \(h^2,1\) indices in the type IIB context. However, let us also note that the aforesaid choice does not automatically satisfy all the (cohomology formulation) Bianchi identities, and even the fluxes counted with lower \(h^2,1\) indices will be further constrained by the following two sets of Bianchi identities,

\[
H_0 R_K + \omega_{a0} Q^a K + \hat{Q}^a_0 \hat{\omega}_{aK} = 0,
\]

\[
H_i R_K + \omega_{ai} Q^a K + \hat{Q}^a_i \hat{\omega}_{aK} = 0.
\]

These identities are \(T\)-dual to those given in Eq. (3.22) corresponding to the type IIA case.

Actually the hallmark of our master formulae of the scalar potential in [105] is the fact that we just need to know the even/odd hodge numbers for reading-off the various pieces of the scalar potential. Using this simplification, and after a bit of reshuffling of terms, the
$T$-dual scalar potential for the type IIB side can be subsequently given as below,

$$V_{\text{IIB}} = \frac{e^{2\phi}}{4V^2 U} \left[ f_0^2 + U f^i G_{ij} f^j + U f_i G^i j f_j + U^2 (f^0)^2 \right] \quad \text{(A.5)}$$

$$+ \frac{e^{2\phi}}{4V^2 U} \left[ h_0^2 + V G^{ab} h_0 a h_0 b + V G_{\alpha \beta} h_0^\alpha h_0^\beta \right.$$

$$+ u^i u^j \left( h_i h_j + V G_{\alpha \beta} h_0^\alpha h_0^\beta + V G^{ab} h_{ai} h_{bj} \right)$$

$$\left. + (U G^{i j} - u^i u^j) \left( \frac{\ell}{2} \bar{h}^i \right) \left( \frac{\ell}{2} \bar{h}^j \right) \right]$$

$$+ U \left( V h_i j^0 - t^\alpha \bar{h}_{\alpha j} \right) \left( \bar{h}_{i K} u^i \right)^{-1} \left( V \bar{h}_{\alpha K}^0 - t^\beta \bar{h}_{\beta K} \right)$$

$$+ \frac{e^{2\phi}}{2V^2} \left[ (f^0 h_0 - f^i h_i) - (f^0 h_0^a - f^i h_0^a) \frac{\ell}{2} \right],$$

where the simplified version of the non-trivial axionic flux orbits are given as below,

$$f_0 = F_0 + v^i F_i + \frac{1}{2} l_{ijk} v^i v^j v^k F^k - \frac{1}{6} l_{ijk} v^i v^j v^k F^0 - \omega a \partial a - \hat{Q}_a^0 \hat{c}_a - c_0 h_0,$$

$$f_i = F_i + l_{ijk} v^j v^k F^k - \frac{1}{2} l_{ijk} v^i v^j v^k F^0 - \omega a \partial a - \hat{Q}_a^i \hat{c}_a - c_0 h_i,$$

$$f^i = F^i - v^i F^0,$$

$$f^0 = - F^0,$$

$$h_0 = H_0 + \omega a h_0^a + \frac{1}{2} \hat{c}_{aab} b^a b^b \hat{Q}_a^0 + v^i h_i,$$

$$h_i = H_i + \omega a h_i^a + \frac{1}{2} \hat{c}_{aab} b^a b^b \hat{Q}_a^i,$$

$$h_{a 0} = \omega a \hat{Q}_0^0 \hat{c}_{aab} b^b + v^j h_{a j},$$

$$h_{ai} = \omega a \hat{Q}_a^i \hat{c}_{aab} b^b,$$

$$h_{a 0}^\alpha = \hat{Q}_a^0 + v^i \hat{Q}_a^i$$

$$h_{a i}^\alpha = \hat{Q}_{ai}$$

$$\hat{h}_{\alpha K} = \omega \hat{h}_K - Q_{\alpha K} \hat{c}_{aab} b^b + \frac{1}{2} \hat{c}_{aab} b^a b^b R_K,$$

$$\hat{h}_K^0 = - R_K.$$

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