Exploring the Predictability of Cryptocurrencies via Bayesian Hidden Markov Models

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Abstract

In this paper, we consider a variety of multi-state Hidden Markov models for predicting and explaining the Bitcoin, Ether and Ripple returns in the presence of state (regime) dynamics. In addition, we examine the effects of several financial, economic and cryptocurrency specific predictors on the cryptocurrency return series. Our results indicate that the 4-states Non-Homogeneous Hidden Markov model has the best one-step-ahead forecasting performance among all the competing models for all three series. The superiority of the predictive densities, over the single regime random walk model, relies on the fact that the states capture alternating periods with distinct returns’ characteristics. In particular, we identify bull, bear and calm regimes for the Bitcoin series, and periods with different profit and risk magnitudes for the Ether and Ripple series. Finally, we observe that conditionally on the hidden states, the predictors have different linear and non-linear effects.

Keywords: Cryptocurrencies, Bitcoin, Ether, Ripple, Hidden Markov Models, Regime Switching models, Bayesian Inference  
JEL: C11, C52, E49

1. Introduction

The growth of cryptocurrency markets made a splash in the world. At present, there are more than one thousand cryptocurrencies that constitute a multi-billion market [Hu et al. (2019)]. The increased popularity of cryptocurrencies along with their peculiar nature, [Dyhrberg et al. (2018)], attract the interest of financial regulators, policymakers and academics, [Corbet et al. (2019)], whereas traders and speculators cast about for predicting their daily or intra-day returns. Their documented safe haven and hedge properties — most relevant in periods with volatile stock markets and inflationary pressures in fiat currencies — render cryptocurrencies increasingly important in optimizing portfolios and diversifying risk, see for example [Selmi et al. (2018); Urquhart and Zhang (2019); Bouri et al. (2020)].

In view of these attractive properties, investors and economists see cryptocurrencies as a new financial instrument which they seek to include in their portfolios. In turn, informed decisions concerning optimal portfolio allocation and asset management require models with good predicting ability [Chen et al. (2020b)]. Similar to forecasting studies about financial assets and exchange rates [McMillan (2020); Panopoulou and Souropanis (2019)], this has generated a vivid literature on the predictability of cryptocurrency returns, ranging from identification of significant explanatory variables, see [Aalborg et al. (2019); Bleher and Dimpfl (2019); Kurka (2019) and Corbet et al. (2019); Katsiampa (2019)] for comprehensive surveys of earlier models, to price prediction with elaborate machine and deep learning models [Chen et al. (2020b,a)]. Under the Bayesian framework, main efforts involve continuous state space models, [Hotz-Behofits et al. (2018)], univariate and multivariate dynamic linear models, model averaging and time-varying vector autoregression models, [Catania et al. (2019)]. In both these articles, the authors show that the time-varying models give significantly improved point and density forecasts, compared to various benchmarks such as the random walk model.

The improved performance of the state space and time varying models is not a surprise, since these models accommodate various characteristics of the cryptocurrency series, such as time varying volatility.

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and time varying mean returns. Accumulating evidence suggest the existence of structural breaks, Mensi et al. (2019); Bouri et al. (2019); Katsiampa (2019); Thies and Molnár (2018), return and volatility jumps, Chaim and Laurini (2018), regime/state switches, Ardia et al. (2019); Koutmos (2019, 2018). On the other hand, regime switching models have been shown to deliver improved forecasting results in exchange rates series and stock market returns, e.g., Panopoulou and Pantelidis (2015); Dias et al. (2015); Yuan (2011) among others.

Stimulated by these results and aiming to contribute to the growing literature about cryptocurrencies’ predictability, we perform a systematic analysis of various multi-state (regime-switching) Hidden Markov (HM) models on the return series of the three largest — in terms of market capitalization — cryptocurrencies, i.e., Bitcoin (BTC), Ether (ETH) and Ripple (XRP). In total, we consider eight discrete state space HM models with exogenous predictors, 2 to 5 hidden states, and both Homogeneous (HHM), i.e., constant, and Non-Homogeneous (NHHM), i.e., non-constant, transition probabilities, as well as the standard 2-state Markov Switching Random Walk (MS-RW) model without exogenous predictors. We benchmark the aforementioned HM models against three single regime models: the Random Walk (RW) model that is commonly used (as a benchmark) in predicting exchange rates, see for example Panopoulou and Pantelidis (2015); Frommel et al. (2005); Yuan (2011); Cheung and Erlendsson (2005), the linear random walk model with all the predictors, often referred to as the Kitchen Sink (KS) model, and the linear AutoRegressive (AR(5)) model with lagged values up to lag 5. All models are estimated using Bayesian MCMC methods.

We examine the impact of regime switches in predicting the return series and the state-dependent (time-varying) effects of several financial, economic and cryptocurrency specific exogenous predictors. The predictor set includes exchange rates of various fiat currencies, stock and volatility indices, commodities, and cryptocurrency specific variables. Following the tradition in the literature, see for example Gelman et al. (2014); Geweke and Amisano (2010); Bergman and Hansson (2005), we use the out-of-sample forecasting performance of the aforementioned models to discriminate between the different empirical models. The statistical evaluation of the models is based on the Continuous Rank Probability Score (CRPS) and Mean Squared Forecast Error (MSFE). Finally, to examine if there is an underlying non-linear correlation between the predictors and the return series through the transition probabilities, we add a stochastic search reversible jump step in the NHHM model with the best forecasting performance. We report the posterior probabilities of inclusion in the hidden states transition equations for each predictor.

Our results reveal that the 4-states NHHM model has the best forecasting performance for all three series, with significant improvements over the single regime models. Based on the in-sample analysis of this model, we observe that the returns of each state present distinct characteristics. Concerning the Bitcoin return series, we find that state 1, the most frequently occurring state, corresponds to a bear regime (i.e., negative returns and high volatility), states 2 and 3 correspond to a bull regime (positive returns and low volatility) but with different kurtosis and state 4 corresponds to a calm regime (returns close to 0 and low volatility). The relation of the BTC returns and volatility within the hidden states is consistent with the asymmetric volatility theory. Regarding the ETH return series, we observe frequent alternations between state 1, the high volatility state, and the low volatility state 2, while states 3 and 4 serve as auxiliary states with low occupancies. Lastly, state 1 of the Ripple series corresponds to periods with extremely high average returns but, as a trade-off, also with high risk. States 2 and 3 are the states with the highest occupancies, while state 4 serves as an auxiliary state. As a general finding, we observe that, unlike conventional exchange rates, see for example Yuan (2011), the hidden states for all coins are not persistent. Enhancing our understanding and prediction of cryptocurrency returns, our preliminary results finally indicate that there exist predictors, such as the US Treasury Yield and the CBOE stock market volatility index, VIX, that have predictive power on all three return series.

The remainder of the paper is structured as follows. Section 2 provides an overview of the data and methodology. In particular, Section 2.1 describes the data along with their transformations and descriptive statistics and Section 2.2 presents the Hidden Markov models and forecasting evaluation criteria. Section 3 presents the empirical findings of the forecasting exercise, i.e., the out-of-sample results and the in-sample analysis of the model with the best performance. Finally, Section 4 summarizes our results and discusses current limitations and possible extensions of this study.

1To determine the number of states we undertook an extensive specification test. Experiments with more than 5 states (not presented here) exhibit worst performance. Even though adding more states may improve the in-sample fit, the decreased parsimony leads to worse predictions Guidolin and Timmermann (2006).
1.1. Other Related Literature

Our model falls into two strands of literature. From a methodological perspective, our model falls into the econometric literature of HM models. Since the seminal work of Hamilton (1989), HM models have been fruitfully applied in diverse areas such as communications engineering and bioinformatics Cappé et al. (2006). In finance, they have been extensively used in predicting and explaining exchange rates Panopoulou and Pantelidis (2015); Lee and Chen (2006); Frömmel et al. (2005); Bollen et al. (2000), stock market returns Dias et al. (2015); Angelidis and Tesseromatis (2009); Guidolin and Timmermann (2006), business cycles Tian and Shen (2019); Chauvet and Hamilton (2006), realized volatility Koki et al. (2020); Liu and Maheu (2018), the behavior of commodities Pereira et al. (2017) and in portfolio allocation Platanakis et al. (2019). The reason for their increased popularity is that they present various attractive features. In particular, the time-varying parameters which are driven by the state variable of the presumed underlying Markov process, lead to models that can accommodate both non-linearities and mean reversions Wu and Zeng (2014); Guidolin and Timmermann (2008). In addition, HM models can act as filtering processes that account for outliers and abrupt changes in financial market behavior Persio and Frigo (2016); Ang and Timmermann (2012) and flexibly approximate general classes of density functions Timmermann (2000).

In the cryptocurrency context, HM models have been applied by Hotz-Behofsits et al. (2018); Catania et al. (2019) as a state space model, by Phillips and Gorse (2017) in the understanding of price bubbles and by Koutmos (2018, 2019) in examining the relation of BTC with conventional financial assets. Bouri et al. (2019); Caporale and Zekokh (2019); Ardia et al. (2019) used the HM setting under GARCH models for modeling the cryptocurrencies volatility. The motivation of these studies lies in the observed features of the cryptocurrencies return and volatility series. In particular, cryptocurrencies series are non-stationary and present non-normalities, heteroskedasticity, volatility clustering, heavy tails and excess kurtosis Katsiampa (2017, 2019); Chaim and Laurini (2018); Thies and Molnár (2018) (among others) document the existence of abrupt price changes and outliers, while Corbet and Katsiampa (2018) show that BTC returns are characterized by an asymmetric mean reverting property. With the aforementioned attractive features of HM models and the characteristics of the cryptocurrency series, it is only natural to ask: do HM models offer improved predictive performance of cryptocurrency returns?

2. Data and Methodology

2.1. The Data

We use the percentage logarithmic end-of-the-day returns, defined as \[y_t = 100 \times (\log(p_t) - \log(p_{t-1}))\], with \(p_t\) be the prices of BTC, ETH and XRP. For each coin, we excluded an initial adjustment market period. In particular, we study the BTC time series for the period ranging from 1/2014 until 11/2019, the ETH series for the period ranging from 9/2015 until 11/2019 and the XRP data series from 1/2015 until 11/2019. Figure 1 displays the series under study. The covariate set is consisted of normalized fiat currencies, i.e., Euros to US Dollars (EUR/USD), Great Britain Pounds to US Dollars (GBP/USD), Chinese Yuan to US Dollars (CNY/USD) and Japanese Yen to US Dollars (JPY/USD), commodities, i.e., Gold and crude Oil normalized future prices, stock indices, i.e. Standard and Poor’s 500 logarithmic returns (SP500), CBOE volatility logarithmic index (VIX), interest rates, i.e., US 10-year Treasury Yield (TY) logarithmic returns and cryptocurrency specific variables, i.e., the blockchain block size (SIZE) as percentage of difference between two consecutive days and the percentage of difference between two consecutive days of Hash Rate (HR).\(^2\) We report some illustrative descriptive statistics of our covariate set in Table 1. Finally, we also include the the lagged 1 autoregressive term of the studied series as a predictive variables.\(^3\) In our extensive experimental study, we added up to 5 lagged autoregressive terms as predictive variables. We did not observed any improvement in the performance and hence we omit the results of these experiments.

\(^2\)The Hash rate is used in BTC and ETH series only

\(^3\)The importance of including autoregressive terms is highlighted in Timmermann (2000) who proves that including autoregressive parameters give rise to cross-product terms that enhance the set of third- and fourth-order moments and the patterns in serial correlation and volatility dynamics that these models can generate and hence provide the basis for very flexible econometric models.

\(^4\)The cryptocurrency price series and blockchain size were downloaded from coinmetrics.io the exchange rates and commodities prices were downloaded from investing.com the Sk&P 500 index, VIX and Treasury Yield from Yahoo Finance and lastly the Hash Rate from quandl.com for BTC and from etherscan.io for ETH.
Figure 1: Daily price series plots for the three cryptocurrencies considered in this study: Bitcoin (upper plot), Ether (middle plot) and Ripple (lower plot). We study the Bitcoin time series for the period ranging from 1/2014 until 11/2019, the Ether series for the period ranging from 9/2015 until 11/2019 and the Ripple data series from 1/2015 until 11/2019.

Table 1: Summary Statistics of the percentage logarithmic return cryptocurrency series and transformed predictors. The first column reports the transformation of each variable. The second column displays the mean. The third column reports the standard deviation. Third to seventh columns display the minimum values, the 5%, 50%, 95% quantiles and the maximum values respectively. Last two columns display the kurtosis and skewness coefficients.

| Variables     | Transf | Mean  | Std.  | Min.   | q05   | p50   | q95   | Max.   | Kurt  | Skew |
|---------------|--------|-------|-------|--------|-------|-------|-------|--------|-------|------|
| Bitcoin       | % log returns | 0.11  | 3.95  | -24.37 | -6.47 | 0.17  | 6.13  | 22.47  | 7.84  | -0.27 |
| Ether         | % log returns | 0.33  | 6.54  | -31.67 | -9.76 | -0.01 | 11.61 | 30.06  | 6.64  | 0.07 |
| Ripple        | % log returns | 0.14  | 7.02  | -63.65 | -8.46 | -0.31 | 10.25 | 100.85 | 38.99 | 2.55 |
| EUR/USD       | normalized | 0     | 1     | -1.42  | -1.20 | -2.24 | 2.05  | 6.30   | 6.16  | 1.50 |
| GBP/USD       | normalized | 0     | 1     | -5.57  | -1.65 | 0.03  | 1.64  | 7.23   | 8.02  | 0.15 |
| CNY/USD       | normalized | 0     | 1     | -16.60 | -1.52 | 0.02  | 1.57  | 6.27   | 41.26 | -2.00 |
| JPY/USD       | normalized | 0     | 1     | -9.70  | -1.52 | 0.04  | 1.44  | 6.42   | 15.13 | -0.59 |
| Gold          | normalized | 0     | 1     | -5.32  | -1.60 | 0     | 1.61  | 9.32   | 9.80  | 0.29 |
| Oil           | normalized | 0     | 1     | -7.93  | -1.22 | 0     | 1.29  | 10.79  | 23.92 | 1.04 |
| SP500         | log returns | 0     | 0.01  | -0.05  | -0.01 | 0     | 0.01  | 0.05   | 9.47  | -0.56 |
| VIX           | log prices  | 2.68  | 0.25  | 2.21   | 2.31  | 2.63  | 3.15  | 3.71   | 3.37  | 0.70 |
| TY            | log returns | 0     | 0.01  | -0.10  | -0.03 | 0     | 0.03  | 0.11   | 7.18  | 0.13 |
| BTC Hash      | % of change  | -1.12 | 14.77 | -138.86| -25.42| -0.72 | 20.81 | 66.80  | 10.58 | -0.88 |
| ETH Hash      | % of change  | 0.42  | 4.02  | -25.50 | -4.21 | 0.37  | 4.82  | 99.90  | 248.92| 9.50 |
| BTC size      | % of change  | -0.82 | 13.61 | -81.26 | -23.04| -0.27 | 20.84 | 45.08  | 5.10  | -0.46 |
| ETH size      | % of change  | -0.52 | 14.63 | -359.96| -16.87| -0.15 | 15.81 | 55.28  | 243.51| -10.24|
| XRP size      | % of change  | -0.83 | 13.76 | -90.94 | -24.63| -0.06 | 20.43 | 51.02  | 6.61  | -0.73 |

2.2. The Econometric framework

In this study, we focus on a widely used class of econometric models, the HM models. In a HM setting, the probability distribution of the studied series $Y_t$ depends on the state of an unobserved (hidden) discrete Markov process, $Z_t$. Let $(Y_t, X_t)$ be pair of a the random process of the assumed cryptocurrency return series $Y_t$, with realization $y_t$ and the set of explanatory variables (predictors) $X_t$ with realization $x_t = (x_{1t}, \ldots, x_{kt})$. Then, given the state $z_t$ the observed process is modeled as $y_t = g(z_t)$, with $g$ a predetermined function. The hidden process $Z_t$ follows a first order finite Markov process with $m < \infty$ states and transition probabilities $P(Z_{t+1} = j \mid Z_t = i) = p_{ij}$, $i, j = 1, \ldots, m$. If $m = 1$, then the model is the standard linear regression model.

We consider the Normal HM models, i.e., conditional on the hidden process marginal distribution of
$Y_t$ is $\mathcal{N}$ormal, 

$$y_t = B_{z_t} X_t + \epsilon_{z_t},$$

with $B_{z_t} = (b_{0z_t}, b_{1z_t}, \ldots, b_{kz_t})$ be the regression coefficients when the latent process at time $t$ is at state $z_t = s, s = 2, \ldots, m$, and $\epsilon_{z_t}$ be the normally distributed random shocks, $\epsilon_{z_t} \sim \mathcal{N}(0, \sigma^2_{z_t})$. The hidden process is determined by the transition probability matrix

$$p(t) = \begin{bmatrix}
p_{11}^{(t)} & p_{12}^{(t)} & \cdots & p_{1m}^{(t)} \\
p_{21}^{(t)} & p_{22}^{(t)} & \cdots & p_{2m}^{(t)} \\
\vdots & \vdots & \ddots & \vdots \\
p_{m1}^{(t)} & p_{m2}^{(t)} & \cdots & p_{mm}^{(t)}
\end{bmatrix},$$

where $p_{ij}^{(t)} = P(Z_{t+1} = j \mid Z_t = i)$ is the probability that at time $t$ the hidden state is $j$ given that at time $t-1$ the hidden state was $i$. If the transition probabilities are time-constant, then the resulting model is a Homogeneous Hidden Markov (HHM) model. However, relaxing the hypothesis of constant probabilities, then the resulting model is the more flexible Non-Homogeneous Hidden Markov (NHHM) model. A graphical representation of the NHHM is shown in Figure 2.

![Graphical representation of the Non-Homogeneous Hidden Markov model.](image)

Figure 2: Graphical representation of the Non-Homogeneous Hidden Markov model.

models, we use recently proposed NHHM of Koki et al. (2020), namely the Non-Homogeneous Pólya-Gamma HM model. In this model, the transition probabilities are modeled using the multinomial link with predictors $X_t$ and multinomial regression coefficients $\beta_{ij} = (\beta_{0,ij}, \beta_{1,ij}, \ldots, \beta_{k,ij})'$, i.e.

$$p_{ij}^{(t)} = \frac{\exp(x_t \beta_{ij})}{m \sum_{l=1}^{m} \exp(x_t \beta_{il})}, i, j = 1, \ldots, m.$$ 

In the proposed model, the authors use a further latent variable scheme to make inference on the multinomial regression coefficients, which is based on a Pólya-Gamma data augmentation scheme (Polson et al. 2013), leading to more accurate and robust inferences.

To make inference on the models’ parameters we use a Bayesian Markov Chain Monte Carlo (MCMC) algorithm, which is consisted of the following steps: (a) A FB algorithm for simulating the hidden states, (b) a Gibbs sampling step for estimating the linear regression coefficients using conditional conjugate analysis, (c) a Gibbs sampling step with a Pólya-Gamma data augmentation scheme for estimating the multinomial regression coefficients and (d) simulation of $L$ one-step-look-ahead forecasts. To study the effects of the predictors on the non-homogeneous transition probabilities, we intercalate between the
fourth (c) and fifth (d) steps a stochastic variable search (via reversible jump) step. We refer to Koki et al. (2020) for a detailed description of these steps.

In addition, we include in our analysis the 2-state Markov-Switching Random Walk (MS-RW) model with drift Engel (1994); Nikolsko-Rzhevskyy and Prodan (2012) and Normal errors, as a simpler and parsimonious regime switching model. By leveraging this model we allow for both the drift term $\mu$ and variance $\sigma^2$ to take two distinct values, i.e.,

$$ y_t \sim \mathcal{N}(\mu_{z_t}, \sigma_{z_t}^2), z_t = 1, 2, $$

where the variable $Z_t$ is governed by the constant transition probabilities $P(Z_t = 1 \mid Z_{t-1} = 1) = p_{11}$ and $P(Z_t = 2 \mid Z_{t-1} = 2) = p_{22}.$

| Model                                | Abbreviation | Predictors | Transition Probabilities | States |
|--------------------------------------|--------------|------------|-------------------------|--------|
| Non-Homogeneous Hidden Markov (NHHM) | X, AR        | X, AR      | multinomial             | 2-5    |
| Homogeneous Hidden Markov (HHM)      | X, AR        | --         | constant                | 2-5    |
| Markov Switching Random Walk (MS-RW) | --           | --         | constant                | 2      |
| Kitchen Sink (KS)                    | (AR(5))      | AR         | --                      | 1      |
| Linear Regression                    | (AR(5))      | AR         | --                      | 1      |
| Random Walk                          | --           | --         | --                      | 1      |

Table 2: Summary of the models of this study. The first two columns show the model and its abbreviation, the third and fourth columns show the assumed relation of the studied time series and the predictors. The fifth column shows the assumed parametrization of the transition probabilities of each model. The last column shows the various number of states we considered for each model.

Summing up, our methodology is the following. First, we study the performance of various HM settings with fixed covariate set, in explaining and predicting the cryptocurrency log-return series. In particular, we consider 9 HM models, i.e., NHHM models with $m = 2, \ldots, 5$ states, HHMs with $m = 2, \ldots, 5$ state and the 2-state MS-RW\footnote{We omitted the m-state MS-RW, $m > 2$ models from this study since they do not offer an improved forecasting performance compared to the 2-state MS-RW.} Following the standard practice, we also implement the Random Walk model (RW) (i.e., a linear model with no covariates), the linear regression model with all the covariates and the autoregressive term, often referred as Kitchen Sink (KS) model and a autoregressive model lagged endogenous variables up to lag 5, AR(5), as single regimes models, leading to 12 in total models for each coin. The 12 models are summarized in Table 2. Then, we choose the model with the out-of-sample (predicting) performance based on the CRPS and MSE. Finally, we focus on the possibly missed hidden effects on the transition probabilities. To this end, we apply a reversible jump stochastic search algorithm on the multinomial regression predictors of the NHHM model with the best predicting performance.

2.3. Performance Evaluation

We assess the performance of the studied models based on their predicting ability. Reflecting the logical positivism of the Bayesian approach stating that a model is as good as its predictions Geweke and Amisano (2010); Guidolin (2011), the predictive accuracy is valued not only for its own sake but rather for comparing different models within the Bayesian framework. Focusing on the accuracy of the predictive density, we rely on two distance-based metrics: the Continuous Rank Probability Score (CRPS) and the Mean Square Error (MSE).

Let $y_p$ be the actual forecasting values with distribution $F_p$. Utilizing the MCMC output, we obtain a sample of the $L$ one-step-look ahead predictions, $\hat{Y}_l, l = 1, \ldots, L,$ from the empirical posterior predictive distribution. For every out-of-sample observation, the CRPS is defined as,

$$ \text{CRPS}(F_{i,p}, y_{i,p}) = \int_{-\infty}^{\infty} (F(\hat{y}_p) - I_{\{\hat{y} \geq y_{i,p}\}})^2 \, d\hat{y}_i, \quad l = 1, \ldots, L. $$

We compute the CRPS numerically, using the identity of Székely and Rizzo (2005)

$$ \text{CRPS} (F_{i,p}, y_{i,p}) = -\frac{1}{2} \mathbb{E} \left[ \hat{Y}_i - \hat{Y}_{i,p} \right] - \mathbb{E} \left[ \hat{Y}_i - y_{i,p} \right], $$

$$ \hat{y}_p = \frac{1}{L} \sum_{l=1}^{L} \mathbb{1}_{\{\hat{Y}_l \geq y_p\}} $$

where $\mathbb{1}_{\{\cdot\}}$ is an indicator function.
were $\hat{Y}_l, \hat{Y}'_l$ are independent replicates from the estimated (empirical) posterior predictive distribution. The MSE for the $l$-th, $l = 1, \ldots, L$, out-of-sample observation is defined as

$$\text{MSE}_l = \frac{1}{N} \sum_{i=1}^{N} (y_{p,l} - \hat{y}_{l,i})^2,$$

where $N$ is the MCMC sample size. We report the the CRPS and MSE for every prediction over all MCMC iterations and the average CRPS and MSE over all observations. The best model among its counterparts, is the one with the lowest CRPS and MSE.

3. Empirical Analysis

3.1. Out-of-Sample analysis

We assess the forecasting performance of the models under scrutiny, i.e., the $m$-states, $m = 2, \ldots, 5$ NHMMs and HHMs, 2-states MS-RW and the benchmarks RW, KS and AR(5). The out-of-sample accuracy is assessed using a sequence of $L = 30$ one-step-ahead predictive densities. In Table 3 we report the CRPS and MSE in parenthesis for 5 randomly chosen out-of-sample points, i.e., $L = 1, 2, 7, 15, 30$. The last column reports the average scores over all the out-of-sample points. Through this exercise, parameter estimates are held fixed.

As a preliminary point, we observe that all the HM models significantly surpass the single regime RW, KS, AR(5) models, while all the single regime models have similar forecasting performance. Even the KS model, which includes all the predictors besides the autoregressive terms, does not improve the forecasting performance over the AR or RW models. These results suggest that the HM models can identify time-varying parametrizations leading to improved forecasting performance, relatively to the forecasting performance of the single regime benchmarks. This finding is in line with previous studies arguing on the necessity of incorporating the structural breaks and regime switches in modeling the BTC return series, see for example Thies and Molnár (2018). In addition, we also confirm the argument on the existence of time-varying effects on BTC cryptocurrencies series, see for example Mensi et al. (2019) and expand it to the ETH and XRP series.

As far as the predicting accuracy among the various HM models, we observe that – based on the average CRPS and MSE scores (last column of Table 3) – the model with the best forecasting performance is the 4-state NHHM for all coins. More specifically, we observe that the 4-state NHHM model delivers the best predicting performance, since it has the lowest CRPS for all the randomly chosen out-of-sample points, with the exception of the 30-th point in the BTC forecasting exercise and the 1-st point of the ETH forecasting exercise where the 5-state HHM and the MS-RW models have better forecasting accuracy for the two coins respectively. However, by collating the resulting MSEs for each point individually with the CRPS, we observe that in some forecasting horizons different models are found to outperform the best models derived by the CRPS. Even though this might seem contradictory, these differences are expected since the CRPS is more robust to outliers and more reliable when assessing the density forecasts, see also Koki et al. (2020) and references therein. Over all coins, the lowest CRPS and the lowest average MSE (best predicting accuracy) are achieved when predicting the BTC return series and the highest CRPS and the highest average MSE (worst predictive accuracy) are achieved when predicting the ETH series.

This forecasting exercise provides empirical evidence that relaxing the hypothesis of constant transition probabilities by allowing the predictors to affect the series non-linearly — through the latent process — improves the forecasting accuracy of the HM models. This is an indication that, besides the conditional time-varying linear correlations between the cryptocurrency return series and predictor set, there exist more complex correlations, such as the underlying non-linear multinomial logistic relationship which can lead to better forecasts.

3.2. In-sample analysis of the models with the best predicting performance

Based on the forecasting accuracy, we treat the 4-state NHHMs as our final model for further analysis for the BTC, ETH and XRP return series. Within our MCMC algorithm and at each iteration, we estimate an in-sample realization of the observed data, i.e., we use the in-sample estimations of the parameters and the states to reproduce the cryptocurrency percentage log return series, often referred to as replicated data or within-sample predictions, Gelman (2003). The derived realized distributions along with the observed series for each coin are shown in Figure 3. The left column shows the in-sample replicated distributions derive using the aforementioned 4-state NHHM and the right column
| Horizon | 1     | 2       | 7       | 15      | 30      | Average |
|---------|-------|---------|---------|---------|---------|---------|
| NHHM_2  | 0.91  | 0.45    | 0.65    | 0.82    | 0.64    | 1.86    |
|         | (8.79)| (8.20)  | (13.30) | (17.07) | (14.80) | (28.80) |
| NHHM_3  | 1.00  | 0.40    | 0.68    | 0.88    | 0.61    | 1.85    |
|         | (8.47)| (8.82)  | (14.13) | (19.42) | (16.04) | (29.26) |
| NHHM_4  | 0.71  | 0.32    | 0.58    | 0.78    | 0.60    | 1.78    |
|         | (7.89)| (6.37)  | (13.58) | (15.62) | (15.54) | (28.00) |
| NHHM_5  | 0.96  | 0.60    | 0.91    | 0.90    | 0.62    | 1.85    |
|         | (15.33)| (14.27) | (25.92) | (18.97) | (15.72) | (30.41) |
| HHM_2   | 0.91  | 0.52    | 0.66    | 0.86    | 0.67    | 1.87    |
|         | (9.67)| (9.33)  | (14.61) | (17.19) | (16.00) | (29.36) |
| HHM_3   | 0.93  | 0.57    | 0.78    | 0.83    | 0.56    | 1.87    |
|         | (15.60)| (15.67) | (18.85) | (17.73) | (15.72) | (30.02) |
| HHM_4   | 0.97  | 0.62    | 0.85    | 0.81    | 0.53    | 1.87    |
|         | (14.61)| (13.86) | (23.06) | (16.73) | (14.85) | (29.43) |
| HHM_5   | 0.91  | 0.49    | 0.75    | 0.86    | 0.62    | 1.85    |
|         | (14.26)| (12.92) | (20.82) | (15.13) | (15.72) | (29.20) |
| MS-RW   | 0.95  | 0.53    | 0.60    | 0.81    | 0.62    | 1.85    |
|         | (9.80)| (8.93)  | (14.00) | (16.05) | (15.50) | (29.50) |
| KS      | 1.10  | 0.95    | 0.90    | 1.09    | 0.92    | 1.92    |
|         | (17.05)| (15.67) | (15.52) | (19.19) | (17.90) | (29.72) |
| AR(5)   | 1.12  | 0.92    | 0.95    | 0.99    | 0.97    | 1.95    |
|         | (17.50)| (15.35) | (15.69) | (16.20) | (18.12) | (30.09) |
| RW      | 1.20  | 1.00    | 1.05    | 1.06    | 0.96    | 1.98    |
|         | (15.80)| (18.50) | (14.89) | (16.15) | (18.85) | (31.12) |

**Table 3:** Continuous Rank Probability Score and Mean Squared Error in parenthesis for all the competing models for the Bitcoin, Ether and Ripple series. The last column reports the average CRPS (MSE) over the whole sequence of 30 one-step ahead predictions. Bold values indicate the lowest CRPS values for each out-of-sample points.

shows the replicated distributions derived using the RW benchmark for all coins. Gray lines show the observed percentage log-return series, yellow lines show the fitted 0.5%-th and 99.5%-th quantiles of the estimated in-sample distribution and the red line shows the 50%-th quantile (median). By visual inspection, we observe that, by identifying the various volatility clusters, the 4-state NHHM models offer substantially improved in-sample performance compared to the in-sample performance of the RW model. The graphical proof of the good in-sample performance of the 4-state NHHM compared to the RW model is substantiated by Table B which shows the overall empirical coverage of the estimated quantiles curves. The first and second rows report the proportion of the observed percentage log-returns that fall out of the empirical quantiles curves for the 4-state NHHM, and the in-sample MSE, respectively, for the 4-state NHHM, while the third and fourth rows report the proportion and in-sample MSE for the RW model.

Table 5 provides a gauge of what drives the documented predictability by showing the posterior
Figure 3: Percentage return series (gray lines) and quantiles of the posterior sample (replicated) empirical distributions for the Bitcoin series (first row), Ether series (second row) and Ripple series (third row). Yellow lines show the 0.5% and 99.5% quantiles of the estimated in-sample distributions and define the 1% credibility region, whereas red lines show the estimated posterior median. Plots on the left are based on the estimated distributions via the 4-state Non-Homogeneous Hidden Markov (HM) model while plots on the right show the estimated distributions as derived from the Random Walk (RW) benchmark model.

![Graphs showing percentage return series for Bitcoin, Ether, and Ripple](image)

Table 4: Empirical coverage of the empirical in-sample distributions using the 4-state NHHM and the RW benchmark for the Bitcoin, Ether and Ripple percentage return series.

|                      | Bitcoin     | Ether       | Ripple      |
|----------------------|-------------|-------------|-------------|
| NHHM                  |             |             |             |
| Proportion           | 0.05 (121/2114) | 0.06 (95/1506) | 0.04 (79/1748) |
| MSE                  | 15.18       | 37.13       | 43.22       |
| RW                   |             |             |             |
| Proportion           | 0.75 (1601/2114) | 0.74 (1114/1506) | 0.72 (1255/1748) |
| MSE                  | 17.03       | 42.28       | 50.01       |

Table 4: Empirical coverage of the empirical in-sample distributions using the 4-state NHHM and the RW benchmark for the Bitcoin, Ether and Ripple percentage return series.

Mean estimates for the linear regression predictors for each state for BTC, ETH and XRP respectively. Predictors that fall into the 10% credibility intervals are marked with asterisk. In addition to the linear regression estimates, the last column of Table 5 shows the posterior probabilities of inclusion for the predictors affecting the transition probabilities, as derived from the stochastic search algorithm on the multinomial regression coefficients. Posterior probabilities of inclusion exceeding 0.4 are marked with bold fonts.

We observe that the majority of the predictors are not statistical significant in the linear regression parametrization, especially for the BTC return series. At this point, it is important to stress that even when the coefficient of an explanatory variable is not statistically different from zero, this does not necessarily mean that the variable has no predictive power for return series, see also Panopoulou and Pantelidis (2015). It is often the case that a variable that is insignificant in-sample has predictive out-of-sample power and vice versa. This argument is strengthened by our extensive experimental study — results not reported here — which shows that if we remove the insignificant predictors, the forecasting accuracy deteriorates. Furthermore, we observe that depending on the hidden state the mean posterior estimates can be markedly different, even change their sign. Regarding the predictors affecting the transition probabilities, we observe that there exist predictors that affect the transition probabilities with high posterior probabilities of inclusion, with the volatility index VIX and Treasury Yield (TY) affecting the transition probabilities for all the cryptocurrency return series.
Table 5: Posterior means estimates of the 4-state Non-Homogeneous Hidden Markov model for the Bitcoin, Ether Ripple percentage return series. The first column specifies the predictors. The second, third, fourth and fifth columns report the posterior mean estimates for each predictor at the first, second and third states respectively. The last row reports the mean estimated residual variance for each state. The last column reports the posterior probabilities of inclusion for the predictors affecting the transition probabilities’ multinomial regression model. These probabilities are calculated by applying a stochastic search reversible jump algorithm within the MCMC scheme. Statistical significance at the 10% level is denoted with * and posterior probabilities exceeding 0.4 are marked with bold fonts.

### BTC

| Predictors | State 1 | State 2 | State 3 | State 4 | Probabilities |
|------------|---------|---------|---------|---------|---------------|
| Intercept  | 11.47   | 6.65    | −0.80   | 0.02    | 0.00          |
| EUR/USD    | 1.02    | 0.43    | −0.15   | −1.59   | 0.00          |
| GBP/USD    | 0.06    | 0.17    | −0.04   | −1.26   | 0.00          |
| CNY/USD    | −0.14   | −0.12   | −0.15   | −1.30   | 0.00          |
| JPY/USD    | −0.37   | 0.08    | −0.16   | −0.78   | 0.00          |
| Gold       | −0.09   | −0.12   | 0.07    | 0.01    | 0.00          |
| Oil        | 0.02    | 0.14    | 0.01    | 0.52    | 0.00          |
| SP500      | −6.73   | −0.85   | 0.32    | 0.08    | 0.00          |
| VIX        | −4.37   | −2.34   | 0.30    | 0.03    | 1.00          |
| TY         | −2.07   | −2.21   | 1.49    | 0.15    | 0.90          |
| Size       | −0.01   | −0.01   | 0.01    | 0.13    | 0.00          |
| Hash       | −0.01   | −0.01   | 0.00    | 0.26    | 0.00          |
| AR(1)      | −0.04   | 0.00    | 0.03    | 0.94*   | 0.00          |

### ETH

| Predictors | State 1 | State 2 | State 3 | State 4 | Probabilities |
|------------|---------|---------|---------|---------|---------------|
| Intercept  | 7.68    | 1.93    | 0.85    | 0.80    | 1.00          |
| EUR/USD    | −0.14   | 0.10    | 0.64    | 0.81    | 1.00          |
| GBP/USD    | 1.78    | 0.51*   | 2.35    | 2.14    | 0.00          |
| CNY/USD    | −0.40   | −0.24   | −0.18   | 0.31    | 0.00          |
| JPY/USD    | −0.14   | 0.31    | 1.49    | 1.54    | 0.00          |
| Gold       | −0.12   | 0.32*   | 1.17    | 1.26    | 0.82          |
| Oil        | −2.02*  | 0.26*   | 0.24    | 0.42    | 1.00          |
| SP500      | −9.14   | −3.52   | 1.62    | 1.91    | 0.11          |
| VIX        | −2.54*  | −0.78*  | −1.01   | −0.78   | 1.00          |
| TY         | −13.45  | 3.75    | 1.45    | 0.31    | 0.84          |
| Size       | 0.07    | 0.21    | 0.10    | 0.03    | 0.00          |
| Hash       | 0.03    | 0.00    | 0.17    | 0.12    | 0.00          |
| AR(1)      | −0.06   | −0.17*  | 0.74    | 0.82    | 0.00          |

### XRP

| Predictors | State 1 | State 2 | State 3 | State 4 | Probabilities |
|------------|---------|---------|---------|---------|---------------|
| Intercept  | 9.30    | 0.25    | 0.87    | 0.96    | 1.00          |
| EUR/USD    | −1.88*  | −0.19   | 0.25*   | 0.07    | 1.00          |
| GBP/USD    | −4.03   | −0.04   | −0.25   | −0.85   | 0.01          |
| CNY/USD    | 3.31    | 0.26    | 0.13    | 0.48    | 0.02          |
| JPY/USD    | 1.36    | 1.13    | 0.02    | 2.12    | 0.70          |
| Gold       | −1.44   | −0.10   | −0.01   | −0.62   | 0.10          |
| Oil        | 0.44    | −0.07   | −0.09   | 0.17*   | 0.06          |
| SP500      | 2.00    | −7.06   | −0.87   | 0.13    | 0.40          |
| VIX        | −1.88   | −0.19   | 0.25    | −0.85   | 1.00          |
| TY         | −22.46  | 1.29    | 3.68    | −0.08   | 0.52          |
| Size       | 0.07    | 0.21    | 0.10    | 0.03    | 0.00          |
| AR(1)      | 0.44    | −0.08   | −0.10*  | 0.18    | 0.00          |

#### 3.3. Hidden States classification and interpretation

Table 5 provides information on the hidden states for each coin; that is, the states’ occupancies as the average time spent at each state $i$, $i = 1, \ldots, 4$, the average returns and the corresponding standard deviation. At a first glance, the hidden process identifies periods with different underlying volatilities for every coin, i.e., periods with high and low volatilities. In more detail, for the BTC series, it identifies periods with negative average returns and high volatilities (state 1), periods with positive returns and...
low volatility (states 2 and 3) and calm periods with average returns close to zero and very low volatility. This segmentation in the return series resembles the bear/turbulent (state 1) and bull (states 2 and 3) markets, while state 4 corresponds to a stable/calm regime. Furthermore, we observe that the states 2 and 3 have similar (almost equal) average returns. The similar average returns and different volatility indicate that the hidden process segments the return series into two subseries with the same skewness but very different kurtosis, see [Timmermann (2000)](#).

Concerning the ETH series, we observe that the highest mean returns occur in the state with the highest volatility. The hidden process alternates between a high volatility and a low volatility regime with almost zero average returns — states 1 and 2 respectively — for the 95% of the overall time, while states 3 and 4 serve as auxiliary states with almost equal average returns but different volatilities. Finally, the hidden process in the XRP series spends most of the time (80%) in the high volatility regime 2 and the low volatility regime 3. We also observe that state 1 has extremely high average returns compared to the returns of states 2 and 3 but is associated with very high risk (high volatility) as a trade-off. Lastly, hidden state 4 serves as an auxiliary state with low occupancy, capturing the extreme values (outliers) of the returns series.

| Coin | State | Occupancies | Average | Std |
|------|-------|-------------|---------|-----|
| BTC  | 1     | 0.35        | -0.46   | 5.91|
|      | 2     | 0.30        | 0.49    | 2.49|
|      | 3     | 0.24        | 0.47    | 1.69|
|      | 4     | 0.11        | 0.02    | 0.53|
| ETH  | 1     | 0.41        | 0.88    | 8.90|
|      | 2     | 0.54        | -0.06   | 2.61|
|      | 3     | 0.02        | 0.16    | 1.13|
|      | 4     | 0.02        | 0.18    | 0.76|
| XRP  | 1     | 0.17        | 3.59    | 14.98|
|      | 2     | 0.35        | 0.76    | 4.46|
|      | 3     | 0.45        | 0.41    | 1.77|
|      | 4     | 0.03        | 6.82    | 0.86|

Table 6: Information on the states as derived from the experiments on the BTC, ETH, XRP return series. First column reports the cryptocurrencies and the second column the different regimes. The third column reports the states’ occupancies, i.e., the average time spend at each regime. The fourth column reports the average returns at each state and finally, the fifth column reports the state’s estimated standard deviation.

The information in Table 6 can be visualized in Figures 4 and 5 which depict the state-switching dynamics of the three cryptocurrencies according to their hidden state classification. Figure 4 illustrates the Bitcoin (upper plot), Ether (middle plot) and Ripple (lower plot) returns conditionally on the realization of the hidden process using the 4-state NHHM model. Gray lines correspond to the percentage log returns, while red, yellow, purple and green dots correspond to the times that the hidden state was in states 1,2,3 and 4, respectively. These graphical representations serve as an easy way to visualize the evolution of the hidden process in reference with the returns for each coin. While frequent alternations between the hidden states are prevalent in all three time-series, the transitional patterns are markedly different. For instance, in the BTC return series, there exist frequent alternations between states 1 and 3 and between 2 and 4, while in the ETH and XRP series they are between states 1 and 2 and between 2 and 3, respectively.

Figure 5 shows the estimated ex ante smoothed probabilities of each state for each time period, i.e., $P(Z_t = m | y_1, ..., y_T)$, $m = 1, \ldots, 4$, over all coins. We observe that hidden state 1 for the BTC series, hidden states 1 and 2 for the ETH series and hidden states 1,2 and 3 for the Ripple series occur with high probability. The identification of these particular hidden states is sound with low probabilities of misclassification. However, we get mixed insights on the occurrence of the other states which are neither high nor low.

4. Discussion

In this work, we modeled the return series of the three largest — in terms of market capitalization — cryptocurrencies, Bitcoin, Ether and Ripple under a Hidden Markov framework. The main motivation of this study relies on the results of recent studies that indicate the existence of structural breaks and
regime/states switches in cryptocurrency series. We, therefore, employed a multi-state Bayesian Hidden Markov methodology with a predefined set of financial and cryptocurrency specific predictors to capture the time-varying characteristics, stylized facts and heteroskedasticity of the cryptocurrencies’ return series.

In line with the literature, we chose the best model among 9 different Hidden Markov models — the standard Markov-Switching Random Walk (MS-RW) model, the Homogeneous Hidden Markov (HHM) and the Non-Homogeneous Hidden Markov (NHHM) models with up to five hidden states — and 3 single regime models — the Random Walk (RW) model, the linear AutoRegressive (AR) model and the Kitchen Sink (KS) model — based on their out-of-sample predictive ability.

The out-of-sample forecasting exercise revealed that the 4-states NHHM model has the best fore-
casting performance for all three series, with significantly improvements over the single regime models. The 4-states NHMM segments the series into four subseries with distinct state-switching dynamics with clear economic interpretation. Considering the Bitcoin return series, we find that the most frequently occurring state 1 corresponds to a bear regime (i.e., negative returns and high volatility), states 2 and 3 correspond to a bull regime (positive returns and low volatility) but with different kurtosis and state 4 corresponds to a calm regime (returns close to 0 and low volatility). Regarding the Ether return series, we observe frequent alternations between the high volatility state 1 and the low volatility state 2, while the states 3 and 4 serve as auxiliary states with low occupancies. Lastly, state 1 of the Ripple series corresponds to periods with extremely high average returns but, as a trade off, also with high risk. States 2 and 3 are the states with the highest occupancies, while state 4 serves as an auxiliary state.

Finally, our results enhance our understanding and prediction of cryptocurrency returns by identifying predictors, such as the US Treasury Yield and VIX with predictive power on all three return series. In line with the existing literature, our results show that we are still unable to find model or a predictor that systematically provides reliable forecasts regarding the specific cryptocurrency series, the out-of-sample forecasting period and forecasting evaluation criteria and therefore investors, fund and portfolio managers, and policy-makers ought to be cautious when using forecasts to make their decisions.

Conflict of interest

The authors declare that they have no conflict of interest.

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