Effect of possible stronger neutrino interaction at $E_{\nu\mu} \sim 10^{11}$ GeV upon the extraction of $\sin^2 \theta_W$ from the neutral-current cross section at NuTeV

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Abstract

The possibility exists that cosmic-ray neutrinos with energies of $\sim 10^{20}$ eV interact in the atmosphere with a cross section at the millibarn level, giving rise to some of the highest-energy air showers. In a specific dynamical model which can give rise to such a cross section, we show that there can be a small effect upon the extraction of the effective value of $\sin^2 \theta_W$ from $\nu_\mu$-hadron, neutral-current cross section data at NuTeV.

The possibility that neutrinos, with energies of the order of $10^{20}$ eV, have stronger interactions with hadrons than those which are given by electroweak theory, has been considered, specifically in connection with the puzzling observation of the highest energy cosmic-ray air shower events.\[1, 2\] Such energetic neutrinos can propagate from sources at cosmological distances (red shifts $\gtrsim 1$), undisturbed by the cosmic microwave background radiation, which photons largely prevent protons of $\gtrsim 0.6 \times 10^{20}$ eV from arriving here.\[3, 4\] Recently\[5\] we have explicitly calculated a $\sigma_{\nu\mu}$ of $\sim 10^{-29}$ cm$^2$ at $E_{\nu\mu} \sim 10^{20}$ eV, in a specific dynamical model which involves the hypothesis of a strong, parity-conserving interaction of $\nu_\mu$ with a hypothetical, point-like component of a $\pi^0$, leading to a very massive neutral lepton $L$, with $m_L \sim 2 \times 10^6$ GeV.\[6\] The essential physical idea is that of neutrino structure at very small distances ($\lesssim 10^{-18}$ cm), due to hadron-like coupling to $L$. For this $m_L$, $E_{\nu\mu}$ must reach $\sim 2 \times 10^{21}$ eV for production of real $L$. However, the amplitude for the diagram in Fig. (1), which involves a virtual $L$, results in a cross section above the electroweak $\sigma_{\nu\mu} \sim 10^{-31}$ cm$^2$, already at $E_{\nu\mu} \sim 10^{20}$ eV.\[7\] For a somewhat smaller $m_L \sim 0.3 \times 10^6$ GeV, the production of real $L$ is possible for $E_{\nu\mu}$ above about $0.5 \times 10^{20}$ eV, which is close to where the GZK cut-off\[8\] is located for protons coming from cosmological distances. In this case $\sigma_{\nu\mu}$ can be of the order of millibarns at $E_{\nu\mu} \sim 10^{20}$ eV. Then, a dip at the GZK cut-off, followed by a “bump-structure”\[9\] near to $10^{20}$ eV could occur in the air-shower.

Footnote: The possibility of an analogous hypothetical structure (with a charged pion) for the muon, can lead to a positive-definite, unusual contribution to $(g_\mu - 2)/2$ at the level of $10^{-9}$, for this value of $m_L$.\[10, 11\] However, in addition to the squared coupling, the magnitude is rather sensitive to possible damping at the structure vertex for virtual momentum near to $m_L$, where the integral gets its main contribution. At present, the significance of a possible experimental deviation from expectation is reduced by the correction of a long-standing sign error in the calculation of a “standard” theoretical contribution.\[12, 13\] In this paper, we consider explicitly, only neutrino, effective neutral-current couplings.
This page is filled with complex scientific text discussing neutrino interactions and their implications. It references experimental results and theoretical considerations, including the role of the Z' boson and the search for neutrino oscillations.

For example, it mentions the NuTeV experiment, which found a sin^2(θ_W) value that was 2% higher than the expected electroweak value. This result suggests that there may be deviations from the standard model at very high energy scales.

The text also discusses the possibility of dark matter and its potential detection through neutrino interactions. It mentions the constraints on the Z' boson mass and the significance of resonant interactions with CMB neutrinos.

Overall, the document is a detailed exploration of neutrino physics, including experimental results, theoretical models, and the implications for our understanding of the universe.
ble line \((j^0)\) in Figs. (2b,c)). In Fig. (2c), two vertices (with strength magnitude \(|gg'| \sim g^2\) and the intermediate state on the neutrino line, characterized by the very high mass \(m_L\), are brought together to an effective form \((g^2/m_L)(\bar{\nu}_\mu \gamma_5 \nu_\mu)/(\bar{q} \gamma_5 \gamma_3 q)\). Similarly for the (coherent) intermediate states in the partonic “black box” on the quark line in Fig. (2b), with an assumed real part for the effective vertex. An effective Lagrangian can be approximately represented by the form

\[
\mathcal{L}_{\text{eff}} = - \left( \frac{\sqrt{g^2}}{m_L m} \right) (\bar{\nu}_\mu \gamma_5 \nu_\mu) (\bar{q} \gamma_5 \gamma_3 q)
\]

where \(q\) represents a \(u\) or \(d\) quark, and \(m\) is an effective mass given by the product of significant numerical factors of \(\pi\) (see Eq. (4) below) and a dynamical mass \(\sim 3\) GeV which characterizes the “black box”. The negative sign before \(\mathcal{L}_{\text{eff}}\) (for \(q = u\)) corresponds to destructive interference of the amplitude with the standard-model amplitude from Fig. (3). The negative sign can arise from the structure of the intermediate states in Fig. (2) and the overall sign of the product of effective vertices. This sign appears to arise naturally when the hadronic black box in Fig. (2b) involves a hard parton scattering (gluon exchange). The order of magnitude of the ratio of the amplitude from Fig. (2c) to that from Fig. (3) is

\[
|R| \sim \left( \frac{g^2}{e^2} \right) \left( \frac{m_z}{m_L} \right) \left( \frac{m_z}{m} \right) \sim 0.002
\]

for \(g^2/(4\pi) \sim 1\), \(m_L \sim 0.3 \times 10^6\) GeV, \(e^2/(4\pi) \approx 1/137\), \(m_z \approx 90\) GeV and with \(m \sim (20\pi^3)(3\) GeV) \(\sim 1.8 \times 10^3\) GeV (calculated in Eq. (4) below).

In Eq. (1), the effective strength is

\[
\left( \frac{\sqrt{g^2}}{m_L m} \right) \sim 0.33 \times 10^{-7} \approx \left( \frac{G_F}{\sqrt{2}} \right) \frac{(0.01)}{2}
\]

The value \(\sim (G_F/\sqrt{2})(0.01)\) has been deduced \(\text{[3]}\) phenomenologically as the approximate necessary strength of an effective four-fermion interaction which is, a priori, required to be \(SU(2)_L\) invariant (we use only a corresponding parity-conserving, axial-axial term involving \(\bar{\pi}_\nu\) and \(\bar{\pi}_w(\bar{\nu}-\bar{d})\)). In the present model, the effective strength in Eq. (3) is directly fixed by the hypothesis of relatively strong, parity-conserving neutrino interactions (thus \(g^2/4\pi \sim 1\) compared to \(e^2/4\pi\), at \(E_\nu \sim 10^{30}\) eV (corresponding to \(m_L \lesssim 10^6\) GeV). The effective mass \(m\) is determined by the dynamical structure of the model, as we now estimate. The neutrino cross section (here assumed to be relevant for p or n in an isoscalar target), calculated approximately from the amplitude in Fig. (2b), is

\[
\Delta \sigma_{\nu_\mu} \sim (g^2)^2 \left\{ \left( \frac{1.22}{(4\pi)^2} \right) \left( \frac{\langle n \rangle}{2\pi^2} \right) \left( \frac{4}{3\pi} \right) \left( \frac{s}{4m_L^2} \right) \right\} \left\{ \left( \frac{4}{3\pi} \right) \left( \frac{s}{4m_L^2} \right) \right\} \sim (8\pi^2)^2 \left( \frac{1}{(1.8\text{ TeV})^2} \right) (4.7 \times 10^{-10})
\]

\[
\sim (0.78 \times 10^{-31}\text{ cm}^2)(4.7 \times 10^{-10}) \approx 0.37 \times 10^{-40}\text{ cm}^2
\]

In order to make clear the origin of the magnitude in this dynamical model, three separate factors are enumerated in detail in Eq. (4). There is the overall strong strength from the two interactions on the neutrino line, represented by the magnitude \(|gg'| \sim g^2 (g^2/4\pi \sim 2\), is used, as in ref. 5). The second factor in curly brackets \(\{\ldots\}\) is kinematical, essentially the phase space. At \(\sqrt{s} \sim 20\) GeV, used in Eq. (4) for illustration, this is a small number of the order of \(10^{-9}\). This number
is controlled by the large mass $m_L \sim 0.3 \times 10^6 \text{GeV}$, whose size is approximately fixed by the possibility of strong neutrino interactions at extremely high energy, $\sim 10^{11} \text{GeV}$. The first factor in curly brackets represents the (inverse) square of an effective mass $m \sim (20 \pi^3) (3 \text{GeV})$; this controls the effective strength of the hadronic “vertex”. The mass has a large kinematic factor ($\sim 20 \pi^4$) which is related to the specific dynamics, i.e., to the integration over virtual momenta, and to the approximate parameterization of the “black box” in Fig. (2b), in terms of a measured cross section for a hard scattering process. For the latter we use a typical jet cross section at relatively low $\sqrt{s}$, $\sigma_j \sim 2 \times 10^{-29} \text{cm}^2$ in a central rapidity interval $|\Delta y| \sim (1 - (-1)) = 2$. We use this as a (fixed) parameter only in a limited range of the integration momentum $k_j \sim 2 \text{GeV}$ to $\sim 3.7 \text{GeV}$; $\sigma_j$ falls away at high loop momentum. With the jet multiplicity $\langle n_j \rangle \equiv 2$, the quantity $((n_j)\sigma_j)^{-1/2}$ translates into a mass of $\sim 3 \text{GeV}$, which can be considered as the intrinsic, mass parameter characteristic of the “black box” in Fig. (2b).

Using the estimate for $\Delta \sigma_{\nu_\mu}$ in Eq. (4), and the empirical $\nu_\mu$ charged-current cross section $\sigma_{\nu_\mu}^{\text{CC}} (E_{\nu_\mu} \sim 200 \text{GeV}) \equiv 1.36 \times 10^{-36} \text{cm}^2$, we calculate the magnitude of the interference term between the amplitudes from Figs. (2,3), in ratio to $\sigma_{\nu_\mu}^{\text{CC}}$,

$$|I| \equiv 2 \sqrt{\frac{\Delta \sigma_{\nu_\mu}}{\sigma_{\nu_\mu}^{\text{CC}}}} \sqrt{\frac{\sigma_{\nu_\mu}^{\text{NC}}}{\sigma_{\nu_\mu}^{\text{CC}}}} \approx 2(0.0052)(0.55) = 0.0057$$

This is approximately independent of the specific $E_{\nu_\mu}$ used above for illustrating the numbers. In Eq. (5), we use $\sim 0.29$ for the ratio of neutral to charged-current cross sections (i.e., as calculated for $(\sin^2 \theta_W)_{\text{LEP}} = 0.223$). A negative interference term of this magnitude corresponds to an effective value of $\sin^2 \theta_W$ in the NuTeV experiment which is about 0.005 larger than the (electroweak) value determined from the LEP measurements.

From $p\rho \to l^+l^- + X$ interactions, CDF has set limits on quantities analogous to $\epsilon \sim 0.01$ in Eq. (3), but referring to effective four-fermion interactions involving quarks and a $\mu^- \mu^+$ (or $e^- e^+$) pair, (approximately, the $|\varepsilon|$ are $\lesssim 0.04$). It is noteworthy that the hypothetical, specific $\nu_\mu$ structure in Fig. (2) does not have a counterpart for $\mu^-$. This is because the scalar “jet” is neutral. Thus, the experimental limits from $q\overline{q} \to \mu^- \mu^+$ are not directly relevant here (as they are for phenomenological interaction forms utilized in ref. 13).

There is a potentially interesting “charge” radius effect for $\nu_\mu$, implied by the hypothetical structure. If $\rho^0$-exchange can occur in place of the $A^0$ in Fig. (2c), a charge radius is generated by replacing the lower vertex by that for $\gamma \to \rho^0 \pi^+ \pi^-$. Thus, the magnitude of the interference term between the amplitudes from Figs. (2,3), in ratio to $\sigma_{\nu_\mu}^{\text{CC}}$,

$$\big|\langle r^2 \rangle_{\nu_\mu}\big| \sim \frac{g^2}{64 \pi^2} \left( \frac{1}{\mu m_L} \right) \sim 1.6 \times 10^{-35} \text{cm}^2$$

The mass parameter $\mu \sim 264 \text{MeV}$, is estimated from the measured partial width $\Gamma(\rho^0 \to \pi^+ \pi^- \gamma) \sim 1.5 \text{MeV}$. Empirically, $|\langle r^2 \rangle_{\nu_\mu}| \lesssim 10^{-32} \text{cm}^2$. More generally, this represents a limit on certain non-standard contributions to $\nu_\mu$ scattering; $\Delta \sigma_{\nu_\mu}$ in Eq. (4) is much less. At the Z-pole, a similar “charge” radius can affect, in principle, the magnitude of the axial-vector (and possibly, vector) effective coupling

$^F_6$ Note that a corresponding typical cross section for $\pi^0$, at low $\sqrt{s}$ in this limited range for transverse momenta, is comparable, about $0.4 \times 10^{-30} \text{cm}^2$. It falls away more rapidly than does $\sigma_j$ at transverse momenta.

$^F_7$ The approximate integration over virtual momenta $Q$, with $\sigma_j$ fixed over a limited range, involves a factor $\ln(Q^2_{\text{max}}/Q^2_{\text{min}})$. This is of order unity; we use $2 \ln(3.7/0.02) \sim 1.22$. If taken as unity, the interference term in Eq. (5) has magnitude $\sim 0.0047$. 

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to $\nu_\mu$. We have, $m_\nu^2 (r^2)_{\nu_\mu} \sim 0.03\%$, which corresponds to a change in this partial width of about $\pm 0.1$ MeV. This means that the effective number of light neutrinos can be less than three. There is $\sim 3\%$ accuracy in the measurement of the $\nu_\mu$-coupling to $Z$ from $\nu_\mu$-electron scattering. The direct measurement of the invisible decay width of the $Z$, is also with $\sim 3\%$ accuracy. There is a fit value with $\sim 0.3\%$ uncertainty.

We summarize the model, the essential hypothesis for which involves the neutrino production by effectively strong interactions, of a very massive neutral lepton at the highest energies. The dynamics involves hypothetical, strong interactions of point-like components of exchanged hadronic entities which couple a neutrino to the heavy lepton $L$; thus, a corresponding effective neutrino substructure at very short distances. The resulting calculated cross section can reach above the millibarn level at $E_\nu \sim 10^{20}$ eV. The enhanced cross section occurs already below threshold, and increases through the threshold for production of real $L$. Clearly, the exchange mechanism in Fig. 1 involves all angular momenta (this is not a unitarity-bounded, S-wave effect, as illustrated in the context of the high-energy model, following Eq. (4) above). At NuTeV $\sqrt{s}$, the relevant effective mass parameter is calculated to be about 4.7 TeV.

In conclusion, we note that while the present NuTeV effect might eventually get an explanation in terms of asymmetries in parton distributions, ideas concerning hypothetical new particles are not necessarily restricted to masses below a few TeV. In the effective four-fermion interaction generated by structure, in Eq. (1), while the overall effective mass parameter is estimated as $\sim \sqrt{m_L g^2} \sim 4.7$ TeV, the particle mass $m_L \sim 300$ TeV, is much higher. This is because the hypothesis is made of a dynamical connection between the possibility of stronger neutrino interaction in the atmosphere, reaching of the order of millibarn cross sections at the highest energies $\sim 10^{11}$ GeV, and a small effect upon $\nu_\mu$-hadron scattering below a few hundred GeV. This relates two presently puzzling phenomena: the cosmic-ray air showers at $10^{20}$ eV and the possible anomalous deviation in the $\sin^2 \theta_W$ extracted from NuTeV data.

### Added note

After completion of this work, authors have called our attention to recent work on large cross sections from brane theory; we include some references. See also \[27\] and \[28\], and the earliest related work \[29\] which we are aware of.

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\[ F^8 \] A limit on the possibility of a small amplitude arising from a hypothetical, point-like component of the pion coupling, to vector bosons, is discussed in the conclusion of \[\[\]. Possible observational consequences are: (1) a partial width for $Z^0 \rightarrow \pi^+ \pi^-$ of the order of an MeV, and (2) a quite small, but negative value, for the $S$ parameter, corresponding to a downward shift in the $Z$ mass of the order of MeV.
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Figure 1: Feynman diagram for a contribution to the interaction cross section of an extremely high energy, cosmic-ray neutrino with an atmospheric nucleon, mediated by a virtual $L^0$. 
Figure 2: For $\nu_\mu$-hadron scattering at $\sqrt{s} \sim 20$ GeV, a schematic sequence leading to the approximate effective four-fermion interaction in Eq. (1), as described in the text and footnotes F4-7. The effective vertices on the neutrino line are assumed to be point-like, up to momenta near to the scale of $m_L$. The loop integration momentum is taken as relevant for jet production, but only a limited range is used in estimating the amplitude for Eq. (4), $2$ GeV $\lesssim |Q| \lesssim 3.7$ GeV, because $\sigma_j$ falls away at high $|Q|$. 
Figure 3: The standard model interaction.