Experimental determination of $m_b(M_Z)$ in DELPHI

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The running mass of the $b$ quark as defined in the $\overline{\text{MS}}$ renormalization scheme, $m_b$, was measured at the $M_Z$ scale using 2.8 million hadronic $Z^0$ decays collected by the DELPHI experiment at LEP. The result is

$$m_b(M_Z) = 2.67 \pm 0.25(\text{stat.}) \pm 0.34(\text{frag.}) \pm 0.27(\text{theo.}) \, \text{GeV}/c^2$$

which differs from that obtained at the $\Upsilon$ scale, by $m_b(M_{\Upsilon}/2) - m_b(M_Z) = 1.49 \pm 0.52 \, \text{GeV}/c^2$. This measurement, performed far from the $b\bar{b}$ production threshold, provides the first experimental observation of the running of the quark masses.

1. Introduction

The quark masses are fundamental parameters of the QCD lagrangian which are not predicted by the theory. Their definition is not unique since quarks are not observed as free particles, but confined inside hadrons. Therefore, the use of dynamical expressions is mandatory in order to determine the masses of the quarks. The perturbative pole mass, $M_q$, and the running mass in the $\overline{\text{MS}}$ scheme, $m_q$, are two definitions among the most frequently used.

In the renormalization procedure of QCD, once the divergent part is subtracted, we deal with finite quantities for the strong coupling constant, $\alpha_s$, and the quark masses, $m_q$. However, these quantities are not fixed parameters, but their actual value depends on the energy scale of the process being considered. In other words, $\alpha_s$ and $m_b$ exhibit a running property.

The running of $\alpha_s$ has been verified many times by different experiments (for a review see [1, 2]). Although the running of the quark masses is a basic to QCD as that of $\alpha_s$, it has never been experimentally tested before. The empirical determination of the quark masses at different scales must be regarded as a fundamental test of QCD. Nowadays, the LEP I data allow measuring the $b$ quark mass at the $M_Z$ scale (far away from the $b\bar{b}$ production threshold), thus testing its running. According to the $m_b$ evolution, dictated by the Renormalization Group Equation (RGE), a change on $m_b$ of $\sim 1.3$ GeV is expected when varying the scale from $M_{\Upsilon}/2$ up to $M_Z$, as $m_b(M_{\Upsilon}/2) \approx 4.16 \pm 0.18 \, \text{GeV}/c^2$ [3].

In $e^+e^- \rightarrow$ hadrons, the three-jet rate is an observable sensitive to the $b$ mass. When testing the universality of $\alpha_s$ by using the three-jet rate, it was observed that due to the mass effects, $b$ quarks radiate a $3$ to $5\%$ less gluons than light quarks do [4]. So, reversing the argument: assuming the $\alpha_s$ universality allows measuring the $b$ quark mass by studying the three-jet rate.

The former Leading Order (LO) calculations of the three-jet cross section in $e^+e^-$ including mass terms [5, 6] are not appropriate in order to evaluate the $b$ quark mass, as these calculations can not discern between $M_b$ and $m_b$. Recently, Next to Leading Order (NLO) expressions for the three-jet rate in $e^+e^-$ are available [7, 8, 9], thus, enabling the measurement of $m_b$.

The observable used in this work is:

$$R^{b\ell}_3(y_c) = \frac{\Gamma^{2\ell_0 \rightarrow b\bar{q}g(y_c)}/\Gamma^{2\ell_0 \rightarrow b\bar{b}}}{\Gamma^{3j}_{3\ell_0} (y_c)/\Gamma^{2\ell_0 \rightarrow b\bar{b}}} = 1 + r_b(\mu) \left( b_0(y_c, r_b) + \frac{\alpha_s(\mu)}{\pi} b_1(y_c, r_b) \right)$$

where $\Gamma^{2\ell_0 \rightarrow b\bar{q}g}$ and $\Gamma^{2\ell_0 \rightarrow b\bar{b}}$ denote the differential three-jet and the total cross sections for $b$ and light quarks ($\ell = u, d, s$), $r_b(\mu) = (m_b(\mu)/M_Z)^2$. The coefficients $b_0$ and $b_1$ are calculated at NLO in reference [8]. $y_c$ is the jet resolution parameter and $\mu$ the characteristic energy scale.
2. Data analysis

The analysis reported was performed over a sample consisting of 2.8 million $Z^0$ hadronic decays recorded by the DELPHI detector at LEP through the years 1992 to 1994.

2.1. Event selection

Those hadronic events that satisfied the flavour tagging (section 2.3) and the jet reconstruction criteria were retained for the analysis.

The jets of the event were reconstructed by means of the Durham jet finding algorithm \cite{10}. By choosing the appropriate $y_c$, it was possible to force the reconstruction of just three jets in every event. Then, a set of quality cuts were applied over each jet (minimum charged multiplicity, enough visible energy, jet direction in the barrel part of DELPHI and overall planar configuration). These conditions had to be simultaneously fulfilled by all three jets.

A total of 1,074,860 and 294,509 events entered in the $\ell$ and $b$ sample respectively.

2.2. Flavour definition

The true flavour of the event was defined as that of the quark coupled to the $Z$ in the $Z \rightarrow q\bar{q}$ vertex, without prejudice to the new flavour production that possibly occurred in the splitting of a bremstrahlung gluon (e.g. $q \rightarrow b\bar{b}$). This is the same criterion adopted in \cite{7}, thus it enables a direct comparison of our result with that NLO calculation.

2.3. Flavour tagging technique

The signed impact parameter of all charged particles in the event was used to set up an algorithm which enables the flavour tagging \cite{11}. The significance of each track was obtained by weighing its impact parameter with its associated error. Thus, a function, $P$, was constructed in order to estimate the probability of having all particles compatible with being generated in the events’ Interaction Point (IP). By construction, light quark events had an uniform distribution of $P$. In the $b\bar{b}$ events, the decay of long lived $B$ hadrons led to particles generated in secondary vertices far away from the IP, besides, biasing $P$ towards low values. Accordingly, $b\bar{b}$ events were selected by requiring $P < 5 \cdot 10^{-3}$ and $\ell\ell$ with $P > 0.2$. The purity attained for the $b$ sample is approximately an 85% and its efficiency a 55%, while for $\ell$ quarks both are about the 80%. The contents on $e\bar{e}$ events in each tagged sample ($b$ and $\ell$) were estimated to be a 10% and a 15% respectively.

On the contrary to the $\ell$ flavour, the $b$ tagging efficiency depends on the number of jets in the event. The mean energy of the $B$ hadrons in a three-jet event is smaller than that in a two-jet event. The same applies to their products and due to the multiple scattering, the error of their impact parameter is larger. Consequently, the probability of having all particles compatible with being produced in the IP is also larger.

The background of the $b$ sample comes mainly from those $\ell$ events having either not identified $V^0$ decays or $\gamma$ conversions in the innermost layers of the detector. The large impact parameter of their products pulls $P$ towards low values.

2.4. Jet rates

The observed three-jet cross section of each tagged sample, $q = b$ and $\ell$, was computed as:

$$ R_{3q}^{\text{obs}}(y_c) = \frac{\Gamma_{Z \rightarrow q\bar{q}}(y_c)}{\Gamma_{\text{tot}}^{Z \rightarrow q\bar{q}}} \frac{1}{R_{3j}^{\text{obs}}} \frac{1}{R_{3q}^{\text{obs}}} $$

The measured value of our observable can be computed simply with the quotient of the reconstructed three-jet rates:

$$ R_{3b}^{\ell,\text{obs}}(y_c) = \frac{R_{3b}^{\text{obs}}(y_c)}{R_{3\ell}^{\text{obs}}(y_c)} $$

This raw value of the observable must be converted into a parton level one which may be compared with the NLO calculations \cite{8}. The correction method accounts for the detector effects, biases introduced in the flavour tagging plus the hadronization effects.

The contribution of each flavour, $R_{3q}^{\text{cor}}$, to the observed three-jet cross section is given by:

$$ R_{3q}^{\text{obs}} = c_q^b \cdot R_{3q}^b + c_q^c \cdot R_{3q}^c + c_q^\ell \cdot R_{3q}^\ell $$

where $c_q^i$ corresponded with the flavour contents for $i = b, c, \ell$ of both tagged samples. These factors were extracted from the simulation.

The reconstruction and parton level three-jet rates of each flavour and sample ($R_{3q}^{\text{obs}}$ and $R_{3q}^{\text{cor}}$...
light quarks: $R_{3q}^i(y_c) = f_{3q}^i(y_c) \cdot g_{3i}(y_c) \cdot R_{3i}^{\text{par}}(y_c)$

where $f_{3q}^i$ corresponds with the factor correcting both: detector acceptance and tagging effects. These factors can be deduced from the modelling of the DELPHI response to the hadronic events. Of course, data sets corresponding to different years of the detector’s operation must be corrected independently. The hadronization factors, $g_{3i}$, can be estimated by comparing the hadron and parton level distribution of the three-jet rate. The impact of the $c$ quark mass in our observable can be safely neglected. As mass effects are proportional to $m_c^2$, the effect of $c$ quarks compared to $b$’s is $(m_c/m_b)^2 \sim 1/10$ times a 10% factor, due to the contents on $c$ flavour of the tagged samples. Hence, the influence of the $c$ mass would be roughly an 1% that of the $b$, i.e. a repercussion below the $0.5\sigma_{\text{obs}}$ in both, numerator and denominator of the observable. Furthermore, the ratio reduces the net effect to negligible levels. According to this, the parton level jet rates of the $c$ quarks are taken equal to those of the light quarks: $R_{3c}^{\text{par}} = R_{3q}^{\text{par}}$. Now, the measured jet rates can be expressed as:

$$R_{3c}^{\text{obs}}(y_c) = A_b(y_c) \cdot R_{3b}^{\text{par}}(y_c) + B_b(y_c) \cdot R_{3t}^{\text{par}}(y_c)$$

$$R_{3q}^{\text{obs}}(y_c) = A_l(y_c) \cdot R_{3b}^{\text{par}}(y_c) + B_l(y_c) \cdot R_{3t}^{\text{par}}(y_c)$$

where $A_q$ and $B_q$ are a redefinition of the original set of parameters: $c_q^i, f_{3q}^i$ and $g_{3b}$. This parametrization allows expressing the corrected observable as:

$$R_{3c}^{\text{bl}}(y_c) = R_{3c}^{\text{par}} = \frac{B_b - B_l \cdot R_{3t}^{\text{bl-obs}}}{A_l \cdot R_{3b}^{\text{bl-obs}} - A_b} + R_{3b}^{\text{par}}$$

The total correction applied to the raw $R_{3c}^{\text{bl}}$ was about the 10%, the bulk of which corresponded to the detector plus tagging effects and an $\sim 1\%$ to the hadronization. The corrected $R_{3c}^{\text{bl}}$ distribution is shown in Figure 1. The experimental points are seen to lay well below 1. So, effectively, $b$ quarks radiate less gluons than light quarks do. In order to quantify this statement, the measurement must be based on a single point, since all data points are highly correlated. Large values of $y_c$ must be avoided in order to keep low the statistical error. Also low values of $y_c$ should be eluded, thus limiting the effects of the higher order terms. Then, $y_c = 0.02$ is chosen and the result is:

$$R_{3c}^{\text{bl}}(0.02) = 0.971 \pm 0.005(\text{stat.}) \pm 0.007(\text{frag.})$$

### 2.5 Error estimation

The correction of the measured $R_{3c}^{\text{bl}}$ induced some systematic uncertainties. The source of systematics considered are the following: fragmentation and simulation.

The fragmentation model uncertainty has been evaluated by the comparison of two models: string fragmentation (JETSET [12]) and cluster decay (HERWIG [13]). Massive samples were used (more than $10^7$ simulated events per generator and flavour). The adopted value of the $R_{3c}^{\text{bl}}$ was the average of those extracted correcting with both models, which differed by about 1%. The fragmentation model uncertainty was taken to be half of difference between them.

The effect of the main fragmentation parameters ($Q_0, \sigma_\eta, \epsilon, a$ and $b$ in JETSET) was studied also with massive statistics ($10^7$ events per flavour and parameter). Each of the above pa-
rameters took the optimum value found in the DELPHI tuning [4] and varied by ±2σ. For yc > 0.01, none of the uncertainties due to the individual parameters exceeded the 2%/os. The total error was obtained by adding quadratically the individual ones, and neglecting the correlation. It represented roughly a 3%/os.

The defects of the DELPHI detector modelling when trying to reproduce the real data were also considered in the analysis and regarded as an additional source of error. The main contribution to this uncertainty came from the limited statistics of the simulated events (4 · 106) with full DELPHI simulation. Thus, the limited knowledge on the purity/background factors, c’i, of the tagged samples entered as the simulation error. This error has a strong dependence on yc as it is linked to the statistics of the three-jet simulated sample. But, for yc = 0.02, it represents a mere 3%/os (already included in the statistical error).

3. The running mass of the b quark

Using the DELPHI result of R3b(yc) and the recent NLO calculations for exactly the same observable [5], the value of the running mass of the b quark at the Z2 scale is found to be:

\[ m_b(M_Z) = 2.67 \pm 0.50 \text{ GeV/c}^2 \]

where the breakdown of the error is listed in figure 3. The theoretical uncertainty has two sources. The first one is the error due to the QCD scale. It has been estimated by varying the scale, μ, in the range: 0.5 ≤ μ/MZ ≤ 2, and represents 0.10 GeV/c². The second one is due to the mass ambiguity. It accounts for the uncertainty of expressing the mass dependence of R3b by either using directly the running mass, m_b, or using the pole mass, M_b, and then run up to M_Z using the RGE. Both practices are licit, but differ on the treatment of the higher order terms (with truncated or resummed expressions). The contribution to the error is 0.25 GeV/c².

The running of m_b is verified when comparing its value at the M_Z with that at the Υ scale. The net change in the b running mass between both scales is (errors added up quadratically):

\[ m_b(M_\Upsilon/2) - m_b(M_Z) = 1.49 \pm 0.52 \text{ GeV/c}^2 \]

This result represents the first experimental evidence of the running property of the b quark mass, in particular, and of the any fermion mass, in general.

REFERENCES

1 S. Bethke, these proceedings
2 S. Martí i García, hep–ex/9704014
3 G. Rodrigo these proceedings
4 DELPHI coll., Phys. Lett. B307 (1993) 221.
5 A. Ballestrero et al. Phys. Lett. B294 (1992) 425.
6 M. Bilenky et al. Nucl. Phys. B439 (1995) 505.
7 G. Rodrigo et al. Phys. Rev. Lett. 79 (1997) 193
8 W. Bernreuther et al. hep-ph/9703305
9 P. Nason and C. Oleari, hep–ph/9705295
10 S. Catani et al. Phys. Lett. B269 (1991) 432.
11 DELPHI coll. Zeit. Phys. C65 (1995) 555
12 T. Sjöstrand Computer Phys. Comm. 82 (1994) 74
13 G. Marchesini et al. Computer Phys. Comm. 67 (1992) 465
14 DELPHI coll. Zeit. Phys. C73 (1996) 11