Supporting Information

Internal gradient distributions: A susceptibility-derived tensor delivering morphologies by magnetic resonance

Gonzalo A. Álvarez, Noam Shemesh, and Lucio Frydman

Supporting Information 1: A formal derivation of the internal gradient distribution tensor

The normalized magnetization arising from an ensemble of non-interacting and equivalent spins under the effects of a sequence of pulses or modulating gradients is

\[ M(t) = \langle e^{-i\phi(t)} \rangle, \]

where the brackets account for an ensemble average over the random phases \( \phi(t) \). For the spin-echo sequences being considered in this work, the average phase \( \langle \phi(t) \rangle \) will be equal to zero. Assuming that the random phase \( \phi(t) \) has a Gaussian distribution [48], \( M(t) = \exp \left\{ -\frac{1}{2} \langle \phi^2(t) \rangle \right\} \), the signal will evidence a decay depending on the attenuation factor \( \beta(t) = \frac{1}{2} \langle \phi^2(t) \rangle \). With most sources of decoherence normalized out by the constant-time, constant-pulses-number, fixed-number-of-gradients nature of the NOGSE sequences assayed [20, 21, 44], we ascribe to diffusion effects as the sole source of this attenuation.

It is then convenient to describe the \( \beta \)-factor in terms of the gradient modulating function \( \vec{G}_{\text{tot}}(t') \) [45–47]:

\[
\beta(TE) = \frac{z^2}{\gamma} \int_{0}^{TE} dt' \int_{0}^{TE} dt'' \vec{G}_{\text{tot}}^\dagger(t') \cdot \langle \vec{r}(t')\vec{r}(t'') \rangle \cdot \vec{G}_{\text{tot}}(t''),
\]

(S.1)

where in the second equation we redefined the gradient modulation function such that \( \vec{G}_{\text{tot}}(t', TE) = 0 \) if \( t' < 0 \) or \( t' > TE \) (i.e., outside the total evolution time range). The evolution is given in terms of a tensorial correlation function reflecting the displacements’ fluctuations \( g(\tau) = \langle \Delta \vec{r}(t')\Delta \vec{r}(t' + \tau) \rangle \); i.e. \( g_{i,j} = \langle \Delta x_i(t')\Delta x_j(t' + \tau) \rangle \) with \( i, j \) representing the spatial axis \( x, y, z \). This correlation function can be related to a diffusion power spectrum \( D(\omega) \) [4, 5, 45, 46] by a Fourier transform: \( \mathcal{F}\mathcal{T}\{g(\tau)\} / \sqrt{2\pi} = D(\omega)/\omega^2 \). In the event of anisotropic diffusion, Eq. (S.1) can thus be recast in its Fourier representation [45–47] as:

\[
\beta(TE) = \frac{z^2}{\gamma} \int_{-\infty}^{\infty} d\omega \vec{G}_{\text{tot}}^\dagger(\omega, TE) \cdot \frac{D(\omega)}{\omega^2} \cdot \vec{G}_{\text{tot}}(\omega, TE),
\]

(S.3)
where \( \vec{G}_{\text{tot}}(\omega, \text{TE}) = \vec{G}(\omega, \text{TE}) + \vec{G}_0(\omega, \text{TE}) \), is the filter function introduced in Eq. (1) of the main text.

Considering the applied gradient modulation \( \vec{G}(t', \text{TE}) \), the internal background gradient modulation \( \vec{G}_0(t', \text{TE}) \), and their respective filter functions \( \vec{G}(\omega, \text{TE}) \) and \( \vec{G}_0(\omega, \text{TE}) \), the argument of the integral defining this attenuation factor can then be expanded as

\[
\vec{G}_{\text{tot}}^\dagger(\omega, \text{TE}) \cdot \frac{\mathbf{D}(\omega)}{\omega^2} \cdot \vec{G}_{\text{tot}}(\omega, \text{TE}) = \left[ \vec{G}_{\text{tot}}^\dagger(\omega, \text{TE}) \cdot \frac{\mathbf{D}(\omega)}{\omega^2} \cdot \vec{G}(\omega, \text{TE}) + \vec{G}_0^\dagger(\omega, \text{TE}) \cdot \frac{\mathbf{D}(\omega)}{\omega^2} \cdot \vec{G}_0(\omega, \text{TE}) \right] + 2\Re \left\{ \vec{G}_{\text{tot}}^\dagger(\omega, \text{TE}) \cdot \frac{\mathbf{D}(\omega)}{\omega^2} \cdot \vec{G}_0(\omega, \text{TE}) \right\}.
\]

This leads to Eq. (2) of the main text, where \( \beta(\text{TE}) = \beta_{\vec{G}^2}(\text{TE}) + \beta_{\vec{G}_0^2}(\text{TE}) + \beta_{\vec{G} \cdot \vec{G}_0}(\text{TE}) \) and the normalized spin magnetization becomes

\[
M(\text{TE}) = M_{\vec{G}^2}(\text{TE}) \times M_{\vec{G}_0^2}(\text{TE}) \times M_{\vec{G} \cdot \vec{G}_0}(\text{TE}).
\]

Assuming a \( \vec{G}(t', \text{TE}) = \vec{G} f(t', \text{TE}) \), involving a strength vector \( \vec{G} \) and a time-dependency \( f(t', \text{TE}) \), then \( \vec{G}(\omega, \text{TE}) = \vec{G} F(\omega, \text{TE}) \) with \( F(\omega, \text{TE}) \) the Fourier transform of \( f(t', \text{TE}) \). The applied gradient diffusion attenuation becomes

\[
M_{\vec{G}^2}(\text{TE}) = \exp \{-\beta_{\vec{G}^2}(\text{TE})\},
\]

where

\[
\beta_{\vec{G}^2}(\text{TE}) = \frac{\gamma^2 G^2}{2} \int_{-\infty}^{\infty} d\omega \frac{D_G(\omega)}{\omega^2} |F(\omega, \text{TE})|^2,
\]

and \( D_G(\omega) = \left[ \vec{G}^\dagger \cdot \mathbf{D}(\omega) \cdot \vec{G} \right] / G^2 \). Likewise, the pure background gradient decay is independent of the applied gradient

\[
M_{\vec{G}_0^2}(\text{TE}) = \exp \{-\beta_{\vec{G}_0^2}(\text{TE})\},
\]

where
\[
\beta_{G, G_0} (TE, \vec{G}) = \frac{\gamma^2 G_0^2}{2} \int_{-\infty}^{\infty} d\omega \frac{D_{G_0} (\omega)}{\omega^2} |F_0 (\omega, TE)|^2 ,
\]
(S.9)
with \( D_{G_0} (\omega) = \left[ \vec{G}_0^\dagger \cdot \mathbf{D} (\omega) \cdot \vec{G}_0 \right] / G_0^2 \), and we have again assumed that \( \vec{G}_0 (t', TE) = \vec{G}_0 f_0 (t', TE) \) and thereby \( \vec{G}_0 (\omega, TE) = \vec{G}_0 F_0 (\omega, TE) \). Finally, the cross-term attenuation will be

\[
\beta_{\vec{G}, \vec{G}_0} (TE, \vec{G}, \vec{G}_0) = \frac{\gamma^2}{2} \vec{G}_0^\dagger \left[ \int_{-\infty}^{\infty} d\omega 2 \text{Re} \left\{ F^\dagger (\omega, TE) \frac{\mathbf{D} (\omega)}{\omega^2} F_0 (\omega, TE) \right\} \right] \cdot \vec{G}_0 = \vec{G}_0^\dagger \cdot \tilde{\mathbf{D}} \cdot \vec{G}_0 ,
\]
(S.10)
where \( \tilde{\mathbf{D}} = \frac{\gamma^2}{2} \int_{-\infty}^{\infty} d\omega 2 \text{Re} \left\{ F^\dagger (\omega, TE) \frac{\mathbf{D} (\omega)}{\omega^2} F_0 (\omega, TE) \right\} \).

Our derivations also assumed that \( \vec{G}_0 \) can be described by a Gaussian distribution. The cross-term contribution to the attenuation factor turns out to be

\[
\beta_{\vec{G}, \vec{G}_0} (TE, \vec{G}, \vec{G}_0) = \vec{G}_0^\dagger \cdot \tilde{\mathbf{D}} \cdot \left\langle \vec{G}_0 \right\rangle + \vec{G}_0^\dagger \cdot \tilde{\mathbf{D}} \cdot \left\langle \Delta \vec{G}_0 \Delta \vec{G}_0 \right\rangle \cdot \tilde{\mathbf{D}} \cdot \vec{G} ,
\]
(S.11)
where \( \Delta \vec{G}_0 = \vec{G}_0 - \left\langle \vec{G}_0 \right\rangle \) and \( \left\langle \Delta \vec{G}_0 \Delta \vec{G}_0 \right\rangle \) is the internal gradient-distribution tensor (IGDT). This second term is always positive, since it is a quadratic term, while the first term depends on the relative sign of the parallel component of \( \left[ \vec{G}_0^\dagger \cdot \tilde{\mathbf{D}} \right] \parallel \) to the background gradient \( G_0 \).

For an isotropic diffusion \( \mathbf{D} (\omega) = D (\omega) \mathbf{I} \), the attenuation factor get the simplified form

\[
\beta_{\vec{G}, \vec{G}_0} (TE, \vec{G}) = \tilde{D}_{iso} \vec{G}_0^\dagger \cdot \left\langle \vec{G}_0 \right\rangle + \tilde{D}_{iso}^2 \vec{G}_0^\dagger \cdot \left\langle \Delta \vec{G}_0 \Delta \vec{G}_0 \right\rangle \cdot \vec{G} ,
\]
(S.12)
where \( \tilde{D}_{iso} = \frac{\gamma^2}{2} \int_{-\infty}^{\infty} d\omega 2 \text{Re} \left\{ F^\dagger (\omega, TE) \frac{D (\omega)}{\omega^2} F_0 (\omega, TE) \right\} \).

As an example on the use of this formalism, we consider the sequence of Fig. 1 of the main text and assume free diffusion to derive Eq. (3-5) of the main text. For free diffusion only the tail of the displacement power spectrum \( D (\omega) \propto 1/\omega^2 \) is important [20, 44]. The purely applied-gradient diffusion term \( M_{G^2} (TE) \) is as derived for a CPMG sequence [44]

\[
M_{G^2} (TE) = \exp \left\{ -\frac{1}{12} \gamma^2 G^2 D_0 \frac{TE^3}{N^2} \right\} ,
\]
(S.13)
where the delay \( x = TE/N \). The pure background gradient decay term is in turn the one that corresponds to a spin-echo modulation [44]

\[
M_{G_0^2} (TE, N) = \exp \left\{ -\frac{1}{12} \gamma^2 G_0^2 D_0 TE^3 \right\} ,
\]
(S.14)
which is independent of $x$. The cross-term signal-decay contribution is calculated from Eq. (S.10) leading to

$$M_{\vec{G},\vec{G}_0}(TE) = \exp \left\{ \frac{1}{4} \gamma^2 \vec{G} \cdot \vec{G}_0 D_0 \frac{TE^3}{N^2} \right\}. \quad (S.15)$$

**Supporting Information 2: sNOGSE/aNOGSE’s: Analytical attenuation expressions for the general case of anisotropic diffusion**

We calculate next the normalized spin signal arising from Eq. (S.5),

$$M^{(s)}_{\text{NOGSE}}(TE) = M^{(s)}_{G_2}(TE) \times M_{G_3}(TE) \times M^{(s)}_{G,G,0}(TE), \quad (S.16)$$

for the symmetric and asymmetric non-uniform gradient spin echo modulations ($^{(s)}$NOGSE) introduced in Fig. 2. As described in the main text,

$$M^{s}_{G_2}(TE) = M^{a}_{G_2}(TE) \quad \text{(S.17)}$$

as a result of

$$F^{s}_{\text{NOGSE}}(\omega, TE) = F^{a}_{\text{NOGSE}}(\omega, TE) \quad \text{(S.18)}$$

in Eq. (S.7). The pure background gradient signal contribution is therefore independent of the applied gradient modulation and direction, providing the same weight for both NOGSE sequences. The cross-term in the attenuation factor for sNOGSE is zero but that for aNOGSE is not, as the products $F^{s}_{\text{NOGSE}}(\omega, TE) F_0(\omega, TE)$ and $F^{a}_{\text{NOGSE}}(\omega, TE) F_0(\omega, TE)$ in Eq. (S.10) are odd and even functions of $\omega$, respectively. This cross-term between the aNOGSE-modulated applied gradient and the background gradient $G_0$ will be

$$\beta_{\vec{G},\vec{G}_0}(TE) = \frac{\gamma^2}{2} \vec{G} \cdot \left[ \int_{-\infty}^{\infty} d\omega 2\text{Re} \left\{ \left( F^{s}_{\text{NOGSE}}(\omega, TE) \right)^\dagger \frac{D(\omega)}{\omega^2} F_0(\omega, TE) \right\} \right] \cdot \vec{G}_0 = \vec{G} \cdot \vec{D} \cdot \vec{G}_0. \quad (S.19)$$

As explained in the main text, the measured spin signal decays for the sNOGSE and aNOGSE sequences as described in Fig. 2e, factor out all non-diffusing sources of decoherence after normalizing them by the single-echo signal [20, 21, 44]. The amplitude of the
NOGSE modulation is then

\[ M_{\text{CPMG}}(TE) / M_{\text{Single-echo}}(TE) = \exp(-\Delta \beta) = \exp\left[-\left(\beta_{\text{CPMG}} - \beta_{\text{Single-echo}}\right)\right], \quad (S.20) \]

where the amplitude contrast of the attenuation factors \( \Delta \beta = \beta_{\text{CPMG}} - \beta_{\text{Single-echo}} \). As the contribution to the attenuation factor that purely depends on the background gradient is independent of the applied gradient modulation its contribution \( \Delta \beta_{G0} \) is null, and the amplitude of the attenuation factors is then

\[ \Delta \beta = \Delta \beta_G + \Delta \beta_{\vec{G} \cdot \vec{G_0}}. \quad (S.21) \]

For the sNOGSE sequence \( \Delta \beta^s = \Delta \beta_G \) as the cross-term is null, and \( \Delta \beta^a = \Delta \beta_G + \Delta \beta_{\vec{G} \cdot \vec{G_0}} \) for the aNOGSE modulation curve. Notice that the contribution of the term that only depends on the applied gradient \( \Delta \beta_G \) is the same for both sequences according to Eqs. (S.17) and (S.18). Then by subtracting \( \Delta \beta^a \) and \( \Delta \beta^s \), the \( \Delta \beta_{\vec{G} \cdot \vec{G_0}} \) cross-term contribution to the amplitude modulation is obtained, where

\[ \Delta \beta_{\vec{G} \cdot \vec{G_0}} = \frac{\gamma^2}{2} \vec{G} \cdot \int_{-\infty}^{\infty} d\omega \, 2Re \left\{ \left[ F_{\text{aNOGSE}}(\omega, TE) - F_{\text{sNOGSE}}(\omega, TE) \right]^{\dagger} \frac{D(\omega)}{\omega^2} F_0(\omega, TE) \right\} \cdot \vec{G}_0 \]

\[ = \vec{G} \cdot \Delta \vec{D} \cdot \vec{G}_0, \quad (S.22) \]

with \( \Delta \vec{D} = \frac{\gamma^2}{2} \int_{-\infty}^{\infty} d\omega \, 2Re \left\{ \left[ F_{\text{aNOGSE}}(\omega, TE) - F_{\text{sNOGSE}}(\omega, TE) \right]^{\dagger} \frac{D(\omega)}{\omega^2} F_0(\omega, TE) \right\} \). Assuming as before a Gaussian distribution for \( G_0 \),

\[ \Delta \beta_{\vec{G} \cdot \vec{G_0}} = \vec{G} \cdot \Delta \vec{D} \cdot \langle \vec{G}_0 \rangle + \vec{G} \cdot \Delta \vec{D} \cdot \langle \Delta \vec{G}_0 \vec{G}_0 \rangle \cdot \Delta \vec{D} \cdot \vec{G}. \quad (S.24) \]

where the first term depends on the relative sign of the parallel component of \( \left[ \vec{G} \cdot D(\omega) \right]_{\parallel} \) to the background gradient \( G_0 \), which depends of the anisotropic restricted-diffusion weighting. Notice that the second term is always positive and it contains the IGDT \( \langle \Delta \vec{G}_0 \Delta \vec{G}_0 \rangle \). This was the expression used to evaluate the results presented in Fig. 4 after being normalized by \( \Delta \beta^s = \Delta \beta_G \) to remove the anisotropic weighting due to restricted diffusion effects. For an isotropic diffusion \( D(\omega) = D(\omega)I \), this gets simplified to

\[ \Delta \beta_{\vec{G} \cdot \vec{G_0}} = \Delta \vec{D}_{\text{iso}} \vec{G} \cdot \langle \vec{G}_0 \rangle + \Delta \vec{D}_{\text{iso}}^2 \vec{G} \cdot \langle \Delta \vec{G}_0 \Delta \vec{G}_0 \rangle \cdot \vec{G}. \quad (S.25) \]

where \( \Delta \vec{D}_{\text{iso}} = \frac{\gamma^2}{2} \int_{-\infty}^{\infty} d\omega \, 2Re \left\{ \left[ F_{\text{aNOGSE}}(\omega, TE) - F_{\text{sNOGSE}}(\omega, TE) \right]^{\dagger} D(\omega) F_0(\omega, TE) \right\} \).
Supporting Information 3: sNOGSE/aNOGSE analytical expressions for the case of free, unrestricted diffusion

For free diffusion only the tail of the displacement power spectrum \( D(\omega) \propto 1/\omega^2 \) will be important, as it is this short-times regime active before restriction effects are seen, that matters \([20, 44]\). The purely applied-gradient diffusion term \( M_{G_2}^{(s)}(TE) \) is then given by \([44] \)

\[
M_{G_2}^{(s)}(TE) = \exp \left\{ -\frac{1}{12} \gamma^2 G_2^2 D_0 \left[ (N - 2)x^3 + 2y^3 \right] \right\},
\]

(S.26)

where \((N - 2)x + 2y = TE_{NOGSE} = TE/2\) (see Fig. 2 of the main text for definitions). The pure background gradient decay term is in turn

\[
M_{G_0}(TE, N) = \exp \left\{ -\frac{1}{12} \gamma^2 G_0^2 D_0 T E^3 \right\},
\]

(S.27)

which is independent of \(x\) and \(y\), and therefore of the applied gradient modulation as was mentioned in the manuscript.

The cross-term signal-decay contribution for sNOGSE is zero as described before, and the one for aNOGSE will be

\[
M_{G_2 G_0}^{sNOGSE}(TE) = \exp \left\{ -\frac{1}{4} \gamma^2 \vec{G} \cdot \vec{G}_0 D_0 T E_{NOGSE} (-1)^{N/2} \left[ 2y^2 - \left( 1 + (-1)^{N/2} \right) x^2 \right] \right\}.
\]

(S.28)

Notice that the sign of the attenuation factor for this cross-term contribution depends of the relative sign of \(G_2 G_0\), where \(G_2\) is the applied component of the \(G\)-gradient that is parallel to the background gradient vector. The extremes of this attenuation arise for \(x = y = T E_{NOGSE}/N\) (CPMG-like modulation)

\[
\beta_{G_2 G_0}^{CPMG}(TE) = -\frac{1}{4} \gamma^2 \vec{G} \cdot \vec{G}_0 D_0 T E_{NOGSE}^3 \frac{T E_{NOGSE}^3}{N^2} \left( (-1)^{N/2} - 1 \right),
\]

(S.29)

and for \(y = T E_{NOGSE}/2\) and \(x = 0\) (single-echo modulation)

\[
\beta_{G_2 G_0}^{Single-echo}(TE) = -\frac{1}{8} \gamma^2 \vec{G} \cdot \vec{G}_0 D_0 T E_{NOGSE}^3 (-1)^{N/2}.
\]

(S.30)

Given the \((-1)^{N/2}\) factor in Eq. (S.28) it follows that if \(N/2\) is even, \(\beta_{G_2 G_0}^{CPMG}(TE) = 0\) and the contrast contribution for the difference of attenuation factors is

\[
\Delta \beta_{G_2 G_0} = \frac{1}{8} \gamma^2 \vec{G} \cdot \vec{G}_0 D_0 T E_{NOGSE}^3;\]

(S.31)
i.e., it depends on the relative sign of $G\parallel$ and $G_0$. If $N/2$ is odd $\beta_{G,G_0}^{CPMG}(TE) \neq 0$; but for $N/2$ odd the attenuation decays with $1/N^2$, and this makes the contrast lower. Assuming a Gaussian distribution for $G_0$,

$$
\beta_{G,G_0}(TE) = \frac{1}{4} \gamma^2 D_0 T E_{NOGSE} (-1)^{N/2} (2y^2 - \left(1 + (-1)^{N/2}\right) x^2) \vec{G} \cdot \langle \vec{G}_0 \rangle \\
+ \frac{1}{32} \gamma^4 D_0^2 T E_{NOGSE}^2 \left[2y^2 - \left(1 + (-1)^{N/2}\right) x^2\right]^2 \left[\vec{G} \cdot \left(\vec{G}_0 - \langle \vec{G}_0 \rangle\right)\right]^2.
$$

The attenuation factor contrast amplitude is then

$$
\Delta \beta_{G,G_0} = \frac{1}{8} \gamma^2 D_0 T E_{NOGSE}^3 \vec{G} \cdot \langle \vec{G}_0 \rangle + \frac{1}{128} \gamma^4 D_0^2 T E_{NOGSE}^6 \vec{G} \cdot \langle \Delta \vec{G}_0 \Delta \vec{G}_0 \rangle \cdot \vec{G},
$$

where $\langle \Delta \vec{G}_0 \Delta \vec{G}_0 \rangle$ is the IGDT.

[1] Callaghan, P. T. *Principles of Nuclear Magnetic Resonance Microscopy* (Oxford University Press, 1993).

[2] Price, W. S. Pulsed-field gradient nuclear magnetic resonance as a tool for studying translational diffusion: Part 1. basic theory. *Concepts Magn. Reson.* 9, 299–336 (1997).

[3] Sen, P. N. Time-dependent diffusion coefficient as a probe of geometry. *Concepts Magn. Reson.* 23, 1–21 (2004).

[4] Grebenkov, D. S. NMR survey of reflected brownian motion. *Rev. Mod. Phys.* 79, 1077–1137 (2007).

[5] Gore, J. C. et al. Characterization of tissue structure at varying length scales using temporal diffusion spectroscopy. *NMR in Biomedicine* 23, 745–756 (2010).

[6] Mitra, P. P., Sen, P. N., Schwartz, L. M. & Le Doussal, P. Diffusion propagator as a probe of the structure of porous media. *Phys. Rev. Lett.* 68, 3555–3558 (1992).

[7] Basser, P., Mattiello, J. & LeBihan, D. *J. Magn. Reson., Series B* 103, 247 (1994).

[8] Basser, P., Mattiello, J. & LeBihan, D. MR diffusion tensor spectroscopy and imaging. *Biophys. J.* 66, 259–267 (1994).

[9] Kukla, V. et al. NMR studies of single-file diffusion in unidimensional channel zeolites. *Science* 272, 702–704 (1996).
[10] Kuchel, P. W., Coy, A. & Stilbs, P. NMR "diffusion-diffraction" of water revealing alignment of erythrocytes in a magnetic field and their dimensions and membrane transport characteristics. *Magn. Reson. Med.* **37**, 637–643 (1997).

[11] Mair, R. W. *et al.* Probing porous media with gas diffusion NMR. *Phys. Rev. Lett.* **83**, 3324–3327 (1999).

[12] Peled, S., Cory, D. G., Raymond, S. A., Kirschner, D. A. & Jolesz, F. A. Water diffusion, $t_2$, and compartmentation in frog sciatic nerve. *Magn. Reson. Med.* **42**, 911–918 (1999).

[13] Song, Y.-Q., Ryu, S. & Sen, P. N. Determining multiple length scales in rocks. *Nature* **406**, 178–181 (2000).

[14] LeBihan, D. Looking into the functional architecture of the brain with diffusion MRI. *Nat. Rev. Neurosci.* **4**, 469–480 (2003).

[15] Song, Y.-Q., Zielinski, L. & Ryu, S. Two-dimensional NMR of diffusion systems. *Phys. Rev. Lett.* **100**, 248002 (2008).

[16] Lawrenz, M. & Finsterbusch, J. Double-wave-vector diffusion-weighted imaging reveals microscopic diffusion anisotropy in the living human brain. *Magn. Reson. Med.* **69**, 1072–1082 (2013).

[17] Hertel, S. A. *et al.* Magnetic-resonance pore imaging of nonsymmetric microscopic pore shapes. *Phys. Rev. E* **92**, 012808 (2015).

[18] Ong, H. H. *et al.* Indirect measurement of regional axon diameter in excised mouse spinal cord with q-space imaging: simulation and experimental studies. *Neuroimage* **40**, 1619–1632 (2008).

[19] Budde, M. D. & Frank, J. A. Neurite beading is sufficient to decrease the apparent diffusion coefficient after ischemic stroke. *Proc. Natl. Acad. Sci. U. S. A.* **107**, 14472–14477 (2010).

[20] Álvarez, G. A., Shemesh, N. & Frydman, L. Coherent dynamical recoupling of diffusion-driven decoherence in magnetic resonance. *Phys. Rev. Lett.* **111**, 080404 (2013).

[21] Shemesh, N., Álvarez, G. A. & Frydman, L. Measuring small compartment dimensions by probing diffusion dynamics via non-uniform oscillating-gradient spin-echo (NOGSE) NMR. *J. Magn. Reson.* **237**, 49–62 (2013).

[22] Assaf, Y. & Basser, P. J. Composite hindered and restricted model of diffusion (CHARMED) MR imaging of the human brain. *Neuroimage* **27**, 48–58 (2005).

[23] Panagiotaki, E. *et al.* Compartment models of the diffusion MR signal in brain white matter:
A taxonomy and comparison. *Neuroimage* **59**, 2241–2254 (2012).

[24] Warren, W. S. *et al.* MR Imaging Contrast Enhancement Based on Intermolecular Zero Quantum Coherences. *Science* **281**, 247–251 (1998).

[25] Sen, P. & Axelrod, S. Inhomogeneity in local magnetic field due to susceptibility contrast. *J. Appl. Phys.* **86**, 4548–4554 (1999).

[26] Faber, C., Pracht, E. & Haase, A. Resolution enhancement in in vivo NMR spectroscopy: detection of intermolecular zero-quantum coherences. *J. Magn. Reson.* **161**, 265–274 (2003).

[27] Pathak, A., Ward, B. & Schmainda, K. A novel technique for modeling susceptibility-based contrast mechanisms for arbitrary microvascular geometries: The finite perturber method. *Neuroimage* **40**, 1130–1143 (2008).

[28] Wharton, S. & Bowtell, R. Whole-brain susceptibility mapping at high field: A comparison of multiple- and single-orientation methods. *Neuroimage* **53**, 515–525 (2010).

[29] Lee, J. *et al.* Sensitivity of MRI resonance frequency to the orientation of brain tissue microstructure. *Proc. Natl. Acad. Sci. U. S. A.* **107**, 5130–5135 (2010).

[30] de Rochefort, L. *et al.* Quantitative susceptibility map reconstruction from MR phase data using bayesian regularization: Validation and application to brain imaging. *Magn. Reson. Med.* **63**, 194–206 (2010).

[31] Li, W., Wu, B., Avram, A. & Liu, C. Magnetic susceptibility anisotropy of human brain in vivo and its molecular underpinnings. *Neuroimage* **59**, 2088–2097 (2012).

[32] Liu, C. & Li, W. Imaging neural architecture of the brain based on its multipole magnetic response. *Neuroimage* **67**, 193–202 (2013).

[33] Chen, W., Foxley, S. & Miller, K. Detecting microstructural properties of white matter based on compartmentalization of magnetic susceptibility. *Neuroimage* **70**, 1–9 (2013).

[34] Liu, C. Susceptibility tensor imaging. *Magn. Reson. Med.* **63**, 1471–1477 (2010).

[35] Lee, J. *et al.* T-2*-based fiber orientation mapping. *Neuroimage* **57**, 225–234 (2011).

[36] Oh, S., Kim, Y., Cho, Z. & Lee, J. Origin of b-0 orientation dependent r*(2) (=1/T*(2)) in white matter. *Neuroimage* **73**, 71–79 (2013).

[37] Liu, C., Li, W., Wu, B., Jiang, Y. & Johnson, G. 3D fiber tractography with susceptibility tensor imaging. *Neuroimage* **59**, 1290–1298 (2012).

[38] Haacke, E., Xu, Y., Cheng, Y. & Reichenbach, J. Susceptibility weighted imaging (SWI). *Magn. Reson. Med.* **52**, 612–618 (2004).
[39] Thomas, B. P. et al. High-Resolution 7t MRI of the Human Hippocampus In Vivo. *J. Magn. Reson. Imaging* **28**, 1266–1272 (2008).

[40] Yao, B. et al. Susceptibility contrast in high field MRI of human brain as a function of tissue iron content. *Neuroimage* **44**, 1259–1266 (2009).

[41] Shmueli, K. et al. Magnetic susceptibility mapping of brain tissue in vivo using MRI phase data. *Magn. Reson. Med.* **62**, 1510–1522 (2009).

[42] Haacke, E. M., Mittal, S., Wu, Z., Neelavalli, J. & Cheng, Y.-C. N. Susceptibility-Weighted Imaging: Technical Aspects and Clinical Applications, Part 1. *Am. J. Neuroradiology* **30**, 19–30 (2009).

[43] Han, S. et al. Magnetic field anisotropy based MR tractography. *J. Magn. Reson.* **212**, 386–393 (2011).

[44] Álvarez, G. A., Shemesh, N. & Frydman, L. Diffusion-assisted selective dynamical recoupling: A new approach to measure background gradients in magnetic resonance. *J. Chem. Phys.* **140**, 084205 (2014).

[45] Stepisnik, J. Time-dependent self-diffusion by NMR spin-echo. *Physica B* **183**, 343–350 (1993).

[46] Callaghan, P. T. & Stepisnik, J. Frequency-domain analysis of spin motion using modulated-gradient NMR. *J. Magn. Reson.* **117**, 118–122 (1995).

[47] Stepisnik, J., Lasic, S., Mohoric, A., Sersa, I. & Sepe, A. Spectral characterization of diffusion in porous media by the modulated gradient spin echo with CPMG sequence. *J. Magn. Reson.* **182**, 195–199 (2006).

[48] Stepisnik, J. Validity limits of gaussian approximation in cumulant expansion for diffusion attenuation of spin echo. *Physica B* **270**, 110–117 (1999).

[49] Zheng, G. & Price, W. Suppression of background gradients in (b-0 gradient-based) NMR diffusion experiments. *Concepts Magn. Reson.* **30A**, 261–277 (2007).

[50] Cho, H., Ryu, S., Ackerman, J. & Song, Y. Visualization of inhomogeneous local magnetic field gradient due to susceptibility contrast. *J. Magn. Reson.* **198**, 88–93 (2009).