On the possible implications of Dark Matter in the rings of Saturn: a conjecture

Alexandre Ciulli\textsuperscript{a} and Sorin Ciulli \textsuperscript{b,1}
\textsuperscript{a}University of Clermont-Auvergne, France
\textsuperscript{b}University of Montpellier, France
E-mail: alexandre.ciulli@hotmail.fr, ciullisori@yahoo.fr

Abstract: In this article we discuss some consequences of the well-known proposition of Fritz Zwicky \cite{1}, published in the nineteen thirties, that Dark Matter ‘mimics’ the inertia-gravitational behaviour of usual matter. In particular, we consider some special dynamical regions such as those of the Ring Systems of the gaseous giants at the edge of the Planetary System. This article is a continuation of an earlier paper \cite{2}, where it was shown that gravitationally interacting particles may remain near the Lagrange Points L4 and L5 for many thousands of years. This provides enough time for the Dark Matter, if present there, to interact with the usual matter. We discuss also a number of questions related to places which might be considered singular in the mathematical sense.

Keywords: Dark Matter inside Solar System, Rings of Saturn.

In memory of Professor Horia Hulubei.

This paper is an extended version of a presentation at a webinar organised by the Romanian Academy November 15, 2021.

\textsuperscript{1}Corresponding author.
1 Introduction. Are the visible Rings of the Giant Planets accompanied by some invisible ones?

Recent findings of the NASA spaceship Cassini [3] have shown that the Rings of Saturn are extremely thin, something like 10 metres across, in comparison to the dimensions of the planet itself and the other nearby astronomical objects. Now according to the daring proposition made by Fritz Zwicky [1] (for many years at the Palomar Observatory, USA) and also of his friend and collaborator, Walter Baade (from the Mount Wilson Observatory), usual matter should be accompanied by a much larger amount of an elusive and invisible material, which should have inertial and gravitational properties similar to usual matter. Further, Jan Henrik Oort [4], Ernst Öpik [5], Vera Rubin [6]) have suggested something similar in order that the galaxies should be stable and do not blow up as a result of the centrifugal forces acting on them – see figure 1 or/and Reference [7]. To explain why this material has not already been seen experimentally, Zwicky and Baade made the simplest possible hypothesis, namely that although this substance might be gravitationally as active as usual matter, electromagnetically it should be absolutely transparent to normal light. So the concept of Dark Matter appeared. Indeed when Astronomers look at the night sky and see just nothing, the sky for them is Dark!

Coming now back to the (visible) Rings of Saturn or to those of the other Gaseous Giants at the edge of our Solar System, according to this conjecture they might be accompanied by some other invisible Rings that share similar, if not identical, positions. Indeed all four planets, Jupiter, Saturn, Uranus and Neptune have some sharp rings around them. Now we should not repeat here the over-simplification which Boltzmann made in the 19th Century in a different context and assume that Dark Matter shares exactly the same position as the usual matter, but might be present in many places in our Solar System. It has been argued that all the Dark Matter around us has been already expelled from our Solar System in the past by some kind of sling effect, like the dust and the other small objects.
swarming in the sky. We do not believe that this is likely to be so, especially in the distant regions of our Solar System where these sling effects which are related to the centrifugal forces are much reduced. Indeed, the very existence of the dust as well as of the other small constituents like pieces of ice and pebbles which can be seen in the images taken by the rocket Cassini [3] in the Rings region, prove that this is not really so. However if there is not enough Dark Matter in Saturn’s Rings region, we can move our whole discussion further, towards the Kuiper Belt. Uranus and particularly Neptune have their own Ring systems which again seem to be extremely sharp. The Kepler velocity around Neptune is approximately 5.4 km/s which is roughly half of the 9.7 km/sec for Saturn (see, for instance, NASA’s planetary data [9]). Since sling effects depend on the centrifugal forces, they are proportional to the square of the velocities (see further the example discussed in the first footnote below), and so the corresponding sling effects will be considerably weaker there.

A direct argument that Dark Matter cannot have been expelled completely by simple sling effects from a region as large as our Planetary System, comes from the existence of the visible Rings themselves, since the visible Rings (NASA, see Cassini [3]) contain small lumps and ice pellets (and powders) which clearly have not been expelled at all. So one could ask why it is that only the Dark Matter has been expelled!

**Figure 1**: Rotation of stars from the Spiral Galaxy NCG 3198 (Van Albada et al. 1985, see [7]). The upper curve with error bars shows the experimental data for the velocities (from Doppler shifts) of the individual stars, while that labelled ‘disk’ represents the Kepler motion one might calculate from the amount of visible matter present in the disk of the galaxy. To recover the observed velocities one has to add an invisible halo containing much more material (Dark Matter) than the visible one. Distances are in kilo-parsecs, velocities in km/sec.
Figure 2: A computed Dark Matter (or usual Matter) particle trajectory around the Lagrange Point L4 of Jupiter [2]. The origins of these symmetry breaking produces the accumulations of loops and knots (extremal points) on the trajectory, which last thousands of years, cannot be attributed to trivial energetic causes, since the trajectory passes above the maximum of the pseudo-potential. They are dynamical consequences of the Coriolis force around L4 and L5.

2 Where Dark Matter might be located

According to what we know about Dark Matter, apart from gravitational effects its intrinsic interaction with usual matter is extremely weak. However, by Zwicky’s conjecture, Dark Matter should have similar inertia-gravitational properties to the usual one and so it may accumulate in various zones of our Solar System, such as in the neighbourhood of some Lagrange points like L4 and L5, where it is known that usual matter also accumulates. See, for example reference [2] where it was shown that the trajectories of usual matter may spend thousands of years in the corresponding Lagrange points of Jupiter (see below figure 2). Hence, according to the ansatz of Zwicky, the dynamical forces that lead to the existence of the visible Rings of Saturn should act in a similar way on Dark Matter, and hence the visible Rings should be accompanied by some totally invisible ones. Hence, Dark Matter could share (almost) identical positions with the visible matter, unless some so far unknown Strong Exclusion Principles forbids this completely. The observational fact that the visible Rings are so thin in comparison with the other cosmological objects that we are used to – 10 metres in some places – suggests that if some Dark Matter is also present there, it should be closely mixed with the usual matter.

There exists also this half not-expressed belief, that in our world new qualities may
Figure 3: Edge-On View of Saturn’s Rings: Cassini’s narrow-angle camera, 2006 (courtesy NASA/JPL-Caltech). Owing to their extreme thinness we may be tempted to call them “singular”, but since they contain observational inaccuracies we shall use the expression ”quasi singular”. The reader should compare their size with that of the Planet.

arise in the neighbourhood of singularities and it as is well known, René Thom [10] has widely advocated these ideas. This is mainly because singularities are usually restricted to small regions of space and hence the various elements have an enhanced opportunity to mix thoroughly.

But who decides when and where singularities occur ? In Physics when you meet an object only a few metres across, while around it are others with dimensions of thousands of kilometres, you may be tempted to call it singular. Anyhow if some Dark Matter were present there, owing to the thinness of the Rings, it would be well mixed with the normal matter. We are in fact in the usual conditions of a Nucleophilic Substitution Reactions $S_N2$, between two substances, where the overall yield is proportional to the product of the densities of the reactants. Owing to the lack of any reliable information on the nuclear interaction lengths between the two kinds of matter, as a first approximation we suppose that the output is proportional to the product of the concentrations.

So far there are no reasonable data concerning the cross sections between these two kinds of matter but, owing to the fact that the Rings are extremely thin, and that their extension is extremely large, by waiting a sufficiently long time we might expect to find some interaction between these constituents. To give some figures on the quantities involved here, we should consider an annulus with an inner radius of approximately 62628 km which is the equatorial radius of Saturn, and an outer radius equal to that of the outermost Ring, estimated at 483000 km. Since the distant Rings F and E have a much lower density, in a first approximation they might be completely neglected, so that we may stop this evaluation much earlier, say, at some 130000 km to consider only the dense Rings (i.e. the Rings A and B, as well as those which are nearest to the Planet, C and D). We will then have to

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Figure 4: A Cassini picture of the rings, in sunset light (courtesy NASA/JPL-Caltech). One can see the waves produced by the shepherd satellites, but also the extremely regular fine lines marked here with dots that are not yet explained in a satisfactory way.

subtract some 10% or 20% from this area since there is a gap between the planet and the first Ring, D, as well as the surface area of the main gaps ENCKE and CASSINI produced by the principal shepherd satellites. The particles produced in these reactions may then be observed by waiting a sufficiently long time using devices placed at the surface of some satellites like Enceladus, where further NASA expeditions seem to be planned.

Owing to the lack of reliable information on the interaction lengths between the two kinds of matter, as a first approximation we shall suppose that the yield of the reaction is proportional to the product of the concentration of the two substances. The particles produced in these reactions may then be observed by waiting sufficient long time, by devices placed at the surface of some co-planar satellites, like Enceladus.

1 As is well known, the (‘classical’) gaps visible at the surface of the Rings are consequences of the fact that if a small object moves on the same orbit as a shepherd satellite (a satellite moving among the Rings) but in the opposite direction, it will be thrown away as the result of the gravitational interaction with this satellite. As an example consider a small object (a golf ball, for instance) with a velocity $V$ coming from the opposite direction along the same orbit as a given shepherd satellite. According to Newton’s laws (in a first approximation we consider only circular orbits) the velocity of this small object will have the same modulus as that of the satellite coming in the opposite direction, so that their relative velocity will be $2V$. If they are close enough, the small object will be attracted by the shepherd satellite, so that it will go around it, and then leave the scene with a velocity $2V$ in their centre of mass system. However, as the satellite is much heavier than the golf ball, this centre of mass will almost coincide with that of the satellite alone. Since the golf ball now moves with a speed $2V$ relative to the satellite, it has a velocity $3V$ with respect to Saturn. As the centrifugal force is proportional to the square of the velocity, this force will be nine times larger than that corresponding to its stable orbit. A factor as large as nine will be enough to expel the small object far away, perhaps as far as the Kuiper Belt.
3 The “Fence Effect”

In figure 4 above, we have deliberately chosen a region which is a little apart from the waves produced by the shepherd satellites. This is to avoid being disturbed by their presence and hence enable us to see better the "crude" interaction of their would be multiple "spectra". See below for explanations. Specifically, we should observe (in figure 4) the very fine lighter lines orthogonal to the radial direction of the Rings, i.e. along the direction of the Rings’ coordinate system, marked with yellow dots. These might be interpreted as a kind of "Zaun Effect" (in German, or “Fence Effect” in English, for explanations see below footnote), between the "spectra" produced by two (or more) different objects (or fluids) which might be simultaneously present there. These phenomena may look at a first glance similar to the usual interferences of two waves, but, as is discussed in footnote 2), its nature extends far beyond the usual interferences. They may be related to different causes, as for instance to the so called moiré patterns produced by two sheets of fine silk put together.

The fine lighter vertical lines might represent a direct proof of the existence of a second, if not of a multiple Ring system instead of a single one. Considering the regular patterns visible in figure 4, we might have the impression that we are observing a carefully printed table from some mathematical treatise on Spherical, or, more correctly, Two Dimensional Cylindrical Functions. It is our belief, which we tried to express in this article, that such special Dynamical Regions like that of the Rings – which are extremely sharp, “quasi singular" – or that of the Lagrange Points discussed in Ref. [2] where the trajectories remain for long periods of time, are zones where these substances mix and where future experiments should be planned.

Some people who try to find various elusive structures in between the fine vertical lines appearing in the figure 4 (for instance among the lines 1&2, 3&4, 4&5 and 7&8, marked by dots), might get the impression that they see the patterns of some Helmholtz-Kelvin instabilities between two intermixed fluids. A brief comment: Helmholtz-Kelvin or similar singularities seen in the flows of various fluids may be recognisable by their form but not by a ready to use analytic algorithm. This situation is somehow similar to that of the examination of rocks ejected by the volcanoes from Mars (or Venus) which fall on Earth as meteorites after a long interplanetary trip. According to our geological colleagues who organise yearly scientific trips to the Alto Plano ATACAMA to collect them, there does

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2To explain this in a simple way, we shall send the reader to Switzerland, a country in which, being a bit wealthier, the intersections between roads of different ranks are always made by means of bridges. The bridges usually have two fences, and the front balusters appearing to an observer approaching the bridge from beneath, a little larger because of perspective, will occult at regular intervals the rear ones. So since in these places only one, and not two, baluster are visible they will appear as lighter fine lines at regular spacings. These lines are also an indirect indication that here we have to consider the existence not only of a single ‘fluid’, but of two or more.

This is the origin of the moiré patterns which do appear when we look to two sheets of fine silk put together, which obviously contain more information than a simple optical interference. Although the example given here has to be considered just as a metonymy, much of the geometric features of the “balusters” can be inferred from the geometrical widths of the fine occlusion lines, or from the (slow) variations of their positions and spacings. So quite a lot of information might be obtained by a careful inspection of these images.
Figure 5: Helmholtz-Kelvin singularity. If by chance you have once looked at the sky overhead and seen the clouds entangled as in this double helix, you have possibly seen a Helmholtz-Kelvin singularity.

not yet exist any algorithm to recognise them among the many other rocks. Only a long human experience permits to accomplish this job. We are probably in the same situation when we try to recognise cosmic singularities looking only at their physical appearance.

Certainly there are sufficient unexplained features in these images to justify the examination of new ones. We have, for instance, not yet mentioned the existence of the radial Spikes described in the NASA reports [3], which “run” on the surface of the Rings. However, here we may be tempted to interpret them as moire effects (see footnote 2), as those discussed in the text. This, at the same time, could be an explanation for their short life nature. Anyhow the rich information brought by the Cassini mission will certainly support research activities for long periods of time.

4 A Powder Dynamics approach.

There are also other facts that might have some importance in the understanding of the formation and the history (the “geology”) of the Rings. There are (a) the multiple parallel stripes or circular annuli, very visible in another excellent NASA picture, figure 6. They are also present as the fine lines, discussed in figure 4 in the previous section, and, very importantly, they have no transparent background (they are not “black”, in contrast to the classical gaps through which you can see the surrounding, dark Space). Then (b), there are the classical gaps produced, see Footnote 1, by sling effect by the shepherd satellites, which have transparent backgrounds and hence they appear black in the pictures.

As the reader may notice in figure 6, the widths of the circular annuli vary, monotonically increasing or decreasing with their positions. A theoretical explanation based on
Powder Dynamics \cite{8} of such effect will be the subject of a subsequent paper. This remark represents an important tool which shows that the "classical" (the "black") gaps were formed at a later date than the circular annuli, since as the reader may verify, they do not interfere at all with the monotonicity of the widths. This is an indication that when the black gap appeared, the "ditches" and "moats" were certainly already there.

Since the Rings of Saturn are extremely thin we find ourselves in a two dimensional situation. Care should however be exercised to introduce the initial conditions correctly. While in three dimensions the forces are obtained by products of the area with the pressure acting on it, in two dimensions we will have to multiply lengths of segments with the corresponding components of the stress tensor. We shall then follow the theory of Mechanics of Powders at Rest for a two-dimensional system presented in \cite{8}. To this end, we start by considering a force acting perpendicular to the side of length $a$ of an infinitesimal triangle at some given point $P$ on the surface of the Rings (see figure 7). This force is related to the $\sigma_{xx}$ component of the stress tensor (we have striven ourselves as much as possible to follow the notations of Mohr). The first of these two subscripts indicates that it is parallel to the $x$-axis and that it acts on the first segment of length $a$ of the triangle from figure 7. If the second index were $y$, this would mean that we are considering the orthogonal component of the force i. e. that which is parallel to this segment $a$. Since the stress tensor is Hermitian (the stress matrix being real and symmetrical) the shear components of the stress acting in
Figure 7: Mohr’s Triangle. Schematic picture of the stresses acting on an infinitesimal triangle at the surface of the Rings. The initial eigenvectors stresses depict with double arrows and are mutually orthogonal, as they should be. The $xx$ and $yy$ components represent the normal stresses while the $xy$ and $yx$ ones the shear ones. We purposely tried as much as possible to stick at the historical notations (i.e. to the “circle of Otto Mohr”).

The orthogonal direction, have to be equal $\sigma_{yx} = \sigma_{xy}$.

Let $\sigma_1$ and $\sigma_2$ denote the initial eigenvectors of the stress tensor acting on this infinitesimal triangle, namely perpendicular on the segments of length $b$ and $c$, producing respectively the forces which act on these sides (represented with double arrows in figure 7). Projecting now everything onto the x axis, the equation of the balance of the forces along the x direction is:

$$a \cdot \sigma_{xx} = b \cdot \sigma_1 \cos(\theta) + c \cdot \sigma_2 \sin(\theta).$$

(4.1)

Both $b$ and $c$ can themselves be expressed in terms of $a$ and the angle $\theta$:

$$b = a \cos(\theta) \quad \text{and} \quad c = a \sin(\theta),$$

(4.2)

that after simplification we obtain the following equation:

$$\sigma_{xx} = \sigma_1 \cdot \cos^2(\theta) + \sigma_2 \cdot \sin^2(\theta).$$

(4.3)

Now, using basic trigonometric identities, we obtain:

$$\sigma_{xx} = \frac{1}{2}(\sigma_1 + \sigma_2) + \frac{1}{2}(\sigma_1 - \sigma_2) \cos(2\theta).$$

(4.4)

All that can also be represented very nicely in a graphical form by means of the circle of Christian Otto Mohr, a nineteenth century German engineer, Professor at the Universities of Stuttgart and Dresden, much beloved by his students, who received for his work the highest distinction from the Fuerst of the State of Sachse.
Considering now the shear component \( \sigma_{xy} \) of the stress tensor which acts along the segment orthogonal to the \( x \) axis, from figure 7 we obtain:

\[
a \cdot \sigma_{xy} = b \cdot \sigma_1 \cdot \sin(\theta) - c \cdot \sigma_2 \cdot \cos(\theta).
\]

(4.5)

The minus sign appears here since the \( y \)-components of the two initial eigenvectors stresses point in opposite directions. From (4.2) and simplifying with \( a \), we have

\[
\sigma_{xy} = (\sigma_1 - \sigma_2) \cdot \cos(\theta) \cdot \sin(\theta),
\]

which means that

\[
\sigma_{xy} = \frac{1}{2}(\sigma_1 - \sigma_2) \cdot \sin(2\theta).
\]

(4.6)

Similarly, \( \sigma_{yy} \) can be computed in an analogous way. Or much more quickly, using the invariance properties of the \textit{Trace}:

\[
\text{Tr}(\sigma) = \sigma_1 + \sigma_2 = \sigma_{xx} + \sigma_{yy}
\]

(4.7)

\[
\sigma_{yy} = \frac{1}{2}(\sigma_1 + \sigma_2) - \frac{1}{2}(\sigma_1 - \sigma_2) \cdot \cos(2\theta)
\]

(4.8)

In a paper in preparation, we shall use these equations to analyse the monotonicity of the widths of the circular strips visible at the surface of the Rings. Before that we should probably say also few words about the microscopic forces acting among the constituent particles of the Rings. This task is relatively simple since the pictures provided by Cassini show clearly that the distances among the particles are rather large. In general the form of the attractive potential derived from the induced dipoles due to the London dispersion forces active there, have a very short range of order \(-const \cdot \frac{1}{r^6}\). The Cassini pictures show clearly that the density of the powders is not very large, the distances between the grains exceeding by much these ranges. There exists of course also a repulsive component, but this has a much shorter range, the usual form given for the Lennard-Jones potential being \((r_0/r)^{12} - (r_0/r)^{6}\). The representation of this repulsive part is not considered to be realistic, an exponential form \(e^{-\frac{r}{r_0}}\) being much closer to the decay of the wave function and hence preferred for the repulsive part. We shall address these questions in our forthcoming paper. And who knows, the prevalent attractive potential acting between the “boulders” (ice pellets, grains of sand) above a certain size existing in the Rings, might be nothing else than the usual Newtonian potential, i.e the \(\frac{1}{r^2}\) force.

5 Some concluding remarks

In summary, according to Zwicky’s classical conjecture Dark Matter mimics the gravitational and the inertial behaviour of usual matter. So it is always interesting to find gaps in the Rings region, even very narrow ones and to look carefully at its borders to see whether
the advanced/retarded wavelet system is present and to verify whether the shepherd satellite is visible or not. If the gap under consideration is a “classical black gap” – see the previous section – usually it should be present!

To our knowledge these features seem to have been somehow overlooked, at least in previous studies, since until now Dark Matter has been studied more at the Cosmic Level. To be fair, however, some of these findings, for instance those discussed in figure 8 below, have the merit to have much enhanced our confidence in the existence of Dark Matter. In the future we shall probably be back in these (almost) singular regions of our Solar System, with suitable devices to be placed on Enceladus or on some other shepherd satellites to make measurements.

The discussions in this paper are based tacitly on the belief of René Thom [10] that ‘Singular’ Dynamical Regions may be able to produce such unusual (very thin) objects like the Rings of Saturn. They are really extremely ‘thin’ in comparison with all the cosmic objects around, so they might represent preferred places where the two kinds of matter could mix, and hence, interact. Another important approach is, of course, that of Sergio Bertolucci et al. [11] (see also references therein) in which one looks at places where also usual matter accumulates naturally, like the interior of the Sun or the large astronomical objects. Indeed according to the conjuncture of Zwicky, Dark Matter follows the patterns of usual one and so these regions will be in the first row where one could find interactions.

Before closing this section, we should like to add also some words which, although lying outside of the main topics of this paper, are interesting by themselves. This is particularly true as NASA apparently plans to return with a new expedition on Enceladus, with a small submarine containing a nuclear source of energy to melt a hole through the glacier crust and to explore the liquid sea beneath it in order to look for possible forms of Life. Indeed, while the northern hemisphere of the small satellite Enceladus has the usual marks (craters) of various impacts, the southern one is absolutely smooth, as after a recent snowfall. Further South, near the South Pole of this moon, Cassini’s cameras have detected the plumes of some Giant Geysers, rising high vertically, since the gravitational field of these moons is rather weak. See figure 9 below. This is direct proof that under the glacial crust we find liquid water at least around zero degrees Celsius. As Saturn is rather far from the Sun, it is clear that this heat is not provided by the Sun, but probably is produced by a mechanism based on the tidal forces made by Saturn which is not too far away.

Acknowledgments

The authors are thankful to the Cassini Team of the NASA Expedition to the Saturn Rings region for the multiple scientific information they have shared with us. The credits of some of the images, especially figures 3, 4, 6, 8 and 9 are related to them, more precisely to Professors Linda and Thomas Spilker from the Jet Propulsion Laboratory, Pasadena, USA..

The authors are very thankful to Professor Guy Auberson from the Charles Coulomb Laboratory, University of Montpellier, FRANCE, and to Professor Michael Pidcock, from Oxford U.K., for long discussions about these subjects. Professor Pidcock played also a
Probably one of the most striking events related to Dark Matter, at the Cosmic level, is the ‘Bullet Cluster Galaxies Event’, recorded by the Chandra NASA X rays embarked telescope, sees in near X-rays and in UV, it was observed also by the Hubble satellite and terrestrial facilities, like the 6.5-meter Magellan telescope from Las Campanas, Chile and the ESO telescope of Paranal Observatory (see also [12]). The right hand image is in false colours, blue for X or UV rays, orange-red for matter. The Dark Matter is partially depicted by naked greenish contours, and it has been recorded, as usual by means of the deformations of the images of the more distant galaxies. One should notice that the blobs of Dark Matter, probably having lesser ‘viscosity’, have overtaken the visible matter, mainly hot gas, in yellow or orange colour. Consequently, we have a succession of four different ‘objects’ – an invisible blob, a visible cluster of galaxies, another cluster of galaxies, and the invisible blob corresponding to this second cluster. [12]

Figure 9: Giant Geyser Plumes over Enceladus Southern hemisphere (courtesy NASA/JPL-Caltech). Their height is certainly a consequence of the weakness of the gravitational field around Enceladus, but they represent also a direct proof that under the ice crust the temperature (produced by tidal frictions due to the attraction of Saturn) is at least 0° Celsius, and hence compatible with Life as we know it. (Source: NASA).
considerable role in the final form of this paper as well as in the letters exchanged with the people from NASA.

Finally we would also like to acknowledge that we are grateful to Professor Julien Lesgourgues from Aix la Chapelle, particularly for discussions related to our first paper, [2], concerning the points of Lagrange.

References

[1] Fritz Zwicky, *Die Rotverschiebung von extragalaktischen Nebeln*. (German) [The Redshift of Extragalactic Nebulae], *Helvetica Physica Acta* 6 (1933).

[2] C. Sebu and S. Ciulli, *Stable Lagrangian Points of Large Planets as Possible Regions where WIMPs could be Sought*, J. Phys. A 61 (2008) 377.

[3] J.P. Lebreton, D.L. Matson, *An overview of the Cassini mission*, *Nouv. Cim. C* 15 (1992) 1137.

[4] Jan Hendrik Oort. *The force exerted by the stellar system in the direction perpendicular to the galactic plane and some related problems*. Bulletin of the Astronomical Institutes of the Netherlands 6 (1932) 249.

[5] Ernst Öpik, *Bull. de la Soc. Astr. de Russie* 21 (1915) 150.

[6] Vera C. Rubin, *The Rotation of Spiral Galaxies*, Science 220(4604) (1983) 1339-1344.

[7] T. S. van Albada, J. N. Bahcall, K. Begeman, and R. Sancisi, *Distribution of dark matter in the spiral galaxy NGC 3198*, The Astrophysical Journal 295 (1985) 305.

[8] K. Rietema, *The Dynamics of Fine Powders*, Elsevier Applied Science, Netherlands (1991).

[9] L. Huber et al., *NASA’S PLANETARY DATA*. PDS, 2014

[10] René Thom, *Stabilité Structurelle et Morphogenèse*, Institut des Hautes Etudes Scientifiques, Inter-Éditions, Paris (1984).

[11] S. Bertolucci, K. Zioutas, S. Hofmann, and M. Maroudas, *The Sun and its Planets as detectors for invisible matter*, Physics of the dark universe 17 (2017) 13-21.

[12] D. Clowe, M. Bradač, A. H. Gonzalez, M. Markevitch, S. W. Randall, C. Jones, and D. Zaritsky, *A Direct Empirical Proof of the Existence of Dark Matter*. The Astrophysical Journal 648(2) (2006) 109.