New Constraint on Weak CP Phase
Rephasing Invariant and Maximal CP Violation

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Abstract

By using the relation between CP-violation phase and the mixing angles in Cabibbo-Kobayashi-Maskawa matrix postulated by us before, the rephasing invariant is recalculated. Furthermore, the problem about maximal CP violation is discussed. We find that the maximal value of Jarlskog’s invariant is about 0.038. And it presents at $\alpha \simeq 71.0^0$, $\beta \simeq 90.2^0$ and $\gamma \simeq 18.8^0$ in triangle db.

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Since the discovery of CP-violation in neutral kaon system in 1964 [1], more than thirty years have passed. Although a great progress has been made during these years [2-6], such as establishing the phenomenological structure of the effects, classifying the CP parameters and constructing the gauge theory models [7-14], our understanding about CP violation is still very poor. On the experimental side, the kaon system remains the only place where CP violation is observed, though the B meson system is the best probe to it as is widely accepted [15-16]. On the theoretical side, the origin of CP violation is not so clear, and the correctness of the standard Cabibbo-Kobayashi-Maskawa mechanism is far from being proved.

In the standard model of three generations, CP violation originates from the phase angle present in the unitary Cabibbo-Kobayashi-Maskawa matrix. Mathematically, it is permitted that, only one phase angle exists in a three by three unitary matrix with the exception of three Eulerian angles. However, the weak CP phase which cannot be eliminated by any means, is introduced somewhat artificially. The requirement that it has nothing to do with the three mixing angles is only due to the mathematical rather than a physical reason. It is naturally to ask whether there is an intrinsic relation between the phase and the three mixing angles.

Recently, we found that the CP-violation phase and the other three mixing angles satisfy the following relation [17-18]

$$\sin \frac{\delta}{2} = \sqrt{\frac{\sin^2 \theta_1 + \sin^2 \theta_2 + \sin^2 \theta_3 - 2(1 - \cos \theta_1 \cos \theta_2 \cos \theta_3)}{2(1 + \cos \theta_1)(1 + \cos \theta_2)(1 + \cos \theta_3)}}$$

(1)

where \( \theta_i \) \((i = 1, 2, 3)\) are the corresponding angles in the standard KM parametrization matrix

$$V_{KM} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix}$$

(2)

with the standard notations \( s_i = \sin \theta_i \) and \( c_i = \cos \theta_i \) are used.

The geometry meaning of Eq.(1) is very clear, \( \delta \) is the solid angle enclosed by three angles \( \theta_1, \theta_2 \) and \( \theta_3 \), or the area to which the solid angle corresponds on a unit sphere. It should be noted that, to make \( \theta_1, \theta_2 \) and \( \theta_3 \) enclose a solid angle, the condition

$$\theta_i + \theta_j > \theta_k \quad (i \neq j \neq k \neq i. \quad i, j, k = 1, 2, 3)$$

(3)

is needed. Now, we find that the CP-violation seems to originate in a geometry reason.

With the discussion on the "maximal" CP violation in various parametrizations of the KM matrix [19-22], It is found [23-26] that all the CP nonconservation effects are proportional
to a universal factor $X_{CP}$ defined as,

$$X_{CP} = s_1^2 c_1 s_2 c_2 s_3 c_3 s_\delta.$$  \hspace{1cm} (4)

$X_{CP}$ is also called the Jarlskog’s invariant in the relevant references.

In fact, only those functions of $V_{KM}$ which are invariant under the rephasing operation of quark fields can be observable. From $V_{KM}$ we can construct [27] nine squared moduli invariants $\Delta^{(2)}_{\alpha\beta} \equiv |(V_{KM})_{\alpha\beta}|^2$ and nine quartic invariant function

$$\Delta^{(4)}_{\alpha\rho} \equiv (V_{KM})_{\beta\sigma}(V_{KM})_{\gamma\tau}(V_{KM}^*)_{\beta\tau}(V_{KM}^*)_{\gamma\sigma}$$ \hspace{1cm} (5)

here, the summation convention for repeated indices is used and $\alpha, \beta, \gamma (\rho, \sigma, \tau)$ are taken cyclic. Owing to the unitary constraint, only four squared-moduli and one quartic term are independent. In the mean time, all the nine quartic term have the same imaginary part, it is just $X_{CP}$ shown in Eq.(4).

Substitue Eq.(1) into Eq.(4), we obtain

$$X_{CP} = s_1^2 c_1 s_2 c_2 s_3 c_3 \frac{(1 + c_1 + c_2 + c_3) \sqrt{s_1^2 + s_2^2 + s_3^2 - 2(1 - c_1 c_2 c_3)}}{(1 + c_1)(1 + c_2)(1 + c_3)\sqrt{s_1^2 + s_2^2 + s_3^2 - 2(1 - c_1 c_2 c_3)}}$$ \hspace{1cm} (6)

This is the rephasing invariant after considering the new constraint on the weak CP phase and the quark mixing angles, based on which, we can further discuss the maximal CP violation in following.

It is not difficult to find out the maximum of $X_{CP}$, it is

$$X_{CP}^{Max} = 0.038296$$ \hspace{1cm} (7)

and presents at

$$c_1 = 0.49666 \quad c_2 = 0.56766$$ \hspace{1cm} (8)

In this case, the Wolfenstein parametrization approximate to the third order of $\lambda$ is invalid. To give a little sense about the CP violation, we calculate the three angles of the triangle ($db$) defined as following [28]

$$\alpha = \arg \left( -\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right)$$ \hspace{1cm} (9)

$$\beta = \arg \left( -\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right)$$ \hspace{1cm} (10)

$$\gamma = \arg \left( -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right)$$ \hspace{1cm} (11)

From Eq.(1), Eq.(2) and Eq.(8), we get

$$\alpha = 71.0^0 \quad \beta = 90.2^0 \quad \gamma = 18.8^0$$ \hspace{1cm} (12)
It is easy to find that is nearly a right triangle with $0.2^0$ deviation. However, nature has not choose this way of CP violation. It deviates very far from the present experimental results [29-30].

As a conclusion, we have recalculated the rephasing invariant with the relation between CP-violation phase and the mixing angles in CKM matrix postulated by us before being used. The problem about maximal CP violation is discussed. We find that the maximal value of Jarlskog’s invariant is about 0.038. And it presents at $\alpha \simeq 71.0^0$, $\beta \simeq 90.2^0$ and $\gamma \simeq 18.8^0$ in the triangle $db$.

Based on the above model-independent results, we can extract some limits on the experiments in $B^0 - \overline{B}^0$ and $D^0 - \overline{D}^0$ system etc. The further work will be reported in the future.

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