Realization of the $1 \rightarrow 3$ optimal phase-covariant quantum cloning machine

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Abstract

The $1 \rightarrow 3$ quantum phase covariant cloning, which optimally clones qubits belonging to the equatorial plane of the Bloch sphere, achieves the fidelity $F_{\text{cov}}^{1\rightarrow3} = 0.833$, larger than for the $1 \rightarrow 3$ universal cloning $F_{\text{univ}}^{1\rightarrow3} = 0.778$. We show how the $1 \rightarrow 3$ phase covariant cloning can be implemented by a smart modification of the standard universal quantum machine by a projection of the output states over the symmetric subspace. A complete experimental realization of the protocol for polarization encoded qubits based on non-linear and linear methods will be discussed.
In the last years a great deal of efforts has been devoted to the realization of the optimal approximations to the quantum cloning and flipping operations over an unknown qubit $|\phi\rangle$. Even if these two processes are unrealizable in their exact forms [1], [2], they can be optimally approximated by the corresponding universal machines, i.e., by the universal quantum cloning machine (UQCM) and the universal-NOT (U-NOT) gate [3]. The optimal quantum cloning machine has been experimentally realized following several approaches, i.e. by exploiting the process of stimulated emission in a quantum-injected optical parametric amplifier (QI-OPA) [4–7], by a quantum network [8] and by acting with projective operators over the symmetric subspaces of many qubits [9,10]. The $N \to M$ UQCM transforms $N$ input qubits in the state $|\phi\rangle$ into $M$ entangled output qubits in the mixed state $\rho_{\text{out}}$. The quality of the resulting copies is quantified by the fidelity parameter $F_{\text{univ}}^{N \to M} = \langle \phi | \rho_{\text{out}} | \phi \rangle = \frac{N+1+\beta}{N+2}$ with $\beta = \frac{N}{M} \leq 1$.

Not only the perfect cloning of unknown qubit is forbidden but also perfect cloning of subsets containing non orthogonal states. This no-go theorem ensures the security of cryptographic protocols as BB84 [11]. Recently state dependent cloning machines have been investigated that are optimal respect to a given ensemble [12]. The partial a-priori knowledge of the state allows to reach a higher fidelity than for the universal cloning. In particular the $N \to M$ phase-covariant quantum cloning machine (PQCM) considers the cloning of $N$ into $M$ output qubits, where the input ones belong to the equatorial plane of the corresponding Poincare’ sphere, i.e. expressed by: $|\phi\rangle = 2^{-1/2} (|0\rangle + e^{i\phi} |1\rangle)$. The values of the optimal fidelities $F_{\text{cov}}^{N \to M}$ for this machine have been found [13]. In the present article we will restrict ourselves to the case in which $N = 1$. For $M$ assuming odd values it is found $F_{\text{cov}}^{1 \to M} = \frac{1}{4} (3 + M^{-1})$ while in the case of even $M$–values $F_{\text{cov}}^{1 \to M} = \frac{1}{2} \left( 1 + \frac{1}{2} \sqrt{1 + 2M^{-1}} \right)$. In particular we have $F_{\text{cov}}^{1 \to 2} = 0.854$ to be compared with $F_{\text{univ}}^{1 \to 2} = 0.833$ and $F_{\text{cov}}^{1 \to 3} = 0.833$ with: $F_{\text{univ}}^{1 \to 3} = 0.778$.

It is worthwhile to enlighten the connections existing between the cloning processes and the theory of quantum measurement [14]. The concept of universal quantum cloning is indeed related to the problem of optimal quantum state estimation [15] since for $M \to \infty$,
\[ \mathcal{F}_{\text{univ}}^{N \rightarrow M} \rightarrow \mathcal{F}_{\text{estim}}^{N} = \frac{N+1}{N+2} \] where \( \mathcal{F}_{\text{estim}}^{N} \) is the optimal fidelity for the state estimation of any ensemble of \( N \) unknown, identically prepared qubits. Likewise, the phase-covariant cloning has a connection with the estimation of an equatorial qubit, that is, with the problem of finding the optimal strategy to estimate the value of the phase \( \phi \) [16], [17]. Precisely, the optimal strategy consists of a POVM corresponding to a Von Neumann measurement of \( N \) input qubits characterized by a set of \( N+1 \) orthogonal projectors and achieves the fidelity \( \mathcal{F}_{\text{phase}}^{N} \) [17]. In general for \( M \rightarrow \infty, \mathcal{F}_{\text{cov}}^{N \rightarrow M} \rightarrow \mathcal{F}_{\text{phase}}^{N} \). For \( N = 1 \) is found:

\[ \mathcal{F}_{\text{cov}}^{1 \rightarrow M} = \mathcal{F}_{\text{phase}}^{1} + \frac{1}{4M} \text{ with } \mathcal{F}_{\text{phase}}^{1} = \frac{3}{4}. \]

To our knowledge, no PQCM device has been implemented experimentally in the domain of Quantum Optics [18,19]. In the present work we report the implementation of a \( 1 \rightarrow 3 \) PQCM by adopting a modified standard \( 1 \rightarrow 2 \) UQCM and by further projecting the output qubits over the symmetric subspace [5,9]. Let the state of the input qubit be expressed by:

\[ |\phi\rangle_{S} = \alpha |0\rangle_{S} + \beta |1\rangle_{S} \]

with real parameters \( \alpha \) and \( \beta \) and \( \alpha^{2} + \beta^{2} = 1 \). The output state of the \( 1 \rightarrow 2 \) UQCM device reads:

\[ |\Sigma\rangle_{SAB} = \sqrt{\frac{2}{3}} |\phi\rangle_{S} |\phi\rangle_{A} |\phi^{\perp}\rangle_{B} - \frac{1}{\sqrt{6}} (|\phi\rangle_{S} |\phi^{\perp}\rangle_{A} + |\phi^{\perp}\rangle_{S} |\phi\rangle_{A}) |\phi^{\perp}\rangle_{B}. \]

The qubits \( S \) and \( A \) are the optimal cloned qubits while the qubit \( B \) is the optimally flipped one. We perform the operation \( U_{B} = \sigma_{Y} \) on the qubit \( B \). This local flipping transformation of \( |\phi\rangle_{B} \) leads to:

\[ |\Upsilon\rangle_{SAB} = (I_{S} \otimes I_{A} \otimes U_{B}) |\Sigma\rangle_{SAB} = \sqrt{\frac{2}{3}} |\phi\rangle_{S} |\phi\rangle_{A} |\phi^{\perp}\rangle_{B} - \frac{1}{\sqrt{6}} (|\phi\rangle_{S} |\phi^{\perp}\rangle_{A} + |\phi^{\perp}\rangle_{S} |\phi\rangle_{A}) |\phi^{\perp}\rangle_{B}. \]

By this non-universal cloning process three asymmetric copies have been obtained: two clones (qubits \( S \) and \( A \)) with fidelity \( 5/6 \), and a third one (qubit \( B \)) with fidelity \( 2/3 \). We may now project \( S, A \) and \( B \) over the symmetric subspace and obtain three symmetric clones with a higher average fidelity. The symmetrization operator \( \Pi_{\text{sym}}^{SAB} \) reads as:

\[ \Pi_{\text{sym}}^{SAB} = |\Pi_{1}\rangle \langle \Pi_{1}| + |\Pi_{2}\rangle \langle \Pi_{2}| + |\Pi_{3}\rangle \langle \Pi_{3}| + |\Pi_{4}\rangle \langle \Pi_{4}| \] where

\[ |\Pi_{1}\rangle = |\phi\rangle_{S} |\phi\rangle_{A} |\phi^{\perp}\rangle_{B}, \]

\[ |\Pi_{2}\rangle = |\phi^{\perp}\rangle_{S} |\phi^{\perp}\rangle_{A} |\phi^{\perp}\rangle_{B}, \]

\[ |\Pi_{3}\rangle = \frac{1}{\sqrt{3}} \left( |\phi\rangle_{S} |\phi^{\perp}\rangle_{A} |\phi^{\perp}\rangle_{B} + |\phi^{\perp}\rangle_{S} |\phi^{\perp}\rangle_{A} |\phi^{\perp}\rangle_{B} + |\phi^{\perp}\rangle_{S} |\phi\rangle_{A} |\phi^{\perp}\rangle_{B} \right) \]

and

\[ |\Pi_{4}\rangle = \frac{1}{\sqrt{3}} \left( |\phi\rangle_{S} |\phi^{\perp}\rangle_{A} |\phi^{\perp}\rangle_{B} + |\phi^{\perp}\rangle_{S} |\phi\rangle_{A} |\phi^{\perp}\rangle_{B} + |\phi^{\perp}\rangle_{S} |\phi^{\perp}\rangle_{A} |\phi^{\perp}\rangle_{B} \right). \]

The symmetric subspace has dimension 4 since three qubits are involved. The probability of success of the
projection is equal to $\frac{8}{9}$. The normalized output state $|\xi\rangle_{SAB} = \Pi_{sym} |\Upsilon\rangle_{SAB}$ is

$$
|\xi\rangle_{SAB} = \frac{\sqrt{3}}{2} |\phi\rangle_S |\phi\rangle_A |\phi\rangle_B - \frac{1}{2\sqrt{3}} (|\phi\rangle_S |\phi^\perp\rangle_A |\phi^\perp\rangle_B + |\phi^\perp\rangle_S |\phi\rangle_A |\phi^\perp\rangle_B + |\phi^\perp\rangle_S |\phi^\perp\rangle_A |\phi\rangle_B)
$$

(2)

Let us now estimate the output density matrices of the qubits $S$, $A$ and $B$

$$
\rho_S = \rho_A = \rho_B = \frac{5}{6} |\phi\rangle \langle \phi| + \frac{1}{6} |\phi^\perp\rangle \langle \phi^\perp|
$$

(3)

This leads to the fidelity $F_{cov}^{1\rightarrow 3} = 5/6$ equal to the optimal one [12,13].

By applying a different unitary operator $U_B$ to the qubit $B$ we can implement the phase-covariant cloning for different equatorial planes. Interestingly, note that by this symmetrization technique a depolarizing channel $E_{dep}(\rho) = \frac{1}{4} (\rho + \sigma_X \rho \sigma_X + \sigma_Y \rho \sigma_Y + \sigma_Z \rho \sigma_Z)$ on channel $B$ transforms immediately the non-universal phase covariant cloning into the universal $1 \rightarrow 3$ UQCM with the overall fidelity $F_{univ}^{1\rightarrow 3} = 7/9$. This represent a relevant new proposal to be implemented within the $1 \rightarrow 2$ UQCM QI-OPA device or other $1 \rightarrow 2$ U-cloning schemes [5,20].

Let us return to the $1 \rightarrow 3$ PQCM. In the present scheme the input qubit, to be injected into a QI-OPA over the spatial mode $k_1$ with wavelength (wl) $\lambda$, is encoded into the polarization ($\pi$) state $|\phi\rangle_{in} = \alpha |H\rangle + \beta |V\rangle$ of a single photon, where $|H\rangle$ and $|V\rangle$ stand for horizontal and vertical polarization: Figure 1. The QI-OPA consisted of a nonlinear (NL) BBO ($\beta$-barium-borate), cut for Type II phase matching and excited by a sequence of UV mode-locked laser pulses having wl. $\lambda_p$. The relevant modes of the NL 3-wave interaction driven by the UV pulses associated with mode $k_p$ were the two spatial modes with wave-vector (wv) $k_i$, $i = 1, 2$, each one supporting the two horizontal and vertical polarizations of the interacting photons. The QI-OPA was $\lambda$-degenerate, i.e. the interacting photons had the same wl’s $\lambda = 2\lambda_p = 795nm$. The NL crystal orientation was set as to realize the insensitivity of the amplification quantum efficiency to any input state $|\phi\rangle_{in}$ i.e. the universality (U) of the ”cloning machine” and of the U-NOT gate [5]. This key property is assured by the squeezing hamiltonian $\hat{H}_{int} = i\chi \hbar (\hat{a}_{1\phi}^\dagger \hat{a}_{2\phi}^\dagger - \hat{a}_{1\phi}^\dagger \hat{a}_{2\phi}^\dagger) + h.c.$ where the field
operator $\hat{a}_{ij}^\dagger$ refers to the state of polarization $j$ ($j = \phi, \phi^\perp$), realized on the two interacting spatial modes $k_i$ ($i = 1, 2$).

Let us consider the injected photon in the mode $k_1$ to have any linear polarization $\overrightarrow{\mathbf{p}} = \phi$. We express this $\overrightarrow{\mathbf{p}}$-state as $\hat{a}_{1\phi}^\dagger |0, 0\rangle_{k_1} = |1, 0\rangle_{k_1}$ where $|m, n\rangle_{k_1}$ represents a product state with $m$ photons of the mode $k_1$ with polarization $\phi$, and $n$ photons with polarization $\phi^\perp$. Assume the input mode $k_2$ to be in the vacuum state $|0, 0\rangle_{k_2}$. The initial $\overrightarrow{\mathbf{p}}$-state of modes $k_i$ reads $|\phi\rangle_{in} = |1, 0\rangle_{k_1} |0, 0\rangle_{k_2}$ and evolves according to the unitary operator $\hat{U} \equiv \exp \left(-i\frac{\hat{H}_{\text{int}}}{\hbar}\right)$. The 1st-order contribution of the output state of the QI-OPA is $\sqrt{\frac{S}{3}} |2, 0\rangle_{k_1} |0, 1\rangle_{k_2} - \sqrt{\frac{S}{3}} |1, 1\rangle_{k_1} |1, 0\rangle_{k_2}$. The above linearization procedure is justified here by the small experimental value of the gain $g \equiv \chi t \approx 0.1$. In this context, the state $|2, 0\rangle_{k_1}$, expressing two photons of the $\phi$ mode $k_1$ in the $\overrightarrow{\mathbf{p}}$-state $\phi$, corresponds to the state $|\phi\phi\rangle$ expressed by the general theory and implies the $L = 2$ cloning of the input $N = 1$ qubit. Contextually with the realization of cloning on mode $k_1$, the vector $|0, 1\rangle_{k_2}$ expresses the single photon state on mode $k_2$ with polarization $\phi^\perp$, i.e. the flipped version of the input qubit. In summary, the qubits $S$ and $A$ are realized by two single photons propagating along mode $k_1$ while the qubit $B$ corresponds to the $\overrightarrow{\mathbf{p}}$-state of the photon on mode $k_2$.

The $U_B = \sigma_Y$ flipping operation on the output mode $k_2$, implemented by means of two $\lambda/2$ waveplates, transformed the QI-OPA output state into: $|\Upsilon\rangle_{SAB} = \sqrt{\frac{S}{3}} |2, 0\rangle_{k_1} |1, 0\rangle_{k_2} - \sqrt{\frac{S}{3}} |1, 1\rangle_{k_1} |0, 1\rangle_{k_2}$. The physical implementation of the projector $\Pi_{\text{sym}}^{SAB}$ on the three photons $\overrightarrow{\mathbf{p}}$-states was carried out by linearly superimposing the modes $k_1$ and $k_2$ on the 50:50 beamsplitter $BS_A$ and then by selecting the case in which the 3 photons emerged from $BS_A$ on the same output mode $k_3$ (or, alternatively on $k_4$) [9]. The $BS_A$ input-output mode relations are expressed by the field operators: $\hat{a}_{1j}^\dagger = 2^{-1/2}(\hat{a}_{1\phi}^\dagger + i\hat{a}_{1\phi^\perp}^\dagger)$; $\hat{a}_{2j}^\dagger = 2^{-1/2}(i\hat{a}_{2\phi}^\dagger + \hat{a}_{2\phi^\perp}^\dagger)$ where the operator $\hat{a}_{ij}^\dagger$ refers to the mode $k_i$ with polarization $j$. The input state of $BS_A$ can be re-written in the following form $\frac{1}{\sqrt{3}} \left( \hat{a}_{1\phi}^{12\phi} + \hat{a}_{1\phi^\perp}^{12\phi^\perp} \right) |0, 0\rangle_{k_1} |0, 0\rangle_{k_2}$. By adopting the previous relations and by considering the case in which 3 photons emerge over the mode $k_3$, the output state is found to be $\frac{1}{2\sqrt{2}} \left( \hat{a}_{3\phi}^{13\phi} + \hat{a}_{3\phi^\perp}^{13\phi^\perp} \right) |0, 0\rangle_{k_3} = \sqrt{\frac{\chi t}{2}} |3, 0\rangle_{k_3} + \frac{1}{2} |1, 2\rangle_{k_3}$. The
output fidelity is $F_{\text{cov}}^{1 \rightarrow 3} = \frac{5}{6}$. Interestingly, the same overall state evolution can also be obtained, with no need of the final $BS_A$ symmetrization, at the output of a QI-OPA with a type II crystal working in a collinear configuration, as proposed by [21]. In this case the interaction Hamiltonian $\hat{H}_{\text{coll}} = i\chi \hbar \left( \hat{a}_H^\dagger \hat{a}_V^\dagger \right) + \text{h.c.}$ acts on a single spatial mode $k$. A fundamental physical property of $\hat{H}_{\text{coll}}$ consists of its rotational invariance under $U(1)$ transformations, that is, under any arbitrary rotation around the $z$-axis. Indeed $\hat{H}_{\text{coll}}$ can be re-expressed as $\frac{1}{2} i \chi \hbar e^{-i\psi} \left( \hat{a}_\psi^\dagger^2 - e^{i2\psi} \hat{a}_\psi^\dagger^2 \right) + \text{h.c.}$ for $\psi \in (0, 2\pi)$ where $\hat{a}_\psi^\dagger = 2^{-1/2}(\hat{a}_H^\dagger + e^{i\psi} \hat{a}_V^\dagger)$ and $\hat{a}_\psi^\dagger = 2^{-1/2}(-e^{-i\psi} \hat{a}_H^\dagger + \hat{a}_V^\dagger)$. Let us consider an injected single photon with $-\pi$-state $|\psi\rangle_{\text{in}} = 2^{-1/2}(|H\rangle + e^{i\psi} |V\rangle) = |1, 0\rangle_k$. The first contribution to the amplified state, $\sqrt{6} |3, 0\rangle_k - \sqrt{2} e^{i2\psi} |1, 2\rangle_k$ is identical to the output state obtained with the device dealt with in the present work up to a phase factor which does not affect the fidelity value.

The UV pump beam with wl $\lambda_p$, back reflected by the spherical mirror $M_p$ with 100% reflectivity and $\mu$-adjustable position $Z$, excited the NL crystal in both directions $-k_p$ and $k_p$, i.e. correspondingly oriented towards the right hand side and the left hand side of Fig.1. A Spontaneous Parametric Down Conversion (SPDC) process excited by the $-k_p$ UV mode created singlet-states of photon polarization ($\pi$). The photon of each SPDC pair emitted over the mode $-k_1$ was back-reflected by a spherical mirror $M$ into the NL crystal and provided the $N = 1$ quantum injection into the OPA excited by the UV beam associated with the back-reflected mode $k_p$. The twin SPDC photon emitted over mode $-k_2$, selected by the ”state analyzer” consisting of the combination (Wave-Plate + Polarizing Beam Splitter: $WP_T + PBS_T$) and detected by $D_T$, provided the ”trigger” of the overall conditional experiment. Because of the EPR non-locality of the emitted singlet, the $\pi$-selection made on $-k_2$ implied deterministically the selection of the input state $|\phi\rangle_{\text{in}}$ on the injection mode $k_1$. By adopting a $\lambda/2$ wave-plate ($WP_T$) with different orientations of the optical axis, the following $|\phi\rangle_{\text{in}}$ states were injected: $|H\rangle$ and $2^{-1/2}(|H\rangle + |V\rangle) = |+\rangle$. A more detailed description of the QI-OPA setup can be found in [5]. The $U_B = \sigma_Y$ flipping operation was implemented by two $\lambda/2$ waveplates (wp), as said. The device $BS_A$ was positioned onto
a motorized translational stage: the position $X = 0$ in Fig. 2 was conventionally assumed to correspond to the best overlap between the interacting photon wavepackets which propagate along $k_1$ and $k_2$.

The output state on mode $k_3$ was analyzed by the setup shown in the inset of Fig. 1: the field on mode $k_4$ was disregarded, for simplicity. The polarization state on mode $k_3$ was analyzed by the combination of the $\lambda/2$ wp $WP_C$ and of the polarizer beam splitter $PBS_C$. For each input $\pi$-state $|\phi\rangle_S$, two different measurements were performed. In a first experiment $WP_C$ was set in order to make $PBS_C$ to transmit $|\phi\rangle$ and reflect $|\phi^\perp\rangle$. The cloned state $|\phi\phi\phi\rangle$ was detected by a coincidence between the detectors $[D_{1C}, D_{2C}, D_{3C}]$ while the state $|\phi\phi\phi^\perp\rangle$, in the ideal case not present, was detected by a coincidence recorded either by the $D$ set $[D_{1C}, D_{2C}, D_{3C}]$, or by $[D_{1C}, D_{3C}, D_{2C}^\ast]$, or by $[D_{2C}, D_{3C}, D_{1C}^\ast]$. In order to detect the contribution due to $|\phi\phi^\perp\phi^\perp\rangle$, $WP_C$ was rotated in order to make $PBS_C$ to transmit $|\phi^\perp\rangle$ and reflect $|\phi\rangle$ and by recording the coincidences by one of the sets $[D_{1C}, D_{2C}, D_{3C}^\ast]$, $[D_{1C}, D_{3C}, D_{2C}^\ast]$, $[D_{2C}, D_{3C}, D_{1C}^\ast]$. The different overall quantum efficiencies have been taken into account in the processing of the experimental data. The precise sequence of the experimental procedures was suggested by the following considerations. Assume the cloning machine turned off, by setting the optical delay $|Z| >> c\tau_{coh}$, i.e., by spoiling the temporal overlap between the injected photon and the UV pump pulse. In this case since the states $|\phi\phi\rangle$ and $|\phi^\perp\phi^\perp\rangle$ are emitted with same probability by the machine, the rate of coincidences due to $|\phi\phi\phi\rangle$ and $|\phi\phi^\perp\phi^\perp\rangle$ were expected to be equal. By turning on the PQCM, i.e., by setting $|Z| << c\tau_{coh}$, the output state (2) was realized showing a factor $R = 3$ enhancement of the counting rate of $|\phi\phi\phi\rangle$ and no enhancement of $|\phi\phi^\perp\phi^\perp\rangle$. In Fig.2 the coincidences data for the different state components are reported versus the delay $Z$ for the two input qubits $|\phi\rangle_{in}$. We may check that the phase covariant cloning process affects only the $|\phi\phi\phi\rangle$ component, as expected. Let us label by the symbol $h$ the output state components as follows: $\{h = 1 \leftrightarrow |\phi\phi^\perp\phi^\perp\rangle, 2 \leftrightarrow |\phi\phi\phi^\perp\rangle, 3 \leftrightarrow |\phi\phi\phi\rangle\}$. For each index $h$, $b_h$ is the average coincidence rate when the cloning machine is turned off , i.e. $|Z| >> c\tau_{coh}$, while the signal-to-noise (S/N) parameter $R_h$ is the ratio between the peak values of the coincidence
rates detected respectively for $Z \simeq 0$ and $|Z| >> c\tau_{coh}$. The optimal values obtained by the above analysis are: $R_3 = 3$, $R_1 = 1$, $b_3 = b_1$ and $b_2 = 0$, $R_2 = 0$. These last values, $h = 2$ are considered since they are actually measured in the experiment: Fig.2. The fidelity has been evaluated by means of the expression $F_{cov}^{1\rightarrow3}(\phi) = (3b_3R_3 + 2b_2R_2 + b_1R_1) \times (3b_3R_3 + 3b_2R_2 + 3b_1R_1)^{-1}$ and by the experimental values of $b_h$, $R_h$. For $|\phi\rangle_{in} = |H\rangle$ and $|\phi\rangle_{in} = |+\rangle$ we have found respectively $R_3 = 2.00 \pm 0.12$ and $R_3 = 1.92 \pm 0.06$ (see Fig.2). We have obtained $F_{cov}^{1\rightarrow3}(|+\rangle) = 0.76 \pm 0.01$, and $F_{cov}^{1\rightarrow3}(|H\rangle) = 0.80 \pm 0.01$, to be compared with the theoretical value 0.83. The fidelity of the cloning $|H\rangle$ is slightly increased by a contribution 0.02 due to an unbalance of the Hamiltonian terms.

For the sake of completeness, we have carried out an experiment setting the pump mirror in the position $Z \simeq 0$ and changing the position $X$ $ob$ $BS_A$. The injected state was $|\phi\rangle_{in} = |+\rangle$. Due to quantum interference, the coincidence rate was enhanced by a factor $V^*$ moving from the position $|X| >> c\tau_{coh}$ to the condition $X \approx 0$. The $|\phi\phi\phi\rangle$ enhancement was found $V_{exp}^* = 1.70 \pm 0.10$, to be compared with the theoretical value $V^* = 2$ while the enhancement of the term $|\phi\phi\rangle$ was found $V_{exp}^* = 2.16 \pm 0.12$, to be compared with the theoretical value $V^* = 3$. These results, not reported in Fig. 2, are a further demonstration of the 3-photon interference in the Hong-Ou-Mandel device.

In conclusion, we have implemented the optimal quantum triplicators for equatorial qubits. The present approach can be extended in a straightforward way to the case of $1 \rightarrow M$ PQCM for $M$ odd. The results are relevant in the modern science of quantum communication as the PQCM is deeply connected to the optimal eavesdropping attack at $BB84$ protocol, which exploits the transmission of quantum states belonging to the $x-z$ plane of the Bloch sphere. [22,11]. The optimal fidelities achievable for equatorial qubits are equal to the ones considered for the four states adopted in $BB84$ [13]. In addition, the phase covariant cloning can be useful to optimally perform different quantum computation tasks adopting qubits belonging to the equatorial subspace [23].

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**Figure Captions**

Figure.1. Schematic diagram of phase-covariant cloner, PQCM made up by a QI-OPA and a Hong-Ou-Mandel interferometer $BS_A$. INSET: measurement setup used for testing the cloning process.

Figure.2. Experimental results of the PQCM for the input qubits $|H\rangle$ and $|+\rangle = 2^{-1/2}(|H\rangle +$
$|V\rangle$). The measurement time of each 4-coincidence experimental datum was $\sim 13000$ s. The different overall detection efficiencies have been taken into account. The solid line represents the best Gaussian fit.
\[ |\Psi_{in}\rangle = |H\rangle \]

\[ |\Psi_{in}\rangle = |+\rangle \]

\[ |H\rangle |H\rangle |H\rangle \]

\[ |+\rangle |+\rangle |+\rangle \]

\[ |H\rangle |V\rangle |V\rangle \]

\[ |+\rangle |-> |-> \]

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