Holography of 3D Asymptotically Flat Black Holes

Reza Fareghbal\textsuperscript{a,\ast} and Seyed Morteza Hosseini\textsuperscript{\dagger}

\textsuperscript{a}Department of Physics, Shahid Beheshti University G.C., Evin, Tehran 19839, Iran and
\textsuperscript{b}School of Particles and Accelerators,
Institute for Research in Fundamental Sciences (IPM),
P.O. Box 19395-5531, Tehran, Iran

Abstract

We study the asymptotically flat rotating hairy black hole solution of a three-dimensional gravity theory which is given by taking flat-space limit (zero cosmological constant limit) of New Massive Gravity (NMG). We propose that the dual field theory of the flat-space limit of NMG can be described by a Contracted Conformal Field Theory (CCFT). Using Flat/CCFT correspondence we construct a stress tensor which yields the conserved charges of the asymptotically flat black hole solution. Furthermore, by taking appropriate limit of the Cardy formula in the parent CFT, we find a Cardy-like formula which reproduces the Wald’s entropy of the 3D asymptotically flat black hole.

\textsuperscript{\ast}Electronic address: fareghbal@sbu.ac.ir
\textsuperscript{\dagger}Electronic address: Morteza.Hosseini@sbu.ac.ir
1. **INTRODUCTION**

Taking the flat-space limit (zero cosmological constant limit) of asymptotically AdS spacetimes results in asymptotically flat geometries. This procedure can be done by taking the $\ell \to \infty$ limit where $\ell$ is the radius of AdS spacetime. From the field theory perspective, one could expect that the $\ell \to \infty$ limit in the bulk theory has a holographic description at the boundary. Recently, it has been argued that the flat-space limit of AdS gravity is dual to the İnönü-Wigner contraction of the boundary CFT \[1, 2\].

The so-called Flat/CCFT correspondence has received a great deal of attention recently. For example, in \[3\] a Cardy-like formula has been obtained for the 2D CCFT which yields the correct entropy of three-dimensional cosmological solution. These asymptotically flat spacetimes can be obtained by taking the flat-space limit, as in \[4\], of non-extremal BTZ black holes. After taking the flat-space limit, the outer horizon of BTZ is mapped to
infinity, however, the value of the inner horizon remains finite and defines the cosmological horizon. The entropy of the cosmological solution has been identified with the area of the cosmological horizon. In the literature (see for example [5]), a modified Cardy formula has been introduced which reproduces the entropy of the inner horizon of BTZ black holes. The CFT origin of this formula has not been well-understood yet but the observation of [6, 7] is that if we accept the modified Cardy formula related to the inner horizon of BTZ and contract it by using appropriate parameters of CCFT, the final result is exactly the CCFT Cardy-like formula which yields the entropy of the cosmological horizon.

Furthermore, in [8], the authors found the correlation functions of CCFT energy-momentum by using the contraction of CFT ones and used it for finding quasi local stress tensor of the asymptotically flat spacetimes which gives the correct conserved charges of these geometries.

The Flat/CCFT correspondence can also propose a dual field theory which lives at the horizon of non-extreme black holes. The idea begins from the appearance of Rindler spacetime in the near horizon limit of non-extreme black holes. If one starts with Rindler-AdS/CFT correspondence [9, 10] and takes the flat-space limit in the bulk, which results in the Rindler spacetime, the boundary field theory is given by the contraction of the parent CFT. This proposal has been used in [11]. Therein, the authors found the Bekenstein-Hawking entropy of the non-extreme BTZ black hole by counting the CCFT microstates.

Moreover, Bagchi et al calculated the entanglement entropy (EE) of a 2D field theory with Galilean conformal symmetry$^1$ recently [12]. The authors used Wilson lines approach and found the holographic EE they computed is in precise agreement with the ones obtained in the field theory side. For an almost complete list of papers related to the Flat/CCFT correspondence see the references of [12].

In (2+1)-dimensional Einstein gravity black holes can exist only in the presence of a negative cosmological constant [13]. In order to find asymptotically flat black holes in three dimensions one has to consider higher derivative gravity theories. The entropy of these black holes can be obtained by Wald’s formula [14]. A successful theory of quantum gravity should be able to give a microscopic description of this semi-classical entropy. An alternative

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$^1$ The group of symmetries of CCFT$_2$ is isomorphic to Galilean Conformal Algebra (GCA) but in higher dimensions these are not the same.
approach for this study is using holography. The Falt/CCFT correspondence as a duality between quantum gravity in the asymptotically flat backgrounds and a field theory with contracted conformal symmetry can be an appropriate context to study this problem. The current paper is in this direction.

We consider a theory of gravity which is given by taking the flat-space limit of NMG \cite{15}. This theory possesses remarkable properties. In \cite{16}, it was shown that it is a ghost-free and power-counting UV finite, three-dimensional gravity. We use the dictionary of Flat/CCFT correspondence for finding quasi local stress tensor of the new type of asymptotically flat black hole \cite{17}. Using the holographic stress tensor along with the Brown and York’s method \cite{18}, we compute the conserved charges of this black hole. We also take limit from the Cardy formula and find a Cardy-like formula for the CCFT and show that this gives agreement with the Wald’s semi-classical approach.

The next sections are devoted to two main parts. In section two we introduce the bulk solutions. We start from NMG and review its asymptotically AdS rotating hairy black hole. Then we take the flat-space limit from the action and its black hole solution and introduce the asymptotically flat rotating hairy black hole with some novel properties. We calculate its entropy using Wald’s formula and check the first law of black hole thermodynamics for it. In section three we argue about the dual boundary theory of the bulk solution. We shortly review the known results about the dual CFT of NMG and then try to contract these results and find a CCFT which is dual to the higher derivative gravity theory of \cite{16}. This work is another check for the correctness of the Flat/CCFT correspondence.

2. BULK SOLUTIONS

2.1. Rotating hairy black hole of NMG

We consider three-dimensional higher derivative gravity theory known as NMG. This theory is described by the action \cite{15}

\[
S = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left[ R - 2\Lambda + \frac{1}{m^2} K \right],
\]  

(2.1)

where

\[
K = R_{\mu\nu}R^{\mu\nu} - \frac{3}{8} R^2.
\]  

(2.2)
The above theory (2.1) for the special case $m^2 \ell^2 = 1/2$ has the following rotating black hole solution [17]

$$ds^2 = -NF dt^2 + \frac{dr^2}{F} + r^2 \left( d\phi + N^\phi dt \right)^2,$$

(2.3)

where $N$, $N^\phi$, and $F$ are given by

$$N = \left[ 1 + \frac{b \ell^2}{4H} \left( 1 - \Xi \right) \right]^2,$$

$$N^\phi = -\frac{a}{2r^2} (4GM - bH),$$

(2.4)

$$F = \frac{H^2}{r^2} \left[ \frac{H^2}{\ell^2} + \frac{b}{2} \left( 1 + \Xi \right) H + \frac{b^2 \ell^2}{16} \left( 1 - \Xi \right)^2 - 4GM \Xi \right],$$

and

$$H = \left[ r^2 - 2GMC \left( 1 - \Xi \right) - \frac{b^2 \ell^4}{16} \left( 1 - \Xi \right)^2 \right]^{1/2},$$

$$\Xi = 1 - a^2/\ell^2.$$

(2.5)

It is labeled by three parameters: the mass $M$, the angular momentum $J = Ma$ and an additional “gravitational hair” parameter $b$. The rotation parameter $a$ is bounded according to $-\ell \leq a \leq \ell$.

The angular velocity of the horizon is

$$\Omega_+ = \frac{1}{a} \left( \Xi^2 - 1 \right).$$

(2.6)

We can associate a Hawking temperature and entropy to it

$$T = \frac{\Xi^{1/2}}{\pi \ell} \sqrt{2G\Delta M \left( 1 + \Xi \right)^{-1}},$$

(2.7)

$$S = \pi \ell \sqrt{\frac{2}{G} \Delta M \left( 1 + \Xi \right)},$$

(2.8)

where

$$\Delta M = M + \frac{b^2 \ell^2}{16G}.$$

(2.9)

These quantities fulfill the relation

$$TdS = \Xi^{1/2} \ dM + \frac{bl^2}{8G} \Xi^{1/2} \ db - \frac{1}{a} \left( 1 - \Xi \right) \Delta M \ da.$$
2.2. The flat-space limit of NMG and its black hole solution

In order to have a well-defined flat-space limit ($\Lambda \to 0$ or $\ell \to \infty$) for (2.1) in the special point $m^2\ell^2 = 1/2$, we need also to scale Newton’s constant to infinity while keeping fixed $\kappa = \ell^2/G$. Thus, the flat-space limit of NMG action (2.1) becomes

$$S = \frac{\kappa}{8\pi} \int d^3x \sqrt{-g} K. \quad (2.11)$$

Moreover, a well-defined flat-space limit for the black hole solution (2.3) needs a scaling of mass parameter $M$ such that $\mu = MG$ remains fixed. The final line element for the asymptotically flat rotating hairy black hole is given by

$$ds^2 = -\mathcal{F}dt^2 + \frac{r^2}{\Delta} dr^2 + a\mathcal{F}dtd\phi + r^2d\phi^2; \quad (2.12)$$

where $\Delta(r)$ and $\mathcal{F}(r)$ are functions of the radial coordinate $r$, given by

$$\Delta = r^2 - \mu a^2 - \left(\frac{a^2 b}{8}\right)^2,$$

$$\mathcal{F} = b\sqrt{\Delta} - 4\mu. \quad (2.13)$$

The Ricci scalar of this asymptotically flat black hole can be written as

$$R = -\frac{16b}{a^2 b + 8\sqrt{\Delta}}. \quad (2.14)$$

One can verify that (2.12) satisfies the equations of motion resulted from the action (2.11). In [16], it was argued that the three-dimensional gravity theory described by (2.11) is ghost-free and finite.

Horizons of (2.12) are at

$$r_+ = \frac{a^2 b}{8} + \frac{4\mu}{b}, \quad r_- = \sqrt{\left(\frac{a^2 b}{8}\right)^2 + \mu a^2}, \quad (2.15)$$

and one can calculate the entropy of the outer horizon by the Wald’s formula. This formula gives the black hole entropy in an arbitrary diffeomorphism invariant theory and is given by

$$S = -\frac{2\pi}{16\pi G} \int_{\Sigma_h} \frac{\delta L}{\delta R_{\alpha\beta\gamma\delta}} \epsilon_{\alpha\beta} \epsilon_{\gamma\delta} \bar{\epsilon}, \quad (2.16)$$

where $L$ is the Lagrangian, and $\bar{\epsilon}$, $\epsilon_{\mu\nu}$, denote the volume form and the binormal vector to the spacelike bifurcation surface $\Sigma_h$, respectively. $\epsilon_{\mu\nu}$ is normalized as $\epsilon_{\mu\nu} \epsilon_{\mu\nu} = -2$. For the
action (2.11) and the asymptotically flat rotating hairy black hole solution (2.12) we obtain
\[
\frac{\partial L}{\partial R_{\alpha \beta \gamma \delta}} = \frac{3}{8} R \left( g^{\alpha \delta} g^\beta \gamma - g^{\alpha \gamma} g^\beta \delta \right) + \frac{1}{2} \left( g^{\alpha \gamma} R^{\beta \delta} - g^{\alpha \delta} R^{\beta \gamma} - g^{\beta \gamma} R^{\alpha \delta} + g^{\beta \delta} R^{\alpha \gamma} \right),
\]
(2.17a)
\[
\epsilon_{\alpha \beta} = - \left( \frac{a^2 F + 4 r^2}{\Delta} \right)^\frac{1}{2} \delta^{t}_t \delta^{r}_r.
\]
(2.17b)
Therefore, the Wald’s entropy for the new type of asymptotically flat black hole becomes
\[
S_{\text{Flat}} = \frac{\pi \kappa b}{2}.
\]
(2.18)

It is instructive to drive the above entropy by taking the flat-space limit of the entropy (2.8). The entropy can therefore be computed as follows:
\[
\lim_{\ell \to \infty} S = \lim_{\ell \to \infty} \pi \ell \sqrt{\frac{2}{G} \Delta M \left( 1 + \Xi^\frac{1}{2} \right)} = \lim_{\ell \to \infty} \frac{\pi \kappa b}{2} \sqrt{1 + \frac{16 \mu}{b^2 \ell^2}} = \frac{\pi \kappa b}{2} = S_{\text{Flat}}.
\]
(2.19)

Now, consider Hawking temperature. From (2.7) it follows that
\[
\lim_{\ell \to \infty} T = \lim_{\ell \to \infty} \frac{1}{\pi \ell} \Xi^{\frac{1}{2}} \sqrt{2 G \Delta M \left( 1 + \Xi^\frac{1}{2} \right)^{-1}},
\]
\[
= \lim_{\ell \to \infty} \frac{b}{4 \pi} \sqrt{1 + \frac{16 \mu}{b^2 \ell^2}} = \frac{b}{4 \pi} = T_{\text{Flat}}.
\]
(2.20)

These quantities fulfill the relation
\[
T_{\text{Flat}} dS_{\text{Flat}} = \frac{b \kappa}{8} db.
\]
(2.21)

This agrees precisely with $\ell \to \infty$ limit of (2.10). A direct calculation by using (2.12) or taking the flat-space limit of (2.6) shows that the angular velocity of the black hole (2.12) at the outer horizon is zero ($\Omega_{\text{Flat}+} = 0$) though it has a non-vanishing angular momentum.

From (2.18) and (2.20), it is clear that the hair parameter $b$ determines the entropy and the temperature of the outer horizon. In the $b \to 0$ limit the hairy black hole (2.12) is reduced to the cosmological solution of [4]. In this limit $r_+$ is mapped to infinity, however, $r_-$ remains finite and defines the cosmological horizon.
3. DUAL BOUNDARY THEORY

3.1. CFT dual to NMG

In [19, 20], it was proposed that NMG has a dual description in terms of a conformal field theory (CFT). The charges associated to the asymptotic symmetries enhance the isometry of asymptotically $AdS_3$ spacetimes to two copies of the Virasoro algebra. The central charges are given by

\[ c_\pm = c = \frac{3\ell}{2G} \left( 1 + \frac{1}{2m^2\ell^2} \right). \]  

(3.1)

At the spacial point $m^2\ell^2 = 1/2$, the central charges being twice the values proposed by Brown and Henneaux for the Einstein gravity with negative cosmological constant [21], i.e.

\[ c = \frac{3\ell}{G}. \]  

(3.2)

The entropy of the black hole (2.3) can be given by the Cardy formula

\[ S = 2\pi \sqrt{\frac{c_+ \Delta_+}{6}} + 2\pi \sqrt{\frac{c_- \Delta_-}{6}}, \]  

(3.3)

where $\Delta_\pm$ are the eigenvalues of the left and right Virasoro generators $L_0^\pm$, and are given by

\[ \Delta_\pm = \frac{1}{2} \Delta M (\ell \pm a). \]  

(3.4)

Using (3.2) and (3.4), this is

\[ S = \pi \ell \sqrt{\frac{2}{G} \left( 1 + \Xi^2 \right) \Delta M}, \]  

(3.5)

in precise agreement with (2.8).

3.2. CCFT dual to the flat-space limit of NMG

In this section we want to propose a dual description for the theory of gravity given by (2.11). To do so, we will use the idea which has been firstly proposed in papers [1, 2]. That is, if we start from AdS/CFT correspondence, the large AdS radius limit in the bulk is equivalent to the contraction of coordinates in the boundary CFT. For our current problem, since the theory (2.11) and the black hole (2.12) are given by taking flat-space limit from the action (2.1) and its black hole solution (2.3), the Flat/CCFT correspondence proposes
a CCFT as the holographic dual of (2.11). To find the appropriate coordinate which must
be contracted in the parent CFT, let us look at the conformal boundary of the black hole
(2.12) for an arbitrary large $\ell$. It could be written as follows:

$$ds^2_{\text{C.B.}} = \frac{r^2}{\kappa^2} \left( -\frac{\kappa^2}{\ell^2} dt^2 + \kappa^2 d\phi^2 \right).$$

We have used $\kappa$ in the conformal factor to make it dimensionless. Moreover, the fact that
$\kappa$ is fixed in our flat-space limit makes the conformal factor well-defined for all large values
of $\ell$. Now, $\ell$ can be absorbed by defining new time coordinate as $\tau = \kappa t/\ell$. The dual CFT
lives on a cylinder with coordinates $(\tau, \phi)$ and radius $\kappa$. Taking $\ell \to \infty$ limit (or $\kappa/\ell \to 0$
limit ), it is obvious that the flat-space limit in the bulk side corresponds to contract time
in the boundary field theory.

### 3.2.1. Symmetries of CCFT

According to the proposal of [1, 2], the symmetries of CCFT realize the group of asymp-
totic symmetries of the asymptotically flat spacetimes at null infinity, namely the BMS
group [22, 25]. The CCFT algebra emerges from the İnönü-Wigner contraction of relativis-
tic conformal symmetries of the parent CFT. Precisely, if we start from two copies of the
Virasoro algebra (as the symmetries of two-dimensional CFT)

$$[L^+_m, L^-_n] = (m - n)L^+_{m+n} + \frac{c}{12} m(m^2 - 1)\delta_{m+n,0},$$

$$[L^-_m, L^-_n] = (m - n)L^-_{m+n} + \frac{c^-}{12} m(m^2 - 1)\delta_{m+n,0},$$

$$[L^+_m, L^-_n] = 0,$$

and define new generators

$$L_n = L^+_n - L^-_{(-n)},$$

$$M_n = \epsilon \left( L^+_n + L^-_{(-n)} \right),$$

then looking at the limit $\epsilon \to 0$, the final algebra reads

$$[L_m, L_n] = (m - n)L_{m+n} + \frac{c}{12} m(m^2 - 1)\delta_{m+n,0},$$

$$[L_m, M_n] = (m - n)M_{n+m} + \frac{cM}{12} m(m^2 - 1)\delta_{m+n,0},$$

$$[M_m, M_n] = 0,$$

(3.9)
where central charges $c_L$ and $c_M$ are given by the linear combination of the parent relativistic central charges

$$c_L = \lim_{\epsilon \to 0} (c^+ - c^-), \quad c_M = \lim_{\epsilon \to 0} \epsilon (c^+ + c^-). \quad (3.10)$$

The algebra (3.9) which is given by the contraction of the Virasoro algebra in the boundary theory is exactly the (centrally extended) BMS$_3$ algebra [23].

We expect the same symmetry for the CCFT dual of (2.11) at the special point $m^2 \ell^2 = 1/2$. The $\epsilon \to 0$ limit in the boundary corresponds to the flat-space limit or more precisely $\kappa/\ell \to 0$ limit in the bulk side. Using (3.2) and (3.10), we find

$$c_L = c_M = 0 \quad (3.11)$$

for the current problem. We will show that although the CCFT algebra has vanishing central charges, it is possible to find a Cardy-like formula for the asymptotic growth of the number of states which reproduces the entropy of the black hole (2.12). To add strength to this claim, let us find more evidence about the correctness of our proposal using CCFT energy-momentum tensor.

### 3.2.2. Quasi local stress tensor

The one-point function of CCFT energy-momentum operator corresponds to quasi local stress tensor of bulk theory. It was argued in [8], the definition (3.8) provides a recipe to calculate the components of stress tensor in the asymptotically flat spacetimes. Therefore, we can write

$$\tilde{T}_{++} + \tilde{T}_{--} = \lim_{\epsilon \to 0} \epsilon (T_{++} + T_{--}) ,$$

$$\tilde{T}_{++} - \tilde{T}_{--} = \lim_{\epsilon \to 0} (T_{++} - T_{--}) ,$$

$$\tilde{T}_{+-} = \lim_{\epsilon \to 0} T_{+-} , \quad (3.12)$$

where $T_{ij}$ and $\tilde{T}_{ij}$ are respectively the stress tensor of asymptotically AdS and flat spacetimes and $x^\pm$ are the light-cone coordinates constructed by non-radial coordinates of the metrics. In the above definition it was assumed that both asymptotically AdS and flat spacetimes are given in the BMS gauge [8].
The non-zero components of the stress tensor at the boundary of the asymptotically AdS black hole (2.3) is given by [26]

\[
T_{tt} = \frac{1}{8\pi G\ell} \left( \frac{b^2\ell^2}{4} + 4MG \right), \\
T_{t\phi} = -\frac{a}{8\pi G\ell} \left( \frac{b^2\ell^2}{4} + 4MG \right), \\
T_{\phi\phi} = \frac{\ell}{8\pi G} \left( \frac{b^2\ell^2}{4} + 4MG \right). \tag{3.13}
\]

The formula (3.12) results in a stress tensor \( \tilde{T}_{ij} \) for the asymptotically flat black hole (2.12) as follows:

\[
\tilde{T}_{tt} = \frac{b^2}{32\pi}, \quad \tilde{T}_{\phi\phi} = \frac{\kappa b^2}{32\pi}, \quad \tilde{T}_{t\phi} = -\frac{ab^2}{32\pi}. \tag{3.14}
\]

Using \( \tilde{T}_{ij} \) we can calculate the conserved charges of the black hole (2.12).

Let us denote the hypersurface of the spacetime where CCFT lives with \( \partial M \). Its line element is given by taking the \( \ell \to \infty \) limit of the conformal boundary (3.6),

\[
ds^2_{\partial M} = \frac{r^2}{\kappa^2} (-dt^2 + \kappa^2 d\phi^2). \tag{3.15}
\]

Following Brown and York’s method [18], the charges associated to a boundary Killing vector \( \xi^\mu \) is given by

\[
Q_\xi = \int_{\Sigma} d\phi \sqrt{\sigma} \xi^\mu n^\nu \tilde{T}_{\mu\nu}, \tag{3.16}
\]

where \( \Sigma \) is the spacelike surface embedded in \( \partial M \) with induced metric \( \sigma_{\mu\nu} \). Moreover, \( n^\mu \) is the timelike unit normal to \( \Sigma \). Using (3.16) and (3.15), the mass and the angular momentum of the asymptotically flat black hole (2.12) are

\[
\mathcal{M} = Q_{\partial_t} = \frac{\kappa b^2}{16}, \quad \mathcal{J} = Q_{\partial_\phi} = -\frac{\kappa ab^2}{16}. \tag{3.17}
\]

It is clear that \( |\mathcal{J}|/\mathcal{M} = a \) as expected.

Given the expressions above, together with (2.21), it is straightforward to check that the first law of black hole thermodynamics is satisfied, i.e.

\[
d\mathcal{M} = T_{\text{Flat}} dS_{\text{Flat}} - \Omega_{\text{Flat}} d\mathcal{J}. \tag{3.18}
\]

3.2.3. Cardy-like formula

If the gravity theory (2.11) has a dual description in terms of a CCFT, then the entropy of the black hole (2.12) must be given by the asymptotic growth of states in the boundary
theory. In [3], the authors found a Cardy-like formula by computing CCFT partition function using saddle point approximation. However, in the recent papers [6, 7] it was shown that the Cardy-like formula of [3] can be obtained if one writes the Cardy formula in terms of CCFT parameters and then takes $\epsilon \to 0$ limit. In the current work we will use the same approach and take $\epsilon \to 0$ limit from the Cardy formula in the parent CFT.

The symmetries of CCFT are given by (3.9). For the current problem $c_L = c_M = 0$, however, the eigenvalues of $L_0$ and $M_0$ are non-zero. We denote the eigenvalues of $L_0$ and $M_0$ by $\Delta_L$ and $\Delta_M$, respectively. Thus, we can write

$$\Delta_L = \lim_{\epsilon \to 0} (\Delta_+ - \Delta_-), \quad \Delta_M = \lim_{\epsilon \to 0} \epsilon (\Delta_+ + \Delta_-).$$

(3.19)

The $\epsilon \to 0$ limit in the boundary corresponds to $\kappa/\ell \to 0$ limit in the bulk. Therefore, using (3.4) and (3.19) one can easily find

$$\Delta_L = \frac{a\kappa b^2}{16}, \quad \Delta_M = \frac{\kappa^2 b^2}{16}.$$  

(3.20)

Let us consider the Cardy formula (3.3) and try to take its $\epsilon \to 0$ limit. For our current problem the relativistic central charges are $c_+ = c_- = 3\epsilon$. Using (3.20), we obtain

$$\lim_{\epsilon \to 0} S_{\text{CFT}} = \lim_{\epsilon \to 0} 2\pi \left( \sqrt{\frac{c_+ \Delta_+}{6}} + \sqrt{\frac{c_- \Delta_-}{6}} \right)$$

$$= \lim_{\epsilon \to 0} \pi \left[ \sqrt{\epsilon \left( \frac{\Delta_M}{\epsilon} + \Delta_L \right)} + \sqrt{\epsilon \left( \frac{\Delta_M}{\epsilon} - \Delta_L \right)} \right]$$

$$= 2\pi \sqrt{\Delta_M} = S_{\text{CCFT}}$$  

(3.21)

This is the Cardy-like formula for the CCFT dual to the flat-space limit of NMG. Inserting (3.20) into (3.21), we finally recover the entropy (2.18):

$$S_{\text{CCFT}} = S_{\text{Flat}},$$  

(3.22)

as we wanted to show.

4. CONCLUSIONS

In this paper, we have provided the first example of a holographic dual of an asymptotically flat black hole solution. Due to the absence of black hole solutions in three-dimensional pure Einstein gravity with vanishing cosmological constant, we have considered
higher derivative gravity theories which admit asymptotically flat black hole solutions. The theory we have investigated is given by the flat-space limit ($\Lambda \to 0$) of NMG. We argued the dual field theory of the black hole solutions of this theory is a CCFT. For this purpose, we have constructed a stress tensor for the asymptotically flat black hole solutions and computed the conserved charges. Furthermore, we have used Flat/CCFT correspondence to find the black hole entropy in terms of the asymptotic growth of the number of CCFT states.

It is interesting to note that, the symmetry algebra of the corresponding CCFT had vanishing central charges though the asymptotic growth of states are non-zero. This remarkable point can be used for finding holographic duals of four-dimensional asymptotically flat space-times. According to the proposal of Flat/CCFT correspondence, the dual of asymptotically flat 4D black holes are field theories with BMS$_4$ symmetry [24, 25]. In [27], authors constructed the field dependent central extension of BMS$_4$ algebra and found that for the Kerr black hole some of the charges involved divergent integrals on the 2-sphere if they used extended BMS algebra with both supertranslations and superrotations. Thus, at first sight it seems counting CCFT$_3$ states would be a problematic issue but our current work shows that counting the asymptotic growth of CCFT states can be done whatever the central charges are.

Although our current study gives a holographic description of asymptotically flat black holes in three-dimensional higher derivative gravity, we believe that the Flat/CCFT correspondence can be extended to find a holographic description of black holes in higher dimensions and specifically four-dimensional Kerr black hole. We hope to explore other intriguing aspects of the relation between asymptotically flat spacetimes and CCFTs in our future works.

**Acknowledgment**

The authors would like to thank Ali Naseh for useful discussions.

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