Weak localisation, interaction effects and the metallic phase in p-SiGe

P.T. Coleridge, A.S. Sachrajda and P. Zawadzki

Institute for Microstructural Sciences, National Research Council, Ottawa, Ontario, K1A OR6, Canada
(21 December 2001)

Magnetoresistance results are presented for p-SiGe samples on the metallic side of the B=0 metal-insulator transition. It was possible to separate the weak localisation and Zeeman interaction effects but the results could not be explained quantitatively within the framework of standard theories for quantum corrections of a weakly interacting 2-dimensional system. Analysis using a theory for interaction corrections at intermediate temperatures, recently proposed by Zala, Narozhny and Aleiner, provided values of the Fermi liquid parameter $F_0$ of order -0.5. Similar values also explain the linear increase of resistance with temperature characteristic of the metallic phase.

PACS numbers: 71.30.+h, 72.20.-i, 73.20.Dx

A. Introduction

Recent experiments indicating the existence of a metallic state and a metal-insulator transition (MIT) in two-dimensional (2-D) semiconductor systems continue to attract attention. There is, as yet, no general consensus about the origin of the metallic behaviour and it remains a controversial topic. The metallic behaviour appears in strongly interacting systems where $r_s$ (the ratio of the interaction energy to the kinetic energy) is large, typically 5 to 20 and the Coulomb interaction energy is by far the largest energy in the problem. While the existence of a MIT in 2-dimensional systems contradicts the well established one parameter scaling theory for non-interacting systems, it is not a priori forbidden for strong interactions and the observation of good scaling behaviour, with symmetry about the critical density, supports the view that this is a genuine transition, driven by the interactions. This is also consistent with predictions of renormalisation group (RG) theories that a low temperature metallic phase can exist when interactions and disorder are both important.

An alternative view is that there is no transition, just a cross-over from weakly localised to strongly localised behaviour, and that the strong interactions do not significantly modify the basic Fermi liquid character. Large values of $r_s$ are almost inevitably associated with low densities and Fermi energies ($k_B T_F$) that are not much larger than the measuring temperatures so effects associated with the small Fermi degeneracy are likely to be significant. It is argued that these can fully account for the metallic-like increase in resistivity with temperature and that weak localisation and interaction effect corrections are present but concealed by the larger semi-classical effects.

Experimental support for this second viewpoint has recently been presented based on results obtained in p-GaAs and p-SiGe. In both cases the B=0 resistivity shows no direct evidence of a lnT dependence, the standard signature of both weak localisation and interaction effects but the magnetoresistance has the characteristic negative peak associated with the destruction, by dephasing, of weak localisation. Also the Hall coefficient shows a lnT dependence, interpreted as evidence for interaction corrections. The magnitude of both these terms is consistent with the standard predictions and it is argued the behaviour is that of an entirely conventional Fermi liquid, despite the large values of $r_s$.

Similar measurements, in p-SiGe samples, are presented here. The experimental data is generally consistent with results obtained in other samples but is more detailed and is interpreted somewhat differently. It is suggested that the behaviour cannot be explained using the standard theories for weakly interacting Fermi liquid systems. Not only are the parameters characterising the interaction too large to justify the assumption of weak interactions but also there are internal inconsistencies in the analysis. At low temperatures the saturation of resistance is generally consistent with the predictions of renormalisation group theories and at higher, intermediate, temperatures the results can be understood within the framework of a new theory for interaction effects. In both cases a large amplitude of spin fluctuations plays an essential role.

B. Theory: weak interactions

Because the data presented below is discussed, initially, in terms of the quantum corrections for weakly interacting systems it is convenient to summarise the standard theoretical treatments, which have been used, historically, to analyse experiments. They start with a semi-classical description of the transport, where the diffusion of electrons at the Fermi level is considered to obey classical dynamics and quantum effects are introduced as small corrections. Drude-Boltzmann theory gives the zero field
conductivity as
\[ \sigma_0 = n_e e^2 \tau / m^* = \frac{g_s k_f e^2}{2} \]

where \( n_e \) is the carrier density, \( \tau \) the transport lifetime, \( m^* \) the effective mass, \( g_s \) the spin degeneracy, \( k_f \) the Fermi wavevector and \( l \) the mean free path. In a magnetic field the conductivity components are given by
\[ \sigma_{xx}(B) = \frac{\sigma_0}{(1 + \mu^2 B^2)}; \quad \sigma_{xy}(B) = \mu B \sigma_{xx}(B) \]  
\[ (2) \]

where the mobility \( \mu = e\tau / m^* \). Quantum interference introduces a weak localisation correction to this conductivity with a logarithmic temperature dependence
\[ \Delta \sigma_{xx}^{wl}(T) = \alpha_p (e^2 / \pi h) \ln(k_B T \tau / h) \]  
\[ (3) \]

where it is assumed the phase breaking time \( \tau_\phi \) varies as \( T^{-p} \). The amplitude \( \alpha \) is expected to be 1 for normal scattering (-0.5 for pure spin-orbit scattering and 0 for spin scattering).

A lnT dependence also results from the Coulomb interaction effect
\[ \Delta \sigma_{xx}^{c}(T) = (1 - \frac{3F^*}{4})(e^2 / \pi h) \ln(k_B T \tau / h). \]  
\[ (4) \]

Two processes, with opposite signs, contribute here. A Hartree term (singlet channel), involving only small momentum transfers, and an exchange term (triplet channel) involving \( F^* \), the Fermi surface average of the screened Coulomb interaction. Increasing \( F^* \) implies an increased tendency towards delocalising behaviour. For Thomas-Fermi screening \( F^* < 1 \) but it may, in principle, be larger. In this case a weakly interacting theory is inappropriate and should be replaced by more sophisticated approaches but with trends that are probably still given correctly by eqn.4. Large values of \( F^* \) imply negative coefficients for the interaction term which may even overcome the weak localisation term and result in a total negative, or delocalising, lnT dependence. Values of \( F^* \) larger than one are often obtained when fitting experimental data.

Application of a magnetic field allows the weak localisation and interaction terms to be separated. At low fields dephasing of the weak localisation term gives a characteristic, negative magnetoresistance
\[ \Delta \sigma_1(B) = \alpha_p e^2 / \pi h \left[ \Psi \left( \frac{1}{2} \right) + \frac{\tau_B}{2\tau_\phi} - \Psi \left( \frac{1}{2} + \frac{\tau_B}{2\tau_\phi} \right) + \ln \left( \frac{\tau_\phi}{\tau} \right) \right] \]  
\[ (5) \]

where \( \Psi \) is the digamma function and \( \tau_B = h / 2eDB \) with \( D \) (the diffusion constant) = \( v_f^2 / 2 \). In terms of the variable \( h = 2\tau_\phi / \tau_B \) (proportional to \( B \) and not to be confused with the Planck constant) this function varies as \( h^2 / 24 \) for small \( h \) and as \( \ln(h) \) for \( 1 < h < \tau_\phi / \tau \). Fitting to the characteristic shape, particularly at small \( B \), allows an experimental determination of \( \alpha \) and \( \tau_\phi \). Although commonly used to analyse experimental data eqn.5 is known to be in error for large fields, when the diffusion limit is no longer valid. In this case, however, it has been shown that the equation still gives a good fit to the data but with smaller values of \( \alpha \) (typically 0.5 -0.7) and fitted values of \( \tau_\phi \) that are in error by 10-20%.

Magnetic fields corresponding to \( h \sim 1 \), sufficient to dephase the weak localisation, are too small to significantly affect the interaction term but higher fields, large enough to produce a Zeeman spin-splitting, modify the triplet part of this term. This results in a positive magnetoresistance given by
\[ \Delta \sigma_2(B) = -(e^2 / \pi h)(F^*/2)G(b) \]  
\[ (6) \]

where \( b = g^* \mu_B B / k_BT \) with \( g^* \) the g-factor of the spins. The function \( G(b) \) is known: for small \( b \) \( G(b) = 0.084b^2 \), for large \( b \) it varies as \( \ln(b^2/1.3) \), and it can be calculated in the intervening region.

In addition to these terms there is also the classical magnetoconductance (eqn.2). For small \( B \), this gives a quadratic correction term
\[ \Delta \sigma_{xx}(B) \approx -\sigma_0 \mu^2 B^2. \]  
\[ (7) \]

For the Hall conductivity \( \sigma_{xy} \) the weak localisation corrections appear a factor of two larger but there are no interaction corrections. Therefore, when the Hall coefficient \( R_H = \rho_{xy} / B \) is obtained by inverting the conductivity tensor the weak localisation terms cancel and
\[ \Delta R_H / R_H = -2\Delta \sigma_{xx}^{ee} / \sigma_{xx}. \]  
\[ (8) \]

The Hall coefficient is therefore expected to have a logarithmic temperature dependence (given by eqns.4 and 8) and a field dependence (eqn.6) but should not include any weak localisation corrections.

If, in low fields, \( \rho_{xx} \) rather than \( \sigma_{xx} \) data is analysed the lnT weak localisation and interaction effect terms appear additively in the same way but the quadratic classical term (eqn.7) is automatically cancelled by the \( \sigma_{xy} \) contribution. At higher fields, when \( \mu B \) becomes significant, the admixture of \( \sigma_{ee} \) and \( \sigma_{xy} \) contributions introduces an additional term, \( \Delta \rho_{xx} = (\mu^2 B^2 - 1)\Delta \sigma_{xx}^{ee} / \sigma_0^2 \), a parabolic negative magnetoresistance with a temperature dependence given by eqn.4.

**C. Samples**

The samples used in this investigation are from a set of p-type modulation doped strained Si-Ge quantum wells which exhibit MIT behaviour. Grown using a low temperature UHV-CVD process they have a Si buffer layer, a 40nm Si,88Ge,12 quantum well, a spacer layer (of variable thickness), a boron doped layer and a thin Si cap. The holes reside in an approximately triangular SiGe quantum well, produced by the asymmetric doping, which is
strained because it is lattice matched to the Si substrate. It has been established that they are in almost pure |\paration{M_{J}}| = 3/2 states, well decoupled from other hole states by strain and confinement. Most of the data presented here comes either from Sample A, with a density (\paration{p_{h}}) of 5.7×10^{11}cm^{-2} which is deep in the metallic phase or from sample B, with a density of 1.2×10^{11}cm^{-2}, which is close to the critical density for the MIT (approximately 1.0×10^{11}cm^{-2}). Values of \paration{\tau_{s}} (defined as \frac{1}{(\pi p_{h})^{1/2}a^{*}}}, where \paratr{a^{*}} is the effective Bohr radius using the semiconductor dielectric constant) are 4 and 6, respectively, for the two samples.

Effective mass values are known only approximately in this system. Measurements from the temperature dependence of the Shubnikov-de Haas oscillations gave values of 0.30\paratr{m_{0}} for sample A and approximately 0.23\paratr{m_{0}} for sample B. These are significantly larger than the band mass of 0.20\paratr{m_{0}}. It is possible this reflects a breakdown of the standard Lifshitz-Kosevitch expression in a system with strong quantum corrections. High field cyclotron resonance measurements have given values as low as 0.18\paratr{m_{0}}.

The g-factor (\paratr{g^{*}}) in p-SiGe is also not well known but is of order 4 in perpendicular fields. In sample A the low field Shubnikov-de Haas (SdH) oscillations appear initially at odd filling factors corresponding to a Zeeman splitting of the Landau levels sufficiently large that \paratr{g^{*}m^{*}/m_{0}} > 1. With \paratr{m^{*}/m_{0}} = 0.25 - 0.30 \paratr{g^{*}} is therefore of order 4. In sample B the SdH oscillations appear initially at even filling factors so \paratr{g^{*}m^{*}/m_{0}} < 1. Using \paratr{m^{*}} = 0.23\paratr{m_{0}} and taking \paratr{g^{*}m^{*}/m_{0}} to be 0.8±0.2 gives \paratr{g^{*}} = 3.6±1.

D. Experimental procedures

Measurements were made using a DC bridge, with current reversal at 15Hz. Two cryostats were used, a dilution refrigerator giving temperatures below 100mK and a sorption pumped He3 system that could achieve sample temperatures down to 270mK. In both cases thermometry was in a field free region, with the sample cooled predominantly by the copper leads. Measurements using a calibrated, field insensitive, thermometer in place of the sample indicated temperature gradients of less than 5mK.

Resistance measurements, especially in sample B, showed an unusual sensitivity to measuring current at the lowest temperatures. Currents as low as 1nA, corresponding to voltages of only a few microvolts, produced detectable metallic-like non-linearities in the I-V characteristics and made accurate low temperature measurements of the zero field resistivity difficult. In addition, sweeping the magnetic field caused abnormally large heating (and cooling) effects. These phenomena are not well understood but are attributed, at least in part, to the proximity of the MIT. Very slow field sweeps were used with frequent checks (making corrections if necessary) to ensure that the measured resistances were the same in static and swept fields. In the He3 system measurements in magnetic fields were also complicated by a superconducting solenoid that had an unexpectedly large degree of trapped flux. The associated hysteresis was, however, found to be reproducible and a calibration procedure could be established that resulted in field errors, around B=0, estimated to be less than 1 mT.

E. Experimental results

Magnetoresistance data from sample A is shown in figure 1. At zero field the temperature dependence of the resistance is characterised by an approximately constant low temperature region followed by a monotonic, metallic-like, increase with temperature. This is characteristic of samples showing a MIT. For this sample, at 20K, the resistance has increased by a factor of roughly two. Over the temperature range shown (0.15 - 2.2K) a logarithmic weak localisation term (eqn.3) would be expected to give a decrease of resistivity of order 100 ohms/square, roughly a factor of two larger and in the opposite sense to the actual experimentally observed change. The well defined low field negative magnetoresistance, with a width that increases with temperature, is attributed to weak localisation; the positive magnetoresistance at higher fields to the Zeeman interaction term.

Attempts to fit the field dependence to a combination of eqns. 5 and 6 failed, even when five separate fitting parameters (including the g-factor) were used. The alternative approach taken was therefore to fit the data to eqn. 5 at the lowest fields, using just three parameters: \paratr{\tau}, \paratr{\tau_{0}} and an amplitude \paratr{\alpha}. The results of these fits are shown in figure 1. Care was taken to establish that the parameters were insensitive to the precise range of B and also that they were not significantly altered when this range was extended and an additional quadratic term included. Values of \paratr{\tau_{0}}, shown in figure 2, are similar to those obtained from other measurements in p-SiGe. Fitted values of \paratr{\alpha} were 0.7 at the lowest temperatures decreasing to 0.6 at higher T.

The residues from these fits are shown in fig.3a. Plotted against B/T (fig. 3b) they collapse onto a single curve which gives some confidence that the fitting procedure has successfully separated the weak localisation term from a Zeeman interaction term which is a function of B/T. This curve is not, however, given by eqn.6. The solid line in fig. 3b is G(b); fitted to the curve for larger values of B/T, with \paratr{F^{*}} = 2.45 and a g-factor of 6.4. This deviates significantly from the data at low values of B/T and the value for \paratr{g^{*}} is somewhat larger than expected. We believe this deviation is not an artefact of the fitting procedure but rather is consistent with the
fact, noted above, that it was impossible to fit individual data curves to a combination of equations 5 and 6.

The most important conclusion, despite the only qualitative agreement between theory and experiment, is the large value of $F^\ast$. This implies that the interactions are strong and that the standard theory is probably inadequate.

Expressions for the Zeeman interaction term have also been obtained within the framework of renormalisation group theory by Finkel’stein for $b \gg 1$,

$$\Delta \sigma_2(B) = -(e^2/\pi h) 2 \left[ \frac{1+\gamma_2}{\gamma_2} \ln(1+\gamma_2) - 1 \right] \ln(b) \quad (9)$$

and by Castellani et al for $b \ll 1$

$$\Delta \sigma_2(B) = -0.084(e^2/\pi h) \gamma_2 (1 + \gamma_2) b^2. \quad (10)$$

Here $\gamma_2$ is a measure of the renormalised interaction strength for the triplet scattering channel. It reduces to $F^\ast/2$ in the limit of weak interactions.

The dashed lines in fig. 3b show fits using these two equations in the high field and low field limits respectively, using $g^\ast = 3.6$ and $\gamma_2 = 2.6$ in both cases. Interpolating between the high field and low field expressions then gives a very reasonable description of the experimental data. The value of $g^\ast$ is perhaps a little small but this is predominantly determined by the low field quadratic regime where experimental errors are relatively large.

It might perhaps be argued that because eqn.5 is invalid at high fields, in the ballistic regime, the subtraction of the weak localisation term introduces an error and the residues should not be attributed solely to a Zeeman term. It has been shown, however, that this equation provides an adequate description of the weak localisation in this regime but with a prefactor $\alpha$ that is less than unity. The values of $\alpha = 0.6 - 0.7$ determined experimentally are consistent with this interpretation and imply that the residues can correctly be identified with the Zeeman term. Further confirmation is provided by the fact that any spurious contribution from the weak localisation term would not have the observed functional dependence on $B/T$.

Magnetoresistance data for sample B is shown in figure 4. Here, again, the zero field increase of resistance with increasing temperature in the opposite sense to that expected for weak localisation. Over the temperature range shown the expected $\ln T$ weak localisation term would correspond to a decrease in resistivity of approximately 2000 ohms/square. The negative magnetoresistance associated with weak localisation is less well defined than in sample A, mainly because the ratio $\tau_\phi/\tau$ is significantly smaller. Because of this and because the Hall coefficient is temperature dependent (see below) it was not possible to just fit $\rho_{xx}$ and rely on the Hall term to cancel the classical quadratic term (eqn.7). Rather it was necessary to explicitly determine and fit the conductivity $\sigma_{xx}$. Shown in figure 5 (and obtained by inverting the measured values of $\rho_{xx}$ and $\rho_{xy}$) this is dominated by a quadratic dependence on field that is the sum of the classical term and the low field Zeeman interaction term (eqns. 6 or 10).

The rather small negative magnetoresistance associated with the weak localisation made it impossible to use the fitting procedure developed for sample A: the equations became ill-conditioned and gave imprecise and interdependent values of $\alpha$ and $\tau_\phi$. The fit is relatively insensitive to the exact value of $\alpha$ (see also Senz et al) so this was fixed (at 0.65) and just three adjustable parameters used: $\tau$, $\tau_\phi$ and a quadratic coefficient. As can be seen in figure 5 these gave very good descriptions of the data provided the fits were restricted to the field region where the Zeeman interaction term is expected to vary quadratically with $B$. Values of $\tau_\phi$ are shown in figure 2.

Subtracting off the weak localisation and classical mobility terms should leave the Zeeman interaction term but there is some uncertainty about exactly how to determine the classical term. In practice $\rho_{xx}$ was obtained at each temperature by adding the amplitude of the weak localisation correction, ie $\alpha(e^2/\pi h) \ln(\tau_\phi/\tau)$, to the measured value and then using this, with the known density, to obtain the mobility. This procedure was applied self-consistently so the value of $\tau$ determined from the mobility was also used in the weak-localisation fit. The Zeeman interaction term obtained in this way is plotted against $B/T$ in figure 6. The errors could be quite large here, for example the classical mobility correction is approximately equal to the Zeeman term at the highest temperatures, but the good collapse onto a single curve suggests they are not, in fact, significant.

The dashed line in figure 6 shows a fit of the data to $G(b)$ assuming a $g$-factor of 3.6. In contrast to sample A (figure 4b), the deviation away from quadratic towards logarithmic behaviour at higher fields appears to be adequately described by eqn. 6 and corresponds to $F^\ast = 1.95$. Alternatively, using eqn. 10 in the quadratic region, $\gamma_2 = 0.6$. These values are significantly smaller than in sample A. Taking eqn. 4 at face-value, and assuming a weak localisation term is present, the value of $F^\ast = 1.95$ is close to the cross-over between a net localising or delocalising $\ln T$ dependence.

According to the standard approach an independent measure of $F^\ast$ can be obtained from the temperature dependence of the Hall coefficient. This should have a $\ln T$ dependence given by eqns.4 and 8, ie be proportional to $(1 - 3F^\ast/4)$. Measurements of $R_H$ at very low fields are complicated by any small magnetic field error. Cubic fits to $\rho_{xy}$ were made in this region and matched onto direct measurements at higher fields. Results are shown in figures 7 and 8.

In sample A the very small, non-monotonic, temperature dependence around $B=0$ is of the same magnitude as the estimated experimental errors. Over the temperature range shown $R_H$ deviates by less than 2% from 1090 ohms/tesla, the value corresponding to a density of $5.7 \times 10^{11}$ cm$^{-2}$ determined from the periodicity of the Shubnikov-de Haas (S-DH) oscillations. Taken at face
value this implies \( F^* \approx 1.33 \), roughly a factor two smaller than the value determined above from the Zeeman interaction term. At higher fields a significant field and temperature dependence develops. Unlike the data shown in figure 3b this is not a good function of \( B/T \) and is also a factor of roughly two smaller than expected for \( F^* \sim 1.3 \). It should be noted, in contrast to measurements recently reported in p-GaAs,[4] that these corrections to the Hall coefficient are not reflected in the Shubnikov-de Haas oscillations. The period of these oscillations, and of the corresponding oscillations in \( \rho_{xy} \) that can be seen in figure 7, has no significant temperature dependence.

In sample B (figure 8) the Hall coefficient at \( B=0 \) shows a clear \( \ln T \) dependence (see inset) and is significantly increased over the value of 5400 ohm/tesla that corresponds to the density of \( 1.16 \times 10^{11} \text{cm}^{-2} \) obtained from the periodicity of the Shubnikov-de Haas oscillations. Although less evident than in sample A, the period of these is again, unaffected by the enhanced Hall coefficient. Using eqns. 4 and 8 the slope of the \( \ln T \) dependence shown in the inset (\( \Delta R_H/R_H = -0.4 \ln(T) \)) corresponds to \( F^* = 1.1 \). The approximately quadratic increase of \( R_H \) with magnetic field is again, a poor function of \( B/T \) and less than half the expected value.

Within RG theory eqn. 4 is modified to

\[
\Delta \sigma_{xx}^{ee}(T) = [4 - 3 \frac{1 + \gamma_2}{\gamma_2} \ln(1 + \gamma_2)][(e^2/\pi h) \ln(k_B T \tau/\hbar)].
\]

(11)

Interpreted using eqn.8 this implies \( \gamma_2 = 0.65 \).

For temperatures such that \( k_B T \tau/\hbar \sim 1 \) the screening function, and therefore the Drude scattering time, becomes temperature dependent. The corresponding temperature dependent conductivity has been calculated by Gold and Dolgopolov[20] as

\[
\sigma(T) = \sigma(0)[1 - C_p(T/T_F)].
\]

(12)

Similar results have also been obtained numerically by Das Sarma[21].

Senz et al on the basis of the analysis of measurements in other p-SiGe samples, have suggested that this can account for the absence of a weak localisation \( \ln T \) term in the zero field conductivity. They argue that such a term is in fact present but when combined with the linear temperature dependent screening result gives an apparent low temperature saturation of resistivity. A similar analysis is presented here: for samples A and B and also for sample C (which has a density very similar to sample B but a significantly larger peak mobility) and sample D which has a very similar density to sample A but where the measurements extend to a much lower temperature.

The solid points in figure 9 show the measured conductivities in these samples, plotted against \( T/T_F \). Open points show the result of subtracting a putative weak-localisation correction \( \alpha \rho(e^2/\pi h) \ln(k_B T \tau/\hbar) \) and the lines are linear fits to the high temperature data. At higher temperatures the modified results, like the raw data, all exhibit the predominantly linear dependence predicted by eqn. 12. The coefficients \( C_p \) are, however, larger than expected. With impurity and interface roughness scattering dominating, \( C_p \) for samples A and D is expected to be 1.2 - 1.5, compared with experimental values of 2.5 and for samples B and C the values of 5.0 and 6.1 shown are two to three times the expected value of about 2. Also, in the adjusted data, there are significant deviations from a linear dependence at the lowest temperatures. The upturn there reflects directly the \( \ln T \) divergence of the weak localisation term subtracted from approximately constant experimental data.

Although in qualitative agreement with the general picture proposed by Senz et al these results do not support their detailed conclusions. The data presented here cannot be explained as just the sum of a \( \ln T \) weak localisation term and a linear temperature dependent screening term. There is either a real saturation of the resistivity at low temperatures or if the weak localisation \( \ln T \) term is present it must be cancelled by another \( \ln T \) term of the opposite sign. Also the magnitude of the linear term is larger than expected.

F. Discussion

When the data presented above is analysed within the framework of the standard theory for weakly interacting systems many of the trends can be understood but there are quantitative disagreements. For much of the data the corrections are not small, as is assumed in the theory, but are comparable to the classical Drude term. The low field negative magnetoresistance is consistent with weak localisation but the expected \( \ln T \) dependence appears to be replaced by a positive, approximately linear, temperature dependence. This linear increase transforms smoothly into insulating behaviour at the critical density, which, as in Si-MOSFETs can be described by a scaling expression.

RG theories predict that in the presence of disorder the Fermi liquid parameter \( \gamma_2 \) measuring the strength of the spin fluctuations becomes temperature dependent. This leads to a decrease in resistance with decreasing temperature that eventually saturates at a finite value. The absence of any \( \ln T \) dependence might possibly be explained if both the weak localisation and interaction terms are to be renormalised in this way. The magnetoresistance of the weak localisation, however, shows little evidence of such behaviour and appears to be entirely conventional. For the values of \( \tau_\phi \) shown in figure 2 the parameter \( k_B T \tau/\hbar \) varies between about 0.02 and 0.3 in which case the dephasing should be determined by inelastic hole-hole scattering with small momentum transfer (sometimes known as Nyquist dephasing) and \( \tau_\phi \) is given by...
\[ \tau_0 = \hbar f(g)/k_B T \]  

where \( f(g) \), with the conductance \( g \) in units of \( e^2/h \), is \( g/\ln(g/2) \) for \( g \gg 1 \) but of order \( g \) as \( g \) approaches 1. These values are shown as lines with \( f(g) = 7.4 \) for sample A (where \( g \approx 15 \)) and 2.6 for sample B (where \( g \approx 2.6 \)). Experimental values of \( \tau_0 \) are four to five times smaller. A similar discrepancy is also seen in Si-MOSFETs, p-GaAs and in other p-SiGe experiments. It is not clear whether this discrepancy is general or confined to systems with large values of \( r_s \).

Interaction effects are clearly large. While the Zeeman effect data in sample A does not have the expected functional form \( F^* \) is at least as large as 2.5. For sample B, where there is a better fit to the theory, \( F^* \) is also large. From the temperature dependence of the Hall coefficient \( F^* \sim 1.2 \) in both samples, roughly a factor two smaller than the values deduced from the Zeeman interaction effect.

If the standard theory is assumed to be valid for these large values of \( F^* \) the absence of a lnT term in \( \sigma_{xx} \) might perhaps be attributed to a cancellation between a positive coefficient for the weak localisation correction and negative coefficient for the interaction correction with the sum, \((ap + 1-\frac{3}{4}F^*)\), vanishing. While this might happen fortuitously in one sample it seems unlikely for two samples where, according to the Zeeman term, the values of \( F^* \) are quite different. It would not explain why, in sample A, the lnT term is absent in both \( \sigma_{xx} \), where the weak localisation is included, and \( R_H \), where it is excluded. Furthermore, in sample B the measured lnT term in \( R_H \) is of the wrong sign to cancel a weak localisation term.

In these samples therefore, with large values of \( r_s \), the standard weakly interacting theory provides an inconsistent and inadequate description of the data. The coulomb interactions are large, probably large enough to overcome weak localisation effects and lead to a net delocalising or metallic-like temperature dependence. Near the critical density for the MIT the contributions to the conductivity from disorder, weak localisation, coulomb interactions and temperature dependent screening effects are all important and all comparable to the classical Drude term so it seems likely that a satisfactory theoretical description of the data must go beyond the weakly interacting theory and must include these effects in combination. A further problem is that in this dilute but relatively clean system the disorder, as measured by \((k_f l)^{-1}\), is large but \( \tau \) is also large. Much of the experimental data is taken in the ballistic regime (when \( k_B T\tau/\hbar \sim 1 \)) whereas the standard theories are formulated in the diffusion limited regime when \( k_B T\tau/\hbar \ll 1 \).

A further caveat, appropriate to p-SiGe, should also be noted. Spin-fluctuations play an important role in any interaction effect theory: in p-SiGe the strong spin-orbit coupling means the “spins” involved are not pure \( S=\pm1/2 \) spin states, but rather a doublet of \( M_J=\pm3/2 \) states. This is in contrast to situation, for example, in Si-MOSFETs and p-GaAs where the spins are much less strongly coupled to the orbital motion.

Renormalisation Group theories, which have already been alluded to, consider some of these problems. They treat, self consistently, the modification of interactions by disorder and predict a metallic-like decrease of resistance with temperature and a low temperature saturation at a finite value. This is qualitatively very similar to what is observed experimentally. The issue of ballistic rather than diffusive motion has recently been addressed by a new theory put forward by Zala, Narozhny and Aleiner (ZNA) which treats coulomb interaction effects in both these regimes (and also the intermediate regime where there is a cross-over). They find, in addition to the standard lnT dependence (cf.eqn 4 and 11.) a linear term with a slope, and even sign, that depends directly on the strength of the spin fluctuations. When only the Hartree part of the interaction is included the standard temperature dependent screening result (eqn.12) is recovered; when the Fock part is added the linear slope can have either sign depending on the strength of the spin fluctuations. The lnT term also changes sign and is in fact given by eqn. 11. That is the theory reproduces the RG result. Because the linear term persists to \( T=0 \) an explicit lnT behaviour is only expected to emerge at very low temperatures.

In RG theories the renormalisation of the spin fluctuation amplitude by disorder becomes important when \( g \), the dimensionless conductivity, approaches one. The ZNA theory is only formulated for small disorder, ie \( g \) large, and does not therefore reproduce the low temperature metallic renormalisation predicted by the RG theories.

In the ZNA theory the interaction strength is characterised by the Fermi liquid parameter \( F_0^\sigma \) describing the renormalisation of the spin susceptibility \( \chi = \chi_0/(1 + F_0^\sigma) \) with a value of -1 corresponding to the ferromagnetic Stoner instability. It is related to \( \gamma_2 \), used in RG theories, by

\[ F_0^\sigma = -\gamma_2/(1 + \gamma_2) \]  

In a subsequent paper Zala, Narozhny and Aleiner have also calculated the Hall coefficient. An important conclusion here is that the ratio of two in eqn.8, relating \( \Delta R_H \) and \( \Delta \sigma_{xx} \), only holds in the \( T=0 \) limit. As \( T \) increases it decreases and is typically less than one when \( k_B T\tau/\hbar \) exceeds about 0.1.

The ZNA theory therefore advances on the “old” theories in two important respects. It explicitly explains the high temperature linear dependence of conductivity (and allows it to be parametrised by a Fermi liquid parameter) and it provides a quantitatively different interpretation for the temperature dependence of the Hall data. It is of interest therefore to re-examine the experimental data presented above using this theory.

Figure 10 compares \( \Delta \sigma_{xx} \), measured in samples A and B, with the ZNA theory. A weak localisation term,
\[(\varepsilon^2/\pi\hbar)\ln(T)\], has been added to the theoretical curves. In all cases there is an arbitrary vertical off-set, both experimentally (because \(\sigma_0\) is not well known) and theoretically (because of the ultra-violet divergence in the \(\ln T\) term). The data has therefore been adjusted to provide a good match of the slopes. Note also that there is an experimental error of order 20\% in \(\tau\), associated with the uncertainties in \(\sigma_0\) and in the effective mass. Values of \(F_0^2\) between about -0.55 and -0.65 (ie \(\gamma_2\) between about 1.2 and 1.8) give a satisfactory fit to the data. As shown in the insert, fits where the weak localisation term has been excluded are somewhat worse.

Figure 11 shows a corresponding comparison for Hall data. For both samples the values of \(F_0^2\) needed to explain the data (-0.3 to -0.5) are significantly smaller than those obtained from the \(\sigma_{xx}\) data.

The values of \(F_0^2\) (and equivalently \(\gamma_2\)) derived from the three separate experimental measurements are summarised in Table I. They are of comparable magnitude, corresponding to an enhancement of the spin susceptibility by a factor of between 1.5 and 3 but are only approximately self-consistent. It should be noted that for the Zeeman term at least some of the measurements were made in the intermediate temperature regime with values of \(T\tau\) sufficiently large that the RG eqns. 9 and 10 used to analyse the data probably need to be replaced by a more general, intermediate temperature, theory (not yet available). With this in mind the values obtained for sample A are in quite reasonable agreement. For sample B, however, there is a disagreement by a factor of order two between the values obtained from the temperature dependence of \(\sigma_{xx}\) and from the Hall coefficient. This is not understood.

Furthermore insight is obtained by fitting the ZNA theory to the data for samples C and D in figure 9 where the absence of a \(\ln T\) term is establish down to quite low temperatures. Better fits (see figure 12) are obtained because of the low temperature cancellation between \(\ln T\) terms of opposite sign from the weak localisation and interaction effects. At higher temperatures, the large linear slopes are also consistent with values of \(F_0^2\) similar to those measured in samples A and B. For sample C, however, there are still deviations from the theory at low temperatures which indicate this might not be the whole story and that the low temperature RG behaviour not contained within the ZNA theory may also play a role. It should also be noted that for samples B and C, which have very similar densities but mobilities differing by a factor two, different values of \(F_0^2\) are required to fit the linear dependence. This implies that \(F_0^2\) is determined not just by the density (ie by the value of \(r_s\)) but also by the degree of disorder.

| Sample | \(\sigma(T)\) | Hall | Zeeman |
|--------|--------------|------|--------|
| A      | -0.55 (1.2)  | -0.45 (.85) | -0.72 (2.0) |
| B      | -0.65 (1.9)  | -0.35 (.55) | -0.38 (0.6) |

### G. Conclusions

Analysis of magneto-transport data in p-SiGe samples, where the values of \(r_s\) are significantly larger than one, has allowed the separate identification of weak localisation and Zeeman effect terms. Attempts to fit the results using the standard theory for quantum corrections in weakly interacting systems fail leads to inconsistencies between the fitting parameters. There are, however, clear indications that interaction effects are strong. At \(B=0\) the \(\ln T\) dependence, expected for both weak localisation and interaction effects, is not observed. Rather there is the large linear increase of resistance with increasing temperature that has been attributed to the MIT transition. The saturation of this, at low temperatures, is consistent with predictions of RG theories.

Analysis of the data using the recent “intermediate temperature” theories of Zala et al provides a much better explanation of the data. Values of the spin triplet interaction parameter, \(F_0^2\) of order -0.5 explain not only the large magnitude of the Zeeman interaction term but also the temperature dependence of the Hall coefficient and the magnitude of the linear temperature dependence of the resistivity. Deviations at the lowest temperatures suggest that there might be a renormalisation of \(F_0^2\), as predicted by Renormalisation Group theories but not including in the ZNA theory.

These results support the view that the MIT type of behaviour observed in this system for clean samples is directly associated with the large Coulomb interactions.

### H. Acknowledgements

H. Lafontaine and R.L. Williams are thanked for the growth of the samples, Y. Feng and J. Lapointe for sample preparation and R. Dudek for technical assistance. Helpful discussions with S. Kravchenko, B. Narozhny and S. Studenikin are acknowledged.
1 for a review see E. Abrahams, S.V. Kravchenko and M.P. Sarachik, Rev. Mod. Phys. 73, 251 (2001); see also cond-mat/0004201

2 E. Abrahams, P.W. Anderson, D.C. Licciardello and T.V. Ramakrishnan, Phys. Rev. Lett. 42, 673 (1979)

3 V. Dobrosavljevic, E. Abrahams, E. Miranda and S. Chakravarty, Phys. Rev. Lett. 79, 455 (1997)

4 D. Simonian, S.V. Kravchenko and M.P. Sarachik, Phys. Rev. B 55, R13421 (1997)

5 For a review see Elihu Abrahams, Ann. Phys. (Leipzig) 8, 7 (1999)

6 A.M. Finkel'stein, Z. Phys. B 56, 189 (1984); A.M. Finkel'stein, Proc. 24th Int. Conf. on The Physics of Semiconductors, Ed. D. Gershoni, p131 (1998)

7 C. Castellani, C. Di Castro, P.A. Lee and M. Ma, Phys. Rev. B 30, 527 (1984)

8 C. Castellani, C. Di Castro and P.A. Lee, Phys. Rev. B 57, R9381 (1998)

9 see eg B.L. Altshuler and D.L. Maslov, Phys. Rev. Lett. 82, 145 (1999); B.L. Altshuler, D.L. Maslov and V.M. Pudalov, preprint cond-mat/9903032, S. Das Sarma and H.Y. Hwang, Phys. Rev. Lett. 83, 164 (1999)

10 M.Y. Simmons, A.R. Hamilton, M. Pepper, E.H. Linfield, P.D. Rose and D.A. Ritchie, Phys. Rev. Lett. 84, 2489 (2000)

11 V. Senz, T. Ihn, T. Heinzel, K. Ensslin, G. Dehlinger, D. Grützmacher and U. Gennser, Phys. Rev. Lett. 85, 4357 (2000)

12 Preliminary reports of some of this work have been published elsewhere: P.T. Coleridge et al, Proc. XXXIV Rencontres de Moriond XXIX, (1999); preprint cond-mat/9909292

13 G. Zala, B.N. Narozhny and I.L. Aleiner, Phys. Rev. B 64, 214204 (2001)

14 B.L. Altshuler and A.G. Aronov, in Electron-Electron Interactions in Disordered Systems, edited by A.L. Efros and M. Pollak, North Holland, Amsterdam, 1985, p1

15 H. Fukuyam, in Electron-Electron Interactions in Disordered Systems, edited by A.L. Efros and M. Pollak, North Holland, Amsterdam, 1985, p155

16 P.A. Lee and T.V. Ramakrishnan, Reviews of Modern Physics 57, 287 (1985)

17 D.J. Bishop, R.C. Dynes and D.C. Tsui, Phys. Rev. B 26, 773 (1982)

18 M.S. Burdiss and C.C. Dean, Phys. Rev. B38, 3269 (1988)

19 S. Hikami, A. Larkin and Y. Nagaoka, Prog. Theor. Phys. 63, 707 (1980)

20 G.M. Minkov, A.V. Germanenko, V.A. Larkinova, S.A. Negashev and I.V. Gornyi, Phys. Rev. B 61, 13164 (2000)

21 A. Houghton, J.R. Senna and S.C. Ying, Phys. Rev. B 25, 2196 (1982)

22 P.T. Coleridge, R.L. Williams, Y. Feng and P. Zawadzki, Phys. Rev. B 56, R12764 (1997)

23 J.C. Hensel and G. Feher, Phys. Rev. 129, 1041 (1963)

24 R. People, Phys. Rev. B 32, 1405 (1985)

25 P.T. Coleridge, A.S. Sachrajda, H. Lafontaine and Y. Feng, Phys. Rev. B 54, 14518 (1996)

26 H. Hasegawa, Phys. Rev. 129, 1029 (1963)

27 S.-H. Song, D.C. Tsui, F.F. Fang, Solid State Comm. 96, 61 (1995)

28 S.L. Wong, D. Kinder, R.J. Nicholas, T. E. Whall and R. Kubiat, Phys. Rev. B 51, 13499 (1995)

29 E. Glaser et al, Phys. Rev. Lett. 65, 1247 (1990)

30 Model AVS-47 resistance bridge. PICOWATT RV-Elektrimikka Oy, Finland

31 C.J. Emelius, T.E. Whall, D.W. Smith, N.L. Mathey, R.A. Kubiat, E.H.C. Parker and M.J. Kearney, Phys. Rev. B 47, 10016 (1993)

32 R. Cheung, L.J. Geerligs, J. Caro, A.H. Verbruggen, K. Werner and S. Radelaar, Physica B 194-196, 1225 (1994)

33 V. Senz, T. Heinzel, Y. Ihn, K. Ensslin, G. Dehlinger, D. Grützmacher and U. Gennser, Phys. Rev. B 61, R5082 (2000)

34 It is important to be sure that any field or temperature dependence measured in the Hall coefficient is not a spurious effect associated with feed-through of a small amount of the $\rho_{xx}$ term. The zero of magnetic field can be accurately determined and in all cases the offset in $\rho_{xx}$ at $B=0$ is subtracted from the $\rho_{xy}$ data before $R_H$ is calculated. In sample B, where feed-through effects are most likely to be important, this off-set corresponds to 0.4% of $\rho_{xx}$ and has the expected temperature dependence. While subtracting this term eliminates most of any feed-through there is the possibility a small B dependent term may remain. Up to 0.2 tesla the maximum observed magnetoresistance in $\rho_{xx}$ is about 2.5% (at 0.28K) corresponding to a contribution to the measured $\rho_{xy}$ values of about 1 ohm. This may be compared with the experimentally observed deviation from linearity of about 85 ohms at 0.2 tesla. At lower fields any B dependent feed-through was eliminated by making cubic fits to the data, over a field range of -0.1 to +0.2 tesla, with the quadratic term set to zero. That is the fit was forced to have the asymmetry required of the Hall resistivity and reject any B dependent $\rho_{xx}$ contribution. A similar analysis for sample A shows even smaller effects so for the field and temperature dependences of $R_H$ reported in figures 7 and 8, only a very small contribution, certainly less than 0.1% of $R_H$, might be attributed to spurious feed-through of a $\rho_{xx}$ term.

35 Y.Y. Prosokryukov, A.K. Savchenko, S.S. Safonov, M. Pepper, M.Y. Simmons and D.A. Ritchie, Phys. Rev. Lett. 86, 4895 (2001)

36 A. Gold and V.T. Dolgopolov, J. Phys. C:Solid State Phys. 18, L463 (1985); A. Gold and V.T. Dolgopolov, Phys. Rev. B 33, 1076 (1986)

37 S. Das Sarma, Phys. Rev. B 33, 5401 (1986); S. Das Sarma and E.H. Hwang, Phys. Rev. Lett. 83, 164 (1999)

38 A.D. Plews, N.L. Mathey, P.J. Phillips, E.H.C. Parker and T.E. Whall, Semicon. Sci Technol. 12, 1231, (1997)

39 V. Senz, U. Dötsch, U. Gennser, T. Ihn, T. Heinzel, K. Ensslin, R. Hartmann, D. Grützmacher, Ann. Physics (Leipzig) 8, SI 237 (1999); also cond-mat/9903367

40 G. Brumthaler, A. Prinz, G. Bauer and V.M. Pudalov, preprint cond-mat/000723

41 B.L. Altshuler, A.G. Aronov and D.E. Khmelnitskii, J.
Phys. C. 15, 7367 (1982); see also I.L. Aleiner, B.L. Altshuler and M.E. Gershenson, Waves Random Media 9, 201 (1999): cond-mat/9808053

42 Private communication, B.N. Narozhny.
43 G. Zala, B.N. Narozhny and I.L. Aleiner, Phys. Rev. B 64, 201201(R) (2001)
44 The theoretical curves shown in figure 10 have been calculated assuming the cross-over functions $f(x)$ and $t(x, F_{\sigma_0}^a)$ are 1. Over the range shown this will lead to small errors, of order $0.1 \ e^2/h$. 
FIG. 1. Magnetoresistance data for sample A at temperatures (shown) between 0.15 and 2.2 K. Lines are fitted to the weak localisation peak (eqn.5) in the low field regime as described in the text.

FIG. 2. Dephasing times, $\tau_\phi$, extracted from fits to the weak localisation peak in sample A (solid points) and sample B (open points). The lines are the values given by eqn.13 with $g(g) = 7.4$ (sample A, short dashes) and 2.6 (sample B, long dashes). For comparison $\tau$, the transport lifetime is approximately 0.9 ps in both samples.

FIG. 3. (a) Residues from the weak localisation fits to the low field magnetoresistance for sample A. Temperatures are respectively 250 (largest resistivity values), 350, 450, 600, 750 and 1000 mK. (b) Data plotted against B/T. Solid line is a fit to $G(b)$ (eqn.6) and the dashed lines fits to eqns. 9 and 10. For $b=1$, B/T is approximately 0.4 tesla/kelvin.

FIG. 4. Magnetoresistance data for sample B at temperatures of 0.28 (lowest values), 0.37, 0.48, 0.65, 0.85 and 1.20 K. At the highest fields and lowest temperatures the Shubnikov-de Haas oscillations are just becoming apparent.

FIG. 5. Magnetic field dependence of the conductivity in sample B derived from the data in figure 4. Dashed lines are fits to eqn.5 plus a quadratic term for $B/T \leq 0.4$ tesla/kelvin.

FIG. 6. Residues data in figure 5 with weak localisation(eqn.5) and classical (eqn.7) terms subtracted off, plotted as a function of B/T. Dashed line is fit to $G(b)$ used to obtain a value for $F$ and (in the quadratic region, with eqn.10) $\gamma_2$. The smooth downturn in the data at the highest fields reflects the onset of the Shubnikov-de Haas oscillations.

FIG. 7. Hall coefficient as a function of field in sample A at temperatures of 0.27, 0.39, 0.58, 0.85 and 1.20 K ($R_H$ decreases with increasing temperature). The low field data, dashed line, is a cubic fit to the $\rho_{xy}$ data matched to direct measurements of $R_H$ at higher fields. The periodicity of the Shubnikov-de Haas oscillations corresponds to a Hall coefficient of 1090 ohms/tesla.

FIG. 8. Hall coefficient as a function of field in sample B at temperatures of 0.28, 0.37, 0.48, 0.65, 0.85 and 1.20 K. Inset shows the B=0 temperature dependence plotted against ln($T$). The low field data, dashed line, is a cubic fit to the $\rho_{xy}$ data matched to direct measurements of $R_H$ at higher fields. The periodicity of the Shubnikov-de Haas oscillations is obtained from experimental values in the low temperature limit.

FIG. 9. Zero field conductivity plotted against $T/T_F$ for: sample A ($T_F = 76$ K), sample B ($T_F = 16$ K), sample C (with a similar density to sample B, $T_F = 17$ K, but a much higher peak mobility of 21,000 cm$^2$/Vs$^{-1}$) and (in the inset) sample D ($T_F = 70$ K) with a density slightly less than sample A. In each case the solid points are measured data and the open points have a lnT weak localisation correction subtracted. The solid lines are linear fits to the high T values with $C_p = 2.5, 5.0, 6.1$ and 2.5 respectively for samples A - D.

FIG. 10. Conductivity correction plotted against $T \tau$, where $\tau$ is the Drude lifetime, compared with the predictions of the ZNA theory for various values of $F_\sigma^\sigma$. A weak localisation term has been added to the theoretical curves. Open points, sample A; solid points, sample B. As noted in the text there is an arbitrary vertical off-set for both the theoretical and experimental curves. The inset shows the same comparison but without a weak localisation term included in the theoretical curves.

FIG. 11. Correction for the Hall data (around B=0) compared with the predictions of the ZNA theory. Open points, sample A; solid points, sample B. The Drude conductivity $\sigma_D$ is obtained from experimental values in the low temperature limit.

FIG. 12. As in figure 10 but for sample C (solid points) and sample D (open points).
$\Delta \rho$ (ohms/square)

$B$ (tesla)

$B/T$ (tesla/kelvin)
