Performance Analysis of TAS/SC as well as TAS/MRC MIMO Techniques under $\alpha$-$\mu$ Fading Channels

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Abstract: Multiple-Input-Multiple-Output (MIMO) system improves performance as well as the capacity of the wireless system. The use of large number of antennas in a MIMO system increases the hardware complexities and also its price. To overcome this, MIMO systems that activate single transmit antenna at a time, namely transmit antenna selection (TAS) is considered in this paper. Selection combining (SC) and Maximal ratio combining (MRC) are carried out at the receiver over $\alpha - \mu$ fading channels. Expressions for outage probability and average bit error rate (ABER) are derived considering TAS/SC as well as TAS/MRC MIMO systems. All the derived expressions are validated by Monte-Carlo simulation results.

Keywords: ABER, Outage Probability, Maximal Ratio Combining (MRC), Selection Combining (SC), Transmit Antenna Selection (TAS)

I. INTRODUCTION

The possible way to get better information carrying capacity is to use multiple antennas at the transmitter resulting in multiple input multiple output (MIMO) [1]. Due to the wide range of RF chains involved in the MIMO system, the complication in hardware in addition to price goes high. To overcome this problem, transmit antenna selection (TAS) can be carried out at the transmitter and to enhance the system performance, selection combining (SC) and maximal ratio combining (MRC) diversity techniques can be implemented at receiver. The channel for the system follows $\alpha - \mu$ fading distribution. The $\alpha - \mu$ fading distribution [2] is a general fading that comprises Nakagami- $m$, Weibull, One-sided Gaussian, Rayleigh fading as a certain case substituting certain values for $\alpha$ and $\mu$. A closed form statement of the BER for coherent modulation technique using moment generating function (MGF) for the $\alpha - \mu$ fading is acquired in [3]. Expressions for the average amount of fading and channel capacity of the $\alpha - \mu$ fading channels are achieved in [4]. In [5], closed form approximation of the summation of i.i.d $\alpha - \mu$ variates are provided and the expressions are also used to obtain the approximation for outage probability and ABER for MRC as well as equal gain combining (EGC). In [6], infinite series expressions are derived for probability density function (PDF) as well as cumulative distribution function (CDF) for the correlated $\alpha - \mu$ fading channels. By using dual branch SC, the expressions for average error probability of coherent phase shift keying (CFSK), binary phase shift keying (BPSK) and also binary differential phase shift keying (BDPSK) are obtained. In [7], analysis of the signal to interference ratio (SIR) over correlated $\alpha - \mu$ fading channels are studied by using triple SC. In [8], asymptotic average symbol error rate for $\alpha - \mu$ fading parameters at high SNR and also for the OSTBC is studied. Performance analysis of TAS system using SC and MRC diversity receiver is rarely available in the literature.

In the paper, explanation of outage probability and ABER for some binary modulations considering an arbitrary number of transmit antennas and receiving antennas over $\alpha - \mu$ fading channels are presented. SC and MRC diversity combining techniques are used in this paper.

The other sections in the paper are arranged as follows: system with channel model is presented in section II. In section III, outage probability for TAS/SC and TAS/MRC is evaluated. ABER performance for TAS / SC and TAS/MRC is given in section IV. In section V, the numerical results with discussions are given. In section VI, the conclusions drawn in the paper are provided.

II. SYSTEM AND CHANNEL MODEL

We examine, $L_r \times L_t$ MIMO system, including $L_r$ transmit antennas and $L_t$ antennas at the receiver side as shown in Figure 1. The transmit antenna which maximizes received SNR at the receiver is taken to send informations. Such a system can be represented as a $(L_t, 1; L_r)$ system. The channel state information is sent back to the transmitter through an error free path. Based on this feedback information, the antenna selector sends the input information to the corresponding transmit antenna. The antenna selector block is an RF switch so it does not require a complete RF chain set for each of the antennas at the transmitter. SC and MRC diversity schemes are implemented at the receiver.
A. SC at the Receiver

SC is the simplest diversity combining scheme. In SC, only the strongest received signal among all the received copies is processed further to detect the information. In TAS/SC system, only the antennas matching the link that provides maximum SNR at receiver are activated. Link that provides the highest SNR at the receiver is set by [9]

$$I_{SC} = \arg \max_{1\leq i \leq L_t} \{ \gamma_{j,i} \}.$$  \hspace{1cm} (1)

Where, $I_{SC}$ denotes the link which gives maximum instantaneous SNR at the receiver and $\gamma_{j,i}$ is instantaneous SNR of the link connecting $i^{th}$ transmit antenna with $j^{th}$ receiving antenna. The advantage of the SC implemented at the receiver is, it requires one RF chain and hence it is easy to implement. In this system, one antenna is selected on the receiver and the instantaneous SNR becomes $\gamma_{j,i} |h_{j,i}|^2$ for an average SNR of $\gamma$. The fading co-efficient from $i^{th}$ transmit antenna to $j^{th}$ receive antenna is denoted by $h_{j,i}$, where $i \in (1,2,...,L_t)$ and $j \in (1,2,...,L_r)$. Using (1) the link that gives the highest received SNR is evaluated by

$$I_{SC} = \arg \max \gamma_{j,i} = \gamma |h_{j,i}|^2.$$  \hspace{1cm} (2)

B. MRC at the Receiver

The received SNR of MRC technique is specified by $\gamma_i = \sum_{j=1}^{L_r} \gamma_{j,i}$, denoting $\gamma_i$ is instantaneous SNR of $i^{th}$ input branch. The transmit antenna that maximizes the receiver SNR is decided by [9]

$$I_{MRC} = \arg \max_{1\leq i \leq L_t} \gamma_{i,j} = \sum_{j=1}^{L_r} \gamma_{i,j}.$$  \hspace{1cm} (3)

In (3), $\gamma_{i,j}$ denotes total received instantaneous SNR when $i^{th}$ transmit antenna is chosen. $I_{MRC}$ is the antenna index that represents transmit antenna which maximizes the receiver SNR. It gives optimum performance in terms of probability of error among all the diversity combining schemes.

The transmit antenna, that maximizes the receiver SNR may be obtained by using (3) as [9]

$$I_{MRC} = \arg \max_{1\leq i \leq L_t} \{ \gamma \sum_{j=1}^{L_r} |h_{j,i}|^2 \}.$$  \hspace{1cm} (4)

We consider the fading coefficient $h_{j,i}$ is $\alpha - \mu$ distributed. The $\alpha - \mu$ fading envelop PDF is specified by [2],

$$f_r(r) = \frac{\alpha \mu}{\Gamma(\mu)} e^{-\frac{\alpha r}{\mu}} \exp \left( -\frac{\alpha}{\mu r} \right).$$  \hspace{1cm} (5)

The $\alpha - \mu$ distributed instantaneous SNR PDF at each receiver antenna may be obtained by scaling of $\alpha - \mu$ distributed random variable in (5) by a factor of average SNR, $\gamma$ [3] and is given as

$$f_r(r) = \frac{\alpha \mu}{\Gamma(\mu)} e^{-\frac{\alpha r}{\mu}} \exp \left( -\frac{\alpha}{\mu r} \right).$$  \hspace{1cm} (6)

Using [10, (3.381.8)], the CDF of the received SNR is clarified as

$$F_r(r) = \frac{1}{\Gamma(\mu)} g \left( \frac{\mu}{\gamma} \frac{\alpha}{\gamma} \right).$$  \hspace{1cm} (7)

where, $g(v,u) = \int_0^u x^{v-1} e^{-x} dx$ is the lower incomplete gamma function. The transmit antenna as well as the receive antenna that maximizes the SNR ($\gamma_{i,t(L_r)}$) at the receiver are selected in TAS/SC system. In TAS/MRC technique, the transmit antenna matching to the highest receiver SNR ($\gamma_{i,t(L_r)}$) is selected. In such type of systems, the PDF of receiver SNR, assuming all $\left( |h_{j,i}| \right)$’s are to be i.i.d can be determined by [11]

$$f_{r_{i,t(L_r)}}(\gamma) = L \left( F_{r_i}(\gamma) \right)^{L-1} f_{r_i}(\gamma).$$  \hspace{1cm} (9)
Substituting the values of $F_{\gamma_L}(\gamma)$ and $f_{\gamma_L}(\gamma)$ from (8) and (6) respectively, we can express (9) as,

$$f_{\gamma_L(L)}(\gamma) = L \left[ \frac{g(\mu, \mu \left[ \frac{\gamma}{\gamma} \right])}{\Gamma(\mu)} \right]^{L-1} \frac{\alpha \mu^{-\frac{\alpha}{2}} \gamma^{\frac{\alpha}{2}} e^{\frac{\alpha}{2} - \frac{\mu}{\gamma}}}{2 \Gamma(\mu) \gamma^{\frac{\alpha}{2}}},$$

(10)

It should be noted that (10) is applicable for TAS/SC as well as TAS/MRC system. In TAS/MRC, $\gamma_L$ is $\alpha - L_\mu$ distributed since it is the summation of $L_\mu$ i.i.d $\alpha - \mu$ square variants [2]. So, in all succeeding expressions, $\mu$ will be represented as $L_\mu$ in the TAS/MRC system. $L = L_\mu L_\gamma$ for TAS/SC system and $L = L_\gamma$ for TAS/MRC system.

III. OUTAGE PROBABILITY

It is the probability while the mutual information is less than the specific threshold SNR, $\gamma_{th}$. The outage probability is stated as

$$P_{out} = \int_{0}^{\gamma_{th}} f_{\gamma_L(L)}(\gamma) d\gamma$$

(11)

Placing the value of (10) in (11) we get,

$$P_{out} = \int_{0}^{\gamma_{th}} \left[ \frac{g(\mu, \mu \left[ \frac{x}{\gamma} \right])}{\Gamma(\mu)} \right]^{L-1} \frac{\alpha \mu^{-\frac{\alpha}{2}} x^{\frac{\alpha}{2}} e^{\frac{\alpha}{2} - \frac{\mu}{x}}}{2 \Gamma(\mu) x^{\frac{\alpha}{2}}} dx$$

(12)

The integral in (12) is solved to obtain an expression for outage probability as

$$P_{out} = \left( \frac{1}{\Gamma(\mu)} \right) \left[ \frac{g(\mu, \mu \left[ \frac{1}{\gamma_{th}} \right])}{\Gamma(\mu)} \right]^{L}$$

(13)

Where, $\gamma_{th} = \frac{\gamma}{\gamma_{th}}$ denotes the normalized average SNR per branch.

IV. AVERAGE BIT ERROR RATE

ABER rests on the fading distributions, including modulation techniques used. The ABER is shown with regard to the CDF of output SNR as [12]

$$p_e = -\int_{0}^{\infty} p_e'(\gamma)F_{\gamma_L(L)}(\gamma)d\gamma,$$

(14)

where, $p_e'(\gamma)$ is conditional error probability that is reported as [12]

$$p_e'(\gamma) = -\frac{e^{\gamma \eta}}{2\Gamma(\eta)} \cdot$$

(15)

Thus, $\Gamma(\eta)$ is the gamma function and in case of a few modulations, the constants $\epsilon$ and $\eta$ values are reported as [12]: $(\epsilon, \eta) = (1, 0.5)$ for BPSK, $(\epsilon, \eta) = (0.5, 0.5)$ for BFSK. Substituting the values of $F_{\gamma_L(L)}(\gamma)$ and $p_e'(\gamma)$ in (15), the ABER can be given as

$$P_{e'} = \frac{\epsilon^\eta \eta^{-\frac{1}{2}} e^{-\frac{\epsilon}{\eta}}}{2\Gamma(\eta)} \times \left[ \frac{1}{\Gamma(\mu)} g(\mu, \mu \left[ \frac{\gamma}{\gamma} \right]) \right]^{L} \cdot$$

(16)

The lower incomplete gamma function in (16) can be represented by its equivalent finite series representation [10, (8.352.6)] as

$$P_{e'} = \frac{\epsilon^\eta}{2\Gamma(\eta)} \left[ \frac{(\mu - 1)!}{\Gamma(\mu)} \right]^{L} \times \left[ \frac{1}{\gamma^{\eta-1}} \sum_{w=0}^{\infty} \left( \frac{\mu}{\gamma} \right)^w \right]^{-L} \cdot$$

(17)

The expression in (17) can be written as,

$$P_{e'} = \frac{\epsilon^\eta}{2\Gamma(\eta)} \left[ \frac{(\mu - 1)!}{\Gamma(\mu)} \right]^{L} \times \int_{0}^{\infty} \gamma^{\eta-1} e^{-\gamma} d\gamma$$

$$- \frac{\epsilon^\eta}{2\Gamma(\eta)} \left[ \frac{(\mu - 1)!}{\Gamma(\mu)} \right]^{L} \int_{0}^{\infty} \gamma^{\eta-1} e^{-\gamma} d\gamma$$

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The ABER expression in (18) can be represented as
\[ p_e = I_1 - I_2. \]  

Where,
\[ I_1 = \frac{e^\eta}{2\Gamma(\eta)} \left[ \frac{(\mu - 1)!}{\Gamma(\mu)} \right]^L \sum_{w=0}^{\mu-1} \left( \frac{\gamma}{\mu} \right)^w \]  

The integration in (20) can be solved using [10, (3.381.4)] as
\[ I_1 = \frac{e^\eta}{2\Gamma(\eta)} \left[ \frac{(\mu - 1)!}{\Gamma(\mu)} \right]^L \frac{1}{e^\eta} \Gamma(\eta). \]  

The expression of (21) can be simplified as
\[ I_1 = \frac{1}{2} \left( \frac{(\mu - 1)!}{\Gamma(\mu)} \right)^L. \]  

Again, from the ABER expression of (19),
\[ I_2 = \frac{e^\eta}{2\Gamma(\eta)} \left[ \frac{(\mu - 1)!}{\Gamma(\mu)} \right]^L \left[ \sum_{w=0}^{\mu-1} \left( \frac{\gamma}{\mu} \right)^w \right]^L \]  

Rearranging the expression of (23),
\[ I_2 = \frac{e^\eta}{2\Gamma(\eta)} \left[ \frac{(\mu - 1)!}{\Gamma(\mu)} \right]^L \sum_{w=0}^{\mu-1} \left( \frac{\mu}{w!} \right)^w \]  

Rearranging the expression of (24),
\[ I_2 = \frac{e^\eta}{2\Gamma(\eta)} \left[ \frac{(\mu - 1)!}{\Gamma(\mu)} \right]^L \left[ \sum_{w=0}^{\mu-1} \left( \frac{\mu}{w!} \right)^w \right]^L \]  

The integral in (24) is of the form [13],
\[ I = \int_0^\infty e^{-\gamma} \frac{e^{-x\gamma}}{\Gamma(\mu)} \frac{(\mu - 1)!}{\Gamma(\mu)} \sum_{w=0}^{\mu-1} \left( \frac{\gamma}{\mu} \right)^w \]  

Where, \( G \) is known as the Meijer-\( G \) function and \( r \) is an integer. Hence, using (25), the integral in (24) can be solved as
\[ I_2 = \left\{ \frac{e^\eta}{2\Gamma(\eta)} \left[ \frac{(\mu - 1)!}{\Gamma(\mu)} \right]^L \sum_{w=0}^{\mu-1} \left( \frac{\mu}{w!} \right)^w \right\} \]  

(26)

Where, \( \bar{\gamma} = \eta + \frac{\alpha wL}{2} \) and by putting the values of (22) and (26) in (19), the final expression of ABER for BPSK modulation can be obtained as
\[ p_e = \frac{1}{2} \left( \frac{(\mu - 1)!}{\Gamma(\mu)} \right)^L \frac{1}{\Gamma(\mu)} \frac{(\mu - 1)!}{\Gamma(\mu)} \sum_{w=0}^{\mu-1} \left( \frac{\mu}{w!} \right)^w \]  

(27)
V. RESULTS AND DISCUSSIONS

Fig. 2. Outage Probability vs. Normalized SNR of TAS/SC system with $L_t = 3, L_r = 2$ and $\gamma = 1$ dB

Fig. 3. Outage Probability vs. Normalized SNR of TAS/MRC with $L_t = 3, L_r = 2$ and $\gamma = 1$ dB

Fig. 4. ABER vs. Average SNR of the TAS/SC system for different $(L_t, L_r)$ with BPSK modulation

Fig. 5. ABER vs. Average SNR of the TAS/MRC system for different $(L_t, L_r)$ with BPSK modulation

Fig. 6. Comparisons of TAS/SC with TAS/MRC system for BPSK modulation considering $L_t = 3, L_r = 2$ and $\gamma = 1$ dB

The numerical presentations derived in the preceding sections, have been evaluated. The results are plotted for different values of transmit antennas $L_t$, receive antennas $L_r$ and fading parameters. In Fig. 2, outage probability vs. normalized SNR per branch curves are plotted for TAS/SC system considering $L = 6$ and $\gamma = 1$ dB. Outage performance becomes better with a rise in the fading parameter $\mu$ for fixed value of $\alpha = 2$. In Fig. 2, the number of transmitting antennas $L_t = 3$ and number of receiving antennas at receiver $L_r = 2$. The outage probability vs. normalized SNR per branch plots are shown in Fig. 3, for the TAS/MRC system with different values of $\mu$ considering $\alpha = 2$. 
Antennas at the transmitter $L_t = 3$ and the number of receiving antennas $L_r = 2$ are considered in Fig. 3. It is observed that with the increment of fading parameter $\mu$, the performance of outage probability increases.

In Fig. 4 and 5, the ABER vs. average SNR per branch have been shown for TAS/SC as well as TAS/MRC systems, respectively, for different values of $\mu$ and $L_r$ considering $\alpha = 2$, $L_r = 3$. It is observed that the performance of ABER becomes better with an increment in the values of fading parameter $\mu$. Similarly, with the increase in the diversity order $L_r$, the ABER performance improves. In Fig. 6, we compare the ABER for TAS/SC with TAS/MRC system for $\alpha = 2$, transmitting antennas $L_t = 3$ and antennas at the receiver $L_r = 2$. We can see that both the slope of the curves improves with an increase in the fading parameter, but TAS/MRC systems provide better performance than TAS/SC systems.

VI. CONCLUSIONS

In the paper, we examine the performance of TAS/SC as well as TAS/MRC systems under $\alpha - \mu$ fading channels. Closed form formulations are obtained for outage probability and ABER with the BPSK modulation. Expressions are derived in the form of incomplete gamma function and Meijer-G function, which are obtainable in mathematical software. The evaluated results of the expressions and computer simulated results are plotted in Figures. These results are in concurrence with each other. It is observed for the outage plots and ABER plots that the performance gets better with increase in the fading parameter. TAS/MRC systems give better performance as compared to TAS/SC systems.

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