Direct observation of quantum phonon fluctuations in a one dimensional Bose gas

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We report the first direct observation of collective quantum fluctuations in a continuous field. Shot-to-shot atom number fluctuations in small sub-volumes of a weakly interacting ultracold atomic 1D cloud are studied using in situ absorption imaging and statistical analysis of the density profiles. In the cloud centers, well in the quantum quasicondensate regime, the ratio of chemical potential to thermal energy is $\mu/k_B T \simeq 4$, and, owing to high resolution, up to 20% of the microscopically observed fluctuations are quantum phonons. Within a non-local analysis at variable observation length, we observe a clear deviation from a classical field prediction, which reveals the emergence of dominant quantum fluctuations at short length scales, as the thermodynamic limit breaks down.

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tive cooling and thermalization for 800 ms, an absorption picture is recorded on a CCD camera. After hundreds of realizations, fluctuations in the density profiles are analyzed as detailed in [15]. For each profile and pixel of length $\Delta = 4.5 \mu m$, we extract the atom number fluctuation $\delta N = N - \langle N \rangle$ where $\langle N \rangle = n\Delta$ is the mean atom number. True atom number variances $\langle \delta N^2 \rangle_m$ are inferred from measured variances $\langle \delta N^2 \rangle_m$ using the thermodynamic relation $\langle \delta N^2 \rangle_m = \kappa_2 \langle \delta N^2 \rangle$, where $\kappa_2$ is a reduction factor due to the finite rms imaging resolution $\delta$. Assuming a gaussian imaging response, $\delta$ is determined precisely from the correlations measured between neighboring pixels [15, 27].

To introduce our data, we show in Fig. 1 a typical picture (a), average profiles, and relative fluctuations, for clouds deep in the 1D regime, with $k_BT/\hbar\omega_{\perp} = 0.11$ (b) and 0.03 (c). As in [15, 21], $T$ is measured accurately by fitting the fluctuations in the cloud centers with Eq. 1 and the Gross-Pitaevski (G-P) quasicondensate EoS in the 1D-3D crossover $\mu = \hbar\omega_{\perp}(\sqrt{1 + 4\alpha a - 1})$ [25] (dashed curve, see also Fig. 3), where $a = 5.3 \mu m$ is the 3D scattering length. At low densities, fluctuations are superpoissonian, i.e., exceed the shot noise (dotted line). They reach a maximum at the transition to the quasicondensate regime, around the density $n_{eq} = \left [ m(k_BT)^2 \right ]/\hbar^2 g^{1/3}$ [21], in good agreement with the exact Yang-Yang solution for the 1D Bose gas [29] (solid curve) [36]. At higher densities, for $\mu > k_BT$ (light grey areas in profiles), quantum fluctuations start dominating thermodynamically and fluctuations turn subpoissonian, as explained in [23]. In this Letter, the important region is the cloud center at $T = 4.7 \mu K$ (zone $\Omega$), where $\langle \delta N^2 \rangle/\langle N \rangle \approx k_BT/\mu$ reaches 0.26, and where we detect quantum phonons microscopically, as explained below.

To analyze fluctuations in the cloud centers, we use Bogoliubov theory, valid for weakly interacting quasicondensates [30], and the 1D G-P EoS $\mu = gn$, where $g = 2\hbar\omega_{\perp}a$ is the 1D coupling constant. This is appropriate since the dimensional correction to the G-P prediction is only 10% in zone $\Omega$, and the 1D interaction parameter $\gamma = g\mu/\hbar^2n \lesssim 0.05$. Bogoliubov excitations have energies $\epsilon_k = h^2k^2/2m\sqrt{1 + 4/k^2\xi^2}$, where $\xi = h/\sqrt{m\mu}$ is the healing length, and thermal occupation numbers $n_k = 1/(e^{\epsilon_k/k_BT} - 1)$. Noting $f_k = 1/(1 + 4/k^2\xi^2)$, the spectrum of density fluctuations is

$$\langle \delta n_k^2 \rangle = nS_k = n(S_k^Q + S_k^T),$$

where $S_k^Q = f_k$ and $S_k^T = 2f_k n_k$ are the quantum and thermal static structure factors, both plotted in Fig. 2c. Note that, in $S_k = 2f_k(n_k + 1/2)$, $S_k^Q$ is the exact analog of the zero-point energy term in the harmonic oscillator spectrum. We also show in Fig. 2d the second order correlation function $g^{(2)}(z) = 1 + f \frac{d^2}{dz^2} e^{ikz}(S_k - 1)$.

Three competing terms determine the fluctuation regimes in quasicondensates. For $k\xi \gg 1$, $S_k^Q \approx 1$ is the trivial autocorrelation "shot noise" [15, 17] of uncorrelated particles, for which $g^{(2)}(z) \approx 1$. At smaller $k$, repulsive interactions give a negative contribution to $S_k - 1$ and to $g^{(2)}(z) - 1$ [33]. Finally, in $S_k^T$, statistical bosonic bunching brings a positive contribution. Let us now focus on the quantum and thermal density correlation lengths $l^Q_k$ and $l^T_k$, respectively defined as the inverse widths of $S_k^Q$ and $S_k^T$. For quantum fluctuations, equally present in all modes $k$, one has $l^Q_k = 1_{\xi}$ [31]. For thermal fluctuations, two cases need be distinguished.

In thermal quasi-condensates ($k_BT \gg \mu$, see Fig. 2a), $S_k^Q$ decays fast for $kl^Q_k \gg 1$, so one has a single $l^Q = l^T_k = l^Q_k$ [25, 31]. However thermal fluctuations domi-
nate, i.e. $n_k \gg 1$, for length scales $L \sim 1/k \gg \lambda_{\text{dB}}$, where $\lambda_{\text{dB}} = \sqrt{2\pi\hbar^2/mk_B T}$ is the thermal de Broglie wavelength. Three regimes are thus present \[16\]. For $L \gg l_c$, thermal phonons dominate; this is the thermodynamic regime, with superpoissonian fluctuations. For $\lambda_{\text{dB}} \ll L \ll l_c$, thermal particles dominate, but correlations are partly lost \[37\]. Finally, for $L \ll \lambda_{\text{dB}}$, only the poissonian quantum shot noise is detected.

In quantum quasicondensates ($k_B T \ll \mu$, see Fig. 2b), noting $c = \sqrt{\mu/m}$ the speed of sound, $n_k$ and $S^T_k$ are both suppressed at length scales lower than \[38\],

$$S^T_k = \frac{\hbar c}{k_B T} = \frac{\mu}{k_B T},$$

which, to our knowledge, has been proposed only recently \[13\] \[16\] \[32\]. The familiar regimes of poissonian shot noise and thermodynamic thermal phonon fluctuations (now subpoissonian) are still present. However, in the range $l_c^Q \ll L \ll l_c^T$, one now has a crucial new regime (highlighted in Fig. 2b), where $n_k \ll 1$ and quantum phonons govern the physics. To compare Bogoliubov theory to our data, we use the imaging transfer function $U_k = \frac{2 \xi}{(1-\cos(k\Delta))} e^{-k^2 \delta^2}$ \[29\], obtained from our gaussian optical response model \[15\] \[27\], and we compute, for $j = Q, T$,

$$\langle \delta N^2 \rangle^j_m = \int \frac{dk}{2\pi} nS^j_k U_k.$$  \hspace{1cm} (5)

$U_k$ is peaked at $k = 0$, and its inverse width is $L_{\text{obs}} \sim \max\{\delta, \Delta\}$. The thermodynamic limit Eq. 2 is thus equivalent to measuring $\langle \delta N^2 \rangle_m = \kappa_2 \langle N \rangle S_0$ \[16\] \[27\], i.e., to only probe the contribution $S_0 = S^Q_0$ of thermal phonons, always proportional to $T$ (see Fig. 2c). In other words, since $S_0 = 1 + \int dz (g^{(2)}(z) - 1)$, a thermodynamic observation probes only the integral of $g^{(2)}(z) - 1$, without resolving its microscopic details (see Fig. 2d).

In Fig. 3, we compare the measured fluctuations $\langle \delta N^2 \rangle_m$ to the spectral 1D Bogoliubov predictions $\langle \delta N^2 \rangle^Q_m$ and $\langle \delta N^2 \rangle^T_m$, and their sum $\langle \delta N^2 \rangle^\text{tot}_m$ (dot-dashed lines). At low densities, one has $l_c^Q \gg L_{\text{obs}}$, which explains that $\langle \delta N^2 \rangle^Q_m$ follows the shot linear noise prediction (see inset to Fig. 3a). At higher densities, $\langle \delta N^2 \rangle^Q_m$ is reduced by repulsive interactions \[39\]. At $T = 18 \text{ nK}$, the ratio $\langle \delta N^2 \rangle^Q_m / \langle \delta N^2 \rangle^\text{tot}_m$ is 7% and cannot be resolved, as in \[23\]. However, at $T = 4.7 \text{ nK}$, it reaches 20% in the cloud center, which now exceeds the experimental uncertainty. Thus, the contribution of quantum fluctuations is here sizeable, i.e. non-negligible, in each pixel, in contrast to the thermodynamic regime. Yet, $\langle \delta N^2 \rangle^\text{tot}_m$ is still in good agreement with the 1D thermodynamic Yang-Yang prediction (solid line) \[10\]. This is because $S^Q_k$ and $S^T_k$ have opposite slopes $\pm k^2/2$ at small $k$ for all $T$ (see Fig. 2c), so that non-thermodynamic (i.e., finite $k$) quantum and thermal contributions cancel each other at first order.

To confirm our detection of quantum fluctuations, we turn to a non-local analysis. Since quantum fluctuations scale only logarithmically with $L_{\text{obs}}$ \[16\] \[23\], thermal fluctuations always dominate for $L_{\text{obs}} \gg l_c^T$, and fluctuations are well described by a classical field model (CFM) that ignores the quantum term $S^Q_k$ \[23\]. On the other hand, a CFM is expected to fail for $L_{\text{obs}} \lesssim l_c^T$. To check this, we focus on the zone $\Omega$, i.e., the 3 central bins $39 < \langle N \rangle < 54$ at $T = 4.7 \text{ nK}$, containing 58% of the atoms (see inset to Fig. 1d and Fig. 3b). There, the profile is the flattest, many data points are available, and error bars are the smallest. The cloud center is indeed the most reliable fraction of the data.

To vary $L_{\text{obs}}$, we merge the imaging pixels in macro-pixels of variable size $n_{\text{pix}} = 1$ to 8, and compute fluctua-
tions accordingly (see Fig. 4a and [27]). Figure 4b shows \( \langle \delta N^2 \rangle / \langle N \rangle \) obtained in the zone \( \Omega \), for \( L_{\text{pix}} = \Delta r_{\text{pix}} = 4.5 \) to 36 \( \mu \)m. The key criterium, as in [13], is to compare the data either to a full Bogoliubov model (FBM) that includes the quantum term (solid line) or to a CFM that ignores it (dashed line). At large \( L_{\text{pix}} \sim L_{\text{obs}} \), both models converge as expected towards the thermodynamic 1D G-P prediction (dot-dashed line), with a logarithmic vanishing of the quantum contribution. At short \( L_{\text{pix}} \), the data clearly deviate from the CFM prediction, which displays a noticeable dip. Note that the gap saturates at small \( L_{\text{pix}} \), due to the finite optical resolution that cuts off high \( k \) fluctuations, which confirms that shot noise is irrelevant in our data.

The theoretical predictions in Fig. 4b are computed at \( T_{\text{1D}} = 5.0 \) nK, that we obtain fitting the FBM to the data, with \( T \) as only free parameter (see inset to Fig. 4b). This fit has an \( \text{rms} \) deviation of only 1.0\%, i.e., much less than the plotted error bars [41] and is thus a very accurate 1D thermometry [42]. On the contrary, the fit to the CFM, yielding \( T_{\text{C}} \sim 5.5 \) nK, has a strong systematic error and an \( \text{rms} \) deviation of 5\% (see inset to Fig. 4b). This clear breakdown of the CFM proves our observation of quantum fluctuations, and reveals the emergence of dominant quantum phonons at short distances.

As for length scales, in the zone \( \Omega \), \( L_{\text{obs}} \sim 0.6 \) \( \mu \)m and \( L_{\text{C}} \sim 2.2 \) \( \mu \)m, while \( L_{\text{obs}} \) is determined by \( \Delta = 4.5 \) \( \mu \)m and \( \delta = 2.8 \) \( \mu \)m. Thus, \( L_{\text{obs}} \sim L_{\text{C}} \) and this explains our unprecedented observations. On the contrary, in [12], at \( T = 33 \) nK, one has \( L_{\text{C}} \sim 0.63 \) \( \mu \)m, and \( L_{\text{obs}} \sim \Delta = 10 \) \( \mu \)m, i.e., \( L_{\text{obs}} > L_{\text{C}} \). All observations in [12] were thus well in the thermal fluctuations regime, indistinguishable from a CFM prediction [13]. In the quantum regime \( z < L_{\text{C}} \), the decay of the first order (phase) correlation function \( g^{(1)}(z) \) is algebraic, whereas, in the thermal regime \( z > L_{\text{C}} \), it is exponential, over a thermal phase correlation length \( L_{\text{C}} = L_{\text{C}} / \sqrt{\gamma} \gg L_{\text{C}} \) [11, 33, 34]. The misinterpretation in [12] came from identifying the quantum regime with \( z < L_{\phi} \).

In summary, we have reported the first microscopic observation of vacuum fluctuations in a continuous field, using a non-local analysis that reveals a clear deviation from a classical field theory. Our observation of emerging dominant quantum phonons is a first microscopic insight into the regime of quasi-long range order, i.e., algebraic decay of \( g^{(1)}(z) \), in the 1D Bose gas [11]. We also demonstrated the possibility of imaging vacuum phonon fluctuations in single density pictures, like Fig. 1a. By further reducing \( T \) and \( L_{\text{obs}} \), one could monitor the full crossover from thermal to quantum fluctuations [27]. Dark solitons, which are defects localized over a length scale \( \sim L_{\text{C}} \), could also be used as sensitive probes for the microscopic physics of quantum fluctuations [32, 33].

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[36] In the cloud wings of Fig. 1c (⟨N⟩ < 30), slight inaccuracies could stem from important density gradients that make the local density approximation questionable [27].
[37] This regime explains the recent observation of fluctuations well below the prediction of Eq. 1 in thermal 2D quasicondensates, with \(L_{\text{obs}} \lesssim \ell_{Qc}^2\) [16, 20, 26].
[38] One obtains \(\ell_{Qc}^2\) requiring that the phonon energy \(\hbar c k \gg k_BT\) (and thus \(n_k \ll 1\)) for wavevectors \(k \gg 1/\ell_{Qc}\).
[39] On the contrary, quantum phase fluctuations are increased by interactions, causing stronger condensate depletion [11].
[40] This is also true for the 1D G-P prediction which is 3% below the exact Yang-Yang prediction. The small correction (at lowest order, \(\sqrt{\gamma}/2\)) stems from quantum fluctuations and is the 1D equivalent of the Lee-Huang-Yang correction in 3D [9, 30].
[41] Error bars clearly overestimate the real noise in Fig. 4.b. They give the statistical uncertainty for each point independently, however all are obtained from the same data set, hence most of the noise is correlated and rejected.
[42] \(T_{1D}^*\) slightly exceeds the previous estimate from the 1D-3D G-P EoS, as expected.