Canonical Quantization for the Light-Front Weyl Gauge

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The canonical quantization on a single light front is performed for the Abelian gauge fields with the Weyl gauge $A_+ = A^- = 0$ coupled with fermion field currents. The analysis is carried separately for 1+1 dimensions and for higher dimensions. The Gauss law, implemented weakly as the condition on states, selects physical subspace with the Poincaré covariance recovered. The perturbative gauge field propagators are found with the ML prescription for their spurious poles. The LF Feynman rules are found and their equivalence with the usual equal-time perturbation for the S-matrix elements is studied for all orders.

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I. INTRODUCTION

In the last decade, there is a resurgence of interest in the formulation of quantum field theory on a null-plane surface of $x^+ = \text{const}$, called after Dirac [1], a light-front (LF) [2]. Though nowadays the basic motivation for it lies in the non-perturbative Hamiltonian field theory [3], there are still some problems with a consistent formulation of the perturbative LF calculations. In this paper the perturbative approach to the quantum electrodynamics will be discussed with a special attention paid to the Abelian gauge vector fields.

Usually, the LF quantization is performed with the light-cone (LC) gauge condition $A_- = A^+ = 0$ being imposed on gauge fields, this removes all non-physical gauge modes from the dynamical system and introduces instantaneous (in $x^+$) interactions of currents in the Hamiltonian [4]. All these effectively lead to the perturbative gauge field propagator with the Cauchy Principal Value (CPV) prescription for the spurious pole at $k_- = 0$.

Recent LF canonical attempts, where one uses two light-fronts $x^+ = \text{const}$ and $x^- = \text{const}$', as the quantization surfaces [5, 6] were only partially successfull. For the free field model, the ML prescription has appeared in the chronologcal product of gauge fields, but one can doubt if this approach can be consistently implemented for interacting fields. We believe that another, more fundamental solution to this problem has to be found.

One possibility is to choose another null axial gauge condition $A_+ = A^- = 0$, which for the reasons that we will explain later, we call the light-front Weyl (LF-Weyl) gauge. In the equal-time formulation both null axial gauges are treated on equal footing, therefore we expect the free gauge field propagator in the form

$$D^\text{LC}_{\mu\nu}(x) = i \langle 0 | \mathcal{T} A_\mu(x) A_\nu(0) | 0 \rangle = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik \cdot x}}{2k_+ k_- - k_+^2 + i\epsilon} \left[ g_{\mu\nu} - \frac{n^\text{LC}_\mu k_\nu + n^\text{LC}_\nu k_\mu}{k_- - \text{CPV}} \right],$$

where the LC gauge vector $n^\text{LC}_\mu = (n^\text{LC}_+ = 1, n^\text{LC}_- = n^\text{LC}_- = 0)$ chooses the LC gauge $n^\text{LC} \cdot A = A_- = 0$. Unfortunately this result is inconsistent with the equal-time canonical quantization [6], where the LC gauge allows for the propagation of non-physical modes and leads to the gauge field propagator with the causal non-covariant distribution $[k_-]_\text{ML} = (k_- + i\text{sgn} k_-)^{-1}$, so-called the Mandelstam-Leibbrandt (ML) prescription [6]. Because the perturbative calculations of the gauge-invariant Wilson loops [7] uniquely indicates the ML prescription as the only consistent regularization, then one finds that the LF canonical quantization is perturbatively inconsistent for the LC gauge.

One possibility is to choose another null axial gauge condition $A_+ = A^- = 0$, which for the reasons that we will explain later, we call the light-front Weyl (LF-Weyl) gauge. In the equal-time formulation both null axial gauges are treated on equal footing, therefore we expect the free gauge field propagator in the form

$$D^\text{Weyl}_{\mu\nu}(x) = i \langle 0 | \mathcal{T} A_\mu(x) A_\nu(0) | 0 \rangle = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik \cdot x}}{2k_+ k_- - k_+^2 + i\epsilon} \left[ g_{\mu\nu} - \frac{n^\text{Weyl}_\mu k_\nu + n^\text{Weyl}_\nu k_\mu}{k_+ + i\text{sgn} k_-} \right],$$

where the LF-Weyl gauge vector $n^\text{Weyl}_\mu = (n^\text{Weyl}_+ = 1, n^\text{Weyl}_- = n^\text{Weyl}_- = 0)$ chooses the LF-Weyl gauge $n^\text{Weyl} \cdot A = A_+ = 0$. However when one LF front $x^+ = \text{const}$ is fixed as a quantization surface, then the LF-
Weyl and the LC gauges are fundamentally different\(^1\); the gauge vector \(n_{\mu}^{LC}\) lies on the quantization surface while \(n_{\mu}^{Weyl}\) is perpendicular to this surface. Recently the LF-Weyl gauge has been successfully implemented in the canonical formulation of the quantum electrodynamics in the finite LF volume \([1]\). In this model, the canonical DLCQ \([12]\) analysis has been carried separately for different subsystems of gauge vector fields and charged fermion fields. This simplifying property of the LF Weyl QED is rather a unique situation the LF approach, contrary to the equal-time intuitions. We connect it with the simple structure of electromagnetic currents, where no derivatives of fermion fields occur. We expect that this simple structure will not be spoiled by the infinite LF volume and in this paper we decided to analyse the case of electrodynamics with fermions. We leave the more involved cases of charged scalar fields and the non-Abelian gauge interactions for the future publications.

Our first aim is to study the structure of the gauge field sector therefore we will start the canonical LF quantization for the models where the fermion currents are expressed by external arbitrary currents. Because the LF Weyl gauge does not fix the gauge symmetry completely, we expect that there will remain dynamical non-physical gauge modes. In the canonical equal-time approach \([13, 14]\) such modes generate the causal LM spurious poles in the propagators and introduce positively non-definite Hilbert space of states. We expect that a similar scenario will also appear in the LF quantization: the gauge field propagator will take the form \([1, 2]\) and the physical states will be chosen by means of the Gauss law implemented as a weak condition on states.

Our second aim is to formulate the LF perturbative Feynman rules for the full QED. As the basic cross-check of the LF formulation we take the formal equivalence of the S-matrix elements calculated according to the LF Feynman rules with the corresponding elements found within the equal-time approach. We use the same functional method of proof, as in \([1, 2]\), for all orders of perturbation in the coupling constant \(e\).

This paper is organized as follows. In Section 2 we analyze the 1+1 dimensional model of the Abelian gauge fields coupled linearly to external currents. The vector gauge field propagators are given with a consistent definition for naively coinciding spurious and physical poles. The higher dimensional model is analysed in Section 3. The infra-red (IR) singularities, encountered in the analysis of the independent gauge modes, are dimensionally regularized and the final expression for the gauge field propagator is found to have a finite limit in 3+1 dimensions. In Section 4 we reintroduce fermion fields and define the perturbative Dyson theory in the interaction representation. The S-matrix elements are proved to be (formally) equivalent to those calculated by the covariant (equal-time) Feynman rules. At the end all these results are discussed and the future investigations are outlined.

All notations and definitions of Green functions are given in Appendix A. In Appendix B the analytical and massive regularizations of IR singularities are presented in some details. Appendix C contains calculations for the Poincaré generators and their commutator relations.

II. ABELIAN GAUGE FIELDS IN 1+1 DIMENSIONS

There are three reasons why we decided to start our analysis with the simple 1+1 dimensional gauge field model. First, inspecting the ML-pole in \([13]\) we find that it contains only the longitudinal momenta \(k_{\pm}\), which are present already in 1+1 dimensions. Second, when we put \(k_\perp \equiv 0\) in \([1, 2]\), two poles: physical and spurious coincide at \(k_+ = 0\). If the LF canonical quantization will correctly reproduce these results, then one can also expect its usefulness also for more physical models in higher dimensions. Third, the Wilson loop calculations for the Yang-Mills fields in \(1+(D-1)\) dimensions show singularity at \(D = 2\) \([17]\). Though we restrict ourselves to the Abelian fields, it may be instructive to compare gauge field sectors in 1 + 1 and higher dimensions.

In this paper our analysis of the quantum electrodynamics in the 1+1 dimensions will be limited to the canonical quantization of Abelian vector gauge field coupled with the external currents \(j^\mu\). These currents describe couplings with the fermion fields \(J^\mu = -e \bar{\psi} \gamma^\mu \psi\) when the fermion dynamics is abandoned. This means that no conservation of \(j^\mu\) is supposed, even in opposite, all its components are treated as the arbitrary functions of space-time.

On can impose the Weyl gauge condition \(A_+ = 0\) explicitly on the gauge field \(A_\mu\) and, as the starting point, one can take the simple Lagrangian density

\[
\mathcal{L}_{Weyl}^{1+1} = \frac{1}{2} (\partial_+ A_-)^2 + A_- j^-.
\]

\(^1\)For example, when the periodic boundary conditions are imposed on the gauge fields, the LC gauge has to be modified \([10]\), while the LF-Weyl gauge remains unchanged.
This reduced model has only one Euler-Lagrange equation

$$\partial_+^2 A_- = j^-, \quad (2.2)$$

which evidently is a dynamical equation and one canonical momentum $\Pi = \partial_+ A_-$. We stress that there are no primary constraints, but instead we notice that the Gauss law

$$G^{1+1} = \partial_- \partial_+ A_- + j^+ = \partial_- \Pi + j^+ = 0 \quad (2.3)$$

is lost. The canonical quantization is immediate and one obtains the canonical Hamiltonian density

$$\mathcal{H}^{1+1}_{can} = \Pi \partial_+ A_- - \mathcal{L}^{1+1}_{Weyl} = \frac{1}{2} (\Pi)^2 - A_- \dot{j}^- \quad (2.4)$$

and the nonvanishing canonical commutator

$$[\Pi(x^+, x^-), A_-(x^+, y^-)] = -i \delta(x^- - y^-). \quad (2.5)$$

They generate the Hamilton equations of motion

$$\partial_+ \Pi = j^- , \quad \partial_+ A_- = \Pi^- \quad (2.6)$$

which are evidently equivalent to the former Euler-Lagrange equation \((2.2)\). The Gauss law \(G^{1+1} = 0\) cannot be imposed strongly as a condition on $\Pi$, because this would be inconsistent with the above Hamiltonian structure. Therefore our quantum theory describes a larger system than the physical electrodynamics and one needs to impose the Gauss law rather as a weak condition on physical states

$$\langle \text{phys} \mid G^{1+1}(x) \mid \text{phys} \rangle = 0. \quad (2.7)$$

The interaction part of \((2.4)\) shows that there is no instant current-current interaction quite contrary to the Hamiltonian in the LC gauge \([18]\). Therefore the perturbative gauge field propagator is given just by the $x^+$-chronological product of free gauge field operators $D_-(x, y) = i \langle 0 \mid T A_-(x) A_-(y) \mid 0 \rangle$. When we put $j^- = 0$ in Eq. \((2.4)\), the free field equations are immediately solved in terms of the $x^+$-independent operators $\pi(x^-)$ and $a_-(x^-)$

$$\Pi(x) = \pi(x^-), \quad A_-(x) = x^+ \pi(x^-) + a_-(x^-), \quad (2.8a)$$

which have to satisfy the commutation relation

$$[\pi(x^-), a_-(y^-)] = -i \delta(x^- - y^-). \quad (2.9)$$

Introducing the Fourier representation for these free fields

$$a_-(x^-) = \int_0^\infty \frac{dk_-}{2\pi} \left[ e^{ik_- x^-} a^\dagger(k_-) + e^{-ik_- x^-} a(k_-) \right] \quad (2.10a)$$

$$\pi(x^-) = \int_0^\infty \frac{dk_-}{2\pi} \left[ e^{ik_- x^-} p^\dagger(k_-) + e^{-ik_- x^-} p(k_-) \right] \quad (2.10b)$$

we get leads to the commutation relations for creation and annihilation operators

$$[a(k_-), p^\dagger(k_-')] = 2i \pi \delta(k_- - k_-') \quad (2.11a)$$

$$[p(k_-), a^\dagger(k_-')] = 2i \pi \delta(k_- - k_-'). \quad (2.11b)$$

Because the above expressions show no singularity at the point $k_- = k_+ = 0$, we find that the propagating massless gauge fields are less singular than the massless scalar fields in $1+1$ dimensions \([20]\). Also we notice that these gauge

\(^2\)In the full QED, the interaction Hamiltonian contains also dynamical fermions $\psi_+^\dagger, \psi_-$, and has a more complicated structure, however with no instantaneous interaction term. The perturbative theory based on the full interaction Hamiltonian can be easily inferred from the results given in the section \([14A]\) by omitting the transverse directions and components.
fields describe non-physical excitations because \( \beta^\dagger = a^\dagger + p^\dagger \) operators can create Fock states with negative metric. The Gupta-Bleuler method can be easily applied here by imposing the weak Gauss law condition \( (2.7) \) or equivalently

\[
p(k_\pm)|\text{phys.}\rangle = 0.
\]

Thus all physical states have zero norm and are created by the operator \( p^\dagger \) acting on the vacuum state \(|0\rangle\). Usually, in the equal-time models, the zero-norm states are also present in the physical subspace while accompanying physical photon states. In the LF picture, the physical photons are described by transverse \( A_\perp \) gauge fields and such fields are evidently absent in 1+1 dimensions, so here we end up with the physical subspace built solely from the zero-norm states.

In the large Hilbert space we immediately find propagators for the independent free fields

\[
\langle 0| T^+ A_-(x)\Pi(y)|0\rangle = \Theta(x^+ - y^+)\langle 0| a_-(x^-)\pi(y^-)|0\rangle + \Theta(y^+ - x^+)\langle 0|\pi(y^-)a_-(x^-)|0\rangle = E_F^1(x-y),
\]

\[
\langle 0| T^+ A_-(x)A_-(y)|0\rangle = x^+ \langle 0| T_\pi(x^-)\pi(y^-)|0\rangle + y^+ \langle 0| T_\pi(y^-)\pi(x^-)|0\rangle = E_F^2(x-y),
\]

where the noncovariant Green functions \( E_F^1(x) \) and \( E_F^2(x) \) are defined in Appendix A and we write a well defined Fourier representation for the gauge field propagator

\[
D_-(x) = i\langle 0| T^+ A_-(x)A_-(0)|0\rangle = -\int_{-\infty}^{\infty} \frac{d^2k}{(2\pi)^2} e^{-ik\cdot x} \frac{1}{[k^\perp + i\epsilon \text{sgn}(k_-)]^2}.
\]

Also we notice a property analogous to that for the LC gauge propagators \[21\]

\[
D_-(x^+, x^-) = 2\partial_- \int_0^{x^+} d\xi D_F(\xi, x^-),
\]

where \( D_F(x) \) is the canonical light-front Feynman propagator for a massless scalar field \[20\]

\[
D_F(x) = -\frac{1}{4\pi} \left[ \ln|x^-| + \frac{i\pi}{2} \text{sgn}(x^+)\text{sgn}(x^-) \right] + f(x^+).
\]

Our conclusion is that in 1+1 dimensions only propagation of non-physical modes is allowed by the Weyl gauge condition and this gives rise to a regular expression for the gauge field propagator. Also we encounter no at \( k_- = 0 \) - quite contrary to the expectations for a 1+1 dimensional model with massless excitations.

### III. Canonical Quantization in D Dimensions

In higher dimensions our model has new features, which were absent in the previous case: the vector fields have their transverse components \( A_i \) and can propagate also in the transverse directions \( x_\perp \). While the first property introduces physical photons, then the second one will lead to the unexpected IR singularities. Therefore anticipating future problems, we choose to work in D dimensions, where 2 coordinates are LF longitudinal \( x_L = (x^\perp) \) and remaining \( d = D-2 \) coordinates are LF transverse \( x_\perp \). The canonical LF quantization will be carried in \( D > 4 \) dimensions, and the the limit \( D \to 4 \) will be taken for all expressions which, being IR-finite, definit the perturbative QED in 3+1 dimensions. Here again one can check that the canonical quantization of the vector gauge fields can be done independently from the fermion sector, therefore the fermion contribution can be first described by the external currents \( j^\mu \). As we did in the previous section, we explicitly implement the Weyl gauge condition \( A_+ = 0 \) in the Lagrangian density

\[
\mathcal{L}_{\text{Weyl}} = \partial_+ A_i (\partial_- A_i - \partial_i A_-) + \frac{1}{2} (\partial_+ A_-)^2 - \frac{1}{4} (\partial_i A_j - \partial_j A_i)^2 + A_- j^+ + A_i j^i.
\]

Our analysis begins with the Euler-Lagrange equations for remaining vector gauge fields

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\(^3\)For notation see Appendix A.
\[ \partial_+ (\partial_+ A_\perp - \partial_j A_j) = j^- , \] (3.2a)

\[ (2\partial_+ \partial_- - \Delta_+^d) A_i = \partial_\perp (\partial_+ A_\perp - \partial_j A_j) + j^i , \] (3.2b)

where \( \Delta_+^d = (\partial_\perp)^2 \) is the Laplace operator in \( d = D - 2 \) dimensions and again the Gauss law \( G^D = 0 \) is lost, where

\[ G^D = \partial_- (\partial_+ A_\perp - \partial_j A_j) - \Delta_+^d A_\perp + j^+ . \] (3.3)

Now we have the primary constraints (in Dirac's nomenclature) connected with the canonical momenta \( \Pi^i = \partial_- A_i - \partial_j A_j \), which are not independent canonical variables\( . \) However these momenta cancel in the canonical Hamiltonian density

\[ \mathcal{H}_{can} = \Pi^- \partial_+ A_\perp + \Pi^i \partial_+ A_i - \mathcal{L}_{Weyl} = \frac{1}{2} \Pi^2 + \frac{1}{2} (\partial_i A_j)^2 + \Pi \partial_i A_i - A_i J^i - A_\perp J^- \] (3.4)

where \( \Pi^- = \partial_+ A_\perp = \Pi + \partial_j A_j \), so one can proceed freely without any further reference to the constrained momenta \( \Pi^i \). The remaining fields: \( A_i, A_\perp \) and \( \Pi \) are the independent canonical variables with their equations of motion

\[ (2\partial_+ \partial_- - \Delta_+^d) A_i = \partial_\perp \Pi + j^i , \] (3.5a)

\[ \partial_+ \Pi = j^- , \] (3.5b)

\[ \partial_+ A_\perp = \Pi + \partial_j A_j , \] (3.5c)

being equivalent to equations (3.2) and the nonzero equal-\( x^+ \) commutators

\[ [\Pi(x^+, \vec{x}), A_\perp(x^+, \vec{y})] = -i\delta^{d+1}(\vec{x} - \vec{y}) \] (3.6a)

\[ [2\partial_- A_i(x^+, \vec{x}), A_j(x^+, \vec{y})] = -i\delta_{ij}\delta^{d+1}(\vec{x} - \vec{y}) \] (3.6b)

where \( \delta^{d+1}(\vec{x}) = \delta(x^-)\delta^{d}(x_\perp) \). The structure of dynamical equations (3.2) indicates our canonical variables are not yet the independent modes, which are very useful in the further formulation. Therefore we propose to introduce the following decomposition of vector fields

\[ A_i = C_i - \partial_\perp [\Delta_+^d]^{-1} (\Pi + 2\partial_j C_j) , \quad A_\perp = C_\perp - \partial_- [\Delta_+^d]^{-1} (\Pi + 2\partial_j C_j) \] (3.7)

which has the form of gauge transformation and its inverse from \( \vec{C} \) to \( \vec{A} \) has the same functional form. The Green function for the Laplace operator is well defined for \( d > 2 \)

\[ [\Delta_+^d]^{-1}(x_\perp) = -\int \frac{d^dk_\perp e^{ik_\perp \cdot x_\perp}}{(2\pi)^d k_\perp^d} = -\frac{1}{4\pi} \frac{\Gamma(\delta/2 - 1)}{\Gamma(1 - \delta)} (\frac{4}{x_\perp^2})^\delta \] (3.8)

and the last analytical expression is regular also for noninteger \( d < 2 \).

There are also other regularizations of the inverse Laplace operator strictly in \( d = 2 \) dimensions [24]: the analytical regularization - when the square pole is changed to \( 2 - 2\delta \) where \( 1 > \delta > 0 \)

\[ [\Delta_\perp^{-1}(x_\perp)] \rightarrow [\Delta_{2-2\delta}^{-1}(x_\perp)] = (-1)^{1-\delta} \int \frac{d^2k_\perp e^{ik_\perp \cdot x_\perp}}{(2\pi)^2 (k_\perp^2)^{1-\delta}} = -\frac{1}{4\pi} \frac{\Gamma(\delta)}{\Gamma(1 - \delta)} \left( \frac{4}{x_\perp^2} \right)^\delta \] (3.9)

and the massive regularization - when the pole at \( k_\perp^2 = 0 \) is shifted by the mass parameter \( m^2 \)

\[ [\Delta_\perp^{-1}(x_\perp)] \rightarrow [\Delta_\perp - m^2]^{-1}(x_\perp) = -\int \frac{d^2k_\perp e^{ik_\perp \cdot x_\perp}}{(2\pi)^2 k_\perp^2 + m^2} = -\frac{1}{2\pi} \Gamma_0(m\sqrt{x_\perp^2}) . \] (3.10)

However our decomposition formula (3.3) cannot be naively regularized in \( d = 2 \) dimensions, because the analytical regularization would bring different dimensions for two terms with \( C_i \), while the massive regularization would lead to the ill-defined inverse transformation (from \( C_i \) to \( A_i \)). The consistent implementation of these two regularizations is presented in Appendix [3], while in this section we will focus on the dimensional regularization.

For the independent modes one easily finds nonvanishing commutators

\[ [\Pi(x^+, \vec{x}), C_\perp(x^+, \vec{y})] = -i\delta^{d+1}(\vec{x} - \vec{y}) , \] (3.11a)

\[ [2\partial_- C_i(x^+, \vec{x}), C_j(x^+, \vec{y})] = -i\delta_{ij}\delta^{d+1}(\vec{x} - \vec{y}) , \] (3.11b)
and effective equations of motion

\begin{align}
(2\partial_+ \partial_- - \Delta_\perp) C_i &= j^i - 2\partial_i[\Delta^d_\perp]^{-1} \ast (\partial_- j^+ + \partial_j j^i), \\
\partial_+ C_- &= -[\Delta^d_\perp]^{-1} \ast (\partial_i j^i + \partial_- j^-) \\
\partial_+ \Pi &= j^-.
\end{align}

(3.12a, 3.12b, 3.12c)

Also the Hamiltonian density \( \mathcal{H}_{\text{eff}} \) can be expressed in terms of these new fields

\[ \mathcal{H}_{\text{eff}} = \frac{1}{2}(\partial_i C_j)^2 - \Pi[\Delta^d_\perp]^{-1} \ast [\partial_i j^i + \partial_- j^-] - C_i [j^i - 2\partial_i[\Delta^d_\perp]^{-1} \ast (\partial_- j^+ + \partial_j j^i)] - C_- j^- , \]

(3.13)

showing again no instantaneous current-current interaction. Therefore, in the interaction picture, the perturbative propagators for independent modes will be given by the \( x^+ \)-chronological products of free fields. The Fourier representation of free fields is defined for positive values of the \( k_- \) momentum

\[ C_-(\vec{x}) = \int_0^\infty \frac{dk_-}{2\pi} \int_0^\infty \frac{dk_\perp}{2\pi} e^{-i\vec{k}_\perp \cdot \vec{x}} c_0(k_\perp) \]

(3.14a)

\[ \Pi(\vec{x}) = \int_0^\infty \frac{dk_-}{2\pi} \int_0^\infty \frac{dk_\perp}{2\pi} e^{-i\vec{k}_\perp \cdot \vec{x}} \bar{p}(\vec{k}) + e^{+i\vec{k}_\perp \cdot \vec{x}} p^i(\vec{k}) \]

(3.14b)

\[ C_i(x) = \int_0^\infty \frac{dk_-}{2\pi} \int_0^\infty \frac{dk_\perp}{2\pi} \frac{dk}{2k_-} e^{-i\vec{k}_\perp \cdot \vec{x}} c_0(k_\perp) \]

(3.14c)

and the commutators for creation and annihilation operators follow from (3.11)

\[ \left[ a(\vec{k}), p^i(\vec{k}) \right] = \left[ a^i(\vec{k}), p(\vec{k}) \right] = i(2\pi)^{d+1} \delta^{d+1}(\vec{k} - \vec{k}') \]

(3.15a)

\[ \left[ c_i(\vec{k}), c_j^\dagger(\vec{k}') \right] = (2\pi)^{d+1} 2k_- \delta_{ij} \delta^{d+1}(\vec{k} - \vec{k}') \]

(3.15b)

where \( \delta^{d+1}(\vec{k}) = \delta_4(k_\perp) \delta(k_-) \). The nonzero propagators for the independent modes can be easily found

\[ \langle 0 | T^+ C_-(\vec{x}) \Pi(\vec{y}) | 0 \rangle = E_F^k(x_L - y_L) \delta^d(x_\perp - y_\perp) , \]

(3.16a)

\[ \langle 0 | T^+ C_i(x) C_j(y) | 0 \rangle = \delta_{ij} D_F^{d+2}(x - y) , \]

(3.16b)

where the covariant Feynman Green function \( D_F^{d+2}(x) \) is defined in Appendix A.3. Next, one can use the decompositions \( \vec{A} \) when calculation the propagators for the fields \( \vec{A} \). For the transverse components \( A_i \) only the contribution of \( C_i \) fields is taken

\[ \langle 0 | T^+ A_i(x) A_j(y) | 0 \rangle = \left( \delta_{ik} - 2\partial_k^x \partial^y_{\vec{k}}[\Delta^d_\perp]^{-1} \ast \right) \left( \delta_{jl} - 2\partial_l^x \partial^y_{\vec{l}}[\Delta^d_\perp]^{-1} \ast \right) D_F^{d+2}(x - y) \delta_{kl} \]

= \delta_{ij} D_F^{d+2}(x - y) , \]

(3.17a)

but for the longitudinal component \( A_- \) there are more terms

\[ \langle 0 | T^+ A_-(x) A_j(y) | 0 \rangle = \partial_j^x [\Delta^d_\perp]^{-1} \ast \left( 2\partial_k^x D^{d+2}_F(x - y) + E_F^k(x_L - y_L) \delta^d(x_\perp - y_\perp) \right) \]

(3.17b)

\[ \langle 0 | T^+ A_-(x) A_-(y) | 0 \rangle = 2\partial_-^x [\Delta^d_\perp]^{-1} \ast \left( 2\partial_k^x D^{d+2}_F(x - y) + E_F^k(x_L - y_L) \delta^d(x_\perp - y_\perp) \right) . \]

(3.17c)

The expression in the above curly brackets can be expressed either as the integral over \( x^+ \) of the covariant Feynman Green function

\[ \langle 0 | T^+ A_-(x) A_i(y) | 0 \rangle = \partial_i \int^{x^+ - y^+}_0 dz^{+} D^{d+2}_F(z^+ + y^+, \vec{x} - \vec{y}) , \]

(3.18a)

\[ \langle 0 | T^+ A_-(x) A_-(y) | 0 \rangle = 2\partial_- \int^{x^+ - y^+}_0 d\xi D^{d+2}_F(z^+ + y^+, \vec{x} - \vec{y}) \]

(3.18b)

\[ \text{The commutators } (3.15a) \text{ show that the operators } a^i \text{ and } p^i \text{ are trivial generalizations of the respective operators in the } 1+1 \text{ dimensions and therefore here again they can create negative norm Fock states and the physical subspace has to be chosen by means of the weak Gauss law.} \]}
or as the Fourier integral with two causal poles
\[
\langle 0\vert T^+A_-(x)A_+(y)\vert 0 \rangle = i \int \frac{d^{d+2}k}{(2\pi)^{d+2}} \frac{e^{-ik\cdot(x-y)}}{k^2 + i\epsilon} \frac{(k_+n_\mu^W + k_\nu n_\nu^W)}{k_+ + i\epsilon \text{sgn}(k_-)}. \tag{3.19}
\]

Finally the Fourier representation can be written in a concise form for all components of \(A_\mu\) field
\[
\langle 0\vert T^+A_\mu(x)A_\nu(y)\vert 0 \rangle = i \int \frac{d^{d+2}k}{(2\pi)^{d+2}} \frac{e^{-ik\cdot(x-y)}}{k^2 + i\epsilon} \left[ -g_{\mu\nu} + \frac{(k_\mu n_\nu^W + k_\nu n_\mu^W)}{k_+ + i\epsilon \text{sgn}(k_-)} \right]. \tag{3.20}
\]

All expressions for propagators have the limit \(d \to 2\) and so we arrive at the canonical LF propagators in 3 + 1 dimensions
\[
\langle 0\vert T^+A_\mu(x)A_\nu(y)\vert 0 \rangle = i \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik\cdot(x-y)}}{k^2 + i\epsilon} \left[ -g_{\mu\nu} + \frac{(k_\mu n_\nu^W + k_\nu n_\mu^W)}{k_+ + i\epsilon \text{sgn}(k_-)} \right]. \tag{3.21}
\]

The relations (3.18), which are evidently valid also in 3 + 1 dimensions, can be shown to be the zeroth order (in powers of \(c\)) terms of the integrated Schwinger-Dyson relations for the full QED in the presence of interactions.

Our above canonical LF analysis gives the same result for the gauge field propagator as the equal-time quantization.

Before going to the interacting model with fermions, let us study the Poincaré covariance in the gauge field sector. We notice that both the choice of a null-surface \(x^+ = \text{const}\) as the quantization surface and by the noncovariant \(\text{LF-Weyl gauge condition } A_+ = 0\) violate the Poincaré covariance. This is quite similar to the equal-time canonical procedure for the temporal gauge condition \(A_0 = 0\) \(\text{[13]},\) where the Poincaré covariance is recovered in the physical subspace selected by the weak Gauss law. In our LF formulation we will check this possibility in 3 + 1 dimensions.\(^5\)

First we check that \(\tilde{A}\) transforms covariantly
\[
[M^{\mu\nu}, A^\lambda(x)] = -i (x^\mu \partial^\nu - x^\nu \partial^\mu) A^\lambda(x) - i \left( g^{\mu\lambda} A^\nu - g^{\nu\lambda} A^\mu \right)(x). \tag{3.22}
\]

Then we analyse the commutator algebra of canonical Poincaré generators
\[
[M^{\mu\nu}, P^\lambda] = ig^{\nu\lambda} P^\mu - ig^{\mu\lambda} P^\nu + i \int d^3 \vec{x} \delta^{\lambda\nu} \left[ g^{\mu\nu} G(x) A^\nu(x) - g^{\nu\nu} G(x) A^\mu(x) \right]_{\text{sym}} \tag{3.23}
\]
\[
[M^{\mu\nu}, M^{\rho\sigma}] = +i \left( g^{\mu\lambda} M^{\nu\rho} - g^{\nu\lambda} M^{\mu\rho} - g^{\mu\rho} M^{\nu\lambda} - g^{\nu\rho} M^{\mu\lambda} \right) + i \int d^3 \vec{x} (x^\mu g^{\nu\tau} - x^\nu g^{\mu\tau}) \left[ g^{\lambda\nu} G(x) A^\tau(x) - g^{\nu\nu} G(x) A^\lambda(x) \right]_{\text{sym}} \nonumber
\]
\[-i \int d^3 \vec{x} (x^\lambda g^{\rho\tau} - x^\rho g^{\lambda\tau}) \left[ g^{\nu\tau} G(x) A^\lambda(x) - g^{\nu\nu} G(x) A^\rho(x) \right]_{\text{sym}}. \tag{3.24}
\]

where \(G = \partial_-(\partial_+ A_+ + \partial_0 A_0) - \Delta A_-\) is the Gauss law operator in the absence of charged currents and the symmetrization for noncommuting \(G\) and \(A^\mu\) operators is imposed. In the physical subspace with states selected by the weak Gauss law \(\langle \text{phys}' \vert G(x) \vert \text{phys} \rangle = 0\) one finds that the anomalous terms vanish
\[
\langle \text{phys}' \vert \left[ g^{\mu\nu} G(x) A^\nu(x) - g^{\nu\nu} G(x) A^\mu(x) \right]_{\text{sym}} \vert \text{phys} \rangle = 0 \tag{3.25}
\]
and one concludes that the Poincaré covariance is recovered here.

\(^5\)These problems will be discussed in a future publication.

\(^6\)In Appendix D we give the definitions of Poincaré generators and present some nontrivial steps of calculations.
IV. INTERACTING THEORY WITH FERMIONS

If one takes the effective Lagrangian density for the gauge sector in the form

\[ \mathcal{L}_{\text{eff gauge}}^{\text{QED}} = \Pi \partial_\alpha A_\alpha + \partial_\alpha (A_\alpha \partial_\alpha A_\alpha) - \frac{1}{2} \Pi^2 - \frac{1}{2} (\partial_\alpha A_\alpha)^2 - \Pi \partial_\alpha A_\alpha + A_\alpha j_\alpha + A_\alpha^j + A_\alpha^j, \] (4.1)

then one can incorporate fermions by resubstituting \( j^\mu \) by the fermion currents \(-e\bar{\psi}\gamma^\mu \partial_\mu \psi\) and adding the kinetic terms for fermions. All this leads to the following Lagrangian density for the interacting theory

\[ \mathcal{L}_{\text{inter}}^{\text{QED}} = \Pi \partial_\alpha A_\alpha + \partial_\alpha (A_\alpha \partial_\alpha A_\alpha) - \frac{1}{2} \Pi^2 - \frac{1}{2} (\partial_\alpha A_\alpha)^2 - \Pi \partial_\alpha A_\alpha + i\sqrt{2} \psi^\dagger \partial_\alpha \psi + \sqrt{2} \psi^\dagger (i\partial_- - eA) \psi - \xi \psi + \psi^\dagger \xi \] (4.2a)

where \( \psi_\pm = \Lambda \psi, \psi^\dagger_\pm = \psi^\dagger \Lambda \) and

\[ \xi = [ -i \partial_\alpha \alpha^\tau + M\beta + eA\alpha^\tau ] \psi_\tau, \xi^\dagger = \psi_\tau^\dagger \left[ i \partial_\alpha \alpha^\tau + M\beta + e\alpha^\tau A_I \right]. \] (4.2b)

As usually in the LF formulation, the fermions \( \psi_- \) and \( \psi^\dagger_- \) satisfy nondynamical nondynamical equations

\[ \sqrt{2} (i\partial_- - eA_-) \psi_- = \xi, \quad -\sqrt{2} (i\partial_- + eA_-) \psi^\dagger_- = \xi^\dagger. \] (4.3)

For solving these equation uniquely, one has to impose some boundary conditions for the dependent fermion fields. Here we choose the simplest and commonly used possibility - the antisymmetric conditions

\[ \lim_{x^\rightarrow -\infty} \psi_-(x) = - \lim_{x^\rightarrow -\infty} \psi_-(x), \quad \lim_{x^\rightarrow -\infty} \psi^\dagger_-(x) = - \lim_{x^\rightarrow -\infty} \psi^\dagger_-(x), \] (4.4)

and this allows us to solve Eqs. (4.3) as

\[ \psi_-(x) = \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} dy^- (i\partial_- - eA_-)^{-1} [x^-, y^-; x^\perp] \xi(y^-, x^\perp), \] (4.5a)

\[ \psi^\dagger_-(x) = \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} dy^- (i\partial_- - eA_-)^{-1} [y^-, x^-; x^\perp] \xi^\dagger(y^-, x^\perp), \] (4.5b)

where the Green function \( (i\partial_- - eA_-)^{-1} [x^-, y^-; x^\perp] \) is defined by the integral equations

\[ (i\partial_- - eA_-)^{-1} [x^-, y^-; x^\perp] = (i\partial_-)^{-1} (x^--y^-) + \int_{-\infty}^{\infty} dz^- (i\partial_-)^{-1} (x^--z^-) eA_-(z^-, x^\perp) \times (i\partial_- - eA_-)^{-1} [z^-, y^-; x^\perp] \] (4.6a)

\[ = (i\partial_-)^{-1} (x^--y^-) + \int_{-\infty}^{\infty} dz^- (i\partial_- - eA_-)^{-1} [x^-, z^-; x^\perp] \times eA_-(z^-, x^\perp) (i\partial_-)^{-1} (z^- - y^-). \] (4.6b)

It is useful to introduce the matrix notation for the integral operators, by leaving the explicite dependence on coordinates, denoting the integration over \( x^- \) variables by asterisks and writing the integral operators as fractions; for example the above equations are written as

\[ \frac{1}{i\partial_- - eA_-} = \frac{1}{i\partial_-} + \frac{1}{i\partial_-} \frac{1}{eA_-} = \frac{1}{i\partial_-} + \frac{1}{i\partial_- - eA_-} \frac{1}{eA_-}. \] (4.7)

These equations can be used for generating the series in the arbitrary powers of e, provided the following integral

\[ \int_{-\infty}^{\infty} dz^- (i\partial_-)^{-1} (x^--z^-) eA_-(z, x^\perp) (i\partial_-)^{-1} (z^- - y^-) \sim \int_{-\infty}^{\infty} dp^- dk^- e^{-ip_-(x^- - y^-)} \bar{A}_-(k^-, x^\perp) \text{ CPV} \left[ \frac{1}{p^-} \right] \] (4.8)
is a well-defined expression. This means that one has to consider $A_-$ fields with their momenta $k_- \neq 0$, otherwise in Eq. (4.8) one would encounter ill-defined expression \((\text{CPV} \left[ \frac{1}{p_-} \right])^2\). Bearing this limitation in mind we can write the infinite series for \((i\partial_- - eA_-)^{-1}\)

$$
\frac{1}{i\partial_- - eA_-} = \frac{1}{i\partial_-} * W_{-1}[\bar{a}] = W_{-1}[\bar{a}] * \frac{1}{i\partial_-}
$$

(4.9a)

where

$$
W_{-1}[\bar{a}] = \sum_{n=0}^{n=\infty} \left( \frac{1}{i\partial_-} eA_- \right) * \ldots * \left( \frac{1}{i\partial_-} eA_- \right),
$$

(4.9b)

$$
W_{-1}[\bar{a}] = \sum_{n=0}^{n=\infty} \left( eA_- \frac{1}{i\partial_-} \right) * \ldots * \left( eA_- \frac{1}{i\partial_-} \right).
$$

(4.9c)

Thus we write the fermion contribution to the Hamiltonian as

$$
H_{fer} = \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} d^d x_\perp \xi^\dagger * \frac{1}{i\partial_-} * W_{-1}[\bar{a}] * \xi,
$$

(4.10)

where the integrand is a local expression in the $x_\perp$ coordinates, and our notation treats $\xi$ as a column matrix and $\xi^\dagger$ as a row matrix.

Thus we end up with the total Hamiltonian, which depends solely on dynamical field variables

$$
H_{total} = \int d^{d+1} \vec{x} \left[ \frac{1}{2} \Pi^2 + \frac{1}{2} (\partial_i A_j)^2 + \Pi \partial_i A_i \right] + \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} d^d x_\perp \xi^\dagger * \frac{1}{i\partial_-} * W_{-1}[\bar{a}] * \xi,
$$

(4.11)

while the nonvanishing equal $x^+$ (anti)commutators have the canonical LF form

$$
\left[ \Pi(x^+, \vec{x}), A_-(x^+, \vec{y}) \right] = -i\delta^{d+1}(\vec{x} - \vec{y}),
$$

(4.12a)

$$
\left[ 2\partial_- A_i(x^+, \vec{x}), A_j(x^+, \vec{y}) \right] = -i\delta_{ij}\delta^{d+1}(\vec{x} - \vec{y}),
$$

(4.12b)

$$
\left\{ \psi^\dagger_+(\vec{x}), \psi_+(\vec{y}) \right\} = \frac{1}{\sqrt{2}} \delta^3(\vec{x} - \vec{y}).
$$

(4.12c)

Thus in the LF Weyl gauge $A_+ = 0$ the quantum electrodynamics of massive fermion matter fields contains no instantaneous interactions of currents and have all (anti)commutator relations (at equal-$x^+$) independent of interactions and c-numbered. All these properties, while being quite normal in the equal-time formulation, are not such in the front description of interacting theories, when the light-front surface probes the fields at the light-like seperated points of space-time.

**A. Perturbation theory**

The perturbation theory is defined most easily in the interaction picture, where all quantum field operators have free dynamics and the interaction appears in the evolution of quantum states. Thus we also choose to work in this representation, however for clarity of further formulas we will omit the usually used subscript $i$. Free evolution follows from a free Hamiltonian, but we notice that the total Hamiltonian \([11]\) depends on the coupling constant $\epsilon$ both non-locally in the expression $W_{-1}[\bar{a}]$ and locally in the fields $x_i$ and $\xi^\dagger$ - compare Eq.(4.2b). Thus we define the free Hamiltonian $H_0$ as the limit

$$
H_0 \equiv \lim_{\epsilon \rightarrow 0} H_{total} = \int d^{d+1} \vec{x} \left[ \frac{1}{2} \Pi^2 + \frac{1}{2} (\partial_i A_j)^2 + \Pi \partial_i A_i \right] + \int d^d x_\perp \frac{1}{\sqrt{2}} \xi^\dagger \frac{1}{i\partial_-} * \xi_0,
$$

(4.13)

where now we have
The perturbative propagators are defined as the chronological products of dynamical free fields

\[ \xi_0 = [-i\partial_t \alpha^i + M\beta] \psi_+, \quad \xi_0^\dagger = \psi_+^\dagger \left[i \partial_t \alpha^i + M\beta \right]. \]  

(4.14)

Thus from the (anti)commutator relations (4.12), which being independent of interactions, remain unchanged, we find the free evolution equations

\[ (2\partial_+ \partial_- - \Delta)A_i = \partial_\mu \Pi, \]
\[ \partial_\nu \Pi = 0, \]
\[ \partial_\nu A_\nu = \Pi + \partial_\nu A_i, \]
\[ (2\partial_+ \partial_- + M^2)\psi_+ = 0, \]
\[ (2\partial_+ \partial_- + M^2)\psi_+^\dagger = 0. \]

(4.15a)

(4.15b)

(4.15c)

(4.15d)

(4.15e)

The perturbative propagators are defined as the chronological products of dynamical free fields

\[ \langle 0 | T^+ A_\mu(x) A_\nu(y) | 0 \rangle = ig_{\mu\nu} D_F(x - y) + (n_\mu \partial_\nu + n_\nu \partial_\mu) (E^0_F * D^{d+2}_F) (x - y), \]
\[ \langle 0 | T^+ \psi_+(x) \psi_+^\dagger(y) | 0 \rangle = i\sqrt{\Delta} \partial_\nu \Delta^{d+2} F^2 (x - y, M^2), \]

(4.16a)

(4.16b)

where for the gauge fields we have repeated our previous analysis, while the result for dynamical fermions is well known [14]. These expressions will produce all nontrivial contractions in the LF version of the Dyson perturbation theory, while the vertices are to be taken from the interaction Hamiltonian, which is defined as the difference between the total and the free Hamiltonians \( H_{\text{int}} = H_{\text{total}} - H_{\text{int}} \). Because all these Hamiltonians are local in the transverse directions we can write the expressions for the density (in \( x_+ \))

\[ \tilde{H}_{\text{int}} = \frac{1}{\sqrt{2}} \xi_0^\dagger * \frac{1}{i\partial_-} \left( eA_\nu \xi_0 \right) * W_{-1} \xi_0 + \frac{e}{\sqrt{2}} \left( \psi_+^\dagger \alpha^i A_i \right) * \frac{1}{i\partial_-} * W_{-1} \xi_0 \]
\[ + \frac{e}{\sqrt{2}} \xi_0^\dagger * \frac{1}{i\partial_-} * W_{-1} \xi_0 \left( \alpha^i A_i \psi_+ \right) + \frac{e^2}{\sqrt{2}} \left( \psi_+^\dagger \alpha^i A_i \right) * \frac{1}{i\partial_-} * W_{-1} \xi_0 \left( \alpha^i A_i \psi_+ \right). \]

(4.17)

This expression looks fairly complicated, but fortunately, for our choice of antisymmetric boundary conditions for \((i\partial^-)^{-1}\), we may conveniently introduce the fields \( \Psi_- \) and \( \Psi_+^\dagger \)

\[ \Psi_- = \frac{1}{\sqrt{2}} \frac{1}{i\partial_-} * \left[-i\partial_t \alpha^i - M\beta\right] \psi_+, \quad \Psi_+^\dagger = \psi_+^\dagger \frac{1}{\sqrt{2}} \left[i \partial_t \alpha^i + M\beta \right] * \frac{1}{i\partial_-}. \]

(4.18)

and further the chronological products for them

\[ \langle 0 | T^+ \Psi_- (x) \psi_+^\dagger(y) | 0 \rangle + \langle 0 | T^+ \psi_+(x) \Psi_+(y) | 0 \rangle = (\partial^+_{\mu} \gamma^i + iM) \gamma^0 \Delta^{d+2}_F (x - y, M^2) \]
\[ \langle 0 | T \psi_-(x) \Psi_-(y) | 0 \rangle = -\partial^+_{\mu} \gamma^0 \Delta^{d+2}_F (x - y, M^2) - \gamma^+ \gamma^0 \frac{1}{2\partial_-} * \delta(x - y). \]

(4.19a)

(4.19b)

Now it is easy to check that for the new fermion fields \( \Psi = \psi_+ + \Psi_- \) and \( \Psi^\dagger = \psi_+^\dagger + \Psi_+^\dagger \) we have a compact expression for the propagator

\[ \langle 0 | T \Psi (x) \Psi(y) | 0 \rangle = - (\partial^+_{\mu} \gamma^i + iM) \Delta_F (x - y, M^2) - \gamma^+ \frac{1}{2\partial_-} * \delta(x - y), \]

(4.20)

while a more tedious algebra for the interaction Hamiltonian (4.17) density gives

\[ \tilde{H}_{\text{int}} = e\Psi \left( \gamma^- A_- + \gamma^i A_i \right) * \left[ 1 + \frac{e}{2} \frac{1}{i\partial_-} * W_{-1} \left( \gamma^- A_- + \gamma^i A_i \right) \right] * \Psi_. \]

(4.21)

These expressions can be compared to those obtained in the LC gauge \( A_- = 0 \). In both cases the chronological product for free fermion fields is the same (4.20) and has one non-covariant non-causal term. Our interaction Hamiltonian (4.21) has the infinite number of non-covariant vertices which are generated by \( W_{-1} \), contrary to the LC gauge Hamiltonian, where there is only one noncovariant term. In the LC gauge case these two non-covariant terms: from the fermion progtator and from the interaction Hamiltonian, cancel each other [14]. A similar cancelation should also appear in the LF Weyl gauge. However, due to the infinite number of non-covariant terms in (4.21), it is an open question if this cancelation will be complete.
B. Structure of the S-matrix in the LF perturbative calculations

Above we have noted that all non-covariant terms are connected with the fermion fields, so in order to prove the equivalence of the LF perturbative theory and the usual equal-time formulation we need to study only fermion contractions. Keeping the gauge fields $A_{\mu}$ as c-numbers, we study Wick’s theorem for the chronological product

$$S_{fer}[^{\bar{\Psi}}, {\bar{\Psi}; A}] = T^+ \exp \left(-ie \int d^d x_{\perp} \bar{\Psi} * {\bar{V}} * \Psi \right),$$

where

$$V = e \left(\gamma^- A_{\perp} + \gamma^j A_j \right) \left[1 + \frac{e}{2} \gamma^+ \frac{1}{i\partial^-} * \mathcal{W}_{-1}[a]\left(\gamma^- A_{\perp} + \gamma^j A_j \right) \right].$$

Using the Schwinger functional technique $[26]$, we derive the one gets the result

$$S_{fer}[^{\bar{\Psi}}, {\bar{\Psi}; A}] = e^{\left(\frac{e}{2} \gamma^+ \frac{1}{i\partial^-} * \mathcal{W}_{-1}[a]\left(\gamma^- A_{\perp} + \gamma^j A_j \right) \right)}$$

where the normal ordering is taken only for fermion fields and

$$\mathcal{S}_{F}(x-y) = -i \langle 0 \left| T^+ \Psi(x) \bar{\Psi}(y) \right| 0 \rangle = (i\gamma^\mu \partial^\mu - m) \Delta_F(x-y, M^2) - \frac{\gamma^+}{2} \frac{1}{i\partial^-} * \delta(x-y).$$

The algebra of Dirac matrices simplifies the contribution for the non-covariant part of the fermion propagator

$$- \frac{\gamma^+}{2} \frac{1}{i\partial^-} * V = - \frac{\gamma^+}{2} \frac{1}{i\partial^-} * \mathcal{W}_{-1}[a]\left(\gamma^- A_{\perp} + \gamma^j A_j \right),$$

and then one easily checks the following factorization property

$$1 - \mathcal{S}_{F} * V = \left[1 - S_{F}^{cov} (\gamma^- A_{\perp} + \gamma^j A_j) \right] * \left[1 + \frac{e}{2} \gamma^+ \frac{1}{i\partial^-} * \mathcal{W}_{-1}[a]\left(\gamma^- A_{\perp} + \gamma^k A_k \right) \right],$$

where $S_{F}^{cov}$ is the covariant part of the fermion propagator. It is also useful to analyse the non-covariant factor

$$\text{Tr} \ln \left[1 + \frac{e}{2} \gamma^+ \frac{1}{i\partial^-} * \mathcal{W}_{-1}[a]\left(\gamma^- A_{\perp} + \gamma^k A_k \right) \right] = \frac{1}{4} \text{tr} 1 \text{Tr ln} \mathcal{W}_{-1}[a^\dagger].$$

Gathering all above results we conclude that almost all non-covariant terms have canceled

$$S_{fer}[^{\bar{\Psi}}, {\bar{\Psi}; A}] = \exp \left[-ie \int d^d x_{\perp} \bar{\Psi} (\gamma^- A_{\perp} + \gamma^k A_k) * \left[1 - S_{F}^{cov} (\gamma^- A_{\perp} + \gamma^k A_k) \right]^{-1} * \Psi \right]:$$

$$\exp \left[\frac{1}{4} \text{tr} 1 \text{Tr ln} \mathcal{W}_{-1}[a^\dagger] \right] \left[\text{Tr ln} \left[1 - S_{F}^{cov} (\gamma^- A_{\perp} + \gamma^k A_k) \right]\right].$$

So far we have used only algebraic properties, specially when keeping track of the various combinatorical factors. Now we may use different arguments connected with the closed loop integrals. We observe that the term $\text{Tr ln} \mathcal{W}_{-1}[a^\dagger]$ contains the integrals of type

$$\int dk \left[ \frac{1}{k} \right]_{CPV} \left[ \frac{1}{k - p_1} \right]_{CPV} \ldots \left[ \frac{1}{k - p_n} \right]_{CPV} = 0 \text{ for all } p_i \neq 0.$$

Thus when $k_{-}$ momenta of all insertions of $A_{-}$ are cut at some nonzero value, then the non-covariant term is field independent may be omitted. In this way we have shown that in the LF perturbative calculation the S-matrix elements are the same as those calculated with covariant (equal-time) rules.
V. DISCUSSION AND PERSPECTIVES

In this paper we have presented the canonical quantization procedure, formulated on a single light-front, applied to the LF-Weyl gauge QED. Special attention was paid to the free gauge field propagators, where the ML prescription arose quite naturally for their spurious poles. This prescription was attributed to the presence of ghostlike modes with negative metric. Further implementing the Gauss law weakly as a condition on states, we have separated the physical subspace with positive semi-definite metric. In this physical subspace the Poincaré covariance was recovered. In the analysis of independent modes IR singularities were encountered and several regularizations were studied - all of them have lead to the same gauge field propagators in $3 + 1$ dimensions. This singularity, specific to the LF formulation, appeared for dimensions $2 < D < 4$ but not for $D = 2$ where the gauge field sector is free from IR problems. We have noticed similarity between the LF-Weyl gauge in the LF approach and the temporal gauge $A_0 = 0$ in the equal-time formulation, which is very encouraging but we are also aware of the limitations. First all this happens for the Abelian gauge theory coupled to fermion currents, second there are no self-interactions with derivative couplings for gauge fields. Unfortunately, for other types of interaction the light-front quantization would be much more complicated. Generally one can have nontrivial q-numbered commutators between the vector gauge fields and the matter fields and these commutators would depend on interactions (contain coupling constants) so the proper perturbative rules could be quite different from those inferred from the free field model. Such things happen in the Abelian model with scalar matter fields, where the LF-Weyl gauge leads to the equal-$x^+$ commutators between scalar matter fields and vector gauge fields which depend on the coupling constant. Similar situation should also occur for the non-Abelian case, where the transverse components $A^\perp_i$ can have q-numbered commutators with the longitudinal ones $A^a$. Such phenomena are usually absent in the equal-time approach and one sees that both formalisms may be quite different for physically relevant models.

Though the above observations form a rather narrow limits for the future LF investigations of the LF-Weyl gauge models, some possibilities still remain. When the non-Abelian gauge fields are analyzed in the finite volume approach (DLCQ) the LF-Weyl gauge can be effectively imposed for the zero mode gauge fields. Also it seems worth to check the another IR regularization for the light-front procedure, where compactification is introduced only for transverse space coordinates $x_\perp$ - this, contrary to the common DLCQ method, would leave the parity symmetry $x^+ \leftrightarrow x^-$ unbroken. For QED one can formally show equivalence of perturbative covariant (equal-time) Feynman rules and the LF rules, provided the appropriate regularization for $(k_-)^{-n}$ poles are introduced. Therefore the renormalizability of QED in the LF-Weyl gauge should be checked within the 'old-fashioned' Hamiltonian perturbation theory.

APPENDIX A: NOTATION

1. Light-front coordinates in D dimensions

In D dimensions we define 2 longitudinal coordinates $x_L = \left( x^\pm = \frac{x^0 \pm x^i}{\sqrt{2}} \right)$ and take $x^+$ as the parameter of dynamical evolution. We denote transverse components $x_\perp = (x^1, \ldots, x^D)$ by the Latin indices $(i,j,\ldots)$. Similarly we define components of any 4-vector. The metric has nonvanishing components $g_{+-} = 1, g_{ij} = -\delta_{ij}$ and the scalar product for any 4-vectors decomposes as $A \cdot B = A^+ B^- + A^- B^+ - A^i B^i$. Partial derivatives are defined as $\partial_\pm = \partial/\partial x^\pm$, $\partial_i = \partial/\partial x^i$. Tensor components are defined analogously e.g. $T^{\pm \mu} = \frac{1}{\sqrt{2}} (T^0 \mu \pm T^1 \mu)$ and summation over repeated indices is understood. Also we introduce the vector notation for coordinates $\vec{x} = (x^- , x_\perp)$, which parameterize the light-front surface $x^+ = \text{const.}$ and for momenta associated with them $\vec{p} = (k_-, k_\perp)$. The product of such 3-vectors decomposes as $\vec{k} \cdot \vec{x} = k_- x^- - k_i x_i$. This vector notation is also used for the components of vector gauge field $\vec{A} = (A_-, A_i)$

2. Dirac matrices

The Dirac matrices $\gamma^\mu$ satisfy anticommutation relation

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu},$$  \hspace{1cm} (A1)
where their components are defined analogously to coordinates e.g. \( \gamma^\pm = \frac{\pm i}{\sqrt{2}} \). Thus \( \gamma^\pm \) are nilpotent matrices \((\gamma^\pm)^2 = 0\).

For the projection operators \( \Lambda_\pm = \frac{1}{\sqrt{2}} \gamma^0 \gamma^\pm = \frac{1}{2} \gamma^\mp \gamma^\pm \), we have useful relations

\[
\begin{align*}
\Lambda_\pm \Lambda_\pm &= \Lambda_\pm ,
\Lambda_+ + \Lambda_- &= 1 , \\
\Lambda_+ \Lambda_- &= \Lambda_- \Lambda_+ = \Lambda_\pm \gamma_\mp = 0 , \\
\gamma^\pm \Lambda_\pm &= \Lambda_\pm \gamma^\pm ,
\gamma^0 \Lambda_\pm &= \Lambda_\pm \gamma^0 ,
\gamma^i \Lambda_\pm &= \Lambda_\pm \gamma^i .
\end{align*}
\]

(A2) (A3) (A4)

We also use the standard non-relativistic notation \( \gamma^0 = \beta \), \( \gamma^i \gamma^i = \alpha^i \).

### 3. Green functions

We define the noncovariant Feynman Green functions \( E_F^1(x_L) \) and \( E_F^2(x_L) \) as

\[
E_F^1(x_L) \triangleq i \int_0^\infty \frac{dk}{2\pi} \left[ \Theta(x^+) e^{-ik\cdot x} - \Theta(-x^+) e^{ik\cdot x} \right] = \frac{1}{2\pi} \frac{1}{x^- - i\epsilon \text{sgn}(x^+)}
\]

(A5)

\[
E_F^2(x_L) \triangleq -x^+ E_F^1(x_L) = -\frac{1}{2\pi} \frac{x^+}{x^- - i\epsilon \text{sgn}(x^+)}
\]

(A6)

These functions can be also represented by the 2-dimensional Fourier integrals

\[
\begin{align*}
E_F^1(x_L) &= -\int_{-\infty}^{\infty} \frac{dk}{(2\pi)^2} \frac{e^{-ik\cdot x_L}}{k_+ + i\epsilon \text{sgn}(k_-)} \\
E_F^2(x_L) &= i \int_{-\infty}^{\infty} \frac{dk}{(2\pi)^2} \frac{e^{-ik\cdot x_L}}{[k_+ + i\epsilon \text{sgn}(k_-)]^2}
\end{align*}
\]

(A7) (A8)

and they satisfy obvious relations: \( E_F^2(0, x^-) = 0 \), \( \partial_+ E_F^2(x_L) = -E_F^1(x_L) \).

In \( D = d + 2 \) dimensions we have the covariant massive Feynman Green function

\[
\Delta_F^{d+2}(x, m^2) \triangleq \int \frac{d^dk}{(2\pi)^d} \left[ \Theta(x^+) e^{-ik\cdot x} + \Theta(-x^+) e^{ik\cdot x} \right] |_{k_+ = \frac{k^2 + m^2}{2\epsilon}}
\]

\[
= i \int \frac{d^{d+2}k}{(2\pi)^{d+2} k^2 - m^2 + i\epsilon}.
\]

(A9)

and its massless limit we denote by \( D_F^{d+2}(x) \triangleq \lim_{m \to 0} \Delta_F^{d+2}(x, m^2) \). The combination of Green functions, which appears in the expression for the longitudinal components of propagator, can be written either as the finite integral over \( x^+ \) coordinate or as the Fourier integral

\[
\begin{align*}
\left[ 2\partial^\xi D_F^{d+2}(x) + E_F^1(x_L) \delta^d(x_L) \right] &= \int_{-\infty}^{\infty} \frac{d^dk}{(2\pi)^d} \int_0^{\infty} \frac{dk}{2\pi} \left[ \Theta(x^+) \left( e^{-i\frac{k^2}{2\epsilon} x^+} - 1 \right) e^{-ik\cdot x} \right] \\
&+ \Theta(-x^+) \left( e^{i\frac{k^2}{2\epsilon} x^-} - 1 \right) e^{ik\cdot x} \right] = \Delta_{\perp} \int_0^{x^+} d\xi D_F^{d+2}(\xi, x) \\
&= \Delta_{\perp} \int \frac{d^{d+2}k}{(2\pi)^{d+2} k^2 + i\epsilon k_+ + i\epsilon \text{sgn}(k_-)}.
\end{align*}
\]

(A10) (A11)

Similar expressions appear for the massive case in 3+1 dimensions

\[
\begin{align*}
\left[ 2\partial^\xi \Delta_F(x) + E_F^1(x_L) \delta^2(x_L) \right] &= \left( \Delta_{\perp} - m^2 \right) \int_0^{x^+} d\xi \Delta_F^2(\xi, x) \\
&= \left( \Delta_{\perp} - m^2 \right) \int \frac{d^dk}{(2\pi)^d} \frac{e^{-ik\cdot x}}{k^2 - m^2 + i\epsilon k_+ + i\epsilon \text{sgn}(k_-)}.
\end{align*}
\]

(A12) (A13)

The analytically regularized inverse Laplace operator in \( d = 2 \) dimensions
\[ [\Delta_{2-2\delta}]^{-1}(x_\perp) = (-1)^{1-\delta} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{e^{ik_\perp \cdot x_\perp}}{|k_\perp|^{2\delta}} = -\frac{1}{4\pi} \frac{\Gamma(\delta)}{\Gamma(1-\delta)} \left( -\frac{4}{x_\perp} \right)^\delta \]  
(A14)
evidently is singular when \( \delta \to 0 \), however its partial derivative
\[ \partial_i [\Delta_{2-2\delta}]^{-1}(x_\perp) = i(-1)^{1-\delta} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{k_i}{|k_\perp|^{2\delta}} e^{ik_\perp \cdot x_\perp} = \frac{1}{2\pi} \frac{\Gamma(1+\delta)}{\Gamma(1-\delta)} \left( -\frac{4}{x_\perp} \right)^\delta \frac{x_i}{x_\perp} \]is already finite for \( \delta \to 0 \)
\[ \partial_i [\Delta_{2}]^{-1}(x_\perp) = \partial_i [\Delta_{\perp}]^{-1}(x_\perp) - i \int \frac{d^2k_\perp}{(2\pi)^2} \frac{k_i}{|k_\perp|^{2\delta}} e^{ik_\perp \cdot x_\perp} = \frac{1}{2\pi} \frac{x_i}{x_\perp}. \]  
(A16)

In Section 3 many convolutions of various Green functions appear and they are all denoted by \(*\) without reference to their variables - hopefully leading to no misunderstandings. Here are some explicit examples of them
\[ [\Delta_\perp^d]^{-1} * \Pi(x) = \int d^dx_\perp [\Delta_\perp^d]^{-1}(x_\perp - y_\perp) \Pi(x - y_\perp) \]
\[ [\Delta_\perp^d]^{-1} * \{ 2\partial_\perp^2 D_\perp^{d+2}(x - y) + E_\perp^1(x_L - y_L)\delta^d(x_\perp - y_\perp) \} = \int d^dw_\perp [\Delta_\perp^d]^{-1}(x_\perp - w_\perp) \]
\begin{equation}
\{ 2\partial_\perp^2 D_\perp^{d+2}(x_L - y_L, w_\perp - y_\perp) + E_\perp^1(x_L - y_L)\delta^d(w_\perp - y_\perp) \}.
\end{equation}

**APPENDIX B: NONLOCAL AND MASSIVE REGULARIZATIONS**

Strictly in \( d = 2 \) dimensions the integral operator \( \partial_i [\Delta_{\perp}]^{-1}(x_\perp) \) exists and the parameterization for \( A_i(x) \)
\[ A_i = C_i - \partial_i [\Delta_{\perp}]^{-1} * (\Pi + 2\partial_j C_j) \]is regular, but one must be careful not to integrate the partial derivative \( \partial_i \) by parts because \( [\Delta_{\perp}]^{-1}(x_\perp) \) is ill-defined. For \( A_- \) field one can introduce the analytical regularization\[^7\]
\[ A_- = C_- - \partial_- [\Delta_{2-2\delta}]^{-1} * (\Pi + 2\partial_j C_j) \]  
(B2)

by rescaling \( A_- \) field
\[ A_- (x) \to [\Delta_{2\delta}]^{-1} * A_- (x) \]  
(B3)
with \( 1 > \delta > 0 \), in the Lagrangian density (3.1), but without changing the source term \( A_- j^- \). Further, without going into details of the canonical procedure, we will present some important points which are different from the respective results in Section 3. The canonical commutator for the pair \( (A_-, \Pi) \) is changed
\[ [\Pi(x^+, \vec{x}), A_(x^+, \vec{y})] = -i\Delta_{\perp}[\Delta_{2-2\delta}]^{-1}(x_\perp - y_\perp)\delta(x^- - y^-). \]  
(B4)

and the independent modes vector fields are given by
\[ C_i = A_i - \partial_i [\Delta_{\perp}]^{-1} * (\Pi + 2\partial_j A_j) \]  
(B5)
\[ C_- = A_- - \partial_- [\Delta_{2-2\delta}]^{-1} * (\Pi + 2\partial_j A_j). \]  
(B6)

Also one commutator for independent modes is changed
\[ [\Pi(x^+, \vec{x}), C_-(x^+, \vec{y})] = -i\Delta_{\perp}[\Delta_{2-2\delta}]^{-1}(x_\perp - y_\perp)\delta(x^- - y^-). \]  
(B7)

\(^7\)See Appendix A for the definition of \( [\Delta_{2-2\delta}]^{-1} \)
One may worry that these modified commutators violate causality because they do not vanish for spatially separated points (in the transverse $x_\perp$ coordinates). In the free field case, one can easily check such observables as components of the field strength $F_{\mu\nu}$. First one expresses them in terms of independent modes

\[ F_{+\mp} = -\Delta_\perp [\Delta_{2-2\delta}]^{-1} \ast \partial_j C_j \]  
\[ 2\partial_\mp F_{\pm j} = (\delta_{ij} \Delta_\perp - 2\partial_i \partial_j) C_j. \]  

and then finds

\[ [2\partial_\pm F_{\pm i}(x^+, \vec{x}), F_{\pm j}(x^+, \vec{y})] = -i\partial^i_\pm (\Delta_\perp)^2 [\Delta_{2-2\delta}]^{-1}(x_\perp - y_\perp)\delta(x^- - y^-) \]  

which means that also for the observables causality is violated when $\delta \neq 0$. Therefore one has the interpretation of this analytical regularization as a mere mathematical trick without deeper physical meaning. Keeping this in mind, one can proceed further with the canonical procedure and define the Fourier representation for $C_-$ and $\Pi$ free fields

\[ C_-(\vec{x}) = \int_{-\infty}^{\infty} \frac{d^2k_\perp}{(2\pi)^2} \int_{0}^{\infty} \frac{dk_\parallel}{2\pi} \left[ e^{-i\vec{k}_\perp \cdot \vec{x}} a(\vec{k}) + e^{+i\vec{k}_\perp \cdot \vec{x}} a^\dagger(\vec{k}) \right] \]  
\[ \Pi(\vec{x}) = \int_{-\infty}^{\infty} \frac{d^2k_\perp}{(2\pi)^2} \int_{0}^{\infty} \frac{dk_\parallel}{2\pi} \left[ e^{-i\vec{k}_\perp \cdot \vec{x}} p(\vec{k}) + e^{+i\vec{k}_\perp \cdot \vec{x}} p^\dagger(\vec{k}) \right] \]

which leads to the commutation relations for the creation and annihilation operators

\[ [a(\vec{k}), p^\dagger(\vec{k}')] = [a^\dagger(\vec{k}), p(\vec{k}')] = i(2\pi)^3(\delta_k^2)^4 \delta(\vec{k} - \vec{k}') \]

where again $a^\dagger(\vec{k})$ and $p^\dagger(\vec{k}')$ operators may create ghostlike states with negative metric.

The perturbative propagators are given by the chronological products of free fields

\[ \langle 0| T C_-(\vec{x})\Pi(\vec{y}) |0\rangle = E_F(x_L - y_L) \Delta_\perp [\Delta_{2-2\delta}]^{-1}(x_\perp - y_\perp) \]  
\[ \langle 0| T C_i(x)C_j(y) |0\rangle = \delta_{ij} D_F^4(x - y). \]

and for the primary gauge fields one obtains the general result

\[ \langle 0| T A_\mu(x)A_\nu(y) |0\rangle = i \int \frac{d^4k}{(2\pi)^4} \frac{e^{-i\vec{k}(x-y)}}{-k^2 + i\epsilon} \left[ g_{\mu\nu} - (k_\perp^2)^4 \frac{\delta\mu\nu + \delta\nu\mu}{k_\perp + i\epsilon} \right] \]

where the scaled momentum $\vec{k}_\mu = (k_+, (k_\perp^2)^4 k_-, k_i)$ shows that for $\delta > 0$ the dimensions of fields $A_-$ and $A_i$ are different. Here again one can easily check the integral properties for propagators

\[ \langle 0| T A_-(x)A_j(0) |0\rangle = \partial_j \Delta_\perp \int_0^{x_+} d\xi [\Delta_{2-2\delta}]^{-1} \ast D_F^4(\xi, \vec{x}) \]  
\[ \langle 0| T A_+(x)A_-(0) |0\rangle = 2\partial_\perp (\Delta_\perp)^2 \int_0^{x_+} d\xi [\Delta_{2-2\delta}]^{-1} \ast [\Delta_{2-2\delta}]^{-1} \ast D_F^4(\xi, \vec{x}). \]

The explicit mass term $\frac{1}{2} m^2 A^\mu A_\mu$ breaks gauge invariance in the Lagrangian density which no longer describes a bona fide gauge theory. However if this mass is treated only as a regularization parameter which will be finally pushed to zero for IR finite objects, then one can take the LF-Weyl gauge and start with the Lagrangian density

\[ \mathcal{L}_{\text{Weyl}}^m = \partial_\mu A_i (\partial_\mu A_i - \partial_i A_\mu) + \frac{1}{2} (\partial_\mu A_-)^2 - \frac{1}{4} (\partial_\mu A_\perp - \partial_i A_\perp)^2 - \frac{m^2}{2} A_\perp^2 + A_\perp j^\perp + A_i j^i. \]

The canonical momenta and commutators for primary canonical variables are the same as in the massless theory and the canonical Hamiltonian density has only one extra term $+m^2 A_\perp^2$. Now the canonical fields decompose into independent modes by the regular invertible transformations

\[ A_\perp = C_\perp - \partial_\perp [m^2 - \Delta_\perp]^{-1} \ast \Pi \]  
\[ A_i = C_i + \partial_i [m^2 - \Delta_\perp]^{-1} \ast [2\partial_j C_j + \Delta_\perp [m^2 - \Delta_\perp]^{-1} \Pi] \]
The basic difference between the massless and the massive cases is the equation for the longitudinal vector field $C_-$
\[
\partial_+ C_- = m^2 [m^2 - \Delta_\perp]^{-1} * \Pi + [m^2 - \Delta_\perp]^{-1} * [\partial_+ j^i - \Delta_\perp [m^2 - \Delta_\perp]^{-1} * \partial_- j^-].
\]
which, for free fields, leads to the Fourier representation with the explicit $x^+$ dependence
\[
\Pi(x) = \int_{-\infty}^{\infty} \frac{d^2 k_\perp}{(2\pi)^2} \int_0^{\infty} \frac{dk_-}{2\pi} \left[ e^{-ik_-x} p(k) + e^{ik_-x} p^+(k) \right]
\]
\[
C_-(x^+, x) = -x^+ m^2 [m^2 - \Delta_\perp]^{-1} * \Pi(x)
\]
\[
+ \int_{-\infty}^{\infty} \frac{d^2 k_\perp}{(2\pi)^2} \int_0^{\infty} \frac{dk_-}{2\pi} \left[ e^{-ik_-x} c_-(k) + e^{ik_-x} c_+^\dagger(k) \right]
\]
and one gets nonzero longitudinal components of the propagator
\[
<0|T C_-(x)C_-(y)|0> = m^2 E^2_F (x_L - y_L)[m^2 - \Delta_\perp]^{-1}(x_\perp - y_\perp).
\]
Also the propagator for $A_-$ fields is modified
\[
<0|T A_-(x)A_-(y)|0> = m^2 E^2_F (x_L - y_L)[m^2 - \Delta_\perp]^{-1}(x_\perp - y_\perp)
+ 2\partial_\perp \Delta_\perp [m^2 - \Delta_\perp]^{-2} * 2\partial_\perp \Delta_\perp^{2+2}(x - y) + E^2_F (x_L - y_L)\delta^2(x_\perp - y_\perp)
\]
\[
= 2\partial_\perp \int_0^{x^+ - y^-} d\xi \Delta_\perp^{2+2}(\xi, \vec{x} - \vec{y}) + m^2 \int_0^{x^+ - y^-} d\xi \int_0^{\xi} d\eta \Delta_\perp^{2+2}(\eta, \vec{x} - \vec{y}).
\]
Finally one derives the Fourier representation of gauge field propagator
\[
<0|T A_\mu(x)A_\nu(0)|0> = i \int \frac{d^4 k}{(2\pi)^4} \frac{e^{ik\cdot x}}{k^2 - m^2 + i\epsilon} \left( -g_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{k^+ + i\epsilon \text{sgn}(k^-)} \right)
\]
\[
+ m^2 \frac{n_\mu n_\nu}{[k^+ + i\epsilon \text{sgn}(k^-)]^2}
\]
where the $n_\mu n_\nu$ term is characteristic for the massive regularization and it smoothly vanishes in the limit $m^2 \to 0$.

**APPENDIX C: POINCARÉ GENERATORS**

The analysis of the Poincaré covariance starts with the definition of the canonical energy-momentum tensor
\[
\mathcal{T}^{\mu\nu} = \partial^\mu A_\nu - \partial^\nu A_\mu + \partial^\sigma A_\nu \frac{\partial}{\partial A_\mu} - g^{\mu\nu} \mathcal{L} = -F^{\mu\lambda} \partial^\nu A_\lambda - g^{\mu\nu} \mathcal{L}
\]
where we take the Lagrangian density $[\mathcal{L}]$ with the LF-Weyl gauge condition explicitly implemented and we confine our discussion to the free Abelian theory in 3+1 dimensions. This canonical tensor may be expressed in terms of the symmetric energy-momentum tensor
\[
\Theta^{\mu\nu} = F^{\mu\lambda} F_{\lambda\nu} - g^{\mu\nu} \mathcal{L}
\]
and one gets
\[
\mathcal{T}^{\mu\nu} = \Theta^{\mu\nu} - \partial_\lambda (F^{\mu\lambda} A_\nu) + \delta^{\mu+} A^\nu G
\]
where $G$ is the Gauss law operator in 3+1 dimensions. Similarly we define the canonical angular momentum tensor
\[
\mathcal{M}^{\mu\nu\rho} = \mathcal{T}^{\mu\nu, x^\rho} - \mathcal{T}^{\mu\nu, x^\rho} + \partial_\mu \frac{\partial \mathcal{L}}{\partial A_\nu} A^\rho - \frac{\partial \mathcal{L}}{\partial A_\mu} A^\nu
\]
and then the generators of the Poincaré transformations
Gathering all these results one obtains the commutator algebra for Poincaré generators (3.23) and (3.24).

\[ P^\mu = \int d^2x_\perp dx^- T^{+\mu} \]  
\[ M^{\mu\nu} = \int d^2x_\perp dx^- M^{+\mu\nu}. \]

We notice that the relation (C3) allows to write the expressions for generators as

\[ P^\mu = \int d^3x T^{+\mu} = \int d^3x \left[ \Theta^{+\mu} + A^\mu G \right] \]  
\[ M^{\mu\nu} = \int d^3x M^{+\mu\nu} = \int d^3x \left[ (\Theta^{+\mu} + A^\mu G) x^\nu - (\Theta^{+\nu} + A^\nu G) x^\mu \right]. \]

where the expected form has regular modifications by the noncovariant terms \( A^\mu G \). These terms arise from the noncovariant gauge condition and they generate \( x^+ \) dependence of the Poincaré generators \( M^{+i} \).

\[ \frac{d}{dx^+} M^{+i} = \int d^2x_\perp dx^- GA^i \]

while other generators are \( x^+ \) independent. For the quantum theory one needs some prescription for noncommuting operators and we choose symmetric product for \( A^\mu G \) and normal ordering for operators appearing in \( \Theta^{\mu\nu} \).

After a tedious but straightforward calculation we find all commutators that we need. First for the Poincaré generators and the independent vector fields \( \vec{A}_\mu = (A_-, A_i) \)

\[ [P^\mu, \vec{A}^\rho(x)] = -i \partial^\mu \vec{A}^\rho(x) \]  
\[ [M^{\mu\nu}, \vec{A}^\lambda(x)] = -i (x^\mu \partial^\nu - x^\nu \partial^\mu) \vec{A}^\lambda(x) - i \left( g^{\mu\lambda} \vec{A}^\nu - g^{\nu\lambda} \vec{A}^\mu \right)(x) \]

so \( \vec{A}^\mu \) transforms covariantly. The noncovariant behaviour appears for the field strength tensor \( F^{\mu\nu} \), where beside the desired relations

\[ [P^\mu, F^{\lambda\rho}(x)] = -i \partial^\mu F^{\lambda\rho}(x) \]  
\[ [M^{\mu\nu}, F^{\lambda}(x)] = -i (x^\mu \partial^\nu - x^\nu \partial^\mu) F^{\lambda}(x) + i g^{\nu\lambda} F^{\mu\rho}(x) + i g^{\rho\lambda} F^{\mu\nu}(x) - i g^{\mu\rho} F^{\lambda\nu}(x) \]

one also gets the anomalous commutator

\[ [M^{+i}, F^{-j}(x)] = -i \left( x^+ \partial^i - x^i \partial^+ \right) F^{-j}(x) - i F^{ij}(x) - i g^{ij} \left( F^{+-}(x) + \frac{1}{2} [\partial_-]^{-1} G(x) \right) \]

which is a direct consequence of (C8). Fortunately the components \( F^{-j} \) are not present in the Poincaré generators and one still has covariant commutators

\[ [M^{\mu\nu}, \Theta^{+\lambda}(x)] = -i (x^\mu \partial^\nu - x^\nu \partial^\mu) \Theta^{+\lambda}(x) + i g^{\nu\lambda} \Theta^{\mu\lambda}(x) + i g^{\rho\lambda} \Theta^{\mu\nu}(x) - i g^{\mu\lambda} \Theta^{+\nu}(x). \]

Finally for the \((G \vec{A}^\mu + \vec{A}^\rho G)\) term one finds

\[ [M^{\mu\nu}, (G \vec{A}^\rho + \vec{A}^\rho G)(x)] = -i (x^\mu \partial^\nu - x^\nu \partial^\mu) (G \vec{A}^\rho + \vec{A}^\rho G)(x) - i \left( g^{\mu\rho} g^{\nu\lambda} - g^{\nu\rho} g^{\mu\lambda} \right) (G \vec{A}^\rho + \vec{A}^\rho G)(x) - i g^{\rho\nu} (G \vec{A}^\rho + \vec{A}^\rho G)(x) \]

Gathering all these results one obtains the commutator algebra for Poincaré generators (3.23) and (3.24).

---

8 The momentum field \( \Pi \) transforms according to the relation \( \Pi = \partial_+ A_- - \partial_i A_i \) and therefore it will be discarded hereafter.

9 Here \( [\partial_-]^{-1} \) may be any real valued Green function.
[1] P.A.M. Dirac, Rev. Mod. Phys. 21, (1949), 392.
[2] The latest review can be found in S.J. Brodsky, H-C. Pauli and S.S. Pinsky, Phys. Rept. 301 (1998) 299.
[3] K.G. Wilson, T.S. Wallhout, A. Harindranath, W.-M. Zhang, R.J. Perry and S.D. Glazek, Phys. Rev. D 49, (1994), 6720; St. D. Glazek, K.G. Wilson, Phys. Rev. D 48, (1993), 5863; ibid. 49, (1994), 4214.
[4] J.B. Kogut, D.E. Soper, Phys. Rev. D 1, (1970), 2901.
[5] A. Bassetto, M. Dalbosco, I. Lazzizzera and R. Soldati, Phys. Rev., D 31, (1985), 2012.
[6] S. Mandelstam, Nucl. Phys. B 213 (1983) 149; G. Leibbrandt, Phys. Rev. D 29 (1984) 1699.
[7] A. Bassetto, in Physical and Nonstandard Gauges, eds. Gaigg et al (Springer, Heidelberg, 1990).
[8] G. McCartor, D.G. Robertson, Z. Phys. C 62, (1994) 349.
[9] R. Soldati, in Theory of Hadrons and Light-Front QCD ed. St. D. Glazek, (World Scientific, Singapore, 1995).
[10] A.C. Kalloniatis and D. Robertson, Phys. Rev and other references in [2].
[11] J. Przeszowski, H. W. L. Naus, A. C. Kalloniatis, Phys. Rev. D 54 (1996) 5135.
[12] H. C. Pauli, S. J. Brodsky, Phys. Rev. D 32, (1985), 1993, 2001.
[13] I. Lazzizzera, Phys. Lett. B 210, (1988), 188.
[14] A. Burnel, Phys. Rev. D 40, (1989), 1221.
[15] S. Schweber, An Introduction to Relativistic Quantum Field Theory, (Harper and Row, Inc., New York 1961).
[16] S.-J. Chang, R. G. Root, T.-M. Yan, Phys. Rev. D 7, (1973), 1333; S.-J. Chang, T.-M. Yan, Phys. Rev. D 7, (1973), 1147.
[17] A. Bassetto, G. Nardelli, Int. J. Mod. Phys. 12 A, (1997), 1075.
[18] t’Hooft, Nucl. Phys. B 75 (1974), 461.
[19] T.-M. Yan, Phys. Rev. D 7, (1973), 1761; 1780.
[20] C. R. Hagen and J. H. Yee, Phys. Rev. D 16, (1976), 1206.
[21] A. Bassetto, I. A. Korchemskaya, G. P. Korchemsky and G. Nardelli, Nucl. Phys. B 408, (1993), 62.
[22] H. Balasin, W. Kummer, O. Piquet and M. Schweda, Phys. Lett. B 287, (1992), 138.
[23] D. Mustaki, S. Pinsky, J. Shigemitsu and K. Wilson, Phys. Rev. D 43, (1991) 3411.
[24] N. Nakanishi, Prog. Theor. Phys. Suppl. 51 (1972) 1.
[25] J. D. Bjorken and S. D. Drell, Relativistic Quantum Fields (McGraw-Hill, New York, 1965).
[26] J. Schwinger, Proc. Natl. Acad. Sci. U.S. 37 (1951) 452; J. L. Anderson, Phys. Rev. 94 (1954) 703; I. Gerstein, R. Jackiw, B. W. Lee and S. Weinberg, Phys. Rev. D 3 (1971) 2486; H. M. Fried, Functional Methods and Models in Quantum Field Theory (MIT Press, Cambridge and London, 1972).