Quantum information processing and multi-qubit entanglement with Josephson-junction charge qubits in a thermal cavity

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Abstract

We propose a scheme to implement the two-qubit quantum phase gate with Josephson-junction charge qubits in a thermal cavity. In this scheme, the photon-number-dependent parts in the time evolution operator are canceled at the special time. Thus the scheme is insensitive to the thermal field. We also demonstrate that the scheme can be used to generate maximally entangled state of many Josephson-junction charge qubits.

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Recently, much attention has been paid to the quantum computers, which are based on the fundamental principle of quantum mechanics. The existence of quantum algorithms for specific problems shows that a quantum computer can in principle provide a tremendous speed up compared to classical computers[1, 2]. This discovery motivated an intensive research into this mathematical concept which is based on quantum logic operations on multi-qubit systems[3]. In order to implement this concept into a real physical system, a quantum system is needed, which makes the storage and the read out of quantum information and the implementation of the required set of quantum gates possible. This system should be scalable and the isolation of the system from the environment should be very well in order to suppress decoherence processes. Several physical systems were suggested to implement the concept of quantum computing: cavity QED systems[4], trapped ion systems[5], nuclear magnetic resonance systems[6]. These systems have the advantage of high quantum coherence, but cannot be integrated easily to form large-scale circuits. There exists better potential to realize large-scale quantum computers by implementation of qubit in solid-state system based on electron spins in quantum dots[7] or nuclear spins of donor atoms in silicon[8].

Recently, superconducting charge[9, 10, 11] and phase qubits[12] have attracted much attention because of possible large-scale integration and relatively high quantum coherence. In this paper, we focus on the superconducting charge qubits. In the
experiment[10], Nakamura et al have demonstrated the coherent oscillations of Cooper
pairs on a superconducting Cooper-pair box. This corresponds to a rotation operation
of a single charge qubit. More recently, Pashkin et al observed the quantum oscillations
in two coupled charge qubits, which demonstrated the feasibility of coupling of multiple
charge qubits [11]. In this experiment, the way of coupling Josephson charge qubits is
to connect two Cooper-pair box via a capacitor. The disadvantage of this coupling is
hard to directly couple two distant charge qubits. In Ref[9], a theoretical scheme was
proposed for coupling charge qubits in terms of the oscillator modes in a LC circuit
formed by an inductance and the qubit capacitors. This scheme requires that the
eigenfrequency $\omega_{LC}$ of the LC circuit is much bigger than the quantum manipulation
frequencies, which not only makes quantum system operate at a low speed but also
limits the allowed number $N$ of the qubits in the circuit because $\omega_{LC}$ scales with $1/\sqrt{N}$.
In Ref[13], a fast scheme was proposed for implementing quantum logic gate by placing
three-level SQUID qubits in a cavity.

In this paper we propose an alternative scheme to implement the quantum-phase
gate by placing two-level charge qubits in a cavity. The distinct feature of the present
scheme is that the photon-number-dependent parts in the evolution operator are can-
celed at the special time and two subsystem (cavity field and charge qubits) are
disentangled. Due to this feature, the scheme is insensitive to the thermal field. We
also demonstrate that the scheme can be used to generate maximally entangled states
of many charge qubits.

We now consider the interaction of $N$ charge qubits with a single-mode cavity. The
associated Hamiltonian of the system is

$$H = H_{\text{photon}} + \sum_{j=1}^{N} H_{C}^{j}$$

(1)

where $H_{\text{photon}} = \omega a^\dagger a$ is the Hamiltonian of the cavity mode, $a$ and $a^\dagger$ are the annihi-
lation and creation operators of the cavity field of frequency $\omega$. $H_{C}^{j}$ is the Hamiltonian of the $j$th charge qubit. The single charge qubit is shown in Fig.1, which has been
proposed in Ref[9]. This charge qubit consists of two symmetric Josephson junctions
in a loop configuration, which can be tuned by an external classical magnetic flux $\Phi_c$
which is controlled by the current through the inductor loop. A controllable gate voltage $V_g$
is coupled to the charge qubit via a gate capacitor $C_g$. If the self-inductance of the loop is low, the charge qubit is described by the Hamiltonian of the form

$$H_{C}^{j} = 4E_C(n_j - n_g)^2 - E_{J0} \cos (\Theta_j - \gamma_j) - E_{J0} \cos (\Theta_j + \gamma_j)$$

(2)

Here $E_C = e^2/2(C_g + 2C_{J0})$ and $n_g = C_gV_g/2e$. $C_{J0}$ and $E_{J0}$ are the capacitance and coupling energy of one Josephson junction. In this paper, we assume that two
Josephson junctions in each charge qubit are same. $n_j$ is the number operator of
Cooper-pair charges on the island, $\Theta_j$ is the phase difference across the Josephson
junction and satisfies the commutation relation $[n_j, \Theta_k] = -i\delta_{jk}$. The interaction
between the charge qubit and the cavity field is contained in $\gamma_j = \frac{\pi}{\Phi_0} \Phi_c + \frac{2\pi}{\Phi_0} \int_{j} A(x) \cdot dl$, where $\Phi_0 = hc/2e$ is the flux quantum and the $A(x)$ is vector potential which arises
from the electromagnetic field of the normal mode of the cavity, and line integral is
taken across the junction. In the Coulomb gauge, this vector potential takes the form
\[ A = \sqrt{\frac{\hbar c^2}{\omega V}}(a + a^\dagger)\hat{e}\] [14], where \( \hat{e} \) is the unit polarization vector of the cavity mode and \( V \) is the volume of the cavity mode. We assumed that the junction dimensions are much smaller than the wavelength of the cavity mode, so that the cavity electric field is approximately uniform within the junction. We define the coupling constant \( g = 2\sqrt{2e} \sqrt{\frac{\pi}{\hbar \omega V}} \hat{e} \cdot \vec{l} \) such that \( \frac{2\pi}{\Phi_0} \int J(x) dl = g(a + a^\dagger) \). The \( \vec{l} \) is the thickness of the insulating layer in one junction.

Now we consider the system in which the charging energy is much larger than the Josephson coupling energy \( E_C \gg E_{J0} \). In this regime, a convenient basis is formed by the charge states, parametrized by the number of Cooper pairs on the island. In this basis, the Hamiltonian(1) reads

\[
H = \omega a^\dagger a + \sum_{j=1}^{N} \sum_{n_j=0}^{\infty} \left[ 4E_C(n_j - n_g)^2 |n_j\rangle \langle n_j| - E_{J0} \cos \left( \frac{\pi}{\Phi_0} \Phi_c + g(a + a^\dagger) \right) (|n_j\rangle \langle n_j + 1| + |n_j + 1\rangle \langle n_j|) \right]
\] (3)

We concentrate on the dimensionless gate charge \( n_g \) near the degeneracy point 1/2 \((n_g \approx 1/2)\), where only two charge states, say \( n_j = 0 \) and \( n_j = 1 \), play a role, all the other charge states, having a much higher energy, can be ignored. In this case, the Hamiltonian(3) can be written as follows

\[
H = \omega a^\dagger a + \Delta \sum_{j=1}^{N} \sigma_{jz} - E_{J0} \cos \left( \frac{\pi}{\Phi_0} \Phi_c + g(a + a^\dagger) \right) \sigma_{jx}
\] (4)

where \( \Delta = 2E_C(2n_g - 1) \), \( \sigma_{jz} \) and \( \sigma_{jx} \) are Pauli operators of the \( j \)th charge qubit and the charge states \( n_j = 0 \) and \( n_j = 1 \) correspond to the spin basis states. In the following, We assume that the dimensionless gate charge \( n_g \) is tuned to be 1/2 and the coupling constant between cavity and junction is weak \( g\sqrt{n+1} \ll 1 \), here \( \bar{n} \) is mean photon number of cavity mode. Under these conditions, we perform the expansion

\[
\cos \left( \frac{\pi}{\Phi_0} \Phi_c + g(a + a^\dagger) \right) \approx \cos \left( \frac{\pi}{\Phi_0} \Phi_c \right) - g(a + a^\dagger) \sin \left( \frac{\pi}{\Phi_0} \Phi_c \right)
\] (5)

and the Hamiltonian(4) can be written as

\[
H = \omega a^\dagger a - E_{J0}[\cos \theta - g(a + a^\dagger) \sin \theta] J_x
\] (6)

where \( \theta = \frac{\pi}{\Phi_0} \Phi_c \) and collective spin operator \( J_x = \sum_{j=1}^{N} \sigma_{ix} \). The parameter \( \theta \) is controllable. The exact time evolution operator of Hamiltonian(6) is

\[
U(t) = \exp[-i\omega a^\dagger a - iE_{J0}gt \sin \theta(a + a^\dagger)J_x] \exp(iE_{J0}t \cos \theta J_x)
\] (7)

which can be rewritten in the form

\[
U(t) = \exp \left[ iJ^2_x \left( \frac{g^2E_{J0}^2 \sin^2 \theta t}{\omega} - \frac{g^2E_{J0}^2 \sin^2 \theta \sin \omega t}{\omega^2} \right) \right] \exp(-i\omega a^\dagger at)
\]
If we choose the interaction time $\tau$ to satisfy the condition $\tau = 2\pi/\omega$, the time evolution operator reduces to

$$U(\tau) = \exp \left( \frac{2i\pi g^2 E_{J_0}^2 \sin^2 \theta}{\omega^2} J_x^2 \right) \exp \left( \frac{2i\pi E_{J_0} \cos \theta}{\omega} J_x \right)$$  \hspace{1cm} (9)$$

It is easy to see, at the time $\tau$, two subsystem are disentangled. The cavity mode is returned to its original state, be it the ground state or any excited state, and we are left with an internal state evolution, which is independent of the cavity state. For two charge qubits, we choose the parameters $g$, $E_{J_0}$, $\theta$ and $\omega$ to satisfy

$$\cos \theta = \frac{g}{\sqrt{g^2 + 4}} \quad \text{and} \quad \frac{\omega}{E_{J_0}} = \frac{8g}{\sqrt{g^2 + 4}}$$  \hspace{1cm} (10)$$

In this case, the time evolution operator becomes

$$U(\tau) = \exp \left( \frac{i\pi}{8} (J_x^2 + 2J_x) \right)$$  \hspace{1cm} (11)$$

It is easy to check that this time evolution operator represents a quantum phase gate

$$| - \rangle_1 | \pm \rangle_2 \rightarrow | - \rangle_1 | \pm \rangle_2$$

$$| + \rangle_1 | - \rangle_2 \rightarrow | + \rangle_1 | - \rangle_2$$

$$| + \rangle_1 | + \rangle_2 \rightarrow - | + \rangle_1 | + \rangle_2$$  \hspace{1cm} (12)$$

in the basis states $| \pm \rangle_i = \frac{1}{\sqrt{2}} (|0\rangle_i \pm |1\rangle_i)$.

We now turn to the problem of generating an entangled state of N charge qubits

$$|\Psi> = \frac{1}{\sqrt{2}} [e^{i\varphi_g} |00 \cdots 0> + e^{i\varphi_e} |11 \cdots 1>].$$  \hspace{1cm} (13)$$

irrespective of N even or odd. Quantum states of this kind were used to improve the frequency standard. Several schemes [15, 16] were proposed to generate this kind of quantum states. We assume that the system was initially prepared in the ground state $|000 \cdots >$. If $N$ is even, we choose the parameters to satisfy $\theta = \pi/2$ and $gE_{J_0}/\omega = 1/4$. The time evolution operator will at the time $\tau = 2\pi/\omega$ be $U(\tau) = \exp \left( \frac{i\pi}{8} J_x^2 \right)$. In Ref.[16] such a kind of time evolution operator was used to generate quantum states of the form(16) with $\varphi_g = -\frac{\pi}{4}$ and $\varphi_e = \frac{\pi}{4} + \frac{N\pi}{2}$. In the case of odd numbers $N$ of charge qubits our scheme makes the generation of maximally entangled quantum states(13) possible by using the time evolution operator (11).

In summary, we proposed a scheme to implement a quantum phase gate or to entangle quantum states of $N$ charge qubits by placing qubits in a cavity. We calculate the exact time evolution and demonstrate how the two subsystems are disentangled at particular time $\tau = 2\pi/\omega$. Thus the photon-number dependent parts in the evolution operator are canceled and scheme is insensitive to the thermal field. Thus, the quantum gate operation speed can be greatly increased, which is important in view of decoherence.


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Figure Captions

Figure 1. Single charge qubit consists of two symmetric Josephson junctions in a loop configuration, which can be tuned by an external classical magnetic flux $\Phi_c$