Effect of odd-multipolarity deformations on fission barriers in superheavy nuclei

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Abstract. The shapes of superheavy nuclei have been investigated using Total-Routhian-Surface calculations in a multidimensional space including both even- and odd-multipolarity deformations. Particularly, we have discussed in detail possible shape coexistence in Fm and No isotopes where normally-deformed rotational bands have been observed experimentally. It is found that the heights of fission barriers can be significantly reduced due to the inclusion of odd-multipolarity deformations. In some neutron-deficient superheavy nuclei, there are shallow superdeformed minima with fission barriers less than 3 MeV.

1. Introduction
The deformation plays an important role in the description of nuclear structure. Due to large mass number, a small change in the shape of a superheavy nucleus would result in a significant change in the property of the nucleus. Various shapes including spherical, axial and triaxial deformations have been predicted in self-consistent energy density functional calculations [1]. In Refs. [2, 3], it was predicted that there would exist superdeformed states in the superheavy mass region. However, it has been argued that superdeformed states may be unstable against fission when higher multipolarity deformations are included [4].

Recently, great progress has been made in the synthesis of superheavy nuclei [5–7]. The experimental information about the shapes of the heaviest nuclei is also increasing. Collective rotational bands in $^{254}$No and its neighbors have been observed with $\beta_2 \approx 0.27$ quadrupole deformations deduced [8–13]. It is an interesting question if the predicted superdeformed states are stable enough to be observed experimentally. The collective rotation at a superdeformed shape requires a certain barrier against fission. Due to large mass and large moment of inertia, the superdeformed shape of the superheavy nucleus can lead to low-lying collective rotational states. In the present work, we have made the systematic calculations of fission barriers for fermium and nobelium isotopes which are being reached experimentally. We have also improved the model calculation [14] by including the odd-multipolarity deformations ($\beta_3, \beta_5$).

2. The model
Total-Routhian-Surface (TRS) calculations [15] have been performed to investigate the shapes and fission barriers of superheavy nuclei. The total Routhian $E^\omega(Z, N; \hat{\beta})$ of a nucleus $(Z, N)$
at the rotational frequency $\omega$ and deformation $\beta$ is calculated as follows [15]

$$E^\omega(Z, N, \beta) = E^{\omega=0}(Z, N, \beta) + \langle \Psi^\omega | \hat{H}^\omega | \Psi^\omega \rangle - \langle \Psi^\omega | \hat{H}^\omega | \Psi^{\omega=0} \rangle,$$

(1)

where $E^{\omega=0}(Z, N, \beta)$ is the total energy at the zero frequency, consisting of the macroscopic liquid-drop energy [16], the microscopic shell correction [17–19] and pairing energy [20]. The last two terms in the bracket represent the change in energy due to the rotation. The total Hamiltonian is written as [15]

$$\hat{H}^\omega = \sum_{ij} [(\langle i | h_{ws} | j \rangle - \lambda \delta_{ij}) a_i^+ a_j - \omega \langle i | j \rangle | j \rangle a_i^+ a_j] - G \sum_{ij, i > j} a_i^+ a_i^+ a_i a_j,$$

(2)

where $G$ is the single-particle pairing strength, and $\lambda$ is the Fermi energy of the pairing calculation. For the single-particle Hamiltonian, $h_{ws}$, a non-axial deformed Woods-Saxon (WS) potential has been adopted [21].

The pairing is treated by the Lipkin-Nogami approach [20] in which the particle number is conserved approximately and thus the spurious pairing phase transition encountered in the BCS calculation can be avoided (see Ref. [20] for the detailed formulation of the cranked Lipkin-Nogami TRS method). Both monopole and quadrupole pairings are considered [22] with the monopole pairing strength $G$ determined by the average gap method [23] and quadruple strengths obtained by restoring the Galilean invariance broken by the seniority pairing force [22, 24, 25]. Pairing correlations are dependent on the rotational frequency and deformation. In order to include such dependence in the TRS, we have run the pairing-deformation-frequency self-consistent TRS calculation, i.e., for any given deformation and frequency, the pairing is self-consistently treated by the Hartree-Fock-Bogolyubov-like equation [20]. At a given frequency, the deformation of a state is determined by minimizing the calculated TRS. For the large deformations of heavy nuclei, the inclusion of reflection-asymmetric deformations would be necessary to obtain the realistic shapes of the nuclei [26–28]. In the present calculations, a large multidimensional space including both even- and odd-multipolarity deformations has been considered. The TRS is calculated in the deformation space $(\beta_2, \beta_3, \beta_4, \beta_5)$.

3. Calculations of shapes and fission barriers

The superheavy nucleus, $^{292}_{118}$, was predicted to have a superdeformed ground state [2, 3]. We have calculated the potential energy surface (PES) as a function of deformations $(\beta_2, \beta_3)$ shown in Fig. 1. At each $(\beta_2, \beta_3)$ point, the potential energy has been minimized with respect to the deformation parameters of $\beta_3$ and $\beta_5$. Our calculated PES shows that the nucleus has a weakly-deformed ground state at $(\beta_2, \beta_3) \approx (0.08, 0.05)$. There are also two coexisting states: one at the deformation $(\beta_2, \beta_3) \approx (-0.14, 0.04)$ and another at the superdeformation $(\beta_2, \beta_3) \approx (0.41, 0.06)$. The two weakly-deformed minima are consistent with calculations by Ćwiok et al. [1] who give small quadrupole moments around the newly discovered nucleus $^{294}_{118}$ [7]. The superdeformed state is separated from the ground state by a barrier of about 6 MeV. For the outer barrier behind the superdeformed minimum, calculations with and without the inclusion of odd-multipolarity deformations are remarkably different. In the calculation with only even-multipolarity deformations, i.e., assuming $\beta_3=0$ and $\beta_5=0$, there is a barrier as high as 5.5 MeV for the fission of the superdeformed state. In the calculation with the inclusion of odd-multipolarity, the height of the barrier is significantly reduced to be 2.1 MeV. On the other hand, we see that the $\beta_3$ deformation increases dramatically along the fission path (as shown in Fig. 1).
Figure 1. Calculated potential energy surface for $^{292}_{118}$. The filled circle, square and diamond indicate the first, second and third minima, respectively. Triangles indicate maxima. The dashed line represents the static fission path. Neighboring contours are at a 500-keV interval. The PES is reflection-symmetric about $\beta_3 = 0$.

At present, the nucleus, $^{292}_{118}$, is too far to be observed for its possible rotational band that is usually used to deduce the information about the deformation. In experiment, collective rotational bands in $^{254,256}_{FM}$ and $^{252,254,254}_{No}$ have been well observed with $\beta_2 \approx 0.27$ deformations deduced [8–13]. As an example, calculated TRS’s for $^{250}_{No}$ in the lattice of ($\beta_2$, $\beta_3$) deformations are shown in Fig. 2. As done in Fig. 1, at each ($\beta_2$, $\beta_3$) point, the Routhian energy has been minimized with respect to the deformation parameters of $\beta_4$ and $\beta_5$. It is seen that there are two coexisting minima at the normal deformation of $\beta_2 \approx 0.25$ and the superdeformation of $\beta_2 \approx 0.70$, respectively. At $\hbar \omega = 0.0$ MeV, the superdeformed minimum is 150 keV lower than the normally-deformed minimum. The height of the fission barrier for the superdeformed minimum is obviously lowered when odd-multipolarity deformations are included, changing from 4.9 MeV (without odd-multipolarity deformations) to 3.0 MeV (with the odd-multipolarity deformations). At $\hbar \omega = 0.15$ MeV (correspondingly $I \sim 24\hbar$), shown in the lower panel of Fig. 2, the height of the fission barrier for the superdeformed minimum is still higher than 2.0 MeV. Therefore, the nucleus $^{250}_{No}$ would be a candidate for possible superdeformed state in the superheavy mass region.

Fig. 3 displays the systematic calculations of static energy curves along fission paths for fermium and nobelium isotopes. The comparison of calculations with and without the inclusion of odd-multipolarity deformations shows that the odd-multipolarity deformation degrees of freedom reduce the heights of the second barriers. In general, the first barriers are about 5 MeV in height. Obtained $\beta_2$ deformations for the first minima are around 0.25, which agrees with the quadrupole deformations of $\beta_2 \approx 0.27$ deduced experimentally in this mass region [8–13]. The second barriers for superdeformed states get reduced with increasing the neutron number. A low barrier implies that the nucleus is easy for spontaneous fission. In $^{254,256}_{FM}$ and $^{258–262}_{No}$, superdeformed minima are too shallow to bind states. In $^{246–252}_{FM}$ and $^{248–256}_{No}$, there are clear superdeformed minima. In the recent work [29] of mean-field and beyond-mean-field calculations, $^{242,244}_{FM}$ and $^{250}_{No}$ were predicted to have superdeformed ground states.
4. Summary

In summary, a multidimensional deformation space including both even- and odd-multipolarity deformations has been used to investigate the possible shape coexistence in the superheavy mass region. The static fission barriers of fermium and nobelium isotopes have been calculated systematically. It is shown that the heights of fission barriers decrease significantly when odd-multipolarity deformations are considered in calculations. For some neutron-deficient fermium and nobelium isotopes, however, there still exist clear superdeformed minima. Further investigations would be needed to answer if the superdeformed minima lead to superdeformed states.

Figure 2. Calculated TRS’s for the nucleus $^{250}$No at $\hbar \omega = 0.0$ MeV (upper) and 0.15 MeV (lower). The filled circle and square indicate the first and second minima, respectively. Triangles indicate maxima. The dashed line represents the static fission path.
Figure 3. Calculated static energy curves for fermium and nobelium isotopes as the function of the $\beta_2$ deformation. At each $\beta_2$ point, the energy has been minimized with respect to $\beta_3$, $\beta_4$, $\beta_5$. Dotted lines represent the calculations without the inclusion of odd-multipolarity deformations. Solid lines represent the calculations with the inclusion of odd-multipolarity deformations.

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References
[1] Ćwiok S, Heenen P H and Nazarewicz W 2005 Nature 433 705
[2] Ren Z 2002 Phys. Rev. C 65 051304(R)
[3] Ren Z and Toki H 2001 Nucl. Phys. A689 691
[4] Muntian I and Sobiczewski A 2004 Phys. Lett. B586 254
[5] Oganessian Yu Ts et al 1999 Nature 400 242
[6] Hofmann S and Münzenberg G 2000 Rev. Mod. Phys. 72 733
[7] Oganessian Yu Ts et al 2006 Phys. Rev. C 74 044602
[8] Julin R 2001 Nucl. Phys. A685 221
[9] Herzberg R D et al 2001 Phys. Rev. C 65 014303
[10] Reiter P et al 1999 Phys. Rev. Lett. 82 509
[11] Reiter P et al 2005 Phys. Rev. Lett. 95 032501
[12] Leino M et al 1999 Eur. Phys. J. A 6 63
[13] Firestone R B and Shirley V S 1996 Table of Isotopes (8th edition, New York: Wiley)
[14] Xu F R, Zhao E G, Wyss R and Walker P M 2004 Phys. Rev. Lett. 92 252501
[15] Nazarewicz W, Wyss R and Johnson A 1989 Nucl. Phys. A503 285
[16] Myers W D and Swiatecki W J 1966 Nucl. Phys. 81 1
[17] Nazarewicz W, Riley M A and Garrett J D 1990 Nucl.Phys. A512 61
[18] Strutinsky V M 1966 Yad. Fiz. 3 614
[19] Strutinsky V M 1967 Nucl. Phys. A95 420
[20] Satula W, Wyss R and Magierski P 1994 Nucl. Phys. A578 45
[21] Nazarewicz W, Dudek J, Bengtsson R, Bengtsson T and Ragnarsson I 1985 Nucl. Phys. A435 397
[22] Satula W and Wyss R 1994 Phys. Rev. C 50 2888
[23] Möller P and Nix J R 1992 Nucl. Phys. A536 20
[24] Sakamoto H and Kishimoto T 1990 Phys. Lett. B245 321
[25] Xu F R, Satula W and Wyss R 2000 Nucl. Phys. A669 119
[26] Möller P 1972 Nucl. Phys. A192 529
[27] Sobieczewski A, Patyk Z, Ćwiok S and Rozmej P 1988 Nucl. Phys. A485 16
[28] Ćwiok S, Rozmej P, Sobieczewski A and Patyk Z 1989 Nucl. Phys. A491 281
[29] Delaroche J P, Girod M, Goutte H and Libert J 2006 Nucl. Phys. A771 103