Critical phenomena in $\mathcal{N} = 2^*$ plasma

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Abstract

We use gauge theory/string theory correspondence to study finite temperature critical behavior of mass deformed $\mathcal{N} = 4$ $SU(N)$ supersymmetric Yang-Mills theory at strong coupling, also known as $\mathcal{N} = 2^*$ gauge theory. For certain range of the mass parameters, $\mathcal{N} = 2^*$ plasma undergoes a second-order phase transition. We compute all the static critical exponents of the model and demonstrate that the transition is of the mean-field theory type. We show that the dynamical critical exponent of the model is $z = 0$, with multiple hydrodynamic relaxation rates at criticality. We point out that the dynamical critical phenomena in $\mathcal{N} = 2^*$ plasma is outside the dynamical universality classes established by Hohenberg and Halperin.

October 2010
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# 1 Introduction

Gauge theory/string theory correspondence [1] presents a solvable framework to study a
large class of strongly interacting four-dimensional gauge theory plasmas. In a nutshell,
solvability of these models comes from ability to approximate a dual string theory
with a corresponding classical supergravity. Unfortunately, real QCD is not any one of
the models studied. It is possible to reach QCD as a particular limit in some of these
models, but in doing so the truncation of the string theory to a supergravity sector
becomes inconsistent. Instead one attempts to discover common/universal features of
strongly coupled gauge theory plasmas, and hopes that real QCD is in the universality
class of the models studied. Typical examples of such universal properties are the
strongly coupled plasma shear and bulk viscosities:

- the shear viscosity $\eta$ to entropy density $s$ ratio [5–8]
  \[ \frac{\eta}{s} = \frac{1}{4\pi}, \]  \hspace{1cm} (1.1)

- the bulk viscosity bound [9]
  \[ \frac{\zeta}{\eta} \geq 2 \left( \frac{1}{3} - c_s^2 \right), \quad c_s^2 = \frac{\partial P}{\partial \epsilon}. \]  \hspace{1cm} (1.2)

Although above properties of strongly coupled plasmas have been observed (or in case
of the shear viscosity derived) in holographic setting, it is not clear why and how
these universalities arise, or how to properly define the corresponding universality class: while the shear viscosity ratio in universal in 2-derivative supergravity\(^1\) (or a phenomenological model of thereof), it can be violated in full string theory \([10–12]\); while the bulk viscosity bound is satisfied in all models of supergravity derived from string theory, it can be violated in some phenomenological models of gauge/gravity correspondence \([13]\).

A more common notion of the 'universality' arises in the theory of continuous critical phenomena. In this paper we follow up the work of \([14–16]\) and focus on static and dynamic properties of strongly coupled non-conformal gauge theory plasma in the vicinity of the second-order phase transition. In \([16]\) it was noticed that there was a tension between the hyperscaling relation among the static critical exponents at the second-order phase transition, and the expectation that in the planar limit the transition should be of the mean-field type, \textit{i.e.}, with vanishing anomalous critical exponent\(^2\).

Direct computation of critical exponents for the second-order phase transition in \(\mathcal{N} = 4\) SYM plasma at finite temperature and the chemical potential for a global \(U(1)\) charge confirmed the vanishing of the anomalous critical exponent. Further, the dynamical critical exponent of this transition was shown to be \(z = 4\), even though the background geometry at criticality did not exhibit a \(z = 4\) Lifshitz-like scaling. In other words, the transition detailed in \([16]\) explicitly showed that the dynamical scaling properties can be “emergent” and should not be necessarily “enforced” on the background geometry of the holographic dual.

Although we restrict our attention here to a second-order phase transition in mass-deformed \(\mathcal{N} = 4\) SYM (also known as \(\mathcal{N} = 2^*\) gauge theory \([2–4]\)), we emphasize that the holographic (static) universality class of this transition includes also a cascading gauge theory \([22]\). In section 2 we review the holographic duality for \(\mathcal{N} = 2^*\) gauge theory plasma. Critical phenomena in \(\mathcal{N} = 2^*\) plasma from both the gauge theory and the dual gravitational perspective is discussed in section 3. Some of the static critical exponents of the second-order phase transition in this plasma, namely \(\{\alpha, \beta, \gamma, \delta\}\), were computed in \([15]\). We directly compute the remaining static critical exponents \(\{\nu, \eta\}\) and the dynamical critical exponent \(z\) of the theory in section 4. We collect all the results in section 5.

\(^1\)This translates into an infinite t’Hooft coupling limit on the gauge theory side.
\(^2\)Contrary to some statements in recent literature (as in \([21]\) for instance), we take a perspective here that for a second-order phase transition to be of a mean-field type the anomalous critical exponent must vanish — whether or not the other critical exponents are integers or not is irrelevant.
2 \( N = 2^*/PW \) holographic duality

In this section we briefly review the main features of the holographic duality between \( \mathcal{N} = 2^* SU(N) \) gauge theory and the Pilch-Warner (PW) geometry of type IIB supergravity. We refer the reader to the original work for further details [2–4, 17–20].

Consider maximally supersymmetric \( \mathcal{N} = 4 \) \( SU(N) \) Yang-Mills theory in the planar limit \( g_Y^2 \to 0, N \to \infty \) with \( \lambda \equiv g_Y^2 N \) kept fixed) and for large \('t\) Hooft coupling \( \lambda \gg 1 \). According to Maldacena correspondence [1] this superconformal theory is equivalent to a classical type IIB supergravity on \( AdS_5 \times S^5 \). A duality between a SYM and a supergravity can be extended (on both sides) away from the conformal point [2–4]. On the gauge theory side, a massive deformation of \( \mathcal{N} = 4 \) superpotential

\[
W_{\mathcal{N}=4} = \frac{2\sqrt{2}}{g_Y^2} \mathrm{Tr} \left( \left[ Q, \tilde{Q} \right] \Phi \right) ,
\]

where \( \{ Q, \tilde{Q}, \Phi \} \) are \( \mathcal{N} = 1 \) adjoint chiral superfields, to

\[
W_{\mathcal{N}=4} \to W_{\mathcal{N}=2^*} = W_{\mathcal{N}=4} + \frac{m}{g_Y^2} \left( \mathrm{Tr} Q^2 + \mathrm{Tr} \tilde{Q}^2 \right) ,
\]

breaks half of the supersymmetries. This mass-deformed theory is known as \( \mathcal{N} = 2^* \) gauge theory. When \( m \neq 0 \), the mass deformation lifts the \( \{ Q, \tilde{Q} \} \) \( \mathcal{N} = 2 \) hypermultiplet moduli directions, resulting in \( (N - 1) \) complex dimensional Coulomb branch parametrized by

\[
\Phi = \mathrm{diag} \left( a_1, a_2, \cdots, a_N \right) , \quad \sum_{i=1}^{N} a_i = 0 .
\]

We study \( \mathcal{N} = 2^* \) gauge theory at a particular point on the Coulomb branch moduli space [3]:

\[
a_i \in [-a_0, a_0] , \quad a_0^2 = \frac{m^2 g_Y^2 N}{\pi} ,
\]

with the (continuous in the large \( N \)-limit) linear number density

\[
\rho(a) = \frac{2}{m^2 g_Y^2} \sqrt{a_0^2 - a^2} , \quad \int_{-a_0}^{a_0} da \, \rho(a) = N .
\]

The reason for such an esoteric choice for a vacuum of the theory is simply because we know a dual holographic description of the theory (as a Pilch-Warner geometry [2]) only at this point [3]. Extending the correspondence to the rest of the moduli space is an important unsolved problem.
Notice that the deformation (2.2) is actually a deformation of a CFT by two different operators: a dimension-2 operator (a mass term for the bosonic components of the \{Q, \bar{Q}\} hypermultiplet) and a dimension-3 operator (a mass term for the fermionic components of the \{Q, \bar{Q}\} hypermultiplet). According to AdS/CFT dictionary [23], a scalar gauge-invariant operator of dimension \(\Delta\) is dual to a scalar field of mass \(m_5^2L^2 = \Delta(\Delta - 4)\) of the five-dimensional dual gravitational description. These two mass-deformation operators are the \(\alpha\) and \(\chi\) scalars of the Pilch-Warner effective action [2]:

\[
S = \int_{\mathcal{M}_5} d\xi^5 \sqrt{-g} \mathcal{L}_5 = \frac{1}{4\pi G_5} \int_{\mathcal{M}_5} d\xi^5 \sqrt{-g} [\frac{1}{4} R - 3(\partial\alpha)^2 - (\partial\chi)^2 - \mathcal{P}] , \tag{2.6}
\]

where the potential\(^3\)

\[
\mathcal{P} = \frac{1}{16} \left[ \frac{1}{3} \left( \frac{\partial W}{\partial \alpha} \right)^2 + \left( \frac{\partial W}{\partial \chi} \right)^2 \right] - \frac{1}{3} W^2 , \tag{2.7}
\]

is a function of \(\alpha\) and \(\chi\), and is determined by the superpotential

\[
W = -e^{-2\alpha} - \frac{1}{2} e^{4\alpha} \cosh(2\chi) . \tag{2.8}
\]

In our conventions, the five-dimensional Newton’s constant is

\[
G_5 \equiv \frac{G_{10}}{2^5 \text{vol}_{S^5}} = \frac{4\pi}{N^2} . \tag{2.9}
\]

In what follows we focus on equilibrium thermal states of \(\mathcal{N} = 2^*\) plasma. Their holographic dual is represented by a regular black brane solution in the effective action (2.6) [17, 19]

\[
ds_5^2 = e^{2A} (-(1 - x)^2 dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + g_{xx} dx^2 , \tag{2.10}
\]

with \(g_{xx} = g_{xx}(x)\), \(A = A(x)\), \(\alpha = \alpha(x)\) and \(\chi = \chi(x)\) being functions of the radial coordinate \(x \in [0, 1]\) only. Note that \(x \to 0_+\) corresponds to the asymptotic \(\text{AdS}_5\) boundary, while \(x \to 1_-\) to a regular Schwarzschild horizon. The temperature and the mass parameters of the plasma are encoded in the asymptotic behavior of the supergravity fields \(\{A, \alpha, \chi\}\). Specifically, near the \(\text{AdS}_5\) boundary we have

\[
e^{\alpha} \equiv \rho = 1 + x^{1/2} (\rho_{10} + \rho_{11} \ln x) + \cdots + x^{k/2} \left( \sum_{i=1}^k \rho_{ki} \ln^i x \right) + \cdots , \tag{2.11}
\]

\(^3\)We set the five-dimensional gauged supergravity coupling to one. This corresponds to setting the radius \(L\) of the five-dimensional sphere in the undeformed metric to 2.
\[ \chi = \chi_0 x^{1/4} \left[ 1 + x^{1/2} \left( \chi_{10} + \chi_{11} \ln x \right) + \cdots + x^{k/2} \left( \sum_{i=1}^{k} \chi_{ki} \ln^i x \right) + \cdots \right], \quad (2.12) \]

\[ a = x^{1/2} \left( a_{10} + a_{11} \ln x \right) + \cdots + x^{k/2} \left( \sum_{i=1}^{k} a_{ki} \ln^i x \right) + \cdots, \quad (2.13) \]

and

\[ \rho = \rho_h + \rho_1 (1-x)^2 + \cdots + \rho_k (1-x)^{2k} + \cdots, \quad (2.14) \]

\[ \chi = \chi_h + \chi_1 (1-x)^2 + \cdots + \chi_k (1-x)^{2k} + \cdots, \quad (2.15) \]

\[ a = a_h + a_1 (1-x)^2 + \cdots + a_k (1-x)^{2k} + \cdots, \quad (2.16) \]

near the regular Schwarzschild horizon. In (2.13), (2.16) \( a(x) \) is defined as

\[ A(x) \equiv \ln \delta_3 - \frac{1}{4} \ln (2x - x^2) + a(x). \quad (2.17) \]

In was shown in [19] that given \( \{ \delta_3, \rho_{11}, \chi_0 \} \), there is a unique singularity-free solution of (2.6) representing the equilibrium state of \( N = 2^* \) plasma. On the gravity side, the coefficients of the leading asymptotics, namely \( \{ \delta_3, \rho_{11}, \chi_0 \} \), determine the remaining 6 parameters (2 in the UV and 4 in the IR) of the solution:

\[ \text{UV : } \{ \rho_{10}, \chi_{10} \}, \]
\[ \text{IR : } \{ \rho_h, \chi_h, a_h, a_1 \}. \quad (2.18) \]

It is possible to unambiguously relate the gravitational and the gauge theory data [19]:

- the plasma temperature \( T \), and the masses \( \{ m_b, m_f \} \) of the bosonic and the fermionic components of the \( N = 2 \) hypermultiplet are given by

\[ T = \frac{\delta_3}{2 \pi} e^{-3a_h}, \quad \frac{m_b^2}{T^2} = 12 \sqrt{2} \pi^2 \rho_{11} e^{6a_h}, \quad \frac{m_f}{T} = 2^{3/4} \pi \chi_0 e^{3a_h}, \quad (2.19) \]

- and the plasma free energy density \( \mathcal{F} \), the energy density \( \mathcal{E} \), and the entropy density \( s \) are given by

\[ \mathcal{F} = -\frac{\delta_3^4}{32 \pi G_5} \left( 1 + \rho_{11}^2 \left( 24 - 96 \ln \delta_3 + 24 \ln 2 \right) - 24 \rho_{10} \rho_{11} + 2 \chi_0^2 \chi_{10} 
+ \chi_0^4 \left( \frac{4}{9} - \frac{2}{3} \ln 2 + \frac{8}{3} \ln \delta_3 \right) \right), \quad (2.20) \]

\[ \mathcal{E} = \mathcal{F} - \frac{1}{8 \pi G_5} \frac{\delta_3^4}{\delta_3^4}, \quad s = \frac{\delta_3^4 e^{3a_h}}{4 G_5}. \]

To recover the \( N = 2 \) supersymmetric PW vacuum (2.4), (2.5) we need to set \( T = 0 \) and fine-tune the masses \( m_b = m_f = m \).
Figure 1: (Colour online) The dimensionless temperature $\frac{m_b}{T}$ (left plot) and the speed of sound $c_s^2$ (right plot) of the strongly coupled $\mathcal{N} = 2^*$ plasma with $m_f = 0$ and $m_b \neq 0$ as a function of the dual gravitation parameter $\rho_{11}$.

3 Critical phenomena in $\mathcal{N} = 2^*$ plasma

The phase diagram of $\mathcal{N} = 2^*$ plasma was studied in details in [19, 20]. It was found there that whenever $m_f^2 < m_b^2$, the theory undergoes a second-order phase transition, with the critical temperature $T_c = T_c \left( \frac{m_f^2}{m_b^2} \right)$. All these transitions are in the same universality class, and thus we can restrict our attention to $m_f = 0, m_b \neq 0$ case:

$$m_f = 0 : \quad \frac{m_b}{T_c} \approx 2.32591.$$  \hspace{1cm} (3.1)

We now recall the main characteristics of this transition [15]:

- The left plot on Figure 1 represents the dependence of the dimensionless temperature $\frac{m_b}{T}$ on the gravitational parameter $\rho_{11}$, see (2.19). The transition is associated with the minimal accessible temperature, to be identified with $T_c$, in the plasma for isotropic and homogeneous equilibrium state\(^4\). For each temperature $T > T_c$ there are two phases — the ”ordered” phase (blue curves), and the ”disordered” phase (red curves). The right plot on Figure 1 represents the square of the speed of sound $c_s^2$ as a function of $\rho_{11}$. Notice that the hydrodynamic modes in the ”disordered phase” are unstable (as $c_s^2 < 0$), and thus must condense. It is tempting to conjecture that the equilibrium state in the plasma at $T < T_c$ breaks translational invariance, and represents the end point of condensation of hydrodynamic modes [24].

\(^4\) This feature of the transition is also observed for the phase transition in $\mathcal{N} = 4$ SYM plasma with a single $U(1) \subset SU(4)_R$ R-symmetry chemical potential [16].
The free energy densities of the stable “ordered” phase (blue curves) \( \Omega_o \) and the unstable “disordered” phase (red curves) \( \Omega_d \) as a function of the gravitational parameter \( \rho_{11} \) (left plot) and the dimensionless temperature \( \frac{m_b}{T} \) (right plot) of \( \mathcal{N} = 2^* \) plasma with \( m_f = 0 \).

It is convenient to recast the critical behavior in \( \mathcal{N} = 2^* \) plasma in that of a 3-dimensional ferromagnet. The thermodynamics of the latter is described by the Gibbs free energy \( \mathcal{W} = \mathcal{W}(t, \mathcal{H}) \) which depends on the reduced temperature \( t = (T - T_c)/T_c \) and the external magnetic field \( \mathcal{H} \). Once we identify

\[
\mathcal{W} \equiv \Omega_o - \Omega_d, \quad \mathcal{H} \equiv m_b, \quad (3.2)
\]

and introduce

\[
\Delta \rho_{11} \equiv \rho_{11} - \rho_{11}^c, \quad |\Delta \rho_{11}| \propto t^{1/2}, \quad \rho_{11}^c = 0.035187(6), \quad (3.3)
\]

we can compute the standard static critical exponents \( \{\alpha, \beta, \gamma, \delta\} \):

\[
c_H = -T \left( \frac{\partial^2 \mathcal{W}}{\partial T^2} \right)_\mathcal{H} \propto |t|^{-\alpha} = \frac{s}{c_s^2} \left| \frac{\text{blue}}{\text{red}} \right| \propto (\Delta \rho_{11})^{-1} \propto t^{-1/2} \Rightarrow \alpha = \frac{1}{2}, \quad (3.4)
\]

\[
\mathcal{M} = -\left( \frac{\partial \mathcal{W}}{\partial \mathcal{H}} \right)_T \propto |t|^\beta \propto - \frac{1}{\Delta \rho_{11}} \partial_{\Delta \rho_{11}} \mathcal{W} \propto - \frac{1}{\Delta \rho_{11}} \partial_{\Delta \rho_{11}} \left( -|\Delta \rho_{11}|^3 \right) \propto -|\Delta \rho_{11}| \propto -t^{1/2} \Rightarrow \beta = \frac{1}{2}, \quad (3.5)
\]

\[
\chi_T = \left( \frac{\partial \mathcal{M}}{\partial \mathcal{H}} \right)_T \propto |t|^{-\gamma} \propto -\partial_t \mathcal{M} \propto \partial_t t^{1/2} \propto t^{-1/2} \Rightarrow \gamma = \frac{1}{2}, \quad (3.6)
\]
\[ M(t = 0) \propto |H - H_c|^{1/\delta} \propto t^{1/\delta} \propto t^{1/2} \Rightarrow \delta = 2. \]  

Thus,
\[{\alpha, \beta, \gamma, \delta} = \left\{ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 2 \right\}. \tag{3.8}\]

The remaining two critical exponents \(\{\nu, \eta\}\) are more difficult to extract as they are related to the scaling properties of the magnetization two-point correlation function at criticality:
\[ G(\vec{r}) = \langle M(\vec{r})M(\vec{0}) \rangle \propto \frac{\partial^2 W}{\partial H(\vec{r})\partial H(\vec{0})}, \tag{3.9}\]
\[ G(\vec{r}) \propto \begin{cases} e^{-|\vec{r}|/\xi}, & t \neq 0 \\ |r|^{-3+2-\eta}, & t = 0 \end{cases}, \text{ with } \xi \propto |t|^{-\nu}, \tag{3.10}\]

where \(\xi\) is the correlation length. Under the static scaling hypothesis,
\[ W(t, H) = \lambda^{-3} W(\lambda^{y_T} t, \lambda^{y_H} H), \quad \tilde{G}(\vec{q}, t, H) = \lambda^{2y_H-3} \tilde{G}(\lambda\vec{q}, \lambda^{y_T} t, \lambda^{y_H} H), \tag{3.11}\]

where \(y_T\) and \(y_H\) are the two independent critical exponents; \(\tilde{G}\) is a spatial Fourier transform of (3.9). The static scaling hypothesis implies 4 scaling relations between \(\{\alpha, \beta, \gamma, \delta, \nu, \eta\}\). In particular, using two of these relations
\[ 2 - \alpha = 3\nu, \quad \gamma = \nu(2 - \eta), \tag{3.12}\]

and (3.8), we find
\[ \{\nu, \eta\}_{scaling} = \left\{ \frac{1}{2}, 1 \right\}. \tag{3.13}\]

Much like in [16], the non-vanishing of the anomalous critical exponent \(\eta\) conflicts with the expectation that for large-N gauge theory plasmas the continuous phase transitions in holographic models should be of mean-field type \(\eta_{mean-field} = 0\).

A relaxation of the system to equilibrium in the vicinity of the critical point is commonly discussed within the theory of the dynamical critical phenomena developed by Hohenberg and Halperin [25]. According to [25] a model is designated to a specific universality class based on the dimensionality, symmetries of the order parameter, the presence of any conserved densities, and any other properties that affect the static critical behavior. A representative of a given dynamical universality class is then characterized by a dynamical critical exponent \(z\). This critical exponent determines
the scaling of the non-equilibrium (time-dependent) two-point correlation function of the order parameter (magnetization in our case) at criticality, i.e.,

\[ \tilde{G}(\omega, \vec{q}, t, \mathcal{H}) = \lambda^{2y_H - 3 + z} \tilde{G}(\lambda^{z} \omega, \lambda \vec{q}, \lambda^{y_H} t, \lambda^{y_H} \mathcal{H}), \]  

(3.14)

for its space-time Fourier transform. The equilibration of a dynamical system is thus characterized by a relaxation time \( \tau \)

\[ \tau \propto \xi^z, \]  

(3.15)

which (for \( z \neq 0 \)) diverges at criticality. The absence of any conserved order parameters puts \( \mathcal{N} = 2^* \) plasma in the universality class of ‘model A’ according to the classification of Hohenberg and Halperin, and predicts

\[ z \bigg|_{\text{prediction}} = 2 + c\eta, \]  

(3.16)

where the constant \( c \) can be computed via renormalization group calculations in \( p = 4 - \epsilon, \epsilon \ll 1 \), spatial dimensions, and \( \eta \) is the anomalous critical exponent.

In the rest of this section we introduce dynamical susceptibility of \( \mathcal{N} = 2^* \) plasma and explain how it can be used to compute the static critical exponents \( \{\nu, \eta\} \) and the dynamical critical exponent \( z \) of its phase transition.

### 3.1 Dynamical susceptibility of \( \mathcal{N} = 2^* \) plasma — gauge theory perspective

Both the critical exponents \( \{\nu, \eta\} \) and the dynamical exponent \( z \) can be extracted from the dynamical susceptibility of the system. Consider the response of the system to the time-dependent inhomogeneous variations of the external magnetic field \( \mathcal{H} \),

\[ \mathcal{H} \to \mathcal{H} + \delta \mathcal{H}(t, \vec{x}), \quad \delta \mathcal{H} = \int \frac{d^3 k}{(2\pi)^3} \int \frac{d\omega}{2\pi} e^{i\vec{k} \cdot \vec{x} - i\omega t} \mathcal{H}_{\omega, \vec{k}}. \]  

(3.17)

At the linearized level the variation of the external magnetic field would produce a corresponding variation in the magnetization \( \delta \mathcal{M}(t, \vec{x}) \) ( \( \mathcal{M}_{\omega, \vec{k}} \) for the Fourier components). Following [25] we introduce the dynamical susceptibility as

\[ \chi_{\omega, \vec{k}} = \left( \frac{\mathcal{M}_{\omega, \vec{k}}}{\mathcal{H}_{\omega, \vec{k}}} \right), \quad \lim_{(\omega, \vec{k}) \to (0, \vec{0})} \chi_{\omega, \vec{k}} = \chi_T = \left( \frac{\partial \mathcal{M}}{\partial \mathcal{H}} \right) \bigg|_T. \]  

(3.18)

By the equipartition theorem, the static susceptibility

\[ \chi_{\vec{k}} \equiv \chi_{\omega = 0, \vec{k}}, \]  

(3.19)
is related to the Fourier transform $\tilde{G}(\vec{k})$ of the magnetization variation two-point correlation function

$$G(\vec{x}) = \langle \delta \mathcal{M}(\vec{x}) \delta \mathcal{M}(\vec{0}) \rangle_{\mathcal{H} = 0}, \quad (3.20)$$
as

$$\tilde{G}(\vec{k}) = T \chi_{\vec{k}}. \quad (3.21)$$

Given the near-critical behavior of the correlation function (3.20) (see (3.10)), (3.21) implies that the static susceptibility $\chi_{\vec{k}}$ has a pole at

$$k^2 \propto -\xi^{-2}, \quad (3.22)$$
in the vicinity, but not right at the critical point. On the other hand, right at the critical point

$$\chi_{\vec{k}} \propto |\vec{k}|^{-2+\eta}. \quad (3.23)$$

The theory of dynamical critical phenomena [25] predicts that in the vicinity of the continuous phase transition, and for $|\vec{k}| \sim \xi^{-1}$ the full dynamical susceptibility $\chi_{\omega,\vec{k}}$ will develop a pole at

$$\omega \sim -i\xi^{-z}, \quad (3.24)$$
with $z$ being the dynamical critical exponent of the system. The frequency in (3.24) (in the hydrodynamic limit) defines a relaxation time $\tau$ as

$$\tau^{-1} \equiv i\omega \propto \xi^{-z}. \quad (3.25)$$

To summarize, following the position of the poles in the static susceptibility $\chi_{\vec{k}}$ as a function of the reduced temperature $t \neq 0$

$$0 = \frac{1}{\chi_{\vec{k}}} \bigg|_{\vec{k}^2 = k^2_*(t)} \quad \Rightarrow \quad k^2_*(t) \propto -\xi^{-2} \propto -t^{2\nu}, \quad (3.26)$$

would determine the critical exponent $\nu$; the critical exponent $\eta$ is determined from the static susceptibility scaling at critical temperature, i.e., $t = 0$, as in (3.23). Likewise, scaling of the pole in the dynamical susceptibility $\chi_{\omega,\vec{k}}$ in the hydrodynamic limit as a function of the reduced temperature $t \neq 0$ determines the dynamical critical exponent $z$:

$$0 = \frac{1}{\chi_{\omega,\vec{k}}} \bigg|_{(\omega = \omega_*(t), \vec{k} = \vec{0})} \quad \Rightarrow \quad i\omega_*(t) \propto t^{-z\nu}. \quad (3.27)$$
3.2 Dynamical susceptibility of $\mathcal{N} = 2^*$ plasma — gravity perspective

In case of $\mathcal{N} = 2^*$ plasma we identify the external magnetic field $\mathcal{H}$ with the bosonic mass $m_b$, (3.2). The variation $\mathcal{H}_{\omega, \vec{k}}$ would correspond to the variation in

$$m_b \rightarrow m_b + \delta m_b(\omega, \vec{k}) e^{i\vec{k} \cdot \vec{x} - i\omega t}, \quad (3.28)$$

which on the gravity side can be induced by the variation in the non-normalizable coefficient $\rho_{11}$ the supergravity scalar $\rho$:

$$\rho_{11} \rightarrow \rho_{11} + \delta \rho_{11}(\omega, \vec{k}) e^{i\vec{k} \cdot \vec{x} - i\omega t}, \quad \delta \rho_{11}(\omega, \vec{k}) \propto \delta m_b(\omega, \vec{k}). \quad (3.29)$$

The variation $\delta \rho_{11}(\omega, \vec{k})$ would produce a linearized response in the normalizable coefficient $\rho_{10}(\omega, \vec{k})$ of the supergravity scalar $\rho$:

$$\rho_{11} \rightarrow \rho_{11} + \delta \rho_{11}(\omega, \vec{k}) e^{i\vec{k} \cdot \vec{x} - i\omega t} \Rightarrow \rho_{10} \rightarrow \rho_{10} + \delta \rho_{10}(\omega, \vec{k}) e^{i\vec{k} \cdot \vec{x} - i\omega t}. \quad (3.30)$$

Thus it is natural to identify the variation $\delta \rho_{10}(\omega, \vec{k})$ with the variation in the magnetization $\mathcal{M}_{\omega, \vec{k}}$:

$$\delta \rho_{10}(\omega, \vec{k}) \propto \mathcal{M}_{\omega, \vec{k}}. \quad (3.31)$$

Finally, the dynamical susceptibility (3.18) is related to the dual gravitational data as

$$\chi_{\omega, \vec{k}} \propto \frac{\delta \rho_{10}(\omega, \vec{k})}{\delta \rho_{11}(\omega, \vec{k})}. \quad (3.32)$$

The identification (3.32) is equivalent to the one made in recent analysis of the holographic critical phenomena [14, 16].

The holographic computation of the susceptibility (3.32) necessitates the analysis of the linearized fluctuations in the gravitational background (2.10). The relevant fluctuations were studied previously in [20, 26]. We briefly review the basic setup of such computations here. Without the loss of generality we can assume that

$$k^i = q \delta^i_3. \quad (3.33)$$

The linearized on-shell fluctuation of the gravitational scalar\(^5\) $\alpha = \ln \rho$,

$$\alpha \rightarrow \alpha + \phi(x) e^{iqx_3 - i\omega t}, \quad (3.34)$$

\(^5\)For a critical phenomena with $m_f = 0$ we can consistently truncate the effective action (2.6) to $\chi = 0$. 

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couples to the on-shell fluctuations in the background metric
\[ g_{\mu\nu} \rightarrow g_{\mu\nu}(x) e^{iqx_3-i\omega t}, \] (3.35)
which form a helicity-0 representation with respect to rotations about \( x_3 \)-axis:
\[ \{ h_{tt}, h_{tx_3}, h_{aa} = h_{x_1x_1} + h_{x_2x_2}, h_{x_3x_3} \} . \] (3.36)
Note that we partially fixed the background diffeomorphisms with
\[ h_{tx} = h_{x_3x} = h_{xx} = 0 . \] (3.37)
Following [26] we introduce the diffeomorphism-invariant linear combinations of fluctuations
\[ Z_H = 4q_\omega H_{tz} + 2H_{xz} - H_{aa} \left( 1 + \frac{q^2}{\omega^2} g_{tt}^t \right) + 2\frac{q^2}{\omega^2} (1-x)^2 H_a , \] (3.38)
\[ Z_\phi = \phi - \frac{\alpha'}{2(\ln g_{x_1x_1})'} H_{aa} , \] (3.39)
where \( g_{tt}(x) \) and \( g_{x_1x_1}(x) \) are the corresponding components of the background metric (2.10), the derivatives are with respect to \( x \) and
\[ h_{tt} = -g_{tt} H_{tt} , \quad h_{tz} = g_{x_1x_1} H_{tz} , \quad h_{aa} = g_{x_1x_1} H_{aa} , \quad h_{x_3x_3} = g_{x_1x_1} H_{x_3x_3} . \] (3.40)
Introduce
\[ w \equiv \frac{\omega}{2\pi T} , \quad q \equiv \frac{q}{2\pi T} . \] (3.41)
The equations of motion for \( \{ Z_H, Z_\phi \} \) take the form
\[ 0 = Z_H'' + C_{11} Z_H' + C_{12} Z_\phi' + C_{13} Z_H + C_{14} Z_\phi , \]
\[ 0 = Z_\phi'' + C_{21} Z_H' + C_{22} Z_\phi' + C_{23} Z_H + C_{24} Z_\phi , \] (3.42)
where the coefficients \( C_{ij} \) are nonlinear functionals of the background fields \( \{ \rho, a \} \) with explicit dependence on \( x \) and \( \{ w, q \} \) [26]:
\[ C_{ij} = C_{ij} \left[ \{ \rho, a \}; x; \{ w, q \} \right] . \] (3.43)
Since the equations (3.42) are homogeneous, we can always set the non-normalizable component of \( Z_\phi \) — which is the diffeomorphism-invariance analog of \( \delta \rho_{11}(\omega, \vec{k}) \) — to
one; the dynamical susceptibility is then proportional to the normalizable component of $Z_\phi$.

We can summarize now the boundary value problem whose solution would determine the dynamical susceptibility. Introducing

$$Z_H = (1 - x)^{-iw} \, z_H(x, w, q),$$
$$Z_\phi = (1 - x)^{-iw} \, q^{-2} \, z_\phi(x, w, q),$$

the equations of motion for $\{z_H, z_\phi\}$ are solved with the following boundary conditions:

$$\lim_{x \to 1} z_H = \lim_{x \to 1} z_\phi = \text{finite},$$
$$z_H = \mathcal{O}(x), \quad z_\phi = (\ln x + Z(w, q))x^{1/2} + \mathcal{O}(x \ln^2 x), \quad \text{as } x \to 0^+. \quad (3.45)$$

The normalizable component $Z$ of $z_\phi$ near the boundary is proportional to the dynamical susceptibility:

$$\chi_{w,q} \propto Z(w, q). \quad (3.46)$$

## 4 Critical exponents $\{\nu, \eta\}$ and $z$ of $\mathcal{N} = 2^*$ plasma

In previous section we explained how the dynamical susceptibility $\chi_{w,q}$ can be used to extract the static critical exponents $\{\nu, \eta\}$ and the dynamical critical exponent $z$ of

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6The $w$– and $q$–dependent rescaling are for convenience in further analysis.
Figure 4: (Colour online) Poles of the static susceptibility in the vicinity of the critical point: $\chi_{w=0,q=q_{c}}^{-1} = 0$. The solid red line is a quadratic fit to data, the dashed green line represents $\rho_{11} = \rho_{11}^{*}$.

a phase transition. We also related this susceptibility to the normalizable component $Z(w,q)$ of the gravitational scalar, which non-normalizable component played the role of the (time-dependent and inhomogeneous) variation of the external magnetic field, see (3.46). Here, without going into the technical details of the analysis of the boundary value problem (3.42)-(3.45), we present the results.

Figure 3 shows the inverse of the static susceptibility at $q = 0$ (blue dots) in the vicinity of the critical point. The solid red line $Z_{fit}^{-1} = Z_{fit}^{-1}(\rho_{11})$ is the best quadratic fit to the data. The vertical green line denotes the critical value of $\rho_{11}$, i.e., $\rho_{11}^{*}$ (3.3), corresponding to critical temperature $T_{c}$, see (3.1). The red line intersects the $\rho_{11}$ axis at $\rho_{11}^{*}$, such that

$$\left. Z_{fit}^{-1} \right|_{\rho_{11}=\rho_{11}^{*}} = 0 \quad \Rightarrow \quad \left| \frac{\rho_{11}^{*}}{\rho_{11}} - 1 \right| = 9.3 \times 10^{-6}, \quad (4.1)$$

in excellent agreement with the critical behavior of $\chi_{T}$ deduced from the thermodynamics (3.6):

$$Z_{fit}^{-1} \propto \Delta \rho_{11} \propto t^{1/2} \quad \iff \quad \chi_{T}^{-1} \propto t^{1/2}. \quad (4.2)$$

Figure 4 presents the poles (blue dots) of the static susceptibility at $q = q_{c}$ in the vicinity of the critical point:

$$\chi_{w=0,q=q_{c}}^{-1} = 0. \quad (4.3)$$
The solid red line represents the best quadratic fit to the data, and the vertical green line denotes the critical value of $\rho_{11}$, \textit{i.e.}, $\rho_{11}^c$, see (3.3). Notice that in the stable phase, \textit{i.e.}, for $\rho_{11} < \rho_{11}^c$, in the vicinity of the phase transition the poles in the static susceptibility are for purely imaginary momenta, which implies the exponential decay of the magnetization density two-point correlation function (3.20). Furthermore, from (3.26) we identify the correlation length as

$$\frac{2\pi T_c}{\xi} \sim -q^2 \sim \frac{1}{|\Delta \rho_{11}|} \sim t^{-1/2}, \quad 0 < \rho_{11} - \rho_{11}^c \ll \rho_{11}^c,$$

where we used the results of the fit and the relation between $\rho_{11}$ and the reduced temperature $t$ (3.3). From (4.4) we extract the (static) critical exponent $\nu$:

$$\xi \sim t^{-\nu} \sim t^{-1/4} \quad \Rightarrow \quad \nu = \frac{1}{4}.$$  \hspace{1cm} (4.5)

Given that the static critical exponent $\alpha = \frac{1}{2}$, (4.5) implies that the hyperscaling relation (3.12) is violated

$$2 - \alpha \neq 3 \nu.$$  \hspace{1cm} (4.6)

Figure 5 shows the inverse of the static susceptibility as a function of $q$ (blue dots) right at the critical point $\rho_{11} = \rho_{11}^c$. The solid red line represents the best quadratic fit to the data

$$(Z_{\text{fit}}^{\text{crit}})^{-1} = -5.55084 \cdot 10^{-6} - 0.30963 \ q^2 - 0.28859 \ q^4 + O(q^6).$$  \hspace{1cm} (4.7)
The red line (4.7) intersects the $q^2$ axis at

$$q_c^2 = -1.8 \cdot 10^{-5},$$

in excellent agreement with the expected value $q_c^2 = 0$ (3.23). The data implies

$$\chi_{m=0,q}^{\text{crit}} \propto Z^{\text{crit}} \propto q^{-2} \iff \chi_{m=0,q}^{\text{crit}} \propto q^{-2+\eta},$$

which determines the anomalous critical exponent $\eta$ as

$$\eta = 0.$$

Finally, we turn to the dynamical critical exponent $z$. According to (3.27), it can be extracted from the scaling of the pole in the dynamical susceptibility in the vicinity of the critical point (in the hydrodynamic limit)

$$0 = \chi_{m=m^*,q}^{-1}, \quad \left. i w \right|_{q=\xi^{-1}} \propto t^{-z\nu}.$$  

A typical behavior of the dynamical susceptibility in a thermodynamically stable phase, i.e., for $\rho_{11} = 0.0351 < \rho_{11}^c$ (blue curve), and a thermodynamically unstable phase, i.e., for $\rho_{11} = 0.0353 > \rho_{11}^c$ (red curve) is presented in Figure 6. We used $q = 10^{-2}$. Notice that in the stable phase, dynamical susceptibility has two separate poles

$$0 = \chi_{m=m^*,q=10^{-2}}^{-1},$$

for $i w > 0$ — according to (3.25) these poles correspond to different relaxation timescales $\tau$ of the system at criticality. In the unstable phase two additional poles appear

Figure 6: (Colour online) The inverse of the dynamical susceptibility of $N = 2^*$ plasma in a stable phase (blue curve) and an unstable phase (red curve) at $q = 10^{-2}$. 
Figure 7: (Colour online) The inverse of the dynamical susceptibility of $\mathcal{N} = 2^*$ plasma in a stable phase (blue curve) and an unstable phase (red curve) at $q = 10^{-2}$ for $|i\omega| \ll 1$.

for small values of $i\omega$. Figure 7 zooms in on the range of $|i\omega| \ll 1$ in the dynamical susceptibility. One of these poles is at negative values of $i\omega_*$, corresponding to a negative relaxation time $\tau$. A detailed analysis show\footnote{Available from authors upon request.} that the negative relaxation time scales as $\tau^{-1} \equiv i\omega_* \propto -q$. Clearly, negative relaxation times signal the instability in the system — this is precisely the instability due to the hydrodynamic (sound-channel) modes which propagate with $c_s^2 \leq 0$ once $\rho_{11} \geq \rho_{11}^c$, see Figure 1. Such instabilities are expected on general grounds: whenever a thermodynamic phase of a system has a negative specific heat, the hydrodynamic modes in the system are unstable \cite{27}. What we demonstrated here is that such instabilities also result in the negative relaxation time: \textit{i.e.}, instead of approaching the equilibrium a system is driven away from it.

We now focus on poles in the dynamical susceptibility at $i\omega > 0$:

$$0 = \chi_{\omega,q}^{-1} \bigg|_{\omega = \{\omega_{*,L}(q), \omega_{*,R}(q)\}},$$

where $L$ and $R$ are the indexes of the two positive poles, such that the corresponding relaxation rates

$$(2\pi T \tau_L(q))^{-1} \equiv i\omega_{*,L}(q) < (2\pi T \tau_R(q))^{-1} \equiv i\omega_{*,R}(q).$$

We performed numerical analysis for different values $q = \{10^{-4}, 10^{-3}, 10^{-2}\}$ — the poles $i\omega_{*,\{L,R\}}(q)$ have a well defined hydrodynamic limit $q \to 0$, which is obtained by \footnote{The reverse is not true: a thermodynamically stable system might still have instabilities \cite{28}.}
Figure 8: (Colour online) The relaxations rates (blue dots) $i\nu_{\ast\{L,R\}}$ ($\{\text{bottom, top}\}$) of $\mathcal{N} = 2^*$ plasma in the vicinity of the critical point. The solid red lines are the quadratic fits to the data, and the dashed green line represents $\rho_{11} = \rho_{11}^\ast$.

computing the susceptibilities (at different values of $\rho_{11}$) strictly at $q = 0$:

$$\lim_{q\to 0} i\nu_{\ast\{L,R\}}(q) = i\nu_{\ast\{L,R\}}(0) \equiv i\nu_{\ast\{L,R\}}. \quad (4.15)$$

The results of such analysis are presented in Figure 8. The blue dots are the relaxation rates of $\mathcal{N} = 2^*$ plasma at criticality, the solid red lines are the best quadratic fits to the data. The top dots/curve corresponds to $i\nu_{\ast,R}$, and the bottom dots/curve corresponds to $i\nu_{\ast,L}$. Once again, the vertical green dashed line corresponds to $\rho_{11} = \rho_{11}^\ast$. The results of the analysis show that in the hydrodynamic limit, both the relaxation rates are finite

$$(2\pi T_c \, \tau_{(L,R)})^{-1} = i\nu_{\ast\{L,R\}} \propto (\Delta \rho_{11})^0 \propto t^0 \propto (2\pi T_c \, \xi^0)^0. \quad (4.16)$$

Thus,

$$\tau_{(L,R)} \propto \xi^z \propto \xi^0 \quad \Rightarrow \quad z = 0. \quad (4.17)$$

In the previous section we pointed out that the critical behavior of $\mathcal{N} = 2^*$ plasma should be identified with that of 'model A' according to dynamical critical phenomena classification in [25]. As such, the dynamical critical exponent $z$ was predicted to be (3.16)

$$z = 2 + c \cdot 0 = 2, \quad (4.18)$$
which differs from the value we obtained \((4.17)\).

5 Conclusions

In this paper, building up on the previous work \([14–16]\), we presented a detailed analysis of the static and dynamic critical phenomena in strongly coupled \(\mathcal{N} = 2^*\) plasma. This model is a string theory derived example of gauge theory/gravity correspondence where one deforms \(\mathcal{N} = 4\) SYM by giving a mass \(m_b\) to bosonic components and a mass \(m_f\) to fermionic components of \(\mathcal{N} = 2\) hypermultiplet. Generically, \(i.e.,\) when \(m_b \neq m_f\), such a deformation completely breaks the supersymmetry. At finite temperature \(\mathcal{N} = 2^*\) gauge theory plasma undergoes a second-order phase transition, provided \(m_f^2 < m_b^2\). This continuous transition is characterized by a terminal temperature \(T_c\), which can be reached within isotropic and homogeneous equilibrium phases. At temperature \(T_c\) the two phases continuously meet, with vanishing speed of sound. One of the phases is always perturbatively unstable — the instabilities reside in the sound-channel hydrodynamic modes which propagate with \(c_s^2 < 0\). Extending \([15]\), we computed the static and the dynamical critical exponents of the transition, as approached from the perturbatively stable phase:

\[
(\alpha, \beta, \gamma, \delta, \nu, \eta; z) = \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{4}, 0; 0 \right). \tag{5.1}
\]

As expected — since the gauge theory is a large-\(N\) model — the transition is of the mean-field theory type with vanishing anomalous static critical exponent \(\eta\). Under the static scaling hypothesis, only two of the critical exponents are independent — thus, the six static critical exponents in (5.1) must satisfy four algebraic constraints. Similar to analysis in \([16]\), we find that only one of these constraints, \(i.e.,\) the hyperscaling relation, is being violated

\[
2 - \alpha \neq 3\nu. \tag{5.2}
\]

Dynamical features of the transition in \(\mathcal{N} = 2^*\) plasma are quite interesting. First of all, the symmetries of the transition identify it as the one in the universality class of ‘model A’, according to classification of Hohenberg and Halperin \([25]\). The latter predicts the dynamical critical exponent as \(z_{\text{prediction}} = 2\), which contradicts direct computations (5.1). Both the stable and the unstable phases have multiple (two) relaxation times which remain finite at the critical point — hence the critical exponent \(z = 0\). Once again, as in analysis in \([15]\), even though the dynamical critical exponent
$z \neq 1$, and thus there is an anisotropy between the time- and the space- coordinates scaling in the two-point (non-equilibrium) correlation functions, the dual gravitational geometry at criticality does not exhibit a Lifshitz-like scaling in the sense of [29]. In fact, if, as suggested by [16], different non-equilibrium correlation functions at criticality have different dynamical exponents $z$, it is not possible to 'by hand' embed the anisotropic scaling of the correlations functions into the symmetric of the dual geometry. The critical phenomena in [16] and the one considered here indicates that anisotropic scaling is rather an emergent phenomena. Second, the unstable phase of $\mathcal{N} = 2^*$ plasma has an additional relaxation rate $\tau_{\text{unstable}}^{-1} \propto -|\vec{k}|$. A negative relaxation rate indicates that rather than approaching the equilibrium, a perturbed system is driven away from it — this is yet another reflection of the instability in the hydrodynamic sector of the theory, which is necessarily linked to a thermodynamic instability of the corresponding plasma phase [27].

In the future, it would be interesting to understand how the general classification of dynamical critical phenomena [25] should be enlarged to incorporate the universality class of $\mathcal{N} = 2^*$ plasma. Probably the most pressing question is the understanding of the equilibrium phases in this universality class (and also the one of the $\mathcal{N} = 4$ SYM plasma with an R-symmetry chemical potential) for $T < T_c$. Such phases can not be homogeneous and isotropic.

Acknowledgments

Research at Perimeter Institute is supported by the Government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Research & Innovation. AB gratefully acknowledges further support by an NSERC Discovery grant and support through the Early Researcher Award program by the Province of Ontario.

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