Entanglement and quantum correlations in the optical parametric oscillator above threshold

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Abstract. We describe several recent experimental studies in the above-threshold optical parametric oscillator (OPO). Seventeen years after the original prediction, entanglement of the twin beams was experimentally demonstrated in 2005. It was followed by the prediction that the OPO should directly produce tripartite pump-signal-idler entangled beams. We also present experimental quantum correlations among these three fields.

The optical parametric oscillator (OPO) is a well-known source of quantum states of light. Above threshold, the generated downconverted beams are called ‘twin beams’ owing to the high level of squeezing in the difference of their intensities. This effect is known since 1987 [1]. In 1988 [2], it was predicted that the twin beams share more than intensity correlations: their phases are also intimately related. Phase anti-correlations, also squeezed, lead to twin beam entanglement. Yet, phase noise is, in general, difficult to measure and this entanglement went unobserved for a long time.

Seventeen years later, we experimentally demonstrated signal-idler entanglement [3] and other groups also succeeded shortly after [4, 5]. In addition to the intensities’ quantum correlation, we observed the quantum anti-correlation of signal and idler phases. Phase quadratures were measured by a self-homodyne technique employing optical cavities [6–8]. In general, as these cavities are detuned from resonance, phase noise of the incident beams is converted to amplitude noise in the reflected ones. The measurement results violated an inequality which is a sufficient condition for demonstrating entanglement [9, 10]. Denoting the amplitude and phase quadratures as $\hat{p}_j$ and $\hat{q}_j$, respectively, where the indices $j \in \{0, 1, 2\}$ refer to pump, signal and idler, in this order, the violated inequality read,

$$\Delta^2 \hat{p}_- + \Delta^2 \hat{q}_+ = 1.41(2) < 2,$$

where $\Delta = |\Delta|$, and $\Delta = 0$. The measurement results violated an inequality which is a sufficient condition for demonstrating entanglement [9, 10]. Denoting the amplitude and phase quadratures as $\hat{p}_j$ and $\hat{q}_j$, respectively, where the indices $j \in \{0, 1, 2\}$ refer to pump, signal and idler, in this order, the violated inequality read,

$$\Delta^2 \hat{p}_- + \Delta^2 \hat{q}_+ = 1.41(2) < 2,$$
A very distinct characteristic of this system is the large frequency difference that can exist between signal and pump beams. This allows for the transfer of quantum information between different regions of the electromagnetic spectrum. As an application, the twin beams could be used to communicate otherwise incompatible quantum hardwares.

While pursuing this effect, we faced novel intriguing behavior in the twin beams [11]. The entanglement observation was restricted to a narrow region of parameters, in opposition to the theoretical model. The effect was observed only very close to the oscillation threshold. We found out empirically that an unaccounted pump excess noise, when included ad hoc in the model, would explain the strange features of the experimental observations (Fig. 3). But we knew our incident pump beam was shot-noise limited. This lead us to investigate the reflected pump noise characteristics. As a result, the measurements indicated that it had excess noise, even when no parametric oscillation was taking place. The origin of the excess noise points at the non-linear crystal, for physical reasons still to be unravelled.
Figure 2. (Color online) General view of experimental values of Eq. (1) obtained for the above-threshold OPO. Measurements performed by our group are designated by the initials ‘LMCAL’. The corresponding references are ‘LMCAL [2005]’, Ref. [3]; ‘LMCAL [2006]’, Ref. [11]; ‘LMCAL [2007]’, Ref. [12]. The references corresponding to measurements performed by other groups are: ‘Peng [2006]’, Ref. [4]; ‘Pfister [2006]’, Ref. [5]; ‘Fabre [2005]’, Ref. [13]. The region below the curved line violates the entanglement inequality of equations (1) and (2), while below the straight line the maximum logarithmic negativity [14], an entanglement quantifier, is strictly positive.

In parallel to these findings, we were investigating the possibility of correlations between the reflected pump beam and the twin beams. The pump field reflected by the OPO had been observed to present quadrature squeezing [15]. Suprisingly, despite the widespread use of this system in quantum optics, its quantum properties as a whole (pump, signal, and idler beams) had not been thoroughly investigated. This study lead us to the theoretical prediction of tripartite entanglement in this system [16]: pump, signal, and idler are inseparable. The different frequencies available are now indeed very much different, increasing the freedom in quantum information color tunability.

Three-color entanglement comes, from a physical standpoint, as a consequence of energy conservation and phase matching. Since pump, signal, and idler optical frequencies must respect \( \omega_0 = \omega_1 + \omega_2 \), it follows that small frequency fluctuations, regarded as phase fluctuations, are connected through the relation \( \delta \varphi_0 = \delta \varphi_1 + \delta \varphi_2 \). This leads to strong quantum correlations among the phase fluctuations of the three fields.

The inequalities to be violated for genuine tripartite entanglement are now given by [17]

\[
S_1 = \Delta^2 (\hat{p}_1 - \hat{p}_2) + \Delta^2 (\hat{q}_1 + \hat{q}_2 - \alpha_0 \hat{q}_0) \geq 4 ,
\]

\[
S_2 = \Delta^2 (\hat{p}_0 + \hat{p}_1) + \Delta^2 (\hat{q}_1 + \alpha_2 \hat{q}_2 - \hat{q}_0) \geq 4 ,
\]

\[
S_3 = \Delta^2 (\hat{p}_0 + \hat{p}_2) + \Delta^2 (\alpha_1 \hat{q}_1 + \hat{q}_2 - \hat{q}_0) \geq 4 ,
\]

where \( \alpha_j \) is a real number that minimizes \( S_j \). Violation of at least two inequalities demonstrates genuine tripartite entanglement.

The first sum of variances, \( S_1 \), is already violated as a consequence of signal-idler entanglement.
Figure 3. (Color online) Noise behavior as a function of $\sigma$. (a) Predictions of the theoretical model, for an input pump beam with amplitude noise $S_p0 = 1.5$ and phase noise $S_q0 = 5.5$; dashed line: $S_q+$; open circles + full line: $S_q-$; full line: $S_q+$/ crosses + full line: $S_p-$. (b) Experimental results for $\sigma$ ranging from 1.06 to 2.2. full circles: $S_p+$; triangles: $S_q-$; open circles: $S_q+$; squares: $S_p-$. The shot noise level is indicated by a dashed line.

Including the pump fluctuations improves its value. Because of symmetry between signal and idler fields, it is found that $S_2 = S_3$. The theoretical predictions for $S_1$ and $S_2$ are given in Fig. 4.

The first experimental evidence of tripartite quantum correlations came for a detuned OPO [18]. In this situation, phase-phase correlations are partially converted to phase-amplitude correlations regarding pump and twin beams. We observed that the pump field amplitude fluctuations were correlated to the phase fluctuations of the twin beams.

We slightly modify inequality (3) to obtain

$$\Delta^2 \hat{p}_- + \Delta^2 (\hat{q}_+ - \alpha_0 \hat{p}_0) \geq 2, \quad \text{with} \quad \alpha_0 = \frac{C_p \hat{q}_+}{\Delta^2 \hat{p}_0}, \quad (6)$$

where $\hat{p}_- = (\hat{p}_1 - \hat{p}_2)/\sqrt{2}$ and $\hat{q}_+ = (\hat{q}_1 + \hat{q}_2)/\sqrt{2}$. This inequality is analogous to expression (1) of Refs. [9, 10] with an additional correction by introducing pump information. This can be made clearer
Figure 4. (Color online) Sum of variances $S_1$ and $S_2 = S_3$ as functions of pump power relative to threshold, $\sigma$, and analysis frequency relative to OPO cavity bandwidth for the twin beams, $\Omega$. $S_1 < 4$ and $S_2 < 4$ demonstrate tripartite entanglement.

Figure 5. (Color online) Noise spectra at 27 MHz, as functions of analysis cavities’ detuning. Sum of twin beam quadratures: full circles + line; difference of twin beam quadratures: open circles + line; sum of twin beam quadratures corrected by correlations with the pump amplitude: full line. Shot noise level is the dashed line. $\sigma = 1.34$.

by writing the second term of the above inequality as

$$\Delta^2 \hat{q}_+'' \equiv \Delta^2 \hat{q}_+ - \beta_0,$$

with

$$\beta_0 = \frac{C^2_{\rho_0 q_+}}{\Delta^2 \rho_0}.$$  \(7\)

where $\beta_0$ is the above-mentioned correction. If $\Delta^2 \hat{q}_+'' < 1$ and $\beta_0 \neq 0$, there is a quantum correlation between $\hat{p}_0$ and $\hat{q}_+.$

The experimental observation of such correlation is presented in Fig. 5. The open circles are the result of subtracting the photocurrents from signal and idler, while the full circles come from their sum. The continuous-line curve adds the correction term $\beta_0$ to the sum curve, demonstrating the desired effect.
The shot noise limited value of $\Delta^2 \hat{q}_+ = 0.99(2)$ becomes squeezed, assuming the value $\Delta^2 \hat{q}_+^r = 0.86(2)$. Using the value $\Delta^2 \hat{p}_- = 0.53(2)$ taken from the difference curve, the generalized criterion of equation (6) assumes the improved value $\Delta^2 \hat{p}_- + \Delta^2 \hat{q}_+^r = 1.39(3) < 2$. Without the correction, the bipartite entanglement between signal and idler would result in $\Delta^2 \hat{p}_- + \Delta^2 \hat{q}_+ = 1.52(3) < 2$.

The behaviors of $\Delta^2 \hat{q}_+$, $\Delta^2 \hat{q}_+^r$ and $\beta_0$ are presented in Fig. 6 as functions of pump power. The exact values obtained for the noises should be lower, presenting more squeezing. The solid curves represent the theoretical predictions including ad hoc pump excess noise in the model. We believe that the reason for the discrepancy comes again from the unexpected noise introduced by the non-linear crystal [11].

As a first step towards the observation of tripartite entanglement, we added an optical cavity to perform phase quadrature measurements of the reflected pump beam. Fig. 7 presents a measurement analogous to the one of Fig. 5 but, instead of correlating pump amplitude to both twins’ quadratures, we study the correlation between the same quadratures for the three beams. This is a measurement of inequality (3), but with each term normalized to the shot noise level, so that inequalities are violated for a sum of variances less than 2. Pump power corresponds to $\sigma = 1.24$.

The sum and difference of the twin beams are considered, and a third term, coming from the correction by the pump, is added to the sum curve. The important region of the difference curve (open circles) is the one relative to the amplitude quadrature (off resonance), while the phase quadrature (half resonance) is the region of interest for the sum of quadratures curve (full circles), according to inequality (3). The correction term, calculated in a similar manner to $\beta_0$ [expression (7)], is added to the sum curve, resulting in the full line curve. Without the correction term, the bipartite inequality of Refs. [9, 10] is recovered.

Once more, the measured noise values are not in agreement with the ordinary theoretical model, since squeezing is always expected in the sum of twin beams phases [16]. As a consequence, the twin beam entanglement is almost inexistent, since $\Delta^2 \hat{p}_- + \Delta^2 \hat{q}_+ \approx 2$. Correcting by the pump phase fluctuations, a great amount of excess noise is removed from $\Delta^2 \hat{q}_+$, resulting in a violation of inequality (3) with the value $\Delta^2 (\hat{p}_1 - \hat{p}_2)/2 + \Delta^2 (\hat{q}_1 + \hat{q}_2 - \alpha_0 \hat{q}_0)/2 = 1.56(3)$ [with the values $\Delta^2 (\hat{p}_1 - \hat{p}_2) = 0.46(1)$ and $\Delta^2 (\hat{q}_1 + \hat{q}_2 - \alpha_0 \hat{q}_0)/2 = 1.10(2)$]. The tripartite quantum correlations seem stronger than in Fig. 5.

The tripartite three-color quantum correlations observed do not suffice to demonstrate full

**Figure 6.** (Color online) Behavior of $\Delta^2 \hat{q}_+$, with and without correction $\beta_0$ owing to correlations with the pump amplitude, as a function of $\sigma$. $\Delta^2 \hat{q}_+$: full squares; $\Delta^2 \hat{q}_+^r$: open circles; $\beta_0$: open triangles. Solid lines correspond to physical model with pump detuning and excess noise as free parameters.
Figure 7. (Color online) Noise spectra at 27 MHz as functions of analysis cavities’ detuning. Sum of twin beam quadratures: full circles + line; difference of twin beam quadratures: open circles + line; sum of two beams’ quadratures corrected by correlations with the remaining third beam: full line. Shot noise level is the dashed line.

inseparability (yet). We believe that tripartite entanglement is currently being hindered by a similar excess noise observed in the bipartite twin beams entanglement [3]. The noise dependences with pump power and OPO cavity characteristics are currently under careful scrutiny in our lab, in order to find ways around this spurious excess noise. Hopefully, experimental three-color entanglement is just a matter of (a short) time.

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