On the Influence of the Characteristic Frequency and Broadband of Seismic Effects on the Vertical Rod Oscillations

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Abstract. The problems of longitudinal oscillations of high vertical rods with orientation to the application of results in the problems of seismic calculations are considered. Mathematical models of free, forced harmonic and forced random oscillations caused by kinematic disturbances of the lower end are obtained. Application of methods of separation of variables and finite differences in combination with visualization of calculations in Matlab environment gave own values and own functions, and then amplitudes and mean square deviations of oscillations. It is shown how the preliminary consideration of harmonic oscillations significantly simplified the solution of the stochastic problem. Influence of characteristic frequency and broadband parameter of input random process on the form and standard deviation of longitudinal oscillations is studied.

1. Introduction
Longitudinal oscillations are dangerous for vertical building structures located near the epicenter of earthquakes [1, 2], as well as for compressor blades, connecting rods of engines and pumps, parts of robotic systems [3, 4]. On the accelerogram of the famous earthquake in Gazli (1976) the maximum vertical velocity ordinate was almost 2 times higher than the maximum horizontal component. In general, vertical accelerations usually make up 60-70% of horizontal accelerations [5].

Figure 1
Often the corresponding design schemes are presented as rods with two concentrated masses (Fig. 1). Let’s consider a vertical rod (Fig.1) of a constant section, length \( l \) from a material with the modulus of elasticity \( E \), with the density of material \( p \), the cross-section area \( S \), bearing on the upper and lower ends of the discrete masses \( M \) and \( M_0 \) and supported by an elastic base with a coefficient of bed with. Seismic effects are represented by a random process of \( v(t) \) acting on the base in a vertical direction.

2. Free fluctuation

It is known that longitudinal oscillations of the continuum part of the rod are described by the differential equation in the partial derivatives of the hyperbolic type [6 - 8]

\[
\ddot{u} - a^2 \ddot{u}'' = 0, \quad a^2 = \frac{E}{\rho}, \quad x \in (0, l), \quad t > - \infty.
\]

The calculation scheme, if \( v(t) \) is ignored, gives the boundary conditions arising from the conditions of fixing the ends of the rod

\[
bu'(0, t) - M_0 \ddot{u}(0, t) - cu(0, t) = 0, \quad b = ES.
\]

\[
bu'(l, t) + M\ddot{u}(l, t) = 0, \quad t > - \infty.
\]

We will write down the solution of the problem (1) - (3) using the method of separation of variables

\[
u(x, t) = X(x) e^{i\omega t},
\]

where \( X(x) \), \( \omega \) is the proper form and frequency of vibration. Substitution (4) in (1) - (3) gives

\[
(i\omega)^2 X - a^2 X'' = 0, \quad x \in (0, l).
\]

\[
bX'(0) + \omega^2 M_0 X(0) - cX(0) = 0, \quad bX'(l) - \omega^2 MX(l) = 0.
\]

The problem (5), (6) is further solved by using the finite difference method [9, 10]. For this purpose, instead of a continuous area of determination of the variable \( x \), let’s introduce a discrete area \( Ih \) in the form of nodes of a uniform grid with the step \( h \)

\[
l_h = \left[ x_i : x_i = (i - 1) h, \quad i = 1, 2, ..., n \right], \quad h = l/(n - 1),
\]

where \( n \) is the number of grid nodes. Following the procedure of the method, we obtain the algebraic equation in the matrix-vector form

\[
A(\omega) X = 0.
\]

Here \( A \) is a square matrix of the order of \( n \), \( X = [X_1, X_2, ..., X_n]^T \) is a transposed vector whose components are rod movements in grid nodes. Non-zero solutions (7) can only exist if

\[
det(A(\omega)) = 0.
\]

The characteristic equation (8) is an algebraic equation relative to \( \omega \) and has many roots of power \( n \). They can be found graphically by building a chart of the \( \det(A(\omega)) \). The points of intersection of the \( \omega \) axis by the graph determine the values of natural frequencies. Then you can define your own vectors \( X_k (k = 1, 2, 3, ..., n) \) in known ways.

Example 1. Fig. 1 shows a rod made of standard steel pipe with length \( l = 20 \) m, diameter \( D = 108 \) mm and wall thickness \( 6 = 4.71 \) mm, with masses \( 10 = 16000 \) kg, \( M = 7000 \) kg. Bedding coefficient of the base with = 2-106 N/m³.
The first three eigenvalues were obtained in the Matlab computational complex environment 
\[ \omega_k = \{9.22; 56.21; 798.79\} \text{ c}^{-1}, \]
the first two of which are shown in Figure 2. They fall into the zone of the usually dominant seismic frequencies and therefore pose a significant risk.

![Figure 2](image1.png)

**Figure 2**

The next task is to find the own vectors of \( X_k \) \((k = 1, 2, 3)\) of matrix \( A \). They are determined from the system of equations (8) at known eigenvalues \( \omega_k \) and are shown in Fig. 3. The first and second forms of oscillation are similar to similar forms of systems with two degrees of freedom. The reason is that there are two heavy concentrated masses. They are the ones who form these forms of oscillation. The third high frequency is determined by the oscillations of the steel continuum part of the system, and the form of oscillations is represented by a curve in the form of a half-wave sinusoid. In this case, the ends of the rod with the discrete masses remain almost motionless.

3. **Forced harmonic vibrations**

Seismic and technogenic disturbances of a stochastic nature can be described for preliminary calculations in the form of a harmonic process \( v(t) = v_0 e^{i \omega t} \). Such a replacement becomes almost adequate if the disturbances are narrowband random processes with a characteristic frequency. Justification for deterministic analysis of seismic reactions is that the amplitudes of harmonic oscillations can become analogues of mean square deviations and the determination of the latter will be significantly simplified.

The mathematical model of free oscillations (1) - (3) proposed above is easily adapted to the kinematically excited forced oscillations. For this purpose, let us add the kinematic perturbation of the foundation as a harmonic process to the mathematical model of the structure (Fig. 1)

\[ v(t) = v_0 e^{i \omega t}. \]  

(9)

With it in mind, the boundary condition (2) will take the form

\[ b \ddot{u}(0, t) - M_0 \dot{u}(0, t) - c \dot{u}(0, t) + c v(t) = 0. \]  

(10)

Add (1), (3)

After applying the finite difference method, will be obtained...
where $A(\omega)$ is the matrix of coefficients of the equation (7) obtained above for free oscillations, $d$ is the vector of the right part
\[ d = \{-2h\omega v_0/b, 0 \ldots 0\}^T. \]

The first element of the first line of the matrix should be changed according to the boundary condition (10), i.e. it will look like
\[ \varepsilon = -3 + 2h(\omega^2 M_0 - c)/b. \]

**Example 2.** Let’s take the rod described in example 1 above. In addition to the initial data, let us add the amplitude of kinematic disturbances $v_0 = 10$ sm.

The results of the calculations are presented in graphs $X(x)$ Fig. 4, constructed at the values of perturbation frequencies $\omega = \{0.1, 3.5, 52, 55, 798.73\}$. The nature of the curves essentially depends on the location of $\omega$ within the spectrum of natural frequencies $\omega_k = \{9.22, 56.21, 798.79\}$ s$^{-1}$.

From the analysis of the curves of rice, 4 it follows that at frequencies of perturbations $\omega < \omega_k$ (curves 1, 2) forms of fluctuations are almost straight lines, deviations of ends of a rod with the concentrated masses of syphase, movements occur on the first form of system from two discrete masses. In these cases, deformations and normal stresses in the sections will be small. As the frequency of perturbations grows, the amplitudes of $X(x)$ oscillations decrease, the oscillations occur in the second form of oscillations of discrete systems (lines 3, 4), and the movements become antiphase. The deformations, and therefore the stresses, will increase. Further, the fluctuations of the ends at $\omega > \omega_k$ (lines 3, 4) become counter-phase. As soon as $\omega$ approaches $\omega_3$, the shape of the curve becomes close to the sinusoid (curve 5). In all cases, the $X(x)$ lines are similar in form to the native functions shown in the fig. 3.
4. Random oscillations
Let now kinematic disturbances be stationary random processes. In the steady-state mode of oscillations, the function $u(x,t)$ will be a centered spatio-temporal random field, stationary in time and heterogeneous in spatial coordinate. Mathematical expectation of such a field will be zero. Let’s find the mean square deviations of the movements of the rod sections $u(x)$.

Let’s consider specifically the input random process with a characteristic frequency. Then the spectral density of random perturbations looks like

$$S(\omega) = \frac{2\alpha \omega^2 \sigma_v^2}{\pi[(\omega^2 - \omega^2)^2 + 4\alpha^2 \omega^2]}; \quad 0^2 = \alpha^2 + \beta^2. \quad (12)$$

Here $\alpha$ - broadband parameter, $\beta$ - characteristic frequency, $\sigma_v$ - standard deviation. Let’s find out the dependence of the dispersion of $D_v(x)$ random oscillations on the characteristic frequency of perturbations. At the same time, the bandwidth factor $\alpha$ is relatively small. Then random perturbations are narrowband and close to harmonic ones. As a consequence, random rod oscillations should also be close to the harmonic oscillations discussed above. To this end, at previous and additional parameter values $\alpha = 0.1 \, s^{-1}; \quad \sigma_v = 10 \, sm$ calculations were performed with increasing values of the parameter $\beta$, coinciding with the deterministic disturbance frequencies $\omega$ in the example above.

$\beta = \{0.1; \quad 3.5; \quad 52; \quad 55.798.7\}.$

When calculating the dispersions, the algorithm and program used for the harmonic oscillation problem are applied. It is taken into account that the dispersion of the random process $v(t)$ appears to be an unrelated integral

$$D_v = \int_{-\infty}^{+\infty} S(\omega)d\omega.$$  

This means that the dispersion is distributed continuously along the entire real numerical axis $R$ by the formula (12), so that each frequency is matched with the elementary dispersion $S(\omega)d\omega$. If we take it instead of the amplitude of the harmonic process $v_0$, then the output of the problem will be obtained.
elementary variance of deviations \( dD_u(\omega, x) \). Subsequent summing up at \( \omega w \) gives the dispersal of \( D_u(x) \) deviations.

For the convenience of comparability with the amplitudes of harmonic oscillations, the results obtained in the form of a function of mean square deviations \( \sigma_u(x) \) in Fig. 5. Taking into account that the standard deviations of a random process can take only positive values, we can say that the curves of the lines are identical in qualitative and quantitative terms to the corresponding amplitude graphs in Figure 4. At the same time, the values of mean square deviations are almost equal to the corresponding amplitudes. The reason is the broadband of the input random process defined by parameter \( \alpha = 0.1 \).

Qualitative conclusions drawn from the rice 4 for harmonic oscillations, remain in force, but in terms of characteristic frequency and standard deviations.

Let us now consider the influence of the input process broadband on random oscillations. At a constant value of the standard deviation of perturbations \( \sigma_v = 10 \text{ sm} \) we will distinguish between two situations:

1. The characteristic frequency is far from the natural frequency (in this case the first one);
2. The characteristic frequency is close to the natural frequency.

In the first case, let's assume the following parameter values \( \alpha \) and \( \beta \):

\[
\alpha = \{0.1; 1; 1.8; 2.8; 3.8\} \text{ s}^{-1}, \quad \beta = 4 \text{ s}^{-1}.
\]

The results of the calculations are shown in the graphs in Fig. 6. It is obvious that the wide bandwidth of kinematic influences strongly influences the dispersion of movements. As the dispersion \( \alpha \) parameter increases, it increases significantly. The reason is this. The characteristic frequency of seismic impact in this case is small compared to the first natural frequency. The first line corresponds to almost harmonic oscillations far from resonance. With increasing broadband perturbations, the area of dominant frequencies in perturbations expands and captures an increasing number of eigenfrequencies. At the same time, the ordinates of the spectral density function \( S(\omega_k) \) at the eigenfrequencies increase, increasing the resonance phenomena.

In the second case

\[
\alpha = \{3.3; 3.7; 4.3; 5.1; 6.3\} \text{ s}^{-1}, \quad \beta = 9 \text{s}^{-1}.
\]

Now the characteristic frequency is very close to its own frequency. Therefore, for small values \( \alpha \), not considered in the example, the oscillations will be almost resonant. The results of the calculations are shown in Fig. 7 and they're the opposite of the previous one. The increase \( \alpha \) leads to the redistribution of the perturbations frequencies to the "blurring" of the dominant frequencies and to tuning from the resonance. The ordinates of the spectral density function \( S(\omega_k) \) at the eigenfrequencies decrease, weakening the resonance phenomena.

As a result, the standard deviation drops.
5. Conclusion
1. The results obtained in the deterministic and stochastic formulation of the problem have many similarities. This makes it possible to make preliminary predictions about random seismic oscillations based on the results of the harmonic oscillations problem.
2. The broadband parameter depending on the proximity to the characteristic frequency of seismic effects affects the fluctuations of the rod in two ways, i.e. it can increase or decrease the fluctuations of the rod.

References
[1] Newmark N, Rosenbluett R 1980 Fundamentals of Earthquake Engineering: Abbreviated Per. (M.: Stroyizdat) 344 p
[2] Kulterbaev Kh P, Baragunova L A, Shogenova M M, Shardanova M A, Abdul Salam I M 2018 Longitudinal Vibrations of Seismic Disturbance Vertical Bar Proceedings of the International Symposium “Engineering and Earth Sciences: Applied and Fundamental Research” (ISEES 2018) Advances in Engineering Research vol 177 pp 515-520
[3] Biederman V L 1979 Applied theory of mechanical vibrations (Moscow: Graduate school) 416 p
[4] Polyakov S I 1983 Seismic Structures of Buildings: Training. Manual for HEIs 2nd edition (Moscow: Higher school) 304 p
[5] 1972 Seismic stability of buildings and structures Under the editorship of I I Goldenblatt (M.: Stroyizdat) 216 p
[6] Ilyin M M, Kolesnikov K S, Saratov Y S 2003 Oscillation theory (MG: MG) 272 p
[7] Tikhonov A N, Samarskiy A A 1977 Equations of mathematical physics (Moscow: Science) 736 p
[8] Kulterbaev H P, Abdul Salam I M 2018 Longitudinal free core oscillations with discrete masses Problems of mechanics and control: Proceedings of the International conference Ed. I G G Goryachev (M. Moscow State University Publishing House) pp 207-210
[9] Karamanskiy T D 1981 Numerical methods of construction mechanics (M.: Stroyizdat) 436 p
[10] Samarskiy A A, Gulin A V 1989 Numerical methods (M.: Science, Main Editorial Board of Physiology and Mathematics) 432 p