High-sensitivity microwave vector detection at extremely low-power levels for low-dimensional electron systems

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We present a high-sensitivity microwave vector detection system for studying the low-dimensional electron system embedded in the gaps of a coplanar waveguide at low temperatures. Using this system, we have achieved 0.005% and 0.001% resolutions in amplitude and phase variations, respectively, at 10 GHz in a magnetotransport measurement on a quantum-wire array with an average signal power less than −75 dBm into the sample at 0.3 K. From the measured phase variation, we can distinguish a very tiny change in the induced dipole moment of each quantum wire.

Coplanar waveguides (CPW’s) have been successfully used as broadband sensors in investigating the high-frequency magnetotransport phenomena of low-dimensional electron systems (LDES’s), such as two-dimensional electron systems (2DES’s), quantum dots (QD’s), and anti quantum dots (QD’s) etc. In these works, a commercial vector network analyzer (VNA) is the major tool to measure the variation of the propagation constant, including the attenuation constant (α) and the phase constant (β), of the CPW that containing the active LDES in the gaps between the metal electrodes. From α and β one can extract the longitudinal conductivity (σxx) (both real and imaginary parts) of the LDES. However, since the microwave power delivered to samples at temperature (T) below few hundred mK must be very low, the resolution of the data becomes very poor, especially for the phase part. Thus in most of the previous studies using CPW sensors, they only presented Re{σxx} data derived from α and discarded the phase part. Even though Kohls et al. and Lewis et al. have addressed the Im{σxx} behavior in the integer quantum Hall (IQH) regime based on other techniques, still, the resolution of Im{σxx} is mediocre due to the constrain of VNA’s. Nevertheless, Im{σxx}, proportional to the change of the real part of dielectric constant, gives important information about the electric polarization, that is of special interest in the case of QD’s, quantum wires (QW’s), or 2DES’s at high magnetic fields (B). Furthermore, the relatively small effective area of QD’s or QW’s compared to 2DES samples leads to a very small signal variation (or dynamic range), that makes the conventional VNA measurement very difficult and impractical.

In this letter, we present a new detection scheme and the instrumental implementation, which can resolve very small variations not only in the amplitude but also the phase of an extremely low-power-level microwave signal traveling through a CPW with LDES’s embedded in the gaps while some external sample parameters, such as the applied magnetic field (B) or T, etc., is changed. The data of a low-T magnetotransport measurement on a QW-array sample manifest the high-resolution capability of this system.

A simplified schematic diagram to illustrate the principle of phase detection by a phase-lock loop (PLL) is depicted in Fig. 1 (a). The CPW sample is connected to a PLL through two semirigid coaxial cables of total length L. The PLL will force the total phase change (∆φ) of the voltage-controlled oscillator (VCO) via

\[ \Delta \phi = \Delta \phi_L + \Delta \phi_s = 0, \]

where \( \Delta \phi_s = -\Delta \phi_L \). Hence \( \Delta \phi_s \) can be obtained directly from the frequency change (∆f) of VCO via

\[ \Delta \phi_s = -\Delta \phi_L = -2\pi \Delta f / v_L = -\Delta \omega / T_L, \]

where \( v_L \) is the phase velocity of the signal in the cable. The result can be expressed as the product of the change of the angular frequency (Δω) and the delay time (τL) of the connecting cables with a different sign.

A complete block diagram, including the pulse handling circuits, the microwave PLL, and the amplitude readout circuit, together with the CPW sample in a cryogenic environment, is shown in Fig. 1 (b). The microwave part of this system is basically a pair of homodyne detectors (mixers) with reference signals of quadrature phase difference. One of the mixers with 0° reference (LO1), used as the phase sensitive detector (PSD), has zero output (IF1) forced by the PLL, while the other one with
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modulation scheme to detect and average the microwave
signal. A short pulse train with a 0.2~2 µs pulse width
and a 0.1~10% duty cycle, provided by a pulse genera-
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the microwave signal path in the PLL.

90' reference (LO2) has an output (IF2) proportional to
the amplitude of the signal.

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modulation scheme to detect and average the microwave
signal. A short pulse train with a 0.2~2 µs pulse width
and a 0.1~10% duty cycle, provided by a pulse genera-
gated by a slow square-wave TTL signal with a
period of 1~10 ms from a lock-in amplifier, modulates
the microwave signal sent to the sample. A time-delayed
pulse with a 0.1~1 µs pulse width triggered by the mod-
ulating pulses controls a sample-and-hold (S&H) circuit
that samples the IF output of the microwave mixer.
The holding capacitor in the S&H circuit is discharged
through an analog switch when the TTL gating signal
is low. Finally the lock-in amplifier reads the output of
the S&H circuit. There are two sets of pulse averaging
circuits, one for the PLL and the other for the amplitude
readout. Note that the time constant of the lock-in am-
plifier for the PLL is about 1~30 ms, in contrast to 300
ms or 1 s for the amplitude readout part. The average
of the PSD output (IF1) is sent to an integrator (loop
filter of PLL) with a time constant of 26.3 ms. The out-
put of the integrator connects to the FM input of the
microwave source (VCO), thus closing the PLL. In fact
this PLL system is modified from what people used in
surface-acoustic-wave detection experiments but with
improved pulse averaging and amplitude detection meth-
ods.

We use three sets of microwave modules to cover the
frequency from about 60 MHz to 18 GHz. The details
of our instrumentations and circuit designs will be pub-
lished elsewhere. To gain an idea of the detection limit,
we tested our system with only an 11 m semirigid ca-
ble connected to the PLL without samples. The input
microwave signal is attenuated down to −70 dBm peak
power, and less than −90 dBm in average. The back-
ground phase fluctuation we obtain in this test is less
than 0.0003° (root-mean-square value) for f≤6 GHz and
0.006° for 6≤f≤18 GHz, which is remarkably low for such
a low-power signal. In fact, the signal power reaching the
low-noise amplifier (LNA) is even lower than the input
value claimed above due to the loss of the cable, which
is about −9 dB at 1 GHz and raises to −41 dB at 10
GHz. This may explain why the noise in phase increases
at high frequencies. The resolution with a low-T sample
loaded is slightly worse due to the loss of the sample and
extra noise from the cryogenic environment. The resolu-
tion of the amplitude readout for a small-variation signal
can be enhanced by the use of the "offset" and "expand"
functions of the lock-in amplifier.

In the following we will present measured results for
a QW array sample to demonstrate the resolving power
of this method. The sample is fabricated from a stan-
dard MBE-grown modulation-doped GaAs/AlGaAs het-
erostructure containing a 2DES, which is 150 nm under
the surface. The 2DES has a mobility of about 1.5
10^{6} cm^{2}/Vs at 4K, and a density of 1.1×10^{11} cm^{-2}. Before
evaporating the Cr/Au(10/300 nm) metal layers for the
CPW pattern, we etch away the 2DES part underneath.
The widths of the center conductor and the gap of the
50 Ω CPW are 36 µm and 23 µm, respectively. A me-
andering pattern is used to increase the effective length
of the CPW. Subsequently we pattern the 2DES left in
the gap into about 7000 identical QW mesas, each of 0.7
µm wide and 20 µm long, by using e-beam lithography
and chemical etching [Fig. 2 (a)]. The QW's, parallel to
the propagating direction of microwave signals, occupy
only about 6 mm in length of the straight sections of the
meandering CPW.

The CPW sample is immersed in liquid ^3He (0.3 K)
with applied B perpendicular to the sample surface.
The total time delay given by the connection cables and mi-
crowave modules is 51.1 ns. From Eq. 1, this time delay
multiplied by Δf gives the phase change (Δφ) of the
CPW sample. Here Δf can be either obtained from the
output voltage of the integrator in the PLL scaled with
the FM deviation setting of the VCO, or measured di-
rectly with a microwave counter. The peak power of the
pulsed microwave signals is −50 dBm (about −66 dBm
in average) at the input end of the semirigid cable.

Figure 2 (b) displays low-field results for both direc-
tions of B. Besides the apparent Shubinkov-de-Hass oss-
cillations, we can observe additional intriguing features
for B below 0.2 T. For frequency higher than few GHz,
the high-B data exhibit behaviors similar to a 2DES,
showing IQH states. The SdH oscillations become less
pronounced at lower f and even completely disappear
below 600 MHz. In Fig. 2 (c), the data are shown up to
B = 11 T. An extra adsorption peak appears at B=6 T
associated with a unique phase change nearby for f=255
MHz, and moves to lower B for higher f, indicating that

FIG. 1: (a) Simplified schematic diagram for phase detection
using a PLL. (b) Block diagram of the vector detection sys-

\[ \Delta f = \frac{4}{\pi} \cdot \frac{B}{E_{\text{W}}} \]
turbations in $\Delta$ and $\Delta \phi$, data are less than 0.003% and 0.001% for $f \lesssim 10$ GHz, and 0.05% and 0.03% for $f \gtrsim 10$ GHz, respectively. Moreover, the 0.3% scale bar in the $\Delta A$ plot is equivalent to only 53 $\mu$A 2D conductivity in average, which is very small compared to the signal levels in previous studies.

The susceptibility $\gamma$ of each QW and $\Delta \phi$ can be related by $\gamma = \Delta \phi \cdot \xi^2 / N \omega Z_0$, where $N$ is the total number of QW’s, $Z_0$ the characteristic impedance of the CPW, and $\xi$ a length scale related to the distribution of the tangential electric field ($E$) on the surface and the geometry of the CPW. The induced dipole moment of each QW segment, $p$, is then $\gamma E$. For our CPW structure, $\xi$ is about 21 $\mu$m. The $\gamma$ value corresponding to the 0.2$^\circ$ scale bar in Fig. 2(c) for 255MHz is about $3 \times 10^{-27}$ F/m$^2$. For a signal of $-51.5$ dBm peak power, we can estimate $p$ accordingly to be about $3 \times 10^{-25}$ Cm, equivalent to about 17 electrons being transferred across a 0.1 $\mu$m effective QW width, assuming a 0.3 $\mu$m depletion length near each edge.

The background noise of the data shown in Fig. 2(b) and (c) is extremely low. The amplitude and phase fluctuations in $\Delta A$ and $\Delta \phi$, data are less than 0.003% and 0.001% for $f \lesssim 10$ GHz, and 0.05% and 0.03% for $f \gtrsim 10$ GHz, respectively. These resolution limits actually depend on the power reaching the LNA, which is $f$ dependent due to the loss of the sample and the semirigid cables. The average power into the sample and into the LNA are about -70 dBm and -76 dBm at $f \sim 1$ GHz, and down to -80 dBm and -107 dBm at 14 GHz, respectively. Moreover, the 0.3% scale bar in the $\Delta A$ plot is equivalent to only 53 $\mu$A 2D conductivity in average, which is very small compared to the signal levels in previous studies.

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Finally, we want to discuss the effect of the cable length and related instrumental considerations. Usually as we increase $L$, the sensitivity in phase is increased according to Eq. 1, and so is the loop gain of the PLL. However, if $L$ is too big, the PLL will have a very small capture range, and the effect of noise and drift in electronic components become serious. In addition, high-frequency signals will suffer a very severe loss.

In conclusion, we have developed and demonstrated a high-sensitivity vector detection system for very low-power microwave signals used in a CPW broadband sensor. This system is a very powerful tool in studying the dynamic behaviors, including the electric polarizations, of LDES’s at low temperatures.

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13. This equation can be derived simply by equating the energy stored in the field for the electric dipole moment of all the QW’s, $N \gamma E^2 / 2$, and the energy increment due to change of the effective capacitance per unit length, $\Delta CV^2 / 2$. The length scale $\xi$ can be shown to be $[2(C_\phi / \phi_0)(V^2 / E^2)]^{1/2}$, where $C_\phi$ is the capacitance of the CPW if the substrate is replaced by air, and $(V^2 / E^2)$ is the average of square of voltage signal over square of tangential field on the surface across the gap of the CPW.