Hard dilepton production in presence of a weakly magnetized hot QCD medium

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(Dated: March 11, 2019)

We have computed hard dilepton production rate from a weakly magnetized deconfined QCD medium within one loop photon self energy by considering one hard and one thermomagnetic resummed quark propagator in the loop. In presence of the magnetic field the resummed propagator leads to four quasiparticle modes. The production of hard dilepton consists of rates when all four quasiquarks originating from the poles of the propagator individually annihilate with a hard quark coming from bare propagator in the loop. Beside these, there are also contributions from mixture of pole and Landau cut part. In weak field approximation, the magnetic field appears as a perturbative correction to the thermal contribution. Since the calculation is very involved as a first effort as well as for simplicity, we obtained the rate up to first order in the magnetic field, i.e., $O[(eB)]$, which causes a marginal improvement over that in absence of magnetic field.

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I. Introduction

Heavy Ion Collisions (HIC) experiments being conducted at Large Hadron Collider (LHC) in CERN and Relativistic Heavy Ion Collider (RHIC) in BNL have ample evidence of production of deconfined QCD matter at extreme conditions of high temperature and density, which is commonly termed as Quark Gluon plasma (QGP). This short-lived deconfined state of QCD matter has been the subject of intense investigation over the past few decades.

In non-central HIC, extremely strong magnetic field of the order of QCD scale is believed to have generated due to the presence of so-called spectator particles which do not participate in the interaction [1, 2]. The presence of external magnetic field is responsible for a bunch of exotic phenomena like chiral magnetic effects [3–5], inverse magnetic catalysis [6, 7], magnetic catalysis [8, 9], superconductivity in the vacuum [10] and many more. Also the thermodynamic properties of hot magnetized deconfined QCD medium has been studied [11–13]. The strength of this magnetic field is very high compared to the temperature scale of the system ($|q_f B| \gg T^2$), where $q_f$ is the charge of quark and $m_\pi$ is the mass of pion. The strength of such field is estimated to be $15m_\pi^2$, but it decays very rapidly with time. So it becomes extremely difficult to analyse the case with arbitrary magnetic field. For the sake of theoretical simplicity, one works in extreme limits of a strong and a weak field regime. Apart from the temperature scale associated with the heat bath, the introduction of background magnetic field invokes another scale into the system. The strong and weak field regimes are recognised by the scale $|q_f B| \gg T^2 \gg m_f^2$ and $T^2 \gg m_f^2 \gg |q_f B|$, respectively.

QGP is a many particle system that shows collectivity and most of its evidences are circumstantial. So, the direct detection of QGP medium is not possible mainly due to two reasons. The first one is the fact that it exists for a very short time and the second one is the color confinement. Thus, one needs to rely on the direct probes like electromagnetic probes, viz., photon and dilepton, and indirect hard probes like bound states of heavy quarks, jets, collective flows etc [14] to extract its properties. One of the most popular theoretical tools is n-points current-current correlations functions that can be related to the photon and dilepton production. The thermal dileptons are considered to be an excellent probe of the QGP medium. The reason is that it interacts only electromagnetically with the medium and leaves the medium without any final state interaction due to its large mean free path. Also it is produced throughout the entire volume of space-time and almost all stages of HIC. But there exists various sources of these emitted dileptons during the evolution of the created fireball. It includes dileptons from Drell-Yan processes [15], bremsstrahlung and absorption of jets by plasma [16], and in the QGP phase. The important parameter used to characterise emitted dilepton spectrum is its invariant mass ($M$) that can be broadly divided in three distinct ranges namely low with $M < M_\phi (= 1.024 \text{GeV})$, intermediate with $M_\phi < M < M_{J/\psi} (= 3.1 \text{GeV})$ and high ($M > M_{J/\psi}$). The intermediate mass range is important for getting QGP signature and in this region the radiation from QGP dominates the mass spectrum [17].

The theoretical calculations of the production rate of dilepton in many different scenario of high temperature and finite chemical potential [18] proceed through the imaginary part of the two point correlation function of photon [19, 20]. One of the earliest seminal works in the framework of Hard Thermal Loop perturbation theory (HTLpt) can be found in [21]. It calculates the rate of production of soft dilepton (lepton pair with momentum scale of the order of $qT$) using the resummed one loop quark propagators and effective vertices. An extensive investigation has also been carried out for small invariant mass in both LO and higher order in early literatures [22–25]. In Ref. [24] the low invariant mass ($M << 1 \text{GeV}$) thermal dilepton rates have been investigated from deconfined QCD phase using perturbative and non-perturbative method. As noted earlier, owing to presence of external magnetic field in non-central heavy ion collision, there are enough motivations to investigate the behaviour of electromagnetic probes under the influence of a background magnetic field [26–28]. Recently there has been some detailed investigation of dilepton rate from one loop photon polarization tensor. In Ref. [29] the production rate of dilepton has been computed using Ritus Eigenfunction method [30]. On the other hand Refs [31, 32] have investigated dilepton production in hot magnetized medium using weak [33] and strong field approximation of quark propagator [8] whereas Ref. [34] has calculated it using the full form of the Schwinger propagator. It has also been computed using effective QCD model in presence of an external magnetic field [35].

The straightforward extension to the case where hard dileptons are considered can be found in Ref. [36]. In this calculation it has been argued that it is sufficient to consider one resummed propagator (i.e., soft) and one hard propagator in one loop photon self energy. The reason is that since the momentum flowing through the external photon line is hard, one of the quark propagator, that must have hard momentum flowing through it, can be taken as bare. But for the other propagator, one should take resummed (i.e., soft) propagator. In this paper we follow the same line in which we use one magnetic field dependent free propagator and one thermomagnetic resummed propagator for
obtaining the hard dilepton rate from a weakly magnetized deconfined QCD medium.\footnote{For having soft dilepton one can use both propagators resummed and effective vertices but the calculation will be extremely involved and complicated. However, as a first effort and also for simplicity, we consider one magnetic field dependent free propagator and one resummed propagator in one loop photon self energy, which itself is an indeed very involved calculation as we will see below.}

The paper is organized as follows. In section II, we have briefly outlined the notation used and also the quark propagator in presence of a weak background field. The dispersion properties of a resummed quark propagator and its spectral density in presence of weakly magnetized hot medium are discussed in section II A. The calculation of dilepton production and results are given in details in section III at zero magnetic field (in sub-section III A) and at weak magnetic field in thermalized background (in sub-section III B). Finally, we conclude in section IV.

II. Notations and Charged Fermion Propagator in Background Magnetic Field within Schwinger Formalism

We begin by defining the following notation for the four vector and the metric tensor:

\[
a^\mu = (a^0, a^1, a^2, a^3), \quad g^{\mu\nu} = \text{diag}(1, -1, -1, -1),
\]

\[
g_0^{\mu\nu} = \text{diag}(1, 0, 0, -1), \quad g_0^{\mu\nu} = \text{diag}(0, -1, -1, 0),
\]

\[
g^{\mu\nu} = g_0^{\mu\nu} + g_0^{\mu\nu}, \quad \phi = \gamma^\mu a_\mu,
\]

\[
a_i = 0^i a^0 - \gamma^3 a^3, \quad a_\perp = (\gamma \cdot a)_\perp = \gamma^1 a^1 + \gamma^2 a^2,
\]

\[
a_\mu = g^{\mu\nu} a^\nu, \quad a_\mu = -g^{\mu\nu} a^\nu.
\]

The Green’s function satisfying the Dirac equation in presence of a magnetic field can be written as

\[
i\partial - Q q_f A_{ext}(x) - m_f)G(x, x') = \delta^{(4)}(x - x'),
\]

where \(A_{ext}\) is the vector potential for an external background magnetic field, \(Q\) is \(\text{sgn}(q_f B)\) and \(q_f\) is the absolute value of the particle’s charge and \(m_f\) is the mass of a particle. This equation can be solved by different methods, as for example, Schwinger’s proper time method \([37]\), Ritus Eigenfunction method \([30]\), Furry’s picture \([38]\) for the case of a constant field pointing in the z direction. The Green’s function in equation (1) can be written as

\[
G(x, x') = \Phi(x, x') \int \frac{d^4 p}{(2\pi)^4} \exp(-ip.x)\tilde{G}(p),
\]

where the prefactor \(\Phi(x, x')\) is a phase factor that breaks both translational and gauge invariance. However, it can be taken as unity by choosing the symmetric gauge of vector potential, \(i.e., A^\alpha_{ext} = \frac{B}{2}(0, y, -x, 0)\). So the momentum space Green’s function vis-a-vis the propagator is written as \([8, 33]\)

\[
\tilde{G}(p) = \exp\left(-\frac{k^2}{q_f B}\right)\sum_{n=0}^{\infty} \frac{D_n(q_f B, K)}{k_0^2 - 2nq_f B - (k_\perp)^2 - m_f^2},
\]

where

\[
D_n(q_f B, K) = (k_\perp + m_f) \left[\left(1 - \text{isgn}(q_f B)\gamma^1\gamma^2\right)L_n\left(\frac{k_\perp^2}{q_f B}\right) - \left(1 + \text{isgn}(q_f B)\gamma^1\gamma^2\right)L_{n-1}\left(\frac{k_\perp^2}{q_f B}\right)\right]
\]

\[
+ 4k_\perp L_1^{(1)}\left(2\frac{k_\perp^2}{q_f B}\right),
\]

where \(L_n^{(1)}(x)\) is the generalized Laguerre polynomial. As stated earlier, we are interested in the domain \(T^2 \gg m_f^2 \gg q_f B\). In this domain, one can approximate the propagator by expanding the sum over \(n\) in equation (3) in power of \(q_f B\) to obtain a simplified form \([33]\) as

\[
S_F(K) = \frac{K + m_f}{K^2 - m_f^2} + i\gamma^2 \frac{1}{(K^2 - m_f^2)^{1/2}} q_f B + \mathcal{O}[(q_f B)^2]
\]

\[
= S_F^{(0)}(K) + S_F^{(1)}(K) + \mathcal{O}[(q_f B)^2],
\]

where \(S_F^{(0)}\) is \(\mathcal{O}[(q_f B)^0]\) and \(S_F^{(1)}\) is \(\mathcal{O}[(q_f B)^1]\) part of the propagator \(S_F\).
A. Dispersion of fermionic modes and Spectral representation

The dispersion behaviour of the resummed fermionic propagator in presence of weak magnetic field is discussed in our earlier work [39]. Here in this section, we shall briefly outline some important results for the sake of clarity that would be useful here. The effective propagator is given by

\[
S^*(K) = \mathcal{P} \cdot \frac{\mathcal{L}(K)}{L(K)^2} \mathcal{P}_+ + K^\perp \frac{\mathcal{R}(K)}{R(K)^2} \mathcal{P}_- ,
\]

with four momentum \(K \equiv (k_0, \mathbf{k}) = (k_0, k_\perp, k_z)\) with \(|\mathbf{k}| \equiv k = \sqrt{k_\perp^2 + k_z^2}\) and the chirality projection operators are given as

\[
\mathcal{P}_\pm = \frac{1}{2} (1 \pm \gamma_5) .
\]

\(\mathcal{L}\) and \(\mathcal{R}\) that appear in Eq. (6) can be written in the rest frame of the heat bath along with the magnetic field in the \(z\)-direction as

\[
\mathcal{L} = [(1 + a(k_0, k))k_0 + b(k_0, k) + b'(k_0, k_\perp, k_z)] \gamma^0 - [(1 + a(k_0, k))k_z + c'(k_0, k_\perp, k_z)] \gamma^3 - (1 + a(k_0, k)) (\gamma \cdot \mathbf{k})_\perp
\]

\[
= g_L^1(k_0, k_\perp, k_z) \gamma^0 - g_L^2(k_0, k_\perp, k_z) (\gamma \cdot \mathbf{k})_\perp - g_L^3(k_0, k_\perp, k_z) \gamma^3 ,
\]

\[
\mathcal{R} = [(1 + a(k_0, k))k_0 + b(k_0, k) - b'(k_0, k_\perp, k_z)] \gamma^0 - [(1 + a(k_0, k))k_z - c'(k_0, k_\perp, k_z)] \gamma^3 - (1 + a(p_0, p)) (\gamma \cdot \mathbf{k})_\perp
\]

\[
= g_R^1(k_0, k_\perp, k_z) \gamma^0 - g_R^2(k_0, k_\perp, k_z) (\gamma \cdot \mathbf{k})_\perp + g_R^3(k_0, k_\perp, k_z) \gamma^3 ,
\]

where \(\mathbf{k} = k/k\). Although, \(g_L^2 = g_R^2\) and \(g_L^3 = g_R^3\), but for the sake of convenience, they are treated separately as \(g_L^i\) and \(g_R^i\).

The pole of the effective fermion propagator \(S^*(K)\) in weak magnetized media gives the dispersion relation of fermionic mode. The dispersion equations are given by

\[
L^2 = L_+ L_- = 0 , \quad R^2 = R_+ R_- = 0 ,
\]

where \(L_+\) and \(R_+\) are, respectively, given by

\[
L_\pm (k_0, k_\perp, k_z) = (1 + a)k_0 + b + b' \mp \left[ (1 + a)k_z + c' \right]^2 + (1 + a)^2 k_z^2 \right]^{1/2} ,
\]

\[
R_\pm (k_0, k_\perp, k_z) = (1 + a)k_0 + b - b' \mp \left[ (1 + a)k_z - c' \right]^2 + (1 + a)^2 k_z^2 \right]^{1/2} .
\]

The form of the structure functions [39] are quoted here as

\[
a = -\frac{m_i^2}{k^2} Q_1 \left( \frac{k_0}{k} \right) ,
\]

\[
b = \frac{m_i^2}{k} \left[ \frac{k_0}{k} Q_1 \left( \frac{k_0}{k} \right) - Q_0 \left( \frac{k_0}{k} \right) \right] ,
\]

\[
b' = 4C_F g^2 M^2(T, m_f, q_f B) \frac{k_z}{k^2} Q_1 \left( \frac{k_0}{k} \right) ,
\]

\[
c' = 4C_F g^2 M^2(T, m_f, q_f B) \frac{1}{k} Q_0 \left( \frac{k_0}{k} \right) ,
\]

along with the thermomagnetic mass is given as [40, 41]

\[
M^2(T, m_f, q_f B) = \frac{q_f B}{16 \pi^2} \left[ \ln(2) - \frac{T}{m_f \pi} \right] ,
\]

\[
\text{with } m_i = \frac{\sqrt{m^2 + (m_f q_f B)^2}}{2} .
\]
and also the thermal mass is given as

\[ m_{th}^2 = \frac{1}{8} C_T g^2 T^2. \] (17)

FIG. 1. This displays the various u-quark dispersion modes. The free dispersion of hard quark q with energy \( \omega = \sqrt{p_z^2 + p_\perp^2} \) with \( p_\perp = m_\pi/2 \) (left panel) and \( m_\pi \) (right panel).

The dispersion solutions [39] are noted below as a function of \( p_\perp \) and \( p_z \) as

\[ L_+ = 0 \implies p_0 = (\omega_{L(+)}, -\omega_{R(-)}), \] (18)
\[ L_- = 0 \implies p_0 = (\omega_{L(-)}, -\omega_{R(+)}), \] (19)
\[ R_+ = 0 \implies p_0 = (\omega_{R(+)}, -\omega_{L(-)}), \] (20)
\[ R_- = 0 \implies p_0 = (\omega_{R(-)}, -\omega_{L(+)}). \] (21)

The corresponding dispersion of various quark modes \( q_{L(+)}, q_{L(-)}, q_{R(+)} \) and \( q_{R(-)} \) with respective frequencies \( \omega_{L(+)}, \omega_{L(-)}, \omega_{R(+)} \) and \( \omega_{R(-)} \) are displayed in Fig. 1. The free dispersion of hard quark \( q \) with energy \( \omega = \sqrt{p_z^2 + p_\perp^2} \) is also displayed. It is clear from Fig. 1 that the processes that we expect will involve one hard and one soft quark since we are using one free (hard) quark propagator in presence of magnetic field and one resummed thermomagnetic quark (soft) propagator in Fig. 2. Now, one can write the various dilepton production processes from the dispersion plot as \( qq_{L(+)} \rightarrow \gamma^* \rightarrow l^+l^- \), \( qq_{L(-)} \rightarrow \gamma^* \rightarrow l^+l^- \), \( qq_{R(+)} \rightarrow \gamma^* \rightarrow l^+l^- \) and \( qq_{R(-)} \rightarrow \gamma^* \rightarrow l^+l^- \). There could also be soft decay processes like \( q_{L(+)} \rightarrow q\gamma^* \rightarrow ql^+l^- \), \( q_{L(-)} \rightarrow q\gamma^* \rightarrow ql^+l^- \), \( q_{R(+)} \rightarrow q\gamma^* \rightarrow ql^+l^- \) and \( q_{R(-)} \rightarrow q\gamma^* \rightarrow ql^+l^- \). We will see below that all of them may not be allowed due to kinematical restrictions. Also beside these processes there will be soft processes from Landau cut contributions. We will discuss these contributions in details later.

B. Spectral function of quark propagator

For computation of dilepton rate, the spectral function of quark propagator is needed. The spectral representation of the effective quark propagator in a hot magnetized medium is obtained in [39]. We briefly outline them here.

Now, the effective propagator in Eq. (6) can be decomposed into six parts by separating out the \( \gamma \) matrices as

\[ S^*(k_0, k_\perp, k_z) = \mathcal{P}_- \gamma^0 \mathcal{P}_+ \frac{g^1_L(k_0, k_\perp, k_z)}{L^2} - \mathcal{P}_-(\gamma \cdot \hat{k}) \mathcal{P}_+ \frac{g^2_L(k_0, k_\perp, k_z)}{L^2} - \mathcal{P}_- \gamma^3 \mathcal{P}_+ \frac{g^3_L(k_0, k_\perp, k_z)}{L^2} \]
\[ + \mathcal{P}_+ \gamma^0 \mathcal{P}_- \frac{g_{K}^i(k_0, k_{\perp}, k_{\parallel})}{R^2} - \mathcal{P}_+ (\gamma \cdot \hat{k}) \mathcal{P}_- \frac{g_{\bar{K}}^i(k_0, k_{\perp}, k_{\parallel})}{R^2} + \mathcal{P}_+ \gamma^3 \mathcal{P}_- \frac{g_{\bar{K}}^i(k_0, k_{\perp}, k_{\parallel})}{R^2}. \]  

(22)

It is discussed earlier that \( L^2 = 0 \) yields four poles, giving four modes with positive and negative energy, \( \omega_{L(\pm)}(k_{\perp}, k_{\parallel}) \) and \( -\omega_{R(\pm)}(k_{\perp}, k_{\parallel}) \), as given in Eqs. (18) and (19). Similarly, \( R^2 = 0 \) also gives four poles, namely \( \omega_{R(\pm)}(k_{\perp}, k_{\parallel}) \) and \( -\omega_{L(\pm)}(k_{\perp}, k_{\parallel}) \), as given in Eqs. (20) and (21). With this information, the spectral representation \([21, 39, 42–44]\) is obtained for the effective propagator in Eq. (22) as

\[
\rho = (\mathcal{P}_- \gamma^0 \mathcal{P}_+) \rho^1_L - (\mathcal{P}_- (\gamma \cdot \hat{k}) \mathcal{P}_+) \rho^2_L - (\mathcal{P}_- \gamma^3 \mathcal{P}_+) \rho^3_L + (\mathcal{P}_+ \gamma^0 \mathcal{P}_-) \rho^1_R - (\mathcal{P}_+ (\gamma \cdot \hat{k}) \mathcal{P}_-) \rho^2_R + (\mathcal{P}_+ \gamma^3 \mathcal{P}_-) \rho^3_R,
\]

(23)

where the spectral function corresponding to each of the term can be written as

\[
\rho^i_L = \frac{1}{\pi} \text{Im} \left( \frac{g_{L}^i}{L^2} \right) = \frac{1}{\pi} \text{Im} \left( F_{L}^i \right)
\]

\[
= Z_{L(\pm)}^i(k_{\perp}, k_{\parallel}) \delta(k_0 - \omega_{L(\pm)}(k_{\perp}, k_{\parallel})) + Z_{L(\mp)}^i(k_{\perp}, k_{\parallel}) \delta(k_0 - \omega_{L(\mp)}(k_{\perp}, k_{\parallel})) + Z_{L(0)}^i(k_{\perp}, k_{\parallel}) \delta(k_0 + \omega_{L(0)}(k_{\perp}, k_{\parallel})) + \beta_{L}^i,
\]

(24)

\[
\rho^i_R = \frac{1}{\pi} \text{Im} \left( \frac{g_{R}^i}{R^2} \right) = \frac{1}{\pi} \text{Im} \left( F_{R}^i \right)
\]

\[
= Z_{R(\mp)}^i(k_{\perp}, k_{\parallel}) \delta(k_0 - \omega_{R(\mp)}(k_{\perp}, k_{\parallel})) + Z_{R(\pm)}^i(k_{\perp}, k_{\parallel}) \delta(k_0 - \omega_{R(\pm)}(k_{\perp}, k_{\parallel})) + Z_{R(0)}^i(k_{\perp}, k_{\parallel}) \delta(k_0 + \omega_{R(0)}(k_{\perp}, k_{\parallel})) + \beta_{R}^i,
\]

(25)

where \( i = 1, 2, 3 \). The delta-functions are originated from the time like domain \( (k_0^2 > k^2) \) whereas the cut parts \( \beta_{L(R)}^i \) are involved with the Landau damping originating from the space-like domain \( (k_0^2 < k^2) \) of the propagator. The residues \( Z_{L(R)} \) are determined at the various poles as

\[
Z_{L(R)}^i \text{sgn of pole } (k_{\perp}, k_{\parallel}) = g_{L(R)}^i(k_0, k_{\perp}, k_{\parallel}) \left. \frac{\partial L^2(R^2)}{\partial k_0} \right|_{k_0 = \text{pole}}^{-1},
\]

(26)

where the expressions of residues can be written \([39]\) in terms of the structure coefficient \( a, b, b' & c' \) and their derivatives.

### III. Dilepton Production

![Feynman Diagram for the production of hard dilepton in presence of weak background magnetic field](image)

FIG. 2. Feynman Diagram for the production of hard dilepton in presence of weak background magnetic field

The differential dilepton production can be written as \([21, 24]\)

\[
\frac{dR}{d^4 x d^4 p} = \frac{\alpha}{12\pi^6} \frac{1}{P^2} \rho_0 - 1 \text{Im} \Pi^\mu_{\mu}(p_0 + i\epsilon, \vec{p}),
\]

(27)

with \( P^2 = p_0^2 - p^2 = M^2 \) where \( M \) is the invariant mass of the dilepton. Now, for simplification we will consider the case with \( p = 0 \).
The expression for one loop self energy can be obtained from the Feynman diagram as

$$\Pi^{\mu\nu}(P) = -N_c e^2 \sum_f \left( \frac{q_f}{e} \right)^2 \sum_K \text{Tr} \left[ \gamma^\mu S_0(K) \gamma^\nu S^*(Q) \right] ,$$  

(28)

where \( N_c = 3 \) is color factor and \( Q \equiv K - P \). In imaginary time formalism the loop integral can be written as

$$\int \frac{d^4 K}{(2\pi)^4} = \sum_K \frac{T \sum k_0}{(2\pi)^3} ,$$  

(29)

A. Dilepton rate at vanishing magnetic field

![Dilepton dispersion relation](image)

**FIG. 3.** Soft (HTL) and hard (free) quark dispersion relation. \( q_+ \) and \( q_- \) are soft quarks coming from HTL resummed propagator and hard quark \( q \) coming from free propagator.

In this section, we first discuss about the dilepton production rate without any external magnetic field. For the purpose, we use one hard quark propagator and HTL resummed soft quark propagator with two modes \[42\] : one quasiquark mode \( q_+ \) with energy \( \omega_+ \) and other one is plasmino mode \( q_- \) with energy \( \omega_- \). The free hard quark is represented by \( q \) with energy \( k \). The corresponding dispersion is shown in Fig. 3. Now, in this case the allowed dilepton production processes coming from pole-pole part are annihilation processes \( qq_+ \rightarrow \gamma^* \rightarrow l^+l^- \) and soft decay process \( q_- \rightarrow q\gamma^* \rightarrow ql^+l^- \). There will also be other processes which are not allowed by energy conservation and kinematical restriction with the photon momentum, \( p = 0 \). In addition there will also be pole-cut contributions as will be discussed below in details. We also note that there is no cut-cut contribution as the spectral function for hard propagator has only pole contribution. Now the one loop photon self energy \( \Pi_\mu^{\nu} \) with one hard propagator \( S_0 \) and one resummed HTL propagator \( S_{\text{HTL}} \) can be written as

$$\Pi_\mu^{\nu} = -N_c e^2 \sum_f \left( \frac{q_f}{e} \right)^2 \sum_K \text{Tr} \left[ \gamma^\mu S_0(K) \gamma^\nu S_{\text{HTL}}(Q) \right]$$

$$= 2N_c e^2 \sum_f \left( \frac{q_f}{e} \right)^2 \sum_k \frac{d_\pm(k_0, q)}{(2\pi)^3} \left[ \frac{1}{D_+(k)} \left( \frac{1}{d_+(q)} + \frac{1}{d_-(-q)} \right) + \frac{1}{D_-(k)} \left( \frac{1}{d_+(q)} + \frac{1}{d_-(-q)} \right) \right] ,$$  

(30)

with

$$d_\pm(q_0, q) = q_0 - q$$

$$D_\pm(k_0, k) = k_0 \mp k - \frac{m_\text{th}^2}{2k} \left[ 1 \mp \frac{k_0}{k} \right] \log \frac{k_0 + k}{k_0 - k} \pm 2 ,$$  

(31)

$$D_\pm(k_0, k) = k_0 \mp k - \frac{m_\text{th}^2}{2k} \left[ 1 \mp \frac{k_0}{k} \right] \log \frac{k_0 + k}{k_0 - k} \pm 2 ,$$  

(32)
Now the imaginary part of Eqn. (30) is obtained as

\[ \text{Im} \Pi_{\mu} = 2N_c e^2 T \sum_f \left( \frac{q_f}{e} \right)^2 \left( e^{E/T} - 1 \right) \]

\[ \times \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega' \delta(E - \omega - \omega') n_F(\omega) n_F(\omega') \pi \left[ (1 - \hat{k} \cdot \hat{q}) (\rho_{+r_-} + \rho_{-r_+}) \right. \]

\[ + (1 + \hat{k} \cdot \hat{q}) (\rho_{+r_+} + \rho_{-r_-}) \],

which at \( p = 0 \) reads as

\[ \text{Im} \Pi_{\mu} = 2N_c e^2 T \pi \sum_f \left( \frac{q_f}{e} \right)^2 \left( e^{E/T} - 1 \right) \]

\[ \times \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega' \delta(E - \omega - \omega') n_F(\omega) n_F(\omega') 2(\rho_{+r_+} + \rho_{-r_-}). \] (34)

The spectral representation of soft and hard propagator read \([42]\), respectively, as

\[ \rho_\pm(\omega, k) = \frac{\omega^2 - k^2}{2m_{th}^2} \left[ \delta(\omega - \omega_\pm(k)) + \delta(\omega + \omega_\mp(k)) \right] + \beta_\pm(\omega, k) \Theta(k^2 - \omega^2), \] (35)

\[ r_\pm(\omega', k) = \delta(\omega' \mp k), \] (36)

with

\[ \beta_\pm(x, y) = \frac{1}{2} \left[ \frac{\beta}{x} \right] - \frac{1}{2} \left[ \frac{\beta}{y} \right] + \log \left[ \frac{x + y}{x - y} \right] \left[ \frac{1}{2} \right] + \left[ \frac{1}{2} \right] \] (37)

where \( x = \omega/m_{th} \) and \( y = k/m_{th} \). The soft spectral function contains pole part coming from the poles of the HTL propagator and Landau cut contribution from space like domain, \( k^2 < \omega^2 \), of the HTL propagator. The hard spectral function has only pole part. So, there will be total four energy conserving \( \delta \)-functions from the pole-pole part as \( \delta(E + \omega_+ + k) \), \( \delta(E - \omega_+ - k) \), \( \delta(E - \omega_- + k) \) and \( \delta(E - \omega_- - k) \). But two processes \( qq_+ \gamma^* \rightarrow \bar{q}_+l^+l^- \) coming respectively from \( \delta(E + \omega_+ + k) \) and \( \delta(E + \omega_- - k) \) are not allowed by the energy conservation. The remaining two allowed processes coming from \( \delta(E - \omega_+ - k) \) and \( \delta(E - \omega_- + k) \) lead to the respective processes \( qq_+ \rightarrow \gamma^* \rightarrow l^+l^- \) and \( q_- \rightarrow q\gamma^* \rightarrow ql^+l^- \) as discussed earlier. The resulting pole-pole part of the dilepton rate is

\[ \left. \frac{dR}{dx^2 dp} \right|_{\text{pole-pole}} = \frac{\alpha}{12\pi^2 E^2 e^{\beta E} - 1} 12\pi e^2 \sum_f \left( \frac{q_f}{e} \right)^2 \left( e^{E/T} - 1 \right) \int \frac{d^3k}{(2\pi)^3} \left[ \frac{\omega_+^2 - k^2}{2m_{th}^2} n_F(\omega_+) n_F(k) \delta(E - \omega_+ - k) + \frac{\omega_-^2 - k^2}{2m_{th}^2} n_F(\omega_-) n_F(-k) \delta(E - \omega_- + k) \right] \]

\[ = \frac{2\alpha}{\pi^2 E^2} \sum_f \left( \frac{q_f}{e} \right)^2 \int k^2 dk \]

\[ \times \left[ \frac{\omega_+^2 - k^2}{2m_{th}^2} n_F(\omega_+) n_F(k) \delta(E - \omega_+ - k) + \frac{\omega_-^2 - k^2}{2m_{th}^2} n_F(\omega_-) n_F(-k) \delta(E - \omega_- + k) \right]. \] (38)

Scaling \( \omega_\pm, k \) with \( m_{th} \) as \( x_\pm = \omega_\pm/m_{th}, E_s = E/m_{th} \) and we get

\[ \left. \frac{dR}{dx^2 dp} \right|_{\text{pole-pole}} = \frac{\alpha^2}{\pi^2 E^2} \sum_f \left( \frac{q_f}{e} \right)^2 \int y^2 dy \left[ \frac{1}{e^{\beta m_{th} x_+} + 1} + \frac{1}{e^{\beta m_{th} y} + 1} \right] \delta(E_s - x_+ - y) \]

\[ + \frac{1}{e^{\beta m_{th} x_-} + 1} + \frac{1}{e^{\beta m_{th} y} + 1} \delta(E_s - x_- + y). \] (39)

Now, the pole-cut part of the rate is obtained as

\[ \left. \frac{dR}{dx^2 dp} \right|_{\text{pole-cut}} = \frac{\alpha}{12\pi^2 E^2 e^{\beta E} - 1} 12\pi e^2 \sum_f \left( \frac{q_f}{e} \right)^2 \left( e^{E/T} - 1 \right) \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} d\omega \]
\[ \times [\beta_+(\omega, k)n_F(\omega)n_F(k)\delta(E - \omega - k) + \beta_-(\omega, k)n_F(\omega)n_F(-k)\delta(E - \omega + k)] \]

\[ = \frac{2\alpha^2}{\pi^4 E_s^2} \sum_f \left( \frac{q_f}{\epsilon} \right)^2 \int y^2 dy \int_{-y}^y dx \]

\[ \times [\beta_+(x, y)n_F(x)n_F(y)\delta(E_s - x - y) + \beta_-(x, y)n_F(x)n_F(-y)\delta(E_s - x + y)]. \]  

(40)

We note that the second term of the pole-cut rate will vanish as the delta function gives the condition \( x = E_s + y \) which lies out of the domain \(-y \leq x \leq y\) and the pole-cut contribution becomes

\[ \left. \frac{dR}{d^4xd^4p} \right|_{\text{pole-cut}} = \frac{2\alpha^2}{\pi^4 E_s^2} \sum_f \left( \frac{q_f}{\epsilon} \right)^2 \int y^2 dy \beta_+(E_s - y, y)n_F(E_s - y)n_F(y)\Theta(2y - E_s). \]  

(41)

It is worth to write the Born rate [24] as

\[ \frac{dR}{d^4xd^4p}\bigg|_{\text{born}} = \sum_f \left( \frac{q_f}{\epsilon} \right)^2 \frac{\alpha^2}{4\pi^2 n_F^2(E/2)}. \]  

(42)

In Fig. 4 we displayed the dilepton rate in absence of magnetic field. For \( E = 0 \) the dilepton rate begins with the transition process \( q_- \rightarrow q\gamma^* \rightarrow ql^+l^- \). This rate begins with a divergence as all plasmino, \( q_- \), modes with higher energy (Fig. 3) prefer to make transition to free quark mode with lower energy and thus density of states diverges. However, this rate decays very fast because the plasmino mode \( q_- \) is exponentially suppressed and merges with the free hard quark mode as shown in Fig. 3. Then the annihilation of one soft (\( q_+ \)) and one hard (\( q \)) mode, \( qq_+ \rightarrow \gamma^* \rightarrow l^+l^- \), begins when \( E = m_{th} \) (as the mass of the hard mode is zero). It then grows with \( E \) and matches with the Bonn rate at large \( E \). The dilepton rate coming from pole-cut part dominates at low \( E \) and falls off below the Bonn rate at large \( E \). The net rate dominates the Bonn rate at low energy.

\section*{B. Dilepton rate at finite magnetic field}

In this section, we shall investigate dilepton production in presence of weak homogeneous background magnetic field. We are concerned about dilepton whose momenta are of the order of \( T \), i.e., \( p_0, p \sim T \). In that case, as discussed, we need to dress just one quark propagator [36] as in figure 2. The bare propagator in weak magnetic field approximation is given in equation (5). The dressed propagator is given in Eq. (6) which, for convenience, is decomposed into two parts as

\[ S^*(K) = S_L^*(K) + S_R^*(K), \]  

(43)
where
\[ S^+_L(K) = \mathcal{P}_- \frac{L}{L^2} \mathcal{P}_+, \quad S^+_R(K) = \mathcal{P}_+ \frac{R}{R^2} \mathcal{P}_-. \] (44)

Now using Eqs. (5) and (43), the one loop photon polarisation tensor in Eq. (28) corresponding to Fig. 2 can be obtained as
\[
\Pi^\mu_\nu(p_0, p) = -N_c e^2 \sum_f \left( \frac{g_f}{e} \right)^2 \sum_K \text{Tr} \left[ \gamma^\mu S^+(K) \gamma_\nu S_F(Q) \right]
\]
\[
= -N_c e^2 \sum_f \left( \frac{g_f}{e} \right)^2 \sum_K \text{Tr} \left[ \gamma^\mu S^+_L(K) \gamma_\nu S^{(0)}_F(Q) \right] - N_c e^2 \sum_f \left( \frac{g_f}{e} \right)^2 \sum_K \text{Tr} \left[ \gamma^\mu S^+_R(K) \gamma_\nu S^{(0)}_F(Q) \right]
\]
\[
- N_c e^2 \sum_f \left( \frac{g_f}{e} \right)^2 \sum_K \text{Tr} \left[ \gamma^\mu S^+_L(K) \gamma_\nu S^{(1)}_F(Q) \right] - N_c e^2 \sum_f \left( \frac{g_f}{e} \right)^2 \sum_K \text{Tr} \left[ \gamma^\mu S^+_R(K) \gamma_\nu S^{(1)}_F(Q) \right]. \] (45)

The result of the Dirac trace is
\[
\text{Tr} \left[ \gamma^\mu S^+(K) \gamma_\nu S_F(Q) \right] = -4 \left[ \frac{L^\mu Q_\nu}{L^2(Q^2 - m_f^2)} + \frac{R^\mu Q_\nu}{R^2(Q^2 - m_f^2)} + q_f B \left\{ \frac{Q^0 L^3 - Q^3 L^0}{L^2(Q^2 - m_f^2)^2} - \frac{Q^0 R^3 - Q^3 R^0}{R^2(Q^2 - m_f^2)^2} \right\} \right], \] (46)

where \( m_f \) is the current quark mass. The components of \( L^\mu = (L^0, L^1, L^2, L^3) \) and \( R^\mu = (R^0, R^1, R^2, R^3) \) are given by
\[
L^0 = [1 + a(k_0, k)] k_0 + b(k_0, k) + b'(k_0, k_\perp, k_z),
\]
\[
L^i = [1 + a(k_0, k)] k_i; \quad i = 1, 2
\]
\[
L^3 = [1 + a(k_0, k)] k_z + c'(k_0, k),
\]
\[
R^0 = [1 + a(k_0, k)] k_0 + b(k_0, k) - b'(k_0, k_\perp, k_z),
\]
\[
R^i = (1 + a(k_0, k)) k_i; \quad i = 1, 2
\]
\[
R^3 = (1 + a(k_0, k)) k_z - c'(k_0, k). \] (47)

Now Eq. (47) can be expressed in terms of \( g^4_{L,R}(i = 1, 2, 3) \) as
\[
L^0 = g^4_L(k_0, k_\perp, k_z),
\]
\[
L^i = g^4_L(k_0, k) k_i; \quad i = 1, 2
\]
\[
L^3 = g^4_L(k_0, k) k^3 + g^4_L(k_0, k),
\]
\[
R^0 = g^4_R(k_0, k_\perp, k_z),
\]
\[
R^i = g^4_R(k_0, k) k_i; \quad i = 1, 2
\]
\[
R^3 = g^4_R(k_0, k) k_z - g^4_R(k_0, k). \] (48)

As discussed in the previous subsection we would investigate the case when the virtual photon is at the rest in the plasma rest frame, i.e., \( p = 0, \ P^\mu = (p_0, 0) \). In this case \( Q^\mu = K^\mu - P^\mu = (k_0 - p_0, k) \). Thus Eq. (45) becomes
\[
\Pi^\mu_\nu(p_0, 0) = 12 e^2 \sum_f \left( \frac{g_f}{e} \right)^2 \sum_K \left[ \frac{L_0(k_0 - p_0) - L \cdot k}{L^2[(k_0 - p_0)^2 - \omega_k^2]} + \frac{R_0(k_0 - p_0) - R \cdot k}{R^2[(k_0 - p_0)^2 - \omega_k^2]} \right]
\]
\[
+ q_f B \left\{ \frac{L_z(k_0 - p_0) - k_z L_0}{L^2[(k_0 - p_0)^2 - \omega_k^2]} - \frac{R_z(k_0 - p_0) - k_z R_0}{R^2[(k_0 - p_0)^2 - \omega_k^2]} \right\}
\]
\[
= 12 e^2 \sum_f \left( \frac{g_f}{e} \right)^2 \sum_K \left[ \frac{(k_0 - p_0)g^4_L - g^4_L k_z^2 - g^4_L k_z^2}{L^2[(k_0 - p_0)^2 - \omega_k^2]} \right]
\]
\[
+ q_f B \frac{k_x g^4_L - (k_0 - p_0)(k_z g^4_L + g^4_L)}{L^2[(k_0 - p_0)^2 - \omega_k^2]} - q_f B \frac{k_y g^4_R - (k_0 - p_0)(k_z g^4_R + g^4_R)}{R^2[(k_0 - p_0)^2 - \omega_k^2]} \right] \]
We take the imaginary part of equation (49) with a decomposition as

\[ \text{Im} \Pi = \sum_{f} \left( \frac{q_{f}}{e} \right)^{2} \int \frac{d^{3}k}{(2\pi)^{3}} \text{Im} T \sum_{k_{0}} \left[ F_{L}^{1}(k_{0}, k_{+}, k_{z}) + F_{R}^{1}(k_{0}, k_{+}, k_{z}) \right] f_{0}^{(0)}(k_{0} - p_{0}, k) \]

\[ + q_{f} B \left\{ k_{z} \left[ (k_{0} - p_{0})^{2} - \omega_{k}^{2} \right] F_{R}^{1} - \hat{k}_{z} \left[ (k_{0} - p_{0})^{2} - \omega_{k}^{2} \right] F_{L}^{1} - \frac{k_{0} - p_{0}}{(k_{0} - p_{0})^{2} - \omega_{k}^{2}} - \frac{k_{0} - p_{0}}{(k_{0} - p_{0})^{2} - \omega_{k}^{2}} \right\} \]

\[ = 12e^{2} \sum_{f} \left( \frac{q_{f}}{e} \right)^{2} \int \frac{d^{3}k}{(2\pi)^{3}} \left[ T \sum_{k_{0}} \left( F_{L}^{1} + F_{R}^{1} \right) f_{0}^{(0)}(k_{0} - p_{0}, k) - k_{z} T \sum_{k_{0}} \left( F_{L}^{1} - F_{R}^{1} \right) f_{1}^{(0)}(k_{0} - p_{0}, k) \right] \]

\[ + q_{f} B \left\{ k_{z} f_{1}^{(0)}(k_{0} - p_{0}, k) - \hat{k}_{z} f_{1}^{(0)}(k_{0} - p_{0}, k) + \hat{k}_{z} f_{1}^{(0)}(k_{0} - p_{0}, k) \right\} \]

\[ = 12e^{2} \sum_{f} \left( \frac{q_{f}}{e} \right)^{2} \int \frac{d^{3}k}{(2\pi)^{3}} \left[ T \sum_{k_{0}} \left( F_{L}^{1} + F_{R}^{1} \right) f_{0}^{(0)}(k_{0} - p_{0}, k) - k_{z} T \sum_{k_{0}} \left( F_{L}^{1} - F_{R}^{1} \right) f_{1}^{(0)}(k_{0} - p_{0}, k) \right] \]

\[ + q_{f} B \left\{ k_{z} T \sum_{k_{0}} \left( F_{L}^{1} - F_{R}^{1} \right) f_{0}^{(0)}(k_{0} - p_{0}, k) - \hat{k}_{z} T \sum_{k_{0}} \left( F_{L}^{1} - F_{R}^{1} \right) f_{1}^{(0)}(k_{0} - p_{0}, k) \right\} \]

\[ = 12e^{2} \sum_{f} \left( \frac{q_{f}}{e} \right)^{2} \int \frac{d^{3}k}{(2\pi)^{3}} \left[ T \sum_{k_{0}} \left( F_{L}^{1} + F_{R}^{1} \right) f_{0}^{(0)}(k_{0} - p_{0}, k) - k_{z} T \sum_{k_{0}} \left( F_{L}^{1} - F_{R}^{1} \right) f_{1}^{(0)}(k_{0} - p_{0}, k) \right] \]

\[ + q_{f} B \left\{ k_{z} T \sum_{k_{0}} \left( F_{L}^{1} - F_{R}^{1} \right) f_{0}^{(0)}(k_{0} - p_{0}, k) - \hat{k}_{z} T \sum_{k_{0}} \left( F_{L}^{1} - F_{R}^{1} \right) f_{1}^{(0)}(k_{0} - p_{0}, k) \right\} \]

\[ \right. \]
Now by applying the Braaten-Pisarski-Yuan prescription [21] the imaginary parts of Eqs. (52)-(57) can be obtained in terms of the spectral function of the propagators (Eqs. (24), (25), (A1), (A2), (A3) and (A4)) as

\[
\text{Im}\Pi^{1\mu}(p'_0, 0) = 12e^2 \sum_f \left( \frac{g_f}{e} \right)^2 \pi (1 - e^{\beta p_0}) \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega' \left[ \rho_L^1(\omega) + \rho_R^1(\omega) \right] \rho_0^1(-\omega') \times n_F(\omega)n_F(\omega')\delta(p_0 - \omega - \omega'), \tag{58}
\]

\[
\text{Im}\Pi^{2\mu}(p'_0, 0) = 12e^2 \sum_f \left( \frac{g_f}{e} \right)^2 \pi (1 - e^{\beta p_0}) \int \frac{d^3k}{(2\pi)^3} k \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega' \left[ \rho_L^2(\omega) + \rho_R^2(\omega) \right] \rho_0^0(-\omega') \times n_F(\omega)n_F(\omega')\delta(p_0 - \omega - \omega'), \tag{59}
\]

\[
\text{Im}\Pi^{3\mu}(p'_0, 0) = 12e^2 \sum_f \left( \frac{g_f}{e} \right)^2 \rho, \pi(1 - e^{\beta p_0}) \int \frac{d^3k}{(2\pi)^3} k \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega' \left[ \rho_L^3(\omega) - \rho_R^3(\omega) \right] \rho_0^0(-\omega') \times n_F(\omega)n_F(\omega')\delta(p_0 - \omega - \omega'), \tag{60}
\]

\[
\text{Im}\Pi^{4\mu}(p'_0, 0) = 12e^2 \sum_f \left( \frac{g_f}{e} \right)^2 q_f B \pi(1 - e^{\beta p_0}) \int \frac{d^3k}{(2\pi)^3} k \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega' \left[ \rho_L^4(\omega) - \rho_R^4(\omega) \right] \rho_1^0(-\omega') \times n_F(\omega)n_F(\omega')\delta(p_0 - \omega - \omega'), \tag{61}
\]

\[
\text{Im}\Pi^{5\mu}(p'_0, 0) = 12e^2 \sum_f \left( \frac{g_f}{e} \right)^2 q_f B \pi(1 - e^{\beta p_0}) \int \frac{d^3k}{(2\pi)^3} k \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega' \left[ \rho_L^5(\omega) + \rho_R^5(\omega) \right] \rho_1^1(-\omega') \times n_F(\omega)n_F(\omega')\delta(p_0 - \omega - \omega'), \tag{62}
\]

\[
\text{Im}\Pi^{6\mu}(p'_0, 0) = 12e^2 \sum_f \left( \frac{g_f}{e} \right)^2 q_f B \pi(1 - e^{\beta p_0}) \int \frac{d^3k}{(2\pi)^3} k \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega' \left[ \rho_L^6(\omega) + \rho_R^6(\omega) \right] \rho_1^1(-\omega') \times n_F(\omega)n_F(\omega')\delta(p_0 - \omega - \omega'). \tag{63}
\]

As before the rate has pole-pole and pole-cut part. There will also be no cut-cut part since the spectral function for a hard quark has only the pole part. Below we compute various contributions.

1. Pole-Pole part

Here to compute the pole-pole contribution of the dilepton rate we divide it by two parts. The contribution coming from free part of $S_F$ and $S^*$ is termed as (a) magnetic field independent part whereas that coming from $\mathcal{O}([q_f B])$ part of $S_F$ and $S^*$ is termed as (b) magnetic field dependent part. Note that we neglect current quark mass $m_f$ so that $\omega_k = k$.

(a) Magnetic field independent part:

Using equation (A11) in (58), we get,

\[
\text{Im}\Pi^{1\mu} = 12e^2 \sum_f \left( \frac{g_f}{e} \right)^2 \pi(1 - e^{\beta p_0}) \frac{1}{(2\pi)^3} \int_0^{2\pi} d\phi \int_0^\infty dk k^2 \int_{-1}^1 d\xi \int_{-\infty}^{\infty} d\omega d\omega' \left[ \rho_L^1(\omega) + \rho_R^1(\omega) \right] \\
\times \left[ -\delta(\omega' + k) - \delta(\omega' - k) \right] n_F(\omega)n_F(\omega')\delta(p_0 - \omega - \omega') \\
= \frac{3e^2}{2\pi} (e^{\beta p_0} - 1) \sum_f \left( \frac{g_f}{e} \right)^2 \int_0^\infty dk k^2 \int_{-1}^1 d\xi \int_{-\infty}^{\infty} d\omega \left[ \rho_L^1(\omega) + \rho_R^1(\omega) \right] n_F(\omega) \\
\times \left[ n_F(-k)\delta(p_0 - \omega + k) + n_F(k)\delta(p_0 - \omega - k) \right]. \tag{64}
\]

Now the spectral functions $\rho_L^1$ and $\rho_R^1$ have pole part as well as cut part. But here we will only use the pole part of the spectral functions. In the pole part there are four terms in $\rho_L^1$ (Eqs. (24) and (25)) out of which the terms with positive sign of pole will survive from energy conservation and we now write them as

\[
\text{Im}\Pi^{1\mu}_{\text{pole-pole}} = \frac{3e^2}{2\pi} (e^{\beta p_0} - 1) \sum_f \left( \frac{g_f}{e} \right)^2 \int_0^{\infty} dk k^2 \int_{-1}^1 d\xi \int_{-\infty}^{\infty} d\omega n_F(\omega) \left[ Z_{L(+)}^1(\omega - \omega_{L(+)}) + Z_{L(-)}^1(\omega - \omega_{L(-)}) \right]
\]
It is easy to see that by parts, these terms will eventually get eliminated. Also using the parity properties of Eq. (A12) in Eq. (60), we obtain

\[ \Im \Pi^{\mu}_{\text{pole}} = \frac{3e^2}{2\pi} \sum_f \left( \frac{q_f}{e} \right)^2 \int_{-\infty}^{\infty} dk \int_{-1}^{1} d\xi n_f(\omega_L) n_f(-\omega_L) \delta(p_0 - \omega) + \delta(p_0 - k) \right) \]

Now, in the similar manner and using the Eq. (A12) in Eq. (59), we get

\[ \Im^{2\mu}_{\text{pole}} = \frac{3e^2}{2\pi} \sum_f \left( \frac{q_f}{e} \right)^2 \left( 1 - e^{\beta p_0} \right) \int_{-\infty}^{\infty} dk \int_{-1}^{1} d\xi \left[ Z^{L+}_{\text{L+}} n_f(\omega_L) n_f(-\omega_L) k + Z^{L-}_{\text{L-}} n_f(\omega_L) n_f(-\omega_L) - k \right] \]

Also using Eq. (A12) in Eq. (60), we obtain

\[ \Im^{3\mu}_{\text{pole}} = \frac{3e^2}{2\pi} \sum_f \left( \frac{q_f}{e} \right)^2 \left( 1 - e^{\beta p_0} \right) \int_{-\infty}^{\infty} dk \int_{-1}^{1} d\xi \left[ Z^{L+}_{\text{L+}} n_f(\omega_L) n_f(-\omega_L) k + Z^{L-}_{\text{L-}} n_f(\omega_L) n_f(-\omega_L) - k \right] \]

(b) Magnetic field dependent part:

We begin by stating that some terms with derivatives of Dirac \( \delta \)-functions are present. But after doing integration by parts, these terms will eventually get eliminated. Also using the parity properties of \( \delta \) function and its derivatives it is easy to see that \( \rho^{(0)}_L(\omega) = -\rho^{(0)}_R(\omega) \). Using equation (A16) in equation (61), we get

\[ \Im^{4\mu}_{\text{pole}} = -\frac{3e^2}{4\pi} \sum_f \left( \frac{q_f}{e} \right)^2 \left( 1 - e^{\beta p_0} \right) q_f B \int_{0}^{\infty} dk \int_{-1}^{1} d\xi \int_{-\infty}^{\infty} d\omega n_f(\omega) n_f(p_0 - \omega) \]

\[ \times \left[ \rho^{(0)}_L(p_0 - \omega) - \rho^{(0)}_R(p_0 - \omega) \right] \]

\[ \times \left[ \rho^{(0)}_L(p_0 - \omega) - \rho^{(0)}_R(p_0 - \omega) \right] \left[ \delta(\omega - k) - \delta(\omega + k) + k \frac{\partial}{\partial \omega} (\delta(\omega - k) + \delta(\omega + k)) \right] \]

\[ \Im^{4\mu}_{\text{pole}} = -\frac{3e^2}{4\pi} \sum_f \left( \frac{q_f}{e} \right)^2 \left( 1 - e^{\beta p_0} \right) q_f B \int_{0}^{\infty} dk \int_{-1}^{1} d\xi \int_{-\infty}^{\infty} d\omega n_f(\omega) n_f(p_0 - \omega) \]

\[ \times \left( \rho^{(0)}_L(p_0 - \omega) - \rho^{(0)}_R(p_0 - \omega) \right) \left[ \delta'(\omega - k) + \delta'(\omega + k) \right] \]
At this point we use partial fraction method to eliminate $\delta'(\omega \pm k)$ and it gives

\[
\text{Im}\Pi^{\mu}_{\rho} \big|_{\text{pole-pole}} = -\frac{3e^2}{4\pi} \sum_f \left( \frac{q_f}{e} \right)^2 (1 - e^{\beta p_0}) q_f B \int_0^\infty \frac{dk}{k} \int_{-1}^1 d\xi \left[ n_F(k)n_F(p_0 - k) \left\{ \rho^1_L(p_0 - k) - \rho^1_R(p_0 - k) \right\} 
- n_F(-k)n_F(p_0 + k) \left\{ \rho^1_L(p_0 + k) - \rho^1_R(p_0 + k) \right\} 
- k \int_{-\infty}^\infty d\omega \frac{\partial}{\partial \omega} \left\{ n_F(\omega)n_F(p_0 - \omega) \left[ \rho^1_L(p_0 - \omega) - \rho^1_R(p_0 - \omega) \right] \right\} \right] 
\left( \delta(\omega - k) + \delta(\omega + k) \right) 
\right]
\]

\[
= -\frac{3e^2}{4\pi} \sum_f \left( \frac{q_f}{e} \right)^2 (1 - e^{\beta p_0}) q_f B \int_0^\infty \frac{dk}{k} \int_{-1}^1 d\xi \left[ n_F(k)n_F(p_0 - k) \left\{ \rho^1_L(p_0 - k) - \rho^1_R(p_0 - k) \right\} 
- n_F(-k)n_F(p_0 + k) \left\{ \rho^1_L(p_0 + k) - \rho^1_R(p_0 + k) \right\} 
- k \int_{-\infty}^\infty d\omega \frac{\partial}{\partial \omega} \left[ n_F(k)n_F(p_0 - \omega) \left[ \rho^1_L(p_0 - \omega) - \rho^1_R(p_0 - \omega) \right] \right] \right] 
\left( \delta(\omega - k) + \delta(\omega + k) \right) 
\right]
\]

\[
= -\frac{3e^2}{4\pi} \sum_f \left( \frac{q_f}{e} \right)^2 (1 - e^{\beta p_0}) q_f B \int_0^\infty \frac{dk}{k} \int_{-1}^1 d\xi \left[ 2n_F(k)n_F(p_0 - k) \left\{ \rho^1_L(p_0 - k) - \rho^1_R(p_0 - k) \right\} 
- n_F(-k)n_F(p_0 + k) \left\{ \rho^1_L(p_0 + k) - \rho^1_R(p_0 + k) \right\} 
- k \int_{-\infty}^\infty d\omega \frac{\partial}{\partial \omega} \left[ n_F(k)n_F(p_0 - \omega) \left[ \rho^1_L(p_0 - \omega) - \rho^1_R(p_0 - \omega) \right] \right] \right] 
\left( \delta(\omega - k) + \delta(\omega + k) \right) 
\right]
\]

The last term \textit{i.e.}, the term that contains derivative with respect to $k$, when integrated out gives the boundary term and it vanishes. Also, by using the properties of $\delta$-function one obtains the pole-pole part as

\[
\text{Im}\Pi^{\mu}_{\rho} \big|_{\text{pole-pole}} = -\frac{3e^2}{2\pi} \sum_f \left( \frac{q_f}{e} \right)^2 (1 - e^{\beta p_0}) q_f B \int_0^\infty \frac{dk}{k} \int_{-1}^1 d\xi \int_0^\infty \frac{d\xi}{\xi} \left[ n_F(k) \left\{ Z_{L(\omega)}^{1+} n_F(\omega_{L(\omega)}) \delta(p_0 - k - \omega_{L(\omega)}) 
+ Z_{L(-\omega)}^{1+} n_F(\omega_{L(-\omega)}) \delta(p_0 - k - \omega_{L(-\omega)}) 
- Z_{R(\omega)}^{1+} n_F(\omega_{R(\omega)}) \delta(p_0 - k - \omega_{R(\omega)}) 
- Z_{R(-\omega)}^{1+} n_F(\omega_{R(-\omega)}) \delta(p_0 - k - \omega_{R(-\omega)}) \right\} 
- n_F(-k) \left\{ Z_{L(\omega)}^{1+} n_F(\omega_{L(\omega)}) \delta(p_0 + k - \omega_{L(\omega)}) 
+ Z_{L(-\omega)}^{1+} n_F(\omega_{L(-\omega)}) \delta(p_0 + k - \omega_{L(-\omega)}) 
- Z_{R(\omega)}^{1+} n_F(\omega_{R(\omega)}) \delta(p_0 + k - \omega_{R(\omega)}) 
- Z_{R(-\omega)}^{1+} n_F(\omega_{R(-\omega)}) \delta(p_0 + k - \omega_{R(-\omega)}) \right\} \right].
\]

Using (A15) in equation (62), we get

\[
\text{Im}\Pi^{\mu}_{\rho} \big|_{\text{pole-pole}} = \frac{3e^2}{4\pi} \sum_f \left( \frac{q_f}{e} \right)^2 (1 - e^{\beta p_0}) q_f B \int_0^\infty \frac{dk}{k} \int_{-1}^1 d\xi \int_0^\infty \frac{d\xi}{\xi} \left[ n_F(k) \left\{ Z_{L(\omega)}^{2+} n_F(\omega_{L(\omega)}) \delta(p_0 - k - \omega_{L(\omega)}) 
+ Z_{L(-\omega)}^{2+} n_F(\omega_{L(-\omega)}) \delta(p_0 - k - \omega_{L(-\omega)}) 
- Z_{R(\omega)}^{2+} n_F(\omega_{R(\omega)}) \delta(p_0 - k - \omega_{R(\omega)}) 
- Z_{R(-\omega)}^{2+} n_F(\omega_{R(-\omega)}) \delta(p_0 - k - \omega_{R(-\omega)}) \right\} 
+ n_F(-k) \left\{ Z_{L(\omega)}^{2+} n_F(\omega_{L(\omega)}) \delta(p_0 + k - \omega_{L(\omega)}) 
+ Z_{L(-\omega)}^{2+} n_F(\omega_{L(-\omega)}) \delta(p_0 + k - \omega_{L(-\omega)}) 
- Z_{R(\omega)}^{2+} n_F(\omega_{R(\omega)}) \delta(p_0 + k - \omega_{R(\omega)}) 
- Z_{R(-\omega)}^{2+} n_F(\omega_{R(-\omega)}) \delta(p_0 + k - \omega_{R(-\omega)}) \right\} \right].
\]
Finally using Eqn. (A15) in Eqn. (63), we get

\[
\text{Im} \Pi^\mu_{\mu | \text{pole-pole}} = \frac{3e^2}{4\pi} \sum_f \left( \frac{q_f}{e} \right)^2 (1 - e^{\beta p_0}) q_f B \int_{-1}^{1} d\xi \int_{0}^{\infty} dk \left[ n_F(k) \left\{ Z_{L(+)}^{2+} n_F(\omega_{L(+)}) \delta(p_0 - k - \omega_{L(+)}) + Z_{R(-)}^{2+} n_F(\omega_{R(-)}) \delta(p_0 - k - \omega_{R(-)}) + Z_{R(+)}^{2+} n_F(\omega_{R(+)}) \delta(p_0 - k - \omega_{R(+)}) \right\} \right. \\
+ \left. n_F(-k) \left\{ Z_{L(-)}^{2+} n_F(\omega_{L(-)}) \delta(p_0 + k - \omega_{L(-)}) + Z_{R(-)}^{2+} n_F(\omega_{R(-)}) \delta(p_0 + k - \omega_{L(-)}) + Z_{R(+)}^{2+} n_F(\omega_{R(+)}) \delta(p_0 + k - \omega_{R(+)}) \right\} \right] \right] .
\]

(72)

(c) Dilepton rate from various processes in pole-pole part in presence of magnetic field:

We note that for numerical computation we make the integration from spherical polar to cylindrical polar through the transformation \( k_\perp = k \sqrt{1 - \xi^2} \), \( k_z = k \xi \). Using (51) and grouping the delta functions together we get the dilepton rates in terms of cylindrical polar coordinate from various processes discussed in subsec. II A as follows:

1. \( q_{L(+)} q \rightarrow \gamma^* \)

\[
\frac{dR}{d^4x d^4p} \left| q_{L(+)} q \rightarrow \gamma^* \right| = \frac{\alpha^2}{2p_0^2 \pi^4} \sum_f \left( \frac{q_f}{e} \right)^2 \int_0^\infty d k_{\perp} \int_{-\infty}^{\infty} d k_z n_F \left( p_0 - \sqrt{k_{\perp}^2 + k_z^2} \right) n_F \left( \sqrt{k_{\perp}^2 + k_z^2} \right) \\
\times \left[ Z_{L(+)}^1 + Z_{L(+)}^2 + \frac{k_z}{\sqrt{k_{\perp}^2 + k_z^2}} Z_{L(+)}^3 + \frac{q_f B}{k_{\perp}^2 + k_z^2} \right] \delta \left( p_0 - \omega_{L(+)}(k_\perp, k_z) - \sqrt{k_{\perp}^2 + k_z^2} \right) .
\]

(73)

2. \( q_{L(-)} q \rightarrow \gamma^* \)

\[
\frac{dR}{d^4x d^4p} \left| q_{L(-)} q \rightarrow \gamma^* \right| = \frac{\alpha^2}{2p_0^2 \pi^4} \sum_f \left( \frac{q_f}{e} \right)^2 \int_0^\infty d k_{\perp} \int_{-\infty}^{\infty} d k_z n_F \left( p_0 - \sqrt{k_{\perp}^2 + k_z^2} \right) n_F \left( \sqrt{k_{\perp}^2 + k_z^2} \right) \\
\times \left[ Z_{L(-)}^1 + Z_{L(-)}^2 + \frac{k_z}{\sqrt{k_{\perp}^2 + k_z^2}} Z_{L(-)}^3 + \frac{q_f B}{k_{\perp}^2 + k_z^2} \right] \delta \left( p_0 - \omega_{L(-)}(k_\perp, k_z) - \sqrt{k_{\perp}^2 + k_z^2} \right) .
\]

(74)

3. \( q_{R(+)} q \rightarrow \gamma^* \)

\[
\frac{dR}{d^4x d^4p} \left| q_{R(+)} q \rightarrow \gamma^* \right| = \frac{\alpha^2}{2p_0^2 \pi^4} \sum_f \left( \frac{q_f}{e} \right)^2 \int_0^\infty d k_{\perp} \int_{-\infty}^{\infty} d k_z n_F \left( p_0 - \sqrt{k_{\perp}^2 + k_z^2} \right) n_F \left( \sqrt{k_{\perp}^2 + k_z^2} \right) \\
\times \left[ Z_{R(+)}^1 + Z_{R(+)}^2 - \frac{k_z}{\sqrt{k_{\perp}^2 + k_z^2}} Z_{R(+)}^3 - \frac{q_f B}{k_{\perp}^2 + k_z^2} \right] \delta \left( p_0 - \omega_{R(+)}(k_\perp, k_z) - \sqrt{k_{\perp}^2 + k_z^2} \right) .
\]

(75)

4. \( q_{R(-)} q \rightarrow \gamma^* \)

\[
\frac{dR}{d^4x d^4p} \left| q_{R(-)} q \rightarrow \gamma^* \right| = \frac{\alpha^2}{2p_0^2 \pi^4} \sum_f \left( \frac{q_f}{e} \right)^2 \int_0^\infty d k_{\perp} \int_{-\infty}^{\infty} d k_z n_F \left( p_0 - \sqrt{k_{\perp}^2 + k_z^2} \right) n_F \left( \sqrt{k_{\perp}^2 + k_z^2} \right) \\
\times \left[ Z_{R(-)}^1 + Z_{R(-)}^2 - \frac{k_z}{\sqrt{k_{\perp}^2 + k_z^2}} Z_{R(-)}^3 - \frac{q_f B}{k_{\perp}^2 + k_z^2} \right] \delta \left( p_0 - \omega_{R(-)}(k_\perp, k_z) - \sqrt{k_{\perp}^2 + k_z^2} \right) .
\]
From the parity symmetry of the dispersion mode, it is possible to show that

\[ \frac{dR}{d^4 x d^4 p} \mid_{q_{L(-)} \to q_{\gamma^*}} = \frac{\alpha^2}{2 \rho_0^2 \pi^2} \sum_f \left( \frac{q_f}{e} \right)^2 \int_0^\infty dk_{\perp} k_{\perp} \int_{-\infty}^{\infty} dk_z n_F \left( \frac{k_{\perp}}{k_{\perp}^2 + k_z^2} - \frac{q_f B}{k_{\perp}^2 + k_z^2} \right) \delta \left( p_0 - \omega_{L(-)}(k_{\perp}, k_z) + \sqrt{k_{\perp}^2 + k_z^2} \right). \] (76)

5. \( q_{L(+)} \to q_{\gamma^*} \)

\[ \frac{dR}{d^4 x d^4 p} \mid_{q_{L(+)} \to q_{\gamma^*}} = \frac{\alpha^2}{2 \rho_0^2 \pi^2} \sum_f \left( \frac{q_f}{e} \right)^2 \int_0^\infty dk_{\perp} k_{\perp} \int_{-\infty}^{\infty} dk_z n_F \left( \frac{k_{\perp}}{k_{\perp}^2 + k_z^2} - \frac{q_f B}{k_{\perp}^2 + k_z^2} \right) \delta \left( p_0 - \omega_{L(+)}(k_{\perp}, k_z) + \sqrt{k_{\perp}^2 + k_z^2} \right). \] (77)

6. \( q_{L(-)} \to q_{\gamma^*} \)

\[ \frac{dR}{d^4 x d^4 p} \mid_{q_{L(-)} \to q_{\gamma^*}} = \frac{\alpha^2}{2 \rho_0^2 \pi^2} \sum_f \left( \frac{q_f}{e} \right)^2 \int_0^\infty dk_{\perp} k_{\perp} \int_{-\infty}^{\infty} dk_z n_F \left( \frac{k_{\perp}}{k_{\perp}^2 + k_z^2} - \frac{q_f B}{k_{\perp}^2 + k_z^2} \right) \delta \left( p_0 - \omega_{L(-)}(k_{\perp}, k_z) + \sqrt{k_{\perp}^2 + k_z^2} \right). \] (78)

7. \( q_{R(+)} \to q_{\gamma^*} \)

\[ \frac{dR}{d^4 x d^4 p} \mid_{q_{R(+)} \to q_{\gamma^*}} = \frac{\alpha^2}{2 \rho_0^2 \pi^2} \sum_f \left( \frac{q_f}{e} \right)^2 \int_0^\infty dk_{\perp} k_{\perp} \int_{-\infty}^{\infty} dk_z n_F \left( \frac{k_{\perp}}{k_{\perp}^2 + k_z^2} - \frac{q_f B}{k_{\perp}^2 + k_z^2} \right) \delta \left( p_0 - \omega_{R(+)}(k_{\perp}, k_z) + \sqrt{k_{\perp}^2 + k_z^2} \right). \] (79)

8. \( q_{R(-)} \to q_{\gamma^*} \)

\[ \frac{dR}{d^4 x d^4 p} \mid_{q_{R(-)} \to q_{\gamma^*}} = \frac{\alpha^2}{2 \rho_0^2 \pi^2} \sum_f \left( \frac{q_f}{e} \right)^2 \int_0^\infty dk_{\perp} k_{\perp} \int_{-\infty}^{\infty} dk_z n_F \left( \frac{k_{\perp}}{k_{\perp}^2 + k_z^2} - \frac{q_f B}{k_{\perp}^2 + k_z^2} \right) \delta \left( p_0 - \omega_{R(-)}(k_{\perp}, k_z) + \sqrt{k_{\perp}^2 + k_z^2} \right). \] (80)

From the parity symmetry of the dispersion mode, it is possible to show that

\[ \frac{dR}{d^4 x d^4 p} \mid_{\omega_{L(+)}(k \to \gamma^*)} = \frac{dR}{d^4 x d^4 p} \mid_{\omega_{R(+)}(k \to \gamma^*)}. \]
\[
\frac{dR}{d^4x d^4p} |_{\omega_L(-)k\rightarrow\gamma^*} = \frac{dR}{d^4x d^4p} |_{\omega_R(-)k\rightarrow\gamma^*}, \\
\frac{dR}{d^4x d^4p} |_{\omega_L(+)+k\gamma^*} = \frac{dR}{d^4x d^4p} |_{\omega_R(+)+k\gamma^*}, \\
\frac{dR}{d^4x d^4p} |_{\omega_L(-)-k\gamma^*} = \frac{dR}{d^4x d^4p} |_{\omega_R(-)-k\gamma^*}.
\]

Finally, the pole-pole contribution of the hard dilepton rate becomes

\[
\frac{dR}{d^4x d^4p} \bigg|_{pp} = 2 \left( \frac{dR}{d^4x d^4p} |_{\omega_L(+)+k\gamma^*} + \frac{dR}{d^4x d^4p} |_{\omega_L(-)-k\gamma^*} + \frac{dR}{d^4x d^4p} |_{\omega_R(+)+k\gamma^*} + \frac{dR}{d^4x d^4p} |_{\omega_R(-)-k\gamma^*} \right).
\]

We note that the various soft decay modes will contribute only to the soft dilepton production at low energy. Since we are interested in hard dilepton production rate, only the annihilation modes will contribute and we will omit those soft decay modes from our considerations. The resulting pole-pole contribution is plotted in Fig. 5. In the left panel the rate is displayed as a function of dilepton energy at \( E = 200 \text{ MeV} \) but for different magnetic fields. In absence of magnetic field \( eB = 0 \) the annihilation between a hard and a soft quarks starts when dilepton energy \( E = m_{th} \) and resembles that of \( q\bar{q} \rightarrow \gamma^* \rightarrow l^+l^- \) as given in Fig. 4. As the magnetic field is turned on all four quasiparticle modes, namely \( \omega_L(+), \omega_L(-), \omega_R(+), \omega_R(-) \), in Fig. 1 separately participate in annihilation with hard quark. As can be seen the dilepton rate begins with little higher energy of the virtual photon. This is because the presence of magnetic field contributes to the thermomagnetic mass which is lower than the thermal mass. As energy of the dilepton increases the rate becomes almost equal to that in absence of magnetic field. In the right panel of Fig. 5 the rate is displayed for various temperatures for a given magnetic filed. At energy up to the \( E = p_0 \approx 2m_{th} \) the rate is found to be almost independent of \( T \) as magnetic field may be the dominant scale there. At energies \( E = p_0 > 2m_{th} \) the rate increases with the increase of \( T \) as \( T \) is the dominant scale in weak field approximation.

![Graph](attachment:graph.png)

**FIG. 5.** Pole pole contribution of dilepton production rate as a function of the energy of dilepton in center of mass reference frame at \( T = 200 \text{ MeV} \) with different magnetic field (left panel) and \( eB = m_{\pi}^2 \) with different temperature (right panel).

### 2. Pole-cut contribution

The presence of \( \Theta \) due to space like momentum in the Landau cut contribution of the spectral function, \( \Theta(k^2 - \omega^2)\beta_L^1(\omega, k_x, k_z) \), immensely simplifies the pole-cut rate. From equation (64) we get,

\[
\text{Im} \Pi_{\mu} \bigg|_{\text{pole-cut}} = \frac{3e^2}{2\pi} (e^{\beta p_0} - 1) \sum_j \left( \frac{g_j}{e} \right)^2 \int_0^\infty dk^2 \int_{-1}^1 d\xi \int_{-\infty}^\infty d\omega \Theta(k^2 - \omega^2) \left[ \beta_L^1(\omega) + \beta_H^1(\omega) \right] n_F(\omega)
\]
We note that the term with \( \delta(p_0 - \omega + k) \) will have no contribution because \( \Theta[k^2 - (p_0 + k)^2] = \Theta[-p_0(p_0 + 2k)] \) will never be satisfied since \( k, p_0 > 0 \). The expression to evaluate pole-cut contribution is

\[
\frac{dR}{d^4xd^4p}_{\text{pole-cut}} = \frac{\alpha^2}{2\pi^4p_0^2} \sum_f \left( \frac{q_f}{m_f}\right)^2 \int_{-1}^{1} d\xi \int_{0}^{\infty} dk n_f(k) n_f(p_0 - k) \Theta(2k - p_0) \\
\times \left[ k^2 \left( \beta^1_L + \beta^1_R + \beta^2_L + \beta^2_R + \xi(\beta^3_L - \beta^3_R) \right) + q_f B \left( \xi(\beta^1_L - \beta^1_R) + \frac{1}{2} \xi(\beta^2_L - \beta^2_R) + \frac{1}{2} (\beta^3_L + \beta^3_R) \right) \right],
\]

where \( \beta^{i_{(L/R)}}(p_0 - k, k_\perp, k^3) \).

In the left panel of Fig. 6 the pole-cut contribution is plotted for various magnetic fields with \( T = 200 \text{ MeV} \). It is found to be independent of \( T \) of the magnetic field. This is because magnetic field appears as a correction in the weak field approximation and we have considered the rate up to \( \mathcal{O}[eB] \). On the other hand, in left panel of Fig. 6, it is plotted for various temperatures for a given magnetic field. The rate is found to be enhanced with increase in temperature as the dominant scale is the thermomagnetic mass in the weak field approximation. Total dilepton rate is obtained by adding the pole-pole contribution from Eq. (82) and the pole-cut contribution from Eq. (84) and is plotted in fig. (7) with a similar behaviour as in Fig. 5.

### IV. Conclusion

In this paper, we have systematically investigated thermal dilepton production from a hot magnetized QCD medium in the weak field approximation. Since we are interested in hard dilepton rate, it is sufficient to use just one resummed and one bare propagator in presence of magnetic field in the photon polarization tensor diagram in Figure 2. We note that the earlier works were carried out using free propagators for both the fermions in the loop in presence of magnetic field. Since we have one resummed propagator, it’s spectral representation contains pole and (Landau) cut contribution. On the other hand, a hard spectral function corresponding to bare propagator has only pole contribution. The dilepton rate contains two types of contributions: pole-pole and pole-cut. As the magnetic field is turned on, all of the four quasiquark modes namely \( \omega_{L(+)} \), \( \omega_{L(-)} \), \( \omega_{R(+)} \), \( \omega_{R(-)} \) individually participate in annihilation with hard quark and contribute to the pole-pole part of the dilepton production. These annihilation processes start at higher energies as the thermomagnetic mass increases in presence of magnetic field. The pole-cut contribution is found to dominate over those annihilation processes at low energies.

In weak field approximation magnetic field appears as a correction to the thermal contributions. Since, for simplicity, we have considered only \( \mathcal{O}[eB] \) correction, the effect of magnetic field in the rate is found to be very marginal here. For having moderate effect of the magnetic field one needs to take into account higher order corrections. On the other
hand, one may consider photon self-energy diagram with two resummed quark propagator along with two effective three-point vertices. In addition a four-point vertex diagram will also contribute. This altogether will present a complete picture of soft dilepton production in one loop order. We also note that in this calculation we have only considered the case where the quarks are affected by the presence of the magnetic field whereas the leptons remain unaffected as they are assumed to be produced at the edge of the fireball. Since dileptons are produced at every stages of the fireball, one should also take into account the modification of the leptons in presence of magnetic field. All these are very interesting prospect but will indeed be very involved calculations.

V. Acknowledgement

The authors would like to acknowledge Aritra Bandyopadhyay and Arghya Mukherjee for useful discussions. PKR was funded by Department of Atomic Energy (DAE), India via the project ALICE/SINP. AD was funded by DAE/ALICE/SINP and partially by School of Physical Science (SPS), NISER. NH was funded by SPS/NISER and MGM was funded by DAE via project TPAES.

A. Spectral Representation of weak field propagator upto $O(q_f B)$

We need to find the spectral representation of $S_B(K)$. To do this we write [33]

$$S_B(K) = \frac{K}{K^2 - m_f^2} + i\gamma^1\gamma^2 \frac{k_f}{(K^2 - m_f^2)^2} q_f B$$

$$= \frac{K}{K^2 - m_f^2} - \frac{k_0\gamma^3 - k^3\gamma_0}{(K^2 - m_f^2)^2} q_f B$$

$$= \frac{k_0}{k_0^2 - \omega_k^2} \gamma^0 - |\vec{k}| \frac{1}{k_0^2 - \omega_k^2} \hat{k} \cdot \gamma - \gamma_5 \left[ \frac{k_0}{(k_0^2 - \omega_k^2)^2} \gamma^3 - \frac{1}{(k_0^2 - \omega_k^2)^2} k^3\gamma_0 \right] q_f B.$$ 

We define the spectral functions as follows

$$\rho_0^{(1)}(k_0, |\vec{k}|) = \frac{1}{\pi} \text{Im} f_0^{(1)}(k_0 + i\epsilon, |\vec{k}|) = \frac{1}{\pi} \text{Im} \frac{k_0 + i\epsilon}{(k_0 + i\epsilon)^2 - \omega_k^2}, \quad (A1)$$

$$\rho_0^{(0)}(k_0, |\vec{k}|) = \frac{1}{\pi} \text{Im} f_0^{(0)}(k_0 + i\epsilon, |\vec{k}|) = \frac{1}{\pi} \text{Im} \frac{1}{(k_0 + i\epsilon)^2 - \omega_k^2}, \quad (A2)$$
\( \rho_{1}^{(1)}(k_0, |\vec{k}|) = \frac{1}{\pi} \text{Im} f_1^{(1)}(k_0 + i\epsilon, |\vec{k}|) = \frac{1}{\pi} \text{Im} \frac{k_0 + i\epsilon}{(k_0 + i\epsilon)^2 - \omega_\vec{k}^2}, \) \hfill (A3) \\
\( \rho_{1}^{(0)}(k_0, |\vec{k}|) = \frac{1}{\pi} \text{Im} f_1^{(0)}(k_0 + i\epsilon, |\vec{k}|) = \frac{1}{\pi} \text{Im} \frac{1}{(k_0 + i\epsilon)^2 - \omega_\vec{k}^2}. \) \hfill (A4)

Now to prove this we need to use [45]

\[
\lim_{\epsilon \to 0} \text{Im} \frac{1}{x + i\epsilon} = -\pi \delta(x), \quad \text{and} \quad \lim_{\epsilon \to 0} \text{Im} \frac{1}{(x + i\epsilon)^2} = \pi \delta'(x),
\]

where \( x, \epsilon \in \mathbb{R}, \epsilon > 0. \)

To prove (A5) and (A6), we use the following limiting representation of Dirac Delta function

\[
\lim_{\epsilon \to 0} \frac{\epsilon}{x^2 + \epsilon^2} = \pi \delta(x).
\] \hfill (A7)

Taking derivative with respect to \( x \) on both sides of equation (A7), we get

\[
\lim_{\epsilon \to 0} \frac{2\epsilon x}{(x^2 + \epsilon^2)^2} = -\pi \delta'(x).
\] \hfill (A8)

Now

\[
\lim_{\epsilon \to 0} \text{Im} \frac{1}{x + i\epsilon} = \frac{1}{2i} \lim_{\epsilon \to 0} \left[ \frac{1}{x + i\epsilon} - \frac{1}{x - i\epsilon} \right] = \frac{1}{2i} \lim_{\epsilon \to 0} \frac{-2i\epsilon}{(x^2 + \epsilon^2)} = - \lim_{\epsilon \to 0} \frac{\epsilon}{x^2 + \epsilon^2} = - \pi \delta(x),
\] \hfill (A9)

and

\[
\lim_{\epsilon \to 0} \text{Im} \frac{1}{(x + i\epsilon)^2} = \frac{1}{2i} \lim_{\epsilon \to 0} \left[ \frac{1}{(x + i\epsilon)^2} - \frac{1}{(x - i\epsilon)^2} \right] = \frac{1}{2i} \lim_{\epsilon \to 0} \frac{-4i\epsilon x}{(x^2 + \epsilon^2)^2} = - \lim_{\epsilon \to 0} \frac{2\epsilon x}{(x^2 + \epsilon^2)^2} = \pi \delta'(x). \] \hfill (A10)

This proves equation (A5) and (A6). With these it is easy to get the spectral representation for the free part:

\[
\rho_{0}^{(1)}(k_0, |\vec{k}|) = \frac{1}{\pi} \text{Im} \frac{1}{2} \left( \frac{1}{k_0 - \omega_k + i\epsilon} + \frac{1}{k_0 + \omega_k + i\epsilon} \right) = - \delta(k_0 + \omega_k) + \delta(k_0 - \omega_k), \] \hfill (A11)

\[
\rho_{0}^{(0)}(k_0, |\vec{k}|) = \frac{1}{\pi} \text{Im} \frac{1}{2\omega_k} \left( \frac{1}{k_0 - \omega_k + i\epsilon} - \frac{1}{k_0 + \omega_k + i\epsilon} \right) = \frac{\delta(k_0 + \omega_k) - \delta(k_0 - \omega_k)}{2\omega_k}. \] \hfill (A12)

Now for the 1st order part, we need to

\[
\frac{k_0}{(k_0^2 - \omega_k^2)^2} = \frac{1}{4\omega_k (k_0 + \omega_k)^2 (k_0 - \omega_k)^2} = \frac{1}{4\omega_k} \frac{(k_0 + \omega_k)^2 - (k_0 - \omega_k)^2}{(k_0 + \omega_k)^2 (k_0 - \omega_k)^2} = \frac{1}{4\omega_k} \left[ \frac{1}{(k_0 - \omega_k)^2} - \frac{1}{(k_0 + \omega_k)^2} \right], \] \hfill (A13)

\[
\frac{1}{(k_0^2 - \omega_k^2)^2} = \frac{1}{4\omega_k^2} \left[ \frac{1}{k_0 - \omega_k} - \frac{1}{k_0 + \omega_k} \right]^2 = \frac{1}{4\omega_k^2} \left[ \frac{1}{(k_0 - \omega_k)^2} + \frac{1}{(k_0 + \omega_k)^2} - \frac{2}{k_0^2 - \omega_k^2} \right] = \frac{1}{4\omega_k^2} \left( \frac{1}{(k_0 - \omega_k)^2} + \frac{1}{(k_0 + \omega_k)^2} - \frac{1}{k_0 - \omega_k} \frac{1}{k_0 + \omega_k} \right). \] \hfill (A14)

Thus

\[
\rho_{1}^{(1)}(k_0, |\vec{k}|) = \frac{1}{\pi} \text{Im} \frac{1}{4\omega_k} \left[ \frac{1}{(k_0 - \omega_k + i\epsilon)^2} - \frac{1}{(k_0 + \omega_k + i\epsilon)^2} \right] = \frac{\delta'(k_0 - \omega_k) - \delta'(k_0 + \omega_k)}{4\omega_k}. \] \hfill (A15)

Also

\[
\rho_{1}^{(0)}(k_0, |\vec{k}|) = \frac{1}{\pi} \text{Im} \frac{1}{4\omega_k} \left[ \frac{1}{(k_0 - \omega_k + i\epsilon)^2} + \frac{1}{(k_0 + \omega_k + i\epsilon)^2} - \frac{1}{\omega_k} \left( \frac{1}{k_0 - \omega_k + i\epsilon} - \frac{1}{k_0 + \omega_k + i\epsilon} \right) \right].
\]
\[ = \frac{1}{4 \omega_k^2} \left\{ \delta\left(k_0 - \omega_k\right) + \delta\left(k_0 + \omega_k\right) + \frac{1}{\omega_k} \left[ \delta(k_0 - \omega_k) - \delta(k_0 + \omega_k) \right] \right\}. \]