Study on $\Xi_{cc} \rightarrow \Xi_c$ and $\Xi_{cc} \rightarrow \Xi'_c$ weak decays in the light-front quark model

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Abstract In this work we study the weak decays of $\Xi_{cc} \rightarrow \Xi_c$ and $\Xi_{cc} \rightarrow \Xi'_c$ in the light-front quark model. Generally, a naive, but reasonable conjecture suggests that the $cc$ subsystem in $\Xi_{cc}$ ($us$ pair in $\Xi'_c$) stands as a diquark with definite spin and color assignments. During the concerned processes, the diquark of the initial state is not a spectator, namely must be broken. A Racah transformation would decompose the original $(cc)q$ into a combination of $c(cq)$ components. Thus we may deal with the decaying $c$ quark alone while keeping the $(cq)$ subsystem as a spectator. With the re-arrangement of the inner structure we calculate the form factors numerically and then obtain the rates of semi-leptonic decays and non-leptonic decays, which will be measured in the future.

1 Introduction

In 2017, the LHCb collaboration observed a doubly charmed baryon $\Xi_{cc}^{++}$ [1] with mass $3621.40 \pm 0.72 \pm 0.27 \pm 0.14$ MeV which indeed was long-expected by physicists of high energy physics. In terms of the constituent quark model there should exist 40 baryon-states of $J^P = 1/2$ and 35 $J^P = 3/2$ states which are composed of the five flavors $u, d, s, c$ and $b$. Most of the light baryons and several heavy baryons with only one heavy quark ($b$ or $c$) have been observed experimentally. Thus the doubly heavy baryons would be the goal of experimental search. One of such states $\Xi_{cc}^{++}$ was reported by the SELEX [2, 3] collaboration with a mass about 3520 MeV, however, it was not confirmed by other collaborations. Recently $\Xi_{cc}^{++}$ has been measured by the LHCb collaboration in the four-body final state $\Lambda_c K^- \pi^+ \pi^+$ [1] and later $\Xi_{cc}^{++}$ was observed via the $\Xi^+ \pi^+$ portal [4].

Theoretically, the structure about the series of $\Xi_{cc}$ has not been fully investigated yet (in this paper we only consider $\Xi_{cc}^{++}$ and the result can be generalized to $\Xi_{cc}^{+}$). For example, its life time and decay rates into several main channels are not well measured yet while on the theoretical aspect a reliable approach to study the decays of doubly heavy baryons still lacks. Therefore, all attempts to deeply investigate $\Xi_{cc}$ from different angles should be valuable. By this study we may determine the inner structure of $\Xi_{cc}$ and its decay behaviors by the light front quark model, consequently the results can be tested and the gained knowledge would be helpful for designing new experiments for searching other baryons with two heavy quarks.

Since the mass of $\Xi_{cc}$ is smaller than the production threshold of $\Lambda_c$ and $D$, it only decays via weak interaction. Apparently $\Xi_{cc}$ should favorably decay into products involving a single-charmed baryon. In order to evaluate its decay rates we first need to know its inner structure. A naive and reasonable conjecture suggests that the subsystem of the two $c$ quarks composes a diquark as a color source for the light quark [5, 6].

In the most works about single charmed-baryons, the two light quarks can be regarded as a light diquark [7, 8]. Thus while dealing with weak decays of such baryons, the diquarks with two light quarks can be safely regarded as spectators which do not undergo any changes during the transitions. The spectator scenario indeed greatly alleviates the theoretical difficulties.

On the contrary, in the decay process of $\Xi_{cc}$, one of the two $c$ quarks in the initial diquark would transit into a lighter quark by emitting gauge bosons and the newly emerged quark $s$ in the final state will be bound with the $u$ quark to form a light diquark subsystem of color anti-triplet $us$ via a recombination from $(cu)s$ into $(us)c$. Namely the picture is that the old diquark is broken and a new diquark emerges during the transition. Anyhow, the diquark [no matter the original $(cc)$ or the final $(us)$] can no longer be treated as a specta-
tor. Therefore the simple quark-diquark picture could not be directly applied to this decay process.

In this paper we will extend the light-front quark model to study the weak decays of $\Xi_{cc}$ where three-body vertex function was obtained in our previous works. The light-front quark model (LFQM) is a relativistic quark model which has been applied to study transitions among mesons and the results agree with the data within reasonable error tolerance [9–25]. We also studied the weak decays of $\Lambda_b$ and $\Sigma_b$ in the heavy-quark-light-diquark picture of baryon [26–29] in this model and our results [26–29] are consistent with those given in literatures. Thus we have a certain confidence that the extension of the light-front quark model to baryon cases is also successful to the leading order at least [26–33].

In Ref. [30] we construct the three-body vertex function which is applied to the decay of heavy baryons. Now we try extending the approach to the concerned process. The transition process can be divided into two steps: first the old diquark of $cc$ is broken and a subsystem of $cu$ serves as a spectator during the transition, and then in the finally produced $\Xi_{cc}$, the subsystem of $cu$ would be broken again and a proper structure of $c(us)$ is reformed via the QCD interaction. As a matter of fact, it is easy to realize as we rearrange the $cc$ diquark-(u) quark structure into a combination of the $cu$ (diquark-like subsystem)-c (quark) structures by a Racah transformation. During the transition the (cu) diquark-like subsystem can be regarded as a spectator. Namely, one c quark transits into an s quark but the other c quark and the u quark are not touched approximately. Then for the second step we also need a Racah transformation to rearrange the (cu)s structure into the (us)c system.

In Refs. [31,32] the authors used the quark-diquark picture to explore the weak decays of doubly charmed baryon. It should be emphasized, the scenario in this paper where three individual quarks are concerned is different from the picture with one-diquark and one quark adopted by the authors of Refs. [31,32].

Indeed, since in $\Xi_{cc}$ the u-quark has a relative momentum with respect to the diquark $cc$ (the distance between two c quarks is small), thus after the recombination, in the subsystem $uc$, between the two constituents $u$ and $c$, there exists a relative momentum. Therefore, rigorously speaking, the subsystem of uc is a diquark-like subsystem. In our work, we have to take into account the momenta carried by all the individual quarks which would undergo some changes during the transition. It is stressed again that in this work, we treat the combination involving one c quark and a u quark as a diquark-like effective subsystem. In other words, $cc$ and $us$ in $\Xi_{cc}$ and $\Xi_{cc}^{(i)}$ possess definite spin and color quantum numbers, so we can transform physical subsystem (diquarks) into other effective subsystems. However, since the subsystem $cu$ is not a diquark, the inner degree of freedom could not be ignored.

This paper is organized as follows: after the introduction, in Sect. 2 we write up the transition amplitude for $\Xi_{cc} \to \Xi_{c}^{(i)}$ in the light-front quark model and give the resultant form factors, then we present our numerical results for $\Xi_{cc} \to \Xi_{c}^{(i)}$ along with all necessary input parameters in Sect. 3. Section 4 is devoted to our conclusion and discussions.

2 $\Xi_{cc} \to \Xi_{c}$ in the light-front quark model

2.1 The vertex functions of $\Xi_{cc}$ and $\Xi_{c}$

In our previous works [26–29], we employed the quark-diquark picture to study the baryon transitions, where the diquark has definite spin and serves as a spectator approximately during the transition process. However in the process $\Xi_{cc}$ and $\Xi_{c}$ the picture is no longer valid. For a generally accepted consideration the two charm quarks in $\Xi_{cc}$ compose a diquark which stands as a color source for the light u quark which is moving around the diquark with a certain relative momentum. The relative orbital angular momentum between the two c quarks is 0, i.e. the cc pair is in an S wave, due to the symmetry requirement the spin of the cc pair must be 1. In Ref. [8] the us-diquark in $\Xi_{c}$ is a scalar diquark whereas in $\Xi_{c}^{(i)}$ it is a vector. In the decay process of $\Xi_{cc}$ the cc diquark must be physically broken and one of the two charm quarks transits into an s quark via weak interaction and a light cu subsystem is formed which becomes a spectator for the decay process. That means neither the diquarks cc in the initial state, nor the us in the final state is spectator. To realize the transition, we mathematically reorder the quark structure of $(cc)u$ into a sum of $\sum_{i}c(cu)_{i}$ where the sum runs over all possible configurations (spin etc.) via a Racah transformation. Because of existence of relative momenta among the quarks, in this work we explore the baryon transition in the three-quark picture where the three quarks are individual subjects and possess their own momenta.

In analog to Refs. [30,34,35] the vertex functions of $\Xi_{cc}$ and $\Xi_{c}$ with total spin $S = 1/2$ and momentum $P$ are

$$|\Xi_{cc}(P, S, S_{c})\rangle = \int \{d^{3}\tilde{p}_{1}\} d^{3}\tilde{p}_{2} | \{d^{3}\tilde{p}_{3}\} 2(2\pi)^{3} \delta^{3}(\tilde{P} - \tilde{p}_{1} - \tilde{p}_{2} - \tilde{p}_{3}) \times \sum_{\lambda_{1},\lambda_{2},\lambda_{3}} \psi_{S_{c}}(\tilde{p}_{1}, \lambda_{1}) \psi_{S_{c}}(\tilde{p}_{2}, \lambda_{2}) \psi_{S_{c}}(\tilde{p}_{3}, \lambda_{3}) \rangle_{c_{u}(P_{1}, \lambda_{1})c_{u}(P_{2}, \lambda_{2})u_{u}(P_{3}, \lambda_{3})},$$

(1)

$$|\Xi_{c}^{(i)}(P, S, S_{c})\rangle = \int \{d^{3}\tilde{p}_{1}\} d^{3}\tilde{p}_{2} | \{d^{3}\tilde{p}_{3}\} 2(2\pi)^{3} \delta^{3}(\tilde{P} - \tilde{p}_{1} - \tilde{p}_{2} - \tilde{p}_{3})$$
\[ \sum_{\lambda_1, \lambda_2, \lambda_3} \psi^{SS}(\vec{p}_1, \lambda_1) C^{\alpha \beta \gamma} F_{c \bar{s} u} \]

\[ s_0(p_1, \lambda_1) c_\beta (p_2, \lambda_2) u_\gamma (p_3, \lambda_3). \]  

(2)

As the spectator approximation cannot be directly applied, dealing with the process seems more complicated. In fact the \( c \) quark which does not undergo a transition and the \( u \) quark play the same role in the transition of \( \Xi_{cc} \rightarrow \Xi_c \), i.e. they are approximately a spectator and their combination can be regarded as an effective subsystem. Actually the \( cc \) and \( us \) are physical subsystems for \( \Xi_{cc} \) and \( \Xi_c \) respectively since they possess definite spin-color quantum number. Baryon is a three-body-system, the total spin can be obtained by different coupling steps and Racah coefficient can transform one to others. By the aforementioned rearrangement of quark flavors the physical states \((cc)u\) and \(c(us)\) are written into sums over effective forms \(c(eu)\) and \(s(eu)\) for \( \Xi_{cc} \) and \( \Xi_c \) respectively. The detailed transformations are [31]

\[ [c^1 c^2]_1[u] = \frac{\sqrt{2}}{2} \left( -\frac{\sqrt{2}}{2} [c^2] [c^1 u]_0 + \frac{1}{2} [c^2] [c^1 u]_1 \right) \]

\[ -\frac{\sqrt{2}}{2} [c^1] [c^2 u]_0 + \frac{1}{2} [c^1] [c^2 u]_1 \]  

(3)

\[ A_0 = \frac{1}{4 \sqrt{P^+ (M_0 m_1 + p_1 \cdot P)(m_2 m_3 M_0^2 + m_3 M_0 p_2 \cdot \bar{P} + m_2 M_0 p_3 \cdot P + p_2 \cdot \bar{P} p_3 \cdot P)} \]

\[ = \frac{1}{4 \sqrt{P^+ M_0^3 (m_1 + e_1)(m_2 + e_2)(m_3 + e_3)}}. \]  

(10)

\[ A_1 = \frac{1}{4 \sqrt{3 P^+ (M_0 m_1 + p_1 \cdot P)(M_0 m_2 + p_2 \cdot \bar{P})(m_0 m_3 + p_3 \cdot \bar{P})}} \]

\[ = \frac{1}{4 \sqrt{3 P^+ M_0^3 (m_1 + e_1)(m_2 + e_2)(m_3 + e_3)}}. \]  

(11)

\[ [s_0]_c = -\frac{1}{2} s [c] u_0 + \frac{\sqrt{3}}{2} s [c] u_1 \]  

(4)

\[ [s_1]_c = \frac{\sqrt{3}}{2} s [c] u_0 + \frac{1}{2} s [c] u_1 \]  

(5)

and then

\[ \psi_{ccu}^{SS}(\vec{p}_1, \lambda_i) = \frac{\sqrt{2}}{2} \left[ \psi_0^{SS}(\vec{p}_1, \lambda_i) + \frac{1}{2} \psi_1^{SS}(\vec{p}_1, \lambda_i) \right] \]

\[ \psi_{c\bar{s}u}^{SS}(\vec{p}_1, \lambda_i) = -\frac{\sqrt{3}}{2} \psi_0^{SS}(\vec{p}_1, \lambda_i) + \frac{\sqrt{3}}{2} \psi_1^{SS}(\vec{p}_1, \lambda_i), \]

\[ \psi_{c\bar{s}u}^{SS}(\vec{p}_1, \lambda_i) = \frac{\sqrt{3}}{2} \psi_0^{SS}(\vec{p}_1, \lambda_i) + \frac{1}{2} \psi_1^{SS}(\vec{p}_1, \lambda_i), \]

with [36]

\[ \psi_{0}^{SS}(\vec{p}_1, \lambda_i) = A_0 \bar{u}(p_3, \lambda_3) \]

\[ \psi_{1}^{SS}(\vec{p}_1, \lambda_i) = A_1 \bar{u}(p_3, \lambda_3) \]

\[ \times \left[ (\vec{P} + M_0) \gamma_5 \right] v(p_2, \lambda_2) \bar{u}(p_1, \lambda_1) u(P, S) \varphi(x_i, k_{i\perp}), \]

\[ \psi_{1}^{SS}(\vec{p}_1, \lambda_i) = A_1 \bar{u}(p_3, \lambda_3) \]

\[ \times \left[ (\vec{P} + M_0) \gamma_5 \gamma_3 \right] v(p_2, \lambda_2) \bar{u}(p_1, \lambda_1) \]

\[ \gamma_3 v(u(P, S) \varphi(x_i, k_{i\perp}), \]

\[ \psi_{c\bar{s}u}^{SS}(\vec{p}_1, \lambda_i) = A_0 \bar{u}(p_3, \lambda_3) \]

\[ \times \left[ (\vec{P} + M_0) \gamma_5 \right] v(p_2, \lambda_2) \bar{u}(p_1, \lambda_1) u(P, S) \varphi(x_i, k_{i\perp}), \]

\[ \psi_{c\bar{s}u}^{SS}(\vec{p}_1, \lambda_i) = A_1 \bar{u}(p_3, \lambda_3) \]

\[ \times \left[ (\vec{P} + M_0) \gamma_5 \gamma_3 \right] v(p_2, \lambda_2) \bar{u}(p_1, \lambda_1) \]

\[ \gamma_3 v(u(P, S) \varphi(x_i, k_{i\perp}), \]

\[ \psi_{c\bar{s}u}^{SS}(\vec{p}_1, \lambda_i) = A_0 \bar{u}(p_3, \lambda_3) \]

\[ \times \left[ (\vec{P} + M_0) \gamma_5 \right] v(p_2, \lambda_2) \bar{u}(p_1, \lambda_1) u(P, S) \varphi(x_i, k_{i\perp}), \]

\[ \psi_{c\bar{s}u}^{SS}(\vec{p}_1, \lambda_i) = A_1 \bar{u}(p_3, \lambda_3) \]

\[ \times \left[ (\vec{P} + M_0) \gamma_5 \gamma_3 \right] v(p_2, \lambda_2) \bar{u}(p_1, \lambda_1) \]

\[ \gamma_3 v(u(P, S) \varphi(x_i, k_{i\perp}), \]

where \( p_1 \) is the momentum of the \( c \) quark which participates in the transition, \( p_2, p_3 \) are the momenta of the spectator quarks \( c \) and \( u \), and \( \lambda_1, \lambda_2, \lambda_3 \) are the helicities of the constituents.

Under the normalization of the state \(|\Xi_{cc}\rangle \) (or \(|\Xi_c^{(i)}\rangle \)),

\[ \langle \Xi_{cc}(P', S', S'_3)|\Xi_{cc}(P, S, S_3)\rangle = 2(2\pi)^3 p^+ \delta^3(\vec{P}' - \vec{P}) \delta S'_3 \delta S \delta S'_3, \]

(8)

and

\[ \int \left( \prod_{i=1}^{3} \frac{dx_i d^2 k_{i\perp}}{2(2\pi)^3} \right) 2(2\pi)^3 \delta (1 - \sum x_i) \delta^2 (\sum k_{i\perp}) \varphi^*(x_i, k_{i\perp}) \varphi(x_i, k_{i\perp}) = 1. \]  

(9)

With a simple manipulation, one can obtain [30]

The spatial wave function is

\[ \varphi(x_i, k_{i\perp}) = \frac{e_1 e_2 e_3}{x_1 x_2 x_3 M_0} \varphi \left( \vec{k}_1, \beta_1 \right) \varphi \left( \vec{k}_2 - \vec{k}_3, \beta_23 \right), \]

(12)

with \( \varphi(\vec{k}, \beta) = 4 \left( \frac{\pi}{\beta^2} \right)^{3/4} \exp \left( -\frac{k^2 - \vec{k}^2}{2\beta^2} \right) \).

2.2 Calculating the form factors of \( \Xi_{cc} \rightarrow \Xi_c \) in LFQM

The leading order Feynman diagram responsible for the weak decay \( \Xi_{cc} \rightarrow \Xi_c \) is shown in Fig. 1. Following the procedures given in Refs. [26,28,34,35] the transition matrix element can be computed with the vertex functions.
of $|\Xi_{cc}(P, S, S_z)\rangle$ and $|\Xi_{c}^\prime(P', S', S_z')\rangle$. The $cu$ subsystem stands as a spectator, i.e. its spin configuration does not change during the transition, so the transition matrix element can be divided into two parts:

$$
\langle \Xi_{c}(P', S'_z) | \bar{s} \gamma^\mu(1-\gamma_5)c | \Xi_{cc}(P, S_z) \rangle \\
= \frac{\sqrt{6}}{4} \langle \Xi_{c}(P', S'_z) | \bar{s} \gamma^\mu(1-\gamma_5)c | \Xi_{cc}(P, S_z) \rangle_0 \\
+ \frac{\sqrt{6}}{4} \langle \Xi_{c}(P', S'_z) | \bar{s} \gamma^\mu(1-\gamma_5)c | \Xi_{cc}(P, S_z) \rangle_1 \tag{13}
$$

with

$$
\langle \Xi_{c}(P', S'_z) | \bar{s} \gamma^\mu(1-\gamma_5)c | \Xi_{cc}(P, S_z) \rangle_0 \\
= \int \left\{ d\tilde{p}_2 \right\} \left\{ d\tilde{p}_3 \right\} \phi_{\Xi_{c}}^\ast(x', k'_\perp) \phi_{\Xi_{c}}(x, k_\perp) \text{Tr} \left\{ (\tilde{P}^0 - M_0) \gamma_5 (p_2 + m_2)(\tilde{P} + M_0) \gamma_5 (p_3 - m_3) \right\} \\
\times \bar{u}(\tilde{P}', S'_z) (\bar{p}_1' + m_1') \gamma^\mu(1-\gamma_5)(\bar{p}_1' + m_1) u(\tilde{P}, S_z), \tag{14}
$$

and

$$
\langle \Xi_{c}(P', S'_z) | \bar{s} \gamma^\mu(1-\gamma_5)c | \Xi_{cc}(P, S_z) \rangle_1 \\
= \int \left\{ d\tilde{p}_2 \right\} \left\{ d\tilde{p}_3 \right\} \phi_{\Xi_{c}}^\ast(x', k'_\perp) \phi_{\Xi_{c}}(x, k_\perp) \text{Tr} \left\{ \gamma_\mu^\ast(\tilde{P}^0 + M_0) \gamma_5 (p_2 + m_2)(\tilde{P} + M_0) \gamma_5 (p_3 - m_3) \right\} \\
\times \bar{u}(\tilde{P}', S'_z) \gamma_{\perp\alpha} \gamma_5 (\bar{p}_1' + m_1') \gamma^\mu(1-\gamma_5)(\bar{p}_1' + m_1) \gamma_{\perp\beta} \gamma_5 u(\tilde{P}, S_z), \tag{15}
$$

where

$$
m_1 = m_c, \quad m_1' = m_s, \quad m_2 = m_c, \quad m_3 = m_u, \\
\gamma_{\perp\alpha} = \gamma_\alpha - \bar{P} \gamma_{\perp\alpha}/M^2, \quad \gamma_{\perp\beta} = \gamma_\beta - \bar{P} \gamma_{\perp\beta}/M^2 \tag{16}
$$

In our earlier paper [30] $f_1^i, g_1^i, f_2^i$ and $g_2^i$ are presented as

$$
f_1^i = \int \frac{ds_1 dk_{1\perp}^2}{(2\pi)^7} \frac{ds_2 dk_{2\perp}^2}{(2\pi)^7} \text{Tr} \left\{ (\bar{P} - M_0) \gamma_\alpha (p_2 + m_2)(\tilde{P} + M_0) \gamma_5 (p_3 - m_3) \right\} \\
\times \frac{1}{M_0^2(m_1 + e_1)(m_2 + e_2)(m_3 + e_3)(m_1' + e_1')(m_2' + e_2')(m_3' + e_3')}. \tag{18}
$$

The form factors for the weak transition $\Xi_{cc} \to \Xi_c$ are defined in the standard way as

$$
\langle \Xi_{c}(P', S', S'_z) | \bar{s} \gamma_{\perp\mu}(1-\gamma_5)c | \Xi_{cc}(P, S, S_z) \rangle \\
= \bar{u}(\tilde{P}', S'_z) \left( \gamma_{\perp\mu} f_1(q^2) + i\sigma_{\mu\nu} q^\nu f_2(q^2) \right) \\
+ \frac{q_{\mu}}{M_{\Xi_{cc}}} f_3(q^2) u(\tilde{P}, S_z), \tag{18}
$$

where $q \equiv P - P'$. For $\langle \Xi_{c}(P', S', S'_z) | \bar{s} \gamma_{\perp\mu}(1-\gamma_5)c | \Xi_{cc}(P, S, S_z) \rangle_0$ and $\langle \Xi_{c}(P', S', S'_z) | \bar{s} \gamma_{\perp\mu}(1-\gamma_5)c | \Xi_{cc}(P, S, S_z) \rangle_1$ the form factors are denoted by $f_1^i, g_1^i, f_2^i$ and $g_2^i$, so we have

$$
f_1 = \frac{\sqrt{6}}{4} f_1 + \frac{\sqrt{6}}{4} f_1^c, \quad g_1 = \frac{\sqrt{6}}{4} g_1 + \frac{\sqrt{6}}{4} g_1^c, \quad f_2 = \frac{\sqrt{6}}{4} f_2 + \frac{\sqrt{6}}{4} f_2^c, \quad g_2 = \frac{\sqrt{6}}{4} g_2 + \frac{\sqrt{6}}{4} g_2^c. \tag{19}
$$
\[ g_1^s = \frac{\int d^2 k_{\perp}^2 \bar{S}(k_{\perp}) \phi_{\Xi}(k_{\perp})}{16 \sqrt{x_1 x_1'}} \times \frac{\phi_{\Xi}(x', k') \phi_{\Xi_1}(x, k_1)}{16 \sqrt{x_1 x_1'}} \times \int \left[ (\bar{P} - M_0) \gamma^+ \gamma_5 (\bar{P} - M_0)_{\gamma^+} \gamma_5 (\bar{p} + m_1') \gamma^+ (\bar{p} + m_1)_{\gamma^+} \gamma_5 \right] \times \frac{8 P^+ P^+}{2 (2\pi)^2}, \]

\[ f_1^s = \frac{-i}{q_1'} \int d^2 k_{\perp}^2 \times \frac{d^2 k_{\perp}^2}{2 (2\pi)^2} \times \frac{\phi_{\Xi}(x', k') \phi_{\Xi_1}(x, k_1)}{16 \sqrt{x_1 x_1'}} \times \int \frac{g_1}{M_{\Xi_1}} \]

\[ g_1^s = \frac{i}{q_1'} \int d^2 k_{\perp}^2 \times \frac{d^2 k_{\perp}^2}{2 (2\pi)^2} \times \frac{\phi_{\Xi}(x', k') \phi_{\Xi_1}(x, k_1)}{16 \sqrt{x_1 x_1'}} \times \int \frac{f_1}{M_{\Xi_1}} \]

\[ f_1^s = \frac{-i}{q_1'} \int d^2 k_{\perp}^2 \times \frac{d^2 k_{\perp}^2}{2 (2\pi)^2} \times \frac{\phi_{\Xi}(x', k') \phi_{\Xi_1}(x, k_1)}{48 \sqrt{x_1 x_1'}} \times \int \frac{1}{M_{\Xi_1}} \]

\[ g_1^s = \frac{i}{q_1'} \int d^2 k_{\perp}^2 \times \frac{d^2 k_{\perp}^2}{2 (2\pi)^2} \times \frac{\phi_{\Xi}(x', k') \phi_{\Xi_1}(x, k_1)}{48 \sqrt{x_1 x_1'}} \times \int \frac{1}{M_{\Xi_1}} \]

3 Numerical results

3.1 The form factors for \( \Xi_{cc} \rightarrow \Xi_c \) and \( \Xi_{cc} \rightarrow \Xi'_c \)

Before we start to evaluate those form factors numerically the parameters in the concerned model are needed to be determined. The masses of quarks given in Ref. [37] are collected in Table 1. The masses of \( \Xi_c \) and \( \Xi'_c \) are taken from [38]. Indeed, we know very little about the parameters \( \beta_1 \) and \( \beta_{23} \) in the wave function of the initial baryon and \( \beta_1' \) and \( \beta_{23}' \) in that of the final baryon. Generally the reciprocal of \( \beta \) is related to the electrical radius of two constituents. Since the strong interaction between \( q \) and \( q' \) is a half of that between \( q \overline{q} \), for a Coulomb-like potential one can expect the electrical radius of \( q(q') \) to be \( 1/\sqrt{2} \) times that of \( q\overline{q} \), i.e. \( \beta_{qq'} \approx \sqrt{2/2} \beta_{q\overline{q}} \). In Ref. [39] considering the binding energy the authors obtained the same results. In our early paper for a compact \( q\overline{q} \) system we find \( \beta_{qq'} = 2.9 \beta_{q\overline{q}} \), i.e. the electrical radius of \( q\overline{q} \) to be 1/2.9 times that of \( q\overline{q} \). In terms of the knowledge the estimate is \( \beta_{1(1u)} \approx 2.9 \beta_{1(1u',1u''}, \beta_{1(1u',1u'')} \approx \sqrt{2} \beta_{1(1u',1u'')} \) where \( \beta_{1(1u), \beta_{1(1u')}, \beta_{1(1u'')}} \) were obtained for the meson case [37]. With these parameters we calculate the form factors and make theoretical predictions on the transition rates.

Since these form factors \( f_i^{\pm(0)} (i = 1, 2) \) and \( g_i^{\pm(0)} (i = 1, 2) \) are evaluated in the frame \( q^+ = 0 \) i.e. \( q^2 = -q_\perp^2 \leq 0 \) (the
Table 1 The quark mass and the parameter \( \beta \) (in units of GeV)

| \( m_c \) | \( m_s \) | \( m_u \) | \( \beta_{\text{[cu]}1} \) | \( \beta_{\text{[cu]}2} \) | \( \beta_{\text{[cu]}3} \) |
|---|---|---|---|---|---|
| 1.5 | 0.5 | 0.25 | 1.898 | 0.760 | 0.656 |

Table 2 The form factors given in the three-parameter form

| \( F \) | \( F(0) \) | \( a \) | \( b \) |
|---|---|---|---|
| \( f_1^i \) | 0.586 | 0.640 | -0.194 |
| \( f_2^i \) | -0.484 | 1.23 | -0.222 |
| \( g_1^i \) | 0.420 | -0.0142 | 0.0748 |
| \( g_2^i \) | 0.228 | 1.02 | -0.101 |
| \( f_1^v \) | 0.610 | 1.18 | -0.0492 |
| \( f_2^v \) | 0.463 | 1.32 | -0.0642 |
| \( g_1^v \) | -0.140 | -0.501 | 0.274 |
| \( g_2^v \) | 0.0673 | 0.00936 | 0.327 |

Table 3 The form factors given in polynomial form

| \( F \) | \( F(0) \) | \( a' \) | \( b' \) | \( c' \) |
|---|---|---|---|---|
| \( f_1^i \) | 0.586 | 1.57 | 1.59 | 0.704 |
| \( f_2^i \) | -0.484 | 2.06 | 2.42 | 1.17 |
| \( g_1^i \) | 0.420 | 0.983 | 0.692 | 0.258 |
| \( g_2^i \) | 0.228 | 1.90 | 2.07 | 0.960 |
| \( f_1^v \) | 0.610 | 2.04 | 2.27 | 1.06 |
| \( f_2^v \) | 0.463 | 2.14 | 2.49 | 1.19 |
| \( g_1^v \) | -0.140 | 0.422 | 0.0931 | 0.00632 |
| \( g_2^v \) | 0.0673 | 0.925 | 0.245 | -0.0862 |

space-like region) one needs to extrapolate them into the time-like region. In Ref. [35] a three-parameter form was employed

\[
F(q^2) = F(0) \left[ 1 - \frac{q^2}{M^2_{\Xi_c}} \right] \left[ 1 - a \left( \frac{q^2}{M^2_{\Xi_c}} \right) + b \left( \frac{q^2}{M^2_{\Xi_c}} \right)^2 \right].
\]  

(22)

where \( F(q^2) \) denotes the form factors \( f_s^{(v)} \) and \( g_s^{(v)} \). Using the form factors in the space-like region we may calculate numerically the parameters \( a, b \) and \( F(0) \) in the un-physical region; namely fix \( F(q^2 \leq 0) \). As discussed in previous section, these form factors are extended into the physical region with \( q^2 \geq 0 \) through Eq. (22). The fitted values of \( a, b \) and \( F(0) \) in the form factors \( f_1, f_2, g_1, \) and \( g_2 \) are presented in Table 2. The dependence of the form factors on \( q^2 \) is depicted in Fig. 2.

Since the form factors \( f_1^i, f_2^i, f_1^v, f_2^v \) rise too quickly after \( q^2 > 6 \text{ GeV} \) which are very different from the results in other \( \frac{1}{2} \rightarrow \frac{1}{2} \) transitions [7, 8, 26, 30, 35], we suggest to use a polynomial to parameterize these form factors

\[
F(q^2) = F(0) \left[ 1 + a' \left( \frac{q^2}{M^2_{\Xi_c}} \right) + b' \left( \frac{q^2}{M^2_{\Xi_c}} \right)^2 + c' \left( \frac{q^2}{M^2_{\Xi_c}} \right)^3 \right].
\]  

(23)

The fitted values of \( a', b', c' \) and \( F(0) \) in the form factors are presented in Table 3. The dependence of the form factors on \( q^2 \) is depicted in Fig. 3. The figures of these form factors are apparently more smooth which are in analog to those for \( \frac{1}{2} \rightarrow \frac{1}{2} \) transition [7, 8, 26, 30, 35]. With the form factors which are polynomials of powers of \( \frac{q^2}{M^2_{\Xi_c}} \), we estimate the decay rates of semi-leptonic and no-leptonic decays.

Fig. 2 The dependence of form factors \( f_1^i, f_2^i, g_1^i, \) and \( g_2^i \) in a three-parameter form on \( q^2 \) (a) and the dependence of the form factors \( f_1^v, f_2^v, g_1^v, \) and \( g_2^v \) on \( q^2 \) (b)
The dependence of form factors $f_{s1}$, $f_{s2}$, $g_{s1}$ and $g_{s2}$ in polynomial form on $q^2$ (a) and The dependence of form factors $f_{v1}$, $f_{v2}$, $g_{v1}$ and $g_{v2}$ on $q^2$ (b).

Differential decay rates $d\Gamma/d\omega$ for the decay $\Xi_{cc} \rightarrow \Xi_c l\bar{\nu}_l$ (a) and $\Xi_{cc} \rightarrow \Xi'_c l\bar{\nu}_l$ (b).

Table 4 The width (in unit $10^{12}$ s$^{-1}$) of $\Xi_{cc} \rightarrow \Xi_c l\bar{\nu}_l$ (left) and $\Xi_{cc} \rightarrow \Xi'_c l\bar{\nu}_l$ (right)

|           | $\Gamma$  | $R$     | $\Gamma$  | $R$     |
|-----------|-----------|---------|-----------|---------|
| This work | $0.100 \pm 0.015$ | $7.14 \pm 0.61$ | $0.0995 \pm 0.0091$ | $1.34 \pm 0.07$ |
| Ref. [31] | $0.173$   | $9.99$  | $0.193$   | $1.42$  |
| Ref. [40] | $0.092 \pm 0.014$ | $22 \pm 8$ | $0.032 \pm 0.006$ | $1.1 \pm 0.2$ |
| Ref. [41] | $0.106$   | $-$     | $0.147$   | $-$     |

3.2 Semi-leptonic decays of $\Xi_{cc} \rightarrow \Xi_c + l\bar{\nu}_l$ and $\Xi_{cc} \rightarrow \Xi'_c + l\bar{\nu}_l$

Using the form factors obtained in last subsection, we evaluate the rate of $\Xi_{cc} \rightarrow \Xi_c l\bar{\nu}_l$ and $\Xi_{cc} \rightarrow \Xi'_c l\bar{\nu}_l$. The differential decay widths $d\Gamma/d\omega (\omega = P \cdot P'/M_M)$ are depicted in Fig. 4. Our predictions on the total decay widths and the ratio of the longitudinal to transverse decay rates $R$ are all listed in Table 4. Deliberately letting the quark masses and all $\beta$s fluctuate up to 5%, one can estimate possible theoretical uncertainties of the numerical results.

In Ref. [31] the authors employ three-parameter parametrization scheme to fix these form factors and their predictions on $\Gamma(\Xi_{cc} \rightarrow \Xi_c l\bar{\nu}_l)$ and $\Gamma(\Xi_{cc} \rightarrow \Xi'_c l\bar{\nu}_l)$ are almost twice larger than our results presented in Table 4. Our estimate on the ratio of longitudinal to transverse decay rates $R$ for $\Xi_{cc} \rightarrow \Xi'_c l\bar{\nu}_l$ is close to 1.42 given in [31] but that of $\Xi_{cc} \rightarrow \Xi_c l\bar{\nu}_l$ slightly deviates from theirs. One also notices that the predictions on $\Gamma(\Xi_{cc} \rightarrow \Xi_c l\bar{\nu}_l)$ are close to each
others in different approaches (not include the prediction in [31]) but those on \(\Gamma(\Xi_{cc} \to \Xi_c' l\bar{\nu})\) deviate from each others a little bit.

3.3 Non-leptonic decays of \(\Xi_{cc} \to \Xi_c + M\) and \(\Xi_{cc} \to \Xi_c' + M\)

On the theoretical aspect, calculating the concerned quantities of the non-leptonic decays seems to be more complicated than for semi-leptonic processes. Our theoretical framework is based on the factorization assumption, namely the hadronic transition matrix element is factorized into a product of two independent hadronic matrix elements of currents,

\[
\langle \Xi_c' (P', S'_c) | H | \Xi_{cc} (P, S_c) \rangle = \frac{G_F V_{cs} V^{*}_{cs}}{\sqrt{2}} \left( \langle M | \bar{q}' \gamma^\mu (1 - \gamma_5) q | 0 \rangle \right) \times \left( \langle \Xi_c' (P', S'_c) | \bar{s} \gamma^\mu (1 - \gamma_5) s | \Xi_{cc} (P, S_c) \rangle \right),
\]

where the term \(\langle M | \bar{q}' \gamma^\mu (1 - \gamma_5) q | 0 \rangle\) is determined by a decay constant and the transition \(\Xi_{cc} \to \Xi_c'\) is evaluated in the previous sections. Since the decay \(\Xi_{cc} \to \Xi_c' + M\) is the so-called color-favored portal, the factorization should be a plausible approximation. The results on these non-leptonic decays can be checked in the coming measurements and the validity degree of the obtained form factors in the doubly charmed baryon would be further examined.

From the results shown in Table 5, we find \(\Xi_{cc} \to \Xi_c' \pi\) and \(\Xi_{cc} \to \Xi_c' \rho\) are the main two-body decay channels for \(\Xi_{cc}\). Especially \(\Gamma(\Xi_{cc} \to \Xi_c' \rho)\) is close to \(\Gamma(\Xi_{cc} \to \Xi_c \pi)\) which should also be observed in LHCb soon. The predictions in other approaches are listed in Table 6. The theoretical predictions on the widths calculated in Ref. [31] are two or three times larger than ours. The results on \(\Gamma(\Xi_{cc} \to \Xi_c \pi)\) and \(\Gamma(\Xi_{cc} \to \Xi_c' K)\) in Ref. [40] are close to ours but there exists still a discrepancy for other channels. This should also be tested in the future more precise measurements.

4 Conclusions and discussions

In this paper we calculate the transition rate of \(\Xi_{cc} \to \Xi_c'\) in the light front quark model. For the baryons \(\Xi_{cc}\) and \(\Xi_c'\) we employ a three-quark picture instead of the quark-diquark one for calculating the transition rates. Generally, two charm quarks constitute a diquark which joins the light quark to constitute the baryon and this widely accepted scenario determines the wave function of \(\Xi_{cc}\). Because two \(c\) quarks are identical heavy flavor particles in a color anti-triplet, it must be a vector boson, whereas, in \(\Xi_c'\) the light \(us\) pair is seen as a diquark. In the concerned process, the diquark in the initial state is different from that in the final state, so that the \(cc\) diquark is no longer a spectator and the diquark picture cannot be directly applied in this case. However in the process the charm quark which does not undergo a transition and the \(u\) quark are approximatively spectators when higher order QCD effects are neglected, so the \(cu\) pair can be regarded as an effective subsystem. Baryon is a three-body-system whose total spin can be obtained through different schemes just in analog to the L–S coupling and \(J–J\) coupling in the quantum mechanics. Making a Racah transformation we can convert one configuration into another. The Racah coefficients of such transformation determines the correlation between the two configuration \((cc)u\) and \(c(uc)\). However, one is noted that the subsystem of \((cu)\) is not a diquark in a rigorous meaning and between the two constituents there exists a relative momentum. Thus in the vertex function of the three-body system there exists an inner degree of free-
dom for the cu subsystem which just manifests by the relative momentum.

We calculate the form factors for the transitions $\Xi_{cc} \rightarrow \Xi_c$ and $\Xi_{cc} \rightarrow \Xi'_c$ in the space-like region. When we extrapolate them to physical region we find three-parameter form is not a good choice, so that we suggest to parameterize these form factors in terms of polynomials. Using these form factors we calculate the rates of semileptonic decays $\Xi_{cc} \rightarrow \Xi_c l \bar{\nu}_l$ and $\Xi_{cc} \rightarrow \Xi'_c l \bar{\nu}_l$. We find that $\Gamma(\Xi_{cc} \rightarrow \Xi_c l \bar{\nu}_l)$ is very close to $\Gamma(\Xi_{cc} \rightarrow \Xi'_c l \bar{\nu}_l)$ but our results on $\Gamma(\Xi_{cc} \rightarrow \Xi_c l \bar{\nu}_l)$ and $\Gamma(\Xi_{cc} \rightarrow \Xi'_c l \bar{\nu}_l)$ are about a half of those in Ref. [31]. The ratio of the longitudinal to transverse decay rates $R$ for $\Xi_{cc} \rightarrow \Xi_c l \bar{\nu}_l$ and $\Xi_{cc} \rightarrow \Xi'_c l \bar{\nu}_l$ are roughly consistent with the predictions of [31]. With the same theoretical framework, we also evaluate the rates of several non-leptonic decays. Our numerical results indicate that the channel $\Xi_{cc} \rightarrow \Xi_c \pi$ has the largest branching ratio for the transition $\Xi_{cc} \rightarrow \Xi'_c$; instead, the channel $\Xi_{cc} \rightarrow \Xi'_c \rho$ is the main channel for the transition $\Xi_{cc} \rightarrow \Xi'_c$. The predictions of Ref. [31] are two or three times larger than ours since in the two approaches the different pictures about the inner structure are adopted. We urge the experimentalists to make more accurate measurements on the channels, and the data would tell us which approach is closer to the reality. Definitely, the theoretical studies on the double-heavy baryons are helpful for getting a better understanding about the quark model and the non-perturbative QCD effects. Especially, the scenarios adopted for investigating the double-charm baryons can be generalized to study the baryons with $bb$ and $bc$ components which will be measured by the LHCb collaboration and other collaborations in the near future.

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Appendix A: Semi-leptonic decays of $B_1 \rightarrow B_2 l \bar{\nu}_l$

The helicity amplitudes are related to the form factors for $B_1 \rightarrow B_2 l \bar{\nu}_l$ through the following expressions [42–44]

$$H^V_{\lambda,0} = \frac{\sqrt{Q^2}}{\sqrt{q^2}} \left( (M_{B_2} + M_{B_1}) f_1 - \frac{q^2}{M_{B_1}} f_2 \right),$$

$$H^V_{\lambda,1} = \frac{\sqrt{Q^2}}{\sqrt{q^2}} \left( -f_1 + \frac{M_{B_1} + M_{B_2}}{M_{B_1}} f_2 \right),$$

$$H^A_{\lambda,0} = \frac{\sqrt{Q^2}}{\sqrt{q^2}} \left( (M_{B_1} - M_{B_2}) g_1 - \frac{q^2}{M_{B_1}} g_2 \right),$$

$$H^A_{\lambda,1} = \frac{\sqrt{Q^2}}{\sqrt{q^2}} \left( -g_1 - \frac{M_{B_1} - M_{B_2}}{M_{B_1}} g_2 \right).$$

where $Q^2 = 2(p \cdot p' \pm M_{B_1} M_{B_2})$ and $M_{B_1}$ ($M_{B_2}$) represents $M_{\Xi_{cc}}$ ($M_{\Xi_{cc}}$). The amplitudes for the negative helicities are obtained in terms of the relation

$$H^V_{-\lambda',-\lambda} = \pm H^V_{\lambda',\lambda},$$

where the upper (lower) index corresponds to $V(A)$. The helicity amplitudes are

$$H_{\lambda',\lambda} = H^V_{\lambda',\lambda} - H^A_{\lambda',\lambda}.$$  

The helicities of the $W$-boson $\lambda$ can be either 0 or 1, which correspond to the longitudinal and transverse polarizations, respectively. The longitudinally ($L$) and transversely ($T$) polarized rates are respectively[42–44]

$$\frac{d\Gamma_L}{dq^2} = \frac{G_F^2 |V_{cb}|^2 q^2 p_c M_{B_2}}{(2\pi)^3 12M_{B_1}} \left[ |H^L_{\frac{1}{2},0}|^2 + |H^L_{-\frac{1}{2},0}|^2 \right],$$

$$\frac{d\Gamma_T}{dq^2} = \frac{G_F^2 |V_{cb}|^2 q^2 p_c M_{B_2}}{(2\pi)^3 12M_{B_1}} \left[ |H^T_{\frac{1}{2},1}|^2 + |H^T_{-\frac{1}{2},-1}|^2 \right].$$

The ratio of the longitudinal to transverse decay rates $R$ is defined by

$$R = \frac{\Gamma_L}{\Gamma_T} = \frac{\int_{q_{min}}^{q_{max}} dq^2 p_c |H^L_{\frac{1}{2},0}|^2 + |H^L_{-\frac{1}{2},0}|^2 \right]}{\int_{q_{min}}^{q_{max}} dq^2 p_c |H^T_{\frac{1}{2},1}|^2 + |H^T_{-\frac{1}{2},-1}|^2 \right]}.\) (A5)$$

Appendix B: $B_1 \rightarrow B_2 M$

In general, the transition amplitude of $B_1 \rightarrow B_2 M$ can be written as

$$M(B_1 \rightarrow B_2 M) = \bar{u}_{B_2} \left( A + B \gamma_5 \right) u_{B_1},$$

$$M(B_1 \rightarrow B_2 V) = \bar{u}_{B_2} e^{i\phi} \left[ A_1' \gamma'_5 + A_2 (p_{\mu}) \gamma_5 + B_1' \gamma_\mu + B_2 (p_{\mu}) \right] u_{B_1},$$

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where $\varepsilon^\alpha$ is the polarization vector of the final vector or axial-vector mesons. Including the effective Wilson coefficient $a_1 = c_1 + c_2/N_c$, the decay amplitudes in the factorization approximation are \[ A = \lambda f_P (M_{B_1} - M_{B_2}) f_1 (M^2), \]
\[ B = \lambda f_P (M_{B_1} + M_{B_2}) g_1 (M^2), \]
\[ A_1 = -\lambda f_P M \left[ g_1 (M^2) + g_2 (M^2) \frac{M_{B_1} - M_{B_2}}{M_{B_1}} \right], \]
\[ A_2 = -2\lambda f_P M \frac{g_2 (M^2)}{M_{B_1}}, \]
\[ B_1 = \lambda f_P M \left[ f_1 (M^2) - f_2 (M^2) \frac{M_{B_1} + M_{B_2}}{M_{B_1}} \right], \]
\[ B_2 = 2\lambda f_P M \frac{f_2 (M^2)}{M_{B_1}}. \] (B2)

where $\lambda = \frac{G_F}{\sqrt{2}} V_{cb} V_{cq}^* a_1$ and $M$ is the meson mass. Replacing $P, V$ by $S$ and $A$ in the above expressions, one can easily obtain similar expressions for scalar and axial-vector mesons. The decay rates of $B_1 \rightarrow B_2 P(S)$ and up-down asymmetries are \[ \Gamma = \frac{p_c}{8\pi} \left[ \frac{(M_{B_1} + M_{B_2})^2 - M^2}{M_{B_1}^2} |A|^2 + \frac{(M_{B_1} - M_{B_2})^2 - m^2}{M_{B_1}^2} |B|^2 \right], \]
\[ \alpha = -\frac{2\kappa \text{Re}(A^* B)}{|A|^2 + \kappa^2 |B|^2}. \] (B3)

where $p_c$ is the $B_2$ momentum in the rest frame of $B_1$ and $m$ is the mass of pseudoscalar (scalar). For $B_1 \rightarrow B_2 V(A)$ decays, the decay rate and up-down asymmetries are \[ \Gamma = \frac{p_c (E_{B_2} + M_{B_1})}{4\pi M_{B_1}} \left[ 2 \left( |S|^2 + |P_2|^2 \right) + \frac{\varepsilon^2}{m^2} \left( |S + D|^2 + |P_1|^2 \right) \right], \]
\[ \alpha = \frac{4m^2 \text{Re}(S^* P_2) + 2\varepsilon^2 \text{Re}(S + D)^* P_1}{2m^2 \left( |S|^2 + |P_2|^2 \right) + \varepsilon^2 \left( |S + D|^2 + |P_1|^2 \right)}. \] (B4)

where $\varepsilon (m)$ is energy (mass) of the vector (axial vector) meson, and \[ S = -A_1, \]
\[ P_1 = -\frac{p_c}{\varepsilon} \left( \frac{M_{B_1} + M_{B_2}}{E_{B_2} + M_{B_2}} B_1 + M_{B_1} B_2 \right), \]
\[ P_2 = \frac{p_c}{E_{B_2} + M_{B_2}} B_1, \]
\[ D = -\frac{p_c^2}{\varepsilon(E_{B_2} + M_{B_2})} (A_1 - M_{B_1} A_2). \] (B5)

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