Reflection of a shock wave from a finely dispersed medium of low concentrations

K I Bashirova¹, C I Mikhaylenko²

¹Ufa State Aviation Technical University, Ufa, Russia
²Mavlyutov Institute of Mechanics, UFRC RAS, Ufa, Russia

E-mail: const@uimech.org

Abstract. A model of a shock tube is prepared to study the behaviour of a shock wave in a layer of light elastic granular particles. Computational experiments are made using the OpenFOAM package. The prepared model allows carrying out simulations for pressure between $10^5$ and $10^4$ Pa and concentration of particles of 20%. Results are obtained on pressures in different time moments, depending on different densities of granular particles. An increase of the density of the granular particles is shown to decrease the blurring of the boundary between the media. It is also noted that a decrease in the density leads to double pressure peaks.

1. Introduction
The shock tube is a long cylinder divided by a diaphragm made of easily collapsing material. Behind the diaphragm there is gas under high pressure. In this way, at the initial moment of time, there are two homogeneous regions with significantly different pressures, temperatures, and densities.

Shock tubes appeared at the end of the 19th century, and since then they have been used as an independent experimental facility for various studies. For example, the possibility of producing gas heated to 10,000 K is exploited in chemical experiments. In astrophysical studies, the spectra of stars are compared with the spectra obtained on shock tubes. Shock waves occur in a supersonic flow, which leads to an increase in the resistance of a moving body and to a temperature change. In this case, the load is very large, which can destroy the aircraft, if appropriate measures are not taken.

Currently, many experimental studies of shock wave propagation have been carried out. Some of them are associated with numerical and analytical analysis of the wave passage through a bubble system [1]. In particular, the effect of gas distribution on attenuation of the wave effect on a wall is studied in [2]. In [3], the transmission and scattering of waves in a bubble layer are numerically analyzed. Significant scattering is shown near the natural frequencies of the bubbles, as well as nonlinear effects of the wave, even at small amplitudes.

In article [4], water foam barriers absorbing blast waves are considered. In the work, the results of numerical simulation are compared with shock tube tests. The effect of the foam on the propagation velocity of the shock wave and pressure after its passage, as well as the distribution of the liquid in the space filled with the medium with foam, is also studied.

In this paper, we describe the studies of the behavior of a shock wave in a layer of light elastic granular particles. The aim of the work is to obtain a correct three-dimensional model of the process, since most studies use one-dimensional or plane-parallel models.
2. The model and methods

2.1. The simulated area

In the paper, a computational simulation of a shock tube is carried out. As a toolbox for the development, the OpenFOAM package is used. OpenFOAM is a free and open-source software designed to solve the problems of continuum mechanics and numerical simulation in hydrodynamics.

A shock tube is considered in the work, the length of which is 60 cm and a diameter is 5 cm. Using the initial conditions, the tube is divided into two areas using the setFields utility: normal (100,000 Pa) and low (10,000 Pa) pressure. In real experimental setups, a thin diaphragm is located at the interface. In accordance with figure 1 at zero time, the normal pressure region occupies the upper third of the total tube volume. In the lower 10 centimeters of the tube there is a granular layer. The volume concentration of the backfill is 0.2. The location of granular layer is also set by the setFields utility. Both phases are considered as interpenetrating continuum. The grain diameter is $3 \cdot 10^{-8}$ m. In the calculations, various values of the density of the dispersed medium are used: from 800 to 3000 kg/m$^3$.

![Figure 1. Initial pressure in the shock tube.](image_url)

To solve this problem, a standard solver of the OpenFOAM twoPhaseEulerFoam package is used. Solver is intended for modeling systems from incompressible fluid phases and dispersed media.

The slip condition for the air velocity on all walls of the tube and that for particles at all boundaries are specified as boundary conditions. For pressure, a zero-gradient condition is specified on the shock tube walls. It is taken into account that each phase is continuous with the chosen approach to solving the problem.

2.2. The mathematical model

To describe the process in the shock tube, we write the standard system of equations for the carrier phase: continuity, impulses, and energy with the equation of state of an ideal gas (1)—(4).

\[
\frac{\partial \rho_{(1)}}{\partial t} = -\frac{\partial}{\partial x_i} \left( \rho_{(1)} v_{(1)i} \right) = 0, \quad (1)
\]

\[
\frac{\partial \left( \rho_{(1)} v_{(1)j} \right)}{\partial t} + \frac{\partial \left( \rho_{(1)} v_{(1)j} v_{(1)i} \right)}{\partial x_i} = -\alpha_{(1)} \frac{\partial p}{\partial x_j} + \frac{\partial \tau_{(1)ij}}{\partial x_i} + \rho_{(1)} g_j + F_{(21)j}, \quad (2)
\]

\[
\frac{\partial \left( \rho_{(1)} e \right)}{\partial t} + \frac{\partial \left( \rho_{(1)} e v_{(1)i} \right)}{\partial x_i} = -\alpha_{(1)} \frac{\partial p v_{(1)i}}{\partial x_i} + \frac{\partial r_{(1)j} v_{(1)i}}{\partial x_j} + \rho_{(1)} v_{(1)i} g_i, \quad (3)
\]

\[
e = \frac{p}{(\gamma-1) \rho_{(1)}}. \quad (4)
\]
We also write the system of equations for the carried granular phase: equations of continuity and impulses (5)—(6):

\[
\frac{\partial \rho_{(2)}}{\partial t} + \frac{\partial (\rho_{(2)}v_{(2)}\rho_{(2)})}{\partial x_i} = 0,
\]

\[
\frac{\partial (\rho_{(2)}v_{(2)}\rho_{(2)}\gamma)}{\partial t} + \frac{\partial (\rho_{(2)}v_{(2)}\gamma \rho_{(2)})}{\partial x_i} = -\alpha_{(2)} \frac{\partial p}{\partial x_i} + \frac{\partial \tau_{(2)}ji}{\partial x_i} + \rho_{(2)} g_j + F_{(12)} j.
\]

The following notations are used in equations (1)—(6). \(\rho_{(k)}\) is the effective density of the k-th phase associated with the true density \(\rho_{(k)0}\) by the relation \(\rho_{(k)} = \alpha_{(k)} \rho_{(k)0}\); \(\alpha_{(k)}\) is the volumetric concentration of the k-th phase, where \(k = 1\) for the carrier phase, \(k = 2\) for the dispersed carried phase; \(v_{(k)i}\) are the velocity vector components of a k-th phase, \(i = 1, 2, 3\); \(p\) is the pressure; \(g_i\) is the acceleration of the gravity vector component; \(E = e + \frac{(\bar{v}_{(1)}^2)}{2}\) is the specific total energy and \(e\) is the specific internal energy of the gas phase; \(\gamma\) is the adiabatic exponent; and \(\tau_{(k)ij}\) is the viscous stress tensor, calculated as it is shown in equation (7):

\[
\tau_{(k)ij} = \mu_{(k)} \left[ \left( \frac{\partial v_{(k)i}}{\partial x_j} + \frac{\partial v_{(k)j}}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \frac{\partial v_{(k)i}}{\partial x_j} \right],
\]

here \(\mu_{(k)}\) is the dynamic viscosity of k-th phase; \(\delta_{ij}\) is the Kronecker symbol; \(F_{(km)i}\) are the components of the vector of interfacial forces acting from the k-th phase to the m-th;

Interfacial forces are related by the ratio \(F^{21} = -F^{12}\), and the force of interphase interaction is given in the equation (8):

\[
F_{(km)i} = \frac{3}{4} \alpha_k \alpha_m \rho_{(2)} C_D \frac{d}{\bar{v}_{(1)} - \bar{v}_{(2)}} |v_{(k)i} - v_{(m)i}|.
\]

Here \(C_D = \frac{4 \pi d^2 \mu_{(1)}}{\alpha_{(1)} \alpha_{(2)} \rho_{(1)} |\bar{v}_{(1)} - \bar{v}_{(2)}|}\) is the drag coefficient calculated based on Stokes force [5]; and \(d\) is a grain diameter.

Viscosity of the granular phase \(\mu_{(2)}\) described by Bagnold’s empirical equation [6]:

\[
\mu_{(2)} = \beta \left( \frac{\alpha_{(1)} - \alpha_{(2)0}}{\alpha_{(2)} - \alpha_{(2)*}} \right)^{\alpha_{(2)0} > \alpha_{(2)0} < \alpha_{(2)*}}.
\]

Here \(\beta\) is the empirical coefficient; \(\alpha_{(2)0}\) is the volumetric concentration of the dispersed phase starting from which the viscosity is nonzero; and \(\alpha_{(2)*}\) is the volumetric concentration, starting from which the granular medium is stationary.

Since the granular medium is modeled by a finite-difference grid, the presented model is sufficient to describe the phenomenon under consideration.

3. Results and discussion

The wave motion in the tube is calculated to be 0.0015 s, which corresponds to the passage of the wave to the lower edge of the setup through a finely dispersed medium [7]. The graphs below correspond to two points in time: during entry into the dispersed layer (see figure 2 (a)) and during passage through the dispersed layer (see figure 2 (b)), but before reflection from the bottom of the tube.

After analyzing the above graphs, we can conclude that light elastic granular particles, due to their movement, accelerate the reflected wave and form a double peak, starting at time point 0.00106. Moreover, the higher is the density of the granules, the closer is the maximum value of the first peak to the same value for the second peak. In addition, a decrease in the density of the medium leads to an increase in pressure at the second peak.

The passage of a shock wave through a finely dispersed scattered medium leads to a significant blurring of the initial interface between the media, as shown in figure 3.
Figure 2. Pressure distribution ($p$) relative to distance ($y$) at time points 0.00106 (a) and 0.00128 (b) for different densities ($\rho$) of the granular medium.
Figure 3. The distribution of the granular medium in the tube after the passage of the shock wave

For lighter granules, the blurring is more significant, therefore, the pressure peak is wider than for heavier granules due to the larger volume filled with granules. To analyze double pressure peaks, we plotted the dependence of the location of the pressure maxima on the density of the finely dispersed medium at time moments of 0.00106 s and 0.00126 s (see figures 4 (a) and (b), respectively).

![Graph](image1.png)

**a**

![Graph](image2.png)

**b**

Figure 4. The location of the peak of maximum pressure depending on the density of the granular medium at time points a) 0.00106, b) 0.00126 s.

At the time instant of 0.00106 s, there are no double peaks; therefore, figure 4 (a) presents one curve of pressure maxima location. Figure 4 (b) shows two curves of the arrangement of pressure maxima corresponding to two maxima. It is seen that with increasing density of the granular medium, the double pressure peaks gradually merge into one, which is explained by the large weight of the granules.

After analyzing the graphs, we can conclude that light elastic granular particles, due to their movement, accelerate the reflected wave and form a double peak, starting at time point of 0.00106 s. Moreover, the higher is the density of the granules, the closer the maximum value of the first peak to the same value for the second peak. Also, a decrease in the density of the medium leads to an increase in pressure at the second peak.
The passage of a shock wave through a finely dispersed scattered medium leads to a substantial blurring of the initial interface of the media. For lighter granules, the blurring is more significant, therefore, the pressure peak is wider than for heavier granules due to the larger volume filled with granules.

Conclusions
In the work, model calculations have been carried out for the passage of a shock wave through a weakly concentrated granular medium using various densities of a granular medium. Comparative graphs have been constructed for a dispersed medium with different densities. An analysis of the results demonstrates that the obtained model accurately shows a decrease in the blurring of the boundary of the media with increasing grain density, which is explained by the large weight. At lower densities, double peaks are shown to appear on the pressure distribution diagrams along the tube and gradually merge into one peak with increasing density.

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