Revisiting D-branes in $AdS_3 \times S^3$

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Abstract

We find the most general supergravity solution in an $AdS^3 \times S^3$ background preserving an $AdS_2 \times S^2$ symmetry and half the supersymmetries. Contrary to previous expectations from boundary state arguments, it is shown that no solutions exist containing localized brane sources.

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I. INTRODUCTION

The AdS/CFT correspondence has led to a much deeper understanding of nonperturbative string theory. However, most analyses of the correspondence have been restricted to the supergravity approximation. It is important to understand more about the truly stringy aspects of the theory.

Major progress was made recently in understanding the quantization of strings in these backgrounds. Much of this progress relies on the fact that the $AdS_3 \times S^3 \times T^4$ background with NS-NS fluxes can be described as a Wess-Zumino-Witten (WZW) theory, where the $AdS_3$ factor corresponds to the noncompact group manifold $SL(2,R)$, and the sphere to a compact group manifold $SU(2)$. This allows the classification of the string states in terms of representations of these groups.

One can also attempt to construct D-branes in $AdS_3 \times S^3$ using the method of conformal field theory boundary states. This was done for the case of compact groups long ago, and the D-branes can be classified. However, the noncompact case is much more subtle.

The basic idea is to treat the D-brane as a boundary state $|B\rangle$ satisfying the condition $(J + J^\dagger)|B\rangle = 0$. (Here $J$ is the worldsheet current of the theory.) This can be solved in the compact case in terms of Ishibashi states. The D-branes are then linear combinations of Ishibashi states that satisfy the Cardy condition. For compact groups, such states can be constructed in a well-understood way. The construction of similar boundary states for $SL(2, R)$ was attempted in the papers; unfortunately, there may be problems with this construction.

We shall study this issue here from another angle, by trying to construct supergravity solutions for D-branes. The boundary state condition implies by semiclassical reasoning that the D-branes must be one-half BPS i.e. they preserve 8 of the 16 supersymmetries of the background. Furthermore, they must wrap $AdS_2 \times S^2$ submanifolds of the total space. We look for such branes by explicitly analyzing the Killing equations, and finding the most general solutions.

Our results are surprising: even at the linearized level, we show that there are no localized branes with $AdS_2 \times S^2$ geometry that preserve half the supersymmetries. There are hence no such half-BPS branes in the $AdS_3 \times S^3$ background. We do find a large class of delocalized solutions, some of which were also previously discussed in the papers. We can write down the solutions to the nonlinear Killing equations in terms of a few functions.

After first reviewing our notation in section II, we present the linearized analysis of the Killing equations in section III and show that consistency requires the charge to be delocalized. We then move to the full nonlinear analysis in section IV and present the most general solution (with delocalized charges) that preserves half the supersymmetries of the $AdS_3 \times S^3$ space. We close with some comments on the relation between this calculation and the boundary state construction in section V.

II. CALCULATING THE SOLUTION

We will consider solutions to Type IIB supergravity, using the conventions of the field equations and invariances. The relevant bosonic field content will consist of the vielbein, complex 3-form field strength $f_3$, real 5-form field strength $f_5$, the complex 1-form dilaton-axion field strength $P$, and the composite connection $Q$. We will consider perturbations around the $AdS_3 \times S^3 \times T^4$ background with NS-NS fluxes turned on. For this background, the metric can be written
in the form
\begin{equation}
\begin{aligned}
\frac{ds^2}{e_{m} d x^{\mu}} (\bar{e}_{\nu}^{\mu} d x^{\nu}) \\
= - d \psi^2 - \cosh^2 \psi (d \omega^2 - \cosh^2 \omega d \tau^2) - d \theta^2 - \sin^2 \theta (d \phi^2 + \sin^2 \phi d \chi^2) - (d x_a) (d x^a)
\end{aligned}
\end{equation}

Here the indices $\psi, \tau, \omega$ parametrize the $AdS_3$, $\theta, \phi, \chi$ parametrize the $S^3$ and $x_a (a = 1..4)$ parametrize the $T^4$. The background NS-NS field strengths are
\begin{equation}
f_{\psi \tau \omega} = f_{\phi \chi \theta} = 1
\end{equation}

(we shall use indices with a tilde to denote curved-space indices, and indices without a tilde to refer to tangent space coordinates.)

This solution preserves 16 supersymmetries, which are encoded in the Killing spinor $\tilde{\epsilon}$. These Killing spinors are subject to the constraint $\gamma_{\psi \tau \omega} e_{\theta} \tilde{\epsilon} = \tilde{\epsilon}$ and satisfy the Killing equation
\begin{equation}
\partial_{\mu} \epsilon - \frac{1}{4} \omega_{\mu}^{pq} \gamma_{pq} \epsilon - \frac{1}{4} Q_{\mu} \epsilon - \frac{1}{192} f_{\mu pq r} \gamma^{pq r} \epsilon - \frac{3}{16} f_{\mu pq} \gamma^p q \epsilon^* + \frac{1}{48} f_{pqr} \gamma_{\mu pq} \epsilon^* = 0
\end{equation}

In addition, the vanishing of the variation of the dilatino gives the equation
\begin{equation}
\frac{1}{2} p_{\mu} \gamma^\mu \epsilon^* + \frac{1}{24} f_{abc} \gamma_{abc} \epsilon = 0
\end{equation}

We will now look for supergravity solutions which preserve 8 of the 16 background supersymmetries. We will also demand that the complete solution respect the symmetries of $AdS_2 \times S^2$.

We may then write the complete metric in the form
\begin{equation}
\begin{aligned}
\frac{ds^2}{M(\psi, \theta)(d \psi^2 + d \theta^2)} - N(\psi, \theta) \cosh^2 \psi (d \omega^2 - \cosh^2 \omega d \tau^2) \\
- P(\psi, \theta) \sin^2 \theta (d \phi^2 + \sin^2 \phi d \chi^2)
\end{aligned}
\end{equation}

We have used a diffeomorphism to set the coefficients of $d \psi^2$ and $d \theta^2$ equal to each other. This is possible because, by construction, the unknown functions in the metric only depend on $\psi, \theta$. We may also perturb all of the NS-NS and RR field strengths, but the demands of $AdS_2 \times S^2$ symmetry require that the perturbed field strengths (in tangent space coordinates) depend only on $\psi$ and $\theta$. Unlike \cite{7}, \cite{8} we will not assume 6d self-duality of the three-form field strength.

Since the solution preserves 8 supersymmetries, there must be a Killing spinor $\epsilon$ which satisfies $\Gamma \epsilon = \epsilon$ for some projector $\Gamma$. Given the symmetries of the problem, the most general projector can be rewritten in the form $\gamma_{\tau \omega} (A(\psi, \theta) - B(\psi, \theta) \gamma_{\psi \phi}) \epsilon$. This projector will act on a Killing spinor of the form
\begin{equation}
\epsilon = [m(\psi, \theta) + n(\psi, \theta) \gamma_{\psi \phi}] \tilde{\epsilon}
\end{equation}

where $\tilde{\epsilon}$ is a Killing spinor of the background and $m$ and $n$ are complex functions. We will not specify $A, B, m$ or $n$, but will leave them as free functions to be solved for by the $\frac{1}{2}$-BPS condition.

We may immediately demonstrate, following \cite{10}, that
\begin{equation}
Z = \frac{e_{\chi}}{e_{\chi}} = P^{-\frac{1}{2}} = \frac{e_{\tau}}{e_{\tau}} = N^{-\frac{1}{2}} = (mm^* + nn^*)^{-1}.
\end{equation}
This is because translation along the $\tau$ and $\chi$ directions are isometries of the full solution. The supersymmetry algebra (in the absence of charges) is

$$\{Q^A, \bar{Q}^B\} = \delta^{AB} P_\mu [(1 + \gamma^{11})\gamma^\mu]_{\alpha\beta}$$ (8)

Thus, the commutator of any two supersymmetries will be a translation. This is valid in any region of space-time where there are no charges (so the supersymmetry algebra is not deformed by central charges).

In particular, since

$$\delta_\epsilon = \epsilon Q + \bar{\epsilon} \bar{Q}$$ (9)

we find

$$\delta_\epsilon \delta_{\epsilon'} - \delta_{\epsilon'} \delta_\epsilon = 2\delta^{AB} [\bar{\epsilon}_A \gamma^\mu \epsilon_B - \bar{\epsilon}'_B \gamma^\mu \epsilon_A] P_\mu$$

$$= 4i\delta^{AB} Im[\bar{\epsilon}_A \gamma^\mu \epsilon_B] P_\mu$$ (10)

where $P_\mu$ is the generator of translations.

A supersymmetry transformation by a Killing spinor will leave the supergravity fields invariant. Therefore, the commutator of two supersymmetry transformations by Killing spinors must be a translation under which the supergravity fields are invariant, i.e. an isometry. Indeed, one can verify that the isometries of translation along $\tau$ and $\chi$ of the background are generated by a commutator of Killing spinor supersymmetry transformations.

The parameter associated with translation along the isometry direction will thus be related to the Killing spinors by the supersymmetry algebra. This translation parameter must be a constant, since the solution is invariant under constant shifts of $\tau$ or $\chi$.

In particular, if $x^\mu$ corresponds either of these two isometry directions (one does not sum over $\mu$), then

$$4Im(\bar{\epsilon} \gamma^\mu \epsilon') \frac{\partial}{\partial x^\mu} = 4Im(\bar{\epsilon} \gamma^\mu \epsilon') e_\mu \frac{\partial}{\partial x^\mu}$$

$$= 4Im(\bar{\epsilon}_o \gamma^\mu \epsilon'_o)(mm^* + nn^*) e_\mu \frac{\partial}{\partial x^\mu}$$

$$= \text{const} \times \frac{\partial}{\partial x^\mu}$$ (11)

where in the last step we have set the anti-commutator of Killing spinors to be a Killing vector. Note that the background solution must also satisfy this constraint. Using this and the boundary condition at infinity, we find

$$(mm^* + nn^*) \frac{e_\mu \bar{\mu}}{\bar{e}_\mu \bar{\mu}} = 1 \implies mm^* + nn^* = (\frac{e_\mu \bar{\mu}}{\bar{e}_\mu \bar{\mu}})^{-1}$$ (12)

### III. KILLING EQUATIONS

We can now substitute the ansatz into the Killing equations. The $\psi$ Killing equation can be written as

$$\gamma^\psi \partial_\psi (m + n \gamma_{\psi\theta}) \bar{\epsilon} - \frac{1}{2} \bar{\omega}_\psi \gamma_{\psi\theta} \epsilon - \frac{1}{4} \bar{Q}_\psi \gamma_{\psi\theta} \epsilon + \frac{i}{8} f_{\tau\omega\phi\chi} \gamma_{\tau\omega\phi\chi} \epsilon - \frac{i}{8} f_{\psi\tau\omega\chi} \gamma_{\psi\tau\omega\chi} \epsilon$$

$$- \frac{3}{8} f_{\psi\tau\omega} \gamma_{\psi\tau\omega} \epsilon^* + \frac{1}{8} f_{\psi\tau\omega} \gamma_{\tau\omega\theta} \epsilon^* - \frac{3}{8} f_{\psi\phi\chi} \gamma_{\psi\phi\chi} \epsilon^* + \frac{1}{8} f_{\phi\chi\theta} \gamma_{\phi\chi\theta} \epsilon^*$$

$$= \gamma^\psi (m + n \gamma_{\psi\theta}) \left( -\frac{1}{2} \bar{\omega}_\psi \gamma_{\psi\theta} \bar{\epsilon} - \frac{3}{8} \bar{f}_{\psi\tau\omega} \gamma_{\tau\omega} \epsilon^* + \frac{1}{8} f_{\phi\chi\theta} \gamma_{\phi\chi\theta} \epsilon^* \right) e_\psi$$ (13)
where we have evaluated derivatives of the background Killing spinor in terms of the background values of the vielbein and field strengths.

We let the $a$ index represent one of the directions along the $T^4$. The Killing equation along this direction is

$$-\frac{1}{2} \omega_a \gamma \psi \epsilon - \frac{1}{2} \omega_a \gamma \theta \epsilon + \frac{i}{8} f^\tau \omega \gamma \psi \theta \epsilon + \frac{i}{8} f^\tau \omega \gamma \psi \theta \epsilon - \frac{1}{8} f^\tau \omega \gamma \psi \theta \epsilon^* + \frac{1}{8} f^\tau \omega \gamma \psi \theta \epsilon^* + \frac{1}{8} f^\tau \omega \gamma \psi \theta \epsilon^* = 0$$

(14)

The other Killing equations are similar. The spinor now satisfies $\gamma \tau \omega \epsilon = \left[ A(\psi, \theta) - B(\psi, \theta) \right] \gamma \psi \theta \epsilon$. Using this, we can simplify the Killing equations; for example, we get

$$\gamma \tau \omega \epsilon^* = -\gamma \psi \theta \epsilon^* = [A \gamma \psi + B \gamma \theta] \epsilon$$

$$\gamma \tau \omega \epsilon^* = \gamma \psi \theta \epsilon^* = [A \gamma \theta - B \gamma \psi] \epsilon.$$  

(15)

Each Killing equation then produces a coefficient of $\gamma \psi \epsilon$ and $\gamma \theta \epsilon$. The coefficients should vanish separately. The Killing equations therefore split into twelve complex algebraic equations, which can be used to solve for the various field strengths and for the vielbein.

The composite connection gauges a local $U(1)$ symmetry under which the Killing spinor is charged. We will thus use a gauge transformation to make $m$ real. The Killing equations in the $a$ direction immediately tell us that $n$ is real and that

$$A = -\frac{\cosh \psi}{\cosh^2 \psi - \sin^2 \theta} (\sinh \psi + i \cos \theta)$$

$$B = \frac{i \sin \theta}{\cosh^2 \psi - \sin^2 \theta} (\sinh \psi + i \cos \theta)$$  

(16)

(Note that this form of the projector was exactly the form found in [7] using $\kappa$-symmetry [11] arguments for a D3-brane source wrapping an $AdS_2 \times S^2$ submanifold.)

We will now look for localized solutions in the linearized approximation. We therefore approximate

$$\frac{e_{\bar{\mu} \mu}}{e_{\bar{\mu} \mu}} = 1 + \delta_{\mu}$$

(17)

We can then solve the Killing equations to find

$$\delta_{\psi} - \delta_{\omega} = -2(\delta_{\omega} + \delta_a) = -\frac{c}{\gamma}$$

(18)

where $\gamma = \sin \theta \cosh \psi$ and $c$ is a constant of integration parameterizing the solution.

We also find

$$f^\tau \omega \theta - f^\psi \phi \theta = \frac{i c}{\gamma} \left( \frac{1}{\gamma} + i \tanh \psi \cot \theta \right)$$

$$f^\tau \omega \theta + f^\phi \psi \theta = 2Z + \frac{c}{\gamma}$$

$$f^\tau \omega \theta + f^\psi \phi \theta = \frac{A}{A^2 + B^2} (-f^\psi \phi \psi + 4i \partial_\theta \log e_{\bar{a}}) - \frac{i B}{A^2 + B^2} (f^\tau \omega \phi \theta + 4i \partial_\psi \log e_{\bar{a}})$$
\[
f_{\psi\tau\omega} - f_{\phi\chi\theta} = \frac{iB}{A^2 + B^2} \left( -f_{\psi\tau\omega\phi\chi} + 4i\partial_\psi \log e_\hat{a}^\hat{a} \right) + \frac{iA}{A^2 + B^2} \left( f_{\tau\omega\phi\chi\theta} + 4i\partial_\tau \log e_\hat{a}^\hat{a} \right)
\]

\[f_{\psi\tau\omega\phi\chi} = iQ_\theta, \quad f_{\tau\omega\phi\chi\theta} = -iQ_\psi
\]

\[
p_\psi(A^2 + B^2) = \frac{i}{2} \left( \frac{1}{1 - \beta^2} \left( f_{\tau\omega\phi\chi\theta} + 4i\partial_\psi \log e_\hat{a}^\hat{a} \right) \right) + \frac{\beta}{2} \left( -f_{\psi\tau\omega\phi\chi} + 4i\partial_\psi \log e_\hat{a}^\hat{a} \right)
\]

\[
p_\theta(A^2 + B^2) = -\frac{-\beta}{1 - \beta^2} \left( f_{\tau\omega\phi\chi\theta} + 4i\partial_\psi \log e_\hat{a}^\hat{a} \right) + \frac{\beta}{1 - \beta^2} \left( -f_{\psi\tau\omega\phi\chi} + 4i\partial_\psi \log e_\hat{a}^\hat{a} \right)
\]

So far $e_\hat{a}^\hat{a}$, $Q_\theta$ and $Q_\psi$ are arbitrary functions. If we demand that all brane sources vanish within any region, then we immediately find from the equations of motion that $e_\hat{a}^\hat{a} = 1$ and that the 5-form field strength $f_5$ vanishes within this region at linear order. The field strengths then become self-dual. The analysis is then identical to the analysis of [7].

Thus, the only solution in which the charge distribution does not fill space-time (to linear order) is the solution found previously in [7]1. The source for this solution is not localized; it is D-string charge delocalized in the $\psi$ direction (though it is localized in $\theta$ at $\theta = 0$). The charge density was found to be $\rho_{\tau\omega} = \frac{2\pi c}{\cosh \psi} \delta(\theta)$. This implies that no solutions with localized charges are possible.

**IV. THE NONLINEAR SOLUTION**

If we do not restrict ourselves to the linearized approximations, the solution is more complicated. It is most conveniently written in terms of the functions:

\[
X = e_\psi^\psi e_\omega^\omega \cosh \psi = e_\theta^\theta e_\omega^\omega \cosh \psi \quad R = e_\hat{a}^\hat{a}
\]

\[
Z = e_\omega^\omega \cosh \psi = e_\tau^\tau \cosh \psi \cosh \omega = e_\phi^\phi \sin \theta = e_\chi^\chi \sin \theta \sin \phi
\]

We find from the Killing equations that

\[
X = \frac{\gamma}{c + \gamma}
\]

\[
ZR = \sqrt{X}
\]

\[
m^2 = Z^{-1}
\]

\[
\frac{1}{2} Q_\theta = \frac{1}{2} X Z f_{\psi\tau\omega\phi\chi}
\]

\[
\frac{1}{2} Q_\psi = -\frac{1}{2} X Z f_{\tau\omega\phi\chi\theta}
\]

The 3-form field strength can also be solved for:

\[
f_{\psi\tau\omega} + f_{\phi\chi\theta} = Z + \frac{Z}{X}
\]

\[
f_{\tau\omega\theta} - f_{\psi\phi\chi} = (Z - \frac{Z}{X})(\tanh \psi \cot \theta - \frac{i}{\sin \theta \cosh \psi})
\]

\[
f_{\tau\omega\theta} + f_{\psi\phi\chi} = \frac{iA}{A^2 + B^2} (-\frac{X}{Z} f_{\psi\tau\omega\phi\chi} + 4i\partial_\tau \log R)
\]

1 Note that $c$ in this paper is equivalent to $\frac{1}{c}$ in the notation of [7].
The dilaton-axion field strength can be written as

\[ p_{\tilde{\psi}}(A^2 + B^2) = \frac{i}{2} \frac{1 + \beta}{1 - \beta^2} \left( \frac{X}{Z} f^{\tau \omega \phi \chi \theta} + 4i \partial_{\tilde{\psi}} \log R \right) \]

\[ + \frac{\beta}{1 - \beta^2} \left( \frac{X}{Z} f^{\psi \tau \omega \phi \chi} + 4i \partial_{\tilde{\theta}} \log R \right) \]

\[ p_{\tilde{\theta}}(A^2 + B^2) = -\frac{i}{2} \frac{1 + \beta}{1 - \beta^2} \left( \frac{X}{Z} f^{\tau \omega \phi \chi \theta} + 4i \partial_{\tilde{\psi}} \log R \right) \]

\[ + \frac{i}{2} \frac{1 + \beta}{1 - \beta^2} \left( \frac{X}{Z} f^{\psi \tau \omega \phi \chi} + 4i \partial_{\tilde{\theta}} \log R \right) \]

(23)

where \( \beta = \frac{\sin \theta}{\cosh \psi} \). Thus, the entire solution is parameterized by the (arbitrary) functions \( R \) and the 5-form field strength \( f_5 \).

Note that we have not yet used the equations of motion to impose additional constraints; we are allowing the possibility of space-filling charge-distributions. The constraints implied by the equations of motion are more difficult to impose beyond linear order. However, if the solutions without space-filling charges are necessarily identical to those found in [7] at linear order, it is difficult to see how they could differ at higher order.

V. CONCLUSIONS AND OPEN QUESTIONS

Our main result is that there are no local states that preserve half the supersymmetry and have geometry \( AdS_2 \times S^2 \). The analysis was simplest in the linearized approximation, and this is already sufficient to show that no such state can exist.

At the nonlinear level, we can still obtain exact results for some fields (in particular, we can solve for \( e_{\tilde{\psi}} \), \( e_{\omega} \), \( \tilde{e}_{\omega} \) exactly). Furthermore, if we demand that there exist a region where sources vanish, then the full non-linear solution necessarily reduces to the solution we had previously found in [7] which has a D-string source which is localized in \( \theta \), but not \( \psi \). It is interesting that one may find non-localized solutions which preserve 8 supersymmetries, but not solutions for localized branes.

This result may seem quite surprising, since the boundary state argument of [6] implies that such a localized \( \frac{1}{2} \)-BPS brane should exist. We believe that the most likely resolution of this paradox is that the D-brane in fact preserves only one-quarter of the supersymmetries. (Such solutions have been found in [12]). In the classical boundary state analysis, it appears that half the supersymmetries are preserved, but interactions must break part of the supersymmetry. This suggests that the boundary state construction in \( AdS_3 \) is somewhat subtle. The true boundary state which satisfies Cardy’s condition must presumably satisfy a deformed version of the boundary condition. This might have relevance to the problems pointed out in [5]. Understanding these issues would be quite interesting.
Another possibility is that the D-brane source is bent in the full solution, and that the embedding manifold is thus deformed away from $AdS_2 \times S^2$. But given the amount of symmetry in the problem, it is difficult to see how the brane geometry could be deformed away from $AdS_2 \times S^2$ at all, let alone at leading order.

If solutions for localized $\frac{1}{2}$-BPS branes had existed, they would have allowed one to examine the puzzling lack of charge quantization in spaces such as $AdS_3 \times S^3$ which asymptotically are curved and have fluxes turned on [13]. Given the absence of such solutions, the best chance for studying the issue of Dirac quantization in this space would be to generate localized $\frac{1}{4}$-BPS brane solutions. We hope to return to this issue in future work.

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