Dynamical Casimir Effect in cavity with $N$-level detector or $N-1$ two-level atoms

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We study the photon generation from vacuum via the Dynamical Casimir Effect in a cavity containing a $N$-level detector in equally-spaced resonant ladder configuration or $N-1$ identical resonant two-level atoms. If the modulation frequency equals exactly twice the unperturbed cavity frequency, the photon growth goes on steadily for odd $N$, while for even $N$ at most $N-2$ photons can be generated. This finding is corroborated by numerical calculations. In the limit $N \to \infty$ we obtain the harmonic oscillator model of the detector, which admits analytical results.

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A possibility of creating quanta of the electromagnetic field from the initial vacuum state in cavities with moving boundaries, called nowadays as the Dynamical Casimir Effect (DCE), was a subject of numerous theoretical studies for a long time: see, e.g., the most recent reviews [1,3]. It was shown [4] that one might expect a significant rate of photons generation inside ideal cavities with resonantly oscillating boundaries. The simplest model describing this effect takes into account a single resonant cavity mode whose frequency is rapidly modulated according to the harmonic law $\omega_t = \omega_0 [1 + \varepsilon \sin(\eta t)]$ with a small modulation depth, $|\varepsilon| \ll 1$. We shall use dimensionless variables, setting $\hbar = 1$. Then the Hamiltonian for the resonance mode has the form [5]

$$H_c = \omega_c n - i \chi_t (a^2 - a^{2\dagger}), \quad \chi_t = (4\omega_t)^{-1} d\omega_t/dt, \quad (1)$$

where $a$ and $a^\dagger$ are the cavity annihilation and creation operators, and $n \equiv a^\dagger a$ is the photon number operator. It is well known that the number of photons created from the initial vacuum state is maximal if the modulation frequency is exactly twice the unperturbed mode frequency, i.e., $\eta = 2$. The mean number of photons $\langle n \rangle$ and the Mandel factor $Q = \langle [\Delta(n)^2] - \langle n \rangle \rangle / \langle n \rangle$ increase with time in this ideal case as (hereafter we use the subscript 0 for the quantities related to the empty cavity)

$$\langle n_0(t) \rangle = \sinh^2(\varepsilon t/2), \quad Q_0(t) = 1 + 2\langle n_0(t) \rangle. \quad (2)$$

The field mode goes to the squeezed vacuum state with the following variances of the field quadrature operators $x = (a + a^\dagger)/\sqrt{2}$ and $p = (a - a^\dagger)/\sqrt{2}$ (the average values of these operators are zero)

$$(\Delta p_0)^2 = \frac{1}{2} e^{-\varepsilon t}, \quad (\Delta x_0)^2 = \frac{1}{2} e^{\varepsilon t} \quad (3)$$

But simple formulas (2) and (3) hold for the ideal empty cavity only. To register the emerging photons one has to couple the field mode to some detector. And here the problem of the back action of the detector on the field arises, because in many realistic cases the coupling between the field and detector can be much stronger than that between the field and vibrating cavity walls. This was noticed in [6], where it was shown that for the simplest model of detector as a two-level “atom”, no photons can be created at all for the modulation frequency $\eta = 2$, if the field atom coupling constant $g$ is much bigger than the frequency modulation amplitude $\varepsilon$. A more detailed investigation of this problem was given recently in [7,8], where different resonant regimes were found and analyzed for different ratios $g/\varepsilon$ (both small and big). On the other hand, in papers [9,10] we have discovered that when the inter-level transitions are resonant with the cavity bare frequency and the modulation frequency equals twice the cavity unperturbed frequency, many photons can be generated for the 3-level atom in ladder configuration or a chain of two 2-level atoms, even if $g \gg \varepsilon$. Therefore, a question naturally arises: how many photons can be generated if one uses the $N$-level atom in the equidistant ladder configuration and the inter-level transitions are resonant with the cavity mode modulated at exactly twice the unperturbed cavity eigenfrequency? We answer this question in a brief report.

If the selected field mode interacts with the $N$-level detector in resonant ladder configuration, the Hamiltonian describing the whole system “field mode + atom” can be taken in the Rotating Wave Approximation (RWA) form

$$H = H_c + \sum_{i=1}^{N} E_i \sigma_{ii} + \sum_{i=1}^{N-1} g_i (a \sigma_{i+1,i} + a^\dagger \sigma_{i,i+1}), \quad (4)$$

where $E_i$ is the energy of the $i$-th atomic eigenstate $|i\rangle$ (in bold), $\sigma_{ij} \equiv |i\rangle \langle j|$ is the generalized Pauli operator, and $g_i$ (assumed to be real) is the coupling parameter between the atomic states $\{|i\rangle, |i+1\rangle\}$ through the cavity field. The condition of validity of RWA is $|g_j| \ll 1$. We consider here the resonant case, so we assume $E_{i+1} - E_i = 1$ for $i = 1, \ldots, N-1$. Our aim is to find out how the photon generation from vacuum is affected by the detector in the weak modulation regime, $|\varepsilon| \ll g_j$ ($j = 1, \ldots, N-1$) for the modulation frequency $\eta = 2$.

To find the wave function of the whole system $|\Psi(t)\rangle$, we make the transformation $|\Psi(t)\rangle = V(t)|\psi(t)\rangle$ with $V(t) = \exp[-it\sum_{i=1}^{N} E_i \sigma_{ii}]$. Then after RWA we get the following Hamiltonian governing the time evolution of $|\psi(t)\rangle$ in the interaction picture (where $\beta \equiv \varepsilon/4$):

$$H_I = -i\beta (a^2 - a^{2\dagger}) + \sum_{i=1}^{N-1} g_i (a \sigma_{i+1,i} + a^\dagger \sigma_{i,i+1}), \quad (5)$$

where $E_i$ is the energy of the $i$-th atomic eigenstate $|i\rangle$ (in bold), $\sigma_{ij} \equiv |i\rangle \langle j|$ is the generalized Pauli operator, and $g_i$ (assumed to be real) is the coupling parameter between the atomic states $\{|i\rangle, |i+1\rangle\}$ through the cavity field. The condition of validity of RWA is $|g_j| \ll 1$. We consider here the resonant case, so we assume $E_{i+1} - E_i = 1$ for $i = 1, \ldots, N-1$. Our aim is to find out how the photon generation from vacuum is affected by the detector in the weak modulation regime, $|\varepsilon| \ll g_j$ ($j = 1, \ldots, N-1$) for the modulation frequency $\eta = 2$.

To find the wave function of the whole system $|\Psi(t)\rangle$, we make the transformation $|\Psi(t)\rangle = V(t)|\psi(t)\rangle$ with $V(t) = \exp[-it\sum_{i=1}^{N} E_i \sigma_{ii}]$. Then after RWA we get the following Hamiltonian governing the time evolution of $|\psi(t)\rangle$ in the interaction picture (where $\beta \equiv \varepsilon/4$):

$$H_I = -i\beta (a^2 - a^{2\dagger}) + \sum_{i=1}^{N-1} g_i (a \sigma_{i+1,i} + a^\dagger \sigma_{i,i+1}), \quad (5)$$
In the special case when \( N \to \infty \) and \( g_j = \sqrt{jg} \) the detector becomes a simple harmonic oscillator (HO) if we associate the atomic level \( |1\rangle \) with the oscillator ground (zero energy) state and make the replacements

\[
\sum_{j=1}^{\infty} \sqrt{j} \sigma_{j+1,j} = b^\dagger, \quad \sum_{j=1}^{\infty} j \sigma_{j,j+1} = b,
\]

where \( b \) and \( b^\dagger \) are the annihilation and creation operators associated with the detector \((b,b^\dagger) = 1\). Then the Heisenberg equations of motion for operators \( a(t) \) and \( b(t) \) can be solved exactly:

\[
a(t) = (C_\beta c_\gamma + \beta S_{\beta s_\gamma}) a_0 - ig C_{\beta s_\gamma} b_0 + (S_{\beta c_\gamma} + \beta C_{\beta s_\gamma}) a_0^\dagger + ig S_{\beta s_\gamma} b_0^\dagger, \quad (6)
\]

\[
b(t) = (C_\beta c_\gamma - \beta S_{\beta s_\gamma}) b_0 - ig C_{\beta s_\gamma} a_0 - (S_{\beta c_\gamma} - \beta C_{\beta s_\gamma}) b_0^\dagger - ig S_{\beta s_\gamma} a_0^\dagger, \quad (7)
\]

where \( \gamma = \sqrt{g^2 - \beta^2}, \ C_\beta \equiv \cosh(\beta t), \ S_\beta \equiv \sinh(\beta t), \ c_\gamma \equiv \cos(\gamma t), \) and \( s_\gamma \equiv \sin(\gamma t) / \gamma \). If \( |\beta| > |g| \), then trigonometrical functions should be replaced by their hyperbolic counterparts with \( \gamma \) replaced by \( \gamma = \sqrt{\beta^2 - g^2} \). The mean number of quanta in the field mode \( \langle n(t) \rangle \) and the mean excitation number of the detector \( \langle n_b(t) \rangle \) for the initial vacuum state are equal to

\[
\langle n(t) \rangle \langle n_b(t) \rangle = S_\beta^2 + \beta S_{\beta s_\gamma} S_{\gamma^2} + \beta^2 C_{\beta s_\gamma} S_{\gamma^2}. \quad (8)
\]

The Mandel factor and quadrature variances of the field mode are

\[
Q(t) = \langle n(t) \rangle + \frac{[S_{2\beta} (1 + 2\beta^2 s_\gamma^2) + 2\beta C_{\beta s_\gamma} S_{\gamma^2}^2]}{4 \langle n(t) \rangle}, \quad (9)
\]

\[
\frac{(\Delta p)^2}{(\Delta x)^2} = e^{2\beta t} \left( \frac{1}{2} + \beta s_\gamma + \beta^2 s_\gamma^2 \right). \quad (10)
\]

If \( g = 0 \), then Eqs. \( (8)-(10) \) go to \( (2)-(4) \). But if \( |g| > |\beta| \), then the rate of photon generation becomes roughly twice smaller than in the empty cavity \[16\]. Moreover, the ratio \( \langle n_b(t) \rangle / \langle n(t) \rangle \) is close to unity if \( |g| > |\beta| \). For \( g > 0 \) and \( \beta t \gg 1 \) we see the exponential growth of the mean photon number with increment \( \epsilon/2 \), which is modulated by some oscillations with the frequency \( 2\gamma \):

\[
\langle n(t) \rangle \approx \frac{1}{4} e^{2\beta t} \left[ 1 + \frac{\beta}{\gamma} \sin(2\gamma t) + \frac{2\beta^2}{\gamma^2} \sin^2(\gamma t) \right]. \quad (11)
\]

For \( t = t_n + \delta t \) with \( 2\gamma t_n = (2n + 1)\pi \) we have the following Taylor expansion of formula (11):

\[
\langle n(t_n + \delta t) \rangle \approx \langle n(t_n) \rangle \left[ 1 + (4/3) g_0^2 (\delta t)^3 \right].
\]

Consequently, the function \( \langle n(t) \rangle \) is practically constant in some neighborhood of \( t_n \). This results in the appearance of almost horizontal “shelves” in the plots of \( \langle n(t) \rangle, Q(t) \) and \( (\Delta p)^2(t) \), which are clearly seen in Figs. 1 and 2 below. For \( \beta t \gg 1 \) Eq. (9) can be simplified as \( Q(t) \approx 2\langle n(t) \rangle \), which is a typical relation for highly squeezed vacuum states. Nonetheless, the state of the field mode is not exactly the vacuum squeezed one, since the uncertainty product \( \Delta \equiv (\Delta p)^2(\Delta x)^2 \) is bigger than the minimal possible value \( 1/4: \Delta = 1/4 + g^2 \beta^2 s_\gamma^2 \) (the covariance between \( x \) and \( p \) quadratures is zero in the case discussed). As soon as the initial state of the system was Gaussian, it remains Gaussian for all times. In such a case, the purity of the field mode equals \[13\]

\[
\rho = \frac{1}{2} + \rho / |\beta|, \quad \rho \equiv (4\Delta)^{-1/2}, \quad \text{where} \ \rho \ \text{is the statistical operator.}
\]

If \( |g| > |\beta| \), the purity is only slightly below unity, oscillating with the frequency \( 2\gamma \). But for \( 0 < |g| < |\beta| \) the purity goes monotonously to zero.
solve the Schrödinger equation for the Hamiltonian \( H \) by expanding the wavefunction in the Fock basis as \(|\psi(t)\rangle = \sum_{j=1}^{N} \sum_{k=0}^{\infty} \beta_{j,k}(t) |j,k\rangle\), where the first index stands for the atomic eigenstate and the second for the Fock state of the cavity field. We get the set of differential equations

\[
\ddot{\beta}_{j,m} = \beta \left( m(m-1) - \frac{1}{2} \sum_{k=1}^{N} \sum_{l=0}^{\infty} \beta_{j,k} \beta_{j,l} \right) - i g_j \sqrt{m} \beta_{j,m-1} - i g_j (m+1) \beta_{j,m+1} \tag{12}
\]

where \( g_0 = g_j \geq N \equiv 0 \) and the dot stands for the time derivative. Hereafter we suppose that \(|\beta| \ll |g_j|\). To understand qualitatively the resulting dynamics in this case we follow the method employed in \([7, 9, 10]\). First we solve the equations \((12)\) for \( \beta = 0 \), obtaining solutions in the form of exponentials with time-varying arguments multiplied by constant coefficients. Then we substitute the solutions obtained back into \((12)\) with \( \beta \neq 0 \), assuming that now the coefficients at the exponentials become time-dependent. Thus we obtain a set of differential equations for these coefficients.

In this paper we assume that the only nonzero initial probability amplitude is \( p_{1,0}(0) = 1 \), so the first equation to be solved is \( p_{1,0}(t) = -\sqrt{2}p_{1,2}(t) \). Our first task is to find the expression for \( p_{1,2}(t) \) when \( \beta = 0 \); it must be obtained from the matrix equation

\[
\dot{X} = -i MX ,
\]

where \( X \equiv (p_{1,2}; p_{2,1}; p_{3,0}) \) and

\[
M = \begin{pmatrix}
0 & \sqrt{2}g_1 & 0 \\
\sqrt{2}g_1 & 0 & g_2 \\
0 & g_2 & 0
\end{pmatrix} .
\tag{14}
\]

For \( N = 2 \) the eigenvalues of matrix \( M \) are \( \pm \sqrt{2}g_1 \), so \( p_{1,2} = \sum_{k=1}^{2} \sqrt{2}g_1 \exp(-i\varphi_2^{(2)} t) \), where \( \Lambda_k^{(2)} \) are constant coefficients and \( \varphi_2^{(2)} \) are the eigenvalues of \( M \). Since these eigenvalues are big, \( |\varphi_2^{(2)}| \gg |\beta| \), no significant coupling between the coefficients \( p_{1,0} \) and \( \Lambda_k^{(2)} \) (with \( k = 1, 2 \)) arises when one puts the ansatz \( p_{1,2} = \sum_{k=1}^{2} \Lambda_k^{(2)}(t) \exp(-i\varphi_2^{(2)} t) \) into Eq. \((12)\) with \( \beta \neq 0 \). So within the RWA reasoning one has \( p_{1,0} \approx 0 \). Thus \( p_{1,0} \) is decoupled from all the other probability amplitudes and the system remains in the initial state \( |1,0\rangle \). On the other hand, for \( N > 2 \) the solution of \((13)\) for \( p_{1,2} = 1 \) \( p_{1,2} = \sum_{k=1}^{N} \Lambda_k^{(2)} \exp(-i\varphi_2^{(2)} t) \) with the third eigenvalue of matrix \( M \) equal to zero, \( \varphi_3^{(2)} = 0 \). Moreover, one can check that \( p_{2,1} \) does not contain the term \( \Lambda_3^{(2)} \), but \( p_{3,0} \) does (exact expressions are given in \([9]\)). Hence, in this case \( p_{1,0} \) is resonantly coupled to \( \Lambda_3^{(2)} \), so \( p_{2,1} \) and \( p_{3,0} \) become populated, while \( p_{2,1} \) is coupled off-resonantly, so it is much smaller than the former two coefficients \((|\Lambda_3^{(2)}| \ll |\Lambda_3^{(2)}| \approx |\Lambda_3^{(2)}|)\). Thus in this case at least four photons can be created for sure.

Now we must see the coupling of the coefficients \( \Lambda_j^{(3)} \) \((j = 1, 2, 3)\) to the next subset, so we have to solve Eq. \((13)\) with \( X = (p_{1,4}; p_{2,3}; p_{3,2}; p_{4,1}; p_{5,0}) \) and

\[
M = \begin{pmatrix}
0 & g_1 \sqrt{4} & 0 & 0 & 0 \\
g_1 \sqrt{4} & 0 & g_2 \sqrt{3} & 0 & 0 \\
0 & g_2 \sqrt{3} & 0 & g_3 \sqrt{2} & 0 \\
0 & 0 & g_3 \sqrt{2} & 0 & g_4 \\
0 & 0 & 0 & g_4 & 0
\end{pmatrix} .
\]

For \( N = 4 \) we have \( p_{1,4} = \sum_{k=1}^{4} \Lambda_k^{(4)} \exp(-i\varphi_k^{(4)} t) \), where all the eigenvalues are different from zero and much larger than \(|\beta|\), being functions of \( g_i \) \((i = 1, 2, 3)\). Therefore \( \Lambda_3^{(3)} \) does not couple resonantly to any \( \Lambda_k^{(4)} \). Thus at most two photons are generated for the four-level atom. For \( N > 4 \) the solution for \( p_{1,4} \) is \( p_{1,4} = \sum_{k=1}^{5} \Lambda_k^{(5)} \exp(-i\varphi_k^{(5)} t) \) with \( \varphi_5^{(5)} = 0 \), and \( \Lambda_5^{(5)} \) also appears in \( p_{3,2} \) and \( p_{5,0} \), but not in \( p_{2,3} \) and \( p_{4,1} \). Thus, the coefficient \( \Lambda_3^{(3)} \) is resonantly coupled to \( \Lambda_5^{(5)} \) and at least four photons can be generated.

Continuing this analysis for higher order matrices, one can easily verify that matrices \( M \) of odd orders always have one null eigenvalue, and the coefficient \( A \) that multiplies the exponential with the null eigenvalue appears in probability amplitudes with odd atomic level and even photon number. On the other hand, all eigenvalues of matrices \( M \) of even orders are different from zero and much larger than \(|\beta|\). Hence we arrive at the general rule: The number of created photons is unlimited for odd numbers of levels, while at most \( N - 2 \) photons can be created if the number of levels \( N \) is even. Moreover, the probability of detecting an odd number of photons is much smaller than the probability of detecting an even number of photons.

These results can be extended straightforwardly to another practical scenario: \((N - 1)\) resonant 2-level atoms interacting with the field mode, when initially all the atoms are in their ground states and the field is in the vacuum state, for the modulation frequency \( \eta = 2 \). It was shown in \([10]\) that for a single atom no photons are generated for this modulation frequency, and for two atoms the photons generated steadily provided the coupling strengths are equal. So here we assume that all the couplings are equal to \( g \). In the interaction picture the Hamiltonian describing this setup is the Tavis-Cummings Hamiltonian \([11, 14]\) with additional parametric amplification term analogous to Eq. \((5)\)

\[
H_I = -i\beta(a^2 - a^2) + g(aS_+ + a^d S_-),
\]

where \( S_\pm = \sum_{j=1}^{N-1} \sigma_j^{(j)} \) are the collective ladder operators and \( \sigma_j^+ = |1_j\rangle\langle 2_j|, \sigma_j^- = |2_j\rangle\langle 1_j| \) are the standard 2-level Pauli operators, where \( |1_j\rangle \) and \( |2_j\rangle \) are the ground and excited states of the \( j \)-th atom \((j = 1, \ldots, N - 1)\), respectively. Writing the wavefunction associated to the Hamiltonian \((15)\) as \(|\psi(t)\rangle = \sum_{j=1}^{N} \sum_{k=0}^{\infty} p_{j,k} |j,k\rangle\), where \(|j\rangle\) denotes the symmetric normalized Dicke state \([15]\) with \((j - 1)\) excitations \((j = 1, \ldots, N)\)
In Fig. 1 we show the behavior of the mean photon number $\langle n \rangle$ for different forms of the coupling strengths $|g|_j$ [18]. We confirmed these results by performing numerical simulations in which we solved the equations (12) using the Runge-Kutta-Verner fifth-order and sixth-order method for the probability amplitudes, where the effective coupling strengths are $g_j \equiv g\sqrt{j(N-j)}$ ($j = 1, \ldots, N$). Thus our statement holds unaltered.

We confirmed these results by performing numerical simulations in which we solved the equations (12) using the Runge-Kutta-Verner fifth-order and sixth-order method for different forms of the coupling strengths $|g|_j$ [18]. In Fig. 1 we show the behavior of the mean photon number $\langle n \rangle$, the Mandel factor $Q$, the squeezed quadrature variance $(\Delta p)^2$ and the probability $P_1$ of finding the detector in the ground state as function of dimensionless time $ct$ for different number of detector's levels $N$. We verified that the probability of finding the detector in an even energy level is always very low (data not shown). In Fig. 1 we set $g_j = \sqrt{j}g$ ($j = 1, \ldots, N-1$), as one expects that for $N \gg 1$ the results should approach the behavior of the case of harmonic oscillator detector (depicted with thick blue line), and the red dashed lines denote the results for the empty cavity according to Eqs. (2)–(3). In Fig. 2 we set $g_j = \sqrt{j}g$ corresponding to $N-1$ two-level atoms. In Fig. 3 we plot the probabilities $P_K$ of detecting $K$ photons as function of time for $g_j = \sqrt{j}g$ (Fig. 3a) and $g_j = \sqrt{j(N-j)}g$ (Fig. 3b) for $N = 12$, to show that at most 10 photons are created, whereas the probabilities of detecting odd photon numbers are very small (the probabilities with $K \geq 12$ are below the horizontal axes). Hence the numerical results are in full agreement with our general rule.

FIG. 2: (Color online) Same as Fig. 1 for $g_j = \sqrt{j}g$.

FIG. 3: (Color online) Probability $P_K$ of detecting $K$ photons as function of dimensionless time $ct$ for $N = 12$, $g = 10^{-2}$ and $\epsilon = 10^{-3}$. a) $g_j = \sqrt{j}g$. b) $g_j = \sqrt{j(N-j)}g$.

In conclusion, we studied the photon pair creation from vacuum via the Dynamical Casimir Effect in a cavity containing a resonant $N$-level detector in equally-spaced ladder configuration or $N-1$ identical resonant two-level atoms. We considered the regime of weak modulation, where the cavity modulation depth is much smaller than the atom-field coupling. We showed that for an odd number of levels pairs of photons are generated steadily as time goes on, while for an even number of levels at most $N - 2$ photons are created. For $N \to \infty$ the difference between even and odd values of $N$ eventually disappears, and for the special choice of the coupling constants we arrived at the harmonic oscillator model of the detector. In this special case the rate of photon generation is roughly twice smaller than for an empty cavity. These findings can be useful for constructing efficient detectors for monitoring the photon generation via Dynamical Casimir Effect, because the detector can be excited without interrupting the photon growth.
Acknowledgments

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[16] A similar effect was discovered for empty cavities with additional symmetry (such as cubical ones), when several cavity modes can be in resonance with the external perturbations and between themselves [11, 12]: in this case other modes play the role of an effective oscillator “detector”.
[17] Notice that we can use the Dicke states only when all coupling coefficients are equal.
[18] The precision of numerical calculations was verified by calculating the total sum of probabilities. In all the cases it was equal to unity with errors below the level of $10^{-10}$. 