Dynamically broken Topcolor at Large-$N$\*\ †

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July, 1999

Abstract

We analyze a model of dynamically broken topcolor in the limit in which the number of colors is large. We show that the second order nature of the phase transition, necessary for the success of topcolor models, passes the nontrivial check of consistency with the large $N$ limit. We also identify and discuss a class of theories that generalizes the topcolor phenomenon to a theory with a richer structure of fermions and global symmetries.

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1 Introduction

More than thirty years after Weinberg's theory of leptons [1], the nature of electroweak symmetry breaking remains obscure. Symmetry breaking by a fundamental Higgs multiplet has been thoroughly studied in perturbation theory. Particularly when combined with the beautiful theoretical idea of supersymmetry, this alternative is quite attractive to theorists [2]. Dynamical symmetry breaking is harder to study because it is necessarily nonperturbative. Nevertheless, this option is highly constrained by precise data on the properties of the $W$ and $Z$ bosons, and by more general considerations such as the suppression of flavor changing neutral currents and the apparent existence of small neutrino masses [3]. Topcolor [4] is one of the few dynamical schemes that still seems viable. In topcolor, the Higgs multiplet exists, but is a composite state of fermions (including the left handed $t$ quark) that carry the strong topcolor interaction. It is necessary in this scheme that the topcolor gauge symmetry is spontaneously broken down to ordinary color (leaving massive, strongly interacting “colorons” that contribute to the binding of the Higgs). A critical component of topcolor is the assumption that the mass squared of the composite Higgs can be tuned to be very small compared to the scale of the topcolor dynamics. This is certainly the case if the electroweak symmetry breaking in the model is a second order phase transition.

Our goal in this note is not to build realistic models, but rather to develop tools that may be useful for model building. Most previous studies of topcolor models have concentrated on the gauge structure and the couplings of the colorons to the $t$ and other quarks. Here, we focus instead on topcolor breaking. We will study a class of theories in which this symmetry breaking is itself dynamical. We analyze these in the limit in which the number of colors is large. We show that the second order nature of the phase transition, necessary for the success of topcolor models, passes the nontrivial check of consistency with the large $N$ limit [5]. We also identify and discuss a class of theories that generalizes the topcolor phenomenon to a theory with a richer structure of fermions and global symmetries.

2 The simple Moose

We begin by considering not a topcolor model, but something simpler — an $SU(N) \times SU(M)$ gauge theory with three multiplets of massless fermions and an $SU(M) \times SU(N)$ global symmetry [6]. The theory is most usefully described by the “Moose” diagram in figure 1.

\[ \begin{array}{c}
M \\
SU(M)
\end{array} \quad \begin{array}{c}
\uparrow \\
A
\end{array} \quad \begin{array}{c}
\quad N \\
\quad SU(N)_{g}
\end{array} \quad \begin{array}{c}
\quad B \\
SU(M)_{g}
\end{array} \quad \begin{array}{c}
\uparrow \\
\quad C
\end{array} \quad \begin{array}{c}
\quad M \\
SU(M)
\end{array} \quad \begin{array}{c}
\uparrow \\
N
\end{array} \]

Figure 1: The basic Moose — in this model, we expect no phase transition as a function of the ratio of coupling strengths.

\footnote{Here and below we indicate gauge symmetries with a subscript $g$.}
The left-handed (LH) fermions are represented by the solid directed lines, labeled $A$, $B$ and $C$. The gauge groups, all $SU(K)$ for some $K$, are represented by circles with the value of $K$ inside. Lines that do not end in circles transform under a global flavor $SU(K)$ symmetry, again with the value of $K$ indicated on the diagram. The Moose encodes the transformation properties of the LH fermions under the various gauge and global symmetries of the model, in this case,

$$SU(M) \times SU(N) \times SU(M) \times SU(N).$$  

(1)

The fermion associated with a directed line in Moose notation transforms like the defining representation if the arrow leads away from the label for the group, and like the complex conjugate of the defining representation if the arrow leads toward to the group label. Thus

- $A$ transforms like $(M, \overline{N}, 1, 1)$
- $B$ transforms like $(1, N, \overline{M}, 1)$
- $C$ transforms like $(1, 1, M, N)$

This theory depends on two parameters — the gauge couplings (or more properly, because of dimensional transmutation \[7\], the $\Lambda$ parameters) of the two gauge groups. We have argued elsewhere \[6\] that independent of the ratio of the gauge couplings the model has an unbroken $SU(M) \times SU(N)$ global symmetry and a multiplet of massless LH fermions transforming like $(M, \overline{N})$ saturating the global anomaly conditions \[8\]. If either of the gauge couplings is much smaller than the other, we are very confident that this is the case because the strong interactions are QCD-like. And there is no reason to expect any phase transition as a function of the ratio of couplings \[9\]. If the couplings are of the same order of magnitude, it makes sense to refer to the massless fermions as composites. It is this region that we will find particularly interesting.

In this note, we will study the limit of this theory in which $N$ and $M$ go to infinity in fixed ratio, with gauge couplings scaling like $1/\sqrt{N}$. We had hoped to be able contribute some additional evidence for the conjecture that there is no phase transition in the theory.

\[2\]

We will not review these arguments here, because we will be going through similar arguments in considerable detail in section \[8\].

Figure 2:
The Moose with a small side chain — in this model, we expect a second order phase transition as a function of the ratio of coupling strengths.

In this note, we will study the limit of this theory in which $N$ and $M$ go to infinity in fixed ratio, with gauge couplings scaling like $1/\sqrt{N}$. We had hoped to be able contribute some additional evidence for the conjecture that there is no phase transition in the theory.

\[2\]
We have been frustrated in this attempt, and will discuss some of the reasons in appendix A. In what follows, we will simply assume that this conjecture is true, and remains true in the limit. We will consider instead the phase structure of related and more interesting models in the same large $N$ limit. In particular, we will discuss below the models described by the Mooses in figures 2 and 3, and the suggest that phase transitions that must occur in these models are second order in the limit that $N$ and $M$ go to infinity with $k$ and $\ell$ fixed. We will show that the states that exist in the large $N$ limit are just those required for the consistency of second order phase transitions in these models. The Moose in figure 2 is the business end of model of a dynamically broken topcolor [4, 10, 11], with only the topcolor (and not the color) gauge symmetry, and without the fields that do not carry topcolor or cause its breakdown. The Moose in figure 3 is a nontrivial generalization of the topcolor phenomenon that is only possible in a theory in which topcolor is dynamically broken.

3 Large $N$ and $M$ for $k = \ell = 0$

Here we discuss the limit of the Moose in figure 1 in which $N$ and $M$ go to infinity with the $\Lambda$ parameters of the gauge groups fixed. The first thing to note is that we must take $N$ and $M$ to infinity at the same rate. Otherwise, one or the other of the gauge couplings would lose asymptotic freedom. Thus we assume that the ratio $M/N$ is fixed in the limit. The limit thus depends both on $M/N$ and on $\Lambda_M/\Lambda_N$, but we will usually regard $M/N$ as fixed and concentrate on the dependence on $\Lambda_M/\Lambda_N$. We will refer to all of this simply as the large $N$ limit.

For finite $M$ and $N$, the model has a single anomaly free $U(1)$ symmetry. Two combinations of the three classical $U(1)$s are broken by the anomalies of the two gauge groups. However, in the large $N$ limit, the gauge anomalies disappear and the classical $U(1)$ symmetries are restored. These $U(1)$ symmetries will not play a crucial role in anything we do, and we will usually ignore them completely.

Let us first consider vacuum to vacuum amplitudes in this large $N$ limit. This is a matrix model, in which we can think of each of the fermions and the gauge bosons as represented by a pair of oppositely directed lines. For the gauge bosons, the two lines refer to the same gauge index, while for the fermions, the two lines are associated with different gauge or

![Diagram](image-url)
global indices. But since \( M \) and \( N \) are going to infinity at the same rate, these differences are irrelevant to the diagrammatics. The leading vacuum to vacuum graphs are therefore those that can be drawn on the surface of a sphere, like pure gluon graphs in QCD.

There are three kinds of interesting bound states in this model that carry the flavor symmetries:

- **Mesons which transform like adjoints under the \( SU(M) \) flavor symmetry** — these are built of \( A\overline{A} \) plus flavor singlet combinations of the fermions;

- **Mesons which transform like adjoints under the \( SU(N) \) flavor symmetry** — these are built of \( C\overline{C} \) plus flavor singlet combinations of the fermions;

- **Baryons which transform like \((M,\overline{N})\) under the \( SU(M) \times SU(N) \) flavor symmetry** — these are built of \( ABC \) plus flavor singlet combinations of the fermions.

Because this is a matrix model, that is, because the number of flavors is going to infinity like the number of colors, we cannot classify the states in terms of their fermion constituents. Arbitrary numbers of all varieties of fermion-antifermion pairs appear in the wave-function in the same order in \( N \). In addition, the bound states are not free in the large \( N \) limit as they are in large \( N \) QCD with a fixed number of flavors. The couplings of the bound states to one another go to zero, but because the number of bound states is going to infinity, the interactions cannot be neglected. However, the flavor structure of the graphs contributing to the propagation and interactions of the particles that carry flavor is still very simple. The leading graphs are those in which all the flavor dependence is associated with the lines running around the outside of a planar diagram. We will now illustrate all this in more detail using the following notation for the fermion lines propagating to the right:

\[
\begin{align*}
A & \quad \cdots \cdots \\
B & \quad \quad \quad \quad \quad \\
C & \quad \quad \quad \quad \quad
\end{align*}
\]

Note that we have defined the direction of the arrow on the \( B \) to be opposite to that of \( A \) and \( C \) so that the lines with their arrows in the same direction are carrying the same kind of gauge charge (\( A \) is an \( \overline{N} \) while \( B \) is an \( N \) of \( SU(N) \) and \( C \) is an \( M \) while \( B \) is an \( \overline{M} \) of \( SU(M) \)). We begin by showing a series of figures that illustrate meson and baryon propagation. We will discuss interactions in the next section.

Figure 4 illustrates the typical planar graphs that contribute to meson propagation in a large \( N \) gauge theory with a finite number of flavors, in this case, illustrated for the \( A\overline{A} \) mesons that are bound by the \( SU(N) \) gauge interactions and carry \( SU(M) \) flavor. The small \( N \)s indicate arbitrary planar dressing of \( SU(N) \) gluons. There are corresponding graphs with a \( C \) loop running around the outside and dressed with \( SU(M) \) gluons that contribute to the propagation of \( C\overline{C} \) mesons.

\[\text{Figure 4}\]

\[\text{In other words, the effective theory describing the bound states is a matrix model in which the number of flavors is going to infinity.}\]
A class of diagrams contributing to the propagation of $A\overline{A}$ adjoint mesons in leading order in large $N$. The $\bullet$s indicate an $\overline{A}T_aA$ interpolating field. The small $N$s indicate arbitrary planar dressing of $SU(N)$ gluons.

Another class of diagrams contributing to the propagation of $A\overline{A}$ adjoint mesons in leading order in large $N$. The internal $A$ loop produces a hole in the planar diagram, but it involves a sum over all $SU(M)$ flavors and thus does not affect the flavor structure. These graphs are of order $M/N = O(1)$ compared to those of figure 4.

Because the number of flavors grows with $N$, internal loops are not suppressed, as illustrated in figure 4. These graphs are suppressed by the large $N$ gluon couplings, but enhanced by the $M$-flavor sum in the internal loop, so they are of order $M/N = O(1)$ compared to those of figure 4. Arbitrary numbers of loops may appear, so that planar graphs for a large number of flavors may actually resemble a slice of Swiss cheese. Note that there is an orientation to the planar gluon graphs determined by the direction of the outside loop. If the external arrows are on the right of the planar gluon dressing as in this case, then to contribute in leading order, all internal $A$ lines must have their arrows on the right in the same way. Thus internal (empty) $A$ loops must circulate in the opposite direction from the external loop. Graphs in which an internal $A$ loop goes around in the other direction are

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4You can see this in detail by putting in some gluon lines and redrawing the diagram in the double-line notation.
non-leading in $N$. The internal $A$ loop involves a sum over all the $SU(M)$ flavors (that is why it contributes in leading order), and thus it has no affect of the flavor structure of the graph, which is entirely associated with the outside line. We will return to this when we discuss interactions below.

![Figure 6](https://example.com/figure6.png)

**Figure 6:** Another class of diagrams contributing to the propagation of $A\overline{A}$ adjoint mesons in leading order in large $N$. The $B$ loop encloses a region full of small $M$s indicating arbitrary planar dressing of $SU(M)$ gluons. Again the flavor structure is unaffected.

The prominence of graphs like those in figure 5 is related to the fact that the meson resonances in a model like this are not narrow, but have unsuppressed decays into multimeson states [12]. We will come back to this also in section 4.

![Figure 7](https://example.com/figure7.png)

**Figure 7:** Another class of diagrams contributing to the propagation of $A\overline{A}$ adjoint mesons in leading order in large $N$. The $C$ loop produces hole in a region dressed with $SU(M)$ gluons. As always, the flavor structure comes entirely from the loop on the outside.

We may also have an internal $B$ loop, as illustrated in figure 6. Because the $B$ carries both the gauge $SU(N)$ and $SU(M)$, it interacts with $N$-gluons on one side and with $M$-gluons on the other, and thus forms a boundary between regions in the planar diagram, as shown. Note that the $M$-region has the opposite orientation, in the sense that the arrows are on the left of the gluon dressing.
Figure 8: A class of diagrams contributing to the propagation of $ABC$ baryons in leading order in large $N$. The $\bullet$s are interpolating fields for the baryon and antibaryon.

A $C$ loop can produce a hole in a regions dressed by $M$-gluons, as illustrated in figure 8. In the internal $C$ loop, as in the $B$ loop, the arrows are on the left of the $M$ region. These diagrams are related to decays of meson resonances into baryon-antibaryon states.

Figure 9: A class of diagrams contributing to the propagation of $\Lambda \Sigma$ singlet mesons in leading order in large $N$.

Finally, the propagation of baryons is described by diagrams like that illustrated in figure 8. Of course, there are more complicated diagrams in which both the $N$ and $M$ regions are full of holes!

The graphs contributing to the propagation of flavor singlet particles are more complicated, because the singlets can couple to the fermion loops inside the planar diagram. These graphs vanish for flavor adjoint mesons, but are leading in $N$ for the flavor singlet case. A simple example is shown in figure 9.
Figure 10: A class of diagrams contributing to the interactions of three $A\overline{A}$ adjoint mesons in leading order in large $N$. These graphs vanish like $1/\sqrt{N}$ as $N \to \infty$, but the large number of meson states sometimes compensates for this suppression so the interactions cannot be ignored. Multi-meson interactions arise when more $\bullet$s appear on the external $A$ loop.

4 Interactions in Large $N$

In this section, we will show that the leading interactions between the bound states in the theory of figure 9 are associated with graphs which, like the propagator graphs in the previous section, are planar and have all their flavor structure on an external loop. It will simplify the discussion of interactions if we begin by establishing an important general principle. **Interactions in leading order involve only a finite number of flavors at a time.** Interactions of states involving many flavors (like singlet states) are suppressed because of large $N$ counting. This may seem counter-intuitive, because the states in which all the flavors participate can give rise to large counting factors from loops. But these factors also appear in the propagator, and so are explicitly divided out in the wave-function renormalization factors. When the interpolating fields for these states then appear in more complicated graphs, the wave function renormalization gives an additional suppression. One way of saying this is that the interactions of singlets and other states involving order $N$ flavors are suppressed by counting factors not only because the number of colors is large, but also because the number of flavors is large.

Let us illustrate this in a simple example. Compare the 3-meson interactions from the class of diagrams shown in figure 10 for two different wave functions: an adjoint field with a flavor wave-function $\frac{1}{2} \lambda_8$ involving only the first three flavors in the global $SU(M)$; and a singlet field, with wave-function $\frac{1}{2} I/\sqrt{M}$, proportional to the identity in the $SU(M)$ flavor space (and thus of course involving all the flavors). Both wave functions also contain the usual $1/\sqrt{N}$ for color. Now in figure 10, the color factors are the same — a factor of $N$ for the loop and three factor of $1/\sqrt{N}$ from the wave-functions. Thus both interactions are suppressed by $1/\sqrt{N}$ as expected. But the flavor factors are different. The adjoint coupling is proportional to $\frac{1}{8} \text{tr} \lambda_8^3 = \mathcal{O}(1)$. But the singlet interactions have an additional suppression
of $\frac{1}{\sqrt{M}}$ tr $I = \mathcal{O}(1/\sqrt{M})$. This is a general feature of such states.

Thus in thinking about interactions, we can completely ignore singlets and other states that have significant contributions from a number of flavors that goes to infinity. This is a great convenience, because it means that we do not have to worry about hidden flavor factors in wave-functions. They are only there if the states are unimportant anyway. When we discuss the interactions below, we will not often remind the reader of this — we will simply assume that we are dealing with states involving only a finite number of flavors.\footnote{See, however, Appendix A, for a discussion of why internal loops cannot be ignored in the effective potential.}

Figure 11: A class of diagrams contributing to the interactions of an $A\overline{A}$ adjoint mesons with a baryon-antibaryon pair in leading order in large $N$.

Figure 10 is related to figure 5. If we cut the diagrams in figure 5 through the internal loop, the cut crosses two regions that look like $A\overline{A}$ meson propagators. Thus the imaginary part will be related to the partial width of the meson state into meson pairs. We see from figure 10 that the individual couplings are small — of order $1/\sqrt{N}$. But because there are order $M$ states that can appear in the final 2-meson state, the contribution to the width is order 1. This is what we expect in a matrix model.

In a similar way, we can construct diagrams that contribute to the interactions of baryons and mesons or multi-baryon states. For example, the diagrams illustrated in figure 11 contribute to the interaction of an $A\overline{A}$ adjoint mesons with a baryon-antibaryon pair in leading order in large $N$. This is related to a cut through the internal loop in figure 4.

In this model, we can show that the $N$ counting for the $ABC$ baryons is actually the same as for the mesons. Each additional baryon in a graph gives a factor of $1/\sqrt{N}$. To see this, note that because the baryon propagator graphs like those in figure 8 have both an $N$ loop and an $M$ loop, the wave-function renormalization factor for a baryon goes like $1/\sqrt{NM} \sim 1/N$, smaller by $1/\sqrt{N}$ than for a meson. However, this is compensated by the $B$ line that emerges from each baryon vertex. This counts like a gluon line because it carries both $N$ and $M$ indices. Thus the extra $1/\sqrt{N}$ in the baryon wave function acts like a
Figure 12: A class of diagrams contributing to the 4-baryon interactions in leading order in large $N$.

Gluon coupling that compensates the extra loop associated with the $B$ line. Adjoint mesons and $ABC$ baryons both act like matrix fields with couplings suppressed by $1/\sqrt{N}$.

Figure 13: A class of diagrams that contribute to the interactions of four $A\overline{A}$ adjoint mesons in leading order in large $N$. These graphs contribute to 4-meson couplings of order $1/N$ (because $M/N^2 = O(1/N)$).

Figure 12 illustrates graphs that contribute to 4-baryon interactions. You can check explicitly that the coupling is order $1/N$, in line with the general argument in the previous paragraph.

In all these leading interaction diagrams, the flavor structure is associated with a single flavor loop around the outside. Flavor loops inside as in figure 13 are OK, but only because they involve a sum over all flavors. If mesons or baryons couple to two separate loops, as in
Figure 14: A class of diagrams that do not contribute to the interactions of four $A\overline{A}$ adjoint mesons in leading order in large $N$. These graphs contribute to 4-meson couplings of order $1/N^2$, because the internal loop does not contribute a factor of $M$.

figure 14, the diagram is suppressed. Note that this implies that there is no coupling between $A\overline{A}$ and $C\overline{C}$ adjoint mesons in leading order in $N$ except in diagrams involving the baryons on the external line, even though both kinds of meson have leading interactions with the $ABC$ baryons.

Before we go on to more complicated and interesting models, it is worth noting that the large $N$ limit that we have discussed in this and the previous section is completely consistent with our assumption that there is no phase transition in this model as a function of the ratio of gauge couplings. The fermion bound states that exist in large $N$ are precisely those required to be massless to saturate the anomaly condition. And the quantum numbers of the massless fermion states match precisely with what we find in the Higgs picture that emerges if one coupling is much larger than the other. The $A\overline{A}$ and $C\overline{C}$ mesons that exist in large $N$ are expected to play no role in global symmetry breaking and there is no reason to expect any of them to be light.

$k \neq 0$

Let us now consider the field theory associated with the moose shown in figure 2 in the limit $N, M \to \infty$ with $M/N$ and $k$ fixed. Define the dimensionless variable

$$\xi_{M/N} \equiv \Lambda_M/\Lambda_N$$

(3)

In this theory, we know that there is a phase transition as a function of $\xi_{M/N}$ because the theory has different symmetry and particle content in the two limits $\xi_{M/N} \gg 1$ and $\xi_{M/N} \ll 1$. To see this explicitly, let us label the fields and symmetries as shown in figure 15.

\[\text{This will be elaborated in the next section.}\]
For $\xi_{M/N} \gg 1$, the $SU(N)_g$ gauge coupling is small at the $SU(M)$ confinement scale, $\Lambda_M$. Thus we can analyze the symmetry breaking produced by the $SU(M)_g$ gauge theory ignoring the effect of the $SU(N)$ interactions. We therefore expect the $B$ and $C$ fermions to condense, breaking the $SU(N)_g \times SU(N)$ symmetry down to a diagonal $SU(N)$ [13]. In the process, the Goldstone bosons associated with the symmetry breaking are eaten by the Higgs mechanism, leaving the $SU(N)_g$ gauge bosons, $G_N$, massive, but light because the gauge coupling is small. This process leaves the global $SU(M+k) \times SU(k)$ symmetry unbroken, and leaves the $A$ and $D$ fields in the theory in the infrared as free massless fermions.

Summarizing,

\[
\text{the unbroken global symmetry} \quad \text{in the theory is} \quad SU(M+k) \times SU(k) \times SU(N)
\]

\[
\text{the light particles} \quad \text{in the theory are:}
\]

- massless LH fermions $A$ transforming like $(M+k, 1, \overline{N})$;
- massless LH fermions $D$ transforming like $(1, \overline{N}, N)$;
- light gauge bosons $G_N$ transforming like $(1, 1, N^2-1)$.

For $\xi_{M/N} \ll 1$, the $SU(M)_g$ gauge coupling is small at the $SU(N)$ confinement scale, $\Lambda_N$. Then we can analyze the symmetry breaking produced by the $SU(N)_g$ gauge theory ignoring the effect of the $SU(M)$ interactions. We therefore expect the $A$ fermions to condense with the $B$ and $D$ fermions. If we could completely ignore the $SU(M)_g$ gauge couplings, the $B$ and $D$ fermions would combine into $M+k$ fermions with a global $SU(M+k)$ symmetry. In this limit, we would have an $SU(M+k) \times SU(M+k)$ symmetry spontaneously broken

\footnote{That is, they interact only through nonrenormalizable interactions.}
down to $SU(M+k)$, producing an $SU(M+k)$ adjoint of Goldstone bosons. However, the $SU(M)_{\text{g}}$ gauge interactions break this down to $SU(M) \times SU(k)$ and eat an $SU(M)$ adjoint of Goldstone bosons, leaving the $SU(M)_{\text{g}}$ gauge bosons, $G_M$, massive, but light because the gauge coupling is small. The $SU(N)$ symmetry is unbroken, and the $C$ fermions are still massless, transforming like $(M, 1, \overline{N})$ under the $SU(M) \times SU(k) \times SU(N)$. The uneaten Goldstone bosons transform like $(1, k^2-1, 1) \oplus (1, 1, 1) \oplus (M, k, 1) \oplus (M, \overline{k}, 1)$ corresponding to the coset space

\[
\frac{SU(M+k) \times SU(k)}{SU(M) \times SU(k)} \tag{5}
\]

Summarizing,

for $\xi_{M/N} \ll 1$

**the unbroken global symmetry** in the theory is

$SU(M) \times SU(k) \times SU(N)$

**the light particles** in the theory are:

- massless LH fermions $C$ transforming like $(M, 1, \overline{N})$;
- light gauge bosons $G_M$ transforming like $(M^2-1, 1, 1)$;

Goldstone bosons in the coset space

\[
\frac{SU(M+k) \times SU(k)}{SU(M) \times SU(k)} \tag{6}
\]

Evidently, (4) and (6) are different, so there must a phase transition between these two regions. The question that we want to address is — What kind is it? We will show that the transition is exactly what we would expect from a second order transition caused by the expectation value of an $A-D$ bound state meson. This is a nontrivial consistency check because the $A-D$ mesons are among the (few) states that we expect in the large $N$ limit.

To see that this works in detail, let us look in detail at the region $\xi_{M/N} \approx 1$, but assuming that the global chiral symmetries are unbroken — that is that we are in a phase continuously connected to (4) for $\xi_{M/N} \gg 1$.

We expect fermion bound states in two channels: $A-B-C$ bound states which transform as $(M+k, 1, \overline{N})$, and a $\overline{C}-\overline{B}-D$ bound states which transform as $(1, k, \overline{N})$. The anomaly conditions are saturated simply if we assume that the lightest state in each channel is a massless LH fermion, so that is what we will assume. That it is possible to saturate the anomaly conditions with the states we expect in large $N$ is a nontrivial check on the assumption that we are in a phase continuously connected to (4). Note further that the anomaly conditions determine uniquely whether the massless states are left handed or right handed. We can usefully write the interpolating fields for these massless states as matrix fields,

\[
\psi_{ABC} \text{ is an } M+k \times N \text{ matrix;}
\]

\[
\psi_{\overline{C}BD} \text{ is an } N \times k \text{ matrix.} \tag{7}
\]
The $A$-$D$ bound state meson can also be described by a matrix field,

$$\phi_{AD}$$

is an $M+k \times k$ matrix. (8)

In terms of these matrix fields, we can write the leading Yukawa coupling between these fermions as

$$\text{tr} \left( \psi_{CBD} \phi_{AD}^\dagger \psi_{ABC} \right)$$

(9)

Note that one reason we have singled out the $A$-$D$ bound state is that it is the only scalar that has a renormalizable Yukawa coupling in leading order in large $N$. In the $k=0$ theory, where the $D$ field does not exist, there are no renormalizable Yukawa couplings at all. This is because the massless fermions in this model are all left-chiral and carry a conserved $U(1)$ quantum number. The $U(1)$ symmetry means that the massless fermions $\psi_L$ must appear in the combination $\overline{\psi}_L \psi_L$, which cannot be used to construct a Lorentz invariant dimension four Yukawa coupling. Of course, the other reason that we have singled out this field is that it is implicated in the phase transition in large $N$, as we will now show more explicitly.

The phase for $\xi_{M/N} \ll 1$ corresponds to a large negative mass squared for the $A$-$D$ mesons. The mesons transform like $(M+k,\bar{k},1)$ under the global symmetries, described by an $M+k \times k$ matrix. If $\phi_{AD}$ develops an expectation value of the form

$$\langle \phi_{AD} \rangle \propto \begin{pmatrix} I \\ 0 \end{pmatrix}$$

(10)

where $I$ is the $k \times k$ identity matrix, it breaks the symmetry down to $SU(M) \times SU(k) \times SU(N)$ producing the Goldstone bosons of $(\mathbf{3})$. The non-Goldstone components of $\phi_{AD}$ get mass due to leading self-couplings of $\phi_{AD}$, but they are light if the expectation value is small. The fermion field $\psi_{ABC}$ breaks up into an $(M,1,\bar{N}) \oplus (1,k,\bar{N})$. At the same time, the Yukawa couplings, $(\mathbf{8})$, gives a small mass to the fermions that transform nontrivially under $SU(k)$, leaving only the $(M,1,\bar{N})$ of massless fermions. If the transition is second order, the complete $\phi_{AD}$ multiplet that exists as $\xi_{M/N}$ approaches 1 from above gets lighter as the transition is approached, and the sign of mass squared changes at the transition, leading to the expectation value $(\mathbf{10})$.

Summarizing,

for $\xi_{M/N}$ not much greater than 1

the **unbroken global symmetry** in the theory is

$$SU(M+k) \times SU(k) \times SU(N)$$

the **light particles** in the theory are:

- massless LH fermions $\psi_{ABC}$ transforming like $(M+k,1,\bar{N})$;
- massless LH fermions $\psi_{CBD}$ transforming like $(1,\bar{k},N)$;
- light scalar bosons $\phi_{AD}$ transforming like $(M+k,\bar{k},1)$.
Let us now think more carefully about whether and how the putative second order phase transition for $\xi_{M/N} \approx 1$ in (11) matches smoothly onto the our reliable calculations of the particle content in the extreme regions, (4) and (6). As $\xi_{M/N}$ falls from a large value, the unbroken global symmetries and the massless fermions match trivially without any physical change. There is a change in our interpretation of the nature of the global symmetries and the fermions, but this is in the eye of the beholder. The $G_N$ gauge bosons in (4) get heavier and heavier and eventually disappear into the hadronic mess at the confinement scale. They have the quantum numbers of an $SU(N)$ $C\overline{C}$ bound state, and look more and more like a strongly interacting bound state as $\xi_{M/N}$ decreases. The $\phi_{AD}$ is more interesting. We can imagine two ways in which this state might arise as $\xi_{M/N}$ falls. It might emerge from the hadronic mess and get lighter with decreasing $\xi_{M/N}$. Or it might be plucked from the $A-D$ fermion-fermion continuum when some interaction in the effective theory becomes sufficiently strong. In this instance, the former picture is not supportable for large $\xi_{M/N}$. The hadronic states that are strongly interacting for large $\xi_{M/N}$ do not carry any $SU(M+k) \times SU(k)$ quantum numbers, and thus the $\phi_{AD}$ cannot be found there for large $\xi_{M/N}$. Either the $\phi_{AD}$ state only develops as $\xi_{M/N}$ approaches 1, where the strong interactions involve all the fermions, or strong interactions must develop at intermediate $\xi_{M/N}$ to bind the meson state at some value of $\xi_{M/N}$. There is an obvious candidate for these strong interactions. The exchange of the massive $G_N$ gauge bosons produce an attractive force in the $A-D$ channel. This is the original inspiration for the physics of topcolor [4]. At low energies, $G_N$ exchange looks like the 4-fermion interaction of the NJL model [15] but it is cut off at the mass of $G_N$, which for large $\xi_{M/N}$ is much smaller than the cut-off in the effective theory, which is of order $\Lambda_M$. In this situation, the attractive force is weak, and the $\phi_{AD}$ is not bound. The second order picture of the phase transition makes sense if when $G_N$ gets sufficiently heavy, the interaction becomes strong enough to bind the meson. This is plausible, and provides an interesting commentary on the NJL picture. Of course, it is not obvious that these two pictures of the $\phi_{AD}$, emerging from the hadronic mess and being bound from the continuum, are fundamentally different. If the binding does not take place until $\xi_{M/N} \approx 1$, these two may simply be complementary descriptions of the same physics [16]. It may be important for topcolor model building to understand these issues better.

Below the transition, as $\xi_{M/N}$ gets smaller and smaller, again the massless fermions match trivially. The massive fermions and scalars get heavier and heavier and approach into the hadronic mess (now the quantum numbers are right to make this sensible). The $G_M$ gauge boson, with the quantum numbers of an $AA$ bound state, emerges from the hadronic mess and gets lighter and lighter.

One more thing is required to show the consistency of the second order phase transition in the large $N$ limit. We must show that the symmetry breaks in the right way when the sign of the $\phi_{AD}$ mass squared term changes.

It follows from our assumption about the $k = 0$ theory that the Swiss cheese slices in the leading order graphs are flavor blind. As we will show in more detail in appendix A, we can’t prove this. We can’t prove that condensates involving an infinite number of quarks don’t affect the Swiss cheese slices. But we have assumed that it is true. It is part of our assumption about the $k = 0$ theory. We know that no such condensates appear in the
extreme limits of the theory (from the Vafa-Witten argument \[13\]). And our smoothness assumption thus implies that do not appear for intermediate $\xi_{M/N}$.

This plus standard large $N$ arguments shows that non-zero $k$ doesn’t change the Swiss cheese slices in leading order in $N$. This is probably obvious. Internal loops are suppressed by powers of $N$. They have leading effects only if these are compensated by flavor sums. But $k$ is finite, so the contributions of the extra quarks to internal flavor loops is nonleading in $N$.

Because the Swiss cheese slices have no flavor structure, the flavor structure of the leading particle interactions is associated with the flavor loop around the outside. Thus if we write an effective field theory in the neighborhood of the phase transition, the effective potential has the Coleman-Witten form in leading order in $N$ \[14\], in which the only leading terms involve singles traces of products of the matrix fields, $\phi_{AD}, \psi_{ABC}$ and $\psi_{C\overline{BD}}$. The parameters in the effective field theory will depend on $\xi_{M/N}$. We have seen that this is consistent with a second order phase transition. All that is required for a second order phase transition is that the mass parameter go through zero for some value of $\xi_{M/N}$.

Generically the second order phase transition will preserve the $SU(k)$ symmetry because of the Coleman-Witten form. In leading order, the potential in the effective low energy theory is

$$\frac{\lambda}{4} \text{tr} \left( \phi_{AD}^\dagger \phi_{AD} \phi_{AD}^\dagger \phi_{AD} \right) + \frac{m^2}{2} \text{tr} \left( \phi_{AD}^\dagger \phi_{AD} \right)$$

(12)

This can be written as

$$\frac{\lambda}{4} \text{tr} \left[ (\phi_{AD}^\dagger \phi_{AD} + \mu^2 I)^2 \right] + \text{constant}$$

(13)

where

$$\mu^2 = \frac{m^2}{\lambda}$$

(14)

and $I$ is the $k \times k$ identity. When $m^2$ is negative, this is obviously minimized for

$$\phi_{AD}^\dagger \phi_{AD} = -\mu^2 I$$

(15)

which implies that vacuum value can be rotated into the form \[14\]. This is necessary for the consistency of the whole scheme, and again it comes out automatically.

Thus there is a plausible picture of a completely smooth evolution of the theory from one extreme to the other as a function of $\xi_{M/N}$. The phase transition can be second order. We regard the arguments in this section as strong evidence that the phase transition is second order for finite $k$ in the large $N$ limit. We must admit that had hoped to find even stronger results. The fact that the Swiss cheese slices that are the background in which all the fermions propagate are independent of $k$ is very suggestive. It certainly means that the there is no evidence of a phase transition in the vacuum energy in leading order. However, we cannot rule out the possibility that a first order transition takes place that only involves an infinitesimal fraction of the fermions and does not have a leading effect on the vacuum energy.

\[8\]We know from large $N$ arguments that condensates involving only a small number of flavors do not appear, but the large $N$ arguments show that they don’t matter anyway.
Let us now perform a similar analysis of the theory shown in figure \( \text{Fig. 3} \). Again, we begin by labeling the fields and symmetries as shown in figure \( \text{Fig. 16} \).

The arguments for the two limiting cases, \( \xi_{M/N} \gg 1 \) and \( \xi_{M/N} \ll 1 \) should by now be familiar, so we will simply summarize the results.

**for \( \xi_{M/N} \gg 1 \)**

*the unbroken global symmetry* in the theory is

\[
SU(M+k) \times SU(k) \times SU(\ell) \times SU(N)
\]

*the light particles* in the theory are:

- massless LH fermions \( A \) transforming like \((M+k, 1, 1, N)\);
- massless LH fermions \( D \) transforming like \((1, \overline{k}, 1, N)\);
- light gauge bosons \( G_N \) transforming like \((1, 1, 1, N^2-1)\).

Goldstone bosons in the coset space

\[
\frac{SU(N+\ell) \times SU(\ell)}{SU(N) \times SU(\ell)}
\]
for $\xi_{M/N} \ll 1$

**the unbroken global symmetry** in the theory is

$$SU(M) \times SU(k) \times SU(\ell) \times SU(N+\ell)$$

**the light particles** in the theory are:

- massless LH fermions $C$ transforming like $(M, 1, 1, N+\ell)$;
- massless LH fermions $E$ transforming like $(M, 1, \ell, 1)$;
- light gauge bosons $G_M$ transforming like $(M^2-1, 1, 1, 1)$;
- Goldstone bosons in the coset space

$$SU(M+k) \times SU(k)$$
$$SU(M) \times SU(k)$$

The arguments of the previous section suggest that we should expect the intermediate region, $\xi_{M/N} \approx 1$, to be continuously connected to each of the extreme regions. The argument is very similar to what we have already seen, but considerably more involved. We consider the light bound states in the confining picture, assuming that the global symmetries are unbroken, or perhaps broken only the vacuum values of composite fields. For $\xi_{M/N} \approx 1$, we expect massless LH fermion bound states in each of the four channels that are allowed in large $N$:

- an $A$-$B$-$C$ bound state which transform as $(M+k, 1, 1, N+\ell)$,
- a $C$-$B$-$D$ bound state which transform as $(1, k, 1, N+\ell)$,
- an $E$-$B$-$A$ bound state which transforms as $(M+k, 1, \ell, 1)$,
- an $D$-$B$-$E$ bound state which transforms as $(1, k, \ell, 1)$.

As before, this is the unique choice that saturates the anomaly conditions. Again, we can usefully write the interpolating fields for these massless states as matrix fields,

$$\psi_{ABC}$$ is an $M+k \times N+\ell$ matrix;
$$\psi_{CBE}$$ is an $N+\ell \times k$ matrix;
$$\psi_{EBA}$$ is an $\ell \times M+k$ matrix;
$$\psi_{DBE}$$ is an $k \times \ell$ matrix.

We also expect light bound state mesons, $A$-$D$ and $E$-$C$. These can also be described by matrix fields,

$$\phi_{AD}$$ is an $M+k \times k$ matrix;
$$\phi_{EC}$$ is an $\ell \times N+\ell$ matrix.
In terms of these matrix fields, we can write the leading Yukawa coupling between these fermions as

\[
\begin{align*}
&\text{tr} \left( \psi_{CBD}^\dagger \phi_{AD}^\dagger \psi_{ABC} \right) \\
&\text{tr} \left( \psi_{EBA}^\dagger \phi_{AD} \psi_{DBE} \right) \\
&\text{tr} \left( \psi_{ABC} \phi_{EC}^\dagger \psi_{EBA} \right) \\
&\text{tr} \left( \psi_{DBE}^\dagger \phi_{EC} \psi_{CBD} \right)
\end{align*}
\]

(20)

Summarizing,

for \( \xi_{M/N} \approx 1 \)

**the unbroken global symmetry** in the theory is

\[SU(M+k) \times SU(k) \times SU(\ell) \times SU(N+\ell)\]

**the light particles** in the theory are:

- massless LH fermions \( \psi_{ABC} \) transforming like \((M+k,1,1,N+\ell)\);
- massless LH fermions \( \psi_{CBD} \) transforming like \((1,\overline{k},1,N+\ell)\);
- massless LH fermions \( \psi_{EBA} \) transforming like \((\overline{M+k},1,\ell,1)\);
- massless LH fermions \( \psi_{DBE} \) transforming like \((1,k,1,\overline{N+\ell})\).

Now let us see how this goes smoothly into the two extreme limits. The situation is fairly symmetrical, so we will just discuss \( \xi_{M/N} \gg 1 \). This corresponds to positive mass squared for \( \phi_{AD} \) and a negative mass squared for \( \phi_{EC} \), which develops a vacuum value of the form

\[\langle \phi_{EC} \rangle \propto \begin{pmatrix} I & 0 \end{pmatrix}\]

(22)

where \( I \) is the \( \ell \times \ell \) identity matrix. This breaks the symmetry down to \( SU(M+k) \times SU(k) \times SU(\ell) \times SU(N) \), and produces the appropriate Goldstone bosons. Under this symmetry, the fermion fields break up as follows:

\[
\begin{align*}
\psi_{ABC} : (M+k,1,1,N+\ell) &\rightarrow (M+k,1,1,\overline{N}) \oplus (M+k,1,\overline{\ell},1) \\
\psi_{CBD} : (1,\overline{k},1,N+\ell) &\rightarrow (1,\overline{k},1,N) \oplus (1,\overline{k},\ell,1) \\
\psi_{EBA} : (\overline{M+k},1,\ell,1) &\rightarrow (\overline{M+k},1,\ell,1) \\
\psi_{DBE} : (1,k,1,\overline{\ell}) &\rightarrow (1,k,1,\overline{\ell})
\end{align*}
\]

(23)

All of the fields that transform nontrivially under the \( SU(\ell) \) get mass due to the Yukawa couplings, (20). The massless fermions are just what we expect from (16).
Thus again, this is consistent with a second order phase transition — or rather, with two second order phase transitions. There is no reason for the mass squares of $\phi_{AD}$ and $\phi_{EC}$ to go to zero at the same value of $\xi_{M/N}$. We do expect that for $\xi_{M/N} \approx 1$, the theory can be described by an effective field theory with both scalars present. But our methods do not tell us which phase transition occurs first. There may be a region near $\xi_{M/N} = 1$ in which both mass squares are positive and the full symmetry is unbroken. Or there may be a region in which both mass squares are negative, and the symmetry is broken down to $SU(M) \times SU(k) \times SU(\ell) \times SU(N)$. We cannot say with the tools we have used here. In either case, however, theories with this structure open up new possibilities for topcolor model building that are only present with dynamical topcolor breaking.

7 Conclusions

We have analyzed a family of chiral gauge theories in the large-$N$ limit. Our aim has been to develop new tools that may be useful in constructing models with dynamically broken topcolor.

The interactions in these models are quite simple in the large-$N$ limit: couplings of mesons and baryons are given by planar diagrams with all interpolating fields on the outermost quark loop. Since the number of flavors is large, the diagrams have a Swiss cheese-like structure, with arbitrary numbers of internal quark loops forming holes in the diagram. Couplings of baryons and mesons are suppressed by the usual factors of $1/\sqrt{N}$. The analysis is simplified by the fact that, contrary to what one might expect, states involving an infinite number of flavors decouple in the large $N$ limit.

The phase structure of these models has proven more difficult to analyze. We have argued that models with $k,\ell \neq 0$ exhibit a simple pattern of chiral symmetry breaking. With the assumption that the $k = \ell = 0$ model does not break its global $SU(M) \times SU(N)$ symmetry, we have shown that the model with $k \neq 0$ has two phases, one with $SU(M+k) \times SU(k) \times SU(N)$ symmetry, and another where condensates break the symmetry to $SU(M) \times SU(k) \times SU(N)$. We used a generalization of the Coleman-Witten argument to show that the $SU(k)$ symmetry does not break. Similar considerations apply to the more complicated case with $k,\ell \neq 0$, where we expect two phase transitions.

For these models to be useful for electroweak symmetry breaking, it is crucial that their phase transitions are second order, or at least weakly first order. We have been unable to conclusively prove that this is the case, but the hypothesis that the phase transitions are second order is consistent with all the available evidence. The effective theories contain obvious candidates for symmetry breaking composite Higgs fields, and simple assumptions about the low energy Higgs potential imply that the phase transitions are indeed second order. Hence it seems likely that these models are relevant to topcolor models of dynamical electroweak symmetry breaking.

Although we have been able to analyze many aspects of these models, our work leaves a number of interesting questions unanswered. In particular, questions about the phase structure of these theories are both interesting and very difficult. It would be very exciting if more definitive conclusions could be drawn. However, it seems reasonable to use models of this kind for topcolor model building. One important feature of these models (already noted
in reference [11]) is that the fields that break the topcolor gauge symmetry also contribute to anomaly matching. The richer structure of the models with \( k \) and \( \ell \) both non-zero, however, are virgin territory, yet to be explored by model builders.
A Confusing condensates

There is something that we found confusing about condensates in the large $N$ limit. Our confusion results from the following apparent paradox: The diagrams that dictate the leading interactions in the model are not the same as those that dictate the vacuum structure of the model. As discussed in the text, scattering of mesons and baryons in the model is given in leading order by Swiss cheese slices where the external fields couple only to the outermost quark loop. Diagrams where the external fields couple to quark lines enclosing a hole in the Swiss cheese are suppressed by the usual $1/N$ factor, and this $1/N$ factor is not compensated by a sum over flavors. On the other hand, the vacuum energy is determined by the sum over Swiss cheese slices with no external lines. If we consider a diagram with insertions of some order parameter (such as, for instance, a quark bilinear), then we must perform a trace on the flavor indices to get the contribution to the vacuum energy. The trace on flavor indices means that the order parameters can be inserted on any quark line in the Swiss cheese. There is no $1/N$ suppression of those diagrams where the order parameter lies on a hole in the Swiss cheese, since the $1/N$ is compensated by the sum over flavors. So for the diagrams that determine the leading interactions, the flavor structure resides only on the outermost quark loop, while for the vacuum energy, the flavor structure can lie on any quark loop.

To add to the confusion, we have argued that states involving an infinite number of flavors decouple in the large $N$ limit. However, when we consider condensates, it is possible that the vacuum values of such fields become larger with $N$ and so do not decouple from the vacuum structure of the theory. It is possible that if an instability develops in such a direction, it is not stabilized by the leading order interactions, and instead is driven to a large value, requiring us to go beyond leading order in $N$ (or more precisely, to include the flavor structure of internal loops in leading order) to determine the vacuum structure.

The presence of the subleading contributions from internal quark loops in the vacuum energy complicates the analysis of the symmetry breaking. If it were not for these subleading contributions, the Coleman-Witten arguments could be used to show that the $SU(N) \times SU(M)$ global symmetry does not break. However, in our case, we can construct simple examples that preserve the flavor symmetry, and other examples which do not. The vacuum energy has the form

$$V(\Phi) \sim \sum \frac{\prod_{\ell=1}^{\ell} \text{Tr} \Phi^{n_i}}{N^\ell - 1},$$

(24)

where $\Phi$ is an order parameter with the flavor quantum numbers of $A\overline{A}$ or $C\overline{C}$. Each trace comes from a separate quark loop, and the powers of $1/N$ come from the usual suppression of internal quark loops. Points of unbroken symmetry have $\Phi = \beta I$ for some $\beta$, possibly zero. Suppose the function $V(\beta I)$ has a minimum for some value $\beta = \beta_*$. Then the point $\Phi = \beta_* I$ is a stationary point of the potential. If we could show that this point is a minimum, then we would have established that there is at least a local minimum with unbroken symmetry. However, there are simple (if highly non-generic) examples where the stationary point is a saddle point rather than a minimum. For instance, take the case $N = M$, and consider the potential

$$V = -\frac{5}{8} m^2 \text{Tr} \Phi^2 + \frac{1}{2N} m^2 (\text{Tr} \Phi)^2 + \frac{1}{16} \text{Tr} \Phi^4.$$

(25)

This has a single stationary point at $\beta_* = m$. But by considering small variations in the
diagonal elements of $\Phi$ in the neighborhood of this point, it can be shown that this is a saddle point, even as $N \to \infty$.

We feel it is unlikely that the $SU(N) \times SU(M)$ flavor symmetry is broken by any condensates. We suspect that the only symmetry-breaking condensates are those that occur in the models with finite $k$ and $l$, and that the low energy theory always preserves (at least) the $SU(N) \times SU(M)$ flavor symmetry. However we have been unable to prove this in general, and the considerations of this appendix seem to indicate that large $N$ and anomaly matching are insufficient to establish the pattern of symmetry breaking.
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