Memory in aged granular media

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(Received ; accepted)

PACS. 05.70.Ln., 81.05.Rm., 75.10.Nr

Abstract. – Stimulated by recent experimental results, we simulate “temperature”-cycling experiments in a model for the compaction of granular media. We report on the existence of two types of memory effects: short-term dependence on the history of the sample, and long-term memory for highly compact (aged) systems. A natural interpretation of these results is provided by the analysis of the density heterogeneities.

The study of the rheology of granular media represents nowadays an important chapter in the more general context of the so-called soft condensed matter [1]. The general questions one is interested in concern the response behaviour of a generic granular medium submitted to an external perturbation. In this spirit it has been recognized that the response of such systems depends in a non-trivial way on the mechanical properties of the grains composing the medium, on the boundary conditions, on the driving procedure (the way to inject energy to perturb the system) and, last but not the least, on the past history of the system, e.g. on the procedures the system has undergone until the moment we study it.

In this context the similarities between granular media and the phenomenology of glassy systems have been exploited. One of the main results has been to recognize that in a compaction procedure, where a certain system increases its density if shaken or tapped, a granular medium displays ageing [2] and its properties, e.g. the two-times correlation function, depend on the time where one performs the measurement [3, 4], i.e. on the age of the system.

The analogy with glassy systems has mainly allowed to export tools and techniques from one field to the other. Typical experiments on glassy systems consists in monitoring the effect of changes imposed to the external temperature on the macroscopic observables [5]. Translated in the framework of granular matter, i.e. of a non-thermal system, this means to analyse the effect of changes in the driving procedure, e.g. in the tapping amplitude $\Gamma$, which, in a typical compaction experiment, represents the control-parameter.
In a previous work [4], we have presented a detailed study, in the framework of a microscopic model for granular media, focusing on the response to small perturbations in the driving rate. We have pointed out the importance of the interplay between the response properties and the spatial structures that spontaneously emerge as a consequence of the dynamics imposed to the system. In particular we have studied the relevance of the densities heterogeneities in the process of granular compaction, and shown that the density is not the only relevant information about the system. Configurations with the same density but reached with different dynamical procedure show completely different rheological properties. This was nicely confirmed by a set of recent experiments [6]: three systems prepared at the same density but in different ways display different behaviours if the same tapping acceleration is applied to them. This feature concerns the coding of the system history in the microscopic configurations.

In the same set of experiments, similar in spirit to the temperature-cycling experiments in spin-glasses [5] the authors of [6] have put in evidence the existence of memory effects in the compaction of granular media. In particular they performed the study of the effect of relatively large and abrupt changes in the tapping acceleration. This is an efficient way to obtain reliable experimental data since the effect of small changes in the tapping acceleration is difficult to detect in real experiments. What was shown in [6] is that, during the compaction, the response to an abrupt change in the tapping acceleration $\Gamma$ is opposite (at least at short times) to what could be expected from the long-time behaviour of the compaction: while at constant $\Gamma$ a higher compaction rate is obtained for larger $\Gamma$, a sudden increase in $\Gamma$ leads to a decompaction, and a sudden decrease gives rise to a temporary increase in the compaction rate. This behaviour is however transient, and the usual compaction rate is recovered after a while: the authors therefore call this effect “short-term memory”. The same qualitative behaviour has been obtained also in experiments of compaction under shear [7].

In order to push forward this kind of studies we have extended our work [4] to study the response to large changes in the driving. In particular we explore the behaviour of the Tetris Model [8, 9] for abrupt changes in the driving rates in various time regimes, that were not considered in the preliminary, time-limited, experiments [6]. We also explore the return to the initial tapping acceleration and the presence or absence of memory, in a way similar to the temperature-cycling experiments in spin-glasses. We perform these temperature-cycling experiments in non-equilibrium (aging) as well as in quasi-stationary (time-translation invariant) situations. We hope in this way to stimulate new and more precise experiments along the same lines.

The model we consider, the so-called Tetris Model [8], consists of particles on a tilted (two-dimensional) square lattice, which has lateral periodic boundary conditions and a closed boundary at the bottom. With respect to its original definition we consider a random version where each particle can be schematized in general as a cross with 4 arms (in general the number of arms is equals to the coordination number of the lattice) of different lengths, denoted by $l_{NE}, l_{NW}, l_{SE}, l_{SW}$, chosen in a random way (see [9] for details).

Gravity and shaking are implemented as follows: the particles diffuse on the lattice, with geometrical constraints, and probability $p_{up}$ to move upwards (with $0 < p_{up} < 1$), and a probability $p_{down} = 1 - p_{up}$ to move downwards. The quantity $x = p_{up}/p_{down}$ can be related to the adimensional acceleration $\Gamma$ used in compaction experiments [10] through the relation $\Gamma \simeq 1/\log(1/\sqrt{x})$. The sizes (width $\times$ height) used ranged from $200 \times 60$ to $20, 40, 60 \times 200$, in order to test various aspect ratios.

The procedure used in the experiments and in our simulations is inspired by classic experiments in spin-glasses. The system is initialized by random addition of particles, and let evolve

(1) the use of closed lateral boundary conditions do not change the results
with a constant external forcing $x_1$ up to a certain time $t_w$. Two copies of the system are then made: one copy is kept evolving with $x_1$ at all times for reference, while the second copy evolves with a different forcing $x_2$ for a time $\Delta t$, after which the forcing is set back to $x_1$. We monitor the mean height, the bulk density (defined as the average density in the lower 50 of the system), and the density profiles of both copies during the whole evolution for various values of the difference $x_1 - x_2$, $t_w$ and $\Delta t$. We recall that the density profiles are defined averaging the density along the horizontal direction, giving thus indications on the heterogeneities along the vertical direction (see figure 1), and allowing to roughly distinguish two regions, the bulk and the interface. Before analyzing the different regimes, let us recall a few general features of the compaction at constant forcing. At short times, the density is an increasing function of the forcing while at long times, there exists an optimal value of $x$ that allows to obtain the maximal density in the fraction of the system considered for the measurement. This phenomenology has been shown to be linked to the presence of heterogeneities in the system [4].

**First case: $x_1 > x_2$.** When, at $t_w$, $x$ is suddenly lowered, the compaction rate of the perturbed system first increases. This behaviour, shown in figure 3 and 4, is in agreement with the experimental results, and it is opposed to the results one would expect looking at the compaction data at constant forcing. This shows that the system has some memory of its history at $t_w$. While a transient, however, this memory is lost: compaction slows down, and the rate of compaction crosses over to the one observed at constant forcing: the curves of the reference and perturbed systems therefore cross.

While the transient, corresponding to the short-term memory, is observed for all values of the parameters investigated, an important difference appears for different $t_w$ at long times: if $t_w$ is small, the system is not very compact, and is able to evolve a lot during $\Delta t$. The system does not display memory for times larger than $t_w + \Delta t$. On the other hand, if the system is already quite compact at $t_w$ (i.e. if it is aged enough), the system evolves very slowly during $\Delta t$, and for $t \geq t_w + \Delta t$ the compaction curves can be translated and superimposed to the reference one, as shown in the inset of figure 4. During $\Delta t$ the system is able to keep memory of its state at $t_w$. This shows that a sufficiently aged system displays a long term memory, in addition to the short-term memory existing at all times.

The interpretation of these results is quite straightforward using the results of [4] and looking at the density profiles along the vertical direction: when $x$ is abruptly lowered, the first effect is that the particles tend to go down, and the interface becomes steeper and more compact. Therefore the density first increases with respect to the unperturbed case. At larger times however, the evolution is slowed down by the creation of a dense layer at the interface, which blocks the bulk rearrangements needed for the compaction [3]. After $t_w + \Delta t$, the increase in the forcing allows to suppress the dense layer, and the compaction can become again fast. Moreover, if $t_w$ is large enough (typically $10^3$ or $10^4$ MC steps), the bulk of the system is already quite compact, and therefore the smaller value of the forcing during $\Delta t$ leads to a compaction of the interface but the bulk almost does not evolve. At $t_w + \Delta t$, the forcing is again increased: the relaxation of the interface being fast, this leads the system back to its state at $t_w$. The inset of Figure 4 illustrates this behaviour: the curves of compaction after $\Delta t$, shifted backwards by $\Delta t$, superimpose with the reference curves. This is not the case for lower values of $t_w$, for which the bulk is not yet highly compact at $t_w$, and evolves a lot even with a small driving $x_2$.

This memory effect, similar to what can be obtained in spin-glasses when the temperature is first lowered then increased back to its previous value, was unfortunately not investigated in [4]: it can indeed occur only at long times. It would certainly be very interesting to have experimental checks on this aspect.

**Second Case: $x_1 < x_2$.** When $x_1 < x_2$ we observe a short-term memory effect, as in the
previous case. First, as the forcing is increased, one observes a decompaction; later on, the fact that \( x \) is larger prevails and the compaction proceeds faster, at the normal rate for constant \( x = x_2 \). The memory of the history up to \( t_w \) is lost after a transient (see figure 4). These features are again similar to the experimental results. Moreover, at long times no memory is kept, since the whole system evolves a lot during \( \Delta t \).

Once again, the study of the density profile allows for an interpretation of these results. As the tapping intensity is increased, the first effect is a decompaction, especially at the interface. The fact that the interface is less compact then allows for a much better compaction of the bulk. Note that this behaviour is closely related to the existence of a response function that is positive at short times and negative at longer times \([4]\). At \( t = t_w + \Delta t \), the bulk has been deeply modified, so the system cannot have any memory of its configuration at \( t_w \).

As a first partial conclusion we see that the interplay between the heterogeneities of the system and the influence of the forcing on different zones of the sample \([4]\) leads to two quite different memory effects. On one hand the short-time memory is always present and it occurs in a way which is counterintuitive with respect to the effect expected on the basis of the behaviour at constant \( x \). On the other hand the long-time memory only exists for a cycle when \( x_2 < x_1 \) and for long times, i.e. for aged systems.

**Perturbation of a stationary state** The numerical experiments mentioned above have been performed during the compaction process, i.e. while the system ages. Let us now focus instead on the behaviour obtained after a slow decrease of the vibration intensity, starting from large forcing: this is the usual way to obtain a very compact system \([10]\) which, under weak driving \( x_1 \), is now in a quasi-stationary state \([4]\), i.e. a state where time-translation invariance holds. This state can now be perturbed by applying a stronger forcing \( x_2 \) for a time interval \( \Delta t \), and the relaxation can be studied when the forcing is set back to the original value \( x_1 \). In \([6]\), such experiments were performed for small \( \Delta t \), showing non-exponential relaxations, slower the larger is \( \Delta t \). We are able here to explore different regimes, varying \( \Delta t \) over four orders of magnitude.

One observes a relaxation occurring in two steps (see figure 5). First one has a rapid decrease (during typically 10 time steps) corresponding to the relaxation of the topmost layers of the interface; this can be checked by looking carefully at the density profiles during the relaxation. Later on a much slower, almost logarithmic, decay occurs corresponding then to the re-compaction of layers under the interface. As \( \Delta t \) grows, deeper and deeper layers get involved during the decompaction process; as \( x \) is lowered again, the first relaxation, which compactifies the interface, makes it more difficult for the lower layers to get as compact as before: the relaxation therefore slows down.

Although the very dense medium, if shaken at constant \( x \), is in a stationary state, the perturbation at larger \( x \) leads to same mechanisms that are also at work during the aging. Therefore, it is not so surprising to find a slow re-compaction process after the perturbation.

The precise form of the relaxation can be expected to be model dependent, at least at short times: for example, in the case of “tapping”, the first, rapid part of the relaxation does not exist. Therefore, we do not investigate it in details, and underline only the most relevant features: even though the system seems in “equilibrium” (as shown in \([4]\), the dense system is stationary and does not display aging), the relaxation is non-exponential and very slow.

In summary, we have shown that the Random Tetris Model reproduces the complex phenomenology appearing in real experiments for large and abrupt changes of the driving: at small times after the change, the reaction of the system is opposite to what could be expected looking at the constant \( x \) compaction curves: if \( x \) is decreased, the system compactifies better, while it gets decompactified for an increase in \( x \). At larger times, the compaction rate crosses over to the one at constant \( x \): this effect was therefore called ‘short-term memory’ \([6]\).
Moreover, we have investigated the return to the previous value of the driving, in various time regimes, and shown that an increase in $x$ erases quickly the memory of the previous configurations while, at long times, a decrease in $x$ can “quench” the system until the previous value of $x$ is restored and the compaction starts again from a configuration statistically equivalent to the one perturbed at $t = t_w$.

It is noticeable that such a behaviour shares some similarities with the phenomena of rejuvenation and memory observed in the context of spin glasses [5]. This feature pushes forward the similarities between (non-thermal) granular media and (thermal) glasses and spin-glasses. For example, note that a trap model [11], widely exploited for the understanding of the phase space of spin glasses, has recently been used to describe granular media [12], as well as a random-walk model [7]. These approaches also reproduce qualitatively the experimental results of [6]; however they describe the behaviour of a system in phase space. It seems to us that the study of the heterogeneities (and therefore of real space models) could be crucial for the understanding of the basic mechanisms underlying the dynamics of these systems. In this respect granular media seem more tractable than spin glasses and detailed experiments about the local rearrangements during compaction could provide valuable information for the understanding of the basic mechanisms at work in dense granular media.

REFERENCES

[1] For a recent introduction to the overall phenomenology see Proceedings of the NATO Advanced Study Institute on Physics of Dry Granular Media, Eds. H. J. Herrmann et al, Kluwer Academic Publishers, Netherlands (1998).

[2] See, for example Chapter 7 of: Struik, L.C.E. Physical Aging in Amorphous Polymers and Other Materials, (Elsevier, Houston, 1978).

[3] M. Nicodemi and A. Coniglio, Phys. Rev. Lett. 82, 916 (1999).

[4] A. Barrat, V. Loreto, J. Phys. A. 33, 4401 (2000).

[5] See e.g. E. Vincent, J. Hammann, M. Ocio, J.-P. Bouchaud, L.F. Cugliandolo, in Spin Glasses and Random Field ed. A. P. Young, (World Scientific, 1997) and references therein.

[6] C. Josserand, A. Tkachenko, D.M. Mueth and H.M. Jaeger, Phys. Rev. Lett. to appear.

[7] S. Luding, M. Nicolas and O. Pouliquen, A minimal model for slow dynamics: Compaction of granular media under vibration or shear, cond-mat/0003172; M. Nicolas, P. Duru and O. Pouliquen, Compaction of a granular material under cyclic shear, cond-mat/0006252.

[8] E. Caglioti, V. Loreto, H.J. Herrmann, and M. Nicodemi, Phys. Rev. Lett. 79, 1575 (1997).

[9] E. Caglioti, S. Krishnamurthy and V. Loreto, Random Tetris Model, unpublished (1999).

[10] Knight, J. B., Fandrich, C. G., Lau, C. N., Jaeger, H. M. and Nagel, S. R. Phys. Rev. E 51, 3957 (1995); Nowak, E. R., Knight, J. B., Povinelli, M., Jaeger, H. M. and Nagel, S. R., Powder Technol. 94, 79-83 (1997); Nowak, E. R., Knight, J. B., Ben-Naim, E., Jaeger, H. M. and Nagel, S. R., Phys. Rev. E 57, 1971-1982 (1998); Jaeger, H. M., in [1].

[11] J.P. Bouchaud, J. Physique I 2, 1705 (1992).

[12] D. Head, cond-mat/000345.
Fig. 1. – Typical density profile obtained after $t = 10^4$ Monte-Carlo steps at constant forcing $x = 0.4$.

Fig. 2. – Density versus time, for a reference case with constant $x = 0.4$, and with $x_1 = 0.4$, $x_2 = 0.1$, for various values of $t_w$ and $\Delta t$. The first effect at $t_w$ is an acceleration of the compaction, but at larger times the curves cross; after $t_w + \Delta t$ the compaction is again fast.
Fig. 3. – Effect of a decrease of the forcing with $x_1 = 0.6$, $x_2 = 0.3$, $t_w = 10^4$, and $\Delta t = 10^4$ and $10^5$: at short times after $t_w$ the mean height $h$ decreases strongly (corresponding to an increase in the compaction, as in figure 1), but remains then almost constant. As $x$ is again increased, $h$ goes back to its value at $t_w$. The long time memory effect is illustrated in the inset: the symbols, corresponding to the data for $\Delta t = 10^4$ and $\Delta t = 10^5$, are shifted by $\Delta t$ and they coincide with the reference curve.

Fig. 4. – Effect of an increase in $x$ (i.e. $x_1 < x_2$), for $x_1 = 0.1$, $x_2 = 0.4$, and various values of $t_w$ and $\Delta t$; the inset shows a zoom of the decompaction effect at short times after the change in $x$. The compaction rate then is increased during $\Delta t$, and decreases again after $t_w + \Delta t$. 
Fig. 5. – Relaxation of the potential energy of the system after a perturbation $\Delta x = 0.1, 0.2$ during $\Delta t$ of a stationary state at $x = 0.1$ obtained after a slow decrease of the vibration intensity, starting from large forcing.