Vacuum energy and cosmological constant:
View from condensed matter

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The condensed matter examples, in which the effective gravity appears in the low-energy corner as one of the collective modes of quantum vacuum, provide a possible answer to the question, why the vacuum energy is so small. This answer comes from the fundamental “trans-Planckian” physics of quantum liquids. In the effective theory of the low energy degrees of freedom the vacuum energy density is proportional to the fourth power of the corresponding “Planck” energy appropriate for this effective theory. However, from the exact “Theory of Everything” of the quantum liquid it follows that its vacuum energy density is exactly zero without fine tuning, if: there are no external forces acting on the liquid; there are no quasiparticles which serve as matter; no space-time curvature; and no boundaries which give rise to the Casimir effect. Each of these four factors perturbs the vacuum state and induces a nonzero value of the vacuum energy density, which is on the order of the energy density of the perturbation. This is the reason, why one must expect that in each epoch the vacuum energy density is on the order of the matter density of the Universe, or/and of its curvature, or/and of the energy density of the smooth component – the quintessence.

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I. INTRODUCTION. THE THEORY OF EVERYTHING IN QUANTUM LIQUIDS.

The Theory of Everything for quantum liquids and solids – “a set of equations capable of describing all phenomena that have been observed” [1] in these quantum systems – is extremely simple. On the “fundamental” level appropriate for quantum liquids and solids, i.e. for all practical purposes, the $^4\text{He}$ or $^3\text{He}$ atoms of these quantum systems can be considered as structureless: the $^4\text{He}$ atoms are the structureless bosons and the $^3\text{He}$ atoms are the structureless fermions with spin 1/2. The Theory of Everything for a collection of a macroscopic number of interacting $^4\text{He}$ or $^3\text{He}$ atoms is contained in the many-body Hamiltonian written in the second quantized form:

$$\mathcal{H} - \mu \mathcal{N} = \int d\mathbf{x} \psi^\dagger(\mathbf{x}) \left[ -\frac{\nabla^2}{2m} - \mu \right] \psi(\mathbf{x}) + \int d\mathbf{x} d\mathbf{y} V(\mathbf{x} - \mathbf{y}) \psi^\dagger(\mathbf{x}) \psi^\dagger(\mathbf{y}) \psi(\mathbf{y}) \psi(\mathbf{x})$$  \hspace{1cm} (1)

where $m$ is the bare mass of the atom; $V(\mathbf{x} - \mathbf{y})$ is the bare interaction between the atoms; $\mu$ is the chemical potential – the Lagrange multiplier which is introduced to take into account the conservation of the number of atoms:
\[ N = \int dx \, \psi^\dagger(x) \psi(x). \] In \(^4\text{He}\), the bosonic quantum field \(\psi(x)\) is the annihilation operator of the \(^4\text{He}\) atoms. In \(^3\text{He}\), \(\psi(x)\) is the fermionic field and the spin indices must be added.

The Hamiltonian \(H\) has very restricted number of symmetries: It is invariant under translations and rotations in 3D space; there is a global \(U(1)\) group originating from the conservation of the number of atoms; \(H\) is invariant under gauge rotation \(\psi(x) \to e^{i\alpha} \psi(x)\) with constant \(\alpha\); in \(^3\text{He}\) in addition, if the relatively weak spin-orbit coupling is neglected, \(H\) is also invariant under separate rotations of spins. At low temperature the phase transition to the superfluid or to the quantum crystal state occurs where some of these symmetries are broken spontaneously. For example, in the \(^3\text{He}-\text{A}\) state all of these symmetries, except for the translational symmetry, are broken.

However, when the temperature and energy decrease further the symmetry becomes gradually enhanced in agreement with the anti-grand-unification scenario \(\text{AGU}\). At low energy the quantum liquid or solid is well described in terms of a dilute system of quasiparticles. These are bosons (phonons) in \(^4\text{He}\) and fermions and bosons in \(^3\text{He}\), which move in the background of the effective gauge and/or gravity fields simulated by the dynamics of the collective modes (Fig. 1). In particular, phonons propagating in the inhomogeneous liquid are described by the effective Lagrangian

\[ L_{\text{effective}} = \sqrt{-g} g^{\mu\nu} \partial_\mu \alpha \partial_\nu \alpha, \] where \(g^{\mu\nu}\) is the effective acoustic metric provided by inhomogeneity and flow of the liquid \(\text{AGU}\).

These quasiparticles serve as the elementary particles of the low-energy effective quantum field theory. They represent the analogue of matter. The type of the effective quantum field theory – the theory of interacting fermionic and bosonic quantum fields – depends on the universality class of the fermionic condensed matter (see review \(\text{AGU}\)). In superfluid \(^3\text{He}-\text{A}\), which belongs to the same universality class as the Standard Model, the effective quantum field theory contains chiral “relativistic” fermions, while the collective bosonic modes interact with these “elementary particles” as gauge fields and gravity (Fig. 1). All these fields emergently arise together with the Lorentz and gauge invariances and with elements of the general covariance from the fermionic Theory of Everything in Eq. (1).

The emergent phenomena do not depend much on the details of the Theory of Everything \(\text{AGU}\), in our case on the details of the pair potential \(V(x - y)\). Of course, the latter determines the universality class in which the system enters at low energy. But once the universality class is established, the physics remains robust to deformations of the pair potential. The details of \(V(x - y)\) influence only the “fundamental” parameters of the effective theory (“speed of light”, “Planck” energy cut-off, etc.) but not the general structure of the theory. The quantum liquids are strongly correlated and strongly interacting systems. That is why, though it is possible to derive the parameters of the effective theory from first principles in Eq. (1), if one has enough computer time and memory, this is a rather difficult task. However, in most cases it is appropriate to consider the “fundamental” parameters as phenomenological.

II. CONDENSED MATTER VIEW ON COSMOLOGICAL CONSTANT PROBLEMS

A. Why is the cosmological constant so small?

The most severe problem in the marriage of gravity and quantum theory is why the vacuum is not gravitating \(\text{AGU}\). The vacuum energy density can be easily estimated: the positive contribution comes from the high momenta, where the energy spectrum of particles is massless, \(E = cp\), the estimation for the flat space with Minkowski metric \(g_{\mu\nu} = \text{diag}(-1, c^{-2}, c^{-2}, c^{-2})\) gives the following energy density of the vacuum:

\[ \rho_\Lambda = \frac{1}{2V} \sum_{\text{bosons}} cp - \frac{1}{V} \sum_{\text{fermions}} cp \sim \pm \frac{1}{c^4} E_{\text{Planck}}^4 \] where \(V\) is the volume of the system. If there is no symmetry between the fermions and bosons (supersymmetry) the cut-off is provided by the Planck energy scale \(E_{\text{Planck}} \sim 10^{19} \text{ GeV}\), with the sign of the vacuum energy being determined by the fermionic and bosonic content of the quantum field theory. In case of supersymmetry, the cut-off is somewhat less, being determined by the scale at which supersymmetry is violated.

If the vacuum energy in Eq. (1) is gravitating, this is in severe contradiction with the experimental observations, which show that \(\rho_\Lambda\) is less than or on the order of \(10^{-120} E_{\text{Planck}}^4 / c^3\). If the vacuum energy is not gravitating, this is in contradiction with the general principle of equivalence, according to which the inertial and gravitating masses must coincide. What can be said about this issue in quantum liquids, where something similar to gravity arises in the low energy corner?
Quantum Fermi liquids:

Normal $^3$He, $^3$He-A, $^3$He-B, normal metal, semiconductor, superconductor, etc

Type of Quantum Field Theory depends on universality class

Universality class of Fermi points

Low temperature limit

Fermionic quasiparticles + Bosonic collective modes = QFT

Low-T limit

Chiral fermions + Gauge fields & gravity = Relativistic QFT

emergent phenomena at low T:

Lorentz invariance
Gauge invariance
General covariance (partly)
Chiral fermions
Gauge fields & gravity

correspondence:

Quantum liquid
Ground state
Chiral fermions
Collective modes
Quanta of modes

Universe
Quantum vacuum
Elementary particles
Gauge fields and gravity
Gauge bosons, gravitons

FIG. 1. Quantum field theories in quantum liquids. If quantum liquid has Fermi points, the relativistic QFT emerges at low $T$. 
Let us first look at the calculation of the vacuum energy. The advantage of the quantum liquid is that we know both the effective theory there and the fundamental Theory of Everything in Eq. (1). That is why we can compare the two approaches. Let us consider for simplicity superfluid $^4$He. The effective theory there contains phonons as elementary bosonic quasiparticles and no fermions. That is why the analogue of Eq. (3) for the vacuum energy is

$$\rho_\Lambda = \frac{1}{2V} \sum_{\text{phonons}} \left( \frac{1}{c^3} E_\text{Debye}^4 \right) = \sqrt{-g} E_\text{Debye}^4 \ , \quad (4)$$

where $c$ is the speed of sound; the “Planck” cut-off is now determined by the Debye temperature $E_\text{Debye} = \hbar c/a$ with $a$ the interatomic distance, which plays the role of the Planck length; $g$ is the determinant of the acoustic metric in Eq. (2). If one ignores an overall conformal factor the space-time interval in acoustic metric provided by the liquid at rest is $ds^2 = dt^2 - c^{-2}dr^2$, so that $\sqrt{-g} = c^{-3}$.

The disadvantages of such calculations of the vacuum energy within the effective field theory are: (i) The result depends on the cut-off procedure; (ii) The result depends on the choice of the zero from which the energy is counted: a shift of the zero level leads to a shift in the vacuum energy. To remove these uncertainties, we must calculate the energy density of the ground state exactly, using the Theory of Everything in Eq. (1):

$$\rho_\Lambda = \frac{1}{V} < \text{vac} | \mathcal{H} - \mu \mathcal{N} | \text{vac} > \ . \quad (5)$$

Note that this energy does not depend on the choice of zero level: the overall shift of the energy in $\mathcal{H}$ is exactly compensated by the shift of the chemical potential $\mu$.

Exact calculation means that not only the low-energy degrees of freedom of the effective theory (phonons) must be taken into account, but all degrees of freedom of the quantum liquid, i.e. including “Planckian” and “trans-Planckian” physics. At first glance, this is an extremely difficult task, to calculate an exact energy of the many-body wave function describing the ground state of the strongly interacting and strongly correlated system of $^4$He atoms in the real liquid. Fortunately the result immediately follows from simple thermodynamic arguments. If there are no external forces acting on the quantum liquid, then at $T = 0$ in the limit of infinite liquid volume $V$ one obtains exact nullification of the energy density:

$$\rho_\Lambda = \frac{1}{V} < \text{vac} | \mathcal{H} - \mu \mathcal{N} | \text{vac} >= 0 \ . \quad (6)$$

The proof is simple. The energy density of the liquid in homogeneous state, $\epsilon = \frac{1}{V} < \text{vac} | \mathcal{H} | \text{vac} > /V$, is a function of its particle density $n = N/V$. The pressure $P$ at $T = 0$ is determined in a usual way as $P = -d(V\epsilon(n))/dV$ where we must take into account that $n = N/V$. Then one obtains the following equation for the external pressure acting on the liquid:

$$P = -\frac{d}{dV} \left( V \epsilon \left( \frac{N}{V} \right) \right) = -\epsilon + n \frac{d\epsilon}{dn} = -\epsilon + \mu n = -\frac{1}{V} < \text{vac} | \mathcal{H} - \mu \mathcal{N} | \text{vac} >= -\rho_\Lambda \ . \quad (7)$$

This can be also seen from the hydrodynamic action for the liquid (see Ref. [5], where it is shown that the pressure at $T = 0$ is equal to the density of the hydrodynamic action). Note that the relation connecting vacuum energy and pressure, $\rho_\Lambda = -P$, is exactly the same as it comes from the variation of Einstein’s cosmological term $S_\Lambda = -\Lambda \sqrt{-g}$:

$$T_{\mu\nu} = \left( 2/\sqrt{-g} \right) \delta S_\Lambda / \delta g^{\mu\nu} = \Lambda g_{\mu\nu} \ .$$

In the absence of external pressure, i.e. at $P = 0$, Eq. (5) gives the zero value for the energy density of the liquid (or solid) in complete equilibrium at $T = 0$.

The only condition which we used is that the liquid exists in equilibrium without external pressure. This condition is fulfilled only for the liquid-like or solid-like states, for which the chemical potential $\mu$ is negative, if it is counted from the energy of the isolated atom. For liquid $^4$He and $^3$He the chemical potentials are really negative, $\mu_4 \sim -7K$ and $\mu_3 \sim -2.5K$ (see review paper Ref. [6]). This condition cannot be fulfilled for gas-like states for which $\mu$ is positive and they cannot exist without an external pressure. Thus the mere assumption that the vacuum of the quantum field theory belongs to the class of states, which can exist in equilibrium without external forces, leads to the nullification of the vacuum energy in equilibrium at $T = 0$.

Thus the first lesson from condensed matter is: the standard contribution to the vacuum energy density from the vacuum fluctuations in sub-Planckian effective theory is, without any fine tuning, exactly canceled by the trans-Planckian degrees of freedom, which are not accessible within the effective theory.
B. Why is the cosmological constant of order of the present mass of the Universe?

We now come to the second problem: Why is the vacuum energy density presently of the same order of magnitude as the energy density of matter $\rho_M$, as is indicated by recent astronomical observations \[1,8\]. While the relation between $\rho_M$ and $\rho_\Lambda$ seems to depend on the details of trans-Planckian physics, the order of magnitude estimation can
be readily obtained. In equilibrium and without matter the vacuum energy is zero. However, the perturbations of the vacuum caused by matter and/or by the inhomogeneity of the metric tensor lead to disbalance. As a result the deviations of the vacuum energy from zero must be on the of order of the perturbations. Let us consider how this happens in condensed matter for different types of perturbations.

1. Vacuum energy from finite temperature

A typical example derived from quantum liquids is the vacuum energy produced by temperature. Let us consider for example the superfluid $^4$He in equilibrium at finite temperature $T$ without external forces. If $T \ll -\mu$ one can neglect the exponentially small evaporation and consider the liquid as in equilibrium. Then the pressure caused by quasiparticles – phonons – which play the role of the hot relativistic matter with equation of state $P_M = (1/3)\rho_M$, must be compensated by the negative vacuum pressure $P_\Lambda = -P_M$ to support the zero value of the external pressure, $P = P_\Lambda + P_M = 0$. In this case one has for the vacuum pressure and vacuum energy density

$$\rho_\Lambda = -P_\Lambda = P_M = \frac{1}{3}\rho_M = \sqrt{-g}\frac{\pi^2}{30\hbar^3}T^4,$$

where $g = -c^{-6}$ is again the determinant of acoustic metric, with $c$ being the speed of sound. In this example the vacuum energy density $\rho_\Lambda$ is positive and always on the order of the energy density of matter. This indicates that the cosmological constant is not actually a constant but is adjusted to the energy density of matter and/or to the other perturbations of the vacuum discussed below.

2. Vacuum energy from Casimir effect

Another example of the induced nonzero vacuum energy density is provided by the boundaries of the system. Let us consider a finite droplet of $^4$He with radius $R$. If this droplet is freely suspended then at $T = 0$ the vacuum pressure $P_\Lambda$ must compensate the pressure caused by the surface tension due to the curvature of the surface. For a spherical droplet one obtains the negative vacuum energy density:

$$\rho_\Lambda = -P_\Lambda = -\frac{2\sigma}{R} \sim -\frac{E_{\text{Debye}}^3}{\hbar^2 c^2 R} \equiv -\sqrt{-g}E_{\text{Planck}}^3 \frac{\hbar c}{R},$$

where $\sigma$ is the surface tension. This is an analogue of the Casimir effect, in which the boundaries of the system produce a nonzero vacuum pressure. The strong cubic dependence of the vacuum pressure on the “Planck” energy $E_{\text{Planck}}^3 \equiv E_{\text{Debye}}^3 \hbar c$ reflects the trans-Planckian origin of the surface tension $\sigma \sim E_{\text{Debye}}^3/a^2$: it is the energy (per unit area) related to the distortion of atoms in the surface layer of atomic size $a$. Such term of order $E_{\text{Planck}}^3/R$ in the Casimir energy has been considered in Ref. [11]; see also Ref. [12], where this vacuum energy is related to the electroweak vacuum energy.

This form of the Casimir energy – the surface energy $4\pi R^2\sigma$ normalized to the volume of the droplet – can serve as an analogue of the quintessence in cosmology [13]. Its equation of state is $P_\sigma = -(2/3)\rho_\sigma$:

$$\rho_\sigma = \frac{4\pi R^2\sigma}{3\pi R^3} = \frac{2\sigma}{R}, \quad P_\sigma = \frac{2\sigma}{R} = -\frac{2}{3}\rho_\sigma.$$

The equilibrium condition within the droplet can be written as $P = P_\Lambda + P_\sigma = 0$. In this case the quintessence is related to the wall – the boundary of the droplet. In cosmology the quintessence with the same equation of state, $< P_\sigma > = -(2/3) < \rho_\sigma >$, is represented by a wall wrapped around the Universe or by a tangled network of cosmic domain walls [14]. The surface tension of the cosmic walls can be much smaller than the Planck scale.

3. Vacuum energy induced by texture

The nonzero vacuum energy density, with a weaker dependence on $E_{\text{Planck}}$, is induced by the inhomogeneity of the vacuum. Let us discuss the vacuum energy density induced by texture in a quantum liquid. We consider here the twist soliton in $^3$He-A. It will be clear later that this texture is related to the Riemann curvature in general relativity.
Within the soliton the field of the unit vector $\hat{1}$ changes as $\hat{1}(z) = \hat{x}\cos \phi(z) + \hat{y}\sin \phi(z)$. The energy of the system in the presence of the soliton consists of the vacuum energy $\rho_{\Lambda}(\phi)$ and gradient energy:

$$\rho = \rho_{\Lambda}(\phi) + \rho_{\text{grad}}, \quad \rho_{\Lambda}(\phi) = \rho_{\Lambda}(\phi = 0) + \frac{K}{\xi_D^2} \sin^2 \phi, \quad \rho_{\text{grad}} = K (\partial_z \phi)^2,$$

(11)

where $\xi_D$ is the so-called dipole length [13].

The solitonic solution of the sine-Gordon equation, $\tan(\phi/2) = e^{z/\xi_D}$, gives the following spatial dependence of the vacuum and gradient energies:

$$\rho_{\Lambda}(z) - \rho_{\Lambda}(\phi = 0) = \rho_{\text{grad}}(z) = \frac{K}{\xi_D^2 \cosh^2(z/\xi_D)}.$$

(12)

Let us consider for simplicity the 1+1 case. Then the equilibrium state of the whole quantum liquid with the texture can be discussed in terms of partial pressures of the vacuum $P_{\Lambda} = -\rho_{\Lambda}$ and inhomogeneity $P_{\text{grad}} = \rho_{\text{grad}}$. The latter equation of state describes the so-called stiff matter in cosmology. In equilibrium the external pressure is zero and thus the positive pressure of the texture (stiff matter) must be compensated by the negative pressure of the vacuum:

$$P = P_{\Lambda}(z) + P_{\text{grad}}(z) = 0.$$

(13)

This equilibrium condition produces another relation between the vacuum and the gradient energy densities

$$\rho_{\Lambda}(z) = -P_{\Lambda}(z) = P_{\text{grad}}(z) = \rho_{\text{grad}}(z).$$

(14)

Comparison with Eq. (12) shows that in equilibrium

$$\rho_{\Lambda}(\phi = 0) = 0.$$

(15)

As before, the main vacuum energy density—the energy density of the bulk liquid far from the soliton—is exactly zero if the liquid is in equilibrium. Within the soliton the vacuum is perturbed, and the vacuum energy is induced being on the order of the energy of the perturbation. In this case $\rho_{\Lambda}(z)$ is equal to the gradient energy density of the texture.

The induced vacuum energy density in Eq. (12) is inversely proportional to the square of the size of the region where the field is concentrated: $\rho_{\Lambda} \sim \sqrt{-g} E_{\text{Planck}}^2 (\hbar c/R)^2$ (in case of soliton $R \sim \xi_D$). Similar behavior for vacuum energy in the interior region of the Schwarzschild black hole was discussed in Ref. [16].

4. Vacuum energy due to Riemann curvature

The vacuum energy $\sim R^{-2}$ has also an analogy in general relativity. If $R$ is the size of the visible Universe, then since $E_{\text{Planck}} = \sqrt{\hbar c/G}$, one obtains $\rho_{\Lambda} \sim c^4/R^2 \sim R/G$, where $R$ is the Riemann curvature. This analogy with general relativity is supported by the observation that the gradient energy of a twisted $\hat{1}$-texture is equivalent to the Einstein curvature term in the action for the effective gravitational field in $^3$He-A [2]::

$$\frac{1}{16\pi G} \int d^3r \sqrt{-g} R = K \int d^3r ((\hat{1} \cdot (\nabla \times \hat{1}))^2.$$

(16)

Here $R$ is the Riemann curvature calculated using the effective metric experienced by fermionic quasiparticles in $^3$He-A, $ds^2 = dt^2 - c^{-2} (\hat{1} \times \hat{r})^2 - c^{-2} (\hat{1} \cdot \hat{r})^2$, with $\hat{1}$ playing the role of the Kasner axis. That is why the nonzero vacuum energy density within the soliton induced by the inhomogeneity of the order parameter is very similar to that caused by the curvature of space-time.

How the nonzero vacuum energy $\rho_{\Lambda}$ is induced by the spatial curvature and matter in general relativity is demonstrated by the solution for the static closed Universe with positive curvature, obtained by Einstein in his work where he first introduced the cosmological term [17]. Let us recall this solution. In the static state of the Universe two equilibrium conditions must be fulfilled:

$$\rho = \rho_M + \rho_{\Lambda} + \rho_R = 0, \quad P = P_M + P_{\Lambda} + P_R = 0.$$

(17)

The first equation in (17) reflects the gravitational equilibrium, which requires that the total mass density must be zero: $\rho = \rho_M + \rho_{\Lambda} + \rho_R = 0$ (actually the “gravineutrality” corresponds to the combination of two equations in (17),
\( \rho + 3P = 0 \), since \( \rho + 3P \) serves as a source of the gravitational field in the Newtonian limit. This gravineutrality is analogous to the electroneutrality in condensed matter. The second equation in (17) is equivalent to the requirement that for the “isolated” Universe the external pressure must be zero: \( P = P_M + P_\Lambda + P_R = 0 \). In addition to matter density \( \rho_M \) and vacuum energy density \( \rho_\Lambda \), the energy density \( \rho_R \) stored in the spatial curvature is added:

\[
\rho_R = \frac{R}{16\pi G} = -\frac{3k}{8\pi G R^2}, \quad P_R = -\frac{1}{3}\rho_R ,
\]

Here \( R \) is the cosmic scale factor in the Friedmann-Robertson-Walker metric, \( ds^2 = dt^2 - R^2(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) \); the parameter \( k = (-1, 0, +1) \) for an open, flat, or closed Universe respectively; and we removed the factor \( \sqrt{-g} \) from the definition of the energy densities.

For the cold Universe with \( P_M = 0 \), the Eqs. (17) give

\[
\rho_\Lambda = \frac{1}{2}\rho_M = -\frac{1}{3}\rho_R = \frac{k}{8\pi G R^2} ,
\]

and for the hot Universe with the equation of state \( P_M = (1/3)\rho_M \),

\[
\rho_\Lambda = \rho_M = -\frac{1}{2}\rho_R = \frac{3k}{16\pi G R^2} .
\]

Since the energy of matter is positive, the static Universe is possible only for positive curvature, \( k = +1 \), i.e. for the closed Universe.

This is a unique example of the equilibrium state, in which the vacuum energy on the order of the energy of matter is obtained within the effective theory of general relativity. It is quite probable that the static states of the Universe are completely contained within the effective theory and are determined by Eqs. (17), which do not depend on the details of the trans-Planckian physics.

However, when the non-stationary Universe is considered, the equation of motion for \( \rho_\Lambda \) must be added, which is beyond the effective theory and must depend on the Planck physics. The connection to the Planck physics can also solve the flatness problem. The Robertson-Walker metric describes the spatially homogeneous space-time as viewed within general relativity. However, if general relativity is the effective theory, the invariance under the coordinate transformations exists only at low energy. For the “Planck” observer the Robertson-Walker metric is viewed as space dependent if \( k \neq 0 \). That is why the condition for the global spatial homogeneity of the Universe both in effective and fundamental theories is \( k = 0 \): the Universe must be flat.

C. Why is the vacuum energy unaffected by the phase transition?

It is commonly believed that the vacuum of the Universe underwent one or several broken symmetry phase transitions. Since each of the transitions is accompanied by a substantial change in the vacuum energy, it is not clear why the vacuum energy is (almost) zero after the last phase transition. In other words, why has the true vacuum has zero energy, while the energies of all other false vacua are enormously big?

The quantum liquid answer to this question also follows from Eq. (17). For simplicity let us assume that the false vacuum is separated from the true vacuum by a large energy barrier, and thus it can exist as a (meta)stable state. Then Eq. (18) can also be applied to the false vacuum, and one obtains the paradoxical result: in the absence of external forces the energy density of the false vacuum in equilibrium must be always the same as the energy density of the true vacuum, i.e. it must also be zero. Moreover, this can be applied even to the unstable vacuum which corresponds to a saddle point of the energy functional, if such a vacuum state can live long enough. There is no paradox, however: after the phase transition to a new state has occurred, the chemical potential \( \mu \) will automatically adjust itself to nullify the energy density of the new vacuum. Thus in an isolated system (the Universe) the vacuum energy density remains zero both above and below the phase transition.

III. DISCUSSION: WHY IS VACUUM NOT GRAVITATING?

We discussed the condensed matter view to the problem, why the vacuum energy is so small, and found that the answer comes from the “fundamental trans-Planckian physics”. In the effective theory of the low energy degrees of freedom the vacuum energy density of a quantum liquid is of order \( E_{\text{Planck}}^4 \) with the corresponding “Planck” energy
appropriate for this effective theory. However, from the exact “Theory of Everything” of the quantum liquid, i.e. from the microscopic physics, it follows that the “trans-Planckian” degrees of freedom exactly cancel the relevant vacuum energy without fine tuning. The vacuum energy density is exactly zero, if the following conditions are fulfilled: (i) there are no external forces acting on the liquid; (ii) there are no quasiparticles (matter) in the liquid; (iii) no curvature or inhomogeneity; and (iv) no boundaries which give rise to the Casimir effect. Each of these four factors perturbs the vacuum state and induces a nonzero value of the vacuum energy density of order of the energy density of the perturbation. This is the reason, why one must expect that in each epoch the vacuum energy density is of order of the matter density of the Universe, and/or of its curvature, and/or of the energy density of the smooth component – the quintessence.

The condensed matter analog of gravity provides a natural explanation, why the effect of cosmological constant is by 120 orders of magnitude smaller than the result based on naive estimation of the vacuum energy. It also shows how the effective cosmological constant of the relative order of \(10^{-120}\) naturally arises as the response to different time-independent perturbations. At the moment it is not clear what happens in a real time-dependent situation of expanding Universe, i.e. how the quantum vacuum self-consistently responds to the time dependent matter field. This requires the knowledge of the dynamics of the vacuum, which probably is well beyond the effective theory of general relativity and requires some new microscopic Lagrangian (see e.g. [8]).

Let us also mention, that the actual problem for cosmology is not why the vacuum energy is zero (or very small), but why the vacuum is not (or almost not) gravitating. These two problems are not necessarily related since in the effective theory the equivalence principle is not the fundamental physical law, and thus does not necessarily hold when applied to the vacuum energy. That is why, one cannot exclude the situation, when the vacuum energy is huge, but it is not gravitating. The condensed matter provides an example of such situation too. If one considers the quantum gas instead of the quantum liquid, one finds that the gas can exists only at positive external pressure, and thus it has the negative energy density. The translation to the relativistic language gives a huge vacuum energy on the order of the Planck energy scale. Nevertheless, even in this situation the equilibrium vacuum is not gravitating, and only small deviations from equilibrium state are gravitating.

The condensed matter analogy gives us examples of how the effective gravity appears as an emergent phenomenon in the low energy corner. In these examples the gravity is not fundamental: it is one of the low energy collective modes of the quantum vacuum. This dynamical mode provides the effective metric (the acoustic metric in \(^4\)He) for the low-energy quasiparticles which serve as an analogue of matter. This gravity does not exist on the microscopic (trans-Planckian) level and appears only in the low energy limit together with the “relativistic” quasiparticles and the acoustics itself.

The vacuum state of the quantum liquid is the outcome of the microscopic interactions of the underlying \(^4\)He or \(^3\)He atoms. These atoms, which live in the “trans-Planckian” world and form the vacuum state there, do not experience the “gravitational” attraction experienced by the low-energy quasiparticles, since the effective gravity simply does not exist at the microscopic scale (we neglect here the real gravitational attraction of the atoms, which is extremely small in quantum liquids). That is why the vacuum energy cannot serve as a source of the effective gravity field: the pure completely equilibrium vacuum is not gravitating. On the other hand, the long-wave-length perturbations of the vacuum are within the sphere of influence of the low-energy effective theory, such perturbations can be the source of the effective gravitational field. Deviations of the vacuum from its equilibrium state, induced by different sources discussed here, are gravitating.

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