COUNTING IRREDUCIBLE BINOMIALS OVER
FINITE FIELDS

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ABSTRACT. We consider various counting questions for irreducible binomials over finite fields. We use various results from analytic number theory to investigate these questions.

1. Introduction

1.1. Background. It is reasonably easy to obtain an asymptotic formula for the total number of irreducible polynomials over the finite field $\mathbb{F}_q$ of $q$ elements, see [8, Theorem 3.25].

Studying irreducible polynomials with some prescribed coefficients is much more difficult, yet remarkable progress has also been achieved in this direction, see [3, 6, 11] and references therein.

Here we consider a special case of this problem and investigate some counting questions concerning irreducible binomials over the finite field $\mathbb{F}_q$ of $q$ elements. More precisely, for an integer $t$ and a prime power $q$, let $N_t(q)$ be the number of irreducible binomials over $\mathbb{F}_q$ of the form $X^t - a \in \mathbb{F}_q[X]$.

We use a well known characterisation of irreducible binomials $X^t - a$ over $\mathbb{F}_q$ of $q$ elements to count the total number of such binomials on average over $q$ or $t$. In fact, we consider several natural regimes, for example, when $t$ is fixed and $q$ varies or when both vary in certain ranges $t \leq T$ and $q \leq Q$. There has always been very active interest in binomials, see [8, Notes to Chapter 3] for a survey of classical results. Furthermore, irreducible binomials have been used in [12] as building blocks for constructing other irreducible polynomials over finite fields, and in [2] for characterising the irreducible factors of $x^n - 1$ (see also [1, 9] and references therein for more recent applications). However, the natural question of investigating the behaviour of $N_t(q)$ has never been addressed in the literature.
Our methods rely on several classical and modern results of analytic number theory; in particular the distribution of primes in arithmetic progressions.

1.2. Notation. As usual, let $\omega(s)$, $\pi(s)$, $\varphi(s)$, $\Lambda(s)$ and $\zeta(s)$ denote the number of distinct prime factors of $s$, the number of prime numbers less than or equal to $s$, the Euler totient function, the von Mangoldt function and the Riemann-zeta function evaluated at $s$ respectively.

For positive integers $Q$ and $s$ we denote the number of primes in arithmetic progression by

$$\pi(Q; s, a) = \sum_{\substack{p \leq Q \atop p \equiv a \pmod{s}}} 1.$$  

We also denote

$$\psi(Q; s, a) = \sum_{\substack{p \leq Q \atop p \equiv a \pmod{s}}} \Lambda(p).$$

The letter $p$ always denotes a prime number whilst the letter $q$ always denotes a prime power.

We recall that the notation $f(x) = O(g(x))$ or $f(x) \ll g(x)$ is equivalent to the assertion that there exists a constant $c > 0$ (which may depend on the real parameter $\varepsilon > 0$) such that $|f(x)| \leq c|g(x)|$ for all $x$. The notation $f(x) = o(g(x))$ is equivalent to the assertion that

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0.$$  

The notation $f(x) \sim g(x)$ is equivalent to the assertion that

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = 1.$$  

We define $\log x$ as $\log x = \max\{\ln x, 2\}$ where $\ln x$ is the natural logarithm. Furthermore, for an integer $k \geq 2$, we define recursively $\log_k x = \log(\log_{k-1} x)$.

Finally, we use $\Sigma^*$ to indicate that the summation is only over square-free arguments in the range of summation.

1.3. Main results. We denote the radical of an integer $t \neq 0$, the largest square-free number that divides $t$, by $\text{rad}(t)$. It is also convenient to define

$$\text{rad}_4(t) = \begin{cases} 
\text{rad}(t) & \text{if } 4 \nmid t, \\
2\text{rad}(t) & \text{otherwise}.
\end{cases}$$
We start with an upper bound on the average value of $N_t(q)$ for a fixed $t$ averaged over $q \leq Q$.

**Theorem 1.** For any fixed $\varepsilon > 0$ uniformly over real $Q$ and positive integers $t$ with $\text{rad}_4(t) \leq Q^{1-\varepsilon}$, we have

$$\sum_{q \leq Q} N_t(q) \leq (1 + o(1)) \frac{Q^2}{\text{rad}_4(t) \log(Q/\text{rad}_4(t))}$$

as $Q \to \infty$.

We also present the following lower bound (which has $\varphi(\text{rad}(t))^2$ instead of the expected $\varphi(\text{rad}(t))$).

**Theorem 2.** There exists an absolute constant $L > 0$ such that uniformly over real $Q$ and positive integers $t$ with $Q \geq t^L$, we have

$$\sum_{q \leq Q} N_t(q) \gg Q^2 \varphi(\text{rad}(t))^2 (\log Q)^2.$$

We also investigate $N_t(q)$ for a fixed $q$ averaged over $t \leq T$.

**Theorem 3.** For any fixed positive $A$ and $\varepsilon$ and a sufficiently large real $q$ and $T$ with

$$T \geq (\log(q - 1))^{(1+\varepsilon)A \log_3 q / \log_4 q},$$

we have

$$\sum_{i \leq T} N_t(q) \leq (q - 1)T / (\log T)^A.$$

Finally, we obtain an asymptotic formula for the double average of $N_t(q)$ over $q \leq Q$ and squarefree $t \leq T$ in a rather wide range of parameters $Q$ and $T$. With more work similar results can also be obtained for the average value of $N_t(q)$ over all integers $t \leq T$. However to exhibit the ideas and simplify the exposition, we limit ourselves to this special case, in particular we recall our notation $\Sigma^\sharp$ from Section 1.2.

**Theorem 4.** For any fixed $\varepsilon > 0$ and any $T \leq Q^{1/2} / (\log Q)^{5/2 + \varepsilon}$, we have

$$\sum_{i \leq T} \sum_{q \leq Q} N_t(q) = (1 + o(1)) \frac{Q^2 \log T}{2\zeta(2) \log Q},$$

as $T \to \infty$. 
It seems difficult to obtain the asymptotic formula of Theorem 4 for larger values of $T$ (even under the Generalised Riemann Hypothesis). However, here we show that a result of Mikawa [10] implies a lower bound of right order of magnitude for values of $T$ of order that may exceed $Q^{1/2}$.

**Theorem 5.** For any fixed $\beta < 17/32$ and $T \leq Q^\beta$, we have

$$\sum_{T \leq t \leq 2T} \sum_{q \leq Q} N_t(q) \gg \frac{Q^2}{\log Q},$$

We note that Theorem 5 means that for a positive proportion of fields $\mathbb{F}_q$ with $q \leq Q$ there is a positive proportion of irreducible binomials whose degrees do not exceed $Q^\beta$.

2. Preparations

2.1. Characterisation of irreducible binomials. Let $\text{ord}_q a$ denote the multiplicative order of $a \in \mathbb{F}_q^*$.

Our main tool is the following characterisation of irreducible binomials (see [8, Theorem 3.75]).

**Lemma 6.** Let $t \geq 2$ be an integer and $a \in \mathbb{F}_q^*$. Then the binomial $x^t - a$ is irreducible in $\mathbb{F}_q[x]$ if and only if the following three conditions are satisfied:

1. $\text{rad}(t) \mid \text{ord}_q a$,
2. $\gcd(t, (q - 1)/\text{ord}_q a) = 1$,
3. if $4 \mid t$ then $q \equiv 1 \pmod{4}$.

**Lemma 7.** Suppose that $q$ is a prime power. Then

$$N_t(q) = \begin{cases} \frac{\varphi(t)}{t} (q - 1), & \text{if } \text{rad}_4(t) \mid (q - 1), \\ 0, & \text{otherwise.} \end{cases}$$

**Proof.** We can assume that $\text{rad}_4(t) \mid (q - 1)$ (or equivalently $\text{rad}(t) \mid (q - 1)$ and if $4 \mid t$ then $q \equiv 1 \pmod{4}$), as in the opposite case the result follows immediately from Lemma 6.

Furthermore, from Lemma 6 we see that

$$N_t(q) = \sum_{a \in \mathbb{F}_q^*} \sum_{\substack{\text{rad}(t) \mid \text{ord}_q a \\ \gcd(t, (q - 1)/\text{ord}_q a) = 1}} 1.$$
Since $\mathbb{F}_q^*$ is a cyclic group, there are $\varphi(\text{ord}_q a)$ elements of $\mathbb{F}_q^*$ that have order equal to $\text{ord}_q a$. Hence, we obtain

$$N_t(q) = \sum_{j \mid (q-1), \text{rad}(t) \mid j, \gcd(t, (q-1)/j) = 1} \varphi(j).$$

We now write $q - 1 = RS$, where $R$ is the largest divisor of $q - 1$ with $\gcd(R, \text{rad}(t)) = 1$ (thus all prime divisors of $S$ also divide $t$). Now, for every integer $j \mid (q-1)$ the conditions $\text{rad}(t) \mid j$ and $\gcd(t, (q-1)/j) = 1$ mean that $j = Sd$ for some $d \mid R$. Since $\gcd(S, R) = 1$, we have

$$N_t(q) = \sum_{d \mid R} \varphi(Sd) = \varphi(S) \sum_{d \mid R} \varphi(d) = \varphi(S) R = \frac{\varphi(t)}{t} SR = \frac{\varphi(t)}{t} (q-1),$$

which concludes the proof.

2.2. Analytic number theory background. We recall a quantitative version of the Linnik theorem, see [7, Corollary 18.8], which is slightly stronger than the form which is usually used.

**Lemma 8.** There is an absolute constant $L$ such that if a positive integer $k$ is sufficiently large and $Q \geq k^L$, then uniformly over all integers $a$ with $\gcd(k, a) = 1$ we have

$$\psi(Q; k, a) \gg Q \frac{1}{\varphi(k) \sqrt{k}}.$$

On average over $k$ we have a much more precise result given by the Bombieri–Vinogradov theorem which we present in the form that follows from the work of Dress, Iwaniec, and Tenenbaum [4] combined with the method of Vaughan [14]:

**Lemma 9.** For any $A > 0$, $\alpha > 3/2$ and $T \leq Q$ we have

$$\sum_{k \leq T} \max_{\gcd(a, t) = 1} \max_{R \leq Q} \left| \pi(R; t, a) - \frac{\pi(R)}{\varphi(t)} \right| \leq Q(\log Q)^{-A} + Q^{1/2}T(\log Q)^\alpha.$$

The following result follows immediately from much more general estimates of Mikawa [10, Bounds (4) and (5)].

**Lemma 10.** For any fixed $\beta < 17/32$, $u \leq z^\beta$ and for all but $o(u)$ integers $k \in [u, 2u]$ we have

$$\pi(2z; k, 1) - \pi(z; k, 1) \gg \frac{z}{\varphi(k) \log z}.$$
We also have a bound on the number \( \rho_T(n) \) of integers \( t \leq T \) with \( \text{rad}(t) \mid n \), which is due to Grigoriev and Tenenbaum [5, Theorem 2.1]. We note that [5, Theorem 2.1] is formulated as a bound on the number of divisors \( t \mid n \) with \( t \leq T \). However, a direct examination of the argument reveals that it actually provides an estimate for the above function \( \rho_T(n) \). In fact, we present it in simpler form given by [5, Corollary 2.3].

**Lemma 11.** For any fixed positive \( A \) and \( \varepsilon \) and a sufficiently large positive integer \( n \) and a real \( T \) with

\[
T \geq (\log n)^{(1+\varepsilon)A \log_3 n / \log_4 n}
\]

we have \( \rho_T(n) \leq T/(\log T)^A \).

3. Proofs of Main Results

3.1. **Proof of Theorem 1.** For the case where \( 4 \nmid t \) we denote \( s = \text{rad}(t) \). Using Lemma 7 we have

\[
\sum_{q \leq Q} N_t(q) = \frac{\varphi(t)}{t} \sum_{q \leq Q} \frac{(q - 1)}{s(q - 1)} = \frac{\varphi(t)}{t} \sum_{q \leq Q} q + O(Q/s).
\]

So, with

\[
\ell = \left\lfloor \frac{\log Q}{\log 2} \right\rfloor \quad \text{and} \quad \lambda = 2\varepsilon^{-1},
\]

we have

\[
\sum_{q \leq Q} q = \sum_{p \leq Q} \frac{p}{s(p-1)} + \sum_{2 \leq r \leq \ell} \sum_{p^r \leq Q} \frac{p^r}{s(p^r-1)}.
\]

Using the Brun-Titchmarsh bound, see [7, Theorem 6.6] and partial summation we obtain

\[
\sum_{p \leq Q} \frac{p}{s(p-1)} \leq (1 + o(1)) \frac{Q^2}{\varphi(s) \log(Q/s)},
\]

provided that \( s/Q \to 0 \).

We now estimate the contribution from other terms with \( r \geq 2 \).

The condition \( s \mid p^r - 1 \) puts \( p \) in at most \( r^{\omega(s)} \) arithmetic progressions modulo \( s \). Extending the summation to all integers \( n \leq Q^{1/r} \) in these progressions, we have

\[
\sum_{p^r \leq Q} p^r \ll r^{\omega(s)} Q(Q^{1/r} s^{-1} + 1).
\]
We use this bound for \( r \leq \lambda \). Since
\[
\omega(s) \ll \frac{\log s}{\log \log(s + 2)},
\]
for \( r \leq \lambda \) we have
\[
r^\omega(s) = \exp\left( O\left( \frac{\log s}{\log \log(s + 2)} \right) \right).
\]
The total contribution from all terms with \( 2 \leq r \leq \lambda \) is at most
\[
\sum_{2 \leq r \leq \lambda} \sum_{p^r \leq Q \atop s|(p^r - 1)} p^r \leq Q^{1/2}s^{-1} + 1 \exp\left( O\left( \frac{\log s}{\log \log(s + 2)} \right) \right)
\]
\[= Q^{1+o(1)}(Q^{1/2}s^{-1} + 1).\]

For \( \lambda \leq r \leq \ell \) we use the trivial bound
\[
\sum_{\lambda \leq r \leq \ell} \sum_{p^r \leq Q \atop s|(p^r - 1)} p^r \leq \ell Q^{1+1/\lambda}.\]

Combining (4) and (5) we see that
\[
\sum_{2 \leq r \leq \ell} \sum_{p^r \leq Q \atop s|(p^r - 1)} p^r \ll Q^{3/2+o(1)}s^{-1} + Q^{1+o(1)} + Q^{1+\varepsilon/2}\log Q
\]
\[\ll Q^{3/2+o(1)}s^{-1},\]
provided that \( s \leq Q^{1-\varepsilon} \) and \( Q \to \infty \). Recalling (1), (2) and (3) and that
\[
\frac{\varphi(t)}{t\varphi(s)} = \frac{1}{s},
\]
we conclude the proof for the case where \( 4 \nmid t \).

In the event that \( 4 \mid t \) then, returning to (1), we have
\[
\sum_{q \leq Q} N_t(q) = \frac{\varphi(t)}{t} \sum_{q \leq Q \atop s|(q-1)} (q - 1) = \frac{\varphi(t)}{t} \sum_{q \leq Q \atop \text{lcm}(4, \text{rad}(t))|(q-1)} (q - 1).
\]
Since \( \text{lcm}(4, \text{rad}(t)) = 2\text{rad}(t) \), the proof now continues as before, replacing \( s \) with \( 2s \).
3.2. Proof of Theorem 2. Combining (1) and (2), we have

\[ \sum_{q \leq Q} N_t(q) \geq \sum_{p \leq Q} N_t(p) = \frac{\varphi(t)}{t} \sum_{p \leq Q} (p - 1) \sum_{\text{rad}(t) | (p-1)} \]

\[ \geq \frac{\varphi(t)}{t} \sum_{p \leq Q} (p - 1), \]

where, as before, \( s = \text{rad}(t) \).

It immediately follows from Lemma 8 that

\[ \pi(Q; 2s, 1) \gg \frac{Q}{\varphi(2s)\sqrt{2s} \log Q} \geq \frac{Q}{\varphi(s)\sqrt{s} \log Q}. \]

Thus

\[ \sum_{p \leq Q} p \geq \sum_{k=1}^{\pi(Q; s, 1)} (2ks + 1) \geq 2s \frac{\pi(Q; s, 1)^2}{2} \gg \frac{Q^2}{\varphi^2(s)(\log Q)^2}. \]

Combining this lower bound with (7) completes the proof.

3.3. Proof of Theorem 3. Fix any positive \( T \) and \( q \). For \( q = 1 \equiv 0 \pmod{4} \) we have, using Lemma 7,

\[ \sum_{t \leq T} N_t(q) = (q - 1) \sum_{t \leq T} \frac{\varphi(t)}{t} \leq (q - 1) \sum_{\text{rad}(t) | (q-1)} 1. \]

For \( q = 1 \not\equiv 0 \pmod{4} \) we have, using Lemma 7,

\[ \sum_{t \leq T} N_t(q) = (q - 1) \sum_{t \leq T} \frac{\varphi(t)}{t} \leq (q - 1) \sum_{\text{rad}(t) | (q-1)} \frac{\varphi(t)}{4t} \]

\[ \leq (q - 1) \sum_{t \leq T} \frac{1}{\text{rad}(t) | (q-1)} \]

Combining (8), (9) and Lemma 11 completes the proof.
3.4. **Proof of Theorem 4.** Using (1), (2) and (6) we have

\[
\sum_{t \leq T} \sum_{q \leq Q} N_t(q) = \sum_{t \leq T} \sum_{q \leq Q} \frac{\varphi(t)}{t} p + O \left( Q^{3/2+o(1)} \sum_{t \leq T} t^{-1} \right) \\
= \sum_{t \leq T} \sum_{p \leq Q} \frac{\varphi(t)}{t} \sum_{t \mid (p-1)} p + O \left( Q^{3/2+o(1)} \right),
\]

as \( T \leq Q^{1/2} \).

Using partial summation we have

\[
\sum_{p \leq Q} p = (Kt + 1) \pi(Kt + 1; t, 1) - t \sum_{1 \leq k \leq K} \pi(kt; t, 1),
\]

where \( K = \lfloor (Q - 1)/t \rfloor \).

We now write

\[
E(Q, t) = \max_{R \leq Q} \left| \pi(R; t, 1) - \frac{\pi(R)}{\varphi(t)} \right|.
\]

With this notation we derive from (11) that

\[
\sum_{p \leq Q} p = \frac{Q \pi(Q)}{\varphi(t)} - \frac{t}{\varphi(t)} \sum_{1 \leq k \leq K} \pi(kt) + O \left( tK E(Q, t) \right).
\]

By the prime number theorem and [7, Corollary 5.29], and noting that for \( 1 \leq k \leq K \) we have \( kt \leq Q \), we also conclude that

\[
\sum_{1 \leq k \leq K} \pi(kt) = t \sum_{1 \leq k \leq K} \frac{k}{\log(kt)} + O(Q^2 \log Q)^{-2})
\]

\[
= t \sum_{K/(\log Q)^2 \leq k \leq K} \frac{k}{\log(kt)} + O(Q^2 \log Q)^{-2}).
\]

Now, for \( K/(\log Q)^2 \leq k \leq K \) we have

\[
\frac{1}{\log(kt)} = \frac{1}{\log Q + O(\log \log Q)} = \frac{1}{\log Q} + O \left( \frac{\log \log Q}{(\log Q)^2} \right).
\]

Therefore

\[
\sum_{1 \leq k \leq K} \pi(kt) = \left( \frac{1}{2} + o(1) \right) t \frac{K^2}{\log Q} = \left( \frac{1}{2} + o(1) \right) Q^2 \frac{t \log Q}{\log Q}.
\]
Substituting this in (12) and using \( \pi(Q) \sim Q/\log Q \), we obtain
\[
\sum_{p \leq Q \atop t|(p-1)} p = \left( \frac{1}{2} + o(1) \right) \frac{Q^2}{\varphi(t) \log Q} + O\left( Q \mathcal{E}(Q,t) \right).
\]

Using this bound in (10) yields
\[
\sum_{t \leq T} \sum_{q \leq Q} N_t(q) = \left( \frac{1}{2} + o(1) \right) \frac{Q^2}{2 \log Q} \sum_{t \leq T} \frac{1}{t} \log Q \sum_{t \leq T} \frac{1}{t} + O\left( Q^{3/2+O(1)} + Q \sum_{t \leq T} \mathcal{E}(Q,t) \right).
\]

By Lemma 9, with \( A = 1 + \varepsilon \) and \( \alpha = 3/2 + \varepsilon/2 \), there is some \( B > 0 \) such that
\[
\sum_{t \leq T} \mathcal{E}(Q,t) \ll Q(\log Q)^{-A} + Q^{1/2} T (\log Q)^{\alpha} \ll Q(\log Q)^{-1-\varepsilon/2}.
\]

Hence
\[(13)\]
\[
\sum_{t \leq T} \sum_{q \leq Q} N_t(q) = \left( \frac{1}{2} + o(1) \right) \frac{Q^2}{2 \log Q} \sum_{t \leq T} \frac{1}{t} \log Q \sum_{t \leq T} \frac{1}{t} + O\left( Q(\log Q)^{-1-\varepsilon/2} \right).
\]

A simple inclusion-exclusion argument leads to the asymptotic formula
\[(14)\]
\[
\sum_{t \leq T} \frac{1}{t} = \left( \frac{1}{\zeta(2)} + o(1) \right) \log T,
\]

see [13] for a much more precise result. Substituting (14) into (13) completes the proof.

3.5. **Proof of Theorem 5.** We proceed as in the proof of Theorem 4 but instead of (10) we write
\[
\sum_{T \leq t \leq 2T} \sum_{q \leq Q} N_t(q) \geq \sum_{T \leq t \leq 2T} \sum_{Q/2 \leq p \leq Q} N_t(p) = \sum_{T \leq t \leq 2T} \sum_{Q/2 \leq p \leq Q \atop t|(p-1)} \frac{\varphi(t)}{t} \sum_{Q/2 \leq p \leq Q} p
\]
\[
\gg Q \sum_{T \leq t \leq 2T} \frac{\varphi(t)}{t} \left( \pi(Q; t, 1) - \pi(Q/2; t, 1) \right).
\]

Using Lemma 10 we easily conclude the proof.

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