Planning of plug-in electric vehicle fast-charging stations considering charging queuing impacts

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Abstract: Fast charging is a promising way for plug-in electric vehicles (PEVs) to get recharged quickly and reduce the impacts of long-lasting charging process on PEV owners’ daily life. Decreasing time during charging PEVs also makes the decision of PEV owners choosing where to charge affected more by the time length of driving towards and waiting in charging stations, raising new requirements for charging facilities planning. In this study, a cost minimisation planning method of PEV fast-charging stations taking influences of queuing and driving time into consideration is proposed and solved by the genetic algorithm-based methodology. An iterative algorithm obtaining the equilibrium of the user’s decision of place to charge is proposed to consider the impacts of waiting and driving time at different charging stations on PEV owners. The effectiveness of the proposed strategy is then verified through the case analysis based on trajectory data of taxis in Beijing, which shows that the proposed methodology has good performances in computation. Weight of time costs and investment restrictions such as the number of charging stations would also influence the planning result.

1 Introduction

With the increasing concern over environmental issues, as a cleaner mode of transport, plug-in electric vehicles (PEVs) have been considered as a promising tool to combat the energy crisis and climate change [1–3]. Governments around the world have released extensive incentive policies to promote the development of PEVs [4], including deploying more public charging stations, offering purchase subsidies etc.

However, although PEVs have a series of notable advantages compared with internal combustion engine vehicles, there are many drivers worrying about that the batteries would run out of energy during the trip due to PEVs’ limited driving range. Therefore, it is essential to locate PEV charging stations and set up charging infrastructure properly to ameliorate their concerns. According to [5], many governments announce to build more public fast-charging stations across the provinces. With the application of fast charging and the increase of charging power, the charging time becomes shorter and more acceptable for PEV owners. For example, a PEV can be fully charged even for only 10–15 min with 300 kW extreme fast charging. Therefore, waiting and driving times, which are then comparable with charging time will be important factors to affect PEV owner’s decision of charging place, which makes differences with the current situation, raising new requirement for charging infrastructure planning.

In recent years, there are a number of studies concerning the underdeveloped refuelling infrastructure and charging problem. Considering that inappropriate locations and capacities of PEV charging stations could have negative effects on the development of PEVs, more attention has been paid to the deployment of charging stations for optimally allocating them. For instance, Frade et al. [6] proposed a maximal covering model to find the optimal location of electric vehicle (EV) charging stations in an urban area to maximise covered demand within a given distance. Sweda and Klahanj [7] presented an agent-based decision support system to enable the strategic deployment of new charging infrastructure. Ip et al. [8] introduced a two-step model: first, congregating the road information into ‘demand clusters’ through hierarchical clustering analysis, and then employing linear programming to conduct the site planning, which considers certain constraints and cost factors.

Pevcek et al. [9] proposed a data-driven approach using predictive analytic to decide optimal charging station locations. Ren et al. [10] established a location model, which details grey decision-making scheme, to minimise the total social cost to solve the quantities and locations of charging stations.

However, the above studies have not taken the interaction between transportation and charging deployment into consideration. In practice, the policies implemented in the transportation system will change the spatial and temporal distribution of PEVs and their energy requirements, therefore affecting the patterns of the power system. According to [11], there are several factors impacting the layout of PEV charging stations, like the performance of a PEV battery, the charging demand and the environment of charging stations. Nie and Ghamami [12] proposed a conceptual optimisation model to select the battery size and charging capacity, attempting to minimise the cost of providing recharging facilities and manufacturing batteries.

Pan et al. [13] developed a two-stage stochastic program to optimally locate charging stations, supporting both the transportation system and the power grid. Liu et al. [14] built a mathematical model to solve the optimal planning problem of charging station, with the combination of the two-step screening method and the modified primal-dual interior-point algorithm. He et al. [15] developed an equilibrium modelling framework with the active-set algorithm, capturing the interactions among availability of public charging opportunities, destination and route choices of plug-in hybrid EVs etc. Zhang et al. [16] presented an integrated planning framework for PEV charging stations in an urban area from the perspective of a social planner, minimising the social costs of the whole PEV charging system. Wang et al. [17] proposed a multi-objective EV-charging station planning method to ensure charging service as well as reduce power losses of distribution systems.

Different from internal combustion engine vehicles, PEVs need more time to refuel (recharge) so that charging congestion might occur at a PEV charging station. However, this factor has not been considered adequately in most literature on PEV charging stations planning. In the limited number of papers referring to the charging congestion, the overall charging process, including arriving, waiting and charging, is usually modelled by the queuing theory [18]. $M/M/s$ is the most widely used queuing model, assuming that
both the arrival process and the charging demand follow a negative exponential distribution and that a vehicle will join the queue unconditionally [19, 20]. For example, Bae and Kwarsinski [21] presented a spatial and temporal model of PEV charging demand for rapid charging stations based on $M/M/s$ queuing theory. Similarly, the $M/M/s$ queuing system is adopted in the research in [22]. In order to apply to the situation under limited waiting spaces, Fan et al. [23] utilised an $M/Gi/s/k$ queuing model to assume a general service time distribution. Zhang et al. [24] proposed the capacitated flow refuelling location model to estimate PEV charging demand, and adopted an $M/Gi/s/k$ queuing system to model each charging station’s serviceability. Yang et al. [25] leveraged an $M/M/s/k$ queuing model to estimate the probability of electric taxis being charged at their dwelling places, but the arrival process of PEVs can be considered as a Poisson process, while the PEVs are served in the order they arrived in, and the size of calling source, i.e. the population from which the PEVs come, is assumed to be infinite because the total PEV population is large enough so that the arrival rate of PEVs will not fluctuate anomalously.

2.2 Mean waiting time

Armed with the queuing model of a PEV charging station, we are now equipped to consider the calculation of the waiting time of PEV charging. Leveraging the approximation for an $M/Gi/s/k$ queuing system developed in [29, 30], we can approximately compute the mean waiting time of PEVs, expressed as below:

$$W^{M/Gi/s/k} = \frac{(1 + \xi) W^{M/Mi/s} W^{M/Ds}}{2 \xi W^{M/Ds} + (1 - \xi) W^{M/Mi/s}}$$  

(1)

where

$$W^{M/Mi/s} = \left[ \sum_{n=0}^{\infty} \frac{(\frac{\lambda}{\mu})^n}{n!} \right]^{\lambda} \times \frac{\lambda^2}{(s-1)!} \sigma^{-1}$$  

(2)

$$W^{M/Ds} = \frac{1}{2} \left[ 1 + H \frac{3\lambda - \lambda^2}{\lambda} \left( 1 - e^{-\frac{s\lambda - 8}{2}} \right) \right]$$  

(3)

$$H = \frac{s-1}{16s} \left( \frac{(10s + 8)^{1/2}}{2} - 2 \right)$$

(4)

$$\xi = \sigma \mu$$

(5)

In the above equations, $W^{M/Gi/s/k}$ is the mean waiting time of an $M/Gi/s/k$ queuing system, and $W^{M/Mi/s}$ and $W^{M/Ds}$ are the mean waiting times of the corresponding $M/M/s$ and $M/D/s$ queuing systems, respectively. $H$ denotes the arrival rate of PEVs, and $\sigma$ are the reciprocal of the mean charging time of PEVs and the standard deviation of charging time, respectively. Note that (1)–(3) hold when the PEV arrival rate is less than the system transmission capacity, i.e. the utilisation factor $\rho$ follows:

$$\rho = \frac{\lambda}{s \mu} < 1$$  

(6)

Interested readers can refer to [29, 30] for the details.

Fig. 1 briefly shows the relationships among mean waiting time, charger number and the utilisation factor $\rho$ (representing different levels of $\lambda$) in one charging station, when adjusting the charger number from 10 to 120 and $\rho$ from 0.7 to 0.98 to observe how the mean waiting time changes. The mean waiting time increases as $\rho$ increases or/and the charger number decreases, and the increasing speed of mean waiting time becomes higher as the utilisation factor rises or/and the charger number decreases.

It should be noted that while calculating the mean waiting time, the PEV arrival rate $\lambda$ is a constant, i.e. the arrival process of PEVs is a homogeneous Poisson process. However, the arrival rate significantly depends on the number of PEVs with charging
demand near the PEV charging station, which varies with time. Thus, in the real-world situation, the PEV arrivals should be equivalent to a non-homogeneous Poisson process with a time-varying \( \lambda \), i.e. \( \lambda(t) \). In this research, we discretely regard \( \lambda \) within an hour as a constant and focus on the mean waiting time in the peak hour. In addition, there might be some ‘overstay’ time after the PEV got fully charged which contributes to the next PEV’s waiting time \[31\]. However, such part of dwelling time is not considered in this paper since PEV owners usually leave right after finishing charging the PEVs without ‘overstay’.

3 GA-based PEV charging station planning method

In this section, a PEV fast-charging station planning model will be first proposed, and then solved by a GA-based methodology containing an iteration algorithm to consider the impacts of time cost on PEV owners.

3.1 Mathematical formulation

To start with, it should be noted that optimisation on the level of single vehicle would bring more variables and rise the computation pressure. It seems more efficient in the infrastructure planning to partition the entire area into cells and treat all PEVs in one cell as a whole.

The siting and sizing of PEV charging stations need to take the monetary cost of equipment investment and the time cost of PEV users’ driving and waiting time into consideration:

\[
\text{min Cost} = \sum \text{INV}_i(s_k) + \sum \text{OPE}_i(\lambda_i, s_k, y_{i,k})
\]  

(7)

where \( s_k \) is the number of chargers in \( k \)th charging station, \( \lambda_i \) is the arrival rate of the PEVs in the \( i \)th cell and \( C_k \) is the set of cells whose PEVs select \( k \)th charging station to refuel. \( \text{INV}_i(s_k) \) denotes the investment cost of the \( k \)th charging station and \( \text{OPE}_i(\lambda_i, s_k) \) is the time cost of the PEVs in the \( i \)th cell. These two parts of cost can be written as follows:

\[
\text{INV}_i(s_k) = \text{inv}^v_i + \text{inv}^a_i s_k
\]  

(8)

\[
\text{OPE}_i(\lambda_i, s_k, y_{i,k}) = c_{\text{time}}(\text{DT}_{i,k} + \text{WT}_k)
\]  

(9)

\( \text{INV}_i(s_k) \) is expressed in a linear form of the charger number \( s_k \), in which \( \text{inv}^v_i \) is the fixed investment cost and the second part is variable investment cost related to the charger number \( s_k \) and \( \text{inv}^a_i \) is the unit investment cost per charger. \( c_{\text{time}} \) is the unit time cost, \( \text{DT}_{i,k} \) is the driving time from the \( i \)th cell to the \( k \)th charging station, \( \text{WT}_k \) is the mean waiting time in the \( k \)th charging station which could be calculated according to (1)-(5).

The charging station planning model should satisfy the following constraints:

\[
\sum_{k} y_{i,k} \leq s_{\text{max}} \quad \forall i \text{, } k
\]  

(10)

\[
\sum_{k} y_{i,k} = 1, \quad \forall i
\]  

(11)

\[
\delta_{i,j} = 1, \quad \forall k
\]  

(12)

\[
\gamma_{i,k} = \sum_{j} y_{j,k} \lambda_i, \quad \forall k
\]  

(13)

\[
\rho_k = \frac{\gamma_{i,k}}{s_k \mu} < 1, \quad \forall k
\]  

(14)

where binary variable \( \delta_{i,j} \) denotes the location of the \( k \)th charging station and \( \delta_{i,j} = 1 \) when building \( k \)th charging station at the \( j \)th cell. \( y_{i,k} \) denotes the charging station choice of the PEVs at the \( i \)th cell and \( y_{i,k} = 1 \) when all PEVs at the \( i \)th cell choose \( k \)th charging station to refuel. \( \lambda_i \) is the arrival rate of the \( k \)th charging station and \( \rho_k \) is the utilisation factor of the \( k \)th charging station. \( s_{\text{max}} \) is the maximal allowed charger number, while \( s_{\text{min}} \) and \( s_{\text{max}} \) are the required minimal and maximal charger numbers in a single charging station, respectively. \( \text{DT}_{i,k} \) is the distance between \( i \)th cell and \( k \)th charging station and \( s_{\text{ave}} \) is the average driving speed for PEVs. Manhattan distance will be applied to calculate \( \text{DT}_{i,k} \) in this paper. Equation (10) ensures that PEVs can only pick those locations where there are charging stations to get charged, while (11) shows the PEVs of a certain cell can only choose one charging station as their destination and (19) shows the selected station is the one with minimal driving and waiting time. Equation (12) guarantees that each charging station can choose only one cell as its location. Equation (13) provides the calculation method for the arrival rate of \( k \)th charging station. Equation (14) guarantees the queuing processes of all charging stations are stable. Equations (15) and (16) provide the charger number limits for a single charging station and all stations, respectively, which are contributed by the economic investment restrictions. Equation (17) shows that the driving time is approximated by the average driving velocity, while (18) shows that the mean waiting time is related to queuing parameters such as \( \lambda_i, s_k, \mu \) and \( \sigma \).

The above optimisation model contains a large number of binary variables and non-linear terms in the waiting time part, forming a mixed-integer non-linear problem. As the waiting time is decided by the charger number and the arrival rate, the latter one is also infected by the number of cells choosing that charging station, which then making the optimisation model an non-deterministic polynomial (NP)-hard problem that cannot be easily and directly solved.

3.2 GA-based algorithm

GA is a classic and effective method to solve NP-hard problems like the aforementioned. In this part, a GA-based algorithm will be proposed to deal with the planning problem. The procedure of GA-based algorithm can be expressed as Fig. 2, and will be described in detail in the following part.

3.2.1 Initialisation: The location coordinates \( (x_i, y_i) \) and size \( s_k \) are set as the individual of the population to be evolved. There are mainly two ways to generate the initial population: one is randomly picking the locations and charger numbers of all charging stations, while the other is based on some demand assignment methods such as \( k \)-means clustering. When randomly generating the locations and sizes, those individuals that might cause the queuing process in some stations unstable should be regenerated. For example, if the total number of chargers do not satisfy supporting the whole charging demand, i.e.

\[
\sum_{k} \lambda_i \geq \mu \sum_{k} s_k
\]  

(20)

the individual needs to be renewed. When utilising \( k \)-means clustering, the location of each charging station can be naturally
chosen as the centroid of every cluster with Manhattan distance, so that the summary of distance from all demand orientation of the same cluster to the charging station would be the shortest. In addition, the number of chargers in each station can also be randomly picked on the basis of making the queuing process stable.

### 3.2.2 Equilibrium and fitness calculation
The fitness value is highly related to the investment decision and the distribution of waiting and driving time, the latter of which is determined by the charging location decision of PEVs in every cell. Therefore, the equilibrium state of different cells choosing their charging destination needs to be calculated. First, in order to make it easy to understand, the definition of optimal equilibrium and equilibrium needs to be clarified, respectively, as follows:

**Definition 1: (Equilibrium)**: In the equilibrium state, no PEV charging demands can find out a better feasible choice for their charging decision.

**Definition 2: (Optimal equilibrium)**: In the optimal equilibrium state, the charging stations which the PEV charging demands in every cell choose exactly the one with the shortest driving and waiting time among all charging stations. Thus PEVs of all cells do not have the intention to change their charging position.

**Remark 1**: Optimal equilibrium is a special kind of equilibrium. In the equilibrium state, some cells might not choose the charging station with the shortest time since they could find an alternative charging station with a shorter time but not able to accept the charging demand of that cell. While in the optimal equilibrium state, there are even no other charging stations with shorter time compared to current choice for every cell, no matter whether the other charging stations can accept addition charging demand.

However, it should be noted that the optimal equilibrium does not always exist since the charging demand in each cell is treated as an entire, even not to mention the calculation of the optimal equilibrium is actually an NP-hard problem. To pursue the optimal solution would probably cost great calculation time while sacrificing the optimality and getting the suboptimal equilibrium solution would be more efficient. Thus, the calculation of equilibrium will be applied in this paper.

As for one cell $i$, whether the PEVs in the cell would choose to switch their charging destination $k$ to another adjacent charging station $k'$ depends on the residual capacity $R_k$ of charging station $k$ and the saved time $t_{ik}^{k,k'}$ after switching. Only if the residual capacity $R_k$ is larger than the charging demand of cell $i$, and the saved time $t_{ik}^{k,k'}$ after switching the charging destination of cell $i$ from station $k$ to $k'$ is positive, the PEVs in cell $i$ have incentives to switch the charging destination to charging station $k'$, which can be expressed as

$$ R_k = \mu s_k - \lambda_i > 0 \quad (21) $$

$$ t_{ik}^{k,k'} = W^{M/4}(\lambda_i s_k + \mu), s_k, \mu, \sigma) + DT_{ik} - WT_i - DT_{ik'} > 0 \quad (22) $$

Thus, constraints (19) can be relaxed and substituted to constraints that no charging stations have incentives to switch based on conditions (21) and (22).

In fact, the equilibrium of charging demand is reached through the gaming process of all PEV users in different cells, and it can be obtained by iterative algorithms. However, several characteristics shown as follows can be utilised in order to accelerate the iteration.

**Theorem 1**: The service area covered by each charging station is a connected area.

**Proof**: If the service area of a certain charging station $k$ is not a connected area and includes more than one isolated part, i.e. there is at least one outer part not covering the charging station. We can pick any cell $i$ from the outer parts. There must be cells not included in the service area on the route from cell $i$ to charging station $k$ since the outer part is not connected with the charging station $k$. Pick any one of those cells $j$, then cell $j$ must have a better charging station $k'$ whose outer part is not connected with the charging station $k$. Thus we have $DT_{ijk} + WT_i > DT_{ijk'} + WT_{k'}$. Since $DT_{ijk} = DT_{ijk} - DT_{ij} > DT_{ijk}$, there is $DT_{ijk} + WT_i > DT_{ijk} + WT_{k'}$, which means choosing the charging station $k'$ for cell $i$ would cost less time and $k'$ would be a better choice. It conflicts with the assumption that $k$ is still the best choice for cells in the outer part. Therefore, the service area cannot include multiple isolated parts and must be a connected area. □

As shown in Theorem 1, it is reasonable to only change the charging location decision of some boundary cells that have adjacent cells choosing different charging stations at each iteration while not breaking the connectivity of the service areas. However, not all boundary cells of the service area for a charging station can change their charging places, because the connectivity of the service area might be broken. We note those boundary cells that can be changed at a certain iteration as switchable cells. In addition, the condition of the switchable cell can be described as follows:

**Lemma 1: (Condition of the switchable cell)**: The boundary cell is switchable if the service areas of the previous charging station and the newly chosen station both remain connected after switching the charging position of the cell.

Fig. 3 shows the typical types of switchable boundary cells in the four-connected graph where a total of eight surrounding cells are shown. In the four-connected graph, only the four cells sharing a common edge with the centre one, i.e. the up, down, left and right cells, are considered as the adjacent cells of the centre cell. Subfigures of Fig. 3 are the typical occasions when the cell has 1–3 adjacent cells of different charging stations, respectively. In Fig. 3, cells of different patterns belong to different charging stations, while blue cells are those deciding ones that need further check in each situation. The cell is switchable if all blue cells belong to the same charging station with it.

The switchable boundary cells can be further divided into multiple types according to whether their charging stations are full and whether there are better choices of charging place, which is specifically listed in Table 1. Only the above types of switchable

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boundary cells can change their charging station decision between two iterations.

The iterative algorithm to obtain the equilibrium is shown in Algorithm 1 (Fig. 4). Boundary cells of Types 2 and 3 will be first cleared through adjustment of these cells, in order to basically guarantee that charging demand in every cell could be satisfied and find a charging station. Then adjustment of Types 1 and 4 is carried out to make PEVs find a better place to get charged. Once a cell of Type 1 or 4 is decided to change its destination, Type 1 cells of its new destination could also be added into the action set \( A \) which contains all candidate cells to be changed at this iteration, since the change of Type 1 cell can shorten the waiting time in the previous charging station. The loop would terminate when there are no Type 1 cells, and final distribution of the charging place decisions and still uncleared Type 4 cells are going to be output.

After getting the equilibrium state, the fitness values of all individuals will be calculated according to the optimisation objective function (7). However, as the optimisation objective is to minimise the total cost, an affine form of the fitness value according to the objective value is proposed to project the objective value to the proper range:

\[
\text{Fitness} = F_0 - \frac{1}{M} \text{Cost} \quad (23)
\]

where \( F_0 \) and \( M \) can be chosen based on case setting or actual parameters to guarantee the fitness value is positive and properly distributed. If a certain individual cannot satisfy the charging demand, i.e. cells of Type 2 or Type 3 cannot be cleared after the maximal allowed number of iteration, the fitness value of that individual would be set as 0.

If the convergence condition is reached, e.g. the variation of optimal fitness value has been within a tolerance range for a certain number of iterations, the iteration loop can be terminated and the optimal individual would be output.

3.2.3 Selection and crossover: Roulette selection and sampling without replacement are utilised here to generate the survived population at each iteration. For each time of selection, two individuals of the current population would be chosen to battle each other, and the one with higher fitness value would survive.

Crossover is the necessary step to get closer to the optimal solution. In order to produce a certain number of kid generations, two parent individuals would be randomly picked from the current population after selection and produce one offspring. These two parent individuals have a certain probability to crossover some parts of the genetic information, e.g. exchange the locations and sizes of some charging stations. The crossover point \( y_{cr} \) and the
randomly choose a new position that has not built a charging hand, one of the parent individuals will be randomly chosen and other parent, and then produce the kid individual. On the other area will substitute these charging stations with the same amount of crossover point

3.2.4 Mutation: During the iteration, certain individuals might happen.

As for changing locations of some charging stations, we can directly inherited to the next generation if the crossover does not happen.

Fig. 6 Location and service area of planned charging stations. Cells marked with stars are places to build fast-charging stations

direction \( d^i \in \{0, 1\} \) are randomly generated. The left part of crossover point \( x^i \) if \( d^i = 0 \) or the right part if \( d^i = 1 \) is the crossover area, where the charging station would be exchanged. The one individual having less charging stations in the crossover area will substitute these charging stations with the same amount of charging stations randomly picked from the crossover area of the other parent, and then produce the kid individual. On the other hand, one of the parent individuals will be randomly chosen and directly inherited to the next generation if the crossover does not happen.

3.2.4 Mutation: During the iteration, certain individuals might have a mutation in their genetic information so that the algorithm could have the ability to jump out of local optimum. There are mainly two kinds of mutation taken into account, i.e. change of location and size of charging stations. Also, it is assumed that only one kind of mutation would take place in one iteration.

As for changing locations of some charging stations, we can randomly choose a new position that has not built a charging station now, or adjust the location of a charging station within its service area, e.g. to the centrist of the service area. As for changing the size of stations, on one hand, the uncleared Type 4 cells can be utilised here to add some chargers to those charging stations still having Type 4 cells or with \( \rho_k \geq 1 \) to make their charging demands satisfied, or some chargers can be removed from those charging stations with a low-utilisation factor \( \rho \). On the other hand, random addition or reduction of chargers in some charging stations could also be applied.

According to Fig. 2, the next iteration will start from the calculation of equilibrium after the mutation completes, until the fitness value finally converges.

4 Case study

In this section, case studies based on real trajectory data in Beijing urban area would be carried out to provide a planning result for PEV fast-charging stations in Beijing and validate the proposed model and algorithm.

4.1 Case settings

Some GPS-based trajectory data of taxis in Beijing urban area is utilised here to generate the charging demand. The data set includes around 29,700 taxis’ travel records in the central area, around within the fifth ring of Beijing from 1–31 July 2016, collected by smartphones or on-board devices. The taxis fleet recorded in the data account for 44% of all the taxis (about 67,000 in total [32]) in Beijing. For each travel record, the information contains the taxi ID, the time and the position (in longitude and latitude). Table 2 gives a sample of the taxi travel records in the data set.

It is assumed that if one electric taxi dwells at a certain location for 30 min, it would have a certain probability, e.g. 40%, to derive the charging demand. The distribution of PEV charging demand in a typical day is shown in Fig. 5, which indicates that the charging demand in the urban centre is relatively high while the distribution of charging demand in the suburbs is relatively sparse. In Fig. 5, the Beijing urban area is divided into 30 \( \times \) 40 cells. Under the division of 30 \( \times \) 40 cells, the side length of each cell is around 1 km, which is appropriate for treating the charging demand in one cell as a whole. Although the number of cells can also be increased or decreased according to the requirement of computation time and accuracy, they should be chosen neither too large to expand the model scale grandly but improve the optimality only slightly nor too small so that the size of a single cell is overly large and the charging demands in each cell might be assigned to improper charging stations. Each built charger costs $10,000. The unit waiting and driving time cost is assumed to be $8.76 per hour, equal to 60 CNY/h at the exchange rate of 6.85 CNY/$ and a monthly salary of 10,560 CNY, which is almost same as the average pay rate in Beijing (10592.25 CNY per month in 2018 [33]). The average driving speed is assumed to be 30 km/h and the mean charging time is set to be 20 min, i.e. 1/\( \mu \) = 3. The total maximal allowed charger number is 1500, and the planned number of charging station is set as 40. For each station, the minimal and maximal number of chargers are set as 10 and 100, respectively. However, it should be noted that the differences in fixed investment cost among different areas are not yet considered here for simplicity, but it can be taken into account easily in the fitness calculation part.

As for parameters of GA-based algorithm, the initial population is set to have 1000 individuals, half of which are generated randomly and the other half are obtained with \( k \)-means clustering. In total, 60% of the whole population would survive at each selection and produce the same amount of offspring with the initial population. The probability to have a crossover is set as 0.3. The probability of mutation is set to be 0.2, with all four kinds of mutation taken into consideration. The convergence condition is that the optimal fitness value varies within the range of 0.1% for >30 consecutive iterations.

4.2 Results analysis

The planning results of PEV fast-charging stations and the choice of charging destination for each cell are shown in Fig. 6. The total investment cost is 18.290 million dollars with a total number of 1029 chargers and 40 fast-charging stations to be built, while the waiting and driving time cost values 3.916 million dollars. The

Table 2 Example of taxi travel records

| Taxi ID | Time          | Longitude | Latitude |
|---------|---------------|-----------|----------|
| 26491   | 20,160,704,141,051 | 116.426285 | 39.921867 |

Fig. 5 Distribution of PEV charging demand in different areas of Beijing

Fig. 6 Distribution of PEV charging demand in different areas of Beijing
Table 3: Planning results

| Station No. | Location size | Waiting time, min | Station No. | Location Size | Waiting time, min | Station No. | Location Size | Waiting time, min |
|-------------|---------------|-------------------|-------------|---------------|-------------------|-------------|---------------|-------------------|
| 1           | (6, 8)        | 10                | 15 (16, 26) | 10            | 3.7               | 29 (23, 26) | 37            | 3.1               |
| 2           | (7, 16)       | 20                | 16 (17, 33) | 30            | 8.4               | 30 (23, 30) | 10            | 3.3               |
| 3           | (6, 28)       | 20                | 17 (17, 16) | 24            | 6.5               | 31 (24, 9)  | 44            | 2.4               |
| 4           | (10, 25)      | 14                | 18 (18, 27) | 19            | 4.1               | 32 (27, 14) | 20            | 2.9               |
| 5           | (11, 13)      | 13                | 19 (20, 24) | 28            | 7.7               | 33 (26, 19) | 21            | 3.9               |
| 6           | (12, 17)      | 52                | 20 (20, 28) | 19            | 6.3               | 34 (25, 24) | 25            | 2.9               |
| 7           | (11, 34)      | 13                | 21 (20, 6)  | 29            | 4.2               | 35 (26, 33) | 44            | 2.5               |
| 8           | (14, 4)       | 11                | 22 (19, 12) | 44            | 7.0               | 36 (27, 12) | 21            | 4.4               |
| 9           | (13, 7)       | 19                | 23 (19, 19) | 16            | 10.5              | 37 (26, 28) | 15            | 3.7               |
| 10          | (14, 10)      | 38                | 24 (21, 18) | 12            | 4.4               | 38 (29, 12) | 23            | 3.0               |
| 11          | (15, 15)      | 54                | 25 (21, 15) | 24            | 4.3               | 39 (28, 24) | 21            | 3.6               |
| 12          | (13, 29)      | 29                | 26 (22, 32) | 38            | 7.0               | 40 (29, 30) | 15            | 6.8               |
| 13          | (15, 21)      | 46                | 27 (21, 37) | 24            | 3.7               | —            | —             | —                 |
| 14          | (15, 24)      | 29                | 28 (24, 17) | 57            | 3.0               | —            | —             | —                 |

Fig. 7: Changing process of optimal fitness value with iterations in the proposed algorithm

Table 4: Comparison between cases with different parameter settings

| Case | Station number | Unit time cost, $/h | Charger number | Investment cost, 10^3 $ | Time cost, 10^3 $ | Total cost, 10^3 $ |
|------|----------------|---------------------|----------------|-------------------------|------------------|-------------------|
| 1    | 40             | 8.76                | 1029           | 18,290                  | 3916             | 22,206            |
| 2    | 40             | 17.52               | 1202           | 20,020                  | 4676             | 24,696            |
| 3    | 30             | 8.76                | 988            | 15,880                  | 3727             | 19,607            |
| 4    | 50             | 8.76                | 1105           | 21,050                  | 4129             | 25,179            |

location, size and mean waiting time of each charging station are shown in Table 3.

As shown in Fig. 6, service areas of those charging stations at the surrounding part are relatively large due to sparse demand distribution in the suburban areas. In central areas of Beijing, especially in some districts with high charging demand, the service areas are relatively concentrated, such as station Nos. 23 and 24. However, it should also be noted that since the calculation of equilibrium only provides a balanced feasible choice of charging destination for all cells and does not guarantee the optimality of users’ choices, the service areas of some charging stations are not quite regular.

The summary of the arrival rate of all cells is 2755.6 vehicles per hour, thus the overall utilisation factor \( \rho = 0.893 \). Actually, the fixed part of investment cost can be neglected since it is decided by the charging station number which is a constant here. After that, the linear part related to charger number is 10,290 million dollars and the time cost is \( \sim 38\% \) of the linear part of investment cost. According to Table 3, the maximal mean waiting time is 0.1749 h, i.e. 10.5 min. For a large number of charging stations, the mean waiting times are between 5 and 10 min, several are <5 min, while few of the charging stations in the surrounding areas even have a waiting time <3 min.

The change in optimal fitness value during the iteration process is shown in Fig. 7, where five epochs in a total of the GA-based algorithm are posted, respectively. It can be observed that the calculation would converge after 140–180 iterations at each epoch and the final optimal fitness value of all epochs are within small differences, validating the effectiveness of the proposed methodology. As for computation time, the program runs on a PC with Inter(R) Core(TM) i7-4790 3.60 GHz processor and 8G memory and costs around 18–25 h per epoch.

4.3 Sensitivity analysis

In order to further analyse the influence of different parameters such as cost and investment restrictions, sensitivity analysis is also carried out. In this part, we will explore the impacts of two factors on the planning decision, respectively: the relationship between investment costs and time costs, and the number of charging stations to build.

The unit time cost is changed while the investment cost stays the same in order to explore the impacts of time costs on planning decision, and the planning result is shown in Table 4 (case 2). It could be observed that both total cost and investment cost rise as unit time cost doubles the value in case 1. The total number of chargers in each station increases to reduce the mean waiting time to save time costs as much as possible. Actually, nearly half of the total time is reduced, making the total time costs only increase slightly compared to case 1. In that situation, the overall utilisation...
factor decreases from 0.893 to 0.764, which indicates that the waiting process is less crowded.

As for the influence of total charging station number, two more cases are studied and the results are listed in Table 4 (cases 3 and 4). As shown in Table 4, with the number of PEV charging stations increasing to 50, the total charger number increases slightly and total time cost rises, while it costs less time and requires less chargers when the total charging station number is limited to 30. In fact, for a given number of chargers, a smaller number of charging stations can achieve less mean waiting time, because centralised chargers are shared by more PEVs and their utilisation is higher. However, a defect of less station is that the distance for a PEV to go to the nearby station becomes longer. However, the driving time here is relatively shorter than waiting time, thus the total time would face a slight decrease.

5 Conclusion

With the increase of charging power for PEVs fast charging and the reduction of charging time, time costs during the trip to charging stations and queuing before getting charged will play more important roles in PEV owner’s decision of charging places. In this paper, a GA-based PEV fast-charging station planning methodology is proposed. In order to consider the influence of waiting and driving time at each charging station on PEV owners, an iterative algorithm is proposed to obtain the equilibrium when the locations and sizes of charging stations are given. Case analysis is carried out with PEV charging demand generated according to real taxi trajectory data in Beijing. Results provide a construction plan of PEV fast-charging stations in Beijing urban area and show that the proposed algorithm has good convergence performance. Sensitivity analysis also indicates that the total number of charging stations and the weight of time costs also have impacts on the final planning decision.

Further studies could be done to deepen the research work in this paper. The future research plan mainly lies in the gaming among different entities and the coordination with the guidance of PEV charging service through pricing in a coupled power and transportation network.

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7 References

[1] Larminie, J., Lowry, J.: ‘Electric vehicle technology explained’ (John Wiley & Sons Ltd., Chichester, England, 2003)
[2] He, F., Yin, Y., Wang, J., et al.: ‘Sustainability SI: optimal prices of electricity at public charging stations for plug-in electric vehicles’, Newt. Spat. Econ., 2016, 16, (1), pp. 131–154
[3] Gardner, L.M., Duell, M., Walter, S.T.: ‘A framework for evaluating the role of electric vehicles in transportation network infrastructure under travel demand variability’, Transp. Res. A, Policy Pract., 2013, 49, pp. 76–90
[4] Song, Y., Xia, Y., Lu, Z.: ‘Integration of plug-in hybrid and electric vehicles: experience from China’. IEEE Power and Energy Society General Meeting, 2010, pp. 1–6
[5] Globe Series.: ‘BC plan for electric car charging stations may unleash vehicles around province’, 2012, https://www.energy-xprt.com/news/bc-plan-for-electric-car-charging-stationsmay-unleash-vehicles-around-province-288665, (Accessed 28 April 2019)
[6] Frade, J., Ribeiro, A., Goncalves, G., et al.: ‘An optimization model for locating electric vehicle charging stations in central urban areas’. Compendium of Papers DVD of TRB 90th Annual Meeting, Transportation Research Board, Washington, DC, USA, 2010
[7] Sweda, T., Khabjan, D.: ‘An agent-based decision support system for electric vehicle charging infrastructure deployment’. Seventh IEEE Vehicle Power and Propulsion Conf., Chicago, IL, USA, 2011
[8] Ip, A., Fong, S., Liu, E.: ‘Optimization for allocating bev recharging stations in urban areas by using hierarchical clustering’. Sixth Int. Conf. on Advanced Information Management and Service, 2010, pp. 460–465
[9] Pevtsev, D., Babic, J., Kayser, M.A., et al.: ‘A data-driven statistical approach for extending electric vehicle charging infrastructure’, Int. J. Energy Res., 2018, 42, (9), pp. 3102–3120
[10] Ren, X., Zhang, H., Hu, R.: ‘Location of electric vehicle charging stations: a perspective using the grey decision-making model’, Transp. Res. A, Policy Pract., 2019, 173, pp. 548–553
[11] Xu, F., Yu, G., Gu, L., et al.: ‘Tentative analysis of layout of electrical vehicle charging stations’. East China Electric Power, 2009, 37, pp. 1677–1682
[12] Nie, Y., Ghamami, M.: ‘A corridor-centric approach to planning electric vehicle charging infrastructure’, Transp. Res. B, Methodol., 2013, 57, pp. 172–190
[13] Pan, F., Bent, R., Berscheid, A., et al.: ‘Locating PHEV exchange stations in v2g’. IEEE Int. Conf. on Smart Grid Communications, Gaithersburg, MD, USA, 2010, pp. 173–179
[14] Liu, Z., Wen, F., Ledwich, G.: ‘Optimal planning of electric-vehicle-charging stations in distribution systems’, IEEE Trans. Power Deliv., 2013, 28, (1), pp. 102–110
[15] He, F., Wu, D., Yin, Y., et al.: ‘Optimal deployment of public charging stations for plug-in hybrid electric vehicles’, Transp. Res. B, Methodol., 2013, 47, pp. 87–101
[16] Zhang, H., Hu, Z., Xu, Z., et al.: ‘An integrated planning framework for different types of phev charging facilities in urban area’, IEEE Trans. Smart Grid, 2016, 7, (5), pp. 2273–2284
[17] Wang, G., Xu, Z., Wen, F., et al.: ‘Traffic-constrained multiojective planning of electric-vehicle-charging stations’, IEEE Trans. Power Deliv., 2013, 28, (4), pp. 2363–2372
[18] Chen, H., Hu, Z., Luo, H., et al.: ‘Design and planning of a multi-charger multiple-port charging system for PEV charging station’, IEEE Trans. Smart Grid, 2019, 10, (1), pp. 173–183
[19] Akbari, H., Fernando, X.: ‘Modeling and optimization of PHEV charging queues’. IEEE 28th Canadian Conf. on Electrical and Computer Engineering (CCECE), Halifax, NS, Canada, 2015, pp. 81–86
[20] Qiu, G., Liu, W., Zhang, J.: ‘Equipment optimization method of electric vehicle fast charging station based on queuing theory’, Appl. Mech. Mater., 2013, 291, pp. 872–877
[21] Bae, S., Kwasinski, A.: ‘Spatial and temporal model of electric vehicle charging demand’, IEEE Trans. Smart Grid, 2012, 3, (1), pp. 394–403
[22] Li, G., Zhang, X.: ‘Modeling of plug-in hybrid electric vehicle charging demand in probabilistic power flow calculations’, IEEE Trans. Smart Grid, 2012, 3, (1), pp. 492–499
[23] Fan, P., Sainbuyar, B., Ren, S.: ‘Operation analysis of fast charging stations with energy demand control of electric vehicles’, IEEE Trans. Smart Grid, 2015, 6, (4), pp. 1819–1826
[24] Zhang, H., Moura, S.J., Hu, Z., et al.: ‘PEV fast-charging station siting and sizing on coupled transportation and power networks’, IEEE Trans. Smart Grid, 2018, 9, (4), pp. 2595–2605
[25] Yang, J., Dong, J., Hu, L.: ‘A data-driven optimization-based approach for siting and sizing of electric taxi charging stations’, Transp. Res. C, Emerg. Technol., 2017, 77, pp. 462–477
[26] Shafiee, S., Fortuni-Furabrad, M., Rastegar, M.: ‘Investigating the impacts of plugin hybrid electric vehicles on power distribution systems’, IEEE Trans. Smart Grid, 2013, 4, (3), pp. 1351–1360
[27] Xu, Z., Hu, Z., Song, Y., et al.: ‘Coordination of PEVs charging across multiple aggregators’, Appl. Energy, 2014, 136, pp. 582–589
[28] Li, M., Jia, Y., Shen, Z., et al.: ‘Improving the electrification rate of the vehicle miles traveled in Beijing: A data-driven approach’, Transp. Res. A, Policy Pract., 2017, 97, pp. 106–120
[29] Kimura, T.: ‘A two-moment approximation for the mean waiting time in the GI/Gi/s queue’, Perform. Eval., 2011, 5, (3), pp. 205–205
[30] Kimura, T.: ‘Approximations for multi-server queues: system interpolations’, Queueing Syst., 1994, 17, (3), pp. 347–382
[31] Zeng, T., Zhang, H., Moura, S.: ‘Solving overstay and stochasticity in PEV charging station planning with real data’, IEEE Trans. Ind. Inf., 2020, 16, (5), pp. 3504–3514
[32] Beijing Trip.: ‘Taxi’, 2018, https://www.beijingtrip.com/transport/taxi.htm. (Accessed 28 April 2019)
[33] Beijing Municipal Bureau of Statistics.: ‘Average wages of employed persons in legal entities’, 2019, http://tjj.beijing.gov.cn/tjj/main/2019-10/tjj/indexch.htm