New Physics Signals through CP Violation in $B \rightarrow \rho \pi$ \textsuperscript{1}

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Abstract

We describe here a method for detecting physics beyond the standard model via CP violation in $B \rightarrow \rho \pi$ decays. Using a Dalitz-plot analysis to obtain $\alpha$, along with an analytical extraction of the various tree ($T$) and penguin ($P$) amplitudes, we obtain a criterion for the absence of new physics (NP). This criterion involves the comparison of the measured $|P/T|$ ratio with its value as predicted by QCD factorization. We show that the detection of NP via this method has a good efficiency when compared with the corresponding technique using $B \rightarrow \pi \pi$ decays.

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1 Introduction

In the Standard Model (SM), CP violation is due to a non-trivial complex phase in the quark mixing matrix, the CKM matrix. This phase is most often described using the unitarity triangle (Fig. 1). Here $\alpha$, $\beta$ and $\gamma$ are the three weak phases that can cause amplitudes contributing to a given decay to interfere in such a way as to create CP violation in this decay [1].

Figure 1: Unitarity triangle, with weak phases definitions

There have been many ways suggested to measure these three phases, the ultimate goal being to overconstrain the unitarity triangle in order to test the validity of the SM description of CP violation. Another way to try and test the SM is to look for a discrepancy between the SM predictions of CP violation in a given decay and the observed values. The method described here is of the latter type. We will review in the next section how $B \to \pi\pi$ has been used to date for both measuring $\alpha$ [2, 3] and detecting physics beyond the SM [4]. Since $B \to \pi\pi$ is not dominated by one amplitude, but involves both tree ($T$) and penguin ($P$) contributions (a situation referred to as penguin pollution), the indirect CP asymmetry does not lead directly to the determination of $\alpha$. However, by using an isospin analysis of $B \to \pi\pi$ decays, [2], one can extract $\alpha$ through geometrical means (up to discrete ambiguities). Moreover, new physics (NP) detection is shown to be possible despite the so-called CKM ambiguity [5], but the technique loses some of its efficiency due to the discrete ambiguities on $\alpha$.

The subsequent section will explain how $B \to \rho\pi$ can itself be used for NP detection through the same method used in $B \to \pi\pi$. Interestingly enough, a Dalitz plot analysis of $B \to \pi\pi\pi$ leads to an unambiguous determination of $\alpha$. Thus,
though quite challenging from the experimental point of view, $B \to \rho \pi$ is shown to be very efficient in the detection of NP.

2 \quad B \to \pi \pi

2.1 \quad B \to \pi \pi$ without New Physics

The $B \to \pi \pi$ system includes three decays (and three CP-conjugate decays): two neutral decays, $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^0\pi^0$, and one charged, $B \to \pi^+\pi^0$. In general, the decays involve a $T$ and a $P$ contribution (see Fig. 2), so that all decay amplitudes take the form

$$A^i = T^i e^{-i\alpha} + P^i, \quad (1)$$

with $(i=-,00,+0)$. (The amplitudes have all been rescaled by $e^{i\beta}$ so as to remove the mixing phase, and $A^{+0}$ has no $P$ contribution.) Although one can measure indirect CP violation ($\text{Im}\lambda$) in the two neutral decays,

$$\text{Im}\lambda = \sin \frac{\bar{A}}{A} = \sin \left( \frac{T e^{i\alpha} + P}{T e^{-i\alpha} + P} \right), \quad (2)$$

these measurements cannot be related to $\alpha$ in a simple way. This is where isospin analysis enters the scene. Decomposing all amplitudes and contributions in term of their isospin content, one obtains the two following triangle relationships:

$$\frac{1}{\sqrt{2}} A^{+-} + A^{00} = A^{+0},$$

Figure 2: Tree and penguin amplitudes
Figure 3: Isospin triangles. $2\alpha$ appears at two places; one (between the $T$ diagrams of $A^{+-}$ and $\bar{A}^{+-}$) would be difficult to obtain, while the other one is obtained through simple geometrical means.

\[ \frac{1}{\sqrt{2}} \bar{A}^{+-} + \bar{A}^{00} = \bar{A}^{+0} . \] (3)

That is, one sees that $A^{+-}$, $A^{00}$ and $A^{+0}$ form a triangle in isospin space (as do their CP-conjugate counterparts). $2\alpha$ is then obtained through simple geometrical means. However, this resolution will give $2\alpha$ up to an eightfold discrete ambiguity (see Fig. 3).

## 2.2 $B \to \pi\pi$ with New Physics

Let us now try and test the sensitivity of $B \to \pi\pi$ to New Physics (NP). It is expected that NP will affect principally flavor-changing neutral current (FCNC) diagrams [6, 7]. Thus both the box diagram of the $B^0 - \bar{B}^0$ mixing and the penguin diagram could be affected, each (in general) in a different way. This would in turn introduce a phase discrepancy between them. The amplitudes would now appear as:

\[ A^i = T^i e^{-i\alpha} + P^i e^{-i\theta_{NP}} . \] (4)

Unfortunately, it has been shown [5] that it is fundamentally impossible to detect a penguin phase, due to the so-called CKM ambiguity. To remove this ambiguity, it is necessary to make an assumption about some hadronic parameters (or some combination of them). One can then detect the presence of $\theta_{NP}$ by comparing the
theoretical SM prediction of this hadronic parameter with its measured value. This analysis was done recently [4]. The criterion obtained is as follows: if $\theta_{NP} = 0$, the observables respect

$$0.05 \leq \frac{1 - \sqrt{1 - (a_{dir}^+)^2 \cos(2\alpha - 2\alpha_{eff})}}{1 - \sqrt{1 - (a_{dir}^-)^2 \cos(2\alpha_{eff})}} \leq 0.5,$$

(5)

where $a_{dir}^+$ and $2\alpha_{eff}$ are the direct and indirect CP asymmetries, respectively. If it is found experimentally that this inequality is not respected, it would indicate that $\theta_{NP} \neq 0$. The main experimental difficulty here is the large number of discrete ambiguities in the extraction of $\alpha$ from $B \to \pi\pi$ decays. NP detection with this channel would thus probably necessitate using $\alpha$ as obtained independently in some other decay.

3 $B \to \rho\pi$

$B \to \rho\pi$ decays offer on this subject an interesting alternative. The $B \to \rho\pi$ system contains five decays (and five CP-conjugate decays): three neutral decays ($B^0 \to \rho^+\pi^-$, $B^0 \to \rho^-\pi^+$ and $B^0 \to \rho^0\pi^0$) and two charged ($B^+ \to \rho^+\pi^0$ and $B^+ \to \rho^0\pi^+$). All amplitudes receive both a tree and a penguin contribution:

$$S_i = T_i e^{-i\alpha} + P_i,$$

(6)

(i = +−, −+, 00, +0, 0+) where the first superscript is for $\rho$ and the second is for $\pi$). Again all amplitudes have been rescaled by $e^{i\beta}$. In this case too, then, we have penguin pollution. Although an isospin analysis would give a pentagon relationship, enabling one to solve geometrically for $2\alpha$, this resolution would be painfully plagued by discrete ambiguities. In this case, as explained in Ref. [8], it is the Dalitz-plot analysis of $B^0 \to \pi^+\pi^-\pi^0$ that saves the day. Since all three neutral amplitudes contribute to $B^0 \to \pi^+\pi^-\pi^0$ (and all three neutral CP-conjugate amplitudes to $B^0 \to \pi^-\pi^+\pi^0$), one can use the interference between them to obtain $2\alpha$ unambiguously.

3.1 $B \to \rho\pi$ without New Physics

In the SM case, the Dalitz plot contains in fact enough information to ensure that one can solve for all theoretical parameters. We define for each decay a branching ratio:

$$B_i = \frac{1}{2}(|S_i|^2 + |\bar{S}_i|^2)$$

(7)

and a direct CP asymmetry:

$$a_i = \frac{|S_i|^2 - |\bar{S}_i|^2}{|S_i|^2 + |\bar{S}_i|^2}.$$

(8)
We also define an indirect CP asymmetry for the neutral decays:

\[ 2\alpha_{\text{eff}}^i = \text{Arg}(\bar{S}_i S_i^*) . \] (9)

The Dalitz-plot analysis ensures that all theoretical parameters can in principle be expressed in terms of these observables. In particular, the penguin and tree amplitudes are solvable analytically. The results we obtain are as follows: for each decay, the ratio of the penguin to the tree amplitude is [9]:

\[ r^i \equiv \left| \frac{P^i}{T^i} \right| = \frac{1 - \sqrt{1 - a_i^2 \cos (2\alpha_{\text{eff}}^i - 2\alpha)}}{1 - \sqrt{1 - a_i^2 \cos 2\alpha_{\text{eff}}^i}} . \] (10)

This expression is the same for the two charged decays, with \(2\alpha_{\text{eff}}^i\) put to zero. The ratio is expressed in terms not only of observables, but also of \(2\alpha_i\); one can simply use for this the value obtained through the Dalitz-plot analysis of the \(B \to \rho\pi\) system.

### 3.2 \(B \to \rho\pi\) with New Physics

Let us now add new physics in the FCNC, just as in the \(B \to \pi\pi\) channel. The amplitudes are modified to:

\[ S^i = T^i e^{-i\alpha} + P^i e^{-i\theta_{NP}} . \] (11)

Just as in the case of \(B \to \pi\pi\), the CKM ambiguity causes \(\theta_{NP}\) to be impossible to measure. However, here too we can remove the CKM ambiguity by making an assumption about hadronic parameters. We are still able to solve for each penguin-to-tree ratio analytically:

\[ r^i = \left| \frac{P^i}{T^i} \right| = \frac{1 - \sqrt{1 - a_i^2 \cos (2\alpha_{\text{eff}}^i - 2\alpha)}}{1 - \sqrt{1 - a_i^2 \cos (2\alpha_{\text{eff}}^i - 2\theta_{NP})}} . \] (12)

One question one can ask is whether there are hidden contributions of \(\theta_{NP}\) in these expressions. Since up to now we’ve used the Dalitz plot to obtain \(2\alpha\), we have to make sure that this analysis still allows its determination if \(\theta_{NP} \neq 0\). The point is that all penguins have been affected in the same way, so that the interference between the neutral decays that allowed the extraction of \(2\alpha\) is left unchanged. Thus, with or without NP, the Dalitz-plot analysis of \(B \to \pi^-\pi^+\pi^0\) gives \(2\alpha\) unambiguously.

Eq. (12) then stands as a testing ground for the presence of new physics. Given the SM prediction of one (or some) of the ratios, we could compare it with its measured value [as given by Eq. (11)] and decide whether it is compatible with \(\theta_{NP} = 0\).
A SM computation of $r^{+-}$ and $r^{-+}$ has recently been made within the framework of QCD factorization [10]. The results are as follows:

$$r^{+-} = 0.10^{+0.06}_{-0.04},$$
$$r^{-+} = 0.10^{+0.09}_{-0.05}. \quad (13)$$

However, as we want to test for the presence of NP and not the accuracy of QCD factorization (or, for instance, our assumption of no electroweak penguins), we must be as conservative as possible regarding the ranges used to compare with data. The above ranges are therefore enlarged to include potential underestimate of errors. Our criterion for the absence of NP will then be:

$$0.05 < r^i < 0.25. \quad (14)$$

or, in terms of observables,

$$0.05 < \frac{1 - \sqrt{1 - a^2_i \cos(2\alpha_{eff}^i - 2\alpha)}}{1 - \sqrt{1 - a^2_i \cos 2\alpha_{eff}^i}} < 0.25 \quad (15)$$

for $i = +-, -+.$

### 3.3 Sensitivity to NP

The last question we want to consider is how restrictive our criterion is. If it happened, for instance, that for every fixed value of $2\alpha$, there existed for most values of $a_i$ a value of $2\alpha_{eff}^i$ such that Eq. (15) is satisfied, then our criterion would effectively be useless. We must therefore test the restrictiveness of Eq. (15).

We first suppose $2\alpha$ to be known unambiguously within a certain range (so as to account for experimental errors in measurements). We then generate randomly-chosen pairs of observables $(2\alpha_{eff}^i, a_i)$, and finally test whether each of these simulated experimental “results” would give a value of $r^i$ that would fit inside the range of Eq. (14). We consider two ranges of values for $2\alpha$: (a) $120^\circ \leq 2\alpha \leq 135^\circ$ and (b) $165^\circ \leq 2\alpha \leq 180^\circ$. We also consider the case where the full Dalitz-plot analysis is not available, but only $\sin 2\alpha$ is measured. In this case, $2\alpha$ is known up to a two-fold ambiguity. Our results are shown in Fig. 4. The darkened regions in $(2\alpha_{eff}^i, a_i)$ parameter-space are consistent with the SM prediction, so that NP is present everywhere in the white regions. Should $2\alpha$ be known unambiguously, only the right-hand dark region remains, leaving even more space for NP.

We therefore see that the analysis of $B \to \rho \pi$ decays can be used to detect NP, should it be present. Moreover, only the $\pi^+\pi^-\pi^0$ final state is necessary to obtain $2\alpha$ – it is not necessary to consider final states with two neutral pions. This is an advantage compared to the detection of NP using $B \to \pi \pi$ decays, since the full isospin analysis does require the two-\pi^0 final state. In addition, since the $P/T$ ratios are expected to be smaller in $B \to \rho \pi$ than in $B \to \pi \pi$, the SM-consistent regions are also smaller in the $B \to \rho \pi$ channel than in the $B \to \pi \pi$ channel.
Figure 4: Regions in $(2\alpha_{\text{eff}}^i, a_i)$ ($i = +-, --$) parameter space consistent with the actual (conservative) QCD factorization prediction on $|P^i/T^i|$. $2\alpha$ is assumed to be known up to a 2-fold ambiguity; should it be obtained unambiguously, only the left-hand region would remain.

4 Conclusion

To summarize, $B \to \rho\pi$ decays can be used to detect a discrepancy between the phase of the mixing diagram and the phase of the penguin diagram. Should such a discrepancy be detected, it would be a clear signal of physics beyond the Standard Model. To carry out this method, it is necessary to make a conservative assumption about some hadronic parameters. QCD factorization provides us with a prediction of $|P^i/T^i|$, $i = +-, --$. The region in $(2\alpha_{\text{eff}}^i, a_i)$ parameter space consistent with this prediction is relatively small. $B \to \rho\pi$ is therefore a particularly useful decay channel to use for the detection of new physics.

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