On Progress in Gravitational Waves Recording

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Abstract. The paper adduces theoretical data and results of experimental studies confirming the existence of gravitational radiation predicted by the general theory of relativity. It also shows that studying of gravitational waves detection is based on equations of general relativity and equations of electrodynamics, and for modeling of gravitational waves space by sources, equations of hydrodynamics are used.

1. Introduction

The possible existence of gravitational waves was predicted by A. Einstein on the basis of solving the equations of general relativity when calculating the power of gravitational radiation [1–3].

The first evidence was received by experimental studies of Joseph H. Taylor, Joel M. Weisberg, et al., who studied the effect of slowing down the period of the binary star system PSR 1913 + 16 due to energy losses on gravitational radiation [4].

Until recently, however, there has remained the main task: the direct recording of gravitational waves from space radiation sources by means of ground-based or space gravitational antennas.

Over the years, several methods have been proposed for recording gravitational radiation [5]. Experimental work began in the 1960s of the 20th century, but before the beginning of the 21st century there was no reliable experimental proof of the ground-based recording of gravitational radiation.

This is due to the fact that gravitational waves have small amplitude, in addition, the proposed detection methods have insufficient sensitivity and are rather complicated in technical implementation.

In general, as history has shown, the projects of laser interference gravitational antennas appeared to be the most promising and technically feasible.
These are broadband gravitational antennas, which offer a lot of opportunities as to the methods of recording of gravitational waves and extracting signals, as well as the use of quantum non-perturbative measurements and the inclusion of gravitational antennas in the network.

The main element of laser interference gravitational antennas is Fabry–Pérot multipath free-mass resonator, on whose such properties as the sensitivity and noise immunity of the entire gravitational antenna largely depends.

The creation of new generation gravitational antennas designed to reliably receive gravitational waves from remote space sources involves the use of high-power lasers, complex computer systems for processing large data arrays, the use of complex seismic protection systems and the solution of other complex engineering and physical problems.

At present large international experience has been gained in the field of creating laser gravitational antennas, which ensured the ground-based recording of gravitational waves from black hole collision [6–7] and neutron star merger [8]. Furthermore, based on almost simultaneous recording of gravitational waves [8] and a short gamma-ray burst [9] from neutron star merger, the gravitational wave propagation velocity was estimated, which appeared to be equal to the speed of light in vacuum with an accuracy of $10^{-15}$ [10].

The experience of gravitational antenna projects by VIRGO (Italy, France), LIGO (USA), TAMA (Japan), CLIO (Japan), GEO-600 (Germany) and OGRAN (Russia) will certainly be used to create more compact and highly sensitive antennas of new generation.

This paper is based on the reports given at the Russian-Italian Meeting "Towards a new generation of gravitational-wave detectors" held at Moscow State University and Bauman Moscow State Technical University on June 19–21, 2008, and the reports presented by participants of the Russian Scientific Meeting “Russian National Projects and “Mega science” Class Projects in the Field of Gravitational Physics” on May 22–24, 2018 in Dubna. These reports contain significant results in the development of methods for recording gravitational waves and, to a large extent, are presented in the proceedings of the International Conference “Physical Interpretations of Relativity Theory”, which is held at Bauman Moscow State Technical University every 2 years (http://www.pirt.info/?lang=eng).

2. Theoretical underpinning for the existence of gravitational waves
Gravitational waves (GW) are space-time curvature disturbances, which propagate at the speed of light. They occur at specific movements of material bodies, leading to inhomogeneous gravity force variation in the environment. Gravitational radiation was predicted by A. Einstein in the general relativity (GR) theory, but so far not detected by direct measurements.

According to GR, space-time is curved around the bodies due to the action of gravity and is represented by a symmetric tensor $g_{\mu\nu}$ with 10 independent components. However, far from the masses (the case of weak gravitational fields), the tensor can be divided into two terms $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ where the first term, i.e. tensor $\eta_{\mu\nu}$, corresponds to the flat space time of the special theory of relativity and has only four components. It is often written in the form
The second tensor $h_{\mu\nu}$ contains information about the curvature caused by the gravitational field, and makes small corrections $|h_{\mu\nu}| \ll 1$. In the case of gravitational disturbances propagating far from their sources, the components of the tensor $h_{\mu\nu}$ can be calculated by the method proposed by A. Einstein [1], similar to that used in electrodynamics for delayed potentials. In the same paper, an expression for the power of gravitational radiation $\frac{dE}{dt}$ was obtained produced by the quadrupole moment of the body

$$\frac{dE}{dt} = -\frac{G}{5c^5} \dot{D}_{ik}^2,$$

where $G$ is gravitation constant, $D_{ik}$ is the tensor of the quadrupole moment, which is defined as the deviation of mass distribution in a given direction, from the spherical one, that is, as the elongation of the body shape in one direction and flatness in the other

$$D_{ik} = \int \rho \left( x_i x_k - \frac{1}{3} \delta_{ik} x^2 \right) dV,$$

where $\rho$ is the body density of the volume $V$, ($i, k = 1, 3$).

In [1, 2], A. Enshtein established the transversality of GW and two degrees of polarization also. Later A. Eddington showed that flat GW, as well as waves coming from a point source, propagate at the speed of light [3]. We can imagine GW as a crease running along a flying flag.

However, estimates of the radiation energy from a rotating rod made in [3] indicated the ineffectiveness of the recording of waves in a laboratory using the equipment of the beginning of the 20th century.

In 1955 Vladimir A. Fock was the first to pay attention to the possibility of using astrophysical catastrophes as a source of powerful gravitational radiation [11]. According to modern calculations, at the two neutron star merger, approximately $10^{45}$ J is released in the form of a burst of gravitational radiation, i.e. about 1% of the total energy $\left( E = mc^2 \right)$ of two stars.

These waves are intensely emitted by compact and massive astrophysical objects located at different distances from the Earth. They can be created, for example, in the collision of black holes or neutron stars, when, as a rule, there is no electromagnetic radiation.

Their existence was confirmed indirectly by observing the motion of binary pulsars. In 1974, Joseph Hooton Taylor Jr. and Russell Alan Hulse published the results of observations of the orbital period of the pulsar, which is a neutron star orbiting around its axis and emitting highly directional electromagnetic radiation fluxes, PSR1913 + 16 [4]. They managed to prove that the period of this
rotation, which is now 3 hours and 45 minutes, is annually reduced by 70 microseconds. This is due to the loss of energy that GW is carrying.

As far as GW recording on Earth is concerned, it is important to represent the finding of potential radiation sources, which essentially can be recorded by ground-based antennas. In [6-8] one can find images of the spatial location of GW sources which can be recorded by the LIGO interferometer and its upgraded version.

It is noteworthy that as early as 1805, Pierre-Simon Laplace in his famous work Traité de Mécanique céleste - "Treatise on celestial mechanics" [12] pointed at the possibility of energy loss by the planet - Sun system, based on the assumption of the final gravity propagation velocity.

The issue of gravitational radiation remained unpopular until the 1960s of the last century, when it became clear that GW can transmit energy and cause the movement of bodies.

An essential feature of a gravitational field was its nonlocal non removability, the existence of tidal forces. Now for its recording, it is necessary to consider the change in the distance between two particles in the reference frame associated with the center of particles mass. It was accepted that GW propagation in vacuum should be considered in the so-called TT-calibration. In it, the field distortion tensor $h_{\mu\nu}^{TT}$ has only two independent components. In this case, the plane GW is split into two components with orthogonal polarizations $h_+ \text{ and } h_\times$.

In addition, the convenience of this calibration was revealed when determining small oscillations of a test particle in the field of a plane GW, since it was necessary to calculate the second derivatives of the distortion tensor with respect to time. It is important that time $t$ measured in the coordinate system with the $TT$-calibration is the proper time $\tau$, and this allows the replacement of absolute derivatives by ordinary ones. The curvature tensor, which is responsible for the oscillations of the test particle and entering its equation of motion, is now proportional to the ordinary second derivative of $h_{\mu\nu}^{TT}$ with respect to time. In the case of an elastic-dissipative connection between test particles, the equations of motion become the equations of forced oscillations of particles with respect to an experimenter located at the center of mass [5].

The calculation of the relative displacement of the position of a particle at rest after the arrival of a plane GW showed a significant difficulty in recording the displacement proportional to the amplitude of the wave – it’s tiny.

In fact, let the field of a flat GW propagating along $Ox$ be described by the metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 + h & 0 & 0 \\ 0 & 0 & -1 - h & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

and create transverse effects along the axes $Ox$ and $Oy$ (here $h=h_\times$). The spatial interval between particles: $dl^2 = (dx_j)^2 - h_{\mu\nu} dx_i dx_j$ will determine the value of the squared distance in one direction.
\[ dl^2 = dl_0^2 - hdl_0^2 = (1 - h)dl_0^2. \]

Therefore, the relative change in the distance between the particles is
\[ \frac{\Delta l}{l} = -\frac{1}{2}h. \]

Along the other transversal axis, the relative change in the distance between the particles will be with the opposite sign.

In interferometers, mirrors on particles are free, and the variation of the relative distance with time is noticeable from the shift of the interference fringes created by electromagnetic waves reflected from the mirrors. In resonant detectors the resonant displacement of the edges of an elastic body is measured under the action of variable tidal force.

The progress achieved in understanding the effects of gravity and ways of describing them has drawn increased attention of both theoretical scientists and experimenters [13, 14] to the problem of GW generating and recording. In [15], the reaction of an elastic quadrupole detector to a flat GW was considered, and in [16] the laser beam passing through the interferometer in the field of a flat GW, was considered. These studies served as the basis for research into the development of gravitational radiation detectors. International projects were set up and partially implemented for the development of various types of GW detectors coming from astrophysical objects (EPLOPER, NAUTILUS, ALLEGRO, NIOBE, LIGO, VIRGO, GEO, AIGO, TAMA, OGRAN). The success of this work, however, confounded expectations.

Despite the first negative search results, scientists did not stop their collaborative work. Theoretical studies and astrophysical observations indicate the possibility of energy loss by complex astrophysical objects, unexplored by gravitational radiation. The GR problems encouraged the development of a geometric approach to vector analysis on curved surfaces, i.e. geometric algebra, geometric calculus, classification of spaces introduced by Aleksey Z. Petrov, Felix Pirani [17], which lead to the classification of gravitational fields. In addition, since GW interact weakly with matter, they can travel long distances in the Universe, carrying valuable information about deep processes in stars and clusters of stars, galactic processes like supernova explosions, the formation of quasars and the formation of active galactic nuclei (AGN). This is their significant difference from the electromagnetic channel through which information is supplied only from the surface of an object (for example, the Sun). Moreover, the penetration of gravitational radiation into clouds of cosmic dust does not affect the GW intensity. Perhaps only the recording of changes in neutrino fluxes can supplement the gravitational information or even compete with it.

On the other hand, numerous factors have shown the expediency of extension studies. Among them: extreme sensitivities and special conditions necessary for measurements, the opportunities of using various achievements in other fields, e.g. laser technology, cryogenics, semiconductor micro-and nanotechnologies, production of ultra-pure optical components and coatings withstanding intense radiation, the development of automated control systems for precision devices located on stability boundary, the ability to work with fast computer complex clusters for processing large databases.

GW interact weakly with matter, are less absorbed compared to electromagnetic radiation, and therefore carry information about the interaction of massive objects and the processes inside them. A
GW stretches and compresses space-and-time. If there are two separated coordinate systems in its field, then the wave causes their relative oscillatory motion. For GW, two polarizations are possible. In the first polarization, the wave during the half-period is compressed vertically and stretched horizontally, in the next half-period - vice versa. The second possible polarization is shifted by 45° with respect to the first one. In time, a GW is a long or short wave packet. Its form contains information about the source. GW frequency range is from $10^{-16}$ to $10^8$ Hz. Modern gravitational antennas (GA) are designed for an interval from $10^{-4}$ to $10^4$ Hz, or in wavelengths from $3 \cdot 10^4$ km to 30 km.

The development of projects and the launch of gravitational-wave antennas pursues the goal of creating a new - non-electromagnetic - channel for obtaining information about the surrounding world.

3. Gravitational Radiation Power Estimation

The estimation of gravitational radiation source can be obtained by simple calculations. According to the laws of conservation of energy and momentum, monopole and dipole gravitational radiation are absent. The next type of radiation is quadrupole [18]. In quadrupole approximation, the static, i.e. Newtonian, gravitational field creates quadrupole acceleration, proportional to $1/r^4$ and which has the form

$$a_{\text{quadrupole}} = G \frac{D}{r^4},$$

where $D$ is quadrupole moment. In the case of dumbbells or a double stars system $D=I=md^2$ ($d$ is the distance between masses $m$).

With electromagnetic or gravitational radiation, the field strength, at distances of much greater wavelength $\lambda$, varies with distance as $1/r$. Therefore, the acceleration from the gravitational field transmitted by the radiation should change as [18-19]

$$a_{\text{quadrupole}}' = G \frac{md^2}{\lambda^3 r}.$$

The intensity (density) of gravitational radiation, which is equal to the square of the acceleration, is referred to the gravitation constant

$$J \sim \frac{1}{G} a_{\text{quadrupole}}^2 = G \frac{m^2 d^4}{\lambda^6 r^2} = G \frac{m^2 d^4}{c^6} \frac{T^6 r^2}{r^2} \sim \frac{G I^2 \omega^6}{c^6 r^2}.$$

The power flow, i.e. module of Umov-Poynting vector, is equal to

$$S_{Gr} = \frac{G I^2 \omega^6}{c^5 r^2}.$$

After summation in all directions, we obtain an estimate of the energy loss due to gravitational radiation

$$\frac{d\varepsilon}{dt} \sim \frac{G I^2 \omega^6}{c^5 r^2}.$$

Using the same approach, we estimate the GW amplitude when $r >> \lambda$. The potential of the quadrupole component of the static field
\[ \Phi_{\text{quadrupole}} \sim G \frac{D}{r^3} \text{(sign "," is omitted)} \]

in the wave zone has the expression

\[ \Phi_{\text{quadrupole}} \sim G \frac{D}{\lambda^2 r} \sim \frac{G D}{c^2 T^2 r}. \]

Therefore, the estimate of the dimensionless quantity \( h \) acting as the amplitude is equal to

\[ h = \frac{\Phi_{\text{quadrupole}}}{c^2} \sim \frac{G \bar{D}}{c^4 r} \sim \frac{r^2}{d \cdot r}, \]

where \( r_s = \frac{2Gm}{c^2} \) is Schwarzschild radius, \( r \) is distance to the source, \( d \) is indicative quadrupole size. From the GW amplitude formula above, for it is easy to get another estimate

\[ h \sim \frac{\Phi_{\text{inside}}}{c^2} \cdot \frac{\Phi_{\text{outside}}}{c^2}, \]

where \( \Phi_{\text{inside}} = \frac{GM}{d} \), \( \Phi_{\text{outside}} = \frac{GM}{r} \).

The GW amplitude is determined by means of a dimensionless quantity [19]

\[ h_{\text{prop}}^T \approx \frac{2G}{c^4 R_0} \frac{d^2}{\nu} \int \rho \left( 3x^\alpha x^\beta - \delta_{\alpha\beta} x^2 \right) dV, \]

where \( G \) is Newtonian constant, \( c \) is speed of light, \( R_0 \) is a distance to the source, \( \rho \) is mass distribution of the source, and the integral is calculated over the volume of the source [9]. The effect that is made on two freely falling particles separated by a distance \( L \) is the variation \( \Delta L \approx 1 / 2 L^2 h_{\text{prop}}^T \), which is a measured value. Usually the GW source amplitude is denoted as \( h = h_{\text{prop}}^T \).

The radiation power of a laboratory generator is around \( 10^{20} \) W [14]. Extremely high-energy particles accelerated in next-generation accelerators are considered potential candidates for laboratory GW, but even in this case it will take many years to achieve a result.

Nowadays, only astrophysical sources can be considered as radiators of gravitational radiation. Let us dwell on the best candidates for GW sources that are likely to be recorded in the coming years.

Historically, the collapse of a star is considered the most studied and real GW radiator. This process results in the formation of a black hole.

The process of GW radiation by a black hole becomes significant if the collapse is spherically asymmetric. The GW amplitude in this case follows the approximate formula [14]

\[ h \approx 5 \times 10^{-21} \left( \frac{\eta}{10^{-2}} \right)^{1/2} \left( \frac{15 \text{ Mpc}}{R_0} \right) \left( \frac{1 \text{ kHz}}{\nu} \right) \left( \frac{10^{-3} \text{ sec}}{\tau} \right)^{1/2}, \]

where \( \eta = \Delta E / M_0 c^2 \) is energy yield radiated in GW, \( R_0 \) is a distance, \( \nu \) is the observed frequency, and \( \tau \approx d / c \) is the time during which a collapse occurs, with the size of the source. The quantity \( \eta \) is a part of the total energy transformed into GW, which is less than 0.2. The rate of explosions for a
galaxy is believed to be 1 time in 40 years, while the pulse duration is about 1 ms, which determines the setting of the detectors [14].

The estimates use the existing statistics of massive X-ray binary systems with candidates for black holes of the Cygnus X-1 type and the number of binary pulsars with black holes. These types of objects are evolutionarily related, at least in one of the channels of the formation of pulsars with black holes, so the black hole formation parameters are limited.

Another class of events far from accurate verification is the dragging of particles in the black hole itself. The dragging of a particle with great force should lead to the GW formation. Moreover, if the particles move in a spiral direction towards the black hole, the radiation intensity is 100 times greater than with radial dragging [20].

There is no doubt that the mechanism for emitting GW which are high-energy in amplitude is the rotation of compressed binary objects, such as neutron stars. With a star diameter of about 10 km, the mutual distance can be so small that very intense radiation occurs before the merger. The high level of GW radiation from binary stars is confirmed by various studies.

The shape of GW bursts for the two neutron stars merger depends on their parameters [21, 22].

The signal amplitude estimate is

\[ h \approx 10^{-23} \left( \frac{100 \text{ Mpc}}{R_0} \right) \left( \frac{M}{M_0} \right)^{2/3} \left( \frac{\mu}{M_0} \right) \left( \frac{v}{100 \text{ Hz}} \right)^{2/3}, \]

where \( M \) and \( \mu \) are the total and reduced mass, respectively and \( M_0 \) is the solar mass [21, 22]. The time covered by the GW with a frequency \( v \) is estimated as

\[ t \approx 7.8 \times \left( \frac{100}{v} \right)^{8/3} \left( \frac{M_0}{M} \right)^{2/3} \left( \frac{M_0}{\mu} \right) \text{ [sec]}. \]

Since the signal-to-noise ratio of the detector is proportional to \( t^{1/2} \), which follows from the two expressions given, and the signal-to-noise ratio increases as \( v^{-2/3} \), then broad-banded detectors at a low frequency are more likely to detect these sources.

Estimation of the frequency of such events gives about 3 events per year in the area of a radius of 35 million light years [23].

Pulsars are known to be rotating neutron stars which have an off-axis magnetic field. They are considered to be the best representative of long-lasting gravitational-wave radiators. The pulsar PSR1913 + 16 is an example of such radiator [24]. Surface protuberance or aspherical shape with ellipticity \( \varepsilon \) could create GW bursts with the amplitude

\[ h \approx 10^{-23} \varepsilon \left( \frac{v}{10 \text{ Hz}} \right)^2 \left( \frac{10 \text{ kpc}}{R_0} \right), \]

where \( v \) is the GW frequency, twice as large as the rotational speed [25]. The upper limits for the GW amplitudes of pulsar radiation in the Vela and Crab nebulae are estimated as \( h \approx 3 \times 10^{-24} \) and \( h \approx 10^{-26} \) respectively [24].

According to another model, the Crab nebula pulsar may have an upper limit \( h \approx 10^{-25} \) [26].
Since the total number of pulsars in the Galaxy was estimated as $10^3$ [27], and some pulsars with GW with a frequency greater than 10 Hz amount to 10%, we can expect several thousand pulsars with GW in the frequency domain of the interference kilometer dimensions.

If a pulsar and a neutron star form a binary system, then the star, with its strong gravitational field, tears off the substance from the surface of the pulsar. This substance is built up around a neutron star, and then moves deep into it. Mass gain can reach the unstable Chandrasekhar-Friedmann-Skuts point, which is characterized by high GW radiation power with amplitude

$$h \approx 2 \times 10^{-28} \left( \frac{300 \text{ Hz}}{v} \right) \left( \frac{F_X}{10^{-17} \text{ J/sm}^2 \text{ sec}} \right)^{1/2},$$

where $F_X$ are emitted x-ray fluxes [28].

The data on the galactic frequency of binary neutron star mergers, derived from the observed statistics of binary radio pulsars, give a value of frequency $8 \times 10^{-6}$ per year, while theoretical estimates of this frequency, obtained from the analysis of the stellar evolution theory, give on average higher rates of mergers: $10^{-4} - 3 \times 10^{-5}$ per year [29-31].

The stochastic GW background created by all sources has an estimated amplitude

$$h \approx 6 \times 10^{-26} \left( \frac{Q_{GW}}{10^{-10}} \right)^{1/2} \left( \frac{100 \text{ Hz}}{v} \right),$$

where $Q_{GW}$ is the ratio of the source energy density for the bandwidth $v$ to the energy density, which is necessary for the Universe to be closed ($10^{15} \text{ J/cm}^3$)[32].

The stochastic background can occur in the interaction of cosmic solitons and stars. The GW amplitude in this case reaches $h \approx 10^{-22}$ at $v \approx 10^{-2}$ Hz and $h \approx 10^{-24}$ at $v \approx 4.5$ Hz[33]. Stochastic GW radiation should lead to the evolution of the sources themselves, for example, pulsars [34].

Seismic data for the last 100 years show that the power density of surface waves excitation on Earth due to the stochastic GW background does not exceed and $6.1 \times 10^{-6} \text{ J m}^{-3} \text{ Hz}$ in the frequency range 0.31 MHz and 1.72 MHz, respectively [35].

One of the mechanisms of supernova explosions considered in the literature — magnetic-rotating — was proposed by Gennady S. Bisnovatyi-Kogan in 1970. The idea of this mechanism is that the shell is dropped off by a magnetic field of a rapidly rotating neutron star (NS). When this happens, the shell accelerates due to the deceleration of the rotation of the neutron star. This idea combines the generation and amplification of magnetic fields and complex three-dimensional hydrodynamics with a strong effect of radiation transfer. Three-dimensional calculations of this scenario are very difficult, and the results of two-dimensional calculations (Ardelyan et al. 1998, 2000) show that the magnetic-rotating mechanism can transfer a few percent of the rotational energy of a compact residue to the kinetic energy of the shell. In this case, the stage at which there is a significant acceleration and the shell is dropped off lasts from 0.01 to 0.1 sec.
To describe the relativistic magneto hydrodynamics of neutron stars and the formation of a super massive neutron star or a black hole based according to the relativistic theory of viscous fluids, various models were used (see reviews [36,37]). These models introduce rather uncertain estimation of the magnetic turbulence effect on their structure and lifetime.

Consider the magneto hydrodynamic model of neutron stars in the framework of general relativity, based on numerical analysis. The model was presented in [38].

The energy-momentum tensor of an ideal fluid corresponding to the neutron star matter is determined as follows

\[ T_{\mu\nu} = \rho h u_\mu u_\nu + p g_{\mu\nu}, \]

where \( \rho \) is density, \( h \) is specific enthalpy, \( u_\mu \) is four-velocity, \( p \) is pressure and \( g_{\mu\nu} \) is space-time metric.

In the case of numerical analysis of the GR equations, space-time is decomposed into spatially similar slices with a normal \( n^\mu \). In this case, the energy-momentum tensor is represented as

\[ T_{\mu\nu} = En_\mu n_\nu + S_{\mu\nu} + S_{\mu} n_\nu + S_{\nu} n_\mu, \]

where

\[ E = T_{\mu\nu} n^\mu n^\nu = \rho h W^2 - p, \]
\[ S_{\mu} = -\gamma_{\mu\alpha} n_\beta T^{\alpha\beta} = \rho h W^2 v_\mu, \]
\[ S_{\mu\nu} = \gamma_{\mu\alpha} \gamma_{\nu\beta} T^{\alpha\beta} = S_{\mu} n_\nu + p \gamma_{\mu\nu}, \]

with the corresponding laws of conservation of energy and momentum

\[ \partial_i (\sqrt{\gamma} S_i) + \partial_j [\alpha \sqrt{\gamma} (S_i n^i + S_i n^j)] = \alpha \sqrt{\gamma} \left( \frac{1}{2} S_{ij} \partial_k n^j + \frac{1}{\alpha} S_{ij} \partial_i \beta_k \right) - E \partial_j \log \alpha, \]
\[ \partial_i (\sqrt{\gamma} E) + \partial_j [\alpha \sqrt{\gamma} (S_i + E n^i)] = \alpha \sqrt{\gamma} (K_{ij} S^j - S^j \partial_j \log \alpha), \]

where \( \gamma_{\mu\alpha} \) is spatial metric, \( v_\mu \) is three-velocity, \( W \) is Lorentz factor, \( \alpha \) is lapse function, \( \beta^i \) is shift vector and \( K_{ij} \) is extrinsic curvature. This system of equations is also complemented by the equation of state and the laws of conservation of baryon and lepton numbers.

After filtering the disturbance modes resolved within the chosen method of numerical simulation, this model yields a spectrum of gravitational waves from the neutron star merger, the spectrum corresponding to the observed one [8]. Calculations show that for the most realistic values of turbulent viscosity, magnetic turbulence effects on physical processes are weak. In Newtonian limit, the considered system of equations goes into Navier-Stokes equations [38].

Without taking into account the effects associated with magnetic turbulence, the hydrodynamics of neutron stars in the framework of general relativity can be simulated on the basis of a relativistic generalization of Euler equation [39].
\[
D_v V^\alpha + V^\mu \nabla_\mu V^\alpha + 2 (\omega^{\alpha\mu} + \theta^{\alpha\mu}) V^\mu - (\alpha_\mu V^\mu + \theta_\mu V^\nu V^\nu) V^\alpha = -\frac{1}{E + p} (\nabla^\alpha p + D_\nu p V^\nu) - a^\alpha,
\]

where \(D_v\) is covariant derivative, \(\omega_{\mu\nu}\) is 2-form determining the shift, \(\theta_{\mu\nu}\) is 2-form determining the rotation, \(\alpha_\mu\) is expansion tensor, \(V^\mu\) is 4-velocity spatial component \(v^\alpha\), \(E\) is energy density, \(p\) is pressure, \(\nabla_{\mu}\) is three-divergence associated with four-divergence \(\nabla_{\mu}\) in the following manner

\[
\nabla_{\mu} A^\mu = \nabla_{\mu} A^\mu + \alpha_\mu A^\mu,
\]

where \(A^\mu\) is a four-vector.

The divergence of the four-velocities field \(\nabla_{\mu} v^\alpha\), in this case, is decomposed into shift, rotation and expansion components

\[
\nabla_{\mu} v^\alpha = \omega_{\alpha\beta} + \theta_{\alpha\beta} - a_{\alpha} v^\beta.
\]

From the generalized relativistic Euler equation, the deviation of a star from a spherical shape or the ellipticity parameter \(E\) is determined, which is used to calculate the amplitude of the gravitational wave signal [39]

\[
h = \frac{16 \pi^2 G}{c^4} \frac{I E}{T^2 r},
\]

where \(I\) is inertia moment, \(T = 2 \pi / \Omega\) is rotation period, \(r\) is stellar distance.

The hydrodynamics of neutron stars and the characteristics of their gravitational waves, excluding magnetic turbulence, are also analyzed on the basis of simpler variations of Euler equation [40].

The above sources determine the basic methods for recording gravitational radiation. At the same time, GW can be recorded by various indirect methods based on the GW influence on some physical objects (for example, by gravitational lensing of stellar images) [41].

The search for GW from astrophysical sources is associated with the solution of various physical and technical tasks [42-44], many of which have already been implemented in modern GA projects and allowed to solve the problem of detection of gravitational radiation [45].

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