ON CP-ODD EFFECTS IN $K_L \to 2\pi$ AND $K^\pm \to \pi^\pm \pi^\pm \pi^\mp$ DECAYS GENERATED BY DIRECT CP VIOLATION

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Abstract

The amplitudes of the $K^\pm \to 3\pi$ and $K \to 2\pi$ decays are expressed in terms of different combinations of one and the same set of CP-conserving and CP-odd parameters. Extracting the magnitudes of these parameters from the data on $K \to 2\pi$ decays, we estimate an expected CP-odd difference between the values of the slope parameters $g^+$ and $g^-$ of the energy distributions of "odd" pions in $K^+ \to \pi^+ \pi^+ \pi^-$ and $K^- \to \pi^- \pi^- \pi^+$ decays.

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1. Introduction

The observation of CP effects in $K^\pm \rightarrow 3\pi$ decays would allow to understand better how the mechanisms of CP violation work.

Now the Collaboration NA48/2 began a search for such effect with accuracy $\delta(g^+g^-) \leq 2 \times 10^{-4}$.

Contrary to the case of $K_L \rightarrow 2\pi$ decay, where CP violates both in $\Delta S = 2$ and $\Delta S = 1$ transitions, in the $K^\pm \rightarrow 3\pi$ decays, only the last (the so-called "direct") CP violation takes place. Experimentally, an existence of the direct CP violation in $K_L \rightarrow 2\pi$ decays, predicted by Standard Model (SM) and characterized by the parameter $\varepsilon'$, is established: $\varepsilon'/\varepsilon = (1.66 \pm 0.16) \times 10^{-3}$.

What is expected for CP effects in $K^+ \rightarrow \pi^+\pi^+\pi^-$ decay? To give an answer, it is necessary to understand the role of the electroweak penguin (EWP) operators in both decays and get rid of the large uncertainties usual for the theoretical calculations. The real scale of these uncertainties is characterized by the following predictions obtained before the above experimental result:

$$\frac{\varepsilon'}{\varepsilon} = (17^{+14}_{-10}) \times 10^{-4} \quad [1], \quad \frac{\varepsilon'}{\varepsilon} = (1.5 \div 31.6) \times 10^{-4} \quad [2].$$

To avoid the uncertainties arising in the theoretical calculation of the ingredients of the theory, we use the following procedure. We express the amplitudes of $K \rightarrow 2\pi$ and $K^\pm \rightarrow 3\pi$ decays in terms of one and the same set of parameters, and calculating $g^+ - g^-$, we use the magnitudes of these parameters extracted from data on $K \rightarrow 2\pi$ decays.

2. The scheme of calculation

A theory of $\Delta S = 1$ non-leptonic decays is based on the effective Lagrangian [3]

$$L(\Delta S = 1) = \sqrt{2} G_F \sin \theta_C \cos \theta_C \sum_i c_i O_i ,$$

where

$$O_1 = \bar{s}_L \gamma_\mu d_L \cdot \bar{u}_L \gamma_\mu u_L - \bar{s}_L \gamma_\mu u_L \cdot \bar{u}_L \gamma_\mu d_L \quad (\{8_f\}, \Delta I = 1/2);$$

$$O_2 = \bar{s}_L \gamma_\mu d_L \cdot \bar{u}_L \gamma_\mu u_L + \bar{s}_L \gamma_\mu u_L \cdot \bar{u}_L \gamma_\mu d_L + 2 \bar{s}_L \gamma_\mu d_L \cdot \bar{d}_L \gamma_\mu d_L$$

$$+ 2 \bar{s}_L \gamma_\mu d_L \cdot \bar{s}_L \gamma_\mu s_L \quad (\{8_d\}, \Delta I = 1/2);$$
\[
O_3 = \bar{s} L \gamma_\mu d_L \cdot \bar{u} \gamma_\mu u_L + \bar{s} \gamma_\mu u_L \cdot \bar{u} \gamma_\mu d_L + 2\bar{s} L \gamma_\mu d_L \cdot \bar{d} L \gamma_\mu d_L \\
- 3\bar{s} L \gamma_\mu d_L \cdot \bar{s} L \gamma_\mu s_L \quad (\{27\}, \Delta I = 1/2); \quad (4)
\]

\[
O_4 = \bar{s} L \gamma_\mu d_L \cdot \bar{u} \gamma_\mu u_L + \bar{s} L \gamma_\mu u_L \cdot \bar{u} L \gamma_\mu d_L - \\
- \bar{s} L \gamma_\mu d_L \cdot \bar{d} L \gamma_\mu d_L \quad (\{27\}, \Delta I = 3/2); \quad (5)
\]

\[
O_5 = \bar{s} L \gamma_\mu \lambda^a d_L (\sum_{q=u,d,s} \bar{q} R \gamma_\mu \lambda^a q_R) \quad (\{8\}, \Delta I = 1/2); \quad (6)
\]

\[
O_6 = \bar{s} L \gamma_\mu d_L (\sum_{q=u,d,s} \bar{q} R \gamma_\mu q_R) \quad (\{8\}, \Delta I = 1/2). \quad (7)
\]

This set is sufficient for calculation of the CP-even parts of the amplitudes under consideration. To calculate the CP-odd parts, it is necessary to add the so-called electroweak contributions originated by the operators \(O_7, O_8\):

\[
O_7 = \frac{3}{2} \bar{s} \gamma_\mu (1 + \gamma_5) d_L (\sum_{q=u,d,s} e_q \bar{q} R \gamma_\mu (1 - \gamma_5) q) \quad (\Delta I = 1/2, 3/2); \quad (8)
\]

\[
O_8 = -12 \sum_{q=u,d,s} e_q (\bar{s} L q_R)(\bar{q} R d_L), \quad e_q = \left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right) \quad (\Delta I = 1/2, 3/2). \quad (9)
\]

The coefficients \(c_{5-8}\) have the imaginary parts necessary for CP violation.

In the case of non-linear realization of chiral symmetry, the bosonization of these operators can be done using the relations [4]

\[
\bar{q}_j (1 + \gamma_5) q_k = -\frac{1}{\sqrt{2}} F_\pi r \left( U - \frac{1}{\Lambda^2} \partial^2 U \right)_{kj}, \quad (10)
\]

\[
\bar{q}_j \gamma_\mu (1 + \gamma_5) q_k = i \left[ (\partial_\mu U) U^\dagger - U \left( \partial_\mu U^\dagger \right) - \frac{r F_\pi}{\sqrt{2} \Lambda^2} \left( m (\partial_\mu U^\dagger) - (\partial_\mu U) m \right) \right]_{kj}, \quad (11)
\]

where

\[F_\pi \approx 93 \text{ MeV}, \quad \Lambda \approx 1 \text{ GeV}, \quad r = 2m^2/(m_u + m_d), \quad m = \text{diag}(m_u, m_d, m_s).\]

\[
U = \frac{F_\pi}{\sqrt{2}} \left( 1 + \frac{i \sqrt{2} \hat{\pi}}{F_\pi} - \frac{\hat{\pi}^2}{F_\pi^2} + a_3 \left( \frac{i \hat{\pi}}{\sqrt{2} F_\pi} \right)^3 + 2(a_3 - 1) \left( \frac{i \hat{\pi}}{\sqrt{2} F_\pi} \right)^4 + ... \right), \quad (12)
\]
where $a_3$ is an arbitrary number and

$$
\hat{\pi} = \begin{pmatrix}
\frac{\pi_0}{\sqrt{3}} + \frac{\pi_8}{\sqrt{6}} + \frac{\pi_3}{\sqrt{2}} & \frac{\pi^+}{\sqrt{3}} + \frac{\pi_8}{\sqrt{6}} - \frac{\pi_3}{\sqrt{2}} & K^+
\pi^- & \frac{\pi_0}{\sqrt{3}} - \frac{2\pi_3}{\sqrt{6}}
K^0 & \pi_0 - \frac{2\pi_3}{\sqrt{6}}
\end{pmatrix}
$$

(13)

The PCAC condition demands $a_3 = 0$ [5] and we adopt this condition, bearing in mind that, on mass shell, the values of the mesonic amplitudes are independent of $a_3$.

Using also the relations between matrices in the color space

$$
\delta_3^a \delta_3^\gamma = \frac{1}{3} \delta_3^a \delta_3^\gamma + \frac{1}{2} \lambda_3^a \lambda_3^\gamma
\lambda_3^a \lambda_3^\gamma = \frac{16}{9} \delta_3^a \delta_3^\gamma - \frac{1}{3} \lambda_3^a \lambda_3^\gamma
$$

and the Fierz transformation relation

$$
\bar{s} \gamma_\mu (1 + \gamma_5) d \cdot \bar{q} \gamma_\mu (1 - \gamma_5) q = -2 \bar{s}(1 - \gamma_5) q \cdot \bar{q}(1 + \gamma_5) d
$$

and representing $M(K \to 2\pi)$ in the form

$$
M(K^0_1 \to \pi^+ \pi^-) = A_0 e^{i\delta_0} - A_2 e^{i\delta_2},
$$

(14)

$$
M(K^0_1 \to \pi^0 \pi^0) = A_0 e^{i\delta_0} + 2A_2 e^{i\delta_2},
$$

(15)

$$
M(K^+ \to \pi^+ \pi^0) = -\frac{3}{2} A_2 e^{i\delta_2},
$$

(16)

where $\delta_0$ and $\delta_2$ are the $S$-wave shifts of $\pi\pi$ scattering in isotopic spin $I = 0, 2$ channels, we obtain

$$
A_0 = G_F F_\pi \sin \theta_C \cos \theta_C \frac{m_K^2 - m_\pi^2}{\sqrt{2}} [c_1 - c_2 - c_3 + \frac{32}{9} \beta (\text{Re}\tilde{c}_5 + i\text{Im}\tilde{c}_5)];
$$

(17)

$$
A_2 = G_F F_\pi \sin \theta_C \cos \theta_C \frac{m_K^2 - m_\pi^2}{\sqrt{2}} \cdot [c_4 + \frac{2}{3} \beta \Lambda^2 \text{Im}\tilde{c}_7 (m_K^2 - m_\pi^2)^{-1}]
$$

(18)

where

$$
\tilde{c}_5 = c_5 + \frac{3}{16} c_6, \quad \tilde{c}_7 = c_7 + 3c_8.
$$

$$
\beta = \frac{2m_\pi^4}{\Lambda^2 (m_u + m_d)^2}.
$$

(19)
The contributions from $\tilde{c}_7 O_7$ into $\text{Re}A_0$ and $\text{Im}A_0$ are small because $\tilde{c}_7/\tilde{c}_5$ is proportional to the electromagnetic constant $\alpha$ and we neglected these corrections. From data on widths of $K \to 2\pi$ decays we obtain

$$c_4 = 0.328; \quad c_1 - c_2 - c_3 + \frac{32}{9} \beta \text{Re}\tilde{c}_5 = -10.13. \quad (20)$$

At $c_1 - c_2 - c_3 = -2.89$ [3], [6] we obtain

$$\frac{32}{9} \beta \text{Re}\tilde{c}_5 = -7.24. \quad (21)$$

From the expression for $A_2$, it is seen that the contribution of the operators $O_7, O_8$ is enlarged by the factor $\Lambda^2/m_K^2$ in comparison with the other operator contribution. The reason is discussed in Appendix.

Using the general relation

$$\varepsilon' = ie^{i(\delta_2 - \delta_0)} \left[ \frac{\text{Im}A_0}{\text{Re}A_0} + \frac{\text{Im}A_2}{\text{Re}A_2} \right] \cdot \left| \frac{A_2}{A_0} \right| \quad (22)$$

and the experimental value $\varepsilon' = (3.78 \pm 0.38) \times 10^{-6}$, we come to the relation

$$- \frac{\text{Im}\tilde{c}_5}{\text{Re}\tilde{c}_5} \left( 1 - \Omega_{\eta,\eta'} + 24.36 \frac{\text{Im}\tilde{c}_7}{\text{Im}\tilde{c}_5} \right) = (1.63 \pm 0.16) \times 10^{-4}, \quad (23)$$

where $\Omega_{\eta,\eta'}$ takes into account the effects of $K^0 \to \pi^0\eta(\eta') \to \pi^0\pi^0$ transitions.

Introducing the notation

$$- \frac{\text{Im}\tilde{c}_5}{\text{Re}\tilde{c}_5} = x \frac{\text{Im}\lambda_t}{s_1}, \quad 24.36 \frac{\text{Im}\tilde{c}_7}{\text{Im}\tilde{c}_5} = -y \quad (24)$$

and using

$$(\text{Im}\lambda_t)/s_1 \approx s_2 s_3 \sin \delta = (1.2 \pm 0.2) \times 10^{-4} \quad (25)$$

we can write Eq.(23) for $\Omega_{\eta,\eta'} = 0.25 \pm 0.08$ in the form

$$x(1 - y) = 0.40 \times (1 \pm 0.22). \quad (26)$$

In the last two equations $s_i$ and $\delta$ are the parameters of CKM matrix. The Eq.(26) depends on the variables $x$ and $y$ representing the contribution of
QCD penguin and relative contribution of EWP, respectively. To move farther, we are enforced to apply to existing theoretical estimates of one of these variables.

In terms of notations in [8-10]

\[ y = \frac{\Pi_2}{\omega / \Pi_0 (1 - \Omega_{\eta,\eta'})}. \]  (27)

According to [8] \( y \approx 0.3 \) and hence \( x = 0.57 \pm 0.12 \). But \( \varepsilon' / \varepsilon = 2.2 \times 10^{-3} \) or by 30\% is larger than the experimental value.

In [10], the central value of \( y \) is \( y \approx 0.5 \) and, consequently, \( x = 0.80 \pm 0.18 \). This result looks as the reliable one. A very close result \( x = 0.71 \pm 0.27 \) can be derived from the result \( (\varepsilon' / \varepsilon)_{\text{EWP}} = (-12 \pm 3) \times 10^{-4} \) [11] comparing it with the experimental value \( (\varepsilon' / \varepsilon)_{\text{exp}} = (16.6 \pm 1.6) \times 10^{-4} \). But it should be noted that the previous estimates of \( x \) were rather different. In particular, according to [12] \( x = 1.4 \pm 0.28 \). An estimate of \( x \) can be extracted also from the papers [13-15] operating with different set of 4-quark operators \( Q_i \), where the combination \( C_6 Q_6 \) corresponds to our \( c_5 O_5 \). From the general representation

\[ C_6(\mu) = z_6(\mu) + (s_2^2 + s_2 s_3 \frac{c_2}{c_1 c_3} \cos \delta) \cdot y_6(\mu) - i s_2 s_3 \frac{c_2}{c_1 c_3} \sin \delta \cdot y_6(\mu) \]

and the calculated magnitudes of \( y_6 \) and \( z_6 \) we find for \( x \approx y_6 / z_6 \):

\[ x \approx 2 \quad \text{at } \Lambda_{\text{QCD}}^{(4)} = 0.35 \text{ GeV}, \mu = 0.8 \text{ GeV}, m_t = 176 \text{ GeV} \quad [13]; \quad (28) \]

\[ x = 2.8 \quad \text{at } \Lambda_{\text{MS}} = 0.3 \text{ GeV}, \mu = 1 \text{ GeV}, m_t = 130 \text{ GeV} \quad [14]; \quad (29) \]

\[ x = 5.5 \quad \text{at } \Lambda_{\text{QCD}}^{(4)} = 0.3 \text{ GeV}, \mu = 1 \text{ GeV}, m_t = 170 \text{ GeV} \quad [15]. \quad (30) \]

Such difference of the theoretical estimates of \( x \) makes very desirable an investigation of CP-effects in \( K^+ \to \pi^+ \pi^\mp \pi^\mp \) decays, where, contrary to \( K_L \to 2\pi \) decays, the EWP contributions increase CP effects.

3. Decay \( K^\pm \to \pi^\pm \pi^\mp \pi^\mp \)

To leading \( p^2 \) approximation

\[ M(K^+ \to \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)) = \kappa [1 + ia_{KM} + \frac{1}{2} g Y (1 + ib_{KM}) + ...], \quad (31) \]
where

\[ \kappa = G_F \sin \theta_C \cos \theta_C m_K^2 c_0 \left( 3 \sqrt{2} \right)^{-1}, \]  

\[ a_{KM} = \left[ \frac{32}{9} \beta \text{Im} \tilde{c}_5 + 4 \beta \text{Im} \tilde{c}_7 \left( \frac{3 \Lambda^2}{m_K^2} + 2 \right) \right] / c_0, \]  

\[ b_{KM} = \left[ \frac{32}{9} \beta \text{Im} \tilde{c}_5 + 8 \beta \text{Im} \tilde{c}_7 \right] / (c_0 + 9 c_4). \]

The last two quantities represent the imaginary parts produced by the Kobayashi-Maskawa phase \( \delta \).

\[ \frac{1}{2} g = -\frac{3 m^2}{2 m_K^2} (1 + 9 c_4 / c_0), \quad Y = (s_3 - s_0) / m^2, \]  

\[ c_0 = c_1 - c_2 - c_3 - c_4 + \frac{32}{9} \beta \text{Re} \tilde{c}_5 = -10.46. \]

As the field \( K^+ \) is the complex one and its phase is arbitrary, we can replace \( K^+ \) by \( K^+ (1 + i a_{KM}) \left( \sqrt{1 + a_{KM}^2} \right)^{-1} \). Then

\[ M(K^+ \to \pi^+ (p_1) \pi^+ (p_2) \pi^- (p_3)) = \kappa [1 + \frac{1}{2} g Y (1 + i (b_{KM} - a_{KM})) + ...]. \]  

Though this expression contains the imaginary CP-odd part, it does not lead to observable CP effects. Such effects arise due to interference between CP-odd imaginary part and the CP-even imaginary part produced by rescattering of the final pions. Then

\[ M(K^+ \to \pi^+ \pi^+ \pi^-) = \kappa [1 + i a + \frac{1}{2} g Y (1 + i b + i (b_{KM} - a_{KM})) + ...] \]

where \( a \) and \( b \) are corresponding CP-even imaginary parts of the amplitude. These parts can be estimated to leading approximation in momenta calculating the imaginary part of the two-pion loop diagrams with

\[ M(\pi^+ (r_2) \pi^- (r_3) \to \pi^+ (p_2) \pi^- (p_3)) = F^-_{\pi} [(p_2 + p_3)^2 + (r_2 - p_2)^2 - 2 m_{\pi}^2], \]  

\[ M(\pi^0 (r_2) \pi^0 (r_3) \to \pi^+ (p_2) \pi^- (p_3)) = F^-_{\pi} [(p_2 + p_3)^2 - m_\pi^2], \]  

\[ M(\pi^+ (r_1) \pi^+ (r_2) \to \pi^+ (p_1) \pi^+ (p_2)) = F^-_{\pi} [(r_1 - p_1)^2 + (r_1 - p_2)^2 - 2 m_\pi^2]. \]

Then we find:

\[ a = 0.12065, \quad b = 0.714. \]
Using the definition
\[ |M(K^+ \rightarrow \pi^+(p_1)\pi^+(p_2)\pi^+(p_3))|^2 \sim [1 + g^2Y + ...] \]
and the results of our calculation
\[ |M(K^+ \rightarrow \pi^+(p_1)\pi^+(p_2)\pi^+(p_3))|^2 \sim [1 + \frac{g}{1 + a^2}Y(1 + ab \pm a(b_K - a_K)) + ...] \]
we find
\[ R_g \equiv \frac{g^+ - g^-}{g^+ + g^-} = \frac{a(b_K - a_K)}{1 + ab}. \]
(41)

At the fixed above numerical values of the parameters and \( \Omega_{\eta,\eta'} = 0.25 \) we obtain to leading \( p^2 \) approximation
\[ (R_g)_{p^2} = 0.030 \frac{\text{Im} \tilde{c}_5}{\text{Re} \tilde{c}_5}(1 - 14.9 \frac{\text{Im} \tilde{c}_7}{\text{Im} \tilde{c}_5}) = -(2.44 \pm 0.44) \times 10^{-5}x \left(1 - \frac{0.13 \pm 0.03}{x}\right). \]
(42)

4. The role of \( p^4 \) and other corrections

The corrections to the result obtained in the conventional chiral theory up to leading \( p^2 \) approximation are of two kinds. The first kind corrections are connected with a necessity to get explanation of the observed enlargement of \( S \)-wave \( I = 0 \) \( \pi\pi \) amplitude. The corrections of the second kind are the \( p^4 \) corrections. As it was argued in [16], [17], the corrections of both kinds be properly estimated in the framework of special linear \( U(3)_L \otimes U(3)_R \) \( \sigma \) model with broken chiral symmetry. The above mentioned enlargement of \( S \) wave in this model is originated by mixing between the \( \bar{q}q \) state and the gluonic state \( (G^a_{\mu\nu})^2 \) states. In such a model
\[ U = \hat{\sigma} + i\hat{\pi} \]
where \( \hat{\sigma} \) is \( 3 \times 3 \) matrix of scalar partners of the mesons of pseudoscalar nonet. The relations between diquark combinations and spinless fields are as given by eqs.(10), (11), but without the terms proportional to \( \Lambda^{-2} \). Such contributions in \( \sigma \) model appear from an expansion of the propagators of the intermediate scalar mesons. The parameter \( \Lambda^2 \) in this model is equal to difference \( m_{a_{(980)}}^2 - m_{\pi}^2 \). The strength of mixing between the isosinglet \( \sigma \) meson and corresponding gluonic state is characterized by the parameter \( \xi \).
If the $p^2$ approximation gives

$$(\kappa)_{p^2} = 1.495 \times 10^{-6}, \quad (g)_{p^2} = -0.172$$

instead of

$$(\kappa)_{\text{exp}} = 1.92 \times 10^{-6}, \quad (g)_{\text{exp}} = -0.2154 \pm 0.0035,$$

the corrected values of these CP-even parameters of $K^+ \rightarrow \pi^+\pi^+\pi^-$ amplitude are closer or practically equal to the experimental ones [16]:

$$(\kappa)_{(p^2+p^4; \xi=-0.225)} = 1.73 \times 10^{-6}, \quad (g)_{(p^2+p^4; \xi=-0.225)} = -0.21.$$ (45)

More information on the parameter $\xi$ can be found in [16],[17]. The expressions for the corrected $\pi\pi \rightarrow \pi\pi$ amplitudes are presented in [17].

Calculating the CP-even imaginary part of the $K^\pm \rightarrow \pi^\pm\pi^\mp\pi^\mp$ amplitude originated by two-pion intermediate states, we obtain

$$a(p^2+p^4; \xi = -0.225) = 0.16265,$$

$$b(p^2+p^4; \xi = -0.225) = 0.762.$$ (47)

An estimate of the parameter $a$ can be obtained also without any calculations using the experimental data on the phase shifts of $\pi\pi$ scattering $\delta^0_0, \delta^2_0, \delta^4_1$. According to definition (38) $a$ is a phase at $s_3 = s_0$. The mean value of the squared energy of $\pi^+\pi^-$ system is

$$\frac{1}{2}[(p_1 + p_3)^2 + (p_2 + p_3)^2] = s_0 + \frac{s_0 - s_3}{2}.$$ (48)

Consequently, $a$ is a phase shift of $\pi^+\pi^-$ scattering at $\sqrt{s} = \sqrt{s_0}$. But the only significant phase shift at $\sqrt{s} = \sqrt{s_0}$ is $\delta^0_0$. The rest phase shifts are very small: $|\delta^2_0(s_0)| < 1.8^\circ$ and $\delta^4_1(s_0) < 0.3^\circ$ [18]. Then, according to Eq.(38), $a \approx \tan \delta^0_0(s_0)$, or $a = 0.13 \pm 0.05$, if $\delta^0_0(s_0) = (7.50 \pm 2.85)^\circ$ [19] and $a = 0.148 \pm 0.018$, if $\delta^0_0(s_0) = (8.4 \pm 1.0)^\circ$ [20]. These results coincide inside the error bars with the result (46). The corrected magnitude of $R_g$ is

$$(R_g)_{(p^2+p^4; \xi=-0.225)} = 0.039 \frac{\text{Im} \tilde{c}_5}{\text{Re} \tilde{c}_5} \left(1 - 11.95 \frac{\text{Im} \tilde{c}_7}{\text{Re} \tilde{c}_7}\right) =$$

$$=-(3.0 \pm 0.5) \times 10^{-5} \times \left(1 - 0.11 \pm 0.025\right).$$ (49)

This result is by 23% larger in absolute magnitude than that calculated in the leading approximation. Therefore, we come to conclusion that the corrections to the result obtained in the framework of conventional chiral theory to the leading approximation are not negligible (23%), but not so large, as it was declared in [21].
5. Conclusion

From Eqs.(22), (26) and (48), it follows that EWP contributions diminish $\varepsilon'/\varepsilon$ and increase $R_g$. The EWP corrections cancell one half of the QCD penguin contribution into $\varepsilon'/\varepsilon$ at $x = 0.8$ and cancell 80% of QCD penguin contribution at $x = 2$. In both cases $\varepsilon'/\varepsilon$ is the same.

In the case of $K^{\pm} \to 3\pi$ decays, the direct influence of EWP corrections themselves on CP effects is not so crucial as in $K_L \to 2\pi$ decays. But if a cancellation between the contribution of QCD and electroweak penguins in $\varepsilon'/\varepsilon$ is large, the factor $x$ in Eq.(48) is also larger than 1. So, for $x = 2$, the predicted $R_g$ must be 2.5 times larger than at $x = 0.8$.

Therefore, measuring $R_g$, one obtains a possibility to determine the true relation between QCD and EWP contributions into CP violation in kaon decays.

Appendix

Here we explain why an expansion of the amplitudes originated by electroweak penguin diagrams begins from the term, independent of momenta and masses of the pseudoscalar mesons.

The operators $O_{7,8}$ can be expressed in terms of colorless diquark combinations in the form

$$O_7 = -\bar{s}(1 - \gamma_5)u \cdot \bar{u}(1 + \gamma_5)d - \frac{3}{8}O_5, \quad O_8 = 3O_7. \quad (A.1)$$

Using eq.(10), we find

$$O_7 = -\frac{F_{\pi}^2r^2}{2}U_{21}U_{13}^* + (\text{terms proportional to } p^2_i(m_i^2)). \quad (A.2)$$

Omitting the terms proportional to derivatives of $U$ and taking in Eq.(12) $a_3 = 0$, we find for the parity-even transitions

$$(O_7)^{P-\text{even}} = -\frac{F_{\pi}^2r^2}{2}\left\{\pi^- K^+ + \frac{1}{2F_{\pi}^2}\left[\pi^- (\frac{2\pi}{\sqrt{3}} + \frac{2\pi_8}{\sqrt{6}}) + K^0 K^-\right]ight\} \times \left[K^+ (\frac{2\pi}{\sqrt{3}} - \frac{\pi_8}{\sqrt{6}} + \frac{\pi_8}{\sqrt{2}}) + \pi^+ K^0\right] + \ldots\} \quad (A.3)$$

This expression does not contain the direct contribution to $K^+ \to 3\pi$ decays, but thanks to the term $\pi^- K^+$, the independent of $p^2_i(m_i^2)$ part of $K^+ \to 3\pi$
amplitude arises. In $p^2$ approximation

$$< \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)|O_7|K^+(k)> = -\frac{F_2^2 r^2}{2(m_K^2 - m^2)} \left[ \frac{s_1 + s_2 - 2m^2}{F_\pi^2} - \frac{s_1 + s_2 - m^2 - m^2_K}{F_\pi^2} \right] = -\frac{r^2}{2}, \quad (A.4)$$

where $s_i = (k - p_i)^2$ and the first term in the brackets describes the $\pi^+(k) \to \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)$ transition. The second term describes the transitions $K^+(k) \to K^+(p_{1,2})\pi^+(p_{2,1})\pi^-(p_3)$.

Therefore, the operator $O_7$ violates the rule, according to which an expansion of the mesonic amplitudes begins from the terms proportional to $p^2_i(m_i^2)$.

It may seem to one that removing the non-diagonal term $-\frac{r^2}{2}(K^- - \pi^-)K^+$ from the effective Lagrangian by redefinition of $K^+$ and $\pi^-$ fields [22], the problem with the constant contribution could be solved. But this is not so.

In our case, the mass part of the effective Lagrangian contains, in particular, the combination

$$-m_\pi^2 \pi^+\pi^- - m_K^2 K^+K^- - \frac{F_2^2 r^2}{2}(\gamma K^+\pi^- + \gamma^* K^-\pi^+), \quad (A.5)$$

where $\gamma = \sqrt{2}G_F \sin \theta_C \cos \theta_C c_7$. The transformations

$$\pi^- \to \pi^- + \beta K^-, \quad K^+ \to K^+ - \beta \pi^+, \quad K^- \to K^- - \beta^* \pi^- \quad \quad (A.6)$$

with

$$\beta = \frac{\gamma^* F_2^2 r^2}{2(m_K^2 - m^2)} \quad (A.7)$$

remove the non-diagonal terms in the linear in $\gamma$ approximation. But the effective Lagrangian of strong interaction generates the sum of the amplitudes

$$< \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)|\pi^+(k)> + < K^+(p_1)\pi^+(p_2)\pi^-(p_3)|K^+(k)> + < K^+(p_2)\pi^+(p_1)\pi^-(p_3)|K^+(k)>$$

which after the transformation (A.6) generates the amplitude

$$< \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)|O_7|K^+(k)> =$$

$$= -\beta \frac{r^2}{2}\left[ \frac{s_1 + s_2 - 2m^2}{F_\pi^2} - \frac{s_1 + s_2 - m^2 - m^2_K}{F_\pi^2} \right] = -\frac{r^2}{2}. \quad (A.9)$$
We have reproduced the result (A.4). The contribution of the operators $O_{7,8}$ to the leading approximation does not depend on $p_i^2(m_i^2)$.

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References
1. S.Bertolini et al., Nucl.Phys.B 514, 93 (1998).
2. T.Hambye et al., Nucl.Phys.B 564, 391 (2000).
3. M.A.Shifman, A.I.Vainshtein and V.I.Zakharov, Zh.Eksp.i Teor.Fiz. 72, 1277 (1977).
4. W.A.Bardeen, A.J.Buras and J.-M.Gerard, Nucl.Phys.B 293, 787 (1987).
5. J.Cronin, Phys.Rev. 161, 1483 (1967).
6. L.B.Okun, Leptons and Quarks (North-Holland Publ.Co. 1982) pp.315,323.
7. A.Ali and D.London, Eur.Phys.J. C 18, 665 (2001).
8. S.Bertolini, J.O.Eeg and M.Fabbrichesi, Phys.Rev. D 63, 056009 (2001).
9. A.J.Buras and J.-M.Gerard, Phys. Lett. B 517, 129 (2001).
10. T.Hambye, S.Peris and E.de Rafael, [hep-ph/0305104] v. 2.
11. J.F.Donoghue and E.Golovich, Phys.Lett. B 478, 172 (2000).
12. M.B.Voloshin, preprint ITEP-22. (Moscow 1981).
13. S.Bertolini et al., preprint SISSA 102/95/EP.
14. A.J.Buras, M.Jamin and M.Lautenbacher, Nucl.Phys.B 408, 209 (1993).
15. S.Bertolini, J.O.Eeg and M.Fabbrichesi, Nucl.Phys. B 449, 197 (1995).
16. E.P.Shabalin, Nucl.Phys.B 409, 87 (1993).
17. E.P.Shabalin, Phys. At. Nucl. 61, 1372 (1998).
18. E.P.Shabalin, Phys.At.Nucl. 63, 594 (2000).
19. L.Rosselet et al., Phys.Rev.D 15, 574 (1977).
20. S.Pislak et al., Phys.Rev.Lett. 87, 221801 (2001).
21. A.A.Bel’kov et al., Phys.Lett.B 300, 283, (1993).
22. G.Feinberg, P.K.Kabir, S.Weinberg, Phys.Rev.Lett. 3, 527 (1959).