A possible mass distribution of primordial black holes implied by LIGO-Virgo

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Received February 15, 2021
Revised March 15, 2021
Accepted March 17, 2021
Published April 20, 2021

Abstract. The LIGO-Virgo Collaboration has so far detected around 90 black holes, some of which have masses larger than what were expected from the collapse of stars. The mass distribution of LIGO-Virgo black holes appears to have a peak at $\sim 30M_{\odot}$ and two tails on the ends. By assuming that they all have a primordial origin, we analyze the GWTC-1 (O1&O2) and GWTC-2 (O3a) datasets by performing maximum likelihood estimation on a broken power law mass function $f(m)$, with the result $f \propto m^{1.2}$ for $m < 35M_{\odot}$ and $f \propto m^{-4}$ for $m > 35M_{\odot}$. This appears to behave better than the popular log-normal mass function. Surprisingly, such a simple and unique distribution can be realized in our previously proposed mechanism of PBH formation, where the black holes are formed by vacuum bubbles that nucleate during inflation via quantum tunneling. Moreover, this mass distribution can also provide an explanation to supermassive black holes formed at high redshifts.

Keywords: gravitational waves / sources, primordial black holes, gravitational wave detectors, Wormholes

ArXiv ePrint: 2101.11098
1 Introduction

Over the past few years, the LIGO-Virgo Collaboration has detected gravitational waves emitted by about 50 inspiraling and merging black hole binaries \cite{1,2}, and many more events are anticipated in the near future. It is intriguing that most of the black holes have masses \( \sim 30M_\odot \), which certainly provides indicative information of the mass distribution of black holes in the universe.

The origin of these black holes is so far unknown. While the possibility that they are ordinary astrophysical black holes from stellar collapse (possibly from different channels) are under active investigations \cite{3–6}, a fascinating speculation is that LIGO-Virgo has detected primordial black holes (PBHs) \cite{7–9}. PBHs are hypothetical black holes formed by deviations from density homogeneity in the early universe, before any large scale structures and galaxies. Their masses can range from the Planck mass (\( \sim 10^{-5} \) g) to many orders of magnitude larger than the solar mass (\( M_\odot \sim 10^{33} \) g). After the release of LIGO-Virgo results, much effort has been dedicated to constrain PBHs and the role they play in dark matter \cite{9–13}. It is now generally recognized that they are unlikely to constitute all dark matter due to the small merger rate inferred by LIGO-Virgo.

It is also of great interest to constrain particular PBH mass distributions with the LIGO-Virgo results \cite{10,14–17}. Among them the log-normal mass function \cite{18} is the most popular because it is a good approximation for various PBH mechanisms \cite{19–21}. However, if all LIGO-Virgo black holes were PBHs, the peak of the log-normal function should be at \( \sim 20M_\odot \), which is incompatible with what was observed. The possibility of LIGO-Virgo black holes being a mixture of two populations of black holes was recently considered in ref. \cite{22}, which concludes that astrophysical black holes that dominate the mass range \( m \lesssim 15M_\odot \) together with PBHs given by a log-normal (or critical collapse) mass function are much more probable than PBHs only.
In this paper, inspired by ref. [22], we perform a maximum likelihood estimation to fit the LIGO-Virgo datasets with a broken power law only. More sophisticated techniques (e.g., Bayesian analysis [23, 24]) are available, but we believe likelihood analysis should suffice for our purposes, given the limited number of detected events and uncertainties in PBH formation. We shall also neglect some details in analyzing the formation of PBHs and binaries, as long as they are not expected to affect the results by orders of magnitude. For instance, following ref. [22], we shall not consider the impact from black hole’s spin and mass accretion.

Apart from the single peak at $\sim 30M_\odot$ in the distribution of LIGO-Virgo black holes, our investigation of the broken power law mass spectrum is mainly motivated by a PBH mechanism we proposed in recent years. In a series of works [25–27], we studied PBHs formed by vacuum bubbles that possibly nucleate during inflation. Depending on its size after inflation ends, a bubble will turn into a black hole in the either subcritical or supercritical regime, and the mass distributions of black holes in these two regimes could obey different power laws. We would like to know if the best-fit parameters from likelihood analysis of LIGO-Virgo data is compatible with this mechanism. Surprisingly, the broken power law mass function implied by the LIGO-Virgo black holes can indeed be realized in our model.

The rest of the paper is organized as follows. In section 2, we will first discuss the merger rate of PBHs and the detection probability of LIGO-Virgo, and then apply maximum likelihood estimation to find the best-fit parameters for the broken power law mass function. In section 3, we will study the mechanism of PBH formation from primordial bubbles and how could produce the LIGO-Virgo black holes. Conclusions will be summarized and discussed in section 4. We set $c = \hbar = G = 1$ throughout the paper.

2 PBHs and LIGO-Virgo events

It will be assumed that all black holes detected by LIGO-Virgo are primordial. They are formed during the radiation era and behave like dark matter, with their number and mass density diluted by Hubble expansion. Two neighboring black holes may collide as their gravitational attraction defeats the Hubble stretching before the radiation-dust equality. Disturbance from the surrounding environment, a typical example being a third nearby black hole exerting a tidal torque, may impede the head-on collision, leading to the formation of an inspiraling binary. If the coalescence time of the binary is comparable to the age of the universe, gravitational waves from the merger could possibly be detected when they reach the earth. Roughly speaking, whether a merger event can be heard depends on the detector’s sensitivity, the masses of the two black holes (source masses), the time when the merger took place (source redshift), and the sky location and orientation of the binary system. In order to estimate how often a merger event can be recorded, we also need the merger rate of the binaries, which is determined by the underlying mechanism of PBH formation.

2.1 PBH mass function and merger rate

We characterize the mass distribution of PBHs by mass function $\psi(m)$, defined by

$$\psi(m) = \frac{m}{\rho_{\text{CDM}}} \frac{dn}{dm}. \quad (2.1)$$

Here $m$ is the black hole mass, $dn$ is the number density of black holes within the mass range $(m, m + dm)$, and $\rho_{\text{CDM}}$ is the energy density of cold dark matter. Since black holes and
dark matter are diluted by cosmic expansion in the same way, $\psi(m)$ is a constant over time. Integrating $\psi(m)$ gives the total fraction of dark matter in PBHs:

$$f_{\text{PBH}} \equiv \frac{\rho_{\text{PBH}}}{\rho_{\text{CDM}}} = \int \psi(m) dm,$$

(2.2)

where $\rho_{\text{PBH}} = \int m dm$ is the energy density of PBHs. If all dark matter is made of PBHs, we have $f_{\text{PBH}} = 1$.

Another function often used to describe the PBH mass distribution is

$$f(m) \equiv m \psi(m),$$

(2.3)

which can be interpreted as the fraction of dark matter in PBHs at $m$ within the mass range $\Delta m \sim m$. This function is particularly useful if the spectrum is relatively broad.

The PBH merger rate in the early universe is well studied in the literature [9–13]. In this paper we take the formula of differential merger rate from ref. [14]:

$$dR \approx \frac{1.6 \times 10^6}{\text{Gpc}^3 \text{yr}^{-1}} f_{\text{PBH}}^{-21/37} T f_{\text{PBH}}^{-34/37} M^{-32/37} \eta^{-34/37} S(\psi, f_{\text{PBH}}, M) \psi(m_1) \psi(m_2) dm_1 dm_2.$$

(2.4)

Integrating $dR$ over all masses gives the number of merger per Gpc$^3$ per year. Here $m_1$ and $m_2$ are the two masses in the binary, $M \equiv m_1 + m_2$ and $\eta \equiv m_1 m_2 / M^2$; $t$ is the time when the merger occurs, and $t_0$ is the present time; $S$ is a suppression factor accounting for the effects from other matter components, including nearby black holes, and $S = \mathcal{O}(1)$ for a not particularly wide spectrum. In our calculations we shall approximate $S$ by a simplified expression given in ref. [22], but setting $S = 1$ would not lead to much difference.

To roughly estimate the largest $f_{\text{PBH}}$ we can have from the LIGO-Virgo results, we can assume a monochromatic mass function with $\psi(m) = f_{\text{PBH}} \delta(m - 30 M_\odot)$, i.e., all PBHs have mass $30 M_\odot$. Setting $t = t_0$ and $S = 1$, we have

$$\int dR \sim 10^5 f_{\text{PBH}}^{53/37} \text{Gpc}^{-3} \text{yr}^{-1}.$$

(2.5)

The merger rate implied by LIGO-Virgo is $\mathcal{O}(10–100) \text{Gpc}^{-3} \text{yr}^{-1}$, which gives $f_{\text{PBH}} \sim 10^{-3}$. Therefore, if all PBHs are of masses around $30 M_\odot$, they can at most contribute to 0.1% of the dark matter.

The distribution of LIGO-Virgo black holes is clearly not monochromatic. In this paper we shall consider the 10 black hole merger events from the GWTC-1 catalog [1] and 34 events from GWTC-2 (discarding GW190719 and GW190909) [2], so there are 88 black holes in our dataset. In figure 1, we depict the black hole numbers in different mass ranges. The distribution appears to have a peak at $m_\ast \sim 30 M_\odot$ and two tails on the ends, with more black holes having masses less than $m_\ast$ than those on the other end. The simplest mass function that can be conceived is a broken power law,

$$\psi(m) = \frac{f_{\text{PBH}}}{m_\ast} (\alpha_1 - \alpha_2)^{-1} \left\{ \begin{array}{ll} (m/m_\ast)^{\alpha_1-1}, & m < m_\ast, \\ (m/m_\ast)^{\alpha_2-1}, & m > m_\ast, \end{array} \right.$$  

(2.6)

which satisfies $\int \psi dm = f_{\text{PBH}}$. We further assume $\alpha_1 > 0$ and $\alpha_2 < 0$ such that $f(m) = m \psi(m)$ has a peak at $m_\ast$, which means most contribution to the PBH energy density comes from black holes with masses around $m_\ast$. In the next subsection, we will analyze the LIGO-Virgo results and find the best-fit parameters for (2.6).
Figure 1. Numbers of detected LIGO-Virgo black holes in different mass ranges, rescaled such that the areas add up to 1. There are 88 black holes in total. The black hole masses are simply taken to be the median values reported in refs. [1, 2]. The distribution certainly depends on the resolution of mass and the choice here is somehow arbitrary. Considering that the measurement uncertainties are $\mathcal{O}(10)M_\odot$ and that a smaller resolution would give a less clear pattern due to the insufficient number of detected events, we believe $\Delta m = 10M_\odot$ is a plausible choice.

2.2 Maximum likelihood estimation

Given the differential merger rate found by eq. (2.4), the expected number of merger signals reaching the earth per unit time within the ranges $(m_1, m_1 + dm_1)$, $(m_2, m_2 + dm_2)$, and $(z, z + dz)$ can be evaluated as [28]

$$dN(m_1, m_2, z) = \frac{1}{1+z} \frac{dV_c}{dz} \frac{dR}{dm_1 dm_2} dm_1 dm_2 dz,$$

(2.7)

where $(1+z)^{-1}$ accounts for the time difference between the source frame and the detector frame, and $V_c$ is the comoving Hubble volume. For later convenience, we define

$$\Lambda(m_1, m_2, z) \equiv \frac{1}{1+z} \frac{dV_c}{dz} \frac{dR}{dm_1 dm_2}$$

(2.8)

In addition, we ought to take into account the fact that not all merger events can be observed by LIGO-Virgo. The detection probability of an event depends on the sensitivity of the instruments, the waveform of the signal as well as the extrinsic parameters of the binary system, i.e., its sky location and orientation. Integrating out the extrinsic parameters for a given detector gives its detection probability $p_{\text{det}}(r)$, with $r \equiv \rho_c/\rho(m_1, m_2, z)$. Here $\rho_c$ is the threshold signal-to-noise ratio above which a signal can be detected, usually taken as $\rho_c = 8$; and $\rho$ is the signal-to-noise ratio for a merger located directly above the detector. $\rho$ is defined by

$$\rho(m_1, m_2, z) = 2 \sqrt{\int \frac{|\tilde{h}(f)|^2}{S_n(f)} df},$$

(2.9)

where $\tilde{h}(f)$ is the Fourier transform of the signal, and $S_n(f)$ is the power spectrum of the detector’s strain noise. In this work, $\tilde{h}(f)$ and $S_n(f)$ are taken from refs. [22, 29] and
where each event is denoted by a superscript mass function. For the GWTC-1 and GWTC-2 datasets, we will be able to attain the constraints on a PBH event \[23, 30\].

The likelihood of a single detected event with source masses \(m_{1,o}, m_{2,o}\) and redshift \(z_o\) is

\[
p_o \propto \frac{p_{\text{det}}(m_{1,o}, m_{2,o}, z_o)\Lambda(m_{1,o}, m_{2,o}, z_o)}{\int p_{\text{det}}(m_1, m_2, z)dN}.
\]

Here we have made an assumption that the source masses and redshift can be perfectly determined by detection. In practice, we take \(m_{1,o}, m_{2,o}\) and \(z_o\) to be the median values of the posterior samples reported by LIGO-Virgo \[1, 2\]. In reality, their values are given with probability distributions resulting from measurement uncertainties.\(^1\) Note that the denominator in eq. (2.12) is \(\propto N_e\). Then the likelihood of all black hole merger events detected by LIGO-Virgo is

\[
\mathcal{L} = p_{\text{Poisson}} \prod_{i=1}^{N_o} p_i^o \propto e^{-N_e} \prod_{i=1}^{N_o} p_{\text{det}}(m_{1,i,o}, m_{2,i,o}, z_{i,o})\Lambda(m_{1,i,o}, m_{2,i,o}, z_{i,o}),
\]

where each event is denoted by a superscript \(i\). By maximizing the log-likelihood \(\ln \mathcal{L}\) with the GWTC-1 and GWTC-2 datasets, we will be able to attain the constraints on a PBH mass function.

There are four parameters in a broken power law (2.6): \(\alpha_1, \alpha_2, m_s\) and \(f_{\text{PBH}}\). Scanning through the relevant part of the parameter space, we found the largest log-likelihood \(\ln \mathcal{L}_{\text{max}}\) at

\[\alpha_1 \approx 1.2, \ \alpha_2 \approx -4, \ m_s \approx 35M_\odot, \ f_{\text{PBH}} \approx 0.0013.\] (2.14)

These results are also partly shown in figure 2. The power law mass function from different PBH mechanisms was discussed in, e.g., refs. \[26, 31, 32\], which suggested \(f(m) \propto m^{-1/2}\) (for large masses). These models are obviously disfavored by LIGO-Virgo.

As a comparison, we did the same analysis for the log-normal mass function

\[
\psi(m) = \frac{f_{\text{PBH}}}{m\sqrt{2\pi\sigma}} \exp \left[ -\frac{\ln^2(m/m_c)}{2\sigma^2} \right].
\] (2.15)

\(^1\)In order to account for the measurement uncertainties, the standard approach to calculate \(p_o\) is to replace the numerator in eq. (2.12) by \(\Lambda(m_{1,o}, m_{2,o}, z_o)/\pi(m_{1,o}, m_{2,o}, z_o)\), where \(m_{1,o}, m_{2,o}\) and \(z_o\) are the posterior samples, \(\pi\) is the corresponding source prior, and the brackets denote an average over all samples of that event \[23, 30\].
The most probable parameters are
\[ m_c \approx 20M_\odot, \sigma \approx 0.6, \ f_{\text{PBH}} \approx 0.0013, \]  

which are consistent with the results found in refs. [10, 14, 22, 24]. The difference between the maximum log-likelihood of broken power law and that of log-normal is \( \ln(\mathcal{L}_{\text{BPL}}/\mathcal{L}_{\text{LN}}) \approx 5 \). However, considering that we are evaluating the likelihood ratio with two different models, and that the broken power law has one more free parameter, we cannot say with much confidence that the broken power law is a better model at the moment in explaining the LIGO-Virgo results.

Nevertheless, using the Metropolis-Hastings algorithm, we have drawn 50000 random samples from the probability distribution \( p_{\text{det}}(m_1, m_2, z) dN \) with the best-fit parameters, both for the broken power law and the log-normal model, and compared the resulting mass distributions with that from LIGO-Virgo black holes. As we can see from figure 3, although it is difficult to compare models under likelihood analysis, the broken power law mass function appears to be a better fit than the log-normal.

In the next section we will discuss a physical mechanism where PBHs could form with such a simple distribution.

3 PBHs from primordial bubbles

In a series of works [25–27], we studied in detail a mechanism of PBH formation where black holes are formed by vacuum bubbles that nucleate during inflation. After inflation ends, a bubble will eventually turn into a black hole in the either subcritical or supercritical regime. PBHs formed in the two regimes might obey different power laws, and the transition region could just be the peak at \( \sim 30M_\odot \) seen by LIGO-Virgo. We shall first briefly describe how these black holes are formed, and then discuss how they are constrained by observations including LIGO-Virgo.

3.1 PBH formation

Vacuum bubbles could nucleate during inflation via quantum tunneling if there is a (positive) “true” vacuum near the inflationary (quasi-de Sitter) vacuum in a multidimensional field
potential. The bubble interior has an energy density \( \rho_b \) smaller than the inflationary energy density \( \rho_i \). A typical bubble expands rapidly, almost at the speed of light. After inflation ends, inflatons outside the bubble turn into radiation with density \( \rho_r \approx \rho_i \), and the bubble will run into the surrounding radiation fluid with a huge Lorentz factor. The bubble continues to grow, but will eventually come to a halt with respect to the Hubble flow, because all the forces acting on the bubble wall, including the interior vacuum pressure, exterior radiation pressure, and the wall’s surface tension, point inwards. A bubble is called “subcritical” if it simply collapses into a black hole after reentering the Hubble horizon. A supercritical bubble, on the other hand, will inflate without bound due to the repulsive vacuum inside. However, instead of consuming our universe, the bubble grows into a baby universe, which is connected to us by a wormhole. For an exterior observer, the wormhole is a spherical object that will eventually turn into a black hole as the “throat” pinches off. Once the relation between the black hole mass in these two regimes and the bubble’s size at the end of inflation is determined, we are able to obtain the mass spectrum of these black holes from the bubbles’ size distribution.

In ref. [26], we considered an ideal scenario where radiation outside the bubble can be completely reflected by the bubble wall, which implies strong interactions between the bubble field and the standard model particles. In this setting, the mass function was found to be \( f \propto m^{-1/2} \) in the supercritical regime. As we know from the previous section, such a distribution is disfavored by the LIGO-Virgo results.

In ref. [27], we studied the other extreme possibility, where the bubble wall is transparent, and radiation can freely flow inside. In this case, the trajectory of the bubble wall before it ceases to grow with respect to the Hubble flow can be estimated by assuming an FRW metric dominated by radiation outside the bubble and matching the spacetimes on two sides of the wall. Let \( r \) be the bubble’s comoving radius. The equation of motion of the bubble wall can be found to be

\[
\ddot{r} + \left(4 - 3a^2r^2\right)H\dot{r} + \frac{2}{a^2r} \left(1 - a^2r^2\right) + \left(\frac{\rho_b}{\sigma} + 6\pi\sigma\right) \frac{(1 - a^2r^2)^{3/2}}{a} = 0, \quad (3.1)
\]

Figure 3. Mass distributions of black holes from LIGO-Virgo (blue), the best-fit broken power law (orange) and log-normal (green).
where the overdot represents the derivative with respect to cosmic time $t$, $H \equiv \dot{a}/a = (2t)^{-1}$ is the Hubble parameter in the exterior FRW universe, and $\sigma$ is the surface tension of the bubble wall. Let $t_i$ be the time when inflation ends, then the scale factor is defined by $a = (t/t_i)^{1/2}$. The “initial” conditions of eq. (3.1) is $r(t_i) = r_i$ and $\dot{r}(t_i) = \left(1 - \gamma_i^{-2}\right)^{1/2}$, where $r_i$ can be smaller or much larger than the Hubble horizon $H_i^{-1}$ at $t_i$, and $\gamma_i$ (regarded as a free parameter) is the Lorentz factor of the bubble wall with respect to the Hubble flow at $t_i$.

Assuming a huge $\gamma_i$, the trajectory of the wall can be approximated by $ar \approx 1$. Let $t_s$ be the time when the wall comes to a stop with respect to the Hubble flow. The bubble’s comoving radius at $t_s$ can then be estimated as

$$r_s \equiv r(t_s) \sim r_i + (a_s - 1) H_i^{-1},$$

(3.2)

where $a_s \equiv (t_s/t_i)^{1/2}$. We are particularly interested in the case where $r_s \gg r_i$. In this case, $r_s \sim a_s H_i^{-1}$, and hence the bubble’s physical radius at $t_s$ can be estimated as

$$R_s \equiv a_s r_s \sim a_s^2 H_i^{-1} \sim t_s,$$

(3.3)

which means the bubble size is comparable to the Hubble horizon at $t_s$.

### 3.1.1 Subcritical bubble

A bubble with sufficiently small $r_i$ would continue to expand after $t_s$, until it reaches a maximum physical size $R_{\text{max}}$. It will then shrink and eventually collapse into a black hole. By eq. (3.3), the bubble reenters the horizon at $\sim t_s$, then its evolution will no longer be affected by Hubble expansion significantly, which means the mass of the bubble itself (excluding radiation inside) will almost conserve after then. When the bubble radius reaches $R_{\text{max}}$, the kinetic energy of the bubble wall vanishes. Furthermore, since the bubble is expected to shrink upon horizon reentry, we have $R_{\text{max}} \sim R_s$. Hence, the mass of the resulting black hole from a subcritical bubble is

$$m \sim \frac{4}{3} \pi \rho_b R_s^3 + 4 \pi \sigma R_s^2 - 8 \pi^2 \sigma^2 R_s^3.$$

(3.4)

Here the three terms can be understood as the volume energy, the surface energy and the surface binding energy, respectively.

As $r_i$ is increased to a critical value, $R_{\text{max}}$ can no longer be reached, because a sufficiently large bubble dominated by its interior vacuum would inflate. When this happens, we enter the supercritical regime.

### 3.1.2 Supercritical bubble

Due to the third term on the left hand side in eq. (3.1), a bubble with a larger $r_i$ experiences a smaller “friction” as it grows, and therefore has a larger $t_s$. By the time $t_s$, the bubble’s vacuum density $\rho_b$ could become even larger than the exterior radiation density $\rho_r(t_s)$ (note that $\rho_r(t_i) = \rho_i > \rho_b$), i.e.,

$$\rho_r(t_s) = \rho_i a_s^{-4} \lesssim \rho_b.$$

(3.5)

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2See discussion in section 4.
Then by eq. (3.3),
\[ R_s \sim a_s^2 H_i^{-1} = \left( \frac{8\pi}{3} \rho_i a_s^{-4} \right)^{-1/2} \gtrsim \left( \frac{8\pi}{3} \rho_b \right)^{-1/2} = H_b^{-1}. \]  

(3.6)

Here $H_b^{-1}$ is the de Sitter horizon associated with the bubble vacuum. $R_s \gtrsim H_b^{-1}$ then implies that the bubble would inflate before $t_s$ and thus create a wormhole, which later turns into a black hole. By the estimate in ref. [27], the black hole mass in this supercritical regime is
\[ m \sim H_i \left( r_s - c_s a_s H_i^{-1} \right)^2, \]

(3.7)

where $c_s$ is the speed of sound in the radiation fluid, and the value of $a_s$ is determined by solving eq. (3.1). By the estimate in eq. (3.3), we have $m \sim H_i r_s^2 \sim a_s^2 H_i^{-1} \approx t_s$, which is the horizon mass at $t_s$.

Therefore, by numerically integrating eq. (3.1) till $\dot{r} = 0$, we are able to find out the black hole masses in the subcritical and the supercritical regimes. These two regimes are expected to be connected by a region where the black hole mass transits smoothly from one regime to the other. In practice, we evaluate both (3.4) and (3.7), and take the smaller one to be the black hole mass for a certain bubble.

### 3.2 Size distribution and mass function

Now that we have the masses of black holes from bubbles with various $r_i$, we need the distribution of $r_i$ in order to obtain the mass function. Bubbles formed earlier expand to larger sizes, but they are rarer due to cosmic expansion. By assuming that bubbles are formed with a vanishing size and that the bubble worldsheet is the future light cone of the nucleation point, one can find the number density of bubbles having radius in the interval $(r_i, r_i + dr_i)$ at $t (t > t_i)$ to be [25]
\[ dn(t) \approx \lambda \frac{dr_i}{a(t) \left( r_i + H_i^{-1} \right)^4}, \]

(3.8)

where $\lambda$ is the dimensionless bubble nucleation rate per Hubble volume per Hubble time. Then by the definition of $f(m)$ (eqs. (2.1) and (2.3)), we have
\[ f(m) \propto \frac{\lambda m^2}{H_i^{3/2} \left( r_i + H_i^{-1} \right)^4 \, dm}. \]

(3.9)

The relation between $r_i$ and $m$ can be found by numerically solving eq. (3.1) and using eqs. (3.4) and (3.7). Therefore, the mass function is completely determined by the following five (independent) parameters: the Lorentz factor of the bubble wall at the end of inflation $\gamma_i$, the bubble nucleation rate $\lambda$, the inflationary density $\rho_i$, the vacuum density of the bubble interior $\rho_b$, and the bubble wall tension $\sigma$. In the following we shall use
\[ \eta_i \sim \rho_i^{1/4}, \quad \eta_b \sim \rho_b^{1/4}, \quad \eta_\sigma \sim \sigma^{1/3}, \]

(3.10)

to characterize $\rho_i$, $\rho_b$ and $\sigma$. They represent the energy scales of inflation, bubble interior and bubble wall, respectively.
Figure 4. Some examples of the mass function $f(m)$ given by (3.9). We have fixed parameters except for the Lorentz factor $\gamma_i$, with $\lambda \approx 10^{-21}, \eta_i \approx 10^{-16} M_{Pl}, \eta_b \approx 6 \times 10^{-21} M_{Pl}, \eta_{\sigma} \approx 4 \times 10^{-15} M_{Pl}$, and $\gamma_i$ varying from $10^4$ to $10^{30}$. For small $\gamma_i$ the maximum of $f(m)$ appears at the transition region. As $\gamma_i$ increases, another peak develops in the subcritical regime. For a sufficiently large $\gamma_i$, supercritical black holes near the transition approximately follow a power law $f \propto m^{-4}$ (the red straight line is $\propto m^{-4}$). For large masses, all spectra approach $f \propto m^{-1/2}$. The mass function in black (the third one from the right) agrees well with the broken power law suggested by LIGO in the relevant mass range, as shown in figure 5.

Depending on the parameter values, the mass function (3.9) can have very different shapes and can be wide or relatively narrow. Several examples of $f(m)$ are shown in figure 4, where we have fixed all other parameters except for the Lorentz factor $\gamma_i$. An intriguing feature relevant to our discussion is that when $\gamma_i$ is sufficiently large, black holes in the supercritical regime near the transition approximately follow a power law $f(m) \propto m^{-4}$ (by a numerical fit to the curves), which is what LIGO-Virgo implies for PBHs with $m > m_* \sim 35 M_\odot$. Moreover, there is a peak in the transition region for some values of $\gamma_i$, and the mass function can be approximated by different power laws on two sides of the peak. Now the question is whether suitable bubble parameters can be found such that subcritical black holes obey $f(m) \propto m^{1/2}$.

### 3.3 Observational constraints

The answer is yes. In fact, there are many sets of bubble parameters that can give the desired mass function. In figure 5 we show an example of $f(m)$ with $\gamma_i \approx 10^{22}, \lambda \approx 10^{-21}, \eta_i = \mathcal{O}(1) \text{ TeV}, \eta_b = \mathcal{O}(0.1) \text{ GeV}$ and $\eta_{\sigma} = \mathcal{O}(10) \text{ TeV}$ (black, solid curve). It agrees well with the broken power law mass function with the parameters in eq. (2.14) (orange, dashed lines). As a comparison, we also show the best-fit log-normal function (green, dotted curve). The light grey and colored areas are regions excluded by different astrophysical observations (see ref. [33] for a recent review). Strictly speaking, these upper bounds are valid only for

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3 Note that other parameters are not less essential than $\gamma_i$. Varying them will give different curves of $f(m)$ (shifting positions on the $f-m$ plane) that look similar to those in figure 4.

4 A semi-analytic discussion of this relation can be found in the appendix. For supercritical bubbles with a huge $\gamma_i$, the mass function near the transition satisfies $f \propto m^{-4.25}$.
PBHs with a very narrow spectrum, and so are not supposed to be used to compare with an extended mass function $f(m)$. (The method of applying these bounds on a broad spectrum was discussed in ref. [34].) Loosely speaking, however, as long as $\psi(m) = f(m)/m$ does not have a plateau over a relatively big range, we can still constrain a model by placing $f(m)$ near the bounds while avoiding hitting them.

The most stringent bound for us in figure 5, which covers the mass range around $30-300M_\odot$, comes from Planck. Ref. [35] studied how disk or spherical accretion of a halo around PBHs could affect CMB, which strongly constrains the population of PBHs in the range around $1-10^4M_\odot$. The broken power law function implied by LIGO-Virgo seems in tension with the model of disk accretion (light blue), but is free from the bound imposed by spherical accretion (light purple). However, we note that the effect of PBH accretion was not taken into account in our discussion. It is pointed out in refs. [36–38] that efficient PBH accretion before the reionization epoch tends to relax the tension with these bounds.\footnote{Although the process of PBH accretion is model-dependent, and is even more uncertain for binary systems, it is generally agreed that PBHs with masses smaller than $O(10)M_\odot$ are not significantly affected by accretion, and so have tiny spins, while larger PBHs generally have larger spins. This is compatible with the LIGO-Virgo results that the correlation between mass and the mean effective spin is negative, whereas the correlation between mass and the spin dispersion is positive [39]. The best-fit of a one-parameter accretion model with the GWTC-1 and GWTC-2 catalogs can be found in ref. [24].}

Besides being able to account for the LIGO-Virgo black holes, another interesting feature of our mass function is that it may provide an explanation to supermassive black holes (SMBHs) at the center of most galaxies [45, 46]. These black holes have masses from $\sim 10^6-10^{10}M_\odot$, which cannot be explained by standard accretion models [47]. Moreover,
observations of quasars indicate that many of them were already in place at high redshifts (see ref. [48] for a review). Recently, a black hole as large as $\sim 10^9 M_\odot$ was discovered at $z \approx 7.6$, which greatly challenges the conventional formation theory of astrophysical black holes [49]. One is then led to speculate that SMBHs were seeded by PBHs, which could have large masses by birth [19, 50–53]. At the present time ($t_0$), the number density of PBHs of mass $\sim m$ is approximately given by

$$n(m) \sim \rho_{\text{CDM}}(t_0) \frac{f(m)}{m} \approx 4 \times 10^{11} \left(\frac{M_\odot}{m}\right) f(m) \text{ Mpc}^{-3}. \quad (3.11)$$

It was shown in ref. [35] that PBHs at $z \sim 6$ with $m \sim 10^2$–$10^4 M_\odot$ can attain sufficient accretion and grow to SMBHs even if $f(m)$ is as small as $10^{-9}$. We can see from figure 5 that the best-fit broken power law mass function gives $f(10^3 M_\odot) \gtrsim 10^{-9}$, which gives $n(10^3 M_\odot) \sim 0.4$ Mpc$^{-3}$. This is larger than the present density of galaxies $\sim 0.1$ Mpc$^{-3}$. Therefore, SMBHs could indeed have been seeded by these PBHs.

4 Conclusions and discussion

In this work we have used maximum likelihood estimation to analyze the GWTC-1 and GWTC-2 datasets released by the LIGO-Virgo Collaboration, assuming that all of the black holes in the 44 confident merger events are PBHs with a mass spectrum $f(m)$ satisfying a broken power law. We found that the best-fit $f(m)$ has a peak at $m_* \approx 35 M_\odot$, consistent with the observations, and that $f \propto m^{1.2}$ for $m < m_*$ and $f \propto m^{-4}$ for $m > m_*$. These black holes can constitute about 0.1% of the dark matter.

Such a unique mass function can be realized in a mechanism of PBH formation we proposed in recent years, where the black holes are formed by vacuum bubbles that could nucleate during inflation. In general, the inflaton field rolls in a multidimensional potential containing a number of minima. Bubble nucleation can occur during inflation if a minimum has energy scale lower than the inflationary scale. After inflation ends, depending on their size, these bubbles will turn into black holes either by simple collapse (subcritical), or by creating a wormhole (supercritical). The resulting mass function of the black holes may obey different power laws in the two regimes, which are connect by a transition region that could possibly be at $m_*$. Surprisingly, if the bubble walls have a sufficiently large Lorentz factor at the end of inflation, which is typically the case, the mass function in the supercritical regime near $m_*$ is indeed approximately given by $f \propto m^{-4}$. With properly chosen parameters (including the Lorentz factor, the energy scales of bubble vacuum, bubble wall and inflation), we can have $f \propto m^{1.2}$ for $m < m_*$. With this mass function, we can also have a sufficient number of seeds ($m \sim 10^3 M_\odot$) that could grow into SMBHs observed at the galactic centers, which is difficult to explain with conventional accretion models. In addition, as we can see in figure 4, it is also possible to estimate the distribution of stupendously large black holes [54] from the large mass end, where $f \propto m^{-1/2}$. This however will not be discussed further in the present work.

Another noticeable feature of our mass function is that it suggests a relatively low inflationary scale. During the slow roll, the ultimate Lorentz factor of the bubble wall for a comoving observer outside the bubble is $\gamma \sim \eta_{\sigma}^2 M_{Pl}/\eta_{\sigma}^{(i3)}$, where $\eta_{\sigma}^{(i)}$ is the energy scale of the wall tension $\sigma$ during inflation [27, 55]. We assumed $\gamma$ to be the Lorentz factor of the wall as it runs into the ambient radiation after inflation, i.e., $\gamma_i = \gamma$. However, at the end of inflation, the inflaton quickly rolls down into our vacuum, which could cause a drastic change.
in $\sigma$ since the shape of the barrier in the field potential could change significantly. Therefore, we regarded $\gamma_i$ and $\sigma$ as two free parameters. With the parameter values that lead to the black solid mass function in figure 5, where the inflationary scale is $\eta_i = \mathcal{O}(1) \text{ TeV}$, we have $\eta_f/\eta_f^{(i)} = \mathcal{O}(100–1000)$. If we require a larger $\eta_i$, the ratio $\eta_f/\eta_f^{(i)}$ is even larger, which seems less likely. Of course, this should be determined by the configuration of the multidimensional potential, which is beyond the scope of the present work.

An inflationary model at the TeV scale was constructed in ref. [56]. In this model of hybrid inflation, the inflaton is directly coupled to the Higgs field such that the symmetry is restored even at a temperatures lower than the electroweak scale.\(^{6}\) As the Higgs potential becomes unstable later, the fields roll down in random directions, leading to non-trivial Higgs configurations. In the presence of CP-violation, this might produce a baryon asymmetry, which is referred to as cold electroweak baryogenesis [57–60], and has been studied extensively with simulations [61–64]. Such a process might also be a source of primordial magnetic fields [65, 66] and stochastic gravitational waves [67].

In hybrid inflation, the inflationary scale could be so small that the expansion rate is much smaller than the typical particle decay rate, hence thermalization occurs almost instantaneously compared to Hubble time [57]. This is compatible with the assumption in our model where the bubble runs into radiation immediately after inflation ends. If this is not the case, a different matter content during reheating would change the scale factor in the equation of motion of the bubble wall (eq. (3.1)). To see how this would affect the resulting mass function, we solved the equation assuming the universe is dust-dominated for the first few Hubble times. It turned out the desired mass function (in figure 5) can be obtained by, e.g., increasing $\gamma_i$ and $\eta_f$ (by a factor $\mathcal{O}(1–10)$) without changing other parameters.

With many more merger events to be detected by LIGO-Virgo in the near future, the mass distribution of black holes can be determined with increasing certainty. Excited as we are, that our mechanism is the only factory of LIGO-Virgo black holes can be ruled out if the mass function turns out to have a very different shape, such as one with a slope (in the log-log plot) much smaller than -4 for large black holes.

A Mass distribution of supercritical black holes

In this appendix, we will find a semi-analytic solution to the bubble wall’s equation of motion for relatively large bubbles, and thus determine the mass function of supercritical black holes, which was found numerically in section 3, and is shown in figure 4.

The bubble wall’s equation of motion after inflation is (eq. (3.1))

$$\ddot{r} + \left(4 - 3a^2 \dot{r}^2\right) H \dot{r} + \frac{2}{a^2 r} \left(1 - a^2 \dot{r}^2\right) + K \frac{\left(1 - a^2 \dot{r}^2\right)^{3/2}}{a} = 0,$$

(A.1)

where we have defined $K \equiv \rho_b/\sigma + 6\pi \sigma$. This equation can be rewritten as

$$\dot{u} + \frac{3}{2t} u + \frac{2\gamma_i}{ar} + K = 0,$$

(A.2)

where $u(t) \equiv \sqrt{\gamma_i^2 - 1}$ and $\gamma \equiv \left(1 - a^2 \dot{r}^2\right)^{-1/2}$. As discussed in section 3, if we assume a huge $\gamma_i$ (or $u(t_i)$), the trajectory of the wall can be approximated by $a\dot{r} \approx 1$, which gives

$$r(t) \approx r_i + 2\sqrt{T_i}(\sqrt{t} - \sqrt{T_i}).$$

(A.3)

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\(^6\)New fields need to be introduced in order to produce bubbles in our model.
Then when the wall comes to a stop with respect to the Hubble flow, we have

\[ r_s \sim r_i + a_s H_i^{-1}, \]  

(A.4)

where we have used \( r_i \gg H_i^{-1} \), which is typically the case for supercritical bubbles in our discussion. In what follows we shall discuss two limits: (a) \( r_i \gg a_s H_i^{-1} \), and (b) \( r_i \ll a_s H_i^{-1} \).

(a) \( r_i \gg a_s H_i^{-1} \). In this limit, \( r_s \sim r_i \). Then by eq. (3.7), the resulting black hole mass is

\[ m \propto r_i^2. \]  

(A.5)

Then it can easily be shown that the mass function (eq. (3.9)) becomes

\[ f(m) \propto m^{2} r_i^4 \frac{dr_i}{dm} \propto m^{-1/2}. \]  

(A.6)

This explains the behavior of \( f(m) \) on the end of large masses, as shown in figure 4.

(b) \( r_i \ll a_s H_i^{-1} \). In this limit, the bubbles are assumed to still be in the supercritical regime, but their initial radius \( r_i \) is negligible compared to \( r_s \sim a_s H_i^{-1} \). By eq. (A.3), we can approximate eq. (A.2) by

\[ \dot{u} + \frac{3}{2t} u + \frac{2u}{r_i \sqrt{t}} \frac{2}{2t} + K = 0. \]  

(A.7)

Now we can roughly divide the time interval between \( t_i \) and \( t_s \) by two stages: before and after \( t \approx r_i^2/4t_i \equiv t_m \), which determines whether \( r_i \sqrt{t} / \sqrt{t_i} \) or \( 2t \) dominates the denominator in the third term of the above equation.

Before \( t_m \), we have

\[ \dot{u} + \frac{3}{2t} u + \frac{2 \sqrt{t_i}}{r_i \sqrt{t}} u + K = 0, \]  

(A.8)

which has an analytic solution

\[ u(t) = \frac{e^{-2 \sqrt{t_i/r_i} / t^{3/2}}}{t^{3/2}} \left[ \frac{\gamma_i t_i^{3/2}}{e^{-4t_i/r_i}} + K \frac{\Gamma(5, -4t_i/r_i) - \Gamma(5, -4\sqrt{t_i/t_i})}{512(\sqrt{t_i/r_i})^5} \right]. \]  

(t < \( t_m \))  

(A.9)

Here we have taken into account the initial condition \( u(t_i) = \gamma_i \). Then at \( t_m \), we have

\[ u(t_m) = \frac{1}{t_m^{3/2}} \left[ \frac{2 \gamma_i t_i^{3/2}}{e^{-4t_i/r_i}} + K \frac{\Gamma(5, -4t_i/r_i) - \Gamma(5, -2)}{512(\sqrt{t_i/r_i})^5} \right]. \]  

(A.10)

After \( t_m \), eq. (A.7) becomes

\[ \dot{u} + \frac{5}{2t} u + K = 0, \]  

(A.11)

with the solution

\[ u(t) = u(t_m) \left( \frac{t_m}{t} \right)^{5/2} + \frac{2}{7} K t \left( \frac{t_m}{t} \right)^{7/2} - 1 \]. \]  

(t > \( t_m \))  

(A.12)
At $t_s$, we have $u(t_s) = 0$. After some algebra, substituting $u(t_m)$ by eq. (A.10) in eq. (A.12) leads to

$$t_s \approx 0.5 \left(\frac{\gamma_i}{K}\right)^{2/7} t_i^{1/7} r_i^{4/7}.$$  \hspace{1cm} (A.13)

In the computations we have assumed the relation $\gamma_i > 10^{-3} Kr_i^5/t_i^4$, which is true in the cases we are interested in. This estimate for $t_s$ has also been verified numerically.

Now by eq. (3.7), we have

$$m \sim t_s \propto r_i^{4/7}.$$  \hspace{1cm} (A.14)

Then the mass function becomes

$$f(m) \propto \frac{m^2 dr_i}{dm} \propto m^{-4.25}.$$  \hspace{1cm} (A.15)

This explains the behavior of $f(m)$ in the supercritical regime near the transition, as shown in figure 4. Note that this relation is but an approximation. For a smaller $\gamma_i$, it was numerically found that the power for $m$ gets slightly larger, giving $f \propto m^{-4}$, as shown in figure 5.

Acknowledgments

I would like to thank Tanmay Vachaspati, Ville Vaskonen, Alex Vilenkin and the anonymous referee for stimulating discussion and comments. This work is supported by the U.S. Department of Energy, Office of High Energy Physics, under Award No. de-sc0019470 at Arizona State University.

References

[1] LIGO Scientific and Virgo collaborations, *GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs*, Phys. Rev. X 9 (2019) 031040 [arXiv:1811.12907] [INSPHERE].

[2] LIGO Scientific and Virgo collaborations, *GWTC-2: Compact Binary Coalescences Observed by LIGO and Virgo During the First Half of the Third Observing Run*, arXiv:2010.14527 [INSPHERE].

[3] K.W.K. Wong, K. Breivik, K. Kremer and T. Callister, *Joint constraints on the field-cluster mixing fraction, common envelope efficiency, and globular cluster radii from a population of binary hole mergers via deep learning*, arXiv:2011.03564 [INSPHERE].

[4] M. Zevin et al., *One Channel to Rule Them All? Constraining the Origins of Binary Black Holes using Multiple Formation Pathways*, Astrophys. J. 910 (2021) 152 [arXiv:2011.10057] [INSPHERE].

[5] C.L. Rodriguez et al., *The Observed Rate of Binary Black Hole Mergers can be Entirely Explained by Globular Clusters*, Res. Notes AAS 5 (2021) 19 [arXiv:2101.07793] [INSPHERE].

[6] M. Fishbach et al., *When are LIGO/Virgo’s Big Black-Hole Mergers?*, arXiv:2101.07699 [INSPHERE].

[7] S. Bird et al., *Did LIGO detect dark matter?*, Phys. Rev. Lett. 116 (2016) 201301 [arXiv:1603.00464] [INSPHERE].

[8] S. Clesse and J. García-Bellido, *The clustering of massive Primordial Black Holes as Dark Matter: measuring their mass distribution with Advanced LIGO*, Phys. Dark Univ. 15 (2017) 142 [arXiv:1603.05234] [INSPHERE].
[9] M. Sasaki, T. Suyama, T. Tanaka and S. Yokoyama, Primordial Black Hole Scenario for the Gravitational-Wave Event GW150914, Phys. Rev. Lett. 117 (2016) 061101 [Erratum ibid. 121 (2018) 059901] [arXiv:1603.08338] [SPIRE].

[10] M. Raidal, V. Vaskonen and H. Veermäe, Gravitational Waves from Primordial Black Hole Mergers, JCAP 09 (2017) 037 [arXiv:1707.01480] [SPIRE].

[11] Y. Ali-Haïmoud, E.D. Kovetz and M. Kamionkowski, Merger rate of primordial black-hole binaries, Phys. Rev. D 97 (2017) 123523 [arXiv:1709.06576] [SPIRE].

[12] V. Vaskonen and H. Veermäe, Lower bound on the primordial black hole merger rate, Phys. Rev. D 101 (2020) 043015 [arXiv:1908.09752] [SPIRE].

[13] J. Garriga and N. Triantafyllou, Enhanced cosmological perturbations and the merger rate of PBH binaries, JCAP 09 (2017) 037 [arXiv:1707.01480] [SPIRE].

[14] M. Raidal, C. Spethmann, V. Vaskonen and H. Veermäe, Formation and Evolution of Primordial Black Hole Binaries in the Early Universe, JCAP 02 (2019) 018 [arXiv:1812.01930] [SPIRE].

[15] Z.-C. Chen and Q.-G. Huang, Merger Rate Distribution of Primordial-Black-Hole Binaries, Astrophys. J. 864 (2018) 61 [arXiv:1801.10327] [SPIRE].

[16] A.D. Gow, C.T. Byrnes, A. Hall and J.A. Peacock, Primordial black hole merger rates: distributions for multiple LIGO observables, JCAP 01 (2020) 031 [arXiv:1911.12685] [SPIRE].

[17] Y. Wu, Merger history of primordial black-hole binaries, Phys. Rev. D 101 (2020) 083008 [arXiv:2001.03833] [SPIRE].

[18] A. Dolgov and J. Silk, Baryon isocurvature fluctuations at small scales and baryonic dark matter, Phys. Rev. D 47 (1993) 4244 [SPIRE].

[19] S. Clesse and J. García-Bellido, Massive Primordial Black Holes from Hybrid Inflation as Dark Matter and the seeds of Galaxies, Phys. Rev. D 92 (2015) 023524 [arXiv:1501.07565] [SPIRE].

[20] S. Blinnikov, A. Dolgov, N.K. Porayko and K. Postnov, Solving puzzles of GW150914 by primordial black holes, JCAP 11 (2016) 036 [arXiv:1611.00541] [SPIRE].

[21] K. Kannike, L. Marzola, M. Raidal and H. Veermäe, Single Field Double Inflation and Primordial Black Holes, JCAP 09 (2017) 020 [arXiv:1705.06225] [SPIRE].

[22] G. Hütsi, M. Raidal, V. Vaskonen and H. Veermäe, Two populations of LIGO-Virgo black holes, JCAP 03 (2021) 068 [arXiv:2012.02786] [SPIRE].

[23] A. Hall, A.D. Gow and C.T. Byrnes, Bayesian analysis of LIGO-Virgo mergers: Primordial vs. astrophysical black hole populations, Phys. Rev. D 102 (2020) 123524 [arXiv:2008.13704] [SPIRE].

[24] K.W.K. Wong et al., Constraining the primordial black hole scenario with Bayesian inference and machine learning: the GWTC-2 gravitational wave catalog, Phys. Rev. D 103 (2021) 023026 [arXiv:2011.01865] [SPIRE].

[25] J. Garriga, A. Vilenkin and J. Zhang, Black holes and the multiverse, JCAP 02 (2016) 064 [arXiv:1512.01819] [SPIRE].

[26] H. Deng and A. Vilenkin, Primordial black hole formation by vacuum bubbles, JCAP 12 (2017) 044 [arXiv:1710.02865] [SPIRE].

[27] H. Deng, Primordial black hole formation by vacuum bubbles. Part II, JCAP 09 (2020) 023 [arXiv:2006.11907] [SPIRE].

[28] M. Dominik et al., Double Compact Objects III: Gravitational Wave Detection Rates, Astrophys. J. 806 (2015) 263 [arXiv:1405.7016] [SPIRE].
[29] P. Ajith et al., *A template bank for gravitational waveforms from coalescing binary black holes. I. Non-spinning binaries*, Phys. Rev. D 77 (2008) 104017 [Erratum ibid. 79 (2009) 129901] [arXiv:0710.2338] [inSPIRE].

[30] I. Mandel, W.M. Farr and J.R. Gair, *Extracting distribution parameters from multiple uncertain observations with selection biases*, Mon. Not. Roy. Astron. Soc. 486 (2019) 1086 [arXiv:1809.02063] [inSPIRE].

[31] H. Deng, J. Garriga and A. Vilenkin, *Primordial black hole and wormhole formation by domain walls*, JCAP 04 (2017) 050 [arXiv:1612.03753] [inSPIRE].

[32] V. De Luca, G. Franciolini and A. Riotto, *On the Primordial Black Hole Mass Function for Broad Spectra*, Phys. Lett. B 807 (2020) 135550 [arXiv:2001.04371] [inSPIRE].

[33] B. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, *Constraints on Primordial Black Holes*, arXiv:2002.12778 [inSPIRE].

[34] B. Carr, M. Raidal, T. Tenkanen, V. Vaskonen and H. Veermäe, *Primordial black hole constraints for extended mass functions*, Phys. Rev. D 96 (2017) 023514 [arXiv:1705.05567] [inSPIRE].

[35] P.D. Serpico, V. Poulin, D. Inman and K. Kohri, *Cosmic microwave background bounds on primordial black holes including dark matter halo accretion*, Phys. Rev. Lett. 126 (2021) 051101 [arXiv:2009.01728] [inSPIRE].

[36] V. De Luca, G. Franciolini, P. Pani and A. Riotto, *GW190521 Mass Gap Event and the Primordial Black Hole Scenario*, Phys. Rev. Lett. 126 (2021) 051101 [arXiv:2009.01728] [inSPIRE].

[37] V. De Luca, V. Desjacques, G. Franciolini, P. Pani and A. Riotto, *Primordial Black Holes Confront LIGO/Virgo data: Current situation*, JCAP 06 (2020) 044 [arXiv:2005.05641] [inSPIRE].

[38] M. Safarzadeh, W.M. Farr and E. Ramirez-Ruiz, *A trend in the effective spin distribution of LIGO binary black holes with mass*, Astrophys. J. 894 (2020) 129 [arXiv:2001.06490] [inSPIRE].

[39] MACHO collaboration, *The MACHO project: Microlensing results from 5.7 years of LMC observations*, Astrophys. J. 542 (2000) 281 [astro-ph/0001272] [inSPIRE].

[40] EROS-2 collaboration, *Limits on the Macho Content of the Galactic Halo from the EROS-2 Survey of the Magellanic Clouds*, Astron. Astrophys. 469 (2007) 387 [astro-ph/0607207] [inSPIRE].

[41] M. Oguri, J.M. Diego, N. Kaiser, P.L. Kelly and T. Broadhurst, *Understanding caustic crossings in giant arcs: characteristic scales, event rates, and constraints on compact dark matter*, Phys. Rev. D 97 (2018) 023518 [arXiv:1710.00148] [inSPIRE].

[42] LIGO Scientific and Virgo collaborations, *Search for Subsolar Mass Ultracompact Binaries in Advanced LIGO’s Second Observing Run*, Phys. Rev. Lett. 123 (2019) 161102 [arXiv:1904.08976] [inSPIRE].

[43] Y. Inoue and A. Kusenko, *New X-ray bound on density of primordial black holes*, JCAP 10 (2017) 034 [arXiv:1705.00791] [inSPIRE].

[44] D. Lynden-Bell, *Galactic nuclei as collapsed old quasars*, Nature 223 (1969) 690 [inSPIRE].

[45] J. Kormendy and D. Richstone, *Inward bound: The search for supermassive black holes in galactic nuclei*, Ann. Rev. Astron. Astrophys. 33 (1995) 581 [inSPIRE].

[46] Z. Haiman, *Constraints from gravitational recoil on the growth of supermassive black holes at high redshift*, Astrophys. J. 613 (2004) 36 [astro-ph/0404196] [inSPIRE].
[48] J. Kormendy and L.C. Ho, Coevolution (Or Not) of Supermassive Black Holes and Host Galaxies, *Ann. Rev. Astron. Astrophys.* 51 (2013) 511 [arXiv:1304.7762] [inSPIRE].

[49] F. Wang et al., A luminous quasar at redshift 7.642, *Astrophys. J. Lett.* 907 (2021) L1.

[50] S.G. Rubin, A.S. Sakharov and M.Y. Khlopov, The formation of primary galactic nuclei during phase transitions in the early universe, *J. Exp. Theor. Phys.* 91 (2001) 921 [hep-ph/0106187] [inSPIRE].

[51] R. Bean and J. Magueijo, Could supermassive black holes be quintessential primordial black holes?, *Phys. Rev. D* 66 (2002) 063505 [astro-ph/0204486] [inSPIRE].

[52] N. Duechting, Supermassive black holes from primordial black hole seeds, *Phys. Rev. D* 70 (2004) 064015 [astro-ph/0406260] [inSPIRE].

[53] B. Carr and J. Silk, Primordial Black Holes as Generators of Cosmic Structures, *Mon. Not. Roy. Astron. Soc.* 478 (2018) 3756 [arXiv:1801.00672] [inSPIRE].

[54] B. Carr, F. Kuhnel and L. Visinelli, Constraints on Stupendously Large Black Holes, *Mon. Not. Roy. Astron. Soc.* 501 (2021) 2029 [arXiv:2008.08077] [inSPIRE].

[55] V.A. Berezin, V.A. Kuzmin and I.I. Tkachev, Dynamics of Bubbles in General Relativity, *Phys. Rev. D* 36 (1987) 2919 [inSPIRE].

[56] E.J. Copeland, D. Lyth, A. Rajantie and M. Trodden, Hybrid inflation and baryogenesis at the TeV scale, *Phys. Rev. D* 64 (2001) 043506 [hep-ph/0103231] [inSPIRE].

[57] J. García-Bellido, D.Y. Grigoriev, A. Kusenko and M.E. Shaposhnikov, Nonequilibrium electroweak baryogenesis from preheating after inflation, *Phys. Rev. D* 60 (1999) 123504 [hep-ph/9902449] [inSPIRE].

[58] L.M. Krauss and M. Trodden, Baryogenesis below the electroweak scale, *Phys. Rev. Lett.* 83 (1999) 1502 [hep-ph/9902420] [inSPIRE].

[59] J. Smit and A. Tranberg, Chern-Simons number asymmetry from CP-violation during tachyonic preheating, in 5th International Conference on Strong and Electroweak Matter, (2002), DOI [hep-ph/0210348] [inSPIRE].

[60] T. Konstandin and G. Servant, Natural Cold Baryogenesis from Strongly Interacting Electroweak Symmetry Breaking, *JCAP* 07 (2011) 024 [arXiv:1104.4793] [inSPIRE].

[61] A. Tranberg and J. Smit, Simulations of cold electroweak baryogenesis: Dependence on Higgs mass and strength of CP-violation, *JHEP* 08 (2006) 012 [hep-ph/0604263] [inSPIRE].

[62] A. Tranberg, J. Smit and M. Hindmarsh, Simulations of cold electroweak baryogenesis: Finite time quenches, *JHEP* 01 (2007) 034 [hep-ph/0610096] [inSPIRE].

[63] Z.-G. Mou, P.M. Saffin and A. Tranberg, Simulations of Cold Electroweak Baryogenesis: Finding the optimal quench time, *JHEP* 07 (2017) 010 [arXiv:1703.01781] [inSPIRE].

[64] Z.-G. Mou, P.M. Saffin and A. Tranberg, Simulations of Cold Electroweak Baryogenesis: Dependence on the source of CP-violation, *JHEP* 05 (2018) 197 [arXiv:1803.07346] [inSPIRE].

[65] A. Diaz-Gil, J. García-Bellido, M. Garcia Perez and A. Gonzalez-Arroyo, Primordial magnetic fields from preheating at the electroweak scale, *JHEP* 07 (2008) 043 [arXiv:0805.4159] [inSPIRE].

[66] Z.-G. Mou, P.M. Saffin and A. Tranberg, Simulations of Cold Electroweak Baryogenesis: Hypercharge U(1) and the creation of helical magnetic fields, *JHEP* 06 (2017) 075 [arXiv:1704.08888] [inSPIRE].

[67] J. García-Bellido, D.G. Figueroa and A. Sastre, A Gravitational Wave Background from Reheating after Hybrid Inflation, *Phys. Rev. D* 77 (2008) 043517 [arXiv:0707.0839] [inSPIRE].