Competitive Mechanism For The Distribution Of Labor Resources In The Transport Objective

Irina Zaitseva1, 2, Oleg Malafeyev3, Yuliya Marenchuk4, Dmitry Kolesov3 and Svetlana Bogdanova1

1 Stavropol State Agrarian University, Zootekhnicheskiy lane 12, Stavropol, 355017, Russia
2 Stavropol branch of the Moscow Pedagogical State University, Dovatortsev str. 66 g, Stavropol, 355042, Russia
3 St. Petersburg State University, Faculty of Applied Mathematics and Control Processes, 7/9 Universitetskaya nab., St. Petersburg, 199034, Russia
4 North Caucasus Federal University
irina.zaitseva.stv@yandex.ru

Abstract. This article proposes a close labor force allocation model based on the use of a competitive mechanism. Many practical economic activity tasks and some economic theory issues are associated with the tasks of determining the optimal variant for solving the problem of the distribution of labor resources. One of these solutions is a compromise set. This article is devoted to the search for this set in a non-antagonistic non-coalition game related to the transport problem of integer programming.

1. Introduction
The study of methods of mathematical programming, especially dynamic programming, becomes necessary for the practical work of an economist. In mathematical economics, the task of the most efficient transfer of labor resources from one point to another is of great importance. Tasks of this type are relevant and constantly arise in various areas of our life, such as economics, industry, etc.

This article discusses decision-making tasks with multiple participants. In such tasks, the compromise value of the income function for each of the participants depends on the decisions made by all other participants [1-5].

The basis of the work is the transport problem of integer programming. In this task, we consider the process of moving labor resources in order to obtain maximum income. The paper considers the game-theoretic version of this problem and for which the static model is built. However, unlike the transportation problem, in the constructed model, the goal of each participant in addition to maximizing their income is to reach a compromise with the other participants. As a result, it is possible that not all participants in the process will receive their maximum possible income. In a static model, a displacement plan will play the role of compromise, satisfying all participants [6-7].
2. Static model

2.1. Formulation Of The Problem

Let’s consider a network $(L, R, K)$, where $L$ is the set of nodes consisting of $N$ production points, $M$ consumption points and intermediate nodes; $R$ is the set of edges; $K$ is the bandwidth function defined on the edges of the network [8].

All $N$ production points produce different goods. We will enumerate all labor resources so that the resource $s = 1, N$ number will correspond to the number of the manufacturer $i$ who produced this product. The quantity of manufactured goods $s$ production point $i$ is given and is denoted by $\sigma_i^s$. In a similar way, the need of a product $s$ is set at each consumption point $j$, denoted as $\delta_j^s$, where $s$ - number of of $j = 1, M$ goods. That is, in the $j$ point of consumption demand in the goods designated by the vector: $(\delta_1^s, \delta_2^s, \ldots, \delta_N^s)$.

At the production point $i$, the costs for the production of a commodity unit are given $s$ and the unit price at the $j$ consumption point $\alpha_j^s$, where $s$ - item number. At the edges of the network, the carrying capacities specified and the cost of transporting a unit of product for each edge of the network are $-d_{(x,y)}$, where $(x,y) \in R$.

Let’s recall the formal definition of a non-cooperative game [9-11].

Definition 1. The system $\Gamma = \{N, \{X_i\}_{i \in N}, \{H_i\}_{i \in N}\}$, in which $N = \{1, 2, \ldots, n\}$ is the set of players, $X_i$ is the set of player strategies, $H_i$ is the player’s win function defined on the set of admissible situations $X_i$ is called non-cooperative game.

We will describe the game-theoretic version of the above model. In accordance with the definition, in order to define a game, it is necessary to determine a plurality of players, a variety of game situations and functions of player income [12].

Consequently, the set of players $P$ are points of consumption $A_i$, $i = 1, N$ and production $B_j$, $j = 1, M$. The set of situations $S$ of the game will be the plans of transportation of the form:

$$\sum_i (A_1 \rightarrow a_{i1}^s \rightarrow B_1, \ldots, A_n \rightarrow a_{iN}^s \rightarrow B_M; \ldots, A_N \rightarrow a_{iN}^s \rightarrow B_M)$$

where $a_{ij}^s$ – is the quantity of the $s$ goods, transported from the $i$ production point to the $j$ consumption point.

The income functions of players are introduced as follows. For production points:

$$H_{A_i}(\Sigma_k) = \sum_{j=1}^M (\delta_j^s \alpha_j^s) - \sigma_i \beta_i - \gamma(p, \delta),$$

where $\sum_{j=1}^M (\delta_j^s \alpha_j^s)$ – revenue from the sale of goods, $\sigma_i \beta_i$ – production costs, $\gamma(p, \delta)$ – the cost of transportation of goods, depending on the route of transportation $p$ and the quantity of goods transported $\delta$.

For consumption points:

$$H_{B_j}(\Sigma_k) = \sum_{i=1}^N \Theta_j (\delta_j^s) - \sum_{i=1}^N \delta_j^s \alpha_j^s,$$
where $\Theta_j (\delta_j^i)$ – is the utility of the quantity of the $s$ product for $j$ consumer and is set by means of the utility function, $\delta_j^i \alpha_j^i$ – the cost of purchasing $\delta_j^i$ units of the $j$ product by the consumption point for the price $\alpha_j^i$.

Let’s recall that the actual function defined in the space of goods and determining the benefit of the consumer from a certain amount of goods is called the utility function [13-15].

Thus, we have non-antagonistic noncooperative game $N + M$

$$\Gamma = \{P, S, H_{A_i}, H_{B_j} \mid i = 1, N; j = 1, M\},$$

where $P$ - a lot of players, the $S$ - set of situations of the game $H_{A_i}$ -income producers $A_i$ functions $i = 1, N$, $H_{B_j}$ - function of income $B_j$, $j = 1, M$ consumers.

In this game we need to find a compromise point, that is, such a situation $\Sigma$, the implementation of which is a compromise between all players [16-18].

2.2. Solution Algorithm

Recall the definition of a compromise point.

Definition 2. Let’s define $X$ - space of situations of the game: $X \rightarrow i, i \in \mathbb{N}$, where $i$ is the essence of the winning players function. Let’s put $M_i = \max \{H_i(x) \mid x \in X\}$. Then compromise point $C_{\mu}$ in the game $\Gamma$ is determined as follows:

$$C_{\mu} = \{x \in X : \max_{i \in \mathbb{N}} (M_i - H_i(x)) \leq \max_{i \in \mathbb{N}} (M_i - H_i(x')) \forall x' \in X\}$$

The algorithm of finding a compromise point follows naturally from the definition [19].

Step 1. Incomes of each player are calculated depending by each situation of the game $\Sigma_k \in S$, $H_i (\Sigma_k)$, where $S$ – plurality of game situations $\Gamma$, $k = 1, 2, ..., \tilde{k}$ -situation number, $r = 1, N + M$ - number of the player.

Step 2. For each player $r = 1, N + M$ magnitudes are calculated by:

$$M_r = \max_{k \in \mathbb{N}} H_r (\Sigma_k), \forall k = 1, 2, ..., \tilde{k},$$

Where $\tilde{k}$ - the number of situations in the $\Gamma$ and $\{ideal vector\}$ are calculating $M = (M_1, ..., M_{N+M})$.

Step 3. For each player $r = 1, N + M$ and for each situation of the game $\Sigma_k \in S$, $k = 1, 2, ..., \tilde{k}$ $\Gamma$ the following deviations from the maximum are calculating $M_r$:

$$\Delta_r (\Sigma_k) = M_r - H_r (\Sigma_k).$$

Step 4. The maximum deflection for all players are calculating $r = 1, N + M$ at each permutation (game situations) $\Sigma_k \in S$, $\alpha(\Sigma_k) = \max_{r \in \mathbb{N}} \Delta_r (\Sigma_k)$, where $k = 1, 2, ..., \tilde{k}$ – number of situations in the game $\Gamma$, $P$ – players multitude of the game $\Gamma$.

Step 5. The minimum of this deflections is calculated $\min_{r \in \mathbb{N}} \alpha(\Sigma_k)$, where $k = 1, 2, ..., \tilde{k}$.

The situation in which this minimum is reached and is a compromise point for all players [20-25].

2.3. Numerical example

Let two manufacturers $A_i, i = 1, 2$ produce two different goods and sell it to three consumers $B_j, j = 1, 2, 3$. The quantity of goods produced $\sigma_i$ and the costs of their production at each production
point $\beta_i$ are known. Also known for the needs of each product and prices for each consumer [26-30].

On the edges of the network given capacity and the cost of transporting a unit of product. The network itself can have an arbitrary appearance, but it should be noted that with the addition of only one edge of the network or another participant in the process, the complexity of the task increases several times.

For example, choose the network of the simplest form $N=2, M=3$.

\[
\begin{align*}
\sigma_1 &= 10, & \sigma_2 &= 8, & \delta_1 &= 1, & \delta_2 &= 2, & \delta_3 &= 3, \\
\beta_1 &= 5.79, & \beta_2 &= 7.24, & \delta_1 &= 6, & \delta_2 &= 5, & \delta_3 &= 1.
\end{align*}
\]

\[
\begin{align*}
K_{AC} &= 6, & d_{AC} &= 2.57, & K_{AD} &= 9, & d_{AD} &= 3.13, & K_{CE} &= 15, & d_{CE} &= 4.9, \\
K_{AC} &= 7, & d_{AC} &= 3.67, & K_{DF} &= 13, & d_{DF} &= 5, & K_{AD} &= 11, & d_{AD} &= 4.26, & K_{EB} &= 5, & d_{EB} &= 1.99, & K_{EB} &= 7, & d_{EB} &= 1.22, \\
K_{FB} &= 4, & d_{FB} &= 3.31, & K_{FB} &= 9, & d_{FB} &= 3.34, & K_{EB} &= 8, & d_{EB} &= 4.1, & K_{FB} &= 9, & d_{FB} &= 1.07.
\end{align*}
\]

The utility functions are: $\Theta_1 (\delta_1^j) = 96,27 \sqrt{3.4 \cdot \delta_1^j}$, $\Theta_2 (\delta_2^j) = 76,99 \sqrt{4.2 \cdot \delta_2^j}$, where $j = 1, M$.

Acting exactly according to the algorithm described above, in the problem thus posed we find a compromise point [31-34].

Thus, in this particular example will be a compromise $\Sigma_{comp}$:

$A_{DFB_1}[1]; A_{DFB_2}[3] + A_{CEB_1}[3]; A_{CEB_1}[3]; A_{CEB_2}[2]; A_{DFB_1}[1] + A_{CEB_2}[4]; A_{CEB_1}[1]$.

The income of the players will be as follows:

\[
\begin{align*}
H_{A_1} (\Sigma_{comp}) &= 309.31, & H_{B_1} (\Sigma_{comp}) &= 256.66, \\
H_{A_2} (\Sigma_{comp}) &= 356.03, & H_{B_2} (\Sigma_{comp}) &= 317.78, & H_{B_3} (\Sigma_{comp}) &= 297.92.
\end{align*}
\]

3. Conclusion

Thus, a description was given of the game model of the transportation problem, in which the parameters do not depend on time. For this model, an algorithm for finding a compromise point is considered, the application of which is illustrated with a specific example.

References

[1] Kirjanen A, Malafeyev O and Redinskikh N 2017 Developing industries in cooperative interaction: Equilibrium and stability in processes with lag Statistics, Optimization and Information Computing.

[2] Kriulina E, Tarasenko N, Miroshnitchenko N, Zaiteva I and Dedyyukhina I 2016 Environmental Management in Agriculture: Problems and Solutions, Research Journal of Pharmaceutical, Biological and Chemical Sciences, no. 7(3), pp. 1908-1912.

[3] Neverova E and Malafeyev O 2015 A model of interaction between anticorruption authority and corruption groups AIP Conference Proceedings, (American Institute of Physics).

[4] Neverova E, Malafeyev O, Alferov G and Smirnova T 2015 Model of interaction between anticorruption authorities and corruption groups in International Conference on "Stability and Control Processes" in Memory of V.I. Zubov, (SCP, Proceedings, SPb), pp. 488-490.

[5] Drozdov G, Malafeyev O and Nemnyugin S 2015 Multicomponent dynamics of competitive
single-sector economy development International Conference on "Stability and Control Processes" in Memory of V.I. Zubov (SCP 2015 – Proceedings).

[6] Kolesin I, Malafeyev O, Andreeva M and Ivanukovich G 2017 Corruption: Taking into account the psychological mimicry of officials AIP Conference Proceedings (American Institute of Physics).

[7] Zaitseva I and Popova M 2013 Technique to study the employment potential of the region: economic-mathematical aspect (World Applied Sciences Journal.), no. 22 (1), pp. 22-25.

[8] Zaitseva I, Kriulina E, Ermakova A, Shevchenko E and Vorokhobina Y 2016 Application of Factor Analysis to Study the Labour Capacity of Stavropol Krai (Research Journal of Pharmaceutical, Biological and Chemical Sciences.), no. 7(4), pp. 2183-2186.

[9] Zaitseva I, Popova M, Ermakova A, Bogdanova S and Rezencov D 2016 Determination Prospects Of Development Labor Potential In Agriculture Stavropol Territory Based On Assessment His Condition (Research Journal of Pharmaceutical, Biological and Chemical Sciences, no. 7(3)), pp. 2592-2595.

[10] Zaitseva I, Malafeyev O, Strekopytvot S, Ermakova A and Shlaev D 2018 Game-theoretical model of labour force training Journal of Theoretical and Applied Information Technology.

[11] Zaitseva I, Malafeyev O, Strekopytvot S, Bondarenko G and Lovyannikov D 2018 Mathematic model of regional economy development by the final result of labor resources AIP Conference Proceedings (American Institute of Physics).

[12] Zaitseva I, Ermakova A, Shlaev D, Shevchenko E and Lugovskoy S 2017 Workforce planning redistribution of the region's results (Research Journal of Pharmaceutical, Biological and Chemical Sciences), 8(1), pp. 1862-1866.

[13] Kostyukov K, Zaitseva I, Bondarenko G, Svechinskaya T and Nechayeva S 2016 Workforce Planning as An Element of Control System (Research Journal of Pharmaceutical, Biological and Chemical Sciences, no. 7(6)), pp. 2315–2319.

[14] Vlasov M, Glebov V, Malafeyev O and Novichkov D 1986 Experimental study of an electron beam in drift space (Soviet journal of communications technology & electronics), 31(3), pp. 145-149.

[15] Malafeyev O and Redinskikh N 2018 Compromise solution in the problem of change state control for the material body exposed to the external medium AIP Conference Proceedings (American Institute of Physics) 1959, 080017 doi.org/10.1063/1.5034734.

[16] Malafeyev O, Redinskikh N, Nemnyugin S, Kolesin I and Zaitseva I 2018 The optimization problem of preventive equipment repair planning AIP Conference Proceedings (American Institute of Physics) 1978, 100013 doi.org/10.1063/1.5043757.

[17] Malafeev O and Nemnyugin S 1996 Generalized dynamic model of a system moving in an external field with stochastic components, Theoretical and Mathematical Physics, no. 107(3), , 770 p.

[18] Malafeev O 1995 On the existence of nash equilibria in a noncooperative n-person game with measures as coefficients Communications in Applied Mathematics and Computational Science, no. 5(4), pp. 689-701.

[19] Malafeev O, Awasthi A, Zaitseva I, Rezenkov D and Bogdanova S 2018 A dynamic model of functioning of a bank AIP Conference Proceedings (American Institute of Physics).

[20] O. Malafeyev, D. Rylow, I. Zaitseva, M. Popova and L Novozhilova, “Game-theoretic model of dispersed material drying process”, AIP Conference Proceedings, American Institute of Physics, 2017.

[21] Malafeev O, Saifulin D, Ivanuikovich G, Marakhov V and Zaitseva I 2017 The model of multi-agent interaction in a transportation problem with a corruption component AIP Conference Proceedings (American Institute of Physics).

[22] Malafeev O, Farvazov K, Zenovich I, Zaitseva I, Kostyukov K and Svechinskaya T 2018 Geopolitical model of investment power station construction project implementation AIP Conference Proceedings (American Institute of Physics).

[23] Malafeev O, Nemnyugin S, Rylow D, Kolpak E and Awasthi A 2017 Corruption dynamics model AIP Conference Proceedings (American Institute of Physics).

[24] Malafeev O 1974 Equilibrium situations in dynamic games Cybernetics, 10(3), pp. 504-513.

[25] Malafeev O 1977 Stationary strategies in differential games USSR Computational Mathematics
and Mathematical Physics, 17(1), pp. 37-46.
[26] Malafeev O 1974 The existence of situations of ε-equilibrium in dynamic games with dependent movements USSR Computational Mathematics and Mathematical Physics, 14(1), pp. 88-99.
[27] Malafeev O and Redinskikh N 2017 Quality estimation of the geopolitical actor development strategy Constructive Nonsmooth Analysis and Related Topics Dedicated to the Memory of V.F. Demyanov, CNSA 2017 – Proceedings.
[28] Malafeev O, Rylov D, Zaitseva I, Ermakova A and Shlaev D 2018 Multistage voting model with alternative elimination AIP Conference Proceedings 1978, 100012 doi.org/10.1063/1.5043756
[29] Kolokoltsov V and Malafeev O 2015 Mean-field-game model of corruption Dynamic Games and Applications, pp. 1-14.
[30] Kolokoltsov V and Malafeev O 2018 Corruption and botnet defense: a mean field game approach International Journal of Game Theory.
[31] Pichugin Y, Malafeev O, Rylov D and Zaitseva I 2018 A statistical method for corrupt agents detection AIP Conference Proceedings (American Institute of Physics) 1978, 100014.
[32] Pichugin Y and Malafeev O 2016 Statistical estimation of corruption indicators in the firm Applied Mathematical Sciences, 10(41-44), pp. 2065-2073.
[33] Malafeev O, Nemnyugin S and Ivaniukovich G 2015 Stochastic models of social-economic dynamics International Conference on "Stability and Control Processes" in Memory of V.I. Zubov, SCP 2015 – Proceedings, 7342178, pp. 483-485.
[34] Malafeev O and Redinskikh N 2016 Stochastic analysis of the dynamics of corrupt hybrid networks Proceedings of 2016 International Conference "Stability and Oscillations of Nonlinear Control Systems" (Pyatnitskiy's Conference), STAB, 7541208.

Acknowledgments
The work is partially supported by the RFBR grant # 18-01-00796.