Neutron stars, as remnants of supernova explosions, are excellent probes of nuclear matter equation of state (EoS) in extreme environments. Neutron-star phenomena, such as the soft γ-ray repeaters and anomalous X-ray pulsars are believed to provide evidence for magnetars. In the interior of these magnetic neutron stars, the magnetic field strength could be as high as $10^{18}$ G. Such strong magnetic fields may affect properties of neutron stars via the additional magnetic pressure and energy density and the population of Landau levels. These additions in turn affect the relative populations of various particles, and the mass-radius relation.

Neutron stars are also one of the most natural laboratories for gravity physics. Different gravity theories would result in different modified Tolman–Oppenheimer–Volkoff (TOV) equations and so one would expect different predictions for mass-radius relation. Imprecision in the terrestrial experiments on EoS is not an important issue here, because these probe densities which are an order of magnitude less than densities predicted in neutron star interior, whereas the curvature around a neutron star is almost 13 orders of magnitude larger than the largest curvature in the Solar System [2].

In this paper [1], we consider the combination of strong magnetic fields and modified gravity. To motivate for this we note that in Kaluza-Klein gravity (the five dimensional unification of gravity and electromagnetism) it may be natural to associate strong electromagnetic fields with modified gravity. We investigate, therefore, the extent that strong interior magnetic fields of neutron stars could be associated with modified gravity.

We adopt a perturbative approach to obtain modified TOV equations in $f(R) = R + aR^2$ gravity: we expand metric functions and hydrodynamic functions $P(r)$ and $\rho(r)$ perturbatively in $a$ and then general relativistic solution is taken as the zeroth order solution of the field equations. This way 4th order differential equations reduce to manageable 2nd order differential equations [3]. Then the effects of both the finite magnetic field and the modified gravity are detailed for various values of the magnetic field and the perturbation parameter $a$ along with a discussion of their physical implications.

Observation of a neutron star with a mass of $m = 1.97 \pm 0.04 M_\odot$ [4] has placed a stringent constraint on many neutron star EoS. There is an inconsistency between this observation and the super-soft EoS [5] obtained by interpreting heavy ion collision data FOPI/GSI [6]. We find that the combination of a strong magnetic field and a negative perturbation due to modified gravity can easily lead to maximum neutron star masses $> 1.97 M_\odot$ even for a very soft nuclear EoS. We note that while magnetic field stiffens the EoS, negative (positive) $a$ values tend to stiffen (soften) EoS (see Fig. 1). Hence, this may provide an alternative means to satisfy the maximum neutron star mass constraint.

![Figure 1: Mass-Radius relation corresponding to an EoS with a nph phase, i.e. including hyperons, with non-zero magnetic field and various values of $a$.](image)

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