Early fault diagnosis of rolling bearings based on signal reconstruction through SDD-SVD and FWEO

Zou Tiangang\textsuperscript{1}, Zhang Jinbao\textsuperscript{1,*}, Sun Yadong\textsuperscript{2}, Zheng Zhenqiong\textsuperscript{1}, Guo Yutong\textsuperscript{1}

\textsuperscript{1}China North Vehicle Research Institute, Beijing 100072, China
\textsuperscript{2}32382 Troops, Beijing 100072, China

*The corresponding author’s e-mail: 13B308008@hit.edu.cn

Abstract. An approach based on the singular spectrum decomposition (SSD)-singular value decomposition (SVD) and frequency weight energy operator (FWEO) was proposed for early fault diagnosis of rolling bearings. Since the interference of heavy noise in the early stage of bearing fault, SSD could eliminate abundant noise, meanwhile adaptively decompose the nonlinear, non-stationary signals into multiple mono-components which had distinct physical meanings. Subsequently, SVD was adopted for de-noising in every mono-components because the noise was distributed throughout the frequency domain. In the following, signal reconstruction was performed based on the selected mono-components with de-noising. Finally, FWEO was employed for envelop analysis and the early fault diagnosis of the rolling bearing is realized. Two groups of experiment data concerning signals of early bearing fault were investigated to valid the effectiveness of the proposed method, and good fault identification effects were obtained.

1. Introduction
Rolling bearings are critical parts of the rotating machinery, and their performance affect the operation precision and safe operation of mechanical equipment, so it is particularly important for early fault diagnosis of bearing. Due to the large amount of interference harmonics and strong noise in the vibration signal of the rolling bearing at the stage of early fault, the vibration signals show nonlinear and non-stationary characteristics [1]. So far, many time-frequency analysis methods have been applied in rolling bearing fault diagnosis, among which the combination of adaptive mode decomposition and envelope demodulation is particularly common [2-3]. The adaptive mode decomposition method, represented by empirical mode decomposition [4], can decompose fault vibration signals of the rolling bearing adaptively into a series of intrinsic mode functions with a single frequency band, and remove the high-frequency noise. Then, envelope demodulation by Hilbert transform [5] or by Teager energy operator (TEO) [6] is carried out for the intrinsic mode functions containing fault information, and the fault spectrum is obtained with fast Fourier transform. Finally, the purpose of early fault diagnosis of rolling bearing is achieved.

To deal with the problems such as mode confusion, false component and boundary effect in empirical mode decomposition, many improved adaptive decomposition methods like local mean decomposition [7], complementary ensemble empirical mode decomposition [8], intrinsic time-scale decomposition [9], the variational mode decomposition [10], and adaptive local filtering decomposition [11] have been investigated by scholars in bearing fault diagnosis. The SSD proposed by Bonizzi [12] can adaptively select the embedded dimension and decompose the original signals...
from high frequency to low frequency into several singular spectrum components (SSC), thus the interference of high frequency noise could be eliminated. At present, the application of this method in the fault diagnosis of rolling bearings has been attractive. For example, Yan X. et al. [13] improved the boundary effect of SSD, and successfully realized the fault diagnosis of rolling bearings combined with 1.5 dimensional energy spectrum. Pang B. et al. [14] differentiated the signal before SSD, and integrated each SSC after decomposition to strengthen the impact characteristics in the signals. Xu Y. et al. [15] used difference spectrum of singular values to improve the reconstruction process and the noise reduction ability of SSD, and useful information was finally effectively extracted. On the basis of tensor singular value analysis, Yi C. et al. [16] combined convex optimization and permutation entropy to estimate the tensor rank and the optimal reconstructed tensor, so that noise interference could be reduced in the signals of rolling bearings and accurate diagnostic results could be obtained.

There are mainly two kinds of envelope demodulation methods, one kind is the Hilbert transform, and the other kind is the methods that are represented by TEO [17]. The improved forms of the latter include high-order analytic energy operator (AEO) [11, 18], FWEO [8, 19] and so on. Compared with Hilbert transform, TEO can enhance the impact signals of the rolling bearing, so as to reduce the noise interference [6]. However, TEO is limited with the condition of single frequency band, and negative values are prone to appear in the process of envelope calculation, which is inconsistent with the theory. High order AEO can eliminate the influence of noise, but the order needs to be determined by observation and comparison. The FWEO can avoid the negative values in the envelop process and well enhance the impact signals of fault.

However, the noise interference covers the whole frequency domain, as well as the effect of early fault diagnosis obtained by the impact signals extracted with adaptive mode decomposition and enhanced in the following process of energy operator envelope demodulation is not outstanding, hence further noise reduction is required. SVD itself has excellent stability and invariability. When appropriate singular values are selected, the random components in the signals could be effectively eliminated through signal reconstruction and the signal-to-noise ratio of the signals is improved [20]. Based on the above analysis, an approach concerning vibration signal reconstruction of the rolling bearing based on SSD and SVD is proposed, which can effectively reduce the interference of full-frequency noise and retain useful information to the maximum extent. Furthermore, the FWEO is used for the secondary enhancement and envelope demodulation of the fault impact signals to obtain a clear envelope spectrum and realize the early fault diagnosis of the rolling bearing.

2. Signal reconstruction

2.1. Singular spectrum decomposition

Singular spectrum decomposition is a popular adaptive decomposition method for non-stationary and nonlinear signals. The signal sequence is decomposed iteratively into a series of singular spectral components with different frequency bands. The basic steps are as follows:

(1) Create a new trajectory matrix. Suppose the embedded dimension and data length of the vibration signals $x(n)$ are $M$ and $N$ respectively, so as to construct matrix $X$ with $M$ rows and $N$ columns, and the $i$th row of matrix $X$ is $x_i = \{x(i), \ldots, x(N), x(1), \ldots, x(i-1)\}$, $i = 1, 2, \ldots, M$, then the matrix $X = [x_1^T, x_2^T, \ldots, x_M^T]^T$.

(2) Adaptive selection of embedded dimension size $M$. Power spectral density (PSD) of the residual component is calculated according to the $j$th iteration, and the residual component is calculated according to the $j$th iteration, and the residual component is

$$v_j(n) = x(n) - \sum_{i=1}^{j-1} v_i(n)$$  \hspace{1cm} (1)

$$v_0(n) = x(n)$$  \hspace{1cm} (2)

The frequency $f_{\text{max}}$ corresponding to the maximum peak value in PSD is estimated. In the first iteration, if the normalized frequency $f_{\text{max}}/f_s$ is less than the established threshold value (set as $10^{-3}$),
the residual quantity is regarded as a general trend term. At this time, $M$ is set as $N/3$, where $F_s$ is the sampling frequency. Otherwise, in the case of iteration number $j > 1$, $M = 1.2 \times F_s / f_{\text{max}}$.

(3) Reconstruct the $j$th SSC component. If a large trend term is detected in the first iteration, only the first left and right eigenvectors are selected to obtain $g^{(i)}(n)$, so that $X_j = \sigma_i u_i v_i^T$. Then $g^{(i)}(n)$ is obtained through diagonal average carried out on $X_j$; Otherwise, for the number of iterations $j > 1$, the frequency component SSC$_j$ is concentrated in the frequency band $[f_{\text{max}} - \Delta f, f_{\text{max}} + \Delta f]$, where $\Delta f$ is half of the main peak bandwidth in the PSD of the residual component $v_j(n)$. Therefore, a subset $I_j = \{i_1, i_2, \ldots, i_p\}$ is created based on the all eigengroups whose left eigenvectors have the prominent peak in the frequency band $[f_{\text{max}} - \Delta f, f_{\text{max}} + \Delta f]$ and the one eigengroup that contributes the most energy to the main peak of the selected component. Then the corresponding SSC is reconstructed by the diagonal averaging calculation with the matrix $X_y = X_{i_1} + \cdots + X_{i_p}$.

(4) Iteration stop condition. The new component sequence $\tilde{g}^{(i)}(n)$ is estimated at each iteration, and the new residual component $v^{(i+1)}(n) = v^{(i)}(n) - \tilde{g}^{(i)}(n)$ is obtained which represents the input of the next iteration $j+1$. Then the standard mean square error between the residual component and the original signals is calculated as:

$$\text{NMSE}^{(i)} = \frac{\sum_{i=1}^{N} (v^{(i+1)}(i))^2}{\sum_{i=1}^{N} (x(i))^2}$$

when NMSE is less than the established threshold value (set as 0.0001 in this paper), the decomposition is terminated. Otherwise, the residual item will be iterated until the iteration stop condition is satisfied and the final SSD processing result is obtained:

$$x(n) = \sum_{k=1}^{m} g^{(i)}(n) + v^{(m+1)}(n)$$

where $m$ is the number of SSCs.

2.2. Singular value decomposition

For the discrete signal sequence $\{h_i, i = 1, 2, \ldots, N\}$, a Hankel matrix $H \in \mathbb{R}^{L \times K}$ could be obtained according to the theory of phase space reconstruction as follows:

$$H = \begin{bmatrix}
  h_1 & h_2 & \cdots & h_K \\
  h_2 & h_3 & \cdots & h_{K+1} \\
  \vdots & \vdots & \ddots & \vdots \\
  h_L & h_{L+1} & \cdots & h_N
\end{bmatrix}$$

(5)

where $N$ is the signal length, and $K = N - L + 1$.

With SVD, the trajectory matrix $H$ could be expressed as:

$$H = USV^T$$

(6)

in which $U \in \mathbb{R}^{L \times L}$, $V \in \mathbb{R}^{K \times K}$ are orthogonal matrixes, $V^T$ is the transposition of $V$; $S = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_s)$ is a diagonal matrix, and $\sigma_1, \sigma_2, \ldots, \sigma_s$ are singular values.

According to the singular value theory, the useful components of the signals corresponds to the first $k$ singular values, while the noise components corresponds to the later smaller singular values. The matrix reconstructed with the first $k$ singular values can achieve the best approximation to $H$ in the sense of Forbeieious norm, thus the noise would be reduced and the Signal to Noise Ratio is enhanced.
The difference spectrum of singular values is defined as the difference between two adjacent singular values:

$$d_i = \sigma_i - \sigma_{i+1}, \ i = 1, 2, \cdots, t$$

(7)

in which $t = \min(L, K) - 1$. According to the definition of the difference spectrum, the larger the difference between two adjacent singular values, the larger the corresponding peak value in the difference spectrum, and the more obvious the characteristics are. The signal noise reduction can be achieved through the signal reconstruction by selecting the appropriate singular value corresponding to the differential spectrum peak.

3. Energy operator for envelop

The general expression of the amplitude modulation-frequency modulation signal is:

$$x(t) = a(t)\sin[\phi(t)]$$

(8)

where $a(t)$ and $\phi(t)$ are time-varying amplitude and phase, respectively.

The analytic version of the signal $x(t)$ is:

$$y(t) = x(t) + j\hat{x}(t) = a(t)e^{j\phi(t)}$$

(9)

where $\hat{x}(t)$ is the Hilbert transform of the signal $x(t)$ and defined as:

$$\hat{x}(t) = x(t) \ast \frac{1}{\pi t} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} \, d\tau$$

(10)

3.1. Teager energy operator

The expression of TEO is shown as follows:

$$\Psi_T [x(t)] = \left[ x'(t) \right]^2 - x(t) x'(t) = a^2(t) \omega^2(t)$$

(11)

where $\omega(t)$ is the instantaneous frequency.

For the discrete-time signal, the time derivative could be approximately expressed by time difference as:

$$\Psi_{Td} [x(n)] = \left[ x(n) \right]^2 - x(n+1)x(n-1)$$

(12)

3.2. Analytic energy operator

The AEO is expressed as follows:

$$\Psi_A [x(t)] = x'(t)\hat{x}(t) - x(t)\hat{x}'(t) = a^2(t)\omega(t)$$

(13)

Then the corresponding discrete expression is:

$$\Psi_{Ad} [x(n)] = x(n+1)\hat{x}(n) - \hat{x}(n+1)x(n)$$

(14)

3.3. Frequency weight energy operator

The expression of FWEO is shown as follows:

$$\Gamma [x(t)] = \hat{x}(t) + jH[\hat{x}(t)] = \hat{x}(t) + H[\hat{x}(t)]$$

(15)

$$\Gamma [x(n)] = \frac{1}{4} \left[ x^2(n+1) + x^2(n-1) + h^2(n+1) + h^2(n-1) \right] +$$

$$\frac{1}{2} \left[ x(n+1) + x(n-1) + h(n+1) + h(n-1) \right]$$

(16)

where $h(n)$ is the discrete Hilbert transform and defined as $h(n) = H[x(n)]$.

From the comparison of three energy operators, it is found that only two or three data points are considered at each time point, which make them easy to calculate.
4. Procedure of early fault diagnosis

Procedure of the early fault diagnosis was as follows:

(1) Rolling bearing fault vibration signals were collected as well as the corresponding information concerning working condition, then the number of vibration signal was set for analysis, and the theoretical fault characteristic frequency of each part of bearings was calculated.

(2) The adaptive mode decomposition of vibration signals were performed with singular spectrum decomposition, and kurtosis values of each intrinsic mode function were calculated.

(3) The intrinsic mode functions with kurtosis value greater than 3 were selected for noise reduction with singular value decomposition, and vibration signals were reconstructed with the sum of SSCs with de-noising.

(4) The envelope demodulation was carried out for the reconstructed signals with the frequency weighted energy operator, and spectrum analysis was conducted with fast Fourier transform.

(5) The theoretical fault characteristic frequency with the frequency in the spectrum were compared, then the fault of the rolling bearing was identified.

5. Experimental verification

In this section, the testing data for verification were from Intelligent Maintenance System in University of Cincinnati. The rotation speed of shaft was controlled at 2000 rpm and a radial load of 6000 lb was placed on the shaft by a spring mechanism. The data length in the save file was 20480 points with sampling rate 20 kHz. Details of the experiment could refer to [21]. Rexnord ZA-115 double row bearings were used in the test with geometry parameters and the fault characteristic frequency listed in Table 1. The ball pass frequency on outer race is calculated as follows:

\[
f_o = 0.5z \left[ 1 - \frac{d}{D} \cos(\alpha) \right] f_r,
\]

where \(d\) is the diameter of the rolling element, \(D\) is the pitch diameter, \(\alpha\) is the contact angle, \(z\) is the number of rolling elements and \(f_r\) the shaft speed.

\[
f_o = 0.5z \left[ 1 - \frac{d}{D} \cos(\alpha) \right] f_r, \quad (17)
\]

Table 1 Structural parameters of bearing Rexnord ZA-2115 and the fault characteristic frequency

| \(d\)  | \(D\)  | \(z\) | \(\alpha\) | \(f_r\) | \(f_o\) |
|------|------|------|-------|-------|-------|
| 8.4 mm | 71.5 mm | 16 | 15.17° | 33.3 Hz | 236.4 Hz |

The first bearing in the second group (bearing 2-1#) was employed for the bearing early fault diagnosis, and the vibration signals as shown in Figure 1(a) with 8192 points in the save file at time 5340 min were selected for study. The corresponding spectrum was shown in Figure 1(b), in which the fault characteristic frequency was not distinct and strongly interfered by noise. The corresponding TEO, AEO and FWEO of the vibration signals were shown from Figure 2(a) to Figure 4(a), respectively. From the comparison, it was shown that the negative values appeared in TEO and AEO, though a better improvement in AEO, while FWEO avoided this situation. In further comparison of the spectra concerning the three operators as shown from Figure 2(b) to Figure 4(b), FWEO could eliminate the noise with frequency above 6000Hz, but its advantage in the extraction of the fault characteristic frequency was not obvious compared with TEO and AEO.
Figure 2 (a) TEO of the original signals and (b) the corresponding spectrum

Figure 3 (a) AEO of the original signals and (b) the corresponding spectrum

Figure 4 (a) FWEO of the original signals and (b) the corresponding spectrum

Figure 5 (a) SSCs by SSD and (b) the corresponding spectra
An adaptive decomposition was performed on the original vibration signals with SSD as shown in Figure 5, and the spectra of all SSCs demonstrated that SSD could efficiently reduce noise with high frequency and interference harmonics with low frequency. To prevent the loss of the fault information, the SSCs with kurtosis value above 3, namely SSC2, SSC3 and SSC4, were selected for signal de-noising. In the process of de-noising through SVD, $L$ was set as $N/2$. The number of singular values (SV) were respectively selected as 62, 12 and 44 according to the difference spectrum (DiffS) as shown in Figure 6. Compared with the constructed signals in Figure 7(a) with the sum of SSC2, SSC3 and SSC4, the constructed signals in Figure 8(a) with the sum of SSC2, SSC3 and SSC4 after de-noising by SVD would be more distinct. FWEOs of the constructed signals without de-noising and with de-noising were shown in Figure 7(b) and Figure 8(b), as well as the corresponding spectra in Figure 9. From the comparison in Figure 9, the fault characteristic frequency of the constructed signals after de-noising was shown clearly and a correct result was obtained.

6. Conclusion
An approach of vibration signal reconstruction based on singular spectrum decomposition and singular value decomposition was proposed, and envelope demodulation with frequency weighted energy...
operator was combined to realize early fault diagnosis of rolling bearings. Through the analysis and comparison of the results concerning the experimental data of rolling bearing, the following conclusions can be drawn:

(1) Singular spectrum decomposition could extract intrinsic mode functions containing rolling bearing fault information and eliminate the interference of the high-frequency noise; while singular value decomposition could eliminate the interference of the low-frequency noise, hence the combination of the two methods could effectively reconstruct the rolling bearing fault signals.

(2) Compared with TEO and AEO, negative values were in the process of the envelope demodulation with FWEO, and the FWEO had more accurate physical meaning and could better reduce the high-frequency noise interference.

(3) The combined method proposed in this paper was an effective method for early fault diagnosis of rolling bearings.

References

[1] Randall R. B., Antoni J. (2011) Rolling element bearing diagnostics-a tutorial. Mechanical Systems and Signal Processing, 25(2): 485-520.

[2] Feng Z., Liang M., Chu F. (2013) Recent advances in time-frequency analysis methods for machinery fault diagnosis: a review with application examples. Mechanical Systems and Signal Processing, 38(1): 165-205.

[3] Feng Z., Zhang D., Zuo M. J. (2017) Adaptive mode decomposition methods and their applications in signal analysis for machinery fault diagnosis: a review with examples. IEEE Access, 5: 24301-24331.

[4] Huang N. E., Shen Z., Long S. R., et al. (1998) The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series. Proceedings of the Royal Society of London Series A-Mathematical Physical and Engineering Sciences, 454(1971): 903-995.

[5] Rai V. K., Mohanty A. R. (2007) Bearing fault diagnosis using FFT of intrinsic mode functions in Hilbert-Huang transform. Mechanical Systems and Signal Processing, 21(6): 2607-2615.

[6] Cheng J., Yu D., Yu Y. (2007) The application of energy operator demodulation approach based on EMD in machinery fault diagnosis. Mechanical Systems and Signal Processing, 21(2): 668-677.

[7] Wang J., Li J., Wan X. (2015) Fault feature extraction method of rolling bearings based on singular value decomposition and local mean decomposition. Journal of Mechanical Engineering, 51(3): 104-110. (in Chinese)

[8] Imaouchen Y., Kedadouche M., Alkama R., et al. (2017) A Frequency-Weighted Energy Operator and complementary ensemble empirical mode decomposition for bearing fault detection. Mechanical Systems and Signal Processing, 82: 103-116.

[9] Yu J., Liu H. (2018) Sparse coding shrinkage in intrinsic time-scale decomposition for weak fault feature extraction of bearings. IEEE Transactions on Instrumentation and Measurement, 67(7): 1579-1592.

[10] Zhao H., Guo S., Gao D. (2016) Fault feature extraction of bearing faults based on singular value decomposition and variational modal decomposition. Journal of Vibration and Shock, 35(22): 183-188. (in Chinese)

[11] Zhang J., Zhao Y., Liu M., et al. (2020) An improved higher-order analytical energy operator with adaptive local iterative filtering for early fault diagnosis of bearings. Journal of Vibroengineering, 22(1): 67-82.

[12] Bonizzi P., Karel J. M. H., Meste O., et al. (2014) Singular spectrum decomposition: a new method for time series decomposition. Advances in Adaptive Data Analysis, 06(04): 1450011.

[13] Yan X., Jia M. (2019) Improved singular spectrum decomposition based 1.5-dimensional energy spectrum for rotating machinery fault diagnosis. Journal of the Brazilian Society of
Mechanical Sciences and Engineering, 41: 50.

[14] Pang B., Tang G., Tian T. (2019) Enhanced singular spectrum decomposition and its application to rolling bearing fault diagnosis. IEEE Access, 7: 87769-87782.

[15] Xu Y., Zhang Z., Ma C., et al. (2019) Improved singular spectrum decomposition and its applications in rolling bearing fault diagnosis. Journal of Vibration Engineering, 32(3): 540-547. (in Chinese)

[16] Yi C., Lv Y., Ge M., et al. (2017) Tensor singular spectrum decomposition algorithm based on permutation entropy for rolling bearing fault diagnosis. Entropy, 19: 139.

[17] Boudraa A. O., Salzenstein F. (2018) Teager-Kaiser energy methods for signal and image analysis: a review. Digital Signal Processing, 78: 338-375.

[18] Faghidi H., Liang M. (2015) Detection of bearing fault detection from heavily contaminated signals: a higher-order analytic energy operator method. Journal of Vibration and Acoustics, 137(4): 041012.

[19] Toole J. M. O., Temko A., Stevenson N. (2014) Assessing instantaneous energy in the EEG: a non-negative, frequency-weighted energy operator. In: 36th Annual International Conference of the IEEE Engineering in Medicine and Biology Society. pp. 3288-3291.

[20] Zhu Q., Liu J., Li Y. (2005) Study on noise reduction in singular value decomposition based on structural risk minimization. Journal of Vibration Engineering, 18(2): 204-207.

[21] Qiu H., Lee J., Lin J., et al. (2006) Wavelet filter-based weak signature detection method and its application on roller bearing prognostics. Journal of Sound and Vibration, 289(4-5): 1066-1090.