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A Novel Dynamic Intuitionistic Fuzzy MADM Approach

Hui Zhou¹, Haojie Gu²

College of Mathematics and Computer Science, Yichun University, Yichun, Jiangxi, 336000, China
huihui7978@126.com

Abstract. This paper explores the multiple attribute decision making problems with dynamic intuitionistic fuzzy information. The notion of intuitionistic fuzzy variable is defined, and one new aggregation operator: dynamic dependent intuitionistic fuzzy Einstein weighted average (DDIFEWA) operator is presented. And based on the D-DIFEWA operator, we develop a procedure to solve the dynamic intuitionistic fuzzy multi-attribute decision making (DIF-MADM) problems where all the decision information about attribute values takes the form of intuitionistic fuzzy numbers and is collected at different periods. Finally, a numerical example is presented to show the applicability of the proposed method for the advanced mathematics teaching effectiveness problem and a sensitivity analysis is conducted to demonstrate efficiency of dynamic evaluation.

1. Introduction

Teacher’s level played a key role in the training of person with higher ability. How objective and impartial evaluation of a teacher’s teaching appears to be particularly important. Fuzzy logic and multi-attribute decision-making (MADM) methods are often used and a more applicable decision-making environment is provided for decision makers.

Most of the current scholars mainly study on single time period of intuitionistic fuzzy multiple attribute decision making problems. When they make comparison of alternatives, only considering single time period of decision information and ignoring the impact of timing characteristics on decision results, and lead to the decision results are not comprehensive and scientific. Some scholars began to study on dynamic intuitionistic fuzzy multiple attribute decision making problems embedded characteristics of sequential information at different time periods based on this idea. The operations, relations and operators in the previous studies can only be used to deal with time independent arguments. However, if we take time into consideration, for example, we collect the argument information at different periods for time pressure, the aggregation operators and their associated weights should not be kept constant. In view of the illegibility of teaching effectiveness evaluation, intuitionistic fuzzy information comes to the basic theory of many teaching evaluation approaches.

This paper explores the fuzzy multi-attribute decision making problems where all the attribute values are expressed in intuitionistic fuzzy numbers collected at different periods (for convenience, we call this kind of problems dynamic intuitionistic fuzzy multi-attribute decision making (DIF-MADM) problems). Motivated by the ideal of dependent aggregation[1], we first introduce the notion of intuitionistic fuzzy variable and develop a dynamic dependent intuitionistic fuzzy Einstein weighted average (DDIFEWA) operator, in which the associated weights only depend on the aggregated intuitionistic fuzzy arguments and can relieve the influence of unfair intuitionistic fuzzy arguments on the aggregated results by assigning low weights to those false and biased ones. In Section 2 one method
to determine the weight vectors associated with the operator is introduced and a procedure for DIF-MADM is developed. Finally, a numerical example for evaluating advanced mathematics teaching with dynamic intuitionistic fuzzy information is given to illustrate the applicability and effectiveness of the proposed approach.

2. Methodology

2.1 Dynamic dependent intuitionistic fuzzy Einstein weighted average operator

Based on [2-5], the definition of the notion of intuitionistic fuzzy variable is firstly given. Then, the Einstein operations on intuitionistic fuzzy sets are introduced and some desirable properties of these operations are analyzed. Motivated by Einstein operations, let the t-norm $T$ and t-conorm $S$ be Einstein product $T$ and Einstein sum $S$ respectively, then the generalised intersection and union on two IFSS $A$ and $B$ become the Einstein product (denoted by $\alpha \otimes \alpha$) and Einstein sum (denoted by $\alpha \oplus \alpha$) on two IVIFS $\alpha_1$ and $\alpha_2$, respectively, as follows [4,6].

Definition 2.1 Let $t$ be a time variable, then we call $\alpha(t) = (\mu_\alpha(t), \nu_\alpha(t))$ an intuitionistic fuzzy variable, where $\mu_\alpha(t) \in [0, 1], \nu_\alpha(t) \in [0, 1], \mu_\alpha(t) + \nu_\alpha(t) \leq 1$ (2.1)

For an intuitionistic fuzzy variable $\alpha(t) = (\mu_\alpha(t), \nu_\alpha(t))$, if $t = t_1, t_2, \cdots, t_p$, then $\alpha(t_1), \alpha(t_2), \cdots, \alpha(t_p)$ indicate $p$ IFNs collected at $p$ different periods. Below we introduce some operations related to IFNs.

Definition 2.2 Let $\alpha(t_1) = (\mu_{\alpha(t_1)}, \nu_{\alpha(t_1)})$ and $\alpha(t_2) = (\mu_{\alpha(t_2)}, \nu_{\alpha(t_2)})$ be two IFNs at two different periods $t_1$ and $t_2$, then $\alpha(t_1) \otimes \alpha(t_2) = \left( \frac{\mu_{\alpha(t_1)} \mu_{\alpha(t_2)}}{1+\left(1-\mu_{\alpha(t_1)}\right)\left(1-\mu_{\alpha(t_2)}\right)}, \frac{\nu_{\alpha(t_1)} + \nu_{\alpha(t_2)}}{1+\left(1-\mu_{\alpha(t_1)}\right)\left(1-\mu_{\alpha(t_2)}\right)} \right)$ (2.2)

Definition 2.3 Let $\alpha(t_1), \alpha(t_2), \cdots, \alpha(t_p)$ be a collection of IFNs collected at $p$ different periods $t_k \ (k = 1, 2, \cdots, p)$, and $\lambda(t) = \left( \lambda(t_1), \lambda(t_2), \cdots, \lambda(t_p) \right)^T$ be the weight vector of the periods $t_k \ (k = 1, 2, \cdots, p)$, then we call $DIFEW \ A_{\lambda(i)}(\alpha(t_1), \alpha(t_2), \cdots, \alpha(t_p)) = \otimes_{\alpha(t_j)}^\lambda \left( \lambda(t_1), \lambda(t_2), \cdots, \lambda(t_p) \right)$ (2.2)

a dynamic intuitionistic fuzzy Einstein weighted averaging (DIFEW) operator.

By definition 2.1, (2.2) can be rewritten as follows: $DIFEW \ A_{\lambda(i)}(\alpha(t_1), \alpha(t_2), \cdots, \alpha(t_p))$

$$= \left( \prod_{j=1}^p \left(1 + \mu_{\alpha(t_j)}\right) \right)^{\lambda(t_j)} - \left( \prod_{j=1}^p \left(1 + \mu_{\alpha(t_j)}\right) \right)^{\lambda(t_j)} + \left( \prod_{j=1}^p \left(1 + \mu_{\alpha(t_j)}\right) \right) \left( \prod_{j=1}^p \left(2 - \nu_{\alpha(t_j)}\right) \right)^{\lambda(t_j)} + \left( \prod_{j=1}^p \left(1 + \mu_{\alpha(t_j)}\right) \right) \left( \prod_{j=1}^p \left(2 - \nu_{\alpha(t_j)}\right) \right)^{\lambda(t_j)}$$

(2.3)

where

$$\lambda\left(t_j\right) \geq 0, \ j = 1, 2, \cdots, p, \ \sum_{j=1}^p \lambda(t_j) = 1$$

(2.4)
2.2A Method to determine the weight vector

**Definition 2.4** Let \( \alpha(t_j) = \left( \mu_{a(t_j)}, \nu_{a(t_j)} \right) \) \( (j = 1, 2, \cdots, p) \) be a collection of intuitionistic fuzzy numbers, the average value of the score function of \( \alpha(t_j) \) is computed as \( s(\alpha(t_j)) = \frac{\sum_{j=1}^{p} s(\alpha(t_j))}{p} \) (2.5)

**Definition 2.5** Let \( \alpha(t_1) = \left( \mu_{a(t_1)}, \nu_{a(t_1)} \right) \) and \( \alpha(t_2) = \left( \mu_{a(t_2)}, \nu_{a(t_2)} \right) \) be two intuitionistic fuzzy numbers, then the Hamming distance between \( \alpha(t_1) = \alpha(t_2) = \left( \mu_{a(t_1)}, \nu_{a(t_1)} \right) \) and \( \alpha(t_2) = \left( \mu_{a(t_2)}, \nu_{a(t_2)} \right) \) is defined as follows:
\[
d(d(\alpha(t_1), \alpha(t_2))) = \frac{\left| \mu_{a(t_1)} - \mu_{a(t_2)} \right| + \left| \nu_{a(t_1)} - \nu_{a(t_2)} \right|}{2} \tag{2.6}
\]

**Definition 2.6** Let \( \alpha(t_j) = \left( \mu_{a(t_j)}, \nu_{a(t_j)} \right) \) \( (j = 1, 2, \cdots, n) \) be a collection of intuitionistic fuzzy numbers, then we call
\[
sim(s(\alpha(t_j)), s(\alpha(t_i))) = 1 - \frac{d(s(\alpha(t_j)), s(\alpha(t_i))))}{\sum_{j=1}^{n} d(s(\alpha(t_j)), s(\alpha(t_i))))} \tag{2.7}
\]
the degree of similarity between the intuitionistic fuzzy values \( s(\alpha(t_j)) \) and the mean \( s(\alpha) \).

In real-life situations, unduly high or unduly low preference values may be assigned to their preferred or repugnant objects by some individuals. In such cases, we shall assign very low weights to these “false” or “biased” opinions, that is to say, the closer a preference value (argument) is to the mid one(s), the more the weight. As a result, based on [5], we define the DIFEWA operator weights as
\[
\omega(t_j) = \frac{\sum_{i=1}^{n} \frac{\sim(s(\alpha(t_j)), s(\alpha(t_1))))}{\sum_{i=1}^{n} \sim(s(\alpha(t_j)), s(\alpha(t_i))))}}{n}, j = 1, 2, \cdots, n \tag{2.8}
\]
Obviously, \( \omega(t_j) \geq 0, \ j = 1, 2, \cdots, n \) and \( \sum_{j=1}^{n} \omega(t_j) = 1 \). Especially, if \( s(\alpha(t_j)) = s(\alpha(t_i)) \), for all \( i, j = 1, 2, \cdots, n \), then by (2.4), we have \( \omega(t_j) = 1/n \), for all \( j = 1, 2, \cdots, n \).

By (2.3), we have \( \text{DDIFEWA} \ A(\alpha_1, \alpha_2, \cdots, \alpha_n) = \bigoplus_{j=1}^{n} \left[ \sum_{j=1}^{n} \frac{\sim(s(\alpha(t_j)), s(\alpha(t_1))))}{\sum_{i=1}^{n} \sim(s(\alpha(t_j)), s(\alpha(t_i))))} \right] \)
should satisfy the attribute
\( \alpha(t_j) \) and should not satisfy the attribute
\( \alpha(t_j) \).

\[ \text{It is called a dynamic dependent intuitionistic fuzzy Einstein weighted average (D-DIFEWA) operator.} \]

**Theorem 2.1** Let \( \alpha(t_j) = (\mu_{\alpha(t_j)}, \nu_{\alpha(t_j)}) \) \( (j = 1, 2, \ldots, n) \) be a collection of intuitionistic fuzzy numbers, and let \( s(\alpha(t)) \) the average value of the score function of \( \alpha(t_j) \) \( (j = 1, 2, \ldots, n) \), if

\[ \text{sim}\{s(\alpha(t_j)), s(\alpha(t))\} \geq \text{sim}\{s(\alpha(t_j)), s(\alpha(t))\}, \text{then }\omega(t_i) \geq \omega(t_j). \]

\[ \text{2.3 Algorithm} \]

In this section, we shall apply DDIFEWA operator to the multiple attribute decision making problems with intuitionistic fuzzy numbers, which are collected at different periods. The following notations are used to depict the considered problems:

- **A**: A discrete set of \( n \) feasible alternatives.
- **G = \{G_1, G_2, \ldots, G_m\}**: A finite set of attributes, whose weight vector is \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \), where \( \omega_j \geq 0, \quad j = (1, 2, \ldots, m), \sum \omega_j = 1 \).
- There are \( p \) periods \( t_k (k = 1, 2, \ldots, p) \), whose weight vector is

\[ \lambda(t) = (\lambda(t_1), \ldots, \lambda(t_p))^T, \quad \text{where } \lambda(t_k) \geq 0, \quad k = (1, 2, \ldots, p), \quad \sum_{k=1}^{p} \lambda(t_k) = 1 \cdot R(t_k) = (r_{G_j}(t_k))_{n \times m} \]: An intuitionistic fuzzy decision matrix of the period \( t_k \), where \( r_{G_j}(t_k) = (\mu_{G_j(t_k)}, \nu_{G_j(t_k)}) \) is an attribute value, denoted by an IFN, \( \mu_{G_j(t_k)} \) indicates the degree that the alternative \( x_i \) should satisfy the attribute \( G_j \) at the period \( t_k \), \( \nu_{G_j(t_k)} \) indicates the degree that the alternative \( x_i \) should not satisfy the attribute \( G_j \) at the period \( t_k \), such that \( \mu_{G_j(t_k)} \in [0, 1], \nu_{G_j(t_k)} \in [0, 1], \mu_{G_j(t_k)} + \nu_{G_j(t_k)} \leq 1, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m \)

Based on the above decision information, a practical procedure is proposed in the following part:

**Step 1.** Utilize the DIFEWA operator:

\[ \prod_{j=1}^{p} \left( 1 + \mu_{\alpha(t_j)} \right) - \prod_{j=1}^{p} \left( 1 - \mu_{\alpha(t_j)} \right), \]

\[ = \prod_{j=1}^{p} \left( 1 + \mu_{\alpha(t_j)} \right) + \prod_{j=1}^{p} \left( 1 - \mu_{\alpha(t_j)} \right) \]

\[ \prod_{j=1}^{p} \left( 2 - \nu(\alpha(t_j)) \right) \]

(2.9)
to aggregate all the intuitionistic fuzzy decision matrices \( R(t_k) = (r_{ij}(t_k))_{m \times n} \) \((k = 1, 2, \ldots, p)\) into a complex intuitionistic fuzzy decision matrix \( R = (r_{ij})_{m \times n} \) , where \( r_{ij} = (\mu_{ij}, \nu_{ij}) \).

\[
\mu_{ij} = \prod_{k=1}^{n} \left(1 + \mu_{ik}(t_k)\right)^{\delta_{ik}(t_k)} - \prod_{k=1}^{n} \left(1 + \mu_{jk}(t_k)\right)^{\delta_{jk}(t_k)}, \quad \nu_{ij} = \prod_{k=1}^{n} \left(2 - \nu(\alpha(t_k))\right)^{\delta_{ik}(t_k)} + \prod_{k=1}^{n} \nu_{jk}(t_k),
\]

**Step 2.** Define \( \alpha^+ = (\alpha^+_1, \alpha^+_2, \cdots, \alpha^+_m)^T \) and \( \alpha^- = (\alpha^-_1, \alpha^-_2, \cdots, \alpha^-_m)^T \) as the intuitionistic fuzzy ideal solution (IFIS) and the intuitionistic fuzzy negative ideal solution (IFNIS), respectively, where \( \alpha^+_i = (1, 0) \) \((i = 1, 2, \cdots, m)\) are the \( m \) largest IFNs, and \( \alpha^-_i = (0, 1) \) \((i = 1, 2, \cdots, m)\) are the \( m \) smallest IFNs. Furthermore, for convenience of depiction, we denote the alternatives \( x_i (i = 1, 2, \cdots, n) \) by

\[
x_i = (r_{i1}, r_{i2}, \cdots, r_{in})^T (i = 1, 2, \cdots, n)
\]

**Step 3.** Calculate the distance between the alternative \( x_i \) and the IFIS \( \alpha^+ \) and the distance between the alternative \( x_i \) and the IFNIS \( \alpha^- \), respectively:

\[
d(x_i, \alpha^+) = \sum_{j=1}^{m} \omega_j d(r_{ij}, \alpha^+_j), \quad d(x_i, \alpha^-) = \sum_{j=1}^{m} \omega_j d(r_{ij}, \alpha^-_j), \quad r_{ij} = (\mu_{ij}, \nu_{ij}), \quad i = 1, 2, \cdots, n, \quad j = 1, 2, \cdots, m.
\]

**Step 4.** Calculate the closeness coefficient of each alternative:

\[
c(x_i) = \frac{d(x_i, \alpha^-)}{d(x_i, \alpha^+) + d(x_i, \alpha^-)}, \quad i = 1, 2, \cdots, n
\]

**Step 5.** Rank all the alternatives \( x_i (i = 1, 2, \cdots, n) \) according to the closeness coefficients \( c(x_i) \) \((i = 1, 2, \cdots, n)\), the greater the value \( c(x_i) \), the better the alternative \( x_i \).

3. A numerical example

This section presents a numerical example to evaluate the advanced mathematics teaching effectiveness with dynamic intuitionistic fuzzy information to illustrate the method proposed in this paper. There are seven possible advanced mathematics instructors \( x_i (i = 1, 2, 3, 4, 5, 6, 7) \). In order to prioritize these advanced mathematics instructors with respect to their teaching effectiveness, a committee has been set up to provide assessment information on them. The attributes which are considered here in assessment of them are: (1) \( G_1 \), is peer evaluation of teaching effectiveness; (2) \( G_2 \), is student evaluation of teaching effectiveness teaching methods; (3) \( G_3 \), is self-evaluation of teaching effectiveness. The committee evaluates the performance of advanced mathematics teachers in the years 2014-2016 according to the attributes \( G_j (j = 1, 2, 3) \), and constructs, respectively, the intuitionistic fuzzy decision matrices...
\( R(t_k)(k=1, 2, 3) \) here, \( t_1 \) denotes the year “2014”, \( t_2 \) denotes the year “2015”, and \( t_3 \) denotes the year “2016” as listed in Tables 1-3. Let \( \lambda(t) = (1/6, 2/6, 3/6)^T \) be the weight vector of the years \( t_k(k=1, 2, 3) \), and \( \omega = (0.3, 0.4, 0.3)^T \) be the weight vector of the attributes \( G_j(j=1, 2, 3) \).

Now we utilize the proposed procedure to prioritize these advanced mathematics instructors:

**Step 1.** Utilize the DDIFEWA operator (2.9) to aggregate all the intuitionistic fuzzy decision matrices \( R(t_k) \) into a complex intuitionistic fuzzy decision matrix \( R \) (see Table 4).

**Step 2.** Denote the IFIS \( \alpha^+ \), IFNIS \( \alpha^- \), and the alternatives \( x_i(i = 1, 2, \cdots, 7) \) by

\[
\alpha^+ = \left( \begin{array}{ccc}
1, 0, 0, & 1, 0, 0, & 1, 0, 0
\end{array} \right)^T,
\alpha^- = \left( \begin{array}{ccc}
0, 1, 0, & 0, 1, 0, & 0, 1, 0
\end{array} \right)^T
\]

and utilize (2.20) to calculate the closeness coefficient of each alternative:

\[
c(x_1) = 0.846, c(x_2) = 0.698, c(x_3) = 0.663, c(x_4) = 0.817, c(x_5) = 0.685, c(x_6) = 0.502, c(x_7) = 0.524
\]

**Step 3.** Rank all the alternatives \( x_i(i = 1, 2, \cdots, 7) \) according to the closeness coefficients \( c(x_1) x_1 \succ x_4 \succ x_2 \succ x_5 \succ x_3 \succ x_7 \succ x_6 \) and thus the advanced mathematics teacher with the most teaching effectiveness is the first advanced mathematics teacher.

**Table 1. Intuitionistic fuzzy decision matrix** \( R(t_1) \)

| \( x_1 \)  | \( x_2 \)  | \( x_3 \)  | \( x_4 \)  | \( x_5 \)  | \( x_6 \)  | \( x_7 \)  |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| (0.8, 0.1, 0.1) | (0.9, 0.1, 0.0) | (0.7, 0.2, 0.1) | (0.7, 0.3, 0.1) | (0.5, 0.4, 0.1) | (0.5, 0.2, 0.3) | (0.5, 0.2, 0.3) |

**Table 2. Intuitionistic fuzzy decision matrix** \( R(t_2) \)

| \( x_1 \)  | \( x_2 \)  | \( x_3 \)  | \( x_4 \)  | \( x_5 \)  | \( x_6 \)  | \( x_7 \)  |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| (0.9, 0.1, 0.0) | (0.8, 0.2, 0.0) | (0.8, 0.1, 0.1) | (0.8, 0.2, 0.0) | (0.8, 0.3, 0.0) | (0.4, 0.6, 0.0) | (0.3, 0.5, 0.2) |

**Table 3. Intuitionistic fuzzy decision matrix** \( R(t_3) \)

| \( x_1 \)  | \( x_2 \)  | \( x_3 \)  | \( x_4 \)  | \( x_5 \)  | \( x_6 \)  | \( x_7 \)  |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| (0.7, 0.1, 0.2) | (0.9, 0.1, 0.0) | (0.9, 0.1, 0.0) | (0.9, 0.1, 0.0) | (0.7, 0.2, 0.1) | (0.5, 0.1, 0.4) | (0.5, 0.5, 0.0) |

**Table 4. Complex Intuitionistic fuzzy decision matrix**

| \( x_1 \)  | \( x_2 \)  | \( x_3 \)  | \( x_4 \)  | \( x_5 \)  | \( x_6 \)  | \( x_7 \)  |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| (0.782, 0.1, 0.0) | (0.856, 0.144, 0.000) | (0.817, 0.117, 0.062) | (0.835, 0.165, 0.000) | (0.534, 0.172, 0.294) | (0.573, 0.226, 0.201) | (0.437, 0.426, 0.137) | (0.729, 0.173, 0.098) | (0.712, 0.141, 0.147) | (0.838, 0.1, 0.062) | (0.776, 0.140, 0.084) | (0.813, 0.187, 0.000) | (0.547, 0.202, 0.251) | (0.724, 0.241, 0.035) | (0.621, 0.196, 0.183) | (0.263, 0.659, 0.078) | (0.420, 0.233, 0.347) | (0.369, 0.251, 0.380) | (0.364, 0.491, 0.145) | (0.589, 0.358, 0.053) | (0.517, 0.483, 0.000) |
4. Conclusion
Further studies will be done to probe into the evaluation of detailed criterion of advanced mathematics teaching effectiveness. As a prospect, the method proposed here could be applied to other problems.

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