Film flows with recirculation: hydrodynamics and heat transfer

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Abstract. Hydrodynamics and heat transfer in film flows along solid surfaces are analyzed by numerical simulation. The current version of the Kolmogorov-Prandtl turbulence model is used to obtain a compact mathematical description focused on engineering applications in power engineering and other heat technologies. Film flows with recirculation are studied as special modes of condensing (or evaporative) devices of power plants. Other possible applications are the problems of evaporative cooling, problems of hydraulic roughness of technical surfaces, technologies of thin-film materials and – in the field of large linear scales - modeling of some natural catastrophic phenomena, such as mudflows. The mathematical description is presented as a system of three ordinary first-order nonlinear differential equations - for distributions of turbulent energy, turbulent energy flux density, flow velocity. The phenomena of laminar – turbulent transition and special modes of “flooding” of the flow are diagnosed. Temperature distributions are calculated and heat fluxes are determined. The Reynolds number of the film and the Stanton number are presented as functions of two dimensionless determining parameters specifying the film thickness and the ratio of the friction stresses at the boundaries.

1. Introduction

The term “film” flow (figure 1) means a one-dimensional stabilized flow along a solid surface in a liquid layer of thickness δ. Velocity and temperature are substantially variable normal to the wall, while longitudinal changes are absent (or negligible). On another boundary surface (interphase boundary in the problems of evaporation / condensation), a dynamic interaction occurs with the external environment, which is specified by the tangential friction stress. The external boundary may also be the plane of symmetry of the stabilized flow in a flat channel, or, to some extent, the external boundary of the boundary layer.

Engineering applications of film flows are very relevant in the energy sector and other heat technologies. First of all, these are condensation and evaporation plants for various purposes. The currently used calculation methods are based on classical solutions for the limiting regimes of a gravitational or shear, laminar or turbulent film, while in practice various combinations of acting factors are always relevant [2,3]. The model presented below solves the problem of a generalized description of film flows with various combinations of acting forces and flow regimes. Actual are results of numerical modeling in the region of laminar-turbulent transition, taking into account possible strong deformations of the main flow.

A special problem is the complex regimes of film flows with recirculation — the return flow under the action of multidirectional current forces (surface friction, gravity, pressure). This combination of forces is natural for film evaporators with a flowing down liquid film and the lifting movement of the
vapor stream. Another example is the operation of the steam generator of a nuclear power plant in condensation mode in the SPOT system during a hypothetical accident, with the potential risk of "flooding".

Possible applications are problems of hydraulic roughness of technical surfaces, technologies for creating thin-film materials, or - in the field of large linear scales - modelling of some natural catastrophic phenomena, such as mudflows.

The central place in the mathematical description of the problem is occupied by the Kolmogorov – Prandtl turbulence model with one differential conservation equation — for turbulent energy (in the “low Reynolds” version, with the necessary corrections for weak turbulence in the near-wall region). [4]). For a problem with a clearly defined single linear scale (film thickness δ), this formulation gives an adequate description of turbulence. The result is a compact mathematical description corresponding to the complexity of the phenomenon under consideration and focused on engineering applications.

The system of ordinary differential equations of the first order determines the distribution over the film thickness of turbulent energy, turbulent energy flux density, flow velocity. Dimensionless formulations are obtained in traditional “wall variables,” with a dynamic velocity (“friction velocity”) as a characteristic scale for the velocity field. The friction stresses on the wall (τW) and on the external surface (τS) are assumed as the parameters of the problem.

Special procedures for numerical integration solve effectively a nonlinear system of differential equations with the singularity at the origin. Distributions of characteristic values over the film thickness (including turbulent transfer coefficients) demonstrate very complex structures of film flows with different friction parameters at the boundaries. The phenomena of laminar - turbulent transition, special modes of "flooding" of the flow are diagnosed. The temperature distributions are calculated (under the assumption that the heat flux density is constant, which is natural for film flows).

The integral (over the film thickness) characteristics are determined, namely, the Reynolds number of the film and the Stanton number as functions of two dimensionless parameters: the dimensionless film thickness and the ratio of the friction stresses at the interface and on the wall. Thus, a generalized representation of hydrodynamics and heat transfer is obtained for various modes of film condensation (or film evaporation), that is, for laminar, turbulent, or transitional flow regimes, with various combinations of friction at the interface, gravity, and longitudinal pressure gradient.

Model extensions are possible to take into account the strong variability of thermophysical properties in special regions of the state or to represent a wider range of generalized Couette-Poiseuille flows.

2. Film flows hydrodynamics

The investigated flat one-dimensional stream (figure 1) moves in the considered region (0 ≤ y ≤ δ) along the x axis (along the solid wall, y = 0) under the action of a longitudinal pressure gradient (dp/dx, further p'(x)), of gravity (acceleration g) and shear stresses (τ). At the right boundary (y = δ), the friction stress τS is assumed to be given. This description corresponds to two-phase stratified flows (ρl, ρv are the liquid and vapor densities), as in the case of film condensation / evaporation.

![Figure 1. Flow pattern.](image-url)
The equation of motion is formulated for a special control volume (figure 1), with its right side onto the interface where the friction stress $\tau_S$ is specified. Gravity is directed vertically along the stream; for inclined surfaces, the appropriate projection must be used. The prototype of the presented model is the problem of film condensation, and the shown direction of forces and the accepted index designations are associated with this.

For the assumed non-inertial flow, the sum of the forces of gravity, pressure, and friction acting on the control volume should be zero. The forces relationship may be different. In the condensation of a fixed vapor, gravity is the main external factor; during condensation of a rapidly moving steam inside the channels, the friction at the phase boundary is great, as well as, possibly, longitudinal pressure drops in a two-phase flow ($p'(x)$). The direction of existing forces may be the opposite.

The heat flux density for film flows is assumed to be constant over the thickness of the layer. Stabilized heat transfer in the channel is simulated by means of a virtual internal heat source (see relations (7), (8) below).

For a steady film flow, a balance of forces should be performed, written down sequentially for a control volume of thickness $(\delta-y)$ with a unit base area normal to the $y$ axis (figure 1), and for a total control volume throughout the film thickness $\delta$, i.e. for $y = 0$:

$$F_{Vol} \cdot (\delta-y) + \left[ \tau_s - \tau(y) \right] = 0;$$
$$F_{Vol} \cdot \delta + \left[ \tau_s - \tau_w \right] = 0.$$  \hspace{1cm} (1)

$$F_{Vol} \equiv g \left( \rho_l - \rho_v \right) - p'(x);$$

The designation $F_{Vol}$ for the total effect of gravity and the longitudinal pressure gradient in the flow gives it a convenient interpretation as a certain volumetric force constant over the cross section of the flow and defined as an external parameter of the problem. The indices "s" and "w" indicate respectively the surface of the film and the wall. The $F_{Vol}$ effect is zero for a pure shift. In other cases, a (negative) pressure gradient and / or gravity (Archimedean force) acts across the cross section. The relationship between the $F_{Vol}$ effect and the characteristic values of shear stresses is given by the last of relations (1). In dimensionless form:

$$\frac{\tau_s}{\tau_w} = R_{sw}; \quad \frac{F_{Vol} \cdot \delta}{\tau_w} = \left( 1 - R_{sw} \right);$$  \hspace{1cm} (2)

$$\frac{\tau(y)}{\tau_w} = \left( 1 - R_{sw} \right) \left( 1 - \frac{y}{\delta} \right) + R_{sw}.$$  \hspace{1cm} (3)

these relations express simple properties of the model flow:

- shear stress varies linearly (3),
- the governing (structural) dimensionless parameter $R_{sw}$ of the generalized flow is the ratio (2) of shear stresses on the flow surface and on the wall (the difference of these stresses is the implicitly specified bulk force acting over the cross section).

Using the one-dimensional formulation of the law of friction for tangential stresses $\tau(y)$, we obtain the differential equation of motion for film flows:

$$\frac{1}{(\tau_w / \rho_f)} \left( v_l + v_r \right) \frac{du(y)}{dy} = \left( 1 - \frac{\tau_s}{\tau_w} \right) \left( 1 - \frac{y}{\delta} \right) + \frac{\tau_s}{\tau_w}; \quad u(0) = 0,$$  \hspace{1cm} (4)

or, in a dimensionless representation, using dynamic variables (“wall variables”, with a dynamic velocity (or “friction velocity”) $u$, as a characteristic scale):
The mnemonic indices "l" and "v" mean "liquid" and "vapor" in the condensate film problem; the default designations are "ρ, ν" for the density and viscosity of the liquid in the film. The quantities «ν T» and «N T» are the turbulent kinematic viscosity and the relative value of this quantity; the corresponding turbulence model is presented below. The friction stress on the wall τ_s is assumed to be a nonzero positive value and is used to construct the scale of reference; relations (2) give the necessary relationship between the values of the dimensional parameters of the problem τ_s, τ_w, F_vol.

When interpreting the results of further calculations, it is convenient to assume that the friction on the wall τ_w is fixed, and the dimensional values of friction on the outer boundary τ_s and the film thickness δ are vary. The result will be the distribution of speed and temperature over the thickness of the film, as well as the integral characteristics (fluid flow in the film, thermal conductivity of the film).

Integration of differential equation (5) determines the velocity distribution $U(Y; D, R_{sw})$ with two parameters: the dimensionless layer thickness $D$ (in another interpretation, the Reynolds number in a special form) and the ratio of the friction stresses on the interface and on the wall $R_{sw}$. Integration over the film cross section determines the liquid flow rate and the Reynolds number $Re_p$ of the film.

The parameter $R_{sw}$ controls the flow structure in the film. Zero value sets the film flow with a free surface (or half of the stabilized flow in the channel). A single value ($R_{sw} = 1$) defines a shear film with a constant friction stress over the cross section. Large positive values correspond to a backpressure flow. Negative $R_{sw}$ values specify special flows with recirculation (reverse flow over some part of the flow cross section). An important result will be finding the value of the parameter $R_{sw}$ at which the flow rate becomes zero: $Re_p = 0$. This is the boundary value when the "flooding" mode sets in - the limiting and possibly dangerous mode in some heat technologies.

For laminar flows ($N_T = 0$), simple parabolic velocity profiles are obtained, which were used, in particular, in test procedures for the numerical implementation of the problem. The task of further analysis is to generalize these results to transitional and turbulent flows in the entire possible range of the structural parameter $R_{sw}$.

### 3. Heat transfer in film flows

The heat transfer in film flows is described by a simple relation for the heat flux (q):

$$ q = -\left(\lambda + \lambda_T\right)\left(\frac{dt}{dy}\right) = \text{const}; $$

Integration within the film thickness gives for the heat transfer coefficient "α" as the thermal conductivity of the film and for the Stanton number (St):

$$ \alpha = \frac{1}{\int_0^\delta \left(\lambda + \lambda_T(y)\right)^{-1} dy}; \quad \text{St} = \frac{\alpha}{\rho c_p u \tau} = \left[D\left(\frac{1}{\text{Pr}} + \frac{1}{\text{Pr}_T} \frac{v_T}{\nu}\right)\left(\frac{1}{\text{Pr}} + \frac{1}{\text{Pr}_T} \frac{v_T}{\nu}\right)^{-1} dY\right]^{-1}. $$

(6)
Similar integration operations within variable limits determine the temperature distribution over the film thickness.

In condensation / evaporation problems, the quantity \( \alpha \) is considered as the thermal conductibility of the film and is used to calculate the condensation / evaporation rate as the density of the transverse mass flow (\( j \ [\text{kg} / (\text{m}^2 \ \text{sec})] \) at the phase boundary: \( j = q/r \), where \( r \) is the heat of phase transformation.

The transverse momentum flux, estimated as the product of the mass flux density and the vapor flow velocity, \( \tau_{s\_asimpt} \approx j \cdot U_{\text{vap}} \), can make a significant contribution to interfacial friction for the processes of intense condensation of fast-moving steam. These estimates illustrate how the selected problem of film flows is included in the complete problem of interphase heat and mass transfer.

A useful extension of the scope of one-dimensional analysis can be heat transfer in a flat channel. For a stabilized flow with fully developed velocity profiles \( u(y) \) and temperature \( t(y) \) (and assuming negligible longitudinal conductive transport), the heat balance equation for the control volume \( (dx \cdot dy \cdot 1) \) is written as follows:

\[
0 = \frac{d}{dy} \left( \lambda + \lambda_T \right) \left( \frac{dt}{dy} \right) - \rho c_P \cdot u(y) \cdot t'_f(x); \tag{7}
\]

The derivative \( t'_f(x) \) of the average temperature of the liquid along the longitudinal coordinate \( x \) is the problem parameter associated (by means of the balance relation) with the set values of the liquid flow rate (calculated per unit channel width) \( G \) and the heat flux density \( q_w \) on the wall:

\[
q_w = G c_P \cdot t'_f(x). \tag{8}
\]

Double integration (7) gives for temperature distribution in the cross section of the flow:

\[
-(\lambda + \lambda_T) \left( \frac{dt}{dy} \right) = \rho c_P \cdot t'_f(x) \cdot \int_y^\delta u(y) \cdot dy;
\]

\[
t(y) - t_w = (-\rho c_P \cdot t'_f(x) \cdot \left[ \int_y^\delta u(y) \cdot dy \right] / (\lambda + \lambda_T(y)) \right) dy. \tag{8}
\]

The hydrodynamics of this flow is described by equation (5), which is solved independently of the thermal problem, since the properties of the coolant are assumed to be constant. This is an optional restriction, and the corresponding generalization (i.e., the joint solution of a complete system of a higher order) does not introduce significant complications into the software implementation.

4. Turbulence model

An adequate description of turbulence for the considered problem with a well-defined unique linear scale (film thickness \( \delta \)) can be a formulation with one differential equation for the turbulent energy “\( k \)” of a unit fluid mass (Kolmogorov-Prandtl model).

In the modern modification [4], with the necessary adjustments for weak turbulence in the near-wall region, the model is represented by the following relationships (for a one-dimensional stationary problem):

\[
0 = v_r \left( \frac{d}{dy} \right)^2 \left[ C_D \cdot f_{\mu} \cdot \frac{k^{3/2}}{l_{\text{mix}}} + 2v \cdot \frac{k}{y^2} \right] + \frac{d}{dy} \left[ v + \frac{v_r}{Pr_r} \right] \left( \frac{dk}{dy} \right); \tag{9}
\]

\[
v_r = C_{\mu} f_{\mu} l_{\text{mix}} k^{1/2} \quad \text{Re}_r = \frac{l_{\text{mix}} k^{1/2}}{v}; \quad f_{\mu} = 1 - \exp(-0.029 \cdot \text{Re}_r); \quad l_{\text{mix}} = \min(k \cdot y, \lambda \cdot \delta);
\]

\[
C_{\mu} = 0.09^{1/4}; \quad C_D = 0.09^{1/4}; \quad Pr_r = 1; \quad \kappa = 0.41; \quad \lambda = 0.085;
\]
where \( y \) is the distance from the solid wall (for a more complex flow geometry, for example, for the generalized Couette-Poiseuille flow between two solid surfaces, this is the minimum distance from the solid walls).

The effects of “weak” turbulence are taken into account by the viscous dissipation operator in (9) and the damping function \((f_\mu)\) in the expression for turbulent viscosity, so the model used belongs to the class of low Reynolds turbulence models.

In dimensionless form:

\[
0 = N_T \left( \frac{dU}{dY} \right)^2 - \left( C_D \cdot f_\mu \cdot \frac{K^{3/2}}{L_{mix}} + 2 \cdot \frac{K}{y^2} \right) - \frac{d}{dY} \left( \frac{1 + N_T}{J_K} \frac{dK}{dY} \right);
\]

\[
Y = \frac{y \cdot u_r}{v}; \quad U = \frac{u}{u_r}; \quad K = \frac{k}{u_r^2}; \quad D = \frac{\delta \cdot u_r}{v}; \quad L_{mix} = \frac{l}{u_r} \cdot \frac{u}{v}; \quad L_{mix} = \min(\kappa \cdot Y, \lambda \cdot D);
\]

\[
\text{Re}_T = \left( \frac{\mu}{\mu} \right)^{1/2}; \quad N_T = \frac{\tau}{\nu} = \frac{C}{\mu} \cdot \frac{1}{J_K}; \quad f_\mu = 1 - \exp(-0.029 \cdot \text{Re}_T).
\]

Here \( K \) and \( J_K \) are the dimensionless quantities of turbulent energy and the diffusion flow of turbulent energy, respectively, \( N_T \) is the dimensionless turbulent viscosity, and \( D \) is the dimensionless film thickness.

5. Differential model

Relations (5) and (10) form a system of ordinary differential equations of the first order for distributions of the flux density of the turbulent energy \( J_K(Y) \), turbulent energy \( K(Y) \) and the flow velocity \( U(Y) \):

\[
\begin{align*}
J'_K(Y) &= N_T(Y) \cdot (U(Y))^2 - \left( C_D \cdot f_\mu \cdot K^{3/2} / L_{mix}(Y) + 2 \cdot K(Y) / Y^2 \right); \\
K'(Y) &= -J_K(Y) / \left( 1 + N_T(Y) \right); \\
U'(Y) &= \left( (1 - R_{SW}) (1 - Y / Y_m) + R_{SW} \right) / \left( 1 + N_T(Y) \right)
\end{align*}
\]

System (11) is a compact universal description of an extensive list of problems classified as generalized (laminar and turbulent, shear and gravitational) film flows (or Couette-Poiseuille flows, with the differences mentioned above for interfaces and solid surfaces, [5, 6]). Variations in the \( R_{SW} \) parameter provide a representation of a variety of flow configurations, including recirculated flows.

Problem (11) was solved under zero boundary conditions on a solid wall (at \( Y = 0 \)), i.e. under the condition of “sticking” for speed and turbulent energy and under the condition of impermeability to the flow of turbulent energy. Since the system of differential equations (11) has a singular point at the origin \((K \to 0 \text{ at } Y \to 0)\), asymptotic expansions of the proposed solutions near the wall were required. Such an expansion for the turbulent energy \( K(Y) \) contains an indefinite constant, for fixing which (in the case of film flows) the condition of zero flow of turbulent energy \( J_K \) at the external boundary \((Y = D)\) is accepted. Next, a special algorithm was used, for the numerical integration of the system with an indefinite coefficient.

The presented algorithm provided an effective count over the entire range of determining parameters, from laminar thin-film flows to developed turbulent flows, with strong changes in values in the immediate vicinity of the wall (in a viscous sublayer).
In the accepted (optional) simplifying assumption that the thermophysical parameters are independent of temperature, the hydrodynamic problem (i.e., finding the distributions (along the transverse \( Y \) coordinate) of velocity, turbulent energy, and turbulent energy flux density) can be solved independently of the thermal problem. The result of the calculations also becomes the distribution of the turbulent transfer coefficient over the film thickness: \( N_T(Y) \); simple calculations make it possible to find further the temperature distribution and thermal conductivity of the film (see (6), (8)).

The structure of the distributions of characteristic quantities over the layer thickness and the list of dimensionless parameters is determined by the following relationships:

\[
J_k = J_k(Y; R_{sw}, D); \quad K = K(Y; R_{sw}, D); \quad U = U(Y; R_{sw}, D); \quad N_T = N_T(Y; R_{sw}, D).
\]  

(12)

The integral characteristics, namely, the Reynolds number of the film \( \text{Re}_F \) (as a measure of the mass flow in the film, see (5)) and the Stanton number \( \text{St} \) (as a measure of the heat transfer through the film and, indirectly, as the local rate of condensation / evaporation, see (6)) are defined as functions of external parameters: ratios of shear stresses \( R_{sw} \) (shear factor), dimensionless film thickness \( D \) and Prandtl number (Pr):

\[
\text{Re}_F = \text{Re}_F(R_{sw}, D); \quad \text{St} = \text{St}(R_{sw}, D, \text{Pr}).
\]  

(13)

The system of differential equations (11) is a compact universal description of film flows. These equations also describe generalized (laminar and turbulent) Couette-Poiseuille flows, i.e. flows between two solid independently moving surfaces - with the corresponding formulation of boundary conditions (for a useful discussion of the problem, see, for example, [5, 6]).

The noted generality of the mathematical model was achieved due to the decomposition of some more general problem. Indeed, interfacial friction \( \tau_s \) is assumed to be a given external parameter, while in a more general formulation this quantity should be defined as a function of special forms of motion at the interphase boundary (taking into account the effects of turbulence, surface tension, and the presence of a transverse mass flow during phase transformations). The solution of such a conjugate two-phase problem can be a further generalization of the basic simple model proposed here, which potentially includes the listed effects of interphase turbulence.

The results of one-dimensional analysis can be used with sufficient accuracy to describe developing film flows, since all the necessary functional relationships between local characteristics (such as mass flow rate and thermal conductivity of the film) are determined.

### 6. Results of numerical analysis: distributions in flow section

The numerical integration of the system of ordinary differential equations (11) determines the distributions (12) of velocity, turbulent energy, and flux density of turbulent energy along the transverse coordinate \( Y \); the distribution of the turbulent transfer coefficient \( N_T \) is also known. These distributions are presented below for some characteristic configurations of film flows controlled by the parameter \( R_{sw} \), including the regime with recirculation (reverse flow over part of the cross section of the film).

As noted above, the film flow at \( R_{sw} = 0 \) and the generalized Poiseuille flow (in the region from the wall to the plane of symmetry) are identical. This made it possible to compare the calculations in this paper and the results of DNS modeling [9] in a wide range of Reynolds numbers of turbulence (or the parameter of dimensionless film thickness \( D \), in a different interpretation). The distributions of turbulent energy, a flux density of this quantity, and turbulent viscosity (figure 2) demonstrate a complex structure of turbulence formed in the processes of its generation, decay and dissipation, as well as diffusion transfer. The obtained distributions reproduce the mode of weak turbulence in the transition region, where the effects of molecular and turbulent transfer are comparable.

The satisfactory agreement can be stated between the calculations according to the phenomenological model of turbulence and the method of direct numerical simulation in a wide range of Reynolds numbers. The continuing agreement for difficult region of the laminar-turbulent transition deserves special mention. A systematic comparison of this kind can be used to optimize the parameters of the adopted turbulence model.
A detailed modeling of the structure of a turbulent flow using system (11) is especially important for modes with recirculation, when simple algebraic turbulence models cannot be used to study the complex flow structure.

The critical mode is presented below (figure 3), in which the total flow rate vanishes, while there is an oncoming moving in the cross section. A significant gradient of velocity, which is preserved over the thickness of the film, ensures the generation of turbulence and the increasing value of the coefficient of turbulent transfer over the entire cross section of the flow. The role of turbulence diffusion in the cross section of the flow is essential.
7. Results of numerical analysis: integral characteristics

The results of a numerical analysis for the integral characteristics of film flows, namely for the Reynolds number of the film and the Stanton number (see (13) and relevant explanations above) are presented in three-dimensional diagrams (figure 4, figure 5).

The $Re_F(D, R_{sw})$ diagram determines the Reynolds number of the film (proportional to the mass flow rate (see (5)) as a function of the structural parameter $R_{sw}$ and the dimensionless film thickness $D$.

The Stanton number $St(D, R_{sw})$ (figure 5) is defined as a quantitative measure of the intensity of heat transfer through the film (see (6)) and, indirectly, as the local rate of condensation / evaporation.

The characteristic changes along the $D$-variable are associated with the laminar-turbulent transition, which can be very sharp for the values of the structural parameter $R_{sw}$ noticeably deviated from zero.

Thus, the complete set of local dynamic and thermal characteristics of film flows is identified under various flow regimes, various acting forces, and various combinations of effects.

A graphical representation of the dependence $Re_F(D, R_{sw})$ demonstrates the phenomenon of “flooding” (figure 4). The horizontal grid plane corresponds to zero flow rate in the film of finite thickness, i.e. represents a crisis situation when non-zero expenses in opposite directions are compared in absolute value:

$$0 = Re_F(R_{sw, flooding}, D)$$

The boundary regime under discussion (i.e. $Re_F = 0$) is of particular interest for vertical film evaporation apparatuses of various heat technologies. Important application is analysis of the operation of a steam generator of WWER-type nuclear power plants in the condensation mode SPOT during a hypothetical accident [1]. Deteriorated modes of this kind can occur in devices with condensation of a gas-vapor mixture in parallel connected vertical or inclined pipes.

The actual problem in the calculation practice of condensing units remains the diagnosis of the laminar-turbulent transition and the calculation of friction and heat transfer in this area. The extrapolation of the characteristics of the laminar flow in the diagram $St (D, R_{sw})$ is represented by a grid surface without color filling (figure 5). For small film thicknesses $D$, the numerical solution reproduces the analytical limit distributions for the laminar flow (which is also a test mode for the software implementation of the mathematical model). It is seen that a change in the flow structure under the influence of the parameter $R_{sw}$ initiates an earlier and increasingly abrupt transition to turbulence, accompanied by a significant increase in the heat transfer intensity.

![Figure 4. Results of a numerical analysis: $Re_F (R_{sw}, D)$.](image)
8. Imitation of roughness

Variations in the parameter $R_{sw}$ provide a wide variety of flow forms, however, it is possible to construct even more complex structures by coupling flows along the interface. An example is imitation of the hydrodynamic roughness of a wall, natural or artificial, created with the aim of intensifying heat transfer.

A general description of the effect consists in the formation of a separated vortex flow behind sufficiently large roughness elements, provided that a turbulent flow takes place and that the characteristic height of the roughness elements exceeds the thickness of the viscous sublayer. Apparently, the simplest model of such a flow can be a composite flow (figure 6), formed

- near-wall recirculation flow with a certain characteristic thickness $\delta R$
- and the main turbulent external flow.

The value of $\delta R$ (or $D_r$ in the dimensionless representation) is an average measure of the roughness height. The distance between the roughness elements noticeably exceeds their height, but not so large as to consider these elements single. In the laminar flow, such a structure will not affect the transverse heat flux; however, for sufficiently large Reynolds numbers, increased velocity gradients in the near-wall zone generate turbulence and, with the participation of the turbulence diffusion mechanism, intensify transport processes over the entire flow cross section.

Consistently for the near-wall region with recirculation and for the main flow (the flow in the channel or the film flow), the following equations of motion are obtained:
In the case of a composite flow, the value of $R_{sw}$ is determined for the recirculation layer near the wall; this value is found by a special algorithm that ensures zero fluid flow in the recirculation layer ($D_r$ or $Y_r$, depending on the context). Further, the friction stress either remains constant in the case of film flow, or decreases to zero, as for the flow in the channel.

The composite region is considered as a single fluid flow with continuous and smooth distributions of velocity, temperature and turbulent characteristics over the entire channel cross section, so that diffusion flows of substances freely penetrate through the conditional boundary between the wall layer and the core of the flow. The interconnection of the flow regions is provided, among other things, by the effects of turbulence diffusion (the quantity $J_K$ is the density of the flow of turbulent energy under the influence of the concentration gradient of this quantity).

The presented roughness simulation suggests some optimal distance between the roughness elements, approximately corresponding to the attachment of the flow to the main wall behind a streamlined single obstacle. When the elements are too close, recirculation degenerates, and if too rare, it is localized only near a single roughness element.

A systematic analysis of the effect of roughness should be the subject of a special parametric study. Here are the results of a single calculation to illustrate the structure of the model flow (figure 7). Flow deformations due to wall recirculation cause increased turbulent energy generation. The effects of turbulence diffusion provide equalization of the turbulent transfer coefficient over the flow cross section. Thanks to these internal mechanisms, a substantial intensification of the transverse heat transfer is provided. It can be expected that the presented simple model of hydrodynamic roughness will be a useful and affordable tool for engineering analysis of the problem of heat transfer intensification.

9. Gravity film condensation

The differential model (11), which represents the class of film flows, may be the basis for a generalized description of heat transfer by film condensation (with (6) together). A certain internal completeness of the base model is very important for this problem, since field experiments are complex and subject to the influence of difficultly controlled additional factors (for example, of small “poisonous” impurities of non-condensable gases).
The results of numerical integration of system (11) obtained above (figure 4, 5) are processed in the traditional system of dimensionless parameters $Nu_{lg}(Re_F)$ adopted for gravitational film condensation problems (see, for example, [3]) and are presented in the diagram (figure 8).

The sloping dashed line (on a logarithmic scale) represents the classic Nusselt’s model of laminar film, and all calculated curves are going together with the Nusselt’s line at relative small values of $Re_F$. The remaining series are the results of calculations according to the model presented here (11). For coolants with Prandtl numbers substantially different from unity, when the temperature and velocity distributions are significantly different, the results need comment.

The $Nu_{lg}(Re_F)$ dependence for coolants with small Prandtl numbers (i.e., for liquid metals) looks paradoxical. The heat transfer predicted by the classical theory under the laminar regime (the oblique dashed straight line on the graph) turns out to be more intense than under the turbulent regime (for the same values of the Reynolds number of the film as a flow rate).

However, this only means that - with relatively weak turbulence in the transition region - its effect on heat transfer is insignificant, since the thermal conductivity is already very high ($Pr << 1$). At the same time, the effective viscosity increases significantly due to the turbulence that arises, therefore, the film thickness significantly increases (at a fixed flow rate) and, therefore, its thermal resistance increases also.

The opposite effect is observed at large Prandtl numbers ($Pr >> 1$), i.e. for viscous low-heat-conducting fluids (upper curves in the graph). Here, a certain increase in the effective viscosity due to the transition remains insignificant, while the effective thermal conductivity increases significantly. The same applies to diffusion processes in liquid films when the values of the diffusion Prandtl number are large.

Such an analysis becomes possible thanks to a model that includes a detailed analysis of the evolution of turbulence in the transition region.

It is important to note that - in a consistent range of definitions - there is satisfactory agreement with the Labuntsov’s model [7] as actual standard for a turbulent condensate film, obtained on the basis of laser Doppler anemometry data of a wall single-phase turbulent flow.

10. Conclusion

A closed formulation of the model of film near-wall flows is obtained that completely determines their local characteristics of hydrodynamics and heat transfer.
In engineering applications (for example, in condensation / evaporation problems), the limiting regimes of gravitational or shear, laminar or turbulent liquid films are investigated in sufficient detail. In practice, however, various mixed or transient modes are common. Therefore, the proposed generalized solution is relevant when the effects of gravity, pressure gradient and interfacial friction are assumed a priori commensurable, and the flow regimes (laminar, turbulent, transitional) remain internal, automatically fixed characteristics of the process.

The evolution of film flows in the longitudinal direction (and, possibly, in time) should be controlled by additional differential balances, taking into account the obtained relations for the rate of condensation (or evaporation), as well as, in special cases, droplet entrainment (or deposition). This wider area of application of the local model can be implemented in the style of the Kutateladze-Leontiev’s integral method of boundary layer.

A natural limitation of the results is that the parameter of the problem \( R_{sw} \) contains interfacial friction as a quantity that, in turn, must be determined (in the complete analysis of the two-phase problem, with flows of both interacting phases contacting along the interphase boundary).

This remark serves as a statement of the complexity of the problem of two-phase heat and mass transfer as a whole, but does not reduce the usefulness of the results obtained. Indeed, the proposed model potentially includes the possibility of assessing the degree of perturbation of the interphase boundary, its special “roughness” - by comparing the turbulent energy at the interface with the energy costs of generating capillary waves and dispersing the liquid. A discussion of the problem of interface instability in phase transitions can be found in [8].

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