Terahertz Bloch oscillator with suppressed electric domains: Effect of elastic scattering

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We theoretically consider the amplification of THz radiation in a superlattice Bloch oscillator. The main dilemma in the realization of THz Bloch oscillator is finding operational conditions which allow simultaneously to achieve gain at THz frequencies and to avoid destructive space-charge instabilities. A possible solution to this dilemma is the extended Limited Space-Charge Accumulation scheme of Kroemer (H. Kroemer, cond-mat/0009311). Within the semiclassical miniband transport approach we extend its range of applicability by considering a difference in the relaxation times for electron velocity and electron energy. The kinetics of electrons and fields establishing a stationary signal in the oscillator is also discussed.

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I. INTRODUCTION

Terahertz radiation (0.3 – 10 THz) has enormous promising applications in very different areas of science and technology such as space astronomy, wideband communications and biosecurity, to name a few (for recent reviews, see [1]). One of the main challenges is to construct a coherent miniature source of THz radiation that can operate at room temperature. Currently many groups worldwide are working in this direction. Along with many experimental and technological problems there are still several fundamental problems to be solved. In particular, the traditional lasing scheme is based on a population inversion between different energy levels. A great recent achievement was the development of quantum cascade lasers that can operate at THz frequencies employing quantum transitions between the energy levels in multiple quantum well heterostructures [2]. These quantum nanodevices require a low temperature to achieve a significant population difference. Continuous improvements in the design of THz quantum cascade lasers allow to increase the temperature of operation above 100 K [3]. However, it would be a quite difficult, if ever possible, to reach population inversion at room temperature. Really, the spacing of the energy levels, which is necessary to emit radiation at frequencies of the order of 1 THz, is comparable with the room temperature in proper units. This simple observation makes it attractive to study other suggestions for nano-devices that do not require population inversion for the generation of THz radiation.

The Bloch oscillator is an inversionless THz laser that is based on Bloch oscillations of miniband electrons in a dc-biased semiconductor superlattice. The continuous operation of a Bloch oscillator has still not been demonstrated in any experiments. The development of a superlattice Bloch oscillator has been initiated by suggestion of Esaki-Tsu [4] and theoretical analysis of Kitorov, Simin and Sindalovskii [5]. The main obstacle in the experimental realization is the formation of high-field electric domains in superlattice.

Nowadays the problem of THz Bloch oscillator again attracts much attention (for review, see [6, 7, 8]). There are several interesting suggestions for modifications of the original scheme of Bloch oscillator, which in principle should allow to reach THz gain in superlattice without formation of destructive electric domains.

In this report, we first briefly review the static electric properties of superlattices and their influence on both small-signal gain and electric instability in section II. In the original part of this report we develop the idea of a large-amplitude THz Bloch oscillator suggested by Kroemer [3]. We consider a dc-biased superlattice subjected to a monochromatic ac probe field, so that the total electric field acting on the miniband electrons is

\[ E = E_{dc} + E_{ac} \cos \omega t. \] (1)

Note that in a real device the probe field with the amplitude \( E_{ac} \) should be a mode of resonator tuned to a desirable THz frequency. It is well known that while a superlattice device should allow strong gain for a small ac field \( (E_{ac} \to 0) \) in the wide range of frequencies from zero up to several THz, simultaneously destructive electric instabilities would arise inside superlattice (see section III). As Kroemer has demonstrated for particular values of superlattice parameters, the ac field with a large enough \( E_{ac} \) can suppress domains but still preserve significant THz gain [3]. In ref. [3] the single scattering constant approximation was used. However, a more realistic approximation allow two different relaxation constants, \( \gamma_v \) for the electron velocity and \( \gamma_e \) for the electron energy.

Using the technique of superlattice balance equations (section III), we re-examine the THz gain and the criterion of electric stability for the case of large \( E_{ac} \). Our main aim is to find how robust the Kroemer scheme of
Bloch oscillator is against the effect of different relaxation constants (section IV). Here our main finding is that this effect does not dramatically change the results obtained within the single scattering constant approximation, if ratio $\gamma_e/\gamma_0 > 0.1$. Moreover, for $\gamma_e/\gamma_0 \geq 0.5$ the differences from the results obtained within the single relaxation time approximation are insignificant.

In section IV we consider the characteristic time scales for the development of space-charge instabilities and for the growth of the ac field due to high-frequency gain in superlattice. We come to the conclusion that it really is possible to suppress a destructive accumulation of charges in the case of THz oscillations with large amplitudes. However, our estimates still demonstrate that it is very difficult to switch the device in the regime of large amplitudes before electric domains would be formed. Discussion devoted to this and other remaining problems in the realization of THz Bloch oscillator is presented in the final section.

II. STATIC ELECTRIC CHARACTERISTIC AND SMALL-SIGNAL GAIN

Let us first review the static electric properties of superlattices using a single relaxation time. Nonlinear electron transport in a superlattice with applied dc bias $E_{dc}$ is a well-studied problem. The dc current through the superlattice depends on the dc bias as [4]

$$I_{ET} = I_{peak} \frac{2 E_{dc}/E_{cr}}{1 + \left( E_{dc}/E_{cr} \right)^2}. \quad (2)$$

Here $E_{cr} = h/(ed\tau)$ is the Esaki-Tsu critical field,

$$I_{peak} = \frac{en_0}{2} \frac{h(2\gamma \tau)}{I_0(2\gamma \tau)} A,$$

is the peak current corresponding to $E_{dc} = E_{cr}$, $\tau$ is the scattering time, $n$ is the density of carriers, $v_0 = \Delta d/(2h)$ is the maximal electron velocity in the first miniband, $\Delta$ is the miniband width, $d$ is the period of superlattice, $T$ is the temperature, $A$ is the cross sectional area of superlattice and $I_1(x)$ and $I_0(x)$ are the modified Bessel functions.

The dependence of $I_{ET}$ on $E_{dc}$ is shown in Fig. 11. For $E_{dc} > E_{cr}$ the current-field characteristic demonstrates negative differential conductance (NDC). Note that $E_{dc}/E_{cr} = \omega_B \tau$ with $\omega_B = edE_{dc}/h$ being the Bloch frequency. Therefore the condition of static NDC is also $\omega_B \tau > 1$. For typical semiconductor superlattices and for the applied dc bias $1 \text{ kV/cm}$, the Bloch frequencies belong to THz range [4].

In the following it is useful to consider the Esaki-Tsu current [3] as a function of voltage drop over one superlattice period

$$I_{ET}(eE_{dc}d) = I_{peak} \frac{2eE_{dc}d/\Gamma}{1 + (eE_{dc}d/\Gamma)^2} \quad (3)$$

with $\Gamma = h/\tau$.

We turn to the consideration of electron transport under the action of combination of dc and ac fields. The time-dependent current induced in superlattice by the field $\mathbf{E}$ can be represented as

$$I(t) = I_0 + \sum_{k=1}^{\infty} \left[ I_{k,\cos} \cos(h\omega t) + I_{k,\sin} \sin(h\omega t) \right]. \quad (4)$$

Absorption of the probe ac field is proportional to the first Fourier component of the time-dependent current $I_{k,\cos}$. Negative absorption or gain corresponds to $I_{k,\cos} < 0$.

We start the discussion of small-signal gain ($E_{ac} \to 0$) with the case of quasistatic interaction $\omega \tau \ll 1$. In this case, it is easy to show that $I_{k,\cos}$ is proportional to the slope of static VI characteristic [8]. Therefore, a choice of working point in the NDC portion of superlattice VI characteristic results in small-signal gain for a quasistatic ac probe field. Obviously, in the quasistatic case the gain does not depend on the frequency. For typical superlattices at room temperature $\tau \approx 100$ fs. Therefore the quasistatic approximation is valid for microwave fields.

For a higher frequency ($\omega \tau \gtrsim 1$), it is still possible to have small-signal gain, but the magnitude of the gain becomes frequency-dependent. Small-signal absorption is also often defined using the frequency-dependent complex conductivity $\sigma(\omega)$: $I_{k,\cos} \cos(\omega t) + I_{k,\sin} \sin(\omega t) = \Re \{ \sigma(\omega)E_{ac}e^{-i\omega t}A \}$. It is easy to see that absorption is proportional to $\Re \sigma(\omega)$. Complex high-frequency conductivity has been first calculated by Kitorov et al. [5]

$$\sigma(\omega) = \frac{1 - i\omega \tau - (E_{dc}/E_{cr})^2}{(1 - i\omega \tau)^2 + (E_{dc}/E_{cr})^2} \sigma_{stat}, \quad (5)$$

where $\sigma_{stat} = I_{ET}/(AE_{dc})$ is the static conductivity of the superlattice. Real and imaginary parts of the conductivities [4] are shown in Fig. 12. Importantly, for $E_{dc} \gg E_{cr}$ gain exists up to frequencies around the Bloch frequency, with even a resonance near $\omega_B$. The physical mechanisms of this high-frequency gain has been studied both in the semiclassical picture with the help of electron bunches [10] and in the quantum mechanical Wannier-Stark picture [11].

Alternatively, the small-signal absorption can be calculated by taking the so-called quantum derivative of the current-field characteristic [3][12] (Fig. 14)

$$I_{I_{k,\cos}} = \frac{I_{ET}(eE_{dc}d + h\omega) - I_{ET}(eE_{dc}d - h\omega)}{2h\omega} e^{E_{ac}d}. \quad (6)$$

For $\omega \tau \to 0$ the difference quotient in the quantum derivative becomes the usual derivative, and the result that small frequency absorption is proportional to $dI_{ET}/dE_{dc}$ is rediscovered. One immediate consequence of the quantum derivative is that in order to get high-frequency gain we must have static NDC (see Fig. 1).
Superlattices with static NDC, similarly as Gunn diodes, are unstable against spatial space-charge fluctuations, which result in formation of high-field electric domains. The possibility of space-charge instability in the superlattice Bloch oscillator was first pointed out by Ktitov et al. themselves \cite{5}. Later, it has been established within a simplified model by Büttiker and Thomas \cite{13} and in most consistent form by Ignatov and Shashkin \cite{14}.

The electric domains are believed to be destructive for high-frequency gain in superlattices. Nevertheless, in series of very interesting experiments French groups did observe gain at reflection of microwaves from long superlattices with \( n \approx 10^{16} \text{ cm}^{-3} \) in a wide range of frequencies up to several hundreds of GHz \cite{15}. It seems that physical reasons for these very interesting experimental observations are still not well understood in the semiconductor community.

On the other hand, for THz waves, Santa-Barbara group recently reported a decrease of absorption, but still not gain, in an array of short superlattices \cite{16}. Experimental techniques for monitoring the domains in superlattices are under development \cite{17}.

Now we turn to consider the regime of large probe field, where high-frequency gain is not necessarily connected to the presence of static NDC.

### III. SUPERLATTICE BALANCE EQUATIONS

Until now we have discussed electron transport in superlattices assuming the existence of a single relaxation time. However, experiments demonstrate that models employing two relaxation times are more realistic. By using the tight-binding approximation and the Boltzmann equation with two scattering times, the following balance equations can be derived \cite{8}

\[
\frac{d}{dt}j(t) + \frac{d^2}{dt^2}E(t)\varepsilon(t) = -\gamma_e j(t) \]

\[
\frac{d}{dt}\varepsilon(t) - E(t)j(t) = -\gamma_{el} [\varepsilon(t) - \varepsilon_{eq}].
\]

Here \( \gamma_e \) and \( \gamma_{el} = \gamma_e + \gamma_{el} \) are the phenomenological scattering constants for electron energy and miniband electron velocity respectively, \( \gamma_{el} \) is the scattering constant describing elastic scattering events, \( j(t) \) is the current density and \( \varepsilon(t) \) is the total miniband energy density of electrons. Electrons are staying at the bottom of miniband for \( \varepsilon = -n\Delta/2 \) and the upper edge is reached if \( \varepsilon = +n\Delta/2 \), where \( n \) is the density of electrons in the first miniband. Average electron energy in thermal equilibrium \( \varepsilon_{eq} \) depends on the temperature, superlattice parameters and carrier density.

The superlattice balance equations have been first introduced by Ignatov and Romanov \cite{18}. Importantly, equations \cite{14} can describe both transient \((t < \gamma_e^{-1})\) and stationary \((t \gg \gamma_e^{-1})\) dynamics of miniband electrons under the action of strong electric fields (The electric field is turned on at \( t = 0 \)). Earlier these equations have been employed in the studies of decaying coherent Bloch oscillations \cite{14}, absolute negative conductance

![Graph](image-url)
and efficiency of THz Bloch oscillator \[21\], as well as in the analysis of such strongly nonlinear phenomena like symmetry-breaking and chaos \[22\].

The solution of equations \[7\] for static field is a scaled Esaki-Tsu characteristic. Its critical field is

\[
E_{cr} = \frac{\hbar}{(ed\tau_{d})},
\]

where \(\tau_{d} = (\sqrt{\gamma_{e}\gamma_{c}})^{-1}\). This new definition of critical field is used in the following and it reduces to the earlier definition, if \(\gamma_{e} = \gamma_{c} = 1/\tau\).

In addition to parameter \(\tau_{d}\), a parameter \(\nu = \gamma_{e}/\gamma_{c}\) is needed to describe the system. If \(\nu = 1\), there is no elastic scattering i.e. \(\gamma_{e} = 0\).

Assuming strong electric field of the form \[1\], the equations \[7\] have time-dependent stationary solution in analytic form if \(\gamma_{e} = \gamma_{c} = 1/\tau\). In this case we find for the Fourier harmonics \[1\] of the current \(I(t) = j(t)A\)

\[
I_{0}^{w} = \sum_{l} J_{l}^{2}(\alpha) I_{ET} (eE_{dc}d + l\hbar\omega),
\]

\[
I_{h}^{w,\cos} = \sum_{l} J_{l}(\alpha) [J_{l+\hbar}(\alpha) + J_{l-\hbar}(\alpha)] I_{ET} (eE_{dc}d + l\hbar\omega),
\]

\[
I_{h}^{w,\sin} = \sum_{l} J_{l}(\alpha) [J_{l+\hbar}(\alpha) - J_{l-\hbar}(\alpha)] K (eE_{dc}d + l\hbar\omega),
\]

where \(J_{l}(x)\) are the Bessel functions, the summation is from \(-\infty\) to \(+\infty\), \(\alpha = eE_{dc}d/(\hbar\omega)\), \(I_{ET}\) is given by equation \[9\] and

\[
K (eE_{d}) = I_{\text{peak}} \frac{1}{1 + (eE_{d}/\Gamma)^{2}}.
\]

The solutions of the balance equations \[9\] are the same as the corresponding expressions for current components found with the help of an exact formal solution of Boltzmann transport equation with a single relaxation time \[23\]. Moreover, it is easy to see that for small ac field \(E_{ac}\) (i.e. for \(\alpha \ll 1\)), \(I_{h}^{w,\cos}\) becomes the quantum derivative defined by eq. \[6\].

Finally, it is useful to find the expression for the dc differential conductivity

\[
\frac{dI_{0}^{w}}{dE_{dc}} = \sum_{l} J_{l}^{2}(\alpha) \sigma_{0} (eE_{dc}d + l\hbar\omega)
\]

\[
= \frac{2I_{\text{peak}}}{E_{cr}} \sum_{l} J_{l}^{2}(\alpha) \frac{1 - (eE_{dc}d + l\hbar\omega)^{2}/\Gamma^{2}}{[1 + (eE_{dc}d + l\hbar\omega)^{2}/\Gamma^{2}]^{2}},
\]

where \(\sigma_{0} (eE_{dc}d)\) is the dc differential conductivity taken from the Esaki-Tsu characteristic.

Formulas \[9\] and \[10\] will be used in the next section in the stability analysis of large-amplitude oscillations in the Bloch oscillator for the case \(\gamma_{e} = \gamma_{c}\). They can also be used to check the numerical solution of the balance equations in the limiting case \(\nu = 1\).

Note that since in the case of static electric field the solution of equations \[4\] was the scaled Esaki-Tsu VI characteristic for all \(\nu\), different scattering rates do not give any qualitative changes to low frequency behavior. This is, however, not necessarily the case for higher frequencies. The quantum derivative result is valid only if \(\nu = 1\), although it works as a good approximation, when \(\nu \approx 1\). Qualitative changes in the high frequency behavior can occur even for small signals, when \(\nu\) is significantly smaller than 1. The changes in large amplitude oscillation dynamics are even more interesting, because the calculations for \(\nu = 1\) indicate possibility for gain and domain suppression \[7\].

**IV. LARGE-SIGNAL GAIN AND STABILITY**

We solved the balance equations \[7\] numerically, found the stationary time-dependent current \[11\], and then calculated the dc current component \(I_{0}\), the absorption \(I_{h}^{w,\cos}\) and the dc differential conductivity

\[
\sigma_{0}^{\nu} = \frac{dI_{0}^{w}}{dE_{dc}}.
\]

For \(\nu = 1\) these functions were found using analytic formulas \[9\] and \[10\]. In this case, we found excellent agreement between these two approaches, as well as with the results of reference \[6\].

According to the formula \[9\], the dc current can be calculated with the help of photon replicas of the Esaki-Tsu characteristic, and when the amplitude of the ac field increases, the so-called Shapiro-like steps occur in VI characteristic [Fig. 3 (\(\nu = 1\))]. Within these step-like structures, regions of positive differential conductivity (PDC) exists [Fig. 3 (b)]. When \(\nu\) decreases, the structures become less pronounced but the region of PDC becomes even larger.

The PDC should be considered as one of the conditions for electric stability of the system \[4\]. This is a sort of extension of the Limited Space Charge Accumulation (LSA) mode of Copeland \[24\], well-known in physics of Gunn diodes, to the case of superlattices and THz frequencies.

On the other hand, even for a large enough ac field \(E_{ac}\) the absorption can stay negative for \(\omega < \omega_{B}\), as can be expected from the electron bunching mechanism \[10\]. Figure 3 shows the absorption \(I_{h}^{w,\cos}\) and the dc differential conductivity as functions of \(E_{ac}\), when \(\omega \tau_{eff} = 10\) and \(\omega B \tau_{eff} = 10.8\). As evident from this figure, there exists a well-defined range of amplitudes \(E_{ac}\) for which the positive differential conductivity and the negative absorption occur simultaneously. When \(\nu\) decreases the range of amplitudes \(E_{ac}\) for negative absorption shrinks and the range of amplitudes for PDC expands.

The ranges of dc bias \(E_{dc}\) and ac amplitude \(E_{ac}\), supporting PDC and gain at the first Shapiro-like step, are presented in Fig. 3 for three different values of \(\nu\), when \(\omega \tau_{eff} = 10\). In this parameter plane, the area of gain and
simultaneous electric stability changes with a decrease of \( \nu \) (cf. (a) and (c)). The change is very small as long as \( \nu \geq 0.4 \).

V. CHARACTERISTIC TIME SCALES

Until now we have considered only stationary transport properties. Here we briefly examine and compare the time scales determining the evolution of the fields in a superlattice. There are the following characteristic times. Stationary VI characteristic is established with the characteristic time \( \tau \). (Single scattering time is assumed for simplicity.) AC field defines the time scale \( 2\pi/\omega \). Field inside an ideal (very high-\( Q \)) resonator is growing with the characteristic time

\[
\tau_g = \frac{2\epsilon\varepsilon_0}{Re[\sigma(\omega)]}. \tag{11}
\]

The characteristic time for domain formation is

\[
\tau_d = \frac{\epsilon\varepsilon_0}{\sigma_0(E_{dc})}, \tag{12}
\]

where \( \sigma_0 \) is the dc differential conductivity.

We are working with a high-frequency fields satisfying \( \omega \tau > 1 \). The condition of LSA, additionally to the requirement for a positive slope of time-averaged VI char-

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**FIG. 3:** Typical current-field characteristic \((\omega \tau_{\text{eff}} = 10, E_{ac} = 10E_{cr})\) demonstrating local structures (a) with positive differential conductivity (b) for \( \omega \tau_{\text{eff}} - 1 < \omega B \tau_{\text{eff}} < \omega \tau_{\text{eff}} + 1 \). (Note that \( \omega_B \tau_{\text{eff}} = E_{dc}/E_{cr} \).) The local structures become weaker, when the ratio of scattering times decreases, but PDC can exist even in a larger range of parameters.

**FIG. 4:** Local PDC and gain can exist simultaneously in a wide range of amplitudes \( E_{ac} \). Dependencies of \( I_1^{\cos}/I_{\text{peak}} \) (a) and \( \sigma_0^2 \) (b) on probe amplitude \( E_{ac} \) are shown, when \( \omega \tau_{\text{eff}} = 10 \) and \( \omega B \tau_{\text{eff}} = \omega \tau_{\text{eff}} + 0.8 \). If \( \nu = 1 \), dc differential conductivity is positive for \( 5E_{cr} \leq E_{ac} \leq 34E_{cr} \) and absorption is negative for \( 0 \leq E_{ac} \leq 27E_{cr} \). This means that gain in conditions of electric stability exists for \( 5 \leq E_{ac} \leq 27E_{cr} \). If \( \nu \) is decreased, the local structures become weaker and the magnitude of the absorption decreases: The gain in conditions of electric stability occurs in a smaller range of \( E_{ac} \) values. The changes are small, if \( \nu > 0.5 \).
FIG. 5: (Color online) Regions of $\sigma_0 > 0$ (grey) and $I_{\text{PDC}}^\omega \cos < 0$ (green) for $\omega \tau_{\text{eff}} = 10$ with different values of $\nu$: (a) $\nu = 0.1$, (b) $\nu = 0.4$ and (c) $\nu = 1.0$. Overlapping of these regions (violet) gives gain without instabilities.

characteristic (PDC), includes also the inequality

$$\omega \tau_d \gg 1.$$  \hfill (13)

It indicates a small (limited) charge accumulation during every THz cycle. Condition (13) can be easily satisfied for typical superlattices. Only in heavy doped superlattices with very wide minibands, $\tau_d$ can be comparable to $\tau$ and therefore $\omega \tau_d = \omega \tau (\tau_d / \tau)$ can approach unity.

On the other hand, increasing the carrier density we are not only increasing the gain but also reducing the time scale for the growth of the field in the resonator (11). Moreover if $\tau_g \ll \tau_d$, then the resonator-mode can reach the minimum amplitude required for switching to local PDC before a domain would be developed. Such a scenario can potentially solve the “device turn-on problem” for Bloch oscillator. The ratio $\tau_g / \tau_d$ depends only on $\omega$ and $\omega_B$, while material parameters are not crucial. However, according to our calculations $\tau_g$ and $\tau_d$ are close to each other for the most interesting case $E_{\text{dc}} \gg E_{\text{cr}}$. This means that the growing space charge domains and the growing ac field cannot be handled separately.

VI. DISCUSSION AND CONCLUSION

In summary, we have shown that THz gain in the conditions of suppressed electric domains is possible in fairly large regions of parameter space, which could allow to build devices which can be used for generation and amplification of THz radiation. Large-signal gain with suppressed domains is preserved with an introduction of two different relaxation times for the electron velocity $\tau_v$ and energy $\tau_e$. In particular, we demonstrated that until $\tau_v / \tau_e \gtrsim 0.5$, the difference from the results obtained using a single $\tau$ is negligible. Note that according to the experiments $\tau_v / \tau_e \gtrsim 0.5$ is a good assumption, although it is not valid for all superlattices.

Quantitatively, the magnitudes of all current components and thus also the gain always decrease with decreasing ratio $\tau_v / \tau_e$.

There remains, however, several important problems, which have not been considered in the present work. The first one is the possible influence of boundary conditions, which according to the computational results of Rieder may turn out to be crucial from the point of view of extended LSA mode. Hopefully, it is still possible to control these boundary conditions at least to some extent in which case the LSA mode can be made to work. Second, we should mention that existence of LSA regime in superlattices at THz frequencies is not completely proven even theoretically. The existence of LSA operational mode is rather well established both theoretically and experimentally in Gunn diodes at microwave frequencies. However, still no experiments devoted to the LSA mode in THz range are performed. From the theory side the consistent derivation of LSA conditions has been done only within quasistatic approximation and thus it is not valid.
at THz frequencies. Therefore, for those who like complete proofs, the existence of extended LSA regime at THz frequencies still continues to be interesting theoretical problem to study. Theoretical research in these and related directions is in progress in Oulu. Of course, experiments would also be very helpful in order to solve the remaining important problems.

Finally, in this work we have mainly focused on large-signal THz gain in superlattices with suppressed space-charge instability. A very important problem that still remains is how to get small-signal gain without domains. One possible solution is to make use of microwave pump to get gain at frequency multiplication in superlattices [30].

More full presentation of our research will be published elsewhere.

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