Anomalous Behavior near $T_c$ and Synchronization of Andreev Reflection in Two-Dimensional Arrays of SNS Junctions

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We have investigated low-temperature transport properties of two-dimensional arrays of superconductor–normal-metal–superconductor (SNS) junctions. It has been found that in two-dimensional arrays of SNS junctions (i) a change in the energy spectrum within an interval of the order of the Thouless energy is observed even when the thermal broadening far exceeds the Thouless energy for a single SNS junction; (ii) the manifestation of the subharmonic energy gap structure (SGS) with high harmonic numbers is possible even if the energy relaxation length is smaller than that required for the realization of a multiple Andreev reflection in a single SNS junction. These results point to the synchronization of a great number of SNS junctions. A mechanism of the SGS origin in two-dimensional arrays of SNS junctions, involving the processes of conventional and crossed Andreev reflection, is proposed.

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Andreev reflection is a microscopic mechanism responsible for the charge transport through a normal-metal–superconductor interface [1]. An electron-like quasiparticle in the normal metal with an energy lower than the superconducting gap ($\Delta$) is reflected from the boundary as a hole-like quasiparticle, while a Cooper pair is transferred to the superconductor. If the normal metal is sandwiched between two superconductors, there is an additional charge transfer mechanism, namely, multiple Andreev reflection (MAR). This mechanism was proposed in [2] to explain the subharmonic energy gap structure (SGS) observed as dips in the differential resistance curves at voltages $eV_n = 2\Delta/n$ ($n$ is an integer). The MAR concept is as follows: owing to sequential Andreev reflections from normal-metal–superconductor (NS) interfaces, a quasiparticle passing through the normal region can accumulate an energy of $2\Delta$, which is sufficient for the transition to single-particle states of the superconductor. Although, by now, ample experimental data on the transport properties of NS and SNS junctions are available [3] and the existing theoretical models fairly well describe the phenomena observed in the experiments [4, 5, 6], the properties of multiply connected SNS systems are poorly investigated. It should be noted that systems consisting of superconducting islands incorporated into a normal metal are spontaneously formed in disordered superconducting films [7]. In view of these circumstances, it is of interest to study the properties of model multiply connected SNS systems with superconducting and normal regions formed in a controlled way.

This paper is devoted to the properties of two-dimensional arrays of SNS junctions fabricated on the basis of a 20-nm-thick PtSi superconducting film (with a critical temperature of $T_c = 0.64$ K) [8]. The transport parameters of the initial PtSi film were as follows: the resistance per square at $T = 4.2$ K was $R_{sq} = 22.8$ $\Omega$, the mean free path was $l = 1.35$ nm, and the diffusion coefficient was $D = 7.2$ cm$^2$/s. The initial samples were fabricated by photolithography in the form of Hall bars 50 $\mu$m wide and 100 $\mu$m long. Then, by electron beam lithography, the initial film was thinned by plasma chemical etching. The period of the structure is 1 $\mu$m, and the island dimensions are 0.8 $\times$ 0.8 $\mu$m. This structure completely covers the whole area of the Hall bar (Fig. a). Thus, the number of islands between the potential terminals is $50 \times 100$. To control the depth of etching, a reference film was etched uniformly over its surface simultaneously with the structure etching. The low-temperature transport measurements were performed by a standard four-probe low-frequency ($\sim$10 Hz) technique. An ac current was within the range 1 – 10 nA.

As the experiment shows, the decrease of film thickness by plasma chemical etching leads to the suppression of the critical temperature. At the same time, in the islands, whose thickness remains equal to that of the initial PtSi film, the suppression of $T_c$ is minor. Hence, there is a certain temperature region within which the thinned film is in the normal (N) state and the islands are in the superconducting (S) state. In this temperature region, the structures under study represent a two-dimensional array of SNS junctions. Moreover, since the superconducting and normal regions of the SNS junctions, fabricated in the aforementioned way, consist of the same...
material, the formation of tunnel barriers at the NS interfaces is excluded and a high transparency of the NS interfaces can be \textit{a priori} expected.

Figure 2 shows the temperature dependence of resistance for two samples that only differ in the film thickness between the islands. The resistance per square of the reference film at $T = 4.2\text{ K}$ is 995 $\Omega$ for sample $S\#1$ and 1483 $\Omega$ for sample $S\#2$. For all structures studied, a noticeable decrease in resistance with decreasing temperature is observed slightly below the temperature $T_c$ of the initial 20-nm-thick PtSi film. As the temperature decreases (Fig. 2), a small decrease in the resistance is first observed at $T \sim 0.61\text{ K}$; then, the resistance increases reaching a maximum and then decreases again. Such an anomalous behavior of the temperature dependence of resistance in the narrow temperature range near $T_c$ was never observed before, and the origin of this behavior is unknown.

To explore the nature of this anomaly, the current–voltage characteristics of the samples have been studied. Figure 3 shows the dependences of the differential resistance ($dV/dI$) on the bias voltage for the sample $S\#1$. The measured values of $dV/dI$ are given per square. The total voltage is divided by the number of junctions (100) in the rows between the potential probes. For a square array, this procedure yields the average voltage across one SNS junction, and this quantity is represented by the abscissa axis in Fig. 3 (below, all dependences of the differential resistance on the bias voltage are plotted in the same coordinate systems). The dependences of $dV/dI$–$V$ are symmetric with respect to the direction of the current. As one can see from the dependences shown in

Fig. 3, an increase in temperature leads to a suppression of the excess conductivity and, at $T \simeq 570\text{ mK}$, the minimum observed at $V = 0$ is replaced by a maximum. It should be emphasized that the change of a minimum to a maximum in the differential resistance dependences at zero bias voltage occurs in the same temperature region within which the anomaly is observed in the temperature dependence of resistance (in Fig. 2 for sample $S\#1$, this interval is indicated by arrows).

Let us estimate the characteristic energy scales for the two-dimensional array of SNS junctions under study. The energy corresponding to the voltage, at which the minimum is suppressed at zero bias and temperatures below 570 mK, is estimated as $eV_{ex} \simeq 3\text{ }\mu\text{eV}$. The presence of the maximum in the differential resistance at $T > 570\text{ mK}$ may be caused by the minimum that occurs in the density of states in the normal region. The theoretical study of the properties of diffusive NS junctions \[9\] shows that the decrease in the density of states is caused...
by the presence of coherent Cooper pairs in the normal metal. According to the theory, at a distance \( x \) from the NS interface, only the pairs from the energy window \( E_T \pm E_{\text{Th}} \) (the coherence window), where \( E_{\text{Th}} = \hbar D/x^2 \) is the Thouless energy, will remain coherent. Therefore, the density of states has a minimum at energies \( E < E_{\text{Th}} \) and a maximum at \( E \approx E_{\text{Th}} \), and value of the density of states in the maximum is higher than that in the normal metal. The experimental study of the density of states as a function of the distance to the NS interface [10] showed a good agreement with the theory. Note that this effect manifests itself in two ways. On the one hand, the presence of coherent Cooper pairs leads to a decrease in the resistance of the normal region (analog of the Maki-Thompson correction), and, on the other hand, the pairing leads to a decrease in the density of single-particle states (an analog of the correction to the density of states in the Cooper channel). These competing contributions may lead to a nonmonotonic temperature dependence of the resistance of SNS junctions [11]. Let us estimate the Thouless energy \( E_{\text{Th}} = \hbar D/L^2 \) for the structure under study, where \( L = 0.2 \mu m \) is the length of the normal region and \( D = 2 \text{ cm}^2/\text{s} \) is the diffusion coefficient for the reference film. Then, the Thouless energy is \( E_{\text{Th}} \approx 3 \mu \text{eV} \), which nearly coincides with the characteristic voltage value (in Fig. 3 this value is indicated by the vertical straight line). However, for the observation of the effects associated with changes in the quasiparticle spectrum at the Thouless energy, the necessary condition is \( kT < E_{\text{Th}} \). In the temperature range of interest, e.g., at \( T = 580 \text{ mK} \), the thermal broadening is \( kT \approx 50 \mu \text{eV} \), which noticeably exceeds the correlation energy \( E_{\text{Th}} \) for the SNS junctions forming the array. Thus, one should expect that the processes determined by \( E_{\text{Th}} \) would not manifest themselves in the differential resistance curves. However, the experimental dependences clearly display the suppression of the maximum at a voltage of about \( E_{\text{Th}}/e \). This fact suggests that the effect observed in the experiment is collective; i.e., the whole array is responsible for its manifestation. Note that the total bias voltage is \( eV_{\text{ex}} \cdot 100 = eV_c = 300 \mu \text{eV} \) and it is much higher than \( kT \). We will return to discussing this issue after presenting results that, in our opinion, also testify to a correlation in the behavior of the array of SNS junctions.

Figure 4a shows the dependences of the differential resistance on the bias voltage per one SNS junction. These dependences have pronounced minima at some voltage values (the minima are marked by \( \alpha, \beta, \) and \( \gamma \) in the plot). When temperature increases, the minima are shifted to lower voltages and, in the interval from 550 to 600 mK, one more minimum (\( \delta \)) appears; at lower temperatures, this minimum is absent. The temperature dependences of the positions of these features are shown in Fig. 4b by different symbols. The solid lines in Fig. 4b represent the temperature dependences of the superconducting energy gap \( 2\Delta(T)/en \), where \( \Delta(T) \) is the temperature dependence of the superconducting energy gap predicted by the BCS microscopic superconductivity theory and \( n \) is an integer (the corresponding values are indicated in Fig. 4b). The exact value of \( \Delta(0) \) for platinum monosilicide is unknown. The estimate by the formula \( \Delta(0) = 1.76kT_c \) \((T_c = 0.88 \text{ K for bulk PtSi} \) [12]) yields the value \( \Delta/e \approx 133 \mu \text{V} \). If we set \( \Delta/e = 126 \mu \text{V} \), the positions of the features in the voltage dependences of the differential resistance will be determined by the condition \( eV = 2\Delta(T)/n \). From Fig. 4b, one can see that the temperature dependences of the positions of minima observed in the experimental differential resistance curves functionally coincide with the dependences \( 2\Delta(T)/en \). From this fact, we can conclude that these features represent the subharmonic energy gap structure (SGS).

However, there are two circumstances clearly indicating that it is difficult to treat the features observed at bias voltages equal to \( 2\Delta(T) \) as the manifestation of the SGS caused by multiple Andreev reflections that occur independently in each SNS junction and that this interpretation is impossible for large values of \( n \). First, the conventional mechanism of the SGS formation in a single SNS junction implies that, for the appearance of a subharmonic of number \( n \), an \( n \)-fold passage of quasiparticles through the normal region is necessary without any energy relaxation. In other words, the energy relaxation length \( (l_r) \) must be no smaller than \( n \cdot L \) [6]. The en-

\[ dV/dI (\Omega) \]

**FIG. 3:** Differential resistance per square versus the bias voltage per SNS junction for sample S#1. All dependences, except for the lower one, are sequentially shifted upwards by 0.5Ω.
energy relaxation length can be estimated from the known phase breaking length $l_\varphi$. In the temperature range of interest, $l_\varphi$ can exceed $l_\varphi$ by no more than an order of magnitude. When a carrier acquires an energy of $2\Delta = 252 \, \mu\text{eV}$, it is heated to a temperature of $\sim 3 \, \text{K}$. According to our magnetotransport measurements, at this temperature, we have $l_\varphi = 40 \, \text{nm}$. Then, the most optimistic estimate yields $l_\varphi \sim 0.4 \, \text{nm}$. This value is much smaller than $14 \times 0.2 \, \mu\text{m} = 2.8 \, \mu\text{m}$, which is necessary for the realization of the subharmonic with $n = 14$. Second, it is difficult to expect that normal regions are fully identical, i.e., characterized by exactly the same resistance. Evidently, we deal with a network of different resistances. Therefore, when a current flows through the structure, the voltages across the normal regions are different. This means that, if, for some of the normal regions, the condition $eV = 2\Delta/n$ is satisfied at a given value of the current, for other regions it will be satisfied at some other values of the current. Only when the resistances are close to each other, it is possible to observe the subharmonic energy gap structure as a result of the statistical averaging; however, this structure will be smeared in proportion to the deviations of actual resistances from the average value. These speculations lead to a natural conclusion that, for a network of random resistances, the observation of the subharmonic energy gap structure is impossible. Contrary to this conclusion, the experiment shows that the SGS still manifests itself, although it is somewhat irregular; i.e., it lacks some of the harmonics. Similar results for an array of SNS junctions of other configuration were obtained in [14]. By now, no theory has been developed to describe multiply connected SNS systems, and, on the basis of the theoretical results obtained for single SNS junctions, it is difficult to explain the observation of subharmonics with numbers reaching $n = 14$.

In considering the properties of single SNS junctions, the superconducting regions are usually assumed to be large and serving as reservoirs for electrons. The fundamental distinction of the array of SNS junctions is the finite size of the superconducting regions ($L_S$). Therefore, it is necessary to take into account the possibility of the mutual influence of the Andreev reflection processes that occur at the interfaces of one superconducting island. In NSN junctions, in addition to the “common” Andreev reflection, when an electron-like excita-
tion is reflected at the interface of the superconductor with transformation into a hole-like excitation (in Fig. 4 process 1: \( e_i \leftrightarrow h_i \)), an additional process of crossed Andreev reflection \( e_i \leftrightarrow h_j \) (process 2) is possible, when the electron-like and hole-like excitations are on different sides of the superconducting region. As a consequence, such a reflection in combination with the common Andreev reflection, which occurs on one side of the superconductor (process 3), leads to the passage of quasiparticles without any energy loss. Note that, for the observation of the \( n \)th subharmonic of Andreev reflection, the realization of this mechanism of charge transport does not require that the voltage across each of the normal regions be exactly equal to \( V_n = 2\Delta/en \). It is sufficient that the total voltage across the chain of normal regions connected in series through superconducting regions be equal to \( 2\Delta/e \):

\[
\sum_{s=1}^{i+n-1} V_s = 2\Delta/e.
\]

Moreover, for the observation of the \( n \)th subharmonic of Andreev reflection, the condition \( l_s > nL \) needs not necessarily be satisfied. It is sufficient that the energy relaxation length be greater than the distance between the superconducting regions \( (l_s > L) \). We also note that the crossed Andreev reflection process makes a considerable contribution to the charge transport through an NSN junction even if the size of the superconducting region is several times greater than the superconducting coherence length \( \xi \). In our case, \( \xi(0) \simeq 70 \text{ nm} \), and at \( T = 100 \text{ mK} \), we have \( L_S/\xi \simeq 10 \).

We believe that this charge transport mechanism is responsible for the features observed in the experiment. Returning to the behavior of the two-dimensional array of SNS junctions near zero bias, we note that the synchronous Andreev reflection on both sides of the superconducting regions should lead to correlated changes of the energy spectra in normal regions by analogy with the formation of minibands and minigaps in semiconductor superlattices.

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[1] A.F. Andreev, Zh. Eksp. Teor. Fiz. 46, 1823 (1964) [Sov. Phys. JETP 19, 1228 (1964)].
[2] T.M. Klapwijk, G.E. Blonder, and M. Tinkham, Physica 109-110B+C, 1657 (1982).
[3] B. Pannetier and H. Courtois, J. of Low Temp. Phys. 118, 599 (2000).
[4] G.E. Blonder, M. Tinkham, and T.M. Klapwijk, Phys. Rev. B25, 4515 (1983).
[5] M. Octavio, M. Tinkham, G.E. Blonder, and T.M. Klapwijk, Phys. Rev. B27, 6739 (1983); K. Flensberg, J. Bindslev Hansen, M. Octavio, ibid. 38, 8707 (1988).
[6] E.V. Bezuglyi, E.N. Bratus’, V.S. Shumeiko, G. Wendin, and H. Takayanagi, Phys. Rev. B62, 14439 (2000).
[7] M.V. Feigel’man, A.I. Larkin, and M.A. Skvortsov, Phys. Rev. Lett. 86, 1869 (2001).
[8] R.A. Donaton, S. Jin, H. Bender, et al, Microelectronic Engineering 37/38, 504 (1997).
[9] B.J. van Wees, P. de Vries, P.H.C. Magoon, and T. M. Klapwijk, Phys. Rev. Lett. 69, 510 (1992).
[10] S. Gueron, Norman O. Birge, D. Esteve, and M.H. Devoret, Phys. Rev. Lett. 77, 3025 (1996).
[11] Anatoly F. Volkov and Hideaki Takayanagi, Phys. Rev. B 56, 11184 (1997).
[12] B.T. Matthias, T.H. Geballe, and V.B. Compton. Rev. Mod. Phys. 35, 1 (1963).
[13] S.I. Dorozhkin, F. Lell, and W. Schoepe, Solid State Comm. 60, 245 (1986).
[14] T.I. Baturina, Z.D. Kvon, and A.E. Plotnikov, Phys. Rev. B 63, 180503(R) (2001).
[15] J.M. Byers and M.E. Flatté, Phys. Rev. Lett. 74, 306 (1995).
[16] G. Deutschner and D. Feinberg, Appl. Phys. Lett. 76, 487 (2000).
[17] Arne Jacobs and Reiner Kümmel, Phys. Rev. B 64, 104515 (2001).
[18] D. Beckman, H.B. Weber, H.v. Löhneysen, Phys. Rev. Lett. 93, 197003 (2004).