Resilience of Electricity Distribution Networks - Part II: Leveraging Microgrids

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Abstract—Recent technological advances in microgrids powered by Distributed Energy Resources (DERs) make them an attractive response mechanism for improving the resilience of electricity distribution networks (DNs) to reliability and security failures. This paper presents an approach to evaluate the value of implementing a timely response using microgrid operations and DER dispatch in the aftermath of a disruption event, which involves strategic compromise of multiple DN components. Firstly, we refine the modeling and resiliency assessment framework in [1] and develop a sequential (bilevel) formulation which models attacker-operator interactions on a radial DN with one or more microgrids. Particularly, the operator response model includes microgrid operations under various islanding configurations (regimes), and single- or multi-master operation of DERs in providing grid-forming services as well as frequency and voltage regulation. Secondly, we introduce a restoration problem in which the operator gradually reconnects the disrupted components over multiple periods to restore the nominal performance of the DN. The first problem, formulated as a bilevel mixed-integer problem, is solved using Benders decomposition method. The second problem, formulated as a multi-period mixed-integer problem, can be solved using a greedy algorithm. Our computational results illustrate the benefit of using microgrids in reducing the post-contingency losses, both immediately after the disruption event and during the restoration process.

Index Terms—Cyber-physical systems, network security, smart grids, bilevel optimization

I. INTRODUCTION

Modern electricity Distribution Networks (DNs) are prone to risks of service interruptions due to the failures of unreliable, and often insecure, cyber and physical components. Recent disruptions caused by natural disasters [2] and security attacks [3, 4] highlight the vulnerability of DNs to cyberphysical failures. In this article, we investigate the use of microgrid technologies, in particular microgrid islanding and dispatch of Distributed Energy Resources (DERs) [4, 5, 6] toward improving DN resilience. Historically, the idea of DER-powered microgrids as a response mechanism has been considered for responding to reliability failures [4, 6]. Indeed, microgrids have been implemented in practice to support the reliability targets of critical facilities such as hospitals, industrial plants, and military bases. However, their technological feasibility (and related operational aspects) in responding to security failures has received limited attention in the existing literature. We address this issue by building on the modeling and resiliency assessment framework in [1], and focus on evaluating the effectiveness of DER-powered microgrids in limiting post-contingency losses after a disruption.

We model the sequential interaction between a DN operator and an external adversary as follows [1]:

$$\mathcal{L}_{\text{lim}} := \max_{d \in D} \min_{u \in \mathcal{U}(d)} L(u, x) \quad \text{s.t.} \quad x \in \mathcal{X}(u), \quad (P1)$$

where $d \in D$ denotes an attacker strategy, $u \in \mathcal{U}(d)$ an operator response strategy, $x \in \mathcal{X}$ the network state, and $L$ the composite loss function. In [1], we argued that cyberphysical disruptions to DNs can lead to operating bound violations and cause uncontrolled or forced disconnects of DN components. Specifically, we modeled the impact of attacker-induced disconnects of DN components as supply-demand disturbances, and the impact of grid-side disturbances as voltage deviations at the substation node. Then, we considered preemptive load control and component disconnects as operator response actions for the generic setting when the attacker’s (resp. operator’s) goal is to maximize (resp. minimize) the post-contingency losses. We introduced $R_{\text{lim}} := 100(1 - \mathcal{L}_{\text{lim}}/\mathcal{L}_{\text{max}})$ as a resilience metric of the DN, where $\mathcal{L}_{\text{max}}$ (chosen for sake of normalization) denotes the operator loss when all DN components are disconnected; see Fig. 1. We showed that the losses incurred by DN operators can be reduced if the available response actions are implemented in a timely manner after a cyberphysical disruption. Finally, we evaluated the value of optimal response as the total reduction in post-contingency losses relative to the no response case, i.e., $R_{\text{lim}} - R_{\text{NR}}$, where $R_{\text{NR}} = 100(1 - \mathcal{L}_{\text{NR}}/\mathcal{L}_{\text{max}})$.

In this paper, we consider another bilevel formulation:

$$\mathcal{L}_{\text{so}} := \max_{d \in \mathcal{D}_m} \min_{u \in \mathcal{U}_m(d)} L_m(u, x) \quad \text{s.t.} \quad x \in \mathcal{X}_m(u), \quad (P2)$$

where the network model $\mathcal{X}_m$, and the loss function $L_m$ are extended to capture the microgrid operations (Sec. II) and DER dispatch and regulation aspects (Sec. III); the set of attacker strategies $\mathcal{D}_m$ and the set of operator strategies $\mathcal{U}_m$ are also modified to capture attacker-operator interactions for DNs with DER-powered microgrids (Sec. IV). The minimax value of (P2) $\mathcal{L}_{\text{so}}$ denotes the worst-case post-contingency loss incurred by the operator for given microgrid and DER capabilities; see...
Fig. 1. Then, analogous to [1], \( R_{\text{MG}} := 100 \left( 1 - \frac{\mathcal{L}_{\text{DG}}}{\mathcal{L}_{\text{max}}} \right) \) can be viewed as a resilience metric of the DN under the extended operator response model. Furthermore, the relative value of timely microgrid response (or equivalently, the improvement in DN resilience due to microgrids) can be evaluated as \( (R_{\text{MG}} - R_{\text{Mm}}) \).

![System performance under various operator response capabilities](image)

Analogous to [1], we consider that a secure Substation Automation (SA) system can detect the disrupted components from changes in measurements of net nodal consumption. By using the knowledge of attack vector the SA can compute and implement the operator response in a timely manner. For our purposes, this is an optimal second-stage response in (P2). Our analysis in this article relies on the premise that such response can be implemented via modern SA systems during disruptions caused by security attacks (besides the classical reliability failures). Furthermore, the continued improvements in SA systems’ attack detection and DN control capabilities can further assist in restoration operations.

Recall that the resiliency of a system also concerns with how quickly it can rebound to its nominal state after a disruption [1, 7]. Microgrids can provide partial demand satisfaction during the system restoration process, especially during the time when the DN is fully disconnected from the TN. We consider an admittedly simple, but practically relevant, multi-period DN restoration problem in which the disrupted DN components are gradually restored over several periods; refer to “DN restoration” in Fig. 1. Our goal in this problem is to compute an operator strategy in each time period (roughly, of the order of few minutes). Such a strategy is comprised of reconnecting disrupted components, modifying the microgrid islanding configuration, and dispatching DERs.

Our modeling approach addresses some key issues regarding microgrid and DER operations. In particular, we allow for the formation of one or more microgrid islands in radial DNs. When all the microgrids are connected to the transmission network (TN), the DN is operating in the grid-connected regime. If none of the microgrids are connected to the TN, then the DN is operating in fully-islanded regime. In our approach, the DN can also operate in a partially-islanded regime, in which some of the microgrids are connected to the TN while other microgrids are not. In both partially- and fully-islanded regimes, each microgrid can operate as an isolated microgrid or as a part of a bigger microgrid. To model power flows in each of the microgrids, we introduce a natural extension of the LinDistFlow equations. The resulting network model captures DN operations in all the above-mentioned regimes (Sec. II). We limit attention to linear power flows mainly for the ease of exposition.

Furthermore, we consider the parallel operation of multiple DERs for the provision of grid-forming services, which involve providing voltage and frequency references, as well as maintaining voltage and frequency within operating bounds (i.e. regulation services). When a microgrid is connected to the TN, the bulk generators provide the grid-forming services. However, when a microgrid is disconnected from the TN, then at least one DER within that microgrid must provide the grid-forming services [5].

Depending on the number of grid-forming DERs within a microgrid, one can consider two modes of DER operation under islanded regimes namely: Single-Master Operation (with a single grid-forming DER) and Multi-Master Operation (with more than one grid-forming DERs) [5].

To the best of our knowledge, ours is the first model which is simple and flexible enough to capture both the single-/multi-master modes of DER operation. In addition to voltage regulation, we also consider frequency regulation, which becomes important for microgrids due to low inertia of the DERs. In particular, by using the appropriate droop control equations, we capture both frequency and voltage regulation aspects resulting from multiple DERs operating in parallel within a microgrid operating in an islanded regime (Sec. III).

We capture the different microgrid regimes as well as DER operating modes using a mixed-integer linear network model. This modeling approach has two major implications. Firstly, it enables us to formulate (P2) as a Bilevel Mixed-Integer Problem (BiMIP). Recall from [1] that (P1) is also a BiMIP, and can be solved using a Benders Decomposition (BD) algorithm. In Sec. IV, we show that the same BD method can be applied to the extended formulation (P2). Secondly, our network model is particularly well-suited for formulating a DN restoration problem as a Mixed-Integer Problem (MIP).

In our restoration problem, the network state in any period only depends on the operator response actions in that period, and the network state in the previous period. We exploit this feature and propose a greedy heuristic that seeks to reconnect the disrupted components in each period such that the post-contingency losses for that period are minimized (Sec. V). Finally, we present the computational results for our implementations of the BD algorithm and the greedy heuristic using modified IEEE 24-, 36-, and 118-node networks.

II. MULTI-MICROGRID DN MODEL

In this section, we develop a model of a radial DN with one or more microgrids. This network model extends the LinDistFlow model [8] to multi-microgrid settings.

We distinguish between two operating stages \( o \) and
c, which denote the pre- and post-contingency stage, respectively. The network is initially in o stage, and after the disturbance event, it enters in the c stage; see Sec. IV for details on disturbance model. Let \( \eta \in \{0, c\} \) denote the operating stage of the network. We define the network state as \( x^n := (p^n, q^n, P^n, Q^n, v^n, f^n)^T \), where each of these entries are themselves row vectors of appropriate dimensions, and are described in Table I.

### Table I: Table of Notations

| DN parameters | Definition |
|---------------|------------|
| \( N \) | set of nodes in DN |
| \( \mathcal{E} \) | set of edges in DN |
| \( M \subseteq \mathcal{E} \) | set of microgrid connecting lines |
| \( N_i' \subseteq N \) | nodes belonging to \( i^{th} \) microgrid |
| \( M_i \subseteq M \) | set of lines which if open isolate the \( i^{th} \) microgrid |
| \( \mathcal{S}_N \subseteq \mathcal{S} \) | set of DERs |
| \( \mathcal{S}_{gi} \subseteq \mathcal{S}_N \) | set of grid-noninteractive DERs |
| \( \mathcal{S}_{gi} \subseteq \mathcal{S}_N \) | set of grid-interactive DERs |
| \( \mathcal{S}_{gi} \subseteq \mathcal{S}_{gi} \) | set of PQ inverter (PQI)-controlled DERs |
| \( j \) | complex square root of \(-1\), \( j = \sqrt{-1} \) |
| \( v_{nom} \) | nominal squared voltage magnitude (1 pu) |
| \( f_{nom} \) | nominal system frequency (1 pu) |
| \( p_i \subseteq \mathcal{E} \) | lines on the path between node i and substn node |
| \( \eta \) | the DN node where the DER \( s \in \mathcal{S} \) is located |
| \( p^n \) | apparent power capability of microsource \( s \in \mathcal{S}_N \) |
| \( q^n \) | lower, upper active power bounds of microsource \( s \in \mathcal{S}_N \) |
| \( q^n \) | lower, upper reactive power bounds of microsource \( s \in \mathcal{S}_N \) |
| \( p^n \) | apparent power capability of storage device \( s \in \mathcal{S}_N \) |
| \( q^n \) | lower, upper active power bounds of microsource \( s \in \mathcal{S}_N \) |
| \( q^n \) | lower, upper reactive power bounds of microsource \( s \in \mathcal{S}_N \) |
| \( p^n \) | if DER \( s \in \mathcal{S}_N \) contributes to grid-forming services |
| \( q^n \) | total power supplied by DER \( s \in \mathcal{S}_N \) |
| \( p^{ref} \) | active, reactive power references of DER \( s \in \mathcal{S}_N \) |
| \( q^{ref} \) | frequency, voltage references of DER \( s \in \mathcal{S}_N \) |
| | Parameters of edge \( (i, j) \in \mathcal{E} \) |
| \( k_{ij} \) | 1 if \( (i, j) \) is switched open; 0 otherwise |
| \( P_{ij} + jQ_{ij} \) | power flowing from node \( i \) to node \( j \) |
| \( r_{ij}, x_{ij} \) | resistance and reactance of line \( (i, j) \in \mathcal{E} \) |

Fig. 2: Multi-microgrid DN model.

Let \( \{N_1, \cdots, N_{|M|}\} \) denote the set of disjoint microgrid subnetworks of the DN, where each \( N_i \) for \( i \in \{1, \cdots, |M|\} \) denotes a connected subnetwork when all connecting lines are open, i.e. \( kt_{mn}^i = 1 \) for all \((m, n) \in M\). For each subnetwork \( N_i \), let \( M_i \subseteq M \) denote the set of connecting lines which need to be open for \( N_i \) to be completely isolated (i.e. autonomously operating). Also, let \( P_i \) denote the set of lines along the path connecting node \( i \) to the substn node. For example, in Fig. 2, the connecting lines for the subnetwork \( N_1 = \{1, 2\} \) are \( M_1 = \{(0, 1), (2, 3), (2, 5)\} \), and \( P_1 = \{(0, 1), (1, 2), (2, 5)\} \).

Now, we describe the constraints related to the power flows, nodal frequencies and load connectivity in microgrids. Unless explicitly stated, the following constraints are valid for any operating stage \( \eta \).

1. **Power flows**: A connecting line permits power flow through it if and only if it is closed. We model this constraint as follows:

   \[
   |P_{ij}^n| \leq (1 - k_{ij}^n) M \quad \forall (i, j) \in M \tag{1a}
   \]

   \[
   |Q_{ij}^n| \leq (1 - k_{ij}^n) M \quad \forall (i, j) \in M \tag{1b}
   \]

   where \( M \) is a large constant.

2. **Voltage drop**: The voltage drop along a non-connected line \((i, j) \notin M\) is given by the standard voltage drop equation of the LinDistFlow model [8]:

   \[
   v_{ij}^n = v_{ij}^0 - 2r_{ij}P_{ij}^n - 2x_{ij}Q_{ij}^n \quad \forall (i, j) \in \mathcal{E}\setminus\mathcal{M}. \tag{2}
   \]

   However, for a connecting line, the voltage drop constraint is active only if it is closed, and is inactive, otherwise, i.e.

   \[
   |v_{ij}^n - (v_{ij}^0 - 2r_{ij}P_{ij}^n - 2x_{ij}Q_{ij}^n)| \leq k_{ij}^n M \quad \forall (i, j) \in M. \tag{3}
   \]

3. **Nodal frequencies**: In islanded regimes, the DER(s) must provide grid-forming and regulation services [5, 9]. Moreover, a microgrid island can have multiple DERs operating in parallel. We assume that DERs can rapidly synchronize their frequencies to a common value with the help of power electronics [5]. This value can be regarded as the island’s frequency. To model that, in
steady state, the nodal frequencies within a microgrid island are identical, we can write:

\[ f_i^n = f_j^n \quad \forall \ i, j \in N_k \text{ and } \forall \ k = 1, \ldots, |\mathcal{M}|, \]

which is equivalent to writing:

\[ f_i^n = f_j^n \quad \forall \ (i, j) \in \mathcal{E} \setminus \mathcal{M}. \quad (4) \]

However, since the DERs have much lower inertia in comparison to a bulk generator, they may not be able to provide frequency regulation that matches the system frequency in the grid-connected regime. As a result, the frequency of a microgrid can be different from the frequency of the TN-connected substation node. Moreover, the system frequencies of two microgrid islands that are not connected to each other, can potentially be different. We model this constraint as follows:

\[ |f_i^n - f_j^n| \leq k c_i^n |\mathcal{M} \quad \forall \ (i, j) \in \mathcal{M}. \quad (5) \]

Finally, we model the constraint that the load gets disconnected (i.e. \( k c_i^n = 1 \)) when on the nodal frequency violates the safe operating bounds:

\[ k c_i^n \geq f_i^n, \quad k c_i^n \geq f_i^n - f_i^n \quad \forall \ i \in N. \quad (6) \]

Note that this constraint becomes relevant especially in the islanded regimes due to the low inertia of the DERs.

III. DISTRIBUTED ENERGY RESOURCES (DERs)

We now introduce a generic taxonomy of DERs that is relevant to microgrid operations and a DER model which captures the two single-/multi-master operating modes of DERs. Please refer to Fig. 3 for the DER taxonomy and Table II for a comparison of various DER types.

![Fig. 3: Basic taxonomy of DERs [10].](image)

**DER classification:** We distinguish DERs in terms of their output behaviour and the service capabilities. First, we distinguish between DERs with fixed output (grid-noninteractive DERs) and DERs whose output is responsive to varying grid conditions (grid-interactive DERs). We denote the sets of grid-noninteractive and grid-interactive DERs by \( S_{\text{gn}} \) and \( S_{\text{gf}} \), respectively. For the sake of clarity, we refer to DERs in set \( S_{\text{gn}} \) as distributed generators (DGs). In this article, we consider that DGs are customer-owned, whereas the grid-interactive DERs (\( S_{\text{gf}} \)) are utility-owned; see Table Ia. Indeed, customer-owned DGs are typically used to support local loads, and their output is set to a fixed value to ensure optimal performance (e.g., efficiency rating) of local loads. Since DG output cannot be changed, they need to be disconnected either by remote means or through autonomous disconnect mechanism to prevent operating bound violations.

In contrast, the utility-owned DERs can stay connected to the TN as zero output source even under significant fluctuations in the network state. Particularly, we assume that utility-owned DERs are fitted with low-voltage and low-frequency ride through (LVRT and LFRT) functionalities. This allows DERs to stay connected to the TN during temporary voltage and frequency bound violations at nodes. Furthermore, the output of utility-owned DERs can be changed by two control mechanisms. In the case of grid-forming DERs (\( S_{\text{gf}} \)), droop-based primary control is activated under specific islanding conditions. In the case of PQ inverter (PQI-) controlled DERs, their active-reactive (PQ) setpoints can be remotely controlled; see Table IIb.

Finally, there are two categories of grid-forming DERs, namely AC-source DERs and DERs connected to the TN via Voltage-Source Inverters (VSIs). Each of these categories contributes to grid-forming services depending upon the specific islanding conditions; see Table Ic. Let \( k r_s^n = 1 \) if the islanding condition for DER \( s \in S_{\text{gf}} \) is satisfied, and \( k r_s^n = 0 \) otherwise. The two main islanding conditions of interest are as follows:

1) An AC-sourced DER contributes to grid-forming services when the microgrid to which it belongs is not connected to the TN. Consider a DER \( s \in S_{\text{ac}} \) and a microgrid \( N_k \) such that \( j(s) = i \in N_k \). Then, DER \( s \) contributes to grid-forming iff \( N_k \) is disconnected from the TN, or equivalently, at least one connecting line along the path connecting node \( i \) to the substation is open, i.e.

\[ k r_s^n = 1 \iff \exists \ (m, n) \in \mathcal{M} \cap \mathcal{P}_i \mid k l_{mn}^n = 1. \]

We formulate this condition using the following mixed-
integer linear constraints:

\[
kr^p_s \geq kl^p_{mn} \quad \forall \, (m,n) \in \mathcal{M} \cap \mathcal{P}_i \\
kr^q_s \leq \sum_{(m,n) \in (\mathcal{M} \cap \mathcal{P}_i)} kl^q_{mn}.
\]

2) The VSI-controlled DERs also contribute to grid-forming services when the microgrid to which they belong operates as an isolated island. Consider a DER \( s \in \mathcal{S}_{si} \) and a microgrid \( \mathcal{N}_i \) such that \( j(s) \in \mathcal{N}_i \). Then, DER \( s \) contributes to grid-forming if and only if all the connecting lines connecting the microgrid \( \mathcal{N}_i \) to the TN or other microgrids are open, i.e.

\[
k^p_s = 1 \iff kl^p_{mn} = 1 \quad \forall \, (m,n) \in \mathcal{M}_i.
\]

We formulate this condition using the following mixed-integer linear constraints:

\[
kr^p_s \geq \left( \sum_{(m,n) \in \mathcal{M}_i} kl^p_{mn} \right) - (|\mathcal{M}_i| - 1) \quad (8a)
\]
\[
kr^q_s \leq kl^q_{mn} \quad \forall \, (m,n) \in \mathcal{M}_i. \quad (8b)
\]

**DER output model:** Next, we describe the output model for the DERs. Each grid-forming DER \( s \in \mathcal{S}_{gf} \) consists of a microsource and a storage device (batteries or flywheels) [5]. The microsource supplies power (quadrants I or II) and the storage device is constrained as follows (see Fig. 4). The total output of the microsource and the storage device is constrained as follows:

\[
G_s [p_s^q \quad q_s^q]^T \leq h_s \quad \forall \, s \in \mathcal{S}_{gf} \quad (9a)
\]
\[
G_s [p_s^q \quad q_s^q]^T + H_s kr^q_s \leq h_s \quad \forall \, s \in \mathcal{S}_{gf}. \quad (9b)
\]

where the \( H_s \) matrix is such that the DER operates in the quadrants III and IV when \( kr^q_s = 0 \); and in the quadrants I and II, when \( kr^q_s = 1 \); see Fig. 4. The total output of the DER is given by:

\[
p_s^q = p_s^q + p_s^e \quad \forall \, s \in \mathcal{S}_{gf}
\]
\[
q_s^q = q_s^q + q_s^e \quad \forall \, s \in \mathcal{S}_{gf}. \quad (10)
\]

On the other hand, PQI-controlled DERs (\( \mathcal{S}_{pq} \)) consist only of microsource, and do not have a storage device. Thus, their output is constrained as in Fig. 4a. We can simply assume that \( \forall \, s \in \mathcal{S}_{pq}, \, p_s^e = q_s^e = 0 \).

**Droop control equations:** We model the regulation services provided by one or more grid-forming DERs using the voltage and frequency droop control equations [9]. This allows the DERs to adjust their active and reactive power outputs based on local voltage and frequency measurements, thus eliminating the need for explicit coordination among DERs (for the purpose of regulation).

The output changes of a grid-forming DER \( s \in \mathcal{S}_{gf} \) depend on the whether or not it is contributing to regulation (i.e. \( kr^q_s = 1 \) or 0) based on the islanding conditions (see (7) and (8)). Then, the classical voltage droop equation can be refined to model the reactive power output of grid-forming DER as follows (see Fig. 5a):

\[
|v_i^q - (v_{ref}^q - mp_s(q^q_{ref} - q^q_s))| \leq (1 - kr^q_s) M \quad \forall \, s \in \mathcal{S}_{gf}, \, i \in \mathcal{N} \mid i = j(s). \quad (11)
\]

Eq. (11) implies that when a DER provides regulation, it contributes more (resp. less) reactive power to as the voltage drops (resp. rises) relative to a reference value.

Similarly, the classical frequency droop control equation can be refined to model the active power output of grid-forming DER as follows (see Fig. 5b):

\[
|f_i - (f_{ref} - mp_s(p^q_{ref} - p^q_s))| \leq (1 - kr^q_s) M \quad \forall \, s \in \mathcal{S}_{gf}, \, i \in \mathcal{N} \mid i = j(s). \quad (12)
\]

Eq. (12) ensures proper power sharing in the sense that DERs can adjust their active power contributions for frequency regulation depending on their individual capacities. The reference setpoints \( (f_{ref}, v_{ref}, p^q_{ref}, q^q_{ref}) \) and the droop coefficients \( (mp_s, mq_s) \) are given constants.

As in [1], we assume that each node has a customer-owned DG without loss of generality. Then, similar to the loads, we model the dependence of DG connectivity on the nodal frequency as follows:

\[
kg^q_i \geq fg_i - f^q_i, \quad kg^q_i \geq f^q_i - fg_i \quad \forall \, i \in \mathcal{N}. \quad (13)
\]

Eq. (13) implies that a DG will disconnect if the corresponding nodal frequency violates safe operating bounds.
The net power consumed at a node \( i \) is the power consumed by the load minus the power generated by the DGs and other grid-interactive DERs at that node, i.e.

\[
p^0_i = p^c_i - p^g_i + \sum_{s \in S_{g,n}(i)=i} p^s_i \quad \forall \ i \in N \quad (14a)
\]

\[
q^0_i = q^c_i - q^g_i + \sum_{s \in S_{g,n}(i)=i} q^s_i \quad \forall \ i \in N \quad (14b)
\]

Finally, we summarize the LinDistFlow and connectivity constraints described in [1] as follows:

\[
P^0_{ij} = \sum_{k \in E} P^0_{jk} + p^0_j \quad \forall \ (i, j) \in E \quad (15)
\]

\[
Q^0_{ij} = \sum_{k \in E} Q^0_{jk} + q^0_j \quad \forall \ (i, j) \in E \quad (16)
\]

\[
p^g_i = (1 - k p^g_i) \eta^g_i \quad \forall \ i \in N \quad (17)
\]

\[
q^g_i = (1 - k q^g_i) \eta^g_i \quad \forall \ i \in N \quad (18)
\]

\[
\beta^{0}_{i} = \beta^{p}_{i} \eta^{p}_{i}, \quad q^{0}_{i} = \beta^{q}_{i} \eta^{q}_{i} \quad \forall \ i \in N \quad (19)
\]

\[
(1 - k c^i_{j}) \beta^{0}_{i} \leq \beta^{0}_{i} \leq (1 - k c^i_{j}) \quad \forall \ i \in N \quad (20)
\]

\[
k c^i_{j} \geq \nu^c_{i} - v^c_{i}, \quad k c^i_{j} \geq v^c_{i} - \nu^c_{i} \quad \forall \ i \in N \quad (21)
\]

\[
k g^i_{j} \geq \nu^q_{i} - v^q_{i}, \quad k g^i_{j} \geq v^q_{i} - \nu^q_{i} \quad \forall \ i \in N \quad (22)
\]

This completes our discussion of multi-regime microgrid network model with parallel operation of DERs.

IV. Bilevel Optimization Problem

A. Attacker-Operator Interaction Model

Recall that in [1], we modeled the sequential interaction between the attacker and operator as a bilevel mixed-integer problem (BiMIP). We now extend this model to include microgrid operations and DER dispatch capabilities. Our revised BiMIP formulation considers multi-regime microgrid operations with multiple DERs/DGs. It also considers frequency disturbances as part of the overall disturbance model.

We consider TN-side disturbance in our attack model because the DN can face significant loss if the attacker targets DN during an active TN failure event. In general, a TN-side disturbance (e.g., failure of a transmission line or bulk generator) can impact the system frequency as well as the substation voltage of the DN, and this can influence the attacker’s strategy. We model the impact of a TN-side failure as a perturbation in the substation voltage and frequency, denoted \( \Delta v_0 \) and \( \Delta f_0 \), respectively. Then, the voltage and frequency at the substation node in the post-contingency stage can be written as follows:

\[
v_0 = v^{\text{nom}} - \Delta v_0. \quad (23)
\]

\[
f_0 = f^{\text{nom}} - \Delta f_0. \quad (24)
\]

For the sake of consistency, we consider the same model of DN-side disruption as in [1], i.e., an attacker-induced compromise of DG management system (DGMS) results in simultaneous disruption of multiple DGs. We model this attack as follows:

\[
k g^c_i \geq d_i \quad \forall \ i \in N. \quad (25)
\]

Let \( k \) denote the maximum number of DGs that the attacker can disrupt. Then, the set of all possible attack strategies, denoted \( D \), is given by \( D_m = \{ d \in \{0, 1\}^N \mid \sum_{i \in N} d_i \leq k \} \).

Unlike DGs (denoted by set \( S_{g,n} \)), the output of utility-owned DERs (set \( S_{g,t} \)) changes depending on the grid conditions. In particular, the DER output either changes autonomously based on the droop control equations, or the DERs are explicitly coordinated by the SA. Hence, the DERs are not vulnerable under our assumed disruption model because they are not controlled by the DGMS.

Remark 1. Note that the above-mentioned disruption model can be extended to other types of attacks, including disruption of loads or circuit breakers. We can model such attacks as follows:

\[
k c^i \geq d^i \quad \forall \ i \in N \quad (26)
\]

\[
k l_{ij} \geq d_{ij} \quad \forall \ (i, j) \in E, \quad (27)
\]

where \( d^i \in \{0, 1\}^N \) and \( d_{ij} \in \{0, 1\}^E \) denote the corresponding attack vectors for loads and DN lines, respectively. We focus only on DG disruptions for the sake of concreteness.

We emphasize that despite its simplicity, our disruption models can be particularized to capture the physical impact of a broad class of security failure scenarios. This class includes Distributed Denial-of-Service (DDoS) attacks on the power grid components that can result in simultaneous failures [12, 13, 14]. Another relevant attack scenario is motivated by the vulnerabilities of Internet-connected customer-side devices (e.g., smart inverters, air conditioners, water heaters), also known as Internet-of-Things (IoT) devices [12]. An adversary can hack into these components via a cyberattack, create an IoT botnet, and can access them via Internet. Indeed, recent work in cyber-security of power systems has identified risk of correlated failures (e.g., simultaneous on/off events) induced/cause by IoT botnets [13]. In our disruption model, the impact of such an attack can be straightforwardly modeled by load/DG/line disconnects, leading to sudden supply-demand disturbance. However, a single point of failure such as a cyberattack on DGMS is perhaps a more likely threat, in comparison to an attack on IoT devices, since the latter requires identification and exploitation of a common mode vulnerability in a large number of spatially distributed IoT devices. Nevertheless, we can also model the impact (in terms of supply-demand disturbance) of such distributed IoT attacks using our disruption model.

Operator response model: Recall that in [1], we considered three operator response capabilities: (a) Remote control by control center during nominal conditions (load control and control of DGs through DGMS); (b) autonomous disconnect of individual components (tripping of DGs or loads under nodal violations in operating conditions); and (c) emergency control by the Substation Automation (SA) system. To capture operator response that also includes microgrid operations, we model another capability: (d) emergency control by the SA involving microgrid islanding and DER dispatch.
Since our attack model is concerned with compromised DGMS, we rule out (a) as a response mechanism. We considered (b) and (c) in [1]; see Fig. 1. Recall that the response model (b) is referred to as no response model. Our underlying assumption is that (c) is not prone to cyberattacks, because distribution utilities are being regulated under NERC CIP standards [15], which provide specific guidelines for secure repermetisation of the substation cyber infrastructure. Analogous to (c), we also consider the response (d) to be executed by the SA, and thus assume that it is secure.

Also recall that both response mechanism (b) and (c) do not consider grid-interactive DERs or microgrid islanding capabilities. In contrast, (d) utilizes both these capabilities, in addition to load control and preemptive disconnection of components. Particularly, we model the operator response (d) as follows: u := (kl, kr, pr, qr, β, kc, kg). Then, the set of all response strategies, denoted \( \mathcal{U}_m \), can be defined as \( \mathcal{U}_m := \{0,1\}^p \times \{0,1\}^5 \times \mathbb{R}^{n_s} \times \{0,1\}^2 \times \{0,1\}^N \). Moreover, given the attacker-induced disruption \( d \), let the set \( \mathcal{U}_m(d) := \{ u \in \mathcal{U}_m \mid Eq. (25) holds \} \) denote the set of feasible response strategies available to the operator after the disruption.

For the sake of simplicity, we consider that in the pre-contingency stage, the DN is in grid-connected regime and all components are connected. That is, there are no microgrid islands (\( kl = 0 \)), and all the loads and DGs are connected to the DN (\( kc = 0 \) and \( kg = 0 \)). Consequently, the grid-forming DERs are not contributing to regulation in the pre-contingency stage, i.e. \( kr = 0 \) for all \( s \in S \). Recall that we also assumed the output of the grid-interactive DERs in mode 3 to be zero, i.e. \( pr = qg = 0 \) for all \( s \in S_3 \). These assumptions allow us straightforwardly to compare the effectiveness of each of the response models (b), (c) and (d).

Post-contingency costs: The post-contingency loss incurred by the operator, denoted \( L_m \), is the sum of following costs: (i) cost due to loss of voltage regulation, (ii) cost of load control, (iii) cost of load shedding, and (iv) cost of islanding:

\[
L_m = W_{VR} \| \mathbf{y} \|^2_{2} + W_{LC} \sum_{s \in S} (1 - \beta_s) \mathbf{p}_{r,s}^\top \mathbf{p}_{r,s} + (W_{LS} - W_{LC}) \sum_{s \in S} \mathbf{kr}_s \mathbf{p}_{r,s} + W_{MG} \sum_{(i,j) \in A} kl_{ij},
\]  

(26)

where \( W_{MG} \) is the cost of a single islanding control action. A typical value for \( W_{MG} \) is a dollar per kilowatt hour; see Table IV for a comparison of the cost coefficients.

For a given operator response \( u \in \mathcal{U}_m \), let \( \mathcal{X}_m(u) \) denote the set of post-contingency states \( x \) that satisfy the constraints (1)-(24). Then, we can restate our bilevel formulation (P2) as follows:

\[
\mathcal{L}_{\text{MG}} := \max_{d \in \mathcal{D}_m} \min_{u \in \mathcal{U}_m(d)} L_m(u, x^c) \quad \text{s.t. } x^c \in \mathcal{X}_m(u).
\]

(P-MG)

Since (P-MG) is a BIMIP with same mathematical structure as the BiMIP in [1], we solve it using the Benders Decomposition algorithm that we developed in [1].

B. Results

Now, we present computational results to: (i) compare the output value of our BD algorithm with the optimal value (generated for small networks by simple enumeration); (ii) compare the DN resilience under response capabilities (b), (c) and (d); and (iii) show the scalability of our approach to realistically large DN network sizes \( N \in \{24, 36, 118\} \); see Fig. 9. We refer the reader to the appendix for the setup of our computational study.

Benders Decomposition vs. Simple Enumeration: We evaluate the ability of our implementation of the BD algorithm to compute optimal attacks in the islanding regime for small (\( N \in \{24, 36\} \)) networks. For each possible cardinality of attack we first compute the optimal attack with maximum loss using simple enumeration. Then we fix the maximum loss as \( L_{\text{target}} \) for BD algorithm. If the BD algorithm can find an attack with the same cardinality, then indeed the BD algorithm has computed the optimal attack. Otherwise, it has computed a suboptimal attack.

Fig. 6: System resilience (\( \mathcal{R} = 100(1 - \mathcal{L}/\mathcal{L}_{\text{max}}) \)) vs. \( k \). Near-optimal performance of BD algorithm.

The results of BD algorithm implemented for solving (P-MG) are shown in Fig. 6. Naturally, the attack cardinality computed by BD algorithm is greater than or equal to the optimal mini-cardinality computed using simple enumeration. In some cases, however, the BD algorithm does not obtain the optimal attack vector. Recall from [1] that the BD algorithm involves iteratively eliminating sub-optimal attack vectors using Benders cuts. Each cut involved an \( \epsilon \) which results in a tradeoff between the accuracy and computational time. For a very small choice of \( \epsilon \), the BD algorithm eliminates exactly one sub-optimal attack vector in each iteration, and performs as worse than simple enumeration. For a large value of \( \epsilon \), relatively more number of attack vectors including optimal attack vectors are eliminated. Hence, the BD algorithm terminates faster although at some loss of optimality. Still, for both 24- and 36-node networks, the BD algorithm computes attack vectors whose cardinalities are at most 8-23% more than the cardinalities of the corresponding optimal attack vectors. Determining a good choice of \( \epsilon \) to achieve better optimality results without substantively degrading the computational performance is beyond the scope of this paper, and is a topic of future research.
Value of timely response: Recall from [1] we used post-contingency loss to define the metric of resilience for no response ($R_{NR}$) and operator response without microgrid capabilities ($R_{Mn}$). In Sec. I, we introduced an analogously defined metric of resilience for operator response involving microgrid islanding and DER dispatch capabilities ($R_{MG}$). Fig. 7 compares the resiliency values for the three cases for varying attack cardinalities, where computation of $R_{MG}$ and $R_{Mn}$ involves using BD algorithm to solve the corresponding BiMIPs, and $R_{NR}$ is computed using Algorithm “Uncontrolled cascade under no response” in [1]. Indeed, under response model (d), the SA triggers microgrid islanding and DER dispatch in a preemptive manner to reduce the impact of attack. This leads to a smaller loss in comparison to using just load control and/or component disconnects (that is, response model (c)). Indeed, our computational results validate that $R_{MG} \geq R_{Mn} \geq R_{NR}$. The difference between the dashed (green) and solid (red) curves in Fig. 7 indicate the value of response (d) relative to response (b). The difference between the dashed (green) and cross-marked (blue) curves indicate the relative value of timely response (d) over response (c).

![Fig. 7: DN resilience under varying attacker-operator interaction scenarios.](image)

(a) $\Delta v_0 = 0$

(b) $\Delta v_0 = 0.02$

Scalability of BD algorithm: We tabulate the performance of the BD algorithm in terms of the computational time and number of iterations required by the BD algorithm to compute min-cardinality attacks for different network sizes and varying values of the resilience metric $R_{target} = 100(1 - \frac{L_{target}}{L_{max}})$; see Table III. We also note the cardinalities of attack vectors output by the BD algorithm as well as the corresponding DN resilience. Note that, $N = 118$ node network has $2^{118}$ possible configuration vectors. Still, with $R_{target} = 80\%$, the BD algorithm computes an attack vector for $N = 118$ nodes in $\approx 1$ minute. In comparison, for $N = 36$ node network, the simple enumeration method took $\approx 6$ hours.

V. MULTI-PERIOD DN RESTORATION

Broadly, the resilience of a system is related to its ability to not only minimize the impact of a disturbance, but also quickly recover from it; see Fig. 1. Our attack model assumes that a compromise of the DGMS leads to remote disconnect of multiple DGs. However, the actual functionality of disconnected DGs is not compromised. In response model (d), we considered that the SA has the ability to detect and obtain the knowledge of the complete attack vector. Moreover, the SA can also control DG connectivity. We now discuss how the SA can restore the disrupted DGs, and bring the DN back to its nominal performance. In this section, we present a simple MIP that models the process of restoring system performance.

We model our DN restoration process entails gradually reconnecting the disrupted DGs, and eventually restoring the grid-connected mode of DN operation. We consider a multi-period horizon $T := \{0, 1, \ldots, T\}$, where $T$ is the maximum of two times: the time when all disrupted DGs can be reconnected plus one, and the time when the TN-disturbance clears. Let a period be denoted by $t \in T$, where each period $t$ is of fixed time duration (say, a few minutes). Furthermore, the operator response at period $t$ is denoted by $u^t$. Then, under the assumed detection and response capabilities of the SA, $t = 0$ coincides with the time of initial post-contingency response, i.e. $u^0 = u^c$. Period $t = T$ denotes the time at which the system performance of the DN is fully restored.

We consider two types of constraints to model the restoration actions of the operator across time periods: the monotonicity constraints and the resource constraints. Consider a period $t \in T$. The monotonicity constraints for period $t$ are as follows.

$$k_l^{ij} - k_l^{ij-1} \leq k_l^{ij-1} \quad \forall \ (i, j) \in M.$$  

(27a)

$$k_q^i - k_q^{i-1} \leq k_q^{i-1} \quad \forall \ i \in N.$$  

(27b)

Eq. (27a) implies that during the restoration process, once a connecting line is closed, it continues to remain closed until the restoration process is completed. Similarly, Eq. (27b) implies that a disconnected DG becomes operational after being reconnected, and then continues to remain operational until the restoration is complete. The monotonicity constraints can be justified based on the practical consideration that frequent changing of

| $R_{target}$ | $N = 24$ | $N = 96$ | $N = 118$ |
|-------------|-----------|-----------|-----------|
| 99          | 98.91, (15), 0.41, 6 | 98.95, (16), 0.37, 5 | 98.95, (18), 2.48, 4 |
| 95          | 91.43, (16), 0.46, 7 | 91.42, (12), 0.51, 7 | 91.28, (15), 3.91, 11 |
| 90          | 82.8, (18), 0.57, 8 | 88.45, (17), 0.91, 11 | 89.74, (20), 10.02, 10 |
| 80          | 78.73, (16), 0.74, 12 | 81.9, (120), 1.24, 14 | 83.49, (29), 28.79, 20 |
| 70          | 78.5, (21), 0.74, 12 | 74.4, (241), 1.74, 15 | 79.5, (40), 67.85, 30 |

Table III: Resilient metric evaluated using BD algorithm for 24-, 36- and 118-node networks. The realized resilience metric can significantly fall short of the target resilience metric ($R_{target} = 100(1 - L_{target}/L_{max})$); for example, when the attack cardinality changes from 6 to 7, the percentage resilience for 24-node network decreases sharply from 98.91% to 91.33%. This means that the 244122122122 node DN is at least 90% (actual value 91.33%) resilient to $k = 7$ cardinality attacks.
states of connecting lines can create large fluctuations in nodal voltages and system frequencies of the microgrids due to the low inertia of DERs. Moreover, the battery life of emergency generators of VSI-controlled DERs would reduce due to frequent changes from charging modes (quadrants III and IV) to discharging modes (quadrants I and II), and vice versa; see Fig. 4.

The resource constraint merely limits the number of DG reconnections. Specifically, we consider that during period $t$, at most $G^t$ DGs can be reconnected, where $G^t$ denotes the restoration budget for that period:

$$\sum_{i \in S_n} k_{gi}^t \geq \sum_{i \in S_n} k_{gi}^{t-1} - G^t.$$  \hfill (28)

Restrictions on number of connecting line closing operations can be similarly considered. Eq. (28) can also be justified in a way similar to that of monotonicity constraints. The operator avoids a large number of simultaneous DG reconnections as they can lead to large voltage and frequency fluctuations.

As stated before, we choose $T$ large enough so that all disrupted DGs can be reconnected before the last period $T$, i.e. $T \geq \max\{T\} \sum_{i=1}^k G^i \geq k \} + 1$. Indeed, the TN-disturbance may clear any time, before or after the DG reconnections. However, since our analysis is focused on determining worst-case resilience of the DN, we assume that the TN-side disturbance clears after the disrupted DGs are fully reconnected. In particular, we assume that the TN-side disturbance ceases to exist at the last time period. We model this as follows:

$$v_0^t = \begin{cases} v_{\text{nom}} - \Delta v_0 & \text{if } t \neq T \\ v_{\text{nom}} & \text{if } t = T \end{cases}$$  \hfill (29a)

$$f_0^t = \begin{cases} f_{\text{nom}} - \Delta f_0 & \text{if } t \neq T \\ f_{\text{nom}} & \text{if } t = T. \end{cases}$$  \hfill (29b)

Let $\mathcal{Y}_m^i(u^{-1})$ denote the feasible set of response strategies for $u^i$, i.e. $\mathcal{Y}_m^i(u^{-1}) = \{u^i \in \mathcal{U}_m \mid \text{such that } (27) - (28) \text{ hold}\}$. Also, given an operator response $u \in \mathcal{U}_m$, let $X_m^i(u)$ denote the set of network states $x^i$ which satisfy the constraints (1)-(22) and (29). Hence, the restoration problem can be posed as follows:

$$L_{\text{res}}(d) := \min\limits_{u^i \in \mathcal{U}_m} \sum_{t \in T} L_m(u^i, x^i)$$  \hfill (P3)

s.t. $u^i \in \mathcal{U}_m(d)$

$$u^i \in \mathcal{Y}_m^i(u^{-1}) \quad \forall \ t = 1, \ldots, T$$

$$x^i \in X_m^i(u^i) \quad \forall \ t = 0, \ldots, T$$

Problem (P3) is a Mixed-Integer Problem (MIP), and can be solved using off-the-shelf MIP solvers. However, due to the large number of binary variables, the computational speed can be very slow for larger networks. In fact, we solve (P3) using a simple greedy algorithm; see Algorithm 1. In each period, the operator simply chooses that response which minimizes the post-contingency loss during that time period subject to the monotonicity and resource constraints. Algorithm 1 is based on the feature that the network state in any period depends only on the operator actions in that period, and the network state in the previous period. The algorithm returns with the operator actions, resulting network state, and corresponding post-contingency loss for each time period.

**Algorithm 1 Greedy Algorithm**

1: $u^0, x^0 \leftarrow \arg\min\limits_{u \in \mathcal{U}_m(d)} L_m(u, x)$ s.t. $x \in X(u)$.
2: for $t = 1, \ldots, T$ do
3: \hspace{0.5cm} $u^t, x^t \leftarrow \arg\min\limits_{u \in \mathcal{Y}_m^i(u^{-1})} L_m(u, x)$ s.t. $x \in X_m^i(u)$.
4: \hspace{0.5cm} $L_t \leftarrow L_m(u^t, x^t)$
5: end for
6: return $\{u^t, x^t, L_t\}_{t \in T}$

Fig. 8 shows the system performance restoration of the DN over multiple time periods for different resource constraints. For each system restoration curve, we chose the $G^t$ to be a constant for all time periods $t \in T$. One can see that after the TN-side and DN-side disturbances, the system performance drops. Then, as disrupted components get connected, the system performance gradually recovers. Also, the post-contingency losses are higher for larger TN-side disturbances. However, as the flexibility to reconnect the disrupted components increases, the system recovers faster.

In order to implement the response computed in (P3), the SA may need to coordinate with the individual microgrid controllers. A detailed description of such a communication architecture is beyond the scope of this paper. However, we refer the reader to [16] for a hierarchical control architecture which can support the coordination between SA and individual microgrid controllers.

**VI. CONCLUDING REMARKS**

In this paper and its companion paper [1], we developed a quantitative framework to evaluate the DN resilience, which is its ability to minimize the impact after a disturbance, as well as to restore the DN performance back to its nominal value. Firstly, we showed that the impact of a broad class of cyberphysical failure scenarios in electricity DNAs can be modeled as DN-side...
disruption of multiple components with or without the presence of TN-side disturbances in substation voltage and frequency. Secondly, we developed a novel network model which captures operations of microgrid(s) under various regimes, and single-/multi-master operation of DERs which provide grid-forming and regulation services. Thirdly, we considered a range of operator response strategies: from load control and component disconnects to microgrid islanding and DER dispatch. Fourthly, we formulated the attacker-operator interactions as bilevel mixed-integer problems (P1) and (P2), and developed a computational approach to solve these problems using Benders decomposition algorithm. Finally, we introduced a problem (P3) about restoration of DN performance over multiple time periods, and presented a greedy algorithm for solving it. Our computational results for (P1)-(P3) show the value of timely response under varying operator capabilities in minimizing the impact of disruption as well as enabling faster system recovery.

Future work includes analyzing and improving the BD and greedy algorithm in terms of the optimality guarantees and computational performance. Also, the class of cyberphysical failures may be extended to include random failures (e.g. weather-induced disturbances). The insights developed through this framework may be useful in designing more resilient DNs by way of strengthening or securing DN components, and/or deploying contingency resources in the face of disruption events.

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Appendix

Setup for computational study: We consider three networks: modified IEEE 24-, 36-, and 118-node networks; see Fig. 9. The set of connecting lines $\mathcal{M}$ are shown with thick edges. The individual microgrid networks $\mathcal{N}_1, \cdots, \mathcal{N}_{|\mathcal{M}|}$ can be obtained by setting $k_{ij} = 1 \forall (i,j) \in \mathcal{M}$. Each line $(i,j) \in \mathcal{E}$ has an identical impedance of $r_{ij} = 0.01, x_{ij} = 0.02$. 50% of the nodes have a DG each and 50% have a load each. Consider a parameter $\alpha = 0.25$. Before the contingency, each DG has active power output of $\bar{p}g_i = \alpha$, and each load has a demand of $\bar{p}c_i = 1.25\alpha$. Thus, we assume 80% DG penetration since the total DG output is 80% of the total demand. The voltage bounds are $\bar{v}c_i = 0.9, \bar{v}c_i = 1.1, \bar{v}g_i = 0.92$ and $\bar{v}g_i = 1.08$. The reactive power values are chosen to be exactly one third that of the corresponding active power value, i.e. a 95% power factor value for each load and DG. The values are chosen such that the total net active power demand in the DN is 0.75 pu, and the lowest voltage in the
network before any contingency is close to $v_g$. The maximum load control parameter is $\beta_i = 0.8$, i.e., at most 20% of each load demand can be curtailed. For the sake of simplicity, we assume that all DGs and loads are homogeneous. $W_{LC} = \frac{100}{P_{c}}$, $W_{VR} = 100$, $W_{FR} = 100$, $W_{LS} = \frac{1000}{P_{c}}$, $W_{MG} = 400$. Each microgrid has one AC-source DER and one VSI-controlled DER. Consider a parameter $\gamma = \frac{\sum_{s=1}^{N}P_{s}}{P_{\text{c}}}$. Then, each VSI-controlled DER has the following parameters: $\forall s \in S_{\text{vsi}}, s_{\text{n}} = s_{\text{e}} = \gamma$, $m_{p_s} = 0.02$, $m_{q_s} = 0.04$; and, each AC-source DER has the following parameters: $\forall s \in S_{\text{ac}}, s_{\text{n}} = s_{\text{e}} = 2\gamma$, $m_{p_s} = 0.1$, $m_{q_s} = 0.2$. These parameters are chosen such that the total capacity of all AC-source and VSI-controlled DERs is 75% of the total demand of all loads. However, this total capacity may not be fully available to meet the demand because (a) the microgrids are not of exact uniform size and topology, and (b) the storage devices supply power only under the specific islanding configurations.

Comments about cost coefficients: The typical values for the parameters in Eq. (26) of the cost terms are listed in Table IV.

| Weights | Typical values                   |
|---------|---------------------------------|
| $W_{LC}$ | $\frac{1}{4} \times 11$ cents per kilowatt hour |
| $W_{VR}$ | $\frac{1}{2} \times 11$ cents per kilowatt hour |
| $W_{LS}$ | 3 dollars per kilowatt hour      |
| $W_{MG}$ | 1 dollar per kilowatt hour       |

TABLE IV: Typical values of cost parameters.
Fig. 9: Modified IEEE test networks. Connecting lines are indicated by thick edges. AC-source (resp. VSI-controlled) DERs are indicated by northwest (resp. vertical) lines.