Analytical investigation of the fuel rod destruction and melt relocation along the surface of the fuel rod

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Abstract. To simulate fuel rod thermal destruction, analytical approaches were developed. Approaches are based on solving the Stefan problem and solving the equation of motion of the molten materials. The paper contains a brief description of the ways to obtain analytical solutions and applicability limits of the solutions. Calculation of the mass of the melt that left the central part of the fuel rod is presented. Proposed analytical solutions can be used only to simulate the first stage of the accidents from melting inception of the cladding to destruction of the whole fuel assembly.

1. Introduction

Processes of the destruction of fuel rods during an accident in nuclear power plants are accompanied by overheating of the surface of the fuel rod and its melting. Melting causes a relocation of the molten cladding and fuel under gravitational forces and friction with the coolant flow. There is heat exchange between the surface of the fuel rod and melt during relocation. Solidification of the melt may occur on the cold surfaces of the fuel pin.

Behavior of a fuel rod during an accident requires an analysis of a large number of physical processes that accompany the process of its destruction. To simulate melt relocation, analytical and numerical approaches are required. Analytical approaches can help to verify numerical codes [1] – [2] that are being developed to simulate severe accidents in the nuclear power plants.

2. Basic approaches for simulating melt relocation

To construct analytical solution we consider the laminar flow of liquid melt spreading along a substrate. The equation of the motion has the following form.

\[ \mu \Delta U = \rho g, \]  

where \( \mu \) is viscosity; \( U \) is velocity of the melt, m/s; \( \rho \) is density of the melt.

We assume that flow has a cylindrical symmetry as a fuel rod. In this case equation (1) can be written in a form:

\[ \frac{\mu}{r} \frac{d}{dr} \left( r \frac{dU}{dr} \right) = \rho g. \]  

The equation can be solved with following boundary condition:
\[ U\bigg|_{r=R} = 0, \quad \mu \frac{dU}{dr} \bigg|_{r=R+\delta} = \tau_c. \tag{3} \]

where \( R \) is external radius of the fuel rod surface; \( \delta \) is melt thickness; \( \tau_c \) is shear stress between coolant and melt. Solving the equations (2) with the boundary condition (3) we can receive:

\[ U = \frac{\rho g}{4\mu} (r^2 - R^2) + \left[ \frac{(R+\delta)}{\mu} \frac{\tau_c}{\mu} - \frac{\rho g (R+\delta)^2}{2\mu} \right] \cdot \ln \frac{r}{R}. \]

The averaged velocity:

\[ \bar{U} = \frac{\rho g}{8\mu} \left( (R+\delta)^2 - R^2 \right) + \frac{R^2}{(R+\delta)^2 - R^2} \left[ \frac{(R+\delta)}{\mu} \frac{\tau_c}{\mu} - \frac{\rho g (R+\delta)^2}{2\mu} \right] \left[ \left( 1 + \frac{\delta}{R} \right) \ln \left( 1 + \frac{\delta}{R} \right) - \frac{1}{2} \left( 1 + \frac{\delta}{R} \right)^2 + \frac{1}{2} \right]. \tag{4} \]

If external fuel rod radius is bigger than melt thickness \( R \gg \delta \) the above expression has a usual form[1]:

\[ \bar{U} = \frac{\tau_c \delta}{2\mu} - \frac{\rho g \delta^2}{3\mu}. \]

The melt thickness profiles can be determined by substituting the averaged velocity into a mass conservation equation.

\[ \frac{\partial S}{\partial t} + \frac{\partial S \bar{U}}{\partial z} = 0, \quad S = \pi \left( (R+\delta)^2 - R^2 \right). \tag{5} \]

In the presented equation we assume that melt has the same temperature along the fuel rod. This assumption is correct for the melt near the melting point. Substituting (4) into (5):

\[ \frac{\partial}{\partial t} \left( (R+\delta)^2 - R^2 \right) + \frac{\partial \frac{\rho g}{8\mu} \left( (R+\delta)^2 - R^2 \right)^2}{\partial z} + \frac{\partial}{\partial z} R^2 \left[ \frac{(R+\delta)}{\mu} \frac{\tau_c}{\mu} - \frac{\rho g (R+\delta)^2}{2\mu} \right] \left[ \left( 1 + \frac{\delta}{R} \right) \ln \left( 1 + \frac{\delta}{R} \right) - \frac{1}{2} \left( 1 + \frac{\delta}{R} \right)^2 + \frac{1}{2} \right] = 0. \tag{6} \]

If condition \( R \gg \delta \) is true then equation (6) transforms to:

\[ \frac{\partial}{\partial t} \left[ \frac{\tau_c \delta^2}{2\mu} - \frac{\rho g \delta^3}{3\mu} \right] + \frac{\partial}{\partial z} \left[ \frac{\tau_c \delta^2}{2\mu} - \frac{\rho g \delta^3}{3\mu} \right] = 0. \tag{7} \]

We can solve the equation (7) by the method of characteristics [4]. In this case:

\[ z - \left[ \frac{\tau_c \delta}{\mu} - \frac{\rho g \delta^2}{\mu} \right] t = z(\delta), \]
$z(\delta)$ is the initial value of the characteristic. For long times ($t \gg 0$), when the initial conditions become small, melt profile can be found from the equation:

$$z - \left( \frac{\tau \delta}{\mu} - \frac{\rho g \delta^2}{\mu} \right) t = 0.$$  \hfill (8)

Solution of the equation (8) has the following form:

$$\delta = \left(1 - \frac{1 - 4\mu \rho g^2}{\tau^2 t} \right) \frac{\tau_c}{2\rho g}.$$  \hfill (9)

The next step is finding the position of the leading edge $z_n$. This position can be found with the help of the condition of the global mass conservation:

$$\int_0^{z_n} \delta(z) \, dz = A_0.$$  \hfill (10)

Using relation (9):

$$z_n - \frac{\tau_c^2 t}{6\mu \rho g} \left(1 - \left(1 - \frac{4\mu \rho g^2}{\tau^2 t} \right)^{3/2}\right) \frac{\tau_c}{2\rho g} = A_0.$$

Non-linear equation (11) can be solved for long times, when $4\mu \rho g^2 / (\tau_c^2 t) \ll 1$. Using the first three terms of the Taylor series expansion, we can obtain:

$$\left(\mu z_n^2 / 2\tau_c t\right) = A_0, \text{ or } z_n = \sqrt{2A_0 \tau_c t / \mu}.$$  \hfill (12)

Equations (9) and (11) or (12) can be used to evaluate melt relocation.

3. Basic approaches for simulating melting of the fuel rod

Calculation of the fuel rod thermal destruction is based on the solving the heat equation with taking into account the melting front motion.

$$\frac{\partial T(r, t)}{\partial t} = \lambda \left( \frac{\partial^2 T(r, t)}{\partial r^2} + \frac{1}{r} \frac{\partial T(r, t)}{\partial r} \right), \quad R_i < r < \eta, \quad \eta = f(t)$$  \hfill (13)

$$T(\eta, t) = T_m, \quad \lambda \frac{\partial T(\eta, t)}{\partial r} \bigg|_{r=R_i} = q, \quad \lambda \frac{\partial^2 T(\eta, t)}{\partial r^2} = \rho L \frac{d\eta}{dt}.$$  \hfill (14)

$\eta$ is melting front position; $\lambda$ is heat conductivity; $L$ is melting enthalpy; $R_i$ is internal radius of the fuel rod.

For constant heat flux, the system of equations (13) – (14) can be solved. The solution for melting front position was found in [5]:

$$\eta = \sqrt{R_i^2 + 2 \frac{R_i}{\rho L} qt}.$$  \hfill (15)

If $R_2$ is external fuel rod radius, then total melting time is equal to:
\[ t = \frac{R_2^2 - R_1^2}{2R_1} \frac{\rho L}{q}. \]  

(16)

If \( l \) is fuel rod length, then total mass \( m(t) \) of the melt can be calculated with help of (15):

\[ m(t) = \pi \cdot l \cdot \frac{R_i}{L} \cdot q t. \]  

(17)

4. Numerical simulation

Some results of the fuel rod melting and melt relocation were obtained with help of the presented relations. Times of the clad melting according to the relation (16) for different heat release are presented in fig.1. As we can see from fig. 1, the melting time is close to 1 s for \( 10^5 \) W/m\(^2\) and equal to 0.2 s for \( 5 \times 10^6 \) W/m\(^2\). Fuel rod diameter was equal to 0.008 m, cladding thickness was equal to 0.0005 m in the presented calculations. Stainless steel 316 was chosen as a material of the cladding.

![Figure 1. Time of the clad melting.](image)

For the fuel rod described above an axial position of the leading edge for coolant velocity of 100 m/s is presented in fig 2. Coolant shear stress has been evaluated by the relation:

\[ \tau_c = \frac{\xi}{8} \rho \cdot U^2, \quad \xi = 0.02. \]  

(18)
As we can see from fig. 2, the leading edge passes the distance that equals 1 m for time longer than 10 s. This is far less than the time of melting. So we can estimate the mass of melt with relation (17) and can calculate the leading edge position and thickness of the melt with relations (11) and (12). Dependence of the melt mass that leaves the fuel rod can be evaluated with the relation:

\[
m(t) = 2\pi R_i \rho \int_{z_i}^{z_f} \left(1 - \frac{1}{\sqrt{1 - \frac{4\mu \rho g z}{\tau_c^2 t}}} \right) \tau_c - dz.
\]  

(19)

Figure 2. Axial position of the leading edge.

For long time we can use the relation (12) to calculate the mass fraction of the melt that left the active part of the fuel rod:

\[
\alpha(t) = \frac{1}{A_0} \left\{ \frac{\mu}{2\tau_c t} \left( z_n - z_r^2 \right) \right\} = \frac{1}{A_0} \left\{ \frac{\mu}{2\tau_c t} \left( 2A_0 \tau_c t \frac{2A_0 \tau_c t}{\mu} - z_r^2 \right) \right\},
\]

(20)

\(z_r\) is upper coordinate of the active part of the fuel rod.

The dependence of the mass fraction is presented in fig. 3. As we can see from fig. 3, the most part of the melt mass leaves the active part of the fuel rod for 5-10 seconds. This time can be used as a characteristic time for estimating the duration of the clad destruction for ULOF accidents in the fast reactors.
Conclusion
Short description of the analytical models that can be used to simulate melting of the fuel rod and melt relocation is presented in the paper. Analytical calculations of the cladding melt relocation were made. The quantity of the fuel mass that left fuel rod was calculated. The presented results can be used to estimate melt relocation during the accidents in nuclear reactors and to verify calculations codes.

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Figure 3. Mass fraction of the melt.