An analytical method for rigid piled-raft foundations subjected coupled loads in layered soils

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ABSTRACT

A simplified analytical approach is proposed to study on the behavior of the piled raft foundation with rigid raft subjected coupled loads in layered soils. Employing the shear displacement method, based on the transfer matrix, the pile-pile interaction under vertical load is analyzed. Adopting the modified subgrade modulus, the pile head-pile head interaction under horizontal load is analyzed through the finite difference method. Base on solutions for stresses and displacement in layered elastic half space, the interaction between pile head and soil surface, soil surface and pile head, soil surface and soil surface subjected coupled loads are taken into account to determine the flexibility matrix of the pile group-soil system. Then the load-settlement relationship of rigid piled raft foundations subjected coupled loads, load and settlement of the piles along the depth are obtained. Comparison among the results of the finite element and the calculated results is carried out, and the feasibility of the present method for the analysis of piled raft foundations and the superiority of the modified subgrade modulus are proved.

Keywords: layered soil, pile raft foundation, coupled loads, interaction

1 INTRODUCTION

The traditional design approach of pile-raft foundation ignores the interaction between the raft and the soil which results in a waste of money by using many unnecessary piles. In this way, the settlement of the foundation could be very small. However, a good design should have advantages in both technical and economic. Thus, the settlement or the differential settlement of the foundation are suggested to be controlled within an acceptable range but a very small value. For this purpose, many researches (Poulos and Davis 1980, Kuwabara 1989, Mendonca and Paiva 2003, Shen et al. 2000, Mu and Huang 2014) were carried out to study the behavior of pile-raft foundations considering the bearing capacity of the raft. And these works mainly focused on pile-raft foundations under vertical loads. Nowadays, more and more pile-raft foundations are used in tower and skyscraper besides traditional buildings. For these buildings, the foundation are subjected to enormous horizontal load. A few literatures (Horikoshi et al. 2003, Zhang and Small 2000) are found to study the responses of pile-raft foundations under pure horizontal loads. However it is found that significant coupled effect occurs between the horizontal load and the vertical load when they are applied onto the pile-raft foundation homogeneously (Davisson and Robinson 1965, Goryunov 1975, Ramaswamy 1974, Anagnostopoulos and Georgiadis 1993). In other words, studying the responses of the pile-raft foundations under pure vertical loads or horizontal loads is not enough. In order to estimate the responses of the pile-raft foundations accurately enough for economical design, it is necessary to find a suitable method to estimate the responses of the pile-raft foundations under the combined load of the vertical load, the horizontal load and the moment. Kitiyodom and Matsumoto[10] used a weighted average modulus to compute the responses of pile-raft foundations in layered soils based on a Winkler model, the method for computing the layered soil system is theoretically imprecise. Poulos and Davis(1980) and Mu and Huang(2014) proposed two method to analysis the responses of pile-raft foundations under combined load in homogeneous soil and layered soil respectively through elastic theory methods. Unfortunately, these methods are limited in calculating the large displacement problem when the soil responses are nonlinear.

Introducing the foundation modulus for the active pile and the passive pile under horizontal loads, an analytical method is proposed herein to calculate the responses of a pile-raft foundation under multi-directional loads in layered soil considering the pile-soil-raft interactions based on the shear
displacement method. And the method is verified through a cases study.

2 METHOD OF RIGID PILE-RAFT FOUNDATION IN LAYERED SOIL

2.1 The flexibility coefficient of pile head and pile head interaction

As shown in Figure 1, the pile with loads on the head is called “active pile” in the two piles system. And the other pile is call “passive pile”. Employing the shear displacement method, the governing equations for the active pile and the passive pile in vertical direction can be written as equation (1) and (2) respectively when the active pile is subjected to the vertical load. Employing the finite difference method and the load transfer matrix, the equations can be solved. Thus, we can obtain the vertical displacements at the head of active pile and passive pile respectively, \( w_{11}(0) \) and \( w_{21}(0) \), when a unit vertical load is applied on the head of active pile.

\[
\frac{d^2w_{11}(z)}{dz^2} - \lambda^2 w_{11}(z) = 0 \quad (1)
\]

\[
\frac{d^2w_{21}(z)}{dz^2} - \lambda^2 [w_{21}(z) - w_s(s, z)] = 0 \quad (2)
\]

where \( \lambda = \sqrt{E_p A_p} \), \( k_s = \frac{2\pi}{\ln(r_m/r_0)} \), \( G \), \( r_0 \) is the radius of piles, \( r_m \) is the effective influencing radius, \( G \) is the shear modulus of soil, \( E_p \) is the elastic modulus of pile, \( A_p \) is the cross-sectional area of piles, \( w_{11}(z) \) is the vertical displacement of active pile at the depth of \( z \), \( w_{21}(z) \) is the additional vertical displacement of the passive pile induced by the active pile at the depth of \( z \), \( w_s(s, z) \) the vertical displacement of the free-field soil at the depth of \( z \) located at the position of the passive pile induced by the active pile, \( s \) is the pile spacing.

Employing the elastic foundation beam method, the governing equations for the active pile and the passive pile in horizontal direction can be written as equation (3) and (4) respectively when the active pile is subjected to the horizontal load or the moment. Through the finite difference method, the displacements at the head of the active pile and the passive pile can be obtained when a unit horizontal load or a unit moment is applied onto the active pile.

\[
\frac{d^4y_{11}(z)}{dz^4} + 4\lambda^4 y_{11}(z) = 0 \quad (3)
\]

\[
\frac{d^4y_{21}(z)}{dz^4} + 4\lambda^4 [y_{21}(z) - y_s(z, s)] = 0 \quad (4)
\]

where \( \lambda_{hi} = \sqrt[4]{k_{hi} E_{hi}} \), \( \lambda_p = \sqrt[4]{k_{p} E_p} \), \( y_{11}(z) \) is the horizontal displacement of the active pile at the depth of \( z \), \( y_{21}(z) \) the additional horizontal displacement of passive pile induced by the active pile at the depth of \( z \), \( z \) is the depth of calculating point, \( y_s(z, s) \) is the horizontal displacement of the free-field soil at the depth of \( z \) located at the position of the passive pile induced by the active pile, \( k_s \) is the modified subgrade modulus considering the influence of the depth which can be calculated by equation (5), \( k_{py} \) is the modified subgrade modulus considering the influence of depth and passive effect, which can be calculated by equation (6).

\[
k_s = \frac{7.5 (1-v)^2}{\eta} \frac{E_s}{3-4v} \frac{1-v'}{1-v} \left( \frac{E_p D^4}{E_p L_p} \right) \quad (5)
\]

\[
k_{py} = \frac{11.5 (1-v)^2}{\eta} \frac{E_s}{3-4v} \frac{1-v'}{1-v} \left( \frac{E_p D^4}{E_p L_p} \right) \quad (6)
\]

where

\[
\eta = \frac{(7-8v) + \frac{2}{R_i} + \frac{3-4v}{R_i^2} + \frac{2}{R_i} \left( \frac{3-4v}{R_i} + \frac{4(1-v')(1-2v)}{R_i R_s + 2 \frac{z}{r}} \right) - \frac{1}{R_s (R_s + 2 \frac{z}{r})}}{7-8v}
\]

\[
* R_s = \sqrt{ \left( \frac{z}{r} \right)^2 + \left( \frac{z}{r} \right)^2 }, r \text{ is the radius of pile, } D \text{ is the diameter of pile, } E_s \text{ is the elastic modulus of the soil, } v \text{ is the poisson’s ratio of the soil, } L_p \text{ is the moment of inertia of the pile, } \frac{E_p D^4}{E_p L_p} = 100 \left( \frac{D}{L} \right)^4 \text{ when } K_r = \frac{E_p I_p}{E_p L_p} \geq 0.01, L \text{ is the length of pile.}
\]

According to the pile-pile interactions defined by Wong and Poulos (2005), the influence coefficient of pile \( j \) to pile \( i \) could be expressed as following:

\[
\alpha_{p_{i j}}^{w_{i j}} = \frac{w_j(0)}{w_j(0)} \quad (7)
\]
where

\[ w_j(0) \] is the vertical displacement at the head of the pile \( i \) when pile \( j \) is subjected to a unit vertical load, \( w_j(0) \) is the vertical displacement at the head of the pile \( j \) when pile \( j \) is subjected to a unit vertical load, \( y_{ij}(0) \) is the horizontal displacement at the head of pile \( i \) when pile \( j \) is subjected to a unit horizontal load, \( y_{ij}(0) \) is the horizontal displacement at the head of the pile \( j \) when pile \( j \) is subjected to a unit horizontal load, \( y_{ij}(0) \) is the horizontal displacement at the head of pile \( i \) when pile \( j \) is subjected to a unit moment, \( \theta_{ij}(0) \) is the rotation angle at the head of pile \( i \) when pile \( j \) is subjected to a unit horizontal load, \( \theta_{ij}(0) \) is the rotation angle at the head of pile \( j \) when pile \( j \) is subjected to a unit horizontal load, \( \theta_{ij}(0) \) is the rotation angle at the head of pile \( i \) when pile \( j \) is subjected to a unit moment, \( \theta_{ij}(0) \) is the rotation angle at the head of the pile \( j \) when pile \( j \) is subjected to a unit moment.

The vertical, horizontal and rotation flexibility coefficients of a single pile can be expressed respectively as:

\[
\begin{align*}
    f_{pp}^{w_{ij}} &= w_{ij}(0),
    f_{pp}^{y_{ij}} &= y_{ij}(0),
    f_{pp}^{\theta_{ij}} &= \theta_{ij}(0),

    f_{pp}^{u_{ij}}(0) &= \sum_{k=1}^{n} u_{ij}^{k}(0) (i = 1,2,...,np) (13)
\end{align*}
\]

where \( n \) is the number of soil surface elements, \( n \) is the element number along the pile, \( f_{pp}^{u_{ij}}(0) \) is the flexibility coefficient of the vertical interaction between the pile head and the soil surface, \( w_{ij}(0) \) is the vertical displacement of the \( j \)th surface induced by the force acting at the \( k \)th node of pile \( j \) when pile \( j \) is subjected to a unit vertical load.

Usually the soil are assumed to be unable to bear moment, thus the flexibility coefficient of horizontal interaction between pile head and soil surface can be expressed as:

\[
\begin{align*}
    f_{pp}^{w_{ij}} &= \sum_{k=1}^{n} u_{ij}^{k}(0) (i = 1,2,...,ns; j = 1,2,...np) (13)
\end{align*}
\]

2.3 The flexibility coefficient of soil surface and pile head interaction

Employing the solution for nonaxisymmetric problems in layered elastic space(Mu and Huang, 2014) the free-field soil displacement at the location of the pile can be obtained, then the vertical displacement, the horizontal displacement and the rotation angle of the pile head can be obtained through equation (2) and (4). The flexibility coefficient of interaction between soil surface and pile head under vertical and horizontal loaded can be expressed as:

\[
\begin{align*}
    f_{ps}^{w_{ij}} &= w_{ij}(0) (i = 1,2,...,np; j = 1,2,...ns),
    f_{ps}^{y_{ij}} &= y_{ij}(0) (i = 1,2,...,np; j = 1,2,...ns),
    f_{ps}^{\theta_{ij}} &= \theta_{ij}(0) (i = 1,2,...,np; j = 1,2,...ns),
\end{align*}
\]

where \( f_{ps}^{w_{ij}} \) is the flexibility coefficient of the vertical interaction between soil surface and pile head, \( f_{ps}^{y_{ij}} \) and \( f_{ps}^{\theta_{ij}} \) are the flexibility coefficients of the horizontal interactions between soil surface and pile head under horizontal loaded, \( w_{ij}(0) \) is the vertical displacement at the head of pile \( i \) when the \( j \)th soil surface element is subject to a unit vertical load, \( y_{ij}(0) \) and \( \theta_{ij}(0) \) are the horizontal displacement and rotation angle at the head of pile \( i \) when the \( j \)th soil surface
element is subject to a unit vertical load.

2.4 The flexibility coefficient of soil surface and soil interaction

The flexibility coefficient of interaction between soil surface and soil can be expressed as:

\[ f^{sv}_{j} = w_{np}(0) \quad (i, j = 1, 2, \ldots, n_{s};) \quad (18) \]

\[ f^{sh}_{j} = u_{np}(0) \quad (i, j = 1, 2, \ldots, n_{s};) \quad (19) \]

where \( f^{sv}_{j} \) is the flexibility coefficient of vertical interaction between soil surface and soil surface, \( f^{sh}_{j} \) is the flexibility coefficient of horizontal interaction between soil surface and soil surface, \( w_{np}(0) \) is the vertical displacement of the \( i \)th soil surface element when the \( j \)th soil surface element is subjected a units vertical load, \( u_{np}(0) \) is the horizontal displacement of the \( i \)th soil surface element when the \( j \)th soil surface element \( j \) is subjected a units horizontal load.

2.5 Analysis of rigid pile-raft foundation

multi-directional load

For the rigid pile-raft foundation, the displacements of pile heads and soil surfaces are strictly restricted by the raft. According to the displacement compatibility conditions between raft and pile head-soil surface system, the government equation of the rigid pile-raft foundation is can be written in matrix form as:

\[
\begin{bmatrix}
D_x & 0 & P_x \\
0 & D_y & P_y \\
A_p & 0 & C_p \\
A_M & 0 & C_M
\end{bmatrix} \begin{bmatrix}
\delta_x \\
\delta_y \\
\delta_w \\
\delta_{\theta}
\end{bmatrix} = \begin{bmatrix}
P_x \\
P_y \\
M_p \\
M_c
\end{bmatrix}
\]

where the dimensions of \( f^{sv}_{i} \) and \( f^{sh}_{i} \) are \((n_{s} + n_{p}) \times (n_{s} + n_{p})\), the dimension of \( f^{sv}_{j} \) is \((n_{s} + n_{p}) \times n_{p}\), the dimension of \( f^{sh}_{j} \) is \(n_{p} \times (n_{s} + n_{p})\), the dimension of \( f^{sv}_{i} \) is \(n_{p} \times n_{p}\).

\[
D_x = \begin{bmatrix}
1 & 0 & x_i \\
0 & M & M \\
1 & 0 & x_{n_{p}+n_{s};i}
\end{bmatrix}
\]

\[
D_y = \begin{bmatrix}
0 & 1 & 0 \\
0 & M & M \\
0 & 1 & x_{n_{p}+n_{s};j}
\end{bmatrix}
\]

\[
D_{\theta} = \begin{bmatrix}
0 & 0 & 1 \\
0 & M & M \\
0 & 0 & 1
\end{bmatrix}
\]

\[ x_i \] is the horizontal coordinate of the element, \( A_p = B_p = [1 \ldots 1]_{(n_{s}+n_{p})} \), \( C_p = [1 \ldots 1]_{n_{p}} \), \( A_M = [x_1 \ldots x_{n_{p}}]_{(n_{s}+n_{p})} \), \( P_{(n_{s}+n_{p})} \) and \( H_{(n_{s}+n_{p})} \) represent the vertical and horizontal internal forces between the raft and the piles or the soil elements, \( M_{(n_{p})} \) represents the internal bending moment between the raft and the piles, \( \delta_w \), \( \delta_y \) and \( \delta_{\theta} \) represent the vertical, horizontal and rotation of the raft, \( P_c \), \( H_c \) and \( M_c \) are the vertical force, horizontal force and bending moment applied on the raft.

3 VALIDATION

As shown in Figure 2, the method herein is used to analyze a pile-raft foundation in a three-layer soil system.

![Fig.2 Configuration of pile raft foundation and soil profiles](image)

The results are normalized as:

\[
I_{sv} = \frac{wE_D}{P_c} \quad (21)
\]

\[
C_w = \frac{F_c}{P_c} \quad (22)
\]

\[
I_{at} = \frac{wE_D}{H_c} \quad (23)
\]

\[
C_{bi} = \frac{F_c}{H_cD} \quad (24)
\]

\[
C_{di} = \frac{F_c}{H_c} \quad (25)
\]

where \( F_c \) is the axial force along the pile, \( F_{bi} \) is the bending moment along the pile, \( F_{di} \) is the shear force along the pile, \( w \) is the settlement along the pile, \( u \) is the horizontal displacement along the pile.

The results from the proposed method are compared with those from FEM and the proposed method with Vesic foundation. Figure 3 shows the responses of the pile-raft foundation under vertical and horizontal load.
separately for the case shown in Figure 2.

From figure 3 (a) ~3 (b), it can be seen that the results from the proposed method herein are almost the same as those from FEM when the foundation is subjected to pure vertical loads. From figure 3 (c) ~ (e), it can be seen that the results from the proposed method are similar to those from the FEM when using visic modulus. The results from the proposed method with modified visic modulus agree with the results from the FEM very well which is much more accurate than using visic modulus.

In order to investigate the responses of pile-raft foundation under multi-directional loads, the case shown in Figure 2 is also analysis using the method proposed herein with modified visic modulus when subjected to a vertical load and a horizontal load simultaneously with the same value of the loads. The results are shown in Figure 4. It can be indicated the calculated results from the proposed method agree very well with the results from FEM when under vertical and horizontal loads simultaneously. The method proposed herein is suitable to evaluate the responses of the pile-raft foundation under multi-directional loads in layered soil.

Comparing figure 3 (c) ~ 3 (e) to 4 (c) ~ 4 (e), it can be seen that the vertical load had no obvious effect on the horizontal responses of the piles in the pile-raft system for the coupled effect on a single pile is ignored and the restriction of the raft on the piles. On the other hand, it can be indicated from Figure 4(a),4(b) and Figure 5(a), 5(b) that the horizontal load influences the vertical responses of piles in the pile-raft system significantly. The horizontal force can change the axial force and vertical displacement of the front pile and the back pile.

In order to investigate the influence of the horizontal load on the vertical responses of the rigid pile-raft foundation, the horizontal load is set as 0, 0.25, 0.5, 1.0 and 2.0 times the vertical loads when calculating the responses of the pile-raft foundation under multi-directional loads. As shown in Figure 5. It can be seen the axial force of the back pile increases with the increase of the horizontal load, while the axial force of the front pile decreases. In this case, the front pile becomes an uplift pile when the horizontal load is larger than 0.5 times the vertical load. And the rotation
of the raft increases with the increase of the horizontal load due to the differential settlements between the front pile and the back pile increase with the increase of horizontal load.

Fig.5 Responses of piles in a pile raft subject to coupled loads

4 CONCLUSION

This paper proposed an analytical method for evaluating the responses of the pile-raft foundation under multi-directional loads in layered soil. And the method is carefully verified through a case study. Then the method is used to evaluating the responses of the pile-raft foundation under multi-directional loads, the following conclusion can be obtained:
(1)The horizontal load can increase the differential displacement of the piles which results in the increase of rotation of the rigid raft. At the same time, the axial force of the front pile and the back pile would redistribute.
(3)The modified visic subgrade modulus should be adopt instead of traditional visic modulus to make the results more accurate.

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