Modeling the Prompt Optical Emission of GRB 180325A: The Evolution of a Spike from the Optical to Gamma Rays

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Abstract

The transition from prompt to afterglow emission is one of the most exciting and least understood phases in gamma-ray bursts (GRBs). Correlations among optical, X-ray, and gamma-ray emission in GRBs have been explored, to attempt to answer whether the earliest optical emission comes from internal and/or external shocks. We present optical photometric observations of GRB 180325A collected with the TAROT and RATIR ground-based telescopes. These observations show two strong optical flashes with separate peaks at ~50 and ~120 s, followed by a temporally extended optical emission. We also present X-rays and gamma-ray observations of GRB 180325A, detected by the Burst Alert Telescope and X-ray Telescope on the Neil Gehrels Swift observatory, which both observed a narrow flash at ~80 s. We show that the prompt gamma-ray and X-ray early emission shares similar temporal and spectral features consistent with internal dissipation within the relativistic outflow (e.g., by internal shocks or magnetic reconnection), while the early optical flashes are likely generated by the reverse shock that decelerates the ejecta as it sweeps up the external medium.

Unified Astronomy Thesaurus concepts: Gamma-ray bursts (629); Theoretical models (2107); Optical observation (1169); Ground telescopes (687)

1. Introduction

Gamma-ray bursts (GRBs) are the most luminous events in the universe. GRBs are classified according to their duration: short GRBs have a duration of $T_{90} \lesssim 2$ s (where $T_{90}$ is defined as the time over which 90% of the gamma-ray photons are detected) and are associated with the coalescence of compact objects (two neutron stars or a neutron star/black hole binary system; Paczynski 1986; Eichler et al. 1989; Abbott et al. 2017), while long GRBs are associated with the collapse of massive stars and the formation of a black hole or a magnetar, and have a typical duration $\gtrsim 2$ s (Woosley 1993; MacFadyen & Woosley 1999; Hjorth et al. 2003).

One of the main features of the prompt GRB phase is its large variability. In the “standard fireball model” of GRBs, the central engine powers a jet with variable velocity. This leads to the formation of internal shocks in which collisions give rise to the observed rapid variability (e.g., Kobayashi et al. 1997). An alternative dissipation mechanism is magnetic reconnection in a Poynting-flux dominated outflow. In addition to this short timescale variability, about 30% of GRBs show X-rays flares (e.g., Burrows et al. 2005; Chincarini et al. 2007; Falcone et al. 2007) that are not associated with bright high-energy (MeV to GeV) emission (Troja et al. 2015). These X-ray flares share several temporal and spectral properties with the gamma-ray pulses observed during the prompt emission (e.g., Nousek et al. 2006; Liang et al. 2006; Krimm et al. 2007; Margutti et al. 2011), suggesting that they may be produced by late engine activity. For instance, both gamma-ray and X-ray sharp peaks are asymmetric, with fast rise and a slower decay (Yi et al. 2017). They both follow a similar hard-to-soft evolution, have spectra typically well fitted by a Band function, and present a similar spectral lag (Chincarini et al. 2010). In addition, the observed small values of $\Delta t/T_p \approx 0.1$ (in which $\Delta t$ is the duration of the pulse and $T_p$ is the time of the peak, see, e.g., Ioka et al. 2005) and the presence of a continuum with a similar slope before and after the X-ray flare, while distinguishing them from the prompt GRB emission, imply that they are not
typically associated with the external shock that is produced when the outflow is decelerated by the surrounding medium. An alternative scenario is that the X-ray flare originates in late-time sporadic magnetic reconnection events (Giannios 2006).

According to the fireball model, the external shock is when the outflow interacts with the circumstellar medium forming a forward and a reverse shock (Meszaros & Rees 1993; Sari & Piran 1995). While the long-lasting “afterglow” emission in wavelengths ranging usually from radio bands to X-rays is attributed to synchrotron radiation from the forward shock, the main observational signature of the reverse shock is typically a strong optical flash observable in the very early stages of the afterglow (e.g., Sari & Piran 1999a; Kobayashi 2000; Fraija 2015). This flash is due to synchrotron emission from the shocked ejecta and is typically expected to produce a single peak in the light curve (e.g., see Fraija et al. 2016; Fraija & Veres 2018; Becerra et al. 2019a). Bright emission requires a moderately magnetized ejecta. Optical flashes or flares are much less common (e.g., Krühler et al. 2009; Li et al. 2012) and typically not correlated with X-ray sharp peaks (e.g., Troja et al. 2015).

In this paper, we present optical observations of GRB 180325A with the TAROT and RATIR ground-based robotic telescopes. These observations show strong optical flashes with two distinct peaks at ~50 s and ~120 s, followed by temporally extended optical emission.

In order to analyze our optical data, we present other optical observations, as well as X-ray and gamma-ray observations with the Burst Alert Telescope (BAT) and X-Ray Telescope (XRT), on the Neil Gehrels Swift Observatory Gehrels et al. (2004). The gamma-ray and X-ray emission show narrow peaks at ~80 s. Using a semianalytic model we show that the gamma-ray and X-ray early emission that shares temporal and spectral features is consistent with production by internal shocks, while the the early optical flash is likely generated by the reverse shock. We also discuss the possibility the multiwavelength peaks have a common origin. We show that this possibility is very unlikely for both observational and theoretical reasons, although it cannot be completely excluded.

This paper is organized as follows. In Section 2, we present the observations made by Swift, TAROT, RATIR, and other telescopes. In Section 3 we present a temporal and spectral analysis of the data. In Section 4 we outline different scenarios to try to explain the observed spikes, using a semianalytic model for internal shocks. In Section 5 we summarize our results and discuss their implications.

2. Observations

2.1. Neil Gehrels Swift Observatory

The Swift/BAT instrument (Barthelmy et al. 2005) triggered on GRB 180225A at $T = 2018$ March 25 01:53:02.84 UTC (trigger 817564). Troja et al. (2018) reported a total duration $T_0 = 120$ s. Swift/BAT observed an initial FRED (fast rise and exponential decay) pulse from $T + 0$ to $T + 10$ s and then a stronger second pulse starting at about $T + 70$ s, peaking at about $T + 80$ s, and lasting until about $T + 110$ s (Lien et al. 2018). The 15–150 keV fluence was $(6.5 \pm 0.2) \times 10^{-9}$ erg cm$^{-2}$ and the 15–350 keV duration $T_{90}$ was $94.1 \pm 1.5$ s (Lien et al. 2018), clearly identifying GRB 180325A as a long GRB.

The Swift/XRT instrument (Burrows et al. 2005) started observing the field at 2018 March 25 01:54:16.2 UTC ($T + 73.4$ s) and detected a bright, fading source at 10:29:42.56 + 24:27:49.0 J2000 with a 90% uncertainty radius of 1.75 (Osborne et al. 2018; Troja et al. 2018).

The Swift/UVOT began to settle observations of the field of GRB 180325A at $T + 82$ s and detected a fading source consistent with the enhanced XRT position (Marshall & Troja 2018).

2.2. TAROT Observations

The TAROT Calern telescope (Klotz et al. 2008) received the BAT GRB position GCN alert packet at 2018 March 25 01:53:17.52 UTC ($T + 14.68$ s). It immediately slewed to the BAT position, and began its first exposure at 01:53:28.94 UTC ($T + 26.10$ s). The observations were taken in the C filter, which is fairly close to $r$ for sources with neutral colors.

The first TAROT image was exposed from $T + 26$ to $T + 86$ s with the tracking speed adjusted to obtain a small trail 11 pixels long. This technique is used to obtain continuous temporal information during the exposure (see, e.g., Klotz et al. 2006). The spatial sampling was 3"/29 pixel$^{-1}$ and the FWHM of stars (perpendicular to the trail) was 2.8 pixels. Subsequent images were taken with standard sidereal tracking.

To calibrate the photometry we use NOMAD-1 1144-0181764 as a reference star and adopted $r = 14.53$ and $r - i = +0.41$ from SDSS/DR9. This star was chosen because its color is very close to the color index $r - i = +0.39$ of the optical counterpart measured about one hour after the trigger by Schweyer & Kann (2018) and Watson et al. (2018). The $r$ magnitude for the reference star was converted into flux density $F_{\text{ref}} = 5.60$ mJy. The flux ratio between the afterglow and the reference star was determined by subtracting a scaled and shifted subframe around the reference star from a subframe around the optical counterpart and minimizing the rms residual.

Table 1 gives TAROT photometry. For each exposure, it gives the initial time $t_i$ and the final time $t_f$ (relative to $T$), the AB $r$ magnitude (not corrected for Galactic extinction), and the $1\sigma$ total uncertainties (including both statistical and systematic contributions). For TAROT, the exposure time is simply $t_f - t_i$.

The TAROT data show nondetections from 26 to 42 s, followed by detections of a flash from 42 to 53 seconds,
followed again by nondetections from 53 to 69 s. After 69 s, TAROT detects a rapidly rising flash, which then fades more slowly.

The first optical flash is not seen in the BAT light curve and unfortunately occurs before the start of XRT observations. The second optical flash is nearly simultaneous with flashes seen in both the BAT and XRT light curves.

### 2.3. RATIR Observations

RATIR is a four-channel simultaneous optical and near-infrared imager mounted on the 1.5 m Harold L. Johnson Telescope at the Observatorio Astronómico Nacional on Sierra Pedro Martir in Baja California, Mexico. RATIR responds autonomously to GRB triggers from the Swift satellite and obtains simultaneous photometry in $riZJ$ or $riYH$ (Butler et al. 2012; Watson et al. 2012; Littlejohns et al. 2015). Unfortunately, the $ZYJH$ detectors were not in service during our observations. Therefore, we only report the observations carried out by $r$ and $i$ filters.

GRB 180325A occurred just before local sunset at the Observatorio Astronómico Nacional, and our observations with RATIR did not start until the end of nautical twilight (Watson et al. 2018). On the first night of 2018 March 25 we observed from 02:54 UTC to 10:55 UTC ($T + 1.02$ to $T + 9.03$ hr) and obtained 288 pairs of simultaneous exposures each of 80 s in $r$ and $i$. On the second night of 2018 March 26 we observed from 03:02 UTC to 10:47 UTC ($T + 25.17$ to $T + 32.90$ hr) and obtained 272 pairs of simultaneous exposures each of 80 s in $r$ and $i$.

Our reduction pipeline performs bias subtraction and flat-field correction, followed by astrometric calibration using the astrometry.net software (Lang et al. 2010), iterative sky-subtraction, coaddition using SWARP (Bertin et al. 2002), and source detection using SEXTRACTOR (Bertin & Arnouts 1996). We calibrate against SDSS. The systematic calibration error is about 1%.

Table 2 gives our RATIR photometry. For each image it gives the initial time $t_0$, the final time $t_1$, the total exposure time $t_{\text{exp}}$, the $r$ and $i$ AB magnitudes (not corrected for Galactic extinction), and their 1σ total uncertainties (including both statistical and systematic contributions). Figure 1 shows the light curve of GRB 180325A from TAROT and RATIR.

### 2.4. Other Observations

Heintz et al. (2018) obtained spectroscopy with NOT/ALFOSC. They identified several absorption features, including Mg II, Fe II, Al II, C IV, and Si II and determined a common redshift of $z = 2.25$ for them. D’Avanzo et al. (2018) obtained spectroscopy with VLT/X-Shooter. They identified absorption and emission lines at $z = 2.248$ and confirmed the presence of a strong double intervening system at $z = 2.041/2.043$.

Zafar et al. (2018) determined that the host galaxy is from the main-sequence of star-forming galaxies using observations at four different epochs with NOT, VLT/X-shooter, and GROND.

Frederiks et al. (2018) estimated the following rest-frame parameters using Konus–Wind: the isotropic photon energy release $E_{\text{iso}}$ is about $2.3 \times 10^{53}$ erg, the peak luminosity $L_{\text{peak}}$ is about $3.2 \times 10^{53}$ erg s$^{-1}$, and the rest-frame peak energy of the time-integrated spectrum, $E_{p,\text{r}}$, is 995 keV.

Further optical observations were reported by Malesani & Fynbo (2018), Strobl et al. (2018), and Schweyer & Kann (2018).

### 3. Spectral and Temporal Analysis

#### 3.1. Spectral Analysis

We constructed the spectral energy distribution (SED) at $T + 10000$ s (just after the start of the afterglow phase) by retrieving the Swift/XRT X-ray from the online repository\(^\text{17}\) (Evans et al. 2009). We added optical observations in the $r$ and $i$ filters of RATIR at $T + 10000$ s. Figure 2 shows the resulting SED.

We fitted the SED with a spectral power law $F_\nu \propto \nu^{-\beta}$, in which $F_\nu$ is the flux density, $\nu$ is the frequency, and $\beta$ is the spectral index. From the X-ray to the optical, the SED can be fitted with a simple power law with a spectral index of $\beta = 0.45 \pm 0.01$. Under our assumption of a thin-shell evolving in the slow-cooling regime with the cooling break above the X-rays (Kobayashi 2000), we would expect the spectral index to be $(p - 1)/2$ or 0.65 $\pm$ 0.13 for $p = 2.32 \pm 0.30$; this value of $p$ is explained in more detail in the next section.

Figure 2 also shows the Xspec model fit, which takes into account the effects of reddening and absorption by the dust (Arnaud 1996). We use a reddening of $E(B - V) = 0.02$ (Marshall & Troja 2018), a redshift of $z = 2.25$ (Heintz et al. 2018), and a column density of $1.77 \times 10^{20}$ cm$^{-2}$ (Page et al. 2018). We obtained a reduced $\chi^2 = 0.86$ with 490 degrees of freedom. The $\beta$ derived from the photon index with Xspec is $\beta = 0.48 \pm 0.01$, which is consistent with the $\beta$ obtained from the simple fit.

#### 3.2. Temporal Analysis

The prompt emission from the GRB detected by Swift/BAT lasted until about $T + 120$ s (Troja et al. 2018). The earliest data from Swift/XRT started at $T + 61$ s. Our optical observations from TAROT and RATIR began at $T + 26.10$ and $T + 3674$ s, respectively, during prompt emission. Therefore, we focus our analysis on the end of prompt emission and the early afterglow of GRB 180325A.

\(^{17}\) http://www.swift.ac.uk/xrt_spectra/
3.2.1. Fitting the Pulses

Figure 3 shows the early emission in gamma rays, X-rays, and in the optical for $t < 200$ s. We fitted these light curves using a model that reproduces the asymmetric shape of the pulses, with a sharper rising phase and a shallower decaying phase. The parameters of best fits are listed in Table 3. This model, proposed by Norris et al. (2005) and Swenson et al. (2013), includes an asymmetric exponential rise and exponential-decay profile, and is given by

$$F(t) = A e^{-\beta t - \gamma t^2}.$$  

(1)

We notice that the lack of observational points at $T + 70 < t < T + 100$ s makes it hard to predict the exact slope of the rising part in the X-rays.

Although, as discussed in Section 4, the optical emission is likely coming from a reverse shock while the X-ray and gamma-ray emission originates from the internal shock, we fitted the temporal width $w$ for the pulse in the optical, X-ray, and gamma-ray bands using the FWHM. The results are shown in Figure 4. Then, we fitted the relationship $w(E) \propto E^{-\alpha}$ and found $\alpha = 0.22 \pm 0.03$ in agreement with $\alpha = 0.3$–0.4 found by Norris et al. (2005) for pulses observed during the prompt emission (between X-rays and gamma rays), and reported by Margutti et al. (2010) for X-ray flares. If all three pulses have a common origin, then our optical observations would extend this empirical relation down by about three orders of magnitude in frequency.

Figure 3 also shows that at $\sim 50$ s there is an earlier optical pulse lasting about 10 s and dimmer by a factor of $\sim 2$ with respect to the late optical peak.
since the BAT trigger, and $\alpha$ is the temporal index. Fits are summarized in Table 4. We can summarize the main stages as:

1. The light curve for $t < 200 \text{ s}$. For $t < 200 \text{ s}$, we see bright spikes in gamma rays, X-rays, and the optical with a peak at about $T + 100 \text{ s}$ (Figure 3). This emission started during the prompt emission phase. Although there is not a completely accepted model to explain these spikes; empirically, we fit two broken power-law segments to the X-ray light curve in order to obtain parameters to describe its behavior. We chose this band because it is the best sampled. The rise has a temporal index of $\alpha_{X,\text{rise}} = -7.11$ for $70 < t < 100 \text{ s}$ and the decay has a temporal index of $\alpha_{X,\text{decay}} = 5.42 \pm 0.26$ for $100 < t < 200 \text{ s}$. Qualitatively, we note the broadening of the spike as a function of the wavelength. These phenomena were previously discussed by Margutti et al. (2010) for X-ray and gamma-ray flares, with the result that high-energy flare profiles rise faster and decay faster. This spike is discussed in more detail in Section 4.

2. The optical curve for $t > 7000 \text{ s}$. We observed a power-law segment described by $F \propto t^{-\alpha_{f,r}}$, with $\alpha_{f,r} = 1.48 \pm 0.18$ for $r$ and $\alpha_{f,i} = 1.49 \pm 0.23$ for $i$. These temporal indexes can be explained under the assumption of a stellar wind medium where the normal decay segment is described by $F \propto t^{-\alpha_{f,i}}$ with $\alpha_{f,i} = (1 - 3p)/4$. Taking $p = 2.32 \pm 0.30$ (see Section 3.1), the expected value is $\alpha_{f,r,i} = 1.52 \pm 0.07$, which is in good agreement with our observed $\alpha_{f,r,i}$.

3. The X-ray light curve for $t > 7000 \text{ s}$. We observed a power-law segment described by $F \propto t^{-\alpha_{f,X}}$ with $\alpha_{f,X} = 1.84 \pm 0.09$ for the Swift/XRT data. We interpret this as the post jet-break decay phase. In this interval, the optical and X-ray light curves decrease steeper than in the plateau phase. We assumed that a nonthermal population of electrons is accelerated by shocks, with a distribution $N(\nu) \propto \nu^{-p}$. We analyzed this epoch in order to provide a restriction on the power-law index $p$. We then assumed

$$\omega(E) \propto E^{-0.22 \pm 0.03}.$$
continuity in this population to explain the optical emission during the earlier phase. Furthermore, from the SED $T + 10,000\,\text{s}$, the spectrum can be fitted with a simple power law with a spectral index of $\beta = 0.45 \pm 0.01$. We suggest that this optical emission arises above the cooling break ($\nu_c < \nu$) in a slow-cooling scenario (Sari et al. 1998), and so expect $F_\nu \propto \nu^{(1-p)/2} \nu^{(1-3p)/4}$. Our observed value of $\alpha_\nu = 1.49 \pm 0.22$ implies $p = 2.32 \pm 0.30$ and subsequently $\beta = 0.66 \pm 0.25$. The value of $p$ is consistent with the range of values typically for GRB afterglows and the predicted value of $\beta$ is consistent with our observation of $\beta = 0.45 \pm 0.01$.

For $300 < t < 7000\,\text{s}$, the optical light curve exhibits a plateau with a temporal index of $\alpha_{\text{plateau}} = 0.46 \pm 0.01$. The plateau phase is usually interpreted as evidence of late time central-engine activity (see, e.g., Kumar & Zhang 2015; Becerra et al. 2019b, 2019c).

4. The Origin of the Spike

4.1. Do the Optical, X-Ray, and Gamma-Ray Spikes Have a Common Origin?

As we see in Figure 3, the sharp peaks become wider at lower frequencies and show a spectral lag, with the high energy light curve rising/dropping on a shorter timescale with respect to the low energy counterpart. This effect has been observed previously in X-ray/$\gamma$ frequencies in X-ray flares (e.g., Chincarini et al. 2010; Margutti et al. 2010) and in gamma-ray pulses during the prompt emission (e.g., Fenimore et al. 1995; Norris et al. 1996). If the spikes at different frequencies share a common origin, our observations here would extend the broadening relation by three orders of magnitude.

Nevertheless, as shown in Figure 5, when the optical emission is included the spectrum is concave at 80 s after the trigger (near the peak of the light curve in X-rays and gamma rays), which is not consistent with standard synchrotron radiation. This suggests that the optical spike and the high energy spikes have different origins. Furthermore, we show below that there is no simple model that can explain the extension of the spectral lag and broadening relations to the optical. In order to explain the spike we consider the following scenarios.

High-latitude emission: Spectral lag and broadening have been explained by considering high-latitude emission (e.g., Ioka & Nakamura 2001; Norris 2002; Dermer 2004; Shen et al. 2005; Genet & Granot 2009; Shenoy et al. 2013). A salient feature of high-latitude emission (e.g., Kumar & Panaitescu 2000; Genet & Granot 2009; Uhm & Zhang 2016) is that the low-energy flux (at energies well below the $E_p$ of the emission along the line of sight) is much lower than the high-energy flux, as it comes from a region of the shock wave in which the flow is moving slightly off-axis with respect to the observer. Observations of GRB 180325A show that the peak in the flux density, $F_{\nu,p}$, is at a remarkably similar amplitude over four orders of magnitude in frequency. This excludes high latitude emission as a possible explanation.

Internal shocks: We assume here that the emission is due to synchrotron radiation in the fast-cooling regime, with $F_{\nu} \propto \nu^{-\alpha}$ for $\nu_c < \nu < \nu_m$, and $F_{\nu} \propto \nu^{-(p+1)/2}$ for $\nu > \nu_m$. As discussed by, e.g., Daigne et al. (2011) and Bošnjak & Daigne (2014), there are three possibilities. (1) Synchrotron radiation in the fast cooling regime, with $\alpha = 1/2$. (2) Fast cooling with $\epsilon_B \ll \epsilon_e$ in the Klein-Nishina (KN) regime, where less energetic electrons suffer less KN suppression and radiate more of their energy into the SSC component (that we do not see) and less of their energy into the synchrotron component (that we do see), resulting in a lower flux at lower frequencies compared to case 1, i.e., a lower $\alpha$: $0 \leq \alpha \leq 1/2$. (3) Marginally fast cooling, $\nu_c \approx \nu_m$, with $\alpha = 1/3$ corresponding to a different power-law segment of the synchrotron spectrum. In all these cases, the flux typically shows some spectral lag and broadening in the X-ray and gamma ray range. However, it is unclear if this behavior can be extended to the optical range, as a drop in the peak energy (due, e.g., to a drop in the microphysical parameters $\epsilon_e$, $\epsilon_B$, or to the shock Lorentz factor, or adiabatic cooling) should lead to a dramatic drop in the flux density. Moreover, the optical emission may also be self-absorbed, leading to a further suppression in its flux.

An accelerating jet coupled with a decaying magnetic field: Uhm & Zhang (2016) interpreted the spectral lag and broadening as due to a decaying magnetic field in an accelerating emitting region (and with a curved emitted photon spectrum in the proper frame). They point out that this is consistent with a Poynting-flux dominated jet accelerated far away from the source (e.g., Drenkhahn & Spruit 2002; Granot et al. 2011). As the synchrotron flux drops with the

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18 Here, $\epsilon_B$ and $\epsilon_e$ are the fractions of the post-shock internal energy going into the magnetic field and into the energy of the nonthermal relativistic electrons, respectively.

19 Self-absorption should be less important in reverse shocks, as their density is much lower.
magnetic field, an increase in the shock velocity keeps the low-energy flux density at a level comparable to the high-energy component.

Following Uhm & Zhang (2016), the peak energy of the observed synchrotron spectrum is $E_p \propto \Gamma B$ (where $\Gamma$ is the Lorentz factor of the radiating material), and the peak flux scales as $F_{\nu,p} \propto N^{1/2} \mu G r^{-5}$, where $N (\propto 1/\Gamma^2)$ is the number of electrons assumed to be constantly injected in the emitting region. Thus, considering a magnetic field in the comoving frame dropping as $B \propto r^{-b}$, and a Lorentz factor increasing as $\Gamma \propto r^s$, Uhm & Zhang (2016) got $F_{\nu,p} \propto r^{s-b} \propto t^{s-b}/(1-2s)$ (where, to derive the last proportionality we employed the relation $t_{\text{obs}} \propto t/\Gamma^2 \propto r/\Gamma^2 \propto r^{1-2s}$) and $F_{\nu,p} \propto t^{s-1}/(1-2s)$. Finally, we obtain $F_{\nu,p} \propto F_{\nu,p,\text{obs}}^{(1-b)/(s-b)}$. As the peak energy should drop by about five orders of magnitude in the emitting region (going from $\sim 200$ keV to $\sim 2$ eV), the exponent $(1-b)/(s-b) \approx 0$ to get a nearly constant flux density as was observed. This implies that $b \approx 1$. For such a drop in the comoving magnetic field, $E_p \propto \Gamma B \propto r^{-1}$ or $E_p \propto r^{-2/3} \propto \Gamma^{-2}$ for $s = 1/3$ that is expected in Poynting-flux dominated models, which implies that the emitting region should extend over $\sim 7.5$ decades (e.g., from $10^{10.5}$ to $10^{18}$ cm), while $\Gamma$ increases by a factor of $\sim 10^{2.5}$. Given that even at the initial radius $\Gamma \gtrsim 10^2$ is typically required because of compactness arguments, this would require an unreasonable large $\Gamma \gtrsim 10^{4.5}$ near the peak time of the optical emission.

In conclusion, while the possibility that the optical emission shares a common origin with X-rays and gamma rays cannot be completely ruled out, it is hard to explain with synchrotron emission. There are further difficulties in explaining the broadening over such a large frequency range. Moreover, a common origin does not naturally explain the convex spectrum shown in Figure 5, which suggests a distinct spectral component in the optical.

4.2. Optical, X-Rays, and Gamma-Rays Originated from Reverse and Internal Shocks

Finally, we consider the possibility that the optical emission originates from the external reverse shock that decelerates the ejecta as it sweeps up the external medium, while the X-rays and gamma rays are from prompt emission (e.g., produced by internal shocks or magnetic reconnection within the ejecta).

A reverse shock component is expected as the expanding relativistic ejecta encounters the circumstellar matter (CBM; be it the ISM for short-hard GRBs or the stellar wind of the massive progenitor star for long-soft GRBs). A forward shock is driven into the CBM, a reverse shock is driven upstream into the unshocked part of the ejecta, and the two shocked regions are separated by a contact discontinuity. A reverse shock is predicted to produce a single peak in the light curve, which may appear similar to a flare (see, e.g., Sari & Piran 1999b; Kobayashi et al. 2007), as the electrons are heated and cool primarily by synchrotron radiation and inverse-Compton scattering of the synchrotron photons (synchrotron self-Compton; SSC). Kobayashi et al. (2007) study the variability and the temporal indexes expected in an X-ray emission created in the reverse shock region. A reverse shock must display a time variability scale of $\Delta t/t \sim 1$ (Kobayashi et al. 2007) and varies as $F_{\nu} \propto t^{(p-1)/4}$ before the peak and $\propto t^{(p-1)/3}$ after the peak. We observe a $\Delta t = 130$ s and $t = 100$ s in the optical band and therefore $\Delta t/t \sim 1.3$, consistent with a reverse shock component.

It is useful to compare the temporal indices expected for this model to our observations in X-rays and gamma rays. We divide the X-ray sharp peak in two regions to fit them with power-law segments. From $T + 70$ s to $T + 200$ s with a peak in $T + 100$ s, we found temporal indexes of $\alpha_{X,\text{rise}} = -7.11 \pm 2.77$ and $\alpha_{X,\text{decay}} = 5.42 \pm 2.54$ for the rise and decay, respectively. Using the value of $p$ described in Section 3 of $p = 2.32 \pm 0.30$ for the afterglow phase, we expect $F_{\nu} \propto t^{5(p-1)/4} = t^{1.65}$ before the peak and $\propto t^{-(3p+1)/3} = t^{-2.65}$ after the peak. The observed values are quite different from those expected theoretically for $\alpha_{X,\text{rise}}$ and $\alpha_{X,\text{decay}}$. Therefore, we can conclude that a reverse shock (at the jet head) is not a plausible mechanism for the X-ray sharp peak of GRB 180325A, which likely originated from internal shocks.

On the other hand, early optical afterglows often have an external reverse shock (RS) origin (Meszaros & Rees 1993). In addition, an RS may explain the temporal behavior of the optical light curve in GRB 180325A. In particular, a peak on a time comparable to and slightly longer than the duration of the prompt GRB emission is expected for a reverse shock that is at least mildly relativistic—the “thick shell” case, and the observed optical light curve may correspond to a mildly relativistic RS (see, e.g., Nakar & Piran 2004).

To reproduce the observations, we assume synchrotron radiation and consider the emission of the two components, i.e., internal and external shocks. Internal shocks result from a nonuniform distribution of the Lorentz factor in the outflow, while by “external shocks” we refer to both the forward shock that propagates into the external medium, and the reverse shock that sweeps back into the ejecta as a result of the deceleration of the jet material. We employ the formalism of Cantó et al. (2013) for the dynamics of internal shocks in relativistic jets with a time-dependent injection velocity and mass-loss rate. This formalism takes into account the loss of energy and momentum due to radiation. Cantó et al. (2013) found an analytic solution for the dynamics of the internal shocks in the case where the material is ejected from the central engine with a step function velocity variation.

We consider the interaction between two shells with normalized velocities $\beta = v/c$, where $c$ is the speed of light, and Lorentz factors $\gamma$. We assume that the first shell is ejected with a slow velocity $\beta_s$ during a time interval $\Delta t_s$, with a mass injection rate $\dot{m}_s$. After a time interval $\Delta t_0 \approx 1$ second, a fast velocity $\beta_f > \beta_s$ and mass injection rate $\dot{m}_f$, is injected during a time interval $\Delta t_f$. The two shells collide after a time $t_{\text{coll}} = \beta_f \Delta t_0 / (\beta_f - \beta_s) \approx 2\gamma_s^2 \Delta t_0 / (1 - \gamma_s^2 / \gamma_f^2)$ (measured from the time when the ejection of the first shell ended) at a distance $x_{\text{coll}} = c / \beta_s t_{\text{coll}}$. At this moment, a two shock structure (the working surface, WS hereafter), is formed at $x_{\text{coll}}$.

In addition, by assuming $\beta_s \ll 1$, the same formalism allows us to study the propagation of the jet head (with the WS made in this case by the forward and reverse shocks). To apply this formalism, one has to substitute in their equations the injection time of the fast material $t_f$ by $t_f - \Delta t_0$. This change does not affect the shell dynamics and is equivalent to a reference frame where $x_{\text{coll}} = t_{\text{coll}} = 0$, and the gas particles are injected at $x_{\text{coll}} = 0$ with a velocity distribution shown in Figure 7. In Appendix A.1 we summarize their solution.
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The interaction between the internal shocks moving into the jet channel leads to the formation of a forward shock, which propagates into the slow medium incorporating this material into the WS, and a reverse shock decelerates and incorporates the fast moving material (in the WS frame, the reverse shock crosses the fast moving material). This double shock structure is similar to the one formed at the external shocks, although in internal shocks both the fast and slow material move with relativistic speeds, while in the external shocks the slow-moving material is moving at a much smaller speed (being the wind of the progenitor star).

Light curves computed by employing the semianalytic model described above (see Appendix A.2) are shown in Figure 6. We take $\xi_e = 0.1$ (external shocks) and $\xi_e = 10^{-3}$ (internal shocks), with $\xi_e$ being the fraction of electrons accelerated at the shock front. The light curves are produced by considering standard synchrotron theory with $\gamma_e = 0.1$, $\gamma_B = 10^{-2}$, and $p = 2.32$ for both components, RS and forward shock (FS).

The high-energy component is due to internal shocks, with Lorentz factors of $\gamma_f = 200$ and $\gamma_s = 30$ for the fast and slow shell respectively. We consider an isotropic energy of $E = 2.8 \times 10^{53}$ erg for both the slow and fast shell. The shells are injected during a time $\Delta \tau_f = \Delta \tau_s = 2$ s, with mass-loss rates $\dot{m}_s = \dot{m}_f$ (sub-indices $s$ and $f$ refer to slow and fast components, respectively). The shells collide at $R \sim 10^{13}$ cm and form a working surface that moves with a Lorentz factor of $\sim 100$. The energy of the fast and slow shells is

$$E_{(s,f)} = \dot{m}_{(s,f)} \Delta \tau_{(s,f)} \gamma_{(s,f)} c^2.$$  

Given the values of $\Delta \tau_{(s,f)}$ and $\gamma_{(s,f)}$ specified above, this equation gives the explicit values of $\dot{m}_{(s,f)}$.

On the other hand, the low-energy component is due to the reverse shock. The slow-moving material has $\gamma_e = 1.0017$ (corresponding to a typical Wolf-Rayet wind velocity of $v_w = 10^8$ cm s$^{-1}$), $\gamma_f = 30$, $\dot{m}_s = 10^{-6} M_\odot$ yr$^{-1}$, $\Delta t_s = \infty$, $\Delta t_f = 1$, $E_f = 4 \times 10^{52}$ erg, and $\dot{m}_f = E_f / (\Delta \tau_f \gamma_f c^2)$.

In the slow-cooling regime, the synchrotron emission depends strongly on the value of $\nu_{\text{ms}}$ (i.e., the peak frequency). Then, to explain the observations with our model we need to choose fitting parameters such that $\nu_{\text{ms}}$ peaks close to optical frequencies in the reverse shock and to gamma frequencies in internal shocks (see Figure 6, bottom panel). This implies that (1) the fraction of accelerated electrons $\xi_e$ in the RS is much larger than in IS (as $\nu_{\text{ms}} \propto 1 / \xi_e^2$), or (2) the jet luminosity/ejecta energy is much larger in the IS than in the RS. In the first case, different values of $\xi_e$ in the IS and RS are typically considered in the literature. Both IS and the RS propagate in the ejecta, but shocks in IS are relativistic while the RS is only mildly relativistic. Then, the acceleration process is much more efficient in RS than in IS.

While the optical light curve shown in Figure 6 accounts for the main optical peak at $\sim 120$ s, it does not account for the earlier optical peak at $\sim 50$ s and the associated temporal variability that is reflected in these two peaks. This variability may potentially arise from the density structure within the ejecta that is encountered by the reverse shock, which is caused by the collision and merger of the original shells that produced the internal shocks responsible for the X-ray and gamma-ray prompt emission. This is in contrast to the model described above for the reverse shock, which assumes a perfectly uniform ejecta shell and therefore leads to a single peak in the light curve. An additional contribution to the time variability of the optical emission from the reverse shock may arise from a variation of the magnetization within the ejecta, where strongly magnetized portions suppress the reverse shock and its emission, while mildly magnetized portions lead to a strong reverse shock with prominent emission.

5. Summary and Discussion

We have presented optical photometry of the very early afterglow of GRB 180325A with the TAROT instrument from $T + 26$ s. We complement these data with RATIR observations up to 30 hr after the burst.

We compare the optical light curve with gamma-ray and X-ray light curves from Swift/BAT and Swift/XRT, respectively. We see an early gamma/X-ray/optical spike from $T + 70$ s, with a peak at about $T + 100$ s, and lasting until about $T + 200$ s. The peak at different times and with not the same duration, turns to GRB 180325A in a very special case of study.

By discussing possible scenarios for its origin we show that the X-ray/gamma-ray spike was likely generated by an internal

Figure 6. Upper panel: light curves (in three energy bands) emitted from the working surface as a result of the interaction between two internal shocks (solid lines) and by the (external) reverse shock (dashed lines). Synchrotron emission is assumed in all the cases. X-rays and gamma rays produced by the RS are negligible, and the optical emission produced by the IS is smaller than the corresponding RS component. The observed increase in flux associated with the IS is likely due to the shells not being uniform in density as assumed in our model. Bottom panel: spectra emitted by the reverse and forward shock formed by internal shocks (at $t = 82$ s, “RS IS” and “FS IS” in the figure), and by the reverse and forward shock at the jet head (at $t = 120$ s, “RS” and “FS” in the figure). The RS dominates the emission in both cases.
shock while the optical spike was likely generated by an 
external reverse shock. Interestingly, optical observations show 
an earlier flare, lasting about 10 s, which is also not associated 
with the high energy emission and is possibly also the result of 
a reverse shock. Observations of fluctuations in the optical 
reverse shock emission are atypical and might be the result of 
density/velocity fluctuations in the material crossing the 
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Appendix
Calculation of Light Curve and Spectrum

Given the dynamical evolution of the forward and reverse 
shocks, we compute the multiwavelength emission by assum-
ing that it is produced by synchrotron radiation. We discuss 
here the dynamical evolution of the working surface, and how 
it is used to compute the emitted radiation.

A.1. Step Function Velocity Variation

Following Cantó et al. (2013), we consider a free-streaming 
flow injected at $x = 0$ with velocity $\beta$ and mass-loss rate $\dot{m}$. We 
assume a velocity variation (see Figure 7) with a slow flow 
injected in the interval $[-\Delta\tau_s, 0]$ and a fast flow injected in the 
interval $[0, \Delta\tau_f]$. A working surface (WS) is formed at the 
position $x_{ws} = 0$ at a time $t = 0$.

![Figure 7. Injection velocity $\beta(\tau)$ as a function of time $\tau$ at the position of the shell collision. The time intervals $\Delta\tau_s$ and $\Delta\tau_f$ represent the duration of the injection of the fast and slow material.](image)

Initially, the WS velocity is constant, and corresponds to a 
Lorentz factor

$$\gamma_{ws0} = \frac{\lambda + 1}{\sqrt{(\lambda/\gamma_f)^2 + 2\lambda(1 - \beta/\beta_f) + 1/\gamma_f^2}}. \tag{3}$$

In this equation one defines

$$\lambda = \frac{br}{a}, \tag{4}$$

where the velocity ratio, the mass-loss ratio, and the gamma 
ratio are given by

$$a = \beta/\beta_s, \quad b = m_f/m_s, \quad r = \gamma_f/\gamma_0. \tag{5}$$

The velocity remains constant during a certain time, which 
depends on which material runs out first. In case I (see 
Figure 8), the constant velocity phase ends when the fast 
material is completely incorporated into the WS, at a time $t_f^I$ 
given by

$$t_f^I = \frac{a(\lambda + 1)}{a - 1} \Delta\tau_f \approx \frac{2\gamma_f^2}{1 - \gamma_f^2/\gamma_f^2} (\lambda + 1) \Delta\tau_f. \tag{6}$$

Afterwards, the WS decelerates such that the Lorentz factor 
$\gamma_{ws}^I(t)$ varies with time as

$$\gamma_{ws}^I = \gamma_f \left(\frac{T^2 + 4\beta_f T + 4}{T^2 - 4}\right) \tag{7}$$

where $T(t)$ is a function of time given by the transcendental 
Equation (23) in Cantó et al. (2013), where $\tilde{t} = t/\Delta\tau_f$. \(^{21}\) The 
deceleration phase ends at a time $t_s$, when the WS completely 
engulfs the slow-moving material. For a free-streaming flow, $t_s$ 
is given by the condition

$$v_s (ts + \Delta\tau_f) = R_{ws}(ts). \tag{8}$$

In case II, the constant velocity phase ends when the slow 
material is completely incorporated into the WS, at a time $t_f^{II}$

\(^{21}\) Note that in Cantó et al. (2013) subscript 1/2 refers to the slow/fast material.
To compute the synchrotron radiation, we assume that a fraction $\chi_e$ of the post-shock electrons are accelerated to relativistic speeds, forming a population of electrons $N(\gamma_e) \propto \gamma_e^{-p}$, where $p$ is the power index of the population of nonthermal electrons accelerated by the shock. The energy of the accelerated electrons and the post-shock magnetic field are taken as a fraction $\epsilon_e$ and $\epsilon_B$ of the post-shock thermal energy. Then, we use standard synchrotron theory to compute the synchrotron emissivity (see, e.g., De Colle et al. 2012) as a function of time.

In synthesis, light curve and spectrum are computed as a function of the following parameters: the Lorentz factors $\gamma_f(t)$ and $\gamma_s(t)$ of the fast and slow unshocked material, the mass-loss rates (or the density) $\dot{m}_f$, $\dot{m}_s$, and the duration of injection of the fast and slow material $\Delta T_f$ and $\Delta T_s$. In addition, the synchrotron radiation calculation is parameterized as a function of the microphysical parameters $\epsilon_e$, $\epsilon_B$, and $p$.

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References

Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2017, ApJL, 848, L13
Arnaud, K. A. 1996, in ASP Conf. Proc. 101, Astronomical Data Analysis Software and Systems V, ed. G. H. Jacoby & J. Barnes (San Francisco, CA: ASP), 17
Barthelmy, S. D., Barbier, L. M., Cummings, J. R., et al. 2005, SSRV, 120, 143
Becerra, R. L., De Colle, F., Watson, A. M., et al. 2019c, ApJ, 887, 254
Becerra, R. L., Watson, A. M., Fraija, N., et al. 2019b, ApJ, 872, 118
Bertin, E., & Arnouts, S. 1996, A&AS, 117, 393
Bertin, E., Mellier, Y., et al. 2002, in ASP Conf. Proc. 281, Astronomical Data Analysis Software and Systems XI, ed. D. A. Bohlender, D. Durand, & T. H. Handley (San Francisco, CA: ASP), 228
Bošnjak, Ž., & Duigne, F. 2014, A&A, 568, A45
Burrows, D. N., Hill, J. E., Nousek, J. A., et al. 2005, SSRV, 120, 165
Butler, N., Klein, C., Fox, O., et al. 2012, Proc. SPIE, 8446, 844610
Cantó, J., Lizano, S., Fernández-López, M., et al. 2013, MNRAS, 430, 2703
Chincarini, G., Mao, J., Margutti, R., et al. 2010, MNRAS, 406, 2113
Chincarini, G., Moretti, A., Romano, P., et al. 2007, ApJ, 671, 1903
Duigne, F., Bošnjak, Ž., & Dubus, G. 2011, A&A, 526, A110
D’Avanzo, P., Bolmer, J., D’Elia, V., et al. 2018, GCN, 22555, 1
De Colle, F., Granot, J., López-Cámara, D., et al. 2012, ApJ, 746, 122
Dermer, C. D. 2004, ApJ, 614, 284
Drenkhahn, G., & Spruit, H. C. 2002, A&A, 391, 1141
Eichler, D., Livio, M., Piran, T., et al. 1989, Natur, 340, 126
Evans, P. A., Beardmore, A. P., Page, K. L., et al. 2009, MNRAS, 397, 1177
Falcone, A. D., Morris, D., Racusin, J., et al. 2007, ApJ, 671, 1921
Fenimore, E. E., in ‘t Zand, J. J. M., Norris, J. P., Bonnell, J. T., & Nemiروف, R. J. 1995, ApJ, 448, L101
Fraija, N. 2015, ApJ, 804, 105
Fraija, N., Lee, W. H., & Veres, P. 2016, ApJ, 818, 190

A.2. Synchrotron Emission

We solve the Taub relativistic shock-jump conditions and compute the post-shock energy density $\epsilon_{ps}$, density $\rho_{ps}$, and velocity $v_{ps}$, as a function of the WS velocity, the pre-shock densities and the velocities of the fast and slow material ejected from the central engine. Thus, the post-shock physical variables will evolve with time as the working surface accelerates or decelerates as discussed in Appendix A.1.

Figure 8. Qualitative temporal evolution of the working surface Lorentz factor for Case I (green line) and Case II (red line). The initial constant velocity for the two cases is shown in blue.

Given by

$$\tau_0^W = \frac{\lambda + 1}{\lambda(a - 1)} \Delta \tau_s \approx \frac{2\gamma_s^2}{1 - \gamma_s^2/\gamma_f^2} \frac{\lambda + 1}{\lambda} \Delta \tau_s \tag{9}$$

The WS then accelerates such that the Lorentz factor $\gamma_{ws}^H(t)$ is

$$\gamma_{ws}^H = \gamma_f \left( \frac{T^2 - 4\beta_T T + 4}{T^2 - 4} \right) \tag{10}$$

where $T(t)$ is given by the transcendental Equation (35) in Cantó et al. (2013), where $t = \tau/\Delta \tau_s$. The acceleration phase ends at a time $t_f$ when the WS completely engulfs the fast moving material, i.e., when

$$v_f(t_f - \Delta \tau_s) = R_{ws}(t_f) \tag{11}$$

Finally, the velocity of the WS becomes constant when all the material has been incorporated (see Figure 8 for a schematic representation of the evolution of the Lorentz factor of the WS). In the case of a fast shell that runs into a static medium with a density $\rho$ and $\beta_s = 0$, since the mass-loss rate is $\dot{m} \propto \rho \beta_f$, the parameter $\lambda$ can be written as $\lambda = (\rho_f \gamma_f/\rho_s)^{1/2}$. Then, the phase of constant velocity has

$$\gamma_{ws \infty} = \frac{\lambda + 1}{\sqrt{(\lambda/\gamma_f)^2 + 2\lambda + 1}} \tag{12}$$

This phase ends at the time $t_i^W$ when all the fast material has been incorporated to the WS. From then on, the WS decelerates continuously with a Lorentz factor given by Equation (7) and approaches asymptotically to rest (see case I in Figure 3 of Cantó et al. 2013).
Frajia, N., & Veres, P. 2018, ApJ, 859, 70
Frederiks, D., Golenetskii, S., Aptekar, R., et al. 2018, GCN, 22546, 1
Gehrels, N., Chincarini, G., Giommi, P., et al. 2004, ApJ, 611, 1005
Genet, F., & Granot, J. 2009, MNRAS, 399, 1328
Giannios, D. 2006, A&A, 455, L5
Granot, J., Komissarov, S. S., & Spitkovsky, A. 2011, MNRAS, 411, 1323
Heintz, K. E., Fynbo, J. P. U., & Malesani, D. 2018, GCN, 22553, 1
Hjorth, J., Sollerman, J., Møller, P., et al. 2003, Natur, 423, 847
Ioka, K., Kobayashi, S., & Zhang, B. 2005, ApJ, 631, 429
Ioka, K., & Nakamura, T. 2001, ApJL, 554, L163
Klotz, A., Boër, M., Eysseric, J., et al. 2008, PASP, 120, 1298
Klotz, A., Gendre, B., Stratta, G., et al. 2006, A&A, 451, L39
Kobayashi, S. 2000, ApJ, 545, 807
Kobayashi, S., Piran, T., & Sari, R. 1997, ApJ, 490, 92
Kobayashi, S., Zhang, B., Mészáros, P., & Burrows, D. 2007, ApJ, 655, 391
Krimm, H. A., Granot, J., Marshall, F., et al. 2007, ApJ, 665, 554
Krühler, T., Greiner, J., McBreen, S., et al. 2009, ApJ, 697, 758
Kumar, P., & Panaitescu, A. 2000, ApJL, 541, L51
Kumar, P., & Zhang, B. 2015, PhR, 561, 1
Lang, D., Hogg, D. W., Mierle, K., Blanton, M., & Roweis, S. 2010, AJ, 139, 1782
Li, L., Liang, E.-W., Tang, Q.-W., et al. 2012, ApJ, 758, 27
Liang, E. W., Zhang, B., O’Brien, P. T., et al. 2006, ApJ, 646, 351
Lien, A. Y., Barthelmy, S. D., Cummings, J. R., et al. 2018, GCN, 22545, 1
Littlejohns, O. M., Butler, N. R., Cucchiara, A., et al. 2015, MNRAS, 449, 2919
MacFadyen, A. I., & Woosley, S. E. 1999, ApJ, 524, 262
Malesani, D., & Fynbo, J. P. U. 2018, GCN, 22551, 1
Margutti, R., Chincarini, G., Granot, J., et al. 2011, MNRAS, 417, 2144
Margutti, R., Guidorzi, C., Chincarini, G., et al. 2010, MNRAS, 406, 2149
Marshall, F. E., & Troja, E. 2018, GCN, 22549, 1
Meszaros, P., & Rees, M. J. 1993, ApJ, 405, 278
Nakar, E., & Piran, T. 2004, MNRAS, 353, 647
Norris, J. P. 2002, ApJ, 579, 386
Norris, J. P., Bonnell, J. T., Kazanas, D., et al. 2005, ApJ, 627, 324
Norris, J. P., Nemiroff, R. J., Bonnell, J. T., et al. 1996, ApJ, 459, 393
Nousek, J. A., Kouveliotou, C., Grupe, D., et al. 2006, ApJ, 642, 389
Osborne, J. P., Beardmore, A. P., Evans, P. A., et al. 2018, GCN, 22539, 1
Paczynski, B. 1986, ApJL, 308, L43
Page, K. L., D’Ai, A., Melandri, A., et al. 2018, GCN, 22540, 1
Sari, R., & Piran, T. 1995, ApJL, 455, L143
Sari, R., & Piran, T. 1999a, ApJL, 517, L109
Sari, R., & Piran, T. 1999b, ApJ, 520, 641
Sari, R., Piran, T., & Narayan, R. 1998, ApJL, 497, L17
Schwegan, T., & Kann, D. A. 2018, GCN, 22544, 1
Shen, R.-F., Song, L.-M., & Li, Z. 2005, MNRAS, 362, 59
Shenoy, A., Sonbas, E., Dermer, C., et al. 2013, ApJ, 778, 3
Strobl, J., Jelinek, M., & Hudec, R. 2018, GCN, 22541, 1
Swenson, C. A., Roming, P. W. A., De Pasquale, M., et al. 2013, ApJ, 774, 2
Troja, E., Piro, L., Vasileiou, V., et al. 2015, ApJ, 803, 10
Troja, E., D’Ai, A., D’Elia, V., et al. 2018, GCN, 22532, 1
Uhm, Z. I., & Zhang, B. 2016, ApJ, 825, 97
Vurm, I., Hascoët, R., & Beloborodov, A. M. 2014, ApJL, 789, L37
Watson, A. M., Richer, M. G., Bloom, J. S., et al. 2012, Proc. SPIE, 8444, 84445L
Watson, A. M., Butler, N., Kutyrev, A., et al. 2018, GCN, 22537, 1
Woosley, S. E. 1993, ApJL, 405, 273
Yi, S.-X., Yu, H., Wang, F. Y., et al. 2017, ApJ, 844, 79
Zafar, T., Heintz, K. E., Fynbo, J. P. U., et al. 2018, ApJL, 860, L21

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