Effect of Thermal Fluctuations in the Resonance Fluorescence of a Three-Level System

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The effect of thermal fluctuations in the resonance fluorescence of a three-level system is studied. The damped three-level system is driven by two strong incident classical fields near resonances frequencies. The simulation of a thermal bath is obtained with a large system of harmonic oscillators that represent the normal modes of the thermal radiation field. The time evolution of the fluorescent light intensities are obtained solving by a iterative method the Heisenberg equations of motion in the integral form. The results show that the time development of the intensity of the fluorescence light is strongly affected by the interaction of the system with the thermal bath.

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I. INTRODUCTION

In the last fifteen years, quantum jumps in three-level systems has been extensively studied \[1\]-\[13\]. The three-level system is a versatile model that has been used to study, among others, the correlations in emission of photons \[4\], resonant fluorescence \[5\], operation of a two-mode laser \[6\], coherent pump dynamics \[7\], squeezing properties of electromagnetic field \[8\], electron shelving \[9\], quantum measurements theory \[10\]. The interest on processes involving atoms with few energy levels recently increased even more with the possibility of to study experimentally non-linear processes with ion trapping.

In this paper we consider the fluorescence resonance of a three-level system with a coherent interaction with two driving fields and an incoherent interaction with a thermal reservoir. When the driving fields are turned on, the system is driven to a new non-equilibrium steady state. If the driven fields are turned off, the system returns to the original equilibrium state with the thermal bath. We assume that the atom is in a cavity where there is a quantized radiation field in thermal equilibrium with the atom and the cavity walls at a certain temperature. In our model of a three-level atom, the allowed transitions are only between the levels 1 and 2 and 1 and 3. The three-level system of this kind is known as the V configuration. We study the system driven by the interaction with two electromagnetic fields of frequencies $\omega_1$ e $\omega_2$, near, respectively, the $\omega_1$ e $\omega_2$ frequencies characteristics of the system. The three-level system also interacts with a radiation field, with which one maintains in thermal equilibrium. We suppose that the normal modes of the radiation field constitutes the thermal bath at a certain temperature. We obtain the equations of motion of the dynamical operators in the Heisenberg formalism in the general form

$$\frac{dx}{dt} = -\beta x + A(t)$$ \hspace{1cm} (1)

that may be recognized as equations that describes a damped system subject to fluctuating forces, accordingly Langevin’s theory of Brownian motion\[22\],\[23\]. In the present problem, these fluctuating forces are represented by

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the non-linear terms in the equation of motion due to the interaction of the system with the thermal bath when \( T \neq 0 \, K \). The role of the fluctuating forces is to bring the system to the thermal equilibrium. The non-linearity of the equations, caused by the saturation of the atomic transitions, enhances the atom-bath interaction.

Even though the works about fluorescence resonance take into account the interaction of the resonant system with a thermal bath, they in general assume the temperature of the bath as \( T = 0 \, K \). Senitzky [17], however, in a elegant way regards the effect of thermal fluctuations in the resonance fluorescence of a two-level atom. But Senitzky treats a more general model for the thermal reservoir than we present here, and his mathematical treatment is more involved.

In this paper we prefer to use the equations of motion of the dynamical operators to calculate the intensity of the fluorescence light. Although this approach is equivalent to the density operator, it has the advantage to facilitate the physical interpretation, because it resembles with the classical treatment given to the Brownian movement, in terms of the Langevin’s equations.

This paper is organized as follows. In Sec. 2 we describe formally the system and obtain the Heisenberg equations of motion in integral form. In Sec. 3 we determine the solution of the system of equations and apply it to the spontaneous emission to obtain the mean intensity of the scattered fields in the fluorescence of the two excited levels. In Sec. 4 we discuss and interpret graphically the results of the Sec. 3. Some details of the calculations are given in Appendix.

II. THE HAMILTONIAN OF THE SYSTEM

We consider here the problem of an atom fixed in space, with three levels and one electron, assuming that transitions occur only between each excited level and the fundamental one. Thus, the three-level system will be described with the help of the following operators

\[
R_{00} = |0\rangle\langle 0|, \quad R_{11} = |1\rangle\langle 1|, \quad R_{22} = |2\rangle\langle 2|, \\
R_{01} = |0\rangle\langle 1|, \quad R_{02} = |0\rangle\langle 2|,
\]

which obey the relation

\[
1 = |0\rangle\langle 0| + |1\rangle\langle 1| + |2\rangle\langle 2|.
\]

The fundamental level is represented by \(|0\rangle\), and the excited levels are represented by \(|1\rangle\) and \(|2\rangle\), respectively. The commutation relations of the \(R_{ij}\) operators are

\[
[R_{01}, R_{01}^\dagger] = R_{00} - R_{11}, \\
[R_{02}, R_{02}^\dagger] = R_{00} - R_{22}, \\
[R_{02}, R_{01}^\dagger] = -R_{12} = 0, \\
[R_{01}, R_{02}^\dagger] = -R_{21} = 0,
\]

where we make the transition rates between the two excited levels vanish. The atom is illuminated with two polarized laser beams; each beam has a frequency close to the characteristic frequency of each excited level of the atom. We also assume that the light beams are intense, and they will be treated classically. Within this point of view, the atom may be assumed as a couple of electric dipoles interacting with the electromagnets fields of the light beams. To account the radiative damping, a thermal bath is simulated with a big system of harmonic oscillators, that perform the role of the normal modes of the thermal radiation field. The Hamiltonian is

\[
H = H_0 + H_I,
\]

where

\[
H_0 = \hbar \omega_1 R_{11} + \hbar \omega_2 R_{22} + \hbar \sum_k \omega_k b_k^\dagger b_k.
\]
\( b_k \) and \( b_k^\dagger \) are the operators corresponding to the modes of the bath (annihilation and creation, respectively), that satisfy

\[
[b_k, b_{k'}^\dagger] = \delta_{kk'}.
\]  

(13)

The interaction Hamiltonian, in the rotating wave approximation, is given by

\[
H_I = \left\{ -\hbar R_{01} \left( \lambda_1 E_1'(t) + i \sum_k g_1(k) b_k \right) - \hbar R_{02} \left( \lambda_2 E_2'(t) + i \sum_k g_2(k) b_k \right) \right\} + h.c.
\]  

(14)

where \( E_1'(t) \) and \( E_2'(t) \) are the driven fields, with frequencies respectively close to \( \omega_1 \) and \( \omega_2 \). \( \lambda_1 \) and \( g_1 \) are coupling constants. The frequencies \( \omega_1 \) and \( \omega_2 \) are assumed quite different, so the driven field tuned with one frequency will not excite electrons to the level corresponding to the other one. The Heisenberg equations of motion of the operators are

\[
\frac{dR_{00}}{dt} = \left\{ -iR_{10} \left( \lambda_1 E_1'(t) + i \sum_k g_1(k) b_k \right) - iR_{20} \left( \lambda_2 E_2'(t) + i \sum_k g_2(k) b_k \right) \right\} + h.c.
\]  

(15)

\[
\frac{dR_{11}}{dt} = \left\{ iR_{10} \left( \lambda_1 E_1'(t) + i \sum_k g_1(k) b_k \right) \right\} + h.c.
\]  

(16)

\[
\frac{dR_{22}}{dt} = \left\{ iR_{20} \left( \lambda_2 E_2'(t) + i \sum_k g_2(k) b_k \right) \right\} + h.c.
\]  

(17)

\[
\frac{dR_{01}}{dt} = -i\omega_1 R_{01} + i(R_{00} - R_{11}) \left( \lambda_1 E_1'(t) + i \sum_k g_1(k) b_k \right)
\]  

(18)

\[
\frac{dR_{02}}{dt} = -i\omega_2 R_{02} + i(R_{00} - R_{22}) \left( \lambda_2 E_2'(t) + i \sum_k g_2(k) b_k \right)
\]  

(19)

\[
\frac{db_k}{dt} = -i\omega_k b_k + g_1^*(k) R_{01} + g_2^*(k) R_{02},
\]  

(20)

where \( h.c. \) means the hermitian conjugate.

Let us define the \( B_1(t) \) operator as

\[
B_1(t) = \sum_k g_1(k) b_k(t),
\]  

(21)

and \( B_2(t) \) as

\[
B_2(t) = \sum_k g_2(k) b_k(t).
\]  

(22)

Integrating the equation (20), we may write

\[
B_1(t) = B_{01}(t) + \sum_k |g_1(k)|^2 \int_0^t R_{01}(t')e^{i\omega_k(t'-t)} dt' + \sum_k g_1(k)g_2^*(k) \int_0^t R_{02}(t')e^{i\omega_k(t'-t)} dt',
\]  

(23)

\[
B_2(t) = B_{02}(t) + \sum_k g_2(k)g_1^*(k) \int_0^t R_{01}(t')e^{i\omega_k(t'-t)} dt' + \sum_k |g_1(k)|^2 \int_0^t R_{02}(t')e^{i\omega_k(t'-t)} dt',
\]  

(24)
where

\[ B_{01}(t) = \sum_k g_1(k) b_k(0) e^{-i\omega_k t}, \quad (25) \]

and

\[ B_{02}(t) = \sum_k g_2(k) b_k(0) e^{-i\omega_k t}. \quad (26) \]

We can now use the expressions (23) and (24) in the equations (17) - (19), to eliminate the \( b_k(t) \) variables of the thermal bath:

\[
\frac{dR_{00}}{dt} = -iR_{10} (\lambda_1 E_1'(t) + iB_1(t)) - iR_{20} (\lambda_2 E_2'(t) + iB_2(t)) + h.c. \quad (27)
\]

\[
\frac{dR_{11}}{dt} = \{iR_{10} (\lambda_1 E_1'(t) + iB_1(t))\} + h.c. \quad (28)
\]

\[
\frac{dR_{22}}{dt} = \{iR_{20} (\lambda_2 E_2'(t) + iB_2(t))\} + h.c. \quad (29)
\]

\[
\frac{dR_{01}}{dt} = -i\omega_1 R_{01} + i (R_{00} - R_{11}) (\lambda_1 E_1'(t) + iB_1(t)), \quad (30)
\]

\[
\frac{dR_{02}}{dt} = -i\omega_2 R_{02} + i (R_{00} - R_{22}) (\lambda_2 E_2'(t) + iB_2(t)). \quad (31)
\]

This is a set of non-linear differential equations that cannot be exactly solved. The non-linearity of these equations is due to the interaction between the three-level system and the radiation fields. They are the Langevin equations for the system and \( B_1(t) \) and \( B_2(t) \), by analogy with the Brownian movement, are the random fluctuating forces acting on the atom. To solve it we can try some approximation. As a first approximation, let us suppose that the interaction is sufficiently weak to be disregarded. Thus, the operators will evolve in time as

\[
R_{01}(t') = R_{01}(t) e^{-i\omega_1(t'-t)}, \quad (32)
\]

\[
R_{02}(t') = R_{02}(t) e^{-i\omega_2(t'-t)}. \quad (33)
\]

With these adiabatic approximations, the equations (23) and (24) becomes

\[
B_1(t) = B_{01}(t) + \sum_k |g_1(k)|^2 R_{01}(t) \int_0^t e^{i(\omega_k - \omega_1)(t'-t)} dt' + \sum_k g_1(k) g_2^\ast(k) R_{02}(t) \int_0^t e^{i(\omega_k - \omega_2)(t'-t)} dt', \quad (34)
\]

\[
B_2(t) = B_{02}(t) + \sum_k |g_2(k)|^2 R_{02}(t) \int_0^t e^{i(\omega_k - \omega_1)(t'-t)} dt' + \sum_k g_1(k) g_2^\ast(k) R_{01}(t) \int_0^t e^{i(\omega_k - \omega_2)(t'-t)} dt'. \quad (35)
\]

Assuming that the reservoir modes are very close, the sums in eqs. (34) and (35) may be substituted by integrations, where the number of modes in the frequency interval \( d\omega' \) is given by \( g(\omega')d\omega' \). With this consideration, and using the known result \( \int_0^\infty e^{i(\omega'-\omega)(t'-t)} dt' = -i \frac{P}{\omega' - \omega} + \pi \delta(\omega' - \omega), \) \( \int_0^\infty e^{i(\omega'-\omega)(t'-t)} dt' = -i \frac{P}{\omega' - \omega} + \pi \delta(\omega' - \omega), \)
where $\mathcal{P}$ is the principal value of the integral, we may write the expressions $B_1(t)$ and $B_2(t)$ as

\[
B_1(t) = B_{01}(t) + \left( \pi \rho(\omega_1) g_1^2(\omega_1) - i\mathcal{P} \int d\omega' \frac{\rho(\omega') g_1^2(\omega')}{\omega' - \omega_1} \right) R_{01}(t) + \left( \pi \rho(\omega_2) g_1(\omega_2) g_2^2(\omega_2) - i\mathcal{P} \int d\omega' \frac{\rho(\omega') g_1(\omega') g_2^2(\omega')}{\omega' - \omega_2} \right) R_{02}(t),
\]

\[(37)\]

\[
B_2(t) = B_{02}(t) + \left( \pi \rho(\omega_2) g_2^2(\omega_2) - i\mathcal{P} \int d\omega' \frac{\rho(\omega') g_2^2(\omega')}{\omega' - \omega_2} \right) R_{02}(t) + \left( \pi \rho(\omega_1) g_2(\omega_1) g_1^2(\omega_1) - i\mathcal{P} \int d\omega' \frac{\rho(\omega') g_2(\omega') g_1^2(\omega')}{\omega' - \omega_1} \right) R_{01}(t).
\]

\[(38)\]

Inserting now the expressions (37) and (38) in the equations of motion (16)-(19) we obtain

\[
\frac{dR_{11}}{dt} = i\lambda_1 R_{10}(t) E_1^*(t) - i\lambda_1^* R_{01}(t) E_1^*(t) - R_{10}(t) B_{01}(t) - B_{01}^* R_{01}(t) - k_1 R_{11}(t),
\]

\[(39)\]

\[
\frac{dR_{22}}{dt} = i\lambda_2 R_{20}(t) E_2^*(t) - i\lambda_2^* R_{02}(t) E_2^*(t) - R_{20}(t) B_{02}(t) - B_{02}^* R_{02}(t) - k_2 R_{22}(t),
\]

\[(40)\]

\[
\frac{dR_{01}}{dt} = -i(\omega_{01} - ik_1/2) R_{01}(t) - (k_{12}(\omega_2) - i\delta\omega_{12}) R_{02}(t) + (R_{11}(t) - R_{00}(t)) (B_{01}(t) - i\lambda_1 E_1^*(t)),
\]

\[(41)\]

\[
\frac{dR_{02}}{dt} = -i(\omega_{02} - ik_2/2) R_{02}(t) - (k_{21}(\omega_1) - i\delta\omega_{21}) R_{01}(t) + (R_{22}(t) - R_{00}(t)) (B_{02}(t) - i\lambda_2 E_2^*(t)).
\]

\[(42)\]

In the above expressions we have used

\[
\omega_{01} = \omega_1 - \mathcal{P} \int d\omega' \frac{\rho(\omega') g_1^2(\omega')}{\omega' - \omega_1},
\]

\[(43)\]

\[
\omega_{02} = \omega_2 - \mathcal{P} \int d\omega' \frac{\rho(\omega') g_2^2(\omega')}{\omega' - \omega_2},
\]

\[(44)\]

\[
k_1 = \pi \rho(\omega_1) g_1^2(\omega_1),
\]

\[(45)\]

\[
k_2 = \pi \rho(\omega_2) g_2^2(\omega_2),
\]

\[(46)\]

\[
k_{12}(\omega_2) = \pi \rho(\omega_2) g_1(\omega_2) g_2^*(\omega_2),
\]

\[(47)\]

\[
k_{21}(\omega_1) = \pi \rho(\omega_1) g_2(\omega_1) g_1^*(\omega_1),
\]

\[(48)\]

\[
\delta\omega_{12}(\omega_2) = \mathcal{P} \int d\omega' \frac{\rho(\omega') g_1(\omega') g_2^*(\omega')}{\omega' - \omega_2},
\]

\[(49)\]
\[ \delta \omega_{21} (\omega_1) = \mathcal{P} \int d\omega' \frac{\rho (\omega') g_2 (\omega') g_1^* (\omega')}{\omega' - \omega_1}. \] (50)

The integral forms of the equations \( (12) \), \( (13) \) are

\[ R_{11} (t) = R_{11} (0) e^{-ik_1 t} + i \lambda_1 \int_0^t R_{10} (t') E_1' (t') e^{ik_1 (t' - t)} dt' - \]

\[ -i \lambda_1^* \int_0^t R_{01} (t') E_1^* (t') e^{ik_1 (t' - t)} dt' - \]

\[ - \int_0^t R_{10} (t') B_{01} (t') e^{ik_1 (t' - t)} dt' - \]

\[ - \int_0^t B_{01}^* (t') R_{01} (t') e^{ik_1 (t' - t)} dt' \] (51)

\[ R_{22} (t) = R_{22} (0) e^{-ik_2 t} + i \lambda_2 \int_0^t R_{20} (t') E_2' (t') e^{ik_2 (t' - t)} dt' - \]

\[ -i \lambda_2^* \int_0^t R_{02} (t') E_2^* (t') e^{ik_2 (t' - t)} dt' - \]

\[ - \int_0^t R_{20} (t') B_{02} (t') e^{ik_2 (t' - t)} dt' - \]

\[ - \int_0^t B_{02}^* (t') R_{02} (t') e^{ik_2 (t' - t)} dt' \] (52)

\[ R_{01} (t) = R_{01} (0) e^{-i(\omega_{01} - ik_1/2) t} - \]

\[ - (k_{12} (\omega_2) - i \delta \omega_{12}) \int_0^t R_{02} (t') e^{i(\omega_{01} - ik_1/2) (t' - t)} dt' + \]

\[ + \int_0^t (R_{11} (t') - R_{00} (t')) (B_{01} (t') - i \lambda_1 E_1' (t')) e^{i(\omega_{01} - ik_1/2) (t' - t)} dt' \] (53)

\[ R_{02} (t) = R_{02} (0) e^{-i(\omega_{02} - ik_2/2) t} - \]

\[ - (k_{21} (\omega_1) - i \delta \omega_{21}) \int_0^t R_{01} (t') e^{i(\omega_{02} - ik_2/2) (t' - t)} dt' + \]

\[ + \int_0^t (R_{22} (t') - R_{00} (t')) (B_{02} (t') - i \lambda_2 E_2' (t')) e^{i(\omega_{02} - ik_2/2) (t' - t)} dt' \] (54)

Supposing that the external fields are monochromatic and plane-polarized, we write

\[ E_1' (t) = E_{01} e^{-i\omega_{11} t}, \] (55)

\[ E_2' (t) = E_{02} e^{-i\omega_{22} t}. \] (56)

Introducing this form of the external fields in eqs. \( (12) \), \( (13) \), and taking the mean values, we obtain

\[ \langle R_{11} (t) \rangle = \langle R_{11} (0) \rangle e^{-k_1 t} + i \lambda_1 E_{01} \int_0^t \langle R_{10} (t') \rangle e^{-i\omega_{11} t'} e^{k_1 (t' - t)} dt' - \]

\[ -i \lambda_1^* E_{01}^* \int_0^t \langle R_{01} (t') \rangle e^{-i\omega_{11} t'} e^{k_1 (t' - t)} dt' - \]

\[ - \int_0^t \langle R_{10} (t') \rangle B_{01} (t') e^{k_1 (t' - t)} dt' - \]

\[ - \int_0^t \langle B_{01}^* (t') \rangle R_{01} (t') e^{k_1 (t' - t)} dt', \] (57)
\[ \langle R_{22}(t) \rangle = \langle R_{22}(0) \rangle e^{-ikz t} + i \lambda_2 E_{02} \int_0^t \langle R_{20}(t') \rangle e^{-i \omega_2 t' e^{ikz(t'-t)} dt' - \]
\[ - i \lambda_2^* E_{02} \int_0^t \langle R_{02}(t') \rangle e^{-i \omega_2 t'} e^{ikz(t'-t)} dt' - \]
\[ - \int_0^t \langle R_{20}(t') B_{02}(t') \rangle e^{ikz(t'-t)} dt' - \]
\[ - \int_0^t \langle B_{02}^\dagger(t') R_{02}(t') \rangle e^{ikz(t'-t)} dt', \]  
(58)

\[ \langle R_{01}(t) \rangle = \langle R_{01}(0) \rangle e^{-i(\omega_{01} - ik_{12}/2)t} - \]
\[ - (k_{12} \omega_2 - i \delta \omega_{12}(\omega_2)) \int_0^t \langle R_{02}(t') \rangle e^{i(\omega_{01} - ik_{12}/2)(t'-t)} dt' + \]
\[ + \int_0^t \langle (2R_{11}(t') + R_{22}(t') - 1) B_{01}(t') \rangle e^{i(\omega_{01} - ik_{12}/2)(t'-t)} dt' + \]
\[ - i \lambda_1 E_{01} \int_0^t \langle (2R_{11}(t') + R_{22}(t') - 1) \rangle e^{-i \omega_{11} t' e^{i(\omega_{01} - ik_{12}/2)(t'-t)} dt', \]  
(59)

\[ \langle R_{02}(t) \rangle = \langle R_{02}(0) \rangle e^{-i(\omega_{02} - ik_{22}/2)t} - \]
\[ - (k_{21} \omega_1 - i \delta \omega_{21}(\omega_1)) \int_0^t \langle R_{01}(t') \rangle e^{i(\omega_{02} - ik_{22}/2)(t'-t)} dt' + \]
\[ + \int_0^t \langle (2R_{22}(t') + R_{11}(t') - 1) B_{02}(t') \rangle e^{i(\omega_{02} - ik_{22}/2)(t'-t)} dt' + \]
\[ - i \lambda_2 E_{02} \int_0^t \langle (2R_{22}(t') + R_{11}(t') - 1) \rangle e^{-i \omega_{22} t' e^{i(\omega_{02} - ik_{22}/2)(t'-t)} dt', \]  
(60)

III. SPONTANEOUS EMISSION AND RESONANT FLUORESCENCE

We follow the procedure outlined in ref. [16] to obtain the solution of the system of integral equations (57) and (60). We use an iterative process: the expressions \( R_{11}, R_{22}, R_{01} \) and \( R_{02} \) are linear in the initial values of the operators and the coefficients of the operators are expressed in terms of powers of \( B_{01}(t) \) and \( B_{02}(t) \). Supposing that the radiation field is in a stationary state and has a Gaussian distribution, the higher-order correlation function can be expressed in terms of the second-order one [16],

\[ \langle B_{0j}^\dagger(t_1) \cdots B_{0j}^\dagger(t_n) B_{0i}(t'_1) \cdots B_{0i}(t'_n) \rangle = \sum \prod_{\lfloor j \rfloor = 1}^n \langle B_{0j}^\dagger(t_j) B_{0j}(t'_j) \rangle. \]  
(61)

The sum must be taken over all permutations \( j = 1, \ldots, n \). As the expressions of the operators depend on the radiations field configuration in earlier times, the only terms that contribute to the series expansion of equations (57) and (58) are those for which the field operators are time ordered,

\[ \min(t_i, t'_i) \geq \max(t_{i+1}, t'_{i+1}), \quad i = 1, 2, \ldots, n. \]

The only term in the sum of the equation (61) that contributes is

\[ \prod_{i=1}^n \langle B_{0j}^\dagger(t_j) B_{0j}(t'_j) \rangle. \]

Thus, terms that contain a different number of factors \( B_{0j}^\dagger(t) \) and \( B_{0j}(t) \) do not contribute to the mean value of the operator. A further simplification can be obtained if we suppose that the radiation field spectrum is dense, flat and
broad. In this case, the correlation functions \( \langle B_{10}^\dagger(t'')B_{10}(t'_1) \rangle \) and \( \langle B_{20}^\dagger(t'')B_{20}(t'_1) \rangle \) can be calculated, by using equations (64) and (65),

\[
\langle B_{10}^\dagger(t'')B_{10}(t'_1) \rangle = \sum_k |g_1(k)|^2 N(\omega) e^{i(\omega_k - \omega_{01})(t'' - t')},
\]

\[
\langle B_{20}^\dagger(t'')B_{20}(t'_1) \rangle = \sum_k |g_2(k)|^2 N(\omega) e^{i(\omega_k - \omega_{02})(t'' - t')},
\]

where

\[
N(\omega) = \langle b_k^\dagger(0)b_k(0) \rangle
\]

is the initial mean number of photons at the \( k \) mode of the radiation field. Substituting the sums in equations (64) and (65) by integrations, we have

\[
\langle B_{10}^\dagger(t'')B_{10}(t'_1) \rangle = \int_0^\infty d\omega \rho(\omega)g_1^2(\omega) N(\omega) e^{i(\omega_k - \omega_{01})(t'' - t')},
\]

\[
\langle B_{20}^\dagger(t'')B_{20}(t'_1) \rangle = \int_0^\infty d\omega \rho(\omega)g_2^2(\omega) N(\omega) e^{i(\omega_k - \omega_{02})(t'' - t')},
\]

and using the properties of the Dirac delta-function and definitions (43) and (46), we obtain

\[
\langle B_{10}^\dagger(t'')B_{10}(t'_1) \rangle = \frac{k_1}{2}N(\omega_1)\delta(t'' - t'),
\]

\[
\langle B_{20}^\dagger(t'')B_{20}(t'_1) \rangle = \frac{k_2}{2}N(\omega_2)\delta(t'' - t').
\]

Now we can calculate the correlations that appear in eqs. (57) and (58). Using relations (67), (68) and (69), in expressions (51) and (52), we obtain the expressions of \( \langle R_{10}(t')B_{01}(t'_1) \rangle \) and \( \langle R_{20}(t')B_{02}(t'_1) \rangle \):

\[
\langle R_{10}(t')B_{01}(t'_1) \rangle = k_1 N(\omega_1)\langle R_{11}(t') \rangle + \frac{k_1}{2}((R_{22}(t')) - 1),
\]

\[
\langle R_{20}(t')B_{02}(t'_1) \rangle = k_2 N(\omega_2)\langle R_{22}(t') \rangle + \frac{k_2}{2}((R_{11}(t')) - 1).
\]

Expressions (69) and (70) are real, and then,

\[
\langle R_{10}(t')B_{01}(t'_1) \rangle = \langle B_{01}^\dagger(t')R_{01}(t'_1) \rangle,
\]

\[
\langle R_{20}(t')B_{02}(t'_1) \rangle = \langle B_{02}^\dagger(t')R_{02}(t'_1) \rangle.
\]

Inserting eqs. (69) and the complex conjugate (54) in eq. (57), we obtain

\[
\langle R_{11}(t) \rangle = \langle R_{11}(0) \rangle e^{-k_1t} + \Gamma_{21}(e^{-z_1t} - e^{-k_1t}) + \Gamma_{21}(e^{-z_1t} - e^{-k_1t}) + \Gamma_{21}(1 - e^{-k_1t}) - 2k_1 N(\omega_1) \int_0^t \langle R_{11}(t') \rangle e^{k_1(t' - t)} dt' - \frac{k_1}{2} \int_0^t \langle R_{22}(t') \rangle e^{k_1(t' - t)} dt' - \frac{\Omega_1^2}{2} \int_0^t dt' e^{-k_1t} e^{-z_1t'} \left\{ \int_0^{t'} \langle R_{11}(t'') \rangle e^{z_1t''} dt'' \right\} - \frac{\Omega_1^2}{2} \int_0^t dt' e^{-k_1t} e^{-z_1t'} \left\{ \int_0^{t'} \langle R_{22}(t'') \rangle e^{z_1t''} dt'' \right\} - \frac{\Omega_1^2}{4} \int_0^t dt' e^{-k_1t} e^{-z_1t'} \left\{ \int_0^{t'} \langle R_{22}(t'') \rangle e^{z_1t''} dt'' \right\} - \frac{\Omega_1^2}{4} \int_0^t dt' e^{-k_1t} e^{-z_1t'} \left\{ \int_0^{t'} \langle R_{22}(t'') \rangle e^{z_1t''} dt'' \right\}.
\]
Inserting the equation (70) and the complex conjugate of eq. (60) in eq. (58), we have

\[ (R_{22}(t)) = \langle R_{22}(0) \rangle e^{-k_2 t} + \Gamma_{22} (e^{-z_2 t} - e^{-k_2 t}) + \Gamma_{22} (e^{-z_2 t} - e^{-k_2 t}) + \Gamma_{12} (1 - e^{-k_2 t}) - 2k_2 N(\omega_2) \int_0^t \langle R_{22}(t') \rangle e^{k_2(t'-t)} dt' - k_2 N(\omega_2) \int_0^t \langle R_{11}(t') \rangle e^{k_2(t'-t)} dt' - \Omega_1^2 2 \int_0^t dt' e^{-k_2 t} e^{z_2 t'} \left\{ \int_0^{t'} \langle R_{22}(t'') \rangle e^{z_2 t''} dt'' \right\} - \Omega_2^2 2 \int_0^t dt' e^{-k_2 t} e^{z_2 t'} \left\{ \int_0^{t'} \langle R_{11}(t'') \rangle e^{z_2 t''} dt'' \right\} - \Omega_2^2 4 \int_0^t dt' e^{-k_2 t} e^{z_2 t'} \left\{ \int_0^{t'} \langle R_{11}(t'') \rangle e^{z_2 t''} dt'' \right\} - \Omega_2^2 4 \int_0^t dt' e^{-k_2 t} e^{z_2 t'} \left\{ \int_0^{t'} \langle R_{11}(t'') \rangle e^{z_2 t''} dt'' \right\} . \]

(74)

In expressions (73) and (74) we use the below definitions

\[ \Gamma_{11} = N(\omega_1) + \frac{\Omega_1^2}{4 \|z_1\|^2}, \]

(75)

\[ \Gamma_{12} = N(\omega_2) + \frac{\Omega_2^2}{4 \|z_2\|^2}. \]

(76)

\[ \Gamma_{21} = \frac{i \lambda_1 E_{01} \langle R_{10}(0) \rangle}{z_1^*} - \frac{\Omega_1^2}{4 \|z_1\|^2}, \]

(77)

\[ \Gamma_{22} = \frac{i \lambda_2 E_{02} \langle R_{20}(0) \rangle}{z_2^*} - \frac{\Omega_2^2}{4 \|z_2\|^2}, \]

(78)

\[ \Omega_1 = 2 \|\lambda_1\| E_{01}, \]

(79)

\[ \Omega_2 = 2 \|\lambda_2\| E_{02}, \]

(80)

\[ z_1 = \frac{k_1}{2} + i(\omega_{11} - \omega_{01}), \]

(81)

\[ z_2 = \frac{k_2}{2} + i(\omega_{21} - \omega_{02}). \]

(82)

When the external fields vanishes, the equations (73) and (74) simplifies

\[ \langle R_{11}(t) \rangle = \langle R_{11}(0) \rangle e^{-k_1 t} + N(\omega_1) (1 - e^{-k_1 t}) - 2k_1 N(\omega_1) \int_0^t \langle R_{11}(t') \rangle e^{k_1(t'-t)} dt' - k_1 N(\omega_1) \int_0^t \langle R_{22}(t') \rangle e^{k_1(t'-t)} dt', \]

(83)
\[
\langle R_{22}(t) \rangle = \langle R_{22}(0) \rangle e^{-k_2 t} + N(\omega_2) \left( 1 - e^{-k_2 t} \right) - \\
-2k_2 N(\omega_2) \int_0^t \langle R_{22}(t') \rangle e^{k_2(t'-t)} dt' - \\
-k_2 N(\omega_2) \int_0^t \langle R_{11}(t') \rangle e^{k_2(t'-t)} dt'.
\]

Integrating these equations, we obtain

\[
\langle R_{11}(t) \rangle = \langle R_{11}(0) \rangle e^{-\overline{k}_1 t} + \frac{k_1}{k_1 - k_2} N(\omega_1) \left( 1 - e^{-\overline{k}_1 t} \right) - \\
-\frac{k_1}{k_1 - k_2} N(\omega_1)\langle R(0) \rangle (2e^{-\overline{k}_1 t} - e^{-k_1 t}),
\]

\[
\langle R_{22}(t) \rangle = \langle R_{22}(0) \rangle e^{-\overline{k}_2 t} + \frac{k_2}{k_2 - k_1} N(\omega_2) \left( 1 - e^{-\overline{k}_2 t} \right) - \\
-\frac{k_2}{k_2 - k_1} N(\omega_2)\langle R(0) \rangle (2e^{-\overline{k}_2 t} - e^{-k_2 t}),
\]

where

\[
\overline{k}_1 = k_1 (1 + 2N(\omega_1)),
\]

\[
\overline{k}_2 = k_2 (1 + 2N(\omega_2)).
\]

If the initial state of the system is given by

\[
\psi = |0\rangle a_0 + |1\rangle a_1 + |2\rangle a_2
\]

with \(|a_0|^2 + |a_1|^2 + |a_2|^2 = 1\), then

\[
\langle R_{11}(t) \rangle = |a_1|^2 e^{-\overline{k}_1 t} + \frac{k_1}{k_1 - k_2} N(\omega_1) \left( 1 - e^{-\overline{k}_1 t} \right) - \\
-\frac{k_1}{k_1 - k_2} N(\omega_1)|a_2|^2 (2e^{-\overline{k}_2 t} - e^{-k_1 t}),
\]

\[
\langle R_{22}(t) \rangle = |a_2|^2 e^{-\overline{k}_2 t} + \frac{k_2}{k_2 - k_1} N(\omega_2) \left( 1 - e^{-\overline{k}_2 t} \right) - \\
-\frac{k_2}{k_2 - k_1} N(\omega_2)|a_1|^2 (2e^{-\overline{k}_1 t} - e^{-k_2 t}).
\]

If the temperature of the bath is zero, and the system is initially in one of the excited states, say, the state \(|1\rangle\), then \(k_1 = \overline{k}_1, a_0 = a_2 = 0\) and

\[
\langle R_{11}(t) \rangle = |a_1|^2 e^{-k_1 t}
\]

which is the expected spontaneous decay of the excited state with lifetime \(1/k_1\). If the temperature is different of zero, the thermal fluctuations enhance the coupling between the system an the field and the decay rate is increased, as we note by the equation \((80)\). To times \(t \gg 1/\overline{k}_1\), the system approaches the saturation regime and then

\[
\langle R_{11}(t = \infty) \rangle = \frac{N(\omega_1)}{1 + 2N(\omega_1)}.
\]
Assuming that the radiation field is in thermal equilibrium with the cavity, we have

\[ N(\omega_1) = \frac{1}{e^{\hbar \omega_1/k_B T} - 1} \]  

(94)

and

\[ N(\omega_2) = \frac{1}{e^{\hbar \omega_2/k_B T} - 1} \]  

(95)

and the equations (31) and (33) becomes

\[ \langle R_{11}(t = \infty) \rangle = \frac{1}{e^{\hbar \omega_1/k_B T} + 1} \]  

(96)

\[ \langle R_{22}(t = \infty) \rangle = \frac{1}{e^{\hbar \omega_2/k_B T} + 1} \]  

(97)

which is the usual Fermi-Dirac distribution.

With the procedure discussed in detail in Appendix, we obtain the solution of integral equations (73) and (74):

\[ \langle R_{11}(t) \rangle = \langle R_{11}(\infty) \rangle + \langle R_{11}(0) \rangle + \sum_{i,j,k=0}^{3} \frac{(f_i + k_1/2)^2 + \Delta \omega_1^2}{(f_i - f_j)(f_i - f_k)} e^{f_i t} + \]

\[ + k_1 N(\omega_1) \sum_{i,j,k=0}^{3} \frac{(f_i + k_1/2)^2 + \Delta \omega_1^2}{(f_i - f_j)(f_i - f_k)} e^{f_i t} + \]

\[ + \frac{i \Omega_1}{2} \sum_{i,j,k=0}^{3} \frac{(f_i + z_1^i)}{(f_i - f_j)(f_i - f_k)} e^{f_i t} - \]

\[ - \frac{i \Omega_1}{2} \sum_{i,j,k=0}^{3} \frac{(f_i + z_1^i)}{(f_i - f_j)(f_i - f_k)} e^{f_i t} + \]

\[ + \frac{\Omega_2^2}{2} \sum_{i,j,k=0}^{3} \frac{(f_i + k_1/2)}{(f_i - f_j)(f_i - f_k)} e^{f_i t} - \]

\[ - k_1 N(\omega_1) \langle R_{22}(0) \rangle + \sum_{i,j,k=0}^{3} \frac{(h_i + k_1/2)^2 + \Delta \omega_1^2}{f(h_i)(h_i - h_j)(h_i - h_k)} e^{h_i t} - \]

\[ - k_1 N(\omega_1) \langle R_{22}(0) \rangle - \sum_{i,j,k=0}^{3} \frac{(f_i + k_2/2)^2 + \Delta \omega_2^2}{(f_i - f_j)(f_i - f_k)} h(f_i) e^{f_i t} - \]

\[ - \frac{\Omega_2^2}{2} \langle R_{22}(0) \rangle - \sum_{i,j,k=0}^{3} \frac{(f_i + k_2/2)^2 + \Delta \omega_2^2}{(f_i - f_j)(f_i - f_k)} h(f_i) e^{h_i t} - \]

\[ - \Omega_1^2 \langle R_{22}(0) \rangle - \sum_{i,j,k=0}^{3} \frac{(h_i + k_1/2)^2 + \Delta \omega_1^2}{f(h_i)(h_i - h_j)(h_i - h_k)} e^{h_i t}. \]  

(98)
and

\[ 
\langle R_{22}(t) \rangle = \langle R_{22}(\infty) \rangle + \langle R_{22}(0) \rangle \sum_{i,j,k=0}^{3} \frac{\left| (h_i + k_2/2)^2 + \Delta \omega_2^2 \right|}{(h_i - h_j)(h_i - h_k)} e^{hi,t} + \\
+k_2 N(\omega_2) \sum_{i,j,k=0}^{3} \frac{\left| (h_i + k_2/2)^2 + \Delta \omega_2^2 \right|}{(h_i - h_j)(h_i - h_k)} e^{hi,t} + \\
+i \Omega_2^2 \left( R_{20}(0) \right) e^{i\phi} \sum_{i,j,k=0}^{3} \frac{\left| (h_i + k_2/2)^2 + \Delta \omega_2^2 \right|}{(h_i - h_j)(h_i - h_k)} e^{hi,t} - \\
-i \Omega_2^2 \left( R_{02}(0) \right) e^{-i\phi} \sum_{i,j,k=0}^{3} \frac{\left| (h_i + k_2/2)^2 + \Delta \omega_2^2 \right|}{(h_i - h_j)(h_i - h_k)} e^{hi,t} + \\
+ \frac{\Omega_2^2}{2} \sum_{i,j,k=0}^{3} \frac{(h_i + k_2/2)}{(h_i - h_j)(h_i - h_k)} e^{hi,t} - \\
-k_2 N(\omega_2) \langle R_{11}(0) \rangle \sum_{i,j,k=0}^{3} \frac{\left| (f_i + k_1/2)^2 + \Delta \omega_1^2 \right|}{h(f_i)(f_i - f_j)(f_i - f_k)} e^{hi,t} - \\
-k_2 N(\omega_2) \langle R_{11}(0) \rangle \sum_{i,j,k=0}^{3} \frac{\left| (f_i + k_1/2)^2 + \Delta \omega_1^2 \right|}{h(f_i)(f_i - f_j)(f_i - f_k)} e^{hi,t} - \\
-\Omega_2^2 \langle R_{11}(0) \rangle \sum_{i,j,k=0}^{3} \frac{(f_i + k_1/2)^2}{h(f_i)(f_i - h_j)(f_i - h_k)} e^{hi,t} - \\
-\Omega_2^2 \langle R_{11}(0) \rangle \sum_{i,j,k=0}^{3} \frac{(f_i + k_1/2)^2}{h(f_i)(f_i - h_j)(f_i - h_k)} e^{hi,t}. \tag{99} 
\]

In the above equations, we have used \( \Delta \omega_1 = \omega_{1l} - \omega_1, \Delta \omega_2 = \omega_{2l} - \omega_2 \) and

\[ 
\langle R_{11}(\infty) \rangle = \frac{(k_1^2/4 + \Delta \omega_1^2) N(\omega_1) + \Omega_1^2/4}{(k_1^2/4 + \Delta \omega_1^2)(1 + 2N(\omega_1)) + \Omega_1^2/2}. \tag{100} 
\]

\[ 
\langle R_{22}(\infty) \rangle = \frac{(k_2^2/4 + \Delta \omega_2^2) N(\omega_2) + \Omega_2^2/4}{(k_2^2/4 + \Delta \omega_2^2)(1 + 2N(\omega_2)) + \Omega_2^2/2}. \tag{101} 
\]

**IV. DISCUSSION OF THE RESULTS**

In the equations (100) and (101), the only terms that remain when \( t \to \infty \) are \( \langle R_{11}(\infty) \rangle \) and \( \langle R_{22}(\infty) \rangle \). The Fig. 3 shows the behavior of \( \langle R_{11}(\infty) \rangle \) as a function of the detuning \( \Delta \omega_1 \) and of the occupation number \( N(\omega_1) \). The system rapidly tends to the maximum intensity of scattered light for occupation numbers \( N(\omega_1) \neq 0 \), and the peak of resonance with the driving field becomes less effective to increasing number of occupation.

We see by an exam of the equations (100) and (101) that one excited level is affected by the other only in transient terms that depend of the initial values of \( \langle R_{11}(t = 0) \rangle, \langle R_{22}(t = 0) \rangle \). Thus, within the second order approximation of the coupling constants we have used, the upper levels are almost independents, and the three-level system behaves as two two-level systems. Some numerical values given by equation (98) for certain combinations of the parameters \( \lambda \)
and $\theta$ for the initial state $\langle R_{11}(0) \rangle = \langle R_{22}(0) \rangle = 0$ and $\langle R_{01}(0) \rangle = \langle R_{10}(0) \rangle = 0$, are given by Figs. 2 - 3. With these initial values the equation (103) becomes

$$\langle R_{11}(t) \rangle = \langle R_{11}(\infty) \rangle + k_1 N(\omega_1) \sum_{i,j,k=0}^{3} \frac{(f_i + k_1/2)^2 + \Delta \omega_1^2}{f_i (f_i - f_j) (f_i - f_k)} e^{f_i t} +$$

$$+ \frac{\Omega_1^2}{2} \sum_{i,j,k=0}^{3} \frac{(f_i + k_1/2)}{f_i (f_i - f_j) (f_i - f_k)} e^{f_i t}$$

(102)

The Fig. 2 shows the effect of the $\lambda_{1,2}$ parameters, that are proportional to the intensities of the driving fields, over the fluorescent light intensities $\langle R_{11}(t) \rangle$ and $\langle R_{22}(t) \rangle$ for Bose occupation numbers $N(w_1) = N(w_2) = 0$ and $\theta_1 = \theta_2 = 1$.

In the Fig. 3 the light intensities $\langle R_{11}(t) \rangle$ and $\langle R_{22}(t) \rangle$ are showed to the same parameters of the Fig. 2, but now with $\theta_1 = 10$ and $\theta_2 = 3$. It can be seen that the evolution in time of the fluorescent light intensity for $N(w_1) = 0$ is increasing oscillatory as the detuning increases. This feature is attenuated for $N(\omega_1) \neq 0$, due to saturation caused by the combined effect of the thermal bath and the driving fields.

The Fig. 4 and Fig. 5 shows that a increase in the Bose occupation numbers $\langle R_{22}(0) \rangle$ for strong fields depends mainly of the intensity of the driven field $\langle R_{11}(0) \rangle$.

By the preceding examples, we see that the presence of the thermal bath modifies strongly the time evolution of the fluorescent light intensity. Thus the spectral density and the intensity correlation of the fluorescent light must too be affected. We will present these calculations in a further paper.

V. APPENDIX

Equations (103) and (104) form a set of integral equations that can be solved by Laplace transformation,

$$\langle R_{11}(s) \rangle = \frac{\langle R_{11}(0) \rangle (s + z_1) (s + z_1^*)}{f(s)} + \Gamma_{21} \frac{k_1 (k_1 - z_1) (s + z_1^*)}{f(s)} +$$

$$\frac{\Gamma_{21}^* (s + z_1) (k_1 - z_1^*)}{f(s)} + k_1 \frac{\Gamma_{11} (s + z_1) (s + z_1^*)}{s f(s)} -$$

$$- \frac{\langle R_{22}(s) \rangle}{f(s)} \left[ k_1 N(\omega_1) (s + z_1) (s + z_1^*) + \Omega_1^2 (s + k_1/2) \right].$$

(103)

$$\langle R_{22}(s) \rangle = \frac{\langle R_{22}(0) \rangle (s + z_2) (s + z_2^*)}{h(s)} + \frac{\Gamma_{22} (k_2 - z_2) (s + z_2^*)}{h(s)} +$$

$$\frac{\Gamma_{22}^* (s + z_2) (k_2 - z_2^*)}{h(s)} + k_2 \frac{\Gamma_{12} (s + z_2) (s + z_2^*)}{s h(s)} -$$

$$- \frac{\langle R_{11}(s) \rangle}{h(s)} \left[ k_2 N(\omega_2) (s + z_2) (s + z_2^*) + \Omega_2^2 (s + k_2/2) \right],$$

(104)

In equations (103) and (104) we have used

$$f(s) = (s + k_1) (s + z_1) (s + z_1^*) + 2 k_1 N(\omega_1) (s + z_1) (s + z_1^*) + \Omega_1^2 (s + k_1/2),$$

(105)

$$h(s) = (s + k_2) (s + z_2) (s + z_2^*) + 2 k_2 N(\omega_2) (s + z_2) (s + z_2^*) + \Omega_2^2 (s + k_2/2)$$

(106)
Substituting (104) in (103), and retaining the terms until second order in the coupling constants, we have

\[
\langle R_{11}(s) \rangle = \frac{\langle R_{11}(0) \rangle (s + z_1) (s + z_1^*)}{f(s)} + \frac{\Gamma_{21} z_1 (s + z_1^*)}{f(s)} + \frac{\Gamma_{21}^* z_1 (s + z_1)}{f(s)} + \frac{k_1 \Gamma_{11} (s + z_1) (s + z_1^*)}{sf(s)} - \frac{\langle R_{22}(0) \rangle (s + z_2) (s + z_2^*)}{f(s) h(s)} [k_1 N(\omega_1) (s + z_1) (s + z_1^*) + \Omega_1^2 (s + k_1/2)].
\]

Taking now the inverse Laplace transform of the above equation,

\[
\langle R_{11}(t) \rangle = \frac{1}{2\pi i} \int_{-i\infty+\alpha}^{i\infty+\alpha} (R_{11}(s)) e^{st} ds, \quad t > 0
\]

where \(\alpha\) is chosen in such a way that all singularities of the integrand fall in the left of the line \(\text{Re} \ s = \alpha\) in the complex plane. Using the residue theorem, we obtain the wanted integral over the line \(\text{Re} \ s = \alpha\). Assuming that we may write

\[
f(s) = (s - f_1) (s - f_2) (s - f_3),
\]

\[
h(s) = (s - h_1) (s - h_2) (s - h_3),
\]

where \(f_1, f_2, f_3\) are the three roots of the cubic equation

\[
s^3 + [1 + N(w_1)] 2k_1 s^2 + \left| [5 + 8N(w_1)] + \theta_1^2 + \lambda_1 \right| \beta_1^2 s^3 + \left| [2 [1 + 2N(w_1)] + 2 [1 + 2N(w_1)] \beta_1^2 + \lambda_1] \beta_1^3 = 0,
\]

and \(h_1, h_2, h_3\) are the three roots of the cubic equation

\[
s^3 + [1 + N(w_2)] 2k_2 s^2 + \left| [5 + 8N(w_2)] + \theta_2^2 + \lambda_2 \right| \beta_2^2 s^3 + \left| [2 [1 + 2N(w_2)] + 2 [1 + 2N(w_2)] \beta_2^2 + \lambda_2] \beta_2^3 = 0,
\]
where \( \theta_i = 2\Delta \omega_i/k_i \), \( \lambda_i = \Omega_i^2/\beta_i^2 \), \( \beta_i = k_i/2 \), with \( i = 1, 2 \). Then we have

\[
\langle R_{11}(t) \rangle = \langle R_{11}(\infty) \rangle + \langle R_{11}(0) \rangle \sum_{i,j,k=0}^{3} \frac{(f_i + k_1/2)^2 + \Delta \omega_i^2}{f_i(f_i - f_j)(f_i - f_k)} e^{f_i t} + \\
+kN(\omega_1) \sum_{i,j,k=0}^{3} \frac{(f_i + k_1/2)^2 + \Delta \omega_i^2}{f_i(f_i - f_j)(f_i - f_k)} e^{f_i t} + \\
+i\Omega_1 \langle R_{10}(0) \rangle e^{-i\phi} \sum_{i,j,k=0}^{3} \frac{(f_i + z_1^2)}{(f_i - f_j)(f_i - f_k)} e^{f_i t} - \\
-i\Omega_1 \langle R_{01}(0) \rangle e^{-i\phi} \sum_{i,j,k=0}^{3} \frac{(f_i + z_1)}{(f_i - f_j)(f_i - f_k)} e^{f_i t} + \\
+\frac{\Omega_1^2}{2} \sum_{i,j,k=0}^{3} \frac{(f_i + k_3/2)}{f_i(f_i - f_j)(f_i - f_k)} e^{f_i t} - \\
-k_1 N(\omega_1) \langle R_{22}(0) \rangle \sum_{i,j,k=0}^{3} \frac{(f_i + k_1/2)^2 + \Delta \omega_i^2}{f_i - f_j)(f_i - f_k) h(f_i) e^{h_i t} - \\
-k_1 N(\omega_1) \langle R_{22}(0) \rangle \sum_{i,j,k=0}^{3} \frac{(h_i + k_1/2)^2 + \Delta \omega_i^2}{f(h_i) (h_i - h_j)(h_i - h_k) e^{h_i t} - \\
-\Omega_1^2 \langle R_{22}(0) \rangle \sum_{i,j,k=0}^{3} \frac{(f_i + k_3/2)^2 + \Delta \omega_i^2}{f(h_i) (f_i - f_j)(f_i - f_k) e^{f_i t} - \\
-\Omega_1^2 \langle R_{22}(0) \rangle \sum_{i,j,k=0}^{3} \frac{(h_i + k_1/2)^2 + \Delta \omega_i^2}{f(h_i) (f_i - f_j)(f_i - f_k) e^{h_i t}.}
\]

With a similar procedure we obtain a solution to \( \langle R_{22}(t) \rangle \).

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1. $(R_{11}(\infty))$ dependence with the occupation number $N(\omega_1)$, the detuning $\Delta \omega_1$ and parameters $\Omega_1 = 0.5$ and $k_1 = 1$ ................................................................. 18
2. Time development of the fluorescent light intensities $(R_{11}(t))$ (points) and $(R_{22}(t))$ (solid line) for Bose occupation numbers $N(w_1) = N(w_2) = 0$ and $\lambda_1 = 0.4, \lambda_2 = 0.3, \theta_1 = \theta_2 = 1$ ........................................ 19
3. Time development of the fluorescent light intensities $(R_{11}(t))$ (points) and $(R_{22}(t))$ (solid line) for Bose occupation numbers $N(w_1) = N(w_2) = 0$ and $\lambda_1 = 0.4, \lambda_2 = 0.3, \theta_1 = 10$ and $\theta_2 = 3$ .............. 20
4. Time development of the fluorescent light intensities $(R_{11}(t))$ (points) and $(R_{22}(t))$ (solid line) for Bose occupation numbers $N(w_1) = N(w_2) = 0.01$ and $\lambda_1 = 0.4, \lambda_2 = 0.3, \theta_1 = 10$ and $\theta_2 = 3$ .................. 21
5. Time development of the fluorescent light intensities $(R_{11}(t))$ (points) and $(R_{22}(t))$ (solid line) for Bose occupation numbers $N(w_1) = N(w_2) = 0.1$ and $\lambda_1 = 0.4, \lambda_2 = 0.3, \theta_1 = 10$ and $\theta_2 = 3$ ............... 22
6. Time development of the fluorescent light intensity $(R_{11}(t))$ for $\lambda_1 = 0.4, \theta_1 = 10$, and increasing occupation numbers $N(\omega_1)$ ............................................................... 23
7. Time development of the fluorescent light intensity $(R_{11}(t))$ for $\lambda_1 = 10, \theta_1 = 10$, and increasing occupation numbers $N(\omega_1)$ ............................................................... 24
FIG. 1: $\langle R_{11}(\infty) \rangle$ dependence with the occupation number $N(\omega_1)$, the detuning $\Delta \omega_1$ and parameters $\Omega_1 = 0.5$ and $k_1 = 1$. 
FIG. 2: Time development of the fluorescent light intensities $\langle R_{11}(t) \rangle$ (points) and $\langle R_{22}(t) \rangle$ (solid line) for Bose occupation numbers $N(w_1) = N(w_2) = 0$ and $\lambda_1 = 0.4$, $\lambda_2 = 0.3$, $\theta_1 = \theta_2 = 1$. 
FIG. 3: Time development of the fluorescent light intensities $\langle R_{11}(t) \rangle$ (points) and $\langle R_{22}(t) \rangle$ (solid line) for Bose occupation numbers $N(w_1) = N(w_2) = 0$ and $\lambda_1 = 0.4$, $\lambda_2 = 0.3$, $\theta_1 = 10$ and $\theta_2 = 3$. 
FIG. 4: Time development of the fluorescent light intensities $\langle R_{11}(t) \rangle$ (points) and $\langle R_{22}(t) \rangle$ (solid line) for Bose occupation numbers $N(w_1) = N(w_2) = 0.01$ and $\lambda_1 = 0.4$, $\lambda_2 = 0.3$, $\theta_1 = 10$ and $\theta_2 = 3$. 
FIG. 5: Time development of the fluorescent light intensities $\langle R_{11}(t) \rangle$ (points) and $\langle R_{22}(t) \rangle$ (solid line) for Bose occupation numbers $N(w_1) = N(w_2) = 0.1$ and $\lambda_1 = 0.4$, $\lambda_2 = 0.3$, $\theta_1 = 10$ and $\theta_2 = 3$. 

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FIG. 6: Time development of the fluorescent light intensity $\langle R_{11}(t) \rangle$ for $\lambda_1 = 0.4$, $\theta_1 = 10$, and increasing occupation numbers $N(\omega_1)$.
FIG. 7: Time development of the fluorescent light intensity \( \langle R_{11}(t) \rangle \) for \( \lambda_1 = 10, \theta_1 = 10 \), and increasing occupation numbers \( N(\omega_1) \).