Spatial averaging and a non-Gaussianity.

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Abstract The spatial averaging used for the splitting of the local scale factor on the homogeneous background and small inhomogeneous perturbation leads to a non-local relationship between locally and globally defined comoving curvature perturbations. We study this relationship within a quasi-homogeneous, nearly spatially flat domain of the Universe. It is shown that, on scales larger than the size of the observed patch, the Fourier components of the locally defined comoving curvature perturbation are suppressed. We have also shown that the statistical properties of local and global comoving curvature perturbations are coincide on a small scale. Several examples are discussed in detail.

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1 Introduction.

The consideration of cosmological inhomogeneities within non-linear perturbation theory has attracted much attention in recent years. Investigation of non-Gaussian features of cosmological perturbations gives new opportunities to test inflationary models, theoretical models of thermal reheating of the Universe. It also provides a possibility to get new cosmological constraints on a number of high energy physics theories.

Currently, the non-linear perturbation theory is rapidly developing. The gradient expansion approach \cite{1,2,3} led to the generalization of the $\delta N$ formalism \cite{4,5,6} to the non-linear case \cite{7,8} that gives a handy tool for an estimation and calculation of non-Gaussianities. The quantum field theory methods are increasingly penetrating into the cosmological perturbation theory. Non-perturbative approaches are under development. For example, there are such techniques as renormalized cosmological perturbation theory \cite{9,10} and the time renormalization group method \cite{11}. However, there are still many unresolved theoretical issues.

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The non-linear generalization of the curvature perturbation implies the need to take into account the finiteness of the observable patch of the Universe in determining the local comoving curvature perturbation. This makes itself evident in the fact that the observer identifies the perturbation as deviation from the spatial average over the observable patch. The consequences of such definition probably related to the problem of infrared divergences in the calculation of n-point correlation functions by perturbation theory [12] are still not well understood. The potential consequences have been considered recently [13, 14] in the context of a landscape picture of the Universe. In this picture, observed statistical properties of perturbations depend on the position of the observable region. A landscape Universe arises in eternal inflation [15, 16], and some of its aspects were discussed in connection with curvaton scenario [17, 18]. However, it is worth noting that several questionable approximations have been applied in the papers [13, 14].

This paper is organized as follow. In Section 2 we consider a quasi-homogeneous domain of the Universe and its observable patch. We introduce globally and locally defined comoving curvature perturbations ($\zeta(x^{\mu})$ and $\zeta_S(x^{\mu})$, correspondingly) and briefly describe the relationship between them. The exact relations between the Fourier components of $\zeta$ and $\zeta_S$ are considered in Section 3. We show that, on a scale much smaller than the size of an observable patch, the values of globally and locally defined comoving curvature perturbations are coincided with good accuracy. Section 4 serves to clarify some issues of application obtained in Section 3 equations. In Section 5 we discuss the obtained discrepancy with some claims of papers [13, 14]. We conclude the paper in Section 6.

2 The comoving curvature perturbation.

Let’s consider a domain of the Universe, which can be described by nearly homogeneous and spatially flat metric. The comoving curvature perturbation $\zeta(x)$ is defined by the local scale factor [7]

$$a(x, t) = a(t)e^{\zeta(x)}.$$  \hspace{1cm} (1)

Equivalently, one can write

$$\zeta(x) = \ln \left( \frac{\tilde{a}}{\bar{a}} \right).$$  \hspace{1cm} (2)

The ambiguity of this definition is eliminated by condition

$$\langle \zeta(x) \rangle_L = \frac{1}{V_L} \int_{V_L} \zeta(x) d^3 x = 0,$$  \hspace{1cm} (3)

where $V_L$ is the comoving three-dimensional volume of the domain.

From equations (1) and (3), it follows that

$$a(t) = e^{\frac{1}{V_L} \int_{V_L} \ln \tilde{a} d^3 x}$$  \hspace{1cm} (4)

and

$$\zeta(x) = \ln \tilde{a} - \frac{1}{V_L} \int_{V_L} \ln \tilde{a} d^3 x.$$  \hspace{1cm} (5)

If the considered domain is infinite, one can use the Fourier integral transformation

$$\zeta(x) = \int \frac{d^3 k}{(2\pi)^3} \tilde{\zeta}_k e^{ikx},$$ \hspace{1cm} (6)

$$\tilde{\zeta}_k = \int \zeta(x) e^{-ikx} d^3 x.$$ \hspace{1cm} (7)
Assuming statistical homogeneity, the \( n \)-point correlators of \( \zeta_k \) are of the form
\[
\langle \zeta_{k_1} \zeta_{k_2} \rangle = (2\pi)^3 \delta^{(3)}(k_1 + k_2)P(\langle k_1 \rangle),
\]
(8)
\[
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = (2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3)B\zeta(\langle k_1 \rangle, k_2, k_3),
\]
(9)
\[
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle = (2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3 + k_4)T\zeta(\langle k_1 \rangle, k_2, k_3, k_4).
\]
(10)

Bispectrum \( B\zeta(\langle k_1 \rangle, k_2, k_3) \) and trispectrum \( T\zeta(\langle k_1 \rangle, k_2, k_3, k_4) \) are usually parameterized in terms of the spectrum \( P(\langle k \rangle) \) and the non-linear parameters \( f_{NL}, \eta_{NL}, g_{NL} \) by
\[
B\zeta(\langle k_1 \rangle, k_2, k_3) = \frac{6}{5} f_{NL}(\langle k_1 \rangle, k_2, k_3) \left( P(\langle k_1 \rangle)P(\langle k_2 \rangle) + P(\langle k_1 \rangle)P(\langle k_3 \rangle) + P(\langle k_2 \rangle)P(\langle k_3 \rangle) \right),
\]
(11)
\[
T\zeta(\langle k_1 \rangle, k_2, k_3, k_4) = \frac{1}{2} \eta_{NL}(\langle k_1 \rangle, k_2, k_3, k_4) \left( P(\langle k_1 \rangle)P(\langle k_2 \rangle)P(\langle k_1 + k_3 \rangle) + \text{cyclic permutations} \right)
\]
+ \[
\frac{54}{25} g_{NL}(\langle k_1, k_2, k_3, k_4 \rangle) \left( P(\langle k_1 \rangle)P(\langle k_2 \rangle)P(\langle k_3 \rangle) \right) + \text{cyclic permutations} \right).
\]
(12)

S

Spatial averaging and a non-Gaussianity. 3

On a sufficiently large scale, it is very convenient to use the ansatz
\[
\zeta(x) = \zeta_G(x) + \frac{3}{2} f_{NL}^2 \langle \zeta_G(x) \rangle + \left( \frac{3}{5} \right)^2 \eta_{NL}^2 \langle \zeta_G(x) \rangle + \ldots,
\]
(13)

where the auxiliary quantity \( \zeta_G(x) \) is a Gaussian random variable and the non-linearity parameters \( f_{NL}^2, \eta_{NL}^2 \) are some dimensionless constants. This expression can be treated as \( \delta \)-Nformula for a simple inflationary model with one single scalar field \( \varphi \)
\[
\zeta(x) = \sum_{n=1}^\infty \frac{N^{(n)}(\langle \varphi(\tau_s) \rangle)}{n!} \delta \varphi^n(\tau_s, x),
\]
(14)

where \( \varphi(\tau) \) is the background scalar field, \( \tau_s \) is some moment immediately after horizon crossing and the scalar field perturbation \( \delta \varphi^n(\tau_s, x) = \zeta_G(x)/N^n \) is assumed Gaussian.

Let us consider the patch \( \Omega_S \) with comoving volume \( V_S \), which is located within the treated above domain. In this patch, one can define the local comoving curvature perturbation, denoted as \( \zeta_S \). Analogically to equation (8), this quantity is given by
\[
\zeta_S(x) = \ln a - \frac{1}{V_S} \int_{\Omega_S} \ln a d^3 x.
\]
(15)

Since the volume averaging does not change the background scale factor \( \langle \langle a(\tau) \rangle \rangle_S = a(\tau) \), one can obtain
\[
\zeta_S(x) = \ln \left( \frac{\bar{a}}{a} \right) - \frac{1}{V_S} \int_{\Omega_S} \ln \left( \frac{\bar{a}}{a} \right) d^3 x = \zeta(x) - \frac{1}{V_S} \int_{\Omega_S} \zeta(x) d^3 x.
\]
(16)

This equation can be rewritten as [14]
\[
\zeta_S(x) = \zeta(x) - \langle \zeta \rangle_S.
\]
(17)

The equation (17) shows that the relationship between \( \zeta_S \) and \( \zeta \) is non-local. To calculate the value of the quantity \( \zeta_S \) at one point, it is necessary to know \( \zeta(x) \) in the whole patch at the same time of this. This non-locality is essential if the region \( \Omega_S \) is not fixed, i.e., if \( \Omega_S = \Omega_S(x) \). In this case, to find \( \zeta_S(x) \) at all points of \( \Omega_S(x_0) \), it is necessary to know the comoving curvature perturbation \( \zeta(x) \) at all points of some larger patch which includes \( \Omega_S(x_0) \).
3 Mapping between $\zeta$ and $\zeta_S$ in the momentum space.

The procedure of spatial averaging plays an important role in the decomposition of the quantity $\tilde{a}(x,t)$ on the homogeneous background and small inhomogeneous perturbation. Here, we consider the relations in Fourier space, which follows from the equation (17).

We use the fact that the operation of spatial averaging is linear and consider only one Fourier mode

$$\zeta(x) = \zeta(k)e^{ikx}. \quad (18)$$

We denote the comoving coordinates of the patch as $x_0$. For example, if the region has a spherical shape, then $x_0$ is the center of this sphere. We also introduce the integration variable $x'$, so that $x = x_0 + x'$. The spatial averaging gives

$$\langle \zeta \rangle_S = \frac{1}{V_S} \int \zeta(k)e^{ik(x_0+x')}d(x_0 + x') = W(k)\zeta(k)e^{ikx_0}, \quad (19)$$

where

$$W(k) = \frac{1}{V_S} \int e^{ikx'} dx'. \quad (20)$$

Thus, the spatial averaging of the function (18) gives a plane wave field with the same wave vector, but with a different amplitude. We obtain the known results

$$\langle \zeta_S \rangle_k = W(k)\zeta_k \quad (21)$$

and

$$[\zeta_S]_k = (1 - W(k))\zeta_k. \quad (22)$$

The equation (22) is exact regardless of the form of the quantity $\zeta(x)$. Although this equation is known, its consequences for cosmological perturbation theory have not been investigated.

In what follows, we assume that the considered patch is Hubble sized at the present time. This choice allows us to get some physical consequences of equation (22).

Let’s denote the characteristic comoving size of the patch as $x_S$. The observer can confidently separate the perturbation from the background if the perturbation experienced several oscillations in the patch. This is carried out for the Fourier modes with wave vector $k$ satisfying the inequality $|k|x_S \gg 2\pi$. Such perturbations we call short wavelength ones. The long wavelength Fourier modes satisfy the opposite condition $|k|x_S \ll 2\pi$.

For short wavelength perturbations, the equations (20), (22) give

$$[\zeta_S]_k \bigg|_{k > k_S} = [\zeta]_k \bigg|_{k > k_S}, \quad (23)$$

where $k = |k|$ and $k_S = 2\pi/x_S$. This equation show that the difference between $[\zeta_S]_k$ and $[\zeta]_k$ can be neglected on a small scale.

Expanding the exponential function in a Taylor series, we get for long wavelength perturbation

$$[\zeta]_k \bigg|_{k \ll k_S} \approx \frac{1}{2} \langle (kx')^2 \rangle_S [\zeta]_k \bigg|_{k \ll k_S} \propto k^2 [\zeta]_k \bigg|_{k \ll k_S}. \quad (24)$$

One can see that quantity $[\zeta_S]_k$ is suppressed on a large scale. This is the consequence of the fact that the observer can not separate surely the long-wavelength perturbation from the background. Perhaps, the equation (24) will allow to solve still existing problem of infrared divergences in the calculation of correlation functions at loop level (see [19] for a
recent review of this topic), although the reformulation of the perturbation theory in terms of locally defined variables is difficult due to the non-locality of the equation (17).

The form of the function \( W(k) \) is of interest over a wide range of scales. Assuming that the observable patch can be approximated as a sphere of radius \( x_S \), a simple calculation gives

\[
W(k) = 3 \left( \frac{\sin(kx_S)}{kx_S} - \frac{\cos(kx_S)}{(kx_S)^2} \right). \tag{25}
\]

Equations (23) and (24) are main results of this work.

Since the quantity \( \zeta(x) \) is statistically homogeneous, there are an expressions which are analogous to the equations (8)-(12). In particular, we have

\[
B_{S^3}(k_1, k_2, k_3) = \frac{6}{5} f_{NL}^{(S)}(k_1, k_2, k_3) \left( P_{S^3}(k_1)P_{S^3}(k_2) + P_{S^3}(k_2)P_{S^3}(k_3) + P_{S^3}(k_3)P_{S^3}(k_1) \right). \tag{26}
\]

For equal-\( k \) (\( k_1 = k_2 = k_3 \equiv k \)), the equations (11), (26) gives

\[
f_{NL}^{(S)}(k_1, k_2, k_3) = \frac{f_{NL}}{1 - W(k)}. \tag{27}
\]

In the squeezed limit (\( k_1 \approx k_2 \equiv k \gg k_3 \)), the mutual relations of \( f_{NL}^{(S)} \) and \( f_{NL} \) depends on the scale. If \( k_1, k_2 \) are infrared and the primordial spectrum is scale-invariant, then

\[
f_{NL}^{(S)}(k_1, k_2, k_3) \approx f_{NL}(k_1, k_2, k_3) - \frac{2}{1 - W(k)} \frac{k}{k_3}. \tag{28}
\]

In both cases, we obtain

\[
f_{NL}^{(S)}(k_1, k_2, k_3) \gg f_{NL}(k_1, k_2, k_3). \tag{29}
\]

Formally, the non-linearity parameter \( f_{NL}^{(S)} \) is enhanced on a large scale due the suppression of \( [\zeta_S]_k \).

4 Examples.

The equation (23) indicates that, on a small scale, the local comoving curvature perturbation \( \zeta \) has the same statistical properties as the quantity \( \zeta \). The only question is whether it is possible to recover the global statistical properties of \( \zeta \) using the observed data sample of \( [\zeta_S]_k \). In other words, are there any reasons for which the local observer can fail to notice the coincidence of statistical properties of \( [\zeta_S]_k \) and \( [\zeta]_k \) on a small scale?

At first, let us consider how to implement the equation (23). For simplicity, we use the ansatz (13), in which we assume that only \( f_{NL}^{(S)} \neq 0 \). The ansatz (13) is reduced to

\[
\zeta(x) = \zeta_G(x) + \frac{3}{5} f_{NL}^{(S)} (\zeta_G^2(x) - \langle \zeta_G^2 \rangle) \tag{30}
\]

and leads to the equation

\[
\zeta(x) = \zeta_G(x) - \langle \zeta_G \rangle s + \frac{3}{5} f_{NL}^{(S)} (\zeta_G^2(x) - \langle \zeta_G^2 \rangle s). \tag{31}
\]

In momentum space it gives

\[
[\zeta_S]_k = [\zeta_G - \langle \zeta_G \rangle s]_k + \frac{3}{5} f_{NL}^{(S)} [\zeta_G^2 - \langle \zeta_G^2 \rangle s]_k. \tag{32}
\]
The volume averaging over the Hubble patch cuts off short wavelength perturbations, so that on a small scale

$$\left[ \zeta_S \right]_k = \left[ \zeta_G \right]_k + \frac{3}{5} f_{NL} \left[ \zeta_S^2 \right]_k .$$  \hspace{1cm} (33)

It coincides with the result \cite{23}.

Observable patch is differs from the large domain by size, and the discrepancy between observed statistical properties of \( \left[ \zeta_S \right]_k \) and theoretical statistical properties of \( \left[ \zeta \right]_k \) may be expected to be due to the influence of infrared perturbations. Let us examine this issue in some detail.

Let's the realization of the auxiliary quantity \( \zeta_G \) has the form

$$\zeta_G = \zeta_1 + \zeta_2 ,$$  \hspace{1cm} (34)

where \( \zeta_1 = \zeta_{k_1} e^{k_1 x} \), \( \zeta_2 = \zeta_{k_2} e^{k_2 x} \) and \( k_1 \ll k_S \) (long wave), \( k_2 \gg k_S \) (short wave). Then the equation \cite{32} yields

$$\left[ \zeta_S \right]_{k_2} = \left[ \zeta_G \right]_{k_2} + \frac{3}{5} f_{NL} \left[ \zeta_S^2 - \langle \zeta_S^2 \rangle S \right]_{k_2} .$$  \hspace{1cm} (35)

In the coordinate space, we obtain from equations \cite{34}, \cite{35}

$$\zeta_2 \left( \langle \zeta_G \rangle \right) S = \left( \zeta_2^2 - \langle \zeta_2^2 \rangle S \right) + 2 \zeta_1 \left( \zeta_2 - \langle \zeta_2 \rangle \right) S .$$  \hspace{1cm} (36)

In the momentum space it gives

$$\left[ \zeta_G^2 - \langle \zeta_G \rangle \right]_{k_2} = \left[ \left( \zeta_2^2 - \langle \zeta_2^2 \rangle S \right) + 2 \zeta_1 \left( \zeta_2 - \langle \zeta_2 \rangle \right) S \right]_{k_1} = 0 .$$  \hspace{1cm} (37)

Using this result one can obtain

$$\left[ \zeta_S \right]_{k_2} = \left[ \zeta_G \right]_{k_2} ,$$  \hspace{1cm} (38)

i. e., the long wavelength Fourier mode with wave vector \( k_1 \) does not affect the mode with wave vector \( k_2 \). It is clear also from consideration of the convolution theorem

$$\left[ \zeta_G^2 \right]_k = \int \left[ \zeta_G \right]_q \left[ \zeta_G \right]_k - \frac{d^3 q}{(2\pi)^3} .$$  \hspace{1cm} (39)

If the quantity \( \zeta_G \) has two modes with wave vectors \( k_1 \) and \( k_2 \), then the quantity \( \zeta_G^2 \) contain modes with wave vectors \( 2k_1, 2k_2, k_1 + k_2 \) only.

To gain influence on mode with the wave vector \( k_2 \), the expansion of \( \zeta_G \) should include mode with \( k_3 = k_2 - k_1 \). However, just like the Fourier mode with \( k_1 \) is indistinguishable from the background, the modes with \( k_2 \) and \( k_3 \) are indistinguishable between themselves. In other words, infrared perturbations do not contribute to the correlation functions of physically distinguishable modes.
5 A landscape Universe.

Recently, in the papers [13], [14], it was made several claims, which seem to contradict the equation (23). However, the authors of [13], [14] have used several approximations which require very careful handling when working in Fourier space. On the other hand, the parameters of ansatz (13) are sensitive to the choice of the auxiliary quantity $\zeta_G$. This can be verified by direct calculation in the simplest case of anzatz (31).

We introduce the notation

$$\zeta_{G,S} = \zeta_G - \bar{\zeta},$$

where $\bar{\zeta} = \langle \zeta_G \rangle_S$. It is important that the quantity $\bar{\zeta}$ can be considered as a constant on a small scale, i.e., one can assume

$$\bar{\zeta}_k \mid_{k > \delta_S} = 0.$$

The equation (31) and the equality

$$\zeta_{G,S}^2 - \langle \zeta_{G,S}^2 \rangle_S = \zeta_G^2 - \langle \zeta_G^2 \rangle - 2 \bar{\zeta} \zeta_{G,S}$$

(42)

gives

$$\zeta_S(x) = \left(1 + \frac{6}{3} f_{NL} \bar{\zeta} \right) \zeta_{G,S} + \frac{3}{3} f_{NL} \left( \zeta_{G,S}^2 - \langle \zeta_{G,S}^2 \rangle_S \right).$$

(43)

This result is in full agreement with the result of the paper [14]. On a small scale, the equations (41), (43) yields now

$$\left[ \zeta_S \right]_k = \left[ \zeta_{G,S} \right]_k + \frac{3}{3} f_{NL} \left[ \zeta_{G,S}^2 \right]_k.$$

(44)

One can make the variable transformation

$$\chi_G = \left(1 + \frac{6}{3} f_{NL} \bar{\zeta} \right) \zeta_{G,S}.$$

(45)

It gives [14]

$$\left[ \zeta_S \right]_k = \left[ \chi_G \right]_k + \frac{3}{3} f_{NL} \left[ \chi_G^2 \right]_k,$$

(46)

where

$$f_{NL} = \frac{f_{NL0}}{\left(1 + \frac{6}{3} f_{NL} \bar{\zeta} \right)^2}. $$

(47)

Consequently, both parameterizations (46) and (33) are possible at once. This can easily be checked on a small scale using the equalities

$$\left[ \zeta_{G,S} \right]_k = \left[ \zeta_{G} \right]_k,$$

$$\left[ \zeta_{G,S}^2 \right]_k = \left[ \zeta_{G}^2 \right]_k - 2 \bar{\zeta} \left[ \zeta_{G} \right]_k.$$

(48)

(49)

which follows from the equations (40) and (42) correspondingly. The parameterizations (33) and (46) yields to the same numerical values for the Fourier components of the local comoving curvature perturbation, as it should be.

It is significant that the convolution theorem (39) allows us to rewrite the equation (49) as

$$\left[ \zeta_{G,S} \right]_k = \int \left[ \zeta_G \right]_q \left[ \zeta_{G} \right]_{k-q} \frac{d^3 q}{(2\pi)^3} - 2 \bar{\zeta} \left[ \zeta_{G} \right]_k.$$

(50)
This expression differs from the approximate one used in the papers [13], [14].

Equations (45), (49) and (47) indicate that the quantities $\chi_G$ and $f_0^{(S)}$ are influenced by the same random parameter $\bar{\zeta}$. Though, the associated uncertainty of parametrization did not affect the numerical value of the local comoving curvature perturbation. One can see that some arbitrariness in definition of the auxiliary function $\chi_G$ is compensated by a corresponding change of the non-linearity parameter $f_0^{(S)}$ without affecting $[\zeta_S]_k$ and its correlation functions. The stochastic quantity $\bar{\zeta}$ also does not affect the calculation of the parameter $f_0^{(S)}$ defined by the equation (26).

6 Conclusions.

We have studied the relationship between locally and globally defined comoving curvature perturbations within a quasi-homogeneous domain of the Universe. It is shown that, on scales larger than the size of the observed patch, the Fourier components of the locally defined comoving curvature perturbation are suppressed. It is shown also that the statistical properties of local and global comoving curvature perturbations coincide on a small scale. Several examples are discussed in detail. We have shown that, in the simplest cases, the long wavelength perturbations do not contribute to the bispectrum and trispectrum on a sub-Hubble scale.

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