New Approach to Find Fixed Point in Extended b-metric Space

Surjeet Singh Chauhan (Gonder) 1 and Kanika Rana 1

1Department of Mathematics, Chandigarh University, Gharuan, Punjab (140413), India.
Email id: surjeetchauhan@yahoo.com

Abstract. A fixed point for a suitable map or operator is identical to the presence of a solution to a theoretical or real-world problem. As a result, fixed points are crucial in many fields of mathematics, science, and engineering. In this paper, we establish new fixed point results on self-mappings in setting of extended b-metric space which can be extended further to give application in real world such as in image processing, computer graphics, Nash equilibrium and many more. Our results extends the corresponding results of Mukheimer et al. [Aimal Mukheimer, Nabil Mlaiki, Kamal Abodayeh, Wàsfi Shantanawi, Non Linear Analysis: Modeling and Control, 24(6), 870-883, 2019.] and Kamran et al. [Tayyab Kamran, Maria Samreen, Qurrat UL Ain, Mathematics, 5(19), 2017, 7 pages.]. Examples are also mentioned to check the authenticity of our results. A solution to Fredholm integral equation is also demonstrated as an application.

Keywords: contractions, metric space, extended b-metric space, fixed point

1. Introduction

Theory related to fixed points is a significant tool of modern mathematics as it helps to find a unique fixed point of multi-valued and single-valued mappings by restricting the condition of the domain of the function. It also helps to find the results of many differential as well as integral equations which can further be used to solve many industrial based problems. Starting with the very renowned result given by S. Banach in 1922[1] which states that “Let A ≠ V be a map from a complete metric space (A, d) into itself satisfying: d(Ax, y) ≤ pd(x, y) known as Banach contraction Principle which is used to find applicable result to find a unique fixed point by using successive approximations”. Since then, many researchers generalizes this principle in various metric spaces see [2-11].

Further, Many variants of metric space have appeared till now. One of the variants of metric space is known as “b-metric space” which was familiarized by Bakhtin [12], he generalizes Banach contraction principle and proved some fixed point results using different contraction conditions in b-metric space. Later, Czerwik [13] extend the work in “b-metric space”. After that several research is done to verify the presence of fixed point in “b-metric” space [14-23]. In 2017, Kamran[24] gave a new direction to this theory by weakened the triangular inequality of “b-metric space” using a function α: A × A → [1, ∞)and proved new results in theory of fixed points. Till now, researchers have extended his idea of work to find a fixed point in this theory. See [25-30].

2. Preliminaries

Below are some important terms which will be beneficial to prove our result.
Example 2.1[13] Assume \( A \neq \emptyset \) and \( g \geq 1 \) is real. If \( \forall q', \varsigma', \xi' \in A \), below conditions holds, a function \( b: A \times A \to \mathbb{R}^+ \) is termed as “b-metric”:

1. \( b(q', \varsigma') = 0 \) if and only if \( q' = \varsigma' \)
2. \( b(q', \varsigma') = b(\varsigma', q') \)
3. \( b(q', \varsigma') \leq g b(q', \varsigma') + b(\varsigma', \xi') \)

The pair \((A, b)\) is termed as “b-metric space”.

Example 2.2[14] Consider \( A \neq \emptyset \) and \( b(q', \varsigma') = (b(q', \varsigma'))^\alpha \) for all \( q', \varsigma' \in A \) where \( \alpha \geq 1 \) is real. Then \((A, b)\) is “b-metric space” having \( g = 2^{\alpha-1} \). Though, \((A, b)\) is not a metric space.

The above example clearly shows that “b-metric space” is indeed greater than a metric space.

Definition 2.2[15] Suppose, \((A, b)\) be a “b-metric space”:

1. \( \{q_n\} \) in \( A \) converge to \( q' \in A \) if for each \( \epsilon > 0 \) there is \( N = N(\epsilon) \in \mathbb{N} \) for which \( b(q_n, q') < \epsilon \) for all \( n > N \), that is \( \lim_{n \to \infty} q_n = q' \).
2. \( \{q_n\} \) in \( A \) is Cauchy such that every \( \epsilon > 0 \), \( N = N(\epsilon) \in \mathbb{N} \) for which \( b(q_n, q_m) < \epsilon \) for all \( n, m \geq N \).

Definition 2.3[24] Consider \( A \neq \emptyset \) and mapping \( \alpha: A \times A \to [1, \infty) \). Function \( \alpha: X \times X \to [0, \infty) \) is an extended b-metric space” if it satisfies the below conditions:

1. \( b_{\alpha}(q, \varsigma) = 0 \) if and only if \( q = \varsigma \);
2. \( b_{\alpha}(q, \varsigma) \geq \alpha(q, \varsigma)[b_{\alpha}(q, \varsigma') + b_{\alpha}(\varsigma', \xi')] \)

The space \((A, \alpha)\) is termed as “extended b-metric space”.

Note that “by making \( \alpha(q, \varsigma) = g \geq 1 \) a constant function, every b-metric space becomes an extended b-metric space”.

Definition 2.4[24] Suppose, we have an “extended b-metric space” \((A, b_{\alpha})\).

1. \( \{q_n\} \) in \( A \) converge to \( q' \in A \) if for each \( \epsilon > 0 \) there is \( N = N(\epsilon) \in \mathbb{N} \) such for which \( b_{\alpha}(q_n, q') < \epsilon \) for all \( n \geq N \).
2. \( \{q_n\} \) in \( A \) is Cauchy such that for every \( \epsilon > 0 \), \( N = N(\epsilon) \in \mathbb{N} \) for which \( b_{\alpha}(q_n, q_m) < \epsilon \) for all \( n \geq N \).

Note that, “if every Cauchy in \( A \) is convergent, then b-metric space is said to be complete”.

Lemma 2.1[16] Suppose \((A, b_{\alpha})\) be an “extended b-metric space” and assume \( \{q_n\} \) in \( A \). Suppose there exists \( p \in (0, 1) \) satisfying

\[ b_{\alpha}(q_n, q_{n+1}) \leq p b_{\alpha}(q_n, q_{n-1}) \]

For any \( n \in \mathbb{N} \). If \( \lim_{n \to \infty} b(q_n, q_m) = \frac{1}{p} \), then \( \{q_n\} \) is Cauchy.

Example 2.3[24] Suppose, \( A = \{1, 2, 3\} \). Define \( \alpha: A \times A \to \mathbb{R}^+ \) and \( b_{\alpha}: A \times A \to \mathbb{R}^+ \) as:

\[ \alpha(q', q'') = 1 + q' + q'' \]
\[ b_{\alpha}(1, 1) = b_{\alpha}(2, 2) = b_{\alpha}(3, 3) = 0 \]
\[ h_{\alpha}(1, 1) = h_{\alpha}(2, 1) = 80, h_{\alpha}(1, 3) = h_{\alpha}(3, 1) = 1000, h_{\alpha}(2, 3) = h_{\alpha}(3, 2) = 600 \]

Then \((A, b_{\alpha})\) is termed as “extended b-metric space”.

2
Definition 2.5[29] Suppose, \((A, b_\alpha)\) and \((A, t_\alpha)\) are “extended b-metric spaces”. If, for each \(\{e_n\}\) in \(A\), \(\{V_{\alpha, e_n}\}\) converges to \(V_{\alpha, e}\) with respect to \(t_\alpha\), then, mapping \(V: A \rightarrow R\) is continuous.

Example 2.4[30] Suppose, \(A = [0, +\infty)\). Define two mappings \(a: A \times A \rightarrow R^+\) and \(b_\alpha: A \times A \rightarrow [0, +\infty)\) as follows:

\[
\alpha(q', \zeta') = 1 + |q'| + |\zeta'| \text{ for every } q', \zeta' \in A \text{ and } b_\alpha(q', \zeta') = \begin{cases} q'^2 + \zeta'^2, & q', \zeta' \notin A, q' \neq \zeta' \\ 0, & q' = \zeta' \end{cases}
\]

Then, \((A, b_\alpha)\) together is an “extended b-metric space”.

Note that, “the above example is not a b-metric space”.

3. Main Result

Theorem 3.1 Consider \((A, b_\alpha)\) to be a “complete extended b-metric space”, and consider \(V\) be a self-map of \(A\). Suppose that there is \(\rho \in [0, 1)\) for which

\[
b_\alpha(Ve', Ve) \leq \frac{\rho}{3} \alpha(e', e)(b_\alpha(e' e'))
\]

For every \(e', e' \in A\). Assume \(e_0' \in A\). We have

\[
\lim_{n-m \to \infty} \sup \alpha(e_n', e_m') \alpha(e_n', e_{n+1}') < \frac{3}{\rho}
\]

Where \(e_j' = V^j e_0'\) for every \(j \in N\). Additionally, assume \(\lim_{n \to \infty} \sup \alpha(e_n', e_n)\) exists for some \(e' \in A\). Then \(V\) has a fixed point.

Besides, a unique fixed point of \(V\) exists, if for some \(e', e' \in A\):

\[
\lim_{n \to \infty} \sup \alpha(V^n e', V^n e') < \frac{3}{\rho}
\]

Where, \(V^n = V^{n-1}(V e)\).

Proof Consider \(e_0' \in A\) be an arbitrary constant. Now formulate a sequence \(\{e_n\}\) in \(A\) in the resulting manner:

\[
e_0', e_1' = V e_0', e_2' = V^2 e_0', e_3' = V^3 e_0', ...\]

Then

\[
\lim_{n \to \infty} \sup \alpha(e_n', e_n') \alpha(e_n', e_{n+1}') < \frac{3}{\rho},
\]

and \(\lim_{n \to \infty} \sup \alpha(e_n', e_n')\) exists for some \(e' \in A\). By expanding (1) \(n\) times, we get

\[
b_\alpha(e_n', e_{n+1}') \leq \frac{\rho}{3} \alpha(e_{n-1}', e_n') b_\alpha(e_{n-1}', e_{n-1}')
\]

\[
\leq \frac{\rho}{3} \alpha(e_{n-1}', e_n') b_\alpha(e_{n-1}', e_{n-1}') \leq \frac{\rho}{3} ... \leq \frac{\rho}{3} \alpha(e_{n-1}', e_n') b_\alpha(e_{n-1}', e_{n-1}')
\]

For \(m, n \in N\) having \(m > n\), we get

\[
b_\alpha(e_n', e_m') \leq \alpha(e_n', e_m') b_\alpha(e_{n-1}', e_{m}') + b_\alpha(e_{n-1}', e_{m}')
\]

\[
\leq \alpha(e_n', e_m') b_\alpha(e_{n-1}', e_{m}') + \alpha(e_{n-1}', e_m') b_\alpha(e_{n-1}', e_{m}') + \alpha(e_{n-1}', e_m') b_\alpha(e_{n-1}', e_{m}') + \alpha(e_{n-1}', e_m') b_\alpha(e_{n-1}', e_{m}')
\]

\[
\leq \alpha(e_n', e_m') b_\alpha(e_{n-1}', e_{m}') + \alpha(e_{n-1}', e_m') b_\alpha(e_{n-1}', e_{m}') + \alpha(e_{n-1}', e_m') b_\alpha(e_{n-1}', e_{m}') + \alpha(e_{n-1}', e_m') b_\alpha(e_{n-1}', e_{m}')
\]

\[
= \alpha(e_n', e_m') b_\alpha(e_{n-1}', e_{m}') + \alpha(e_{n-1}', e_m') b_\alpha(e_{n-1}', e_{m}') + \alpha(e_{n-1}', e_m') b_\alpha(e_{n-1}', e_{m}') + \alpha(e_{n-1}', e_m') b_\alpha(e_{n-1}', e_{m}')
\]

\[
(2)
\]
Combining (2) and (3), we get

\[ b_{n+1}(e_{n+1}, e_m) \leq \alpha(\xi_{n+1}, e_m) \alpha(\xi_n, \xi_{n+1}) b_{\alpha}(\xi_n, \xi_{n+1}) \]

(4)

Suppose

\[ \alpha(e_{n+1}, e_m) \leq \sum_{i=1}^{n+1} \prod_{j=1}^{i} \alpha(e_j, e_m) \prod_{j=1}^{i-1} \alpha(e_{t-j}, e_t) s^i b_{\alpha}(e_0, e_1) \]

And

\[ \alpha(e_{n+1}, e_m) \leq \prod_{j=1}^{n+1} \alpha(e_j, e_m) \prod_{j=1}^{n+1} \alpha(e_{t-j}, e_t) s^{n+1} b_{\alpha}(e_0, e_1) \]

Thus, we get a conclusion that

\[ \lim_{n \to \infty} \frac{\alpha(e_{n+1}, e_m)}{\alpha(e_n, e_m)} = \frac{\prod_{j=1}^{n+1} \alpha(e_j, e_m) \prod_{j=1}^{n+1} \alpha(e_{t-j}, e_t) s^{n+1} b_{\alpha}(e_0, e_1)}{\prod_{j=1}^{n} \alpha(e_j, e_m) \prod_{j=1}^{n} \alpha(e_{t-j}, e_t) s^{n} b_{\alpha}(e_0, e_1)} \]

for, \( s = \frac{p}{2} \)

We get

\[ \lim_{i \to \infty} \frac{r_{i+1}}{r_i} = \lim_{i \to \infty} \frac{\alpha(e_{i+1}, e_m) \alpha(e_i, e_{i+1})}{\alpha(e_i, e_m)} s < 1 \]

Thus, we get a conclusion that

\[ \sum_{i=0}^{\infty} \frac{1}{i!} s^i b_{\alpha}(e_0, e_1) < \infty \]

Hence,

\[ \left\{ \sum_{i=0}^{\infty} \frac{1}{i!} s^i b_{\alpha}(e_0, e_1) \right\} \]

is Cauchy in \( \mathbb{R} \) and using (4), we conclude \( \{\xi_n\} \) is also Cauchy in \( A \).

Subsequently, \( A \) is complete means \( \{\xi_n\} \) converges to \( u' \in A \).

Then by using triangular inequality and (1), we get

\[ b_{\alpha}(V u', u') \leq \alpha(V u', u') [b_{\alpha}(V u', q_{n+1}) + b_{\alpha}(q_{n+1}, u')] = \alpha(V u', u') [b_{\alpha}(V u', q_{n+1}) + b_{\alpha}(q_{n+1}, u')] \leq \alpha(V u', u') \frac{p}{2} b_{\alpha}(u', q_{n+1}) + b_{\alpha}(u', q_{n+1}) + b_{\alpha}(u', u') \]

Letting \( n \to \infty \) in above inequalities, we get \( b_{\alpha}(V u', u') \leq 0 \). Hence, \( b_{\alpha}(V u', u') = 0 \). So \( u' \) is a fixed point of \( V \).

Next stage is to prove the uniqueness, for this suppose \( u' \) and \( v' \) be two distinct fixed points of \( V \). Then,

\[ b_{\alpha}(u', v') = b_{\alpha}(V u', V v') \leq \frac{p}{2} \alpha(v', u') b_{\alpha}(v', u') = \frac{p}{2} \alpha(v', u') b_{\alpha}(v', u') \]

Letting \( n \to \infty \) in above inequalities, we get \( b_{\alpha}(u', v') < b_{\alpha}(u', v') \), which contradicts. So we conclude that \( V \) has a unique fixed point.

Now consider \( V \) is continuous in Theorem 3.1 resulting in,

**Theorem 3.2** Consider \( (A, d_{\alpha}) \) be a “complete extended b-metric space”, and suppose \( V \) be a self-map of \( A \). Assume there is \( p \in [0, 1) \) such that
For every \( g', \zeta' \in A \). Assume for \( g'_0 \in A \). We have
\[
\lim_{n,m \to \infty} \sup \alpha(g'_n, q'_m) \alpha(g'_n, q'_{n+1}) < \frac{\beta}{\gamma}.
\]
Where \( q'_j = VJ q'_t \) for all \( j \in N \). If \( V \) is continuous, then \( h_x(g', \zeta') = 0 \).

**Proof** By applying similar conditions as used in Theorem 3.1, we construct a Cauchy \( \{ q'_n \} \). Subsequently, \( A \) is complete, \( \{ q'_n \} \) converges to any \( u' \in A \) and \( V \) is continuous, we have \( Vq'_n \to Vu' \).

By using Triangular inequality, we get
\[
b_n(v'u', u') \leq \alpha(v'u', u') [b_n(v'u', q'_n+1) + b_n(q'_n+1, q'_s)]
= \alpha(v'u', q'_s) [b_n(v'u', q'_n+1) + b_n(q'_n, q'_s)]
\]
Letting \( n \to \infty \) above, we get \( b_n(g', \zeta') \to 0 \) and hence \( g' \) is a fixed point of \( V \).

**Corollary 3.1** Consider a “complete b-metric space” \( (A, d) \) with \( \beta \geq 1 \). Suppose \( V \) be a self-map. Assume that there exists \( p \in [0,1) \) for which
\[
b(Vq', q'_s) < \frac{\beta}{\gamma} h(u', \zeta')
\]
For every \( g', \zeta' \in A \). Suppose \( g'_0 \in A \). Then \( V \) has a unique fixed point.

**Proof** By defining \( u : A \times A \to [1, \infty] \) and \( \alpha(g', \zeta') = \beta \) in theorem 3.1. We get a unique fixed point of \( V \).

**Theorem 3.3** Consider \( (A, b_d) \) be a “complete extended b-metric” space, where \( b_d \) is a continuous functional and assume \( V \) be a self-map of \( A \). Suppose there exists \( p \in [0,1) \) such that
\[
b_d(v'u', q'_s) \leq \frac{\beta}{\gamma} b_d(g', \zeta')
\]
For all \( g', \zeta' \in A \). Assume for \( g'_0 \in A \). We have
\[
\lim_{n,m \to \infty} \alpha(g'_n, q'_m) < \frac{\beta}{\gamma}
\]
Where \( q'_j = VJ q'_t \) for all \( j \in N \). Then \( V \) has one fixed point.

**Proof** Consider \( q'_0 \in A \) be an arbitrary constant. Now formulate a sequence \( \{ q'_n \} \) in \( A \) in the resulting manner:
\[
e_1, e_2 = V^2 e_0, e_3 = V^3 e_0, ...
\]
Then by using (1) \( n \) times, we get
\[
b_d(g'_n, q'_s) \leq s^n b_d(e_0, e_1)
\]
(5)

By using triangular inequality and (5), for \( m > n \) we have;
\[
\alpha(g'_n, q'_m) \leq \alpha(g'_n, q'_m) s^n b_d(e_0, e_1) + \alpha(g'_n, q'_m) s^{n+1} b_d(e_0, e_1) + ... + \alpha(g'_n, q'_m) s^{m-1} b_d(e_0, e_1) + \alpha(g'_n, q'_m) s^m
\]
Letting \( n \to \infty \) we get a conclusion that \( \{q_n^1\} \) is a Cauchy and \( \mathcal{A} \) is complete. Suppose \( q_n^1 \to u^1 \), where \( u^1 \in \mathcal{A} \) we have

\[
\begin{align*}
   b_n(V u^1, u^1) & \leq a(V u^1, u^1)[b_n(V u^1, q_n^1) + b_n(q_n^1, u^1)] \\
   & \leq a(V u^1, u^1)[b_n(u^1, q_n^1) + b_n(q_n^1, u^1)] \\
   & \leq 0 \quad \text{as} \quad n \to \infty \\
   b_n(V u^1, u^1) & = 0
\end{align*}
\]

Hence \( u^1 \) is the unique fixed point of \( V \).

**Example 3.1** Suppose, \( \mathcal{A} = [0, \frac{1}{2}] \). Define two mappings \( \alpha : \mathcal{A} \times \mathcal{A} \to \mathbb{R}^+ \) and \( b_n : \mathcal{A} \times \mathcal{A} \to [0, \infty) \) as follows:

\[
\alpha(q^1, q^2) - 0 = (q^1 - q^2)^2
\]

For all \( q^1, q^2 \in \mathcal{A} \) and \( b_n(q^1, q^2) = (q^1 - q^2)^2 \)

Define \( V : \mathcal{A} \to \mathcal{A} \) by \( V q^1 = \frac{q^1}{2} \). Then we have

\[
\left( \frac{q^1}{2} - q^2 \right)^2 \leq \frac{1}{2} \left( |q^1| + |q^2| \right) (q^1 - q^2)^2
\]

Note that for each \( q^1 \in \mathcal{A} \), \( V^n q^1 = \frac{q^1}{2^n} \). Thus we obtain

\[
\lim_{n \to \infty} \sup_{m \to \infty} (V^n q^1, V^m q^1) < 9
\]

Hence, the above hypothesis of theorem 3.1 are fulfilled. Therefore, \( V \) has a unique fixed point.

**Example 3.2** Suppose, \( \mathcal{A} = [0, \frac{1}{2}] \). Define two mappings \( \alpha : \mathcal{A} \times \mathcal{A} \to \mathbb{R}^+ \) and \( b_n : \mathcal{A} \times \mathcal{A} \to [0, \infty) \) as follows:

\[
\alpha(q^1, q^2) = 2 \quad \text{for all} \quad q^1, q^2 \in \mathcal{A} \quad \text{and} \quad b_n(q^1, q^2) = (q^1 - q^2)^2
\]

Define \( V : \mathcal{A} \to \mathcal{A} \) by \( V q^1 = \frac{q^1}{4} \). Then we have

\[
\left( \frac{q^1}{4} - q^2 \right)^4 \leq \frac{1}{25} (q^1 - q^2)^4
\]

Note that for each \( q^1 \in \mathcal{A} \), \( V^n q^1 = \frac{q^1}{4^n} \). Thus we obtain

\[
\lim_{n \to \infty} \lim_{m \to \infty} (V^n q^1, V^m q^1) < 25
\]

The above hypothesis of theorem 3.3 are fulfilled. Therefore, \( V \) has a unique fixed point.

4. Application

In the application section we provide a solution to Fredholm Integral equation. Consider a set \( \mathcal{A} = C([l', m'], \mathbb{R}] \) and the following Fredholm Integral equation.

\[
q^1(v) - \int_{l'}^{m'} \varphi(v, w, q^1(w)) \, dw + f(v), \quad v, w \in [l', m'] \tag{6}
\]

Where, \( f : [l', m'] \to [0, \infty) \) and \( G : [l', m'] \times [l', m'] \to [0, \infty) \) are continuous functions.

Now, consider \( \mathcal{A} \) be complete extended b-metric space having \( b_\alpha = \sup_{x \in [l', m']} |G(v, w)| \) with \( \alpha(q^1, q^2) = 2 + |q^1| + |q^2| \) and \( m : \mathcal{A} \times \mathcal{A} \to \mathbb{R}^+ \).

Consider \( V : \mathcal{A} \to \mathcal{A} \) given by:

\[
V q^1(v) = \int_{l'}^{m'} \varphi(v, w, V q^1(w)) \, dw + f(v), \quad v, w \in [l', m']
\]

Where, \( f : [l', m'] \to [0, \infty) \) and \( G : [l', m'] \times [l', m'] \to [0, \infty) \) are continuous functions.

Further assume that,

\[
|G(v, w, q^1(w))| \, dw - G(v, w, V q^1(w)) \, dw \leq \frac{1}{2} |q^1(w) - V q^1(w)|
\]

Then integral solution has a solution.

Now, for any
Hence, all the hypothesis of Theorem 3.3 are fulfilled.

5. Conclusion

A solution to the Fredholm integral equation is also given to validate the preceding conclusions, which are frequently encountered in electrostatic, low frequency electromagnetic difficulties. Moreover, the production of photo-realistic images is also an application of the above type Fredholm equations, in which the Fredholm equation is utilised to represent transverse of light from virtual light sources to the image plane. We also discuss the relevant existing results in “extended b-metric space”. Then we prove new fixed point results followed by examples. Clearly, by letting $Q(x', y') - Q$, where $Q \geq 1$, comparable findings in “b-metric space” can also be attained. On the other hand, by modifying the condition on $p$ one can attain new results in this metric space. Also, theorem 3.3 is an analogue of Banach contraction Principle by taking $\frac{p}{Q} = p$, where $p \in [0,1)$. One can also prove another set of examples by restricting the condition on the domain of the function. In the end, we conclude that theory of fixed points can be extended in “b-metric space” for some applications as well and that the analogue of many known results can also be obtained in this literature.

References

[1] Banach S. 1922, Sur les operations dans les ensembles et leur application aux equation sitegrales, Fundam. Math., 3,133–181.
[2] Mlaiki N., Mukheimer A., Rohen Y., Souayah, N. and Abdeljawad T. 2017, Fixed point theorems for alphapsi-contractive mapping in Sh-metric spaces, J. Math. Anal., 8, 40–46.
[3] Choudhury B.S., Metiya N. and Postolache M. 2013, A generalized weak contraction principle with applications to coupled coincidence point problems, Fixed Point Theory Appl, Article No. 152., 21 pages.
[4] Chandok S., Postolache M. 2013, Fixed point theorem for weakly Chatterjea-type cyclic contractions, Fixed Point Theory Appl., Article No. 28, 9 pages.
[5] Mukheimer A., 2014, Common fixed point theorems for a pair of mappings in complex valued b-metric spaces, Adv. Fixed Point Theory, 4, 344–354.
[6] MlekiN. Mukheimer A. 2014, Some common fixed point theorems in complex valued b-metric spaces, Hindawi Pub. Corp, Sci. World J., Article. ID 587825.
[7] Miandaragh M. A. , Postolache M. and Rezapour,S. 2013, Some approximate fixed point results for generalized α-contractive mappings, U. Politeh Buch. Ser. A, 75, 3–10.
[8] Ali M.U., Kamran T. and Postolache M. 2015, Fixed point theorems for multivalued G-contractions in Hausdorff b-Gauge spaces, J. Nonlinear Sci., 8, 847–855.
[9] Shatanawi W., Postolache M. 2013, Common fixed point theorems for dominating and weak annihilator mappings in ordered metric spaces. Fixed Point Theory Appl., Article No. 271.
[10] Kamran T., Postolache M., Fahninuddin M. and Ali M. 2016, Fixed point theorems on generalized metric space endowed with graph, J. Nonlinear Sci. Appl., 9, 4277–4285.
[11] Abodayeh K., Mlaiki N., Abdeljawad T. and Shatanawi W. 2016, Relations between partial metric spaces and M-metric, Caristi-Kirk theorem in M-metric type spaces, J. Math. Anal.,7, 1–12.
[12] Bakhtin A. 1989, The contraction mapping principle in almost metric spaces, Funct. Anal., Gos. Ped. Inst. Unionovsk, 30, 26–37.
[13] Czerwik S. 1993, Contractive mappings in b-metric spaces, Acta Math. Inform. Univ. Ostrav., 1, 5–11.
[14] Aghajani A., Abbas M. and Roshan J.R. 2014, Common fixed point of generalized weak contractive mappings in partially ordered b-metric space. Math Slovaca., 64, 941-960.
[15] Berinde V. 1993, Generalized contractions in quasimetric spaces. Semin. Fixed Pt The Bab Bolyai Univ, 3, 3–9, 1993.
[16] Alqahtani B., Fulga A. and Karapinar E. 2018, Non-unique fixed point results in extended b-metric space, Mathematics, 6, 11 pages.
[17] Kamran T., Postolache M., Ali M.U. and Kiran Q. 2016, Feng and Liu type F-contraction in b-metric spaces with application to integral equations, J. Math. Anal., 7, 18–27.
[18] Ali M.U., Kamran T. and Postolache M. 2017, Solution of Volterra integral inclusion in b-metric spaces via new fixed point theorem, Nonlinear Anal. Mod. Cont., 22, 17-30.
[19] Alsamir H., Salimi M., Noorani M.D., Shatanawi W. and Shaddad F.,2016 Generalized Berinde-type ($\eta, \xi, \theta$) contractive mapping in b-metric spaces with an application, J. Math. Anal., 7, 1–12.
[20] Shatanawi W. 2016, Fixed and Common Fixed Point for Mapping Satisfying Some Nonlinear Contraction in b-metric Spaces, J. Math. Anal., 7, 1–12.

[21] Mukheimer A. 2014, α − ψ − φ-contractive mappings in ordered partial b-metric spaces, J. Nonlinear Sci. Appl., 7, 168–179.

[22] Khan M.S., Singh Y., Maniu G. and M. Postolache 2017, On generalized convex contractions of type-2 in b-metric and 2-metric spaces, J. Nonlinear Sci. Appl., 10, 2902–2913.

[23] Shatanawi W., Pitea A. and Lazovic R. 2014, Contraction conditions using comparison functions on b-metric spaces, Fixed Point Theory and Appl, 11 pages.

[24] Kamran T., Samreen M., and ULAin Q. 2017, A generalization of b-metric space and some fixed point theorems, Mathematics, 5, 7 pages.

[25] Samreen M., Kamran T. and Postolache M. 2018, Extended b-metric space, extended b-comparison function and nonlinear contractions, U. Politeh. Buch. Ser. A, 80, 21-28.

[26] Fahed S., Asif M. and Umar M. 2021, Extension of Fixed-Point Theorems In Extended B-Metric Spaces, Int. J. of Sci. and Research Pub., 11, 153-156.

[27] Mukheimer A., Malaiki N., Abodayeh K. and Shatanawi W. 2019, New theorems on extended b-metric space under new contractions, Non Lin Anal: Mod. and Cont., 24, 870-883.

[28] Alqahtani B., A. Fulga A., Karapınar E. and Raksocevi V. 2019, Contractions with rational inequalities in the extended b-metric space, J. of Inequal. And Appl., 11 pages.

[29] Aydi H., Felhi A., Kamran T., Karainar E. and Ali M.U.2019, On nonlinear contractions in new extended b-metric space, Appl. and Appl. Math, 14, 537-547.

[30] Huang H., Singh Y.M., Khan M. S. and Radenovic S. 2021, Rational type contractions in extended b-metric space, Symmetry, 19 Pages.

