Light sterile neutrinos from flavor symmetries

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Abstract. We study the realization of eV-scale sterile neutrinos in flavor symmetry theories. In particular, we focus on a popular flavor $A_4$ model, and point out that, eV-scale sterile neutrinos can easily be accommodated, while the exact tri-bimaximal mixing pattern is perturbed due to active-sterile mixing. We also illustrate the effects of light sterile neutrinos on the effective mass in neutrino-less double beta decay.

1. Introduction

Despite the successful achievements of neutrino oscillation experiments, there are experimental anomalies that cannot be explained within the standard three neutrino framework. In particular, the possible presence of sterile neutrinos points towards non-standard neutrino physics. The issue of the LSND and MiniBooNE results together with a recent re-evaluation of the anti-neutrino spectra of nuclear reactors [1–3] hint towards the presence of one or two sterile neutrino states with masses at the eV scale. Moreover, the light-element abundances from precision cosmology and Big Bang nucleosynthesis favor extra radiation in the Universe, which could be interpreted with the help of additional sterile neutrinos [4].

Although sterile neutrinos do not directly enter the weak interactions by definition, their admixture with active neutrinos would modify the neutrino flavor mixing and lead to rich experimental phenomena, e.g., the active-sterile oscillations [5], the contributions to the neutrino-less double beta decay amplitude, and the corrections to the beta decay spectra. From the theoretical side, a straightforward extension to the Standard Model to accommodate sterile neutrinos is the seesaw mechanism, in which right-handed neutrinos play the role of sterile neutrinos, whereas they are typically assumed to be many order of magnitude heavier than the Standard Model scale. Possible attempts yielding low-scale sterile neutrinos include the split seesaw mechanism in extra dimensions [6], the Froggatt-Nielsen (FN) mechanism [7, 8], flavor symmetries (e.g., Ref [9–11]), and the minimal extended type I seesaw [12]. Another kind of approaches frequently employed in explaining the neutrino mixing are flavor symmetries, e.g., models based on the tetrahedral group $A_4$. In this work, we illustrate the effects of light sterile neutrinos in $0\nu\beta\beta$, and then study a model combining two of the above-mentioned ideas, i.e., the FN mechanism and flavor symmetry, giving a non-trivial origin of eV-scale sterile neutrinos.

2. Sterile neutrinos in neutrino-less double beta decay

Assuming that there are $n_s = n - 3$ sterile neutrinos, the neutrino mass matrix then takes an $n \times n$ form, and can be diagonalized by using a unitary transformation as $m_\nu = U \text{diag}(m_1, m_2, ..., m_n)U^T$. The neutrino flavor eigenstates $\nu_f$ are therefore related to their mass

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eigenstates \( \nu_i \) via \( \nu_f = \sum_i U_{fi} \nu_i \). In the 4 \times 4 case (namely, \( n_s = 1 \)), \( U \) is parameterized by \( U = R_{34} \tilde{R}_{24} R_{14} R_{23} R_{13} R_{12} P \), where \( R_{ij} \) denote the rotations in \( ij \) space, and \( P = \text{diag}(1, e^{i\alpha/2}, e^{i(\beta/2+\delta_{13})}, e^{i(\gamma/2+\delta_{14})}) \) contains the three Majorana phases \( \alpha, \beta \) and \( \gamma \). Note that the sterile neutrino masses could be either heavier or lighter than the active neutrinos. For the latter case, the active neutrino masses should be around the eV scale, which induces more tensions with cosmological observations on the total neutrino mass. We will thus in the following parts only consider the case \( m_{\text{sterile}} \gg m_{\text{active}} \).

In the four neutrino mixing case, the effective neutrino mass in 0\( \nu/\beta\beta \) is given by

\[
\langle m_{ee} \rangle_{4\nu} = \left| c_{14}^2 \langle m_{ee} \rangle_{3\nu} + s_{14}^2 \sqrt{\Delta m^2_{41}} e^{i\gamma} \right|,
\]

where \( \langle m_{ee} \rangle_{3\nu} = c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 e^{i\alpha} + s_{13}^2 c_{14}^2 m_3 e^{i\beta} + s_{14}^2 m_4 e^{i\gamma} \) is the standard expression for three active neutrinos. By using data from Ref. [2], the allowed range of \( \langle m_{ee} \rangle_{4\nu} \) as a function of the lightest \( m_{\text{light}} \) are shown in Fig. 1. One observes from the plot that, in the presence of one sterile neutrino, the standard behavior can be completely mixed up. In particular, if the lightest neutrino mass is reasonably small, e.g. \( m_{\text{light}} < 0.01 \text{ eV} \), the allowed range of \( \langle m_{ee} \rangle_{4\nu} \) is dominated by the term \( s_{14}^2 \sqrt{\Delta m^2_{41}} \simeq 0.031 \text{ eV} \), which means that \( \langle m_{ee} \rangle_{4\nu} \) cannot vanish in the normal ordering case (the contribution of the two light active neutrinos cannot cancel that of the sterile neutrino). However, in the inverted ordering \( \langle m_{ee} \rangle_{4\nu} \) can vanish even for very small active neutrino masses. The effective mass can also be zero in the regime where the active neutrinos are quasi-degenerate (\( m_{\text{light}} > 0.1 \text{ eV} \)). This modified behavior of \( \langle m_{ee} \rangle \) is of particular interest in the inverted ordering case, since the usual lower bound on the effective mass (cf. the solid and dashed lines in the right panel of Fig. 1) is no longer valid (see, e.g., Ref. [13] for detailed discussions). A similar situation holds for two sterile neutrino case. If future 0\( \nu/\beta\beta \) experiments measure a tiny effective mass and the neutrino mass hierarchy is confirmed to be inverted from long-baseline neutrino oscillations, the sterile neutrino hypothesis would be an attractive explanation for this inconsistency.
allows the hierarchies by powers of the small parameter $a = 2x_a \frac{\Lambda^2}{\Lambda_3}$, where $\Lambda$ is the cut-off scale, and the dots stand for higher dimensional operators. If one chooses the real basis for $A_4$, along with the flavon VEV alignments $\langle \xi \rangle = u$, $\langle \varphi \rangle = (v, 0, 0)$ and $\langle \varphi' \rangle = (v', v', v')$, then the charged-lepton mass matrix is diagonal, and the $4 \times 4$ neutrino mass matrix reads

$$M_{\nu}^{4 \times 4} = \begin{pmatrix}
a + \frac{2d}{3} & -\frac{d}{3} & -\frac{d}{3} & e \\
\cdot & \frac{2d}{3} & a - \frac{d}{3} & e \\
\cdot & \cdot & 2d & e \\
\cdot & \cdot & \cdot & m_s
\end{pmatrix},$$

(3)

where $a = 2x_a \frac{\mu^2}{\Lambda_3^2}$, $d = 2x_d \frac{\nu^2}{\Lambda_3^2}$, $e = \sqrt{2}x_e \frac{\nu'\nu}{\Lambda_3^2}$, and $m_s = x_s \frac{\tau^2}{\Lambda_3}$ have dimensions of mass. Note that the active-sterile neutrino couplings and the sterile neutrino mass are both suppressed by powers of $a$.

Assuming $v'/\Lambda \approx v/\Lambda \approx u/\Lambda \approx \langle \theta \rangle /\Lambda \approx 10^{-1.5}$ together with the cut-off scale $\Lambda \approx 10^{12.5}$ GeV, one can estimate that $a \sim d \sim e \sim 0.1$ eV, and $m_s \approx 10^{0.5}$ eV with the assumption that the Yukawa couplings $x_{a,d,e}$ are of order 1. As a result, the choice of the mass scales allows the hierarchies $a \sim d \sim e < m_s$. We can then approximately diagonalize $M_{\nu}^{4 \times 4}$ (i.e., $M_{\nu}^{4 \times 4} = U \text{diag}(m_1, m_2, m_3, m_4)U^T$), and obtain

$$U \approx \begin{pmatrix}
\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 & 0 \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\
0 & 0 & \frac{1}{\sqrt{3}} & 0
\end{pmatrix} + \begin{pmatrix}
0 & 0 & 0 & \frac{e}{m_s} \\
0 & 0 & 0 & \frac{m_s}{\sqrt{2}m_2} \\
0 & 0 & 0 & \frac{m_s}{\sqrt{2}m_2} \\
0 & 0 & \frac{3e^2}{m_s} & 0
\end{pmatrix} + \begin{pmatrix}
0 & -\frac{\sqrt{3}e^2}{2m_2} & 0 & 0 \\
0 & 0 & -\frac{\sqrt{3}e^2}{2m_2} & 0 \\
0 & -\frac{\sqrt{3}e^2}{2m_2} & 0 & 0 \\
0 & 0 & 0 & -\frac{3e^2}{2m_s^2}
\end{pmatrix},$$

(4)

### Table 1.

| Field      | L | $e^c$ | $\mu^c$ | $\tau^c$ | $h_{u,d}$ | $\varphi$ | $\varphi'$ | $\xi$ | $\nu_s$ |
|------------|---|-------|---------|----------|-----------|-----------|------------|-------|---------|
| $SU(2)_L$  | 2 | 1     | 1       | 1        | 2         | 1         | 1          | 1     | 1       |
| $A_4$      | $\frac{3}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |
| $Z_3$      | $\omega$ | $\omega^2$ | $\omega^2$ | $\omega^2$ | $1$ | $1$ | $\omega$ | $\omega$ | $1$ |
| $U(1)_{FN}$ | $-3$ | 1 | 0 | - | - | - | - | - | 6 |

3. Flavor $A_4$ model of sterile neutrinos

We proceed to show the realization of light sterile neutrinos in a modified flavor $A_4$ symmetry model [14], which accommodates both the tri-bimaximal mixing and one or two light sterile neutrinos. The relevant particle assignments are shown in Table 1, where a flavon $\theta$ carrying a negative unit of the FN charge is also introduced, suppressing the corresponding mass term by powers of the small parameter $\langle \theta \rangle /\Lambda \equiv \lambda < 1$.

With the above particle assignments, along with the $A_4$ multiplication rules, we arrive at the invariant Lagrangian

$$L_Y = \frac{y_e}{\Lambda} \langle \varphi L \rangle h_d + \frac{y_{\mu}}{\Lambda} \langle \varphi L \rangle h_d + \frac{y_{\tau}}{\Lambda} \langle \varphi L \rangle h_d + \frac{x_a}{\Lambda^2} \langle \varphi' L_h u_h \rangle + \frac{x_d}{\Lambda^2} \langle \varphi' L_h u_h \rangle + \frac{x_f}{\Lambda^2} \langle \varphi' L_h u_h \rangle + \frac{x_s}{\Lambda} \langle \varphi \varphi \rangle + \text{h.c.} + \ldots,$$

(2)
where the first term reproduces the tri-bimaximal mixing, while the higher order corrections second and third terms are suppressed by the ratio $e/m_s$. Furthermore, the eigenvalues are

$$
\begin{align*}
m_1 &= a + d, & m_2 &= a - \frac{3e^2}{m_s}, & m_3 &= -a + d, & m_4 &= m_s + \frac{3e^2}{m_s}.
\end{align*}
$$

(5)

One immediately observes that the active neutrino mass spectrum is slightly modified by the sterile neutrino, while $m_4 \simeq m_s$ is stabilized at the eV scale. Compared to the parametrization of $U$, we obtain

$$
\sin^2 \theta_{12} \simeq \frac{1}{3} - \frac{2}{3} \left( \frac{e}{m_s} \right)^2, \quad \sin^2 \theta_{23} \simeq \frac{1}{2} + \frac{1}{2} \left( \frac{e}{m_s} \right)^2,
$$

(6)

and a vanishing $\theta_{13}$. In addition, the active-sterile mixing angles are of similar magnitude and given by

$$
\sin^2 \theta_{14} \simeq \sin^2 \theta_{24} \simeq \sin^2 \theta_{34} \simeq \left( \frac{e}{m_s} \right)^2.
$$

(7)

Therefore, the desired active-sterile mixing $\sin^2 \theta_{14} \sim 0.01$ could be obtained by taking, e.g., $m_s \sim 1$ eV and $e \sim 0.1$ eV.

Note that, the flavor model under discussion can be easily generalized to a seesaw framework. A detailed survey could be useful and will be elaborated on in future.

4. Conclusion

We have pointed out that the presence of light sterile neutrinos would significantly change the effective mass measured in neutrino-less double beta decay experiments. In particular, the sterile neutrino contributions to the effective mass could lead to a vanishing $\langle m_{ee} \rangle$, even if three active neutrinos feature an inverted mass ordering, which is clearly different from the predictions of the standard three neutrino picture. We also presented a flavor $A_4$ model, which accommodates both a light sterile neutrino and deviations from the exact tri-bimaximal mixing. We further stress that a keV sterile neutrino playing the role of warm dark matter could also be included in the flavor $A_4$ model by choosing smaller FN charges.

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References

[1] Mention G et al 2011 Phys. Rev. D 83 073006 (Preprint arXiv:1101.2755 [hep-ex])
[2] Kopp J, Maltoni M and Schwetz T 2011 Phys. Rev. Lett. 107 (Preprint arXiv:1103.4570 [hep-ph])
[3] Giunti C and Laveder M 2011 Preprint arXiv:1109.4033 [hep-ph]
[4] Hamann J, Hannestad S, Raffelt G G, Wong Y Y Y 2011 Preprint arXiv:1108.4136 [astro-ph.CO]
[5] Zhang H 2007 Mod. Phys. Lett. A 22 1341-1348 (Preprint hep-ph/0606040)
[6] Kusenko A, Takahashi F and Yanagida T T 2010 Phys. Lett. B 693 144-148 (Preprint arXiv:1006.1731 [hep-ph])
[7] Froggatt C D and Nielsen H B 1979 Nucl. Phys. B 147 277
[8] Merle A and Niro V 2011 JCAP 1107 023 (Preprint arXiv:1105.5136 [hep-ph])
[9] Mohapatra R N 2001 Phys. Rev. D 64 091301 (Preprint hep-ph/0107264)
[10] Shaposhnikov M 2007 Nucl. Phys. B 763 49-59 (Preprint hep-ph/0605047)
[11] Lindner M, Merle A and Niro V 2011 JCAP 1101 034 (Preprint arXiv:1011.4950 [hep-ph])
[12] Barry J, Rodejohann W and Zhang H 2011 JHEP 1107 091 (Preprint arXiv:1105.3911 [hep-ph])
[13] Rodejohann W 2011 Preprint arXiv:1106.1334 [hep-ph]
[14] Altarelli G and Feruglio F 2005 Nucl. Phys. B 720 64-88 (Preprint hep-ph/0504165)