Far-infrared laser action from parabolic quantum dots matrix

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Abstract. In this paper we present results of calculations for quantum dots matrix acting as an active medium in novelty proposal of far-infrared laser. The proposal is based on the pumping laser by rapid (nonadiabatic) switching on \textit{in-plane} electric field which allows us to obtain population inversion. The numerical analysis of electron-photon system kinetics was performed for various electric fields and temperatures. These calculation utilises the method of solving the Cauchy problem for infinite chain of linear differential equations. Also the contribution of dynamics of non-radiative transitions mediated by the phonons has been taken account. The obtained results indicate that by the properly chosen of QDs parameters it is possible to reduce the rate of these transitions in the degree that allows us to obtain good laser efficiency at low temperatures.

1. Introduction

The quantum dots (QDs) \cite{1} also called ‘artificial atoms’ offer an excellent opportunities to engineering the energy levels separations. Thus they are very promising nanostructures that could be used in the producing of coherent electromagnetic radiation in the infrared region of spectrum. Such radiation could be applied in telecommunications (the near-infrared radiation), in remotely detection of various substances - especially in the detectors of molecules of explosive materials - as the spectrum of rotational and vibrational modes of such particles is spanned in the infrared region. Also important application of such radiation is the area of investigations of solid state properties in high magnetic fields (the Larmor frequency in such fields lies in the far-infrared region). In lasers based on QD’s the emission of photons is obtained either by recombination of exciton trapped in QD or by intraband transition of the electron to the lower energetic level in QD. For such devices the emission in meV range is attainable \cite{2}.

In this paper we present novel proposal of a laser based on the quantum dots matrix with harmonic confining potential placed between plates of capacitor and acted as lasing medium. The population inversion is obtained by rapid (nonadiabatically) switching on the \textit{in-plane} electric field. In the case under consideration (i. e. the parabolic confining potential) we deal with infinite number of energetic levels which resulting in cascade amplification effect. Electron-phonon coupling and its influence on the state of the system was taken into account. This interaction could lead to diminish laser action because it causes the nonradiative transitions which could prevent the maintain of population inversion during sufficiently long time. The
obtained results indicates that the properly choosing of QD’s thickness results in reductions of this interaction which yields in adequate lifetime of electronic states on excited levels.

2. System of Quantum Dots
The lasing medium is material consists the matrix of planar QDs with parabolically shaped confining potential. The spacing between levels in the z direction is much more greater than in the plane of the dots so for small energies of excitations only the \( x - y \) plane could be considered (i.e. the 2D harmonic oscillator). The QDs in matrix are assumed to be placed sparse the interaction between them could be neglected. Let us assume that each dot contains one electron in its ground state with wavefunction \( \Psi_{n=0m=0}(\vec{r}) \) and energy \( E_{0}^{(0)} = \hbar\omega \) where \( \omega \) is the characteristic frequency of oscillator the superscript \( (0) \) indicates the situation without electric field and \( n, m \) are principal and magnetic quantum numbers respectively. In order to obtain a population inversion of QDs matrix, an in-plane electric field is rapidly (meaning switching time \( \Delta t \ll \omega^{-1} \)) switched on, which leads to shifted confining potential (both the axis of symmetry of potential and energetic levels). This gives the eigenstates and the eigenergies of the form \( \Psi_{nm}(\vec{r}) = \Psi_{nn}(\vec{r} - \vec{r}_0) \) and \( E_0 = \hbar\omega - e^2E^2/(2\mu\omega^2) \) respectively (here \( \vec{r}_0 = c\vec{E}/(\mu\omega^2) \)) and \( \mu \) is the electron rest mass. The stationary states in the presence of in-plane electric field do not coincide with the original states of QDs, and their overlaps define the transition probabilities [3]. The probability of finding an electron on \( n-th \) level reads:

\[
\bar{w}_n = \frac{1}{n!} \left( \frac{x_0^2}{2} \right)^n e^{-x_0^2/2},
\]

where \( x_0^2 = (e^2E^2)/\mu\hbar\omega^3 \) (one can notice that \( \bar{n} = x_0^2/2 \) for averaged number of occupied states), and dimensionless variable \( x_0 = r_0/l_0 \), where \( l_0^2 = \hbar/\langle \mu\omega \rangle \) was introduced. The above distribution of electron states depends on the magnitude of applied in-plane electric field via the relation \( x_0 = eE/\sqrt{\mu\hbar\omega^3} \).

3. Coupling between electrons and photons
In order to obtain stimulated emission coupling of electron system with photons is necessary. Due to the energy conservation principle the photons energy have to coincide with the interlevel spacing in QDs (i.e. \( \hbar\omega \)). The Fermi golden rule and the dipole approximation leads to the probability of spontaneous transition of electron in QD from the \( n-th \) state to the state \( n'-th \) per time unit in the form: \( W_n^m = A [n\delta_{n',n-1} + (n+1)\delta_{n',n+1}] \) where \( A = 4/3 \left( \omega^3/\hbar\delta^3 \right) e^2l_0^2 \) is the probability of spontaneous emission and the \( m-th \) degeneracy (due to absence of magnetic field) has been taken account. The time evolution of number of QDs with electrons on any energetic level could be described by the infinite chain of linear differential equations:

\[
\frac{dn_k}{dt} = -n_k (b+c) + bn_{k+1} + cn_{k-1},
\]

where \( k = 1, 2, 3, \ldots \) and \( b = A(\nu + 1), c = A\nu, \nu(\omega) = [\exp(\hbar\omega/k_BT) - 1]^{-1} \) is the equilibrium distribution of photons at the temperature \( T \). To the above system equation for conservation of number of electrons: \( N = n_0(t) + \sum_{k=1}^\infty n_k(t) \) and the initial conditions: \( n_0(0) = Nw_0 \) have to be added. The time dependent inverse-filling factor (i.e. the difference between QDs in excited states and in the ground state) could be written in the form [5]:

\[
\Delta n(t) = 2N_0(t) - N = 2N_0(t) - N \left\{ 2\frac{e^2E}{\mu\hbar} + 4\int_{-1}^1 d\omega \sqrt{1 - \omega^2} e^{-((b+c-2\omega\sqrt{\delta})t)}v_0(\omega) \right. \\
\times \sum_{m=0}^\infty \left( \frac{\delta}{\omega} \right)^{m/2} \left( 1 - \sum_{l=0}^m w_l \right) - \left( \frac{\delta}{\omega} \right)^{1+m/2} v_m(\omega) \right\},
\]
where \( v_m(\omega) \) are the Chebyshev polynomials of the second kind. To calculate the value of the integral presented in the above formula the numerical estimation has been performed.

Our aim was to find the \( \Delta n \) dependency on time and to estimate the parameter \( t_0 \) which is determined by the condition \( \Delta n(t_0) = 0 \) (i.e. when the laser should be pumped again - the \textit{in-plane} electric field have to be turning off and turning on nonadiabatically). The parameters of QDs used in calculations we assumed as follows: Energy spacing \( 4.15 \text{ meV} \), which corresponds to parabolic conising parameter \( \omega = 6.28 \times 10^{12} \text{ Hz} \) and wavelength \( \lambda = 300 \mu \text{m} \), the related characteristic dimensions of QDs \( l_0 = \sqrt{h/(\mu \omega)} = 16 \text{ nm} \). Using those values the maximal operating temperature \( T_{\text{max}} \approx 70 \text{ K} \) and the rate of spontaneous emission \( A = 7350 \text{ s}^{-1} \).

Obtained results (presented on fig. 1 a)) indicated that the characteristic scale for considered system is \( t_0 \sim 10^{-5} t_0 - 10^{-4} \tau_0 \) so the frequency of repeating cycle of turning on and turning off \textit{in-plane} electric field is of order \( 10^8 - 10^9 \text{ s}^{-1} \) which is technically attainable.

The very small scale of switching on the electric field (\( \sim 10^{-12} \text{ s} \)) needs ultra-high frequency techniques. The main restriction is the limitation of the capacity (up to single pF) because the RC time. To avoid considerable inhomogenity of time-dependent (during the switching time) electric field inside capacitor its radius should be well smaller than 2.405c/\( \omega \). This limits the radius of capacitor to the few \( \mu \text{m} \) range.

4. Electron-phonon interactions and dissipative transitions

The non-radiative transitions in QDs mediated by phonons are strongly suppressed [6], [7] (so-called \textit{phonon-bottleneck effect}) due to the \( \delta \)-shaped density of states in such structures. In medium such as GaAs both LA and LO modes couple to electron degrees of freedom. Due to the energy range which we deal with (\( \sim 3 - 6 \text{ meV} \)) that lies well below LO phonon energy (\( \sim 36 \text{ meV} \) in GaAs) only the LA mode couples to the electrons. The Fermi golden rule gives the rate of electronic transitions due to the electron-phonon interactions:

\[
\tau_{i\to f}^{-1} = \frac{\hbar^2 \alpha(q_0)}{\pi \hbar^2 c} \left( n_B + \frac{1}{2} \pm \frac{1}{2} \right) \int_0^{\pi/2} \sin \vartheta d\vartheta F_z^2 (l_z q_0 \cos \vartheta) G_{if}^2 (l_0 q_0 \sin \vartheta),
\]

where the \( i \) and \( f \) denotes the initial and final states respectively, \( n_B \) denotes the Bose-Einstein distribution for phonons and \( q_0 \) corresponds to the wave number of phonon with energy \( \hbar \omega \). The + and − signs corresponds to emission and absorption of phonon respectively. The function \( G_{if} \) has the form:

\[
G_{if}(l_0 q) = \sum_{j=0}^{n_f} \sum_{l=0}^{n_f} \frac{(-1)^{j+l}}{j!} \left( \frac{n_f^t + |m_f^t|}{n_f - j} \right) \left( \frac{n_f^l + |m_f^l|}{n_f^l - l} \right) \Gamma(\zeta_{if} + j + l + 1) \left( \frac{\zeta_{if} + j + l + 1}{2!} \right)^{m_f^t - m_f^l}
\]

where \( \zeta_{if} = \left( |m_f^t| + |m_f^l| + m_f^t - m_f^l \right) /2 \). \( n_f \) and \( m_f \) are radial and magnetic number respectively. The factor \( \alpha \) corresponds to the specific kind of phonons and for LA phonons it reads:

\[
\alpha(q) = \hbar q / (2 \rho_{\text{mat}} c_s) \sigma^2\]

where \( \rho_{\text{mat}}, c_s \) and \( \sigma \) denotes the density of material, the speed of sound in the material and the deformation potential constant respectively. In the calculations we have used material parameters for GaAs, i.e. \( \rho_{\text{mat}} = 5320 \text{ kg/m}^3 \), \( c_s = 5150 \text{ m/s} \) and \( \sigma = 6.8 \text{ eV} \). The above formula shows that for thicker QDs the lifetime of excited electronic states is substantially elongated, so by properly chosen QDs parameters the strength of electron-phonon coupling could be reduced.

It should be emphasized that the increasing of QD thickness have to be limited because of the fact that for very thick QDs the model of calculations of 2D harmonic potential should be expanded to the full 3D model. However e.g. for QDs with the ratio \( l_z/l_0 \approx 1/3 \) the energetic
Figure 1. a) The dependence of dimensionless time $\tau_0 = -2it_0\sqrt{bc}$ (indicates period of time in which inverse filling factor reaches 0) on magnitudes of applied *in-plane* electric field for various temperatures and b) the numerical results for time $t_{\text{relax}}$ after which the electrons on the first excited level falls down due to the interaction of phonons and its dependence on temperature and $\beta = l_z/l_0$.

5. Conclusions
Summarizing, we have presented the theoretical model of the far-infrared laser with active medium consisted the parabolic planar quantum dot matrix. We have assumed the parabolic shape of QDs confining potential and one electron trapped in each dot. The population inversion has been obtained by the nonadiabatically turning on the *in-plane* electric field which allows for greater occupation of excited states than the ground state (i.e. the population inversion). The rate of non-radiative transitions caused by phonons were analyzed and the numerical calculation has been performed. Obtained results shows that for thick dots the lifetime of excited electrons could be long enough to obtain reasonable laser efficiency.

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