A Complexity Approach for Steganalysis

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Résumé

In this proposal for the Journées Codes et Stéganographie 2012, we define a new rigorous approach for steganalysis based on the complexity theory. It is similar to the definitions of security that can be found for hash functions, PRNG, and so on. We propose here a notion of secure hiding and we give a first secure hiding scheme.

1 Introduction

Robustness and security are two major concerns in information hiding. These two concerns have been defined in [6] as follows. “Robust watermarking is a mechanism to create a communication channel that is multiplexed into original content [...]. It is required that, firstly, the perceptual degradation of the marked content [...] is minimal and, secondly, that the capacity of the watermark channel degrades as a smooth function of the degradation of the marked content. [...]. Watermarking security refers to the inability by unauthorized users to have access to the raw watermarking channel. [...] to remove, detect and estimate, write or modify the raw watermarking bits.”

In the framework of watermarking and steganography, security has seen several important developments since the last decade [1, 4, 7]. The first fundamental work in security was made by Cachin in the context of steganography [2]. Cachin interprets the attempts of an attacker to distinguish between an innocent image and a stego-content as a hypothesis testing problem. In this document, the basic properties of a stegosystem are defined using the notions of entropy, mutual information, and relative entropy. Mittelholzer, inspired by the work of Cachin, proposed the first theoretical framework for analyzing the security of a watermarking scheme [8].

These efforts to bring a theoretical framework for security in steganography and watermarking have been followed up by Kalker, who tries to clarify the concepts (robustness vs. security), and the classifications of watermarking attacks [9]. This work has been deepened by Furon et al., who have translated Kerckhoffs’ principle (Alice and Bob shall only rely on some previously shared secret for privacy), from cryptography to data hiding [5]. They used Diffie and Hellman methodology, and Shannon’s cryptographic framework [10], to classify the watermarking attacks into categories, according to the type of information Eve has access to [4, 9], namely: Watermarked Only Attack (WOA), Known

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Message Attack (KMA), Known Original Attack (KOA), and Constant-Message Attack (CMA). Levels of security have been recently defined in these setups. The highest level of security in WOA is called stego-security [3], recalled below.

In the prisoner problem of Simmons [11], Alice and Bob are in jail, and they want to, possibly, devise an escape plan by exchanging hidden messages in innocent-looking cover contents. These messages are to be conveyed to one another by a common warden, Eve, who over-drops all contents and can choose to interrupt the communication if they appear to be stego-contents. The stego-security, defined in this framework, is the highest security level in WOA setup [3].

To recall it, we need the following notations:

– \( K \) is the set of embedding keys,
– \( p(X) \) is the probabilistic model of \( N_0 \) initial host contents,
– \( p(Y|K_1) \) is the probabilistic model of \( N_0 \) watermarked contents.

Furthermore, it is supposed in this context that each host content has been watermarked with the same secret key \( K_1 \) and the same embedding function \( e \).

It is now possible to define the notion of stego-security:

**Definition 1 (Stego-Security)** The embedding function \( e \) is **stego-secure** if and only if:

\[
\forall K_1 \in K, p(Y|K_1) = p(X).
\]

## 2 Toward a Cryptographically Secure Hiding

### 2.1 Introduction

Almost all branches in cryptology have a complexity approach for security. For instance, in a cryptographic context, a pseudorandom number generator (PRNG) is a deterministic algorithm \( G \) transforming strings into strings and such that, for any seed \( k \) of length \( k \), \( G(k) \) (the output of \( G \) on the input \( k \)) has size \( \ell_G(k) \) with \( \ell_G(k) > k \). The notion of secure PRNGs can now be defined as follows.

**Definition 2** A cryptographic PRNG \( G \) is secure if for any probabilistic polynomial time algorithm \( D \), for any positive polynomial \( p \), and for all sufficiently large \( k \)’s,

\[
| \Pr[D(G(U_k)) = 1] - \Pr[D(U_{\ell_G(k)}) = 1] | < \frac{1}{p(k)},
\]

where \( U_r \) is the uniform distribution over \( \{0, 1\}^r \) and the probabilities are taken over \( U_N, U_{\ell_G(N)} \) as well as over the internal coin tosses of \( D \).

Intuitively, it means that no polynomial-time algorithm can make a distinction, with a non-negligible probability, between a truly random generator and \( G \).

Inspired by these kind of definitions, we propose what follows.

### 2.2 Definition of a stegosystem

**Definition 3 (Stegosystem)** Let \( \mathcal{A} \) an alphabet and \( \mathcal{S}, \mathcal{M}, \mathcal{K} \) three sets of words on \( \mathcal{A} \) called respectively the sets of supports, messages, and keys.

A **stegosystem** on \( (\mathcal{S}, \mathcal{M}, \mathcal{K}) \) is a tuple \( (I, \mathcal{E}, \text{inv}) \) such that:
- \( \mathcal{I} : \mathcal{S} \times \mathcal{M} \times \mathcal{K} \rightarrow \mathcal{S}, (s, m, k) \mapsto \mathcal{I}(s, m, k) = s' \),
- \( \mathcal{E} : \mathcal{S} \times \mathcal{K} \rightarrow \mathcal{M}, (s, k) \mapsto \mathcal{E}(s, k) = m' \),
- \( \text{inv} : \mathcal{K} \rightarrow \mathcal{K}, \text{s.t. } \forall k \in \mathcal{K}, \forall (s, m) \in \mathcal{S} \times \mathcal{M}, \mathcal{E}(\mathcal{I}(s, m, k), \text{inv}(k)) = m \).
- \( \mathcal{I}(s, m, k) \) and \( \mathcal{E}(c, k') \) can be computed in polynomial time.

\( \mathcal{I} \) is called the insertion function, \( \mathcal{E} \) the extraction function, \( s \) the host content, \( m \) the hidden message, \( k \) the embedding key, \( k' = \text{inv}(k) \) the extraction key, and \( s' \) the stego-content. If \( \forall k \in \mathcal{K}, k = \text{inv}(k) \), the stegosystem is symmetric, otherwise it is asymmetric.

**Definition 4 (Bounded stegosystem)** Let \( f : \mathbb{N} \rightarrow \mathbb{N} \). A stegosystem \( (\mathcal{I}, \mathcal{E}, \text{inv}) \) of \( (\mathcal{S, M, K}) \) is \( f \)-bounded if \( \text{dom}(\mathcal{I}) \subseteq \bigcup_{n \in \mathbb{N}} \mathfrak{A}^n \times \mathfrak{A}^{f(n)} \times \mathcal{K} \).

**Definition 5 (Probability set)** A probability set \( \mathcal{X} = \{(S_n, P_n), n \in \mathbb{N}\} \) on \( \mathfrak{A} \) is an infinite family of couples of finite sets \( S_n \subseteq \mathfrak{A}^* \) together with their probability distributions \( P_n \), such that for every \( n \in \mathbb{N} \), there exists \( r \in \mathbb{N} \) such that each element of \( S_n \) is in \( \mathfrak{A}^r \). The integer \( r \) is denoted \( \ell(S_n) \). Moreover it is required that \( \ell(S_n) \) is bounded by a polynomial in \( n \).

### 2.3 Cryptographically secure hiding

**Definition 6 (Secure hiding)** Let \( f : \mathbb{N} \rightarrow \mathbb{N} \), \( (\mathcal{I}, \mathcal{E}, \text{inv}) \) a stegosystem of \( (\mathcal{S, M, K}) \), and \( \mathcal{X} = \{(S_n, P_n), n \in \mathbb{N}\} \) a probability set. \( \mathcal{I} \) is a \( f \)-secure hiding for \( \mathcal{X} \) if it is \( f \)-bounded and if for every positive polynomial \( p \), for any probabilistic polynomial-time algorithm \( \mathcal{D} \), for any \( k \in \mathcal{K} \), for all sufficiently large \( i \)'s, for all \( m \in \mathfrak{A}^{f(\ell(S_i))} \),

\[
\left| \Pr(\mathcal{D}(\mathcal{I}(S_i, m, k)) = 1) - \Pr(\mathcal{D}(S_i) = 1) \right| < \frac{1}{p(i)} + \frac{1}{p(k)} \quad (1)
\]

where the probabilities are taken over the distribution \( \mathcal{X} \) as well as over the coin tosses of \( \mathcal{D} \) and where \( \varepsilon(k) \rightarrow 0 \) when the size of \( k \) grows up.

Intuitively, it means that there is no polynomial-time probabilistic algorithm being able to distinguish the host contents from the stego-contents.

**Proposition 1** If \( f_1 \leq f_2 \) one can compute from any \( f_2 \)-secure hiding stegosystem for \( \mathcal{X} \) a \( f_2 \)-secure hiding stegosystem for \( \mathcal{X} \).

**Example 1** Assume that \( \mathfrak{A} = \{0, 1\} \), and that
- there exists \( \alpha \in \mathbb{N} \) such that for every \( n \), \( |S_n| = 2^{\ell(S_n) - \alpha} \) and,
- for each \( n \), there exists \( s_n \in \{0, 1\}^* \) such that \( S_n = \{s_n \cdot w \mid w \in \{0, 1\}^{\ell(S_n) - \alpha}\} \), where \( \cdot \) is the concatenation product and,
- for each \( n \), each \( s \in S_n \), \( P_n(s) = 1/|S_n| \).

Let \( G \) be a cryptographic PRNG. We consider that \( \mathcal{I}(s, m, k) \) is defined for \( s \in S_n \) and \( m \in \{0, 1\}^{\ell(S_n) - \alpha} \), for \( k \) such that the length of \( G(k) \) is \( \ell(S_n) - \alpha \), by \( I(s, m, k) = s_n \cdot (m \oplus G(k)) \). Let \( f_0 \) be the function mapping \( n \) into \( n/\alpha \).

The symmetric stegosystem defined by \( \mathcal{I} \) and where \( \mathcal{E}(s_n, k) = w \oplus G(k) \) is a \( f_0 \)-secure hiding stegosystem for \( \mathcal{X} \).
3 Conclusion

We thus intend to propose to these Journées Codes et Stéganographie 2012 this new rigorous approach for steganalysis based on the complexity theory. It is inspired by the definitions of security that can usually be found in other branches of cryptology. We have proposed a notion of secure hiding and presented a first secure hiding scheme. Our intention is to prove this result during the presentation, and to give further developments, by adapting such a notion to define using complexity what is a robust watermarking, and what is a message that cannot be extracted.

Références

[1] Mauro Barni, Franco Bartolini, and Teddy Furon. A general framework for robust watermarking security. *Signal Processing*, 83(10):2069–2084, 2003. Special issue on Security of Data Hiding Technologies, invited paper.

[2] Christian Cachin. An information-theoretic model for steganography. In *Information Hiding*, volume 1525 of *Lecture Notes in Computer Science*, pages 306–318. Springer Berlin / Heidelberg, 1998.

[3] F. Cayre and P. Bas. Kerckhoffs-based embedding security classes for wa data hiding. *IEEE Transactions on Information Forensics and Security*, 3(1):1–15, 2008.

[4] F. Cayre, C. Fontaine, and T. Furon. Watermarking security : theory and practice. *IEEE Transactions on Signal Processing*, 53(10):3976–3987, 2005.

[5] T. Furon. Security analysis, 2002. European Project IST-1999-10987 CERTIMARK, Deliverable D.5.5.

[6] T. Kalker. Considerations on watermarking security. pages 201–206, 2001.

[7] Andrew D. Ker. Batch steganography and pooled steganalysis. In Jan Camenisch, Christian S. Collberg, Neil F. Johnson, and Phil Sallee, editors, *Information Hiding*, volume 4437 of *Lecture Notes in Computer Science*, pages 265–281, Alexandria, VA, USA, July 2006. Springer.

[8] Thomas Mittelholzer. An information-theoretic approach to steganography and watermarking. In Andreas Pfitzmann, editor, *Information Hiding*, volume 1768 of *Lecture Notes in Computer Science*, pages 1–16, Dresden, Germany, September 29 - October 1. 1999. Springer.

[9] Luis Perez-Freire, F. Pérez-gonzalez, and Pedro Comesana. Secret dither estimation in lattice-quantization data hiding : A set-membership approach. In Edward J. Delp and Ping W. Wong, editors, *Security, Steganography, and Watermarking of Multimedia Contents*, San Jose, California, USA, January 2006. SPIE.

[10] Claude E. Shannon. Communication theory of secrecy systems. *Bell Systems Technical Journal*, 28:656–715, 1949.

[11] Gustavus J. Simmons. The prisoners’ problem and the subliminal channel. In *Advances in Cryptology, Proc. CRYPTO’83*, pages 51–67, 1984.