A New Modeling of Periodic Inspection Interval Optimization in Computer System with Failure Interaction

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ABSTRACT

The hidden failure is commonly occur in different industries with increasing operational complexity. The component of computers is an example for hidden failure. This component called soft component. In this study two component repairable systems with failure interaction are considered. The failure of these components can be soft or hard. The hard failure causes the system stop, while the soft failure does not, but it increases the system operating costs too. The periodic inspections have a fix cost to system. When the first component fails, it remains in a failed state until the next inspection time. Therefore, if the first component failed in each inspection interval, a down time penalty cost is incurred. The cost is proportional to the elapsed time from failure time to its detection at inspection time. The short inspection interval increases the number of inspection and causes the extra cost for system. As well, long inspection interval causes the greater cost due to long elapsed time between real occurrence of the failure time and the failure detection (penalty cost). On a finite time horizon, the objective of current study is to figure out the optimal inspection interval for the soft failure component to minimize the expected total cost. Therefore, an adapted sample problem is resolved and numerical results are presented.

Key words: Periodic inspection interval, optimization, repairable system, failure interaction, computer failure

INTRODUCTION

Now-a-days, the repairable systems caused complexity in evaluation reliability of systems. While the component of repairable system was failed, it may have been an increasing impact on the other component failure rate. The interactions between components can be categorized into economic, structural and stochastic dependences. The economic dependence implies that grouping maintenance actions either save costs (economies of scale) or result in higher costs (because of, e.g., high down-time costs) as compared with individual maintenance. The stochastic dependence occurs when the condition of components influences the lifetime distribution of other components. The structural dependence applies if components structurally form a part, so that maintenance of a failed component implies maintenance of working components (Cho and Parlar, 1991; Nicolai and Dekker, 2008; Thomas, 1986).
The failure interaction between components can be defined as (Murthy and Nguyen, 1985):

**Type I:** Failure interaction implies that the failure of component 1 can induce a failure of the other component with probability \( p \) which has no effect on the other component with probability \( 1-p \)

**Type II:** Failure interaction implies that the failure of component 1 can induce a failure of component 2 with probability \( p \), whereas, each failure of component 2 acts as a shock to component 1, without inducing an instantaneous failure, but affecting its failure rate.

**Type III:** Failure interaction implies that the failure of each component affects the failure rate of the other component. That is, each failure of one of the components acts as a shock to the other component.

The quantity of \( p \) is usually uncertain in this type of failure interaction. Therefore, employing expert judgment is so effective in this regard. The expert judgment numerously applied to MADM problems (Tyagi et al., 2014; Franek and Kresta, 2014; Asamoah et al., 2012).

The scheduling optimization problems are usually divided in two classes: The first, the optimization model that needs to heuristic algorithms (Doostparast et al., 2014; Chen et al., 2014) which attempts to find the optimal inspection number in each period. Because of long distance between two inspections in one period (between zeros to period time), this model is not appropriate for inspection interval. But, it is appropriate for gaining optimum stock inventory. The second, the optimization models which their aim is to find the correct inspection interval time. The second class is more appropriate for inspection interval time problems. In this class, the problems modeled based on cost (Taghipour and Banjevic, 2011, 2012; Taghipour et al., 2010; Golmakani and Moakedi, 2012a, b).

In previous studies, the repairable system has been considered to obtain optimum inspection interval based on cost and soft-hard failure concept (Taghipour and Banjevic, 2012, 2011; Taghipour et al., 2010; Golmakani and Moakedi, 2012a, b). They calculated the cost for different modes of inspections (1, 2, 3, ..., 10 periodically) and then selected the mode with the minimum cost.

In complex repairable system usually components are not independent in which failure/repair of one component have shock on other components (Murthy and Nguyen, 1985; Murthy and Wilson, 1994; Golmakani and Moakedi, 2012a, b). Inspection interval with failure interaction for two and multi components have been studied by Golmakani and Moakedi (2012a, b). They considered a two-component system, the capacitor bank (first component) and the transformer (second component) for a distribution substation in an electric power distribution system. Also, for the failure interaction, Satow and Osaki (2003) proposed a two-parameter (\( T, k \)) replacement model for a two-component system with shock damage interaction. The system is replaced preventively whenever the total damage of component 2 exceeds \( k \) or the age of the system reaches time \( T \). Zequeira and Berenguer (2005) studied inspection policies for a two-component parallel standby system with failure interaction and compared staggered and non-staggered inspections through numerical examples considering constant hazard rates.

The model was presented by Taghipour et al. (2010) for the ‘probability that the first component doesn’t fail’ and ‘expected survival time of the first component’ is required calculating of multi-integrals. This multi-integral values are negligible, which is possibly ignored in calculations. To simplify and avoid from the error of calculation, the Bayesian theory is applied and proposed the new model. The proposed model reduced calculations significantly and culminate the close results.
In this study, a simple two-component repairable system with failure interaction is considered. The failure interaction between components of this system is as follow, failure of the first component is soft (soft failures do not make the system stop, but can reduce the system’s performance and increase the system operating costs) and the second one is hard (hard failures make the system stop). Any failure in the second component acts as a shock to the first component and increases its failure rate. The first component is periodically inspected and if a failure is observed, it will be repaired. The second component’s failure is detected as soon as it occurs and it will be repaired. The repair of the second component restores it as good as new (perfect repair). The objective is to find the optimal inspection interval for the first component with failure interaction so that minimizes the expected total cost on a finite time horizon.

INSPECTION OPTIMIZATION MODEL

Problem definition: Here, a system consists of two-repairable component with failure interaction is considered. The failure of the first component is soft and the second component is hard. The first component is periodically inspected and if a failure is observed during the inspection. Thus, for the soft component there is a time delay between a real occurrence of failure and its detection. It is assumed that the long time delay have greater cost to the system. The hard failure of the second component is detected immediately as soon as it occurs and the second component is perfectly repaired. As well, the second component is not inspected.

It is assumed that the first component’s failure have increasing failure rate (Non-Homogeneous Poisson Process (NHPP)) and the second component’s failures have a constant failure rate (Homogeneous Poisson Process (HPP)). In addition to the above assumptions, we make the following assumptions:

- The inspections of the first component is minimal repair
- The inspection time of the first component as well as the time required to repair for both components are ignored and treated as being zero
- The soft failure of the first component cannot convert to hard failure
- The cost resulting from the second component includes only the cost associated with its perfect repair. Since the failure rate of this component is assumed constant, its corresponding cost per unit time is constant and, thus, it is not included in the optimization model

The defined parameters and variables are encapsulated in Table 1.

Proposed model: As noted, the second component failure accelerates the failure of the first component, but the first component failure does not affect the second component failure. The accelerating effect (p) is not synergistic. The 1/p number of the second component failure extended the first component failure rate to $2\lambda_j(x)$. The $\lambda_j(x)$ is given by Eq. 1:

$$\lambda_j(x) = \lambda_i(x|N_j(x) = j) = (1 + jp)\lambda_n(x), j = 0,1,...$$

(1)

where, $\lambda_n(x)$ is the failure rate of the first component, if the second component doesn’t fail until time x.
Table 1: Defined variables

| Parameters | Description |
|------------|-------------|
| λ₁(x)     | The average failure rate of the first component at time x |
| λ₂(x)     | The failure rate of the first component at time x, provided that the number of failures of the second component from the beginning of the planning horizon until time x is equal to j; j = 0, 1, 2,... |
| ((k-1)τ, kτ] | kth inspection interval in the cycle T, k = 1, 2,..., n |
| N_j(x)    | A random variable representing the number of failures of the second component from the beginning of the planning horizon until time x |
| E[C^T]    | The expected total cost of the first component in the cycle T |
| E[C^T_{(k-1)τ,kτ}] | The expected total cost of the first component in kth inspection interval of the cycle T, i.e., From a scheduled inspection at kτ over time period ((k-1)τ, kτ] |
| τ         | The time between two consecutive inspections, τ = T/n |
| p^t(t)    | The probability that the first component does doesn’t fail in kth inspection interval of the cycle T with t inspection interval, provided that we know that its age at the beginning to the cycle T is equal to t and that it is as good as new at that time |
| C^t       | The cost of each inspection of the first component |
| C^r       | The cost of each minimal repair of the first component |
| C^d       | The downtime penalty cost associated with the first component per each unit of elapsed time from the soft failure of the first component to its detection at the inspection time |
| λ₂        | The failure rate of the second component |
| p         | The ratio of the first component’s failure of the second component due to the occurrence of the second component failure. For example, The 1/p number of second component failure increases the first component failure to j₁(x) |
| T         | The planning horizon length (e.g., 1 year) which is known and fixed |
| n         | The number of inspections to be performed on the first component during the cycle T |
| t         | The initial age of the first component at the beginning of the cycle T |

It is assumed that the second component’s failures occur according to HPP with a constant failure rate. Thus, we have Eq. 2:

\[
p(N_j(x) = j) = \frac{(\lambda_2 \times x)^j e^{-\lambda_2 \times x}}{j!} \text{, } j = 0, 1, ...
\]  

(2)

The expected failure rate of the first component at time x, \( \lambda_1(x) \), depends on the number of the second component’s failures \( j = 0, 1, 2,... \). Thus, from Eq. 1 and 2, \( \lambda_1(x) \), is given by Eq. 3:

\[
\lambda_1(x) = \sum_{j=0}^{\infty} \lambda_1(x|N_j(x) = j)\cdot p(N_j(x) = j) = \sum_{j=0}^{\infty} \left[ \lambda_0^j(x)(1 + jp) \right] \cdot \left( \frac{\lambda_2 \times x)^j e^{-\lambda_2 \times x}}{j!} \right)
\]

\[
= e^{-\lambda_2 \times x} \lambda_0^0(x) \left[ \sum_{j=0}^{\infty} \left( \frac{\lambda_2 \times x)^j}{j!} \right) + \sum_{j=0}^{\infty} j\left( \frac{\lambda_2 \times x)^j}{j!} \right) \right] 
= e^{-\lambda_2 \times x} \lambda_0^0(x) \left[ e^{\lambda_2 \times x} + p(\lambda_2 \times x)e^{\lambda_2 \times x} \right] = \lambda_0^0(x) \left[ 1 + p(\lambda_2 \times x) \right]
\]  

(3)

The cumulative distribution function is given by Eq. 4 and simplification by Eq. 5-7:

\[
F(x) = 1 - e^{-\int \lambda_1(x) dx} \text{, } 0 \leq x \leq \tau
\]  

(4)

\[
F(x) = 1 - e^{-\int \frac{(\lambda_2 \times x)^j}{j!} \cdot [1+p(\lambda_2 \times x)] dx} = 1 - e^{-\int \frac{(\lambda_2 \times x)^j}{j!} \cdot [1+p(\lambda_2 \times x)] dx} = 1 - e^{-\int \left( \frac{x^j \cdot e^{\lambda_2 \times x}}{j!} \right) dx}
\]  

(5)
Assume that:
\[
\zeta = \frac{1}{\theta^\beta} \quad \text{and} \quad \zeta' = \frac{\mu_2}{\beta + 1} \tag{6}
\]

Then Eq. 6 given by:
\[
F(x) = 1 - e^{-\left(\theta^\beta + \mu_2(\beta + 1)\right)x} \tag{7}
\]

The cycle \( T \) is the planning horizon (e.g., 1 year) which is fixed. In the cycle \( T \), the first component is inspected at times, \( k\tau (k = 1, 2, \ldots, n) \), where \( T = n\tau \). Failures of the two components are perfectly repaired if failure occurs. We assume that inspection and possible repairs are also fulfilled at the end of the cycle \( T \) (last inspection is on the end of cycle \( T \)), that is, for \( k = n \). The objective is to find the optimal inspection interval that can minimize the expected total cost of the first component incurred over the cycle \( T \). When the first component fails, it remains in a failed state until the next inspection time. Therefore, if the first component failed in each inspection interval, a downtime penalty cost is incurred. The cost is proportional to the elapsed time from failure time to its detection at inspection time. Thus, the costs for resulting from the first component in each of the inspections \( k \), \( k = 1, 2, \ldots, n \) includes the cost of inspection, \( C_1 \), the cost of repair if found failed, \( C_1^+ \), and the penalty cost for the elapsed time for the failure, \( C_1^- \), thus, the expected cost occurred in the inspection \( k \) in the cycle \( T \) is given by Eq. 8:
\[
E[C_1^T_k] = \sum_{k=1}^{n} E[C_1^{(k-1)n:k]} = \left(\frac{T}{\tau}\right)C_1^- + \sum_{k=1}^{n} C_1^+ [1 - p_1^*(t)] + \sum_{k=1}^{n} \left[ \tau (1 - p_1^*(t)) \right] = \left(\frac{T}{\tau}\right)C_1^- + \left(C_1^+ + \tau C_1^+\right) \left[ \frac{T}{\tau} - \sum_{k=1}^{n} p_1^*(t) \right] \tag{8}
\]

Bayesian theory is used to obtain \( p_1^*(t) \). Due to different times of inspection intervals, the probability of \( p_1^*(t) \) depends on \( p_1^{k-1}(t) \). The Bayesian approach to obtain \( p_1^*(t) \) is given by Eq. 9:
\[
p_1(k) = p_1^*(t|\text{safe in } p_1^{k-1}(t))p_1^{k-1}(t) + p_1^*(t|\text{unsafe in } p_1^{k-1}(t))(1 - p_1^{k-1}(t)), \quad k = 1, \ldots, T/\tau \tag{9}
\]

It is possible to rewrite the Eq. 9 as follows:
\[
p_1^+(t) = p_1^{k-1}(t) \left[ 1 - F(x)|_{x=k-1} \right] + (1 - p_1^{k-1}(t)) \left[ 1 - F(x)|_{x=k} \right] \tag{10}
\]

For the different inspection intervals, \( F(x)|_{x=k} \) indicates the probability of the first component failure at \([k-1] \tau, k\tau \] interval, when the first component is on safety condition in the last interval. As well, \( F(x)|_{x=k} \) indicates that the probability of the first component failure at \((0, \tau)\) (due to perfect repair) interval, when the first component is on unsafety condition in the last interval. According to Eq. 7, \( p_1^*(t) \) can be simplified as follows in Eq. 11:
\[
p_1^*(t) = p_1^{k-1}(t) \left[ e^{-\left(\mu_2(\beta + 1) \theta^\beta x\right)} - e^{-\left(\mu_2(\beta + 1) \theta^\beta (k-1)\tau\right)} \right] + (1 - p_1^{k-1}(t)) \left[ e^{-\left(\mu_2(\beta + 1) \theta^\beta x\right)} \right], \quad k = 1, \ldots, T/\tau \tag{11}
\]
For example, for two inspection frequency given in Eq. 12 and 13:

\[
p_{T_1/2}(t) = 1 \times \left[ e^{-\left(\frac{(T_1/2T_2^1)}{T_2} + \frac{(T_1/2T_2^2)}{T_2} + \frac{(T_1/2T_2^3)}{T_2} + \frac{(T_1/2T_2^4)}{T_2} + \frac{(T_1/2T_2^5)}{T_2} \right)} \right] + (1-1)\left[ e^{-\frac{(T_1/2T_2^6)}{T_2}} \right] \tag{12}
\]

\[
p_{T_2}(t) = p_{T_2}(t) \left[ e^{-\left(\frac{(T_2/2T_2^1)}{T_2} + \frac{(T_2/2T_2^2)}{T_2} + \frac{(T_2/2T_2^3)}{T_2} + \frac{(T_2/2T_2^4)}{T_2} + \frac{(T_2/2T_2^5)}{T_2} \right)} \right] + (1-p_{T_2}(t)) \left[ e^{-\frac{(T_2/2T_2^6)}{T_2}} \right] \tag{13}
\]

**Numerical example:** The numerical example which is considered in this study referred to Golmakani and Moakedi (2012a). They considered a two-component system, a capacitor bank (first component) and a transformer (second component) in a distribution substation in an electric power distribution system. The power capacitors are often used in distribution systems to supply reactive volt-amperes to the system. When it applied to a system or circuit having a lagging power factor, several beneficial results are obtained. These results include power factor increase, voltage increase, system loss reduction and release of electric system capacity. In this case study, if the transformer doesn’t fail until month x, the failure rate of the capacitor bank is estimated as:

\[
\lambda_1(x) = \frac{\beta x^\beta - 1}{\theta}
\]

where, \( \beta = 1.8, \theta = 10 \) and other parameters are as following; \( T = 12, \lambda_2 = 1/6 \) per month, \( p = 1/10, C^S = 2000, C^d = 7500 \). The downtime penalty cost of the capacitor bank (cost of delay in detecting the capacitor bank’s failure) per each unit of elapsed time from its failure to its detection at the inspection is \( C^P = 12000 \) per month.

**DISCUSSION**

The expected total cost of the capacitor bank is calculated by the proposed model in which MATLAB software is employed to increase the correctness of calculation.

The results from Table 2 can indicate the improvement of \( p(T) \) with decreasing inspection interval time (\( \tau \)) and fixed inspection number (\( k \)) (the value of each column is on increasing). As well, for each fixed inspection interval time (\( \tau \)) and increasing inspection number (\( k \)), the \( p(T) \) it’s on decreasing. Increasing the number of inspections increases the inspection costs and reduces the downtime penalty cost. The contrast between these two costs caused the non-strict total cost plot. The optimal inspection interval obtain for 8 inspection frequencies and it’s related to \( \tau = 1.5 \).

From Eq. 8, total expected cost for each fixed inspection interval time (\( \tau \)) with inspection cost, repair cost and downtime penalty cost are shown in Fig. 1. The plots of inspection and repair cost have ascending trend. Contrary of them, the downtime penalty cost have decreasing trend.

In the model proposed in this study, the expected total cost associated to the component is formulated in terms of number inspections per cycle. Then, the expected cost is evaluated for different number of inspections in a cycle to identify the optimal one. The model can be used in different industries where the assumptions made in the model are applicable.
Fig. 1: Costs resulting for different inspection frequencies $K = 1, 2, ..., 12$

Table 2: $p_i^k(t)$ results for $\tau = 12, 6, 4, ..., 1$ and different $k$

| Sub-inspection intervals | K = 1 | K = 2 | K = 3 | K = 4 | K = 5 | K = 6 | K = 7 | K = 8 | K = 9 | K = 10 | K = 11 | K = 12 | Cost*10000 |
|--------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------------|
| $\tau = 12$              | 0.2259|       |       |       |       |       |       |       |       |       |       |       | 1.1728      |
| $\tau = 6$               | 0.6617| 0.4498|       |       |       |       |       |       |       |       |       |       | 0.7464      |
| $\tau = 4$               | 0.8214| 0.6428| 0.5862|       |       |       |       |       |       |       |       |       | 0.5871      |
| $\tau = 3$               | 0.8900| 0.7596| 0.6941| 0.7263|       |       |       |       |       |       |       |       | 0.5190      |
| $\tau = 2.4$             | 0.9292| 0.8292| 0.7706| 0.7843| 0.6914|       |       |       |       |       |       |       | 0.4888      |
| $\tau = 2$               | 0.9457| 0.8728| 0.8234| 0.8268| 0.7520| 0.7246|       |       |       |       |       |       | 0.4656      |
| $\tau = 1.7143$          | 0.9586| 0.9016| 0.8605| 0.8583| 0.7981| 0.7730| 0.7509|       |       |       |       |       | 0.4573      |
| $\tau = 1.5$             | 0.9674| 0.9216| 0.8872| 0.8822| 0.8332| 0.8108| 0.7906| 0.7723|       |       |       |       | 0.4554      |
| $\tau = 1.3333$          | 0.9735| 0.9359| 0.9070| 0.9005| 0.8602| 0.8404| 0.8223| 0.8056| 0.7902|       |       |       | 0.4579      |
| $\tau = 1.2$             | 0.9781| 0.9467| 0.9229| 0.9149| 0.8813| 0.8638| 0.8477| 0.8326| 0.8186| 0.8053|       |       | 0.4635      |
| $\tau = 1.0909$          | 0.9815| 0.9548| 0.9336| 0.9264| 0.8981| 0.8826| 0.8681| 0.8546| 0.8419| 0.8298| 0.8184|       | 0.4715      |
| $\tau = 1$               | 0.9842| 0.9613| 0.9428| 0.7263| 0.9115| 0.8978| 0.8849| 0.8727| 0.8612| 0.8502| 0.8398| 0.8298| 0.4813      |

The costs results (inspection, repair, downtime and expected total cost) of this proposed model has same trend in compare to Golmakani and Moakedi (2012a).

CONCLUSION

In the model proposed in this study, the expected total cost associated to the first component is formulated in terms of number inspections per cycle. Then, the expected cost is evaluated for different number of inspections in a cycle to identify the optimal one. The model can be used in different industries where the assumptions made in the model are applicable. For example: (1) In the steel industry, the sensor is used to combines two materials (A and B) for production of alloys. If the sensor fails, its failure is not visible and alloy is produced with low quality (cost). The combinatory sensor is affected by the failure of adjacent parts, (2) Intelligent condition monitoring cameras are used to detect defects fibers, when the camera fails, lines continued to work, but its performance comes down and dust can reduce visibility and increase failure to the camera and (3) As well, in injector of a four-cylinder car, if one of the four injectors fails, the car is moving but with low performance (more fuel consumption) and failure of filter can increase the failure of the injection.
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