Abstract—In this paper, assuming multi-antenna transmitter and receivers, we consider multicast beamformer design for the weighted max-min-fairness (WMMF) problem in a multi-stream multi-group communication setup. Unlike the single-stream scenario, the WMMF objective in this setup is not equivalent to maximizing the minimum weighted SINR due to the summation over the rates of multiple streams. Therefore, the non-convex problem at hand is first approximated with a convex one and then solved using Karush-Kuhn-Tucker (KKT) conditions. Then, a practically appealing closed-form solution is derived, as a function of dual variables, for both transmit and receive beamformers. Finally, we use an iterative solution based on the sub-gradient method to solve for the mutually coupled and interdependent dual variables. The proposed solution does not rely on generic solvers and does not require any bisection loop for finding the achievable rate of various streams. As a result, it significantly outperforms the state-of-art in terms of computational cost and convergence speed.

Index Terms—Multi-stream multi-group communications; Multicast beamforming; Weighted max-min fairness

I. INTRODUCTION

Wireless data communication is now playing a significant role in our everyday lives, and this importance will continue to grow with recent innovative applications such as autonomous driving and mobile immersive viewing. One of the key enablers for coping with the constantly growing mobile data traffic is the emergence of multi-antenna communications, which enables additional spatial degrees of freedom (DoF), and hence, higher spectral efficiencies to be achieved [1]. Moreover, when the users’ requests are correlated, one can benefit from multi-antenna multicast techniques to serve multiple users within a group with the same content [2]. In multicasting, a single beamformer is used for a group of users, resulting in potentially higher bandwidth efficiency and transmission rate. With the recent emergence of applications with correlating requests such as venue casting [3] and mobile immersive viewing [4], designing efficient multicasting techniques has also gained much attention from the research community.

In the basic multicasting setup, a single group of users is interested in the same common message. This basic setup is studied thoroughly in [2], where beamformers are designed to minimize the total transmit power subject to a given signal-to-interference-plus-noise ratio (SINR) at each user. It is shown that this problem, referred to as the quality of service (QoS) problem, is NP-hard and can be solved efficiently using the semi-definite-relaxation (SDR) method.

An interesting extension to the basic multicast setup is proposed as the multi-group multi-casting (MGMC) problem, where various data streams are multicast to different groups of users simultaneously. Solving this problem through SDR methods under different assumptions (e.g., total or per-user power constraint and centralized or decentralized settings) is extensively studied in the literature [5]–[8]. However, SDR-based approaches are computationally complex, and hence, alternative methods with reduced complexity have also gained interest. In [9], successive-convex-approximation (SCA) is used to design beamformers for both QoS and max-min-fairness (MMF) problems in an orthogonal frequency division multiplexing (OFDM) setup, and the solution is found iteratively using first-order Taylor expressions of SINR terms. Similarly, in [10], another solution based on KKT conditions is proposed for QoS and MMF problems to make beamformer design complexity (almost) independent of the antenna count. From another perspective, other works in the literature have proposed semi-closed-form beamforming solutions to remove the dependency on generic solvers, hence potentially reducing the beamformer design complexity. For example, in [11], a descent direction method is used to solve the QoS problem in a basic multicast setup, and in [12], [13], alternating direction method of multipliers (ADMM) is utilized to solve both QoS and MMF problems for the MGMC setup. Similar works in this context are proposed also in [14]–[16].

In this paper, we address the MGMC beamformer design problem in a multiple-input multiple-output (MIMO) setup where both the transmitter and receivers have multiple antennas. In such a setup, multi-stream communication becomes possible, and the MMF objective no longer corresponds to maximizing the minimum SINR value as is generally considered in the literature. Instead, one has to maximize the sum of \( \log(1 + \text{SINR}) \) rate terms over multiple streams while guaranteeing the decodability of each stream at every user. Due to these reasons, the state-of-art works such as [19]–[13] are no longer suitable as they would require excessively long convergence times (due to inter-dependent bisection loops.
for calculating the rate of multiple streams). To address this issue, we first formulate the non-convex WMMF problem for the considered system model, and then use SCA to propose an approximate equivalent problem that is convex on either transmitter or receiver side but not jointly. For this convex problem, we use KKT conditions to derive optimal beamformer expression in terms of dual variables. However, the dual variables are highly coupled and interdependent, preventing a closed-form solution. To solve this problem, we compute dual variables using a fast-converging iterative algorithm based on the sub-gradient method. Interestingly, the proposed algorithm outperforms other existing works even in the presence of single-antenna users, where the problem becomes equivalent to max-min SINR. Simulation results show the superiority of the proposed algorithm over the state-of-the-art in terms of computation time and complexity.

Throughout the rest of the paper, we use bold-face lowercase- and upper-case letters to represent vectors and matrices, respectively. By $|K|$, we mean the set $\{1, 2, ..., K\}$. Other notations are defined as they are used throughout the text.

II. SYSTEM MODEL

A. Network Setup

We consider a multi-group multicasting system where a single server with $N_T$ transmitting antennas serves $K$ multi-antenna users. Every user $k \in [K]$ has $N_k$ receiving antennas. The user set is divided into $G$ non-overlapping groups, such that every user appears in exactly one group and the users within the same group request the same multicast message. Let us use $K_g$ to denote the set of user indices in multicast group $g \in [G]$. We use $W_g \in \mathbb{C}^{N_x \times L_g}$ to denote the pre-coder matrix for users in $K_g$, where $L_g$ represents the maximum number of independent streams that can be transmitted to these users ($L_g$ is a function of $N_T, N_k$ and $K$). The columns of $W_g$ are stream-specific transmit beamformers for users in $K_g$, i.e., $W_g = \{W_{g,1}, \ldots, W_{g,L_g}\}$, where $W_{g,l}$ is the beamforming vector for the $l$-th stream of group $g$. The channel matrix between user $k \in [K]$ and the transmitter is denoted by $H_k \in \mathbb{C}^{N_k \times N_T}$ and it is assumed to be perfectly known at the transmitter. Then, the received signal at user $k \in K_g$ can be written as

$$y_k = H_k W_g d_g + \sum_{g \neq g'} H_k W_{g'} d_{g'} + z_k,$$

where $d_g = [d_{g,1}, \ldots, d_{g,L_g}]^T \in \mathbb{C}^{L_g}$ is the transmitted data vector to multicast group $g \in [G]$ with $E(d_g d_g^H) = I_{L_g}$, and $z_k \sim \mathcal{CN}(0, \sigma_z^2 I_{N_k})$ is the additive white Gaussian noise at user $k$ with noise variance $\sigma_z^2$. The estimate of $d_{g,l}$ at user $k$ is given by $d_{k,l} = u_{k,l}^H y_k$, where $u_{k,l} \in \mathbb{C}^{N_k}$ denotes the corresponding linear receive beamforming vector. The mean-squared error (MSE) for stream $l$ of user $k \in K_g$ can then be written as

$$\epsilon_{k,l}(W, u_{k,l}) = |1 - u_{k,l}^H H_k w_{g,l}|^2 + \sum_{g \neq g'} \sum_{j \in [L_g]} |u_{k,l}^H H_k w_{g',j}|^2 + \sigma_z^2 \|u_{k,l}\|^2,$$

where $W := [W_1, \ldots, W_G]$. Note that (2) is convex on $\{u_{k,l}\}$ or $\{w_{g,l}\}$ but not on both at the same time. Finally, the SINR for stream $l$ at user $k \in K_g$ can be written as

$$\gamma_{k,l} = \frac{\|u_{k,l}^H H_k w_{g,l}\|^2}{\sum_{g \neq g'} \sum_{j \in [L_g]} |u_{k,l}^H H_k w_{g',j}|^2 + \sigma_z^2 \|u_{k,l}\|^2}. \tag{3}$$

B. Problem Formulation

First, we consider linear beamformers to simplify the beamforming process. Then, in Section III-C, we remove this assumption and consider a more general model. The performance gap of these two models is compared in Section IV.

The objective is to achieve the weighted fairness among different groups. For every group $g \in [G]$, we aim to maximize the sum-rate over all its $L_g$ streams. Moreover, as each data stream $d_{g,l}$ is requested by all users in group $g$, its corresponding transmission rate should be assigned such that every user in $K_g$ can decode it. Hence, we need to solve

$$S_0 : \max_{w_{g,l}, u_{k,l}, g \in [G]} \min_{l \in [L_g]} \alpha_g \sum_{k \in K_g} \min \log(1 + \gamma_{k,l}) \tag{4a}$$

s.t. $\sum_{g \in [G]} \sum_{l \in [L_g]} \|w_{g,l}\|^2 \leq P_T,$ \tag{4b}

where $\alpha_g$ is the associated weight for group $g$ and $P_T$ is the total available power at the transmitter. Problem $S_0$ is non-convex on both receive and transmit beamformers, but following similar steps as in [17], can be solved for $u_{k,l}$ (while fixing $w_{g,l}$). The result is the standard linear minimum mean-square error (MMSE) receiver

$$u_{k,l} = (H_k w_g H_k^H + \sigma_z^2 I_k)^{-1} H_k w_{g,l}. \tag{5}$$

Using (5) in (2), the SINR terms in (3) can be written as

$$\gamma_{k,l} = \frac{1}{\epsilon_{k,l} - 1}, \forall (k, l). \tag{6a}$$

Thus, writing the rate expression in (4) in terms of MSE and relaxing the objective, the problem $S_0$ can be reformulated as

$$S_1 : \max_{w_{g,l}, r_{g,l}} \sum_{l \in [L_g]} r_{g,l} \tag{6a}$$

s.t. $r_{g,l} \leq \alpha_g \sum_{l \in [L_g]} r_{g,l}, \forall g \in [G], \tag{6b}$

$r_{g,l} \leq \log(\epsilon_{k,l}^{-1}), \forall (g, k) \in K_g, l \in [L_g], \tag{6c}$

and the power constraint in (4b).

Note that MSE constraint (6c) in $S_1$ is still non-convex. To relax this constraint, we use auxiliary variables $t_{k,l}$ satisfying

$$\epsilon_{k,l} \leq [f(t_{k,l})]^{-1}, \tag{7}$$

where $f(t_{k,l})$ is a monotonic and continuously differentiable function that is Lipschitz continuous (hence, has finite first-order approximation coefficients), log-concave on its domain (i.e., $t \in [x_1 f(x) \in [1, \infty)]$), and equipped with convex multiplicative inverse (i.e., $f(t)^{-1}$ is convex on $t \in [x_1 f(x) \in [1, \infty]]$). The domain of $f(\cdot)$ is dictated by the range of MSE values, i.e., $\epsilon_{k,l} \in (0, 1], \forall (k, l)$. There are different classes of functions satisfying these conditions (c.f. [17]). For convenience, in this paper we assume $f(t_{k,l}) = 2^{t_{k,l}}$. Applying (7) into (6c) the problem $S_1$ can be written as

$$S_2 : \max_{w_{g,l}, r_{g,l}, t_{k,l}} \sum_{l \in [L_g]} r_{g,l} \tag{8a}$$
we compute receive beamformers but not jointly), it can be directly handled by generic solvers such as CVX. In this paper, we consider the CVX-based solution for WMMF.

The dual variables \( \mu, \zeta, v_k, \) and \( \lambda_k, \) are related to the power, common rate, stream-specific rate, and MSE constraints respectively.

**Theorem 1.** With fixed receive beamformers \( \mathbf{u}_{k,l} \) and the auxiliary function \( f(t_{k,l}) = 2^{k,l}, \) the following primal and dual variables satisfy the KKT conditions at the optimal point

\[
\mathbf{w}_{g,l}^* = \mathbf{H}^* \mathbf{U}^* \mathbf{H} + \mu^* \mathbf{I} = \sum_{k \in K_g} \lambda_{k,l}^* \mathbf{H}_{k,l}^* \mathbf{u}_{k,l};
\]

where \( \lambda_{k,l}^* = \lambda_{k,l}^* (\mathbf{W}^*, \mathbf{u}_{k,l}), \mu^* = \mu (\lambda_{k,l}^*, \mathbf{u}_{k,l}), \mathbf{H} := [H_1, \ldots, H_K], \mathbf{u}_{k,l} \) is calculated using (5), and \( \mathbf{U} \) is a block-diagonal matrix with elements \( \mathbf{U}_k, \) where \( k \in [K] \) and \( \mathbf{U}_k = [\sqrt{\lambda_{k,l}^*} \mathbf{u}_{k,l}^1, \ldots, \sqrt{\lambda_{k,l}^*} \mathbf{u}_{k,l}^L]. \) Note that (12a) and (12e) each represent a set of \( \sum_{g \in \mathcal{G}} |L_g| \) conditions.

**Proof.** The condition (12a) on optimal transmit beamformers results from the stationary KKT condition with respect to \( \mathbf{w}_{g,l}, \) i.e., \( \nabla_{\mathbf{w}_{g,l}} \mathcal{L}(\mathbf{w}_{g,l}) = \mathbf{0}. \) Similarly, conditions (12d), (12c), and (12e) result from stationary KKT conditions with respect to \( r_{g,l}, r_c, \) and \( \lambda_{k,l}, \) respectively (note that to achieve (12d), we have to once use (12a) to replace \( \zeta_g^* \)). To derive optimal stream-specific rates in (12c), we first update \( t_{k,l} \) using \( t_{k,l} = \sum_{g \in \mathcal{G}} v_g^* (r_{g,l} - \log(f(t_{k,l}))) \). Then, we use complementary slackness on (8b) and sum over all the users within group \( g, \) to get \( \sum_{k \in K_g} v_g^* (r_{g,l} - \log(f(t_{k,l}))) = 0. \) This results in

\[
r_{g,l} = \frac{-\sum_{k \in K_g} v_g^* (r_{g,l} - \log(\epsilon_{k,l}))}{\alpha_g \zeta_g^*};
\]

where \( \epsilon_{k,l} = \epsilon_{k,l} (\mathbf{W}^*, \mathbf{u}_{k,l}), \) \( \alpha_g = \alpha_g (\lambda_{k,l}^*, \mathbf{u}_{k,l}), \mathbf{W} := [W_{1}, \ldots, W_K], \) and \( \mathbf{u}_{k,l} \) is calculated using (5).
which can then yield (12b) by simply replacing $\zeta^*_g$ with (12d). Similarly, we can use complementary slackness on (6b) and sum over all the groups, i.e., $\sum_{g \in G} \alpha_g (r_c - \alpha_g \sum_{l=1}^{L_g} r_{g,l}) = 0$, and then replace $r_{g,l}$ form (13) to get (12c). Finally, the dual variable $\mu^*$ is derived using similar steps as in (16). To save space, the steps are not repeated here and are left for the extended version of this paper.

From (12a)-(12g), we can see that there exist closed-form solutions for all variables except for $v_{k,l}$. From (12a), it can be seen that variables $v_{k,l}$ are interdependent, and hence, proposing a closed-form solution is infeasible. However, we can still use sub-gradient method to update $v_{k,l}$ (c.f. [17], [18]). The following Lemma clarifies this procedure.

**Lemma 1.** The gradient of $\mathcal{L}(\cdot)$ in (11) with respect to $v_{k,l}$ at the point given by $\overline{\tau}_g, \overline{\tau}_{g,l}$, and $\epsilon_{k,l}$ can be written as

$$\nabla_{v_{k,l}} \mathcal{L}(\cdot) = \frac{\tau_c - \alpha_g \sum_{l \in [L_g]} \tau_{g,l}}{\alpha_g L_g} + \overline{\tau}_{g,l} + \log (\epsilon_{k,l}) .$$  \hspace{1cm} (14)

**Proof.** Since (12a) is true for any $l \in [L_g]$, we can sum its both sides over all $L_g$ streams to get

$$\zeta_g = \frac{\sum_{k \in K_g} \sum_{l \in [L_g]} v_{k,l}}{\alpha_g L_g} .$$  \hspace{1cm} (15)

Now, we can replace $\zeta_g$ in (11) with its equivalent in (15), and take the derivative with respect to $v_{k,l}$ to get (14). \hspace{1cm} \square

Using Lemma 1, the sub-gradient update for dual variables $v_{k,l}$ can be done using

$$v_{k,l}^{(n)} = [v_{k,l}^{(n-1)} + \beta \nabla_{v_{k,l}} \mathcal{L}(\overline{\tau}_c, \overline{\tau}_{g,l}, \epsilon_{k,l})]^{+}, \forall (k, g, l) \hspace{1cm} (16)$$

where $\beta > 0$ is the step size and $[x]^{+} := \max(x, 0)$.

In Algorithm 2, we have outlined the general procedure of the proposed iterative solution. As an additional explanation, we first choose a set of random transmit beamformers $w_{g,l}$ such that the power constraint in (4b) is met. Also, assuming zero common rate for each stream (i.e., $\epsilon_{k,l} = 1, \forall (k, l)$), we initialize rate and MSE dual variables $(\lambda_{k,l}, \epsilon_{k,l})$ with $\alpha_g$ (so that (12a) (12f)) are satisfied. Then, we iteratively compute primal and dual variables using (5), (12a) (16) until the convergence is met. In Algorithm 2, the maximum value for inner and outer loop iterations for SCA and sub-gradient updates are denoted by $I_{in}$ and $I_{out}$, respectively. It is worth noting that since (12g) is valid only for optimal $\lambda^*_{k,l}$ values, it may not satisfy the power constraint in (4b) at every iteration. Thus, in each iteration, we have to use the bisection method to compute a $\mu$ value satisfying (4b). Moreover, from the complexity perspective, the dominant term in the proposed iterative solution is the inversion of the $N_T \times N_T$ matrix in (12a), which requires the complexity of $O((N_T + \sum_{g \in [G]} |K_g| L_g)N_T^2)$. As a result, the complexity of the proposed method scales linearly with the number of users in the network (or the number of streams), making it suitable for large networks.

**Algorithm 2: Iterative algorithm for WMMF**

**Result:** $w_{g,l}; u_{k,l}; \lambda_{k,l}; r_{g,l}; r_c; \mu; \epsilon_{k,l}$

Set $i \leftarrow 0; v_{g,l}^{(0)} \leftarrow \frac{\alpha_g}{\tau_c}, \lambda_{k,l}^{(0)} \leftarrow \frac{\alpha_g}{\tau_c}, \forall (l, k)$

Choose random vectors for $w_{g,l}$ such that (4b) is met; while convergence not met and $i < I_{out}$ do

Set $i \leftarrow i + 1$

Solve $w_{g,l}$ from (12a), using $u_{k,l}, \lambda_{k,l}^{(i-1)}, \mu$; \hspace{1cm} (17)

Compute $r_c$ from (12a), using $w_{g,l}, u_{k,l}$

Compute $r_{g,l}$ from (12b), using $\epsilon_{k,l}, v_{k,l}^{(i-1)}$

Compute $r_{g,l}$ from (12a), using $\epsilon_{k,l}, v_{k,l}^{(i)}$

Update $v_{k,l}^{(i)}$ from (16), using $r_c, v_{k,l}^{(i-1)}, r_{g,l}, \epsilon_{k,l}$

Normalize $v_{k,l}^{(i)}$ by $\sum_{g \in [G]} \alpha_g \sum_{k \in K_g} v_{k,l}^{(i)}$

This is done to satisfy (12c).

Update $\lambda_{k,l}^{(i)}$ from (12f), using $v_{k,l}^{(i)}$

end

Using (20) in (19) and following similar steps as in section II-B the problem $\mathcal{S}_0$ can be iteratively approximated with

C. Upper-bound

In section III, we modeled the transmitted signal for group $g$ as $x_g = W_g d_g$. As a result, the maximum rank of the transmit covariance matrix $K_{x_g} = E\{x_g x_g^H\} = E\{W_g d_g d_g^H W_g^H\} = W_g I_{L_g} W_g^H$ was limited to $L_g$. Here we propose a different approach, where we relax the rank limitation on $K_{x_g}$ and remove the linear per-stream decodability requirement in (8b). Therefore, in general, this relaxation would require encoding across spatial dimensions, and hence, non-linear receiver processing not considered in this paper. However, we can use the result as an upper-bound on the performance and compare it with the iterative solution proposed in Section III-B.

For the upper-bound approach, we consider a generalized transmission vector $\hat{x}_g$ for group $g$, with a generalized covariance matrix $\hat{K}_{x_g}$. Then, the received signal model in (1) can be re-written as

$$\hat{y}_k = H_k \hat{x}_g + \sum_{g \neq g} H_k \hat{x}_g + z_k ,$$  \hspace{1cm} (17)

and the achievable sum-rate of user $k \in K_g$ during the transmission of $\hat{x}_g$ is

$$R_k = \log |I + Q_{g}^{-1} H_k \hat{K}_{x_g} H_k^H| ,$$  \hspace{1cm} (18)

where $Q_g := \sum_{g \neq g} H_k \hat{K}_{x_g} H_k^H + \sigma_g^2 I$. Accordingly, the WMMF problem would change to

$$\hat{S}_0 : \max_{K_{x_g}} \min_{g \in [G]} \alpha_g R_k \hspace{1cm} (19a)$$

s.t. \hspace{0.5cm} $\sum_{g \in [G]} \text{Trace}(\hat{K}_{x_g}) \leq P_T$ . \hspace{1cm} (19b)

The objective function in $\hat{S}_0$ is not convex. However, one can show that it can be written as the difference of convex functions as

$$R_k = \log |I + \sum_{g \in [G]} H_k \hat{K}_{x_g} H_k^H + \sigma_g^2 I| - \log |Q_g| , \hspace{1cm} \forall k .$$  \hspace{1cm} (20)

Using (20) in (19) and following similar steps as in section II-B the problem $\mathcal{S}_0$ can be iteratively approximated with
the following convex problem
\[
\hat{S}_1 : \max_{\mathbf{K}_{x_g}, R} R \\
\text{s.t.} \sum_{g \neq g} \text{Trace} \left( \mathbf{Q}_g^{-1} \mathbf{H}_g^H \left( \mathbf{K}_{x_g} - \mathbf{R}_{x_g} \right) \mathbf{H}_k \right) + \log |\mathbf{Q}_g| \\
- \log \left[ \sum_{g \in [G]} \mathbf{H}_g \mathbf{K}_{x_g} \mathbf{H}_k^H + \sigma_k^2 \mathbf{I} \right] + \frac{R}{\alpha_g} \leq 0, \forall (g, k), \\
\sum_{g \in [G]} \text{Trace}(\mathbf{K}_{x_g}) \leq P_T ,
\]
where \( \mathbf{Q}_g := \sum_{g \neq g} \mathbf{H}_g \mathbf{R}_x \mathbf{H}_k^H + \sigma_k^2 \mathbf{I} \), and \( \mathbf{R}_{x_g} \) denotes the covariance matrix of the transmitted signal to group \( g \) at the previous iteration. Since (21) is convex, it can be directly handled by generic solvers such as CVX. The required procedure is quite similar to what we followed in Section III-A, and is outlined in Algorithm 3. Finding the solution to problem \( \hat{S}_1 \) follows a similar approach to the SDR method proposed in [5], and hence, its computation complexity is in the order of \( \mathcal{O}((N_T + \sum_{k \in [K]} N_k)^6) \).

Algorithm 3: CVX-based solution for upper-bound

Result: \( \mathbf{K}_{x_g}, R \)

Choose random \( \mathbf{K}_{x_g} \) matrices such that (21c) is met;

while convergence is not met do

    set \( \mathbf{K}_{x_g} \leftarrow \mathbf{K}_{x_g}, \forall g \),

    update \( \mathbf{K}_{x_g}, R \) using \( \hat{\mathbf{K}}_{x_g} \), from (21);

end

IV. SIMULATION RESULTS

We use MATLAB simulations to compare the complexity and performance of the proposed beamforming solution with state-of-the-art. We consider a downlink communication setup with equal-sized groups with uniform group priority (i.e., \( \alpha_g = 1, \forall g \)) and AWGN noise with unit variance (\( N_0 = 1 \)). The users are assumed to have the same number of \( N_R \) receive antennas, and the maximum number of transmitted streams for each group (i.e., \( L_g \)) is also considered to be equal to \( N_R \). The step size \( \beta \) is set to \( 10^{-2} \). We also consider the scheme proposed in [13] as a baseline for the single-antenna receiver scenario (the per-antenna power constraint in [14] is relaxed in our simulations). Note that in all figures, the numbers on the arrows show the actual simulation time averaged over all the realisations. SDPT3 solver is selected for CVX, and all the simulations are performed on the same hardware platform.

Fig. 1 compares the proposed iterative method with the CVX-based solutions, in terms of the achievable rate and the required convergence time, for different number of receive antennas at each user. As illustrated in the figure, both CVX-based and the proposed iterative solutions converge to the same optimal point but with considerably different convergence times. Moreover, it can be seen that the iterative solution performs fairly close to the upper-bound solution (which requires non-linear receiver implementation). However, the gap between the upper-bound and the linear solutions increases with the number of receive antennas \( N_R \), as the rank of the transmit covariance matrix also increases with the same rate. It is also worth mentioning that due to the linearly growing complexity with respect to the number of receive antennas, the iterative solution is quite fast even for a large number of antennas, whereas CVX-based solutions are highly time-consuming.

Fig. 2 compares the performance of various methods for different number of transmit antennas. As can be seen, the proposed iterative method is significantly superior compared to the CVX-based solution, in terms of the required convergence time. Moreover, the gap between the linear solutions (CVX-based and iterative solutions) and the non-linear upper-bound method does not increase by the number of transmit antennas. In other words, the rank of the transmit covariance matrix is highly limited by the number of receive antennas (in the simulated scenario, this rank is almost always equal to \( N_R \)). It is also worth noting that the convergence time of the proposed method is more sensitive to the number of transmit antennas (compared to the number of receive antennas), confirming the previous discussions in Section III-B.

Fig. 3 depicts the achievable rate versus the available signal-to-noise-ratio (SNR) for the interference-limited scenario where the number of transmit antennas is not enough to support \( N_R = 2 \) streams for each group. Since the beamforming vectors are initialized such that \( L_g = N_R \) streams can be transmitted for each group, linear solutions need more iterations to converge to the optimal single-stream solution. This effect is more prominent in high-SNR communications, as the interference terms are more dominant in this regime. Although the increased number of iterations results in an increase in the convergence time, still the iterative solution outperforms the CVX-based solution by a large margin.

Finally, in Fig. 4 we have compared the convergence time and total iteration count (i.e., the total number of transmit and receive beamformer updates) for the proposed iterative method, the CVX-based solution, and the ADMM-bisection method in [13]. We have chosen the ADMM-bisection as it has the best performance in our simulations (e.g., compared with [8], [6], [9], [10]). As can be seen, even though the proposed solution is not originally designed for the single-antenna receiver scenario, its required iterations is smaller than the CVX-based solution and much smaller than the ADMM-based solution (Note that the iteration counts of all the methods are increasing with the available SNR. However, due to the different scaling, it is less visible for iterative and CVX-based solutions). This is because in our solution, the achievable rate is calculated in a closed-form, and hence, there is no need for bisection over the rate (as is the case for the ADMM-based method as well as other works in the literature such as [10]). Similarly, the required convergence time of our iterative solution is smaller than the ADMM-based method and much smaller than the CVX-based method.
In this paper, we proposed a low-complexity iterative method for the multi-stream multi-group multicasting problem with the weighted max-min fairness objective. In this method, the original non-convex and NP-hard problem is solved up to a locally optimal point by iterating between receive and transmit beamformer updates. Using KKT conditions on the Lagrangian function, the optimal beamformer structure is derived as a function of dual variables. As some of the dual variables are highly-coupled and interdependent, an iterative sub-gradient method is used to find them efficiently. Finally, by finding the common achievable rate, the problem is solved directly without relying on bisection over the common rate. Simulation results show that the proposed algorithm finds optimal beamformers much faster than generic solver-based methods. Potential extensions include non-perfect channel state information at the transmitter (CSIT) and cell-free joint transmission over multiple transmitters.

V. CONCLUSION AND FUTURE WORK

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