A Fuzzy-Decomposition Grey Modeling Procedure for Management Decision Analysis

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1. Introduction

Decision-making is the heart of administration and one of the most important tasks for managers [1, 2]. To cope with the increasingly fierce market competition environment, managers need to quickly respond to business problems and maintain business advantages, which require timely and correct decisions. However, uncertain events and uncontrollable factors often make decisions invalid and affect business performance. Forecasting analysis can help managers grasp the future development trends to reduce the impact of uncertainty [3] and then make meaningful decisions.

In various management situations, decisions that require an immediate response are the more difficult tasks for managers. Real-time grasp of the situation through a limited amount of data can enable managers to make appropriate treatments and obtain effective control and management [4]. Analyzing the occurrence of a new disease is another example. If the government can make a correct decision sooner, the possible harm and impact of the new disease on the country will be reduced. Decision-making should not wait until more people with infections appear; the more immediate decision-making can bring higher management value. Therefore, using a smaller amount of data to build a prediction model has more practical value.

Machine learning algorithms and statistical theory have been widely used in knowledge extraction for management, but these approaches are generally developed based on a large number of samples [5, 6]. These typical approaches may not provide satisfactory predictions when the sample size is small [7]. Multivariate analysis is another popular prediction method; its prediction accuracy depends on the choice of independent variables. If the established causal model cannot effectively explain the variation of the dependent variable, a prediction model with high variance will...
be produced. In contrast, time series models only require historical data to predict their future trends [8]. However, it usually requires a large number of observations to get accurate prediction results. In all the above methods, the key factor that affects the prediction performance is the sample size, which limits their applicability in certain prediction situations. Therefore, how to use limited data to extract useful information to assist managers to make immediate decisions has great management significance and practical value [9].

Grey system theory was proposed by Deng [10], which mainly studies the uncertainty of insufficient information caused by limited samples. It provides strong technical support for solving small-data-set analysis and decision-making problems [11]. The typical first-order one-variable grey model, abbreviated as the typical GM (1, 1), is one of the most commonly used grey methods due to its convenience and easy calculation [12], which can use only four data points to predict the future trend of a time series with a favorable result [13]. The model and its extensions have succeeded applied to the fields of business [14], energy [15–18], environment [19–21], industry [22, 23], engineering [24], medicine [25], hydraulics [26], economics [27–30], and many other domains.

To further improve it, this study proposed a fuzzy-decomposition modeling procedure for enhancing the prediction quality of typical GM (1, 1). This new method can build a robust model based on Latent Information (LI) function [31] and bring a better forecasting accuracy than the typical GM (1, 1).

To confirm the validity of the proposed fuzzy-decomposition modeling procedure, one real case was selected to implement the experimental analysis for confirming the effectiveness and practical value investigating; the data is the total demand of thin-film transistor liquid crystal display (TFT-LCD) panels provided by a leading manufacturer in Taiwan. Experimental results showed that the fuzzy-decomposition modeling procedure can significantly improve the accuracy of the typical GM (1, 1) and is a useful decision analysis tool for managers.

The remaining parts of this paper are systematically presented as follows. Section 2 introduces the typical GM (1, 1), LI function, and the proposed modeling procedure. Section 3 addresses the data analysis and comparison which is applied to one real case. Finally, the conclusions are presented in Section 4.

2. Methodology

When the sample size is small, the main challenge is how to effectively extract useful information for modeling. Therefore, a fuzzy-decomposition modeling procedure was developed for this issue. The following subsections will introduce the two main components of the modeling procedure and the detailed steps of the proposed method.

2.1. The Typical GM (1, 1). Among the existing grey approaches, the typical GM (1, 1) is the simplest and widely used model. It can use four data to build a model and bring favorable performance. Its primary means is to use the generating operation to weaken the randomness of the data and then recognize the inherent regular pattern of the data to establish a fitting model [32]. At present, it is one of the important methods to solve the problem of small-sample analysis. The modeling process of typical GM (1, 1) is as follows:

(1) Suppose that the original nonnegative data series is 
\[ X^{(0)} = \{ x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n) \} \]
where \( n \) is the sample size, and \( n \geq 4 \). Let \( x^{(0)}(k) \) represents the datum at the \( k \)th phase.

(2) Form an accumulated generating series \( X^{(1)} = \{ x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n) \} \):
\[ x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \quad k = 1, 2, \ldots, n. \]  

(3) Calculate the background values \( Z^{(1)} = \{ z^{(1)}(1), z^{(1)}(2), \ldots, z^{(1)}(n) \} \):
\[ z^{(1)}(k) = \frac{1}{2} \left[ x^{(1)}(k-1) + x^{(1)}(k) \right], \quad k = 2, 3, \ldots, n. \]

(4) Formulate the grey differential equation:
\[ x^{(0)}(0) + ax^{(1)}(k) = b. \]  

(5) Expand equation (3) as a vector-matrix form such as equation (4); let \( Y = [x^{(0)}(0), x^{(0)}(3), \ldots, x^{(0)}(n)]^T \),
\[ \bar{a} = \begin{bmatrix} a \\ b \end{bmatrix}, \text{ and } \mathbf{B} = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}. \]

(6) Solve the ordinary differential equation \[ dx^{(1)}(t)/dt + ax^{(1)}(t) = b \] with the initial condition \( x^{(0)}(1) = x^{(1)}(1) \) to build the grey forecasting model:
\[ \ddot{x}^{(1)}(k + 1) = \left[ x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} + \frac{b}{a}, \]
\[ x^{(0)}(k) = \ddot{x}^{(1)}(k + 1) - \dddot{x}^{(1)}(k). \]
(7) Obtain the desired forecasting output with equation (6).

2.2. LI Function. When the sample size is small, the training process of a prediction model usually cannot be implemented effectively. This problem could be solved by increasing the amount of information used, while also enhancing the stability of the forecasting results. Therefore, in this study, we selected the LI function [31] to analyze the data behavior and then combined the obtained fuzzy data from the analysis with the typical GM (1, 1) to develop a robust modeling procedure.

The LI function was proposed by Chang et al. [31]; its development concept is to appropriately expand the possible margins of data by using four statistical indexes to fill the information gap. The degree of expansion of the data range in the LI function depends on the samples size. Specifically, when there are a large number of observations available, the overall outline of the data is clearer due to more information, so it is necessary to substantially expand the data range. On the contrary, if there is no adequate information, the data range must be greatly expanded. Therefore, in the LI function, the degree of expansion of the data range is calculated by the division operation between the range and the number of samples, and the ratio of expansion of the data range toward the left or right is determined by the skewness. The upper bound (UB) and the lower bound (LB) are the expanded boundaries, which are combined with the central tendency (CT) to form the LI function. The LI function is essentially a fuzzy membership function, whose value falls between 0 and 1, and the magnitude of the value represents the possibility of potential data occurrence. The complete process for forming the LI function is as follows:

1. Suppose that the time series obtained is \(X = \{x_1, x_2, \ldots, x_n\}\); in the set \(X\), the element with the minimal value is \(x_{\text{min}}\), and the element with the maximal value is \(x_{\text{max}}\).
2. Calculate the range \(R\):
   \[
   R = x_{\text{max}} - x_{\text{min}}. 
   \]  
3. Determine the CT:
   \[
   CT = \frac{\sum_{i=1}^{n} i x_i}{\sum_{i=1}^{n} i}, \quad i = 1, 2, \ldots, n. 
   \]  
4. Calculate the central location (CL) of the existing data:
   \[
   CL = \frac{x_{\text{min}} + x_{\text{max}}}{2}. 
   \]  
5. Count the number of elements in the subset composed of data with values larger than CL, denoted by \(N^+\); count the number of elements in the subset composed of data with values less than CL, denoted by \(N^-\).

6. Compute the increasing tendency (IT) and the decreasing tendency (DT):
   \[
   \begin{align*}
   IT & = \frac{N^+}{N^+ + N^-}, \\
   DT & = \frac{N^-}{N^+ + N^-}.
   \end{align*}
   \]  
7. Expand the domain range asymmetrically using IT and DT and then use equation (11) to obtain the extended UB and LB.
   \[
   \begin{align*}
   UB & = x_{\text{max}} + IT \times \frac{R}{n}, \\
   LB & = x_{\text{min}} - DT \times \frac{R}{n}.
   \end{align*}
   \]  
8. Combine CT, UB, and LB to generate a triangular LI function (Figure 1 is one of the possible shapes). Here, the LI value of CT is set to 1, and the LI value of the boundaries (UB and LB) is set to 0.
9. Obtain the LI values of existing data using equation (12) (the properties of the generated triangle).

\[
\text{LI value} = \begin{cases} 
\frac{x - LB}{CT - LB}, & \text{if } x < CT, \\
1, & \text{if } x = CT, \\
\frac{UB - x}{UB - CT}, & \text{if } x > CT.
\end{cases}
\]  

2.3. Fuzzy-Decomposition Modeling Procedure. A time series usually contains many elements; decomposition can identify these elements and separate the time series data into basic components. The decomposed components are relatively easy to grasp, which will be helpful for model fitting. The concrete realization of this idea in the paper is to decompose the time series into three subseries through the LI function (data fuzzification); they are the upper series, central series, and lower series. These three subseries are, respectively, predicted using the grey model. Finally, simply set the weights of the three series according to the possibility to combine the estimated values of the three series into a single final predicted value (defuzzification). The computational process is detailed as follows:

1. Denote \(X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\}\) as the initial series, where \(n\) is the sample size, and \(n \geq 4\). Let \(x^{(0)}(k)\) represents the datum at the \(k\)th phase.
2. Decompose the initial series into three subseries using LI function, referring to Section 2.2; these three series are \(X^{(0)}_{UB}\), \(X^{(0)}_{CT}\), and \(X^{(0)}_{LB}\).
3.2. A Modeling Example of the Proposed Modeling Procedure. Here, the first four samples in the case of TFT-LCD are used as an example to illustrate the calculation details of the proposed procedure. That is, the actual demand from January 2010 to April 2010 is used to establish a model to predict the demand in May 2010, specifically, using series \( X^{(0)} = \{1.135, 1.000, 1.231, 1.277\} \) as the inputs and then separating it into three derived series, namely, \( X^{(0)}_{UB} = \{1.135, 1.169, 1.282, 1.312\} \), \( X^{(0)}_{CT} = \{1.135, 1.045, 1.138, 1.194\} \), and \( X^{(0)}_{LB} = \{1.135, 0.9663, 0.974, 0.965\} \). The typical GM (1, 1) is used to build models for these three derived series and obtain their estimated values in the fifth period; they are \( x^{(0)}_{UB}(5) = 1.4019 \), \( x^{(0)}_{CT}(5) = 1.2811 \), and \( x^{(0)}_{LB}(5) = 0.9678 \). The three estimated values are combined through the weighted average method to create the final predicted value \( \bar{x}^{(0)}(5) = 1.2330 \).

3.3. Performance Evaluation and Comparison. To verify the improvement effect of the proposed modeling procedure, the typical GM (1, 1) is used for comparison. The typical GM (1, 1) is one of the most popular models in grey system theory due to its ease of use. In the experiment, the typical GM (1, 1) is built from the same data, and the results are
To overcome this uncertainty, enterprises must have appropriate forecasting techniques. Predictive analysis can help managers grasp the future development trends to ease the impact of uncertainty and then make meaningful decisions. In various management contexts, decisions that need immediate response are the more hard tasks for managers. Real-time grasp of the situation through a limited amount of data enables managers to carry out appropriate processing and obtain effective management and control. Therefore, it is of greater practical value to establish forecasting models under small data sets.

Grey system theory can build models with small data sets, which meets the needs of enterprise decision analysis. To further improve it, a fuzzy-decomposition modeling procedure based on LI function was developed in this study for enhancing the prediction performance of typical GM (1, 1). This method fits the model better by decomposing the series into three subsseries and then obtains a more robust prediction output. Through the verification of the monthly demand for TFT-LCD panels, the experimental results showed that the proposed method has good predictive performance under small data sets. From the comparisons, it can be found that the proposed procedure outperforms the typical GM (1, 1) in the empirical case. Obviously, the proposed method has important practical value and is an effective tool to assist decision analysis with limited samples.

The proposed method maintains the features of the grey approach, which is a consistent model without stochastic processes; given the same input, only one result will be obtained, rather than a different result each time. If a model is built with small training samples in a stochastic process, managers would bear the risk of uncertainty when making decisions. In the future, the proposed method can be applied to solve the forecasting problems in other fields, such as medicine, energy, finance, engineering, and transportation, to further confirm its effectiveness. In addition, using more training samples to verify the predictive power of the proposed method is also a feasible direction for future work. Finally, researchers may use optimization algorithms to find a more appropriate defuzzification weight to acquire a better grey model.

### Data Availability

The data used in the experiment are listed in this article; anyone can use these data by citing this article.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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