Robust Downlink Power Control in Wireless Cellular Systems

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A serious shortcoming of current downlink power control methods is that their performance may be severely degraded when the downlink channel information is known imprecisely at the transmitter. In this paper, a computationally and implementationally simple centralized downlink power control method is proposed for cellular wireless communication systems using code division multiple access (CDMA) or space division multiple access (SDMA). Our method provides a substantially improved robustness against imperfect knowledge of the wireless channel by means of maintaining the required quality of service for the worst-case channel uncertainty. In the SDMA case, the proposed technique can be straightforwardly combined with any of the existing transmit beamforming methods. Simulation results validate substantial robustness improvements achieved by our approach.

Keywords and phrases: power control, cellular system, CDMA, SDMA, downlink beamforming.

1. INTRODUCTION

Power control is an intelligent way of adjusting the transmitted powers in cellular systems so that the total transmitted power is minimized but, at the same time, the user signal-to-interference-plus-noise ratios (SINRs) satisfy the system quality of service (QoS) requirements [1].

Depending on the location where the decision on how to adjust the transmitted powers is made, the power control algorithms can be divided into two groups: centralized and noncentralized (distributed) techniques. In distributed power control, local measurements are used to evaluate the transmitted power for each user so that all users finally meet the QoS requirements [2, 3]. In centralized power control, users channel information is sent to the central unit which computes the desired transmitted powers for each user [4, 5].

Downlink beamforming and power control techniques have been a recent focus of intensive studies in application to cellular communication systems [4, 6, 7, 8, 9, 10, 11, 12, 13, 14]. The user SINR criterion has been adopted in these papers to optimize the transmitted powers and beamformer weights to ensure that the QoS requirements are satisfied for all users. For example, in [8, 13], the problem of optimal centralized power control and downlink beamforming is considered in the case when the exact downlink channel information is available at the base stations. Several other works consider simpler suboptimal power control and/or beamforming methods [7, 9, 10, 14].

However, a serious shortcoming of all centralized power control methods is that they assume the exact knowledge of the user downlink channel at the transmitter and, as a result, can be quite sensitive to imprecise channel knowledge [11]. In practical situations, the downlink channel may be uncertain and, in the general case when base stations are equipped with antenna arrays, the downlink channel correlation (DCC) matrices may be subject to substantial errors.
As the DCC matrices are estimated at base stations by means of uplink channel measurements or through some feedback from the users, in the time division duplex (TDD) mode such errors may be caused by channel variability, user mobility, finite training data length effects, and so forth. Furthermore, downlink channel errors are quite typical for the case when the frequency division duplex (FDD) mode is employed and there is no channel feedback (or imperfect/outdated feedback) from the users [11].

In the presence of downlink channel errors, the QoS constraints can be violated and, therefore, the existing power control techniques can break down in performance. Therefore, the robustness of downlink power control and transmit beamforming algorithms appears to be of high importance in this case. The problem of robust transmit beamforming has been originally addressed in [9, 12] in application to space division multiple access (SDMA) systems where modifications of the method of [8] are considered which are robust to downlink channel errors. Further extension of the approach of [12] to the case when each mobile user employs multiple antennas is considered in [15]. Several robust beamforming algorithms have been considered by the authors of [9, 12] but these algorithms are computationally and implementationally quite expensive. Below, we develop another alternative and simpler way to incorporate robustness into the problem of downlink transmissions. We consider the robust downlink power control problem separately and, in the SDMA context, apply it to the case when all beamforming vectors are preliminarily obtained by any of the nonrobust existing methods. A new closed-form solution to this problem is proposed for cellular wireless communication systems which may (or may not) use multisensor antenna arrays at base stations. An improved robustness against imperfect knowledge of the downlink channel is achieved in our technique by means of maintaining the required QoS constraints for the worst-case channel uncertainty.

We demonstrate via extensive computer simulations that in the SDMA case, even if the downlink beamforming vectors are obtained in a nonrobust and noncentralized way, using our robust power control in combination with such simple beamforming strategies substantially improves the robustness of the whole system making it comparable to that of the robust centralized beamforming method of [12]. Moreover, we show that these robustness improvements can be achieved at nearly the same total transmitted power as in the method of [12]. At the same time, as will be clarified in Section 3, the proposed approach can be implemented in a much simpler way than the method of [12] because our technique requires much less computations and much lower communication rates between the system base stations and the central unit than the technique of [12].

2. BACKGROUND

Consider a cellular wireless communication system with \( M \) cells (base stations) and \( K \) cochannel users. Let \( P_k \) be the transmitted power for the \( k \)th user, \( \sigma_k^2 \) its noise variance, and \( h_{k,m} \) \((h_{k,m} = 0 \text{ for all } 1 \leq k \leq K \text{ and } 1 \leq m \leq M) \) the channel gain (loss) between the \( k \)th user and \( m \)th cell site (CS) as illustrated in Figure 1. Assuming that the \( k \)th (desired) user is assigned to the CS with the index \( c(k) \) \((1 \leq c(k) \leq M) \), its receive SINR can be expressed as follows:

\[
\text{SINR}_k = \frac{\text{desired user power}}{\text{noise + interference power}} = \frac{P_k h_{k,c(k)}}{\sigma_k^2 + \sum_{m=1,m \neq k}^K P_m h_{m,c(m)} |a_k^H a_m|^2}.
\] (1)

Two representative examples that can be described by (1) are code division multiple access (CDMA) and SDMA systems.

2.1. CDMA system

Consider a CDMA cellular system where each base station uses a single antenna for signal transmission. In such a system, the information for the \( k \)th desired user is transmitted by the \( c(k) \)th base station using the spreading code vector \( s_k \). In the flat-fading case, the user receive SINR can be written as [3, 16]

\[
\text{SINR}_k = \frac{P_k G_{k,c(k)} |a_k^H s_k|^2}{\sigma_k^2 + \sum_{m=1,m \neq k}^K P_m G_{k,c(m)} |a_k^H s_m|^2}.
\] (2)

where \( a_k \) is the receive filter coefficient vector of the \( k \)th user, \( \beta_k^2 \) is the noise power at the input of this filter, \( \sigma_k^2 = \beta_k^2 |a_k^H a_k|^2 \) is the noise power at its output, \( G_{k,c(m)} \) is the average channel gain (loss) between the CS with the index \( c(m) \) and the \( k \)th user, and \((\cdot)^H\) denotes the Hermitian transpose.

Comparing (1) with (2), we see that (2) corresponds to (1) if

\[
h_{k,c(m)} = G_{k,c(m)} |a_k^H s_m|^2.
\] (3)

Therefore, (1) can be used to characterize the user receive SINR in CDMA systems.

2.2. SDMA system

In SDMA systems, the base stations are equipped with antenna arrays. In such a system, the receive SINR for the \( k \)th
user can be expressed as \([6, 9, 13]\)

\[
\text{SINR}_k = \frac{P_k w_k^H R_{k,c(m)} w_k}{\sigma_k^2 + \sum_{m=1; m \neq k}^K P_m w_m^H R_{k,c(m)} w_m},
\]

(4)

where \(w_k\) is the normalized \((\|w_k\|^2 = 1)\) beamformer weight vector for the \(k\)th user, \(\sigma_k^2\) is the noise power of the \(k\)th user, \(R_{k,c(m)}\) is the DCC matrix between the CS with the index \(c(m)\) and the \(k\)th user, and \(\| \cdot \|\) denotes the Euclidean norm of a vector or the Frobenius norm of a matrix.

Comparing (1) with (4), we see that they coincide if

\[
h_{k,c(m)} = w_m^H R_{k,c(m)} w_m.
\]

(5)

Hence, (1) can be also used to characterize the user receive SINR in SDMA systems.

### 2.3. Conventional power control

The goal of power control is to find all \(P_k > 0\) such that the total transmitted power

\[
P = \sum_{k=1}^K P_k
\]

(6)

is minimized while a certain required QoS is guaranteed for each user [2]. The QoS for the \(k\)th user can be defined by means of its receive SINR as

\[
\text{SINR}_k \geq \gamma_k \quad \text{for} \quad 1 \leq k \leq K,
\]

(7)

where \(\gamma_k\) are predefined constants.

Note that the total transmitted power (6) is minimized when all constraints in (7) become equalities. This statement can be proven by contradiction as follows. Assume that the transmitted power is minimized while some of inequalities in (7) remain strict. For example, let such an inequality be strict for the \(n\)th user. Then, we have

\[
\text{SINR}_n = \frac{P_n h_{n,c(n)}}{\sigma_n^2 + \sum_{m=1; m \neq n}^K P_m h_{m,c(m)}} > \gamma_n.
\]

(8)

This inequality can be transformed to equality if we reduce the transmitted power \(P_n\) as

\[
P_n := \alpha P_n.
\]

(9)

where \(\alpha \triangleq \gamma_n/\text{SINR}_n < 1\). This operation does not violate any of the QoS constraints for the other users (note that with reduction of the transmitted power \(P_n\), the produced interference to the other users is reduced). However, this means that the total transmitted power (6) can be further reduced, which is an obvious contradiction. Therefore, all the QoS constraints (7) must be satisfied as equalities if (6) is minimized.

Using this result, the problem of minimizing the total transmitted power (6) subject to the QoS constraints can be written as [8]

\[
\min_{P_k} \sum_{k=1}^K P_k \quad \text{s.t.} \quad \frac{P_k h_{k,c(k)}}{\sigma_k^2 + \sum_{m=1; m \neq k}^K P_m h_{k,c(m)}} = \gamma_k.
\]

(10)

We can rewrite the constraints in (10) as

\[
P_k \frac{h_{k,c(k)}^T}{\gamma_k} - \sum_{m=1; m \neq k}^K P_m h_{c(m)} = \sigma_k^2.
\]

(11)

In matrix form, (11) can be expressed as

\[
H(h,g)p = n,
\]

(12)

where

\[
p = \begin{bmatrix} P_1, \ldots, P_K \end{bmatrix}^T,
\]

\[
n = \begin{bmatrix} \sigma_1^2, \ldots, \sigma_K^2 \end{bmatrix}^T.
\]

(13)

are the \(K \times 1\) vectors of the transmitted powers and noise powers, respectively; \(h\) is the vector containing all channel coefficients \(h_{i,c(l)}\), \(i,l = 1, \ldots, K\); \(g = [\gamma_1, \ldots, \gamma_K]^T\);

\[
[H(h,g)]_{i,l} = \begin{cases} h_{i,c(l)} & \text{for } i = l, \\ \gamma_l & \text{for } i \neq l, \\ -h_{i,c(l)} & \text{for } i \neq l, 
\end{cases}
\]

(14)

and \((\cdot)^T\) stands for the transpose. Using (12), the optimal transmitted powers can be computed as [2, 4]

\[
p = [H(h,g)]^{-1} n.
\]

(15)

Note that all transmitted powers must be positive and, therefore, the positiveness of \(P_k\) has to be checked for all \(k = 1, \ldots, K\). If \(P_k \leq 0\) for some values of \(k\), then the underlying problem is infeasible. To make the problem feasible, either the parameters \(\gamma_k\) should be decreased or the number of users should be reduced.

### 3. Robust Downlink Power Control

In practice, the downlink channel coefficients \(h_{i,c(l)}\) are known imprecisely because of an imperfect (quantized or outdated) feedback, channel estimation errors, channel variability, user mobility, and so forth. As a result of such imperfect knowledge of the downlink channel, the QoS constraints (7) may become violated for some of the users. Therefore, the robustness of downlink power control and transmit beamforming algorithms appears to be of primary importance in practical systems.
known error. We assume that each downlink channel error \( h_{k,m} \) is the presumed downlink channel gain (loss), \( h_{k,m} \) is its actual value, and scalar quantity \( e_{k,m} \) is the unknown error. We assume that each downlink channel error \( e_{k,m} \) is bounded by some known constant:

\[
|e_{k,m}| \leq \delta h_{k,m} \triangleq \delta k_{m}, \quad m = 1, \ldots, K. \tag{17}
\]

Here, the known value of \( \delta < 1 \) (or, equivalently, of \( \delta k_{m} < h_{k,m} \)) determines the maximal expected amount of uncertainty in the channel coefficient.

It should be stressed that the channel error itself is assumed to be unknown but only some tight upper bound on the absolute value of this error should be known. A proper value of \( \delta \) can be easily deduced using preliminary knowledge of the type of feedback imperfections and/or coarse knowledge of the channel type and its main characteristics (e.g., using channel simulators and the results of channel measurement campaigns [17, 18]).

We modify the QoS conditions (7) to incorporate the robustness against unknown but bounded channel errors. Instead of (7) (which is formulated for the ideal error-free downlink channel case), we require the QoS conditions to be satisfied for all possible mismatched downlink channel errors. That is, for the \( k \)th user (\( k = 1, \ldots, K \)), we require that

\[
P_k \frac{\min_{e_{k,m}} (\tilde{h}_{k,(k)} + e_{k,(k)})}{\sigma_k^2 + \sum_{l \neq k} P_l (\tilde{h}_{k,(l)} + e_{k,(l)})} \geq \gamma_k
\]

for all \( e_{k,m} \) that are bounded as \( |e_{k,m}| \leq \delta k_{m} \).

Note that (18) is equivalent to the worst-case QoS constraint which should be satisfied for the worst-case SINR of the \( k \)th user (\( k = 1, \ldots, K \)). This constraint can be rewritten as

\[
\min_{|e_{k,m}| \leq \delta k_{m}} P_k \frac{\tilde{h}_{k,(k)} + e_{k,(k)}}{\sigma_k^2 + \sum_{l \neq k} P_l (\tilde{h}_{k,(l)} + e_{k,(l)})} \geq \gamma_k
\]

for all \( e_{k,m} \) that are bounded as \( |e_{k,m}| \leq \delta k_{m} \).

Unfortunately, the numerator and denominator of (19) are not independent in the case when \( c(l) = c(k) \) for any \( l \neq k \), that is, when the \( k \)th user and any of the remaining users are assigned to the same base station. In this case, the complexity of (19) does not allow us to obtain any closed-form solution. Therefore, we strengthen the QoS constraints (19) by replacing the worst-case user SINR by its lower bound in each of them. The left-hand side of (19) can be lower-bounded by

\[
P_k \frac{\min_{e_{k,m}} (\tilde{h}_{k,(k)} + e_{k,(k)})}{\sigma_k^2 + \sum_{l \neq k} P_l (\tilde{h}_{k,(l)} + e_{k,(l)})} \geq \gamma_k
\]

Replacing the worst-case user SINR in (19) by its lower bound (20) and taking into account that the total transmitted power is minimized when the inequality constraints become equalities, we obtain the following robust QoS constraint for the \( k \)th user:

\[
P_k \frac{\tilde{h}_{k,(k)} - e_{k,(k)}}{\sigma_k^2 + \sum_{l \neq k} P_l (\tilde{h}_{k,(l)} + e_{k,(l)})} = \gamma_k
\]

The solution to K linear equations in (21) is given by

\[
p_{\text{rob}} = [\mathbf{H}(\mathbf{h}, \mathbf{g}) - \mathbf{\Delta}(\delta, \mathbf{g})]^{-1} \mathbf{n}, \tag{22}
\]

where \( \mathbf{h} \) is the vector of all presumed channel values \( \tilde{h}_{k,(l)} \) and the \((i, l)\)th element of \( \mathbf{\Delta}(\delta, \mathbf{g}) \) is defined as

\[
[\mathbf{\Delta}(\delta, \mathbf{g})]_{ij} = \left\{ \begin{array}{ll}
\delta_{c(l)} & \text{for } i = l, \\
\gamma_i & \text{for } i \neq l.
\end{array} \right.
\]

Equations (22) and (23) are the core of the proposed robust power control algorithm. Similar to (15), all transmitted powers (all elements of the vector \( p_{\text{rob}} \) in (22)) must be positive. Once the optimal powers are computed, the positiveness of all \( P_k \) has to be checked. The fact that \( P_k \leq 0 \) for some values of \( k \) shows that the underlying problem is infeasible and the parameters \( \gamma_k \) should be changed (decreased) to warrant feasibility.

In what follows, we show how in the SDMA case the robust power control algorithm (22) can be combined with any of transmit beamforming algorithms, for example, that in [14]. The algorithm (22) can be directly applied to CDMA cellular systems as well.

3.2. Matrix formulation

In this subsection, we consider the SDMA case when each base station is equipped by an antenna array. In this case, it is reasonable to model the error in the DCC matrices rather than in the channel values [19]. Using this approach, we have

\[
\mathbf{R}_{k,m} = \tilde{\mathbf{R}}_{k,m} + \mathbf{E}_{k,m}, \quad m = 1, \ldots, K, \tag{24}
\]

where \( \tilde{\mathbf{R}}_{k,m} \) is the presumed DCC matrix, \( \mathbf{R}_{k,m} \) is its actual (mismatched) value, and \( \mathbf{E}_{k,m} \) is the unknown DCC matrix error. We assume that the Frobenius norm of each error matrix \( \mathbf{E}_{k,m} \) is bounded by some known constant:

\[
||\mathbf{E}_{k,m}|| \leq \epsilon ||\mathbf{R}_{k,m}|| \triangleq \delta k_{m}, \quad m = 1, \ldots, K. \tag{25}
\]
Here, the known value of $\epsilon$ (or, equivalently, of $\epsilon_{k,c(m)}$) determines the maximal expected amount of uncertainty in the DCC matrix. We stress here that the matrix $E_{k,c(m)}$ itself is unknown but only some tight upper bound on its Frobenius norm is known.

We modify the QoS conditions (7) to incorporate the robustness against unknown but norm-bounded DCC matrix errors. Similarly to Section 3.1, we require the QoS constraints to be satisfied for the worst-case SINR of each user. This QoS constraint for the $k$th user can be written as

$$\frac{P_k w_k^H (R_{k,c(k)} + E_{k,c(k)}) w_k}{\sigma_k^2 + \sum_{l=1; l \neq k}^K P_l w_l^H (R_{k,c(l)} + E_{k,c(l)}) w_l} \geq \gamma_k \quad \forall \|E_{k,c(m)}\| \leq \epsilon_{k,c(m)}, \quad m = 1, \ldots, K,$$

where $w_k$ is the normalized ($\|w_k\|^2 = 1$) beamformer weight vector for the $k$th user.

This constraint can be rewritten as

$$\min_{\{E_{k,c(m)}\}_{m=1}^M} \frac{P_k w_k^H (R_{k,c(k)} + E_{k,c(k)}) w_k}{\sigma_k^2 + \sum_{l=1; l \neq k}^K P_l w_l^H (R_{k,c(l)} + E_{k,c(l)}) w_l} \geq \gamma_k,$$

where the norms of all $E_{k,c(m)}$ ($m = 1, \ldots, K$) are bounded according to (25).

Unfortunately, the complexity of (27) does not allow us to obtain any closed-form solution. Therefore, we strengthen the QoS constraints (27) by replacing the worst-case user SINR by its lower bound in each of them. Then, the left-hand side of (27) can be lower-bounded by

$$\frac{P_k \min_{E_{k,c(k)}} w_k^H (R_{k,c(k)} + E_{k,c(k)}) w_k}{\sigma_k^2 + \sum_{l=1; l \neq k}^K P_l \max_{E_{k,c(l)}} w_l^H (R_{k,c(l)} + E_{k,c(l)}) w_l} \geq \gamma_k.$$

To simplify (28), we will make use of the following results.

**Lemma 1.** For any $n \times 1$ nonzero vector $x$ and $n \times n$ matrix $A$ which is unitarily similar to a diagonal matrix, the following inequality holds:

$$\frac{|x^H A x|}{x^H x} \leq \|A\|,$$

which is satisfied as equality if and only if $A = \xi x x^H$, where $\xi$ is an arbitrary scalar.

See the appendix for the proof.

**Corollary 1.** If $A$ is a Hermitian matrix and $x$ is any given normalized vector ($\|x\| = 1$), then

$$\max_{A : \|A\| \leq \rho} x^H A x = \rho,$$

$$\min_{A : \|A\| \leq \rho} x^H A x = -\rho.$$

Replacing the worst-case user SINR in (27) by its lower bound (28), using Corollary 1 and the equalities $\|w_k\| = 1$ ($k = 1, \ldots, K$), and taking into account that the total transmitted power is minimized when the inequality constraints become equalities, we obtain the following robust QoS constraint for the $k$th user:

$$\frac{P_k (w_k^H \tilde{R}_{k,c(k)} w_k - \epsilon_{k,c(k)})}{\sigma_k^2 + \sum_{l=1; l \neq k}^K P_l (w_l^H \tilde{R}_{k,c(l)} w_l + \epsilon_{k,c(l)})} = \gamma_k.$$

The solution to $K$ linear equations in (31) is given by

$$p_{\text{rob}} = [H(\tilde{h}, \epsilon) - \Delta(\epsilon, \epsilon)]^{-1} n,$$

where $h$ is the vector containing all $h_{m,c(n)} = w_n^H \tilde{R}_{m,c(n)} w_n$ and $\Delta(\epsilon, \epsilon)$ is similar to (23), that is,

$$[\Delta(\epsilon, \epsilon)]_{i,j} = \begin{cases} \epsilon_{c(x(i))} & \text{for } i = j, \\ \epsilon_{c(x(j))} & \text{for } i \neq j. \end{cases}$$

Comparing the constraints in (21) and (31), we see that they fully coincide if $h_{k,c(l)} = w_k^H \tilde{R}_{k,c(l)} w_l$ and $\delta_{k,c(l)} = \epsilon_{k,c(l)}$. This means that although the approaches in Section 3.1 and in this section are derived using different (scalar and matrix) error models, they lead to the same system of linear equations and, therefore, correspond to the same solution.

It is important to stress that the proposed robust power control method (32) can be efficiently combined with any (even noncentralized and/or nonrobust) transmit beamforming method. In the next section, it will be demonstrated via computer simulations that the robustness provided by (32) is, in fact, sufficient to combat channel estimation errors irrespectively of the fact that the transmit beamformer used is noncentralized and/or nonrobust. Note that in the aforementioned situation (where our centralized power control method is combined with a noncentralized downlink beamformer), only the $K^2$ real-valued scalars $w_k^H \tilde{R}_{k,c(l)} w_l$ ($l, k = 1, \ldots, K$) should be transmitted from the base stations to the central unit, while in the case when the robust centralized beamforming method of [12] is used, all elements of $K^2$ complex matrices $R_{k,c(l)}$ ($l, k = 1, \ldots, K$) are required to be transmitted from the base stations to the central unit. Moreover, the noncentralized beamforming techniques which can be combined with the proposed robust power control method have much lower computational cost than the robust downlink beamforming method of [12] because noncentralized beamformers (e.g., that of [14]) can be computed in a closed form, whereas the centralized robust beamformer of [12] requires computationally demanding optimization techniques such as iterative optimization [8, 13] or semidefinite programming [9].

Therefore, the robustness against channel estimation errors can be achieved in the proposed technique at substantially reduced computational cost and much lower communication rate between the base stations and the central unit.
than the robustness of centralized beamforming methods. As a result, our robust technique can be implemented in a much simpler way than the robust centralized transmit beamforming method proposed in [12]. At the same time, as will be seen in the next section, such simple implementation of the proposed technique provides nearly the same performance and requires nearly the same total transmitted power as the method of [12].

4. SIMULATION RESULTS

In our computer simulations, we consider a TDD cellular system with $K = 16$ cochannel users and $M = 7$ cells. Each base station is assumed to have a transmit uniform circular array of eight omnidirectional sensors spaced half a wavelength apart. The geometry of the simulated scenario is shown in Figure 2. The signal power attenuation is assumed to be proportional to $r^{-4}$, where $r$ is the distance from a base station to a user. The users are assumed to be incoherently locally scattered sources [17, 18, 20, 21] with uniform angular distributions characterized by corresponding central angles and angular spreads. The presumed user central angles and angular spreads are fixed throughout the simulations. Also, the presumed and the true angular spreads are the same for each user and are selected randomly from the interval $[1^\circ, 6^\circ]$ (these randomly selected values do not change from run to run). However, to model DCC matrix errors, the true user central angles are mismatched with respect to the presumed ones. These user location mismatches vary in each simulation run where the true user central angles are randomly selected from the interval $[-\Delta \theta, \Delta \theta]$ around the corresponding presumed central angles. This situation may correspond, for example, to the practical case of outdated feedback.

We assume that the required receive SINR is identical for each user so that $y_i = y_i; i = 1, \ldots, K$. Also, the noise powers are assumed to be the same for each user, that is, $\sigma_i^2 = \sigma^2$ ($i = 1, \ldots, K$) with $\sigma^2 = -20$ dB.

All our simulation results are averaged over $1000$ simulation runs. The performances of the conventional and the proposed power control methods (15) and (32) are compared through simulations with the performance of the robust beamforming method of [12].

To compute the transmit weight vectors $w_k$ ($k = 1, \ldots, K$), four beamforming methods are used which are referred to in our figures as Methods 1 to 4, respectively.

In Method 1, the beamforming weight vector is computed in a nonrobust and noncentralized way. In this method, the ratio of the desired user received signal power to the produced interference power at the other users is maximized for each particular desired user [14]. This is equivalent to computing each $w_k$ as the solution to the following maximization problem:

$$w_k = \arg \max_w \frac{w^H \tilde{R}_{k,k}(k) w}{w^H \sum_{m=1; m \neq k} R_{m,m}(k) w}.$$  \hfill (34)

The solution to the above problem is given by [14]

$$w_k = \mathcal{P}\left\{ \sum_{m=1; m \neq k} \tilde{R}_{m,k}(k) + \varepsilon_{m,k}(k) \right\}^{-1} \tilde{R}_{k,k}(k),$$  \hfill (35)

where $\mathcal{P}\{ \cdot \}$ denotes the principal eigenvector. Note that this algorithm itself does not necessarily satisfy the QoS constraints (7).

Method 2 corresponds to the centralized iterative beamforming technique of [8].

Method 3 is the centralized robust beamformer of [12] which is implemented using the iterative technique of [8].

Method 4 represents a robust modification of the noncentralized beamformer (35). The proposed modification is to introduce an additional robustness against DCC matrix errors by applying the idea of [12] to (35). As a result, our Method 4 corresponds to the following noncentralized diagonally loaded beamformer:

$$w_k = \mathcal{P}\left\{ \sum_{m=1; m \neq k} \tilde{R}_{m,k}(k) + \varepsilon_{m,k}(k) \right\}^{-1} \left( \tilde{R}_{k,k}(k) - \varepsilon_{k,k}(k) \right).$$  \hfill (36)

Throughout all the examples, Methods 1, 2, and 4 are used in combination with the power control algorithms tested, whereas Method 3 is used separately (because it inherently involves power control feature already). Also, Method 3 uses the same robustness parameters as the proposed robust power control method (32) and Method 4.

It is important to emphasize that our power control technique combined with Method 2 neither will have simpler implementation than Method 3, nor can be expected to outperform this method in the case of channel estimation errors. The only reason we include Method 2 in our simulations is to provide a comprehensive and fair comparison of the performance of our robust power control algorithm when used in combination with different centralized and noncentralized beamforming techniques.

Figure 2: Simulated scenario with $M = 7$ cells and $K = 16$ cochannel users. Positions of base stations and users are indicated by $\ast$ and $\cdot$, respectively.
The achieved minimal user SINR as well as the required user SINR (which corresponds to the ideal case when the DCC matrices are exactly known) of the conventional power control algorithm (15) are shown in Figures 3a and 3b versus $\gamma$. This figure illustrates that, because of DCC matrix errors, the minimal user SINR of the algorithm (15) for all values of $\gamma$ is substantially lower than what is required by QoS constraints, specially for large uncertainty of the downlink channel.

In Figures 4a and 4b, the probability $p(\text{SINR} < \gamma)$ of the receive SINR being less than $\gamma$ is shown for the conventional power control method (15) versus $\gamma$ for $\Delta \theta = 1^\circ$ and $\Delta \theta = 4^\circ$, respectively. In this figure, the required SINR is $\gamma_c = 5$ dB. As can be seen, for $\Delta \theta = 4^\circ$, more than 70% and 68% of users have their receive SINRs less than $\gamma_c$ in the cases where Method 1 and Method 2 are used, respectively. In other words, the QoS constraints are not satisfied for the major part of users.

Figures 5a and 5b present the achieved minimal user SINRs versus $\epsilon$ for our robust power control method (32) in the cases $\Delta \theta = 1^\circ$ and $\Delta \theta = 4^\circ$, respectively. In this figure, we assume that the system-required SINR is $\gamma_c = 5$ dB.

Figure 6 is similar to Figure 5 except for the fact that in Figure 6 the value of the system-required SINR is $\gamma_c = 7$ dB. From these figures, it follows that, as expected, higher values of the robustness parameter $\epsilon$ are required in situations with larger channel estimation errors. It can be also seen from Figures 5 and 6 that the optimal choice of $\epsilon$ is nearly
the same for all the robust methods tested. This motivates the use of identical values of $\varepsilon$ for all the robust methods used in simulations. Interestingly, the actual worst cases for $\Delta \theta = 1^\circ$ and $\Delta \theta = 4^\circ$ correspond to the values $\varepsilon = 0.07$ and $\varepsilon = 0.28$, respectively, which are substantially larger than the values of $\varepsilon$ sufficient for providing good robustness (see Figures 5 and 6). This can be explained by the fact that worst-case designs may be overly conservative (i.e., the actual worst case may occur very rarely) and, therefore, smaller values of robustness parameter $\varepsilon$ are sufficient to achieve satisfactory robustness.

The probability $p(\text{SINR} < \gamma)$ of the conventional and robust power control techniques combined with Method 1 is shown in Figure 7 versus $\gamma$. In this figure, $\gamma = 5$ dB. For the sake of comparison of the nonrobust and robust power control approaches, the values $\varepsilon = 0$ and $\varepsilon = 0.01$ are tested in this figure.

Figure 8 is similar to Figure 7 except for the fact that beamforming Method 2 is used instead of Method 1.

Figure 9 displays the probability $p(\text{SINR} < \gamma)$ of Method 3 versus $\gamma$. Similar to the previous two figures, $\gamma = 5$ dB and the values $\varepsilon = 0$ and $\varepsilon = 0.01$ are tested.
5 dB) as a function of $\varepsilon$ of the robust beamforming algorithm of [12] (Method 3).

Method 2 versus $\gamma$ the robust and nonrobust power control methods combined with noncentralized Method 4 is very similar to that of our robust power control method combined with noncentralized beamforming Methods 1 and 4 is used instead of Method 1. In the latter case, the excess power is less than 1 dB.

As can be observed from Figure 11, for $\Delta \theta = 4^\circ$, this probability drops from approximately 70% (for nonrobust methods with $\varepsilon = 0$) to less than 20% (when using our robust power control technique along with beamforming Methods 1, 2, and 4, as well as robust beamforming Method 3; all with $\varepsilon = 0.01$).

Figures 7–10 display the probability $P(\text{SINR} < \gamma)$ for the robust and nonrobust power control methods combined with Methods 1 and 4 versus $\gamma$. For $\gamma_s = 5$ dB, the probability drops from approximately 70% (for nonrobust techniques) to less than 20% (when using our robust power control technique combined with noncentralized beamforming Methods 1 and 4).

From Figure 12, it can be seen that for a wide range of the parameter $\varepsilon$ and for both values of $\gamma_s$ used, the transmitted power of our robust power control technique combined with Methods 2 and 4 is nearly identical to that of the robust beamforming technique of [12] (Method 3). This implies that although our robust power control algorithm (when combined with noncentralized Method 4) can be implemented in a much simpler way than Method 3, there is no advantage in the total transmitted power of Method 3. However, there is such an advantage if our robust power control technique is combined with Method 1. In the latter case, the excess transmitted power with respect to Method 3 for low values of $\varepsilon$ is a few decibels only. For example, if $\varepsilon = 0.01$, then the excess power is less than 1.5 dB. Interestingly, the transmitted power of our robust power control algorithm
combined with Method 4 decreases when the parameter $\varepsilon$ is increased from zero to moderate values (e.g., to $\varepsilon = 0.01$).

Comparing the results of Figure 12 with those of Figure 11, we can see that in the robust power control and beamforming methods tested, quite substantial robustness improvements in terms of the percent of QoS-supported users can be achieved only at a slight increase of the transmitted power.

5. CONCLUSIONS

A new centralized downlink power control algorithm is proposed for cellular wireless communication systems. Our technique provides a substantially improved robustness against imperfect knowledge of the wireless channel by means of maintaining the required QoS for the worst-case channel uncertainty.

Simulation results have validated a substantially improved robustness of our algorithm as compared to the conventional power control approach and demonstrated that such improvements are achieved at the price of only a slight increase of the total transmitted power.

It has been shown that, when combined with the simple noncentralized beamformer of [14] and its simple robust (diagonal loading-based) modification, the proposed centralized robust power control algorithm has nearly the same performance and transmitted power requirements as the popular robust beamforming algorithm of [12]. At the same time, the overall implementation of the former algorithm is much simpler than that of [12].

Figure 11: Probability of the user received SINR being less than $\gamma_\text{c} = 5$ dB for the proposed robust power control technique and for Method 3 versus $\varepsilon$: (a) $\Delta \theta = 1^\circ$; (b) $\Delta \theta = 4^\circ$.

Figure 12: Required transmitted power for the proposed robust power control technique and for Method 3 versus $\varepsilon$: (a) $\gamma_\text{c} = 5$ dB; (b) $\gamma_\text{c} = 7$ dB.
APPENDIX

Proof of Lemma 1. Since $A$ and $Λ$ = diag($λ_1, \ldots, λ_n$) are unitarily similar, there exists a unitary matrix $Q$ such that $A = QAQ^H = \sum_{i=1}^{n} λ_i q_i q_i^H [22]$, where $q_i$ is the $i$th column of $Q$. Here, we assume that $|λ_1| \geq \cdots \geq |λ_n|$. Since $Q$ is an $n \times n$ unitary matrix, any $n$ dimensional vector $x$ can be expressed as $x = \sum_{i=1}^{n} α_i q_i$, where $α_i = q_i^H x$. Accordingly, $x^H A x = \sum_{i=1}^{n} |α_i|^2$ and

$$|x^H A x| = |x^H \left( \sum_{i=1}^{n} λ_i q_i q_i^H \right) x|$$

$$= \left| \sum_{i=1}^{n} λ_i x^H q_i q_i^H x \right|$$

$$= \left| \sum_{i=1}^{n} λ_i |α_i|^2 \right|$$

$$\leq \sum_{i=1}^{n} |λ_i| |α_i|^2 \tag{A.1}$$

$$\leq |λ_1| \sum_{i=1}^{n} |α_i|^2$$

$$= |λ_1| x^H x$$

$$\leq x^H x \left| \sum_{i=1}^{n} |λ_i|^2 \right|^{1/2}$$

$$= x^H x \|A\|_2$$

and the lemma is proved.

It can be readily verified that (A.1) jointly become equalities if and only if $A = ξ xx^H$, where $ξ$ is an arbitrary scalar.

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