Generalized analysis on $B \to K^*\rho$
within and beyond the Standard Model

– Can it help understand the $B \to K\pi$ puzzle?

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Abstract
We study $B \to K^*\rho$ modes that are analogues of the much studied $B \to K\pi$ modes with $B$ decaying to two vector mesons instead of pseudoscalar mesons, using topological amplitudes in the quark diagram approach. We show how $B \to K^*\rho$ modes can be used to obtain many more observables than those for $B \to K\pi$ modes, even though the quark level subprocesses of both modes are exactly the same. All the theoretical parameters (except for the weak phase $\gamma$), such as the magnitudes of the topological amplitudes and their strong phases, can be determined in terms of the observables without any model-dependent assumption. We demonstrate how $B \to K^*\rho$ can also be used to verify if there exist any relations between theoretical parameters, such as the hierarchy relations between the topological amplitudes and possible relations between the strong phases. Conversely, if there exist reliable theoretical estimates of amplitudes and strong phases, the presence of New physics could be probed. We show that if the tree and color-suppressed tree are related to the electroweak penguins and color-suppressed electroweak penguins, it is not only possible to verify the validity of such relations but also to have a clean measurement of New Physics parameters. We also present a numerical study to examine which of the observables are more sensitive to New Physics.
I. INTRODUCTION

The study of two-body hadronic $B$ decays provides good opportunities to test the Standard Model (SM) and further to probe possible new physics (NP) effects beyond the SM. Large numbers of $B$ mesons have been produced at the B factories enabling accurate measurements of branching ratios and direct CP asymmetry for many modes. The $B \to VV$ modes, where $V$ denotes a vector meson, have the advantage that they provide many more observables, compared with those being measured in $B \to PP$ (e.g., $B \to K\pi$) or $B \to VP$ (e.g., $B \to K^*\pi$) modes, where $P$ denotes a pseudoscalar meson, due to spins of the final state vector mesons. Since the first observation of $B \to K^*\phi$ by CLEO Collaboration [1], several $B$ decays to two charmless vector mesons, such as $B \to K^*\rho$ and $B \to \rho\rho$, have been reported by BABAR and BELLE Collaboration [2, 3, 4, 5, 6, 7]. In fact, polarization measurements for several such modes have already been reported.

Recent experimental results [2, 3, 8] for the $B \to K\pi$ mode, show deviations from SM expectations; the discrepancy, commonly being referred to as the “$B \to K\pi$ puzzle.” The dominant quark level subprocesses for $B \to K\pi$ decays are $b \to s\bar{q}q$ ($q = u, d$) penguin processes which are potentially sensitive to NP effects. Many efforts have been made to resolve the puzzle [9, 10]. A model-independent study shows that the experimental data strongly indicate large enhancements of both the electroweak (EW) penguin and the color-suppressed tree contributions [10]. The $B \to K\pi$ modes have certain inherent limitations. The four $B \to K\pi$ decay modes can experimentally yield at most 9 observables: four each of the branching ratios and direct CP asymmetries and one time-dependent CP asymmetry. Clearly, the 9 observables are insufficient to determine all the 12 theoretical parameters [10] needed to describe these decay modes. One hence needs to make some assumptions. Traditionally, assumptions have often been made on sizes of the topological amplitudes as well as on the strong phases of the different topologies.

The $B \to K^*\rho$ modes are the $B \to VV$ analogues of $B \to K\pi$ modes, in the sense that the quark level processes of both modes are exactly the same. Thus, it is expected that if there appear any NP effects through $B \to K\pi$, then similar NP effects will appear through $B \to K^*\rho$ as well. However, the study of $B \to VV$ modes necessitates performing an angular analysis in order to obtain the helicity amplitudes. While angular analysis is often regarded as an additional complication needed due to the presence of both CP-even
and CP-odd components that dilute the time dependent CP asymmetry, it can provide an impressive gain in terms of the large number of observables. We note that preliminary polarization measurement for $B^+ \to K^{*+}\rho^0$, $B^+ \to K^{*0}\rho^+$ and $B^0 \to K^{*0}\rho^0$ have already been done [2, 3, 4, 7].

In this work, we study $B \to K^*\rho$ decays in a model-independent approach. We will show that in comparison to the $B \to K\pi$ modes that yield 9 observables, the $B \to K^*\rho$ modes result in a total of 35 independent observables. A theoretical description of $B \to K^*\rho$, however, requires 36 independent parameters, which is still one short of the number of possible observables. While the large number of observables may not seem like a distinct advantage at first, we will argue that they provide valuable insights into resolving the “$B \to K\pi$ puzzle.” Our goal is two-fold. First, we try to determine all the relevant theoretical parameters describing the decay amplitudes of $B \to K^*\rho$ in a model-independent way in terms of experimental observables. These determined theoretical parameters can be compared with the corresponding model estimates. The information will be very useful for improving model calculations, such as those based on QCD factorization [11], perturbative QCD [12], and so on. Second, we try to suggest certain tests of the SM that may reveal NP effects if they appear in $B \to K^*\rho$ decays. Any indication of NP effects in $B \to K^*\rho$ will provide valuable hints on possible NP effects in $B \to K\pi$ decays. For our goal, the decay amplitudes of $B \to K^*\rho$ are decomposed into linear combinations of the topological amplitudes in the quark diagram approach [13]. We then focus on how to extract all the theoretical parameters, including the magnitudes of the topological amplitudes and their strong phases, in terms of experimental observables. As it turns out, all the parameters can be determined in analytic forms. We also propose tests of conventional hierarchy relations between the topological amplitudes and of possible relations between the relevant strong phases within the SM. A breakdown of these relations may indicate possible NP contributions appearing in $B \to K^*\rho$ decays, as well as in the analogous mode $B \to K\pi$. One could hence verify if NP is the source of the “$B \to K\pi$ puzzle.”

The paper is organized as follows. A general formalism for $B \to K^*\rho$ is presented in Sec. [II]. In Sec. [III] we explicitly show that it is possible to obtain analytic solutions to all the theoretical parameters in terms of observables. In Sec. [VI] we discuss how to examine the conventional hierarchy of the topological amplitudes and possible relations of their strong phases. We conclude in Sec. [VII].
II. FORMALISM FOR $B \to K^* \rho$ DECAYS

The decay amplitudes for four $B \to K^* \rho$ modes can be written in terms of the topological amplitudes in the quark diagram approach as

\[
A_\lambda^{0+} \equiv A_\lambda(B^+ \to K^{*0}(\rho^+)) = V_{ub}^*V_{us}A_\lambda' + V_{tb}^*V_{ts}P_{\lambda}',
\]

\[
A_\lambda^{+0} \equiv A_\lambda(B^+ \to K^{*+}(\rho^0)) = -\frac{1}{\sqrt{2}} \left[ V_{ub}^*V_{us}(T_\lambda' + C_\lambda' + A_\lambda') + V_{tb}^*V_{ts}(P_\lambda' + P_{EW}' + P_{EW}^C_\lambda) \right],
\]

\[
A_\lambda^- \equiv A_\lambda(B^0 \to K^{*-}(\rho^-)) = -\frac{1}{\sqrt{2}} \left[ V_{ub}^*V_{us}T_\lambda + V_{tb}^*V_{ts}(P_\lambda' + P_{EW}' + P_{EW}^C_\lambda) \right],
\]

\[
A_\lambda^{00} \equiv A_\lambda(B^0 \to K^{*0}(\rho^0)) = -\frac{1}{\sqrt{2}} \left[ V_{ub}^*V_{us}C_\lambda - V_{tb}^*V_{ts}(P_\lambda - P_{EW}^C_\lambda) \right],
\]

where $V_{ij}$ ($i = u, t; \ j = s, b$) are Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and the subscript $\lambda = \{0, \|, \perp\}$ denotes the helicity of the amplitudes. The amplitudes $T'$, $C'$, $A'$, $P'$, $P_{EW}'$, and $P_{EW}^C_\lambda$ are defined as

\[
T' \equiv T + P_{uc} + E_{uc},
\]

\[
C' \equiv C - P_{uc} - E_{uc},
\]

\[
A' \equiv A + P_{uc} + E_{uc},
\]

\[
P' \equiv P_{tc} + E_{tc} - \frac{1}{3} P_{EW}^C + \frac{2}{3} E_{EW},
\]

\[
P_{EW}' \equiv P_{EW}^C + E_{EW},
\]

\[
P_{EW}^C_\lambda \equiv P_{EW}^C - E_{EW}^C,
\]

where $P_{ic} \equiv P_i - P_c$ and $E_{ic} \equiv E_i - E_c$ ($i = u, t$). The topological amplitude $T$ is a color-favored tree amplitude, $C$ is a color-suppressed tree, $A$ is an annihilation, $P_j$ ($j = u, c, t$) is a QCD penguin, $E_j$ is a penguin exchange, $P_{EW}$ is a color-favored electroweak (EW) penguin, $P_{EW}^C$ is a color-suppressed EW penguin, $E_{EW}^C$ is a color-suppressed EW penguin exchange diagram. We follow and generalize the notation used in Ref. [10].

The relative sizes among these topological amplitudes are roughly estimated [13] as

\[
1 : |V_{tb}^*V_{ts} P_{tc}|,
\]

\[
\mathcal{O}(\bar{\lambda}) : |V_{ub}^*V_{us} T|, |V_{tb}^*V_{ts} P_{EW}|,
\]

\[
\mathcal{O}(\bar{\lambda}^2) : |V_{ub}^*V_{us} C|, |V_{tb}^*V_{ts} P_{EW}^C|,
\]

\[
\mathcal{O}(\bar{\lambda}^3) : |V_{ub}^*V_{us} A|, |V_{ub}^*V_{us} P_{uc}|,
\]

(11)
where $\bar{\lambda} \sim 0.2$. For the relative size of $|V_{ub}^* V_{us} P_{uc}|$, one can roughly estimate that
\[ \frac{|V_{ub}^* V_{us} P_{uc}|}{|V_{tb}^* V_{ts} P_{tc}|} \sim \chi^2 \frac{|P_{uc}|}{|P_{tc}|}. \] (12)

Note that $|P_u|$ and $|P_c|$ are smaller than $|P_t|$ \[14\], and more precisely it can be estimated that $0.2 < \frac{|P_{uc}|}{|P_{tc}|} < 0.4$ within the perturbative calculation \[15\]. Therefore, we assume $|(V_{ub}^* V_{us} P_{uc})/(V_{tb}^* V_{ts} P_{tc})| \sim \mathcal{O}(\bar{\lambda}^3)$ for our analysis.

Now we re-express Eqs. (1)–(4) as
\begin{align}
A_{\lambda}^{0+} &= e^{i\gamma} \tilde{A}_\lambda e^{i\delta_\lambda^A} - \tilde{P}_\lambda e^{i\delta_\lambda^P}, \quad (13) \\
A_{\lambda}^{+0} &= -\frac{1}{\sqrt{2}} \left[ e^{i\gamma} (\tilde{T}_\lambda e^{i\delta_\lambda^T} + \tilde{C}_\lambda e^{i\delta_\lambda^C} + \tilde{A}_\lambda e^{i\delta_\lambda^A}) \\
&- (\tilde{P}_\lambda e^{i\delta_\lambda^P} + \tilde{P}_\lambda e^{i\delta_\lambda^{PEW}} + \tilde{P}_\lambda e^{i\delta_\lambda^{CEW}}) \right], \quad (14) \\
A_{\lambda}^{+0} &= \left[ e^{i\gamma} \tilde{T}_\lambda e^{i\delta_\lambda^T} - (\tilde{P}_\lambda e^{i\delta_\lambda^P} + \tilde{P}_\lambda e^{i\delta_\lambda^{CEW}}) \right], \quad (15) \\
A_{\lambda}^{00} &= -\frac{1}{\sqrt{2}} \left[ e^{i\gamma} \tilde{C}_\lambda e^{i\delta_\lambda^C} + (\tilde{P}_\lambda e^{i\delta_\lambda^P} - \tilde{P}_\lambda e^{i\delta_\lambda^{PEW}}) \right], \quad (16)
\end{align}

where the $\gamma$ and $\delta_\lambda$’s are the weak phase and the relevant strong phases, respectively. We note that isospin symmetry relates the amplitudes for these 4 decay modes and their conjugate modes by the relations:
\begin{align}
\frac{1}{\sqrt{2}} (A_{\lambda}^{0+} - A_{\lambda}^{+0}) &= A_{\lambda}^{00} - A_{\lambda}^{+0}, \quad (17) \\
\frac{1}{\sqrt{2}} (A_{\lambda}^{0+} - A_{\lambda}^{+0}) &= A_{\lambda}^{00} - A_{\lambda}^{+0}. \quad (18)
\end{align}

It should be emphasized that as mentioned in Ref. \[16\], for the $B \to K\pi$ mode, the above expressions in Eqs. (13)–(16) for the decay amplitudes describe not only the SM contributions but also any possible NP effects that contribute to the amplitude. Consider for instance the contribution of NP with an amplitude $N_\lambda e^{i\delta_\lambda} e^{i\phi_{NP}}$. This amplitude may be re-expressed using reparametrization invariance \[17\] as a sum of two contributions with one term having no weak phase and the other term having a weak phase $\gamma$, i.e. $N_\lambda e^{i\delta_\lambda} e^{i\phi_{NP}} \equiv N_1^\lambda e^{i\delta_\lambda} + N_2^\lambda e^{i\delta_\lambda} e^{i\gamma}$, where $N_1^\lambda$ and $N_2^\lambda$ are determined purely in terms of $\phi_{NP}$ and $\gamma$. As an explicit example let us consider NP contributing via the EW penguin to amplitudes in Eqs. (14) and (16). Using reparametrization invariance it can easily be absorbed by redefining the amplitudes $\tilde{P}_\lambda^{PEW}$ and $\tilde{C}_\lambda$, so that the amplitudes in Eqs. (13) and (16) retain the same form. In general NP contributing to any of the topological amplitudes can be easily absorbed so that the amplitudes in Eqs. (13)–(16) retain the same form.
The amplitudes for \( B \to K^* \rho \) involve three helicities for each of the modes. These amplitudes and their conjugates, involving the three helicities, are expressed as

\[
Amp(B \to K^* \rho) = A_0 g_0 + A_\parallel g_\parallel + i A_{\perp} g_{\perp},
\]

\[
Amp(\bar{B} \to K^* \rho) = \bar{A}_0 g_0 + \bar{A}_\parallel g_\parallel - i \bar{A}_{\perp} g_{\perp},
\]

where the \( g_\lambda \) are the coefficients of the helicity amplitudes written in the linear polarization basis. The \( g_\lambda \) depend only on the angles describing the kinematics \[18\]. The helicity amplitudes (and their conjugate amplitudes) for the four \( K^* \rho \) modes are denoted by \( A_{\lambda^0}^{(0)}, A_{\lambda^0}^{(\pm)}, A_{\lambda^0}^{(0)}, (A_{\lambda^0}^{(0)}, A_{\lambda^0}^{(0)}, A_{\lambda^0}^{(0)}, A_{\lambda^0}^{(0)}). \) Thus, the number of amplitudes is three times that for the \( B \to K \pi \) modes. In contrast to the \( B \to K \pi \) case, one can in principle measure many more observables in the \( B \to K^* \rho \) case. Without including the interference terms between helicities, one would have three times the number of observables in comparison to the \( K \pi \) modes, \textit{i.e.}, 27 observables. However, many more of observables result from the interference terms between the helicities. Let us examine in detail the number of observables available in \( B \to K^* \rho \).

The time dependent decay for \( B \to f \), where \( f \) is one of the \( K^* \rho \) final states, may be expressed as

\[
\Gamma(B(t) \to f) = e^{-i \Gamma t} \sum_{\lambda \leq \sigma} (\Lambda_{\lambda \sigma}^f \pm \Sigma_{\lambda \sigma}^f \cos(\Delta Mt) \mp \rho_{\lambda \sigma}^f \sin(\Delta Mt)) g_\lambda g_\sigma, \tag{20}
\]

where

\[
B_{\lambda}^f \equiv \Lambda_{\lambda \lambda}^f = \frac{1}{2} (|A_\parallel^f|^2 + |\bar{A}_{\parallel}^f|^2), \quad \Sigma_{\lambda \lambda}^f = \frac{1}{2} (|A_\parallel^f|^2 - |\bar{A}_{\parallel}^f|^2),
\]

\[
\Lambda_{\perp \perp}^f = -\text{Im}(A_\perp^f A_\perp^{f*} - \bar{A}_{\perp}^f \bar{A}_{\perp}^{f*}), \quad \Lambda_{\| \|}^f = \text{Re}(A_\|^f A_\|^{f*} + \bar{A}_{\|}^f \bar{A}_{\|}^{f*}),
\]

\[
\Sigma_{\perp \perp}^f = -\text{Im}(A_\perp^f A_\perp^{f*} + \bar{A}_{\perp}^f \bar{A}_{\perp}^{f*}), \quad \Sigma_{\| \|}^f = \text{Re}(A_\|^f A_\|^{f*} - \bar{A}_{\|}^f \bar{A}_{\|}^{f*}),
\]

\[
\rho_{\perp \perp}^f = \text{Re}(e^{-i \phi_{\perp \perp}^f} [A_\perp^f \bar{A}_{\perp}^{f*} + A_{\perp}^f \bar{A}_{\perp}^{f*}]), \quad \rho_{\| \|}^f = \text{Im}(e^{-i \phi_{\| \|}^f} [A_\|^f \bar{A}_{\|}^{f*} + A_{\|}^f \bar{A}_{\|}^{f*}]),
\]

\[
\rho_{\perp \perp}^f = -\text{Im}(e^{-i \phi_{\perp \perp}^f} [A_\perp^f \bar{A}_{\perp}^{f*} + A_{\perp}^f \bar{A}_{\perp}^{f*}]), \quad \rho_{\| \|}^f = -\text{Im}(e^{-i \phi_{\| \|}^f} [A_\|^f \bar{A}_{\|}^{f*} + A_{\|}^f \bar{A}_{\|}^{f*}]),
\]

\[
(\lambda, \sigma = \{0, \|, \perp\}, \quad i = \{0, \|\}). \tag{21}
\]

Only for the CP eigenstate \( K^{*0} \rho^0 \) one can measure all these 18 observables. The other 3 modes are not CP eigenstates so that time dependent asymmetry cannot be measured. For each of these modes only \( \Lambda_{\lambda \sigma}^f \) and \( \Sigma_{\lambda \sigma}^f \) can be measured, resulting in a total of 12
observables for each of the 3 modes: $B^0 \rightarrow K^{*+}\rho^-$, $B^+ \rightarrow K^{*0}\rho^+$ and $B^+ \rightarrow K^{*+}\rho^0$. This results in a total of 54 observables. However, due to the isospin relations in Eqs. (17) and (18), the number of independent amplitudes is 18. This results in a total of 35 independent informations related to 18 magnitudes of the amplitudes and their 17 relative phases at best. Thus, only 35 of the above 54 observables can be independent.

The modes $B \rightarrow K^\ast \rho$ can be described theoretically using isospin in a manner analogous to the $B \rightarrow K\pi$ modes. Since there are three helicity states, the amplitudes corresponding to the different topologies carry a helicity index and may be denoted by $\tilde{T}_\lambda$, $\tilde{C}_\lambda$, $\tilde{A}_\lambda$, $\tilde{P}_\lambda$, $\tilde{P}_{EW}$, and $\tilde{P}_{CEW}$. There are hence 18 amplitudes each with its own strong phase denoted by $\delta_T^\lambda$, $\delta_C^\lambda$, $\delta_A^\lambda$, $\delta_P^\lambda$, $\delta_{EW}^\lambda$ and $\delta_{CEW}^\lambda$, respectively. Since only relative strong phases can be measured, the number of strong phases may be reduced to 17. Thus the theoretical description requires 36 parameters: 18 (real) amplitudes, 17 strong phases and $\gamma$. Despite the large number of observables in the $K^\ast \rho$ case, we still have one more parameter than the observables.

In the next section, we discuss how to determine all the theoretical parameters, such as the magnitudes and strong phases of the topological amplitudes, in term of the observables.

### III. EXTRACTING CONTRIBUTIONS OF VARIOUS TOPOLOGIES

The $B \rightarrow K^\ast \rho$ modes are described by a total of 36 parameters. However, as discussed above, one can obtain a maximum of 35 independent informations from the measurements. Therefore, it is only possible to solve for the parameters with respect to one unknown parameter namely $\gamma$. It is well known that the weak phase $\gamma$ can be measured through certain $B$ decay processes, such as $B \rightarrow D^{(*)}K^{(*)}[2,3]$. In this section we present analytic solutions to all the parameters with respect to $\gamma$. To simplify expression we introduce some new notation. We define

$$y^j_\lambda = \sqrt{1 - \left(\frac{\Sigma^j_\lambda}{A^j_\lambda}\right)^2}, \quad (22)$$

$$\alpha^i_{\lambda} = \arg(A^i_\lambda), \quad \bar{\alpha}^{ij}_\lambda = \arg(\bar{A}^{ij}_\lambda), \quad (23)$$

$$A^{ij}_\lambda = |A^{ij}_\lambda|, \quad \bar{A}^{ij}_\lambda = |\bar{A}^{ij}_\lambda|, \quad (24)$$

where $(ij) = (0+), (+0), (+-), (00)$ and $\lambda = \{0, ||, \perp\}$.

For illustration, we divide our task of finding the analytic solutions into two steps as follows. We first find the phases $\alpha^{ij}_\lambda$ and $\bar{\alpha}^{ij}_\lambda$ in terms of observables. Then, using the $\alpha^{ij}_\lambda$ and
\( \alpha_{ij} \), we determine the amplitudes \( \tilde{T}_\lambda, \tilde{C}_\lambda, \tilde{A}_\lambda, \tilde{P}_\lambda, \tilde{P}_{\lambda EW}, \tilde{P}_{\lambda EW C} \) as well as the strong phases \( \delta_T^\lambda, \delta_C^\lambda, \delta_A^\lambda, \delta_P^\lambda, \delta_{EW}^\lambda, \delta_{CEW}^\lambda \) given in Eqs. (13)−(16).

We begin by considering the decay amplitudes of the \( K^{*0}\rho^+ \) mode shown in Eq (13). Because the theoretical estimation of the annihilation contribution is very small (\( |\tilde{A}/\tilde{P}| \sim O(\bar{\lambda}^3) \) where \( \bar{\lambda} \sim 0.2 \)), one can safely neglect it \([19]\). After neglecting the annihilation terms, we obtain

\[
\tilde{P}_\lambda = A_{\lambda}^{0+}, \tag{25}
\]

\[
\delta_P^\lambda = \alpha_{\lambda}^{0+} - \pi, \tag{26}
\]

\[
\bar{\alpha}_{\lambda}^{0+} = \alpha_{\lambda}^{0+}. \tag{27}
\]

These relations imply that the direct CP asymmetry of the \( K^{*0}\rho^+ \) mode vanishes: \( \Sigma_{0+}^{\lambda} = 0 \) or \( y_{0+}^{\lambda} = 1 \).

We set \( \alpha_0^{0+} = \pi \) (or \( \delta_0^P = 0 \)) without loss of generality. Then the phases \( \alpha_{\perp}^{0+} \) and \( \alpha_{\parallel}^{0+} \) can be obtained from the relative phases \( (\alpha_{\parallel}^{0+} - \alpha_0^{0+}) \) and \( (\alpha_{\perp}^{0+} - \alpha_0^{0+}) \) that are determined from the angular analysis through the measurement of \( A_{\perp 0}^{0+}, \Sigma_{\parallel 0}^{0+}, A_{\parallel 0}^{0+}, \Sigma_{\perp 0}^{0+} \). Subsequently all the \( \delta_{\lambda}^T \) and \( \bar{\alpha}_{\lambda}^{0+} \) for \( \lambda = \{0, \parallel, \perp\} \) are determined from Eqs. (26) and (27), up to a discrete ambiguity. This ambiguity can be removed by using theoretical estimates \([19]\). From now on, for the sake of convenience, we re-parameterize for each helicity state every relevant phase, such as \( \delta_{T}^\lambda, \delta_{C}^\lambda, \alpha_{\parallel}^{+-}, \) etc., as the relative phase to \( \delta_P^\lambda \). For instance, the strong phase \( \delta_{||}^\lambda \) is understood as \( (\delta_{||}^\lambda - \delta_P^\lambda) \).

As a next step, we use the isospin analysis to determine \( \alpha_{ij}^\lambda, \bar{\alpha}_{ij}^\lambda \) in terms of the observables. The isospin relations between the decay amplitudes for \( B \rightarrow K^{*}\rho \) and their conjugate modes are the same as those given in Eqs. (17) and (18). Eq. (17) can be rewritten as

\[
\frac{1}{\sqrt{2}} \left( A_{\lambda}^{0+} e^{i\alpha_0^{0+}} - A_{\lambda}^{+-} e^{i\alpha_{\parallel}^{-+}} \right) = A_{\lambda}^{00} e^{i\alpha_{00}^0} - A_{\lambda}^{+0} e^{i\alpha_{+0}^0}, \tag{28}
\]

where \( A_{ij}^{\lambda} \equiv A_{\lambda}^{ij} e^{i\alpha_{ij}^\lambda} \) and \( \alpha_0^{0+} = \pi \). In Eq. (28) we note that all the magnitudes \( A_{ij}^{\lambda} \) of the decay amplitudes are directly measured and the relative phases \( (\alpha_{ij}^\lambda - \alpha_0^{ij}) \) and \( (\alpha_{ij}^{0+} - \alpha_0^{ij}) \) are also measured. Thus, for the three helicity states \( \lambda = \{0, \parallel, \perp\} \), the relevant three isospin relations given in Eq. (28) are described by only three independent parameters in total, \( \alpha_{ij}^{+-}, \alpha_0^{00} \) and \( \alpha_0^{+0} \), which are to be determined. Since we have 3 independent complex equations with these 3 real parameters for \( \lambda = \{0, \parallel, \perp\} \), we can solve these equations to determine the parameters \( \alpha_0^{0+}, \alpha_0^{+-} \) and \( \alpha_0^{+0} \). As a result, all the 12
we reexpress Eqs. (15) and (16) as

\[ \frac{1}{\sqrt{2}} (\hat{A}_\lambda^{0+} e^{i\alpha_{\lambda}^{0+}} - \hat{A}_\lambda^{+} e^{i\alpha_{\lambda}^{+}} - \hat{A}_\lambda^{-} e^{i\alpha_{\lambda}^{-}}) = \hat{A}_\lambda^{00} e^{i\alpha_{\lambda}^{00}} - \hat{A}_\lambda^{0+} e^{i\alpha_{\lambda}^{0+}}, \]  

(29)

where \( \hat{A}_\lambda^{ij} \equiv A_\lambda^{ij} e^{i\alpha_{\lambda}^{ij}} \) and \( \alpha_{00}^{0+} = \alpha_{00}^{0+} = \pi \ (\lambda = \{0, \|, \perp\}) \). Thus, in Eq. (29) for \( \lambda = \{0, \|, \perp\} \) there are only three independent real parameters \( \alpha_{00}^{0+}, \alpha_{00}^{00} \) and \( \alpha_{00}^{0+} \) that can be determined by solving the three independent complex equations. Consequently, all the 12 \( \hat{A}_\lambda^{ij} \) and the 12 \( \alpha_{\lambda}^{ij} \) are also completely determined.

Now let us define the following useful parameters:

\[ X_\lambda e^{i\delta_{\lambda}^{X}} = A_\lambda^{+} e^{i\alpha_{\lambda}^{+}} - \tilde{P}_\lambda, \quad \bar{X}_\lambda e^{i\delta_{\lambda}^{X}} = \bar{A}_\lambda^{+} e^{i\alpha_{\lambda}^{+}} - \tilde{\bar{P}}_\lambda, \]  

(30)

\[ Y_\lambda e^{i\delta_{\lambda}^{Y}} = \sqrt{2} A_\lambda^{00} e^{i\alpha_{\lambda}^{00}} + \bar{P}_\lambda, \quad \bar{Y}_\lambda e^{i\delta_{\lambda}^{Y}} = \sqrt{2} A_\lambda^{00} e^{i\alpha_{\lambda}^{00}} + \bar{\bar{P}}_\lambda. \]  

(31)

Since everything on the right-hand side of these equations has been found, one can determine for each helicity state all the 8 parameters \( X_\lambda, \bar{X}_\lambda, Y_\lambda, \bar{Y}_\lambda, \delta_{\lambda}^{X}, \bar{\delta}_{\lambda}^{X}, \delta_{\lambda}^{Y}, \bar{\delta}_{\lambda}^{Y} \) on the left-hand side in terms of the known parameters by directly solving the 4 complex equations. Then we reexpress Eqs. (15) and (16) as

\[ X_\lambda e^{i\delta_{\lambda}^{X}} = -e^{i\gamma} \tilde{T}_\lambda e^{i\delta_{\lambda}^{T}} + \tilde{P}_{C,\lambda} e^{i\delta_{\lambda}^{C,EW}}, \]  

(32)

\[ Y_\lambda e^{i\delta_{\lambda}^{Y}} = -e^{i\gamma} \tilde{C}_\lambda e^{i\delta_{\lambda}^{C}} + \tilde{P}_{\lambda} e^{i\delta_{\lambda}^{EW}}. \]  

(33)

For each \( \lambda \), these two complex equations together with their CP conjugate mode equations (i.e., 8 real equations) include 8 real parameters (the magnitudes and strong phases of 4 topological amplitudes) that need to be determined in terms of the observables. It is straightforward to obtain the magnitudes of the topological amplitudes:

\[ \tilde{T}_\lambda = \left[ \frac{B_\lambda^{+}}{2 \sin^2 \gamma} \right] \left[ 1 - y_{\lambda}^{++} \cos(\tilde{\alpha}_{\lambda}^{++} - \tilde{\alpha}_{\lambda}^{++}) \right], \]  

(34)

\[ \tilde{C}_\lambda = \left[ \frac{B_\lambda^{00}}{2 \sin^2 \gamma} \right] \left[ 1 - y_{\lambda}^{00} \cos(\tilde{\alpha}_{\lambda}^{00} - \tilde{\alpha}_{\lambda}^{00}) \right], \]  

(35)

\[ \tilde{P}_{C,\lambda}^{EW} = \left[ \frac{1}{4 \sin^2 \gamma} \right] \left[ X_\lambda^2 + \bar{X}_\lambda^2 - 2X_\lambda \bar{X}_\lambda \cos(\delta_{\lambda}^{X} - \delta_{\lambda}^{X} + 2\gamma) \right], \]  

(36)

\[ \tilde{P}_{\lambda}^{EW} = \left[ \frac{1}{4 \sin^2 \gamma} \right] \left[ Y_\lambda^2 + \bar{Y}_\lambda^2 - 2Y_\lambda \bar{Y}_\lambda \cos(\delta_{\lambda}^{Y} - \delta_{\lambda}^{Y} + 2\gamma) \right]. \]  

(37)
And the strong phases are

\begin{align*}
\tan \delta^T_\lambda &= -\frac{\bar{A}^-_\lambda \cos \alpha^+_\lambda - A^+_\lambda \cos \alpha^+_-\lambda}{\bar{A}^-_\lambda \sin \alpha^+_\lambda - A^+_\lambda \sin \alpha^+_-\lambda}, \\
\tan \delta^C_\lambda &= -\frac{\bar{A}^{00}_\lambda \cos \alpha^{00}_\lambda - A^{00}_\lambda \cos \alpha^{00}_\lambda}{\bar{A}^{00}_\lambda \sin \alpha^{00}_\lambda - A^{00}_\lambda \sin \alpha^{00}_\lambda}, \\
\tan \delta^{CEW}_\lambda &= -\frac{\bar{A}^{+1}_\lambda \cos(\bar{\alpha}^+_\lambda + \gamma) - A^+_\lambda \cos(\alpha^+_-\lambda - \gamma)}{\bar{A}^-_\lambda \sin(\bar{\alpha}^+_\lambda + \gamma) - A^+_\lambda \sin(\alpha^+_-\lambda - \gamma) - 2A^{+1}_\lambda \sin \gamma}, \\
\tan \delta^{EW}_\lambda &= -\frac{A^{00}_\lambda \cos(\bar{\alpha}^{00}_\lambda + \gamma) - A^{00}_\lambda \cos(\alpha^{00}_\lambda - \gamma) + \sqrt{2} A^{0+}_\lambda \sin \gamma}{\bar{A}^{00}_\lambda \sin(\bar{\alpha}^{00}_\lambda + \gamma) - A^{00}_\lambda \sin(\alpha^{00}_\lambda - \gamma) + \sqrt{2} A^{0+}_\lambda \sin \gamma}.
\end{align*}

We have shown that all the hadronic parameters can be cast in terms of the observables and only one unknown parameter \( \gamma \). If we measure the \( \gamma \) from somewhere else, then we can achieve a model-independent understanding of which hadronic parameter is dominating in these modes.

Future experiments are important to provide the necessary information to extract each hadronic parameter. For instance, the parameters, such as the color-suppressed tree \((\tilde{C}_\lambda)\) and the EW penguin \((\tilde{P}_{EW}^{^C})\) amplitudes, can be determined by using the relevant observables expected to be measured in the near future and the formulas given in Eqs. (35) and (37). Then, by comparing the determined parameters with theoretical predictions, one can further investigate possible NP effects appearing in \( B \to K^\ast \rho \) decay processes [20]. In the next section we discuss in details how the determination of these parameters in terms of the observables can be used to verify the hierarchy relations between the topological amplitudes that are conventionally assumed to be true in the SM. We also discuss ways to test the validity of assumptions equating the strong phases of a certain set of topological amplitudes.

IV. TESTING THE HIERARCHY OF TOPOLOGICAL AMPLITUDES AND POSSIBLE RELATIONS BETWEEN THEIR STRONG PHASES

In last section we have estimated all the topological amplitudes and strong phases purely in terms of the observables and \( \gamma \). Having obtained these relations it is straightforward to conclude that if there exist any relations between the theoretical parameters they must also result in relations among the observables.

We first derive certain relations between the observables that test the conventional hier-
arch between the topological amplitudes within the SM. It may be expected that

\[ \frac{\tilde{T}_\lambda}{P_\lambda} \approx \frac{\tilde{P}^{EW}_{\lambda}}{P} \approx \tilde{\lambda}, \quad \frac{\tilde{C}_\lambda}{P_\lambda} \approx \frac{\tilde{P}^{EW}_{C\lambda}}{P} \approx \tilde{\lambda}^2 \]  

in analogy to the expectations [21] for the modes \( B \rightarrow K\pi \), as the two modes are topologically equivalent.

The topological amplitudes \( \tilde{P}_\lambda, \tilde{A}_\lambda, \tilde{T}_\lambda, \tilde{C}_\lambda, \tilde{P}^{EW}_{\lambda} \) and \( \tilde{P}^{EW}_{C\lambda} \) have been expressed in terms of the observables and \( \gamma \) in the previous section. It is therefore easy to see that there must exist a relation between the observables and \( \gamma \) that must hold as a consequence of the hierarchy between the topological amplitudes. The relations (42) indicate the hierarchy relation \( \tilde{P}_\lambda > \tilde{T}_\lambda \approx \tilde{P}^{EW}_{\lambda} > \tilde{C}_\lambda \approx \tilde{P}^{EW}_{C\lambda} \) which must hold within the SM.

A simple approach would be to test the hierarchy \( \tilde{P}_\lambda > \tilde{T}_\lambda \approx \tilde{C}_\lambda \), which would imply the following relation:

\[ 2 \sin^2 \gamma B^{0+}_\lambda > B^{+-}_\lambda [1 - y^{+-}_\lambda \cos(\tilde{\alpha}^{+-}_\lambda - \alpha^{+-}_\lambda)] > 2B^{00}_\lambda [1 - y^{00}_\lambda \cos(\tilde{\alpha}^{00}_\lambda - \alpha^{00}_\lambda)] \]  

To test the hierarchy \( \tilde{P}^{EW}_{C\lambda} > \tilde{P}^{EW}_{\lambda} \), one can test the following relation:

\[ Y^2_\lambda + \tilde{Y}^2_\lambda - 2Y_\lambda \tilde{Y}_\lambda \cos(\tilde{\delta}^Y_\lambda - \delta^Y_\lambda + 2\gamma) > X^2_\lambda + \tilde{X}^2_\lambda - 2X_\lambda \tilde{X}_\lambda \cos(\tilde{\delta}^X_\lambda - \delta^X_\lambda + 2\gamma) \]  

Besides the above relation, simple tests verifying the hierarchy of \( \tilde{P}^{EW}_{\lambda} \) and \( \tilde{P}^{EW}_{C\lambda} \) can be derived. Assuming that \( \tilde{P}^{EW}_{C\lambda} = \tilde{\lambda}^2 \tilde{P}_\lambda \) in Eq. (15), it can be shown that

\[ \frac{B^{0+}_\lambda [1 - y^{+-}_\lambda \cos(\tilde{\alpha}^{+-}_\lambda - \alpha^{+-}_\lambda + 2\gamma)]}{2 \sin^2 \gamma B^{00}_\lambda} = 1 + \mathcal{O}(\tilde{\lambda}^2) \]  

Similarly assuming that \( P^{EW}_\lambda = \tilde{\lambda} P_\lambda \) in Eq. (16), it can be found that

\[ \frac{B^{00}_\lambda [1 - y^{00}_\lambda \cos(\tilde{\alpha}^{00}_\lambda - \alpha^{00}_\lambda + 2\gamma)]}{\sin^2 \gamma B^{0+}_\lambda} = 1 + \mathcal{O}(\tilde{\lambda}) \]  

The relations \( \tilde{T}_\lambda \approx \tilde{P}^{EW}_{\lambda} \) and \( \tilde{C}_\lambda \approx \tilde{P}^{EW}_{C\lambda} \) would imply that

\[ 2B^{+-}_\lambda [1 - y^{+-}_\lambda \cos(\tilde{\alpha}^{+-}_\lambda - \alpha^{+-}_\lambda)] \approx Y^2_\lambda + \tilde{Y}^2_\lambda - 2Y_\lambda \tilde{Y}_\lambda \cos(\tilde{\delta}^Y_\lambda - \delta^Y_\lambda + 2\gamma) \]  

\[ 4B^{00}_\lambda [1 - y^{00}_\lambda \cos(\tilde{\alpha}^{00}_\lambda - \alpha^{00}_\lambda)] \approx X^2_\lambda + \tilde{X}^2_\lambda - 2X_\lambda \tilde{X}_\lambda \cos(\tilde{\delta}^X_\lambda - \delta^X_\lambda + 2\gamma) \]  

Testing the hierarchy \( \tilde{T}_\lambda > \tilde{P}^{EW}_{C\lambda} \) and \( \tilde{P}^{EW}_{\lambda} > \tilde{C}_\lambda \) is rather simple. We note that \( \tilde{T}_\lambda \) and \( \tilde{C}_\lambda \) can be rewritten as

\[ \tilde{T}_\lambda = \sqrt{\frac{1}{4 \sin^2 \gamma} \left[ X^2_\lambda + \tilde{X}^2_\lambda - 2X_\lambda \tilde{X}_\lambda \cos(\tilde{\delta}^X_\lambda - \delta^X_\lambda) \right]} \]  

\[ \tilde{C}_\lambda = \sqrt{\frac{1}{4 \sin^2 \gamma} \left[ Y^2_\lambda + \tilde{Y}^2_\lambda - 2Y_\lambda \tilde{Y}_\lambda \cos(\tilde{\delta}^Y_\lambda - \delta^Y_\lambda) \right]} \]  

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Comparing these equations with Eqs. (36) and (37), we find that

\[ \tilde{T}_\lambda > \tilde{P}_{C,\lambda}^{\text{EW}} \implies \sin \left( \delta^X - \delta^X + \gamma \right) < 0 , \]
\[ \tilde{P}_\lambda^{\text{EW}} > \tilde{C}_\lambda \implies \sin \left( \delta^Y - \delta^Y + \gamma \right) > 0 . \]

The relations (51) and (52) will provide a litmus test to verify the hierarchy assumption between the magnitudes of topological amplitudes. Since the decay modes \( B \to K\pi \) and \( B \to K^*\rho \) are equivalent at quark level, the same hierarchy relation is expected to hold in \( B \to K\pi \) modes.

Now let us move our focus on to the strong phases. One can derive several relations purely in terms of the observables and \( \gamma \) by assuming relations between the strong phases of the topological amplitudes. These relations would be very important in verifying the assumptions often made between these strong phases, such as \( \delta^C_\lambda \approx \delta^{\text{CEW}}_\lambda \approx \delta^P_\lambda \) and \( \delta^{\text{EW}}_\lambda \approx \delta^T_\lambda \), which are expected to hold within the SM. We discuss only a few such relations that test some common assumptions being made on the strong phases [22].

We first consider the implication of the interesting relation \( \delta^C_\lambda = \delta^P_\lambda \). In our convention \( \delta^P_\lambda = 0 \), it means \( \delta^C_\lambda = 0 \). So from Eq. (39) we get the relation

\[ \tilde{A}^{00}_\lambda \cos \tilde{\alpha}^{00}_\lambda = A^{00}_\lambda \cos \alpha^{00}_\lambda . \]

In fact, this relation can be obtained directly from Eq. (16). If \( \delta^C_\lambda = 0 \), the real part of the amplitude \( A^{00}_\lambda \) is the same as that of its CP conjugate amplitude \( \tilde{A}^{00}_\lambda \), which is just the restatement of the relation (53). Since the topological amplitude \( \tilde{C}_\lambda \) is estimated to be very small (\( \tilde{C}_\lambda = \tilde{\lambda}^2 \tilde{P}_\lambda \)) in the SM, it is also expected from Eq. (16) that in the SM the direct CP asymmetry in the \( K^{*0}\rho^0 \) mode almost vanishes. Similarly the assumption \( \delta^{\text{CEW}}_\lambda = \delta^T_\lambda \) leads to the relation

\[ \tilde{A}^{+-}_\lambda \cos (\tilde{\alpha}^{+-}_\lambda + \gamma) = A^{+-}_\lambda \cos (\alpha^{+-}_\lambda - \gamma) , \]

obtained from Eq. (40). Finally, from the assumption \( \delta^T_\lambda = \delta^{\text{EW}}_\lambda \), we get the relation

\[ - \frac{\tilde{A}^{+-}_\lambda \cos \tilde{\alpha}^{+-}_\lambda - A^{+-}_\lambda \cos \alpha^{+-}_\lambda}{\tilde{A}^{+-}_\lambda \sin \tilde{\alpha}^{+-}_\lambda - A^{+-}_\lambda \sin \alpha^{+-}_\lambda} = - \frac{\tilde{A}^{00}_\lambda \sin \tilde{\alpha}^{00}_\lambda + A^{00}_\lambda \sin \alpha^{00}_\lambda}{2A^{00}_\lambda \cos \alpha^{00}_\lambda + \sqrt{2}A^{0+}_\lambda} \left[ 1 + \mathcal{O}(\tilde{\lambda}^2) \right] , \]

where we have used \( \tilde{C}_\lambda = \tilde{\lambda}^2 \tilde{P}_\lambda \).

Before concluding this section we note that the validity of the several relations derived above, or the degree to which they fail to hold, will shed light on the possible origins of the “\( B \to K\pi \) puzzle,” and hence help in uncovering possible NP contributions.
In the previous section we derived relations that test possible relations between topological amplitudes and strong phases. If we instead assume that these relations hold within the SM, a violation of these relations would signal NP. In Sec. II we showed that NP contributing to any of the topological amplitudes can be easily absorbed so that the amplitudes in Eqs. (13) – (16) retain the same form, making it impossible to have a clean signal of NP. However, if there exist relations between the amplitudes or strong phases, the $B \rightarrow K^* \rho$ amplitudes would differ from the SM form in the presence of NP. Since the number of independent SM parameters is reduced, one may now, not only be able to see signals of NP but also solve for NP parameters. In this section we consider two cases to explore this possibility.

We first discuss the consequence of relations between $\tilde{P}^{\text{EW}}_{\lambda} e^{i\delta^{\text{EW}}_{\lambda}}$ and $\tilde{T}_{\lambda} e^{i\delta^{T}_{\lambda}}$, and $\tilde{P}^{\text{EW}}_{C,\lambda} e^{i\delta^{\text{CEW}}_{\lambda}}$ and $\tilde{C}_{\lambda} e^{i\delta^{C}_{\lambda}}$ that are expected to hold in the SM. We next consider the case where the strong phases are related in the SM.

Let us consider NP contributing via the EW penguins, to amplitudes in Eqs. (14) and (16). We assume that NP contributes with an amplitude $N_{\lambda} e^{i\delta^{NP}_{\lambda}} = N_{1,2} e^{i\gamma_{NP}}$. We have shown that using angular-analysis we can solve for all the parameters in Eqs. (13) – (16). In particular we can measure $\tilde{P}^{\text{EW}}_{\lambda}$, $\tilde{P}^{\text{EW}}_{C,\lambda}$, $\tilde{T}_{\lambda}$, $\tilde{C}_{\lambda}$, $\delta^{\text{EW}}_{\lambda}$, $\delta^{\text{CEW}}_{\lambda}$, $\delta^{T}_{\lambda}$, $\delta^{C}_{\lambda}$ in terms of $\gamma$. Note that these measured values include any NP contributions that may be present. In fact, if NP contributes via the EW penguins, the SM amplitudes (defined by calligraphic characters)– $\tilde{P}^{\text{EW}}_{\lambda}$ and $\tilde{C}_{\lambda}$ are the only ones modified by NP, but they cannot themselves be measured. The other amplitudes are unmodified by NP and hence we need not distinguish the SM amplitudes from the amplitudes defined in Eqs. (13) – (16).

In the SM, to a good approximation [23], using flavor SU(3) the $\Delta I = 3/2$ parts of the tree and electroweak penguin Hamiltonians are simply related by

$$\mathcal{H}_{\Delta I=3/2}^{\text{EW}} = -\frac{3}{2} \left[ \frac{c_9 + c_{10}}{c_1 + c_2} \right] \frac{V_{ub}^* V_{ts}}{V_{us} V_{ts}} \mathcal{H}_{\Delta I=3/2}^{\text{tree}}.$$  

Hence, $\tilde{P}^{\text{EW}}_{\lambda} e^{i\delta^{\text{EW}}_{\lambda}}$ and $\tilde{P}^{\text{EW}}_{C,\lambda} e^{i\delta^{\text{CEW}}_{\lambda}}$ are related to $\tilde{T}_{\lambda} e^{i\delta^{T}_{\lambda}}$ and $\tilde{C}_{\lambda} e^{i\delta^{C}_{\lambda}}$:

$$\tilde{P}^{\text{EW}}_{\lambda} e^{i\delta^{\text{EW}}_{\lambda}} \simeq \frac{3}{2} \left[ \frac{c_9 + c_{10}}{c_1 + c_2} \right] \frac{V_{ub}^* V_{ts}}{V_{us} V_{ts}} \tilde{T}_{\lambda} e^{i\delta^{T}_{\lambda}} \equiv \zeta \tilde{T}_{\lambda} e^{i\delta^{T}_{\lambda}},$$

$$\tilde{P}^{\text{EW}}_{C,\lambda} e^{i\delta^{\text{CEW}}_{\lambda}} \simeq \frac{3}{2} \left[ \frac{c_9 + c_{10}}{c_1 + c_2} \right] \frac{V_{ub}^* V_{ts}}{V_{us} V_{ts}} \tilde{C}_{\lambda} e^{i\delta^{C}_{\lambda}} \equiv \zeta \tilde{C}_{\lambda} e^{i\delta^{C}_{\lambda}}.$$  

(57)
where the $c_i$ are Wilson coefficients \cite{24}. Using the SM relations in Eq. (57), it is easy to see that

\begin{align}
\zeta C_\lambda e^{i\delta_C} &= \zeta \tilde C_\lambda e^{i\delta_C} + \zeta N_2^\lambda e^{i\delta_N}, \\
&= \tilde P_{C,\lambda} e^{i\delta_{CEW}} + \zeta N_2^\lambda e^{i\delta_N}, \\
\tilde P_{EW}^\lambda e^{i\delta_{EW}^\lambda} &= \tilde P_{EW}^\lambda e^{i\delta_{EW}^\lambda} + \zeta N_1^\lambda e^{i\delta_N}, \\
&= \zeta \tilde T_\lambda e^{i\delta_T^\lambda} + N_1^\lambda e^{i\delta_N}.
\end{align}

Eqs. (58) and (59) form four relations in terms of only three unknowns $N_1^\lambda$, $N_2^\lambda$ and $\delta_N^\lambda$ for each $\lambda$. Hence $N_1^\lambda$, $N_2^\lambda$ and $\delta_N^\lambda$ can easily be solved. $\phi_{NP}$ the weak phase of NP can be obtained from the relation

\begin{equation}
\frac{N_1^\lambda}{N_2^\lambda} = \frac{\sin(\gamma - \phi_{NP})}{\sin \phi_{NP}}.
\end{equation}

In fact, under the assumptions made there are enough observables even to solve for $\zeta$, enabling us not only to measure NP but also test the SU(3) assumption.

Instead of assuming that the SM amplitudes are related by Eq. (57), we next assume that in the SM the strong phases are related such that $\delta_C^\lambda = \delta_{CEW}^\lambda = \delta_P^\lambda$ and $\delta_{EW}^\lambda = \delta_T^\lambda$. It is then easy to conclude that

\begin{align}
\tilde P_{EW}^\lambda e^{i\delta_{EW}^\lambda} &= \tilde P_{EW}^\lambda e^{i\delta_{EW}^\lambda} + N_1^\lambda e^{i\delta_N}, \\
\tilde C_\lambda e^{i\delta_C} &= \tilde C_\lambda e^{i\delta_P} + N_2^\lambda e^{i\delta_N}.
\end{align}

We again have four equations, but now in terms of five unknowns $\tilde P_{EW}^\lambda$, $\tilde C_\lambda$, $N_1^\lambda$, $N_2^\lambda$ and $\delta_N^\lambda$. Hence if the strong phases are related in the SM, the failure of the relations $\delta_C^\lambda = \delta_{CEW}^\lambda = \delta_P^\lambda$ and $\delta_{EW}^\lambda = \delta_T^\lambda$ can be tested, but it is not possible to solve for any of the NP parameter. Hence, the assumptions on strong phases do not allow a clean test of NP; the failure of the relations between strong phases could be due to NP or simply due to hadronic effects within the SM. The assumptions of Eq. (57) in the case discussed previously allow a clean test of NP as the assumption can be verified independent of possible NP contributions.

We emphasize that due to reparameterization invariance it is in general impossible to have a clean signal (i.e. model-independent signal) of NP \cite{16}. Furthermore, tests of NP are based on relations between the amplitudes and strong phases of topological amplitudes. It is not always possible to independently test the hadronic assumption and at the same time cleanly measure the NP parameters. However, we do demonstrate that if the tree and
TABLE I: Experimental data on the CP-averaged branching ratios ($B$ in units of $10^{-6}$), the longitudinal polarization fractions ($f_L$), and the direct CP asymmetries ($A_{CP}$) for $B \to K^*\rho$ modes [2].

| Mode                  | $B$ ($10^{-6}$) | $f_L$     | $A_{CP}$     |
|-----------------------|-----------------|-----------|--------------|
| $B^+ \to K^{*0}\rho^+$ | 9.2 ± 1.5       | 0.48 ± 0.08 | -0.01 ± 0.16 |
| $B^+ \to K^{++}\rho^0$ | 3.6 ± 1.9       | 0.96$^{+0.06}_{-0.16}$ | 0.20$^{+0.32}_{-0.29}$ |
| $B^0 \to K^{*+}\rho^-$ | 5.4 ± 3.9       | -         | -            |
| $B^+ \to K^{*0}\rho^0$ | 5.6 ± 1.6       | 0.57 ± 0.12 | 0.09 ± 0.19  |

color-suppressed tree are related to the electroweak penguins and color-suppressed electroweak penguins by the well known relations of Eq. (57), it is possible not only to verify the validity of these relations but also to have a clean measurement of New Physics parameters. It would be worth doing an angular analysis in $B \to K^*\rho$ not only to establish cleanly the validity of the relations in Eq. (57) but also at the same time to cleanly probe for NP.

VI. NUMERICAL ANALYSIS

In this section we perform numerical analysis to investigate how much sensitive to possible NP effects each observable for $B \to K^*\rho$ decays could be. As discussed in Sec. V, we consider NP contributing via the EW penguins. For simplicity we further assume that additional information on the theoretical parameters is given from somewhere, for instance, from future theoretical estimates, so that the SM amplitudes $\tilde{C}_\lambda$ and $\tilde{P}^{EW}_\lambda$ are known. Therefore, the SM amplitude $\tilde{P}^{EW}_\lambda$ is the only one modified by NP and the amplitudes $\tilde{C}_\lambda$ and $\tilde{P}^{EW}_\lambda$ in Eqs. (14) and (16) can be expressed explicitly in terms of the SM and NP amplitudes:

$$\tilde{C}_\lambda = \tilde{C}_\lambda , \quad (63)$$

$$\tilde{P}^{EW}_\lambda e^{i\delta^{EW}_\lambda} = \tilde{P}^{EW}_\lambda e^{i\delta^{EW}_\lambda} + \tilde{P}^{\prime EW}_\lambda e^{i\delta^{\prime EW}_\lambda} e^{i\phi_{EW}} , \quad (64)$$

where $\delta^{\prime EW}_\lambda$ and $\phi_{EW}$ are the strong and weak phases of the NP amplitudes, respectively.

We first summarize the present status of the experimental results on $B \to K^*\rho$ modes in Table II [2]. So far only two modes $B^+ \to K^{*0}\rho^+$ and $B^+ \to K^{++}\rho^0$ have been observed. For illustration, we perform a numerical study to make predictions on the physical observables. Because those data shown in Table II are currently the only available ones, we follow the following procedures: (i) In order to determine the theoretical parameters in a reasonable way,
we adopt the $\chi^2$ minimization technique and use the currently available data as constraints on the parameters. We first consider only the dominant strong penguin contributions and neglect all the other topological amplitudes. Then, we use five known observables given in Table I and try to fit the dominant strong penguin amplitudes $\tilde{P}_\lambda$ ($\lambda = 0, \perp, ||$) and their phases $\delta_\lambda^P$ with $\delta_0^P \equiv 0$. The degrees of freedom (d.o.f) for this fit is 0. As a next step, we assume that the SM amplitudes, such as $\tilde{T}$, $\tilde{C}$, $\tilde{P}^{EW}$, $\tilde{A}$, follow the conventional hierarchy as in $B \to K\pi$ within the SM: for instance, in the pQCD approach \cite{25}, $\tilde{T}/\tilde{P} = 0.15$, $\tilde{P}^{EW}/\tilde{P} = 0.12$, $\tilde{C}/\tilde{P} = 0.04$, $\tilde{A}/\tilde{P} = 0.005$. Their phases are set to be small. (ii) Using the parameters determined in the previous step, we calculate all the 35 observables within the SM. We use $\gamma = 62^\circ$. (iii) To investigate the possible NP effects, we consider two different cases: a case with sizable but relatively small NP effects and another case with relatively large NP effects. For the former case, we assume $r_{EW} = 0.12$ and $r'_{EW} = 0.05$, while for the latter, we assume $r_{EW} = 0.12$ and $r'_{EW} = 0.20$, where $r_{EW} \equiv \tilde{P}^{EW}/\tilde{P}$ and $r'_{EW} \equiv \tilde{P}'^{EW}/\tilde{P}$. In both cases, the strong phases are set to be $\delta^P_\lambda = \delta^{EW}_\lambda$. The NP weak phase is chosen to be $\phi^{EW} = 90^\circ$, which is consistent with that used to explain the $B \to K\pi$ puzzle \cite{26}.

The results are presented in Figs. 1 and 2. Here the observables $\phi_{ij}^{\parallel, \perp}$ and $\overline{\phi}_{ij}^{\parallel, \perp}$ defined as the relative phases $\arg \left( A_{ij}^{\parallel, \perp}/A_0^{ij} \right)$ and $\arg \left( \overline{A}_{ij}^{\parallel, \perp}/\overline{A}_0^{ij} \right)$ respectively, are included, since they have been measured through $B^0 \to \phi K^{*0}$ \cite{5}. The uncertainties depicted in the figures have been assumed just for illustration as follows. The BRs are expected to be measured accurately so that their errors have been assumed to be 5%. The other observables including the direct CP asymmetries, the polarization fractions, etc, except the observables $\rho_{ii}$, have been assumed to be measured with 10% errors. The uncertainties in $\rho_{ii}$ have been assumed to be 20%.

In Fig. 1 the physical observables are predicted within the SM as well as with the NP effects of $r'_{EW} = 0.05$. Here the notation $B_{r^{ij}}$ denotes the branching ratio for $B \to K^{*i}\rho^j$, and others are similarly defined. The SM values are shown as the “thin” bars and the numbers in the left column. The predictions with the NP contributions are shown as the “thick” bars and the numbers in the right column. It is clearly shown that certain observables are very sensitive to the NP effects in $B \to K^{*}\rho$ decays: for instance, $A_{CP}^+, \Lambda_{\perp \perp}, \Lambda_{\perp 0}, \Sigma_{\perp \perp}, \Sigma_{|| ||}, \Sigma_{00}$, and $\Sigma_{|| 0}$. In particular, it is interesting to note that the direct CP asymmetry for the mode $B^+ \to K^{*+}\rho^0$, which has been already observed but with large errors yet, is sensitive to the NP effect. Fig. 2 shows predictions with the NP effects of $r'_{EW} = 0.20$. Here the SM values
are the same as those in Fig. [1]. We see that for many observables, the predictions with the NP contributions are very much off the SM ones. This is the expected result, due to the large NP effects. Thus, if any anomalously large NP effects, e.g., a large color-suppressed tree contribution, appears in $B \rightarrow K^*\rho$ decays, one can easily find them through those observables. Obviously if more (precise) experimental data are available in the future, all the predictions shown in Fig. [1] and [2] will be able to become more reliable.

VII. CONCLUSIONS

We have performed a detailed study of the $B \rightarrow K^*\rho$ decays using a model-independent approach. It was shown that $B \rightarrow K^*\rho$ modes have a distinct advantage due the large number of independent observables that can be measured. In comparison to the $B \rightarrow K\pi$ modes that yield only 9 independent observables, the $B \rightarrow K^*\rho$ modes result in as many as 35 independent observables. Since $B \rightarrow K\pi$ and $B \rightarrow K^*\rho$ have the same quark level subprocess, the study of $B \rightarrow K^*\rho$ may well shed light on the well known “$B \rightarrow K\pi$ puzzle.” The relevant decay amplitudes were decomposed into linear combinations of the topological amplitudes with their respective strong phases assuming isospin. We point out that the amplitude written this way are the most general ones and included contributions not only from the SM but also any NP that might exist. We obtain explicit model-independent expressions for all the topological amplitudes and their strong phases in terms of observables and the weak phase $\gamma$. With $\gamma$ measured using other modes, our results are the first in literature to estimate the topological amplitudes and strong phases purely in terms of observables, for the $B \rightarrow K\pi$ analogous modes. We further suggest clean tests to verify if there exist any hierarchy relations among topological amplitudes analogous to the ones conventionally assumed to exists for $B \rightarrow K\pi$ in the SM. In addition we present tests that would verify any equality between the strong phases of the topological amplitudes. A model independent understanding of the relative sizes of the topological amplitudes and relations between their strong phases could provide valuable insights into NP searches. While it is not in general possible to independently test the hadronic assumption and at the same time cleanly measure the NP parameters, we show one example where it is possible to do both. We demonstrate that if the tree and color-suppressed tree are related to the electroweak penguins and color-suppressed electroweak penguins, it is not only possible
to verify the validity of such relations but also to cleanly measure New Physics parameters. We also present a numerical study to examine which of the observables are more sensitive to New Physics.

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[1] T. Bergfeld et al. [CLEO Collaboration], Phys. Rev. Lett. 81, 272 (1998) [arXiv:hep-ex/9803018].
[2] Heavy Flavor Averaging Group, [http://www.slac.stanford.edu/xorg/hfag/] (2006).
[3] W. M. Yao et al. [Particle Data Group], J. Phys. G 33, 1 (2006).
[4] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 91, 171802 (2003) [arXiv:hep-ex/0307026].
[5] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 93, 231804 (2004) [arXiv:hep-ex/0408017].
[6] B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0607097; B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0607098; B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0607092; B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 74, 051102 (2006) [arXiv:hep-ex/0605017]; J. Zhang et al. [BELLE Collaboration], Phys. Rev. Lett. 91, 221801 (2003) [arXiv:hep-ex/0306007]; K. F. Chen et al. [BELLE Collaboration], Phys. Rev. Lett. 94, 221804 (2005) [arXiv:hep-ex/0505013]; A. Somov et al., Phys. Rev. Lett. 96, 171801 (2006) [arXiv:hep-ex/0601024].
[7] K. Abe et al. [BELLE-Collaboration], Phys. Rev. Lett. 95, 141801 (2005) [arXiv:hep-ex/0408102]; B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 97, 201801 (2006) [arXiv:hep-ex/0607057].
[8] K. Abe et al., arXiv:hep-ex/0507045. K. Abe et al. [Belle Collaboration], arXiv:hep-ex/0609006; K. Abe., arXiv:hep-ex/0609015; K. Abe et al. [Belle Collaboration], arXiv:hep-ex/0608049; B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0607096; B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0607106; B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 97, 171805 (2006) [arXiv:hep-ex/0608036].
[9] S. Mishima and T. Yoshikawa, Phys. Rev. D 70, 094024 (2004) [arXiv:hep-ph/0408090]; Y. L. Wu and Y. F. Zhou, Phys. Rev. D 71, 021701 (2005) [arXiv:hep-ph/0409221]; A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Acta Phys. Polon. B 36, 2015 (2005) [arXiv:hep-ph/041007]; X. G. He and B. H. J. McKellar, arXiv:hep-ph/0410098; S. Baek, P. Hamel, D. London, A. Datta and D. A. Suprun, Phys. Rev. D 71, 057502.
T. Carruthers and B. H. J. McKellar, arXiv:hep-ph/0412202.
S. Nandi and A. Kundu, arXiv:hep-ph/0407061.
T. Morozumi, Z. H. Xiong and T. Yoshikawa, arXiv:hep-ph/0408297.

[10] C. S. Kim, S. Oh and C. Yu, Phys. Rev. D 72, 074005 (2005) arXiv:hep-ph/0505060.

[11] M. Beneke and M. Neubert, Nucl. Phys. B 675, 333 (2003) arXiv:hep-ph/0308039.

[12] Y. Y. Keum, H. N. Li and A. I. Sanda, Phys. Rev. D 63, 054008 (2001) arXiv:hep-ph/0004173.

[13] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D 50, 4529 (1994) arXiv:hep-ph/9404283
M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D 52, 6374 (1995) arXiv:hep-ph/9504327.

[14] S. Baek, F. J. Botella, D. London and J. P. Silva, Phys. Rev. D 72, 114007 (2005) arXiv:hep-ph/0509322.

[15] A. J. Buras and R. Fleischer, Phys. Lett. B 341, 379 (1995) arXiv:hep-ph/9409244.

[16] M. Imbeault, D. London, C. Sharma, N. Sinha and R. Sinha arXiv:hep-ph/0608169.

[17] F. J. Botella and J. P. Silva, Phys. Rev. D 71, 094008 (2005) arXiv:hep-ph/0503136.

[18] N. Sinha and R. Sinha, Phys. Rev. Lett. 80, 3706 (1998) arXiv:hep-ph/9712502.

[19] In this case, the number of parameters is still the same as that of informations. As 6 parameters
\((\tilde{A}_\lambda, \tilde{\delta}^\lambda_\lambda)\) are neglected, the number of informations are also reduced by 6 (i.e., \(\tilde{A}_{\lambda}^{0+}, \tilde{\alpha}_{\lambda}^{0+}\)) due
to the vanishing direct CP asymmetry of the \(K^{(*)0}\rho^+\) mode.

[20] D. London, N. Sinha and R. Sinha, Phys. Rev. D 69, 114013 (2004) arXiv:hep-ph/0402214.

[21] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D 50, 4529 (1994),
Phys. Rev. D 52, 6374 (1995).

[22] More relations between observables under various assumptions on strong phases are also avail-
able. Furthermore, though the results shown in Secs. III and IV have been obtained by neglect-
ing the annihilation contribution, more general results including the annihilation contribution
are also available in detail. Interested readers may contact one of the authors.

[23] M. Neubert and J. L. Rosner, Phys. Lett. B 441, 403 (1998), Phys. Rev. Lett. 81, 5076 (1998);
M. Gronau, D. Pirjol and T. M. Yan, Phys. Rev. D 60, 034021 (1999) [Erratum-ibid. D 69,
119901 (2004)]; M. Imbeault, A. L. Lemerle, V. Page and D. London, Phys. Rev. Lett. 92,
081801 (2004).

[24] See, for example, G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. 68, 1125
(1996).

[25] H. n. Li, S. Mishima and A. I. Sanda, Phys. Rev. D 72, 114005 (2005) [arXiv:hep-ph/0508041].

[26] C. S. Kim, Sechul Oh, and Yeo Woong Yoon, [arXiv:hep-ph/0706xxx].
APPENDIX A: DETERMINATION OF $A^f_\lambda$ AND $\bar{A}^f_\lambda$ WITH OBSERVABLES

1. Determination of the magnitude $A^f_\lambda$ and $\bar{A}^f_\lambda$

The branching ratios (BRs) and direct CP asymmetries of the decay modes $B \to K^* \rho$ are measured experimentally [2, 3]. Using the measured values of BRs and direct CP asymmetry of each helicity for the decay modes, $A^f_\lambda$ ($\equiv |A^f_\lambda|$) can be determined straightforwardly. The direct CP asymmetry is defined as $a^f_\lambda \equiv \Sigma^f_\lambda$, where $\Sigma^f_\lambda$ and $B^f_\lambda$ are defined in Eq. (21), and $f$ is one of the final states of $K^* \rho$. Therefore, $A^f_\lambda$ and $\bar{A}^f_\lambda$ can be written as

$$ (A^f_\lambda)^2 = B^f_\lambda + \Sigma^f_\lambda \quad \text{and} \quad (\bar{A}^f_\lambda)^2 = B^f_\lambda - \Sigma^f_\lambda. \quad (A1) $$

2. Determination of the phases of $A^f_\lambda$ and $\bar{A}^f_\lambda$

Let us first try to find out the phases $\alpha^ij_\lambda$ of $A^f_\lambda$. Since the relative phases ($\alpha^ij_\parallel - \alpha^ij_0$) and ($\alpha^ij_\perp - \alpha^ij_0$) can be measured in experiment, one needs to determine only $\alpha^ij_\parallel$. We express the three equations of Eq. (28) explicitly with three unknown parameters $\alpha^0_0$ and $\alpha^0_\pm$:

$$ \frac{1}{\sqrt{2}} \left( A^0_0 e^{i\pi} - A^0_\pm e^{i\alpha^0_\pm} \right) = A^0_0 e^{i\alpha^0_0} - A^0_\pm e^{i\alpha^0_\pm}, $$
$$ \frac{1}{\sqrt{2}} \left[ A^0_\parallel e^{i\pi} - A^\pm_\parallel e^{i(\alpha^0_\parallel + \phi^0_\parallel)} \right] = A^0_\parallel e^{i(\alpha^0_0 + \phi^0_\parallel)} - A^\pm_\parallel e^{i(\alpha^0_\pm + \phi^0_\parallel)}, $$
$$ \frac{1}{\sqrt{2}} \left( A^0_\perp e^{i\pi} - A^\pm_\perp e^{i(\alpha^0_\perp + \phi^0_\perp)} \right) = A^0_\perp e^{i(\alpha^0_0 + \phi^0_\perp)} - A^\pm_\perp e^{i(\alpha^0_\pm + \phi^0_\perp)}, \quad (A2) $$

where $\phi^0_\parallel$ and $\phi^0_\perp$ are defined in terms of the observables $\phi^ij_\parallel (\equiv \alpha^ij_\parallel - \alpha^ij_0)$ and $\phi^ij_\perp (\equiv \alpha^ij_\perp - \alpha^ij_0)$ such that

$$ \phi^ij_\parallel = \phi^ij - \phi^0_\parallel \quad \text{and} \quad \phi^ij_\perp = \phi^ij - \phi^0_\perp. \quad (A3) $$

Here we remind that in our convention each phase $\alpha^ij_\parallel(\perp)$ has been defined as the relative phase to $\phi^P_\parallel(\perp) = \alpha^0_\parallel(\perp) - \alpha^0_0 \equiv \phi^0_\parallel(\perp)$. Then we can re-write Eq. (A2) as the matrix equation

$$ SX = A, \quad (A4) $$
where the matrix $\mathbf{S}$ and the column vectors $\mathbf{X}$ and $\mathbf{A}$ are given by

$$
\mathbf{S} = \begin{pmatrix}
\frac{1}{\sqrt{2}}A_{0}^{++} & 0 & A_{0}^{00} & 0 & -A_{0}^{-0} & 0 \\
0 & \frac{1}{\sqrt{2}}A_{0}^{-+} & 0 & A_{0}^{00} & 0 & -A_{0}^{-0} \\
\frac{1}{\sqrt{2}}A_{\|}^{+-} \cos \tilde{\phi}_{\|}^{+} & -\frac{1}{\sqrt{2}}A_{\|}^{+-} \sin \tilde{\phi}_{\|}^{+} & A_{\|}^{00} \cos \tilde{\phi}_{\|}^{0} & A_{\|}^{00} \sin \tilde{\phi}_{\|}^{0} & -A_{\|}^{00} \cos \tilde{\phi}_{\|}^{0} & A_{\|}^{00} \sin \tilde{\phi}_{\|}^{0} \\
\frac{1}{\sqrt{2}}A_{\perp}^{+-} \sin \tilde{\phi}_{\perp}^{+} & -\frac{1}{\sqrt{2}}A_{\perp}^{+-} \cos \tilde{\phi}_{\perp}^{+} & A_{\perp}^{00} \sin \tilde{\phi}_{\perp}^{0} & A_{\perp}^{00} \cos \tilde{\phi}_{\perp}^{0} & -A_{\perp}^{00} \sin \tilde{\phi}_{\perp}^{0} & A_{\perp}^{00} \cos \tilde{\phi}_{\perp}^{0} \\
\frac{1}{\sqrt{2}}A_{\perp}^{+-} \cos \tilde{\phi}_{\perp}^{+} & -\frac{1}{\sqrt{2}}A_{\perp}^{+-} \sin \tilde{\phi}_{\perp}^{+} & A_{\perp}^{00} \cos \tilde{\phi}_{\perp}^{0} & A_{\perp}^{00} \sin \tilde{\phi}_{\perp}^{0} & -A_{\perp}^{00} \cos \tilde{\phi}_{\perp}^{0} & A_{\perp}^{00} \sin \tilde{\phi}_{\perp}^{0} \\
\frac{1}{\sqrt{2}}A_{\perp}^{+-} \sin \tilde{\phi}_{\perp}^{+} & -\frac{1}{\sqrt{2}}A_{\perp}^{+-} \cos \tilde{\phi}_{\perp}^{+} & A_{\perp}^{00} \sin \tilde{\phi}_{\perp}^{0} & A_{\perp}^{00} \cos \tilde{\phi}_{\perp}^{0} & -A_{\perp}^{00} \sin \tilde{\phi}_{\perp}^{0} & A_{\perp}^{00} \cos \tilde{\phi}_{\perp}^{0}
\end{pmatrix},
$$

$$
\mathbf{X} = \left( \begin{array}{cccc}
\cos \alpha_{0}^{+} & \sin \alpha_{0}^{+} & \cos \alpha_{0}^{0} & \sin \alpha_{0}^{0} \\
\cos \alpha_{0}^{-} & \sin \alpha_{0}^{-} & \cos \alpha_{0}^{0} & \sin \alpha_{0}^{0} \\
\cos \alpha_{0}^{0} & \sin \alpha_{0}^{0} & \cos \alpha_{0}^{0} & \sin \alpha_{0}^{0}
\end{array} \right)^{T},
$$

$$
\mathbf{A} = \left( \begin{array}{cccc}
-\frac{1}{\sqrt{2}}A_{0}^{0+} & 0 & -\frac{1}{\sqrt{2}}A_{\|}^{0+} & 0 \\
-\frac{1}{\sqrt{2}}A_{\perp}^{0+} & 0 & -\frac{1}{\sqrt{2}}A_{\perp}^{0+}
\end{array} \right)^{T},
$$

where $\mathbf{X}$ is the column vector to be determined. One can easily solve this matrix equation by calculating the inverse matrix of $\mathbf{S}$. The solution $\mathbf{X}$ is given by

$$
\mathbf{X} = \mathbf{S}^{-1} \mathbf{A}.
$$

We note that in Eq. (A6) both cosine and sine of each phase $\alpha_{0}^{+}, \alpha_{0}^{0}, \alpha_{0}^{-}$ can be determined, which results in removing discrete ambiguities associated with trigonometrical functions of the solution.

By using exactly the same method as above, one can also find the phases $\bar{\alpha}_{ij}$ of the CP conjugate amplitudes $\bar{A}_{\ell}^{+}$. 

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FIG. 1: The case of $r_{EW} = 0.12$ and $r'_{EW} = 0.05$, where $r_{EW} = \bar{P}^{EW}/\bar{P}$ and $r'_{EW} = \bar{P}'^{EW}/\bar{P}'$. The SM values are shown as the “thin” bars and the numbers in the left column. The predictions with the NP contributions are shown as the “thick” bars and the numbers in the right column.
FIG. 2: The case of $r_{EW} = 0.12$ and $r'_{EW} = 0.20$, where $r_{EW} \equiv \tilde{F}_{EW}/\tilde{P}$ and $r'_{EW} \equiv \tilde{F}'_{EW}/\tilde{P}$.

The SM values are shown as the “thin” bars and the numbers in the left column. The predictions with the NP contributions are shown as the “thick” bars and the numbers in the right column.