Determination of the refractive index profile and surface topography of optically smooth objects using interference of optical vortices

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Abstract. In this paper, we present the results of the propagational dynamics of vortex beams in the scope of their possible applications for interferometric non-contact robust and precision optical surface profilometry with nanoscale longitudinal resolution. The result of coaxial superposition of the reference plane wave with singly charged vortex beams represents a dynamically changing intensity distribution. The nature of this changes, namely, rotational effects of intensity zeros, allows to determine directly the optical path difference which is introduced by the surfaces and internal structure of test object. We have proposed the experimental setup for examination of reflecting and transmitting objects.

1. Introduction

Since J. Nye and M. Berry demonstrated the basic elements of phase singularities in the wave fronts of optical beams about half a century ago [1], optical vortices have found wide application in many areas of scientific research: microscopy and profilometry [2], optical communication [3], and manipulation of micro-objects [4–6], and astronomy [7]. The rapid development of this direction was made possible by the unique properties acquired by a light beam in an optical vortex: topological charge, orbital angular momentum, and a well-defined minimum intensity corresponding to the singularity localization [8]. These properties have allowed vortices to be used as markers and highly sensitive probing beams in studies of changes in small quantities such as the refractive index of the medium [9], the absolute width of the surface ridges and grooves [10] with an accuracy of tens of nanometers, and to improve capabilities of a quantitative phase imaging [11].

The use of optical vortices for certain metrological tasks has been known for two decades. The first works in this area show promising results in surface examination [11,12] carrying with phase singularities in speckle fields, caused by various surfaces. Another advantage of singular beams is their application in combination with interferometric techniques, which provide high precision measurements of samples with various shapes and sizes in a non-invasive manner [13].

In general, interferometric approach utilizes two types of superimposed optical beams: fundamental Gaussian beam and vortex one, which produce typical “fork-like” and spiral interference patterns. However, both transmissive and reflective specimens may be examined with optical vortices. In
manuscripts [14–16] authors demonstrated a new interferometric solution for visualization and characterization of micro-structures called Optical Vortex Scanning Microscope where the sample is scanned by movable vortex inside the beam. In the recent research [17,18] authors develop this imaging system in combination with various test samples, such as transmissive square grooves. The accuracy and resolution of this approach can be greatly enhanced by digital processing of interferograms, in particular, in [19] authors demonstrated the ability of deep neural networks to localize vortex points with subpixel accuracy. Given solution allows to obtain not only high longitudinal but also transverse resolution of the scanning vortex microscope up to few nanometers.

Additionally, the interferometric method of vortex phase analysis has been successfully applied to optical characterization of phase shifts introduced by metamaterials [20] and the refractive index of liquid solutions with an accuracy significantly exceeding the capabilities of ordinary refractometers [9]. At the same time, the design of the optical system based on a spatial light modulator is quite simple and vibration-resistant [21]. An pioneering research which employs array of self-interfered optical vortices was demonstrated in manuscript [22], where authors proposed a new concept of full-field topographic microscopy enabling a reference-free displacement and 3D shape measurement of reflective samples.

In our previous study [2,23–25] we demonstrated the capabilities of array-organised self-interfered singular beams for measurements of transparent specimens. Analysis of the evolution of interference pattern is an established method for extracting detailed information about a sample surface or its thickness. Nevertheless, in complex case of transparent object with certain surface profile, it is hard to extract information about thickness and refraction index variation separately without tomographic and holographic measurements [26].

In this paper, we describe a new, from our perspective, interferometric measurement approach for simultaneous examination of reflective and transmissive specimens, their surface relief and the refractive index variation as the endogenous feature.

2. The interferometry by optical vortices

The new approach in recognition of surface geometry and refractive index variation is based on optical path difference and, as result, on phase delay introduced in probe vortex beam in reference to the Gaussian beam. In case of singular and non-vortex beam superposition with near equal beam waists and energies, the interference pattern observed contain local minima which exhibit angular positioning in dependence of geometric phase changes. Let us consider this process in brief.

The dynamic vortex-beam superposition may be considered as the propagation of a monochromatic beam with a wavelength \( \lambda \) along z-axis, where dynamic interference pattern is generated by superposition of vortex beam with a fundamental Gaussian beam, which is written as slowly varying complex amplitude \( E_{\psi}(x,y,z) \):

\[
E_{\psi}(x,y,z) = \frac{1}{1-iz/\zeta_0} \exp \left[ -\frac{r^2}{\omega_0^2(1-iz/\zeta_0)} \right],
\]

where \( \zeta_0 = k_0 \omega_0^2 / 2 \), \( \omega_0 \) is the radius of the waist of the Gaussian beam, \( r = \sqrt{x^2+y^2} \), \( k_0 = 2\pi/\lambda \)

is a wavenumber in vacuum and \((x,y)\) is the Cartesian coordinates of the beam in the plane \( z = 0 \). A particular solution of paraxial wave equation for vortex beam may be expressed in terms of Laguerre-Gaussian beam [27]:

\[
E_{LG}(x,y,z) = \left( \frac{r}{w} \right)^|m| L_m^{|m|} \left( \frac{r^2}{w^2} \right) \cdot \exp \left( -\frac{r^2}{w^2} \right) \cdot \exp \left( -\frac{ik_0 r^2}{2z(1+z_R^2/z^2)} \right) \times
\]
\[
\times \exp(-il\phi) \cdot \exp \left( i(2m+l-1)\arctan \left( \frac{z}{z_R} \right) \right) \cdot \exp(-ikz)
\]
here $w$ is the radius of the waist of the Laguerre-Gaussian beam, $l$ – azimuthal index, $m$ – radial index, $L_m^l$ – is Laguerre polynomial, $\varphi$ – azimuthal coordinate and current beam radius is expressed as $w = \omega_0 \left[ 1 + \left( \frac{z}{z_R} \right)^2 \right]^{1/2}$. Note, the generated vortex bundle in experiment, particularly, can be represented by a Kummer beam instead of Laguerre-Gaussian mode. As it was shown in [28,29], a better analytical approach for vortex beam field shaped with typical “fork” hologram and spiral phase plates corresponds to Kummer function, especially on short distances from vortex beam shaper. In far field both approximations in terms of Kummer or Laguerre-Gaussian beams will not affect the interference pattern, namely the dynamics of intensity minima (figure 1, a). Hence, for far field vortex beam, we describe its evolution with Laguerre-Gaussian mode without loss of generality.

Topological charge of vortex beam is introduced as: $l = \pm 1, \pm 2, \ldots$. The vortex position, by definition, is described by the equation $\text{Re} \hat{E}_l(x, y, z) = \text{Im} \hat{E}_l(x, y, z) = 0$. By definition, the distribution of the field intensity of interfering waves, for instance, with amplitude factor described by Eq. (1) and (2) is described by expression:

$$I(x, y, z) = E_L^2(x, y, z) + E_G^2(x, y, z) + E_L(x, y, z)E_G(x, y, z).$$

(3)

Numerical simulation of the interference pattern in accordance with the second term in expression (2) for a beam carrying optical vortex (figure 1, a) indicates a characteristic spiral in the case of axisymmetric interference of beams and in the form of a “fork” for interference fringes obtained by adding slightly inclined beams, as shown in the figure 1 (c, d). The phases of the centered vortex and Gaussian beam are shown in frames (a, b) at the bottom.

![Figure 1. The vortex beam intensity distribution (a) in longitudinal section across z-axis, Gaussian beam amplitude (b); axial interference pattern Laguerre-Gaussian and Gaussian beam with 10$\omega_0$ (c) and their interference in case of small tilt (d).](image)

From the other hand, in case of near equal intensities and beam waists, the shape of interference pattern slightly different from mention above. Now we consider only coaxial interference, when both Laguerre-Gaussian and Gaussian beams axis coincide.

Nevertheless, the position of the minimum intensity in the beam field depends on the ratio of the intensities of interfering beams. In the figure 2 (top row) we shown the result of experimental registration of intensity distribution of superposed Laguerre-Gaussian ($l = \pm 1$) and Gaussian beams in Mach-Zehnder interferometer. As a result, it is noticeable dependence of the vortex displacement from the beam axis on the intensity ratio. Maximal vortex shift was observed when amplitude values are equal. A decrease in the intensity of a Gaussian beam entails a smaller shift of the vortex and its gradual return to the beam axis. By adjusting the intensity ratio of the interfering beams, it becomes possible to precisely control the position of the optical vortex.
\[ \frac{E_G}{E_{LG}} = 0.05 \quad \frac{E_G}{E_{LG}} = 0.15 \quad \frac{E_G}{E_{LG}} = 0.25 \quad \frac{E_G}{E_{LG}} = 0.35 \quad \frac{E_G}{E_{LG}} = 0.5 \quad \frac{E_G}{E_{LG}} = 0.6 \quad \frac{E_G}{E_{LG}} = 0.8 \]

Figure 2. Vortex beam superposition with Gaussian beam under different amplitude ratios. The vortex shift in transverse plain. Intensity distributions for single charged \( (l = \pm 1) \) and double charged \( (l = \pm 2) \) optical vortices. Red dashed line indicates geometrical center of the beam. Frame dimensions are 500×500 μm.

Another example of the displaced optical vortex shaping is shown in figure 2 (bottom row) for the case of an initial singular beam with a double topological charge. A distinctive feature of the evolution of the intensity distribution in this case is the appearance of two intensity zeros in the vicinity of the beam axis. A change in the amplitude ratios of the beams from smaller to larger corresponds to a proportional decrease in the distance between the minima.

The dynamics of the obtained interference patterns, in contrast to amplitude ratios, depending on the optical path differences between beams, deserves special attention. Due to the superposition of the fronts of the helicoidal and plane waves, a characteristic circular motion of the vortex around the beam axis arises. If the ratio of the beam amplitudes is closer to unity, the larger the radius of rotation of the singularity becomes. In this case, it is worth noting the different number of total revolutions performed during the interference of beams with different topological charges. Experimental examination of this process is shown in figure 3. The first and second rows show the intensity distributions and the phase of superposition of a single and double charged vortices and Gaussian beams depending on the optical path difference \( Z_z = z + \Delta \). From the presented result, it can be concluded that there is one complete revolution from 0 to 360 degrees for a single charged vortex and from 0 to 180 degrees for a beam with a vortex of a double topological charge. Here we mean a revolution until the start and end positions coincide.

Figure 3. Vortex beam revolving under different optical path differences \( Z = z + \Delta \).

The presented dependence of the vortex shift on the amplitude ratio gives the possibility to reveal the optimal conditions for the formation of the beam field with a displaced vortex for unambiguously
programmable recognizing of vortex position by zero intensity location. From performed experiments we propose a new approach of phase delay $\Delta \varphi(x, y)$ extracting directly from intensity distribution of interfering beams in terms of angular positions of intensity minima.

It is known that the dependence of the optical path difference and relative phase delay $\Delta \varphi(x, y)$ for the transmissive objects is proportional to the refractive index:

$$\Delta l_{\text{trns}}(x, y) = \frac{\lambda \Delta \varphi(x, y)}{2 \pi} \frac{1}{n(x, y) - n_0},$$

where $n_0$ is a refractive index of surrounding media. In contrast, optical path difference for reflecting surfaces may be written as:

$$\Delta l_{\text{refl}}(x, y) = \frac{\lambda \Delta \varphi(x, y)}{4 \pi}.$$

Here we have taken into account the double optical path of reflected beam by $4\pi$ in the denominator.

In the general case, the transparent object under study may, in addition to the complex relief geometry, have internal inhomogeneities of the refractive index. The approaches described earlier allow us to determine with high accuracy either changes in the refractive index with unchanged geometry or surface topography if the refractive index is constant. We propose an optical scheme (figure 4), which allow to separate the contribution to the optical path difference from the geometry of the surfaces bounding the specimen and the spatial distribution of the refractive index. Note that this method makes it possible to estimate changes in relief relative to the initial "base" measurement as marked (1) point in figure 4 inset, which solves the profilometric problem, and, in part, perform the localization of internal inhomogeneities (changes in the refractive index, cavities, and defects) in the material of the object under test.

Figure 4 presents the general layout of a setup that is implied in the numerical calculation of superposed vortex beams arrays. Proposed setup supposes studying of both reflecting and transparent objects.

**Figure 4.** The scheme of experimental set-up for reflecting and transparent objects. He-Ne: Helium-Neon Laser; $\lambda/4$ is the quarter-wave plate; SPP: spiral phase plate; M: mirrors; L: micro objective $8 \times$ lenses; PBS – polarizing beamsplitter; BS: non-polarising beamsplitters; CMOS are digital cameras, P – polarizers.
The Gaussian beam from Helium-Neon (He-Ne) laser has been converted into circularly polarized one and spitted into probe vortex beam which is shaped with spiral phase plate (SPP) with topological charge $l = -1$ and passed through optical system of lens ($L_1$ and $L_2$) and the specimen. Probe beam passes through the three Mach-Zehnder interferometers: (i) back-reflected $y$-polarised by polarization beam splitter (green-coloured line); (ii) passing through beam (purple line) with $y$-polarization; and (iii) second reflected beam from another side of specimen (orange line). As a result, each pair of probe vortex beam and reference Gaussian beam (red line) superposed at the output of beamsplitters $BS_1$, $BS_3$ and $BS_4$ then filmed with CMOS cameras. Polarizing beamsplitter allow to avoid unwanted interference and to observe the superposition of the reference and object beams directly on the camera matrix. Additional polarizers in front of cameras allow to avoid incorrect interference and cut-off orthogonally polarized speckle.

The simplest way for rapid and computationally non-intensive analysis is using of direct data from image sensors and cameras with minimal adjustment and transformations. For this purpose, we propose employing of highly sensitive probe vortex beams superposition with the reference one. The method of determining the angle of intensity minima rotation as an interference pattern is quite simple. The post-processing application determines the centre of the dark area surrounded by maximal intensity. The same angle evaluation process is applied to each scanning step (steps (1) and (2) in figure 4 inset). The result of extracting the phase shift according to the proposed scheme makes it possible to achieve a determination of surface step heights $\Delta l_1$, $\Delta l_2$ and thickness $\Delta l_0$ with vertical resolution of up to 8 nm with an experimentally distinguishable rotation of the single intensity minimum up to 3 degrees for a He-Ne laser source with $\lambda = 632.8$ nm and less than 5 nm for sources with a shorter wavelength.

The above approach is applicable for reflective surfaces, as well as for the case of transparent objects with possible internal inhomogeneities of the refractive index. In the case of transparent specimens, an additional third beam (purple line in figure 4) is used. The interferogram processing program is configured in such a way as to ensure the equality of the optical path difference arising when the probing beam passes through the object and the optical path difference due to the geometry of both object surfaces. If the equality is violated, the magnitude of the phase difference corresponds to a local change in the refractive index.

A complete bump map is formed in this scheme by step-by-step scanning in the (x, y) plane with an array of probing beams simultaneously (figure 5). Using the sample holder with transverse movement, it is possible to examine and visualize the entire surface, for example, optical wedge, without limiting the linear dimensions, and obtaining a profilogram can be provided with one pass along the selected direction.

**Figure 5.** The result of surface reconstruction with scanning vortex profilometry.
By subtracting the relative difference between the optical and geometric paths of the transmitted and reflected beams, information on the local thickness and the relative distribution of the refractive index of the sample medium may be extracted. Note that the sample thickness and surface topography can only be determined relative to the adjacent position during scanning, in step-by-step mode. In this case, the disadvantage of such a system, as well as of a similar one based on interferometry with optical vortices, is the impossibility to determine the absolute thickness of the object or the depth of surface topography. However, an undoubted advantage is high-precision profilometry and defectoscopy of transparent and reflecting objects, allowing to determine local deviations in the refractive index of the medium and surface roughness, separately determining the contribution of each of these factors in the interferogram.

3. Conclusions
In the presented work, we analyzed the possibilities of extracting data on the surface profile of the studied sample in the reflection and transmittance mode. The peculiarity of the proposed method is the simultaneous use of three interferometric measurements using singular beams. The phase features of these beams make it possible to determine changes in the geometry of the sample surface on the nanometer scale as well as the refractive index. The proposed approach provides detection of inhomogeneities in the internal structure of the studied object, which may significantly expand the capabilities of interferometry-based metrological tools.

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