Research on Kalman Filter for One-dimensional Discrete Data

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Abstract. Kalman filter processes the input and observation signals with noise on the basis of linear state space representation to obtain the system state or real signal. In the one-dimensional model, due to the lack of multi-dimensional description of the target, the prior estimate of the state is often the measured value of the previous moment. When the target state changes, the divergence phenomenon will occur. Aiming at the problem that the one-dimensional traditional Kalman filter lacks the target observation dimension, which leads to the divergence or imprecision of the filter, this paper focuses on improving the estimation method of the target state, and proposes a real-time prediction model based on the cascade structure. This model can improve the response of Kalman filter to the change of target state and dynamically adjust the Kalman iterative domain to improve the measurement accuracy. The digital signal filtering simulation is carried out and the performance of the filter is verified based on LabVIEW. Experimental results show that the algorithm can maintain the accuracy and real-time performance of filtering when only one dimension observation results are obtained.

1. Introduction

1.1. Virtual Instrument Development Environment - LabVIEW
LabVIEW is the abbreviation of Laboratory Virtual Instrument Engineering Workbench, namely Laboratory Virtual Instrument Engineering Platform. It is a powerful and flexible instrument and analysis software application development tool, which plays an important role in the field of experimental measurement, industrial automation and data analysis[1]. With the construction of the virtual instrument development platform, users can easily build their own virtual instruments or test systems to meet the test needs of different fields[2]. At present, LabVIEW virtual instruments have been used in the monitoring and control of process variables in various industrial applications[3].

1.2. Kalman Algorithm
Kalman filter applied in the time domain method, invented by the Hungarian mathematician Kalman[4]. By establishing the linear system state equation, the estimated value and the observed value are fused to get the optimal estimate of the system state[5]. Kalman filter has been widely used in sensor and data prediction due to its strong real-time performance and high accuracy. In the actual measurement process, people in many application fields have proposed solutions to the inaccuracy of prior state modeling according to their respective specific background: Li G et al. proposed a method for state...
estimation using multiple UKF models[6], Jiang C et al. proposed that the SAGE -Husa method was used to update the noise and double Kalman algorithm was used for state estimation to suppress the divergence of Kalman filter[7]. George Galanis et al. proposed a one-dimensional Kalman filter algorithm and NGP model for temperature prediction[8].

Although these methods play a certain role, but the premise is that the prediction noise and observation noise parts are known and do not change with the state. These methods are difficult in the only one-dimensional observation model. In view of the above analysis, a real-time prediction model based on cascade structure is proposed for one-dimensional model, which is used to monitor the updating and iteration of the filter to the target state. Finally, the dynamic simulation is carried out by the signal generator, and the filtered results and analysis are given.

2. Fundamental principle of Kalman filter

Suppose there is a dynamic data $X_t$ that changes with time $t$, where $X_t$ is the state of the dynamic system at time $t$:

$$
X_k = F_k \cdot X_{k-1} + W_k \\
Z_k = H_k \cdot X_k + V_k
$$

(1)

(2)

In the formula, $X_k$ is a prior estimate and is the state predicted by the system, while $Z_k$ is the state observed by the system. $W_k$ and $V_k$ are estimated noise and observed noise of the system respectively, and $F_k$ and $H_k$ are coefficient state matrices at time $t$. At the same time, before the filter is run, it must be determined that the noise $W_k$ and $V_k$ are uncorrelated and that $X_k$ and $Z_k$ are independent of each other:

$$
\text{Cov}(W_k, V_k) = 0, \text{Cov}(X_k, Z_k) = 0
$$

(3)

According to the estimated state $X$ and observed value $Z$, a recursive estimation method is proposed by Kalman filter on time series. The algorithm can be roughly described as follows:

$$
X_{k/k-1} = F_k \cdot X_{k-1}
$$

(4)

$$
P_{k/k-1} = F_k \cdot P_{k-1} \cdot F_k^T + W_k
$$

(5)

Formula (4) and (5) are called prediction equation, in which $X_{k/k-1}$ is the estimated value at time $t-1$ and $X_{k-1}$ is the prediction of the priori estimate state at time $t$. $P_{k/k-1}$ and $P_{k-1}$ are the covariance transfer matrices of $X_k$ and $X_{k-1}$ respectively, with the purpose of predicting the error of the priori estimate $X_k$ at time $t$. When the new observed state $Z_k$ is detected, the value of $X_k$ at the current time $t$ can be calculated:

$$
X_k = X_{k/k} = X_{k/k-1} + K_k \cdot (Z_k - H_k \cdot X_{k/k-1})
$$

(6)

In the formula, $K_k$ is the key coefficient in the filtering algorithm, also known as the Kalman gain. The appropriate value of $K_k$ can make the Kalman filter adapt to different situations more easily and determine the updating speed of the pre-estimated calibration. Its calculation formula is as follows:

$$
K_k = P_{k/k-1} \cdot H_k^T \cdot (H_k \cdot P_{k/k-1} \cdot H_k^T + V_k)^{-1}
$$

(7)

It can be seen from the formula that the prior estimate has a great influence on the value of $K_k$. Finally, Kalman gives the update formula of the covariance matrix $P_k$ of the unknown state $X_k$:

$$
P_k = (1 - K_k \cdot H_k) \cdot P_{k/k-1}
$$

(8)

Equations (6), (7) and (8) are also referred to as the renewal equation.
3. Filter design

3.1. One-dimensional Kalman filter

In order to study the filtering performance of one-dimensional discrete Kalman filter, the one-dimensional model of Kalman filter is firstly extended. \( W_k \) and \( V_k \) in Equation (1) and Equation (2) determine the Kalman gain coefficient. When the accuracy of \( W_k \) and \( V_k \) is not accurate, Kalman filter can still get a relatively ideal estimate value, but it will also affect the recursion speed and filtering effect. In the one-dimensional model:

**Prediction:**

\[
X_{k|k-1} = X_{k-1} + W_k \tag{9}
\]

\[
P_{k|k-1} = P_{k-1} + W_k \tag{10}
\]

Formula (10) is the state transfer formula of the prior estimate error. \( P_{k|k-1} \) is the error of the prior estimate \( X_k \) at time \( t \).

**Update:**

\[
X_k = X_{k|k} = X_{k|k-1} + K_k \cdot (Z_k - X_{k|k-1}) \tag{11}
\]

\[
K_k = P_{k|k-1} \cdot (P_{k|k-1} + V_k)^{-1} \tag{12}
\]

\[
P_k = (I - K_k) \cdot P_{k|k-1} \tag{13}
\]

Due to the lack of multi-dimensional description of the target observation, a rough prediction can be constructed: the prior estimate of the state at time \( t \) is the target observation value at the previous time, and there is no error caused by state transition. The following formula can be obtained:

\[
P_{k|k-1} = P_{k-1} + W_k = W_k \tag{14}
\]

Calculate the variance of \( X_k \):

\[
Var(X_k) = Var(K_k \cdot Z_k + (1 - K_k) \cdot X_{k|k-1}) \tag{15}
\]

\( X_k \) and \( Z_k \) are independent of each other:

\[
Var(X_k) = K_k^2 \cdot V_k + (1 - K_k)^2 \cdot W_k \tag{16}
\]

According to formula (16), it is easy to observe the value of \( K_k \) when \( Var(X_k) \) is the minimum:

\[
K_k = W_k \cdot (W_k + V_k)^{-1} \tag{17}
\]

Equation (17) can also be obtained from Equation (10), Equation (12) and Equation (14).

3.2. Real-time prediction model

Equation (9) updates the estimated state at time \( t \), so that the Kalman filter can be iterated continuously to obtain the optimal state estimate of the target. As can be seen from Equations (11), (12) and (13), as the number of iterations increases, the Kalman filter will output a signal that tends to be stable. This is because the variance \( P_k \) of the target state will decrease with time. At this time, the new observed value \( Z_k \) is once again input into the system, which only plays a minor role in the calibration of the filter results. When the state of the observed target changes, the divergence of the traditional Kalman filter will occur. In the traditional model, the judgment of the observed noise and the estimated noise does not get real-time feedback, and its filtering effect often depends on the prior estimation of the target state[9]. In order to solve the divergence phenomenon caused by the lack of other observation methods
for the description of the target state in the one-dimensional model, a real-time prediction model is introduced, as shown in Figure 1.

![Real-time prediction model based on cascade structure](image)

Figure 1. Real-time prediction model based on cascade structure.

The prediction model is divided into the following steps:

- Construct the convergence judgment for the target state by obtaining a set of observation sequences of target signals:
  \[
  \tilde{B}_k = \{Z_{k-3}, Z_{k-2}, Z_{k-1}, Z_k\}
  \]  
  (18)

- Calculate the mean, variance and standard deviation of the observation series:
  \[
  \bar{Z}_k = \frac{1}{4} \sum_{i=k-3}^{k} Z_i
  \]  
  (19)
  \[
  \sigma_k^2 = \frac{1}{4} \sum_{i=k-3}^{k} (Z_i - \bar{Z}_k)^2
  \]  
  (20)
  \[
  \sigma_k = \sqrt{\frac{1}{4} \sum_{i=k-3}^{k} (Z_i - \bar{Z}_k)^2}
  \]  
  (21)
  \[
  p \times \sigma_k < Z_k - X_{k-1}
  \]  
  (22)

As the convergence judgment basis, where the convergence factor \( P \geq 0 \) is the adjustable coefficient, representing the confidence interval of discrete signal error. When the above formula holds, the filter thinks that the target has been in a new state and starts to make an optimal estimation of the state at this time according to experience:

- A priori estimate of the current state:
  \[
  X_{k/k-1} = Z_{k-1}
  \]  
  (23)

- Predicted observation noise:
\begin{align*}
V_k &= W_{k-1} \quad (24)
\end{align*}

- Update the prior estimated noise. Since the target reaches a new state, the predicted noise \( W_k \) at the time \( t-1 \) is judged by the prediction model:
\begin{align*}
W_k &= (Z_k - Z_{k-1})^2 \quad (25)
\end{align*}

- Predicting the state at time \( t \):
\begin{align*}
K_k &= W_k \cdot (W_k + V_k)^{-1} \quad (26) \\
P_k &= (I - K_k) \cdot P_{k/k-1} \quad (27) \\
X_k &= X_{k/k} = X_{k/k-1} + K_k \cdot (Z_k - X_{k/k-1}) \quad (28)
\end{align*}

Forecast time \( t \) state of formula (24), formula (25) to filter the target error of time \( t \), the kalman filter estimate of the state of \( t - 1 \) time already no longer believe, according to the experience of the state of \( t - 1 \) time to forecast the new state, and this time the value of the target state update forecast state to calculate the current moment the optimal estimation of target state.

If the judgment basis in Equation (22) is not satisfied, it does not indicate that the target state is still changing at this time, the filter stops the iteration process, tracks the target, and updates the estimated state of the Kalman filter at the same time:
\begin{align*}
X_k &= Z_k \quad (29) \\
W_k &= \sigma_k^2 \quad (30)
\end{align*}

The Kalman filter with the prediction model does not rely on a large amount of prior knowledge and multi-dimensional observation data, but only needs the prediction model to provide the change of target state for the Kalman filter. Compared with the traditional one-dimensional Kalman filter, the number of iteration steps is shown in Fig. 2.

![Figure 2. Real-time prediction model based on cascade structure.](image)

Compared with the filter with a prediction model, the traditional Kalman filter can not update the observed variance and estimated variance, so the filter will take the initial estimated state as the prediction for iteration. When the prior estimate is inaccurate or the target state changes, divergent phenomenon will occur, as shown in Figure 3.
Figure 3. The function of prediction model to filter calibration

It can be seen from this that the Kalman filter with the prediction model can update in time when the target state changes. When the target reaches a new state, it will iterate the number of steps. Some values of $P$ can be set according to the experience of Gaussian distribution, as shown in Figure 4.

Figure 4. Gaussian distribution model

The convergence factor $P$ makes it possible for Kalman filter to apply to various signal characteristics. As the confidence interval $P \cdot \sigma$ increases, the filter will expand the iteration region of the current state. Reducing the value of $P$ will increase the response of Kalman filter to state change. As for the value of $P$, setting different $P$ values for signals with different characteristics will improve the filtering results of signals.

4. Experiment and result analysis

4.1. Simulation experiment

The hysteria of state response of traditional one-dimensional Kalman filter has been explained in Section 3.2, which will not be discussed in this chapter. Through the simulation signal module of LabVIEW, a random signal generator with Gaussian noise of 0.2 is set [20-23]. The program panel design is shown in Figure 5.

Figure 5. Gaussian distribution model
A one-dimensional Kalman filter, a sliding median filter and a mean filter with a real-time prediction model are established[10]. The sliding window of the prediction model, the mean filter and the median filter is set as 4, and the initial convergence factor $p$ of the Kalman filter is set as 4.2. A filtering simulation experiment is carried out for a group of collected data. The filtering effect of the four filters on the square wave signal is shown in Figure 6.

Figure 6. Gaussian distribution model

It can be observed that one-dimensional Kalman filter has a better reduction effect on square wave containing Gaussian noise than median filter and mean filter, and its determination of target change is a dynamic prediction process, which is different from the limiting filter. Median filtering responds to the step state only when the wave peak signal is in the middle of the sequence, while the mean filtering presents an obvious "slope" change in the reduction of the opposite wave. Kalman filter carries out incomplete statistics on noise through real-time prediction model, so one-dimensional Kalman filter has a higher response to square wave signal. In the duration period of the square wave peak, the prediction model does not detect the change of the target state. At this time, the Kalman filter iterates in the confidence interval to complete the optimal estimation of the target. Therefore, the filtering effect is smoother and the degree of data dispersion is smaller in the duration of the peak.

4.2. Practical application

A digital thermopile sensor was used to collect the temperature of the blackbody set at 40℃. The voltage signal of thermopile is collected into the upper computer software through ADC. Figure 7 records the data processing results of the three-type filters.

Figure 7. Gaussian distribution model
Fig. 6 shows the filtering effect of three kinds of filters on noise signals. Compared with the other two kinds of filters, the one-dimensional Kalman filter has a more convergent and less discrete data processing result. In the initial stage, the filter will choose the number of iterations according to the convergence judgment conditions and constantly modify the results of the filter. Jumping occurs in the 27th sampling process because the judgment conditions in Equation (22) are not satisfied in the calculation process. At this time, the filter mistakenly believes that the target state has changed, uses mean filtering and updates the observation error at this moment. When the prediction model judges that the target state is no longer changing, the Kalman filter starts to repeat the previous iteration steps. In the case of jump, appropriately increasing the value of $p$ to expand the confidence interval can effectively avoid it, but it will also increase the response time of the filter to the change of target state.

|                | Kalman | Mean | Media |
|----------------|--------|------|-------|
| STDEVP         | 7.89E-03 | 1.32E-02 | 1.32E-02 |
| VAR            | 6.42E-05 | 1.79E-04 | 1.81E-04 |
| RMSE           | 1.01E-01 | 1.05E-01 | 1.06E-01 |
| MSE            | 1.02E-02 | 1.11E-02 | 1.12E-02 |

As shown in Table 1, by comparing the simulation analysis from the four aspects of quasi error, variance, root mean square error and mean square error, it can be seen that the Kalman filter has a smaller degree of data dispersion and a better denoising effect under the same conditions. The performance of median filter is close to that of mean filter.

5. Conclusions
This paper focuses on the prediction modeling of one-dimensional discrete data using Kalman filter algorithm. After analyzing the problems of traditional Kalman filter algorithm, a real-time prediction model based on cascade structure is proposed. It effectively solves the problem that the Kalman filter lacks the target observation dimension, which leads to the divergence or inaccuracy of the filter. The practical application results show that the improved Kalman filter is more competitive than the traditional filter and has the best denoising effect on temperature signal compared with the median filter and the mean filter. The future research will concentrate on the improvement of the prediction model to quickly distinguish the variation and noise so as to get an accurate signal.

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