CORRELATION FUNCTIONS
OF THE APM CLUSTERS OF GALAXIES

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ABSTRACT

We have found that the two-point correlation function of the APM clusters of galaxies has an amplitude much higher than that claimed by the APM group. As the richness limit increases from \( R = 53 \) to 80, the correlation length increases from 17.5 to 28.9 \( h^{-1}\)Mpc. This indicates that the richness dependence of the APM cluster correlation function is also much stronger than what the APM group has reported. The richness dependence can be represented by a fitting formula \( r_0 = 0.53d_c + 0.01 \), which is consistent with the Bahcall’s formula \( r_0 = 0.4d_c \). We have tried to find the possible reason for discrepancies. However, our estimates for the APM cluster correlation function are found to be robust against variation of the method of calculation and of sample definition.

Key Words: cosmology: large-scale structure of universe — galaxies: clusters of — galaxies: clustering

I. INTRODUCTION

Rich clusters of galaxies have been used for studies of large-scale structure on scales \( \simeq 10 \sim 100 h^{-1}\)Mpc. The spatial two-point correlation function for the Abell clusters has been estimated by many authors (see the review by Bahcall 1988; also Huchra et al. 1990; Postman et al. 1992; Peacock & West 1992), which has been found to be consistent in shape with the power law form

\[
\xi_{cc}(r) = \left(\frac{r}{r_0}\right)^{-\gamma}
\]

with the correlation length \( r_0 \simeq 20 \sim 26 h^{-1}\)Mpc and with the power law index \( \gamma \simeq 2 \). More recently new catalogs of clusters have been obtained by automated selecting algorithm from the Edinburgh-Durham Southern Galaxy Catalog (Lumsden et al. 1992), and from the APM Galaxy Survey (Dalton et al. 1997). Redshift surveys of these new clusters (Collins et al. 1995; Dalton et al. 1994a) have been used to estimate the spatial two-point correlation function (Nichol et al. 1992; Dalton et al. 1994b). The correlation length \( r_0 \) measured for these new clusters is reported to be lower than that for the Abell clusters, that is \( 14 \lesssim r_0 \lesssim 16 h^{-1}\)Mpc.

It has been argued that the counting radius \( (r_c = 1.5 h^{-1}\)Mpc) for the Abell clusters is so large that the catalog contains projection effects, which cause artificial line-of-sight correlations (Sutherland 1988; Efstathiou et al. 1992). In addition to this, it has been recognized that the intrinsically subjective nature of the Abell catalog causes problems in homogeneity and statistical completeness (Postman et al. 1986). However, Bahcall & West (1992) have suggested that the discrepancy between the Abell clusters and the APM clusters can be accounted by the richness dependence of cluster correlation amplitudes (Bahcall 1988; Bahcall & West 1992)

\[
r_0 = 0.4d_c,
\]

where \( d_c \) is the mean intercluster separation (Here \( d_c = n_c^{-1/3} \), where \( n_c \) is the mean space density of clusters). But Croft et al. (1997) have analyzed richness subsamples of the APM clusters and argued that there is only a
weak dependence of correlation amplitude with cluster richness, and that this disagrees with results from the Abell clusters.

We describe our statistical richness subsamples drawn from the APM cluster catalog in Section II. The method of calculation of the spatial two-point correlation function and our results are presented in Section III. In Section IV we discuss many possibilities which might affect the estimate of the correlation function.

II. THE APM CLUSTER CATALOG

Dalton et al. (1997) have published the catalog of APM clusters which is complete over richness range $R \geq 40$ and the characteristic magnitude range $17.5 \leq m_x \leq 19.4$ (the distance estimation $m_x$ corresponds to Abell’s $m_{10}$). The catalog covers the region of the sky $21^h \lesssim \alpha \lesssim 5^h$ and $-72.5^\circ \leq \delta \leq -17.5^\circ$ of the APM Galaxy Survey (Maddox et al. 1990; see also Loveday et al. 1996) and contains 957 clusters (see Fig. 1). The exact APM survey field definition can be found in the Astronomical Data Center [http://adc.gsfc.nasa.gov/]. Fig. 1 shows the distribution of 957 APM clusters (open circles), 185 APM fields (open squares), and 1456 holes (filled squares) in the $(\alpha, \delta)$ plane (Loveday et al. 1996). Holes are the regions contaminated by big bright objects.

Although the 957 sample is complete in itself, it contains only 374 clusters with measured redshift. In the range of $R \geq 53$ and $17.5 \leq m_x \leq 19.2$, however, the completeness of the redshift sample is 90.9% and increases up to 100% at higher richness limits. So we generate four statistical subsamples with $R \geq 53$, $R \geq 60$, $R \geq 70$, and $R \geq 80$, which contains 213, 149, 76, and 43 clusters, respectively. We have further restricted our APM samples to the declination range $-65^\circ \leq \delta \leq -25^\circ$ to reduce the strong dependence of number of clusters in declination (see below). To convert the redshifts to comoving distances we assumed the Einstein-de Sitter universe with $\Omega_0 = 1$.

III. CLUSTER CORRELATION FUNCTION

We estimate the spatial two-point correlation functions using the Hamilton’s estimator (1993)

$$\xi_{cc}(r) = \frac{(DD)(RR)}{(DR)^2} \frac{4N_rN_c}{(N_c - 1)(N_r - 1)} - 1,$$

which is evaluated to be less affected by uncertainties in the selection function for $\xi_{cc} < 1$ (Dalton et al. 1994b). In equation (3), DD is the number of pairs in the sample with $N_c$ clusters, RR is the number of pairs in a random sample with $N_r$ random points, and DR is the cluster-random pair count. The numerical factor 4 in the normalization term of pair counts accounts for the fact that we count each DD or RR pair only once. The uncertainties of $\xi_{cc}$ are computed from the simple equation $\delta \xi_{cc} = (1 + \xi_{cc})/\sqrt{DD}$, which is easy to calculate but may underestimate the cosmic variance in comparison to simulations (Croft & Efstathiou 1994).

Croft et al. (1997) have estimated the correlation function of the APM clusters located in the distance range of $50 \ h^{-1}\text{Mpc} \leq r \leq 500 \ h^{-1}\text{Mpc}$. They used a selection function obtained by smoothing the distribution of clusters in redshift space over $40/\sqrt{2} \ h^{-1}\text{Mpc}$ by a Gaussian filter (dashed lines in Fig. 2). The random catalog is generated in accordance with this radial selection function within the sample boundaries. Equal weight is given to clusters and random points in the calculation of correlation functions.

We have calculated the correlation function of the APM clusters in the exactly same way that Croft et al. (1997) have followed. Our estimates of correlation functions are plotted in Fig. 3 for four subsamples; the $R \geq 53$ (filled dots), $R \geq 60$ (triangles), $R \geq 70$ (squares), and $R \geq 80$ (open circles) samples. The correlation lengths are $17.5^{+1.4}_{-1.3}$, $23.4^{+2.0}_{-2.0}$, $25.1^{+3.7}_{-3.7}$, and $28.9^{+7.4}_{-7.1} \ h^{-1}\text{Mpc}$, respectively. For a comparison the fitting lines to Croft et al. (1997)’s $R \geq 50$ and $R \geq 70$ samples are drawn as solid and dashed lines, respectively. It can be seen that there are clear discrepancies between our results and Croft et al (1997)’s.
Fig. 1. The distribution of 957 APM clusters (open circles), 185 APM fields (open squares), and 1456 holes (filled squares). The long dashed lines represent the galactic latitude.

Fig. 2. Distributions of clusters with distance for subsamples with $R \geq 53$ and $R \geq 70$. Short dashed curves are distributions smoothed by a $40/\sqrt{2}\, h^{-1}\, \text{Mpc}$ Gaussian. Solid lines are the selection function obtained by eq. (4).

Amplitudes of correlation functions calculated by us are much higher both for $R \geq 53$ and $R \geq 80$ samples. Furthermore, the richness dependence is much stronger. The richness dependence of correlation functions of our APM subsamples can be described by a formula $r_0 = 0.53d_c + 0.01$. This richness dependence is stronger even than that reported for the Abell clusters (Bahcall & West 1992); $r_0 = 0.4d_c$, quite contrary to Croft et al. (1997)'s claim.

IV. DISCUSSION

We have looked for various possibilities that could explain the discrepancies between our and Croft et al. (1997)'s estimates of the APM cluster correlation function.
First, we changed the method calculating the selection function. Instead of using the distribution of clusters in redshift space smoothed over \(40/\sqrt{2} \, h^{-1} \text{Mpc}\), we calculated the selection function from the conventional formula

\[
S(r) = \sum_{i} \frac{\Omega}{3} \frac{1}{D_{\text{max},i}^3 - D_{\text{min},i}^3} \quad \text{inside which the } i\text{-th cluster could be included.}
\]

where \(\Omega\) is the solid angle of the survey region. \(D_{\text{max},i}\) and \(D_{\text{min},i}\) are the maximum and minimum distances to which the \(i\)-th cluster could be included in the sample for the faint and bright magnitude limits of the survey 

\[
m_{\text{lim,upp}} = 19.2 \quad \text{and} \quad m_{\text{lim,low}} = 17.5.
\]

The selection functions calculated by this formula are shown in Fig. 2 for the \(R \geq 53\) and \(R \geq 70\) samples. We have calculated the correlation functions of the APM subsamples using these new selection functions. However, there is practically no change in the amplitude and in the richness dependence of the cluster correlation functions.

Second, we limited the distance range of clusters to \(170 \, h^{-1} \text{Mpc} < r < 330 \, h^{-1} \text{Mpc}\) instead of \(50 \, h^{-1} \text{Mpc} < r < 500 \, h^{-1} \text{Mpc}\) to eliminate the possible dominance of shot noises. Again this did not affect our results.

Third, we changed the way to give weights to clusters and random points in the calculation of correlation functions. Instead of giving equal weights we gave weights equal to the inverse of the selection function. To reduce the shot noise we limited the subsamples to the distance range of \(170 \, h^{-1} \text{Mpc} < r < 330 \, h^{-1} \text{Mpc}\). This time, amplitudes of the correlation functions became slightly higher than those shown in Fig. 3. But this scheme still gives results which are inconsistent with Croft et al. (1997)'s weak cluster correlation function.

Finally, we inspected the distribution of clusters in declination and galactic latitude spaces. If the sample is affected by large galactic obscuration or airmass variation across the sky, a selection function varying on the sky should be taken into account. Fig. 4 shows the number of clusters in declination (squares) or in galactic latitude (circles) strips which have equal areas on the sky. Points are located at the centers of strips. It can be shown that, while there is no large fluctuation in the number of clusters in the galactic latitude space, there is a rather strong variation in the declination space for the \(R \geq 53\) and \(R \geq 70\) subsamples.
Large number of clusters are concentrated in the declination strip centered at $\delta = -48^\circ$. We first varied the declination limits from the original range $-72^\circ 5 \lesssim \delta \lesssim -17^\circ 5$ to narrower ranges in the subsample with $R \geq 53$. The correlation function at short separations did not change but became somewhat steeper. However, the decrease in the amplitude of the correlation function at large separations is small, leaving the correlation function still within the $1\sigma$ error limits. We then used the smoothed distribution of clusters in declination to generate the random catalog. This eliminates the effect of the strong variation of number of clusters with declination. The resulting correlation function has again somewhat lower amplitude at large separations, but the changes are again within the error limits.

All these tests indicate that our estimate for the APM cluster correlation function is robust. It remains to be seen what has made APM group’s results different from ours.

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