Physics of Antiproton Nuclear Interactions near Threshold

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Antiproton-nucleus optical potentials fitted to $\bar{p}$-atom level shifts and widths are used to calculate the recently reported very low energy ($p_L < 100$ MeV/c) $\bar{p}$ cross sections for annihilation on light nuclei. The apparent suppression of annihilation upon increasing the atomic charge $Z$ and mass number $A$ is resolved as due to the strong effective repulsion produced by the very absorptive optical potential which keeps the $\bar{p}$-nucleus wavefunction substantially outside the nuclear surface, so that the resulting reaction cross section saturates as function of the strength of $\text{Im } V_{\text{opt}}$. This feature, for $E > 0$, parallels the recent prediction, for $E < 0$, that the level widths of $\bar{p}$ atoms saturate and, hence, that $\bar{p}$ deeply bound atomic states are relatively narrow. Predictions are made for $\bar{p}$ annihilation cross sections over the entire periodic table at these very low energies and the systematics of the calculated cross sections as function of $A$, $Z$ and $E$ are discussed and explained in terms of a Coulomb-modified strong-absorption model. Finally, optical potentials which fit simultaneously low-energy $\bar{p}-^4\text{He}$ observables for $E < 0$ as well as for $E > 0$ are used to assess the reliability of extracting Coulomb modified $\bar{p}$ nuclear scattering lengths directly from the data.

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I. INTRODUCTION

Antiproton annihilation cross sections at very low energies ($p_L < 100$ MeV/c) have been reported for light nuclei by the OBELIX collaboration [1–3]. At these energies the total $\bar{p}$ reaction cross section consists only of $\bar{p}$ annihilation. Whereas at relatively higher energies ($p_L \approx 200 - 600$ MeV/c) the $\bar{p}$ annihilation cross sections exhibit the well known $A^{2/3}$ strong absorption dependence on the nuclear target mass number $A$, these cross sections at very low energies have defied any simple, obvious regularity. It has been demonstrated that the ‘expected’ $ZA^{1/3}$ dependence on the atomic charge $Z$ and mass number $A$ is badly violated [3,4]. Antiproton annihilation cross sections at very low energies simply do not rise with $A$ as fast as is anticipated.

Here I report a recent study of low energy $\bar{p}$ annihilation on nuclei, using the optical model approach [3,4]. Optical potentials have been very successful in describing strong interaction effects in hadronic atoms [5], including $\bar{p}$ atoms [6]. It has been noted, for pions, that the total reaction cross sections at low energies are directly related to the atomic-state widths, and that once a suitable optical potential is constructed by reasonably fitting it to the atomic level shifts and widths in the negative energy bound state domain, these total reaction cross sections...
sections are reliably calculable [9,10]. The recent publications [1–3] of experimental results of total cross sections for $\bar{p}$ annihilation on nuclei at very low energies raise the intriguing possibility of connecting these two energy regimes in a systematic way also for antiprotons. However, most of the data on annihilation cross sections for $\bar{p}$ are for very light nuclei, where the concept of a rather universal optical potential that depends on $A$ and $Z$ only through the nuclear densities is questionable. For this reason, in the present work, optical potentials are used mostly for crossing the $E = 0$ borderline within the same atomic mass range, from bound states to scattering. These potentials are strongly absorptive, which leads to a remarkable saturation of the total reaction cross section with increasing $A$.

This review is organized as follows. The saturation phenomenon in $\bar{p}$ atoms, and for $\bar{p}$ total reaction cross sections, is described and discussed in Sect. II. Calculational results are given, demonstrating the success of using a unified optical potential methodology across the $\bar{p}$-nucleus threshold. The $A$ and $Z$ dependence of these low energy annihilation cross sections at $p_L = 57$ MeV/c is discussed. The Coulomb modified strong absorption model at very low energies is described in Sect. III and, particularly, how it successfully reproduces, due to the Coulomb focussing effect, the optical potential reaction cross sections. Lastly, in Sect. IV optical potentials which fit simultaneously low energy $\bar{p}^{-4}$He observables for $E < 0$, as well as for $E > 0$, are used to assess the reliability of extracting $\bar{p}$ nuclear scattering lengths directly from the data.

II. SATURATION OF $\bar{P}$ ATOMIC WIDTHS AND OF TOTAL REACTION CROSS SECTIONS

Antiprotonic and $K^-$-atom optical potentials are strongly absorptive [7]. This strong absorptivity has recently been shown [11,12] to lead to a saturation of the widths of atomic states upon increasing the absorptivity of $V_{\text{opt}}$, and to the prediction of relatively narrow deeply bound atomic states. Figures 1 and 2 show calculated $\bar{p}$ and $K^-$ atomic energy levels in Pb, respectively, for several values of $l$. The bars stand for the full width $\Gamma$ of the level and the centers of the bars correspond to the binding energies $\text{Re } B$. It is seen that the calculated widths saturate at about 2 MeV. The dependence on the model is found to be negligibly small, affecting the calculated widths by less than 5%, provided the optical potential was fitted to the known part of the spectrum throughout the periodic table. For this heavy nucleus the levels are quite close to each other although the spectrum is still well defined. The 9$k\bar{p}$ level [13] and the 7$i\bar{K}^-$ level [14] in Pb are the last observed in the respective X-ray spectra. The close analogy between bound-state widths and total reaction cross sections is best demonstrated, assuming for simplicity a Schrödinger-type equation, by comparing the following expressions for the width $\Gamma$ and for the total reaction cross section at positive energies with each other:

$$\frac{\Gamma}{2} = - \frac{\int \text{Im}V_{\text{opt}}(r)|\psi(r)|^2 dr}{\int |\psi(r)|^2 dr}, \quad \sigma_R = -\frac{2}{\hbar v} \int |\chi(r)|^2 \text{Im}V_{\text{opt}}(r) dr,$$

(1)

where $\psi(r)$ is the $\bar{p}$ full atomic wavefunction and $\chi(r)$ is the $\bar{p}$ - nucleus elastic scattering wavefunction; $v$ is the c.m. velocity.

A standard $V_{\text{opt}}$, to be used in the above expressions, is given by the ‘$t\rho$’ form [7]
where $m$ is the nucleon mass, $\mu$ is the reduced $\bar{p}$-nucleus mass, $b_0$ is a complex parameter obtained from fits to the data and $\rho(r)$ is the nuclear density distribution normalized to $A$. Although the effect of the $(A - 1)/A$ factor may often be absorbed into the fitted parameters, as is the case in this section, for very light $\bar{p}$ atoms one must include the effect of the underlying $\bar{p}N$ interaction, at least by folding a phenomenological interaction form factor into the nuclear density. Thus, the $2\bar{p}$ level shift and width data in $^3$He \cite{2,3} are well fitted by folding a Gaussian with a range parameter of 1.4 fm into the nuclear density distribution. The resulting potential, with $b_0 = 0.49 + i3.0$ fm, is highly absorptive. Using this potential (a) to calculate the total reaction (annihilation) cross section for 57 MeV/c $\bar{p}$ on $^4$He, the calculated cross section is 901 mb, in excellent agreement with the reported value of 915$\pm$39 mb \cite{4}.

Figure 3 demonstrates the extreme strong absorption conditions which are relevant to the $\bar{p}$ nucleus interaction at very low energies (and for $\bar{p}$ atoms). It shows calculated reaction cross sections for $\bar{p}$ at 57 MeV/c on $^4$He and Ne as function of the strength $\text{Im} \, b_0$ of the imaginary part of the potential (a) described above, with the rightmost edge corresponding to its nominal value $\text{Im} \, b_0 = 3.0$ fm. It is seen that as long as the absorptivity ($\text{Im} \, b_0$) is very weak, less than 1% of its nominal value, $\sigma_R$ is approximately linear in $\text{Im} \, b_0$, which according to Eq. (1) means that the $\bar{p}$ wavefunction depends weakly on $\text{Im} \, b_0$. However, already at below 5% of the nominal value of $\text{Im} \, b_0$ the reaction cross sections begin to saturate, much the same as for the widths of deeply bound $\bar{p}$ atomic states \cite{11,12}. The mechanism is the same in both cases, namely exclusion of the wavefunction from the nucleus due to the absorption, which reduces dramatically the overlap with the imaginary potential in the integrals of Eq. (1). The onset of saturation is determined approximately by the strength parameter.
\[2\mu(\text{Im } V_{\text{opt}})R^2,\] where \(R\) is the radius of the nucleus. Thus, saturation of \(\sigma_R\) in Ne starts at a smaller absorptivity than its onset in \(^4\text{He}\). The Ne/\(^4\text{He}\) ratio of \(\sigma_R\) values changes, due to this effect, from about 15 in the perturbative regime of very weak absorptivity to about 3 in the strong absorption regime. However, the precise, detailed pattern of the change also depends on \(\text{Re } V_{\text{opt}}\) which may enhance or reduce the exclusion of the \(\bar{p}\) wavefunction from the nucleus.

Figure 3 shows calculated \(\bar{p}\) reaction cross sections at 57 MeV/c across the periodic table. The dot-dashed line is for the above mentioned potential (a) which is expected to be valid only in the immediate vicinity of He. The dashed line (b) is for the first potential from Table 7 of Ref. [7] which fits \(\bar{p}\) atom data over the whole periodic table, starting with carbon. This potential is not expected to fit data for very light nuclei, and indeed it does not fit the \(^4\text{He}\) annihilation cross section. However, it is noteworthy that the two potentials predict almost the same cross sections for \(A > 20\) and certainly display a very similar dependence on \(A\). Also shown in the figure is the recent experimental result [8] for Ne, with a very limited accuracy. Furthermore, for \(A > 20\) the points along the solid line are the calculated \(\bar{n}\) - nucleus total reaction cross sections, obtained from potential (b) by switching off the Coulomb interaction. The solid line is a fit to an \(A^{1/3}\) power law which appears to be appropriate to strong absorption of uncharged particles at very low energies. Comparing the dashed line for negatively charged particles with the solid line for uncharged particles, it is clear that the \(\sigma_R\) values obtained by including the Coulomb interaction are considerably enhanced at this very low energy with respect to the ones obtained without it. This is due to the Coulomb focussing effects which are discussed below.

Comprehensive measurements of \(\bar{n}\)-nucleus total annihilation cross sections on six targets
from C to Pb at \( p_L = 50 - 400 \text{ MeV/c} \) have been recently reported \[16\] in terms of a factorized dependence on \( p_L \) and \( A \):

\[
\sigma_{\text{ann}}(p_L, A) = \sigma_0(p_L) A^x, \quad x = 0.6527 \pm 0.0044,
\]

where \( \sigma_0(p_L) \) is given explicitly as a monotonically decreasing function of \( p_L \). This essentially uniform \( A^{2/3} \) behavior, which normally is expected only at considerably higher energies, is puzzling. Furthermore, the reported \( \bar{n} \) cross sections for a given target, particularly at the lower end of the energy range, are too high to be compatible with the \( \bar{p} \)-atom and \( \bar{p} \)-nucleus phenomenology considered in this review. For example, Eq. (3) gives about \( 4000 \pm 600 \text{ mb} \) for \( \bar{n} \) annihilation at \( p_L = 57 \text{ MeV/c} \) on Ne, roughly three times the value shown in Fig. 4 for \( \bar{n} \)-Ne using potential (b), and substantially higher than the \( \bar{p} \)-Ne annihilation cross section \[3\] of \( 2210 \pm 1105 \text{ mb} \). This persists, although to a lesser extent, also at higher energies. Thus, at \( p_L = 192.8 \text{ MeV/c} \) Eq. (3) gives about \( 1175 \pm 125 \text{ mb} \) for \( \bar{n} \)-Ne, whereas the measured annihilation cross section for \( \bar{p} \)-Ne at this energy is only \( 956 \pm 47 \text{ mb} \) \[17\].

### III. COULOMB FOCUSSING EFFECTS ON TOTAL REACTION CROSS SECTIONS AT LOW ENERGIES

The behavior of the calculated \( \bar{p} \)-nucleus total reaction cross sections at very low energies may be explained in terms of a simple Coulomb-modified strong absorption model \[6\] as follows. Recall that an attractive Coulomb potential causes focussing of partial-wave trajectories onto the nucleus, an effect which at low energies may be evaluated semiclassically, as recognized a long time ago by Blair in connection with \( \alpha \) particle reactions on nuclei \[18\] (see Ref. \[19\] for the applicability of the semiclassical approximation to Coulomb scattering). Assuming that complete absorption occurs in all partial waves for which the distance of closest approach is smaller than \( R \), one gets the following relation between the Coulomb-modified \( l_{\text{max}} \) and \( R \):

\[
(l_{\text{max}} + \frac{1}{2})^2 \approx (kR)^2(1 + \frac{2\eta}{kR}) ,
\]

where \( \eta \) is the standard Coulomb parameter \[19\]. At very low energies, \( 2\eta >> kR \), and therefore \( l_{\text{max}} >> kR \) due to the focussing effect of \( V_c \). The total reaction cross section in the strong absorption limit is then given by

\[
\sigma_R = \frac{\pi}{k^2} \sum (2l + 1) \approx \frac{\pi}{k^2} \left(l_{\text{max}} + \frac{1}{2}\right)^2 \approx \pi R^2 \left(1 + \frac{2\eta}{kR}\right) = \pi R^2 \left(1 + \frac{2mZe^2}{\hbar^2 k_L kR}\right) ,
\]

where \( k_L \) and \( k \) are the laboratory and c.m. wave numbers, respectively. The second term within the brackets represents the Coulomb focussing effect and at very low energies it becomes dominant \[3\], thus leading to a \( Z A^{1/3} \) dependence of the cross section if \( R = r_0 A^{1/3} \). At high energies, where the Coulomb focussing effects become insignificant, the well known strong absorption value \( \pi R^2 \) is obtained, giving rise to the expected \( A^{2/3} \) dependence. In order to use expression (5), one needs to define an equivalent radius \( R \) that will properly represent the \( \bar{p} \)-nucleus interaction at the particular energy. For a density-folded \( \bar{p} \) optical
TABLE I. Measured $\bar{p}$ shifts ($\varepsilon$) and widths ($\Gamma$) in eV [15], and $\bar{p}$ annihilation cross sections (in mb) on $^3$He [20], $^4$He [2] ($p_L$ in MeV/c). The calculations [6] use potential (c).

|         | $\varepsilon_{2p}$ | $\Gamma_{2p}$ | $\sigma_{ann}$ (47.0) | $\sigma_{ann}$ (55.0) | $\sigma_{ann}$ (70.4) |
|---------|-------------------|---------------|----------------------|----------------------|----------------------|
| $^3$He  | calc.             | -12           | 33                   | —                    | 1038                 |
| $^3$He  | exp.              | $-17 \pm 4$   | $25 \pm 9$          | —                    | $1850 \pm 700$       |
| $^4$He  | calc.             | -19           | 42                   | 1116                 | —                    |
| $^4$He  | exp.              | $-18 \pm 2$   | $45 \pm 5$          | $979 \pm 145$        | $827 \pm 38$         |

potential which fits the atomic data for $A > 12$, the following parameterization [6] holds to a few percent for the whole range of very low energies up to $p_L \sim 100$ MeV/c:

$$R = \frac{7}{6}(1.840 + 1.120A^{1/3}) \text{ fm} \ .$$

Equations (5,6) reproduce explicitly, for antiprotons, the dependence of the calculated total reaction cross sections on the three relevant parameters ($Z, A$ and energy).

IV. THE ANTIPROTON - HELIUM SYSTEM

In this section I specialize to very light nuclear systems. Excluding the deuteron for which the optical model approach obviously is not well suited, the next targets to consider are the He isotopes [4]. The $(A - 1)/A$ factor in Eq. (2) is now included. Since the $2p$ and the $3d$ width data are incompatible with each other within an optical potential approach, as noted already in Ref. [15], the $3d$ width data were excluded from the fitting procedure, partly on the grounds that the $d$-wave contribution to the very low energy $\bar{p}$ annihilation cross sections is almost negligible compared to the dominant $s$ and $p$ waves (see below). The resulting density-folded optical potential, referred to as (c), has the following parameters: 1.8 fm for the range parameter of the two-body Gaussian interaction folded in with the He densities, and a strength parameter $b_0 = -0.26 + i2.07$ fm. Again, its real part plays only a minor role. The $2p$ level shifts and widths, as well as the $\bar{p}$ total annihilation cross sections calculated using the optical potential (c) are shown in Table I, where the measured values are also given, including a recent report of $\bar{p}$ annihilation on $^3$He [20].

Protasov et al. [21] have recently fitted the two low-energy $\bar{p} - ^4$He total annihilation cross sections listed in the table, using the scattering-length approximation expressions for $s$, $p$ and $d$ waves [22]. The input $p$- and $d$-wave Coulomb-modified scattering ‘lengths’ were derived using the Trueman formula [23] from the $2p$ shift and $2p$ and $3d$ widths. The $s$-wave Coulomb-modified scattering length $\tilde{a}_0$ was left as a fitting parameter, since the $\bar{p}$ atomic $1s$ level shift and width in He are not known experimentally. Assuming a value of $1.0 \pm 0.5$ fm for Re $\tilde{a}_0$, the annihilation cross sections were fitted by

$$\text{Im } \tilde{a}_0 = -0.36 \pm 0.03(\text{stat.})^{+0.19}_{-0.11}(\text{syst.}) \text{ fm} \ .$$

In contrast, the $\bar{p} - ^4$He potential (c), which is also fitted to essentially the same data set, yields numerically the following values:

$$\text{Re } \tilde{a}_0 = 1.851 \text{ fm} \ , \ \text{Im } \tilde{a}_0 = -0.630 \text{ fm} \ .$$
TABLE II. Composition of $\bar{p} - ^4\text{He}$ total annihilation cross section in mb

| $p_L = 57$ MeV/c | $l = 0$ | $l = 1$ | $l = 2$ | $l = 3$ | sum |
|------------------|--------|--------|--------|--------|-----|
| Protasov et al. [21] | 280.3 | 652.5 | 16.2 | 949    |     |
| potential (c) [3]   | 395.8 | 500.3 | 49.8  | 1.5    | 949 |
| experiment [3]       |        |        |       | 915 ± 32 |

It is clear that the model independence claimed by Protasov et al. [21] is violated by this specific example and, therefore, the determination of $a_0$ is not model independent. In order to study the origin of the above discrepancy, the partial wave contributions to the calculated cross section at $p_L = 57$ MeV/c are shown in Table [1]. The most important contributions are due to the $s$- and $p$ waves, for which the two calculations disagree badly with each other. In particular, the $p$-wave contributions differ from each other, although sharing practically the same value of the Coulomb-modified $p$-wave ‘scattering length’ (more traditionally called ‘scattering volume’) $\tilde{a}_1$. Whereas at $-20$ keV, for the $2p$ atomic state, the $l = 1 \bar{p} - ^4\text{He}$ dynamics is well determined by $\tilde{a}_1$ alone, over the energy range of $1 - 3$ MeV, corresponding to the annihilation measurements, it depends on more than just this ‘scattering length’. The effective range term, and perhaps higher order terms in the effective range expansion at low energies, become equally important. Indeed, it was verified for potential (c) that the variation of the Coulomb-modified $l = 1$ scattering phase shift is not reproduced in this energy range by specifying the ‘scattering length’ alone. Once the $p$-wave contributions to the total annihilation cross section differ by as much as is observed in the table, the $s$-wave contributions must also differ from each other. Therefore, the prediction for the Coulomb-modified $s$-wave scattering length using potential (c) is necessarily different from that of Ref. [21] which is based on an unjustified $p$-wave contribution. Such a difference would not occur for the $\bar{p}p$ system, which at the appropriate low energies is largely controlled by $s$ waves [22].

Regarding the behavior of the $\bar{p}$-nucleus $s$-wave scattering lengths as function of $A$, say for $A > 10$, the (plain strong-interaction) scattering lengths follow a simple geometrical picture, as borne out by a fit to $\bar{p}$-atom data [24] which has been recently updated [3]:

$$\text{Re } a_0 = (1.54 \pm 0.03)A^{0.311 \pm 0.005} \text{ fm} \quad \text{Im } a_0 = -1.00 \pm 0.04 \text{ fm}.$$  

Thus, $\text{Re } a_0$ varies roughly as the nuclear radius $R$, whereas $\text{Im } a_0$ is roughly constant over the periodic table. However, the Coulomb-modified scattering lengths do not show such a clear geometrical picture [3].

V. SUMMARY

In conclusion, it was shown that the recently reported annihilation cross sections for $\bar{p}$ on $^4\text{He}$ and $\text{Ne}$ at $57 \text{ MeV/c}$ are reproduced very well by strongly absorptive optical potentials which fit $\bar{p}$ atomic data. For these potentials, the asymptotic $ZA^{1/3}$ dependence of the total reaction cross section follows from the Coulomb focussing effect at very low energies. The full dependence on $E$, $Z$ and $A$ was derived using a Coulomb-modified strong absorption model.
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