Optical quantum memory with generalized timereversible atom–light interactions

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We examine a quantum memory scheme based on controllable dephasing of atom–light coherence of a non-resonant, inhomogeneously broadened Raman transition. We show that it generalizes the physical conditions for time-reversible interaction between light and atomic ensembles from weak to strong electric and from linear to non-linear interactions. We also develop a unified framework for different realizations exploiting either controlled reversible inhomogeneous broadening or atom–light frequency combs, and discuss new aspects related to storage and manipulation of quantum states.

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The study of the time-reversible evolution of physical systems was central in the development of thermodynamics and statistical mechanics [1], and CPT symmetry [2]. Furthermore, reversible interaction underpins reversible transfer of quantum states between light and atomic ensembles, i.e. quantum memory (QM), which is key for quantum repeaters [3] and all-optical quantum computing [4].

Approaches to QM exploit atomic states in cavities [5], non-resonant Raman transitions [6], electron–atomically induced transparency [7–10], electromagnetic coupling techniques [11–14], electron–phonon interactions [15], and photon–light–light interactions [16]. The latter is of particular interest in the present context, as the equations of motion include a hidden time-reversal symmetry [16]. However, there have so far only been discussed for weak electric fields. Here we generalize the physical conditions allowing for time-reversal symmetry in the mapping of quantum states between light and atomic ensembles to fields of arbitrary strength and non-linear interactions. The scheme exploits reversible dephasing of atomic coherence of a non-resonant, inhomogeneously broadened Raman transition (RAM an echo quantum memory (REQM)). It has been proposed in [22], further developed in [23,24], and most recently presented in [25]. We also compare the conditions for time-reversibility for storage of strong fields with those for weak fields, which naturally lead to a unified framework for realizations of REQM based on controlled reversible inhomogeneous broadening (CRIB) [18] or atomic frequency combs (AFC) [22]. Our findings show new light on time-reversibility in the interaction between light and inhomogeneously broadened atomic ensembles, and also pave the road to storage of macroscopic light fields in nano-sized atomic media. Furthermore, we discuss with the example of frequency conversion how REQM enables controlled manipulation of quantum light fields.

The scheme: Energy and temporal diagram s of the interaction scheme are depicted in Fig. 1. At time t = 0 the probe light field, $E_1(t)$ with duration $t_1$, carrier frequency $\omega_1$, and spectral width $W_1 = t_1^{-1}$ enters the medium with three-level atom (labeled by $j$) prepared in the long-lived level $j = 1$. Along the $+z$ direction, the atom is simultaneously exposed to an intense control (writing) field propagating along wavevector $k_1$ with carrier frequency $\omega_1$ and Rabi frequency $\Omega_1$ ($t_1$). It is reduced to zero after absorption of the probe field. The probe and writing fields are assumed to be in Raman resonance $|j = 1\rangle$ with sufficiently large spectral detuning $|s| = 1$, from the $1\rightarrow 3$ transition.

We take the 1$\rightarrow$2 and 1$\rightarrow$3 transitions to feature inhomogeneous broadenings (IB), resulting in a total dephasing of each atom $j$ from the center of the Raman transition given by $\Omega_1(t) = \Omega_{1\text{rot}} + \omega_1$, with $\Omega_{1\text{rot}} = \frac{1}{\sqrt{2}}\kappa_{\text{rot}} + \frac{1}{2}\kappa_{\text{cont}}$, $\kappa_j = (k = 2;3)$. The indices refer to total and natural dephasing, and possibly detuning induced in a controlled way using, e.g., external electric fields. We take the IB of the Raman transition to be large enough to absorb all spectral components of the probe field, and assume the natural IB on the 1$\rightarrow$2 transition to be negligible (as is usually the case for hyperfine ground states of rare-earth-ion doped crystals [26]). $f_1(t)$ is related to the Rabi frequency of the intense light field, $f_1(t) = f = \frac{\Omega_1}{\kappa}$, as well as to the Stark shift $\Omega_1$ induced by the control field, $\Omega_1(t) = \Omega_1^{-1}$. It can exceed unity by orders of magnitude.

The atom–light interaction leads to excitation of atomic coherence, which rapidly decays due to IB. To retrieve the stored field, we apply a phase (or mode) matching operation, and launch at time $t > t_1$ a second control (reading) pulse propagating roughly in $z$ direction, and with wave vector, carrier and Rabi frequencies $K_2$, $\omega_2$ and $\Omega_2$ ($\Omega_2$), respectively. (See Fig. 1.) In addition, we either actively invert the Raman an broadening, as in CRIB [18] (now RECRI B) or, in the case of a generalization of AFC [22] (now REAFC), simply wait until the atom–light coherence automatically rephases. The probe field, at frequency $\Omega_2$, $\Omega_2$, $\Omega_2$, will then be re-emitted at time $t_1$ as an echo in the backward ($-z$) direction. In the following we will limit our discussion to the case where the spectral width and duration of the echo are equal to the ones of the probe field: $\Omega_1(t) = \Omega_1(t_1) = \Omega_1$.

We emphasize that the memory’s storage bandwidth, which depends on the IB of the Raman transition, does not only rely on material properties but also on the con-
trol eld via $\delta \omega$. A lo, the direct Ram an transfer allows us using atom ic m onials with short oherence time $\tau$.[27,28]. These two features result in a larger choice of m onials compared to photon-echo quantum memories with direct Ram an transfer.

Basic equations: We describe the interaction between the three-level atom s, probe and echo eils $E_j(z)$ = $\lambda_j(z)expf(z)$g, and control eils $\sim (\omega;x)$ = $\omega\exp(1)(\omega';x)g$ using the Hamiltonian $H = H_a + H_f + \hat{V}_a + \hat{V}_c$. $H_a$ labels storage and recall, and the indices $a;f;e$ and $c$ denote the Hamiltonian for the atom s, quantum light eils, and interaction between quantum and classical light eils and atom s, resp.: 

$$H_a = \sum_{j=1}^{\infty} (\sum_{n=1}^{\infty} E_j(z)F_j(z))$$

$$H_f = \sum_{j=1}^{\infty} \sum_{n=1}^{\infty} (z)F_j(z))$$

$$\hat{V}_a = \sum_{j=1}^{\infty} \sum_{n=1}^{\infty} (\lambda_j(z)expf(z)g)$$

$$\hat{V}_c = \sum_{j=1}^{\infty} \sum_{n=1}^{\infty} (\lambda_j(z)expf(z)g)$$

$\lambda_j(z)$ are the refractive indexes and group velocities for the probe and echo eils in the absence of interaction with the three-level atom s, and $g$ is the photon-atom coupling constant.

In the following we use the Heisenberg picture and derive the equations of motion for the slowly varying operators for the light eils $E_j(z)$ and atom ic coherences ($R_{m,j}$) between states $m$ and $n$: 

$$\dot{A}_j(z) = \frac{1}{\lambda_j(z)expf(z)g}) - \frac{1}{\lambda_j(z)expf(z)g})$$

where $\lambda_j(z) = \lambda_j(z)expf(z)g)$, $E_j(z) = \sqrt{j} \lambda_j(z)expf(z)g)$, $\lambda_j(z) = \lambda_j(z)expf(z)g)$, and $\lambda_j(z) = \lambda_j(z)expf(z)g)$.

We also assume $j = \frac{1}{2}$ and $\frac{1}{2}$, and $\frac{1}{2}$. Being an obvious condition for the e-reversibility, we ignore all atom ic decay as well as irreversible atom ic dephasing, and assume that the probe eld is completely absorbed in the atom ic eils. To simplify the expressions, we use a coordinate system moving with the probe, or echo eils, resp. For absorption ($t < t_0$, $1$), we use $\omega = \omega + v$, and for retrieval ($t > t_0$, $2$), we have $\omega = \omega + v$, where $Z = Z$. Finally, we use new quantum eils $E_j(z) = (1)(\lambda_j(z)expf(z)g)$, and after adiabatic elimination of the excited atom ic coherences $R_{13,j}(t) = \frac{1}{\lambda_j(z)expf(z)g}$, and $R_{32,j}(t) = \frac{1}{\lambda_j(z)expf(z)g}$, we nd:

$$\frac{\partial}{\partial \lambda_j(z)expf(z)g}) = \frac{1}{\lambda_j(z)expf(z)g})$$

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where $\lambda_j(z)expf(z)g)$ and $\lambda_j(z)expf(z)g)$ with atom ic density $n_0$, and cross section of the probe (echo) eils $S$. $B = \gamma d_{12}d_{31}$, $G \gamma d_{12}$, $G \gamma d_{12}$, and $G \gamma d_{12}$ describe the B of the m onial transition. All variables in Eqs. [4] depend on $\omega$ and $Z$. We emphasize that these equations hold for arbitrary numbers of atom s and photons. Compared to the usual Maxwell-Bloch equations [6], they contain additional non-linear term s, i.e. Stark shifts in the evolution of the atom ic coherence $R_{13,j}$, and a population dependent term in the evolution of the light eils $E_j$. Their analytic solution and analysis for the e-reversal symmetry has not been considered before.

General reversibility of quantum dynam cs: We now show that Eqs. [4] allow for the e-reversal evolution (i.e. storage and retrieval of the probe eil) despite the non-linear term s. Indeed, the equations for retrieval coincide with the equations for absorption for the e-reversed echo emission (i.e. $2$ $\omega$ and $\omega$) if the following strong eil conditions of reversibility are satisfied:

$$\lambda_j(z)expf(z)g) = \lambda_j(z)expf(z)g)$$

It is found from $R_{12,j}(t) = \lambda_j(z)expf(z)g)$, where $\omega$ denotes a m onial after complete probe absorption, and by expressing $R_{12,j}$ and $R_{32,j}$ through $F_{12}$. Is a
phase factor that contributes to the global phase of the echo signal. Note the absence of a similar equality for the light-echo operators, which is due to complete absorption of the probe echo.

ii) $1 = 2$ and $f_1(1) = f_2(0, 2)$, i.e., equal coupling between the atom:ic coherence of the Raman transition and the probe and echo fields, respectively, and $j_1(1)j_2(0, 2)$ i.e., temporal reversibility of the Rabi frequencies of the writing and reading echoes (see Fig. 1).

$$j_{R \text{rot} 02} + j_{R \text{rot} 01} = 0,$$ i.e., rephasing of atomic coherence when reversing the IB, similar to CRIB [2]. The last equality sign requires meeting conditions (ii) and (iv).

iv) $2 = 1$, and $1_{2} = 1_{3} + 1_{4}$, i.e., anti-correlated spectral detunings of the light fields. This condition completely determines $1_{2}$ for a given $1_{3}$, and $1_{2} = 1_{3} + 1_{4}$, where we used that $1_{3} = 1_{4}$.

We note that the time-reversibility hidden in Eqs. [1] results in a temporally reversed replica of arbitrary probe echoes in the echo field, which is equivalent to the standard, linearized system of Maxwell-Bloch equations [16, 18]. Thus any quantum state encoded into the input light field can be stored and recalled with unit efficiency and fidelity, despite the nonlinear interactions. At the same time, the deterministic reversibility enables new possibilities for the realization of optical quantum memories, e.g., storage of macroscopic light fields in nanoscale memories where the usual weak echo approximation may no longer be valid.

Storage of weak probe echoes: It is interesting to consider the previous conditions for time-reversibility with the conditions in the case of weak probe echoes, where $R_{11}^1 = 1$, and $R_{22}^2 = 0$. In the following we also assume that the Stark shift due to the presence of the probe and echo fields is so small compared to the spectral width of the stored light: $\frac{\Delta \omega}{\Delta \omega_{\text{IB}}} < 1$, where $\Delta \omega$ denotes the expectation value, $j_{R \text{rot} 12} = j_{R \text{rot} 12}$, and we use new variables $\left( \frac{1}{2} R_{12}^2, \frac{1}{2} R_{12}^2 \right)$ expf(1) $\frac{1}{2} R_{12}^2$ in Eqs. [1], where $\expf(1)$ is a modified exponential function.

\[ \frac{1}{2} R_{12}^2 = \frac{1}{2} R_{12}^2 + \frac{1}{2} R_{12}^2 \quad (1) \]

(2)

where the centers of the Raman transitions are shifted by $f(1)$ (see Fig. 1). For a probe echo with symmetric shape in time, and a Raman broadening that is symmetric in frequency, these linearized equations describe again a time-reversed evolution of the atom-light system, but this time under weak echo conditions of reversibility [30]:

iv) $c(R_{1}^1 R_{2}^2) = c_{1}[1 + n_{1} + c(1 + 1 = 2)]$, which is found from $R_{12}^1 R_{12}^2 = \expf i \phi R_{12}^1 R_{12}^2$, as explained in (i). Note that this generalized mode matching condition coincides with (i) if conditions (ii) and (iv) are met. Yet, in the case of weak probe echoes, these requirements are relaxed, e.g., due to the lack of the probe (echo) induced Stark shift. In particular, this allows for continuous frequency conversion of the echo compared to the probe. Assuming resonance with the center of the IB, Raman transition and constant Stark shifts, we find $1_{1} = 1_{1} + 1_{2}$, and $1_{2} = 1_{1} + 1_{2}$, similar to (ii), but without the necessity to individually equalize each variable.

v) $f_1(1) = 2f_2(0, 2)$, similar to (ii), but without the necessity to individually equalize each variable.

Thus any quantum state encoded into the input light field can be stored and recalled with unit efficiency and fidelity, despite the nonlinear interactions. At the same time, the deterministic reversibility enables new possibilities for the realization of optical quantum memories, e.g., storage of macroscopic light fields in nanoscale memories where the usual weak echo approximation may no longer be valid.
the two protocols decrease.

Conclusion: We have shown that REQM generalized time-reversibility in photon-echo quantum memory storage of strong light fields and non-linear interactions. Forthomre one must use AFC and CRIB in an extension to optical resonant Raman transitions, and allows using atoms in materials with short optical coherence times. It also allows exploiting additional degrees of freedom such as wave vectors, and carrier and Rabi frequencies of the control fields. This leads to the possibility to hence Raman in IB, i.e. allow larger storage bandwidth, and to a large variety of manipulations of light fields, e.g. frequency conversion and generalized quantum com pression. Note that CRIBM [25] and AFC [23] have recently been demonstrated in atomic vapor and rare-earth-ion doped crystals, resp., hence REQM can be explored with present technology.

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FIG. 2: Quantum efficiency for single mode storage as a function of $x = \frac{C R E C R I B}{C R E A F C}$ for resonant optical depths $\phi = 50; 200; 1000$ (bottom to top sets of two traces).
try in the probe field shape is not required in the case of $k = 0$. It is included here for a unified presentation.

[31] S. A. Moiseev and W. Titel, (in preparation).