Fractional quantization of ballistic conductance in 1D hole systems

M. Rosenau da Costa,1 I.A. Shelykh,1,2 and N.T. Bagraev3

1International Center for Condensed Matter Physics, Universidade de Brasilia, Caixa Postal 04667, 70910-900, Brasilia-DF, Brazil
2St. Petersburg State Polytechnical University, 195251, St. Petersburg, Russia
3A.F. Ioffe Physico-Technical Institute of RAS, 194021, St. Petersburg, Russia

We analyze the fractional quantization of the ballistic conductance associated with the light and heavy holes bands in Si, Ge and GaAs systems. It is shown that the formation of the localized hole state in the region of the quantum point contact connecting two quasi-1D hole leads modifies drastically the conductance pattern. Exchange interaction between localized and propagating holes results in the fractional quantization of the ballistic conductance different from those in electronic systems. The value of the conductance at the additional plateaux depends on the offset between the bands of the light and heavy holes, \( \Delta \), and the sign of the exchange interaction constant. For \( \Delta = 0 \) and ferromagnetic exchange interaction, we observe additional plateaux around the values 7\( e^2/4h \), 3\( e^2/4h \) and 15\( e^2/4h \), while antiferromagnetic interaction plateaux are formed around \( e^2/4h \), \( e^2/4h \) and \( 9e^2/4h \). For large \( \Delta \), the single plateau is formed at \( e^2/h \).

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The progress in nanotechnology allowed the fabrication of the quasi-one dimensional mesoscopic components in which the transport of the carriers has a ballistic character and is not accompanied by the Joule losses. The conductance through these objects, \( G \), is proportional to the number of the open 1D propagation modes, \( N \), which can be modulated by the application of the perpendicular gate voltage, \( V_g \). Tuning \( V_g \) one observes ballistic conductance staircase with plateaux at integer multiples of the conductance quantum \( G_0 = 2e^2/h \), where the factor 2 reflects the spin degeneration of the bands. This result can be understood in frameworks of the model of non-interacting electrons.

However, in the region of small carrier concentrations, an additional plateau is observed near 0.7\( G_0 \). In external magnetic field it evolves smoothly into the spin-polarized conductance plateau at 0.5\( G_0 \)\(^{11,12}\) thus indicating that "0.7 feature" can be connected with spin. Later experiments have shown a zero-bias peak (ZBP) in the differential conductance and a peculiar temperature dependence of "0.7 feature"\(^2\), which led to the proposal that it could be a Kondo-type phenomenon connected with dynamical spin polarization. However, more recent experiments demonstrated the absence of any ZBP at least for some experimental configurations\(^3\) thus contradicting the Kondo interpretation and supporting the models based on the static spin polarization of the electrons in the region of the quantum point contact (QPC) leading to the appearance of the spin gap\(^8,10,11\). The formation of the spin polarized state in quantum wires and quantum point contacts was predicted theoretically\(^12,13,14\) and obtained additional support from recent experimental studies\(^8,15\).

Within the model of the spontaneous spin polarization of the contact the appearence of the "0.7 feature" can be explained as follows. Suppose that QPC contains a single localized electron which affects the propagating carriers via exchange interaction. Since the latter is defined by the mutual orientation of their spins, the transmission coefficient through the QPC with a magnetic moment is spin-dependent. Besides, if the triplet state energy is lower than the singlet state one, the potential barrier formed by the QPC region for the carrier in the singlet configuration is higher than for the triplet state. Therefore, at small concentration of carriers, the ingoing electron in the triplet configuration passes the QPC freely, while the carriers in the singlet configuration are reflected, thereby defining the principal contribution of the triplet pairs to the total conductivity. In zero magnetic fields the probability of the realization of the triplet configuration is equal to 3/4 against 1/4 for the singlet one, and thus the QPC conductance in the regime considered reads \( G = 3/4G_0 \)\(^16\). On the contrary, if the singlet configuration is energetically preferable, the conductance should be equal to \( G = 1/4G_0 \). The application of the external magnetic field leads to the spin polarization of both the propagating and localised carriers thus giving rise to the conductance value equal to \( G = 1/2G_0 \) in accordance with experimental data. If more than one unpaired electrons is localized on the QPC, the value of the conductance on the additional plateau decreases\(^12\), which can explain the evolution of "0.7 feature" into "0.5 feature" with the increase of the length of the QPC\(^18\).

In this work we analyze the ballistic conductance associated with the holes bands in Si, Ge and GaAs quasi 1D systems. We consider the transmission of propagating hole states facing effective potential barriers, generated by a spin-dependent interaction with a localized hole, supposedly, present in the region of the QPC\(^19\). As it was mentioned above for electrons this model qualitatively reproduces the major characteristics observed in the 0.7 experiments\(^10\). Due to the difference of spin structure for electrons and holes one can expect that the effect of exchange interaction on ballistic conductance will be qualitatively different in these two cases.

The valence bands of bulk Si, Ge and GaAs present two branches: the heavy hole band, with spins \( J_{zh}^h = \pm 3/2 \), and the light hole band, with spins \( J_{zh}^l = \pm 1/2 \). In quasi-1D systems due to the effects of confinement the energetic splitting \( \Delta \) appears between these two bands, which depends on the width of the quantum wire, the difference in the effective masses of the light and heavy holes, \( m_l \) and \( m_h \), and strains. In our further consideration we will take account of only the lowest bands of the light and heavy holes\(^21\). We suppose also that the propagating and localized holes interact only in the region of the QPC having the length \( L \). The Hamiltonian of the system...
can be thus cast in the form:

\[
H = \begin{cases}
\frac{\hbar^2 k^2}{2m_{hh, in}} + V_{dir}, & x < 0, x > L \\
\frac{\hbar^2 k^2}{2m_{hh, in}} + V_{dir} + V_{ex} J_p J_l, & x \in [0, L]
\end{cases}
\] (1)

where the indices \( p \) and \( l \) correspond to the propagating and localized holes respectively, \( V_{dir} > 0 \) is the matrix element of the direct interaction and \( V_{ex} \) is the matrix element of the exchange interaction (\( V_{ex} < 0 \) for ferromagnetic interacting and \( V_{ex} > 0 \) for antiferromagnetic interaction). It should be noted that many-body correlations of the Kondo type can lead to the temperature-dependent renormalization of the exchange interactions.

The general expression for the conductance of the system at zero temperature is given by

\[
G(E_F) = \frac{N e^2}{h} \sum \alpha_{m_p}(E_F) \alpha_{m_l}(E_F) \times [A(E_F)_{m_p,m_l \rightarrow m_p',m_l'}] \delta_{m_p,m_p'} \delta_{m_l,m_l'},
\] (4)

where \( E_{hh, lh} = \frac{\hbar^2 k^2}{2m_{hh, lh}} + V_{dir} \).

The general expression for the conductance of the system at zero temperature is given by

\[
H^{(\pm 2)} = \begin{pmatrix}
E_{hh}^0 + \frac{3}{4} V_{ex} + \Delta & \frac{3}{4} V_{ex} \\
\frac{3}{4} V_{ex} & E_{hh}^0 + \frac{3}{4} V_{ex} + \Delta
\end{pmatrix}, \quad H^{(\pm 1)} =
\begin{pmatrix}
E_{hh}^0 + \frac{3}{4} V_{ex} + \Delta & V_{ex} \sqrt{3} \\
V_{ex} \sqrt{3} & E_{hh}^0 + \frac{3}{4} V_{ex} + 2 \Delta \quad 0 \\
0 & V_{ex} \sqrt{3} \\
0 & E_{hh}^0 + \frac{3}{4} V_{ex} + 2 \Delta \quad 0
\end{pmatrix},
\]

\[
H^{(0)} =
\begin{pmatrix}
E_{hh}^0 + \frac{9}{4} V_{ex} \\
\frac{3}{4} V_{ex} + 2 \Delta & E_{hh}^0 + \frac{9}{4} V_{ex} \\
0 & 0 \\
0 & 0
\end{pmatrix}, \quad H^{(\pm 3)} = E_{hh}^0 + \frac{9}{4} V_{ex},
\] (3)

where \( E_{hh, lh} = \frac{\hbar^2 k^2}{2m_{hh, lh}} + V_{dir} \).

The transmission amplitudes in this case can be determined from the procedure, which we show for instance when the initial spin configuration of the propagating and localized holes corresponds to the state \( I \rangle \pm \frac{1}{2}, + \frac{1}{2} \rangle \). The wavefunction of the propagating hole reads

\[
\Psi_I = \begin{pmatrix}
1 \\
0
\end{pmatrix} e^{i k_{hh} x} + B \begin{pmatrix}
\frac{1}{2} + \frac{1}{2} \\
\frac{1}{2} - \frac{1}{2}
\end{pmatrix}, \quad \Psi_{II} = \begin{pmatrix}
1 \\
0
\end{pmatrix} e^{-i k_{hh} x}, \quad x < 0,
\]

\[
\Psi_{II} = \begin{pmatrix}
X_1 \left( C_1 e^{i k_{hh} x} + C_1 e^{-i k_{hh} x} \right) + X_2 \left( C_2 e^{i k_{hh} x} + C_2 e^{-i k_{hh} x} \right), \quad x \in [0, L],
\end{pmatrix}
\]

\[
\Psi_{III} = \begin{pmatrix}
0 \\
1
\end{pmatrix} A \begin{pmatrix}
\frac{1}{2} + \frac{1}{2} \\
\frac{1}{2} - \frac{1}{2}
\end{pmatrix}, \quad \Psi_{III} = \begin{pmatrix}
0 \\
1
\end{pmatrix} A \begin{pmatrix}
\frac{1}{2} + \frac{1}{2} \\
\frac{1}{2} - \frac{1}{2}
\end{pmatrix}, \quad x > L,
\]

where

\[
\begin{pmatrix}
1 \\
0
\end{pmatrix} = \begin{pmatrix}
\frac{3}{2} \\
1
\end{pmatrix}, \quad \begin{pmatrix}
0 \\
1
\end{pmatrix} = \begin{pmatrix}
\frac{1}{2} \\
\frac{1}{2}
\end{pmatrix}, \quad k_{hh, lh} = \sqrt{\frac{2m_{hh, lh} E_F}{\hbar^2}},
\]

\( X_{1,2} \) are the eigenvectors of the matrix \( H^{(\pm 2)} \) and the wavevectors \( k_{1,2} \) are determined self consistently in such a way that the eigenenergies, \( e^{(\pm 2)}(k_{1,2}) \), correspond to the total energy of the system, \( E_F + \Delta \) (once that in the considered case we initially have a propagating heavy hole, with energy \( E_F \), and a localized light hole, with energy \( \Delta \)). The transmission and reflection amplitudes, \( A \) and \( B \) are found from the continuity conditions at \( x = 0, x = L \). Analogous procedures must be
Figure 1: Steps of the quantum conductance staircase vs the chemical potential of carriers in a Si QPC for the ferromagnetic case. We considered the standard values of the Si light and heavy holes effective masses: \( m_{lh} = 0.16 m_e \), \( m_{hh} = 0.49 m_e \). The different lines correspond to three different values of \( \Delta (\text{meV}) \): 0, 0.45, 10. The direct and exchange interaction are estimated as \( V_{ex} \approx -0.5 \text{meV} \) and \( V_{dir} \approx 1 \text{meV} \), the length of the contact as \( L = 50 \mu m \) and we considered the temperature \( T = 0.5 \text{K} \). The vertical gray lines correspond to the values of the heights of the effective potential barriers, \( V_{dir} - 3V_{ex}/4 \), \( V_{dir} - 11V_{ex}/4 \) and \( V_{dir} - 15V_{ex}/4 \), whereas the dashed horizontal lines correspond to the values \( 7e^2/4h \), \( 3e^2/h \) and \( 15e^2/4h \). The inset shows a band structure of the 1D holes.

Figure 2: Steps of the quantum conductance staircase vs the chemical potential of carriers in a Si QPC for the antiferromagnetic case. We used the same parameters of Fig.(1), only changing for an antiferromagnetic exchange interaction, \( V_{ex} \approx 0.5 \text{meV} \), and supposing \( V_{dir} \approx 2 \text{meV} \). The vertical gray lines correspond to the values of the heights of the effective potential barriers, \( V_{dir} - 15V_{ex}/4 \), \( V_{dir} - 11V_{ex}/4 \), \( V_{dir} - 3V_{ex}/4 \) and \( V_{dir} + 9V_{ex}/4 \), whereas the dashed horizontal lines correspond to the values \( e^2/4h \), \( e^2/h \) and \( 9e^2/4h \).

is quantized in units of \( 4e^2/h \), one easily obtains the values of the fractional conductance of the interacting system mentioned above.

The situation is different if the offset between the bands of light and heavy holes is not negligible. This version is illustrated when \( \Delta \rightarrow \infty \) that should be realized in extremely narrow wires. In this case both localized and propagating carriers are heavy holes, and thus the conductance of non-interacting system is quantized in the units of \( 2e^2/h \). The spins of the pair of holes can be either parallel or antiparallel. Bearing in mind that in the absence of the external magnetic field both configurations have equal probability and spin-flip processes are blocked because of the large offset of the intermediate light hole states, one obtains just one additional plateau in the fractional conductance corresponding to \( G = e^2/h \). The situation is thus different from the case of electrons where spin-flip processes are allowed and additional plateau is formed at \( G = 3e^2/2h \).

When \( \Delta \) is finite, it is clearly that the situation is beyond the two considered extreme versions, as it is also illustrated in Figs.(1) and (2). Both probabilities of realization of initial states and transmission amplitudes are seen to be strongly energy dependent.

The quantization of the ballistic conductance associated with holes in Ge and GaAs structures is expected to be qualitatively the same as in Si because of the similarity of the spin structure of the valence band in these materials. However, it can be qualitatively different in IV-VI semiconductors such as PbTe, PbSe and PbS where the electron-hole symmetry holds.

In addition, we also analyzed the effects of an applied external magnetic field parallel to the structure growth axis which produces the Zeeman splitting of the light and heavy hole bands, \( H_{lh}^{th,hh} = -g_{lh}^{th,hh} \mu_B J_z B \), where \( \mu_B \) is the Bohr mag-
parameter $\Delta$ light and heavy hole subbands will be split with the energies of the bottom of the subbands, in addition to the $\Delta$ splitting between them, increases the energy dependence of the initial state’s probabilities of realization and of the scattering processes. Therefore, under a moderate magnetic field, the relative width of the steps in the quantum conductance tends to be reduced, increasing in number and changing in energy. This behavior is illustrated by the plotted conductance for $B = 1.5T$ in Fig.(3). However, if the external magnetic field is strong enough such that the heights of the spin effective potential barrier are smaller than the splitting between the different subbands, then the conductance of the system tends to show plateaux at the integer values of $e^2/h$.

In the case of $B = 3.0T$, in Fig.(3), the plateaux at $e^2/h$ and $3e^2/h$ are very clear, while the plateau at $2e^2/h$ is absent, because the splitting between the second and third subbands is smaller than the heights of the spin effective potential barriers.

In conclusion, we have considered the effect of the exchange interaction on the fractional quantization of the ballistic conductance of the hole systems. It results in the fractional quantization of the ballistic conductance different from those in electronic systems. The value of the conductance at the additional plateaux depends on the offset between the bands of the light and heavy holes $\Delta$, the sign of the exchange interaction on the fractional quantization of the ballistic conductance different from those in electronic systems.

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