Gravitational wave detection with single-laser atom interferometers

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Abstract We present a new general design approach of a broad-band detector of gravitational radiation that relies on two atom interferometers separated by a distance $L$. In this scheme, only one arm and one laser will be used for operating the two atom interferometers. We consider atoms in the atom interferometers not only as perfect inertial reference sensors, but also as highly stable clocks. Atomic coherence is intrinsically stable and can be many orders of magnitude more stable than a laser. The unique one-laser configuration allows us to then apply time-delay interferometry to the responses of the two atom interferometers, thereby canceling the laser phase fluctuations while preserving the gravitational wave signal in the resulting data set. Our approach appears very promising. We plan to investigate further its practicality and detailed sensitivity analysis.

Keywords Gravitational waves · Atom Interferometry · Time-Delay Interferometry

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1 Introduction

The detection of gravitational radiation is one of the most challenging efforts in physics in this century. A successful observation will not only represent a great triumph in
experimental physics, but will also provide a new observational tool for obtaining better and deeper understandings about its sources, as well as a unique test of the proposed relativistic theories of gravity [1].

Non-resonant detectors of gravitational radiation (with frequency content $0 < f < f_0$) have one or more arms with coherent trains of electromagnetic waves (of nominal frequency $\nu_0 \gg f_0$), or beams, and at points where these intersect, relative fluctuations of frequency or phase are measured (homodyne detection). Frequency fluctuations in a narrow Fourier band can alternatively be described as fluctuating sideband amplitudes. Interference of two or more beams, produced and monitored by a (nonlinear) device such as a photo detector, exhibits these sidebands as a low frequency signal again with frequency content $0 < f < f_0$. The observed low frequency signal is due to frequency variations of the sources of the beams about $\nu_0$ along their propagation paths, including motions of the sources and any mirrors (or amplifying microwave or optical transponders) that do any beam folding, to temporal variations of the index of refraction along the beams, and, according to general relativity, to any time-variable gravitational fields present, such as the transverse traceless metric curvature of a passing plane gravitational wave train. To observe these gravitational fields in this way, it is thus necessary to control, or monitor, all other sources of relative frequency fluctuations, and, in the data analysis, to optimally use algorithms based on the different characteristic interferometer responses to gravitational waves (the signal) and to other sources (the noise).

In present single-spacecraft microwave Doppler tracking observations, for instance, many of the noise sources can be either reduced or calibrated by implementing appropriate frequency links and by using specialized electronics, so the fundamental limitation is imposed by the frequency fluctuations inherent to the reference clock that controls the microwave system. Hydrogen maser clocks, currently used in Doppler tracking experiments, achieve their best performance at about 1000 seconds integration time, with a fractional frequency stability of a few parts in $10^{-16}$. This is the reason why these one-arm interferometers in space are most sensitive to millihertz gravitational waves. This integration time is also comparable to the microwave propagation (or “storage”) time $2L/c$ to spacecraft en route to the outer solar system ($L \approx 3AU$), so these one-arm, one-bounce, interferometers have near-optimum response to gravitational radiation, and a simple antenna pattern.

By comparing phases of split beams propagated along equal but non-parallel arms, common frequency fluctuations from the source of the beams can be removed directly and gravitational wave signals at levels many orders of magnitude lower can be detected. Especially for interferometers that use light generated by presently available lasers, which have frequency stability of roughly a few parts in $10^{-12}/\sqrt{\text{Hz}}$ to $10^{-18}/\sqrt{\text{Hz}}$ (in the millihertz and kilohertz bands respectively) it is essential to be able to remove these fluctuations when searching for gravitational waves of dimensionless amplitude less than $10^{-20}$ in the millihertz band [2,3]. Combined with the fact that plane gravitational waves have a spin-two polarization symmetry, this implies that the customary right-angled Michelson configuration is optimal. The response to gravitational waves is then maximized in Earth-based systems by having many bounces in each arm [5,6].

The frequency band in which a ground-based interferometer can be made most sensitive to gravitational waves [1] ranges from about a few tens of Hertz to about a few kilohertz, with arm lengths ranging from a few tens of meters to a few kilometers. Space-based interferometers, such as the coherent microwave tracking of interplanetary
spacecraft and the proposed Laser Interferometer Space Antenna (LISA) mission are most sensitive to millihertz gravitational waves and have arm lengths ranging from $10^6$ to $10^8$ kilometers.

Recently, atom-wave interferometers have been proposed as potentially new gravitational waves detectors, as their technology has reached a high-level of maturity in providing extremely sensitive inertial sensors. The discussions on how to use atom interferometers as gravitational wave detectors have mainly focused on two fundamental types of proposed approaches. The first relies on exploiting the atom wave and the interferometers directly. In this scheme, atoms in an atom-wave interferometer correspond to photons in a Michelson laser interferometer, and the effects of a gravitational wave signal are measured by monitoring the phase (momentum) changes of the atoms due to the GW. Although this design has been shown of not providing any sensitivity improvements over optical interferometers, it stimulated more thoughts on the subject. The result was an alternative design in which atom interferometers (AI) are used as local inertial sensors and laser beams (which are used for operating the AIs) imprint on the atoms the phase fluctuations generated by a gravitational wave signal propagating across the detector. This approach is fundamentally the same as using AI for gravity gradient measurements, which has been pursued both on the ground and in space.

Along the line of using atoms as test masses in a gravitational wave detector, we present in this paper an alternative configuration in which atoms are treated as local free-falling clocks. Consider a detector configuration with two ensembles of atoms separated by a distance $L$, in which only a single laser beam is used to operate them. The laser interrogates the atoms similar to a local oscillator laser interacting with atoms in an optical clock. The results give the phase differences between the laser and the highly coherent atomic internal oscillations. As the laser phase fluctuations enter into the responses of the two phase difference measurements at times separated by the one-way-light-time, $L$ (units in which the speed of light $c = 1$), we show that the laser phase fluctuations can be exactly canceled (while retaining the gravitational wave signal) by applying time-delay interferometry (TDI) to the phase measurement data.

The rest of the paper is organized as follows. In Section 2 we consider a detector configuration with two AIs separated by a distance $L$, in which only a laser beam is used to operate them. In Section 3 we turn to the problem of canceling the laser phase fluctuations by using the data from the two AIs. Since the laser phase noise enters into the responses of the two AIs at times separated by the one-way-light-time $L$ (units in which the speed of light $c = 1$), we show that it can exactly be canceled (while retaining the gravitational wave signal) by applying time-delay interferometry (TDI) to the AIs data. In Sections 4 we make some further considerations about how to enhance the sensitivity of these single-laser AIs by large-momentum-transfer techniques, while in Section 5 we present our conclusions and remarks on future prospects for this gravitational wave detector design.

## 2 Laser phase measurements with atom interferometry

The fundamental limitation of a one-arm Doppler measurement configuration (such as that of interplanetary spacecraft tracking experiments) is determined by the frequency stability of the “clock” that defines the frequency of the electromagnetic link. The most stable clocks are presently optical atomic clocks. Optical clocks have already
Fig. 1 A laser of nominal frequency $\nu_0$ shines on two AIs, separated by a distance $L$, at space-times events $(z_1, t_1)$ and $(z_2, t_2)$.

shown stabilities of about $10^{-17}$ over 1000 sec integration time \[14\]. This is accomplished by frequency-locking a highly stabilized laser to an atomic transition as an ideal passive frequency standard. The intrinsic atomic coherence is only limited by its natural lifetime. External perturbations cause additional frequency fluctuations, which may be controlled to a level of $10^{-18}$ and lower \[15,16\].

These considerations imply that we might use atoms directly as ideal local reference oscillators for gravitational wave detection. To illustrate this point, let’s assume to have two identical atomic systems that are free-falling. The two atomic systems, A and B, are placed $L$ light-seconds apart. A coherent laser is near the atomic system A, as shown in Fig. 1. Like in a conventional atomic clock, the laser excites the atomic systems through a Rabi (or Ramsey) interrogation method. In essence, the laser frequency and phase are compared to that of the atomic internal clock, i.e. the oscillation between two atomic energy levels of the atom. In Figure 1, the laser shines from the right-hand-side of our proposed experimental setup, its field interacts with the atomic system A at time $t_1$ (which has an interrogation time $T$), and then it reaches the atomic system B (that has the same interrogation time as A) at time $t_2$ ($L$ seconds later). This results in two phase differences containing the laser phase noise at space-times $(z_1, t_1)$ and $(z_2, t_2)$ and noises of the local atomic coherence over the interrogation time $T$. We will show in Section 3 that (i) the laser phase noise can be canceled exactly by applying TDI \[13\] to the data measured by the two AIs, and (ii) the resulting new data set retains sensitivity to the gravitational wave signal and is now limited by the stability of the passive atomic clocks, which can be several orders of magnitude smaller than that of the laser.

It is important to point out here that making a “stand-alone” atomic clock requires a local oscillator (typically a laser) whose stability then becomes the main limitation of the atomic standard stability over the interrogation time. Currently, the best approach is to use as local oscillator a laser that is frequency locked to a highly stabilized Fabry-Perot cavity. This approach however has already reached its fundamental limit due to thermal noise \[17\]. Our approach of coupling two passive atomic standards with the same laser completely circumvents the laser stability requirement. It is in fact easy to see that the laser phase noise cancellation is valid even in the large phase noise
3 Single-laser atom interferometers as free-falling test masses

One of the key requirements in interferometric gravitational wave experiments is for the local reference frames to be as much inertially free as possible. This is to reduce any non-gravitational forces and local gravitational disturbances that can cause changes in the laser phase. Ground-based interferometers achieve a high-level of seismic isolation of their mirrors by using either passive or active isolation systems. Space-based detectors instead, such as LISA, achieve inertial isolation by using highly sophisticated drag-free test masses. Although in principle one could trap atoms in such test masses, it is more practical to rely on laser-cooled atoms in ultra-high vacuum as alternative drag-free test masses and directly use them as reference sensors.

Although an ensemble of laser-cooled free atoms would still have a finite velocity distribution that would degrade the fringe contrast because of inhomogeneous Doppler dephasing, there exist techniques (such as spin echo, for instance) that allow one to compensate for inhomogeneous dephasing effect. This is in fact discussed in relation with Doppler-free Ramsey-Borde interferometers, which have been successfully used as optical clocks with free-falling atoms. Since this technique requires a pair of counter-propagating laser beams, it cannot be applied to our proposed design since the phase fluctuations from the two lasers cannot be canceled both at the same time.

To overcome this problem, recall that, in an atom interferometer with $\frac{\pi}{2} - \pi - \frac{\pi}{2}$ stimulated Raman pulse sequence, the resulting phase shift measured by the atom interferometer is equal to $\Delta \Phi = k_{eff} a T^2 + \phi(t_1) - 2\phi(t_2) + \phi(t_3)$, where $k_{eff}$ is the effective wave-number of the Raman lasers, $a$ is the acceleration of the atoms, and $\phi$ is the phase difference of the two Raman lasers. This result is independent of the individual atom velocities. All $\frac{\pi}{2} - \pi - \frac{\pi}{2}$ type atom interferometers considered so far use stimulated Raman transitions with a pair of counter-propagating laser beams. However, this lack of velocity independence is not due to the presence of counter-propagating beams, but results from the $\pi$ phase reversal taking place at the middle of the interferometer, quite similarly to spin echo in nuclear magnetic resonance. In fact, it can be shown that one can achieve a $\frac{\pi}{2} - \pi - \frac{\pi}{2}$ sequence in an atom interferometer with only one laser from one direction only, as sketched in Fig. 2. In this case, the two atomic levels involved will be an optical clock transition. The resulting phase of the optical transition based atom interferometer will be exactly the same as in stimulated Raman transitions: $\Delta \Phi = k a T^2 + \phi(t_1) - 2\phi(t_2) + \phi(t_3)$, where now $k$ is the wave number and $\phi$ is the phase of the laser.

For an ensemble of free-falling atoms the measured interferometer phase is equal to $\Delta \Phi = \phi(t_1) - 2\phi(t_1 + T) + \phi(t_1 + 2T)$. This expression indicates that such an atom interferometer can compare the phase of the laser against the atomic internal clock coherence. This phase difference measurement can then be used as part of our proposed gravitational wave detector design, as we mathematically show below.
3.1 The gravitational wave signal

Let us assume a plane polarized gravitational wave (GW) propagating across the experimental setup shown in Fig. 3. As a result it will leave an imprint on the phases of the light and the atoms, and appear as additional phase fluctuations at the output of the two AIs. The phase fluctuations induced by the GW on the light propagating along "the arm" \( L \) of our experimental setup represent the dominant contribution from the GW to the overall GW phase fluctuations measured by the atom interferometers \[12\]. This is because the characteristic size \( T \) of the AIs is much smaller than the arm-length \( L \). In what follows, therefore, we will disregard the effects of the wave on the phase of the atoms. This means that the main GW signal will appear at the output of AI B, and the GW signal at the output of AI A can be assumed to be null.

If we introduce a set of Cartesian orthogonal coordinates \((X, Y, Z)\) in which the wave is propagating along the \( Z \)-axis and \((X, Y)\) are two orthogonal axes in the plane of the wave (see Figure 3), then the frequency of the laser at time \( t \), \( \nu(t) \), at the beam splitter of AI B is related to the nominal frequency \( \nu_0 \) of the laser and the gravitational wave's amplitudes, \( h_+ (t) \) and \( h_\times (t) \), by the following relationship \[22,13\]:

\[
\frac{1}{2\pi\nu_0} \frac{d}{dt} \delta\phi_B(t) \equiv \frac{\nu(t) - \nu_0}{\nu_0} = \frac{1 - \frac{\mu}{2}}{2} [h(t - (1 + \mu)L) - h(t)] ,
\]

where \( h(t) \) is equal to

\[
h(t) = h_+ (t) \cos(2\phi) + h_\times (t) \sin(2\phi) ,
\]
Coherent laser light is transmitted along arm $L$ from AI A to AI B. The plane gravitational-wave signal is propagating along the $Z$ direction, and the $(X,Y)$ coordinates are defined in the plane of the wave. In these coordinates the direction of propagation of the laser light is described by the two polar angles $(\theta, \phi)$. We have denoted with $\delta \phi_h(t)$ the phase fluctuations of the light due to the GW. The wave’s two amplitudes are defined with respect to the $(X,Y)$ axis, $(\theta, \phi)$ are the polar angles describing the location of AI B with respect to the $(X,Y,Z)$ coordinates, and $\mu$ is equal to $\cos \theta$.

From Eq. (1) we deduce that gravitational wave pulses of duration longer than the one-way-light-time $L$ give a Doppler response that, to first order, tends to zero with the distance $L$. This system essentially acts as a pass-band device, in which the low-frequency limit $f_l$ is roughly equal to $(L)^{-1}$ Hz, and the high-frequency limit $f_H$ is set by the atom shot noise in the AIs.

If we assume the arm of the AIs to be equal to $T$, we find that the phase shifts due to the GW signal at the outputs of the AIs, $R_{1h}(t)$ and $R_{2h}(t)$, are equal to

$$R_{1h}(t) = 0,$$  
$$R_{2h}(t) = -[\delta \phi_h(t) - 2 \delta \phi_h(t-T) + \delta \phi_h(t-2T)].$$

The additional time-signature of the gravitational wave signal in the response $R_{2h}(t)$ follows from the fact that the light of the laser affects the atoms of AI B at times $t$, $t-T$, and $t-2T$; these are the instances when the phase fluctuations of the wave get imprinted on the phase of the atom.
3.2 The noise responses

Since the main goal of this paper is to show that it is possible to combine the data from the two AIs in such a way to exactly cancel the phase fluctuations of the laser and retain those from the GW, in this section we will focus on how the laser phase fluctuations, \( P(t) \), enter into the responses of the two AIs.

As an initial AI laser pulse excites an atom, it establishes the atomic internal oscillation and imprints its “instantaneous” phase onto the atomic oscillation. While the subsequent phase fluctuation \( P(t) \) of the laser is governed by the laser stability, the atomic coherence may be perturbed by external environment and can be characterized by equivalent phase fluctuations \( P_i(t), i = 1, 2 \) indicating atoms in different AIs. Subsequent AI laser pulses result in interference between the laser and the atomic coherence.

The responses of the AIs to the laser phase noise, \( P(t) \), and the phase fluctuations of the radiation emitted by an atom, \( P_i(t) \), can be written in the following form

\[
R_{1\text{rad}}(t) = [P_1(t) - 2 P_1(t - T) + P_1(t - 2T)] - [P(t) - 2 P(t - T) + P(t - 2T)] \quad (5)
\]

\[
R_{2\text{rad}}(t) = [P_2(t) - 2 P_2(t - T) + P_2(t - 2T)] - [P(t - L) - 2 P(t - T - L) + P(t - 2T - L)] \quad (6)
\]

Note that the phase fluctuations of the laser at time \( t \) at AI B were generated \( L \) seconds earlier, and for this reason they appear in \( R_{2\text{rad}}(t) \) time-delayed by \( L \).

By combining Eqs. (3, 4, 5, 6), and denoting with \( N_1, N_2 \) the phase fluctuations due to all other noises affecting the responses of the two AIs, \( R_1(t) \) and \( R_2(t) \), we get

\[
R_1(t) = [P_1(t) - 2 P_1(t - T) + P_1(t - 2T)] - [P(t) - 2 P(t - T) + P(t - 2T)] + N_1(t) \quad (7)
\]

\[
R_2(t) = -[\delta \phi_h(t) - 2 \delta \phi_h(t - T) + \delta \phi_h(t - 2T)] + [P_2(t) - 2 P_2(t - T) + P_2(t - 2T)] \\
- [P(t - L) - 2 P(t - T - L) + P(t - 2T - L)] + N_2(t) \quad (8)
\]

By noticing that the laser phase fluctuations enter into the response of AI B at a time-shifted by \( L \) seconds, it is easy to see that the following linear combination of the two responses \( R_1 \) and \( R_2 \) cancels them

\[
\Delta R(t) \equiv R_1(t - L) - R_2(t) = [\delta \phi_h(t) - 2 \delta \phi_h(t - T) + \delta \phi_h(t - 2T)] + [P_1(t - L) \\
- 2 P_1(t - T - L) + P_1(t - 2T - L)] - [P_2(t) - 2 P_2(t - T) + P_2(t - 2T)] \\
+ N_1(t - L) - N_2(t) \quad (9)
\]

If we now denote with \( \widetilde{\Delta R}(f) \) the Fourier transform of \( \Delta R(t) \), and substitute into it the Fourier transform of \( \delta \phi_h(t) \) from Eq. (6), we get the following expression of the laser noise-free combination \( \widetilde{\Delta R}(f) \) in terms of the remaining noises and the gravitational wave amplitude \( h \) (defined in Eq. 4) in terms of the wave’s two amplitudes, \( \tilde{h}_+(t) \) and \( \tilde{h}_x(t) \)

\[
\widetilde{\Delta R}(f) = \left[ \frac{\mu_0}{fT} \left( \frac{1 - \mu}{2} \right) \tilde{h}(f) \left[ e^{2\pi ifL} - 1 \right] + \tilde{P}_1(f) e^{2\pi ifL} - \tilde{P}_2(f) \right] \\
\times \left[ 1 - e^{2\pi ifT} \right] \left[ e^{2\pi ifL} - \tilde{N}_1(f) \right] - \left[ e^{2\pi ifL} - \tilde{N}_2(f) \right] \quad (10)
\]

Note that the resulting one-arm interferometric response to a GW signal, given in Eq. (10), is much simpler than that of an unequal-arm laser interferometer [13].
4 Discussion

We have shown that atoms in their free-falling state can be used in one-way precision laser interferometry experiments. This is possible because (i) atomic internal state oscillations can be used as highly stable clocks for laser phase comparison measurements and, (ii) by relying on our “one-way” configuration, the laser phase fluctuations can be exactly canceled by applying TDI to the data measured by the atom interferometers.

It is instructive to look at the sensitivity of our measurement scheme for gravitational wave detection, since now the fundamental limitation becomes the phase measurement resolution of the atom interferometers. In its simplest operational configuration, the phase measurement resolution is determined by the quantum projection noise of atoms, commonly known as the atom number shot noise. If \( N \) is the total number of atoms in an AI, the resulting phase noise measurement is equal to \( \frac{1}{\sqrt{N}} \). If we follow the rather optimistic assumption of operating our AIs with \( 10^8 \) atoms/s [12], the resulting measurement (shot) noise would be \( \approx 10^{-4} \text{rad}/\sqrt{\text{Hz}} \). Of course this is not as small as the shot noise at the photo detector of an optical interferometer (in the case of LISA, for instance, this is \( \approx 10^{-6} \text{rad}/\sqrt{\text{Hz}} \)) simply because the typical number of photons impinging on a photo detector is easily larger than the number of atoms in an AI.

In order to compensate for the low number of atoms, AIs with large momentum transfer (LMT) have been proposed [12,24]. In LMT schemes, an equivalent of multiple photon transitions between the same pair of atomic energy levels is performed, resulting in perhaps as much as 1000-photon momentum transfer [12]. This increases the effective laser wave number to \( k_{\text{eff}} \equiv Nk \) (or \( \lambda_{\text{eff}} = \frac{\lambda}{N} \)), quite analogous to a many-bounce laser interferometer. The measurement sensitivity is increased without a higher phase resolution. However, in order to accumulate the photon momentum transfer to atoms, counter-propagating laser beams are used, which would unfortunately break our one-way laser noise cancellation scheme. In order to improve the AI sensitivity by implementing some alternative LMT scheme (and still preserve the ability of canceling the laser phase noise) one would have to consider high-order multi-photon transitions in an atomic or even nuclear system characterized by large energy separations. The feasibility of such implementation remains to be investigated.

5 Conclusion

The idea of using atoms as proof masses for a gravitational wave detector is certainly very interesting, and deserves further studies. We have presented the design concept of a one-way laser interferometer detector of gravitational radiation that treats atoms both as proof-masses as well as ultra-stable clocks. Our discussion has in fact focused on the perspective of using the atomic internal oscillations as ideal local reference clocks. The salient feature of the one-way phase measurement configuration we have discussed is the exact cancellation of the laser phase noise by applying TDI to the AIs data. Although the detection sensitivity of our detector design might be limited by the available signal to noise ratio due to the atom shot noise, moderate sensitivity gains might be obtainable by using multi-photon transition. We plan to further study in a forthcoming article this problem together with a detailed noise analysis of the detector design presented here.
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