Exact Ground State Properties of Disordered Ising-Systems

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Exact ground states are calculated with an integer optimization algorithm for two and three dimensional site-diluted Ising antiferromagnets in a field (DAFF) and random field Ising ferromagnets (RFIM), the latter with Gaussian- and bimodal-distributed random fields. We investigate the structure and the size-distribution of the domains of the ground state and compare it to earlier results from Monte Carlo simulations for finite temperature.

Although DAFF and RFIM are thought to be in the same universality class we found essential differences between these systems as far as the domain properties are concerned. For the DAFF the ground states consist of fractal domains with a broad size distribution that can be described by a power law with exponential cut-off. For the RFIM the limiting case of the size distribution and structure of the domains for strong random fields is the size distribution and structure of the clusters of the percolation problem with a field dependent lower cut-off. Consequently, the domains are fractal and in three dimensions nearly all spins belong to two large infinite domains of up- and down spins - the system is in a two-domain state. The fractal dimensions for the DAFF and the RFIM agree. The DAFF ground state properties agree with results from MC simulation in the whole whereas there are essential differences between our exact ground states calculations and earlier MC simulations for the RFIM.

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I. INTRODUCTION

Many aspects of the influence of random field disorder on a system of interacting spins are still not well understood (for a review see \[1\]). Since it has been argued that the diluted Ising antiferromagnet in a homogenous field (DAFF) and the random field Ising ferromagnet (RFIM) are in the same universality class \[2\] experimental investigations focus on the DAFF \[4\] as an experimental realization of a system with random field disorder while theorists usually focus on the RFIM \[5\].

However, there have been investigations on the domain structures of the RFIM \[6\] and the DAFF \[7,8\], both based on MC simulations which suggest that there might be essential differences between DAFF and RFIM, at least as far as the domain structure is concerned. In the limit of strong disorder for the RFIM the domains are fractal in three and compact in two dimensions while the domains of the DAFF have a fractal structure both in two and three dimensions. Additionally, due to the fractality of the domains essential deviations from Imry-Ma-type \[9\] behavior which is thought to be valid for small disorder have been found.

In order to prove how far the results from MC simulations which are out of equilibrium for these systems with frozen dynamics \[10\] can be transferred to the equilibrium $T = 0$ case we performed exact ground state calculations on DAFF and RFIM with an integer optimization algorithm. With this method we get exact information on equilibrium properties at zero temperature. We investigate the distribution of domain sizes and the structure of the domains and we compare our findings for the DAFF and the RFIM. Hopefully, the results may help to understand the critical behavior of these systems as well as dynamical aspects \[11\].

The Hamiltonian of the DAFF in units of the nearest neighbor coupling constant $J$ is

$$H = \sum_{\langle ij \rangle} \epsilon_i \sigma_i \sigma_j - B \sum_i \epsilon_i \sigma_i$$

with the uniform field $B > 0$ on all sites of the quadratic $L \times L$ respectively cubic $L \times L \times L$ lattice. Here $\epsilon_i = \pm 1$ denotes Ising spins and a fraction $p$ of the sites is occupied with a spin (quenched disorder: $\epsilon_i = 0, 1$). The phase diagram of the 2d-DAFF consists of an antiferromagnetic low temperature phase for magnetic field $B = 0$ and a disordered phase for all finite values of $B$. In three dimensions there exists a long-range ordered phase \[12,13\] for magnetic fields $B < B_c$ smaller than the critical fields $B_c$ which is $B_c \approx 1.4$ for a dilution of $p = 0.5$ \[1\]. For higher fields and low temperatures the DAFF develops a frozen disordered domain state in two and three dimensions. This domain state has many of the characteristics of a spin glass, as for instance a remanent magnetisation and an irreversibility line scaling like the deAlmeida-Thouless line \[10\].

The Hamiltonian of the RFIM in units of the nearest neighbor coupling constant $J$ is

$$H = - \sum_{\langle ij \rangle} \sigma_i \sigma_j - \sum_i B_i \sigma_i.$$  

Here, all sites are occupied and the random fields $B_i$ are taken from a Gaussian (Gaussian-RFIM) respectively bimodal ($\pm \Delta$-RFIM) probability distribution corresponding to...
\[ P(B_i) = \frac{1}{\sqrt{2\pi}\Delta} e^{-(B_i/\sqrt{2}\Delta)^2} \]  \hspace{1cm} (3)

and

\[ P(B_i) = \frac{1}{2}(\delta(B_i - \Delta) + \delta(B_i + \Delta)), \]  \hspace{1cm} (4)

respectively.

As in the case of the DAFF the phase diagram of the 2d system consists of a long-range ordered low temperature phase for zero random fields and a disordered phase for all finite values of the random field \[ \Delta \]. In three dimensions for weak random fields and low temperatures there exists a long-range ordered phase \[ \Delta \]. The critical value for the random field at which long-range order disappears was found to be \( \Delta_c = 2.35 \) at zero temperature \[ \Delta \]. Even for the RFIM a possible glassy transition has been discussed \[ \Delta \].

We calculate exact ground states using an optimization algorithm well known in graph-theory. The Ising-system is mapped on an equivalent transport network, and the maximum flow is calculated using the Ford-Fulkerson algorithm \[ \Delta \]. In order to investigate the distribution and the structure of domains we perform a cluster analysis with a suitable adjusted Hoshen-Kopelman type algorithm \[ \Delta \]. This algorithm pieces the system into domains, where a domain is defined as a group of spins which are connected and antiferromagnetically (DAFF) respectively ferromagnetically (RFIM) ordered.

The investigations for the DAFF are carried out for systems of size \( 400 \times 400 \) with an average over 50 different dilution configurations with \( p = 0.7 \) for each value of \( B \).

For some structural aspects even ground states of size up to \( 700 \times 700 \) were calculated. Systems of size \( 50 \times 50 \times 50 \) averaged over 40 dilution configurations with \( p = 0.5 \) are used for \( d = 3 \). The calculation of ground states for the RFIM is more time consuming. Here we considered \( 300 \times 300 \) systems with 50 different random field configurations in 2d and 40 systems of size \( 30 \times 30 \times 30 \) in 3d.

II. DISTRIBUTION OF DOMAIN-SIZES

A. DAFF

In two dimensions for \( B > 0 \) no long-range order exists and the ground state of the system consists of antiferromagnetic domains of finite size. Within a ground state for each domain the domain-wall energy \( E_w \), i.e. the number of broken bonds, must be smaller than the field-energy \( Bm_D \), where \( m_D \) is the absolute value of the magnetisation of the domain which is not zero in these antiferromagnetic domains due to the dilution. Therefore, only domains with \( m_D \geq E_w/|B| \) exist in the system. The minimum wall energy of a domain is one broken bond, \( E_w = 1 \). Additionally, it is always \( V_D \geq m_D \), where \( V_D \) is the size of the domain (number of spins), so that with \( V_D \geq 1/|B| \) we get a rigorous lower cut-off for possible domain sizes. This relation indicates that except from the isolated spin-clusters only large domains can be realized for very weak fields. Due to this finite size effect we consider here fields \( B \geq 0.5 \) only and hence, all domain sizes \( V_D \geq 3 \) exist.

As has been shown earlier \[ \Delta \] for the two dimensional case the distribution of domains in a DAFF is well described by a power law with an exponential cut-off,

\[ N_D(V_D) \sim V_D^{-\delta} \exp(-V_D/V_0). \]  \hspace{1cm} (5)

where \( V_0 \) depends strongly on the field. Because of the inaccuracy in the determination of \( V_0 \) no significant \( p \)-dependence can be observed for this quantity. We increased the numerical effort in order to investigate a possible concentration and field dependence of \( \delta \). We find \( \delta \) weakly decreasing for increasing field but no \( p \)-dependence. The results are summarized in Tab. \[ \Delta \].

Fig. \[ \Delta \] shows the size-distribution of the domain state of the three dimensional DAFF for different fields. For the largest value of the field a reasonable fit to Eq. \[ \Delta \] can be carried out yielding \( \delta = 1.8 \pm 0.3 \) and \( V_0 = 50 \pm 20 \). However, a detailed investigation of the field or concentration dependence of \( V_0 \) and \( \delta \) is not possible with satisfactory precision for the following reasons: As has been shown earlier \[ \Delta \] for \( B \approx B_c \) the domain state consists mainly of two large interpenetrating domains (for \( B = 4 \) these domains contain approximately 80% of the spins).

In Fig. \[ \Delta \] the two isolated peaks in the size distribution corresponding to the two percolating domains are not shown but due to the existence of these two infinite domains there are only few data for domains of finite size so that the statistics for these domains is rather bad. In the limit of very high fields the domains become very small. For \( B > 6 \) all spins are polarized by the field, i.e. all domains have size 1. Hence, there is only a small region of values of the magnetic field where a broad distribution of domain sizes exist. As we will show in the next subsection the existence of two infinite domains has even more dramatic consequences for the RFIM. Here, there are no values of random fields where relevant domains of finite size can exist.

B. RFIM

In the limit of strong random fields \( \Delta > 2d \ (\pm\Delta \text{-RFIM}) \) respectively \( \Delta \to \infty \) (Gauss-RFIM) all spins follow the direction of the random field. Hence, the ferromagnetic domains are the clusters of sites with uniform random field signs. In this limit the domains of the RFIM correspond to clusters of the percolation problem \[ \Delta \] with a concentration of 50%.

The distribution of domain-sizes for different \( \Delta \) is shown in Fig. \[ \Delta \] for the two dimensional \( \pm\Delta \)-RFIM. For comparison the size-distribution for the percolation problem with a concentration of 50% is also shown. With increasing \( \Delta \), the domain-size distribution approaches the
cluster-size distribution of the percolation problem which follows Eq. (5) with $\delta = 1.55 \pm 0.05$ and $V_0 = 120 \pm 20$.

For lower random fields there is a striking difference between the Gaussian-RFIM on the one hand and the $\pm \Delta$-RFIM on the other hand: for the latter system there is a lower cut-off for the possible domain-sizes depending on the strength of the random field $\Delta$. This cut-off can be seen in Fig. 3 and it can be determined analytically: Similar to the arguments given for the DAFF for each domain the relation $E_w < E_B = \sum_{i=1}^{V} \sigma_i B_i$ must hold. The positive domain-wall energy has to be compensated by the negative field energy. In the most favorable case all random fields acting on the spins of a domain have the same sign. $E_B = V_D \Delta$ so that for the smallest possible domains we get $V_D \geq E_w / \Delta$. With this relation as starting point a lower cut-off can be determined as the sizes of those domains which have a minimum surface. For large domain sizes the minimum surface of a domain is the surface of a circle (2d), $E_w \sim V_D^{1/2}$, respectively sphere (3d), $E_w \sim V_D^{2/3}$. For smaller sizes, the domain with the minimum surface $V_{min}$ can be determined numerically.

Fig. 4 shows this dependence of $V_{min}$ from $\Delta$ for the $\pm \Delta$-RFIM in two dimensions. In addition, the minimum domain-volumes of the ground states of the RFIM for different $\Delta$ are depicted. For $\Delta \leq 1$ there is a strong deviation of the observed domains of minimum size from the theoretical curve. Obviously, this is due to the fact that for larger domains it is less probable that all random fields within a domain with minimum surface have the same orientation. Hence, either the field energy $E_B$ is overestimated or the shapes of the domains are more complicated than those with minimum surface. The first possibility would correspond to a crossover to a behavior following the Imry-Ma-argument [8], where $E_B$ is thought to scale with the root of the domain size due to simple statistical fluctuations of random fields within a domain. However, in the range of fields where this crossover may occur the minimum domain size is larger than the system sizes that we can investigate with numerical methods.

Fig. 5 shows the volumes of the two largest domains in the ground state of the $\pm \Delta$ and the Gaussian-RFIM. For lower fields $\Delta < 0.8$ the order parameter is finite. Obviously, this is the region where finite size effects lead to wrong results for the system sizes we investigated since in this regime minimum domain sizes are larger than the system. Therefore, in the following we consider only results for $\Delta > 0.8$.

An additional, striking feature of Fig. 3 is the jump in the data for the $\pm \Delta$-RFIM at $\Delta = 2$. Considering Fig. 4 (right) one realizes $\Delta = 2$ as a special value: The random fields are marked with $+$ and $-$ signs and the arrows represent the spin-orientations ($\uparrow$-spins are oriented parallel to a positive $\Delta$). The energy of the center spin with $\sigma = \pm 1$ is

$$E_\sigma = 2\sigma - \sigma \Delta.$$  \hspace{1cm} (6)

In the ground state it is $\sigma = +1$ for $\Delta > 2$ and the bold drawn domain wall is realized. $\Delta < 2$ favors $\sigma = -1$ and the center spin belongs to the down-domain. Generally, for $\Delta > 2$ the domain wall arranges in such a way that the sign of the random fields changes at each position of a domain wall, i.e. the domain walls run along the surfaces of the random field clusters (clusters of sites with uniform random field sign). Fig. 5 (right) demonstrates that nevertheless even for $\Delta > 2$ domains larger than these random field clusters can exist. A cluster with a magnetisation that is oriented antiparallel to the random fields is enclosed such that its surface lies within the domain. Hence, for $\Delta > 4$ each domain is a random field cluster while for $2 < \Delta < 4$ each domain wall is a part of the surfaces of the random field clusters. Consequently, the domains in this range of $\Delta$ are larger. In three dimensions each spin has six next neighbors. Here, the corresponding effect occurs at $\Delta = 4$.

Both, for $d = 2$ and $d = 3$ the RFIM approaches the percolation problem with a concentration of 50% for large $\Delta$, but while for $d = 2$ in this case there exists no percolating cluster of random fields in three dimensions 50% is above the percolation threshold. This means that as mentioned above for the case of the DAFF in three dimensions one will always find two infinite interpenetrating domains within the disordered phase whose antiparallel oriented magnetisations cause the order-parameter to vanish. The normalized volumes of the largest and second-largest domain for the $d = 3 \pm \Delta$-RFIM are shown in Fig. 6. Adding the sizes of these two largest domains we found that more than 97% of the spins of the system belong to these two domains in the whole range of random fields. In contrast to $d = 2$ there is no broad range of domain-sizes for $d = 3$ and in contrast to the DAFF there is no range of fields where a relevant number of domains of finite size exist. For this reason it is senseless to analyze the distribution of the domains in more detail.

III. DOMAIN STRUCTURES

A. DAFF

In a DAFF domains are composed in such a way that as many spins as possible are oriented parallel to the applied field with a minimum number of broken bonds. Hence, domain walls favorably run along vacancies to minimize the wall energy. These two competing requirements for minimizing the total energy lead to highly non-trivial structures. Fig. 8 shows the antiferromagnetic domains in the ground state. Here, the configuration of occupied sites and vacancies is kept fixed, while the external field $B$ is tuned. In order to investigate the structural properties of the domains we consider the following quantities and corresponding scaling relations:

- volume $V_D$: number of spins
• surface $S_D$: sum of all unsatisfied bonds (broken or "open" due to vacancies) with $S_D \sim V_D^{1/D_{sf}}$

• radius of gyration $R_D$, i. e. root of mean squared distance of all spins of a domain with $V_D \sim R_D^{D}$

• absolute value of the magnetisation of a domain $m_D$ with $m_D \sim V_D^{\theta}$

• wall energy $E_W$, sum of all broken bonds with $E_W \sim V_D^\theta$

For large domains the scaling relations above hold and the exponents are determined by analyzing all domains in the ground states and taking an average of the quantities above for all domains of equal size $V_D$. In Fig. 1 we showed an example for the two dimensional case. In the meantime we investigated the three dimensional system also. Our resulting exponents are summarized in Tab. 3 and compared to the values resulting from MC simulations [4].

The domains are fractal with fractal dimension $D_v = 1.65$ for $d = 2$ and $D_v = 2.14$ for $d = 3$. This fractality is also reflected in the proportionality of surface and volume ($D_{sf} \approx 1$). A great part of the surface is inside the domains. Slight deviations of exponents from exact ground states from those resulting from MC simulation are due to a systematic inaccuracy which is not reflected by the error-bars representing the fluctuations of the data only. Probably, a small, systematic error is due to the fact that the data are slightly curved in a log-log plot so that the linear range for large domains can hardly be determined (see also the corresponding Fig. 1 for the RFIM). From comparison with the corresponding Figures 3, 4 it is found that in spite of similar exponents the values $m_d(V_D)$ are larger for exact ground states - the domains of the ground states are obviously better optimized.

In good agreement with MC simulations is $\theta \approx 1$ both in two and three dimensions. Altogether the fractal structures for strong disorder lead to deviations from Imry-Ma-type arguments [4] based on compact domains. Here, a statistically distribution of vacancies in a domain would yield $\theta \approx 1/2$ which can clearly be ruled out by our data for the range of fields that we investigated. A field dependence of any of the exponents could not be found.

\section*{B. RFIM}

As we showed in the previous section there is no broad distribution of domains for the three dimensional RFIM - domains of finite size are irrelevant for the ground state. Therefore, in this subsection we restrict ourselves to the case $d = 2$.

Fig. 8 shows the ferromagnetic domain configuration of a $\pm\Delta$-RFIM in the ground state. The configuration of signs of the random fields is kept fixed, while the value $\Delta$ of the field is tuned. As for the DAFF we study the structural quantities volume, surface and radius of the domains. Since there is no dilution the surface and the domain wall energy are identical. Also, the magnetisation and the volume of the domains are identical. We study the following quantities:

• local random field fluctuations $b_D$, which is the absolute value of the sum of the random fields of a domain

• random field fluctuations of all sites of a domain along the domain wall inside the domain $b_{in}$

• random field fluctuations of all sites of a domain along the domain wall outside the domain $b_{out}$

Fig. 9 shows $b_{in}/S_D$ and $b_{out}/S_D$ each multiplied with the sign of the magnetisation of the considered domain for the $\pm\Delta$-RFIM. As we demonstrated with Fig. 5 for $\Delta > 2$ the domain walls run exactly along the boundaries of random field clusters. Hence, in this range of fields it is $|b_{in}/S_D| = |b_{out}/S_D| = 1$. Even for smaller fields the domain walls are presumably located along these random field surfaces and for large domains these fluctuations are proportional to the surface of the domains.

Corresponding to the DAFF the exponents $D_v$, $D_{vf}$ and $\theta$ with $b_D \sim V_D^\theta$ are determined from appropriate fits (Fig. 10). In Tab. 3 the exponents are summarized. The exponents resulting from MC are from [1].

The investigations are carried out for relative strong random fields, $\Delta = 3.01$ (Gaussian-RFIM) and $\Delta = 4.01$ ($\pm\Delta$-RFIM), because then - as explained in the previous chapter - a broad range of domain sizes exists. It is found that the exponents do not depend on the kind of random field distributions within the given precision.

In agreement with the findings for the DAFF the domains are fractal with nearly the same exponents $D_v \approx 1.6$ and $D_{sf} \approx 1$. Also, it is $\theta \approx 1$ indicating that the random fields are distributed non-statistically within a domain. These results are in contradiction to the earlier investigations using MC methods [4] where results were found corresponding to more compact domains. We interpret this discrepancy as being due to the fact that the MC simulations were carried out with $\Delta = 1$ and $T/k_B = 1$. For this value of $\Delta$ only very large domains can exist in the ground state. Therefore, the broad range of analyzed domains in [4] results obviously from thermal fluctuations. For larger fields and lower temperature we would expect agreement of these two methods.

Note that all exponents found for the fractal behavior of the domains of the RFIM as well as the DAFF are very close to those for the lattice animals of the percolation problem [10]. This is plausible since as mentioned before the domains of the RFIM in the limit of high random fields have the same distribution and structure like the clusters from the percolation problem. As we discussed in the previous subsection systematic inaccuracies are not reflected by the error-bars and systematic error can be due to a slight curvature of the data.
Our results deviate once more from the assumption of Imry-Ma-type random field fluctuations \( H \) for compact domain structures with \( \theta = 1/2 \) and \( b_{\text{in/out}} \sim L^{1/2} \). However, no \( \Delta \)-dependence for our exponents can be determined as indication for a crossover to Imry-Ma behavior within the range of fields that we investigated.

**IV. CONCLUSIONS**

We investigated the distribution and structure of the domains of the ground state of DAFF and RFIM in two and three dimensions. Since except for the Gaussian RFIM there is always a lower cut-off for the minimum domain size one is restricted to rather large disorder due to finite size effects, i. e. high dilution and fields of the order of the spin-spin coupling constant.

The domain-size distribution for the DAFF can be well described by a power law with a field dependent exponential cut-off for a broad range of the applied fields in 2d and for strong fields \( (B > 3) \) in 3d. For the RFIM with increasing random field strength there is a continuous transition to the cluster-size distribution of the percolation problem which also follows a power law with an exponential cut-off for both Gaussian- and bimodal distribution of the random fields. For the latter a non trivial minimum domain-size is found, that can be estimated based on energetic considerations.

Investigating structural properties we have found that the domains are fractal for the DAFF \( (d = 2, 3) \) as well as for the RFIM \( (d = 2) \) with the same fractal exponents. Surprisingly, the domain state of the RFIM in 3d is found to consist mainly \( (> 97\%) \) of two infinite interpenetrating domains of opposite phase in the whole range of random fields for which the long range order is broken. In this sense, the phase diagram of the RFIM at \( T = 0 \) consists only of two regions, a one-domain state (long-range order below the critical field) and this two-domain state.

While for the RFIM the results for the structural scaling exponents differ essentially from earlier findings at finite temperature carried out using MC simulations, our results are in good agreement with Monte Carlo simulations for the DAFF both in two and three dimensions.

The magnetisation of the domains and the distribution of random fields within the domains, respectively, strongly deviates from the assumptions of the Imry-Ma argument which however can still thought to be correct in the limit of small disorder. A more detailed investigation of the structure of domains for weak random fields is desirable. Unfortunately, for this the numerics exceeds current capacities.

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| $B$ | $\delta$ |
|-----|--------|
| 0.6 | 1.76 ± 0.06 |
| 0.9 | 1.76 ± 0.06 |
| 1.2 | 1.68 ± 0.04 |
| 1.4 | 1.61 ± 0.06 |
| 1.5 | 1.58 ± 0.05 |
| 2.2 | 1.44 ± 0.03 |
| 2.5 | 1.42 ± 0.06 |
| 2.8 | 1.37 ± 0.04 |
| 3.5 | 1.32 ± 0.05 |

TABLE I. Field dependence for the exponent $\delta$ of the domain-size distribution for the 2d DAFF

| $\theta$ | $\sigma$ | $D_v$ | $D_{vf}$ |
|--------|--------|------|--------|
| 1.00 ± 0.01 | 0.996 ± 0.001 | 0.97 ± 0.01 | 0.99 ± 0.01 |
| 0.98 ± 0.01 | 0.995 ± 0.001 | 0.96 ± 0.02 | 0.98 ± 0.03 |
| 1.001 ± 0.002 | 1.000 ± 0.001 | 1.00 ± 0.01 | 1.00 ± 0.01 |
| 1.56 ± 0.03 | 2.0 ± 0.1 | 1.64 ± 0.06 | 2.14 ± 0.08 |

TABLE II. Structural exponents for the DAFF

| $\pm \Delta$ | Gaussian | Exact Ground States | $\pm \Delta$ | Gaussian |
|--------|--------|--------|--------|--------|
| $D_v$ | - | - | 1.65 ± 0.08 | 1.67 ± 0.08 |
| $D_{vf}$ | 0.59 ± 0.04 | - | 0.98 ± 0.05 | 0.97 ± 0.05 |
| $\theta$ | 0.66 ± 0.04 | - | 1.00 ± 0.01 | 1.00 ± 0.01 |

TABLE III. Structural exponents for the 2d RFIM; $\Delta = 4.01$ for $\pm \Delta$- and $\Delta = 3.01$ for Gaussian-distributed random fields

FIG. 1. $N_D$ versus $V_D$ for the DAFF in three dimensions. For $B = 4.01$ a fit to Eq. $\mathcal{B}$ is also shown as solid line.

FIG. 2. Domain-size distributions for the two dimensional $\pm \Delta$-RFIM (Impulses) and size-distribution of the random field clusters (open symbols)

FIG. 3. Minimum domain size for given values $\Delta$ of the field of a $\pm \Delta$-RFIM in two dimensions

FIG. 4. Largest (open symbols) and second largest (filled symbols) domain size divided by the system size $L \times L$ for the $\pm \Delta$-RFIM in two dimensions

FIG. 5. $\pm \Delta$-RFIM; left: cut of a domain wall; right: a domain for $\Delta > 2$ consisting of multiple random field clusters

FIG. 6. Largest and second-largest domain size corresponding to Fig. $\mathcal{B}$ for the RFIM in three dimensions.

FIG. 7. Ground states of a $200 \times 200$ DAFF with a fixed dilution configuration for increasing fields $B = 0.8, 1.4, 2.2$ (from above). The two antiferromagnetic phases are represented black and white the vacancies are also black.

FIG. 8. Ground states of a $200 \times 200 \pm \Delta$-RFIM for a fixed configuration of random fields and $\Delta = 1.0, 1.1, 1.5$ (from above). The two ferromagnetic phases are represented black and white.

FIG. 9. Random field fluctuations $b_{out}$ and $b_{in}$ of the domain surfaces each multiplied with the sign of the domain-magnetisation and divided by the surface $S_D$ for the two dimensional $\pm \Delta$-RFIM.

FIG. 10. Figures for the determination of the structural exponents $\theta$, $D_v$ and $D_{vf}$ for the two dimensional $\pm \Delta$-RFIM.
Figure 1:
$\Delta=2.1$

$\Delta=2.5$

$\Delta=3.5$

Figure 2:
Figure 3:
Figure 4:
Figure 5:
Figure 6:
Figure 9:
Figure 10: