Light Pseudoscalar and Axial Spectroscopy using AdS/QCD Modified Soft Wall Model

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We study light pseudoscalar and axial states using a soft wall model that is modified by introducing two extra parameters: an UV-cutoff and an anomalous term in the conformal dimension of the fields involved such that meson fields with different parities are easily distinguishable. We find that 23 of these light mesonic states, including the scalars and vector states analyzed with one of our previous results, are fitted within the RMS error bounds given for this model, whose parameters are given by the quadratic dilaton profile, the UV locus $z_0$ and the anomalous term $\Delta P$. Our results show that Regge trajectories for pseudoscalar and axial mesons are linear in the radial quantum number $n$, as expected for these sort of regular $q\bar{q}$ mesons.

I. INTRODUCTION

Throughout the last twenty years, the AdS/CFT correspondence [1–3] has been used to study a wide range of non-perturbative phenomena with significant success; some examples are given by low-energy QCD vacuum properties such as the Quark-Gluon Plasma State, Confinement, Meson Spectra and Chiral Symmetry Breaking. In order to study all of them, two approaches can be employed: the first one, called Top-Down, takes an AdS-like gravity that is holographically equivalent in its conformal boundary to a low-energy QCD theory. The second one, also known as Bottom-Up, considers a four-dimensional QCD theory and looks for its holografical dual gravity theory.

Both of the approaches mentioned above are opposite paths that lead to noticeable results, as happens with Top-Down models and Quark-Gluon Plasma properties [4–6]. On the other hand, Bottom-Up approaches such as Soft-Wall models are very useful to describe hadron spectra, along with Chiral Symmetry Breaking effects, as we will describe below, but first of all, let us describe some properties of Chiral Symmetry, which is related to the impossibility of distinguishing between left and right projections of massless fermions.

At zero temperature, and considering an unbroken chiral symmetry, both the QCD vacuum and its action are invariant under chiral symmetry group transformations. If this symmetry is broken as a consequence of either a nonzero quark condensate or by the introduction of quark masses, a set of light scalar/vector and pseudoscalar/axial mesons are generated [7]. The importance of chiral symmetry breaking relies on the possibility of distinguishing states with even and odd parity numbers. This nonperturbative QCD phenomenon can be described via three different mechanisms [8]: Dynamical Symmetry Breaking, Spontaneous Symmetry Breaking and Explicit Symmetry Breaking.

The last two mechanisms lately mentioned are present both in Low-Energy-QCD Effective Field Theories and Models and in AdS/QCD nonconformal approaches, such as the Hard-Wall [9] and Soft-Wall model [10]. These models are based on the breaking of conformal symmetry either by imposing a D-Brane (Hard-Wall) or by introducing a static quadratic dilaton field (Soft-Wall). Both of these approaches can be also obtained through Top-Down models such as those given in [11, 12].

Dilaton potentials in graviton-dilaton models are directly related with confinement. In order to consider chiral symmetry breaking effects, meson multiplets and an auxiliary scalar field are to be introduced; such scalar field contains all the information of the the scalar quark condensate, which is the most appropriate order parameter for studying chiral symmetry effects. In this way, a gluon-scalar condensate vacuum background term can be obtained for the respective 5D action. This term contains a gluon condensate that describes Regge trajectories for $\pi$’s, $a_1$’s, $\rho$’s and $f_0$’s (when discarding either the $f_0(500)$ or the $f_0(980)$ state) since their slopes are proportional to this quantity.

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In cases where the auxiliary field acquires a nonzero vacuum expectation value, it is asserted that chiral symmetry $SU(N_f)_L \otimes SU(N_f)_R$ (with $N_f = 2, 3$) is spontaneously broken; this fact cannot be seen in the usual soft-wall model when taking the chiral limit since the quark condensate has a null contribution, something that can be fixed after adding nonlinear terms to the auxiliary field potential [13], whose quartic contribution allows to separate explicit and spontaneous chiral symmetry breaking effects and also to obtain a proper description of scalar, vector and axial resonances [13].

An alternative way of introducing chiral symmetry breaking effects considers a changeable mass parameter in the model that depends on the holographic coordinate $z$. In this case, there is no need to add nonlinear terms to the auxiliary field potential since this $z$-dependence introduces anomalous dimensions in the meson fields in such a way that their spectra can be properly described through Regge trajectories, as happens with the $f^0_i$’s (including the $f^0(500)$), $\rho$’s and $a_1$’s [10]. These sort of models allow to consider different solutions for the auxiliary scalar field, along with a set of proper parameters that are to be fixed; in any of those cases, the particle spectra are well described when compared with experimental bounds [17]. In these models, there is no need to deal with nonregular state mesons such as tetraquark and molecular mesonic states, and the dilaton profile is the usual quadratic $z$-dependent form. Besides, all of these Regge trajectories are linear in $n$, where $n$ is the radial quantum number, with their slopes given by the respective dilaton profile; these results, particularly the ones concerning $a_1$’s and $\rho$’s, coincide with the linear trajectories obtained in $(n, M^2)$ plots, with all of them having the same slope parameter [18] ($f_0$ scalar particles are also plotted as linear Regge trajectories here, however, the $f_0(500)$ state has to be discarded for not being a $q\bar{q}$ state). More complex dilaton profiles can be studied by introducing quartic coupling constants that depend on the $z$ coordinate, in such a way that $f^0_i$’s, $\pi$’s, $\rho$’s and $a_1$’s are well described [10].

Effective Models are useful tools that can be used to describe meson spectra and their masses, along with chiral symmetry breaking effects; one example is given by the Extended Linear Sigma Model where scalar, pseudoscalar, vector and axial mesons are introduced as multiplets of an $U(3)_L \otimes U(3)_R$ symmetry group, and their masses are analyzed via proper parameters such as strange/nonstrange quark condensates and coupling constants coming from the model itself. This model has to be fitted to a set of experimental quantities involved in meson decays; the results are quite remarkable (considering that radiative correction effects are not introduced), specially those regarding the masses of the $\eta$ and $a_1$ particles, whose errors are between 3-7% [20]. A similar treatment is considered to analyze the decay widths of $a_1$ mesons within an $U(2)_R \otimes U(2)_L$-symmetric extended Linear Sigma Model; in this case, the mass of this field (along with another physical quantities) is taken as an experimental input parameter to fit this decay so that it is to be compared with the PDG value given to that date [21].

The present work is organized as follows: in section II we present the holographic approach for a modified soft wall model that includes scalar, vector, pseudoscalar and axial mesons. We distinguish these states in a straightforward way by introducing an anomalous term in the conformal dimension of the respective meson, thus avoiding to consider any other auxiliary scalar field; in this way, their solutions hold with well-known stability criteria. Since our main focus is describing pseudoscalar and axial mesons, their associated quantities such as two-point functions and their respective pole structure are showed in section III. We present and discuss our main results in section IV, and we finally show our conclusions in section V.

II. HOLOGRAPHIC SETUP

We will follow the holographic prescription given in [22]. As a starting point, we will define the usual AdS Poincare patch with an extra UV cutoff as

$$dS^2 = g_{MN} \, dx^M \, dx^N = \frac{R^2}{z^2} \left[ dz^2 + \eta_{\mu\nu} \, dx^\mu \, dx^\nu \right] \Theta (z - z_0),$$

where $\Theta (x)$ is the Heaviside step function and $z_0$ is the UV cut-off. The associated action reads

$$I = I_{\text{Scalar}} + I_{\text{Vector}},$$

with

$$I_{\text{Scalar}} = -\frac{1}{2g_5^2} \int d^5x \sqrt{-g} \, e^{-\Phi(z)} \left[ g^{MN} \partial_M S \partial_N S + \tilde{M}_5^2 \, S^2 \right],$$

$$I_{\text{Vector}} = -\frac{1}{2g_V^2} \int d^5x \sqrt{-g} \, e^{-\Phi(z)} \left[ \frac{1}{2} F_{MN} F^{MN} + \tilde{M}_5^2 \, g^{MN} A_M A_N \right].$$
where $F_{MN} = \partial_M A_N - \partial_N A_M$ is the field strenght related to the U(1) field $A_M (z, x^\mu)$, the coupling $g_{S(V)}$ is a constant that fixes units on the scalar (vector) sector, and $M_5$ ($\tilde{M}_5$) is the bulk mass that fixes the hadronic identity for scalar (vector) states. For example, in the usual AdS/QCD approach, $M_5^2 R^2 = -3$ for scalar mesons and $\tilde{M}_5^2 R^2 = 0$ for vector mesons. $\Phi (z) = \kappa^2 z^2$ is the static quadratic dilaton profile.

These values for the bulk masses come from the holographic dictionary: the conformal dimension $\Delta$ of the bulk operators is fixed to be equal to $\Delta$, unless otherwise stated. We propose to introduce an extra anomalous dimension $\Delta_P$ modifying the bulk mass as

$$\Delta = \Delta_{\text{Phys}} + \Delta_P. \quad (5)$$

The idea of using anomalous dimensions in AdS/QCD models has been previously employed as a form to introduce changes in bulk masses (see for example in [16, 17, 24–28]). It is allowed even when bulk masses depend on the holographic coordinate $z$. Here we use it to introduce the parity information of the mesonic state by hand, making a different choice of $\Delta_P$ for light pseudoscalar mesons, and axial mesons, we will modify this idea by introducing anomalous dimensions.

We can infer the equations of motion for light pseudoscalar and axial mesons from the action (3). These mesons are equal to scalar and vector mesons except for their parity behavior: their quark content is similar to that present in ordinary scalar mesons (even states) are invariant under parity transformations, we can set $\Delta_P = 0$, thus obtaining the results exposed in [22]. Now it is necessary to fix the proper value of $\Delta_P$ for the $\eta$'s. In order to do so, we keep in mind the Breitenholner–Freedman limit. For scalar fields in AdS$_5$, we need to impose $M_5^2 R^2 \geq -4$ to generate stable solutions [29, 31]. With this constrain, a possible (and also suggestive) fixing for $\Delta_P$ could be $-1$, implying that for light pseudoscalar mesons the bulk mass should be shifted to $M_5^2 R^2 = -4$. This choice allows us to distinguish between even and odd parity states since chiral symmetry is broken and thus, mesons with different parities cannot be degenerated.

The equation of motion for the pseudoscalar mesons is obtained by doing variations on the action (4). After Fourier transforming and imposing the on-shell mass condition $q^2 = m_n^2$, we arrive to the following expression:

$$\partial_z \left[ e^{-\kappa^2 z^2} \partial_z S \right] + e^{-\kappa^2 z^2} q^2 S + 4 e^{-\Phi} S = 0. \quad (7)$$

According to the well-established holographic recipe for hadrons [10, 32–34], it is customary to obtain the mass spectrum from the pole expansion of the 2-point function constructed with the solutions of eq. (7). We will focus on this method hereafter.

A. Holographic pseudo-scalar mesons

We will focus on the $\eta$ orbital Regge trajectory for studying light pseudoscalar mesons, and the $a_1$ family for the axial vector sector. In order to introduce the holographic description of these particles, we will start from the definition (5), giving us the following expression for the bulk mass:

$$M_5^2 R^2 = (\Delta_{\text{Phys}} + \Delta_P) (\Delta_{\text{Phys}} + \Delta_P - 4). \quad (6)$$

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B. Holographic Axial Mesons

When dealing with axial mesons, the action is given by (4). The bulk mass for the vector field in AdS (including its anomalous dimension) reads

$$\tilde{M}_5^2 R^2 = (\Delta_{\text{Phys}} + \Delta_P)(\Delta_{\text{Phys}} + \Delta_P - 4) + 3.$$ \hfill (8)

For the vector case, we can follow a quantum mechanical analysis like the one performed in [35] to address the stability of the solutions. In our case the condition $\tilde{M}_5^2 R^2 \geq -1$ (placed by hand) will give rise to stable solutions. So, fixing $\Delta_P = -1$ fulfills this stability condition. A comment at this point: the $\Delta_P$ is expected to be independent of the spin character of the meson since it just deals with the parity of the mesonic state that is being considered.

Bearing this in mind, we can construct the equations of motion for the vector field $A_M$ associated to axial mesons as given below

$$\frac{1}{\sqrt{-g}} \partial_M \left[ \sqrt{-g} e^{-\kappa^2 z^2} g^{MR} g^{NP} F_{RP} \right] + e^{-\kappa^2 z^2} g^{MN} A_N = 0,$$ \hfill (9)

where we have used $\tilde{M}_5^2 R^2 = -1$.

As usual, we impose the gauge fixing $A_z = 0$ since no holographic information related to the $z$-coordinate should appear at the boundary $z_0$. Therefore, after Fourier transforming the equation, the vector transverse modes $A(z,q)$ are read from the equation

$$\partial_z \left[ \frac{e^{-\kappa^2 z^2}}{z} \partial_z A \right] + q^2 \frac{e^{-\kappa^2 z^2}}{z} A + \frac{e^{-\kappa^2 z^2}}{z^3} A = 0.$$ \hfill (10)

Following the standard holographic recipe, we will obtain from this equation the bulk to boundary propagator and use it to calculate the 2-point function. Mass spectrum comes from the pole expansion of this quantity. We will describe this procedure in the next section for both pseudoscalar and axial mesons.

Table I summarizes the $\Delta_P$ choice for each of the cases considered.

| Meson Identity   | $\Delta_P$ | $\tilde{M}_5^2 R^2$ |
|------------------|------------|---------------------|
| Scalar meson     | 0          | -3                  |
| Vector meson     | 0          | 0                   |
| Pseudoscalar meson| -1        | -4                  |
| Axial vector meson| -1        | -1                  |

Table I: This table summarizes the fixing of $\Delta_P$ and the value of $\tilde{M}_5^2 R^2$ on each case of interest. Notice that pseudoscalar and axial cases are allowed by the stability conditions given in [29–31, 35]. Scalar and vector cases were discussed in [22].

C. Bulk to boundary propagators

According to the holographic prescription [22], the non-normalizable modes generate the QFT operators; in our case, these modes create light pseudoscalar and axial mesons. They can be summarized in a single equation as

$$\partial_z \left[ e^{-B(z)} \partial_z \psi(z,q) \right] + q^2 e^{-B(z)} \psi(z) - \frac{\tilde{M}_5^2 R^2}{z^2} e^{-B(z)} \psi(z) = 0,$$ \hfill (11)

where $\psi(z,q)$ is a generic field. In the last equation we have done the following definition:

$$B(z) = \kappa^2 z^2 + \alpha \log z.$$ \hfill (12)

Fixing $\alpha = 3$ and $\tilde{M}_5^2 R^2 = -4$ produces the pseudoscalar case given in the expression (7). If we fix now $\alpha = 1$ and $\tilde{M}_5^2 R^2 = -1$, we have the axial equation (11).
The QFT operators that create mesons are given by the generic field $\psi$ as

$$\psi(z, q) = \tilde{\psi}_0(q) V(z, q) \ .$$

(13)

where $\tilde{\psi}_0$ is the Schwinger source (that can be scalar or vector) and $V(z, q)$ is the bulk to boundary propagator that satisfies the Dirichlet condition $V(z_0, q) = 1$ at the boundary locus $z_0$ [36] for the mesonic states considered so far.

Using these definitions, we arrive to an expression for the bulk to boundary propagator equation of motion given by

$$\partial_z \left[ e^{-B(z)} \partial_z V \right] + q^2 e^{-B(z)} V - \frac{M^2 R^2}{z^2} e^{-B(z)} V = 0. \quad (14)$$

After solving this equation for pseudoscalar and axial mesons, we obtain the respective pseudoscalar and axial bulk to boundary propagators in terms of Kummer confluent hypergeometric functions:

$$V_{\text{Pseudo}}(z, q) = \frac{z^2}{z_0^2} \frac{1}{F_1 \left( 1 - \frac{q^2}{4\kappa^2}, 1, \kappa^2 z_0^2 \right)} \ .$$

$$V_{\text{Axial}}(z, q) = \frac{z}{z_0} \frac{1}{F_1 \left( 1 - \frac{q^2}{4\kappa^2}, 1, \kappa^2 z_0^2 \right)} \ .$$

(15) and (16)

We will construct the pseudoscalar and axial 2-point functions in the next section using the solutions given by (15) and (16).

III. HOLOGRAPHIC 2-POINT FUNCTION AND POLE EXPANSION FOR PSEUDOSCALAR AND AXIAL MESONS

In general, the 2-point function for a given boundary operator is given by the second derivative of the on-shell boundary action [3, 23, 37, 38]. For our purposes, the on-shell boundary action for pseudoscalars and axial mesons can be summarized as

$$I_{\text{On-shell}} = \frac{1}{K} \int d^4x \sqrt{-\gamma} e^{-\kappa^2 z^2} F(\gamma) \tilde{\psi}_0(-q) \tilde{\psi}_0(q) g^{zz} V(-q, z) \partial_z V(q, z) \hat{n}_z \bigg|_{z_0} \ .$$

(17)

where $\gamma$ is the induced metric on the boundary at $z_0$ and $K$ is a constant that fixes units; for pseudoscalars is $g^2_S$ and for axial vectors is $g^2_A$. $F(\gamma)$ is a geometric function that gives the correct covariant structure with the sources. For the pseudoscalar case, $F = 1$, whilst for axial vector mesons $F = \gamma^{\mu\nu}$. $\hat{n}_z$ is a unitary vector normal to the boundary given by $\hat{n}_z = \frac{1}{\sqrt{g}} (1, 0, 0, 0, 0)$. The induced metric $\gamma$ is given by

$$dS^2 = \gamma_{\mu\nu} dx^\mu dx^\nu = \frac{R^2}{z^2} \left[ -dt^2 + d\vec{x}^2 \right] \bigg|_{z_0} \ .$$

(18)

The 2-point functions are calculated as

$$\Pi(q^2) = \frac{\delta^2 I_{\text{On-shell}}}{\delta \tilde{\psi}_0(-q) \delta \tilde{\psi}_0(q)} = \frac{1}{K} \sqrt{-\gamma} e^{-\kappa^2 z^2} F(\gamma) g^{zz} V(-q, z) \partial_z V(q, z) \hat{n}_z \bigg|_{z_0} \ .$$

(19)

Applying this machinery to the bulk to boundary propagators given in equations (15) and (16), we deduce the following 2-point functions:
\[ \Pi_{\text{Pseudo}}(q^2) = -\frac{R_3}{g_5^2} \frac{e^{-\kappa^2 z_0^2}}{z_0^3} \left[ \frac{2}{z_0} + \left( 1 - \frac{q^2}{4 \kappa^2} \right) \frac{2 \kappa^2 z_0}{1 F_1 \left( 1 - \frac{q^2}{4 \kappa^2}, 2, \kappa^2 z_0^2 \right)} \right] . \] (20)

\[ \Pi_{\text{Axial}}(q^2) = -\frac{R}{2 g_V^2} \frac{e^{-\kappa^2 z_0^2}}{z_0} \left[ \frac{1}{z_0} + \left( \frac{1}{2} - \frac{q^2}{4 \kappa^2} \right) \frac{2 \kappa^2 z_0}{1 F_1 \left( \frac{1}{2} - \frac{q^2}{4 \kappa^2}, 2, \kappa^2 z_0^2 \right)} \right] . \] (21)

In the last expression for axial vector mesons, we have dropped out the \( \eta_{\mu\nu} \) metric that appears by the derivation with respect to the vector sources.

From the expressions given for the 2-point functions for pseudoscalar (20) and axial mesons (21), we can obtain the respective mass spectra. It is worth to note that their structure depends solely on the zeroes of the hypergeometric functions \( _1F_1(1 - q^2/4\kappa^2, 1, \kappa^2 z_0^2) \) and \( _1F_1(1/2 - q^2/4\kappa^2, 1, \kappa^2 z_0^2) \).

### A. Pseudoscalar Mesons spectrum

The poles of 2-point function (20) give rise to the masses of the particles. These poles can be read from the roots of the Kummer confluent hypergeometric \( _1F_1(a, b, x) \) at the denominator as

\[ _1F_1 \left( 1 - \chi_n, 1, \kappa^2 z_0^2 \right) = 0, \] (22)

where \( \chi_n = M_n^2/4\kappa^2 \) is the root spectrum and \( M_n^2 = q^2 \) are the masses of the pseudoscalar mesons. Thus, the mass spectrum is given by

\[ M_n^2 = 4 \kappa^2 \chi_n (\kappa, z_0, \Delta P) . \] (23)

The result above assures that the mass spectrum above is a linear radial Regge trajectory defined by the parameters \( z_0, \kappa \) and \( \Delta P \). In general, the roots of the Kummer confluent hypergeometric function are increased with radial excitation number \( n \), so the masses also increase with \( n \), as we expected.

Notice that if we fix \( \Delta P = 0 \), we obtain \( M_n^2 R^2 = -3 \), thus giving the light scalar meson spectrum discussed in [22]. Results for the fitting of the \( \eta \) trajectory are summarized in table \[ \text{II} \].

### B. Axial Mesons

After making the same procedure as in the pseudoscalar case, we conclude that the poles of (21) hold with the following equality:

\[ _1F_1 \left( \frac{1}{2} - \chi_n, 1, \kappa^2 z_0^2 \right) = 0. \] (24)

When taking the on-shell limit \( \tilde{M}_n^2/4\kappa^2 = \tilde{\chi}_n \) in the equation above, we deduce that the axial mass spectrum is

\[ \tilde{M}_n^2 = 4 \kappa^2 \tilde{\chi}_n (\kappa, z_0, \Delta P) , \] (25)

where we have used \( \tilde{\chi}_n \) to denote the zeroes of the hypergeometric function involved here. If the limit \( M_n^2 R^2 = 0 \) is chosen, we obtain the vector meson case.

In the next section, we show our theoretical results for the pseudoscalar \( \eta \) and axial \( a_1 \) multiplets obtained from equations (23) and (25).
C. Connection with the soft wall model

The standard soft wall model is motivated by the idea of obtaining discrete linear spectra from a confining potential. From the phenomenological point of view, a linear mass spectrum is an indication of confinement on the model, thus, the choice of a quadratic dilaton $\Phi(z) = c^2 z^2$ effectively generates linear Regge trajectories. In the soft wall model case, the $c$ parameter is related to the Regge slope, which is connected to the nature of the strong interaction inside mesons. Therefore, it is expected to be the same for all the trajectories.

It is interesting to point out that, unlike the soft wall model case \[35\] , the mass spectrum is not given by integers but $1_F^1$ function zeroes. In the soft wall model, the most general mass spectrum is given by

$$M_n^2 = 4c^2 \left(n + \frac{1}{4} + \frac{\alpha}{4} + \frac{1}{4} \sqrt{1 + 4M_2^2/R^2 + 2\alpha + \alpha^2}\right),$$

where $c$ is the slope in the quadratic dilaton profile, $n$ is an integer starting from zero and $\alpha$ is related to the spin characteristic for the hadronic states, labeled by the bulk mass $M_2^2R^2$. For example, after fixing $M_2^2R^2 = -3$ and $\alpha = 3$ (scalars), we recover the usual scalar meson trajectory $M_n^2 = 4c^2(n+3/2)$ exposed in \[35, 39\]. If we fix $\alpha = 1$ (vectors) and $M_2^2R^2 = 0$ instead, we obtain the vector meson trajectory $M_n^2 = 4c^2(n+1)$ reported in \[10\].

In our case, we could try to apply the modified bulk mass to investigate the pseudo and axial mesons spectra. in the case of the $\eta$ trajectory, fixing $M_2^2R^2 = -4$ implies $M_n^2 = 4c^2(n+1)$. On the other hand, for the axial mesons $M_2^2R^2 = -1$ gives $M_n^2 = 4c^2(n+1/2)$. These mass spectra are summarized in tables IV and V.

IV. RESULTS

The spectra obtained for the light pseudoscalar and axial mesons are defined as the set of roots of the Kummer hypergeometrical function $1_F^1$ constructed from the parameters $z_0$ and $\kappa$ after fixing $\Delta P$ to be $-1$. At this point, one natural question arises: what would be the best choice for $\kappa$ and $z_0$?

Since we are dealing with the light pseudoscalar and axial sectors, it is expected that the choice of parameters used in the scalar model \[22\] could apply here. The main differences come on the parity behavior of the pseudoscalar mesons, as we commented before, and the fact that chiral symmetry is broken since the quark content has changed: now we are including bound states with two light $u$ and $d$ quarks, along with a nondegenerated and heavier $s$ quark.

As it was established in \[22\], $\kappa$ should be related to the quark content, and $z_0$ is connected to the nature of the strong interactions. Notice that compared to the soft wall model, the dilaton parameter $\kappa$ has a different meaning.

We will fix our parameters as

$$z_0 = 5.0 \text{ GeV}^{-1},$$
$$\kappa = 0.45 \text{ GeV},$$
$$\Delta P = -1 \text{ (for odd parity mesons).}$$

We choice to hold the same value for $\kappa$ as the one used in \[22\] since the mesonic states considered here have total strangeness equal to zero.

In the table III we present the numerical results for $\eta$ mesons obtained with the holographic description given above compared to the experimental data. It is important to notice that we take the first state in the radial trajectory as the $\eta$ instead of the $\eta'$ meson.

| $\eta$ mesons | $M_{\text{Exp}}$ (MeV) | $M_{\text{Th}}$ (MeV) | $\% M$ |
|---------------|-------------------------|------------------------|-------|
| $\eta(550)$   | $547.86 \pm 0.017$      | $975.25$               | 43.8  |
| $\eta(1295)$  | $1294 \pm 4$           | $1233.6$               | 4.90  |
| $\eta(1405)$  | $1408.8 \pm 1.8$       | $1455.3$               | 3.18  |
| $\eta(1475)$  | $1476 \pm 4$           | $1652.9$               | 10.65 |
| $\eta(1760)$  | $1760 \pm 11$          | $1829.2$               | 3.78  |
| $\eta(2225)$  | $2216 \pm 21$          | $1992.7$               | 11.3  |

Table II: Mass spectrum for $\eta$ pseudoscalar mesons with $\kappa = 0.45 \text{ GeV}$ and $z_0 = 5.0 \text{ GeV}^{-1}$. Experimental values are obtained from \[41\]. For the $\eta(1760)$ and $\eta(2225)$ states, their masses are taken from \[42, 43\].
Table III: Mass spectrum for $a_1$ axial mesons with $\kappa = 0.45$ GeV and $z_0 = 5.0$ GeV$^{-1}$. Experimental values are obtained from [41]. For the $a_1(1420)$ state, its mass is read from [44].

| $a_1$ mesons | $M_{\text{Exp}}$ (MeV) | $M_{\text{Th}}$ (MeV) | %M |
|--------------|-------------------------|-----------------------|-----|
| $a_1(1260)$  | $1230 \pm 40$          | 808.96                | 52.2|
| $a_1(1420)$  | $1414^{+15}_{-13}$    | 1114.7                | 26.9|
| $a_1(1640)$  | $1654 \pm 19$          | 1351.3                | 22.4|

Table IV: Mass spectrum for $\eta$ trajectory calculated from the soft wall model spectrum $M^2_{\eta} = 4c^2(n+1)$ with $c = 0.388$ GeV and $M^2 R^2 = -4$. Experimental values are obtained from [41]. For the $\eta(1760)$ and $\eta(2225)$ states, their masses are read from [42, 43].

| $\eta$ mesons | $M_{\text{Exp}}$ (MeV) | $M_{\text{SWM}}$ (MeV) | %M |
|---------------|-------------------------|------------------------|-----|
| $\eta(550)$   | $547.86 \pm 0.017$      | 800.0                  | 31.3|
| $\eta(1295)$  | $1294 \pm 4$            | 1131.7                 | 14.4|
| $\eta(1405)$  | $1408.8 \pm 1.8$        | 1385.6                 | 1.61 |
| $\eta(1475)$  | $1476 \pm 4$            | 1600.0                 | 7.75 |
| $\eta(1760)$  | $1760 \pm 11$           | 1788.8                 | 1.61 |
| $\eta(2225)$  | $2216 \pm 21$           | 1959.5                 | 13.8 |

Table V: Mass spectrum for $a_1$ trajectory calculated from the soft wall model spectrum $M^2_{a_1} = 4c^2(n+1/2)$ with $c = 0.388$ GeV and $M^2 R^2 = -1$. Experimental values are obtained from [41]. For the $a_1(1420)$ state, its mass is read from [44].

| $a_1$ mesons | $M_{\text{Exp}}$ (MeV) | $M_{\text{SWM}}$ (MeV) | %M |
|--------------|-------------------------|------------------------|-----|
| $a_1(1260)$  | $1230 \pm 40$          | 274.36                 | 348 |
| $a_1(1420)$  | $1414^{+15}_{-13}$    | 475.20                 | 198 |
| $a_1(1640)$  | $1654 \pm 19$          | 613.48                 | 170 |

If we compare our first state of the fitted trajectory to the $\eta'$ [18], we obtain a very good correspondence since its associated error is close to 1.77%. It is appropriate to recall that the $\eta'$ state appears when we consider the composite particles built from the fundamental and antifundamental representations of SU(3). In the approach presented here we do not consider chiral effects directly since our goal is to fit the mass spectrum. Chiral approaches on the AdS/QCD models are treated in previous works such as [16, 38, 40].

We show our predictions for three axial mesons in Table III. The lightest state $a_1(1260)$ is described within an error of 52.2% for this model, whilst the errors obtained for the other two axial mesons are 26.9 and 22.4%, respectively. We do not show higher states since they have already been discarded from the PDG listings [41].

We construct the mass spectra for pseudoscalar and axial mesons applying the shifting in the bulk mass in tables IV and V. In these cases, trajectories are linear, however, they show a strong deviation with respect to experimental data in the case of axial mesons. In the pseudoscalar sector, the soft wall model gives a good fitting of these states, except for the first state, $n = 0$, that has an error near to 31%. Therefore, we conclude that both models do not describe properly the first state. In the case of the soft wall model, there is no other parameter carrying information about the broken chiral symmetry in these states besides the choice made for $M^2_a$.

In the case of the modified version, the parameter $\kappa$, as it was introduced originally in [34], is considered as flavor dependent and related to the quark content. Therefore, it is expected that the $\kappa$ parameter used here should be sensitive to the chiral symmetry since now we have $s$ quarks in the mesonic constituent bag. What could be a proper value for $\kappa$ under these conditions is a question that should be addressed by considering a mechanism for chiral symmetry breaking in this model.

Following this discussion about universality of the parameters used, it is interesting to point out that $z_0$ was chosen to be related to the nature of the strong interactions, or in other words, to show that the constituents are interacting inside the particle. Since the states described here are purely mesonic, it is expected that the choice of $z_0$ would be valid for light scalar, pseudo scalar, vector and axial mesons.

Following [34], we can test the predictability of the model developed here with the RMS error for estimating $N$ parameters using $N_p$ parameters as
\[ \delta_{RMS} = \sqrt{\frac{1}{N - N_p} \sum_{i}^{N} \left( \frac{\delta O_i}{O_i} \right)^2}, \]

where \( O_i \) is the experimental mean value of an operator with an absolute uncertainty \( \delta O_i \). In our case, the model we examine can fit 23 states, which can be distinguished by 6 pseudoscalars, 3 axials, 8 scalars and 6 vectors (we take here our previous results given in [22]). Therefore, after considering 3 parameters given by \( \kappa, z_0 \) and \( \Delta P \), the value we obtain for \( \delta_{RMS} \) is close to 21.5%.

Tables IV and V show the results obtained for the usual Soft-Wall model when taking a dilaton profile \( c = 0.388 \text{ GeV} \). We can see that the results for the \( \eta \) particles have some similarities with the poles showed in table III; nevertheless, the axial particles found in the usual Soft-Wall model are way too far from the experimental bounds. Because of this, we do not show the RMS error for the usual Soft-Wall model since the errors obtained for axial states are too large.

V. CONCLUSIONS

In this work we have used the modified soft wall model with an extra UV cutoff, an approach that has proven to be a good approximation to study mass spectra of heavy quarkonium [34] and in the lightest scalar and vector meson sector [22]. We have fitted the radial trajectory of the \( \eta \) pseudoscalar mesons, as we showed in table II with \( \delta_{RMS} = 21.5 \% \). The first excited state, \( \eta(550) \), was not well fitted by the model. However, the \( \eta'(975) \) meson is well fitted with an error of 1.71%. On the other hand, two of the axial states that we obtained here using our heuristical approach agree within an error of 22-27% when compared with the available experimental data. The lightest of these axial states is fitted up to 52.2% of error, a value that is close to that obtained for the lightest \( \eta \) state. For both of these cases, we have taken \( I = J = 0 \) for the \( \eta \)'s and \( I = 0, J = 1 \) for the axial particles so that we only had to consider one Regge trajectory for each multiplet. Furthermore, both of these trajectories are linear in the orbital number since the separation between two consecutive poles in the two-point function is almost constant. This is an expected result since both \( \eta \) and \( a_1 \) particles are regular \( q\bar{q} \) mesons and these sort of particles have a linear Regge trajectory in the \((n, M^2)\) plane [18].

Comparing our results with those given in other Effective Models such as that given in [20], where twenty-one phenomenological input parameters are to be taken to properly fit the model due to the huge amount of mesons considered, we infer that the small amount of theoretical parameters taken in our approach must have had an important influence in our results for the \( \eta \) state, besides avoiding us to obtain a more precise value for the \( a_1(1260) \) mass. Something similar happens with nonconformal models without chiral symmetry breaking effects that describe the lowest orbital states of scalar and vector mesons, where the lowest state of these last ones has an error of approximately 20.5% [22].

We also have contrasted the pseudoscalar and axial states obtained from the usual Soft-Wall model approach; as we have seen, modifying the AdS Poincare patch with an UV cutoff allows us to improve the description of the \( a_1 \) orbital trajectory, as well as the one associated to \( \eta \) states; furthermore, and thanks to the fact that we have described both scalar and vector states using this modified model, we have obtained a \( \delta_{RMS} \) value that lies within the expected bounds of these sort of models, something that could not be attained with the usual Soft-Wall approach.

As it has been showed so far, chiral symmetry breaking effects have been introduced without considering an auxiliar scalar field; this phenomenon occurs due to the introduction and evaluation of proper limits of massive and parity parameters for both pseudoscalar and axial mesons, thus giving an straightforward way of distinguishing them from scalar and vector states. Since these terms have been introduced by hand, we deduce that chiral symmetry is explicitly broken in this model, as happens when including quark mass contributions in effective theories such as Chiral Perturbation Theory [45].

Other nonconformal holographic approaches also take into account more parameters such as quark masses and condensates to fit meson spectra, with both of these parameters coming from the auxiliar scalar field that describes chiral symmetry breaking [13–17]. Although having more parameters is very useful to minimize fitting errors, we did not take them into account since our mechanism for symmetry breaking does not have any information of quark structure or constitution for mesons.

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