Spherically symmetric solution in a space–time with torsion

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Abstract By using the method of group analysis, we obtain a new exact evolving and spherically symmetric solution of the Einstein–Cartan equations of motion, corresponding to a space–time threaded with a three-form Kalb–Ramond field strength. The solution describes in its more generic form, a space–time which scalar curvature vanishes for large distances and for large time. In static conditions, it reduces to a classical wormhole solution and to an exact solution with a localized scalar field and a torsion kink, already reported in literature. In the process we have found evidence towards the construction of more new solutions.

Keywords Spherically symmetric solution · Space–time with torsion · Group analysis
1 Introduction

Unification of all fundamental forces in nature has represented a difficult task along the last few decades. The most promising models are encoded within the context of string theory. For instance, there are models in heterotic string theories in which grand unifications theories are present, while in type II superstring theories, partial unification is an available option in several model-dependent scenarios, where there exists the possibility to study geometric aspects of general relativity at the microscopic level (see [1] and references therein).

In order to construct effective four-dimensional theories with a minimal (non-zero) number of supersymmetric generators, critical type II superstrings are typically compactified on Calabi-Yau (CY) threefolds, which break three-quarters of the total supersymmetry. Roughly speaking, this follows from the fact that, in the absence of other supergravity fields, the ten-dimensional gravitino satisfies the constraint \( \delta \Psi^A_M = \nabla_M \epsilon^A = 0 \), implying the presence of two four-dimensional covariantly constant spinors. Under these fluxless conditions, it is possible to construct a maximally symmetric four-dimensional theory (Minkowski, de Sitter or anti-de Sitter). However, fluxless compactification scenarios suffer from the moduli stabilization problem.

Moduli are stabilized, once one includes fluxes in the compactification process. Nevertheless, fluxes back-react and their presence modifies the internal geometry, forcing us to depart from the nice and smooth CY geometry. Even more, the presence of fluxes contributes to the ten-dimensional gravitino variation by

\[
\delta \Psi^A_M = \nabla_M \epsilon^A + \kappa_M \epsilon^A ,
\]

where \( \kappa_M \) encodes the flux contributions: it vanishes in the fluxless case. Notice however, that to preserve two supersymmetries in the effective four-dimensional theory, the variation of the gravitino must vanish, implying that the four-dimensional component of the supersymmetric parameter is not covariantly constant with respect to the Levi-Civita equation, but with respect to a connection with torsion. It is then clear that, the effective four-dimensional supergravity theories obtained from the compactification process, must be different from the theories constructed in the fluxless case. Such effective theories receive the name of gauged supergravities. For more details, see [2] and [3].

One interesting question, concerns the existence of spherically symmetric solutions in the context of gauged supergravities [4,5] and in particular, the specific case in which a Kalb–Ramond (KR) three-form flux is extended in the effective four-dimensional space–time. The presence of KR fluxes implies the study of gravitational theories in curved backgrounds with torsion [6]. Torsion appears as an antisymmetric tensorial piece in the Christoffel connection symbol in Einstein–Cartan (EC) theory. The simplest level of torsion theory can provide a classical background for quantum matter fields and a relationship with spin [7–11]. This is important since, an alternative way to study geometric aspects of general relativity at the microscopic level, is given by the EC theory [12,13], which is characterized by the presence of a spin angular momentum in addition to the mass. The main task to include torsion in space–time is to unify...