ON THE ABSENCE OF SHEAR FROM COMPLETE EINSTEIN RINGS AND THE STABILITY OF GEOMETRY

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ABSTRACT

Concordance cosmology points to a universe of zero mean curvature, due to the inflation mechanism which occurred soon after the big bang, while along a relatively small number of lower redshift light paths where lensing events are observed, space is positively curved. How do we know that geometry and topology are robust rather than in a state of chaos? The phenomenon of cosmic shear provides an effective way of mapping curvature fluctuations, because it affects any light rays whether they intercept mass clumps or not. Moreover, shear depends to lowest order only on the total mass density of clumps and not on their mass function. We discuss a range of astrophysical applications of the principal manifestation of shear—the distortion of images. It will be shown that the quickest way of testing the existence of shear in the near universe is to look at the shape of Einstein rings. The fact that most of these rings are circular to a large extent means, statistically speaking, shear occurs at a much lower level than the expectation based on our current understanding of the inhomogeneous universe. While inflation may account for the mean geometry, it offers no means of stabilizing it against the fluctuations caused by nonlinear matter clumping at low redshift. Either this clumping is actually much less severe or the physical mechanism responsible for shaping the large-scale curvature has been active not only during the very early epochs, but also at all subsequent times. Might it be the vital “interface” between expansion on Hubble distances and gravity on cluster scales and beneath?

Subject headings: cosmological parameters — galaxies: halos — gravitational lensing — large-scale structure of universe — quasars: individual

1. INTRODUCTION

In the prevailing $\Lambda$CDM cosmological model the observational evidence for departures from the standard theory of gravity (general relativity) at large distance scales, be they in the form of flat rotation curves for galaxies or accelerated expansion between galaxies, is explained in terms of an “extension” of the theory to postulate two foreign ingredients: dark matter and dark energy. Apart from the lack of direct detection of a single molecule or quantum from either “dark” components and especially despite decades of large investments in the search for dark matter, the delicate balance of proportions between the two components that manifests itself in the observed global flatness of space necessitates yet another postulate, viz., that the early universe underwent a brief period of extremely rapid expansion called the inflation epoch.

None of the three new postulates have been verified in the laboratory, and one should also add to the list the assumptions of seed density perturbations (quantum fluctuations) in the early universe, the big bang singularity, which relies heavily on the existence of a minimum (Planck) scale in time, and the Hubble expansion itself. These are all independently postulated axioms, and the last (the expansion of space) has recently been deemed unverifiable by any terrestrial apparatus (Chodorowski 2007).

Since astronomers did not enjoy their status as pioneers of modern science through Newton’s habit of invoking unknowns to explain unknowns, it seems reasonable to ask whether the surprises presented to us by the cosmological data are due in fact to a breakdown in our understanding of the nature of the fundamental forces at work over very long ranges. Indeed, while general relativity was comparatively well tested over stellar distances scales, the evidence for its validity over galactic scales and beyond are scarce and highly indirect. It would be very important, e.g., to be able to test if there really is the expected statistical effect on the propagation of light by the gravity of “embedded” mass clumps and the expansion of space in between clumps.

Let us examine more closely this last point, as it is the subject of the present paper. What is the conventional understanding? The gravitational effect of a mass structure on light may be quantified in terms of the impact parameter $b$ between a bundle of light rays and the center of mass of the distribution. If this point of closest approach was reached during redshift $z$ and at a comoving distance $x$ from the observer, the average fractional magnification $\eta$ of a light source at distance $x_s > x$ away, to which the rays look back, is given by the standard formula

$$\eta = \frac{(x_s - x)x}{2(1+z)x_s} \left( \frac{\psi}{b} + \frac{d\psi}{db} \right),$$

where $\psi$ is the deflection angle. The two terms on the right-hand side correspond respectively to half the fractional increase in the linear size of the image (in angular space and relative to the image in the absence of the mass $m$) along the directions perpendicular and parallel to $b$.

If, under what we classify here as phenomenon 1, the light rays pass through a mass clump, in general we will have $\eta > 0$, i.e., the source will be magnified. These are the lensing events. They tend to be observationally dramatic, although their occurrence frequency is relatively low. If the light passes in between clumps, two phenomena are at play: (2) the shear effect of all the nonintercepting clumps and (3) demagnification in the underdense (or void) regions.

The scenario of particular interest to us in this paper is, however, case 2. When our small bundle of light rays passes $b$ (or skirts) the clump, then $\psi = 4Gm/(c^2b)$, so that $\psi/b$ is positive while $d\psi/db = -\psi/b$ is negative, i.e., the resulting zero magnification ($\eta = 0$) is due to squeezing of the source image along the
b-direction and stretching of it in the perpendicular direction. This is known as shear, or weak lensing, and because in principle any clump, no matter how far, can affect the light signal in question, it means the shearing of a distant source is a cumulative (hence large) effect of numerous clumps and can occur no matter in which direction the source lies. Thus in case 2 one is dealing with both a significant and frequent phenomenon, which is why a weak-lensing survey of many background quasars (e.g., Dodelson et al. 2006) represents a powerful technique of probing global geometry. We shall therefore focus on the phenomenon of shear for the rest of the paper by investigating its principal manifestation of direct image distortion.

2. THE STATISTICAL EFFECT OF SHEAR: HOW FOREGROUND GALAXIES CAN AFFECT THE APPEARANCE OF DISTANT SOURCES

At low redshifts any passing light can be sheared by the large-scale inhomogeneity of primordial matter and the distribution of nonlinear virialized structures over smaller distances. We first examine the former. Denote the gravitational perturbation of an otherwise zero curvature Friedmann-Robertson-Walker universe (as inferred from WMAP1 and WMAP3, viz., Bennett et al. 2003; Spergel et al. 2007) by \( \Phi(x, y) \), where the x-axis is aligned with the light path and y is a vector along some direction transverse to x. The correlation function between the deflection angles \( \delta y(\theta')/x \) and \( \delta y(\theta'')/x \) of two light rays making small angles \( \theta' \) and \( \theta'' \) with respect to the x-axis may be written (see Lieu & Mittaz 2008 for details) as

\[
C_y(|\theta' - \theta''|) = \frac{1}{x^2} \left( \frac{\delta y(\theta') \delta y(\theta'')}{x} \right) = \frac{4}{c^4 k^2} \int_0^\infty dx' (x - x') \int_0^\infty dx'' (x - x'') \times \left( \nabla^i \Phi(x', \theta' x') \nabla^j \Phi(x'', \theta'' x'') \right).
\]

We can calculate the integrals by expressing the integrand in terms of the matter power spectrum \( P(k) \),

\[
\left( \nabla^i \Phi(r') \nabla^j \Phi(r') \right) = \frac{9 \Omega_m H_0^2}{32 \pi^2} \int \frac{d^3 k}{k^3} k_k e^{ikr} P(k),
\]

where \( r = r' - r'' \) and

\[
P(k) = \frac{8 \pi^2}{9 \Omega_m H_0^2} \frac{d}{d \ln k} (\delta \Phi_k)^2,
\]

with \( \delta \Phi_k \) being the standard deviation of the potential over length scales \( 2\pi/k \). The ensuing functional form of \( C(\theta) = C_n(\theta) \equiv \left( \frac{\delta y(x) \delta y(y)}{x^2} \right) \) [where the repeated \( i \) index implies summation over the two y directions (0, 1) and (0, 0, 1), transverse to the light path vector (1, 0, 0) = \( \hat{x} \) ] is

\[
C(\theta) = C_0 + \frac{1}{2} C_2 \theta^2 + O(\theta^4),
\]

when expanded as a Taylor series in \( \theta \).

The random relative deflection between the two rays is then given by

\[
(\delta \theta)^2 = \frac{1}{x^2} \left[ \delta y \left( \frac{1}{2} \theta \right) \right] = 2 [C(0) - C(\theta)] = C_2 \theta^2 + O(\theta^4)
\]

and has the \( C_2 \) coefficient as its leading term, viz., \( \delta \theta \approx C_2 \theta \), clearly indicating that the \( C_0 \) (constant) term relates only to absolute deflection of the two rays. The calculation of \( C_2 \), and hence \( \delta \theta \), was done in Lieu & Mittaz (2008). The result points to a small shear effect, \( \delta \theta \approx C_2 \approx 1 \%, \) for a primordial spectrum \( P(k) \) derived from WMAP1/2dFGRS. This is also consistent with previous conclusions reached by Seljak (1996) and Lewis & Challinor (2006).

Obviously, the above treatment does not take into account the role of nonlinear matter clumping, in particular, galaxies in the near universe. We therefore proceed to calculate the same lowest order effect of relative deflection, i.e., \( \theta \sim \theta \), due to a random ensemble of nearby galaxies. The validity criterion of using a Poisson clump distribution was already enumerated in detail in § 3 of Lieu (2008); in short, the two rays must always be separated by lengths that are small compared with the typical value of the minimum impact parameter \( y_{\text{min}} \) at which each ray skirts the galaxies. The test for this form of shear that we shall make generally involves rays that satisfy this requirement.

Let us first work out carefully the effect of one mass clump. Referring to Figure 1, the deflection angles of two neighboring light rays with the mass clump \( m \) positioned at \( y \) and \( y + \delta y \) relative to the points of closest approach of the rays are given by

\[
\alpha = \frac{4 Gm}{c^2 y^2} y, \quad \alpha' = \frac{4 Gm}{c^2 (y + \delta y)^2} (y + \delta y).
\]

Denoting the angle between \( y \) and \( \delta y \) as \( \vartheta \), the differential deflection may be written as

\[
\delta \alpha = \frac{4 Gm \delta y}{c^2 y^2} - 2 \cos \vartheta \delta y.
\]

The variance in \( \delta \alpha \) may now be calculated. It is

\[
(\delta \alpha)^2 = \frac{\left( \frac{4 Gm \delta y}{c^2 y^2} \right)^2}{c^2 y^2},
\]

and is independent of \( \vartheta \). If the rays originated from two points that subtend the angle \( \vartheta \) at the observer \( O \), then equation (7) will
once again give the value of $\delta y$ for a mass clump at comoving distance $x$ from O. Moreover, we may also write $(\delta \theta)^2 = (\delta \alpha)^2$ as the variance of the random excursion of the angular separation $\theta$ between the actual images of these two sources. Thus, we arrive at the equation

$$ (\delta \theta)^2 = \left( \frac{4 G m x \theta}{c^2 y^2} \right)^2, \quad (8) $$

for the shear distortion of the shape of extended sources, if $\theta$ is the angle subtended at O by two boundary points of the source.

Our final step is to derive the total variance by integrating $(\delta \theta)^2 2 \pi n dy dx$, where $n$ is the non-evolving number density of clumps (for the effect of evolution, see the end of this section), down the light path $y$ and over all impact parameters $x$ from $y = y_{\text{min}}$ upward. Care should be taken here, however, because for deflections at finite $z$ the impact parameter scales as $y/(1 + z)$, where $y$ is the comoving distance of closest approach, which means $(\delta \alpha)^2 \sim (1 + z)^2$. If the two light rays originated from points of the same comoving distance $D$ from O, one obtains in this way

$$ (\delta \theta)^2 = \frac{16 \pi G^2 m^2 n \mathcal{D}^3 \theta^2}{c^4 y_{\text{min}}^2}, \quad (9) $$

where $n$ is the number density of clumps and

$$ \mathcal{D}^3 = \int_0^D x^2(1 + z(x))^2 dx $$

$$ = \left( \frac{c}{H_0} \right)^3 \int_0^1 dz' \frac{(1 + z')^2}{E(z')} \left[ \int_0^z \frac{dz''}{E(z'')} \right]^2, \quad (10) $$

with the function $E(z)$ being defined as

$$ E(z) = \left[ \Omega_m (1 + z)^3 + \Omega_\Lambda \right]^{1/2}. \quad (11) $$

We see in equation (9) the phenomenon of random walk, viz., $\delta \theta \sim \sqrt{n}$, due to the accumulation of relative deflections along the light path from our statistical ensemble of clumps. There is however a divergence with respect to $y_{\text{min}}$, in the sense that the theoretical lower limit of $y$ is zero. In practice we conservatively set $y_{\text{min}}$ at the value where, throughout the entire light path, the expected number of uniformly and randomly distributed clumps having this impact parameter is on average equal to one. Thus, we find that for a typical direction to some source, $y_{\text{min}} = \frac{1}{\sqrt{\pi n D}}$. \quad (12)

It follows that the percentage variation in the angular separation $\theta$ between the images of the two points can be expressed as

$$ \frac{\delta \theta}{\theta} = \frac{3H_0^2}{2c^2} \Omega_m D^{1/2} \mathcal{D}^{3/2}, \quad (13) $$

after employing the definition that $Gmn$, which may also be written as $\sum G_{mm}$, under the scenario of a spread in the mass and number density of galaxies, equals $3H_0^2 \Omega_m/(8\pi)$, with $\Omega_m$ being the total mass density of clumps as a fraction of the critical density. The implication of this last step is that our final answer does not depend on the details of the mass function of the clumps, but only on the total fraction of the matter density, $\Omega_m$, belonging to all these clumps.

For application to strong-lensing observations the calculation has to be divided into two parts. The contribution to $(\delta \theta)^2$ from galaxies in the foreground region between O and the lensing plane is obtained from equation (9) with $D$ replaced by $D_l$, as

$$ (\delta \theta)^2 = \frac{(4 \pi G m)^2}{c^4} n^2 D_l \mathcal{D}_l^3 \theta^2, \quad (14) $$

where $\mathcal{D}_l$ is as in equation (10) with $D_l$ (or $z_l$) as the upper integration limit. Next, the contribution from galaxies lying behind the lens and in front of the source is calculated in a likewise manner; in particular, $y_{\text{min}}$ is again from equation (12) with the substitution $D \rightarrow D_h$. The total variance is then given by

$$ \frac{\delta \theta}{\theta} = \frac{3H_0^2}{2c^2} \Omega_m D_l^{1/2} \left( \mathcal{D}_l^3 + \frac{D_l}{D_h} \mathcal{D}_h^{3/2} \right), \quad (15) $$

with

$$ \mathcal{D}_h^3 = \int_{D_h}^{D_l} (D_l - x)^2 (1 + z)^2 dx $$

$$ = \frac{c}{H_0} \int_{z_l}^{z_i} dz (1 + z)^2 \left[ D_s - \frac{c}{H_0} \int_{z}^{z_i} dz' \right]^2, \quad (16) $$

when this variance is also cast as a fractional deviation.

An important (and tacit) assumption underlying equation (15) is that $y_{\text{min}}$ should always remain greater than the size of a galaxy plus its halo before we can defend our neglect of clump evolution along the light path. Between $z = 0$ and $z \sim 3$ strongly lensed sources are found, evolution causes the halo of a galaxy that is virialized at $z = 0$ to become less compact at higher $z$, but most of the matter in the present-day halo of this galaxy would still have “turned around” (see Eke et al. 1996) by $\frac{1}{2} z \sim 3$. Thus, the light path near the source may legitimately be considered as being affected by the same population of random clumps as that in $z = 0$, unless the comoving scale height of the mass distribution of the $z > 0$ galaxies exceeds $y_{\text{min}}$.

To be more quantitative, one may start with the observed density of galaxies $n = 0.17 \ h^{-1} = 0.06$ Mpc$^{-3}$ for $h = 0.7$ (Ramella et al. 1999), to estimate that throughout the 3 Gpc comoving distance between $z = 0$ and 1 a typical light ray is within $y_{\text{min}} \approx 40$ kpc from a galaxy, which may be taken as an isothermal sphere of circular velocity $\sim 250$ km s$^{-1}$, i.e., the cutoff radius is then 20 kpc. Even taking into account the fact that at higher $z$ a virialized system was effectively larger by the factor $1 + z$, at $z = 1$ the average comoving galaxy radius then becomes 40 kpc, which is barely equal to $y_{\text{min}}$. Thus, as mentioned above the passing light generally misses all the galaxies. If, however, evolution causes the galactic halo to lie beyond the 20($1 + z$) kpc virial radius in the past, so that $y_{\text{min}}$ falls within the halo scale height back then, the light rays would have been weakly lensed at that part of their journey, and all shear effects will be reduced from the level calculated above, because the

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1 Thus, e.g., in an Einstein–de Sitter universe a galaxy just virializing today at $t = t_0$ would have turned around at $z = t_0/2$, or $z \approx 0.7$, when the (turnaround) radius was $\approx 3.3$ times larger than the $z = 0$ virial radius, i.e., the $t = t_0/2$ sphere that contains one virial mass at $t = t_0$ was a factor of 3.3 greater in radius then, and all the matter within it already belonged to the clump. Most galaxies that exist today would have virialized at $z > 0$; hence, their turnaround epochs were at $z > 0.7$. If dark energy is invoked to accelerate the expansion, this would push the turnaround epoch to even higher $z$, because it would take longer for the clump to collapse and virialize. Thus, it is reasonable to assume that when one looks back at $z \leq 3$ most galaxies were equally massive, just a few times bigger in size.
mass that affected the light falls short of the galaxy’s total mass. Such a violation of our present assumption could take place on the far side of the strong-lensing plane where \( z \approx 1 \), thereby lowering \( \delta \theta / \theta \) to a value given only by the first term of equation (15). On the near side of the strong-lensing plane, evolution is less important because galaxies generally virialize well ahead of the \( z = 0 \) epoch. This theoretical (modeling) result is also corrobated by observations, which indicate that galaxies indeed exhibit no evidence for evolution at least up to redshifts \( z \approx 1 \) (Ofek et al. 2003).

We close this section with a point of fundamental physics. Whether the cause be primordial matter or nonlinear clumping, the relative deflection between two neighboring rays as calculated above originates from the first-order \( (C_2) \) term in the Taylor expansion of equation (16) and is the same order effect as the incoherent time delay of Lieu (2008). In fact, by means of the light reciprocity theorem it was shown (Lieu & Mittaz 2008) that the formulae for relative deflection and incoherent delay are interconvertible and that the same statement applies to absolute deflection and coherent delay. Although the latter pair are both zeroth-order effects, they are much harder to observe, as already explained in Lieu (2008).

3. THE SHAPE OF DISTANT GALAXIES; SUPERLUMINAL MOTION IN QUASARS

Here we discuss two astrophysical applications of \( \S \) 2 in the context of \( \Lambda \)CDM cosmology, where

\[
\Omega_m = 0.3, \quad \Omega_\Lambda = 0.7, \quad h = 0.7, \quad \Omega_{cd} = 0.15. \tag{17}
\]

The setting of \( \Omega_{cd} = 0.15 \) is fully consistent with the expectation of the standard model, which assumes that half the baryons, hence, approximately the same fraction for dark matter also, of the low-\( z \) universe resides in galaxies and their halos—mass clumps that may completely be distributed as field galaxies or partly congregated into groups (see Fukugita 2004; Fukugita et al. 1998). In fact, the observed properties of galaxies given in \( \S \) 2 do indeed yield \( \Omega_m = 0.15 \).

In the first application we consider the appearance of resolved sources at \( z \approx 1 \). According to equations (15) and (17), \( \delta \theta / \theta \geq 13% \) at \( z \approx 1 \). Since this is caused by shear, viz., the light rays in question have not directly been lensed, there is no magnification (\( \S \) 1). Hence, if the percentage change in the angular size of the image along one direction has the typical value of \( +13\% \), the same for the orthogonal direction must be \( -13\% \) (i.e., between the two axes \( \delta \theta / \theta \) are correlated) to conserve total solid angle subtended by the image at us. This means the aspect ratio of the resulting distorted image reaches 26% at \( z = 1 \) and larger at \( z > 1 \). Now, most \( z \approx 1 \) sources we detect are quasars, i.e., elliptical galaxies to “begin with.” One could ask if the ellipticity is completely due to shear. Nevertheless, an aspect ratio \( \geq 26\% \) is quite large, so that even ahead of a statistical analysis at high resolution, one could already query whether such a level of shear really exists. Thus, apart from time delay observations, this represents another potential challenge to all the cosmological models, with \( \Lambda \)CDM in particular.

Next, we turn to the problem of quasar superluminal motion, which is inferred from the angular speed at which blobs of ejected material move away from the central active galactic nucleus engine; by means of the distance to the quasar as derived from its redshift, this angular speed is often converted to a physical speed that exceeds \( c \). When the redshift of the quasar is high, however, caution is needed in the conversion, because if in equation (15) \( \delta \theta / \theta \) is no longer \( \ll 1 \), the true angular speed could be substantially larger or smaller than the observed value, depending on which way the apparent motion of the blob is being sheared by the foreground matter. In fact, \( \delta \theta / \theta \) is the percentage error in the superluminal speed (note that because superluminal motion typically involves \( \theta \sim \text{milliarcseconds} \), i.e., the light rays being sheared are very close to each other, the assumptions underlying the validity of eq. (15) hold exceptionally well). Thus, the point raised here starts to be relevant for quasars of \( z \approx 1 \), where \( \delta \theta / \theta \approx 13\% \) as before.

As the improvement of sensitivity and resolution may lead to the discovery of quasar superluminal motion at higher redshifts and (likely) with ever increasing jet speeds, examples being Bouchy et al. (1998) on 1338+381 at \( z = 3.1 \) and \( v_j / c \approx 27 / h \) and Frey et al. (2002) on 1351–018 at \( z = 3.7 \) and \( v_j / c \approx 9.2 / h \), it is an expectation, based on the cosmological effect of shear, that not all of the apparent largeness of \( v_j / c \) is due to relativistic distortions at the source. An interesting future pursuit worthy of consideration is to correlate \( v_j / c \) with \( z \) to see if there is more scatter at high redshifts. If so, this could be indicative of the presence of shear.

4. WHY ARE COMPLETE EINSTEIN RINGS A CHALLENGE TO COSMOLOGICAL MODELS?

Finally, the third application of \( \S \) 3. We return to the question raised in \( \S \) 1, on whether existing data can already be used to clinch cosmological models on the problem of global geometry. We hold the view that the most effective test currently available is still the weak-lensing distortion of images of distant sources mentioned in \( \S \) 1. The new point to be made in this work, however, is that unlike the primordial matter distribution the shear effect of foreground galaxies is severe for \( z \approx 1 \) sources, i.e., one should not need such a large sample of background emitters to detect it. Nevertheless, the usual difficulty is in finding circularly symmetric patterns to start with, so that one knows that any apparent elongation is not an intrinsic property of the object being looked at.

For the above reasons, the Einstein rings of well-aligned strong-lensing configurations play a unique role in satisfying our requirement, because the intrinsic shape of such a pattern is circular, or quite nearly so. While the zeroth-order “\( C_0 \)” term or absolute deflection at constant \( \theta / \theta \) by mass inhomogeneities can affect the existence of an Einstein ring by bringing misaligned source-lens observer arrangements into alignment (i.e., in a smooth universe the same Einstein ring seen somewhere in the sky would not even have been observable as the optical components involved are intrinsically noncollinear), the higher order effect of relative deflection \( (\delta \theta / \theta) \) between two neighboring rays calculated in \( \S \) 2 plays the role of distorting the ring via cosmic shear, as illustrated in Figure 2.

We therefore focus our attention on a very recent set of well-observed Einstein ring images, starting with the best candidate, J0332–3357, where \( z_s = 0.986 \) and \( z_l = 3.773 \) (Cabanac et al. 2005) or \( D_L = 3.271 \) Gpc, \( D_A = 7.012 \) Gpc, and \( D_0 = D_s = D_l \) in the flat \( \Lambda \)CDM cosmology of equation (17). Substituting these numbers into equation (15), one obtains \( \delta \theta / \theta \approx 25\% \) or an aspect ratio of 50%. It is evident without any further analysis necessary that the J0332–3357 Einstein ring is much too circular to accommodate such a significant ellipticity.

Could the J0332–3357 observation simply be a statistical anomaly? There has recently been a wave\(^2\) of Einstein ring detections, such as J073728.45+321618.5, J232120.93–093910.2, and

\(^2\) See the images on http://hubblesite.org/newscenter/archive/releases/2005/32/image/a/.
J163028.15+452036.2. Together with the more historical B1938+666 (King et al. 1998), all these lensing systems have characteristic parameters values $D_l \approx D_{ls} \approx 3.3 \text{ Gpc}$ ($z_l \approx 1$, $z_L \approx 3.5$ in the cosmology of eq. [17]), i.e., the resulting $\delta \theta/\theta$ of shear is at a comparable level as that for J0332−3357 and can be cast in a convenient form as (noting that the $D$ and $D_s$ values are related to each other once a cosmology is chosen)

$$\frac{\delta \theta}{\theta} = 0.23 \left( \frac{h}{0.7} \right)^2 \left( \frac{\Omega_{cl}}{0.15} \right) \left( \frac{D_l}{3.3 \text{ Gpc}} \right)^{1/2} \times \left( \frac{\Omega_{cl}^3}{35.7 \text{ Gpc}^3} \right) + \left( \frac{D_l}{D_s} \right) \left( \frac{\Omega_{cl}^3}{71.3 \text{ Gpc}^3} \right)^{1/2}.$$ (18)

Thus, these images should also be sheared with an aspect ratio similar to that of J0332−3357, yet none of them are observed to exhibit this behavior. As a guide to the eyes, we show in Figure 3 an ellipse with 50% aspect ratio. It is fair to say that no continuous and nearly complete Einstein rings have been found to suffer from so much distortion.

Since our prediction on shear, equation (15), lies with the fact that it depends simply on the mean mass density of clumped matter $\Omega_{cl}$ (which cannot differ too greatly from $\Omega_{cl} \approx \Omega_m/2$ or else structure formation will be in jeopardy; see §2 and 3) regardless of, e.g., the number density of clumps or the mass distribution of individual clumps, this goes to highlight just how robust the prediction is. Nevertheless, equation (15) has caveats and these were stated in §2, where we contended that the details of clumping are irrelevant provided the universe did not have too high a degree of homogeneity at any time during the light propagation. Specifically, a milder shear prediction can be reached by appealing to the increased size of galaxies and groups at higher redshifts as the universe turned too smooth (§2). In a revised prediction which is probably overconservative, one could take into account the shear contribution, in the manner calculated in this section and §3, only from those clumps lying within the foreground universe between the (strong) lensing plane at $z \approx 1$ and the observer, i.e., considering the universe behind the lensing plane as completely homogeneous. According to §3, the expected aspect ratio of an Einstein ring would then be $\sim 26\%$. Such a distortion is also depicted in Figure 3 to demonstrate that it is still too large to reflect reality—among the aforementioned Einstein rings, only J140228.21+632133.5 exhibits a commensurate ellipticity.

5. CONCLUSION: THE QUESTION OF GEOMETRY

When light propagates through an inhomogeneous universe there are three possible outcomes: (1) if the rays intercept mass clumps there will be magnification, (2) if they only skirt a clump

Fig. 2.—Two strong-lensing images (which could be part of an Einstein ring) are displaced relatively to each other as their associated light paths are perturbed by external mass clumps lying at distances far larger than the separation between the paths. This relative displacement is also accompanied by an incoherent (random) difference in the arrival times of two photons emitted simultaneously and propagating down the two paths. In fact, both the relative deflection and incoherent delay are first-order effects. Moreover, they are manifestations of the same underlying phenomenon, viz., shear.

Fig. 3.—Distortion of a perfect Einstein ring (leftmost circle) by shear. Clumps lying in the foreground, i.e., between us and the lensing plane, could deform the ring’s appearance to become like the first ellipse. If background clumps are also taken into account and their evolution is neglected, the ring will be stretched into the shape of the second ellipse. In reality, the orientation of the two ellipses is, of course, random.
there will be shear, and (3) if they intercept underdense voids there will be demagnification. When the mean density is critical, the effects of outcome 1 and 3 cancel statistically, and in this sense, one can say that the “mean” global geometry of space is determined solely by $\Omega$. In terms of occurrence rate, however, class 1 events are relatively rare, and although class 3 events are more frequent, each event brings about a small change which is hard to measure. On the other hand, for any bundle of light rays case 2 applies to all clumps with impact parameters less than a Hubble radius, and the sum total of their contributions is a rather large shear which has observable consequences, such as the appearance of high-redshift quasars and systems with superluminal motion.

But perhaps the most important manifestation of shear is to be found in the apparent shape of the (intrinsically circular) Einstein rings, as these should be stretched into randomly oriented ellipses with an aspect ratio $\lesssim 25\%$. Observed rings are, however, usually much less affected, so unless the near universe is much more homogeneous than expected, this poses a formidable challenge to cosmological models, viz., if geometry is shaped solely by inflation rather than some physical mechanism that operates at all epochs, why are such fluctuations as shear not seen in the Einstein rings?

It is therefore not entirely inconceivable that the puzzles of cosmology in general and global geometry in particular lie hand in hand. If the latter is solved, we will be much closer to developing the correct model of the former. The current distinction between the concepts of “dark energy” and “dark matter” merely represents our on-going effort in seeking a complete understanding of the missing physical mechanism(s) that bridge the gap between gravity on smaller scales and expansion of space on Hubble scales. Although at present an improbable scenario, the stability of a certain prescribed geometry might have been enforced by an ongoing tension between two opposing mechanisms, i.e., the gravitational fields of mass clumps are not simply “embedded” phenomena in an otherwise decoupled “cosmic substratum” of uniform and accelerated expansion.

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