Can cosmology detect hierarchical neutrino masses?

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We have carefully analysed the potential of future Cosmic Microwave Background (CMB) and Large Scale Structure (LSS) measurements to probe neutrino masses. We perform a Fisher matrix analysis on a 9-dimensional cosmological parameter space and find that data from the Planck CMB experiment combined with the Sloan Digital Sky Survey (SDSS) can measure a neutrino mass of 0.12 eV at 95% conf. This is almost at the level of the 0.06 eV mass suggested by current neutrino oscillation data. A future galaxy survey with an order of magnitude larger survey volume than the SDSS would allow for a neutrino mass determination of 0.03-0.05 eV (95% conf.).

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I. INTRODUCTION

The absolute value of neutrino masses are very difficult to measure experimentally. On the other hand, mass differences between neutrino mass eigenstates, \( \delta m^2 \), can be measured in neutrino oscillation experiments. Observations of atmospheric neutrinos suggest a squared mass difference of \( \delta m^2 \approx 3 \times 10^{-3} \text{ eV}^2 \), \[1, 2, 3\]. While there are still several viable solutions to the solar neutrino problem the so-called large mixing angle solution gives by far the best fit with \( \Delta m^2 \), \[1, 2, 3\].

In the simplest case where neutrino masses are hierarchical these results suggest that \( m_1 \approx 0, m_2 \approx \delta m_{\text{solar}}, \) and \( m_3 \approx \delta m_{\text{atmospheric}}. \) If the hierarchy is inverted \[1, 2, 3\], one instead finds \( m_3 \approx 0, m_2 \approx \delta m_{\text{atmospheric}}, \) and \( m_1 \approx \delta m_{\text{atmospheric}}. \) However, it is also possible that neutrino masses are degenerate \[1, 2, 3\]. In which case oscillation experiments are not useful for determining the absolute mass scale.

Experiments which rely on kinematical effects of the neutrino mass offer the strongest probe of this overall mass scale. Tritium decay measurements have been able to put an upper limit on the electron neutrino mass of \( 2.2 \text{ eV (95% conf.)} \) \[27\]. However, cosmology at present yields an even stronger limit which is also based on the kinematics of neutrino mass. Neutrinos decouple at a temperature of 1-2 MeV in the early universe, shortly before electron-positron annihilation. Therefore their temperature is lower than the photon temperature by a factor \((4/11)^{1/3}\). This again means that the total neutrino number density is related to the photon number density by

\[
\frac{n_{\nu}}{n_{\gamma}} = \frac{9}{11},
\]

Massive neutrinos with masses \( m \gg T_0 \approx 2.4 \times 10^{-4} \text{ eV} \) are non-relativistic at present and therefore contribute to the cosmological matter density \[24, 25, 26\].

\[
\Omega_{\nu}h^2 = \frac{\sum m_{\nu}}{92.5 \text{ eV}},
\]

calculated for a present day photon temperature \( T_0 = 2.728 \text{ K} \). Here, \( \sum m_{\nu} = m_1 + m_2 + m_3 \). However, because they are so light these neutrinos free stream on a scale of roughly \( k \approx 0.03 m_{\nu} \Omega_{m}^{1/2} \text{ h Mpc}^{-1} \) \[27, 28, 29\]. Below this scale neutrino perturbations are completely erased and therefore the matter power spectrum is suppressed, roughly by \( \Delta P/P \sim -8 \Omega_{\nu}/\Omega_{m} \) \[27\].

This power spectrum suppression allows for a determination of the neutrino mass from measurements of the matter power spectrum on large scales. This matter spectrum is related to the galaxy correlation spectrum measured in large scale structure (LSS) surveys via the bias parameter, \( b^2(k) \equiv P_g(k)/P_m(k) \). Such analyses have been performed several times before \[30, 31\], most recently using data from the 2dF galaxy survey \[32, 33, 34\]. These investigations find mass limits of 1.5-3 eV, depending on assumptions about the cosmological parameter space.

In a seminal paper it was calculated by Eisenstein, Hu and Tegmark that future CMB and LSS experiments could push the bound on the sum of neutrino masses down to about 0.3 eV \[27\]. This calculation was done using a Fisher matrix analysis on a 6-dimensional cosmological parameter space and a fairly crude approximation of the MAP CMB data.

In the present paper we discuss the prospects for neutrino mass detection in more detail. First we discuss the Fisher matrix technique in some detail and then we move on to discuss future data sets from CMB and LSS.
II. FISHER MATRIX ANALYSIS

Measuring neutrino masses from cosmological data is quite involved since for both CMB and LSS the power spectra depend on a plethora of different parameters in addition to the neutrino mass. Furthermore, since the CMB and matter power spectra depend on many different parameters one might worry that an analysis which is too restricted in parameter space could give spuriously strong limits on a given parameter. Therefore, it is desirable to study possible parameter degeneracies in a simple way before embarking on a full numerical likelihood analysis.

It is possible to estimate the precision with which the cosmological model parameters can be extracted from a given hypothetical data set. The starting point for any parameter extraction is the vector of data points, $x$. This can be in the form of the raw data, or in compressed form, typically the power spectrum ($C_l$ for CMB and $P(k)$ for LSS).

Each data point has contributions from both signal and noise, $x = x_{\text{signal}} + x_{\text{noise}}$. If both signal and noise are Gaussian distributed it is possible to build a likelihood function from the measured data which has the following form [30]

$$L(\Theta) \propto \exp \left( -\frac{1}{2} x^T [C(\Theta)^{-1}] x \right),$$

where $\Theta = (\Omega, \Omega_b, H_0, n_s, \tau, \ldots)$ is a vector describing the given point in model parameter space and $C(\Theta) = \langle x x^T \rangle$ is the data covariance matrix. In the following we shall always work with data in the form of a set of power spectrum coefficients, $x_i$, which can be either $C_l$ or $P(k)$.

If the data points are uncorrelated so that the data covariance matrix is diagonal, the likelihood function can be reduced to $L \propto e^{-\chi^2/2}$, where

$$\chi^2 = \sum_{i=1}^{N_{\text{max}}} \frac{(x_{i,\text{obs}} - x_{i,\text{theory}})^2}{\sigma(x_i)^2},$$

is a $\chi^2$-statistics and $N_{\text{max}}$ is the number of power spectrum data points [30].

The maximum likelihood is an unbiased estimator, which means that

$$\langle \Theta \rangle = \Theta_0.$$  \hfill (5)

Here $\Theta_0$ indicates the true parameter vector of the underlying cosmological model and $\langle \Theta \rangle$ is the average estimate of parameters from maximizing the likelihood function.

The likelihood function should thus peak at $\Theta \simeq \Theta_0$, and we can expand it to second order around this value. The first order derivatives are zero, and the expression is thus

$$\chi^2 = \chi^2_{\text{min}} + \sum_{i,j} (\theta_i - \theta) \left( \sum_{k=1}^{N_{\text{max}}} \frac{1}{\sigma(x_k)^2} \left[ \frac{\partial x_k}{\partial \theta_i} \frac{\partial x_k}{\partial \theta_j} - \langle x_k, \text{obs} \rangle \frac{\partial^2 x_k}{\partial \theta_i \partial \theta_j} \right] \right) (\theta_j - \theta),$$  \hfill (6)

where $i, j$ indicate elements in the parameter vector $\Theta$.

The second term in the second derivative can be expected to be very small because $\langle x_k, \text{obs} \rangle$ is in essence just a random measurement error which should average out.

The remaining term is usually referred to as the Fisher information matrix

$$F_{ij} = \frac{\partial^2 \chi^2}{\partial \theta_i \partial \theta_j} = \sum_{k=1}^{N_{\text{max}}} \frac{1}{\sigma(x_k)^2} \frac{\partial x_k}{\partial \theta_i} \frac{\partial x_k}{\partial \theta_j}.$$  \hfill (7)

The Fisher matrix is closely related to the precision with which the parameters, $\theta_i$, can be determined. If all free parameters are to be determined from the data alone without any priors then it follows from the Cramer-Rao inequality [32] that

$$\sigma(\theta_i) = \sqrt{(F^{-1})_{ii}},$$  \hfill (8)

for an optimal unbiased estimator, such as the maximum likelihood [33].

In order to estimate how degenerate parameter $i$ is with another parameter, $j$, one can calculate how $\sigma(\theta_i)$ changes if parameter $j$ is kept fixed instead of free in the analysis. Starting from the $2 \times 2$ sub-matrix

$$S_{ij} = (F^{-1})_{ij},$$  \hfill (9)

one then finds

$$\sigma_{\text{fixed}}(\theta_i) = \sqrt{\frac{1}{(S^{-1})_{ii}}}$$  \hfill (10)

with another parameter, $k$. This is almost equivalent to the mass of the heaviest mass eigenstate, $m_3$. In the remainder of the paper we therefore use $m_\nu$ and $\sum m_\nu$ interchangeably, except where otherwise stated.
We therefore define the quantity

\[ r_{ij} = \frac{\sigma_j \text{fixed}(\theta_i)}{\sigma(\theta_i)} \leq 1 \]  \hspace{1cm} (11)

as a measure of the degeneracy between parameters \( i \) and \( j \).

In the next two sections we discuss how to calculate the contributions to the Fisher matrix from LSS and CMB data respectively.

**III. LSS DATA**

At present data from the first very large scale precision galaxy surveys are becoming available. The 2dF survey has measured about 250,000 galaxies and the Sloan Digital Sky Survey [38] is currently in the process of measuring up to 10^6 galaxy redshifts.

With surveys of this scale and precision it will be possible to obtain quite precise limits on the neutrino mass. However, even for large scale surveys the problem is that massive neutrinos mainly affect the smaller scales where there are potential problems with non-linearity.

At present analyses are usually carried out using an effective cut in \( k \)-space at \( k = 0.2h \text{ Mpc}^{-1} \). This cut is placed roughly where the quantity \( \Delta^2 \), defined as

\[ \Delta^2 = \frac{V}{(2\pi)^2} 4\pi k^3 |\delta k|^2 \propto k^3 P(k), \]  \hspace{1cm} (12)

is equal to 1 in linear theory. Here, \( P(k) \) is the power spectrum of fluctuations. In most cases this corresponds reasonably well to the point where \( \Delta^2_{\text{Non-Linear}} - \Delta^2_{\text{Linear}} \sim \Delta^2_{\text{Linear}} \), and the reason for the cut-off is then that poorly understood non-linear effects begin to dominate the power spectrum at scales smaller than the cut-off.

However, for problems involving the detection of neutrino masses in the 0.1 eV range one must be even more careful. For instance, for \( m_\nu = 0.1 \text{ eV} \) the power spectrum suppression is only \( \Delta P/P \sim 0.06 \) so that any effect on the power spectrum at this level which is not well understood can impair the mass detection ability.

From simulations Peacock and Dodds [39] were able to derive the following approximate relation between the linear and non-linear spectra

\[ \Delta^2_{\text{NL}} = f_{\text{NL}}[\Delta^2_{L}], \]  \hspace{1cm} (13)

with

\[ f_{\text{NL}}[x] = \left( \frac{1 + B\beta x + [Ax]^{\alpha/\beta}}{1 + ([Ax]^{\alpha/\beta}g^3(\Omega)/[V x^{1/2}])^{\beta}} \right)^{\frac{1}{\beta}}, \]  \hspace{1cm} (14)

and

\[ A = 0.482(1 + n/3)^{-0.947} \]  \hspace{1cm} (15)
\[ B = 0.226(1 + n/3)^{-1.778} \]  \hspace{1cm} (16)

At the large scales we are interested in one finds that

\[ \frac{\Delta^2_{\text{Non-Linear}} - \Delta^2_{\text{Linear}}}{\Delta^2_{\text{Linear}}} \rightarrow Q \Delta^2_{\text{Linear}}, \]  \hspace{1cm} (21)

where

\[ Q = \frac{B\beta + A^{\alpha}(\alpha - \frac{1}{2})g^3(\Omega)V^{-\beta}}{\beta}. \]  \hspace{1cm} (22)

We take \( \Omega_m = 0.3 \) and \( \Omega_\Lambda = 0.7 \) as values for the matter and vacuum energy densities respectively.

In the above equations \( n \) is the effective spectral index of the linear power spectrum. At very small scales \( n \sim -3 \) and at scales beyond the horizon at matter-radiation equality \( n \sim 1 \). Fig. 1 shows \( Q(n) \). For the scales we are interested in \( n \sim 0 - 1 \) so that \( Q \sim 0.2 \).

In order to have a reasonably clean interpretation of the data a \( k \)-space cut should be made roughly where

\[ \frac{|\Delta^2_{\text{Non-Linear}} - \Delta^2_{\text{Linear}}|}{\Delta^2_{\text{Linear}}} \simeq \left| \frac{\Delta P(k)|_{m_\nu}}{P(k)} \right| \]  \hspace{1cm} (23)

For a neutrino mass of 0.1 eV this means that \( Q\Delta^2_{\text{Linear}}|_{k_{\text{cut}}} \simeq 0.06 \), or that \( \Delta^2_{\text{Linear}}|_{k_{\text{cut}}} \simeq 0.3 \). This requirement leads to an estimate of \( k_{\text{cut}} \simeq 0.1h \text{ Mpc}^{-1} \), which in fact is not far from the \( k_{\text{cut}} = 0.2h \text{ Mpc}^{-1} \) often used in present analyses.
A. $k$-dependent bias

In all present parameter estimation analyses it is assumed that the bias parameter, $b^2(k) \equiv P_g(k)/P_m(k)$, is independent of the scale, $k$.

However, many independent simulations find that this bias is in fact quite strongly scale dependent\cite{40,41} in the non-linear regime. In the linear regime bias is expected to be constant, and the asymptotic value $b_{\infty} = \lim_{k \to 0} b(k)$ is reached at a scale of roughly $k \approx 0.1 - 0.2 h \, \text{Mpc}^{-1}$. This means that at scales larger than $k_{\text{cut}}$ bias should be very close to scale-independent, and that we can therefore use a single parameter, $b$, to describe it.

B. Mock LSS surveys

For purposes of parameter estimation the most important parameter in galaxy surveys is the effective volume, defined as

$$ V_{\text{eff}} = \int \frac{\bar{n}(r)P(k)}{1 + \bar{n}(r)P(k)} \, d^3r. \quad (24) $$

In the above equation $n(r)$ is the selection function. The simple interpretation of $V$ is that it is the volume available for measuring power at wavenumber $k$.

In the following we shall assume that the survey is volume limited, meaning that the selection function is constant throughout the survey volume. If the survey is flux limited the selection function is much more complicated. In the region where $P(k) \gtrsim 1/\bar{n}$, $V_{\text{eff}}$ is independent of $k$ and equal to the total survey volume.

Essentially this means that, with certain restrictions, we can use just one free parameter, $V_{\text{eff}}$, to describe a hypothetical galaxy survey.

It was shown in Ref.\cite{12} that the contribution to the Fisher matrix from such a galaxy survey can be written as

$$ F_{ij} \approx 2\pi \int_{k_{\text{min}}}^{k_{\text{max}}} \frac{\partial \ln P(k)}{\partial \theta_i} \frac{\partial \ln P(k)}{\partial \theta_j} w(k) \, dk, \quad (25) $$

where the weight-function is $w(k) = V_{\text{eff}}/\lambda^3$ and $\lambda = 2\pi/k$. The upper limit of the integral should be taken to be $k_{\text{cut}}$, discussed above. In principle the lower limit, $k_{\text{min}}$, should be zero but at large scales the assumption that $P(k) \gtrsim 1/\bar{n}$ breaks down. However, by far the most of the weight in the above integral comes from $k$ close to the upper limit. Therefore, as long as the $k$ where $P(k) = 1/\bar{n}$ is much smaller than $k_{\text{max}}$ the error from taking $V_{\text{eff}}$ and $k_{\text{min}} = 0$ is quite small.

It should be noted that the above integral expression is quite crude. However, it offers a very simple way of estimating parameter estimation errors from galaxy surveys. The error arising from it can be of order a factor 2, leading to an error in the estimated $\sigma(\theta_i)$ of order $2^{1/2}$.

Instead of $V_{\text{eff}}$ we use $\lambda_{\text{eff}} = (3V_{\text{eff}}/4\pi)^{1/3}$ as the free parameter. As discussed in Ref.\cite{12} the SDSS BRG survey\cite{35} has an effective volume of roughly $(1 h^{-1} \, \text{Gpc})^3$, corresponding to $\lambda_{\text{eff}} \approx 620 h^{-1} \, \text{Mpc}$. Note that the number of independent Fourier modes on a given scale, $k$, enclosed within the survey volume is proportional to $V_{\text{eff}}$. Therefore it essentially corresponds to the factor $(2l + 1)$ for the CMB measurements which measures the number of $m$-modes for a given $l$. In that sense both $V_{\text{eff}}$ and $(2l + 1)$ are a measure of the lack of ergodicity in the given data set.

IV. CMB DATA

The CMB temperature fluctuations are conveniently described in terms of the spherical harmonics power spectrum

$$ C_l = \langle |a_{lm}|^2 \rangle, \quad (26) $$

where

$$ \frac{\Delta T}{T}(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi). \quad (27) $$

Since Thomson scattering polarizes light there are additional powerspectra coming from the polarization anisotropies. The polarization can be divided into a curl-free ($E$) and a curl ($B$) component, yielding four independent power spectra: $C_{E,l}, C_{E,l}, C_{B,l}$ and $C_{ET,l}$. In the present work we neglect the curl-free component since it is usually tiny compared with the other contributions. However, for some purposes, such as the detection of tensor fluctuation modes, it is essential. Altogether we focus on the three spectra $C_{TT}, C_{EE}$ and $C_{ET}$. The initial measurement of CMB anisotropies by COBE\cite{43} initiated the development of a new generation of high-precision CMB experiments. At present there are data sets on the temperature anisotropy from BOOMERANG\cite{44}, MAXIMA\cite{45}, DASI\cite{46}, VSA\cite{47}, CBI\cite{48}, and several other experiments, as well as polarization data from DASI\cite{46}.

In the near future data from the MAP and Planck satellites will become available. This data offers full sky coverage combined with very small pixel noise due to long integration times.

A. Foregrounds

There are several sources of foreground contamination in the CMB data. Within the galaxy these are primarily synchrotron and free-free emission, as well as dust. Extragalactic contamination is mainly due to point sources and the SZ-effect.

Several papers deal with the issue of foregrounds for future CMB experiments. In the present work we rely on the calculations by Tegmark et al.\cite{43}. Their estimate is
that for the temperature anisotropies there is little effect from foregrounds at the important frequencies around 100 GHz. The most likely source of contamination at high \(l\) is from point sources, but this effect is unlikely to be dominant for \(l \lesssim 2500\).

For the polarization anisotropy the problem with foregrounds is more severe. For \(E\)-type polarization the foreground contamination from point sources is likely to be dominant already beyond \(l \sim 1500\).

In conclusion, no CMB experiment is likely to retrieve information on the primary spectrum beyond \(l \sim 2500\) for temperature and \(l \sim 1500\) for polarization. In the present work we assume that the \(E-T\) cross-correlation is subject to the same foreground contamination as the \(E\)-polarization.

Another important foreground issue is the question of secondary CMB anisotropy generation from non-linearity, i.e. weak lensing and the Rees-Sciama effect (and of course the SZ effect discussed above). These effects have been estimated to be extremely small for the range of \(l\)-values we are interested in here (below 2500), but can be dominant on smaller scales.

Finally, there is a possible effect from inhomogeneous reionization. In all present CMB parameter estimation analyses reionization is treated as a single parameter which is either the optical depth due to reionization, \(\tau\), or the redshift of reionization, \(z_r\). However, the reionization must to some extent have been inhomogeneous and this could affect the arcminute scale CMB anisotropy. It has been estimated for instance by Gruzinov and Hu [50] that this patchy reionization is unlikely to be important for parameter estimation, but whether this is really the case is not yet clear.

For lack of a better description we consider reionization in the standard homogeneous picture using \(\tau\) as a free parameter, and note that this simplification is unlikely to cause any serious problems [54].

### B. Mock CMB experiments

In view of the discussion above we use the following prescription for a mock CMB experiment. We assume it to be cosmic variance (as opposed to foreground) limited up to some maximum \(l\)-value. This value can, however, be different for temperature and polarization detection. Therefore a given hypothetical experiment can be described by only two free parameters, \(l_{T,\text{max}}\) and \(l_{P,\text{max}}\). For all experiments it will be the case that \(l_{T,\text{max}} \geq l_{P,\text{max}}\).

In this picture the MAP data will be well described by \(l_{T,\text{max}} \simeq 1000\) and \(l_{P,\text{max}} = 0\), and the Planck data by \(l_{T,\text{max}} \simeq 2500\) and \(l_{P,\text{max}} = 1500\). In some sense Planck can therefore be regarded as the “ultimate” CMB experiment because it measures all of the power spectrum parameter space not dominated by foregrounds.

The contribution to the Fisher matrix from such a CMB experiment is then

\[
F_{ij} = \sum_{l=2}^{l_{p,\text{max}}} \sum_{X,Y} \frac{\partial C_{l,X}}{\partial \theta_i} \text{Cov}^{-1}(C_{l,X}, C_{l,Y}) \frac{\partial C_{l,Y}}{\partial \theta_j} + \sum_{l=1}^{l_{T,\text{max}}} \frac{\partial C_{l,T}}{\partial \theta_i} \text{Cov}^{-1}(C_{l,T}, C_{l,T}) \frac{\partial C_{l,T}}{\partial \theta_j}
\]

where \(X, Y = T, E, TE\).

The covariance matrices are given by [31]

\[
\text{Cov}(C_{l,T}, C_{l,T}) = \frac{2}{(2l + 1)} C_{l,T}^2 \quad (29)
\]

\[
\text{Cov}(C_{l,E}, C_{l,E}) = \frac{2}{(2l + 1)} C_{l,E}^2 \quad (30)
\]

\[
\text{Cov}(C_{l,TE}, C_{l,TE}) = \frac{2}{(2l + 1)} [C_{l,T}^2 + C_{l,T} C_{l,E}] \quad (31)
\]

\[
\text{Cov}(C_{l,T}, C_{l,E}) = \frac{2}{(2l + 1)} C_{l,T E} \quad (32)
\]

\[
\text{Cov}(C_{l,T}, C_{l,TE}) = \frac{2}{(2l + 1)} C_{l,T E} \quad (33)
\]

\[
\text{Cov}(C_{l,E}, C_{l,TE}) = \frac{2}{(2l + 1)} C_{l,E} C_{l,TE} \quad (34)
\]

It should be noted here that this approximation relies on the assumption of 4\(\pi\) sky coverage and no pixel noise up to the maximum \(l\). Even though these assumptions are not realised in any real experiment they are sufficiently accurate for estimating the parameter estimation accuracy of a given experiment.

### V. NUMERICAL RESULTS

In order to calculate estimated 1\(\sigma\) errors on the various cosmological parameters we need to apply the Fisher matrix analysis to a specific cosmological model.

We choose as the reference model the generic \(\Lambda\)CDM model with the following free parameters: \(\Omega_m\), the matter density, \(\Omega_A\), the vacuum energy density, \(\Omega_b\), the baryon density, \(H_0\), the Hubble parameter, \(n_s\), the spectral index of the primordial perturbation spectrum, \(\tau\), the optical depth to reionization, \(Q\), the spectrum normalization, \(b\), the bias parameter, and \(m_\nu\), the neutrino mass. The reference model has the following parameters: \(\Omega_m = 0.3\), \(\Omega_A = 0.7\), \(\Omega_b h^2 = 0.02\), \(H_0 = 70\) km s\(^{-1}\) Mpc\(^{-1}\), \(n_s = 1\), \(\tau = 0\), and \(m_\nu = 0.07\) eV.

#### A. CMB data only

Even though the canonical view of cosmological neutrino mass determination is that CMB in itself can achieve little because the CMB spectrum is insensitive to \(m_\nu\), experiments such as Planck will in fact be able to probe \(m_\nu\) precisely.
FIG. 2: The change in $C_l$ due to a neutrino mass of 0.07 eV. The shaded band is the cosmic variance error of $\Delta C_l/C_l = 1/\sqrt{2l+1}$.

As an example of the effect of massive neutrinos on the CMB we show the change in temperature and $E$-type polarization spectra for a neutrino mass of 0.07 eV in Fig. 2.

In Fig. 3 we show the expected 1σ error bar in a determination of $m_\nu$ from a hypothetical future CMB experiment, without invoking any other data whatsoever.

The hatched region beyond $l_{P,\text{max}} = 1500$ corresponds to the region where the CMB signal is dominated by foregrounds, and therefore unlikely to be retrieved in any measurement. The thick line corresponds to $\sigma(m_\nu) = 0.07$ eV, i.e. the central value suggested by current oscillation analyses provided that neutrino masses are hierarchical.

As can be seen from the figure the Planck satellite in itself without any additional information could be able to provide a marginal detection of a non-zero $m_\nu$. However, an unambiguous result from CMB alone seems unlikely and in fact the argument should rather be reversed, as was emphasized in Ref. [27]: Even a neutrino mass as small as 0.07 eV will have a significant effect on the power spectrum at high $l$ and can bias the estimation of other parameters unless accounted for.

FIG. 3: Estimated 1σ error bars on $m_\nu$ from the Fisher matrix analysis using only CMB data. The thick solid line corresponds to $m_\nu = 0.07$ eV, and the hatched region above $l_{P,\text{max}} = 1500$ is the region where the CMB signal is dominated by foregrounds. The blank triangle in the upper left corner is the region where $l_{P,\text{max}} > l_{T,\text{max}}$.

B. The addition of LSS data

Adding data from LSS surveys markedly improves the ability to detect a non-zero $m_\nu$.

In Figs. 4 and 5 we show how the expected 1σ error bar is changed for the MAP and Planck experiments when LSS data is added.

In the near future there will be data available from the Sloan Digital Sky Survey, for which the Bright Red Galaxy (BRG) survey in our language corresponds to $\lambda \simeq 620h^{-1}$ Mpc. In Ref. [27] the expected 1σ error on $m_\nu$ was calculated for the case of combining the SDSS BRG survey and the MAP satellite data, taking the $k$-space cut to be at $k_{\text{cut}} = 0.2h$ Mpc$^{-1}$. For this specific case the error was estimated to be $\sigma(m_\nu) \simeq 0.25 - 0.3$ eV. From our calculation we find for the same case an expected error of 0.23 eV, which can be ascribed to the fact that our estimated MAP precision is higher than that used in Ref. [27].

For the case of Planck + SDSS we estimate the 1σ error to be of the order 0.06 eV. This means that adding the SDSS data to the Planck data only improves the detection threshold marginally (from 0.07 eV to 0.06 eV).

In order to achieve a 2σ detection of a neutrino mass of 0.06 eV, which is the most favoured value using the present data, it is necessary to combine the Planck data with data from a galaxy survey with $\lambda \sim 3h^{-1}$ Gpc (if $k_{\text{cut}} = 0.1h$ Mpc$^{-1}$). Such a survey would contain a large fraction of the present Hubble volume and is probably not feasible. If, on the other hand, numerical simulations could provide the ability to disentangle the non-linear effects beyond $k \sim 0.1h$ Mpc$^{-1}$ a much smaller survey would suffice. For instance if $k_{\text{cut}} = 0.5h$ Mpc$^{-1}$ the survey volume needed for a 2σ detection would be “only” 1500 $h^{-1}$ Mpc.
Finally, we note that all of these calculations were done for the case where there is no prior information on any cosmological parameters. If the neutrino mass were the only free parameter, SDSS+Planck could achieve a $1\sigma$ accuracy of 0.003 eV, corresponding to a many-sigma detection of a neutrino with mass of order 0.06 eV. This is of course wishful thinking in the sense that no other experiment is likely to yield such information on the cosmological parameters.

In Fig. 6 we show a matrix of the parameter $r_{ij}$ for the case of Planck+SDSS data. We note that the main parameter degeneracies in this case are with $H_0$ and $\Omega_b$, not with $\Omega_m$ and $\Lambda$ as with the present data. The estimated $1\sigma$ errors on these two parameters from the Fisher matrix analysis are $\Delta H_0/H_0 = 0.018$ and $\Delta \Omega_b/\Omega_b = 0.025$. If it were possible to measure these two parameters more precisely by other means the expected precision of the neutrino mass determination could be improved by almost a factor 2, to a level of 0.03 eV for SDSS+Planck.

VI. DISCUSSION

We have discussed in detail the prospects for detecting neutrino masses with CMB and large scale structure observations, extending the pioneering calculation in Ref. [27].

From the Sloan Digital Sky Survey and the Planck data we estimated that it would be possible to obtain a $1\sigma$ error bar on the neutrino mass of roughly 0.06 eV. It should be noted that this estimate is probably fairly safe in the sense that it requires only robust assumptions. For instance it requires only LSS data at scales larger than $k \approx 0.1 \, h \, Mpc^{-1}$ where effects of non-linearity are negligible.

This data would allow for a marginal detection of the neutrino mass if the neutrino mass is 0.06 eV, the value favoured by current oscillation data. Maltoni et al. [3] find that the most massive mass eigenstate should be at $m \sim 0.04 - 0.07$ eV (assuming the LMA Solar solution). Therefore it seems unlikely that neutrino masses will be detected by SDSS+Planck data, unless additional information can be retrieved. Perhaps the most promising prospect for this is that detailed numerical simulations will allow for a disentanglement of the non-linear effects in LSS data at high $k$. If SDSS data up to $k \approx 0.5 \, h \, Mpc^{-1}$ could be used the error would diminish to 0.04 eV.

However, in order to achieve an unambiguous detection of a neutrino mass of 0.06 eV a much larger survey than the SDSS is needed. Whether such a survey becomes feasible in the future remains to be seen, but with improvement in detector technology and the easier access to 8 meter class telescopes it may not be impossible.

It should also be noted that in the case of an inverted mass hierarchy there will be two mass eigenstates of almost degenerate mass, so that the atmospheric neutrino mass...
data indicates a sum of order 0.12 eV. Such a scenario would be detectable at the 2σ level with Planck+SDSS.

All of the mass constraints discussed here should be compared with the expected precision of other, planned experiments. The KATRIN tritium endpoint experiment [54] is planned to measure the effective electron neutrino mass, \( m_{\nu_e} = \left( \sum |U_{e j}|^2 m_j^2 \right)^{1/2} \), to an accuracy of 0.35 eV (95% conf.). This is better than the 95% accuracy expected from MAP+SDSS (0.45 eV at 95% conf.), but not competitive with the 0.12 eV (95% conf.) expected from SDSS+Planck. However, it is possible that tritium decay experiments can be pushed to the limit of \( m_{\nu_e} \lesssim 0.06 \) eV, which would indicate hierarchical masses.

If neutrinos are Majorana particles processes such as neutrinoless double beta decay will become possible. The detection of \( 2 \beta 0 \) decays probes the mass combination

\[
    m_{ee} = \sum_j |U_{ej}|^2 m_j^2
\]

and has led to a current upper bound of 0.27 eV [3]. Future experiments, such as GENIUS [24], could take the accuracy to 0.01 eV, much better than the expected accuracy from cosmology. However, this requires that neutrinos are Majorana particles and that there is no cancellation of terms in the expression for \( m_{ee} \).

In conclusion, using LSS and CMB data which will become available within the next 8-10 years and reasonable assumptions it should become possible to achieve a 95% confidence limit on the mass of the heaviest neutrino mass eigenstate of 0.12 eV. This is almost at the level expected if neutrino masses are hierarchical, and with a future generation of LSS survey it seems feasible to detect neutrino masses as low as 0.03-0.06 eV at 95% confidence. The timescale for such future surveys is comparable to, or shorter than, the timescale for building tritium endpoint experiments beyond KATRIN. Therefore there is good reason to be optimistic regarding the future potential of cosmology to probe neutrino masses.

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