Two-Coupled Chains with Spin-Anisotropic Backward Scattering

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(Received 16 March 1998; revised manuscript received 16 April 1998)

By applying renormalization group method to the bosonized Hamiltonian of two-coupled chains with repulsive intrachain interaction, we have examined a role of backward scattering with a spin-anisotropy which competes with interchain hopping. From calculation of a dominant state in the limit of low energy, it is found that superconducting state moves into spin density wave state when the anisotropy becomes larger than a critical value. Further phase diagram is shown on the plane of $g$-ology.

PACS numbers: 71.10.Hf, 74.70.Kn, 75.30.Fv, 75.30.Gw

keywords: d-wave superconductor, fluctuation effects, Hubbard model, organic superconductor, spin density wave

I. INTRODUCTION

In low dimensional electron systems, which consist of an array of one-dimensional chains with repulsive interaction, a noticeable phase diagram has been found where spin density wave (SDW) state competes with superconducting (SC) state. Among these materials, the organic conductors, (TMTSF)$_2$X salts, have shown the SC state with $d$-wave like pairing. Such a SC state is located close to the SDW state which exhibits one-dimensional fluctuation originating in intrachain interaction and a small interchain electron hopping. In addition to phenomena arising from the repulsive intrachain interaction, these materials exhibit anisotropy of spin susceptibility in the SDW state with the easy axis (b-direction) perpendicular to the conducting chain although anisotropy is invisible above the SDW transition temperature.

For studying the competition between the SC state and the SDW state, we examine two-coupled chains with spin-anisotropic interaction. The electronic state which is obtained in a single chain with repulsive interaction is quite different from those of quasi-one-dimensional system of these conductors in which the SC state shows $d$-wave like pairing denoting electrons between chains. Therefore, we need at least two chains treated correctly although infinite number of chains gives rise to a phase transition at finite temperatures. Actually such a SC state in case of weak coupling of Hubbard model has been obtained not only for two-coupled chains but also for three-coupled chains and speculated for arbitrary number of coupled-chains. It seems that the dominant state in two-coupled chains is relevant to electronic states for the above systems if we note that electronic states of quasi-one-dimensional conductors may be obtained as a result of developing these short range correlations perpendicular to chain.

Two-coupled chains with spin-isotropic interaction have been studied extensively. In case of forward scattering described by Tomonaga-Luttinger model, the ground state given by the density wave is the same as that of a single chain except for removing degeneracy of in-phase and out-of-phase pairings. On the other hand, two-chains of Hubbard model with repulsive backward scattering leads to the ground state given by the $d$-wave like SC state while that of a single chain is the SDW state. Numerical studies for the Hubbard model also show an enhancement of SC state for some ranges of parameters of interaction, interchain hopping and filling. Analysis in terms of the bosonization indicates the fact that such a SC state appears due to formation of a gap of the total spin fluctuation, which is triggered by the relevance of transverse charge fluctuation. As for the spin-anisotropic interaction, the one-dimensional model has been explored to study the SDW state of above organic conductors.

In the present paper, the effect of spin-anisotropic backward scattering on the electronic state of two-coupled chains is examined by improving the previous method of the renormalization group, which is based on the bosonization. In section 2, formulation for renormalization group equations is given where the backward scattering with parallel spins is distinguished from that with opposite spins to examine the spin-anisotropic interaction. In section 3, renormalization group equations are calculated numerically and the most dominant state in the limit of energy is examined where phase diagram of SC state and SDW state are shown on the plane of coupling constants and interchain hopping. Discussion is given in section 4.

II. FORMULATION

We consider the Hamiltonian for two-coupled chains given by

$$
\mathcal{H} = \sum_{k,p,\sigma,i} \epsilon_{k,p} a_{k,p,\sigma,i}^{\dagger} a_{k,p,\sigma,i} - t \sum_{k,p,\sigma} \left[ a_{k,p,\sigma,1}^{\dagger} a_{k,p,\sigma,2} + h.c. \right]
$$

arXiv:cond-mat/9805270v2 [cond-mat.str-el] 23 May 1998
of the interchain hopping. Coupling constants are given by
\[
\gamma = \frac{\pi v_F}{\sqrt{2} \gamma} \quad \text{and} \quad \gamma_\perp = \frac{\pi v_F}{\sqrt{2} \gamma_\perp},
\]
and
\[
\gamma = \frac{\pi v_F}{\sqrt{2} \gamma} \quad \text{and} \quad \gamma_\parallel = \frac{\pi v_F}{\sqrt{2} \gamma_\parallel},
\]
respectively.

The quantity \( a_{k,p,\sigma,i} \) denotes a creation operator for the electron with momentum \( k \), spin \( \sigma (= \uparrow, \downarrow \text{ or } +, -) \) where \( p = + (\text{or} -) \) represents the right-going (left-going) state and \( i (= 1, 2) \) denotes chain index. In Eq. \((2.1)\), \( \epsilon_{k,p}(= v_F (pk - kF)) \) is the kinetic energy with Fermi velocity \( v_F \) and Fermi momentum \( k_F \). The energy, \( t \), is that of the interchain hopping. Coupling constants \( g_{\perp} \) and \( g_{\parallel} \) represent the intrachain interaction of the backward scattering with parallel and opposite spins respectively and \( g_2 \) is the coupling constant of the forward scattering.

By use of the unitary transformation given by \( c_{k,p,\sigma,\mu} = (-\mu a_{k,p,\sigma,\mu} + a_{k,p,\sigma,\mu})/\sqrt{2} (\mu = \pm) \), the first and the second terms in Eq. \((2.2)\) are diagonalized resulting in the separation of the Fermi wave number, \( k_{F\mu} \equiv k_F - \mu t/v_F \). Based on the bosonization around the new Fermi point, we introduce the phase variables, \( \theta_+ \) and \( \phi_+ (\hat{\theta}_+ \text{ and } \hat{\phi}_+) \), expressing fluctuations of the total (transverse) charge density and spin density respectively. These variables are defined by \( \sum_{\mu \neq 0} \frac{2\pi i}{(qL)} e^{-\alpha q/2 - iq\tau} c_{k+q,p,\sigma,\mu} c_{k,p,\sigma,\mu} = (\theta_+ + \phi_+ + \phi_{-} + \phi_- + \phi_{\perp} + \phi_{\parallel} + \phi_{\perp} + \phi_{\parallel})/2\sqrt{2} \).

They satisfy the relation, \( \theta_+(x), \theta_-(x') = [\phi_+(x), \phi_-(x')] = [\hat{\phi}_+(x), \hat{\phi}_-(x')] = i \pi \text{sgn}(x - x') \) where the suffix - denotes the canonically conjugate variable. The field operator is also expressed as \( \psi_{\mu,\sigma,\mu}(x) = L^{-1/2} \sum_{\mu} e^{ikx} c_{k,p,\sigma,\mu} = 1/\sqrt{2\pi} \epsilon \exp(ipk_F x + i\Theta_{p,\sigma,\mu}) \exp(i\pi \Sigma_{p,\sigma,\mu}) \) where \( \Theta_{p,\sigma,\mu} = p\theta_+ + \sigma(p\phi_+ + \phi_-) + \mu(p\phi_- + \theta_-) + \sigma(\mu(p\phi_- + \phi_-))/2\sqrt{2} \) and \( \alpha \) is of the order of the lattice constant. For keeping the anticommutation relation, the phase factor, \( \pi \Sigma_{p,\sigma,\mu} \), is chosen as \( \Sigma_{2n+\tau} = N_1 + \cdots + N_{2n+\tau} \).

The index \( \mu \) is defined by \( i = (p, \sigma, \mu) \) where \((+, +, +) = 1, (+, +, -) = 2, (+, -,-) = 3, (-, +, -) = 4, (-, +, +) = 5, (-, -,-) = 6, (-, -,-) = 7, (-, -,-) = 8 \) respectively. Such a choice of \( \Sigma_{p,\sigma,\mu} \) leads to a conservation of a sign of interactions. Thus Eq. \((2.1)\) is rewritten as
\[
\mathcal{H} = \frac{v_F}{4\pi} \int dx \left\{ \frac{1}{K_{\theta}} (\partial \theta_+)^2 + K_{\theta} (\partial \theta_-)^2 \right\} + \frac{v_F}{4\pi} \int dx \left\{ \frac{1}{K_{\phi}} (\partial \phi_+)^2 + K_{\phi} (\partial \phi_-)^2 \right\}
\]
\[
+ \frac{1}{2\pi^2 a^2} \int dx \left[ g_a \cos \left( \sqrt{2} \theta_+ - 4tx/v_F \right) \cos \sqrt{2} \phi_+ + g_b \cos \left( \sqrt{2} \theta_- - 4tx/v_F \right) \cos \sqrt{2} \phi_- \right]
\]
\[
+ g_c \cos \sqrt{2} \theta_+ \cos \sqrt{2} \phi_+ + g_d \cos \sqrt{2} \theta_- \cos \sqrt{2} \phi_- \right]
\]
\[
+ g_e \cos \sqrt{2} \phi_+ \cos \sqrt{2} \phi_- \right]
\]
\[
R_A(x_1 - x_2, \tau_1 - \tau_2) \equiv \left\{ T_A(x_1, \tau_1) O_A(x_2, \tau_2) \right\}, \quad \text{where} \quad \tau_1 \text{ is the imaginary time and } O_A \text{ denotes operator for order parameter of SDW and SC states. In order to calculate second order renormalization group equations for all the coupling constants, we introduce } g_\phi, g_\theta, \text{ and } g_\phi, \text{ given by}
\]
\[
K_{\phi} = 1 + g_\phi/2v_F \quad \text{where} \quad g_\phi = g_\parallel = 0. \quad \text{By making use of } G_\phi = g_\phi/2v_F, \text{ renormalization group equations are obtained as}
\]
\[
d\frac{d}{d\ell} G_\phi(l) = -\frac{1}{2} \left( \hat{G}_\phi^2(l) J_0(y(l)) + G_\phi^2(l) + G_\phi^2(l) + G_\phi^2(l) \right),
\]
\[
d\frac{d}{d\ell} G_\theta^2(l) = \frac{1}{2} \left( -G_\theta^2(l) J_0(y(l)) + G_\theta^2(l) J_0(y(l)) + G_\theta^2(l) + G_\theta^2(l) - g_\theta^2(l) J_0(y(l)) + g_\theta^2(l) \right),
\]
where the detailed derivation is shown in Appendix A. In the above equations, \( \hat{t}(l) = t(l)/(v_F \alpha^{-1}) \), \( y(l) = 4\hat{t}(l) \) and \( J_n(y(l)) \) \((n = 0, 1)\) is the \( n \)-th order Bessel function. Initial conditions are given by \( G_z(0) = g_z/2\pi v_F \) and \( \hat{t}(0) = t/(v_F \alpha^{-1}) \). These equations retain the SU(2) symmetry for \( g_{\parallel} = g_{\perp} = g_2 \). In Eqs. (2.7)-(2.14), the bilinear term with respect to \( G_z(l) \) \((z = a \sim h)\) does exist for two-coupled chains with backward scattering. Such a term, which has been obtained by improving the previous calculation, is crucial to reproduce the one-dimensional renormalization equations in the limit of small \( t \). We examine order parameters for the possible states in case of repulsive interaction, which are given by

\[
O_{\text{LSDW}} = \sum_{\sigma} \left\{ \psi_{+\sigma,1}^{\dagger} \psi_{-\sigma,-2} - \psi_{+\sigma,2}^{\dagger} \psi_{-\sigma,-1} \right\}
\]

\[
O_{\text{CDW}} = \sum_{\sigma} \left\{ \psi_{+\sigma,1}^{\dagger} \psi_{-\sigma,2} - \psi_{+\sigma,2}^{\dagger} \psi_{-\sigma,1} \right\}
\]

\[
O_{\text{TS}} = \sum_{\sigma} \left\{ \psi_{+\sigma,1}^{\dagger} \psi_{-\sigma,2} + \psi_{+\sigma,2}^{\dagger} \psi_{-\sigma,1} \right\}
\]

where \( \psi_{p,\sigma,i}(x) = (1/\sqrt{L}) \sum_k e^{ikx} a_{k,p,\sigma,i} \). In Eqs. (2.16)-(2.20), \( \text{LSDW}_{\downarrow,\text{out}} \) (TSDW\( \downarrow,\text{out} \)) and CDW\( \downarrow,\text{out} \) denote longitudinal (transverse) SDW with intrachain and out-of-phase pairing and charge density wave (CDW) with intrachain and out-of-phase pairing while SS\( \downarrow,\text{in} \) (TS\( \downarrow,\text{in} \)) represents the singlet (triplet) SC state with interchain and in-phase pairing. Since there is a symmetry with respect to \( g_{\parallel} = 2g_2 \) in Eq. (2.4), the case with \( 2g_2 > g_{\parallel} > 0 \) is investigated explicitly where the relevant order parameters in terms of phase variables are reexpressed as
We study only 2kF-SDW and SC states and do not consider 4kF-CDW found for \(K_g < 1/2\), of which denotes the range beyond that of the conventional Hubbard model.

**III. SS State VS. SDW State**

We examine the state in the limit of low energy by calculating Eqs. (2.4)-(2.13) where the corresponding energy is given by \(\epsilon_F\). The numerical calculation is performed with the fixed \(t/\epsilon_F = 0.1\) or 0.01 and several choices of coupling constants in the region of \(2g_2 > g_{11} > 0\). The quantity \(\epsilon_F(= \nu_F \alpha^{-1})\) denotes a Fermi energy and the coupling constant, which has the magnitude of the order of the bandwidth, is given by \(\pi v_F\). Equations (2.4), (2.5) and (2.6) show that coupling constants corresponding to the total spin, the transverse charge and the transverse spin are renormalized leading to the excitation gap for the relevant case while that of the total charge is unrenormalized. We find that, among Eqs. (2.21), (2.22) and (2.23), LSDW (TSDW) state becomes dominant for the relevant \(\phi_+, \theta_-\) and \(\phi_-, \theta_+\) (\(\phi_-, \theta_-\) and \(\phi_+, \theta_+\)) and SC state becomes dominant for the relevant \(\phi_+, \theta_-\) and \(\phi_-, \theta_+\).

![FIG. 1: The l-dependence of \(K_\phi(l)\), \(K_\theta(l)\), \(K_{\phi_+}(l)\) and \(t(l)/\epsilon_F\) for \(t/\epsilon_F = 0.1\) and \(g_{11}/2\pi v_F = g_{11}/2\pi v_F = g_2/2\pi v_F = 0.4\). Here \(K_\nu(l) = 1 + G_\nu(l)\). The dotted curves denote \(K_\nu(l)\) of one-dimensional case given by \(t = 0\).](image1.png)

First the case with spin-isotropic interaction is shown in Fig. 1 where \(g_{11}/2\pi v_F = g_{11}/2\pi v_F = g_2/2\pi v_F = 0.4\). In this case, response function of LSDW is the same as that of TSDW for \(g_{11} = g_{11}\) due to the SU(2) symmetry for the spin rotation. The quantity \(t(l)\) is always relevant i.e. increases steeply for the present choice of parameters. Renormalization of coupling constants shows that \(G_\nu(l)\) and \(G_\theta(l)\) become relevant while \(G_\phi(l)\) and \(G_\phi(l)\) become irrelevant due to the spatially oscillating term in Eq. (2.2) and both \(|G_\phi(l)/G_\phi(l)|\) and \(|G_\phi(l)/G_\phi(l)|\) reduce to zero. Since \(K_\phi(l)\) and \(K_\phi(l)\) decrease and \(K_\phi(l)\) increases, it turns out that the fluctu-ations for \(\phi_+, \phi_+\) and \(\theta_+\) become relevant. The resultant excitation gap leads to a locking of phases at \(\sqrt{2} \phi_+ = 0\), \(\sqrt{2} \phi_+ = 0\) and \(\sqrt{2} \phi_+ = \pi\) (or \(\sqrt{2} \phi_+ = \pi\) and \(\sqrt{2} \phi_+ = \pi\) because the signs of the renormalized coupling constants for large \(l\) are given by \(G_\phi(l) > 0\), \(G_\phi(l) < 0\) and \(G_\phi(l) > 0\)). Thus the dominant state in the limit of low energy is given by SC state with the singlet pairing of interchain and in-phase. Such a result of SC state is consistent with the previous one.

For parameters used in Fig. 1, SS (SDW) state becomes dominant compared to SDW (SS) state when the energy is smaller (larger) than a magnitude of \(\epsilon_F \exp[-6.4]\). For the comparison, the result for a single chain (i.e., \(t = 0\)) is shown by the dotted curve where \(K_\phi(l) = K_\phi(l) = 1\) and \(K_\phi(l)\) decreases to 1. The difference between the solid curve and the dotted curve is attributable to the interchain hopping and appears for \(l \gtrsim 1\), which corresponds to \(4t(l)/\epsilon_F \gtrsim 1\).

![FIG. 2: The l-dependence of \(K_\phi(l)\) with the fixed \(g_{11}/g_{11} = 1, 1.02, 1.04\) and 1.06 where \(t/\epsilon_F = 0.1\) and \(g_{11}/2\pi v_F = g_2/2\pi v_F = 0.4\).](image2.png)

Next the state for the spin-anisotropic interaction is examined by varying the magnitude of the backward scattering with opposite spins, \(g_{11}\). The case of \(g_{11} > 1\) is calculated where response function for LSDW becomes larger than that for TSDW. The case of \(g_{11} < 1\) is discussed in section 4. In Fig. 2, the l-dependence of \(K_\phi(l)\) is shown by choosing \(g_{11}/g_{11} = 1, 1.02, 1.04\) and 1.06 where \(g_{11}/2\pi v_F = g_2/2\pi v_F = 0.4\) and \(t/\epsilon_F = 0.1\). There are two kinds of fixed points depending on the
magnitude of $g_{1\perp}/g_{1\parallel}$. The quantity $K_\perp(l)$ with large $l$ decreases for $g_{1\perp}/g_{1\parallel} < 1.035$ (case (I)) but increases otherwise (case (II)) although $K_\perp(l)$ with small $l$ always decreases. In case (I), the behavior of renormalization is similar to that of the spin-isotropic case and then SC state becomes a dominant state. In case (II), the rapid increase of $K_\perp(l)$ shows the relevance of $\phi_-$ instead of $\phi_+$ and then leads to the reduction of $|G_e(l)/G_d(l)|$ and $|G_{\phi}(l)/G_h(l)|$ which is opposite to the case (I). From Eq. (2.21), the dominant state in the limit of low energy for the case (II) is given by LSDW with intrachain and out-of-phase pairing since the signs of relevant couplings are given by $G_d(l) < 0$, $G_f(l) < 0$ and $G_h(l) < 0$ and then phases are locked by $\sqrt{2}\phi_- = 0$, $\sqrt{2}\phi_+ = 0$ and $\sqrt{2}\theta_- = 0$ (or $\sqrt{2}\phi_- = \pi$, $\sqrt{2}\phi_+ = \pi$ and $\sqrt{2}\theta_- = \pi$). In the one-dimensional case, the dominant state with $g_{1\perp} > g_{1\parallel}$ is given by LSDW due to the excitation gap induced in the spin fluctuation. For two-coupled chains with $g_{1\perp} > g_{1\parallel}$, the difference between response function of LSDW and that of TSDW is enhanced for $l \gtrsim |\ln(t^*/\epsilon_F)|$, where $t^*$ is renormalized as $t$ due to onedimensional fluctuation. In the case (II), the dominant state is given by LSDW for all the energies while a crossover from LSDW to SC state with decreasing energy is obtained in the case (I).

![FIG. 3: Phase diagram on the plane of $g$ and $g_{1\perp}/g_{1\parallel}$ where where $g = g_2 = g_{1\parallel}$. The regions (I) and (II) correspond to SC state SDW state respectively and solid (dotted) curve denotes the boundary for $t/\epsilon_F = 0.1$ ($t/\epsilon_F = 0.01$). In the inset, phase diagram on the plane of $t$ and $g_{1\perp}/g_{1\parallel}$ with the fixed $g/2\pi\nu_F = 0.4$ is shown.](image)

In Fig. 3, the ground state is shown on the plane of $g$ and $g_{1\perp}/g_{1\parallel}$ with the fixed $t/\epsilon_F = 0.1$ (solid curve) and 0.01 (dashed curve) where $g = g_2 = g_{1\parallel}$. The regions (I) and (II) denote SS, SDW, TS and CDW states which are given by Eqs. (2.17), (2.16), (2.20) and (2.19) respectively. In the inset, the phase diagram of SS (I) and SDW (II) state on the plane of $g$ and $t$ in the case of $g_2 = g_{1\parallel} = g$ is shown where the solid curve and dashed curve denote the boundaries for $g_{1\perp}/g_{1\parallel} = 1.04$ and 1.02 respectively.

In Fig. 4, the phase diagram is shown on the plane of $g_2$ and $g_{1\parallel}$ with the fixed $g_{1\perp}/g_{1\parallel} = 1.04$ and $t/\epsilon_F = 0.1$. The regions (I), (II), (III) and (IV) correspond to states for SS, LSDW, TS and CDW, which are given by Eqs. (2.17), (2.16), (2.20) and (2.19) respectively. There is a symmetry given by $g_2 \rightarrow g_{1\parallel}$ - $g_2$ with the fixed $g_{1\perp}$ for the magnitude of response functions where response function of LSDW becomes equal to that of TS at $g_{1\parallel} = 2g_2$. In the inset, the phase diagram of SS (I) and LSDW (II) on the plane of $g$ - $t$ is shown for $g_{1\perp}/g_{1\parallel} = 1.04$ (solid curve) and $g_{1\perp}/g_{1\parallel} = 1.02$ (dashed curve). Since these curves are well reproduced by the formula, $t/\epsilon_F = \exp[-C_1 - C_2 \cdot 2\pi\nu_F/g]$, with $C_1 \approx 1.3(1.2)$ and $C_2 \approx 2.6(1.2)$ for $g_{1\perp}/g_{1\parallel} = 1.04(1.02)$, it is found that the boundary is determined by the competition between the interchain hopping and the spin gap induced by the spin-anisotropy interaction.

SC state moves to the LSDW state with increasing $g$. Such a critical value decreases with increasing $g_{1\perp}/g_{1\parallel}$ and/or decreasing $t/\epsilon_F$. The inset shows a phase diagram of SC state (I) and LSDW state (II) on the plane of $t$ and $g_{1\perp}/g_{1\parallel}$ with the fixed $g/2\pi\nu_F = 0.4$ where the SC (LSDW) state is obtained with increasing interchain hopping (anisotropy of interaction). For SC state, there is an energy gain by the $g_0$-term which competes with the $g_{\perp}$-term and then becomes relevant for energy smaller than $t$. On the other hand, the energy gain for LSDW comes from the $g_0$-term, which is smaller but cooperates with the $g_{\perp}$-term for all the energies.

![FIG. 4: Phase diagram for two-coupled chains on the plane of $g_2$ and $g_{1\parallel}$ where $t/\epsilon_F = 0.1$ and $g_{1\perp}/g_{1\parallel} = 1.04$. Regions (I), (II), (III) and (IV) denote SS, SDW, TS and CDW states which are given by Eqs. (2.17), (2.16), (2.20) and (2.19) respectively. In the inset, the phase diagram of SS (I) and SDW (II) state on the plane of $g$ and $t$ in the case of $g_2 = g_{1\parallel} = g$ is shown where the solid curve and dashed curve denote the boundaries for $g_{1\perp}/g_{1\parallel} = 1.04$ and 1.02 respectively.](image)
IV. DISCUSSION

By applying the renormalization group method within the second order, we have investigated the dominant state in the limit of low energy for two-coupled chains with the spin-anisotropic backward scattering. The phase diagram where SDW and SS states are calculated as the function of $g_{1\perp}$, $g_2$ and $t$ shows a noticeable interplay of the interchain hopping and the intrachain interaction.

In the previous section, we have examined the case $g_{1\perp} > g_{1\parallel}$, which leads to a competition between SC state and SDW state. For $g_{1\perp} < g_{1\parallel}$, we have found that results are always similar to those of Fig. 1 and then the dominant state is given by SC state. The difference between the case of $g_{1\perp} > g_{1\parallel}$ and that of $g_{1\perp} < g_{1\parallel}$ originates in the fact that the renormalization for $g_{1\perp} > g_{1\parallel}$ ($g_{1\perp} < g_{1\parallel}$) leads to a strong (weak) coupling coupling regime in a one-dimensional model.

The case of $g_{1\perp} > g_{1\parallel}$, which leads to LSDW state, denotes interactions which enhance the anisotropy along the direction parallel to the quantized axis. When the anisotropy is large for the perpendicular direction, one can expect TSDW in a similar way by adding interactions with the perpendicular anisotropy, which have been treated in the one-dimensional model.

We compare the present result with that of Fabrizio who obtained the SC state for two-coupled chains of the Hubbard model by use of a diagrammatic method. The relation between our coupling constants and those of him is given by $G_0 = g^\parallel + g_f^\parallel - g_f^{(2)}, G_\phi = g^\parallel + g_f^\parallel + g_f^{(2)} = g^\parallel + g_f^{(2)}, G_\sigma = g^\parallel - g_f^{(2)}, G_a = 2g_b, G_d = 2g_t, G_e = 2g_b, G_f = -2g_f^{(2)}, G_g = -2g_f^{(2)}$ and $G_h = -2g_f^{(2)}$. He has derived eight equations by taking account of the SU(2) symmetry in the spin space, with the condition, $g^\parallel = g_f^{(1)} - g_f^{(2)}$. We have obtained twelve renormalization group equations, i.e., Eqs. (2.4)-(2.14), for coupling constants which become equal to his results within the second order under the assumption of the symmetry. Our renormalization group equations has been derived to cover the space where the spin-anisotropic interaction can be treated.

Finally, we comment on the anisotropy of SDW state of organic conductors where the experimental evidence has been shown by spin susceptibility and the origin has been discussed in terms of spin-orbit coupling and dipole-dipole interaction. In the present paper, we have shown in Figs. 3 and 4 that a transition from SDW state to SC state occurs in the presence of spin-anisotropic backward scattering by the increase of the interchain hopping and/or the decrease of the renormalized coupling constant. Although such a transition might be expected under pressure resulting in the increase of band width and then $\nu_F$, the critical value for $(g_{1\perp} - g_{1\parallel})/g_{1\parallel}$, which is evaluated by the moderate choices of parameters (i.e., $g = \pi \nu_F$ and $t/eg = 0.1$), is much larger than that found in the experiment. It is rather plausible to consider that the transition from SDW state to SC state in organic conductors comes from the suppression of the nesting condition. However the results of the present calculation indicate a reasonable magnitude of spin-anisotropic interaction for the competition between SDW state and SC state, which may be found in quasi-one-dimensional systems.

Acknowledgment

The authors thank to H. Yoshioka for useful discussions. This work was partially supported by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports and Culture, (No.09640429) Japan.

APPENDIX A: DERIVATION OF RENORMALIZATION GROUP EQUATIONS

By treating the nonlinear terms in Eq. (2.3) as the perturbation, the response function for $\phi_\pm$ field is calculated up to the third order as

$$
\left\langle T_e(i/\sqrt{2})\phi_+(x_1,r_1)e^{-i(\sqrt{2})\phi_+(x_2,r_2)} \right\rangle
= e^{-(K_\phi/2)U(r_1-r_2)}
+ \frac{1}{2!} \frac{1}{(4\pi)^2} \sum_{e=\pm 1} \int \frac{d^2r_3}{\alpha^2} \frac{d^2r_4}{\alpha^2} e^{-(K_\phi/2)U(r_1-r_2)e^{-(2K_\phi)U(r_3-r_4)}}
\times \left\{ G_e^{(2)} e^{-(K_\phi)U(r_3-r_4)} \cos 2q_0(x_3-x_4) + G_i^{(2)} e^{-(2/K_\phi)U(r_3-r_4)} + G_g^{(2)} e^{-(2K_\phi)U(r_3-r_4)} + G_h^{(2)} e^{-(2/K_\phi)U(r_3-r_4)} \right\}
- \frac{1}{3!} \frac{1}{(4\pi)^3} \sum_{e=\pm 1} \int \frac{d^2r_3}{\alpha^2} \frac{d^2r_4}{\alpha^2} \frac{d^2r_5}{\alpha^2} e^{-(K_\phi/2)U(r_1-r_2)e^{-(2K_\phi)U(r_3-r_4)}}
$$
where \( U(r) = \ln \left( \frac{x^2 + \nu_F \tau^2}{\alpha} \right) \), \( d^2 r = \nu_F \, dx \, d\tau \), \( x = x_1 - x_2 \) and \( \tau = \tau_1 - \tau_2 \). The quantity \( \nu_F \) is replaced by \( \nu_F \) and \( \nu_F = 2t/\nu_F \).

In order to obtain scaling equations of the coupling constants up to the second order, we need the response functions expanded up to the third order of the nonlinear terms, while these third terms are absent in one-dimensional case. These results originate in the nonlinear terms including both \( \phi_+ \) and \( \tilde{\theta}_\pm \) (or \( \phi_+ \) and \( \tilde{\phi}_\pm \)) in Eq. (2.2). By putting \( r_5 = r_4 + r \) and \( r_5 = r_3 + r \), and expanding near \( r = 0 \), we find

\[
\text{Eq. (A1)} = e^{-(K_\phi/2)U(r_1 - r_2)} + \frac{1}{(4\pi)^2} \sum \int \frac{d^2 r_3}{\alpha^2} \frac{d^2 r_4}{\alpha^2} e^{-(K_\phi/2)U(r_1 - r_2)} e^{-2K_\phi U(r_3 - r_4)} \times \left( \exp \left[ eK_\phi \left( U(r_1 - r_3) - U(r_1 - r_4) - U(r_2 - r_3) + U(r_2 - r_4) \right) \right] - 1 \right) \times \left\{ G_\phi^{\text{eff}} e^{-2K_\phi U(r_3 - r_4)} \cos 2q_0(x_3 - x_4) + G_f^{\text{eff}} e^{-(2/K_\phi)U(r_3 - r_4)} + G_g^{\text{eff}} e^{-(2/K_\phi)U(r_3 - r_4)} \right\} .
\]

The quantities \( G_\phi^{\text{eff}} \) are given by

\[
G_\phi^{\text{eff}} = G_\phi^2 - 2G_\phi G_e G_g \int \frac{dr}{\alpha} \left( r^2 \right)^{-1-2K_\phi} - 2G_\phi G_e G_h \int \frac{dr}{\alpha} \left( r^2 \right)^{-1-2K_\phi},
\]

\[
G_f^{\text{eff}} = G_f^2 - 2G_f G_e G_g \int \frac{dr}{\alpha} \left( r^2 \right)^{-1-2K_\phi} - 2G_f G_e G_h \int \frac{dr}{\alpha} \left( r^2 \right)^{-1-2K_\phi},
\]

\[
G_g^{\text{eff}} = G_g^2 - 2G_g G_e G_g \int \frac{dr}{\alpha} \left( r^2 \right)^{-1-2K_\phi} \cdot J_0(2q_0 r) - 2G_g G_e G_g \int \frac{dr}{\alpha} \left( r^2 \right)^{-1-2K_\phi},
\]

\[
G_h^{\text{eff}} = G_h^2 - 2G_h G_e G_g \int \frac{dr}{\alpha} \left( r^2 \right)^{-1-2K_\phi} \cdot J_0(2q_0 r) - 2G_h G_e G_g \int \frac{dr}{\alpha} \left( r^2 \right)^{-1-2K_\phi},
\]

where \( r = (x^2 + (\nu_F \tau)^2)^{1/2} \) and the second and third terms of r.h.s. in Eqs. (3.3)-(3.6) are obtained by exponentiating the third order terms of Eq. (A1). From Eq. (3.3), the quantity \( K_\phi^{\text{eff}} \) is derived as

\[
K_\phi^{\text{eff}} = K_\phi - \frac{1}{2} G_\phi^{\text{eff}} \int \frac{dr}{\alpha} \left( r^2 \right)^{-3-2K_\phi} J_0(2q_0 r) U(r_1 - r_2) - \frac{1}{2} G_\phi^{\text{eff}} \int \frac{dr}{\alpha} \left( r^2 \right)^{-3-2K_\phi} U(r_1 - r_2)
\]

For \( \alpha \to \alpha' = \alpha e^{i\Delta \phi} \), these quantities are scaled as \( K_\phi^{\text{eff}}(K_{\phi'}, g', q_0', \alpha') = K_\phi^{\text{eff}}(K_{\phi'}, g, q_0, \alpha) \al e^{i\Delta \phi} \), \( G_\phi^{\text{eff}}(K_{\phi'}, g', q_0', \alpha') = G_\phi^{\text{eff}}(K_{\phi'}, g, q_0, \alpha) \al e^{i\Delta \phi} \), \( G_f^{\text{eff}}(K_{\phi'}, g', q_0', \alpha') = G_f^{\text{eff}}(K_{\phi'}, g, q_0, \alpha) \al e^{i\Delta \phi} \), \( G_g^{\text{eff}}(K_{\phi'}, g', q_0', \alpha') = G_g^{\text{eff}}(K_{\phi'}, g, q_0, \alpha) \al e^{i\Delta \phi} \), and \( G_h^{\text{eff}}(K_{\phi'}, g', q_0', \alpha') = G_h^{\text{eff}}(K_{\phi'}, g, q_0, \alpha) \al e^{i\Delta \phi} \), where \( K_{\phi'}, G_{\phi'} \) and \( q_0' \) denote renormalized quantities. From this infinitesimal transform and putting \( K_{\phi'}(l) \pm 1 = 1 \pm G_{\phi}(l) \), the renormalization equations for the coupling constants, Eqs. (2.4), (2.11), (2.12), (2.13) and (2.14) are given by Eqs. (A7), (A8), (A9), (A10), (A11) and (A12).
respectively. In a similar way, renormalization group equation for \( \widetilde{K}_\theta(l) (\widetilde{K}_\phi(l)) \) is calculated from the response function for \( \widetilde{\theta} (\widetilde{\phi}) \) field, and Eqs. (2.7)-(2.10) are obtained from the response function for both \( \theta \) and \( \phi \) fields.

The renormalization equation for \( t(l) \) is calculated as follows. The quantity, \( \Delta n \), which denotes the actual difference of the density between two bands, having \( k_{F+} \) and \( k_{F-} \), is given by

\[
\Delta n \equiv 2\Delta k \alpha
\equiv 2(k_{F+} - k_{F-})\alpha + \frac{T}{L} \int dx \, d\tau \left\langle \widetilde{k}_{F+} - \widetilde{k}_{F-} \right\rangle \alpha,
\tag{A8}
\]

where \( (\widetilde{k}_{F+} - \widetilde{k}_{F-})/(2\pi/L) \times 2 \equiv 1/(2L) \sum_{p,\sigma,\mu} \mu \rho_{p,\sigma,\mu} \) and \( \rho_{p,\sigma,\mu}(x,\tau) = \psi_{p,\sigma,\mu}^\dagger(x,\tau) \psi_{p,\sigma,\mu}(x,\tau) \).

By using the phase variable, \( \partial_x \widetilde{\theta}_+(x,\tau) = (\pi/\sqrt{2}) \sum_{p,\sigma,\mu} \mu \rho_{p,\sigma,\mu}(x,\tau) \), the quantity \( \Delta n \) is expressed as

\[
\Delta n = -2q_0 \alpha + \frac{1}{\sqrt{2} L} \alpha \int dx \, d\tau \left\langle \partial_x \widetilde{\theta}_+(x,\tau) \right\rangle \, .
\tag{A9}
\]

By assuming the scaling relation for Eq. (A9) in a way similar to ref. 21, the renormalization equation for \( t(l) \), Eq. (2.15), is derived.

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