On the NLO Power Correction to Photon-Pion Transition Form Factor

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We propose a perturbative evaluation for the next-to-leading-order (NLO) $O(1/Q^4)$ power correction to the photon pion transition form factor $F_{\pi\gamma}(Q^2)$. The effects of the NLO power correction are analyzed.

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I. INTRODUCTION

The photon-pion transition process $\gamma^*\pi \rightarrow \gamma$ provides a good example for tests of QCD. This is because, at tree level in PQCD, there only involve electro-magnetic interactions. The short distance interactions can be calculated in a definite way. The amplitude for the process $\gamma^*\pi \rightarrow \gamma$ can be expressed as

$$M(\gamma^*\pi \rightarrow \gamma) = -ie^2\epsilon_{\mu\nu\rho\lambda}P^\mu P^\nu P^\rho P^\lambda F_{\pi\gamma}(Q^2),$$

where $P_1$ means the pion momentum, $P_2$ and $\epsilon^\lambda$ denote the momentum and polarization of the real photon and $Q^2 = -(P_2 - P_1)^2$ is the virtuality of the virtual photon. The QCD dynamics are contained in the form factor $F_{\pi\gamma}(Q^2)$. In PQCD, the leading form factor $F_{\pi\gamma}(Q^2)$ can be expressed as

$$F_{\pi\gamma}(Q^2) = 4C_\pi \int_0^1 dx \frac{\phi_2(x)}{Q^2 x(1-x)}$$

with $C_\pi$ the charge factor and $\phi_2(x)$ the pion wave function. In the high energy limit $Q^2 \rightarrow \infty$, the asymptotic of the form factor $F_{\pi\gamma}$ can be evaluated and have an extremely simple form

$$F_{\pi\gamma}(Q^2) \bigg|_{Q^2 \rightarrow \infty} = \frac{2f_\pi}{Q^2}$$

that it is only in terms of the pion decay constant, $f_\pi = 93$ MeV and $Q^2$. However, the above asymptotic is about 15% lower than the upper end of the experimental data. There have been many explanations about this difference between the theory and the experiment. For example, the inclusion of NLO correction in $\alpha_s$ and the transverse structure of the pion wave function. It has also been shown that the data can also be described by the Brodsky-Lepage interpolating formula for both $Q^2 \rightarrow \infty$ and $Q^2 \rightarrow 0$ of $F_{\pi\gamma}(Q^2)$

$$F_{\pi\gamma}^{BL}(Q^2) = \frac{2f_\pi}{s_0 + Q^2}$$

where $s_0 = 8\pi^2 f_\pi^2 \approx 0.68$ GeV$^2$. This implies that the NLO power correction, the $O(1/Q^4)$ correction, might be important.

In this paper, we shall present a perturbative approach to calculate the NLO power correction for form factor $F_{\pi\gamma}(Q^2)$. The calculations are related to the terminology of the collinear expansion. The expansion has the following features: (1) it preserves individual gauge invariance of the soft function and the hard function; (2) it can systematically separate the leading twist (LT) contributions from the next-to-leading twist (NLT) contributions; (3) it encounters the high twist contributions from non-collinear partons, wrong spin projection and higher Fock states; (4) it is a twist-by-twist expansion and free from twist mixing problem; (5) it is a Feynman diagram approach such that the parton picture for high twist contributions is preserved.

Our main results are summarized as follows. We shall employ collinear expansion to derive the LT and NLT contributions for tree diagrams of processes $\gamma^*\pi \rightarrow \gamma$. The NLT contributions to $F_{\pi\gamma}(Q^2)$ involve four NLT pion DAs. Two of them come from non-vanishing masses of the valence quarks. By the help of equations of motion, the four DAs are reduced to two DAs. The asymptotic forms for the remaining DAs are employed. The theory appears in a good agreement with the data. The organization of the remaining text is as follows. We sketch the collinear expansion for the process $\gamma^*\pi \rightarrow \gamma$ in Sec. II. The explicit formula up to order $O(Q^{-4})$ of the process $\gamma^*\pi \rightarrow \gamma$ are given in Sec. III. Sec. IV is devoted to conclusions.

II. COLLINEAR EXPANSION AND FACTORIZATION

We sketch the procedures of performing collinear expansion for $\gamma^*\pi \rightarrow \gamma$.

A. Tree Level Collinear Expansion

Let $\sigma = \sigma_p(k) \otimes \phi(k)$ represent the lowest order amplitude for $\gamma^*(q)\pi(P_1) \rightarrow \gamma(P_2)$ as depicted in Fig. 1(a) and (b). The $\sigma_p(k)$ denotes the amplitude for partonic subprocess and the $\phi(k)$ represents the meson DA. The $\otimes$ means convolution integral over the loop momentum $k$.

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and the traces over the color indices and spin indices. If we assign
the momentum of the final state photon in the minus light-cone
direction, \( P_2 = Q^2n/2 \), and the momentum of the light
meson in the plus light-cone direction, \( P_1 = p \), then the leading
configuration for the process can be a function of the collinear
momentum \( k = xp \) with \( x = k \cdot n \) the fraction of the momentum of the
meson carried by the partons. \( p \) and \( n \) represent light-like
vectors in + and - directions and satisfy \( n \cdot p = 1 \). The first
step is to perform a Taylor expansion for the parton amplitude
\[
\sigma_p(k) = \overline{\sigma}_p(k = xp) + (\overline{\sigma}_p)_{\alpha}(x, x)w'^{\alpha}_\alpha k^{\alpha} + \cdots
\]
where we have assumed the low energy theorem
\[
\frac{\partial}{\partial k^{\alpha}} \sigma_p(k) \bigg|_{k=xp} = (\overline{\sigma}_p)_{\alpha}(x, x) .
\]
and have employed \( w'^{\alpha}_{\alpha} k^{\alpha} = (k - xp)^{\alpha} \) and \( w'^{\alpha}_{\alpha} =
\gamma^{\alpha}_{\alpha} - p^{\alpha}n_{\alpha} \). The leading term \( \overline{\sigma}_p \otimes \phi \) contains leading,
next-to-leading and higher twist contributions in accord
with the spin structures of the leading parton amplitude
\( \overline{\sigma}_p \). That is those terms are proportional to \( \not{p} \) or
\( \not{q} \). The terms proportional to \( \not{p} \) would project collinear
Qq pair from the meson, while those terms proportional to
\( \not{q} \) would not diminish only when the Qq pair carry non-
collinear momenta. The second step is to substitute the
leading parton amplitude \( \overline{\sigma}_p \) into the convolution
integral with the meson wave function \( \phi \) to extract the NLT
contribution \( \overline{\sigma}_p \otimes \phi_1 \) from the leading one \( \overline{\sigma}_p \otimes \phi_0 \)
\[
\overline{\sigma}_p \otimes \phi = \overline{\sigma}_p \otimes \phi_0 + \overline{\sigma}_p \otimes \phi_1 + \cdots,
\]
where \( \phi_0 \) and \( \phi_1 \) denote the leading and next-to-leading
meson DAs, respectively. The high twist DA \( \phi_1 \) contains both
short distance and long distance contributions. The short
distance contributions of \( \phi_1 \) arise from the non-
collinear components of \( k \). By the equations of
motion, the non-collinear components of \( k \) will induce one quark-
gluon vertex \( i\gamma_{\alpha} \) and one special propagator \( i\not{k}/2x_1 \). Because that the special propagator is not propagating,
the quark-gluon vertex and the special propagator should be
included into the hard function, \( \overline{\sigma}_p \). In this way, we
may factorize \( \phi_1 \) as \( \phi_1 \approx (\phi^H_1)_{\alpha}w'^{\alpha}_{\alpha}(\phi^S_1)^{\alpha'} \)
and absorb the short distance piece \( (\phi^H_1)_{\alpha} \) into \( \overline{\sigma}_p \). It leads to the third
step
\[
\overline{\sigma}_p \otimes \phi_1 = \overline{\sigma}_p \otimes ((\phi^H_1)_{\alpha} \cdot w'^{\alpha}_{\alpha}(\phi^S_1)^{\alpha'})
= (\overline{\sigma}_p \otimes \phi^H_1)_{\alpha} \cdot w'^{\alpha}_{\alpha}(\phi^S_1)^{\alpha'},
\]
where \( (\phi^S_1)^{\alpha'} \) containing covariant derivative \( D^{\alpha'} = i\partial^{\alpha'} - ga^{\alpha'} \) is implied. Notice that the light-cone gauge
\( n \cdot A = 0 \) assures \( w'^{\alpha}_{\alpha} A^{\alpha} = A^{\alpha} \). The second term of
Eq. (5) also contribute to high twist corrections, as it con-
volutes with \( \phi_0 \). The momentum factor \( k^{\alpha} \) will be ab-
sorbed by \( \phi_0 \) to become a coordinate derivative, denoted
as \( k^{\alpha}\phi_0 = \phi^1_{\alpha,p} \). Consider another NLT contributions
\( \sigma_1 \approx (\overline{\sigma}_p)_{\alpha}w'^{\alpha}_{\alpha}(\phi^0_{1,A}) \) from Fig. 1(c) and (d), where \( \phi^0_{1,A} \)
contains gauge fields. We have employed the approximation
that \( (\overline{\sigma}_p)_{\alpha} \otimes \phi^0_{1,A} \) is the leading term of \( \sigma_1 \). This
comes to the fourth step:
\[
(\overline{\sigma}_p)_{\alpha} \otimes w'^{\alpha}_{\alpha}(\phi^0_{1,a}) + (\overline{\sigma}_p)_{\alpha} \otimes w'^{\alpha}_{\alpha}(\phi^0_{1,A}) \equiv (\overline{\sigma}_p)_{1} \otimes (\phi^S_1) ,
\]
where we have employed \( \phi^0_{1,a} + \phi^0_{1,A} = (\phi^S_1)^{\alpha} \). However, it will be found that \( (\overline{\sigma}_p)_1 \) diminishes as convoluting with twist-4 DA \( \phi^1_1 \) (see below definition). Up to NLO, we may drop the \( (\overline{\sigma}_p)_1 \) term and arrive at the factorization
for tree amplitudes
\[
\sigma_0 + \sigma_1 \approx \overline{\sigma}_p \otimes \phi_0 + (\overline{\sigma}_p \otimes \phi^H_1) \otimes \phi_1
\]
where \( \phi_1 \) means \( \phi^S_1 \). There involves only one NLT DA \( \phi_1 \) for NLO power corrections.
To proceed, we need to consider the factorizations of
the spin indices, the color indices and the momentum
integrals over loop partons. For factorization of spin
dindices, we employ the expansion of the meson DA into
its spin components as
\[
\phi_{0,1} = \sum_F \phi^F_{0,1} \Gamma \]
where \( \Gamma \) means Dirac matrix \( \Gamma = 1, \gamma^\mu, \gamma^\mu \gamma_5, \sigma^{\mu\nu} \). The
factorization of the color indices take the convention that
the color indices of the parton amplitudes are extract and
attributed to the meson DAs. The factorization of the
momentum integral is performed by making use of the
fact that the leading parton amplitudes depend only on
the momentum fraction variables \( x_i \). The identity can always be used
\[
\int_0^1 dx_i \delta(x_i - k_i \cdot n) = 1 .
\]
The choice of the lowest twist components \( \phi^0_{0,1} \) of \( \phi_{0,1} \)
is made by employing the power counting. Consider \( \pi \) meson whose high twist DA \( \phi^{\mu_1,..,\mu_F;\alpha_1..,\alpha_B} \) has the
fermion index \( F \) and the boson index \( B \). The fermion
index \( F \) arise from the spin index factorization for \( 2F \)
fermion lines connecting DA and parton amplitude and
the boson index \( B \) denotes the \( n_D \) power of momenta in
previous collinear expansion and the \( n_G \) gluon lines as
\( B = n_D + n_G \). We may write
\[
\phi^{\mu_1,..,\mu_F;\alpha_1..,\alpha_B} = \sum_i A^{\tau_i - 1} e^{\mu_1,..,\mu_F;\alpha_1..,\alpha_B} \phi^i
\]
where \( A \) denotes a small scale associated with DA. Spin
polarizers \( e_i \) denote the combination of vectors \( p^\mu, n^\mu \)
and \( \gamma^\mu_\perp \). Variable \( \tau_i \) represents the twist of DA \( \phi^i \). The
restrictions over projector \( e^{\mu_1,..,\mu_F;\alpha_1..,\alpha_B} \) are
\[
n_{\alpha_j} \epsilon^{\mu_1,..,\mu_F;\alpha_1..,\alpha_j..,\alpha_B} = 0
\]
which are due to the fact that polarizers $e_i$ are always projected by $w_{\alpha i}^\ast$. The dimension of $\phi^{\mu_1\cdots\mu_F,\alpha_1\cdots\alpha_B}$ is determined by dimensional analysis
\begin{equation}
    d(\phi) = 3F + B - 1 \tag{14}
\end{equation}
By equating the dimensions of both sides of Eq. (12), one can derive the minimum of $\tau_i$
\begin{equation}
    \tau_{i\text{min}} = 2F + B + \frac{1}{2}[1 - (-1)^B] . \tag{15}
\end{equation}
It is obvious from Eq. (13) that there are only finite numbers of fermion lines, gluon lines and derivatives contributes to a given power of $1/Q^2$.

B. Collinear Expansion for Arbitrary Loop Orders

The factorization theorem for the NLO power correction should be proven in order to have a confident PQCD formalism. Before this can be done, we can at least show that the collinear expansion is compatible with the conventional approach for proving the factorization theorem. The conventional approach is based on the factorization of the soft divergences from the ultraviolet divergences arising from the radiative corrections. The soft divergences are shown to be cancelled or absorbed by the meson wave function such that the parton amplitude is free from the soft divergences. The ultraviolet divergences can be absorbed by the parton amplitude. We shall assume that the factorization of the soft and ultraviolet divergences can be done upto NLO power correction. Then, we can show that the the collinear expansion for tree diagrams can be straightforwardly extended to those diagrams containing arbitrary loop corrections. The starting point is to notice that the collinear expansion for the one loop corrections in the collinear region of the radiative gluons can be written down as
\begin{equation}
    (\sigma_0^{(0)} + \sigma_1^{(0)} + \sigma_0^{(1)} + \sigma_1^{(1)}) \bigg|_{\text{collinear gluons}} \\
    \approx \sum_{j=0,1} \sigma_p^{(0)} \otimes \phi_0^{(j)} + \sigma_p^{(0)} \otimes (\phi_1^H)^{(0)} \otimes \phi_1^{(j)} 
\end{equation}
where superscript $(0), (1)$ denote tree and one loop corrections, respectively. This is because, as the collinear gluons with momentum $l \sim (Q, \lambda^2/Q, \lambda)$ go through the fermion lines, the valence fermion momenta behave similarly to those in the tree level expansion. This leads to the fact that the collinear expansions for one loop amplitudes in collinear region can be performed just like for tree amplitudes. The soft gluon corrections can not affect the collinear expansion. The cancellations of double logarithms are assured in light-cone gauge by adding ladder and self energy diagrams. The one loop corrected parton amplitudes are determined by subtracting the amplitudes in collinear and soft regions from the full one loop amplitudes. Following the standard considerations, the LT parton amplitude $\tilde{\sigma}_p^{(0)}$ and NLT parton amplitude $\tilde{\sigma}_p^{(0)} \cdot (\phi_1^H)^{(0)}$ are infrared finite, and the soft divergences are absorbed by $\phi_0^{(1)}$ and $\phi_1^{(1)}$. The one loop factorization is derived up to NLT order
\begin{equation}
    (\sigma_0^{(0)} + \sigma_1^{(0)} + \sigma_0^{(1)} + \sigma_1^{(1)}) \\
    \approx \left[ \sum_{i=0}^1 \sigma_p^{(i)} \otimes \sum_{j=0}^1 \phi_0^{(j)} \right] \\
    + \left[ \sum_{i=0}^1 \sum_{j=0}^1 \sigma_p^{(i)} \cdot (\phi_1^H)^{(i-j)} \right] \otimes \sum_{k=0}^1 \phi_1^{(k)} . \tag{17}
\end{equation}
The generalization to arbitrary loop orders can be obtained by iteration. Suppose that
\begin{equation}
    \sigma = (\sigma_p)_0 \otimes \phi_0 + (\sigma_p)_1 \otimes \phi_1 \tag{18}
\end{equation}
where
\begin{equation}
    (\sigma_p)_1 = \sum_{i=0}^N (\sigma_p)_1^{(i)} , \tag{19}
\end{equation}
where
\begin{equation}
    (\sigma_p)_1^{(i)} = \sum_{j=0}^N \sigma_p^{(j)} \cdot (\phi_1^H)^{(i-j)} . \tag{20}
\end{equation}
The above factorization still holds for $N + 1$ order corrections, since the collinear gluons cannot attach to the parton amplitudes. The remaining proof of factorization requires the cancellations of double logarithms of soft divergences, the single soft logarithms absorbed by pion DAs and the infrared finiteness of the parton amplitudes. This can be achieved by standard analysis (see e.g. [9]) and it is left to other publish [9].

III. $O(1/Q^4)$ CONTRIBUTIONS OF $\gamma^* \pi \rightarrow \gamma$

The lowest-order diagrams are displayed in Fig. 1. By applying the previous collinear expansion to separate the LT and NLT contributions in a factorized form, we may write the result as
\begin{equation}
    M(\gamma^* \pi \rightarrow \gamma) = -ie^2 \epsilon_{\mu\nu\alpha\beta} \frac{P_1^\alpha P_2^\beta e^\lambda}{2} F_{\pi\gamma}(Q^2) , \tag{21}
\end{equation}
where $e^\lambda$ denotes the polarization vector of the final state photon. The leading order of $F_{\pi\gamma}(Q^2)$ is calculated from Fig.1(a) and (b) as
\[ F_{\gamma\gamma}^{\pi}(Q^2) = 4C_\pi \int_0^1 dx \frac{\phi_2(x)}{Q^2 x (1 - x)}, \]  
(22)

where the charge factor \( C_\pi = (e_u^2 - e_d^2)/\sqrt{2} \). \( e_u \) and \( e_d \) mean the charges of \( u \) and \( d \) quark in units of the elementary charge. The NLO of \( F_{\gamma\gamma}(Q^2) \) is evaluated from Fig.2

\[ F_{\gamma\gamma}^{\text{NLO}}(Q^2) = -16C_\pi \int_0^1 dx \frac{[G(x) + \tilde{G}(x)(1 - 2x)]}{Q^2 x (1 - x)}. \]  
(23)

The relevant DAs are expressed explicitly as follows

\[ \phi_2(x) = -\frac{1}{4} \int_0^\infty \frac{d\lambda}{(2\pi)} e^{i\lambda x} \langle 0 | \bar{q}(0) \gamma_5 \not{q} (\lambda n) | \pi(P_1) \rangle \]  
(24)

\[ G(x) = -\frac{1}{8} \int_0^1 dx \int_0^\infty \frac{d\lambda}{(2\pi)} \frac{d\eta}{(2\pi)} e^{i\eta(x_1 - x)} e^{i\lambda x} \times \langle 0 | \bar{q}(0) \gamma_5 \alpha \beta D(x) (\not{q} + \not{n}) q (\lambda n) | \pi(P_1) \rangle, \]  
(25)

\[ \tilde{G}(x) = -\frac{i}{8} \int_0^1 dx \int_0^\infty \frac{d\lambda}{(2\pi)} \frac{d\eta}{(2\pi)} e^{i\eta(x_1 - x)} e^{i\lambda x} \times \langle 0 | \bar{q}(0) \gamma_5 \alpha \beta D(x) (\not{q} + \not{n}) q (\lambda n) | \pi(P_1) \rangle. \]  
(26)

The tensors \( \epsilon^{a\beta}_{\perp} \) and \( d^{a\beta}_{\perp} \) are defined as \( \epsilon^{a\beta}_{\perp} = \epsilon^{a\beta\gamma} \lambda p_\gamma n_\lambda \) and \( d^{a\beta}_{\perp} = p^\alpha n^\beta + n^\alpha p^\beta - g^{a\beta} \).

The nonvanishing valence quark mass can also contribute to NLO correction. We use the scheme that the partons involved in the hard function are massless. This does not affect the final conclusion. By assigning new contributions from the quark mass operator \( m \), we get the result

\[ F_{\gamma\gamma}^{\text{NLO}}(Q^2)|_{m_q \neq 0} = -8C_\pi \int_0^1 dx \frac{H(x) + \tilde{H}(x)(1 - 2x)}{Q^2 x (1 - x)}. \]  
(27)

where two twist-4 DAs are introduced

\[ H(x) = -\frac{i}{16} \int_0^\infty \frac{d\lambda}{(2\pi)} e^{i\lambda x} \langle 0 | \bar{q}(0) m_{\alpha \beta} q (\lambda n) | \pi(P_1) \rangle \]  
(28)

\[ \tilde{H}(x) = -\frac{i}{4} \int_0^\infty \frac{d\lambda}{(2\pi)} e^{i\lambda x} \langle 0 | \bar{q}(0) m_{\gamma \beta} q (\lambda n) | \pi(P_1) \rangle. \]  
(29)

Note that \( H(x) \) and \( \tilde{H}(x) \) are related to the conventional twist-3 pion DA \( \phi_2(x) \) and \( \phi_3(x) \) [1] by a factor \( m_0 \), the average quark mass.

The DAs \( G, \tilde{G}, H \) and \( \tilde{H} \) are dependent and reducible under equations of motion to

\[ G'(x) = -\frac{1}{16} \int_0^1 dx \int_0^\infty \frac{d\lambda}{(2\pi)} \frac{d\eta}{(2\pi)} e^{i\eta(x_1 - x)} e^{i\lambda x} \times \langle 0 | \bar{q}(0) \gamma_\alpha D(x) (\not{q} + \not{n}) q (\lambda n) | \pi(P_1) \rangle, \]  
(30)

\[ G'(x) = -\frac{i}{16} \int_0^1 dx \int_0^\infty \frac{d\lambda}{(2\pi)} \frac{d\eta}{(2\pi)} e^{i\eta(x_1 - x)} e^{i\lambda x} \times \langle 0 | \bar{q}(0) \gamma_\alpha D(x) (\not{q} + \not{n}) q (\lambda n) | \pi(P_1) \rangle. \]  
(31)

Due to the factor \( 1 - 2x \) for \( \tilde{G}' \), \( G' \) becomes dominate. The normalization of \( \phi_2(x) \) is fixed from process \( \pi \to \mu \nu \) such that \( \phi_2^{\text{AS}}(x) = 3f_x x(1 - x)/\sqrt{2} \) for asymptotic (AS) model and \( \phi_2^{\text{CS}}(x) = 15f_x x(1 - x)(1 - 2x)/\sqrt{2} \) for Chernyak-Zhitnitsky (CZ) model [2]. Similarly, the normalization of \( G'(x) \) is determined from the axial anomaly \( \pi \to 2\gamma \) [3] to yield \( G'^{\text{AS}}(x) = 3\sqrt{2}\pi^2 f_x^2 x(1 - x) \) for AS model and \( G'^{\text{CZ}}(x) = 15\sqrt{2}\pi^2 f_x^2 x(1 - x)(1 - 2x)^2 \) for CZ model. The comparison between the experiment and our result indicates that the data is in more favor of model than of CZ model (see Fig.3). This is consistent with that conclusion made by Jakob et al. [4].

### IV. CONCLUSIONS

We have shown that the collinear expansion for \( \gamma^* \pi \to \gamma \) can be systematically performed. The \( O(Q^{-4}) \) power correction for \( F_{\gamma\gamma}(Q^2) \) has been evaluated in terms of four twist-4 DAs. The effects of NLO power correction are estimated to account for the data [3]. Applications of the collinear expansion to other processes can be straightforwardly performed.

The other sources of power correction may also be important, such as the renormalon. The investigation of this kind of power correction is beyond the scope of this paper.

We have also limited ourselves to the tree level. The factorization theorem for the NLO power correction should be proven if we require a confident PQCD formalism. As we have shown, the collinear expansion is compatible with the conventional approach for proving the factorization theorem. The collinear expansion can be performed order by order for radiative corrections.

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[1] S.J. Brodsky and G.P. Lepage, Phys. Lett. B 87, 359 (1979); Phys. Rev. Lett. 43, 545 (1979); Phys. Rev. D 22, 2157 (1980).
[2] CLEO Collaboration (J. Gronberg et. al.), Phys. Rev. D 57, 33 (1998).
[3] P. Kroll and M. Raulfs, Phys. Lett. B 387, 848 (1996).
[4] R. Jakob et al., J. Phys. G 22, 45 (1996).
[5] F.-G. Cao et al., Phys. Rev. D 53, 6582 (1996).
[6] S.J. Brodsky and G.P. Lepage, Phys. Rev. D 24, 1808 (1980).
[7] R.K.Ellis, W.Furmanski and R. Petrozio, Nucl. Phys. B 207,1(1982); B 212, 29(1983); J. Qiu, Phys. Rev. D 42,30(1990).
[8] T.W. Yeh, contribution to the Proceedings of the Fourth International Workshop on B Physics and CP Violation, Ise-Shima, Japan, February 18-23, 2001.
[9] T.W. Yeh (in preparation).
[10] V.M. Braun and I.E. Filyanov, Z. Phys. C 48, 239(1990).
[11] V.L. Chernyak and A.R. Zhitnitsky, Phys. Rep. 112, 173 (1984).

Figure Caption

Fig.1 The leading order diagrams for $\gamma^*\pi \rightarrow \gamma$. The cross symbol means the vertex of the virtual photon.
Fig.2 The next-to-leading-twist (NLT) diagrams for $\gamma^*\pi \rightarrow \gamma$. The propagator with one bar means the special propagator.
Fig.3 The pion-photon transition form factor $Q^2F_{\gamma\pi}(Q^2)$ calculated with different distribution amplitudes: the asymptotic (solid line) and Chernyak-Zhitnitsky (dash line). The experimental data are taken from [2].
Fig. 1
Fig. 2
