Isovector pairing effect on the moments of inertia of proton-rich heated nuclei.

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Abstract. The perpendicular and parallel moments of inertia are calculated as a function of the temperature by taking into account the isovector pairing. The used single-particles energies are those of a deformed Woods-Saxon mean-field. The obtained results are compared to their homologues of the conventional Finite Temperature BCS (FTBCS) theory. With this aim, the generalized gap equations have been solved for even-even heated deformed nuclei such as $Z = 30 - 38$ and $N - Z = 0, 2, 4$. The isovector pairing effect leads to a change in the behavior of the perpendicular and parallel moments of inertia. Moreover, there is a non-negligible discrepancy between the perpendicular and parallel moments of inertia values calculated within the two models when $T < T_{cnp}$ ($T_{cnp}$ being the critical temperature beyond which the neutron-proton (np) gap parameter vanishes). Beyond this temperature, a discrepancy between the two models persists. It is due to the shift of the critical temperatures of the proton ($T_{cpp}$) and neutron ($T_{cnn}$) systems when evaluated with and without inclusion of the isovector pairing effect.

1. Introduction
The neutron-proton (np) pairing correlations [1, 2, 3, 4] in proton-rich nuclei plays a very significant role in the evaluation of physical observables. Indeed, it is already well established that in $N \simeq Z$ nuclei, the neutron and proton Fermi levels are close to each other and therefore pairing, and in particular np pairing, correlations are expected to play an important role in their structure. One also expects that they play a significant role in the evaluation of the moment of inertia. Indeed, this observable has been the subject of many studies because it is involved in the description of rotation motion of nuclei. The nuclear moment of inertia has been evaluated by taking into account pairing between like-particles correlations at zero as well as at finite temperature [5, 6, 7, 8]. It has also been calculated at zero temperature including isovector pairing correlations [9, 10, 11, 12]. However, only a few studies of the np pairing correlations effect on the moment of inertia have been performed at finite temperature. Recently, expressions of temperature-dependent perpendicular and parallel moments of inertia including np pairing effects [13] have been derived using the cranking method as well as isovector temperature-dependent gap equations [14, 15]. Numerical calculations have been carried out within the
framework of the schematic Richardson model [16] as well as for even-even $N = Z$ nuclei such as $30 \leq Z \leq 38$, using the single-particles energies and eigen-states of a deformed Woods-Saxon mean field [13]. The aim of the present work is to complete this study by considering even-even nuclei such as $N - Z = 0, 2, 4$. The formalism is briefly recalled in next section. Numerical results are presented and discussed in section 3.

2. Formalism

Let us consider a system constituted by $N$ neutrons and $Z$ protons which is cranked around the $O_i$ ($i = x, z$) axis ($O_z$ being the symmetry axis) of a rotating frame. The grand-partition function of a such a hot nuclear system is given by:

$$Z = Tr \left\{ \exp(-\beta[H - \sum_t \lambda_t N_t - \hbar \omega J_i]) \right\} \quad i = x, z$$

(1)

where $\beta$ is the inverse of the temperature $T$ and $H$ is the Hamiltonian of the system given, in the second quantization and isotopic spin formalism, in the isovector pairing case, by [11, 12, 13, 14, 15, 16]:

$$H = \sum_{\nu t} \varepsilon_{\nu t}(a_{\nu t}^\dagger a_{\nu t} + a_{\nu t}^\dagger a_{\bar{\nu} t}) - \sum_{t t'} G_{t t'} \sum_{\nu, \mu > 0} \left( a_{\nu t}^\dagger a_{\mu t'}^\dagger a_{\mu t} a_{\nu t'} + a_{\nu t}^\dagger a_{\mu t'}^\dagger a_{\nu t} a_{\mu t'} \right)$$

(2)

The subscript $t$ corresponds to the isotopic spin component ($t = n, p$), $a_{\nu t}^\dagger$ and $a_{\nu t}$ respectively represent the creation and annihilation operators of the particle in the state $|\nu t\rangle$, of energy $\varepsilon_{\nu t}$; $|\bar{\nu} t\rangle$ is the time-reverse of $|\nu t\rangle$, $G_{t t'}$ characterizes the pairing strength. The neutron and proton are supposed to occupy the same energy levels. In all that follows, it is assumed that the single-particles energies are independent from the temperature [17]. $\lambda_t(t = n, p)$ are Lagrange parameters which represent the Fermi level energies and the $N_t$ are the particle-number operators given by:

$$N_t = \sum_{\nu > 0} \left( a_{\nu t}^\dagger a_{\nu t} + a_{\nu t}^\dagger a_{\bar{\nu} t} \right) \quad t = n, p$$

(3)

$\omega$ is the rotation frequency and $J_i$ is the $i$ projection of the angular momentum.

The usual Inglis [18] expression of the energy may be easily generalized in order to include the temperature effects [5] as well as the np pairing correlations, that is [13, 16]:

$$E = \left( -\frac{\partial \ln Z}{\partial \beta} \right)_{\beta = \text{cte}}$$

(4)

Its expansion to the second order in $\omega$ is given by:

$$E \simeq E_0 - \omega^2 \hbar^2 \int_0^\beta \langle J_i(\beta) J_i(\chi) \rangle d\chi, \quad \text{with} \quad i = x, z$$

(5)

where

$$J_i(\kappa) = e^{iH'} J_i e^{-iH'} \quad \text{and} \quad \langle J_i(\beta) J_i(\chi) \rangle = \frac{T_r e^{-\beta H'} J_i(\beta) J_i(\chi)}{T_r e^{-\beta H'}} \quad \kappa = \beta, \chi \quad i = x, z$$

(6)

$J_i(\kappa)$ is the Heisenberg transform of $J_i$ and the thermal average in Eq. (5) is evaluated using the grand-canonical ensemble associated to the Hamiltonian without rotation $H'$ given by:

$$H' = H - \sum_t \lambda_t N_t$$

(7)
This thermal average value may easily be determined using the quasiparticle representation. In the latter, the auxiliary Hamiltonian has been approximately diagonalized by means of the Feynman path integral technique [14] and using the Hubbard-Stratonovich transformation [19]. In fact, the diagonal form of the Hamiltonian $H'$ becomes:

$$H' = \sum_{\nu>0, \tau=1,2} E_{\nu\tau} \left( \alpha_{\nu\tau}^+ \alpha_{\nu\tau} - \alpha_{\nu\tau}^+ \alpha_{\nu\tau}^\dagger \right) + \sum_{\nu>0, t} \tilde{\varepsilon}_{\nu t}$$

where we set

$$\tilde{\varepsilon}_{\nu t} = (\varepsilon_{\nu t} - \lambda_t) - (G_{tt} + G_{np})/2 \quad t = p, n$$

$\alpha_{\nu\tau}^\dagger$ (respectively $\alpha_{\nu\tau}$) is the creation (respectively annihilation) operator of a quasiparticle of $\tau (\tau = 1, 2)$ type, given by the generalized Bogoliubov-Valatin transformation:

$$\alpha_{\nu\tau}^\dagger = \sum_{\nu>0, t} \left( u_{\nu\tau t} a_{\mu t}^\dagger + v_{\nu\tau t} a_{\mu t} \right) \quad \tau = 1, 2$$

The corresponding energies $E_{\nu\tau}$ are defined in Ref. [14, 15]. As $E = E_0 - \frac{1}{2} \Im \omega^2$ ($\Im$ being the moment of inertia), the perpendicular and parallel moments of inertia, are then respectively defined by:

$$\Im_\bot^{np} = 2 \hbar^2 \int_0^\beta \langle J_{x(z)}(\beta) J_{x(z)}(\chi) \rangle_0 d\chi$$

One has, after some algebra, for the perpendicular moment of inertia:

$$\Im_\bot^{np} = \hbar^2 \sum_{\mu\tau\tau' t' t''} \langle \mu t | J_{x} | \mu t' \rangle \langle \mu t' | J_{x} | \mu t'' \rangle$$

$$\left\{ \left[ u_{\nu\tau t} u_{\nu\tau' t'} - v_{\nu\tau' t} u_{\nu\tau t'} \right] \left[ v_{\nu\tau' t} u_{\nu\tau' t'} - v_{\nu\tau' t'} u_{\nu\tau t'} \right] \frac{\tanh \left( \frac{\beta E_{\nu\tau}'}{2} \right) + \tanh \left( \frac{\beta E_{\nu\tau'}}{2} \right)}{E_{\nu\tau} + E_{\mu\tau'}} \right\}$$

$$+ \left[ u_{\nu\tau t} u_{\nu\tau' t'} + v_{\nu\tau' t} u_{\nu\tau t'} \right] \left[ u_{\nu\tau' t} u_{\nu\tau' t'} + v_{\nu\tau' t'} u_{\nu\tau t'} \right] \frac{\tanh \left( \frac{\beta E_{\nu\tau}'}{2} \right) - \tanh \left( \frac{\beta E_{\nu\tau'}}{2} \right)}{E_{\mu\tau'} - E_{\nu\tau}}$$

(12)

In the same way, the parallel moment of inertia $\Im_{||}^{np}$ takes the following form:

$$\Im_{||}^{np} = \hbar^2 \sum_{\nu\tau\tau' t' t''} \langle \nu t | J_{z} | \nu t' \rangle \langle \nu t' | J_{z} | \nu t'' \rangle$$

$$\left\{ \left[ u_{\nu\tau t} u_{\nu\tau' t'} + v_{\nu\tau' t} u_{\nu\tau t'} \right] \left[ u_{\nu\tau' t} u_{\nu\tau' t'} + v_{\nu\tau' t'} u_{\nu\tau t'} \right] \frac{\tanh \left( \frac{\beta E_{\nu\tau}}{2} \right) - \tanh \left( \frac{\beta E_{\nu\tau'}}{2} \right)}{E_{\nu\tau} - E_{\nu\tau'}} \right\}$$

$$+ \left[ u_{\nu\tau t} v_{\nu\tau' t'} - u_{\nu\tau' t} v_{\nu\tau t'} \right] \left[ u_{\nu\tau' t} v_{\nu\tau' t'} - u_{\nu\tau' t'} v_{\nu\tau t'} \right] \frac{\tanh \left( \frac{\beta E_{\nu\tau}}{2} \right) + \tanh \left( \frac{\beta E_{\nu\tau'}}{2} \right)}{E_{\nu\tau} + E_{\nu\tau'}}$$

(13)

If the np pairing effects vanish, Eqs. (12) and (13) reduce to the sum of the moments of inertia of the neutron and proton systems considered separately in the framework of the pairing between like-particles [5, 6].
3. Numerical results-Discussion

3.1. Pairing gap parameters

The previously described formalism has been numerically applied using the single-particles and eigen-states of a deformed Woods-Saxon mean-field. We used the parameters of Ref. [20] with a maximum number of shells \(N_{\text{max}} = 12\). The ground state deformations are those of the Möller Table [21]. We considered even-even nuclei such as \(N - Z = 0, 2, 4\). Indeed, in such nuclei, it had been shown that np pairing effect is non-negligible [3, 4, 12] at zero temperature. In the present work, we considered only nuclei with a deformed ground-state. In what follows, we present the results that correspond to Zirconium and Strontium nuclei chosen as an example. The pairing-strength values \(G_{\text{pp}}, G_{\text{nn}}\) and \(G_{\text{np}}\) have been deduced from the \(\Delta_{\text{pp}}, \Delta_{\text{nn}}\) and \(\Delta_{\text{np}}\) values at zero temperature. The latter are obtained using the odd-even mass differences [1].

Figures 1 and 2 show the variations of the \(\Delta_{\text{pp}}, \Delta_{\text{np}}\) and \(\Delta_{\text{nn}}\) gap parameters as a function of the temperature as well as those of the perpendicular and parallel moments of inertia. The values obtained within the framework of the conventional \(\text{FTBCS}\) method are also given in the same figures. In the latter case, the pairing-strength values \(G_{\text{pp}}\) and \(G_{\text{nn}}\) have also been chosen in such a way to reproduce the \(\Delta_{\text{pp}}\) and \(\Delta_{\text{nn}}\) values at zero temperature.

As it is pointed out in Refs. [13, 15], the \(\Delta_{\text{np}}(T)\) parameter has a behavior which is similar to that of the conventional \(\text{FTBCS}\) \(\Delta_{\text{pp}}, \Delta_{\text{nn}}\) gap parameters, i.e. a quasi-constant value on a given interval, and then a decreasing until it vanishes at \(T = T_{\text{cnp}}\) (\(T_{\text{cnp}}\) being the critical value of the temperature).

Dealing with \(\Delta_{\text{pp}}\) and \(\Delta_{\text{nn}}\) of the present model, one observes a plateau in the region where \(\Delta_{\text{np}}\) is constant and then a sudden variation in the vicinity of \(T_{\text{cnp}}\) which is a consequence of the sudden increasing of \(\Delta_{\text{np}}\). This behavior is also observed for all studied nuclei. The phase transition when \(\Delta_{\text{np}} = 0\) (i.e. \(T = T_{\text{cnp}}\)) is sharp. As in the pairing between like-particles case, this abrupt phase transition could be smoothed out if one includes the quantum fluctuations that have been neglected in the partition function [14]. It could be also smoothed out by including a particle number-projection [23, 24, 25].

When \(T = T_{\text{cnp}}\) (i.e. \(\Delta_{\text{np}} = 0\)), the behavior of \(\Delta_{\text{pp}}\) and \(\Delta_{\text{nn}}\) is similar to that of the \(\text{FTBCS}\) method. However, there exists a shift between the proton and neutron critical temperature values \(T_{\text{pp}}\) and \(T_{\text{nn}}\) of the present model and those of the \(\text{FTBCS}\) one. The discrepancy varies from about 34% (32%) for \(T_{\text{pp}}\) (\(T_{\text{cnp}}\)) in the \(^{60}\)Zn case to about 13.5% (5%) for \(T_{\text{pp}}\) (\(T_{\text{cnp}}\)) in the \(^{76}\)Sr case.

Moreover, one notices that for each studied element (for a given \(Z\)), the critical temperature \(T_{\text{cnp}}\) is as weak as the \((N - Z)\) value is important. Indeed, for the Strontium for example, the critical temperatures \(T_{\text{cnp}}\) varies between 0.156MeV for the \(^{76}\)Sr and 0.070MeV for the \(^{80}\)Sr. Its value is lowered by about 55.13%. It is well established that at zero temperature \(\Delta_{\text{np}}\) rapidly decreases as a function of \((N - Z)\) and practically vanishes when \((N - Z) > 8\) [3, 4, 12]. At finite temperature, the present study shows that for a given \(Z\), the critical temperatures \(T_{\text{cnp}}\) are as weak as \((N - Z)\) is important and the np pairing effect practically disappears with the increasing values of \(N\) for a given \(Z\).

3.2. Moments of inertia

Panels (b) and (c) in Figs. 1 and 2 show respectively the variations of the perpendicular and parallel moments of inertia as a function of temperature for the previously cited nuclei (\(Z = 30\) and \(Z = 38\) with \(N - Z = 0, 2, 4\)). The values obtained using the conventional \(\text{FTBCS}\) method (i.e. within the framework of pairing between like-particles) are also given in the same figures.

With regard to the perpendicular moment of inertia \(\mathbb{I}_{\perp}\), one observes a plateau in the region where \(\Delta_{\text{np}}\) is constant (i.e. \(0 \leq T \leq T_{\text{cnp}}\)) followed by a sudden variation in the vicinity of \(T_{\text{cnp}}\) which is a consequence of the sudden decreasing of \(\Delta_{\text{np}}\). This behavior is also observed for all studied nuclei. Besides, the relative discrepancy between the present model values and the
Figure 1. Variation of the various gap parameters (a), the perpendicular moment of inertia (b) and the parallel moment of inertia (c) as a function of the temperature for the nucleus $^{60}$Zn, $^{62}$Zn and $^{64}$Zn. Straight lines refer to the present model and dashed lines to the conventional FT BCS model.

FTBCS ones, defined by $\left| \Im_{\perp}^{np} - \Im_{\perp}^{FTBCS} \right| / \Im_{\perp}^{FTBCS}$ varies from about 32% in the $^{78}$Sr case to 1.14% in the $^{64}$Zn case. Its average value for all considered nuclei is of the order of 12.23% in this region.

When $T > T_{cnp}$, the present model curve and the FT BCS one behave likely. However, there is a quasi-constant shift between the two curves. Indeed, although the gap parameters [13, 14, 16, 15] as well as the expression (12) of $\Im_{\perp}^{np}$ reduce to their FTBCS homologues when $\Delta_{np}$ vanishes, the pairing-strength values are different in each case. This is the reason why $\Delta_{pp}, \Delta_{nn}, T_{cpp}, T_{cnn}$ and $\Im_{\perp}$ are different from their homologues of the FTBCS model. One notes that the maximum value of the relative discrepancy varies from one nucleus to another. It varies from 33.72% in the $^{60}$Zn case to 1.91% in the $^{80}$Sr case. It is about 11.46% on average in this region for all studied nuclei.

When there is no more pairing effect, i.e. when $\Delta_{pp} = \Delta_{nn} = 0$, the two curves join and then correspond to the rigid body values.

Dealing with the parallel moment of inertia $\Im_{\parallel}$, the np pairing effect is not visible in the
region $0 \leq T \leq T_{cnp}$ for all the studied nuclei. In fact, the parallel moment of inertia is nil in this region and hence there is no difference between the present model previsions and those of the conventional $FTBCS$ method. This fact is due to the weak values of $T_{cnp}$.

When $T > T_{cnp}$, the behavior of the parallel moment of inertia $\mathcal{I}_\parallel$ is similar to that of the $FTBCS$ method. However, one observes, as in the perpendicular moment of inertia case, a shift between the two curves which is due to the previously discussed shift between the critical temperatures. The maximum value of the relative discrepancy between the present model and the $FTBCS$ one, defined by $|\mathcal{I}_\parallel^{\text{np}} - \mathcal{I}_\parallel^{\text{FTBCS}}|/\mathcal{I}_\parallel^{\text{FTBCS}}$, differs from one nucleus to another. It is about 38.84\% in the $^{60}\text{Zn}$ case and about 2.16\% in the $^{80}\text{Sr}$ one. It is of the order of 13.51\% on average for all the studied nuclei.

Naturally, both models join when $T > T_{cnp}$ since, in this case, there is no more pairing effect ($\Delta_{pp} = \Delta_{nn} = 0$) and the obtained values are those of the rigid body.

4. Conclusion
The temperature-dependent perpendicular and parallel moments of inertia including isovector pairing effect have been calculated as a function of the temperature. The used single-particles
energies are those of a deformed Woods-Saxon mean-field. The obtained results have been compared to their homologues of the conventional \textit{FTBCS} theory. It has been shown that:

(i) In the region where $0 \leq T \leq T_{cnp}$, regarding the perpendicular moment of inertia $\mathcal{I}_\perp$, the study shows that the np pairing effect leads to a maximum discrepancy of 12.23\% on average for all the considered nuclei. Dealing with the parallel moment of inertia $\mathcal{I}_\parallel$, in this region, it has been shown that there is no difference between the previsions of both models since for all the considered nuclei $\mathcal{I}_\parallel$ is nil. In fact, $T_{cnp}$ is small and the np pairing effect on $\mathcal{I}_\parallel$ is not visible.

(ii) In the region where $T > T_{cnp}$, in spite of the fact that there is no more np pairing, there exists a discrepancy between the present model values and those of the FTBCS method, which is due to the shift between the critical temperatures. The maximum relative discrepancy between the present model results and the \textit{FTBCS} ones is of the order of 11.46\% on average for the perpendicular moment of inertia and about 13.51\% on average for the parallel moment of inertia for the six studied nuclei.

Finally, it has been noticed that, at finite temperature, for a given $Z$, the critical temperatures $T_{cnp}$ are as weak as $(N-Z)$ is important and the np pairing effect practically disappears with increasing values of $N$.

As a conclusion, the isovector pairing effect on both perpendicular and parallel moments of inertia is non negligible at finite temperature. These correlations must be taken into account in study of warm rotating nuclei in the $N \simeq Z$ region.

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