Buckling and Free Vibration Analysis of Laminated Sandwich Beams under Hygrothermal Conditions

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Abstract. In present work, an attempt has been made for carrying out free vibration and buckling analysis of laminated sandwich beams under hygrothermal conditions. The analysis is carried out using fourth order zigzag theory based on finite element formulation. The efficiency of proposed model is validated by comparing the present results with those available in literature. Geometric properties and loading condition widely affect the behavior of the laminated sandwich beams.

1. Introduction
Behavior of sandwich structure is widely affected by the presence of moisture concentration and thermal conditions [1-3]. With increase in temperature and moisture concentration, stiffness of laminated composite structure decreases [4]. Several review works are available in literature regarding the hygrothermal based analysis of laminated sandwich structures [5-8].

Several theories are available in literature for studying the hygrothermal based behavior of laminated sandwich structures. These are classical laminated theory, shear deformation theories (SDT) and layerwise theories. First-order shear deformation theory (FSDT) requires shear correction factor. Higher-order shear deformation theories (HSDT) are not able to predict continuous transverse shear stresses at interfaces. Layerwise theories (LWT) are much more accurate than SDTs.

In present work, free vibration and buckling analysis of sandwich beams is carried out under hygrothermal conditions using recently proposed higher-order zigzag theory (HOZT) which is a form of LWT [9].

2. Mathematical Formulation
Considering in-plane and transverse displacement fields as

\[ U_{(x,z)} = u_{(x,z)}^{(0)} + zq_{(x,z)} + z^2\mu_{(x,z)} + z^3\xi_{(x,z)} + z^4\psi_{(x,z)} + \sum_{i=1}^{N-1} \left( z - z_{i}^{(u)} \right) H \left( z - z_{i}^{(u)} \right) \Phi_{i}^{(x,z)} \]

(2.1)

where, \( \varphi \) is rotation, \( u_{(x,z)}^{(0)} \) is displacement of reference plane, \( \mu, \xi \) and \( \psi \) are higher-order unknowns. \( H \left( z - z_{i}^{(u)} \right) \) and \( H \left( -z + z_{j}^{(l)} \right) \) are unit step function and \( \Phi \) is the slope of corresponding layer.

Stress-strain relationship for orthotropic material is written as

\[ \left[ \sigma \right]_{(l)} = \left[ C \right]_{k} \left[ e_{net} \right]_{(l)} \]

(2.2)

Applying the following conditions: At \( z = h/2 \): \( U_{(x)} = u^{(u)}, U_{(z)} = w^{(u)}, \sigma_{xz} = 0; \) At \( z = -h/2 \): \( U_{(x)} = u^{(l)}, U_{(z)} = w^{(l)}, \sigma_{xz} = 0. \) At interfaces: \( \sigma_{xx}^{(l)} = \sigma_{xx}^{(l+1)} \), the unknowns can be written in form of displacement as

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\{P\} = [N]\{\bar{a}\} \quad (2.3)

where,
\[
\{P\} = \begin{bmatrix}
\mu(x) \xi(x) \psi(x) & \mu(x) \xi(x) \psi(x) & \cdots & \mu(x) \xi(x) \psi(x) \\
\phi_1^{x(u)} & \phi_1^{x(u)} & \cdots & \phi_1^{x(u)} \\
\phi_2^{x(u)} & \phi_2^{x(u)} & \cdots & \phi_2^{x(u)} \\
\cdots & \cdots & \cdots & \cdots \\
\phi_{N(x)}^{x(u)} & \phi_{N(x)}^{x(u)} & \cdots & \phi_{N(x)}^{x(u)}
\end{bmatrix}
\begin{bmatrix}
\phi_1^{x(1)} & \phi_1^{x(1)} & \cdots & \phi_1^{x(1)} \\
\phi_2^{x(1)} & \phi_2^{x(1)} & \cdots & \phi_2^{x(1)} \\
\cdots & \cdots & \cdots & \cdots \\
\phi_{N(x)}^{x(1)} & \phi_{N(x)}^{x(1)} & \cdots & \phi_{N(x)}^{x(1)}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial}{\partial x}u^{(0)} & \frac{\partial}{\partial x}u^{(1)} & \cdots & \frac{\partial}{\partial x}u^{(N(x))} \\
\frac{\partial}{\partial x}w^{(0)} & \frac{\partial}{\partial x}w^{(1)} & \cdots & \frac{\partial}{\partial x}w^{(N(x))}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial}{\partial x} \frac{\partial}{\partial x} u^{(0)} & \frac{\partial}{\partial x} \frac{\partial}{\partial x} u^{(1)} & \cdots & \frac{\partial}{\partial x} \frac{\partial}{\partial x} u^{(N(x))} \\
\frac{\partial}{\partial x} \frac{\partial}{\partial x} w^{(0)} & \frac{\partial}{\partial x} \frac{\partial}{\partial x} w^{(1)} & \cdots & \frac{\partial}{\partial x} \frac{\partial}{\partial x} w^{(N(x))}
\end{bmatrix}
\]

\{\bar{a}\} = \begin{bmatrix}
\{u^{(0)}\} & \{w^{(0)}\} & \{u^{(1)}\} & \{w^{(1)}\} & \cdots & \{u^{(N(x))}\} & \{w^{(N(x))}\}
\end{bmatrix}

All the derivative terms are easily explained in form of displacements; hence, the present formulation is free from C-1 requirement conditions.

Utilizing Equation (2.3), Equation (2.1) can be re-written as
\[
U^{(x)} = f^{(1)}u^{(0)} + f^{(2)}w^{(0)} + f^{(3)}\phi^{(x)} + f^{(4)}\phi^{(x)} + f^{(5)}u^{(u)} + f^{(6)}w^{(u)} + f^{(7)}u^{(l)} + f^{(8)}w^{(l)} \quad (2.4)
\]
\[
U^{(y)} = g^{(1)}u^{(0)} + g^{(2)}w^{(0)} + g^{(3)}\phi^{(x)} + g^{(4)}\phi^{(x)} + g^{(5)}u^{(u)} + g^{(6)}w^{(u)} + g^{(7)}u^{(l)} + g^{(8)}w^{(l)} \quad (2.5)
\]

where, \(f\) and \(g\) are function of material properties, thickness and unit step function.

Using 3 node beam element having eight degrees of freedom per node, displacement vector can be written as
\[
\{\delta\} = [N] \{\delta_i\} \quad (2.6)
\]

Writing strain-displacement relationship in linear range, and with the help of Equations (2.1) - (2.3), the strains can be written in form of unknowns as:
\[
\{\varepsilon\} = \begin{bmatrix}
\frac{\partial u^{(x)}}{\partial x} \\
\frac{\partial u^{(x)}}{\partial x} \\
\frac{\partial u^{(x)}}{\partial x} + \frac{\partial u^{(x)}}{\partial x}
\end{bmatrix}
\quad \text{or}
\{\varepsilon\} = [H]\{\varepsilon\} \quad (2.7)
\]

where,
\[
\{\varepsilon\} = \begin{bmatrix}
u^{(0)} w^{(0)} \phi^{(x)} \phi^{(x)} u^{(u)} w^{(u)} u^{(l)} w^{(l)} & \frac{\partial u^{(0)}}{\partial x} & \frac{\partial u^{(0)}}{\partial x} & \frac{\partial u^{(0)}}{\partial x} & \frac{\partial u^{(0)}}{\partial x} & \frac{\partial u^{(0)}}{\partial x} & \frac{\partial u^{(0)}}{\partial x} & \frac{\partial u^{(0)}}{\partial x} \\
\frac{\partial w^{(0)}}{\partial x} & \frac{\partial w^{(0)}}{\partial x} & \frac{\partial w^{(0)}}{\partial x} & \frac{\partial w^{(0)}}{\partial x} & \frac{\partial w^{(0)}}{\partial x} & \frac{\partial w^{(0)}}{\partial x} & \frac{\partial w^{(0)}}{\partial x} & \frac{\partial w^{(0)}}{\partial x}
\end{bmatrix}
\]

and the elements contained in \([H]\) are function of unit step function and thickness coordinate.

Using Equation (2.6), in above Equation \{\varepsilon\} can be written with the help of unknowns (structural deformation)
\[
\{\varepsilon\} = [B]\{\delta\} \quad (2.8)
\]

where, \([B]\) is the strain-displacement relationship on cartesian coordinates.

For buckling study:
Writing strain-displacement relationship in linear range, and with the help of Equations (2.1) - (2.6), the strains can be written in form of unknowns as:
\[
\{\varepsilon\} = \{\varepsilon\}_\text{Linear} + \{\varepsilon\}_\text{Non-linear} \quad (2.9)
\]

where,
\[
\{\varepsilon\}_\text{Linear} = \begin{bmatrix}
\frac{\partial u^{(x)}}{\partial x} & \frac{\partial u^{(x)}}{\partial x} & \frac{\partial u^{(x)}}{\partial x} \\
\frac{\partial w^{(0)}}{\partial x} & \frac{\partial w^{(0)}}{\partial x} & \frac{\partial w^{(0)}}{\partial x}
\end{bmatrix}
\quad \text{or}
\{\varepsilon\}_\text{Linear} = [H]\{\varepsilon\}_\text{Linear},
\]
\[
\{\varepsilon\}_\text{Non-linear} = \frac{1}{2} \left( \frac{\partial u^{(x)}}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial u^{(x)}}{\partial x} \right)^2 \left[ A_G \right] \{\Lambda\} = \frac{1}{2} [H_G] [B] \{\delta\},
\]
\[
\{\varepsilon\} = \{u^{(0)} \} w^{(0)} \phi^{(x)} u^{(u)} w^{(u)} u^{(l)} w^{(l)} & \frac{\partial u^{(0)}}{\partial x} & \frac{\partial u^{(0)}}{\partial x} & \frac{\partial u^{(0)}}{\partial x} & \frac{\partial u^{(0)}}{\partial x} \\
\frac{\partial w^{(0)}}{\partial x} & \frac{\partial w^{(0)}}{\partial x} & \frac{\partial w^{(0)}}{\partial x} & \frac{\partial w^{(0)}}{\partial x} & \frac{\partial w^{(0)}}{\partial x} & \frac{\partial w^{(0)}}{\partial x} & \frac{\partial w^{(0)}}{\partial x} & \frac{\partial w^{(0)}}{\partial x}
\end{bmatrix}
\]
and the elements contained in \([H],[A_G] [H_G]\) are function of unit step function and thickness coordinate and \([B]\) is the strain-displacement relationship in cartesian coordinates.

With the matrix \([B]\) in the above equation, the geometric stiffness matrix \([K_G]\) can be derived and may be written as
\[
[K_G] = \sum_{k=1}^{n} \int [B]^T [S^k] [B] dx dz \quad (2.10)
\]

where \([S^k]\) is the in-plane stress components of the kth layer can be written as
\[
[S^k] = \begin{bmatrix}
\sigma_{xx} & 0 \\
0 & 0 
\end{bmatrix}
\]
For calculating buckling load under hygrothermal loading, the last term appearing in Equation (2.16) vanishes because no mechanical load is expected on the beam under hygrothermal conditions. Applying, principle of minimum potential energy, the governing equation for bending studies are written as:
\[
[K_e] \{\delta\} = \{p_{hygrothermal}\}
\]  
(2.11)
For the thermal buckling analysis, in the first step, a static problem is solved as per governing Equation (2.11) to calculate thermal stresses at the Gauss points of different elements for assumed temperature/moisture concentration rise or fall. Finally, these thermal stresses are used to form the matrix \([S^k]\) of the geometric stiffness matrix and the linear thermal buckling problem is solved as eigen value problem using equation (2.12)
\[
[K_e] \{\delta\} = \lambda[K_{ge}] \{\delta\}
\]  
(2.12)
where \(\lambda\) is buckling load factor.

For free vibration study:
At any point within the plate, displacement due to free vibration can be written as:
\[
\{f\} = -\omega^2 \left\{ U(x) \right\}_{(x)}
\]  
(2.13)
or
\[
\{f\} = -\omega^2 \left\{ F \right\}[f]
\]  
(2.14)
where the matrix \([F]\) is similar to the \([H]\) which is contain terms in form of \(z\) and unit step function and
\[
\{f\} = [C]\{\delta\}
\]  
(2.15)
Using the Equations (2.22) – (2.24), the consistent elemental mass matrix for an element can be stated as:
\[
[m^e_i] = \sum_{i=1}^{n_u+l_f} \rho_i [C]^T [F]^T [F] [C] dx dz = \int [C]^T[L][C] dx
\]  
(2.16)
where \(\rho_i\) is the mass density of the \(i\)th layer and \([C]\) is the shape function matrix and the matrix \([L]\) is:
\[
[L] = \sum_{i=1}^{n_u+l_f} \rho_i [F]^T [F] dz
\]  
(2.17)
Elemental stiffness, mass and load matrix are assembled to form corresponding global matrices by taking into account the behavior of all the elements. Finally, thermal vibration problem is solved as eigen value problem by taking different temperatures.

The equation of thermal vibration may be written using Hamilton’s principle as,
\[
[K']\{\Gamma\} = \omega^2 [M]\{\Gamma\}
\]  
(2.18)
in which \([K']\) is the reduced stiffness matrix, \([M]\) mass matrix, \(\{\Gamma\}\) is eigen vector containing mode shapes and \(\omega\) is the frequency of thermal vibration respectively.

### 3. Results and Discussion
Buckling analysis:
In this section, results for buckling analysis of laminated composite and sandwich beams under hygrothermal conditions are reported. Material used in present study have following material properties: \(E_1/E_3 = 20, G_{13} = 0.6E_3, \nu_{12} = 0.25, \alpha_{33}/\alpha_{11} = 3\). 3-layered \((0°/90°/0°)\) laminated composite beam with different end conditions is analysed under thermal loading subjected to equal rise/fall of temperature for different values of length-to-thickness ratio \((l/h)\). Results for dimensionless critical buckling load \((\phi = T_{cr} l^2 \alpha_1 / h^2)\) are reported in Table 1. Present results are found to be in good agreement when compared with those reported by Aydogdu [10] using HSDT and Lee [11] using WLT. It can be seen that with increase in thickness of beam, % error decreases i.e., present model is able to predict the buckling behavior of thick beams with good accuracy. It can be seen that for c-c end condition, non-dimensional buckling load is maximum while the same is found to be minimum for h-h (hinge-hinge) condition. With increase in thickness of beam, non-dimensional critical buckling load decreases.
Table 1. Variation of non-dimensional critical buckling load ($\phi$) 3-layered laminated composite beam subjected to thermal loading.

| l/h | Source       | H-H | H-C | C-C |
|-----|--------------|-----|-----|-----|
| 10  | Present      | 0.8015 | 1.2566 | 1.8561 |
|     | HSDT [10]    | 0.790 | 1.230 | 1.797 |
|     | LWT [11]     | 0.8229 | 1.2803 | 1.8711 |
|     | % Error$^1$ | -2.60 | -1.85 | -0.801 |
| 50  | Present      | 1.0532 | 2.1359 | 4.1532 |
|     | HSDT [10]    | 1.049 | 2.110 | 4.030 |
|     | LWT [11]     | 1.0921 | 2.1967 | 4.1963 |
|     | % Error$^1$ | -3.56 | -2.767 | -1.02 |

$^1\%$ Error = \(\frac{\text{Present results} - \text{LWT results}}{\text{LWT results}} \times 100\)

Table 2 shows non-dimensional critical buckling load for c-c laminated composite beam with same material properties as used in previous example for different ratio of elastic modulus ($E_1/E_3$) and thermal expansion coefficient ($\alpha_{33}/\alpha_{11}$). Present results are compared with those given by Aydogdu [10] using HSDT and Vosoughi [12] using FSDT and are found to be in good agreement. It can be seen that the behavior of beam depends upon both $E_1/E_3$ and $\alpha_{33}/\alpha_{11}$. For thin beam ($l/h=50$), with increase in value of $E_1/E_3$, non-dimensional critical buckling load also increases. However, for moderately thick beam, value of $\phi$ depends upon value of $\alpha_{33}/\alpha_{11}$ which attains maximum value at $E_1/E_3=30$.

Table 2. Variation of non-dimensional critical buckling load ($\phi$) 3-layered laminated composite beam subjected to thermal loading with c-c end condition.

| l/h | $E_1/E_3$ | Source       | $\alpha_{33}/\alpha_{11}$ |
|-----|-----------|--------------|--------------------------|
|     | 3         | 10           | 20                       | 50 | 100 |
| 10  | 20        | Present      | 1.5893 | 1.2318 | 0.9882 | 0.5937 | 0.3657 |
|     |           | HSDT [10]    | 1.511 | 1.223 | 0.961 | 0.585 | 0.354 |
|     |           | FSDT [12]    | 1.8121 | 1.4677 | - | - | - |
| 30  | 20        | Present      | 1.5021 | 1.3254 | 1.1987 | 0.6790 | 0.4452 |
|     |           | HSDT [10]    | 1.397 | 1.202 | 1.001 | 0.668 | 0.429 |
|     |           | FSDT [12]    | 1.4602 | 1.2565 | - | - | - |
| 40  | 20        | Present      | 1.1883 | 1.0511 | 0.9126 | 0.6500 | 0.4409 |
|     |           | HSDT [10]    | 1.169 | 1.040 | 0.898 | 0.638 | 0.429 |
|     |           | FSDT [12]    | 1.2167 | 1.0826 | - | - | - |
| 50  | 20        | Present      | 2.5531 | 2.0712 | 1.6352 | 0.9946 | 0.6023 |
|     |           | HSDT [10]    | 2.542 | 2.060 | 1.620 | 0.986 | 0.597 |
| 30  | 20        | Present      | 2.6010 | 2.2399 | 1.8701 | 1.2515 | 0.8080 |
|     |           | HSDT [10]    | 2.590 | 2.230 | 1.859 | 1.240 | 0.797 |
| 40  | 20        | Present      | 2.6012 | 2.3148 | 2.0116 | 1.4253 | 0.9631 |
|     |           | HSDT [10]    | 2.590 | 2.305 | 1.992 | 1.414 | 0.952 |

Figure 1 shows variation of non-dimensional critical buckling load for 3-layered c-c laminated composite beam (0°/90°/0°) for different values of l/h at different temperature and moisture concentration. Temperature and moisture concentration dependent material property is used during study as given in Table 3.
Table 3. Material property $E_2 = E_3, G_{13} = G_{12}, G_{23} = 0.5 G_{13}, \nu_{13} = 0.3, \beta_{11} = 0, \beta_{33} = 0.44, \alpha_{11} = -3 \times 10^{-6}/K, \alpha_{33} = 28.1 \times 10^{-6}/K$. 

| Elastic Moduli (GPa) | Moisture concentration (%) |
|----------------------|-----------------------------|
|                      | 0  | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 |
| $E_1$                | 130| 130  | 130  | 130  | 130  | 130  | 130  |
| $E_3$                | 9.5| 9.25 | 9.0  | 8.75 | 8.5  | 8.5  | 8.5  |
| $G_{13}$             | 6  | 6    | 6    | 6    | 6    | 6    | 6    |

| Elastic Moduli (GPa) | Temperature (°C) |
|----------------------|------------------|
|                      | 25  | 50  | 75  | 100 | 125 | 150 |
| $E_1$                | 130 | 130 | 130 | 130 | 130 | 130 |
| $E_3$                | 9.5 | 8.5 | 8.0 | 7.5 | 7.0 | 6.75|
| $G_{13}$             | 6   | 6   | 5.5 | 5   | 4.75| 4.5 |

Figure 1. Variation of non-dimensional critical buckling load for 3-layered c-c laminated composite beam ($0°/90°/0°$) with different values of $l/h$ at different (a) temperature and (b) moisture concentration.

Free vibration:
4-layered angle-ply laminated composite beam ($\theta^0/-\theta^0/-\theta^0/\theta^0$): In this example, a three-layered laminated composite beam is analysed under thermal and hygro conditions. Material properties used are: $E_1/E_3 = 15, G_{13} = 0.5 E_3, \nu_{13} = 0.3, \alpha_{11}/\alpha_{33} = 0.016, \beta_{33} = 0.44$. All layers are assumed to be made up of equal thickness. Results for non-dimensional natural frequencies ($\tilde{\lambda}$) are reported in Table 4. Beam is subjected to equal rise/fall of temperature/moisture concentration at top and bottom surfaces. Present results are found to be in good agreement when compared with the results obtained using ABAQUS. It can be seen that non-dimensional natural frequencies are more hygro loading as compared to thermal loading. With increase in angle of ply, non-dimensional natural frequency also decreases.

Laminated sandwich beam: In this example, 3-layered laminated sandwich beam ($0°$/core/$0°$) simply supported at its edges is analysed for free vibration condition under hygro-thermal loading. Face layer is assumed to be made up of material $E_1/E_3 = 25, G_{13} = 0.5 E_3, \nu_{13} = 0.25, \alpha_{11}/\alpha_{33} = 1E-6, \beta_{33} = 0.33, \rho = 1000$ and core layer of material $E_1/E_3 = 0.08, G_{13} = 0.12 E_3, \nu_{13} = 0.12, \alpha_{11}/\alpha_{33} = 1E-7, \beta_{33} = 0.5, \rho = 100$. Results for non-dimensional natural frequency are reported in Table 5. It can be seen that with increase in thickness of beam, non-dimensional natural frequency decreases.
Table 4. Variation of non-dimensional natural frequency ($\bar{\lambda}$) for 4-layered laminated composite beam ($\theta^0/\theta^0/\theta^0/\theta^0$) subjected to hygro-thermal loading (l/h=15) ($\Delta c=1$, $\Delta T=1$).

| End condition | Source | $\theta^0$ |
|---------------|--------|------------|
|               |        | 0° | 15° | 30° | 45° | 60° | 75° | 90° |
| Thermal loading ($\bar{\lambda} = \omega l^2 \frac{a_1}{a_3} \sqrt{\frac{\rho}{E_i h^2}}$) |
| c-c           | Present | 0.7674<sup>-1</sup> | 0.7196<sup>-1</sup> | 0.6077<sup>-1</sup> | 0.5595<sup>-1</sup> | 0.3237<sup>-1</sup> | 0.2542<sup>-1</sup> | 0.2608<sup>-1</sup> |
|               | ABAQUS  | 0.7593<sup>-1</sup> | 0.7288<sup>-1</sup> | 0.6181<sup>-1</sup> | 0.5672<sup>-1</sup> | 0.3310<sup>-1</sup> | 0.2633<sup>-1</sup> | 0.2692<sup>-1</sup> |
| c-s           | Present | 0.6235<sup>-1</sup> | 0.5361<sup>-1</sup> | 0.4481<sup>-1</sup> | 0.3622<sup>-1</sup> | 0.2436<sup>-1</sup> | 0.1871<sup>-1</sup> | 0.2100<sup>-1</sup> |
|               | ABAQUS  | 0.6351<sup>-1</sup> | 0.5774<sup>-1</sup> | 0.4581<sup>-1</sup> | 0.3737<sup>-1</sup> | 0.2520<sup>-1</sup> | 0.1993<sup>-1</sup> | 0.2261<sup>-1</sup> |
| Hygro loading ($\bar{\lambda} = \omega l^2 \beta_2 \sqrt{\frac{\rho}{E_i h^2}}$) |
| c-c           | Present | 2.1103 | 1.9789 | 1.6712 | 1.5388 | 0.8904 | 0.6992 | 0.7172 |
|               | ABAQUS  | 2.1231 | 2.0793 | 1.7557 | 1.6516 | 0.9913 | 0.7767 | 0.7935 |
| c-s           | Present | 1.7147 | 1.4743 | 1.2324 | 0.9960 | 0.6701 | 0.5147 | 0.5775 |
|               | ABAQUS  | 1.7935 | 1.5134 | 1.2991 | 1.0983 | 0.7638 | 0.6080 | 0.6334 |

Table 5. Variation of non-dimensional natural frequency ($\bar{\lambda}$) for 3-layered laminated sandwich beam (0°/core/0°) subjected to hygro-thermal loading.

| End condition | l/h |
|---------------|-----|
|               | 100 | 50  | 20  | 10  | 5  |
| Thermal loading ($\bar{\lambda} = \omega l^2 \frac{a_{11}}{a_{33}} \sqrt{\frac{\rho}{E_i h^2}}$) |
| s-s           | 3.3041 | 3.1979 | 2.6733 | 1.9134 | 1.2379 |
| c-c           | 7.1902 | 6.3352 | 4.1579 | 2.6775 | 1.7140 |
| Hygro loading ($\bar{\lambda} = \omega l^2 \beta_3 \sqrt{\frac{\rho}{E_i h^2}}$) |
| s-s           | 1.0903 | 1.0553 | 0.8821 | 0.6314 | 0.4085 |
| c-c           | 2.3727 | 2.0906 | 1.3721 | 0.8835 | 0.5656 |

4. Conclusion
A new C-0 HOZT is proposed in this work including transverse deformation under hygro-thermo-mechanical loading for laminated composite and sandwich beams. The theory satisfies transverse stress continuity condition along with zero transverse stresses at top and bottom of plate. Proposed theory is free from any penalty function or C-1 requirements. Three-noded C-0 FE having 8 d.o.f. per node is used. During the analysis, following conclusion are drawn:

a. Inclusion of transverse displacement field during analysis of laminated composite and sandwich beams helps to predict behavior of the same more efficiently especially for thick one.
b. Introduction of zigzag field also helps in predicting behavior of laminate composite and sandwich structures more efficiently especially for thicker one.
c. Thick beams are more susceptible buckle under thermal conditions as compared to thin beams.
d. Change in moisture concentration have no significant effect on buckling load for both thin and thick laminated composite beams.
e. Increase in thickness of laminated composite and sandwich beams, non-dimensional natural frequency decreases.
Hence, it is seen that the environmental condition widely affects the behavior of laminated composite and sandwich beams. Therefore, the present model can be applied for various research, industrial applications for laminated composite and sandwich plates during analysis stage.

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