Chord-Conditioned Melody Choralization with Controllable Harmonicity and Polyphonicity

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Abstract

Melody choralization, i.e. generating a four-part choral based on a user-given melody, has long been closely associated with J.S. Bach chorales. Previous neural network-based systems rarely focus on chorale generation conditioned on a chord progression, and none of them realised controllable melody choralization. To enable neural networks to learn the general principles of counterpoint from Bach’s chorales, we first design a music representation that encoded chord symbols for chord conditioning. We then propose DeepChoir, a melody choralization system, which can generate a four-part chorale for a given melody conditioned on a chord progression. Furthermore, with the improved density sampling, a user can control the extent of harmonicity and polyphonicity for the chorale generated by DeepChoir. Experimental results reveal the effectiveness of our data representation and the controllability of DeepChoir over harmonicity and polyphonicity. The code and generated samples (chorales, folk songs and a symphony) of DeepChoir, and the dataset we use now are available at https://github.com/sander-wood/deepchoir.

1 Introduction

Counterpoint is one of the most important compositional techniques in music history that developed during the common practice period (1650 to 1900) and nowadays is widely used in music like pop music, country music and big band. It is about how to establish a relationship between two or more parts that are harmonically interdependent yet independent in rhythm. A successful practice of counterpoint should ensure that the several parts with different rhythms are in harmony (constrained by the given chord progression).

Johann Sebastian Bach four-part (soprano, alto, tenor and bass) chorales, in short JSB Chorales, have enjoyed great importance since they were first composed, and are the supreme examples of the study of harmony and counterpoint. These concise musical gems demonstrate unique contrapuntal skill in the fusion of the vertical and horizontal aspects of music into a coherent and beautiful whole.

In this paper, we focus on the process of automatically composing a four-part chorale based on a melody and refer to it as a made-up word, i.e. melody choralization as shown in Fig. 1. In addition, harmonicity indicates the degree to which a given chord progression successfully harmonises a chorale, while polyphonicity describes how rhythmically independent the melody is from the rest of the parts.

Studies of melody choralization [14; 15; 7] have long been associated with JSB Chorales, and yielded promising results. For example, BachBot [16] generated-chorales is almost indistinguishable from that of Bach according to their experiment results. DeepBach [8], a system aimed at modelling polyphonic music, on the other hand, successfully applied to Monteverdi five-part madrigals and Palestrina masses even if they focus on JSB Chorales. As for non-neural network solutions like BacHMMachine [22], a probabilistic framework that integrates compositional principles, is capable of generating musically convincing chorales. However, the studies of melody choralization have been so far, rarely applied in practice, and just sort of in the air for the following limitations.
One of the most obvious limitations of melody choralization systems is that the generated music always being typically Baroque, regardless of the style of the melody given by users. This is because almost every melody choralization system is studied on JSB Chorales which are all composed during the Baroque period (17th–18th century). However, as chord progressions greatly determine the style of music, it is possible to generate chorales that follow the style of the chords provided by users if chord conditioning is supported.

Furthermore, as most melody choralization systems do not support controllable generation, users of them are not involved in the composition of the music at all, nor can they guide systems in generating what kind of music. For example, Bach Doodle [11], a melody choralization system designed by Google Magenta, is one of the most approachable neural network-powered music composition tools. However, all the user can do is draw a melody and click the “harmonise” button to generate a chorale.

In this paper, our goal is to provide a controllable tool that can generate a chorale or more general, contrapuntal music based on a melody with chords. We propose DeepChoir, a chord-conditioned melody choralization system with controllable harmonicity and polyphony. Our music representation uses MIDI pitch to encode notes while chords are represented as chromagrams. We further add fermata and beat information to enable the system to understand the distribution of notes more clearly. Furthermore, we improve the density sampling proposed in [18] to achieve controllable harmonicity and polyphonicity in the melody choralization task. With controllable harmonicity, users can intuitively guide the system on how strictly to follow the given chord conditions. On the other hand, as independence in rhythm is a crucial feature of contrapuntal music, with controllable polyphony, even systems like DeepChoir that trained on JSB Chorales can generate homophonic music (as shown in Fig. 2b) if users prefer it to polyphonic music.

2 Related Work
2.1 Melody Choralization
Most non-agnostic approaches (requiring music knowledge) are based on Markovian models. Yi et al. [20] use a factored Markov decision processes planner to generate classical four-part chorales based on the input melody. However, as they stated, the generated chorales are not very sophisticated. Kaliakatsos-Papakostas et al. introduce a hierarchical modelling approach in [13] using a hidden Markov model (HMM), motivated by the fact that some parts of a phrase (like the cadence) or a piece are characteristic of its idiomatic identity. Allan et al. propose an approach [1] based on HMMs, which represents chords as lists of intervals that form the states of the Markov models. These non-agnostic methods produce interesting chorale-like textures, even if they all use domain knowledge and require a considerable effort to generate rules with lots of subjective choices.

Unlike those methods mentioned above, agnostic ones do not require domain knowledge but rely on iterative training to discover the compositional patterns inherent in music. Hild et al. propose HARMONET [9], a melody choralization system capable of producing four-part chorales in the style of Bach. Boulanger-Lewandowski et al. present an RNN-based model [2] that can learn harmonic and rhythmic probabilistic rules from polyphonic music scores. Huang et al. present a convolutional approach [10] to model contrapuntal music based on the orderless NADE framework.

2.2 Controllable Music Generation
For users, a non-controllable music generation system is not very different from a random music player. As a rising number of researchers are aware of this, in recent years, there has been a growing interest in controllable music generation [21] to make users more involved in the music creation process.

Chen et al. propose SurpriseNet [4], a model based on Conditional Variational Auto-Encoder (CVAE) and Bi-LSTM, which combines a surprise contour from the transition probability in a Markov chain to achieve a user-controlled melody harmonization task. Wu et al. introduce AutoHarmonizer [18], a harmonic rhythm-controllable melody harmonization system, which can generate denser or sparser chord progressions with the use of density sampling. Chen et al. present Music SketchNet [3], a new framework to explore decoupling latent variables in music generation. Di et al. establish three rhythmic relations between video and background music, and propose a Controllable Music Transformer (CMT) [6] for video background music generation. In [17], Wang et al. demonstrate an algorithm to disentangle polyphonic music representation into chord and texture.
3 Methodology

This section details how we realise the melody choralization system DeepChoir with controllable harmonicity and polyphony. We first introduce the data representation we use. We then describe the structure of DeepChoir. Finally, we illustrate the improved density sampling for controllable harmonicity and polyphony.

3.1 Data Representation

Previous works [10; 12] encode chorales based on piano-roll which divides notes into multiple fixed small intervals. For works studying JSB Chorales, their data representations usually encode a sign called fermata, which is used extensively in JSB Chorales and functions similarly to a full stop in natural language for marking the end of a phrase. As shown in Fig. 3c and Fig. 3d, the two most representative data representations are those of DeepBach [8] and BachBot [16], both of which encode the notes of the four parts as well as fermata information. The major difference between the two is that DeepBach encodes each chorale into multiple sequences, whereas BachBot encodes it into a single sequence. Although systems based on both data representations can produce convincing polyphonic music, they ignore the following information.

- **Time Signature**: since time signatures of most JSB Chorales are 4/4, the neglect of time signature information does not have a serious impact. However, without encoded time signatures, it is impossible to distinguish between the rhythmic patterns of chorales with different time signatures.

- **Chord Symbol**: chord conditioning cannot be supported without encoding chord information. As the JSB Chorales Dataset (included in the music21 toolkit [5]) does not contain chord symbols, it is impossible to encode chord information without pre-processing. In Sec. 4.1, we describe how we pre-processed the JSB Chorales Dataset to extract chord symbols from four-part chorales.

As shown in Fig. 3b, our representation takes into account the two types of information mentioned above and encodes each chorale into seven sequences as follows.

- **Soprano, Alto, Tenor and Bass Sequences**: use 130-dimensional one-hot vectors representing four parts of chorales, with a time resolution of sixteenth notes. The first 128 dimensions correspond to the 128 different pitches in MIDI. The 129th dimension represents rests, while the 130th dimension represents holds.

- **Fermata Sequence**: uses a boolean value to indicate whether there is a fermata symbol on the current note.

- **Beat Sequence**: uses a boolean value to indicate whether there is a fermata symbol on the current note.

- **Chord Sequence**: encoded chords as chromagrams (12-dimensional multi-hot vector). Each dimension corresponds to an activated pitch class of a chord.

The improvement in our data representation over DeepBach seems to be technically incremental, but the difference in music is very significant. Since our beat sequence is encoded based on time signatures, it contains more rhythmic information than the subdivision list used by DeepBach, which only includes indexes of sub-beats within a quarter note. On the other hand, the introduction of the chord sequence provides the foundation for DeepChoir to support chord conditioning and controllable harmonicity.

3.2 DeepChoir

The goal of DeepChoir is to infer the alto, tenor and bass sequences given the soprano, fermata, beat and chord sequences.

As shown in Fig. 4, DeepChoir uses three neural networks: two Deep Bi-directional Recurrent Neural Networks, one as the soprano encoder, summing up information from the soprano sequence, and another as the condition encoder, sum-
ming up information from the fermata sequence, the beat sequence and the chord sequence. Both of them have a time distributed input layer with time scope \( \Delta t \) (in our implementation \( \Delta t = 128 \)). The outputs of these two encoders are then merged and passed as the input of the chorale decoder with three time distributed output layers (also have a time scope of \( \Delta t \)), whose outputs are the alto sequence, the tenor sequence and the bass sequence.

### 3.3 Improved Density Sampling

Wu et al. proposed density sampling in [18], which achieved the harmonic rhythm-controllable melody harmonization task via modifying the log probability of the holding token:

\[
\begin{align*}
p^*_h &= p_h \cdot \tan(\frac{\pi d}{t}) , \\
p^*_l &= (p_l - p^-_h) \cdot \frac{p_l}{\sum p_h} + p_l ,
\end{align*}
\]

where \( d \in (0,1) \) is the density parameter, \( p_h \) and \( p_l \) are the original and the updated probability of the holding token, while \( p^*_h \) and \( p^*_l \) are the original and the updated probabilities of non-holding tokens \( i \in \set{h} \). As stated by the author, “the density sampling ... is not only for harmonic rhythms, but for more diverse and controllable generation.” However, when we apply it for controllable harmonicity and polyphonicity, we are faced with two challenges.

The first challenge is how to modify the probabilities of multiple tokens simultaneously. For controllable harmonicity, we need to increase (or decrease) the probabilities of all the chord tones at the same time, to make the generated three parts are more (or less) in harmony with the melody. However, this is impossible for the original density sampling as it only supports modifying the probability of one given token.

The second challenge comes from controlling multiple attributes of the generated music at once. The original density sampling controls only one attribute of the generated music, whereas if using it several times directly to control multiple ones, it would cause the previous modification of the probability distribution overwritten by the later one.

To address these challenges mentioned above, we make two improvements to density sampling below.

- If multiple tokens are specified, modify the sum of the probabilities of these tokens instead. Subsequently, all the specified tokens are updated according to the modified sum of probabilities calculated previously.

- If multiple density sampling is required, then the probabilities of tokens that have been changed in previous turns are frozen.

The improved density sampling can be formalised as follows.

\[
\begin{align*}
p^*_S &= (1 - p_F) \cdot p_S \cdot \tan(\frac{\pi d}{t}) , \\
p^*_l &= p_l \cdot \frac{p_S}{\sum p_S} , & t \in S , \\
p^*_l &= (p_S - p^*_S) \cdot \frac{p_l}{\sum p_S} + p_l , & i \notin S \cup F ,
\end{align*}
\]

where \( p^*_t \) indicates it is an updated probability (otherwise it is the original one), \( S \) is the set of specified tokens of the current turn, \( F \) is the set of frozen tokens (whose probabilities have been changed in previous turns), \( p_S \) is the sum of the probabilities of tokens in \( S \), while \( p_F \) is the sum of the probabilities of tokens in \( F \), and \( \sum p_S \) is the sum of the probabilities of tokens that are neither in \( S \) nor in \( F \).

In our implementation, the probability of the holding token is first modified according to polyphonicity \( d_p \) in the first turn, and then the probabilities of all chord tones are changed according to harmonicity \( d_h \) in the second turn.

For controllable polyphonicity, at time step \( t \), the density \( d \) depends on whether the \( l \)-th token of the soprano sequence \( S_l \) is a holding token. The smaller the \( d_p \) is, the higher the probability of the holding token when \( S_l \) is a holding token, and thus the more consistent the rhythm of the generated three parts and the melody:

\[
d = \begin{cases} 
    d_p , & \text{if } S_l \text{ is a holding token} \\
    1 - d_p , & \text{elsewise}
\end{cases}
\]

For controllable harmonicity, we set \( d = 1 - d_h \) to make it more intuitive. At time step \( t \), the probabilities of all chord tones (according to the \( l \)-th token of the chord sequence \( c_l \)) are changed: the greater the \( d_h \) is, the smaller the \( d \) is, and thus more likely to sample chord tones, which leads to the generated three parts are more in harmony with the melody.

### 4 Experiments

This section describes the experiments we conducted. Sec. 4.1 introduces how we pre-process the JSB Chorales dataset. Sec. 4.2 reveals the effectiveness of our data representation. Finally, Sec. 4.3 provides the metrics we use to evaluate the controllability of DeepChoir and analyses the results comparison of among the validation set of JSB Chorales and DeepChoir in nine different settings.
4.1 Dataset

Since the original JSB Chorales Dataset has no chord progressions and the workload of carrying out harmonic analysis manually is too large, we perform the following automated pre-processing to add chord symbols.

- **Flattening**: all repeat barlines are removed by flattening each score to make them more machine-readable.
- **Chordify**: a tool in music21 [5] for simplifying a complex score with multiple parts into a succession of chords in one part.
- **Labelling**: we first move all the chords to the closed position, and then label the chordified chords as chord symbols. Finally, all chord symbols on beats of the soprano part are kept.

After removing a few scores that cannot be properly chordified, we ended up with a total of 366 chorales for training (90%) and validation (10%). This chordified JSB Chorales Dataset is now available at [GitHub](https://github.com/sander-wood/deepchoir).

4.2 Effectiveness of Data Representations

We compare our data representation with DeepBach [8] and BachBot [16] by evaluating the validation accuracy of the generated three parts. For a fair comparison, we trained DeepChoir with these three different music representations. The original DeepBach uses several models corresponding to the different parts, while the original BachBot generates chorales in SATB order. Now with the architecture of DeepChoir, all the three parts are generated simultaneously. As shown in Table 1, the representation of DeepBach generally has a marginal improvement over the BachBot one, which can be attributed to the subdivision list introduced by DeepBach. On the other hand, the music representation we introduced in Sec. 3.1 has a significant improvement over the other two. As the JSB Chorales Dataset contains a smaller variety of time signatures (predominant by 4/4), this performance gain is mainly attributed to the chord sequence rather than the beat sequence. Therefore, we can conclude that with chords provided by Bach, the chorales generated by DeepChoir are more Bach-like than other melody choralization systems. More generally, users of our system can obtain chorales that are more like Bach than other melody choralization systems. More specifically, users of our system can obtain chorales that are more like Bach than other melody choralization systems.

| Method      | Alto | Tenor | Bass |
|-------------|------|-------|------|
| BachBot     | 81.91| 81.35 | 80.83|
| DeepBach    | 81.88| 81.98 | 81.17|
| DeepChoir   | **85.03** | **84.04** | **86.32** |

Table 1: Results of the different data representations on the validation accuracy of the generated three parts.

4.3 Controllability of DeepChoir

Metrics

For evaluations of the controllability of DeepChoir over harmonicity and polyphonicity, we use five metrics.

- **Chord Tone to non-Chord Tone Ratio (CTnCTR)**: computes the ratio of the number of the chord tones ($n_c$) and proper non-chord tones ($n_p$), to the number of the non-chord tones ($n_n$):

$$CTnCTR = \frac{n_c + n_p}{n_c + n_n},$$

CTnCTR equals one when there are no non-chord tones at all, or all non-chord tones are proper.

- **Pitch Consonance Score (PCS)**: calculates a consonance score which is computed based on the musical interval between the pitch of the melody notes and the chord notes.

- **Melody-Chord Tonal Distance (MCTD)**: the average of the tonal distance between every melody note and its corresponding chord label, which calculates the closeness between a melody note and a chord label.

As the above metrics do not measure polyphonicity, we further propose two new metrics as follows.

- **Voice Alignment Rate (VAR)**: calculate (frame-wise) the extent of the two voices (i.e. parts) are in rhythmic alignment. The lower the VAR indicates the more independent the rhythm between voices.

- **Onset Alignment Ratio (OAR)**: similar to VAR, but calculate how independent the note onsets are between voices.

Analyses

To demonstrate the controllability of DeepChoir, we select three values for harmonicity $d_h$ and polyphonicity $d_p$: the small one 0.1, the medium one 0.5, which does not change probability distributions, and the large one 0.9. The 38 sopranos extracted from the validation set choralized by DeepChoir in 9 different settings are used to evaluate the effectiveness of the improved density sampling.

Since JSB Chorales Dataset (validation) is the ground truth and DeepChoir ($d_h = 0.5$, $d_p = 0.5$) is equivalent to removing density sampling (i.e. the ablation of density sampling), there is no need to introduce other baselines.

Table 2 gives the results in five metrics introduced in Sec. 4.2 to ground truth and our system in various settings.

For the three harmonicity metrics (CTnCTR, PCS, and MCTD), the best performance is achieved by DeepChoir ($d_h = 0.9$, $d_p = 0.1$). With this setting, the system generates music with a high harmonicity but low polyphonicity, which is typical of homophonic music. The worst performer on these metrics is DeepChoir ($d_h = 0.1$, $d_p = 0.9$), with a low harmonicity but high polyphonicity. Although each part
of polyphonic music is rhythmically independent, it is necessary to keep them in harmony. However, the music generated in the setting of \(d_h = 0.1\) while \(d_p = 0.9\) is not polyphonic music, but more like failed exercises composed by beginners who are learning polyphonic music with an overemphasis on polyphonicity while ignoring harmonicity.

As for the two polyphonicity metrics (VAR and OAR), the best performance is achieved by DeepChoir \((d_h = 0.1, d_p = 0.9)\), which is the same one that achieved the worst result on harmonicity metrics. Contrary to our intuition, it is DeepChoir \((d_h = 0.1, d_p = 0.1)\) and not DeepChoir \((d_h = 0.9, d_p = 0.1)\) that is the worst on the polyphonicity metrics, although the former is only marginally worse than the latter.

From Table 2 we can observe two general phenomena: 1) the higher the value of harmonicity \(d_h\) or polyphonicity \(d_p\), the better DeepChoir performs on the corresponding metrics, and 2) the higher value of polyphonicity \(d_p\) (or harmonicity \(d_h\)) with equal harmonicity \(d_h\) (or polyphonicity \(d_p\)), the worse (although not very significant) DeepChoir performs on the corresponding metrics. The first phenomenon illustrates the effectiveness of the improved density sampling, while the second is consistent with reality: generally, homophonic music is more harmonious than polyphonic music, but the rhythms between the parts are less independent. Note that harmonicity and polyphonicity are not incompatible, as DeepChoir achieves better performance in all metrics for \(d_h = 0.9\) and \(d_p = 0.9\) than for \(d_h = 0.5\) and \(d_p = 0.5\).

In addition, according to Table 2, we observe that the music composed by Bach is slightly more dissonant than the music generated by DeepChoir \((d_h = 0.5, d_p = 0.5)\), and the rhythms between the parts are more independent. This suggests that a \(d_h\) slightly less than 0.5 and a \(d_p\) slightly more than 0.5 might lead to DeepChoir generating music in the Bach style with greater fidelity.

We developed a publicly accessible musical discrimination test at GitHub\(^2\) with all the music we used for evaluations, which can be used to have an intuitive experience of how much (or how little) the music generated by DeepChoir resembles Bach’s one at different settings.

| Metrics | JSB Chorales (validation) | DeepChoir \((d_h=0.1, d_p=0.1)\) | DeepChoir \((d_h=0.1, d_p=0.5)\) | DeepChoir \((d_h=0.5, d_p=0.1)\) | DeepChoir \((d_h=0.5, d_p=0.5)\) | DeepChoir \((d_h=0.9, d_p=0.1)\) | DeepChoir \((d_h=0.9, d_p=0.5)\) | DeepChoir \((d_h=0.9, d_p=0.9)\) |
|---------|-----------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| CTnCTR↑ | 0.8721                      | 0.8482                          | 0.7519                          | 0.4804                          | 0.9660                          | 0.9165                          | 0.6987                          | 0.9981                          |
| PCS↑    | 0.6581                      | 0.6903                          | 0.6001                          | 0.1361                          | 0.7755                          | 0.7303                          | 0.3260                          | 0.8041                          |
| MCTD↓   | 0.9973                      | 0.9636                          | 1.0298                          | 1.4758                          | 0.8866                          | 0.9160                          | 1.2812                          | 0.8578                          |
| VAR↓    | 0.9100                      | 0.9930                          | 0.9467                          | 0.7879                          | 0.9924                          | 0.9496                          | 0.7184                          | 0.9928                          |
| OAR↓    | 0.9087                      | 0.9708                          | 0.9160                          | 0.3331                          | 0.9686                          | 0.9126                          | 0.3544                          | 0.9707                          |

5 Conclusions

In this paper, we proposed DeepChoir, a chord-conditioned melody chorализation with controllable harmonicity and polyphonicity. Although this system is trained/validated on the JSB Chorales Dataset, its purpose is not to generate counterfeits in the style of Bach, but to learn the general principles of counterpoint from his music compositions.

Instead of simply encoding chorales as four sequences corresponding to four parts, we further introduce fermata, beat and chord sequences to provide the system with a better understanding of the pattern of note distribution. Based on the density sampling proposed in [18], we improve and apply it for controllable harmonicity and polyphonicity. The experimental results prove the effectiveness of our data representation and demonstrate that the improved density sampling can significantly modify the attributes of the generated chorales. Controllable harmonicity on top of chord conditioning gives users the ability to steer the system to how strictly to follow a given chord progression. As for controllable polyphonicity, which enables the system to generate music other than polyphonic music, greatly broadens the scope of DeepChoir’s application.

In the future, we plan to develop a plug-in based on the MuseScore music editor that allows users to call DeepChoir to generate a four-part chorale based on a given melody and chord progression.

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\(^2\)https://sander-wood.github.io/deepchoir/test
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