Macroscopic and mesoscopic matter waves

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Abstract

It has been shown earlier [3,4,6] that matter waves which are known to lie typically in the range of a few Ångstrom, can also manifest in the macrodomain with a wave length of a few centimeters, for electrons propagating along a magnetic field. This followed from the predictions of a probability amplitude theory by the author[1,2] in the classical macrodomain of the dynamics of charged particles in a magnetic field. It is shown in this paper that this case constitutes only a special case of a generic situation whereby composite systems such as atoms and molecules in their highly excited internal states, can exhibit matter wave manifestation in macro and mesodomains. The wave length of these waves is determined, not by the mass of the particle as in the case of the de Broglie wave, but by the frequency $\omega$, associated with the internal state of excitation, and is given by a nonquantal expression, $\lambda = 2\pi v/\omega$, $v$ being the velocity of the particle. For the electrons in a magnetic field the frequency corresponds to the gyrofrequency, $\Omega$ and the nonquantal wave length is given by $\lambda = 2\pi v_{\parallel}/\Omega; v_{\parallel}$ being the velocity of electrons along the magnetic field.

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1. Introduction

The de Broglie matter waves associated with quantum particles have a wavelength typically a few Angstroms ($\lambda = \hbar/mv$) essentially because of the small value of $\hbar$. The question may be asked however, whether matter can exhibit its wave aspect in the macrodimensions as well, not in the sense of macroscopic correlated quantum systems such as superfluids or superconductors, but in the manner and spirit of de Broglie waves associated with single particles. Following the development of the concept of macroscopic matter waves through the theory of Ref.[1], reinforced by a more recent work by the author [2], we had demonstrated the existence of such a wave behavior for electrons propagating along a magnetic field having a wavelength independent of $\hbar$ and typically in the range of a few centimeters [3,4]. We wish to show here that such a wave manifestation is not entirely peculiar to this system but is a generic property of composite bound systems in their highly excited internal states approaching the classical limit. The wavelength of these new matter waves is related, not to the masses of these particles as in the case of de Broglie waves, but to the frequency associated with their internal state of excitation. This is an entirely new wave manifestation of matter not hitherto pointed out. We also predict the existence of such macroscopic or mesoscopic waves with atoms and molecules.

We first present here the development of the concept of macroscopic matter waves in the context of charged particle dynamics in a magnetic field, as expounded in the theories of Ref.[1,2] which is not widely known. After having captured the essence of such a behaviour for this system we shall show later how one could extend this concept to other systems, essentially composite systems such as atoms and molecules.

Even though the concept of the macroscopic matter waves in relation to the charged particle dynamics along a magnetic field actually followed from the theories of Ref[1,2], we present here first a direct quantum mechanical derivation of the macroscopic form of the wave function which is consistent with the form obtained from Ref[1,2] and which can account for the rather astonishing observations reported [3,4] in this connection, which were predicted by the theory. We later extend these considerations to other systems such as
atoms and molecules, and in fact any composite system.

2. Macroscopic wave function and matter waves for charged particles in magnetic field

A charged particle in a magnetic field in the classical mechanical domain corresponds in quantum mechanics to a particle in a Landau level with a very large quantum number. If \( E_\nu \) be the energy of a Landau level so that

\[
E_\nu = (\nu + \frac{1}{2})\hbar \Omega, \tag{1}
\]

where \( \Omega = (eB/mc) \) is the gyrofrequency in the magnetic field \( B \), then \( \nu \gg 1 \) corresponds to the classical limit and \( \nu \hbar = \mu \) defines the gyroaction, which classically has the form \( \mu = \frac{1}{2}mv_\perp^2/\Omega \), \( v_\perp \) is the component of velocity perpendicular to the magnetic field).

Consider now the propagation of an electron beam of a given energy injected into the magnetic field with a small pitch angle \( \delta \), so that \( v_\perp = v \sin \delta \), and \( v_\parallel = v \cos \delta \), \( v_\parallel \) being the velocity parallel to the magnetic field. The electrons in this beam are then in a group of Landau levels sharply peaked around the quantum number \( \nu = E_\perp/\hbar \Omega = \frac{1}{2}mv_\perp^2/\hbar \Omega \). For a typical laboratory situation, if we choose \( E = 1 \) keV, and a magnetic field \( B = 100 \) g, then \( \nu \approx 10^8 \), which is clearly \( \gg 1 \).

A charged particle in a magnetic field can be described by the Hamiltonian

\[
H = \frac{P_\parallel^2}{2m} + \frac{P_\perp^2}{2m} + \frac{1}{2}m\Omega^2\xi^2, \tag{2}
\]

where \( \xi \) represents the coordinate perpendicular to the magnetic field and the potential energy term \( \frac{1}{2}m\Omega^2\xi^2 \) represents the harmonic oscillator term corresponding to the Landau gyro-oscillations, while the first term represents the free motion along the magnetic field. As we shall see later, this Hamiltonian will also describe, with some modification, the oscillatory motion of a diatomic molecule.

Let \( \chi_\nu \) represent the Landau eigenfunctions which are essentially the harmonic oscillator wave functions [5]. Let there be a scatterer in the path of the electron beam, a small obstacle,
like the wires in grid through which the electron beam may be made to pass. The scattering which is assumed to be elastic may kick the electrons from the Landau level $\nu$ to $\nu \pm l$, where $\nu \gg l > 1$. If $\tilde{H}$ be the perturbation Hamiltonian which describes the scattering, then the transition amplitude for the process is given by

$$\beta_{\nu}^{(l)} \equiv \langle \nu - l \mid \tilde{H} \mid \nu \rangle = \int d\xi \chi_{\nu-l}(\xi) \tilde{H} \chi_{\nu}(\xi)$$  \hspace{1cm} (3)

where $\xi$ is the coordinate normal to the magnetic field representing the coordinate of the Landau gyro-oscillator.

Let $\phi_{\nu}$ represent the complete wave function of the particle in a magnetic field including a plane wave corresponding to its free motion along the magnetic field, so that

$$\phi_{\nu} = \chi_{\nu}(\xi)e^{i\kappa_{\nu}x}$$  \hspace{1cm} (4)

where

$$\kappa_{\nu} = \frac{1}{\hbar} \left[ 2m(E - \nu \hbar \Omega) \right]^{1/2}$$  \hspace{1cm} (5)

and $x$ is the coordinate along the magnetic field, while $E$ is the total energy of the particle. The transition amplitude including the eigenfunction along the magnetic field is given by

$$\alpha_{\nu}^{(l)} = \int d\xi \phi_{\nu-l}^* \tilde{H} \phi_{\nu} = \beta_{\nu}^{(l)} \exp\left[ i (\kappa_{\nu} - \kappa_{\nu-l}) x \right]$$  \hspace{1cm} (6)

Now making use of the assumption $l \ll \nu$, we expand $\kappa_{\nu-l}$ around $\kappa_{\nu}$ using the expression (5) which gives

$$\kappa_{\nu} - \kappa_{\nu-l} \simeq l \frac{\partial \kappa_{\nu}}{\partial \nu} = \frac{l \Omega}{v}$$  \hspace{1cm} (7)

where $v$ is the velocity of the particle along the magnetic field

$$v = \left[ \frac{2}{m} \left( E - \nu \hbar \Omega \right) \right]^{1/2}$$  \hspace{1cm} (8)
The difference \( (\nu - \nu - l) = l\Omega/v \) represents the change in the wave number \( \kappa \) of the plane wave, in consequence of the change in the Landau level quantum number from \( \nu \) to \( \nu - l \) due to the elastic scattering off the obstacle. The transition amplitude is thus given by

\[ \alpha^{(l)}_\nu = \beta^{(l)}_\nu \exp [i(l\Omega/v)x] \] (9)

This transition amplitude is again a wave function, representing a plane wave by virtue of the exponential factor. But as we notice, it is clearly independent of \( \hbar \). Since it is derived directly from the quantum mechanical wave function (4), it represents a matter wave with a wave length \( \lambda = 2\pi v/\Omega \). For an electron energy parallel to the magnetic field \( E_\parallel = 500eV \) and a magnetic field \( B = 100g \), we find \( \lambda \approx 5 \text{ cm} \). Thus this matter wave length falls in the macrodomain in contrast to the usual de Broglie wave length which is generally in the Å range.

It may be mentioned that these ideas have been more formally expressed in a recent paper by the author[2] where he has derived a set of Schrödinger-like equations starting from the quantum mechanic Schrödinger equation (in its path integral representation) for the charged particle dynamics in a magnetic field. These are

\[ i\mu \frac{\partial \Psi(l)}{\partial t} = -\left(\frac{\mu}{l}\right)^2 \frac{\partial^2 \Psi(l)}{\partial x^2} + (\mu \Omega) \Psi(l), \quad l = 1, 2, .. \] (10)

where \( \mu \), which is the gyroaction and has been shown in typical laboratory conditions to be \( \sim 10^8 \hbar \), is a classical object and appears in the role of \( \hbar \) in these equations. In terms of the notation of the foregoing treatment \( \mu = \nu \hbar \) with \( \nu \gg 1 \). Furthermore, the wave functions \( \Psi(l) \) of these equations are actually the transition amplitudes as defined above, for the quantum mechanical state with a large Landau quantum number \( \nu \) to the one with quantum number \( \nu - l \), induced by a perturbation. The number \( l \) labels this wave function as \( \Psi(l) \).

By virtue of the large (classical) value of \( \mu \) which appears in the place of \( \hbar \), these equations for the amplitude functions \( \Psi(l) \) describe matter wave phenomena in the macrodomain of
classical mechanics. This is essentially equivalent to what has been demonstrated above in Eq.(9), in more direct manner.

The wave function of Eq.(9), as also the Schrödinger-like equations, predict the matter wave phenomena with the wave length of a few centimeters for the charged particles moving along a magnetic field. The one-dimensional matter wave interference phenomena which correspond to these macroscopic wave functions of the form (9) have indeed been observed by the author and his coworkers [3,4].

The experimental results reported in Ref. [3,4] exhibit the existence of discrete energy bands (the maxima and minima) in the transmission of electrons along a magnetic field, when the energy from a gun is swept as they transit from the latter along a magnetic field to a detector plate a distance $L_p$ away. These bands which are rather unexpected in the parameter domain of the experiments where classical mechanical equations of motion are supposed to operate, have been identified as the interference maxima and minima in the energy domain, with a (nonquantal) macroscopic wave length $\lambda = 2\pi v_{||}/\Omega$, in accordance with the form (9) of the macroscopic wave function (where $v_{||}$ is the electron velocity along the magnetic field and $\Omega = eB/mc$, the electron-gyrofrequency in the magnetic field $B$). The interpeak separation of the transmission bands (in energy) are found to be inversely proportional to the distance $L_p$, so that the latter corresponds to a frequency as the energy is swept. The experiments of Ref.[3,4] thus confirm the predictions of the theory on the existence of the macroscopic form of the matter waves.

We have also found the existence of beat phenomena [6] in these experiments - a modulating beat structure of the already reported discrete energy band structure when the two “frequencies” in the system are close together. In the presence of a grounded grid at a distance $L_g$ from the electron gun, one has two frequencies in the system corresponding to the $L_p$ and $L_g$ and the frequency of the observed beats in the transmitted signal is found to correspond to the difference $(L_p - L_g)$, where $(L_p - L_g) \ll L_p$. This is just what occurs in other wave phenomena as well. These observed beats thus constitute a further, even tighter evidence for the wave behaviour of particles moving along a magnetic field.
3. Macroscopic matter waves for composite systems in their high internal state of excitation.

Having discussed the concept of macroscopic matter waves for charged particles in a magnetic field in the last section, whose wave manifestations in the macrodomain have also been observed, we now extend these considerations to other composite systems such as atoms and molecules in their internal state of excitation. We first discuss some gedanken experiments to point out the possible macroscopic wave manifestations of these systems and then discuss the possibility of carrying out real experiments to observe these manifestations associated with such composite systems.

A. Diatomic molecule in a highly excited vibrational state

First we consider a diatomic molecule in a highly excited vibrational state ignoring for the moment its rotational and electronic degrees of freedom. Such a system is described by the same Hamiltonian as (2) except for changing the 'parallel' momentum $p_\parallel$ to the momentum $P$ of the centre of mass $M$ of the diatomic molecule and changing the 'perpendicular' momentum $p_\perp$ to the momentum $p$ of the reduces mass $m$, and identifying $\xi$ as the reduced mass coordinate, so that we have the Hamiltonian as

$$H_{DA}^\nu = \frac{P^2}{2M} + \frac{p^2}{2m} + \frac{1}{2}m\omega^2\xi^2 \quad (11)$$

where we have now the vibrational frequency $\omega$ of the diatomic molecule, and we have of course ignored the anharmonic terms for simplicity.

If we employ a similar notation as before then the eigenfunction for the system with the Hamiltonian $H_{DA}^\nu$ corresponding to the free motion of the centre of mass with momentum $P$, and vibrational state $\nu$ is given by

$$\psi(P, \nu) = A_1 e^{iP \cdot X/h} \chi_\nu(\xi), \quad (12)$$

with the total energy $E$ given by

$$E = \frac{P^2}{2M} + \hbar \omega \left( \nu + \frac{1}{2} \right), \quad (13)$$
where $\chi_\nu(\xi)$ are the normalized harmonic oscillator wave functions.

Consider now a beam of such particles with a given momentum $P$ and in a highly excited vibrational state $\nu \gg 1$, which can be prepared using appropriate laser techniques. Let the beam be scattered by a grid of scatterers $G_1$ in its path, with small transverse dimensions. Assume that the scattering is elastic with respect to the total energy $E$ of the particle, and the scattering changes only its internal vibrational state to $\nu'$. So, the final state after the scattering is

$$\Psi' = A_1' e^{iP' \cdot X/\hbar} \chi_{\nu'}(\xi)$$  \hspace{1cm} (14)

where $|\nu' - \nu| \ll \nu$. If then $\tilde{H}(\xi)$ is the perturbing Hamiltonian which causes the scattering, then the transition amplitude is given by

$$\alpha_{\nu'\nu} = \langle \nu' | \tilde{H} | \nu \rangle = A_1 A_1' \exp \left[ -i (P' - P) \cdot X/\hbar \right] \int d\xi \chi_{\nu'}(\xi) \tilde{H}(\xi) \chi_\nu(\xi)$$  \hspace{1cm} (15)

where $A_1$ and $A_1'$ are appropriate normalization constants.

The change $\Delta E_i$ in the internal energy $E_i$ is $\Delta E_i = \hbar \omega (\nu' - \nu) = \hbar \omega l$, where $l \ll \nu$. Taking $P$ and $P'$ to be predominantly along a given $X$-direction, or alternatively, choosing for simplicity $P'$ to be in the direction of $P$ (one can do so experimentally) we have, because of the elastic nature of the scattering, the expressions: $P = [2M (E - \nu \hbar \omega)]^{1/2}$ and $P' = [2M (E - \nu' \hbar \omega)]^{1/2}$

On expanding $P'$ around $P$ for $l \ll \nu$, one gets to lowest order in $l \hbar$ : $P' - P \approx [2 (E - \nu \hbar \omega) / M]^{-1/2} l \hbar \omega$. Using this in (15) yields the transition amplitude as

$$\alpha_{\nu'\nu}^{(1)}(X) = A_1 A_1' \beta_{\nu'\nu} \exp \left[ \frac{i \omega (X - X_1)}{v} \right]$$  \hspace{1cm} (16)

where $\beta_{\nu'\nu}$ is the matrix element of the perturbation $\tilde{H}(\xi)$ between the oscillator states $\nu'$ and $\nu$ and $v$ is the magnitude of beam velocity $v = [2 (E - \nu \hbar \omega) / M]^{1/2}$. Note that while the expression for $v$ does involve $\hbar$, but in the limit $\nu \gg 1$, $\nu \hbar$ is actually a classical object $I = \nu \hbar, (\nu \gg 1)$. Also from the experimental point of view $v$ is simply the beam speed of the particles and it is not relevant what its expression is in terms the total energy $E$ and the
internal state energy $E_i = I\omega$. Similar remarks apply to the other cases as well discussed later.

The form of the perturbation $\tilde{H}$ is left general enough, and may be given any specific form as required. The important thing to note is that the transition amplitude for the translational centre of mass degree of freedom has the form $\exp \left[ il\omega X / v \right]$ and is independent of $\hbar$. Note that in general the scattering at the grid $G_1$ could lead to different values of $l = 1, 2, 3, \cdots$. But we keep only any one value for the simplicity of discussion.

Consider next another grid $G_2$ of scatterers located at the point $X_2$ in the path of the beam, the scattering (transition) amplitude from this grid is given by

$$\alpha_{\nu', \nu}^{(2)}(X) = A_2 A'_2 \beta_{\nu', \nu} \exp \left[ il\omega (X - X_2) / v \right]$$ (17)

where $A_2$ and $A'_2$ are again appropriate normalization constants.

Note that the expressions (16) and (17) represent wave functions corresponding to a wave number $k_l = l\omega / v = lk$ which is the $l^{th}$ harmonic of the basic wave number $k = \omega / v$, the corresponding wave length being $\lambda = 2\pi v / \omega$. This is clearly independent of $\hbar$, and could lie in the macro or meso-domain. For a typical diatomic molecule, the vibrational wave number is $(\omega / 2\pi c) \approx 2 \times 10^3 cm^{-1}$. Taking a modest value of beam velocity $v \approx 10^8 cm^{-1}$, this gives $\lambda \approx 0.1 \mu$. This is about three orders of magnitude larger than the typical de Broglie wave length of a few Å.

One can now look for interference between the waves given by (16) and (17) originating at the scatterer grids at $X_1$ and $X_2$. At a point $X$ downstream of the grids at $X_1$ and $X_2$ the total amplitude is given by

$$\alpha = \alpha_{\nu', \nu}^{(1)} + \alpha_{\nu', \nu}^{(2)} = e^{i kX} \beta_{\nu', \nu} \left[ A_1 A'_1 e^{-i kX_1} + A_2 A'_2 e^{-i kX_2} \right]$$ (18)

whence the intensity of the scattered particles is given by

$$| \alpha |^2 = | \beta_{\nu', \nu} |^2 \left\{ (A_1 A'_1)^2 + (A_2 A'_2)^2 + 2 (A_1 A'_1 A_2 A'_2) \cos [lk (X_1 - X_2)] \right\}$$ (19)

This therefore describes interference maxima and minima through the $\cos [lk (X_1 - X_2)]$ term which is independent of $\hbar$ and hence belongs to a nonquantal domain. Such interference
effects should be present in the experimental arrangement described above. This is analogous to the double slit interference, the grids at $X_1$ and $X_2$ corresponding to the two slits, but now in one dimension. One can check the validity of the expression (19), by working with different diatomic molecule to vary $\omega$, and different beam velocities to check the dependence on $v$, as well as different values of $(X_1 - X_2)$.

Though we have considered only one value of $l$, there would, in general exist many values of $l = 1, 2, 3...$ in the excitation spectrum of the vibrational states of the diatomic molecule as a result of scattering off the grids $G_1$ and $G_2$. However, $l = 1$ would be the most dominant one, being closet in energy to the central quantum number $\nu$. There would thus exist interference maxima, most dominant ones for $l = 1$ and successively subdominant ones for $l = 2, 3, ...$. On the other hand, the higher harmonics may lead to appropriate change of shape of the peaks corresponding to the fundamental.

There would arise, however, other more practical difficulties, if one were to move from the plane of the gedanken experiments to more real ones. One of them would be the preparation of the vibrational states with quantum numbers strongly peaked around a $\nu, (\nu \gg 1)$. This we assume can be carried out with appropriate laser techniques. There may be other technical difficulties, as for instance, the preparation of a beam of such excited molecules with a given well defined velocity. While there will certainly be such experimental difficulties, our main objective here is to point out the existence, in principle, of the macroscopic and mesoscopic matter waves rather than to address all the practical difficulties, which we hope can be overcome with some experimental ingenuity.

**B. Charged particles in a magnetic field**

If we apply these considerations to the case of charged particle in a magnetic field, we have the following correspondence $\omega \Rightarrow \Omega = eB/mc$, and $v \Rightarrow v_\parallel = \left[2 \left(\mathcal{E} - \mu \Omega\right)/m\right]^{1/2}$

In the case of the charged particle in a magnetic field, the bound vibrational state of the diatomic corresponds to the Landau state (bound to the magnetic field in the perpendicular direction) and the free motion of the centre of mass corresponds to the free motion of the charged particle along the magnetic field. The wave number $k$ now has the form $k = \Omega/v_\parallel$
with the wave length $\lambda = 2\pi v\|/\Omega$. This is precisely the wave length which follows from the form (9) of the macroscopic wave function and Eq.(10) of the formalism[1,2] corresponding to the mode $l = 1$. The discrete energy bands observed in the experimental results reported [3,4] as well as the beat structure observed more recently [6] are indeed a manifestation of matter waves in macro-dimension, for $l = 1$. In fact the experimental results have indeed exhibited the dependence of the locations of the interference maxima quite decisively on the parameters $(X_1 - X_2)$, and $\Omega = eB/mc$ and $v$ through $k = \Omega/v$ in the manner required by the expression (19). Note that the mode number $l$ in Eq.(1) corresponds precisely to the harmonic number $l$ in Eq.(9) and (19). This thus vindicates the formalism of Ref.[1,2] whose predictions motivated these experiments in the first place.

It may be noted that for a magnetic field of around 150 g and $v\| = 10^9\, cm/s$, the wave length $\lambda = 2\pi v\|/\Omega \approx 2.6\, cm$. This is clearly of macroscopic dimensions.

C. Rotational and vibrational states of a diatomic molecule

If we next think of the case of rotational states of a diatomic molecule, they do not obviously belong to the generic Hamiltonian (11). Yet with some difference such a system can also be shown to exhibit matter waves in macro-dimensions through wave amplitudes of the form (16) and (17). We recall that the origin of the form of the amplitude (16) is that the quantity $(P - P')/\hbar$ becomes independent of $\hbar$ to lowest order, when $P$ and $P'$ are substituted for. For the case of rotational states, $E_j = \hbar^2 j(j+1) \simeq \hbar^2 j^2$ for highly excited states. Then $P = \left[2M \left(E - \hbar^2 j^2\right)\right]^{1/2}$ and $P' = \left[2M \left(E - \hbar^2 j'^2\right)\right]^{1/2}$. This gives $P - P' \simeq 2\hbar^2 j l \left[2 \left(E - \hbar^2 j^2\right) / M\right]^{-1/2}$.

If we now define $J = \hbar j (j \gg 1)$, as the angular momentum value in the large quantum number limit, we get $(P - P')/\hbar \simeq (2JK/v_l)$, where $(j' - j = l \ll j)$ and where

$$v = \left[2 \left(E - \hbar^2 j^2\right) / M\right]^{1/2}.$$  It may be remarked that while $J = \hbar j$ and likewise $v$ may not appear to be independent of $\hbar$, they could be considered to be effectively so in the large quantum number limit $j \gg 1$, as in this limit $J = \hbar j$ could be considered a classical angular momentum. We thus see that looked at this way, $(P - P')/\hbar$ may be taken to be
independent of $\hbar$, and the wave amplitude $\alpha_{j'j}$ takes the form

$$\alpha_{j'j} = A_1 A'_1 \gamma_{j'j} \exp \left[ il \frac{2KJ}{v} (X - X_1) \right]$$

$$= A_1 A'_1 \gamma_{j'j} \exp \left[ i \frac{\omega_j}{v} (X - X_1) \right]$$  (20)

where $KJ = J(2mR^2)^{-1}$, $m$ being the reduced mass, and $R$ the internuclear distance. $2KJ$ is then $J/mR^2$ and is of the dimension of angular velocity. Thus we have written $\omega_j = J/mR^2$ in the second line of the above equation. The amplitude $\alpha_{j'j}$ is then again of the form (16), where $\gamma_{j'j}$ has a meaning corresponding to that for $\beta_{\nu'\nu}$ in (16). The amplitude (20) is again $\hbar$ independent and corresponds to a wave length of meso or macrodimension. However, the important difference between this case and the vibrational case is that while $\omega$ in the latter case is independent of the quantum state, $\omega_j$ increases linearly with the quantum number $j$.

In general, however, the scattering by the grid of scatterers would induce transitions in vibrational as well as in rotational states. One would then have a general wave amplitude of the form:

$$\alpha_{j'j; \nu'\nu} = A_1 A'_1 \Gamma_{j'j; \nu'\nu} \exp \left[ i \frac{\nu}{v} \left( l_{\nu} \omega_\nu + l'_{j'j} \omega_{j'j} \right) (X - X_1) \right]$$

(21)

where now we mean by $\omega_\nu$, the vibrational frequency and by $\omega_{j'}$, the rotational frequency. We define the corresponding wave numbers as $k_\nu = \omega_\nu/v$ and $k_{j'} = \omega_{j'}/v$ where $v$ is now

$$v = \left[ 2 \left( E - \nu \hbar \omega_\nu - K_{j'}^2 \hbar^2 \right) / M \right]^{1/2}$$

We thus have a more general expression than (19) for this case involving both the vibrational and rotational excitations

$$| \alpha |^2 = | \Gamma_{j'j; \nu'\nu} |^2 \left\{ (A_1 A'_1)^2 + A_2 A'_2 \right\}^2 + 2(A_1 A'_1 A_2 A'_2) \cos \left[ \left( l_{\nu} k_{\nu} + l'_{j'j} k_{j'j} \right) (X_1 - X_2) \right] \right\}, \quad (22)$$

As we know, the vibrational frequency of a diatomic molecule $\omega_\nu \gg \omega_j$, the rotational frequency. Therefore, analogously to the case with the vibrational-rotational spectrum of the
diatomic molecules, the spacing of the fringes corresponding to the rotational matter wave number $k_j = \omega_j/v$ would be correspondingly greater compared to those relating to the vibrational matter wave number $k_\nu = \omega_\nu/v$. Here the vibrational fringes will be contained within the rotational fringes, contrary to the case of the vibrational-rotational spectrum. It would thus be possible to differentiate the two sets of fringes with respect to the variation of the spatial separation $(X_1 - X_2)$.

D. Rydberg states of an atom

One may also consider the Rydberg states of an atom for a similar discussion, which presents a rather interesting case. The energy levels for this case are given by

$$E_n = -\frac{me^4}{2\hbar^2 n^2}.$$  

Using the expressions $P = [2M (E - E_n)]^{1/2}$ and $P' = [2M (E - E_{n'})]^{1/2}$, we find $P - P' \approx \frac{2E_n}{nv}$ where $v = [2 (E - E_n) / M]^{1/2}$ is again the magnitude of the centre of mass velocity in the highly excited state $n$, with the total energy being $E$, and where $l = |n' - n| \ll n$ is assumed. Using the foregoing expression for $P' - P$ in the expression corresponding to (16) gives

$$\alpha_{n'n} = A_1 A'_1 \Delta_{n'n} \exp \left[\frac{2E_n}{nv\hbar} (X - X_1)\right].$$  

where $\Delta_{n'n}$ is the scattering (transition) amplitude for a perturbation $\tilde{H}$ between the Rydberg states $n'$ and $n$. The expression (23), of course, involves $\hbar$ unlike the expressions (16) and (17) for the harmonic internal degree of freedom, which are independent of $\hbar$. But because of the presence of $n$ in the denominator of the exponent of (23), the effective wave number

$$k_n = \frac{2E_n}{v(n\hbar)} = \frac{\omega_n}{v} ; \quad \omega_n = \frac{2E_n}{n\hbar},$$

can be small enough to be of macroscopic or mesoscopic dimensions if $n$ is taken to be sufficiently large. For $n = 100$, $\omega_n \approx 6.6 \times 10^{10}$ rad s$^{-1}$, and $k_\parallel \approx 660$ cm$^{-1}$ for $v = 10^8$ cm s$^{-1}$. This gives $\lambda_n = 2\pi/k_n \approx 10^{-2}$ cm, which is of macroscopic, or if one prefers, of mesoscopic dimensions.
Discussion and summary

The concept of macroscopic and mesoscopic matter waves as distinct from the deBroglie matter waves that has been presented above, first arose in relation to the dynamics of charged particles in a magnetic field as described by a probability amplitude theory [represented by a set of Schrödinger-like equations] given by the author[1] in 1985. This was obtained as a Hilbert space representation of the classical Liouville equation for the system, and by virtue of its amplitude character, it predicted the existence of matter wave phenomena with a macroscopic wave length, namely the one-dimensional interference effects. The role of $\hbar$ in these equations is enacted by a macroscopic action $\mu = \nu \hbar, (\nu \gg 1)$.

Recently the author has been able to derive [2] the same set of Schrödinger-like equations starting from the quantum mechanic Schrödinger equation. It is the macroscopic magnitude of $\mu$ (which replaces $\hbar$) that is responsible for the macroscopic matter waves in charged particle dynamics along a magnetic field. Indeed the one-dimensional interference effects with macroscopic matter waves with wave length typically in the range $\sim 5cm$ have been reported by the author and his collaborators [3,4,6]

A close examination of the derivation of Ref.[2] gave the author the clue that such macroscopic and mesoscopic matter waves should be possible with, in fact, any composite system, such as atoms and molecules in their highly excited internal states. The discussion presented in this paper is an exposition these concepts as applied to diatomic molecules and Rydberg atoms as illustrative examples.

As already mentioned, the interesting point about these matter waves is that their wave length is not related to the mass of the particle as with the deBroglie waves, but rather to the frequency $\omega$ associated with the highly excited state of its internal degree of freedom—vibrational, rotational for a molecule, and Landau gyro-oscillations for charged particles in a magnetic field, or the frequency associated with atomic levels in an atom. and is independent of the Planck quantum in this correspondence limit. In a sense these waves may be regarded as classical counterpart of the quantum deBroglie waves.
It is also clear from the above discussion that such kind of meso or macrodimension matter waves are generically possible. It would be interesting to try to observe them with atoms and diatomic molecules or even nuclei, just as we have observed them in the system of charged particles in a magnetic field [3,4,6]

The existence of these waves has already been experimentally demonstrated [3,4,6] for electrons propagating along a magnetic field, following initially its predictions by a quantum like theory given by the author [1,2] which actually stimulated the entire line of investigation culminating in the findings presented here. We now predict the existence of such waves for other systems, atoms and molecules, or any other composite systems which could be looked for experimentally.

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