Bang-Bang control over a direct current to direct current converter

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Abstract. The following document presents an optimal control technique to control power converters direct current to direct current using the Bang-Bang control technique, for example a Buck power reducing converter will be shown. Subsequently and using an approach based on the principle of the maximum of Pontryagin, an optimized switched control technique (bang-bang) is designed and implemented that allows to obtain a performance comparable to the on-off action in terms of dynamic characteristics. Additional tests allow quantifying the optimality of the proposed technique and verifying the performance of the controlled system in energy terms, showing that an optimal switched control has, in addition to the minimization of the functional cost (minimum error energy), a lower incidence in the generation of high frequency switching noise, compared to conventional technique. As one of the most important aspects highlighted in the results corresponds to the simplicity of design and the high efficiency of the Bang-Bang controllers, making the output voltage practically no ripples.

1. Introduction
The law of controlling a problem with bounded inputs is known as the Bang-Bang control. This problem is of special interest, since the fastest admissible response of a dynamic system is achieved, considering the limitations of its actuators [1-4]. All dynamic systems, if they wish, are exposed to a control law which can vary according to the characteristics of each model, but always looking to be able to control (reference signal) the system in the shortest time and, if possible, perform it optimally subject to some restrictions. The Bang-Bang control theory assumes a continuous system, with the bounded control action |u(t)| ≤ Umax, for all t ∈ [0, T] as shown in Equation (1).

\[ \dot{x}(t) = Ax(t) + Bu(t). \]

(1)

Associated with a cost index in Equation (2).

\[ J = \int_0^T dt = T. \]

(2)

Being free T; the optimal control problem is to find an acceptable u(t) control that minimizes the cost index J and satisfies the constraint |u(t)| ≤ Umax for all time instants and leads to the system from the initial state x(0), known to the final state x(T) = 0, also known. The Hamiltonian for the linear minimum time problem is Equation (3).
\( H(x(t), u(t), \lambda(t)) = 1 + \lambda^T(t)(Ax(t) + Bu(t)). \) \( (3) \)

If the optimality conditions apply and the minimum principle of Pontryagin will be taken in Equation (4) to Equation (6).

\[
\dot{\lambda}^*(t) = -\frac{\partial H}{\partial x(t)} = -A^T\lambda^*(t), \tag{4}
\]

\[
\dot{x}^*(t) = \frac{\partial H}{\partial \lambda(t)} = Ax^*(t) + Bu(t), \tag{5}
\]

\[
\min H\left(x^*(t), u(t), \dot{\lambda}^*(t)\right) = \min 1 + \left(\lambda^*(t)\right)^T(Ax(t) + Bu(t)). \tag{6}
\]

From Equation (6) it is follows that the control action that minimizes \( (\lambda^*(t)^T Bu(t)) \) must be selected, since it is the only term in which the control action appears in the Hamiltonian explicitly. If we analyze it for each of the inputs \( u(t) \), the condition is the minimization of \( (\lambda^*(t))^Tb_i(t) \), being \( b_i \) each of the columns of \( B \) \([5-7]\). Therefore, it can be reasoned that the control action to be selected is

\[
u(t) = -\text{signo}(B^T\lambda^*(t)). \tag{7} \]

Physically the sign function can be implemented by means of a relay, which switches to the maximum control action depending on the sign of \( B^T\lambda^*(t) \). Precisely for this reason, this optimal control law is called Bang-Bang. Figure 1 shows an example of the exchange function control law.

\[
\lambda^*(t) = e^{-At}\lambda^*(0). \tag{8} \]

Which replacing in the control law Equation (7) an open loop control law is obtained, Equation (9).

\[
u(t) = -\text{signo}(B^Te^{-At}\lambda^*(0)). \tag{9} \]

Whose main drawback is that \( \lambda^*(0) \) is unknown. In a closed loop, a relationship between the state and the co-state must be sought, so that it can be expressed as Equation (10).
\[ \lambda^*(t) = h(x(t)). \] (10)

However, there is no general analytical solution to obtain this function instead, so the Bang-Bang control in closed loop must be particularized for each case [8-12].

2. Application

Consider the Buck power converter circuit shown in Figure 2, it is intended to bring the system from a known initial state to a final state in the shortest possible time [8]. The cost function associated with this problem is given by Equation (11).

\[ J = \int_{t_0}^{t_f} dt = t_f - t_0. \] (11)

Subjecting \(-1 < u(t) < 1\) and null desired end conditions for the state vector, starting from arbitrary initial conditions. In this way, the form taken by the lagrangian of the system \(L(x, u) = 1\) is evident, from this it is possible to formulate the Hamiltonian of the system which is described by Equation (12).

\[ H(x, u, \lambda) = 1 + \lambda^T f(x, u). \] (12)

That in the case of a system described in the form \(\dot{x} = Ax(t) + Bu(t)\) and by means of the maximum theorem it is an optimal solution of the type of Equation (7). The system model shown in Figure 2 can be written in equations of state such as Equation (13).

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = 
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} + Bu(t).
\] (13)

Figure 2. Buck power converter circuit.

The previous Figure 2 contains an element of the power electronics that distorts the output signal as well as a switch that introduces unbounded variations during its operation. Table 1 shows the model data of the type converter Buck, where do you have to Equation (14).

| Parameter          | Value       | Parameter          | Value       | Parameter          | Value   |
|--------------------|-------------|--------------------|-------------|--------------------|---------|
| Input voltage      | 24 V        | Switching frequency| 10 kHz      | Capacitance        | mF      |
| Output voltage     | 12 V        | PWM useful cycle   | 50%         | Inductive resistance| 0.2 Ω   |
| Load resistance    | 10 Ω        | Inductance         | 300 mH      | Capacitive resistance| 0.1 Ω  |

\[
A = \begin{bmatrix}
-(R_L + \frac{R R_C}{R+R_C})L & -\frac{R}{(R+R_C)L} \\
\frac{R}{(R+R_C)C} & -\frac{1}{(R+R_C)C}
\end{bmatrix}; \quad B = \begin{bmatrix}
\frac{V_{in}}{L} \\
0
\end{bmatrix},
\] (14)
where $x_1(t)$ represents the inductor current, and $x_2(t)$ the capacitor voltage and $u(t)$ the value of the useful cycle of the pulse width modulated switching signal, then to solve the minimum time problem is equivalent to zeroing the error of the state vector, with respect to the desired steady state values. In this way the following functional [11] can be formulated Equation (15).

$$J = \int_{t_0}^{t_f} \frac{(x_{2d} - x_2)^2}{2} dt.$$  \hspace{1cm} (15)

To regulate the output voltage in the converter (where $x_2(t)d = 12V$), with the restrictions on the control signal given by Equation (16); considering a useful percentage cycle.

$$0 \leq u(t) \leq 1.$$ \hspace{1cm} (16)

3. Results

Next, we will present a group of simulations carried out with MATLAB software associated with the output voltage when the system without the control action and when it is. Figure 3 shows the output voltage of the Buck step-down converter without the Bang-Bang control action and with a disturbance at the output. In this figure the noise that occurs.

![Figure 3. Drive output voltage without control action.](image)

From the simulation in Figure 4 and Figure 5, associated with the action of the Bang-Bang controller, we can see that the output voltage of the converter under the action of said control is smoothed and the oscillations are reduced, thereby guaranteeing the efficiency of this system.

![Figure 4. Output voltage of the converter under the Bang-Bang control action.](image)
In the simulation corresponding to Figure 6 a PID control over the converter is shown. As a final result visible in this graph, it is observed that the Bang-Bang control makes the system reach the reference in the shortest possible time, while the PID control increases it due to its multiple undulations.

The previous results showed how the circuit works without the control action, as can be seen in Figure 3, in which an overshoot is observed due to the current switching; and later it is observed how the control action corrects the inconveniences produced by the diode and the commutation.

4. Conclusions
In the results it is observed that the Bang-Bang control presents a control action which causes the system to reach the desired state which in this case is the output voltage of the converter in the shortest time possible making the output voltage not present as many oscillations as seen in a PID controller Figure 6. Another important aspect in Bang-Bang controllers is the simplicity of design and high efficiency, making the output voltage virtually undulating. Bang-Bang controllers are an effective solution to many optimization problems such as power converters, minimum or maximum temperature problems in which it is important for the system to respond quickly in the shortest possible time.

Bang-Bang controllers are an effective solution to many optimization problems such as power converters, minimum or maximum temperature problems in which it is important for the system to respond quickly in the shortest possible time. The Bang-Bang control technique allows to implement optimal controllers on non-linear systems, which makes it possible to tackle very difficult control problems.

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