Study on the Vibrant Angular Velocity Measurement Algorithm Based on Planar Sensor Array

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Abstract. To solve the complex vibration testing six degree-of-freedom rigid space problem, this paper designs a sensor array configuration model and corresponding algorithm. The model integrating the method of integration and the method of the square calculates the angular velocity data. Through the analysis of the inner link of data in sensor output equation, the calculating data updates and iterations constantly, maintaining understanding of real-time and accuracy of calculation data. At the same time it can solve the problem of the error accumulation of integral method and extraction of the problem of misjudgement of symbols. The simulation results show that the error is always within the range of 10^-4 in 1000 seconds.

1. Introduction
Modern platform space vibrant condition is very complex. Real-time tests and records its vibrate parameters for condition monitoring of platform, system virtual prototype designing and key components system modal analysis has very important significance [1]. The platform has many degrees of vibration freedom, large dynamic range, complex frequency components, and the vibrant state is the coupling of multi-dimensional vibration and angular vibration [2]. Unidimensional sensor testing technology can only track the movement of a single state under the condition of the vibration of the small angular velocity gyroscope responses under the condition in low frequency and it is so expensive. They are not adapted to the vibrant measurement of space [3]. The accelerometer has the characteristics of low cost, large range of testing and sensitive reaction, which provides a new way for the measurement of vibrant parameters [4]. This paper tries to configure multiple acceleration sensors in the method of plane array to detect six degrees of freedom vibrant parameters. Through the simulation analysis, we found the solution adapted to the multi degrees of freedom, large dynamic complex vibrant environment, has a good applied prospect [5].

2. Sensor array testing scheme
Installation of configurational model was shown in Figure 1. The O - XYZ is a coordinate system as establish [6]. Three triaxial acceleration sensor must be in the same position with coordinate system. The installed position of three triaxial acceleration sensors is overlapping with coordinate axis.
Figure 1. Schematic diagram of planar accelerometer array installation.

The positional coordinate matrix \( r \) and the sensitive direction coordinate matrix \( \theta \) of 9 sensors are compared with the O - XYZ.

\[
\begin{pmatrix}
    l_1 & l_1 & l_1 & 0 & 0 & 0 & -l_3 & -l_3 & -l_3 \\
    0 & 0 & 0 & l_2 & l_2 & l_2 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
    1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
    0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\
    0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 1
\end{pmatrix}
\]

(1)

According to the kinematics equation, the output of any accelerometer based on the linear accelerometer is obtained [7].

\[
f = J \begin{pmatrix}
    \dot{\Omega} \\
    A
\end{pmatrix} + \begin{pmatrix}
    \theta_1^2 \Omega^2 r_1 \\
    \vdots \\
    \theta_N^2 \Omega^2 r_N
\end{pmatrix}
\]

(2)

\( F \) -- Accelerated signals measured on the accelerometers;
\( J \) -- configurational matrix;
\( \dot{\omega} \) -- angular acceleration of the O - XYZ;
\( \Omega \) -- the anti-symmetric matrix;
\( A \) -- the linear acceleration in the direction of the three axes of the vector coordinate system.
When equation 2 was fully expanded, it can get angular velocity, angular acceleration and linear acceleration of the O – XYZ. Here is the nine accelerated sensors’ equation for testing of vibration.

\[
\dot{\omega} = \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix}, \quad A = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}
\]

\[
\Omega^2 = \begin{bmatrix} -\omega_x^2 - \omega_y^2 & \omega_x \omega_y & \omega_x \omega_z \\ \omega_x \omega_y & -\omega_y^2 - \omega_z^2 & \omega_y \omega_z \\ \omega_x \omega_z & \omega_y \omega_z & -\omega_z^2 - \omega_x^2 \end{bmatrix}
\]

\[
J = \begin{bmatrix} J_1^T \\ J_2^T \end{bmatrix}, J_2 = \theta
\]

\[
r = [r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, r_9, r_10, r_11, r_12, r_13, r_14, r_15, r_16, r_17, r_18, r_19, r_20, r_21, r_22, r_23, r_24, r_25, r_26, r_27, r_28, r_29, r_30, r_31, r_32, r_33, r_34, r_35, r_36]
\]

\[
\theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{10}, \theta_{11}, \theta_{12}, \theta_{13}, \theta_{14}, \theta_{15}, \theta_{16}, \theta_{17}, \theta_{18}, \theta_{19}, \theta_{20}, \theta_{21}, \theta_{22}, \theta_{23}, \theta_{24}, \theta_{25}, \theta_{26}, \theta_{27}, \theta_{28}, \theta_{29}, \theta_{30}, \theta_{31}, \theta_{32}, \theta_{33}, \theta_{34}, \theta_{35}, \theta_{36}]
\]

\[
J_1 = [r_1 \times \theta_1, r_2 \times \theta_2, \ldots, r_9 \times \theta_9]
\]

(3)

According to the formula, the correlation expression of the related parameters represented in six degrees of freedom can be calculated.

\[
\begin{align*}
f_1 &= A_x - I_x (\omega_x^2 + \omega_y^2) \\
f_2 &= A_y + I_y (\omega_x \omega_y) \\
f_3 &= A_z - I_z (\omega_x \omega_z) \\
f_4 &= A_x + I_x (\omega_x \omega_y - \omega_z) \\
f_5 &= A_y - I_y (\omega_x^2 + \omega_z^2) \\
f_6 &= -A_x - I_x (\omega_y + \omega_z) \\
f_7 &= -A_y - I_y (\omega_x^2 + \omega_y^2) \\
f_8 &= -A_z + I_z (\omega_y + \omega_z) \\
f_9 &= A_z - I_z (\omega_x - \omega_y)
\end{align*}
\]

(4)
3. The design for method of angular velocity

Applying the squared of angular velocity, angular acceleration of Z axis calculating by equation 5, we can obtain three axes angular velocity by applying to the integral method and the combination algorithm [8].

3.1. Integral method

In the case of the initial angular velocity of the O - XYZ known, the angular velocity of the O - XYZ is calculated according to the integration [9-10]. The initial parameters of the angular velocity of the O - XYZ are set, the sampling period is relatively short, and it is estimated that the time interval of the sampling interval is uniformly varying from the time interval of the sampling interval. Given the Z axis angular acceleration, the direct integral algorithm can be obtained, as shown below:

\[
\begin{align*}
\omega_z^2 - \omega^2 &= \frac{l_z(f_z - f_1) - l_z(f_1 + f_z) + l_z(f_i + f_z)}{l_z(l_i + l_z)} \\
\omega_x^2 + \omega_y^2 &= \frac{l_x(f_x - f_2) - l_x(f_2 + f_x)}{l_x(l_i + l_x)} \\
\dot{\omega}_z &= \frac{l_z(f_z - f_1) - l_z(f_1 + f_z) - f_z}{2l_z(l_i + l_z)} \\
\dot{\omega}_x &= \frac{l_x(f_x - f_2) - l_x(f_2 + f_x)}{2l_x(l_i + l_x)} \\
\omega_z^{\text{2}} - \omega^2 &= \frac{l_z(f_z - f_1) - l_z(f_1 + f_z) + l_z(f_i + f_z)}{l_z(l_i + l_z)} \\
\end{align*}
\]

(5)

3.2. Open method

According to the square item in the formula, it can be obtained, but cannot directly obtain the symbol of the absolute value of three axes. In this paper, the expression of the solution of the method is
expressed as the symbol sign, sign and sign, which is the absolute value of the angular velocity in the open method.

\[
\begin{align*}
\omega_x(t) &= \text{sign}(\omega_x(t)) \times |\omega_x(t)| \\
\omega_y(t) &= \text{sign}(\omega_y(t)) \times |\omega_y(t)| \\
\omega_z(t) &= \text{sign}(\omega_z(t)) \times |\omega_z(t)|
\end{align*}
\]

(7)

Open method for calculation of angular velocity, process, calculating numerical not accumulate over time, avoids the production of large error, but its application is integral method calculating the value of the symbol, there is still a symbol misjudgment phenomenon, with the accumulation of time, symbol mistakes continue to increase the influence of the results.

3.3. Combination algorithm

Through analysis and comparison of various calculating methods for acceleration of plane array configurations designed an integral method, the combination of square root method algorithm, calculation of angular velocity, , the application of open method for Z axis angular velocity calculating value of the algorithm. According to related no gyro strapdown inertial navigation system alignment technology set X axis and Y axis initial angular velocity for, its plug type (4), obtain, and application, integral operation, and the integral value symbols, symbols, and its product as angular velocity calculating values. Combination algorithm based on the update of the initial value, to avoid the error integral method to accumulate over time, and continue to achieve accurate angular velocity of absolute value, through the open method combining both can obtain accurate angular velocity and settlement value of the and do not accumulate over time.

4. The simulation analysis

In order to verify the feasibility of the angular velocity algorithm for plane configuration design, the model is compared and analyzed. In the simulation, the sampling frequency is 1kHz, the sampling time is 1000s, the constant error of the acceleration sensor is 0.0001g, and the mean variance of random noise is 0.001g.

The angular velocity parameter is set to

\[
\begin{align*}
\omega_x &= 5\sin(100t) \\
\omega_y &= 5\cos(80t) \\
\omega_z &= 5\sin(60t)
\end{align*}
\]

(8)

The line acceleration parameter is set to.

\[
\begin{align*}
A_x &= 600\cos(60t) \\
A_y &= 800\cos(80t) \\
A_z &= 1000\cos(100t)
\end{align*}
\]

(9)

The three algorithms are simulated, and the X axis and Y-axis angular velocity errors are analyzed respectively, as shown in Figure.2 - Figure. 4.
The angular velocity error of Figure 2 algorithm and the angular velocity error of the algorithm can be seen from the graph 3, and the error of the angular velocity is increasing over time. When the test time is short, the error is small, but when time increases, its error increases exponentially. Therefore, the algorithm cannot be applied to the attitude calculation of plane acceleration sensor array.
As can be seen from the simulation curve of Figure 3, angular velocity error will no longer increase with time accumulation to 10rad/s. Simulation shows that although open method to solve the single integral method of error accumulation problem, but the open method of the integral gain symbol component to its symbol misjudgment phenomenon with the serious time, difficult to get the real angular velocity calculating values. Therefore, it is difficult to apply integral method to new configurations.

As can be seen from the Figure 4 combination algorithm simulation, the angular velocity error for calculating combination algorithm stability in magnitude, did not appear the phenomenon of error accumulated over time, also won't appear misjudgment phenomenon, and always maintain a high accuracy of calculating results, meet the range of allowable error. The simulation shows that the combination algorithm can solve the problem of the sign misjudgment of the improved method by updating the initial value, so as to ensure the accuracy of the data. The proposed algorithm provides a reliable initial solution data for the next step, which is an effective method.

Acknowledgments
In order to realize the engineering application of sensor array technology in the field of complex vibration testing of moving body, it is necessary to design the sensor configuration scheme that meets the requirements. In this paper, a planar sensor array is introduced, and the angular velocity calculation is the key to the vibration test of sensor array. According to the new configuration, three solutions are designed by studying traditional solutions, where the error of integral method is serious with time, and the error of the method is inevitable. Combination algorithm effectively avoids the integral error accumulation and small, the angular velocity of misjudgement of symbols, in a long period of time can still maintain a high level of accuracy in the process of the simulation calculation, engineering implementation of sensor array vibration test technology has a certain significance.

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