Anisotropy of partially self-absorbed jets and the jet of Cyg X-1

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Accepted 2016 August 17. Received 2016 August 2; in original form 2016 June 10

ABSTRACT

We study the angular dependence of the flux from partially synchrotron self-absorbed conical jets (proposed by Blandford & Königl). We consider the jet viewed from either a side or close to on axis, and in the latter case, either from the jet top or bottom. We derive analytical formulae for the flux in each of these cases, and find the exact solution for an arbitrary angle numerically. We find that the maximum of the emission occurs when the jet is viewed from top on-axis, which is contrast to a previous result, which found the maximum at some intermediate angle and null emission on-axis. We then calculate the ratio of the jet-to-counterjet emission for this model, which depends on the viewing angle and the index of power-law electrons.

We apply our results to the black-hole binary Cyg X-1. Given the jet-to-counterjet flux ratio of $\gtrsim 50$ found observationally and the current estimates of the inclination, we find the jet velocity to be $\lesssim 0.8c$. We also point out that when the projection effect is taken into account, the radio observations imply the jet half-opening angle of $\lesssim 1^\circ$, a half of the value given before. When combined with the existing estimates of $\Gamma_j$, the jet half-opening angle is low, $\lesssim 1/\Gamma_j$, and much lower than values observed in blazars, unless $\Gamma_j$ is much higher than currently estimated.

Key words: acceleration of particles–galaxies: jets–radiation mechanisms: non-thermal–radio continuum: stars–stars: individual: Cyg X-1–stars: jets.

1 INTRODUCTION

The radio emission of jets in the hard state of black-hole binaries and of extragalactic parsec-scale radio sources is often flat in the $\mathrm{d}F/\mathrm{d}E$ representation, $\propto E^\alpha$, with $\alpha \sim 0$, where $E$ is the photon energy, e.g., Cawthorne (1991), Fender et al. (2000), Healey et al. (2007). The emission of this type of jet is usually interpreted as due to the partially self-absorbed synchrotron process in a continuous conical jet (Blandford & Königl 1979, hereafter BK79). The emission at a given energy is self-absorbed from its onset up to a certain height of $z \propto E^{-1}$, and it is optically thin at higher $h$. The jet becomes optically thin at all $z$ at energies above the synchrotron break energy, $E_b$.

The model of BK79 is also assumed in an important method of measuring magnetic fields of extragalactic jets from core shifts, i.e., angular displacements with frequency of the observed maxima of jet radio emission (Lobanov 1998; Pushkarev et al. 2012). This method can also be used to measure the jet power (Zamaninasab et al. 2014; Zdziarski et al. 2015).

The angular emissivity pattern of such continuous, partially optically-thick jets differs from that of optically-thin jets, either steady-state or in the form of moving blobs (Lind & Blandford 1985; Sikora et al. 1997). The case of the angular dependence of the emission from partially self-absorbed jets was considered by Cawthorne (1991). Here, we re-examine this problem, and obtain different results at viewing angles close to the jet axis. The cause of this discrepancy is that the usual approximation that we view the jet from a side in the comoving frame breaks down in that case.

We also consider implications of our results for the ratio of the jet-to-counterjet emission. We then apply our results to the compact radio jet observed from Cyg X-1 in its hard spectral state.

2 DEFINITIONS

We develop here the formalism of Zdziarski, Lubiński & Sikora (2012), hereafter ZLS12. We use the notation similar to that of ZLS12, except that now the Doppler factor is $\delta_j$, the dimensionless jet length, $\xi$, is in units of the distance of the position of the onset of the synchrotron emission, $z_0$, as in Zdziarski et al. (2014a), and we take into account the effect of cosmological redshift, which we denote as $z_c$. Also, the jet radius is given now accurately as $z \tan \theta_j$ rather than by $z_\theta$, where $z$ is the distance from the jet origin and $\theta_j$ is the half-opening angle. We consider the model of a conical jet moving with a constant bulk velocity, $\beta_j c$, and emitting synchrotron radiation (BK79).

We are concerned with the angular distribution of the emission of such jets. We observe the jet at an angle, $\iota$, with respect to the jet
The top solid and bottom dashed (red) and the two middle (black) curves are for $\Gamma_j = 10$ and $5/3 (\beta_i = 0.8)$, respectively. We see that at large $\Gamma_j$, the jet-frame viewing angle for most values of $i$ is close to $180^\circ$. In the comoving frame moves in a changing environment, in spite of its comoving position being constant. We perform our calculations in the observer frame, in which the jet is stationary (though its medium is moving). We do it using the emission and absorption coefficients transformed from the comoving frame, see below.

The steady-state electron distribution per unit volume in the comoving frame at a given point (defined in the observer frame) is assumed to be a of a power-law form, $N(\gamma) \approx K\gamma^{-p}$, $p > 1$, (2)

where $K$ is the normalization and $\gamma$ is the electron Lorentz factor. Following BK79, we assume conservation of the electron distribution along the jet, and conservation of the energy flux in the toroidal component of the magnetic field. We also define the dimensional energy, $E$, in the observer’s frame and a dimensionless one, $\epsilon$, in the jet frame. Thus, we have

$$K = K_0\xi^{-2}, \quad B = B_0\xi^{-1}, \quad \xi \equiv z/z_0, \quad \epsilon = \frac{(1 + z_0)E}{\delta\mu e c^2}. \quad (3)$$

where $z_0$ corresponds to the onset of emission.

For calculation of the synchrotron emission and absorption, we assume the magnetic field is tangled (Heinz & Begelman 2000), which implies the emission in the local frame is isotropic. The emission coefficient from isotropic relativistic power-law electrons per unit volume at the jet frame can then be written as,

$$j_3(\epsilon, \xi) \approx \frac{C_1(\sigma_T c K B_0^2)}{48\pi^2} \frac{1}{\epsilon^{1+p}}, \quad (4)$$

where $B_0 = m_e^2c^2/\hbar$ is the critical magnetic field, $\hbar$ is the reduced Planck constant, $e$ is the electron charge, and $C_1(p) \sim 1$ ($= 1$ for $p = 3$) follows from averaging over the pitch angle, see, e.g., ZLS12. The emission coefficient in the local observer frame at $E$ equals $\delta_j^2 j_3(\epsilon, \xi)$ (e.g., Ghisellini 2000). The synchrotron self-absorption coefficient averaged over the pitch angle for a power-law electron distribution can be expressed as,

$$\alpha(\epsilon, \xi) = \frac{C_2\sigma_T c K}{2\alpha_i} \frac{1}{\epsilon^{1+p}} e^{-\epsilon \tau_\alpha} = \alpha_0\epsilon^{-\alpha_0} e^{-\epsilon \tau_\alpha}, \quad (5)$$

$$\alpha_0 \equiv \frac{C_2\sigma_T c K}{2\alpha_i} \frac{B_0}{B_0} \frac{1}{\epsilon^{1+p}}, \quad (6)$$

where $\alpha_i$ is the fine-structure constant, and the constant $C_2(p) (= 1$ for $p = 3$) is given, e.g., in ZLS12. The absorption coefficient in the
local observer frame at $E$ equals $\delta_i^1 \alpha_i E(\xi, \xi)$. The source function, $f(x)/\alpha_i$, is then,

$$S(\xi, \xi) = S_0 \delta_i^1/\xi^{1/2}, \quad S_0 = \frac{C_{10} c B_{10}^2}{24\pi^2 C_j B_{0j}^2},$$

which becomes $\delta_i^1 S(E, \xi)$ in the observer frame.

### 3 THE JET ANGULAR EMISSIVITY PATTERN

In order to obtain the spectrum observed from the jet, we need to calculate the emission towards the observer from a given location of the jet in the observer frame and then integrate it over the jet projected area. We can do it by integrating the radiative transfer equation over the line of sight, e.g., equation (1.29) in Rybicki & Lightman (1979), and then integrate the solution over the projected area. In the conical geometry and for an arbitrary angle, this can be done only numerically, which we do in Appendix A. However, if the viewing angle in the observer frame is $\sim 90^\circ$, see the case (iii) in Fig. 2, we can approximate the jet locally as a cylinder, and neglect the variation of $K$ and $B$ along the line of sight. The above approximation also implies that the source function is constant along the line of sight. In this approximation, the radiative transfer equation solves for the observed intensity as $\tau$, the optical depth to synchrotron self-absorption integrated over the entire line of sight,

$$\tau(\xi, z, x) = \frac{2\alpha_i(\xi, z)}{\delta_i^1 \sin i} \left[ z \tan \theta_j \xi^2 - x^2 \right]^{1/2},$$

where $x$ is the distance perpendicular to both the jet axis and the line of sight. The flux (for either jet or counterjet) is then given by (ZLS12)

$$\frac{dF}{dE} = \frac{(1 + \xi^2 \beta_0^2) \sin i}{m_e c^2 D^2} \int_0^\infty dz \int_{z \tan \theta_j}^{\infty} \int_{\xi^2 \sin i}^{\xi \sin i} d\xi \left[ 1 - \exp(-\tau) \right],$$

where $D$ is the luminosity distance. Since uppermost parts of the jet usually contribute weakly to the total flux, we assume here the jet extends to infinity. Hereafter, the redshift terms are included, which becomes $\delta_i^1 S(E, \xi)$ in the observer frame.

### Anisotropy of partially self-absorbed jets

The integral defined in ZLS12, which we integrate here analytically,

$$C_i(p) \equiv \int_0^\infty d\xi \xi^2 \int \frac{d\psi}{(1 - \exp(-\tau) \xi^2 \sin i)^{1/2}} \left[ 1 - \exp(-\tau) \xi^2 \sin i \right],$$

where $\tau = \Gamma(1 + \psi)/(1 - \beta)$ is the Doppler factor at $i = 0$. We note that equation (12) is equivalent to the limit of $i \to 0$ of the expression for $\tau$ in the side-view case, see, e.g., equation (19) of ZLS12. Namely, in that case, $dr = \alpha_i dx/(\delta_i \sin i)$. We have $\sin i = dx/(dx^2 + dz^2)^{1/2}$. Thus, $dr = \alpha_i (dx^2 + dz^2)^{1/2}/\delta_j$, which, for $i = 0$, equals $dr = \alpha_i dx/\delta_j$, as above. We also note that although $\tau$ is invariant between different frames, it does depend on $\delta_j$.
where $i$ is the approximation of equation (10), which is exact at $90^\circ$.

The solid (red) curve in Fig. 3 shows an example of the exact angular distributions of a partially optically-thick jet, calculated in Appendix A, which reaches the values of the analytical formulæ at $i = 0^\circ$ and $180^\circ$. We see that the flux seen on axis is substantially higher than the maximum of the flux in the large-angle approximation, shown by the dashed (black) curve. This appears to be a new result; e.g., Cawthorne (1991) derived the maximum flux and the corresponding viewing angle using the large-angle approximation (as for the dashed curve in Fig. 3), while we find the overall maximum to occur at $i = 0^\circ$, with the flux decrease with the decreasing $i$ of that approximation being spurious.

We then compare the values of the synchrotron break energy, $E_c$. For the top view, we solve $\tan i = 1$ with equation (12),

$E_c(i = 0) = \frac{Gm_e c^2}{1 + z_c} \left( \frac{2m_e c \delta z_0}{(4 + p)\delta p} \right) \frac{1}{1 + 4 \delta_j \sin i} \Gamma \left( \frac{1}{p - 4} \right) . \quad (17)$

In the side-view case, we have (ZLS12),

$E_c(i \sim 90^\circ) = \frac{Gm_e c^2}{1 + z_c} \left( \frac{2m_e c \delta z_0}{(4 + p)\delta p} \right) \frac{1}{1 + 4 \delta_j \sin i} \Gamma \left( \frac{1}{p - 4} \right) . \quad (18)$

With the same assumptions as above, we obtain with equation (17),

$E_c(i \sim 0) \approx E_c(i \sim 90^\circ) = \frac{1}{(1 + \beta_j) \Gamma \left( \frac{1}{p - 4} \right) \left[ (4 + p)\alpha_i \right] } . \quad (19)$

This ratio is typically $\sim 1$, e.g., it is $\sim 1.5$ for $\beta_j = 1$, $p = 2$, $a = 0.2$. 

### 4 THE COUNTERJET AND LARGE ANGLE EMISSION

The emission of the counterjet corresponds to the viewing angle of $\pi - i$. We note that $\sin(\pi - i) = \sin i$, and $\cos(\pi - i) = -\cos i$. The Doppler factor of the counterjet is then

$\delta_j = \frac{1}{\Gamma_j(1 + \beta_j \cos i)} . \quad (20)$

Equation (10) then applies to the counterjet emission with the substitution of $\delta_j \rightarrow \delta_j$. When we consider the flux ratio between the jet and counterjet in the side-view approximation, the angular dependencies on $\sin i$ in equation (10) cancel each other.

Thus, we find the jet to counterjet flux ratio under the assumption of both components being intrinsically symmetric of

$R \equiv \frac{\delta_j}{\delta_j} \left( \frac{\Delta \varepsilon_j}{\Delta \varepsilon_j} \right) \frac{1 + \beta_j \cos i}{1 - \beta_j \cos i} . \quad (21)$

The power-law index of this dependence changes from 2 for $p = 1$ to $\sim 3$ for $p \gg 1$, different from the index of 2 for an optically thin jet with $a = 0$. The total emission is, obviously, the sum of the fluxes from the jet and the counterjet. We note that equation (22) of ZLS12 implies an incorrect ratio, due to the neglect of the different
value of $E_i$ between the jet and counterjet. Equation (21) can be solved for $\beta_j$:

$$\beta_j = \frac{1}{\cos i} \frac{R^{\pi j}}{R^{\pi j} + 1}.$$

(22)

We now consider the case of the counterjet emission at $i \sim 0$. At this approximation, we have considered the emission from the top of the jet, equations (13–14). However, this counterjet emission corresponds to that from the jet bottom, $i \sim 180^\circ$, see the case (ii) in Fig. 2, for which those equations are not applicable. We can still use the approximation of a ray along the jet axis, but the direction of the emission is opposite. This also means that we cannot multiply the optical depth by different $\tau$ depending on the distance from the axis. Using radiative transfer, we write

$$\frac{dF}{dE} = \frac{2\pi(1 + \xi)\delta_0}{m_e c^2 D^2} \int_0^\infty drr^{(r)} \int_0^{(r)} d\tau' S(\tau') \exp(\tau' - \tau),$$

(23)

where $S(\tau')$, given by equation (12), corresponds to the intersection of the line of sight with the jet boundary at the radial distance $r$. Then the optical depth along the ray emitted at $i = 180^\circ$ is $\tau = \tau'$, i.e., it is measured from the intersection with the boundary. We can now change the variable of the outer integration to $\tau$ using equation (12). The resulting double integral can be calculated analytically, and the final result is

$$\frac{dF}{dE} \propto \left[ \frac{\pi c^2 \tau r \delta_0 B_{\psi}}{(4 + p)\alpha_0 \alpha_0 B_R} \right] \left( \cot \frac{5\pi}{4 + p} - \cot \frac{4 + p}{4 + p} \right) \left( \frac{1}{\Gamma(\frac{1}{p-1})} \right).$$

(24)

We can obtain the jet-to-counterjet flux ratio for this case using equation (14). We find it is almost the same, within $\sim 10$ per cent, as that given by equation (21). This shows that equation (21) is valid quite generally, in spite of the underlying approximation broken at $i$ substantially different from $90^\circ$. Naturally, equation (24) with a replacement of $\delta_0 \rightarrow \delta_j$ also gives the jet emission at $i \sim 180^\circ$.

We also note that we have neglected here a possible (and likely in a range of angles) obscuration of the counterjet emission at $i \sim 0$, by the accretion disc and stellar wind. Also, the jet will suffer synchrotron-absorbed reprocess the counterjet synchrotron emission.

## 5 Application to Cyg X-1

Stirling et al. (2001) have obtained radio maps of the black-hole binary Cyg X-1 using VLBA and VLA at 8.4 GHz. They have found no evidence for the presence of a counterjet, and constrained the flux ratio to $R \gtrsim 50$. They assumed $i = 40^\circ$ and optically-thin emission in calculating constraints on the jet velocity. Currently, a lower value of $i$ appears to be the inclination of the orbit of Cyg X-1, in particular Orosz et al. (2011) found $i \approx 27 \pm 1^\circ$. On the other hand, Ziolkowski (2014), considering also the evolutionary status of the system, found $i \approx 29.5^\circ$ as the most likely range. For this range, equation (22) at $p = 2$ gives $\beta_j \gtrsim 0.82^{+0.03}_{-0.01}$, corresponding to $\Gamma_j \gtrsim 1.75^{+0.25}_{-0.11}$. The value of $p$ has been constrained by broad-band spectral models of ZLS12, Malyshov, Zdziarski & Chernyakova (2013) and Zdziarski et al. (2014b), which give the most likely range of $1.4 \lesssim p \lesssim 2.5$. The resulting limits on $\beta_j$, which are relatively insensitive to the value of $p$, as illustrated in Fig. 4. A caveat for this result is that the jet is likely aligned with the black-hole rotation axis, which may be misaligned with the normal to the binary plane. In this case, the jet inclination may not be given by the above constraints. Possible indications for this to be the case are given by the results of Tomisick et al. (2014) and Walton et al. (2016), who have fitted X-ray data from observations of Cyg X-1 in the soft spectral state (in which the inner disc is expected to extend to the innermost stable orbit) by NuSTAR and Suzaku, and NuSTAR, respectively. Their fits gave the inclination of the inner disc substantially larger than $30^\circ$, namely $\approx 70^\circ$ and $\approx 40^\circ$, respectively, which would indicate the plane of the inner disc inclined with respect to the orbital axis. We caution, however, that this may be due to the limitations of the used models. Tomisick et al. (2014) used the Compton reflection model of Ross & Fabian (2005), which averages over all viewing angles. Then, Walton et al. (2016) used the model of García & Kallman (2010), which calculates angle-dependent reflection, but both models convolve their reflection spectra using the relativistic code of Dauser et al. (2010), which neglects a number of important effects, as discussed in Niedźwiecki, Zdziarski & Szanecki (2016). Thus, we consider the issue of the misalignment to be open.

Other constraints on the jet velocity in Cyg X-1 are by Gleissner et al. (2004), who claimed $\beta_j \lesssim 0.7$ or so based on non-detection of short-time scale correlations between radio and X-ray emission, though this limit appears model-dependent. Then Malzac, Belmont & Fabian (2009) estimated $0.3 \lesssim \beta_j \lesssim 0.8$, which also relies on a number of assumptions. Still, if we accept those constraints at face value, the most likely jet velocity in Cyg X-1 becomes $\beta_j \approx 0.8$.

Stirling et al. (2001) also constrained the half-opening jet angle of the projection of the jet on the sky as $\lesssim 2^\circ$. We note here that the actual half-opening angle is the one after de-projection, i.e., multiplied by $\sin i$ (e.g., Königl 1981). Given that $i \approx 30^\circ$, $\theta_j \lesssim 1^\circ$. Since the lower limit on $\Gamma_j$ is rather low, see above, this upper limit on the half-opening angle implies a very small factor $a \equiv \Gamma_j \tan \theta_j$, unless $\Gamma_j$ is much higher than the lower limit of Stirling et al. (2001) obtained from the absence of an observed counterjet. If $\beta_j = 0.8$ ($\Gamma_j = 5/3$), $a \lesssim 0.03$. This implies an extremely efficient collimation mechanism, e.g., that related to a very low jet magnetization in the models of Tchekhovskoy et al. (2009) and Komissarov et al.
imply the jet half-opening angle of the projection effect
\[ \beta \gtrsim \text{arbitrary viewing angle} \]
found observationally \[ \Gamma \gtrsim 50 \]
also calculated the emission corresponding to the emission angle of 
\[ i \approx 180^\circ, \]
when the jet is viewed on axis, with \( i = 0 \), e.g., Cawthorne (1991). However, we point out that the above approximation breaks down in the low-\( i \) regime, since the jet is no longer viewed sideways in the comoving frame. We have considered another limiting case, of the jet viewed on axis. We have found an analytical solution of the radiative transfer integrated over the jet cross section in that case. We have found on axis. We have found an analytical solution of the radiative transfer integrated over the jet cross section in that case. We have found out that this emission is rather strong, corresponding to the global maximum of the flux as a function of the viewing angle. We have also calculated the emission corresponding to the emission angle of \( i \approx 180^\circ \), which also corresponds to the counterjet emission in the case of \( i = 0 \). Then, we have obtained the general exact solution at an arbitrary viewing angle numerically, described in Appendix A.

We have applied our results to the black-hole binary Cyg X-1. Given the jet-to-counterjet flux ratio of \( \gtrsim 50 \) found observationally (Stirling et al. 2001) and the current estimate of the inclination of \( i \approx 29.2^\circ \) (Orosz et al. 2011; Ziolkowski 2014), we have found \( \beta \gtrsim 0.8, \Gamma_i \gtrsim 1.6 \). Combining it with other published constraints, the most likely value is \( \beta \gtrsim 0.8 \). We have also pointed out that when the projection effect is taken into account, the radio observations imply the jet half-opening angle of \( \theta_i \lesssim 1^\circ \), a half of the value given by Stirling et al. (2001). If \( \Gamma_i \) is not much above the counterjet limit, the opening angle is \( \theta_i \ll 1/\Gamma_i \), and much lower than the values typically observed in blazars.

6 CONCLUSIONS
We have studied the extended synchrotron jet model, originally proposed by BK79. We have considered three limiting analytical approximations to the flux vs. the viewing angle. In one, usually assumed, the jet is viewed sideways in the comoving frame. This approximation implies that the flux becomes null when the jet is viewed on axis, with \( i = 0 \), e.g., Cawthorne (1991). However, we point out that the above approximation breaks down in the low-\( i \) regime, since the jet is no longer viewed sideways in the comoving frame. We have considered another limiting case, of the jet viewed on axis. We have found an analytical solution of the radiative transfer integrated over the jet cross section in that case. We have found out that this emission is rather strong, corresponding to the global maximum of the flux as a function of the viewing angle. We have also calculated the emission corresponding to the emission angle of \( i \approx 180^\circ \), which also corresponds to the counterjet emission in the case of \( i = 0 \). Then, we have obtained the general exact solution at an arbitrary viewing angle numerically, described in Appendix A.

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ACKNOWLEDGMENTS
We thank Marek Sikora for valuable discussions and the referee for valuable comments and suggestions. This research has been supported in part by the Polish National Science Centre grants 2012/04/M/ST9/00780, 2013/10/M/ST9/00729 and 2015/18/A/ST9/00746.

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APPENDIX A: EXACT CALCULATION OF THE ANGULAR DEPENDENCE

As stated in Section 3, we first obtain the intensity emitted by a given point of the jet projection in a given direction, and then integrate it over the projected area. In order to efficiently deal with the geometry of an inclined cone and its projection, we set up two coordinate systems in the observer frame. One is the usual jet coordinate system, with the $z$ axis along the jet axis, and the $x$ and $y$ axes orthogonal to it. The other system has the origin at the top of the jet, at the height $Z$, at $(0, 0, Z)$ in the jet coordinates, and it is rotated clockwise by the viewing angle, $i$. This corresponds to an anticlockwise rotation of the jet itself. Since the optically-thin jet emission declines fast with the distance, the results are almost independent of the assumed value of $Z$ as long as it is much larger than the height at which the emission at a given frequency becomes optically thin. The coordinates in the rotated system are marked by the subscript $i$. The transformations from the jet system to the rotated one and the reverse one are

$$
\begin{pmatrix}
  x_i \\
  y_i \\
  z_i
\end{pmatrix} = R_i
\begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix}, \quad R_i =
\begin{bmatrix}
  1 & -\sin i & \cos i \\
  0 & 1 & 0 \\
  -\cos i & -\sin i & 0
\end{bmatrix}, \quad \text{and}
\begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix} = R_i^{-1}
\begin{pmatrix}
  x_i \\
  y_i \\
  z_i
\end{pmatrix},
$$

(A1)

respectively, where $R_i$ is the three-dimensional clockwise rotation matrix fixing the $x$ axis.

The way we calculate the intensity depends on the viewing direction. For viewing from either the top or the side, $i < \pi - \theta_i$, $\tau$ is taken to be 0 when the ray leaves the jet, and integrated backward to its maximum value at the ray origin. For viewing the jet from the back, we follow the usual convention of the radiative transfer, and take $\tau$ to be 0 at the origin and integrate it to its maximum value at which the ray leaves the jet. This corresponds to the expressions for the intensity in the jet frame (see Section 2) of

$$
I = \int_{\tau_1}^{\tau_2} c^{-1} S(\tau') d\tau', \quad i \in (0, \pi - \theta_i),
$$

(A2)

where $c$ is the running length measured along the ray. We then change the variable of integration from $\tau'$ to the height, $z$, measured in the jet coordinates as it is the only geometric quantity that the source and absorption functions vary with, the other quantities being constant. This gives us

$$
I = \int_{\tau_1}^{\tau_2} c^{-1} S(\tau') d\tau', \quad i \in (\pi - \theta_i, \pi),
$$

(A3)

$$
\tau(z) = \frac{2\alpha_{\text{opt}} \pi c \sqrt{\frac{\tan^2 \theta_i - \tan^2 \theta_j}{\tan^2 \theta_j}}}{(p + 4) \beta_i} \left(z + \sqrt{z^2 - z_1^2}\right) \times \left(\cos^2 \frac{\pi}{2}, i \in (0, \pi - \theta_i)\right) + \left(\cos^2 \frac{\pi}{2}, i \in (\pi - \theta_i, \pi), \right),
$$

(A4)

where the boundaries $z_1$ and $z_2$ are derived below. Since we are concerned with the emission in the partially optically-thick regime, we calculate the emission of the entire jet, down to $z = 0$, as in Sections 3 and 4, and do not impose in numerical calculations $z_{1,2} \geq z_0$. From equation (A4), we can readily deduce the value of $z$ corresponding to $\tau = 1$.

Given that we calculate the intensity in the rotated coordinate system, we have to find the heights of the intersections as functions of $x_i$ and $y_i$. For a given a point, $(x, y, z)$, in the jet coordinates and the viewing angle, $i$, we can define a straight line at that angle that passes through the point. We need to find the intersections of this line with the cone, given by

$$
z = \cot \theta_i \sqrt{x^2 + y^2}.
$$

(A5)

The intersections with the cone can be found by squaring both sides and solving the resulting quadratic equation. However, squaring the equation for the cone also gives us a second cone flipped over the $x$-$y$ plane (the counterjet). When the viewing angle is less than the jet opening angle, the first intersection is on the second cone and when it is greater than $\pi - \theta_i$, the second intersection will be on the second cone, so the functions are assigned the value of infinity at these angles so that these cases can be separated. The equations are then written in terms of the rotated coordinate system using the transformation of equation (A1) with $z_i = 0$, since the area will be projected onto the $x_i$-$y_i$ plane. This yields,

$$
y_1(x_i, y_i, i) = \frac{-\omega \cot \theta_i (Z - y_i \sin i) - y_i \cos i \cot^2 \theta_i}{\cot^2 i - \cot^2 \theta_i}, \quad i \in (0, \theta_i),
$$

(A6)

$$
y_2(x_i, y_i, i) = \frac{\omega \cot \theta_i (Z - y_i \sin i) + y_i \cos i \cot^2 \theta_i}{\cot^2 i - \cot^2 \theta_i}, \quad i \in (0, \pi - \theta_i),
$$

(A7)

$$
\omega = \cot \theta_i \sqrt{(Z - y_i \sin i)^2 - 2y_i(Z - y_i \sin i) \cos i + (x_i^2 \cos^2 \theta_i + y_i^2) \cot^2 \theta_i - x_i^2 \cot^2 \theta_i},
$$

(A8)

The heights of the intersections in the jet coordinates are then given by

$$
z_1(x_i, y_i, i) = \min \left(\cot \theta_i \sqrt{y_i^2 + x_i^2}, Z \right),
$$

(A9)

$$
z_2(x_i, y_i, i) = \min \left(\cot \theta_i \sqrt{y_i^2 + x_i^2}, Z \right).
$$

(A10)

In the next step, we integrate the intensity over the jet projected area in the rotated coordinates. Depending on the orientation of the
observer with respect to the jet, we have three different cases, illustrated in Fig. A1. (i) \( i < \theta_j \), for which the projected area changes from a circle at \( i = 0 \) but it contracts into an ellipse as \( i \) gets larger. (ii) \( \pi \geq i > \pi - \theta_j \), which is very similar but the jet is viewed from the back. (iii) \( i \in (\theta_j, \pi - \theta_j) \), in which case the projected area is shaped similarly to an ice cream cone. Two examples of the changes of the projected area with \( i \) are shown in Fig. A2.

In the cases (i) and (ii), the boundaries are easy to calculate; we can just apply the transformation of equation (A1) to the equation for a circle in the jet coordinates at height \( Z \). This yields

\[
\frac{dF_1}{dE} = \frac{1 + \frac{Z}{r_m \epsilon^{2} D^2}}{m_e c^2 D^2} \int_{-\frac{Z}{\tan \theta_j \cos i}}^{\frac{Z}{\tan \theta_j \cos i}} \int_{-\sqrt{\frac{Z^2 - (y_i/\cos i)^2}{\tan^2 \theta_j}}}^{\sqrt{\frac{Z^2 - (y_i/\cos i)^2}{\tan^2 \theta_j}}} I \, dx \, dy \tag{A11}
\]

\[
\frac{dF_2}{dE} = \frac{1 + \frac{Z}{r_m \epsilon^{2} D^2}}{m_e c^2 D^2} \int_{-\frac{Z}{\tan \theta_j \cos (i-\pi)}}^{\frac{Z}{\tan \theta_j \cos (i-\pi)}} \int_{-\sqrt{\frac{Z^2 - (y_i/\cos (i-\pi))^2}{\tan^2 \theta_j}}}^{\sqrt{\frac{Z^2 - (y_i/\cos (i-\pi))^2}{\tan^2 \theta_j}}} I \, dx \, dy \tag{A12}
\]

in the cases (i) and (ii), respectively. We can see that these equations become identical to the flux given by equations (13) and (23) in the cases of \( i = 0 \) and \( i = \pi \), respectively, for \( Z \rightarrow \infty \).

In the case (iii), the integral must be split into two parts in order to accommodate the change of the domain from triangular to a part of an ellipse. Directly finding how the edge of the cone transforms is difficult but it is easy to find where the base of the jet is in the rotated coordinate system. We also know the shape of the ellipse and that the edge must intersect exactly once tangentially, so we can form the line using these two points. The intersections at the ellipse are given by \( (v, u) \) and \( (v, -u) \) in the \( x_i\)-\( y_i \) plane, where

\[
u = Z \cos i \cot \tan \theta_j, \tag{A13}
\]

\[
u = Z \tan \theta_j \sqrt{1 - \cot^2 i \tan^2 \theta_j}. \tag{A14}
\]
Figure A3. The dependence of the flux on the viewing angle for $\theta_j = 22.5^\circ$, $S_0 = \alpha_0 = \epsilon = z_0 = 1$, $p = 3$, $Z = 10^4$ (which is much larger than the value of $z$ corresponding to $\tau = 1$, which implies the results being virtually independent of $Z$), and $\beta_j = 0$ (left) and $0.8$ (middle and right). The middle and right panels have the linear and logarithmic vertical axes, respectively. The upper curves show the exact model for the case (i) at $i < \theta_1$ (blue), case (iii) at $i \in (\theta_j, \pi - \theta_j)$ (red) and case (ii) at $i < \theta_j$ (green). The lower (black) curves shows the cylindrical model for the flux, equations (9), (10). We see the two regimes connect smoothly in the exact model and the cylindrical approximation becomes more accurate for increasing viewing angle.

The edge lines, giving some of the integration limits in equation (A17) below, are then given by,

\begin{align}
    p(y_i) &= \frac{u_j v_i}{v \cos^2 i} - \frac{zu \tan i}{v \cos^2 i}, \\
    q(y_i) &= \frac{-u_j v_i}{v \cos^2 i} + \frac{zu \tan i}{v \cos^2 i}.
\end{align}

Past the intersection, the domain is the remaining part of the ellipse although now a modulus sign is needed to ensure that we are always integrating to the furthest point on the ellipse even when the viewing angle passes $\pi/2$ and $\cos i$ changes sign. This leads us to the final equation,

\begin{equation}
    \frac{dF_3}{dE} = \frac{1 + z_0}{m_e c^2 D^2} \int_0^{\sin\theta} \int_P \int_{-Z \tan\theta \cos i}^{\sqrt{Z^2 - (ey/c)^2}} I dx dy, \\
    \int_{\sqrt{Z^2 - (ey/c)^2}}^{\sqrt{Z^2 - (ey/c)^2}} I dx dy, \quad \text{(A17)}
\end{equation}

This equation becomes identical to the flux of equation (9) in the case of $i = \pi/2$ for $Z \to \infty$. Fig. A3 shows two examples of this solution, and compares it to the cylindrical approximation, equations (9) and (10).