On the ‘snow-plow’ phase of supernovae interacting with dense circumstellar media
(Research Note)

Takashi J. Moriya¹,²

¹ Argelander Institute for Astronomy, University of Bonn, Auf dem Hügel 71, D-53121 Bonn, Germany
² Research Center for the Early Universe, Graduate School of Science, University of Tokyo, Hongo 7-3-1, Bunkyo, Tokyo, Japan

e-mail: moriyat@astro.uni-bonn.de

ABSTRACT

Recently, Moriya et al. (2013) developed an analytic bolometric light curve model for supernovae interacting with dense circumstellar media. Because of the dense circumstellar medium, the shocked region is assumed to be radiative and make a thin dense shell. The model is based on the conservation of momentum in the shocked dense shell. However, the analytic model was mentioned to neglect the ‘snow-plow’ phase of the shocked dense shell. Ofek et al. (2014). The ‘snow-plow’ or momentum-conserving phase refers to the period in which the momentum injection from the supernova ejecta is almost terminated and the radiative shocked dense shell keeps moving due only to the momentum previously provided by the supernova ejecta. In this Note, I clarify that the analytic model of Moriya et al. (2013) does take the ‘snow-plow’ phase into account and the criticism of Ofek et al. (2014) is incorrect. In addition, Ofek et al. (2014) related the sudden luminosity break observed in the light curve of Type IIn SN 2010jl to the transition to the ‘snow-plow’ phase. However, I argue that the sudden transition to the ‘snow-plow’ phase is not consistent with the luminosity break observed in SN 2010jl. The luminosity break is likely to be related to other phenomena like the dense shell exiting the dense part of the circumstellar medium.

Key words. supernovae: general – supernovae: individual (SN 2010jl)

1. Introduction

Mass loss of massive stars has critical roles in many aspects of astrophysical phenomena (e.g., Smith 2014). One of the outstanding problems regarding the stellar mass loss is the existence of supernova (SN) progenitors having extremely high mass-loss rates immediately before their explosion. Especially, Type IIn SNe are known to show the signatures of the dense circumstellar medium (CSM) which are presumed to be related to the extreme mass-loss activities of the progenitors immediately before their explosion (e.g., Fransson et al. 2002; Taddia et al. 2013; Kieze et al. 2013). Recently, Moriya et al. (2013 M13 hereafter) developed a bolometric light curve (LC) model for SNe interacting with the dense CSM. The model is used to interpret many Type IIn SN LCs to estimate the general properties of the mass loss of Type IIn SN progenitors (Moriya et al. 2014). Similar LC models are suggested previously for the CSM from the steady mass loss (e.g., Svirski et al. 2013; Wood-Vasey et al. 2004) but M13 considered more general cases. Ofek et al. (2014) have also developed a similar LC model to the M13 model to interpret the LC of SN 2010jl, which is one of the most well-observed Type IIn SNe so far (see Fransson et al. 2013; Ofek et al. 2014 and the references therein). One interesting feature in the LC of SN 2010jl is the luminosity break observed at around 320 days after the first detection of the SN (Fransson et al. 2013; Ofek et al. 2014). Ofek et al. (2014) related the break to the transition of the forward shock propagating in the dense CSM to the ‘snow-plow’ or momentum-conserving phase (Svirski et al. 2012). If the shocked region by the SN explosion is radiative as is the case in Type IIn SNe, the shocked region makes a thin dense shell (e.g., Chevalier & Fransson 1994). The ‘snow-plow’ phase corresponds to the phase when the momentum supply from the SN ejecta to the shell is almost terminated. Roughly speaking, the shell enters the ‘snow-plow’ phase when the mass of the shocked dense CSM is comparable to the mass of the SN ejecta. Ofek et al. (2014) mentioned that the ‘snow-plow’ phase is neglected in the model of M13.

In this Note, I will first clarify that the criticism of Ofek et al. (2014) on the M13 LC model regarding the ‘snow-plow’ or momentum-conserving phase is incorrect. I will show that the ‘snow-plow’ phase is properly taken into account in the M13 model. I confirm this by following the evolution of the shocked dense shell numerically. Finally, I discuss the luminosity break observed in SN 2010jl and argue that the sudden luminosity break observed in SN 2010jl is inconsistent with the transition to the ‘snow-plow’ phase.

2. Light curve models for interacting SNe

2.1. M13 analytic model

I briefly summarize the M13 analytic LC model. More detailed and complete discussion of the model is in M13. The model assumes that the shocked SN ejecta and shocked dense CSM create a thin dense shell because of the efficient radiative cooling. Thus, the model assumes that the location of the shocked region can be expressed with a single radius $r_{sh}(t)$. Under this assumption, the evolution of $r_{sh}(t)$ is governed by the conservation of...
momentum,
\[ M_{\text{sh}} \frac{d\rho_{\text{sh}}}{dt} = 4\pi r_{\text{sh}}^2 \left[ \rho_{\text{ej}} (v_{\text{ej}} - v_{\text{sh}})^2 - \rho_{\text{csm}} (v_{\text{sh}} - v_{\text{csm}})^2 \right], \]  
(1)

where \( M_{\text{sh}} \) is the mass of the shell which is the sum of the mass of the shocked SN ejecta and shocked dense CSM, \( v_{\text{sh}} \) is the velocity of the shell, \( \rho_{\text{ej}} \) is the density of the SN ejecta entering the shell, \( \rho_{\text{csm}} \) is the density of the CSM entering the shell, and \( v_{\text{csm}} \) is the CSM velocity. The first term on the right-hand side of Equation (1) is \( \rho_{\text{ej}}\)\( v_{\text{ej}}\)\( v_{\text{sh}}\)\( (v_{\text{ej}} - v_{\text{sh}})^2 \), represents the momentum provided by the SN ejecta to the shell. The second term \(-4\pi r_{\text{sh}}^2 \rho_{\text{csm}} (v_{\text{sh}} - v_{\text{csm}})^2 \) is the momentum provided by the dense CSM. The M13 LC model is obtained solely by solving Equation (1).

The SN ejecta is assumed to have two density structures, \( \rho_{\text{ej}} \propto r^{-\alpha} \) outside and \( \rho_{\text{ej}} \propto r^{-\delta} \) inside. The outer part of SN ejecta continues to enter the shell until the time \( t_i \) (see M13 for details). After \( t_i \), the inner part of the SN ejecta starts to enter the shell. The evolution of the shell before \( t_i \) can be followed analytically but the general analytic solution does not exist after \( t_i \). As is discussed in M13, long after \( t_i \) when most of the SN ejecta has entered the shell, little momentum is provided from the SN ejecta and the equation for the conservation of momentum can be approximated as (Equation 10 in M13)
\[ M_{\text{sh}} \frac{d^2r_{\text{sh}}}{dt^2} = 4\pi r_{\text{sh}}^2 \rho_{\text{ej}} (v_{\text{ej}} - v_{\text{sh}})^2 \]  
(2)

Equation (2) clearly corresponds to the phase when the radiatively cooling shell continues to move with the momentum initially provided by the SN ejecta without any additional momentum supply from the SN ejecta, i.e., the ‘snow-plow’ phase. M13 suggested that the evolution of the shell can be approximated by Equation (2) long after \( t_i \) when most of the SN ejecta is shocked by the shell.

For the case of the steady mass loss (\( \rho_{\text{csm}} \propto r^{-\alpha} \)), Equation (2) is shown to have the analytic solution in M13 and they provided the asymptotic analytic bolometric LC of the interacting SNe corresponding to this ‘snow-plow’ phase (Equation 29 in M13)
\[ L = \frac{\epsilon}{2} \frac{M}{v_{\text{csm}}} \left( \frac{2E_{\text{ej}}}{M_{\text{ej}}} \right)^{\frac{\alpha}{\alpha - 1}} \left[ 1 + \frac{M}{v_{\text{csm}} M_{\text{ej}}} \left( \frac{2E_{\text{ej}}}{M_{\text{ej}}} \right) \frac{t}{t_{\text{i}}} \right]^{\frac{\alpha - 1}{\alpha}}, \]  
(3)

where \( \epsilon \) is the conversion efficiency from the kinetic energy to radiation, \( M \) is the mass-loss rate, \( E_{\text{ej}} \) is the kinetic energy of the SN ejecta, and \( M_{\text{ej}} \) is the mass of the SN ejecta. Svirski et al. (2012) earlier showed that the bolometric luminosities of the interacting SNe eventually follow \( L \propto r^{-1.5} \) in the ‘snow-plow’ phase. As can be clearly seen from Equation (3), the M13 analytic solution approaches \( L \propto r^{-1.5} \) after the following condition is satisfied
\[ M \rho_{\text{csm}} \sim M_{\text{ej}}, \]  
(4)

If the radius of the shell can be roughly approximated as \( r_{\text{sh}} \sim (2E_{\text{ej}}/M_{\text{ej}})^{\frac{1}{2}} \) as is assumed in Svirski et al. (2012), the condition (4) corresponds to
\[ M_{\text{csm}} \sim M_{\text{ej}}, \]  
(5)

where \( M_{\text{csm}} = M \rho_{\text{csm}} \) is the mass of the shocked CSM. Thus, the M13 model clearly takes the ‘snow-plow’ phase into account. Equation (3) is shown to be a good approximation for the luminosity evolution at the transitional phase to the ‘snow-plow’ phase as well in the next section.

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![Fig. 1. Bolometric LCs of the numerical model and M13 asymptotic model. The asymptotic model corresponds to the ‘snow-plow’ phase (Equation 3). When the mass of the CSM swept by the shocked shell is sufficiently larger than the SN ejecta mass, the two LCs follows \( L \propto r^{-1.5} \) as is expected in the ‘snow-plow’ phase (Svirski et al. 2012). The bolometric LC of SN IIb 2010jl obtained by Fransson et al. (2013) is shown for comparison.](image)

2.2. Numerical confirmation

To confirm that the M13 analytic model properly takes the ‘snow-plow’ phase into account, I numerically solve the equation for the conservation of momentum (Equation (1)). The CSM from the steady mass loss with \( M = 0.1 M_\odot \) yr\(^{-1} \) and \( v_{\text{csm}} = 100 \) km s\(^{-1} \) is put outside the homologously expanding SN ejecta with the two density structure components with \( n = 10 \) and \( \delta = 0 \). The SN ejecta mass and energy are assumed to be 10 \( M_\odot \) and 3 \times 10\(^{51} \) erg, respectively. The kinetic energy of the SN ejecta is chosen to match the early bolometric luminosity of SN 2010jl with \( \epsilon = 0.5 \) (see Section 5 for the discussion of SN 2010jl).

Figure 1 presents the bolometric LC obtained numerically as well as the asymptotic solution or the solution at the ‘snow-plow’ phase derived by M13 (Equation (3)). The blue part of the LC corresponds to the phase when the SN density structure entering the shocked shell is \( \rho_{\text{ej}} \propto r^{-\delta} \) \( (t < t_i) \) and the red part to \( \rho_{\text{ej}} \propto r^{-\alpha} \) \( (t > t_i) \). The transition time \( t_i = 61 \) days expected from the M13 model matches that obtained in the numerical model. The asymptotic LC model of M13 which corresponds to the ‘snow-plow’ phase (Equation (3)) starts to match the numerical LC relatively soon after \( t_i \). Both analytic and asymptotic models approach \( L \propto r^{-1.5} \) in the ‘snow-plow’ phase as is expected (Svirski et al. 2012). An important feature to note is that there is no sudden transition to the ‘snow-plow’ phase. The bolometric LC gradually starts to follow \( L \propto r^{-1.5} \) with time as the fraction of the shocked CSM mass to the shocked SN ejecta mass increases (see Figure 2).

3. SN 2010jl

I have clarified that the M13 bolometric LC model takes the ‘snow-plow’ or momentum-conserving phase into account in the previous section. Ofek et al. (2014) suggested that the luminosity break observed in SN 2010jl is related to the transition to the ‘snow-plow’ phase and the time of the break corresponds to when the mass of the shocked CSM and SN ejecta gets compa-
rable. However, as we have shown in the previous section, the transition to the ‘snow-plow’ phase is not expected to occur with the small timescale observed in SN 2010jl.

Since the numerical model presented in the previous section satisfies $M_{ej} = M_{csm}$ at around 1000 days since the explosion, I show another model in which $M_{ej} = M_{csm}$ is satisfied at around 350 days when the luminosity break of SN 2010jl is observed. Figure 3 shows the result. The ejecta mass is reduced to $M_{ej} = 5 M_\odot$ in the new model to satisfy $M_{ej} = M_{csm}$ at around 350 days. The conversion efficiency $\epsilon$ is also reduced to 0.3 to match the luminosity of SN 2010jl but the other parameters are kept the same. The transition to the ‘snow-plow’ phase does not make a luminosity break as is observed in SN 2010jl.

The sudden transition to the ‘snow-plow’ phase may occur if a high-density circumstellar shell exists on top of the dense CSM considered in the model. If the shocked shell suddenly encounters a high-density (and thus massive) shell, the shocked shell can be suddenly decelerated and can suddenly turn to the ‘snow-plow’ phase. However, when the shocked shell encounters the high-density shell, a sudden bolometric luminosity increase is expected rather than a break in the luminosity. This is because the increased density makes the deceleration of the shocked shell more efficient and more kinetic energy will be converted to radiation. Thus, the sudden transition to the ‘snow-plow’ phase by the collision to the high-density shell should be accompanied by the sudden luminosity increase which is not observed in SN 2010jl. This means that the sudden luminosity break observed in SN 2010jl is not likely to be related to the transition to the ‘snow-plow’ phase.

Alternative causes for the sudden luminosity decline in SN 2010jl were also discussed in Ofek et al. (2014); Fransson et al. (2013). One possibility is that the shocked shell gets out of the dense part of CSM at the time of the transition. Also, the CSM density slope may have suddenly become steep. As is noted by the previous works, the luminosity follows $L \propto r^{-3}$ after the break but no clear reason for this has been suggested. The CSM density structure should be steeper than $\rho_{csm} \propto r^{-3}$ if the break is due to the transition to the different density slope (M13).

4. Conclusion

I have shown that the analytic LC model of M13 which is developed to model SNe interacting with dense CSM takes the ‘snow-plow’ phase, or momentum-conserving phase, of the shocked dense shell into account. The model was criticized for neglecting it by Ofek et al. (2014) but the criticism is incorrect. The M13 LC model can reproduce the LC behavior expected in the ‘snow-plow’ phase well ($L \propto r^{-1.5}$ in the steady wind, see Svirski et al. 2012). The analytic LC model is also compared to the numerical model and the analytic model is shown to reproduce thenumerical result well.

The luminosity break observed in SN 2010jl was related to the transition to the ‘snow-plow’ phase by Ofek et al. (2014). However, the sudden luminosity break observed in SN 2010jl is not expected from the transition to the ‘snow-plow’ phase. The transition is expected to occur gradually as the shocked shell accumulates the CSM mass. The sudden transition may occur if there is a high-density shell in the CSM but the collision of the shocked shell to the high-density shell is expected to be accompanied by the luminosity increase, not the break. Thus, the luminosity break is not likely to be related to the transition to the ‘snow-plow’ phase.

Fig. 2. Mass of the shocked CSM and SN ejecta and their fraction obtained by the numerical model presented in Section 2.2 and Figure 1.

Fig. 3. Top: Numerical bolometric LC model in which $M_{ej} = M_{csm}$ is satisfied at around 350 days since the explosion, when the luminosity break is observed in SN 2010jl. Bottom: Evolution of the shocked mass and the mass fraction in the model presented in the top panel.
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