Traffic flow stochastic model \((2 \times 2)\) with discrete set of states and continuous time

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The present paper proposes a stochastic model of the traffic flow. This model has a discrete set of states and the continuous time. The model is a generalization of the discrete stochastic model that has been considered in a previous paper of this authors collective, for the case of a mixed traffic flow which consists of ”fast” and ”slow” vehicles. Moreover the continuous time model allows to developed the manner for estimation of characteristics of the traffic flow.

1. Introduction

The analysis of the traffic flow and problems of the traffic control becomes more pressing last years. Deterministic (hydrodynamic) models, that do not take into account some substantial factors as availability of several lanes, the different characteristics of traffic participants and so on, parallel with stochastic models are considered. In stochastic models the road is treated as a set of cells that proportional to the dynamic distance and a discrete time unit. The traffic flow velocity consists of the deterministic component connected with dynamic distance and a stochastic component that individual behavior into account. So it has been studied the particles of different types movement
on a set of cells. The dimension of this set is $N \times m$, $N >> 1$, $m > 1$.

The discrete model of traffic flow on a one-lane road has been considered in [1,2]. The road is represented by a set of subsequent cells. In each cell there is one or no particle (vehicle). The transitions of particles are possible only at discrete time units $n\Delta$. Suppose that a cell is occupied by a particle and the cell following ahead is vacant. Then with probability depending on the particle type (vehicle velocity) transition of the particle forwards to the following cell occurs. The steady characteristics of the considered system have been investigated. The multilane simulation models have been considered in [3]. The analysis of traffic flow characteristics on the basis of simulation has been presented in [4, 5]. Traffic flow mathematical models have been investigated in [6, 7]. A deterministic model has been investigated in [6]. It has been considered a stochastic model in terms of the queuing theory in [7].

A two-lane traffic flow discrete stochastic model, that is a generalisation of the models, considered in [1,2], has been developed in [8]. In this model a road is represented by two closed sequences of cells (lanes). The transitions of particles are possible only at discrete time units $n\Delta$. If the cell following ahead is busy and the two adjacent cells are empty then with the appropriate probability the particle is rebuilt in adjacent lane with propulsion. In [8] the approximate method has been elaborated for evaluation of the steady state probabilities of considered system fragments and such characteristics as the particle flow rate (traffic flow rate) and the changes of lane rate. The exactness of proposed approximations has been estimated by means of simulation modelling.

In the present paper the model is investigated which differs from the model considered in [8] by that in this model transitions of particles occur at arbitrary instants with an assigned intensity.
There are two types of particles characterized by the intensity of particle transitions. Thus the model takes into account the availability in the traffic flow of "fast" and "slow" vehicles.

The continuous time model is the limit case of the discrete time model when $\Delta \to 0$ and accordingly the transition probabilities $p \to 0$.

The method used for calculation of the studied characteristics is similar to the method proposed in [8]. It is proved that the continuous time model allows to deal with less complicated calculations than the appropriate discrete time model. It is explained by that in a continuous time model every change of states of a cells set is realized by a transition of the only particle.

2. Model description

Suppose that there are $N \times 2$ cells. Two indexes correspond to each cell. The value of the first index can be equal to $1, 2, \ldots, N$ and the second index (lane index) can be equal 1 or 2. So the pair of numbers $(i, j)$, $i = 1, 2, \ldots, N$, $j = 1, 2$, corresponds to the cell. There are $M_1$ particles of the first type and $M_2$ particles of the second type. At each instant each particle occupies one cell and no cell contains more than one particle.

Suppose that the value of the first index, which equal to $N+1$, is identified with 1 and the second index, which is equal to $j+1$, is identified with 1 when $j = 2$.

The particle movement is submitted to the next rules. If at time $t$ a particle of the $k$th type ($k = 1, 2$) occupies the cell $(i, j)$ and the cell $(i + 1, j)$ is vacant then during the time interval $(t, t + \Delta)$ with probability $\mu_k \Delta + o(\Delta), \Delta \to 0$, the particle passes from the cell $(i, j)$ to the cell $(i + 1, j)$ (i.e. the particle move forward remaining in the same lane). If the particle of the $k$th type occupies the cell $(i, j)$, the cell $(i + 1, j)$ is busy, and the cells $(i, j + 1), (i + 1, j + 1)$ are empty then during the time interval...
\((t, t + \Delta)\) with probability \(\mu_k \Delta + o(\Delta)\) the particle moves to the cell \((i + 1, j + 1)\), i.e. moves forward with change of lane. In the other cases the particle is not replaced. The probability of two or more transitions during the time interval is the infinitesimal of higher order than \(\Delta\).

3. Evaluation of the steady state probabilities

Let us consider the four cells with the first index, which is equal to 1 and 2. For approximate calculation of the steady states of the four cells we use method similar to the method which has been elaborated in [8].

The mean idea of the approximated method is that the considered four cells are described by a Markov process with discrete set of state and continuous time. The transition intensity for each pair of the process is calculated provided that in every cell, adjacent to the four cells, \(((N, 1), (N, 2), (3, 1), (3, 2))\) with probability \(\rho_1 = m_1/(2N)\), that is independent on the other cells states, there is a particle of the first type, with the probability \(\rho_2 = m_2/(2N)\) the cell is occupied by a particle of the second type and with probability \(1 - \rho\) \((\rho = \rho_1 + \rho_2)\) the cell is empty.

Taking into account that the configurations of particles, symmetrical respecting to line between the lanes, are characterized by just the same probabilities, we compound them to one state. There are 45 different states of the four cells shown in fig. 1. Suppose that equality \(\theta(i, j) = k, k = 1, 2\), means that the cell \((i, j)\) contains the particle of the \(k\)th type. The equality \(\theta(i, j) = 0\) means that the particle \((i, j)\) is empty. Then, for example, the states \(E_1\) and \(E_2\) are described as \(E_1 = \{\theta(1, 1) = \theta(1, 2) = \theta(2, 1) = \theta(2, 2) = 0\}\), \(E_2 = \{\theta(1, 1) = \theta(1, 2) = \theta(2, 1) = 0, \theta(2, 2) = 1\}\) \(\cup\{\theta(1, 1) = \theta(1, 2) = \theta(2, 2) = 0, \theta(2, 1) = 1\}\).

Let \(P_i(t)\) be the probability that at time the state of the four cells is \(E_i\). The considered Markov process satisfies the condi-
Figure 1: States of the four cells

The system for steady-states probabilities consists of the 45 equations, corresponding to 45 system states, and the normal-
izing equation
\[ \sum_{i=1}^{45} p_i = 1. \] (3)

4. Macro-characteristics of flow

According to the Markov processes theory each equation for the appropriate state is a consequence of the other ones. The system of the 45 equations and the normalizing equation has the only solution \( p_i, i = 1, \ldots, 4. \) This solution yields the approximate values of the steady state probabilities of the four cells.

As it has been mentioned above the density \( \rho_k \) of the flow of particles of the \( k \)th type flow satisfies the relations \( \rho_k = \frac{m_k}{2N} \), \( k = 1, 2 \). On the other hand
\[
\rho_1 = \frac{1}{2} (p_4 + 2p_9 + p_{10} + p_{12} + p_{13} + p_{16} + p_{17} + p_{20} + p_{21} + p_{22} + p_{23} + 2p_{28} + 2p_{29} + p_{30} + p_{31} + p_{32} + p_{33} + 2p_{36} + 2p_{37} + 2p_{38} + p_{39} + p_{40} + p_{41} + p_{42}),
\]
\[
\rho_2 = \frac{1}{2} (p_5 + p_{10} + 2p_{11} + p_{14} + p_{15} + p_{16} + p_{17} + p_{20} + p_{21} + p_{22} + p_{23} + p_{26} + p_{27} + p_{30} + p_{31} + p_{32} + p_{33} + 2p_{34} + 2p_{35} + p_{39} + p_{40} + p_{41} + p_{42} + 2p_{43} + 2p_{44} + 2p_{45}).
\] (4)

Let us prove the relations (4) and (5). The pair of cells \((i, 1), (i, 2)\) is identical with the pair \((j, 1), (j, 2), 1 \leq i, j \leq N.\) So the mean number of particles of \( k \)th type \((k = 1, 2)\) in each pair equals \( M_k/N, \) i.e. equals \( 2\rho_k. \) On the other hand the mean number of the particles of the \( k \)th type in the pair of cells is equal to the sum of probabilities of the states of the considered four cells in that there is exactly one particle in the pair of cells and the double probabilities of the states with two particles in this pair of cells. Hence the equations (4) and (5) are true.

Let \( \overline{\rho}_k \) be the value of \( \rho_k \) which satisfies the relations (4) and (5) if in this relations the approximate values \( p_i \) substitute for exact values \( p_i. \)
Let mean number $q_k$ of particles of the $k$th type ($k = 1, 2$) that leave the pair of cells (1,1), (1,2) be called the particles of $k$th type flow intensity and mean number $h_k$ of particles of the $k$th type leaving the pair cells with a change of lane be called the changes intensity of the particles of the $k$th type.

There is one particle, which can move, in the pair of cells (1,1) and (1,2), when the state of the four cells is one from the states $E_4, E_{10}, E_{12}, E_{16}, E_{17}, E_{28}, E_{29}, E_{30}, E_{31}$. If the state of the four cells is $E_9$ then there are two particles which can move. Hence the value of $q_1$ can be calculated as

$$q_1 = (p_4 + 2p_9 + p_{10} + p_{12} + p_{13} + p_{16} + p_{17} + p_{28} + p_{29} + p_{30} + p_{31}) \mu_1.$$  \hfill (6)

Similarly we obtain the relations

$$h_1 = (p_{12} + p_{13}) \mu_1,$$ \hfill (7)

$$q_2 = (p_5 + p_{10} + 2p_{11} + p_{14} + p_{15} + p_{18} + p_{19} + p_{32} + p_{33} + p_{34} + p_{35}) \mu_2,$$ \hfill (8)

$$h_2 = (p_{14} + p_{15}) \mu_2.$$ \hfill (9)

Let $\bar{q}_k, \bar{h}_k$ ($k = 1, 2$) be values of $q_k$ and $h_k$, which are obtained if in equations (6)–(9) $p_i$ substitutes for $\overline{p}_i$.

If $\mu_1 > \mu_2$ then $\bar{q}_1$ is some greater than the value $q_1$ obtained by simulation and in contrast $\bar{q}_2$ is some smaller then the value $q_2$ obtained by simulation. As it be shown below the difference between the calculated approximate values and data obtained by simulation is smaller if values $\tilde{q}_k, \tilde{h}_k$ determined by means of the correcting coefficient substitute for values $q_k, h_k$

$$\tilde{q}_k = \left(\rho_k / \bar{\rho}_k\right)\bar{q}_k, \quad k = 1, 2,$$ \hfill (10)

$$\tilde{h}_k = \left(\rho_k / \bar{\rho}_k\right)\bar{h}_k, \quad k = 1, 2.$$ \hfill (11)

Let us explain why the correcting coefficient improves the approximation. Let the ratio of the mean number of transitions of
particles of the $k$th type to number of the particles of this type be called the velocity of the particles of the $k$th type. Let the velocity of particles of the $k$th type be $v_k$. Then

$$v_k = q_k/(2\rho_k), \ k = 1, 2. \quad (12)$$

Because of the relation $q_k = 2\rho_k \cdot v_k$ the obtained approximate value of the particles of the $k$th type differs from the exact value by the coefficient $\bar{q}_2/\rho$. Coefficient $\rho/\bar{q}_2$ compensates the error.

Suppose

$$\tilde{v}_k = \tilde{q}_k/(2\rho_k), \quad (13)$$
$$\tilde{v}_k = \tilde{q}_k/(2\rho_k). \quad (14)$$

5. Approximations by means of Bernoulli scheme

Let us consider another manner of evaluation of the steady-state probabilities and the macro-characteristics that is more simple than the manner described above. Assume that the probability of the state of each cell is independent of the states of other cells, and the cells with probability $\rho_1$ contains a particle of the first type, with probability $\rho_2$ contains a particle of the second type and with probability $1 - \rho$ the cell is empty. Let $\hat{p}_i$ ($i = 1, 2, \ldots, 45$), $\hat{v}_k$, $\hat{q}_k$, $\hat{h}_k$ ($k = 1, 2$) be values of the appropriate characteristics that are obtained when these assumptions are used.

Then $\hat{p}_1 = (1 - \rho)^4$, $\hat{p}_2 = \hat{p}_4 = 2\rho_1(1 - \rho)^3$, $\hat{p}_3 = \hat{p}_5 = 2\rho_2(1 - \rho)^3$, $\hat{p}_6 = \hat{p}_9 = \rho_1^2(1 - \rho)^2$, $\hat{p}_7 = \hat{p}_{10} = \hat{p}_{13} = \hat{p}_{14} = \hat{p}_{17} = \hat{p}_{18} = 2\rho_1\rho_2(1 - \rho)^2$, $\hat{p}_8 = \hat{p}_{11} = \rho_2^2(1 - \rho)^2$, $\hat{p}_{12} = \hat{p}_{16} = 2\rho_1^2(1 - \rho)^2$, $\hat{p}_{15} = \hat{p}_{19} = 2\rho_2^2(1 - \rho)^2$, $\hat{p}_{20} = \hat{p}_{28} = 2\rho_1^3(1 - \rho)$, $\hat{p}_{21} = \hat{p}_{22} = \hat{p}_{24} = \hat{p}_{29} = \hat{p}_{30} = \hat{p}_{32} = 2\rho_1\rho_2(1 - \rho)$, $\hat{p}_{23} = \hat{p}_{25} = \hat{p}_{26} = \hat{p}_{31} = \hat{p}_{33} = \hat{p}_{34} = 2\rho_1^2\rho_2^2(1 - \rho)$, $\hat{p}_{27} = \hat{p}_{35} = 2\rho_2^3(1 - \rho)$, $\hat{p}_{36} = \rho_1^4$, $\hat{p}_{37} = \hat{p}_{39} = 2\rho_1^3\rho_2$, $\hat{p}_{38} = \hat{p}_{43} = \rho_1^2\rho_2^2$, $\hat{p}_{40} = \hat{p}_{41} = 2\rho_1^2\rho_2$, $\hat{p}_{42} = \hat{p}_{44} = 2\rho_1\rho_2^3$, $\hat{p}_{45} = \rho_2^4$, $\tilde{v}_k = (1 - \rho)(1 + \rho - \rho^2)\mu_k$, $k = 1, 2$,
\[ \hat{q}_k = 2\rho_k(1 - \rho)(1 + \rho - \rho^2)\mu_k, \quad k = 1, 2, \quad \hat{h}_k = 2\rho_k\rho(1 - \rho)^2\mu_k, \quad k = 1, 2. \]

6. Sistem solution and results of calculation

Suppose \( \rho_1 = \rho_2 = 0.25. \) The calculated approximate values are represented in table 1. These values are compared to values obtained by simulation. The symbols for values which have been obtained by simulations are marked by star. The duration of the simulation interval is equal to 12000 time units, \( N = 500. \)

We do not present the tables of calculations for \( \rho_1 = 0.54, \rho_2 = 0.06 \) and \( \rho_1 = 0.08, \rho_2 = 0.72. \)

The dependence of state probabilities on state numbers is represented on fig 2-10.

\[ \text{Figure 2: State probabilities for } \rho_1 = \rho_2 = 0.25, \mu_1 = 2, \mu_2 = 1 \]

The calculated values of macro-characteristics are represented in table 2.

7. Analysis of approximation accuracy

Let us compare the values \( \tilde{q}_k, \tilde{v}_k, \tilde{h}_k, \tilde{q}_k, \tilde{v}_k, \tilde{h}_k, \hat{q}_k, \hat{v}_k, \hat{h}_k \) with the values obtained by simulation.
Table 1: State probabilities, $\rho_1 = \rho_2 = 0.25$

| $i$ | $\hat{p}_i$ | $p_{i1} \mu_1 = 2, \mu_2 = 1$ | $p_{i1} \mu_1 = 3, \mu_2 = 1$ | $p_{i1} \mu_1 = 4, \mu_2 = 1$ |
|-----|------------|--------------------------------|--------------------------------|--------------------------------|
| 1   | 0.091      | 0.098                          | 0.063                          | 0.097                          |
| 2   | 0.064      | 0.049                          | 0.063                          | 0.069                          |
| 3   | 0.053      | 0.072                          | 0.063                          | 0.047                          |
| 4   | 0.061      | 0.032                          | 0.063                          | 0.062                          |
| 5   | 0.064      | 0.087                          | 0.063                          | 0.067                          |
| 6   | 0.026      | 0.023                          | 0.016                          | 0.029                          |
| 7   | 0.041      | 0.040                          | 0.031                          | 0.039                          |
| 8   | 0.014      | 0.019                          | 0.016                          | 0.008                          |
| 9   | 0.022      | 0.011                          | 0.016                          | 0.021                          |
| 10  | 0.045      | 0.042                          | 0.031                          | 0.045                          |
| 11  | 0.023      | 0.032                          | 0.016                          | 0.023                          |
| 12  | 0.012      | 0.010                          | 0.031                          | 0.013                          |
| 13  | 0.015      | 0.014                          | 0.031                          | 0.019                          |
| 14  | 0.008      | 0.008                          | 0.031                          | 0.005                          |
| 15  | 0.011      | 0.012                          | 0.031                          | 0.010                          |
| 16  | 0.033      | 0.023                          | 0.031                          | 0.033                          |
| 17  | 0.025      | 0.023                          | 0.031                          | 0.019                          |
| 18  | 0.034      | 0.036                          | 0.031                          | 0.036                          |
| 19  | 0.025      | 0.037                          | 0.031                          | 0.018                          |
| 20  | 0.023      | 0.025                          | 0.016                          | 0.021                          |
| 21  | 0.014      | 0.015                          | 0.016                          | 0.013                          |
| 22  | 0.025      | 0.023                          | 0.016                          | 0.036                          |
| 23  | 0.018      | 0.017                          | 0.016                          | 0.017                          |
| 24  | 0.010      | 0.014                          | 0.016                          | 0.006                          |
| 25  | 0.007      | 0.010                          | 0.016                          | 0.005                          |
| 26  | 0.013      | 0.012                          | 0.016                          | 0.011                          |
| 27  | 0.009      | 0.011                          | 0.016                          | 0.005                          |
| 28  | 0.016      | 0.014                          | 0.016                          | 0.016                          |
| 29  | 0.023      | 0.018                          | 0.016                          | 0.030                          |
| 30  | 0.009      | 0.009                          | 0.016                          | 0.005                          |
| 31  | 0.012      | 0.009                          | 0.016                          | 0.010                          |
| 32  | 0.017      | 0.021                          | 0.016                          | 0.018                          |
| 33  | 0.024      | 0.023                          | 0.016                          | 0.032                          |
| 34  | 0.009      | 0.008                          | 0.016                          | 0.005                          |
| 35  | 0.013      | 0.015                          | 0.016                          | 0.010                          |
| 36  | 0.007      | 0.009                          | 0.004                          | 0.008                          |
| 37  | 0.020      | 0.017                          | 0.008                          | 0.030                          |
| 38  | 0.013      | 0.008                          | 0.004                          | 0.025                          |
| 39  | 0.007      | 0.014                          | 0.008                          | 0.004                          |
| 40  | 0.010      | 0.009                          | 0.008                          | 0.009                          |
| 41  | 0.010      | 0.012                          | 0.008                          | 0.007                          |
| 42  | 0.014      | 0.008                          | 0.008                          | 0.014                          |
| 43  | 0.002      | 0.003                          | 0.004                          | 0.001                          |
| 44  | 0.005      | 0.005                          | 0.008                          | 0.002                          |
| 45  | 0.003      | 0.003                          | 0.004                          | 0.002                          |
Table 2: Macro-characteristics (the error that is relative to values obtained by simulation)

|       | $\mu_1 = 2$, $\mu_2 = 1$ | $\mu_1 = 3$, $\mu_2 = 1$ | $\mu_1 = 4$, $\mu_2 = 1$ |
|-------|---------------------------|---------------------------|---------------------------|
| $k$   |                           |                           |                           |
| $q_k^*$ | 0.432 (19) | 0.353 | 0.507 | 0.375 | 0.544 | 0.366 |
| $\tilde{q}_k$ | 0.337 (37) | 0.717 (42) | 0.361 (4) | 0.923 (70) | 0.372 (2) |
| $\hat{q}_k$ | 0.313 (-12) | 0.938 (85) | 0.313 (-17) | 1.250 (130) | 0.313 (-15) |
| $\check{q}_k$ | 0.296 (-16) | 0.879 (74) | 0.294 (-22) | 1.176 (116) | 0.292 (-20) |
| $\hat{h}_k$ | 0.048 | 0.063 | 0.063 | 0.17 | 0.076 | 0.17 |
| $\tilde{h}_k$ | 0.022 (6) | 0.076 (20) | 0.021 (23) | 0.101 (32) | 0.219 (12) |
| $\check{h}_k$ | 0.063 (212) | 0.188 (198) | 0.063 (268) | 0.250 (229) | 0.263 (286) |
| $\hat{h}_k$ | 0.019 (-5) | 0.093 (47) | 0.017 (0) | 0.128 (69) | 0.105 (-12) |
| $v_k^*$ | 0.864 | 0.706 | 1.014 | 0.750 | 1.088 | 0.732 |
| $\tilde{v}_k$ | 1.024 (19) | 0.674 (-5) | 1.434 (42) | 0.722 (-4) | 1.846 (70) | 0.743 (2) |
| $\check{v}_k$ | 0.625 (-12) | 1.875 (85) | 0.625 (-17) | 2.500 (130) | 0.625 (-15) |
| $\hat{v}_k$ | 1.180 (37) | 0.592 (-16) | 1.758 (74) | 0.588 (-22) | 2.352 (116) | 0.584 (-20) |

$\rho_1 = \rho_2 = 0.25$

|       | $\mu_1 = 2$, $\mu_2 = 1$ | $\mu_1 = 3$, $\mu_2 = 1$ | $\mu_1 = 4$, $\mu_2 = 1$ |
|-------|---------------------------|---------------------------|---------------------------|
| $k$   |                           |                           |                           |
| $q_k^*$ | 0.1024 | 0.083 | 1.335 | 0.92 | 1.624 | 0.101 |
| $\tilde{q}_k$ | 0.076 (3) | 0.076 (-8) | 1.465 (10) | 0.084 (-8) | 1.944 (20) | 0.090 (-11) |
| $\hat{q}_k$ | 1.071 (+4) | 0.060 (-28) | 1.607 (+20) | 0.060 (-35) | 2.143 (+32) | 0.060 (-41) |
| $\check{q}_k$ | 1.026 (+19) | 0.058 (-30) | 1.530 (+15) | 0.057 (-38) | 2.048 (+26) | 0.056 (-45) |
| $\hat{h}_k$ | 0.076 | 0.003 | 0.099 | 0.002 | 0.128 | 0.001 |
| $\tilde{h}_k$ | 0.094 (+14) | 0.004 (+30) | 0.121 (+22) | 0.003 (+48) | 0.159 (+24) | 0.003 (+220) |
| $\check{h}_k$ | 0.207 (+172) | 0.012 (+284) | 0.311 (+214) | 0.012 (+476) | 0.415 (+224) | 0.012 (+1052) |
| $\hat{h}_k$ | 0.082 (+8) | 0.003 (0) | 0.126 (+27) | 0.002 (0) | 0.168 (+31) | 0.002 (+100) |
| $v_k^*$ | 0.948 | 0.692 | 1.236 | 0.767 | 1.504 | 0.842 |
| $\tilde{v}_k$ | 0.915 (-3) | 0.630 (-8) | 1.356 (+10) | 0.704 (-8) | 1.800 (+20) | 0.747 (-11) |
| $\check{v}_k$ | 0.992 (+4) | 0.496 (-28) | 1.488 (+20) | 0.496 (-35) | 1.984 (+32) | 0.496 (-41) |
| $\hat{v}_k$ | 0.950 (+19) | 0.483 (-30) | 1.417 (+15) | 0.475 (-38) | 1.896 (+26) | 0.467 (-45) |

$\rho_1 = 0.54$, $\rho_2 = 0.06$

|       | $\mu_1 = 2$, $\mu_2 = 1$ | $\mu_1 = 3$, $\mu_2 = 1$ | $\mu_1 = 4$, $\mu_2 = 1$ |
|-------|---------------------------|---------------------------|---------------------------|
| $k$   |                           |                           |                           |
| $q_k^*$ | 0.044 | 0.334 | 0.039 | 0.344 | 0.048 | 0.346 |
| $\tilde{q}_k$ | 0.045 (+2) | 0.351 (+5) | 0.050 (+28) | 0.371 (+8) | 0.056 (+17) | 0.374 (+8) |
| $\hat{q}_k$ | 0.074 (+69) | 0.334 (0) | 0.111 (+186) | 0.334 (-3) | 0.149 (+210) | 0.334 (-3) |
| $\check{q}_k$ | 0.072 (+64) | 0.328 (-2) | 0.105 (+169) | 0.325 (-5) | 0.140 (+192) | 0.305 (-12) |
| $\hat{h}_k$ | 0.002 | 0.015 | 0.003 | 0.017 | 0.004 | 0.016 |
| $\tilde{h}_k$ | 0.004 (+87) | 0.020 (+36) | 0.004 (+43) | 0.023 (+34) | 0.006 (+60) | 0.025 (+54) |
| $\check{h}_k$ | 0.010 (+412) | 0.046 (+207) | 0.015 (+412) | 0.046 (+171) | 0.021 (+412) | 0.046 (+188) |
| $\hat{h}_k$ | 0.006 (+200) | 0.019 (+27) | 0.009 (+200) | 0.020 (+18) | 0.016 (+300) | 0.020 (+25) |
| $v_k^*$ | 0.275 | 0.232 | 0.244 | 0.239 | 0.300 | 0.240 |
| $\tilde{v}_k$ | 0.280 (+2) | 0.244 (+5) | 0.313 (+28) | 0.258 (+8) | 0.350 (+17) | 0.260 (+8) |
| $\check{v}_k$ | 0.464 (+69) | 0.232 (0) | 0.696 (+186) | 0.232 (-3) | 0.928 (+210) | 0.232 (-3) |
| $\hat{v}_k$ | 0.450 (+64) | 0.228 (-2) | 0.656 (+169) | 0.226 (-5) | 0.875 (+192) | 0.212 (-12) |
Suppose $\rho_1 = \rho_2 = 0.25$. The values $\tilde{q}_1 (\tilde{v}_1)$ exceeds the values $q^*_1 (v^*_1)$. The relative error makes up the values from 19% to 70% on the three considered set of data. The value $\tilde{q}_1 (v_1)$ exceeds the value $q^*_1 (v^*_1)$. The difference $\tilde{q}_1$ and $q^*_1$ makes up from 37% to 116% of $q^*_1$. The value $\hat{q}_1 (\hat{v}_1)$ exceeds the value $q^*_1 (v^*_1)$. The relative error make up from 45% to 113%.

In the other cases the approximate values calculated according to formulas (10), (11), (14) are also nearer to value obtained by simulation than the values calculated by the two others manners, or in some cases the formulas (10), (11), (14) yields approximation only few worse.

In each the three cases the value of $\tilde{q}_2 (\tilde{v}_2)$ difference the values
Figure 5: State probabilities for $\rho_1 = 0.54$, $\rho_2 = 0.06$, $\mu_1 = 2$, $\mu_2 = 1$

Figure 6: State probabilities for $\rho_1 = 0.54$, $\rho_2 = 0.06$, $\mu_1 = 3$, $\mu_2 = 1$

$q_2^* (v_2^*)$ is not greater than 5% of $q_2^*$. The value $\tilde{h}_1$ difference from
the value $h_1^*$ not greater than 32% of $h_1^*$. The value $h_2^*$ exceeds
$h_2^*$. The relative error is not greater than 23%.

Usually the relative error is the better the smaller is the ratio
of $\mu_1$ to $\mu_2$ ($\mu_1 > \mu_2$).

The values $\tilde{p}_1$, $\tilde{p}_2$, $\ldots$, $\tilde{p}_{45}$ that satisfies the system of linear
equations usually differs from the values obtained simulation less
than values calculated by means of Bernoulli scheme.

The analysis of calculation for the cases $\rho_1 = 0.54$, $\rho_2 = 0.06$, and
$\rho_2 = 0.06$, $\rho_1 = 0.72$, also shows that meaning $\hat{v}_k$, $\hat{q}_k$, $\hat{h}_k$, are
usually more close differs from the values obtained by simulation
less than values $\tilde{v}_k$, $\tilde{q}_k$, $\tilde{h}_k$, $\hat{v}_k$, $\hat{q}_k$, $\hat{h}_k$. 

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Figure 7: State probabilities for $\rho_1 = 0.54$, $\rho_2 = 0.06$, $\mu_1 = 4$, $\mu_2 = 1$

Figure 8: State probabilities for $\rho_1 = 0.08$, $\rho_2 = 0.72$, $\mu_1 = 2$, $\mu_2 = 1$

For $\rho_1 = 0.54$, $\rho_2 = 0.06$, the largest difference between $\tilde{q}_k$ ($\tilde{v}_k$) and $q^*_k$ ($v^*_k$) corresponds to the case $k = 1$, $\mu_1 = 4$, $\mu_2 = 1$. In this case the value $\tilde{q}_1$ exceeds the value $q^*_2$. The relative error is not greater than 20%.

For $\mu_1 = 0.08$, $\mu_2 = 0.72$ the largest difference between $\tilde{q}_k$ ($\tilde{v}_k$) $q^*_k$ ($v^*_k$) is reached in the case $k = 1$, $\mu_1 = 3$, $\mu_2 = 1$. In this case the value $\tilde{q}_1$ ($\tilde{v}_1^*$) exceeds values $q^*_1$ ($v^*_1$). The relative error is not greater than 28%.
8. Analysis of results of calculations of macro-characteristics

If Bernoulli scheme approximation is used then calculated values of macro-characteristics are proportional to the particle transitions intensity.

On the other hand it is clear intuitively that the fast particles occur more often than slow ones in the states in that their transitions are prevented by other particles. Hence the ratio of velocity ($v_k$) of particle to its transition intensity ($\mu_k$) for the fast particles is less than for the slow particles. It is confirmed both by simulation and calculations. The difference between $v_2/\mu_2$
and \( v_1/\mu_1 \) is the greater the greater the \( \mu_1/\mu_2 \) is. Because of that for fixed \( \mu_2 \) both value \( v^* \) and value \( \bar{v}_1 \) calculated according to formulas (10), (14) increases slower than linearly if \( \mu_1 \) increases. But it can be noticed that for the valued obtained by simulation the slowing down of increasing of the velocity of the particle of the first type with increasing of \( \mu_1 \) is more obvious. It can be noticed also if \( \mu_2 \) is fixed and \( \mu_1 \) increases then the values \( \bar{v}_2 \) and \( v^*_2 \) increases slowly and the value \( \bar{v}_2 \) is constant. For fixed \( \mu_2 \) and increasing \( \mu_1 \) the values \( \bar{h}_1 \) and \( h^*_1 \) increase approximately linearly. The ratio of \( \bar{h}_k/q_k \) to \( h^*_k/q^*_k \) for the fast particles is greater than for slow ones (moving forward the fast particles change lane more often than slow particles).

9. Conclusion

It is presented a model of the traffic flow on two lanes. An approximate manner is elaborated for calculation the steady state probabilities and macro-characteristics of model. The results of calculations by means of the elaborated method are compared with results of simulation. This comparison shows that the presented method yield rather good approximation and describes the qualitative behavior of the considered characteristics.
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