Maximally minimal preons in four dimensions

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Abstract
Killing spinors of \(N = 2, D = 4\) supergravity are examined using the spinorial geometry method, in which spinors are written as differential forms. By making use of methods developed in Gran \(\textit{et al.}\) (2007 \textit{J. High Energy Phys.} JHEP 02(2007)044 (Preprint hep-th/0606049)) to analyse preons in type IIB supergravity, we show that there are no simply connected solutions preserving exactly \(3/4\) of the supersymmetry.

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1. Introduction

The classification of supersymmetric solutions has been an active area of research due to the importance of these solutions in string and M-theory. Many years ago, Tod was able to find all metrics admitting supercovariantly constant spinors in \(N = 2, D = 4\) ungauged minimal supergravity \([2]\). In recent years and motivated by the work of \([2]\), progress has been made in the classification of supersymmetric solutions and in particular for lower dimensional gauged supergravity theories \([3–6]\). The basic idea in this classification is to construct differential forms as bilinears from the supercovariantly constant spinor. The algebraic and differential equations satisfied by these forms are then used to deduce the metric and the bosonic fields of the supergravity theory.

In our present work, we will focus on the classification of supersymmetric solutions of \(N = 2, D = 4\) gauged supergravity. In the light of the AdS/CFT correspondence \([7]\), these solutions can shed light on CFT in three dimensions. From the CFT point of view, solutions preserving fractions of supersymmetry can be regarded as an expansion of the theory around non-zero vacuum expectation values of certain operators. Moreover, the classification of supersymmetric solutions is also relevant for the construction of microstates for supersymmetric black holes \([8]\).
The classification of lightlike and timelike solutions preserving fractions of supersymmetry of minimal gauged supergravity in four dimensions has been performed in [5, 6]. In particular, in [6] it was shown that a configuration which admits a null Killing spinor can be either 1/4 or 1/2 but not exactly 3/4 supersymmetric.

In this paper, we will show that, as in the five-dimensional case [4], there are no simply connected 3/4-supersymmetric solutions in the theory of $N = 2$, $D = 4$ gauged supergravity irrespective of the nature of the solutions. In our analysis, it will be particularly useful to consider the spinors as differential forms [9–11]. This method of writing spinors as forms has been used to classify solutions of supergravity theories in 10 and 11 dimensions (see, for example, [1, 12–14]).

The plan of the paper is as follows. In section 2, the theory of $N = 2$, $D = 4$ gauged minimal supergravity is presented and it is shown how spinors of the theory can be written as differential forms. The gauge freedom present in the theory is used to reduce a spinor to one of three ‘canonical’ forms. A Spin(3, 1)-invariant non-degenerate bilinear form $B$ on the space of spinors is also defined. In section 3, it is shown that solutions preserving 3/4 of the supersymmetry can be placed into three classes according to the canonical form of the spinor which is orthogonal (with respect to $B$) to the Killing spinors. This method of characterizing supersymmetric solutions by the spinors which are orthogonal to the Killing spinors was originally developed in [1] where it was used to show that there are no preons in type IIB supergravity. The integrability conditions of the Killing spinor equations, for all three possible types of solution, fix the gauge field strengths to vanish and constrain the spacetime geometry to be locally isometric to AdS4. Furthermore, AdS4 is a maximally supersymmetric solution of $N = 2$, $D = 4$ minimal gauged supergravity. It therefore follows that there can be no simply connected exactly 3/4-supersymmetric solutions of $N = 2$, $D = 4$ minimal gauged supergravity.

2. Supersymmetric solutions of $N = 2$ supergravity

2.1. Minimal $N = 2$ gauged supergravity

In this section, we summarize some of the properties of the minimal gauged $N = 2$ supergravity theory in four dimensions and also describe how to write Killing spinors of this theory as differential forms. The bosonic action of minimal $N = 2$, $D = 4$ gauged supergravity is [15, 16]

$$S = \int d^4x \sqrt{-g} \left( \frac{1}{4} R - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{3}{2\ell^2} \right),$$

(1)

where $\ell$ is a nonzero real constant. The metric has signature $(-, +, +, +)$. We shall consider solutions which preserve some proportion of the supersymmetry; hence, there are Killing spinors $\epsilon$ satisfying the Killing spinor equation

$$D_\mu \epsilon = \nabla_\mu \epsilon + \frac{1}{2\ell} \gamma_\mu \epsilon + i F_{\nu_1\nu_2} \gamma_{\nu_1\nu_2} \gamma_\mu \epsilon = -\frac{i}{\ell} A_\mu \epsilon = 0,$$

(2)

where

$$\nabla_\mu = \partial_\mu + \frac{i}{4} \omega_{\mu, \nu_1\nu_2} Y^{\nu_1\nu_2}$$

(3)

and $F = dA$ is the $U(1)$ gauge field strength. Maximally supersymmetric solutions of this theory have $F = 0$ and are locally isometric to AdS4. More generally, supersymmetric solutions must preserve either 2, 4, 6 or 8 of the supersymmetries, because the Killing spinor equation (2) is linear over $\mathbb{C}$. Hence, preonic solutions which for this theory would preserve...
exactly $7/8$ of the supersymmetry are not possible. Preons have been examined in ten- and eleven-dimensional supergravity theories [1, 17–21]. In addition, many examples of $1/4$- and $1/2$-supersymmetric solutions of $N = 2$, $D = 4$ gauged supergravity are known [5, 6, 22, 23]. It is therefore natural to examine whether it is possible to construct solutions preserving exactly $3/4$ of the supersymmetry.

2.2. Spinors in four dimensions

The spinors $\epsilon$ appearing in the Killing spinor equation (2) are Dirac spinors. Following [9–11], these spinors can be written as complexified forms on $\mathbb{R}^2$; if $\Delta$ denotes the space of Dirac spinors then $\Delta = \Lambda^*(\mathbb{R}^2) \otimes \mathbb{C}$. A generic spinor $\eta$ can therefore be written as

$$\eta = \lambda + \mu^i e^i + \sigma e^{12},$$

(4)

where $e^1, e^2$ are 1-forms on $\mathbb{R}^2$ and $i = 1, 2$; $e^{12} = e^1 \wedge e^2$; $\lambda, \mu^i$ and $\sigma$ are complex functions.

The action of $\gamma$-matrices on these forms is given by

$$\gamma_0 = -e^2 \wedge +i e^2$$
$$\gamma_1 = e^1 \wedge +i e^1$$
$$\gamma_2 = e^2 \wedge +i e^2$$
$$\gamma_3 = i(e^1 \wedge -i e^1).$$

(5)

$\gamma_5$ is defined by

$$\gamma_5 = i\gamma_{0123}$$

(6)

and satisfies

$$\gamma_5^1 = 1, \quad \gamma_5 e^{12} = e^{12}, \quad \gamma_5 e^i = -e^i, \quad i = 1, 2.$$ 

(7)

The charge conjugation operator $C$ is defined by

$$C 1 = -e^{12}, \quad C e^{12} = 1, \quad C e^i = -\epsilon_{ij} e^j, \quad i = 1, 2$$

(8)

where $\epsilon_{ij} = e^{ij}$ is antisymmetric with $\epsilon_{12} = 1$. We also use the convention $\epsilon_{0123} = 1$.

We note the useful identities

$$(\gamma_\mu)^* = \gamma_0 C \gamma_\mu \gamma_0 C$$

(9)

and

$$C \gamma_\mu^* = \gamma_\mu C \quad C \gamma_0^* = -\gamma_0 C$$

(10)

and

$$(\gamma_\mu)_0^{ab} = -(\gamma_0^*)_{ba}, \quad (\gamma_\mu)_0^{ab} = (\gamma_0^*)_{ba}$$

(11)

for $m = 1, 2, 3$; where $(\gamma_\mu)_0^{ab} \equiv \delta_{ac} (\gamma_\mu)^{cb}$.

It will be particularly useful to complexify the gamma operators via

$$\gamma_+ = \frac{1}{\sqrt{2}}(\gamma_2 + \gamma_0) = \sqrt{2}i e^2,$$
$$\gamma_- = \frac{1}{\sqrt{2}}(\gamma_2 - \gamma_0) = \sqrt{2}e^2 \wedge$$
$$\gamma_1 = \frac{1}{\sqrt{2}}(\gamma_1 + i\gamma_3) = \sqrt{2}e^1,$$
$$\gamma_1 = \frac{1}{\sqrt{2}}(\gamma_1 - i\gamma_3) = \sqrt{2}e^1 \wedge,$$

(12)

where the metric components in the null basis are given by $g_{+} = 1$, $g_{11} = 1$. 

Maximally minimal preons in four dimensions 3261
2.3. Gauge transformations and canonical spinors

There are two types of gauge transformation which can be used to simplify the Killing spinors of this theory. First, there are local $U(1)$ gauge transformations of the type

$$\epsilon \rightarrow e^{i\theta} \epsilon$$

for real functions $\theta$, and there are also local Spin$(3, 1)$ gauge transformations of the form

$$\epsilon \rightarrow e^{\frac{i}{2} f^{\mu\nu} \gamma_{\mu\nu}}$$

for real functions $f^{\mu\nu}$.

Note in particular that $\gamma_{12}, \gamma_{13}, \gamma_{23}$ generate $SU(2)$ transformations which act (simultaneously) on both $1, e_1, e_2$. In particular, $\gamma_{13}$ acts via

$$1 \rightarrow e^{i\theta} 1, \quad e_1 \rightarrow e^{-i\theta} e_1, \quad e_2 \rightarrow e^{i\theta} e_2, \quad e_{12} \rightarrow e^{-i\theta} e_{12}$$

for $\theta \in \mathbb{R}$. Furthermore, $\gamma_{02}$ generates a scaling of the form

$$1 \rightarrow e^{x} 1, \quad e_1 \rightarrow e^{x} e_1, \quad e_2 \rightarrow e^{-x} e_2, \quad e_{12} \rightarrow e^{-x} e_{12}$$

for $x \in \mathbb{R}$.

Applying the $SU(2)$ transformation on a general spinor of the form

$$\epsilon = \lambda 1 + \mu^1 e^1 + \sigma e_{12}$$

allows us to set $\sigma = 0$ and $\lambda \in \mathbb{R}$ so that

$$\epsilon = \lambda 1 + \mu^1 e^1 + \mu^2 e^2.$$  

(17)

There are then various cases to consider.

First, suppose that $\mu^2 \neq 0$. Then consider the Spin$(3, 1)$ gauge transformation generated by $\gamma_{01} - \gamma_{12}$ and $\gamma_{03} + \gamma_{23}$, which acts via

$$1 \rightarrow 1, \quad e_1 \rightarrow e^1, \quad e_2 \rightarrow -2(y + ix)e^1 + e^2, \quad e_{12} \rightarrow 2(y - ix)1 + e_{12}$$

(19)

where $x, y \in \mathbb{R}$ are two gauge parameters. This transformation can be used to set $\mu^1 = 0$, leaving

$$\epsilon = \lambda 1 + \mu^2 e^2.$$  

(20)

If $\lambda \neq 0$, we can use the scaling generated by $\gamma_{02}$ to obtain

$$\epsilon = 1 + \mu^2 e^2.$$  

(21)

However, if $\lambda = 0$, then by combining the scaling generated by $\gamma_{02}$ with a $SU(2)$ transformation generated by $\gamma_{13}$ we can take

$$\epsilon = e^2.$$  

(22)

If instead $\mu^2 = 0$, then there again two cases. If $\lambda \neq 0$, then by combining the scaling generated by $\gamma_{02}$ with a $SU(2)$ transformation generated by $\gamma_{13}$ we can set

$$\epsilon = 1 + \mu^1 e^1,$$

(23)

where $\mu^1 \in \mathbb{C}$.

If however $\lambda = 0$, then by using a $SU(2)$ transformation together with the scaling generated by $\gamma_{02}$, the spinor can be written as (22).

So, one can always use Spin$(3, 1)$ gauge transformations to write a single spinor as

$$\epsilon = e^2$$

(24)

or

$$\epsilon = 1 + \alpha e^1$$

(25)

or

$$\epsilon = 1 + \beta e^2$$

(26)

for some functions $\alpha, \beta \in \mathbb{C}$. 
2.4. A Spin(3, 1) invariant inner product on spinors

In order to analyse the 3/4-supersymmetric solutions it is necessary to construct a non-degenerate inner product on the space of spinors. We first define a Hermitian inner product on the space of spinors via

\[ \langle z^0 + z^1 e^1 + z^2 e^2 + z^3 e^{12}, w^0 + w^1 e^1 + w^2 e^2 + w^3 e^{12} \rangle = \bar{z}^q w^q \]  

summing over \( q = 0, 1, 2, 3 \). However, \( \langle , \rangle \) is not Spin(3, 1) gauge invariant. To rectify this, we define an inner product \( B \) by

\[ B(\eta, \epsilon) = \langle C \eta^*, \epsilon \rangle, \]

then it is straightforward to show that \( B \) satisfies

\[ B(\eta, \epsilon) + B(\epsilon, \eta) = 0 \]
\[ B(\gamma_{\mu} \eta, \epsilon) - B(\eta, \gamma_{\mu} \epsilon) = 0 \]
\[ B(\gamma_{\mu \nu} \eta, \epsilon) + B(\eta, \gamma_{\mu \nu} \epsilon) = 0 \]

for all spinors \( \eta, \epsilon \).

The last of the above constraints implies that \( B \) is Spin(3, 1) invariant. Note that \( B \) is linear over \( \mathbb{C} \) in both arguments. The inner product \( B \) is non-degenerate: if \( B(\epsilon, \eta) = 0 \) for all \( \eta \) then \( \epsilon = 0 \).

To show the Spin(3, 1) invariance of \( B \) we consider

\[ B(\gamma_{\mu \nu} \eta, \epsilon) = \langle CY_{\mu \nu} \eta^*, \epsilon \rangle. \]

Then for \( m, n = 1, 2, 3 \),

\[ B(\gamma_{mn} \eta, \epsilon) = (\gamma_{mn})^a_{\ b} (C \eta)^b \epsilon^a \]
\[ = -(\gamma_{mn})^a_{\ b} (C \eta)^b \epsilon^a \]
\[ = -B(\eta, \gamma_{mn} \epsilon) \]

and

\[ B(\gamma_{0n} \eta, \epsilon) = -(\gamma_{0n})^a_{\ b} (C \eta)^b \epsilon^a \]
\[ = -(\gamma_{0n})^a_{\ b} (C \eta)^b \epsilon^a \]
\[ = -B(\eta, \gamma_{0n} \epsilon). \]

We have then verified the Spin(3, 1) invariance of the product.

3. 3/4-supersymmetric solutions

We now proceed to examine solutions preserving six out of the eight allowed supersymmetries. This implies the existence of three Killing spinors, which we shall denote by \( \epsilon_0, \epsilon_1, \epsilon_2 \), which are linearly independent over \( \mathbb{C} \). More precisely, it is assumed that there is some open neighbourhood \( U \) such that at every point in \( U \), \( \epsilon_0, \epsilon_1, \epsilon_2 \) are linearly independent over \( \mathbb{C} \).

Suppose we denote the span (over \( \mathbb{C} \)) of \( \epsilon_0, \epsilon_1, \epsilon_2 \) by \( W \). Any complex three-dimensional subspace of \( \mathbb{C}^4 \) can be uniquely specified by its one (complex) dimensional orthogonal complement with respect to the standard inner product on \( \mathbb{C}^4 \). It follows that one can specify \( W \) via its orthogonal complement with respect to \( B \). If the one-dimensional \( B \)-orthogonal subspace to \( W \) is spanned by \( \epsilon \), one has

\[ W = W_\epsilon = \{ \psi \in \Delta : B(\psi, \epsilon) = 0 \} \]

(33)
for some fixed non-vanishing \( \epsilon \in \Delta \). As \( B \) is \( \text{Spin}(3, 1) \) invariant, it will be most convenient to use \( \text{Spin}(3, 1) \) gauge transformations in order to write the spinor \( \epsilon \) in one of its canonical forms.

If \( \epsilon = 1 + \alpha e^1 \) then \( W \) is spanned by \( \eta_0 = 1, \eta_1 = e^1 \) and \( \eta_2 = e^2 - \alpha e^2 \). If \( \epsilon = 1 + \beta e^2 \) then \( W \) is spanned by \( \eta_0 = 1, \eta_1 = e^2 \) and \( \eta_2 = e^1 + \beta e^1 \). If \( \epsilon = e^2 \) then \( W \) is spanned by \( \eta_0 = 1, \eta_1 = e^2 \) and \( \eta_2 = e^2 \).

In all cases the Killing spinors \( \epsilon_0, \epsilon_1, \epsilon_2 \) are related to the spinors \( \eta_A \) for \( A = 0, 1, 2 \) via

\[
\epsilon_A = z_A \epsilon_B \eta_B ,
\]

where \( z \) is a complex \( 3 \times 3 \) matrix such that \( \det z \neq 0 \).

In order to analyse the solutions we shall consider the integrability conditions associated with the Killing spinor equations (2). These can be written as

\[
\left[ \frac{1}{\ell} * F_{\mu \nu} \gamma_5 - \frac{1}{\ell} F_{\mu \nu} - i \left( \frac{1}{\ell} F^{\nu\gamma}_{[\mu} Y^{\gamma\nu]} \right) - i (\nabla_{[\mu} F_{\nu]} \gamma_5) \right.
\]

\[
+ \left( \frac{1}{2\ell} Y_{\mu\nu} + \frac{1}{4} R^{\nu\gamma\nu}_{\mu\nu} Y_{\nu12} + \frac{1}{4} F_{\nu12} F^{\nu\gamma\nu}\gamma_{\nu12} - F^{\nu12} F_{\nu12} \right) \epsilon = 0
\]

for \( A = 0, 1, 2 \). This constraint is equivalent to

\[
\tilde{R}_{\mu\nu} \eta_A = 0,
\]

where

\[
\tilde{R}_{\mu\nu} \eta_A = \left( \frac{1}{2} \left( S^{2}_{\mu\nu} \right)^{\gamma \nu_{12}} + \frac{1}{2} \left( T^{2}_{\mu\nu} \right)^{\gamma \nu_{12}} + i (T^{1}_{\mu\nu} \gamma_{5}) + V^{1}_{\mu\nu} \right) \eta_A + (V^{5}_{\mu\nu} \gamma_{5}) = \frac{1}{\ell} * F_{\mu\nu} \eta_A
\]

for \( A = 0, 1, 2 \), with

\[
S^{2}_{\mu\nu} = \frac{1}{2\ell} \delta^{[\nu}_{\mu} \delta_{\nu]}^{\gamma_{12}} + \frac{1}{4} R^{\nu\gamma\nu}_{\mu\nu} \delta^{[\nu}_{\mu} \delta_{\nu]}^{\gamma_{12}} - F^{\nu\gamma\nu} F_{\nu12} \delta_{\nu12}
\]

\[
T^{2}_{\mu\nu} = \frac{1}{\ell} F^{[\nu}_{\mu} \delta^{\gamma_{12}}_{\gamma_{12}}
\]

\[
T^{1}_{\mu\nu} = \frac{1}{\ell} \nabla_{[\mu} F_{\nu]}^{\nu_{12}}
\]

\[
V^{1}_{\mu\nu} = \frac{1}{2} \delta^{[\nu}_{\mu} \nabla_{[\nu} F_{\gamma\nu]}^{\nu_{2}}
\]

\[
V^{5}_{\mu\nu} = \frac{1}{\ell} * F_{\mu\nu}.
\]

In all cases, we shall show that the integrability condition \( \tilde{R}_{\mu\nu} \eta_A = 0 \) for \( A = 0, 1, 2 \) can be used to obtain constraints that are sufficient to fix \( F = 0 \), and so \( T^{1} = T^{2} = V^{1} = V^{5} = 0 \). Furthermore, in all cases, the integrability conditions then imply that \( S^{2} = 0 \) or, equivalently,

\[
R_{\mu\nu\nu_{12}} = - \frac{2}{\ell^{2}} \delta_{\mu}^{[\nu} \delta_{\nu]}^{\gamma_{12}}.
\]

This implies that the spacetime geometry is locally isometric to AdS. However, it is known that AdS is a maximally supersymmetric solution of this theory and that all maximally supersymmetric solutions must be locally isometric to AdS. Hence, there can be no simply connected solutions preserving exactly \( 3/4 \) of the supersymmetry.
In the following analysis, we present the integrability constraints used to prove this for all possible types of 3/4-supersymmetric solutions, according to whether the Killing spinors $\epsilon_A$ are orthogonal to $1 + \alpha e^1$ or $1 + \beta e^2$ or $e^2$.

In what follows, it will be convenient to suppress the $\mu \nu$ indices in the tensors $S^2, T^2, T^1, V^1, V^5$ and $F$.

### 3.1. Minimal solutions with B-orthogonal spinors to $1 + \alpha e^1$

The integrability constraints obtained by requiring that $\tilde{R}_{\mu \nu}1 = 0$ are

\begin{align*}
(S^2)^{+} + (S^2)^{11} + V^5 + i(T^2)^{+} + i(T^2)^{11} - \frac{i}{\ell} F &= 0, \\
it(T^1)^{+} + (V^1)^{+} &= 0 \\
it(T^1)^{+} + (V^1)^{-} &= 0 \\
(S^2)^{-1} + i(T^2)^{-1} &= 0,
\end{align*}

the integrability constraints obtained by requiring that $\tilde{R}_{\mu \nu}e^1 = 0$ are

\begin{align*}
it(T^1)^{+} - (V^1)^{+} &= 0 \\
(S^2)^{+} - (S^2)^{11} + i(T^2)^{+} - i(T^2)^{11} - V^5 - \frac{i}{\ell} F &= 0 \\
(S^2)^{-1} + i(T^2)^{-1} &= 0 \\
-i(T^1)^{+} - (V^1)^{-} &= 0
\end{align*}

and the integrability constraints obtained by requiring that $\tilde{R}_{\mu \nu}(e^2 - \alpha e^{12}) = 0$ are

\begin{align*}
-\sqrt{2\alpha}(S^2)^{+1} - i\sqrt{2\alpha}(T^2)^{+1} + i(T^1)^{+} - (V^1)^{+} &= 0 \\
-\sqrt{2}(S^2)^{+1} - \sqrt{2}i(T^2)^{+1} + i\alpha(T^1)^{+} + \alpha(V^1)^{+} &= 0 \\
-(S^2)^{+} + (S^2)^{11} - i(T^2)^{+} + i(T^2)^{11} - i\sqrt{2}\alpha(T^1)^{1} - \sqrt{2}\alpha(V^1)^{1} - V^5 - \frac{i}{\ell} F &= 0 \\
\alpha(S^2)^{+} + \alpha(S^2)^{11} + i\alpha(T^2)^{+} + i\alpha(T^2)^{11} + i\sqrt{2}(T^1)^{1} - \sqrt{2}(V^1)^{1} - \alpha(V^5) + \frac{i\alpha}{\ell} F &= 0.
\end{align*}

By taking the real and imaginary parts of (40) we see that

\begin{align*}
(S^2)^{+} + V^5 + i(T^2)^{11} &= 0 \\
(S^2)^{11} + i(T^2)^{+} + i(T^2)^{+} - \frac{i}{\ell} F &= 0 \\
-(S^2)^{11} + i(T^2)^{-} - \frac{i}{\ell} F &= 0.
\end{align*}

and in the same way (45) yields

\begin{align*}
(S^2)^{+} - V^5 - i(T^2)^{11} &= 0 \\
-(S^2)^{11} + i(T^2)^{-} - \frac{i}{\ell} F &= 0.
\end{align*}

These equations imply that

\begin{align*}
(S^2)^{+} &= (S^2)^{11} = 0 \\
it(T^2)^{+} - \frac{i}{\ell} F &= 0.
\end{align*}
From equations (43) and (46) we see
\[(S^2)^{-1} = (T^2)^{-1} = 0 \quad (58)\]
and from (42) and (47)
\[(T^1)^{-1} = (V^1)^{-1} = 0. \quad (59)\]

Note that, upon comparison with (38), imposing (57) forces all components of \(F\) to vanish.

Hence \(F = V^1 = V^5 = T^1 = T^2 = 0\), and by the above constraints it follows that \(S^2 = 0\) also. This implies that the spacetime geometry is locally isometric to AdS_4.

3.2. Minimal solutions with B-orthogonal spinors to \(1 + \beta e^2\)

The integrability constraints obtained by requiring that \(\tilde{R}_{\mu\nu} = 0\) give
\[(S^2)^{+-} + (S^2)^{11} + V^5 + i(T^2)^{+-} + i(T^2)^{11} - \frac{i}{\ell} F = 0 \quad (60)\]
\[i(T^1)^1 + (V^1)^1 = 0 \quad (61)\]
\[i(T^1)^- + (V^1)^- = 0 \quad (62)\]
\[(S^2)^{-1} + i(T^2)^{-1} = 0 \quad (63)\]
as before. The constraints that follow from \(\tilde{R}_{\mu\nu} e^2 = 0\) are
\[i(T^1)^+ - (V^1)^+ = 0 \quad (64)\]
\[-(S^2)^y + i(T^2)^y = 0 \quad (65)\]
\[-(S^2)^{-} + i(T^2)^{-} + i(T^2)^{11} - V^5 - \frac{i}{\ell} F = 0 \quad (66)\]
\[i(T^1)^1 - (V^1)^1 = 0. \quad (67)\]

Lastly, the integrability constraints arising from \(\tilde{R}_{\mu\nu} (e^1 + \beta e^{12}) = 0\) are
\[\sqrt{2} \beta (S^2)^y + \sqrt{2} i \beta (T^2)^y + i(T^1)^1 - (V^1)^1 = 0 \quad (68)\]
\[(S^2)^{+-} - (S^2)^{11} + i(T^2)^{-} - i(T^2)^{11} - \sqrt{2} i \beta (T^1)^y - \sqrt{2} \beta (V^1)^y - V^5 - \frac{i}{\ell} F = 0 \quad (69)\]
\[\sqrt{2} (S^2)^{-1} + \sqrt{2} (T^2)^{-1} + i \beta (T^1)^1 + \beta (V^1)^1 = 0 \quad (70)\]
\[-\beta (S^2)^{+-} - \beta (S^2)^{11} - i \beta (T^2)^{-} - i \beta (T^2)^{11} - \sqrt{2} i (T^1)^y + \sqrt{2} (V^1)^y + \beta V^5 - \frac{i \beta}{\ell} F = 0. \quad (71)\]

By taking the real and imaginary parts of (60) and (66), respectively, we see that
\[(S^2)^{+-} + V^5 + i(T^2)^{11} = 0 \quad (72)\]
\[(S^2)^{11} + i(T^2)^{+-} - \frac{i}{\ell} F = 0 \quad (73)\]
and
\[-(S^2)^{+-} - V^5 + i(T^2)^{11} = 0 \quad (74)\]
\[(S^2)^{11} - i(T^2)^{+-} - \frac{i}{\ell} F = 0. \quad (75)\]
These equations imply that
\[(T^2)^+ = (T^2)^{11} = 0 \tag{76}\]
\[-(S^2)^+ - V^5 = 0 \tag{77}\]
\[(S^2)^{11} - i\frac{\ell}{V} F = 0. \tag{78}\]

Comparing (61) and (67) we find that
\[(T^1)^1 = (V^1)^1 = 0 \tag{79}\]
and from (62) and (64)
\[(T^1)^- = (V^1)^- = (V^1)^* = 0. \tag{80}\]

Substituting these results into (69), we find
\[(S^2)^+ - V^5 = 0 \tag{81}\]
\[-(S^2)^{11} - i\frac{\ell}{V} F = 0. \tag{82}\]

In this case, we note that imposing both (78) and (82) forces all components of \(F\) to vanish. Hence \(F = V^1 = V^5 = T^1 = T^2 = 0\), and by the above constraints it follows that \(S^2 = 0\) also. This implies that the spacetime geometry is locally isometric to AdS4.

3.3. Minimal solutions with B-orthogonal spinors to \(e^2\)

The integrability constraints obtained by requiring that \(\tilde{R}_{\mu\nu} 1 = 0\) are given by
\[(S^2)^+ + (S^2)^{11} + V^5 + i(T^2)^+ + i(T^2)^{11} - i\frac{\ell}{V} F = 0 \tag{83}\]
\[i(T^1)^1 + (V^1)^1 = 0 \tag{84}\]
\[i(T^1)^- + (V^1)^- = 0 \tag{85}\]
\[(S^2)^- + i(T^2)^- = 0 \tag{86}\]
as before. The constraints that follow from \(\tilde{R}_{\mu\nu} e^2 = 0\) are
\[i(T^1)^+ - (V^1)^* = 0 \tag{87}\]
\[-(S^2)^+ - i(T^2)^+ = 0 \tag{88}\]
\[-(S^2)^{11} + i(T^2)^{11} + i(T^2)^+ - V^5 - i\frac{\ell}{V} F = 0 \tag{89}\]
\[i(T^1)^1 - (V^1)^1 = 0. \tag{90}\]

Lastly, the integrability constraints arising from \(\tilde{R}_{\mu\nu} e^{12} = 0\) are
\[(S^2)^{11} + i(T^2)^{11} = 0. \tag{91}\]
\[i(T^1)^+ + (V^1)^* = 0 \tag{92}\]
\[i(T^1)^1 + (V^1)^1 = 0 \tag{93}\]
\[-(S^2)^+ - (S^2)^{11} - i(T^2)^+ - i(T^2)^{11} + V^5 - i\frac{\ell}{V} F = 0. \tag{94}\]
By taking the real and imaginary parts of (83) and (89), respectively, we see that

\[(S^2)^{++} + V^5 + i(T^2)^{1\bar{1}} = 0\]  
\[ (S^2)^{1\bar{1}} + i(T^2)^{++} - \frac{i}{\ell} F = 0 \]

and

\[-(S^2)^{++} - V^5 + i(T^2)^{1\bar{1}} = 0\]  
\[ (S^2)^{1\bar{1}} - i(T^2)^{++} - \frac{i}{\ell} F = 0. \]

These equations imply that

\[(T^2)^{++} = (T^2)^{1\bar{1}} = 0\]  
\[-(S^2)^{++} - V^5 = 0\]  
\[ (S^2)^{1\bar{1}} - \frac{i}{\ell} F = 0. \]

Comparing (84) and (90) we find that

\[(T^1)^\bar{1} = (V^1)^\bar{1} = 0\]

and from (85) and (87)

\[(T^1)^- = (V^1)^- = (T^1)^+ = (V^1)^+ = 0. \]

Substituting these results into (94), we find

\[(S^2)^{++} - V^5 = 0\]  
\[-(S^2)^{1\bar{1}} - \frac{i}{\ell} F = 0. \]

In this case, we note that imposing both (101) and (105) forces all components of \( F \) to vanish. Hence \( F = V^1 = V^5 = T^1 = T^2 = 0 \), and by the above constraints it follows that \( S^2 = 0 \) also. This implies that the spacetime geometry is locally isometric to AdS4.

4. Conclusion

In conclusion, we have completed the work of [6] and studied configurations preserving only 3/4 of supersymmetry for the theory of \( N = 2, D = 4 \) minimal gauged supergravity. In our analysis, we have employed the method of writing spinors of the theory as differential forms. Using the gauge symmetries of the spinors, one is able to place solutions preserving 3/4 of supersymmetry into three classes. Furthermore, using the integrability conditions of the Killing spinor equations coming from the vanishing of the gravitino supersymmetric variations, it was shown that the gauge field strengths must vanish. This means that the spacetime geometry is locally isometric to AdS4. Hence, solutions which preserve 3/4 of the supersymmetry are locally maximally supersymmetric. Therefore, there can be no simply connected exactly 3/4-supersymmetric solutions.

One subtlety which remains to be addressed is whether there exist non-simply connected solutions preserving 3/4 of the supersymmetry for which \( F = 0 \), and the spacetime geometry is some quotient of AdS4 by a discrete subgroup of the symmetry group Spin(3, 2).
For example, in the analysis of preons in $D = 11$ supergravity, it was proven in [24] that all solutions preserving $31/32$ of the supersymmetry must be locally isometric to maximally supersymmetric solutions, which proves that there can be no simply connected preons in 11 dimensions. Then, in [25], it was shown that no quotient of a maximally supersymmetric solution by a discrete subgroup of its symmetry group can preserve $31/32$ of the supersymmetry. It would be interesting to see if $3/4$-supersymmetric quotients of AdS$_4$ can be excluded using similar reasoning.

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