FERMI EXCITATIONS IN HOT AND DENSE QUARK-GLUON PLASMA

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Abstract

The Fermi excitations in hot and dense quark-gluon plasma are studied in the Feynman gauge using the temperature Green function technique. We find the four well-separated branches for the case $m = 0$ and establish the additional splitting between them (the four different masses) when $m \neq 0$. The long wavelength limit of these excitations is found in the general case of the massive fermions at finite temperature and densities to give the exact one-loop spectrum. Simultaneously the many known results are reproduced as its different limits.

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1 Introduction

At present the interest has essentially grown to study the quark excitation spectra (in particular the quark masses) in hot QCD theory at finite fermion densities. In the region of the large temperatures and densities normally there are no massless particles (all quarks and gluons, at least, have the dynamical masses $m \sim gT$ [1]) and these masses being not small influence qualitatively on many properties of the quark-gluon medium. Moreover the quarks being the Fermi particles are arranged under or near the Fermi sphere and the appearance of the diquark pairs is very probably for a rather wide temperature region. On the other hand this coupling is not evident at once since the particle and hole excitations have the completely different effective masses and other properties that is especially pronounced for large $m$. There are a lot paper (starting from [2,3]) which in many aspects discuss the problem of the mass generations but today this scenario is well studied only in what concerns the dynamical quark mass which can be calculated perturbatively. The effective quark mass is usually calculated nonperturbatively to generate the chiral phase transition for hot and dense quark-gluon plasma and this question is open for discussion. Usually these investigations are rather complicated and frequently are accompanied by numerical calculations [4,5,6] where many of details are screening.

The goal of this paper is to find analytically the Fermi excitations in hot and dense quark-gluon plasma and investigate their peculiarities. We use the standard temperature Green function technique and fix the Feynman gauge for explicit calculations. The case of a zero damping are only considered and we concentrate our attention to solve the fermion dispersion relation in a more complete form. We find the four well-separated branches for the case $m = 0$ and establish the additional splitting between them (the four different masses) when $m \neq 0$. The long wavelength limit of these excitations is investigated in the general case of the massive fermions at finite temperature and densities to give the exact one-loop spectrum. We find the dynamical quark masses (here only two values) and determine the four effective quark masses which define the above mentioned splitting. This splitting show that the particle excitations and holes have different masses and that takes place for any $T, \mu$-values if bare $m \neq 0$. Simultaneously a number of known results are reproduces as the different limits of the solution found to demonstrate its reliability.
2 QCD Lagrangian and quark self-energy

The QCD Lagrangian in covariant gauges has the form

\[
\mathcal{L} = -\frac{1}{4} G_{\mu \nu}^a G^{a \mu \nu} + N_f \bar{\psi} \left[ \gamma_\mu (\partial_\mu - \frac{i}{2} g \lambda^a V^a_\mu) + m \right] \psi \\
- \mu N_f \bar{\psi} \gamma_4 \psi + \frac{1}{2 \alpha} (\partial_\mu V^a_\mu)^2 + \tilde{C}^a (\partial_\mu \delta^{ab} + gf^{abc} V^c_\mu) \partial_\mu C^b
\]

(1)

where \( G_{\mu \nu}^a = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + gf^{abc} V^b_\mu V^c_\nu \) is the Yang-Mills field strength; \( V_\mu \) is a non-Abelian gauge field; \( \psi (\text{and } \bar{\psi}) \) are the quark fields in the SU(N)-fundamental representation (\( \frac{1}{2} \lambda^a \) are its generators and \( f^{abc} \) are the SU(N)-structure constants) and \( C^a (\text{and } \tilde{C}^a) \) are the ghost Fermi fields. In Eq.(1) \( \mu \) and \( m \) are the quark chemical potential and bare quark mass, respectively, \( N_f \) is the number of quarks flavours and \( \alpha \) is the gauge fixing parameter (\( \alpha = 1 \) for the Feynman gauge). The metric is chosen to be Euclidean and \( \gamma^2 = 1 \).

Our starting point is the exact Schwinger-Dyson equation for the temperature quark Green function

\[
G^{-1}(q) = G_0^{-1} + \Sigma(q)
\]

(2)

where the quark self-energy has the standard representation

\[
\Sigma(q) = \frac{N^2 - 1}{2N} g^2 \beta \sum_{p_4} \int \frac{d^3 p}{(2\pi)^3} D_{\mu \nu}(p - q) \gamma_\mu G(p) \Gamma_\nu(p, q|p - q)
\]

(3)

Here we calculate \( \Sigma \) only in the one-loop approximation using the bare Green functions in Eq.(3) and fixing the Feynman gauge. The bare functions in accordance with Eq.(1) have the form

\[
\Gamma^0_\mu(p, q|p - q) = \gamma_\mu, \quad G^0(p) = \frac{-i \gamma_\mu \hat{p}_\mu + m}{\hat{p}^2 + m^2}, \quad D^0_{\mu \nu}(p) = \frac{\delta_{\mu \nu}}{\hat{p}^2}
\]

(4)

where \( \hat{p} = \{(p_4 + i \mu), \mathbf{p}\} \) is the convenient notation. The summation over the spinor indices are performed within Eq.(3) using the \( \gamma \)-matrix algebra and the result is found to be

\[
\Sigma(q) = \frac{N^2 - 1}{N} g^2 \beta \sum_{p_4} \int \frac{d^3 p}{(2\pi)^3} \frac{i \gamma_\mu \hat{p}_\mu + 2m}{(\hat{p}^2 + m^2) (p - q)^2}
\]

(5)
Now we introduce two new functions and rewrite Eq. (5) as follows

$$\Sigma(q) = i\gamma_\mu K_\mu(q) + m \, Z(q)$$  \hspace{1cm} (6)

where $K_\mu(q) = q_\mu \, a(q) + i u_\mu \, b(q)$. Here $u_\mu = \{1, 0\}$ is the standard medium vector and all functions separately depend on $q_4$ and $|q|$. The decomposition (6) is not completed [3,7] but all other functions are not generated in the one-loop approximation. Using decomposition (6) we transform Eq. (2) into the form

$$G(q) = -i\gamma_\mu (\hat{q}_\mu + K_\mu) + m \left(1 + Z\right) \frac{1}{(\hat{q}_\mu + K_\mu)^2 + m^2 \left(1 + Z\right)^2}$$  \hspace{1cm} (7)

which is more convenient further. Setting the determinant of Eq. (7) to zero, we find the dispersion equation

$$(\hat{q}_\mu + K_\mu)^2 + m^2 \left(1 + Z\right)^2 = 0$$  \hspace{1cm} (8)

which determines the Fermi excitation spectra after the standard analytic continuation.

3 Calculations of the quark self-energy

Now the summation over the Fermi frequencies $p_4 = 2\pi T (n + 1/2)$ in Eq. (5) is performed with the aid of the usual prescription [8] and all terms are collected in the convenient form using the simple algebraic transformations. The result is given by

$$\Sigma(q) = -\frac{g^2 (N^2 - 1)}{N} \int \frac{d^3 p}{2(2\pi)^3} \left\{ \frac{1}{\epsilon_p} \frac{n_p^+ \left[ \gamma_4 \epsilon_p + (i\gamma_\mu + 2m) \right]}{\left[ q_4 + i(\mu + \epsilon_p) \right]^2 + (q - p)^2} + \frac{n_p^B \left( |p| + \mu - iq_4 \right) \gamma_4 - \left[ i\gamma(q - p) + 2m \right]}{\left[ q_4 + i(\mu + |p|) \right]^2 + \epsilon_{p-q}^2} \right\} - \left[ h.c.(m, \mu) \rightarrow - (m, \mu) \right]$$  \hspace{1cm} (9)

where $\epsilon_p = \sqrt{p^2 + m^2}$ is the bare quark energy; $n_p^B = \{\exp\beta |p| - 1\}^{-1}$ and $n_p^\pm = \{\exp\beta (\epsilon_p \pm \mu) + 1\}^{-1}$ are the Bose and Fermi occupation number, respectively.
Before the last integration over the angles being performed it is desirable to separate the functions \(K(q)\) and \(Z(q)\) to simplify all further calculations. Using that \(\text{Tr}\Sigma(q)/4 = m\ Z(q)\) we find the function \(Z(q)\)

\[
Z(q) = -\frac{g^2(N^2 - 1)}{N} \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{1}{\epsilon_p} \left[ \frac{n^+_p}{[q_4 + i(\mu + \epsilon_p)]^2 + (q - p)^2} \right] \right. \\
- \frac{n^B_p}{|p|} \left[ \frac{1}{[q_4 + i(\mu + |p|)]^2 + \epsilon^2_{p-q}} \right] + \left[ h.c.(\mu \rightarrow -\mu) \right] \right\}
\]

(10)

and then analogously we find the vector \(K_\mu(q)\). At first, \(\text{Tr}\gamma_4\Sigma(q)/4 = iK_4(q)\) reproduces the \(K_4\)-function

\[
iK_4(q) = -\frac{g^2(N^2 - 1)}{N} \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{n^+_p}{[q_4 + i(\mu + \epsilon_p)]^2 + (q - p)^2} \right. \\
+ \frac{n^B_p}{|p|} \left[ \frac{1}{[q_4 + i(\mu + |p|)]^2 + \epsilon^2_{p-q}} \right] - \left[ h.c.(\mu \rightarrow -\mu) \right] \right\}
\]

(11)

and then \(\text{Tr}\gamma_n\Sigma(q)/4 = iK_n(q)\) gives the vector \(K_n\) \((n=1,2,3)\)

\[
K_n(q) = -\frac{N^2 - 1}{N} g^2 \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{1}{\epsilon_p} \left[ \frac{n^+_p}{[q_4 + i(\mu + \epsilon_p)]^2 + (q - p)^2} \right] \right. \\
- \frac{n^B_p}{|p|} \left[ \frac{1}{[q_4 + i(\mu + |p|)]^2 + \epsilon^2_{q-p}} \right] + \left[ h.c.(\mu \rightarrow -\mu) \right] \right\}
\]

(12)

All the further calculations are simple but rather complicated when the integration over angles within Eqs.(10)-(12) is performed. Here we use the two standard integrals and introduce the new notations to simplify the final result. These notations are as follows

\[
a^\pm_F = \frac{q^2 - m^2 - (iq_4 - \mu)^2 \pm 2\epsilon_p (iq_4 - \mu) - 2|p||q|}{q^2 - m^2 - (iq_4 - \mu)^2 \pm 2\epsilon_p (iq_4 - \mu) + 2|p||q|}
\]

(13)

for integrals with the fermion distribution function and analogously

\[
a^\pm_B = \frac{q^2 + m^2 - (iq_4 - \mu)^2 \pm 2|p|(iq_4 - \mu) - 2|p||q|}{q^2 + m^2 - (iq_4 - \mu)^2 \pm 2|p|(iq_4 - \mu) + 2|p||q|}
\]

(14)

and for integrals with boson one.
After all algebra is made the function $Z(q)$ has the form

$$Z(q, q) = \frac{g^2(N^2 - 1)}{N} \int_0^\infty \frac{d|p|}{4\pi^2} \frac{|p|}{2|q|} \left\{ \frac{1}{\epsilon_p} \left[ \frac{n^+_p + n^-_p}{2} \log(a^+_F a^-_F) + \frac{n^+_p - n^-_p}{2} \log(a^+_B a^-_B) \right] \right\}$$

and the similar form has the function $K_4(q)$

$$iK_4(q, q) = \frac{g^2(N^2 - 1)}{N} \int_0^\infty \frac{d|p|}{8\pi^2} \frac{|p|}{2|q|} \left\{ \frac{1}{\epsilon_p} \left[ \frac{n^+_p + n^-_p}{2} \log(a^+_F a^-_F) + \frac{n^+_p - n^-_p}{2} \log(a^+_B a^-_B) \right] \right\}$$

The more complicated calculations are necessary to obtained the vector $K_n(q)$ where $n = 1, 2, 3$. Here we use a definition $K_n(q) = q_n K(q)$ and after that the scalar $K$-function will be calculated. The result has the form

$$q^2 K(q, q) = \frac{g^2(N^2 - 1)}{N} \int_0^\infty \frac{d|p|}{4\pi^2} \left\{ \frac{p^2}{|p|} \left[ \frac{n^+_p - n^-_p}{2} \right] \left[ \frac{1}{8|p||q|} \left( h_F \ln(a^+_F a^-_F) + d_F \ln(a^+_F a^-_F) \right) \right] + n^+_B |p| \left[ 1 + \frac{1}{8|p||q|} \left( h_B \ln(a^+_B a^-_B) + d_B \ln(a^+_B a^-_B) \right) \right] \right\}$$

where $h_F = q^2 - m^2 - (iq_4 - \mu)^2$ and $d_F = 2\epsilon_p (iq_4 - \mu)$ (the analogously $h_B = q^2 + m^2 - (iq_4 - \mu)^2$ and $d_B = 2|p|(iq_4 - \mu)$) are the new notations.

4 The quark excitation spectrum

Here we use Eq.(8) to find the spectrum of the Fermi excitations of hot quark-gluon plasma in the presence of background for the different values of the $T$, $\mu$ and $m$ parameters. The cases of a zero damping are only considered and due to this fact our analytical continuation is trivial.
4.1 The spectrum for all \( |q| \) in the case \( m = 0 \)

This is a more simple case which can be exactly considered in the analytic form. Here we find the Fermi excitation spectrum for all \( |q| \neq 0 \) within the standard high temperature technique keeping \( \mu \neq 0 \). To this end we expand Eqs.(15)-(17) taking only the leading \( T^2 \)-term with the \( \mu/T \)-corrections and solve the dispersion equation selfconsistently exploiting everywhere the new mass shell. Our starting point is the expressions for \( a_F^\pm \) and \( a_B^\pm \) quantities which being equal for the case \( m = 0 \) determine all functions which are necessary to expand. Using the integration of the \( a^\pm \) quantities over \( |p| \) we replace \( |p| \) to \( |p|T \) and find the following expansions

\[
\ln(a^+a^-) = -\frac{2|q|}{|p|} + O\left(\frac{1}{T^2}\right), \quad \ln\frac{a^+}{a^-} = 2\ln\frac{\xi - 1}{\xi + 1} + O\left(\frac{1}{T^2}\right) \tag{18}
\]

which are able to simplify essentially the further calculations. Here \( \xi = (iq_4 - \mu)/|q| \) is a convenient variable.

For our case the dispersion equation (8) can be presented as follows

\[
[ (iq_4 - \mu) - \bar{K}_4]^2 = q^2 (1 + K)^2 \tag{19}
\]

and at once to solve it as usual

\[
(iq_4 - \mu) - \bar{K}_4 = \eta|q| (1 + K) \tag{20}
\]

where \( \eta = \pm 1 \) and we use the definition \( K_4 = i\bar{K}_4 \). All functions within Eq.(20) should be calculated with the aid of the expansions (18) before we solved Eq.(20) explicitly.

These calculations are easily performed and one finds the following result

\[
K(q_4, q) = \frac{IK}{q^2} \left( 1 + \frac{\xi}{2} \ln\frac{\xi - 1}{\xi + 1} \right) + IB \left( \xi - \frac{1}{2} (1 - \xi^2) \ln\frac{\xi - 1}{\xi + 1} \right) \tag{21}
\]

for the function \( K \) and analogously for another function

\[
-\bar{K}_4(q_4, q) = \frac{IK}{2|q|} \ln\frac{\xi - 1}{\xi + 1} + IB \tag{22}
\]

Here the integrals are defined to be

\[
IK = \frac{g^2 (N^2 - 1)}{N} \int_0^\infty \frac{d|p|}{4\pi^2} |p| \left[ \frac{n^+_p + n^-_p}{2} + n^B_p \right] \tag{23}
\]

\[
IB = -\frac{g^2 (N^2 - 1)}{N} \int_0^\infty \frac{d|p|}{4\pi^2} \frac{n^+_p - n^-_p}{2} \tag{24}
\]
Now one should put the expressions found above into Eq.(20) and perform a number of the algebraic transformations to find $\omega = \xi |q|$. Here $\omega = (iq_4 - \mu)$ and our variable is $\xi$ which is a more convenient than $|q|$. The latter should be excluded with the aid of Eq.(20). The result is the quadratic equation with respect to $\omega$

$$\omega^2 \left( \xi - \eta [1 + I_B B(\xi)] \right) + \omega \xi I_B + I_K \xi^2 A(\xi) = 0$$

(25)

where the functions $A(\xi)$ and $B(\xi)$ are given by

$$A(\xi) = \eta \left( 1 + \frac{\xi - \eta}{2} \ln \frac{\xi - 1}{\xi + 1} \right)$$

$$B(\xi) = \xi - \frac{1}{2} (1 - \xi^2) \ln \frac{\xi - 1}{\xi + 1}$$

(26)

Keeping the accuracy of calculations our solution of Eq.(25) is found to be

$$E(\xi) = \mu - \frac{\xi I_B}{2(\xi - \eta)} \pm \xi \sqrt{I_K \left( \frac{\eta}{\xi - \eta} + \frac{\eta}{2} \ln \frac{\xi - 1}{\xi + 1} \right)}$$

(27)

which presents the four branches of the Fermi excitations in the medium. Here $\eta = \pm 1$ and we return the physical variable $E = ip_4$. Eq.(27) is a more general one-loop result for the case $m = 0$ and contains all known ones. For example, if $\mu = 0$, the well-known spectrum found in paper [2]

$$\omega^2(\xi) = \xi^2 \omega_0^2 \left( \frac{\eta}{\xi - \eta} + \frac{\eta}{2} \ln \frac{\xi - 1}{\xi + 1} \right)$$

(28)

is in agreement with Eq.(27) if one puts $\mu = 0$ and squares it. In Eq.(28) $\omega_0^2 = g^2 T^2 / 6$ as it is given by Eq.(23) for $N = 3$. Within Eq.(27) $1 < \xi < \infty$ and the long wavelength limit corresponds to $\xi \to \infty$. For this limit one finds the very simple formula

$$E = \mu - \frac{I_B}{2} \pm \sqrt{I_K}$$

(29)

which, indeed, is correct for any $T$ (and for the case $T = 0$ as well). We demonstrate this interesting fact in the next section although, in principle,
this situation is known. Eq.(29) represents the more general expression for
the dynamical quark mass when \( m = 0 \)

\[
M^2 = \frac{g^2 (N^2 - 1)}{N} \int_0^\infty \frac{d|\mathbf{p}|}{4\pi^2} |\mathbf{p}| \left[ \frac{n_\mathbf{p}^+ + n_\mathbf{p}^-}{2} + n_\mathbf{p}^B \right]
\] (30)

which for many special cases is well-known. For example, when \( T = 0 \) one
finds the dynamical mass as follows [9]

\[
M^2 = \frac{N^2 - 1}{N} \frac{g^2 \mu^2}{16\pi^2}
\] (31)

and in another limit \( \mu = 0 \) this expression was calculated in paper [2].

4.2 The long wavelength limit with \( m \neq 0 \)

This is a rather important limit which determines the effective quark mass
taking into account all radiative corrections: here only the one-loop corrections. We present our result keeping the perturbative accuracy and establish
the additional splitting between the branches found above.

Our starting point is Eq.(8) which for the case \( m \neq 0 \) it is useful to
rewrite as follows

\[
 iq_4 (1 + a) = \mu + b + \eta \sqrt{m^2 (1 + Z)^2 + q^2 (1 + a)}
\] (32)

where \( \eta = \pm 1 \) and all functions should be calculated by using Eq.(6) and its
decomposition. These calculations yield the simple result

\[
a(q) = \frac{(qK(q)) - (uq)(uK(q))}{q^2 - (uq)^2} , \quad ib(q) = (uK(q)) - (qu)a(q)
\] (33)

and now Eq.(32) can be transformed in the form

\[
iq_4 = \mu - i(uK(q)) + \eta \sqrt{m^2 (1 + Z)^2 + q^2 + (qK(q))}
\] (34)

which is convenient for the further calculations in the case studied. Below
Eq.(34) will be presented in a more explicit form in the long wavelength
limit.
If the limit \(|q| = 0\) is only considered all functions which determine Eq.(34) can be simplify by doing the necessary expansions within Eqs.(15)-(17) or exploiting directly Eqs.(10)-(12) in the \(|q| = 0\) limit. After all algebraic transformations being made all terms are collected to reproduce the final result in the convenient form.

The function \(K_4(q, 0)\) is found to be

\[
-i(uK)(q, 0) = \frac{g^2(N^2 - 1)}{N} \int_0^\infty \frac{d|p|}{4\pi^2} \left\{ \frac{4|p|^2 \epsilon_p}{4\epsilon_p^2 - (iq_4 - \mu)^2[1 + \frac{m^2}{(iq_4 - \mu)^2}]} \right\} \left[ \frac{n^+_p + n^-_p}{2} + \frac{(iq_4 - \mu)}{2\epsilon_p} \left( 1 + \frac{m^2}{(iq_4 - \mu)^2} \right) \frac{n^+_p - n^-_p}{2} \right]
\]

(35)

and the function \(Z(q, 0)\) which determines the mass renormalization can be presented as follows

\[
Z(q, 0) = -\frac{g^2(N^2 - 1)}{N} \int_0^\infty \frac{d|p|}{4\pi^2} \left\{ \frac{4|p|^2}{4\epsilon_p^2 (iq_4 - \mu)^2 - [(iq_4 - \mu)^2 + m^2]^2} \right\} \left[ \frac{(iq_4 - \mu)^2 + m^2}{\epsilon_p} \frac{n^+_p + n^-_p}{2} + 2(iq_4 - \mu) \frac{n^+_p - n^-_p}{2} \right]
\]

(36)

The more complicated calculations yield that \((qK)(q, 0) = 0\).

Now our problem is to solve Eq.(34) explicitly. To this end we should put Eq.(35)-(36) on the new mass shell \((iq_4 - \mu) = \omega\) and in this form substitute these integrals into Eq.(34). However in this case the arisen equation is very complicated and can be used only for numerical calculations [4]. On the other hand keeping the perturbative accuracy the obtained integrals are possible to simplify considering the quantity \(\omega^2 - m^2 \approx 0\) inside them. In doing so,
the result has the form
\[ -iK_4(q_4, 0) = \frac{I_A}{\omega} - I_B \] (37)
\[ Z(q_4, 0) = -2I_Z + 2\frac{I_B}{\omega} \] (38)
where the new integrals are:
\[ I_A = \frac{g^2(N^2 - 1)}{N} \int_0^\infty \frac{d|p|}{4\pi^2} \left[ \epsilon_p \frac{n_p^+ + n_p^-}{2} + |p| n_p^B \right] \]
\[ I_Z = \frac{g^2(N^2 - 1)}{N} \int_0^\infty \frac{d|p|}{4\pi^2} \frac{n_p^+ + n_p^-}{2\epsilon_p} \] (39)
and \( I_B \) was earlier determined by Eq.(24). Now the dispersion equation is the simple quadratic equation
\[ \omega^2 - \omega \left[ \eta m(1 - 2I_Z) - I_B \right] - (I_A + 2\eta mI_B) = 0 \] (40)
whose solution reproduces the final result. This result has the form
\[ \omega = \frac{1}{2} \left[ \eta m(1 - 2I_Z) - I_B \right] \pm \sqrt{\left[ \eta m(1 - 2I_Z) - I_B \right]^2 + (I_A + 2\eta mI_B)} \] (41)
which gives the additional splitting of the branches found above. This splitting show that the particle excitations and holes have different masses and that takes place for any \( T, \mu \)-values if the bare mass \( m \neq 0 \). In Eq.(41) all parameters are free and this solution extends the previously known result [6] taking into account all one-loop corrections. The dynamical quark mass is defined to be
\[ M^2 = I_A + 2\eta mI_B \] (42)
and this expression is the more general one-loop result for this quantity. The quark mass shift has a rather complicated form
\[ \delta m = -\frac{1}{2} \left[ \eta m(1 + 2I_Z) + I_B \right] \pm \sqrt{\left[ \eta m(1 - 2I_Z) - I_B \right]^2 + (I_A + 2\eta mI_B)} \] (43)
and is different for each branches as it mentioned above. This means that a question about phenomenological quark mass remains open and requires the additional studying.

In the limit \( T = 0 \) all integrals in Eq.(41) are exactly calculated. For example within the standard QCD (where \( N = 3 \)) one has

\[
I_Z = \frac{g^2}{3\pi^2} \ln \frac{\mu + \sqrt{\mu^2 - m^2}}{m}; \quad I_B = \frac{2g^2}{3\pi^2} \sqrt{\mu^2 - m^2}
\]

\[
I_A = \frac{g^2}{6\pi^2} \left( \mu \sqrt{\mu^2 - m^2} + m^2 \ln \frac{\mu + \sqrt{\mu^2 - m^2}}{m} \right)
\]

(44)

and the spectrum can be investigated by using the simple algebraic transformations. In particular case when \( m = 0 \) the spectrum has a rather simple form

\[
E = \mu \left( 1 - \frac{g^2}{3\pi^2} \right) \pm \sqrt{\frac{g^2\mu^2}{6\pi^2}}
\]

(45)

which presents the one-loop result for the massless quark excitation in the cold medium. In the case \( m = 0 \) Eq.(41) coincides with Eq.(29) as it was promised. The found equality proves Eq.(29) and this remark is very useful for the practical calculations.

5 Conclusion

To summarize we have solved the fermion dispersion relation in the general case of the massive fermions at finite temperature and densities. We find that Fermi excitations in medium for \( m = 0 \) have the four well-separated branches: two of them present the particle excitations and two other correspond to the antiparticles ones. The additional splitting of the branches mentioned above is found for the case \( m \neq 0 \) which demonstrate that the effective mass for all branches are different for any \( T, \mu \)-values if only bare \( m \neq 0 \). The dynamical mass have only two different values in the general case \( m \neq 0 \) and an unique value in other cases. Here we see the problem with the definition of the phenomenological quark mass since there is a question which branch it presents. It is also essential that the particles excitations and the hole ones have a different behaviour with respect of the bare mass and there is a
problem to find any correlations between them. Another problem concerns
the gauge dependence of all results found here and in other papers (see
e.g. [10]). This is a serious problem and it is not excluded that a number
of the one-loop results are, indeed, gauge dependent. In the first place it
concerns a damping (where resummation is necessary [11]) and probably all
the next-to-leading order terms where the same scale $\sim g^2 T$ is essential.
However, in our opinion the long wavelength limit of any excitation spectra
when the damping is not important should be gauge independent. This
statement is in agreement with [12] but there is a different result [13] where
even $T^2$-term is gauge dependent. So today the problem remains and in
this situation the Feynman gauge used here is a more reliable one to have
correct result. This problem is still pronounced for many nonperturbative
calculations which usually use the axial gauge (see e.g. [5,14]). Moreover at
present the peculiarities of many singular gauges are exploited to produce
the physical results [15]. However these calculations being very complicated
requires the special attention and should be discussed in a separate paper.

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