Ocean Reverberation Suppression via Matrix Completion with Sensor Failure

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Abstract

In the ocean environment, reverberation is a common and strong interference which significantly degrades the performance of target bearing estimation. Meanwhile, sensor failure is inevitable in actual sonar deployment as the underwater scene is complicated. Therefore, it is a challenge for target detection in the ocean reverberation with sensor failure. To address this issue, we propose an improved approach based on the principle of low rank characteristics of matrix in this paper. Firstly, we utilize Hankel structured matrix to counteract the problem of sensor failure. Then, the algorithm of matrix completion (MC) based on $\ell_1$-norm and $\ell_2$-norm are exploited to recover the true signal matrix from the corrupted received matrix. Numerical results demonstrate that compared with other related methods, the $\ell_1$-norm provides better capability in the probability of recovery and shows the best robustness of bearing estimation with the narrow beam width and low sidelobe. Moreover, the proposed approach verifies the performances of reverberation suppression and achieves high resolution of target detection.

**Keywords:** direction of arrival (DOA), MC, low rank, ocean reverberation, sensor failure.

I Introduction

There exist various irregular scatterers in the ocean, such as marine lives, sediment particles, air bubbles, water masses, as well as roughness of sea surface and seafloor. The inhomogeneity generated by all these random scatterers forms a scattering field [1]. Similar to the long, slow and quivering sounds, ocean reverberation caused by scattering is one of the most important physical phenomena of underwater acoustics. According to the scattering sources, ocean reverberation can be divided into three categories, i.e., volume reverberation, sea surface reverberation, and seafloor reverberation [2]. Besides the ambient noise, ship noise, and so on, reverberation is known to be a common and strong interference in source localization. It is generated along with the transmit signal, and hence is closely related to the transmit signal itself. Additionally, it is also related to the propagation characteristics of sound field. Usually, the target signal is overwhelmed by reverberation, which severely limits the effective working distance and degrades the performance of sonar system. Fig. 1 shows the DOA estimation with a vertical line array (VLA) in ocean reverberation environment. As reverberation is formed by the superposition of a large
number of random scatterers at the received sonar, it is a random process. In general, it is challenging for target detection to suppress reverberation under reverberation background.

![Diagram of DOA estimation with a VLA in ocean reverberation environment.](image)

**Fig. 1**: DOA estimation with a VLA in ocean reverberation environment.

Many works have investigated reverberation suppression to improve the constant false alarm probability, especially for weak targets in the ocean. When combining the reverberation with transmit signals, the transmit waveform can be designed to reduce the influence of reverberation on the target, but it still needs to be analyzed by signal processing algorithms to extract the actual waveforms information [3, 4]. At the receiver, the reception directivity can be improved by increasing the azimuth direction aperture for reverberation suppression [5, 6]. According to the theory of normal mode, the acoustic signals can be composed of different modes. The reverberation usually has a high-order mode, and the filtering of normal mode can improve the signal-to-reverberation ratio (SRR), where a group or a low-order mode is chosen to enhance the target signal. The disadvantage of this method is that the choice of the normal mode heavily depends on the ocean environment and source depth [7, 8]. Besides, the background of reverberation can be normalized by adaptive beamforming algorithm [5, 9, 10]. However, it requires efficient estimation of the correlation matrix, and the estimation error will increase because of the instability. In addition, the classic matched processing method can be performed to use the transmit signal as a reference for coherent compression. Nevertheless, this method only performs well in white noise background. Therefore, a generalized matched filtering method [11] is firstly modified to whiten the reverberation by using the parametric model filtering with a recursive procedure. Meanwhile, the copy of sound field and the optimum focusing filtering are mainly used to reduce the mismatch error. Furthermore, the concept of time-reversal in acoustics is also derived from the method of phase conjugate in the frequency domain in optics. [12] used the time-reversal process to focus the acoustic energy on the target position, thereby effectively enhancing the target echo to improve the SRR. [13, 14] proposed a time-reversal processing based on reverberation
groove method and carried out an experimental demonstration. However, the zero point of reverberation in the specified distance and the excitation of the time-reversal array are not set. Due to the sparse property in detection circumstances, compressed sensing (CS) is utilized for target estimation [15]. It is demonstrated that the results of estimation achieved by CS beamforming have a higher azimuth resolution and higher detection ability than the conventional beamforming methods [16–18]. The detection of sparse target can be implemented in fractional Fourier transform, which exhibits an improved localization performance [19, 20]. Moreover, based on the difference between the reverberation and target signal, the method of principal component analysis (PCA) is used to suppress the reverberation. It does not require prior statistical knowledge and deterministic models, and hence it is highly adaptable [21, 22].

Note that the array processing algorithms in the ocean are generally based on the assumption of absence of errors in the hydrophone. Undesirably, in practical applications, due to the effects of hydrophone technologies, deployment methods, device aging and ocean environment, corrupted hydrophones with sensor failure are inevitable. Fig. 1 shows two corrupted hydrophones (marked with blue) in the VLA. The corrupted hydrophones will destroy the geometric distribution structure of the array. For example, a corrupted element will make the uniform line array lose the geometric symmetry, thereby reducing the array gain. The most direct method is to replace the corrupted hydrophones. However, it is difficult to implement when the sonar array has been placed in water because the acoustic data cannot be easily obtained in the experiment. Accordingly, reconstruction algorithms of the acoustic data can be used to reduce the influence of the corrupted sonar array and avoid the human and material resources incurred by replacing the hydrophones. Moreover, if the probability of corrupted array is taken into account in the early stage of data processing, the effect on array signal processing can be prevented.

For the corrupted array, there are the weight re-optimization of the remaining normal array elements and the reconstruction of contaminated array signals. According to the desired array pattern, the weight of the remaining normal array elements can be re-optimized, so that the directional pattern formed by the effective elements is as close as possible to the original one. The sidelobe level caused by the corrupted array elements can be suppressed by conjugate gradient method [23], quadratic constrained linear optimization [24], maintaining fixed nulls [25], adaptive weight reconstruction [26], and orthogonal methods [27]. To overcome the limitations on a long distance and huge computing quantity, these methods of weight re-optimization are not easy to achieve as all the weights must be readjusted. The basic idea of signal reconstruction is to use the received data of the normal array elements to reconstruct the data of the
corrupted array elements by interpolation or optimization algorithms. For a uniform linear array, the signal received by adjacent array elements has only a fixed phase difference, so the output of corrupted array elements can shift the phase according to the normal array element output. This method is relatively simple for single source but complicated for multiple sources. Generally, adjacent [28] or all [29] normal array elements can be used to reconstruct the output signal. Besides bilinear programming [30], cumulants [31], inverse Fourier transform [32], and global optimization algorithms (such as genetic algorithm [33], particle swarm optimization algorithm [34] and neural network [35]) have been successfully applied to reconstruct the corrupted elements.

In the absence and corruption of data, it is known that PCA is a common approach to data analysis and dimensionality reduction, e.g., information extraction such as DOA estimation. However, if the entries in the observation matrix are severely damaged, the error between the matrix obtained by PCA and the original matrix will be very large. This is mainly because PCA is sensitive to outliers. The accurate calculation of principal components in the presence of outliers is called robust principal component analysis (RPCA) [36, 37]. The matrix completion (MC) [38] theory proposed by Candès et al. is an important method for data analysis and processing after the CS theory. CS and MC are the main components of principal component pursuit (PCP) [39, 40], which can be used to solve RPCA. When the data matrix is severely damaged or corrupted by noise, MC is capable of the near-perfect recovery of matrices if the original matrices have low rank and non-coherent characteristics. Motivated by the advantages of MC techniques, in this paper, we propose to utilize the low-rank matrix factorization to recover the acoustic data caused by sonar sensor failure in ocean reverberation. Note that the conventional techniques for MC depend on the assumption of Gaussian noise; however, in the ocean environment, the background of noise is usually non-Gaussian. [41] have proved that $\ell_p$-norm with $0 < p < 2$ shows a better performance than the Frobenius norm in the presence of outliers. Thus, with the reverberation signal in simulation, $\ell_1$-norm and $\ell_2$-norm are utilized to reply to reverberation suppression, and Hankel structured matrix is employed to deal with sensor failure.

The remainder of this paper is organized as follows. Section II presents the problem formulation. Section III shows the proposed solution based on $\ell_p$-norm. Section IV introduces the application of our algorithm under reverberation and the superiority over other related algorithms. Finally, Section V draws conclusions.
II Problem Formulation

In order to model sensor failure in sonar array, the following two cases illustrated in Fig. 2 will be investigated:

(a) Some hydrophones are failed during the entire sampling period, and the missing positions of channel are randomly selected.

(b) Some hydrophones are failed during the entire sampling period, and the missing positions of channel are randomly and continuously selected.

Fig. 2: The model of sensor failure in sonar array.

Assume there are \( r \) underwater targets from unknown distinct direction of \( \{\theta_1, \cdots, \theta_r\} \) impinging on the VLA which has \( N \) sensors with array spacing of half-wave. Moreover, we use \( A(\theta) \in \mathbb{C}^{N \times r} \) to denote the steering matrix of all the steering vectors. It is also assumed that the DOAs of the source are not time-varying and the number of snapshots is \( M \). Then, we have

\[
X = (A(\theta)S + R)^T. \tag{1}
\]

In (1), \( X = [x_1, x_2, \ldots, x_N] \in \mathbb{C}^{M \times N} \) is the array data matrix, and the \( n \)th element \( x_n \) of \( X \) denotes the samples of sensor \( n \). Moreover, \( S = [s_1, s_2, \ldots, s_M] \in \mathbb{C}^{r \times M} \) is the signal matrix, where \( s_m \) represents
the signal vector at time instant \( m \). \( \mathbf{R} \in \mathbb{C}^{N \times M} \) is the measured reverberation matrix. In practical systems, we usually have \( r < N < M \). Therefore, \( \mathbf{M} = \mathbf{A}(\theta) \mathbf{S} \in \mathbb{C}^{N \times M} \) is a low-rank matrix. When the array is corrupted with sensor failure, the incomplete measurement matrix can be written as

\[
\mathbf{X} = \mathbf{B} \odot (\mathbf{M} + \mathbf{R})^T.
\]

In (2), \( \mathbf{B} \) is a binary matrix composed of 0 and 1 at the positions associated with normal and corrupted data, respectively. \( \odot \) denotes element-wise product. In this paper, we assume \( \mathbf{B} \) has been judged by the received acoustic data. Moreover, we define the operator \( \mathcal{P}_\Omega \) as \([\mathcal{P}_\Omega(\mathbf{X})]_{i,j} = X_{i,j} \) if \((i, j) \in \Omega\) and 0 otherwise, where \( \Omega \in \{1, \ldots, M\} \times \{1, \ldots, N\} \) contains the indices of observed entries.

From the MC theory we know that when the entries of a certain row or column of a matrix are completely missing, generally, the missing entries cannot be directly recovered based on other entries. One solution is converting the matrix into a Hankel structured matrix, and then applying the MC theory to recover the missing entries. Let \( \mathcal{H}_{n_1} : \mathbf{X} \in \mathbb{C}^{M \times N} \rightarrow \mathbf{Z} \in \mathbb{C}^{M n_1 \times n_2} \) denote a linear operator, mapping the data matrix into the corresponding Hankel matrix [42]:

\[
\mathcal{H}_{n_1}(\mathbf{X}) = \begin{bmatrix}
x_1 & x_2 & \cdots & x_{n_2} \\
x_2 & x_3 & \cdots & x_{n_2+1} \\
\vdots & \vdots & \ddots & \vdots \\
x_{n_1} & x_{n_1+1} & \cdots & x_N
\end{bmatrix},
\]

where \( n_1 + n_2 = N + 1 \), and we usually take \( n_1 = N/2 \). It is not difficult to find that even if some columns of the original data matrix are missing, in the new reshaped Hankel matrix, the coordinates of the missing entries have been all shuffled, so the situation of all missing entries of some columns will no longer occur. In our paper, for simplicity, we use \( \mathcal{H}(\mathbf{X}) \) instead of \( \mathcal{H}_{n_1}(\mathbf{X}) \). For example, we assume that an original matrix is

\[
\mathbf{X} = \begin{bmatrix}
1 & 4 & 7 & 10 \\
2 & 5 & 8 & 11 \\
3 & 6 & 9 & 12
\end{bmatrix}.
\]
If column 2 of $X$ corrupts, the unobserved entries over $\Omega$ is $\{(1, 2), (2, 2), (3, 2)\}$. Then, we have

$$\mathcal{P}_\Omega(X) = \begin{bmatrix} 1 & 7 & 10 \\ 2 & 8 & 11 \\ 3 & 9 & 12 \end{bmatrix},$$

whose Hankel structured matrix is

$$\mathcal{H}(\mathcal{P}_\Omega(X)) = \begin{bmatrix} 1 & 7 \\ 2 & 8 \\ 3 & 9 \\ 7 & 10 \\ 8 & 11 \\ 9 & 12 \end{bmatrix}.$$

After the Hankel transformation, $\Omega \rightarrow \Omega'$, the unobserved entries over $\Omega'$ are $\{(1, 2), (2, 2), (3, 2), (4, 1), (5, 1), (6, 1)\}$. Accordingly, the inverse Hankel transformation of $\mathcal{H}$, i.e., $\mathcal{H}^\dagger$, $\mathcal{H}^\dagger(Z) \in \mathbb{C}^{M \times N}$ [42] is given by

$$(\mathcal{H}^\dagger(Z))_{i,j} = \frac{1}{w_j} \sum_{k_1+k_2=j+1} Z_{(k_1-1)n_c+i,k_2},$$

where $w_j$ is the number of elements in the $j$th anti-diagonal. For example, the 3rd anti-diagonal contains three elements over $\{(3, 1), (2, 2), (1, 3)\}$, so $w_3 = 3$; and the $N$th anti-diagonal contains only one elements, so $w_N = 1$.

The task of MC in this paper aims at recovering $M$ from $\mathcal{P}_\Omega(X)$, and the corresponding optimization problem is formulated as

$$\min_{M,R} \| \mathcal{H}(\mathcal{P}_\Omega(X)) - \mathcal{H}(\mathcal{P}_\Omega(M^T)) \|^p_p,$$

s.t. $\text{rank}(\mathcal{H}(M^T)) \leq r$.

where $\| \cdot \|^p_p$ is the $l_p$ norm of a matrix, i.e.,

$$\|E\|_p = \left( \sum_{(i,j) \in \Omega} \|E_{i,j}\|^p \right)^{1/p}.$$
Algorithm 1: Hankel Structured MC Based on $\ell_p$-norm

Input: $X_\Omega$, $\Omega$, $B$, $r$ and the iteration times $K$.

1. Preprocess: $X \rightarrow H(X)$.

2. $\Omega \in \mathbb{C}^{M \times N} \rightarrow \Omega' = H(B) \in \mathbb{C}^{Mn_1 \times n_2}$.

3. Initialize: Randomly initialize $U^0 \in \mathbb{R}^{Mn_1 \times r}$.

4. Determine $\{I_{jj}\}_{jj=1}^{n_2}$ and $\{J_{ii}\}_{ii=1}^{Mn_1}$ according to $\Omega'$.

5. for $k = 0, 1, \ldots, K$ do

6. for $jj = 1, 2, \ldots, n_2$ do

7. $(v_{jj})^{k+1} \leftarrow \arg \min_{v_{jj}} \| b_{I_{jj}} - U^k_{I_{jj}} v_{jj} \|_p$.

8. end for

9. for $ii = 1, 2, \ldots, Mn_1$ do

10. $(u_{ii}^T)^{k+1} \leftarrow \arg \min_{u_{ii}^T} \| b_{J_{ii}}^T - u_{ii}^T V^{k+1} \|_p$.

11. end for

12. end for

Output: $M = H^\dagger(U^{k+1} V^{k+1})$.

two situations $p = 1$ and $p = 2$ in this paper. As the solution procedure is executed based on the Hankel structure, problem (8) is equivalent to

$$\min_{M, R} \| (H(X))_{\Omega'} - (H(M^T))_{\Omega'} \|_p^p,$$

s.t. rank($H^\dagger(M^T)$) $\leq r$. (10)

The minimization with the observed entries is made a transition from $\Omega$ to $\Omega'$.

III Hankel structured MC based $\ell_p$-norm

Low rank matrix factorization, which is considered as the low rank property of matrix, has been used to void full singular value decomposition (SVD). Therefore, optimization problem (10) can be transformed into [41]

$$\min_{U, V} f_p(U, V) := \| (H(X))_{\Omega'} - (UV)_{\Omega'} \|_p^p,$$ (11)

where $U \in \mathbb{C}^{Mn_1 \times r}$ and $V \in \mathbb{C}^{r \times n_2}$. Because $r \ll Mn_1 n_2$, the complexity of MC will be greatly reduced. Furthermore, the smaller the rank $r$, the higher the accuracy of MC. The error matrix after Hankel structured processing can be rewritten as $E_{\Omega'} = (H(X))_{\Omega'} - (UV)_{\Omega'}$. It is worth noting that the inverse Hankel transformation in (7) needs to be operated with $\hat{M} = (H^\dagger(UV))^T$ when $U$ and $V$ have been determined.
The strategy of alternating minimization with a relaxation as a bi-convex problem is adopted to solve $U$ and $V$ respectively according to:

$$V^{k+1} = \arg\min_V \| (\mathcal{H}(X))_{\Omega'} - (U^k V)_{\Omega'} \|_p,$$

$$U^{k+1} = \arg\min_U \| (\mathcal{H}(X))_{\Omega'} - (U V^{k+1})_{\Omega'} \|_p.$$

In the first step, we solve $V$ by fixing $U$, i.e.,

$$\min_V f_p(V) := \| (\mathcal{H}(X))_{\Omega'} - (U V)_{\Omega'} \|_p,$$

where $(\cdot)^k$ is omitted for simplicity. Let $u_{ii}^T \in \mathbb{C}^r$ and $v_{jj} \in \mathbb{C}^r$ denote the $ii$th row of $U$ and the $jj$th column of $V$, respectively, where $ii = 1, ..., Mn_1$ and $jj = 1, ..., n_2$. Then, problem (14) can be expressed as follow:

$$\min_V f_p(V) := \sum_{(ii,jj) \in \Omega'} \left\| (\mathcal{H}(X))_{ii,jj} - u_{ii}^T v_{jj} \right\|_p.$$

According to the number of columns in $V$, problem (15) can be decomposed into $n_2$ independent subproblems:

$$\min_{v_{jj}} f_p(v_{jj}) := \sum_{ii \in \mathcal{I}_{jj}} \left\| (\mathcal{H}(X))_{ii,jj} - u_{ii}^T v_{jj} \right\|_p.$$  

Here we use $\mathcal{I}_{jj} = \{jj_1, ..., jj_{|\mathcal{I}_{jj}|}\}$ and $|\mathcal{I}_{jj}|$ to denote the $jj$th column of $\Omega'$ and its cardinality, respectively, where $|\mathcal{I}_{jj}| > r$. From the Hankel structured matrix given in (6), we know that only the $(1,1)$, $(2,1)$ and $(3,1)$ can be observed when $jj = 1$, so we have $\mathcal{I}_1 = \{1,2,3\}$ and $\mathcal{I}_2 = \{4,5,6\}$.

Furthermore, we respectively define a matrix $U_{\mathcal{I}_{jj}} \in \mathbb{C}^{\mathcal{I}_{jj} \times r}$ that contains the observed row indexed by $\mathcal{I}_{jj}$:

$$U_{\mathcal{I}_{jj}} = \begin{bmatrix} u_{jj_1}^T \\ \vdots \\ u_{jj_{|\mathcal{I}_{jj}|}}^T \end{bmatrix},$$

and a vector $b_{\mathcal{I}_{jj}} = [X_{jj_1,jj}, ..., X_{jj_{|\mathcal{I}_{jj}}}]^T \in \mathbb{C}^{\mathcal{I}_{jj}}$. Then, problem (16) is equivalent to

$$\min_{v_{jj}} f_p(v_{jj}) := \| b_{\mathcal{I}_{jj}} - U_{\mathcal{I}_{jj}} v_{jj} \|_p.$$  

When $p = 2$, problem (18) becomes a least squares problem and can be solved by $v_{jj} = U_{\mathcal{I}_{jj}}^T b_{\mathcal{I}_{jj}}$, and
the implementary complexity of the $jj$th iteration is $\mathcal{O}(|I_{jj}| r^2)$. When $p = 1$, problem (18) can be solved by the iteratively reweighted least squares (IRLS) algorithm [43]. We use $N_{IRLS}$ to definite the number of iteration of IRLS, and the complexity of $jj$-iteration here is $\mathcal{O}(|I_{jj}| r^2 N_{IRLS})$; moreover, for the $n_2$ independent subproblems, we have $\sum_{jj=1}^{n_2} |I_{jj}| = |\Omega'|$, so the overall complexity is $\mathcal{O}(|\Omega'| r^2 N_{IRLS})$.

As the acoustic data is complex-valued, the real-valued transforms need to be preprocessed in the above algorithm. Then, the real-valued vector of complex-valued $b_{I_{jj}} \in \mathbb{C}^{I_{jj}}$ is

$$b_{I_{jj}} = \begin{bmatrix} \Re\{b_{I_{jj}}\} \\ \Im\{b_{I_{jj}}\} \end{bmatrix} \in \mathbb{R}^{2I_{jj}}. \quad (19)$$

Besides, the real-valued matrix of complex-valued $U_{I_{jj}} \in \mathbb{C}^{I_{jj} \times r}$ is

$$U_{I_{jj}} = \begin{bmatrix} \Re\{U_{I_{jj}}\} - \Im\{U_{I_{jj}}\} \\ \Im\{U_{I_{jj}}\} \Re\{U_{I_{jj}}\} \end{bmatrix} \in \mathbb{R}^{(2I_{jj}) \times (2r)}. \quad (20)$$

Similarly, the $ii$th row of $U$ can be also solved trough $M_{n_1}$ independent subproblems:

$$\min_{u_{ii}} f_p(u_{ii}^T) := \|b_{J_{ii}}^T - u_{ii}^T V_{J_{jj}}\|_p^p, \quad (21)$$

where $V_{J_{ii}} \in \mathbb{C}^{r \times J_{ii}}$ and $b_{J_{ii}}^T = [X_{ii,ii,1}, ..., X_{ii,ii,J_{ii}}]^T \in \mathbb{C}^{J_{ii}}$, and $J_{ii} = \{ii_1, ..., ii_{J_{ii}}\} \in \{1, ..., n_2\}$ denotes the column for the $ii$th row in $\Omega'$. In (6), when $ii = 1$, we have $J_1 = J_2 = J_3 = \{1, 3\}$ and $J_4 = J_5 = J_6 = \{2, 3\}$.

The proposed algorithm consists of two stages. The first stage preprocesses $X$ into $\mathcal{H}(X)$, and then converts $\Omega$ into $\Omega'$. The second stage fulfills the MC with $\ell_p$-norm in $\Omega'$. The overall complexity of the proposed algorithm is $\mathcal{O}(|\Omega'| r^2 N_{IRLS} K)$, where $K$ is the iteration times.

### IV Applications

According to the central limit theorem in statistics, when the number of scattered waves is large enough, the instantaneous value of reverberation will meet the Gaussian distribution, and the phase will obey the uniform distribution. However, the improvement of the resolution of sonar makes the central limit theorem no longer meet, which leads a fact that the number of effective scatterers in each spatial processing unit is greatly reduced. Therefore, the envelope of reverberation $R_e(x)$ of high-resolution sonar has serious
smearing. Therefore, it is with skewness and no longer obeys the Gaussian distribution. The statistical properties have been investigated by some experimental data, and in this paper we choose the $K$ distribution to describe the reverberation [44–46] as a representative, which can be determined by different degrees of freedom $n$. Moreover, the phase $R_p(\varphi)$ can still be described by the uniform distribution:

$$R_p(\varphi) = \begin{cases} \frac{1}{2\pi}, & \text{if } \varphi \in (0, 2\pi) \\ 0, & \text{else} \end{cases} \quad (23)$$

Accordingly, the reverberation signal $R$ in (1) can be obtained by

$$R = R_e(x)e^{jR_p(\varphi)}. \quad (24)$$

In this paper, we use the SRR defined as follow to compare the level of the target signal with the reverberation:

$$SSR = 10 \log_{10} \left( \frac{\sigma^2_s}{\sigma^2_n} \right) \text{dB}, \quad (25)$$

where $\sigma^2_s$ and $\sigma^2_n$ are the variances of signal and reverberation, respectively.

![Fig. 3: The normalized RMSE versus iteration number.](image)

Moreover, the normalized root mean square error (RMSE) defined as follows is exploited to measure
the performance based on 100 independent trials:

$$\text{RMSE}(\hat{M}) = \sqrt{E \left\{ \frac{\| \hat{M} - M \|^2_F}{\| M \|^2_F} \right\}}, \quad (26)$$

where $\hat{M}$ is the result obtained by equations (12) and (13). A typical experimental setting is $N = 20$, $M = 100$, and $r = 2$. We compare the proposed algorithm respectively based on $\ell_1$-norm and $\ell_2$-norm with OptSpace [47] and structured alternating projection (SAP) [42] to recover $M$ from $\mathcal{P}_\Omega(X)$.

Fig. 3 plots the normalized RMSE versus the iteration times. We set SSR = 10, and the sensor failure percentage is 30%, and the degrees of freedom of K distribution is $n = 0.1$. We can observe that the OptSpace algorithm exhibits a high error without falling. Moreover, the SAP and $\ell_2$-norm begin to converge and they have almost same the RMSEs. Besides, they also provide robust performances in the ocean reverberation. However, their performances are still worse than that of of the $\ell_1$-norm. Finally, it
can be observed that the algorithms of SAP, $\ell_1$- and $\ell_2$-norms converge within ten iterations.

Fig. 4 illustrates the SSR with respect to the probability of recovery. For each realization, 100 independent trials were evaluated. Firstly, if the normalized RMSE is smaller than 0.15, we think that the trial is successful and observed as white region, and dark region otherwise. Moreover, we can observe that the algorithm based on $\ell_1$-norm has the best performance than others in the ocean reverberation. Besides, the algorithm of OptSpace is only valid when the SSR is above 10 dB and the sensor failure percentage
is smaller than 10%. It is also worth noting that the SAP and algorithm based on \( \ell_2 \)-norm have similar performances. Therefore, the MC algorithm based on \( \ell_1 \)-norm is suitable for the reverberation suppression.

In this paper, the performance of DOA estimation is measured in terms of the RMSE:

\[
\text{RMSE}(\hat{\theta}) = \sqrt{\frac{1}{100r} \sum_{p=1}^{r} \sum_{q=1}^{100} (\hat{\theta}_{q,p} - \theta_q)^2},
\]

where \( \hat{\theta} \) is the estimated DOA.

Fig. 5 shows the normalized spectrum versus bearing angle in degrees. Due to the effect of sensor failure and reverberation interference on the original data, the normalized spectrum appears false peaks. Hence, the beam pattern is sensitive to the contaminated signal. Besides, the main lobe of the OptSpace algorithm converges to the true signal orientation, but there are certain apparent estimation errors. Thirdly, the algorithms of SAP and \( \ell_2 \)-norm have obvious performance improvements, and both of them can make more accurate spectrum estimation and have almost the same performances. Fourthly, the algorithm based on \( \ell_1 \)-norm exhibits the narrowest beamwidth and lowest sidelobe compared with other algorithms. Most importantly, the algorithm based on \( \ell_1 \)-norm has the best robustness of spectrum estimation in the ocean reverberation with sensor failure.

According to Fig. 4, we choose the sensor failure percentage less than 50% for the comparison of DOA estimation because of the high probability of recovery. Fig. 6 shows the results of RMSE versus SSR and sensor failure percentage, where the threshold of normalized RMSE of DOA is chosen as 0.15. Firstly, if the RMSE is smaller than 0.15, the result can be observed as white region and dark region otherwise. Furthermore, the original data with sensor failure and reverberation interference cause large errors along with different SSRs, in which the all normalized RMSEs definitely exceed 0.15 and show a whole dark region in Fig. 6 (a). From Fig. 6 (b)∼(e) it can also be observed that the MC algorithms of OptSpace, SAP and \( \ell_p \)-norm with the receiving acoustic data have a constructive influence on the DOA estimation. Thirdly, when the SSR is above 5 dB, there are all almost no errors of DOA with the involved algorithms. As for the SSR less than 5 dB, the errors of DOA achieved by the algorithm with \( \ell_1 \)-norm will be less. Accordingly, the MC algorithm with \( \ell_1 \)-norm is suitable for source location in the ocean reverberation environment with sensor failure, especially for low SSR.
V Conclusion

This paper proposed an approach with improved DOA estimation performance in the presence of ocean reverberation with sonar sensor failure. First, the model of sensor failure is established, and the corrupted column is existing. So the Hankel structured matrix is transformed for the received incomplete matrix. With the low rank factorization, the algorithms of $\ell_1$- and $\ell_2$- norms are developed to recover the signal matrix for DOA estimation. According the statistical property of the ocean reverberation, the instantaneous value of reverberation is model as the K distribution and the phase is modeled as uniform distribution. The proposed approach can converge to the ground-truth matrix ultimately. The results show that the algorithm based on $\ell_1$-norm is suitable for reverberation suppression because of the narrow beam width and low sidelobe. The robustness of DOA estimation in the ocean reverberation with sensor failure can be improved by the matrix completion with $\ell_1$-norm minimization.

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