Self-Sovereign Personal Cryptocurrencies: Foundations for Grassroots Cryptoeconomy

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The ecosystem of cryptocurrencies benefits the few and exacerbates economic inequality. Here, we aim to offer an egalitarian and inclusive alternative by morphing the concepts, tools and technologies developed by the cryptocurrencies ecosystem, together with the distributed computing technology that enabled it, into novel foundations for grassroots cryptoeconomy. The foundations are layered, with each layer consisting of a cryptoeconomic component and a computational component, supporting the layers above.

The cryptoeconomic layers include, from the ground up: (i) Self-sovereign personal cryptocurrencies, a novel concept, that achieves liquidity through personal currency exchange and equality of coin-value through coin redemption among liquid personal currencies; (ii) non-fungible tokens (NFTs) trade network, an abstract counterpart of asset-transfer objects; both layers are augmented with novel consensus-free multisignature accounts; and (iii) digital social contracts/democratic DAOs, which are consensus-based smart contracts autonomously executed by their vetted participants, rather than by third-party miners. The economic layers are designed so that they can be implemented by their corresponding computational layers – an established hierarchy in distributed computing: (i) dissemination with leader-based double-spending prevention; (ii) reliable broadcast, supermajority-based double-spending prevention; and (iii) ordering, based on blockchain consensus, State-Machine Replication, or Byzantine Atomic Broadcast. Importantly, the first two layers are consensus-free, whereas the third layer—like mainstream cryptocurrencies—is consensus-based.

Trust is a key differentiator between mainstream cryptocurrencies, which purport to be trustless, and grassroots cryptoeconomy, which is trust-based: Personal cryptocurrencies assume trust in the person issuing a currency; multi-signature accounts assume trust in the specified supermajority of the designated signatories; the NFT trade network assumes trust in two-thirds of the traders; and consensus-based digital social contracts assume trust in two-thirds of the parties to the contract.

Mathematical uniformity within the layered architecture is achieved by specifying all components—both cryptoeconomic and computational—as distributed multiagent transition systems. This facilitates specifying the horizontal implementations of cryptoeconomic components by computational components as mappings among the transition systems that specify them, in line with the vertical implementations of each computational component by the component below it. The vertical implementations are known, and so are horizontal implementations at the consensus-based third layer and the implementation of asset-transfer objects by reliable broadcast.

Here, our focus is the cryptoeconomic components of the novel consensus-free zone – self-sovereign personal cryptocurrencies, an NFT trade network, and their multisignature accounts: We formalize them; demonstrate their economic functionality via use cases; present and prove correct horizontal implementations of them by their corresponding computational components; and compose the horizontal implementations with the vertical ones into double-spending-resilient implementations – leader-based for the personal cryptocurrencies and supermajority-based for the NFT trade network.
A key message of this paper is the coupling of the economic and computational features of the novel consensus-free zone: rich economic functionality with lean computational implementations. They make the proposed layered architecture, inclusive of the consensus-based layer, promising foundations for grassroots cryptoeconomy.

ACM Reference Format:
Ehud Shapiro, Weizmann Institute of Science and Columbia University. 2022. Self-Sovereign Personal Cryptocurrencies: Foundations for Grassroots Cryptoeconomy. 1, 1 (February 2022), 31 pages. https://doi.org/10.1145/nnnnnnn.nnnnnnn

1 INTRODUCTION AND RELATED WORK

Mainstream cryptocurrencies—based on proof of work [24] or stake [20]—grant participants power and wealth in accordance with their capital investment, thus benefiting the few and exacerbating economic inequality. Community cryptocurrencies attempt to redress this: They aim to achieve social goals such as distributive justice and Universal Basic Income (UBI) [3, 7, 18], for example by egalitarian coin minting [30]. But, like mainstream cryptocurrencies, they presently rely on execution by third parties – miners that require remuneration for their service. Hence, neither mainstream cryptocurrencies nor present community cryptocurrencies address a key requirement of a grassroots cryptoeconomy – bootstrap without external capital or credit and without reliance on third-party computing services.

Smartphones are getting ever more powerful, cheap, and prevalent, and the networks that connect them are getting ever more speedy and ubiquitous. This unprecedented computing power and connectivity at the hands of the many can and should be put to a good use – cutting the umbilical cord that today connects the multitudes to surveillance-capitalism-based services (e.g. Facebook), autocratic ‘sharing-economy’ apps (e.g. Uber, Airbnb), and plutocratic cryptoeconomy platforms (e.g. DeFi on mainstream cryptocurrencies), to be supplanted by grassroots, peer-to-peer and community-based digital conduct – social, economic, and political.

The aim of this work is to provide conceptual and mathematical foundations for the cryptoeconomic components of this egalitarian and democratic vision. The novel foundations for grassroots cryptoeconomy presented here can be viewed as the result of morphing the concepts, tools and technologies developed by the cryptocurrencies ecosystem, together with the distributed computing technology that enabled it, to serve this vision.

The foundations are layered, as shown in Figure 1, with each layer consisting of a cryptoeconomic component and a computational component, both supporting the layers above. The cryptoeconomic components include, from the ground up:

1) Self-sovereign personal cryptocurrencies, a novel concept. The key defining characteristic of personal cryptocurrencies is coin redemption, which is the obligation of a person to redeem any personal coin it has minted with any personal coin it holds. It provides for chain payments and arbitrage, and hence equalizes the value of all personal coins with sufficient mutual liquidity, allowing personal cryptocurrencies to form the basis of grassroots economies, with community cryptocurrencies [30] as an emergent phenomena. Fault-resilience and transaction-finality are provided by each agent regarding its own personal currency. A liquid grassroots economy can emerge via mutual credit lines created via personal coin exchange [17], forsaking initial capital or external credit.

Self-sovereignty implies that the responsibility for the economic integrity and the computational integrity of a personal currency resides with the person; as the value of a personal currency depends on such integrity, every person is incentivized to maintain them. In Section 3 we explore all the above, as well as discuss how personal cryptocurrencies address questions of creditworthiness, insolvency, bankruptcy, duplicate (sybil) identities, and death.
(2) Non-fungible tokens (NFTs) trade network, an abstract counterpart of asset-transfer objects [15]. While both mainstream and community cryptocurrencies require total ordering for their realization and hence employ Nakamoto consensus/Byzantine Atomic Broadcast, NFT trade can make do with the strictly-weaker equivocation exclusion [16]. Both layers are augmented with novel consensus-free multisignature accounts. We show that adding multisig accounts can enhance personal cryptocurrencies with much of the functionality of smart contracts, but in a grassroots way without depending on anonymous third parties and without paying them a fee (gas) for their services.

(3) Consensus-based digital social contracts [5] are smart contracts autonomously executed by their vetted participants, rather than by third-party miners. Community applications on top of personal cryptocurrencies can be realized via consensus-based digital social contracts. In Section 5 we discuss how a community credit union, with its own community currency, can be established based on the personal cryptocurrencies of its participants; and how it can effectively support Direct Philanthropy and UBI – both external third-party funded and internal community funded.

The economic layers are designed so that they can be implemented by their corresponding computational layers – an established hierarchy [16] in distributed computing:

(1) Dissemination (e.g., [10]) with leader-based double-spending prevention [31].
(2) Reliable broadcast [4], supermajority-based double-spending prevention [8, 15, 32].
(3) Ordering, based on blockchain consensus, State-Machine Replication, or Byzantine Atomic Broadcast [6, 19, 39].
Importantly, as shown in Figure 1, the first two layers are consensus-free, whereas the third layer—like mainstream cryptocurrencies—is consensus-based.

Trust is a key differentiator between mainstream cryptocurrencies, which purport to be trustless, and grassroots cryptoeconomy, which is trust-based: Personal cryptocurrencies assume trust in the person issuing a currency; multi-signature accounts assume trust in the specified supermajority of the designated signatories; the NFT trade network assumes trust in two-thirds of the traders; and consensus-based digital social contracts assume trust in two-thirds of the parties to the contract.

Mathematical uniformity within the layered architecture is achieved by specifying all components—cryptoeconomic and computational—as distributed multiagent transition systems [31]. This facilitates specifying the horizontal implementations of cryptoeconomic components by computational components as mappings among their respective multiagent transition systems, in line with the vertical implementations of each computational component by the component below. The vertical implementations are known, and so are horizontal implementations at the consensus-based third layer and the implementation of asset-transfer objects [15] by reliable broadcast [4]. This uniformity, while very convenient mathematically, is not essential: The cryptoeconomic components can be realized by computational components different from the ones presented here, and it may turn out that in practice some other protocol stack, e.g. asset-transfer objects and their underlying protocols, may be more useful.

Here, we focus on the cryptoeconomic components of the novel consensus-free zone – personal cryptocurrencies, an NFT trade network, and their multisignature accounts: We formalize them; demonstrate their economic functionality via use cases; present and prove correct implementations of them by their corresponding computational components; and compose the horizontal implementations with the vertical ones into consensus-free double-spending-resilient implementations – leader-based for the personal cryptocurrencies and supermajority-based for the NFT trade network.

A key message of this paper is the coupling of the economic and computational features of the novel consensus-free zone: rich economic functionality with lean computational implementations. They make the proposed layered architecture, inclusive of the consensus-based layer, promising foundations for grassroots cryptoeconomy.

1.1 Outline
Section 1.2 includes the needed mathematical background on multiagent transition systems [31]. The grassroots cryptoeconomy is presented layer by layer (Fig. 1). Each layer is presented informally, followed by use cases, formal definition, and description and proof of correctness of an implementation (proofs are relegated to Appendix 7 due to space limitations). A key contribution of this paper is self-sovereign personal cryptocurrencies, which are also the base-layer of the grassroots cryptoeconomy. However, personal cryptocurrencies are best defined as an instance of the abstract NFT trade network. Hence, description commences in Section 2 with the middle layer 2 of the NFT trade network; followed by layer 1 in Section 3 with self-sovereign personal cryptocurrencies; Section 4 discusses adding multisignature accounts to layers 1 and 2; and Section 5 discusses informally layer 3 – consensus-based digital social contracts. Section 6 concludes.

First reading would be more productive and pleasant if focused on informal descriptions and use cases, reserving the preliminaries, specifications, implementations, and proofs for a second reading.

1.2 Preliminaries: Multiagent Transition Systems
This work employs the mathematical framework of multiagent transition systems [31] to define the cryptoeconomic components, to describe their implementations, and to prove them correct.
and fault-resilient. Here is some mathematical background on which the presentation depends; additional background needed for the proofs is relegated to Appendix 7.1.

**Definition 1** (Transition System with Faults). Given a set $S$, referred to as *states*, the *transitions* over $S$ are all pairs $(s, s') \in S^2$, also written $s \rightarrow s'$. A *transition system* $TS = (S, s_0, T)$ consists of a set of states $S$, an initial state $s_0 \in S$, and set of (correct) transitions $T \subseteq S^2$. A *computation* of $TS$ is a sequence of transitions $s \rightarrow s' \rightarrow \cdots$, and a *run* of $TS$ is a computation that starts from $s_0$. A transition $s \rightarrow s' \in T$ is *correct*, and a computation of correct transitions is *correct*. A transition in $S^2 \setminus T$ is *faulty*, and a computation is *faulty* if it includes a faulty transition. We denote by $s \rightarrow s' \in T$ the existence of a correct computation (empty if $s = s'$) from $s$ to $s'$.

It is often possible to associate a partial order with a distributed system, wrt which the local state of each agent only increases.

**Definition 2** (Monotonic Transition System). Given a partial order $\leq$ on $S$, a transition system $(S, s_0, T)$ is *monotonic* with respect to $\leq$ if $s \rightarrow s' \in T$ implies $s \leq s'$.

**Definition 3** (Multiagent, Centralized, Distributed, Synchronous & Asynchronous Transition Systems). Given agents $\Pi$, a transition system $TS = (C, c_0, T)$ is:

1. *multiagent* over $\Pi$ if there is a partition $C^2 = \bigcup_{p \in \Pi} C_p^2$ such that each transition $c \rightarrow c' \in C_p^2$, $p \in \Pi$, referred to as a p-*transition*, is effected by $p$.
2. *distributed* if, in addition,
   a. there is a set $S$ for which $C = S^\Pi$, in which case $S$ is referred to as local states, $C$ as configurations over $\Pi$ and $S$, and $c_p \in S$ as the local state of $p$ in $c$, for any configuration $c \in C$ and agent $p \in \Pi$; and
   b. every p-transition $c \rightarrow c' \in C_p^2$ only affects $p$’s local state, namely $c_p \neq c'_p$ and $c_q' = c_q$ for all $q \neq p \in \Pi$.
   
   Else $TS$ is *centralized*.
   
   A partial order $\leq$ on local states $S$ naturally extends to configurations: $c \leq c'$ if $c_p \leq c'_p$ for every $p \in \Pi$.
3. *asynchronous*, or *Einsteinian*, if, in addition,
   a. there is a partial order $\preceq$ on $S$ wrt which $TS$ is monotonic, and
   b. for every $p$-transition $c \rightarrow c' \in T$ and for every $d, d' \in C$ that satisfies the following asynchrony condition, $T$ also includes the p-transition $d \rightarrow d'$.

   **Asynchrony condition**: $c \preceq d$, $c_p = d_p$, $c'_p = d'_p$, and $d_q = d'_q$ for every $q \neq p \in \Pi$.
   
   Else $TS$ is synchronous, or *Newtonian*.

**Definition 4** (Implementation: Safety, Completeness, Liveness, Correctness). Given two transition systems $TS = (S, s_0, T)$ and $TS' = (S', s'_0, T')$ an *implementation* of $TS$ by $TS'$ is a function $\sigma : S' \rightarrow S$ where $\sigma(s'_0) = s_0$. The implementation $\sigma$ is:

1. *Safe* if $s'_0 \xrightarrow{\cdot} y \rightarrow y' \in T'$ implies that $s_0 \xrightarrow{\cdot} x \rightarrow x' \in T$ for some $x = \sigma(y)$ and $x' = \sigma(y')$ in $S$. If $x = x'$ then the $T'$ transition $y \rightarrow y'$ *stutters*, else it *activates* $T$.
2. *Complete* if $s_0 \xrightarrow{\cdot} x \rightarrow x' \in T$, implies that $s'_0 \xrightarrow{\cdot} y \rightarrow y' \in T'$ for some $y, y' \in S'$. Such that $x = \sigma(y)$ and $x' = \sigma(y')$.
3. *Live* if $s'_0 \xrightarrow{\cdot} y \in T'$ implies that there is $y' \in S'$, $y' \neq y$, such that $y \xrightarrow{\cdot} y' \in T'$ and $s_0 \xrightarrow{\cdot} \sigma(y) \xrightarrow{\cdot} \sigma(y') \in T$, $\sigma(y) \neq \sigma(y')$.

An implementation is *correct* if it is safe, complete and live.

**Lemma 1** *(Composing Implementations)*. The composition of correct implementations is correct.
2 LAYER 2: AN NFT TRADE NETWORK

We begin with defining layer 2, the NFT trade network, as it provides the mathematical basis for the personal cryptocurrencies network of layer 1 (Section 3), as well as for extending both layers with multisignature accounts (Section 4).

2.1 Informal Description

The goal of the NFT Trade Network is to support the trade of NFTs among agents. NFTs emerged in the specific context of the Ethereum blockchain, to contrast them with the coins of the cryptocurrency that are fungible (interchangeable). Here we consider an abstract notion of NFTs. We assume a set of agents, each equipped with a a single and unique cryptographic key pair [29], and consider NFTs, which are structures with two components:

1. **Object**: A string, which may be a digital representation of a number, piece of text, document, picture, or movie, cryptographically signed by an agent, referred to as its creator.
2. **Provenance**: A list of holders, starting from the creator of the object, each signing an act of transfer of the NFT received to the next agent in the list, ending in its ultimate holder.

Any agent may turn any string it signs into an NFT it holds. We assume agents do so without infringing on copyright or other rights other agents may have in various strings. Besides creation, an agent may transfer an NFT it holds to another agent, the recipient, which becomes the new holder of the NFT and is also added to the provenance of the NFT. An agent may transfer only an NFT it holds. A transfer specifies the NFT to be transferred, the recipient, and some metadata (e.g. invoice, purchase order, contract), and is signed by the sender.

Layer 2 assumes a given ‘community of traders’, a supermajority of which may finalize a trade. Of course there could be multiple NFT trade networks operating in parallel, each with its own traders community. A variant in which a supermajority of only a subset of the ‘traders’, say only the ‘artists’, finalize trades is also possible; this subset can be appointed, elected, sampled, or be self-appointed upon network creation [31]. In that sense, the NFT trade network is permissioned and the parties are vetted by an ‘off-chain’ external authority or process. This is in different from mainstream cryptocurrencies, which are permissionless and use ‘on-chain’ vetting such as proof of work or stake, and also different from layer 3 consensus-based digital social contracts (Section 5), which may use on-chain vetting such as a democratic admission process or an algorithmic one [25].

2.2 Use Cases

As NFTs are all the rage right now [11], we skip discussing use cases and go straight to their math.

2.3 Specification

We assume a set of agents $\Pi$, each equipped with a unique and single key pair, and let $S$ denote the set of all strings.

**Definition 5** (NFT, Transfer, Object, Provenance, Ultimate Holder). The construct $x \overset{p,s}{\rightarrow} q$, signed by $p$, records a transfer by the sender $p$ of the payload $x$ and metadata $s$ to the recipient $q$.

The set of NFTs over $S$, $T(S)$ is defined inductively as follows:

1. **Initial NFT**: $s \overset{p,\bot}{\rightarrow} p \in T(S)$ for every agent $p \in \Pi$ and every object $s \in S \times \Pi$ signed by $p$.
2. **Transfer NFT**: If $x \in T(S)$ with recipient $p$ then $y = x \overset{p,s}{\rightarrow} q \in T(S)$ for every $q \in \Pi$ and string $s \in S \cup \{\bot\}$.
The history of an NFT \( x \in \mathcal{T}(S) \) is the sequence of NFTs \( x_1, x_2, \ldots, x_k, k \geq 1 \), where \( x_1 \in \mathcal{T}(S) \) is an initial NFT, \( x_k = x \), and for each \( i \in [k-1], x_i \in \mathcal{T}(S) \) is the sequence of \( x_{i+1} \). The provenance of \( x \) is the sequence \( p_1, \ldots, p_k \) where \( p_i \) is the recipient of \( x_i \), also referred to as its holder.

For example, the string representation of `coin201` is an NFT with \( p \)-signed string `coin201` as an object, transferred by \( p \) to \( p' \) for love and then transferred by \( p' \) to \( p'' \) to fulfill purchase order 157, with provenance \( p, p', p'' \) and with \( p'' \) being its holder.

The metadata \( s \) is either a string or undefined \( \perp \), in which case it can be omitted. But in a standard use case it could be a request to obtain a product or service for which the payment was made, a sales quote, or a purchase order. In a more complex example, a contract \( c \) signed by the parties \( p \) and \( q \) to exchange NFT \( x \) for NFT \( y \), in which case \( s \) may be `transferred according to contract \( c \)'.

For the transfer of \( y \) from \( q \) to \( p \), \( s \) could be `acknowledging receipt of \( x \) and fulfilling my obligation according to \( c \')`. The metadata \( s \) could be encrypted with \( q \)'s public key, for privacy; \( p \) could still prove the cleartext of \( s \) if this is subsequently needed for resolving a disagreement with \( q \). In Section 3, personal cryptocurrencies employ the metadata \( s \) to initiate and respond to redemption claims.

**Definition 6** (NFT Consistency, Double-spending, Consistent and Complete Sets, Holdings). Two sequences are **consistent** if one is a prefix of the other. Two NFTs with the same object are **consistent** if their provenances are consistent, else **inconsistent**. Two inconsistent NFTs constitute a double-spending by the last agent common to both provenances.

A set \( X \subset \mathcal{T} \) of NFTs is **consistent** if it does not include a double-spending; it is **complete** if for every NFT \( x \in X \), \( X \) includes the history of \( x \).

A consistent set of NFTs \( X \) defines a holdings partial function on signed objects \( X : S \times \Pi \mapsto \Pi \), as follows. The holder of a \( p \)-signed object \( s \in S \times \Pi \) is the holder of the maximal-provenance NFT in \( X \) with signed object \( s \), if there is one, else undefined \( \perp \).

The essence of the following NFT trade network specification is that agents can only transfer NFTs that they hold, resulting in configurations that are complete and consistent, and in particular do not include double-spending and hence induce well-defined holdings. Section 2.4 presents a fault-resilient implementation that excludes double-spending by faulty agents, provided there are not too many of them. The following definition of the NFT trade network employs the notion of a distributed multiagent transition system (Def. 3).

**Definition 7** (NT: NFT Trade Network). An **NFT trade network** is a distributed multiagent transition system \( NT = (CT, c0, TT) \), with sequences of NFTs \( S := \mathcal{T}(S)^\ast \) over \( S \) as local states, configurations \( CT := S^{\Pi}, \) initial configurations \( c0 := \{A\}^{\Pi} \), and \( p \)-transitions \( TT_p \subset CT^2 \) for every \( p \in \Pi \) being all pairs \( c \rightarrow c' \) where \( c'_p = c_p \cdot y \) for any NFT \( y \), provided the set of NFTs in \( c'_p \) is consistent.

In the following we overload an NT local state \( c_p \) to mean the set of NFTs in the sequence, and an NT configuration \( c \) to denote the union of these local sets.

**Proposition 1** (Consistency and Completeness of NT Configurations). Every configuration of every correct NT run is consistent and complete.

Hence the holder of each object of each NFT in each NT configuration \( c \) is well defined, to be the holder of the maximal-provenance NFT with this object in \( c \). We note the following:

**Observation 1.** NT is asynchronous.

The following causality relation \( > \) on NT configurations is the `declarative' counterpart Lamport's notion of 'happened before' [21].
**Definition 8** (Causality relation $>$. Given an NT configuration $c \in CT$, we define a causality relation $>$ on $c$ to be the minimal strict partial order satisfying:

1. **Same Agent:** NFTs in the local state of an agent are ordered by $>$ according to their sequence order. For any $p \in \Pi$ and any two NFTs $x, x' \in c_p$, if $x$ precedes $x'$ in $c_p$ then $x > x'$.

2. **Same Object:** Any two NFTs with the same object are ordered according to their provenance. Namely, for any two NFTs $x, x' \in c$ with the same object and with provenances $v, v'$ respectively, if $v$ is a prefix of $v'$ then $x > x'$.

For any $x, x' \in c$, we say that $x$ caused $x'$ in $c$ if $x > x'$.

This notion of causality declarative as it relates to a data structure created by the run, rather than to the run itself. It will have a couple of uses below.

### 2.4 Consensus-Free, Equivocation-Resilient Implementation of the NFT Trade Network

**Theorem 1** (Equivocation-resilient Implementation of NT). The blocklace dissemination protocol (Def. 32) can implement correctly the NFT trade network NT with resilience to equivocations by less than one third of the agents.

The proof is in Appendix 7.2. Here we give some background and intuition. A blocklace (Def. 24) is a partially-ordered generalization of the totally-ordered blockchain. Each block in a blocklace includes a set of pointers to previous blocks, not just one as in a blockchain. As such, each blocklace defines a DAG, and a correct execution of a blocklace protocol (see below) results in a DAG-encoding of a partial order that includes the causality relation $>$ (Def. 8), with each NFT-encoding block (except initial ones) having a same-object and a same-agent pointers. A blocklace realization of an NT configuration (Def. 7) is illustrated in Figure 2.

![Fig. 2. An NFT-Consistent Blocklace: Five agents, three objects. Each column represents the blocks of the agent $p_i$ named above. Same-agent pointers are in grey, same-object pointers connect identically-colored blocks. The sequence of NFTs of each agent can be read bottom up ($p_1, \ldots, p_5$). The provenance of each NFT can be read following the same-colored blocks, starting from the block with the object ($a, b, c$).](image-url)
The proof of Theorem 1 in Appendix 7.2 employs instances of two blocklace-based protocols – blocklace consistency (Def. 32) and blocklace dissemination (Def. 33), and follows the arrows in Figure 1. It shows a horizontal mapping – a correct implementation of the NFT trade network by the NFT-consistent blocklace protocol; and a vertical mapping – an equivocation-resilient implementation of the NFT-consistent blocklace protocol by the NFT-dissemination blocklace protocol (with supermajority-based finality). It then argues that the two implementations can be composed, proving the Theorem.

3 LAYER 1: SELF-SOVEREIGN PERSONAL CRYPTOCURRENCIES

3.1 Related Work

The idea of a personal cryptocurrency was floated almost a decade ago [27]. Circles [7] developed a UBI-based personal currency system that is not self-sovereign in several respects: It has an enforced uniform minting rate of personal coins, an enforced 1:1-exchange rate among currencies of people who trust each other, and consensus-based execution by third parties on an Ethereum platform. More recently, a platform for ‘celebrity coins’ named Promify was launched [26]. It enabled artists/celebrities (defined as having at least 10,000 followers on their social media account) to create their personal coins, and sell their fans exclusive or preferred access to content or merchandise owned by the artist, in exchange for said coins. The economic framework and computational support for these coins is provided by the Promify platform, in exchange for transaction fees and appreciation of their underlying coin [26]. We share the basic concept that the primary economic function of a personal currency is to purchase goods and services from the person issuing the currency, and Promify provides a proof of the viability of this concept. Yet, celebrity coins controlled by a third party do not seem a relevant starting point for the design of self-sovereign personal cryptocurrencies for grassroots economy.

The term ‘self-sovereign’ was proposed in the context of personal digital identity [2], with the idea that the person, rather than an organization or the government, should be the sovereign of their digital identity [37]. It is now being developed into a set of standards by the W3C [36]. The notion of self-sovereign personal cryptocurrency extends this idea to the crypto realm.

Economically, self-sovereign personal cryptocurrencies are more similar to national fiat currencies than to mainstream cryptocurrencies, in that their sovereign—the state or the person—has complete control on minting new coins. Thus, the following aphorism holds for both states and people: 'everyone can create money; the problem is to get it accepted’ [23]. Hence, the dynamics of personal cryptocurrencies is dictated first and foremost by their self-sovereignty, with basic concepts of international monetary economics [33] such as foreign debt, trade balance, and currency velocity being directly relevant to the analysis of personal cryptocurrencies, as discussed below (#11).

In that sense a personal cryptocurrency is complementary to the original Ripple idea [13], of money as IOUs. Here, a coin is a unit of credit that can be used either to purchase goods or services from the creditor, or to swap it with another unit of credit owned by the creditor. The effect of redemption claims is similar to coin equality among trusting members of Circles UBI [7, 18], but different in that it is not UBI based. Once mutual credit lines are established via personal cryptocurrency exchanges, the result is effectively identical to Trustlines [17] and credit networks [9, 14], allowing the body of research on liquidity in credit networks to carry over to personal cryptocurrency networks.
3.2 Informal Description

Personal cryptocurrencies are a ‘special case’ (formally, an instance, Def. 19) of the NFT trade network. Personal coins are NFTs with their objects being signed strings of the form $\text{coin}_i$, $i \in \mathbb{N}$. In addition, we introduce a special type of coin transfer, referred to as a redemption claim from agent $p$ to agent $q$, which transfers a $q$-coin from $p$ to $q$ as in an ordinary NFT transfer, requests in return from $q$ an $r$-coin for some $r$ in a provided list of agents $R$ (with the request encoded in the metadata). An agent receiving a redemption claim with a list of agents $R$ must promptly respond to it, positively if it can, by returning an $r$-coin for any $r \in R$, else negatively, returning the coin just received.

The sole characteristic of personal cryptocurrencies, beyond the more abstract NFT trade network, is the notion of coin redemption, which:

(1) **Economics**: Makes personal coins tradeable and of equal value given sufficient liquidity, and

(2) **Computation**: Enables a consensus-free, self-sovereign implementation of fault resilience and transaction finality.

In general, an NFT may have intrinsic value that transcends the individual that has created it, for example if it is a piece of (digital) art, or a deed-of-transfer of a real-world asset (e.g. a car or a house). Hence, the successful transfer of a general NFT should not depend on the person that created it. To achieve that, the realization of an NFT trade network requires an a priori bound on the number or fraction of faulty traders and approvals by non-faulty traders. On the other hand, the personal coin NFTs of different people are the same, except for their signatures, and the value of each is derived solely from the behavioral integrity of its creator. Hence, there is no reason to separate the responsibility for the economic integrity and the computational integrity of a personal cryptocurrency: Both reside with the person. Thus, personal coins can have a simple, open, decentralized implementation that does not depend on other agents being non-faulty. Fault resilience is realized by each agent providing transaction finality to their currency holders, as each person is the sovereign over its currency and the final authority over its holdings.

Moreover, a person has no economic incentive whatsoever to double-spend its own currency, as anyhow it can mint as many of its own coins as it wishes; eventually, double-spending one’s own currency will be exposed and undermine the person’s credibility for no economic gain. Even if agent $p$ is coerced to cover up a double-spend of $p$-coins by another agent $q$, it would be less damaging for $p$, and the same economic gain for $q$, if $p$ mints additional $p$-coins and hand them over to $q$ rather than approve a double-spend of $p$-coins by $q$.

3.3 The Economy of Personal Cryptocurrencies

Before specifying personal cryptocurrencies in Section 3.4 below, we describe the economic implications of their design by illustrating a plausible scenario of its use. We believe, based on the concepts, scenario and use cases presented, that a deployment of personal cryptocurrencies could result in the emergence of a thriving, open, decentralized, liquid and efficient grassroots economy:

(1) **Minting**: Each agent $p$ mints (i.e. creates NFTs with) its own $p$-coins, as many as it pleases.

(2) **Selling Goods and Services**: Each agent $p$ announces ‘off-chain’, say via social media, the prices of goods and services it offers, possibly as $p$-signed NFTs, but in any case denominated in $p$-coins. Examples of NFTs describing goods and services that $p$ may announce and price include ‘babysitting for you this Friday evening’ for 5 $p$-coins, ‘the use of my apartment this weekend’ for 120 $p$-coins, ‘a lift to the beach this morning’ for 3 $p$-coins, ‘use of my car tomorrow’ for 80
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𝑝-coins, or ‘ownership of my car’ for 20,000 𝑝-coins. Possibly (but not necessarily) following some ‘off-chain’ discussion and negotiation, 𝑝 and 𝑞 may agree to the purchase of such a good or service NFT of 𝑝, which represents an ‘off-chain’ commitment by 𝑝, in return for some 𝑝-coins. We note that a trade of personal coins with an ‘off-chain’ asset represented by an NFT cannot be atomic: It requires trust and entails a risk, borne by the payer and/or the payee of the personal coins, depending on the nature of the trade and of the asset traded. Such a risk can be reduced using a multisignature escrow account shared with a trusted third party, as is common and discussed in Section 4, #3b below.

(3) Selling Personal Coins: An agent 𝑝 may also offer its 𝑝-coins for sale, for example to those who wish to procure 𝑝’s goods or services but do not have the needed 𝑝-coins. For example, it may offer 10 𝑝-coins per 1 dollar. As, presumably, the transfer of 1 dollar takes place off-chain, such a trade also cannot be atomic, requiring trust and entailing a risk on the payer and/or the payee, depending whether the off-chain transfer of 1 dollar occurs before or after the on-chain transfer of 10 𝑝-coins. Here, the risk can be mitigated using trusted intermediaries (e.g., exchanges), which could be wholly or partially off-chain, or by using on-chain multisignature accounts with trusted third parties, as discussed in Section 4.

(4) Redemption Claims: An discussed, a redemption claim to 𝑝 must be addressed by 𝑝 promptly. Hence, a 𝑝-coin is like a promissory note by 𝑝, subject to redemption at a 1:1-exchange rate with any other personal coin held by 𝑝 at the time of redemption. Note that redemption could be an unfriendly, even hostile, act. If agent 𝑝 holds a 𝑞-coin and needs an 𝑟-coin held by 𝑞, it can of course choose to redeem its 𝑞-coin from 𝑞 in return for an 𝑟-coin. A more friendly way for 𝑝 to obtain an 𝑟-coin from 𝑞 would be to ask 𝑞 what would it want in return? Perhaps, for 𝑞 to retain its level of credit, it may ask 𝑝 to trade some other 𝑟’-coin that 𝑝 holds. Following off-chain discussion, 𝑝 and 𝑞 could agree to trade an 𝑟’-coin for an 𝑟-coin, with both parties feeling comfortable with the trade.

(5) Forsaking External Capital and Bank Credit: In a healthy economic community, each person’s consumption and production are more-or-less aligned, so that nobody goes insolvent and nobody hogs cash. But, even such a community cannot function without liquidity, lest any temporary decline in personal productivity would result in the person going broke. Liquidity can be provided by external capital (e.g., inheritance, investor) or external credit (e.g., bank). The notion of personal cryptocurrency allows the establishment of mutual lines of credit by trading personal coins, as explained next, forsaking the need for external capital or external credit for an economic community to form and thrive.

(6) Mutual Lines of Credit: Two agents can form mutual lines of credit of size 𝑛, increasing the liquidity of each at the risk of exposure to the other, by trading 𝑛 personal coins of one with the other. Such coins can be redeemed at any time against any personal coins held by the other agent, if solvent, as explained above. A mutual line of credit that results from the exchange of personal coins is essentially the same concept as a trustline in Trustlines Network [17]. Note that a mutual line of credit requires mutual agreement, which can be negotiated off-chain with an asymmetric result: A high-liquidity agent may charge a premium for a mutual line of credit with a credit-less newcomer; a premium that may turn into profit once the newcomer establishes independent liquidity. See discussion of creditworthiness (#11) below.

(7) Arbitrage: Coin redemption has an important ramification: It enables ‘chain-redemptions’, explained next, which in turn cause all high-liquidity personal coins to converge to equal value. Given sufficient liquidity in a community, any gap in value among personal coins could be closed by arbitrage using chain redemptions.

(8) Chain Redemption: Assume that there is 1-liquidity from 𝑝 to 𝑞, defined by the existence of a sequence of agents 𝑝₀, . . . 𝑝ₖ, 𝑝₀ = 𝑝, 𝑝ₖ = 𝑞, where each 𝑝ᵢ, 0 ≤ 𝑖 < 𝑘 holds a 𝑝ᵢ₊₁-coin
Then, if the price of a $q$-coin is higher than the price of a $p_1$-coin, then $p$ can initiate an arbitrage trade, realized via a chain of $k-1$ redemptions, where the $i^{th}$ redemption claim by $p$ to $p_i$, $0 < i < k$, transfers the coin $c_i$ in return for the $p_{i+1}$-coin $c_{i+1}$.

(9) **Chain Payments:** A chain redemption can be used for a chain payment as follows. Assume that $p$ wants to pay a $q$-coin to $q$, but does not hold any. If there is 1-liquidity from $p$ to $q$, then $p$ can trade a coin it holds for a $q$-coin via chain-redemption as described above, then use the $q$-coin it now holds to pay $q$.

(10) **Liquidity:** Credit networks have been studies extensively [9, 14], and have inspired the initial design of some cryptocurrencies [13]. The key instrument of credit networks is chain payments. The fundamental difference between credit networks and personal cryptocurrencies, is manifest when liquidity is constrained: Credit networks assume an outside measure of value (e.g. a fiat currency) in which credit units are given, whereas personal cryptocurrencies, being self-sovereign, are not linked a priori or during their lifetime to any external unit of value. As a result, the value of coins of a liquidity-constrained personal cryptocurrency may naturally decrease. However, as discussed above (#6), mutual lines of credit can be established via the exchange of personal coins, which together with coin redemptions provide for chain payments, just as in credit networks. Hence, much of the body of knowledge regarding liquidity in credit networks can be readily applied to personal cryptocurrencies, with the caveat that lack of liquidity does not only cause chain payment failures, as in credit networks, but also affects the value of the liquidity-constrained personal cryptocurrencies involved.

(11) **Creditworthiness:** Determining the creditworthiness of a person is an art. Still, personal cryptocurrencies offer several objective measures for assessing the creditworthiness of an agent $p$, inspired by the analogue concepts of international monetary economics [33]:

(a) The foreign debt, or more precisely, and conversely, the net interpersonal investment position of $p$: The difference between the number of non-$p$ personal coins held by $p$ and the number of $p$-coins not held by $p$ (namely, in circulation). Larger is better.

(b) The trade balance of $p$: The difference between the number of personal coins transferred to $p$ and the number of personal coins $p$ transferred from $p$ (in any denomination) over a given period. Larger is better.

(c) The velocity of the currency of $p$: The number of $p$-coins transferred divided by the average number of $p$-coins in circulation, during a given period. Larger is considered better.

These three measures provide a rich foundation for objectively assessing the creditworthiness of an agent, e.g., by a bank that assesses the risk of granting a credit line or a loan to an agent, as well as by an agent that negotiates a premium in a mutual credit line with another agent. This direct analogy between personal cryptocurrencies and international monetary economy is a consequence of—and an attestation to—the self-sovereignty of personal cryptocurrencies.

(12) **Insolvency:** An agent $p$ may reach insolvency—not holding any non-$p$ coins of others—by spending all the credit it has received without generating sufficient income in return. The dynamics of insolvency offers a "built-in" natural path for recovering from it: The longer an agent $p$ is insolvent, its creditors, namely the holders of $p$-coins, will have less faith in—or patience for$-p$, and hence may offer to sell $p$-coins at an ever-growing discount. The lower the price of $p$-coins, the easier it would be for $p$ to recover from its insolvency. Since $p$ prices its goods and services in $p$-coins, the lower the value of $p$-coins, the higher $p$ can price its goods and services in $p$-coins and still be competitive. The higher the price of $p$’s goods and services, the faster is $p$’s road to recovery from insolvency. Note that those who would take a hit in such a path to recovery from insolvency are the creditors of $p$ – as presumably they would all end up selling their $p$-coins at a steep discount.
(13) **Bankruptcy and Multiple Identities**: An insolvent person may opt to escape its anguish by abandoning its digital identity and starting afresh, not unlike bankruptcy. However, bankruptcy is a regulated process that aims to mitigate the damage to creditors, whereas shedding one’s digital identity and assuming another is akin to fleeing the country in order to escape creditors. In both cases, the act leaves one’s creditors high and dry. To avert that, a mechanism to deal with multiple identities (Sybils) of the same person is needed. While the notion of sybil identities and sybil-resilience is deep and wide [25, 28, 34, 35], we note that an essential aspect of it, as it relates to personal cryptocurrencies, must be equating all identities of a person for the purpose of redemption: If it is determined (how and by whom – that’s the big question) that $p$ and $q$ are identities of the same person, then any $p$-coin must be redeemed by $q$ and vice versa. This would allow creditors to pursue the assets held by the credit-recipient under any of its multiple identities. We expect the market to punish people who try to escape their creditors by creating multiple identities even before a system of penalties and fines for doing so is established.

(14) **Death**: Any individual faces inevitable death, following which it cannot consume or produce. However, its computational agent should still redeem its coins until either all its outstanding coins are redeemed or it becomes insolvent. For now, we leave open the question of inheritance and the management of the estate, noting that multisig accounts, discussed in Section 4 #5 below, seem well-suited to implement any method devised to address them.

This completes the list of economic functions and use cases of personal cryptocurrencies. It is our hope that this long and rich list would convince the reader of the utility of personal cryptocurrencies, despite their formal and computational simplicity, demonstrated next.

### 3.4 Specification

3.4.1 **Personal Coins and Their Redemption.** We define personal cryptocurrencies, PC, as an instance (Def. 19) of the NFT trade network NT.

**Definition 9** (Coin Object, $p$-Coin, Personal Coin, Personal Coin NFT). Let $C$ denote the set of all strings of the form $\text{coin}_i, i \in \mathbb{N}$, and such a string signed by an agent $p \in \Pi$, resulting in $c \in C \times \Pi$, is referred to as a *coin object*, specifically as a $p$-coin. Given $R \subseteq \Pi$, an $R$-coin is an $r$-coin for some $r \in R$. A *personal coin* is a $\Pi$-coin. We let $C$ denote the set of all personal coins, with $T(C)$ being the set of personal-coin NFTs.

When this does not cause confusion, we may refer to ‘an NFT with a $p$-coin as an object’ simply as a $p$-coin. The key extension of personal cryptocurrencies over general NFTs is the notion of coin redemption. A redemption claim from $p$ to $q$ is an NFT transfer of a $q$-coin from $p$ to $q$, with the metadata including a request to transfer in return an $R$-coin, for some $R \subseteq \Pi$. More formally:

**Definition 10** (Redemption Claim, Response, Settled/Outstanding). A *redemption claim* against an agent $q$ is an NFT with a $q$-coin as an object, metadata $r(R), R \subseteq \Pi$, and form $x = (x' \xrightarrow{p,r(R)} q)$, where $p$ is the holder of $x'$. A *response* to such a redemption claim $x$ is a personal-coin NFT of the form $y = (y' \xrightarrow{q,x} p)$, where:

1. $y'$ is an $R$-coin with $q$ as its holder, in which case the response is **positive**, or
2. $y' = x$, in which case the response is **negative**.

Given a set of personal coin NFTs $X \subset T(C)$, we say that a redemption claim $x \in X$ is *settled* in $X$ if $x$ has a response $y \in X$, else the redemption claim is *outstanding*.

The informal definition of PC is thus: PC is like NT, except that states include personal-coin NFTs and an agent must respond to a redemption claim against it promptly, positively if it can.
This informal definition begs the question of time and causality: What does ‘promptly’ mean? There are at least two different interpretations, affording vastly-different implementations: Newtonian/synchronous and Einsteinian/asynchronous. We present them in turn.

### 3.4.2 Synchronous Personal Cryptocurrencies

The following definition employs the notion of a synchronous distributed multiagent transition system (Def. 3).

**Definition 11** (PCs: Synchronous Personal Cryptocurrencies). A *synchronous personal cryptocurrencies* transition system PCs = (CCs, c0, TCs) is an instance of NT = (CT, c0, TT) where CCs ⊂ CT has all NT configurations over \( \mathcal{T}(C) \), and TCs ⊂ TT consists of all TT transitions over CCs that satisfy the *synchronous redemption condition*: A p-transition from configuration c must be a response to a redemption claim in c against q if c has any. Furthermore, the response to a redemption claim \( x \xrightarrow{q,R,p} q \in c \) must be positive if q is the holder of an R-coin in c.

The multiagent transition system PCs is synchronous, or ‘Newtonian’, as manifest in the synchronous redemption condition, where an agent cannot ignore a redemption claim by another agent once issued. This condition can be satisfied if all acts of all agents are totally-ordered, as in a blockchain. But, the whole point of our effort is to build a consensus-free foundation for grassroots economy, which avoids the overhead of total ordering and the plutocratic ecosystem of mainstream cryptocurrencies. Hence, given this goal, synchronous personal cryptocurrencies are a non-starter.

### 3.4.3 Asynchronous Personal Cryptocurrencies

To specify asynchronous personal cryptocurrencies, we provide a causal (Def. 8)—or Einsteinian ‘light-cone’—interpretation to the notion of ‘promptly’, allowing an agent to ignore outstanding redemption claims as long as it can ‘plausibly deny’ having observed them. In other words, in a correct asynchronous realization of redemption claims an agent may perform any transfer that is not causality related to an outstanding redemption claim against it; q cannot make a transfer if it is causally related to an outstanding redemption claim against q and is not a response to an outstanding redemption claim against q.

With this notion \( \succ \) of causality, we can define the asynchronous personal cryptocurrencies transition systems PC as an instance of NT: First, we define PC configurations to be NT configurations over personal coins that satisfy the asynchronous redemption condition, and then define PC transitions to be NT transitions restricted to PC configurations.

**Definition 12** (PC Configurations). Consider the set of NT configurations \( CT(C) \) over sequences of NFTs with personal coins as objects \( S := \mathcal{T}(C)^* \), and let \( \succ_c \) be the causality relation over each configuration \( c \in CT(C) \). Then a personal-coin NFT \( x \in c \) violates the asynchronous redemption condition if:

1. \( x \) is caused by a redemption claim \( y \in c \), \( y \succ_c x \) that is not resolved in \( c \) and \( x \) not a response to a redemption claim in \( c \).
2. \( x \) has sender \( p \) that responds negatively to a redemption claim that requests an R-coin, even though \( p \) holds an R-coin in the configuration \( c'_p \) that precedes \( x \) in \( c_p \).

\( c \) satisfies the *asynchronous redemption condition* if it does not include a personal-coin NFT that violates it. The personal cryptocurrencies configurations \( CC \) includes all configurations in \( CT(C) \) that satisfy the asynchronous redemption condition.

**Observation 2 (Completeness of CC).** If \( c \leq_N T \ c' \) and \( c', c' \in CC \) then \( c \in CC \)

**Definition 13** (PC: Personal Cryptocurrencies). The *personal cryptocurrencies* transition system PC = (CC, c0, TC) is an instance of NT= (CT, c0, TT) with configurations CC ⊂ CT as specified by Def. 12 and correct transitions TC ⊂ TT consisting of all TT transitions over CC.
Note that in PC an agent \( p \) can make a transfer that is causally-related to an outstanding redemption claim against it, only if the transfer is a response to this or another such outstanding redemption claim. This implies that agent \( p \) can still transfer to others any personal coins that it held prior to any redemption claim issued against \( p \). However, once agents that issued redemption claims against \( p \) interact, eventually all transfers by all agents will be causally related to redemption claims, and \( p \) will face the stark choice of either responding to these claims or be paralyzed.

More importantly, as awareness of \( p \) dodging redemption claims spreads, a ‘bank run’ on \( p \) may ensue, with all holders of \( p \)-coins asking to redeem them. This would definitely result in severe loss of liquidity and decrease in creditworthiness for \( p \), and possibly also in \( p \)’s insolvency. Hence, delaying responses to redemption claims, even if permitted by the protocol, most probably will not be practiced in the normal course of business.

### 3.5 Implementing Self-Sovereign Personal Cryptocurrencies with Leader-Based Finality

As PC is an instance of NT, it can also be provided with a supermajority-based equivocation-resilient implementation by the blocklace protocol stack. However, the design of self-sovereign personal cryptocurrencies was geared from the outset for leader-based finality, where each agent \( p \) finalizes all transfers of \( p \)-coins, as the term ‘self-soverignty’ implies. Hence this is the direction employed by the following implementation and its proof.

**Theorem 2 (Leader-based equivocation-resilient implementation of PC).** The blocklace dissemination protocol \( P_3 \) (Def. 33) can implement the personal cryptocurrencies network PC, with an implementation that is resilient to equivocation by non-leaders; furthermore, if a leader \( r \in R \) does not equivocate in a computation of the implementation, then there is no double-spending of any \( r \)-coin in the computation.

The key idea in employing here leader-based finality is to define \( p \) to be the leader of all blocks that encode transactions in \( p \)-coins. Note that, in general, these will not be \( p \)-blocks, as in a healthy cryptoeconomy other agents will trade in \( p \)-coins; still, such blocks will be finalized by their leader \( p \). The underlying blocklace dissemination protocol ensures that \( p \) will observe, and subsequently acknowledge and approve (if not a double-spending), all blocks encoding \( p \)-coin transactions.

With this in mind, the first part of the theorem states that if all issuers of personal cryptocurrencies do not equivocate, then the behavior of the ‘users’ of their cryptocurrencies cannot result in equivocation. The second part claims that if a leader \( p \) does not equivocate, then the subset of the blocklace under its leadership, namely all transfers of \( p \)-coins, will not include equivocations. In other words, \( p \)-coins cannot be double-spent without the collusion of \( p \), which is precisely the behavior we aimed for in the informal discussions above. The proofs is in Appendix 7.3.

### 4 Adding Consensus-Free Multisignature Accounts

#### 4.1 Informal Description

Personal cryptocurrencies can realize much of the functionality of smart contracts just by extending them with multisignature accounts, or multisig for short. A multisig account does not have its own currency, but may hold personal coins (or NFTs). Any agent may create a multisig account with any set of signatories it is acquainted with, namely knows their public keys, and specify its signature rights, in the form of \( k \)-out-of-\( n \) signatures. Any agent may transfer coins it holds to a multisig account, but the transfer from a multisig account requires signatures as specified during its creation.

We show here that multisig accounts, with all their benefits and use cases, can be incorporated in a consensus-free cryptoeconomic system—both NFTs and personal cryptocurrencies—by an implementation that is resilient to double-spending without ordering. This is possible since a
double-spending of an NFT from a multisig account is resolved no differently from the double-spending by a sole agent – by supermajority approval of the traders for an NFT, and by the agent $p$ in case of a $p$-coin NFT.

Multisig NFTs are richer than simple, single-signature NFTs, in several respects:

1. Conflicting transfers or confirmations by an agent do not necessarily end up in double-spending; at least $2k - n$ signatories have to sign self-contradictory transfers in order to double-spend from the account. Hence we use the term **equivocation**, typically used for miners, rather than double-spending, for signatories signing self-contradictory transfers.

2. A ‘fork’, namely two different multisig NFTs with a shared initial history, may arise even if signatories are not equivocating, simply due to the asynchronous nature of the distributed system. Such forks may be benign, where the signatories agree on the transfer to be issued, but fail to order them, or hostile, when there is disagreement among the signatories, and different signatories issue and sign conflicting transfers.

3. Hostile forks may result in a deadlocked NFT, where neither branch can gather sufficient signatures to confirm a transfer. On the other hand, benign forks can always be resolved by signatories repeating the same act on both forks (this does not constitute an equivocation).

4. Multisig NFT forks, whether benign or hostile, will not result in double-spending the NFT unless $2k - n$ of them equivocate. In other words, conflicting histories of a multisig NFT will not result in conflicting provenances provided less than $2k - n$ of the signatories equivocate.

All these considerations are reflected in the formal specification of multisig NFTs below. While both the NT and PC can be similarly extended with multisignature accounts, we present use cases of multisig accounts for personal cryptocurrencies, as they are more convincing. Following that, we show a formal specification of multisig accounts for NFTs, as they are more abstract and hence simpler. We then discuss double-spending/equivocation-resilient implementations of both.

### 4.2 Use Cases of Personal Cryptocurrencies with Multisignature Accounts

With multisig accounts, the following functions or use cases can be added to personal cryptocurrencies, importantly – without an underlying consensus protocol or a smart-contract engine, thus greatly increasing the utility of personal cryptocurrencies for grassroots economies. Ironically, these functions were described in a document [38] promoting the introduction of multisig accounts in Bitcoin. Today, we would not expect anyone to use a multisig Bitcoin wallet for a petty cash account (#1), as the cost of a single Bitcoin transaction would eat the entire wallet; but, this use case, as well as the other use cases listed next, could very-well be meaningful for personal cryptocurrencies.

1. **1-of-2**: Live-in partners petty cash joint account – either signature is sufficient to spend funds.
2. **2-of-2**:
   a. A couple’s savings account – both signatures are required to spend the funds, preventing one spouse from spending the money without the approval of the other.
   b. A single agent two-factor authentication account – one private key is on the primary computer, the other on the smartphone. Funds cannot be spent without a signature from both devices. Thus, an attacker must gain access to both devices in order to steal the funds.
3. **2-of-3**:
   a. Parents’ savings account for child – the child can spend the money with the approval of either parent, and money cannot be taken away from the child unless both parents agree.
   b. Buyer-seller with trustless escrow – buyer transfers money into the account with the buyer, the seller and a third-party arbitrator as signatories. If the transaction goes smoothly, then both buyer and seller sign the transaction to transfer the money to the seller. If something goes wrong, they can sign a transfer that refunds the buyer. If they cannot agree, they both
appeal to the third-party who will arbitrate and provide a second signature to the party that it deems to deserve it. The arbitrator cannot steal the money as they have only one signature.

(c) A group of three partners with a joint operation – funds cannot be spent unless any two of the partners agrees. Of course, bigger multisig accounts for bigger partnerships are possible.

(4) 3-of-5: Donation account – five trusted people from a project are signatories. Three are required to spend the money but anybody can donate to the project’s account. Reduces risk of embezzlement, hacking/malware, or loss due to a single person losing interest in the project.

(5) 1 or 3-of-4:
(a) Key backup – the primary owner can use the account as desired, but if that owner loses their private keys, they can recover with the help of 3 of the other 4 trusted friends.
(b) Living Will – the primary owner can use the account as desired. In case of the owner’s demise, any three of the trustees can execute the owner’s will.

(6) Any k-of-n: The signatories are elected or appointed trustees of an account of a democratic community that conducts its affairs ‘off-chain’, for example a community credit union. Such a multisig account can be a layer 2 consensus-free stepping-stone to the layer3 consensus-based democratic distributed autonomous organizations (DAOs), described in Section 5, which conduct their affairs democratically ‘on-chain’.

More complex setups are also possible.

4.3 Specification of an NFT Trade Network with Multisignature Accounts

Both the NFT trade network NT and the personal cryptocurrencies network PC can be similarly extended with multisignature accounts. To simplify the presentation, we (i) extend NT, not PC, with multisignature accounts, resulting in NT+; PC with multisignature accounts, PC+, is an instance of NT+; (ii) consider only the k-out-of-n signature scheme; more complicated multisignature schemes can be similarly defined; and (iii) use naive signature aggregation: all signatories independently sign with their private key the same transfer NFT. Subsequent optimizations are possible [22].

Definition 14 (Multisignature Accounts). A multisig account has the form \((p, name, k, R)\), where \(p \in \Pi\) is the account creator, \(name \in S\) is the account name, \(R \subseteq \Pi\) are its signatories, \(k \leq |R|\) is the signature threshold, and \(h = hash((p, name, k, R)) \in \mathcal{H}\) is its identifier. The default account of agent \(p \in \Pi\) is \((p, \Lambda, 1, \{p\})\), namely an unnamed account created by \(p\) with \(p\) as its sole signatory, and we use an agent \(p \in \Pi\) as a synonym to its default account identifier \(h_p := hash((p, \Lambda, 1, \{p\}))\).

The use of the hash value \(h\), rather than the pair \((p, name)\), as account identifier is a defense against \(p\) creating multiple accounts with the same name but with different (even disjoint) signatories. Next, we enhance to notion of NFTs to include multisig accounts as holders.

Definition 15 (Multisig NFTs, Holder, History, Provenance, Equivocation). The construct \(x \xrightarrow{p,s} h\), signed by \(p\), records a transfer by the sender \(p\) of the payload \(x\) and metadata \(s\) to the destination \(h\).

The set of multisig NFTs over \(S\), \(MT(S)\) and their holders are defined inductively:

1. Initial NFT: \(y = s \xrightarrow{p,\perp} p \in MT(S)\) for every agent \(p \in \Pi\) and every object signed by \(p\), \(s \in S \times \Pi\), in which case \(p\) is the holder of \(y\).

2. Transfer NFT: If \(x \in MT(S)\) with holder \(h = hash((p, name, k, R))\), then \(y = x \xrightarrow{r,s} h' \in MT(S)\) for every \(r \in R\), destination account \(h' \in \mathcal{H}\) and string \(s \in S\), with \(x\) referred to as the predecessor of \(y\). If \(x\) has predecessors \(x_1, \ldots, x_j, j \geq 0\), recursively, with \(x_j\) the immediate predecessor of \(x\), all of which are held by \(h\), and \(|\{q : x_{i+1} = x_i \xrightarrow{q,s} h', i \in [j - 1]\}| \geq k - 1\), then the holder of \(y\) is \(h'\), else it is \(h\).
The history of a multisig NFT $x \in NT(S)$ is defined to be the sequence $x_1, x_2, \ldots, x_k$, $k \geq 1$, where $x_1 \in NT(S)$ is an initial multisig NFT, $x_k = x$, and for each $i \in [k-1]$, $x_i$ is the predecessor of $x_{i+1}$, and the provenance of $x$ is the sequence $h_1, h_2, \ldots, h_k$, where $h_i \in H \cup \Pi$ is the holder of $x_i$. Two multisig NFTs $y, y'$ with the same sender $p$ and different destinations form an equivocation if their histories share a prefix $x$, and the holder of $x$ is $p$ in the suffixes $y \setminus x$ and $y' \setminus x$.

Note that the provenance of a multisig NFT may contain repetitions, as the holder does not change until $k$ signatories say so. An equivocation by $p$ constitutes an evidence of an attempt by $p$ to double-spend $x$. Note that $y, y'$ may form a fork or be consistent (one a prefix of the other).

We can now define a multisig NFT trade network. To do so we add to the transfer act $\mapsto$ an act for creating a multisig account $\text{multisig}(h)$, for $h \in H$. If issued by agent $p$ for $h = \text{hash}((p, \text{name}, k, R))$, it creates a multisignature account $(p, \text{name}, k, R)$.

We assume that negotiations between $p$ and the signatories $R$ appointed by $p$ happens off-chain; $p$ can demonstrate that the account $h$ it has created is valid and functional by transferring a coin to $h$ and transferring it back from $h$ to itself, expecting the signatories in $R$ to confirm the transfer.

Clearly, signatories can double-spend. A key insight of our consensus-free multisig accounts is that prevention of double-spending is the same for sole agents and for multisig accounts: By supermajority-approval in the NFT trade network, and by approval of the leader $q$ for the transfer of a $q$-coin from a multisig account in the personal cryptocurrencies network.

**Definition 16 (NT$: Multisig NFT Trade Network).** A multisig NFT trade network is a distributed multiagent transition system $NT^+ = (CT^+, c_0, TT^+)$, with sequences of NFTs $S^+ := MT(S)^*$ over $S$ as local states, configurations $CT^+ := (S^+)\Pi$, initial configurations having empty local states $c_0 := \{\lambda\}\Pi$, and $p$-transitions $TT^+_p \subset (CT^+)^2$ for every $p \in \Pi$, being all pairs $c \rightarrow c'$ satisfying $c'_p = c_p \cdot y$ for any multisig NFT $y$, provided that:

1. **Initial NFT:** $y = s \xrightarrow{p,1} p \in MT(S)$ for every object $s \in S \times \Pi$ signed by $p$, or
2. **Transfer multisig NFT:** $y = x \xrightarrow{p,s} q \in MT(S)$ for every $q \in \Pi$ and string $s$, provided that $y$ does not constitute an equivocation with any NFT in $c_p$.

### 4.4 Equivocation-Resilient Implementations of Multisig NFT Trade Network and Multisig Personal Currencies

The definitions above point to way to enhancing the implementations of NT and PC to support multisignature accounts. The key insight is that while signatories of a multisig account may equivocate, resulting in double-spending of the account, the resolution of double-spending is unchanged from the basic NT and PC protocols: In NT, an NFT transfer from a multisig account, signed by $k$-out-of-$n$ signatories of the account, is finalized if approved by a supermajority of the traders. Hence, as long as less than one third of the traders equivocate there will be no double-spending of an NFT, whether such is attempted by its holding agent or by the signatories of the holding multisig account. In PC, a transfer of a $p$-coin from a multisig account, signed by $k$-out-of-$n$ signatories of the account, is finalized if approved by $p$. As long as $p$ does not equivocate there will be no double-spending of $p$-coins, whether such is attempted by its holding agent or by the signatories of the holding multisig accounts. The rest are details, covered in Appendix 7.4.

### 5 LAYER 3: DIGITAL SOCIAL CONTRACTS FOR DEMOCRATIC DAOS

Many desired economic functions can be realized by smart contracts. In their full generality, the execution of smart contracts requires total ordering of all acts by all parties, as achieved by the blockchain consensus protocol of mainstream cryptocurrencies. Can such functions be realized in a way congenial to grassroots economies?
In mainstream cryptocurrencies, the parties to a smart contract must remunerate the miners who execute it, which undermines capital-free bootstrap. An alternative, termed digital social contracts [5], extends the notion of self-sovereignty from personal cryptocurrencies to smart contracts, in proposing that the vetted parties to a contract also execute it, rather than third-party miners. Consensus-based digital social contracts can be realized by any of the ‘permissioned’ blockchain-consensus protocols, using State-Machine Replication [39] or Byzantine Atomic Broadcast [19]. These could be the basis for truly-autonomous democratic alternatives to ordinary DAOs [12], democratic DAOs in the service of a grassroots economy:

1. **Self-Sovereign Community**: A fundamental capability of a consensus-based digital social contract is vetting the parties to the contract. This can be done by the contract creator, by an initial ‘on-chain’ constitution agreed among the founders [1], or by a sybil-resilient community-building algorithmic protocol [25].

2. **Community Bank**: A community bank, or credit union, may streamline liquidity and simplify payments within a grassroots cryptoeconomic community that employs personal cryptocurrencies. The bank issues its own currency and provides credit to its members at its discretion (e.g., employing the objective measures for creditworthiness, Section 3.3 #11), in return for the members accepting its coins. It allows each member $p$ to deposit $p$-coins and draw bank coins, up to $p$’s credit limit, and in return requires $p$ to accept bank coins as ‘legal tender’, namely as a substitute to $p$-coins in payments to $p$. The deposited personal coins are the bank’s collateral, and any agent $p$ may redeem bank coins, first to its own $p$-coins held by the bank and any remaining balance to other personal coins held by the bank. Note that such a bank does not (necessarily) need initial capital, and can operate at a capital adequacy ratio of 1, provided that its members perish while solvent. Obviously, some people are more credit-worthy than others, hence the bank should use discretion in providing credit. Additional measures by the bank to increase its capital may include charging interest on drawn credit as a function of risk and transaction fees. By introducing a consensus protocol among bank members, or among elected or sampled trustees [5], the bank can be realized as a democratic DAO, owned, operated and democratically-governed by its members.

3. **Outside Support for a Community Currency**: A higher exchange rate of a community currency with other currencies, e.g., a fiat money, will have a positive impact on the economic well-being of community members, as well as support the adoption and use of the community currency within the community. Government or philanthropic support for the exchange rate of a community currency may be achieved with little cash outlays compared to direct payments to citizens. For example, a commitment to buy $n$ community coins a day at market rate up to $\text{US}1/\text{coin}$ will cost the buyer at most $\text{US}n/\text{day}$, but may cost much less (possibly 0), depending on the market exchange rate of the community currency. Also, it will result in the buyer having a tradeable asset that might even appreciate in value if the community is economically healthy. In general, such a commitment will help support a $\text{US}1/\text{coin}$ exchange rate for the community currency, boosting the economic well-being of all productive members of the community.

4. **UBI and Direct Philanthropy**: Another form of government or philanthropic support for a grassroots cryptoeconomic community, still different from and more effective than cash allowances in fiat money, is granting credit in community coins: This supports not only the individual recipient but also the community at large, providing it with internal liquidity. Such support can be realized by the community bank increasing credit to members over time, say by one coin a day for each member, even if the member is not credit worthy (Section 3.3, #11). The bank thus provides a form of UBI for those who need it, and a kind of basic pension fund...
for those who do not need additional credit at present. To offset write-offs associated with members that perish while insolvent, the bank will need donations to maintain its capital adequacy, from inside or outside the community. This is how a community bank can offer an alternative form of community-based UBI [3, 7, 17, 18].

(5) Digital Cooperatives: The ethos of earthly cooperatives include autonomy and democratic conduct. As such, consensus-based digital social contracts offer a natural embodiment for their digital counterparts: members’ owned, operated and governed digital cooperatives. We envision digital cooperatives for social networks that offer a democratic and income-sharing alternative to Facebook, a true sharing-economy drivers-and-passengers-owned cooperative that may offer a beneficial alternative to Uber, and a true sharing-economy owners-and-renters-owned cooperative that would be preferable to Airbnb.

6 CONCLUSIONS
We have presented a long-term vision for an grassroots, inclusive and egalitarian cryptoeconomy, and offered a first step towards its realization: We specified a layered architecture that consists of cryptoeconomic components with a novel consensus-free zone, discussed their uses cases, presented corresponding computational components that can implement them and proved the formal implementations correct and equivocation-resilient. Much remains to be done, including analytic, computational and experimental analysis of the cryptoeconomic components, as well as proof-of-concept implementation and deployment.

ACKNOWLEDGEMENTS
I thank Nimrod Talmon, Gal Shahaf and Ouri Poupko for discussions and feedback. Ehud Shapiro is the Incumbent of The Harry Weinrebe Professorial Chair of Computer Science and Biology at the Weizmann Institute and a visiting scholar at Columbia University.
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7 Appendix

7.1 Appendix for Section 1.2

This appendix section includes mathematical background from [31] needed for the proofs in the following sections of this appendix. It is best to skip it and consult it when referenced in these proofs.

7.1.1 Multiagent transition systems.

Definition 17 (Monotonic & Monotonically-Complete Transition System). Given a partial order $\preceq$ on $S$, a transition system $(S, s_0, T)$ is monotonic with respect to $\preceq$ if $s \rightarrow s' \in T$ implies $s \preceq s'$, and monotonically-complete wrt $\preceq$ if, in addition, $s_0 \rightarrow s \in T$ and $s \preceq s'$ implies that $s \rightarrow s' \in T$.

Definition 18 (Order-Preserving Implementation). Let transition systems $TS = (S, s_0, T)$ and $TS' = (S', s'_0, T')$ be monotonic wrt the partial orders $\preceq$ and $\preceq'$, respectively. Then an implementation $\sigma : S' \rightarrow S$ of TS by TS' is order-preserving wrt $\preceq$ and $\preceq'$ if:

1. **Up condition:** $y_1 \preceq' y_2$ implies that $\sigma(y_1) \preceq \sigma(y_2)$
2. **Down condition:** $s_0 \rightarrow x_1 \in T$, $x_1 \preceq x_2$ implies that there are $y_1, y_2 \in S'$ such that $x_1 = \sigma(y_1)$, $x_2 = \sigma(y_2)$, $s'_0 \rightarrow y_1 \in T'$ and $y_1 \preceq' y_2$.

Lemma 2 (Correct Implementation Among Monotonically-Complete Transition Systems). Assume two transition systems $TS = (S, s_0, T)$ and $TS' = (S', s'_0, T')$, each monotonically-complete wrt the unbounded partial orders $\preceq$ and $\preceq'$, respectively, and an implementation $\sigma : S' \rightarrow S$ among them. If $\sigma$ is order-preserving then it is correct.

Observation 3 (Representative Implementation State). Assume TS and TS' as in Lemma 2 and an implementation $\sigma : S' \rightarrow S$ that satisfies the Up condition of Definition 18. If there is a function $\hat{\sigma} : S \rightarrow S'$ such that $x \simeq \sigma(\hat{\sigma}(x))$ for every $x \in S$, and $x_1 \preceq x_2$ implies that $\hat{\sigma}(x_1) \preceq' \hat{\sigma}(x_2)$, then $\sigma$ also satisfies the Down condition.

Definition 19 (Transition System Instance). Given a transition system $TS = (S, s_0, T)$, then transition system $TS' = (S', s'_0, T')$ is an instance of $TS$ if $s'_0 = s_0$, $S' \subseteq S$ and $T' \subseteq T$.

Lemma 3 (Restricting a Correct Implementation to an Instance). Let $\sigma : C2 \rightarrow S1$ be an order-preserving implementation of $T1 = (S1, s1, T1)$ by $T2 = (C2, s2, T2)$, monotonically-complete respectively with $\preceq$s1 and $\preceq$s2. Let $T1' = (S1', s1, T1')$ be an instance of $T1$ and $T1'' = (C2', s2, T2')$ be the instance of $T2$ defined by $C2' := \{s \in C2 : \sigma(s) \in S1'\}$, with $T2' := T2/C2'$, and assume that both instances are also monotonically-complete wrt $\preceq$s1 and $\preceq$s2, respectively. If $y_1 \rightarrow y_2 \in T1''$ and $\sigma(y_1) \in S1'$ implies that $\sigma(y_2) \in S1'$ then the restriction of $\sigma$ to $C2'$ is a correct implementation of $T1'$ by $T1''$.

Definition 20 (Fault Resilience). Given multiagent transition systems $TS = (C, c_0, T)$, $TS' = (C', c'_0, T')$ over $\Pi$, and a set of faulty transitions $F \subset C^{\sum} \setminus T'$, a correct implementation $\sigma : C' \rightarrow C$ is F-resilient if:

1. **Safety:** For any $TS'$ run $r' = c'_0 \rightarrow c'_1 \rightarrow \ldots \in T' \cup F$, there is a correct $TS$ run $r := \sigma(c'_0) \rightarrow \sigma(c'_1) \rightarrow \ldots$, namely $\sigma(r') \in T$, and
2. **Liveness:** For any $TS'$ run $c'_1 \rightarrow c'_2 \in T' \cup F$ there is a correct $TS'$ computation that does not include $F$-transitions $c'_1 \rightarrow c'_2 \in T'$ that activates $TS$, namely such that $\sigma(c'_1) \neq \sigma(c'_2)$.

Lemma 4 (Composing Fault-Resilient Implementations). Assume transition systems $T1 = (S1, s1, T1)$, $T2 = (S2, s2, T2)$, $T3 = (S3, s3, T3)$, correct implementations $\sigma_{21} : S2 \rightarrow S1$ and $\sigma_{32} : S3 \rightarrow S2$, and let $\sigma_{31} := $ $\sigma_{21} \circ \sigma_{32}$. Then:
(1) If σ₂ is resilient to F₃ ⊆ S³ \ T3, then σ₃₁ is resilient to F₃.
(2) If σ₂ is resilient to F₂ ⊆ S² \ T₂, and F₃ ⊆ S³ \ T₃ satisfies σ₂(F₃) ⊆ F₂, then σ₃₁ is resilient to F₃.
(3) These two types of fault-resilience can be combined for greater resilience: If σ₂₁ is F₂-resilient, σ₂ is F₃-resilient, and σ₂₂(F₃′) ⊆ F₂, then σ₃₁ is resilient to F₃ ∪ F₃′.

Definition 21 (Can Implement). Given multiagent transition systems TS = (C, c₀, T), TS′ = (C′, c₀′, T′) over Π, TS can implement TS correctly if there is an instance TS'' = (S'', c₀′′, T'') of TS′ and a correct implementation σ : S'' → S of TS by TS''; and given F ⊆ C² \ T', it can do so with F-resilience if σ is F-resilient.

Lemma 5 (Fault-Resilience of Order-Preserving Implementations). Assume multiagent transition systems TS = (S, c₀, T) and TS′ = (S', c₀', T'), each monotonically-complete wrt the unbounded partial orders ≤ and ≤', and an order-preserving correct implementation σ : S' → S of TS by TS', and let F' ⊆ S² \ T'. If for every TS' run r' = c₀' → c₁' → c₂' ∈ T' ∪ F',

1. Safety: σ(c'_₀) ≤ σ(c'_₁), and
2. Liveness: there is a correct TS' computation c'_₂ → c'_₃ ∈ T' for which σ(c'_₂) < σ(c'_₃),
then σ is F'-resilient.

7.1.2 The blocklace. In the following, we assume a given set of payloads Α.

Definition 22 (Block). A block b = (p, a, H), referred to as a p-block, p ∈ Π, with a ∈ Α being the payload of b, and H is a (possibly empty) finite set of hash pointers to blocks, namely for each h ∈ H, h = hash(b') for some block b'. Such a hash pointer h is a q-pointer if b' is a q-block. The set H may have at most one q-pointer for any miner q ∈ Π, and if H has no self-edge then b is called initial. The depth of b, depth(b), is the maximal length of any path emanating from b.

Note that hash being cryptographic implies that a circle of pointers cannot be effectively computed.

Definition 23 (Dangling Pointer, Grounded). A hash pointer h = hash(b) for some block b is dangling in B if b ∉ B. A set of blocks B is grounded if no block b ∈ B has a pointer dangling in B.

The non-dangling pointers of a set of blocks B induce finite-degree directed graph (B, E), E ⊆ B×B, with blocks B as vertices and directed edges (b, b') ∈ E if b, b' ∈ B and b includes a hash pointer to b'. We overload B to also mean its induced graph (B, E).

Definition 24 (Blocklace). Let B be the maximal set of blocks over Α and hash for which the induced directed graph (B, E) is acyclic. A blocklace over Α is a set of blocks B ⊆ B.

The two key blocklace notions used in our protocols are acknowledgement and approval, defined next.

Definition 25 (>, Acknowledge). Given a blocklace B ⊆ B, the strict partial order >ₜ is defined by b' >ₜ b if B has a non-empty path of directed edges from b' to b (B is omitted if B = B). Given a blocklace B, b' acknowledges b in B if b' >ₜ b. Miner p acknowledges b via B if there is a p-block b' ∈ B that acknowledges b, and a group of miners Q ⊆ Π acknowledge b via B if for every miner p ∈ Q there is a p-block b' ∈ B that acknowledges b.

Definition 26 (Closure, Tip). The closure of b ∈ B wrt > is the set [b] := {b' ∈ B : b ≥ b'}. The closure of B ⊆ B wrt > is the set [B] := ∪_{b ∈ B}[b]. A block b ∈ B is a tip of B if [b] = [B] ∪ {b}.

Note that a set of blocks is grounded iff it includes its closure (and thus is identical to it):
Observation 4. \( B \subset \mathcal{B} \) is grounded iff \( \mathcal{B} \subseteq B \).

With this background, we can define the basic notion of a pair of blocks forming a double-act (called double-spending if the two payloads are conflicting asset-transfer instructions, or equivocation if the blocks were created by a miner).

**Definition 27 (Double-Act, Double-Actor, Consistent Blocklace).** A pair of \( p \)-blocks \( b \neq b' \in \mathcal{B} \), \( p \in \Pi \), are a double-act of \( p \) if they are not consistent wrt \( \succ \), namely \( b' \not\succ b \) and \( b \not\succ b' \). An agent \( p \) is a double-actor in \( B \) if \( [B] \) has a a double-act of \( p \). A blocklace \( B \) is consistent if it is grounded and does not include double-acts.

Namely, a pair of \( p \)-blocks are a double-act of \( p \) if they do not acknowledge each other in \( B \). In particular, two initial \( p \)-blocks constitute a double-act by \( p \). As \( p \)-blocks are cryptographically signed by \( p \), an double-act of \( p \) is a volitional fault of \( p \). Also, note that an inspection of a blocklace \( B \) can conclude that \( p \) is not a double-actor in \( B \) only if \( B \) is grounded, lest a dangling pointer in \( B \) points to a yet-to-be-uncovered equivocating \( p \)-block.

![Fig. 3. Observing a Double-Acts: Initial blocks are at the bottom. Assume \( b_1, b_2 \) are an equivocation (Def. 27) by the red agent. According to the figure, \( b'' \) approves \( b_2 \) (Def. 28). However, since \( b' \) acknowledges \( b'' \) (Def. 25) it also acknowledges \( b_2 \) and hence does not approve the double-acting \( b_1 \) (nor \( b_2 \)).](image-url)

**Definition 28 (Conflict, Approval).** We assume a given conflict relation \( \gg \) over \( \mathcal{B} \), and say that \( b \) conflicts with \( b' \) if \( b \gg b' \). Given a blocklace \( B \), a block \( b \) approves \( b' \) in \( B \) if \( b \) acknowledges \( b' \) in \( B \) and does not acknowledge any block \( b'' \) in \( B \) that conflicts with \( b' \). An agent \( p \) approves \( b' \) in \( B \) if there is a \( p \)-block \( b \) that approves \( b' \) in \( B \), in which case we also say that \( p \) approves \( b' \) in \( B \) via \( b \). A set of agents \( Q \subseteq P \) approve \( b' \) via \( B' \) in \( B \) if every agent \( p \in Q \) approves \( b' \) in \( B \) via some \( p \)-block \( b \in B', \ B' \subseteq B \).

Here we employ the conflict relation \( \gg \) over \( \mathcal{B} \) only for double-acts, namely conflicting blocks by the same agent. However, other applications of the blocklace may use the notion of conflict and approval more generally, for example conflicting acts by different agents on the same object. Hence we retain the more general notion of conflict and approval.

**Observation 5.** Approval is monotonic wrt \( \supset \).

Namely, if \( b \) or \( p \) approve \( b' \) in \( B \) they also approve \( b' \) in \( B' \supseteq B \).

A key observation is that an agent cannot approve a double-act of another agent or, more generally, a conflict, without being a double-actor itself (Fig. 3):
Observation 6. [Approving a Conflict] If agent $p \in \Pi$ approves a conflict $b_1 > b_2$ in a blocklace $B \subseteq B$, then $p$ is a double-actor in $B$.

We note that once an agent $p$ observes a double-act $b, b'$ by another agent $q$, it would not approve any subsequent block $b''$ by $q$ that acknowledges say $b$ but not $b'$, as such a block would constitute a double-act with the block $b'$.

Lemma 6 (No Supermajority Approval for Conflict). If there are at most $f$ double-actors in a blocklace $B \subseteq B$ with a conflict $b > b' \in B$, then not both $b, b'$ have supermajority approval in $B$.

Definition 29 (Supermajority-Based Finality). A block $b \in B$ is final by supermajority in $B$ if the set of blocks that approve $b$ in $B$ is a supermajority. The final subset of $B$ is the set $\phi(B) := \{ b \in B : b$ is final in $B\}$.

From the definition of finality and Lemma 6 we conclude:

Observation 7 (Finality Excludes Conflict). $\phi(B)$ is consistent.

And conclude that finality by supermajority is indeed final:

Observation 8 (Finality is Monotonic). Let $B \subseteq B'$ be two blocklaces. Then $\phi(B) \subseteq \phi(B')$.

For some peer-to-peer applications it is more appropriate to assign the responsibility for finality of specific blocks to specific agents. For those, the notion of leader-based finality is useful:

Definition 30 (Leader-Based Finality). Let $R \subseteq \Pi$ be a nonempty set of agents, referred to as leaders, and assume a partition of $B$ to the leaders, $B = \bigcup_{r \in R} B_r$ (the partition need not be congruent with the creators of the blocks). Let $B$ be a blocklace. Then a block $b \in B$ is final by its leader $r \in R$ in $B$ if $b \in B_r$ and $B$ has an $r$-block that approves $b$. The $r$-final subset of $B$ is the set $\phi^r(B) := \{ b \in B' : b$ is final by leader $r \in R$ in $B\}$, and $\phi^R := \bigcup_{r \in R} \phi^r(B)$.

The two observations above regarding finality by supermajority hold also for finality by leaders, but in a more refined way. From the definition of finality and Lemma 6 we conclude:

Observation 9 (Leader-Based Finality Excludes Conflict). If a leader $r \in R$ does not equivocate in $B$ then $\phi^r(B)$ is consistent.

And conclude that finality by a correct leader is indeed final:

Observation 10 (Finality by a Correct Leader is Monotonic). Let $B \subseteq B'$ be two blocklaces in which $r \in R$ does not equivocate. Then $\phi^r(B) \subseteq \phi^r(B')$.

7.1.3 Blocklace protocol stack.

Definition 31 (Blocklace Configuration). A blocklace configuration over $\Pi$ and $\mathcal{A}$ is a configuration over $\Pi$ and $B(\mathcal{A})$. The initial blocklace configuration is empty, $c_0 := \{0\}^\Pi$.

We may overload a blocklace configuration $c$ to also denote the union of its local blocklaces $\bigcup_{p \in P} c_p$; in particular, we say that $c$ is consistent if the union of its local blocklaces is.

Definition 32 (P2: Blocklace Consistency Protocol). The P2 blocklace consistency protocol is a transition system $P2 = (C2, c_0, T2)$, with states $C2$ being all consistent blocklace configurations over $\Pi$ and $\mathcal{A}$, and transitions $T2$ being all $p$-transitions $c \rightarrow c'$, $p \in \Pi$, where $c'_p := c_p \cup \{b\}$ with a $p$-block $b \in B(\mathcal{A})$ such that:

1. **Grounded**: $[b] \subseteq c', and
2. **No double-act**: $b$ acknowledges every $p$-block in $c_p$. 

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The definition ensures that:

**Observation 11.** A correct \( P2 \) run only produces consistent blocklace configurations.

**Observation 12.** Define \( c \preceq_c c' \) if \( c, c' \in C2 \) are consistent and \( c \subseteq c' \). Then \( P2 \) is monotonically-complete wrt \( \preceq \) (Definition 17).

**Definition 33** (P3: Blocklace Dissemination Protocol). The P3 blocklace dissemination protocol is a transition system \( P3 = (C3, c0, T3) \), with states \( C3 \) all blocklace configurations over \( \Pi \) and \( \mathcal{A} \), transitions \( T3 \) all \( p \)-transitions, \( p \in \Pi \), where \( c \rightarrow c', c'_p := c_p \cup \{ b \} \), and:

1. **\( p \)-Acts:** \( b \in B(\mathcal{A}) \) is a \( p \)-block such that \( \{ b \} = c_p \cup \{ b \} \), or
2. **\( p \)-Delivers:** \( b \in c_q \setminus c_p, \{ b \} \subseteq c_p \cup \{ b \} \).

We also consider specifically the following faulty transitions:

3. **\( p \)-Equivocates:** \( b \in B(\mathcal{A}) \) is a \( p \)-block such that \( \{ b \} \subseteq c_p \cup \{ b \} \)

and refer to an agent taking such a transition as **equivocator**.

**Theorem 3.** Protocol \( P3 \) can implement \( P2 \) correctly with resilience to less than \( f \) equivocators.

### 7.2 Appendix for Section 2

**Proof of Proposition 1.** Let \( r = c_0 \rightarrow c_1 \rightarrow \ldots \) be a correct \( NT \) run. We prove by induction on the index of the configurations of \( r \). The initial configuration \( c_0 \) is empty and therefore vacuously consistent and complete. Assume that \( c \in r \) is consistent and complete, and consider a \( p \)-transition \( c \rightarrow c' \in TT \). \( c'_p = c_p \cdot y, y = (x \xrightarrow{p,s} q), q \in \Pi \). The new configuration \( c' \) is consistent since \( c \) is consistent by the inductive assumption, and the requirement that the set of NFTs of \( p \) following a \( p \)-transition is consistent. \( c' \) is complete since \( c \) is complete by the inductive assumption and \( y \), the only new NFT in \( c' \), extends the provenance of \( x \) by one agent, \( q \).

**Proof of Observation 1.** \( NT \) satisfies the asynchrony condition as the requirements on a \( p \)-transition refer solely to \( p \)'s local state.

We recall the blocklace consistency protocol \( P2 \) and the blocklace dissemination protocol \( P3 \) from [31]. We show a correct implementation of \( NT \) by \( P2 \) (Theorem 4), then compose it with the equivocation-resilient implementation of \( P2 \) by \( P3 \) from [31], resulting in an equivocation-resilient implementation of \( NT \) by \( P3 \) (Theorem 1). The proof follows the framework established in [31] for proving the correctness of implementations among distributed multiagent transition systems.

**Proof Outline for Theorem 4.** We define an instance of \( P2 \), termed \( P2T \) (Def. 37), with a partial order \( \leq_{2T} \), and prove that \( P2T \) is monotonically-complete wrt \( \leq_{2T} \) (Prop. 4). We define an implementation \( \sigma_T \) of \( NT \) by \( P2T \) (Def. 38), and prove that \( \sigma_T \) is order-preserving wrt \( \leq_{2T} \) and \( \leq_{NT} \) (Prop. 5). As \( NT \) is monotonically-complete wrt \( \leq_{NT} \) (Ob. 2), the conditions for Lemma 2 of [31] are fulfilled, thus completing the proof.

**Definition 34** (\( \leq_{NT} \)). The partial order \( \leq_{NT} \) is defined by \( c \preceq_{NT} c' \) if \( c, c' \in CT \) are consistent and complete, and \( c_p \) is a prefix of \( c'_p \), for every \( p \in \Pi \).

**Proposition 2.** \( NT \) is monotonically-complete with respect to \( \leq_{NT} \).

**Proof.** \( NT \) is monotonic wrt \( \leq_{NT} \) since according to Observation 1 all configurations in every run of \( NT \) are consistent and complete, and the result of a \( p \)-transition \( c \rightarrow c' \in TT \) is that \( c_p \) is a prefix of \( c'_p \), with other local states intact. To prove that \( NT \) is monotonically-complete, consider two NT configurations \( c, c' \in CT, c \preceq_{NT} c' \), with a correct computation \( r = c_0 \rightarrow c \in TT \). By Proposition 1 \( c, c' \) are consistent and complete. Let \( \succ \) be the causality relation over \( c' \), and let \( x_1, \ldots, x_n \) be an
order of $c' \setminus c$, with $p_i$ being the sender of NFT $x_i$. Let $c = c_1 \rightarrow c_2 \rightarrow \ldots \rightarrow c_k = c'$ be an NT computation, where $c_1 \rightarrow c_j' \in TC$ being the $p_i$-transition $c'_{p_i} = c_{p_i} \cup \{x_i\}$. Every configuration in the computation satisfies the consistency and completeness by the ordering of the NFTs according to the causality relation. Hence the computation is a correct $TT$ computation from $c$ to $c'$, establishing that NT is monotonically-complete.

We assume a collision-free cryptographic hash function $hash : S \mapsto H$, with a range $H$ disjoint from $\Pi$. When the hash function is applied to a finite mathematical structure, the intention is that it is applied to a canonical string-representation of that structure.

**Definition 35 (NFT Blocklace, Consistent).** The set $\mathcal{T}B \subset \mathcal{B}$ of NFT blocks is defined inductively to be the minimal set that includes all blocks $b = (p, a, H)$ with payload $a = (x \xrightarrow{p,s} q)$, $p, q \in \Pi$ and:

1. **Initial NFT Block:** $p = q$, $s = \bot$, and $x \in S$ is a string, in which case $b$ is an initial NFT $b$-block, or
2. **Transfer NFT:** $s \in S$, $x = hash(b')$ referred to as a same-object hash pointer, where $b'$ is an NFT block with payload $(x' \xrightarrow{p',s'} p)$ and $b$ is a transfer NFT $p$-block.

The set $\mathcal{T}A$ of NFT block payloads is $\mathcal{T}A := \{a : (p, a, H) \in \mathcal{T}B\}$. An NFT blocklace $B$ is a blocklace of NFT blocks $B \subseteq \mathcal{B}T$. A provenance path in an NFT blocklace $B$ is a path of same-object pointers. An NFT blocklace $B$ is $NFT$-consistent if:

1. The blocklace $B$ is consistent (Def. 27), and hence grounded, and
2. $B$ has at most one initial NFT $p$-block with object $x$, for any object $x \in S$ and agent $p \in \Pi$, and
3. $B$ has at most one transfer NFT $p$-block with same-object pointer $h$, for any hash pointer $h$ and agent $p \in \Pi$.

Note that for a blocklace to be consistent, all $p$-blocks by the same agent $p$, except for the initial $p$-block, must have a same-agent pointer in addition to a same-object pointer. Also, by Def. 36, each block that does not encode an initial NFT must have a same-object pointer to the block encoding the preceding NFT. This two-pointer blocklace structure encodes precisely the ‘same agent/same object’ causality relation $\rightarrow$ (Def. 8).

**Definition 36 (Blocklace Encoding of NFTs).** Given an NFT block $b = (p, a, H)$ with payload $a = (x \xrightarrow{p,s} q)$, the NFT $y$ encoded by $b$ is defined recursively to be:

1. **Initial NFT:** $y = a$ if $p = q$, $s = \bot$ and $x \in S$, or
2. **Transfer NFT:** $y = (y' \xrightarrow{p',s'} q)$, if $x = hash(b')$ is a same-object pointer and $y'$ is the NFT encoded by $b'$.

Recall that a blocklace is consistent if it does not include a double-act. The added requirements of a single initial $p$-block for every object and a single transfer $p$-block for every same-object hash pointer ensure that the provenance paths of all NFTs encoding the same $p$-signed object end in the same initial $p$-block, thus preventing the double-spending of an initial NFT by $p$.

**Observation 13 (Complete NFT Blocklace).** The set of NFTs encoded by an NFT-consistent blocklace is complete and consistent.

**Proof.** Given a NFT-consistent blocklace $B$ and a block $b \in B$ that encodes NFT $x = (x' \xrightarrow{q,s} p)$, then by Def. 36, the block pointed to by $b$ encodes $x'$. By assumption that $B$ is consistent it is also grounded, hence $b' \in B$. It does not include NFTs with inconsistent provenances by the definition of NFT-consistent blocklace.
Next we define P2T, an instance of the blocklace consistency protocol P2 = (C2, c0, T2) [31].

**Definition 37** (P2T: NFT Blocklace Consistency Protocol). The P2T NFT blocklace consistency protocol P2T = (C2T, c0, T2T) is an instance of P2, with configurations C2T ⊂ C2 being all NFT-consistent blocklace configurations over Π and T.A, and transitions T2T ⊂ T2 being all p-transitions c → c′ ∈ T2p, p ∈ Π, where c′ := cP ∪ {b} with an NFT p-block b = (p, y, H) ∈ B(T.A), provided that c′ is an NFT-consistent blocklace.

Figure 2 presents a possible P2T configuration c, where each NFT transfer x ∈ cP by agent p is represented by a block b with two special pointers: A same-object pointer to the block b′ representing the predecessor NFT of x, unless b is an initial NFT in which case the block includes the object, and a same-agent pointer to the block b′′ representing the NFT transfer of p that precedes x in cP, unless b is the initial block of p.

**Proposition 3.** Every configuration of every correct run of P2T run is an NFT-consistent blocklace.

**Proof.** By induction on configurations index of the run, similar to the proof of Proposition 1. □

We define a mapping σT from C2T configurations of NFT blocklaces to NT configurations:

**Definition 38** (σT : C2T ↦ CT). Let c′ ∈ C2T be a P2T configuration. Then c := σT(c′) is defined as follows. For each p ∈ Π, cP has the NFTs represented by blocks in c′P (which are all p-blocks by definition of P2T), ordered according to their same-agent pointers. Since the blocklace c′ is consistent, the ordering is total.

**Definition 39** (≤2T). The partial order ≤2T is defined by c ≤2T c′ if c, c′ ∈ C2T and σT(c) ≤T σT(c′).

Recall Def. 17.

**Proposition 4.** P2T is monotonically-complete with respect to ≤2T.

**Proof.** Assume a P2T p-transition c → c′ ∈ T2T that adds the NFT p-block b to cP. Then cP ≤ cP ∪ {b} = c′P, and ∀q ≠ p : cP = c′P, hence c ≤2T c′. Consider a P2T computation c0 → c ∈ T2T and some consistent c′ for which c ≤2T c′. Since c is NFT-consistent, there is an ordering b1, b2, . . . bk of c′ \ c satisfying b1 > b{i+1}, i ∈ [k − 1]. Let c1′ := c, and for every i ∈ [k − 1] and p-block bi, define the p-transition c′i → c′i+1 to be the p-transition c′P → c′P ∪ {bi} = c′P. Consider some i ∈ [k − 1]. If bi is an initial NFT p′-block, then by assumption of NFT-consistency of c′, it is the first such p′-block and therefore the conditions for Initial NFT transition c′i → c′i+1 are fulfilled. If bi is a transfer NFT block, then by Observation 13 the set of NFTs encoded by c′ is complete, and therefore the predecessor b′ of bi is encoded in c′, and either b′ ∈ c or b′ = b j for some j < i, since b′ > bi, and therefore the conditions for Transfer NFT transition c′i → c′i+1 are fulfilled. In either case c′i → c′i+1 ∈ T2T. Hence c = c1 → c2 . . . → ck = c′ ∈ T2T, concluding that P2T is monotonically-complete wrt ≤2T. □

**Proposition 5.** σT order-preserving wrt ≤2T and ≤NT.

According to Observation 3, identifying an inverse δT to σT can greatly simplify the proof.

**Definition 40** (δT : CT ↦ C2T). Given a configuration c ∈ CT and a computation r = c0 → . . . → c ∈ TT, δT(c) is defined inductively as follows: δT(c0) := 0T. If c → c′ ∈ r is a p-transition c′ := c′P · y then δT(c′) := δT(c) ∪ {b}, where b is the NFT block encoding of y, with a same-agent hash pointer pointing to the previous p-block in c, if any, and a same-object hash pointer pointing to a block b′ ∈ δT(c) encoding the predecessor of y, which exists since c is complete.
Observation 14. Given $c, c' \in CT$ such that $c_0 \rightarrow^* c$ and $c \preceq_T c'$, then $\sigma(\hat{\sigma}_T(c)) = c$ and $\hat{\sigma}_T(c) \preceq_{2T} \hat{\sigma}_T(c')$.

Proof of Proposition 5. According to Observation 14, the function $\hat{\sigma}(T)$ satisfied the conditions of Lemma 3.

This completes the proof of Theorem 4.

We are now ready to add equivocation-resilience.

Theorem 4 (P3 implements NT with equivocation-resilience). The blocklace dissemination protocol P3 can implement correctly the NFT trade network NT with resilience to equivocations by less than one third of the agents.

Proof Outline. Protocol P3 can implement protocol P2 with equivocation-resilience (Theorem 5 in [31]). Protocol P2 can implement NT (Theorem 4). As P2 implements NT via an instance P2T, we define the corresponding instance P3T of P3, and verify (Lemma 7) that the implementation $\sigma_3 : P3 \mapsto P2$, restricted to P2T, satisfies the conditions for Lemma 3, concluding that $\sigma_3$ restricted to P2T implements P2C with equivocation resilience. Lemma 4 specifies conditions under which the composition of these two implementations is an equivocation-resilient implementation of NT by P2T. We verify (Lemma 8) that these conditions hold, completing the proof.

Lemma 7 (P3T implements P2T). The implementation $\sigma_3 : P3 \mapsto P2$, restricted to P3T, implements P2T.

Lemma 8 ($\sigma_T \circ \sigma_3$ is Equivocation Resilient). The transition systems NT, P2T, P3T, and the implementations among them $\sigma_T$ and $\sigma_3$, satisfy the conditions of Lemma 4 for resilience of $\sigma_T \circ \sigma_3$ to equivocations by less than a third of the agents.

7.3 Appendix for Section 3

Proof of Observation 2. If $c' \in CC$ then it satisfies the asynchronous redemption condition, namely it does not include NFTs that violate the asynchronous redemption condition, which implies that the NFTs in $c$, being a subset of those in $c'$, to not include such violations, namely $c \in CC$.

As PC is an instance of NT, it can also be implemented by P2T under certain conditions stated and verified next.

Theorem 5 (P2T can implement PC). The blocklace consistency protocol P2 can implement correctly the personal cryptocurrencies network PC.

Proof Outline. As PC is an instance of NT, and P2T can implement NT via $\sigma_T$, Lemma 3 may be applied. Proposition 6 verifies that the conditions of the Lemma hold. Hence P2T can implement PC.

Observation 15. PC is monotonically-complete with respect to $\preceq_{NT}$.

Proof. As an instance of NT, PC computations are also NT computations, and hence the monotonicity of NT (Ob. 2) implies the monotonicity of PC.

To prove that PC is monotonically-complete, consider two PC configurations $c, c' \in CC, c \preceq_{NT} c'$, with a correct computation $r = c_0 \rightarrow^* c \in TC$. Due to the monotonic-completeness of NT, there is a computation $c \rightarrow^* c' \in TT$. Due to Ob. 2, all its configurations are in CC, and hence the computation is also in TC.

Observation 16. Let $P2T' = (C2T', c_0, T2T')$ be the instance of P2T defined by $C2T' := \{ c \in P2T : \sigma_T(c) \in CC \}$, with $T2T' := T2T/C2T'. Then P2T' is monotonically-complete wrt $\preceq_{2T}$.
Proof. As an instance of P2T, P2T′ computations are also P2T computations, and hence the monotonicity of P2T (Ob. 4) implies the monotonicity of P2T.

To prove that P2T′ is monotonically-complete, consider two PC configurations \( c, c' \in C2T, c \leq_{2T} c' \), with a correct computation \( r = c0 \rightarrow c \in T2T \). Due to the monotonic-completeness of P2T, there is a computation \( r = c^* \rightarrow c' \in T2T \). Due by definition of P2T′, \( \sigma_T(c), \sigma_T(c') \in CC \); by Observation 2, all the configurations of \( \sigma_T(r) \) are in \( CC \), hence by definition of P2T′, all configurations of \( r \) are in \( C2T' \), which makes it monotonically-complete wrt \( \leq_{2T} \). □

Proposition 6. Lemma 3 can be applied to show that P2T can implementation PC.

Proof. It was shown above that \( \sigma_T : C2T \rightarrow CT \) is an order-preserving implementation of \( NT = (CT, c0, TT) \) by P2T= (C2T, c0, P2T), which are monotonically-complete respectively with \( \leq_T \) and \( \leq_{2T} \), respectively, and that PC= (CC, s0, TC) is an instance of NT. Let P2T′ = (C2T′, c0, T2T′) be the instance of P2T defined by \( C2T' := \{ c \in P2T : \sigma_T(c) \in CC \} \), with T2T′ := T2T/C2T′. We have to show that:

1. Both instances PC and P2T′ are also monotonically-complete wrt \( \leq_T \) and \( \leq_{2T} \), respectively. The first claim is argued in Ob. 15, the second in Ob. 16.

2. \( y_1 \rightarrow y_2 \in T2T' \& \sigma_T(y_1) \in CC \) implies that \( \sigma_T(y_2) \in CC \). As T2T′ was defined to be T2T restricted to states that are mapped by \( \sigma_T \) to CC, the requirement holds.

These are sufficient conditions for Lemma 3 to apply and to conclude that the restriction of \( \sigma_T \) to \( C2T' \) is a correct implementation of PC by P2T′. □

The next step is identify the instance of P3T that is mapped to P2T′, and then compose the mapping from it to P2T′ with \( \sigma_T \). However, for PC we wish a different notion of fault resilience: Not supermajority-based, but leader-based, where each agent \( p \) is the leader of the NFTs of \( p \)-objects. We employ the leader-based finality mapping \( \sigma_R^3 \) of P3 to P2.

Proof Outline of Theorem 2. The proof is similar to the proof of Theorem 1, with one change: The use of a leader-based finality mapping \( \sigma_R^3 \), that maps each NFT block with \( r \)-coin to the leader \( r \in R \), instead of the supermajority-based finality mapping \( \sigma_3 \), with Proposition providing the needed conclusion. □

7.4 Appendix for Section 4

Theorem 6 (Equivocation-resilient Implementation of NT+). The blocklace dissemination protocol (Def. 32) can implement correctly the NFT trade network with multisig accounts NT+ with resilience to equivocations by less than one third of the agents.

Proof Outline. The proof is essentially the same as the proof of Theorem 1. The key insight is that the added complexity of deciding whether a transfer is confirmed (with \( k \)-out-of-\( n \) signatures) is orthogonal to equivocation exclusion, which is the same in both implementations and both theorems. □