Bjorken unpolarized and polarized sum rules: 
comparative analysis of large-$N_F$ expansions

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ABSTRACT

Analytical all-orders results are presented for the one-renormalon-chain contributions to
the Bjorken unpolarized sum rule for the $F_1$ structure function of $\nu N$ deep-inelastic scattering in the large-$N_F$ limit. The feasibility of estimating higher order perturbative QCD corrections, by the process of naive nonabelianization (NNA), is studied, in anticipation of measurement of this sum rule at a Neutrino Factory. A comparison is made with similar estimates obtained for the Bjorken polarized sum rule. Application of the NNA procedure to correlators of quark vector and scalar currents, in the euclidean region, is compared with recent analytical results for the $O(\alpha_s^4 N_F^2)$ terms.

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1 Introduction

The differential cross-section for neutrino (anti-neutrino) nucleon deep-inelastic scattering (DIS) is parametrized by three structure functions in the familiar formula

\[
\frac{d^2 \sigma}{dx dy} = \frac{G_F^2 M_N E_\nu}{\pi(1 + Q^2/M_N^2)^2} \left[ y^2 x F_1 + \left(1 - y - \frac{M_N xy}{2E_\mu}\right) F_2 \pm \left(y - \frac{1}{2}y^2\right) x F_3 \right]
\]

with \(0 \leq x \leq 1, y = E_{\text{had}}/E_\nu, 0 \leq y \leq 1/(1 + xM_N/2E_\nu),\) and a \(+(-)\) sign applying to neutrino (anti-neutrino) beams.

Analyses of data from previous \(\nu N\) DIS experiments concentrated on the extraction of \(F_2\) and \(xF_3\) (see e.g. the works of Ref.\[1\]) and the ratio \(R = (1 + 4M_N^2 x/Q^2) F_2/(2xF_1)\). (For a recent extraction in \(\nu N\) DIS, see Ref.\[2\].) However, there are already at least two attempts to obtain direct information about \(F_1\) \[3\]. Such efforts are interesting from several points of view. They allow comparison with models for the \(O(1/Q^2)\) contributions to both \(F_1\) and \(xF_3\), obtained within the framework of an infrared renormalon approach, which predicts similar \(x\)-dependence of these two power corrections \[4\]. A second point of interest is comparison with QCD sum rule estimates \[5\] for the twist-4 matrix element introduced in Ref.\[6\] which gives an \(O(1/Q^2)\) correction to the unpolarized Bjorken sum rule (Bjunp SR)

\[
C_{\text{Bjunp}}(Q^2) = \int_0^1 dx \left[ F_1^{\nu p}(x, Q^2) - F_1^{\nu n}(x, Q^2) \right]
\]

with a corresponding twist-4 term for the Bjorken polarized sum rule estimated in Ref.\[7\].

In this note, we present analytical all-orders results for the one-renormalon-chain contributions to \(C_{\text{Bjunp}}\), obtained within the large-\(N_F\) expansion, and then apply the simplistic procedure of naive nonabelianization (NNA), proposed in Ref.\[8\]. A comparison is made with the polarized Bjorken sum rule, and with the application of NNA to vector and scalar correlators. We hope that these considerations will encourage theoretical study of the Bjorken unpolarized sum rule, whose experimental study may be enabled by \(\nu N\) DIS data from a future Neutrino Factory, operating in a region of medium \(Q^2\), with \(N_F = 3,4\) active flavours. (For active consideration of such a facility, see Ref.\[9\].)

2 Large-\(N_F\) series

Here we study radiative corrections to Bjunp SR induced in the large-\(N_F\) limit. Using methods developed for the calculations in Refs.\[10, 11, 12\], the large-\(N_F\) limit of the perturbative part of Eq.(2) was obtained with the result:

\[
C_{\text{Bjunp}} = 1 + \frac{C_F}{T_F N_F} \sum_{n=1}^\infty U_n \left( T_F N_F \pi_s \right)^n + O(1/N_F^2)
\]

\[
U_n = \lim_{\delta \to 0} \left( - \frac{4}{3} \frac{d}{d\delta} \right)^{n-1} U(\delta)
\]

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where $\bar{\alpha}_s = \alpha_s(\mu^2 = Q^2)/4\pi$ is the coupling in the $\overline{\text{MS}}$-scheme, $C_F = 4/3$, $T_F = 1/2$ and

$$U(\delta) = -\frac{2 \exp(5\delta/3)}{(1 - \delta)(1 - \delta^2/4)}.$$ (4)

This expression produces the following large-$N_F$ $\overline{\text{MS}}$-scheme series for the Bjump SR

$$\sum_{n<10} U_n x^n = -2x + \frac{64}{9} x^2 - \frac{2480}{81} x^3 + \frac{113920}{729} x^4 - \frac{6195968}{6561} x^5 + \frac{395898880}{59049} x^6$$
$$-\frac{29418752000}{531441} x^7 + \frac{2510236057600}{4782969} x^8 - \frac{242876551331840}{43046721} x^9.$$ (5)

It should be noted that in this new result the order $x^2$ and $x^3$-terms are in agreement with the corresponding parts of the total expression for the $\bar{\alpha}_s^2$-correction to the Bjump SR [13] and its $\bar{\alpha}_s^3$-term, calculated in the $\overline{\text{MS}}$-scheme in Ref.[14]. Notice also that the series of Eq.(5) has sign-alternating behaviour with factorially increasing coefficients. This pattern is explained by the dominant role of the renormalon at $\delta = 1$, which is generated by a single chain of quark-loop insertions into the corresponding one-loop QCD diagrams for this characteristic of $\nu N$ DIS.

It is interesting to compare the above expressions to the analogous ones, obtained in Ref. [11], for the coefficient function $C_{\text{Bjp}}(Q^2)$ in the Bjorken polarized sum rule (Bjp SR)

$$\int_0^1 dx \left[ g_1^p(x, Q^2) - g_1^n(x, Q^2) \right] = \frac{1}{3} \left| \frac{g_A}{g_V} \right| C_{\text{Bjp}}(Q^2).$$ (6)

The large $N_F$-expression for the Bjp SR can be obtained from the following equation [11]

$$C_{\text{Bjp}} = 1 + \frac{C_F}{T_F N_F} \sum_{n=1}^{\infty} K_n \left( T_F N_F \bar{\alpha}_s \right)^n + O(1/N_F^2)$$

$$K_n = \lim_{\delta \to 0} \left( -\frac{4}{3} \frac{d}{d\delta} \right)^{n-1} K(\delta)$$ (7)

where

$$K(\delta) = \left( \frac{3 + \delta}{2(1 + \delta)} \right) U(\delta) = \frac{(3 + \delta) \exp(5\delta/3)}{(1 - \delta^2)(1 - \delta^2/4)}.$$ (8)

The all-orders large-$N_F$ result of Eq.(8) was given in Ref.[11]. The expansion up to order $O(\alpha_s^3 N_F^8)$ was also given in Ref.[11] in the $\overline{\text{MS}}$-scheme and reads:

$$\sum_{n<10} K_n x^n = -3x + 8x^2 - \frac{920}{27} x^3 + \frac{38720}{243} x^4 - \frac{238976}{243} x^5 + \frac{130862080}{19683} x^6$$
$$-\frac{10038092800}{177147} x^7 + \frac{274593587200}{531441} x^8 - \frac{82519099473920}{14348907} x^9.$$ (9)

As in the case of Eq.(5), this series is dominated by the renormalon at $\delta = 1$, which has the same reside in each sum rule. (Note that the renormalon in $K(\delta)$ at $\delta = -1$ is suppressed by a factor $\frac{1}{2} \exp(-10/3) = 0.018$, relative to the dominant renormalon at $\delta = 1$.)

In the next section we apply naive nonabelianization to the large-$N_F$ series for both unpolarized and polarized Bjorken sum rules, in an attempt to estimate higher-order perturbative coefficients.
3 Application of NNA to euclidean sum rules

By naive nonabelianization (NNA) we simply mean the substitution $N_F \rightarrow -\frac{3}{2} \beta_0 = N_F - \frac{3}{2} \beta_0$ in the leading terms of the large-$N_F$ expansion. The hope is that terms of lower order in $N_F$ may be roughly estimated by assuming that they follow the leading terms with weights generated by the one-loop term of the QCD beta function, $\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F N_F$. In a variety of cases (see e.g. Refs. [8, 12, 15, 16, 17]) this simplistic procedure gives reasonable estimates of known higher-order perturbative coefficients in different physical quantities (see Ref. [18] for a review). As a rule, the signs of contributions from sets of diagrams with fewer quark loops are correctly predicted and often the actual magnitudes of coefficients of lower powers of $N_F$ are within a factor of 2 of the NNA estimates. In this section we apply this procedure to the Bjorken unpolarized and polarized sum rules, where $Q^2 > 0$ is, of course, in the euclidean region.

We expand the perturbative contributions to Eq.(2) in powers of $\alpha_s/\pi = 4 \pi a_s$

$$C_{\text{Bjup}} = 1 + \sum_{n \geq 1} d_n \left( \frac{\alpha_s}{\pi} \right)^n$$

with known results $d_1 = -\frac{2}{3}$ and

$$d_2 = -3.8333 + 0.29630 N_F$$
$$d_3 = -36.155 + 6.3313 N_F - 0.15947 N_F^2$$

(11)

from Refs. [13, 14], in the \text{MS}-scheme. Taking the input of Eq.(5), naive nonabelianization yields

$$d_2^{\text{NNA}} = -4.8889 + 0.29630 N_F$$
$$d_3^{\text{NNA}} = -43.414 + 5.2623 N_F - 0.15947 N_F^2$$
$$d_4^{\text{NNA}} = -457.02 + 83.094 N_F - 5.0360 N_F^2 + 0.10174 N_F^3$$

(12)

obtained from the exact results for the terms with highest powers of $N_F$. One can observe reasonable agreement between the estimates and exact results for the coefficients of $N_F^0$ in $d_2$ and of $N_F^1$ in $d_3$. Moreover, even the estimate, $-43.414$, of the $N_F^0$ term in $d_3$ has the correct sign and a magnitude close to the known value, $-36.155$.

We now perform a similar analysis for the Bjp SR perturbative series $C_{\text{Bjp}}(Q^2) = 1 + \sum_{n \geq 1} \overline{d}_n (\alpha_s/\pi)^n$. The exact known results are $\overline{d}_1 = -1$ and

$$\overline{d}_2 = -4.5833 + 0.33333 N_F$$
$$\overline{d}_3 = -41.440 + 7.6073 N_F - 0.17747 N_F^2$$

(13)

obtained at $O(\alpha_s^2)$ in Ref. [15] and at $O(\alpha_s^3)$ in Ref. [20]. The NNA estimates are

$$\overline{d}_2^{\text{NNA}} = -5.5 + 0.33333 N_F$$
$$\overline{d}_3^{\text{NNA}} = -48.316 + 5.8565 N_F - 0.17747 N_F^2$$
$$\overline{d}_4^{\text{NNA}} = -466.00 + 84.728 N_F - 5.1350 N_F^2 + 0.10374 N_F^3$$

(14)
with $\bar{d}_2$ and $\bar{d}_3$ in tolerable agreement with the exact results of Eq.(13).

More ambitious estimates are presented in Tables 1 and 2, where we have evaluated Eq.(12) and Eq.(14) for $N_F = 3, 4, 5$ active flavours and compared these estimates with exact results from Eq.(11) and Eq.(13), or with estimates in Ref.[21] at $O(\alpha_s^4)$, obtained by a method of effective charges [22]. (The latter are in fair agreement with Padé estimates in Ref.[23].)

| Order | $N_F = 3$ | $N_F = 4$ | $N_F = 5$ |
|-------|-----------|-----------|-----------|
| $(\frac{\alpha_s}{\pi})^2$ (NNA) | -4 | -3.70 | -3.40 |
| $(\frac{\alpha_s}{\pi})^2$ (exact) | -2.94 | -2.65 | -2.35 |
| $(\frac{\alpha_s}{\pi})^3$ (NNA) | -29.1 | -24.9 | -21.1 |
| $(\frac{\alpha_s}{\pi})^3$ (exact) | -18.6 | -13.4 | -8.5 |
| $(\frac{\alpha_s}{\pi})^4$ (NNA) | -250 | -199 | -155 |
| $(\frac{\alpha_s}{\pi})^4$ (exact) | -133 | -76 | -29 |

**Table 1.** The $N_F$-dependence of the NNA expressions for Bjump SR and their comparison with the results of explicit calculations and $O(\alpha_s^4)$ estimates of Ref.[21].

| Order | $N_F = 3$ | $N_F = 4$ | $N_F = 5$ |
|-------|-----------|-----------|-----------|
| $(\frac{\alpha_s}{\pi})^2$ (NNA) | -4.5 | -4.17 | -3.83 |
| $(\frac{\alpha_s}{\pi})^2$ (exact) | -3.58 | -3.25 | -2.92 |
| $(\frac{\alpha_s}{\pi})^3$ (NNA) | -32.3 | -27.7 | -23.5 |
| $(\frac{\alpha_s}{\pi})^3$ (exact) | -20.2 | -13.8 | -7.8 |
| $(\frac{\alpha_s}{\pi})^4$ (NNA) | -255 | -203 | -158 |
| $(\frac{\alpha_s}{\pi})^4$ (Ref.[21]) | -130 | -68 | -18 |

**Table 2.** The $N_F$-dependence of the NNA expressions for Bjp SR and their comparison with the results of explicit calculations and $O(\alpha_s^4)$ estimates of Ref.[21].

Here one is pushing NNA rather hard, since there are substantial cancellations between powers of $N_F$ with alternating signs, which are most pronounced at $N_F = 5$. Nevertheless, there is agreement to within a factor of 2 for $N_F = 3, 4$ at order $\alpha_s^2$ and $\alpha_s^3$.

The similarities between Tables 1 and 2 are rather striking: the NNA estimates, the known values, and the effective-charges estimates in the two sum rules follow very similar patterns. Indeed this was observed in the renormalization-group invariant analysis of known results in Ref.[24], where this similarity between polarized and unpolarized sum rules appeared to be somewhat mysterious. From our point of view there is a simple-minded explanation: the residues of the dominant infrared renormalon, at $\delta = 1$, in the Borel transforms of Eq.(4) and Eq.(8) are identical. Thus one may attribute the close similarities in the full perturbative structures to the rough success of NNA, which assumes that the overall trends are driven by this renormalon.

However, in the concluding section we remind the reader of two important effects that suggest caution in relying on estimates obtained from a single chain of quark loops.
4 Application of NNA to euclidean correlators

To sharpen our understanding of the limitations of NNA, we reconsider analyses of the vector \[14\] and scalar \[12\] correlators, of light-quark currents, following the recent and impressive calculations of the \(O(\alpha_s^4 N_F^2)\) terms reported in Ref.\[25\].

In the vector channel we study the Adler function

\[
D(Q^2) = Q^2 \int_0^\infty \frac{R(s)}{(s+Q^2)^2} ds = 3 \sum_F Q_F^2 \left[ 1 + \sum_{n \geq 1} d_n^V (\frac{\alpha_s}{\pi})^n \right] \tag{15}
\]

where \(R(s)\) is the well-known \(e^+ e^-\) ratio, \(Q_F\) are the quarks charges, and the known perturbative results, at large \(Q^2 > 0\) in the euclidean region, are \(d_1^V = 1\) and

\[
\begin{align*}
    d_2^V &= 1.9857 - 0.11530 N_F \\
    d_3^V &= 18.243 - 4.2158 N_F + 0.086207 N_F^2 \\
    d_4^V &= d_{4,0}^V + d_{4,1}^V N_F + 1.8753 N_F^2 - 0.010093 N_F^3
\end{align*}
\tag{16}
\]

taking \(d_2^V\) from Ref.\[26\] and \(d_3^V\) from Ref.\[27\], with neglect of the term in the vector correlator with two quark loops and a three-gluon intermediate state. The \(O(\alpha_s^4 N_F^2)\) term in \(d_4^V\) was obtained in Ref.\[28\], and the \(O(\alpha_s^4 N_F^2)\) term was recently published in Ref.\[25\]. The corresponding NNA estimates are

\[
\begin{align*}
    d_2^{V,NNA} &= 1.9024 - 0.11530 N_F \\
    d_3^{V,NNA} &= 23.470 - 2.8448 N_F + 0.086207 N_F^2 \\
    d_4^{V,NNA} &= 45.338 - 8.2433 N_F + 0.49959 N_F^2 - 0.010093 N_F^3
\end{align*}
\tag{17}
\]

While the pattern of agreement of signs and rough magnitudes is comparable to that for the sum rules it is notable that the \(O(N_F)\) term in \(d_3^V\) exceeds the NNA estimate by a factor 1.5 and the \(O(N_F^2)\) term in \(d_4^V\) exceeds the estimate by a factor 1.8753/0.49959 \(\approx 3.8\).

Proceeding to the scalar correlator, again in the euclidean region, we study the following analogue of Eq.(15)

\[
D^S(Q^2) = Q^2 \int_0^\infty \frac{R^S(s)}{(s+Q^2)^2} ds = 3 \left[ m(Q^2) \right]^2 \left[ 1 + \sum_{n \geq 1} d_n^S (\frac{\alpha_s}{\pi})^n \right] \tag{18}
\]

with \(d_1^S = \frac{47}{3}\) and

\[
\begin{align*}
    d_2^S &= 51.567 - 1.9070 N_F \\
    d_3^S &= 648.71 - 63.742 N_F + 0.92913 N_F^2 \\
    d_4^S &= d_{4,0}^S + d_{4,1}^S N_F + 54.783 N_F^2 - 0.45374 N_F^3
\end{align*}
\tag{19}
\]

taking \(d_2^S\) from Ref.\[29\], \(d_3^S\) from Ref.\[30\], the \(O(\alpha_s^4 N_F^3)\) term in \(d_4^S\) from Ref.\[12\], and the \(O(\alpha_s^4 N_F^2)\) term from Ref.\[25\]. We choose this comparator so as to make contact with the analyses of Refs.\[31, 24\], notwithstanding the comment made in Ref.\[12\] that the necessity
of a second subtraction in the dispersion relation for the scalar correlator makes such a construct infrared unsafe. The corresponding NNA estimates are

\[
\begin{align*}
    d_{2,\text{NNA}}^S &= 31.465 - 1.9070 N_F \\
    d_{3,\text{NNA}}^S &= 252.96 - 30.661 N_F + 0.92913 N_F^2 \\
    d_{4,\text{NNA}}^S &= 2038.3 - 370.60 N_F + 22.460 N_F^2 - 0.45374 N_F^3
\end{align*}
\]  

(20)

again with a fair pattern of agreement in signs and magnitudes. The \(O(N_F)\) term in \(d_3^S\) exceeds the NNA estimate by a factor 2.1 and the \(O(N_F^2)\) term in \(d_4^S\) exceeds the estimate by a factor of \(54.783/22.460 \approx 2.4\)

Two remarks are in order. First, one notes that the absolute sizes of the radiative corrections in the scalar channel are much larger than those in the vector channel. This may be regarded as a success for the NNA estimator, which attributes this trend to the inadequately subtracted dispersion relation of Eq.(18), taken as the object of study in Refs.[31, 25]. The failure to make a second subtraction in the scalar channel results in an infrared renormalon at \(\delta = 1\), whereas no such renormalon can appear in Adler’s correctly subtracted dispersion relation of Eq.(15). Moreover, this disparity between the scalar and vector channels is already apparent in the \(O(\alpha_s)\) diagrams, into which quark loops are inserted, where the coefficient \(d_1^S = \frac{17}{4}\) is almost 6 times larger than \(d_1^V = 1\).

As in the previous section, this renormalon analysis has value only if the known radiative corrections follow the pattern suggested by NNA. Fortunately this is again the case, in the euclidean region. We remark that the very large \(O(\alpha_s^4 N_F^2)\) coefficient \(d_{1,2}^S = 54.783\) is better approximated by NNA than is the far smaller coefficient \(d_{1,2}^V = 1.8753\). It thus appears that NNA is a useful indicator of significant euclidean radiative corrections.

The second important observation is that a very different picture emerges if one chooses the comparator \(R^S(s)\), which is the discontinuity of the scalar correlator across the cut in the minkowskian (timelike) region \(-Q^2 = s > 0\). The modest success of NNA in the euclidean region does not ensure comparable success in the minkowskian region, because of the numerically large terms involving powers of \(\pi^2\) and coefficients of the beta function, \(\beta(\alpha_s)\), and the mass anomalous dimension, \(\gamma_m(\alpha_s)\), noted in [31]. These easily computed effects of analytic continuation do not naively nonabelianize. Moreover, the infrared renormalon at \(\delta = 1\) in Eq.(18) is absent from the scalar imaginary part \(R^S(s)\). Accordingly, NNA behaves poorly in this comparator. For example, it was noted in Ref.[25] that the exact \(O(\alpha_s^4 N_F^2)\) term in \(R^S(s)\) exceeds that obtained by applying NNA to the exact \(O(\alpha_s^4 N_F^2)\) term of Ref.[12] by a factor of \(9.6848/1.0128 \approx 10\).

Thus we learn from the analytical calculations of Refs.[12, 25] that NNA estimates for the scalar correlator perform better in the euclidean region, where we have already noted tolerable agreement with exact results on sum rules.

5 Conclusions

In summary:
1. The new all-orders results for the leading terms in the large-$N_F$ expansion of the
perturbative contributions to the Bjorken unpolarized sum rule, when naively non-
abelianized, offer an explanation of the observation of Ref. [24] that the actual radiative
corrections in this sum rule closely follow those of the Bjorken polarized sum rule.
Within the simplistic, but here relatively successful, framework of NNA, we
attribute this parallelism to the equality of the residues of the dominant infrared renormalon, at $\delta = 1$, in the Borel transforms of Eq.(4) and Eq.(8).

2. The pattern of NNA estimates for the vector and scalar correlators, in the euclidean region, also suggests a leading role for infrared renormalons. In this case, it helps one to understand why the actual scalar radiative corrections are so much larger than those in the vector case. To attribute this to NNA, it is necessary for the procedure to have reasonable success in each channel. In fact we find that it is slightly more successful in the scalar case, notwithstanding conspicuous failure after analytic continuation to the minkowskian region.

3. Since NNA yields correct signs and sensible magnitudes for all known coefficients of terms with lower powers of $N_F$, in all four euclidean analyses considered in this work, we believe that its sign predictions in Eqs.(12,14,17,20), for unknown $O(\alpha_s^4)$ terms, are probably reliable. In particular, we expect that in Eqs.(16,19) further dedicated calculation will yield negative signs for $d_{4,1}^V$ and positive signs for the $d_{4,0}^V$ coefficients that result from purely gluonic radiative corrections.

4. It is important to recognize the intrinsic limitations of NNA. First we remark that the $N_F$ dependence of QCD radiative corrections results not only from insertions of quark loops in gluon propagators; in addition there are diagrams with quark loops inserted at gluonic vertices. It seems unreasonable to expect NNA to mimic contributions such as that in Fig.1(a) of Ref.[24], where a quark loop modifies a three-gluon vertex within a second quark loop. More significantly, perhaps, one first encounters at $O(\alpha_s^4)$ the effects of quark loops in different gluon propagators. As remarked in Ref.[32], multiple renormalon chains modify the asymptotic behaviour at next-to-leading order in $1/N_F$, notably in the $O(\alpha_s^4 N_F^2)$ terms. Therefore, it would appear more prudent to use the final row in each of the Tables 1 and 2 as estimates for DIS sum rules. Moreover, in the vector correlator, the effective-charges analysis of Ref.[21] appears to be in good agreement with the partial analytical results of Ref.[25].

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