Homogeneous Poisson process in daily case of covid-19

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Abstract. Since December 2019, an increasing number of new pneumonia cases have emerged in Wuhan, China. The rise of the spread of diseases caused by the Corona Virus Disease (covid-19) which has been established as a pandemic by WHO on March 12, 2020, gave rise to so much anxiety and speculation from various parties. The case of covid-19 positive patients Daily can be calculated by the homogeneous Poisson process. A Poisson process with a constant rate (λ) is called a homogeneous Poisson process. The average number of positive patients of Covid-19 from January 24, 2020, to April 16, 2020, is still very large. The chances of not having cases of covid-19 positive patients from January 24, 2020 to April 16, 2020 are very small so there will always be covid-19 cases every. Therefore, elements of society and government must consider handling and preventing the Covid-19 case.

1. Introduction
Since December 2019, an increasing number of new pneumonia cases have appeared in Wuhan, China [1]. The etiology of this infection has been reported as a new coronavirus (2019-nCov) and the disease has been named Corona Virus Disease 2019 (covid-19) by WHO. This virus has spread rapidly throughout China and at least 23 other countries such as Thailand, South Korea, Japan and the United States. On February 11, more than 44,000 have been confirmed cases of covid-19 and more than 1,100 deaths have been reported in China [2]. Coronavirus is transmitted from human to human that occurs in the environment around who are closely related to patients or carriers of covid-19 [3].

The rise of the spread of disease caused by the covid-19 virus which was declared a pandemic by WHO on March 12, 2020, gave birth to a lot of anxiety and speculation from various parties. Even though the pandemic SOP was immediately followed up by the government, the various issues and news spreads with a variety of possibilities are increasingly worrying the public. The flow of sea and air transportation was canceled, some universities replaced the lecture system with online lectures [4, 5].

The covid-19 cases that have occurred since early December have increased from time to time every day. The number of cases of covid-19 patients can be analyzed based on the number of cases positive covid-19 at this time. Therefore, it can be analyzed using a homogeneous Poisson process. Homogeneous Poisson process is a Poisson process with rate (λ) which is time dependent [6–8].

Several studies on the homogeneous Poisson process have been carried out, namely the characterization of the homogeneous Poisson process carried out by Vidmar [9]. Then, research has been carried out by Utami which focuses on the numerical study of the distribution of the expected value estimator function and the multiple Poisson process variance function with the exponential intensity of the linear function [10]. Furthermore, Deniz also conducted research in the field of economics, namely on the Poisson process with random intensity for financial stability [11]. Based on the explanations and research that has been done, this paper describes the homogeneous Poisson process and its application to the many daily cases of covid-19 positive patients in the world.

2. Method
2.1. Counting process
If X(t) or Xᵢ represents the number of events that happen during an interval, then this process is called the counting process of the stochastic process X(t), t ≥ 0 [6–8]. Some examples of the counting
process are the number of positive cases of covid-19 hospitalized at time interval \( t \), the number of medical personnel who came to the hospital at time \( t \), or the number of covid-19 patients who recovered at the time interval \( t \).

The properties that must be fulfilled in the counting process \( X(t), t \geq 0 \) are as follows \([6–8]\):

i. \( X(t) \geq 0 \).

ii. \( X(t) \) is integer.

iii. If \( s < t \), then \( X(s) < X(t) \).

iv. For \( s < t \), the number of phenomena that happen at a time interval is given by \( X(s) - X(t) \).

2.2. Poisson process

**Definition 1** \([6–8]\). The process of calculating \( \{X(t), t \geq 0\} \) is said to be a Poisson process with a rate (parameter) if it fulfils:

i. \( X(0) = 0 \)

ii. Process has independent increment and stationary increment.

iii. For \( h \) short time intervals then \( P(X(h) = 1) = \lambda h + o(h) \).

iv. \( P(X(h) \geq 2) = o(h) \).

Based on Definition 1 it is obtained:

\[
\sum_{k=0}^{\infty} P_i(t) = 1.
\]

Using the stationarity of the Poisson process, obtained:

\[
P_i(t) = P[N(s+t) - N(s) = k] = P[N(t) = k | N(0) = 0], \forall t \geq 0 \text{ and } s \geq 0.
\]

Applying equations ii, iii and iv from Definition 1, obtained:

\[
P_i(t + h) = P_i(t)P_i(h) = P_i(t)[1 - \lambda h + o(h)] \text{ and }
\]

\[
P_i(t + h) = P_i(t)[\lambda h + o(h)] + P_i(t)[1 - \lambda h + o(h)] + \sum_{k=2}^{\infty} P_i(t) o(h) k = 1, 2, 3, \ldots.
\]

From Eq. (3) and Definition 1, obtained:

\[
P_i(t) = \frac{(\lambda t)^i}{i!} e^{-\lambda t}, k = 0, 1, 2, 3, \ldots
\]

Eq. (8) is the probability mass function of the Poisson distribution with parameters \( \lambda t \) \([6]\).
Definition 2 [6–8]. The process of calculating \( \{X(t), t \geq 0\} \) is said to be a Poisson process with a rate (parameter) if it fulfills:

i. \( X(0) = 0 \).

ii. Process has independent increment.

iii. The probability of occurrence \( k \) for each interval \( t \) is given:

\[
P(N(t + s) - N(s) = k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, (k = 0, 1, 2, 3, \ldots), \forall s, t \geq 0.
\]

then \( N(t + s) - N(s) \sim POI(\lambda t) \).

Let \( X(t) \) be a Poisson random variable with parameter \( \lambda t > 0 \), so obtained

\[
P(X(t) = n) = e^{-\lambda t} \frac{(\lambda t)^n}{n!}, n = 0, 1, 2, \ldots .
\] (9)

Then, determine the expectation and variance of the Poisson random variable \( X(t) \) as follows:

\[
E[X(t)] = \sum_{n=0}^{\infty} n e^{-\lambda t} \frac{(\lambda t)^n}{n!} = e^{-\lambda t} \lambda t \sum_{n=1}^{\infty} \frac{(\lambda t)^{n-1}}{(n-1)!} = e^{-\lambda t} \lambda t = e^{-\lambda t} \lambda t e^{\lambda t} = \lambda t . \] (10)

Using the same method, it can be determined \( E\left[ X(t)^2 \right] = (\lambda t)^2 + \lambda t \), so that the variance of \( X(t) \) is obtained as follows:

\[
\text{var}\left[ X(t) \right] = E\left[ X(t)^2 \right] - (E\left[ X(t) \right])^2 = (\lambda t)^2 + \lambda t - \lambda t = \lambda t . \] (11)

2.3. Proposed approach

The data used in this study is data on covid-19 positive patient around the world. This data was taken from website https://www.worldometers.info/coronavirus/coronavirus-cases/#daily-cases on January 24 to April 16, 2020. Steps were taken in calculating the homogeneous Poisson process are:

i. Retrieve daily data on covid-19 positive patients through the website https://www.worldometers.info/coronavirus/coronavirus-cases/#daily-cases.

ii. Counting positive patients of covid-19 every day from January 24, 2020 to April 16, 2020.

iii. Looking for descriptive statistics from daily data on covid-19 positive patients.

iv. Calculating the average and variance cases of covid-19 positive patients.

v. Calculating the homogeneous Poisson process on the daily data of covid-19 positive patients.

vi. Interpretation results.

3. Result and Discussion

3.1. Statistics descriptive

Daily case data for covid-19 positive patients around the world from January 24, 2020 to April 16, 2020 can be depicted in graphical form with the help of Microsoft Excel in Fig.1 below:

![Figure 1. Daily cases of covid-19 positive patients around the world](image-url)
Based on Figure 1, cases of covid-19 positive patients around the world have fluctuated. On the 71st day, (April 3, 2020) cases of covid-19 positive patients experienced a significant increase reaching 96,236 people and an increase from the previous cases of 15,938 people. Furthermore, descriptive statistics for positive patients of covid-19 are presented in Table 1 below:

| Mean   | Min  | Max     | St. Deviation |
|--------|------|---------|--------------|
| 25,732 | 476  | 96,236  | 32,557       |

Based on Table 1, the average daily cases of covid-19 positive patients from January 24 to April 16, 2020, were 25,732 people. Most cases of covid-19 positive patients occurred on April 3, 2020, as many as 96,236 people. The least number of cases of covid-19 positive patients occurred on January 24, 2020, as many as 476 people. The case of covid-19 positive patients on January 24, 2020, was the beginning of the covid-19 case in Wuhan, China whose data was captured on [https://www.worldometers.info/coronavirus/].

3.2. Poisson process of daily cases of covid-19 positive patients

Daily cases of covid-19 positive patients follow the Poisson process at the following rates:

\[ \lambda = \frac{\sum_{i=1}^{84} X_i}{84} = \frac{472 + 698 + ... + 83,483 + 90,254}{84} = 25,732 \text{ people per day} \]

The number of cases of Covid-19 patients is 25,732 people per day from January 24, 2020 to April 16, 2020 with assumption \( t = (0,24] \). Then, Expectations and variances of daily cases of covid-19 positive patients from January 24, 2020 to April 16, 2020:

\[ E(X(t)) = \lambda t = 25,732 \times 84 = 2,161,523 \]

\[ Var(X(t)) = \lambda t = 25,732 \times 84 = 2,161,523 \]

The average number of covid-19 positive patients from January 24, 2020 to April 16, 2020 was 2,161,523 people. The variance of covid-19 positive patients from January 24, 2020 to April 16, 2020 is 2,161,523 people. The number of cases of covid-19 positive patients from January 24, 2020 to April 16, 2020, was 2,161,523 people. Chances of not having cases of covid-19 positive patients from January 24, 2020 to April 16, 2020:

\[ P_k(t) = \frac{(\lambda t)^k}{k!} e^{-(\lambda t)} \]

\[ p(x = 0) = \frac{(\lambda t)^k}{k!} e^{-(\lambda t)} = \frac{(2,161,523)^0}{0!} e^{-2,161,523} = 3.0803 \times 10^{-938738} \]

The chances of not having cases of covid-19 positive patients from January 24, 2020 to April 16, 2020 are very small. It is meaning that it is impossible on January 24, 2020 to April 16, 2020 there will be no cases of covid-19 positive patients or there will always be covid-19 cases every day. This is evidenced by:
Table 2. Number of cases of positive patients of covid-19 from April 17, 2020, to April 21, 2020

| Date          | Number of cases of positive patients of Covid-19 (Person) |
|---------------|----------------------------------------------------------|
| April 17, 2020| 87,119                                                   |
| April 18, 2020| 81,760                                                   |
| April 19, 2020| 76,022                                                   |
| April 20, 2020| 73,955                                                   |
| April 21, 2020| 75,979                                                   |

Based on Table 2, from April 17, 2020, to April 21, 2020, the average number of cases of positive Covid-19 patients was 78,967 people per day. There were still quite large positive cases of covid-19. Therefore, the public must always be aware of the spread of the coronavirus. Also, the homogeneous Poisson process can calculate the probability of covid-19 occurring in the future.

4. Conclusion
The homogeneous Poisson process is a Poisson process with a constant rate ($\lambda$). The biggest cases of covid-19 positive patients were 96,236 cases that occurred on April 3, 2020. The smallest cases of covid-19 positive patients were 476 cases that occurred on January 24, 2020. The average number of cases of covid-19 positive patients every day is 25,732 cases. The chances of not having cases of covid-19 positive patients from January 23, 2020 to March 17, 2020 are very small, meaning that there will always be covid-19 cases every day. Therefore, the public must always be aware of the spread of the coronavirus.

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