Assessment of simulation adequacy of construction machine transmission dynamic systems

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Abstract Procedure and theoretical dependences are given for numerical methods for assessing adequacy of construction machinery mechanical transmission gear models, which are linear oscillatory systems.

One of the main ways to solve the problem of extending construction machinery survival time is development of effective systems and methods for monitoring technical condition parameters of machines and gears under operating conditions with no need for their components and assemblies to be disassembled. Domestic and foreign practice shows that significant reduction in labour and material costs for machines maintenance and repair can be achieved by vibro-acoustic diagnostic systems using [1, 2, 3].

Spur gears are the most popular of construction machine transmissions due to their advantages over other types of mechanical gears [1].

Dynamic forces that are generated in them due to various defects are disturbing effects on the dynamic transmission system. Thus, calculation of its vibro-acoustic parameters requires a mathematical model of this dynamic system available. Accuracy of determined gear oscillation parameters depends on the type of adopted models of dynamic system and disturbance. Therefore, choice of the model type should be consistent with objectives of research conducted with the chosen model type [1, 2, 3].

Any mechanical structure is a dynamic system that converts input action into output reaction.

For linear systems of constant parameters, relationship between $x(t)$ input signal and $y(t)$ output is given by $h(\tau)$ impulse response and $H(f)$ frequency response. $H(f)$ frequency response, which describes linear system properties in the frequency domain, is defined as the Fourier transform of $h(\tau)$ function [4]

$$H(f) = \int_{-\infty}^{\infty} h(\tau) e^{-j2\pi ft} d\tau$$

(1)

Frequency response is a complex function that can be presented through a module and argument

$$H(f) = |H(f)| e^{-j\phi(f)}.$$  (2)

Frequency response module $|H(f)|$ is conventionally called an amplitude or amplitude-frequency characteristics, and argument $\phi(f)$ is called a phase response.

For a linear system with constant parameters, the frequency response $H(f)$ depends only on
frequency and does not depend on time and type of input signal.

When assessing adequacy of mechanical oscillatory system simulation, coincided Eigen frequency values and their amplitudes, i.e. amplitude-frequency characteristics of the real object and its model are of primary interest.

The Green functions are used to describe the linear systems behaviour in the time domain, which allows expressing a response of the system to an external action using the convolution equation (the Duhamel integral). The Green functions are a core of the Fredholm integral equation

\[ y(f) = \int_{0}^{\infty} h(\tau)x(t-\tau) d\tau \quad (3) \]

where \( h(\tau) = 0 \) at \( \tau < 0 \), if the system is physically feasible.

By substituting \( x(t)=\delta(t) \) into Eq. (3), where \( \delta(t) \) is a delta function, we obtain, in accordance with its filtering properties, \( y(t)=h(t) \). Thus, the core of the Fredholm integral equation is a reaction to the delta pulse of the linear system.

Fourier delta function transformation

\[
\int_{0}^{\infty} \delta(\tau)e^{-j2\pi f \tau} d\tau = 1 \quad (4)
\]

Therefore, when applying a single pulse \( x(t) = \delta(t) \) to the input, taking into account that in this case \( X(f) = 1 \), the expression for the response Fourier transformation has the following form

\[ Y(f) = H(f) \quad (5) \]

where \( X(f) \) and \( Y(f) \) is the Fourier transformation of input \( x(t) \) and output \( y(t) \) signals, respectively.

That is, when a disturbance system is fed to the input in a single pulse, the Fourier transform of the response will be the system frequency response. Thus, it is possible to reveal the internal dynamic properties of the mechanical oscillatory system by acting on its input with a shock pulse — a disturbance that is closest to the delta function in its nature. Herewith, having determined the response spectral density, it is possible to obtain amplitude characteristic of the linear oscillatory system as a “black box”, which allows to perform comparative analysis and simulation adequacy assessment of various mechanical dynamic systems.

An important engineering application of the frequency response can be obtained by taking the Fourier transform from both sides of equation (3)

\[ Y(f) = H(f)X(f) \quad (6) \]

where \( S_x(f) \) and \( S_y(f) \) are spectral densities of the stationary process input and output signals, respectively.

This ratio allows to calculate \( |H(f)| \) by using known \( S_x(f) \) and \( S_y(f) \), i.e. unambiguously determine the amplitude-frequency characteristic of the linear mechanical dynamic system.

There are several ways of experimental studying of dynamic properties of mechanical oscillatory systems, which are supplying a known input action and recording an output response [5]. One of the most universal and effective types of input action, as was shown above, is a pulsed input action that excites complex oscillations in a wide range of frequencies of the oscillatory system. Pulse bandwidth is determined by its duration, which is the most important characteristic. For complete identification and evaluation of internal properties of the mechanical oscillatory system, we need to have the perturbation spectrum no less than the system natural oscillation spectrum in which all essential frequencies are positioned.

In case of impact interaction of various metal parts with curvilinear surfaces, a shock pulse is
generated, having a sinusoidal or cosine-shaped form. For that sort of a pulse shape [6, 7], 95...99% of
the energy is concentrated in interval \(0 < f_u \leq \frac{(1.5\ldots1.7)}{\tau_u}\), where \(\tau_u\) is a duration of the shock pulse.

That is, the oscillatory system can have oscillations excited with a frequency of no more than
\(f_u = \frac{(1.5\ldots1.7)}{\tau_u}\).

In addition to the bandwidth of the oscillations excited, the pulse duration has a direct impact on
the spectrum uniformity in this frequency band. Herewith, the input effect amplitude and, therefore,
vibrations at the output do not have any fundamental significance. Their values are chosen within the
range of linear characteristics of the object, but significantly above the interference level.

Thus, for a qualitative and quantitative assessment of the internal intrinsic properties of mechanical
oscillatory systems and their models, which determine dynamic characteristics of these systems, and to
assess adequacy of description of the relationship between input and output processes, information is
needed on parameters of their amplitude characteristics.

The above theoretical prerequisites were practically implemented in surveys of the construction
machine mechanical transmission spur gear element vibrations.

The main component of the dynamic gearing system, which directly perceives external disturbance,
transforms and transfers it into the body elements, is shafts with gear wheels. Development of a
mathematical model of a shaft of this type based on the oscillations classical theory is extremely
difficult to complete due to impossibility of taking into account all the factors affecting the
propagation of various types of waves and their interaction. This resulted in using of one of the most
effective and modern numerical methods, the finite element method (FEM), for modelling of a gear
shaft dynamic system. The FEM is successfully used to solve many spatial problems associated with
structures calculation.

Numerical surveys of the gear wheel shaft vibro-acoustic parameters were carried out using the
IMPULS software package that is designed for calculation of fast-varying processes in complex
spatial structures [8].

Experimental surveys were performed on a developed laboratory machine with a set of measuring
and recording equipment. A primary shaft of the mechanical gear box of the middle class earth-
moving vehicle was chosen as an object of theoretical and experimental research. Purpose of these
surveys was verification of adequacy of the developed mathematical model of real structure.

A sinusoidal pulse of unit amplitude with the duration of \(\tau_u = 0.2\) ms, and a hammer strike with
an initial velocity \(V_0 = 0.3\) m/s were taken in the laboratory setup to determine and compare internal
dynamic characteristics of the oscillating system of a shaft with a gear wheel as an input power effect
in the model. Such input effect parameters provide excitation of natural frequencies with the upper
limit of up to 7.5...8.5 kHz and spectrum uniformity equivalent to the spectrum of the delta function
with the maximal error of up to 5% for \(f = 0...6000\) Hz frequency band, and 10% for \(f = 0...8500\)
Hz [6]. Comparison and analysis of dynamic characteristics, and verification of the mathematical
model adequacy were carried out using values of vibration accelerations of the gear tooth, which are
directed along the horizontal axis, perpendicular to the shaft axis.

Impact of a shock impulse on a tooth wheel results in origination and propagation of transverse and
longitudinal waves in the shaft, causing various types of oscillations of this shaft. Thus, a
characteristic feature of shock excitation is occurrence of complex damped oscillations, which, like the
input action, are not a stationary process. That is, there is a non-stationary \(x(t)\) input process that
physically exists only for a finite time interval \(\tau_u\) and a corresponding \(y(t)\) output process that is
different from zero on \(0 \leq t \leq T\) interval.

[4] shows that relations between input and output characteristics of transient processes are identical
to relation (6) for stationary processes. Only spectral densities of “energy” are used instead of spectral
densities of “power”. Theoretically, this implies that averaging required for obtaining estimates of transient process energy spectra can be obtained as a result of the experiment multiple repetitions. However, [4] shows a possibility of obtaining significant results with sufficient accuracy for practical calculations in a single experiment that was shown if the signal-to-noise ratio is large, which is the case in many real oscillatory systems under shock excitation.

In accordance with the definition, the function of spectral energy density for \( f \) frequencies within the range \( f = 0 \) to \( f = f_n \), which is computed using the finite Fourier transform, is

\[
S_e(f) = 2E[X_T(f)]^2
\]  

(7)

where \( E[\cdot] \) is averaging by an existing group of \( n_d \) (transient) realizations \( |X_T(f)|^2 \) at \( f \) fixed frequency. \( X_T(f) \) magnitude is a finite Fourier transform of the original function \( x(t) \) defined on a time interval of length \( t_u \).

Functions of spectral energy density and power spectral density of the same transient process of \( T \) duration are related by relation [4]

\[
G_e(f) = TS_e(f)
\]

(8)

Spectral analysis of the sampled transient implementation was carried out using the discrete Fourier transform of \( x_n \) equidistant sequence of samples with \( \Delta t \) sampling interval and zero mean. This sequence is given by \( x_n = x(n, \Delta t) \) where \( n = 0, 1, 2, \ldots, N - 1 \) at the beginning of the countdown \( t_0 = 0 \).

Converted sequence is determined by expression [9]

\[
X_k = \Delta t \sum_{n=0}^{N} x_n \exp\left(-j\frac{2\pi kn}{N}\right)
\]

(9)

for discrete frequencies \( f_k = \frac{k}{T} = \frac{k}{N\Delta t} \), \( k = 0, 1, 2, \ldots, N - 1 \).

Then the energy spectrum density is written as

\[
S(f_k) = |X_k|^2
\]

(10)

When the signal amplitude changes in volts, \( S(f_k) \) unit of measure will be \( \frac{v^2}{Hz^2} \).

\( f_d = \frac{1}{\Delta t} \) sampling frequency was: \( f_d^* = 25 \text{ kHz} \) for experimental surveys; and \( f_d^w = 100 \text{ kHz} \) for theoretical surveys.

The Figure shows the one-sided spectral energy densities of vibration accelerations of free damped oscillations of a shaft with gear, obtained from the formulas presented above.

Analysis of the above results shows that:

- the main frequencies of various types and forms of natural oscillations of the investigated shaft with gear are positioned within the low-frequency – 250…1000 Hz, mid-frequency – 3000…4500 Hz and high frequency - 7000…8500 Hz ranges;
- the minimal difference in the obtained estimates of frequency characteristics of the mathematical model and experiment is observed in the mid-frequency area. Thus, the spectral density peaks are: for the model \( f_{n3}^m = 3.13 \text{ kHz} \) and \( f_{n4}^m = 4.24 \text{ kHz} \); for the experiment \( f_{n3}^* = 3.08 \text{ kHz} \) and \( f_{n4}^* = 4.22 \text{ kHz} \), the difference is \( \Delta f_{n3} = 1.6 \% \); \( \Delta f_{n4} = 0.5 \% \), respectively. Difference in values of other peak frequencies is more significant.
Figure 1. Spectral densities of vibration acceleration energy of free damped oscillations of a shaft with gear: a) theoretical; b) experimental

Thus, according to the results of a comparative analysis of characteristics of the model and the object, an assessment of its adequacy can be carried out, as well as clarification of the model parameters for the purpose of improvement.

Conclusion

The presented procedure and theoretical dependencies for numerical methods for assessing the adequacy of gear models of mechanical transmissions of construction machines using test input and spectral analysis of the response allows analysing any linear dynamic systems as a “black box” without describing their internal structure.

Based on the finite element method, the mathematical model of a shaft with a gear wheel of a construction machine gearing adequately describes for the tasks of vibro-acoustic diagnostics, dynamic properties of its mechanical oscillatory system and allows replacing expensive full-scale tests with model studies.

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