Price Interpretability of Prediction Markets: A Convergence Analysis

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Information, often scattered among the crowd, is valuable yet hard to aggregate. Throughout the centuries, researchers and practitioners have invented various tools in an attempt to piece them together. Classical mechanisms like polls, surveys, and brainstorming work just fine. However, they lack the commitment to let people put their money where their mouth is. In the last century, prediction market, a mechanism designed specifically for information elicitation and aggregation, has gained much popularity. Success stories of prediction markets can be found in a variety of applications such as election prediction, corporate consensus pooling, box office forecasting, and online recreational sports/game betting.

Many works in the literature have been devoted to the designing of a proper implementation of prediction markets. However, relatively few have tried to explain its apparent predicting efficiency, and even fewer have studied the evolutionary behavior of a prediction market in price forming. This paper aims to complement the current analysis of prediction markets in the literature by shedding light on the following questions.

(1) Under what conditions can we expect a prediction market to reach a consensus? That is, when does the trading position of each trader converge? When does the price process converge?

(2) If the trading position and the price sequence converge in the end, does the convergent point depend on the sequence of trading?

(3) How to interpret the limiting price? What is the relation between such limiting price with each trader’s belief and risk attitude?

Following the spirit of [5], we construct our dynamic prediction market model with multiple securities and a finite number of myopic risk-averse traders with heterogeneous beliefs. These traders interact with one algorithmic market maker to optimize their welfare repeatedly and sequentially. The main contributions of this work are the following:

A unified prediction market-making mechanism. Many prediction markets introduce an algorithmic market maker to address the illiquidity issue. Our work proposes a multivariate utility (MU)-based mechanism that unifies several commonly used mechanisms [1, 2, 4] in a consistent framework. We show that this framework is equivalent to the classical cost function-based market-making mechanism. One prominent feature of our framework is that it only requires the MU function to be monotone and concave, while the cost function-based mechanism requires an additional translation invariance property. The pricing of an incoming order in a MU-based mechanism is a simple line search that can be efficiently computed. Moreover, our MU framework helps to significantly simplify the modeling and analysis of the underlying trading process.

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A convergence scheme under general conditions. With the help of MU-based mechanism, we are able to reformulate the original bi-level trading decision problem at each trade into a standard constrained convex program. Under mild conditions, we prove the compactness of the set of all feasible market states generated by the trading process. We then show that the traders’ wealth processes converge to some limiting wealth distribution which associates with the Pareto efficient frontier defined by all the market participants’ utilities. Such a convergence result is established in three steps: (a) we show that there exists a sub-sequence of the utility value generated from the trading sequence that converges to some limiting utility level due to the compactness of the feasible market state set; (b) we then prove that the limiting wealth allocation of such a convergent subsequence is a Pareto optimal allocation; (c) finally, we show that any convergent subsequence of the original bounded sequence converges to the same limit. Such a convergence result is fundamental to understanding the effectiveness of prediction markets. Moreover, it helps us bypass the difficulty of analyzing the transient behavior of the price dynamics but can instead examine the limiting price directly.

Interpretation of the limiting price in various types of markets. Our price analysis builds upon the Pareto optimality of the limiting wealth allocation. For markets where each agent (including the market maker) has an exponential utility, we show that the limiting price is the geometric mean of agent beliefs. For markets where each participant’s utility is characterized by the risk measure (see, e.g., [3] for a detailed discussion), we derive a set of conditions that will ensure convergence and construct a special family of risk measures under which the limiting price can converge to a weighted power mean of agent beliefs. A remarkable feature of the preceding two special utility classes (exponential and risk measure) is that the limiting price can be solely dependent on risk parameters while independent of initial wealth allocation and realized trading order. This feature testifies to the importance of the risk attitude in the price forming process. For markets based on hyperbolic absolute risk aversion (HARA) utilities, we show that the limiting price is also a risk-adjusted weighted power mean of agent beliefs, even though the trading order will affect the aggregation weights. Again, risk attitudes are essential in determining the functional form of limiting price. We further propose an approximation scheme for the limiting price under the HARA utility family. The numerical experiments demonstrate that our approximation scheme works well in predicting the limiting prices.

The full paper is available at https://arxiv.org/abs/2205.08913.

CCS Concepts: • Applied computing → Decision analysis; • Theory of computation → Exact and approximate computation of equilibria.

Additional Key Words and Phrases: information elicitation, prediction markets

ACM Reference Format:
Dian Yu, Jianjun Gao, Weiping Wu, and Zizhuo Wang. 2022. Price Interpretability of Prediction Markets: A Convergence Analysis. In Proceedings of the 23rd ACM Conference on Economics and Computation (EC ’22), July 11–15, 2022, Boulder, CO, USA. ACM, New York, NY, USA, 2 pages. https://doi.org/10.1145/3490486.3538347

ACKNOWLEDGMENTS
The second author acknowledges support from the National Natural Science Foundation of China [Grant NSFC-71971132, 72192832]. The fourth author acknowledges support from the National Natural Science Foundation of China [Grant NSFC-72150002].

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