Multiple point principle as a mechanism for the suppression of FCNC and CP–violation phenomena in the 2HDM

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Multiple point principle as a mechanism for the suppression of FCNC and CP–violation phenomena in the 2HDM

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Abstract. We argue that multiple point principle (MPP) can be used to ensure CP conservation and the absence of flavour changing neutral currents within the two Higgs doublet model (2HDM). We also discuss Higgs phenomenology in the MPP inspired 2HDM.

PACS. 12.60.Fr Extensions of electroweak Higgs sector – 14.80.Bn Standard-model Higgs bosons

1 Introduction

In spite of the success of the standard model (SM) in accounting for existing experimental data, there are several well–motivated theoretical reasons (like the hierarchy problem, the presence of dark matter in the Universe, observed neutrino oscillations etc.) to expect new physics beyond it. New physics beyond the SM generally introduces new sources of CP violation and gives rise to flavour changing neutral current (FCNC) transitions that are forbidden at the tree level in the SM. Indeed, the violation of CP invariance and the existence of tree–level flavor–changing neutral currents are generic features of new physics beyond the SM. Although one can eliminate the violation of CP invariance and tree–level FCNC transitions in the 2HDM by imposing discrete $Z_2$ symmetry, such a symmetry leads to the formation of domain walls in the early Universe Ref. [1] which create unacceptably large anisotropies in the cosmic microwave background radiation Ref. [2].

Here, instead of the custodial $Z_2$ symmetry, we use the multiple point principle (MPP) to suppress FCNC and CP–violation effects in the 2HDM. The MPP postulates the existence of the maximal number of phases with the same energy density which are allowed by a given theory Refs. [3]–[4]. The application of the multiple point principle to the SM leads to the remarkable prediction for the top quark (pole) and Higgs boson masses Ref. [5]:

$$M_t = 173 \pm 5 \text{ GeV}, \quad M_H = 135 \pm 9 \text{ GeV},$$

which are consistent with current experimental data. In previous papers (see Refs. [6]–[7]) the MPP assumption has been adapted to models based on ($N = 1$) local supersymmetry – supergravity, that allowed an explanation for the small deviation of the cosmological constant from zero. Recently we also considered the application of the MPP to the SUSY inspired 2HDM of type II Ref. [8]. Here we extend this analysis to the general 2HDM and discuss the quasi–fixed point scenario in the MPP inspired 2HDM.

2 MPP conditions

The self–consistent implementation of the MPP in the 2HDM can be achieved if, at some high energy scale $\Lambda$, the following set of degenerate vacua is realized (see...
where \( \Phi_1^2 + \Phi_2^2 = \Lambda^2 \) and \( \omega \) is an arbitrary parameter. According to the MPP, vacua at the scale \( \Lambda \) must have approximately the same energy density as the physical one. This means that all couplings at the MPP scale \( \Lambda \) have to be adjusted, so that the energy density of the MPP scale vacua vanishes with relatively high accuracy. When \( \Lambda \gg v \) the mass terms in the 2HDM effective potential can be safely ignored, which simplifies our analysis. Then, in compliance with the MPP assumption, the \( \lambda_i(\Lambda) \) should be adjusted so that an appropriate cancellation takes place among the quartic terms in the Higgs effective potential (11).

MPP implies that the quartic part of the Higgs potential (11) goes to zero for all possible values of the phase \( \omega \). At the same time for minima to exist, one has also to ensure that all partial derivatives of this part of the Higgs potential (11) vanish for any choice of \( \omega \) near the scale \( \Lambda \). These two requirements are fulfilled only if

\[
\begin{align*}
\lambda_3(\Lambda) &= \lambda_6(\Lambda) = \lambda_7(\Lambda) = 0, \\
\beta_{\lambda_3}(\Lambda) &= \beta_{\lambda_6}(\Lambda) \Phi_1^2 + \beta_{\lambda_7}(\Lambda) \Phi_2^2 = 0, \\
\tilde{\lambda}(\Phi) &= \beta_{\lambda_3}(\Lambda) = 0,
\end{align*}
\]

(4–6)

where \( \tilde{\lambda}(\Phi) = \sqrt{\lambda_3(\Phi)\lambda_2(\Phi) + \lambda_3(\Phi) + \lambda_4(\Phi)} \) while \( \beta_{\lambda_i}(\Phi) = \frac{d\lambda_i(\Phi)}{d\ln \Phi} \) is the beta function for \( \lambda_i(\Phi) \), i.e. we assume here that the \( m_i^2 \) and \( \lambda_i \) depend only on the overall Higgs norm \( \Phi = \sqrt{\Phi_1^2 + \Phi_2^2} \).

Eqs. (4–6) represent a complete set of the MPP conditions. These conditions are satisfied identically in the minimal SUSY model (MSSM) at any scale lying higher than the masses of the superparticles. The MPP conditions (11–13) should be supplemented by the vacuum stability requirements:

\[
\lambda_1(\Phi) > 0, \quad \lambda_2(\Phi) > 0, \quad \tilde{\lambda}(\Phi) > 0,
\]

(7)

which must be satisfied everywhere between the MPP and electroweak scales. Otherwise another minimum of the Higgs effective potential arises at some intermediate scale, destabilising the physical and MPP scale vacua. Taking into account the MPP conditions (4–6) and substituting the vacuum configuration (3) into the quartic part of the 2HDM scalar potential, one finds that near the MPP scale vacua:

\[
\Phi_1 = \Lambda \cos \gamma, \quad \Phi_2 = \Lambda \sin \gamma, \quad \tan \gamma = \left( \frac{\lambda_1}{\lambda_2} \right)^{1/4}.
\]

(8)

It is also worth noting that the set of degenerate vacua (3) is realised only when \( \lambda_4(\Lambda) \) is negative.

### 3 MPP and custodial symmetries

The MPP conditions constrain the couplings of the Higgs fields to fermions. The observed mass hierarchy of quarks and charged leptons implies that the Yukawa interactions in the quark and lepton sectors have a hierarchical structure. Assuming that the Yukawa couplings of the quarks and leptons of the third generation are considerably larger than quark and lepton Yukawa couplings of the first two generations, the 2HDM Lagrangian describing the interactions of quarks and leptons with the Higgs doublets \( H_1 \) and \( H_2 \) reduces to

\[
\mathcal{L}' \simeq h_1(H_2\tilde{e}Q)^{\dagger} \tilde{e}_R + g_6(H_2^2 Q)^{\dagger} \tilde{b}_R + g_7(H_1^2 L)^{\dagger} \tilde{\tau}_R \quad (9)
\]

\[+g_4(H_1\tilde{e}Q)^{\dagger} \tilde{e}_R + h_0(H_1^2 Q)^{\dagger} \tilde{b}_R + h_7(H_1^2 L)^{\dagger} \tilde{\tau}_R + h.c.,\]

where \( Q \) and \( L \) are left–handed doublets of quarks and leptons of the third generation, while \( \tau_R, \tilde{\tau}_R \) and \( h_R \) are right–handed \( SU(2)_W \) singlet components of \( \tau–\) and \( b– \) quarks. It is always possible to choose a basis in the field space in which \( g_1(\Lambda) = 0 \). In this basis the MPP conditions (9) are fulfilled simultaneously only if

\[
\begin{align*}
(I) & \quad h_1(\Lambda) = h_2(\Lambda) = 0; \\
(II) & \quad g_6(\Lambda) = g_7(\Lambda) = 0; \\
(III) & \quad h_0(\Lambda) = g_7(\Lambda) = 0; \\
(IV) & \quad g_4(\Lambda) = h_7(\Lambda) = 0.
\end{align*}
\]

(10)

The solutions (I) – (IV) correspond to the 2HDM Model I (where only \( H_1 \) couples to the fermions) and Model II (where the couplings of \( H_1 \) and \( H_2 \) to quarks and leptons are the same as in the MSSM) Yukawa couplings and their leptonic variations.

Usually the existence of a large set of degenerate vacua is associated with an enlarged global symmetry of the Lagrangian of the considered model. The 2HDM is not an exception. In all models (I – IV) the quartic part of the Higgs effective potential (11) and \( \mathcal{L}' \) are invariant under extra \( U(1) \) (Peccei–Quinn) symmetry transformations, which prevent the appearance of FCNC transitions at the tree level.

The generalisation to the three family case requires more accurate consideration. When three generations of quarks and leptons have non–negligible couplings to the Higgs doublets, all SM bosons and fermions contribute to the Higgs effective potential which is convenient to present in the following form

\[
V_{\text{eff}}(H_1, H_2) = \sum_{n=0}^{\infty} V_n(H_1, H_2), \quad (11)
\]

where \( V_0 \) corresponds to the tree–level Higgs potential, while \( V_n \) represents the \( n–\)loop contribution to \( V_{\text{eff}} \). In the one–loop approximation we have

\[
V_1 = \frac{1}{64\pi^2} \text{Str} |M|^4 \left[ \log \frac{|M|^2}{\mu^2} - C \right]. \quad (12)
\]

Here the supertrace operator counts positively (negatively) the number of degrees of freedom for the different bosonic (fermionic) fields, \( C \) is a diagonal matrix which depends on the renormalization scheme while \( \mu \) is a renormalization scale. Before we restricted our consideration to the leading log approximation, i.e. we replaced \( \log \frac{|M|^2}{\mu^2} \) by \( \log \frac{\Phi^2}{\mu^2} \) and summed all leading logs.

Refs. [8]–[9]
using the renormalisation group equations. A more accurate analysis requires us to include all terms that are proportional \( \delta^4 \) in Eqs. \((11)\) and \((12)\).

The independence of the full Higgs effective potential \((11)\) on \( \omega \) at the scale \( \Lambda \) implies that the Lagrangian for the Higgs–fermion interaction is invariant under symmetry transformations (see Ref. [9]):

\[
H_1 \rightarrow e^{i\alpha} H_1, \quad d_R^i \rightarrow e^{i\alpha} d_R^i, \quad u_R^i \rightarrow e^{i\alpha} u_R^i, \quad \varepsilon_{Ri} \rightarrow e^{-i\alpha} \varepsilon_{Ri}, \quad \lambda_{Ri} \rightarrow \lambda_{Ri},
\]

\[
H_2 \rightarrow e^{i\beta} H_2, \quad d_R^i \rightarrow e^{-i\beta} d_R^i, \quad u_R^i \rightarrow e^{i\beta} u_R^i, \quad \varepsilon_{Ri} \rightarrow e^{-i\beta} \varepsilon_{Ri}, \quad \lambda_{Ri} \rightarrow \lambda_{Ri}, \quad \lambda_{6i} \rightarrow \lambda_{6i}, \quad \lambda_{7i} \rightarrow \lambda_{7i},
\]

where \( u_R^i, d_R^i, \varepsilon_{Ri} \) are right-handed quarks and leptons which couple to \( H_1 \) while \( u_R^r, d_R^r, \varepsilon_{Ri} \) are right-handed fermions that interact with \( H_2 \). The renormalisation group (RG) flow of Yukawa couplings does not spoil the invariance of the Lagrangian, describing the interactions of quarks and leptons with the Higgs bosons under the custodial symmetry transformations \((13)\).

The two global \( U(1) \) symmetries \((13)\) guarantee that each fermion eigenstate couples to only one Higgs doublet, which in turn ensures the absence of tree-level FCNC transitions. Thus MPP provides a reliable mechanism for the suppression of FCNC processes. The MPP solutions based on the custodial symmetries \((13)\) may be considered as generalisations of the well-known Peccei–Quinn symmetric solution of the FCNC problem in the 2HDM.

Being spontaneously broken at the electroweak scale, the custodial symmetries \((13)\) give rise to a massless axion which allows us to avoid CP–violation in the 2HDM, entirely eliminating the \( \theta \)–term in QCD. The mixing term \( m_3^2(H_1^\dagger H_2) \) in the Higgs effective potential \((11)\), which is not forbidden by the MPP, softly breaks the extra \( U(1) \) global symmetry. It spoils the solution of the strong CP problem in QCD, but does not create new sources of CP–violation or FCNC transitions. Indeed, in the Higgs sector of the general 2HDM only imaginary parts of \( m_3^2, \lambda_3, \lambda_6, \) and \( \lambda_7 \) cause CP–nonconservation. MPP suppresses the Higgs couplings, which are responsible for the violation of the CP–invariance. At the same time the complex phase of \( m_3^2 \) can be easily absorbed by the appropriate redefinition of the Higgs fields. In such a way MPP protects the CP–invariance within the two Higgs doublet extension of the SM. Tree–level FCNC transitions also do not emerge after the soft breakdown of the custodial symmetries \((13)\), simply because the structure of the interactions of quarks and leptons with the Higgs doublets remains intact.

There is one important feature that may distinguish the MPP inspired 2HDM from other two Higgs doublet models, where softly broken Peccei–Quinn (or \( Z_2 \)) custodial symmetry is postulated. In the MPP inspired 2HDM the Higgs and Yukawa couplings which violate custodial symmetries can have non–zero values, because the multiple point principle does not imply that physical and MPP scale vacua should be exactly degenerate. Since we ignore all mass terms in the 2HDM potential \((1)\) during the derivation of the MPP conditions, one can expect to get the degeneracy of vacua with the accuracy \( O(\nu^2\Lambda^2) \). This determines the allowed interval of variations of \( \lambda_3(\Lambda), \lambda_6, \lambda_7(\Lambda) \) and custodial symmetry violating Yukawa couplings \( g_i \):

\[
|\lambda_3(\Lambda)| \simeq |\lambda_6(\Lambda)| \simeq |\lambda_7(\Lambda)| \leq \frac{\nu^2}{\Lambda^2}, \quad |g_i(\Lambda)| \leq (4\pi)^2 \frac{\nu^2}{\Lambda^2}.
\]

Custodial symmetry violating Yukawa and Higgs couplings being set small at the scale \( \Lambda \) does not change much at any scale below \( \Lambda \). If \( \Lambda \) is quite close to the Planck scale then \( \lambda_5, \lambda_6, \) and \( g_i \) are extremely suppressed at the electroweak scale, so that FCNC and CP–violation effects could not be observed in the nearest future. However if \( \Lambda \simeq 100 \text{ TeV} \) then custodial symmetry violating Yukawa couplings may induce non–diagonal flavour transitions, which give rise to new channels of rare decays of heavy quarks and leptons that can be detected at future experiments.

4 Higgs phenomenology

At moderate values of the ratios \( v_2/v_1 \), where \( v_2 \) and \( v_1 \) are vacuum expectation values of \( H_2 \) and \( H_1 \) in the physical vacuum, the MPP conditions \((6)\) can be rewritten in the following form:

\[
\lambda_3(\Lambda) = -\sqrt{\lambda_1(\Lambda)\lambda_2(\Lambda)} - \lambda_4(\Lambda),
\]

\[
\lambda_2^2(\Lambda) = \frac{6h_1^2(\Lambda)\lambda_1(\Lambda)}{\left(\sqrt{\lambda_1(\Lambda)} + \sqrt{\lambda_2(\Lambda)}\right)^2} - 2\lambda_1(\Lambda)\lambda_2(\Lambda) - \frac{3}{8} \left(3g_2^2(\Lambda) + 2g_2^2(\Lambda)g_1^2(\Lambda) + g_1^4(\Lambda)\right).
\]

Thus, in the MPP inspired 2HDM, \( \lambda_3 \) and \( \lambda_4 \) are not independent parameters. As a result the considered 2HDM has less free parameters than the 2HDM of type II and therefore can be regarded as a minimal non–supersymmetric two Higgs doublet extension of the SM.

As follows from Eq. \((15)\), the RG flow of all couplings in the MPP inspired 2HDM is determined by \( \lambda_1(\Lambda), \lambda_2(\Lambda) \) and \( h_i(\Lambda) \). When \( h_i(\Lambda) > 1 \) the solutions of the RG equations for the top quark Yukawa coupling are concentrated in the vicinity of the quasi–fixed point at the electroweak scale. The value of the ratio \( \tan \beta \) that corresponds to the quasi–fixed point scenario depends mainly on the MPP scale \( \Lambda \). It varies from 1.1 to 0.5 when \( \Lambda \) changes from \( M_{Pl} \) to 10 TeV \([10]\). At large values of \( h_i(\Lambda) \), the MPP and vacuum stability conditions constrain \( \lambda_i(\Lambda) \) very strongly. Our numerical studies show that, for \( \Lambda = M_{Pl} \) and \( \lambda_1(M_{Pl}) = \lambda_2(M_{Pl}) = \lambda_0 \), the ratio \( \lambda_0/h_i(M_{Pl}) \) can vary only within a very narrow interval from 0.79 to 0.87 if \( h_i(\Lambda) > 1.5 \). This ensures the convergence of the solutions of the RG equations for \( \lambda_i(\mu) \) to the quasi–fixed points.

The Higgs spectrum of the two Higgs doublet extension of the SM contains two charged and three neutral scalar states. Because in the MPP inspired 2HDM
The lightest Higgs scalar in the considered case is predominantly a SM–like Higgs boson, since its coupling to a Z pair is rather close to the SM one. Nevertheless at low MPP scales the quasi–fixed point scenario leads to large values of the coupling of the lightest Higgs scalar to the top quark, resulting in the enhanced production of this particle at hadron colliders (see Fig. 2). Thus the analysis of production and decay rates of the SM–like Higgs boson at the LHC should make possible the distinction between the quasi–fixed point scenario in the MPP inspired 2HDM with low scale λ, the SM and the MSSM even if extra Higgs states are relatively heavy, i.e. $m_A \approx 500 – 700$ GeV.

5 Conclusions

We have considered the application of the multiple point principle to the non–supersymmetric two-Higgs doublet extension of the SM. A complete set of the MPP conditions that leads to the realisation of the MPP assumption has been presented. We have shown that the existence of a large set of degenerate vacua at some high energy scale Λ, caused by the MPP, results in approximate custodial symmetries which suppress FCNC and CP violating phenomena in the 2HDM.

The spectrum and couplings of the Higgs bosons in the MPP inspired 2HDM have been also studied. When $h_i(\Lambda) > 1$ the solutions of the RG equations in the considered model converge to the quasi–fixed point, leading to stringent restrictions on the lightest Higgs boson mass. In the quasi–fixed point scenario the Higgs couplings to the $t$–quark can be significantly larger than in the SM, which allows us to test this scenario at hadron colliders.

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