On scattering of CMB radiation on wormholes: kinetic SZ-effect

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The problem of scattering of CMB radiation on wormholes is considered. It is shown that a static gas of wormholes does not perturb the spectrum of CMB. In the first order by $v/c$ the presence of peculiar velocities gives rise to the dipole contribution in $\Delta T/T$, which corresponds to the well-known kinetic Sunyaev-Zel’dovich effect. In next orders there appears a more complicated dependence of the perturbed CMB spectrum on peculiar velocities. We also discuss some peculiar features of the scattering on a single wormhole.

I. INTRODUCTION

As it was shown recently all features of cold dark matter models (CDM) can be reproduced by the presence of a gas of wormholes \cite{1,2}. At very large scales wormholes behave exactly like very heavy particles, while at smaller subgalactic scales wormholes strongly interact with baryons and cure the problem of cusps. Moreover, there are some strong theoretical arguments, which come from lattice quantum gravity, that the topological structure of our Universe should have fractal properties \cite{3}. Therefore, we may claim that up to date wormholes give the best candidate for dark matter particles. The final choice between different dark matter candidates requires the direct observation of effects related to wormholes.

Presumably cosmological wormholes are not very large (otherwise they would be directly seen on the sky). In the present paper we consider the scattering of CMB radiation on wormholes and show that they can be observed by means of the kinematic Sunyaev-Zel’dovich effect (kSZ) \cite{4,5,6}. KSZ signal is long used to study peculiar motions of galaxy clusters and groups and has already long history, e.g., see \cite{6,7} and references therein. It has a universal nature, i.e., it is produced by any kind of matter which scatters CMB (not only by hot gas). In this respect, it is rather difficult separate the contribution of wormholes into kSZ from that of the electron gas in clusters and groups. Therefore, we think that one has to look for such an effect in those spots on the sky where the baryonic matter is absent, e.g., in voids where the number density of wormholes should have the biggest value and the leading contribution will come from wormholes alone. Indeed, if we accept the fractal topological structure and the gas of wormholes as the basic DM candidate, then in voids wormholes push out (or replace) baryons, which explains why there is no galaxies in voids. There remains also the possibility to study peculiar features of the scattering of CMB on wormholes.

As it was demonstrated recently in \cite{2} stable cosmological wormholes have throat sections in the form of tori. In the present paper we however restrict to the simplest spherically symmetric wormholes. The spherical wormholes have a more symmetric form and can be viewed as a torus averaged over its orientations. Upon averaging out, some peculiar features of interest will disappear (we present them elsewhere), nevertheless, the basic kSZ effect remains.

II. CROSS-SECTIONS AND KSZ-EFFECT

The scattering of signals on spherical wormholes has been already studied e.g., \cite{8,10} see also recent papers \cite{11,12}. The basic two important features is the generation of a specific interference picture (upon scattering on a single wormhole) and the generation of a diffuse halo around any discrete source. Unfortunately, both features are not so good to use them in the searching for wormholes (the first one gives too weak signal, while the second feature may have various interpretations).

Consider first the case of a static gas of wormholes, i.e. in the absence of peculiar motions. The spherical wormhole can be considered as a couple of conjugated spherical mirrors, when a relict photon falls on one mirror it is emitted, upon the scattering, from the second (conjugated) mirror. The cross-section of such a process has been described by us earlier in \cite{13} and can be summarized as follows. Let an incident plane wave (a set of photons) falls on one throat. Then the scattered signal has the two parts. First part represents the standard diffraction (which corresponds to the absorption of CMB photons on the throat) and forms a very narrow beam along the direction of the propagation. This is the so-called scattering forward which is described by the cross-section

$$\frac{d\sigma_{\text{absor}}}{d\Omega} = \sigma_0 \frac{(ka)^2}{4\pi} \left| \frac{2J_1(ka \sin \chi)}{ka \sin \chi} \right|^2,$$  

(1)

where $\sigma_0 = \pi a^2$, $a$ is the radius of the throat, $k$ is the wave vector, and $\chi$ is the angle from the direction of propagation of the incident photons, and $J_1$ is the Bessel function. Together with this part the second throat emits an omnidirectional isotropic flux with the cross-section

$$\frac{d\sigma_{\text{emit}}}{d\Omega} = \sigma_0 \frac{1}{4\pi}.$$  

(2)

It is easy to check that the total cross-sections coincide

$$\int d\sigma_{\text{absor}} d\Omega = \int d\sigma_{\text{emit}} d\Omega = \sigma_0$$.
which means the conservation law for the number of photons (the number of absorbed and emitted photons coincides). This is enough to understand what is going on with CMB in the presence of the gas of wormholes. For the static gas (in the absence of peculiar motions) every wormhole throat end absorbs photons as the absolutely black body, while the second end of the throat re-radiates them in an isotropic manner (with the same black body spectrum). It is clear that there will not appear any distortion of the CMB spectrum at all. In other words, we may say that in the absence of peculiar motions the distortion of the spectrum does not occur.

Consider now the presence of peculiar motions. The motion of one end of the wormhole throat with respect to CMB causes the angle dependence of the incident radiation with the temperature

\[ T_1 = \frac{T_\gamma}{\sqrt{1 - \beta_1^2}} (1 + \beta_1 \cos \theta_1) \approx T_\gamma (1 - \frac{1}{2} \beta_1^2 \cos \theta_1 + ...) \]

where \( \beta_1 = V_1/c \) is the velocity of the throat end and \( \beta_1 \cos \theta_1 = \left( \frac{\beta_1 \vec{n}}{||\vec{n}||} \right) \), \( \vec{n} \) is the direction for incident photons. Therefore, the absorbed radiation has the spectrum

\[ \rho(T_1) = \rho(T_\gamma) + \frac{d\rho(T_\gamma)}{dT} \Delta T_1 + \frac{1}{2} \frac{d^2\rho(T_\gamma)}{dT^2} \Delta T_1^2 + ... , \]

where \( \rho(T_\gamma) \) is the standard Planckian spectrum and \( \Delta T_1 (\beta_1 \cos \theta_1) = T_1 - T_\gamma \). It turns out that in the first order by \( \beta_1 \) such an anisotropy gives no contribution in the re-radiated photons and does not contribute in the distortion of the spectrum. Indeed, in the reference system in which the second end of the throat is at rest we have the isotropic flux \( \rho(T_\gamma) \) and, therefore, integrating over the incident angle \( \theta_1 \) we find \( \int \Delta T (\cos \theta_1) d\Omega = 0 \). This means that in the first order by \( \beta_1 \) the second end of the throat radiates (in the rest reference system) the same black body radiation with the same temperature \( T_\gamma \). In next orders by \( \beta_1 \) there appears a non-vanishing contribution to the distortion of the spectrum \( \rho(T_1) - \rho(T_\gamma) \). However, in next orders a more important features will appear, when we consider the actual wormhole sections in the form of tori. Therefore, we leave next orders for the future research.

Consider now the re-radiation of the absorbed CMB photons. In the first order by \( \beta_2 = V_2/c \) (\( V_2 \) is the peculiar velocity of the second end of the wormhole throat) it radiates the black body radiation with the apparent surface temperature (brightness)

\[ T_2 \approx T_\gamma (1 + \beta_2 \cos \theta_2 + ...) \tag{3} \]

where \( \beta_2 \cos \theta_2 = \left( \frac{\beta_2 \vec{m}}{||\vec{m}||} \right) \) and \( \vec{m} \) is the unit vector pointing out to the observer. This is exactly the kinematic Sunyaev-Zel’dovich effect.

Consider a collection (cloud) of wormhole throats. To obtain the net energy-momentum transfer between the CMB radiation and the gas of wormholes we have to average over the wormhole distribution. On average CMB photons undergo \( \tau_w \) scatterings, where \( \tau_w \) is the projected cloud optical depth due to the scattering. If \( n(r) \) is the number density of wormholes measured from the center of the cloud, then \( \tau_w \) is given by

\[ \tau_w = \pi a^2 \int n(r) dl \]

where the integration is taken along the line of sight and

\[ a^2 = \frac{1}{n} \int a^2 n(r,a) da. \]

Here \( n(r,a) \) is the number density of wormholes depending on the throat radius \( a \). The optical depth \( \tau_w \) may be also interpreted as follows. Let \( L \) be the characteristic size of the cloud of wormholes. Then on the sky it will cover the surface \( S \sim L^2 \), while the portion of this surface covered by wormhole throats is given by

\[ \tau_w = \frac{N \pi a^2}{L^2} = \pi a^2 \pi L, \]

where \( N \) is the number of wormhole throats in the cloud and \( \pi \) is the mean density. Since all wormhole throats have the surface brightness \( \beta_2 \) which is different from that of CMB, the parameter \( \tau_w \) defines (together with the peculiar velocity of the cloud \( \beta_2 \)) the surface brightness of the cloud itself.

III. THE SCATTERING OF CMB ON A SINGLE WORMHOLE

In the case of a single wormhole we should account for the two important features. The first feature is the fact that a stable cosmological wormhole has the throat section in the form of a torus. We point out that under the stable cosmological wormhole we mean here the wormhole, which does not require the presence of any form of exotic matter (save baryons), but which is involved in the cosmological expansion. Stability of such a wormhole is described by the standard Lifshitz theory and from the qualitative standpoint it does not differ from the development of cosmological primordial perturbations. Therefore, if such a wormhole is sufficiently big, then the simplest way to find it is to look for the direct imprints on CMB map. Indeed, by means of kSZ effect a wormhole should produce a ring on CMB map which has slightly different from the background temperature. In particular, it was reported recently in [14], that there are, with confidence level 99.7 per cent, such ring-type structures in the observed cosmic microwave background. We hope that such structures could be imprints of cosmological wormholes indeed.

The second important feature is that the scattering forward (i.e. absorption of CMB photons) produces much bigger effect (since \( kR \gg 1, ka \gg 1 \), where \( k \) is the wave-vector, \( R \) is the largest, and \( a \) is the smallest radiiuses of the torus respectively). This effect corresponds
to the standard diffraction on the torus-like obstacle. In the approximation \( \mu = a/R \ll 1 \), where \( a \) is the smallest radius of the torus, we may use the flat screen approximation.

Let the orientation of the torus (the normal to the torus direction) be along the \( Oz \) axis, i.e. \( m = (0,0,1) \). The cross-section depends on the two groups of angle variables, i.e. the two unit vectors \( n_0(\phi_0, \theta_0) \) and \( n(\phi, \theta) \). The vector \( n_0 = (\cos \phi_0 \sin \theta_0, \sin \phi_0 \sin \theta_0, \cos \theta_0) \) points to the direction of the incident photon (i.e., the wave vector is \( k_0 = \frac{\omega}{c}n_0 \)), while the vector \( n \) corresponds to the scattered photons. Then the cross-section is given by

\[
\frac{d\sigma}{d\Omega} = \sigma_R \sin^2 \theta_0 \frac{(kR)^2}{4\pi} \left( 1 + \cos^2 \theta \right) |F|^2,
\]

where \( \sigma_R = \pi R^2 \), and the function \( F \) is

\[
F = (1 + \mu) \frac{2 J_1((1 + \mu) x)}{(1 + \mu) x} - (1 - \mu) \frac{2 J_1((1 - \mu) x)}{(1 - \mu) x}
\]

where \( x = kR\xi \). We also denote

\[
\xi = (\sin^2 \theta + \sin^2 \theta_0 - 2 \sin \theta \sin \theta_0 \cos(\phi - \phi_0))^{1/2}
\]

and \( J_n(x) \) are the Bessel functions. We also averaged \( \sigma \) over polarizations. Let us expand the kernel \( F \) by the small parameter \( \mu \ll 1 \) which gives

\[
F \approx 2\mu \left( x \left( \frac{J_1(x)}{x} \right) + \frac{2 J_1(x)}{x} \right).
\]

Using the property \( (J_\nu(x)/x^\nu)' = -J_{\nu+1}(x)/x^\nu \) we find

\[
F \approx 2\mu \left( -2J_2(x) + \frac{2 J_1(x)}{x} \right)
\]

and from the identity \( J_2(x) = \frac{2}{x}J_1(x) - J_0(x) \) we get \( F \approx 4\mu J_0(x) \) which gives

\[
\frac{d\sigma}{d\Omega} = 8\sigma_R \frac{(ka)^2}{4\pi} \left( 1 - \cos^2 \theta_0 \right) \left( 1 + \cos^2 \theta \right) |J_0(kR\xi)|^2.
\]

Since \( kR \gg 1 \), the above expression shows the presence of specific ring-type oscillations in the cross-section. Indeed, if we consider the normal fall of the incident photons, i.e., \( \theta_0 = 0 \), then we find \( \frac{d\sigma}{d\Omega} \sim (1 + \cos^2 \theta)J_0(kR\theta) \).

### IV. DISCUSSIONS

Now the basic question arises where we should look for wormholes? This problem resembles the traditional trial in Russian folklore ("go there, we do not know where and bring us that we do not know what"). What we should find out it is more or less fair. When we will have enough sensitivity to see effects of a single wormhole, we may observe a specific features of the scattering of CMB on a wormhole section in the form of a torus (specific rings, etc.). In this respect preliminary result in [14] looks very optimistic though it requires an independent confirmation. The more straightforward way is to observe the collective KSZ effect discussed above. There we meet two basic problems. First one is the unfair predictions of such an effect. Indeed, we know a little about the density of wormholes \( n_w \) and the characteristic size of sections \( \pi \) (or \( \sigma_0 = \pi \sigma^2 \)). The fractal distribution of galaxies and the behavior of dark matter in galaxies may fix the two another parameters [3] by means of measuring the empirical Green function

\[
G_{emp} = \frac{-4\pi}{k^2 (1 + (Rk)^{-\alpha})}
\]

which describes deviation from the Newton’s law (at small scales \( (Rk) \gg 1 \) it gives the standard Newton’s law, while at large scales \( Rk \ll 1 \) it transforms to the fractal law, or logarithmic behavior). We point out that the logarithmic correction observed in galaxies corresponds to the value \( \alpha \approx 1 \) and \( R \sim 5Kpc \) [13]. However, it is still not quite clear how we may extract the safe estimate for the optical depth \( \tau_w \). The gas of wormholes may be described by at least tree scale parameters (the characteristic size of the throat section \( \pi \), the number density \( n_w \), and the characteristic distance between wormhole entrances \( \ell \)), therefore, the two parameters \( (R \) and \( \alpha \) are not enough to fix all parameters of the gas of wormholes and define \( \tau_w \). In other words, this problem requires the further study.

The second problem closely relates to the problem where we should look for wormhole effects. In galaxies and clusters (as well in the hot X-ray gas) the KSZ effect based on wormholes mixes with that on other sorts of matter (dust, hot gas, etc.). Therefore, the best way to look for wormhole effects is to search them in voids, where the baryonic matter is less dense.

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