Cross-scale identification for friction and wear boundaries of tooth flank of high-feed milling cutter

Bin Jiang, Junwei Jia, Peiyi Zhao, Feifei Li and Lili Fan

Abstract
In the milling process, under the high-efficiency and intermittent cutting loads, the instantaneous posture of the milling cutter and the instantaneous cutter-workpiece engagement are in an unstable state. It is difficult to accurately identify the friction and wear boundary of the cutter tooth flank. It has become an urgent problem to be solved in the life evaluation of milling cutters. The characteristics of the instantaneous posture variation of the milling cutter and the distribution of the thermal-stress mechanical coupling field of the flank of cutter tooth under the vibration were studied. Using the cross-scale load transfer method, the cross-scale relationship between the macro and micro friction loads on the cutter tooth flank was realized. The number of broken valence bonds, the sudden change of potential energy and the attenuation of elastic modulus of the super-cell on the flank of cutter tooth were studied. The mesoscopic scale identification method of the friction and wear boundary feature points of the tooth flank was proposed. The instantaneous and cumulative boundary distribution of friction and wear on the flank of cutter tooth were obtained, and verified by experiments.

Keywords
Milling cutter, flank of cutter tooth, friction and wear boundary, cross-scale identification

Introduction
High-feed milling cutter is a typical high efficiency cutting tool, which is widely used in the machining of large structural parts. In the process of high efficiency milling, under the milling cutter cutting into and out of the workpiece, the instantaneous contact relationship between the milling cutter tooth and the workpiece is in an unstable state. At the same time, the variation of vibration in the cutting process makes the friction process of the flank of the cutter tooth is unstable. It results in the ambiguity of the friction and wear boundaries, and the difficulty in assessing the life of milling cutter.

Under the combined action of vibration and cutting load, the friction and wear of the flank of high-feed milling cutter tooth are oriented from the super-cell damages in mesoscope. Then evolved to the macroscope, forming a macroscopic friction and wear area. Therefore, studying the deformation and damage of the super-cell on the flank of the cutter tooth, cross-scale identification method of flank friction and wear boundary was proposed. It had important significance for revealing the formation and evolution of friction and wear of high-feed milling cutter.
Scholars at home and abroad have made many contributions in milling cutter friction and wear state, boundary identification, and cross-scale research on milling cutter wear. Zhu et al.\textsuperscript{10–13} proposed a method for detecting cutter tooth wear based on the cutter tooth wear area. The results showed that the predicted wear area can summarize the cutter tooth wear degree and effectively prevent cutter tooth wear. Wu et al.\textsuperscript{14} conducted wear experiments in micro-milling. They observed the wear characteristics of the milling cutter, and analyzed the wear mechanism. The results showed that the cutter wear was concentrated on the tip of the milling cutter. A wear band was formed on the bottom surface of the milling cutter. Altan et al.\textsuperscript{15–17} studied the effectiveness of cutting parameters on each wear area in the flank wear curve of the milling cutter to improve the service life of the cutter tooth. The results showed that in the initial wear area of the milling cutter, the wear is most seriously affected by the feed rate. In the steady-state cutter tooth wear area, the wear is most seriously affected by the cutting speed. Chang et al.\textsuperscript{18} studied the evolution of the surface integrity of the cutter tooth at different wear stages. The results showed that the surface integrity and wear resistance of the treated cutter tooth were improved. The change of the cutter tooth wear was consistent with the change of the coating surface integrity. Scholars at home and abroad have conducted extensive research on the friction and wear of milling cutters, which has laid a foundation for the study of structural damage at small scales.

This study proposed a method for calculating the instantaneous milling posture of the flank of cutter tooth under vibration. According to the thermal-stress coupling field distribution of the flank in the milling process, a method for calculating the characteristic velocity vector of the flank was proposed. The stresses of the feature points of the flank was obtained. According to the cross-scale method, the effective correlation between the macroscopic and mesoscopic load was realized. The variation of the super-cell during the deformation and damage process were studied, and the local and integrity damage criteria were constructed. The feature points of the friction and wear boundary was identified. The instantaneous and cumulative friction and wear boundaries were obtained. The calculation results of the cumulative and cumulative friction and wear boundary on the flank face and the experimental results were compared. Then the accuracy of the cross-scale identification method of the friction and wear boundary on the flank face was verified by experiments.

High-feed milling cutter flank feature point trajectory and milling vibration experiments

The high-feed milling cutter and its tooth structure are shown in Figure 1.

In Figure 1, $o_x$, $x$, $y$, $z$ is the coordinate system of the milling cutter structure. Where $o_x$ is the cutter tooth rotation center, the cutter origin. $o_x$, $y$, $z$ is cutter tooth coordinate. The coordinate origin $o_i$ is the tip point of the $i$th cutter tooth of the milling cutter. $x_i$ axis parallel to the lower boundary of the cutter flank. $y_i$ axis passes the tip of the cutter tooth and is perpendicular to the lower boundary of the cutter flank. $z_i$ axis is perpendicular to $x_i$ and $y_i$ axis. $r_i$ is the nominal milling radius of any cutter tooth of the milling cutter. $r_{\text{max}}$ is the maximum turning radius of milling cutter tooth. $\theta_i$ is the angle between milling cutter tooth. $\alpha_i$ is the milling cutter tooth installation angle. $\Delta z_i$ is the axial error of the cutter tooth. $\Delta r_i$ is the radial error of milling cutter tooth. $D$ is the diameter of milling cutter handle. $H(x_i, y_i, z_i) = 0$ is the flank of cutter tooth equation.
The $y_{q}$ axis is an axis perpendicular to the installation and positioning plane, which is along the radial direction of the cutter tooth of the milling cutter and is tangent to the tip point of the cutter tooth near the milling cutter center. The $x_{q}$ axis is the axis perpendicular to the $y_{q}$ axis, and it lies in the mounting positioning plane. $x_{1}-x_{N}$ was divided equidistantly along the $x_{q}$ direction. The curve $y_{1}-y_{N}$ were divided along the $y_{q}$ direction with an equidistant offset. The feature point of the left end point of the flank along the $x_{q}$ axis was marked as feature point $o_{a1}$-$o_{AN}$. The feature point along the opposite direction of the $y_{q}$ axis were marked as $o_{a1}$-$o_{AM}$ in turn, as shown in Figure 2.

During the milling process, the cutting attitude of the milling cutter was offset due to the influence of the milling vibration. The milling cutter cutting reference coordinate and cutting attitude model under the vibration were established, as shown in Figure 3.

In Figure 2, $o_{c}x_{g}y_{g}z_{g}$ is the workpiece coordinate, where $x_{g}$ is consistent with the feed speed direction of the milling cutter, $y_{g}$ parallel to cut width, and $z_{g}$ is consistent with the cutting depth direction. The cutting coordinate $o_{0}$-$x_{0}y_{0}z_{0}$ of the milling cutter without vibration is established with the rotation center $o_{0}$ of the milling cutter under the vibration-free condition as the origin, where the $x_{0}$, $y_{0}$, and $z_{0}$ are respectively parallel to the $x_{g}$, $y_{g}$, and $z_{g}$ of the workpiece coordinate. Under the action of milling vibration, the origin of the milling cutter coordinate becomes $o_{d}$. The cutting coordinate $o_{c}x_{d}y_{d}z_{d}$ of milling cutter with vibration is established, where $x_{d}$, $y_{d}$, and $z_{d}$ axis are the offsets of $x_{0}$ axis, $y_{0}$ axis, and $z_{0}$ caused by vibration. $o_{c}x_{c}y_{c}z_{c}$ is the milling cutter structure coordinate, where $o_{c}$ is the cutter tooth rotation center point, $x_{c}$ is $o_{c}$ to the cutter tooth milling radius maximum tip point, $z_{c}$ parallel to $z_{g}$, $y_{c}$ perpendicular to $x_{c}$ axis and $z_{c}$, $o_{0}$ respectively are the starting point and end point of the milling cutter overhang. $l$ is the milling cutter overhang. $\theta(t)$ is the instantaneous position angle when the milling cutter cuts into the workpiece. $\theta_{c}(t)$ is the instantaneous attitude angle of milling cutter under vibration, $e_{oa}(t)$ is the projection of the $z_{d}$ on the plane $z_{d}\theta(t)x_{0}$, $e_{ob}(t)$ is the projection of the $z_{d}$ on the plane $z_{d}\theta(t)y_{0}$. Where the angle between $e_{oa}(t)$ and $z_{d}$ axis is $\theta_{oa}(t)$, and the angle between $e_{bc}$ and $z_{0}$ axis is $\theta_{bc}(t)$; $A_{c}(t)$, $A_{d}(t)$ are the vibration displacements on $x_{0}$, $y_{0}$, and $z_{0}$ axes respectively.

From Figure 3, the trajectory equation of the feature point of the flank in the workpiece coordinate is:

$$[x_{g} \ y_{g} \ z_{g} \ 1]^{T} = M_{1}M_{2}T_{2}T_{1}M_{1}T_{1}[x_{q} \ y_{q} \ z_{q} \ 1]^{T}$$ (1)

In the equation, $M_{1}$, $M_{2}$, and $M_{3}$ are translation matrices, and $T_{1}$, $T_{2}$, $T_{3}$, and $T_{4}$ are rotation matrices, as shown in equations (2) to (4):
A high-feed milling cutter with a diameter of 32 mm was used to conduct the milling experiment of titanium alloy. Before the milling experiments, the axial error and radial error of the cutter tooth were measured. The milling experiment parameters are shown in Table 1.

In Figure 4, the parameters of seven sampling cycles are shown in Table 2.

According to Figure 4 and Table 2, the vibration acceleration signals of seven cycles are extracted, as shown in Figure 5.

It can be seen from Figure 5 that the change characteristics of the milling vibration signal in different milling cycles are not the same, which directly affects the contact relationship between the cutter tooth and the workpiece, resulting in unstable friction and wear area of the flank of cutter tooth.

**Distribution of thermal-stress coupling field on flank of high-feed milling cutter tooth**

The finite element model and boundary conditions of the milling cutter were constructed by the above models and experimental results. Then the thermal-stress coupling field analysis of the milling cutter was carried out. It is found that the distribution area of the thermal-stress coupling field on the flank was more than 2.56 mm in $y_q$. Therefore, the origin of coordinates was defined as $q_o'$ along $y_q$. The distribution of the thermal-stress coupling field of the finite element cutter tooth at different instantaneous position angles was intercepted in each cycle. Among them, the thermal-stress coupling field distribution of cutter tooth 1 at different instantaneous position angles in cycle 3 of Table 2 is shown in Figure 6.

According to Figure 6, under the cutter tooth installation angle and the cutter tooth edge structure, with
Figure 5. Vibration time domain signals of cycle 1 to cycle 7: (a) cycle 1, (b) cycle 2, (c) cycle 3, (d) cycle 4, (e) cycle 5, (f) cycle 6, and (g) cycle 7.

Figure 6. Thermal-stress coupling field of cutter tooth 1 (cycle 3): (a) $\varphi = 5^\circ$, (b) $\varphi = 31^\circ$, (c) $\varphi = 57^\circ$, and (d) $\varphi = 85^\circ$. 
the increase of the cutter tooth instantaneous position angle in 1 milling cycle, the cutter tooth participated in the cutting area from the inside to the outside along the radial direction of the milling cutter.

According to the distribution of the thermal-stress coupling field of the cutter tooth, the stress range of the cutter tooth is relatively large in the cutter tooth mid-cutting stage of the milling cycle. Taking the third milling cycle of tooth 1 as an example, extract the equivalent stress area above 2000 MPa in the instantaneous thermal-stress coupling field on the flank of the cutter tooth, as shown in Figure 7.

According to Figure 7, due to the influence of the cutter tooth structure, the force position of the cutter tooth was biased toward the inner side of the cutter tooth when it initially cut into the workpiece. In a milling cycle, the boundary of the equivalent stress distribution was shifted from the inside to the outside along the radial direction of the milling cutter. At different instantaneous position angles in the same cycle, the propagation range of the effective stress curves become larger and then smaller.

The 2000 MPa effective stress curves of the instantaneous thermal-stress coupling field of the flank under the conditions of different cutting cycles of different tooth were extracted, as shown in Figures 8 and 10.

According to Figures 8 to 10, under the influence of milling vibration and cutter tooth errors, the instantaneous posture of milling cutter and cutter tooth and cutter contact state were different. The effective stress distribution states of milling cutter tooth under different cycle conditions of the same cutter tooth load were different. There were obvious differences in the boundary distribution state of effective stress on the flank of different tooth.
Calculation method of instantaneous characteristic velocity vector and stress of flank of cutter tooth

For the feature point in the equivalent stress curve, the characteristic velocity vector model of the flank feature point relative to the machining transition surface was established, as shown in Figure 11.

In the Figure 11, \( G(x_g(t), y_g(t), z_g(t)) = 0 \) is the machining transition surface equation. \( H(x_g(t), y_g(t), z_g(t)) = 0 \) is the flank equation of the cutter tooth. \( P(x_g(t), y_g(t), z_g(t)) = 0 \) is the equation of the common tangent plane between the flank of the passing tooth and the machining transition surface.

According to equation (1), the instantaneous movement speed \( v_s' \) of the feature point of the cutter tooth flank relative to the workpiece was obtained along the \( x_g, y_g, z_g \) components \( v_{sx}, v_{sy}, v_{sz} \), respectively, as shown in equation (5).

\[
v_{sx}(t) = \frac{\partial x_g(t)}{\partial t}, \quad v_{sy}(t) = \frac{\partial y_g(t)}{\partial t}, \quad v_{sz}(t) = \frac{\partial z_g(t)}{\partial t}
\]

Using \( v_{sx}, v_{sy}, v_{sz} \), the instantaneous movement speed \( v_s' \) of the cutter tooth relative to the workpiece was obtained, as shown in equation (6).

\[
v_s = \sqrt{v_{sx}^2 + v_{sy}^2 + v_{sz}^2}
\]

According to equations (5) and (6), the instantaneous movement speed \( v_s' \) in the workpiece coordinate system could be obtained.

Using equations (2) to (4), the cutter tooth flank equation \( H(x_g(t), y_g(t), z_g(t)) = 0 \) in the workpiece coordinate system and the machining transition surface equation \( G(x_g(t), y_g(t), z_g(t)) = 0 \) simultaneously, the common tangent equation \( P(x_g(t), y_g(t), z_g(t)) = 0 \) was obtained, as shown in equation (7).

\[
P(x_g(t), y_g(t), z_g(t)) = \begin{cases} H(x_g(t), y_g(t), z_g(t)) = 0 \\ G(x_g(t), y_g(t), z_g(t)) = 0 \end{cases}
\]

In the cutter tooth coordinate system, the normal vector \( P_j \) of the feature point of the cutter tooth flank was solved by using the cutter tooth flank equation. Using equations (2) to (4), \( P_j' \) is converted to the workpiece coordinate system, denoted as \( P_j' \). In the workpiece coordinate system, the angle between the normal vector and the \( z_i \) axis of the tool tooth coordinate system is \( \theta_j \).

\[
\theta_j = \arccos \frac{P_j' \cdot z_i'}{|P_j'| \cdot |z_i'|}
\]
Figure 12. Macroscopic stress calculation model of tooth flank: (a) flank feature point normal vector, (b) normal stress and tangential stress, and (c) feature point stress decomposition.

Where, $\mathbf{z}_j$ is the unit vector of the $z_j$ axis in the workpiece coordinate system.

The characteristic velocity vector $\mathbf{v}_m = \mathbf{v}_s - v_{ij} \cdot \left( \frac{\mathbf{p}_j}{\mathbf{p}_j} \right)$ is the projection of the instantaneous movement speed $\mathbf{v}_s$ on the common tangent plane, which can be expressed as:

$$\mathbf{v}_m = \mathbf{v}_s - v_{ij} \cdot \left( \frac{\mathbf{p}_j}{\mathbf{p}_j} \right)$$

$$v_{ij} = \left| \mathbf{v}_s \right| \cdot \cos \theta_{js}$$

$$\theta_{js} = \arccos \left( \frac{\mathbf{p}_j \cdot \mathbf{v}_s}{\left| \mathbf{p}_j \right| \cdot \left| \mathbf{v}_s \right|} \right)$$

Where, $\theta_{js}$ is the angle between the instantaneous movement speed vector $\mathbf{v}_s$ and the normal vector $\mathbf{P}_j$.

The feature point stress component was projected on the characteristic velocity vector direction, and the tangential stress in the characteristic velocity direction of the feature point on the flank of the cutter tooth was calculated, as shown in Figure 12.

From Figure 12, using the finite element stress extraction results, the normal stress $\sigma_N$ of the characteristic point of the flank of the cutter tooth was calculated, as shown in equation (12).

$$\sigma_N = \sigma_x \cdot \cos \theta_{px} + \sigma_y \cdot \cos \theta_{py} + \sigma_z \cdot \cos \theta_{pz}$$

$$\theta_{px} = \arccos \left( \frac{\mathbf{p}_j \cdot \sigma_x}{\left| \mathbf{p}_j \right| \cdot \left| \sigma_x \right|} \right), \quad \theta_{py} = \arccos \left( \frac{\mathbf{p}_j \cdot \sigma_y}{\left| \mathbf{p}_j \right| \cdot \left| \sigma_y \right|} \right),$$

$$\theta_{pz} = \arccos \left( \frac{\mathbf{p}_j \cdot \sigma_z}{\left| \mathbf{p}_j \right| \cdot \left| \sigma_z \right|} \right)$$

Where, $\theta_{px}$, $\theta_{py}$, and $\theta_{pz}$ are the instantaneous equivalent stress vectors of the feature point of the flank of the cutter tooth along the reverse direction of $x_g$, the reverse direction of $y_g$, and the direction of $z_g$, respectively.

From Figure 12, using the equivalent stress extracted by the finite element, the tangential stress $\tau_m$ in the characteristic velocity direction of the characteristic point of the flank of the cutter tooth was calculated, as shown in equation (14).

$$\tau_m = \sigma_x \cdot \cos \theta_{mx} + \sigma_y \cdot \cos \theta_{my} + \sigma_z \cdot \cos \theta_{mz}$$

$$\theta_{mx} = \arccos \left( \frac{\mathbf{v}_y \cdot \sigma_x}{\left| \mathbf{v}_y \right| \cdot \left| \sigma_x \right|} \right), \quad \theta_{my} = \arccos \left( \frac{\mathbf{v}_z \cdot \sigma_y}{\left| \mathbf{v}_z \right| \cdot \left| \sigma_y \right|} \right),$$

$$\theta_{mz} = \arccos \left( \frac{\mathbf{v}_x \cdot \sigma_z}{\left| \mathbf{v}_x \right| \cdot \left| \sigma_z \right|} \right)$$

Where, $\theta_{mx}$, $\theta_{my}$, and $\theta_{mz}$ are the angles between the characteristic velocity vector and $\sigma_x$, $\sigma_y$, and $\sigma_z$, respectively.

Super-cell model and its failure criterion of the feature point of the flank of the cutter tooth

In order to study the influence of cutting load on the super-cell (expansion of the original cell to form new repeating units) structure of flank of the high-feed cutter tooth, the element content and proportion of flank of the cutter tooth were detected by scanning electron microscope and energy dispersive spectrometer, as shown in Figure 13. The appropriate lattice parameters of TiAlN were determined and the super-cell model of TiAlN coating on the flank of the cutter tooth was established by using ICSD Findit software.
According to the minimum energy theory, the super-cell model on the flank of the cutter tooth was optimized to reduce the stress inside the atomic group and make the super-cell reach a stable state. High temperature relaxation method and fast condensation method were adopted to further optimize the super-cell model to eliminate unreasonable atomic configuration. The optimized super-cell model is shown in Figure 14.

The Bridge Domain cross-scale transfer method (BD) was introduced. Molecular dynamics theory was used to describe the selected area on the flank of the cutter tooth. And Lagrange method was used to describe the unselected area of flank of cutter tooth. The BD cross-scale coupling model is shown in Figure 15.

By using the above model and the cross-scale transfer theory, the macro scale cutting load could be transferred to atoms in mesoscale crystal cells. The motion of atoms at mesoscale could reflect the friction and wear characteristics of flank of cutter tooth. The effectiveness of the communication and transfer of macro scale and mesoscale load was realized.

Figure 13. Analysis results of energy spectrum of flank of the cutter tooth: (a) cutter tooth before milling experiment and (b) cutter tooth after milling experiment.

Figure 14. Optimized super-cell model.
In order to identify the critical value of potential energy when an atom crosses the boundary of the super-cell and reaches the potential energy mutation value of integrity failure, and the super-cell was loaded. The super-cell was subjected to a bidirectional normal stress of 2.23 GPa and a tangential stress in characteristic velocity direction of 0.92 GPa, as shown in Figure 16(a). Two-way normal stress of 2.84 GPa, tangential stress of 1.46 GPa, as shown in Figure 16(b). The bidirectional normal stress was 4.26 GPa, and the tangential stress was 2.21 GPa, as shown in Figure 16(c). The variation of failure parameters of super-cell is shown in Figure 17.

As shown in Figure 17(b), there is no bond rupture occurs between atoms of super-cell with 2.23 GPa of $\tau_m$ and 0.92 GPa of $\sigma_N$, so the super-cell does not form local damage. When the potential energy of super-cell reaches $\approx 147,000$ kcal/mol, atoms begin to leave the boundary of super-cell, and the super-cell structure forms local destruction. With the continuous application of load, when the potential energy difference between the two adjacent mutation points of the potential energy curve reaches $5500$ kcal/mol, the integrity of the super-cell structure is destroyed.

Based on the variation of the characteristic parameters of super-cell failure, the local failure $D_p$ on the flank of the cutter tooth was identified according to equation (16), and the local failure criterion of super-cell was constructed.

$$D_{e0} = e(t) - e(t_0) - \Delta e_p$$  \hspace{1cm} (16)

Where, $t_0$ is the initial moment when the super-cell is loaded on the flank of the cutter tooth, $e(t_0)$ is the potential energy value of the super-cell at $t_0$, $e(t)$ is the instantaneous potential energy value of super-cell; $\Delta e_0$ is the change value of potential energy in $t_0-t$ time interval. $\Delta e_p$ is the critical potential energy value of the super-cell boundary corresponding to the formation of local destruction of the super-cell.

When the local failure of the super-cell has been formed, the integrity of the super-cell can be determined according to equation (17):
\[ D_a = \begin{cases} \Delta e_w = e(t_{w+1}) - e(t_w) > \Delta e_a & \\ \Delta E_0 = E(t) - E(t_0) > \Delta E_a \end{cases} \tag{17} \]

Where, \( D_a \) is the integrity of the super-cell on the flank of cutter tooth. \( \Delta e_w \) is the potential energy change of super-cell in the time interval from \( t_w \) to \( t_{w+1} \). \( \Delta e_a \) is the critical value of potential energy mutation corresponding to the integrity failure of super-cell. \( E(t_0) \) is the elastic modulus of super-cell at \( t_0 \). \( E(t) \) is the instantaneous elastic modulus of super-cell. \( \Delta E_0 \) is the change of elastic modulus of super-cell in \( t_0 - t \) time interval. \( \Delta E_a \) is the critical value of elastic modulus attenuation corresponding to the integrity failure of super-cell.

**Mesoscopic failure identification and instantaneous boundary construction of feature point on flank cutter tooth**

In order to reveal the friction and wear of the flank cutter tooth, the feature point of the flank cutter tooth were identified by using the super-cell damage evaluation method of the flank cutter tooth. The local and integrity damage feature points of the super-cell on the flank were marked with different colors.

The lowest integrity damage feature point along the \( y_q \) axis on each line \( x_1 - x_N \) was select. The set of feature points of the boundary of the super-cell local damage feature point boundary on the flank of the tooth flank is obtained, as shown in equation (19):

\[ U_P = \{ \sigma_P(x_q, y_q) | x_1 < x_q < x_N, y_1 < y_q < y_M \} \tag{19} \]

In order to analyze the variation of super-cell damage within the boundary range of super-cell damage feature points. The paper taked the period as 3 and the instantaneous position angle as \( 31^\circ \) as an example. The feature point within the boundary range of the integrity damage feature point was (5.16, 3.67). The feature point within the boundary range of the integrity damage feature point and the local damage feature point was (5.16, 3.29). The characteristic parameters of super-cell damage at the same position were extracted for the remaining two teeth, and the results are shown in Figures 18 to 20.

It can be seen from Figures 18 to 20 that the super-cell damage structure was mainly related to three mesoscopic characteristic parameters. In small-scale time, as the load continues to be applied, the valence bonds of the super-cell were broken, causing a sudden change in the potential energy. Its mesoscopic structure began to form damage, and the elastic modulus began to decay. When the elastic modulus decay reaches the critical
Figure 18. Characteristic parameters of super-cell damage of cycle 3 and tooth 1 ($\varphi = 31^\circ$): (a) potential energy, (b) elastic modulus, and (c) valence bond breaking.

Figure 19. Characteristic parameters of super-cell damage of cycle 3 and tooth 2 ($\varphi = 31^\circ$): (a) potential energy, (b) elastic modulus, and (c) valence bond breaking.
value of its structural integrity damage, it means that the integrity of the super-cell structure was destroyed.

It can be known from equations (18) and (19). In \( U_{Ax} \), along the \( x_q \) direction, the feature point with the smallest \( y_q \) coordinate value was selected to form the boundary of the super-cell integrity damage feature point. In the \( U_{P} \), the feature point with the smallest \( y_q \) coordinate value is selected to form the boundary of the feature point of the local damage of the super-cell.

Using the above method, the number of valence bond fractures, the value of potential energy, and the distribution of elastic modulus of the super-cell damage in Figure 7(b) were extracted for fitting. Two types of instantaneous boundaries were obtained, and the results are shown in Figure 21.

It can be seen from Figure 21 that the characteristic parameters distribution of the super-cell integrity damage feature point boundary and the local damage feature point boundary were obviously different. The boundary of the integrity damage feature point were larger than the local damage feature point boundary. The above method can be used to characterize small-scale friction and wear boundary.

According to the meshing method of the flank surface, the flank and the feature point were selected to identify the damage of the feature points within the boundary of the supercell damage feature points. Stress loading was performed on the super-cell of the feature point of the flank of cutter tooth. The super-cell damage of feature points in a milling cycle were determined. Figure 22 shows the boundary of the super-cell damage feature point of tooth 1 and cycle 3.

It can be seen from Figure 22 that the boundary of the super-cell damage feature point on the flank of cutter tooth was related to the integrity of the super-cell structure. When the super-cell is loaded, its structure was partially damaged and its integrity was damaged. By accumulating the damage of the super-cell structure of the flank in a milling cycle, the super-cell integrity damage and local damage feature point boundary of a single milling cycle were obtained.

In order to analyze the changes of the super-cell integrity damage and local damage feature point boundary between different cutter teeth. The same milling cycle as tooth 1 was selected, and the feature points of tooth 2 and tooth 3 stress were applied. The results are shown in Figures 23 and 24.

It can be seen from Figures 23 and 24 that the boundary of the super-cell integrity damage and local damage feature point on the flank of cutter tooth were different, but the overall distribution area was relatively close. Due to the existence of milling vibration and the installation angle of the cutter tooth, with the increase of the instantaneous position angle of the
cutter tooth, the part of the cutting edge that participates in cutting shifts from the inner side of the cutter tooth to the outer side. The scale of super-cell damage was much smaller than the detection scale of friction and wear in the experiment. The extent of the boundary of the super-cell integrity damage and local damage feature point may be larger than the experimental results of friction and wear boundary.

**Cumulative friction and wear boundary of flank of cutter tooth and its experimental verification**

To reveal the correctness of the cross-scale identification method of the friction and wear boundary of the flank of cutter tooth, the test results of the wear of the experimental scheme in Table 1 were obtained, as shown in Figure 25.

According to the entire milling process, using the above-mentioned friction and wear boundary identification method, the thermal-stress coupling field of the three cutter teeth were analyzed. The super-cell damage feature point boundary of a single instantaneous position angle in one milling cycle were obtained. The super-cell failure feature point boundary of all instantaneous position angles of a single milling cycle were accumulated to obtain the super-cell failure feature point boundaries of one milling cycle. To get the cumulative solution boundary of friction and wear of a cutter tooth, accumulation the boundary of the super-cell failure feature points formed by the seven milling cycles divided in Table 2, as shown in Figures 26 to 28.

In the Figure, $\omega_{sf}$ is the characteristic point of failure of the super-cell integrity at the leftmost end of the tooth flank. $\omega_{sf}$ is the leftmost local damage feature point. $\omega_{sf}$ is the rightmost integrity damage feature point. $\omega_{sf}$ is the rightmost local destruction feature point. $\omega_{sd}$ is the lowest integrity damage feature point. $\omega_{sd}$ is the bottommost local damage feature point. $l_f$ is the friction and wear width of the tooth flank, and is the distance between the leftmost integrity failure feature point and the rightmost integrity failure feature point along the $x_q$ axis. $l_f$ is the local failure

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**Figure 21.** Super-cell characteristic parameters distributed surface of tooth 1 in cycle 3 ($\phi=31^\circ$): (a) number of valence bond fractures, (b) potential energy value, and (c) Elastic modulus.

**Figure 22.** Super-cell damage characteristic points boundary of cycle 3 and tooth 1.
width of the tooth flank, and is the distance between the leftmost local failure feature point of the tooth and the rightmost local failure feature point along the $x_q$ axis. $h_{fw}$ is the friction and wear height of the flank of the cutter tooth, which is the distance between the bottom integrity failure point and the midpoint of the cutter tooth cutting edge along the $y_q$ axis. $h_f$ is the local failure height of the tooth flank, which is the distance between the bottommost local failure feature point and the midpoint of the tooth cutting edge along the $y_q$ axis.

It can be seen from Figures 26 to 28 that, along the $x_q$ direction, comparing curve 1 and curve 3, the left and right boundaries were relatively close. Comparing curve 2 and curve 3, there was a certain difference between the left and right boundaries. Although curve
1 exhibited similar distribution characteristics to curve 2 and curve 3, there were still some differences along the $y_q$ direction. The reason was that the measurement scale of curve 2 was 0.3 nm, and the overall measurement scale of the super-cell was 7 nm, which is much smaller than the measurement scale of curve 1. The micro-scale structural damage caused by friction can be identified in a larger range, so that the range of curve 2 was larger than that of curve 3. And the range of curve 3 was larger than that of curve 1.

Using the gray correlation analysis method, the correlation analysis was carried out on the distribution characteristics of curve 1, curve 2, and curve 3. Among them, $\xi_1$ was the gray correlation degree between curve 1 and curve 3. $\xi_2$ was the gray correlation degree between curve 1 and curve 2.

From the Table 3, the correlation value between the solution and experimental of the friction and wear boundary of each milling cutter was above 0.72. The results showed that the solution and experimental of the friction and wear boundary had similar distribution characteristics. The cross-scale identification method of milling cutter friction and wear boundary can identify the smaller-scale wear caused by the friction on the flank of the cutter tooth, which was beneficial to improve the evaluation accuracy of milling cutter service life.

### Conclusions

1. The instantaneous cutting posture and thermal-stress coupling fields on tooth flank of milling cutter have been analyzed. The result showed that, the contact stress distribution of the flank was shifted from the inside to the outside of the tooth in each cutting cycle. And the equivalent stress boundary propagated cyclically from the inside to the outside along the radial direction of the milling cutter. A method for calculating the instantaneous characteristic velocity vector, normal stress and shear stress in the characteristic velocity direction of the flank of cutter tooth was proposed. The variation of the instantaneous contact relationship between the flank of cutter tooth and the machining transition surface under vibration was revealed. The cross-scale correlation of the macro-mesoscopic friction force on flank of cutter tooth was realized by the cross-scale transfer method.
(2) A mesoscopic identification method for the feature points of friction and wear boundary on the flank of cutter tooth was proposed. This method utilized the number of valence bond fractures, potential energy abrupt change, and elastic modulus decay characteristics of the super-cell. The local and integrity damages in mesoscope of the tooth flank under the effects of shear stress in the characteristic velocity direction and normal stress were identified. The results showed that the boundaries formed by super-cell integrity damage feature points on flank were significantly larger than that by local damage feature points. These two boundaries reflect the instantaneous distribution of two different degrees of friction and wear on the flank of the cutter tooth, respectively.

(3) The boundary distribution of instantaneous friction and wear on the flank was revealed. The cumulative friction and wear distribution of tooth flank were obtained by using the feature point of the maximum value of the instantaneous boundary in the whole cycle of milling cutter. The results showed that the range of the cumulative friction and wear solution boundaries were larger than that in experiments. But correlation degree between two kinds of boundaries was greater than 0.7, they had similar distribution characteristics. This method could be used to identify the formation of the friction and wear boundaries of the flank and the distribution of small-scale wear.

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ORCID iD
Peiyi Zhao https://orcid.org/0000-0002-7004-5023

References
1. Zhang X, Sui H, Zhang D, et al. Feasibility study of high speed ultrasonic vibration cutting titanium alloy. J Mech Eng 2017; 53: 120–127.
2. Zhang J, Liu C, Zheng F, et al. Study on tool forced vibration suppression based on milling dynamics. J Mech Eng 2018; 54: 94–99.
3. Sun L, Ling C, Chen H, et al. Multiscale mechanical method in structural integrity analysis. J Mech Eng Sci 2021; 57: 106–121.
4. Zhang X and Jin W. Finite element simulation of high speed turning of titanium alloy based on DEFORM 3D. Tool Eng 2017; 51: 45–48.
5. Matuszewski M, Mikolajczyk T, Pimenov DY, et al. Influence of structure isotropy of machined surface on the wear process. Int J Adv Manuf Technol 2017; 88: 2477–2483.
6. Kaczmarek J. A method of multiscale modelling considered as a way leading to unified mechanics of materials. Acta Mech 2015; 226: 1419–1443.
7. Zhu K and Li G. Theoretical modeling and experimental study of micro milling force based on tool wear mapping relationship. J Mech Eng Sci 2021; 57: 246–259.
8. Ren C, Liu T, Siddique A, et al. High-speed visualizing and mesoscale modeling for deformation and damage of 3D angle-interlock woven composites subjected to transverse impacts. Int J Mech Sci 2018; 140: 119–132.
9. Zheng Z and Li Z. A multiscale computational method with inheriting simulation of moving trans-scale boundary for damage-induced structural deterioration. Eng Comput 2017; 34: 1677–1699.
10. Zhu K and Yu X. The monitoring of micro milling tool wear conditions by wear area estimation. Mech Syst Signal Process 2017; 93: 80–91.
11. Shan Z and Zhu F. Tool wear mechanism and prediction model of sand milling with PCD tool. Chin J Mech Eng 2018; 54: 124–132.
12. Liu J, Jiang X and Zhang D. Study on chip characteristics and tool wear of high-speed rotary ultrasonic elliptical vibration side milling of titanium alloy. J Mech Eng Sci 2019; 55: 186–194.
13. Mikolajczyk T, Nowicki K, Klodowski A, et al. Neural network approach for automatic image analysis of cutting edge wear. Mech Syst Signal Process 2017; 88: 100–110.
14. Wu X, Li L, He N, et al. Study on the tool wear and its effect of PCD tool in micro milling of tungsten carbide. Int J Refract Metals Hard Mater 2018; 77: 61–67.
15. Altan E, Çalışkan O and Uysal A. Investigation into the effectiveness of cutting parameters on wear regions of the flank wear curve and associated cutting tool life improvement. Int J Mater Prod Technol 2018; 57: 54–70.
16. Lu Z, Ma P, Xiao J, et al. Online monitoring of tool wear state in machining process based on machine tool information. China Mech Eng 2019; 30: 220–225.
17. Pimenov DY, Guzeev VI, Mikolajczyk T, et al. A study of the influence of processing parameters and tool wear on elastic displacements of the technological system under face milling. Int J Adv Manuf Technol 2017; 92: 4473–4486.
18. Chang K, Zheng G, Cheng X, et al. Surface integrity evolution and wear evolution of the micro-blasted coated tool in high-speed turning of Ti6Al4V. Int J Adv Manuf Technol 2021; 115: 603–616.
19. Zhang L, Sun Y, Wang G, et al. Research on MD simulation for diamond tool cutting iron. *Mol Simul* 2021; 47: 46–57.

20. Liu C, He W, Chu J, et al. Molecular dynamics simulation on cutting mechanism in the hybrid machining process of single-crystal silicon. *Nanoscale Res Lett* 2021; 16: 66.

21. Wang Z, Zhao P, Guo Y, et al. Molecular dynamics simulation of gradient nanocrystalline copper nanocutting process. *Mater Rev* 2019; 33: 419–423.

22. Hao Z, Lou Z and Fan Y. Study on phase transformation in cutting Ni-base superalloy based on molecular dynamics method. *J Mech Eng Sci* 2021; 235: 16–39.