Inherent characteristic analysis of a dual power split gear train

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Abstract. The dynamic model of a dual power split coupling gear transmission is established by lumped mass method. The time-varying meshing stiffness of the model was solved by using the loaded tooth contact analysis (LTCA) techniques. Through using numerical calculation method to solve the dynamics equation, the natural frequencies and natural modes is obtained. The results show that the quadruple frequency phenomenon is appeared, and the distribution of natural frequency is reasonable. It is meet the design requirements. The relative amplitude is the largest between the nodes at 23 and 29 in the vibration mode curve of the fundamental frequency. The method above provides a theoretical basis and data support for its dynamic performance optimization design.

1. Introduction
A dual power-split transmission adopts the power split technology, it can realize the power split in four ways. The system vibration and dynamic is of important in the application, it is directly related to the transmission system reliability and security.

Many researchers had already done lots of analysis on the load sharing systems with elastic supports at abroad and home. Kahraman [1] analyzed the overall deformation of the planetary gear train in dynamics; Bartelmus [2] analyzed the dynamic characteristics of planetary gear box under the influence of external load factor; Ambarisha [3] analyzed the nonlinear dynamic characteristics of planetary gear train by using the finite element method; Kiracofe and Parker [4] analyzed the vibration model of planetary gear train; Dong [5] analyzed the load-sharing characteristics of double power diversion gear train based on the deformation coordination.

However, much of recent research in power-split transmission system minimize the influence of gear surface tooth contact; most of these just adopt equivalent average mesh stiffness of a gear pair to express progress of load-sharing research and could not accurately reflect a real meshing process. In this paper, a real meshing process of gear pair is dispersed into a limited meshing point, according to the method of theoretical analysis of tooth contact analysis (TCA) and loaded tooth contact analysis (LTCA). Statics characteristic of each meshing position is analyzed, and the mechanical properties are obtained. This method can improve accuracy of the calculation.

In this paper, the bending-torsion coupling dynamic model is established by using the lumped parameter method, and the inherent frequencies and corresponding vibration mode is obtained under a given power and speed. It will provide certain reference basis for the dynamic optimization of design of the dual power split transmission system.
2. Dynamics Model Building

An analysis of the structure of a dual split-path transmission is shown in Fig. 1. The input I-stage pinion meshes with two gears, offering two paths to transfer power to the gear of II-stage, and the II-stage pinion, respectively, meshes with two II-stage idle gear, thereby offering four paths to transfer power to the II-stage gear and output.

The lumped parameter method is used to establish the bending-torsion coupling dynamic model, which is shown in Fig. 2. \( T_{in} \) is input torque, \( T_{out} \) is output torque, \( k_{ij} \) is time-varying mesh stiffness, \( c_{ij} \) is meshing damping, \( \phi_i \) and \( \phi_j \) is the torsion angle, \( e_{ij}(t) \) is integrated error, \( M_i \) is moment of inertia concentrated mass, \( k_{24} \) and \( k_{35} \) is torsional rigidity, \( c_{24} \) and \( c_{35} \) is torsional damper.

The meshing force and the damping force can be represented by

\[
P_{ij} = k_{ij}(r_{bi} \phi_i - r_{bj} \phi_j) + (x_j - x_i) \cos \zeta_{ij} + (y_j - y_i) \sin \zeta_{ij} +
(\Delta A_{xi} - \Delta A_{xj}) \cos \zeta_{ij} + (\Delta A_{yi} - \Delta A_{yj}) \sin \zeta_{ij} - e_{ij}(t) \]
\[
D_{ij} = c_{ij}(r_{bi} \phi_i - r_{bj} \phi_j) + (\dot{x}_j - \dot{x}_i) \cos \zeta_{ij} + (\dot{y}_j - \dot{y}_i) \sin \zeta_{ij} +
(\Delta A\dot{x}_i - \Delta A\dot{x}_j) \cos \zeta_{ij} + (\Delta A\dot{y}_i - \Delta A\dot{y}_j) \sin \zeta_{ij} - \dot{e}_{ij}(t)
\]

Where, \( x_i \) and \( x_j \) are displacement deformation along the x-axis, \( y_i \) and \( y_j \) are displacement deformation along the y-axis, \( \Delta A_{xi} \) and \( \Delta A_{xj} \) are the amplitudes of errors along the x-axis, \( \Delta A_{yi} \) and \( \Delta A_{yj} \) are the amplitudes of errors along the y-axis, respectively for the pinion \( i \) and gear \( j \), \( \zeta_{ij} \) is actual operating pressure positive angle of the line of action down from x-axis, \( r_{bi} \) and \( r_{bj} \) is the base radius.

The system lateral-bending vibration dynamics differential equation can be represented by

\[
\sum [(P_{ij} + D_{ij}) \cdot \cos \zeta_{ij}] + M_i \ddot{x}_i + C_{xi} \dot{x}_i + K_{xi} x_i = 0
\]
\[
\sum [(P_{ij} + D_{ij}) \cdot \sin \zeta_{ij}] + M_j \ddot{y}_j + C_{yi} \dot{y}_j + K_{yi} y_j = 0
\]

Where, \( K_{xi} \) and \( K_{yi} \) are the equivalent supporting rigidity of gear \( i \), while \( C_{xi} \) and \( C_{yi} \) are the equivalent bearing damping. \( \ddot{x}_i, \dot{x}_i, \) and \( x_i \) are the transverse vibration acceleration, velocity and displacement in the x direction respectively; \( \ddot{y}_j, \dot{y}_j, y_j \) are the transverse vibration acceleration, velocity and displacement in the y direction, respectively.

The direction of rotation differential equation can be represented by
\( m_{eq,i} \ddot{u}_i + (P_{r2} + D_{r2} + P_{r3} + D_{r3}) = \frac{T_m}{n_1}, \)
\( m_{eq,2} \ddot{u}_2 - (P_{r2} + D_{r2}) - c_{24} \left( \frac{\ddot{u}_2 - \ddot{u}_4}{n_2 - n_4} \right) - k_{24} \left( \frac{u_2 - u_4}{n_2 - n_4} \right) = 0, \)
\( m_{eq,3} \ddot{u}_3 - (P_{r3} + D_{r3}) - c_{35} \left( \frac{\ddot{u}_3 - \ddot{u}_5}{n_3 - n_5} \right) - k_{35} \left( \frac{u_3 - u_5}{n_3 - n_5} \right) = 0, \)
\( m_{eq,4} \ddot{u}_4 + (P_{r4} + D_{r4}) + (P_{r7} + D_{r7}) + \left( \frac{\ddot{u}_2 - \ddot{u}_4}{n_2 - n_4} \right) = 0, \)
\( m_{eq,5} \ddot{u}_5 + (P_{r5} + D_{r5}) + (P_{r6} + D_{r6}) + c_{35} \left( \frac{\ddot{u}_3 - \ddot{u}_5}{n_3 - n_5} \right) + k_{35} \left( \frac{u_3 - u_5}{n_3 - n_5} \right) = 0, \)
\( m_{eq,6} \ddot{u}_6 - (P_{r6} + D_{r6}) + (P_{r10} + D_{r10}) = 0, \)
\( m_{eq,7} \ddot{u}_7 - (P_{r7} + D_{r7}) + (P_{r10} + D_{r10}) = 0, \)
\( m_{eq,8} \ddot{u}_8 - (P_{r8} + D_{r8}) + (P_{r10} + D_{r10}) - (P_{r10} + D_{r10}) = -\frac{T_{out}}{n_{50}}, \)
\( m_{eq,9} \ddot{u}_9 - (P_{r9} + D_{r9}) + (P_{r11} + D_{r11}) - (P_{r11} + D_{r11}) = 0, \)
\( m_{eq,10} \ddot{u}_{10} - (P_{r10} + D_{r10}) - (P_{r10} + D_{r10}) = 0, \)
\( \sum_{j=1}^{n} p_{j1} = p_{j} \quad (j = 1, 2, \ldots, n) \)
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Where, \( m_{eq,i} \) is the equivalent mass, \( m_{eq,i} = I_i/r_i^2 \). The angular displacement of generalized coordinates is translate into the line displacement, \( u = r_i \dot{\phi}_i \).

The variable-step four order Runge-Kutta method was used to solve the Equations (2) and (3), at the same time, the definition of dimensional time \( t = \tau \omega_n \), gives a displacement nominal scale \( b_c \). The Equations (2) and (3) are processed by the dimensional normalization.

3. Time-varying Mesh Stiffness

Under the load \( P \), the driving gear goes through an approach \( Z \) and the contact load becomes distributed due to tooth deformation. Torque balance and displacement after the tooth contact deformation coordination conditions is described by

\[ F_k = p_k = \sum_{j=1}^{n} p_{j1} = p_{j} \quad (j = 1, 2, \ldots, n) \]

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Where, \( k = 1, II, \ldots, n \)

\[ d_j = [d_{1j}, d_{2j}, \ldots, d_{nj} \ldots, d_{nj}]^T ; [Z] = [Z_k] = [1, 1, 1, \ldots, 1]^T ; [f]_k \] is the normal flexibility coefficient matrix of the gear pair \( k \).

The corresponding angular transmission error \( \Delta \phi_j (T_{ij}(k)) \) under load for the contact position is determined by reversing Eq. (4). Functional relations between \( \Delta \phi_j (T_{ij}(k)) \) and \( T_{ij}(k) \) are expressed as

\[ \Delta \phi_j (T_{ij}(k)) = \delta_1 (T_{ij}(k)) + \delta_2 [T_{ij}(k)] + \delta_3 [T_{ij}(k)] = a + bT_j(k) + cT_j(k)^{2/3} \]

\( \delta_1, \delta_2 \) and \( \delta_3 \) are the deformations of torsional angle caused by above-mentioned three cases; \( T_{ij}(k) \) is the torque of the \( k \)-th meshing position that gear \( j \) is relative to pinion \( i \) in a meshing cycle; \( a, b \) and \( c \) are constants. By solving equations (5), we can obtain the coefficient of \( a, b \) and \( c \). And then, functional relations between loaded transmission errors and some nominal load of \( T_{ij}(k) \) may be proposed, and the calculation curves are supplied in a meshing cycle.
The time-varying mesh stiffness is represented as

$$K_g(k) = [T_g(k)/(r_0\cos\alpha_0)]\cdot[1/(r_0\Delta\varphi_g(T_g(k)))]$$  \hspace{1cm} (6)$$

The mesh stiffness could reflect real meshing elastic properties at the meshing position more directly. The discrete value of meshing stiffness is fitted by the polynomial and through the Fourier series transformation to spread out into a periodic function.

4. Inherent characteristic analysis

Input power is 25 000 kW, input speed is 7 000 r/min. Here, gear parameters are shown in Table 1. The bearing stiffness parameters are shown in Table 2.

| Gear | Teeth z | Module m/n/mm | Face with b/mm | Pressure angle $\alpha/\degree$ | Helix angle $\beta/\degree$ |
|------|---------|---------------|----------------|-------------------------------|--------------------------|
| 1    | 42      | 3             | 62             | 20                            | 15.48                    |
| 2–3  | 117     | 3             | 62             | 20                            | 15.48                    |
| 4–5  | 24      | 4             | 96             | 20                            | 15.28                    |
| 6–9  | 51      | 4             | 96             | 20                            | 15.28                    |
| 10   | 198     | 4             | 96             | 20                            | 15.28                    |

According to the method mentioned in the section 3, the time-varying mesh stiffness activation curve is shown in Fig.3.

![Fig. 3 Curves of the time-varying mesh stiffness activation](image)

The expression of dynamic differential equations is written in matrix form:

$$[M]\{X^t\} + [C]\{X^t\} + ([K(t)] - e(t))\{X\} = \{P_e\}$$  \hspace{1cm} (7)$$

The inherent characteristics of problem can be converted into solving the following equation of eigenvalue problem:

$$\omega^2[M] = [\tilde{K}]$$  \hspace{1cm} (8)$$

$[M]$ is system mass matrix, $[\tilde{K}]$ is stiffness matrix. Solving the equation of eigenvalue is the natural frequency of the system, and calculated system 30 order natural frequency results are shown in Table 2, including zero frequency system exist rigid body displacement.

| Order | Inherent frequency | Order | Inherent frequency | Order | Inherent frequency | Order | Inherent frequency | Order | Inherent frequency |
|-------|--------------------|-------|--------------------|-------|--------------------|-------|--------------------|-------|--------------------|
| 1     | 0                  | 7     | 199.877            | 13    | 468.116            | 19    | 1018.471           | 25    | 2084.417           |
| 2     | 142.802            | 8     | 273.048            | 14    | 523.136            | 20    | 1018.471           | 26    | 2084.418           |
| 3     | 152.281            | 9     | 322.915            | 15    | 824.536            | 21    | 1748.084           | 27    | 3774.596           |
As seen from Table 3, the system of fundamental frequency is 142.802 Hz, and the dual power split transmission system with symmetric structure, the equal value, the natural frequency of the system with double frequency, and the complexity and particularity of this system, the quadruple frequency phenomenon.

Corresponding to various degrees of freedom vibration model as shown in figure 4:

![Fig.4 each natural principal mode of system](image)

It is can be seen from the Fig.4 the node between 23 and 29 of the largest relative amplitude. According to above analysis, the rational distribution of the torsional vibration natural frequency meet the design requirements of system. For further into the action of load characteristic analysis to provide certain theory basis and data support.
5. Conclusions

By means of lumped mass method, a dynamic model of dual power split transmission system is established. The time-domain dynamic load and frequency response of the system are obtained. This method can provide a reference for the other power split transmission. Time-varying mesh stiffness based on LTCA is analyzed by using the Fourier series transformation, the periodic function was used, and spread out it into a periodic function, improved the calculation accuracy.

Acknowledgment

National Natural Science Foundation of China(51705390), Supported by the Natural Science Foundation of Shaanxi Provincial (2018JQ5029), Xi’an science and technology plan project 2017075CG/RC038(XAGY008), Supported by the Program for Innovative Science and Research Team of Xi’an Technological University.

References

[1] Kahraman A., Kharazi A. A., Umrani M.. (2003) A Deformable Body Dynamic Analysis of Planetary Gears with Thin Rims, Journal of Sound and Vibration. 262: 752-768.
[2] Bartelmus W., Zimroz R.. (2009)Vibration Condition Monitoring of Planetary Gearbox Under Varying External Load, Mechanical Systems and Single Processing. 23: 246-257.
[3] Ambarisha V. K., Parker R. G.. (2007) Nonlinear dynamics of planetary gears using analytical and finite element models, Journal of Sound and Vibration. 302: 577-595.
[4] Kiracofe D. R., Parker R. G., (2007) Structured vibration modes of general compound planetary gear systems, ASME, Journal of Vibration and Acoustics. 129: 1-16.
[5] Dong H., Fang Z. D.. (2012) Load-Sharing Characteristics of Gear Train with Dual Power Split Based on Deflection Compatibility, Journal of South China University of Technology (Natural Science Edition). 40: 18-23.