Neutron Star–Neutron Star and Neutron Star–Black Hole Mergers: Multiband Observations and Early Warnings

Chang Liu1,2 and Lijing Shao2,3

1 Department of Astronomy, School of Physics, Peking University, Beijing 100871, People’s Republic of China
2 Kavli Institute for Astronomy and Astrophysics, Peking University, Beijing 100871, People’s Republic of China; lishao@gkaa.pku.edu.cn
3 National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, People’s Republic of China

Received 2021 August 19; revised 2021 November 2; accepted 2021 November 22; published 2022 February 21

Abstract

The detections of gravitational waves (GWs) from binary neutron star systems and neutron star–black hole systems provide new insights into dense matter properties in extreme conditions and associated high-energy astrophysical processes. However, currently, information about the neutron star equation of state (EoS) is extracted with very limited precision. Meanwhile, the fruitful results from the serendipitous discovery of the γ-ray burst alongside GW170817 show the necessity of early warning alerts. Accurate measurements of the matter effects and sky location could be achieved by joint GW detection from space and ground. In our work, based on two example cases, GW170817 and GW200105, we use the Fisher information matrix analysis to investigate the multiband synergy between the space-borne decihertz GW detectors and the ground-based Einstein Telescope (ET). We especially focus on the parameters pertaining to the spin-induced quadrupole moment, tidal deformability, and sky localization. We demonstrate that (i) only with the help of multiband observations we can constrain the quadrupole parameter; and (ii) with the inclusion of decihertz GW detectors, the errors of tidal deformability would be a few times smaller, indicating that many more EoSs could be excluded; (iii) with the inclusion of ET, the sky localization improves by about 1 order of magnitude. Furthermore, we have systematically compared the different limits from four planned decihertz detectors and adopting two widely used waveform models.

Unified Astronomy Thesaurus concepts: Gravitational wave astronomy (675); Neutron stars (1108); Gravitational wave detectors (676); Compact binary stars (283)

1. Introduction

Until now, more than 50 gravitational wave (GW) events have been published by the LIGO/Virgo Collaboration (Abbott et al. 2019a, 2021a, 2021b), in which the majority is from binary black hole (BBH) mergers. In comparison, the GW signals from binary neutron star (BNS) systems (Abbott et al. 2017a, 2020) and neutron star–black hole (NSBH) systems (Abbott et al. 2021c) are rare but of special interests, as they could help us comprehend high-density nuclear matter (Abbott et al. 2018), improve views about astrophysical processes under extreme conditions (Abbott et al. 2017b), and understand compact object populations (Abbott et al. 2021d).

Extracting BNS and NSBH properties solely from GW signals is crucial for GW astronomy, which highly depends on the accuracy of the waveform. Two dominant finite-size effects distinguish neutron stars (NSs) from black holes (BHs): (i) the deformation due to NS’s own rotation and (ii) due to the companion’s tidal field. They enter the waveform as a self-spin term (Poisson 1998) and tidal term (Flanagan & Hinderer 2008; Vines et al. 2011), respectively. With the accurate waveform model (Dietrich et al. 2019a, 2019b), we could constrain the equation of state (EoS)-dependent spin-induced quadrupole moment and tidal deformability, and pick out the correct EoS model (Read et al. 2009; Hinderer et al. 2010; Agathos et al. 2015), thus informing the low-energy quantum chromodynamics and quark confinement behaviors. Moreover, we could test the nature of BHs (Krishnendu et al. 2019b; Narikawa et al. 2021), distinguish BNS models from BBH models (Gralla 2018; Krishnendu et al. 2019a; Chen et al. 2020), and test alternative gravity theories (Sennett et al. 2017; Shao et al. 2017; Shao 2019).

In addition to the GW signal, short γ-ray burst (GRB) GRB 170817A was found right after the peak of the first BNS inspiral GW170817 (Abbott et al. 2017a, 2017c). Together with the following counterparts in the X-ray, ultraviolet, optical, infrared, and radio bands, simultaneous detections of GWs and electromagnetic (EM) signals initiate a new era of multimessenger astronomy with precious information (Abbott et al. 2017d). In the meantime, EM signals also call for a better localization ability from GW detectors. Scientists have explored the future localization abilities of LIGO/Virgo detectors (Nitz et al. 2020; Magee et al. 2021), as well as the third generation (3G) detectors including the Europe-led Einstein Telescope (ET; Hild et al. 2011) and the US-led Cosmic Explorer (CE; Abbott et al. 2017e), using the post-Newtonian (PN) waveform (Chen et al. 2018; Zhao & Wen 2018) with precession (Tsutsui et al. 2021), eccentricity (Ma et al. 2017; Pan et al. 2019), and tidal effects (Wang et al. 2020).

For the discovered LIGO/Virgo sources, the angular resolution of the 3G GW detectors can be as accurate as a few degrees (see, e.g., Zhao & Wen 2018). On the other hand, the space-borne detectors could localize sources within arcminutes (Takahashi & Nakamura 2003; Nair & Tanaka 2018). To obtain even better constraints, the multiband observations could be a win-win solution for both space-borne and ground-based detectors.

The BNS and NSBH signals can hardly reach the signal-to-noise ratio (S/N) threshold of the millihertz-band space-borne detectors such as LISA (Amaro-Seoane et al. 2017) and will spend more than a few years before coalescence even if they do. Therefore, we direct our attention on the decihertz detectors, e.g., Decihertz Observatories (DOs; Sedda et al. 2020, 2021).
and DECihertz laser Interferometer Gravitational Wave Observatory (DECIGO; Yagi & Seto 2011; Kawamura et al. 2019). Because of their shorter arm length, decihertz detectors are sensitive in the frequency range of 0.01–10 Hz. DOs have two LISA-like proposals, the ambitious DO-Optimal and the less challenging DO-Conservative. DECIGO also has two designs. B-DECIGO is a primordial version of DECIGO consisting of one LISA-like detector, while the complete design of DECIGO consists of four independent LISA-like detectors and uses the Fabry–Perot cavity to achieve a much lower noise level. As shown in early studies, the joint detection of BNSs and NSBHs with decihertz detectors and ground-based detectors will improve the parameter precision prominently (Nair et al. 2016; Nakamura et al. 2016; Liu et al. 2020; Nakano et al. 2021). Isoyama et al. (2018) and Nair & Tanaka (2018) have shown the precision improvement specially focusing on BNS finite-size effects and the angular resolution. In this work, we extend the study of Isoyama et al. (2018) by constraining both finite-size effects and localization parameters simultaneously, for both BNS and NSBH systems. Comparing to previous works, we use the updated sensitivity curves and detector designs. For the first time, we give the parameter errors and the multiband improvement distributions on the sky maps. Due to the need of early warnings, we further include the multiband sky localization as a function of time. Moreover, our work gives a systematic analysis on parameter correlations, illustrates the capability of different detectors, and compares the implementation of different waveforms. Our work enables a better understanding of joint observations and could provide more information for different observing scenarios.

In this work, with the help of the Fisher matrix analysis, we investigate the multiband measurement uncertainties considering the complete parameter space including spin, tidal, self spin, and location parameters. We give the sky distributions of multiband enhancement for quadrupole–monopole parameters, tidal deformabilities, and angular resolutions, as well as the premerger localization precision as a function of the inspiraling expansion, we measure the redshifted mass 

And early warning alerts; Section 3.4 compares the PE measurements given by different decihertz detectors; and Section 3.5 compares the limits imposed by using PN waveform and phenomenological waveform. In Section 4, we discuss constraints on the NS’s EoS, and finally, in Section 5, we briefly summarize our work. Throughout this paper, we use geometrized units in which \( G = c = 1 \).

2. Method

In this section, we first introduce the waveforms used in the following calculations in Section 2.1; then we introduce the detectors we use and their responses to GWs in Section 2.2, and at last in Section 2.3, we briefly summarize the PE method and list the physical properties of the specific example systems that we explore.

2.1. Waveform Construction

We model the GW signal using the Fourier domain, restricted PN approximation (Buonanno et al. 2009). With the Fourier representation computed using the stationary phase approximation, the source-frame strain is

\[
\tilde{h}_+ (f) = A_F^{-7/6} e^{i \Psi_{\text{NS}} (f)} ,
\]

(1)

where the amplitude is

\[
A_F = \sqrt{5/24} \pi^{-2/3} M^5/6/D_L ,
\]

in which \( D_L \) is the luminosity distance of the source, and \( M = m_1 + m_2 \) and the symmetric mass ratio \( \eta = m_1 m_2/M^2 \). Due to the cosmological expansion, we measure the redshifted mass \( m_{2E} = (1 + z)m_{2,0} \) of the two compact objects, where \( z \) is the redshift calculated from \( D_L \), and \( m_{2,0}^S \) are the source-frame component masses with \( m_1^S \geq m_2^S \) by default. Note that we include amplitude’s dependence on the inclination angle in the pattern function (see Section 2.2).

The phase \( \Psi_{\text{NS}} (f) \) in the waveform is

\[
\Psi_{\text{NS}} (f) = 2 \pi f \psi_n - \phi_c - \frac{\pi}{4} + \frac{3}{128 m_0^3} (\Psi_{\text{PP+spin}}^{3.5\, \mathrm{PN}} + \Psi_{\text{QM}}^{2.3\, \mathrm{PN}} + \Psi_{\text{tidal}}^{2.6\, \mathrm{PN}}) ,
\]

(3)

where \( \psi_n \) and \( \phi_c \) are the time and orbital phase at coalescence, respectively, and \( \nu = (\pi M f)^{1/3} \) is the orbital velocity. Note that terms with \( O(\nu^{2n}) \) correspond to the \( n \)th PN order.

Apart from the BBH baseline waveform, we consider two matter effects specially generated by NSs: the spin-induced and tidal-induced deformations, which respectively count for the second and third terms in the bracket of Equation (3). In total, the GW phase contains three parts, as elaborated below.

1. The point particle term, \( \Psi_{\text{PP+spin}}^{3.5\, \mathrm{PN}} \) (Arun et al. 2009; Mishra et al. 2016), is kept up to 3.5 PN. Because we consider the nonprecessing case, \( \Psi_{\text{PP+spin}}^{3.5\, \mathrm{PN}} \) also contains the aligned spin effect characterized by the dimensionless spin parameters projected to the angular momentum direction, \( \psi_n = S_r \times \hat{L}/(m_0^3 \eta^2) \), where \( S_r \) is the spin angular momentum, and \( \hat{L} \) is the unit normal of the orbital plane, expressed later in Equation (17). \( \Psi_{\text{PP+spin}}^{3.5\, \mathrm{PN}} \) includes the linear spin–orbit effects up to the 3.5 PN order, quadratic...
in-spin (spin–spin) effects to the 3 PN order, and cubic-in-spin (spin–spin–spin) effects to the (leading) 3.5 PN order.

2. The second term is the quadrupole–monopole term, \( \psi_{2,3.5\text{PN}}^Q \). The spin-induced quadrupole moment \( Q_i = -\kappa_i \mathcal{Q}_i (m_i^5) \) is a measure of the degree of the oblateness due to NS’s rotation, where \( \kappa_i \) is a dimensionless quadrupole parameter with \( \kappa_i \approx 2–20 \) for NSs and \( \kappa_i = 1 \) for BHs (Narikawa et al. 2021). This finite size effect that depends on NS’s EoS enters the GW signal as an order-\( v^6 \) correction through the quadrupole–monopole interaction (Poisson 1998; Mikoczi et al. 2005), and we include them up to the 3.5 PN order by (Krishnendu et al. 2017; Dietrich et al. 2019a; Nagar et al. 2019).

\[
\psi_{2,3.5\text{PN}}^Q = -25 Q_i v^4 + \left\{(156354260\eta) Q \frac{-2215\delta M}{24} \right\} v^6 + \left\{\frac{375}{2} (\chi_s + \chi_a \delta M) - 200\pi - 10\eta \chi_s \right\} Q - \frac{1985}{6} (\chi_a + \chi_a \delta M) \delta Q \right\} v^7, \tag{4}
\]

where

\[
\bar{Q} = \frac{2m_1^2 \chi_1^2 (\kappa_1 - 1) + 2m_2^2 \chi_2^2 (\kappa_2 - 1)}{M^2}, \tag{5}
\]

\[
\delta \bar{Q} = \frac{-2m_1^2 \chi_1^2 (\kappa_1 - 1) + 2m_2^2 \chi_2^2 (\kappa_2 - 1)}{M^2}, \tag{6}
\]

with \( \delta M = (m_1 - m_2)/M \) and \( \chi_{a,s} = (\chi_1 \pm \chi_2)/2 \). \( \bar{Q} \) is the combination of individual quadrupole parameters \( \kappa_i \), to which GW detectors are most sensitive, while \( \delta \bar{Q} \) is the subdominant parameter. We find that it is difficult to simultaneously constrain \( \kappa_1 \) and \( \kappa_2 \), or \( \bar{Q} \) and \( \delta \bar{Q} \), due to the strong degeneracies among the quadrupole parameters and the spin parameters. Therefore, we only constrain the leading term \( \bar{Q} \), and we will refer to it as the “quadrupole term” in the following analyses.

3. The last term is the tidal term \( \psi_{5,6\text{PN}}^{\text{tidal}} \) (Flanagan & Hinderer 2008; Vines et al. 2011). At the last stages of the inspiral, the quadrupolar tidal field \( \mathcal{E}_{ij} \) of one compact object induces a quadrupole moment \( Q_{ij} \) to the other component. To the leading order in the adiabatic approximation, \( Q_{ij} = -\lambda \mathcal{E}_{ij} \), where \( \lambda \) is the tidal Love number that takes the form \( \lambda = \kappa_i m_i^8 \), EoS\( = 2k_2 R^2 (m_i^5) / 3 \), with \( k_2 \) being the second Love number, and \( R(m_i^5) \) is the NS radius as a function of its mass. Both \( k_2 \) and \( R(m_i^5) \) are EoS-dependent. The deformation effect enters the GW phase from 5 PN through the dimensionless tidal deformability \( \Lambda \) and \( \delta \Lambda \) are

\[
\Lambda = \frac{8}{13} ((1 + 7\eta - 31\eta^2)(\Lambda_1 + \Lambda_2) + (1 + 9\eta - 11\eta^2)(\Lambda_1 - \Lambda_2)) \delta M, \tag{8}
\]

\[
\delta \Lambda = \frac{1}{2} \left\{ \left( \frac{1 - 132\eta}{1319} \right) \left( \frac{8944}{1319} \eta^2 \right) (\Lambda_1 + \Lambda_2) \delta M + \left( \frac{1 - 1591\eta}{1319} \right) \left( \frac{32850}{1319} \eta^2 + \frac{3380}{1319} \eta^3 \right) (\Lambda_1 - \Lambda_2) \right\}. \tag{9}
\]

Similar to \( \bar{Q} \), the tidal phase is dominated by the leading term characterized by \( \Lambda \), and the contribution from \( \delta \Lambda \) is small. Hence, we exclude the estimation of \( \delta \Lambda \) in our work. We refer to \( \Lambda \) as the “tidal deformability” of the system throughout this work. It is worth noting that BHs have zero tidal deformability (Binnington & Poisson 2009), and for asymmetric NSBHs \( (\Lambda_1 = 0, m_2 \ll m_i) \) or very massive BNSs \( (\Lambda_1, \Lambda_2 \rightarrow 0) \), \( \Lambda \) will be small and thus they would be indistinguishable from BBHs \( (\Lambda = 0) \). For equal mass systems, \( \delta \Lambda = 0 \).

Essentially, in matched filtering, since we do not know the true EoS, we search for the quadrupole parameters and tidal parameters independently. Nevertheless, with the universal Q-Love relations (Yagi & Yunes 2013, 2017), one can prescribe the quadrupole moments through the tidal deformability without the knowledge of the correct EoS, therefore reducing the number of parameters to infer. Because our purpose is to constrain the EoS, we use \( \bar{Q} \) and \( \Lambda \) as separate parameters to estimate. There are waveforms that use the universal relation, such as the phenomenological waveforms (Dietrich et al. 2019b, 2019a), which we will discuss in Section 3.5. In that specific section, we will constrain only \( \Lambda \).

2.2. Detector Responses and Sensitivities

After having the source-frame waveform in the last subsection, we now construct the detector responses and obtain the detector-frame waveform. For the space-borne detectors, we use the method in Section 2.1 of Liu et al. (2020) to model their responses. The basic idea is as follows. The signal received by the detector is

\[
\tilde{h}(f) = [F^+(f) \tilde{h}_+(f) + F^\times(f) \tilde{h}_\times(f)] e^{-i\omega(f)}, \tag{10}
\]

where the location-dependent pattern functions are

\[
F^+(\theta, \phi, \psi, \iota) = \frac{(1 + \cos^2 \iota)}{2} \times \left[ \frac{1}{2} (1 + \cos^2 \theta) \cos 2\phi \cos 2\psi - \cos \theta \sin 2\phi \sin 2\psi \right], \tag{11}
\]

\[
F^\times(\theta, \phi, \psi, \iota) = \cos \iota \times \left[ \frac{1}{2} (1 + \cos^2 \theta) \cos 2\phi \sin 2\psi + \cos \theta \sin 2\phi \cos 2\psi \right]. \tag{12}
\]

The \( \{\theta, \phi, \psi, \iota\} \) are the time-varying source direction angles (\( \theta, \phi \)), polarization angle (\( \psi \)), and inclination angle (\( \iota \)) in the detector frame. Since we know the orbital motion of the detector, the way to construct the response is to use the fixed \( \{\tilde{\theta}_S, \tilde{\phi}_S, \tilde{\theta}_L, \tilde{\phi}_L\} \), which are the source direction and angular
momentum direction in the Solar system barycentric frame, and the time $t$ to substitute $\{\theta, \phi, \psi, t\}$ (see details in Section 2 of Liu et al. (2020)). The last term of Equation (10) is the Doppler phase correction (Cutler 1998),

$$\varphi_D(t) = 2\pi R \sin \bar{\theta}_S \cos [\Phi_{\text{space}}(t) - \bar{\phi}_S],$$

where $R = 1$ au is the orbital radius of the detector, and $\Phi_{\text{space}}(t)$ is the azimuthal angle of the detector around the Sun. The BNS and NSBH signals normally last more than one day in 3G ground-based detectors, so they also have time-varying $F^+$, $F^-$, and $\varphi_D$. For ground-based detectors, we follow the same logic as with space-borne detectors to construct their responses. The difference between them is in the arm direction, which could change, with a rotation angle $\phi_0$, varying from day in 3G ground-based detectors, so they also have time-

To explore the multiband enhancement, for the decihertz observatories, we choose four designs, namely, B-DECIGO, DECIGO, DO-Conservative, and DO-Optimal, and we use ET as a representative of the hectohertz ground-based detector. Below we give details on the equivalent number of detectors, as a representative of the hectohertz ground-based detector.

1. For DO-Optimal and DO-Conservative, we use two effective detectors and triangular LISA-like orbits. Their sensitivity curves are taken from Sedda et al. (2020), which we treat as the averaged PSD over $\theta, \phi, \psi$, and detector numbers. The frequency range is $[f_{\text{low}}, f_{\text{high}}] = [10^{-3}, 10]$ Hz.

2. For B-DECIGO, we use two effective detectors and a triangular LISA-like orbit. The sensitivity curve is taken from Equation (20) of Isoyama et al. (2018), and the frequency range is $[f_{\text{low}}, f_{\text{high}}] = [10^{-3}, 100]$ Hz.

3. For DECIGO, we use eight effective detectors with four triangular LISA-like interferometers located from one another by 120° separation on their heliocentric orbits. The sensitivity curve is taken from Equation (5) of Yagi & Seto (2011), and the frequency range is $[f_{\text{low}}, f_{\text{high}}] = [10^{-3}, 100]$ Hz.

4. For ET, we adopt the final design ET-D, which has three triangular detectors and will be possibly placed in Italy; so we set the latitude of ET $\delta = 0.7615$. The sensitivity curve is taken from Hild et al. (2011), and the frequency range is $[f_{\text{low}}, f_{\text{high}}] = [1, 10^4]$ Hz.

The sky-averaged noise curves of these GW detectors are given in Figure 1. Throughout the paper, we mainly study the PE using the synergy of B-DECIGO and ET as a fiducial scenario. We will make comparison with the other three space-borne detectors specifically in Section 3.4.

2.3. PE Method and Source Selection

We use matched filtering to estimate the binary parameters (Finn 1992; Cutler & Flanagan 1994). The noise weighted inner product between two signals, $h(t)$ and $g(t)$, is defined as

$$\langle h|g \rangle = \int_{f_{\text{in}}}^{f_{\text{out}}} \frac{\tilde{h}^*(f) \tilde{g}(f) + \tilde{g}^*(f) \tilde{h}(f)}{S_n(f)} \text{d}f,$$

where $S_n(f)$ is the noise PSD of the detector; the frequency range $f_{\text{in}}$ and $f_{\text{out}}$ are determined by the detector’s limitation and the property of the signal by $f_{\text{in}} = \max(f_{\text{in yr}}, f_{\text{in low}})$ and $f_{\text{out}} = \min(f_{\text{SCSO}}, f_{\text{high}})$, where $f_{\text{in yr}} = (a_{\text{obs}}/5)^{3/8} M^{-5/8} / 8\pi$ with $a_{\text{obs}} = 4$ yr is the GW frequency 4 years before the merger, and $f_{\text{SCSO}} = (6^{1/2} \pi M)^{-1}$ is the GW frequency at the innermost stable circular orbit (ISCO) of a Schwarzschild metric with mass $M$. We list $f_{\text{in}}$ and $f_{\text{out}}$ for different sources in Table 1.

The $S/N$ for a signal $h$ is given by $\rho = \sqrt{\langle h|h \rangle}$. In the limit of large $S/N$s, supposing that the noise is stationary and Gaussian, the Fisher matrix method (Finn 1992) is a fast way to estimate parameter statistical errors. We denote a collection of
parameters in a vector, $\Xi$. The element of the Fisher matrix $\Gamma_{ab}$ is then given by $\Gamma_{ab} = \langle \partial h / \partial \Xi^a \partial h / \partial \Xi^b \rangle$, where $h$ is the detector-frame GW strain, i.e., Equation (10). The error vector, $\Delta \Xi$, has a multivariate Gaussian probability distribution (Vallisneri 2008), $p(\Delta \Xi) \propto \exp (-\Delta \Xi^T \Gamma^{-1} \Delta \Xi/2)$, where $\Delta \Xi^a = \Xi^a - \bar{\Xi}^a$, with $\bar{\Xi}$ the maximum-likelihood parameter determined by the matched filtering. The variance–covariance matrix element is given by $(\delta \Xi^a \delta \Xi^b) = (\Gamma^{-1})_{ab}$; then an estimate of the rms, $\Delta \Xi^a$, and the cross correlation between $\Xi^a$ and $\Xi^b$, $c_{ab}$, are $\Delta \Xi^a = \sqrt{\Delta \Xi^a}$ and $c_{ab} = (\delta \Xi^a \delta \Xi^b) / \Delta \Xi^a \Delta \Xi^b$, respectively. The angular resolution $\Delta \xi$ is defined as $\Delta \xi = 2\pi / (\Delta \mu \Delta \delta \phi_2)$, where $\mu_2 = \cos \phi_2$ (Barack & Cutler 2004; Lang & Hughes 2008). Finally, to estimate the parameter precision from joint observations, we add the Fisher matrices from both detectors together as $\Gamma_{ab}^{\text{joint}} = \Gamma_{ab}^{\text{space}} + \Gamma_{ab}^{\text{ground}}$ (Cutler & Flanagan 1994).

Now we turn to source selection. Because we are interested in both BNS and NSBH systems, we choose our fiducial values from the properties of (i) the BNS inspiral GW170817, and (ii) the NSBH merger GW200105. Meanwhile, we take reasonable values for the poorly measured parameters, such as $\chi_{1,2}$, $\kappa_{1,2}$, and $\Lambda_{1,2}$. We list source properties in Table 1. Furthermore, we also select three fixed locations for later comparisons: (I) $\cos \phi_2 = 0$ and $\phi_3 = 2.0\pi$, (II) $\cos \phi_2 = 0.271$ and $\phi_3 = 0$, and (III) $\cos \phi_2 = 0.936$ and $\phi_3 = 4.768$. We will refer to the BNS system at location I/II/III as “BNS I/II/III” and the NSBH analog as “NSBH I/II/III” in the following analyses. As we will see, location I has a large S/N and location III has precise sky localization.

Finally, we define three parameter sets for the convenience of exposition: (i) the intrinsic parameter set,

$$\Xi^{\text{int}} = \{ M, \eta, \chi, \delta, \bar{\phi}, \bar{\Lambda} \}; \quad (22)$$

(ii) the extrinsic parameter set,

$$\Xi^{\text{ext}} = \{ t_0, \chi_0, \bar{L}_h, \bar{\phi}_5, \bar{\theta}_5, \bar{\phi}_6, \bar{\theta}_6 \}; \quad (23)$$

and (iii) the localization parameter set which is a subset of $\Xi^{\text{ext}}$,

$$\Xi^{\text{loc}} \subset \{ \bar{\theta}_5, \bar{\phi}_5, \bar{\theta}_6, \bar{\phi}_6, \bar{L}_h \} \subset \Xi^{\text{ext}}. \quad (24)$$

As a short summary, the parameters that we put into the waveforms are

$$\Xi^{\text{input}} = \{ m_1, m_2, \chi_1, \chi_2, \kappa_1, \kappa_2, \Lambda_1, \Lambda_2 \} \cup \Xi^0, \quad (25)$$

whereas the parameters we estimate are

$$\Xi^{\text{PE}} = \Xi^{\text{int}} \cup \Xi^0. \quad (26)$$

For the spin parameters, we choose only to estimate $\chi$, mainly for two reasons: (i) when simultaneously estimating $\lambda$ and $\chi_5$, or $\lambda$ and $\chi_2$, the correlations between them, as well as with $Q$, become larger than 0.9999 such that the Fisher matrix will be rather singular, while estimating $\chi$ is slightly uncorrelated than estimating $\chi_1$, $\chi_2$, or $\chi_5$; (ii) from the formation channel point of view, a BNS system often consists of a rapidly spinning, recycled pulsar and a slowly rotating, second-born pulsar whose $\chi$ is very close to zero (Tauris et al. 2017), so estimating one of the spin parameter is sufficient to constrain such a system within an astrophysical setting for field binaries.

It is worth noting that when the contribution of $\delta \bar{\Lambda}$ grows, the omission of $\delta \bar{\Lambda}$ in the estimation could lead to overestimated constraints on $\Delta \bar{\Lambda}$. On the other hand, the lack of prior knowledge in our consideration could underestimate the parameter errors. Quantitatively, we have checked that both kinds of effects on the uncertainties are less than 1 order of magnitude.

In calculating the Fisher matrix, the analytical expressions for the partial derivative $\partial h / \partial \Xi^i$ are usually not available. We decide to calculate the partial derivatives of $\tilde{h}(f)$ with respect to $t_0$, $\phi_5$, $\bar{\phi}$, and $\bar{\Lambda}$ analytically, and calculate the partial derivatives of $\bar{h}(f)$ with respect to the rest of the parameters numerically. For the latter, we adopt a numerical scheme that $\partial h / \partial \Xi = [\tilde{h}(\Xi + \delta \Xi) - \bar{h}(\Xi - \delta \Xi)]/2\delta \Xi$, and we have

\begin{table}
\centering
\begin{tabular}{cccc}
\hline
 & GW170817-like & GW200105-like \\
\hline
$\bar{m}_1$ ($M_{\odot}$) & 1.46 & 8.9 \\
$\bar{m}_2$ ($M_{\odot}$) & 1.27 & 1.9 \\
$\chi_1$ & 0.0469 & 0.125 \\
$\chi_2$ & 0.002 & 0.004 \\
$\kappa_1$ & 9 & 1 \\
$\kappa_2$ & 10 & 3 \\
$\Lambda_1$ & 675 & 0 \\
$\Lambda_2$ & 951 & 237 \\
$\bar{\Lambda}$ & 793 & 2.81 \\
$Q$ & $1.01 \times 10^{-2}$ & $1.98 \times 10^{-6}$ \\
$\cos \delta \bar{\phi}_5$ & -0.65 & -0.65 \\
$\phi_5$ (rad) & 5.016 & 5.016 \\
$L_h$ (Mpc) & 40 & 280 \\
$\bar{L}_h$ & 0.01 & 0.06 \\
$t_{\text{obs,DECIGO/DO}}$ (yr) & 4 yr & 4 yr \\
$t_{\text{obs,ET}}$ (yr) & 5.6 d & 0.90 d \\
$f_{\text{obs,DECIGO/DO}}$ (Hz) & 0.124 & 0.0622 \\
$f_{\text{obs,ET}}$ (Hz) & 1.0 & 1.0 \\
$f_{\text{out,DECIGO}}$ (Hz) & 100 & 100 \\
f& $f_{\text{out,ET}}$ (Hz) & 1595 & 384.1 \\
\hline
\end{tabular}
\caption{Properties of the GW170817-like BNS System and the GW200105-like NSBH System Explored in the Paper}
\end{table}
In this section, we present our PE results focusing on the constraints of the quadrupole parameter, tidal deformability, and localization precision.

We show the dependence of the S/N on the source sky position for B-DECIGO and ET, respectively, in the large and small sky maps in Figure 2. For the convenience of reading Figure 2, as well as the following Figures 4–6, and 8, we explain the common characteristics of such sky maps here. The plots are based on ecliptic coordinates, showing the S/N or PE precision as a function of the source’s sky location. The red star in each sky map marks the direction of the source’s angular momentum \( \mathbf{L} \), and the red “×”/“+” marks the lowest/highest values in each map. Roman numbers I, II, and III mark the source locations I, II, and III.

From Figure 2 we see that the S/N is larger when the source is either face on or face off, which is intuitive. The slight deviation of the maximum point location is caused by the detector’s orbit. Meanwhile, the minimum point is perpendicular to this direction. In later analyses, we show that the S/N is not the dominant factor for the PE precision, especially for localization.

We also tabulate the complete PE results in Table 2 for the example sources whose properties are listed in Table 1. The table gives a comparison of the PE results for different sky locations, different detectors, and different systems. We will analyze them in detail in the following subsections.

In Section 3.1, we present the parameter correlations from single/joint detection for BNS/NSBH systems. Based on different features of \( \Sigma_{\text{int}} \) and \( \Sigma_{\text{ext}} \), we analyze the constraints on them in Sections 3.2 and 3.3, respectively. More specifically, in Section 3.2, we show the multiband improvements on \( \Sigma_{\text{int}} \), with Section 3.2.1 focusing on \( \Delta \bar{Q} \) and Section 3.2.2 focusing on \( \Delta \bar{\Lambda} \), and discuss the different characteristics of BNS/NSBH systems. In Section 3.3, we show the features of \( \Sigma_{\text{ext}} \), where Section 3.3.1 investigates the localization ability from space, ground, and multiband observations, and Section 3.3.2 displays the evolution of angular resolution with the passing of observing time, which provides information for early warning alerts. In Section 3.4, we analyze the different constraints of using DOs/B-DECIGO/DECIGO jointly with ET, and in Section 3.5, we briefly compare our results with the PE results obtained using another phenomenological waveform, i.e., IMRPhenomPv2_NRTidalv2.
Based on the above arguments, considering the difference between intrinsic and extrinsic parameters, we will analyze them separately in the following two subsections.

3.2. Estimation on Intrinsic Parameters

In this subsection, we illustrate how multiband observations improve intrinsic parameters in \( \Xi \) int, especially focusing on the quadrupole parameter \( \tilde{Q} \) and the tidal deformability \( \tilde{\Lambda} \). Before digging into them in Sections 3.2.1 and 3.2.2, respectively, we present some general characteristics first.

Columns 4–8 in Table 2 show the precision of parameters in \( \Xi \) int for the selected sources. From the first part of the table, we find that, with the change in location, the parameter precision is approximately inversely proportional to the S/N. However, unlike the relation between the S/N and distance \( D_L \), this inverse relation is not exact. For example, using B-DECIGO (ET), the S/N is 2.13 (4.65) times higher at location I than at location II, but the parameter precision improvement for \( (\Delta_M/M, \Delta \eta, \Delta \chi_{\text{in}}, \Delta \tilde{Q}, \Delta \tilde{\Lambda}) \) is \( (2.17, 2.02, 1.90, 1.84, 1.55) \) \((3.29, 3.56, 4.31, 3.81, 4.47)\)) times instead. The deviation of such scaling with the S/N indicates the impact of source direction and orientation on estimating parameters in \( \Xi \) int.

During the investigation, we notice that the change in the fiducial values of \( \tilde{Q} \) and \( \tilde{\Lambda} \) within reasonable ranges will not affect the PE results significantly. In general, with increasing fiducial values, the estimated errors decrease slightly within 1 order of magnitude.

We have also verified that all the parameters in \( \Xi \) int follow the same distribution patterns on the sky maps for each detector, similar to those of \( \Delta \tilde{Q} \) in Figure 4 and \( \Delta \tilde{\Lambda} \) in Figure 5. Nevertheless, the precision distribution for extrinsic parameters are different from one another.

In addition, we have executed a Fisher matrix analysis without estimating the localization parameters \( (\tilde{\theta}_s, \tilde{\chi}_s) \), which can be seen as the PE precision when the EM counterparts are observed and they provide precise sky location of the sources. We found, in that case, that all errors for parameters in \( \Xi \) int are about 80% (99%) of the errors in Table 2 for BNS II using B-DECIGO (ET), which is insignificant compared to the improvement of the multiband observations with the consideration of localization.

3.2.1. Quadrupole Parameter

We now focus on the quadrupole parameter and show how joint detections benefit observations from space and ground in a mutual way.

In Figure 4(a), we show the combined constraints from “B-DECIGO+ET”. The relative error \( \Delta \tilde{Q}_{\text{B+ET}}/\tilde{Q} \) could reach down to 0.1 with the worst cases < 0.8, which indicates that \( \tilde{Q} \) is measurable no matter of the GW source direction. In contrast, the small sky maps in panels (b) and (c) of Figure 4 reveal that individual observations from neither B-DECIGO nor ET can fully identify this parameter. Comparing the small map in Figure 4(c) with Figure 4(a), we demonstrate that the sky distribution of the combined precision, \( \Delta \tilde{Q}_{\text{B+ET}}/\tilde{Q} \), as well as other intrinsic parameters as we have verified, is dominated by ET’s distribution pattern, \( \Delta \tilde{Q}_{\text{ET}}/\tilde{Q} \).

The large sky maps in panels (b) and (c) of Figure 4 stress the multiband effects by plotting the relative improvement in comparison with using B-DECIGO or ET alone. We notice that their distribution patterns are complementary. The color bar value shows that the joint detection measures 10–50 times better than B-DECIGO alone and 6–12 times better than ET alone. We also notice that the enhancement is less evident (the yellow

| System | Detector | S/N | \( \Delta L/M \) \((10^{-6})\) | \( \Delta \eta/\eta \) \((10^{-5})\) | \( \Delta \chi_{\text{in}}/\chi_{\text{in}} \) \((10^{-6})\) | \( \Delta \tilde{Q}/\tilde{Q} \) | \( \Delta \tilde{\Lambda}/\tilde{\Lambda} \) | \( \Delta \tau_t \) \((\text{ms})\) | \( \Delta \phi_i \) \((10^{-1})\) | \( \Delta D_{L}/D_{L} \) \((10^{-2})\) | \( \Delta \Omega \) \((\text{arcmin}^2)\) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| BNS I | ET | 1340 | 0.58 | 1.5 | 0.64 | 1.4 | 0.0053 | 0.13 | 0.11 | 0.71 | 2.55 (deg²) |
| | B-DEC | 212 | 0.023 | 0.59 | 4.5 | 5.6 | 27 | 12 | 5.4 | 3.3 | 9.08 |
| | B-DEC+ET | 1360 | 0.0037 | 0.050 | 0.18 | 0.16 | 0.0040 | 0.040 | 0.10 | 0.49 | 0.431 |
| BNS II | ET | 286 | 1.9 | 5.2 | 2.8 | 5.2 | 0.024 | 0.26 | 0.23 | 0.96 | 0.803 (deg²) |
| | B-DEC | 99.6 | 0.051 | 1.2 | 8.6 | 10 | 42 | 20 | 9.2 | 1.0 | 0.309 |
| | B-DEC+ET | 305 | 0.0092 | 0.13 | 0.55 | 0.49 | 0.017 | 0.028 | 0.14 | 0.45 | 0.0128 |
| BNS III | ET | 374 | 1.6 | 4.4 | 3.3 | 4.2 | 0.013 | 0.11 | 0.12 | 1.0 | 0.2068 (deg²) |
| | B-DEC | 95.2 | 0.050 | 1.2 | 8.5 | 42 | 20 | 9.3 | 1.7 | 0.0797 |
| | B-DEC+ET | 386 | 0.0088 | 0.13 | 0.51 | 0.44 | 0.014 | 0.023 | 0.13 | 0.57 | 0.00320 |
| NSBH II | ET | 105 | 54 | 48 | 5.7 | >10⁵ | 8.7 | 3.8 | 3.5 | 13 | 184 (deg²) |
| | B-DEC | 56.9 | 0.3 | 26 | 5.7 | >10⁴ | 1400 | 170 | 11 | 2.7 | 32.8 |
| | B-DEC+ET | 112 | 0.079 | 0.42 | 0.56 | >10³ | 3.3 | 0.19 | 0.29 | 1.2 | 0.354 |

Note. The first two columns label the source type, location, and detectors used. The table includes three parts: (i) the first part illustrates the effect of different locations for BNSs I, II, and III; (ii) the second part illustrates the effect of different detectors for BNS II; and (iii) the third part illustrates results for NSBH II for comparison with the corresponding BNS results. When only ET is used, \( \Delta \Omega \) is given in units of square degrees.
region) usually at the location where the former detector measures pretty well and the later joined detector measures poorly. There is a small region near the antialigned direction of $\hat{L}$ that has a relative lower improvement, which is caused by a degeneracy of location parameters. Such degeneracy gets worse when joining B-DECIGO with ET; therefore, it leads to a worse PE improvement (see triangle markers in Figure 6).

Comparing the distribution of PE errors, e.g., in Figure 4 (a), with the S/N in Figure 2, we find that the former has larger yellow areas, which also indicates an imperfect inverse relation between PE errors and the S/N. Even if some locations have a lower S/N, they still yield relative precise PE results.

From the measurement of the quadrupole parameter, we see the striking advantage of multiband detection, because $\Delta \hat{Q}$ determined by a space-borne detector or a ground-based detector alone is too large to yield any constraints, while multiband observations enable pretty good limits on $\Delta \hat{Q}$.

### 3.2.2. Tidal Deformability

We now investigate the multiband constraints on the tidal deformability, as well as the comparison between BNS and NSBH systems. In Figure 5, we show the joint detection errors and multiband enhancement relative to using only ET.

Figure 5(a) displays the distribution of $\Delta \Lambda_{B+ET}/\Lambda$ for the BNS system as a function of the sky location, which gives a value ranging between $3 \times 10^{-1}$ to $2 \times 10^{-2}$. The small sky map indicates that multiband limits are dominated by ET’s value, with 1–2 times tighter when B-DECIGO joins in. In contrast to other intrinsic parameters, the tidal effect starts from 5 PN and contributes largely at the very last stage of inspiral in the ET band; therefore, ET plays a leading role in constraining $\Lambda$.

Figure 5(b) is the NSBH analog to Figure 5(a). Comparing the large maps in both panels, we see that the BNS system yields a tighter relative error, $\Delta \Lambda_{B+ET}/\Lambda$, while the NSBH...
Such difference partially comes from the fact that BNS has a larger tidal deformability parameter $\tilde{\Lambda}$, which leads to a larger contribution of the tidal term in the GW phase. The detectors are thus more sensitive to $\tilde{\Lambda}$. Another reason is that the BNS signal has a longer time (5.6 days) in ET than NSBH (0.9 days), which helps it accumulate more GW cycles; hence ET could extract more information on the tidal parameter.

The improvement ratio $\Delta \tilde{\Lambda}_{ET}/\Delta \tilde{\Lambda}_{B-DECIGO}$, on the other hand, is more significant in the NSBH system with more information gained from B-DECIGO. From the NSBH II, we notice that even when B-DECIGO measures $\Delta \tilde{\Lambda}_B/\tilde{\Lambda}$ larger than $10^3$, it still helps reduce ET's uncertainty by about 3 times in multiband observation. Such improvement benefits from the precise measurements of B-DECIGO on other parameters, which helps ET to break the degeneracies among them. This is similar to the measurement of the dipole radiation parameter in Zhao et al. (2021). The above analysis of comparing BNS and NSBH systems could extend to other parameters as well. To keep the discussion brief, we do not give too much detail here.

3.3. Estimation on Extrinsic Parameters

In this subsection, we demonstrate how multiband observations improve extrinsic parameters in $\Xi^{int}$, especially focusing on the angular resolution $\Delta \Omega$. Same as the last subsection, we begin with some general characteristics of $\Xi^{int}$ and then concentrate in Sections 3.3.1 and 3.3.2 on $\Delta \Omega$ and its improvement over time, respectively, which is important for EM follow-ups. It is worth noting that, in contrast to the intrinsic parameters in $\Xi^{int}$, the sky distribution pattern of each parameter in $\Xi^{ext}$ is not the same, especially among parameters $D_L$, $\Delta \Omega$, and $\Delta \vec{r}$, though we only show the distribution of $\Delta \Omega$ in this study.

The time at coalescence $t_c$ is an intriguing parameter that correlates with both intrinsic and extrinsic parameters, as can be seen in the correlation matrices in Figure 3. Therefore, measurement of $t_c$ will be affected by many aspects. For example, if we know the location of the source from EM observations—which means there is no need to estimate $\psi$ and $\tilde{\psi}$—unlike $\Xi^{int}$, the precision of $t_c$ will improve enormously by nearly 1 order of magnitude. Moreover, although whether or not to include the Earth’s orbit in the Doppler phase $\varphi_{D}(t)$ does not affect the PE results of all other parameters, $\Delta D_L$ will be influenced by about 1 order of magnitude.

Also, unlike the nearly inverse relation between intrinsic parameters $\Xi^{int}$ and the S/N, the value $\Delta \Omega$ is largely unrelated with the S/N. From column 12 in Table 2 we see that, among the three locations, location I has the largest localization area but the highest S/N. Comparing B-DECIGO’s PE results of BNS II and BNS III, they have similar S/Ns, $\Delta \Xi^{int}$, $\Delta \psi$, and $\Delta \tilde{\psi}$, but significant differences in $\Delta D_L$ and $\Delta \Omega$. The precision $\Delta D_L$ is codetermined by the S/N and the sky location of the source.

3.3.1. Sky Localization

Due to the need of an accurate sky location for successful EM follow-ups, we pay close attention to the precision of $\Delta \Omega$. We present the multiband sky localization improvement as well as constraints from individual detectors for the BNS system in

$$\left[ \frac{\Delta \tilde{\Lambda}_{B+ET}}{\Lambda} \right]_{\text{BNS}} = 0.47 \left( \frac{D_L}{40 \text{ Mpc}} \right).$$  (27)
Small: the improvement of constraining the meanings of the marks, including "x", "X", "+", and Roman numbers. We mark the error ΔΩ. The Astrophysical Journal, 926:158 (15pp), 2022 February 20

Figure 6. The reason why we display the improvement ΔΩ_{B}/ΔΩ_{B+ET} rather than ΔΩ_{B+ET} is that the distribution of ΔΩ_{B+ET} is completely dominated by ΔΩ_{B}'s pattern, which means that the uncertainty distribution of ΔΩ_{B+ET} is extremely close to the right small map in Figure 6. Note that, in these sky maps of ΔΩ, we smooth the values on the map over a few pixels to eliminate the discrete effect caused by a limited number of points. The smoothing leads to a decrease in the maximum value in the B-DECIGO plot but does not affect the discussions presented here.

Comparing the two small maps in Figure 6, we notice that B-DECIGO can localize down to 10^{-2} arcmin, which is orders of magnitude better than is ET. ET, albeit less powerful, still exceeds most ground-based observatories for it has three individual detectors and a lower cutoff frequency, which prolong the signal's active time in the sensitive band. The localization precisions, as well as sky distributions, are mainly determined by the motions and orbital baselines of the two detectors. The multiband improvement ΔΩ_{B}/ΔΩ_{B+ET} ranges from 10 to 50, which is far better than that of ΔΩ_{ET}/ΔΩ_{B+ET}. Considering that both ΔΩ_{ET} and ΔΩ_{B} are 3–4 orders of magnitude worse than ΔΩ_{ET} and ΔΩ_{B+ET}, such a huge improvement in sky localization benefits from the two distinct orbits of B-DECIGO and ET.

From the B-DECIGO plot in the lower right of Figure 6, we notice large errors in the ecliptic plane. This is a unique effect for space-borne detectors as it also shows up in Figure 8 for DOs and DECIGO. Such a phenomenon results from a combination of detector orials and the signal duration. We first clarify that ΔΩ is codetermined by Δμ_{S} and Δμ_{E}, and this “line-like” effect comes from the characteristic of μ_{S}.

When the ecliptic polar angle of the source μ_{E} changes from 0°/180° (the two poles) to 90° (the ecliptic plane), the partial derivative of ∂h/∂μ_{S} becomes smaller across 3 orders of magnitude. With the weight S_{n} in Equation (21), the integral Γ_{μ_{E}μ_{S}} becomes smaller by 8 orders of magnitude, which makes the error Δμ_{S} range from 10^{-2} to 10^{-3}. Gradually from the two poles to the ecliptic plane. While Δμ_{E} remains around 10^{-3}, the angular resolution is determined by the less accurate Δμ_{S}; accordingly, large localization errors appear near the ecliptic plane.

However, such a “line-like” effect will eventually vanish with the increase in the source masses, thanks to another characteristic from μ_{S}. With the increment in mass, the GW source merges at a lower frequency, which makes the GW signal exist in the detector’s band for a shorter time—the signal enters the detectors only few months or days before coalescence for dechertzeters. This is a key factor because ∂h/∂μ_{S}, which depends strongly on the secular evolution of the detector orbit, is orders of magnitude lower after one month before the merger. Without information on the long-term orbital modulation, the integral Γ_{μ_{E}μ_{S}} is tremendously smaller, which leads to a higher Δμ_{S}. While Δμ_{S} still varies greatly with the latitude, it no longer dominates the value of ΔΩ. Therefore, the “line-like” effect vanishes.

We have also verified that, for sources whose total mass M > 10^{3}M_{⊙}, this bad measurement along the ecliptic plane disappears. From a mathematical point of view, the whole process is a competition between ∂h/∂μ_{S}, ∂h/∂μ_{E}, and S_{n}.

We mark the locations where the maximum correlation in the off-diagonal elements of the correction matrix is over 0.9995 in each map in Figure 6 by red triangles. We illustrate them briefly in Section 3.1 and conclude that ET is slightly better than B-DECIGO in breaking such degeneracy, and joint observations deepen this effect by enlarging the regions covered by red triangles.

3.3.2. Early Warnings

Prompt communication of source location from the GW detection to EM facilities is crucial for multimessenger follow-ups. While the accurate sky area derived from the complete GW signal is promising, the premerger alert is equally important.

We show the localization precision of the BNS systems as a function of the frequency in Figure 7 and mark the time before coalescence at the top of the plot. We find that, with the joining in of ET, the localization precision ΔΩ gradually narrows. The shape of how ΔΩ improves depends on the detectors’ sensitivity curves and designs. In addition, the same shape of
the dashed and the dotted blue lines reveals that the location of the source does not affect the variation trend of the multiband improvement.

With nearly 4 yr time, the space-borne detector already has a stable localization area ∼arcmin². ET starts the observation about 5 days before the merger. It gradually narrows down its own localization area until a few minutes before the merger and soon becomes stable, as shown in the light green lines. However, after ET joins the observation, it will not improve the sky area of the space-borne detectors in the first few days. One day before the merger, the joint detection begins to take effect; ΔΩ gradually drops again and finally has an improvement of 1–2 orders of magnitude for DOs and B-DECIGO.

DO-Conservative and B-DECIGO have similar single detector angular resolutions but different multiband improvements. “B-DECIGO+ET” is about 1 order of magnitude better than “DO-Conservative+ET”, which is caused by the better sensitivity of B-DECIGO in the high-frequency band. Furthermore, from the blue and dark green dashed lines in Figure 7, we discover that the variation trend of ΔΩ with the frequency is different: B-DECIGO drops more sharply, though B-DECIGO and DOs have the same orbital configuration. The reason might be that, at a frequency larger than 0.1 Hz, the BNS signal is at the lowest noise region, the so-called sweet point, of B-DECIGO’s sensitivity curve. Therefore, it gains more information from the source after this frequency, while DOs get more information before this point.

The multiband enhancement of DECIGO, on the other hand, is different from the other three decihertz detectors. Benefiting from four LISA-like designs, it can localize precisely to ∼10⁻⁶ arcmin². The sky area keeps shrinking until minutes before the merger, and the inclusion of ET barely improves the precision. We will discuss further the comparisons between different decihertz detectors in the next subsection.

3.4. Comparison Between Different Decihertz Detectors

In this subsection, we compare the multiband results by combining ET with different decihertz observatories. In Figure 8, we plot the DO-Optimal and DECIGO analogs to Figures 4(b) and 6 in order to highlight their similarities and differences.

For the sky localization ability, from the small maps in Figures 8(a), (b), and 6, we see that the sky distribution is similar, except that DECIGO appears more clumpy with more apparent delimitations. DOs have similar designs with B-DECIGO; thus the localization precision is approximately ∝1/S/N². Contrarily, DECIGO has exceedingly good localization precision down to 10⁻⁶ arcmin² because of multiple interferometers. This distinction is also reflected in the large maps in Figures 8(a) and (b), where the difference in multiband enhancement is displayed. From the localization point of view, DECIGO’s ability is sufficient by itself.

For intrinsic parameters, based on the measurement on ΔΩ/Ω, we stress three points. (i) Comparing the small map in Figure 8(c) with the small map in Figure 4(b), although DO-Optimal yields larger S/Ns, its measurements on ΔΩ do not exceed those of B-DECIGO, because DOs have a relatively poor performance at high frequencies. This situation also applies for λ values that enter at a higher PN order. (ii) Comparing the small map in Figure 8(d) with the small map in Figure 4(b), we notice that, with the help of
four interferometers, DECIGO alone could discriminate the quadrupole parameter. Moreover, the uncertainty dispersion on the sky map reduces significantly. (iii) Comparing the large map in Figure 8(d) with the small map in Figure 4(c), we see that the relative improvement just follows the distribution of \(\Delta \hat{Q}_{\text{ET}}\), which indicates that DECIGO has reached its ceiling in detecting \(\Delta \hat{Q}\).

3.5. Comparison Between Different Waveforms

After investigating the PE results constrained by different detectors, we extend our method to other waveform models and confirm that the PN waveform and phenomenological waveform could yield similar statistical errors.

We adopted two templates implemented in the LIGO Scientific Collaboration Algorithm Library (LAL; LIGO Scientific Collaboration 2018), TaylorF2 and IMRPhenomPv2_NRTidalv2 (Husa et al. 2016; Dietrich et al. 2019a). TaylorF2 is the same with ours except that it includes a tidal correction up to 7 PN. IMRPhenomPv2_NRTidalv2 uses the precessing phenomenological BBH waveform baseline, augmented with a tidal prescription “NRTidalv2”. Other than the PN approximation, “NRTidalv2” uses a numerical-relativity-based closed-form tidal phase contribution with a tidal amplitude correction, as well as an inclusion of spin–spin and cubic-in-spin effects up to 3.5 PN (Dietrich et al. 2017, 2019b, 2019a). Furthermore, it implements the universal relations to relate the tidal deformability to the spin-induced quadrupole–monopole terms and therefore reduces the parameters \(\kappa_{1,2}\). To compare our results with it, we change the parameter set to \(\{M, \eta, \chi, \tilde{\Lambda}\} \cup \Xi^{\text{ext}}\) and inject only the spin-aligned waveforms in IMRPhenomPv2_NRTidalv2.

In Figure 9, we briefly compare the multiband parameter errors of BNS II detected by “B-DECIGO+ET” adopting “our TaylorF2” in Section 2.1, TaylorF2 in LAL, and IMRPhenomPv2_NRTidalv2. We exhibit the results of four parameters, i.e., \(\chi, \tilde{\Lambda}, \Delta \hat{\Lambda}, \) and \(\phi_r\), that vary among the three waveforms. For single-detector detections, such variation is more evident in B-DECIGO and less obvious in ET, and the variation of the combined errors we present here is a mixture of B-DECIGO’s and ET’s trends. We have checked that the parameters that do not appear in the figure are consistent across all three waveform models. We also find that, without assessing \(\hat{Q}\), \(\Delta \hat{\Lambda}\) has been tightened marginally by 30%, from 1.7% to 1.3%.

We notice that our lower-order TaylorF2 and TaylorF2 in LAL are very close to each other in the uncertainties and

\[
\Xi^{\text{PE}} \equiv \{M, \eta, \chi, \tilde{\Lambda}\} \cup \Xi^{\text{ext}}
\]
We could estimate the precision of $c_0$ and then obtain the limitation on $\Delta \Lambda$. We take the EoS AP4 as an example to illustrate the multiband constraints. The fiducial values of $[c_0, c_1, c_2]$ of AP4 are $[4.09, -3.87, -2.8] \times 10^{-24} s^5$, respectively. By assuming that both NSs are described by the same EoS, we use AP4’s tidal deformability as the fiducial values for the BNS system at location II and derive $\Delta \Delta_{\text{ET}}/\tilde{\Lambda} = 4.3\%$ and $\Delta \tilde{\Delta}_{\text{ET}}/\tilde{\Lambda} = 7.0\%$. Using Equation (30), we give our multiband constraints on the uncertainty of $\Lambda_{1,2}$.

$$\Delta \Lambda_1 = 19 \left( \frac{D_L}{40 \text{ Mpc}} \right).$$

$$\Delta \Lambda_2 = 10 \left( \frac{D_L}{40 \text{ Mpc}} \right).$$

In Figure 10, we show the multiband constraints on $\Lambda_{1,2}$. At mass $m_1^3 = 1.46 M_\odot$, we plot the 1$\sigma$ constraints by “B-DECIGO+ET” in black and by ET in gray. We find that joint detection can almost rule out all the wrong EoSs in the plot, while ET alone cannot. To be more illustrative, we show the 10$\sigma$ constraint at mass $m_1^3 = 1.27 M_\odot$, which equals the projected results of such a source but at $\sim$10 times further. We find that, although many EoSs are within the error bar, multiband observations eliminate nearly half more EoSs than ET alone. We have to admit that space-borne detector’s help on reducing the errors on $\tilde{\Lambda}$ is not comparable to a reduction in the luminosity distance or a change of source location, as could be seen in Table 2. However, the multiband improvement is a human endeavor other than nature’s choice of GW sources. With its help, we are one step closer to understanding the NS structure and its EoS. In addition, we should be cautious when treating the uncertainties on $\Lambda_{1,2}$, since our assumption is in the limit of $\delta \Delta \rightarrow 0$, which will break down when the mass ratio of two components grows. In that case, an estimation of $\delta \Delta$ is needed, and if no prior is provided such an estimation will lead to an uncertainty on $\Delta \Lambda_{1,2}$ larger by 1 order of magnitude. However, if some physically motivated prior (e.g., from the universal relation and so on) is included, the effect will not be large. Overall, our result should be viewed as reasonable but optimistic. Meanwhile, as we have...
carefully checked, the multiband improvement factor basically keeps the same level in both cases, with or without estimating $\delta \Lambda$.

5. Summary

In this paper, we adopted a multiband detection strategy, namely, using both decihertz GW detectors and the ET, to jointly observe two classes of coalescing binary systems. We take a GW170817-like BNS system and a GW200105-like NSBH system as examples. We analyzed their PE uncertainties, presented the sky distributions of the parameter constraints, and discussed the synergy effects in detail. Here we give a brief summary of our key findings.

1. Assuming the joint B-DECIGO and ET detection and adopting a PN waveform, we found that joint detection could break the strong correlation between the quadrupole parameter and spin parameter that occurs when the source is observed by a decihertz detector alone. The joint detection also breaks the strong correlations among localization parameters that occur when the source is observed by the ET alone.

2. We have shown that only the joint detection could effectively measure the quadrupole parameter for the BNS system. While tidal deformability is detectable by ET alone, joint detection still gives an improvement of a factor of 1–3. Tidal deformability measurements of NSBHs are dozens of times worse than those of BNSs, mainly because of NSBHs' indistinctive tidal contribution to the GW phase.

3. Combining with the EoS information, we constrained the individual tidal deformability and demonstrated that multiband observations could rule out many more EoSs than the use of ET alone, which would greatly help to understand the behaviors of supranuclear dense matters at low temperatures.

4. We made comparisons of using different detectors and waveforms. We concluded that DECIGO, which is made up of four independent LISA-like detectors, is remarkably different from the other three space-borne decihertz detectors because of its more complex design. We also showed that the IMRPhenomPv2_NRTidalv2 waveform constrains the parameters looser than the PN waveform, but the difference is small.

5. BNS systems and some NSBH systems are expected to be accompanied by EM counterpart signals. Rapid alerts of source localization are crucial for successful multimessenger follow-ups for some EM wave bands. We demonstrated that multiband detections could narrow down the arcmin-level resolution from space-borne decihertz detectors alone by another dozens of times. Meanwhile, joint detections start to take effect about one day before the merger, which makes multiband premerger alerts promising.

In the first three observing runs of the LIGO/Virgo/KAGRA Collaborations, the latencies of public alerts on GW candidates take minutes or hours after the preliminary detection of GW events (Abbott et al. 2019b). With the inclusion of space-borne decihertz detectors, EM facilities can be forewarned minutes or hours before the merger with an accuracy in sky localization better than a squared arcminute, which will enable a deeper and more targeted search for the EM counterpart by telescopes with narrow fields of view, such as Swift's X-Ray Telescope (Burrows et al. 2005) and the James Webb Space Telescope (Kalirai 2018). This hence will maximize the science return in an unprecedented way.

Throughout the paper, priors are ignored in our calculations, or equivalently priors are all considered uniform in each parameter, which leads to underestimated parameter errors. This can be improved if we have more informed priors from astrophysical or other considerations, for example, from the limit of Kerr spin in general relativity or the universal relations of NSs. In addition, a full Bayesian analysis could incorporate various kinds of prior knowledge and further test effects from different physical priors.

The Fisher information matrix method, which can be viewed as a fast but approximate version of parameter inference, however, has several caveats. It could be problematic when the parameter dimension grows (Harry & Lundgren 2021), in particular when including complex parameter corrections other than the simple Gaussian ones (Vallisneri 2008; Smith et al. 2021). In that case, very high correlations might appear and ruin the predictive ability of the Fisher matrix. Though parameters in our parameter set are not highly correlated, the complex relation between intrinsic parameters, for example, between $\Lambda$ and $\delta \Lambda$, still makes us cautious viewing the results. Our analysis is thus only preliminary and indicative. Nevertheless, we have tested our method with the noise curve and system parameters of GW170817 and found consistent results for both tidal deformability and masses with those reported in the dedicated LIGO/Virgo analyses (Abbott et al. 2017a, 2019c). In addition, we have tested the validity of using the Fisher analysis in our scenarios using the likelihood ratio that was proposed by Vallisneri (2008); we found that the linear signal approximation is satisfied, and the use of the Fisher matrix is valid. Moreover, we focus on the statistical errors of the multiband improvement. Nonetheless, such enhancement could be hampered by the systematic errors that result from, e.g., inaccurate waveform modeling (Isoyama et al. 2018; Gamba et al. 2021). A full Bayesian analysis with consideration of systematic errors will help to address these issues better.

In conclusion, multiband observations of BNS/NSBH systems will complement the single wave band detections in providing new insight into nuclear matter under extreme conditions, the origins of high-energy astrophysical phenomena, and so on. We foresee that, with future multiband detection, our understanding of astrophysics and fundamental physics will make great progresses and pin down unsolved important puzzles.

We thank Yong Gao and Junjie Zhao for useful discussions and the anonymous referee for critical comments. This work was supported by the National Natural Science Foundation of China (11975027, 11991053, 11721303), the National Ska Program of China (2020SKA0120300), the Young Elite Scientists Sponsorship Program by the China Association for Science and Technology (2018QNRC001), the Max Planck Partner Group Program funded by the Max Planck Society, and the High-Performance Computing Platform of Peking University. Some of the results in this paper have been produced using the healpy and HEALPix packages (Górski et al. 2005; Zonca et al. 2019).

Facilities: DECIGO, DO, ET.

Software: PyCBC (Nitz et al. 2020), HEALPix (Górski et al. 2005; Zonca et al. 2019).
