MEASUREMENTS OF SOLAR IRRADIANCE AND EFFECTIVE TEMPERATURE AS A PROBE OF SOLAR INTERIOR MAGNETIC FIELDS

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Received 2000 May 4; accepted 2000 November 1

ABSTRACT

We argue that a variety of solar data suggest that the activity-cycle timescale variability of the total irradiance is produced by structural adjustments of the solar interior. Assuming these adjustments are induced by variations of internal magnetic fields, we use measurements of the total irradiance and effective temperature over the period from 1978 to 1992 to infer the magnitude and location of the magnetic field. Using an updated stellar evolution model, which includes magnetic fields, we find that the observations can be explained by fields with peak values ranging from 120 to 2.3 kG, located in the convection zone between 0.959 and 0.997 $R_\odot$, respectively. The corresponding maximal radius changes are 17 km when the magnetic field is located at 0.959 $R_\odot$ and 3 km when it is located at 0.997 $R_\odot$. At these depths, the $W$-parameter (defined by $\Delta \ln R/\Delta \ln L$, where $R$ and $L$ are the radius and luminosity) ranges from 0.02 to 0.006. All these predictions are consistent with helioseismology and recent measurements carried out by the Michelson Doppler Imager experiment on the Solar and Heliospheric Observatory.

Subject headings: Sun: interior — Sun: magnetic fields

1. INTRODUCTION

Direct satellite measurements of total solar irradiance (Willson & Hudson 1991; Fröhlich & Lean 1998) show that it varies almost in phase with the solar activity cycle and that its relative variation in amplitude is about 0.1%. Although the most common explanation of this change involves the effect of magnetic regions and network (superposed on an otherwise constant solar photosphere), an alternative explanation is that the cyclic variation is caused primarily by structural changes of the solar interior. Two arguments support this second possibility. First, direct measurements of the spectrum of $p$-mode oscillations show variations (Libbrecht & Woodard 1990) that can be best explained in terms of structural changes (e.g., changes in pressure, density, radius; Lydon, Guenther, & Sofia 1996; Antia, Chitre, & Thompson 2000; Balmforth, Gough, & Merryfield 1996). Second, measurements of the “effective temperature” by Gray & Livingston (1997b) also vary nearly in phase with the solar cycle. This variation is obtained by monitoring the neutral carbon 5380 Å line in the solar flux spectrum. They present several arguments supporting their contention that what they measure is the gas temperature in the deep photosphere, free from the influence of sunspots, faculae, and small-scale magnetic flux tubes. Their main argument rests on their simultaneous monitoring of C I $\lambda$5380, Fe I $\lambda$5379, and Ti II $\lambda$5381. They use the ratios of spectral line depths, C I $\lambda$5380 to Fe I $\lambda$5379 and to Ti II $\lambda$5381, as temperature indicators. Since these three lines have different excitation potentials (1.57 eV for Ti II $\lambda$5381, 3.69 eV for Fe I $\lambda$5379, and 7.68 eV for C I $\lambda$5380), the consistency of the results for the two line depth ratios shows that the temperature indicators are free from the influence of variations of faculae and small-scale magnetic flux tubes that would be expected to affect each line in a different way. See Gray & Livingston (1997b) for the entire discussion. For the gas temperature to change, the energy flow from the interior must also change. Structural adjustment will be able to accomplish this. Since the relative radius variation $\Delta \ln R$ is very small (Emilio et al. 2000),

$$\Delta \ln L \approx 4 \Delta \ln T,$$  

where $L$ and $T$ are luminosity and temperature, respectively. The temperature change measured by Gray & Livingston (1997b) is $1.5 \pm 0.2$ K in one cycle, which corresponds to $\Delta \ln T \sim 0.025\%$ and $\Delta \ln L \sim 0.1\%$. This is approximately the entire cyclic variation of the total irradiance.

The most obvious way the solar structure may change is in response to variations of the internal magnetic field during the dynamo buildup and decay. In order to compute this effect, we introduce magnetic variables as new stellar structure variables (Lydon & Sofia 1995) in addition to the conventional ones (Guenther et al. 1992). Since the magnetic field is not a scalar, we have to use at least two variables to describe it: the magnetic energy density $\chi$ (Lydon & Sofia 1995) and the ratio of the magnetic pressure to the magnetic energy, $\gamma - 1$ (Endal, Sofia, & Twigg 1985). This method was developed by Lydon & Sofia in 1995, in which $\gamma$ was treated as a parameter, and it was subsequently applied by Lydon, Guenther, & Sofia (1996) to explain the observed variation of solar $p$-mode oscillations with the solar cycle (Libbrecht & Woodard 1990). Here we want to update this method as follows:

1. Treating both $\chi$ and $\gamma$ as new structure variables, as done for $\chi$ in Lydon & Sofia (1995);
2. Taking into account the influence of magnetic fields on radiative opacities;
3. Taking into account all time-dependent contributions to the equations of stellar structure (since we want to treat short timescales);
4. Modifying the radiative-loss assumption of a convective element (as discussed in § 3);
5. Updating the code by using the new version of Yale stellar evolution code (YREC7); and
6. Removing the perfect-gas approximation, which was assumed when calculating some first- and second-

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order derivatives associated with magnetic fields in Lydon & Sofia (1995). Because luminosity, radius, and temperature variations are sensitive to the location, intensity, orientation, and distribution of the perturbation magnetic field \( B = (8\pi \rho)^{1/2} \) (\( \rho \) is the gas density), we can use the measured yearly averaged irradiance, temperature, and radius variations to determine the solar interior magnetic field as a function of mass and time.

2. SOLAR IRRADIANCE AND EFFECTIVE TEMPERATURE RECORDS

Space-based solar irradiance measurements have been reviewed by Fröhlich & Lean (1998), and a composite solar irradiance record for the period from 1978 to the present can be downloading from the National Geophysics Data Center Web site.\(^2\) In our magnetic perturbation model calculations, we use 1 year as the time step since variations on the shorter timescales may be attributed to the effect of magnetic regions and network. We compare the results with the yearly means of total solar irradiance, as shown in Figure 1. The standard variances range from 0.1 to 0.5 W m\(^{-2}\).

The spectroscopic temperature variations of the Sun, measured by Gray & Livingston (1997b) over the period from 1978 to 1992, are shown in Figure 2 (reproduced from their Fig. 9). The points are the yearly means, the zero point being chosen arbitrarily.

3. METHOD

3.1. Definition of Variables

Standard solar models use pressure, temperature, radius, and luminosity as the structure variables and mass \( M_s \), interior to a radius \( r \) as the independent variable. To self-consistently take into account magnetic effects, Lydon & Sofia (1995) introduced two new magnetic structure variables to model the Sun: the magnetic energy per unit mass, \( \chi \), and the effective ratio of specific heats for the magnetic perturbation, \( \gamma \). The former describes the magnetic perturbation strength, and the latter describes the tensor feature of the magnetic pressure. In general, the determination of \( \chi \) and \( \gamma \) requires a comprehensive understanding of turbulent dynamics in the solar convective zone (Brummell, Cattaneo, & Toomre 1995), an undertaking that is impractical at present. Therefore we take \( \chi \) and \( \gamma \) as free parameters (functions) that need to be determined at this stage instead of using dynamo equations to determine them.

Our aim is to use the measured solar irradiance and effective temperature variations given in Figures 1 and 2 to determine \( \chi(M_D, t) \) and \( \gamma(M_D, t) \), where \( M_D \) and \( t \) are mass depth and time, respectively. Using \( \chi(M_D, t) \), we can obtain the solar interior magnetic field \( B(M_D, t) = (8\pi \rho)^{1/2} \). Since these observations are not enough to determine the two new variables uniquely, we assume they have a Gaussian-like shape and also utilize helioseismic (Antia et al. 2000) and radius (Emilio et al. 2000) observations as additional constraints. The mass depth \( M_D \) is defined as

\[
M_D = \log \left[ 1 - M_s/(1 M_\odot) \right].
\]

This replaces the mass variable \( M_r \) when we want to describe the location, orientation, and form of all magnetic perturbations.

For our purpose, we can use the toroidal \((B_t)\) and poloidal \((B_p)\) components to express a magnetic field vector \( B = (B_t, B_p) \). The magnetic energy density variable \( \chi \) can be written as

\[
\chi = (B^2/2\pi)/\rho,
\]

where \( B = (B_t^2 + B_p^2)^{1/2} \) is the magnitude of the magnetic field vector. The magnetic field direction variable \( \gamma \) can be written as

\[
\gamma = (2B_t^2 + B_p^2)/B^2.
\]

The magnetic pressure \( P_\chi \) can be defined in terms of the new magnetic variables as

\[
P_\chi = (\gamma - 1)\chi\rho.
\]

The value of \( \gamma \) depends on the field geometry. Parallel to the field lines, \( P_\gamma \) is zero, so \( \gamma = 1 \), whereas perpendicular to the field lines, \( P_\gamma \) is a maximum, so \( \gamma = 2 \).

3.2. Consequences on the Equations of Stellar Structure

The continuity equation remains the same. The hydrostatic equation is replaced by the equation of motion, which, including the Lorentz force, is

\[
\rho \frac{\partial^2 r}{\partial t^2} = -VP - \frac{GM_p}{r^2} \rho \dot{r} + \frac{1}{4\pi} (\nabla \times B) \times B,
\]
where $G$ is the gravitational constant. The radial component is
\[
\rho \frac{d^2 r}{dt^2} = - \frac{\partial P}{\partial r} - \frac{GM_r}{r^2} - \frac{\rho - \partial P}{\partial r}.
\]

The magnetic field has both a magnetic tension and a magnetic pressure. This can be seen by writing the Lorentz force, \((1/4\pi)(\nabla \times B) \times B\), as the sum of the magnetic tension, \((B \cdot \nabla)B/4\pi\), and the magnetic pressure, \(-\nabla(B^2/8\pi)\). The former is anisotropic, and the latter is isotropic. Both contribute to the last term of the radial motion equation and are incorporated into $P_x$. For example, for a purely radial magnetic field, the magnetic tension force, \((1/4\pi)B_r B_r/\partial r = (1/8\pi)B_r^2/\partial r = (1/8\pi)B_r^2/\partial r\), which cancels the magnetic pressure force. As a result, $P_x = 0$ in this case.

Defining
\[
P_T = P + P_x
\]
as the total pressure, the equation of radial motion becomes
\[
\frac{\partial P_T}{\partial M_r} = - \frac{GM_r}{4\pi r^2} - \frac{1}{4\pi r^2} \frac{\partial^2 r}{\partial t^2},
\]
where the last term represents the inertial force. Since $\rho = \rho(T, P_T, T, \chi, \gamma)$,
\[
\frac{d\rho}{\rho} = \alpha \frac{dP_T}{P_T} - \frac{\delta}{T} \frac{dT}{\chi} - \frac{v}{\chi} \frac{d\chi}{\gamma} - \frac{v' \gamma}{\gamma} \frac{d\gamma}{\gamma},
\]
where
\[
\alpha = \frac{\partial \ln \rho}{\partial \ln P_T} \text{ at constant } T, \chi, \gamma,
\]
\[
\delta = - \frac{\partial \ln P}{\partial \ln T} \text{ at constant } P_T, \chi, \gamma,
\]
\[
v = - \frac{\partial \ln P}{\partial \ln \chi} \text{ at constant } P_T, T, \gamma,
\]
\[
v' = - \frac{\partial \ln P}{\partial \ln \gamma} \text{ at constant } P_T, T, \chi.
\]

Since we use the one-dimensional stellar evolution model, we cannot model the transverse component of the equation of motion. To handle this transverse component, we need at least a two-dimensional stellar evolution model. Since such a model is not available yet, we consider only the radial motion equation in this paper.

The energy conservation equation becomes
\[
\frac{\partial L}{\partial M_r} = \epsilon - T \frac{dS_T}{dt} - \frac{1}{\rho} \frac{\partial u}{\partial t}.
\]

Here
\[
T dS_T = dQ_T = dU + P dV + d\chi
\]
\[
= dU_T + P_T dV - (\gamma - 1) \frac{\chi}{V} dV
\]
\[
= c_p dT - \frac{\delta}{\rho} dP_T + \left(1 + \frac{P_T \delta \gamma}{\rho \chi} \right) d\chi + \frac{P_T \delta \gamma}{\rho \chi} d\gamma.
\]

is the first law of thermodynamics including magnetic fields. The total internal energy $U_T = U + \chi$ and the total entropy $S_T = S + \chi/T$; $c_p$ is the specific heat at constant pressure. If $\gamma$ is constant, then $d\gamma = 0$, and equation (8) reduces to equation (75) in Lydon & Sofia (1995). The symbol $u = aT^4$ is the radiation energy density, where $a$ is the radiation constant; $\partial u/\partial t$ appears in the full energy conservation equation,
\[
\frac{\partial u}{\partial t} + \nabla \cdot F = \rho (\epsilon - T) \frac{dS_T}{dt}.
\]

$F = u\nu$ is the radiation energy flux vector, and $\nu$ is the photon diffusion velocity.

The equation of transport of energy by radiation,
\[
\frac{\partial T}{\partial M_r} = - \frac{3}{64\pi^2 ac r^2 T^3},
\]
does not change in form, but the convection temperature gradient, $V$, with a magnetic field, is different from that without it. Lydon & Sofia (1995) have demonstrated how to account for magnetic effects in the mixing-length theory and, hence, obtain $V$, provided $\gamma$ is constant. Their method can also be used to derive $V$ when $\gamma$ is variable, as we are about to do here. See Appendix $A$ for an estimate of this effect.

The equation of energy transport by convection,
\[
\frac{\partial T}{\partial M_r} = - \frac{T GM_r}{4\pi r^2} \nabla V,
\]
does not change form either, but the convection temperature gradient, $V$, with a magnetic field, is different from that without it. Lydon & Sofia (1995) have demonstrated how to account for magnetic effects in the mixing-length theory and, hence, obtain $V$, provided $\gamma$ is constant. Their method can also be used to derive $V$ when $\gamma$ is variable, as we are about to do here. See Appendix $B$ for the concrete expression for $V$ in our case.

### 3.3. Numerical Implementation

The stellar evolution code solves the stellar structure equations (such as eqs. [6], [7], and [9] or [10]), and the continuity equation, $\partial T/\partial M_r = 1/(4\pi r^2 \rho)$ with suitable initial and boundary conditions. In the model, $M_r$ and $r$ are the independent variables, and the conventional structure variables are $P_T, T, r$, and $L$, while the magnetic variables are $\chi$ and $\gamma$.

To follow the yearly variations of solar global parameters with a stellar evolution code requires a time step no larger than 1 yr. Such a small time step requires a very precise code. For example, because $L_0$ has increased by about 30% during the lifetime of the Sun, the relative mean rate of change is about $10^{-10}$ yr$^{-1}$. To achieve such a high precision is a challenge. Fortunately, the Yale stellar evolution code (YREC) meets this need, as shown before (Lydon & Sofia 1995). We follow Lydon & Sofia (1995) by modifying YREC7, a new version released in 1999 May, in order to accommodate the magnetic effects described above. The reason YREC permits a small time step and achieves such a high numerical precision is that it uses analytical formulae (Prather 1976), rather than numerical methods, to solve the linearized stellar structure equations.

To accommodate the various magnetic effects described above, we must first write a routine to specify $\chi$ and $\gamma$. We assume $\chi = \chi_{\nu}(t)F(M_0)$. The maximum magnetic energy density $\chi_{\nu}(t)$ is to be determined from solar activity indices. The yearly averaged sunspot number, $R_0$, is the most widely used solar activity index. From numerical experiments, we
find that the results are sensitive to the function form of $B_m$ on $R_Z$. If the maximum magnetic field in the solar interior, $B_m$, is related to $R_Z$ via
\[ B_m = B_0 (190 + [1 + \log (1 + R_Z)]^5), \] (11)
then by adjusting $B_0$, we can nearly match the measured cyclic variations of irradiance and effective temperature. The reason such a functional form of $B_m$ is chosen is that $B \sim 20$ kG, at a depth of $M_D = -4.25$ ($r = 0.96 R_\odot$) in 1996. This result is inferred from helioseismology (Antia et al. 2000) when $R_Z$ was at a minimum. Using this prescription for $B_m$, the value of $B_m$ is about twice as large at the maximum of solar cycle as it is at the minimum.

$F(M_D)$ specifies the distribution of $\gamma$ and is to be determined by fitting the measured irradiance and effective temperature variations. $F(M_D)$ has infinite degrees of freedom and thus cannot be determined uniquely by observational results, which have finite degrees of freedom. However, we can remove this degeneracy by assuming a field shape of the form
\[ F(M_D; M_{Dc}, \sigma) = \exp \left[ -\frac{1}{2} (M_D - M_{Dc})^2/\sigma^2 \right], \] (12)
where $M_{Dc}$ specifies the location and $\sigma$ specifies its width. This Gaussian profile allows us to pinpoint the location of the required magnetic field by using observations of cyclic variations of irradiance and effective temperature.

We either fix $\gamma$, or use
\[ \gamma(M_D) = 1 + (B/B_m)^{1/5}(R_Z/200) \] (13)
to express the spatial and temporal variations of the direction of magnetic fields. This implies that the more intense the magnetic field, the larger its toroidal component. The profile of $\gamma$ is arbitrary, but its value must lie between 1 and 2. Although the detailed results are sensitive to the functional form of $\gamma$, the qualitative features are same. We find that the magnetic effects are maximized when $\gamma = 2$, so we set $\gamma = 2$ in subsequent computations. The main effect of a smaller $\gamma$ is to increase the magnetic field required to reproduce the observed variation of the Sun.

As the magnetic fields that cause the solar activity cycle are believed to exist outside of the solar core (where the thermonuclear reactions occur), we do not consider the influence of magnetic fields on the energy generation rate. Magnetic fields affect the equation of state in the following way. If $\beta$, the ratio of the gas pressure to the total pressure when magnetic fields are present, is defined as
\[ \beta = 1 - \frac{a T_0^{4/3}}{P_T} - \frac{\gamma - 1 \chi \rho}{P_T}, \] (14)
($a$ is the radiation constant), then the density of a gas can be determined from
\[ \beta P_T = \rho_T (1 + E)/\mu_a. \] (15)
$\rho$ is the gas constant, $E$ is the number of free electrons per nucleus (determined by solving the Saha equation), and $\mu_a$ is the mean atomic weight per atom (Prather 1976). Note that $E$ also depends on $\beta$. In the standard case, since $\chi = 0$, $\beta$ does not depend on $\rho$. But in the nonstandard case discussed in this paper, $\beta$ depends on $\rho$. Consequently, not only $\rho$, but also its first derivatives $\dot{\chi}$, $\dot{\chi}$, and $\chi'$, and its second derivatives with respect to $T$ and $P_T$, all need to be calculated by iteration (which is tedious). In the original implementation of this method by Lydon & Sofia (1995), the second derivatives were approximated by assuming a perfect gas, which needs no iteration. In this code upgrade, we modify the equation-of-state routine so that we can accurately calculate all these thermodynamic variables by iteration. Although this improves the numerical precision, it does not make any qualitative difference. $\V_x$ and $\V_y$ (defined in Appendix B) are also calculated by numerical derivatives, as are those time derivatives that appear in the equations of stellar structure.

The qualitative difference comes from $f'$ (see Appendix B), which represents the influence of the magnetic field on radiative loss of a convection element. The convection temperature gradient is now given by
\[ \V = \V_{ad} + (y'/V'_\gamma \gamma^2 \gamma_0 C (1 + y'/V')) - A_m, \] (16)
where
\[ A_m = f'([\nu/a]V_x + (\nu/a)V_y)\V_{ad}. \] (17)
All symbols used here are defined in Appendix B. When $f' = 1$ and $\gamma$ is fixed, equation (17) reduces to
\[ A_m = ([\nu/a]V_x V_{ad}. \] (18)

However, under those assumptions, the calculated cyclic variation of effective temperature is reversed in phase from the solar activity, contrary to the observations (Gray & Livingston 1997b). To get a proper fit to the observations, it is necessary to assume $f' = 3$, a value that will be used in all our subsequent calculations.

We use equation (A2) to compute the global magnetic effect on the radiative opacity, and we find that this has little influence on the effective temperature. On the other hand, the need of $f' = 3$ implies that the energy loss of a convective element caused by turbulence associated with the local magnetic field is much more efficient than the radiative-loss enhancement of the convective element caused by the global magnetic field.

We do not redistribute the mass grid points, nor do we predict stellar structure variables for the next time step, when the time step is smaller than a solar cycle period since these operations will generate an error much larger than $10^{-10}$ yr$^{-1}$ for luminosity. All convergence criteria for iteration are tightened by requiring the relative error to be no larger than $6 \times 10^{-16}$ or so. In this modified code, the matching point is at $M_D \approx -8.6$.

The conventional model parameters are the mixing-length parameter (the ratio of the mixing length to the pressure scale height, usually denoted as $\alpha$-parameter), the initial helium mass abundance ($Y_0$), and the initial heavy-element mass abundance ($Z$). The magnetic parameters introduced in this paper are the peak mass depth $M_{Dc}$ of the magnetic energy density $\chi$, the width of $\chi$ (i.e., $\sigma$), the magnetic strength parameter $B_0$, and the turbulent loss parameter of energy of a convective element $f'$. It is well known that the conventional parameters can be determined by present observations of solar radius and luminosity. Through numerical experiments (i.e., model calculations), we find that the magnetic parameters ($M_{Dc}$, $\sigma$, $B_0$, and $f'$) can be determined by the observed cyclic variation of solar radius, luminosity, and effective temperature. We fix $f' = 3$ in all subsequent runs, as required by the observed variation of effective temperature. The value $\gamma = 2$ is fixed to maximize the magnetic effects. To probe the observationally required location, strength, and profile of the solar interior magnetic field, we change $M_{Dc}$, $\sigma$, and $B_0$ from run to run.
4. CONSTRAINTS ON THE SOLAR INTERIOR
MAGNETIC FIELD

4.1. The Overshoot Layer
At present, it is generally believed that the magnetic field is stored only in the subadiabatically stratified overshoot layer in the form of flux tubes, which may break loose because of kink instabilities, when a threshold of 10^5 G is exceeded (Caligar et al. 1995, 1998). Figure 3 depicts V – V_{ad} versus M_{Dc}, in which the subadiabatically stratified overshoot layer approximately corresponds to the mass depth range –1.68 ≤ M_{Dc} < –1.2. M_{Dc} = –1.68 marks the base of the convection zone of our Sun. Therefore, the maximum yearly averaged field strength B_{m} in the overshoot layer during the solar cycle should be smaller than the threshold. The field with M_{Dc} = –1.45 and σ < 0.2 can be considered to be confined in the overshoot layer. However, even if the threshold value of B is used, the resultant irradiance and effective temperature variations are negligible.

Only when we abandon this threshold constraint (e.g., using a field larger than 2.6 × 10^6 G) can we get irradiance variations comparable to the observations, in magnitude and phase; however, even then, the effective temperature variation is much smaller and in antiphase with the solar cycle. Besides, the resulting relative radius variation is larger than 0.03%. Since the observations reveal that both irradiance and effective temperature variations are in phase with the solar cycle, and the relative radius variation should be much smaller than 0.01% (Emilio et al. 2000), we conclude that the magnetic field that gives rise to the observed variations of irradiance and effective temperature (shown in Figs. 1 and 2) cannot be completely confined in the overshoot layer.

4.2. The Convection Zone
The above arguments lead us to investigate the possibility that the magnetic field is located totally within the convection zone. We set σ = 0.5 in this region. In order to reproduce the observed cyclic variation of irradiance and effective temperature, B_{0} will increase if σ decreases, and vice versa. We choose M_{Dc} from the interval (–8.6, –1.68], which covers the convection zone of our numerical solar model. For each selected value for M_{Dc}, we change B_{0} to reproduce the observed cyclic variations of solar irradiance and effective temperature by using our solar evolution code.

4.2.1. The Lower Part
We find that the lower part of the convection zone, –1.68 ≥ M_{Dc} > –4.2, is ruled out because it produces radius changes that are too large (Δln R > 0.01%) and/or because it requires too strong a magnetic field (B > 5 × 10^4 G).

4.2.2. The Intermediate Part
For the next region up, –4.2 > M_{Dc} > –7.8, or 0.95 R_{⊙} < R < 0.997 R_{⊙}, we find that our model can reproduce the observed cyclic variations of irradiance and effective temperature by using a possible equipartition magnetic field, B_{m} < 5 × 10^4 G. In this case, the amplitude of the radius changes is from 17 to 3 km, and W = Δln R/Δln L ranges from 0.02 to 0.006.

To pinpoint the location of the required magnetic fields in the allowed region of the solar convection zone (M_{Dc} ∈ [–7.8, –4.2]), we need more information. We obtain the allowed region by using the observational information for solar irradiance and effective temperature cyclic variations and by assuming the corresponding solar radius variation Δln R to be much smaller than 0.01%. If we have the direct observational information for Δln R or the W-parameter, we can pinpoint the location of solar interior magnetic fields. Since we do not have simultaneous observational information for the radius variation in the period ranging from 1978 to 1992, we must invoke some physical considerations.

Figure 4 shows the ratio of radiative to adiabatic temperature gradients as a function of mass depth in the Sun, from which we can see that the layer with M_{Dc} ranging from –4 to –8 is the most unstable convective region. In this layer, superstrong plasma turbulence is inevitable since the radiative gradient V_{rad} is much larger than the adiabatic temperature gradient V_{ad}. Therefore, this may suggest a turbulence generation mechanism for solar magnetic fields. Strong plasma turbulence can generate strong small-scale magnetic fields, which form small magnetic cells with almost random orientation. The residual field of the previous cycle and differential rotation tend to align and

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**Fig. 3.** Difference between temperature gradient and adiabatic gradient as a function of the mass depth in the solar interior. The dashed line marks the location of the lower boundary surface of the convection zone.

**Fig. 4.** Ratio of radiative to adiabatic temperature gradients as a function of mass depth in the solar interior. The dashed line marks the location of the lower boundary surface of the convective zone.
bundle these random magnetic "needles" to form flux tubes.

The solid curve in Figure 5 (choosing $M_{dc} = -6.5$, $\sigma = 0.5$, $B_0 = 7$ G) shows the corresponding magnetic field distribution in the solar interior in 1989, near solar maximum. Figure 6 compares the calculated (solid curve) and measured (dash-dotted curve) cyclic irradiance variation while Figure 7 compares the calculated (solid curve) and measured (dash-dotted curve) cyclic effective temperature variation. The irradiance fit ($\chi^2 = 2$) is better than the effective temperature fit ($\chi^2 = 47$). The computed radius variation is $5 \times 10^{-6}$, and the $W$-parameter is almost constant (about $4 \times 10^{-3}$). Figure 8 shows the calculated structure change generated by the field in 1989 (solid curve).

Although we can relate the magnetic field generation to the plasma turbulence in this region, the magnetic field maintenance here faces the old problem of magnetic buoyancy. How can we get around this problem? Observations and simulations suggest that the actual dynamics within the solar convection zone is extremely intricate (Brummell et al. 1995; Kim & Chan 1998; Nordlund 1999). The velocities and magnetic fields are complex, exhibiting large-scale structure and ordered behavior amid rapidly varying and intense small-scale turbulence. In fact, numerical simulations of the solar outer convection zone (Kim & Chan 1998; Nordlund 1999) indicate a major presence of downward-moving plumes. These ordered downdrafts may gather small magnetic flux tubes, generated in the extremely unstable layer, to form larger flux tubes and carry them to the deeper layer. These downdrafts may also push down the magnetic flux tubes to balance the magnetic buoyancy to

![Fig. 5. Three possible distributions of inferred magnetic field in the solar interior in 1989 according to the measured irradiance and photospheric temperature cyclic variations given in Figs. 1 and 2. The vertical line indicates the base of the convection zone.](image)

![Fig. 6. Comparison between the measured (dot-dashed curve) and calculated solar irradiance variations. The line style for the calculated value is the same as that in Fig. 5.](image)

![Fig. 7. Same as Fig. 6, but for photospheric temperature variation.](image)

![Fig. 8. Structural changes caused by the magnetic field distributions given in Fig. 5: relative pressure, temperature, radius, and luminosity changes (from top to bottom). The vertical line indicates the base of the convection zone. The line style for the calculated values is the same as that in Fig. 5.](image)
form a magnetized layer below the most unstable convective region.

One possibility is that the downdrafts gather and carry magnetic flux tubes, generated by turbulence, to depths in the convective envelope until some sort of equipartition is reached. In fact, Antia et al. (2000) have explored this possibility. They employ the observed splittings of solar oscillation frequencies to separate the effects of interior solar rotation and to estimate the contribution from a large-scale magnetic field. After subtracting the estimated contribution from rotation, there is some residual signal in the even splitting coefficients. This may be explained by a magnetic field and free-free transition absorption. We can estimate this effect in terms of the mean free paths with and without a magnetic field, \( M_D = -4.25 \) (\( r = 0.96 R_\odot \)) in 1996. Since the density near \( M_D = -4.25 \) is of order \( 4 \times 10^{-3} \), and the downward velocity for the plumes is of order \( 5 \times 10^4 \) cm s\(^{-1}\), the estimated dynamical pressure of the plumes, \( \rho v^2 \), is equal to or larger than \( 10^7 \) dyn cm\(^{-2}\). The size of \( \rho v^2 \) is comparable to the magnetic pressure, \( B^2/8\pi \), corresponding to a field strength of 20–30 kG. This demonstrates that a stable magnetized layer in the convection zone proper may form when the complexity of convective motion is taken into account. If we wish to reproduce the observed temperature and irradiance variations by the magnetic field at the depth indicated by helioseismology, we find that \( B_0 \) ranges from 20 to 47 kG during a solar cycle (choosing \( B_0 = 90 \) G). Therefore, in Figures 5–8 we also show this case (\( M_{Dc} = -4.25 \), \( \sigma = 0.5 \); dotted curves). The irradiance fit (\( \chi^2 = 1 \)) is also better than the effective temperature fit (\( \chi^2 = 43 \)), the \( W \)-parameter is equal to about \( 2 \times 10^{-2} \), and the predicted cyclic variation of the solar radius is equal to about \( 2 \times 10^{-5} \), which can be tested by measuring the \( W \)-parameter.

In fact, using Michelson Doppler Imager (MDI)/SOHO (Solar and Heliospheric Observatory) data obtained between 1996 April 19 and 1998 June 24, Emilio et al. (2000) found \( W \leq 2 \times 10^{-2} \), which is consistent with the above prediction. This shows that the second case is in agreement with all relevant precise observations, including solar irradiance, effective temperature, radius, and \( p \)-mode oscillation observations. The first case is ruled out by the \( W \)-parameter inferred from the observation.

### 4.2.3. The Upper Part

The upper part of the convection zone, \( M_{Dc} < -7.8 \), is ruled out because the resulting irradiance and effective temperature variations are in antiphase with the solar cycle, which is in conflict with the observations. In all cases, to produce the observed luminosity and effective temperature variations, the required magnetic field must increase with depth. The resulting \( W \)-parameter also increases with depth. This is in agreement with early studies (Däppen 1983; Endal et al. 1985).

### 4.3. The Extended Layer

Another possibility is that these magnetic flux tubes are carried downward into the stable, subadiabatically stratified overshoot layer to provide a seed field for the dynamo operating at the base of the convection zone. Therefore, the magnetic field extends from the convection zone to the overshoot layer. The following magnetic field distribution mimics this case:

\[
F = \begin{cases} 
    h(700), & \text{if } M_D \geq M_{Dc}, \\
    h(1) - 0.325(M_D - M_{Dc}), & \text{if } M_D \leq M_{Dc}, \\
    h(1) + 1.3 \exp \left[ -2(M_D + 5.55)^2 \right], & \text{if } M_D \leq -5.55,
\end{cases}
\]

where \( h(m) = (M_D/M_{Dc})^m \exp \left[ -m(M_D/M_{Dc} - 1) \right] \). Choosing \( M_{Dc} = -1.55 \) in equation (19) and \( B_0 = 250 \) G in equation (11), we also get a good fit to Figures 1 (\( \chi^2 = 1 \)) and 2 (\( \chi^2 = 40 \)). The magnetic field profile is depicted as a dashed curve in Figure 5. This case is also represented by dashed curves in Figures 6–8. The predicted \( W \)-parameter is about \( 2 \times 10^{-2} \). It should be noted that what contributes to the observed cyclic variations of solar irradiance and effective temperature in this case is the field located within the convection zone proper, while the contribution of the field confined in the overshoot layer is negligible. MDI/SOHO data do not rule out this possibility.

### 5. Conclusions

It is possible to locate solar interior magnetic fields using the observed cyclic variations of three global solar parameters, such as luminosity, temperature, and radius at the surface of the Sun. This provides an alternative to helioseismology as a probe of the solar interior magnetic fields. Simultaneous measurements of solar total irradiance and effective temperature can select only an allowed range of solar internal magnetic fields, which in terms of magnitude and location are consistent with helioseismic observations and recent MDI experiments on SOHO.

Although the observed cyclic variations of solar irradiance, effective temperature, radius, and \( p \)-mode oscillation frequencies require a magnetic field component between 20 and 47 kG, peaked at \( r = 0.96 R_\odot \) (within the convection zone proper), a stronger component of about 300 kG buried in the overshoot layer beneath the base of the convection zone cannot be ruled out since the contribution of the latter to those observations is negligible.

This work was supported in part by a grant from the National Aeronautics and Space Administration and in part by the Natural Science Foundation of China (project 19675064).

### APPENDIX A

**AN ESTIMATE OF MAGNETIC EFFECTS ON THE RADIATIVE OPACITY**

A magnetic field affects the absorption processes associated with free electrons, such as electron scattering absorption, \( \kappa_{ee} \), and free-free transition absorption, \( \kappa_{ff} \). We can estimate this effect in terms of the mean free paths with and without a magnetic field, \( l_B \) and \( l_0 \):

\[
\kappa_{ee}/\kappa_{ee} \propto (l_B/l_0)^2 \approx 1 + \frac{1}{2} (\gamma - 1) \tau_0^2 \Omega_e^2 .
\]

(A1)
Here $1/\tau_e$ is the collision frequency between electrons, $\kappa_e = \kappa_{ee} + \kappa_{et}$ is the absorption component that will be affected by the magnetic field, and $\Omega_e$ is the electron cyclotron frequency. Magnetic fields also affect the bound-bound absorption, but this effect cannot be estimated easily by means of the classical method. Nevertheless, the quantum effect must be much smaller than the classical one described above.

The influence of magnetic fields on the radiative opacity can be treated only approximately since we use only opacity tables. We assume that we can decompose the total opacity coefficient $\kappa$ into two parts, $\kappa = \kappa_e + \kappa_1$, since $\kappa_{ee}$ and $\kappa_{et}$ do not or only weakly depend on frequency (before taking the Rosseland mean). We use the opacity subroutine provided by YREC7 to calculate $\kappa_1$ and use equations (17.2) and (17.5) in Kippenhahn & Weigert (1990) to calculate $\kappa_e$. The opacity corrected for magnetic fields, $\kappa'$, can be expressed approximately as follows:

$$\kappa' = \kappa - \kappa_e \frac{(\gamma - 1)\sigma T_e^2 \Omega_e^2}{4 + (\gamma - 1)\sigma T_e^2 \Omega_e^2}. \quad (A2)$$

The approximation originates from the fact that the absorption due to free-free transitions depends on frequency as $\kappa_e \propto v^{-3}$.

**APPENDIX B**

**CONVECTION TEMPERATURE GRADIENT**

When both $\chi$ and $\gamma$ are considered to be variables, the convection temperature gradient $\nabla = (\partial \ln T/\partial \ln P_T)_S$ ($S$ stands for “surroundings”) can be expressed as follows:

$$\nabla = \nabla_{\text{ad}} + (y'/V'\gamma_0 C)(1 + y'/V') - A_m, \quad (B1)$$

where the initial $\chi$ of the convection element is assumed to remain frozen in the surrounding material. $\nabla_{\text{ad}} = (\partial \ln T/\partial \ln P_T)_S$ ($S$ stands for entropy) is the adiabatic gradient, and $y'$ is obtained by solving the cubic algebraic equation

$$0 = 2Ay'^3 + V'y'^2 + V'^2y' - V'. \quad (B2)$$

Here $\gamma_0$, $C$, $V'$, $A_m$, and $A$ are defined as

$$\gamma_0 = [(e_p)/(2acT^3)][(1 + (1/3)\omega^2/\omega)], \quad (B3)$$

$$C = (\gamma/\mu m^2)/8H_p, \quad (B4)$$

$$A_m = f'[(v/\alpha)\nabla_x + (v'/\alpha)\nabla_0]\nabla_{\text{ad}}, \quad (B5)$$

$$V' = 1/\gamma_0 C^{1/2}(\nabla_{\text{rad}} - \nabla_{\text{ad}} + A_m)^{1/2}, \quad (B6)$$

$$A = (9/8)[\omega^2/(3 + \omega_0)], \quad (B7)$$

Here $\omega = \kappa \ell_m$, $g$ is the gravitational acceleration, $H_p$ is the pressure scale height, $l_m$ is the mixing length, and $V_{\text{rad}}$ is the radiative temperature gradient; $f'$ is a dimensionless parameter that determines the influence of the magnetic field on the radiative loss of a convective element, $\nabla_x = (\partial \ln x/\partial \ln P_T)_S$, and $\nabla_y = (\partial \ln \gamma/\partial \ln P_T)_S$. In general, magnetic fields tend to inhibit convection.

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