Super-horizon evolution and the fate of $f_{NL}$

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Abstract. Local form primordial non-Gaussianity is a powerful probe of the number of effective degrees of freedom during the inflationary epoch. In this paper, we have described the recent work, which attempts to further narrow down the set of multi-field models that can result in large local non-Gaussianities. This is done by imposing the constraint that any iso-curvature produced by multi-field inflation be eliminated before the end of the inflationary epoch. In the restricted class of models studied, where non-Gaussianity is produced by a sharp turn in field space and eliminated by a short phase of single field inflation, it is found that adiabaticity and a large local $f_{NL}$ are incompatible.

1. Introduction
The inflationary paradigm has proved successful in providing a theory of the very early universe that reproduces the primordial spectrum observed through CMB and other measurements. However, the space of inflationary models is large, and exploring it is a challenging task. One valuable tool for such an exploration is primordial non-Gaussianity, the deviation from a purely Gaussian spectrum that can arise in some inflationary models.

Non-Gaussianity comes in many guises, but perhaps the easiest way to study it is through the 3-point curvature correlation function, and (as observations become more precise) through other higher order correlation functions. The power spectrum, bispectrum (3-point function) and non-Gaussianity parameter $f_{NL}$ are defined as follows:

\[
\langle \zeta_{k_1} \zeta_{k_2} \rangle \equiv (2\pi)^3 \delta^{(3)}(k_1 + k_2) \frac{2\pi^2}{k_1^3} P_\zeta(k_1),
\]

\[
\langle \delta \phi^I_{k_1} \delta \phi^J_{k_2} \rangle \equiv (2\pi)^3 \delta^{IJ} \delta^{(3)}(k_1 + k_2) \frac{2\pi^2}{k_1^3} P_* (k_1),
\]

\[
P_* (k) = \frac{H^2}{4\pi^2},
\]

\[
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \equiv (2\pi)^3 \delta^{(3)} \left( \sum_i k_i \right) B_\zeta (k_1, k_2, k_3), \quad \text{and}
\]

\[
\frac{6}{5} f_{NL} \equiv \frac{\prod_i k_i^3}{\sum_i k_i^4} \frac{B_\zeta}{4\pi^4 \sum_{i \neq j} P_{\zeta k_i k_j}}.
\]

In standard single field inflation, with a canonical kinetic term and a Bunch-Davies vacuum, Maldacena [1] has shown that the parameter $f_{NL}$ must be small. As a result, a non-zero
measurement of $f_{NL}$ can be a valuable probe into the nature of the inflation epoch, by indicating a departure from the simplest scenario.

The momentum-conserving delta function in the definition of the bispectrum means that the domain of $f_{NL}$ can be described by the set of triangles in momentum space. Moreover, different triangles are indicative of different particular deviations from canonical, single field inflation \[2\]. In particular, an $f_{NL}$ signal that peaks for squeezed triangles ($k_1 \sim k_2 \gg k_3$) is indicative of multi-field inflation \[3\]. Such signals are denoted by $f_{loc}^{\text{local}}$.

Current experimental bounds on $f_{NL}$ are given by $-5 < f_{loc}^{\text{local}} < 5$ (WMAP-7+SDSS), $-214 < f_{loc}^{\text{equil}} < 266$ (WMAP-7) and $-410 < f_{loc}^{\text{orthog}} < 6$ (WMAP-7). The equilateral shape has $k_1 \sim k_2 \sim k_3$ and can indicate a non-canonical kinetic term during inflation. The orthogonal shape is constructed to be orthogonal to the other two shapes.

As mentioned above, the only known models that produce a large local non-Gaussianity are multi-field models. In addition, in all of these models, such non-Gaussianity is produced by super-horizon evolution of the curvature perturbation. Super-horizon evolution occurs for modes for which $k/aH \ll 1$ and can only happen in multi-field models. A number of scenarios exist with a large $f_{loc}^{\text{local}}$ arising from super-horizon evolution, including the curvature scenario, modulated reheating and multi-field models with sharp turns in field space \[4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\].

As well as the possibility of a measurable $f_{loc}^{\text{local}}$, multi-field models of inflation can also give rise to iso-curvature perturbations. Since (a) primordial iso-curvature is unobserved, and (b) the super-horizon evolution of the curvature perturbation will continue until the iso-curvature is driven to zero, it is important that the evolution of the perturbations be tracked until a purely adiabatic spectrum is obtained. From this point, the primordial perturbations will be time-independent, until they are once again inside the horizon.

The work described herein (carried out in detail in \[19\] and \[20\]) has studied a class of two field models, where a large local non-Gaussianity is produced as the inflaton undergoes a sharp turn in field space. To match the observed universe and preserve the super-horizon constancy of the curvature perturbation, the residual iso-curvature is eliminated by a short period (a few e-foldings) of effectively single field inflation, which takes place immediately before the end of the inflationary era. In these models (with some assumptions about the form of the potential), it is found that $f_{loc}^{\text{local}}$, and all other local form non-Gaussianities are driven to zero as the iso-curvature is eliminated. This suggests that local non-Gaussianity and an adiabatic spectrum are difficult to achieve simultaneously in this class of models.

2. The model and results

The action in the models studied is:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} m_p^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - W(\vec{\phi}) \right]. \quad (2)$$

Using $\phi$ and $\chi$ for the two scalar fields, the slow roll equations of motion are (assuming $|\dot{H}| \ll H^2$ and $|\dot{\phi}_a| \ll H|\phi_a|$):

$$3H \dot{\phi} \simeq -\partial_\phi W,$$

$$3H \dot{\chi} \simeq -\partial_\chi W, \quad \text{and} \quad H^2 \simeq \frac{1}{3m_p^2} W. \quad (3)$$

The potential $W$ can be any function of the form $F(U(\phi) + V(\chi))$, but for clarity, the treatment here is restricted to $W = (U(\phi) + V(\chi))^\gamma$. 

2
The evolution of the perturbations at scales larger than the Hubble radius has been studied using the $\delta N$ formalism [21, 22, 23, 24]. This gives the local contribution to $f_{NL}$ as:

$$\frac{6}{5} f_{NL}^{(4)} = \sum_{IJ} \frac{N_{I,I} N_{J,J} N_{I,J}}{\left( \sum_{K} N_{K,K}^2 \right)}.$$  \hspace{1cm} (4)

$I$ and $J$ denote derivatives with respect to the fields, and $N$ is the background number of e-foldings, given by:

$$N = \int_{*}^{c} H dt.$$ \hspace{1cm} (5)

In terms of the slow roll parameters, the values of the potential at horizon exit, and at the end of inflaton, one obtains the following result for $f_{NL}$:

$$\frac{6}{5} f_{NL}^{(4)} = \frac{2}{\gamma} \left( \frac{x_{\phi}^2}{c_{\epsilon}^2} \left[ 1 - \left( \frac{2\epsilon_{\phi}}{c_{\epsilon}} - \gamma + 1 \right) x_{\phi} \right] + \frac{y_{\phi}^2}{c_{\epsilon}^2} \left[ 1 - \left( \frac{2\epsilon_{\phi}}{c_{\epsilon}} - \gamma + 1 \right) y_{\phi} \right] \right) \left( \frac{x_{\phi}^2}{c_{\epsilon}^2} + \frac{y_{\phi}^2}{c_{\epsilon}^2} \right)^2 + \frac{2}{\gamma} \left( \frac{(U_{\phi} + V_{\phi})^2}{(U_{\phi} + V_{\phi})} \left( \frac{x_{\phi}}{c_{\epsilon}} - \frac{y_{\phi}}{c_{\epsilon}} \right)^2 \frac{2\epsilon_{\phi} x_{\phi}}{c_{\epsilon}} \left( \frac{2\epsilon_{\phi} - 1}{c_{\epsilon}} \right) \right) \right). \hspace{1cm} (6)

This can be usefully expressed in terms of the leading contributions as:

$$\frac{6}{5} f_{NL}^{(4)} \sim \mathcal{O}(\epsilon_{\phi}) + \mathcal{O}(1) \times \frac{\epsilon_{\phi} c_{\epsilon}}{c_{\epsilon}} \eta_{c}^{ss}.$$ \hspace{1cm} (7)

It can be shown that to eliminate iso-curvature prior to the end of inflation, the parameter $\eta_{c}^{ss}$ must become large ($> 1$) for a few e-foldings [19], this in turn results in the exponential suppression of note only in the iso-curvature ($\delta s$), but also $f_{NL}^{(4)}$, which are given by:

$$|\delta s| \sim \exp \left[-\frac{3}{2} \int H dt \right],$$  
$$f_{NL}^{(4)} \sim \mathcal{O}(\epsilon_{\phi}) + \mathcal{O}(1) \times \eta^{ss} \exp \left[-2 \int C_{\eta} H \eta^{ss} dt \right].$$ \hspace{1cm} (8)

$C_{\eta}$ is a number ($> 1$), whose value depends on the particular direction of the inflaton as $\eta^{ss}$ becomes larger than 1.

A similar exponential suppression is found for more general potentials [19] and higher order (but still local form) non-Gaussianity [20].

3. Conclusions

Local non-Gaussianities are a powerful probe of the inflationary model space. They are know to be small in single field models, and so their detection would be indication of multiple degrees of freedom during inflation. The work described here suggests that as well as ruling out single field models, a detection of $f_{NL}^{local}$ would also be incompatible with a subset of multi-field models, where the non-Gaussianity arises from the super-horizon evolution of curvature during inflation, and where iso-curvature is eliminated through a period of effectively single field inflation. Of course, such models only form a small subset of possible ways to produce a large local primordial non-Gaussianity, and it remains an outstanding challenge to see if the class of models for which $f_{NL}^{local}$ and adiabaticity are mutually exclusive can be expanded.
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