Erratum

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N Metwally

Department of Mathematics, College of Science, Zallaq, Bahrain University, Bahrain
Department of Mathematics, Faculty of Science, Aswan University, Aswan, Egypt

E-mail: Nmetwally@uob.edu.bh and Nmetwally@gmail.com

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An error occurred in the title of section 2.2 in the above article. The title should read as follows: 2.2. Accelerating the communication quantum state.
Letter

Estimation of teleported and gained parameters in a non-inertial frame

N Metwally

Department of Mathematics, College of Science, Zallaq, Bahrain University, Bahrain
Department of Mathematics, Faculty of Science, Aswan University, Aswan, Egypt

E-mail: Nmetwally@uob.edu.bh and Nmetwally@gmail.com

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Abstract

Quantum Fisher information is introduced as a measure of estimating the teleported information between two users, one of which is uniformly accelerated. We show that the final teleported state depends on the initial parameters, in addition to the gained parameters during the teleportation process. The estimation degree of these parameters depends on the value of the acceleration, the used single mode approximation (within/beyond), the type of encoded information (classic/quantum) in the teleported state, and the entanglement of the initial communication channel. The estimation degree of the parameters can be maximized if the partners teleport classical information.

Keywords: estimation, Unruh acceleration, Fisher information

(Some figures may appear in colour only in the online journal)

1. Introduction

In the context of classical estimation theory, Fisher information represents a key value that is used to estimate any parameter [1, 2]. Similarly, quantum Fisher information may be used as a measure of estimation in quantum information theory [3]. As an example, the optimal parameter estimation of the Pauli channels is discussed by Ruppert et al in [4]. Zheng et al [5] have used Fisher information to estimate the channel parameter of a two-qubit state, where each qubit interacts independently with its own environment. The possibility of estimating multi-quantum parameters is discussed by Yue et al in [6].

Due to its importance in estimation theory, there are many studies that have quantified Fisher information for different models. For example, the quantum Fisher information of the GHZ state in a decoherence channel is quantified by Ma et al in [7]. The relation between the fidelity susceptibility and quantum Fisher information is investigated by Liu et al in [8]. The dynamic of quantum Fisher information in the Ising model is discussed in [9]. Altintas [10] studied the dynamics of the quantum Fisher information of a steady state in a noisy environment. Recently, He et al [11] studied the possibility of enhancing quantum Fisher information by using uncollapsing measurements. Xiao et al [12] proposed a scheme to enhance teleported quantum Fisher information by utilizing partial measurements. Liu et al [13] introduced a new expression of the quantum Fisher information for a general system.

There are some limited studies that have been done in the context of a non-inertial frame. For example, Yao et al [14] have investigated the performance of quantum Fisher information under the Unruh–Hawking effect. The effect of the observer’s acceleration on a parameter estimation protocol using NOON states is discussed by Hosler and Kok in [15]. The dynamics of Fisher information and skew information for the Unruh effect within and without external noise is discussed by Banerjee et al in [16]. Recently, Metwally [17] discussed the effect of Unruh acceleration on Fisher information for different classes of maximum and partially entangled states.

Therefore, we are motivated to discuss the possibility of estimating the teleported parameters by means of quantum teleportation. In this proposal, it is assumed that only one user (Alice) stays in the inertial frame, while the second user (Bob) is accelerated with uniform acceleration.
The manuscript is organized as follows: in section 2, we describe the different types of initial states that Alice and Bob share, and the relation between the Minkowski and Rindler spaces is reviewed. The dynamics of the final state is obtained analytically. This accelerated state is used as a quantum communication channel to teleport the unknown state from Alice to Bob, as described in section 3. In section 4, we estimate the teleported and the gained parameters by evaluating the Fisher information corresponding to these parameters. Finally, we summarize our results in section 5.

2. The suggested proposal

It is assumed that Alice and Bob share an initially prepared two-qubit Bell state or X-state. It is considered that only one qubit (Alice’s) is at rest, while Bob’s qubit is accelerated with uniform Unruh acceleration. Alice’s task is teleporting an unknown state to Bob by using Bennett’s protocol [18, 19]. Bob will use this teleported state to estimate the initial parameters of the teleported state as well as the gained parameters during the teleportation process by quantifying the Fisher information [3] corresponding to these parameters. In this context, we aim to investigate the effect of the Unruh acceleration and the initial state settings of the communication channel between the partners on the precision of the estimation.

Let the users Alice and Bob share a self-transposed class of two-qubits [20] as,

\[ \rho_{ab} = \frac{1}{4} \left( I_{4 \times 4} + \sum_{i,j} \sigma_i^{(a)} \otimes \sigma_j^{(b)} \right), \tag{1} \]

where \( I_{4 \times 4} \) is an identity matrix and \( \sigma_i^{(a)} = (\sigma_i^{(x)}, \sigma_i^{(y)}, \sigma_i^{(z)}) \), \( i = a, b \) are the Pauli operators of Alice and Bob’s qubits, respectively. The dyadic \( \sigma_i^{(a)} \) is a 3 x 3 matrix, with its elements defined as \( c_{ij} = \text{tr}[\sigma_i^{(a)} \sigma_j^{(b)}] \) [20]. From the state (1), different important states may be considered. However, if we set \( c_{11} = c_{22} = 0 \) and \( c_{ij} \neq 0, i \neq j \), one obtains what is called an X-state. Moreover, if \( c_{11} = c_{22} = c_{33} = -1 \) and \( c_{ij} = 0, i \neq j \), one gets a maximum Bell state: the singlet state \( | \psi^- \rangle \). Also, what is called a Werner state can be obtained if \( c_{11} = c_{22} = c_{33} = -F \) and \( c_{ij} = 0, i \neq j \). The dynamics of all these states in the non-inertial frame is discussed by Metwally in [21].

2.1. The Unruh effect

It has been shown that from the perspective of inertial observers, the Minkowski coordinates are the most suitable choice for describing the Dirac qubit. On the other hand, from the perspective of non-inertial observers, the Rindler coordinates are the most adequate coordinates for describing the Dirac qubits. Now, we assume that Alice’s qubit moves in the inertial frame and that it is described by the Minkowski coordinates \((t, z)\), while Bob’s qubit is uniformly accelerated with a constant acceleration \(a\). Therefore, Bob’s qubit can be described by using the Rindler coordinates \((\tau, \eta)\), such that \( \eta = \sqrt{\tau^2 - 3}, -\infty < \eta < \infty \) and \(-\infty < \tau < \infty\). These transformations define two regions in space-time: the first region \(I\) for \(|\tau| < z\) and the second region \(II\) for \(z < -|\tau|\). The accelerated qubit moves in a parabola in the first region \(I\) defined by \( \eta = 1/a\), \( a\) is the uniform acceleration, while the anti-accelerated qubit moves in the parabola \( \eta = -1/a\) in the second region II [22, 23]. To describe a Minkowski state in terms of Rindler’s space, one has to use the Bogoliubov transformation,

\[ \nu_k = \cos rC_k^{(\downarrow)} - e^{-i\phi} \sin rD_k^{(\uparrow)}, \quad \mu_k^{(\dagger)} = e^{i\phi} \sin rC_k^{(\uparrow)} + \cos rD_k^{(\downarrow)}, \tag{2} \]

where \( \nu_k \) and \( \mu_k^{(\dagger)} \) represent the annihilation and creation operators in Minkowski space such that [22]

\[ \nu_k |0\rangle_M = 0, \quad \mu_k^{(\dagger)} |0\rangle_M = 0, \quad \nu_k^{(\dagger)} |0\rangle_M = |1\rangle_M, \quad \mu_k^{(\dagger)} |0\rangle_M = |1\rangle_M, \]  \[ (3) \]

with \( C_k^{(\dagger)} \) and \( D_k^{(\dagger)} \) representing the annihilation and creation operators in the regions \(I\) and \(II\), respectively. The parameter \( \phi \) is an unimportant phase that can be absorbed into the definition of the operators. The operators \( \nu_k \) mix a particle (fermion) in region \(I\) and an anti-particle (anti-fermion) in region \(II\). On a computational basis, \( |0\rangle_k \) and \( |1\rangle_k \), and the operators \( \nu_k \) may be written in terms of the Rindler–Fock states as [23–25],

\[ |0\rangle_k = q^*_R |0\rangle^\perp I_0 |0\rangle_H + \sin r_h |1\rangle^\perp I_0 |1\rangle_H, \quad |1\rangle_k = q_R |1\rangle^\perp I_0 |0\rangle_H + q^*_L |0\rangle^\perp I_0 |1\rangle_H, \]  \[ (4) \]

where \( q_R \) and \( q_L \) are complex numbers with \( |q_R|^2 + |q_L|^2 = 1 \), and the dimensionless parameter \( r \) is given from \( \tan \omega \tau = e^{-\pi r}, \quad a \in [0, \infty), \quad r \in [0, \pi/4] \). \( \omega \) is the frequency of the traveling qubits, and \( c \) is the speed of light.

2.2. Accelerating the communication quantum state

Let us consider that the users Alice and Bob share the X-state. As described above, Alice’s qubit remains stationary, while Bob’s is accelerated with uniform acceleration. Bob is
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causally disconnected from the second region II, therefore all accessible information is encoded in Alice and Bob’s qubit in the first region I. Consequently, by tracing out all the modes in the second region, the final state between Alice and Bob is given by

$$\rho_{ab} = B_0 |00\rangle\langle 00| + B_1 |01\rangle\langle 10| + B_2 |10\rangle\langle 01| + B_3 |11\rangle\langle 11|,$$

where

$$B_0 = A_1 \cos^2 r + A_2 |q_L|, \quad B_1 = A_3 |q_R| \sin r + A_4 |q_L| \sin r,$$

and $|q_L|^2 + |q_R|^2 = 1$. $A_1 = 1 + c_0$, $A_2 = 1 - c_0$, $A_3 = c_1 + c_2$ and $A_4 = c_3 - c_2$.

In the next section, Alice and Bob will use the state (5) as a communication channel to teleport an unknown state from one to the other using the Bennett protocol [18].

3. Quantum teleportation

Now, the users share the accelerated state (5) and Alice is asked to send the unknown state to Bob. The users perform the scheme which is described in figure 2. Let us assume that the unknown state given to Alice is defined by,

$$\rho_u = |\alpha|^2 |0\rangle\langle 0| + |\beta|^2 |1\rangle\langle 1| + \beta \alpha^* |0\rangle\langle 1| + |\alpha|^2 |1\rangle\langle 0|,$$

and $|\alpha|^2 + |\beta|^2 = 1$.

Alice and Bob perform the teleportation protocol [18] by using the following steps:

1. Alice performs a CNOT operation between her own qubit and the one given to her, followed by the Hadamard gate on the given qubit.
2. Alice performs measurements on the two qubits on her hand and sends her results to Bob via the classical communication channel.
3. According to the received results, Bob performs the required operations to get the teleported state.

Alice and Bob perform the steps (1–3) to teleport the unknown state $\rho_u$. Finally, if Alice measures 00, then Bob will obtain the state [26].

Figure 2. A quantum circuit to teleport an unknown state between Alice and Bob. A source supplies the users with an entangled state $\rho_{ab}$, where $\rho_a$ and $\rho_b$ are sent to Alice and Bob, respectively. Moreover, Alice is supplied with unknown information coded in $\rho_u$ to be teleported to Bob. Alice performs a local operation (CNOT and Hadamard gate, $H$) between the two qubits on her hand, followed by Bell measurements (BM). These measurements are sent via a classical channel (double lines) to Bob, who performs a unitary operation (UO) on his own qubit to get $\rho_u$.

Figure 3. The dynamics of the Fisher information $F_\theta(\theta, r)$ at a fixed value of the phase parameter $\phi = \pi/4$ against the Unruh parameter $r$ for a system is initially prepared in the state $\rho_{\phi'}$, where (a) WSMA, i.e. $q_R = 1$ and $q_L = 0$, and (b) BSMA with $q_R = q_L = 1/\sqrt{2}$. 

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Let us assume that the coefficients \( \alpha = \cos(\theta/2) \) and \( \beta = \sin(\theta/2)e^{i\phi} \), where the parameters \( \theta \in [0, \pi] \) and \( \phi \in [0, 2\pi] \) are the weight and the phase, respectively. Then (9) can be written explicitly as,

\[
\begin{align*}
\rho_{\text{Bob}} &= \rho_{00}|0\rangle\langle 0| + \rho_{01}|0\rangle\langle 1| + \rho_{10}|1\rangle\langle 0| + \rho_{11}|1\rangle\langle 1|.
\end{align*}
\]

Now, we have all the details to estimate the weight (\( \theta \)), the phase (\( \phi \)), and the Unruh acceleration (\( r \)) parameters by calculating the Fisher information corresponding to each parameter, as we shall see in the next sections.

4. Fisher information

It is clear that the final teleported state depends on the initial parameters, the weight and the phase parameters, as well as the Unruh parameter, which is gained during the teleportation process. The main task of the following sections is to estimate these parameters by calculating the Fisher information.

It is well known that any single mixed qubit can be described by its Bloch vector as,

\[
\begin{align*}
\rho &= \frac{1}{2} \left| \cos^2 r(1 + c_{11}\cos \theta) + |q_1|^2(1 - c_{11}\cos \theta) \right|, \\
\rho_{01} &= \frac{1}{8} \left| c_{11}\cos \phi(q_6^* \cos r + q_4 \sin r) + ic_{22}\sin \phi(q_6^* \cos r - q_4 \sin r) \right|, \\
\rho_{10} &= \frac{1}{8} \left| c_{11}\cos \phi(q_6^* \sin r + q_4 \cos r) + ic_{22}\sin \phi(q_6^* \sin r - q_4 \cos r) \right|, \\
\rho_{11} &= \frac{1}{8} \left| \sin^2 r(1 + c_{11}\cos \theta) + |q_1|^2(1 - c_{11}\cos \theta) \right|.
\end{align*}
\]

where \( \mathbf{s} = (s_x, s_y, s_z) \) is the Bloch vector and \( \mathbf{\sigma} = (\sigma_x, \sigma_y, \sigma_z)^T \) are the Pauli operators. The Fisher information for a mixed state with respect to a parameter \( \kappa \), which will be estimated, can be described by means of the Bloch vector as [27].
\[ \mathcal{F}_\theta = \left| \frac{\partial^2}{\partial \alpha^2} \right| + \frac{1}{1 - |\mathcal{F}|} \left( \frac{\partial \mathcal{F}}{\partial \alpha} \right)^2, \]

while for a pure state, namely $|\mathcal{F}| = 1$, the Fisher information \[ \mathcal{F}_\theta = \left| \frac{\partial^2}{\partial \alpha^2} \right|. \]

Now, to quantify the amount of teleported Fisher information which is contained in the state (8), we describe it by means of its Bloch vector as,

\[ \rho_{\text{Bob}} = \frac{1}{2}(1 + s_x \sigma_x + s_y \sigma_y + s_z \sigma_z), \]

where $s_i = Tr(\rho_{\text{Bob}} \sigma_i), i = x, y \text{ and } z$.

\[ s_x = \frac{\sin \theta}{8} \{ c_{31} \cos \phi [(q_{R}^* + q_{L}) r + (q_{L}^* + q_{R}) \sin r] + ic_{23} \sin \phi [(q_{R}^* - q_{L}) r + (q_{L}^* - q_{R}) \sin r] \}, \]

\[ s_y = \frac{\sin \theta}{8} \{ c_{32} \sin \phi [(q_{L}^* + q_{R}) r - (q_{R}^* + q_{L}) \sin r] + ic_{14} \cos \phi [(q_{R}^* - q_{L}) r + (q_{L}^* - q_{R}) \sin r] \}, \]

\[ s_z = \frac{1}{8} \cos 2r(1 + c_{33} \cos \theta) + (|q_{L}|^2 - |q_{R}|^2)(1 - c_{33} \cos \theta). \]

4.1. Estimation of the weight parameter

In this investigation, we assume that the users initially share a maximum entangled Bell-type state, $\rho_{\phi^+}$ and $\rho_{\phi^-}$, or a partially entangled state (an $X$-state). In this context, we shall estimate the teleported weight and phase parameters by calculating the Fisher information with respect to these two parameters.

Figure 3 displays the behavior of the Fisher information $\mathcal{F}_\theta(\theta, r)$ at a fixed value of the phase parameter $\phi = \pi/4$ of the teleported state (13). In figure 3(a), we set $q_R = 1$ and $q_L = 0$, i.e. within the single mode approximation (WSMA). The general behavior shows that the Fisher information decreases as $r$ increases. The effect of the initial weight parameter $\theta$ appears clearly at large values of $r$, where $\mathcal{F}_\theta(\theta, r)$ decreases as $r$ increases to reach its minimum values at $\theta = \pi/2$. However, for further values of $\theta$, the Fisher information increases gradually to reach its maximum value at $\theta = \pi$.

This behavior changes dramatically when we consider the Unruh effect beyond the single mode approximation (BSMA), i.e. the single qubit has a right and a left component, where we set $q_R = q_L = 1/\sqrt{2}$. As described in figure 3(b), the Fisher information increases as $r$ increases. Moreover, as $\theta$ increases,
the Fisher information decreases to reach its minimum values at $\pi/2$. Then, as the weight parameter increases, $\mathcal{F}_0(\theta, r)$ increases gradually to reach its maximum values. These maximum values depend on the Unruh acceleration, where the maximization of $\mathcal{F}_0(\theta, r)$ is depicted at large values of $r$.

The effect of the phase parameter $\phi$ on the dynamics of the Fisher information $\mathcal{F}_\phi(\theta, r)$ at a fixed value of the Unruh acceleration $r = \pi/8$ is depicted in figure 4. The behavior of the Fisher information within the single mode approximation is displayed in figure 4(a). It is clear that the phase parameter has a negligible effect on the behavior of $\mathcal{F}_\phi(\theta, r)$. On the other hand, the phenomenon of the sudden decay of the Fisher information is displayed as soon as $\theta$ increases. However, at $\theta = \pi/2$, $\mathcal{F}_\phi(\theta, r)$ vanishes completely. At further values of $\theta$, the Fisher information is re-birth to reach its maximum value at $\theta = \pi$. Figure 4(b) shows the behavior of $\mathcal{F}_\phi(\theta, r)$, but in a contour description, where it clearly displays the values of $\theta$ that maximize or minimize $\mathcal{F}_\phi(\theta, r)$.

Figure 4(c) displays the behavior of the Fisher information $\mathcal{F}_\phi(\theta, r)$ against the phase parameter, $\phi$ BSMA, where we fixed the value of Unruh acceleration, $r = \pi/8$. In this case, the phase parameter has a completely different effect.

It is evident that as $\phi$ increases, $\mathcal{F}_\phi(\theta, \phi)$ decreases gradually to reach its minimum values at $\phi = \pi$. For further values of $\phi$ the Fisher information vanishes completely to be re-birth for $\phi \in [3\pi/4, 2\pi]$. These results are displayed in figure 4(d), where the values of $\theta$ and $\phi$, which maximize and minimize $\mathcal{F}_\phi$, are seen clearly.

Figures 5(a) and (b) are devoted to the investigation of the effect of the phase parameter, $\phi$, on the Fisher information $\mathcal{F}_\phi(\theta, \phi)$ BSMA at a fixed $r = \pi/8$, where the users initially share the singled state, $\rho_{\psi}$. It is evident that $\mathcal{F}_\phi(\theta, \phi)$ increases as $\phi$ increases in the interval $[0, \pi]$ to reach its maximum values at $\phi = 3\pi/4$, then it decreases again at further values of $\phi$. On the other hand, as $\theta$ increases, $\mathcal{F}_\phi(\theta, \phi)$ decreases gradually to vanish completely at $\theta \in [\pi/4, 3\pi/4]$. For further values of $\theta$, $\mathcal{F}_\phi(\theta, \phi)$ is re-birth to reach its maximum bounds at $\theta = \pi$.

Figures 5(c) and (d) display the behavior of the $\mathcal{F}_\phi(\theta, \phi)$ WSMA for a system that is initially prepared in the $X$-state, where we set $c_{11} = -0.9$, $c_{22} = -0.8$, $c_{33} = -0.7$. In this case, the effect of the phase parameter, $\phi$ is different from that depicted in figures 5(a) and (b), where $\mathcal{F}_\phi(\theta, \phi)$ reaches its maximum values at $\phi = 0, \pi, 2\pi$. Moreover, $\mathcal{F}_\phi(\theta, \phi)$ does not vanish completely for any value of $\phi \in [0, 2\pi]$. The
vanishing phenomenon of the Fisher information is due to the weight parameter, where it decays suddenly as $\theta$ increases. From figure 4(d), it is clear that $F(\theta, \phi)$ vanishes completely at $\theta \in [3\pi/8, 5\pi/8]$. For further values of $\theta$, the Fisher information is re-birth to reach its maximum bounds at $\theta = \pi$.

From figures 3–5, one concludes that the entanglement of the initial communication channel between the users plays an important role on the teleported Fisher information, where the upper bounds of the teleported Fisher information are large if the users initially use a maximum entangled communication channel. The phenomenon of the sudden decay of Fisher information is depicted WSMA as the Unruh acceleration increases. Moreover, the suddenly increasing phenomenon of the Fisher information is displayed for large values of Unruh acceleration BSMA, while the gradually increasing behavior is also displayed BSMA.

Moreover, the degree of estimating the teleported weight parameter $\theta$ depends on the type of information which is encoded in the teleported state. It is clear that at $\theta = 0$ or $\pi$, $F(\theta, \phi)$ is the maximum, where at these values the initial teleported state is reduced to be $|\psi\rangle = |0\rangle$ and $|\psi\rangle = e^{i\phi}|1\rangle$, respectively, which means that the state carries only classical information. On the other hand, the minimum values of the estimation degree appear at $\theta = \pi/2$, where the initial teleported state is defined by $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle \mp e^{i\phi}|1\rangle)$, which means that the initial state carries quantum information.

4.2. Estimating the phase parameter

The behavior of the Fisher information $F(\phi, r)$ at a fixed value $\theta = \pi/4$ with respect to the phase parameter is shown in figure 6. Figures 6(a) and (b) display the effect of the Unruh acceleration on the teleported Fisher information within/beyond the single mode approximation, respectively. The general behavior shows that $F(\phi, r)$ decreases as $r$ increases. The phenomenon of the sudden changes of Fisher information appears within/beyond the single mode approximation. Within the single mode approximation, the upper bounds of the Fisher information $F(\phi, r)$ are larger than those depicted for the BSMA.
Figures 6(c) and (d) show the behavior of $\phi \theta F$, at a fixed value of the Unruh acceleration, $r = \pi/8$, within/beyond the single mode approximation, respectively. In figure 4(c), the effect of the weight parameter decreases as $\phi$ increases. The maximum values of $\phi \theta F$ are depicted at $\phi = \pi$ and $2\pi$, while the weight parameter may be arbitrary.

This effect is dramatically changed in the presence of the BSMA. It is evident that at small values of $\phi$, $\phi \theta F$ decreases as $\theta$ increases to reach its minimum value for the first time at $\theta = \pi/2$. For further values of $\theta$, $\phi \theta F$ increases gradually to reach its maximum values at $\theta = \pi$. The effect of the weight parameter completely disappears as $\phi$ increases. However, as $\phi$ increases in the interval $[\pi/2, 3\pi/4]$, the Fisher information $\phi \theta F$ almost vanishes completely. However, for further values of $\phi$, $\phi \theta F$ increases gradually to reach its maximum values at $\phi = \pi$. This behavior is repeated at larger values of $\phi$.

Figures 7(a) and (b) are devoted to the investigation of the behavior of $\phi \theta F$ at a fixed value of $\phi = \pi/4$ for a system that is initially prepared in the Bell state $\rho_\phi$. (a) WSMA, and (b) BSMA, and figures (c) and (d) represent the Fisher information, $\phi \theta F$ at a fixed value of $\theta = \pi/4$ (c) WSMA, and (b) BSMA.

4.3. Estimation of the Unruh acceleration

1. Numerical investigation

The final teleported state (8) not only depends on the initial parameters, but also on the Unruh acceleration parameter. Therefore, it is important to estimate this parameter by quantifying the corresponding Fisher information $\phi \theta F$. In figure 8, we investigate the effect of the initial parameters $\theta$ and $\phi$ on the behavior of the Fisher information $\phi \theta F$.
\[ \mathcal{F}_r(\theta, r) \]

within/beyond the single mode approximation. The effect of the parameter \( \theta \) is displayed in figures 8(a) and (b), where we set \( \phi = \pi/4 \). It is manifest that \( \mathcal{F}_r(\theta, r) \) increases as \( r \) increases to reach its upper bounds at \( \theta = 0 \). For further values of \( \theta \) the Fisher information \( \mathcal{F}_r \) decreases to vanish completely. For any arbitrary value of \( \theta \) and small values of \( r \), \( \mathcal{F}_r \) is almost zero. However, this behavior is changed if the BSMA is considered (figure 8(b)), where \( \mathcal{F}_r \) increases gradually as \( \theta \) increases to reach its maximum values at \( \theta = \pi/2 \). For further values of \( \theta \), the Fisher information \( \mathcal{F}_r \) decreases gradually to disappear at \( \theta = \pi \).

The effect of the phase parameter on \( \mathcal{F}_r(\phi, r) \) is described in figures 8(c) and (d), where we set \( \theta = \pi/4 \). It is clear that within the single mode approximation \( \mathcal{F}_r(\phi, r) \) increases as \( r \) increases, where the phase parameter has a feeble effect. On the other hand, beyond the single mode approximation \( \mathcal{F}_r(\phi, r) \) increases gradually as \( \phi \) increases, and the maximum values are reached at \( \phi = 3\pi/2 \). The upper bound of the Fisher information depicted in figure 7(c) is smaller than that shown in figures 7(a) and (b).

2. An analytical solution: special case

In this section we analytically evaluate the teleported Fisher information with respect to the Unruh acceleration (\( \mathcal{F}_r \)) for a simple case. In this context, we assume that the teleported state is polarized on the z-axis, i.e. it is defined by the Bloch vector \( s = (0, 0, s_z) \), where \( s_z \) is given from (14). In this case, the teleported Fisher information is given by,

\[ \mathcal{F}_r = \frac{\sin 2r^2}{4} \left[ 1 + c_3^2 + \frac{(1 + c_3^2) \cos^2 2r}{4 - (1 + c_3^2) \cos^2 2r} \right] \]

From (15) it is clear that \( \mathcal{F}_r \) depends only on the initial state and the value of the acceleration. Moreover, if Alice and Bob initially share \( \rho_{s_z} \), then \( \mathcal{F}_r = \frac{1}{2} \sin^2 2r(2 + \cos^2 2r)/(2 - \cos^2 2r) \), while if the users initially share \( \rho_{\phi} \), then \( \mathcal{F}_r = \frac{1}{2} \sin^2 2r \).

5. Conclusion

In this contribution, we estimate the initial teleported parameters and the gained parameter during the teleportation process. The users, Alice and Bob, initially share a communication channel of the self-transposed class, which may be a maximum entangled Bell state or an X-state. It is considered that only Bob’s qubit is accelerated, while Alice’s qubit is in the inertial frame. The final accelerated state between Alice and Bob is used as a quantum channel to teleport the unknown state from one to the other. The final teleported state depends on the initial parameters in addition to the Unruh acceleration parameter. Fisher information is used as a measure of estimating the initial and the gained parameters, where it is calculated corresponding to each parameter within and beyond the single mode approximations.

The possibility of estimating the teleported parameters within(beyond) the single mode approximation decreases/increases as the Unruh acceleration increases. The maximum values of estimation depend on the estimated parameter and the approximation mode. For estimating the teleported weight parameter at a particular value of the Unruh acceleration, the phase parameter has a slight effect within the single mode approximation, while this effect is large beyond the single mode approximation. Similar behavior is depicted when the teleported phase parameter is teleported, i.e. the weight parameter has a slight/large effect within/beyond the single mode approximation, respectively.

Estimating the gained parameter (Unruh acceleration) is discussed within/beyond the single mode approximation. It is clear that the degree of estimation increases as the weight parameter increases, to reach its maximum values when the initial teleported state encodes only classical information. For small values of acceleration, the weight and the phase parameters have a slight effect on the degree of estimating the acceleration parameter within the single mode approximation. The maximum value of estimating the gained parameter for the arbitrary weight parameter is larger than that depicted for the arbitrary value of the phase parameter.

The effect of the different classes of the initial states is discussed, where we show that using different classes of Bell states causes a shift of the maximum and minimum bounds of the Fisher information. Moreover, there are some extra tops that appear for the singlet state. The estimation degree of the teleported parameter using the X-state is similar to that predicted for the singlet state, but with smaller upper bounds, where the degree of estimation depends on the initial entanglement of the communication channel.

In conclusion, it is possible to estimate the teleported and the gained parameters by means of Fisher information. The maximum values of the estimation degree depend on the used approximation, the entanglement of the initial state between the partners, and the structure of the initial teleported state. One may estimate these parameters with a large probability if the users teleport classical information.

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