Electromagnetic Characterization of Shielded Spherical Gyromagnetic Resonators

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Abstract—The electromagnetic (EM) properties of shielded gyromagnetic spheres are characterized with the aid of an electrodynamic model formulated for the transverse electric (TE) modes of an isotropic sphere, in a similar way as dielectric resonators are commonly analyzed. Broadband characteristics of geometric and energy filling factors are presented. A new measure of the magnetic losses tangent is introduced which is positive despite the intrinsic permeability of the gyromagnetic medium being negative, where existing definitions yield a nonphysical negative value. The sample size dependence of those quantities is also assessed. The conclusions from the analysis are applied to investigate and de-embed ferromagnetic linewidth errors which can occur in broadband measurements of magnetic garnet spheres. Magnetic anisotropy fields and the gyromagnetic g-factor are also estimated from the measurement data. Experiments were performed in a two-port subwavelength cavity, covering a frequency range of up to 28 GHz. Additionally, two-port rectangular resonant cavity linewidth measurements at a few modes have been conducted. Good agreement between the two methods is found if the systematic error is accounted for.

Index Terms—Ferrimagnetic materials, garnets, gyromagnetism, magnetic losses, microwave measurement, yttrium compounds.

I. INTRODUCTION

ELECTRODYNAMIC analysis of structures such as resonant cavities [1] or circulators [2] loaded with gyrotropic samples constitutes a contemporary research area [3]. Resonant modes in spherical cavities concentrically loaded with magnetized gyromagnetic spheres have been analyzed using various approaches, including perturbation methods [4], Mie scattering theory [5], the coupled field surface volume integral equation method [6], or the extended boundary condition method [7]. In general, such resonant modes are hybrid [8]. However, recent research [1], [9], [10] has shown that dominant modes in the gyromagnetic sphere can be analyzed as transverse electric (TE) modes of an isotropic one. This has led to the finding that the observed dominant mode (corresponding to TE_{101}), which is otherwise known as Kittel’s resonance and distinct from ferromagnetic resonance [11]–[14], is in fact a magnetic plasmon resonance (MPR) mode with a scalar effective relative permeability of the sphere \( \mu_r \approx -2 \) [9], which is the dual case of electric plasmons occurring in isotropic dielectric spheres [15]. The fundamental condition under which the gyromagnetic sphere can be treated as an isotropic one is that the microwave magnetic field components are perpendicular to the static field bias, which is satisfied sufficiently close as comparisons with magnetostatic [16] and approximate electrodynamic calculations [17], [18] have shown [10]. These advances have paved the way for improving the accuracy of magnetic property characterization of gyromagnetic materials, most notably microwave losses, represented by the Gilbert damping factor, \( \alpha \), which is directly related to the ferromagnetic linewidth, \( \Delta H \). It is a major parameter of the following permeability tensor of the gyromagnetic medium magnetized along the z-axis:

\[
\tilde{\mu} = \mu_0 \begin{bmatrix} \mu & i\kappa & 0 \\ -i\kappa & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix}.
\] (1)

Methods enabling the determination of all permeability tensor components in (1) of gyromagnetic materials (most notably ferrites) of cylindrical [19] and spherical [20] shapes are available. However, they require the use of a pair of cavities or a somewhat complex measurement setup to excite clockwise and counterclockwise polarized electromagnetic (EM) fields. The EM field pattern of eigenoscillations of a magnetized gyromagnetic sphere whose properties are described by (1) rotates clockwise about the z-axis [9] with Larmor’s frequency. Consequently, one can assume that the effective permeability of the sphere in a synchronously rotating coordinate system is [9]

\[
\mu_r = \mu_r' - j \mu_r'' = \mu + \kappa
\] (2)
which corresponds to the effective permeability for a clockwise-polarized magnetic field perpendicular to the z-axis. In the low-loss regime ($\alpha \ll 1$), the diagonal and off-diagonal components $\mu$ and $\kappa$ in (1) can be expressed as follows [21]:

$$
\mu = 1 + \frac{H_r + i\alpha \hat{\omega}}{H_r^2 - \omega^2 + 2i\alpha H_r \hat{\omega}}
$$

$$
\kappa = \frac{\hat{\omega}}{H_r^2 - \omega^2 + 2i\alpha H_r \hat{\omega}}
$$

where $H_r$ and $\hat{\omega}$ denote, respectively, the internal magnetic field bias, $H_{int}$, and complex frequency, $\hat{f} = f + i(f/2Q_0)$, normalized in the following way:

$$
H_r = H_{int}/(4\pi M_s)
$$

$$
\hat{\omega} = \hat{f}/(\gamma 4\pi M_s)
$$

where $4\pi M_s$ denotes the saturation magnetization [22, p. 455] of the sample and $Q_0$ is the unloaded quality factor of the resonant system containing the sample. For a perfectly magnetically isotropic sphere, the internal static magnetic field bias, $H_{int}$, is associated with the externally applied static field bias, $H_{ext}$, by

$$
H_{int} = H_{ext} - \frac{4\pi M_s}{3}.
$$

The Gilbert damping factor, $\alpha$, is defined as [23, p. 32]

$$
\alpha = \frac{\Delta H_{int}}{2H_{int}}
$$

The coefficient $\gamma$ in (6) is the gyromagnetic ratio of the sample. Instead of $\gamma$, the $g$-factor is frequently used, which is related to the electron spin gyromagnetic ratio, $\gamma_e$, and the electron spin $g$-factor, $g_e$, which are both constants, by

$$
g = g_e \frac{\gamma_e}{\gamma}
$$

The intrinsic $g$-factor of a ferrite material given by (9) is independent of magnetic bias but is dependent, e.g., on the temperature [24]. The relationship between the MPR frequency and the magnetic bias can be approximately expressed as [17]

$$
f = \gamma (H_{ext} + H_a) - \frac{4\pi^2}{90}4\pi M_s \gamma (\epsilon_r + 5) \left(\frac{df}{c_0}\right)^2
$$

where $H_a$ is the effective anisotropy field, dependent on the orientation of the bias field with respect to the anisotropy axes, $d$ is the sample diameter, $\epsilon_r$ its dielectric constant, and $c_0$ the speed of an EM wave in vacuum. One should keep in mind that using (10) one cannot analyze the influence of adjacent conductors or dielectrics, however these factors can be considered via a magnetodynamic transcendental equation (TDE) [10], [25] formulated for the shielded isotropic sphere.

It is quite common in the literature to assume without any explicit comment that the ferromagnetic linewidth is related to the static magnetic field outside the sample, $H_{ext}$, in the low-loss regime ($\alpha \ll 1$). It can be, therefore, named as the extrinsic linewidth, $\Delta H_{ext}$, which is in general non-linear with respect to the resonance frequency, mainly due to shape-dependent demagnetization factors [26]. In this work, the focus is given to the intrinsic ferromagnetic linewidth related to the internal static magnetic field, $\Delta H_{int}$, as it depends on the medium properties only. The relation between the two linewidths, assuming that the sample diameter is much smaller than the free-space wavelength ($d \ll \lambda_0$), is given by

$$
\Delta H_{int} = \Delta H_{ext} \left(1 + \frac{4\pi M_s}{3\hat{\omega}}\right)^{-1}.
$$

It follows from the TDE results [26], [27] that if $d \ll \lambda_0$, the intrinsic ($\Delta H_{int}$) and extrinsic ($\Delta H_{ext}$) ferromagnetic linewidths can be obtained as [26]:

$$
\Delta H_{int} = \frac{H_{int}}{Q}
$$

$$
\Delta H_{ext} = \frac{f}{\gamma Q}
$$

where $Q$ is the measured MPR mode $Q$-factor. It should be noted, however, that the $Q$-factor is influenced not only by the magnetic losses that are of interest but also by dielectric losses and conduction losses in the cavity walls, so their contribution should be de-embedded. In addition, (12) and (13) gradually lose their validity as the electrical diameter of the samples is increased. It thus follows that both (12) and (13) in fact denote loaded ferromagnetic linewidths. Measurements of these linewidths are reported in [28], together with an assessment of measurement errors. One of the major goals of this article, however, is to investigate the properties of spherical gyromagnetic resonator parameters such as the geometric factor, $G$, accounting for conduction losses in the cavity walls, energy filling factors, and the magnetic loss tangent $\tan \delta_m$. The motivation to investigate these parameters is twofold. On the one hand, fundamental questions pertaining to the gyromagnetic sample size dependence of $G$, the frequency dependence of the energy filling factors, or the sign of the loss tangent under negative permeability remain unanswered. On the other hand, the influence of these parameters on the loaded ferromagnetic linewidth has not been thoroughly investigated thus far either. The remaining part of this article is organized as follows. Section II is a summary of the experimental setup. Section III introduces electromagnetic (EM) analysis of energy filling and geometric factors, as well as the loss tangent and their impact on the extraction of the ferromagnetic linewidth. Only the dominant mode exhibiting negative effective permeability is analyzed. Properties of gyromagnetic spheres under conditions where their effective permeability is positive have been recently described in [29]. The extraction of the linewidth and $g$-factor from raw sample measurement data is addressed in Section IV, and a discussion of the extracted quantities is provided in Section V.

**II. Experiments**

Experiments were performed on one monocrystalline Ga-doped yttrium iron garnet (YIG) sphere of a relatively small diameter $d = 0.305$ mm and one polycrystalline Ca,N-doped YIG of a relatively large diameter $d = 1.42$ mm. A priori known parameters of the samples are summarized in Table 1. Henceforth, the two samples will be differentiated by their chemical composition. Experiments were performed in a cylindrical subwavelength cavity made of brass (see Fig. 1)
TABLE I
SUMMARY OF THE PARAMETERS OF THE INVESTIGATED SAMPLES AS SPECIFIED BY THEIR VENDORS. $4\pi M_s$ - SATURATION MAGNETIZATION, $\varepsilon_\text{s}$ - SAMPLE DIELECTRIC CONSTANT [30]–[32]

| Sample           | $4\pi M_s$ (G) | Diameter (mm) | $\varepsilon_\text{s}$ (μm) |
|------------------|---------------|---------------|-----------------------------|
| monocristalline Ga-YIG | 1300          | 0.305         | 16                          |
| polycristalline Ca,V:YIG | 1850          | 1.42          | 14.8                        |

Fig. 1. Brass subwavelength cavity with adjustable coupling loops used for broadband experiments [28].

Fig. 2. TE$_{102}$ rectangular resonant cavity operating at ca. 5 GHz [28].

Fig. 3. TE$_{102}$ rectangular resonant cavity operating at ca. 10 GHz.

loops was controlled in such a way that the sample was weakly coupled ($|S_{21}| < -40$ dB). At weak coupling, the difference between the loaded ($Q$) and unloaded ($Q_0$) factors amounts to only a few percent, dropping below 1% for $|S_{21}| < -45$ dB [33]. Therefore, in measurement practice, the loaded $Q$-factor can be equated with the unloaded one. In the resonant cavity experiments, the static magnetic field was set to such a value for which strong coupling between the sample and cavity modes occurred, leading to the appearance of two resonant modes (hybrid magnon-polariton modes) [34]. The average $Q$-factor of the hybrid modes was extracted. Both samples were measured at the TE$_{102}$ modes of each of the two cavities. The Ca,V:YIG sample was additionally measured at the TE$_{104}$ (16.5 GHz) and TE$_{106}$ (23.4 GHz) modes of the 10-GHz cavity.

III. EM ANALYSIS

A. Spherical Cavity Model

The $g$-factor and effective anisotropy field $H_a$ were fit by adjustment of the parameters of an electrodynamic TDE to the $f$ versus $H_{\text{ext}}$ data measured using the subwavelength cavity shown in Fig. 1. The utilized TDE was formulated for an isotropic sphere of relative permeability $\mu_r$ and permittivity $\varepsilon_r$ surrounded by a dielectric medium enclosed in a concentric spherical perfect electric conductor (PEC) cavity whose model is depicted in Fig. 5. Admittedly, such a TDE is not rigorously applied to the cylindrical subwavelength cavity experiments due to cavity shape mismatch, as it can be seen by comparing its axial cross section shown in Fig. 6 with Fig. 5. Such an approach was prompted by the lack of a rigorous EM model of a cylindrical cavity loaded with a spherical gyromagnetic sample. Due to the fact that the vast majority of EM energy is concentrated in the sample, a hypothesis has been put forward that the aforementioned mismatch can be suppressed by adjusting the radius of the concentric spherical cavity, $R_2$, and the dielectric constant of the medium surrounding the sample, $\varepsilon_d$. In addition to these two parameters, $g$ and $H_a$
factor, and electric loss tangent of the sample. Similar measures will be considered hereafter in relation to the spherical gyromagnetic resonators. As has already been confirmed experimentally [27], magnetic energy stored in a gyromagnetic resonator at the MPR mode can be orders of magnitude larger than the electric one. However, variations in energy shares between the sample and its exterior have not been investigated quantitatively so far. To fill this gap, the spectral dependence of the energy filling factors has been computed with the aid of the electrodynamic TDE assuming the effective model parameters determined as described in Section III-A and provided in Tables I and II. The energy filling factors are defined as follows (see also Fig. 5):

\[
p_{e}^{in} = \frac{\iint V_{i} \varepsilon_{0} \varepsilon_{r} |E(v)|^{2} \, dv}{\iint V_{i} \mu_{0} |H(v)|^{2} + \varepsilon_{0} |E(v)|^{2} \, dv} = \frac{W_{e}^{in}}{W_{total}} \tag{14}
\]

\[
p_{e}^{out} = \frac{\iint V_{i} \varepsilon_{0} \varepsilon_{d} |E(v)|^{2} \, dv}{\iint V_{i} \mu_{0} |H(v)|^{2} + \varepsilon_{0} |E(v)|^{2} \, dv} = \frac{W_{e}^{out}}{W_{total}} \tag{15}
\]

\[
p_{m}^{in} = \frac{\iint V_{i} \mu_{0} \mu_{eff} |H(v)|^{2} \, dv}{\iint V_{i} \mu_{0} |H(v)|^{2} + \varepsilon_{0} |E(v)|^{2} \, dv} = \frac{W_{m}^{in}}{W_{total}} \tag{16}
\]

\[
p_{m}^{out} = \frac{\iint V_{i} \mu_{0} |H(v)|^{2} \, dv}{\iint V_{i} \mu_{0} |H(v)|^{2} + \varepsilon_{0} |E(v)|^{2} \, dv} = \frac{W_{m}^{out}}{W_{total}} \tag{17}
\]

where \(v\) is a variable denoting the volume, \(V_{i}\) is the total volume of the cavity, \(V_{i}\) is the volume of the sample, and \(V_{2}\) is the volume exterior to the sample (\(V_{1} + V_{2} = V_{i}\)). The vacuum permittivity (permeability) is denoted by \(\varepsilon_{0} (\mu_{0})\). The relative permittivity of the dielectric medium is denoted as \(\varepsilon_{r}\), and \(\varepsilon_{d}\), respectively. \(\mu_{eff}\) is the effective permeability of the gyromagnetic sphere which must be accounted for in stored energy computations [27], [35]

\[
\mu_{eff} = \frac{\varepsilon_{0} (\mu_{r})}{\varepsilon_{0} (\mu_{r}) + \alpha} = \frac{H_{r}}{(H_{r} - \omega)^{2}} \tag{18}
\]

where \(\omega = \text{Re}(\dot{\omega})\), and \(\alpha = 0\) was assumed. The stored energy is denoted by \(W\), the subscript \(m (e)\) stands for the magnetic (electric) energy, and the superscript \(i (o)\) denotes the volume inside (outside) the sample. The distribution of the electric and magnetic field components used in (14)–(17) was determined via analytical calculations assuming the isotropic loaded cavity model of Fig. 5.

The dependence of electric and magnetic energy filling factors, inside and outside of the gyromagnetic sample computed for the estimated parameters from Table I, are presented in Figs. 7 and 8. It can be seen that the vast majority of the EM energy of the resonant system is stored in the magnetic field, mostly inside the sample. As the resonance frequency increases, the magnetic energy outside the gyromagnetic sample is converted into magnetic and electric energy within the sample indicating enhanced focusing of EM fields inside the sample.

Computation shows that increasing the sample diameter decreases the maximum value of the magnetic energy filling factor \(p_{m}^{out}\) and increases the share of the electric energy both inside and outside of the sample. However, for spheres of diameters up to ca. 1 mm which are used in microwave experiments with the gyromagnetic sample placed inside a quartz tube. Fig. 6. Axial cross section of the subwavelength cylindrical cavity used in experiments with the gyromagnetic sample placed inside a quartz tube.

**Fig. 5.** Model of a gyromagnetic sample in a concentric spherical PEC cavity filled with a dielectric medium. \(p_{e}^{in}, p_{e}^{out}, p_{m}^{in}, p_{m}^{out}\) are the electric and magnetic energy filling factors defined in (14)–(17). Distribution of the electric field is also illustrated. It increases linearly inside the sphere and decays exponentially beyond. Similarly, the microwave magnetic field components decay exponentially, but are nearly constant inside the sample. See [9] for details, including the field components of the electrodynamic model.

were subjected to the optimization algorithm and the obtained fits were much closer than those obtained via an alternative approach, as in (10). The values of the estimated effective parameters are repeated after the conference paper [28] in Table II. More detailed information, including fit plots are available therein [28].

### B. Properties of Spherical Gyromagnetic Resonators

For dielectric resonators, which are widely applicable, e.g., in microwave material characterization, the most important figures of merit are the electric energy filling factor, geometric

### TABLE II

**Summary of the Fit Effective Parameters of the Investigated Samples. \(\varepsilon_{d}\)-Dielectric Constant of the Medium Surrounding the Sample, \(R_{2}\)-Radius of the Metal Cavity, \(H_{o}\)-Anisotropy Field**

| Sample        | \(R_{2}\) (mm) | \(\varepsilon_{d}\) | g-factor | \(H_{o}\) (Oe) |
|---------------|----------------|---------------------|-----------|--------------|
| monocristalline Ga:YIG | 1.5          | 2.5                 | 2.014     | 57           |
| polycristalline Ca:V:YIG | 2.78         | 3.48                | 2.014     | 0            |
Fig. 7. Energy filling factors computed for the Ga:YIG sphere. The sample’s magnetic energy filling factor attains a value of 0.95 at the frequency 16.55 GHz, as indicated in the upper legend.

Fig. 8. Energy filling factors computed for the Ca, V:YIG sphere. The sample’s magnetic energy filling factor attains a value of 0.9 at the frequency 28.45 GHz, as indicated in the upper legend.

In practice, this share does not exceed a few percent. This is the reason why dielectric losses can usually be neglected when characterizing magnetic losses of gyromagnetic spheres. It should also be emphasized that the filling factors depend negligibly on the ferromagnetic linewidth. The confinement of magnetic energy away from the cavity walls [36], which is key for the assessment of conduction losses, is characterized by the geometric factor of a resonant cavity, given by the formula

$$G = Q_c R_s$$

(19)

where $Q_c$ is the $Q$-factor due to electric losses in the metallic cavity walls, and $R_s$ the surface resistance of the walls. Fig. 9 depicts $G$ and $Q_c$ versus the resonance frequency for the two spheres under study. $Q_c$ was computed via numerical integration of the magnetic field tangential to the cavity walls [37]. As evidenced by the slope of the curves plotted in Fig. 10, the geometric factor $G$ drops proportionally to the volume of the sample. Similarly, as in the case of energy filling factors, $Q_c$ and $G$ depend negligibly on $\Delta H$. Only for sample diameter of over a few mm does the rate of decrease of $G$ diminish (see Fig. 10).

In addition to energy filling factors and geometric factors, another fundamental parameter of interest of dielectric resonators is the loss tangent of the insert ($\tan \delta_e$) associated with the electric energy filling factor ($p_e$) and $Q$-factor due to dielectric losses ($Q_e$) by

$$Q_e^{-1} = p_e \tan \delta_e.$$ 

(20)

One of the objectives of this article is to define a similar expression for a gyromagnetic resonator and investigate the character of its terms. To this end, the following definition of the magnetic loss tangent can be introduced:

$$\tan \delta_m = \frac{W_m^{loss}}{W_m^{in}}$$

(21)
and the filling factor

$$p = 2p_m^\text{in} = \frac{2W_m^\text{in}}{W_{\text{total}}}$$  \hspace{1cm} (22)

where $W_m^\text{loss}$ is the average (magnetic) energy dissipated in the sphere. In dielectric resonators, where energies stored in the electric and magnetic fields are equal, the electric energy filling factor varies from 0 (negligibly small sample) up to 0.5 (homogeneous filling of the cavity with the sample).

In the spherical gyromagnetic resonators, however, the dominant share of magnetic energy makes the upper limit of the magnetic energy filling factor, $p_m^\text{in}$, approach unity. Consequently, the filling factor $p$ varies in the 0–2 range. It has been chosen to associate the factor of 2 with the filling factor $p$ rather than with $\tan\delta_m$ in order to conform with the definition of the dielectric loss tangent. It is crucial to note that according to definition (21), $\tan\delta_m > 0$, while applying a standard definition for a scalar permeability $\mu_d = \mu_d^\prime - j\mu_d^\prime\prime$, \[38\]

$$\tan\delta_m = \frac{\mu_d^\prime}{\mu_d^\prime\prime}$$  \hspace{1cm} (23)

to the tensor (1) components will yield $\tan\delta_m < 0$ since $\mu^\prime$, $\kappa^\prime$, $\mu^\prime\prime$ are all negative at the MPR resonance. The dependence of the filling factor $p$ on the magnetic field for a few sample diameters is depicted in Fig. 11. It can be seen that the smaller the diameter of the sample, the lower the filling factor for higher magnetic bias values. The decrease in the filling factor with an increase in the sphere diameter becomes appreciable at higher magnetic bias values.

In the limiting case of electrically small samples, for which $\omega = H + (1/3)$ is a very good approximation, (21) simplifies to (see also [1], [27])

$$\tan\delta_m = \frac{\alpha}{\frac{1}{\omega} + \frac{1}{H_r} - \frac{1}{(H_r - \omega)^2}} = \frac{H_r}{\omega} - \frac{\omega}{H_r}$$  \hspace{1cm} (24)

Results of computations demonstrated in Fig. 11 show that, for electrically small samples the filling factor $p$, defined by (22), approaches the quantity

$$p = \frac{2H_r + \frac{1}{\alpha}}{H_r + \frac{1}{\alpha}}.$$  \hspace{1cm} (25)

Consequently, the $Q$-factor due to magnetic losses becomes

$$Q_m = (p\tan\delta_m)^{-1} = (2\alpha)^{-1} = \frac{H_{\text{int}}}{\Delta H}.$$  \hspace{1cm} (26)

The loss tangent $\tan\delta_m$ also exhibits size dependence, albeit its relative deviations are usually a few times smaller than the deviations of $p$, as can be concluded upon comparing Figs. 12 and 13. The relative deviations of $p$ and $\tan\delta_m$ have been computed via

$$\delta(p) = \frac{H_r + \frac{1}{\alpha} - W_{\text{total}}}{H_r + \frac{1}{\alpha}} - 1$$  \hspace{1cm} (27)

and

$$\delta(\tan\delta_m) = \frac{\alpha H_r - \frac{1}{\alpha} - W_m^\text{in}}{\alpha H_r - \frac{1}{\alpha}} - 1$$  \hspace{1cm} (28)
respectively. It can be also noted that the deviations of \( \tan \delta_m \) and \( p \) for the Ga:YIG sample have opposite signs and can therefore compensate each other.

**IV. Determination of Magnetic Parameters**

Measurement methods of the \( g \)-factor of spherical specimens are mostly based on a single-mode cavity approach \cite{39}, which requires knowledge of the anisotropy field. It can be obtained via resonance frequency versus angle measurements, but magnetically oriented spheres mounted on rods are necessary \cite{1}. With broadband methods, \( g \) and \( H_a \) can be in principle determined simultaneously, e.g., by measuring the frequency spacing of so-called magnetostatic modes with \([1,1,0]\) and \([4,4,0]\) mode orders at low magnetic bias values \cite{24}, where the \([1,1,0]\) mode is the MPR mode. However, this method may require strong coupling to the sphere, as the \([4,4,0]\) mode is much more difficult to couple with than the \([1,1,0]\) mode. Consequently, the measured loaded \( Q \)-factor can deviate significantly from the unloaded \( Q_0 \) factor, thus, deteriorating the accuracy of \( \Delta H \). This can be solved either by calibrating the system and unloading the \( Q_0 \) factor or by running the measurement again on \([1,1,0]\) mode only with reduced coupling. Another broadband method that allows determining \( g \), \( H_a \), \( \Delta H \) simultaneously is the cross-guide coupler method \cite{40}, in which the sample under test is placed in a coupling hole between two orthogonal waveguides or triplate transmission lines. However, it is a technique which is quick rather than accurate.

In Section IV, the \( g \)-factor estimated with the aid of the TDE is compared with the direct calculation of the effective \( g \)-factor from raw subwavelength cavity experimental data points for adjacent magnetic fields:

\[
g = \frac{g_e f^{(i+1)} - f^{(i)}}{\gamma_e H^{(i+1)}_{\text{ext}} - H^{(i)}_{\text{ext}}} \quad (29)
\]

It should be emphasized that (29) is the effective (apparent) \( g \)-factor \cite{41–43}, not to be confused with (9). Comparisons of the \( g \)-factor with the effective \( g \)-factors for each of the investigated spheres are shown in Figs. 14 and 15. For the Ga:YIG sphere, the raw data calculation points in Fig. 14 extrapolated to zero magnetic bias are visibly very close to the value estimated using the TDE. The downward slope is a manifestation of the influence of the size of the sphere on the resonance frequency, described approximately by the quadratic term of (10). The same but much more pronounced effect can be seen in Fig. 15, the reasons being that the Ca,V:YIG sample diameter and \( 4\pi M_s \) are significantly greater (see Table I). Although it may seem at first glance that the difference between the TDE-estimated \( g \)-factor and the raw data calculation of the effective \( g \)-factor is large, it can be attributed to a small effective anisotropy field of ca. 7 Oe which, as explained in Section V later in the text, is difficult to properly account for. The obtained value of 7 Oe is a result of the following estimation: The resonant frequency \( f = \gamma H_{\text{ext}} \) of a magnetically isotropic sample at \( H_{\text{ext}} = 1000 \) Oe with \( g = 2.014 \) is the same as the resonant frequency of an anisotropic sample with \( H_a = 7 \) Oe and \( g = 2 \) at the same bias field.

**Fig. 14.** Effective \( g \)-factors computed for the Ga:YIG sphere from raw data via (29) (blue dots) compared with the \( g \)-factor value estimated via the TDE. The black dashed line marks the falling trend of the effective \( g \)-factor due to the size effect.

**Fig. 15.** Effective \( g \)-factors computed for the Ca,V:YIG sphere from raw data via (29) (blue dots) compared with the \( g \)-factor value estimated via the TDE.

As has been mentioned in Section I, \( \Delta H \) computed via (12) is an apparent linewidth. It consists of two terms

\[
\Delta H_{\text{apparent}} = \frac{H_{\text{int}}}{Q_m} + \frac{H_{\text{int}}}{Q_c} \quad (30)
\]

as does the relative error

\[
\delta(\Delta H) = \frac{\Delta H_{\text{apparent}}}{\Delta H} - 1 = \left( \frac{H_{\text{int}}}{Q_m \Delta H} - 1 \right) + \frac{H_{\text{int}}}{Q_c \Delta H}. \quad (31)
\]

The error term associated with \( Q_m \) is negative and arises for electrically large samples as (26) loses its validity and it is virtually independent of \( \Delta H \). While this error is almost nonexistent for the Ga:YIG sample, it reaches ca. \(-7\%\) for the Ca,V:YIG sample (see Fig. 16). Computations for up to \( \Delta H_{\text{int}} = 20 \) Oe show that the relative error does not depend on the value of the linewidth. To keep the second error term associated with losses in the cavity walls as small as possible, \( Q_c \) should be maximized, which can be achieved by minimizing \( R_s \) (maximizing the conductivity) and
maximizing the geometric $G$-factor (decreasing the sample size). Computations show that $Q_c$ is virtually independent of $\Delta H$. As it should be noticed, the two relative linewidth errors have opposite signs and therefore may compensate each other. Systematic linewidth errors can be computed via the TDE and used to de-embed the intrinsic linewidth from the apparent linewidth computed via (12). Figs. 17 and 18 show the estimated and de-embedded linewidths versus $H_{\text{int}}$ for the Ca,V:YIG and Ga:YIG samples, respectively. Fig. 19 depicts the Gilbert damping factors (8) for both samples.

V. DISCUSSION

The effective anisotropy field obtained for the Ga:YIG sample, $H_a = 57$ Oe, is in satisfactory agreement with the literature. Based on [44], the interpolated value of the first-order cubic anisotropy constant for Ga:YIG at 295 K amounts to $K_1 = -477.5$ J/m$^3$, which leads to a magnetocrystalline anisotropy field of $H_a = (4/3)(|K_1|/M_s) = 61.5$ Oe for the easy magnetization axis aligned with the external magnetic bias. A relatively small difference of 4.5 Oe can be attributed to, e.g., temperature variations or other small contributions to the effective anisotropy field. Additional experiments consisting in controlled rotation of the quartz rod with the Ca,V:YIG sample rotating freely inside confirm that the anisotropy field is at least a few times lower than for the Ga:YIG sample because observed shifts in the resonance frequency are correspondingly lower. Small effective anisotropy fields can occur in polycrystalline spheres due to elastic stress induced in the manufacturing process [45]. The apparent discrepancy may be due to the fact that there is a positive correlation between the cavity radius parameter and the anisotropy field since the increase of each of them lowers the value of $\omega - H_a$.

We conclude that the proposed method may not be suitable for measuring low anisotropy fields.

Regarding the estimated $g$-factors, it is well known that the rough proportionality for the Gilbert damping factor holds [41], [46]

$$a \propto (g - 2)^2.$$
In view of (32), the obtained g-factors, both equal to 2.014, are plausible given that the minimum Gilbert damping factors for both samples are of the order of $1 \times 10^{-4}$ as shown in Fig. 19.

The spike in the linewidth of the Ca,V:YIG sample visible in Fig. 17 in the frequency range of ca. 3–5 GHz is most likely attributable to the Buffler peak [47]. It is a little-known sharp increase in the linewidth observed in spherical samples only and attributed to the two-magnon scattering loss mechanism. It occurs as the resonance frequency moves through the spin-waveband crossover frequency $f_X$ equal to

$$f_X = \frac{2}{3^3} 4\pi M_s$$

(33)

which for the studied sample amounts to $f_X \approx 3.47$ GHz. Measurements below $f_X$ require reducing power levels below a critical threshold to avoid Suhl instability [10].

Deviations of $\Delta H_{int}$ for the Ca,V:YIG sample (see Fig. 17) from a straight line are partly due to the presence of other modes in the sample which influence the $Q$-factor of the measured dominant mode. Some of these modes can also be analyzed using the magnetodynamic TDE. Details can be found in [25].

A few final remarks can be made about the strengths and weaknesses of the described subwavelength cavity method compared to other broadband methods. Its accuracy of the $Q$-factor determination [21].

VI. CONCLUSION

The main systematic error source present in the described ferromagnetic linewidth measurements is primarily due to the size dependence of the magnetic energy filling factor. The resulting negative relative error is independent of ferromagnetic linewidth and increases in absolute value with increasing electrical size of the sample. The error due to losses in the cavity walls increases as the internal magnetic field and sample diameter increase, and as the linewidth decreases. Moreover, good agreement between ferromagnetic linewidths measured via direct $Q$-factor measurements and rectangular cavity-assisted measurements in different cavities and at different cavity modes in the strong-coupling regime has been demonstrated if the systematic errors are accounted for.

Although the electrical size-dependent variations in the $Q$-factor due to magnetic losses are mostly attributable to magnetic energy filling factor changes, the magnetic loss tangent is not free from size dependence either. In this respect, there seems to be no advantage in characterizing magnetic losses via tan $\delta_m$ over $\Delta H$, which can be obtained as a size-independent parameter.

The authors believe that the presented characterization of spherical gyromagnetic resonators using a rigorous approach well-known for dielectric resonators, including the newly introduced positive magnetic loss tangent, will facilitate the development of a coherent theory of resonant modes in gyrofluid media.

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