Profiling of the flat-tool for manufacturing worms with circular eccentric profile by cold forming

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Abstract. The incremental pumps are encountered in pharmaceutical and cosmetics industries. One of the most representative categories of these pumps is formed by the progressing cavity pumps. The helical surfaces with circular profile, eccentrically positioned relative to worm axis, are frequently used in their rotors construction. In this specific case, the constructive dimensions of the worms allow their manufacturing by cold forming, with flat-tools similar to screwing dies. In this paper, an analytical algorithm for profiling such flat-tool is proposed.

The algorithm starts from the analytical definition of the helical cylindrical surface with constant pitch of the generated worm, and it lays on the "Minimum distance method". The equations of the gearing surface between worm and flat-tool are found, and, based on this, the active surface of the tool is determined. A MatLab application developed for implementing the profiling algorithm is presented. The results obtained after running the application in the case of a particular generated surface are also included.

1. Introduction
Progressing cavity pumps belong to the group of rotating positive displacement pumps. They are self-priming, valve-less, and due to their high process liability and suction capacity they are often used for the continuous, gentle conveyance and precise dosing - in proportion to speed - of difficult media [1].

Among their principal benefits, one can mention:
- Flow in proportion to speed;
- Precise dosing;
- Continuous, gentle and low-pulsation flow;
- Large free ball passage (up to 150 mm);
- High suction capacity;
- Low life-cycle costs due to long lifetime and meantime between failure;
- Reverse rotation and flow.

In what concerns their functioning, the delivery principle is based on a rotor, 1, which turns in an oscillating motion within a fixed stator, 2, see figure 1. The coordinated, spiral geometry of both components creates delivery chambers in which the medium is transported from the suction to the pressure side. Due to the design of rotor and stator, hardly any pulsation or shearing forces are affecting the fluid. Instead, the medium is transported gently and continuously. Consistency and especially the viscosity are not a factor for the delivered flow with this displacement technology; the transported quantity is determined by the speed alone and – in combination with a frequency inverter –
can be regulated conveniently and with precision. This enables an accuracy of between five and three percent to be achieved, while small dispensers even achieve one percent.

![Progressing cavity pump](image)

**Figure 1.** Progressing cavity pump [1].

The cylindrical helical surface with constant pitch of the progressing cavity pump rotor can be generated by milling, with disc-tools having specific profile, or by slotting, with cutters having their active surface delimited by cylindrical surfaces. The profiles of the mentioned types of tools can be found on the base of Nikolaev theorem [2-5].

Another solution for machining the progressing cavity pump rotor is turning with rotating cutter, which is a highly productive process. In this case, the tool specific profile can be determined by "Minimum distance" method [5], applied for finding profiles associated to rolling centrodes.

The "Minimum distance" method resulted from a new approach of well-known Willis theorem [3], applied in the case of profiles associated to a couple of rolling centrodes \((C_1, C_2)\), see figure 2). According to the method, the envelop of a profile associated to a couple of rolling centrodes is the locus of the profile points for which, in the successive rolling positions, the distance \(d\) to gearing pole \(P\) (meaning the point of tangency between the centrodes) is minimum.

![Conjugated profiles & rolling centrodes](image)

**Figure 2.** Conjugated profiles & rolling centrodes [5].

The threads manufacturing by cold forming [6, 7] presents some notable benefits relative to their cutting, such as smoother flanks, higher resistance to wear and fatigue, higher productivity. Flat or
cylindrical tools can be used in this purpose. In both cases, the active surface of the tool is the envelop of thread cylindrical helical surface, in the specific generating motion.

In this paper, an analytical algorithm for profiling such flat-tool, enabling the manufacturing of progressing cavity pump rotor is proposed. The algorithm starts from the analytical definition of the helical cylindrical surface with constant pitch of the generated worm, and it lays on the “Minimum distance method”. In the next section, the equations of the helical surface of mentioned pump rotor are deducted. The third section is dedicated to profiling of the flat-tool active surface. The fourth section presents a numerical application, based on a MatLab application, developed for implementing the profiling algorithm. The last section is for conclusion.

2. Equations of the pump rotor surface

The equations defining the helical surface of the pump rotor can be obtained after defining the following reference systems (figure 3):

- \( xyz \), meaning the global system, fix and having its origin \( O \) onto rotor axis,
- \( X_1Y_1Z_1 \) – local system, having the origin \( O_1 \) into the center of the eccentrical circle, and
- \( XYZ \) – local system, having the origin \( O \) onto rotor axis and executing a helical motion of \( \varphi \) – angular and \( p \) – helical parameters, along rotor axis.

![Figure 3. The eccentrical circle generating the helical surface of the rotor.](image)

The co-ordinates of the generic point from the eccentrical circle are:

\[
\begin{align*}
X_1 &= -e + R \cos \beta ; \\
Y_1 &= R \sin \beta , \\
Z_1 &= 0 ,
\end{align*}
\]

where \( \beta \) means an angular variable, \( \beta \in [0, 2\pi] \).

The helical motion of \( XYZ \) reference system has the following equation, written with matrices:

\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix} = \begin{pmatrix}
\cos \varphi & -\sin \varphi & 0 \\
\sin \varphi & \cos \varphi & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
-e + R \cos \beta \\
R \sin \beta \\
0
\end{pmatrix} + \begin{pmatrix}
0 \\
0 \\
p \cdot \varphi
\end{pmatrix} .
\]

After development and mathematical calculus, relation (2) becomes:

\[
\begin{align*}
X &= -e \cos \varphi + R \cos(\beta + \varphi) ; \\
\Sigma & \begin{pmatrix}
Y \\
Z
\end{pmatrix} = -e \sin \varphi + R \sin(\beta + \varphi) ; \\
Z &= p \cdot \varphi ,
\end{align*}
\]

meaning the equations of rotor helical surface, \( \Sigma \).
3. Profiling of the flat tool active surface

The manufacturing of progressing cavity pump rotor by cold forming with flat-tools is accomplished by rolling the cylindrical workpiece between a couple of profiled flat tools (see figure 4). Corresponding to this situation, a cylindrical axoid $A_1$, of $R_{rp}$ radius is considered for the workpiece, while each tool has a plane as axoid, $A_2$ and $A_2'$, respectively. The $A_1$ axoid executes a rotation motion of $\varphi_1$ parameter (figures 4 and 5), while $A_2$ and $A_2'$ have translation motions, of $\lambda$ parameter. The rolling motion requires to keep always the rolling condition:

$$\lambda = R_{rp} \cdot \varphi_1$$  \hspace{1cm} (4)

Figure 4. Manufacturing scheme.  \hspace{1cm} Figure 5. Transversal section of rolling axoids.

Because both flat-tools have identical profile, only one of them is further considered, together with its axoid, $A_2$, as depicted in figure 5. The active surface of the flat-tool results as envelop of the surfaces family generated in the motion of rotor surface $\Sigma$ relative to the local system $\xi\eta\zeta$, attached to $A_2$ axoid.

The equation of workpiece motion relative to considered flat-tool is:

$$\xi = \omega_3^T(\varphi) \cdot X - a$$, \hspace{0.5cm} with \hspace{0.5cm} $a = \begin{pmatrix} -R_{rp} \\ -R_{rp} \cdot \varphi \\ 0 \end{pmatrix}$  \hspace{1cm} (5)

In relation (5) $\omega_3$ is the well-known matrix of coordinates transform for rotation around $Oz$ axis. If here $X$ means the vector formed by the coordinates of the generic point from $\Sigma$ surface, calculated with (3), then (5) gives the equations of the surfaces family generated in the motion of rotor surface, depending on the parameters $\beta$, $\varphi$ and $\varphi_1$. The envelop of this family results by associating to (5) the enveloping condition, which, according to Gohman theorem [8] has the expression:

$$| \begin{bmatrix} \xi_\beta' \\ \xi_\varphi' \\ \xi_{\varphi_1}' \end{bmatrix} | = 0.$$  \hspace{1cm} (6)

In relation (6), $\xi$, $\eta$ and $\zeta$ mean the components of the vector $\xi$ calculated with (5). The ensemble formed by equation (5) and condition (6) defines the flat-tool active profile. As it can be noticed, the finding of profile in this manner is relative complicated from calculus point of view. Instead, we suggest a simplified profiling algorithm, consisting in two steps:
• The flat-tool profile is determined, at first, in a plane section, corresponding to $\varphi = 0$, in equation (5), by using a numerical method (e.g. the “Minimum distance” method, [5]) for finding the generated profile envelop. Thus, after imposing the $\varphi = 0$ condition, (5) becomes:

$$
\begin{align*}
\xi &= -e \cos \varphi_1 + R \cos(\varphi_1 + \beta) + R_{rp}; \\
\eta &= -e \sin \varphi_1 + R \sin(\varphi_1 + \beta) + R_{rp} \cdot \varphi_1.
\end{align*}
$$

(7)

According to “Minimum distance” method, in the case of generating with the rack-tool (applicable when profiling the flat-tool for manufacturing by rolling) the enveloping condition has the shape:

$$
\xi \cdot \xi' + (\eta - R_{rp} \cdot \varphi_1)\eta' = 0.
$$

(8)

After replacing the expressions for $\xi$ and $\eta$ and some mathematical calculus, relation (8) becomes:

$$
\varphi_1 = \sin^{-1}(e \sin \beta / R_{rp}) - \beta.
$$

(9)

• The flat-tool 3D profile is generated by extruding the plane profile from above after a direction inclined with the angle $\alpha$ of the generatrix line of the tool cylindrical surface (see figure 6).

Figure 6. The generating of flat-tool 3D profile.

The angle $\alpha$ can be easily calculated with:

$$
\alpha = \tan^{-1}(R_{rp} / p).
$$

(10)

4. Numerical application

The feasibility of the algorithm for profiling the flat-tool that can be used for generating the rotor of progressing cavity pump was tested by running a numerical application, further presented. The application addresses the case of the rotor having $R = 10$ mm, $e = 3$ mm and $p = 10$ mm.

| Crt. no. | X [mm] | Y [mm] | Z [mm] |
|----------|--------|--------|--------|
| 1        | 7      | 0      | 0      |
| 2        | 6.6574 | 2.1631 | 3.1416 |
| 3        | 5.6631 | 4.1145 | 6.2832 |
| 4        | 4.1145 | 5.6631 | 9.4248 |
| . . . . . | . . . . | . . . . | . . . . |
| 429      | 10.5172| -7.6412| 25.1327|
| 430      | 12.3637| -4.0172| 28.2743|

| Crt. no. | X [mm] | Y [mm] | Z [mm] |
|----------|--------|--------|--------|
| 431      | 13     | 0      | 31.4159|
| 432      | 12.3637| 4.0172 | 34.5575|
| 858      | 4.1145 | -5.6631| 53.4071|
| 859      | 5.6631 | -4.1145| 56.5487|
| 860      | 6.6574 | -2.1631| 59.6903|
| 861      | 7      | 0      | 62.8319|
The coordinates of the points defining rotor surface were calculated with the help of a dedicated MatLab application, on the base of relation (3). Because the rotor profile is periodic, it was determined and drawn only for a length equal to its helical surface pitch. The value of $\beta$ angle was discretized between 0 and $2\pi$, with the increment of $\pi/20$, while the value of $\varphi$, in the same interval, with the increment of $\pi/10$. Hereby, the coordinates of 861 points from rotor surface were found, some of them sampled in Table 1. The shape of the corresponding rotor surface is depicted in figure 7.

![Figure 7. Profile of the pump helical rotor.](image)

The flat-tool profile in the plane section corresponding to $\varphi = 0$ was then determined, with the help of another dedicated MatLab application, on the base of relations (7) and (9), by discretizing the values of $\beta$ angle in 101 equidistant points between 0 and $2\pi$. Some of the points belonging to flat-tool profile are sampled in Table 2, while the profile resulted by joining these points is depicted in figure 8.

| Crt. no. | $\xi$ [mm] | $\eta$ [mm] | Crt. no. | $\xi$ [mm] | $\eta$ [mm] | Crt. no. | $\xi$ [mm] | $\eta$ [mm] |
|---------|------------|------------|---------|------------|------------|---------|------------|------------|
| 1       | 6          | -40.8407   | 46      | 0.2457     | -3.1706    | 91      | 5.7362     | 31.6670    |
| 2       | 5.9975     | -39.9224   | 47      | 0.1583     | -2.5282    | 92      | 5.7890     | 32.5825    |
| 3       | 5.9901     | -39.0041   | 48      | 0.0895     | -1.8912    | 93      | 5.8352     | 33.4986    |
| 4       | 5.9777     | -38.0859   | 49      | 0.0399     | -1.2585    | 94      | 5.8752     | 34.4153    |
| 5       | 5.9602     | -37.1679   | 50      | 0.0100     | -0.6285    | 95      | 5.9092     | 35.3325    |
| 6       | 5.9374     | -36.2501   | 51      | 0          | 0          | 96      | 5.9374     | 36.2501    |
| 7       | 5.9092     | -35.3325   | 52      | 0.0100     | 0.6285     | 97      | 5.9602     | 37.1679    |
| 8       | 5.8752     | -34.4153   | 53      | 0.0399     | 1.2585     | 98      | 5.9777     | 38.0859    |
| 9       | 5.8352     | -33.4986   | 54      | 0.0896     | 1.8912     | 99      | 5.9901     | 39.0041    |
| 10      | 5.7890     | -32.5825   | 55      | 0.1583     | 2.5282     | 100     | 5.9975     | 39.9224    |
|         |            |            |         |            |            |         |            |            |
|         |            |            |         |            |            |         |            |            |
|         |            |            |         |            |            |         |            |            |

*Table 2. Calculated coordinates of the points defining flat-tool profile (excerpt).*
Figure 8. The flat-tool profile in the plane section corresponding to $\varphi = 0$.

The solid model of the flat-tool (depicted in figure 9) has been generated in Catia graphical environment after calculating with relation (10) the value of angle $\alpha$, which, in the addressed case is of 52.4314 degrees.

Figure 9. The flat-tool 3-D model.

The flat-tool is provided with an engagement surface, having plane shape and inclined with 30° relative to feed direction (depicted in grey, figure 9). According to [6, 7], its length, measured into feed direction is equal to $\pi R_{rp}$, which means 31.4 mm, while the active surface length is equal to $6\pi R_{rp}$ (188.4 m), measured into the same direction.

5. Conclusion
The paper presents an algorithm that can be used in order to profile the flat-tool used for manufacturing the helical rotor of progressing cavity pumps by cold forming. In this purpose, the geometrical model of the rotor has been built, at first. Then the equations of the surfaces family described by the rotor in its motion relative to the tool reference system have been found. By associating to these the enveloping condition, written on the base of the “Minimum distance” method, the flat-tool active profile has been obtained. The profiling algorithm feasibility was tested by running a numerical application, with the help of MatLab soft. The results are showing that the profiling algorithm is fast, robust and precise, which recommend its application in solving actual profiling problems industrial practice.
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