Gravity in the quantum lab

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\textbf{ABSTRACT}

At the beginning of the previous century, Newtonian mechanics was advanced by two new revolutionary theories, Quantum Mechanics (QM) and General Relativity (GR). Both theories have transformed our view of physical phenomena, with QM accurately predicting the results of experiments taking place at small length scales, and GR correctly describing observations at larger length scales. However, despite the impressive predictive power of each theory in their respective regimes, their unification still remains unresolved. Theories and proposals for their unification exist but we are lacking experimental guidance towards the true unifying theory. Probing GR at small length scales where quantum effects become relevant is particularly problematic but recently there has been a growing interest in probing the opposite regime, QM at large scales where relativistic effects are important. This is principally because experimental techniques in quantum physics have developed rapidly in recent years with the promise of quantum technologies. Here we review recent advances in experimental and theoretical work on quantum experiments that will be able to probe relativistic effects of gravity on quantum properties. In particular, we emphasise the importance of using the framework of Quantum Field Theory in Curved Spacetime (QFTCS) in describing these experiments. For example, recent theoretical work using QFTCS has illustrated that these quantum experiments could also be used to enhance measurements of gravitational effects, such as Gravitational Waves (GWs). Verification of such enhancements, as well as other QFTCS predictions in quantum experiments, would provide the first direct validation of this limiting case of quantum gravity.
1. Introduction

We all have an intuitive notion of what time and space are. In our experience, time flows and we see objects occupying a place in space. The physics of these scales of space and time that we experience are described by Newton’s laws. The fundamental workings of amazing technologies, such as computers and mobile phones, are all based on these classical laws. However, at different space and time scales, reality seems very different and is not in harmony with our experience.

At small scales, Quantum Mechanics (QM) dominates where objects appear to no longer deterministically travel along exact points in space and are instead described by a wave function that evolves in time and only tells us the probability amplitudes for the possible results of measurements. The wavefunction obeys the superposition principle, which explains the famous double-slit experiment where it can appear as if a particle has travelled through both slits at the same time and then interfered with itself. However, things can get even stranger for multipartite systems: such systems can be entangled so that they are described by a single wavefunction that is not a product of the wavefunctions of the individual parts. This means that the parts are not independent of one another, irrespective of the distance between them. This requires the rejection of either the principle of local realism or non-superluminal signalling, and it is the former that is chosen in conventional QM. However, although superluminal transmissions of classical information is not possible, entanglement does allow for correlations beyond those allowed by classical theory. This is the principle reason for the development of quantum technology, such as quantum computers, that promise superior performance over classical counterparts for certain tasks. QM has been tested to incredible precision but there is still the question of when and why physical systems appear to lose their quantum properties and start behaving classically, such as no superpositions of everyday objects. Many ideas have been proposed. For example, it has been suggested that QM breaks down at some scale due to an objective collapse of the wavefunction through a non-linear stochastic modification of the Schrödinger equation such that superpositions of macroscopic systems are untenable (see e.g. [3]).
Let us now discuss physics at large length scales. In GR, space and time form part of a more general structure that is four-dimensional and is called spacetime. Contrary to our experience, where space and time are absolute notions, spacetime is relative. Einstein taught us that the notion of spacetime could help us understand gravity as the curvature of spacetime produced by the presence of energy. However, although the theory has been verified to incredible accuracy within certain regimes, we are still lacking experiments that help us understand General Relativity (GR) at small lengths or very large energies where quantum effects become relevant. An alternative approach is to study quantum effects at large scales [4]. For example, one could study whether entanglement is affected by the curvature of spacetime (for a review see e.g. [5]). Indeed, quantum experimental techniques have been developing rapidly over the last few decades, including the control of quantum systems for which Haroche and Wineland were awarded the 2012 Nobel Prize in Physics, and are now even starting to approach regimes where general relativistic effects become important. For example, quantum states are currently being sent between Earth and satellite stations where GR already needs to be accounted for in classical systems, such as the Global Positioning System (GPS), to determine time and positions accurately [6–8]. As well as space-based quantum experiments, general relativistic effects are also seen in high precision quantum experiments such as those that are able to observe time dilation at distances as small as a few cms where previous experiments assumed spacetime to be flat [9].

However, we currently know very little about how gravity and motion might affect quantum properties, such as quantum coherence and entanglement. In order to predict the results of quantum experiments at relativistic regimes we require a consistent theoretical framework. Fortunately, we do have a theory that enables the study of the overlap of these theories at low energies. This is Quantum Field Theory in Curved Spacetime (QFTCS) [10,134–137] and describes the behaviour of quantum fields in a classical general relativistic spacetime (see Section 3 and Appendix 1 for a short introduction to QFTCS). Predictions of the theory include Hawking radiation [11,12] and particle creation by an expanding universe [13,14] but such predictions still await direct experimental demonstration. Indeed, so far only a flat, but not Minkowski, spacetime effect has been observed, the Dynamical Casimir Effect (DCE) [15,16]. Due to obvious difficulties in demonstrating Hawking radiation and particle creation by an expanding Universe directly, several groups have been working on analogue demonstrations of these effects. Here researchers find ways of reproducing the basic features of the theory in systems such as BECs, water waves, non-linear optical waveguides and superconducting circuits. These experiments cannot strictly falsify QFTCS since they are based on analogue models, instead they are probing the mathematics of the theory and can only find potential clues to the breakdown of the physical theory.
However, new proposals using QFTCS have identified real spacetime effects on quantum fields that could, in principle, be within experimental reach (see e.g. [17–19]). As well as enabling a number of experiments in the overlap of quantum theory and GR, developments in this regime will ultimately also lead to technologies that are compatible with spacetime while exploiting both quantum and relativistic effects. These new relativistic quantum devices are being actively investigated in the research field of relativistic quantum information and metrology. In this field, research groups try to develop quantum technologies that are compatible with relativity, and some ideas for using relativistic effects to improve quantum technologies, such as quantum measurement technologies [17], have surfaced. For example, recent results have suggested that the phononic field of a Bose–Einstein Condensate (BEC) is particularly well-suited to measuring spacetime effects [17,18,20]. This recent observation enables the possibility of developing microscopic devices capable of probing local spacetime effects. For instance, a microscopic Gravitational Wave (GW) detector has been proposed in which GWs can be detected from how they modify the quantum state of phonons in BECs. GWs are spacetime distortions produced by certain accelerated energy distributions, such as spiralling black holes, which, after almost one hundred years since they were first predicted, were finally observed for the first time in 2015 by the Laser Interferometer Gravitational-Wave Observatory (LIGO) in a milestone moment in physics [21]. GWs promise to provide deeper insights into our Universe by opening up a new window with which to observe it. The proposed microscopic GW detector using phonons of BECs would enable the detection of higher frequencies than those observable by LIGO and, due to its physical and economical practicality, allow for multiple GW detectors to be used by various experimental groups around the world. This would mean improved validation of the signal through combined measurements, as well as superior positioning of the source of the signal.

In this review we analyse recent advances in experimental and theoretical work on quantum experiments that will be able to test relativistic effects of gravity on quantum properties. In particular, we look at how QFTCS has been used in describing such experiments and what open problems remain with this approach. While this is not an exhaustive review of theoretical and experimental works related to relativistic effects on quantum properties, we hope that it provides an overview of the current status of this exciting field, and that it may inspire new research.

The review is organized as follows. In Section 2 we review quantum experiments that are approaching regimes in which relativistic gravitational effects become relevant. In general, these experiments have been designed and described using non-relativistic QM. However, in Section 3 we discuss the motivation for moving to a QFTCS framework to describe such experiments, and how this could also lead to new possibilities in technology as well as answering fundamental questions relevant to quantum gravity. The theoretical work that has illustrated
the need for using QFTCS is reviewed in more detail in Sections 4 and 5. Specifically, in Section 4 we review recent works in QFTCS that have shown how the performance of quantum information protocols can be affected by relativistic effects, requiring corrections to the predicted outcomes of the experiments. The measurement of such effects would represent a direct experimental confirmation of QFTCS and, in the same section, we additionally review proposals that have been purposely designed to illustrate these effects using current technology. Section 5 then reviews recent proposals based on QFTCS that demonstrate how a new type of technology could be developed that exploits relativistic effects, in contrast to current quantum technologies that would see the effects as a hindrance to their design. As well as providing greater performance than their non-relativistic counterparts, the implementation of such devices would also provide an observation of QFTCS effects and offer further insight into the interplay of quantum physics and GR. Finally, in Section 6, we conclude this review and discuss future prospects.

2. Quantum experiments approaching relativistic regimes

In this section we review quantum experiments that are approaching regimes where quantum properties will be modified by relativistic effects of gravity. Generically, these quantum experiments are approaching these regimes because they involve large distances and implement high precision techniques. Studying relativistic effects, however, has not been the principal motivation behind the development of most of these quantum experiments. Instead the experiments are primarily designed to advance quantum technology and its applications, such as the quantum internet, navigation, geophysics and GW astronomy. Otherwise they have been designed to advance our understanding of fundamental physics, such as modifications to quantum theory at large scales.

2.1. Long-range quantum communication experiments

A primary goal of quantum technology is to develop a global quantum communications network and, ultimately, a quantum internet, which would provide an exponential speed-up in distributed computation and be impenetrable to hackers. To achieve this goal it is likely that quantum systems will need to operate over large distances such as that of the Earth to satellites. Over these large distances it is clear that the Earth’s gravitational field is not constant. In fact these are length scales where the relativistic nature of the Earth’s gravitational field already needs to be taken into account in established technologies such as GPS signalling. Therefore, one would expect that relativistic effects would have to be considered in long range quantum experiments. In Section 4, we review theoretical works which demonstrate that these effects could be crucial to the workings of such experimental setups.

An important step in the realisation of a global quantum communications network was made in 2011 when the group led by Anton Zeilinger distributed
entanglement across a 144 km free-space link between Canary islands La Palma and Tenerife [22,23]. This set the record for the distance achieved for quantum entanglement and teleportation, and provided the benchmark for an efficient quantum repeater, which could be placed at the heart of a global quantum-communication network. In collaboration with the Chinese Academy of Sciences (CAS), the group is now planning to demonstrate satellite-based quantum teleportation, which is anticipated to take place later this year.

Quantum transmission is also being investigated between the Earth and a satellite, where clearly the influence of the gravitational field has to be taken into account. In particular, in 2015 a single photon exchange was performed from a medium Earth orbit satellite to the ground station at the Matera Laser Ranging Observatory [6,7]. In this set-up, a weak laser beam was transmitted to a satellite, which back reflected single photons. The set-up was therefore simulating a single photon source on a satellite suitable for quantum cryptography experiments. The repetition rate of the laser pulses was 100 MHz and the satellite was at more than 7000 km of slant distance. As well as providing a further step towards a global communication network, this experiment also opens up the potential for fundamental tests of QM with moving frames [7]. Furthermore, with an upgrade of the detector on the ground, the group claims that it would be possible to achieve quantum communication for up to 23 000 km [7]. Most importantly, in 2015 the group also demonstrated interference at the single photon level along satellite-ground channels [8]. Here the relative movement between the detector and source introduces an additional phase, and a gravitational phase shift of about 2 mrad was detected in accordance with [24].

Many other groups are working on quantum communication between Earth and satellite links. For example, the Institute for Quantum Optics and Quantum Information (IQOQI) in Austria is planning on performing quantum optics experiments, such as a Bell-type experiment with an entangled pair of photons, using an optical ground station (OGS) and the International Space Station (ISS) [25]; the quantum information processing group (QIV) at the Max Planck Institute for the Science of Light (MPL) is investigating quantum communication experiments between the Teide Observatory on Tenerife and the Alphasat I-XL satellite at around a separation of 36 000 km [26]; the Centre for Quantum Technologies (CQT) group at the National University of Singapore, together with the Satellite Research Centre at Singapore’s Nanyang Technological University, is investigating a Small Photon-Entangling Quantum System (SPEQS) that will be set into orbit on a type of small satellite known as a CubeSat and be used as a testbed for technology for future quantum communication networks [27–29]; the National Institute of Information and Communications Technology (NICIT) in Japan is studying a microsatellite mission called SOCRATES with a goal to experiment with Quantum Key Distribution (QKD) techniques using a Small Optical Transponder (SOTA) on board a small satellite and an optical ground station located at NICT [30]; the Institute for Quantum Computing (IQC) at the
University of Waterloo, in collaboration with industry partners, have proposed a microsatellite mission Quantum Encryption and Science Satellite (QEYSSat) to demonstrate the generation of encryption keys through the creation of quantum links between ground and space, and to conduct fundamental science investigations of long-distance quantum entanglement [31]; and the Chinese Academy of Sciences (CAS) is investing heavily in projects on long-distance satellite and ground quantum communications [32], for example, see [33].

2.2. Quantum clocks

The previous section reviewed quantum experiments that operate over large enough distances such that effects from the gravitational field of the Earth start to play a role. However, with increasing precision of an experiment, the length scales over which one must consider general relativistic effects will significantly reduce. This was dramatically demonstrated in the optical atomic clock experiment performed by the group of David Wineland where the time dilation due to the Earth’s gravitational field was observed over a relative change of height of just 33 cm [9]. The experiment was achieved by comparing the frequencies of two Al\(^+\) clocks with frequency uncertainties of \(10^{-17} - 10^{-18}\) at two sample height differences. However, the most accurate time keepers today are optical lattice clocks, with a \(^{87}\)Sr optical lattice clock showing a total uncertainty of \(2.1 \times 10^{-18}\) in fractional frequency units [34]. Repeating a similar experiment with this precision would correspond to a measurable gravitational time dilation for a height change of just 2 cm [34].

In addition to time dilation due to the Earth’s gravitational field, the aluminium-based atomic clock was also used in a separate experiment to simulate the twin paradox and to observe time dilation for relative speeds of less than \(10 \text{ ms}^{-1}\). For this experiment the two ion clocks were placed in different labs, spaced 75 m apart and connected via a stabilized optical fibre. One atom, the one corresponding to the travelling twin, was then set into an oscillating motion around its equilibrium position. Time dilation for this motion leads to a fractional frequency shift for the moving clock, which was found to be in agreement with that expected from relativity.

It is remarkable that general relativistic effects can be seen at such small length scales where one would naturally expect spacetime to appear completely flat, illustrating the power and precision of current technology. Furthermore, these length scales are expected to reduce in the near-term with future major developments such as quantum clocks and implementations in space. Here by quantum clocks we mean clocks that utilise quantum theory in the ‘second revolutionary’ sense [35]. That is, clocks whose fundamental principles are based on quantum not classical concepts rather than clocks that rely on an understanding of quantum theory in a similar way to how quantum theory is used to understand the working of transistors in classical computers. This doesn’t require the clock to just utilize entanglement, instead any quantum property
such as quantum coherence and squeezing can be utilized. Such quantum clocks are able to operate close to the Heisenberg limit [36], the ultimate fundamental bound imposed by QM, enabling a performance enhancement over non-quantum clocks. By implementing quantum or atomic clocks in space-based operations, the performance would further be enhanced since noise would be significantly reduced, potentially allowing for new applications in fundamental physics, geophysics, astronomy and navigation.

The results from the above experiments can so far be explained by modelling the clocks as classical point-like, time-keeping systems in a relativistic gravitational field. However, in Section 4 it is argued that, as the precision continues to improve, and as quantum correlations such as entanglement begin to be utilized, relativistic effects will start to influence the quantum properties of quantum clocks such that the results of experiments cannot be solely explained using this semiclassical approach. As well as modifying the precision of quantum clocks, this would also enable such clocks to be used as probes to study relativistic effects on quantum properties. In principle, the most appropriate and accurate theory for describing these quantum clock experiments that operate in relativistic regimes is QFTCS rather than non-relativistic QM since time is absolute in the Schrödinger equation, which is a notion that is incompatible with the concept of time dilation.

2.3. Atom interferometers

In the last few decades quantum technology, specifically atom-interferometry, has transformed the field of gravitational metrology. In particular, atom interferometers have found major applications in precise measurements of gravitational acceleration [37–39], gravity gradients [40,41] and as gyroscopes [42,43]. These experiments all utilise the wave nature of atoms by sending them through an interferometer and measuring the difference in phase induced by different gravitational potentials, an effect that was first demonstrated with neutrons in the Colella–Overhauser–Werner (COW) experiment in 1975 [44]. For example, in a trend-setting experiment by Tino (LENS) [45], the change in the gravity gradient of the gravitational field induced by test masses arranged around the experimental setup was measured using three atomic interferometers in parallel. While this experiment enables highly accurate measurements of the gravitational field, this is still different from measuring purely relativistic effects since it can also be fully explained using a Newtonian framework. The experiment used cold $^{87}$Rb atoms and relied on the combination of three vertical Mach–Zehnder interferometers implemented via Raman beams. Three clouds of cold atoms with a temperature of $\sim 4 \mu K$ were prepared in a superposition of the $F = 1$ and $F = 2$ states using counter-propagating Raman beams. The three clouds were then launched to different heights such that each trajectory corresponded to an interferometer with a different length. They obtained a gravity curvature of $\approx 1.4 \times 10^{-5} \text{ s}^{-2} \text{ m}^{-1}$ for the arranged test masses, which are well-characterized
tungsten weights with a total mass of about 519 kg, and this agreed with a Monte-Carlo simulation to an accuracy of $10^{-3}$.

Another example is the large scale Matter-Wave laser Interferometer Gravitation Antenna (MIGA), which consists of several spatially separated atom interferometers horizontally aligned and interrogated by a common laser field inside a 200-meter-long optical cavity. This detector is currently being built in a low-noise underground laboratory in France [46,47] and is designed to provide measurements of sub Hertz variations of the gravitational strain tensor. These light-pulse atom interferometers have demonstrated great potential as highly accurate quantum sensors, and recently a representation-free description of such devices was achieved that should allow future studies to consider arbitrary trajectories and varying orientations of the interferometer setup in non-inertial frames of reference [48].

Note that these atom interferometer experiments are testing for the effects of gravity on quantum properties of the system, namely quantum coherence. To date only effects that are fully accountable using Newtonian gravity have been observed. However, atom interferometry is now starting to reach a precision such that it could, in principle, be used to test GR [49–52]. Measurable effects of GR could include non-linear effects; velocity dependent forces; the Lense–Thirring effect; and GWs, the latter of which will be discussed in the following section.

In [50,51], atom interferometry tests of GR were motivated by comparing theoretically several GR and non-relativistic effects to determine if the former could be distinguishable from the latter. Light-pulse atom interferometry was promoted to a relativistic setting with both the atoms and the light treated relativistically and all coordinate dependencies removed. In particular, a semiclassical description was used where the free propagation of the atoms and the light was treated classically such that, in analogy with non-relativistic calculations, the propagation phase is proportional to the classical (but now GR) action, whereas the atom–light interaction is interpreted using non-relativistic QM but described in a covariant manner. The atom interferometry was assumed to be placed in a Schwarzschild metric created by the Earth with a post-Newtonian expansion of the metric utilized since the Earth’s gravitational field is weak. This post-Newtonian expansion was parameterised such that any effects beyond GR could be investigated. Relativistic effects that were studied include non-linear terms, gravity from the atom’s kinetic energy, and the falling of light. The non-linear and velocity dependent effects were found to be measurable with an effective strength of $10^{-15} \text{g}$, and it was argued that such small accelerations could still be observable by implementing particular set-ups such that these relativistic effects can be isolated from the total phase shift. For example, magnetic shielding, a rotation servo to null the Earth’s rotation rate, strategically placed masses and multiple atom interferometers in different configurations could be employed. Furthermore, with advancements such as using entangled states and increasing the length of atom interferometers, it was argued that it would be possible to
improve the precision by several orders of magnitude and exceed that of present astrophysical tests of GWs. In [51], other metrics were also assumed, such as those created by a rotating planet and the expansion of the Universe, to determine if additional effects could be measurable (e.g. the Lense–Thirring effect, which was also discussed in [49]).

Using atom interferometry to test GR would, in principle, demonstrate relativistic effects on quantum properties since they utilise quantum coherence. However, atom interferometers could also be used to illustrate how GR affects quantum properties in a rather different setting: in [24] an atom interferometer experiment was proposed where time dilation induced by different gravitational potentials in the two arms of the experiment should lead to a loss in visibility of the interference pattern for the two paths. What makes this experiment unlike standard atom interferometer set-ups exploring gravity is that an intrinsic time-evolving degree of freedom of the matter-wave is used as a clock [24,53], making time a ‘which path’ witness. There must, therefore, be a loss in the contrast of quantum interference due to Bohr’s complementarity principle, leading to an interesting interplay between spatial and temporal degrees of freedom in the interference process. A proof-of-principle analogue experiment was carried out for this type of proposal last year [54]. This experiment consisted of a $^{87}$Rb BEC where the $m_F = 1$ and $m_F = 2$ sublevels of the $F = 2$ hyperfine state were used for the clock. This clock ticks too slowly to observe time dilation and instead a vertical magnetic-field gradient is used to simulate a gravitational field by driving the clocks out of phase. As anticipated, when the clocks are made to tick at different rates, there is a loss in the visibility of the interference pattern.

To test GR, however, accuracy and stability of the clocks will have to be greatly improved [55]. Once this is achieved it would allow for experiments able to explore aspects of the interplay of GR and QM, such as the differing roles of time in the two theories, and if gravity has any role in the transference from the quantum to the classical world [56].

Note that all of these proposals use a framework of non-relativistic QM evolving via the Schrödinger equation but with time, space, and any external gravitational fields displaying properties of GR. For example, an atom would be described as a quantum system that evolves via the Schrödinger equation (and not as an excitation of a quantum field) but with the temporal variable that enters into the Schrödinger equation undergoing time dilation due to an external non-uniform gravitational field. This fundamentally differs to the proposals outlined in Sections 4 and 5 which use the framework of QFTCS and consider extended rather than point-like systems.

Although tests that can distinguish GR from Newtonian gravity have so far not been carried out with atom interferometers, these devices have been used to test the Equivalence Principle (EP) [57–60]. These have been laboratory-based investigations, but space missions with significantly improved precision have also been proposed. For example, the STE-QUEST satellite mission,$^4$ was
designed to test for the universality of free fall of matter waves by comparing the trajectory of two Bose–Einstein condensates of $^{85}$Rb and $^{87}$Rb [61]; and the QTEST (Quantum Test of the Equivalence principle and Space Time) mission has been designed for the International Space Station (ISS) and will use two rubidium isotope gases with a precision that is two orders of magnitude superior to the current limit on EP violations, and six orders of magnitude better than similar quantum experiments demonstrated in laboratories [62].

### 2.4. Gravitational wave detectors using quantum systems

GWs were finally discovered by LIGO in September 2015 after decades of experimental work carried out by more than a thousand scientists and engineers [21]. The GWs were detected in the interference pattern created by a laser interferometer that used a Michelson interferometer with Gires–Tournois etalon arms of 4 km length. The device is currently sensitive enough to be able to detect a change in the distance between the solar system and the nearest star to just the thickness of a human hair, corresponding to a sensitivity of $10^{-23}$ Hz$^{-1/2}$. However, plans are currently under way to upgrade the device to further improve the precision. This will be made possible by, for example, exploiting quantum properties such as squeezing the laser light [63,64]. In this case a quantum system would be used to probe purely relativistic effects with breathtaking accuracy.

Apart from LIGO there are many other schemes designed to measure GWs. Typically these either employ large optical interferometers similar to LIGO, or follow the pioneering experiments of Weber by utilising large metal (Weber) bars [65]. In the past few years there have also been proposals to use atom interferometers for the detection of GWs. For example, very recently the group of P. Bouyer has suggested using a chain of atom interferometer based gradiometers for the detection of GWs with frequencies between 0.3 and 3 Hz [66]. This particular regime has so far been inaccessible to earth-based GW detectors due to the influence of fluctuations in the terrestrial gravitational field, the so-called Newtonian noise (NN). The suggested experiment overcomes this noise by utilising an array of atom interferometers in an appropriate configuration such that the noise can be rejected and the GW signal can be extracted by averaging over several realisations of the NN. The separation chosen between the individual atom interferometers determines the Newtonian noise rejection efficiency. For a baseline length of 16.3 km, 80 gradiometers, and separations of $\delta = 200$ m, they predict an optimum sensitivity of $3 \times 10^{-23}$ Hz$^{-1/2}$ for a frequency of 2 Hz.

There have also been proposals to use atom interferometers in space for detecting GWs [67,68]. For example, the proposal in [68] positions two light-pulse atom interferometers in separated spacecrafts with the relative acceleration measured using a Mach–Zehnder type configuration. The interferometers are operated with local lasers, which are phase-stabilized to a third laser beam linking the two spacecrafts. With separations of two spacecrafts on the order of $10^6$ m, a passing GW would then affect the laser beam linking the two spacecrafts to a
measurable level by creating a relative phase-shift between the two light pulse atom-interferometers. This set-up would reach a sensitivity of $10^{-20} \text{ Hz}^{-1/2}$ at low frequencies (mHz).

In addition to atom interferometers, superconducting microwave cavities have also been investigated for the detection of GWs. For example, in [69] a Microwave Apparatus for Gravitational Waves Observation (MAGO) was proposed that is based on the concept that a GW could induce an energy transfer between two levels of an electromagnetic resonator [69], where the two levels are obtained in this case by coupling two high frequency spherical superconducting cavities. Such a set-up was suggested to be able to achieve a strain sensitivity of $10^{-23} \text{ Hz}^{-1/2}$ for GW frequencies of $10^3 - 10^4 \text{ Hz}$ if it could be cooled to mK temperatures, which does not seem unreasonable considering that much larger Weber bar devices have now been cooled to such temperatures [70]. The detector would be tunable in resonant frequency and bandwidth, although the latter must be traded with the maximum sensitivity that could be achieved. For example, a maximum sensitivity of $1.6 \times 10^{-21} \text{ Hz}^{-1/2}$ at 4kHz and bandwidth of only 1 Hz was estimated for a potential prototype operating at $T = 0.05 \text{ K}$, but it is claimed that the set-up could be modified to allow for a much more reasonable bandwidth of 350 Hz and maximum sensitivity of $6 \times 10^{-21} \text{ Hz}^{-1/2}$ at 2 kHz. Due to the small size ($\approx 0.5 \text{ m}$) and cheapness of such detectors, and the fact that the sensitivity is independent of frequency within a broad frequency range ($10^3-10^4 \text{ Hz}$), it might also be possible to create ‘xylophone’ configurations of many short broadband detectors operating at different resonant frequencies (see e.g. [71]). Xylophone configurations have also been suggested for Weber bar systems, but these are larger and generally more expensive devices.

### 2.5. Massive quantum systems

The previous sections reviewed quantum experiments that involve macroscopic distances such that relativistic effects from the Earth’s gravitational field on quantum properties should be detectable. For example, in Section 2.1 we discussed proposals to distribute entanglement between Earth and satellite based stations and, in Section 2.2, we reviewed how quantum clocks are reaching a precision such that relativistic effects could, in principle, be detected over macroscopic distances of the order of centimeters. With further developments, these experiments will effectively be able to probe low-energy consequences of quantum gravity where matter is quantized but the gravitational field in which they move is described by GR. This regime is expected to be well-described by QTFCS and so, if observed effects are not in agreement with this theory’s predictions, then this would challenge our current perceptions of gravity at the quantum scale. So far we’ve only been able to test how Newtonian gravity affects quantum systems, and this has been in agreement with our expectations [38,44].

This approach to investigating quantum gravity in the lab is in contrast to that where one would attempt to probe microscopic (Planck length) effects of
gravity by, for example, sending elementary quantum particles to Planck scale energies. Instead, rather than taking gravity to microscopic regimes where the quantum world dominates, the experiments reviewed in the previous sections are bringing quantum systems to a macroscopic regime where gravity dominates. In these experiments, quantum systems are brought to length scales where the effects from Earth’s gravitational field are enhanced.\textsuperscript{5} However, another way to enhance gravitational effects is to increase the rest mass of the quantum system. One could then, in principle, measure the gravitational field generated by a quantum system in the lab. In particular, this would enable an experiment to measure the gravitational field that is generated by a quantum system in a spatial superposition. From a quantum gravity point of view this should create a superposition of gravitational fields so that a nearby quantum system would be put into a superposition of motions \textsuperscript{[72]}. Measuring this system would, therefore, inform us about a quantum theory of gravity. In particular, it could rule out theories in which gravity is predicted to be fundamentally classical and not quantum. Unfortunately, performing this experiment is extremely challenging since, on the one hand, the gravitational field of small masses is difficult to detect and, on the other hand, the larger the mass of the system the more difficult it is to prepare in a spatial superposition. However, there has been significant experimental progress on both sides of this problem in recent years, particularly with increasing the mass of systems in a superposition of states.

The current mass record for an object placed in a spatial quantum superposition over distances comparable to its size is 10,000 AMU, which was demonstrated in macromolecule interferometry \textsuperscript{[73,74]}. There has been considerable interest in recent years in whether the use of levitated quantum nano-mechanical oscillators could significantly improve this mass record \textsuperscript{[75–78]}. Quantum objects with masses as large as $10^{11}$ and $10^{13}$ AMU have been prepared but so far the delocalisations achieved have been much smaller than their size \textsuperscript{[79–82]}. However, proposals exist to significantly extend the delocalisation of such systems. For example, in \textsuperscript{[78]} it is argued that a micron-sized superconducting sphere of mass $10^{13}$ AMU could be prepared in a spatial superposition of the order of half a micrometer. A principle aim of all these experiments investigating massive quantum systems is to determine if QM can be taken to macroscopic scales or whether it breaks down at some point due to a fundamental objective modification that results in a non-unitary evolution of the system and an effective collapse of the wavefunction. In fact it has been suggested that such a process could be required for gravity to be reconciled with QM \textsuperscript{[83–86]}, with motivation coming from, for instance, the non-linearity of GR or the potential granularity of spacetime. In particular, QFTCS with the semiclassical Einstein equations, and its non-relativistic counterpart the Schrödinger-Newton equation, have inspired gravitationally induced collapse mechanisms \textsuperscript{[86–89]}. More quantum gravity-like scenarios have also been suggested, such as predictions that the wavefunction cannot be sustained in a superposition if the gravitational self-
energy of the difference between the states is greater than the Planck energy [86],
or if gravitational fluctuations from granularity of spacetime or the cosmic micro-
wave background may be responsible [90]. Furthermore, low-energy QM in the
presence of the Earth’s gravitational field, but with no additional extensions, has
been considered, with an inherent decoherence process originating from time
dilation [56].

It is possible that micro-mechanical devices could also help with the other
side of the experimental challenge: measuring the gravitational field of very small
masses [91,92]. In particular, in [92] a proof-of-principle experiment was out-
lined where a micro-mechanical device is resonantly driven by the gravitational
field generated by a nearby oscillating source mass of just a few grams. This source
mass would be around three orders of magnitude smaller than what has been
measured so far. However, although this would represent substantial progress,
there is still some way to go before the mass scale begins to close in on what can be
put into a superposition. Therefore, a top-down approach has been taken and the
initial objective of this mechanical setup is to measure Newton’s constant G. This
constant currently has the greatest uncertainty of all the fundamental constants
with different groups obtaining values that are outside each other’s error bars. A
revolutionary new way of measuring G may, therefore, provide information on
why the measurements differ so wildly and enable better characterisation of the
systematic errors in these measurements.

Although the primary objective of this experimental approach is to measure
effects that can be accounted for by, at first classical, and eventually quantum,
Newtonian gravity, the incredible precision of these setups, and their expected
rapid progress, potentially opens up the possibility to measure relativistic effects
of gravity. So far GR has only been tested with astronomical source masses, such
as the Earth and far-away stars and galaxies, whose motion we have no control
over. However, laboratory based experiments would provide completely new
tests of GR since they would offer control over the motion and properties of the
source mass. By further using quantum systems as test masses, one might then
be able to measure relativistic effects on certain quantum properties in a highly
controlled setting.

2.6. Quantum experiments probing the dynamical Casimir effect

If the boundary conditions, dispersion law or background of a quantum field
are quickly varied in time such that the system is unable to adiabatically follow
the instantaneous ground state, then correlated pairs of excitations are created
out of the vacuum state. This is the DCE [15,16], which is a clear relativistic
effect and is typically described in terms of the production of photons from
the electromagnetic vacuum between two mirrors when one of the mirrors
undergoes a rapid acceleration. This effect has received numerous theoretical
studies since its incarnation but it was only recently tested experimentally. This
is principally because, in its typical formulation, the rate of photon production
is only non-negligible when the acceleration is extremely large, which makes the use of massive mirrors particularly challenging [46,93]. However, the DCE is not just restricted to a set-up of moving mirrors, and this has led to a number of alternative experimental proposals.

In 2011, the DCE was observed for the first time by modulating the electrical properties of a cavity [94]. The system consisted of a coplanar waveguide (CPW) terminated by a superconducting quantum interference device (SQUID), and the boundary conditions were rapidly changed by modulating the inductance of the SQUID. The changing inductance can be described as a change in the electrical length of the transmission line and provides the same time-dependent boundary condition as the idealized moving mirror [16,95]. The effective boundary can be changed rapidly enough in this case such that photon production from the vacuum is experimentally detectable.

In the same year, the DCE was also demonstrated using a Josephson metamaterial consisting of an array of 250 SQUIDs embedded in a microwave cavity [96,97]. In this set-up the background in which the field propagates was quickly changed by periodically changing the index of refraction, resulting in the generation of quantum-correlated photons from the vacuum. This experiment and that of [94] have demonstrated the potential power of utilising relativistic effects in quantum information tasks, which is discussed further in Section 5. However, in order to use these effects for quantum computing, more complex states need to be generated [98]. Recently, a step towards this has been performed by generating coherence as well as squeezing correlations in tripartite states by double parametric pumping of a superconducting microwave cavity in the ground state [98]. Furthermore, in [99] continuous variable Gaussian cluster states were considered to be generated by simulating relativistic motion in superconducting circuits. Proposals for simulating the DCE in other systems have also considered. In particular, an acoustic analogue of the DCE was investigated in a BEC but the temperature was too high to observe quantum effects [100].

As well as paving the way for quantum information tasks based on the DCE and similar processes, these experiments have also demonstrated the potential of SQUIDs and BECs for simulating effects of curved spacetime on quantum properties since the equivalence principle links accelerations and gravity. Proposals for experiments that simulate QFTCS effects in this way are discussed further in Section 4.

3. Quantum field theory in curved spacetime

In this section we briefly review QFTCS and introduce the motivating factors behind using this framework to analyse experiments that are approaching relativistic regimes, such as those reviewed in Section 2.
3.1. Background

Quantum Field Theory (QFT) is currently our best attempt at merging special relativity and quantum physics into a coherent framework, and QFTCS represents the first step in extending this highly successful theory to curved spacetime [101]. In QFTCS, matter obeys QFT but the spacetime in which the quantum field of matter propagates is classical, in the sense that it is not affected by it. That is, spacetime is taken to obey the rules of GR but with its curvature pre-assumed and not influenced by quantum matter, whereas matter obeys QFT but is influenced by the curvature of the classical spacetime. This theory is expected to offer an effective description of the interplay of quantum physics and GR at low-energies i.e. below the Planck scale, and is therefore a limiting case of quantum gravity. Predictions of this theory, such as the Hawking effect [11,12] and particle creation from an expanding Universe [13,14], are among the most dazzling and challenging of modern physics, and in the past few decades an overwhelming amount of research has been dedicated to understanding how to demonstrate these predictions. This has principally included astrophysical and cosmological studies but there has recently been a growing interest in complementing these studies with laboratory based setups. As yet there have been no direct measurements of QFTCS but quantum experiments operating at relativistic gravity regimes offer hope that such tests could be realized in the near future. In Appendix 1, we provide a very brief mathematical introduction to this theory for the interested reader.

3.2. Motivation

The quantum experiments that were reviewed in Section 2, with the exception of those in Section 2.6, are all based on non-relativistic QM. That is, their dynamics are described by the Schrödinger equation, which is inherently non-relativistic. However, as these experiments approach relativistic regimes, it will be appropriate to describe these systems in terms of QFT and, as relativistic gravity becomes relevant, in terms of QFTCS. Experimental signatures of QFTCS will then enable us to learn about the effects of gravity and motion on quantum resources, such as entanglement, at low-energies. From this we can also formulate predictions about how the performance and precision of future technologies will be modified by relativistic effects. Such effects will need to be corrected for or new methods must be found to minimize them. In recent theoretical studies, corrections due to relativistic effects have been analysed using QFTCS for quantum technological systems that involve long-range quantum communication protocols, quantum teleportation and quantum clocks. Section 4 reviews these recent theoretical studies.

3.3. QFTCS in experiments and technology

Observing influences of QFTCS effects on quantum technologies would provide the first measurements of QFTCS and would thus give credence to predicted ef-
ffects such as Hawking radiation. As well as providing corrections to existing non-
relativistic protocol designs, our improved understanding of these effects could 
also open up the possibility for developing new technologies. Examples of such
relativistic quantum technologies have been developed by applying Quantum
Information Theory (QIT) techniques to QFTCS. Proposed technological devices 
include an accelerometer; a device that can measure gravitational properties of 
the Earth; and a GW detector. Realisation of these devices would represent the 
first time that the theory of GR has been used in the fundamental function of a 
technology, thus opening up a new era in technology. Section 5 is dedicated to 
reviewing these proposed relativistic quantum devices.

4. Relativistic effects of gravity on quantum properties

In this section we review recent studies that have utilized a QFTCS framework 
to show that the performance of a range of existing quantum technologies 
will be modified by relativistic effects of gravity. Such quantum technologies 
include long-distance quantum communication protocols that use and do not 
use entanglement, as well as quantum clocks. These are all reviewed in Sections
4.1–4.3.

4.1. Gravity and motion affect entanglement

In [102], a quantum information experiment was theorized that involves the 
entanglement of two scalar quantum fields. Each scalar field is placed in a cavity 
and the full system is initially prepared in the maximally entangled Bell state
$|\psi\rangle = (|0\rangle|0\rangle + |1\rangle\_k|1\rangle\_k)/\sqrt{2}$ where $|0\rangle$ denotes the vacuum and $|1\rangle\_k$ denotes 
the one-particle state of mode $k$. After the experiment is prepared, one of the 
cavities undergoes a non-uniform acceleration for a certain period of time, and 
this period of non-uniform acceleration was found to cause the entanglement 
between the cavities to degrade. The origin of this loss in entanglement is a pure 
QFTCS effect, in fact the same mechanism as particle generation in the dynamical 
Casimir and Unruh-Hawking radiation effects.

This theoretical experiment illustrates how quantum information protocols 
that involve entanglement, such as quantum teleportation, can be affected by 
non-uniform accelerations and, from the equivalence principle, by changing 
gravitational fields. Scenarios of this type are relevant to the types of experiments 
reviewed in Section 2.1 where quantum communication and teleportation pro-
tocols take place over large distances where the gravitational field of the Earth 
is non-constant, and to experiments proposed to measure gravitational effects 
using entangled systems, which we discussed in Section 2.3. Although presently 
unobservable in these quantum technology experiments, with increasing exper-
imental precision and distances, this degradation in entanglement will become 
relevant. Furthermore, these quantum experiments have not primarily been 
designed to look for such effects and so it is possible that, with experiments that
Figure 1. Experiment proposed in [20]. Two BECs inside separate satellites are entangled while both are in the same circular low Earth orbit. One of them then undergoes an acceleration for a finite time in order to change to a different circular orbit, which translates in a change of gravitational field felt by the satellite during this process. The entanglement between the BECs is found to be degraded to an observable level. Source: Figure taken from [20].

are purposely designed, degradation in entanglement could be observed for non-uniform accelerations or gravitational fields. Such an experiment was proposed in [20] where it was shown that, by entangling the phononic modes of two BECs that are placed in two separate satellites, the entanglement would be degraded to an observable degree if one of the satellites subsequently moved to a new orbit. In fact a degradation in entanglement as large as 20% was predicted. An illustration of the experiment is presented in Figure 1 and the degradation in entanglement for different orbits is illustrated in Figure 2. The reason that this effect is predicted to be currently observable using BECs and not, for example, with a cavity of light, is that the speed of propagation of phonons of BECs is considerably smaller than that of light [20]. This lower propagation speed enables the acceleration required to induce the effects to be considerably smaller and within experimental regimes since the parameter that defines the strength of the effect is not the acceleration but rather the dimensionless quantity $h = aL/c^2$, $a$ is the proper acceleration at the centre of the cavity, $L$ is the proper length of the cavity, and $c$ is the speed of light for photons or the speed of sound for phonons.

The effects of relativistic motion on quantum teleportation can also be simulated in the laboratory using experiments based on superconducting resonators with tunable boundary conditions [105]. The superconducting experiments re-
Figure 2. The degradation in entanglement as measured by negativity $N$ for the experiment illustrated in Figure 1. $\delta \phi$ is the difference in gravitational field strength between the initial and final orbits. Three different values of acceleration of the satellite are given: $a = 10^{-3}$ ms$^{-2}$ (solid, blue), $a = 2 \times 10^{-3}$ ms$^{-2}$ (red, dashed), $a = 3 \times 10^{-3}$ ms$^{-2}$ (black, dotted). The speed of sound of the BEC is $c_s = 10^{-2}$ ms$^{-1}$ and the length of the cavity $L = 100 \mu m$ such that the fundamental mode has angular frequency $\Omega_1 = 2\pi \times 50$ Hz. The initial squeezing was taken to be $r = 0.5$. Source: Figure taken from [20].

viewed in Section 2.6 proved the relation between a single SQUID and a relativistically moving mirror. To simulate a relativistically moving cavity, one instead has to use two SQUIDS [105]. The experimental platform implemented in this case consists of arrays of hundreds of SQUIDs, and the refractive index for the microwave photons can be controlled externally by bias magnetic fields applied to the SQUID loops. Therefore, with appropriate sample design, the effective spacetime metric of the photonic excitations can be modulated either locally (below the wavelength size) or globally. By simulating accelerations of a quantum field cavity, and thus gravitational fields, it was shown that the relativistic effects on a quantum teleportation protocol would be sizeable for realistic experimental parameters.

4.2. Gravity and motion affect quantum communication protocols

Experiments and proposals were reviewed in Section 2.1 that send pulses of light from the Earth to a satellite in a quantum communication protocol that could one day be used for a global quantum communications network. Treating this protocol within a QFTCS framework, it has been shown that the photons, which are never monochromatic in practice and so are modelled as peaked wavepackets, are red-shifted and broadened, losing energy as they travel through the Earth’s non-uniform gravitational field [19]. The satellite will, therefore, receive a wavepacket with a different frequency distribution to the one prepared
and sent from the Earth. This means that, as illustrated in Figure 3, gravity will cause the quantum communication scheme to differ from one performed in flat spacetime, effectively adding additional noise to the scheme.

For protocols that require users to perform only local operations and exchange quantum systems, these effects will be challenging to compensate for since there is not simply a linear shift of frequencies but also a broadening of the wavepacket. In particular, for an Earth-to-space Quantum key distribution (QKD) protocol that relies on entanglement to share a secret key, gravity would induce an additional contribution to its quantum bit error rate (defined as the ratio of exchanged error bits and the total number of shifted key bits) of 0.7% if a photon wavepacket with realistic peak frequency $625 \text{ THz}$ and bandwidth $\sigma = 1 \text{ MHz}$ is sent between the two stations [19] that are 400 km apart. However, if there is an exchange of additional systems, such as local oscillators (LOs), then it may be possible to implement a protocol that is unaffected by the curvature. In this way the LO will be affected by the space-time curvature in the same way as the quantum wavepacket sent between the two stations. The receiving station will then use the LO as a reference beam for matching and detection of the input signal using homodyne detection. This then compensates for the change in the frequency profile so that the key rate of such a scheme will be unaffected by gravity. However, the exchange of additional resources between the two parties can, in principle, have a substantial impact on the complexity and performance of any quantum communication protocols [19].

### 4.3. Gravity and motion affect the precision of quantum clocks

As reviewed in Section 2.2, clocks that are based on optical transitions of ions or neutral atoms in optical lattices are achieving unprecedented levels of precision, and further improvements are expected with the development of quantum clocks. Current designs of atomic and quantum clocks are formulated within the framework of non-relativistic QM, and the measured relativistic effects such as time dilation can be explained by treating the clocks as point-like classical systems. However, according to QFTCS, as clocks that exploit quantum properties, such as entanglement, become more precise then relativistic effects on quantum properties will need to be taken into account. This was illustrated in [106], which considered a quantum field description of a quantum clock where in this case a single mode of a scalar quantum field constrained to a cavity was used to model a clock, with the phase of the mode used as the pointer of the clock. The single mode was considered to be in a highly non-classical state such as a squeezed state or a coherent state with a low number of photons. This fundamental model is both necessarily quantum and relativistic, i.e. all quantities are compatible with Lorentz invariance and the notions of probability. Furthermore, its motion through spacetime can be properly described. For example, for uniform acceleration it was found that the rate of the cavity clock (the frequency) would be modified. This is due to the fact that, unlike previous
Figure 3. A quantum communication protocol performed between the Earth and a satellite. Here Alice, located on Earth, prepares and sends photons to Bob who is located on a satellite. The photon that is created by Alice has certain characteristics, such as peak frequency and bandwidth, that change once the photon is received by Bob since he is at a different gravitational potential to Alice. Source: Figure taken from [19].

studies which assumed the clock to be point-like, in this approach the clock has a length and different points in the cavity experience different proper times. This effect can of course be understood classically. However, for changes in acceleration there is an additional phase shift that therefore affects the precision of the clock. This phase shift is due to mode-mixing and particle creation from changes in acceleration, and is a QFTCS effect.

How these general relativistic effects modify the precision of the clock was analysed by considering the change in the Quantum fisher Information (QFI) for estimating the phase. The QFI provides the ultimate bound on the estimation of the parameter of interest, and is discussed in more detail in Section 5.1. The results are given in Figure 4 for when the single mode is in a squeezed state. It is found, for example, that for values of $h = 0.1$ where $h := aL/c^2$, $a$ is the proper acceleration for an observer in the centre of the cavity, and $L$ is the
Figure 4. Ratio of the transformed and original quantum fisher information for phase estimation of the quantum clock described in Section 4.3 as a function of \( h \). The initial state of the clock is taken to be the single-mode squeezed vacuum with \( N = 1 \) (blue), \( N = 5 \) (red) and \( N = 10 \) (green) photons. Source: Figure taken from [106].

proper length \( L \) of the cavity, the QFI would reduce by 0.7% if the single mode is in a squeezed state with an average of \( N = 10 \) photons. Since a quantum clock is never stationary on Earth due to local effects such as seismic activity, this loss in precision would have to be accounted for in future developments of ultra-precise quantum clocks. However, so far an idealistic version of a quantum clock has been analysed and what is still lacking is how QFTCS effects can alter the precision of more realistic quantum clocks. For example, the measurement readout of time needs to be taken into account, noise needs to be considered, and only Gaussian states have been analysed in these models of quantum clocks. Furthermore, only effects from non-uniformly accelerating quantum clocks have been investigated rather than the movement of clocks in a curved spacetime.

In [106] it was shown that these results could, in principle, be simulated in the laboratory using superconducting quantum circuits. Furthermore, by implementing two clocks with superconducting circuits, one would also be able to simulate the twin paradox scenario with velocities as large as a few percent of the speed of light [107]. Such velocities would allow one to investigate QFTCS effects, and theoretical results indicate that the time dilation in the paradox is altered due to quantum particle creation and for different cavity lengths [107].

5. **Using quantum properties to measure relativistic effects of gravity**

The previous section considered how relativistic effects of gravity can modify current quantum technologies such that their performance is degraded, requiring corrections to be made to their predicted outcomes. In this section we instead look at the potential benefits of the relativistic effects. An immediate possibility is to exploit the observations of relativistic effects on quantum properties to measure aspects of spacetime. This notion has culminated in the creation of a new research field, relativistic quantum metrology, which investigates how
devices based on QFTCS can be used to study the relativistic nature of spacetime and how these new quantum devices can enhance measurements of properties of spacetime.

5.1. **Relativistic quantum metrology**

In quantum metrology, quantum properties are exploited to achieve greater statistical precision than purely classical strategies when determining a parameter of the system that is not an observable [108]. Examples of such parameters include temperature, time, acceleration, and coupling strengths. As with any measurement strategy, we can divide the process into the following phases: input of the initial state, evolution through a channel, and measurement of the output state. The parameter to be estimated typically controls the evolution of the input state and is encoded in the output state. One then looks for the most optimal measurement strategy achievable such that the error in the estimation process is minimised.

In classical physics, the error of a measurement can, in general, scale at most as $1/\sqrt{MN}$ on average, where $N$ is the number of input probes used in each estimation process, and $M$ is the number of times this estimation process is repeated. This scaling is just a consequence of the central limit theorem. However, it has been shown that using quantum states one can achieve what is known as the quantum or Heisenberg scaling $1/\sqrt{MN}$ where $N$ is, for example, the number of entangled input probes used in each estimation process, and $M$ is again the number of times this process is repeated [109]. There is, therefore, an enhancement by a factor of $\sqrt{N}$ in the quantum case, and the overall scaling is a consequence of the Heisenberg uncertainty relation. This improved scaling can become extremely advantageous when the available number of input probes is high, and provides a signature of genuine quantum effects.

Quantum metrology is being investigated in advanced technological proposals aimed at ultra-precise measurements of gravity such as the gravitational potential and the detection of GWs [64,110,111]. These systems have been designed using non-relativistic QM and thus utilise non-relativistic quantum properties. However, recently quantum metrology has been applied to the field of QFTCS to create a new line of research ‘relativistic quantum metrology’ that investigates measuring spacetime quantities such as relative distances, proper times and gravitational field strengths by exploiting relativistic effects on quantum systems [17,112–115]. Excitingly, this has led to the development of novel ‘relativistic’ quantum devices that, in certain cases, appear to improve upon the precision of their non-relativistic counterparts [17,18]. Proposed relativistic quantum devices include an accelerometer [17]; a GW detector [18]; and a device that can measure Schwarzschild properties of the Earth [116,117], which are all reviewed in the following subsections. Other investigations include how two entangled atoms could be used to detect spacetime curvature through the Casimir–Polder interaction [118], how entanglement can enhance the precision of the detection of the
Unruh effect using accelerated probes [118–120], and how the expansion rate of the universe in the Robertson-Walker metric for Dirac fields can be estimated [121].

5.2. Measuring Schwarzschild properties of the Earth

In Section 4.2 we discussed how the Earth’s gravitational field can affect Earth-to-satellite quantum communication protocols requiring corrections to be made to the detection methods in comparison to those for protocols carried out in flat spacetime. In this way these effects could be viewed as a hindrance to carrying out a quantum communication protocol, effectively adding additional noise. However, since the effects depend on parameters of the background curvature, it is possible to utilise them in order to carry out precision measurements of the background spacetime using quantum techniques. Specifically, relativistic quantum metrology techniques could be used to measure, for example, the Schwarzschild radius of the Earth, the angular momentum of the Earth, or the relative distance between the Earth and a satellite [116,117]. This is performed by sending a wavepacket of light from the Earth to a satellite and then comparing the wavepacket’s final state to the state of a wavepacket that has not undergone the trajectory. As discussed in Section 4.2, the background curvature becomes imprinted in the propagating photon or light pulse so that its final state will differ to that expected from propagating in flat spacetime.

The effects of spacetime on the photon can be treated as a channel and one is able to employ a relativistic quantum metrology scheme to estimate the desired parameters. For example, a two-mode squeezed state, which contains quantum correlations, could be sent from the Earth to the satellite and the final state is effectively compared to the initial state to obtain an estimate of the spacetime parameters. A lower bound can then be obtained on the error of the estimation from the number of times the processes is repeated and on the squeezing that can be chosen for the initial state. For peak frequencies of 400 THz, bandwidths of 1 MHz, a repetition rate of $10^{10}$ Hz, an integration time of 1 s, a squeezing of 13.0 dB, and an Earth-satellite distance of $3.6 \times 10^6$ m (which is typical for geostationary satellites), one obtains a relative error of $\Delta r_s/r_s \sim 4.8 \times 10^{-5}$ on the Schwarzschild’s radius $r_s$. This is about four orders of magnitude off the state-of-the-art that is achieved using classical technologies [124]. Therefore, for this type of quantum experiment to outperform the current classical techniques one would require an enhancement on the repetition rate, being able to integrate such amount of measurements over a longer period of time, an enhancement in the squeezing that can be achieved for $10^2$ THz frequencies, or a combination of these. For example, for a repetition rate of $10^{16}$ Hz and an integration time of 1 minute, it may be possible to improve the current state-of-the-art by one order of magnitude. Given the amount of interest in space-based quantum protocols, and the rate of development of the technologies [7], it is conceivable perhaps that experiments of this kind will be feasible within the next decade.
5.3. An accelerometer

In a BEC, phonons propagate on an effective metric created by the condensate [17]. This is utilised in analogue gravity experiments where the condensate is manipulated such that the phonons think they are propagating on a curved background. However, as well as depending on BEC parameters such as condensate density and velocity flows, this effective metric also depends on the real spacetime metric [125,126]. In fact it depends on the real metric in a conformal way such that, if the velocity flows are neglected, the phonons behave like unpolarised photons propagating in a relativistic field but at a highly reduced velocity. In [17] this was utilised to demonstrate that, by oscillating a BEC constrained to an ideal one-dimensional, uniform cavity, phonons will be created in a dynamical Casimir-like process. However, unlike for an electromagnetic cavity, the frequencies of oscillations required to create measurable effects from the dynamical Casimir effect are very small, being that of typical phonon frequencies i.e. 10–1000 Hz.

By creating a squeezed state of the phonons, allowing the phonons to freely propagate for a period of time, and then measuring the final state, one is able to apply relativistic metrology techniques to obtain an estimate on the non-uniform motion of the one-dimensional cavity. This, therefore, enables the design of a BEC accelerometer, and it should be possible to also design a BEC gravimeter using similar principles. Intriguingly, by comparing the accuracy of this relativistic device, calculated from the QFI, to the state-of-the-art, an improvement by two orders of magnitude is predicted, demonstrating the potential in exploiting relativistic effects on quantum properties. This improvement in performance is essentially coming from the enhancement to the dynamical Casimir effect for accelerations of interest due to the reduced velocity of phonons when compared to photons.

5.4. A gravitational wave detector

The proposal for the accelerometer reviewed in the previous section used the fact that real spacetime distortions can create phonons of a BEC. From the equivalence principle, it should also be possible to use this in the design of a GW detector. This was illustrated in [18] where it was shown that GWs will resonate with modes of the phononic field. Since phonons can have frequencies of order 10–1000 Hz in BEC experiments, the GWs that could, in principle, be detected would be those with a similar frequency range. Such GWs are predicted to be generated by the merging of binary neutron stars and black holes systems, as well as potentially from continuous sources such as rotating neutron stars or pulsars. The frequency range of 10–1000 Hz overlaps with that of LIGO but, while the strain sensitivity of LIGO is optimal \((10^{-23} \text{ Hz}^{-1/2})\) in the range of 100 Hz and decreases at higher frequencies, the sensitivity of the device proposed in [18] improves at higher frequencies. Furthermore, initial calculations using rel-
ativistic quantum metrology have shown that the strain sensitivity could surpass that of LIGO by orders of magnitude, allowing for the detection of many more GW systems. The detector proposed in [18] would also be considerably smaller, cheaper, and simpler to build than the laser interferometers currently used in GW detection. Among other benefits, this would enable many experimental groups to employ GW detectors, allowing for better determination of true GW signals through near-simultaneous measurements and precise positioning of the source of the signals.

Note that the GW detectors described in Section 2.4 that are also partly based on BECs are designed using a fundamentally different concept to the GW detector described here, and work in a completely separate frequency regime. These are atom interferometers and make use of non-relativistic QM and phase estimation techniques applied to the condensate atoms such that one can, in principle, detect GWs with frequencies between $10^{-5}$ to $10^{-2}$ Hz with strain sensitivities of the order of $10^{-18}$ Hz$^{-1/2}$. In contrast, the GW detector described here uses state discrimination techniques applied to the phononic excitations of the BEC in a QFTCS description, which should allow for strain sensitivities of the order of $10^{-27}$ Hz$^{-1/2}$ [18]. Furthermore, whereas the frequency range of the GWs detected by the atom interferometers correspond to waves generated by small mass binary systems, the GW detector described here could, theoretically, detect GWs in the kHz regime, which would deepen our understanding of neutron stars by gathering the information necessary to describe their equation of state. This information would further our understanding of the origins, evolution and extent of our universe since it would enable cosmologists to compute distances that are key in the study of the cosmological constant and of dark matter.

There is an interesting similarity between this GW detector and that of Weber bars. These are, generically, large metal objects of cylindrical or spherical shape, and were in fact the first type of GW detectors to be built. In the lab frame [128,129] GWs should displace the atoms of the metal object creating elastic oscillations of the material. Therefore, sound waves, and at the quantum level, phonons, will be created in the metal object. If the GW frequency matches one of the modes of the object, such as the fundamental mode, a resonance occurs. The sound waves are then amplified into measurable electrical signals using a transducer. The GW frequencies that cause resonance in these devices can also cause resonance in the BEC detector since the lower speed of sound in the BEC ($\sim 10^{-2} - 10^{-3}$ ms$^{-1}$) compared to those of Weber bars ($\sim 10^{3}$ ms$^{-1}$) is roughly compensated for by the much smaller length (micrometres compared to metres).\(^\text{10}\) It would thus be tempting to argue that the BEC detector is just a Weber bar using a different material. However, there are a few major differences between the two detectors. Most importantly, the BEC detector is a quantum device in that it utilizes quantum correlations. That is, the input state of the phonons contains quantum correlations and quantum metrology techniques are used to estimate the GW strain given the effect the GW has on the quantum
correlations of the phonons. This allows for performances that are, in principle, unachievable in classical systems [18]. A BEC represents a perfect system to utilise such quantum effects since the temperatures can be of order nano-Kelvin and so the thermal population of phonons is negligible. In contrast, the lowest temperatures that have been achieved for Weber bars are of order micro-Kelvin [70]. Further differences between the two devices include the fact that in a BEC the ground state is macroscopically occupied and so the phonons of this system also perceive an effective spacetime generated by the condensate. The phonon resonance is also different in that it is a parametric resonance in [18] in contrast to a direct resonance in Weber bars (although parametric amplification can be used in the measurement procedure). The BEC is also kept inside an electromagnetic cavity, such that, in the lab frame, the boundaries of the BEC can be considered rigid to the GW to a good approximation.

6. Conclusion

Quantum experiments have undergone extraordinary developments in recent years. These experiments are constantly probing and pushing science into new and unknown territories, such as investigating entanglement over terrestrial distances and superpositions over macroscopically sized objects. Excitingly, the constant progress provided by the successful accomplishments of these experiments implies that they are rapidly approaching a regime in fundamental physics that has yet to be explored. This is the regime in which general relativistic effects of gravity will begin to influence quantum properties such as entanglement. The fact that quantum experiments are starting to approach such a regime is most clearly illustrated by the experiments that are attempting to prepare an entangled state between an Earth based station and a satellite for proof-of-principle experiments of a global quantum communications network. Here distances are being used where GR already has to be taken into account in classical technology such as GPS.

In this article we have reviewed the current types of quantum experiments that are beginning to approach this relativistic regime, and have emphasized that the most appropriate theory for describing such experiments will be QFTCS. As reviewed in Section 4, recent work that has applied QFTCS to QIT has shown that relativistic effects of gravity can have a detrimental impact on the performance of quantum devices, requiring corrections in the predicted outcomes of quantum experiments. However, as discussed in Section 5, rather than just having a negative impact, relativistic effects can also be exploited to design quantum devices that can directly measure spacetime effects. One such promising quantum device is a GW detector that is built from a BEC and operates in the high frequency regime where one expects to see binary black hole and neutron star mergers as well as rotating neutron stars and pulsars. Interestingly, the strain sensitivity of this device is expected to be at least comparable to LIGO but
advantages would include the fact that it would be smaller, cheaper and simpler to build, enabling many experimental groups to implement such detectors. This would then allow for pinpoint accuracy in the positioning of the sources, enabling a high performance GW telescope. As reviewed in Section 5.1, this GW detector is just one example of how QFTCS can be utilized to develop new microscopic quantum devices capable of probing spacetime and the overlap of quantum physics and GR. In particular, these devices would be capable of observing real QFTCS effects for the first time, in contrast to analogue gravity experiments that search for only analogue effects of QFTCS, such as acoustic Hawking radiation [130]. They would also offer a controlled setting and repeatability since they are laboratory based, in contrast to astrophysical and cosmology searches for signatures of the overlap of quantum physics and GR, such as observing primordial GWs in the cosmic microwave background [131–133]. Any violation of QFTCS seen in these experiments would offer a tantalising signature of the need for a radically new approach to quantum gravity.

In summary, we expect to soon be embarking upon a new revolution in physics where we will observe the interplay of the quantum world and GR for the first time. This interplay would involve classical GR affecting quantum properties in a laboratory setting. The effects must be predictions of a limiting case of quantum gravity, and QFTCS is currently our best attempt at this. Observing predictions of QFTCS could, therefore, provide guidance for theories of quantum gravity and the microscopic structure of spacetime.

Notes

1. For example, a computer relies on classical information theory and stores information in binary bits so that, even though an understanding beyond Newtonian physics is required to manipulate certain components of the technology that are used to carry out the classical information tasks, the computer is fundamentally based on classical concepts.

2. The de Broglie-Bohm pilot wave theory is an interpretation of QM that chooses superluminal signalling but issues include extending the theory naturally to a quantum field theory and finding natural initial conditions [1,2].

3. To determine whether relativistic effects are important one must compare a relevant combination of parameters with the speed of light $c$. Such combinations include $v^2/c^2$ where $v$ is the characteristic speed of the system of interest; $aL/c^2$ where $a$ is the characteristic acceleration of the system, and $L$ is the characteristic length of the system; and $L^2/T^2c^2$ where $T$ is the characteristic time of the system.

4. Although not ultimately selected by the European Space Agency (ESA)

5. Even the quantum clock experiments of Section 2.2 are working at macroscopic scales but the length scales (cms) are much smaller than those of the experiments in Section 2.1 due to their precision.

6. A levitation version of this experiment using two cold superconducting spheres moving along parallel magnetic waveguides has also been considered in [78].

7. For uniform accelerations see e.g. [103,104].

8. This Gbit exchange regime is expected to be implemented in future QKD protocols [122,123].
9. See [127] for an example of a BEC constrained to a uniform potential of a box trap.

10. For an approximately one-dimensional BEC or Weber bar, the mode frequencies are given by $\omega_n = n\pi c_s/L$ where $L$ is the length of the object.

11. Namely, every contracted index is a short notion for a summation. Explicitly, $A_\mu B^\mu := \sum_{\nu=0}^{\nu=3} g_{\mu \nu} A_\mu B^\nu$. This can be easily generalised to quantities, or tensors, with more indices.

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Appendix 1. A brief introduction to the theory of quantum fields in curved spacetime

Here we provide a brief introduction to the basic aspects of QFTCS. This theory is many decades old and has been extensively studied. By no means will we be exhaustive here, and we refer the interested reader to the standard literature [10,134–137].

The basic element of QFTCS is the quantum field, an operator-valued function defined on a manifold, called spacetime. The quantum field can be bosonic, with integer spin-statistics, or fermionic, with odd spin-statistics. For this review we focus only on bosonic fields $\hat{\Phi}$. Given a $3 + 1$ spacetime with coordinates $x^\mu = (x^0, x^1, \ldots, x^3)$, the field takes ‘values’ $\Phi(x^\mu)$ for each spacetime point.

The theory of QFTCS relies on the following important assumptions:

(i) To a very good approximation, gravity can be treated as classical for the calculations of interest,
(ii) Matter fields are quantum objects that propagate and interact ‘on a manifold’,
(iii) The manifold is endowed by a metric $g$ that is a particular solution to the Einstein field equations in GR with a specific background energy-mass distribution from a macroscopic body, such as the Earth. This background energy-mass distribution is independent of the matter fields.

The fundamental gravitational quantity is the metric $g$, a symmetric and non-degenerate tensor with components $g_{\mu\nu}$. The metric defines lengths between neighbouring points in curved spacetime through the infinitesimal line element $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$. Here we assume Einstein’s summation convention. The inverse metric $g^{-1}$ has components $g^{\mu\nu}$ and is defined by $g_{\mu\nu} g^{\nu\rho} = \delta^\rho_\mu$, where $\delta^\rho_\mu$ is the Kronecker delta.

Flat spacetime is characterised by $g_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(-1,1,1,1)$, and we recall that for each point $P$ in the manifold there always exist local coordinates $y^\rho$ where $g_{\mu\nu}(y^\rho) \equiv \text{diag}(-1,1,1,1)$. This is the expression for the locality principle of GR, which states that locally the spacetime is always flat. We also note that four-vectors $\xi$ with components $\xi^\mu$ can be classified depending on the sign of their length. We say that the vector is (i) timelike, when $\xi^\mu \xi_\mu < 0$; (ii) null, when $\xi^\mu \xi_\mu = 0$; and (iii) spacelike, when $\xi^\mu \xi_\mu > 0$. A timelike vector, for example, is tangent to trajectories followed by (pointlike) physical objects. Such a vector is orthogonal to a surface of points that are causally disconnected, i.e. that cannot exchange information, which is known as a Cauchy-hypersurface. This concept plays an important role in establishing the initial conditions of a problem. We note that, while in flat spacetime the derivatives read $\partial_\mu := \partial/\partial x^\mu$, in curved spacetime the situation is more complicated. One needs to replace all standard derivatives $\partial_\mu$ with covariant derivatives $\nabla_\mu$, whose action of functions, four-vectors and tensors is different. For example, if $f$ is a scalar function, then $\nabla_\mu f = \partial_\mu f$, while if $\xi$ is a four-vector with components $\xi^\rho$, one has $\nabla_\mu \xi^\rho := \partial_\mu \xi^\rho - \Gamma^\rho_\mu_\sigma \xi^\sigma$. Here, $\Gamma^\rho_\mu_\sigma$ are the Christoffel symbols and they are defined in terms of derivatives of the metric and products of the metric elements. Standard expressions can be found in the literature [138–142].

Bosonic fields include spin-0 fields, known as scalar fields; spin-1 fields, such as the electromagnetic field; and spin-2 fields, which describe gravitons. To each corresponding classical field is associated a specific Lagrangian density, which is the generalisation of the standard Lagrangian for classical systems and their degrees of freedom. For example, for a classical scalar field $\Phi$, the Lagrangian $L$ of the field reads $L := \int d^3x L(\Phi, \partial_\mu \Phi)$, while the action $A$ reads $A := \int d^4x L(\Phi, \partial_\mu \Phi)$ where $L(\Phi, \partial_\mu \Phi)$ is the Lagrangian density. The equations of motion for the scalar field can then be obtained by the usual least-action principle.
Table A1. Field equations and Lagrangians for a free and real scalar (spin-0) field $\Phi$, and a source-free electromagnetic (spin-1) field $A_\mu$ [143]. The parameter $m$ is the (bare) mass of the field and $F_{\mu\nu} := \nabla_\mu A_\nu - \nabla_\nu A_\mu$ is the electromagnetic field tensor, with $A_\mu$ the covariant generalisation of the standard vector-potential in classical electrodynamics.

| Spin | Lagrangian $\mathcal{L}$ | Field Equation |
|------|--------------------------|----------------|
| 0    | $\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2} \frac{m^2 c^2}{\hbar^2} \Phi^2$ | $(g^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi - \frac{m^2 c^2}{\hbar^2}) \Phi = 0$ |
| 1    | $\mathcal{L} = -\frac{1}{16} g^{\sigma\rho} g^{\mu\nu} F_{\mu\nu} F_{\sigma\rho}$ | $g^{\sigma\rho} \nabla_\sigma F_{\mu\nu} = 0$ |

This is easily generalised to fields of spin greater than 0 [143]. In Table A1 we present explicit expressions of the Lagrangians, and the corresponding equations of motion, for the two cases of free (real) scalar (spin-0) fields and source-free electromagnetic (spin-1) fields, which we denote by $\Phi$ and $A_\mu$ respectively. We restrict ourselves to these two types of fields since they are the most relevant to us.

There are fundamental differences between the electromagnetic field and scalar field. However, for all purposes, it can be shown that massless (i.e. $m = 0$), uncharged (i.e. $\Phi = \Phi^\dagger$) scalar fields behave, to very good approximation, as single polarisation modes of the electromagnetic field. We can therefore focus our attention principally on the scalar field $\Phi$, with extensions to the full electromagnetic field only necessary and paramount when designing concrete experiments.

The field equation for the massive scalar field is $(g^{\mu\nu} \nabla_\mu \nabla_\nu - \frac{m^2 c^2}{\hbar^2}) \Phi = 0$. The massless case can be obtained by setting $m = 0$. Equivalently, the field equation can be written as

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi) - \frac{m^2 c^2}{\hbar^2} \Phi = 0,$$

where we have defined $g := \text{det}(g_{\mu\nu})$, see [10]. It is common to introduce the operator $\Box := \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu)$ for convenience of notation, which allows us to rewrite (A2) as

$$\Box \Phi = 0.$$ 

Solutions to (A2) are almost never completely analytical. Even in the simplest spacetimes, such as Schwarzschild blackhole spacetime [144], full analytical solutions cannot be obtained. The situation improves when the spacetime is $1 + 1$ dimensional. This occurs due to the conformal invariance of all $1 + 1$ dimensional spacetimes, which allows for simple solutions [10]. Reducing the standard $3 + 1$ dimensional problem to a $1 + 1$ dimensional problem can provide great insight into relevant physics, often sacrificing only quantitative predictions to gain a qualitative understanding of the physics.

### A.1. QFT of scalar fields in curved spacetime

In general, however, the aim is to understand scenarios where the full dimensionality of spacetime cannot be ignored. In this case, it is possible to obtain (partial) analytical results if the spacetime is equipped with (at least) one timelike Killing vector. A Killing vector $\xi$ is defined by the Killing equation $\mathcal{L}_\xi (g) = 0$, where $\mathcal{L}_\xi$ defines the Lie derivative with respect to the vector field $\xi$. Physically, this means that the timelike vector field $\xi$ defines a preferred notion of time, and that the metric components are independent of this time. This, in turn, allows one to define a meaningful (and ‘constant’) notion of frequency and energy, which is crucial for a meaningful definition of particles, as we will see in the following.
It is possible, at least formally, to write the general solution of (A2) as
\[
\Phi = \int d\Sigma [a_k u_k + b_k u_k^*],
\]  
where \( \Sigma \) is a chosen spacelike hypersurface, \( k \) collects all parameters for the Fourier decomposition, typically 3, and \( u_k \) are modes that satisfy \( (\Box - m^2 c^2 / \hbar^2) u_k = 0 \). The coefficients \( a_k \) are Fourier coefficients, which we will soon promote to operators.

There is no a priori way to choose the spacelike hypersurface \( \Sigma \). If we wish to decompose the field \( \Phi \) on a different hypersurface \( \Sigma' \) in the same fashion as done above, we would obtain \( \Phi = \int d\Sigma' [a'_k u'_k + b'_k u'_k^*] \), and there would be no ‘natural’ and intuitive way of relating the modes \( u_k \) to the modes \( u'_k \). This is a basic problem in GR and QFTCS, namely that in a generic spacetime, there is no preferred time or space and that two different choices might be both perfectly viable and physically unrelated. However, if the spacetime has a timelike Killing vector field \( \xi \), the situation changes. In this case we can foliate the whole spacetime with hypersurfaces \( \Sigma \) all orthogonal to \( \xi \) and labelled by a particular value \( \xi \) of the parameter given by \( \xi = \partial_\xi \). This means that \( \xi \) gives us a preferred time coordinate and we can choose an orthonormal set of modes \( u_k \) that satisfies \( i \partial_\xi u_k(\xi, x, y, z) = \omega_k u_k(\xi, x, y, z) \), where \( \omega_k \) is a particular eigenvalue that we can identify with the frequency of the mode \( u_k \), and \( (x, y, z) \) is some appropriate coordinate choice of the foliation \( \Sigma \). The symmetries of the scalar field Lagrangian \( \mathcal{L} \) allow us to introduce the conserved inner product \( \langle,\rangle \), defined by \( \langle u_k, v'_k \rangle := i \int dx\,dy\,dz\, [u_k \partial_\xi v'_k - (\partial_\xi u_k) v'_k] \). We can then decompose our field as
\[
\Phi = \int d^3k \{a_k u_k + b_k u_k^*\},
\]  
where we have chosen the orthonormality conditions \( \langle u_k, u'_k \rangle = \delta^3(k - k') \) and \( \langle u_k, u'_k \rangle = -\delta^3(k - k') \). Here \( \delta^3(k - k') := \delta(k_x - k'_x) \delta(k_y - k'_y) \delta(k_z - k'_z) \) and \( \delta(k - q) \) is the Dirac delta. We note that it is always possible to divide a complete set of orthonormal modes in those with positive and negative norm, and this is one of the most striking differences between standard QM and QFTCS.

At this point, we can proceed with the usual canonical quantisation procedure. We do not provide all of the details but we mention that we promote \( a_k \) and \( b_k \) to bosonic annihilation operators \( \hat{a}_k \) and \( \hat{b}_k \). These must satisfy the canonical commutation algebra \( [\hat{a}_k, \hat{a}_k^\dagger] = [\hat{b}_k, \hat{b}_k^\dagger] = \delta^3(k - k') \), while all other commutators vanish. The operators \( \hat{a}_k \) and \( \hat{b}_k \) define the vacuum state \( |0\rangle \) through \( \hat{a}_k |0\rangle = \hat{b}_k |0\rangle = 0 \). Given the normalisation of the modes, we identify \( |1_k\rangle_a := \hat{a}_k^\dagger |0\rangle \) as a one particle state, and \( |1_k\rangle_b := \hat{b}_k^\dagger |0\rangle \) as a one antiparticle state. We note that, if the field is a real scalar field, \( \Phi = \Phi^\dagger \), then it is possible to show that the field decomposition becomes \( \hat{\Phi} = \int d^3k \{\hat{a}_k u_k + \hat{a}_k^\dagger u_k^*\} \). From now on we specialise to real scalar fields, for a matter of convenience, and we drop the label \( a \) and \( b \) in the particle states.

We have introduced the notion of one particle states \( |1_k\rangle \). We have also noted that these particles are associated to modes that satisfy an eigenvalue equation, namely \( i \partial_\xi u_k = \omega_k u_k \). The physical interpretation is that we are able to define, at any instant of time \( \xi \), a notion of one particle state \( |1_\xi\rangle \) that is equivalent and consistent with the definition of one particle state at any other instant of time \( \xi' \). For example, a state \( |1_\xi\rangle \) that contains one particle of momentum \( k \) at time \( \xi_0 \), will contain the same particle at any later time \( \xi > \xi_0 \). Any observer that measures time using the coordinate \( \xi \) will agree that the state has one particle with momentum \( k \). We now proceed to analyse situations where agreement between different observers is not guaranteed, nor should be expected, in QFTCS.

### A.2. Bogoliubov transformations

We have argued that, in general, the field decompositions are inequivalent and with little, if no, connection between them. However, we have also seen that a Killing vector allows us to define
a meaningful decomposition that maintains its ‘character’ in time. It can occur, and it is not so uncommon, that there exists more than one Killing vector, for example $\xi$ and $\mu$. In this case, we can obtain two decompositions of the field $\Phi$, namely $\Phi = \int d^3k [\hat{a}_k u_k + \hat{a}_k^\dagger u_k^s]$ and $\Phi = \int d^3k [\hat{a}'_k u'_k + \hat{a}'_k^\dagger u'^s_k]$, where $i \partial_\xi u_k(\xi, x, y, z) = \omega_k u_k(\xi, x, y, z)$ and $i \partial_\mu u_k(\mu, x', y', z') = \omega'_k u'_k(\mu, x', y', z')$ respectively. We note that there are two a priori different vacua to be considered. The first is $|0\rangle$, which satisfies $\hat{a}_k |0\rangle = 0$ for all $k$, while the second is $|0'\rangle$, which satisfies $\hat{a}'_k |0'\rangle = 0$ for all $k$. The two Killing vector fields allow us to define two different, although perhaps inequivalent, notions of time and, therefore, of particles.

To understand the relationship between the different notions of particles, we introduce the Bogoliubov transformations [10], defined by the Bogoliubov coefficients

$$\alpha_{kk'} := (u_k, u'_k) \quad \beta_{kk'} := (u_k, u'^s_k),$$

which satisfy the Bogoliubov identities $\int dq (\alpha_{kq} \alpha^*_{k'q} - \beta_{kq} \beta^*_{k'q}) = \delta^3(k-k')$ and $\int dq (\alpha_{kq} \beta^*_{k'q} - \beta_{kq} \alpha^*_{k'q}) = 0$, and must be calculated at a common spacelike hypersurface. We can then expand the modes of one basis on the other as $v_k = \int dq (\alpha_{kq} \hat{a}_q + \beta_{kq} \hat{a}_q^\dagger)$ and analogously for $v'_k$. The inverse transformations allow us to relate the creation and annihilation operators to each other, namely $\hat{a}'_k = \int dq (\alpha'_{kq} \hat{a}_q - \beta_{kq} \hat{a}_q^\dagger)$, and analogously for $\hat{a}'_{k'}$.

These considerations allow us to provide a simple, yet powerful, example of an application of the theory. Let us work in the Heisenberg picture and assume that, at $\xi = 0$, an observer has prepared the field in the vacuum state $|0\rangle$ of the operators $\hat{a}_k$. The field will stay in this vacuum state at later times, and such an observer will not detect any particle. In fact, if such an observer is given a detector that measures a wavepacket $\hat{a}_{k_0} := \int d^3k F(k, k_0) \hat{a}_k$, where $k_0$ is a peak frequency of detection and $\int d^3k |F(k, k_0)|^2 = 1$, then the number count will be $N := \langle 0 | \hat{a}_{k_0} \hat{a}_{k_0} | 0 \rangle = 0$. However, let us assume that a second observer, who measures time through the parameter $\mu$, decides to also perform a measurement and see if he agrees with the first observer. This observer also has a detector that detects the same type of wavepacket $\hat{a}'_{k_0} := \int d^3k F(k, k_0) \hat{a}'_k$. This observer computes the number count

$$N' := \langle 0 | \hat{a}'_{k_0} \hat{a}'_{k_0} | 0 \rangle = \int d^3k d^3k' d^3q F^*(k, k_0) F(k', k_0) \beta_{kq} \beta^*_{k'q},$$

which is the standard result when grey body factors, or wavepackets, are taken into account. When the wavepackets are very peaked, i.e. $F^*(k, k_0) \sim \delta^3(k - k_0)$, this reduces to the well-known formula $N' \sim \int d^3q |\beta_{kq}|^2$, which explicitly indicates that the second observer will not necessarily agree on the particle content of the initial state of the field.

These results are well-known in literature and are at the core of many of the most celebrated predictions of QFTCS, such as black hole evaporation [12], the Unruh effect [145–147], particle creation due to an expanding universe [148] and the dynamical Casimir effect [16, 149]. We conclude by emphasising that the treatment presented here aims at giving the reader a taste of the flavour of the theory. Rigorous and in depth presentations can be found in the standard literature [10, 134–137].