On the outage performance of decode-and-forward based relay ordering in cognitive wireless sensor networks

S. Devipriya1 · J. Martin Leo Manickam1 · X. Anita2

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Abstract
In this paper, the issue of secondary network access in cognitive wireless sensor networks is investigated. An efficient scheme (in terms of both power and spectrum) named decode-and-forward based multi-relay scheme with relay ordering (DF-MRRO) is proposed for secondary network access. More important, this scheme is used to enhance the performance of secondary network. In an effort to assess the performance, the received signal-to interference-plus-noise ratio of both the primary and secondary links are derived, from which new exact closed-form expressions for the outage probability of primary and secondary networks are derived. Moreover, the diversity order is calculated for both the primary and secondary network. It is also shown that full diversity order can be achieved if the condition of power requirement is satisfied. Finally, simulation results obtained for the outage performance of the proposed scheme in primary and secondary networks prove the effectiveness of the proposed scheme. It also demonstrates that a maximum of 7 dB SNR gain can be achieved in the proposed DF-MRRO scheme.

Keywords Cooperative communication · Cognitive wireless sensor network · Decode-and-forward protocol · Maximal ratio combining · Outage probability · Relay ordering · Relay selection

1 Introduction
Cognitive radio has received significant attention because it has been considered as a key technology for solving spectrum utilization and congestion issues [1, 2]. It is a well-known fact that most of the licensed spectrum remains unused in practice at different times. Without interrupting the transmission of primary network, unlicensed secondary users can have access to transmit on the licensed band of primary network. The concept of overlay cognitive networks is utilized in this paper. Overlay cognitive networks allow concurrent primary and secondary transmissions [3].

In addition to improve spectrum utilization, secondary user can also act as a relay for primary link [4]. Adopting the concept of cognitive radio in wireless sensor network (WSN) improves the efficiency of WSN [5]. Significant advantages of cognitive wireless sensor networks (CWSN) are high spectrum utilization, extended lifetime and good network efficiency [6–8]. As a candidate technology for future wireless systems, Cooperative communication is the promising solution to realize spatial diversity, coverage extension and mitigation of channel impairments. Cooperative transmission scheme emerges as an efficient strategy to reduce the hardware complexity of multiple-input-multiple-output (MIMO) systems. Virtual antenna pattern can be designed with the help of a relay. Different relay schemes have been explored in [9–12]. The performance of secondary users in a cognitive network was investigated in [13]. Salameh et al. investigated several performance measures such as interruption probability and mean delay for secondary users. Yuolong Zuo et al. [14] showed that the usage of cooperative relays in cognitive radio systems can appreciably handle the problems in spectrum sensing and secondary transmissions.

1.1 Related works
Incorporating the concept of relay ordering into overlay cognitive radio networks enhance the system performance. A detailed study of cognitive radio network was found in...
[15]. In [16], authors found that the order of relaying has a large impact on the performance of multi-hop network. Zhihang et al. [16] proposed an algorithm for relay ordering which considerably reduce the complexity. Nevertheless, this concept of relay ordering has not yet been extensively investigated for cognitive networks. Some of the relay selection strategies were explored in [17–21].

Diversity analysis was done for relay selection schemes in [18]. It is worth emphasizing that full diversity order can be achieved for two-way wireless relay networks.

Authors in [20] have derived the outage probability, block error rate, diversity order and sum rate of relay selection techniques. Atapattu et al. [20] also provide a guideline about the requirement of overhead and feedback for the implementation of relay selection schemes. Krikidis et al. [21] extensively studied the impact of finite buffer in decode-and-forward cooperative systems. Interestingly, it is known that diversity order is equal to twice the number of relays for large buffer sizes. MinChul et al. [22] showed that the outage performance of cooperative network can be substantially enhanced by combining relay selection and relay ordering. Results in [22] proved that both spectrum and energy efficiency can be significantly increased. Tran et al. [23] found that secondary spectrum access can be greatly improved by Hybrid Decode-and-amplify secondary access (HDASA) protocol. Author derived a tight lower and upper bound of outage probability for HDASA protocol. This study is further extended to a framework where the selected secondary node alone forward the information of primary network [24]. In [24], author proposed a protocol named, Decode-and-forward scheme with relay ordering for secondary spectrum access (DFROSA) which help to minimize the required transmit power. Secondary spectrum access was explored for overlay networks in [23, 24]. Jiaafar et al. [25] is the pioneering work in the field of underlay cognitive radio networks.

In [25], author presented a scheme where the spectrum access to the secondary users can be achieved when primary quality of service (QoS) is not satisfied. Secondary user control method was proposed in cognitive radio networks [26], where a jamming signal was injected to degrade the performance of unauthorized secondary users. With the target of increasing the average sum rate of secondary users, optimal spectrum access was done using Poisson point processes [27]. Dynamic spectrum access was introduced in [28] to improve the spectrum utilization of secondary users. Random and reservation channel schemes were designed for finite primary users and infinite secondary users using Markov models. The concept of CR-NOMA was explored for an underlay networks and its outage probability was studied in [29]. A threshold based sensing transmission framework was developed in [30] to optimize the secondary user access in cognitive radio networks. To reduce the complexity of spectrum sensing, a novel machine learning assisted intelligent cooperative spectrum sensing model has been proposed for heterogeneous 5G systems [31]. A reinforcement learning-based spectrum sensing technique has been developed in [32] and it is also analyzed that this proposed sensing method outperforms cooperative sensing technique. In [33], authors developed a cluster based sensing network for cooperative non-orthogonal multiple access (NOMA) systems.

1.2 Contributions

Most of the previous publications focus only on the problem of resource allocation of primary transmissions. Moreover, the location of primary and secondary users is designed using different random distribution models. But, the significance of relay ordering has not been considered effectively for overlay CWSN. This motivated our work. As a result, we proposed an efficient relay ordering scheme for overlay networks, where the best selected secondary user can assist the primary transmission. Decode-and-Forward relaying technique is utilized in this paper. Our focus is not only at improving the performance of primary transmissions, but rather at providing more opportunities for secondary access.

The major contributions of this paper are summarized as follows:

In this paper, we explicitly model a cooperative scheme for multi-relay cognitive WSN which aid in improving the access for secondary users. The proposed scheme is similar to [23–25] by the fact that it serves the primary network in a certain sense to offer more opportunities for secondary network. However, the secondary network encounter low interference from primary network and it is more likely that relay ordering helps to offer better signal-to-noise ratio (SNR).

- To the best of our knowledge, the outage performance of multi-relay CWSN for relay ordering cooperative scheme has not been investigated yet.
- We derive closed-form expressions for the outage probabilities of primary and secondary networks and diversity order analysis is then performed for the proposed scheme.
- We analyze the outage performance of the proposed cooperative scheme for both primary and secondary networks and the effect of various parameters (distance and number of relays) are investigated on its outage performance.

The rest of the paper is organized as follows. The next section describes the system model and the proposed cooperative scheme. Sections 3 and 4 detail the analysis of outage probability and diversity order. Section 5 presents analytical and Monte Carlo simulation results. Section 6 concludes the paper.
2 System model

Consider a cognitive wireless sensor network (CWSN) in which a primary network comprised of a primary transmitter (PTO), and a primary receiver (PRO) coexist with a secondary network comprised of a set of secondary transmitters and receivers \{ST\textsubscript{j}, SR\textsubscript{j}\} where j = 1, 2, as illustrated in Fig. 1. The centralized control unit (CCU) in the secondary network has the knowledge of the position of ST\textsubscript{j} and SR\textsubscript{j} which can act as relays for primary network. All the nodes are equipped with a single antenna and operates in a half-duplex mode. It is assumed that the channel coefficients over all the channel links are assumed as Rayleigh fading. The channel links are quasi-static, in which the fading coefficients are constant within the coherence time. Moreover, all the channel links are assumed to be independent and non-identically distributed random variables (i.i.d). The noise associated with every channel is modeled as additive white Gaussian noise (AWGN) having zero mean and unit variance.

Assume that the data transmission is performed over three successive time slots. At the first time slot, PTO broadcasts a signal \(x_1\) to PRO. Due to the broadcast nature of the channel, \(x_1\) will also be received by ST\textsubscript{1} and SR\textsubscript{1} (j = 1, 2). At the second time slot, if ST\textsubscript{1} can decode, it combines this signal with its own information signal \(P_{ST1}\) and forwards to PRO which is received by ST\textsubscript{2} and SR\textsubscript{2}. Otherwise, ST\textsubscript{1} remain silent. At the third time slot, if ST\textsubscript{2} can decode \(x_1\), it combines this signal with PST\textsubscript{2} and forwards to PRO. PRO combines all the signals received during three time slots using MRC. Without loss of generality, we assumed that primary transmission (direct path) was considered successful and maximal ratio combining (MRC) is implemented at the primary (or secondary) receiver to combine all the received signal from different links. Meanwhile, we assume that destination have a perfect knowledge of channel state information (CSI) of all the channel links involved in the primary and the secondary networks. To guarantee the primary QoS requirement, interference power at PRO must be kept below a tolerable interference constraint \(Q\). The transmit power of the secondary network are limited by.

\[ P_S \leq \min \left\{ \frac{P_{max}^S}{\left[ \frac{Q}{|a_j|} \right]} \right\} \quad \text{and} \quad P_R \leq \min \left\{ \frac{P_{max}^R}{\left[ \frac{Q}{|b_k|} \right]} \right\}. \]

where \(P_{max}^S\) and \(P_{max}^R\) are the maximum available powers of ST\textsubscript{j} and SR\textsubscript{j} (j = 1, 2) respectively. The inter-relay channel is assumed to be reciprocal. We also assume that the secondary network can able to synchronize itself to the time-slotted primary network transmissions. However, the method of achieving synchronization was beyond the scope of this paper, some of them are explored in [34–37].

2.1 Preliminaries

Let \(h_0, h_{11}, h_{12}, h_{21}, h_{22}, h_{31}, h_{41}, h_{42}\) and \(h_5\) denote the fading coefficients of the links PTO-PRO, PTO-ST\textsubscript{1}, PTO-ST\textsubscript{2}, ST\textsubscript{1}-PRo, ST\textsubscript{2}-PRo, PT\textsubscript{O}-SR\textsubscript{1}, PT\textsubscript{O}-SR\textsubscript{2}, ST\textsubscript{1}-SR\textsubscript{1}, ST\textsubscript{1}-SR\textsubscript{2} and ST\textsubscript{1}-ST\textsubscript{2} respectively, as illustrated in Fig. 1. The variance of the channel \(h_{mn}\) is given as \(\lambda = d_{mn}^{-\beta}\) where \(d_{mn}\) is the distance between nodes m and n and \(\beta\) is the path-loss exponent.

2.2 Proposed DF-MRRO scheme

A transmission frame of the network consists of three equal length time slots. The best relay which has the maximum end-to-end SNR is selected based on the following max-min criterion:

\[ \text{ST}_\text{best} = \arg \max \max \left( \Gamma_{(PT,ST_j)}, \Gamma_{(ST_j,PRO)} \right) \]

where \(\text{best}\) is the index of the selected secondary node based on the channel conditions, \(\Gamma_{(PT,ST_j)}\) is the received signal-to-interference-plus-noise ratio (SINR) signal-to-interference-plus-noise ratio at ST\textsubscript{j} from PTO, and \(\Gamma_{(ST_j,PRO)}\) is the received SINR at PRO from ST\textsubscript{j}.

At the first time slot, the selected relay first decodes \(x_1\) and combines it with its own information signal and then forwards it to PRO at the second time slot, which was received by ST\textsubscript{1} and ST\textsubscript{2}. At the first time slot, the received signal at ST\textsubscript{1} is given by

\[ y_{ST1} = \sqrt{P_i} x_{11} + n_{ST1} \]

where \(P\) is the transmit power of PTO, \(x_{11}\) is the information symbol of PTO and \(n_{ST1}\) is the AWGN at ST\textsubscript{1}. ST\textsubscript{1} decodes \(x_1\) and combines it with its own information signal which is given by

\[ x_{ST1} = \sqrt{\alpha_1} P x_1 + \sqrt{(1-\alpha_1)} P P_{ST1} \]

where \(\alpha_1 P\) and \((1-\alpha_1) P\) are the fractions of the transmit power which is allocated for \(x_1\) and \(P_{ST1}\) respectively at ST\textsubscript{1}. During the second time slot, the received signal at ST\textsubscript{2} is given by

![Fig. 1 System model](image-url)
\[ y_{1,ST_2} = \sqrt{P_x} h_{12} + n_{ST_2} \]
\[ y_{2,ST_2} = x_{ST_1} h_5 + n_{ST_2} \]
\[ y_{2,ST_2} = \left[ \sqrt{x_1 P_x} + \sqrt{(1-x_1) P_{PP,ST_1}} \right] h_5 + n_{ST_2} \]

where \( n_{ST_2} \) is the AWGN at ST_2. ST_2 combines the signal received in first and second time slots using MRC. The combined signal at SR_2 is given by
\[ x_{SR_2} = \sqrt{\alpha_2} P x + \sqrt{(1-\alpha_2) P_{PP,ST_2}} \]

where \( \alpha_2 P \) and \( (1-\alpha_2)P \) are the fractions of the transmit power allocated for \( x_s \) and \( P_{PP,ST_2} \), respectively at ST_2. SR_1 receives two signal from PT_O (first time slot) and ST_1 (second time slot) which can be expressed as follows:
\[ y_{1,SR_1} = \sqrt{P_x} h_{31} + n_{SR_1} \]
\[ y_{2,SR_1} = x_{ST_1} h_{41} + n_{SR_1} \]

where \( n_{SR_1} \) is the AWGN at SR_1. Similarly, SR_2 receives two signals from PT_O (first time slot) and ST_2 (third time slot) which can be expressed as follows:
\[ y_{1,SR_2} = \sqrt{P_x} h_{32} + n_{SR_2} \]
\[ y_{2,SR_2} = x_{ST_2} h_{42} + n_{SR_2} \]
\[ y_{2,SR_2} = \left[ \sqrt{\alpha_2 P_x} + \sqrt{(1-\alpha_2) P_{PP,ST_2}} \right] h_{42} + n_{SR_2} \]

The received SINR at destination node PR_O (from PT_O, ST_1 and ST_2) are given as
\[ y_{1,PPO} = \sqrt{P_x} h_{O2} + n_{PPO} \]
\[ y_{1,PPO} = x_{ST_1} h_{21} + n_{PPO} \]
\[ y_{1,PPO} = \left[ \sqrt{x_1 P_x} + \sqrt{(1-x_1) P_{PP,ST_1}} \right] h_{21} + n_{PPO} \]
\[ y_{2,PPO} = x_{ST_2} h_{22} + n_{PPO} \]
\[ y_{2,PPO} = \left[ \sqrt{x_2 P_x} + \sqrt{(1-x_2) P_{PP,ST_2}} \right] h_{22} + n_{PPO} \]

where \( n_{PPO} \) is the AWGN at PR_O. The received SINR at PR_O for the link ST_1-PR_O (to decode \( x_s \)) is given as
\[ \Gamma_{(ST_1,PPO)} = \frac{x_1 P |h_{21}|^2}{(1-x_1) P |h_{21}|^2 + \sigma^2} \]

The received SINR at PR for the link ST_2-PR_O (to decode \( x_s \)) is given as
\[ \Gamma_{(ST_2,PPO)} = \frac{x_2 P |h_{22}|^2}{(1-x_2) P |h_{22}|^2 + \sigma^2} \]

where
\[ \gamma_{th_1} = \frac{2^M - 1}{2^{R_1} - 1}, \gamma_{th_2} = \frac{2^M - 1}{2^{R_2} - 1} \]

and
\[ \gamma_{th_3} = \frac{2^M - 1}{2^{R_3} - 1} \]

are the thresholds of received SINR and R, R_1 and R_2 represent the target rate of PR_O, ST_1, ST_2 respectively. Therefore, the closed-form expression for outage probability at PR_O is given in Eq. (24).

\[ \Pr(\Gamma_{(ST_1,PPO)} \geq \gamma_{th_1}) = \exp(\theta_1) \]
\[ \Pr(\Gamma_{(ST_2,PPO)} \geq \gamma_{th_2}) = \exp(\theta_2) \]
\[ \Pr(\Gamma_{(ST_3,PPO)} \geq \gamma_{th_3}) = \exp(\theta_3) \]
Similarly, the outage events at ST j (j = 1, 2) occur in three cases, as follows: ST j fails to decode $x_t$ during first phase and ST 2 fails to decode $x_t$ even if ST 1 successfully decode $x_t$ during second phase. It is not necessary that ST 2 act as a relay for ST 1. Hence the outage event for PST 1 is not considered. ST 1 is not in outage only when

$$Pr\left(\Gamma_{(PTO,ST_1)} \geq \gamma_{th_1}\right) = \exp(\theta_4)$$

and ST 2 is not in outage only when

$$Pr\left(\Gamma_{(ST_1,ST_2)} \geq \gamma_{th_2}\right) = \exp(\theta_5)$$

The closed-form expressions for outage probability of ST 1 and ST 2 are given as

$$p_{out}^{ST_1} = 1 - Pr\left(\Gamma_{(PTO,ST_1)} \geq \gamma_{th_1}\right) = 1 - \exp(\theta_4)$$

$$p_{out}^{ST_2} = \left(1 - p_{out}^{ST_1}\right) \left(1 - Pr\left(\Gamma_{(ST_1,ST_2)} \geq \gamma_{th_2}\right)\right) = \left[1 - \exp(\theta_4)\right]\left[1 - \exp(\theta_5)\right]$$

$$p_{out}^{PR} = \left(1 - p_{out}^{ST_1}\right) \left(1 - p_{out}^{ST_2}\right) \left[1 - Pr\left(\Gamma_{(PTO,PR)} \geq \gamma_{th_3}\right) Pr\left(\Gamma_{(ST_1,PR)} \geq \gamma_{th_2}\right)\right]$$

$$p_{out}^{PSR_{out}} = \exp(\theta_6)\left[1 - \left(1 - \exp(\theta_4)\right)\left(1 - \exp(\theta_5)\right)\right]$$

$$p_{out}^{PSR_1} = \left[1 - Pr\left(\Gamma_{(PTO,SR_1)} \geq \gamma_{th_1}\right) Pr\left(\Gamma_{(ST_1,SR_1,PST_1)} \geq \gamma_{th_3}\right)\right]$$

$$p_{out}^{PSR_2} = \left[1 - \exp(\theta_6)\left[1 - \exp(\theta_4)\right]\left(1 - \exp(\theta_5)\right)\right]$$

$$p_{out}^{PSR_3} = \left[1 - \exp(\theta_6)\left[1 - \exp(\theta_4)\right]\left(1 - \exp(\theta_5)\right)\right]$$

where

$$\theta_6 = -\gamma_{th_3}/\lambda_3$$

$$\theta_5 = -\gamma_{th_5}/\lambda_3$$

$$\theta_4 = -\gamma_{th_4}/\lambda_4$$

$$\theta_3 = -\gamma_{th_1}/\lambda_1$$

$$\gamma_{th_2} = \gamma_{th_3} - 1$$

### 3.3 Outage probability at SR 1 (secondary RX pair)

An outage at SR 1 occurs in three cases: SR 1 fails to decode $x_t$ during first phase (from PT O), SR 1 fails to decode $x_t$ during second phase (from ST 1) even if ST 1 successfully decode $x_t$, and SR 1 fails to decode PST 1 (from ST 1). SR 1 is not in outage only when

$$Pr\left(\Gamma_{(PTO,SR_1)} \geq \gamma_{th_1}\right) = \exp(\theta_6)$$

$$Pr\left(\Gamma_{(ST_1,SR_1,PST_1)} \geq \gamma_{th_3}\right) = \exp(\theta_5)$$

$$Pr\left(\Gamma_{(ST_1,SR_1,PST_1)} \geq \gamma_{th_2}\right) = \exp(\theta_4)$$

where $\gamma_{th_k} = \gamma_{th_k} - 1$ is the received SINR at SR 1 and $R_k$ represents the target rate of SR_k. The closed-form expression for outage probability at SR 1 is given in Eq. (25).
Where
\[
\theta_6 = \frac{-\gamma_{th_a} \sigma^2}{P_{\lambda_6}}
\]
\[
\theta_7 = \frac{-\gamma_{th_a} \sigma^2}{(x_1 - (1 - x_1) \gamma_{th_a})P_{\lambda_6}}
\]
\[
\theta_8 = \frac{-\gamma_{th_a} \sigma^2}{(x_1 - (1 - x_1) \gamma_{th_a})P_{\lambda_6}}
\]

Similarly, the outage events at SR_2 occurs in the following three cases: SR_2 fails to decode x_4 during first phase (from PT_0), SR_2 fails to decode x_4 (from ST_1-ST_2 during third phase) even if ST_1 and ST_2 can successfully decode x_4 and SR_2 fails to decode PST_2 (from ST_2) during second phase. SR_2 is not in outage only when
\[
Pr(\Gamma_{(PT_0,SR_2)} \geq \gamma_{th_a}) = \exp(\theta_6)
\]
\[
Pr(\Gamma_{(ST_2,SR_2-PST_2)} \geq \gamma_{th_a}) = \exp(\theta_10)
\]
\[
Pr(\Gamma_{(PT_0,ST_2,SR_2-x_4)} \geq \gamma_{th_a}) = \exp(\theta_11)
\]
\[
Pr(\Gamma_{(PT_0,ST_1,ST_2,SR_2-x_4)} \geq \gamma_{th_a}) = \exp(\theta_12)
\]

The closed-form expression for outage probability at SR_1 is given in Eq. (26).

Where
\[
\theta_9 = -\gamma_{th_a} \sigma^2 / P_{\lambda_9}
\]
\[
\theta_{10} = \frac{-\gamma_{th_a} \sigma^2}{((1 - x_2) - x_2 \gamma_{th_a})P_{\lambda_9}}
\]
\[
\theta_{11} = \frac{-\gamma_{th_a} \sigma^2}{(x_2 - (1 - x_2) \gamma_{th_a})P_{\lambda_9}}
\]
\[
\theta_{12} = \frac{-\gamma_{th_a} \sigma^2}{(x_2 - (1 - x_2) \gamma_{th_a})P_{\lambda_9} + (x_2 - (1 - x_1) \gamma_{th_a})P_{\lambda_9}}
\]

where \(\gamma_{th_a} = 2^{M_a} - 1\) is the received SINR at SR_2 and \(P_{\lambda_9}\) represents the target rate of SR_2.

4 Diversity order analysis

We considered a cooperative relay ordering scheme for CWSN with N secondary transmitter and receiver pairs and derive the diversity order in the following propositions. Assume that there N secondary Tx-Rx pairs. Among N pairs, there are C set of secondary transmitters which can decode primary information signal correctly and D set of secondary transmitters which will decode incorrectly. Note that there are \(C_j\) and \(D_j\) are random variables. Also assume that there are y non-diversity relays which belong to set C and x–y non-diversity relays which belong to set D. Let \(C_j = \{ST_{p_1}, ST_{p_2}, \ldots, ST_{p_n}\}\), \(D_j = \{ST_{q_1}, ST_{q_2}, \ldots, ST_{q_n}\}\)

There are \(2^{i-1}\) possible sets of \(C_j\) and the probability of each set \(C_j\) can be calculated as follows: When the received SNR at STP_1 is greater than the threshold \(\gamma_{th_a}\), decoding will be successful. The probability of this case is given as
\[
Pr(\Gamma_{PT_0,ST_{p_1}} \geq \gamma_{th_a}) = \frac{P_{|h|_1}^2}{\sigma^2}
\]

Therefore, the probability for each set \(C_j\) is calculated as
\[
P(C_j) = \prod_{j=1}^{n} Pr(\Gamma_{PT_0,ST_{p_j}} \geq \gamma_{th_a})
\]

When \(n = 1,\) and
\[
P(C_j) = \exp\left(-\frac{\gamma_{th_a} \sigma^2}{P_{\lambda_1}}\right)
\]

The CDF of the random variable \(\Gamma_{PT_0,ST_{p_j}}\) can be calculated as \(F_{\Gamma_{PT_0,ST_{p_j}}} = 1 - \exp(-\theta_j)\), where \(\theta_j = \frac{\gamma_{th_a} \sigma^2}{P_{\lambda_j}}, j = 1, \ldots, n\). Similarly, the probability of unsuccessful decoding at STP_1 is calculated as follows:where
\[
P(D_j) = \prod_{j=n+1}^{N-x} Pr(\Gamma_{PT_0,ST_{p_j}} < \gamma_{th_a})
\]

The probability for each set \(D_j\) can be generally expressed as follows
\[
P(D_j) = \left[1 - \exp\left(-\frac{\gamma_{th_a} \sigma^2}{P_{\lambda_j}}\right)\right] \prod_{j=n+1}^{N-x} Pr(\Gamma_{PT_0,ST_{p_j}} < \gamma_{th_a})
\]

The total outage probability of the primary network is given as
\[
P_{out\ primary} = \sum_{D} \prod(C_j) P(D_j) \prod_{out\ PT_0}^P
\]

Similarly, the total outage probability at SR_j is given as:
\[
P_{out\ SR_j} = \sum_{C_j} \prod(C_j) P(D_j) \left[1 - \left(P_{out\ ST_j}^{SR_j} \left(P_{out\ ST_j}^{SR_j}\right) \right]\right]
\]

where \(P_{out\ ST_j}^{SR_j}\) and \(P_{out\ ST_j}^{SR_j}\) are the outage probabilities at \(ST_j\) and \(SR_j\) (to decode \(x_s\)) respectively and \(P_{out\ ST_j}^{SR_j}\) is the outage probability that the \(SR_j\) fails to decode \(PST_j\).

\[
P_{out\ ST_j}^{SR_j} = \left[1 - \exp\left(-\frac{\gamma_{th_a} \sigma^2}{P_{\lambda_j}^{PST_j}}\right)\right] \prod_{b=1}^{n} D_{j=SR_j}\]

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Table 1 Different cases (different position of secondary terminals) used in simulation

| Different cases | Location of secondary transmitters and relays |
|-----------------|-----------------------------------------------|
| Case 1          | Equal distance between neighbouring nodes in the link PT0-ST1-ST2-PR0 |
| Case 2          | ST1 and ST2 are randomly distributed between PT0 and PR0 |
| Case 3          | SR1 is closer to PT0 than SR2 |
| Case 4          | SR2 is closer to PT0 than SR1 |
| Case 5          | ST1-SR1 link is shorter than ST2-SR2 link |

Therefore, the total outage probability of the secondary network is given as:

\[ P_{\text{out,Secondary}} = \prod_{j=1}^{N} P_{\text{out,SR}_j} \]

**Proposition 1** Each secondary transmitter node \( ST_j \) assigns a fraction of the transmit power \( P \) to the primary network and its own information signal. If it does not satisfy the condition

\[ \frac{\alpha_j}{1 - \alpha_j} \leq \gamma_{th} \]

then, \( ST_j \) is a non-diversity relay. Among \( N \) selected secondary transmitters, if there are \( x \) non-diversity relays, the achievable diversity gain of the primary network is \( N + 1 - x \). The proof of the above proposition is given in “Appendix 1”.

**Proposition 2** If there are \( N \) secondary transmitter–receiver pairs, the achievable diversity gain of the secondary network is \( N \). (i.e. Diversity order of 1 can be achieved for each \( ST-SR \) link). The proof of the above proposition is given in “Appendix 2”.

### 5 Analytical and simulation results

In this section, we present simulation results to verify our new analytical closed-form expressions for multi-relay cooperative scheme with relay ordering of cognitive wireless sensor networks. All the figures show that the obtained curves are in precise agreement with the Monte Carlo simulations. We assume a path loss exponent \( \beta = 3 \). We also assume that the target rate of primary and secondary network is constant (\( R = R_1 = R_2 = R_3 = R_4 = 1 \text{ bits/s/Hz} \)). Outage performance of primary and secondary network are evaluated for the following five different cases which depict the position of secondary terminals (Table 1).

Figure 3 demonstrates the outage performance of primary network as a function of SNR (dB) for the first two cases. Performance of case 1 is better than case 2 since the hop distance is shorter in case 2. 4 dB SNR gain can be achieved in case 1. Figure 4 shows the outage performance of secondary network for the last three cases when \( \alpha_1 = 0.8 \) and \( \alpha_2 = 0.8 \). The performance of case 3 is better for SR1 since it is more close to PT0 whereas the performance of case 4 is better for SR2 since it is more close to PT0.

Figure 5 compares the outage probability of primary and secondary network. It is shown that the performance of primary network is better than secondary network.

Figure 6 illustrates the impact of number of relays for primary and secondary network. As the number of relays increases, outage probability becomes slightly better for both primary and secondary network. This is expected because the decoding sets of larger number of relays may contain more number of nodes which will support either the primary or the secondary network. SNR gain of 7 dB can be achieved for \( N = 3 \).
Figure 7 demonstrates the influence of power allocation for primary and secondary network. Outage performance is evaluated for three different combinations of $a_1$ and $a_2$. Figure 7 substantiates that the performance of the secondary network which is greatly influenced by the value of $a_1$ and $a_2$. When the value of $a_1$ and $a_2$ increases, the performance of the secondary network improves. Similarly, the performance of the primary network also improves for the increasing value of $a_1$ and $a_2$.

Figure 8a, b describe the outage performance of primary and secondary user for different values of distance between ST1 and ST2. If this distance increases, both ST1 and ST2 will move closer to primary users and there is no additional benefit achieved from these relays. Thus, the outage performance is poor. If both the relays ST1 and ST2 are exactly at the middle of primary users ($d = 0.5$), performance will be improved. It is clear from these figures that when all the primary and secondary users are at almost equal distance, outage performance of all the users can be improved. Better performances of both primary and secondary network can be achieved only when both ST1 and ST2 are at the middle of PT0-PR0 link.

Figure 9a illustrates the throughput of relay based cognitive network. It is inferred from the figure that the achievable throughput of primary and secondary user increases as a function of SNR. Throughput of primary user is higher than secondary user because of its limited transmit power constraint.

Impact of number of relays on throughput has been investigated in Fig. 9b. It is shown that the throughput improves for increasing values of number of relays.
This paper focuses on the multi-relay cooperative scheme with relay ordering for CWSN. This framework is well suited for the enhancement of the secondary network due to its cooperation with the primary network and also for the interference mitigation of the primary network. We have presented the closed-form expressions for outage probability of both primary and secondary network. It is shown that the analytical results agree with the simulation results perfectly. We speculate that the diversity order is upper bounded by $N^{1}$ and lower bounded by $N^{1-p}$, which is well proved by numerical results. Extending the proposed relay scheme for Non-orthogonal multiple access (NOMA) based heterogeneous IOT network can be the subject of future research.
Appendix 1

At high SNR, Eq. (28) can be approximated as

\[ P(C_j) = \prod_{b=1}^{N-n} \lambda_{PTO, ST, y_k} \]

\[ P(D_j) = \begin{cases} \zeta_{1,C}(7)^{-(N-n)}, & m \leq y + 1 \\ \zeta_{2,C}(7)^{-(N-n)(m-x)}, & m > y + 1 \end{cases} \]

\[ P_{out}^{PR} = \zeta_{3,C}(7)^{-(n-y+1)} \]

where

\[ \zeta_{1,C} = \prod_{b=1}^{N-n} \lambda_{PTO, ST, y_k} \]

\[ \zeta_{2,C} = \prod_{b=1}^{N-n} \lambda_{PTO, ST, y_k} \frac{m!}{(m+1)!} \lambda_{PTO, ST, y_k} \]

\[ \zeta_{3,C} = \lambda_{PTO, PR, y_k} \]

The outage probability of primary link for each possible set of C can be approximated as

\[ P(C)P(D)P_{out}^{PR} = \begin{cases} \zeta_{1,C}(7)^{-(N-n+1)}, & m \leq y + 1 \\ \zeta_{1,C}(7)^{-(N-n)(m-y+1)}, & m > y + 1 \end{cases} \]

If all secondary transmitters can decode successfully, i.e. \( C = \{ST_1, ST_2, \ldots, ST_N\} \) and \( D = \{0\} \), the outage probability can be written as

\[ P(C)P(D)P_{out}^{PR} = 1 - \exp\left(-\lambda_{PTO, PR, R} \prod_{j=1}^{N} \exp\left(-\lambda_{ST, PR, R}\right)\right) \]

At high SNR, the above equation can be approximated as

\[ P(C)P(D)P_{out}^{PR} = \zeta_{4,C}(7)^{-(N-x+1)} \]

where

\[ \zeta_{4,C} = \lambda_{PTO, PR, y_k} \prod_{j=x+1}^{N} \lambda_{ST, PR, y_k} \]

Moreover, we have

\((N-n)(m-y) + n - y + 1 \geq N - x + 1\)

Hence, outage probability at high SNR can be given as

\[ P_{out}^{primary} = \sum_{D} P(C_j)P(D_j)P_{out}^{PR} = \zeta(7)^{-(N-x+1)} \]

The diversity order is calculated as follows: Diversity Order

\[ \lim_{\gamma \to \infty} \frac{-\log P_{out}^{primary}}{\log(\gamma)} = \lim_{\gamma \to \infty} \frac{-\log \left(\zeta(7)^{-(N-x+1)}\right)}{\log(\gamma)} = N - x + 1. \]

Appendix 2

At high SNR, Eqs. (28) and (30) can be approximated as

\[ P(C_j) = 1 \]

\[ P(D_j) = \begin{cases} \zeta_{1,C}(7)^{-(N-n)}, & m \leq y + 1 \\ \zeta_{2,C}(7)^{-(N-n)(m-x)}, & m > y + 1 \end{cases} \]

\[ P_{out}^{PR} = \zeta_{3,C}(7)^{-(n-y+1)} \]
\[
P(D_j) = \begin{cases} 
\gamma_1 c_1 (\gamma_1)^{-1}, & m \leq y + 1 \\
\gamma_2 c_2 (\gamma_2)^{-1}, & m > y + 1 
\end{cases} \tag{42}
\]

where

\[
\gamma_1 c_1 = \prod_{b=1}^{j-n-1} \lambda_{PT_0-ST_0} g_k \\
\gamma_2 c_2 = \prod_{g=y+1}^{j-n-1} \lambda_{PT_0-ST_0} g_k \prod_{g=y+1}^{N} \lambda_{ST_0-ST_0} g_k 
\]

The outage probability for each possible set of \( C_j \) is approximated as

\[
P_{\text{out}}^{SR} = \sum_c P(c_j) P(D_j) \left( 1 - \left( 1 - P_{\text{out}}^{SR} \right) \left( 1 - P_{\text{out}c_1}^{SR} \right) \left( 1 - P_{\text{out}c_2}^{SR} \right) \right)
\]

\[
P_{\text{out}}^{SR} = \begin{cases} 
\gamma_1 c_1 \gamma_3 c_3 (\gamma_1)^{-1}, & m \leq y + 1 \\
\gamma_2 c_2 \gamma_4 c_4 (\gamma_2)^{-1}, & m > y + 1 
\end{cases} \tag{43}
\]

where

\[
\gamma_3 c_3 = (\lambda_{PT_0-ST} + \lambda_{PT_0-SR} + \lambda_{ST_0-SR} + \lambda_{ST_0-SR}) g_k \\
\gamma_4 c_4 = \lambda_{ST_0-SR} / (1 - g_k) \gamma_k
\]

If all the secondary transmitters \( ST_1, ST_2, ..., ST_{j-1} \) can decode the primary signal \( x_s \) successfully, the above equation can be approximated as

\[
P_{\text{out}}^{SR} = \gamma_j^{-1} \tag{44}
\]

where \( \gamma_j \) is a constant. The outage probability of secondary system can be calculated as

\[
P_{\text{out secondary}} = \prod_{j=1}^{N} P_{\text{out}}^{SR} \approx (\gamma_j)^{-N} \prod_{j=1}^{N} \gamma_j \tag{45}
\]

From the definition of diversity order, the diversity gain of \( ST_1-SR_1 \) link and secondary system are calculated as 1 and \( N \) respectively.

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**Declarations**

**Conflict of interest** The authors declare that they have no conflict of interest.

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S. Devipriya received the B.E. degree in Electronics and Communication Engineering from Periyar Maniammai University in 2012 and M.E. degree in communication systems from SSN college of Engineering, Chennai, in 2014. She is currently pursuing the Ph.D. degree in Information and Communication Engineering at Anna University, Chennai, India. Since 2014, she has been an Assistant Professor with the department of Electronics and Communication Engineering, St. Joseph’s College of Engineering, Chennai, India. Her research interests include cooperative communication, signal estimation, NOMA techniques, deep learning based resource allocation in wireless UAV communication.

J. Martin Leo Manickam is working as a Professor in the Department Electronics and Communication Engineering at St. Joseph’s College of Engineering, Chennai. He acquired B.E. degree in Electronics and Communication Engineering from Alagappa Chettiar College of Engineering and Technology in 1995. He received M.E. Degree in Optical Communication and Ph.D. degree in the Faculty of Information and communication Engineering from the College of Engineering, Anna University, Chennai. He has over 24 years of experience in teaching and guiding projects for Undergraduate and post graduate students. Under his guidance, eleven
scholars had got Ph.D. degree and 6 research scholars are pursuing their Ph.D. programme. He has to his credit 80 publications in national/international conferences and journals. His areas of interest include Mobile Ad hoc Networks, Wireless Sensor Networks, Cognitive radio and Network Security. He is a fellow in IIEI and IETE.

X. Anita received her B.E degree in computer science and engineering from Madurai Kamaraj University, Madurai, India, in 2003 and her M.E degree in computer science and Engineering from Anna University, Chennai, India, in 2008. She has obtained her Ph.D. from Anna University, Chennai, India in 2015. She is presently working as an Assistant Professor (Sr) in the School of Computer Science and Engineering, VIT Chennai, India. Her research interests include data analytics, image processing, she has 20 publications in national/international journals and conferences.