Abstract. In the case with a large splitting between the squark and slepton masses, the supersymmetric identities which enforce the equality of the gauge and gaugino couplings are violated in the effective theory below the squark mass threshold. We compute the full one-loop (s)quark corrected slepton production cross-sections. We find that the one-loop corrected slepton production cross-sections can depend on the squark mass strongly, up to $9\% \times \log_{10}(M_{\tilde{Q}}/m_{\tilde{\ell}})$. We investigate the squark mass sensitivity of the slepton cross-section measurements at a future linear collider. For sneutrino production accessible at $\sqrt{s} = 500$ GeV there can be sensitivity to squark masses at or larger than 1 TeV.

1. Introduction

Supersymmetry is an attractive possibility beyond the standard model. Because of the relations supersymmetry imposes among the dimensionless couplings, the quadratic divergences in the Higgs sector are cut-off by the superpartner mass scale. The cancellations stabilize the hierarchy between the Planck scale and the weak scale. The minimal supersymmetric standard model (MSSM) is consistent with gauge coupling unification suggested by grand unified theories. Also, it is interesting that one of the hallmarks of supersymmetry, a light Higgs boson ($m_h \lesssim 130$ GeV), is favored by global fits to precision electroweak data.

In this contribution we examine the prospect of testing supersymmetry via a precise measurement of the lepton-slepton-gaugino vertex. The linear collider provides a suitably clean experimental environment. Among the relations which account for the cancellations of quadratic divergences, supersymmetry relates the lepton-slepton-gaugino coupling to the usual gauge coupling.

Although bare (or DR) couplings enjoy the relations imposed by supersymmetry, the effective gauge and gaugino couplings are not equal because supersymmetry is broken. In particular, all non-singlet nondegenerate supermultiplets such as the quark-squark supermultiplets contribute to the splitting. Hence, measurements of the type we consider here not only provide for detailed tests of supersymmetry, but can also elucidate important features of the scale and pattern of supersymmetry breaking.

For example, the (s)quark contribution to the splitting of the U(1) and SU(2) gaugino/gauge (s)lepton couplings grows logarithmically with the squark mass, as

$$\frac{\delta g_Y}{g_Y} \simeq \frac{11g_Y^2}{48\pi^2} \ln \left( \frac{M_{\tilde{Q}}}{m_{\tilde{\ell}}} \right), \quad \frac{\delta g_2}{g_2} \simeq \frac{3g_2^2}{16\pi^2} \ln \left( \frac{M_{\tilde{Q}}}{m_{\tilde{\ell}}} \right).$$

This correction is obtained by evolving the couplings according to the renormalization group equations (RGE’s) of the effective theory below the squark mass threshold. When $M_{\tilde{Q}}/m_{\tilde{\ell}} \simeq 10$ the correction...
In this section we discuss the calculation of the cross-section of $e^+e^- \rightarrow \tilde{\ell}\tilde{\ell}^*$, for (a) $s$-channel and (b) $t$-channel amplitudes.

![Feynman graphs](image)

Figure 1. Feynman graphs of the one-loop quarks-squark corrections to the processes $e^-e^+ \rightarrow \tilde{\ell}\tilde{\ell}^*$, for (a) $s$-channel and (b) $t$-channel amplitudes.

to the SU(2) (U(1)) coupling is about 2% (0.7%). This gives rise to an enhancement of the $t$-channel slepton or gaugino production cross-section of about 8% (2.8%). If large statistics are available and systematic errors can be controlled, we can (assuming the MSSM) constrain the squark mass scale by this measurement.

We restrict our attention to the measurement of the first generation lepton-slepton-gaugino coupling at an $e^-e^+$ linear collider. Much study has been undertaken to determine how accurately we can expect to measure these couplings. We perform a full one-loop calculation of the slepton production cross-section within the MSSM. We include only $(s)quark loops in the calculation, because the correction is enhanced by a color factor and the number of generations. The remaining corrections are small, and if we did include them we expect our conclusions would not change.

In this contribution, we only discuss our calculation only briefly in section 2. We point out that, to a good approximation, the one-loop $t$-channel amplitudes can be rewritten in the same form as the tree-level amplitudes, with the replacement of the tree-level parameters with renormalized effective parameters. Hence we introduce the effective coupling, the effective masses, and the effective mixing matrix. In section 3 we discuss our numerical results, and show how well we can measure the squark loop correction to the coupling, and thereby constrain the squark mass, assuming both slepton and chargino production are possible. We show the statistical significance of the results by combining our knowledge of the superpartner masses and cross-sections. The uncertainty in the slepton mass measurement is quite important in this analysis. In the last section, section 4, we give our conclusions.

2. calculation

In this section we discuss the calculation of the cross-section of $e^+e^- \rightarrow \tilde{\ell}\tilde{\ell}^*$, including one-loop $(s)quark corrections. The full result is explicitly given in Ref.[10]; here we restrict ourselves to outline the general features of the calculation.

The tree level slepton productions proceed through $s$-channel exchange of $Z$ and $\gamma$, and $t$-channel exchange of neutralinos or charginos ($\tilde{\chi}_i$). To evaluate the one-loop amplitude, we treat all the parameters appearing in this tree-level expression as running $\overline{\text{MS}}$ quantities, and add the contributions from the one-loop diagrams (see Fig. 1). Note that the $(s)quark loop corrections do not give rise to external wave-function renormalization.

To avoid a complexities of several gauge interactions, let us consider $e^+e^- \rightarrow \tilde{\nu}_R\tilde{\nu}_R$ production. If the $\sqrt{s} \gg m_Z$, the process approximately proceeds through $s$-channel exchange of $B$ boson which couples to hypercharge, and $t$-channel exchange of $\tilde{B}$, superpartner of $B$ boson. The $s$-channel amplitude below squark mass threshold is very well approximated by effective coupling $g_{Y,\nu}^{\tilde{\nu}_R}(Q = \sqrt{s})$. $g_{Y,\nu}^{\tilde{\nu}_R}(Q)$ in MSSM is related to $g_Y^{\tilde{\nu}_R}$ as $\alpha_Y^{\tilde{\nu}_R} = \alpha_Y^{\overline{\text{MS}}} (1 + \Sigma^\prime(Q) + \Sigma^\prime(Q))$ where $\Sigma^\prime(Q)$ and $\Sigma^\prime(Q)$ is the gauge two point function from quark and squark loops respectively and $Q$ is renormalization scale. The $g_{Y,\nu}^{\tilde{\nu}_R}(Q)$ is equivalent to the $t$-channel coupling $g_{Y,\nu}^{\overline{\text{DR}}}(Q)$, but we do not directly measure the coupling. The $t$-channel amplitude is the sum of tree level contribution and 1-loop contribution shown in Fig1.(b). The difference $S_Y^\nu \equiv (g_{Y,\nu}^{\tilde{\nu}_R} - g_{Y,\nu}^{\overline{\text{DR}}})/g_{Y,\nu}^{\overline{\text{DR}}}$ is the correction to the SUSY relation(Fig.2).
The leading logarithms of the corrections $S^e$ at $Q = m_\tilde{t}$ are exactly those of Eq. (1), showing that the RGE approach of Ref. [11] gives the proper results.

Although both s-channel and t-channel diagrams receive one loop corrections from squark and quark loops, only t-channel amplitude receive physically interesting correction. Notice in our approximation, s-channel amplitude receives oblique correction of the gauge two point function only, which is common to the $e^+e^-$ collision at Z pole. The measurement at Z pole fixes the s-channel amplitude of slepton production, thanks to the gauge symmetry.

In Ref [10], we also show the one-loop corrected t-channel amplitude can be well approximated by a tree-level form. For $e^+e^- \to \tilde{e}_R \tilde{e}_R$, we find

$$M_{RR}^t = 2 \tilde{v}_Y \left[ - \frac{1}{4} \sum_{i=1}^4 g_{e\tilde{e}_R \tilde{b}_i} \frac{N_{i1} N_{i1}(p^2)}{p^2 - m_i^2} P_R \right] u ,$$

where $p$ is t-channel momentum and $g_{e\tilde{e}_R \tilde{b}_i}(p^2)$ is the effective bino coupling defined as

$$g_{e\tilde{e}_R \tilde{b}_i}(p^2) = \tilde{g}_Y(Q) \left( 1 - \frac{1}{2} \Sigma_{11}^L(Q, p^2) \right) ,$$

and $\Sigma_{11}^L(p^2)$ is the bino-bino component of the neutralino two-point function, and $\tilde{g}_Y(Q)$ is the DR coupling. The $N_{ij}$ and $\overline{\eta}_{ij}$ are the effective neutralino mixing matrix and neutralino masses obtained by diagonalizing the effective neutralino mass matrix $\overline{\eta}_{ij}$, $\tilde{g}_i$, $N_{ij}$, and $\overline{\eta}_{ij}$ are physical scale independent quantities to $O(\alpha)$. $\tilde{m}_i(\tilde{m}_i)$ is the pole mass of neutralinos. We also checked numerically the expression Eq. (2) reproduce the full result very well.

3. Numerical results

3.1. $m_{\tilde{Q}}$ Dependence of Various Cross Sections

We next show the numerical dependence of the one-loop corrected cross-sections of $e^-e^+ \to \tilde{e}_i \tilde{e}_j^*$ ($\tilde{e}_i = (\tilde{e}_L, \tilde{e}_R, \tilde{e}_c)$) on the squark mass. We consider the case where the initial electron is completely longitudinally polarized. We therefore treat the following eight modes,

$$e_{\tilde{L}}^- e^+ \to \tilde{e}_L^+ e_L^- , \quad e_{\tilde{R}}^- e^+ \to \tilde{e}_R^+ e_R^- , \quad \tilde{e}_c^+ \tilde{\nu}_c ,$$

and

$$e_{\tilde{L}}^- e^+ \to \tilde{e}_L^+ e_L^- , \quad e_{\tilde{R}}^- e^+ \to \tilde{e}_R e_R^+ , \quad \tilde{e}_c^+ \tilde{\nu}_c .$$

The production involves the t channel exchange of chargino and neutralino, which depends of gaugino mass parameter $M_1, M_2$, higgsino mass parameter $\mu$, and $\tan \beta$. We take the three pole masses ($m_{\chi^0_k}, m_{\chi^+_k}, m_{\chi^-_k}$), and $\tan \beta(M_Z)$ as inputs. We assume $|\mu| \gg M_Z$, in which case $M_{1\text{eff}} \simeq m_{\chi^0_1}, M_{2\text{eff}} \simeq m_{\chi^+_1}$, and $|\mu\text{eff}| \simeq m_{\chi^-_3}$ hold, where $M_{1\text{eff}}, M_{2\text{eff}}$, and $-\mu\text{eff}$ are the (1,1), (2,2), and (3,4) elements of the effective neutralino mass matrix $\overline{\eta}_{ij}$, defined in [11].

We show in Fig. 3 the $M_{\tilde{Q}}$ dependence of the cross-sections for left handed electron beam. Here we normalize the cross-sections to the tree-level values defined in Ref. [10]. The one-loop corrected cross-sections of the modes which have a t-channel contribution are similar to tree-level ones at
handed electron beam, and larger than the other sparticle production cross-section. We focus solely on the \( \tilde{\chi}_s \)fermions \( \tilde{M}_s \)the corrected cross-sections (see Fig. 2). In contrast, the remaining channels, which have only \(-\)well constrained input parameters. For 20 fb\(^{-1}\) contributions, show very little \( \text{CL} = 0.95 \). The measurement of the end point energies determine the masses. Recently, Baer et al. \[4\] performed a MC study for the case that left-handed sfermions are produced and dominantly decay into a gaugino-like \( \chi \). The decay mode \( \tilde{\nu}_e \tilde{\nu}_e^* \rightarrow e^- e^+ \tilde{\chi}_1^\pm \tilde{\chi}_1 \rightarrow e^- e^+ \mu 2 j (\nu_\mu, 2 \tilde{\chi}_1^0) \) is background free and the measured electron endpoint energies allow for a 1% measurement of \( m_{\tilde{\chi}_1^\pm} \) and \( m_\mu \).

The results of Ref. \[4\] encourage us to consider their example point 3. The chosen parameter set corresponds to \( m_{\tilde{\nu}_e} = 207 \) GeV, \( m_{\tilde{\chi}_1^\pm} = 96 \) GeV, \( m_{\tilde{\chi}_1^0} = 45 \) GeV and \( m_{\tilde{\chi}_2^0} = 270 \) GeV, and the lightest chargino and neutralinos are gaugino-like. Their study suggests that we can take \( m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_1^0}, m_{\tilde{\nu}_e} \) as well constrained input parameters. For 20 fb\(^{-1}\) of luminosity, their MC simulations show that at 68\% CL, \( (\delta m_{\tilde{\chi}_1^\pm}, \delta m_{\tilde{\nu}_e}) = (1.5 \text{ GeV}, 2.5 \text{ GeV}) \).

In the following we estimate the statistical significance of the radiative correction to the production cross-section. We focus solely on the \( \tilde{\nu}_e \) production cross-section, because it is larger than 1 pb for a left-handed electron beam, and larger than the other sparticle production cross-sections at \( \sqrt{s} = 500 \) GeV.

We would first like to provide a feel for the sensitivity to the squark mass scale and tan \( \beta \) in the ideal case where we ignore the slepton and gaugino mass uncertainties. In Fig. 4(left) we show the statistical significance of the loop correction by plotting contours of constant cross-section. Here we fix the sneutrino mass and determine \( \mu, M_1 \) and \( M_2 \) by fixing the one-loop corrected masses \( m_{\tilde{\nu}_e}^0, m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0} \), and \( m_{\tilde{\chi}_1^\pm} \). We plot the contours corresponding to the number of standard deviations of the fluctuation of the accepted number of events. The 1-\( \sigma \) fluctuation corresponds to \( \sqrt{N_{\text{input}}} \).
nominal value of the number of events at $M_{\tilde{Q}} = 1000$ GeV and $\tan \beta(M_Z) = 4$. The accepted number of events $N$ is given by

$$N = A \cdot \sigma(e^+e^+ \to \tilde{\nu}_e \tilde{\nu}_e) \times (\text{BR}(\tilde{\nu}_e \to e^{\mp}_{1,2} \chi^{\pm}_1)) \times 100 \text{ fb}^{-1}. \quad (5)$$

Here we took $\text{BR}(\tilde{\nu}_e \to e^{\mp}_{1,2} \chi^{\pm}_1) = 0.6$ and overall acceptance $A = 0.28$. The number of accepted events at our nominal point $N_{\text{input}}$ is about 12800 for $\mu < 0$.

![contours of constant $\sigma(e^+e^+ \to \tilde{\nu}_e \tilde{\nu}_e)$](image)

Figure 4. (left): The constraint on $M_{\tilde{Q}}$ and $\tan \beta$ coming from $\sigma(e^+e^+ \to \tilde{\nu}_e \tilde{\nu}_e \to e^{\mp}_{1,2} \chi^{\pm}_1 \chi^{\mp}_1)$, with $\int d\ell_1 d\ell_2 = 100$ fb$^{-1}$. The central value is taken as $M_{\tilde{Q}} = 1000$ GeV and $\tan \beta(M_Z) = 4$. $m_{\tilde{\chi}_i^0} = 270$ GeV, $\mu < 0$ (right): $\Delta \chi^2_{\text{min}}$ vs. $M_{\tilde{Q}}$ with $\mu < 0$ and fixed $\tan \beta$, $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_2^0}$, but allowing $m_{\tilde{\chi}_1^\pm}$ and $m_{\tilde{\nu}_e}$ to vary freely.

We show the contours for $\mu < 0$ in Fig. 4(left). If $\tan \beta$ is well measured, $M_{\tilde{Q}}$ is constrained to $M_{\tilde{Q}} = 1000^{+270}_{-280}$ GeV at 1-$\sigma$ significance. If instead we assume the constraint $2 < \tan \beta < 8$, the mild $\tan \beta$ dependence yields $700 < M_{\tilde{Q}} < 1900$ GeV. In the case $\mu > 0$, the mixing of chargino gives rise to significant $\tan \beta$ dependence. Increasing the squark mass can be compensated for by decreasing $\tan \beta$, and measuring sneutrino production then determines a region of the $\tilde{M}_{\tilde{Q}}$, $\tan \beta$ plane.

We now turn to the effect of the mass uncertainties. The sneutrino production cross-section depends on the masses $m_{\tilde{\nu}_e}$, $m_{\tilde{\chi}_1^0}$, $m_{\tilde{\chi}_3^0}$ and $m_{\tilde{\chi}_1^\pm}$. Of these masses the cross-section is most sensitive to the sneutrino mass. All of the same chirality scalar production cross-sections suffer from the strong $\beta_i^2$ kinematic dependence. Near threshold this results in an especially large sensitivity to the final-state mass. Although a simple statistical scale-up of the results of Ref. [4] implies a sneutrino mass uncertainty of only 0.3%, this nevertheless leads to a significant degradation in our ability to constrain the squark mass scale. (Systematic errors might be the limiting factor here.)

Note, however, that the measurement of the sneutrino mass in Ref. [4] was obtained by studying a small fraction of the total sneutrino decay modes. The mode they studied amounts to only about 4% of the total sneutrino decays. Using other modes, such as $e^- e^+ 4j(2\chi^{0}_1)$, might reduce the mass error even further. Because the $\tilde{\nu}_e$ production cross-section is significantly larger than the other slepton cross-sections, isolating the various sneutrino signatures is less affected by SUSY backgrounds such as $e^- e^+ \to e^- L \to e^- e^+ 4j(2\chi^{0}_1)$.

Now we show the constraint on the squark mass $M_{\tilde{Q}}$ after taking into account the uncertainty of the masses $\delta m_{\tilde{\nu}_e}$ and $\delta m_{\tilde{\chi}_1^\pm}$ for $\mu < 0$ case. The effect of $\delta m_{\tilde{\chi}_3^0}$ turns out to be negligible for the case, and we assume it is possible to distinguish sign of $\mu$ by measuring heavier ino mass differences at $\sqrt{s} > 2m_{\tilde{\chi}_3^0}$. In Fig. 4(right) we plot $\Delta \chi^2_{\text{min}}$ against $M_{\tilde{Q}}$, where $\Delta \chi^2_{\text{min}}$ is a minimum of $\Delta \chi^2$ with respect to variations in $m_{\tilde{\chi}_1^\pm}$ and $m_{\tilde{\nu}_e}$. The region of $M_{\tilde{Q}}$ where $\sqrt{\Delta \chi^2_{\text{min}}} < 1, 2, ..., 3$ corresponds to 1, 2, ..., $3\sigma$ error of the squark mass when the chargino and sneutrino mass uncertainties are taken into account. The sneutrino mass uncertainty reduces the sensitivity of the production cross-section to $M_{\tilde{Q}}$ considerably, because the effect of increasing $M_{\tilde{Q}}$ can be compensated for by a small increase in $m_{\tilde{\nu}_e}$. On the other hand, we do not find any significant effect due to non-zero $\delta m_{\tilde{\chi}_1^\pm}$.

From Fig. 4(right) we see that in this case, even with the sneutrino mass uncertainty, we can reasonably constrain the squark mass scale. For example, at the 1-$\sigma$ level with $M_{\tilde{Q}} = 1$ TeV, we constrain $M_{\tilde{Q}}$ to $1^{+1.2}_{-0.5}$ TeV, using the naive scale up (from 20 fb$^{-1}$ to 100 fb$^{-1}$) of the statistical
errors of Ref. [4]. This corresponds to the difference between the gauge and gaugino effective couplings, $\delta g_2/g_2 = 0.011 \pm 0.006$. This can be compared to the estimate of the constraint $\delta g_2/g_2 = \pm 0.02$ from the chargino production measurement [2]. Such comparisons are sensitive to different choices of parameter space and other assumptions. If we reduce the mass uncertainties by a factor of 2, we find the interesting constraint $600 < M_{\tilde{Q}} < 1500$ GeV.

4. conclusions

Supersymmetry is a beautiful symmetry which relates bosons and fermions. If we wish to determine whether this symmetry is realized in nature, the relations imposed between particles and their superpartners must be confirmed by experiment. Of course, discovering a particle with the quantum numbers of a superpartner is the first very important step in this procedure. An equally important test, though, is the confirmation of the hard relations imposed by supersymmetry, for example, the equivalence of the gauge and gaugino couplings.

It has been argued that a next generation linear collider would be an excellent tool to verify supersymmetry in this respect. Production cross-sections such as $\sigma(e^- e^+ \rightarrow \tilde{\ell} \tilde{\ell}^*)$ and $\sigma(e^- e^+ \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_i^\pm)$ involve the $t$-channel exchange of gauginos or sleptons, so they depend on gaugino couplings [5, 3].

In this paper, we approached this problem from a somewhat different direction. Because supersymmetry must be badly broken by soft breaking terms, the tree-level relations of the couplings are also broken, by radiative corrections. The corrections are logarithmically sensitive to the splitting of the supersymmetry multiplets. To quantify this, we have calculated the full one-loop correction due to $(s)quark$ loops of the slepton production cross-sections. The difference between the effective lepton-slepton-gaugino couplings $g_{\tilde{G}}^{\text{eff}}$ and the effective gauge couplings $g_i^{\text{eff}}$ is given by a coupling factor times $\log M_{\tilde{Q}}/m_{\tilde{\ell}}$.

We gave an explicit example which illustrates that the statistics at the future linear collider may be enough to constrain the squark mass scale through the measurement of the slepton production cross-section. We found, with $1\sigma$ significance, $M_{\tilde{Q}}$ could be constrained to $1^{+1.2}_{-0.5}$ GeV by the measurement of the sneutrino production cross-section. We found this constraint in the $\mu < 0$ case where we took into account the errors (based on existing MC simulation) of the sneutrino and light chargino masses, but assumed $\tan \beta$ was well constrained by other measurements.

The mass of the sleptons and gauginos must be measured very precisely in order to successfully constrain the squark mass scale via production cross-section measurements. In order to determine the ultimate sensitivity of this procedure, a thorough study of the systematic uncertainties in the slepton mass measurements is necessary.

It is important to note that the constraint on the squark mass scale can be stronger than the one presented in this paper. Here, we estimated the sensitivity to squark mass scale by utilizing sneutrino production followed by its decay into a chargino and an electron. Depending on the spectrum and center-of-mass energy, there will typically be many other production processes which involve $t$-channel exchange of gauginos or sleptons, and all those amplitudes have $\log M_{\tilde{Q}}$ corrections.

The constraint on the squark mass we have realized here could be unique in the sense that this information may not be available at the LHC. Even if the LHC squark production rate is large, the gluino could be produced in even larger numbers, creating a large irreducible background to the squark signal. A large gluino background could make the extraction of the squark mass from kinematical variables difficult.

On the other hand, if information on the squark masses is obtained at the LHC, we would have rather accurate predictions for the gaugino couplings. In this case, the measurement of the production cross-sections we considered here would constrain new supersymmetry-breaking physics with standard model gauge quantum numbers. In a sense, the study proposed here is similar in nature to studies performed at LEP and SLC. The physics of gauge boson two-point functions has been studied extensively at LEP and SLC, and it has provided strong constraints on new physics. Similarly, a future LC and the LHC might provide precision studies of the gaugino two-point functions, to realize a supersymmetric version of new precision tests.
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