Sub-Kelvin Parametric Feedback Cooling of a Laser-Trapped Nanoparticle

Jan Gieseler\textsuperscript{1}, Bradley Deutsch\textsuperscript{3}, Romain Quidant\textsuperscript{1,2} and Lukas Novotny\textsuperscript{3}

\textsuperscript{1} ICFO-Institut de Ciencies Fotoniques, Mediterranean Technology Park, 08860 Castelldefels (Barcelona), Spain
\textsuperscript{2} ICREA-Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain and
\textsuperscript{3} Institute of Optics, University of Rochester, Rochester, NY 14627, USA

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Recent experiments have demonstrated the ability to optically cool a macroscopic mechanical oscillator to its quantum ground state \cite{1} by means of dynamic backaction \cite{2, 3}. Such experiments allow quantum mechanics to be tested with mesoscopic objects \cite{1, 4} and represent an essential step toward quantum optical memories, transducers, and amplifiers \cite{5, 6}. Most oscillators considered so far are rigidly connected to their thermal environment, fundamentally limiting their mechanical Q-factors and requiring cryogenic precooling to liquid helium temperatures. Here we demonstrate parametric feedback cooling of a laser-trapped nanoparticle which is entirely isolated from the thermal bath. The lack of a clamping mechanism provides robust decoupling from internal vibrations and makes it possible to cool the nanoparticle in all degrees of freedom by means of a single laser beam. Compared to laser-trapped microspheres \cite{9}, nanoparticles have the advantage of higher resonance frequencies and lower recoil heating, which are favorable conditions for quantum ground state cooling.

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The interaction between light and matter sets ultimate limits on the accuracy of optical measurements. Vladimir B. Braginsky predicted that the finite response time of light in an optical interferometer can lead to mechanical instabilities \cite{4} and impose limits on the precision of laser-based gravitational interferometers. Later, it was demonstrated that this “dynamic back-action mechanism” can be used to reduce the oscillation amplitude of a mechanical system and to effectively cool it below the temperature of the environment \cite{2, 10–14} and even to its quantum ground state \cite{1}. In addition to the fascinating possibility of observing the quantum behavior of a mesoscopic system, many applications have been proposed for such systems ranging from detection of exotic forces \cite{15–17} to the generation of non-classical states of light and matter \cite{18, 19}.

Most of the mechanical systems studied so far are directly connected to their thermal environment, which imposes limits to thermalization and decoherence. As a consequence, clamped systems require cryogenic precooling. A laser-trapped particle in ultrahigh vacuum, by contrast, has no physical contact to the environment \cite{20, 21}, which makes it a promising system for ground state cooling even at room temperatures \cite{18, 19}. First experiments along this line have recently been carried out with micron-sized glass beads trapped in two counter-propagating laser beams \cite{9}. Cooling to milli-Kelvin temperatures has been achieved by an active feedback employing three additional laser beams. However, alignment inaccuracies limit the minimum temperature that can be reached with this technique. Furthermore, the feedback cooling scheme is based on optical scattering forces, which necessarily leads to recoil heating and therefore limits the lowest attainable temperature. To eliminate recoil heating as the limiting factor for ground state cooling one requires considerably smaller mechanical systems, such as dielectric nanoparticles \cite{18, 19}. Here we use the optical gradient force both to trap and cool a fused silica nanoparticle to milli-Kelvin temperatures in high vacuum, thereby avoiding the limitations of alignment and recoil heating. The same laser that is used for trapping is also used to cool the nanoparticle to sub-Kelvin temperatures in all three spatial degrees of freedom.

FIG. 1: Trapping of a nanoparticle. (a) Photograph of light scattered from a trapped fused-silica nanoparticle (arrow). The object to the right is the outline of the objective lens that focuses the trapping laser. (b) Time trace of the particle’s x coordinate (transverse to the optical axis) at 2 mBar pressure.
In our experiments we use a single laser beam of wavelength $\lambda = 1064\text{ nm}$, focused by an NA=0.8 lens mounted in a vacuum chamber, to trap fused silica particles of radius $R = 50-100\text{ nm}$ by means of the optical gradient force. Radiation pressure slightly displaces the trapped particle from the geometrical focus of the laser beam in direction of beam propagation. For particles much smaller than the wavelength, the polarizability scales as $\alpha \propto R^3$ and the gradient force dominates over the scattering force. Scattered light from the particle is measured interferometrically with three separate photodetectors that render the particle’s motion in the $x$, $y$, and $z$ directions (Supplementary Information). This phase-sensitive detection scheme makes use of balanced detection and yields a noise floor of $1.5 \pm \sqrt{Hz}$. Fig. 1 shows a photograph of a trapped nanoparticle along with a typical time trace of the particle’s $x$ coordinate. Trapping times of more 24 hours have been achieved.

To control and stabilize the particle’s motion in the optical trap we implemented an active feedback loop, similar to the original proposal by Ashkin [20, 21]. All three spatial degrees of freedom are controlled with the same laser used for trapping. To cool the center-of-mass motion of the particle we employ a parametric feedback scheme, similar to parametric amplification of laser fields [22] and stabilization of nanomechanical oscillators [23]. After trapping a single nanoparticle at ambient temperature and pressure we evacuate the vacuum chamber in order to reach the desired vacuum level. At ambient pressure the particle’s motion is dominated by the viscous force (Stokes force) due to the random impact of gas molecules. However, as shown in Fig. 1(b), the inertial force dominates in a vacuum of a few millibars as the particle’s motion becomes ballistic [24].

Parametric feedback is activated as soon as we enter the ballistic regime. In a time-domain picture, the feedback loop hinders the particle’s motion by increasing the trap stiffness whenever the particle moves away from the trap center and reducing it when the particle falls back toward the trap. In the frequency domain, this corresponds to a modulation at twice the trap frequency with an appropriate phase shift. Fig. 2 illustrates this feedback mechanism. To obtain a signal at twice the oscillation frequency we multiply the particle’s position $x(t)$ with it’s time derivative. The resulting signal $x(t)\dot{x}(t)$ is then phase-shifted by a controlled amount in order to counteract the particle’s oscillation. Note that depending on the latency of the feedback loop we can achieve damping or amplification of the particle’s oscillation. In the absence of active feedback, the particle’s oscillation naturally locks to the modulation phase in such a way as to achieve amplification [22] (Supplementary Information). Cooling therefore requires active feedback to adjust the modulation phase constantly.

In our cooling scheme, frequency doubling and phase shifting is done independently for each of the photodetector signals $x$, $y$ and $z$. Since the three directions are spectrally separated (see Fig. 3(b)), there is no cross-coupling between the three signals, that is, modulating one of the signals does not affect the other signals. Therefore, it is possible to sum up all three feedback signals and use the result to drive a single Pockels cell that modulates the power $P$ of the trapping laser. Thus, using a single beam we are able to effectively cool all spatial degrees of freedom.

For small oscillation amplitudes, the trapping potential is harmonic and the three spatial dimensions are decoupled. Each direction can be characterized by a frequency $\Omega_0$, which is defined by the particle mass $m$ and the trap stiffness $k_{\text{trap}}$ as $\Omega_0 = \sqrt{k_{\text{trap}}/m}$. The equation of motion for the particle’s motion in $x$ direction (polarization direction) is

$$\ddot{x}(t) + \Gamma_0 \dot{x}(t) + \Omega_0 x(t) = \frac{1}{m} [F_{\text{fluct}}(t) + F_{\text{opt}}(t)], \tag{1}$$

where $F_{\text{fluct}}$ is a random Langevin force that satisfies $\langle F_{\text{fluct}}(t) F_{\text{fluct}}(t') \rangle = 2m\Gamma_0 k_B T \delta(t-t')$ according to the fluctuation-dissipation theorem. $F_{\text{opt}}(t) = \Delta k_{\text{trap}}(t) x(t)$ is a time-varying, non-conservative optical force introduced by parametric feedback. It leads to shifts $\delta \Omega$ in the particle’s natural damping rate $\Gamma_0$ and oscillation frequency $\Omega_0$, respectively. Similar equations and considerations hold for the particle’s motion in $y$ and $z$ directions.

We first consider the particle’s dynamics with the feedback loop deactivated. For small oscillation amplitudes,
the particle experiences a harmonic trapping potential with a trap stiffness $k_{\text{trap}}$, which is a linear function of $P$. In the paraxial and dipole approximations (small particle limit, weak focusing) the transverse trap stiffness is calculated as (Supplementary Information)

$$k_{\text{trap}} = 4\pi^3 \alpha P \frac{NA^4}{c\varepsilon_0 \lambda^4},$$

(2)

where $NA$ is the numerical aperture of the focused beam, $\lambda$ is the wavelength, and $\alpha$ is the particle polarizability. A similar expression holds for the longitudinal trap stiffness. For the parameters used in our experiments we find that the particle’s oscillation frequency in $x$ direction is $f_0^{(x)} = (k_{\text{trap}}/m)^{1/2}/(2\pi) = 127\, \text{kHz}$. For the axial oscillation frequency we find $f_0^{(z)} = 39\, \text{kHz}$ and for the $y$ direction we measure $f_0^{(y)} = 138\, \text{kHz}$. The different oscillation frequencies in $x$ and $y$ directions originate from the symmetry of the laser focus [25]. The linear dependence of the trap stiffness on laser power has been verified for all three directions. Fig. 3(a) shows the measurements for a trapped $R = 73\, \text{nm}$ particle at a pressure of $P_{\text{gas}} = 7.8\, \text{mBar}$. For the same particle, we show in Fig. 3(b) the spectral densities of the $x$, $y$, and $z$ motions. The $y$ detection channel picks up some of the motion in $x$ direction, but this coupling becomes negligible at lower pressures when the linewidths become narrower. This spectral separation makes it possible to cool the laser-trapped nanoparticle with a single beam.

Once a particle has been trapped, the interaction with the background gas thermalizes its energy with the environment and, according to the fluctuation-dissipation theorem, damps the particle’s motion with the rate $\Gamma_0$ in Eq. (1). From kinetic theory we find that [26]

$$\Gamma_0 = \frac{6\pi\eta R}{m} 0.619 \frac{0.619 + Kn}{(1 + c_K)},$$

(3)

where $c_K = 0.31Kn/(0.785 + 1.152Kn + Kn^2)$, $\eta$ the viscosity coefficient of air and $Kn = \overline{l}/R$ is the Knudsen number. When the mean free path $\overline{l} \propto 1/P_{\text{gas}}$ is much larger than the radius of the particle, $\Gamma_0$ becomes proportional to $P_{\text{gas}}$. Fig. 4 shows the measured value of $\Gamma_0$ for all three directions as a function of pressure. The deviations from the theoretical curve at low pressures is due to laser power noise. For ambient pressure we find $\Gamma_0 \approx 635\, \text{kHz}$. In ultrahigh vacuum ($P_{\text{gas}} = 10^{-9}\, \text{mBar}$) the damping constant becomes $\Gamma_0 \approx 635\, \text{kHz}$, which corresponds to a quality factor of $Q = \Omega_0/\Gamma_0 = 10^{12}$.

Activation of the parametric feedback loop gives rise to additional damping $\delta\Gamma$ and a frequency shift $\delta\Omega$. The resulting spectral line shapes are defined by the power spectral density $S_{\chi}(\Omega)$, which follows from Eq. (1) as

$$S_{\chi}(\Omega) = \int_{-\infty}^{\infty} \langle x(t)x(t') \rangle \, dt' = \frac{k_BT}{\pi m} \frac{\Gamma_0}{\Omega_0^2} \frac{\Gamma_0}{(\Omega_0 + \delta\Omega)^2 - \Omega^2}.$$

(4)

Integrating both sides over $\Omega$ yields the mean square displacement

$$\langle x^2 \rangle = \langle x(0)x(0) \rangle = \frac{k_BT}{m(\Omega_0 + \delta\Omega)^2} \frac{\Gamma_0}{\Gamma_0 + \delta\Omega}.$$

(5)

According to the equipartition principle, the center-of-mass temperature $T_{\text{cm}}$ follows from $k_BT_{\text{cm}} = m(\Omega_0 + \delta\Omega)^2(\langle x^2 \rangle)$. Considering that $\delta\Omega \ll \Omega_0$ we obtain

$$T_{\text{cm}} = T \frac{\Gamma_0}{\Gamma_0 + \delta\Gamma}.$$

(6)

FIG. 3: Trap stiffness and spectral densities. (a) Normalized trap stiffness in the $x$, $y$, and $z$ directions as a function of laser power $P$. Dots are experimental data and the solid line is a linear fit. (b) Spectral densities of the $x$, $y$, and $z$ motions. The trapped particle has a radius of $R = 73\, \text{nm}$ and the pressure is $P_{\text{gas}} = 7.8\, \text{mBar}$. The spectral separation of the resonances makes it possible to feedback-cool the trapped particle with a single laser beam. The resonance frequencies are $f_0 = 39\, \text{kHz}$, $127\, \text{kHz}$ and $138\, \text{kHz}$, and the quality factors are $Q = 6.8, 19.9$ and 20.7. The solid curves are fits according Eq. (4) and the data on the bottom correspond to the noise floor.

FIG. 4: Damping rate as a function of gas pressure. The damping rate $\Gamma_0$ decreases linearly with pressure $P_{\text{gas}}$. The dashed line is a fit according to Eq. (6).
where $T$ is the equilibrium temperature in the absence of the parametric feedback ($\delta T = 0$). Thus, the temperature of the parametric feedback can be raised or lowered, depending on the sign of $\delta T$ in Eq. (6).

The experimental results of parametric feedback cooling are shown in Fig. 5 which depicts the dependence of the center-of-mass temperature $T_{cm}$ on pressure. The cooling action of the feedback loop competes with reheating due to collisions with air molecules, ultimately setting a minimum achievable temperature for each pressure value. Since the area under the lineshape defined in Eq. (4) is proportional to pressure value. Since the area under the lineshape in Fig. 5a, we find that the temperature will be reached at ultrahigh vacuum ($10^{-11}$ mBar), provided that the $T_{cm} = 100$ mK while maintaining the trapped particle.

In the quantum limit, a mechanical oscillator exhibits discrete states separated in energy by $\hbar (\Omega_0 + \delta \Omega) \approx \hbar \Omega_0$. The mean thermal occupancy is

$$\langle n \rangle = \frac{k_B T_{cm}}{\hbar \Omega_0}.$$

In order to resolve the quantum ground state we require $\langle n \rangle < 1$. For a $127$ kHz oscillator, this condition implies $T_{cm} \sim 6 \mu$K. According to Eq. (4), a low pressure implies a low damping rate and thus, extrapolating Fig. 5a, we find that this temperature will be reached at ultrahigh vacuum ($10^{-11}$ mBar), provided that the particle oscillation can be measured and the feedback remains operational. Alternatively, we can increase the feedback gain and therefore $\delta T$, which will result in a steeper slope of the curve in Fig. 5a. However, a high feedback gain requires a good signal-to-noise ratio (SNR). The current noise floor is $1.5$ pm/$\sqrt{Hz}$ (c.f. Fig. 3b), but it can be significantly lowered by laser stabilization and background suppression. In fact, the current limiting factor is low-frequency laser power noise, which according to Eq. (2) introduces fluctuations in the trap stiffness. Therefore, at a pressure of $10^{-9}$ mBar the spectral widths of the oscillation modes saturate at $\approx 1\text{kHz}$. We used no power stabilization in the experiments reported here, but by using advanced techniques it is possible to reduce relative power fluctuations down to $5 \times 10^{-9}/\sqrt{Hz}$ [27]. Thus, by reducing laser power fluctuations and pumping to lower pressures we should be able to cool a nanoparticle to considerably lower temperatures.

Feedback cooling is fundamentally limited by the fact that the particle's position has to be measured in order to operate the feedback loop. Therefore, the measurement uncertainty of $x$, $y$, and $z$ introduced by shot-noise will limit the lowest attainable temperature $T_{cm}$. The measurement uncertainty follows from the uncertainty principle $\Delta x \Delta p \geq \hbar / 2$, where $\Delta p = \Delta n \hbar k$, $\Delta n$ being the uncertainty in photon number and $k = 2 \pi / \lambda$. For shot noise $\Delta n \propto N^{1/2}$, where $N$ is the mean photon number $N = P \Delta t / (\hbar k c)$. In terms of the bandwidth $B = 1 / \Delta t$ we obtain $\Delta x \geq [\hbar c \lambda B / (8 \pi P)]^{1/2}$. Thus, the measurement uncertainty is determined by the bandwidth $B$ and the signal power $P$ at the detector. For a $R = 73$ nm nanoparticle and the parameters used in our experiments we find $\Delta x \geq 6.75 \text{ pm}$, which corresponds to a center-of-mass temperature of $T_{cm} = 7.1$ mK. Thus, in principle, parametric feedback should allow us to cool a laser-trapped nanoparticle close to its quantum ground state.

The measurement uncertainty $\Delta x$ can be reduced by increasing the signal power at the detector, for example by using a larger particle size $R$ and hence a larger scattering cross-section $\sigma_{\text{scatt}} = k^4 |\alpha|^2 / (6\pi x_o^2)$. However, strong scattering introduces recoil heating, which can become the limiting factor for reaching the quantum ground state. In analogy to atomic trapping, the transition rate $\Gamma_{\text{recoil}}$ between consecutive harmonic oscillator states is calculated as [18, 28]

$$\Gamma_{\text{recoil}} = \frac{2}{5} \left[ \frac{\hbar k^2 / 2m}{\Omega_0} \right] \left[ \frac{I_0 \sigma_{\text{scatt}}}{\hbar \omega} \right],$$

where $I_0$ is the laser intensity at the focus. The last term in brackets corresponds to the photon scattering rate. Comparing $\Gamma_{\text{recoil}}$ with the frequency of a center-of-mass oscillation $\Omega_0$ we find that there is one recoil event per $\sim 10$ oscillations in the current configuration. Thus, the
trapped nanoparticle can coherently evolve for many oscillation periods. The number of coherent oscillations in between recoil events $N_{\text{osc}}$ scales with the ratio $(\lambda/R)^2$. Thus, small particles and long wavelengths are favorable.

Our discussion highlights the tradeoff between measurement uncertainty and recoil heating. A nanoparticle of size of $R = 73\,\text{nm}$ is a good compromise between the two limiting factors. Notice that $\Gamma_{\text{recoil}}$ and the photon scattering rate differ by a factor of $\sim 10^{-9}$, and hence most of the scattered photons do not alter the center-of-mass state of the particle. The possibility of observing the particle without destroying its quantum coherence is a critical advantage over atomic trapping and cooling experiments. Finally, parametric cooling should work even without continuously tracking $x(t)$ as long as the frequency and the phase of the center-of-mass oscillation are known.

In conclusion, we have demonstrated that an optically trapped nanoparticle can be efficiently cooled in all three dimensions by parametric feedback. The parametric feedback makes use of a single laser beam and is therefore not limited by alignment inaccuracies of additional cooling lasers. Theoretical considerations show that center-of-mass temperatures close to the quantum ground state are within reach. To fully exploit the quantum coherence of a laser-trapped nanoparticle, parametric feedback cooling can be combined with passive dynamical back-action cooling and, for example, by use of optical cavities or electronic resonators. The results shown here also hold promise for ultrasensitive detection and sensing. The ultrahigh quality factors and small oscillation amplitudes yield force sensitivities on the order of $10^{-20}\,\text{N}/\sqrt{\text{Hz}}$, which outperforms most other ultrasensitive force measurement techniques by orders of magnitude, and can find applications for the detection of single electron or nuclear spins, Casimir forces and vacuum friction, phase transitions, and non-Newtonian gravity-like forces.

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Correspondence and requests for materials should be addressed to L.N. (email: lukas.novotny@rochester.edu) or R.Q. (email: romain.quidant@icfo.es)

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