Einsteinian Field Theory as a Program in Fundamental Physics

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(Dated: April 18, 2004)

I summarize here the logic that leads us to a program for the Theory of the Total Field in Einstein’s sense. The purpose is to show that this theory is a logical culmination of the developments of (fundamental) physical concepts and, hence, to initiate a discussion of these issues.

PACS numbers: 01.55+b; 01.65+g; 01.70+w

I. INTRODUCTION

Our purpose is to analyze in one place Einstein’s reasoning that led him to the program of a field-theory that, in his own words, is: Continuous functions in the four-dimensional [continuum] as basic concepts of the theory [1].

Einstein had summarized these reasons in [1]. However, we reconsider these from a different perspective, particularly, of [2]. Then, we also modify Einstein’s program suitably.

Furthermore, von Laue summarized in [3] the developments related to conservation postulates in Physics. In the present article, we shall often use this excellent, historically as well as scientifically important [13], article to illustrate and substantiate our physical arguments.

II. INERTIA, ENERGY AND CONSERVATION LAWS

To begin with, and following von Laue, consider Galileo’s famous analysis leading to the concept of an inertia of a physical body.

Galileo observed that a body falling on the surface of the earth from a certain altitude must obtain precisely that velocity which it requires to return to its former level. Any deviation from this law would but be able to furnish us a method for making the body ascend by means of its own gravity, an obvious impossibility.

Then, following Galileo, let a body ascend, after it has fallen downwards a certain distance, on an inclined plane. The lower the inclination of the inclined plane toward the horizontal, the longer the path the body requires to obtain its former level on the plane. Galileo verified this experimentally. Then, if the inclined plane were made horizontal, we may conclude that the body will keep moving on it to infinity with undiminished velocity.

This last conclusion is not any experimental result but an inference to be drawn by imagining an infinite horizontal plane tangential to the surface of the earth, an obvious impossibility.

Now, continuing with the earlier analysis, the simplest form of the law of inertia was obtained by Galileo, that the velocity of each (force-free) body is maintained in direction and magnitude, both. Here, the word inertia refers to the tendency of a physical body to oppose a change in its state of motion. An inertial mass is then a measure of the inertia of a physical body.

Further, in those times, collisions were considered as the simplest form of interaction of physical bodies. For \( m \) as the inertial mass and \( v \) as the velocity of a body relative to an observer, Descartes formed the quantity of motion, \( m v \), the linear momentum, and asserted that it is conserved in a collision. He, however, considered velocity and, hence, also the linear momentum as scalar quantities, for us today, an erroneous notion.

Huygens, on the other hand, realized correctly that the sum (formed with correct signs) of \( m v \) has the same value before and after a perfectly elastic collision, and that the sum of \( m v^2 \) is also conserved in such a collision.

Here entered Newton. He adopted the geometric approach in defining appropriate physical quantities for physical bodies. Newton therefore conceptualized velocity and, hence, linear momentum also, as vector quantities. From this, Newton developed (difference) calculus needed to deal with the notion of velocity of a body as a tangent to its path. On this basis, Newton then proposed his famous three laws of motion.

The concept of a mass point was introduced by Newton as it was needed in his geometric approach. (If not mass point, then what would move with the tangential velocity along the one-dimensional path?) We note, however, that, for Galileo, the notion of an inertial mass was that of the measure of the inertia of a physical body.
In Newton’s *Principia*, two pronouncements appear: first, the rate of change of vector of (linear) momentum of a mass point per unit time equals vector of the force acting on it, and the second, the forces between two mass points are equal and opposite. Consequently, the interaction of an arbitrary number of mass points never changes their total momentum, it remains constant for any system not subject to external forces.

Then was formulated the law of conservation of angular momentum which is one of the important consequences of the law of conservation of linear momentum. In Newton’s formulation, it is also based on the concept of a mass point because the angular momentum is a vector quantity: $\ell = \vec{r} \times \vec{p}$ where $\vec{r}$ is the radius vector of the mass point relative to a given fixed point, $\vec{p} = m\vec{v}$ is the vector of its linear momentum and we use the cross product of vectors $\vec{r}$ and $\vec{p}$ to obtain the angular momentum vector $\ell$ of the mass point.

Simultaneously, Newton also formulated his famous (inverse square) law of gravitation, furnishing us the gravitational force of attraction between two mass points. Here, Newton introduced a new notion of the source properties of a particle. Precisely, if $M$ is the source or gravitational mass of one particle and $m$ is the inertial mass of another particle located at distance $d$ from the first particle, then the gravitational force of attraction (produced by $M$ and acting on $m$) is given by the famous expression:

$$F_g = -G \frac{mM}{d^2} \hat{d}$$

where $G$ is Newton’s constant of gravitation and $\hat{d}$ is a unit vector along the line joining the two particles with origin of the coordinates at the location of the gravitational mass $M$.

It is essential to distinguish between the inertial mass and the gravitational mass of the newtonian particle. These two are conceptions of very different physical origins.

However, as first shown by Galileo’s experiments at the leaning tower of Pisa, the inertial and the gravitational mass of a physical body are equal to a high degree of accuracy. That is an experimental result. But, the fact that these two quantities are equal is to be recognized as an assumption of the newtonian theory. (This recognition played a crucial role in the formulation of the General Theory of Relativity.)

Newton’s three laws of motion and his law of gravitation then provided us a complete solution to the problem of motions (mechanics) of physical bodies as mechanical systems, as collections of newtonian particles.

Here, the newtonian third law of motion, the law of equality of action-reaction pair, and his law of gravitation show very distinctly that his mechanics assumes action at a distance. Of course, there is nothing objectionable in this and it is beside the point as to whether Newton himself considered his law of gravitation as some sort of approximation to be replaced eventually by a law incorporating finite speeds of propagation.

On the basis of the mathematical formulation developed by him, Newton could then calculate the planetary motions. Newtonian mechanics obtained its major confirmation from Kepler’s astronomical observations. Moreover, all the other great achievements of Newton’s work, the theory of tides, the equilibrium configurations of rotating bodies, the calculation of the speed of sound etc. lent credence to various laws of conservation, of linear momentum, of angular momentum and, hence, also to this monumental newtonian formulation of mechanics of physical bodies.

There however existed one problem with the newtonian framework, that of optics - the theory of light. Newton’s corpuscular theory for light did not explain every phenomenon of light, and Newton recognized this. In particular, the existence of umbra and penumbra indicated that the light corpuscles penetrated “forbidden” region in the shadows behind objects illuminated by light. Forbidden region exists as Newton’s laws show that a particle of light should not move there.

These problems of optical phenomena got neglected and did not hinder in any way with the scientific developments of mathematical character, of those times, in newtonian mechanics. Some, in particular, Huygens, however considered these and developed the wave theory of light which could explain the optical phenomena.

Primarily, one of the simplest forms of a general law of Nature is to assert the conservation of some particular physical quantity. That a given physical quantity is really subject to a conservation principle is, however, to be decided only by experimentations. Different experimentations then confirmed the conservation principles obtainable from the newtonian mechanics.

That is why the impact of the newtonian formulation of mechanics completely overshadowed the developments in Physics for the next few centuries. That the mathematical structure of the newtonian mechanics, erected by many others after Newton, required no new experiments or observations is testimony enough to say that the physical foundation laid by Newton was completely sufficient to support it. This led the physicists of those times to (erroneous) conviction that all of physics could be reduced to newtonian mechanics.
Of course, the newtonian concept of a mass point had been the basis of these developments because Newton’s laws of motion presuppose it. As we have already noted, this mass point is a notion derived from the (point-wise) geometric approach to (one-dimensional) paths of physical bodies. Then, we have also noted earlier that the galilean notion of inertial mass is simply that of the measure of the inertia of a physical body.

The concept of kinetic energy, \( \frac{1}{2}mv^2 \), was then proposed and developed by Bernoulli (who gave us the word “energy”) and Euler. That a change in this quantity in a closed mechanical system did not at all result in a reduction in its “capacity of action” was emphasized by them.

The stimulus for a generalization of the concept of energy to include other forms of “energy” existed from the experiences with the thermal processes. That the kinetic energy or mechanical work could be lost while the temperature of the bodies under consideration increased was known. Thus, the fact that kinetic energy could be transformed to heat was known and that led to thermodynamical considerations of conservation postulates.

J R Mayer provided the reasonably accurate theoretical value for the mechanical equivalent of heat and L A Colding obtained almost the same value in his experiments involving friction.

Independently of Mayer, Helmholtz developed the principle of conservation of energy and its implications. Helmholtz derived his expressions for energy directly from the impossibility of the perpetuum mobile. He, then, reached the concept of potential energy for mechanical systems, of the potential energy of a body experiencing gravitational force, of a charged body experiencing electric force, of a magnetized body experiencing magnetic force etc. His energy considerations applied to the production of currents in galvanic cells, thermocouple, electromagnetic induction etc.

Historically, Helmholtz’s considerations were not generally accepted in the beginning. However, G J Jacobi emphasized that in them we essentially obtain the logical continuation of the earlier ideas behind the science of newtonian mechanics. The initial opposition to Helmholtz’s ideas gradually disappeared and “energy considerations” became important in Physics.

In the words of William Thomson (Lord Kelvin of Largs), we finally had: We denote as energy of a material system in a certain state the contribution of all effects (measured in mechanical units of work) produced outside the system when it passes in an arbitrary manner from its state to a reference state which has been defined ad hoc. The words “in an arbitrary manner” embody the physical law of the conservation of energy.

Thus, the law of conservation of energy was, gradually, found to hold beyond the sphere of newtonian mechanics. The same could also be called the situation with the law of conservation of linear momentum, to begin with. It took a gradual while for this law to emerge out of the sphere of newtonian mechanics but that needed new conceptions in the form of locality of actions.

Although Newton’s laws involved action at a distance, the collisions of a specific mass point with the others in its immediate vicinity in a mechanical system (of closely packed mass points) are thinkable as local phenomena. A local disturbance could then propagate in such a mechanical system to other regions from the region of its origin. These considerations lead us to the calculation of the speed of sound in such fluid configurations. This is exactly how finite speeds of propagation obtain in Newton’s theory, although all its basic laws are based on the action at a distance.

Concept of locality of action and related ideas developed from those of fluid properties of matter. Considerations of closely packed newtonian particles led to development of concepts of density, pressure, fluxes, stresses etc.

We define density of newtonian particles (inertial masses) as a function of space coordinates and integrate it over the volume under consideration to obtain the inertial mass contained within that volume. Motions of newtonian particles lead us to concepts of momentum transfer, pressure, etc. Here, the mathematical procedure for switching from the particle picture to the fluid picture is a well-defined one, we note.

From the viewpoint of present mathematics, different physical quantities are measurable functions defined over the underlying [continuum] space. Measure Theory tells us as to how to perform then the integration (averaging) procedures involved in above mentioned considerations.

The principle of local action (as opposed to action at a distance) and that of finite velocity of propagation of disturbances of physical quantities, even in a vacuum, meaning a region with no (newtonian) particles present in it, first gained prominence in electromagnetism. (It is this connotation of the word vacuum that is generally taken to be implied by it and will be used here.)

Considerations of static electricity prompted Coulomb to propose the (inverse square) law of electrostatic force between two charged bodies. Similar to the occurrence of inertial/gravitational masses in Newton’s law of gravitation, Coulomb introduced the electric charge as a measure/source of the quantity of electricity in his law of force between electrically charged bodies. Similar considerations were also used in magnetism.
The motion of a charged physical body along a (point-wise) geometric path was again the pivotal concept behind these laws. Therefore, of necessity, the electric charge became the intrinsic property of a newtonian point-particle.

At this point, we therefore note that the newtonian particle is then endowed with two distinct source attributes or properties, gravitational mass and electric charge. The gravitational mass acts as a source of its gravitational force (defined as per Newton’s law of gravitation) while the electric charge acts as a source of its electric force (defined as per Coulomb’s law).

It then gradually emerged that the motion of a charged body results into the (simultaneous) existence of its magnetic force along with its electric force. Ampere’s experiments and his laws of magnetism associated with current (of charges) were the reasons behind this realization.

J C Maxwell then connected all these empirical laws to provide a sound mathematical foundation to the theory of electromagnetism. Maxwell’s theory then provided us the prediction of electromagnetic radiation, an electromagnetic disturbance propagating from the region of the source to other regions at the speed of light.

Predictions of Maxwell’s theory of electromagnetism were confirmed in numerous experiments. In particular, contributions of Faraday were noteworthy. Also, Hertz’s spectacular confirmation of the existence of electromagnetic radiation lent due credence to Maxwell’s theory.

For electromagnetic processes, Helmholtz’s energy methods yield merely a formula for the total energy. This is as long as one believed in action at a distance without a transmitting medium. But, the question of localization of action or disturbance then lacked meaning.

It is against this background that Michael Faraday developed the concept of field as the medium transmitting such localized action or disturbance. The field was considered as a change in the physical state of a system which was essentially located in the dielectric. It is equally necessary to invoke the same conceptions for even the empty space between the carriers of electric charge, electric currents and magnets.

Note that Maxwell’s theory does provide the energy density (of the field) which is composed additively of an electric and a magnetic term: \( \frac{1}{2} (E^2 + B^2) \), where \( E \) is the electric field strength and \( B \) is the magnetic field strength. It is a necessary supplement of the field concept, field has energy associated with, and inseparable from, it. The question therefore arose of the nature of physical processes involving this energy of the field of Faraday’s conception.

J H Poynting then introduced the notion of a flux of electromagnetic energy entirely on the basis of the mathematical formalism of Maxwell’s theory. He showed that there is a flux of electromagnetic energy whenever an electric and a magnetic field are present at the same time.

With this recognition, it is now possible to determine the route by which the chemical energy, which in the galvanic cell is transformed into electromagnetic energy, gets to wire that completes the circuit, where that energy gets converted into heat. Similarly, we can also trace the energy flow in a circuit involving an electric motor that transforms it to mechanical work.

It was then recognized that the electromagnetic energy must also exist in the space intervening its emitter and its absorber. Emitter loses energy while the absorber gains energy only on the absorption of radiation. Then, during the transit of the electromagnetic radiation from the time of its emission to the time of its absorption, the sum total of all energies can be constant only if we take into account the energy of radiation.

Similar considerations also apply for the law of momentum. The emitter of electromagnetic radiation experiences the opposite force of the absorber of radiation. But, during the transit of radiation, the electromagnetic radiation must carry the momentum with it and “deposit” that momentum at the absorber. In fact, Maxwell showed that a body which absorbs a light ray experiences a force in the direction of that ray. This must also hold for all electromagnetic fields.

Henri Poincaré showed that, if \( S \) denotes the magnitude of the flux of electromagnetic energy, then the field must contain momentum of the magnitude \( S/c^2 \) per unit volume where \( c \) is the speed of light. Electromagnetic momentum was then shown to be observable not only in phenomena with light and heat but also with static fields.

As a matter of historical interest, this approach was the cause of considerable difficulties and took a long time to gain acceptance. The chief reasons behind the difficulties were the required generalizations of the laws of conservation of energy and of momentum.

Of certain importance is the conclusion of the inertia of electromagnetic energy that follows from Poincaré’s expression \( S/c^2 \). If we displace a carrier of electric charge, then the motion of the corresponding electric field gives rise to a magnetic field, and their coexistence leads both to a current of energy and momentum. This additional momentum represents an additional inertial mass in the system under consideration. For an electron, this electromagnetic mass is of the same order of magnitude as the observed mass.
The newtonian picture of a mechanical system of closely packed mass points had always been at the background of these electromagnetic considerations. Then, the question arises: what constitutes field. It must be newtonian particles, each one of some electromagnetic inertia, making up the medium or the field. This medium was the ether. Interactions of these ethereal particles would then provide us the mechanical interpretation of Maxwell’s equations.

Also, the ether was required to be incompressible since the electromagnetic radiation was only of transverse type. But, if a body moved through ether there must be observable effects of the presence of ether on its motion. Such observational effects were fruitlessly sought.

Ultimately, one got used to the concept of the “field” existing independently of newtonian particles. Thus emerged the field-particle dualism. A material particle in Newton’s sense and the (electromagnetic) field as a continuum existed side by side with the material particle acting as a source of its field. The newtonian (source) particle appeared here as a singularity of the (electromagnetic) field it generated around it.

We owe this clear formulation of the field and the particle dualism to H A Lorentz. In this formulation of Lorentz, the newtonian action at a distance gets replaced by that of the field which also represented radiation.

Disturbing here are two facts: firstly, kinetic energy (of a newtonian particle) and the energy of the field appear as physically unrelated entities, and secondly, the field energy carried inertia but not that of a newtonian particle.

An obvious question is then of the nature of the inertia of the field energy. But, it is thinkable that the inertia of field energy is the same as the inertia of a newtonian particle. Then, the concept of a newtonian particle would be simply that of a region of special density of field energy. In that case one could hope to deduce the concept of a particle and its equations of motion from that of only the equations of the field.

Einstein then comments about these ideas as: H A Lorentz knew this very well. However, Maxwell’s equations did not permit the derivations of the equilibrium of the electricity which constitutes a particle. Only other, non-linear field equations could possibly accomplish such a thing. But no method existed by which this kind of field equations could be discovered without deteriorating into adventurous arbitrariness.

In other words, Lorentz was aware of the above mentioned disturbing facts and clearly recognized that the total inertia of a newtonian particle could possess origin in the field conception.

However, the problem Lorentz faced was that of the linearity of Maxwell’s equations. Solutions of (linear) Maxwell’s equations obey superposition principle. Then, one could always superpose required number of solutions to obtain the solution of any assumed field configuration. The newtonian particle would still continue to be the singularity of the final field configuration. Therefore, there were no means here of removing all together the newtonian particle that had the nature of the singularity of the field it generated.

Some non-linear field equations could conceivably possess singularity-free solutions for the field. Solutions of such (non-linear) field equations would also not obey the superposition principle. Then, one could hope that these (intrinsically non-linear) equations for the (total) field would permit some appropriate treatment of newtonian particles as singularity-free regions of concentrated field energy. But, an obviously vexing question was now that of the appropriate (non-linear) field equations of this type. There did not exist (with Lorentz) any physical guidelines (principles) for getting to these (non-linear) field equations.

This however does not mean that such an approach is an impossibility. In fact, it is a logical continuation of the newtonian framework in a definite sense. This is what led Einstein to express the following optimism.

Einstein remarks that: In any case one could believe that it would be possible by and by to find a new and secure foundation for all of physics upon the path which had been so successfully begun by Faraday and Maxwell. —

Then, we recall here that the galilean notion of (inertial) mass of a physical body is that of the measure of the tendency of a physical body to oppose a change in its state of motion. It is the (point-wise) geometric approach of Newton that actually gave us the concept of a mass point or a newtonian particle in Physics.

By giving up the point-wise geometric picture of Newton as basic approach (but retaining it in some appropriate form), one may still hope to treat inertia in the galilean sense of it being a measure of the tendency of a physical body, an extended region of the field energy, to oppose a change in its state of motion. This is then the meaning of Einstein’s above mentioned optimism.

The question is, of course, of reaching the (non-linear) field equations of appropriate nature without venturing into meaningless arbitrariness. We then need some definite physical principles to reach to the non-linear field equations of the desired type. This is what we turn to. However, we note that the route to appropriate field equations had been long and difficult.
Now, as separate remarks, we firstly recall that the equality of the inertial mass and the gravitational mass is certainly an assumption of the newtonian mechanics. Newton’s theory then does not offer any explanation of this experimental result. This is apart from the fact that Newton’s theory does not provide satisfactory explanations of the optical phenomena. Huygens’s wave theory of light then has existence separate, meaning independent, from that of Newton’s theory.

Secondly, from a geometric point of view, all coordinate systems are among themselves logically equivalent. But, Newton himself had realized (the bucket experiment) that the validity of his laws of motion (for example, the law of inertia) is restricted to only certain types of such reference systems, the inertial frames of reference. Why this special status to inertial frames of reference among all the possible others? This fact therefore needed an explanation.

Newton’s analysis (of the bucket experiment) then showed, quite to his own dislike, that this explanation required the introduction of absolute space as an omnipresent active participant in all mechanical events, but the one which is not affected by the masses and their motions. Else, his laws, in particular, the law of inertia, could not have any physical content.

Newton himself could not resolve this impasse and nor could any one else in his own times. Later on, it was also thought for some while that the ether provided this absolute space of Newton’s theory. But, the ether, as a mechanical system made up of newtonian mass points, must get affected by motions of masses.

It was then soon realized that, since the masses and their motions did not affect the absolute space, there could not be any way of establishing the existence of the absolute space. The absolute space then came to possess the ghostly existence in the newtonian framework.

Furthermore, in Newton’s theory, there are essentially two distinct concepts: firstly, that of the law of motion and, secondly, that of the law of force. The law of motion is then empty of content without the law of force. One may then be tempted to ask if any (arbitrary) law of force could work. But, Newton’s law of gravitation and Coulomb’s law of electrostatic attraction possess, both, only a specific form: that of the inverse-square type. The newtonian theoretical framework but provided no explanation of this fact.

In essence, we therefore gradually came to recognize that various of the basic concepts of the newtonian theoretical framework needed to be replaced by suitable others.

III. SPECIAL RELATIVITY AND SPACETIME

It is then History that Einstein realized: many of the simultaneous problems of electromagnetism and galilean principle of relativity of newtonian mechanics could be resolved by invoking a principle of the constancy of the speed of light in all the (inertial) frames of reference.

Maxwell’s equations do not permit a situation of spatially oscillatory electromagnetic wave, the “standing” electromagnetic wave. An electromagnetic wave always travels and, in vacuum, with the speed of light. Moreover, various experiments did not indicate the existence of any such standing electromagnetic wave. Then, Maxwell’s equations had experimental verification of certain importance. Therefore, the galilean principle of relativity (the galilean coordinate transformations) needed to be modified suitably so that no standing electromagnetic wave could be observed by any inertial observer.

That is to say, what are needed here are some “new” transformation laws (of coordinates) which keep Maxwell’s equations invariant. Then, no standing electromagnetic wave would be obtained in all the (inertial) frames of reference if it cannot be obtained in one frame of reference, that of Maxwell’s standard equations.

The coordinate transformations needed for this purpose were already available to Einstein in the form of the Lorentz transformations. This situation then led him to understand in a clear manner what the spatial coordinates and the temporal duration of events meant in the (new) formulation with the Lorentz transformations.

Einstein’s analysis led him to abandon the notion of the absolute simultaneity of events and to the demand that the Laws of Physics remain invariant under the Lorentz transformations. This is similar in spirit to the demand of the newtonian theory that the Laws of Physics be invariant under the galilean transformations.

Minkowski’s important contributions then made it possible to write the Laws of Physics in a mathematical form that ensured their invariance under the Lorentz transformations.

Now, to the criticism of the basis of the Special Theory of Relativity.

Firstly, this theory provides us a (point-wise) geometric description of physical bodies as mass points. This is evident from the laws of motion in Special Relativity.

Secondly, the law of the force is also to be separately stated and, hence, the relevant criticism of the newtonian mechanics also applies.
Thirdly, as Einstein himself recognized, the special theory of relativity treats, essentially independently, two kinds of physical things. (1) measuring rods and clocks and (2) all other things, like the electro-magnetic field, the material point etc. This is unsatisfactory.

Surely, measuring rods and even clocks must be made up of material points. Then, measuring apparatuses would also have to be represented as objects consisting of moving material configurations. Their treatment in Special Relativity is then not a consistent one.

Next, the special status of the inertial frames of reference among all the possible others remains unexplained in Special Relativity. Therefore, the corresponding criticism of the newtonian theory (absolute space) also applies to the theory of special relativity as well.

Next, this theory does not explain the (observed) equality of the inertial mass and the gravitational mass, although optical phenomena are explained by way of Maxwell’s equations in a consistent manner in this theory.

Clearly, then, the theory of special relativity is not an entirely satisfactory one since it leaves so much completely unexplained.

This, however, does not mean that it is not a step forward in the direction of some satisfactory theory. There are certain noteworthy and important achievements of Special Relativity.

In this theory, the inertial mass $m$ of a closed system is identical with its energy $E$, i.e., $E = mc^2$ where $c$ is the speed of light in vacuum. Further, the inertial mass $m$ is dependent not only on the rest mass of the particle but also on its velocity:

$$m = m_o / \sqrt{1 - v^2/c^2}$$

where $m_o$ is the rest mass of the particle. This variation of $m$ with velocity is a direct consequence of the Lorentz transformations.

Conceptually, if a physical body were to “move” in a “field” surrounding it, that body should “display” opposition to a change in its state of motion and this opposition may be expected to depend on its “state of motion” - the velocity that body possesses. Recall that the inertial mass of a body is a measure of its tendency to oppose a change in its state of motion.

As a consequence, the principles of conservation of linear momentum and the conservation of energy are here fused into one thing.

It also clearly recognizes that the coordinates have no absolute physical meaning. This recognition is a very significant and important step away from that of Newton’s theory.

Anyway, it was very clear to Einstein that the Theory of Special Relativity was only a transition from the newtonian framework to incorporate electromagnetism in a consistent way. That the special relativistic laws are linear laws obeying superposition principle for their solutions was indication enough for him that it is only a transitory phase in the formulation of an appropriate (non-linear) theory of the field.

However, what emerged in the special theory of relativity is the Minkowskian four-dimensional continuum indicating clearly that the coordinates have no absolute physical meaning. But, differences of coordinates had some definite physical meaning in this theory, that of physical length and of duration of physical time.

It then gradually became clear to Einstein that even the differences of coordinates need not possess any absolute physical meaning. It was not any “easy to come by” realization. This is what we turn to next.

IV. GENERAL RELATIVITY AND CURVED SPACETIME

There were many problems with the theory of special relativity and one could easily have chosen to rectify them individually starting from different possible perspectives. However, as is well known, Einstein chose to concentrate only on one of them: the problem of the equality of the inertial and the gravitational mass.

This fact is not just mere luck. Einstein’s line of argument has been clearly stated by him as the following one. Firstly, one can see very clearly that, in the special theory of relativity, if the inertial mass depended on the velocity of a material point, then its gravitational mass would also depend on its velocity (kinetic energy) in exactly the same manner if, of course, the equality of the inertial and the gravitational mass holds. This equality is but known experimentally to be true to a high degree of accuracy.

Then, the weight of a physical body would depend on its total energy in a precise manner since the inertial mass now includes also the kinetic energy of the body. In essence, the acceleration of a material system falling freely in a given gravitational field is then independent of the nature (total energy) of the falling system.

It then occurred to Einstein that: In a gravitational field (of small spatial extent) things behave as they do in a space free of gravitation, if one introduces in it, in place of an “inertial system,” a reference system which is accelerated relative to an inertial system.
The above is a statement of the equivalence principle, that represents the equality of the inertial and the gravitational mass. Then, in a real gravitational field, it is therefore possible to regard an appropriate reference frame, the frame of reference of a falling body, as an “inertial” frame of reference. The concept of the inertial frame becomes completely vacuous, then.

The equivalence principle then implies that the demand of the Lorentz invariance of physical laws is too narrow. Therefore, it is necessary to demand that the physical laws be invariant relative to non-linear transformations of coordinates in the four-dimensional continuum. This is then the principle of general covariance.

Locally, meaning in a small region of the four-dimensional continuum, the laws of special relativity must, however, hold in an approximation. In the context of modern terminology, a general four-dimensional spacetime manifold is then required to be locally Lorentzian. The Lorentz group is therefore a subgroup of the group of general coordinate transformations.

Einstein then raised the following two questions: Of which mathematical type are the variables (functions of the coordinates) which permit the expression of the physical properties of the space (“structure”)? Only after that: Which equations are satisfied by those variables?

Interestingly, he writes below these questions in that: The answer to these questions is today by no means certain. (That was in 1949, some decades after the (standard) field equations of general relativity were given by him!)

We now trace here the path that Einstein chose to reach the standard (Einstein) field equations of general relativity.

Although we do not, to begin with, know which mathematical type are the variables of the theory, we do know with certainty one special case, the case of “no gravitational field” - the spacetime of the special theory of relativity.

The line element of the Minkowski space is

$$ds^2 = -dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$$

for a properly chosen coordinate system. It is a measurable distance between events.

Now, referring to an arbitrary coordinate system, the same (Minkowski) line element (metric) is expressible as

$$ds^2 = g_{ij}dx_idx_j$$

where $g_{ij}$ are the (real) metric coefficients forming a symmetric tensor of rank two and $i, j$ take values from 1 to 4. (Here, we have also used the (Einstein) summation convention.)

If the first derivatives of $g_{ij}$ do not vanish after the coordinate transformation, then there exists a special gravitational field in the (new) coordinate system in the sense that it is “accelerated” with respect to the earlier coordinate system.

Detailed mathematical investigations of metric spaces were carried out by Riemann and various relevant conceptions existed before Einstein began his search for the variables of the physical properties of the space structure.

From Riemann’s investigations of metric spaces, the Minkowski spacetime can be uniquely characterized as the one for which the Riemann curvature tensor vanishes. That is, the Minkowski spacetime is a flat manifold. Then, it can also be seen easily that the path of a mass point (not acted upon by any force) in the Minkowski spacetime is a straight line and also a geodesic of the Minkowski spacetime. A geodesic (straight line) is then already a characterization of the Law of Motion in the (flat) Minkowski spacetime.

In Einstein’s words: "The universal law of physical space must now be a generalization of the law just characterized.”

He then assumes that there are two steps of generalization, namely,

(a) pure gravitational field

(b) general field (in which quantities corresponding somehow to the electromagnetic field occur, too).

The situation of (a), that of the pure gravitational field, then is characterized by a symmetric (Riemannian) metric (tensor of rank two) for which the Riemann curvature tensor does not vanish. Now, if the universal field law is required to be of the second order of differentiation and also linear in the second derivatives of the metric coefficients $g_{ij}$, then only the vanishing of the Ricci tensor comes under consideration as an equation for the (vacuum) field.

It may also be noted that the geodesics of the metric can then be taken to represent the law of motion of the material point. Therefore, the law of motion (of a mass point) is already incorporated in that for the field.

Einstein then goes on elaborating further: It seemed hopeless to me at that time to venture the attempt of representing the total field (b) and to ascertain field-laws for it. I preferred, therefore, to set up a preliminary formal frame for the representation of the entire physical reality; this was necessary in order to be able to investigate, at least preliminarily, the usefulness of the basic ideas of general relativity.
Now, let us therefore consider, following Einstein\footnote{1}, this preliminary (formal) formulation of the (non-linear) field theory based on the principle of general covariance.

Then, in Newton’s theory, the gravitational field obeys the law (Poisson’s equation):
\[ \nabla^2 \Phi = 4\pi G \rho \]
where \( \Phi \) denotes the gravitational potential and \( \rho \) denotes the density of the sources generating that gravitational potential.

In general relativity, it is the Ricci tensor which takes the place of \( \nabla^2 \Phi \). Therefore, Einstein proposed that the preliminary formulation of general relativity be based on the equations
\[ R_{ij} - \frac{1}{2} R g_{ij} = -\kappa T_{ij} \]
where \( R_{ij} \) denotes the Ricci tensor, \( R \) denotes the Ricci scalar, \( \kappa \) denotes a proportionality constant and, importantly, \( T_{ij} \) denotes the energy-momentum tensor of matter. In so far as these equations are concerned, the energy-momentum tensor does not contain the energy (or inertia) of the pure gravitational field.

The form of the left hand side of these equations (geometric part) is chosen to be such that its divergence in the sense of absolute differential calculus vanishes since a similar divergence of the right hand side (matter part) must vanish from conservation principles.

In this connection, Einstein expresses\footnote{1} his judgement and concerns about these preliminary equations as: The right side is a formal condensation of all things whose comprehension in the sense of a field theory is still problematic. Not for a moment, of course, did I doubt that this formulation was merely a makeshift in order to give the general principle of relativity a preliminary closed expression. For it was essentially not anything more than a theory of the gravitational field, which was somewhat artificially isolated from a total field of as yet unknown structure.

Then, keeping in mind these important remarks, let us consider these two preliminary formulations of the above field equations.

It is then History that within a few months of Einstein’s publication of these field equations, K Schwarzschild obtained (under heroic circumstances) the first solution of these highly non-linear differential equations in a spherically symmetric case of pure gravitational field. The Schwarzschild solution, a spherically symmetric spacetime geometry, represents the exterior or the vacuum gravitational field of a point mass.

Further solutions of these field equations (and history of some) can be found in\footnote{1}.

However, noteworthy for us here are also the Kerr-Newman spacetimes which represent a mass point with electric charge and spin. Of course, when the electric charge and the spin are vanishing, the Kerr-Newman spacetimes reduce to the Schwarzschild spacetime.

We shall refer to this “preliminary formulation” as the “standard general relativity” since, over the decades, it has indeed become one of the standard formulations of general relativity.

Since our purpose here is not to consider the history of these developments but rather that of considering, partially, the history of the development of associated ideas, we shall now turn to the criticism of this “preliminary formulation” or the standard general relativity.

V. CRITICISM OF STANDARD GENERAL RELATIVITY

We recall that Einstein himself was never at ease with this standard general relativity. In\footnote{1}, he says: If anything in the theory as sketched - apart from the demand of the invariance of the equations under the group of the continuous coordinate transformations - can possibly make the claim to final significance, then it is the theory of the limiting case of the pure gravitational field and its relation to the metric structure of space.

However, “matter” is not any part of the theory of the “vacuum” or pure gravitational field. Einstein, of course, recognized this but hinted only that it is the relation of the metric structure with the field which holds in this case can possibly make the claim to final significance.

Since physical objects must be made up of material particles, there simply cannot be physical objects, hence, measuring rods and clocks, in this case of the pure gravitational field. Therefore, the criticism that Einstein himself levelled against the special relativity applies here.

It also does not therefore make “real sense” to say that the geodesics of the field provide the law of motion for the material points since these are not the part of the equations of the pure gravitational field. However, such a relation may be expected in the “final theory” of the general field, the case (b) referred to earlier.

Because of this problem, we also cannot claim that the equivalence principle has been incorporated in the theory of the pure gravitational field. That is, the equality of the inertial and the gravitational mass is not explainable on the basis of the theory of the pure gravitational field. This fact then invites the same criticism as that of the special theory of relativity.
Furthermore, because of the same problem, it also cannot be claimed that the formalism incorporates the law of force, which must be separately stated. Then, again, this fact invites the same criticism as that of the special theory of relativity and of the newtonian theory.

Next, the inertia associated with the field energy is also not the inertia of the material particle in this framework. This is then very similar to the case of Maxwell’s electromagnetism. Therefore, the relevant criticism of the field-particle dualism in that theory also applies to this case of the pure gravitational field.

But, what is more offending here is that, in this case of the pure gravitational field, there definitely are the laws for the field, but there simply cannot be laws for the material particles generating that gravitational field.

To see this, notice that a mass point in this case is, necessarily, a curvature singularity of the spacetime manifold.

A point-particle has existence only at one spatial location and everywhere else, except at this special location, it is the non-singular gravitational field of the particle that “exists”. In its spherically symmetric spacetime geometry, Schwarzschild or the Reissner-Nordstrom geometry, all the points, except for the location of the mass-point, are then “equivalent” to each other in the sense of matter. That is, there does not exist a material particle at any of these points.

This gravitational field only diverges at the location of the mass-point unless, perhaps, we include the self-field in some way, which, of course, is not the case with the equations of the pure gravitational field. Hence, this geometry should not “include” the location of the mass-point.

Then, any mass-point is a curvature singularity of the manifold describing its “exterior” gravitational field which, like in the newtonian case, diverges at the mass point.

Addition of other source attributes of a physical body, like charge, spin etc., does not change this situation. The Kerr-Newman family is obtainable (in the case of the pure gravitational field) and the point-mass with charge and spin is a spacetime singularity in it.

The issue of the inadequacy of such a description of a physical body would have arisen if the Kerr-Newman family of spacetimes were not obtainable for the case (a) of the pure gravitational field. We could then have said that the pure mass-point is an inadequate description of a “physical body” and the addition of other attributes “takes us away from the case (a)” to some new situation, the case (b) of the general field, with some new description of a physical body.

Restricting to the case (a) of the pure gravitational field, we then note that, strictly speaking, a path of a mass point is not a geodesic since the mathematical structure defining a geodesic breaks down at every point along the path of the singularity. We may try to circumvent this problem in the following ways.

We may surround the singular trajectory of a point-particle by an appropriately “small” worldtube, remove the singular trajectory and call this tube the “geodesic” of the particle. But, the new spacetime is not the original spacetime and, by the physical basis of general relativity, it corresponds to different gravitational source. Therefore, this procedure is not at all satisfactory.

We may, along a geodesic, change the spatial coordinate label(s), by which we “identify the singularity”, with respect to the time label of the spacetime geometry and may term this change the “motion” of a particle along a geodesic. However, this is only some “special” simultaneous relabelling of the coordinates. But, a singularity in the spacetime does not “move” at all.

In the case (a) of the pure gravitational field, the motion of a particle must be a “singular trajectory” in the spacetime if a particle, the singularity, “moves” at all. It is also equally clear that any singular trajectory is not a geodesic in the spacetime since such a trajectory is not the part of the smooth spacetime geometry.

Then, strictly speaking, geodesics of a spacetime do not provide the law of motion of particles. Thus, strictly speaking, we have to specify separately the law of motion for the particles. It is also clear that the equations of pure gravitational field do not (strictly speaking, cannot) specify this law of motion for the particles.

But, mathematically speaking, there simply cannot be any laws for the motion of a geometric singularity! That is why, in the case (a) of the pure gravitational field, we have only the (non-linear) equations for the field but no equations of motion for the sources of that field.

In Newton’s theory, the laws of motion for the particles were Newton’s laws of motion, but there were none for the field. The field laws had to be postulated separately in the newtonian framework and it was possible to formulate such laws for the field independently. This is what Maxwell’s electromagnetism achieved.

Then, the case (a) of the pure gravitational field leads us to a situation which is the other extreme of that of Newton’s theory. We have the laws for the field in this case and there are none for the particles. But, it is more offending that, with it, we cannot even formulate separate laws for the motion of the mass points or particles.
To further clarify this situation, let us consider a mass point executing a simple harmonic motion along a straight line, the $X$-axis, say, in relation to two other mass points located some distance away from the first one along the $X$-axis. At $t = 0$, let the first mass point be located at, say, $x = 0$ and the second one at, say, $x = x_2$ and the third one at, say, $x = x_3$.

To be able to describe the simple harmonic motion of the first mass point in relation to other mass points under consideration, we need a spacetime having two singular curves at $x = x_2$ and at $x = x_3$ which run parallel to the time axis (in the Minkowski-type diagram) and having another singular curve, of sinusoidal character, running along the time axis.

It is then thinkable that such a solution of the field equations of the pure gravitational field exists and is obtainable explicitly. This is however not the issue under consideration here.

The issue is of the “cause” behind the harmonic motion of a curvature singularity, and there cannot be any laws providing us this cause of the motion. The solution to the field equations of pure gravitational field is then “ghostly.”

That is why, if we continue to consider, with complete disregard to this fundamental difficulty, the case (a) of the pure gravitational field, we are bound to face inconsistencies. That this is indeed the case can be seen as under.

Further, if there existed other particles, then the spacetime geometry would be different from that of a single point-particle, now having singularities at every location of particles in it. Then, a physical body is, for our considerations, a collection of curvature singularities.

Now, we may want to replace a collection of particles by that of their smooth (fluid) distribution. To achieve this, let us attach a “weight function” to each mass-point in our collection. Each of these weight-functions must “diverge at a suitable rate” at its mass-point to balance the field-singularity at that location. Then, the resulting “weighted-sum” should provide a smooth “volume-measure” as well as a smooth source density.

Mathematically, $d\mu = \rho(x) \, d\mu_o$, where $\mu$ is the volume-measure of the smooth geometry, $\mu_o$ that of the geometry with singularities and $\rho(x)$ is the required source density.

But, any smooth $\rho(x)$ is impossible if the geometry of $\mu_o$-measure has curvature singularities. To keep $\mu$ smooth, $\rho(x)$ must diverge at the singularities. Thus, associated difficulties arise for constructing a smooth spacetime here.

Now, ignore also these difficulties and consider a spherical star, using the fluid approximation, with a smooth spherical spacetime.

Consider two copies of such a star separated by a very large distance today. Now, the spacetime of two stars taken together is not globally spherically symmetric even though the spacetimes of single stars are globally spherically symmetric. However, the evolution of each star would be “weakly” affected by the other distant star, an expectation justifiable on general considerations.

Let stars collapse. Then, if we had some definite outcome in the collapse of the star, exactly the same outcome is observable at locations of each star in the above (symmetric) situation.

We may add further copies of the same star, separated by the same large distance from original stars and from each other. If stars collapse, the same end-result will be seen for each of the stars in this, arbitrary, situation too.

Now, if gravitational collapse leads to a naked singularity for the spherical spacetime, two totally inequivalent descriptions obtain for a particle, that it is sometimes naked and is sometimes covered by a horizon, an inconsistency.

There are then enough indications for us here that the case of the pure gravitational field is an internally inconsistent one to consider.

We then note that Einstein, in recognition of some of these problems, wrote in (14, p. 675) that: Maxwell’s theory of the electric field remained a torso, because it was unable to set up laws for the behavior of electric density, without which there can, of course, be no such thing as an electromagnetic field. Analogously the general theory of relativity furnished then a field theory of gravitation, but no theory of the field-creating masses. (These remarks presuppose it as self-evident that a field-theory may not contain any singularities, i.e., any positions or parts in space in which the field-laws are not valid.)

However, any considerations of the pure gravitational field in general relativity provide us the equations for the field but, in complete contrast to Maxwell’s theory, there clearly can never be any, not even faintest, possibility of our formulating any dynamical laws for the sources of that field, may those be even independent of these field laws. These considerations provide us, similar to that of Newton’s absolute space, not just a mere torso but a ghost 14 for us to deal with.

Recall that, since the masses and their motions did not affect Newton’s absolute space, no means are possible of establishing the existence of the absolute space. Then, the absolute space has the ghostly existence in the newtonian framework. Similarly, the field equations of the pure gravitational field also possess a ghostly existence since the laws of motions of the sources of the pure gravitational field are an impossibility here.
Without the laws for the motion of sources (generating the field under consideration), there is no meaning to the equations of the pure gravitational field because these equations for the field do not permit any understanding of the physical situations, as the above example shows.

Now, we may be tempted to imagine that the solutions of the equations of the pure gravitational field will be able to approximate the “true” situation in some useful way.

Unfortunately, this optimism ignores some very important aspects of the general theory of relativity, those related to the fact that its solutions do not follow any superposition principle. It may then be noticed that the geometry with (smooth) matter fields cannot be approximated by the collection of those representing the pure gravitational field since the latter would possess the curvature singularities at the locations of sources in it while the former geometry would not.

Some (apparently) physically reasonable results could also be obtained using some solutions of the equations of the pure gravitational field. For example, the bending of light, the perihelion precession, the prediction of gravitational radiation etc. The observations may also “agree” with such results. However, such cases are the “pathological situations” of definite kind.

Just as we cannot conclude the correctness of the newtonian corpuscular theory of light on the basis of the explanation it provides for the existence of penumbra on the assumption of “some suitable hypothetical force” acting on the light corpuscles making them enter the shadow of an object illuminated by light, we also cannot conclude that the solutions of the equations of the pure gravitational field provide us physically reasonable explanations of the observed phenomena.

The meaning of the phrase “the equations of the pure gravitational field are ghostly” is then this above. Clearly, any such physical explanations of observed phenomena must, therefore, be based on the case (b) mentioned earlier.

Of course, it may happen in some situations that the mathematical expression for some phenomena obtained on the basis of case (b) is “identical” with that of the case of some solution of the equations of the pure gravitational field. But, that is for the case (b) to tell us.

However, this does not mean that such solutions of the pure gravitational field are some “good approximation” to the corresponding situation of case (b). Just as the explanation of the formation of penumbra on the basis of some suitable force acting on light corpuscles is “ghostly,” the solutions of the equations of the pure gravitational field also remain “ghostly.”

In essence, one must then wait for the case (b) to provide us such physically acceptable explanations of various observed phenomena.

Such considerations of fundamental difficulties then prompt us to abandon the case (a) of the pure gravitational field or of the field-particle dualism in General Relativity, i.e., the notion of a (curvature) singularity as a mass point or particle. This is as far as the case (a) of the pure gravitational field is concerned, then.

(We must therefore, unassumingly, seek the pardon of all those whose sincere as well as herculean efforts gave us the many solutions of these highly non-linear equations of the pure gravitational field. Unfortunately, being ghostly, these equations cannot, however, further our understanding of the Nature for the above reasons.)

At this place, we may also note that any (open or concealed) increasing of the number of dimensions from four does not change this situation. Clearly, all of the above fundamental problems associated with a (curvature) singularity will arise in higher dimensions in exactly the same manner as holds for the case of the pure gravitational field which we have considered above.

Therefore, unless there are some other specific reasons, it does not appear compelling to consider higher dimensional situations.

Restricting to four dimensions, we then consider next the situation of Einstein’s “preliminary equations” containing matter in the form of the energy-momentum tensor.

We then recall here the concern expressed by Einstein regarding these equations. The point of Einstein’s concern [1] is that of the comprehension of the concept of an energy-momentum tensor in the sense of a field theory.

In this connection, we then note that the energy-momentum tensor is (usually) obtained on the basis of only the particle considerations. Recall that to obtain the related basic physical quantities we consider a collection of particles. By defining various physical quantities (such as, for example, the flux of particles across a surface), we average these quantities over this collection of particles. These related (averaged) quantities then help us define the energy-momentum tensor.

(For a mass point as a singularity of the spacetime, the notion of its motion is not mathematically available. Therefore, it is clear that we would not be able to define, for example, the flux of singularities across a surface. Hence, the energy-momentum tensor is not obtainable by considering particles as spacetime singularities.)

Since there cannot be any point-particle, it is not clear what we mean by a particle in General Relativity. It is only after we have specified this concept
that the question of defining physical quantities and averaging them over a collection of particles can be tackled.

Clearly, difficulties arise with the comprehension of the energy-momentum tensor because it is not clearly specified in the “field theoretical framework” of general relativity what exactly we mean by a “particle” (which, of course, cannot be a singular-particle now). Moreover, it is also not clear as to how one can unambiguously define the concept of a particle in this “field-theoretic framework” of geometric character, except perhaps as an extended region of energy.

Then, the energy-momentum tensor of the Einstein field equations is not a well-defined concept to begin with. Therefore, it is unclear whether the Einstein field equations can be formulated without first specifying what we mean by a particle. It is then equally unclear as to what the solutions of these equations mean in the absence of a clear formulation of the concept of a particle (which, now, cannot be a singular-particle).

However, it should be clear by now that only smooth (singularity-free) spacetime geometries in General Relativity need to be considered since the energy density should be non-vanishing everywhere in a spacetime. Many such solutions of Einstein’s makeshift equations, spacetime geometries, are available in the literature [4].

The issue therefore arises of extracting physical results out of these spacetimes, in particular, in the absence of the availability of the notion of particle. Then, which of these smooth geometries are to be considered relevant to this Physics without the field-particle dualism?

It may, however, be noted at this stage that there simply cannot be any considerations of “singular-particles” and “appropriate fluid description approximating some collection of particles” in all these spacetimes because of the reasons already considered by us earlier.

It may be stressed once again that such considerations are not justifiable in any manner. To stress the same thing again, we note that the concept of energy-momentum tensor is itself not yet defined by us because we do not have any well-defined notion of what a particle is in this framework. This is the primary reason behind the current situation of the above kind.

However, the makeshift equations, the Einstein field equations, may provide us a useful starting point. This is true provided we use other physical principles to some advantage. We therefore need to explore this issue further.

We will, for the time being, postpone these considerations of smooth spacetimes.

VI. QUANTUM AND ITS THEORY

Historically, the theory of the quantum owes its origin to the dualism of a particle and a wave. It can be traced to Newton’s times.

As seen earlier, Newton’s geometric considerations led him to postulate a mass point and his laws (of mechanics and gravitation) are primarily based on this conception of physical bodies. If every physical body were to follow these laws, then it must be possible to treat it using the concept of a mass point.

Therefore, Newton proposed the corpuscular theory for light. That a ray of light propagates in a straight line, that it is reflected from a surface (of a mirror) etc. are then explainable on the basis of the corpuscular hypothesis.

However, in Newton’s own experiments in optics, it emerged that the corpuscular hypothesis does not explain, in a natural manner, all the phenomena displayed by light.

As an example, in the situation of an object illuminated by light, umbra and penumbra form. As a corpuscle, light is not expected to penetrate the region forming the shadow of the object. Then, the existence of penumbra needed some corpuscular explanation.

In another experiment, Newton observed “Newton’s rings”. This experiment shows that corpuscles of light gather in some regions forming the bright rings while they do not at all gather in other (dark) regions. This fact also needed some corpuscular explanation.

On the basis of Newton’s laws, it then follows that some force (acting on light) is causing this behavior of the light corpuscles. However, this force then acts on only “some” corpuscles to pull them to the bright regions but not on some other light corpuscles (which escape in a straight path behind the object, for example).

Furthermore, in the case of Newton’s rings, there are more than one bright/dark such regions. Then, the force acting on the light must be (comparatively) “larger” for some corpuscles and “smaller” for some other corpuscles.

Such an explanation is definitely thinkable. But, an important question is now that of the simplicity of this explanation. The simplicity of a theory has always been the driving impetus behind the scientific investigations.

It was clear to Newton that this explanation is not appealing. It requires us to postulate the “switching on and off” of the force whose strength is also different for different corpuscles. What causes such a behavior of this force? Is there some universal rational explanation for this?
There did not exist in the newtonian framework any conceivable explanations of such behavior of the force under consideration here.

Huygens, on the other hand, considered these phenomena from an entirely different perspective. His wave theory of light postulated undulatory behavior for light on the basis of the continuum postulate. Similar to waves on the surface of ocean, he imagined light as a wave phenomenon in some (unknown) medium.

Huygens’s wave theory of light then provided satisfactory explanations of the optical phenomena on the basis of constructive and destructive interference of the waves. The wave theory of light then gained wide acceptance.

This situation persisted till almost the end of the nineteenth century. Newton’s corpuscular theory for light was then forgotten or, at least, not considered seriously.

Incorporation of various optical phenomena in Maxwell’s electromagnetism was a climactic stage for the wave theory. It showed that light consists of oscillations of electric and magnetic fields transverse to the direction of a propagating light ray. The polarization of light then received an explanation here. (We also note that Newton’s corpuscular theory had no explanation whatsoever for this peculiar behavior of light.)

It then came as a real surprise that the corpuscular behavior of light surfaced in some experiments again, of special significance is the photo-electric effect discovered by Hertz in 1887.

In Hertz’s experiments, a metal is subjected to an incident radiation. Such a metal ejects electrons if the frequency of incident radiation is above a certain threshold which depends on the metal properties. The kinetic energy of ejected electrons does not, however, depend upon the intensity of the radiation but only on the difference of the frequency of incident radiation and the threshold frequency of the metal.

In the beginning, it was not at all clear as to how one could explain this Hertzian photo-electric effect in any manner. G Kirchhoff’s investigations into radiation had also provided us various laws regarding the behavior of heat radiation interacting with matter. Efforts then began of providing explanations of these empirical laws on the basis of Maxwell’s theory.

No one realized that a stage was slowly getting prepared for a fundamental crisis with Maxwell’s theory of electromagnetism in this era of hectic experimental activities.

In 1896, Max Planck’s remarkable intuition in dealing with deep physical problems suddenly brought into focus the seriousness of this crisis with classical theories.

These investigations are all the more remarkable in that they were based primarily on only the theories regarding the behavior of heat radiation interacting with matter.

In what follows, we (partly) adopt Einstein’s (method of) exposition of these theoretical developments, because his exposition remarkably clearly describes the (theoretical) nature of this fundamental crisis.

Then, let us first note that Kirchhoff had concluded, on thermodynamical grounds, that the energy density and the spectral composition of radiation in a cavity (Hohlraum) with impenetrable walls of absolute temperature \( T \) is independent of the material of the walls. That is to say, the monochromatic density, \( \varrho \), of radiation is some universal function of the frequency \( \nu \) of radiation and of the absolute temperature \( T \).

Thus arose the problem of determining this universal function \( \varrho(\nu, T) \).

As per Maxwell’s theory, radiation exerts pressure on the walls of the cavity and this pressure is determined by the total energy density. From this, Boltzmann then concluded that the entire energy density of radiation, \( f \varrho \, d\nu \), is proportional to \( T^4 \) and thereby provided a theoretical explanation of Stefan’s empirical law on the basis of Maxwell’s theory. W Wien then used ingenious thermodynamical arguments, also using Maxwell’s theory, to deduce that

\[
\varrho \sim \nu^3 f \left( \frac{\nu}{T} \right)
\]

Clearly, \( f \) is a universal function of only one variable \( \nu/T \) and its theoretical determination is of definite importance.

Basing his faith on the empirical form of the function \( f \), Planck firstly succeeded in reaching the following form for \( \varrho \)

\[
\varrho = \frac{8\pi \hbar \nu^3}{c^3} \frac{1}{e^{\hbar \nu/kT} - 1}
\]

whereby he had two universal constants \( k \) and \( \hbar \) in the expression for the monochromatic energy density \( \varrho \) of the radiation.

If this formula were correct, it permitted the calculation of the average energy \( E \) of an oscillator interacting with the radiation in the cavity:

\[
E = \frac{\hbar \nu}{e^{\hbar \nu/kT} - 1}
\]

For fixed \( \nu \) but high temperature, this gives \( E = kT \) - an expression obtainable from the kinetic theory of gases. From this theory, we have \( E = (R/N)T \) where \( R \) is the gas constant and \( N \) is the famous Avogadro’s number.
Then, \( N = R/k \) and its numerical value agreed reasonably well with that of the kinetic theory of gases. Therefore, Planck's investigations were in agreement with the size of the atom since Avogadro's number tells us about it.

Using Boltzmann's (and Gibbs's) entropy methods, Planck then partitioned the total energy into a large but finite number of identical bins of size \( \epsilon \) and asked in how many different ways can \( \epsilon \) be divided among the oscillators. This number would then furnish the entropy and, hence, the temperature of the cavity-radiation system.

As is well known, Planck obtained the aforementioned radiation formula if \( \epsilon = h \nu \). His expression then yields correctly the Rayleigh-Jeans, the Stefan-Boltzmann, the Wien laws.

However, Planck's reasoning camouflaged the fact that his derivation demands that energy can be emitted and absorbed by an oscillator only in *quanta* of energy \( \epsilon = h \nu \).

Planck's formula implies that the energy of any arbitrary mechanical system capable of oscillations can be transferred only in "packets" or these quanta. The same is also the situation with the entropy of radiation. This contradicts fundamentally the laws of Newton's mechanics as well as those of Maxwell's electrodynamics.

Then, we note that Planck's formula is *compatible* with Maxwell's electrodynamics, although it is not a necessary consequence of its equations. Therefore, the contradiction with Newton's mechanics is (more) fundamental here than that with the electrodynamics.

This was clear to Einstein soon after the appearance of Planck's fundamental work. Although he had no ideas on what framework(s) should substitute Newton's and Maxwell's theories, Einstein's intuition nonetheless permitted him to apply Planck's formula to explain the photo-electric effect in a remarkably simple fashion.

If the implication of Planck's reasoning were correct, then an electron can absorb the energy of radiation only in quanta. An electron (bound to an atom) could then be set free (ejected) on absorption of energy of the incident radiation only if the frequency of incident radiation were larger than certain minimum corresponding to the binding energy of the electron. There is a threshold then for the frequency of incident radiation below which electron ejection does not occur in this hertzian photo-electric effect.

N Bohr's remarkable insights developed the quantum theoretical explanations for the empirical laws of atomic spectra. Sommerfeld then also included special relativistic considerations in Bohr's theory of the atom. These inputs were of radical theoretical nature indeed.

Einstein followed with interest these works and, in his own turn, revealed the deep connection between Planck's formula and Bohr's law of frequencies, thereby introducing the probabilities of quantum transitions of atomic systems. This is where the Einstein coefficients for induced and spontaneous transitions made their inroad.

This is the return, in a sense, of Newton's corpuscle - a particle of light and, hence, of the wave-particle dualism (for light). We will however not go into details of other developments here. Instead, we turn to the next important step that was taken for the concept of a quantum.

Next, L de Broglie proposed that if light, primarily an electromagnetic wave in Maxwell's theory, displays particle-like phenomena, then particles (of Newton's type) should display wave-like phenomena in order that the basic symmetry of wave versus particle is maintained.

This radical proposition, of course, needed experimental confirmation and it was soon obtained in diffraction experiments.

Einstein then generalized S N Bose's statistical methods (for light particles) to particles indistinguishable from each other. This goes in the name of the Bose-Einstein statistics and the particles obeying these statistical methods are now called as the Bosons.

The fundamental difference between the statistical properties of like and unlike particles is intimately connected with the circumstance that, due to Heisenberg's indeterminacy relations, the possibility of distinguishing between like particles, with the help of the continuity of their motion in space and time, is getting lost. Pauli's analytical mind grasped this fundamental issue immediately and that is what led him to (Pauli's) exclusion principle for electrons.

Fermi and Dirac then showed that electrons, in particular, follow a different statistical method since they obey Pauli's exclusion principle. This goes in the name of the Fermi-Dirac statistics and the particles obeying these statistical methods are now called as the Fermions.

It is still a deep mystery as to whether the particles of Nature are only of these two types, Bosons and Fermions. If yes, why.

The need for replacement(s) of Newton's and of Maxwell's theories became urgent.

E Schrödinger then formulated the Wave Mechanics and, almost simultaneously, W Heisenberg formulated the Matrix Mechanics for the quantum. These works provided the bridge between the particle and wave conceptions. M Born then provided the "probability interpretation" of Schrödinger's Wave Mechanics and Bohr supported it with his complementarity arguments.
The phenomenon of barrier penetration (tunnelling) was also discovered in such considerations. Special relativistic considerations were taken up by P A M Dirac and others. These provided us the concept of particle and anti-particle pair. The implications of such considerations were “confirmed” in numerous experiments. Today, these considerations are the basis of numerous technological equipments and also of their theories.

The method of second or field quantization was subsequently developed. The primary thesis of this method is that we should be able to count the number of quanta, if these are really the packets of energy. It is this (second quantization) method that has come to be recognized as the genuine theory of the quantum.

The theory of the quantum that developed as a result was mathematically satisfactory but extremely counter-intuitive. We then note here that many vigorous discussions and efforts were needed to reconcile the results, in particular, those concerning Heisenberg’s indeterminacy relations, with the physical intuition. Rather than entering these historical details, we then refer to various excellent essays, in particular, Bohr’s article and Einstein’s Reply to Criticisms in [1].

VII. CRITICISM OF THE THEORY OF THE QUANTUM

Let us first recollect that Einstein, in spite of his initial resistance as reported in Bohr’s essay in [1], finally agreed [3] to the correctness of Bohr’s indeterminacy relations. However, he differed from most other of his contemporary physicists on the issue of probability being the only basis of understanding the entire physical world.

In Einstein’s own words [3] (my curly brackets): ... On the strength of the successes of this theory they {Born, Pauli, Heitler, Bohr, and Margenau} consider it proved that a theoretically complete description of a system can, in essence, involve only statistical assertions concerning the measurable quantities of this system. They are apparently all of the opinion that Heisenberg’s indeterminacy relation (the correctness of which is, from my own point of view, rightfully regarded as finally demonstrated) is essentially prejudicial in favor of the character of all thinkable reasonable physical theories in the mentioned sense. ...

He further hastened to add to the above that: ... I am, in fact, firmly convinced that the essentially statistical character of contemporary quantum theory is solely to be ascribed to the fact that this [theory] operates with an incomplete description of physical systems.

Still, it is undoubtable that the self-consistent formalism of the Quantum Theory provides us the theoretical explanations of variety of experimental results. In fact, the amount of experimental data it supports is so enormous, so unparalleled in the history of Physics, that there must be some element of the finality in its formalism.

Einstein did not want to leave behind any doubts that he did not recognize the importance of the contributions from the Quantum Theory. So, he added further: ... This theory is until now the only one which unites the corpuscular and undulatory dual character of matter in a logically satisfactory fashion; and the (testable) relations, which are contained in it, are, within the natural limits fixed by the indeterminacy relation, complete. The formal relations which are given in this theory - i.e., its entire mathematical formalism - will probably have to be contained, in the form of logical inference, in every useful future theory.

Then, Einstein went on to tell us what exactly it is that does not satisfy him with this Theory of the Quantum: What does not satisfy me in that theory, from the standpoint of principle, is its attitude towards that which appears to me to be the programmatic aim of all of physics: the complete description of any (individual) real situation (as it supposedly exists irrespective of any act of observation or substantiation).

In what follow we recall (although not in a verbatim manner) Einstein’s arguments in support of his above statement [1].

Let us then consider a radioactive atom as a physical system. For practical purposes, we can consider that it is located exactly at a point of the coordinate system. We may also neglect the motion of the residual atom after its radioactive disintegration process in which a (comparatively light) particle is emitted by the atom. Then, following Gamow’s theory, we may replace the rest of the atom by a potential barrier which surrounds the particle to be emitted. The radioactive disintegration process is then the “tunnelling” of the particle out of this potential barrier.

We then solve Schrödinger’s equation and obtain Schrödinger’s Ψ-function which is initially nonzero only inside the potential barrier, but which, with time, becomes non-vanishing outside the barrier. Essentially, the Ψ-function yields the probability of finding the initially “trapped” particle to be outside of the trapping barrier, at some later time, in some specific portion of the space outside of that potential barrier or the atom.

However, this does not imply any assertion of the time-instant of the disintegration of the radioactive atom. That is, no observable exists in the quantum theory for this time-instant.
Einstein then raised the question: Can this theoretical description be taken as the complete description of the disintegration of a single individual atom? The immediately plausible answer is: No, he wrote. For we are inclined to assume the existence of an instant of the disintegration and such a definite value of the time-instant is not implied by the $\Psi$-function.

Einstein then answered for a quantum theorist: This alleged difficulty arises from the fact that one postulates something not observable as “real.” That is to say, this difficulty arises because one is assuming the (reality of) time-instant of the disintegration of an individual atom that is not an observable of its (quantum) theory.

Einstein then phrased the question as: Is it, within the framework of our theoretical total construction, reasonable to assume the existence of a definite time-instant of the disintegration of an individual atom?

Then, if one takes the viewpoint that the description in terms of a $\Psi$-function refers only to an ideal systematic totality (ensemble) but not to an individual system, then one may assume the existence of the time-instant of disintegration of an individual atom.

But, if one represents the assumption that the description in terms of the $\Psi$-function is a complete description of an individual system, then one must reject the existence of the time-instant of an individual atom and can justifiably point to the fact that a determination of the exact time-instant of disintegration is not possible for an individual atom. Any such attempt would mean disturbances (of the atom) of such nature which would then destroy the very phenomenon whose time-instant we are trying to determine.

Now, following Schrödinger’s reasoning, one may construct a contraption which kills a cat (macroscopic object) sitting close to the radioactive atom only if the decay particle is emitted by the atom. One may then ask: Is the cat alive or dead at some later instant of time? Here, one would expect to get an answer with certainty since a beam of torch-light falling on the cat cannot be expected to disturb (kill it, if alive) its state. But, the theory of the quantum can only tell us the probability of the cat being alive or dead.

Then, to begin with we have the $\Psi$-function for an alive cat. With time, the $\Psi$-function is a superposition of two components, one for the alive cat and one for the dead cat. It is only when an observer makes an observation (of the cat) that the $\Psi$-function reduces or collapses to one of (these two of) its components. This collapse of the $\Psi$-function represents the interference of an observer with the system under observation.

Surely, the quantum theory as represented by Schrödinger’s $\Psi$-function is self-consistent. Now, if this probabilistic theory of the quantum is of universal character, that is to say, if the basic laws of nature are intrinsically probabilistic in character, then this formalism applies to microscopic as well as macroscopic systems.

Some of the macroscopic systems constitute experimental equipments and, hence, their behavior is also probabilistic in character then. By associating Schrödinger’s $\Psi$-function with every physical system, the theory of the quantum treats then measuring apparatuses and all other things on an equal footing in its formalism.

But, a question can then be asked as to when, within this formalism of the quantum theory, can we say that the “measuring apparatus” has made its observation.

It is evident that this question is related to the collapse of the $\Psi$-function because there is no another way of answering it within the realm of the quantum theory. Therefore, we have to say that the system makes an observation only when the collapse of the $\Psi$-function takes place to one of its various components, to that which uniquely corresponds to the value of the observable measured by the macroscopic apparatus.

Then, we can consider an apparatus which has been “suitably prepared” to measure a specific physical observable of a certain physical system. But, let us not interfere with this apparatus in any manner whatsoever and let us even not attempt to ‘look’ at the reading of the apparatus. But, now, let us ask a question as to whether the apparatus has “performed” the measurement of that quantity if were prepared to measure by “prearrangement of some special kind.”

For example, we prepare the apparatus to locate the position of an electron in Heisenberg’s microscope and place a charge-coupled device (CCD) at the eyepiece so that a photon, reflected after the collision with an electron, makes a mark on it. This action in Heisenberg’s microscope takes place, but, we choose not to look at that mark made by that photon on the CCD plate.

Then, the question is: whether this apparatus has performed the measurement of the position of an electron, particularly when we have not looked at the CCD plate.

Evidently, if the formalism of the quantum theory were to tell us that the above apparatus has “never” performed any observation, then the result of the observation is not known to any observer, that is to say, the collapse of the $\Psi$-function has not occurred for any observer. This is indicative of the importance of an observer in the framework of the quantum theory.
On the other hand, if we say that an observation has been performed in the above prearrangement with Heisenberg’s microscope, then the $\Psi$-function will always be in some collapsed state because the phenomenon occurring in Heisenberg’s microscope (for that matter, any other apparatus) occurs everywhere in the space. This then results in an obvious absurdity with the entire formalism of the quantum theory.

Therefore, we are forced to assume here that no observation is made in the concerned prearrangement with Heisenberg’s microscope or with any other prearranged apparatus. An observer is therefore of definite importance in this interpretation of the quantum theory.

Now, an observer is also made up of the same thing (matter) that an apparatus is made up of. Hence, the above situation also applies to an observer (as a measuring apparatus, may be of special kind). It is therefore not specified by the quantum theory as to what it means by an observer and, hence, what precisely constitutes an act of observation within its formalism.

Surely, although complicated constructs, as is an apparatus or an observer, are not explicitly describable, one can ascribe a corresponding total $\Psi$-function to them, thereby rendering the phenomena of the macroscopic world also probabilistic in character. Surely, the measuring apparatuses are then at par with everything the quantum theory treats. However, some questions of serious scientific concern then arise.

Clearly, an observer is then given an exceptional importance in the theory of the quantum but this theory does not tell us what constitutes an observer and how does an observer act to collapse the $\Psi$-function on having made an observation of some (quantum) system.

Then, the issue arises as to where, in some appropriate sense, does the collapse (of the $\Psi$-function) occur? How is it to be described in some understandable language? Many such questions can be raised.

In response to such questions, we may, for example, then propose that the collapse occurs in the mind of a conscious observer. Such an approach but transgresses the obvious limits of the scientific inquiry and, hence, invites the corresponding criticism. This is then the problem of the collapse of the $\Psi$-function.

To this date, there do not appear to exist any genuinely satisfactory resolutions of such paradoxes of the quantum theory.

Schrödinger’s Cat Paradox and other paradoxes therefore highlight such problematic issues of the quantum theory if this theory were assumed to be universal of character.

This is surely then a problematic issue for the quantum theory (description using Schrödinger’s $\Psi$-function) if it is to be of universal character, that is, if its laws are to be universally applicable to all the physical systems.

Hence, from the point of view of a complete theory forming the basis of the totality of physical phenomena, in a sense similar to that of Newton’s theory (supposedly) forming the basis for the totality of physical phenomena, we therefore note the following important lacunae with the theory of the quantum described above.

Firstly, the theory of the quantum, as it stands even today, does not incorporate the explanations needed for the equality of the inertial and the gravitational mass of a physical body, which is to be taken as an experimental result.

Secondly, this theory separates artificially “an observer” (as definable in the theory of the quantum) and all other things that it treats since there is no consistent demarkation line between what constitutes “an act of observation” in this theoretical framework. The considered example of the “prearranged” Heisenberg’s microscope highlights precisely this issue.

In this connection, it cannot be forgotten that the measuring apparatuses are also made up of the same things that the formalism of the theory is supposed to treat. Therefore, their treatment in a theory must be at par with everything else that the theory intends to treat, this is if the theory claims universality of its formalism. Then, we would not expect to find any serious paradoxical situations raised in the theory.

Then, we note that, in relation to the standard viewpoint regarding the probabilistic interpretation of the laws of the quantum theory, serious paradoxical situations have been constructed and the resolutions of these paradoxical situations have not been achieved. Clearly, the emphasis of these paradoxes is on whether the quantum theory is of universal character, whether the basic laws of nature are probabilistic of character.

If we assume that the quantum theory is of universal character, then we end up with the paradoxical situations of serious character, with the quantum theory not offering us any satisfactory resolutions of these paradoxes.

On the other hand, if we assume that it is not of universal character, then we must search for an alternative formulation which evidently cannot be based on the (probabilistic) formalism of the quantum theory. That is to say, the laws as are applicable to all the physical systems cannot then be probabilistic of character. Question is then of such an alternative formulation for the description of the totality of physical phenomena.
It may therefore be noted here that various paradoxes of the quantum theory then indicate that [1]:

The attempt to conceive the quantum-theoretical description as the complete description of the individual systems leads to unnatural theoretical interpretations, which become immediately unnecessary if one accepts the interpretation that the description refers to ensembles of systems and not to individual systems.

Einstein [1] then continued: There exists, however, a simple psychological reason for the fact that this most nearly obvious interpretation is being shunned. For if the statistical quantum theory does not pretend to describe the individual system (and its development in time) completely, it appears unavoidable to look elsewhere for a complete description of the individual system; in doing so it would be clear from the very beginning that the elements of such a description are not contained within the conceptual scheme of the statistical quantum theory. With this one would admit that, in principle, this scheme could not serve as the basis of theoretical physics. Assuming the success of efforts to accomplish a complete physical description, the statistical quantum theory would, within the framework of future physics, take an approximately analogous position to the statistical mechanics within the framework of classical mechanics. ...

Perhaps, there is an element of truth in these (last quoted) Einstein’s opinions. Next, we shall see that a general relativistic theory of the total field does offer such a possibility.

VIII. A THEORY OF THE TOTAL FIELD

Let us then return to the considerations of the smooth spacetime geometries in general relativity that are obtainable on the basis of Einstein’s makeshift field equations.

As noted before, in this case, to extract physical results in the absence of any conception of a particle, which cannot be a point-particle, we need to employ other physical principles to some advantage. That this is indeed possible is what is the subject of the present section.

It is of course not very clear to begin with as to which physical principles to use to extract meaningful results in this situation. However, we must look for some hints here.

To begin with, as we have seen before, considerations of the “pure or vacuum gravitational field” lead us to an internal inconsistency in the formalism. Then, we note that the energy density of “matter” must be non-vanishing at every spatial location in any such (smooth) spacetime.

Also, the volume-form, in Cartan’s sense, is well-defined at every location in this (smooth) spacetime. Consequently, just as it was permissible to attribute the concept of inertia to a point of space in the newtonian situation, we should also be able to define the concept of inertia for (every) spatial location of the (smooth) spacetime, of course, in only some non-singular sense.

Then, a well-behaved (Cartan’s) volume-form can be expected to allow us appropriate definitions of non-vanishing “gravitational,” “inertial” and also “total” mass for (every) point of the space in such a spacetime. (See later.)

Next, the case (b) under consideration is, clearly, that of the general field in which quantities corresponding somehow to an electric field occur as well. Therefore, a point of the space in such a spacetime can also be prescribed an electric charge as a source attribute of a physical body in a manner similar to the case of the total mass attribute.

This must also be the case with all the permissible source attributes of a physical body since the same procedure can be expected to work for each of such source attributes.

Thus, it must be possible to incorporate all the source attributes of a physical body in it. Then, in this spacetime, physical bodies are concentrated total energy [1] and would everywhere in space be describable as singularity-free. We may also look at a physical body in the newtonian, non-singular, sense of a point particle possessing the source attributes as outlined above.

Next, any local motion of a physical body can, clearly, be a change in the local energy distribution in this spacetime.

However, the global properties of the spacetime must not change with any “local” motions of a physical body in it. Otherwise, infinite speeds of communication exist in the spacetime. This is, physically speaking, undesirable.

A spacetime for which all the spatial properties are arbitrary has these desired properties. (See below.) The required spacetime is given by [8]:

\[ ds^2 = -P^2 Q^2 R^2 dt^2 + P^2 Q^2 R^2 B^2 dx^2 + P^2 Q^2 R^2 C^2 dy^2 + P^2 Q^2 R^2 D^2 dz^2 \]

where \( P \equiv P(x) \), \( Q \equiv Q(y) \), \( R \equiv R(z) \), \( B \equiv B(t) \), \( C \equiv C(t) \), \( D \equiv D(t) \). We also use \( P' = dP/dx \), \( Q = dQ/\,dy \) and \( R = dR/dz \).

Now, consider an energy-momentum tensor (with heat flux) for the fluid in the spacetime and Einstein’s (makeshift) field equations using it. The energy-density in the spacetime of [1] then varies as \( \rho \propto 1/P^2 Q^2 R^2 \) and is seen [6] to be arbitrary because the field equations do not determine the spatial functions \( P, Q, R \).
But, we must remember that we do not know whether the energy-momentum tensor is \emph{ab initio} definable one for this spacetime under consideration. (However, see later.) Thus, it is not truly justifiable to use it to obtain the Einstein makeshift field equations for \textbf{1}.

However, what we realize here is that it is also the 3-space, any constant-time section of \textbf{1}, that is endowed with various properties such as curvature, pseudo-metric nature etc.

Let us recall here that the characteristic of Newtonian Physics is that it ascribes independent and real existence to space and time as well as to matter. In this Newtonian formulation, space and time play a dual role.

Firstly, they play the role of a background for things happening physically. Secondly, they also provide us the inertial systems which happen to be advantageous to describe the law of inertia. Therefore, if matter were to be somehow removed completely, the space and time of the Newtonian framework would “remain” behind.

Let us also recall Descartes’s opposition to consider space as independent of material objects \textbf{7}: space is identical with extension, but extension is connected with physical bodies; thus there should be no space without physical bodies and hence no empty space \textbf{10}.

We then notice that this expectation is indeed true of the spatial sections of \textbf{1}. Physical bodies are “extended regions of space” in it.

The issue, however, remains of incorporating time in the framework of \textbf{1}, in particular, in the absence of a well-defined concept of a particle. Here, Einstein’s makeshift equations provide us the required clue to this issue.

The makeshift equations essentially provide the laws for the temporal evolution of physical bodies in \textbf{1}. Then, the makeshift equations can, in turn, be determined on the basis of the temporal evolution of physical bodies in \textbf{1}.

It may now be stressed that the makeshift equations are based on the as-yet undefined concept of the energy-momentum tensor for matter fields, undefined because it is not clear as to how to define the concept of a physical particle in this theoretical framework as yet.

It is therefore logically compelling to investigate whether the second alternative, that of determining the makeshift equations on the basis of the temporal evolution of physical bodies, extended space in \textbf{1}, is the realizable one.

In fact, this last approach is really the (logically) appropriate one since the temporal evolution of “physical bodies” in \textbf{1} is a mathematically well-definable concept (dynamical systems) while the makeshift equations are not.

We therefore abandon \textbf{2} the four-dimensional spacetime in favor of a three-dimensional pseudo-Riemannian manifold admitting a pseudo-metric, called the Einstein pseudo-metric, given as:

$$dt^2 = P^2 Q^2 R^2 dx^2 + P^2 Q^2 R^2 dy^2 + P^2 Q^2 R^2 dz^2$$

where, as before, $P = P(x)$, $Q = Q(y)$, $R = R(z)$ and $P' = dP/dx$, $Q' = dQ/dy$, $R' = dR/dz$. We denote the space of \textbf{2} by the symbol \textbf{B}. The three spatial functions $P, Q, R$ are \emph{initial data} for the space \textbf{B}. The vanishing of any of these spatial functions is a \emph{curvature singularity}, and constancy (over a range) is a \emph{degeneracy} of \textbf{2}.

A particular choice of functions, say, $P_o, Q_o, R_o$ is a specific spatial distribution of energy in the space of \textbf{2}. As some “concentrated” energy “moves” in the space, we have the original set of functions changing to the “new” set of corresponding functions, say, $P_1, Q_1, R_1$.

Then, “motion” as described above is, basically, a \emph{change of one set of initial data to another set of initial data} with “time”.

Clearly, we are considering the isometries of \textbf{2} while considering “motion” of this kind. Then, we will remain within the group of the isometries of \textbf{2} by restricting to the triplets of \emph{nowhere-vanishing} functions $P, Q, R$. We also do not consider any degenerate situations for \textbf{2}.

If we denote by \textbf{d}, a metric function canonically \textbf{8} obtainable \textbf{7} from the pseudo-metric \textbf{2}, then the space $(\mathbb{B}, d)$ is an uncountable, separable, complete metric space. If $\Gamma$ denotes the metric topology induced by $d$ on \textbf{B}, then $(\mathbb{B}, \Gamma)$ is a Polish topological space. Further, we also obtain a Standard Borel Space $(\mathbb{B}, \mathcal{B})$ where $\mathcal{B}$ denotes the Borel $\sigma$-algebra of the subsets of \textbf{B}, the smallest one containing all the open subsets of $(\mathbb{B}, \Gamma)$ \textbf{9}.

But, the Einstein pseudo-metric \textbf{2} is a metric function on certain “open” sets, to be called the P-sets, of its Polish topology $\Gamma$. A P-set of $(\mathbb{B}, d)$ is therefore never a singleton subset, $\{x : x \in \mathbb{B}\}$, of the space $\mathbb{B}$. Note also that every open set of $(\mathbb{B}, \Gamma)$ is not a P-set of $(\mathbb{B}, d)$.

Now, the differential of the volume-measure on $\mathbb{B}$, defined by \textbf{2}, is

$$d\mu = P^2 Q^2 R^2 \left( \frac{dP}{dx} \frac{dQ}{dy} \frac{dR}{dz} \right) dx dy dz$$

This differential of the volume-measure vanishes when any of the derivatives, of $P, Q, R$ with respect to their arguments, vanishes. (Functions $P, Q, R$ are non-vanishing over $\mathbb{B}$.)
A P-set of the space $\mathbb{B}$ is then also thinkable as the interior of a region of $\mathbb{B}$ for which the differential of the volume-measure, $m$, is vanishing on its boundary while it being non-vanishing at any of its interior points.

Any two P-sets, $P_i$ and $P_j$, $i, j \in \mathbb{N}, i \neq j$, are, consequently, pairwise disjoint sets of $\mathbb{B}$. Also, each P-set is, in its own right, an uncountable, complete, separable, metric space.

A P-set is the mathematically simplest form of “localized” total energy in the space $\mathbb{B}$. This suggests that we should use set-theoretic concepts for it. One such concept is of a suitable (Lebesgue) measure definable on sets.

We then recall that the Galilean concept of the (inertial) mass of a physical body is that of the measure of its inertia. Therefore, some appropriate measure definable for a P-set is the property of inertia of a physical body, a P-set in question. So also should be the case with the gravitational mass of a physical body. Such should also be the case with other relevant properties of physical bodies, for example, its electric charge.

Thus, we associate with every attribute of a physical body, a suitable class of (Lebesgue) measures on such P-sets. Therefore, a P-set is a physical particle, always an extended body, since a P-set cannot be a singleton set of $(\mathbb{B}, d)$.

Now, in the absence of the field-particle dualism, the field and the source-particle are indistinguishable. The source-properties are then also the field-properties. Thus, a P-set (the total field) and the measures on P-sets (sources) are, then, are amalgamated into one thing here, ie, are indistinguishable from each other, in a sense.

Therefore, various source-properties (measures) change only when the field (P-set) changes. This union of the field and the source-properties is then clearly perceptible here.

Moreover, a given measure can be (Haar) integrated over the underlying P-set in question. The integration procedure is always a well-defined one here as a P-set, being an “open” subset of a continuum, is a non-empty locally compact Hausdorff group in an obvious sense.

The (Haar) integral provides then an “averaged quantity characteristic of a P-set” under question. Of course, this is a property of the entire P-set under consideration.

For example, let us define an almost-everywhere finite-valued positive-definite measurable function, $\rho$, on $(\mathbb{B}, \mathcal{B})$ and call it the energy density. Integrating it over the volume of a P-set, the resultant quantity can be called a total mass, $m_p$, of that P-set under consideration. The total mass, $m_p$, is a property of that entire P-set and, hence, of every point of that P-set.

Clearly, to define the notions of “gravitational mass” and of “inertial mass” of a P-set, we need to consider the “motion” of a P-set and also an appropriate notion of the “force” acting on that P-set. Since we are yet to define any of these associated notions, we call the integrated energy density as, simply, the total mass of the P-set.)

We note that every point of the P-set is then thinkable as having these averaged properties of the P-set and, in this precise mathematically nonsingular sense, is thinkable as a point-particle possessing those averaged properties. It is in this nonsingular sense that we can recover the notion of a point particle in the present framework. That this is indeed permissible in a mathematically precise sense is then an indication of the internal consistency of the present approach.

Clearly, the “location” of the mass $m_p$ will be intrinsically indeterminate over the size of that P-set because the averaged property is also the property of every point of the set under consideration. We may then associate a Dirac $\delta$-distribution with the mass $m_p$ over that P-set.

Thus, “averaging a given measure” over any P-set and associating a Dirac $\delta$-distribution with that averaged measure, an intrinsic indeterminacy of location over the size of that P-set is obtained for that averaged measure.

Now, any two P-sets of the same cardinality, belonging either to the same metric-space $(\mathbb{B}, d)$ or to two different metric-spaces $(\mathbb{B}, d_1)$ and $(\mathbb{B}, d_2)$, are Borel-isomorphic [10]. Then, copies (P-sets) of a physical particle are indistinguishable from each other except for their spatial locations.

Let us reserve the word particle for a P-set since it is the simplest form of “localized” energy in the present framework.

Then, in a precise mathematical sense, sets can be touching and that describes our intuitive notion of touching physical bodies. Of course, the corresponding point particles are then “touching” within the limits of the sizes of the corresponding P-sets. (The size of a P-set then acts in the manner of the de Broglie wavelength for a point particle, a point of the P-set.)

Further, if a P-set splits into two or more P-sets, we have the process of creation of particles since the measures are now definable individually over the split parts, two or more P-sets. On the other hand, if two or more P-sets unite to become a single P-set, we have the process of annihilation of particles since the measures are now definable over a single P-set.

Clearly, the laws of creation and annihilation of particles will require of us to specify the corresponding transformations causing the splitting and the merger of the P-sets.
Now, we call as an object a region of \( \mathbb{B} \) bounded by the vanishing of \( \mathcal{B} \) but containing interior points for which it vanishes (so such a region is not a P-set). Such a region of \( \mathbb{B} \) is then a collection of P-sets. But, a P-set is a particle. Therefore, an object is a collection of particles.

Objects may also unite to become a single object or an object may also split into two or more than two objects under transformations of P-sets. We may then also think of the corresponding laws for these processes involving objects.

Obviously, various concepts such as the density of particles, a flux of particles across some surface etc. are then well definable in terms of the transformations of P-sets and the effects of these transformations on the measures definable over the P-sets under consideration.

Then, such “averaging procedures” are well-defined over any collection of P-sets and, also, of objects. Thus, we may, in a mathematically meaningful way, define a suitable “energy-momentum tensor” and some relation between the averaged quantities, an “equation of state” defining appropriately the “state of the fluid matter” under consideration.

(Such conceptions require however the notion of transformations of P-sets and objects. Moreover, this averaging is a “sum total” of the effects of various such transformations of P-sets and objects and, hence, will require corresponding mathematical machinery. This is, then, the premise of the ergodic theory. Recall that \((\mathbb{B}, \mathcal{B})\) is a Standard Borel Space.)

Einstein’s makeshift field equations are then definable in the sense (only) of these averages. Therefore, Einstein’s makeshift equations are “obtainable” on the basis of the temporal evolution of points of the space \( \mathbb{B} \), physical particles as elements of the 3-space of \( \mathbb{B} \). This is also the sense in which Descartes’s conceptions are then realizable in the present formalism. However, we will not pursue this obvious issue of details here.

Now, we can “count” P-sets and, also, objects. This precise mathematical notion of countability of P-sets and objects here then agrees well with our very general experience that arbitrary physical objects (chairs, stones, persons etc.) are “countable” in Nature in an obvious sense.

Moreover, the metric of \((\mathbb{B}, d)\) allows us the precise definition of the sizes of P-sets and objects. Then, given an object of specific size, we may use it as a measuring rod to measure “distance” between two other objects.

A measurable, one-one map of \( \mathbb{B} \) onto itself is a Borel automorphism. Now, the Borel automorphisms of \((\mathbb{B}, \mathcal{B})\), forming a group, are natural for us to consider here.

Let a Borel automorphism \( \phi \) of \( \mathbb{B} \) act to take a point \( x_1 \mapsto y_1 \) and point \( x_2 \mapsto y_2 \) where \( x_1 \in P_1 \) and \( x_2 \in P_2 \); \( P_1, P_2 \) being P-sets. Let \( y_1 \in P_1' \) and \( y_2 \in P_2' \) with \( P_1' \) and \( P_2' \) being the images of \( P_1 \) and \( P_2 \) under \( \phi \). Then, the canonical distance “d” between \( P_1 \) and \( P_2 \) can evidently change under the action of \( \phi \) continuously (with respect to the corresponding Polish topologies).

A Borel automorphism of \((\mathbb{B}, \mathcal{B})\) then induces an associated transformation of \((\mathbb{B}, d)\), say, to \((\mathbb{B}, d')\), and that “moves” P-sets about in \( \mathbb{B} \), since (suitably defined) distance between the P-sets can change under that Borel automorphism.

We call this the “physical” distance separating P-sets (as extended bodies). We also (naturally) define distance separating objects.

Now, the Borel automorphisms of \( \mathbb{B} \) can be classified as follows:

1. those which preserve measures defined on a specific P-set and
2. those which do not preserve measures defined on a specific P-set

Note that we are restricting our attention to only a specific P-set/Object and not every P-set/Object is under consideration here.

Measure-preserving Borel automorphisms then “transform” a P-set maintaining its characteristic classes of (Lebesgue) measures on a P-set, its physical properties.

Non-measure-preserving Borel automorphisms change the characteristic classes of Lebesgue measures (physical properties) of a P-set while “transforming” it. Evidently, such considerations also apply to objects.

It is therefore permissible that a particular periodic Borel automorphism leads to an oscillatory motion of a P-set or an object while preserving its class of characteristic measures.

We can then think of an object undergoing periodic motion as a (physically realizable) time-measuring clock. Such an object undergoing oscillatory motion then “measures” the time-parameter of the corresponding (periodic) Borel automorphism since the period of the motion of such an object is precisely the period of the corresponding Borel automorphism.

Then, within the present formalism, a measuring clock is therefore any P-set or an object undergoing periodic motion.

In the physical world, we do measure distances and construct clocks in this manner. Then, crucially, the present formalism represents measuring apparatuses, measuring rods and measuring clocks, on par with every other thing that the formalism intends to treat.
Such considerations then suggest an appropriate distance function, physical distance, on the family of all P-sets/objects of the space (B, d). More than one such distance function will be definable, depending obviously on the collection of P-sets or objects that we may be considering in the form of a measuring rod or measuring clock.

This above is permissible since we are dealing here with a continuum which is a standard Borel space with Polish topology. Relevant mathematical results can be found in [9].

A Borel automorphism of (B, B) may change the physical distance resulting into “relative motion” of objects. We also note here that the sets invariant under the specific Borel automorphism are characteristic of that automorphism. Hence, such sets will then have their distance “fixed” under that Borel automorphism and will be stationary relative to each other.

On a different note, an automorphism, keeping invariant a chain of objects separating two other objects, can describe the situation of two or more relatively stationary objects.

Effects of the Borel automorphisms of (B, B) on the physical distance are then motions of physical bodies. Furthermore, various physical phenomena will then be manifestations of relevant properties of such automorphisms.

As an example, a joint manifestation of Borel automorphisms of the space (B, B) and the association of a Dirac δ-distribution of an integrated measure with the points of a P-set is a candidate reason behind Heisenberg’s indeterminacy relations in the present continuum formulation [1]. However, details regarding these considerations are outside the limits of the present article.

But, intuitively, let it suffice to say that as the size of the P-set gets smaller and smaller we “know” the position of the point-particle (of integrated characteristics of a P-set) more and more accurately. (But, recall that a P-set is never a singleton subset of B. So, complete positional localization is not permissible.)

Now, that P-set “transforms” as a result of our efforts to “determine any of its characteristic measures” since these “efforts or experimental arrangements” are also Borel automorphisms, not necessarily the members of the class of Borel automorphisms keeping invariant that P-set (as well as the class of its characteristic measures).

Hence, any Borel automorphism (as an experimental arrangement) purporting to “determine” a characteristic measure of that P-set changes, in effect, the very quantity that it is trying to determine. This peculiarity of the present continuum description then leads to Heisenberg’s (corresponding) indeterminacy relation.

Then, in the present continuum description, it is indeed possible to explain the origin of Heisenberg’s indeterminacy relations. The present continuum description provides us therefore an origin of indeterminacy relations, an alternative to their probabilistic origin.

This circumstance is then extremely encouraging indeed. Essentially, it tells us that one of the fundamental characteristics, Heisenberg’s indeterminacy relations, of the theory of the quantum has an, indeed plausible, explanation in a general relativistic theory of the continuum!

Notice then that, in the present considerations, we began with none of the fundamental considerations of the concept of a quantum. But, the present continuum formalism unfolds itself before us in such a manner that one of the basic characteristics of the conception of a quantum emerges out of the present formalism.

Notice also that, in the present continuum description, we have essentially done away with the “singular nature” of the particles and, hence, also with the unsatisfactory dualism of the field and the source particle. Furthermore, we have, simultaneously, well-defined laws of motion (Borel automorphisms) for the field and also for the well-defined conception of a particle (of integrated measure characteristics).

Any particulate character or undulatory character perceptible in a given physical phenomenon is then attributable to properties of Borel automorphisms of the space B, more precisely to an interplay of Borel automorphisms simultaneously acting on the space B.

We refrain here from elaborating further on this issue. However, intuitively, the particulate nature would be perceptible in a phenomenon if the classical newtonian concepts (eg., momentum) hold in some useful way. Otherwise, the undulatory nature of the (total) field would be perceptible in that phenomenon. Any phenomenon is, of course, a result of the simultaneous interplay of Borel automorphisms of B here.

All these achievements of the present general relativistic (continuum) formulation cannot be without any element of the finality then.

The contention here is then the following: that the set of classes of (Lebesgue) measures on P-sets of (B, d) (as various attributes of a physical particle) and the group of Borel automorphisms of (B, B) (resulting into dynamics of P-sets) are, both, sufficiently large as to encompass the entire diversity of physical phenomena.

In a definite sense, this approach then provides the theory of the total field of Einstein’s conception [1]. It is therefore also a continuum theory of everything in that sense.
The question naturally arises as to where, in the present formulation, are the various constants of Nature such as Newton’s constant of gravitation, Planck’s constant, constant speed of light etc. Here, we only note that these constants arise from relations of physical conceptions definable in this formulation. In some definite sense, this present situation is describable as obtaining Constants without Constants.

As an example, we need to analyze here as to how one obtains Newton’s law of gravitation in the present formalism in order to obtain Newton’s constant of gravitation. For this, we will need to consider the “gravitational mass” measure of P-sets, and also analyze the tendency of this P-set to oppose a change in its state of motion, its inertia as per the conception of Galileo.

Any such analysis is, of course, based on the appropriate subgroup of the group of Borel automorphisms of $(\mathbb{B}, \mathcal{B})$ as well as the association of a Dirac $\delta$-distribution (of an “averaged measure”) with a P-set. Such considerations are, of course, beyond the scope of the present article.

However, some definite conclusions are obtainable from very general considerations of the present formalism and, we now turn to a few such considerations.

Now, a Borel automorphism of $(\mathbb{B}, \mathcal{B})$ cannot lead to a singularity of $\mathbb{B}$ and, hence, to any kind of “naked singularities” from which some null trajectory would reach other points of the manifold. Since the present formalism is logically compelling, this would then mean that the naked singularities would not be obtainable in the Physics without a field-particle dualism.

Evidently, the Universe also does not originate or end in any singularity as there also cannot be any such singularity of the space $\mathbb{B}$ in this framework. But, an Expanding Universe or a portion of $\mathbb{B}$ thereof, that may be hotter in the past than today but having no origin, singular or otherwise, is thinkable in it.

This can be seen from the fact that certain Borel automorphisms may move the P-sets about in the space $\mathbb{B}$ in such a manner that these P-sets may move away from each other, while simultaneously pushing some other P-sets closer to each other. This is a very complicated picture but definitely thinkable nonetheless.

Moreover, Borel automorphisms of $(\mathbb{B}, \mathcal{B})$ form a group. Thus, we can always cross any 2-surface both ways. To see this intuitively, let a Borel automorphism of $(\mathbb{B}, \mathcal{B})$ produce motion of an object “into” the given 2-surface. However, the inverse of that Borel automorphism, producing motion pulling that object “out” of that 2-surface, exists always is the point here.

Therefore, there does not arise any one-way membrane in the present formalism. It therefore does not seem that the concept of a black hole is any relevant to this Physics without the field-particle dualism. As noted earlier, the concept of a black hole is a child of the “ghostly” equations of the pure gravitational field.

Now, a P-set, evidently, can be of any size and, hence, objects can also be of any size. Therefore, a one-way membrane (black hole) is not but, ultracompact objects are of relevance to this Physics without field-particle dualism.

As one more example, we consider Schrödinger’s cat paradox. In the present formalism, some Borel automorphism of base space $\mathbb{B}$ “causes” the disintegration of the atom while the “measurement” of this time-instant can take place using the “suitable structure” on the family of the P-sets of $\mathbb{B}$. Therefore, the time-instant of disintegration of a radioactive atom is “well-defined as a parameter” of that Borel automorphism and, simultaneously, there is also the Heisenberg-type indeterminacy involved in its measurement.

There is then some kind of Two-Time formalism in consideration here. The time of disintegration of a radioactive atom as the parameter of a Borel automorphism of $\mathbb{B}$ can then be different than that which is determinable.

Recall that we began with no considerations of the quantum conceptions. The disintegration of a radioactive atom is a quantum consideration and the probabilistic interpretation leads us to a “fuzzy” description of macroscopic system - a cat. That a possible resolution naturally arises here is then remarkable indeed.

We now turn to some other aspects which were so beautifully explained and elucidated by von Laue - those related to conservation principles in physical theories.

As von Laue expressed it: Mass is nothing but a form of energy which can occasionally be changed into another form. Up to now our entire conception of the nature of matter depended on mass. Whatever has mass, - so we thought -, has individuality; hypothetically at least we can follow its fate throughout time. But, this does not hold for the elementary particles.

These remarks are then understandable in the present formalism since the individuality of particles is that of the P-sets.

In the present formalism, a particle is a point of the P-set of the space $\mathbb{B}$ with associated integrated measures defined on that P-set. As a Borel automorphism of the space $\mathbb{B}$ changes that P-set, the integrated properties also change and, hence, the initial particle changes into another particle(s), since integrated measures change.
Of course, we then need to discover various laws of such transformations of particles into one another in the present formalism. But, it is clear at the outset that these will crucially depend on the structure of the group of Borel automorphisms of the space $\mathcal{B}$.

Von Laue concluded his article with the following relevant remarks: ... Can the notions of momentum and energy be transferred into every physics of the future? The uncertainty relations of W. Heisenberg according to which we cannot precisely determine location and momentum of a particle at the same time - a law of nature precludes this - can, for every physicist who believes in the relation of cause and effect, only have the meaning that at least one of the two notions, location and momentum, is deficient for a description of the facts. Modern physics, however, does not yet know any substitute for them.

We may then conclude the present section with the following remarks.

Here, the notion of energy is then that of the integrated measure defined on a $\mathcal{P}$-set of the space $\mathcal{B}$. The notion of the momentum of a particle (as a point of the $\mathcal{P}$-set of $\mathcal{B}$ with associated integrated measures on that $\mathcal{P}$-set) is then that of the appropriately defined notion of the rate of change of the physical distance under the action of a Borel automorphism of $\mathcal{B}$, including evidently any changes that may occur to measures definable on that $\mathcal{P}$-set. Therefore, the notions of energy and momentum of a particle are certainly (well-) definable in the present formalism.

Further, none of these two notions is any deficient for a description of the facts since Heisenberg’s indeterminacy relations are also “explainable” within the present formalism. This explanation crucially hinges on the fact that the points of the space $\mathcal{B}$ can never be particles since these, as singleton subsets of the space $\mathcal{B}$, are never the $\mathcal{P}$-sets. It is only in the sense of associating the measures integrated over a $\mathcal{P}$-set that the points of the space $\mathcal{B}$ are particles.

The measurable location of a particle is essentially a different conception and that depends on the physical distance definable on the class of all $\mathcal{P}$-sets of the space $\mathcal{B}$. The measurable momentum of a particle is also dependent on the notion of the physical distance changing under the action of a Borel automorphism of $\mathcal{B}$.

Therefore, it does seem possible to combine the virtues of the theory of the quantum and of the general theory of relativity in a formalism that does possess the notions of energy and momentum, both. Then, this formalism will further unite more number of conservation laws than those already united in special relativity.

Now, as seen earlier, physical processes occur as a result of the (combined effects of) the Borel automorphisms of the space $\mathcal{B}$ and these include the processes of quantum character. Then, it is thinkable that “some suitable statistical description” is obtainable by “approximating” the effects of these Borel automorphisms acting on $\mathcal{B}$ to produce the concerned physical processes of quantum character. This, intuitively speaking, can be considered as the probabilistic description of the involved physical processes.

Hence, at this point, we also note that, in relation to the present formalism, the theory of the quantum as represented by Schrödinger’s $\Psi$-function can be expected to assume a place which is similar to that of the usual statistical mechanics within the realm of the classical newtonian theory. (Details of these considerations are, once again, outside the scope of the present article.)

Provided that the description of physical systems of Nature based on a continuum is permissible, the approach [2] followed here in Section VIII is also a logically compelling approach as this article discussed.

Of course, many mathematical, physical details and their implications need to be worked out before we can “test” the above described theoretical framework vis-a-vis experimentation or astronomical observations. But, confidence may be voiced that it will stand tests of experiments and/or observations since it is a logically compelling approach as discussed here.

A comment on the mathematical methods would not be out of place here. Then, we note that the mathematical formalism of the ergodic theory is what is of immediate use for the physical framework of the present section. This much is already clear from the present considerations.

However, it is not entirely satisfactory to use the present methods of ergodic theory. One of the primary reasons for this state of affairs is the inability of the present methods in ergodic theory to let us handle, in a physical sense, the $\mathcal{P}$-sets. Some newer methods are then required here.

IX. CONCLUDING REMARKS

In the present article, we outlined the reasons behind the logically compelling character of the approach adopted in Section VIII.

In doing so, we also critically examined the reasons behind the “failures” of other approaches to General Relativity based on the equations of the pure gravitational field and Einstein’s makeshift equations for the matter fields.
In particular, the equations of the pure gravitational field are “ghostly” in the sense of Newton’s Absolute Space or in the sense of this approach providing only the equations for the field without any possibility for the equations of motion for the sources of the field. These equations cannot therefore provide the means to verify or test predictions in any manner whatsoever.

Then, any solution of these field equations of the pure gravitational field is “ghostly” as well. Any conclusion of a physical nature obtained using these equations is therefore dubious.

Further, Einstein’s makeshift equations for general field are based on an ill-defined concept of the energy-momentum tensor. This is mainly because various concepts leading us to the notion of the energy-momentum tensor are based primarily on the concept of a point-particle which is not defined \textit{ab initio} in a (dynamical) geometric approach such as the one postulated and adopted by general relativity. Clearly, a point-particle as a spacetime singularity leads to the impossibility of the definition of the energy-momentum tensor.

The pivotal problem with the solutions of the makeshift field equations is then that of their physical interpretation in view of some continuum description of physical systems. We cannot consider these solutions as describing some “fluid” matter since the conceptions of fluid properties are ill-defined to begin with. Therefore, it follows that not all the solutions of Einstein’s makeshift field equations are necessarily useful for the continuum description of physical bodies. Of particular mention is the Friedmann-Lemaitre-Robertson-Walker (FLRW) geometry of the standard model of the big bang cosmology.

(Therefore, we must, unassumingly, also seek the pardon of all those whose sincere and herculean efforts provided us the many solutions \textsuperscript{2} of Einstein’s makeshift field equations. Clearly, these equations are based on ill-defined considerations. Furthermore, the solutions of these highly non-linear partial differential equations do not follow any superposition principle. Therefore, it also follows that even if there existed some particular spacetime geometry useful for some specific continuum description of physical systems, it would not be a superposition of other spacetime geometries. Other spacetime geometries are then rendered unphysical in this sense.)

However, it also follows that only the smooth spacetime geometries are the ones that need to be considered if the description of physical systems is permissible on the basis of a continuum. The question then arises of choosing some particular spacetime geometry that can provide us this continuum description of physical systems.

Clearly, the answer to this question requires use of some physical principles, not contained within the framework of standard general relativity (the framework of Einstein’s makeshift equations) since this theory permits many different smooth spacetime geometries containing matter fields. This is what we considered in Section \textbf{VIII}.

In Section \textbf{VI} somewhat separated section from the earlier considerations, we then considered developments related to the theory of the quantum conception. In this section, we recalled Einstein’s exposition of the relevant developments of the concept of a quantum. In particular, we noted the precise nature of problems with the (classical) newtonian theories. We also noted that the contradiction with the laws of newtonian mechanics is of more fundamental nature than that with the laws of Maxwell’s electromagnetism.

The theory of the quantum is based primarily on the probabilistic conceptions. Schrödinger’s \(\Psi\)-function then provides us only the probability of any physical event. As is well known, this then leads to an indeterminacy in the simultaneous measurement of canonically conjugate (classical) variables. This is the probabilistic origin of Heisenberg’s indeterminacy relations. This probabilistic nature of the theory of the quantum leads to paradoxical situations of serious concern if every physical system, microscopic or macroscopic notwithstanding, obeyed probabilistic laws. We then also considered, in Section \textbf{VII} various objections of such serious nature related to this interpretation of the theory of the quantum. In particular, some of the serious objections raised by Einstein \textsuperscript{10} were quoted in details.

In view of the considerations of these objections, it is then possible to adopt the view that the theory of the quantum, Schrödinger’s \(\Psi\)-function, represents an ensemble(s) of systems.

With this above point of view, the theory of the quantum may then be expected to assume, in relation to an appropriate general relativistic theory, one based on the principle of general covariance, for the case (b) of general fields, a place similar to that of the statistical mechanics within the realm of the classical newtonian framework. Then, various paradoxical situations arising in the theory of the quantum as embodied in Schrödinger’s \(\Psi\)-function evidently disappear. Einstein \textsuperscript{10} had, very clearly, perceived this situation with the theory of the quantum.

With this viewpoint, within the theory of the quantum as embodied in Schrödinger’s \(\Psi\)-function, there is obviously no possibility of obtaining any non-probabilistic theoretical framework for the description of physical phenomena. Einstein had also clearly recognized this aspect.
Therefore, we had to look “elsewhere” for the theoretical framework (obeying the principle of general covariance) that would provide us the theory of the case (b) of general fields. This is what Einstein meant in his remarks which were quoted in the context of the relevant discussions.

Therefore, on the basis of some few (physically reasonable) principles (observed to hold in Nature), we developed the approach [2] outlined in Section VIII. This theoretical framework then obeys the principle of general covariance, in a round about manner, since the group of Borel automorphisms of the space \( \mathcal{B} \) is very large.

We then saw that this formalism provides us a clear possibility of explaining the laws of the quantum realm while simultaneously treating the concept of a particle in a non-singular manner. It also shows us the possibility of visualizing the theory of the quantum as embodied in Schrödinger’s \( \Psi \)-function to be of similar character to the usual statistical mechanics.

In particular, the (continuum) formalism of Section VIII provides [11] a non-probabilistic explanation for Heisenberg’s indeterminacy relations. Furthermore, it also allows us satisfactory resolutions of different (serious) paradoxical situations faced by the theory of the quantum as embodied in Schrödinger’s \( \Psi \)-function.

Moreover, it is then also clear that many of the current ideas in theoretical cosmology need drastic modifications. Notably, there is no singularity of the space(time) in the past as well as in the future in the present framework.

The physical matter in the universe can only be “rearranged” in the present framework (of Section VIII) that is a complete field theory. This fact can be expected to have important implications and consequences [12] for our understanding of the cosmological phenomena.

So also our models of some (galactic as well as extragalactic) high energy sources need drastic changes. There does not arise a black hole or a naked singularity in the present framework. Therefore, our models of astronomical sources cannot be based on these conceptions.

In summary, Einstein had been one of the proponents of the ensemble interpretation of the quantum theory. We then recall here that, to emphasize Einstein’s contributions to developments in the theory of the quantum, Max Born wrote [1] about Einstein that: He has seen more clearly than anyone before him the statistical background of the laws of physics, and he was a pioneer in the struggle for conquering the wilderness of quantum phenomena. Yet later, when out of his own work, a synthesis of statistical and quantum principles emerged which seemed to be acceptable to almost all physicists, he kept himself aloof and sceptical. Many of us regard this as a tragedy - for him, as he gropes his way in loneliness, and for us who miss our leader and standard-bearer.

However, as a result of our combined efforts, with so many in the past wrestling with and weakening to a large extent the difficult problems of grasping the Nature, we have been able to see a little beyond their vision.

It was, of course, extremely difficult for the physicists of earlier times, Einstein and others included, to imagine the current path [2], the formalism of the Section VII in those times, turbulent times of vigorous theoretical and experimental activities. In fact, many of the mathematical conceptions of Section VIII specifically those of the ergodic theory, were not even available to them. But, from our present considerations, it then should be clear that Born’s leader, Einstein, had indeed the right intuition all along.

The physically complete framework of the Section VIII is also in conformity with the relevant ideas of Descartes II [11]. Clearly, this field-theoretic program of Section VIII is then also: the complete description of any (individual) real situation (as it supposedly exists irrespective of any act of observation or substantiation).

Then, a stage can be said to have been certainly reached in the history of Physics, once again since Newton’s times, in that we have a physically complete framework in the formalism of Section VIII for describing the totality of physical phenomena in Einstein’s sense.

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[1] Einstein A (1970) in Albert Einstein: Philosopher-Scientist (Ed. P. A Schilpp, La Salle: Open Court Publishing Company - The Library of Living Philosophers. Vol. VII). See, in particular, Autobiographical Notes, Reply to Criticisms and other relevant essays.

Pais A (1982) Subtle is the Lord ... The science and the life of Albert Einstein (Oxford: Clarendon Press) and references therein

[2] See, for details, Wagh S M (2004) Some fundamental issues in General Relativity and their resolution Database: gr-qc/0402003 and references therein

See also, Wagh S M (2004) A Theory of the Total Field, to be submitted.

[3] von Laue M (1970) essay in Albert Einstein:
Philosopher-Scientist (Ed. P A Schlipp, La Salle: Open Court Publishing Company - The Library of Living Philosophers, Vol. VII).

Kramer D, Stephani H, MacCallum M A H and Herlt E (1980) Exact Solutions of Einstein’s Field Equations (Cambridge University Press, Cambridge) and references therein

C W Misner, K S Thorne and J A Wheeler (1973) Gravitation (W H Freeman) and references therein

Einstein A (1970) in Albert Einstein: Philosopher-Scientist (Ed. P A Schlipp, La Salle: Open Court Publishing Company - The Library of Living Philosophers, Vol. VII) (Remarks concerning the essays brought together in this co-operative volume) p. 666

Wagh S M (2002) Classical formulation of Cosmic Censorship Hypothesis, Database: gr-qc/0201041

Einstein A (1968) Relativity: The Special and the General Theory (Methuen & Co. Ltd, London) (See, in particular, Appendix V: Relativity and the Problem of Space.)

See, also, Lorentz H A, Einstein A, Minkowski H, Weyl W (1952) The Principle of Relativity: A collection of original papers on the special and general theory of relativity. Notes by A Sommerfeld (Dover, New York)

Joshi K D (1983) Introduction to General Topology (Wiley Eastern, New Delhi) and references therein

Nadkarni M G (1995) Basic Ergodic Theory (Texts and Readings in Mathematics - 6: Hindustan Book Agency, New Delhi) and references therein

Parthasarathy K R (1967) Probability measures of Metric spaces (Academic Press, New York) and references therein

Wagh S M (2004) On the continuum origin of Heisenberg’s indeterminacy relations Database: physics/0404066

Wagh S M (2004) On cosmological considerations in a theory of the total field (in preparation)

Einstein commented on this essay as: Max von Laue: An historical investigation of the development of the conservation postulates, which, in my opinion, is of lasting value. I think it would be worth while to make this essay easily accessible to students by way of independent publication. —

Therefore, just as various theoretical conclusions based on the existence of the absolute space of Newton’s theory were unjustified, destined to be failure from the very beginning, the solutions to the equations of the pure gravitational field are similarly not justifiable. Clearly, a black hole is one of the many “children” of the “ghost” of the field equations of the pure gravitational field and, hence, is itself a ghost. Similar is the case with a “naked singularity.”

Then, anything done employing such “children of the ghost” is dubious. For example, the works of the author in “The energetics of black holes in electromagnetic fields by the Penrose process” (Wagh, S.M. and Dadhich, N.) Physics Reports, vol. 183, p. 137 (1989) (and references therein) are, simply, dubious, since these works use this “ghost” of the equations of the pure gravitational field in every possible manner. A naked singularity, it too being dubious, is then not useful for any astrophysical purposes.

It is then evident that efforts (for example, see, McClintock J E, Narayan R, Rybicki G B (2004) Database: astro-ph/0403251 and references therein) to establish the existence of a black hole horizon on the basis of observations of astronomical sources are not justifiable. The corresponding observations must then admit explanations of some entirely different nature.

See also Section VIII.

There certainly are (philosophical) weaknesses of such an argument. However, we shall not enter the relevant issues here. It suffices for our purpose here to say that “the space must be indistinguishable from physical bodies.”

We define an equivalence relation “∼” such that x ∼ y iff ℓ(x, y) = 0 where ℓ is the pseudo-metric distance defined on the space X. Denote by Y the set of all equivalence classes of X under the equivalence relation ∼. If A, B ∈ Y are two equivalence classes, then let e(A, B) = ℓ(x, y) where x ∈ A and y ∈ B. The (metric) function e on Y is the canonical distance.

This is a field-theoretic comprehension, in a definite sense, of the energy-momentum tensor.

Recall that Einstein regarded the correctness of Heisenberg’s indeterminacy relations as being “finally demonstrated”.

[1] See also Section VIII.

[2] There certainly are (philosophical) weaknesses of such an argument. However, we shall not enter the relevant issues here. It suffices for our purpose here to say that “the space must be indistinguishable from physical bodies.”

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[4] This is a field-theoretic comprehension, in a definite sense, of the energy-momentum tensor.

[5] Recall that Einstein regarded the correctness of Heisenberg’s indeterminacy relations as being “finally demonstrated”.

[6] Kramer D, Stefani H, MacCallum M A H and Herlt E (1980) Exact Solutions of Einstein’s Field Equations (Cambridge University Press, Cambridge) and references therein

[7] C W Misner, K S Thorne and J A Wheeler (1973) Gravitation (W H Freeman) and references therein

[8] Einstein A (1970) in Albert Einstein: Philosopher-Scientist (Ed. P A Schlipp, La Salle: Open Court Publishing Company - The Library of Living Philosophers, Vol. VII) (Remarks concerning the essays brought together in this co-operative volume) p. 666

[9] Wagh S M (2002) Classical formulation of Cosmic Censorship Hypothesis, Database: gr-qc/0201041

[10] Einstein A (1968) Relativity: The Special and the General Theory (Methuen & Co. Ltd, London) (See, in particular, Appendix V: Relativity and the Problem of Space.)

[11] See, also, Lorentz H A, Einstein A, Minkowski H, Weyl W (1952) The Principle of Relativity: A collection of original papers on the special and general theory of relativity. Notes by A Sommerfeld (Dover, New York)

[12] Joshi K D (1983) Introduction to General Topology (Wiley Eastern, New Delhi) and references therein

[13] Nadkarni M G (1995) Basic Ergodic Theory (Texts and Readings in Mathematics - 6: Hindustan Book Agency, New Delhi) and references therein

[14] Parthasarathy K R (1967) Probability measures of Metric spaces (Academic Press, New York) and references therein

[15] Wagh S M (2004) On the continuum origin of Heisenberg’s indeterminacy relations Database: physics/0404066

[16] Wagh S M (2004) On cosmological considerations in a theory of the total field (in preparation)

[17] Einstein commented on this essay as: Max von Laue: An historical investigation of the development of the conservation postulates, which, in my opinion, is of lasting value. I think it would be worth while to make this essay easily accessible to students by way of independent publication. —

[18] Then, anything done employing such “children of the ghost” is dubious. For example, the works of the author in “The energetics of black holes in electromagnetic fields by the Penrose process” (Wagh, S.M. and Dadhich, N.) Physics Reports, vol. 183, p. 137 (1989) (and references therein) are, simply, dubious, since these works use this “ghost” of the equations of the pure gravitational field in every possible manner. A naked singularity, it too being dubious, is then not useful for any astrophysical purposes.

[19] It is then evident that efforts (for example, see, McClintock J E, Narayan R, Rybicki G B (2004) Database: astro-ph/0403251 and references therein) to establish the existence of a black hole horizon on the basis of observations of astronomical sources are not justifiable. The corresponding observations must then admit explanations of some entirely different nature.