Condition for reducing the beam squint of left-handed leaky wave antennas composed of CRLH transmission lines

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Abstract: The unit cell of a composite right-/left-handed (CRLH) rod-shaped leaky wave antenna (LWA) is represented by a lossless T-type circuit, and the circuit parameters that suppress the beam squint with respect to the frequency fluctuation are investigated. As a result, it was found that reducing the right-handed circuit parameter values contributes to suppressing the beam squint. In addition, it was found that the beam squint depends on the ratio of right-handed inductance and left-handed inductance when the CRLH transmission line was formed on the basis of a conventional transmission line, and a ratio condition that minimizes the beam squint was derived.

Keywords: Metamaterial, CRLH transmission line, leaky wave antenna, beam squint, left handed.

Classification: Antennas and propagation

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1 Introduction

A left-handed rod antenna was proposed and introduced as a small cell base station antenna [1]. This antenna was developed for the 3.5 GHz band and was configured by a leaky wave antenna (LWA) composed of a composite right-/left-handed transmission line (CRLH-TL). The problem to expand the operating band to 3.7 GHz band, which has been assigned for 5G [2], with LWAs is beam squint [3]. A method for reducing the beam squint of an LWA by adjusting the circuit parameters of the CRLH-TL has been reported [4]. In [4], it is stated that the beam squint of an LWA can be effectively reduced when LWA operations are performed in the right-handed region, and the reduction in the beam squint in a right-handed LWA is investigated. Other methods to reduce the beam squint have also been studied [5]-[9]. However, the LWA used for the small cell base station reported in [1] operates in the left-handed region and it is desirable to add nothing to make the antenna inconspicuous.

This letter investigates how much the beam squint of the LWA composed of CRLH-TL can be reduced by changing only the CRLH-TL configuration when the LWA operates in the left-handed region. This study focuses on expanding the operating bandwidth of the LWA from the 3.5 GHz band to the 3.7 GHz band and examines the beam squint characteristics with the equivalent circuit of the CRLH-TL. The conditions required for beam squint reduction are discussed. Finally, the beam squint characteristics of the LWA considering the feasibility of CRLH-TL are investigated. The condition that minimizes the beam squint in this situation is derived.

2 Circuit parameters that reduce the beam squint

A symmetric T-type circuit shown in Fig. 1 is used as the equivalent circuit of the CRLH-TL unit cell. The parameters are based on the antenna structure given in [1]. $p$ is 14 mm in this study. The transmission phase $\beta p$ of the two-
port network is given by the ABCD parameters, as shown in the following equation [10]:

$$\beta_p = \cos^{-1}\left(\frac{A + D}{2}\right)$$  \hspace{1cm} (1)

and, the main beam direction $\theta_t$ of the LWA is given by

$$\theta_t = \sin^{-1}\left(\frac{\beta_p}{k_0 p}\right)$$ \hspace{1cm} (2)

where $k_0$ is the wavenumber in free space, $\theta_t$ is based on the broadside direction of the LWA, and the positive $\theta_t$ direction is defined as downward. A random search was performed to examine the beam squint reduction characteristics. $\beta_p$ values at 3.5 GHz and 3.8 GHz are considered representative of the 3.5 GHz and 3.7 GHz bands in this study. Although the equivalent circuit has four parameters, due to Eq. (1) and (2), the number of independent parameters is reduced to three when the main beam direction is specified. In this study, $L_R, C_R$, and $L_L$ as shown in Fig. 1 were randomly set, and $C_L$ was defined to satisfy $\theta_t$ of 30 degrees at 3.5 GHz, which corresponds to $\beta_p$ of 29.4 degrees. The number of trials is 10,000.

![Fig. 1. Equivalent circuit of CRLH-TL unit cell.](image)

Fig. 2(a) shows the dependency of the $L_R$ and $L_L$ on the $\beta_p$ difference $\Delta\beta_p$, which is defined by $\beta_p_{3.5GHz} - \beta_p_{3.8GHz}$, and Fig. 2(b) shows the dependency of the $C_R$ and $C_L$ on the $\Delta\beta_p$. These are shown only when operating in the left-handed region. The y-axis range of capacitance is limited to a maximum of 10 pF to clarify the difference between $C_R$ and $C_L$. The blue and red circles represent the right- and left-handed parameters. Fig. 2(c) shows the relationship between $\beta_p$ and $\omega$ when the parameters are optimized. By optimizing both the right- and left-handed parameters, $\Delta\beta_p$ is reduced to approximately 2.5 degrees. The corresponding beam squint $\Delta\theta_t$ is approximately 5 degrees, which can be calculated by Eq. (2). The optimized right-handed parameters, $L_R = 0.022$ nH and $C_R = 0.018$ pF in this simulation, are smaller than the values of the LWA in [1], which were 4.5 nH and 0.32 pF, respectively. It is also seen from Figs. 2(a) and (b) that the right-handed parameters tend to be reduced to reduce the beam squint due to frequency changes.
3 Discussion on the conditions required for circuit parameters to suppress the beam squint

Fig. 2(d) shows the relationship between frequency $\omega$ and transmission phase $\beta_p$ of the right-handed, left-handed, and CRLH transmission lines, where $\omega_{se} = 1/\sqrt{L_RC_L}$ and $\omega_{sh} = 1/\sqrt{L_CL_R}$. The cutoff frequency $\omega_{CL}$ is given by

$$\omega_{cL} = \omega_0 \sqrt{\frac{\left[k + (2/\omega_L)^2\right]\omega_0^2 - \sqrt{\left[k + (2/\omega_L)^2\right]^2\omega_0^4 - 4}}{2}} \tag{3}$$

where $\omega_0 = \sqrt{\omega_{se}\omega_{sh}}$, $k = L_RC_L + L_CL_R$, $\omega_L = 1/\sqrt{L_CL_R}$. $\beta_p$ of the CRLH-TL appears as the combination of the left-handed and right-handed transmission line contributions. To reduce $\Delta\beta_p$, since $\beta_p$ of the considered LWA operates relatively close to the $\omega$ axis, it is necessary to increase $\omega_{se}$ and $\omega_{sh}$, and reduce $\omega_{CL}$. $\omega_{se}$ increases as $L_R$ and $C_L$ decrease, and $\omega_{sh}$ increases as $L_L$ and $C_R$ decrease. When $\omega_{se}$ and $\omega_{sh}$ increase, $\omega_0$ also increases. As $\omega_0$ increases, the value of the term $\left[k + (2/\omega_L)^2\right]\omega_0^2$ in Eq. (3) increases. If the second term of the numerator on the right side of Eq. (3) can be approximated to $\left[k + (2/\omega_L)^2\right]\omega_0^2$, the cutoff frequency $\omega_{CL}$ approaches zero. The term $\left[k + (2/\omega_L)^2\right]\omega_0^2$ can also be increased by decreasing $\omega_L$. $\omega_L$ decreases as $C_L$ and $L_L$ increase. However, to increase $\omega_{se}$ and $\omega_{sh}$, it is necessary to
decrease \(L_R\) and \(C_R\). \([k + (2/\omega_L)^2] \omega_0^2\) also increases as \(k\) increases. However, since \(k\) depends on \(L_RC_L\) and \(L_DC_R\), considering the above condition, \(k\) is smaller than \(2/\omega_L^2\). Therefore, if \(L_R\) and \(C_R\) are small and \(C_L\) and \(L_L\) are made large, the cutoff frequency becomes low and the resonance frequency becomes high. Then, the fluctuation of \(\beta p\) according to the frequency change is considered to be small. The random search results shown in Figs 2(a) and (b) follow this discussion.

### 4 Consideration of the feasibility of the CRLH-TL

To consider the feasibility of CRLH-TL, the right-handed circuit parameters are assigned as parameters given on the transmission line. The right-handed circuit parameters of transmission lines satisfy \(2L_RC_R = \varepsilon \mu p^2\) [11], where \(\varepsilon\) and \(\mu\) represent the permittivity and permeability of the medium that configures transmission line. The product of the propagation constant and the length of the transmission line unit cell \(\gamma = \sqrt{2ZY}\), where \(Z = j\omega L_R + \frac{1}{j\omega C_L}\) and \(Y = j\omega C_R + \frac{1}{j\omega L_L}\). From the above relations, \((\beta p)^2\) can be expressed as

\[
(\beta p)^2 = \omega^2 \varepsilon \mu p^2 + \frac{2L_R}{L_L} + \frac{\varepsilon \mu p^2}{C_L} \frac{1}{L_R} \quad (4)
\]

Here, \(\beta p\) at frequency \(f_1\) (or \(\omega_1\)) is specified. Let the transmission phase of the unit cell at \(f_1\) and \(f_2\) be \(\beta_1 p\) and \(\beta_2 p\), \(M = (\beta_1 p)^2 - \omega_1^2 \varepsilon \mu p^2\), and \(L_R/L_L\) be \(x\), \((\beta p)^2\) can be expressed as a function of \(x\) from Eq. (4) as follows:

\[
(\beta p)^2 = \omega^2 \varepsilon \mu p^2 + \frac{2(M + 2x)}{\omega_1^2 (2 - N/x)} - 2 \left( x + \frac{N M + 2x}{2} - N \right) \quad (5)
\]

where \(N = \omega_1^2 \varepsilon \mu p^2\). In the range of \(0 \leq |\beta_2| \leq |\beta_1|\) in the left-handed region, \(x\), which minimizes \(|\beta_2 p - \beta_1 p| \equiv \Delta \beta p\), is equal to \(x\), which minimizes \(|(\beta_2 p)^2 - (\beta_1 p)^2| \equiv \Delta (\beta p)^2\). \(\Delta (\beta p)^2\) between frequencies \(\omega_2\) and \(\omega_1\) is expressed as

\[
\Delta (\beta p)^2 = (\omega_2^2 - \omega_1^2) \varepsilon \mu p^2 + 2 \left( \frac{\omega_1^2}{\omega_2^2} - 1 \right) \frac{M x + 2x^2}{2x - N} \quad (6)
\]

The derivative of \(\Delta (\beta p)^2\) is

\[
\frac{d\Delta (\beta p)^2}{dx} = \frac{2}{(2x - N)^2} \left( \frac{\omega_1^2}{\omega_2^2} - 1 \right) \left( 4x^2 - 4Nx - MN \right) \quad (7)
\]

\(x\), which minimizes \(\Delta (\beta p)^2\), is given by the solution of equation \(4x^2 - 4Nx - MN = 0\). The solution is

\[
x = \frac{N \pm \sqrt{N^2 + MN}}{2} \quad (8)
\]

From Eqs. (7) and (8), it can be found that the minimum \(\Delta (\beta p)^2\) is determined by \(x\) (the ratio of \(L_R\) to \(L_L\)) and the condition that minimizing \(\Delta (\beta p)^2\) does not depend on the frequency \(\omega_2\). When the CRLH-TL is configured in a vacuum and the parameters \(p\) and \(\beta p\) at 3.5 GHz are the same as those
described in Section 2, $x$, which minimizes $\Delta(\beta p)^2$, is obtained as 0.79 and 0.26 from Eq. (8).

Fig. 3 shows the dependency of $\beta p$ at 3.8 GHz calculated with Eq. (1). In Fig. 3, $L_R$ is fixed to 1 nH and $C_R$ is determined by $2L_RC_R = \epsilon\mu p^2$.

$L_L$ changes from 0.1 nH to 6 nH and $C_L$ is determined to satisfy $\beta p$ at 3.5 GHz to be 29.6 degrees, and $p$ is 14 mm. $\beta p$ at 3.5 GHz is also plotted in Fig. 3. In the right-handed operation, $\beta p$ at 3.8 GHz is greater than $\beta p$ at 3.5 GHz. In the left-handed region, $\beta p$ at 3.8 GHz increases from an $L_L$ of 0.8 nH and the peak of $\beta p$ is at 1.27 nH, which corresponds to an $x$ of 0.79. The value agrees with the solution in Eq. (8). The peak value of $\beta p$ is 17.34 degrees and $\Delta \beta p$ is 12.27 degrees in Fig. 3. The minimum $\Delta \beta p$ when $x$ is 0.79 calculated with the square root of Eq. (4) at 3.8 GHz is 12.3 degrees, which also agrees with the simulation that uses Eq. (1). The corresponding main beam is shifted at least to 16.0 degrees at 3.8 GHz only by adjusting the equivalent circuit parameters of the CRLH-TL. The beam shift is increased by 9 degrees compared to if the three circuit parameters are freely set.

5 Conclusion

This letter investigated the conditions for reducing the beam squint of an LWA composed of an CRLH-TL that operates in the left-handed region. A symmetric T-type equivalent circuit of the CRLH-TL was used for the analysis. As a result, it was found that it is desirable to set the right-handed parameter small in order to reduce the beam squint. In addition, it was found that when CRLH-TL is formed based on conventional TL, the condition that minimizes the beam squint is determined by the ratio between $L_R$ and $L_L$. The ratio between $L_R$ and $L_L$, that minimizes the beam squint, does not depend on the frequency, and a lower limit exists to reduce the beam squint of LWAs composed of CRLH-TLs operating in the left-handed region. Therefore, other phase control parameters such as a transmit array are required to reduce the beam squint.