Zak phase reconstruction enabled by thermal motion of atoms

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Ultracold atoms provide a flexible platform for simulating topological phases. However, they are intrinsically prone to thermal noise, which can overwhelm topological response and hamper promised applications. On the other hand, topological invariants can be reconstructed from the energy spectra of particles subjected to a static force, based on the polarization relation between Wannier-Stark ladders and Zak phases. Such spectroscopic methods circumvent the obstacle to measuring topological invariants in thermal atoms. Here we reconstruct Zak phases from the anti-crossing between Wannier-Stark ladders in room-temperature superradiance lattices. Without dynamical manipulation routinely needed in ultracold atoms, we evaluate Zak phases directly from the absorption spectra. The thermal atomic motion, an essential source of noise in cold atoms, plays a key role in extracting Zak phases. Our approach paves the way of measuring topological invariants and developing their applications in room-temperature atoms.

Topological matter has promising applications in noise resilient devices and quantum information processing\cite{1,2}, thanks to the robust topological response guaranteed by global geometric quantities of the Bloch bands, namely, the topological invariants\cite{3–6}. These invariants only change stepwisely when the bulk goes through a topological phase transition, which involves band gap closing and reopening. Characterizing topological invariants is a central task in synthesizing and simulating topological phases. They are usually measured by the response from gapless edge states based on the bulk-edge correspondence. However, edges are not always available in atomic quantum simulators\cite{7,8}. New methods in measuring topological invariants in bulk are highly wanted. Along this line techniques based on reciprocal-space interference\cite{9,10}, quench dynamics\cite{11,12}, and Hall transport\cite{13,14} have been developed in atomic simulators.

Previous experiments of determining topological invariants from bulk response in ultracold atoms rely on dynamic evolution or adiabatic manipulation\cite{9–14}. On the other hand, spectroscopic methods\cite{15,19} have been proposed to measure the topological invariants by applying a constant force which turns the Bloch energy bands into Wannier-Stark ladders (WSLs)\cite{20} with equidistant discrete energies. In one-dimensional (1D) systems, the energies of the WSLs are determined by Wannier centers (WCs)\cite{21} reflecting the Zak phases\cite{3} of the energy bands\cite{22,24}. Such spectroscopic methods circumvent the requirements of atom cooling and enable the measurement of topological invariants in thermal atoms.

Here we report the reconstruction of Zak phases of the Rice-Mele (RM) model\cite{25} through the anti-crossing between the WSLs of superradiance lattices\cite{26,28}, which are momentum-space lattices of timed Dicke states\cite{29}. The Doppler shift of moving atoms introduces an effective force in the superradiance lattice. Different velocity groups of thermal atoms provide a set of continuous values of the effective force, which results in WSLs with energies determined by the Zak phases\cite{15,17}. The absorption spectra of superradiance lattices are characterized by peaks and dips due to the anti-crossing between the WSLs from different energy bands. We obtain the Zak phases from the locations of the anti-crossing points. Two celebrated versions of the RM model, the Semenoff insulator and the Su-Shrieffer-Heeger (SSH) model\cite{30}, are investigated in detail. We also demonstrate the Zak phase reconstruction for general RM models. Our approach can be generalized to identify topological invariants in higher dimensions\cite{31,32}.

We perform the experiment with the hyperfine levels of the \textsuperscript{87}Rb D1 line in a standing-wave-coupled EIT configuration, as shown in Fig. 1(a) (see the complete setup in Supplemental Material\cite{33}). A weak probe field propagating in $x$ direction couples the ground state $|g\rangle \equiv |5^2S_{1/2}, F = 1\rangle$ to the excited state $|b\rangle \equiv |5^2P_{1/2}, F = 2\rangle$. The excited state is also coupled to a metastable state $|a\rangle \equiv |5^2S_{1/2}, F = 2\rangle$ by two strong counter-propagating light fields, forming a 1D bipartite superradiance lattice\cite{26,27}. The absorption spectra of the probe field is used to obtain the Zak phases of the superradiance lattices.

In order to clarify the effect of atomic motion, we start by analysing the contribution from atoms with different velocities in $x$ direction. For atoms in each velocity group, the opposite Doppler shifts of the two coupling fields lead to a linear potential in momentum-space\cite{26,10} (we set $\hbar = 1$ and see details in Supplemental
Material [26],
\[
H_f = \delta \sum_j [2j \hat{a}_j^{\dagger} \hat{a}_{2j} + (2j + 1) \hat{b}_{2j+1}^{\dagger} \hat{b}_{2j+1}],
\]
(1)
where the Doppler shift \(\delta \approx k_c v \approx k_p v\) with \(k_p, k_c\) being the probe (coupling) field wave vector amplitude and \(v\) being the velocity of the atoms in \(x\) direction, \(\hat{d}^{\dagger}_j = \sqrt{1/N} \sum_m |d_m\rangle \langle g_m| \exp(ik_j x_m)\) \((d = a, b)\) is the creation operator of the timed-Dicke state [29] \(|\hat{d}_j\rangle \equiv \hat{d}^{\dagger}_j|g_1,g_2,\ldots,g_N\rangle\) with wave vector \(k_j = k_p + (j - 1) k_c \approx j k_c\) \((j\) is an integer), \(m\) labels the \(m\)th atom at the position \(x_m\), \(N\) is the total number of atoms within the velocity range \([v - \Gamma/2k_c, v + \Gamma/2k_c]\) where \(v\) satisfies the Maxwell distribution and \(\Gamma\) is the decay rate of the state \(|b\rangle\).

The total Hamiltonian \(H = H_s + H_p + H_f\), where \(H_s\) and \(H_p\) are interaction Hamiltonians involving the coupling and probe fields, respectively. \(H_s\) is the tight-binding Hamiltonian of the RM superradiance lattices [26,28], as shown in Fig. 1(b),
\[
H_s = \sum_j \Delta_e \hat{a}_j^{\dagger} \hat{a}_{2j} + \hat{a}_2^{\dagger} (\Omega_1 \hat{b}_{2j+1} + \Omega_2 \hat{b}_{2j-1}) + H.c.\]
(2)
where \(\Omega_1\) and \(\Omega_2\) are the Rabi frequencies of the co-propagating and counter-propagating coupling fields, and \(\Delta_e = \nu_e - \omega_{ba}\) with \(\nu_e\) being the coupling field frequency and \(\omega_{ba}\) being the transition frequency between states \(|a\rangle\) and \(|b\rangle\). The timed Dicke state \(|\tilde{b}_1\rangle\) in the superradiance lattice can be created from the ground state by \(H_p = \sqrt{N} \Omega_p e^{-i \Delta_p t} \hat{b}_1^{\dagger} + H.c.\), where the probe detuning \(\Delta_p = \nu_p - \omega_{bg}\) with \(\nu_p\) being the probe field frequency and \(\omega_{bg}\) being the transition frequency between states \(|b\rangle\) and \(|g\rangle\).

The WCs are the expectation positions of the Wannier functions [24]. For the Hamiltonian \(H_s\), the WCs in the \(n\)th unit cell \(r^{(n)}_{\pm}\) for the upper (+) and lower (−) bands are related to the geometric Zak phases by (in unit of distance between neighboring lattice sites) [22,23],
\[
r^{(n)}_{\pm} = 2n + \theta_{\pm}/\pi,
\]
(3)
where the Zak phases \(\theta_{\pm} \equiv i \int dx \langle u_{\pm}(x)|\partial_x|u_{\pm}(x)\rangle\) with \(|u_{\pm}(x)\rangle\) being the periodic Bloch functions in real space and the integration is over the whole Brillouin zone. Therefore, the Zak phases are the fractional parts of the corresponding WCs [22,23] as schematically depicted in Fig. 1(c)-(e). The values of the Zak phases are gauge dependent due to the freedom in choosing the origin of the unit cell [9]. Throughout this work, the gauge is uniquely fixed by the zero energy point of \(H_f\) (see Fig. 1(c)).

In the presence of a weak static force described by the Hamiltonian \(H_f\), the energy spectra split into discrete WSLs [15,17,40,41].

\[
E^{(n)}_{\pm}(\delta) = \varepsilon_{\pm} + r^{(n)}_{\pm} \delta,
\]
(4)
where \(\varepsilon_{\pm}\) denote the energies of the Bloch band centers (bc), defined as the average band energies of \(H_s\). From Eqs. (3) and (4), the Zak phases are obtained by \(\theta_{\pm} \equiv (\partial E^{(n)}_{\pm}/\partial \delta - 2n \pi)\).

The relation in Eq. (4) can be seen from the absorption spectra in Fig. 2(a)-(c) as functions of \(\delta\) for three different RM lattices, namely, the Semenoff insulator with \(\theta_{\pm} = \pi, \theta_{\pm} = 0\) (Fig. 2(a)), and two topologically distinct phases of SSH model with half-integer Zak phases \(\theta_{\pm} = 0.5\pi\) (Fig. 2(b)) and \(\theta_{\pm} = 1.5\pi\) (Fig. 2(c)). In the weak force regime where the coupling between the WSLs from different bands are negligible, the spectra...
Wannier-Stark ladders and the absorption spectra of the superradiance lattices. (a)-(c) The absorption spectra as functions of the Doppler shift $\delta$ and probe detuning $\Delta_p$. The white dashed lines indicate the uncoupled WSLs in Eq. (4). The highlighted numbers denote the values of $r_{[n]}$ (square) and $r_{[n]}$ (round) of the corresponding WSLs. The cyan arrows mark the anti-crossing $ac_{0,-1}$. (a) and (d) $\Omega_1 = \Omega_2 = 120$ MHz and $\Delta_\gamma = 298$ MHz; (b) and (e) $\Omega_1 = 125$ MHz, $\Omega_1/\Omega_2 = 5.3$, and $\Delta_\gamma = 0$; (c) and (f) $\Omega_2 = 138$ MHz, $\Omega_1/\Omega_2 = 0.17$, and $\Delta_\gamma = 0$. (d)-(f) The absorption spectra of the room-temperature superradiance lattices. The experimental data and numerical simulation (obtained by integrating $\delta$ in (a)-(c)) are shaded with blue and red colors, respectively. The dips and peaks in the spectra capture the locations of the band centers (bc) and anti-crossing points (ac_{0,-n}). The blue and red dashed lines show the dimerized limits of $\Omega_2 = 0$ and $\Omega_1 = 0$. It is schematically shown that a $\delta$ corresponds to atoms with a certain velocity and the absorption spectra are obtained by integrating $\delta$.

Figure 2. Wannier-Stark ladders and the absorption spectra of the superradiance lattices. (a)-(c) The absorption spectra as functions of the Doppler shift $\delta$ and probe detuning $\Delta_p$. The white dashed lines indicate the uncoupled WSLs in Eq. (4). The highlighted numbers denote the values of $r_{[n]}$ (square) and $r_{[n]}$ (round) of the corresponding WSLs. The cyan arrows mark the anti-crossing $ac_{0,-1}$. (a) and (d) $\Omega_1 = \Omega_2 = 120$ MHz and $\Delta_\gamma = 298$ MHz; (b) and (e) $\Omega_1 = 125$ MHz, $\Omega_1/\Omega_2 = 5.3$, and $\Delta_\gamma = 0$; (c) and (f) $\Omega_2 = 138$ MHz, $\Omega_1/\Omega_2 = 0.17$, and $\Delta_\gamma = 0$. (d)-(f) The absorption spectra of the room-temperature superradiance lattices. The experimental data and numerical simulation (obtained by integrating $\delta$ in (a)-(c)) are shaded with blue and red colors, respectively. The dips and peaks in the spectra capture the locations of the band centers (bc) and anti-crossing points (ac_{0,-n}). The blue and red dashed lines show the dimerized limits of $\Omega_2 = 0$ and $\Omega_1 = 0$. It is schematically shown that a $\delta$ corresponds to atoms with a certain velocity and the absorption spectra are obtained by integrating $\delta$.

When a pair of WSLs from different bands ($E_{[n]}^+ \neq E_{[n]}^-$) have the same energy for a large $\delta$, the interband coupling removes their degeneracy and results in an anti-crossing [11, 12] denoted by $ac_{n,m}$. Their positions in the energy-force diagram can be estimated by the degeneracy points of the uncoupled WSLs [13] satisfying $E_{[n]}^+ = E_{[n]}^- = \Delta_{n,m}$, where $\Delta_{n,m}$ is the probe detuning of the corresponding anti-crossing point. The anti-crossing $ac_{0,-1}$, marked by the arrow in Fig. 2(a), is due to the coupling between the WSLs $E_{[0]}^+ = \varepsilon_+ + \delta$ and $E_{[-1]}^- = \varepsilon_+ - 2\delta$, from which we obtain $\delta = (\varepsilon_+ - \varepsilon_-)/3$ and $\Delta_{0,-1} = (\varepsilon_+ - 2\varepsilon_-)/3$. On the other hand, for the half-integer topological invariants of the SSH models, the prominent anti-crossing $ac_{0,-1}$ locates at $\delta = (\varepsilon_+ - \varepsilon_-)/2$ and $\Delta_{0,-1} = (\varepsilon_+ + 3\varepsilon_-)/4$ in Fig. 2(b) and $\Delta_{0,-1} = (3\varepsilon_+ - \varepsilon_-)/4$ in Fig. 2(c). The values of $\Delta_{n,m}$ obtained from the absorption spectra are the key to extract the Zak phases.

The absorption of the superradiance lattice is contributed by all atoms with velocities in Maxwell distribution [14]. We obtain Fig. 2(d)-(f) from the WSLs in Fig. 2(a)-(c) by integrating $\delta$, which has a distribution width about 500 MHz and covers all relevant values for the Zak phase reconstruction. The anti-crossings and band centers modify the PDOS drastically and their signatures can be easily picked out in the absorption spectra, as shown in Fig. 2(d)-(f). In particular, we notice that the anti-crossing leads to the decrease of PDOS, yielding a dip in the absorption spectra. The band centers are characterized by a dip in most cases or a peak (for the upper band center when $\Delta > 0$), due to the modification of the PDOS of the state $|\psi_1\rangle$ resulted from Wannier-Stark localization [15].

In order to reconstruct the Zak phases, we shall quantify the common features in the absorption spectra of lattices with the same Zak phase. In Fig. 2(a), we main-
In Fig. 3(c), we obtain two equations of superradiance lattices with \( \Delta \) for \((\text{points})\) compared with the normalized ratio of WCs (dashed lines) for \((\text{data sets})\) the band centers and anti-crossings.

\[
\begin{align*}
\text{In conclusion, we realize the spectroscopic reconstruction of the Zak phases of superradiance lattices. Without trapping atoms or controlling their velocities [9, 11, 12], we take advantage of the atomic thermal motion [36, 17] and locate the anti-crossing points of the WSLs, which give out the values of the Zak phases. This method can be directly generalized to measure Chern numbers in two dimensional lattices [17, 31] by introducing more coupling fields [18, 50] in the current framework as well as in photonic lattices [31] and synthetic dimensions [32, 53]. Our results push forward the room-temperature quantum simulation of topological phases and pave the way to detect multipole moments in higher-order topological insulators [19, 54, 35].}
\end{align*}
\]

The authors thank Luqi Yuan for useful discussions. This work was supported by the National Key Research and Development Program of China (Grants R0_{-2} and conclude that \( \theta_- \approx \pi \) and \( \theta_+ \approx 0 \), as shown in Fig. 3(c).

For the SSH models, we keep \( \Delta_c = 0 \) and tune the Rabi frequencies of the two coupling fields from almost dimerization to the topological phase transition point. The Zak phases are maintained the same while the anti-crossing energy gaps increase from top to bottom in Fig. 3(b). The measured \( R_{n,m} \) in Fig. 3(d) agree well with their expected values and the reconstructed Zak phases are obtained, \( \theta_\pm \approx 0.5\pi \), as shown in Fig. 3(f).

For a general RM Hamiltonian, i.e., \( \hat{H}_s \) with \( \Omega_1 \neq \Omega_2 \) and \( \Delta_c \neq 0 \), the Zak phases are neither integers nor half-integers. Along the yellow line in the phase diagram in Fig. 4(a), we measure the absorption spectra to obtain \( R_{0,-1} \) and \( R_{-1,0} \) for each coupling field detuning \( \Delta_c \), as shown in Fig. 4(b), and accordingly reconstruct the Zak phases in Fig. 4(c), in comparison with the theoretical prediction as indicated by the dashed lines.
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