Protostellar Jets Driven by a Disorganized Magnetic Field

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Abstract. We have proposed a viscoelastic model of the Maxwell stresses due to the disorganized magnetic field in MRI-driven MHD turbulence. Viscoelastic fluids in the laboratory are known to produce jet-like structures under the action of a rotating sphere. Here we argue that a similar mechanism may help explain jets in protostellar systems. Such jets would be driven not by large-scale organized magnetic fields, but by the mean-field stresses of small-scale tangled magnetic fields.

1. Introduction

Broadly speaking, the theory of jet acceleration and collimation has, over the past decades, progressed from hydrodynamical models to models incorporating magnetic fields. These models typically involve acceleration by what may be called ordered, large-scale fields. The long-range effects of ordered fields enable, for example, the coupling of a disk wind or jet to an accretion disk, which is a convenient source of power.

In contrast, the predominant role of magnetic fields in accretion disks is presumed to be the creation of a large effective turbulent viscosity, through the action of the Balbus-Hawley magnetorotational instability (MRI). The field produced by the MRI, as shown in simulations, has a significant contribution due to what might be characterized as a disorganized, or tangled, field. That a tangled field can have such important dynamical consequences as angular momentum transport in accretion disks leads one to ask what other consequences such a field might have. We propose that one consequence of this tangled field may be the driving of an axial outflow, i.e. a jet.

2. Turbulent Maxwell Stresses

We break the magnetic field into mean and fluctuating parts, and the mean is a spatial average over some sufficiently small length scale so as to not blur out dynamically important details. Using an overbar to denote averaging, we write $B_i = \bar{B}_i + B'_i$, where $B_i$ is a component of the total field, and $B'_i$ is the fluctuating part. The field $B'$ by definition must have structure on scales equal to or less than the scale of averaging. By a “tangled field” we mean that the dominant contribution to the magnetic field is from the small-scale fluctuating component; in other words, $\bar{B}^2 \ll \overline{B^2}$, or, equivalently, $(\overline{B'})^2 \gg \overline{B^2}$. In what
follows we will assume that the mean field is negligible, and thus \( B_i = B'_i \). While the mean field may be zero, the mean stress will not be for \( B' \neq 0 \).

The force on a fluid element due to the small-scale component of the magnetic field (\( i.e. B' \)) may be written as the divergence of the turbulent Maxwell stress tensor \( \tau_{ij}^{\text{turb}} \),

\[
\mathbf{f}^{\text{mag}}_i = -\partial_j \tau_{ij}^{\text{turb}},
\]

(1)

where the turbulent Maxwell stress tensor is given by

\[
4\pi \tau_{ij}^{\text{turb}} = B'_i B'_j - \frac{1}{2}(B')^2 \delta_{ij} = 4\pi M_{ij} - 2\pi M_{kk} \delta_{ij}
\]

(2)

and we have defined \( 4\pi M_{ij} \equiv B'_i B'_j \). Due to the MRI, the turbulent Maxwell stress tensor in an accretion disk has a large off-diagonal component, which (along with the Reynolds stress) is interpreted as an effective viscous stress; furthermore, there is a large on-diagonal azimuthal component, which represents the more-or-less passive advection of the field by the background shear. The importance of this latter stress is the focus of this paper.

3. Viscoelastic Models of MHD Turbulence in Disks

As has been pointed out by Ogilvie (2001) and by Williams (2001), there is a potentially interesting analogy between a tangled field in MHD turbulence and tangled polymers in solution. We emphasize here that in both cases there is a stochastic element which tends to isotropize the system — random turbulent motions of the fluid in the case of MHD turbulence and thermal Brownian motion in the case of polymers in solution — and in both cases the action of large-scale (\( i.e. \) mean-field) shear acts to destroy this isotropy by aligning the filaments, be they polymers or magnetic field lines.

To be more explicit, we turn to the vector advection-diffusion equation for the field:

\[
\partial_t B_i = B_j \partial_j v_i - v_j \partial_j B_i - B_i \partial_j v_j + \eta \partial_{jj} B_i.
\]

(3)

As with the magnetic field, let us decompose the velocity field into a mean and fluctuating part, \( v_i = \bar{v}_i + v'_i \). From equation (3) we may derive the advection equation for the turbulent Maxwell stress tensor. Upon averaging, there will appear various cross-correlation terms as well as dissipative terms (abbreviated as \( cc. \) and \( diss. \)), none of which we write explicitly; they will be modeled below.

For simplicity, we write the transport equation for the tensor \( M_{ij} \) instead of for \( \tau_{ij}^{\text{turb}} \); the latter may easily be obtained from the former. We obtain:

\[
(D_t M)_{ij} \equiv (\partial_t + \bar{v}_k \partial_k) M_{ij} - (\partial_k \bar{v}_i) M_{kj} - M_{ik} (\partial_k \bar{v}_j) + 2(\partial_k \bar{v}_k) M_{ij} = cc. + diss.
\]

(4)

The notation \( D_t \) is chosen to emphasize that the tensor advective operator defined above (known as the upper-convected invariant derivative when the flow is divergenceless) is an extension of the more familiar scalar advective derivative, \( \partial_t + v_i \partial_i \), which is often written \( D_t \). If set equal to zero, this operator acting on \( M_{ij} \) would cause the stress in a steady shear flow to grow without bound.
This is the “elastic” part of the response of the magnetized fluid to shear. So long as the dissipation time scale is shorter than the compressive time scale, the compressibility effects should be negligible.

Of course the stress does not grow without bound, and this may be modeled by the inclusion of a relaxation term. The simplest relaxation term is the stress $M_{ij}$ itself, divided by a relaxation time $s$. This will appear as a dissipative term, and we need yet an additional term to act as a source term.

The simplest source term is the effective viscous stress tensor; this is the so-called Maxwell model (Ogilvie 2001). Alternatively, one may assume explicitly that random turbulent motions will tend to isotropize the stress tensor (Williams 2001). Lastly, one may assume that the Maxwell stresses are produced in proportion to the Reynolds stresses (this paper). In this last case, we expect that there may be a useful model for the Maxwell and Reynolds stresses, loosely derivable from first principles, in which there appear coupled equations for the evolution of the two stress tensors. We leave this project to a future paper. The above three models are:

Model A:  
$$s(D_{[\epsilon]}^{[s]}M)_{ij} + M_{ij} = a(\partial_i \bar{v}_j + \partial_j \bar{v}_i)$$  
(5)

Model B:  
$$s(D_{[\epsilon]}^{[s]}M)_{ij} + M_{ij} = a\delta_{ij}$$  
(6)

Model C:  
$$s(D_{[\epsilon]}^{[s]}M)_{ij} + M_{ij} = aR_{ij}$$  
(7)

Model C is not a complete model in the absence of a further relation to close the system of equations, but we may still compare its predictions with simulations. In all three cases, we have two free parameters, namely the relaxation time $s$ and the coefficient $a$ for the source term that appears on the right. These determine six parameters, the three diagonal and the three independent off-diagonal components of the symmetric matrix $M_{ij}$. Note that Ogilvie (2001) considers a Maxwell model for the full turbulent stress (up to magnetic pressure) $T_{ij} = M_{ij} - R_{ij}$, where $R_{ij}$ is the Reynolds stress; we will refer to this model as model A'.

A nice set of simulations of the MRI are those of Hawley, Gammie, & Balbus (1995) and we use their results for comparison of the models. It should be noted that none of these models as they stand include the effects of a mean field, whereas all the runs of Hawley et al. start with a mean seed field to get the MRI going. Due to flux conservation across the boundaries of the simulation box, the saturation of the MRI is not wholly independent of these initial conditions.

In the table below we compare best-fit models with results from Table 4 of Hawley et al., in which the seed field is azimuthal. In our table the parameters $W$ (for Weissenberg) and $b$ are, respectively, $s$ and $a$ in eqns. (7–9), normalized to the rate of shear. Note that models A, B, and C are fit to $M_{ij}$, whereas model A' is fit to $M_{ij} - R_{ij}$. For ease of comparison, we normalize all data to $-M_{\epsilon g}$, except for purposes of fitting parameter $b$. The reduced chi-square is artificially small in all cases as we have estimated the relative uncertainty in quantities by using the spatial variance as given in Hawley et al.; the relative goodness-of-fit is essentially unchanged if one performs a simple least-squares fit. Similar results are obtained in comparing the models to Table 2 of that paper, for which the seed field is vertical; most notable in that case is that the relaxation time obtained for all models is roughly $2/3$ the relaxation time in the case of an azimuthal seed field.
Table 1. Fits of Models A, B, C and $A'$ to Hawley et al.

| qty or index | $M_{ij}$ | A      | B      | C      | $M_{ij} - R_{ij}$ | $A'$ |
|--------------|----------|--------|--------|--------|-------------------|------|
| $rr$         | 0.511    | 0.000  | 0.249  | 0.516  | -0.452            | 0.000|
| $r\theta$    | -1.000   | -1.000 | -0.741 | -0.921 | -1.243            | -1.243|
| $r\theta$    | 3.953    | 3.961  | 4.153  | 4.054  | 3.389             | 3.386|
| $rr$         | -0.027   | 0.000  | -0.007 | -0.007 | -0.040            | 0.000|
| $z\theta$    | 0.173    | 0.000  | 0.249  | 0.178  | -0.159            | 0.000|
| We           | -1.971   | 2.977  | 2.036  | -1.363 | 1.363             |      |
| b            | -0.030   | 0.007  | 0.536  | -0.037 | 0.037             |      |
| $\chi^2/\nu$ |   -      | 0.135  | 0.038  | 0.003  | -      | 0.038 |

As can be seen, the models provide reasonable agreement with simulations, with model C providing the best fit, although as noted it is not a fully predictive model. In all three cases the dominant feature is the creation of a significant streamwise stress $M_{\theta\theta}$ by the advection of $M_{r\theta}$ (or $T_{\theta\theta}$ and $T_{r\theta}$, respectively, in the case of model $A'$, although it should be noted that the operator $D_i^{[a]}$ does not appear in the advection of the Reynolds stress $R_{ij}$). The creation of a large streamwise stress $M_{\theta\theta}$ is a reliable feature of any model for MHD turbulence in disks that includes a treatment of the advection of turbulent Maxwell stresses, so long as the relaxation time is not very much faster than the shear time scale. Furthermore, so long as the turbulent magnetic energy density dominates the turbulent kinetic energy (as it does here) and the relaxation time is sufficiently long, $M_{\theta\theta}$ will dominate the azimuthal component of the full turbulent stress tensor.

4. Azimuthal Shear Flow and Hoop-Stresses

The streamwise stress $M_{\theta\theta}$ is an azimuthal hoop-stress. This hoop-stress creates an inwards force

$$f^\text{hoop}_r = -\frac{M_{\theta\theta}}{r},$$

much like a circularly-stretched rubber band. This is true both in the astrophysical context, as well as in the circular shear flow of a viscoelastic fluid in the laboratory. In fact, a rubber band is an apt analogy, as a rubber band is itself composed of a linked polymer matrix, and the stress in anisotropically-stretched rubber corresponds to a statistical alignment of these polymers.

One of the more startling laboratory demonstrations of the dynamical effects of these hoop stresses is the reversal of the secondary flow of a viscoelastic fluid in the neighborhood of a spinning sphere. Through ordinary viscous forces, the sphere induces an azimuthal shear flow. The inertial term $v_i \partial_j v_j$ in the momentum-conservation equation, in an ordinary Newtonian fluid, flings material out centrifugally along the equator; this causes a pressure drop near the sphere, which sucks fluid in along the poles. If the elasticity of the fluid is sufficiently strong, however, the above-mentioned hoop-stresses dominate the centrifugal forces, and the secondary flow is reversed. Fluid is pulled in along
the equator, creating a pressure rise near the sphere; this increased pressure drives an outward jet-like flow along the axis of rotation of the sphere.

5. Application to Protostellar Jets

We propose that essentially the same mechanism responsible for these laboratory jet-like structures may be responsible for protostellar jet formation, in particular in those cases in which evidence does not preclude a disk that extends to the photosphere of the nascent star. We specifically concentrate on protostellar jets because we believe the presence of a relatively firm central object (as opposed to a black hole) will help the hoop-stresses drive an outflow, as well as collimate it, which has also been proposed recently by Li (2002).

For a thick inner disk, we find that if the disk thickness exceeds the stellar radius by more than a factor of a few, then the radial force \( f_{\text{hoop}} \) can dominate the centrifugal, gravitational, and magnetic pressure forces, depending on the viscosity. This force is directed radially inward in cylindrical, not spherical, coordinates. Following the laboratory analogy, this force causes a pressure build-up near the surface of the star that has an outlet in the axial direction and which, we conjecture, will drive an axial outflow. Note that there is net work performed by these hoop-stresses on a fluid element that spirals in equatorially and is then expelled axially, as there are no hoop-stresses along the axis.

The current level of understanding of thick disks is far too rudimentary to provide anything other than a very rough estimate of the forces and energetics. In analogy to thin disks, we write the viscosity as

\[
\nu = \alpha L^2 \left( \frac{\partial \Omega}{\partial \ln r} \right),
\]

where it is assumed that \( \alpha \sim 1 \). Our preliminary results, as given in Williams (2001), are as follows: We assume that the star is embedded in the thick disk, that is, the thickness of the disk \( H \) is greater than the stellar radius \( R_* \). If we assume that the appropriate \( L \) in eqn. (9) is \( R_* \), then it is not clear if the hoop stresses are sufficient to power an outflow. On the other hand, if we assume that the appropriate \( L \) is the thickness of the disk, then the hoop stresses are more than sufficient, so long as the jet does not lose too much energy to viscous dissipation on its way out of the thick disk.

References

Hawley, J. F., Gammie, C. F., & Balbus, S. A. 1995, ApJ, 440, 742
Giesekus, H. 1963 in Fourth International Congress on Rheology, Pt. 1, ed. E. H. Lee & A. L. Copley (New York: Wiley), 249
Li, L.-X. 2002, ApJ, 564, 108L, astro-ph/0108469
Ogilvie, G. I. 2001, MNRAS, 325, 231, astro-ph/0102245
Thomas, R. H., & Walters, K. 1964, Quart. Journ. Mech. and Applied Math, 17, 39
Williams, P. T. 2001, xxx.lanl.gov, astro-ph/0111603