A New Statistical Parameter for Identifying the Main Transition Velocities in Bubble Columns*

The identification of the main flow regime boundaries in bubble columns is essential since the degrees of mixing and mass and heat transfer vary with the flow regime. In this work, a new statistical parameter was extracted from the time series of the cross-sectional averaged gas holdup. The measurements were performed in bubble columns by means of conductivity wire-mesh sensors at very high sampling frequency. The columns were operated with an air/deionized water system under ambient conditions. As a flow regime indicator, a new dimensionless statistical parameter called “relative maximum number of visits in a region” was introduced. This new parameter is a function of the difference between the maximum numbers of visits in a region, calculated from two different division schemes of the signal range.

Keywords: Bubble columns, Conductivity wire-mesh sensor, Flow regime identification, Gas holdup, Hydrodynamics

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1 Introduction

Bubble columns find broad application as reactors and separation units in the chemical, mining, pharmaceutical, and biotechnological industries. These gas-liquid contactors consist of a discontinuous gas phase (in the form of bubbles) moving relative to a continuous phase (a liquid or slurry). They can be single-staged or multi-staged, liquid-batch or continuous, operated co-currently or counter-currently. Bubble columns have a wide range of applications such as absorption, stripping, catalytic slurry reactions (such as Fischer-Tropsch synthesis), bioreactions, coal liquefaction, fermentation, etc. Comprehensive information about bubble columns and their applications is available in [1]. In bubble columns, the rate of transport of the gas phase to the liquid phase often limits the productivity and is therefore a critical design criterion. The uncertainty in today’s bubble column design arises from a lack of fundamental understanding of their complex flow characteristics.

Bubble dynamics and hydrodynamic regimes indirectly influence the scale-up and design of bubble column reactors. The main transition velocities $U_{\text{trans}}^{(1)}$ are important design parameters for bubble columns. A thorough knowledge of these parameters is necessary for a proper scale-up of bubble columns. In particular, the degree of mixing and the heat and mass transfer – and thus, the whole bubble column performance (especially the achievable conversion) – depend on the prevailing flow regime. Therefore, it is important to identify and to be able to predict the conditions under which the transition from bubbly flow to the churn-turbulent flow regime occurs. An example of the effect of the prevailing flow regime on the hydrodynamic parameters can be drawn from the gas holdup correlations. In general, the exponent of $U_G$ varies from 0.7 to 1.2 in the gas holdup correlations recommended for the bubbly flow regime, whereas in the churn-turbulent flow regime the effect of $U_G$ is less pronounced and correlations reveal exponent values of 0.4–0.7. In addition, the first $U_{\text{trans}}$ value (distinguishing the end of the bubbly flow regime and the onset of the transition regime) plays an important role in the calculation of the equilibrium (large) bubble size and the large bubble holdup, which determine the mass transfer rates [2]. So, the development of a reliable method for its identification is essential.

Only for aqueous liquid systems such as water, electrolyte solutions, etc., approximate flow regime maps (for instance, presented in [1]) are currently available. These steady empirical flow regime maps predict a certain flow pattern for given volume flow rates of liquid and gas. However, they are neither able to predict the change of the flow pattern along the flow path (in case of stationary flows) nor the time- and space-dependent flow structure (in case of transient flows). For the design and

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1) List of symbols at the end of the paper.
scale-up of bubble columns, the effects of the fluid properties and the nature of the gas distributor on the flow regime boundaries require detailed examination.

### 1.1 Main Flow Regimes in Bubble Columns

In bubble column reactors, the hydrodynamics, transport and mixing properties such as pressure drop, holdups of various phases, fluid-fluid interfacial areas, and interphase mass and heat transfer coefficients depend strongly on the prevailing flow regime. The gas-liquid flow pattern is affected by the bubble formation and coalescence rates. As the superficial gas velocity \( U_G \) increases, three different flow regimes are being formed:

- **Bubbly flow (homogeneous) regime**: This regime is characterized by relatively small and rather uniform bubbles with equal radial bubble and gas holdup distributions. There is a gentle agitation of the gas-liquid dispersion and insignificant bubble coalescence.

- **Transition flow regime**: This regime is characterized by a widened bubble size distribution and local liquid circulation patterns. It is formed because the transition from bubbly flow to the churn-turbulent flow regime is a gradual process.

- **Churn-turbulent (heterogeneous) regime**: At \( U_G \) values higher than 0.05 m s\(^{-1}\), the homogeneous gas-liquid dispersion cannot be maintained and an unsteady flow pattern (with channeling) occurs. The churn-turbulent flow regime is characterized by large bubbles (formed due to coalescence) moving with high rise velocities (\( \geq 0.8 \) m s\(^{-1}\)) in the presence of small bubbles [3]. The large bubbles take the form of spherical caps with a very mobile and flexible interface. These large bubbles can grow up to a diameter of about 0.15 m [1]. The churn-turbulent flow regime is most commonly encountered in industrial bubble columns [4]. It is characterized by the establishment of gross circulation patterns, the existence of a pronounced radial gas holdup profile, and vigorous mixing.

The transition from bubbly flow to the churn-turbulent flow regime is usually accompanied by a sharp increase in the interstitial gas velocity (the mean rise velocity of the bubbles in a bubble swarm).

It is noteworthy that Franz et al. [5] described the flow in a bubble column as a wide distribution of vortices following in a stochastic sequence. The authors divided the gas-liquid dispersion into three zones: A region of homogeneous turbulence (swarm turbulence) exists in the core of the column and bubble coalescence prevails especially at high \( U_G \) values; in the annulus of the column, a relatively bubble-free zone exists and the flow there occurs in downward direction; between these two regions there is a zone with a high velocity gradient and free anisotropic turbulence and most of the re-dispersion of the gas phase takes place there. Franz et al. [5] also confirmed the existence of a stable axially asymmetric flow structure in the bubble column.

### 1.2 Previous Methods for Flow Regime Identification

Different methods for the differentiation of the boundaries of the main flow regimes in bubble columns have been proposed in the literature. The most important ones are summarized in Tab. 1. They are all well-known and standard methods.

In addition, Bhole and Joshi [18] used the theory of linear stability in order to identify the main transition velocity. It has been also demonstrated that computational fluid dynamics (CFD) simulations [19–21] are capable of identifying the main transition velocities. Olmos et al. [22] applied different techniques (statistical, frequency, fractal, and chaos analyses) to laser Doppler anemometry (LDA) data for flow regime identification in bubble columns.

It is noteworthy that even minute traces of contaminants in the tap water can change the transition between flow regimes [23]. Furthermore, it has been reported that porous spargers with mean pore sizes of less than 150 \( \mu \)m generally produce bubbly flows up to \( U_G \) values of about 0.05–0.08 m s\(^{-1}\), while for perforated plates with orifice diameters larger than \( 1 \times 10^{-3} \) m, bubbly flow may not occur in case of pure liquids.

In some works [7, 17] only the first transition velocity was identified, while in other papers the identification criterion was unclear. For instance, Lin et al. [8] used a local maximum in some cases, whereas in other cases they used a local minimum. The information entropy theory [17] is also not very sensitive to every flow regime transition. Thus, new powerful and reliable methods should be developed that are able to precisely identify the two main transition velocities.

**Table 1. Overview of the different methods applied for flow regime identification in bubble columns.**

| Literature source                   | Applied method       | Measured signal       |
|------------------------------------|----------------------|-----------------------|
| Lockett and Kirkpatrick [6]        | drift-flux analysis  | pressure fluctuations  |
| Letzel et al. [7]                  | nonlinear chaos analysis | absolute pressure   |
| Lin et al. [8]                     | nonlinear chaos analysis | differential pressure |
| Ajbar et al. [9]                   | nonlinear chaos analysis | acoustic time series |
| Nedeltchev et al. [10]             | nonlinear chaos analysis | particle trajectories |
| Nedeltchev et al. [11]             | nonlinear chaos analysis | photon counts        |
| Nedeltchev et al. [12]             | nonlinear chaos analysis | differential pressure |
| Kikuchi et al. [13]                | nonlinear chaos analysis | bubble frequency     |
| Drahoś et al. [14]                 | Hurst analysis       | pressure time series  |
| Vial et al. [15], Gourich et al. [16]| statistical analysis | pressure fluctuations |
| Letzel et al. [7], Ajbar et al. [9]| spectral analysis    | pressure fluctuations |
| Nedeltchev and Shaikh [17]         | information entropy theory | photon counts        |
2 The Essence of the New Method

Nedeltchev et al. [24] found that, when the range of the gas holdup time series signal was divided into different regions (with progressively increasing heights proportional to the division step of 0.25, i.e., 1×0.25, 2×0.25, 3×0.25, 4×0.25, etc., shown in Fig. 1a), the maximum number of visits \( N_v^{\text{max}} \) (step 0.25) (see the grey box in Fig. 1a) in one of these regions was capable of identifying the two main transition velocities \( U_{\text{trans}} \) in the bubble column operation. The only disadvantage of this approach is that the \( N_v^{\text{max}} \) (step 0.25) values depended on the division scheme, i.e., the selection of the division step.

In order to eliminate this dependence on the selection of the division scheme, the range of each signal was divided in two different ways (with steps of 0.25 and 0.5), which accordingly yielded two different \( N_v^{\text{max}} \) values. At first, both the minimum and maximum values of the gas holdup signal (measured as percentage) were determined and then the signal range was divided into different regions (with progressively increasing heights) by using the division step of 0.25 as explained above (Fig. 1a). The most frequently visited region (marked by a grey box) is indicated as \( N_v^{\text{max}} \) (step 0.25). Secondly, the same signal range was divided into different regions by means of step 0.5 (with progressively increasing heights: 1×0.5, 2×0.5, 3×0.5, 4×0.5, etc.; shown in Fig. 1b). When a bigger division step is used, the maximum number of visits in a region \( N_v^{\text{max}} \) increases. In addition, the division scheme with a smaller step is characterized by more regions. In the second division pattern, the most frequently visited region is indicated as \( N_v^{\text{max}} \) (step 0.5). The new dimensionless parameter \( N_r^{\text{max}} \) (also called "relative maximum number of visits in a region") is a function of both step-dependent \( N_v^{\text{max}} \) values:

\[
N_r^{\text{max}}(1) = \frac{N_v^{\text{max}}(\text{step } 0.5) - N_v^{\text{max}}(\text{step } 0.25)}{(0.5 - 0.25)} \quad (1)
\]

In this form, the new dimensionless parameter \( N_r^{\text{max}} \) is step independent and it can be used as a powerful tool for flow regime identification.

In order to demonstrate that it is really a step-independent parameter, the \( N_v^{\text{max}} \) values were also calculated at two different steps: 0.5 and 0.1. In this case, Eq. (1) was redefined as follows:

\[
N_r^{\text{max}}(2) = \frac{N_v^{\text{max}}(\text{step } 0.5) - N_v^{\text{max}}(\text{step } 0.1)}{(0.5 - 0.1)} \quad (2)
\]

The transition velocities \( U_{\text{trans}} \), identified by means of \( N_r^{\text{max}}(1) \) and \( N_r^{\text{max}}(2) \) will be compared and it will be shown that the new index does not depend on the preselection of any other parameter. It is worth noting that, when the division step becomes too small (for instance, equal to 0.1), counting of the number of visits in each region becomes very complicated since there are many values lying on the boundaries between two different regions. So, in principle, the division step should not be too small.

3 Experimental Setup

The small bubble column had an inner diameter (i.d.) of 0.15 m and was equipped with a perforated plate distributor (14 holes, \( \Omega 4\times10^{-3} \text{m} \), open area (OA) = 1%). The large bubble column had an i.d. of 0.4 m and was equipped with a similar type of perforated plate distributor (101 holes, \( \Omega 4\times10^{-3} \text{m}, \text{OA} = 1% \)). Both bubble columns were operated with an air/deionized water system under ambient conditions. The clear liquid height in both columns was adjusted at 2.0 m.

The time series (60 000 points) of the cross-sectional averaged gas holdup (recorded as percentage) in both bubble columns were measured by means of conductivity wire-mesh sensors (Fig. 2).

![Figure 1](example.com) Example of the cross-sectional averaged gas holdup time series data (40 000 points, 20 s) obtained by the wire-mesh sensor (f = 2000 Hz) in the large bubble column (Dc = 0.40 m) for a step size equal to (a) 0.25, (b) 0.5, at \( U_G = 0.023 \text{m s}^{-1} \).

![Figure 2](example.com) Photograph of the applied wire-mesh sensor with an i.d. of 0.15 m.

The wire-mesh sensors consisted of two electrode planes each with 24 (in the case of the small column) or 64 (in the case of the large column) stainless-steel wires of \( 0.2\times10^{-3} \text{m} \) and \( 6.125\times10^{-3} \text{m} \) distance between the wires. The distance between the planes was \( 4.0\times10^{-3} \text{m} \) and the wires from differ-
ent planes crossed each other at right angles. This arrangement gave 576 (in the small column) or 4096 (in the large column) crossing points, 78% thereof inside the circular cross-section of the columns. One plane of the electrodes acted as a transmitter, the other one as a receiver. The transmitter electrodes were activated by a multiplexing circuit in successive order, and signals derived from the current measured at the receiver electrodes were stored. After one multiplexing cycle, a two-dimensional matrix of values was available, reflecting the conductivities between all crossing points of the electrodes of the two perpendicular planes. The signals of the matrix were converted into gas holdup data based on proper calibration measurements on the liquid-flooded and empty column, respectively. The wire-mesh sensor technology is described in more detail by Prasser et al. [25]. The wire-mesh sensor was always installed at 1.3 m above the gas distributor.

In the case of organic liquids (with very low electrical conductivity), a capacitance wire-mesh sensor can be used. Such a capacitance wire-mesh sensor has already been designed and manufactured at our institute. Our conductivity and capacitance wire-mesh sensors have a simple design and they can be fitted into any industrial bubble column. The sensors can be built in many different design types.

4 Results and Discussion

In the narrow (0.15 m i.d.) bubble column, two well-pronounced local minima (Fig. 3) in the profiles of the new parameters, \( N_r^{\text{max}}(1) \) and \( N_r^{\text{max}}(2) \), were identified. At \( U_G = 0.034 \text{ m s}^{-1} \), the end of the gas maldistribution regime was observed. This regime existed due to the relatively large size (Ø 4 \( \times 10^{-3} \text{ m} \)) of the hole openings. The first \( U_{\text{trans}} \) value is very close to the one calculated theoretically (0.029 \text{ m s}^{-1}) by the correlation of Reilly et al. [26]. The physical meaning of the first transition velocity is to show at which superficial gas velocity \( U_G \) the gas maldistribution regime discontinues. The onset of the churn-turbulent flow regime was identified (based on a local minimum) at \( U_G = 0.089 \text{ m s}^{-1} \). The second transition velocity indicates the onset of bubble coalescence (the formation of many spherical-cap bubbles in the core). It is worth noting that in the transition flow regime both profiles exhibited a monotonous declining trend.

The results in Fig. 3 show that the new parameter \( N_r^{\text{max}} \) is not very sensitive to the scheme of division (provided that the difference between the two division steps is not smaller than 0.25). The two local minima in \( N_r^{\text{max}}(1) \) and \( N_r^{\text{max}}(2) \) occurred at the same \( U_G \) values. However, the local minima were more pronounced at the smaller step difference (0.5–0.25; see Eq. 1). The well-pronounced local minima of the new parameter \( N_r^{\text{max}} \) exhibit that the signal is better (or more equally) distributed between its minimum and maximum values at these critical \( U_G \) values, and that is why the \( N_r^{\text{max}} \) values are the lowest. In other words, the degree of order of the signal improves at each transition velocity \( U_{\text{trans}} \).

It is very important to mention that the two \( U_{\text{trans}} \) values identified in Fig. 3 can also be identified based on the \( N_v^{\text{max}} \) (step 0.5) profile (Fig. 4a). The same identification criterion (local minimum) was used. In addition, the maximum information entropies \( IE_{\text{max}} \) were estimated on the basis of this data. The algorithm described in [17] was used. The only difference was that, in the estimation of the probability, the height of the largest region multiplied by 60 000 visits was used in the denominator. Fig. 3b shows two local minima occurring at 0.034 and 0.089 \text{ m s}^{-1}, and they are in full agreement with both the \( N_r^{\text{max}} \) and \( N_v^{\text{max}} \) (step 0.5) results.
It is noteworthy that such two well-pronounced local minima cannot be identified in the profiles of many other parameters derived on the basis of different theories (Kolmogorov entropy [7], metric entropy [8], largest Lyapunov exponent [8], correlation dimension [8], mutual information [8], maximum information entropy [17], average absolute deviation, mean, etc.). Usually, the researchers mix the identification criteria and sometimes use the local minimum and sometimes the local maximum for the sake of flow regime identification.

In order to illustrate the existence of the gas maldistribution regime, several photographs were taken at different \( U_G \) values in the vicinity of the gas sparger, as shown in Fig. 5. The photographs show that bubbles were formed at all distributor openings for the first time at \( U_G = 0.04 \text{ m s}^{-1} \) (case d). At lower \( U_G \) values (cases a–c), a clear gas maldistribution was observed. So, the new parameter \( N_r^{\text{max}} \) can correctly identify the end of the gas maldistribution regime.

Fig. 6 shows that the new parameter \( N_r^{\text{max}} \) can also identify the two \( U_{\text{trans}} \) values in the large bubble column (0.4 m i.d.). The first \( U_{\text{trans}} \) value is distinguishable (based on the well-pronounced local minimum) at \( U_G = 0.034 \text{ m s}^{-1} \). Both the profiles of \( N_r^{\text{max}}(1) \) and \( N_r^{\text{max}}(2) \) exhibited a well-pronounced minimum at this critical velocity. A comparison between the results given in Figs. 3 and 6 reveals that the column diameter \( D_c \) does not have an effect on the first \( U_{\text{trans}} \) value, which is in agreement with the empirical formulas available in the literature [26, 27].

The second \( U_{\text{trans}} \) value was identified at \( U_G = 0.078 \text{ m s}^{-1} \) (see the second local minimum in both the \( N_r^{\text{max}}(1) \) and \( N_r^{\text{max}}(2) \) profiles in Fig. 6). This result means that the increase in \( D_c \) shifts the onset of the churn-turbulent flow regime to somewhat lower \( U_G \) values, and thus, the transition regime becomes slightly narrower.

Fig. 7 a shows that the \( N_r^{\text{max}} \) (step 0.5) profile in the large column yields identical \( U_{\text{trans}} \) values to the ones identified in Fig. 6. Two well-pronounced local minima were observed at \( U_G = 0.034 \) and \( 0.078 \text{ m s}^{-1} \). The comparisons between Figs. 3 and 4 and Figs. 5 and 6 imply that the correct division step (yielding the correct \( U_{\text{trans}} \) values) is equal to 0.5. This conclusion can be reached only with the help of the new parameter \( N_r^{\text{max}} \). Based on the \( N_r^{\text{max}} \) (step 0.5) data, the maximum information entropies \( IE_{\text{max}} \) [17] were calculated. Fig. 7 b shows that two well-pronounced local minima identified the same \( U_{\text{trans}} \) values (0.034 and 0.078 m s\(^{-1}\)).

Fig. 8 shows several photographs of the bubble formation process in the vicinity of the gas sparger at different \( U_G \) values. It is obvious that gas maldistribution prevailed up to \( U_G = 0.03 \text{ m s}^{-1} \), and above this critical value, bubble formation from all distributor holes was observed. Therefore, the results shown in Figs. 6 and 7 for the first \( U_{\text{trans}} \) value are in agreement with the flow visualization in the large bubble column.

The results regarding the transition velocities identified in both columns are summarized in Tab. 2. As was mentioned above, the column diameter \( D_c \) has an effect only on the second \( U_{\text{trans}} \) value. Nedeltchev et al. [24] identified the two transition velocities \( U_{\text{trans}}(0.022 \text{ and } 0.112 \text{ m s}^{-1}) \) in the narrow column on the basis of the local minima in both the Kolmogorov entropy and the \( N_r^{\text{max}} \) (step 0.25) profiles. The photographs in Fig. 5 prove that both parameters identify the first \( U_{\text{trans}} \) value somewhat earlier than the real case. Nedeltchev and Schubert [28] introduced another parameter (average/(3 \( \times \) (average absolute deviation)), which precisely identified the first \( U_{\text{trans}} \) value (0.034 m s\(^{-1}\)) in the narrow bubble column. However, in the bigger column, the new parameter identified the first \( U_{\text{trans}} \) value at a somewhat higher value (0.045 m s\(^{-1}\)). So, the new parameter \( (N_r^{\text{max}}(1) \text{ or } N_r^{\text{max}}(2)) \) defined in this article is the most reliable and precise one.

In summary, the main findings are the values of the two main transition velocities in both the small and the large bubble column. Both transition velocities are important design parameters for bubble columns. On the basis of a new and original parameter, it is shown that the column diameter affects only the second critical velocity. The paper shows that perforated plates...
with hole openings of $4 \times 10^{-3}$ m generate a gas maldistribution regime, so the hole openings should be (much) smaller than $4 \times 10^{-3}$ m. By means of the new parameter, the boundaries of the gas maldistribution regime can be identified.

It is noteworthy that the new method relies on experimental data and that the results are not directly applicable to a different industrial column equipped with a different gas distributor and/or operated with an organic liquid. For this purpose, new measurements should be performed and further analyzed by means of the new parameter.

5 Conclusions

A new dimensionless statistical parameter, $N_r^{\text{max}}$, was defined on the basis of the division of the signal range into different regions by means of different steps. In the small bubble column (0.15 m i.d.), two local minima in the $N_r^{\text{max}}$ profile were observed at $U_G = 0.034$ and 0.089 m s$^{-1}$. They identified the two main transition velocities $U_{\text{trans}}$. In the large column (0.4 m i.d.), the $N_r^{\text{max}}$ profile exhibited two local minima at $U_G = 0.034$ and 0.078 m s$^{-1}$. As the column diameter increased, the first $U_{\text{trans}}$ remained the same, whereas the second $U_{\text{trans}}$ decreased to some extent. For both bubble columns, the effect of the division scheme on the $N_r^{\text{max}}$ profiles was found to be insignificant. The results were supported by photographs taken in both columns.

On the basis of the profiles of the new statistical parameter $N_r^{\text{max}}$ in both columns, it was determined that, when the signal was divided into different regions with a step of 0.5, both $U_{\text{trans}}$ values were accurately identified. The maximum information entropies extracted from this data can also be used for flow regime identification.

In the case of the small column, a comparison among the $U_{\text{trans}}$ values identified by the new parameter $N_r^{\text{max}}$, the Kolmogorov entropy, and another statistical parameter was performed. It was found that only the transition velocities identified by the new parameter $N_r^{\text{max}}$ were in agreement with the flow visualization images in both columns.
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Symbols used

- \( D_c \) column diameter
- \( f_s \) sampling frequency
- \( E_{\text{max}} \) maximum information entropy
- \( N_v \) maximum number of visits in a single region
- \( N_r \) relative maximum number of visits in a region
- \( N_r^{\text{max}}(1) \) relative maximum number of visits in a region at steps 0.5 and 0.25, Eq. (1)
- \( N_r^{\text{max}}(2) \) relative maximum number of visits in a region at steps 0.5 and 0.10, Eq. (2)
- \( N_v \) maximum number of visits in a region at step 0.5
- \( N_v^{\text{max}}(\text{step 0.5}) \) maximum number of visits in a region at step 0.5
- \( N_v^{\text{max}}(\text{step 0.25}) \) maximum number of visits in a region at step 0.25
- \( N_v^{\text{max}}(\text{step 0.1}) \) maximum number of visits in a region at step 0.1
- \( U_G \) superficial gas velocity
- \( U_{\text{trans}} \) transition gas velocity

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