Relativistic independence bounds nonlocality

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Abstract

If Nature allowed nonlocal correlations other than those predicted by quantum mechanics, would that contradict some physical principle? Various approaches have been put forward in the past two decades in an attempt to single out quantum nonlocality. However, none of them can explain the set of quantum correlations arising in the simplest scenarios. Here it is shown that generalized uncertainty relations, as well as a specific notion of locality give rise to both familiar and new characterizations of quantum correlations. In particular, we identify a condition, relativistic independence, which states that uncertainty relations are local in the sense that they cannot be influenced by other experimenters’ choices of measuring instruments. We prove that theories with nonlocal correlations stronger than the quantum ones do not satisfy this notion of locality and therefore they either violate the underlying generalized uncertainty relations or allow experimenters to nonlocally tamper with the uncertainty relations of their peers.

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I. INTRODUCTION

Quantum mechanics stands out in enabling strong, nonlocal correlations between remote parties. On the one hand, these quantum correlations cannot in any way be explained by models of classical physics. On the other hand, quantum theory remains rather elusive about their physical origin [1–3]. What if Nature allowed nonlocal correlations other than those predicted by quantum mechanics – would that break any known physical principle? This question becomes all more important when the predictions of quantum mechanics are experimentally verified time and again.

Initially it was speculated that those correlations excluded by quantum mechanics violate relativistic causality – *the principle which dictates that experiments can be influenced only by events in their past light cone, and influence events only in their future light cone*. But then it was shown that other theories may exist whose correlations, while not realizable in quantum mechanics, are nevertheless non-signaling and are hence consistent with relativistic causality [1].

Over the past 20 years, many efforts have been invested in a line of research aimed at quantitatively deriving the strength of quantum correlations from basic principles. For example, it was shown that violations of the Bell–CHSH inequality [4] beyond the quantum limit, known as Tsirelson’s bound, are inconsistent with the uncertainty principle [5]. Popescu-Rohrlich–boxes (PR–boxes), the hypothetical models achieving the maximal violation of the Bell–CHSH inequality [1], would allow distributed computation to be performed with only one bit of communication [6], which looks unlikely but does not violate any known physical law. Similarly, in stronger-than-quantum nonlocal theories some computations exceed reasonable performance limits [7], and there is no sensible measure of mutual information between pairs of systems [8]. Finally, it was shown that superquantum nonlocality does not permit classical physics to emerge in the limit of infinitely many microscopic systems [9–11], and also violates the exclusiveness of local measurement outcomes in multipartite settings [12]. However, none of these and other principles that have been proposed [2] can explain the set of one- and two-point correlators that fully characterize the quantum probability distributions witnessed in the simplest bipartite two-outcome scenario.

A consequence of relativistic causality within the framework of probabilistic theories is known as the no-signaling condition – the local probability distributions of one experimenter (marginal probabilities) are independent of another experimenter’s choices [1]. While the no-signaling condition is insufficient to single out quantum correlations, it is shown here that an analogous requirement applicable in conjunction with generalized uncertainty relations is satisfied exclusively by
quantum mechanical correlations.

II. RESULTS

In what follows we first assume (Subsection II A) that generalized uncertainty relations are valid within the theory in question. Such uncertainty relations broaden the meaning of uncertainty beyond the realm of quantum mechanics, and give rise to the Schrödinger-Robertson uncertainty relation when applied to the latter. Then in Subsection II B we assume in addition a certain form of independence we name relativistic independence, meaning here that local uncertainty relations cannot be affected at a distance. The above assumptions accord well with experimental observations, yet generalize the underlying theoretical model beyond the quantum formalism.

A. Generalized uncertainty relations

Three experimenters, Alice, Bob, and Charlie, perform an experiment, where each of them owns a measuring device. On each such device a knob determines its mode of operation, either “0” or “1”, which allows measuring two physical variables, \( A_0/A_1 \) on Alice’s side, \( B_0/B_1 \) on Bob’s side, and \( C_0/C_1 \) on Charlie’s side. Alice and Bob are close to one another and so they use the readings from all their devices to empirically evaluate the variances, \( \Delta^2_{A_i}, \Delta^2_{B_j} \), and the covariances, \( C(A_i, B_j) \) def \( = E_{A_iB_j} - E_{A_i}E_{B_j} \), where \( E_{A_i}, E_{B_j} \), and \( E_{A_iB_j} \) are the respective one- and two-point correlators. Charlie, on the other hand, is far from them. See Figure 1.

Assume that measurements of physical variables are generally inflicted with uncertainty. Not only does this uncertainty affect pairs of local measurements performed by individual experimenters, it also governs any number of measurements performed by groups of remote experimenters. In our tripartite setting, for example, the measurements of Alice, Bob, and Charlie are assumed to be jointly governed by the generalized uncertainty relation,

\[
\Lambda_{ABC} \overset{\text{def}}{=} \begin{bmatrix}
\Lambda_C & C(B, C)^T & C(A, C)^T \\
C(B, C) & \Lambda_B & C(A, B)^T \\
C(A, C) & C(A, B) & \Lambda_A \\
\end{bmatrix} \succeq 0
\]  

which means that \( \Lambda_{ABC} \) is a positive semidefinite matrix. Here, \( C(A, B), C(A, C), \) and \( C(B, C) \) are the empirical covariance matrices of Alice-Bob, Alice-Charlie, and Bob-Charlie measurements. The diagonal submatrices, e.g., \( \Lambda_A \), represent the uncertainty relations governing the
individual experimenters. Below and in Materials and Methods, (1) is shown to imply the quantum mechanical Schrödinger-Robertson uncertainty relations [13], as well as their multipartite non-quantum generalizations. Moreover, in local hidden variables theories where all measurement outcomes preexist, (1) coincides with a covariance matrix, which is by construction positive semidefinite and represents the uncertainty of $A_i$, $B_j$, and $C_k$, hence the natural generalization to other theories.

Provided that Bob measured $B_j$ and Charlie measured $C_k$, the system as a whole is governed by a submatrix of $\Lambda_{ABC}$,

$$
\Lambda_{ABC}^{jk} = \begin{bmatrix}
\Delta^2_{C_k} & C(C_k, B_j) & C(C_k, A_1) & C(C_k, A_0) \\
C(C_k, B_j) & \Delta^2_{B_j} & C(B_j, A_1) & C(B_j, A_0) \\
C(C_k, A_1) & C(B_j, A_1) & \Delta^2_{A_1} & r_{jk} \\
C(C_k, A_0) & C(B_j, A_0) & r_{jk} & \Delta^2_{A_0}
\end{bmatrix} \succeq 0
$$

Here, $r_{jk}$ is a real number whose value guarantees that $\Lambda_{ABC}^{jk} \succeq 0$. Therefore, it generally depends not only on Alice’s choices but also on Bob’s $j$ and Charlie’s $k$. The lower $2 \times 2$ submatrix in (2), which is henceforth denoted as the positive-semidefinite $\Lambda_A^{jk}$, implies that Alice’s measurements satisfy $\Delta^2_{A_0} \Delta^2_{A_1} \geq r^2_{jk}$, as well as other uncertainty relations that depend on $r_{jk}$ rather than $r^2_{jk}$, i.e., $u^T \Lambda_A^{jk} u \geq 0$, where $u$ is any two-dimensional real-valued vector.

Local hidden variables theories, quantum mechanics, and non-quantum theories such as the hypothetical PR–boxes [1] obey (2). Moreover, they provide different closed forms for this $r_{jk}$, which in general we are unable to assume. In local hidden variables theories, where $A_0$ and $A_1$ are classical random variables whose joint probability distribution is well-defined, (2) holds for $r_{jk} = C(A_0, A_1)$, which is independent of $j$ and $k$. In quantum mechanics the Schrödinger-Robertson uncertainty relations show that $r_{jk}$ depends exclusively on Alice’s self-adjoint operators, in particular their commutator and anti-commutator. If Alice and Charlie share a PR–box then $r_{jk} = (-1)^k$, which, in contrast to the two other theories, depends on $k$.

### B. Independence

In the above setting, Bob and Charlie may be able to nonlocally tamper with Alice’s uncertainty relation, $\Lambda_A^{jk} \succeq 0$, through their $j$ and $k$. Prohibiting this by requiring that Alice’s uncertainty
Figure 1: An illustration of relativistic independence in a tripartite scenario. In a theory obeying generalized uncertainty relations (shown in the bottom right corner in the form of a certain positive-semidefinite matrix), relativistic independence (RI) prevents Bob and Charlie from influencing Alice’s uncertainty relations, e.g., \( \Delta^2_{A_0} \Delta^2_{A_1} \geq r_{jk}^2 \), through their choices \( j \) and \( k \), i.e. \( r_{jk} = r \). Here, \( \rho_{ij}^{AB} = C(A_i, B_j) \), \( \rho_{ik}^{AC} = C(A_i, C_k) \), and \( \rho_{jk}^{BC} = C(B_j, C_k) \), illustrated by the arrows are the covariances of Alice-Bob, Alice-Charlie, and Bob-Charlie measurements, respectively. In the quantum mechanical formalism a similar matrix inequality gives rise to the Schrödinger-Robertson uncertainty relations of Alice’s self-adjoint operators \( \hat{A}_0 \) and \( \hat{A}_1 \), as well as between the nonlocal Alice-Bob operators, \( \hat{A}_0 \hat{B}_j \) and \( \hat{A}_1 \hat{B}_j \). See Materials and Methods.

relation as a whole, i.e., the trio \( \Delta_{A_0}, \Delta_{A_1}, \) and \( r_{jk} \), would be independent of Bob’s \( j \) and Charlie’s \( k \) leads to the set of quantum mechanical one- and two-point correlators. This condition is named henceforth relativistic independence (RI).

By RI, the Alice-Bob system, which is governed by the lower \( 3 \times 3 \) submatrix of \( \Delta^j_{ABC} \), satisfies \( \Lambda^j_k \overset{\text{def}}{=} \Lambda_A \), for \( r_{jk} \overset{\text{def}}{=} r \). Swapping the roles of Alice and Bob, where Alice measures \( A_i \),
RI similarly implies $\Lambda_{ik} = \Lambda_{B}$, for $\bar{r}_{ik} = \bar{r}$. In other words, RI means

$$
\begin{bmatrix}
    \Delta_{Bj}^2 & C(B_j, A_1) & C(B_j, A_0) \\
    C(B_j, A_1) & \Delta_{A_1}^2 & r \\
    C(B_j, A_0) & r & \Delta_{A_0}^2
\end{bmatrix} \succeq 0,
$$

(3)

$$
\begin{bmatrix}
    \Delta_{A_i}^2 & C(A_i, B_1) & C(A_i, B_0) \\
    C(A_i, B_1) & \Delta_{B_1}^2 & \bar{r} \\
    C(A_i, B_0) & \bar{r} & \Delta_{B_0}^2
\end{bmatrix} \succeq 0,
$$

for $i, j \in \{0, 1\}$. RI (3) and no-signaling are distinct and do not follow from one another. The no-signaling condition, for example, dictates that the (marginal) probability distributions of Alice’s measurements, and therefore also $\Delta_{A_0}^2$ and $\Delta_{A_1}^2$, are independent of Bob’s choices. RI, on the other hand, implies that $\Lambda_A$ in its entirety must be independent of Bob’s choices, which may hold whether or not Alice’s marginal probabilities are independent of $j$. The relationship between the two conditions is discussed in more detail in the Materials and Methods section.

PR–boxes satisfy the no-signaling condition but violate RI (see Materials and Methods). Moreover, as stated below, RI (3) is satisfied exclusively by the quantum mechanical bipartite one- and two-point correlators.

**Theorem 1.** The conditions (3) imply

$$
|\rho_{00}\rho_{10} - \rho_{01}\rho_{11}| \leq \sum_{j=0,1} \sqrt{(1 - \rho_{0j}^2)(1 - \rho_{1j}^2)}
$$

and

$$
|\rho_{00}\rho_{01} - \rho_{10}\rho_{11}| \leq \sum_{i=0,1} \sqrt{(1 - \rho_{i0}^2)(1 - \rho_{i1}^2)}
$$

(4)

where $\rho_{ij} \equiv C(A_i, B_j)/(\Delta_{A_i}\Delta_{B_j})$, is the Pearson correlation coefficient between $A_i$ and $B_j$.

It is known that any four correlators, $E_{A_iB_j}$, must satisfy (4) if they are to describe the nonlocality present in a physically realizable quantum mechanical pair of systems [3]. In addition, all the sets of such correlators permitted by (4) are possible within quantum mechanics. This result was proven when assuming quantum mechanics and vanishing one-point correlators, $E_{A_i} = E_{B_j} = 0$, independently by Tsirelson, Landau, and Massanes [14–16]. More recently, (4) has been derived for the case of binary measurements from the first level of the NPA hierarchy [17]. We show without assuming any of these that this bound (in the form of Landau) originates from RI (3). Moreover, it is now clear that (4) must hold not only for binary but also for other, both discrete
and continuous, variables. Consequently, Tsirelson’s $2\sqrt{2}$ bound on the Bell-CHSH parameter $[4]$, $\mathcal{B}_{AB} \equiv \varrho_{00} + \varrho_{10} + \varrho_{01} - \varrho_{11}$, applies to any type of measurement. For example, Alice’s and Bob’s measurements may be the position and momentum of some wavefunction.

Quantum theory satisfies the RI condition (3) and is therefore subject to (4). Furthermore, in the case of binary $\pm 1$ measurements whose one-point correlators vanish, the first Alice-Bob uncertainty relation in (3) is given in quantum mechanics by the Schrödinger-Robertson uncertainty relations of $\hat{A}_0 \hat{B}_j$ and $\hat{A}_1 \hat{B}_j$, where $\hat{A}_i$ and $\hat{B}_j$ are Alice’s and Bob’s self-adjoint operators. See Materials and Methods for the proof of this theorem and for further details.

Surprisingly, within the quantum formalism (4) is a special case of another bound with two extra terms.

**Theorem 2.** In quantum theory, where the Alice and Bob measurements are represented by the self-adjoint operators $\hat{A}_i$ and $\hat{B}_j$, the following holds,

$$\begin{align*}
|\varrho_{00}\varrho_{10} - \varrho_{01}\varrho_{11}| &\leq \sum_{j=0,1} \sqrt{\frac{(1 - \varrho_{00}^2)(1 - \varrho_{10}^2) - \eta_{\hat{A}}^2}{(1 - \varrho_{00}^2)(1 - \varrho_{11}^2) - \eta_{\hat{B}}^2}} \\
|\varrho_{00}\varrho_{01} - \varrho_{01}\varrho_{11}| &\leq \sum_{i=0,1} \sqrt{\frac{(1 - \varrho_{00}^2)(1 - \varrho_{01}^2) - \eta_{\hat{A}}^2}{(1 - \varrho_{00}^2)(1 - \varrho_{11}^2) - \eta_{\hat{B}}^2}}
\end{align*}$$

(5)

where $\varrho_{ij} \equiv \left(\langle \hat{A}_i \hat{B}_j \rangle - \langle \hat{A}_i \rangle \langle \hat{B}_j \rangle \right) / (\Delta_{\hat{A}_i} \Delta_{\hat{B}_j})$, and $\eta_{\hat{X}} \equiv \frac{1}{2\Delta_{\hat{X}}^2} \langle [\hat{X}_0, \hat{X}_1] \rangle / (\Delta_{\hat{X}_0} \Delta_{\hat{X}_1})$, with $\hat{X}$ being either $\hat{A}$ or $\hat{B}$. Here, $[\hat{X}_0, \hat{X}_1] \equiv \hat{X}_0 \hat{X}_1 - \hat{X}_1 \hat{X}_0$ is the commutator of $\hat{X}_0$ and $\hat{X}_1$, and $\Delta_{\hat{X}}^2 = \langle \hat{X}^2 \rangle - \langle \hat{X} \rangle^2$ is the variance of $\hat{X}$. The $\langle \cdot \rangle$ is the quantum-mechanical expectation. Note that $\frac{1}{2\Delta_{\hat{X}}^2} [\hat{X}_0, \hat{X}_1]$ is self-adjoint and is therefore an observable. Moreover, $|\eta_{\hat{X}}| \leq 1$, where $|\eta_{\hat{X}}| = 1$ only if the Robertson uncertainty relation of $\hat{X}_0$ and $\hat{X}_1$ is saturated.

The proof of this theorem is given in Materials and Methods.

C. Local uncertainty relations and nonlocal correlations

The geometry of bipartite RI in Hilbert space is illustrated in Figure 2. The left picture in this figure is the geometry underlying the first bound in (5). This bound arises from the two uncertainty relations (3), which from within quantum mechanics coincide with the Schrödinger-Robertson uncertainty relations of $\hat{A}_0 \hat{B}_j$ and $\hat{A}_1 \hat{B}_j$ in the special case of binary measurements. In other cases, (3) may be viewed as a generalization of the Schrödinger-Robertson uncertainty relations. As shown in Materials and Methods, inside Hilbert space (3) describe two circles in the complex plane, one for $j = 0$ (red) and another for $j = 1$ (yellow). The circles are centered at $\varrho_{0j}\varrho_{1j}$, and
Figure 2: Geometry of bipartite relativistic independence in Hilbert space, the bounds (5). The $\eta_A$ is as defined in Theorem 2, and $\nu_A \equiv \left( \frac{1}{2} \langle \{ 0, 1 \} \rangle - \langle 0 \rangle \langle 1 \rangle \right) / \left( \Delta_{00} \Delta_{11} \right)$, where $\{ X, Y \}$ is the anticommutator. Using these definitions the Schrödinger-Robertson uncertainty relation between Alice’s observables is $\nu_A^2 + \eta_A^2 \leq 1$, hence the pair of bluish unit disks. Bob’s choice, $j = 1$ or $j = 0$, further confines Alice’s uncertainty, the $\eta_A$ and $\nu_A$, to one of the circles, the yellow or the red, respectively. The extent and location of these circles is determined by the nonlocal covariances, $\varrho_{ij}$, Quantum mechanics satisfies relativistic independence and thus keeps Alice’s uncertainty relations independent of Bob’s choices, i.e., by allowing only those covariances for which the red and yellow circles intersect. Tsirelson’s bound is an extreme configuration where these circles intersect at the origin.

Their respective radiuses are $\sigma_{0j} \sigma_{1j}$, where $\sigma_{ij}^2 = 1 - \varrho_{ij}^2$. Alice’s local uncertainty relations are confined to one or another circle depending on Bob’s choice $j$. Quantum mechanics satisfies RI and thus keeps Alice’s uncertainty relations independent of Bob’s choice, i.e., by allowing only those covariances $\varrho_{ij}$ for which the red and yellow circles intersect. Tsirelson’s bound (the right picture), for example, is attained when the region of intersection collapses to a single point at the origin.

RI implies that the extent of nonlocality is governed by local uncertainty relations. The interplay between nonlocality as quantified by the Bell-CHSH parameter, $B$, and Heisenberg uncertainty where $\hat{A}_0 = \hat{x}$ and $\hat{A}_1 = \hat{p}$, are the position and momentum operators (See Materials and Methods for the complete derivation), is

$$\left( \frac{B}{2\sqrt{2}} \right)^2 + \left( \frac{\hbar/2}{\Delta_x \Delta_p} \right)^2 \leq 1. \quad (6)$$
It is known that a complete characterization of the set of quantum correlations must follow from inherently multipartite principles [19]. Indeed, as shown in the Materials and Methods section, RI applies to any number of parties with any number of measuring devices. This allows, for example, deriving a generalization of (4) for the Alice-Bob, Alice-Charlie, and Bob-Charlie one- and two-point correlators in a tripartite scenario. The property known as monogamy of correlations, the \(|B_{AB}| + |B_{AC}| \leq 4\), follows as a special case of this inequality. In the same section, it is shown that the correlators in local hidden variable theories can be similarly bounded by a variant of RI.

III. DISCUSSION

Within a class of theories obeying generalized uncertainty relations, relativistic independence was shown to reproduce the complete quantum mechanical characterization of the bipartite correlations in two-outcome scenarios, and potentially in much more general cases as straightforward corollaries of our approach. To fully characterize the set of quantum correlations would generally require analyzing the uncertainty relation (1) in an elaborate multipartite setting, accounting for all the parties’ cross-correlations and assuming relativistic independence (this point, as well as some other technical issues, are discussed in detail within the Materials and Methods section). All these imply that stronger-than-quantum nonlocal theories may either be incompatible with the uncertainty relations analyzed above or allow experimenters to nonlocally tamper with the uncertainty relations of other experimenters.

IV. MATERIALS AND METHODS

A. No-signaling and relativistic independence

A consequence of relativistic causality in probabilistic theories is the no-signaling condition [1]. Consider the Bell-CHSH setting where \(a\) and \(b\) are the outcomes of Alice’s and Bob’s measurements. The joint probability of these outcomes when Alice measured using device \(i\) and Bob using device \(j\) is denoted as \(p(a, b \mid i, j)\). No-signaling states that one experimenter’s marginal
probabilities are independent of another experimenter’s choices, namely,
\[
\sum_b p(a, b \mid i, 0) = \sum_b p(a, b \mid i, 1) \overset{\text{def}}{=} p(a \mid i)
\]
\[
\sum_a p(a, b \mid 0, j) = \sum_a p(a, b \mid 1, j) \overset{\text{def}}{=} p(b \mid j)
\]
(7)

Of course it means that one experimenter’s precision is independent of another experimenter’s choices,
\[
\Delta^2_{A_i} = E_{a^2|i,j} - E_{a^2|i,j}^2 = \sum_{a,b} a^2 p(a, b \mid i, j) - \left( \sum_{a,b} a p(a, b \mid i, j) \right)^2 = \sum_a a^2 p(a \mid i) - (\sum_a a p(a \mid i))^2
\]
\[
\Delta^2_{B_j} = E_{b^2|i,j} - E_{b^2|i,j}^2 = \sum_{a,b} b^2 p(a, b \mid i, j) - \left( \sum_{a,b} b p(a, b \mid i, j) \right)^2 = \sum_b b^2 p(b \mid j) - (\sum_b b p(b \mid j))^2
\]
(8)

The no-signaling condition thus implies that the variances of one experimenter in the Alice-Bob uncertainty relations (3) are independent of the other experimenter’s choices.

Relativistic independence implies that one experimenter’s uncertainty relation is altogether independent of the other experimenter’s choices, i.e., that \( \Lambda_A \) as a whole, and therefore also \( r_j \), are independent of \( j \). This does not necessarily imply the no-signaling condition as there may exist, for example, marginal distributions \( p(a \mid i, j) \) that depend on Bob’s \( j \) whose variances, \( \Delta^2_{A_i} \), are nevertheless independent of this \( j \). This shows that relativistic independence does not at all require us to assume the no-signaling condition.

B. Popescu–Rohrlich boxes violate relativistic independence

Consider a tripartite setting where Bob and Charlie are uncorrelated, \( C(B_j, C_k) = 0 \), and Alice and Charlie share a PR-box [1]. The PR-boxes define, \( E_{A_i, C_k} = (-1)^k \), and \( E_{A_i} = 0 \), \( E_{C_k} = 0 \). The variances are thus, \( \Delta^2_{A_i} = E_{A_i^2} - E_{A_i}^2 = 1 \) and \( \Delta^2_{C_k} = E_{C_k^2} - E_{C_k}^2 = 1 \), and the covariances are \( C(A_1, C_k) = (-1)^k \) and \( C(A_0, C_k) = 1 \). In this case, a permutation of (2) reads

\[
\Lambda_{PR}^{jk} \overset{\text{def}}{=} \begin{bmatrix}
\Delta_{B_j}^2 & 0 & C(A_1, B_j) & C(A_0, B_j) \\
0 & 1 & 1 & (-1)^k \\
C(A_1, B_j) & 1 & 1 & r_{jk} \\
C(A_0, B_j) & (-1)^k & r_{jk} & 1
\end{bmatrix} \succeq 0
\]
(9)
Namely,

\[
M^{-1} \mathcal{N}^{jk}_{PR} M^{-1} = \begin{bmatrix}
1 & 0 & \varrho_{1j}^{AB} & \varrho_{0j}^{AB} \\
0 & 1 & 1 & (-1)^k \\
\varrho_{1j}^{AB} & 1 & 1 & r_{jk} \\
\varrho_{0j}^{AB} & (-1)^k & r_{jk} & 1
\end{bmatrix} \geq 0 \quad (10)
\]

where \( M \) is a diagonal matrix whose (non-vanishing) terms are all ones but \( \Delta_{Bj} \). By the Schur complement condition for positive semidefiniteness, \((10)\) is equivalent to

\[
\begin{bmatrix}
1 & r_{jk} \\
r_{jk} & 1
\end{bmatrix} \succeq \begin{bmatrix}
\varrho_{1j}^{AB} & 1 \\
\varrho_{0j}^{AB} & (-1)^k
\end{bmatrix}^T \begin{bmatrix}
\varrho_{1j}^{AB} & \varrho_{0j}^{AB} \\
\varrho_{1j}^{AB} & \varrho_{0j}^{AB}
\end{bmatrix} + \begin{bmatrix}
1 & (-1)^k \\
(-1)^k & 1
\end{bmatrix} \quad (11)
\]

which renders \( \varrho_{ij}^{AB} = 0 \) (positive-semidefiniteness of the matrix obtained by subtracting the right-hand side from the left-hand side implies the non-negativity of its diagonal entries from which this result follows). The inequality \((11)\) is equivalent to \(-[r_{jk} - (-1)^k]^2 \geq 0\), and only holds for \( r_{jk} = (-1)^k \). Such a theory therefore violates relativistic independence.

But the PR-box example teaches us something profound. In this model, complementarity (i.e., the inability to measure both local variables in the same experiment) must be assumed in both Alice’s and Charlie’s ends, for otherwise Alice, for example, may evaluate,

\[
A_0 A_1 = (A_0 C_k)(A_1 C_k) = C(A_0, C_k)C(A_1, C_k) = (-1)^0(-1)^k = (-1)^k = r_{jk} \quad (12)
\]

from which she could tell Charlie’s choice \( k \). Lack of complementarity immediately leads to signaling in the case of PR-boxes, but as we have seen, the weaker assumption of uncertainty leads to a problem with relativistic independence.

C. Schrödinger-Robertson uncertainty relations and the generalized uncertainty relations (1), (2), and (3)

Let \( \hat{A}_i \) and \( \hat{B}_j \) be self-adjoint operators with \( \pm 1 \) eigenvalues and \( \langle \hat{A}_i \rangle = \langle \hat{B}_j \rangle = 0 \), whose product, \( \hat{A}_i \hat{B}_j \) is similarly self-adjoint. The Schrödinger-Robertson uncertainty relations of the corresponding products, \( \hat{A}_0 \hat{B}_j \) and \( \hat{A}_1 \hat{B}_j \),

\[
\Delta^2_{\hat{A}_0 \hat{B}_j} \Delta^2_{\hat{A}_1 \hat{B}_j} \geq \left( \frac{1}{2} \langle \{ \hat{A}_0, \hat{A}_1 \} \rangle - C(\hat{A}_0, \hat{B}_j)C(\hat{A}_1, \hat{B}_j) \right)^2 + \left( \frac{1}{2\pi} \langle [\hat{A}_0, \hat{A}_1] \rangle \right)^2 \quad (13)
\]
where $C(\hat{A}_i, \hat{B}_j) = \langle \hat{A}_i \hat{B}_j \rangle$, and the variance, $\Delta^2_{\hat{A}_i \hat{B}_j} = 1 - C(\hat{A}_i, \hat{B}_j)^2$, can alternatively be written as

$$
\begin{bmatrix}
1 & \langle \hat{A}_0 \hat{A}_1 \rangle \\
\langle \hat{A}_1 \hat{A}_0 \rangle & 1
\end{bmatrix} \succeq \begin{bmatrix}
C(\hat{A}_1, \hat{B}_j)^2 & C(\hat{A}_1, \hat{B}_j) C(\hat{A}_0, \hat{B}_j) \\
C(\hat{A}_1, \hat{B}_j) C(\hat{A}_0, \hat{B}_j) & C(\hat{A}_0, \hat{B}_j)^2
\end{bmatrix}
$$

(14)

By the Schur complement condition for positive semidefiniteness this is equivalent to

$$
\begin{bmatrix}
\Delta^2_{\hat{B}_j} & C(\hat{A}_1, \hat{B}_j) & C(\hat{A}_0, \hat{B}_j) \\
C(\hat{A}_1, \hat{B}_j) & \Delta^2_{\hat{A}_1} & \langle \hat{A}_0 \hat{A}_1 \rangle \\
C(\hat{A}_0, \hat{B}_j) & \langle \hat{A}_1 \hat{A}_0 \rangle & \Delta^2_{\hat{A}_0}
\end{bmatrix} \succeq 0
$$

(15)

because $\Delta^2_{\hat{B}_j} = \langle \hat{B}_j^2 \rangle - \langle \hat{B}_j \rangle^2 = 1$ and $\Delta^2_{\hat{A}_i} = \langle \hat{A}_i^2 \rangle - \langle \hat{A}_i \rangle^2 = 1$. This in turn implies

$$
\begin{bmatrix}
\Delta^2_{\hat{B}_j} & C(\hat{A}_1, \hat{B}_j) & C(\hat{A}_0, \hat{B}_j) \\
C(\hat{A}_1, \hat{B}_j) & \Delta^2_{\hat{A}_1} & r \\
C(\hat{A}_0, \hat{B}_j) & r & \Delta^2_{\hat{A}_0}
\end{bmatrix} \succeq 0
$$

(16)

with $r = \langle \{\hat{A}_0, \hat{A}_1\} \rangle / 2$. The inequalities in (3) generalize the uncertainty relation (16) to arbitrary measurements. The inequalities (11) and (2) further extend (16) to include the remaining measurements of Alice, Bob and Charlie.

**D. Proof of Theorem 1**

By the Schur complement condition for positive semidefiniteness the first condition in (3) is equivalent to $\Lambda_A \succeq \Delta^{-2}_{\hat{B}_j} C(A, B_j) C(A, B_j)^T$. This can be normalized,

$$
M^{-1} \Lambda_A M^{-1} = \begin{bmatrix}
1 & r' \\
r' & 1
\end{bmatrix} \succeq \begin{bmatrix}
\varrho_{ij}^2 & \varrho_{ij} \varrho_{1j} \\
\varrho_{ij} \varrho_{1j} & \varrho_{0j}^2
\end{bmatrix} = \Delta^{-2}_{\hat{B}_j} M^{-1} \begin{bmatrix}
C(A_1, B_j) \\
C(A_0, B_1)
\end{bmatrix} \begin{bmatrix}
C(A_1, B_j) & C(A_0, B_1)
\end{bmatrix} M^{-1}
$$

(17)

where, $r' \equiv \frac{r}{\Delta_{\hat{A}_1} \Delta_{\hat{A}_0}}$, and $M$ is a diagonal matrix whose (non-vanishing) entries are $\Delta_{\hat{A}_1}$, and $\Delta_{\hat{A}_0}$. This condition is equivalent to

$$
|r' - \varrho_{ij} \varrho_{1j}| \leq \sqrt{(1 - \varrho_{ij}^2)(1 - \varrho_{1j}^2)}
$$

(18)

which follows from the non-negative determinant of the matrix obtained by subtracting the right-hand side from the left-hand side in (17). This together with the triangle inequality yield

$$
|\varrho_{00} \varrho_{10} - r' + r' - \varrho_{01} \varrho_{11}| \leq |r' - \varrho_{00} \varrho_{10}| + |r' - \varrho_{01} \varrho_{11}| \leq \sum_{j=0,1} \sqrt{(1 - \varrho_{0j}^2)(1 - \varrho_{1j}^2)}
$$

(19)
The second inequality in (4) is similarly obtained by swapping the roles of Alice and Bob, i.e., from the second relativistic independence condition in (3).

E. Proof of Theorem

In the Hilbert space formulation of quantum mechanics Alice’s measurements are represented by the self-adjoint operators \( \hat{A}_0 \) and \( \hat{A}_1 \). Similarly, Bob’s measurements are represented by the self-adjoint operators \( \hat{B}_j \). The Schrödinger-Robertson uncertainty relations of \( \hat{A}_0 \) and \( \hat{A}_1 \) is

\[
\Delta_{A_0}^2 \Delta_{A_1}^2 \geq \left( \frac{1}{2} \langle \{ \hat{A}_0, \hat{A}_1 \} \rangle - \langle \hat{A}_0 \rangle \langle \hat{A}_1 \rangle \right)^2 + \left( \frac{1}{2l} \langle [\hat{A}_0, \hat{A}_1] \rangle \right)^2
\]

(20)

where \( \Delta_{A_i}^2 = \langle \hat{A}_i^2 \rangle - \langle \hat{A}_i \rangle^2 \) is the variance of \( \hat{A}_i \). This may alternatively be written as

\[
\Lambda_{\hat{A}} = \begin{bmatrix} \Delta_{A_1}^2 & r_Q \\ r_Q^* & \Delta_{A_0}^2 \end{bmatrix} \succeq 0
\]

(21)

where \( r_Q \triangleq \langle \hat{A}_1 \hat{A}_0 \rangle - \langle \hat{A}_1 \rangle \langle \hat{A}_0 \rangle \) with \( r_Q^* \) being its complex conjugate. It can be recognized that this leads to Alice’s part in the generalized uncertainty relation (2) where \( r_{jk} = (r_Q + r_Q^*)/2 \) is independent of \( j \) and \( k \).

We shall show that the relativistic independence condition, the first inequality in (3), holds in Hilbert space. This condition tells that

\[
\Lambda_{\hat{A}\hat{B}} = \begin{bmatrix} \Delta_{B_j}^2 & \langle \hat{A}_1 \hat{B}_j \rangle - \langle \hat{A}_1 \rangle \langle \hat{B}_j \rangle & \langle \hat{A}_1 \hat{B}_j \rangle - \langle \hat{A}_1 \rangle \langle \hat{B}_j \rangle - \langle \hat{A}_0 \rangle \langle \hat{B}_j \rangle \\
\langle \hat{A}_1 \hat{B}_j \rangle - \langle \hat{A}_1 \rangle \langle \hat{B}_j \rangle & \Delta_{A_1}^2 & \langle \hat{A}_1 \hat{A}_0 \rangle - \langle \hat{A}_1 \rangle \langle \hat{A}_0 \rangle \\
\langle \hat{A}_0 \hat{B}_j \rangle - \langle \hat{A}_0 \rangle \langle \hat{B}_j \rangle & \langle \hat{A}_0 \hat{B}_j \rangle - \langle \hat{A}_0 \rangle \langle \hat{B}_j \rangle & \Delta_{A_0}^2 \end{bmatrix}
\]

(22)

where \( \Delta_{B_j}^2 = \langle \hat{B}_j^2 \rangle - \langle \hat{B}_j \rangle^2 \), is a positive semidefinite matrix. Let \( U^* = [u_1, u_2, u_3] \) be any \( 3 \times 1 \) complex-valued vector, and denote \( |\phi\rangle \) the underlying state. Note that

\[
U^* \Lambda_{\hat{A}\hat{B}} U = V^* V \succeq 0
\]

(23)

where

\[
V \triangleq u_1 \left( \hat{B}_j - \langle \hat{B}_j \rangle \right) |\phi\rangle + u_2 \left( \hat{A}_1 - \langle \hat{A}_1 \rangle \right) |\phi\rangle + u_3 \left( \hat{A}_0 - \langle \hat{A}_0 \rangle \right) |\phi\rangle
\]

(24)

which shows that \( \Lambda_{\hat{A}\hat{B}} \succeq 0 \), and therefore (3) hold.
In what follows we show that $\Lambda_{\hat{A}\hat{B}} \succeq 0$ implies the first bound in (5). Note that

$$M^{-1} \Lambda_{\hat{A}\hat{B}} M^{-1} = \begin{bmatrix} 1 & \varrho_{1j} & \varrho_{0j} \\ \varrho_{1j} & 1 & \frac{r_0}{\Delta_{\hat{A}_1} \Delta_{\hat{A}_0}} \\ \varrho_{0j} & \frac{r_0}{\Delta_{\hat{A}_1} \Delta_{\hat{A}_0}} & 1 \end{bmatrix} \succeq 0, \quad j = 0, 1$$

(25)

where $M$ is a diagonal matrix whose (non-vanishing) entries are $\Delta_{\hat{B}_j}$, $\Delta_{\hat{A}_1}$, and $\Delta_{\hat{A}_0}$. By the Schur complement condition for positive semidefiniteness (25) is equivalent to

$$\begin{bmatrix} 1 - \varrho_{1j}^2 & \frac{r_0}{\Delta_{\hat{A}_1} \Delta_{\hat{A}_0}} - \varrho_{1j} \varrho_{0j} \\ \frac{r_0}{\Delta_{\hat{A}_1} \Delta_{\hat{A}_0}} - \varrho_{1j} \varrho_{0j} & 1 - \varrho_{0j}^2 \end{bmatrix} \succeq 0, \quad j = 0, 1$$

(26)

This in turn is equivalent to the requirement that the determinant of this matrix is nonnegative, i.e., that

$$(1 - \varrho_{1j}^2) (1 - \varrho_{0j}^2) \geq \left( \frac{\langle \hat{A}_0, \hat{A}_1 \rangle / 2 - \langle \hat{A}_0 \rangle \langle \hat{A}_1 \rangle}{\Delta_{\hat{A}_1} \Delta_{\hat{A}_0}} - \varrho_{0j} \varrho_{1j} \right)^2 + \left( \frac{1}{2} \frac{\langle [\hat{A}_0, \hat{A}_1] \rangle}{\Delta_{\hat{A}_1} \Delta_{\hat{A}_0}} \right)^2, \quad j = 0, 1$$

(27)

namely,

$$\sqrt{(1 - \varrho_{1j}^2) (1 - \varrho_{0j}^2)} - \eta_{\hat{A}}^2 \geq \left| \frac{\langle \hat{A}_0, \hat{A}_1 \rangle / 2 - \langle \hat{A}_0 \rangle \langle \hat{A}_1 \rangle}{\Delta_{\hat{A}_1} \Delta_{\hat{A}_0}} - \varrho_{0j} \varrho_{1j} \right|, \quad j = 0, 1$$

(28)

where $\eta_{\hat{A}}$ is as defined in the theorem. This together with the triangle inequality implies the first bound in the theorem,

$$|\varrho_{00} \varrho_{10} - \varrho_{01} \varrho_{11}| \leq \sum_{j=0,1} \left| \frac{\langle \hat{A}_0, \hat{A}_1 \rangle / 2 - \langle \hat{A}_0 \rangle \langle \hat{A}_1 \rangle}{\Delta_{\hat{A}_1} \Delta_{\hat{A}_0}} - \varrho_{0j} \varrho_{1j} \right| \leq \sum_{j=0,1} \sqrt{(1 - \varrho_{1j}^2) (1 - \varrho_{0j}^2)} - \eta_{\hat{A}}^2$$

(29)

The remaining bound similarly follows from the second relativistic independence condition in (3).

It was previously noted that for the case where $\hat{A}_i^2 = \hat{B}_j^2 = I$ and $\langle A_i \rangle = \langle B_j \rangle = 0$, the inequality (27) coincides with the Schrödinger-Robertson uncertainty relations of $\hat{A}_0 \hat{B}_j$ and $\hat{A}_1 \hat{B}_j$, the inequality (13).

F. Nonlocality and Heisenberg uncertainty

An interesting corollary of Theorem 2 is that there is a bound, a generalization of Tsirelson’s $2\sqrt{2}$, for different values of $\eta_{\hat{A}}$ and $\eta_{\hat{B}}$. In particular,

$$|\mathcal{B}| \leq 2\sqrt{2} \sqrt{1 - \max\{\eta_{\hat{A}}^2, \eta_{\hat{B}}^2\}}$$

(30)
A geometrical view of this bound is given in Figure 2. Application of (30) to $\hat{A}_0 = \hat{x}$ and $\hat{A}_1 = \hat{p}$, the position and momentum operators, yields

$$|B| \leq 2\sqrt{2} \sqrt{1 - \left(\frac{\hbar/2}{\Delta_x \Delta_p}\right)^2}$$

which follows from the definition of $\eta_A$ and the identity $[\hat{x}, \hat{p}] = i\hbar$. This elucidates the interplay between the extent of nonlocality and the Heisenberg uncertainty principle. The greater the uncertainty $\Delta_x \Delta_p$, the stronger the nonlocality may get, where Tsirelson’s $2\sqrt{2}$ corresponds to the limit $\Delta_x \Delta_p \to \infty$.

More generally, relativistic independence implies a close relationship between nonlocality as quantified by the Bell-CHSH parameter and the uncertainty parameter $r$ in (3). This is summarized in the next theorem.

**Theorem 3.** By relativistic independence

$$\left(\frac{B}{2\sqrt{2}}\right)^2 + |r'|^2 \leq 1$$

where as before, $r' \overset{\text{def}}{=} \frac{r}{\Delta_A \Delta_A}$. In quantum mechanics where $r = r_Q$ in (21) this relation assumes an explicit form

$$\left(\frac{B}{2\sqrt{2}}\right)^2 + \left|\frac{\langle A_0 A_1 \rangle - \langle A_0 \rangle \langle A_1 \rangle}{\Delta_A \Delta_A}\right|^2 \leq 1$$

**Proof.** Assume that $\varrho_{ij} = (-1)^i j$, a configuration underlying the maximal Bell-CHSH parameter, i.e., $B = 4\varrho$. Relativistic independence (3) implies (18), which in this case yields

$$[r' - (-1)^i j \varrho^2]^2 \leq (1 - \varrho^2)^2$$

That is,

$$|r'|^2 + \left(\frac{B}{2\sqrt{2}}\right)^2 - 2(-1)^i j \varrho \leq 1$$

where we have used the identity $\varrho = B/4$. Averaging (35) for $j = 0$ and $j = 1$ implies the theorem.

**G. Locality from relativistic independence**

The preceding sections forged a theory-free notion of nonlocality in the form of correlators that satisfy relativistic independence. Can locality (as appearing in classical statistical theories), which
is normally defined by means of Bell inequalities, be similarly characterized? We will show that locality is in some sense a variant of relativistic independence.

The first relativistic independence condition in (3) may alternatively be written as

\[ M^{-1} \Lambda \Lambda A^{-1} - \tilde{R}_0 \tilde{R}_0^T \begin{bmatrix} 0_{2 \times 2} & M^{-1} \Lambda \Lambda A^{-1} - \tilde{R}_1 \tilde{R}_1^T \end{bmatrix} \succeq 0 \]  

(36)

where \( M \) is a diagonal matrix whose (non-vanishing) entries are \( \Delta_{A_0} \) and \( \Delta_{A_1} \), and \( \tilde{R}_j^T = [\varrho_{0j}, \varrho_{1j}] \). Relativistic independence may further restrict the underlying correlators when the off-diagonal blocks do not vanish. Locality is implied, for example, by

\[ M^{\mathcal{L}} \overset{\text{def}}{=} \begin{bmatrix} M^{-1} \Lambda \Lambda A^{-1} - \tilde{R}_0 \tilde{R}_0^T & \tilde{R}_0 \tilde{R}_1^T \\ \tilde{R}_1 \tilde{R}_0^T & M^{-1} \Lambda \Lambda A^{-1} - \tilde{R}_1 \tilde{R}_1^T \end{bmatrix} \succeq 0 \]  

(37)

In particular,

\[ u M^L u^T = 4 - \mathcal{B}^2 \geq 0 \]  

(38)

where \( u = [1, 1, 1, -1] \), and \( \mathcal{B} \overset{\text{def}}{=} \varrho_{00} + \varrho_{10} + \varrho_{01} - \varrho_{11} \) is the Bell-CHSH parameter.

The non-vanishing off-diagonal matrices in (37) essentially render the underlying uncertainty relations of both experimenters ineffective. To see how, note that the matrix in (37) (but not that in (36)) is the covariance of the four products \( A_i B_j \), \( i, j = 0, 1 \), where \( A_i \) and \( B_j \) are Alice’s and Bob’s measurement outcomes. Therefore, the joint probabilities of \( A_0 \) and \( A_1 \), and of \( B_0 \) and \( B_1 \) exist and the correlators satisfy the Bell-CHSH inequality. As mentioned in the main text, here the parameter \( r = C(A_0, A_1), \Delta_{A_0}^2 \Delta_{A_1}^2 \geq r^2 \). However, this form of the uncertainty relation cannot be saturated but for the trivial case of deterministic \( A_0 \) and \( A_1 \).

**H. Relativistic independence in general multipartite settings**

Suppose that some experimenters are located at spacetime region \( S \) and some others at spacetime region \( T \). Each experimenter has an arbitrary number of measuring devices. We shall denote by \( S_i \) and \( T_j \) the vectors of measurements in \( S \) and \( T \), where the indices \( i \) and \( j \) represent sets of choices of measuring devices in each region. As in the bipartite case, we may write \( \Lambda_S(i) \) and \( \Lambda_T(j) \) for the uncertainty relations underlying the sets of measurements \( i \) in \( S \) and \( j \) in \( T \). The covariances between \( S_i \) and \( T_j \) may similarly be expressed by a matrix \( R \).

Relativistic independence dictates that uncertainty relations in \( S \) are independent of choices in \( T \). Therefore, \( S \) is independent of whether \( j = 0 \) or \( j = 1 \) in \( T \). This is expressed mathematically
by
\[
\begin{bmatrix}
\Lambda_T(0) & R_0^T \\
R_0 & \Lambda_S
\end{bmatrix} \succeq 0, \quad \begin{bmatrix}
\Lambda_T(1) & R_1^T \\
R_1 & \Lambda_S
\end{bmatrix} \succeq 0
\]  

(39)

But also in the converse direction, uncertainty relations in \( T \) are independent of choices in \( S \),
\[
\begin{bmatrix}
\Lambda_T & R_T^0 \\
R_0 & \Lambda_S(0)
\end{bmatrix} \succeq 0, \quad \begin{bmatrix}
\Lambda_T & R_T^1 \\
R_1 & \Lambda_S(1)
\end{bmatrix} \succeq 0
\]  

(40)

Below we use these to derive a bound on the quantum mechanical, Alice-Bob, Alice-Charlie, and Bob-Charlie, one- and two-point correlators. The relation thus obtained generalizes (4) in this tripartite setting.

We note that (39) and (40) do not represent the most general approach for characterizing non-local correlations. Nevertheless, they facilitate analyses and in particular the derivation of the theorems that follow. A complete characterization of the set of quantum correlations would require analyzing (1) in a general multipartite setting. In such a case the cross-correlations between the \( S \) and \( T \) subsets would have to be accounted for. To some degree this is practiced in the derivation of Theorem 4 where it is assumed that Bob and Charlie are correlated. Disconnecting them by making their correlations zero leads to the well known monogamy relation in Theorem 5.

In the tripartite case, where Alice in \( S \) measures either \( A_0 \) or \( A_1 \), and Bob and Charlie in \( T \) measure \((B_l, C_k)\) or \((B_l', C_k')\), relativistic independence (39) holds for
\[
\begin{align*}
\Lambda_T(0) & \defeq \begin{bmatrix}
\Delta_{C_k}^2 & C(C_k, B_l) \\
C(C_k, B_l) & \Delta_{B_l}^2
\end{bmatrix}, \quad \Lambda_T(1) \defeq \begin{bmatrix}
\Delta_{C_k'}^2 & C(C_k', B_l') \\
C(C_k', B_l') & \Delta_{B_l'}^2
\end{bmatrix}, \\
\Lambda_S & \defeq \begin{bmatrix}
\Delta_{A_1}^2 & r \\
r & \Delta_{A_0}^2
\end{bmatrix}
\end{align*}
\]  

(41)

where
\[
R_0^T = \begin{bmatrix} 
C(A_1, C_k) & C(A_0, C_k) \\
C(A_1, B_l) & C(A_0, B_l)
\end{bmatrix}, \quad R_1^T = \begin{bmatrix} 
C(A_1, C_k') & C(A_0, C_k') \\
C(A_1, B_l') & C(A_0, B_l')
\end{bmatrix}
\]  

(42)

**Theorem 4.** The relativistic independence condition (39) with the matrices in (41) and (42) imply
\[
|\zeta_{01}(l, k) - \zeta_{01}(l', k')| \leq \sqrt{(1 - \zeta_{11}(l, k))(1 - \zeta_{00}(l, k)) + (1 - \zeta_{11}(l', k'))(1 - \zeta_{00}(l', k'))}
\]  

(43)

where,
\[
\zeta_{ij}(l, k) \defeq \frac{\varrho_{ij}^{AC} \varrho_{ij}^{BC} - \varrho_{ij}^{BC} \varrho_{ij}^{AC} - \varrho_{ij}^{BC} \varrho_{ij}^{AB} + \varrho_{ij}^{AB} \varrho_{ij}^{AC}}{(1 - \varrho_{lk}^{BC})^2}
\]  

(44)

and \( \varrho_{ij}^{XY} \defeq C(X_i, Y_j)/(\Delta_{X_i} \Delta_{Y_j}) \). Note that letting \( \varrho^{AC} = \varrho^{BC} = 0 \) in (43) recovers the bound on the Alice-Bob correlators, the first inequality in (4).
and similarly for \( k' \) and \( l' \). This is equivalent to

\[
M^{-1} \Lambda_{ABC} M^{-1} = \begin{bmatrix}
1 & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} \\
\varrho_{lk} & 1 & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} \\
\varrho_{lk} & \varrho_{lk} & 1 & \varrho_{lk} & \varrho_{lk} \\
\varrho_{lk} & \varrho_{lk} & \varrho_{lk} & 1 & \varrho_{lk} \\
\varrho_{lk} & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} & 1
\end{bmatrix} \succeq 0
\]

(46)

where \( r' \equiv r/(\Delta_{A_1} \Delta_{A_0}) \), and \( M \) is a diagonal matrix whose (non-vanishing) entries are \( \Delta_{C_k} \), \( \Delta_{B_l} \), \( \Delta_{A_1} \), and \( \Delta_{A_0} \). By the Schur complement condition for positive semidefiniteness, (46) is equivalent to

\[
\begin{bmatrix}
1 & r' \\
r' & 1
\end{bmatrix} \succeq \begin{bmatrix}
\varrho_{lk} & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} \\
\varrho_{lk} & 1 & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} \\
\varrho_{lk} & \varrho_{lk} & 1 & \varrho_{lk} & \varrho_{lk} \\
\varrho_{lk} & \varrho_{lk} & \varrho_{lk} & 1 & \varrho_{lk} \\
\varrho_{lk} & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} & 1
\end{bmatrix} \begin{bmatrix}
1 & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} \\
\varrho_{lk} & 1 & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} \\
\varrho_{lk} & \varrho_{lk} & 1 & \varrho_{lk} & \varrho_{lk} \\
\varrho_{lk} & \varrho_{lk} & \varrho_{lk} & 1 & \varrho_{lk} \\
\varrho_{lk} & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} & 1
\end{bmatrix}^{-1} \begin{bmatrix}
\varrho_{lk} & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} \\
\varrho_{lk} & 1 & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} \\
\varrho_{lk} & \varrho_{lk} & 1 & \varrho_{lk} & \varrho_{lk} \\
\varrho_{lk} & \varrho_{lk} & \varrho_{lk} & 1 & \varrho_{lk} \\
\varrho_{lk} & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} & 1
\end{bmatrix}
\]

(47)

which holds if and only if the determinant of the matrix obtained by subtracting the right-hand side from the left-hand side in (47) is nonnegative. Carrying out this calculation for \( k, l \) and then for \( k', l' \), and invoking the triangle inequality yield (43).

The next theorem shows that the bound (43) implies monogamy of correlations. This means that breaking of monogamy necessarily violates relativistic independence.

**Theorem 5.** If Charlie and Bob are uncorrelated, \( C(C_k, B_j) = 0 \), then by relativistic independence

\[
|\mathcal{B}_{AB}^2 + \mathcal{B}_{AC}^2| \leq 8
\]

(48)

and therefore also \( |\mathcal{B}_{AB}| + |\mathcal{B}_{AC}| \leq 4 \), where both Bell-CHSH parameters, \( \mathcal{B}_{AB} \) and \( \mathcal{B}_{AC} \), are for the same pair, \( A_0 \), \( A_1 \).

**Proof.** Substituting \( \varrho_{lk}^{BC} = 0 \) in (47) implies

\[
2(1 \pm r') = u^T \begin{bmatrix}
1 & r' \\
r' & 1
\end{bmatrix} u \geq u^T \begin{bmatrix}
\varrho_{lk} & \varrho_{lk} & \varrho_{lk} & \varrho_{lk} \\
\varrho_{lk} & 1 & \varrho_{lk} & \varrho_{lk} \\
\varrho_{lk} & \varrho_{lk} & 1 & \varrho_{lk} \\
\varrho_{lk} & \varrho_{lk} & \varrho_{lk} & 1
\end{bmatrix} u = [\varrho_{0j}^{AB} \pm \varrho_{1j}^{AB}]^2 + [\varrho_{0k}^{AC} \pm \varrho_{1k}^{AC}]^2
\]

(49)
for \( u^T = [1, \pm 1] \). Therefore,
\[
4 \geq [\varrho_{00}^{AB} \pm \varrho_{10}^{AB}]^2 + [\varrho_{00}^{AC} \pm \varrho_{10}^{AC}]^2 + [\varrho_{01}^{AB} \pm \varrho_{11}^{AB}]^2 + [\varrho_{11}^{AC} \pm \varrho_{11}^{AC}]^2 \geq \frac{1}{2} \mathcal{R}_{AB}^2 + \frac{1}{2} \mathcal{R}_{AC}^2 \tag{50}
\]
from which the theorem follows.

I. Monogamy of correlations in general multipartite settings

The above result is a special case of the more general scenario where any number of experimenters are correlated with Alice but uncorrelated among themselves. Suppose there are \( n \) experimenters whose measurements are uncorrelated, \( C(M^k_i, M^l_j) = 0 \), where \( M^k_i \) stands in for the \( k \)th physical variable measured by the \( i \)th experimenter. In this case the generalized uncertainty relations underlying Alice measurements \( A_0, A_1, \) and the \( n \) other measurements \( M^1_{i1}, \ldots, M^n_{in} \) are described by
\[
\begin{bmatrix}
1 & \varrho_{0,1}^0 & \varrho_{1,1}^0 \\
\vdots & \vdots & \vdots \\
\varrho_{0,n}^n & \varrho_{1,n}^n & 1 \\
1 & r'_{i1,...,in} & 1
\end{bmatrix} \succcurlyeq 0 
\tag{51}
\]
where, \( \varrho_{i,k}^s \overset{\text{def}}{=} C(A_i, M^k_i)/(\Delta A_i \Delta M^k_i) \). This matrix is obtained as an extension of (2) following a normalization similar to the one in previous sections. In this case, Alice’s uncertainty relations are governed by the parameter \( r'_{i1,...,in} \) which may depend on the choices of all of the other experimenters.

**Theorem 6.** Relativistic independence implies
\[
\sum_{s=1}^{n} |\mathcal{B}_s| \leq \sqrt{2n} \left( \sqrt{1 + r'} + \sqrt{1 - r'} \right) \leq 2\sqrt{2n} 
\tag{52}
\]
where \( \mathcal{B}_s \overset{\text{def}}{=} \varrho_{0,i}^s + \varrho_{1,i}^s + \varrho_{0,j}^s - \varrho_{1,j}^s \) is the Bell-CHSH parameter of Alice and the \( s \)th experimenter. Tsirelson’s bound and the monogamy property of correlations follow from this inequality as special cases for \( n = 1 \) and \( n = 2 \), respectively.

**Proof.** If relativistic independence holds then \( r'_{i1,...,in} = r'_{j1,...,jn} = r' \). By the Schur complement condition for positive semidefiniteness, (51) is equivalent to
\[
\begin{bmatrix}
1 & r' \\
r' & 1
\end{bmatrix} \succeq \sum_{s=1}^{n} \begin{bmatrix} \varrho_{0,i}^s & \varrho_{1,i}^s \\ \varrho_{1,i}^s & \varrho_{0,i}^s \end{bmatrix} \begin{bmatrix} \varrho_{0,i}^s & \varrho_{1,i}^s \\ \varrho_{1,i}^s & \varrho_{0,i}^s \end{bmatrix} \tag{53}
\]

and similarly,

\[
\begin{bmatrix}
1 & r' \\
-r' & 1
\end{bmatrix} \succeq \sum_{s=1}^{n} \begin{bmatrix}
\varrho^{s}_{0,j_s} & \varrho^{s}_{1,j_s} \\
\varrho^{s}_{1,j_s} & \varrho^{s}_{0,j_s}
\end{bmatrix}
\]

(54)

Both (53) and (54) imply

\[
2(1 \pm r') \geq \sum_{s=1}^{n} (\varrho^{s}_{0,i_s} \pm \varrho^{s}_{1,i_s})^2, \quad 2(1 \pm r') \geq \sum_{s=1}^{n} (\varrho^{s}_{0,j_s} \pm \varrho^{s}_{1,j_s})^2
\]

(55)

which are obtained similarly to (49). By norm equivalence,

\[
2n(1 \pm r') \geq \left( \sum_{s=1}^{n} |\varrho^{s}_{0,i_s} \pm \varrho^{s}_{1,i_s}| \right)^2, \quad 2n(1 \pm r') \geq \left( \sum_{s=1}^{n} |\varrho^{s}_{0,j_s} \pm \varrho^{s}_{1,j_s}| \right)^2
\]

(56)

Finally, invoking the triangle inequality

\[
\sum_{s=1}^{n} |\mathcal{B}_s| \leq \sum_{s=1}^{n} \left| \varrho^{s}_{0,i_s} + \varrho^{s}_{1,i_s} \right| + \left| \varrho^{s}_{0,j_s} - \varrho^{s}_{1,j_s} \right| \leq \sqrt{2n(1+r')} + \sqrt{2n(1-r')} \leq 2\sqrt{2n}
\]

(57)

J. Tighter than Schrödinger-Robertson uncertainty relations following from (3)

Alice’s uncertainty relations are represented by the $2 \times 2$ lower submatrix $\Lambda_A$ in the generalized uncertainty relation (3). This shows that (3) is more stringent than any uncertainty relation derived exclusively from $\Lambda_A \succeq 0$. Consider, for example, a generalized uncertainty relation of the form

\[
\begin{bmatrix}
\Lambda_D & C \\
C^T & \Lambda_A
\end{bmatrix} \succeq 0
\]

(58)

where $D$ is an invertible $n \times n$ matrix, and $C$ is $n \times 2$ cross-covariance matrix. By the Schur complement condition for positive semidefiniteness this inequality is equivalent to $\Lambda_A \succeq C^T \Lambda_D^{-1} C$, which unless $C$ vanishes is tighter than $\Lambda_A \succeq 0$.

As shown in the preceding sections, from within quantum mechanics the inequality $\Lambda_A \succeq 0$, which follows from the lower $2 \times 2$ submatrix in (2) and (3), is equivalent to the Schrödinger-Robertson uncertainty relations underlying Alice’s observables $\hat{A}_0$ and $\hat{A}_1$. That quantum mechanics obey generalized uncertainty relations like (3), and more generally (58), implies that any uncertainty relation derived from $\Lambda_A \succeq 0$ makes only a small part of the story. There are many more restrictions arising from our approach all of which are tighter than the Schrödinger-Robertson uncertainty relation that are obeyed by Alice’s observables. One such uncertainty relation is given below.
Let $D = \hat{A}_i^m$, where $\hat{A}_i$ is one of Alice’s observables, $i = 0, 1$, and $m$ is an integer, $m > 1$. From within quantum mechanics, the generalized uncertainty (58) is now given by

$$
\begin{bmatrix}
\Delta^2_{\hat{A}_i^m} & C(\hat{A}_i^m, \hat{A}_1) & C(\hat{A}_i^m, \hat{A}_0) \\
C(\hat{A}_1, \hat{A}_i^m) & \Delta^2_{\hat{A}_1} & C(\hat{A}_1, \hat{A}_0) \\
C(\hat{A}_0, \hat{A}_i^m) & C(\hat{A}_0, \hat{A}_1) & \Delta^2_{\hat{A}_0}
\end{bmatrix} \succeq 0
given by

(59)

where $C(\hat{A}_i, \hat{A}_j) \equiv \langle \hat{A}_i \hat{A}_j \rangle - \langle \hat{A}_i \rangle \langle \hat{A}_j \rangle$. The quantities $\Delta^2_{\hat{A}_i^m}$ and $C(\hat{A}_i^m, \hat{A}_1)$ in (59) involve higher statistical moments of the underlying observables. The inequality (59) is equivalent to

$$
\Lambda_A = \begin{bmatrix}
\Delta^2_{\hat{A}_1} & C(\hat{A}_1, \hat{A}_0) \\
C(\hat{A}_0, \hat{A}_1) & \Delta^2_{\hat{A}_0}
\end{bmatrix} \succeq \Delta^2_{\hat{A}_i^m} \begin{bmatrix}
C(\hat{A}_1, \hat{A}_i^m) \\
C(\hat{A}_0, \hat{A}_i^m)
\end{bmatrix} \begin{bmatrix}
C(\hat{A}_i^m, \hat{A}_1) \\
C(\hat{A}_i^m, \hat{A}_0)
\end{bmatrix}
given by the Schur complement condition for positive semidefiniteness. Let $v^T \equiv [1, \pm 1]/\sqrt{2}$ and note that

$$
v^T \Lambda_A v = \frac{1}{2} \Delta^2_{\hat{A}_1} + \frac{1}{2} \Delta^2_{\hat{A}_0} \pm \left[ \frac{1}{2} \langle \{\hat{A}_1, \hat{A}_0\} \rangle - \langle \hat{A}_1 \rangle \langle \hat{A}_0 \rangle \right] \geq \frac{1}{2 \Delta^2_{\hat{A}_i^m}} \left| C(\hat{A}_1, \hat{A}_i^m) \pm C(\hat{A}_0, \hat{A}_i^m) \right|^2
$$

(61)

Therefore,

$$
\Delta^2_{\hat{A}_1} + \Delta^2_{\hat{A}_0} \geq 2 \left[ \frac{1}{2} \langle \{\hat{A}_1, \hat{A}_0\} \rangle - \langle \hat{A}_1 \rangle \langle \hat{A}_0 \rangle \right] + \frac{1}{2 \Delta^2_{\hat{A}_i^m}} \left| C(\hat{A}_1, \hat{A}_i^m) \pm C(\hat{A}_0, \hat{A}_i^m) \right|^2
$$

(62)

This uncertainty relation is to be contrasted with

$$
\Delta^2_{\hat{A}_1} + \Delta^2_{\hat{A}_0} \geq 2 \left| \frac{1}{2} \langle \{\hat{A}_1, \hat{A}_0\} \rangle - \langle \hat{A}_1 \rangle \langle \hat{A}_0 \rangle \right|
given by

(63)

which follows from $\Lambda_A \succeq 0$ using similar arguments. Note also that much like the Maccone-Pati uncertainty relations [20], these additive inequalities do not become trivial in the case where the state coincides with an eigenvector of one of the observables.

**K. The measurability of $r_j$ in a bipartite setting**

In what follows we examine relativistic independence from a different perspective. As mentioned in the main text, this condition may be viewed as the requirement that one experimenter’s uncertainty relations are independent of another experimenters’ choices. We claim that if it weren’t so, relativistic causality would have been necessarily violated. Our argument is based on the measurability of $r_j$ in Alice’s $\Lambda_A^j$. 

21
Lemma 1. There exists an $r_{jk}$ which is independent of $j$ and $k$ such that (2) holds with $C(C_k, B_j) = 0$ if and only if the four intervals $[d_{jk}(-), d_{jk}(+)]$, $j, k \in \{0, 1\}$, with the $d_{jk}(-)$ and $d_{jk}(+)$ given below, all intersect.

$$d_{jk}(\pm) \overset{\text{def}}{=} g_{0j}^{AB} g_{1j}^{AB} + g_{0k}^{AC} g_{1k}^{AC} \pm \sqrt{[1 - (g_{0j}^{AB})^2 - (g_{0k}^{AC})^2] [1 - (g_{1j}^{AB})^2 - (g_{1k}^{AC})^2]} \quad (64)$$

Proof. The inequality (2) may be written as

$$M^{-1} A_{ABC}^{jk} M^{-1} = \begin{bmatrix} 1 & r'_{jk} \\ r'_{jk} & 1 \end{bmatrix} - \begin{bmatrix} g_{1k}^{AC} & g_{1k}^{AB} \\ g_{0k}^{AC} & g_{0k}^{AB} \end{bmatrix} \begin{bmatrix} g_{1k}^{AC} & g_{1k}^{AB} \\ g_{0k}^{AC} & g_{0k}^{AB} \end{bmatrix}^T \geq 0 \quad (65)$$

where $r'_{jk} \equiv r_{jk}/(\Delta_{A_0}\Delta_{A_1})$, and $M$ is a diagonal matrix whose non-vanishing entries are $\Delta_{C_k}$, $\Delta_{B_j}$, $\Delta_{A_1}$, and $\Delta_{A_0}$. As $g_{jk}^{BC} = C(C_k, B_j)/(\Delta_{B_j}\Delta_{C_k}) = 0$, the Schur complement condition for positive semidefiniteness implies that (65) is equivalent to

$$\begin{bmatrix} 1 & r'_{jk} \\ r'_{jk} & 1 \end{bmatrix} - \begin{bmatrix} g_{1k}^{AC} & g_{1k}^{AB} \\ g_{0k}^{AC} & g_{0k}^{AB} \end{bmatrix} \begin{bmatrix} g_{1k}^{AC} & g_{1k}^{AB} \\ g_{0k}^{AC} & g_{0k}^{AB} \end{bmatrix}^T \geq 0 \quad (66)$$

which holds if and only if the diagonal entries obey, $1 - (g_{1j}^{AB})^2 - (g_{1k}^{AC})^2 \geq 0$, $i = 0, 1$, and the determinant of this matrix satisfies

$$\sqrt{[1 - (g_{0j}^{AB})^2 - (g_{0k}^{AC})^2] [1 - (g_{1j}^{AB})^2 - (g_{1k}^{AC})^2]} - (r'_{jk} - g_{0j}^{AB} g_{1j}^{AB} - g_{0k}^{AC} g_{1k}^{AC})^2 \geq 0 \quad (67)$$

Namely, (66) holds if and only if

$$|r'_{jk} - g_{0j}^{AB} g_{1j}^{AB} - g_{0k}^{AC} g_{1k}^{AC}| \leq \sqrt{[1 - (g_{0j}^{AB})^2 - (g_{0k}^{AC})^2] [1 - (g_{1j}^{AB})^2 - (g_{1k}^{AC})^2]} \quad (68)$$

for $j, k \in \{0, 1\}$. It thus follows that $r'_{jk} \in [d_{jk}(-), d_{jk}(+)]$. If these intervals all intersect then there is $r$ and $r' \overset{\text{def}}{=} r/\Delta_{A_0}\Delta_{A_1}$ which are independent of $j, k$ such that $r'_{jk} = r'$. In particular,

$$\max_{j,k} d_{jk}(-) \leq r' \leq \min_{j,k} d_{jk}(+) \quad (69)$$

Conversely, if there is such $r'_{jk} = r'$ then the underlying intervals necessarily intersect.

Lemma 1 shows that in the absence of Charlie, $g_{ik}^{AC} = g_{jk}^{BC} = 0$, the parameter $r_j$ in a bipartite Alice-Bob setting satisfies

$$g_{0j} g_{1j} - \sqrt{(1 - g_{0j}^2)(1 - g_{1j}^2)} \leq r'_{j} \leq g_{0j} g_{1j} + \sqrt{(1 - g_{0j}^2)(1 - g_{1j}^2)} \quad (70)$$
where \( g_{ij} = C(A_i, B_j)/(\Delta_{A_i}\Delta_{B_j}) \), and \( r'_j \equiv r_j/(\Delta_{A_0}\Delta_{A_1}) \).

Let \( D_j \) be the range of admissible \( r_j \) in (70). Unless \( D_0 \cap D_1 \neq \emptyset \), relativistic independence cannot be satisfied. We shall show that whenever the two intervals \( D_0 \) and \( D_1 \) do not intersect, in which case relativistic independence fails, signaling takes place. Define

\[
\epsilon \equiv \min_{w_j \in D_j} |w_0 - w_1|
\]  

(71)

It can be recognized that this \( \epsilon \) is the smallest of the four possible numbers

\[
\epsilon = |\varrho_{00}\varrho_{10} - \varrho_{01}\varrho_{11}| \pm \sqrt{(1 - \varrho_{00}^2)(1 - \varrho_{10}^2) \pm\sqrt{(1 - \varrho_{01}^2)(1 - \varrho_{11}^2)}}
\]  

(72)

Assume now that the intervals \( D_0 \) and \( D_1 \) do not intersect and thus \( \epsilon > 0 \). Here is a procedure that Alice may in principle follow for detecting a signal from Bob using her local measurements. Let \( \tau \) be a set of local parameters describing Alice’s non-trivial system (for practical reasons \( \tau \) can be discretized). The precision is represented for any physical variable \( A \) by the variance \( \Delta^2_A(\tau) \). This \( \Delta^2_A(\tau) \) can be evaluated empirically by measuring \( A \) in many trials of an experiment while reproducing time and again the same set \( \tau \).

For any real parameter \( \theta \in [-\pi, \pi] \), Alice is able to evaluate

\[
g(\theta, \tau) \equiv \cos(\theta)^2 \Delta_{A_0}(\tau)/\Delta_{A_1}(\tau) + \sin(\theta)^2 \Delta_{A_1}(\tau)/\Delta_{A_0}(\tau)
\]  

(73)

Her uncertainty relation (1) dictates that this quantity is bounded from below

\[
\min_{\tau} g(\theta, \tau) \geq \max \{0, r'_j \sin(2\theta)\}
\]  

(74)

which follows from \([\cos \theta, -\sin \theta] \Lambda^2_A[\cos \theta, -\sin \theta]^T \geq 0\). That Alice may reach \( r'_j \) means that for some \( \theta \) there exists a subset of parameters \( \tau^* \) saturating (74),

\[
\min_{\tau} g(\theta, \tau) = g(\theta, \tau^*) = r'_j \sin(2\theta)
\]  

(75)

which also implies that \( \Lambda^2_A \) is a singular matrix and therefore \( \Delta^2_{A_0}(\tau^*)\Delta^2_{A_1}(\tau^*) = r^2_j \).

Suppose that Alice and Bob agree in advance to repeat the underlying experiment \( N \) times, for a sufficiently large \( N \). Alice may choose a new set \( \tau \) and a device with which to measure in the beginning of each trial. All this time Bob uses only one of his devices, say the \( j \)-th one. Using the measurement outcomes from all these trails, Alice may approximate \( g(\theta, \tau) \) for each \( \tau \) in the domain of these parameters. According to (75) Alice may then evaluate \( \hat{r}'_j \), an estimate of \( r'_j \), using
the approximated minimum of \( g(\theta, \tau) \). In practice, her estimate is accurate up to an error term, \( \delta_j \) of the order \( \mathcal{O}(1/\sqrt{N}) \), i.e., \( \tilde{r}'_j = r'_j + \delta_j \). It now follows that for sufficiently large \( N \),

\[
|\tilde{r}'_0 - \tilde{r}'_1| = |r'_0 - r'_1 + \delta_0 - \delta_1| \geq |\epsilon + \mathcal{O}(1/\sqrt{N})|
\]  

(76)

Alice may therefore be able to evaluate a number whose magnitude is as large as \( \epsilon \) and whose sign tells whether Bob measured first using \( j = 0 \) and then using \( j = 1 \) or the opposite. Of course, if independence holds, in which case \( \epsilon = 0 \), Alice will not detect any signal from Bob via her local uncertainty relations.

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