Bistability, Switching Waves, and Dissipative Solitons in a Non-Resonantly Excited Semiconductor Microcavity with Saturatable Absorption

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We consider a system of exciton polaritons in the specially designed semiconductor optical microcavity which has an embedded saturable absorber in the heterostructure growth direction. It results in nonlinear behavior of the system with the formation of bistability of the condensate particles number on the non-resonant (electrical or optical) pump intensity. We demonstrate a bright spatial dissipative exciton-polariton soliton of a new kind in the system under non-resonant excitation. The soliton width lies in the sub-micron range and it can be an order (or even several orders) of magnitude smaller than a typical width of regular optical dissipative solitons. The stability of the solutions is examined and the properties of the soliton investigated with account of non-linear self-scattering processes.

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Introduction. — Dissipative solitons (DSs) have been in the focus of laser optics research for decades [1–8] due to their significant fundamental properties and possibility of application in information processing systems [9]. One of the possible realizations of DSs can be found in structures with the so-called “saturable absorption” [10, 11] where solitons can be attributed desired characteristics by the geometry and composition design [12–14]. Such an implementation, however, has several fundamental drawbacks in the framework of classical optics, one of which is large value of the effective soliton width [5] which amounts to dozens of micrometers (µm). In this Letter, we will address dissipative solitons appearing in a system of cavity exciton polaritons.

Exciton-polaritons (later polaritons) represent half-light half-matter bosonic quasi-particles appearing in high-quality semiconductor microcavities. Their photonic component allows high-velocity propagation of the polariton wavepackets, on one hand; strong non-linear self-interaction inherited from the excitonic counterpart gives the possibility of control and manipulation, on the other hand. Polaritons have proven to be highly promising entities from both the fundamental and application-oriented points of view, besides, thanks to recent progress in technology, high-Q microcavities of any geometry are routinely produced (see e.g. [15]). Indeed, the state-of-the-art technology allows to create various semiconductor heterostructures with precise control of parameters and etch any spatial patterns in the lateral directions, thus producing confining potentials of various kinds [16–19] that results in a growing number of theoretical proposals [20–27].

One of the main advantages of microcavities with respect to classical optical systems in “availability” of a nonlinear Kerr-like media with extremely large exciton-mediated response of the system which allows to reduce, first, the required input power and, second, the characteristic size of spatial formations, both by order(s) of magnitude [28–30]. Another reason why polaritons attract growing interest of the scientific community is possibility of their quasi-Bose-Einstein condensation (BEC) [31], similar to BEC of quasiparticles in other mesoscopic systems [32–42].

In a wide range of situations, the system of exciton polaritons (along with other BEC quasicondensates) can be described by a time- and coordinate-dependent macroscopic wavefunction (also called the order parameter) which dynamics is governed by the Gross-Pitaevskii-type equation (GPE) [43]. (It should be noted, that this approach has certain drawbacks. However, we will not concentrate on them since GPE is suitable to demonstrate the desired significant effects which we plan to consider within the scope of current Letter.) In GPE, there is a nonlinear term associated with self-interactions between particles. It plays crucial role in dynamics and stability of spatial formations [44] and allows to observe regimes with similar dynamical behavior as in classical Kerr media with such formations as stationary and moving optical dark and bright solitons [45–49].

In work [47], dissipative exciton-polariton solitons were demonstrated in the scheme of driven nonlinear interferometer. Solitons in such sketch require a coherent holding radiation and have low contrast due to the background. In this Letter, we propose a new type of spatial bright DSs originating from the bistable dissipative dynamics of particles in the microcavity with embedded...
saturable absorption. In this case, pump is nonresonant (of incoherent nature) and the background is absent. We develop a theory applicable to one-dimensional (1D) dynamics, which can be easily generalised to the case of two-dimensional (2D) polariton structures.

**Bistability at non-resonant excitation.**—We consider a system sketched in Fig. 1 that consist of a incoherently pumped semiconductor microcavity with embedded saturated absorber (SA). In the framework of the parabolic dispersion approximation, the equations of motion for the polaritons field and the reservoir occupation number read \[48, 49\]

\[
\begin{align*}
\frac{i\hbar}{2m} \nabla \psi(r, t) & = \psi(r, t) + \alpha |\psi(r, t)|^2 \psi(r, t) \quad (1) \\
\frac{\partial n_R(r, t)}{\partial t} & = -(\gamma_R + R|\psi(r, t)|^2)n_R(r, t) + P(r),
\end{align*}
\]

where in our model we introduce the dependence

\[
\gamma_c = \gamma_c(\psi(r, t))^2 = \gamma_0 \left(1 + \frac{\beta}{1 + \sigma |\psi(r, t)|^2}\right) \quad (2)
\]

of the polariton inverse lifetime on their density, \(|\psi(r, t)|^2\), brought by the saturated absorber. Qualitatively, such nonlinear term tends to increase the lifetime in the regions of high particle density and thus provides spatially-dependent lifetime enhancing formation of localised solutions. Here \(\beta\) and \(\sigma\) are the parameters describing the SA which we will choose phenomenological for the moment. Formula (2) has been adapted by us from the classical laser optics \[50\] where it is used as a saturation of the electric field vector in the description of a laser operation. Since the field operator, \(\psi(r, t)\), is linearly connected with the quantized electric field, we yield that the classical formula should be compatible with the quantum case which we consider. Therefore, though \(\beta\) and \(\sigma\) are taken phenomenological, Eq. (2) is microscopic.

In the steady state (when \(n_R = \text{Const}(t)\), \(\psi(r, t) = \psi_0(r)\exp(-i\omega t)\) and thus \(|\psi(r, t)|^2 = |\psi_0|^2\)), assuming homogeneous excitation of the system \(|P(r)| = P_0\) and thus \(n_R = n_{R0} = \text{Const}(t)\), we obtain the following equations:

\[
\begin{align*}
\hbar \omega \psi_0 & = \alpha |\psi_0|^2 \psi_0 + \frac{i\hbar}{2} (R n_{R0} - \gamma_c |\psi_0|^2) \psi_0; \quad (3) \\
0 & = -(\gamma_R + R|\psi_0|^2) n_{R0} + P_0, \\
0 & = -(\gamma_R + R|\psi_0|^2)^2 n_{R0} + P_0. \quad (6)
\end{align*}
\]

Eq. (4) fixes the chemical potential of the particles, \(\mu = \alpha |\psi_0|^2\); Eq. (5) is the gain condition which defines the homogeneous polariton occupation directly linked to the reservoir particle number in a steady state by Eq. (6). Apart from the trivial (no-lasing) solution \(|\psi_0|^2 = 0\), we find

\[
(|\psi_0|^2)_{1, 2} = -B \pm \sqrt{B^2 - 4AC},
\]

where \(A = \gamma_0 \sigma R\), \(B = -(RP \sigma - \gamma_0 \gamma_R \sigma - \gamma_0 R - \gamma_0 \beta R)\), \(C = -(RP - \gamma_0 \gamma_R - \gamma_0 \gamma_R \beta)\). These solutions forming

**FIG. 2: Bistability of solutions in the steady state:** dependence of the polariton field intensity, \(|\psi_0|^2\), on the intensity of non-resonant pump, \(P_0\). We used the following parameters in the calculations: \(\gamma_0 = 0.01 \text{ ps}^{-1}\), \(R = 5 \cdot 10^{-6} \text{ ps}^{-1}\), \(\gamma_R = 1/400 \text{ ps}^{-1}\), \(\beta = 5\), \(\sigma = 0.1\). The 0-branch corresponding to the third, trivial solution is not plotted in figure.
the bistability curve are presented in Fig. 2. It should be noted, that the quadratic equation (7) impose certain restrictions,

$$\gamma_0(\sigma R + R + \beta R) < P_0 < \frac{\gamma_0 \sigma (1 + \beta)}{\sigma R}. \tag{8}$$

Further we investigate the stability of the solutions in more detail.

Linear stability analysis.— The first solution of the cubic equation in (3) is trivial (the so-called generation-free or no-lasing solution): $\psi_0 = 0$. Further, in (1) let us assume $|\psi|^2 \to 0$. Then we can neglect the non-linear terms to find:

$$\frac{d|\psi|}{dt} = \frac{1}{2} \left( \frac{P_0 R}{\gamma_R} - \gamma_0 (1 + \beta) \right) |\psi|,$$

from where we yield the condition of stability:

$$P_0 R - \gamma_R \gamma_0 (1 + \beta) < 0. \tag{9}$$

In order to investigate the regimes of generation, we introduce $a, b, \delta n \to 0$ treated as independent small variables,

$$\psi = \psi_0 e^{-i\omega t} (1 + a e^{i k_\perp x + \gamma t} + b^* e^{-i k_\perp x + \gamma^* t}); \tag{10}$$

$$n_R = n_{R0} (1 + (\delta n) e^{i k_\perp x + \gamma t} + (\delta n^*) e^{-i k_\perp x + \gamma^* t}).$$

Then, substituting these terms in Eq. (1) with account of (2) and linearizing over the small variables, we obtain the closed system of linear algebraic equations on $a, b^*$, and $\delta n$ (see the Supplemental Material [51] for details). The determinant of that system includes the parameter $\gamma$ which we are interested in. To find $\gamma$, one should put the determinant equal to zero and solve the resulting cubic equation. The analytical solutions are quite cumbersome, therefore we don’t present them here explicitly. The stability conditions are given by the Ljenar-Shipar’s criterion. Negative or zero real part of $\gamma$ for all $k_\perp \in [0, \infty]$ in some range of pumps in the vicinity of $P_0$ gives stable solution in this region meaning that the switching waves are possible between the regions corresponding to two stable solutions in such region. The velocity of propagation of a switching wave, $v$, evidently depends on $P_0$ vanishing at the so-called Maxwell’s value of pump, $P_M$. In the vicinity of this value, a spatial soliton can be formed [3].

Results and discussion.— The results of numerical modelling based on Eqs. (1), (2) are presented in Figs. 3, 4 for the cases $\alpha = 0$ and $\alpha \neq 0$, respectively. Initially, we create a Gaussian (with width 10 $\mu$m) density profile in the centre of the sample by a short strong laser pulse thus preparing two spatial regions with different particle concentration. Further, the system evolves under the background homogenious non-resonant excitation, $P$.

Figure 3 demonstrates the formation of the DS in the absence of polariton-polariton interaction ($\alpha = 0$). The stability examination shows existence of the switching

**FIG. 3:** DS formation and behavior in the absence of the particles’ self-scattering, $\alpha = 0$. Parameters of the calculation are taken from Fig. 2; the intensity of pump is $P = 42$ ps$^{-1}$. The colormap shows the polariton density. The total time interval is split into two regions: soliton formation (0-200 ps); soliton existence (200 ps-ever). Upper left-most inset shows the DS profile (with the width $\approx 10 \mu$m) in the steady state. Upper right-most inset illustrates the phase distribution. Lower inset illustrates the switching wave velocity dependence on the intensity of pump. The Maxwell point, $P_M$, corresponds to the value of pump at which the switching waves stop ($v = 0$).

**FIG. 4:** DS dynamics with account of the particles’ self-scattering, $\alpha > 0$. Parameters of the calculation are the same as in Figs. 2, 3; the intensity of pump is $P = 48$ ps$^{-1}$. Inset shows the DS profile (with the width less than 1 $\mu$m) in the steady state. See main text for details.
waves in nearly the whole range of pumps corresponding to the bistability region. The bright soliton forms in balance of (i) gain and dispersion terms which favor spreading of particles, on one hand, and (ii) nonlinear losses favoring localisation and a peaked landscape formation, on the other hand. There can be found a range of pump intensities, $P_{\text{min}} < P_M < P_{\text{max}}$, at which the switching waves stop and thus we achieve the regime where the DS exists. At $P < P_{\text{min}}$ the system collapses towards the no-lasing solution, whereas at $P > P_{\text{max}}$ the final state represents homogenous profile with high particle concentration (see the Supplemental Material for details).

We affirm and unleash here that the nonlinear properties of the system and the soliton formation are stemming from the nonlinear losses term only since other nonlinear mechanisms are switched off. The width of the DS is $\approx 10 \, \mu m$, having the same order of magnitude as classical optical solitons.

Switching on the particle interaction drastically changes the situation, see Fig. 4. We use $\alpha = 0.5 \cdot 6 \cdot E_b a_B^2 / dx/w$, where $E_b$ is the exciton binding energy; $a_B$ is its Bohr radius; $dx, w$ are the 1D discretization element width and the width of the 1D microwire (0.5 and 2 $\mu m$, correspondingly). We observe remarkable alteration of the DS properties (compare Fig. 4 and Fig. 3). The DS formation is now the result of the fourfold competition of the (i) gain, dispersion, and particle repulsion (since $\alpha > 0$) and (ii) nonlinear losses. Obviously, $P_M$ is displaced. The benefit of interactions is that we have much narrower DS profile with the healing length $\xi = h/\sqrt{2m^* \tau}$ and soliton width lying in the sub-micron range. This squeezing of the spatial formation is the direct result of the “ancillary” particle-particle interaction which leads to the consolidation of the bright spatial DS.

It should additionally be noted, that variation of the initial ($t = 0$) profile, allows one to create various spatial patterns in the steady state ranging from multipeaks to dark solitons (see Fig. 5 for the illustration of a multipeak structure).

Besides, an asymmetry in the initial condition may lead to the DS propagation along the microwire. In transversely 2D schemes, various soliton complexes may be formed, with motion determined by the complex symmetry, like in [52]. It can also be mentioned, that in the case of modulation of the cavity length, solitons may acquire additional features intermediate between those of conservative and dissipative solitons, similar to [53, 54]. However, we leave these effects beyond the scope of current Letter.

Conclusion.— We have considered a system of interacting exciton polaritons in the semiconductor microcavity with an embedded saturated absorber. It has been shown that this design may result in nonlinear behavior of the system with the formation of bistability in the dependence of the condensate particles concentration on the intensity of pump. Besides, we have demonstrated a new type of spatial dissipative soliton which width lies in the sub-micron range and comprehensively investigated its properties with account of the non-linear particle self-scattering.

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