An FLRW interacting dark energy model of the Universe

Anirudh Pradhan\textsuperscript{1}, G. K. Goswami\textsuperscript{2}, A. Beesham\textsuperscript{3}, Archana Dixit\textsuperscript{4}

\textsuperscript{1} Department of Mathematics, Institute of Applied Sciences and Humanities, G L A University, Mathura-281 406, Uttar Pradesh, India
E-mail: pradhan.anirudh@gmail.com

\textsuperscript{2} Department of Mathematics, Kalyan P G College, Bhilai-490006, India
Email: gk.goswami9@gmail.com

\textsuperscript{3} Department of Mathematical Sciences, University of Zululand, Kwa-Dlangezwa 3886, South Africa
E-mail: beeshama@unizulu.ac.za

\textsuperscript{4} Department of Mathematics, Institute of Applied Sciences and Humanities, G L A University, Mathura-281 406, Uttar Pradesh, India
E-mail: archana.dixit@gla.ac.in

New Astronomy 78 (2020) 101368

Abstract

In this paper, we have presented an FLRW universe containing two-fluids (baryonic and dark energy), with a deceleration parameter (DP) having a transition from past decelerating to the present accelerating universe. In this model, dark energy (DE) interacts with dust to produce a new law for the density. As per our model, our universe is at present in a phantom phase after passing through a quintessence phase in the past. The physical importance of the two-fluid scenario is described in various aspects. The model is shown to satisfy current observational constraints such as recent Planck results. Various cosmological parameters relating to the history of the universe have been investigated.

PACS No.: 98.80.Jk; 95.36.+x; 98.80.-k
Keywords: FLRW universe; Observational parameters; Phantom; Quintessence.

1 Introduction

A cosmological model must satisfy the basic cosmological principle (CP) which says that at any time the universe is spatially homogeneous and isotropic. There is no privileged position in the universe. The Friedmann-Lemaitre-Robertson-Walker (FLRW) model satisfies the CP. This was manifest in an expanding and decelerating universe filled with a perfect fluid. However, the latest findings on observational grounds during the last three decades by various cosmological missions \cite{1}−\cite{17} confirm that universe has an accelerating expansion at present. It is believed that there is a bizarre form of dark energy (DE) with negative pressure prevailing all over the universe which is responsible for the said acceleration. In $\Lambda$CDM cosmology \cite{18} \cite{19}, the $\Lambda$-term is used as a candidate of DE with equation of state $p_{\Lambda} = -\rho_{\Lambda} = \frac{-\Lambda c^4}{8\pi G}$. However, the model suffers from, inter alia, fine tuning and cosmic coincidence problems \cite{20}. Any acceptable cosmological model must explain the accelerating universe.

As of now, many models and theories such as quintessence, phantom, $k$-essence, holographic DE models, $f(R)$ and $f(R, T)$ theories have been proposed to explain the acceleration in the universe. One may refer to the review article \cite{15} for a brief introduction to these models and theories.

Of late, many authors \cite{21}−\cite{27} presented DE models in which the DE is considered in a conventional manner as a fluid with an EoS parameter $\omega_{de} = \frac{p_{de}}{\rho_{de}}$. It is assumed that our universe is filled with two types of perfect fluids of which one is a baryonic fluid (BF) which has positive pressure and creates deceleration in the universe. The other is a DE fluid which has negative pressure and creates acceleration in the universe. Both fluids have different EoS parameters. The EoS for baryonic matter has been solved by cosmologists...
by providing the phases of the universe like stiff matter, radiation dominated and present dust dominated universe, but the determination of the EoS for DE is an important problem in observational cosmology at present. The present value of $\omega_{de}$ is observationally estimated nearly equal to $-1$. In the quintessence model, $-1 \leq \omega_{de} < 0$ whereas in the phantom model $\omega_{de} \leq -1$. Latest surveys \cite{28}\textendash\cite{32} rule out the possibility of $\omega_{de} \ll -1$, but $\omega_{de}$ may be little less than $-1$. But we are facing fine tuning and coincidence problems \cite{33}. So we need a dynamical DE with an effective EoS, $\omega_{(de)} = p_{(de)}/\rho_{(de)} < -1/3$. The two types of surveys SDSS and WMAP \cite{9} and \cite{34} provide limits on $\omega_{(de)}$ as $-1.67 < \omega_{de} < -0.62$ and $-1.33 < \omega_{de} < -0.79$, respectively.

It is worthwhile to mention here that various researchers \cite{35}\textendash\cite{39} proposed that DE may interact with BF, so they have developed both types of interacting and non-interacting models of the universe. Recently it has been discovered that allowing and interaction between DE and dark matter(DM) offers an attractive alternative to the standard model of the cosmology \cite{40}\textendash\cite{41}. In these works the motivation to study interacting DE model arises from high energy physics. In recent work Risaliti and Lusso \cite{42} and Riess et al \cite{43} stated that a rigid $\Lambda$ is ruled out by $4\sigma$ and allowing for running vacuum favored phantom type DE ($\omega < -1$) and $\Lambda$ CDM is claimed to be ruled out by $4.4\sigma$ motivating the study of interacting DE models. Interacting DE models \cite{44}\textendash\cite{51} lead to the idea that DE and DM do not evolve separately but interact with each other non gravitationally (see recent review \cite{52} and references there in.).

Motivated from above discussion, in this paper, we have presented an FLRW universe containing two-fluids (baryonic and dark energy), with a deceleration parameter (DP) having a transition from past decelerating to the present accelerating universe As per our model, universe is at present in a phantom phase after passing through a quintessence phase in the past. The model is shown to satisfy current observational constraints such as Planck’s latest observational results \cite{17}. Various cosmological parameters relating to the history of the universe have been investigated.

Our paper is structured as follows: In Sec. 2, we set the initial field equations. In Sec. 3, we have described the results and physical properties of interacting DE model. Finally, Sec. 4 is devoted to our conclusions.

2 Field equations

The FLRW space-time (in units $c = 1$) is given by

\[ ds^2 = dt^2 - a(t)^2 \left( \left[ \frac{dr^2}{1 + kr^2} \right] + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right), \]

where $a(t)$ stands for the scale factor and $k$ is the curvature parameter. The stress-energy tensor $T_{ij} = T_{ij}(m) + T_{ij}(de)$, where $T_{ij}(m) = (\rho_m + p_m) u_i u_j - p_m g_{ij}$ and $T_{ij}(de) = (\rho_{de} + p_{de}) u_i u_j - p_{de} g_{ij}$. We assume that DE interacts with and transforms energy to baryonic matter. We follow \cite{arXiv:1905.10801} and 1906.00450 to get Einstein field equations (EFEs) for the FLRW metric \cite{1} are as follows.

\[ H^2 (1 - \Omega_{de}) = H_0^2 \left[ (\Omega_m)_0 \left( \frac{a_0}{a} \right)^{3(1 - \sigma)} + (\Omega_k)_0 \left( \frac{a_0}{a} \right)^2 \right], \]

and

\[ 2q = 1 + 3\omega_{de} \Omega_{de} + 3 \frac{H_0^2}{H^2} \omega_k (\Omega_k)_0 \left( \frac{a_0}{a} \right)^2, \]

where symbols have their usual meanings.

3 Results and disussions

In the above, we have found two field equations \cite{29} and \cite{43} in five unknown variables $a$, $H$, $q$, $\Omega_{de}$ and $\omega_{de}$. Therefore, for a complete solution, we need three more relations involving these variables. Many researchers \cite{44}\textendash\cite{55} have considered constant DP which is not valid from present observations. The DP $q$ may be taken as time dependent as supported by many observations like SN Ia \cite{5}\textendash\cite{28} and CMB anisotropies \cite{7}\textendash\cite{8}. From
these observations, we observe that $z < 0.5$ for the present accelerated phase whereas $z > 0.5$ for the early decelerating phase. Furthermore, the corrected red shift $z_t = 0.43 \pm 0.07$ by (1 $\sigma$) c.1. [8] from $z_t = 0.46 \pm 0.13$ at (1 $\sigma$) c.1. [28] as of late found by the High-Z Supernova Search (HZSNS) group. The Supernova Legacy Survey (SNLS) [29], and additionally the one as of late incorporated by Knop et al [33], yields $z_t \sim 0.6(1 \sigma)$ in better concurrence with the flat $\Lambda$CDM model ($z_t = (2\Omega_m/\Omega_m)^{1/3} - 1 \sim 0.66$). In this way, the DP, which by theory is the rate with which the universe decelerates, must show signature flipping [56] from $q \alpha 0$. For these discussions, we observe that $z < \alpha$. From this, we calculate $\dot{a} = -\left(1 + \alpha\right) \exp\left[-\left(1 + \alpha\right) t - \frac{1}{\beta} \frac{l}{\beta}\right]$, provided $\alpha \neq -1$.

From above equation, we have $\frac{\ddot{a}}{a} + \beta \frac{\dot{a}}{a} + \alpha = 0$, which on solving, yields

$$a = \exp\left[-\frac{1 + \alpha}{\beta} t - \frac{1}{\beta} \frac{l}{\beta}\right],$$

provided $\alpha \neq -1$.

Here $l$ is a constant of integration.

From this, we calculate

$$\dot{a} = -\left(1 + \alpha\right) \exp\left[-\left(1 + \alpha\right) t - \frac{1}{\beta} \frac{l}{\beta}\right],$$

$$\ddot{a} = \left(1 + \alpha\right)^2 \exp\left[-\left(1 + \alpha\right) t - \frac{1}{\beta} \frac{l}{\beta}\right].$$

Putting above values in Eq. (4), we obtain the DP value as $q = -1$. Similarly we also observed that $q = -1$ for $\alpha = 0$.

For $\alpha = -1$, we have to find another solution. In this case Eq. (12) reduces to

$$q = -\frac{\ddot{a}}{a} = -1 + \beta H,$$

which yields the following differential equation:

$$\frac{\ddot{a}}{a} + \beta \frac{\dot{a}}{a} - 1 = 0.$$  

The solution of above equation is found to be

$$a = \exp\left[\frac{1}{\beta} \sqrt{2\beta t + k}\right],$$

where $k$ is an integrating constant.

Since we are interested to study the cosmic decelerated-accelerated transit universe, so we only consider the later case for which $\alpha = -1$.

The derivation of Eq. (5) can also be seen in [65]. Now we determine the constants $\beta$ and $k$ on the basis of the latest observational findings due to Planck [17]. The values of the cosmological parameters at present are as follows. ($\Omega_m)_0 = 0.30, (\Omega_k)_0 = \pm 0.005, (\omega_{de})_0 = -1, (\Omega_{de})_0 = 0.70 \pm 0.005, H_0 = 0.07Gyr^{-1} q_0 \approx -0.55, t_0 = 13.72Gyr$. Eq. (7) provides following differential equation

$$(1 + z)H_z = \beta H^2 = H(1 + q) = \frac{\beta}{2\beta t + k},$$

where we have used $\frac{\dot{a}}{a} = 1 + z, \dot{z} = -(1 + z)H$ and $H_z = \frac{dH}{dz}$. From Eq. (6) and the Planck results, we get the value of constants $\beta$ and $k$ as

$$k = 27.6816 Gyr^2, \beta = 6.42857 Gyr$$
Integrating Eq. (6), we get
\[
H^{-1} = A - \beta \log(1 + z),
\]
where \(A\) is constant of integration. As \(H_0 = 0.07\), \(A = 100/7\). So, we get following solution
\[
H = \frac{7}{100 - 45 \log(1 + z)} \text{Gyr}^{-1}, \quad q = \frac{45}{100 - 45 \log(1 + z)} - 1.
\]

(i) Hubble function \(H\):

The determination of the two physical quantities \(H_0\) and \(q\) plays an important role to describe the evolution of the universe. \(H_0\) provides us the rate of expansion of the universe which in turn helps in estimating the age of the universe, whereas the DP \(q\) describes the decelerating or accelerating phases during the evolution of the universe. From the last two decades, many attempts have been made to estimate the value of the Hubble function \([27], [67] - [69]\). For detailed discussions, readers are referred to Kumar \([27]\). We present the following figures 1, 2 & 3 to illustrate the solution Eq. (8). Various researchers \([15, 16], [70] - [76]\) have estimated values of the Hubble function at different red-shifts using a differential age approach and galaxy clustering method [see \([70]\) for list of 38 Hubble function parameters]. We obtain \(\chi^2\) from the following formula
\[
\chi^2 = \sum_{i=1}^{i=38} \left[\frac{(H_{\text{th}}(i) - H_{\text{ob}}(i))^2}{\sigma(i)^2}\right],
\]
where \(H_{\text{th}}(i)\)'s are theoretical values of Hubble function parameter as per Eq. (8) and \(\sigma(i)\)'s are errors in the observed values of \(H(z)\). It comes to \(\chi^2 = 33.22\) i.e. 87.43 over 38 data’s, which shows best fit in theory and observation. From figure 1, we observe that \(H\) increases with the increase of red shift. In this figure, cross signs are 31 observed values of the Hubble function \(H_{\text{ob}}\) with corrections, whereas the linear curve is the theoretical graph of the Hubble function \(H\) as per our model. Figure 2 plots the variation of red shift \(z\) versus \(t\), which shows that in the early universe the red shift was more than at present.

(ii) Transition from deceleration to acceleration:

Now we can obtain the DP ‘\(q\)’ in term of red shift ‘\(z\)’ by using Eq. (8). We present figure 3 to illustrate the solution. This describes the phase variation of the universe from deceleration to acceleration. We see that at present our universe is undergoing an accelerating phase. It has begun at the transit red shift \(z_t = 2.395\), i.e., at the time \(T_t = 1.034\) Giga year. It was decelerating before time \(T_t\)

(iii) DE Parameter \(\Omega_{de}\) and EoS \(\omega_{de}\)

Now, from Eqs. \([24]\, [33]\) and energy conservation equations, the density parameter \(\Omega_{de}\) and EoS parameter \(\omega_{de}\) for DE are given by the following equations and are solved numerically.
\[
H^2 \Omega_{de} = H^2 - (\Omega_m)_{0}H_0^2 (1 + z)^{3(1-\sigma)}
\]

\[
\omega_{de} = \frac{H^2 (2\alpha H + 2\beta - 1)}{3[H^2 - H_0^2 (\Omega_m)_{0}(1 + z)^{3(1-\sigma)}]}.
\]

where we have taken \((\Omega_k)_{0} = 0\) for the present spatially flat universe. We would take \(\sigma = 0.243\) for numerical solutions to match with latest observations. We solve Eqs. \([9]\, [10]\) with the help of Eq. (8) and present
Figures 4 and 5: Plot of $\Omega_{de}$ versus red shift ($z$)(left) and plot of $\omega_{de}$ versus $z$ (right).

Phantom phase ($0 \leq z \leq 3.665$), quintessence phase $3.665 \leq z \leq 3.74$ and deceleration phase $z \geq 3.74$

the following figures 3 and 4 to illustrate the solution.

Our model envisages that at present we are living in a phantom phase $\omega_{(de)} \leq -1$. In the past at $z = 2.77$ $\omega_{(de)} = -25.4947$ was minimum, and then it started increasing. This phase remains for the period ($0 \leq z \leq 3.665$). Our universe entered into a quintessence phase at $z = 3.665$ where $\omega_{de}$ comes up to $-0.333123$. As per our model, the period for the quintessence phase is the following $3.665 \leq z \leq 3.74$.

DE favors deceleration at $z \geq 3.665$. We look carefully Figs. 4 and 5 in context of Fig. 3. In fact as per Fig. 3, the transition red shift is $z_{tr} = 2.395$. As we expressed in our explanation, dark energy will begin its roll of opposing deceleration and favoring acceleration during $0 \leq z \leq 2.395$. Before, i.e., $z \geq 2.395$, universe is decelerating, so dark energy as well as $\omega_{de}$ have no physical rolls. We may say that the validity of Figs. 4 and 5 is only during the said tenure. During this DE always increases with time. As per our model, the present ratio of DE is 0.7. It decreases over the past, attains a minimum value $\Omega_{de} = 0.005$ at $z = 2.747$, and then it again increases with red shift.

(iv) Distance modulus $\mu$ and Apparent Magnitude $m_b$:

The distance modulus $\mu$ and apparent magnitude $m_b$ [18] are derived as

$$\mu = m_b - M = 5\log_{10} \left( \frac{D_L}{Mpc} \right) + 25 = 25 + 5\log_{10} \left( \frac{c(1+z)}{H_0} \int_0^z \frac{dz}{h(z)} \right)$$

$$m_b = 16.08 + 5\log_{10} \left[ \frac{1+z}{0.026} \int_0^z \frac{dz}{h(z)} \right].$$

We solve Eqs. (11) − (12) with the help of Eq. (8). Our theoretical results have been compared with SNe Ia related union 2.1 compilation 581 data [14], and the derived model was found to be in good agreement with current observational constraints. The following figures 6 depict the closeness of observational and theoretical results, thereby justifying our model. In order to get quantitative closeness of theory and observation, we obtain $\chi^2$ from the following formula

$$\chi^2 = \sum_{i=1}^{LengthSNe1aData} \frac{(\mu_{th}(i) - \mu_{obs(i)})^2}{\sigma_{SNe1a(i)}^2}$$

where $\mu_{th}(i)$’s are theoretical values of distance modulus as per Eq. (12) and $\sigma_{SNe1a(i)}$’s are errors in the observed values of $\mu$. It comes to $\chi^2 = 562.227$ i.e. 96.7% over 581 data’s, which shows best fit in theory and observation.
Figure 6: Plot of distance modulus ($\mu = M - m_b$) versus red shift ($z$). Crosses are SNe Ia related union 2.1 compilation 581 data’s with possible corrections

4 Conclusion:

In the present paper, we have presented an FLRW universe filled with two fluids (baryonic and dark energy), by assuming a scale factor as a linear function of the Hubble function. This results in a time-dependent DP having a transition from past decelerating to the present accelerating universe. The main findings of our model are itemized point-wise as follows.

- The expansion of the universe is governed by an expansion law $a(t) = (\beta H - 1) = \exp \frac{\sqrt{2\beta} + k}{\beta}$, where $\beta = 6.42857$ Gyr and $k = 27.6816$ Gyr$^2$. This describes the transition from deceleration to acceleration.

- Our model is based on the recent observational findings due to the Planck results [17]. The model agrees with present cosmological parameters.

$(\Omega_m)_0 = 0.30$ ($\Omega_k)_0 = 0.005$, $\left(\omega_{de}\right)_0 = -1$, $(\Omega_{de})_0 = 0.70 \pm 0.005$, $H_0 = 0.07$ Gyr$^{-1}$, $q_0 = 0.055$ and present age $t_0 = 13.72$ Gyr.

- At present our universe is undergoing an accelerating phase. It has begun at the transit red shift $z_t = 2.395$, i.e., at the time $T_t = 1.034$ Gigayear. It was decelerating before time $T_t$.

- Our model has a variable EOS $\omega_{de}$ for the DE density. Our model envisages that at present we are living in the phantom phase $\omega_{de} \leq -1$. In the past at $z = 2.77$ $\omega_{de} = -25.4947$ was minimum, then it started increasing. This phase remains for the period $(0 \leq z \leq 3.665)$. Our universe entered into a quintessence phase at $z = 3.665$ where $\omega_{de}$ comes up to $-0.333123$. As per our model, the period for the quintessence phase is the following

$$3.665 \leq z \leq 3.74.$$  

DE favors deceleration at $z \geq 3.665$. As per our model, the present ratio of DE is 0.7. It decreases over the past, attains a minimum value $\Omega_{de} = 0.005$ at $z = 2.747$, and then it again increases with red shift.

- The DE interacts with dust matter in our model, giving rise to a new density law for dust as $\rho_m = \left(\rho_m\right)_0 \left(\frac{a}{a_0}\right)^{3(1-\sigma)}$, where $\sigma$ is a constant which has been assigned the value 0.243 to match with observations.

In a nutshell, we believe that our study will pave the way to more research in future, in particular, in the area of the early universe, inflation and galaxy formation, etc. The proposed hybrid expansion law may help in investigations of hidden matter like dark matter, dark energy and black holes.

Acknowledgement

The authors (G. K. Goswami & A. Pradhan) sincerely acknowledge the Inter-University Centre for Astronomy and Astrophysics (IUCAA), Pune, India for providing facilities where part of this work was completed during a visit.
References

[1] Perlmutter, S. et al., 1998. Discovery of a supernova explosion at half the age of the Universe, Nature 391, 51.

[2] Perlmutter, S. et al., 1999. Measurements of Ω and Λ from 42 high-redshift supernovae, Astrophys. J. 517, 5.

[3] Riess, A.G. et al., 1998. Observational evidence from supernovae for an accelerating universe and a cosmological constant, Astron. J. 116, 1009.

[4] Tonry, J.L. et al., 2003. Cosmological results from high-z supernovae, Astrophys. J. 594, 1.

[5] Riess, A.G. et al., 1998. Observational evidence from supernovae for an accelerating universe and a cosmological constant, Astron. J. 116, 1009.

[6] de Bernardis, P. et al., 2000. A flat universe from high-resolution maps of the cosmic microwave background radiation, Nature 404, 955-959.

[7] Hanany, S. et al., 2000. MAXIMA-1: a measurement of the cosmic microwave background anisotropy on angular scales of 10°-5, Astrophys. J. 545, L5-L9.

[8] Spergel, D.N. et al. [WMAP collaboration], 2003. First year wilkinson microwave anisotropy probe (WMAP) observations determination of cosmological parameters, Astrophys. J. Suppl. 148, 175.

[9] Tegmark, M. et al. [SDSS collaboration], 2004. Cosmological parameters from SDSS and WMAP, Phys. Rev. D 69, 103501.

[10] Seljak, U. et al., 2005. Cosmological parameter analysis including SDSS Lyα forest and galaxy bias constraints on the primordial spectrum of fluctuations neutrino mass and dark energy, Phys. Rev. D 2005, 71.

[11] Adelman-McCarthy, J. K. et al., 2006. The fourth data release of the sloan digital sky survey, Astrophys. J. Suppl. 162, 38.

[12] Bennett, C.L. et al., 2003. First year wilkinson microwave anisotropy probe (WMAP) observations preliminary maps and basic results, The Astrophys. J. Suppl. 148, 1-43.

[13] Allen, S.W. et al., 2004. Constraints on dark energy from chandra observations of the largest relaxed galaxy clusters, Mon. Not. R. Astron. Soc. 353, 457.

[14] Suzuki, N. et al., 2012. The Hubble space telescope cluster supernova survey V improving the dark-energy constraints above z > 1 and building an early-type-hosted supernova sample, Astrophys. J. 746, 85-115.

[15] Delubac, T. et al. [BOSS Collaboration], 2015. Baryon acoustic oscillations in the Lyα forest of BOSS DR11 quasars, Astron. Astrophys. 574, A59.

[16] Blake, C. et al. [The WiggleZ Dark Energy Survey], 2012. The wiggleZ dark energy survey joint measurements of the expansion and growth history at z < 1, Mon. Not. R. Astron. Soc. 425, 405-414.

[17] Ade, P.A.R. et al. [Planck Collaboration], 2016. Planck 2015 results XIV dark energy and modified gravity, Astron. Astrophys. 594, A14.

[18] E J Copeland et al, Int. J. Mod. Phys. D 15, 1753 (2006)

[19] Ø Grøn and S Hervik, Einstein’s general theory of relativity with modern applications in cosmology (Springer Publication, 2007)

[20] Weinberg, S., 1989. The cosmological constant problem, Rev. Mod. Phys. 61, 1.

[21] Carroll, S.M., Hoffman, M., 2003. Can the dark energy equation-of-state parameter ω be less than −1, Phys. Rev. D 68, 023509.
[22] Amirhashchi, H., Pradhan, A., Saha, B., 2011. An interacting two-fluid scenario for dark energy in an FRW universe, Chin. Phys. Lett. 28, 039801.

[23] Amirhashchi, H., Pradhan, A., Saha, B., 2011. An interacting and non-interacting two-fluid dark energy models in FRW universe with time dependent deceleration parameter, Int. J. Theor. Phys. 50, 3529.

[24] Pradhan, A., Amirhashchi, H., Saha, B., 2011. An interacting and non-interacting two-fluid scenario for dark energy in FRW universe with constant deceleration parameter, Astrophys. Space Sci. 333, 343.

[25] Saha, B., Amirhashchi, H., Pradhan, A., 2012. Two-fluid scenario for dark energy models in an FRW universe-revisited, Astrophys. Space Sci. 342, 257.

[26] Pradhan, A., 2014. Two-fluid atmosphere from decelerating to accelerating Friedmann-Robertson-Walker dark energy models, Indian J. Phys. 88, 215.

[27] Kumar, S., 2011. Some FRW models of accelerating universe with dark energy, Astrophys. Space Sci. 332, 449.

[28] Riess, A.G. et al., 2004. Type Ia supernova discoveries at $z > 1$ from the Hubble Space Telescope: Evidence for past deceleration and constraints on dark energy evolution, Astrophys. J. 607, 665, astro-ph/0402512.

[29] Astier, P. et al., 2006. The supernova legacy survey measurement of $\Omega_m$, $\Omega_\Lambda$ and $\omega$ from the first year data set, Astron. Astrophys. 447, 31.

[30] Eisenstein, D.J. et al., 2005. Detection of the baryon acoustic peak in the large-scale correlation function of SDSS luminous red galaxies, Astrophys. J. 633, 560.

[31] MacTavish, C.J. et al., 2006. Cosmological parameters from the 2003 flight of BOOMERANG, Astrophys. J. 647, 799.

[32] Komatsu, E. et al., 2009. Five-year Wilkinson microwave anisotropy probe observations: Likelihoods and parameters from the wmap data, Astrophys. J. Suppl. Ser. 180, 330.

[33] Knop, R.K. et al., 2003. New constraints on $\Omega_m$, $\Omega_\Lambda$ and $\omega$ from an independent set of 11 high-redshift supernovae observed with the hubble space telescope, Astrophys. J. 598, 102.

[34] Hinshaw, G. et al., 2009. Nine-year Wilkinson microwave anisotropy probe (WMAP) observations cosmological parameter results, Astrophys. J. Suppl. 180, 225.

[35] Amendola, L., Campos, G.C., Rosenfeld, R., 2007. Consequences of dark matter-dark energy interaction on cosmological parameters derived from type Ia supernova data, Phys. Rev. D 75, 083506.

[36] Guo, Z.K., Ohta, N., Tsujikawa, S., 2007. Probing the coupling between dark components of the universe, Phys. Rev. D 76, 023508.

[37] Sahoo, P.K., Tripathi, S.K., Sahoo, P., 2018. A periodical varying deceleration parameter in $f(R, T)$ gravity, Mod. Phys. Lett. A 33, 1850193.

[38] Aktas, C., 2019. Various dark energy models for $G$ and $\lambda$ in $f(R, T)$ modified gravity, Mod. Phys. Lett. A 34, 1950098.

[39] Varshney, G., Sharma, U.K., Pradhan, A., 2019. Statefinder diagnosis for interacting Tsallis holographic dark energy models with $\omega - \omega'$ pair, New Astronomy, 70, 36.

[40] Sola, J., Gomej-Valent, A., 2015. The $\Lambda$CDM cosmology: From inflation to dark energy through running $\Lambda$, Int. J. Mod. Phys. D 24, 1541003.

[41] Begue, D., Stahl, C., Xue, S.S., 2019. A model of interacting dark fluids tested with supernovae and baryon acoustic oscillations data, Nucl. Phys. B 940, 312.

[42] Risaliti, G., Lusso, E., 2019. Cosmological constraints from the Hubble diagram of quasars at high redshifts, Nat. Astron. 3, 272.
[43] Riess, A.G., et al, 2019. Magellanic Cloud Cepheid Standards Provide a 1% foundation for the determination of the Hubble constant and stronger evidence for Physics beyond ΛCDM, [arXiv:1903.07603[astro-ph.CO]].

[44] Gavela, M.B., Hernandez, D., Lopez Honoreza, L., Menac, O., Rigoliad, S., 2009. Dark coupling, JCAP 0907, 034.

[45] Faraoni, V., Dent, J.B., Saridakis, E.N., 2014. Covariantizing the interaction between dark energy and dark matter, Phys. Rev. D 90, 063510.

[46] Salvatelli, V., Said, N., Bruni, M., Melchiorri, A., Wands, D., 2014. Indications of a late-time interaction in the dark sector, Phys. Rev. Lett. 113, 181301.

[47] Xue, S.S., 2015. How universe evolves with cosmological and gravitational constants, Nucl. Phys. B 897, 326.

[48] Ferriera, E.G.M., 2017. Evidence for interacting dark energy from BOSS, Phys. Rev. D 95, 043520.

[49] Koivisto, T.S., Saridakis, E.N., Tamanini, N., 2015. Scalar-Fluid theories: cosmological perturbations and large-scale structure, JCAP 1509, 047.

[50] Kumar, S., and R C Nunes, R.C., 2016. Probing the interaction between dark matter and dark energy in the presence of massive neutrinos, Phys. Rev. D 94, 123511.

[51] S Kumar and R C Nunes, Phys. Rev. D 96, 103511 (2017)

[52] Wang, B., Abdulla, E., Atrio-Barandela, F., Pavon, D., 2016. Dark matter and dark energy interactions: theoretical challenges, cosmological implications and observational signatures, Rept. Prog. Phys. 79, 096901.

[53] Berman, M.S., 1983. A special law of variation for Hubble’s parameter, II Nuovo Cimento B 74, 1971.

[54] Berman, M.S., Gomide, F.M., 1988. Cosmological models with constant deceleration parameter, Gen. Relativ. Gravit. 20, 191.

[55] Pradhan, A., Amirhashi, H., Saha, B., 2011. Bianchi type-I anisotropic dark energy model with constant deceleration parameter, Int. J. Theor. Phys. 50, 2923.

[56] Riess, A.G., et al., 2001. The farthest known supernova: support for an accelerating universe and a glimpse of the epoch of deceleration, Astrophys. J. 560, 49.

[57] Riess, A.G., et al., 2007. New Hubble space telescope discoveries of type Ia supernovae at z ≥ 1: Narrowing constraints on the early behavior of dark energy, Astrophys. J. 659, 98.

[58] Padmanabhan, T., Roychowdhury, T., 2003. A theoretician's analysis of the supernova data and the limitations in determining the nature of dark energy, Mon. Not. R. Astron. Soc. 344, 823.

[59] Amendola, L., 2003. Acceleration at z > 1?, Mon. Not. R. Astron. Soc. 342, 221.

[60] Sharma, U.K., Pradhan, A., 2019. Diagnosis Tsallis holographic dark energy models with statefinder and ω − ω′ pair, Mod. Phys. Lett. A 34, 1950101.

[61] Dixit, A., Sharma, U.K., Pradhan, A., 2019. Tsallis holographic dark energy in FRW universe with time varying deceleration parameter, New Astronomy, 73, 101281.

[62] Goswami, G.K., Pradhan, A., Beesham, A., 2019. Pramana-J. Phys. 93, 89, [arXiv:1906.00450[gr-qc]].

[63] Dixit, A., Zia, R., Pradhan, A., 2019. Anisotropic bulk viscous string cosmological models of the Universe under a time-dependent deceleration parameter, Pramana-J. Phys. https://doi.org/10.1007/s12043-019-1884-2, arXiv:1906.05515[physics.gen-ph]

[64] Tiwari, R.K., Singh, R., Shukla, B.K., 2015. A Cosmological model with variable deceleration parameter, African Rev. Phys. 10, 0048.
[65] Sharma, U.K., Zia, R., Pradhan, A., Beesham, A., 2019. Stability of cosmological models in modified $f(R,T)$-gravity with $\Lambda(T)$, Res. Astron. Astrophys. 19, 55.

[66] Tiwari, R.K., Beesham, A., Shukla, B.K., 2018. Scenario of two-fluid dark energy models in Bianchi type-III Universe, Int. J. Geo. Methods Mod. Phys. 15, 1850189.

[67] Jarosik, N., et al., 2010. Seven-year Wilkinson microwave anisotropy probe (WMAP*) observations: sky maps, systematic errors, and basic results, Astrophys. Journ. Suppl. 192, 14.

[68] Riess, A.G., et al., 2011. A 3% Solution: Determination of the Hubble constant with the Hubble space telescope and wide field camera 3, Astrophys. Journ. 730, 119.

[69] Beutler, F., et al., 2011. The 6dF Galaxy survey: baryon acoustic oscillations and the local Hubble constant, Mon. Not. R. Astron. Soc. 416, 3017.

[70] Zhang, C. et al., 2014. Four new observational $H(z)$ data from luminous red galaxies in the sloan digital sky survey data release seven, Res. Astron. Astrophys. 14, 1221.

[71] Stern, D. et al., 2010. Cosmic chronometers constraining the equation of state of dark energy I $H(z)$ measurements, Jour. Cosmo. Astropart. Phys. 02, 008.

[72] Moresco, M., 2015. Raising the bar new constraints on the hubble parameter with cosmic chronometers at $z \sim 2$, Mon. Not. R. Astron. Soc. 450, L16.

[73] Simon, J. et al., 2005. Constraints on the redshift dependence of the dark energy potential, Phys. Rev. D 71, 123001.

[74] Benitez, N. et al., 2002. The magnification of SN 1997ff the farthest known supernova, Astrophys. J. 577, L1.

[75] Turner, M., Riess, A.G., 2002. Do type Ia supernovae provide direct evidence for past deceleration of the universe?, Astrophys. J. 569, 18.

[76] Farooq, O. et al., 2017. Hubble parameter measurement constraints on the redshift of the deceleration-acceleration transition: dynamical dark energy ans space curvature, Astrophys. J. 835, 26.

[77] Liddle, A. R., Lyth, D.H., 2000. Cosmological inflation and large-scale structure (Cambridge University Press).