DYNAMICAL Friction AND THE EVOLUTION OF SATELLITES IN VIRIALIZED HALOS: THE THEORY OF LINEAR RESPONSE

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ABSTRACT

The evolution of a small satellite inside a more massive truncated isothermal spherical halo is studied using both the theory of linear response for dynamical friction and N-body simulations. The analytical approach includes the effects of the gravitational wake, the tidal deformation, and the shift of the barycenter of the primary, thereby unifying the local and global interpretations of dynamical friction. The N-body simulations follow the evolution of both rigid and live satellites within larger systems. Sizes, masses, orbital energies, and eccentricities are chosen as expected in hierarchical clustering models for the formation of structures. Results from this coupled approach are applicable to a vast range of astrophysical problems, from galaxies in galaxy clusters to small satellites of individual galaxies. The main contribution to the drag results from the gravitational pull of the overdensity region trailing the satellite's path, since the stellar response to the external perturbation remains correlated over a time shorter than the typical orbital period. The analytical approach and the N-body experiments demonstrate that there is no significant circularization of the orbits and that the dynamical friction timescale is weakly dependent of the circularity, \( e \). While the theory and the N-body simulations give a complete description of the orbital decay of satellites, a good fitting formula for the orbital decay time is

\[
\tau_{DF} = 1.2 \frac{J_{\text{cir}} r_{\text{cir}}}{(GM_{\text{sat}}/e) \ln(M_{\text{halo}}/M_{\text{sat}})} e^{0.4},
\]

where \( J_{\text{cir}} \) and \( r_{\text{cir}} \) are, respectively, the initial orbital angular momentum and the radius of the circular orbit with the same energy as the actual orbit. Tidal stripping can reduce the satellite's mass by 60% after the first pericentric passage, increasing the orbital decay time. The \( e \) factor takes that effect into account and should be removed in the simplified case of rigid satellites. In cosmologically relevant situations, our model gives orbital decay times larger by a factor of 2 than most previous estimates. For peripheral orbits in which the apocenter is larger than the virial radius of the primary decay, the tidal field and the shift of the barycenter become important. In this case, \( \tau_{DF} \) needs to be further increased by at least \( \simeq 50\% \). The final fate of a satellite is determined by its robustness against the effect of tides. While low-density satellites are disrupted over a time comparable to the decay time of their rigid counterparts, satellites with small cores can survive up to a Hubble time within the primary, regardless of the initial choice of orbital parameters. Dwarf spheroidal satellites of the Milky Way, such as Sgr A and Fornax, have already suffered mass stripping, and with their present masses, the sinking times exceed 10 Gyr even if they are on very eccentric orbits.

Subject headings: celestial mechanics, stellar dynamics — methods: n-body simulations

1. INTRODUCTION

Dynamical friction is a fundamental physical process that drives the evolution of most cosmological structures, from satellites in galaxies to galaxies in large clusters. The satellites or the galaxies can decay toward the center of the halos as friction causes the loss of orbital energy and angular momentum. This braking force is a force of back-reaction resulting from the global distortion of the stellar density field induced in the primary by the satellite's gravity.

The theory of linear response (TLR) is an ideal tool for studying the dynamics of sinking satellites in spherical halos. Recently explored by Colpi & Pallavicini (1998, hereafter CP), this formalism is an alternative to other perturbative techniques developed to overcome the limits of Chandrasekhar's theory of dynamical friction (Chandrasekhar 1943), which is restricted to infinite, uniform, non–self-gravitating stellar backgrounds. In TLR, the force can either be related to the density changes, or be viewed as a direct manifestation of the fluctuation-dissipation theorem (CP; Nelson & Tremaine 1997; Bekenstein & Maoz 1992; Kandrup 1981). In the last interpretation, the fluctuations of the microscopic two-body force (between the satellite and the particles) add to give a nonvanishing drag. The force is the result of a time integral that preserves memory of the actual dynamics of the satellite and of the dynamics of the stars as derived from their Hamiltonians. The stellar reaction to the external perturbation and the amount of dissipation that follows depend closely on the (self)-correlation properties of the underlying stellar field (Colpi 1998, hereafter C98).

TLR has been successful in describing the decay of satellites spiraling outside their primary spherical halos, and in describing their deceleration in short-lived penetrating encounters. In the first case, global tidal deformations...
Excited during orbital motion are responsible for the loss of stability of the satellite’s orbit (C98). In flybys (CP), the deceleration is viewed as a superposition of two effects: the gravitational pull from the overdensity region that forms just behind the satellite’s trail and the brake from extended tides.

Cosmological simulations aimed at studying the building up of cosmic structures (Ghigna et al. 1998; Tormen, Diaferio, & Syer 1998; Kravtsov & Klypin 1998) show that the majority of the satellite’s (or substructure) orbits have rather large eccentricities, and apocenters that rarely exceed twice the virial radius of the primary halo (defined as the radius at which the average density inside the halo is 200 times the critical one). In this work we consider the fate of satellites placed on cosmologically relevant orbits, using TLR and N-body simulations as a tool for exploring the decay in a self-gravitating spherical halo.

TLR is a major advance over alternative analytical studies (Weinberg 1986, 1989; Ségui & Dupraz 1994), since it embraces in a relatively simple way all aspects of the gravitational interaction of the satellites with the collisionless background: the wake and the tides in a self-gravitating system, and the shift of the stellar center of mass. We then explore the dependence of the sinking times as a function of the orbital parameters and the possible role played by mass stripping on their evolution. A previous analysis was carried out by Lacey & Cole (1993) in their semianalytical treatment of the merging of cosmic structures. Their aim was to clarify the role of dynamical friction in determining the merging rate of the luminous part of galaxies as opposed to the merging rate of their dark matter halos. A dependence of the decay time on eccentricity was introduced by simply fitting the decay time curve obtained using Chandrasekhar’s formula for a pointlike satellite and for a fixed orbital energy (the orbital energy was such that a circular orbit had a radius equal to the virial radius of the primary halo). The authors found that satellites on nearly circular orbits decay on a timescale that is almost 3 times longer than that of satellites on very eccentric orbits. However, satellites accreting onto larger halos are usually on more tightly bound orbits according to cosmological simulations, i.e., even radial orbits have apocenters close to the virial radius of the main halo (in the Lacey & Cole case, the apocenter of radial orbits was well outside the virial radius). A more appropriate choice of the orbital parameters is thus needed to determine to what extent the sinking times depend on the eccentricity of the orbits. Moreover, dark matter halos of individual satellites would not dissolve immediately; Navarro, Frenk, & White (1995), using N-body/smoothed particle hydrodynamics (SPH) simulations, have shown that discarding the dark matter mass can lead to an overestimate of the sinking times of the baryonic cores. It is thus evident that a complete description of the orbital evolution of satellites must take into account the effect of tidal stripping on the decay times of satellites. It is also necessary to discover whether the orbits circularize during the decay, this being another long-standing issue. If orbits do circularize, we expect the initial distribution of orbital eccentricities of the satellites to be substantially altered, and this could prolong their lifetimes in the primary halos, provided that a substantial mismatch between decay times on eccentric and circular orbits really does exist.

Recently, van den Bosch et al. (1999; hereafter vBLLS) carried out a series of N-body simulations of satellites placed inside a nonsingular tidally limited spherical halo. They did not include tidal stripping, treating the satellites as rigid spheres. To study the orbital evolution of satellites in a fully self-consistent way and to complement the results obtained with TLR, we have performed a number of high-resolution N-body simulations with the parallel binary tree-code PKDGRAV (J. Stadel & T. R. Quinn 1999, in preparation; Dikaikos & Stadel 1996; vBLLS) for both rigid and deformable satellites.

The structure of the paper is as follows. In § 2 we give a brief description of the theory of linear response for dynamical friction, setting the framework for the calculation carried out in § 3. In § 4 we illustrate the results of our semianalytical study and compare them with vBLLS. In § 5 we explore the self-correlation properties of the equilibrium stellar system in terms of the correlation timescale and study the fading of the density wake in a uniform infinite medium. In § 6 we extend our analysis to the important case of nonrigid satellites. Section 7 contains our conclusions.

2. THE THEORY OF LINEAR RESPONSE FOR DYNAMICAL FRICTION

A satellite bound to a primary galaxy experiences in its motion a dissipative force that results from the collective response of the background to its perturbation. In TLR, the response depends only on the properties of the underlying matter field in its unperturbed state; the (self)-correlations existing among the particles ultimately lead to energy dissipation.

Under the hypothesis that the N stars (of mass m) or dark matter particles are in virial spherical equilibrium, the drag force $F_\Delta$ on a satellite described as a pointlike object of mass M reads

$$F_\Delta(t) = GMm \sum_{i=1}^{N} \int_{t_0}^{t} ds \int d\Gamma \left[ \nabla_{p(s)} f_0 \cdot \frac{R(s) - r(s)}{R(s) - r(s)^3} \right] \times \left[ GMm \sum_{j=1}^{N} \frac{R(t) - r(t)}{R(t) - r(t)^3} \right],$$  \hspace{1cm} (1)$$

where $d\Gamma$ is the elementary volume in the 6N-dimensional phase space ($\Gamma$) of the stars in the galaxy, and $f_0$ is the N-point equilibrium distribution function (we drop the subscript ”s” hereafter to follow more closely the notation of CP, and denote the total halo mass $M_{\text{halo}}$ as $N m$). The drag on $M$ is a consequence of a memory effect that develops with time and involves a suitable phase space average of the microscopic two-body force. It requires a knowledge of the dynamics of the N stars [r(s), p(s)], as determined by the unperturbed Hamiltonian, over the whole interaction time from $t_0$ (when the perturbation is turned on) to the current time t. The distribution function, $f_0$, incorporates the properties of the system in virial equilibrium.

Dark matter halos in virial equilibrium can be regarded as an assembly of collisionless particles subject to a mean field potential $\Psi_0$ that can be computed by simultaneously solving the Poisson and Boltzmann equations. The distribution function can thus be written in terms of the one-particle phase space density, $f^{sep}(r,p)$. Under this hypothesis, and because of the statistical independence of the particles, all cross-correlation terms cancel identically in the limit of $N \gg 1$. Only the self-correlation properties of the collision-
less background survive to yield
\[ F_A(t) = (GM)^2 N m^2 \int_{t_0}^t ds \int d_3 \mathbf{r} d_3 \mathbf{p} \]
\[ \times \left\{ V_{\mathbf{u}(t)} f^{op} \cdot \left[ \frac{R(s) - r(s)}{[R(s) - r(s)]^3} - \int d_3 r' n_0(r') \frac{R(s) - r'}{[R(s) - r']^3} \right] \right\} \times \frac{R(t) - r(t)}{[R(t) - r(t)]^3}. \] (2)

The new term appearing in square brackets [involving the equilibrium background density \( n_0(r) \)] represents, at a given time \( s \), the mean force acting on \( M \) resulting from the system as a whole; it accounts for the shift of the center of mass of the galaxy during the encounter. The recoil of the halo (due to linear momentum conservation) is a coherent shift of all the orbits of the background particles, giving rise to a global correlation among them. TLR, as it is formulated, can account for that shift naturally and permits the use of the one-particle distribution function, \( f^{op} \), for the system in virial equilibrium (see CP and C98). Thus, \( F_A \) in the form of equation (2) is the force as measured in the noninertial reference frame comoving with the halo’s center of mass.

In the context of the fluctuation-dissipation theorem, the braking force can be seen as an integral over time of the correlation function of a fluctuating component of the microscopic force,
\[ F_A(t) = \int_{t_0}^t ds K^q(t - s) = \int_{t_0}^t ds \int d_3 \mathbf{r} d_3 \mathbf{p} V_{\mathbf{u}(t)} f^{op} T^{ba}, \] (3)
where the self-correlation tensor reads
\[ T^{ba} \equiv (GM)^2 N m^2 \left[ \frac{R^a(s) - r^a(s)}{[R(s) - r(s)]^3} - \int d_3 r' n_0(r') \frac{R^b(s) - r^b}{[R(s) - r']^3} \right] \times \frac{R^a(t) - r^a(t)}{[R(t) - r(t)]^3}. \] (4)

The correlation function \( K^q(t - s) \) introduces a timescale \( \tau^* \) characterizing the rise time of the force \( F_A(t) \); it is the scale over which the stars redistribute the satellite’s orbital energy into the internal degrees of freedom of the system.

The interpretation of \( F_A \) in terms of a global time-dependent density deformation is also possible within TLR, noting that equation (2) can be written formally as
\[ F_A = -GMN m \int d_3 \mathbf{r} \Delta n(r, t) \frac{R(t) - r}{[R(t) - r]^3}, \] (5)
where the function \( \Delta n(r, t) \) maps the response, i.e., the time-dependent changes in the density field \( n_0(r) + \Delta n(r, t) \) resulting from the superposition (memory) of disturbances created by the satellite over the entire evolution; the function \( \Delta n(r, t) \) can be derived by comparing equation (5) with equation (2) (see also CP for details).

Equation (2) applies when the interaction potential between \( M \) and the stars is weak relative to the mean field potential, \( \Psi_0 \), of the equilibrium system (when in isolation). This is the reason why only the properties of the halo in virial equilibrium are required to evaluate \( F_A \). As a consequence, \( F_A \) is accurate to second order in the coupling constant \( G \). Higher order terms would describe the self-gravity of the response, i.e., the modification in the self-interaction potential due to the external perturbation driven by \( M \).

Equation (2) can describe the sinking of satellites moving on arbitrary orbits, even outside the primary halo. Previous semianalytical studies focused on purely circular orbits (Weinberg 1986) to explore the role of resonances and on almost radial orbits to explore the transient nature of the interaction (Seguin & Dupraz 1994).

3. TLR: THE FORCE OF BACK-REACTION IN A SPHERICALLY SYMMETRIC GALACTIC HALO

In a nonuniform collisionless background, the back-reaction force on \( M \) results, in the high-speed limit, from the combined action of a global tidal response related to the density gradients (absent in an infinite uniform medium) and from the development of an extended wake forming behind the satellite’s path that contributes mostly to its deceleration. The force acquires a component along \( \mathbf{R} \) as symmetry around \( V \) is lost, the underlying system being nonhomogeneous.

To estimate the drag in the domain where the satellite maintains a velocity \( V \) (determined primarily by the mean field potential \( \Psi_0 \) of the unperturbed background) comparable to the background velocity dispersion, we avoid separating out the tidal and frictional contributions as aspects of a unique process. Exploiting the time independence of the distribution function \( f^{op} \) and of the phase-space volume \( d_3 r d_3 v \) (hereafter \( f^{op} \) will be considered as a function of \( r \) and \( v = p/m \) and is normalized accordingly), the drag force (eq. [2]) can be equivalently written as
\[ F_A = (GM)^2 N m \int_{t_0}^t ds \int d_3 \mathbf{r} d_3 \mathbf{v} V_{\mathbf{u}(t)} f^{op} \left( \mathbf{r}, \mathbf{v} \right) \]
\[ \times \left\{ V_{\mathbf{u}(t)} \phi(|\mathbf{R}(s) - \mathbf{r}|) - \int \sum_3 d_3 r' n_0(r') V_{\mathbf{u}(t)} \phi(|\mathbf{R}(s) - r'|) \right\} \times V_{\mathbf{u}(t)} \phi(|\mathbf{R}(t) - (r(t) - s)|), \] (6)
where \( \phi \) is proportional to the Newtonian gravitational potential,
\[ \phi(|\mathbf{R} - \mathbf{r}|) \equiv -\frac{1}{|\mathbf{R} - \mathbf{r}|}. \] (7)
In equation (6), \( \mathbf{R} \) denotes the satellite position vector relative to the halo’s center of mass, and is computed self-consistently following the actual dynamics of the satellite (which now acquires the reduced mass \( \mu \)).

Because of the difficulty of including the dynamics of the stars as determined by the unperturbed Hamiltonian, we are led to approximate their motion as linear, giving
\[ r(t-s) = r + (t-s)\mathbf{v}. \] (8)
We will compare our model with \( N \)-body simulations (described in § 4) to test indirectly the validity of such an approximation. In neglecting the acceleration of the stars, i.e., their “curvature,” during the interaction of the satellite, we introduce a simplification that will prove to be satisfactory.

The shortcoming of TLR is its inability to describe short-distance encounters, since it is derived from a linear analysis expanded to first order in the perturbation. For a pointlike satellite moving in an infinite uniform medium, these encounters lead to a minimum impact parameter that is determined uniquely by \( V \) and the background velocity dispersion, \( \sigma \). Satellites have finite size, and as in \( N \)-body simulations, the short-distance two-body interaction \( \phi \) is
smoothed, introducing in the microscopic gravitational potential a softening length \( \epsilon \). The necessity of a closer comparison with numerical simulations led us to consider the spline kernel potential, \( \phi_{sp} \) (Hernquist \& Katz 1989), as the interaction potential between the satellite and the stars, reducing to the Newtonian form (eq. [7]) at 2\( \epsilon \).

The introduction of the softening length in the computation of the force accounts for the finite size of \( M \), permitting an unambiguous comparison with the numerical simulations by vBLLS. The drag force depends on the response of the stars and, in turn, on the characteristics of their equilibrium state, which is described below.

Dark halos are often modeled as truncated nonsingular isothermal spheres with a core (vBLLS; Hernquist 1993): accordingly, their density profile,

\[
 n_0(r) = \frac{1}{2\pi^{3/2}} \frac{\exp \left( -r^2 / r_t^2 \right)}{gr_t^3} \left( \frac{r}{r_t} \right)^{-\frac{1}{2}} \left( 1 - \text{erf} \left( \frac{r}{r_t} \right) \right),
\]

(9)

declines exponentially at radii exceeding the tidal (or truncation) radius, \( r_t \). The homogenous core of radius \( r_c \) is surrounded by a region where \( n_0(r) \propto r^{-2} \), as in a singular isothermal sphere. The constant

\[
g = 1 - \frac{1}{\sqrt{2}} \left( \frac{r_c}{r_0} \right) \exp \left( \frac{r_c^2}{r_0^2} \right) \left( 1 - \text{erf} \left( \frac{r_c}{r_0} \right) \right)
\]

(10)
is introduced to guarantee that \( \int dr \rho(r) = 1 \).

The one-dimensional background velocity dispersion, \( \sigma \), is computed according to the second-order Jeans equation,

\[
\sigma^2(r) = \frac{\left( \frac{c}{r_0} \right)}{\int_0^\infty dr' n_0(r') \frac{4\pi G}{r'^2} \int_0^r dr'' (r'')^2 n_0(r'')},
\]

(11)

and is a local function of \( r \). For \( r_c \to 0 \),

\[
\sigma^2(r) = \frac{G M}{gr_t^2} \left( \frac{r}{r_t} \right)^2 \exp \left( \frac{r^2}{r_t^2} \right) \int_{\sqrt{2}\sigma}^\infty dx \exp \left( -x^2 \right) x^{-4} \text{erf} (x),
\]

(12)
giving \( \sigma^2 \approx G M / (gr, \pi^{1/2}) \), at \( r \ll r_c \). The back-reaction force on \( M \) is derived under the hypothesis that the one-particle distribution function is isotropic and Gaussian in the velocity space, with \( \sigma^2 \) computed according to equation (12):

\[
f_{\text{op}}(r, v) = n_0(r) \frac{1}{2m\sigma^2} \exp \left( -\frac{v^2}{2\sigma^2} \right).
\]

(13)

This choice is dictated not only by simplicity arguments but by the fact that collisionless systems with these characteristics are found to be nearly in equilibrium (Hernquist 1993; vBLLS) and hence are viable for describing the unperturbed system equilibrium state required by TLR. Given \( f_{\text{op}} \), we compute \( F_\Delta \) from equation (6). Not all multiple integrals of equation (6) can be carried out analytically; only those over the velocity phase space are evaluated, and the complex expression of the drag is reported in the Appendix.

The evolution of a satellite of reduced mass \( \mu = Mn_0 / (M + Nm) \) is followed by solving for the equations of motion in the reference frame comoving with the center of mass of the primary halo:

\[
\frac{d^2 \mathbf{R}(t)}{dt^2} = -G N m \frac{R(t)}{|R(t)|^3} \int_{|r-r_0(t)|} d_s r'n_0(r') + F_\Delta.
\]

(14)

The mass ratio, \( M/Nm \), and the cusp, \( \epsilon \), entering the effective potential, \( \phi_{sp} \), are the only parameters of the model.

4. THE SINKING OF RIGID SATELLITES

In this section we explore the evolution of satellite orbits, comparing results obtained using TLR with our N-body runs and, where possible, with those of vBLLS. As in vBLLS, the primary system, scaled to the Milky Way’s halo, has a mass of \( 10^{12} M_\odot \), a tidal radius \( r_t = 200 \text{ kpc} \), and a core radius \( r_c = r_t / 50 \). In the N-body simulations, the primary halo has \( 10^4 \) particles; it was first evolved in isolation for 10 Gyr, and the stability of the density profile was verified. The satellite is 50 times lighter than the primary \( (M = Nm/50) \). Its mass distribution is described by a rigid spline-softerned potential with a length scale of 3.4 kpc, comparable to the effective radius of the Large Magellanic Cloud \( (\epsilon = 0.0172 r_t) \). With these choices, the time unit, \( T_0 = (G N m / r_t^3)^{-1/2} \), is 1.34 Gyr.

4.1. The Dynamical Friction Decay Time and the Evolution of the Eccentricity for Rigid Satellites

Cosmological simulations have shown that in the hierarchical clustering scenario, most of satellite’s orbits have pericenter varying between \( 0.2 < r_{\text{peric}} / r_t < 0.5 \) and apocenter \( r_{\text{apo}} < 2r_t \) (Ghigna et al. 1998). More loosely bound orbits are unlikely, since their apocenter can exceed the turnaround radius (about \( 2r_t \)) of the major overdensity that produced the primary halo. Moreover, orbits are found to be quite eccentric on average, with a typical apocenter to pericenter distance ratio of \( \sim 6-8 \), corresponding to eccentricities between 0.6 and 0.8. Below, we focus on orbits with reference circular radii \( r_{\text{cir}} \) (determining the initial energy) in the range \( 0.5 \leq r_{\text{cir}} / r_t \leq 1 \) (to fulfill the above inequalities). We are thus able to study the dependence of \( t_{\text{DF}} \) on both eccentricity and orbital energy.

Figure 1 shows the dynamical evolution of the satellite set on bound orbits having initially equal energy but different eccentricities, \( e_{\text{orb}} = 0.8 \) (top left), 0.6 (top right), 0.3 (bottom left), and 0 (bottom right). The radius of the circular orbit at the onset of evolution is \( r_{\text{cir}} / r_t = 0.5 \), as in vBLLS; the selected runs coincide with models 3, 4, 5, and 6 of vBLLS (see their Table 1). To characterize the decay and quantify the results, we show in Figure 2 the angular momentum as a function of time (in Gyr); filled circles show the results of the N-body simulations carried out by vBLLS. The agreement between theory and N-body simulations is excellent to a few percent.

The analysis of more loosely bound orbits has been carried out using both TLR (for \( r_{\text{cir}} / r_t = 0.8 \) and 1) and N-body simulations (\( r_{\text{cir}} / r_t = 1 \)). The results show once more an excellent agreement between theory and simulations (Fig. 3). Minor differences might be caused by the limited resolution of our N-body galaxy model; in particular, the potential sampling may be exceedingly poor in the outer region of the halo, where the satellites now spends a lot more time. We have tested this hypothesis by performing a high-resolution simulation with \( 10^5 \) particles; in this case there is a closer agreement (Fig. 3), proving the potential of the theory with respect to costly N-body simulations.

To evaluate the decay time of a satellite moving within a singular isothermal sphere, Lacey & Cole (1993) proposed the following general expression to incorporate the dependence of the sinking time on the initial eccentricity and
FIG. 1.—Collection of orbits in the plane (x, y) computed within TLR, for $r_{\text{cir}}/r_t = 0.5$ and initial eccentricities $e_{\text{orb}} = 0.8$ (top left), 0.6 (top right), 0.3 (bottom left), and 0 (bottom right). Length is in units of $r_t$. (15)

In this formula, $V_{\text{cir}}$ is the circular velocity of the satellite, $r_{\text{cir}}$ is the radius of the circular orbit having the same energy as the actual orbit, and the circularity $e \equiv J(E)/J_{\text{cir}}(E)$ is the ratio between the orbital angular momentum and that of the circular orbit having the same energy. E. Lacey & Cole (1993) suggested a value of $\alpha = 0.78$ for the dependence on eccentricity. However, as already noted by vBLLS, the decay time, $\tau_{\text{DF}}$, depends more weakly on the initial eccentricity, and for the case of $r_{\text{cir}}/r_t = 0.5$, vBLLS proposed a best fit of the form $\tau_{\text{DF}} \propto e^\alpha$, with exponent $\alpha = 0.53$.

Both TLR and our N-body simulations show that $\alpha$ depends on the energy of the orbit and that the value of the scale $T_e$ deviates slightly from the one inferred using Chandrasekhar’s formula. For the cosmologically relevant orbits (those with $r_{\text{cir}}/r_t = 0.5$), the TLR approach (supported by our set of N-body simulations), gives a $\tau_{\text{DF}} \propto e^\alpha$ with exponent $\alpha = 0.4$.

For the case of orbits outside the halo (C98), satellites on wider orbits not only have longer decay times, but the mismatch between very eccentric and nearly circular orbits becomes increasingly larger with decreasing binding energy. A continuity in the behavior of $\tau_{\text{DF}}$ should therefore exist when moving from peripheral orbits to internal ones. Figure 4 shows $\tau_{\text{DF}}(e, r_{\text{cir}})$, computed using TLR, for $r_{\text{cir}}/r_t = 0.5$, 0.8, and 1. Circles show eccentricities 0(e = 1), 0.3, 0.6, 0.8, and 0.85. As mentioned above, the best fit gives a slope $\alpha = 0.4$ for $r_{\text{cir}}/r_t = 0.5$. We find that for the typical eccentric orbits (Fig. 4) occurring in current structure formation scenarios, the dynamical friction timescale is longer by a factor of 1.5–2 than previous estimates (Lacey & Cole 1993), a result that may significantly affect the statistics of the satellites in galaxy halos and of galaxies in galaxy clusters.

One remaining problem is whether orbits tend to circularize under the effect of dynamical friction. This could be true if satellites lose energy faster than angular momentum. It has recently been shown (Ghigna et al. 1998) that satellites inside larger halos have a distribution of orbital eccentricities quite indistinguishable from that of the diffused
dark matter component. This strongly suggests that orbits of satellites do not evolve significantly under the effects of tidal stripping and dynamical friction. We can now support this numerical result within TLR, showing that bound orbits are not subject to any significant circularization, as illustrated in Figure 5; this result also holds when exploring the evolution of live satellites. Only when the satellite happens to fall from the very far reaches of the halo (in grazing encounters) does TLR predict some degree of circularization (C98), consistent with numerical simulations by Bontekoe & van Albada (1987).

5. The self-correlation properties of the fluctuating microscopic force

5.1. The Self-Correlation Time

An important result of this study is that two independent calculations, i.e., a semianalytic theory and a set of N-body simulations, give equivalent results. The quite stringent agreement between the two approaches confirms the applicability of TLR in describing the sinking of satellites in spherical nonhomogeneous halos, neglecting the actual stellar dynamics.

The question thus naturally arises: What role is played by the self-gravity of the background in determining the extent of the drag? Equation (6) contains many aspects of the self-gravity of the collisionless background (in its unperturbed state): (1) the equilibrium dynamics that establish the strength of the self-correlation properties of the system; (2) the density profile, \( n_0(r) \); (3) the virial relation that links \( n_0(r) \) to the dispersion velocity, \( \sigma(r) \), and ultimately, (4) the shift of the system barycenter. The braking torque depends on all these quantities, which are interrelated.

The dynamics of the stars in the unperturbed potential, \( \Psi_0 \), is expected to be important in the determination of \( F_\alpha \) if their response to the external perturbation remains correlated for a time \( \tau^* \) longer than the typical radial period \( T_r = 2\pi r_{_{\text{cir}}}/V_{_{\text{cir}}} \) (where \( V_{_{\text{cir}}} \) is the circular velocity in the primary halo), which is of a few time units, for the mean field potential \( \Psi_0 \) generated by the density distribution of equation (9). The value of \( \tau^* \) can be estimated using equations (3) and (4); Figure 6 shows the cumulative function

\[
P^*(s) \equiv \int_{s_0}^{s} ds' K^*(t - s')
\]

at four selected times (\( t = 2, 3, 4, 6, \) and 8 time units) during

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**Fig. 2.** Orbital angular momentum \( J/\Gamma_0 \) as a function of time for \( r_{_{\text{cir}}}/r_c = 0.5 \) and eccentricities \( e_{_{\text{orb}}} = 0.8 \) (top left), 0.6 (top right), 0.3 (bottom left), and 0 (bottom right). Solid lines, results of TLR; filled circles, results of vBLLS; dot-dashed lines, results of our N-body runs.
orbital evolution, for $e_{\text{orb}} = 0.6$ and $r_{\text{circ}}/r_t = 0.8$. We find that the rise of the correlation function, $K^*$, is rather rapid as it occurs over a time $\tau^*$, which is only $\approx 1.5 T_0$, i.e., a fraction of the typical radial time $T_r$. On this scale, $\tau^*$ stars thus follow a dynamics that can be approximated as free; this verifies the internal consistency of our calculation and explains the equivalence between the analytical and numerical calculations.

A finite $\tau^*$ does not necessarily imply $\tau^* < T_r$. The self-correlation timescale can exceed $T_r$; memory can be maintained over many orbital periods, and resonant transfer of energy can dramatically accelerate the decay. As found in C98, a satellite moving outside a spherical halo experiences a drag resulting from the global tidal deformations excited by the satellite itself. The drag force was generally found to be

$$F^a_{\Delta} = -(GM)^2 \frac{N_{\text{tot}}}{\sigma^2} O^{abc}(t) \int_{t_0}^t ds \, \mathcal{B}(t-s) Q^{bc}(s), \quad (17)$$

where the tensors $Q$ and $O$ represent the quadrupolar and octupolar terms leading the multipole expansion of the interaction potential (Prugniel & Combes 1992 recognized the importance of these terms in their numerical simulations). In equation (17), $\mathcal{B}(t-s)$ is a four-point self-correlation function of the type $\langle x^* x(t-s) y y(t-s) \rangle$, gauging the degree of correlation in the equilibrium dynamics of the stars. For a purely harmonic interaction potential of proper frequency $\omega_0$, the function $\mathcal{B}(t-s)$ was found to scale as $\sin [2 \omega_0 (t-s)]$. Interestingly, we here note that for such a potential, the correlation timescale is infinite. To illustrate this property, let us consider a more general expression of the self-correlation function,

$$\mathcal{B}(t-s) \approx \int_{-\infty}^{+\infty} d\omega \sin [2 \omega (t-s)] \exp \left[ -\frac{(\omega - \omega_0)^2}{\sigma^2} \right],$$

obtained by weighting the sinusoidal function that describes the periodic nature of the orbits of the background particles with a Gaussian centered about $\omega_0$ with dispersion $\sigma_\omega$. We have included such a dispersion to mimic the characteristic spread in the frequencies of the background-particle
motions. The integral can be evaluated straightforwardly,

\[ \mathcal{B}(t-s) \propto \sigma_{av} \sin [2\omega_0(t-s)] \exp \left\{ -\left[ \sigma_{av}(t-s) \right]^2 \right\} , \]

(19)

to show that \( \mathcal{B} \) acquires an intrinsic cutoff timescale

\[ \tau^* = \frac{1}{\sigma_{av}} , \]

(20)

which is determined by the dispersion (\( \sigma_{av} \)) in the orbital frequencies of the equilibrium system. If the distribution is sharply peaked (as for the harmonic potential), \( \tau^* \) is exceedingly large, and the inclusion of the actual dynamics is essential in determining the drag. As illustrated by equation (20), \( \tau^* \) is finite when more frequencies are contributing to the dynamics in the virial spherical system. Hence, the “richness” in the spectral decomposition of the orbits is an indirect measure of the self-correlation time, a quantity that must be compared with the typical radial period time, \( T_r \), and with the characteristic time of interaction, \( T_{in} \) (of the order of \( T_r \) for the cases explored in this paper), between the satellite and the stars to determine the importance of the real dynamics in affecting the drag.

5.2. On Global Tides, the Wake, and the Shift of the Barycenter

One long-standing question is whether dynamical friction in self-gravitating backgrounds is a local or global process. As shown in C98, the satellite excites a tidal deformation when orbiting outside the halo; this deformation is clearly global, since it involves the whole galaxy volume. A global response is also excited in short-lived flybys deep across the halo, giving a force along \( V \) and \( R \) that is proportional to the background density gradients (CP). In addition to such a global response, a back-reaction force arises as a result of the overdensity that the satellite excites along its path (eqs. [42] and [47] of CP). This is usually referred to as the “local” contribution to the drag, depending on intensive quantities such as the background density \( n_0 \), despite the presence of a Coulomb logarithm that accounts for those “distant” encounters that are effective for the satellite’s drag. These examples (CP) illustrate that the global tidal field and the wake are aspects of the response that are simultaneously present; they can be clearly distinguished in the high-velocity limit.

When considering the interaction along bound orbits inside the halo, the two contributions are technically difficult to separate out. Nonetheless, an approximate estimate of the degree of locality of the response can be inferred by

Fig. 4.—Dynamical friction timescale, \( \tau_{df} \) (in Gyr), vs. circularity, \( \epsilon = J_0/J_{cir} \). \( \tau_{df} \) refers to the time at which \( J/J_0 = 0.1 \). Filled circles connected with solid lines denote the decay times computed within TLR, for \( r_{cir}/r_t \leq 0.5 \) (bottom), 0.8 (middle), and 1 (top). Open circles (connected with dot-dashed lines) show the \( \tau \) with \( a = 0.4 \) (bottom), \( a = 0.45 \) (middle), and \( a = 0.78 \) (top). The dashed line denotes the fit by Lacey & Cole (1993), given by eq. (15).

Fig. 5.—Eccentricity as a function of time \( t \), for \( r_{cir}/r_t \) equal to 0.5 (left) and 1 (right), with initial \( e_{\text{init}} \) = 0.8, 0.6, and 0.3. Filled circles are from TLR, open circles are from the \( N \)-body runs. Eccentricity is computed at each pericentric passage considering a complete cycle along the actual satellite’s orbit.
Fig. 6.—Correlation integrals $P(s)$ (solid line) and $P_t(s)$ (dot-dashed line), as defined in eq. (16), with $s$ varying from $t = 0$ up to $t$, with time in dimensionless units. A view of the orbit (with $r_{\text{in}}/r_t = 0.8$ and $e_{\text{in}} = 0.6$) in the $(x, y)$ plane is drawn, where filled circles denote the times $t/T_0$ equal to 2, 3, 4, 6, and 8, at which the functions (proportional to the components of the drag force and expressed in dimensionless units) are computed. The integrals quantify the memory effect and provide a measure of the self-correlation timescale ($\tau^*$).

selecting from the force $F_\Lambda$ its component along $V$ ($F_V$) and comparing it with the frictional force,

$$F_\infty = -4\pi(GM)^2nm_0\ln \Lambda \left[ \text{erf}(x) - \frac{2x}{\pi^{1/2}} \exp(-x^2) \right] \frac{V}{|V|},$$

from an infinite, homogeneous, non–self-gravitating collisionless background (Chandrasekhar 1943; Binney & Tremaine 1987); $x$ is equal to $|V|/\sqrt{2\sigma}$.

Figure 7 shows that $F_V$ (filled circles linked by solid line) is at maximum just after each pericentric passage, the lag being a manifestation of the memory effect. $F_V$ accounts for nearly 80%–90% of the total force that also has a component along $R$.

The force $F_\infty$ (Fig. 7, dashed line) is computed using the value of the density $n_0$ and the dispersion velocity $\sigma$ at the (“local”) instantaneous satellite’s position, and setting $\ln \Lambda = \ln(r_t/\epsilon)$, a value that is close to $\ln(Nm/M)$. In $F_\infty$, the time lag is absent, and a closer analysis of the two forces reveals that $F_\infty$ would predict a more rapid sinking than $F_\Lambda$. A time-dependent Coulomb logarithm can better fit the orbit and the evolution of the angular momentum, particularly in two regions: at the periphery where $\ln \Lambda \sim \ln \left[ \ln (R_{\text{apo}} - R(t))/\epsilon \right]$, and close to the halo’s center, where $\ln \Lambda \sim \ln \left[ \ln (R(t))/\epsilon \right]$ (see Fig. 7, dot-dashed line). In general, we find that it is difficult to accurately reproduce the evolution: slow the component $F_\Lambda$ along $R$ gives a nonnegligible contribution; related to the tides and to the spatial inhomogeneities, this component varies in each single path.

Customarily, $\ln \Lambda$ gives an indication of the interval of background-particle impact parameters for which the encounter is effective. Can we have an intuitive understanding of the fits introduced above? As a guideline, let us consider the temporal evolution of the density perturbation in a uniform background: $\Delta n$ is found to result from the composition of disturbances that originate at earlier times $s$,

$$\Delta n(r, t) = \sqrt{\frac{2}{\pi}} \frac{GMn_0}{\sigma} \int_0^{t-\tau} \frac{ds}{(t-s)^{3/2}} \exp \left\{ - \frac{1}{2} \Gamma_s[R(s) - r]^2 \right\},$$

which are Gaussian in space, spherically symmetric about $R(s)$, and have a characteristic length

$$\lambda_s = \Gamma_s^{-1/2} = \sigma(t-s).$$

(In eq. [22], $\tau_s$ is introduced to mimic short-distance encounters yielding a finite minimum impact parameter.) Since the characteristic scale length becomes increasingly small as $s \rightarrow t - \tau_s$, the deformation is large primarily in the vicinity of the satellite, where $|R(s) - r| \ll \lambda_s$. At earlier times, $s \ll t - \tau_s$, the Gaussian disturbance has a wider extension, indicating that the density perturbation broadens in space and weakens in magnitude, being a transient structure. (Only in the high-speed limit, i.e., $\sigma/V \rightarrow 0$, does the overdensity develop in a sharp edge, a shock that never broadens as stars behave as a cool continuum, i.e., as dust.) In an infinite medium, the decay of the overdensity is not sufficiently rapid to make the drag finite, and this is why a cutoff distance, of the order of $r_s$, is introduced in the expression of $\ln \Lambda$ (eq. [21] is derived from eq. [6], and is a test of TLR).

In a spherical halo, the wake develops only as soon as the satellite enters the stellar medium, so the maximum impact parameter initially varies with time, as suggested by our first fitting formula. Later, the wake spatially widens while fading across the medium, in analogy with equation (22), at a rate that increases with decreasing distance $r$, being $\propto \sigma$ (eqs. [12] and [22]). In bound nonuniform systems, there is a tendency to erase the memory of the perturbation more rapidly than in an infinite medium, yielding to a weaker drag and to a force along $V$ that is influenced more by the
local properties of the background. This is likely a consequence of $\tau^*$ being smaller than the internal dynamical time. Nonetheless, the actual dynamics of the satellite can be determined only within TLR, which gives the description of the full stellar response (including tides and the effect of a nonuniform background).

The coherent shift of the halo’s center of mass is an important aspect of the response (White 1983; Weinberg 1989; Hernquist & Weinberg 1989; Prugniel & Combes 1992; Séguin & Dupraz 1994, 1996; Cora, Muzzio, & Vergne 1997). Its inclusion accounts for the correct estimate of the global large-scale density deformations induced by the satellite; pinning the center of mass of the primary (i.e., not including the shift) would result in more intense tides that instead are not excited in a real encounter. We have verified that this correction becomes important when the satellite is set on progressively wider orbits of low eccentricity. For the vBLLS models, the correction on the sinking times accounts for about 10%. It is larger ($\sim$40%) when the satellite mass increases (we explored a few cases with $M/Nm = 0.08$ and $r_{\text{cir}}/r_t = 1$) and goes always in the direction of reducing the extent of the drag. Weinberg included in his formalism the shift of the barycenter of the primary system coupling the Boltzmann equation for $f^\text{opt}$ to the Poisson equation for the density perturbation. This approach makes the calculations too complex and does not allow for a simple expansion of the drag force in powers of the coupling constant $G$.

6. THE SINKING OF THE DEFORMABLE SATELLITES: HOW DOES TIDAL STRIPPING AFFECT ORBITAL EVOLUTION?

In the previous sections we have carried out a detailed study of dynamical friction using TLR and $N$-body simulations to gain insight into the physical mechanisms that cause the braking of a satellite and its subsequent orbital decay. In the cases explored in § 4, the satellite was treated as rigid-body, while real satellites are deformable systems, comprising a small luminous component hosted by a massive and extended dark matter halo.

The outer part of the dark matter halo can be strongly damaged by the tidal field of the primary, while experiencing dynamical friction. As a consequence, a reasonable picture of the evolution of satellites must take into account the role of tidal forces as well as dynamical friction. Here we determine how mass stripping affects evolution.

Among the satellites of the Milky Way, a few have experienced at least one or two close pericenter passages and have suffered mass loss by the global tidal field of the Milky Way; a clear example is Sgr A, which at present is on the verge of being disrupted (Ibata & Lewis 1998). Using $N$-body simulations, we have followed the evolution of satellites, described as spherical halos, to explore the interplay between mass stripping and orbital decay due to dynamical friction. We then tried to fit the numerical results within the framework presented in the previous sections.

6.1. Initial Conditions

Large cosmological $N$-body simulations within the cold dark matter (CDM) framework show that (satellite) halos have density profiles that can be fitted by the so-called NFW or Hernquist density profiles (Navarro et al. 1996, 1997; Ghigna et al. 1998). These profiles have a central cusp and fall more steeply than the isostructural profile at large radii. However, the resulting rotation curves are in conflict with those observed for dwarf galaxies and low surface brightness galaxies, which exhibit a large core in the center (Moore et al. 1999b; Persic & Salucci 1997). Feedback due to mass outflows of baryons as a consequence of supernova-driven winds have been invoked to reconcile this discrepancy (Navarro et al. 1996; Gelato & Sommer-Larsen 1999), but the solution of this problem still awaits (Burkert & Silk 1999). We thus employ truncated isothermal profiles with cores to model satellite halos, since these allow good fits with observed rotation curves for different galaxies (de Blok & McClure 1997). The primary galaxy is represented by the same model used in the simulations described in § 4.

We build three different models for the satellite; these have the same virial mass, $M$, which is 0.02$N$, but differ in the value of the concentration, $c$, where $c$ is the ratio between the satellite’s tidal radius, $r_t$, and its core radius, $r_c$. This parameter sets the value of the central density as $\rho_0 \propto c^{-2}$. Tidal damage of the satellites’ halos should be basically related to the ratio between their own central density and that of the primary halo; for this reason, the concentration of the satellite’s density profile could play an important role in deciding its final fate. A general result of hierarchical clustering is that lower mass halos are on average denser because they formed earlier, when the background density of the universe was higher. An analysis of cosmological simulations shows that the characteristic halo density scales as $M^{-2}$ (Syer & White 1998). The value of $c$ is related to the slope of the power spectrum on the scale of interest, and is $\sim 0.33$ for galaxy-sized halos in a standard CDM cosmogony; according to this estimate, an LMC-like satellite should have a central density $\sim 4$ times higher than that of the Milky Way.

The reference model (S1) for our dark matter satellite is simply a rescaled version of the primary galaxy, according to the relations $r_t/r_c = (M/Nm)^{1/3}$ and $V_{\text{cir}}^2/V_{\text{circ}}^2 = (M/Nm)^{1/3}$ (White & Frenk 1991).

The resulting satellite has a circular velocity, $V_{\text{cir}}$, of about 50 km s$^{-1}$, very close to that of the LMC. The other two models have concentrations that are 2 times (model S2) and 3 times (model S3) that of the reference model S1. We use 10,000 particles for the satellite models. One simulation has been rerun with 50,000 particles as a test, giving practically identical results. We employ the same system of units as in vBLLS, along with the same time step and softening for the primary system. The softening for the satellite scales as $(M/Nm)^{1/3}$ relative to the primary. We have considered only orbits with $e_{\text{orb}} = 0.8$ and 0.6 for $r_{\text{conf}}/r_t = 1$ and 0.5. Models S2 and S3 have been run only for the most destructive encounter, i.e., $r_{\text{conf}}/r_t = 0.5$ and $e_{\text{orb}} = 0.8$, respectively.

6.2. The Fate of Galaxy Satellites

Our results allow for a clear interpretation of the interplay between dynamical friction and mass loss due to the tidal field of the primary. Satellites lose on average about 60% of their mass after the first pericentric passage (at 1.5 Gyr), while their orbital angular momentum has decreased by no more than 20% (as illustrated in Fig. 8); this means that tidal stripping is always more efficient than dynamical friction. Their final fate, however, depends on their initial concentration and, although less sensitively, on the orbital parameters. Model S1 is disrupted over a time comparable to the dynamical friction decay time, $t_{\text{DF}}$, of its rigid counterpart just after the second pericenter passage. (In a
test with $5 \times 10^4$ particles on a $r_{\text{crit}}/r_s = 0.5$ and $e_{\text{orb}} = 0.8$ orbit, the satellite [model S1] was disrupted at nearly the same time as in the runs with $10^4$ particles. This proves that the resolution used does not significantly affect the physical interpretation of our results.) Satellites with a high density contrast (S2 and S3) relative to the primary central density can survive even the third pericenter passage (at 5 Gyr), despite being on eccentric, tightly bound orbits. They will then suffer disruption along their orbit, after 6–8 Gyr (performing a total of 4–5 pericenter passages). One must bear in mind that these high-concentration satellites more likely correspond to the halos of small galaxies found in cosmological CDM simulations. The case of model S1 could instead represent the halos of low surface brightness satellites that typically have large cores and still need to find an explanation in a cosmological context. All satellite halos survive much longer than 10 Gyr, regardless of their concentration, if they move on the peripheral orbits (with $r_{\text{crit}}/r_s = 1$). In these cases, dynamical friction almost switches off, as a result of mass loss, when the satellite is still far from the densest region of the primary. As for the rigid case, orbital decay is not accompanied by significant circularization.

The reduced effectiveness of dynamical friction as a result of mass loss has important consequences for the merging of the baryonic components inhabiting dark matter halos. In principle, the loss of orbital angular momentum implies a decrease in the sinking time, depending on $r_{\text{crit}}^2$. On the other hand, the mass loss implies an increase in the sinking time, which scales as $M^{-1}$. The simulations have shown that substantial mass loss already occurs at the first pericentric passage, when the angular momentum has not yet significantly decreased. The satellite orbit has thus not decayed sufficiently (i.e., $r_{\text{crit}}$ has only slightly decreased) to counterbalance the reduction in mass, and the overall result is that the orbital decay will be considerably slowed down. The central region would survive the subsequent disruption of the outer dark matter halo, being more compact (Mayer et al. 1999; Navarro et al. 1995; Ghigna et al. 1998), and would then decay on a very long timescale. A decoupled orbital evolution of dark and baryonic components was suggested by Lacey & Cole (1993) in their semianalytical treatment of galaxy merging rates. In that case, it was implicitly assumed that satellites immediately lose their dark matter halos, finding themselves on a bound orbit at the periphery of the primary halo; the dynamical friction time was then computed with equation (15), adopting merely the baryonic mass for the mass of the satellite. Navarro et al. (1995) instead were led to suggest that satellite galaxies merge at the center of the primary halo on a timescale determined by their initial total mass (baryonic + dark); they came to this conclusion by directly comparing the prediction of equation (15) with the sinking times of gaseous cores in N-body/SPH simulations of galaxy formation. Their distribution of merging times showed, however, a large scatter with respect to the analytical prediction; moreover, a significant group of satellites existed with sinking times 2–3 times larger than the analytical estimate. These turned out to be satellites with an initial mass of less than 0.1 of their primary halo, i.e., they were in the mass range of typical galaxy satellites. Our results suggest that the higher efficiency of tidal stripping with respect to dynamical friction is responsible for such an increase of merging times for galaxy satellites.

We can now give an estimate of the merging time that incorporates mass loss as well as orbital decay. We can approximate the mass-loss curve obtained for satellites moving on eccentric, tightly bound orbits (which are the most likely for satellites) with an exponentially decreasing function of time given by

$$M(t) = M_d \exp (-t/T_r) + M_b ,$$

where $M_d$ is the dark mass of the satellite, corresponding roughly to its initial total mass, $M_b$ is the baryonic mass, which we assume to be less than 0.1 of the total mass, and $T_r$ is the orbital radial period. The satellites continue to lose mass at every pericenter passage; however, most of their mass is stripped on the first orbit, and this occurs independently of their concentration. Moreover, at this time the orbital parameters are still very close to the initial ones due

![Figure 8](image-url)
to the low efficiency of the orbital decay. We then use the tidally limited mass at the first pericentric passage, \( (\dot{M}/d) \), as the "effective" mass for the satellite in equation (15). The merging time of the satellite galaxies, \( \tau_m \), can then be computed using

\[
\tau_m = 1.2 \frac{J_{\text{cir}} r_{\text{cir}}}{(GM_d/e) \ln (M_{\text{halo}}/M_d)} e^{-0.4},
\]

(25)

where \( J_{\text{cir}} \) and \( r_{\text{cir}} \) are, respectively, the initial orbital angular momentum and the radius of the circular orbit with the same energy as the orbit on which the satellite is placed. This formula updates the one by Lacey & Cole (1993), incorporating both the different normalization factor and the eccentricity dependence, as well as the "delaying" effect due to tidal stripping. The reliability of this prescription was tested by running a simulation with a rigid satellite on an orbit \( e_{\text{orb}} = 0.8 \) with an initial mass reduced by a factor of \( 1/e \) with respect to the standard mass, and then comparing the angular momentum loss in this case with that occurring in the corresponding run with a deformable satellite (Fig. 9). These are remarkably close, while much more angular momentum is lost by the rigid satellite with the standard mass, which suffers complete orbital decay (Fig. 9). Our estimate suggests that the satellite galaxies would merge on a timescale almost 2–3 times larger than previously estimated with equation (15). In addition, our \( N \)-body simulations indicate that the survival time of the dark matter halos of the satellites falls between \( \tau_{\text{DF}} \) and \( \tau_m \), with more concentrated halos surviving for a longer time. However, the presence of a baryonic core inside the halo can prolong the lifetime of the central part of the halo itself, because the overall potential well becomes deeper (Mayer et al. 1999). Only very large satellites (i.e., satellites with a mass greater than 0.1 that of the primary) could decay on a short timescale compared to the Hubble time, since the rate of orbital angular momentum loss would be comparable to the rate of mass loss (this being related only to the ratio of the central densities of the two systems and not to their masses). Such events could have occurred in the building up of our galaxy, at \( z > 1 \), when the progenitor halo at that time probably accreted large lumps on orbits with small pericenters (because the virial radius of the primary was smaller), a condition that should further reduce the dynamical friction timescale (see also Kravtsov & Klypin 1998). Satellites accreted after that epoch had not enough time to decay and merge, and these are the present-day satellites of the Milky Way and Andromeda.

7. DISCUSSION AND CONCLUSION

In this paper we present, for the first time, a unified view of the physical process responsible for the braking of a satellite in a self-gravitating spherical stellar system. We have shown that TLR embraces all aspects of the gravitational interaction of a massive object with a background of lighter (dark matter) self-gravitating particles.

We have found that the characteristic dynamical friction time for satellites with mass ratios less than 0.1 (moving on cosmologically relevant orbits) in a galaxy such as our Milky Way is sufficiently long that satellites have not yet merged with the stellar disk. As first noted by Ghigna et al. (1998), orbital decay is not followed by a significant circularization when the satellite happens to orbit well inside the primary halo. The expected equilibrium distribution of eccentricities in a spherical potential is skewed toward high \( e_{\text{orb}} \approx 0.6–0.7 \) (see vBLLS), and dynamical friction plays no role in modifying such a distribution. We have in addition shown that rigid satellites on eccentric orbits have sinking times only slightly shorter than those of satellites on circular orbits; previous analyses instead predicted a wider spread, implying a much shorter lifetime in the primary halo. Tidal stripping is more efficient than dynamical friction for the typical masses of galaxy satellites. We have shown that this should considerably prolong the lifetime of the baryonic lumps inhabiting satellite halos with masses up to 0.1 that of the primary halo, so that they will wander along their orbit for at least a Hubble time. This has important implications for many issues concerning the evolution of galaxies. We indeed expect that those satellites that entered the primary halo after \( z \sim 1 \) cannot have yet decayed to the center, while their dark matter halo has already been substantially stripped. These correspond to the present-day satellites of spiral galaxies, such as those populating our Local Group.

Our results show that the disk is unlikely to have suffered any late merging event or penetrating encounter with a typical satellite. The overall picture that emerges is that substructure survives longer than previously believed, which nicely agrees with numerical findings in large cosmological simulations (Ghigna et al. 1998; Tormen et al. 1998; Klypin et al. 1999). In general, the pericenters of the orbits of satellites are reduced by no more than a factor of 2 in about 7 Gyr, which is approximately the time that passes between \( z = 1 \) and \( z = 0 \). This large population of almost indestructible satellites could have a dramatic effect on the dynamics of spiral galaxies disks. Work by Moore et al. (1999a) suggests that the cumulative effect of many nearby encounters between this numerous population of small satellites on almost radial orbits and a galactic disk would
heat its stellar component considerably on a timescale of a few Gyr, even without merging with it.

Dynamical friction and tidal stripping are among the main dynamical mechanisms involved during the formation of cosmic structures. Our results provide a detailed description of these processes, as well as giving theoretical support for understanding much of the underlying physics. Numerical simulations and theoretical models are now converging toward a common picture in which CDM models create a wealth of long-lived substructure in dark matter halos. It now remains to investigate its effects and observational evidence, so to provide tight constraints on theories of galaxy formation and evolution.

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APPENDIX

In this Appendix we briefly describe the semianalytical calculation for the drag force, $F_d(t)$, acting on the satellite at time $t$ (eq. [6]). The force is computed under the hypothesis expressed in equation (8), considering a Gaussian distribution function in the velocity space for which $V_v f^{op} = -(\sigma / \sigma^2) f^{op}$. Because of the complexity of the various expressions, we introduce the vector

$$X \equiv R(t) - r,$$

(A1)

and define

$$V_{R(s)} \phi \equiv V_{R(s)} \phi(|R(s) - r|) - \int d_3 r' n_0(r') V_{R(s)} \phi(|R(s) - r'|).$$

(A2)

The integral on the velocity space can be written in a simplified form, so the force reads

$$F^b_d(t) = -(GM)^2 N m \frac{1}{2\pi} \int_0^t ds \left[ \int d_3 r n_0(r) \frac{\partial \phi}{\partial R^a(s)} \frac{\partial^2}{\partial R^a(t)} \frac{1}{|R(t) - r|} \right]$$

$$\times \int_0^\infty dy y \phi(y) \left\{ \exp \left[ -\frac{1}{2} \Gamma(X - y)^2 \right] - \exp \left[ -\frac{1}{2} \Gamma(X + y)^2 \right] \right\},$$

(A5)

Equation (A5) involves integrals over the physical volume of the halo, over time $s$, and over $y$. The integral in the $y$ variable can be computed analytically, given the expression of the potential $\phi(y)$. According to our analysis in § 3, the function $\phi$ is just the spline kernel potential, $\phi_{sp}$, introduced by Hernquist & Katz (1989; see their Appendix), which reduces to the Newtonian potential ($\phi = -1/y$) at distances larger than $2c$; this potential is written as an expansion in powers of $y$. In equation (A5) we thus need to calculate integrals of the form

$$\int_\text{inf}^{\text{sup}} dy y^n \left\{ \exp \left[ -\frac{1}{2} \Gamma(X - y)^2 \right] - \exp \left[ -\frac{1}{2} \Gamma(X + y)^2 \right] \right\},$$

(A6)

where $n \geq 0$; the domain of integration (inf, sup) is uniquely defined by the form of $\phi_{sp}$. If we introduce the functions

$$B_d(a, \Gamma) \equiv B_d(a, X, \Gamma) - B_d(0, -X, \Gamma),$$

(A7)

where

$$B_d(a, X, \Gamma) \equiv \int_0^a dy y^n \exp \left[ -\frac{1}{2} \Gamma(X + y)^2 \right]$$

(A8)

and

$$B_d(a, -X, \Gamma) \equiv \int_0^a dy y^n \exp \left[ -\frac{1}{2} \Gamma(y - X)^2 \right],$$

(A9)

we can express $B_d(a, \Gamma)$ as a linear combination of error functions, and the following recurrence relations apply:

$$B_d(a, X, \Gamma) = \left( \frac{\pi}{2\Gamma} \right)^{1/2} \left\{ \text{erf} \left[ \sqrt{\Gamma} (X + a) \right] - \text{erf} \left( \sqrt{\Gamma} X \right) \right\},$$

(A10)
\[ B_1(a, X, \Gamma) = -X B_0(a, X, \Gamma) - \frac{1}{\Gamma} \left\{ \exp \left[ -\frac{\Gamma}{2} (X + a)^2 \right] - \exp \left( -\frac{\Gamma}{2} X^2 \right) \right\}, \quad (A11) \]

\[ B_{n+1}(a, X, \Gamma) = -X B_n(a, X, \Gamma) + \frac{n}{\Gamma} B_{n-1}(a, X, \Gamma) - \frac{a^n}{\Gamma} \exp \left[ -\frac{\Gamma}{2} (X + a)^2 \right]. \quad (A12) \]

The integral on the \( y \) variable in equation (A5) is constructed using equations (A6)–(A12).

Given the above relations, we can also calculate the first and second derivatives of \( B, B' \) and \( B'' \), relative to \( X \); after a number of simple but long steps, we can express the drag force as

\[
F^B = -(GM)^2 N \left( \frac{1}{2\pi} \right)^{1/2} \int d_3 r \frac{n_0(r)}{\sigma} \times \int ds \nabla_n \phi \left\{ \frac{3X^2a^b}{X^5} - \frac{\delta^b}{X^3} \right\} \sum_n c_n \left[ B_n([d], \Gamma) - X B_n([d], \Gamma) \right] + \frac{X^2a^b}{X^5} \sum_n c_n B'_n([d], \Gamma), \quad (A13) \]

where \( B_n([d], \Gamma) \) briefly denotes that the functions of equations (A6)–(A9) are computed over the whole domain \((0, \infty)\), which is divided into three parts: \((0, \epsilon), (\epsilon, 2\epsilon), \) and \((2\epsilon, +\infty)\) (see Hernquist & Katz 1989). The coefficient \( c_n \) in equation (A13) contains the \( n \)th power of the softening length \( \epsilon \), as shown in the expression of \( \phi_{sp} \). The time and spatial integrals are computed numerically, using standard procedures, given the density and velocity dispersion profiles (eqs. [9] and [12]).

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