Loss revives bistable state near the exceptional point in a non-Hermitian microwave photonic meta-molecule

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Abstract
By exploring the extraordinary property of exceptional points (EPs) in non-Hermitian systems, we here demonstrate that losses can play a constructive role in controlling bistable states. We experimentally realize the EP in a non-Hermitian meta-molecule of coupled resonators in a microwave regime. By increasing the loss, we first observe the bistable state suppression at the weak-dissipative regime, and then the bistable state recovery in the strong-dissipative regime. Both the experimental and theoretical analysis demonstrate that the revival of bistable states results from the revival of the field intensity after the system encounters EPs, in spite of the increasing loss. Our results provide an alternative way to controlling and manifesting bistable systems so as to achieve flexible photonic devices not limited to the microwave regime.

1. Introduction

Naturally, dissipation is always eliminated or neglected in normal physical systems due to its disadvantages, such as wasting energy, reducing field intensity and increasing threshold. In recent years, however, researchers have discovered nontrivial physics in non-Hermitian dissipative systems described by complex eigenvalues and nonorthogonal eigenstates that depart from the conventional Hermitian model [1]. When steering the parameters of such a dissipative system, it encounters a different type of singularity where the eigenvalues and the corresponding eigenstates may coalesce by a square root branch point, which has been dubbed an exceptional point (EP) by Kato [2–4]. Also, an EP may emerge in purely dielectric crystals [5], even in topological photonic systems such as lattices of gyrotropic materials [6] or lattices of resonators coupled with metamaterial elements [7]. Spectral coalesces crucially determine unusual transport properties [8] and the topological structure of eigenmodes [9,10] in the vicinity of EPs such as counter-intuitive features of lasing [11–17], chiral modes [18] and repulsion of energy levels [19]. In particular, the physics of EPs have even been associated with parity-time ($PT$) symmetry [20], where losses and gains are balanced [11,21–23] with effects such as coherent perfect absorption [24,25], loss-induced transparency [8] and unidirectional invisibility [26]. Additionally, non-Hermitian stochastic dynamics have so far been studied in the context of microwave [4,9,27,10,18,28,29], optical [10,11,24], atomic [30–32] and electron waves [10,33,34].

Very recently, a two coupled waveguides [12,21–23] and resonators model [11,24,35] has attracted enormous attention as it can demonstrate properties of optical systems visually. Other recent studies in the lasing regime have also provided examples, like a pump-induced lasing death [36], spontaneous $PT$ symmetry breaking [22] and enhancement of the laser linewidth [37]. Notably, the photonic ‘meta-molecule’ in the microwave regime constructed by metamaterial units (meta-atom) [24,38,39] triggers a unique view in investigating the internal physics of such systems.
In this paper, we first show how to simulate an EP in a non-Hermitian microwave photonic meta-molecule made of coupled resonators. Then, by increasing the dissipative loss to the resonator solely, the system undergoes a non-ideal \( PT \) phase transition, which is shown clearly from the experimental transmission. With the introduction of the nonlinearity on one resonator, we observe the bistable state of this system, manifesting as the transmission hysteresis when scanning in power and frequency respectively. Further, the evolution of the measured intensity reveals that the revival of the bistable state results from the revival of the field strength in this system, which also matches the theoretical and simulated results. The investigations enrich the manifestation of EP physics, and help to realize such anomalous phenomena and offer a supporting platform for researching bistable state systems.

2. Theoretical and experimental results

Our system is based on a microwave photonic molecule, as shown in figure 1(a). This molecule model is constructed by a pair of coupled atoms, a 'bright' atom (the lower green sphere) and a 'dark' atom (the upper orange sphere). The 'bright' atom with resonant frequency \( \omega_1 \) can be easily excited by an incoming wave \( S_0 e^{-i\omega t} \), while the 'dark' atom with resonant frequency \( \omega_2 \) can only be excited by the near-field from the bright atom through a coupling parameter \( \kappa \). When considering the radiative loss \( \gamma_1, \gamma_2 \) and dissipative loss \( \Gamma_1, \Gamma_2 \) of the two atoms, the meta-molecule can be regarded as a non-Hermitian system. To mimic the above-mentioned microwave photonic molecule exactly, we designed the experimental system with two coupled ring resonators in figure 1(b). The lower resonator \( a_1 \) (green) connected to the transmission line \( TL_1 \) denotes the 'bright' meta-atom that is excited directly by the incoming wave from Port1, while the upper resonator \( a_2 \) (orange) connected to \( TL_2 \) denotes the 'dark' meta-atom that can be excited by the 'bright' meta-atom. The lumped elements loaded on two meta-atoms can tune experimental parameters; for instance, the tunable resistors can change the dissipative loss of meta-atoms and the tunable capacitors can modulate the resonant frequency of each meta-atom. The element in the red circle on the 'bright' meta-atom depicts a varactor, which can induce the bistable state in the second experiment, but its equivalent capacitance is almost fixed throughout this work (see section 3 for more details). As a consequence, we can use the following linear-coupled mode theory to describe that system:

\[
\frac{dn_1}{dr} = (-i\omega_1 - \gamma_1 - \Gamma_1) n_1 + i\kappa n_2 + \sqrt{\gamma_1} S_0 e^{-i\omega t},
\]

\[
\frac{dn_2}{dr} = (-i\omega_2 - \gamma_2 - \Gamma_2) n_2 + i\kappa n_1.
\]

To simplify the theoretical model, the resonant frequency of the two meta-atoms was set to \( \omega_1 = \omega_2 = \omega_0 \) via the tunable capacitor. In this basis, the effective Hamiltonian of the system can be written in a matrix format, as

![Figure 1. (a) Model of meta-molecule constructed by 'bright' and 'dark' meta-atoms. (b) Experimental sample constructed by coupled resonators \( a_1 \) (‘bright’ meta-atom) and \( a_2 \) (‘dark’ meta-atom) with the transmission line couplers \( TL_1 \) and \( TL_2 \).](image-url)
of which the characteristic equation can be calculated from $|\omega I - M| = 0$. Consequently, the eigenfrequencies of the modes formed by these coupled meta-atoms are obtained as

$$\omega_{\pm} = \omega_0 - iX \pm \sqrt{X^2 - Y^2},$$

where $X = (\gamma_1 + \Gamma_1 + \Gamma_2 + \Gamma_2)/2$, $Y = (\gamma_1 + \Gamma_1 - \gamma_2 - \Gamma_2)/2$. The evolution of the eigenfrequencies is calculated in figure 2(a). The first panel depicts the real parts of the eigenfrequencies, which indicates that the two modes coalesce at an EP with increasing dissipative loss $\Gamma_2$ in the ‘dark’ meta-atom. After undergoing the EP, their imaginary parts are repelled, shown in the second panel, which results in an increasing imaginary part for one of the eigenfrequencies and a decreasing imaginary part for the other. As a consequence, one of the modes becomes less lossy, while the other becomes more lossy.

To uncover the clear physics of field intensity revival, we substitute $a_k = A_k e^{-i\omega t}$ and $\frac{da_k}{dt} = -i\omega A_k e^{-i\omega t} + \frac{dA_k}{dt} e^{-i\omega t} (k = 1, 2)$ in equations (1), (2), and we obtain

$$\frac{dA_1}{dt} = (i\Delta_1 - \gamma_1 - \Gamma_1)A_1 + i\kappa A_2 + \sqrt{\gamma_1} S_{in},$$

$$\frac{dA_2}{dt} = (i\Delta_2 - \gamma_2 - \Gamma_2)A_2 + i\kappa A_1,$$

where $\Delta_1 = \omega - \omega_1$ and $\Delta_2 = \omega - \omega_2$ are the detuning between the resonant frequency and the frequency of the incoming wave $S_{in}$. Further, we can solve equations (5), (6) at steady state and find the intra-cavity fields $A_1$ and $A_2$

$$A_1 = \frac{-\sqrt{\gamma_1}(\gamma_2 + \Gamma_1 - i\Delta_0)}{\kappa^2 + (\gamma_1 + \Gamma_1 - i\Delta_0)(\gamma_2 + \Gamma_1 - i\Delta_0)} S_{in},$$

$$A_2 = \frac{i\kappa \sqrt{\gamma_1}}{\kappa^2 + (\gamma_1 + \Gamma_1 - i\Delta_0)(\gamma_2 + \Gamma_1 - i\Delta_0)} S_{in}.$$

Here, $\Delta_0 = \Delta_{1,2} = \omega - \omega_0$ because the resonant frequencies of the two meta-atoms are set as identical in our scheme. Based on equations (4), (7), and (8) we can calculate the field intensity $I_{1,2} = |A_{1,2}|^2$ at eigenfrequencies with the parameters fitted from our system. Figure 2(b) shows the evolution of field intensities (normalized by the total field intensity at $\Gamma_2 = 0$) at modes $\omega_{\pm}$. At first, the field intensities in both meta-atoms are nearly equal, and they both decrease with the enlarging dissipative loss in the ‘dark’ meta-atom. When the system is in the vicinity of the EP, the field intensity in the ‘bright’ meta-atom recovers, which however does not occur in the ‘dark’ one. After passing the EP, the field intensity revives completely, even with the enormous loss in the...
microwave photonic meta-molecule. Next, we are going to demonstrate such counterintuitive features in our experiments.

In the first set of experiments, we investigate the evolution of eigenfrequencies and transmission $T_{1\rightarrow 2}$ by continuously increasing $G_2$ (experimentally, increasing the resistor $R_2$). The transmittance $T_{1\rightarrow 2}$ of the system from the input Port 1 to output Port 2 can be analytically obtained from equation (7) as

$$T_{1\rightarrow 2} = \left| \frac{S_{in} - \sqrt{\gamma_1 A_1}}{S_{in}} \right|^2 = \left| 1 + \frac{\gamma_1 (\gamma_2 + \Gamma_2 - i\Delta'_2)}{\kappa^2 + (\gamma_1 + \Gamma_1 - i\Delta_0) (\gamma_2 + \Gamma_2 - i\Delta'_2)} \right|^2. \quad (9)$$

As a consequence, we depicted both the theoretical and experimental results (shown in figure 3(a)) and found that they matched very well. The coupling between the two meta-atoms as well their losses leads to the formation of two eigenmodes with complex eigenfrequencies. At the beginning, both resistors are set as $R_1 = R_2 = 0 \ \Omega$ such that $\Gamma_1 \approx \Gamma_2 \approx 0 \ \text{GHz}$; namely, the system is in the weak-dissipation regime (i) $|Y| < \kappa$. In this regime, the eigenmodes have different resonant frequencies (mode splitting), which are determined by the value of the real part of the square root in equation (4). As we increase the dissipative loss $\Gamma_2$ in the ‘dark’ meta-atom, the real part of the square root declines rapidly until it reaches the special EP, where (ii) $|Y| = \kappa$ and the eigenmodes coalesce therein, where $\Gamma_2 = 0.14 \ \text{GHz}$. With a further increase of $\Gamma_2$, the system enters the strong-dissipation regime, quantified by (iii) $|Y| > \kappa$ and the square root values as a pair of pure conjugated imaginary numbers, which leads to the conjugate eigenmodes with resonant frequencies of the same real value. The corresponding simulated field intensity distribution of the three regimes is exemplified in figure 3(b), which clearly demonstrates the revival of the field intensity on the ‘bright’ meta-atom $a_1$. We set the varactor in the red dash

![Figure 3. (a) Measured (solid line) and calculated (dash line) transmission $T_{1\rightarrow 2}$ (from Port 1 to Port 2) in three regimes with increasing dissipative loss $\Gamma_2$. (i) weak-dissipation regime (blue), (ii) middle-dissipation regime (yellow) and (iii) strong-dissipation regime (red) are depicted distinctly in the bottom panel. Solid lines denote the calculated real parts of the eigenfrequencies and dots denote the experimentally measured resonant frequencies. (b) The corresponding simulated field intensity in ‘bright’ ($a_1$) and ‘dark’ ($a_2$) meta-atoms. The red dash circle on $a_1$ indicates the place where the varactor (or the capacitor) is loaded. The field intensity revival in the ‘bright’ meta-atom $a_1$ is clear.](image-url)
circle in the following experiment to yield the bistable state. Remarkably, the revival of the field intensity in the ‘bright’ meta-atom, especially in the place where the varactor is mounted, can be observed directly.

To further demonstrate the effect of revival on the bistable state, we design the second set of experiments, where the capacitor in the previous experiment is replaced by a nonlinear element, varactor, with equivalent capacitance. In this configuration, a bistable state can be stimulated with a strong field intensity above the threshold. This bistable behavior merely results from the nonlinearity of the varactor. Figure 4(a) shows the effect of loss on the frequency sweeping curves. The forward (blue line) and backward (green line) sweeping curves separate from each other (in the top panel), but then coalesce when the dissipative loss in the ‘dark’ meta-atom approaches a critical value where the system is near the EP (in the middle panel). After further increasing loss to achieve a high level dissipation, the system passes through the EP, and we can then observe the curves separate distinctly again (in the bottom panel). Note, the data are measured at mode $w_-$ while at this mode one of the magnetic coupled meta-atoms has a nonlinear varactor. The bistable state does not result from the mode switching between $w_+$ and $w_-$, but is a consequence of the nonlinear response of the varactor. Actually, we also find a similar result at the power sweeping curves (not shown), which indicates that the bistable state indeed changes from attenuation to revival merely by increasing the loss.

The bistable state recovery in the frequency sweeping curves is due to the loss-assisted field intensity revival in the vicinity of an EP. To support this result, we measured the field intensity in the ‘bright’ meta-atom and the power threshold of the bistable state, thanks to the varactor. As shown in figure 4(b), the larger the intensity, the lower the threshold. When initially increasing loss, the normalized intensity declines at first which raises the threshold. With a further increase in loss, so that the system passes through the EP, the increasing field intensity decreases the threshold. When we draw a dash line at our input power $-2$ dBm, the value of this dash line first becomes larger than the power threshold line, which means the bistable state is present. Then with increasing dissipative loss $\Gamma_2$, the threshold becomes larger than the input power and we found that the bistable state vanished in this regime; while enlarging the loss, the threshold reduces again and the bistable state revives. Moreover, we found that the final threshold can be even smaller than the initial value (in the absence of loss) when the loss is large enough. The experimental results are well consistent with the theoretical results in equation (7) and figure 2(b), which describe a loss-assisted bistable state (field intensity) revival in a non-Hermitian microwave photonic molecule with an EP. The opposite evolution trends between the normalized intensity and bistable threshold tell us that the bistable loop is indeed induced by the nonlinear varactor, which is different from the mechanism of parameter loops in other topological works with linear elements [40, 41].

Figure 4. (a) Measured $T_{\rightarrow 2}$ forward and backward frequency sweeping results with increasing dissipative loss $\Gamma_2$ and fixed power on $-2$ dBm. Note, we measured the mode $w_-$ because the bistable state induced by a varactor usually lowers the resonant frequency. (b) Loss-induced evolution of the power threshold (blue circle) and normalized field intensity (red triangle) in the meta-molecule. The blue dash line, with the same input power $-2$ dBm, indicates a non-monotone compared with the power threshold.
3 Methods

The samples were all fabricated on copper-clad 0.787-mm thick Rogers RT5880 substrates using laser direct structuring technology (LPKF ProtoLaser 200). We used equivalent capacitors $C_0 = 2.65 \, \text{pf}$ in both the ‘bright’ and ‘dark’ meta-atoms. To stimulate the bistable state, we put a varactor on $a_1$ whose capacitance $C_0 = 2.65 \, \text{pf}$, but in the first set of experiments, an equivalent capacitor was used instead of a varactor. Two tunable resistors were loaded on resonators whose values could be changed from 0–100 Ω. An additional tunable capacitor was set on $a_2$ to fix the resonant frequency error induced by manufacturing the sample. The distributive capacitance and inductance could be designed with different geometrical structure parameters. The two identical resonators were given as $a = 10 \, \text{mm}, d = 8 \, \text{mm}, w = 0.2 \, \text{mm}, s = 0.2 \, \text{mm}, g_2 = 0.8 \, \text{mm}, g_3 = 5 \, \text{mm}$. The four ports were fixed to transmission lines by SMA linkers whose impedance was equal to 50 Ω. Transmission spectra $T_{1 \rightarrow 2}$ and $T_{1 \rightarrow 4}$ (the same as $T_{3 \rightarrow 1}$) were obtained directly from a microwave vector network analyzer (Agilent N5222A). In addition, a commercial software package (CST Microwave Studio) was used in designing the samples.

4. Conclusions

All of these observations contrast with what one would expect in conventional systems, where a higher loss usually causes a higher bistable threshold and thus is detrimental to the bistable behavior. Now, surprisingly, in the vicinity of an EP in our system, considerable loss will reversely reduce the bistable threshold, playing a constructive role in recovering the bistable behavior. Less loss is detrimental to the optimization of a bistable state system with low threshold. As is well-known, a varactor can exhibit bistable behavior once the strength of the field intensity exceeds some critical value. Thus, the field intensity on the varactor of the resonator plays a key role. When the loss exceeds a critical value, the field strength of one eigenmode mostly locates in one of the subsystems with less loss (‘bright’ meta-atom in our present set-up) so as to avoid the dissipation. Thus, the total field can build up more strongly, which then aids the recovery of the bistable behavior. The observations may also provide an alternative way to control and manifest the effect of loss in bistable state systems or other physical systems reliant on an EP. We believe the results could pave the way for developing devices in switching, memory, sensors, as well as other logic operations, for all-optical communication and computing.

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