Analysis of the coherent and turbulent stresses of a numerically simulated rough wall pipe

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Abstract. A turbulent rough wall flow in a pipe is simulated using direct numerical simulation (DNS) where the roughness elements consist of explicitly gridded three-dimensional sinusoids. Two groups of simulations were conducted where the roughness semi-amplitude \( h^+ \) and the roughness wavelength \( \lambda^+ \) are systematically varied. The triple decomposition is applied to the velocity to separate the coherent and turbulent components. The coherent or dispersive component arises due to the roughness and depends on the topological features of the surface. The turbulent stress on the other hand, scales with the friction Reynolds number. For the case with the largest roughness wavelength, large secondary flows are observed which are similar to that of duct flows. The occurrence of these large secondary flows is due to the spanwise heterogeneity of the roughness which has a spacing approximately equal to the boundary layer thickness \( \delta \).

1. Introduction
Typically, a turbulent quantity can be decomposed into its mean and fluctuating component

\[ u_i = \bar{u}_i + u''_i \] (1)

where \( \bar{u}_i \) is the time-averaged mean and \( u''_i \) is the fluctuation about the time-averaged mean (also known as the turbulent fluctuation) for component \( i \). This decomposition is known as the Reynolds decomposition and is commonly used to analyse the turbulent fluctuations in a flow. However, in a rough wall flow, variations due to the unevenness of the spatial geometry must be taken into account when analysing the turbulent flow data. Therefore, a triple decomposition is applied to the turbulent quantity where it is decomposed into three components \([1, 2]\),

\[ u_i = U_i + \tilde{u}_i + u''_i. \] (2)

Here, \( U_i = \langle \bar{u}_i \rangle \) is the spatial and temporal averaged mean, which is also known as the global mean and \( \tilde{u}_i = \bar{u}_i - U_i \) is the spatial variation of the time-averaged flow around individual roughness elements (also known as the coherent or dispersive component). The sum of \( \tilde{u}_i + u''_i = u'_i \) is the total fluctuation and contains both the turbulent and coherent fluctuations. For a smooth wall, \( \tilde{u}_i = 0 \) and therefore \( U_i = \bar{u}_i \). The triple decomposition is a result from double averaging, where the governing equations are averaged over time and then
in space. This technique has been used to analyse plant canopy flows (urban-canopy model) [3, 4, 5] and also in open-channel flow over rough beds [6, 7, 8] where the rough surfaces are spatially inhomogeneous.

When conducting laboratory experiments, point measurement techniques such as hot wire anemometry and laser Doppler velocimetry gather data locally in space and only measures the turbulent fluctuations of the flow. This can result in misinterpretation of the data especially when conducting rough wall experiments where large spatial variation of velocity might exist in the flow. In this paper, we will investigate the importance of the coherent and turbulent fluctuations in a turbulent rough wall flow.

2. Computational setup

DNS is conducted in a pipe using CDP which is a second-order, energy conserving finite volume code. The rough wall pipe has a Cartesian ‘O-grid’ mesh and a body conforming grid is used to explicitly represent the roughness elements. The pipe has a length of \( L_x = 4\pi R_0 \) where \( R_0 \) is the reference radius. Further details of the numerical method used can be found in [9].

Throughout the paper, the roughness cases are identified by the following identifying code

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c}
\hline
\text{Case Symbol} & \text{Symbol} & N_{r,\theta} & N_x & N_{\lambda_x} & \Delta r^+ & \Delta x^+ & k_a^+ & ES & Re_t & Re_D & \Delta U^+ \\
\hline
\text{Smooth} & \odot & 94752 & 1152 & - & 0.23 & 5.8 & 0.0 & 538 & 18729 & 0.0 \\
20_141 & \bullet & 104400 & 1152 & 24 & 0.14 & 4.4 & 8.1 & 541 & 12114 & 6.4 \\
40_283 & \bullet & 104400 & 1152 & 48 & 0.13 & 4.1 & 16.2 & 534 & 9027 & 9.1 \\
60_424 & \bullet & 108720 & 1152 & 72 & 0.15 & 4.0 & 24.3 & 543 & 7870 & 10.5 \\
\hline
\end{array}
\]

**Table 1.** Details of the computational mesh and mean flow properties of the cases simulated. \( N_{r,\theta} \) is the number of elements in an \((r, \theta)\) plane, \( N_x \) the number of elements in the streamwise direction and \( N_{\lambda_x} \) the number of elements per roughness wavelength. \( \Delta r^+ \) and \( \Delta x^+ \) are the radial and streamwise mean viscous grid spacings at the wall calculated using the local friction velocity \( U_r \). The largest cells are located at the centre of the pipe where \( \Delta r^+ \approx \Delta r \theta^+ \approx \Delta x^+ \). \( k_a^+ \) is the average roughness height and \( ES \) is the effective slope of the roughness (see [9]). \( Re_t \) and \( Re_D \) are the friction and bulk Reynolds numbers and \( \Delta U^+ \) is the Hama roughness function.

**S1: Geometrically increasing roughness (fixed \( h/\lambda \))**

**S2: Decreasing roughness wavelength \( \lambda^+ \) (fixed \( h/\lambda \))**
3. Mean velocity profile

The mean velocity profile of the rough cases are plotted in figure 1. The downward shift in the mean velocity profile of the rough wall is quantified by the roughness function \( \Delta U^+ \) which is measured 200 viscous wall units above the crest of the roughness elements. The roughness function increases with increasing roughness height and also with decreasing roughness wavelength. However decreasing the roughness wavelength from \( \lambda^+ = 424 \) to 212 only results in a marginal increase in the \( \Delta U^+ \) as the surface is in the dense regime (\( ES = 2\Lambda = 0.72 \)). From [9], it is found that case 60,424 is in the fully rough regime. Therefore, it is deduced that case 60,212 which has a higher \( ES \) (or solidity) is also in the fully rough regime.

![Figure 1](image)

Figure 1. Streamwise velocity profile for (a) geometrically scaled roughness cases (group \( S_1 \)) and (b) roughness cases with fixed \( h^+ \) and varying wavelength \( \lambda^+ \) (group \( S_2 \)). Dash-dotted lines show \( U^+ = y^+ \) and \( U^+ = (1/\kappa) \log (y^+) + C \), \( \kappa = 0.40 \) and \( C = 5.3 \). Symbols are as in table 1.

4. Stress profiles

The various fluctuating terms will be compared in this section using the triple decomposition method introduced in § 1. The stress profiles for the azimuthal turbulent \( u_{θ,rms}^+ \), total \( u_{θ,rms}^{\prime} \) and coherent \( \tilde{u}_{θ,rms}^{\prime} \) fluctuations are plotted in figure 2.

In figures 2 (a, d), the occurrence of a secondary peak is observed within the roughness canopy \( (y \leq h) \). This peak increases with increasing \( h^+ \) (group \( S_1 \)) and also with increasing roughness wavelength \( \lambda^+ \) (group \( S_2 \)). The secondary peak is contributed by the coherent (or dispersive) fluctuating component of the flow which is time independent (see figures 2 (c, f)).

The peak of the azimuthal coherent stress profile is located at a wall-normal location which ranges from \( y^+ \approx 10 - 20 \) and has a value which is comparable to the azimuthal turbulent stress. The peak in the azimuthal coherent stress profile is due to the channelling of the flow around the three-dimensional sinusoidal roughness elements. However, the coherent stress profiles quickly reduces to zero above the crest of the roughness elements (expect for case 60,848). This indicates that the effects of the azimuthal coherent stresses to the flow are small. However, for case of
60.848, the azimuthal coherent stress profile extends more than 200 viscous wall units above the crest of the roughness elements. This is because there are large stationary features occurring in the flow and this will be investigated further in § 5.

\[
u' + \theta, \text{rms} \quad u'' + \theta, \text{rms} \quad \tilde{u} + \theta, \text{rms}
\]

\[
y + h
\]

**Figure 2.** Azimuthal velocity \((a, d)\) total (solid lines), \((b, e)\) turbulent (dash lines) and \((c, f)\) dispersive (dash-dotted lines) stress for group \(S1\) (left) and \(S2\) (right). Dashed vertical line shows the wall-normal location of the crest of the largest roughness in the group \((y^+ = h^+)\).

The stress profiles for the total (figures 2\((a, d)\)) and turbulent (figures 2\((b, e)\)) fluctuation in the azimuthal direction for the rough cases collapses with the smooth wall in the outer region of the flow. Good collapse in the outer region of the flow is also obtained for the stress profiles of the total and turbulent fluctuations in the streamwise and radial direction (not shown here). Good collapse is observed as the total fluctuation and turbulent stresses scale with the friction Reynolds numbers. Therefore, our findings support Townsend’s outer-layer similarity hypothesis when considering the second-order statistics. However, when analysing the contours of the streamwise premultiplied energy spectra, it is observed that the collapse occurs at a much larger wall-normal location as energy is redistributed to the wavelength corresponding to the roughness elements and its harmonics [10].

In general, the maximum coherent stresses in all three dimensions (streamwise and radial components, not shown here) increases with increasing geometrical scale \((S1)\) and increasing wavelength \((S2)\). In addition, with increasing \(h^+\) and \(\lambda^+\), the influence of the coherent
component is felt further away from the roughness and for case 60,848, extends even into the edge of the outer layer. As for the turbulent stresses, the profiles for the rough cases above the roughness elements are not too dissimilar with the smooth case profiles as the turbulent stresses scales predominantly with the friction Reynolds number.

5. Streamwise vorticity
The azimuthal coherent fluctuation stress profile for case 60,848 extends far above the crest of the roughness elements due to large secondary flows. Isosurfaces of the time-averaged streamwise vorticity $\omega$ of the unwrapped surface for case 60,848 are plotted in figures 3(a, b). There is strong streamwise vorticity near the surface of the roughness which alternates between positive and negative values in the streamwise direction due to the topological changes of the three-dimensional sinusoidal roughness (figure 3(a)). Plotting isosurfaces of the time-averaged streamwise vorticity where $|\omega| = 0.01$ in figure 3(b), we observe large secondary flows which extend up to 200 viscous units from the wall. Figures 3(c, d) shows the contours of the time-averaged streamwise vorticity overlay with the time-averaged azimuthal and radial velocity vectors along cross sectional slice I where the surface is locally rough and II where the surface is locally smooth. The zoom in view of these contours clearly shows the occurrence of these secondary flows which are similar to that observed in duct flows [11]. The sketch in figure 3(e) illustrates the orientation of the secondary flow which alternates in the spanwise direction. Fluid is pushed upwards from the wall along the locally rough streamwise plane and is pushed downwards in the locally smooth streamwise plane. Secondary flows are also observed in the other rough cases (not shown here) but is confined to regions near the crest of the roughness. Large secondary flows occur in case 60,848 due to the spanwise heterogeneity of the roughness elements which are spaced approximately equal to the boundary layer thickness $\delta^+ = R_0^+ = 540 \approx L_s^+ = 424$ (for a pipe, the $\delta$ is fixed and is equal to the reference radius of the pipe $R_0$). This is consistent with the findings of [12] who found that $\delta$-spaced roughness elements induces secondary flows on the scale of $\delta$.

6. Conclusion
DNS in a rough wall pipe with explicitly gridded three-dimensional sinusoidal roughnesses is simulated at moderate Reynolds numbers where the height $h^+$ and the wavelength $\lambda$ of the roughness elements are systematically altered. Increasing the roughness height and/or roughness wavelength results in an increase in the coherent fluctuation stresses. For case 60,848, the coherent fluctuation stress profile extends much further above the crest of the roughness elements compared to the other rough cases due to the occurrence of large time invariant secondary flows. These large secondary flows develop in the pipe due to the spanwise heterogeneity of the roughness which has a spanwise spacing approximately equal to the boundary layer thickness.

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Figure 3. Isosurfaces of the time-averaged streamwise vorticity of the unwrapped rough wall pipe for case 60.848 are shown in (a) and (b) where $|\omega^+| = 0.1$ and $|\omega^+| = 0.01$ respectively. Contours of the time-averaged streamwise vorticity $\omega^+$ for case 60.848 at cross-sectional (a) plane I (locally rough), (b) plane II (locally smooth). Overlaid on top of the contour is the time-averaged azimuthal and radial velocity vectors. Black solid box shows the zoom in view. Flow is into the page. The cross-sectional sketch of the unwrapped pipe in (c) illustrates the secondary flow occurring between the roughness elements where the solid vertical and dash-dotted lines are streamwise planes which are locally rough and smooth respectively. Grey dashed line represents the roughness element located downstream.