Resonant pumping of polaritonic SSH chains

Yury Krivosenko, Ivan Iorsh, and Ivan Shelykh

E-mail: y.krivosenko@gmail.com
ITMO University, St. Petersburg 197101, Russia

Abstract. An optical-lattice realization of a Su-Schrieffer-Heeger chain is theoretically investigated. The topological state is controlled by the relative phase of the lasers constructing the lattice. The hopping amplitudes are calculated within the harmonic approximation to the optical potential in the vicinity of its minima. Finally, we examine polaritons loaded into adiabatically shaken optical lattices with damping and pumping and demonstrate that, in the case of resonant pumping, the system manifests different quantitative and qualitative behaviour for topologically trivial and non-trivial phases.

1. Introduction

A topological insulator (TI) is a state of matter that behaves as a typical insulator in the bulk but can host midgap states localized at its boundary [1] (the so-called edge states). The minimalistic model demonstrating topological properties was initially suggested by Su, Schrieffer, and Heeger (SSH) [2]. The system can be presented as a dimerized linear chain with each unit cell consisting of two sites with staggered intra- \(t_1\) and inter-cell \(t_2\) hopping amplitudes.

Due to the simplicity and rich properties, the SSH model and its different generalizations have been extensively studied during the recent years (in the field of electronics [3,4] and waveguide photonics [5,6]). Periodically modulated (Floquet-engineered [7,8]) SSH chains have been investigated as well with respect to topological properties and phase transitions [9,11]. Floquet engineering is known to be achievable not only in the field of charged particles, but in the field of cold atoms physics as well. There, a possible realization of Floquet engineering is utilization of cyclically modulated (shaken) optical lattices (OL) [12,14].

In this research, we theoretically investigate the resonant pumping of the SSH chain modelled as an adiabatically shaken OL loaded with exciton-polaritons in microcavities. The topology is adjusted by tuning the relative phase of the lasers constructing the optical potential.

2. Theory

Behaviour of the SSH chain is described by the non-stationary Schrödinger equation

\[
H_{SSH} |\psi(t)\rangle = i \frac{\partial}{\partial t} |\psi(t)\rangle,
\]

where the tight-binding SSH Hamiltonian in real space is [1]:

\[
H_{SSH} = \sum_n t_1 |n\rangle\langle n| \otimes \hat{\sigma}_x + \sum_n t_2 \left( |n+1\rangle\langle n| \otimes \frac{\hat{\sigma}_x + i\hat{\sigma}_y}{2} + \text{h.c.} \right).
\]
Here, the external \((n)\) and internal \((\alpha)\) states are separated by means of a tensor product: \(|n, \alpha\rangle \rightarrow |n\rangle \otimes |\alpha\rangle\). \(\hbar = 1\) is accepted throughout the paper, \(\sigma_i\) are Pauli matrices.

To solve (1), \(\psi(t)\) is conventionally sought in the form

\[
|\psi(t)\rangle = \sum_n |n\rangle \otimes \left( A_n(t) \frac{1}{B_n(t)} \right)
\]

(3)

(the explicit notation of the dependence on time is further omitted). Then, we insert (2) and (3) into (1) and move to a typically polaritonic domain: we add damping and pumping to the system (as in (15)) by means of the damping exponential factor \(\gamma\) (for all the sites) and by the pumping term \(P \exp(\pm i\delta \omega t + \phi_0)\) placed into the equation relative to the pumped site \(|n_0\rangle \otimes |C\rangle\) (\(C\) equals \(A\) or \(B\)), whereupon we obtain the following set of equations

\[
i \partial_t A_n = t_1 B_n + t_2 B_{n-1} - i \gamma A_n + \varepsilon A_n + \delta_{A,C} \delta_{n,n_0} P \exp(\pm i\delta \omega t + \phi_0), \quad (4a)
\]

\[
i \partial_t B_n = t_1 A_n + t_2 A_{n+1} - i \gamma B_n + \varepsilon B_n + \delta_{B,C} \delta_{n,n_0} P \exp(\pm i\delta \omega t + \phi_0), \quad (4b)
\]

where \(P\), \(\omega_p\), and \(\phi_0\) denote the amplitude, frequency, and relative phase of the pumping field, respectively, and \(\varepsilon\), in the field of photonics, represents the cavity mode energy. Resonant pumping would mean the equality \(\varepsilon = \omega_p\).

Further, we utilize the fact that the optical potential profile and thus the topological phase can be controlled by the relative phase of the lasers forming the OL (16). Consider an OL formed by three laser fields: \(E_{\alpha} e^{ikx+i\alpha}\), \(E_{\alpha} e^{-ikx}\), and \(E_{3\alpha} e^{3ikx}\), where \(\alpha\) is the relative phase achieved by the frequency detuning \((\delta \omega)\): \(\alpha(t) = \delta \omega t\). The optical potential produced by them is

\[
V_{OL}(x,t) = V_0 \left| e^{ikx+i\alpha(t)} + e^{-ikx} + e^{3ikx} \right|^2 = V_0 (3 + 4 \cos \kappa x \cos \alpha(t) + 2 \cos 2\kappa x).
\]

(5)

Here, \(\kappa = 2k\), and \(V_0\) is the dimensional constant. Within the potential, the intracell, \(\Delta x_1\), and intercell, \(\Delta x_2\), distances between the neighbour minima (which are attributed to the sites) are:

\[
\Delta x_1(\alpha) = x_{n,B} - x_{n,B} = \frac{2 \arccos \cos \alpha}{\kappa}, \quad \Delta x_2(\alpha) = \frac{2\pi}{\kappa} - \Delta x_1.
\]

(6)

In order to evaluate the hopping amplitudes \(t_1 = t(x_{n,A} \rightarrow x_{n,B})\) and \(t_2 = t(x_{n,B} \rightarrow x_{n+1,A})\), we use the harmonic approximation for the optical potential (15) in the vicinity of its minima. That leads to the magnitudes of the vibration frequency, \(\omega\), and the zero-vibrations amplitude, \(r_0\):

\[
\omega(\alpha) = \sqrt{\frac{2V_0 \kappa^2 (4 - \cos^2 \alpha)}{\mu}}, \quad r_0(\alpha) = \sqrt{\frac{1}{\mu \omega(\alpha)}} = \left[ 2V_0 \mu \kappa^2 (4 - \cos^2 \alpha) \right]^{-1/4}, \quad (7)
\]

where \(\mu\) is the oscillator reduced mass. The vibrations frequency \(\omega\) remains the same for all the minima. The localized Wannier states are taken as the zero vibrational level harmonic wavefunctions centred at the corresponding minima. In this case, the hopping amplitudes can be calculated as \(t_{1\text{eff}} = t(\Delta x_1)\) and \(t_{2\text{eff}} = t(\Delta x_2)\) with

\[
t(\Delta x_i) = \int_{-\infty}^{+\infty} dx \chi_0(x + \Delta x_i) \frac{\mu \omega^3 x^2}{2} \chi_0(x) = \frac{\omega}{2} e^{-\Delta_i^2} \left[ \Delta_i^2 + \frac{1}{2} \right],
\]

(8)

where \(\Delta_i = \Delta_i(\alpha) = \Delta x_i(\alpha) / 2r_0(\alpha)\) (see (6), (7)), and \(\omega\) is a function of \(\alpha\) too (7).
3. Results and discussion

The dependences of $\Delta_{x(2)}$ and $\omega$ on $\alpha$ after a certain algebra disclose that

$$\Delta_1(\alpha) = \frac{\Delta x_1(\alpha)}{2 \tau_0(\alpha)} = \arccos \left( \frac{\cos \alpha}{2} \right) \cdot \left[ 2 V_0 \mu k^2 (4 - \cos^2 \alpha) \right]^{1/4} = \Delta_2(\alpha - \pi), \quad (9a)$$

and consequently

$$t_2(\alpha) = t_1(\pi - \alpha). \quad (9b)$$

Here, we focus on the adiabatic regime of lattice shaking: $\delta \omega \ll t_1, t_2$. Fig. 1 represents the numerical solution of the adiabatic dynamic problem for $\alpha = 0, \pi/4, \pi, 3\pi/4$ (see panels A, B, C, and D, respectively) at the zero initial conditions. The pumped site is the terminal one ($|0, A\rangle$). In the whole figure, green and red colours refer to topologically trivial and non-trivial chain configurations, respectively. Panels E and F display the delocalization of the excitation

![Figure 1](image-url)

**Figure 1.** Adiabatic pumping. A–D: evolution of the sites occupation of the SSH chain subject to the resonant pumping of the terminal (first) site for the values of relative phase $\alpha = 0$ (A), $\pi/4$ (B), $\pi$ (C), and $3\pi/4$ (D). Darker colors correspond to greater values, the color normalization is power-law with the exponent 0.2. E and F: the delocalization of the excitation. G and H: the total occupations of the chain versus time ($t$). Panels E and G correspond to A and C, F and H – to B and D. I: the band structure dependent on $\alpha$, the green and red labels serve to associate values $\alpha$ with relative A–D subplots.
(the dispersion), panels G and H – temporal dependences of the total occupation of the chain: A and C are summated into E, B and D – into F. On panel I, the band structure as function of $\alpha$ is demonstrated, the vertical lines and adjacent characters indicate the values $\alpha$ taken for the relative A–D figures. The parameters used for the simulations were: $\omega_p = \varepsilon = 1$ (resonant pumping), $V_0\kappa^2 = 2\mu = 0.5$, $\gamma = 0.0075$.

From this figure, we can see that the sites occupations of the chain differ both qualitatively and quantitatively for trivial and non-trivial phases. The non-trivial chain is pumped more intensively to a considerable extent (as there exist the edge state that can respond to and capture the pumping). For the same reason, the delocalization of the excitation is (at time scales of several tens of pumping periods) is lower for the non-trivial chain but this difference finally vanishes. Moreover, the occupation pattern of the trivial chain (subplots A and B) clearly demonstrates the features of the solution corresponding to the no-pumping scheme with non-zero initial conditions when only the terminal state is initially occupied.

4. Conclusion

The main aim of the present study was to theoretically examine the topological properties of the Su-Schrieffer-Heeger chain subject to pumping. The chain was modelled by the condensate of non-interacting polaritons in the adiabatically shaken optical lattice. The topological phase of the chain was managed by the relative phases of the lasers constructing the lattice, the corresponding intersite hopping amplitudes were analytically calculated within the harmonic approximation to the local site potential. The chains were resonantly pumped at their terminal sites. We have shown that, under the considered conditions, the topologically trivial and non-trivial chains can exhibit perceptible behaviour.

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