Emergence of a Brunnian neutron state

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(Dated: March 19, 2018)

We discuss a quantum-statistical feature of non-relativistic identical fermions whose interaction is predominantly attractive at low energies. Specifically, we consider exotic, multi-neutron nuclei. From the enhancement of an arbitrarily small $P$-wave interaction between two nucleons, we infer the existence of a particle-stable nucleus composed entirely of neutrons. While we cannot specify the number of neutrons in the system, we predict that none of its subsystems is bound. The independence of this deduction from the short-distance structure of the nuclear interaction and its consistency with deuteron, triton, and helium-4 properties is established.

Overture: The inter-nuclear interaction is not known accurately enough to make reliable predictions about the existence of stable, multi-neutron clusters. An improved understanding of such exotic structures transcends to equally intriguing systems of contemporary theoretical and experimental interest, e.g., neutron stars and neutron-rich nuclei near and beyond the drip line of stability. With the major impediment for advancement in this sector being the extreme difficulties in obtaining data of similar accuracy for the uncharged and unstable neutrons as for nuclei which incorporate protons, it was, unsurprisingly, recent experiments [3–7] which reinvigorated research in this field. Prior to these modern approaches which draw from the decade-long development of high-precision nuclear interaction models and effective field theories, universal mechanisms and phenomena related to multi neutrons were envisioned by A. Migdal [10]. In particular, he considered the formation of stable di-neutrons ($^2n$) in the force field of a lithium-9 core.

Here, we replace this stabilizing nuclear core with a set of neutrons which is unstable in isolation. We show that it can be expected to be glued together with orbiting neutrons, and as such, constitutes a neutron analog of the molecular hydrogen ion. Naturally, we are also constrained by the uncertainty in two-neutron data, but employing an effective field theory (EFT) allows us to assess the sensitivity of the neutron states with respect to precisely such modifications of the nuclear interaction which reflect this ignorance while still being consistent in the description of small nuclei for which data is accurate. Furthermore, the EFT framework allows for a generalization of the result to a class of equal-mass fermionic systems with an attractive short-range potential of insufficient strength to form a bound dimer, and hence the main conjecture of the article, namely, the emergence of a bound state without bound subsystems, can be tested with non-nuclear particles.

Pertinent to this work, W. Heisenberg’s original approximation of an isospin-independent nuclear force is sufficient. All differences between multi-neutron states and ordinary nuclei can in this approximation be attributed to quantum statistics: the demand of a totally anti-symmetric wave function reduces the dimension of the state space available to multiple neutrons severely compared to a mixture of protons and neutrons. The two-nucleon (NN) configuration of strongest nuclear attraction, supporting the deuteron, is thereby forbidden for two neutrons. The formation of larger neutron states by an agglomeration on a bound seed, like the deuteron, is thus inhibited. There is, however, precedent for bound systems which contain only unbound subsystems. In the hydrogen molecule, a single electron combines with a system of otherwise unbound protons. Even more apt is the helium-6 nucleus, which can be understood as a three-body system with a helium-4 core plus two neutrons. While neither helium-5 nor the two neutrons form bound states [6], the compound does. It is this note, we demonstrate that neutrons behave similarly in the sense that there exists a critical number ($\mathfrak{a}$) of neutrons which are bound together by the addition of another neutron. The accuracy of our study does not allow for a prediction of $\mathfrak{a}$ but the logical conclusion guarantees $\mathfrak{a} < \infty$ and thereby the existence of a bound, pure neutron state [4].

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2At the scale of strong interaction processes, i.e., we do not consider electro-weak effects.
To that end, we specify the interaction between two neutrons, first. Subsequently, we test for its analogy to the Coulomb interaction in the hydrogen molecule, namely, whether it bears the potential to affect binding of larger systems despite of its failure to bind two, three, four, up to \(\Sigma - 1\) neutrons. We confirm this molecular character \([13]\) of the nuclear interaction by quantifying the enhancement of an arbitrarily small disturbance of the NN potential in the three and four-neutron systems. For the tri-neutron, the \(J^\pi = \frac{1}{2}^+\) configuration is identified as the channel most amenable to experimental detection and \(0^+\) for the tetra-neutron. From the invariance of the enhancement of the \(P\)-wave interaction with respect to a renormalization-group transformation, and under the assumption that the increase in attraction is correlated with the number of interacting pairs in the system, we infer the Brunnian character of neutrons.

**Interaction:** To describe the interaction between neutrons, we invoke a minimal theory that provides reliable uncertainty estimates for its predictions.\(^1\) With this theory, we can guarantee consistency of our results for few-neutron systems with all other observables it is appropriately used for.

The Hamiltonian comprises iso-spin and momentum independent contact terms with

\[
\hat{H} = \sum_i^A \frac{-\nabla_i^2}{2m} + \sum_{i<j}^A \left[ c_S^A \hat{P}_S^{ij} + c_T^A \hat{P}_T^{ij} \right].
\]

In this form,\(^2\) the theory has been successful in its description of ground-state properties of nuclei with up to four constituents. To our knowledge, the consideration of shallow few-neutron states is the first application of EFT(\(\pi\)) to nuclear properties of a *molecular* character. Previous studies highlighted the sensitivity of multi-neutron states to details of the nuclear interaction, and hence the systematic uncertainty analysis of the EFT framework is the most important reason for its usage here. We parametrize part of this uncertainty with a spin-orbit interaction:

\[
\hat{V}_s = \epsilon c_L^A \mathbf{L} \cdot \mathbf{S},
\]

another part with the cutoff regulator \(\Lambda\). As we anticipate a molecular binding mechanism for neutrons, this choice suggests itself given the prominent role of the term in refinements of atomic and nuclear shell models. Specifically, we vary \(\epsilon\) such that the theory still predicts those nuclear processes for which it devised with the required accuracy, namely, the deuteron, triton, helium-3, and \(\alpha\)-particle binding energies, as well as NN \(P\)-wave phase shifts (inset of Fig. 2). The effect on the latter is shown in Fig. 1 where we compare the EFT phases (dashed) for an \(\epsilon\) large enough to produce an unphysical, deeply-bound tetra-neutron with neutron-proton data (solid). The EFT-predicted phases are small relative to data for \(\epsilon = 0\) (not shown). The enhancement in the \(J = 0\) channel (blue dashed) induced by a non-zero \(\epsilon\) is dominant, and the theory clearly does not reproduce the attractive/repulsive character of the other channels well. However, data and the \(\epsilon\)-enhanced EFT yield similar results for the sum of the three channels, whose smallness traditionally explains the insignificance of the \(P\)-waves in non-exotic nuclei. In contrast to Refs. \([4, 18, 19]\), where all \(P\)-waves were enhanced democratically and bound multi-neutrons emerged only after di-neutrons were bound, the selective enhancement induced here by the short-range spin-orbit interaction binds tetra- and trinucleons before di-neutrons.

**Effective interaction(s):** In order to assess whether this particular interaction exhibits molecular dynamics of neutrons, we appeal to a non-canonical variant of the Born-Oppenheimer method. Instead of fixing the relative distance \(R\) between the unbound two neutrons, considering the third’s motion in this two-center field, and subsequently treating \(R\) as a vary-

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1. For details on this pion-less effective field theory (EFT(\(\pi\))) see Refs. \([14, 15]\).
2. \(\Lambda\) parameterizes the regularization, the operator \(\hat{P}_{S(T)}^A\) projects onto spin-singlet/triplet antisymmetric NN states. For details see, e.g., Ref. \([17]\), and the supplemental material to this article.

\[\sum_{i<j}^A \left[ c_S^A \hat{P}_S^{ij} + c_T^A \hat{P}_T^{ij} \right].\]
evoke is weak relative to lattice QCD predictions [23]. If we evoke the attraction between two neutrons, where the latter is a shallow resonance, it is significant in comparison with $\alpha$ such that the two-neutron phases approach the QCD value [2], a bound $^3n$ emerges, while the deuteron, the triton and helion, and the $\alpha$ are unaffected by this variation. We use this $^3n$ state as a target for a fourth neutron to test for an enhancement of the two-neutron interaction acting in three or four-neutron systems.

Indeed, under similar conditions, i.e., a tri-neutron bound by about 16 MeV, the effective interaction of the neutron projectile with the $^3n$ is visibly stronger than with a $^2n$ target (compare solid blue line to hatched band in Fig. 2). Under such conditions, a bound $^4n$ is present in the spectrum, with a large enough energy not to affect the elastic scattering process. Yet, its existence is crucial to notice because if $\epsilon$ is decreased below the critical value $\epsilon(3n)$, where no bound tri-neutron exists, this teta-neutron remains stable. It disintegrates into four free neutrons at some $\epsilon(4n) \ll \epsilon(3n)$.

In essence, the above shows that the effective $P$-wave interaction between a single neutron and (i) another neutron, (ii) a di-neutron, and (iii) a tri-neutron, becomes increasingly attractive. By deducing from the uncertainty in two-neutron $S$-wave scattering-length extractions – in lattice calculations and experiments – an even less accurately constrained $P$-wave interaction, we find bound $^3n$’s and $^4n$’s sustainable within this uncertainty. The formation of these states is independent of the existence of a bound $^1S_0$ di-neutron‡ and hence the scenario of a bound tetra-neutron with none of its subsystems bound realizes a Brunnian nuclear state.

It remains to translate this effect to physical systems. In other words, we release the fixed-node condition, as imposed through the stronger attraction between neutrons at a pion mass of 806 MeV, and calibrate the interaction strengths $c_{S,T}$ to the deuteron and the virtual singlet state with $k \approx -i25$ MeV. We constrain $\epsilon$ with experimental $P$-wave data [24] as shown in Fig. 2.

In the following analysis of the $^2n(3P_0)$, the $^3n(1/2^-)$, and the the $^4n(0^-)$ states, which were identified as most sensitive to the arbitrarily small spin-orbit disturbance, we find the same signature of the molecular character of the $^2n$ interaction as for the heavy-quark system. Namely, critical short-distance behavior appears in the three systems at well spaced values of $\epsilon$.

This means, e.g., that for $\epsilon \approx \epsilon(3n)$, a shallowly bound tri-neutron can exist, but no di- or tetra-neutron. The latter are either unbound or deeply bound. Furthermore, we find our results with a neutron target also consistent with the helion/triton reactions in Ref. [22].

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\[ i^2 = \text{We find our results with a neutron target also consistent with the helion/triton reactions in Ref. [22].} \]
more, the hierarchy
\[ \epsilon(4n) < \epsilon(3n) < \epsilon(2n) \quad , \]
which encodes the appearance of bound states in larger systems for weaker two-neutron interactions, is found independent of \( \Lambda \), the parameter which complements \( \epsilon \) in the uncertainty assessment. This independence implies that a weak attraction between two neutrons becomes more attractive between three and even stronger between four neutrons as a consequence of the physical deuteron binding energy, the two-nucleon singlet scattering length, and the specific nucleon mass. With the assumption that the enhancement is proportional to the overlap of the two-neutron state most sensitive to the weak attraction, and thereby proportional to the number of two-neutron pairs in this configuration – above, we considered the \( 3P_0 \) state – the emergence of a bound \( 2 \)-neutron state follows.

How does this prediction of a bound neutron nucleus relate to previous theoretical work? On the one hand, the most advanced – in terms of the accuracy of the employed interaction models and the precision of the numerical method – theoretical investigations do not find neutron nuclei bound. Consistently, calculations of equations of state of neutron- and asymmetric nuclear matter with modern high-accuracy interaction models do not show characteristics of emergent, bound neutron clusters (compare, e.g., Refs. [29–32] and [33, 34]). On the other hand, Ref. [34] compiles earlier work based on the already mentioned intuitive analogy between the formation of bound neutron drops and liquid helium, which suggested relatively large bound neutron nuclei with \( \varepsilon > 64 \). In Ref. [18], it was shown more recently how a similar modification of the nuclear interaction, namely a selective enhancement of a single \( P \)-wave channel, can lead to bound tri-neutrons without severe consequences for two-nucleon observables. From these analyses and our work, two classes of modifications of nuclear interaction models can be identified. Modifications of the first class bind neutron nuclei but affect ordinary nuclei. The resultant unphysical models modify either the \( 1S_0 \) component of the NN interaction, enhance the \( P \)-wave NN channels democratically, or they introduce \( T = \frac{3}{2} \) three-nucleon or \( T = 2 \) four-nucleon interactions. Modifications of the second class, selectively enhance the interaction in two-neutron channels with an odd relative angular momentum subject to constraints by data on ordinary nuclei. The specific operator which induces this behavior is irrelevant. In the EFT framework, any operator which modifies the short-distance structure of the observables of interest consistently with the accuracy of the considered order of the EFT expansion is admissible. A future realization of a second class modification with a high-accuracy model can resolve the seeming contradiction in the theoretical analyses.

Relating to experiment, we stress that neither this nor all previous work predict the precise number of neutrons necessary to bind a neutron nucleus. It is the sheer existence which is predicted. While a direct detection of such a potentially large droplet might be as hard as resolving the conflicting measurements on triand tetra-neutrons or resonance, the inference of the peculiar Brunnian character can be tested in fermionic systems in which a modification of the interaction of the described type is feasible. Loss rates associated with the formation of bound multi-fermion clusters for a finely tuned two-fermion \( P \)-wave interaction are signatures of the effect.

The critical number \( \varepsilon \) can be predicted with available data which is correlated to the two-fermion \( P \)-waves but more accurately measured. Nuclear \( P \)-wave observables, like the first excited state of helium-4, the resonant states of helium-5, and/or the instability of beryllium-8 suggest themselves. Assuming the NN \( P \)-waves to be constrained by any of these systems, future work will enable a prediction of the minimal neutron number \( \varepsilon \).

Before summarizing, it is in order to list to important assumptions. (i) Isospin-breaking effects must lead to similar or larger deviations of the \( 2n P \)-wave phases from the respective neutron-proton phase shifts. Evidence for this is provided by the relatively small shift of the summed phases necessary to yield bound multi-neutron. (ii) The increasing probability to find neutron pair in relative \( P \)-waves in larger systems and the change induced in the two-neutron interaction by the background of the other neutrons must eventually accumulate to an interaction corresponding to a critical \( \epsilon \). We have shown this explicitly for the most crucial numbers, from the shell-model perspective, three and

\begin{footnotesize}
\begin{itemize}
\item \footnote{Experimental findings of such shallow structures are reported in Refs. [1, 2, 54, 55] in contrast to negative measurements in Refs. [25, 26]. (We list only work known to us and not reviewed in Refs. [42–44].)}
\item \footnote{Consistent with Ref. [42], the \( \epsilon \) tuning does not yield an accumulation of shallow \( 3P \) states.}
\item \footnote{Exemplary were experiments on cold-atom gases [45, where a magnetic field was used to tune the atom-atom interaction to detect the Efimov effect.}
\item \footnote{The success of such an approach in the description of bulk features of the nuclear chart has been demonstrated in Ref. [14].}
\end{itemize}
\end{footnotesize}
four. Under these two assumptions, we predict a multi-neutron state with binding energy of order 10 MeV and thereby of a scale comparable to the deuteron and triton ground states, and the bonding energy per nucleon of larger nuclei in a narrow window around the respective critical $\epsilon(n^2)$. For other $\epsilon$, despite practically invariant NN phase shifts, the shallow neutron clusters become very deeply bound and thus decouple from processes at the nuclear scale and might also escape a detection in numerical calculations.

**Epilogue:** We fine tune the nuclear interaction such that it supports bound multi-neutron states with energies of order 10-100 MeV without sacrificing the usefulness of the theory for its description of ordinary nuclei. The binding mechanism is attributed to a specific component of the two-fermion interaction that becomes increasingly attractive with the number of particles in the system. For the three and four-neutron system, we find the enhancement factors sufficiently different to infer that only one of the two can be shallowly bound, while the other is either unbound or so deeply bound that its coupling to nuclear processes can be disregarded. We extrapolate this finding to larger systems and deduce the existence of a finite number of neutrons which is bound, while any of its isolated sub-configurations is unbound. The employed interaction model suggests the emergence of such Brunnian states in any two-component fermionic system with an entirely attractive two-body potential with a characteristically large scale relative to its range.

**ACKNOWLEDGEMENTS**

I owe thanks to B. C. Tiburzi for his careful reading of and comments on the manuscript. Support comes from the National Science Foundation under Grant No. PHY15-15738.

**I. APPENDIX**

**Interaction:** In table [I] we list the low-energy constants (LEC) used in our calculations. The triplet constant $c^T$ is calibrated to the deuteron binding energy, $B_D = 2.22$ MeV in the physical world, and $B_D = 19.5$ MeV in the heavy-pion world. The singlet LEC $c^S$ is fitted to the $S$-wave neutron-proton singlet scattering length $a_S = -23.8$ fm for physical quarks and to the binding energy of the singlet neutron-proton ground state at $m_\pi \approx 806$ MeV, namely $B_S = 16$ MeV.

The three-nucleon contact operator

$$\hat{V}_3 = \sum_{i<j<k}^{A} d_3(\Lambda) \left( \frac{1}{2} - \frac{1}{6} r_{ij} \cdot \tau_i \right) e^{-\frac{\Lambda^2}{2} (r_{ij}^2 + r_{ik}^2)} ,$$

projected onto the spin-doublet channel, is included in the calculations to assess the invariance of the triton and helium-4 binding energies with respect to the disturbance induced through the spin-orbit interaction. Its strength $d(\Lambda)_3$ is fitted to match the triton's binding energy $B_{3\pi} = 8.48$ MeV at the physical and $B_{3\pi} = 53.9$ MeV at $m_\pi \approx 806$ MeV.

The superscript $\Lambda$ parameterizes a Gaussian regulator of the singular contact interaction and a substitution

$$c^A \rightarrow c(\Lambda) e^{-\frac{\Lambda^2}{2} \tau^2}$$

is understood with the relative distance $r$ between two interacting particles. We employ interactions with $\Lambda \in [6, 12]$ fm$^{-1}$ because lower cutoffs were shown to unjustly cut out modes at large pion masses which are necessary for the formation of the different bound states. To observe a convergent behavior it is unnecessary to go beyond 12 fm$^{-1}$ where numerical convergence is harder to demonstrate than for values closer to the physical scales.

**TABLE I.** The LECs $c_T(\Lambda)$, $d_3(\Lambda)$ [GeV] for physical ($m_\pi = 140$ MeV) and lattice ($m_\pi = 806$ MeV) nuclei for various values of the momentum cutoff $\Lambda$ [fm$^{-1}$].

| $m_\pi$ | $\Lambda$ | $c_T(\Lambda)$ | $c_S(\Lambda)$ | $d_3(\Lambda)$ |
|---------|----------|---------------|----------------|---------------|
| 140     |  2       | -0.1423       | -0.1063        |  0.06849      |
|        |  4       | -0.5051       | -0.4350        |  0.6778       |
|        |  6       | -1.091        | -0.9863        |  2.653        |
|        |  8       | -1.899        | -1.760         |  7.816        |
|        | 10       | -2.929        | -2.757         |  20.48        |
|        | 12       | -4.182        | -3.976         |  50.94        |
|        | 15       | -6.480        | -6.222         |  195.6        |
| 806    |  2       | -0.1480       | -0.1382        |  0.07102      |
|        |  4       | -0.4046       | -0.3885        |  0.3539       |
|        |  6       | -0.7892       | -0.7668        |  1.001        |
|        |  8       | -1.302        | -1.273         |  2.221        |
|        | 10       | -1.942        | -1.907         |  4.308        |
|        | 12       | -2.710        | -2.670         |  7.712        |
|        | 15       | -4.103        | -4.052         |  16.84        |

**Resonating groups:** A version [18] of Wheeler’s resonating-group method [49] is used to solve the stationary Schrödinger equation. For the considered systems, the general two-fragment ansatz for the wave
function,
\[ \Psi = \mathcal{A}\left\{ \sum_i \phi_i^{(i)}(R_i) \right\}, \tag{6} \]

simplifies because the second fragment \( \phi_{II} \) is identified with a single neutron. The first fragment \( \phi_I \) is either another neutron, a \( {}^2S_0 \) dineutron, or a \( \frac{1}{2}^+ \) trinucleon. For the latter, \( i \) indexes all spin coupling schemes of three spin–1/2 neutrons which couple with a total angular momentum \( L = 1 \) to \( J = 1/2 \). In the partial-wave expansion of the inter-cluster relative-motion function
\[ F(R) = \frac{1}{R} \sum_i f_i(R) P_l(\cos \theta_R) \tag{7} \]
we find sufficiently converged results with \( l \leq 1 \). In addition to these functions, which describe asymptotically non-interacting fragments,
\( \hat{H} \Psi = \left[ E_{\text{c.m.}} - B^{(3)}(n) \right] \Psi \), so-called distortion channels are included in the sum over \( i \). For those, the functions \( F^{(i)} \) do not approximate a free relative motion, but allow for any deviation from the “frozen” configuration when fragments are close and interact strongly. Each fragment is set up in an antisymmetric state, and the cross-fragment antisymmetrizer \( \mathcal{A} \) excludes, e.g., all dineutron-dineutron configurations with positive parity from the \( 4n \) \( 0^+ \) channel. We expand the radial functions \( f_i \), which appear also in the cluster-internal functions \( \phi \), in a Gaussian basis whose parameters are optimized for each cutoff parameter \( \Lambda \). A typical dimension of a variational space in which results converge is found to be of \( \mathcal{O}(10^4) \) because of this tailoring. For the scattering problem, a variant of the Kohn-Hulthén variational principle is invoked, and bound-state energies are inferred from a diagonalization of \( \hat{H} \) in the basis comprised of the element space of \( i \) in Eq.\((6) \) (spin/orbital angular-momentum coupling schemes, Gaussian parameters).

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