Abstract
Lateral collision is one of the top two accidents in the world and often occurs in the lane change. Trajectory optimisation is an effective method to solve traffic conflicts in the lane-changing process. However, current trajectory optimisation methods are not friendly to human-computer-based driving assistance. This study proposes a time-dependent lanechange trajectory optimisation considering comfort and efficiency. First, spacing constraints between the lane-changing vehicle and surrounding vehicles are determined and quantified. Second, lane-changing trajectory data are obtained by driving simulation experiments and extracted by data features. Third, a quintic multinomial model of lane-changing trajectory is proposed. The results reveal that the predicted trajectory is close to the observed value. Then, the obtained trajectory is optimised by the objective function considering lane-changing efficiency and comfort. Finally, a case study is used to demonstrate the application of the model. When a lane-changing vehicle changes a lane at an initial speed of 90 km/h to the faster lane at the terminal speed of 110 km/h, the optimal lane-changing time is 3.4 s, with the lateral acceleration 1.79 m/s² and the maximum yaw angle 0.081 rad.

1 INTRODUCTION
Lateral and rear-end collisions are the two most collision manoeuvres in the world. According to the national highway traffic safety administration (NHTSA) traffic accident data from 2014 to 2018, rear-end collisions accounted for the highest proportion, reaching 40%. The second was angle collision, accounting for 24%, of which the proportion of lateral collisions was 13% [1]. In China, lateral collisions have a higher percentage. The 2018 traffic accident data revealed that sideswipe accounted for the highest proportion, reaching 42.7%, followed by rear-end collision, accounting for 7.95%.

Car following is a process of following but not exceeding obstacles, during which there is only one potential traffic conflict, while lane-changing is a process of surpassing obstacles to reach the ideal position, and there are multiple potential traffic conflicts. This is also the reason why side-impact accidents occur frequently in China.

The surrounding traffic environment influences the lane-changing process of the lane-changing vehicle. In the lane-changing process, drivers need to analyse and make a reasonable decision according to the spacing and time gap between the lane-changing vehicle and surrounding vehicles. Once a wrong judgment is made on lane-changing spacing or starting time, lateral collisions will occur.

To avoid the occurrence of side impacts, it is necessary to determine which vehicles have restrictions on the lane-changing vehicle and their influence mode during the lane-changing process. The lane-changing trajectory is an important entry point for studying the lane-changing process. An optimal lane-changing trajectory can minimise traffic conflicts during the lane-changing process.

This study focuses on the lane change trajectory optimisation for lateral collision avoidance.

1.1 Safety constraints in lane change
A lane-changing process contains four stages: (1) Preparation stage, (2) lane departure stage, (3) target lane adjustment stage, and (4) recovery stage as displayed in Figure 1. The following

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conditions [2, 3] should be met to complete lane change and avoid lateral collisions.

1. No rear-end collision or lateral collision occurs between the subjective vehicle (SV) and the leading vehicle (LV; in the current lane).
2. When SV enters the target lane and adjusts, no rear-end collision or lateral collision occurs between SV and the leading vehicle in the adjacent lane (ALV).
3. After SV enters the target lane, no lateral collision occurs between the following vehicle in the adjacent lane (AFV) and SV.

1.2 Lane-changing trajectory model

The commonly used models of lane-changing trajectory include cosine curve [4], trapezoid lateral acceleration curve [5], hyperbolic tangent curve [6], B-spline curve [7], and so forth. The cosine curve has smooth curvature and the hyperbolic tangent curve has smooth lateral acceleration. Nevertheless, both of them cannot replan the trajectory according to a real-time traffic environment [8]. The trapezoid acceleration model worked for the lateral acceleration control but cannot adjust the longitudinal velocity of lane-changing vehicles. The lane-changing trajectory of the B-spline curve is smooth and continuous. However, due to the number of parameters, the solution may not be optimal.

Piazzi and Bianco [9] proposed a quintic spline curve for trajectory planning [10]. They found that the flatness of the quintic polynomial curve could increase the robustness of the lane-changing trajectory. The multinomial curve [11, 12] was adopted in the lane-changing trajectory because the multinomial function can adjust the order to achieve the desired performance [13]. For example, if the constant of the first derivative of a cubic polynomial trajectory was zero, the planned lateral velocity (in the geodetic coordinate system) was smooth [14]. If the constant of the first and second derivatives of the quintic polynomial was zero, the lateral curvature of the trajectory was smooth [15, 16]. The higher-order multinomial function could smooth the variations of variables and, indeed, more input information was necessary. Additionally, the multinomial method could also realise the path re-planning of lane change [17]; therefore, it was more suitable for lane-changing trajectory function.

With the development of automatic driving, lane-changing trajectory optimisation has ushered in new development and application [18, 19]. Displacement, lateral velocity, lateral acceleration and lane-changing time were key optimisation variables of lane change trajectory [20–22]. Most lane-changing trajectory planning methods based on numerical optimisation relied on other methods to find the optimal trajectory [23, 24]. Zhang et al. [25] used the time-dependent cubic polynomial equation and presented a cost function considering the driving comfort and efficiency to optimise the lane-changing trajectory. Suh et al. [26] limited the maximum lateral acceleration of the vehicle in their model to ensure lateral comfort. Additionally, the maximum curvature was also a key variable to control the trajectory [27, 28].

Although lane-changing trajectory was affected by many factors, the lateral velocity, lateral acceleration, displacement were not friendly for human-computer interaction [29]. The lateral velocity and acceleration of lane change were very small. Drivers were not sensitive to these variables and it was difficult to adjust operations accordingly. In contrast, time was a sensitive and easily acceptable indicator for people. As it happened, the lane-changing trajectory could be written as a time-dependent function. Time-based driving assistance and trajectory adjustment would be more practical. In the case of certain lateral displacement, drivers only need to be told when to start and end changing lanes to ensure the lane-changing track optimally.

The determination of the optimal lane-changing time is the key to determine the optimal lane-changing trajectory. In this study, a lane-changing trajectory optimisation method is proposed to avoid lateral collisions. First, the safety constraints between the lane-changing vehicle and surrounding vehicles before and after lane change need to be determined and quantified. Second, the lane-changing trajectory and related parameters will be obtained by driving simulation experiments. Third, a time-dependent lane-changing trajectory function is proposed and optimised by an objective optimisation function considering lane-changing comfort and efficiency. Finally, a case study is used to verify the application of the model. This study is organised as follows: Safety constraints in lane change are characterised in Section 2. Section 3 describes the data preparation of lane-changing data. The time-dependent lane-changing trajectory model and the objective optimisation function of trajectory are established in Sections 4 and 5, respectively. Section 6 demonstrates the results of a case study of the model and discusses the applications and limitations. Section 7 is the conclusion.

2 CHARACTERISATION OF SAFETY CONSTRAINTS IN LANE CHANGE

In the lane-changing analysis, the vehicle model can be simplified to an ellipse model [30, 31] as shown in Figure 2 below: The major axis of the ellipse is Lx, the minor axis is Ly, the yaw angle is ϕ, the width of the vehicle is w and the length is L.

The three safety constraints of lane change are shown in Figures 3(a) to (c), respectively. In Figure 3(a), the minimum safety distance between SV and LV is \( D_{\text{MIN}}(SV, LV) \), and the longitudinal displacement of SV is \( D(SV, LV) \). At the collision...
FIGURE 2  Ellipse vehicle model in lane-changing analysis

\[
D(SV, LV) = \max\{X_{SV} - X_{LV} + L + w \sin \varphi \}
\]

= \max \left\{ \int_0^t \int_0^\lambda [a_{SV} (\tau) - a_{LV} (\tau)] d\tau d\lambda + (v_{SV} (0) - v_{LV} (0)) t + L + w \sin \varphi \right\},
\]

\[ t \in (t_0, t_f) \quad (2) \]

In Figure 3(c), the minimum safety distance between SV and ALV is \( D_{MSS}(SV, ALV) \), and the longitudinal displacement of SV is \( D(SV, ALV) \). At the collision point, the lateral displacement of the left front point of SV is \( H - w \), where \( H \) is the lateral displacement of SV. The minimum safety distance shall meet the conditions as follows:

\[
D_{MSS}(AFV, SV) = \max\{X_{AFV} - X_{LV} + L + w \sin \varphi \}
\]

= \max \left\{ \int_0^t \int_0^\lambda [a_{AFV} (\tau) - a_{SV} (\tau)] d\tau d\lambda + (v_{AFV} (0) - v_{SV} (0)) t + L + w \sin \varphi \right\},
\]

\[ t \in (t_0, t_f) \quad (3) \]

where \( t_0, t_3, t_f \) denote lane-changing start time, critical collision time, lane-changing end time, respectively. \( X, v, a \) denote longitudinal displacement, velocity and acceleration, respectively. \( \lambda, \tau \) denote integration variables.

3 | LANE-CHANGING DATA PREPARATION

3.1 | Data acquisition

3.1.1 | Apparatus

The Tongji University driving simulator was applied to implement experiments and collect data. In the driving simulator, a fully instrumented Renault Megane III vehicle cab is mounted on an eight degree-of-freedom motion system with an X-Y range of 20 × 5 m (Figure 4). An immersive five-projector system provides a front image view of 250 and 40° at 1400 × 1050 resolution refreshed at 60 Hz. Liquid crystal display (LCD) monitors provide rear views at the central and side mirror positions. SCANeR™ studio software displays the simulated highway scenario and control a force feedback system that acquires data from the steering wheel, pedals and gear shift lever.
3.1.2 | Scenario

The test drivers changed lanes according to the requirements. They would experience five conditions: Free left lane change, free right lane change, restricted left lane change, restricted right lane change and continuous lane change. Among these schemes, the free lane-changing task was directed by roadside sign, while the restricted lane-changing task was guided by conical buckets before and after the operation area. The experimental schemes were shown in the left column in Figure 5, and the scenarios are shown in the right column.

3.1.3 | Participants

Thirty-four qualified participants (28 males, 6 females) were recruited from the university population, with 24 to 50 (mean = 30.2, S.D. = 4.5) years of age and six to 13 years of driving experience (mean = 7.2, S.D. = 3.6). The qualification included: (1) Driver license with at least 8000 km of driving experience; (2) average annual driving distance of at least 2000 km; (3) no criminal records, mental illness and drug use.

3.1.4 | Driving tasks and pre-tests

Before the driving simulation experiment, the drivers were informed of the driving task: After identifying the sign or conical buckets, turn on the left or right turn signal and change lanes. Before the formal test, pilots had 10 min to conduct the pre-test to adapt to the cockpit environment and control system. Drivers were observed whether having any discomfort reaction. If the driver was normal, the formal experiment would start after 5 min of rest. The formal experiment lasted for about 20 min.

3.1.5 | Data acquisition

When drivers were experimenting, their operation data were collected simultaneously. The operation data included longitudinal and lateral velocity, acceleration, throttle and brake pedal percentage, steering angle, yaw angle and lane departure (Table 1).

3.2 | Extraction of lane-changing data

The original data contained all driving states, including lane-changing and non-lane-changing states. To get the lane-changing law, the lane-changing data needs to be extracted from the original data first, but different indicators reflect different features of the lane change. Therefore, a feed-forward neural network is applied for pattern recognition to extract the lane-changing data. The operation parameters are illustrated in Figure 6.

Among these parameters, the longitudinal speed variation is not affected by lane-changing mode, and there is no uniform law (acceleration, deceleration, first increase then decrease, first decrease then increase are all present). The lateral speed and steering wheel angle have apparent characteristics similar to sine or cosine function when changing lanes. Lane departure also shows distinct step-downs with a lane change. Therefore, the lateral operation parameters are used to identify the starting and ending points of lane change. The seven lateral parameters...
TABLE 1  Operation data acquisition

| Category                  | Symbol | Parameters                      | Description                                                        |
|---------------------------|--------|---------------------------------|--------------------------------------------------------------------|
| Longitudinal driving data | \( v_x \) | Longitudinal speed (m/s\(^{-1}\)) | Tangential vector of speed                                        |
|                           | \( a_x \) | Longitudinal acceleration (m/s\(^{-2}\)) | Tangential vector of acceleration                                 |
|                           | \( \eta_{TH} \) | Throttle (%)                     | Percentage of the gas pedal                                       |
|                           | \( \eta_{BR} \) | Brake (%)                        | Percentage of the brake pedal                                     |
| Lateral driving data      | \( v_y \) | Lateral speed (m/s\(^{-1}\))     | The normal vector of speed, left turn, positive; right turn, negative |
|                           | \( a_y \) | Lateral acceleration (m/s\(^{-2}\)) | The normal vector of acceleration, left turn, positive; right turn, negative |
|                           | \( \phi \) | Yaw angle (\(^\circ\))          | One of the Euler angles describing the heading of a vehicle       |
|                           | \( \theta_{sw} \) | Steering angle (\(^\circ\))     | Rotating angle of the steering wheel (left turn, positive; right turn, negative) |
|                           | \( l_d \) | Lane departure (m)              | Displacement of the vehicle from the centreline of the current lane (left departure, positive; right departure, negative) |

FIGURE 6  Operation parameters of the lane-changing manoeuvre

(lateral velocity, lateral acceleration, steering angle, steering wheel angular rate, yaw angle, yaw rate, lane departure) can all reflect lane-changing characteristics. To determine the best accurate results, a feed-forward neural network [32, 33] is established for identification.

A two-layer feed-forward neural network is established in which lateral operation parameters are as input (Figure 7). The model consists of an input layer, two hidden layers and an output layer. The input layer denotes one of the seven lateral parameters mentioned above. Three neurons in the output layer denote three manoeuvres including left lane-changing, lane-keeping and right lane-changing. There are six neurons in the hidden layer and the number of neurons is determined by test to enable recognition results optimal. The proportion of training, verification and test set is 7:1.5:1.5, and the result of lane change of training sample is manually identified according to video.

Running the model in Figure 7 seven times, the recognition rates for the seven lateral parameters can be obtained in Table 2. The results indicate that the recognition rate of the indicators reaches more than 80% except for steering wheel angular rate, and the highest recognition rate is the yaw angle, with an accuracy rate of 88.76%. Therefore, the lane-changing data are extracted by the yaw angle from time-series data.

4 | LANE-CHANGING TRAJECTORY MODEL

4.1 | Lane-changing trajectory features

The lane departure, lateral speed and lateral acceleration of left and right lane change are plotted in Figures 8 to 10. The line
The focus of this article is to find the common law of all lane-changing manoeuvres. There are indeed subtle differences between left lane-changing and right lane-changing. It appears that due to China’s right-hand driving traffic rules, left lane changes are steeper in trajectory, especially when crossing lane lines, while right lane changes are smoother.

The overall changes of lane departure reveal the following characteristics: (i) The preparation phase and the recovery phase change slowly, while the lane departure stage and target lane adjustment phase changed rapidly. (ii) The trajectories before and after the lane change are not completely symmetrical. In the first half of lane change, the trajectories are similar and the paths are concentrated, while the differences are apparent in the second half no matter the lane-changing mode.

The lateral velocity is drawn in Figure 9 after filtering. The lateral velocity curve of left lane change changes from negative to positive, which presents a pattern similar to the negative sine function. The change of lateral velocity on the right lane change is just opposite from positive to negative, which shows a pattern similar to the positive sine function. The extreme values of lateral velocity are only \( \pm 3 \times 10^{-3} \) m/s. Drivers’ trajectories are different, and there is a gap of about 2 s in lane-changing time distribution. The lateral velocity variations reveal that the value at the initial and terminal time is 0.

The noise of lateral acceleration data is small compared to lateral velocity (Figure 10). The data trends display an inverted pattern to the lateral velocity, in which the extreme values reach \( \pm 0.8 \) m/s\(^2\). The lateral acceleration suggests that the value of the lateral acceleration is 0 at the initial and terminal time.

4.2 The quintic multinomial lane-changing trajectory

The quintic multinomial lane change trajectory is defined as follows:

\[
\begin{align*}
    x(t) &= a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \\
    y(t) &= b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5
\end{align*}
\]

where \( x \) denotes longitudinal displacement, \( y \) denotes lateral displacement, \( a_0, a_1, \ldots, a_5 \) and \( b_0, b_1, \ldots, b_5 \) denote multinomial coefficient.
The lateral acceleration in the lane-changing process is given as follows:

\[ y(t) = a_1 + 2a_2t + 3a_3t^2 + 4a_4t^3 + 5a_5t^4 \]  
\[ \dot{y}(t) = 2a_2 + 6a_3t + 12a_4t^2 + 20a_5t^3 \]  

FIGURE 10   Lateral acceleration in the lane-changing process

According to the lane change data in Figures 8 to 10, the following assumptions are made for the lane change trajectory.

(i) The longitudinal displacement of the starting point equals zero, \( x(t_0) = 0 \).
(ii) Vehicles move along the centreline of the current lane before and after lane change, namely, the initial lateral displacement equals zero, and the terminal displacement equals the lane width, \( y(t_0) = 0, \dot{y}(t_f) = w \).
(iii) The initial longitudinal velocity of the starting point is \( v_0 \), \( \dot{x}(t_0) = v_0 \); the lateral velocity of the starting point is zero, \( \dot{y}(t_0) = 0 \).
(iv) The longitudinal and lateral acceleration of the starting point is zero, \( \ddot{x}(t_0) = 0, \ddot{y}(t_0) = 0 \).
(v) The longitudinal velocity at the terminal point is \( v_d \), \( \dot{x}(t_f) = v_d \); the lateral velocity at the terminal point is zero, \( \dot{y}(t_f) = 0 \).
(vi) The longitudinal and lateral acceleration of the terminal point is zero, \( \ddot{x}(t_f) = 0, \ddot{y}(t_f) = 0 \).

The lateral velocity and acceleration of the lane change trajectory are given as follows:

\[ \dot{y}(t) = a_1 + 2a_2t + 3a_3t^2 + 4a_4t^3 + 5a_5t^4 \]  
\[ \ddot{y}(t) = 2a_2 + 6a_3t + 12a_4t^2 + 20a_5t^3 \]  

FIGURE 11   Comparison of predicted and observed lane-changing trajectories

There are six constraints in \( y \)-direction and five constraints in \( x \)-direction. According to the 11 initial constraints, the constants \( a_i \) \( (i = 0 \sim 5) \) and \( b_i \) \( (i = 0 \sim 5) \) are determined as follows:

\[ \begin{align*}  
  a_0 &= a_1 = a_2 = 0 \\
  a_3 &= \frac{10w}{t_f^3}, \quad a_4 = \frac{-15w}{t_f^4}, \quad a_5 = \frac{6w}{t_f^5} \\
  b_1 &= b_2 = b_3 = 0 \\
  b_0 &= v_0, \quad b_4 = \frac{v_d - v_0}{2t_f^3} 
\end{align*} \]  

The constants are brought into Equation (4), and the lane change trajectory is rewritten as follows:

\[ \begin{align*}  
  \dot{x}(t) &= v_0t + \frac{10w - v_0}{t_f^3} t^3 + \frac{2v_0 - v_d}{2t_f^3} t^4 \\
  \dot{y}(t) &= 10 \frac{w}{t_f^3} t^3 - 15 \frac{w}{t_f^4} t^4 + 6 \frac{w}{t_f^5} t^5 
\end{align*} \]  

FIGURE 11   Comparison of predicted and observed lane-changing trajectories

The actual lane width \( w = 3.75 \text{ m} \) and lane change time \( t_f = 5 \text{ s} \) are brought into Equation (7), lane change trajectory is plotted in Figure 11 below. It can be seen from the diagram that the model well describes the trajectory of the lane change.

5 | TIME-DEPENDENT LANE CHANGE TRAJECTORY OPTIMIZATION

5.1 | Influence of lane-changing time on lane-changing parameters

The lane-changing trajectory of SV is constrained by LV, ALV and AFV. Therefore, the key to lateral collision avoidance is to seek an optimal lane change trajectory.

For example, SV changes a lane from the middle lane to the faster lane on a three-lane freeway. When the lane change occurs, the speed of the current lane is 25 m/s and the speed of the faster lane is 30 m/s, which means the initial speed of SV is 25 m/s and the terminal speed of SV is 30 m/s. The lane width...
of the freeway equals 3.75 m, the vehicle length equals 4.5 m and the vehicle width equals 1.8 m.

The trajectory of lane change can be obtained as follows:

\[
\begin{align*}
\dot{x}(t) &= 25t + \frac{5}{t_f^2}t^3 - 2.5t^4 \\
\dot{y}(t) &= \frac{37.5}{t_f^2}t^3 - 56.25\frac{1}{t_f^4}t^4 + 22.5\frac{1}{t_f^5}t^5 \\
\ddot{y}(t) &= 112.5\frac{1}{t_f^3}t^2 - 225\frac{1}{t_f^4}t^3 + 112.5\frac{1}{t_f^5}t^4 \\
\dddot{y}(t) &= 225\frac{1}{t_f^3}t - 675\frac{1}{t_f^4}t^2 + 450\frac{1}{t_f^5}t^3
\end{align*}
\] (8)

The key parameters are affected by the lane-changing trajectory including longitudinal and lateral displacement, lateral acceleration and the maximum yaw angle. According to Equation (8), the lane-changing trajectory is only affected by the lane-changing time \( t_f \). The influence of lane-changing time on lane-changing parameters are analysed in the following sections.

5.1.1 Influence on longitudinal displacement

The lane-changing trajectories under different \( t_f \) are plotted in Figure 12, where longitudinal displacement and lateral displacement are set as \( x \) and \( y \) axes, respectively. Different from the previous three pictures, the line colours in the latter pictures denote different lane-changing time. In Figure 12, with the increase of \( t_f \), the lateral displacement remains the value of lane width. When \( t_f \) equals 2 s, the longitudinal displacement is only 70 m; when \( t_f \) equals 8 s, the longitudinal displacement reaches 230 m.

5.1.2 Influence on lateral acceleration

According to Equation (8), the lateral acceleration \( a_y \) of SV can be obtained and displayed in Figure 13. The image shows that the lateral acceleration amplitude decreases with the increase of lane-changing time \( t_f \). When \( t_f \) equals 2 s, \( a_y \) reaches 5.41 m/s\(^2\); when \( t_f \) equals 8 s, \( a_y \) reaches only 0.34 m/s\(^2\). Lateral acceleration \( a_y \) must be controlled when \( D_{\text{MC4}} \) is required to prevent the rollover accident. In the premise of safety, better comfort and high lane-changing efficiency are in pursuit.

5.1.3 Influence on maximum yaw angle

According to Equation (8), \( \varphi = \arctan(\dot{y}/\dot{x}) \). The variation of maximum yaw angle with lane-changing time is illustrated in Table 3 and Figure 14. It reveals that the moment of \( \varphi_{\text{max}} \) is slightly before the midpoint of lane-changing time. The maximum yaw \( \varphi_{\text{max}} \) increases with the shortening of \( t_f \). When \( t_f \) equals 2 s, \( \varphi_{\text{max}} \) reaches 7.6°, and when \( t_f \) equals 8 s, \( \varphi_{\text{max}} \)
reaches 1.83°. Meanwhile, the maximum yaw under any lane-changing time is obtained at the same moment of 0.48 \( \psi \).

### 5.1.4 Influence on \( D_{MS} \)

As the speed of ALV in the target lane is higher than the speed of SV in the lane-changing process, it is considered that ALV does not affect SV as long as there is a safe \( D(SV', ALV) \) at the beginning of lane change. However, LV and AFV still threaten the lane change of SV. The minimum safe lane-changing distance between SV and the two vehicles are obtained as follows (taking \( t_f = 2 \)s as an example):

\[
D_{MS}(SV', LV) \geq X_{SV'} - X_{LV} + L + W' \sin \varphi
\]

\[
= \max \left\{ \int_0^t \int_0^\lambda \left[ a_{SV'}(\tau) - a_{LV}(\tau) \right] d\tau d\lambda + (v_{SV'}(0) - v_{LV}(0))t + L + W' \sin \varphi \right\}, \: t \in (t_f, t_{c2})
\]

\[
= -25 \times 0.93 + 24 + 4.5 + 1.8 \times \sin 0.133 = 5.5 \text{ m}
\]

\[ D_{MS}(AFV', SV') = \max \{X_{AFV'} - X_{SV'} + L \cos \varphi\} \]

\[
= \max \left\{ \int_0^t \int_0^\lambda \left[ a_{AFV'}(\tau) - a_{SV'}(\tau) \right] d\tau d\lambda + (v_{AFV'}(0) - v_{SV'}(0))t + 2L \cos \varphi \right\}, \: t \in (t_f, t_{c2})
\]

\[
= 30 \times (2 - 0.93) - 26 + 4.5 \times \cos 0.133 = 10.3 \text{ m}
\]

The results of the minimum safe lane-changing distance under the remaining lane-changing time are listed in Table 4. The data suggest that the minimum safe distance increases with the growth of lane-changing time. Moreover, \( D_{MS}(AFV', SV') \) is always higher than \( D_{MS}(SV', LV') \). When \( t_f \) equals 2 s, they are 5.5 m and 10.3 m, respectively. Moreover, when \( t_f \) equals 8 s, they are 7.6 m and 19.5 m, respectively.

### 5.2 The objective function of lane-changing time

According to Equation (8), lane-changing trajectory and lateral velocity are only affected by the lane-changing time \( t_f \). The value of \( t_f \) is mostly given by experience, which is not conducive to the flexible application in the complex traffic environment. Lateral acceleration \( a_y \) has an effect on lane-changing safety and comfort, and lane-changing time \( t_f \) influences lane-changing efficiency.

Therefore, the objective function \( J(t_f) \) is established to reduce the lateral acceleration and lane-changing time. Therefore, \( t_f \) is optimised and the objective function is defined as follows:

\[
\min J(t_f) = \frac{\tau_1}{t_f} \left| a_y \right| + \frac{\tau_2}{t_f} \frac{t_f}{t_f \max}
\]

The maximum longitudinal acceleration \( a_x \), lateral acceleration \( a_y \) and minimum safety distance of lane change \( D_{MS} \) should meet the constraint conditions reflecting comfort and safety, respectively. The constraint conditions are obtained as follows:

\[
\min J(t_f)
\]

s.t.

\[
\left\{ \begin{array}{l}
\left| a_y(t) \right| < a_y \max \\
\left| a_x(t) \right| < a_x \max \\
D_{MS}(SV', LV') \geq L + d \\
D_{MS}(SV', AFV') \geq 0 \\
D_{MS}(ARV', SV') \geq 0
\end{array} \right.
\]

where \( \tau_1, \tau_2 \) are weight coefficient, and \( \tau_1 + \tau_2 = 1 \). \( \tau_1 = \max \{D_{MS}(SV', LV')/D(SV', LV'), D_{MS}(SV', ALV')/D(SV', ALV'), D_{MS}(AFV', SV')/D(AFV', SV')\} \). The maximum term reveals the most extreme safe distance in lane change that determines the value of \( \tau_2 \); then \( \tau_1 = 1 - \tau_2 \).
The value of $\tau_1, \tau_2$ indicates comfort and efficiency during lane change. Their values are determined by the state of surrounding vehicles. Lane-changing efficiency should be focused on when the longitudinal distance between SV and surrounding vehicles is small and much comfort cannot be realised. However, comfort can be focused on and reduce the proportion of lane-changing efficiency when the longitudinal distance between SV and surrounding vehicles is significant.

6 | RESULTS AND DISCUSSIONS

Assume that the lane-changing vehicle changes a lane from the middle lane at the speed of 90 km/h to the faster lane at the terminal speed of 110 km/h. The initial distance $D(SV, LV) = 10.0 m$ and $D(AFV, SV) = 20.0 m$ at the lane-changing starting moment.

In this case, the lane-changing trajectory function is as shown in Equation (8). Then, the value of $\tau_1$ and $\tau_2$ need to be calculated according to Equation (11). Take $t_f = 2 s$ as an example and the value of $\tau_2$ is calculated in Equation (13) below.

$$\tau_2 = \max \left\{ D_{\text{MS}}(SV, LV)/D(SV, LV), D_{\text{MS}}(AFV, SV)/D(AFV, SV) \right\} = 0.55$$

Thus, the value of $\tau_1$ can also be obtained $\tau_1 = 1 - \tau_2 = 0.45$. The optimisation problem is solved by iteration according to Equation (12). The results are illustrated in Figure 15, and the optimal lane-changing time $t_f = 3.47 s$ is obtained.

At the condition of $v_i = 25 m/s, v_f = 30 m/s, D(SV, LV) = 10 m, D(AFV, SV) = 20 m$, the optimal lane-changing time $t_f$ equals 3.4 s. Meanwhile, the maximum later acceleration $a_{\text{max}}$ equals 1.79 m/s$^2$, the maximum yaw angle $\varphi_{\text{max}}$ equals 0.081 rad, the minimum safety distance from LV $D_{\text{MS}}(SV, LV) = 5.98 m$, the minimum safety distance from AFV $D_{\text{MS}}(AFV, SV) = 11 m$. The comparison of lane-changing parameters when $t_f = 3.4 s, t_f = 2 s$ and $t_f = 8 s$ are shown in Table 5.

It can be seen that the two safety distances at $t_f = 3.4 s$ are safer than those at $t_f = 2 s$ and more efficient than those at $t_f = 8 s$.

This trajectory optimisation method can automatically select the proportion of efficiency and comfort according to the spacing relationship with surrounding vehicles, which is not available in other methods. Therefore, the global optimal lane change time and trajectory parameters can be obtained. This method can be applied in the driving assistance system to assist the driver in decision-making and operation adjustment. In the future, if the driver’s style can be added, this method will be more customised.

7 | CONCLUSION

This study proposes a novel method of lane change trajectory optimisation, which aims to solve the traffic conflict caused by the contradiction between lane-changing safety and efficiency. First, by quantifying the environmental traffic constraints of the lane-changing vehicle, the constraints of trajectory optimisation are obtained. Then, the lane change trajectory data is obtained by driving simulation experiments, and a quintic polynomial lane change trajectory model based on time is established. Third, the objective function of lane-changing efficiency and comfort is established to optimise lane-changing trajectory. Finally, an example is given to illustrate the application of the model. Some key findings can be drawn as follows:

1. Three kinds of safety constraints between SV and LV, AFV and SV, SV and ALV are quantified, expressed as $D_{\text{MS}}(SV, LV), D_{\text{MS}}(AFV, SV)$ and $D_{\text{MS}}(SV, ALV)$, respectively.
2. A time-dependent quintic multinomial lane-changing trajectory is proposed. According to the law of operation data acquired by driving simulation experiments, the coefficients of the multinomial are determined, and the predicted trajectory is close to the actual value.
3. The objective function of lane-changing time is established based on the constraint of lane-changing comfort and efficiency, characterised by lateral acceleration and lane-changing time, respectively.
4. A case study is applied to verify the model. Assume that the lane-changing vehicle changes a lane from the middle lane at the speed of 90 km/h to the faster lane at the terminal speed of 110 km/h. In this case, the optimal $t_f$ equals 3.4 s, $a_{\text{max}}$ equals 1.79 m/s$^2$, $\varphi_{\text{max}}$ equals 0.081 rad, $D_{\text{MS}}(SV, LV)$ equals 5.98 m and $D_{\text{MS}}(AFV, SV)$ equals 11 m.

The novel lane-changing trajectory optimisation model can be applied in driving assistance. It is the development trend that the contradiction between comfort and efficiency of

| $t_f$ (s) | $a_{\text{max}}$(m/s$^2$) | $\varphi_{\text{max}}$(rad) | $D_{\text{MS}}(SV, LV)$(m) | $D_{\text{MS}}(AFV, SV)$(m) |
|----------|-----------------|-----------------|-----------------|-----------------|
| 2        | 5.4             | 0.133           | 5.515           | 10.257          |
| 3.4      | 1.79            | 0.081           | 5.98            | 11.00           |
| 8        | 0.34            | 0.032           | 7.662           | 19.525          |
lane-changing solved by machines and provide decision-making. In the future, the parameters in the model can be customised according to different driving styles (aggressive, ordinary and conservative), which can achieve better results.

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