Changes in solar oscillation frequencies during the current activity maximum: analysis and interpretation

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Abstract. We describe systematic changes in the centroid frequencies and the splitting coefficients as found using data from MDI on board SOHO, covering cycle 23. The data allow us to construct a seismic map of the evolving solar activity – covering all latitudes. At lower latitudes, the temporal evolution closely tracks that of butterfly diagram. The additional information from higher latitudes in the map is of a significant activity in the polar region, peaking at activity minimum in 1996. The most plausible source of solar oscillation frequency changes over the solar cycle is the evolution of the radial component of the small-scale magnetic field. The amplitude of the required mean field changes is \( \sim 100 \) G at the photosphere, and increasing going inward.

1. Phenomenology of frequency variations

Using satellite intensity data from cycle 21, Woodard & Noyes (1985) first noted that p-mode frequencies increase with increasing solar activity. This discovery was confirmed by a number of investigations made during cycle 22. In particular, it was then established that the most significant changes occur in the antisymmetric part of the spectrum of oscillation multiplets – that is, the part that reflects the asphericity of the sun (Kuhn, 1988; and Libbrecht & Woodard, 1990). The dominant part of the multiplet structure is symmetric and arises from advection by rotation.

Oscillation data, from the on-going spatially resolved observations, are represented as centroid frequencies, \( \bar{\nu}_{\ell n} \) and splitting coefficients, \( a_{2k,\ell n} \), in the following expression for the frequencies of the individual modes

\[
\nu_{\ell nm} = \bar{\nu}_{\ell n} + \sum_{k=1} a_{k,\ell n} P_{k,\ell}.
\]  

The quantities \( P \) are orthogonal polynomials of \( m \) defined for \( 2k \leq \ell \) (see Ritzwoller and Lavely 1991 and Schou, et al. 1994).

We use MDI data, which contains coefficients up to \( k = 18 \) for about 2000 p- and f-modes with \( \ell \leq 300 \). There are 24 sets of data corresponding to 72 day long measurements done between May 1996, when the sun was at its activity minimum, to June 2001, when the sun was in it high activity phase.
The relative changes of solar frequencies are of order $10^{-4}$, which does is comparable to the individual measurement errors. Significant rates of change are obtained by binning the data or by assuming, as Libbrecht and Woodard (1990) did, that the changes scale inversely to mode inertia, which what is expected if the activity related changes acts only near the surface. Following this idea, we write the frequency changes in the form

$$\bar{\nu}_{\ell,n} - (\bar{\nu}_{\ell,n})_{\text{min}} = \frac{\gamma_0}{I_{\ell,n}},$$

where the subscript “min” refers to measurements made at solar minimum. We assume the same scaling for the variable part of the $a$ coefficients of even orders, and thus, we write

$$a_{2k,\ell,n} = a_{2k,\ell,n;\text{rot}} + C_{k,\ell} \frac{\gamma_k}{I_{\ell,n}},$$

The numerical factor $C_{k,\ell}$ (see Dziembowski, et al, 1999, for the explicit expression and its justification) was introduced to make each $\gamma_k$ an unbiased probe of the $P_{2k}$ distortion of the sun. Here, we removed the constant contribution from the centrifugal force, which is the only non-negligible effect of rotation.

The constant values of $\gamma_k$ inferred from the p-mode data show systematic evolution as solar activity varies. The coefficients, $\gamma_k$, up to $k = 9$ are in excellent correlation with the corresponding coefficients from the Legendre polynomial expansion of the Ca II K line intensities, which are regarded as a good proxy for the magnetic field (Dziembowski et al. 2000).

For p-modes, we expect that the $\gamma$’s do not depend on $\ell$, if their source is localized near the surface. *A priori*, we might expect a significant frequency dependence, but it is not very strong. This tells us something the precise localization of the source. For the f-modes, where we have the approximate proportionality, $\nu \propto \sqrt{\ell}$, a component of $\Delta \nu$ was determined, which is also $\propto \sqrt{\ell}$ and grows with increasing activity. This component may be interpreted as evidence for a contraction of the sun’s outer layers as activity rises. Such contraction would take place if the increase of the field were dominated by its radial component (Dziembowski, Goode & Schou, 2001)

2. Seismic map of the sun’s activity

Having determined the $\gamma_k$ coefficients as functions of time, we can construct a seismic map of varying solar activity. To do this, we determine the quantity,

$$\gamma(\theta, t) = \sum_{k=0}^\infty \gamma_k(t) P_{2k}(\cos \theta).$$

In Figure 1, we show the changes in $\gamma$ averaged into bins 5°-wide in latitude. At lower latitudes, we recognize features that are well-known from the butterfly diagram: activity appearing first at $35 - 45^\circ$, and gradually moving toward the equator. However, there is also unexpectedly large activity in the polar regions. This decreases with the rise of activity at the lower latitudes. This result is not an entirely surprising one, because it has been known for some time that the polar field flips at activity maximum. Furthermore, Moreno-Insertis & Solanki
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Figure 1. Zonal averaged values of $\gamma$ in $\mu$Hz. The ranges of the latitude are given in the legend, and the overall zero-point is arbitrary.

(1999) have already found that their models for low $\ell$ mode frequency behavior, during the activity cycle requires, fields at high-latitude.

3. The cause of the frequency changes

There is ample evidence that the behavior of the $\gamma$s in some way reflects changes in the magnetic field in the outermost layers of the sun. We would like is to infer more about the field: its intensity and its behavior as a function of the depth below the photosphere. For this it is essential to consider the role of changes in the temperature and turbulent velocity induced by the magnetic field. A significant role of the latter effect may be eliminated (Dziembowski and Goode, in preparation).

Following Goldreich et al. (1991), we consider a small-scale, random magnetic field as the primary cause of the frequency changes in which we allow the vertical ($r$) component to be statistically different from the two horizontal ($\theta$ and $\phi$) ones. In this, the averaged values $\overline{B^2}$ are treated as functions of depth, and slowly varying functions of the co-latitude. The latter dependence is represented in the form of a truncated Legendre polynomial series,

$$\overline{B_i B_j} = \delta_{ij} \sum_{k=0}^{\infty} \left[ \delta_{jr} M_{r,k}(r) + \frac{1}{2} M_{H,k}(\delta_{j\theta} + \delta_{j\phi}) P_{2k}(\cos \theta) \right], \quad (5)$$

where we included only seismically detectable (symmetric about equator) terms.

Each of the $k$-components gives rise to a $P_{2k}$ distortion of the sun’s structure. For $k > 0$, the hydrostatic equilibrium condition suffices to determine the distortion of all the thermodynamical parameters.
In the evaluation of the $\gamma$s, which in general must be treated as functions of both $\nu$ and $\ell$, we use the variational principle for stellar oscillations in which we treat the effects of the magnetic field as a small perturbation. This principle, with use of hydrostatic equilibrium and equations for adiabatic oscillations, leads to

$$\gamma_k = \int \left( K_{k,T} \frac{\Delta T}{T} + K_{k,r}^B M_{r,k} + K_{k,H}^B M_{H,k} \right) dD,$$  \hspace{1cm}(6)

where $D$ is depth. All the kernels, $K$, may be explicitly expressed in terms of parameters of the standard solar model and the radial eigenfunctions of its p-modes. If the magnetic perturbation is significant only in the layers well-above the lower turning points of all p-modes considered, then the kernels are both $k$ and $\ell$ independent.

Goldreich et al. (1991) considered only changes in centroid frequencies. They pointed out that to explain the frequency increase between 1986 and 1988 a 1% decrease of photospheric temperature is needed ($K_{k,T}$ is always $< 0$). Regarding this requirement as being incompatible with observations, they adopted the changing magnetic field as the sole cause of the frequency increase; they found that the field increase must be about 250 G at the photosphere, and steadily growing to about 1 kG at a depth of 10 Mm. Their numbers refer to the case of a statistically isotropic field ($M_{H,k} = 2M_{r,k}$). A much more modest field increase ($< 100$ G at the photosphere) would result for an inwardly growing, pure radial field. A similar result was obtained by us (Dziembowski, Goode & Schou, 2001) from our analysis of the MDI data.

For $k > 0$, the term involving temperature change may be eliminated. In the outer layers, the signs of the modified kernel are $K_{k,r}^B > 0$ and $K_{k,H}^B < 0$. Thus, the most economical requirement is a pure radial field. With the assumption that all the frequency changes are due to an increase of the radial component of the small-scale magnetic fields, the plots in Fig. 1 may be scaled from $\mu$Hz to Gauss using the factor $\approx (190 G)^2 / 1 \mu$Hz.

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