Partons in the chiral periphery of the nucleon

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Abstract. We introduce transverse densities in the study of parton dynamics in the proton’s peripheral region. We calculate these densities using chiral perturbation theory (χPT) and parametrize the long distance structure of the nucleon in a model independent framework in which we identify chiral ($b \sim O(1/M_\pi)$) and molecular ($b \sim O(M_N^2/(M_\pi^2))$) parametrical regions. Through the light cone formulation of the nucleon’s electromagnetic current in χPT, one calculates transverse densities from local products of light cone wave functions of a pion-nucleon system. These products are understood as 2-dimensional parton distributions and as such are universal in processes that probe the nucleon’s periphery. This universality and also that of the corresponding transverse density can be tested and used in experimental and phenomenological studies of reactions such as high energy proton-nucleon/nucleus collisions and in electron proton scattering as well as in low $Q^2$ extraction of baryonic form factors.

1. Introduction
The internal structure of the nucleon has been investigated for decades mainly by extracting structure functions from deep inelastic scattering experiments at dedicated facilities. What these structure functions can tell us about the nucleon relies on how they are interpreted through the parton model as superposition of parton distributions in which what the electromagnetic or electro-weak probe sees is the structure of the nucleon in the infinite momentum frame as a collection of quasi-free point-like particles. The parton model predicted the scale invariance of these distributions at high momentum transfer, away from the resonance region, and quantum chromodynamics (QCD) provided corrections to this approximation by way of evolution equations through which parton distributions take into account the fact that the internal dynamics of the nucleon is ultimately dictated by a relativistic quantum field theory. However, despite their universality, and the acknowledgment of QCD, a first principle calculation of these distributions has been prevented by the nonperturbative nature of hadrons as relativistic multiparticle systems governed by QCD. These limitations significantly reduce the quantitative insight into the origin of the global properties of the nucleon that can be gained through QCD alone, and model dependent descriptions supported by phenomenology have to be invoked.

At low energies however, the dynamical description of the strong interaction can be approached by means of effective theories. Chiral effective field theory in particular provides a general framework which arises from the chiral invariance of the QCD Lagrangian with massless quarks. The spontaneous breaking of this symmetry permits a quantitative description of hadron interactions through perturbative expansions dictated by power counting schemes on the pion mass or momentum. One can also make use of χPT for quantifying certain features of the structure of the nucleon that are relevant at the momentum or spatial scale characteristic of
chiral dynamics. As opposed to what happens at high momentum, the structure of the nucleon at large distance scales can be addressed in a model independent manner through $\chi$PT. We define the chiral periphery as the region in impact parameter space in which the internal dynamics of the nucleon is governed by leading terms in $\chi$PT.

We identify the chiral periphery through transverse densities, which are shown to profile the nucleon’s spatial structure in characteristic parametric regions. In the nucleon’s periphery, these transverse densities are computed from form factors in turn obtained from invariant $\chi$PT, or as it will be explained below, from pion-nucleon wave-functions derived in the light front formulation. The latter allows a mechanical interpretation of transverse densities, giving them their quantum classical meaning in impact parameter space which in turns directly connects the associated experimentally observable form factor to a space-time picture of the nucleon as a relativistic multiparticle system, a feature that form factors by themselves cannot provide.

2. Transverse densities and parametric regions
For the electromagnetic form factors $F_1$ and $F_2$ that covariantly expand the electromagnetic current, the corresponding transverse densities are defined through [1],

$$\rho_{1,2}(b) \equiv \int \frac{d^2\Delta}{(2\pi)^2} e^{-i\Delta \cdot b} F_{1,2}(-\Delta^2).$$

The above equation is modified by using dispersion relations that use the analytic properties of the form factors. Then, to calculate the corresponding transverse densities, it suffices with computing the imaginary part of the form factors or spectral functions along the 2-pion cut in the positive real axis of the complex $t = \Delta^2$ plane [2, 3],

$$\rho_{1,2}(b) \equiv \int_{-\infty}^{\infty} \frac{dt}{4M_\pi^2} K_0(\sqrt{tb}) \frac{\text{Im} F_{1,2}(t)}{\pi},$$

in which $K_0(\sqrt{tb})$ is a modified Bessel function which filters out contributions from large momentum modes ($\sqrt{t} > 1/b$) to the transverse density parameterized by $b$. Transverse densities then become a tool for identifying the kinematic structure and potentially the relevant internal dynamics of the nucleon at a particular region in impact parameter space.

Pion current contribution to the nucleon’s electromagnetic form factors $F_1$ and $F_2$ can be calculated in invariant perturbation theory using a general chiral Lagrangian (see, e.g., Ref.[4]),

$$\mathcal{L}_{\pi N} = -\frac{g_A}{2F_\pi} \bar{N} \gamma^\mu \gamma_5 \tau N \cdot \partial_\mu \pi - \frac{1}{4F_\pi^2} \bar{N} \gamma^\mu \tau N \times \partial_\mu \pi.$$

The leading terms are generated by a pion nucleon axial vector coupling and by a contact term as illustrated in Fig.1.

Figure 1. Leading pion current diagrams in computing the nucleon electromagnetic form factors in $\chi$PT. (a) Pion loop with axial vector $N\pi N$ coupling. Equivalent to pseudoscalar coupling plus contact term with strength $-g_A^2$. (b) Pion loop diagram with contact coupling with strength 1.
In Ref. [2], by computing transverse densities through \( \chi PT \) and Eq. (2), the internal transverse profile of the nucleon is segmented in three parametric regions (see Fig. 2): Non chiral \( (b < O(M_\pi^{-1})) \), chiral \( (b \sim O(M_\pi^{-1})) \) and molecular \( (b \sim O(M_N^2/M_\pi^2)) \). The non-chiral region is characterized by high momentum kinematics and thus cannot be studied through \( \chi EFT \).

Expansions in the pion's momentum converge in the chiral region while the molecular region arises from a sub threshold singularity \( (t_{sub} = 4M_\pi^2 - M_N^4/M_\pi^2) \) in the corresponding spectral function which limits the convergence of kinematic expansions around the two pion threshold.

The perturbative nature of the chiral region supports a picture of quasi-free relativistic pions populating the chiral periphery of the nucleon. Furthermore, it is found also that in the chiral region the spin-dependent \( (\tilde{\rho}_2, \text{see Eq.} 5) \) and spin independent \( (\rho_1) \) components of the light-cone transverse current density are of the same parametric order and that in general they obey an inequality that guarantees the positive definiteness of the light-cone pion current. These features reinforce a mechanical picture in which the nucleon is composed by a nucleon-like core weakly interacting with a relativistic pion [5, 6].

### 3. Partonic pions

Form factors can be written as overlap of light-cone wave functions in internal transverse momentum space [7]. In light-front dynamics, one studies the evolution of multiparticle systems in light-cone time \( (x^+ = ct + z) \) [8]. It makes possible the study of such dynamical systems in a relativistic framework. In the chiral region, the light-cone wavefunctions can be written as pion-nucleon components in a Fock expansion of the nucleon wave function with pseudoscalar coupling \( (\bar{N}\gamma_5\tau N \cdot \pi) \) (see e.g., Ref.[9]). If the axial-vector coupling provided by the general chiral Lagrangian is used instead in light-cone perturbation theory, one needs to include a contact term correction which contributes of the order of 10% in the chiral region and that is said to account for states with higher masses than the nucleon-like spectator considered here [11].

Using \( \pi N \) light-cone wavefunctions in relative transverse position space computed with a pseudoscalar coupling, the transverse densities are found to take the following form [6],

\[
\rho_1 = \frac{1}{2\pi} \int \frac{dy}{2y} \left[ \psi_1^2 \left( y, \frac{b}{y} \right) + \psi_2^2 \left( y, \frac{b}{y} \right) \right] \\
\tilde{\rho}_2 = \frac{\partial \rho_2(b)}{\partial b} \frac{1}{2M_N}
\]

**Figure 2.** Parametric regions of the nucleon in transverse space. Transverse densities define regions in impact parameter space with characteristic dynamics which controls the relevant kinematics. The structure of the non-chiral core is fitted by phenomenology for instance through vector meson dominance models of the \( \gamma^*N \) interaction. The chiral region is described through model independent \( \chi PT \). In the molecular region \( \chi PT \) cannot be truncated. It is dominated by the dynamics near the two pion threshold, and because of the large scale distances involved it is mainly only of theoretical interest.
In which \( y = k^+/p^+ \) is the fraction of the total light-cone momentum carried by the pion in the \( \pi N \)-system, \( \bar{y} = 1 - y \), and \( \psi_{1,2}(y,b/\bar{y}) \) are the spin non-flip and spin-flip amplitudes of the \( \pi N \) component of the nucleon light-cone wave function. Their explicit form in the invariant formalism as well as various properties that hint to a first quantized approach to this multiparticle system are highlighted at length in Ref.[2]. In these forms (Eq.5), transverse densities are thus recognized in the classical quantum mechanical sense as probability densities in impact parameter space. This association is in close connection with a partonic interpretation of pions in the nucleon’s chiral periphery. This is made explicit by identifying the integrand of the above representations with general parton distributions of pions in impact parameter representation [10]. The explicit form agrees with that obtained through the pion light-ray operator in Ref. [11], in which it is argued that gluon GPDs of the nucleon at low \( x \) can be computed from gluon GPDs in pions in convolution with pion GPDs in nucleons, and that using the universality of these pion GPDs, gluon GPDs in pions can be extracted from experiments on diffractive hard pion emission in photon or vector meson production off the nucleon since these processes can be factorized (see Fig.3). Further applications and studies of the universal properties of these distributions can also be found in experimental studies of high energy \( NN \) exclusive reactions where the periphery can be selected from the kinematics of the final states for instance by selecting high transverse momentum pion-pairs produced in high energy, low \( t \sim O(M^2_\pi) \), \( NN \) collisions [2].

**Figure 3.** Transverse parton densities in processes at hard kinematics. (Left) Factorization of gluon parton distributions in the nucleon’s peripheral region. (Right) Leading chiral contribution to diffractive meson production with hard pion emission.

4. Outlook

We presented a partonic picture of the nucleon’s periphery which arises from the onset of chiral dynamics. Parton distributions, or more precisely general peripheral parton distributions of pions can be computed from the pion-nucleon light cone wave functions obtained starting from \( \chi PT \). Additionally, the moments of these distributions are directly related to distributions of kinematic contributions from parton species to the nucleon’s intrinsic physical properties, which opens the possibility of quantifying the peripheral contributions to these properties and of constraining QCD inspired model predictions of these properties as they approach to the chiral limit. Of particular interest is the distribution of orbital angular momentum, motivated by Ji’s sum rule [12] connection with GPDs which are experimentally measurable in for instance deeply virtual Compton scattering or time like Compton scattering reactions which are part of the nucleon structure program at PANDA [13].
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