The $e^+e^-$ pair production at $\mu^+\mu^-$ collider

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Abstract

The main source of $e^+e^-$ pair at $\mu^+\mu^-$ collider is incoherent process $\mu^+\mu^- \rightarrow \mu^+\mu^-e^+e^-$ with cross section about 10 mb (at muon energy 2 TeV). The corresponding distributions are discussed briefly. These pairs give the dangerous background.

The coherent production of $e^+e^-$ pairs here is negligibly small.

The possible $\mu^+\mu^-$ collider is discussed widely now (see e.g. [1]) with the expected set of parameters:

$$E = 2 \text{ TeV} \; (\gamma_\mu = 2 \cdot 10^4); \quad L = 2 \cdot 10^{35} \text{ cm}^{-2}\text{s}^{-1};$$

$$N = 2 \cdot 10^{12}; \sigma_{x,y} = 3\mu m; \sigma_z = 3\text{mm}; f_{\text{rep}} = 30\text{s}^{-1}; \quad B \sim \frac{eN}{2\sigma_x\sigma_z} \sim 0.5 \cdot 10^3 T. \quad (1)$$

Here $B$ is the characteristic magnetic field of bunch at the collision point.

The $e^+e^-$ pair production in this collision seems to be source of dangerous background [2]. We consider two mechanisms of this production — standard incoherent production, $\mu^+\mu^- \rightarrow \mu^+\mu^-e^+e^-$, and coherent production on the collective electromagnetic field of the oncoming bunch.

1 Incoherent production, $\mu^+\mu^- \rightarrow \mu^+\mu^-e^+e^-$

The production of $e^+e^-$ pairs at the collision of charged particles was considered first 60 years ago [3]. The asymptotical behaviour of total cross section was obtained in this paper. The numerous papers related to the more detail and precise treatment of this process are reviewed in ref. [4].

The similar process $pp \rightarrow ppe^+e^-$ have been considered in details in ref. [5]. Both the precise equations and the detailed qualitative description of the main features of process with numerical estimates were presented in this paper (to use this process for the luminosity monitoring at ISR). The equations there are valid for our case with the evident renotations, some numerical estimates should be reconsidered due to much higher Lorentz-factor in our case.

We present here some general characteristics of the process for the preliminary rough estimates.

1) The total cross section of the process is [3]:

$$\sigma = \frac{28\alpha^4}{27\pi m_e^2} \left[ (l - 2.12)^3 + 2.2(l - 2.12) + 0.4 \right] \approx 10 \text{ mb}. \quad (2)$$
2) The main mechanism of process is two–photon production of $e^+e^-$ pairs via collision of two virtual photons, emitted by muons. Main features of process can be obtained with the equivalent photon (Weizsacker–Williams) approximation (EPA). The spectra of virtual photons are obtained from the analysis of Feynman diagrams. Their dependence on the energy $\omega$ and virtuality $Q^2$ has the form

$$dn(\omega, Q^2) = \frac{\alpha}{\pi} \frac{dx}{x} \frac{dQ^2}{Q^2} \left[ 1 - x + \frac{x^2}{2} - (1 - x)\frac{Q^2_{\min}}{Q^2} \right]; \quad x = \frac{\omega}{E},$$

where $Q^2_{\min} = \frac{m_e^2 \mu}{1 - x}$.

Last inequality is obtained easily from kinematics.

The cross section of subprocess $\gamma\gamma \rightarrow e^+e^-$ has maximum at the effective mass squared of produced system $W^2 \approx 8m_e^2$. Therefore, the effective mass of produced $e^+e^-$ system is near the threshold, the transverse momenta of produced particles are $\sim m_e$. Besides, this cross section decreases quickly with the growth of virtuality above $m_e^2$. In other words, the main contribution to the cross section is given by the region

$$m_e^2 > Q^2 > Q^2_{\min}.$$

Using eq. (4), we obtain from here limitation for the energy of photon for the process:

$$x = \frac{\omega}{E} < \frac{m_e}{m_\mu} \Rightarrow \omega < \gamma m_e.$$

Therefore, — in accordance with the "naive" expectations — the Lorentz–factor of produced $e^+e^-$ pair cannot be higher than that of the initial muon.

Besides, the number of equivalent photons for this production is obtained by integration of eq. (3) over virtuality, it is

$$dn(\omega) = \frac{2\alpha}{\pi} \frac{dx}{x} \left[ \ln \left( \frac{m_e}{m_\mu x} \right) - \frac{1}{2} \right].$$

Note, that this quantity is much lower than that for the two–photon production of muons or hadrons, which is

$$\sim (2\alpha/\pi)(dx/x) \ln(1/x).$$

The source of this difference is the much higher upper limit of effective virtualities for these processes, which determined by the much higher scale of the $Q^2$ dependence for these subprocess. (For more detail discussion see ref. [4].)

3) The produced pairs distributed uniformly in the rapidity scale. The distribution over the total energy of pair $\epsilon$ is

$$d\sigma = \begin{cases} \frac{56\alpha^4}{9\pi m_e^2} \frac{dk_z}{\epsilon} \left[ \ln^2 \gamma - \ln^2 \frac{\epsilon}{m_e} \right] & \text{at } \epsilon < \gamma m_e; \\ \sim \frac{\alpha^4}{m_e^2} \frac{d\epsilon}{\epsilon} \left( \frac{m_e}{\epsilon} \right)^2 & \text{at } \epsilon > \gamma m_e. \end{cases}$$

Here $k_z$ is the longitudinal momentum of the pair, $|k_z| \approx \epsilon$.

Mean energy of pair is $\sim 2m_e\gamma/\ln \gamma \sim 2$ GeV. In accordance with eq. (5), the number of pair produced is about $10^5$ per bunch collision, i.e. about $10^8$ during the life of bunch.
Therefore, the entire energy losses due to the discussed process are about $2 \cdot 10^{-6}\%$, i.e. negligible.

4) The distribution over the energy of one electron $\epsilon_1$, emitted along the motion of initial $\mu^+$, has the form

$$d\sigma = \frac{56\alpha^4}{9\pi m_e^2} \frac{dk_{1z}}{\epsilon_1} \left[ \ln^2 \gamma - \ln^2 \frac{\epsilon_1}{m_e} \right] \quad (\epsilon_1 \gg m_e).$$  \hspace{1cm} (9)

Here $k_{1z}$ is the longitudinal momentum of the electron. The effective mass of produced pairs is near the threshold and their total transverse momentum is very low. Therefore, the main part of produced electrons moves initially precisely along the beam.

However, as it was pointed out by Palmer, the created electrons are deflected by the magnetic field of opposite beam. It is easily seen that the electrons with the energy $< 100$ MeV have Larmor radius $R = \epsilon_1/B$ less than 1 mm, and they are invisible in the detectors. The electrons with the energy $100$ MeV $< \epsilon_1 < 1$ GeV have Larmor radius between 1mm and 1cm, they are dangerous for the vertex detectors. Some part of these electrons can reach main detector. In accordance with eq. (9), the corresponding cross section is about 1 mb for each direction (along $\mu^−$ and along $\mu^+$), we expect about 10 000 of these electrons (or positrons) per bunch crossing. Last, electrons with highest energy $1$ GeV $< \epsilon_1 < 10$ GeV are deflected at the angle $\beta = e^2 \bar{N}/(2\sigma_x \epsilon_1) > 50$ mrad, to the main detector. The corresponding cross section is about 0.4 mb for each direction, with 4 000 electrons and total energy flux about 10 TeV per bunch crossing.

The more detail studies of these problems are necessary.

2 Coherent production

The coherent production was considered in the paper [2]. It is based on the following facts: The electromagnetic field of bunch $B$ is rather large. Its ratio to the Schwinger critical field for the creation of $e^+e^−$ pairs from vacua $B_c$ in our case is small, $B/B_c \approx 2 \cdot 10^{-7}$. But the quantity $(E/m_e)(B/B_c) \sim 0.4$.

The idea of the paper [2] looks very nice: Photons (either real or virtual) from one bunch which traverse the electromagnetic field of other bunch would turn into $e^+e^−$ pairs. The rate of this process is determined by the parameter (related to electrons only!)

$$\chi = \frac{\omega}{m_e B_c} \left( B_c = \frac{m_e^2 c^3}{e\hbar} \approx 4.4 \cdot 10^{13} \text{ G} \right). \hspace{1cm} (10)$$

The probability of pair creation per unit time can be written as $\hbar = c = 1$ \[8\]

$$\frac{3\sqrt{3}}{16\sqrt{2}} \frac{\alpha m^2}{\omega} \chi \exp \left( -\frac{8}{3\chi} \right). \hspace{1cm} (11)$$

The photons with energy $\omega \approx E$ with $\chi \sim 0.4$ exist in the spectrum of equivalent (virtual) photons, for these photons large enough production of $e^+e^-$ pairs is expected. The number of produced $e^+e^-$ pairs is obtained by convolution of this probability with the spectrum (7).

Unfortunately, the last point is inexact. Indeed, the spectrum of equivalent photons depends strongly on the nature of both particle created and system produced. The simple
equation (3) for the spectrum is valid at \( Q^2 < Q_{\text{max}}^2 \). The quantity \( Q_{\text{max}}^2 \) determined in our case by the nature of subprocess considered, it is \( \hat{s}/4 \) where \( \hat{s} \) is the effective mass of the produced system. The main contribution is from region near the threshold. Therefore, \( Q_{\text{max}}^2 \sim m_e^2 \). (High value of \( Q^2 \) corresponds to small size of production region.)

At \( Q^2 > Q_{\text{max}}^2 \) the quantity \( d\eta \) decreases quickly, the contribution of this region into a flux of equivalent photos is negligible. The inequality \( Q_{\text{max}}^2 < Q_{\text{min}}^2 \) results in limitation for the virtual photon energy (3) and spectrum (3), which differs strongly from the spectrum (7). For this spectrum the parameter \( \chi \) is limited from above by the value \( \chi_{\text{max}} = \gamma B/B_c \approx 0.002 \). It means that the coherent production of \( e^+e^- \) pairs is negligibly small.

The same very conclusion can be explained by other words:

The spectrum (3) has no relation to the \( e^+e^- \) production. In fact, the virtual photon is described by both energy \( \omega \) and virtuality \( Q^2 \geq Q_{\text{min}}^2 \) (3,4). The virtuality \( Q^2 \) (if it is higher than \( m_e^2 \)) determine the scale of production region, so that for such photon the critical field for \( e^+e^- \) pair production become \( \sim (Q^2/m_e^4)B_c \). The more precise interpolation equation, obtained from the analysis of corresponding dependence for the \( e^+e^- \) production on real photon, is

\[
B_c \Rightarrow \frac{m_e^2 + Q^2/6}{m_e^2} B_c.
\]

The probability of \( e^+e^- \) coherent production should be obtained by the convolution of the spectrum (3) with the probability (11) related to this critical field. Using for \( Q^2 \) its least value for given photon energy (3), we obtain that the maximal value of the quantity \( \chi \) is reached at \( x \approx 2.5(m_e/m_\mu) \), it is \( \sim \gamma (B/B_c) \approx 0.002 \).

I am very thankful to K.J. Kim, N. Mokhov, R. Palmer, A. Sessler, A. Skrinsky V. Serbo, K. Yokoya, M. Zolotarev for useful discussions. This work is supported by grants of Russian Foundation of Fundamental Investigations and INTAS–93–1180.

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