Switching Current Distributions in Josephson Junctions at Low Temperatures Resulting From Noise Enhanced Thermal Activation

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Experiments on the distributions of switching currents in Josephson junctions are sensitive probes of the mechanism by which a junction changes abruptly to a finite voltage state. At low temperatures data exhibit smooth and gradual deviations from the expectations of the classical theory of thermal activation over the barrier in the tilted washboard potential. In this paper it is shown that if a very small proportion of the noise energy entering the apparatus at room temperature survives filtering and reaches the sample, it can enhance the escape rate sufficiently to replicate experimental observations of the temperature dependence of the switching bias. This conjecture is successfully tested against published experimental data.

I. INTRODUCTION

When the bias current applied to a Josephson junction is gradually increased, the junction will eventually switch to a non-zero voltage state. For repeated trials carried out at the same temperature, each commencing at zero bias current, this switching occurs at slightly different values of bias current. Accumulated escape data form a switching current distribution (SCD) whose peak defines the most probable value for the escape bias. For temperatures above about 100 mK, these characteristics are well described by the classical theory of thermally activated escape out of the washboard potential, as discussed in the next section.

When dilution refrigerators became available, experiments were possible down to base temperatures as small as a few millikelvin. Leggett [1] had predicted that below a “crossover temperature” a Josephson junction would enter a macroscopic quantum state. Voss and Webb [2] were the first to claim confirmation of this conjecture with evidence based on observations reaching 5 mK and interpreted with the hypothesis of macroscopic quantum tunneling (MQT) as the new mechanism for escape from the well.

But as has already been pointed out [3], [4] the data in swept bias experiments [2], [5], [6] do not, on very close inspection, exhibit the temperature independence inherent in MQT theory. In fact SCD escape peaks always retained some slight temperature dependence. Therefore this crucial attribute of peak freezing had not been observed, thus casting doubt on the conjecture of a crossover.

At such low temperatures, the SCD peak behavior was instead observed to deviate smoothly from the classical prediction, and this fact requires an explanation. A modified classical thermal escape rate is proposed here and is successfully tested against data from three independent experiments.

II. THERMAL ACTIVATION

Thermal activation from a potential well at a given temperature is governed, in the low damping regime, by the well known escape rate due to Kramers [7].

\[ \Gamma = f_J \exp \left( -\frac{\Delta U}{k_B T} \right) \]  

where \( \Delta U \) is the barrier height and, for a Josephson junction, \( f_J \) is the plasma frequency \( f_J = f_{J0} \left( 1 - \eta^2 \right)^{1/4} \), with \( \eta = I/I_C \) denoting the normalized bias current. The barrier height in the Josephson washboard potential is given by

\[ \Delta U = 2E_J \left( \sqrt{1 - \eta^2} - \eta \cos^{-1} \eta \right) \]  

and \( E_J = \hbar I_C / 2e \) is the Josephson energy.

Fig.1 displays the output from an algorithm-based simulation [8], [9] (see [10]) carried out with the experimental parameters of Oelsner et al. [6] and the escape rate given in Eq.(1). Also shown are the data points from this experiment, digitized from Fig.1 in [6] and converted to a linear temperature scale. The gradual peeling away of
FIG. 1: Temperature dependence of peak positions in switching current distributions obtained from a simulation of the experiment of Oelsner et al. (solid line) using the escape rate given in Eq.(1), together with experimental data points (squares). The shaded area highlights the peeling away from the Kramers thermal activation prediction that occurs at the lowest temperatures.

Experimental data from the expectation based on the standard escape rate represents a deviation from classical thermal activation at the lowest temperatures. Such smooth peeling away is also observed in the experiments of Voss and Webb [2] and Yu et al. [5] - see Fig. 2.

FIG. 2: Experimental data (squares) and simulation results (lines) from Voss & Webb (left panel) and Yu et al. (right panel).

With linear temperature scales, it is very apparent that the deviation of experimental data from standard thermal activation characteristics is gradual and smooth, without any sign of an abrupt crossover.

III. ESCAPE RATES

The escape rate $\Gamma$ is a static function that can be evaluated directly from Eqs. (1 and 2). In Fig. 3 escape rates are calculated with the parameters of Oelsner et al. [6]. Each plot represents the evolution of $\Gamma$ over the course of a bias sweep at some specified temperature. As can be seen, the escape rate remains negligible until, in the neighbourhood of some bias, $\Gamma$ increases very rapidly. This upswing is comparatively gradual at higher temperatures and becomes more abrupt as the temperature is lowered. This property is the cause of SCD peaks becoming sharper as temperature...
FIG. 3: Escape rate $\Gamma$ as a function of normalized bias $\eta$ for temperatures $T = 0.200K$ (diamonds), $T = 0.100K$ (squares), and $T = 0.010K$ (circles). The junction critical current and zero bias plasma frequency were taken from Oelsner et al.

decreases. As simulations reveal, the peak in a switching current distribution is located close to the bias value at this abrupt upswing.

However, in Fig.3 the characteristic at $T = 10mK$ has an onset at about $\eta = 0.985$ which, it turns out, is too high; the experimental peak is closer to $\eta = 0.964$. Escape rates extracted from numerical solutions of the Langevin equation [11] at this temperature matched the curve in Fig. 3 and thus confirmed that the simple Kramers expression is still valid even down to such a low temperature [12]. Therefore, the origin of the discrepancy must be sought elsewhere.

IV. ENHANCED THERMAL ACTIVATION

For high temperatures, simulation results closely match experimental data. But as indicated in the previous section, discrepancies exist at the lowest temperatures. This is illustrated in Fig.4 which is based on the same experiments [6] used for Fig. 1.

FIG. 4: Low temperature region of the simulation shown in Fig.1 together with data points (squares) from the experiment of Oelsner et al. The arrows indicate the required lowering of the simulated peak positions at these temperatures.

Clearly, the simulation based on the escape rate in Eq.[1] does not agree with observations at the lowest temperatures. As indicated by the arrows in Fig.4, the simulation predicts SCD peaks lying above the observed switching current distributions, therefore an enhanced escape rate is implied.
The following ansatz is proposed:

\[ \Gamma = f_J \exp \left( -\varepsilon \frac{\Delta U}{k_B T} \right) \]  

(3)

where the parameter \( \varepsilon \) will be 1.0 for standard thermal activation and \( \varepsilon < 1 \) when there is an increase in the escape rate. So thermal activation would be enhanced for \( 0 < \varepsilon < 1 \).

To bring simulation into agreement with observation, the following sequence was followed: select a temperature; begin with \( \varepsilon = 1 \) and run a simulation to find the SCD preak position \( \eta_P \); iteratively decrease the value of the parameter \( \varepsilon \) in small steps, performing simulated bias sweeps and stop when the simulated peak closely matches the experimental value for that temperature; repeat for each temperature in the experimental dataset.

This iterative process was carried out for the experiments of Voss and Webb [2], Yu et al. [5], and Oelsner et al. [6]. The results are displayed in Fig.5.

![Figure 5](image.png)

**FIG. 5:** Escape rate enhancement parameter \( \varepsilon \) as a function of bath temperature for three experiments: Yu et al. (diamond), Voss & Webb (circles), Oelsner et al. (triangles).

The line at \( \varepsilon = 1.0 \) indicates when simulations based on Eq.(1) will accurately match the observations.

The procedure is illustrated by the following example based on the data of Oelsner et al. [6]. As noted earlier with respect to Fig.3, the 10mK escape rate ‘switched on’ at too high a bias current. From Fig. 5 it can be seen that at this value of \( T \), \( \varepsilon = 0.28 \) is appropriate to correct this problem. Fig.6 illustrates the change from an unmodified \( \Gamma \) as defined in Eq.(1) to a modified expression Eq.(3) - the upswing point can be seen to shift down to agree with the experimental SCD position of \( \eta_P = 0.964 \).

The values of \( \varepsilon \) determined in Fig.5 for the experiment of Oelsner et al. [6] were applied in a full simulation covering a broader range of temperatures. The results are shown in Fig.7.

As can be seen, the simulation precisely matches the experimental data, even below 50mK.

V. EXTERNAL NOISE

No experimental system consisting of a sample chamber in a dilution refrigerator and shielded from the room temperature world by means of isolation stages [13] and powder filters [14] can actually achieve a perfect noise free state. Some noise energy is bound to reach the sample chamber. The following two quotes bear on this issue:

1. “Moreover, the available concepts often provide inadequate filtering to operate at temperatures below 10mK” from [15]
2. "A residual noise temperature $T_N$, which can be minimized but never fully eliminated in this kind of experiments, can also lead to some ambiguities at low temperatures" from [10].

Therefore consider a fixed residual noise energy $E_N$ which combines with the thermal energy $E_T = k_B T_B$, where $T_B$ is the bath temperature, to activate escapes from the Josephson potential well. Then from Eq.(3),

$$\frac{\varepsilon}{k_B T_B} = \frac{1}{k_B T_B + E_N}$$

Hence,

$$\varepsilon = \frac{T_B}{T_B + E_N/k_B}$$

(4)

note that the noise factor $E_N/k_B$ has dimensions of temperature. This expression quantifies the inherent temperature dependence of $\varepsilon$ in terms of both bath temperature and $E_N/k_B$. Clearly $\varepsilon \rightarrow 1$ as $E_N \rightarrow 0$. Fig.8 shows $\varepsilon(T_B)$ curves for various values of $E_N/k_B$.

There are two procedures with which the noise factor $E_N/k_B$ may be estimated from experimental data.

1. From Eq.(4) $\varepsilon$ will equal 1/2 when the bath temperature is equal to the noise factor; hence $T_B = E_N/k_B$. Considering the experimental data for Oelsner et al. [6] in Fig.5 $\varepsilon = 0.50$ for $T_B \approx 20mK$ and so the noise factor for that experiment must be $E_N/k_B \approx 20mK$. 

FIG. 6: Thermal activation escape rates, with and without the correction factor $\varepsilon$, for the parameters of Oelsner et al.

FIG. 7: Final results combining a simulation that uses $\varepsilon$, and experimental data from Oelsner et al.
2. Even if the bath temperature were to reach zero, the energy from noise would still be available to enable escapes from the well. Data points for Oelsner et al shown in Fig.4 extrapolate to a limiting SCD peak of $\eta_P \approx 0.965$ at $T_B = 0$. It is then only necessary to examine the simulation characteristic $\eta_P$ vs $T$ as shown in Fig.1 (solid line) to obtain a temperature corresponding such a peak position; hence $E_N/k_B \approx 30 mK$.

From Fig.5 it is apparent that the residual noise in Oelsner et al. is the lowest of the three experiments.

VI. SUMMARY

At the lowest temperatures, experiments exhibit a smooth transition away from the predictions of the standard Kramers’ escape rate, Eq.(1). Figure 4 clearly indicates that an increase of the escape rate, specified by $\varepsilon$ in Eq.(3), is needed to match simulations with experimental data at these low temperatures. The procedure to determine $\varepsilon$ is based on a fast algorithm (10) that simulates a swept bias experiment and yields the SCD peak for any selected temperature. The obvious consistency of this approach over the three experiments [2],[5],[6], as indicated in Fig.5, strongly supports this classical model.

The enhancement factor $\varepsilon$ has been shown to result from the combination of two sources of energy that drive escapes from the Josephson potential well: the sample at temperature $T$ and residual external noise at an equivalent temperature $E_N/k_B$. For the test case of Oelsner et al. (6) the temperature dependence of $\varepsilon$ derived from this model (Fig.5) is in good agreement with the empirical temperature dependence (Fig.5) obtained from matching simulations to experimental data.

Deviations of observed SCD peaks from predictions based on standard escape rates, at low temperatures, have until now been interpreted as a sign of a crossover from a classical to a macroscopic quantum state of the Josephson junction. However as shown in Fig.5, the near perfect agreement with experiment of predictions based solely on noise enhanced thermal activation serve as clear evidence that classical activation remains the mechanism by which a Josephson junction switches to a finite voltage (running) state.

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