Probing CP Violation Via Higgs Decays to Four Leptons

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Abstract

Since decays to four leptons is widely considered a promising way to search for the Higgs particle, we show how the same final state can also be used to search for signals of CP nonconservation. Energy asymmetries and triple correlations are related to parameters in the underlying CP violating effective interaction at the $H^0-W-W$ and $H^0-Z-Z$ vertex. Expected size of the effects are shown to be small for both the Standard Model and its extension with an extra Higgs doublet.

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The search for the Higgs particle is clearly one of the top priorities in particle physics. In view of the remarkable successes of the Standard Model (SM), some manifestation of the Higgs boson should exist, responsible for the spontaneously breaking of the gauge group $SU(2)_L \times U(1)$ down to $U(1)$. Another important problem in particle physics is the origin of CP violation. In the SM, CP violation is accounted for by the Kobayashi-Maskawa (KM) phase [1]. However, there is considerable interest in searching for sources of CP-violation other than the KM phase. For example, it is generally believed that the KM mechanism alone cannot produce sufficient baryon asymmetry in the Universe [2]. A number of extensions of the SM have CP violation other than the KM phase. Tests of CP nonconservation will therefore test theories beyond the SM.

Once the Higgs particle is discovered, its properties will have to be investigated vigorously. In particular its role in CP nonconservation will undoubtedly get a close scrutiny. Since a very promising way to search for the Higgs is through its decay to four leptons, in this paper we show how that final state can also, simultaneously, be used to study the CP properties of the Higgs particle. We will thus consider the following processes: (I) $H^0 \rightarrow Z^*Z^* \rightarrow \ell^+\ell^-\bar{\nu}\nu$; (II) $H^0 \rightarrow W^*W^* \rightarrow \ell^+\nu\ell'^-\bar{\nu}'$; (III) $H^0 \rightarrow Z^*Z^* \rightarrow \ell^+\ell^-\ell'^-\ell'^-$. Here $\ell$ and $\ell'$ stand for either the electron or the muon, and $\nu$ and $\nu'$ stand for any species of neutrino allowed by lepton flavor conservation and $W^*$ and $Z^*$ can be either on- or off-shell. Even though we will keep our formalism general, we are primarily interested in the heavy Higgs mass region ($m_H \geq 2m_W$) in this paper since the corresponding branching fractions are larger. Also these processes are expected to have considerable background problems in the intermediate mass region ($m_Z \leq m_H \leq 2m_W$) [4]. In the heavy mass region the branching fractions are approximately $8 \times 10^{-3}$, $2 \times 10^{-2}$ and $10^{-3}$ [5], respectively, for processes (I)-(III) where we have summed over electron and muon final states.

CP-odd observables that we will discuss are the energy asymmetries and the CP-odd angular correlations of the charged leptons. Indeed these are the only
CP-odd observables that can be constructed for the above processes if we assume that polarizations of the final state leptons are not observed. There are several different energy asymmetries that can be defined. The first one is:

$$\delta_1 = \frac{< E_+ - E_- >}{< E_+ + E_- >} \quad (1)$$

This is suitable for process (I), where $E_{\pm}$ stands for the energy of positively and negatively charged leptons this paper). Strictly speaking, the energy asymmetry for process (II) is meaningful only when $\ell^+$ is the antiparticle of $\ell^-$. However, assuming lepton universality and setting all lepton masses to zero, we can also define a kind of “flavor-blind” energy asymmetry. Thus $\delta_2$, the energy asymmetry for process (II), is unambiguously defined, in analogy to eq.(1). Two independent energy asymmetries can also be defined for process (III), i.e.

$$\delta_3 = \frac{< E_+ - E_- >}{< E_+ + E_- >} \quad \text{and} \quad \delta_3' = \frac{< E_+ + E'_+ - E_- - E'_- >}{< E_+ + E'_+ + E_- + E'_- >}. \quad (2)$$

We recall that for any of the energy asymmetries to receive a nonvanishing contribution the amplitude for the process must have an absorptive part as required by CPT invariance since the energy asymmetry is a CP-odd but naive T-even object.

CP noninvariance can also show up in the angular correlation of the two decay planes defined by the momenta of the final state leptons. This is a straightforward generalization of Yang’s parity test. In the case of only four particles in the final state, all their momenta need to be tracked down to determine the angular correlations. Thus the CP-odd angular correlation is only useful in the process (III), i.e. $H^0 \to \ell^+\ell^-\ell^+\ell^-$. The angular correlation between the decay planes can be parameterized as

$$\frac{d\Gamma}{d\phi} = \frac{\Gamma}{2\pi} [1 + \lambda_1 \cos \phi + \lambda_2 \cos 2\phi + \lambda_3 \sin \phi + \lambda_4 \sin 2\phi], \quad (3)$$

where $\Gamma$ is the partial decay width for process (III) and $\phi$ is the angle between the two decay planes. A nonvanishing $\lambda_1$ indicates parity is violated, whereas nonvanishing $\lambda_3$ and/or $\lambda_4$ are indications of CP nonconservation. CP violating
angular correlation effects will be completely washed out for two identical lepton pairs in the final state, since there is ambiguity in identifying $\phi$ and $-\phi$. However, both parity and CP violating effects can be picked up if two lepton pairs are different, thus we will only consider non-identical lepton pairs in process (III) when we consider angular correlations.

It is easy to show that $\lambda_3$ is related to the usual CP-odd triple product correlation. Let $\vec{p}_1$ and $\vec{p}_2$ be momenta of one pair of leptons coupled to one $Z$-boson and $\vec{k}_1$ and $\vec{k}_2$ be momenta of the other pair of leptons. A CP-odd triple product $\Omega$ can be defined as:

$$\Omega \equiv (\vec{p}_1 - \vec{p}_2) \cdot (\vec{k}_1 \times \vec{k}_2).$$

(4)

In the rest frame of Higgs, $\Omega$ is the only independent CP odd triple product that can be formed by using lepton momenta only. Then a measure of CP asymmetry $A_{CP}$ defined as

$$A_{CP} \equiv \frac{\Gamma(\Omega > 0) - \Gamma(\Omega < 0)}{\Gamma(\Omega > 0) + \Gamma(\Omega < 0)}$$

(5)

is just $2\lambda_3/\pi$.

To simplify the calculations we will consistently neglect lepton masses. Then the most general tensor structure of the $H^0VV$ ($V$ stands for either the $Z$ or the $W$) vertex relevant for decays to massless leptons assumes the form

$$m_V \left[ g_V g_{\mu\nu} + \sqrt{2} G_F \varsigma_V (q_1 \cdot q_2 g_{\mu\nu} - q_{1\nu} q_{2\mu}) + \sqrt{2} G_F \vartheta_V \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \right].$$

(6)

Here $m_V$ stands for the mass of $Z$ or $W$; $G_F$ is the Fermi constant, $q_1$ is the momentum of one of the vector bosons coupled to the lepton current $j_1^\mu$; and $q_2$ is the momentum of the other vector boson coupled to the lepton current $j_2^\nu$. We have inserted $m_V$ and $\sqrt{2} G_F$ in eq.(6) to make $g_V$, $\varsigma_V$ and $\vartheta_V$ dimensionless. Coefficients $g_V$, $\varsigma_V$, $\vartheta_V$ are functions of $q_1^2$ and $q_2^2$ in general. The origins of different terms in eq.(6) can be identified as follows. The first term is the familiar $H^0VV$ tree level coupling accompanying the Higgs mechanism for giving masses to the gauge bosons. The second term is from the dimension-5 operator $H^0FF$ ($F$ being the field strength of the vector filed $V$) which can be generated by “integrating out”
heavier particles in the theory. The last term is from another dimension-5 operator 
$H^0 F \tilde{F}$, $\tilde{F}$ being the conjugate of $F$. Note that CP is violated if both $\varsigma_V$ and $\vartheta_V$ are 
simultaneously present since $FF$ and $F\tilde{F}$ have opposite transformation properties 
under CP. $\vartheta_V$ and $\varrho_V$ cannot coexist either if CP were a good symmetry. 

In all the calculations in this paper, only the first term in eq.(6), i.e. the tree 
level coupling, and its interference with the third term in eq.(6) will be kept. The reason for this is that we are interested in numerically retaining only the 
most significant terms that reflect CP violation. It is not difficult to see that 
this procedure is justified for our purpose because both $\varsigma_V$ and $\vartheta_V$ in eq.(6) are 
radiatively induced, and $\varsigma_V$ is unimportant because it conserves CP and we are 
only interested in CP violating effects in this paper.

First we discuss the energy asymmetries. It is straightforward to show that 
$$ \delta_1 = \delta_3 = \delta'_3 = -\frac{8c_V c_A}{c_V^2 + c_A^2} \delta_Z, \quad \text{and} \quad \delta_2 = 4\delta_W, $$
where $c_V = -1 + 4 \sin^2 \theta_W$ and $c_A = 1$ are respectively the vector and the axial-
vector coupling constants of the $Z$-boson to the charged-leptons; $\delta_Z$ and $\delta_W$ are 
defined as $[10]$,

$$\delta_V = \sqrt{2}GF \frac{\int dq_1^2 dq_2^2 \varrho_V |q|^3 q_1^2 q_2^2 \Delta_V}{\int dq_1^2 dq_2^2 \varrho_V |q| (3q_1^2 q_2^2 + m_H^2 |q|^2) \Delta_V}. \tag{7}$$

In eq.(7), $|\vec{q}|$ is the magnitude of the spatial momentum of either of the gauge 
baons in the rest frame of the Higgs boson,
$$ |\vec{q}| = \sqrt{m_H^4 + q_1^4 + q_2^4 - 2(m_H q_1^2 + m_H q_2^2 + q_1^2 q_2^2)} \over 2m_H, \tag{8} $$
where $m_H$ is the Higgs mass; $\Delta_V$ is from the propagators of the two gauge bosons,
$$ \Delta_V = \frac{1}{[(q_1^2 - m_V^2)^2 + m_V^2 \Gamma_V^2] \Gamma_V^2 \Gamma_V^2}, $$
$\Gamma_V$ being the total width of the gauge boson $V$. The integrations in both the 
umerator and the denominator of eq.(7) are over the region $\sqrt{q_1^2} + \sqrt{q_2^2} < m_H$.

In general, $\vartheta_V$ is a function of both $q_1^2$ and $q_2^2$. However, we will assume $\vartheta_V$ to be 
a constant here. This approximation is justified if $\vartheta_V$ is a slowly varying function
of $q_1^2$ and $q_2^2$. Furthermore, the integrands in eq.(7) are peaked in the region where either of the propagators can be on-shell. As we have indicated earlier, we are mostly interested in the case when both the vector bosons are on-shell since the branching ratios are larger. Thus it is not a bad approximation to replace the function $\vartheta_V$ by its value at $q_1^2$ and/or $q_2^2$ set equal to $m_V^2$. Using the narrow width approximation for both vector bosons, eq.(7) becomes

$$\delta_V = \frac{\text{Im} \vartheta_V \sqrt{2} G_F (m_H^2 - 4 m_W^2)}{12 + \frac{m_H^2}{m_V^2} - 4}. \quad (9)$$

Both the exact result eq.(7) and the on-shell approximation eq.(9) for $\delta_1$ and $\delta_2$ are plotted in Fig. 1. From the figure we see that $\delta_1$ is about $10^{-3} \times \frac{\text{Im} \varrho_Z}{\varrho_Z}$ and $\delta_2$ is about $10^{-2} \times \frac{\text{Im} \varrho_W}{\varrho_W}$.

Now we discuss the CP-odd angular correlations. It is straightforward to show that the differential partial decay rate is

$$\frac{d^3 \Gamma}{dq_1^2 dq_2^2 d\phi} = \frac{(c_V^2 + c_A^2)^2 m_Z^2 |\vec{q}| \Delta Z}{128 (2\pi)^6 m_Z^4} \left\{ q_Z^2 \left[ \frac{8}{3} q_1^2 q_2^2 + \frac{8}{9} m_H^2 |\vec{q}|^2 \right] + \frac{\pi^2 (c_V c_A)^2}{(c_V^2 + c_A^2)^2} q_1 \cdot q_2 \sqrt{q_1^2 q_2^2 \cos \phi + \frac{4}{9} q_1^2 q_2^2 \cos 2\phi} \right\} - \sqrt{2} G_F q_Z \text{Re}(\vartheta_Z) m_H |\vec{q}|$$

$$\left[ \frac{\pi^2 (c_V c_A)^2}{(c_V^2 + c_A^2)^2} q_1 \cdot q_2 \sqrt{q_1^2 q_2^2 \sin \phi + \frac{8}{9} q_1^2 q_2^2 \sin 2\phi} \right]. \quad (10)$$

It is obvious from the above equation that the parity violating coefficient $\lambda_1$ is suppressed by a factor of $\pi^2 (c_V c_A)^2 / (c_V^2 + c_A^2)^2 \sim 10^{-2}$ (see also Ref. [8, 11]). In addition to this suppression, $\lambda_3$ is further suppressed by the ratio $\text{Re}(\vartheta_Z) / \varrho_Z$ rendering $\lambda_3$ extremely small. The single differential decay rate with respect to $\phi$ can be obtained by numerically integrating eq.(10). $\lambda_3$ and $\lambda_4$ are plotted in Fig. 2, again, we have assumed that $\varrho_V$ and $\vartheta_V$ are constants. The units in Fig. 2 are taken to be $\frac{\text{Re} \varrho_Z}{\varrho_Z}$. From the figure we see that $\lambda_3$ stays approximately constant for $m_H > 200 \text{GeV}$ at $6 \times 10^{-4}$; $\lambda_4$ peaks around $m_H = 190 \text{GeV}$, it varies between $8 \times 10^{-4}$ and $8 \times 10^{-3}$.

In the SM, $H^0$ is a scalar particle. It couples to $Z$- and $W$-bosons at tree level, with $\varrho_Z = g / \cos \theta_W$ and $\varrho_W = g$. $HFF$ is induced at one loop level, whereas
CP violating interaction $H F \tilde{F}$ does not arise till two-loop order for $W$-bosons and at three-loops for $Z$-bosons. In addition to the suppression by powers of $4\pi$ associated with these loops, CP violation in the SM will necessarily involve product of small mixing angles and also perhaps small ratio of masses. It is thus clear that in the SM $\vartheta_V$ and consequently the asymmetry parameters discussed above are all expected to be extremely small.

On the other hand, many extensions of the SM have other sources of CP violation besides the KM phase which may enhance the rate of CP violating decay of the Higgs particle significantly. As an illustration we will consider a two-Higgs-doublet model with softly symmetry breaking term [12]. The relevant Yukawa coupling in such a model is taken to be

$$- L_Y = \lambda_{ij} \bar{Q}_i L \tilde{\varphi}_2 U_R^j + g_{ij} \bar{Q}_i L \phi_1 D_R^j + h.c. \quad (11)$$

There are three spin-0 neutral bosons $\varphi^0_1$, $\varphi^0_2$ and $\varphi^0_3$ in the theory. If there are no CP violation from the scalar sector, two of them will be CP-even and the other one is CP-odd. In the presence of CP violation, they mix through their mass matrix. Let us call the three neutral mass eigenstates $H^0_1$, $H^0_2$ and $H^0_3$, then

$$H^0_i = O_{ij} \varphi_j. \quad (12)$$

Where $O_{ij}$ are the matrix elements of an orthogonal matrix that diagonalizes the mass matrix. $\varrho_Z$ and $\varrho_W$ arise from the tree-level coupling of $H^0_i$ to two $Z$ and $W$-bosons, while coefficients $\vartheta_Z$ and $\vartheta_W$ arise through the top quark loop shown in Fig. 3. It is easy to show that

$$\left( \frac{\vartheta_Z}{\varrho_Z} \right)_i = \frac{3m_t^2 \kappa_i}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \frac{2\tilde{c}_a^2(x+y) + \tilde{c}_v^2 - \tilde{c}_a^2}{q_1^2(x^2 - x) + q_2^2(y^2 - y) - 2xyq_1q_2 + m_t^2}, \quad (13)$$

$$\left( \frac{\vartheta_W}{\varrho_W} \right)_i = \frac{12m_t^2 \kappa_i}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \frac{(x+y)}{q_1^2(x^2 - x) + q_2^2(y^2 - y) - 2xyq_1q_2 + m_t^2} \quad (14)$$

where $\tilde{c}_a = -1$ and $\tilde{c}_v = 1 - 8/3 \sin^2 \theta_W$ are axial-vector and vector coupling constants of $Z$ to top-quark; $\kappa_i = \cot \beta O_{i3}/O_{i1}$, where $\cot \beta = |v_1|/|v_2|$, $|v_2|$ and $|v_1|$ are vacuum expectation values of $\phi_2$ and $\phi_1$ respectively. In Fig. 4 we show
the both the real and the imaginary part of $\frac{\theta}{\varphi^2}$ and $\frac{\theta}{\varphi}$ for on-shell $W'$s and $Z'$s, with $m_t = 130 \text{GeV}$.

It is clear that both eq.(13) and eq.(14) will develop an imaginary part only when $m_H > 2m_t$. Thus, within the context of the two-Higgs-doublet model with soft symmetry breaking, energy asymmetry is a useful observable only when $m_H > 2m_t$. Nevertheless, the angular correlation is useful both for $m_H > 2m_t$ and also for $m_H < 2m_t$. In this model, $\kappa_i$ is completely undetermined, although one may invoke some naturalness argument that it should not be too different from order one. Assuming $\kappa_i$ is of order one, then we see that typically $\delta_1 \sim 10^{-5}$ and $\delta_2 \sim 10^{-4}$; $\lambda_3 \sim 10^{-6}$ and $\lambda_4 \sim 10^{-5}$. Thus in this model process (II) appears the most promising. Indeed, from Figs. 1 and 4 we see that $\delta_2$ can be as large as $1.5 \times 10^{-3}$ for $m_H$ about 300GeV. Assuming that the branching ratio ($BR$) for this process is about $3 \times 10^{-2}$, as it is in the SM, we see that the number of Higgs needed to see such an asymmetry, given roughly by $(\delta_2^2 \times BR)^{-1}$, is about $10^7$. This number is about a factor of forty larger than the expected number of Higgs at the hadron supercolliders, based again on the SM \cite{4}. Consequently, at least in this extension of the Standard Model the resulting asymmetries appear too small to be observable. On the other hand, there is a large uncertainty in these estimates as many of the relevant parameters in these models have not been pinned down. Furthermore, our analysis is completely general so it may be useful to study what other extensions of the SM will yield for these asymmetries. The virtue of these tests of CP nonconservation is that they can be done at little or no extra cost.

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Figure Captions

**Figure 1** $\delta_1$ from eq.(7) (solid line) and from the on-shell approximation (dotted line) in the units of $\text{Im} \, \vartheta_Z \varrho_Z$. $\delta_2$ from eq.(7) (dashed line) and the on-shell result (dash-dotted line) in the units of $\text{Im} \, \vartheta_W \varrho_W$.

**Figure 2** $\lambda_3$ (solid line) and $\lambda_4$ (dashed line) in the units of $\text{Re} \vartheta_Z \varrho_Z$.

**Figure 3** One-loop Feynman diagram contributing to $\vartheta_V$ in the two-Higgs-doublet model.

**Figure 4** Real (solid line) and imaginary (dotted line) part of $(\vartheta_Z \varrho_Z)_i$, and real (dashed line) and imaginary (dash-dotted line) part of $(\vartheta_W \varrho_W)_i$. They are all in the units of $\kappa_i$ and $m_t$ is taken to be 130 GeV.