An Ultra-Broadband Unidirectional Coaxial Waveguide Based on YIG

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Abstract—The planar physical model of ultra-broadband unidirectional waveguide based on surface magnetoplasmons (SMPs) has been derived and calculated in detail, but the coaxial physical model of ultra-broadband unidirectional waveguide based on SMPs has not been reported. Based on the gyromagnetic properties of Ferrite (taken yttrium iron garnet as an example, abbreviated as YIG), a novel ultra-broadband unidirectional coaxial waveguide is proposed in this paper. The basic model of the waveguide is a multilayer coaxial waveguide system composed of metal-layer-YIG-yig-metal wire. The magnetization vectors of two middle YIGs are equal and opposite. Theoretical analysis and simulation results show that the waveguide supports two unidirectional transmissions, and both unidirectional bands have excellent properties of immune scattering and back reflection. The waveguide system has the characteristics of simple structure, immune scattering, and ultra-broadband unidirectional band, which is expected to be used in all-photon communication system.

1. INTRODUCTION

Unidirectional electromagnetic modes only allow one-way transmission and have the ability of immune reverse transmission. Even if there are defects or bends, the propagation of the unidirectional electromagnetic mode is not affected by backscattering. Unidirectional electromagnetic modes have important potential applications in electronic communication, controlling optical unidirectional and quantum information processing. There are two ways to realize the unidirectional electromagnetic modes. One is OWEM (One-Way Edge Mode) [1–5], and the other is unidirectional SMP (surface magnetoplasmon) [6–8]. OWEM is a surface wave confined at the boundary of magneto-optical photonic crystal. Its main characteristic is that the group velocity only refers to a single direction determined by the direction of external constant magnetic field. However, the complex structure makes it hard to achieve integrated optics. In addition, their working frequency band is exceedingly narrow due to the sensibility of OWEM to frequency. Unidirectional waveguide based on SMPs which generally requires one layer of dielectric plate and one layer of magnetic material is more likely to achieve. However, in addition to supporting SMPs, this kind of waveguide also supports the conventional mode of propagation in the medium. The dispersion curve of the conventional mode is located within the light lines and can usually propagate in both directions. Owning to the existence of this bidirectional mode, SMP is particularly prone to lose unidirectional propagation characteristics once the surface of the material is rough or curved. Later, researchers found that changing the thickness of the dielectric layer can adjust some conventional modes so that unidirectional SMPs are not affected by the conventional mode and have the properties of immune scattering.

A unidirectional waveguide with a ferrite-ferrite (YIG-YIG, YY) structure has attracted the attention of many researchers because of its simple structure and immune scattering [9, 10]. The working
band of this structure is seriously limited because of the bulk mode of magneto-optical materials. In the microwave band, the metal can be approximately a perfect electrical conductor, and the bulk mode can be suppressed by the method of metal cutting off the medium. On this foundation, a new ultra-broadband unidirectional coaxial waveguide based on YIG material is proposed in this paper. Its basic model is a multilayer waveguide system of metal-YIG-YIG-metal wire. The two layers of YIG have equal but opposite magnetizations. The bulk mode of magneto-optical material can be compressed by metal layer and metal wire. In addition to the inherent photonic bandgap of gyromagnetic materials, a new photonic bandgap supporting unidirectional electromagnetic mode appears in the structure, which expands the working band of unidirectional waveguide compared with a YY structure. Like the unidirectional modes supported by the inherent photonic bandgap of gyromagnetic materials, the emerging unidirectional modes also have the properties of immune scattering and back reflection. Further, different from the dielectric layer used in the YY structure, the dielectric layer of this new waveguide is very thin, which is more conducive to engineering implementation.

2. PHYSICAL MODEL AND DISPERSION ANALYSIS OF COAXIAL UNIDIRECTIONAL WAVEGUIDE

The planar physical model of ultra-broadband unidirectional waveguide based on surface magnetoplasmons (SMPs) has been derived and calculated in detail [11, 12], but the coaxial physical model of ultra-broadband unidirectional waveguide based on SMPs has not been reported. The coaxial physical model of ultra-broadband unidirectional waveguide based on SMPs is shown in Fig. 1. In this axisymmetric structure, two YIG layers with thicknesses of $d^+$ and $d_-$ are sandwiched between the thin metal wire and metal film. In the layers of YIG, the magnetization direction is $\theta^+$ or $\theta^-$, i.e., $M^+ = M_r \theta^+$, $M^- = -M_r \theta^-$. In the cylindrical coordinates, the (relative) permeability tensor of YIG layer is expressed as:

$$\bar{\mu}(\omega) = \begin{bmatrix} \mu_1 & 0 & \pm i \mu_2 \\ 0 & 1 & 0 \\ \mp i \mu_2 & 0 & \mu_1 \end{bmatrix}$$  

(1)

with

$$\mu_1 = 1 + \frac{\omega_m (\omega_0 - i \nu \omega)}{(\omega_0 - i \nu \omega)^2 - \omega^2}$$  

(2)

$$\mu_2 = \frac{\omega \omega_m}{(\omega_0 - i \nu \omega)^2 - \omega^2}$$  

(3)

Figure 1. Physical model of coaxial unidirectional waveguide.

In Eqs. (2) and (3), $\omega$ is the angular frequency, $\omega_m = \mu_0 \gamma M_r$ the characteristic frequency ($\mu_0$ is the vacuum permeability, and $\gamma$ is the gyromagnetic ratio), and $\nu$ the damping coefficient of YIG material. The signs $\pm$ in Eq. (1) denote magnetization $M^+_r$ and $M^-_r$, respectively. In order to study the attenuation characteristics of waveguide propagation mode, we take $\nu = 0$ when analyzing the dispersion
where $k$ is the propagation constant. From Maxwell’s equation $\nabla \times E = j\omega \mu_0 \mu H$, one can express all the magnetic ($H$) field components in terms of $e_\phi(r)$:

\begin{align}
H_r(z, r) &= -\frac{1}{\omega \mu_1 \mu_v} \left( \mu_2 \frac{\partial e_\phi(r)}{r \partial r} r + \mu_1 k e_\phi(r) \right) \exp(ikz) \\
H_z(z, r) &= \frac{j}{\omega \mu_1 \mu_v} \left( \mu_1 \frac{\partial e_\phi(r)}{r \partial r} + \mu_2 k e_\phi(r) \right) \exp(ikz)
\end{align}

where $\mu_v = \mu_1 - \mu^2_2/\mu_1$. In turn, from Maxwell’s equation $\nabla \times H = -j\omega \epsilon_0 m E$, $e_\phi(r)$ satisfies:

\[\frac{\partial^2 e_\phi(r)}{\partial r^2} + \frac{1}{r} \frac{\partial e_\phi(r)}{\partial r} - \frac{1}{r^2} e_\phi(r) \frac{\mu_2 k}{\mu_1 r} e_\phi(r) - \left( k^2 - \epsilon_m \mu_0 \eta_0^2 \right) e_\phi(r) = 0\]

where $\epsilon_m$ is the relative permittivity of YIG. Taking $e_\phi(r) = r^{-1/2} f(x)$, $x = \beta r$, $\beta = 2\sqrt{k^2 - \epsilon_m \mu_0 \eta_0^2}$, Eq. (7) can be transformed into Whittaker equation:

\[f''(x) + \left( -\frac{1}{4} - \frac{\alpha}{x} - \frac{3}{4x^2} \right) f(x) = 0\]

where $\alpha = \mu_2 k/\mu_1 \beta$, there exist two independent solutions of differential Eq. (8), i.e., the Whittaker’s functions $M_{a,b}(x)$ and $W_{a,b}(x)$ with $b = 1$. Note that when $x \to 0$, $M_{a,b}(x) \to 0$, and $W_{a,b}(x) \to \infty$. Suppose that SMP mode is mainly distributed near the interface between the two remanence layers, and the central metal wire is very thin, the influence of SMPs can be ignored. Therefore, the electric field component of YIG inner layer is:

\[E_\phi = r^{-1/2} AM_{a,b}(\beta r) \exp(izk)\]

It can also be solved that when the magnetization of YIG outer layer is $M_r = -M_\phi$, the electric field component in YIG outer layer is

\[E_\phi = r^{-1/2} [B_1 M_{-a,b}(\beta r) + B_2 W_{-a,b}(\beta r)] \exp(izk)\]

Since $E_\phi$ disappears at the boundary of the outermost metal layer, it can be obtained that $B_3 = -[M_{-a,b}(\beta R)/W_{-a,b}(\beta R)B_1]$, where $R \approx d_+ + d_-$. The magnetic field of SMPs has nonzero components $H_r$ and $H_z$, which can be derived directly from $E_\phi$. The $Z$ component ($H_z$) of the magnetic field is:

\[H_z = -\frac{ir^{-1/2}A}{\omega \mu_v} [(\kappa k - \mu/2r) M_{a,b}(\beta r) - \mu \beta M'_{a,b}(\beta r)] \exp(izk)\]

in the inner YIG layer, and it becomes:

\[H_z = -\frac{ir^{-1/2}A}{\omega \mu_v} \left\{ [B_1 M_{-a,b}(\beta r) - B_2 W_{-a,b}(\beta r)] - \mu \beta \left[ B_1 M'_{a,b}(\beta r) - B_2 W'_{a,b}(\beta r) \right] \right\} \exp(izk)\]

in the outer layer of YIG. The derivatives $M'_{a,b}$ and $W'_{a,b}$ of Whittaker functions in Eqs. (11) and (12) satisfy the recursive relationships

\begin{align}
M'_{a,b}(x) &= \left( \frac{1}{2} - \frac{a}{x} \right) M_{a,b}(x) + \frac{1}{x} \left( \frac{1}{2} + b + a \right) M_{a+1,b}(x) \\
W'_{a,b}(x) &= \left( \frac{1}{2} - \frac{a}{x} \right) W_{a,b}(x) - \frac{1}{x} W_{a+1,b}(x)
\end{align}
Since the electric field component $E_\phi$ and the magnetic component $H_z$ (the boundary conditions) are continuous at the YIG interface $r \approx d_+$, the dispersion relationship of SMPs can be obtained

\[
W_{-a,1}(\beta r_2) \left[ 2a\beta r_1 + 2a \right] M_{-a,1}(\beta r_1) + \left( \frac{3}{2} + a \right) M_{a+1,1}(\beta r_1) M_{-a,1}(\beta r_1) + \left( \frac{3}{2} - a \right) M_{-a+1,1}(\beta r_1) \right] \\
+ (\beta r_1) W_{-a,1}(\beta r_1) \left[ (2a\beta r_1 + 2a) M_{a,1}(\beta r_1) + \left( \frac{3}{2} + a \right) M_{a+1,1}(\beta r_1) \right] + M_{-a+1,1}(\beta r_1) = 0 \quad (14)
\]

In order to verify the above theoretical analysis, we numerically calculate the dispersion curve of guiding mode in the proposed coaxial waveguide. The basic physical parameters of the system are $d_+ = d_- = 6$ mm, $\nu = 0$ and $\omega_0 = 0.5\omega_{in}$, $\omega_{in} = 10\pi \times 10^9$ rad/s.

The dispersion curves of SMP are plotted as red lines in Fig. 2(a), showing three types of SMP sustained in our system. The highest SMP in Fig. 2(a) is similar to the bound mode in [12], and its modal field almost distributes in the YIG material. Therefore, this SMP is regarded as a bulk mode, indicated as the grey-shaded areas in Fig. 2(a). The middle SMP located at the photonic bandgap is a typical surface mode, whose field is tightly sustained at the material surface. Moreover, this type of SMP is a one-way mode and only allowed to propagate in one direction, since its dispersion curve only possesses a positive slope. The lowest SMP is of our particular interest, which is not observed in a YIG-YIG structure. Evidently, the newly observed SMP is located at the bandgap below the lower bulk-mode zone, which starts from zero point ($k = 0$) in other systems. However, in our system, the bulk modes of magneto-optical material are compressed by the introduced metal layers, thus the cutoff of the bulk-mode zone lifts up, leaving us an opportunity to broaden the frequency band of SMP. Similar to the second type of SMP, the dispersion of the third SMP also has a single slope in a region. We refer to the dispersion bands of the one-way SMPs as completely unidirectional propagating bands, see the green and yellow shaded areas. Note that in the YIG-YIG waveguide, the unidirectional propagating region usually exists only in the photonic bandgap (corresponding to the completely unidirectional band 2), while our coaxial waveguide supports two completely unidirectional bands. Figs. 2 (b)–(d) show the modal field of bulk mode and SMPs in two unidirectional regions, respectively. For SMPs, the electric field locates at the interface of YIG layers, while for bulk mode, the electric field mainly distributes in the outer layer of YIG.

![Figure 2](image-url)  
**Figure 2.** (a) Dispersion curve in metal layer-YIG-YIG-wire coaxial waveguide. The modal field of (b) bulk mode, (c) SMP in completely unidirectional band 2, and (d) SMP in completely unidirectional band 1, respectively.

### 3. SIMULATION VERIFICATION

To verify the unidirectional propagating characteristics of SMPs in the coaxial waveguide, we use finite element method to simulate the transmission of electromagnetic wave in the coaxial waveguide. In the simulation, a ring current source is placed at the position of $r = d_+$ and close to the YIG interface.
to excite the beam. The frequency is set to 3.2 GHz (completely unidirectional band 1); the length of the waveguide is 60 mm; and the thicknesses of the YIG layers are \(d_\ominus = d_\oplus = 6\) mm. Fig. 3(a) shows the electric field amplitudes obtained by simulation. The excited SMPs only propagate forward along YIG-YIG interface (at \(r = d_\oplus\)) as expected. Similarly, we simulate the waveguide transmission with the operating frequency at 6.2 GHz (completely unidirectional band 2), as shown in Fig. 3(b). Obviously, the one-way propagating property is observed again. Therefore, SMPs in the two associated frequency bands have unidirectional transmission characteristics, and these simulated results are consistent with our analysis in Section 2. For comparison, we also calculated the waveguide transmission in the bulk-mode zone, as shown in Fig. 3(c). In Fig. 3(c), electromagnetic waves are transmitted on the left and right sides of the excitation source, which is a typical two-way propagation behavior.

In order to further validate the robustness of unidirectional SMPs, an air-filled hollow ring is placed on the YIG interface as an obstacle in the coaxial waveguide. Figs. 4(a) and (b) show the electric field simulation diagram when obstacles are introduced, corresponding to 3.2 GHz and 6.2 GHz, respectively. The results show that in the metal-YIG\(^+\)-YIG\(^-\)-metal coaxial structure, the electric fields at the frequencies of 3.2 GHz and 6.2 GHz perfectly bypass the obstacles without any scattering and back reflection, which is completely consistent with our theoretical analysis again. Figs. 4(c) and 4(d) correspond to the real part of the electric fields in Figs. 4(a) and 4(b), respectively. We can more clearly observe that our coaxial waveguide is indeed immune to any scattering and back reflection even if there are obstacles in the waveguide at two operating frequencies. Hence, it can be concluded that unidirectional SMPs in the proposed coaxial waveguides can be immune to backscattering.

Assuming that the unidirectional coaxial waveguide is conical in the direction of wave transmission. In the completely unidirectional bands, there is no backward wave in our waveguide; therefore, the wave must be completely blocked at the conical tip without any backscattering. To confirm our idea, we conducted the wave transmission in a conical structure. For simplifying the model, the thicknesses of the inner and outer YIG layers are considered equal, i.e., \(d = d_\oplus = d_\ominus\), and the thickness of YIG layer decreases linearly from 6 mm to 0.01 mm over a length of 60 mm. In the simulation, the wave was input into the conical waveguide from the large port with \(d = 6\) mm, and then output into free space from the small port with \(d = 0.01\) mm. The ferrite loss of 0.001 is taken into account. The working frequency is set at \(f = 3.2\) GHz. The simulated magnetic \((H)\) and electric \((E)\) field amplitudes are, respectively, exhibited in Figs. 5(a) and 5(b). Obviously, the \(H\) and \(E\) field amplitudes are extremely

Figure 3. Simulated electric field amplitudes at operating frequency (a) 3.2 GHz, (b) 6.2 GHz and (c) 12 GHz.

Figure 4. Electric field amplitudes at (a) 3.2 GHz (b) 6.2 GHz. (c), (d) The real part of (a) and (b).
Figure 5. Simulated field amplitudes for focusing. The wave frequency is 3.2 GHz. (a) $H$ field. (b) $E$ field. Distributions of (c) $H$ and (d) $E$ field amplitudes along a line at the ferrite interface.

enhanced near the conical tip. This is more clearly shown in Figs. 5(c) and 5(d). Figs. 5(c) and 5(d) show the distributions of $H$- and $E$-field amplitudes along a line at the YIG-YIG interface. We should point out that such a huge field enhancement never occurs in a conventional tapered waveguide, where the propagating wave in it will be (partially) reflected back at the tip.

4. CONCLUSION

Based on the spin magnetic properties of ferrite materials, a new ultra-broadband unidirectional coaxial waveguide is proposed in this paper. Theoretical analysis and numerical results show that the introduced metal layer compresses the bulk mode of magneto-optical material. With the decrease of YIG thickness to a certain extent, a new photonic bandgap will appear in the waveguide system. Like the inherent photonic bandgap of YIG, the photonic bandgap also supports unidirectional electromagnetic mode, resulting in a significant increase in the unidirectional working band of the waveguide. The simulation results further show that even if there are obstacles, the unidirectional electromagnetic modes supported by the two photonic bandgaps are completely unidirectional modes, which can be completely immune to rough surface scattering and back reflection. The waveguide system has the characteristics of simple structure, immune scattering, and ultra-broadband unidirectional working frequency, which is an effective way to realize all-photon communication.

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