The relative distances to the Virgo, Fornax, and Coma clusters of galaxies through the $D_n - \sigma$ and the Fundamental Plane relations

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ABSTRACT

We derive the relative distances to the Virgo, Fornax, and Coma clusters of galaxies by applying the $D_n - \sigma$ and the Fundamental Plane (FP) relations to the data of the homogeneous samples of early–type galaxies studied by Caon et al. (1990, 1994), Lucey et al. (1991a,b), and Jørgensen et al. (1992, 1995a,b). The two distance indicators give consistent results, the relative distance moduli to Fornax and Coma with respect to Virgo being $\Delta \mu_{AV} = (0.45 \pm 0.15)$ mag and $\Delta \mu_{CV} = (3.55 \pm 0.15)$ mag respectively. The formal error on $\Delta \mu$ may be as small as 0.07 mag ($\sim 3\%$ in distance), provided that all the sources of bias are taken into account and a correct statistical approach is used. Unfortunately, much of the actual uncertainty in the relative distance of the clusters ($\sim 12 – 15\%$), is due to the existence of systematic departures in the measurements of the velocity dispersions among the various datasets, and to the corrections for aperture effects.

The above result for the Fornax cluster is supported by the $L - \sigma - \mu$ relation and, with lesser accuracy, by the $\log(m) - \log(r_e)$ relations. Our value of $\Delta \mu_{BF}$ is in fair agreement with the one derived using planetary nebulae and SNe–Ia, while it is in open contrast with that coming from surface brightness fluctuations, globular clusters luminosity function, and infrared Tully-Fisher relation. In our data Coma appears slightly nearer than indicated by the other distance indicators, but now a better agreement with the Tully-Fisher relation seems to exist.

We show that for the galaxies of the Virgo and Fornax clusters the residuals of the $D_n - \sigma$ relation do not correlate with the effective surface brightness $\langle \mu_e \rangle$. There is also no correlation of the residuals of the $D_n - \sigma$ and FP relations with the total luminosity of the galaxies, with the ellipticity and with the isophotal shape parameter $a_4$. Instead, a correlation seems to exist with the maximum rotation velocity of the galaxies, $V_m$, with the $(V/\sigma)$ ratio, and with the exponent $m$ of the $r^{1/m}$ fit to the major axis light profiles of the galaxies. If confirmed, these effects introduce a systematic bias in both relations when used as distance indicators.

Key words: Galaxies: clusters: Virgo, Fornax, and Coma – galaxies: distances – galaxies: elliptical and lenticular – galaxies: kinematics and dynamics

1 INTRODUCTION

Empirical scaling laws built using the main structural parameters of galaxies such as characteristic radii, mean surface brightnesses, and central velocity dispersions, have become quite fashionable in the last decade. In particular, the relations holding for early-type galaxies are believed to be useful tools for the understanding of the structure, formation, and evolution of the kinematically hot stellar systems (hereafter early-type galaxies), and to be also quite accurate distance indicators (DIs) for these objects.

One of such relations is the so-called Fundamental Plane (hereafter FP; Djorgovski & Davis 1987), which, in its original formulation, binds the central velocity dispersion, $\sigma_0$, and the average surface brightness within the ef-
effective isophote (that encircling half of the total light of a galaxy in a given photometric band), \( \langle \mu \rangle_e \), to the mean radius of this same isophote, named effective radius \( r_e \):

\[
\log(r_e) = a \log(\sigma_0) + b(\mu)_e + c.
\]

A second scaling law is the \( D_n-\sigma \) relation (Dressler et al. 1987): \( D_n \propto \sigma_n^2 \), where \( D_n \) is the diameter of the circular aperture within which the average corrected surface brightness equals a fixed value \( \langle \mu \rangle_n \), usually 20.75 B mag arcsec\(^{-2} \), and \( \gamma \) is a constant of the order of 3/2. To a first order approximation the \( D_n-\sigma \) relation is an edge-on view of the FP (Dressler et al. 1987). In this case the surface brightness parameter is hidden in the definition of \( D_n \).

The slope and the small thickness measured for the FP and the \( D_n-\sigma \) relations imply a narrow range of density structure, velocity anisotropy, and mean age among elliptical galaxies, which are excellent properties for DIs. Actually these relations have provided much of the present information on the large scale flows in the local universe (Lynden-Bell et al. 1988).

Two complementary strategies have been developed. On the one hand the enormous progress in the observational techniques and data reduction procedures has produced larger and larger samples and increasingly better distance determinations of galaxies. On the other hand much effort has been put in the last years in quantifying internal errors, biases, calibration uncertainties, second parameter corrections, environmental influences, and behaviour of the residuals in both relations; see e.g. Lucey et al. (1991a; b = LGCT), Jacoby et al. (1992), Jørgensen et al. (1992; = JFK, 1995a,b), Davies et al. (1993), Saglia et al. (1993), van Albada et al. (1993). In conclusion, from the original 32% uncertainty given by the Faber & Jackson (1976; FJ) relation, the error has been progressively reduced by the coming of the \( D_n-\sigma \) and of the FP relations, being nowadays as low as 10% in the relative distances between clusters (Jacoby et al. 1992).

Despite this success, a number of problems are still open. Which of the two relations, \( D_n-\sigma \) and FP, is more suitable as DI? Or, in other words, which of the two is less affected by observational errors and biases? Can S0 galaxies be included in these relations? Many papers have addressed the above questions with contradictory results.

JFK state that the FP should be preferred, because the \( D_n-\sigma \) relation has a larger scatter and suffers from a surface brightness bias. Van Albada et al. (1993) find that the \( D_n-\sigma \) relation has the same accuracy of the FP, provided that a proper surface brightness correction is applied. Lucey et al. (1991a) and LGCT derive contradictory evidences for such a bias in their samples, and Saglia et al. (1993) show that the scatter around both relations can be reduced by rejecting E galaxies with inner disk components. Finally, Davies et al. (1993) point out that only galaxies in a restricted range of surface brightness should be used in the \( D_n-\sigma \) relation in order to avoid contaminations.

We argue here that the \( D_n-\sigma \) and the FP relations have substantially the same accuracy – at least as long as the distance to nearby clusters is concerned – by applying them to the measure of the relative distances to the Fornax and Coma clusters with respect to Virgo. The motivation of this work is twofold. Firstly, the relative distances to these clusters are far from being accurately known (cf. Section 3), and secondly, today we can exploit an accurate and homogeneous data-set of photometric and spectroscopic measurements for the galaxies of these three clusters: Caon et al. (1990, 1994, hereafter C\(^2\)D), D’Onofrio et al. (1995), LGCT, JFK, and Jørgensen et al. (1995a, b). The study of a volume-limited sample of galaxies has the advantage that one can look at the intrinsic properties of the \( D_n-\sigma \) and FP relations by means of objects which are probably coeval, and reduce as much as possible the principal sources of errors, in particular those related to the unknown distances of the galaxies.

The paper is organized as follows. Section 2 reviews the current determinations of the relative distances to the Virgo, Fornax, and Coma clusters, as measured by the relevant DIs. Our determinations of the relative distances to the three clusters using the \( D_n-\sigma \) and FP relations are presented in Sections 3 and 4 respectively, together with a discussion of the errors intrinsic to both methods and a comparison of the results with other DIs. In Section 5 we analyze the behaviour of the residuals of the \( D_n-\sigma \) and FP relations by exploiting the advantage of our volume-limited, accurate, and homogeneous (in the methodological approach and in the data analysis) sample of galaxies, and we perform some further tests on the \( D_n-\sigma \) relation by varying the definitions of \( D_n \) and \( \sigma \). In Section 6 we present the \( \log(m)-\log(r_e) \) relation used as DI for the Fornax cluster. Conclusions are drawn in Section 7.

2 THE RELATIVE DISTANCES TO THE VIRGO, FORNAX, AND COMA CLUSTERS

In his review of the extragalactic distance scale, de Vaucouleurs (1993) showed that the differential distance modulus (\( \Delta \mu \)) between the Fornax and the Virgo cluster is, on average, \( \Delta \mu_{FV} = +0.16 \) and +0.38 mag for the long and for the short distance scale respectively. In other words, according to this study, both scales place Virgo closer than Fornax.

The trend is not confirmed by all DIs. Table 1 lists the measurements of \( \Delta \mu_{FV} = \mu_F - \mu_V \), sorted according to the method used; we have essentially ignored the old measurements, already revisited by de Vaucouleurs. A quick look to the Table shows that the inner ring diameters of spirals, the globular cluster luminosity function (GCLF), the surface brightness fluctuations (SBF), the L–\( \sigma \)–\( \mu \) relation, the IR Tully–Fisher relation (IR-TF), and the scale length of dE galaxies, place the Fornax cluster closer than Virgo, in marked contrast with the results from the brightest cluster members (BCM), the \( D_n-\sigma \) relation, the planetary nebulae luminosity function (PNLF), and the SNe–Ia. Taking a plain average, the \( \Delta \mu_{FV} \) turns into a salomonical ~ 0.0, with a r.m.s. of 0.24 mag.

A few comments to Table 1 are in order. The SBF method gives distances systematically lower by 10-30% than the \( D_n-\sigma \) relation, probably due to the adopted calibration (Sandage & Tammann 1995). The values of \( \Delta \mu \) obtained from the analysis of the GCLF strongly depend on the morphological types of the selected galaxies and on metallic-ity effects, while those provided by the PNLF method are in good agreement, within the errors, with the \( D_n-\sigma \) relation (McMillan et al. 1993). For both methods however, the number of galaxies in common with our \( D_n-\sigma \) relation is quite small, and a conclusive comparison cannot be made.

The filamentary and complex structure of the Virgo cluster...
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velocity dispersion profiles $\sigma_V$ homogeneous. We have measured the rotation curves for the above cited effects.

as explained in C2

corrections are very small for the galaxies in both clusters, the samples are respectively complete (and volume-limited) down to $18.3$ Mpc

For the photometric parameters of the Coma galaxies we made use of the data of LGCT, JFK, and Jørgensen et al. (1995a), which have a similar degree of homogeneity and accuracy needed for the analysis of these problems.

3.1 The samples

The $C^2D$ set of B-band photometric parameters for Virgo E and non-barred S0 galaxies is 80% complete down to $B_T = 14$ mag (according to the catalog of Binggeli et al., 1985, hereafter BST), and that of Fornax is 100% complete down to $B_T = 15$ mag (according to the membership list of Ferguson, 1989). Assuming the same distance of 18.3 Mpc to both clusters, the samples are respectively complete (and volume-limited) down to $M_B = -17.3$ and $M_B = -16.3$ mag.

The $D_n$, diameters, effective radii $r_e$, and effective mean surface brightnesses $⟨μ⟩_e$ are given by $C^2D$ with an accuracy of about 5%, 20%, and $\sim 0.3$ mag ($\sim 30\%$) respectively. The above parameters have not been corrected for seeing, galactic absorption, internal extinction and redshift. All these corrections are very small for the galaxies in both clusters, as explained in $C^2D$.

For the photometric parameters of the Coma galaxies we made use of the data of LGCT, JFK, and Jørgensen et al. (1995a), which have a similar degree of homogeneity and accuracy as $C^2D$. These data have been corrected by the authors for the above cited effects.

Unfortunately, the spectroscopic data are far less homogeneous. We have measured the rotation curves $V(r)$ and velocity dispersion profiles $σ(r)$ of 15 galaxies of the $C^2D$ Fornax sample (D’Onofrio et al. 1995), while for the Virgo galaxies we collected central velocity dispersion measurements from the literature: Whitmore et al. (1985, WMT),

| $Δ\mu CV$ | Method | Reference | Notes |
|-----------|--------|-----------|-------|
| +3.70 ± 0.17 | B and H TF | Giraud 1986 | Hybrid form of the TF relation |
| +3.76 ± 0.12 | L-σ | Lucey 1986 | Through fixed metric apertures |
| +3.75 ± 0.18 | $V_{26}$-σ | Dressler 1984 | $V_{26}$ from Sandage & Visvanathan (1978) |
| +4.07 ± 0.75 | Reduced radii | Gudehus 1991 | Mean error for a bright isolated galaxy |
| +3.42 ± 0.22 | B-band TF | Rood & Williams 1992 | For M31-like galaxies |
| +3.70 ± 0.09 | Mean of 3 | Bower et al. 1992 | Mean of CM diagram, colour-diameter and FJ relations |
| +3.59 ± 0.06 | CM diag. for E | Bower et al. 1992 | As above for E galaxies only |
| +3.80 ± 0.06 | Mult. analysis | Vader 1986 | Comparison between the mass-luminosity relations |
| +3.60 ± 0.17 | IR CM diagram | Christensen 1991 | Unbiased value for spiral galaxies |
| +3.90 ± 0.10 | Mean of 9 | Tammann & Sandage 1985 | Mean of 9 determinations from the literature |
| +4.18 ± 0.19 | Mean of 3 | Sandage & Tammann 1982 | See Table 4 of Capaccioli et al. (1990) |
| +3.60 ± 0.40 | B-band TF | Fukugita et al. 1993 | Virgo centre placed at 15 Mpc |

Figure 1. Panel a) Distribution of the kinematical subset of the Virgo galaxies of the $C^2D$ sample in the log $D_n$–log $σ$ plane. The solid line gives the fit to the “best sample”, obtained by rejecting the objects deviating largely from the mean relation. Panel b) The same diagram for the Virgo (“best sample”) and Fornax early-type galaxies. The dashed line is the fit of minimum scatter for the galaxies of the Fornax cluster, adopting the same slope of the Virgo relation.
4649 (Franx et al. 1989), NGC 4459 (Peterson 1978) and NGC 4473 (Young et al. 1978).

For the galaxies of the Coma cluster we used the central velocity dispersions quoted by LGCT, JFK (who homogenized the data of Faber et al. 1989 and Dressler 1987), and Jørgensen et al. (1995b, who increased the JFK sample and obtained average velocity dispersions from a fixed and a variable, within r2/4, aperture of the slit).

The accuracy of the kinematical data is more difficult to evaluate. Much of the offset in the distance determinations is likely caused by systematic differences in the σ measurements. Such differences are difficult to evaluate due to the heterogenous nature of the samples and of the methods adopted to gauge the central velocity dispersion.

According to Jacoby et al. (1992) the errors on σ likely range between 6 and 14%. It is clear that such an estimate does not account for possible local deviations from the overall dynamical behaviour due for instance to singularities, which may cause the appearance of central spikes in the velocity dispersion profile (which behave as systematic errors on σ).

Of the 52 early-type galaxies in the Virgo sample (29 S0 and 23 E), 13 S0s and 4 Es have no σ0, 3 S0s have σ0 < 100 km s−1, and 4 S0s and 2 Es have anomalous high/low redshifts with respect to the mean of the cluster (c20 = 1179 ± 17, Sandage & Tammann 1995). One galaxy (NGC 4370) has been eliminated because its central surface brightness is fainter than μB = 20.75 (implying Dn = 0).

In Fornax we have 12 S0s and 16 Es: 7 S0s and 7 Es have no measurements of σ0, and 2 S0s have σ0 < 100 km s−1. There are no peculiar redshifts.

3.2 The Dn−σ relation for the Virgo and Fornax clusters

Fig. 1 plots the Dn−σ relation for the C2D sample. The upper panel shows only the Virgo galaxies, with different symbols according to their morphology: open circles for Es and triangles for S0s. Here the central velocity dispersions of the Virgo galaxies are the plain averages of the data found in the literature.

It is apparent that E and S0 galaxies follow the same relation. There are, however, 8 Virgo galaxies that largely deviate from the mean relation (indicated by the solid line; see below).

The discrepant galaxies have peculiar redshifts and/or high inclination to the line of sight. Three of them have σ0 < 100 km s−1 and, following the recipe of the 7-Sam have been discarded. Of the 9 S0s that follow the mean relation, 3 have apparent flattening higher than S0(5) (according to the BST classification) and one of them has a peculiar redshift. Two galaxies fitting the mean relation are rounder than S0(5) but have discrepant redshifts. Therefore it seems that inclination and redshift do not determine unequivocally the galaxies that follow the Dn−σ relation and are not useful selection criteria for establishing a “representative” sample.

Table 3a presents the morphological type, the axis ratio, the recession velocity and the BST membership code for the Virgo galaxies which deviate significantly from the mean Dn−σ relation, and Table 3b the same data for those S0s that instead fit the relation. In the following we will reject the 8 discrepant galaxies when computing the Dn−σ and FP relations (the same galaxies, with the possible exception of NGC 4200, also show a large scatter with respect to the FP; they are likely foreground and background objects.

The solid line in Fig. 1 has been computed through a linear least-square fit of the best Virgo sample, ignoring errors on both coordinates. We get:

\[ \log(D_n) = (1.26 \pm 0.06) \log(\sigma) - (1.11 \pm 0.14) \]

with a r.m.s. of 0.06 in log(Dn), corresponding to a 15% uncertainty in the average distance.

With a more robust fit which takes into account the errors on both coordinates (Fasano & Vio 1988), it is:

\[ \log(D_n) = (1.31 \pm 0.07) \log(\sigma) - (1.24 \pm 0.16) \]

and the same r.m.s. fit as above.

Taking into account the above quoted errors, the intrinsic scatter of the Dn−σ relation is 0.05, corresponding to a distance uncertainty of ~ 12% per galaxy.

In the lower panel of Fig. 1 the Virgo and Fornax galaxies are indicated with different symbols. The solid line is our eq. 1.

If we assume that the Dn−σ relation for the Fornax

\* Note that the errors on both coordinates (Δ log(Dn) ~ 0.02 and Δ log(σ) ~ 0.04) are approximately equal, and the use of a least square fit that weights both variables does not produce a significantly different result.

Table 3a: Virgo galaxies that do not fit the Dn−σ relation

| Ident. | Morph. | (b/a)25 | cz      | Membership |
|--------|--------|---------|---------|------------|
| NGC 4168 | E2    | 0.79    | 2316    | –          |
| NGC 4200 | S0(4) | 0.61    | 2376    | –          |
| NGC 4261 | E2    | 0.80    | 2200    | –          |
| NGC 4270 | S0(6) | 0.46    | 2347    | –          |
| NGC 4281 | S0(6) | 0.56    | 2711    | –          |
| NGC 4342 | S0(7) | 0.54    | 714     | –          |
| NGC 4417 | S0(7) | 0.62    | 832     | M          |
| NGC 4550 | S0(7) | 0.31    | 378     | M          |

Table 3b: Virgo S0s fitting the Dn−σ relation

| Ident. | Morph. | (b/a)25 | cz      | Membership |
|--------|--------|---------|---------|------------|
| NGC 4339 | S0(0) | 0.86    | 1287    | –          |
| NGC 4377 | S0(3) | 0.85    | 1371    | M          |
| NGC 4459 | S0(2) | 0.84    | 1210    | M          |
| NGC 4526 | S0(6) | 0.37    | 533     | M          |
| NGC 4552 | S0(0) | 0.85    | 321     | M          |
| NGC 4570 | S0(7) | 0.31    | 1730    | M          |
| NGC 4578 | S0(4) | 0.70    | 2284    | M          |
| NGC 4638 | S0(7) | 0.70    | 1147    | M          |
| NGC 4649 | S0(2) | 0.81    | 1144    | M          |

Notes to Tables 3a,b: Morphological types, recession velocities and membership are from Binggeli et al. (1985); axis ratios at μB = 25 are from C2D.
cluster has the same slope as that for Virgo (a reasonable approximation if the environment does not influence the $D_n - \sigma$ relation), the fit providing the minimum scatter for the Fornax galaxies gives $\Delta \log(D_n) = 0.09$, and a ratio $D_n(Virgo)/D_n(Fornax) = 1.23$, corresponding to $\Delta \mu_{FV} = 0.45$ mag.

This result disagrees with Davies et al. (1993), who found $D_n(Virgo)/D_n(Fornax) = 1.07$, and a $\Delta \mu_{FV} = 0.15$ mag. The origin of such a discrepancy (15% in distance) is not in the values of $D_n$, since the two samples compare quite well, but rather in the values of $\sigma$. For 7 of the 8 Fornax galaxies we have in common, the values of $\sigma_0$ are larger in our sample. The discrepancy between the spectroscopic data of the two samples increases for the objects with $\sigma_0 > 150$ km s$^{-1}$ (D’Onofrio et al. 1995).

The non-homogeneity of the velocity dispersion data is also clear from Fig. 2. Note that the scatter for the Virgo objects is lower than for Fornax: the r.m.s. of the least square fit of the two clusters is 0.06 for Virgo and 0.13 for Fornax.

This is at variance with the expectation based on the morphology of the two clusters: diffuse (Virgo) and compact (Fornax), according to BST and Ferguson (1989). If one assumes that the two clusters have spherical symmetry, an intrinsic $\sim 7\%$ and $\sim 11\%$ scatter is expected for Virgo and Fornax respectively, according to their angular sizes.

One might speculate that the small scatter of the Virgo data is due to the averaging of several measurements of $\sigma$, obtained by different authors using different methods of data analysis on spectra taken with different slit apertures. What does it happen if we do not average the central velocity dispersion measurements? This is shown in Fig. 2, where, for each source of $\sigma_0$ data, we report the slope, intercept, and r.m.s. scatter of the best fit for the corresponding Virgo galaxies and the resulting $\Delta \mu_{FV}$ computed adopting the slope derived for Virgo.

This experiment tells us two things. The first is that the small scatter of the Virgo data is independent of the databases (each catalog of velocity dispersions is made of averaged and homogenized data). If we use, however, the 12 galaxies of the sub-sample with the best observed rotation curves and velocity dispersion profiles, the r.m.s. scatter from the best fit increases to 0.08 (20%). The second is that the relative distance moduli between the two clusters span a large interval, varying with the properties of the adopted Virgo sample. The null value of $\Delta \mu_{FV}$ found in our comparison with Pierce (1989) demonstrates this effect.

Since most of the Virgo data come from the 7-Sam sample, we decided to concentrate on the errors introduced by the aperture sizes on the relative distance between Virgo and Fornax, adopting the central velocity dispersions of their Lick and LCO-HI measurements ($4'' \times 1''\,5$ and $4'' \times 4''$ slit aperture respectively). The two data-sets for precisely the same galaxies, give $\Delta \mu_{FV} = 0.35$ and 0.55 respectively. This means that an accurate distance between the clusters could be obtained only after an homogenous comparison of the $\sigma$ measurements, corrected for the aperture size effects. A more quantitative idea of such corrections has been provided by Jørgensen et al. (1995b).

Finally, we calculate the distance between Fornax and Virgo through the $L - \sigma$ relation using the $C^2D$ B-magnitudes for the galaxies of the two clusters and the velocity dispersions from the catalog of McElroy (1995). We obtained $\Delta \mu_{FV} = 0.43$ mag, in good agreement with the $D_n - \sigma$ relation.

### 3.3 The $D_n - \sigma$ relation for the Coma cluster

We used the data given by LGCT, JFK and Jørgensen et al. (1995a,b) to derive the distance to the Coma cluster. Unfortunately, the $D_n$ diameters measured by LGCT are in the V band (instead of B), at the mean surface brightness level of 19.8 mag arcsec$^{-2}$. However, since the average $B - V$ colour of elliptical galaxies is approximately constant and equal to 0.95 mag, we can equally compare the two data sets. The same happens for the $D_n$ values of Jørgensen et al. (1995a) which are in the $r$ band. In that case, using the photometric parameters in the $B$ and $r$ bands listed in JFK, we derived the relations: $\log(r_n^B) = 0.968 \log(r_n^r) + 0.017$ and $\log(r_n^B) = 1.006 \log(r_n^r) + 0.026$. The new parameters of Jørgensen, together with those given by LGCT, have been corrected by the authors for cosmological and seeing effects.

For the LGCT and JFK samples the line of minimum scatter with respect to eq. 2 (Fig. 3) gives $\Delta \log(D_n) = 0.68$ and $\Delta \mu_{FCV} = 0.69$ respectively, corresponding to a $\Delta \mu_{CV}$ of 3.4 and 3.45 mag. Considering a 5% aperture correction to the observed $\sigma_0$ passing from the Virgo and Coma galaxies (as in Davies et al. 1987), $\Delta \mu_{CV}$ turns out to be $\sim 3.55$ mag. We note that the use of the LGCT seeing corrected parameters does not influence very much the final result.

We also calculated the relative distance between Coma and Virgo using the photometric parameters and the Coma velocity dispersions quoted by Jørgensen et al. (1995b), who give the average $\sigma$ within a fixed aperture of $3''\,4$ and within a variable aperture $r_e/4$, which is connected to the physical size of the galaxies. It was impossible for us to get the same variables for the Virgo galaxies, since this requires the velocity dispersion profiles. We found $\Delta \mu_{CV} = 3.45$ and 3.55 mag respectively in the two cases, confirming the 12-15% distance uncertainty due to aperture effects. Since the $r_e/4$ aperture is generally lower than the $3''\,4$ slit for the Coma galaxies, we conclude that, by enlarging the aperture, one obtains a lower average value of $\sigma$ and consequently a lower differential distance modulus.

It is also interesting to compare the $D_n - \sigma$ results with the $V_{26} - \sigma$ relation of Dressler (1984, hereafter AD), who found $\Delta \mu_{CV} = 3.75$ mag. His result is particularly important here because Dressler considered two different slit apertures for the galaxies in Virgo and Coma in order to reduce as much as possible the aperture effect. The $\sim 0.25$ mag difference of the two methods can be explained either by a systematic difference in $D_n$ (15%) or in $\sigma_0$ (10%).

We first compared the magnitudes of the Virgo galaxies in common with Dressler (he used the data of Sandage & Visvanathan 1978). From the growth curves of the $C^2D$ sample, we derived the $B_{26}$ magnitudes of our galaxies, and compared them with Dressler’s data by transforming his $V_{26}$ magnitudes in the $B$-band. We adopted the color equation of Sandage & Visvanathan (1978): $B - V = 1.25(b - V) + 0.22$. We obtained $B_{our} = 1.03 B_{Dre} - 0.80$. Being only a

Excluding NGC 1316, the galaxy that has the largest discrepant value of $\sigma_0$, the r.m.s. lowers to 0.10.
The discrepancy between the $D_n$–σ and the $L$–σ relations can also be due to a systematic ∼10% offset in the velocity dispersions. Part of this offset ($\Delta \log(\sigma) \approx 0.028$) has been found by LGCT in their comparison with the data of Dressler (unfortunately however, such a correction to the $\sigma$ values of Dressler brings the result of the $B$–σ relation to $\Delta \mu_{CV} = 4$). By comparing the central velocity dispersions of the Coma galaxies from LGCT, Dressler (1984), and McElroy (1995, hereafter MeE), we found $\Delta \log \sigma_0(McE - LGCT) \approx -0.017$ and $\Delta \log \sigma_0(MeE - AD) \approx 0.014$. For the Virgo galaxies we made the comparisons between Dressler (1984), McElroy (1995) and Davoust et al. (1985, hereafter DPV) obtaining $\Delta \log \sigma_0(DPV - AD) \approx -0.015$ and $\Delta \log \sigma_0(DP V - AD) \approx 0.04$ (note that the DPV data have been homogenized by the authors taking into account aperture effects, method of reduction, etc.). This means that the aperture effects can account for the observed difference. The large differential distance modulus is likely the result of the low central velocity dispersion that Dressler got for the Virgo galaxies.

Looking at the data of the 7-Sam and of DPV, we discovered that in 70 ÷ 75% of the cases the use of a small aperture (slit width over face on corrected diameter $D(0)$ less than 0.02) produces an higher central velocity dispersion ($\sigma_l/\sigma_s = 0.85$), where the subscripts here stay for "large aperture" and "small aperture". This effect can be easily responsible of the scatter observed in $\Delta \mu_{CV}$, since it gives ∼15% difference in the central velocity dispersions.

A further possible explanation of the difference between the $L$–σ and $D_n$–σ relation resides in the surface brightness bias which affects the $L$–σ relation. To test this effect we corrected the total $B$ magnitudes of the galaxies adopting the relation $\Delta B = -0.3(\mu)_e + 6.49$, which represents the trend of the residuals as a function of the effective surface brightness in our data. The result is that we obtain a change in the slope and in the scatter of the $B$–σ relation, but the differential modulus is not affected.

The distance to Coma derived here is in conclusion
slightly smaller than the results of other DIs. The responsibility of this difference is likely in the adopted values of the central velocity dispersions. Within the uncertainty, however, the adopted value is in good agreement with the B-band TF relation (Fukugita et al. 1993), with the $D_n$-$\sigma$ relation of Dressler et al. (1987), with the SN–Ia (Capaccioli et al. 1990), and with Bower et al. (1992), who used a combination of three methods: CM diagram, colour-diameter relation, and FJ relation.

Another possible source of error, discussed by LGCT and Lucey et al. (1991a), is the surface brightness bias which may affect the $D_n$-$\sigma$ relation of the Coma galaxies. Such a bias is evident in the LGCT data looking at the residuals of the $D_n$-$\sigma$ relation for the galaxies in the central region of the Coma cluster with respect to those in the outer parts. We checked the influence of this bias on $\Delta \mu_{CV}$ by selecting objects in the core of the cluster and outside, but we did not find any significant difference between the two samples.

3.4 The aperture effects on the central velocity dispersion

Since aperture effects play a major role in determining the measured central velocity dispersion, we perform another test to better investigate this problem.

In Fig. 4 (bottom panel) we plot the difference between the velocity dispersions for the same galaxy measured through two apertures of different radius $r_s < r_l$ (small and large slit apertures) versus the aperture size increment in units of the galaxy effective radius. The data are for the Virgo galaxies: the velocity dispersions have been measured from slit widths of 2", 4" and 16" and are taken from the 7-Sam and Dressler (1984) samples. The solid line represents a fit obtained for the galaxies of our Fornax sample. For each galaxy we calculate the difference between the central velocity dispersion and the luminosity weighted velocity dispersions within increasing fractions of the effective radius. The average values for 13 galaxies can be fitted by the relation: $\sigma_0 - \sigma_{ap} = 18.3 \times \log(r_{ap}/r_e) + 38.3$ with a correlation coefficient of 0.97 and a rms of 2.57.

The upper panel reports the data presented in the catalog of Davoust et al. (1985). Here the authors listed the width of the slit normalized to the corrected face-on diameter of the galaxies $D(0)$.

The figure shows that when the difference among the apertures is larger than $\sim 0.1$ $r_e$, the value of $\sigma$ obtained through the small aperture is systematically larger than that given by the large slit. It should be emphasized that for the smaller galaxies this effect is much more evident (the slit covers a larger fraction of the effective radius).

The scatter around zero at smaller $\Delta \xi = (r_l - r_s)/r_e$ is possibly explained by the 10-15% error of the individual measurements, but it may have also another explanation. According to Ciotti et al. (1996) and Ciotti & Lanzoni (1996), the projected central velocity dispersion depends on the fraction of effective radius of the galaxy covered by the slit width and on the surface brightness distribution of the galaxy as whole. The ratio of the central to the virial $\sigma$ as a function of $r/r_e$ has approximately a gaussian shape, with a peak centered between 0.1 and 0.3$r_e$. The shape and position of the peak depend on the exponent $m$ of the $r^{1/m}$ law, which best fits the light distribution of early-type galaxies (Caon et al. 1993). When $m$ is greater than 4, the curve becomes progressively monotonic decreasing. This particular behaviour can be responsible of the observed scatter around zero in Fig. In fact, according to the fraction of $r_e$ covered by the slit one can get different $\sigma$. Only for the galaxies with...
m > 4 the difference between the central velocity dispersion obtained from the small (σ_{m}) and large (σ_{l}) apertures is always greater than zero. Interestingly enough we verified that for the Virgo galaxies with m > 4 we always have σ_{m} - σ_{l} > 0. Anyway, taking into account the uncertainties on the measured σ, the prediction of this model cannot be presently confirmed.

By applying the aperture correction to the data of Dressler (1984) for the Virgo and Coma clusters the resulting D_{n} - σ relation is log(D_{n}) = 1.53 log(σ) - 1.79 and the differential distance modulus Δμ_{CV} = 3.685. The same correction applied to the data of the 7-Sam (for Virgo) and to our Fornax sample gives log(D_{n}) = 1.53 log(σ) - 1.81 and Δμ_{FV} = 0.35. Notice the change of the slope from 1.26 to 1.53, and the better agreement of the differential distance moduli with the average literature values.

3.5 The error analysis

As already stated, the limits of the above results in the determination of the differential distance moduli are in: 1) the a priori assumption that the slope of the D_{n} - σ relation is the same for the three clusters, 2) the choice of the “representative” sample of objects for the whole cluster, 3) the accuracy of the data (in particular σ_{0}), and 4) the definition of the variables which enter the relation: e.g. σ_{0} or the average σ within a prefixed radius.

Before turning our attention to the errors associated to the use of the D_{n} - σ relation, we have checked the assumed invariance of the slope d log(D_{n})/d log σ. To this end we have shifted the Fornax and Coma galaxies to the distance to the Virgo cluster using the above calculated Δ log(D_{n}), and fitted the whole distribution again. The result:

log(D_{n}) = (1.24 ± 0.06) log σ_{0} - (1.08 ± 0.15),

with r.m.s. = 0.08, shown in Fig. 3, is in excellent agreement with eq. 3.

Let us now analyze the fitting procedure. Isobe et al. (1990) and Feigelson & Babu (1992) developed five methods for applying linear regression fits to a bivariate data distribution with unknown or negligible measurement errors. The advantage of their method is that the uncertainties in the slope and intercept of the fitted distribution, and in the process of comparative calibration, are evaluated through resampling procedures: jackknife and bootstrap error analysis. The bootstrap error analysis is based “on the distribution of slopes and intercepts of a large number of data sets constructed by random sampling of the observed data set with replacement”. The jackknife error analysis is performed in a similar way but with only N synthetic data sets, each containing (N - 1) points from the original data set, leaving out one observation in sequence.

Following their approach we used the standard linear regression analysis program SLOPES, kindly provided by the authors), with the distance dependent variable (log(D_{n})) as the Y variable. This choice has the effect of minimizing the residuals in D_{n}. The slopes for the Virgo, Fornax, and Coma samples range from 1.16 to 1.37, with different errors for the different samples. For the Fornax cluster the errors in the intercept (0.69) and in the slope (0.31) are much larger than for the other two clusters, due to the large scatter of the data and to the small number of galaxies.

Which is the error that affects the intercept offset? Assuming that the distance to the calibrating cluster is known with high accuracy (i.e. that intercept and slope of the fit are error free), the error in the relative distance comes approximately from the uncertainties in log(D_{n}) (E_{D} = Δ log(D_{n})) and in log σ (E_{σ} = Δ log(σ)) for the new sample of galaxies. Since we are looking for the fit that minimizes the residuals of the new sample, and we assume that the slope is the same as that of the calibrating sample, we can set the maximum shift in the intercept equal to ~ √E_{D}^{2} + E_{σ}^{2}. Being the errors in both coordinates equal to 0.02 (minimum error), the minimum shift of the intercept is ~ 0.03, corresponding to an error in Δμ of 0.14 mag or 6% in distance.

A more correct estimate of such errors comes from the program CALIB developed by Isobe et al. (1990) and Feigelson & Babu (1992), which predicts the value of the intercept (and its uncertainty) in the fit of a new sample of galaxies, from the linear regression of a calibrating sample, without the assumption that the two slopes are equal.

In summary this procedure gives Δ log(D_{n}) = 0.093 ± 0.015 and Δ log(D_{n}) = 0.095 ± 0.06, assuming Virgo and Fornax respectively as calibrators, and Δ log(D_{n}) = 0.693 ± 0.013 and Δ log(D_{n}) = 0.690 ± 0.016 for Coma and Virgo. A 0.015 uncertainty in log(D_{n}) corresponds to ~ 3% error in the relative distance to the clusters. This is the minimum uncertainty attainable with such data for the relative distance between the two clusters.

4 THE FUNDAMENTAL PLANE RELATION

For the Virgo galaxies of the C2D sample, the FP relation shown in Fig. 5 writes:

$$\log(r_v) = (1.26 \pm 0.09) \log(\sigma) + (0.28 \pm 0.01)(\mu)_v - (7.31 \pm 0.28).$$

Figure 5. The fit to the full data-set of Virgo, Fornax, and Coma galaxies (C2D, LGCT, and JFK samples, for a total of 128 objects), after shifting Fornax and Coma to the distance to Virgo. Symbols are the same as in the previous figures.
The values of the coefficients are derived here through a multivariate statistical analysis. The r.m.s. error of the fit is 0.06, which translates into a 15% uncertainty in the distance to a single object, the same obtained with the 0.06, which translates into a 15% uncertainty in the distance for Fornax and Coma respectively, assuming the same

eq 4, the dotted and dashed lines represent the fit of minimum scatter for Fornax and Coma respectively, assuming the same slope of Virgo.

The FP for the Virgo (open circles), Fornax (filled circles), and Coma (open triangles) galaxies. The solid line is our eq. \[ \text{the dotted and dashed lines represent the fit of minimum scatter for Fornax and Coma respectively, assuming the same slope of Virgo.} \]

The FP method gives exactly the same relative distance to Virgo obtained through the

atisfaction has been claimed by many authors (e.g. Lynden-Bell et al. 1988, LGCT, JFK, van Albada et al. 1993). We have looked for such a bias using the D_n–σ relation modified by adding a term in surface brightness, as proposed by van Albada et al. (1993):

\[ \log(D_n) = a \log(\sigma) + b \Delta \mu + c \Delta \mu^2, \]

where \( \Delta \mu = \langle \mu \rangle - 20.75 \). For our Virgo sample we obtain \( a = 1.356, b = -0.008, \) and \( c = 0.002, \) with a r.m.s. scatter of 0.06. The multivariate statistical analysis performed on the three variables \( \log(D_n), \log(\sigma), \) and \( \Delta \mu, \) shows that 95% of the total variance is enclosed in the correlation between the first two variables. For comparison the coefficients found by van Albada et al. (1993) with the data of Faber et al. (1989) are \( a = 1.35, b = 0.047, \) and \( c = -0.019. \) With our data the contribution of the terms in surface brightness is very low.

We show in Fig. 3 the distribution of the galaxies of the C²D and JFK samples in the D_n–σ–Δμ diagram. Again the offsets among the clusters distributions are of the same order as in the classical D_n–σ and FP relations.

The trend of the residuals \( \Delta \log(D_n) \) in the D_n–σ relation with respect to \( \langle \mu \rangle \) is shown in Fig. 3 for the galaxies of the C²D sample. Here we have explored different definitions of \( D_n \) by varying the average surface brightness \( \langle \mu \rangle_n \) from 19.75 to 22.75 mag arcsec^{-2}. It is apparent that there is no clear trend of the residuals with \( \langle \mu \rangle_n, \) in agreement with LGCT, but not with JFK. The r.m.s. scatter of the residuals is similar in all cases and equal to 0.09.

Following Dressler et al. (1987), who showed that \( D_n/2r_e \propto (I_e)^{0.8} \) for r^{1/4} galaxies, D’Onofrio et al. (1994; their eq. 8 and Fig. 1) derived the relation between \( D_n/2r_e \) and \( \langle \mu \rangle_n \) for objects with r^{1/4} light profiles. Here m is the exponent that provides the best fit to the major axis light profiles for the early–type galaxies of the C²D sample, and ranges from 1 to > 10, increasing systematically with the galaxy luminosity.

Under simple assumptions, the inverse relation \( \langle \mu \rangle_n - \langle \mu \rangle \) versus \( D_n/2r_e \) can be written:

\[ \log(D_n/2r_e) = \sum_{k=0} a_k(m)[\langle \mu \rangle_n - \langle \mu \rangle]^k \]

where \( \langle \mu \rangle_n \) is the average surface brightness within a prefixed isophote (in general \( \langle \mu \rangle_n = 20.75 \)), and the coefficients \( a_k(m) \) are functions of m.

Writing the observed D_n–σ relation as:

\[ \log(D_n) = a' \log(\sigma) + b' \]

5 TESTING THE D_n–σ AND FP RELATIONS

The second part of this paper is dedicated to a critical analysis of the D_n–σ and FP relations, taking advantage of our well defined (homogeneous and volume-limited) sample of galaxies in Virgo and Fornax.

5.1 The surface brightness bias in the D_n–σ relation for the Virgo and Fornax clusters

The existence of a surface brightness bias in the D_n–σ relation has been claimed by many authors (e.g. Lynden-Bell et al. 1988, LGCT, JFK, van Albada et al. 1993). We have looked for such a bias using the D_n–σ relation modified by adding a term in surface brightness, as proposed by van Albada et al. (1993):

\[ \log(D_n) = a \log(\sigma) + b \Delta \mu + c \Delta \mu^2, \]

This means that the FP method gives exactly the same relative distance to Virgo obtained through the D_n–σ relation. The same result is obtained for Coma if we adopt the FP relation derived by JFK.

Now, by setting \( X = 1.26 \log(\sigma) + 0.28 \langle \mu \rangle_e - 7.31 \) and \( Y = \log(r_e), \) and running again the program SLOPES and CALIB, we obtain: \( \Delta \log(r_e^{FV}) = 0.06 \pm 0.01 \) and \( \Delta \log(r_e^{CV}) = 0.07 \pm 0.05, \) using Virgo and Fornax respectively as the calibrating samples, \( \Delta \log(r_e^{FV}) = 0.69 \pm 0.01 \) and \( \Delta \log(r_e^{CV}) = 0.70 \pm 0.02, \) using instead Virgo and Coma. Again the uncertainty in \( \log(r_e) \) corresponds to a \( \sim 3\% \) error in the relative distance to the clusters. The results of these programs are summarized in Table 4.

If we shift Fornax and Coma to the distance of Virgo we obtain for the whole sample of 65 galaxies: \( \log(r_e) = 1.02 \times X + 0.07 \) with r.m.s. = 0.09, comparable to the intrinsic scatter of the relation, and corresponding to an uncertainty of \( \sim 23\% \) in distance. This is a value somewhat larger than previously reported in the literature.

The distance moduli to Fornax and Coma relative to Virgo, based on the FP relation, now span the interval 0.3 ≤ \( \Delta \mu_{FV} \) ≤ 0.45 mag, and 3.45 ≤ \( \Delta \mu_{CV} \) ≤ 3.50 mag, depending on the fitting method that is used.

\[ \log(D_n) = a \log(\sigma) + b' \]

\[ \sum_{k=0} a_k(m)[\langle \mu \rangle_n - \langle \mu \rangle]^k \]

\[ \log(D_n/2r_e) = \sum_{k=0} a_k(m)[\langle \mu \rangle_n - \langle \mu \rangle]^k \]
and the FP equation as:

$$\log(R_c) = a \log \sigma + b(\mu)_c + c,$$

the difference between eqs. 6 and 7 can be expressed by the formula:

$$\Delta \log(D_n) = (a - a') \log \sigma + [b + a_1(m)](\mu)_c + \sum_{k=2}^{\infty} a_k(m)[(\mu)_c - (\mu)_n]^k + \text{const},$$

where $\text{const} = a_0(m) + c - b' = 20.75 a_1(m)$. The values of the first three coefficients $a_k$ for $m = 4$ are approximately: $a_0 = 1.2 \times 10^{-3}$, $a_1 = -2.7 \times 10^{-1}$, $a_3 = -1.5 \times 10^{-2}$, vanishing for large values of $k$.

Looking at the values of the slope and of the intercept of the $D_n - \sigma$ and FP relations, one sees that all the coefficients in eq. 6 are very small (e.g. $[a - a'] \sim 0.05$, assuming respectively the slopes in eq. 6 and eq. 7, $[b + a_1(4)] = 0.007$, and so on) for each value of $m$. For this reason the dependence on $(\mu)_c$ of the residuals of the $D_n - \sigma$ is almost completely cancelled out, and this is confirmed also in our Fig. 6.

5.2 The behaviour of the residuals in the $D_n - \sigma$ and the Fundamental Plane relations

A test of the behaviour of the residuals of the $D_n - \sigma$ and FP relations is presented in Fig. 6b, where the residuals $\Delta \log(D_n)$ and $\Delta \log(r_e)$ for the C/D galaxies are plotted versus the galaxy luminosity $M_B$, the ellipticity at the effective radius $e$, the normalized coefficient $a_0$ of the Fourier expansion of the residual of the isophotes with respect to the best fitting ellipse (Bender & Möllenhoff 1987), the logarithm of the exponent $m_a$ of the $r^{1/m}$ law that best fits the major axis light profiles of the galaxies, the logarithm of the maximum observed rotation velocity along the major axis of the galaxies $V_a$, and the logarithm of the sum $V_a^2 + \sigma_R^2$.

No clear trend of the residuals is seen in almost all cases, with the possible exception of the galaxies with large values of $m_a$ and low maximum rotation velocities $V_a$: galaxies with $V_a < 60$ km s$^{-1}$ have preferably negative residuals, the opposite being true for the faster rotators. Unfortunately this result rests on a small number of objects. The $V_a$ values for the Fornax galaxies have been derived from the rotation curves measured by D’Onofrio et al. (1995), while for Virgo we used the sub-sample of 12 objects with the best observed rotation curves and velocity dispersion profiles available in the literature.

Notably, the small asymmetry in the residuals disappears if we consider the quantity $V_a^2 + \sigma_R^2$, which is more closely related than $V_a$ to the total kinetic energy.

The dependence of the residuals on the kinematical properties of the galaxies is more clearly seen in Fig. 7, where $\Delta \log(D_n)$ is plotted versus the ratio $V/\sigma$ between the ordered and random motions, and versus the anisotropy parameter $(V/\sigma)^*$. (cf. Davies et al. 1983), which is a measure of the departure of an object from the rotational support. In the lower plots $V/\sigma$ is computed using the highest values assumed by the rotational velocity and the velocity dispersion in the observed range, while in the upper plots it is the ratio of the values of $V$ and $\sigma$ measured at the galactocentric distance 0.3$a_c$, where $a_c$ is the effective semi-major axis of the galaxies.

There is possibly a trend in the residuals: we find $\Delta \log(D_n) = 0.09 \log(V_a/\sigma) + 0.06$ with a r.m.s. equal to 0.1 and a correlation coefficient of 0.34. From the t-Schmidt distribution the (null) hypothesis of no correlation between the two variables can be rejected at the 95% confidence level. This mild correlation, however, is not completely attributable to the presence of the S0 galaxies, but rather is due to the different distribution of the galaxies dominated by anisotropic pressure $(\log(V_a/\sigma)^* < -0.25)$ with respect to those supported by rotation $(\log(V_a/\sigma)^* > -0.25)$. In fact, if we eliminate the S0 galaxies, a slight trend is still apparent: $\Delta \log(D_n) = 0.06 \log(V_a/\sigma) + 0.02$, with a correlation coefficient equals to 0.21. The trend in the residuals disappears in the correlation with $\log(V/\sigma)/a_{3a}$, when the S0 are eliminated. At large radii only the S0 galaxies, that are largely supported by rotation, have positive residuals. In the upper plot only few Virgo galaxies have rotation curves and velocity dispersion profiles extended to 0.3$a_c$.

In the above discussion, we did not take into account projection effects in determining $V_a$, since this would require the knowledge of the intrinsic structure of each galaxy. This is of course incorrect. However, Binney (1978, 1980) showed that for oblate ellipticals with Hubble luminosity profiles, the ratio $(V^2)/\sigma^2$ can be approximated to better than 20% by $V_0^2/\sigma_c^2$, where $V_0$ is the true maximum equatorial rotation velocity and $\sigma_c$ is the velocity dispersion measured at the centre of the galaxy, and that $V_{eq}^2/\sigma_c^2 \equiv (16/\pi^2) V_0^2/\sigma_0^2$, where $V_0$ and $\sigma_0$ are the projected values for an edge-on oblate galaxy. Since we can assume a random orientation of the rotation axes with respect to the line of sight, the mean projection of $V_0^2$ along this line is $V_m^2 = 2/3 V_0^2$, and we can believe that, to a first order approximation, the use of $V_m$ and $\sigma_0$ should not produce a systematic error of $(V^2)/\sigma^2$.

Projection effects along the line of sight are also impor-
Figure 8. Residuals of the $D_n - \sigma$ relation versus the average surface brightness $\langle \mu \rangle_e$ for the galaxies of the C2D sample. Different choices of $\langle \mu \rangle_e$ are used in each box (see text). Open and filled symbols represent Virgo and Fornax objects respectively. Circles and triangles are E and S0 galaxies respectively. The dashed lines are best fits to the plotted distributions.

In this case rotation velocities are generally underestimated (by a factor 1.5-2), while velocity dispersions remain almost unchanged (Busarello et al. 1992). We have not tried here any deprojection since this would require the knowledge of the rotation axis of the galaxy with respect to the line of sight. The measured $V/\sigma$ ratios are therefore lower limits.

Unfortunately the small number of data does not permit a full analysis of the effects of rotation on the $D_n - \sigma$ and FP relations. We can only state that our result is along the same line of that of Saglia et al. (1993), who claim that the presence of disk components inside ellipticals produce a scatter in the FP, and in agreement with that of Prugniel & Simien (1994), who found the same correlation of $V/\sigma$ with the residuals of the FP, using a sample of galaxies which is not volume-limited.

Indeed, despite the poor correlation, it seems that the kinematical structure of the galaxies plays an important role in the $D_n - \sigma$ and the FP relations, in particular for what concerns the thickness. The amount of the detected deviations is in agreement with the prediction of the Virial theorem, when the contribution of rotation to the global equilibrium of elliptical galaxies is taken into account.

Fig. 11 shows that the rotationally supported galaxies provide a mean relative distance modulus for the Fornax cluster with respect to Virgo which is lower ($\Delta \mu_{FV} = 0.2$ mag) than that obtained from the galaxies dominated by velocity anisotropy ($\Delta \mu_{FV} = 1.0$ mag). Actually, this result is uncertain due to the large scatter of the Fornax galaxies and the small number of objects, and should be considered here only as an example of the bias that can be introduced by the rotation effects. If confirmed by future data, this source of bias should be taken into account in the $D_n - \sigma$ and the FP relations when used as DIs of high redshift clusters. The misclassification of S0 galaxies would in fact introduce a systematic bias.

Can we correct the $D_n - \sigma$ for such effect? A simple way could be the following. Let us write that $D_n \propto (\sigma^2)\alpha$, which means that the diameter of a galaxy is strictly related to the central velocity dispersion. If the contribution of the rotation cannot be neglected, we can write that $D'_n \propto (\sigma^2 + \beta V^2)^{\alpha'}$, where $\beta$ parametrizes the degree of rotational support. Taking the logarithm of these quantities and assuming that $\alpha' = \alpha \pm \Delta \alpha$, one obtains:

$$\Delta \log(D_n) = (\alpha \pm \Delta \alpha) \log(1 + \beta k^2),$$

(10)
Figure 9. The residuals $\Delta \log(D_n)$ and $\Delta \log(r_e)$ of the $D_n - \sigma$ and FP relations with respect to the absolute magnitude $M_B$, the ellipticity at $r_e$, $\log(m_a)$, the boxy/disky parameter $(a_4/a) \times 100$, $\log(V_m^2 + \sigma^2)$, and the maximum observed rotation velocity along the major axis of the galaxies $\log(V_m)$. The Virgo and Fornax galaxies are indicated by open and filled symbols respectively.
where \( k = V/\sigma \).

Unfortunately we do not know the correct values for the parameters \( \alpha \) and \( \beta \). The rotational support, parametrized by \( \beta \), is different for each galaxy, but a mean values for early-type galaxies could be within 0.5 and 1. The same interval is probably spanned by \( \alpha \) (for the classical \( D_n - \sigma \) relation \( \alpha \sim 0.6 \)). Using the approximation \( \alpha = 1 \) and \( \beta = 0.5 \), one obtains the correction curves plotted in Fig. 12.

After applying this correction, the trend of the residuals of the \( D_n - \sigma \) relation disappears, but the slope of the fitted distribution increases from 1.26 to 1.37, thus affecting the distance determinations of the clusters. Unfortunately the r.m.s. of the fitted distribution does not decrease significantly.

By considering only the sample of galaxies possessing both \( \sigma_0 \) and \( V_n \), we find that these two variables are not mutually correlated and that \( \log(V_n/\sigma)^* \) does not correlate with \( M_B \), while it correlates with \( \langle \mu \rangle_n \) (as in Wyse & Jones 1984). It is also interesting to note that the residuals of the \( D_n - \sigma \) and FP relations, \( \Delta \log(D_n) \) and \( \Delta \log(r_e) \), are mutually correlated. We find \( \Delta \log(r_e) = 0.94 \Delta \log(D_n) + 0.05 \) with an r.m.s. of 0.036. This is an obvious consequence of the Dressler et al. relation \( D_n/2r_e \propto \langle I \rangle_e^{0.8} \). The two relations are therefore almost equivalent when used as DIs for nearby clusters.

### 5.3 Further test on the \( D_n - \sigma \) relation

Do variations in the definition of \( D_n \) (i.e. in the surface brightness level \( \langle \mu \rangle_n \)) affect the \( D_n - \sigma \)? We performed an experiment with the Virgo and Fornax C2D sample by varying the value of \( D_n \), i.e. by assuming that \( \langle \mu \rangle_n \) changes from 18.75 to 22.75 mag in steps of 1 mag. In the case of Virgo the slope is found to have small variations, from 1.21 to 1.31, while for Fornax the variation is larger, and range from 1.19 to 1.37. This is due to the larger spread in the central velocity dispersions. The r.m.s. of the fit varies from 0.07 to 0.11 for Virgo and from 0.13 to 0.16 for Fornax. The best fits occur at \( \mu_B = 19.75 \) and 20.75 mag arcsec\(^{-2} \). All these expressions for the \( D_n - \sigma \) relation provide approximately the same \( \Delta \log(D_n) \) (it varies from 0.09 to 0.11), that is the same \( \Delta \mu \).

In another experiment we have acted on the definition of \( \sigma \), adopting once the value of the velocity dispersion mea-
Relative distances to the Virgo, Fornax, and Coma clusters

Figure 11. The $D_n - \sigma$ relation for the Virgo and Fornax clusters. The galaxies of Fornax are plotted with different symbols according to the value of $\log(V/\sigma)_{\text{max}}$. Two Fornax galaxies with $\sigma_0 < 100$ km s$^{-1}$ have been added here to increase the number of datapoints. The larger scatter of the Fornax data with respect to Virgo is more evident in this plot.

Figure 12. The solid line is the correction curve for the $D_n$ diameters according to eq. (12) with $\alpha = 1$ and $\beta = 0.5$. The dashed lines give the curves for $\Delta \alpha = \pm 0.25$ and the same $\beta$.

measured at the centre, $\sigma_0$, and another time the average within a given radial range. The motivation for this experiment comes from the Virial Theorem, since we want to look for the value of $\sigma$ that best represents $\langle \sigma \rangle$.

We decided to average $\sigma$ within circles of radius $r_n = D_n/2$ and $0.3a_e$, where $a_e$ is the effective semi-major axis. In each case we took plain averages, weighted means for the surface brightness, and means weighted for the rotational contribution. Unfortunately, this test was possible only for the 15 galaxies of the Fornax cluster which have accurate extended rotation curves, velocity dispersion profiles, and luminosity profiles. We found that the slopes of the $D_n - \sigma$ relation obtained in each case vary in the interval 1.34–1.62 (to be compared with the slope of 1.26 achieved using the central velocity dispersion), that is it changes by an amount larger that the error of the fit, and increases with the radius out to which the average is computed. Approximately equal r.m.s. fits are obtained using the central velocity dispersions $\sigma_0$ and the value averaged within $0.3a_e$. Slightly larger errors are derived with the means within $r_n$.

These results are an indication that early-type galaxies deviate from the pure homology and that the $D_n - \sigma$ and the FP relations have a slope and an intrinsic scatter connected to the spatial and dynamical structure of the galaxies.

One might try to use the quantity $(V_0^2 + \sigma^2)$ instead of $\sigma$ in order to account for the above mentioned bias introduced by the different kinematical properties of the galaxies. The result of such an experiment is that the slope of the relation changes from $\sim 1.23$ to $\sim 1.37$, but the r.m.s. of the fit does not show a significant improvement.

6 THE $\log(m) - \log(r_n)$ RELATION

We finally present another method to derive the relative distance to the Virgo and Fornax clusters. This method is based on the $\log(m) - \log(r_n)$ relation (Caon et al. 1993, Young & Currie 1994) and is shown in Fig. 12. Here $m_a$ is the exponent of the $r^{1/m}$ Sersic’ law best fitting the major axis surface brightness distribution of the galaxies, and $R_n$ is the effective radius in kpc. Although the scatter in this best fitting is quite large (the r.m.s. of the fit is $0.28$ and the correlation coefficient is 0.7), the intercept offset $\Delta \log R_n$ turns out to be $0.089$, corresponding to $\Delta \mu_{PV} = 0.44$ mag, in excellent agreement with the $D_n - \sigma$ and FP relations. Such agreement is remarkable although probably fortuitous given the large scatter of the data in this relation.

7 CONCLUSION

The numerical results obtained in this work fitting the $D_n - \sigma$ and FP relations are summarized in Table 4. From these data we derive $\Delta \mu_{PV} = 0.45$ mag and $\Delta \mu_{CV} = 3.45$ mag respectively for the relative distance moduli to Fornax and Coma with respect to the Virgo cluster. Taking into account the corrections for the aperture effects $\Delta \mu_{PV}$ is closer to 0.35 mag and $\Delta \mu_{CV}$ to 3.55 mag. Our distance to Fornax is slightly larger than derived with the other DIs, but it is nearly equal to that derived with the BCM method (see Table 1).

The inner ring diameters of spirals, the globular cluster luminosity function (GCLF), the surface brightness fluctuations (SBF), the $L - \sigma - \mu$ relation, the IR Tully–Fisher relation (IR-TF), and the scale length of dE galaxies, are in open contrast with this determination, while the planetary nebulae luminosity function (PNLF), and the SNe–Ia are consistent.

For the Coma cluster we derived a distance smaller than...
Table 4

| Method | a         | b         | c         | rms | \(\Delta \mu_{FW}\) | \(\Delta \mu_{CV}\) | Notes |
|--------|-----------|-----------|-----------|-----|-----------------|-----------------|-------|
| \(D_n - \sigma\) | 1.26 ± 0.06 | -1.11 ± 0.14 | 0.06 | 0.55 | 1               |
| \(D_m - \sigma\) | 1.31 ± 0.07 | -1.24 ± 0.16 | 0.06 | 0.45 | 2               |
| \(D_n - \sigma\) | 1.24 ± 0.06 | -1.08 ± 0.15 | 0.08 | 0.45 | 3               |
| \(D_m - \sigma\) | 1.26 ± 0.06 | -1.11 ± 0.14 | 0.06 | 0.46 | 4               |
| \(D_n - \sigma\) | 1.37 ± 0.31 | -1.46 ± 0.09 | 0.13 | 0.47 | 5               |
| \(D_m - \sigma\) | 1.16 ± 0.08 | -1.59 ± 0.19 | 0.08 | 3.45 | 6               |
| \(L - \sigma - \mu\) | 1.35 ± 0.08 | -0.008 ± 0.178 | 0.002 ± 0.016 | 0.08 | 3.45 | 7             |
| FP     | 1.26 ± 0.09 | 0.28 ± 0.01 | -7.31 ± 0.28 | 0.06 | 0.45 | 8               |
| FP     | 1.01 ± 0.03 | 0.07 ± 0.04 | 0.06 | 0.30 | 9               |
| FP     | 0.85 ± 0.10 | 0.23 ± 0.13 | 0.13 | 0.35 | 10              |
| FP     | 1.15 ± 0.04 | -0.84 ± 0.06 | 0.06 | 3.50 | 11              |
| \(\log(m) - \log(r_e)\) | 1.16 ± 0.25 | -0.160 ± 0.14 | 0.28 | 0.44 | 12              |

Notes: 1) Linear least square fit for the Virgo galaxies and fit of minimum scatter for Fornax adopting the same slope of Virgo. 2) Fit to the Virgo galaxies taking into account the errors on both coordinates. 3) Fit of minimum scatter for Fornax and Coma using the fit of the combined sample of 128 galaxies. 4) Output of program CALIB using Virgo as calibrator. 5) Output of program CALIB using Fornax as calibrator. 6) Output of program CALIB using Coma as calibrator. 7) Coefficients of the \(L - \sigma - \mu\) relation from the multivariate statistical analysis. Fit of minimum scatter for Fornax and Coma, adopting the same slope. 8) Coefficients of the FP relation from the multivariate statistical analysis. Fit of minimum scatter for Fornax and Coma, adopting the same slope. 9) Output of program CALIB using Virgo as calibrator. 10) Output of program CALIB using Fornax as calibrator. 11) Output of program CALIB using Coma as calibrator. 12) Coefficients of the fit for the Virgo galaxies. Fit of minimum scatter for Fornax, adopting the same slope.

Figure 13. The \(\log(m) - \log(r_e)\) relation for the Virgo and Fornax galaxies. \(r_e\) is the effective radius of the galaxies in kpc. The solid line gives the best fit for the Virgo galaxies. The dashed line is the fit of minimum scatter for the Fornax sample.

Given by the other DIs, but now a better agreement exists with the TF relation.

Probably our estimates can be improved when new accurate and homogeneous central velocity dispersions and rotation velocities will be available for a complete sample of Virgo and Fornax early-type galaxies. Actually, the large intrinsic scatter we found for the \(D_n - \sigma\) and FP relation of Fornax is at variance with previous determinations. We believe that the problem is in the measurements of \(\sigma\): the large spread of our central velocity dispersions being caused by projection effects, \(\sigma\) anomalies, and the presence of inner sub-components such as disks or central black holes. In this context, the large uncertainty on \(\Delta \mu\) resides probably in the different procedures used for measuring the velocity dispersions and in the corrections adopted for taking into account the effects of the aperture sizes.

By applying an average aperture correction to the central velocity dispersions, we obtain differential distance moduli in better agreement with the literature data, and we find that the tilt of the \(D_n - \sigma\) relation increases.

Looking at the data presented in this paper we conclude that the \(D_n - \sigma\) and the FP relation give substantially the same result. An advantage of the \(D_n - \sigma\) relation over the FP is that \(D_n\) can be measured with high photometric accuracy, while the effective radius \(r_e\) and \(\langle \mu \rangle_e\) may depend on the actual shape of the luminosity profiles and on the error in the total luminosity of the galaxies. The errors in \(r_e\) and \(\langle \mu \rangle_e\) are in fact usually correlated (Capaccioli et al. 1992).

At variance with van Albada et al. (1993), we do not find any need for adding a surface brightness correction to the \(D_n - \sigma\) relation, at least for the galaxies of the C2D sample. The inverse \((D_n/A_e) - \langle \mu \rangle_e\) relation of Dressler et al. (1987), coupled with the observed \(D_n - \sigma\) and FP relations, has indicated that the residuals of the \(D_n - \sigma\) relation depend only slightly on \(\langle \mu \rangle_e\), since all the coefficients in eq. 3 are approximately null.

The residuals in both the \(D_n - \sigma\) and FP relations are found to be independent of the intrinsic luminosity of the galaxies, the ellipticity, the mean surface brightness and the boxiness/disksiness parameter \(a_4\). A trend of the residuals, on the other hand, is present with the exponent \(m\) of the \(r^{1/m}\) Sersic’s law, with the maximum rotation velocity of the galaxies and with the \(V_{\text{rot}}/\sigma\) ratio. The last result is in agreement with Busarello et al. (1992) and Prugniel & Simien (1994), who first pointed out the contribution of rotation to the global equilibrium of elliptical galaxies.

The correlation of the residuals of the \(D_n - \sigma\) relation...
with the \((V/\sigma)\) ratio, with the anisotropy parameter \((V/\sigma)^*\), and with the exponent \(m_0\) of the major axis light profiles of the galaxies are all indications of the non-homologous nature of the early-type galaxies.

Further improvements call for more accurate kinematical measurements, for a critical estimate of the different methods of data reductions, for a standard correction of the aperture effects, and for a deeper analysis of the kinematics of each individual galaxy. Even at this stage, we are probably legitimated to say that most of the observed deviations from the FP are likely due to departures from the pure homology in the density distribution and in the dynamical structure of the early-type galaxies.

An important step for the use of the \(D_n-\sigma\) and the FP relations as DIs will be that of defining a “representative sample” of galaxies, of using standard selection criteria, and data with the same degree of homogeneity and accuracy, as well as variables properly and univocally determined.

In closing we want to remark that, while our analysis disagrees with Saglia’s et al. (1993) for what concerns the dependence of the residuals on the ellipticity and on \(\alpha_4\), nonetheless we back their conclusion: the kinematical structure of the early-type galaxies, if not properly taken into account, may produce biased cluster distances, when the \(D_n-\sigma\) and FP relations are used as DIs without care.

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