Motif-based Rule Discovery for Predicting
Real-valued Time Series*†‡

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Abstract

Time series prediction is of great significance in many applications and has attracted extensive attention from the data mining community. Existing work suggests that for many problems, the shape in the current time series may correlate an upcoming shape in the same or another series. Therefore, it is a promising strategy to associate two recurring patterns as a rule’s antecedent and consequent: the occurrence of the antecedent can foretell the occurrence of the consequent, and the learned shape of consequent will give accurate predictions. Earlier work employs symbolization methods, but the symbolized representation maintains too little information of the original series to mine valid rules. The state-of-the-art work, though directly manipulating the series, fails to segment the series precisely for seeking antecedents/consequents, resulting in inaccurate rules in common scenarios. In this paper, we propose a novel motif-based rule discovery method, which utilizes motif discovery to accurately extract frequently occurring consecutive subsequences, i.e. motifs, as antecedents/consequents. It then investigates the underlying relationships between motifs by matching motifs as rule candidates and ranking them based on the similarities. Experimental results on real open datasets show that the proposed approach outperforms the baseline method by 23.9%. Furthermore, it extends the applicability from single time series to multiple ones.

Introduction

The prediction of real-valued time series is a topic of great significance and colossal enthusiasm in the data mining community, and has been applied extensively in many research areas (Hamilton 1994; Esling and Agon 2012). Most of the work in literature aims at modeling the dependencies among variables, and forecasting the next few values of a series based on the current values (Makridakis, Wheelwright, and Hyndman 2008). However, other work suggests that for many problems it is the shape of the current pattern rather than actual values that makes the prediction (Das et al. 1998). For clarity, we call the latter one, forecasting by shape, rule-based prediction, which is the subject of this paper.

Informally, a rule $A \rightarrow B$ associates a pattern $A$ as the antecedent with a pattern $B$ as the consequent in an upper-bounding time interval $\tau$. The rule-based prediction works as follows: if a shape resembling $A$ is observed in a series, then a shape resembling $B$ is supposed to appear in the same or another series, within the time interval $\tau$. Usually, a rule refers to an “authentic rule”, an underlying relationship between two events implied by the two shapes.

In most work, firstly the subsequence clustering method is employed to symbolize/discretize the series, and then the symbolic series rule discovery method is applied to find rules from real-valued series (Das et al. 1998). However, such work has met limited success and ends up discovering spurious rules (e.g. “good” rules discovered from random walk data), because the symbol representation by common symbolization methods is independent with the raw real-valued series (Keogh and Lin 2005). In conclusion, the symbolized time series maintains little information about the original series to mine valid rules.

The state-of-the-art work (Shokoohi-Yekta et al. 2015) directly manipulates the series under the assumption that a rule is contained in a subsequence. It first selects a subsequence and then splits it into a rule’s antecedent and consequent. However, usually there is an interval between a rule’s antecedent and consequent. The splitting method will append the extra interval series to the antecedent/consequent, which fails to segment precisely for antecedents/consequents and results in rules with a lower prediction performance. Furthermore, it cannot be applied to find rules from multiple series, i.e. a shape in one series predicting a shape in another series.

Only when the observed object presents repeatability, can predictions be valid/reasonable. We believe that a serviceable rule should appear repeatedly in a real-valued time series and therefore its antecedent and consequent must be recurring patterns of the series. Recent studies (Vahdatpour, Amini, and Sarrafzadeh 2009; Brown et al. 2013; Mueen 2014) show that Motifs, frequently occurring patterns in time series (Patel et al. 2002), contain key information about the series. Therefore, we could utilize motif dis-

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1, 2, 3 A formal definition is given in the Preliminaries section.
discovery methods to find all distinct motifs which accurately represent antecedent and consequent candidates of rules.

However, three challenges are confronted: (1) there could be quite a few (overlapping) motifs in a single series (see Figure 1), let alone the dataset containing multiple series. How to screen for effective motifs as antecedents and consequents? (2) there could be many instances for two given effective motifs and different combinations of them will lead to different instances of a rule (see Figure 2). How to identify the best combinations for the underlying rule? (3) How to rank different rules given the instances of each rule?

In this paper, we present a rule-based prediction method for real-valued time series from the perspective of motifs. For each pair of motifs as a candidate rule, a heuristic-matching-based scoring algorithm is developed to investigate the connection between them.

We summarize contributions of this paper as follows:

- We develop a novel heuristic matching algorithm in search of the best combinations of motif instances. We also put forward a score function to assess each rule candidate on the training data by leveraging Minimum Description Length (MDL) principle.
- We evaluate our work on real open datasets, and the experimental results show that our method outperforms state-of-the-art work by 23.9% on average. Moreover, rules in multiple series can also be discovered.

Related Work

Early work extracts association rules (a.k.a. sequential patterns, etc.) from discrete or symbol series in the sequential database (Agrawal, Faloutsos, and Swami 1993; Agrawal and Srikant 1995; Mannila, Toivonen, and Verkamo 1997). Das et al. are the first to study the problem of finding rules from real-valued time series in a symbolization-based perspective (Das et al. 1998), followed by a series of work (Sang Hyun, Wesley, and others 2001; Wu, Salzberg, and Zhang 2004).

However, it has been widely accepted that few work dealing with real-valued series discovers valid rules for prediction. Because the symbolized series by common symbolization methods, including subsequence clustering and Piecewise Linear Approximation (PLA), cannot effectively represent the original series (Struzik 2003; Keogh and Lin 2005; Shokoohi-Yekta et al. 2015). Eamonn Keogh et al. (2005) demonstrate that clustering of time series subsequences is meaningless, resulting from the independency between the clustering centers and the input. Shokoohi-Yekta et al. (2015) point out that two time series can be differ only by a small discrepancy, yet have completely different PLA representations. Therefore, the symbolized series is irrelevant with the original real-valued one and the failure of finding valid rules is doomed.

The state-of-the-art work directly manipulates the real-valued series (Shokoohi-Yekta et al. 2015). Their method is found on the assumption that a rule is contained in a subsequence, which splits a subsequence into the rule’s antecedent and consequent. Usually, there is an interval between a rule’s antecedent/consequent, and the splitting method will append the extra series to the antecedent/consequent. The complicated diversity of the intervals could result in rules with bad prediction performance. Besides, the splitting method cannot be applied to discover rules from two series either, since a motif is a subsequence in a single series. Their work employs motif discovery mainly for an accelerating purpose to locate the position of a potential rule, while our method leverages motif discovery to segment patterns of most interest, extracting antecedent/consequent candidates precisely.

Preliminaries

In this paper, we consider the problem of rule discovery from real-valued time series for rule-based prediction. Without loss of generality, the rule discovery problem is exemplified using two time series $T_A$ and $T_B$, since finding rules from over two time series can be treated pairwisely. It can
also be applied to the situation of only one series by letting \( T_A = T_B \). We begin with the definition of a rule and its instance.

**Definition 1 (Rule).** A rule \( r \) is a 4-tuple \((m_A, m_B, \tau, \theta)\), \( m_A, m_B \) are the subsequences of two real-valued time series \( T_A, T_B \), respectively, and also the antecedent and consequent of the rule \( r \). \( \tau \) is a non-negative value\(^2\) indicating the max length of time interval between the \( m_A \) and \( m_B \). \( \theta \) is a non-negative value as the trigger threshold for firing the rule.

**Definition 2 (Instance).** A rule \( r \)'s instance \( e \) is a 3-tuple \((\tilde{m}_A, \tilde{m}_B, \tilde{\tau})\). \( \tilde{m}_A \) is the \( m_A \)-like subsequence observed in series \( T_A \) subject to \( d(\tilde{m}_A, m_A) < \theta \). \( \tilde{m}_B \) is the \( m_B \)-like subsequence observed later than \( \tilde{m}_A \) in series \( T_B \). \( \tilde{\tau} \) is the time interval between \( \tilde{m}_A \) and \( \tilde{m}_B \).

By the definitions, given a rule \( r = (m_A, m_B, \tau, \theta) \), if a subsequence \( \tilde{m}_A \) of \( T_A \) is observed and \( d(\tilde{m}_A, m_A) < \theta \), the rule \( r \) is fired and a subsequence of \( \tilde{m}_B \) similar with \( m_B \) is supposed to be observed from \( T_B \) within the subsequent \( \tau \)-time interval\(^2\). Notice that there are no constraints for \( \tilde{\tau} \) and \( d(\tilde{m}_B, m_B) \) in Definition 2, since an instance with \( \tilde{\tau} > \tau \) or \( d(\tilde{m}_B, m_B) \gg 0 \) can be viewed as a “bad” instance for \( r \). An example is shown in Figure 2.

Intuitively, if a rule is good, it must have many supporting instances in the time series, and then \( \tilde{m}_A \) and \( \tilde{m}_B \) will be frequently occurring patterns. By the definition of the motif (Patel et al. 2002), they are actually the motifs of \( T_A \) and \( T_B \), respectively. Inspired by that, we make the following assumption.

**Assumption 1.** If \( r = (m_A, m_B, \tau, \theta) \) is a rule, then \( m_A \in M_A, m_B \in M_B \), where \( M_A \) and \( M_B \) are motif sets of \( T_A \) and \( T_B \).

Based on the assumption, the rule discovery problem can be formulated as follows. Given two time series \( T_A, T_B \) with their motif sets \( M_A \) and \( M_B \), find top-K\(^4\) rules \( r = (m_a, m_b, \tau, \theta) \), where \( m_a \in M_A \) and \( m_b \in M_B \). Efficient algorithms for motif discovery have already been developed, and we could choose the widely-applied MK algorithm. (Mueen et al. 2009)

Directly, we present the top-level algorithm 1. It first finds the motif set for each series in line 1 by MK algorithm. Line 2 sorts and returns the top K motifs according to the domain knowledge of the series. Then it traverses every pair of motifs, and scores the corresponding rule in line 3 to 4. It finally returns the best K rules.

### Methodologies

Given a pair of motif \( m_a \) and \( m_b \), the score algorithm aims at evaluating a rule candidate \( r \) based on the instances in the training data. We will use the example in Figure 2 to illustrate. The scoring approach consists of three steps:

1. **Find out all \( m_A, m_B \)-like patterns in \( T_A, T_B \).** In this step, we use the sliding window method to select similar patterns by setting a threshold. In Figure 2, three \( m_A \)-like patterns \( m_A^{(i)} (i = 1, 2, 3) \) and four \( m_B \)-like patterns \( m_B^{(i)} (i = 1, \ldots, 4) \) are discovered;

2. **Match \( m_A, m_B \)-like patterns into rule instances, and search for the matching result that can support the rule most.** A brutal search is not only intractable but also lack of robustness. Instead, we propose a heuristic matching algorithm according to the belief that a rule is preferred when it has (1) many instances and (2) a short max-length time interval. The lines (both dotted and solid) in Figure 3 are all possible instances and the solid ones \( \{e_{1,2}, e_{2,3}\} \) are the best instances, because it has the most number of instances (i.e. 2), and the average length of time intervals is also the smallest.

3. **Score each instance with respect to the rule \( r \), and then integrate to the final score.** In this step, instead of Euclidean distance, we use bit-saved number to evaluate each instance based on MDL principle (Grünwald 2007) illustrated in Figure 4. We also consider the ratio of antecedents being matched, i.e. 2/3. In the example, the two best instances \( \{e_{1,2}, e_{2,3}\} \) are evaluated respectively based on MDL principle. The final score for \( r \) is the sum of \( \{e_{1,2}, e_{2,3}\} \)’s results multiplied by 2/3.

### Step 1. Motif-like Pattern Discovery

MK algorithm (Mueen et al. 2009) returns pairs of subsequences which are very similar with each other as motifs. In this step, given a motif \( m \) (a pair of subsequences) of a time series \( T \), we need to search for all \( m \)-like non-overlapped subsequences in \( T \).

A direct approach based on sliding window method is as follows: (1) search all subsequences of the same length with \( m \) and calculate the distance between them and \( m \); (2) set an appropriate threshold \( \theta' \) to filter the subsequences with distance smaller than \( \theta' \); (3) sort the remaining subsequences by distance and remove the overlap ones with larger distances. After that, the motif-like patterns are chosen as the non-overlapped subsequences with small distance.

The rule threshold \( \theta \) is set as the threshold for selecting antecedent-like subsequences. The complexity of sliding window method is determined by the number of windows and the distance computation procedure. In this step, the complexity is \( O(|m| \cdot |T|) \).

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\(^2\)In real applications, \( \tau \) is a predefined value according to the series, which avoids nonsense rules with infinity max time interval.

\(^3\)In fact, \( \tilde{m}_A \) is observed after \( \tilde{\tau} \) time.

\(^4\)For concise, the footnotes of all Ks are omitted throughout this paper. In fact, Ks can be different.
Solve the following 0-1 integer programming problem:

Given a non-complete weighted graph \( G(M_A \cup M_B, E) \), find the instance set \( S \subseteq E \) subject to (1) \(|S|\) is maximized, and (2) \( W(S) \), the total weight of \( S \), is minimized.

One cannot simply apply algorithms solving assignment problem due to a parallel constraint. Concretely speaking, for any two instances, if the antecedent-like pattern in one instance appears earlier than the other’s, then its corresponding consequent-like pattern must also come earlier than the other’s. In the graph, this constraint requires no crossed edges in \( S \), as is illustrated in Figure 3.

Suppose that the max \(|S|\) is known as \( s \) somehow, we can solve the following 0-1 integer programming problem:

\[
\begin{align*}
\text{minimize} & \quad \sum_i w_{i,j}x_{i,j} \\
\text{subject to} & \quad \sum_i x_{i,j} = s \\
& \quad \sum_i x_{i,j} \leq 1, \sum_j x_{i,j} \leq 1 \\
& \quad x_{i,j} + x_{k,l} \leq 1, \forall i > k \text{ and } l < j \\
& \quad x_{i,j} \in \{0, 1\}
\end{align*}
\]

The optimization variables \( x_{i,j} \)'s are 0, 1 variables, constrained by (1e), each of which represents the selection of corresponding instance \( e_{i,j} \). (1b) restricts that \(|S|\) is maximized as \( s \). (1c) requires that at most one edge can be chosen in the graph \( G \) with respect to the same vertex. (1d) refers to the parallel constraint.

Now consider how to solve for \( s \). It is not the classical problem of maximum unweighted bipartite matching due to the parallel constraint and therefore it cannot be easily solved by max/min flow algorithms. We formulate it as another optimization problem:

\[
\begin{align*}
\text{maximize} & \quad \sum_i x_{i,j} \\
\text{subject to} & \quad \sum_i x_{i,j} \leq 1, \sum_j x_{i,j} \leq 1 \\
& \quad x_{i,j} + x_{k,l} \leq 1, \forall i > k \text{ and } l < j \\
& \quad x_{i,j} \in \{0, 1\}
\end{align*}
\]

The optimization problems are both 0-1 integer programming, which are NP-hard generally. Existing solvers (e.g. Matlab Optimization Toolbox) based on cutting plane method can handle these problems within a tolerable time.

\section*{Step 2. Heuristic Matching}

The patterns found in step 1 can be combined into pairs as instances of a rule. This step aims at finding the best instance set that support the rule, which should satisfy the following conditions: (1) its cardinality is the largest among all possible sets; (2) the average length of interjacent intervals of instances is the smallest. The two condition come from the belief about what a good rule is.

**Modeling.** To formulate the problem concretely, we introduce the following notations and construct a weighted bipartite graph.

\[
\begin{align*}
\tilde{M}_A &= \{\tilde{m}_A^{(1)}, \ldots, \tilde{m}_A^{(p)}\}, \quad \tilde{M}_B &= \{\tilde{m}_B^{(1)}, \ldots, \tilde{m}_B^{(q)}\}, \\
E &= \{e_{i,j} = (\tilde{m}_A^{(i)}, \tilde{m}_B^{(j)}) | 1 \leq i \leq p, 1 \leq j \leq q, 0 < t(\tilde{m}_B^{(j)}) - t(\tilde{m}_A^{(i)}) < \tau\},
\end{align*}
\]

where function \( t(\cdot) \) returns the occurrence time of the pattern. \( E \) is the set of all feasible instances, since the antecedent must appear before the consequent and the interval between them cannot be too large. It imposes a structure on the set \( E \) that given \( \tilde{m}_A^{(i)} \), for \( \forall j \) such that \( \tau > t(\tilde{m}_B^{(j)}) - t(\tilde{m}_A^{(i)}) > 0 \), then \( e_{i,j} \in E \).

Besides, let \( w_{i,j} = t(\tilde{m}_B^{(j)}) - t(\tilde{m}_A^{(i)}) \) measure the length of interjacent interval of the instance \( e_{i,j} \).

\[
\begin{align*}
\tilde{M}_A, \tilde{M}_B, E \text{ make up a weighted bipartite graph } G = (M_A \cup M_B, E). \quad \text{Figure 3 shows the graph } G \text{ of the example in Figure 2.}
\end{align*}
\]

**Optimization.** Using the notation introduced above, we restate the heuristic matching process as: Given a non-complete weighted graph \( G(M_A \cup M_B, E) \), find the instance set \( S \subseteq E \) subject to (1) \(|S|\) is maximized, and (2) \( W(S) \), the total weight of \( S \), is minimized.

One cannot simply apply algorithms solving assignment problem due to a parallel constraint. Concretely speaking, for any two instances, if the antecedent-like pattern in one instance appears earlier than the other’s, then its corresponding consequent-like pattern must also come earlier than the other’s. In the graph, this constraint requires no crossed edges in \( S \), as is illustrated in Figure 3.

Suppose that the max \(|S|\) is known as \( s \) somehow, we can solve the following 0-1 integer programming problem:

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& \quad x_{i,j} + x_{k,l} \leq 1, \forall i > k \text{ and } l < j \\
& \quad x_{i,j} \in \{0, 1\}
\end{align*}
\]

The best subset \( S \) is known as the rule's antecedent \( m_A \) and consequent \( m_B \), respectively.

As is illustrated in Figure 3.\( \tilde{M}_A, \tilde{M}_B, E \) make up a weighted bipartite graph \( G = (M_A \cup M_B, E) \). Figure 3 shows the graph \( G \) of the example in Figure 2.

\section*{Step 3. MDL-based Instance Scoring}

In this step, given the best instance set \( S \), we first evaluate each instance by the similarity between it and the rule \( r \) made up by \( m_A \) and \( m_B \), and then aggregate the results to the score of rule \( r \) for further comparison.

Intuitively, the more similar the shape in the instance is with respect to the rule’s consequent, the better the instance can support the rule. The Euclidean distance is the most widely accepted measure for similarity. However, the length of consequent varies in different rules, where Euclidean metric cannot fairly judge the differences between subsequences of different length.

Inspired by the Minimum Description Length principle that any regularity in a given set of data can be used to compress the data, i.e. to describe it using fewer symbols than needed to describe the data literally (Grünewald 2007), we take the rule’s consequent as the regularity and measure how many bits can be saved by compressing the shape in the instances according to the regularity using Huffman coding. An example is shown in Figure 4.

To use MDL principle, we digitize the series first, and denote \( dl(\cdot) \) as the coding length. The digitization loses little information of the raw data according to Yekta et al. 2015.
The number of bits saved for instance $e$ by encoding $\tilde{m}_B$ with respect to $r$’s consequent $m_B$ is as below:

$$\text{bit\_saved}(e, r) = dl(e, m_B) - dl(e, \tilde{m}_B | r, m_B)$$  

(3)

We further take the ratio of antecedent-like pattern being matched into consideration. Intuitively, when the ratio is too small, indicating the number of matched instances is much less than the times that the antecedent is fired, the rule shouldn’t be considered a good rule. Therefore, the final score for a rule $r$ is:

$$\text{score}(r) = \frac{s}{|M_A|} \left( \sum_{e \in S} \text{bit\_saved}(e, r) - dl(r, m_B) \right)$$  

(4)

**Experiment Evaluation**

We evaluate our method on real open datasets. Top rules discovered by our method and the baseline method from the same training data are compared and analyzed. In addition, we also validate the applicability of our method on multiple series.

**Experiment Setup**

The baseline method (Y15) is the state-of-the-art work by Yekta et al. The experiment is conducted on two open metering datasets. One is Almanac of Minutely Power dataset (AMPds), mainly containing electricity stream data at one minute intervals per meter for 2 years of monitoring in a single house (Makonin et al. 2013; 2016). The other is UK Domestic Appliance-Level Electricity (UK-DALE) dataset, which records both the whole-house mains power demand every six seconds as well as power demand from individual appliances every six seconds from five houses (Kelly and Knottenbelt 2015).

**Settings.** Two groups of experiments are performed to (1) evaluate the prediction performance of our method and (2) validate the applicability of multiple series.

- **On Single Series.** We utilize the aggregate power series of clothes washer and clothes dryer for 1 year from AMPds, which is also used by Y15 as the experiment dataset. The series of first month is used to discover rules, while the rest is used for testing. We select 5 top rules by each method and evaluate the prediction performance on the test data;

- **One Multiple Series.** We attempt to discover rules from the separated power series of total 52 appliances from the house 1 of UK-DALE dataset, such as boiler, laptop, etc. We run our method on each pair of the series to search for valid rules.

**Metric.** To measure the prediction performance of rules, we adopt the same metric $Q$ proposed by Yekta et al. as:

$$Q(r) = \frac{\sum_{i=1}^{N} d(m_B, u_i)}{\sum_{i=1}^{N} d(m_B, v_i)},$$

(5)

where $N$ is the total firing number of the rule $r$ in the test data, and $d(m_B, x)$ is Euclidean distance between the consequent $m_B$ with the the shape beginning by position $x$. $u_i$ and $v_i$ are the $i$-th firing position and the $i$-th randomly chosen position, respectively. The denominator is used to normalize $Q$ to a value between 0 and 1. The final $Q$ is averaged after 1000 measurements.

A smaller $Q$ indicates a better prediction performance. The $Q$ close to 1 suggests the rule $r$ is no better than random guessing and $Q$ close to 0 indicates that the rule $r$ captures the structure in the data and predicts accurately.

**On Single Series**

To compare Y15 with our method, we select top 5 rules discovered by each method from the training data, and then evaluate them on the test data.

**Result.** The top 5 ranked rules’ $Q$s are listed in Table 1. The top rules discovered by our methods are better than those by Y15. Specifically, our method outperforms Y15 by 23.9% on average.

**Comparison.** To demonstrate the reason why the rules discovered by our method outperform those by Y15, we take a close look at the 5-th top rule, $r_Y$ (by Y15) and $r_O$ (by this paper), and scrutinize their prediction results on the test data in Figure 5.

As is shown in Figure 5a and 5b, the 5-th top rules $r_Y$ and $r_O$ are quite different from each other though they are describing the same thing\(^5\). $r_Y$ comes from a splitting point at 10%-th of a 120-long subsequence, whose antecedent is 12

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\(^5\)Actually, they both imply the fact that the clothes dryer is often used after the clothes washer.

|      | 1      | 2      | 3      | 4      | 5      | Mean |
|------|--------|--------|--------|--------|--------|------|
| Y15  | 0.389  | 0.436  | 0.398  | 0.481  | 0.424  | 0.426|
| MBP  | 0.340  | 0.299  | 0.337  | 0.310  | 0.341  | 0.324|

Table 1: The prediction performance $Q$ of top 5 rules on the test data.
in length and consequent is 108 in length, whereas $r_o$ takes two motifs as its antecedent and consequent, whose lengths are 50 and 30 respectively. In contrast, $r_Y$ has more reasonable antecedent and consequent. To illustrate, consider the case in Figure 5c and 5d.

The antecedents of both rules present a good match and trigger both rules around the same position. However, $r_o$ gives an accurate prediction, while $r_Y$ predicts a shape with a clear discrepancy to the real series. The interval before $r_Y$’s consequent is so long that the consequent misses the best matching position. Intuitively, $r_Y$’s consequent can be viewed as $r_o$’s consequent appended by a piece of noise series, which results in the mismatch of the consequent.

The inaccurate prediction has its root in the splitting method of Y15, which inevitably adds some extra noises to the “real” antecedent/consequent, because the splitting method cannot position the boundaries of the antecedent/consequent. Our method, however, directly finds the key part of the series, i.e. motif, as the rule’s antecedent/consequent.

Additionally, any rule discovered by Y15 is a split of a motif. Since the split parts are also frequent patterns in the series, they can be discovered as motifs, i.e. the elements of $M_A$ and $M_B$. Therefore, rules discovered by Y15 can also be found by our method.

**Discussion.** In the electricity datasets, zero series$^6$ is recognized as motif by MK algorithm because it is also a frequent pattern (though meaningless). To avoid such motifs, we sort the discovered motifs by the “roughness” of the shape and choose the top ones. Commonly, the sorting process, mentioned in the line 2 of Algorithm 1, is relevant with the characteristics of the time series.

**On Multiple Series**
We attempt to discover rules in multiple appliance series from the UK-DALE datasets, which include 52 kinds of appliances. The original data is sampled every six seconds, and we resample it per minute for efficiency consideration.

**Result.** A serviceable rule $r_E$ discovered is from the power series of hair dryer and straighteners, the antecedent and consequent of which are shown in Figure 6.

The rule $r_E$ describes the relationship between the usage of hair dryer and straightener. An interesting fact is that the rule’s antecedents and consequents are interchangeable, coinciding the common sense that the two appliances, hair dryer and straightener, are often used at the same time. To illustrate $r_E$ concretely, we list an overview of 4 power series in Figure 7, including hair dryer, straightener, bread-
maker and laptop. The series of hair dryer and straightener are well matched, whereas the rest combinations are ranked relatively lower.

**Conclusion and Discussion**

In this paper, we have introduced a novel rule-based prediction method for real-valued time series from the perspective of motifs. It first leverages motif discovery to segment the time series precisely for seeking recurring patterns as antecedents/consequents, and then investigates the underlying temporal relationships between motifs by combing motifs as rule candidates and ranking them based on the similarities. The motif-based approach possesses a much better prediction performance than the state-of-the-art work. Furthermore, it is applicable for rule discovery from multiple series.

For future work there are a couple issues need to be addressed. In this paper, we experiment our method on the electricity datasets for the convenience of examining and explaining the rule and its performance. However, our method is based on a general assumption and is supposed to be applicable for different sources of time series. In the future, we attempt to collect more real-valued time series with groundtruth and then conduct extensive experiments on them to evaluate our method. Inter alia, it is possible to generalize the rule representation to allow more expressive logical connectives, e.g. $m_1 \cap m_2 \rightarrow m_3$ or $m_1 \cup m_2 \rightarrow m_3$. The interactive occurrence of multiple antecedents may foretell the future; or several antecedents predict a same consequent.

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