Nonlinear Phase-Quantized Constant-Envelope Precoding for Massive MU-MIMO-OFDM

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Abstract—We propose a novel algorithm for phase-quantized constant-envelope precoding in the massive multi-user (MU) multiple-input multiple-output (MIMO) downlink. Specifically, we extend the nonlinear squared-infinity norm Douglas-Rachford splitting (SQUID) precoder to systems that use oversampling digital-to-analog converters (DACs) at the base station (BS) and orthogonal frequency-division multiplexing (OFDM) to communicate over frequency-selective channels. We demonstrate that SQUID is able to generate constant-envelope signals, which enables the use of power-efficient analog radio-frequency circuitry at the BS. By quantizing the phase of the resulting constant-envelope signal, we obtain a finite-cardinality signal that can be synthesized by low-resolution (e.g., 1-bit) DACs. We use error-rate simulations to demonstrate the superiority of SQUID over linear precoders for massive MU-MIMO-OFDM.

I. INTRODUCTION

Massive multi-user (MU) multiple-input multiple-output (MIMO) systems are expected to be a key technology component of fifth-generation (5G) wireless networks, scaling traditional radio frequency (RF) front-end architectures to BSs with hundreds of antenna elements leads to a prohibitive growth in circuit power consumption, system costs, and hardware complexity. Hence, a successful deployment of massive MU-MIMO requires inexpensive, power-efficient, and low-complexity hardware components, which, in turn, will limit the capacity of the system due to signal-quality degradation.

A. Precoding with Low-PAR/Constant-Envelope Waveforms

In the massive MU-MIMO downlink (the BS transmits data to the UEs), precoding must be used to reduce MU interference. Unfortunately, precoding generates time-domain signals with high peak-to-average power ratio (PAR) [3]; this fact is further aggravated in systems that use orthogonal frequency-division multiplexing (OFDM) to facilitate communication over wideband frequency-selective channels [4]. For such high-PAR waveforms, one has to operate the power amplifiers (PAs) in the linear regime to prevent significant signal-quality degradation, which results in high PA power consumption [4].

To mitigate the high-PAR issue, a convex-optimization based PAR-aware massive MU-MIMO-OFDM precoder was proposed in [5]. This precoder achieves significantly lower PAR compared to linear precoders at the cost of a negligible reduction in BER performance. A constant-envelope precoder for the massive MU-MIMO-OFDM case was proposed [6]; its design ensures that the precoded signal has equal amplitude on all antennas. This precoder enables PAs to operate in the nonlinear regime, allowing for highly efficient analog circuitry. Recently, the authors of [7] designed a precoder for the frequency-flat case that outputs a constant-envelope signal with eight phases. This precoder requires the digital-to-analog converters (DACs) at the BS to generate only eight outputs, which enables the use of power-efficient converter architectures, and reduces the interconnect bandwidth between the baseband processing unit to the radio unit.

B. Precoding with 1-Bit DACs

Motivated by potential power savings and reduced interconnect bandwidths, the use of 1-bit DACs in the massive MU-MIMO downlink has recently attracted significant attention. Specifically, linear precoders followed by quantization have been recently proposed for precoding in massive MU-MIMO-OFDM systems that use oversampling DACs [8], [9]. These methods achieve low bit-error rates (BERs) and high sum-rate throughputs over frequency-selective channels with OFDM, despite the adverse impact of the 1-bit DACs. In contrast, nonlinear precoders (the precoder depends on the instantaneous realizations of the information symbols) are known to significantly outperform linear-quantized precoders [10]–[14], but have been proposed exclusively for frequency-flat channels and single-carrier transmission.

C. Contributions

We propose a nonlinear phase-quantized constant-envelope precoder for a massive MU-MIMO-OFDM downlink operating over frequency-selective channels. The proposed precoder, which builds upon the squared-infinity-norm Douglas-Rachford splitting (SQUID) algorithm put forward in [12], Sec. IV-B, can be used to precode the transmitted signal for systems that use 1-bit DACs at the BS. In contrast to previous works [10]–[14] that propose nonlinear precoders for the case of Nyquist-
rate sampling 1-bit DACs and for the frequency-flat case, we consider nonlinear precoding for oversampling DACs with OFDM. We analyze the computational complexity of our algorithm and demonstrate the efficacy of the proposed precoder via numerical simulations.

D. Notation

Lowercase and uppercase boldface letters to denote vectors and matrices, respectively. The $M \times N$ all-zeros matrix and the $M \times M$ identity matrix are denoted by $0_{M \times N}$ and $I_M$, respectively. The unitary $N \times N$ discrete Fourier transform (DFT) matrix is denoted by $F_N$. The $l_\infty$-norm of $a = [a_1, \ldots, a_M]^T$ is $\|a\|_\infty = \max_{\ell=1,\ldots,M}|a_\ell|$; the $l_\infty$-norm is $\|a\|_\infty = \max \{ |\Re\{a\}|_\infty, |\Im\{a\}|_\infty \}$. We use $\|a\|_2$ and $\|A\|_F$ to denote the $l_2$-norm of vector $a$ and the Frobenius norm of matrix $A$, respectively. If $A$ is an $M \times N$ matrix, then vec$(A)$ is a $MN$-dimensional vector obtained by column-wise vectorization of $A$. The phase of $a \in \mathbb{C}$ is denoted by arg$(a)$; the sign of $r \in \mathbb{R}$ is denoted by $\text{sgn}(r) \in \{-1, +1\}$. The floor function $[r]$ produces the largest integer less than or equal to $r$. The complex-valued circularly symmetric Gaussian distribution with covariance matrix $K \in \mathbb{C}^{M \times M}$ is denoted by $CN(0_{M \times 1}, K)$. The expected value of $A$ is $\mathbb{E}[A]$.

II. SYSTEM MODEL

We consider the single-cell massive MU-MIMO-OFDM downlink system illustrated in Fig. 1. The system operates over a wideband channel where OFDM is used to deal with the frequency-selectiveness of the channel. Let $B$ denote the number of BS antennas and $U$ the number of single-antenna UEs. At the BS, the frequency-domain information symbols are mapped to the antenna array by a precoder. At each BS antenna, the precoded signal is mapped to time domain through an inverse discrete Fourier transform (IDFT) before being passed to a pair of finite-resolution DACs, which generate the in-phase and quadrature components of the transmitted time-domain signal. We assume no other RF impairments and perfect synchronization between the BS and the UEs.

A. Channel Input-Output Relation

Under these assumptions, the received signal $y_n \in \mathbb{C}^U$ for the $U$ UEs can be written as

$$y_n = \sum_{\ell=0}^{L-1} H_{\ell} x_{n-\ell} + w_n$$

(1)

at discrete time instants $n = 0, \ldots, N-1$. Here, $x_n$ is the $B$-dimensional transmit signal at discrete time $n$ and $N$ is the size of the IDFT. The vector $w_n \sim CN(0_{U \times 1}, N_0 I_U)$ denotes the i.i.d. additive white Gaussian noise (AWGN) at the UEs at discrete time $n$. Here, $N_0$ is the noise power and we define the signal-to-noise ratio (SNR) as $SNR = 1/N_0$. The matrix $H_{\ell} \in \mathbb{C}^{U \times B}$ is the $\ell$th tap of the frequency-selective channel ($\ell = 0, \ldots, L-1$). We assume that the realizations of $\{H_{\ell}\}_{\ell=0}^{L-1}$ remain constant for the duration of the OFDM symbol and that they are perfectly known to the BS. Let $X = [x_0, \ldots, x_{N-1}]^T$, $Y = [y_0, \ldots, y_{N-1}]^T$, and $W = [w_0, \ldots, w_{N-1}]^T$. Furthermore, let $\hat{X} = X F_N^H$, $\hat{Y} = Y F_N^H$, and $\hat{W} = W F_N^H$. Finally, let $\hat{H}_{\ell} = \sum_{\ell=0}^{L-1} H_{\ell} e^{-j \pi \ell N / L}$. A cyclic prefix of length $L-1$ is prepended to the transmit signal at the BS. After removing the cyclic prefix and the after a DFT at the UEs, the received signal for the $U$ UEs on the $k$th subcarrier can be written as

$$\hat{y}_k = \hat{H}_k \hat{x}_k + \hat{w}_k$$

(2)

for $k = 0, \ldots, N-1$. Here, $\hat{x}_k$, $\hat{y}_k$, and $\hat{w}_k$ correspond to the $k$th column of $\hat{X}$, $\hat{Y}$, and $\hat{W}$, respectively.

B. Precoding, Quantization, and OFDM Parameters

We use the disjoint sets $\mathcal{I}$ and $\mathcal{G}$, where $|\mathcal{I}| + |\mathcal{G}| = N$, to denote the set of subcarriers associated with information symbols (occupied subcarriers) and zeros (guard subcarriers), respectively. We define the number of occupied subcarriers as $S = |\mathcal{I}|$, and the oversampling ratio as $N/S$. Let $s_k = [s_{1,k}, \ldots, s_{U,k}]^T$ denote the symbol vector associated with the $k$th subcarrier ($k = 0, \ldots, N-1$). It holds that $s_{u,k} \in \mathcal{O}$ for all $k \in \mathcal{I}$, where $\mathcal{O}$ is the set of constellation points, and that $s_{u,k} = 0$ for all $k \in \mathcal{G}$. In this paper, we
assume that $O$ represents a quadrature amplitude modulation (QAM) constellation (e.g., 16-QAM).

The precoder uses the available transmit-side channel-state information to map the symbols $S = \{s_0, \ldots, s_{N-1}\} \in \mathbb{C}^{U \times N}$ to the transmitted signal $X$, such that it satisfies the average power constraint $E_p[\|X\|_2^2] = S$. Furthermore, due to the finite resolution of real-world DACs we require that $X \in \mathbb{C}^{2N \times 2N}$, where $X_p$ is the set of values that are supported by the DACs’ transcoder. Here, $p > 0$ is the number of phase bits and $2^p$ is the number of allowed phases per antenna. We restrict the transmitted signal to have constant envelope, irrespectively of the value of $p$, i.e., $|x|^2 = P_{\text{ant}}$ for $x \in X_p$, where $P_{\text{ant}} = S/(BN)$ is the per-antenna transmit power, which ensures that the average power constraint is satisfied. Specifically, for $p < \infty$, the $n$th element $(m = 0, \ldots, 2^p - 1)$ of $X_p$ is given by $\frac{\sqrt{P_{\text{ant}}}}{\sqrt{2^p}} \beta \sin((\pi + 2\pi m)/2^p)$. We define $X_{\infty} = \{x \in \mathbb{C} : |x|^2 = P_{\text{ant}}\}$.

Linear precoding is an attractive solution for massive MU-MIMO-OFDM as it offers low complexity and competitive performance for large antenna arrays [13]. In this paper, we shall benchmark the performance of our precoding algorithm against the Wiener-filter (WF) precoder given by

$$X_{\text{WF}} = P_p(\tilde{Z}_{\text{WF}}F_{\text{NN}}^H),$$

where the $k$th column of $\tilde{Z}_{\text{WF}}$ is given by

$$\tilde{z}_{\text{WF}}^k = \beta_{\text{WF}} \hat{H}_k^H \left( \hat{H}_k \hat{H}_k^H + U N_k I_k \right)^{-1} \beta_{\text{WF}}$$

for $k \in I$, and by $\tilde{z}_{\text{WF}}^k = 0_{B \times 1}$ for $k \in G$. Here, the constant $\beta_{\text{WF}} \in \mathbb{R}^+$ ensures that $E_p[\|X_{\text{WF}}F_{\text{NN}}^H\|_F^2] = S$. The time-domain precoded signal in (3) is, prior to transmission, quantized by the function $\mathcal{P}_p(\cdot) : \mathbb{C}^{B \times N} \rightarrow \mathbb{C}^{B \times N}$, which is applied entry-wise to the matrix $X_{\text{WF}}$, so that the transmitted signal matches the transcoder in the DACs. Specifically,

$$\mathcal{P}_p(z) = \begin{cases} \sqrt{P_{\text{ant}}} \beta_{\text{WF}} \left( \frac{\beta_{\text{WF}} |z|}{\sqrt{P_{\text{ant}}}} \right)^{\frac{1}{4}}, & p < \infty \\ \frac{P_{\text{ant}}}{2} \left( \text{sgn}(\Re(z)) + j \text{sgn}(\Im(z)) \right), & p \rightarrow \infty. \end{cases}$$

Note that for the 2-phase-bit case, we retrieve the 1-bit-DAC setup studied in [3, 9] for which the in-phase and quadrature components of the per-antenna transmitted signal are generated independently by a pair of 1-bit DACs and it holds that

$$\mathcal{P}_2(z) = \sqrt{\frac{P_{\text{ant}}}{2} \left( \text{sgn}(\Re(z)) + j \text{sgn}(\Im(z)) \right) \cdot \max(|\Re(z)|, |\Im(z)|)}.$$ The 1-phase-bit case, on the other hand, corresponds to the case when there is only a single 1-bit DAC per antenna, i.e., the transmitted signal has no in-phase component.

### III. NONLINEAR CONSTANT-ENVELOPE PRECODING

As in [12–14], we focus on nonlinear precoding techniques that minimize the mean square error (MSE) at the UEs given a phase-quantized constant-envelope constraint. Let $MSE_{u,k} = E_{w_{u,k}}[|s_{u,k} - \beta^u_{\text{WF}}|]$, denote the MSE between the transmitted and received symbols for the $u$th UE and on the $k$th subcarrier. Here, $\beta_{\text{WF}}$ is the $u$th element of the vector $\beta_{\text{WF}}$ and $\beta \in \mathbb{R}^+$ is a constant that takes into account the channel gain. With these definitions, we write the sum-MSE over the $U$ UEs and over the $S$ occupied subcarriers as follows:

$$\sum_{u=1}^U \sum_{k \in I} MSE_{u,k} = \sum_{k \in I} E_{w_{\text{WF}}}[|s_k - \beta_k|^2] = \sum_{k \in I} |s_k - \beta_k|^2 + \beta^2 \sum_{k \in I} MSE_{u,k}.$$

Recall that $\beta_k$ is the $k$th column of $\hat{X}$. We can now define the sum-MSE-optimal precoding problem (PP) as follows:

$$\begin{array}{rl} \text{minimize} & \sum_{k \in I} |s_k - \beta_k|^2 + \beta^2 \sum_{k \in I} MSE_{u,k} \\ \text{subject to} & X = \hat{X} F_{\text{NN}}^H, \\ \end{array}$$

where $\gamma = BUN N_0$. Similarly to [12], by setting $\hat{B} = \beta \hat{X}$ and by dropping the nonconvex constraint $X \in X_p$, we obtain the following convex relaxation of (10):

$$\begin{array}{rl} \text{minimize} & \sum_{k \in I} |s_k - \beta_k|^2 + \beta^2 \gamma \|\text{vec}(X)\|_\infty^2 \\ \text{subject to} & X = \hat{X} F_{\text{NN}}^H, \\ \end{array}$$

In Section III-A, we shall show that the problem (P$^\infty_2$) can be solved efficiently. Note that for the 2-phase-bit case, it holds that $\|\text{vec}(X)\|_\infty = 2\|\text{vec}(X)\|_\infty$. In this case, it turns out that one achieves better performance by solving instead the following optimization problem:

$$\begin{array}{rl} \text{minimize} & \sum_{k \in I} |s_k - \beta_k|^2 + 2\gamma \|\text{vec}(X)\|_\infty^2 \\ \text{subject to} & X = \hat{X} F_{\text{NN}}^H, \\ \end{array}$$

In Section III-A we shall discuss the implications of this slight modification of the precoding problem.

#### A. SQUID: Squared-Infinity Norm Douglas-Rachford Splitting

Douglas-Rachford splitting [16] is an efficient iterative scheme to solve convex optimization problems of the form

$$\begin{array}{rl} \text{minimize} & f(\hat{B}) + g(\hat{B}) \\ \text{subject to} & \hat{B} \in \mathbb{C}^{B \times N}, \end{array}$$

where $f(\cdot)$ and $g(\cdot)$ are closed convex functions, which have proximal operators [17] defined as follows:

$$\text{prox}_\lambda f(V) = \arg \min_{\hat{B} \in \mathbb{C}^{B \times N}} \|\hat{B} - V\|_2^2,$$

$$\text{prox}_\lambda g(V) = \arg \min_{\hat{B} \in \mathbb{C}^{B \times N}} \|\hat{B} - V\|_2^2.$$
By starting at any $\hat{A}^{(0)}$, $\hat{B}^{(0)}$, and $\hat{C}^{(0)}$, Douglas-Rachford splitting solves problems of the form \((14)\) by repeating for \(t = 1, \ldots, T\), where \(T\) is the maximum number of iterations, the following iterative procedure:

\[
\hat{A}^{(t)} = \text{prox}_f \left( 2\hat{B}^{(t-1)} - \hat{C}^{(t-1)} \right) \tag{17}
\]

\[
\hat{B}^{(t)} = \text{prox}_g \left( \hat{C}^{(t-1)} + \hat{A}^{(t)} - \hat{B}^{(t-1)} \right) \tag{18}
\]

\[
\hat{C}^{(t)} = \hat{C}^{(t-1)} + \hat{A}^{(t)} - \hat{B}^{(t)} \tag{19}
\]

We now outline the SQUID precoder, which builds on the algorithm proposed in [12] Sec. IV-B and performs Douglas-Rachford splitting to solve the problems \((P_{\infty}^\ell)\) and \((P_{\infty}^2)\). Let \(f(B) = \sum_{k \in \mathcal{I}} \| s_k - \hat{H}_k b_k \|_2^2 \). For the problem \((P_{\infty}^2)\) we use that \(g(B) = \gamma \| \text{vec}(B F_H^H) \|_2^2 \), and for the problem \((P_{\infty}^\ell)\) we use that \(g(B) = 2\gamma \| \text{vec}(B F_H^H) \|_2^2 \). For both cases, the proximal operator for \(g(\cdot)\) in \((16)\) can be computed using [12] Alg. 1. For the proximal operator of \(f(\cdot)\), we note that the objective function in \((15)\) is separable in the columns of \(\hat{B}\) and that the \(k\)th column of \(\hat{A}^{(t)}\) in \((17)\) can be computed as

\[
d_k^{(t)} = \left( \hat{H}_k^H \hat{H}_k + \frac{1}{8} I_N \right)^{-1} \left( \hat{H}_k^H s_k + \hat{b}_k^{(t-1)} - \frac{1}{2} \hat{c}_k^{(t-1)} \right) \tag{20}
\]

\[
d_k = \left( \hat{H}_k^H s_k - Q_k \hat{H}_k \hat{H}_k^H s_k \right). \tag{23}
\]

We can now solve the problems \((P_{\infty}^\ell)\) and \((P_{\infty}^2)\) by using the iterative procedure outlined in Algorithm 1.

**Algorithm 1 (SQUID):** Compute \(Q_k\) and \(d_k\) for \(k \in \mathcal{I}\) using \((22)\) and \((23)\). Initialize \(\hat{A}^{(0)} = 0_{B \times N}\), \(\hat{B}^{(0)} = 0_{B \times N}\), and \(\hat{C}^{(0)} = 0_{B \times N}\). Then, for every iteration \(t = 1, 2, \ldots, T\) compute the following three quantities:

\[
d_k^{(t)} = \left\{ \begin{array}{ll} R_k \left( 2B_k^{(t-1)} - c_k^{(t-1)} \right) + d_k, & k \in \mathcal{I} \\ 2B_k^{(t-1)} - c_k^{(t-1)}, & k \in \mathcal{G} \end{array} \right. \tag{24}
\]

\[
\hat{B}^{(t)} = \text{prox}_g \left( \hat{C}^{(t-1)} + \hat{A}^{(t)} - \hat{B}^{(t-1)} \right) F_N \tag{25}
\]

\[
\hat{C}^{(t)} = \hat{C}^{(t-1)} + \hat{A}^{(t)} - \hat{B}^{(t)} \tag{26}
\]

Here, \(R_k = (I_B - Q_k \hat{H}_k)\). In \((25)\), prox\(_g(\cdot)\) is computed using [12] Alg. 1. After the last iteration, we find the transmitted signal \(X^{(T)}\) by quantizing \(\hat{B}^{(T)} F_N^H\) to the constant-envelope alphabet \(A_p^{B \times N}\) using \((5)\).

Fig. 2 shows the time-domain SQUID output \(\hat{B}^{(20)} F_N^H\) after \(T = 20\) iterations before and after quantization. Fig. 2b shows the SQUID output for the problem \((P_{\infty}^\ell)\) before and after 2-phase-bit quantization. We see that the \(\ell_\infty\)-norm constrains the SQUID output to a box in the complex plane, which limits the error caused by the quantizer. Fig. 2c shows the output for the problem \((P_{\infty}^2)\) before and after 3-phase-bit quantization. Here, we slightly modified the problem to force the real part of the output to the proximal operator \(\text{prox}_g(\cdot)\) to zero. This constrains the SQUID output to a line in the complex plane, which is suitable for quantization using only one phase-bit.

### B. Computational Complexity

Table I shows the required number of complex-valued multiplications for SQUID and WF precoding, which are found by counting the number of operations as is detailed below.

1) **WF Precoding:** Computing \(H\) for all \(k \in \mathcal{I}\) requires \(S(B(U^2 + U) + U^3)\) complex-valued multiplications. Here, we have assumed that computing the regularized pseudo-inverse in \((4)\) requires \(BU^2 + U^3\) complex multiplications. At each antenna element, the frequency-domain precoded vector is converted to time-domain via an IDFT. This operation requires \(BN \log_2 N\) complex-valued multiplications. By adding these two numbers, we get the total complexity for WF precoding as in Table I.

2) **SQUID:** Computing the matrices \(Q_k\) (another pseudo-inverse) in \((22)\) for \(k \in \mathcal{I}\) requires \(S(B(U^2 + U) + U^3)\) complex-valued multiplications and computing the vectors \(v_k\) for \(k \in \mathcal{I}\) in \((23)\) requires an additional \(3SU\) complex-valued multiplications. Hence, the preprocessing complexity of SQUID is \(S(B(U^2 + 3U) + U^3)\). Computing the vectors \(a_k^{(t)}\) for \(k \in \mathcal{I}\) in \((24)\) requires \(2SU\) complex-valued multiplications per occupied subcarrier and per iteration. Furthermore, computing the proximal operator in \((25)\) requires an additional \(B(1 + N \log_2 N)\) complex-valued multiplications (if the FFT algorithm is used to compute the IDFT and DFT). Hence, the per-iteration complexity of SQUID is \(B(2SU + 1 + N \log_2 N)\). By adding the preprocessing and per-iteration complexity, we get the total complexity for SQUID as in Table I.

| Precoder | Number of complex-valued multiplications |
|----------|------------------------------------------|
| WF       | \(S(B(U^2 + U) + U^3) + \frac{1}{2} BN \log_2 N\) |
| SQUID    | \(S(B(U^2 + 3U) + U^3) + TB(2SU + 1 + N \log_2 N)\) |

1The simulation parameters are discussed in Section IV-A.
2By setting \(S = 1\) and \(N = 1\) for the problem \((P_{\infty}^\ell)\), Algorithm 1 reduces to the single-carrier SQUID from [12] Sec. IV-B.
3For the single-carrier case (i.e., \(N = 1\) and \(S = 1\)), the per-iteration complexity of SQUID reduces to \(2BU + B\).
IV. NUMERICAL RESULTS

We will now present numerical simulation results for the SQUID precoder in the massive MU-MIMO-OFDM downlink.

A. Simulation Parameters

Due to space constraints, we focus on a selected set of system parameters.

Specifically, we consider the case in which the BS is equipped with \( B = 128 \) antennas and serve \( U = 16 \) simultaneous UEs. We consider long term evolution (LTE)-inspired OFDM parameters \(^{[18]}\) with \( S = 1200 \) occupied subcarriers and where \( N = 4096 \) (the oversampling ratio is \( N/S = 4096/1200 \approx 3.41 \)). The subcarrier spacing is \( \Delta f = 15 \text{ kHz} \) and the sampling rate is \( f_s = N\Delta f = 61.44 \text{ MHz} \). The set of occupied subcarriers is \( \mathcal{I} = \{1, 2, \ldots, 600, 3497, 3498, \ldots, 4096\} \) and the set of guard subcarriers is \( \mathcal{G} = \{0, 1, \ldots, 4096\} \setminus \mathcal{I} \). We assume Rayleigh fading, for which the entries of \( \{H_{u, k}\}_{u,k=0}^{4096} \) are i.i.d. \( \mathcal{CN}(0, 1/L) \). The number of taps is \( L = 4 \). We furthermore assume that the \( u \)th UE scales the received signal for each OFDM symbol as in \(^{[13]}\)

\[
\beta_u = \frac{1}{\sqrt{\frac{1}{\pi} \sum_{k \in \mathcal{I}} |y_k|^2 - N_0}} \approx \beta \tag{27}
\]

to obtain an estimate \( \hat{s}_{u,k} = \beta_u \hat{y}_{u,k} \) of \( s_{u,k} \) for \( k \in \mathcal{I} \).

B. Convergence and Complexity of SQUID

We start by investigating the convergence of SQUID for the 2-phase-bit case. Fig. 3 shows the complementary cumulative distribution function (CCDF) of the error-vector magnitude (EVM), with 16-QAM signaling for \( SNR = 10 \text{ dB} \). Here, the EVM for the \( u \)th UE \( (u = 1, \ldots, U) \) is defined as

\[
EVM_u = \sqrt{\frac{\sum_{k \in \mathcal{I}} |s_{u,k} - \beta_u \hat{h}_{u,k}^T \hat{x}_k|^2}{\sum_{k \in \mathcal{I}} |s_{u,k}|^2}},
\]

where \( \hat{h}_{u,k}^T \) is the \( u \)th row of \( \hat{H} \) and where \( \beta_u \) is given by \(^{[27]}\). For reference, we also show the CCDF of the EVM with WF precoding for the 2-phase-bit case and for the infinite-resolution case (i.e., when \( X_{WF} = Z_{WF} F_N^H \in \mathbb{C}^{B \times N} \), respectively).\(^{[5]}\)

Interestingly, we see that SQUID with only one iteration significantly outperforms WF precoding in terms of EVM. Furthermore, we see from Table II that for \( T = 1 \) the complexity of SQUID is only slightly (about 30%) higher than that of WF precoding. We also see from Fig. 3 that by increasing the number of SQUID iterations from 20 to 100, we attain only marginal EVM gains. In what follows, we set the number of iterations to \( T = 20 \). In this case, SQUID requires less than six times more complex-valued multiplications compared to the WF precoder (assuming the parameters in Section IV-A).

C. Error-Rate Performance

1) Uncoded BER: Fig. 5a shows the uncoded BER with 4-QAM for \( p \)-phase-bit \((p \in \{1, 2, 3\})\) SQUID/WF precoding as a function of the SNR. We also show the uncoded BER with infinite-resolution WF precoding. We assume that the UEs perform symbol-wise nearest-neighbor decoding (i.e., each UE maps the received signal to the nearest constellation point in \( C \)). We note that SQUID outperforms WF precoding for all

\(^{[4]}\)Our simulation framework will be made available for download from GitHub upon (possible) acceptance of the paper. An implementation of the 1-bit SQUID precoder for the frequency-flat single-carrier case is available on GitHub: https://github.com/quantizedmassimimo/1bit_precoding

\(^{[5]}\)In the infinite-resolution case, the WF precoder \( X_{WF} = Z_{WF} F_N^H \in \mathbb{C}^{B \times N} \) is the solution to the problem \((PP) \) \(^{[19]}\).
At the BS, the information bits are encoded using a rate-5/6 convolutional code. Each codeword is randomly interleaved over 4800 occupied subcarriers and is then modulated and transmitted over an OFDM symbol. To detect the information bits, each UE performs soft-input max-log BCJR decoding. We note that low values of SNR and for any number of phase bits. Interestingly, low uncoded BERs are supported with 1-phase-bit SQUID.

2) **Coded BER**: Fig. 4B shows the coded BER with 16-QAM for p-phase-bit (p ∈ {1, 2, 3, ∞}) SQUID as a function of the SNR. The gap is approximately 3 dB with 2-phase-bit SQUID for a BER of 10^-9.

3-phase-bit WF
3-phase-bit SQUID
∞-phase-bit SQUID
Inf.-res. WF
2-phase-bit SQUID
2-phase-bit WF
1-phase-bit SQUID
1-phase-bit WF

SNR [dB] coded BER

(a) Uncoded BER with 4-QAM. SQUID outperforms WF precoding irrespectively of the SNR value and for any number of phase bits.

Fig. 4. Uncoded/coded BER as a function of SNR and the number of phase bits; B = 128, U = 16, S = 1200, and N = 4096. SQUID significantly outperforms linear precoders and approaches infinite resolution performance: the gap is approximately 3 dB with 2-phase-bit SQUID for a BER of 10^-9.

(b) Coded BER with 16-QAM. Low coded BERs are supported with SQUID and 3-phase-bits approach the performance of ∞-phase-bits.

![Coded BER with 16-QAM](image)

V. CONCLUSIONS

We have proposed a nonlinear phase-quantized precoder for the massive MU-MIMO-OFDM downlink. The precoder builds upon the SQUID algorithm in [13] and supports OFDM and oversampling DACs. SQUID is shown to offer superior error-rate performance to linear precoders such as WF precoding at an increased computational complexity. The constant-envelope transmit signals generated by SQUID enable energy-efficient PAs, which is in stark contrast to the infinite-precision WF precoder whose PAR typically ranges between 11 dB and 13 dB. Furthermore, for 2-phase-bit SQUID the amount of raw data that has to be processed is 15.7 Gbit/s. In contrast, a traditional system that uses high-resolution DACs, e.g., 12-bit, must sustain raw baseband data rates that exceed 188 Gbit/s for the parameters considered in this paper.

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