Best-fit parameters of MDM model from Abell-ACO power spectra and mass function

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ABSTRACT

The possibility of determining MDM model parameters on the basis of observable data on the Abell-ACO power spectrum and mass function is analysed. It is shown that spectrum area corresponding to these data is sensitive enough to such MDM model parameters as neutrino mass $m_{\nu}$, number species of massive neutrino $N_{\nu}$, baryon content $\Omega_{b}$ and Hubble constant $h \equiv H_0/100$ km/s/Mpc. The $\chi^2$ minimization method was used for their determination. If all these parameters are under searching then observable data on the Abell-ACO power spectrum and mass function prefer models which have parameters in the range $\Omega_{\nu} \sim (0.4 - 0.5)$, low $\Omega_{b} \leq 0.01$ and $h \sim (0.4 - 0.6)$. The best-fit parameters are as follows: $N_{\nu} = 3$, $m_{\nu} = 4.4eV$, $h = 0.56$, $\Omega_{b} \leq 0.01$. The high-$\Omega_{b} \sim 0.4 - 0.5$ solutions are obtained when mass of neutrino is fixed and $\leq 3eV$.

To explain the observable excessive power at $k \approx 0.05h/Mpc$ the peak of Gaussian form was introduced in primordial power spectrum. Its parameters (amplitude, position and width) were determined along with the MDM model parameters. It decreases $\chi^2$, increases the bulk motions, but does not change essentially the best-fit MDM parameters.

It is shown also that models with the median $\Omega_{\nu} \sim 0.2 - 0.3$ ($m_{\nu} \sim 2.5$, $N_{\nu} \sim 2 - 3$) and $\Omega_{b} = 0.024/h^2$, which match constraints arising from cosmological nucleosynthesis and high redshift objects, are not ruled out by these data ($\Delta\chi^2 < 1$).

Subject headings: Large Scale Structure: Abell-ACO power spectrum, mass function, Mixed Dark Matter models, initial power spectra, best-fit cosmological parameters
1. Introduction

The observable data on large scale structure of the Universe obtained during last years and coming from current experiments and observational program give a possibility to determine more exactly the parameters of cosmological models and the nature of the dark matter. Up till now the most certain data are about the largest scale inhomogeneities of the current particle horizon of the order of $\sim 7000 h^{-1}$ Mpc ($h \equiv H_0/100 \, km/s/Mpc$, $H_0$ is today Hubble constant) which are obtained from the study of all-sky temperature fluctuations of cosmic microwave background (CMB) with $\sim 10^\circ$ angular resolution by the space experiment COBE (Smoot et al. 1992, Bennett et al. 1994, Bennett et al. 1996). According to them the primordial power spectrum of density fluctuations is approximately scale invariant $P_{\nu}(k) = A k^n$ with $n = 1.1 \pm 0.2$ that well agrees with the predictions of standard inflation model of the Early Universe ($n = 1, \Omega_0 = 1$). Besides, they most certainly determine the amplitude of a linear power spectrum (or normalization constant $A$) which does not depend on any transition processes, nonlinearity effects and other phenomena connected with the last stages of large scale structure formation. On the contrary, the CMB temperature fluctuations at degree and sub-degree scales as well as the space distributions of the cluster of galaxies, galaxies, quasars, Lyman-α clouds, etc. are defined by those processes and also depend essentially on the nature of the dark matter. Theoretically it is taken into account by introducing the transfer function $T(k)$ which transforms the primordial (post-inflation spectrum) into the postrecombination (initial) one - $P(k) = P_{\nu}(k)T^2(k)$, which defines all characteristics of the large scale structure of the Universe. The transfer function depends also on the curvature of the Universe or the present energy density in units of critical density, $\Omega_0$, vacuum energy density or cosmological constant $\Omega_\Lambda$, content of baryons $\Omega_b$, and values of the Hubble constant.

The theory of a large scale structure formation is so far advanced today that all these dependencies can be accurately calculated for the fixed model by public available codes (e.g. CMBfast one by Seljak & Zaldarriaga 1996). The actual problem now is the determination of the nature of the dark matter and the rest of the above mentioned parameters by means of comparison of theoretically predicted and observable characteristics of the large scale structure of the Universe.

As most advanced candidates for the dark matter are cold dark matter (CDM), particles like axions, hot dark matter (HDM), particles like massive neutrinos with $m_\nu \sim 1-20 eV$ and baryon low luminosity compact objects. The last ones can not dominate as it results from the cosmological nucleosynthesis constraints ($\Omega_b h^2 \leq 0.024$, Tytler et al. 1996, Songalia et al. 1997, Schramm and Turner 1997) and observation of microlensing events in the experiments like MA-CHO, DUO, etc. The pure HDM model conflicts with the existence of high redshift objects, the pure CDM one, on the contrary, overpredicts them. Therefore mixed dark matter model (CDM+HDM+baryons) with $\Omega_{HDM} \equiv \Omega_\nu \leq 0.3$ looks more viable. The advantage of these models is a small number of free parameters. But today it is understood already that models with the minimal number of free parameters, such as a standard cold dark matter (sCDM, one parameter) or a standard cold plus hot mixed dark matter (sMDM, two parameters) only marginally match the observable data. A better agreement between theoretical predictions and observable data is achieved in the models with a larger number of free parameters (tilted CDM, open CDM, CDM or MDM with the cosmological term, see review in Valdarnini et al. 1998 and references therein).

The oscillations of solar and atmospheric neutrinos registered by SuperKamiokande experiment show that the difference of rest masses between $\tau$- and $\mu$-neutrinos is $0.02 < \Delta m_{\tau \mu} < 0.08 eV$ (Fukuda et al. 1998, Primack & Gross 1998). It also gives a lower limit for the mass of neutrino $m_\nu \geq 0.02 eV$ and does not exclude models with cosmologically significant values $\sim 1-20 eV$. Therefore, at least two species of neutrinos can have approximately equal masses in this range. Some versions of elementary particle theories predict $m_{\nu_s} \approx m_{\nu_e} \approx 2.5 eV$ and $m_{\nu_s} \approx m_{\nu_\tau} \sim 10^{-5} eV$, where $\nu_e$, $\nu_\tau$, $\nu_\mu$ and $\nu_s$ denote the electron, $\tau$-, $\mu$- and sterile neutrinos accordingly (e.g. Berezhiani et al. 1995). The strongest upper limit for the neutrino mass comes from the data on a large scale structure of our Universe: $m_\nu / 93 h^2 \leq 0.3$ (Holtzman 1989, Davis Summers & Schlegel 1992, Schaefer & Shafi 1992, Van Dalen & Schaefer 1992, Novosyadlyj 1994, Pogosyan & Starobinsky 1993, Ma 1996, Valdarnini et al. 1998), that for $h = 0.8$ (the upper observable limit for $h$) gives $\sum m_{\nu_i} \leq 18 eV$. It is interesting that the upper limit for the mass of electron neutrino obtained from supernova star burst SN1987A neutrino signal
is approximately the same $m_\nu \leq 20 eV$.

Is it possible to find the best fit neutrino mass from experimental data on a large scale structure of the Universe? The problem is that it must be determined together with other large number uncertainties parameters such as $h, \Omega_m, \Omega_b$, etc. Here we study the possibility of finding them by $\chi^2$ minimization method. Realization of such a task became possible in principle after the appearance in literature of accurate analytical approximations of transfer function for mixed dark matter model in at least 4-dimension space of the above mentioned cosmological parameters $T(k; \Omega_b, m_\nu, N_\nu, h)$ (Eisenstein & Hu 1997, Novosyadlyj et al. 1998). That is why that even CMBfast codes are too bulky and slow yet for searching the cosmological parameters by the method of minimization of $\chi^2$, like Levenberg-Marquardt one (see Press et al. 1992).

The next problem is a choice of the observable data suitable for the solution of this task. They must be enough accurate, sensitive to those parameters and not too dependent on the model assumptions about the formation and nature of objects. The most sensitive to the presence of neutrino component are scales of order and smaller of its free-streaming (or Jeans) scale $k \geq k_f(z) = 8 (10^{15} m_\nu) / \sqrt{1+z} h^{-1}$ Mpc because perturbations at these scales are suppressed and it is imprinted in the transfer function of the HDM component. At $z \sim 0$ for cosmologically significant neutrino masses it is approximately galaxy clusters scale. The power spectrum reconstructed from space distributions of galaxies is distorted significantly by nonlinearity effects the accounting of which is model dependent (Peacock & Dodds 1994). The models of the formation of smaller scale structures or high redshift objects (e.g. Lyman-$\alpha$ clouds, quasars etc.) contain the additional assumptions and parameters which makes their using rather problematic in such an approach. The CMB temperature anisotropy at subdegree angular scales (first and second acoustic peaks) has minimal additional assumptions (e.g. secondary ionization) but its sensibility to the presence of neutrino component is low $(\leq 10\%)$. Dodelson et al. 1995). These data are sensitive and suitable for determination by $\chi^2$ minimization methods other set of parameters such as tilt of primordial spectrum $n$, $\Omega_b$, $h$, $\Omega_m$, $\Omega_L$ or/and parameters of scaling seed models of structure formation (see Lineweaver & Barbosa 1997, Durrer et al. 1997).

The data on Abell-ACO power spectrum and function mass of rich clusters seem to be suitable for determining the best fit values of $m_\nu$ and $N_\nu$ because they do not depend on above mentioned additional assumptions.

The data on rich clusters power spectrum (Einasto et al. 1997) were used by Eisenstein et al. 1997 and Atro-Barandela et al. 1997 for analyzing $\sim 100 h^{-1}$ Mpc clustering. The first collaboration group tried to explain the narrow peak in the power spectrum at $\sim 100 h^{-1}$ Mpc scale by baryonic acoustic oscillations in low- and high-$\Omega_0$ models $(\Omega_0 = \Omega_{CDM} + \Omega_b)$. In both cases such an approach needs very high content of baryons $\Omega_b$ $(> 0.3)$, that is essentially out of the cosmological nucleosynthesis constraints. The second one has shown that this feature is in agreement with Saskatoon data (Netterfield et al. 1997) on $\Delta T/T$ power spectrum at subdegree angular scales. They have concluded that these data prefer models with built-in scale in the primordial power spectrum which can be generated in the more complicated inflation scenario (e.g. double one).

For reducing the number of free parameters we restrict ourselves to analysis within the framework of the matter dominated universe and standard inflation scenario: $\Omega_m = \Omega_0 = 1$, $n = 1$ without the tensor mode of cosmological perturbations. The free parameters in our task will be baryon content $\Omega_b$, dimensionless Hubble constant $h$, neutrino mass $m_\nu$, and number species of neutrinos with equal masses $N_\nu$.

The outline of this paper is as follows: the observable data which will be used here are described in Section 2. The method of determination of parameters and its testing are described in Sect. 3. Results of best fit finding of parameters under different combination of free and fixed ones are presented in Sect. 4. Discussion of results and conclusions are given in Sect. 5 and 6 accordingly.

2. Experimental data set

The most favorable data for the search of best fit cosmological parameters are real power spectrum reconstructed from redshift-space distribution of Abell-ACO clusters of galaxies (Einasto et al. 1997, Refsdal et al. 1997). It is biased linear spectrum reliably estimated for $0.03 \leq k \leq 0.2 h/Mpc$ whose position of maximum $(k_{\text{max}} \approx 0.05h/Mpc)$, inclination before and after it are sensitive to baryon content $\Omega_b$, Hubble constant $h$, neutrino mass $m_\nu$ and number species...
of massive neutrinos $N_{\nu}$ (see Fig.1-4). Here in numerical calculations the data of last estimation of power spectrum by Retzlaff et al. 1997 will be used. All the sources of systematic and statistical uncertainties as well as window function and differences between Abell and ACO parts of sample have been accurately taken into account there. The values of the Abell-ACO power spectrum for 13 values of $kP_{A+ACO}(k_j)$ ($j = 1, 13$) and their $1\sigma$ errors are presented in Table 1 and are shown in figures.

Other observable data which will be used here are constraints of amplitude of the fluctuation power spectrum at cluster scale derived from cluster mass and X-ray temperature functions. It is usually formulated as a constraint for density fluctuations in top-hat sphere of $8h^{-1}$ Mpc radius, $\sigma_8$, which can be easy calculated for the given initial power spectrum $P(k)$:

$$\sigma_8^2 = \frac{1}{2\pi^2} \int_0^{\infty} k^2P(k; \Omega_b, h, m_\nu, N_\nu) W^2(8k/h) dk,$$

where $W(x) = 3(sinx - xcosx)/x^3$ is Fourier transformation of top-hat window function. The different collaboration groups gave similar results which are in the range of $\sigma_8 \sim 0.5 - 0.7$. The new optical determination of the mass function of nearby galaxy clusters (Girardi et al. 1998) gives median values: $\sigma_8 = 0.60 \pm 0.04$. It matches very well the cluster X-ray temperature function (Viana & Liddle 1996). For taking into account the data of other authors I shall be more conservative and will use it with $3\sigma$ error bars instead of $1\sigma$ one. But, as we will see, it does not rule out predicted $\sigma_8$ value from the $1\sigma$ limit of the observable one by Girardi et al. 1998 for best fit parameters determined here.

The COBE 4-year data will be used here for normalization of power spectra. A useful fit for them is the amplitude of density perturbation of the horizon crossing scale $\delta_h$, which for a flat model with the $n = 1$ equals $\delta_h = 1.94 \cdot 10^{-5}$ (Liddle et al. 1990, Bunn and White 1997). Taking into account the definition of $\delta_h$ (Liddle et al. 1990) and the power spectrum, the normalization constant $A$ is calculated as

$$A = 2\pi^2\delta_h^2(3000/h)^4 Mpc^4.$$  

| No | $k_j$ | $y_j$ | $\Delta y_j$ |
|----|------|------|-------------|
| 1  | 0.030| 9.312 \times 10^4 | $\pm 59723.65$ |
| 2  | 0.035| 1.037 \cdot 10^5 | $\pm 65488.2$ |
| 3  | 0.040| 1.039 \cdot 10^5 | $\pm 58014.15$ |
| 4  | 0.047| 1.258 \cdot 10^5 | $\pm 51005.75$ |
| 5  | 0.054| 1.448 \cdot 10^5 | $\pm 68638.6$ |
| 6  | 0.062| 1.016 \cdot 10^5 | $\pm 39184.6$ |
| 7  | 0.072| 8.098 \cdot 10^4 | $\pm 25179.7$ |
| 8  | 0.083| 5.444 \cdot 10^4 | $\pm 21925.45$ |
| 9  | 0.096| 5.303 \cdot 10^4 | $\pm 24914.75$ |
| 10 | 0.111| 3.853 \cdot 10^4 | $\pm 13344.5$ |
| 11 | 0.131| 2.031 \cdot 10^4 | $\pm 8546.35$ |
| 12 | 0.151| 2.039 \cdot 10^4 | $\pm 9804.3$ |
| 13 | 0.171| 1.691 \cdot 10^4 | $\pm 9383.21$ |
| 14 | $\sigma_8$| 0.60 | $\pm 0.12$ |

### 3. Method and its testing

The Abell-ACO power spectrum is connected with matter one by means of the cluster biasing parameter $b_{cl}$:

$$P_{A+ACO}(k) = b_{cl}^2 P(k; \Omega_b, h, m_\nu, N_\nu).$$

For fixed parameters $\Omega_b$, $h$, $m_\nu$, $N_\nu$ and $b_{cl}$ the values of $P_{A+ACO}(k_j)$ are calculated for the same $k_j$ as in Table 1 and $\sigma_8$ according to (1). Let’s denote them by $y_j$ ($j = 1, ..., 14$), where $y_1, ..., y_{13}$ correspond $P_{A+ACO}(k_1), ..., P_{A+ACO}(k_{13})$, and $y_{14}$ is $\sigma_8$. Their deviation from observable data set (noted by the tilde) can be described by $\chi^2$:

$$\chi^2 = \sum_{j=1}^{14} \frac{(\tilde{y}_j - y_j)^2}{\Delta \tilde{y}_j},$$

where $\tilde{y}_j$ and $\Delta \tilde{y}_j$ are experimental data set and their dispersion accordingly. Then parameters $\Omega_b$, $h$, $m_\nu$, $N_\nu$ and $b_{cl}$ or some part from them can be determined by minimizing $\chi^2$ using Levenberg-Marquard method (Press et al. 1992). The derivatives of predicted values on search parameters which are required by this method will be calculated numerically. The step for their calculation was experimentally assorted and is $10^{-3}$ of the values for all parameters.

The analytical approximation of MDM transfer function will be used in the form:

$$T_{MDM}(k; \Omega_b, h, m_\nu, N_\nu; z) = T_{CDM+b}(k; \Omega_b, h; z)D(k; \Omega_b, h, \Omega_\nu, N_\nu; z),$$
where \( T_{CDM+b}(k; \Omega_b, h; z) \) is the transfer function by Eisenstein & Hu 1997, for CDM+baryon system (\( z \) is redshift), the correction factor for the HDM component \( D(k) \) was used in the form given by Novosyadlyj et al. 1998. It is correct in a sufficiently wide range of search parameters (for a more detailed analysis of its accuracy see in Novosyadlyj et al. 1998). We suppose the scale invariant primordial power spectrum because the initial power spectra of MDM models now is as follows: 

\[
P_{MDM}(k) = AkT_{MDM}^2(k; \Omega_b, h, m_\nu, N_\nu; z)
\]

The method was tested in the following way. I calculated the MDM power spectrum for the given parameters (e.g. \( \Omega_b = 0.15, \Omega_\nu = 0.2, N_\nu = 1, h = 0.5 \)) using CMBFast code, normalized to 4-year COBE data, calculated \( \sigma_8 \) and interpolated \( P(k) \) for the same \( k_j \) (\( j = 1, ..., 13 \)) which are in Table 1. Then I have took cluster biasing parameter \( b_{cl} = 3 \) and calculated model \( \tilde{P}_{A+ACO}(k_j) \). The 'experimental' errors for them as well as for \( \sigma_8 \) I have suggested to be the same as relative errors from Table 1. These model experimental data like the ones in Table 1 were used for search of parameters \( \Omega_b, h, \Omega_\nu \), and \( b_{cl} \) (\( N_\nu \) is fixed and the same). The initial (or start) values of the parameters I have put as random deviated from the given ones. In all cases the code found all the given parameters with high accuracy.

### 4. Dependence of density fluctuations power spectra at cluster scale on cosmological parameters

Before finding of the best-fit parameters let’s look how the power spectrum of density fluctuations at cluster scale depends on search parameters. For this we leave only \( b_{cl} \) as a free parameter and fix the remaining ones. In Fig.1 such a dependence of rich cluster power spectra on \( \Omega_\nu \) is shown for \( h = 0.5, \Omega_b = 0.05 \) and \( N_\nu = 1 \). The r.m.s. of density fluctuations in the top-hat sphere of 8h\(^{-1}\) Mpc radius in models with \( \Omega_\nu = 0.1, 0.2, 0.3, 0.4 \) are \( \sigma_8 = 0.93, 0.81, 0.75, 0.71 \) accordingly. The best-fit values of \( b_{cl} \) are presented in the caption of Fig.1. The deviations of the predicted rich cluster power spectra and mass function in these models from the observable ones are correspondingly \( \chi^2 = 17.3, 9.88, 6.64, 5.33 \). Therefore, for the MDM model with \( h = 0.5, \Omega_b = 0.05 \) and \( N_\nu = 1 \) Abell-ACO power spectrum and mass function prefer high \( \Omega_\nu \) (\( \sim 0.3 \) to \( 0.4 \)).

Now we repeat the same calculations for different number species of massive neutrinos \( N_\nu = 1, 2, 3 \) and fixed \( \Omega_\nu = 0.2 \) (Fig.2). The \( \sigma_8 \)’s for these 3 models are 0.81, 0.73, 0.68 accordingly, the corresponding deviations of predicted rich cluster power spectra and mass functions from the observable ones respectively are \( \chi^2 = 9.88, 6.48, 5.54 \). So, the MDM model with three species of equal mass neutrino is preferable.

In the first two cases (\( h \) fixed and equal 0.5) the mass of neutrino was different for differing \( \Omega_\nu \) (\( N_\nu \) fixed) and \( N_\nu \) (\( \Omega_\nu \) fixed) because they are connected by relations

\[
m_\nu = 93\Omega_\nu h^2/N_\nu.
\]

Let’s fix the neutrino mass (\( m_\nu = 2.5eV \)), suggest that \( N_\nu = 2 \) and repeat calculations for different \( h = 0.5, 0.6, 0.7 \). The results are shown in Fig.3. \( \sigma_8 \) for these 3 models are following 0.71, 0.98, 1.24. The \( \chi^2 \) for all points of power spectrum and \( \sigma_8 \) are 5.72, 19.9 and 42.6 accordingly. Therefore, when neutrino mass is fixed (by laboratory experiments for example) the data prefer low \( h \).

Similarly, one shall calculate rich cluster power spectra for different \( \Omega_b \) when the rest of the parameters are fixed. The results for \( \Omega_b = 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 \) are presented in Fig.4. The corresponding \( \sigma_8 \)’s are following 0.71, 0.64, 0.58, 0.53, 0.48, 0.44, the characteristics of deviations of the predicted val-
Fig. 2.— The rich cluster power spectrum for MDM models with a varying number of species of massive neutrino \( N_\nu \) (\( \Omega_\nu, \Omega_b \) and \( h \) are fixed). The filled circles are the same as in Fig.1. The best-fit biasing parameters for models with \( N_\nu = 1, 2, 3 \) are \( b_{cl} = 2.8, 3.1, 3.3 \) accordingly.

Fig. 3.— The rich cluster power spectrum for MDM models with a varying \( h \) (\( m_\nu, N_\nu \) and \( \Omega_\nu \) are fixed). The filled circles are the same as in Fig.1. The best-fit biasing parameters for models with \( h = 0.5, 0.6 \) and 0.7 are \( b_{cl} = 3.2, 2.5, 2.1 \) accordingly.

As we see the theoretically predicted values of the chosen data are sensitive to search parameters \( m_\nu, N_\nu, \Omega_b \) and \( h \). It is interesting now where the global minimum of \( \chi^2 \) in space of these parameters is when all or a part of them are free.

5. Results

The searching of \( m_\nu, N_\nu, \Omega_b \) and \( h \) by \( \chi^2 \) Levenberg-Marquardt minimization method can be realized in the following way. We shall put \( m_\nu, \Omega_b, h \) and \( b_{cl} \) or part of them free and find the minimum of \( \chi^2 \) for \( N_\nu = 1, 2, 3 \) in a series. The lowest value from them will be suggested as minimum of \( \chi^2 \) for each set of free parameters. This is because the \( N_\nu \) possesses the discrete value.

The key point is narrowing the range of search parameter values. The analytical approximation of the MDM power spectra used here is accurate enough in the following range of parameters: \( 0.3 \leq h \leq 0.7, \Omega_\nu \leq 0.5, \Omega_b \leq 0.3, N_\nu \leq 3 \) \cite{Novosyadlyj et al. 1998}. By the upper and lower boundaries of \( h, \Omega_\nu \) and \( \Omega_b \) availability of the used analytical approximation we admeasure the range of search values of these parameters. We make these boundaries as ‘mirror walls’.

5.1. All parameters are free

The minima of \( \chi^2 \) in a 4-dimensional space of parameters \( \Omega_\nu, \Omega_b, h \) and \( b_{cl} \) for models with 1, 2 and 3 species of massive neutrinos are achieved for the set of parameters presented in Table 2. The spectra for them are shown in Fig.5 and \( \sigma_8 \)'s are presented in the Table 2. (The accuracy of analytical approximation of MDM spectra is better than 5%).

As we can see \( \chi^2 \) is few times lower than the formal degree of freedom, \( d = n - m \), where \( n \) is the number of data points, \( m \) is the number of free parameters. The reason is that not all the points of the Abell-ACO power spectrum presented in Table 1 are independent. The numerical experiment has shown that the minimal number of points which determine the same MDM parameters is \( \approx 7 \) (odd points of \( P_A(k_i) \) in Table 1, for example). Indeed, such a spectrum can be described by amplitude and inclination at small and large scale ranges and the second order curve at the peak (or maximum) range. It means that real \( d \approx 3 - 4 \).

Therefore, in the 5-dimensional space of free parameters \( (\Omega_\nu, N_\nu, \Omega_b, h \) and \( b_{cl} \) \) the global minimum of \( \chi^2 \) is achieved for the MDM model with 3 sorts of mas-
sive neutrinos. It has the lowest $m_{\nu}$ and the highest $h$ which better matches the data on immediate measurements of Hubble constant. However, it is unexpected that the found $\Omega_{b}$ is so high and $\Omega_{b}$ is so low. They contradict the data on high redshift objects and nucleosynthesis constraint ($0.007 \leq \Omega_{b}h^{2} \leq 0.024$). Therefore, the MDM models with so high a $\Omega_{b}$ also have a problem with the galaxy formation, $\sigma_{0} \sim 1$ for them. Let’s analyze the cases with additional constraints which can lead us out of this difficulty.

### 5.2. Coordination with nucleosynthesis constraint

The increasing of baryon content can decrease this difficulty (see Eisenstein et al. 1997). We shall fix baryon content by the upper limit which is resulted from the nucleosynthesis constraint $\Omega_{b}h^{2} = 0.024$ and keep up the rest parameters as free. The found best-fit parameters are in the Table 3, rich power spectrum for the case with 3 sorts of massive neutrino is shown in Fig.6 (dotted line). The spectra for the cases with 1 and 2 sorts are close to this one. As we can see $\Omega_{\nu}$ increases when $\Omega_{b}$ decreases and the minima of $\chi^{2}$ are achieved at high $\Omega_{\nu}$ again. But they are quite close to the corresponding minima from the previous table.

### Table 2: Best-fit parameters of MDM models with 1, 2 and 3 sorts of massive neutrinos for Abell-ACO power spectrum by Retzlaff et al. 1997 and mass function by Girardi et al. 1998.

| $N_{\nu}$ | $\chi^{2}$ | $\Omega_{\nu}$ ($m_{\nu}$) | $\Omega_{b}$ | $h$ | $b_{cl}$ | $\sigma_{8}$ |
|----------|------------|------------------|-----------|----|--------|---------|
| 1        | 2.07       | 0.44 (7.2)       | 0.0006    | 0.42| 3.49   | 0.55    |
| 2        | 1.77       | 0.47 (5.5)       | 0.0014    | 0.50| 3.37   | 0.56    |
| 3        | 1.66       | 0.47 (4.9)       | 0.0021    | 0.58| 3.29   | 0.57    |

### Table 3: Best-fit parameters of MDM models with 1, 2 and 3 sorts of massive neutrinos for Abell-ACO power spectrum by Retzlaff et al. 1997 and mass function by Girardi et al. 1998 when baryon content is fixed by nucleosynthesis constraint ($\Omega_{b}h^{2} = 0.024$).

| $N_{\nu}$ | $\chi^{2}$ | $\Omega_{\nu}$ ($m_{\nu}$) | $\Omega_{b}$ | $h$ | $b_{cl}$ | $\sigma_{8}$ |
|----------|------------|------------------|-----------|----|--------|---------|
| 1        | 2.58       | 0.42 (8.3)       | 0.12      | 0.46| 3.56   | 0.54    |
| 2        | 2.02       | 0.46 (6.5)       | 0.08      | 0.55| 3.37   | 0.56    |
| 3        | 1.82       | 0.48 (5.7)       | 0.06      | 0.62| 3.27   | 0.57    |
of MDM spectra for so high a Ω_b because the performance of analytical approximation are close to this one.

Fig. 6.— The rich cluster power spectrum of MDM models with 3 sorts of massive neutrinos and best-fit parameters for the cases when all parameters are free (solid line), when baryon content Ω_b is fixed by nucleosynthesis constraint (dotted line), when mass of neutrino m_ν = 2.5eV is fixed (dashed line) and when both Ω_b and m_ν are fixed (dashed dotted line). The filed circles are the same as in Fig.5.

5.3. When the mass of neutrino is known

An interesting question ensuing from last two items is: which best-fit values of Ω_b and h can be obtained from these data on the Abell-ACO power spectrum and mass function in the case when mass of neutrino is determined by any physical or astrophysical experiments and is known. Let’s assume that m_ν is fixed but the number of species N_ν is unknown. We fix Ω_b by relation (4) and the rest of parameters leave free. The search in such an approach was unsuccessful because it halted in the upper limit of Ω_b=0.3. When this ‘mirror wall’ was removed the solutions were found but with extremely high content of baryons for which an accuracy of analytical approximation for MDM spectra is worse (∼ 15 – 20%). Results for m_ν=2.5eV and 3eV are presented in Table 4. The rich cluster power spectrum for m_ν = 2.5eV and N_ν = 3 is shown in Fig.6 (dashed line). The spectra for 1 and 2 sorts are close to this one.

The χ^2’s in all cases here are lower than in Table 2 because the performance of analytical approximation of MDM spectra for so high a Ω_b and h is essentially worse than in the allowance range. Therefore we can not conclude that the global minimum of χ^2 in the 4-dimension space of parameters m_ν, N_ν, Ω_b and h is in the range of high Ω_b and h. It is in point with the parameters which are in the last row of Table 2. But we certainly conclude that when m_ν ∼ 2 – 3eV the minimum is absent in the range of Ω_b ≤ 0.3, 0.3 ≤ h ≤ 0.7. Therefore the Abell-ACO power spectrum and mass function among the MDM models with m_ν ≤ 4eV and N_ν ≤ 3 prefer Ω_b > 0.3 and h ∼ 0.8 that agrees well with the results by Eisenstein et al. 1997.

5.4. Ω_b and m_ν are fixed

One can look now which h is preferable by Abell-ACO power spectrum and mass function when neutrino mass and baryon content are fixed by the other observable constraints or theoretical arguments. Let’s put that m_ν = 2.5eV (Ω_ν = m_ν N_ν/93h^2) and Ω_b = 0.024/h^2 is fixed by the upper limit of nucleosynthesis constraint. Only h and b_{cl} are free parameters. Their best-fit values found for 1, 2 and 3 sorts of massive neutrino are presented in the Table 5. The rich cluster power spectrum for N_ν = 3 MDM model with those parameters is shown in Fig.6 (dashed line). The spectra for 1 and 2 sorts are close to this one.

As we see in the MDM model with 3 sorts of 2.5eV neutrinos the best-fit value of h and σ_8 are closer to the corresponding observable data than in models with 1 or 2 sorts.

Table 4: Best-fit parameters of MDM models with 1, 2 and 3 sorts of massive neutrinos for Abell-ACO power spectrum by Retzlaff et al. 1997 and mass function by Girardi et al. 1998 when neutrino mass is fixed (m_ν=2.5 and 3.0eV, Ω_ν = m_ν N_ν/93h^2).

| N_ν | χ^2 | Ω_ν (m_ν) | Ω_b | h | b_{cl} | σ_8 |
|-----|-----|-----------|-----|---|-------|-----|
| 1   | 1.53| 0.05 (2.5)| 0.47| 0.75| 3.22  | 0.65|
| 2   | 1.44| 0.09 (2.5)| 0.45| 0.79| 3.21  | 0.65|
| 3   | 1.39| 0.12 (2.5)| 0.43| 0.82| 3.20  | 0.65|
| 1   | 1.51| 0.06 (3.0)| 0.47| 0.75| 3.21  | 0.65|
| 2   | 1.42| 0.10 (3.0)| 0.44| 0.79| 3.20  | 0.65|
| 3   | 1.37| 0.14 (3.0)| 0.42| 0.83| 3.19  | 0.65|
Table 5: Best-fit parameters of MDM models with 1, 2 and 3 sorts of massive neutrinos for Abell-ACO power spectrum by Retzlaff et al. 1997 and mass function by Girardi et al. 1998 when baryon content and neutrino mass are fixed: $\Omega_{b} = 0.024/h^{2}$, $m_{\nu} = 2.5eV$ ($\Omega_{\nu} = m_{\nu}/93h^{2}$).

| $N_{\nu}$ | $\chi^{2}$ | $\Omega_{\nu} (m_{\nu})$ | $\Omega_{b}$ | $h$ | $b_{cd}$ | $\sigma_{8}$ |
|-----------|------------|---------------------------|-------------|-----|---------|-----------|
| 1         | 4.20       | 0.16 (2.5)                | 0.15        | 0.41| 3.90    | 0.55      |
| 2         | 3.31       | 0.24 (2.5)                | 0.11        | 0.47| 3.76    | 0.56      |
| 3         | 2.85       | 0.29 (2.5)                | 0.09        | 0.52| 3.69    | 0.56      |

6. Discussion

Rich cluster power spectra of models with the best fit parameters are within the error bars of the corresponding experimental data (Fig.5-6). But none of them explains the peak at $k \approx 0.05h/Mpc$ that corresponds to the linear scale $\approx 120h^{-1}$ Mpc. It has excess power at $\approx 50\%$ in comparison with the best-fit model and $\approx 30\%$ in comparison with the high-$\Omega_{b}$ one. It is more prominent yet in the data by Einasto et al. 1997. Apparently, it is a real feature of the power spectrum. The necessity of a similar feature in the power spectrum was argued earlier by the explanation of Great Attractor phenomenon (Hnatyk et al. 1995, Novosyadlyj 1996). A sample of the Abell-ACO clusters of galaxies used by Retzlaff et al. 1997 is placed in 60° double-cone with the axis pointing towards the Milky Way pole. The Great Attractor, on the contrary, placed in the plane of our galaxy. Therefore, they are an independent experimental demonstration of the reality of those peak. Other important arguments for its validity come from pencil-beam redshift survey by Broadhurst et al. 1990 and from 2-dimensional power spectrum of the Las Campanas Redshift Survey (Landy et al. 1996). The angular correlations in the APM survey (Gaztanaga & Bandila 1997) and high-redshift absorption lines in quasar spectrum (Quashnock et al. 1996) also show similar features at these scales. It was shown also by Atrio-Barandela et al. 1997 that this $\approx 120h^{-1}$ Mpc peak well agree with Saskatoon data on the $\Delta T/T$ power spectrum. Therefore, the data used here on rich cluster power spectrum are based on the surveys which represent a fair sample of $\approx 120h^{-1}$ Mpc structures and that peak is significant despite the large error bars of experimental data.

Obviously, that turnabout to open ($\Omega_{b} < 1$) models or flat with cosmological term ($\Omega_{b} + \Omega_{\Lambda} = 1$) does not improve the situation with the explanation of that peak in our approach. It is because the maximum of power spectra in those models is shifted to larger scales in comparison with matter dominated flat models analyzed here. Explaining of it by baryonic acoustic oscillations calls for extremely high content of baryons that disagree with nucleosynthesis constraint (see Eisenstein et al. 1997). Therefore we face a necessity to consider models with a built-in scale in the primordial power spectrum again.

Let’s determine the parameters of this peak. The comparison of rich cluster power spectrum predicted by the MDM model with the best-fit parameters (Table 2) with the observable one showed that the peak has approximately the Gaussian form. Therefore we approximate it by the function $p(k) = 1 + a_{p}exp(2(k_{p} - k)^{2}/w_{p}^{2})$, where $a_{p}$, $k_{p}$ and $w_{p}$ are amplitude, center and width of the peak accordingly. We set the power spectrum in the form of $P_{MDM+\nu}(k) = P_{MDM}(k; \Omega_{b}, h, m_{\nu}, N_{\nu})p(k; a_{p}, k_{p}, w_{p})$, and repeat previous calculations with additional free parameters $a_{p}$, $k_{p}$ and $w_{p}$.

It should seem that this peak causes such high best-fit values of $\Omega_{\nu}$ or $\Omega_{b}$ in Tables 2-4. The results of the search for best-fit parameters in the 8-dimensional space of the MDM+peak model parameters showed that it is not so, that well agrees with the numerical results by Retzlaff et al. 1997. The introducing of the peak really decreases the $\chi^{2}$ but the MDM model parameters are changed weakly. It is because they are determined mainly by the inclination of the Abell-ACO power spectrum after the peak and $\sigma_{8}$ as the most accurate value of the data set used here. The models with 3 sorts of massive neutrinos are preferable like in the previous cases. In Table 9 the best-fit parameters of the MDM models with 3 sorts of massive neutrino as well as best-fit parameters of the peak are presented for 4 cases: all the MDM parameters were free (1st row), baryon content $\Omega_{b}$ was fixed by the upper limit of nucleosynthesis constraint (2), neutrino mass was fixed at $m_{\nu} = 2.5eV$ (3), $\Omega_{b}$ and $m_{\nu}$ were fixed (4). The $\chi^{2}$ for them are 0.81, 0.86, 1.11, 1.04 accordingly. In all the cases except (3) the $\sigma_{8} = 0.6$, in (3) case the $\sigma_{8} = 0.66$. The rich cluster power spectrum for these cases are shown in Fig.7.

The introducing of such a peak increases the predicted bulk velocities in a top-hat sphere of the radius
Table 6: Best-fit parameters of MDM+peak models with 3 sorts of massive neutrinos for Abell-ACO power spectrum by Retzlaff et al. 1997 and mass function by Girardi et al. 1998. The fixed parameters are noted by () ($\Omega_b = 0.024/h^2$, $m_\nu = 2.5\text{eV}$).

| $m_\nu$ | $\Omega_b$ | $h$ | $b_\text{cl}$ | $k_p$ | $a_p$ | $w_p$ |
|---------|------------|-----|---------------|-------|-------|-------|
| 4.6     | 0.01       | 0.58| 3.14         | 0.056 | 0.46  | 0.011 |
| 5.0     | 0.064(*)   | 0.61| 3.11         | 0.056 | 0.47  | 0.012 |
| 2.5(*)  | 0.424      | 0.82| 3.16         | 0.054 | 0.34  | 0.007 |
| 2.5(*)  | 0.084(*)   | 0.53| 3.33         | 0.060 | 0.63  | 0.013 |

Fig. 7.— The rich cluster power spectrum of MDM+peak models with 3 sorts of massive neutrino and best-fit parameters from Table 6. The filed circles are the same as in Fig.5.

R whose r.m.s. values can be calculated according to

$$V_R^2 = \frac{H_0^2}{2\pi^2} \int_0^\infty dk P_{MDM}(k)W^2(kR),$$

where $W(kR)$ is the Fourier transform of this sphere. So, for $R = 50h^{-1}\ \text{Mpc}$ it increases from $340\text{km/s}$ to $360\text{km/s}$ for the best-fit model (3rd row of Table 2, 1st row of Table 6) and from $330\text{km/s}$ to $345\text{km/s}$ for a model with fixed $m_\nu$ and $\Omega_b$ (3rd row of Table 5, the last row of Table 6). The observable value of bulk velocity for this scale is $V_{50} = 375 \pm 85\text{km/s}$, which follows from Mark III POTENT results (Kolatt & Dekel 1997). Therefore, this peak is preferable also by the data on large scale peculiar velocity of galaxies and Great Attractor like structures. However, the models with high values of $\Omega_\nu \sim 0.4 - 0.5$ ($m_\nu \sim 4 - 7\text{eV}$), which are best-fit ones for the Abell-ACO data, have problems with the explanation of galaxy scale structures and high redshift objects. But models with median $\Omega_\nu \sim 0.2 - 0.3$ ($m_\nu \sim 2.5$, $N_\nu \sim 2 - 3$) are not ruled out by these data ($\Delta \chi^2 < 1$). On the contrary, the CDM model with $\Omega_b \leq 0.2$ and $h \geq 0.5$ is ruled out by these data at a high confidence level because for them $\Delta \chi^2 \leq 15$.

At last it must be noted that primordial spectrum feature like this peak is inherent for double inflation models (Kofman et al. 1985, Kofman & Linde 1987, Kofman & Pogosyan 1988, Gottlober et al. 1991, Polarski & Starobinsky 1992) and inflationary model wherein an inflation field evolves through a kink in the potential (Starobinsky 1992). Both classes of these models were confronted with the observational data on the Abell-ACO power spectrum by Lesgourgues et al. 1997 and Retzlaff et al. 1997 accordingly.

7. Conclusions

The Abell-ACO power spectrum by Retzlaff et al. 1997 and mass function by Girardi et al. 1998 in the parameter space of the MDM model ($\Omega_b = 1$) prefer a region with high $\Omega_\nu \sim 0.4 - 0.5$, low $\Omega_b \leq 0.01$ and $h \sim 0.4 - 0.6$. The best-fit parameters are as follows: $N_\nu = 3$, $m_\nu = 4.4\text{eV}$, $h = 0.56$, $\Omega_b \leq 0.01$. Unfortunately, experimental uncertainties of the data used here for the determination of these parameters give no chance to rule out models with a different set of parameters at a sufficiently high confidence level. The MDM models with baryon content at the upper limit of the nucleosynthesis constraint ($\Omega_b h^2 = 0.024$) do not outstep $\Delta \chi^2 = 1$ of best-fit model (see Table 3). The high-$\Omega_b$ ($\sim 0.4 - 0.5$) solutions are obtained when neutrino mass are fixed and $\leq 3\text{eV}$.

Introducing artificially into the primordial power spectrum a peak of Gaussian form decreases the $\chi^2$, increases the bulk motions but does not change essentially the best-fit parameters of the MDM models. It means that determinative for these parameters is mainly inclination of the Abell-ACO power spectrum at the scales smaller than the scale of the peak position and $\bar{\sigma}_8$ as the most accurate value of the data set used here.

Hereby, the power spectrum of the Abell-ACO clusters of galaxies and mass function are a sensitive test for the MDM model parameters. But more accurate data on power spectrum of matter density fluctuations are necessary for more certain determination.
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