Estimation of loads' main statistics for hot materials silo

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Abstract. The paper presents a statistical description of the values of random variables of pressure and temperature as the most important loads on wall of silo, in which hot materials, such as cement, clinker, fly ash, and raw meal are stored. An attempt to statistically describe the pressure and temperature and their additive share in loads are clearly present in the standards and guidelines for reinforced concrete (RC) silo design. The authors conducted many experiments and measurements of pressure and temperature that impact on the silo walls. For example, in a silo for a hot cement, variable pressure and temperature of the silo wall were measured simultaneously. In this example, coincidence coefficients for temperature and pressure were calculated using many statistical methods. For the pressure, a statistical determination of characteristic values was proposed.

1. Introduction
The need for proper evaluation of all load’s sources acting on building structure is well defined in Codes and Recommendations [1]. The authors of this paper as members of research team in Faculty of Civil Engineering in Wroclaw University of Science and Technology, carried out research on reinforced concrete (RC) silos for many granular materials as cement, rape, wheat, limestone powder, fly ash [2-5] for more than 30 years. Various load cases applied simultaneously to RC structures were also widely analyzed and described in technical literature [6] with special regard to the impact of formworks [7], thermodynamic influences [8] and chemical [9] or geodynamic hazards [10].

In addition to pressure, thermal load is an important load because hot materials were stored in these silos. The researches conducted at Wroclaw University of Science and Technology included evaluation of value of pressure, temperature field and temperature gradient in the silo wall. Pressure and thermals affect the silo at random [11, 12]. But they are also random quantities as strength and quantity of the reinforced bars, strength of concrete, thickness of the silo wall, vertical and horizontal distances between bars, value of imperfections, thickness of the silo wall, quality of lap splices, corrosion grade etc. Last but not least, random displacement of support (settlement of foundations) should also be considered [13]. All of these factors have influence on silo designing. In particular, the frequency of these loads should be considered to account for the probability of their simultaneous occurrence. In this way coincidence factor is evaluated. [14].

Because it is obvious that pressure of stored material and thermal impact against silo wall is random value than very therefore it is important with what probability that their greatest values occur together. A typical graph of pressure measured on cement silos (figure 1) is shown on the figure 2. The figure 3 shows the temperature distribution along the height and circumference.
The pressure and temperature were tested simultaneously and their values can be determined at that time at the measured measuring points. In the European Code [1] the maximum values of both quantities are added directly during filling of the silo but during silo emptying thermal influences are reduced by 40%.

2. Pressure as a random value.
The following equation (1) must be fulfilled considering the reliability requirement [14]:

\[ A_{s_{\text{max}}} \leq R_{\text{min}} \]

where: \( A_{s_{\text{max}}} \) – denotes multi componential action effects, that is, for example, force in horizontal reinforced bars in case of calculating wall strength,
\( R_{\text{min}} \) – is the minimum resistance, i.e. bar strength in case of calculating wall strength.
\( A_{s_{\text{max}}} = f(\Delta T, p_h, \text{etc}) \)

is a function of many parameters:
\( \Delta T \) - temperature gradient across the silo wall,
\( p_h \) – horizontal pressure caused by stored material.

Horizontal pressure is a random function of many parameters

\[ p_h = f(\gamma, \mu, D, K\alpha, z, \phi_u, T, w, h, \ldots) \]

as:
\( \gamma \) – volumetric weight,
\( \mu \) – coefficient of material friction against the wall,
\( D \) – silo diameter
\( K\alpha = p_h/p_v \) – quotient of horizontal to vertical pressure, described by function \( K\alpha = \tan^2(45^\circ - \phi/2) \)
\( z \) – calculation level of charge
\( \phi \) – internal friction factor of the material
\( T \) – generally, the temperature field in the silo material and wall,
\( w \) – humidity,
\( h \) – silo height,
\( t \) – time.

The material pressure consists of two deterministic and random parts. Deterministic depends on the height of material in the filling silo, the eccentricity of filling and emptying the silo, material parameters such as density. Jansen applied this part in the equilibrium equation of stored material. This deterministic part of pressure is constantly subject to numerical calculations. The random part of pressure is part of the variable pressure and is subject to statistical description. As a rule, these descriptions of random value are subjected to Gaussian normal distributions or in the case of searching for extreme values for Gumbel distributions. For the distribution of minimal, e.g. strength of reinforcement, Weibull random distribution is used. As a rule, however, as it is used in the silo norm [1], the pressure is determined by simple equations containing the influence of random variability.

Due to the need to adjust Jansen's formula of pressure values measured experimentally on technical facilities, and taking into account the variability of material parameters in the silo, an adjustment of these parameters was introduced in [1]. From the material parameters such as: apparent density or bulk weight of bulk material, internal friction angle, specific density of bulk material, friction coefficient of bulk material against the wall, linear factor binding vertical and horizontal pressure (where the angle of internal friction plays the most important role), sample shear strength, cohesion, porosity, compressibility, humidity, which play an important role in the formation of material flow channels and also in the calculation of pressure value in [1], only the first 4 of the above mentioned were selected for statistical analysis.

The statistical description of the changes in the values of these parameters was limited to providing a formula for calculating characteristic values associated with the mean value and the coefficient of variation. It is similar to the method for calculating partial safety factors \( \gamma_f \) used for loads [14].
Figure 1. Silo for cement with placed in bottom and wall sensors, \( h_c = 38 \) m, \( d_c = 17 \) m, \( e_0 = 8 \) m

Figure 2. Experimentally measured pressure of cement against silo wall

Limit values (e.g. characteristic values) are described according to the formula with used so-called “method of probability quantile “\( k \”

\[
x^k = \mu_x \pm k \cdot \sigma_x
\]

(3)

\( \mu_x \) – mean value, \( \sigma_x \) – standard deviation,
\( k = (1.7 : 3.0) \) – number indicating the quantile of probability of expected characteristic value occurrence.

\[

\nu_x = \frac{\sigma_x}{\mu_x}
\]

(4)

is coefficient of variation:
\( \nu_x = (0.05 : 0.30) \) – is the average range of the coefficient of variation,
e.g. \( \nu_x = (0.05 : 0.10) \) – low variability of the described parameter,
\( \nu_x = (0.10 : 0.30) \) – average variability of the described parameter.
By simplifying the recording of the 'characteristic' value, the following are obtained for the design purposes:

- upper characteristic value:
  \[ x^{k,up} = \mu_x \left(1 + k \frac{\sigma_x}{\mu_x}\right) = \mu_x \left(1 + k \nu_x\right) = \mu_x a_x \]  
  \[ (5) \]

- lower characteristic value
  \[ x^{k,low} = \mu_x \left(1 - k \frac{\sigma_x}{\mu_x}\right) = \mu_x \left(1 - k \nu_x\right) = \mu_x / a_x \]  
  \[ (6) \]

The experiment is therefore set up to determine the dimensionless coefficient \( \alpha \) for specific material and shape of the silo, as well as the conditions of use, such as storage, filling, and emptying. This formula includes specific density that can be determined experimentally with high accuracy, especially in the case of inorganic materials. The use of apparent density \( \gamma_o \) is not very accurate, because it changes in a large range of values (see Table 1), for example at different levels of the silo. Therefore, it seems appropriate to supplement the values included in the tables with specific density values sometime in the future. Table 5 shows the variability of the \( \alpha_x \) coefficient from 1.06 to 1.28, which results in a change in the value of material parameters in the range from 12% to 64%. The higher and lower characteristic values calculated in this way by means of the mean value \( \mu_x \) and the coefficient \( a_x \) differ from the classically calculated ones by no more than 5%. The values of parameters \( \mu_x, a_x \) for numerous materials are given in [1]. When calculating the \( p_{hb}, p_v, p_w \) pressure, those \( x^k \) values should be used, i.e. \( x^{k,up} \) or \( x^{k,low} \), which give a larger result of the calculation of horizontal \( p_{hb} \), vertical \( p_v \) or tangential \( p_w \) pressure.

**Table 1.** Selected materials with statistically variable parameter values for pressure calculations [1]

| Table E1 – Particulate solids properties |
|------------------------------------------|
| **Type of particulate solid** | **Unit weight** | **Angle of internal friction** | **Lateral pressure ratio** | **Wall friction coefficient** (wall type D3) | **Patch load reference factor** |
|------------------------------------------|
| Cement                                  | Lower 13.0 | Upper 16.0 | Mean 36.0 | 1.22 | 0.54 | 1.20 | 0.51 | 1.07 | 0.50 |
| Cement clinker                          | 15.0     | 18.0       | 40.0    | 1.02 | 0.38 | 1.31 | 0.62 | 1.07 | 0.70 |
| Fly ash                                 | 8.0      | 15.0       | 35.0    | 1.16 | 0.46 | 1.20 | 0.72 | 1.07 | 0.50 |
| Limestone powder                        | 11.0     | 13.0       | 30.0    | 1.22 | 0.54 | 1.22 | 0.56 | 1.07 | 0.50 |
This method is quite reliable for calculating pressure at the bottom of the silo during chimney flows. In the upper parts of slender silos, changes, e.g. in the density of the stored material or the angle of internal friction are very random; it can even be assumed that these quantities, in statistical terms, are ‘perfectly disordered,’ which in turn are described by uniform or multimodal distributions. The values of the coefficients of variation of material parameters are the measure of dispersion. At high coefficients of variation, the $k$ coefficient increases from 1.72 (Gaussian distribution) even to 3 (the 0.98 quantile), and for strong skew distributions even $k = 3.5$ is used. In the ECI [1] additional changes in material pressure are included by the addition patch loads this is local pressure. For $k=1.64$ and mean value of $\mu_x=0.2$ coefficient $a_x=1.32$ and for $k=3$ $a_x=1.6$. For $k=1.64$ and mean value of $\mu_x=0.1$ coefficient $a_x=1.16$ and for $k=3$ $a_x=1.3$.

Borcz [3, 4] offered suggestions for recording test results in an invariant form, i.e. in a dimensionless form that is the simplified method of calculating the pressure and it rather covers the distribution of the maximum values obtained in the experimental measurement of pressure. For example, horizontal pressure at the tested location can be represented by the formula:

$$p = \alpha \gamma_w g D = \alpha \left( \frac{z}{D} \cdot \frac{\gamma_o}{\gamma_w} \right) \gamma_w g D$$

(7)

where:
- $\gamma_w$ - specific density,
- $g$ - gravitational acceleration,
- $D$ - silo diameter,
- $z/D$ - determines the depth of sensor location in the silo.

Experimental results were presented in the form of a coefficient $\alpha$ diagrams and it could be used for geometrically similar silos [3]. This method was used for identical silo as used in experiment.

3. The distribution of temperature in silo wall where hot materials are stored or subjected to daily temperature changes

The temperature in the silo wall from the hot material stored is accompanied by pressure. Its uneven distribution on the silo wall surface and in thickness causes additional compressive forces and bending moments. But the maximum pressure value is usually not accompanied by a maximum temperature value. During storage, the material cools in contact with the wall and the thermal effect on internal forces is reduced over time. Therefore, in many standards, a coefficient is proposed to reduce the share of temperature in the loads of silo walls.

Having data of pressure $p_h$, gradient of temperature $\Delta T$ we can compute inner forces $N (p_h, \Delta T)$ i.e. axial forces and $M (p_h, \Delta T)$ i.e. bending moments and then reinforced bars area, separately $A_x (p_h, \Delta T)$, $A_y (p_h)$, $A_z (\Delta T)$.

![Figure 4](image_url) Inner forces in silo wall.

Taking into account only main loads acting against silo wall we can write eq. (2) in a simple form treated $A_{x_{\text{max}}}$ as a sum of two factors affecting the silo wall:
where:

- $A_{si}$ – is the effect of pressure, the area of horizontal bars, here $A_s(p_h)$,
- $A_{s2}$ – is the effect of temperature, here $A_s(\Delta T)$,
- $A_{max}$ – is total area, here $A_s(p_h, \Delta T)$,
- $f_{yd}$ – strength of bars,
- $\phi_i$ – coincidence factor.

We can write eq. (8) in another form:

$$A_s(p_h, \Delta T) \leq A_s(p_h^*) + A_s(\Delta T^*)$$

in the case when $p_h$ and $\Delta T$ are measured simultaneously but $p_h^*, \Delta T^*$ are measured independently - for example $p_h^*$, $\Delta T^*$ are standard quantity (maximum, characteristic or design values). Than equation (9) can be written as:

$$A_s(p_h, \Delta T) = A_s(p_h^*) + \varphi \cdot A_s(\Delta T^*)$$

(10)

In this formula we can estimate proportional part of $\Delta T$ in this two components of loads. Silo standards [1] proposes to reduce second, third components of loads about 30%, 40% respectively. But in the case of silo for hot materials, temperature is very important load which causes big bending of walls in two directions, vertical and horizontal.

For silo with cracks we observe than bending moments decrease proportional to the width of crack. Than the value of computing temperature gradient can be reduced:

$$A_s(p_h, \Delta T) = A_s(p_h^*) + \varphi \cdot \beta \cdot A_s(\Delta T^*)$$

(11)

where: $\beta$ influence of cracks presence, $\beta=(0.5\div1.0)$.

4. Experimental research.

Pressure and temperature field were measured on cement silo [4, 5]. Cement silo has 42.00 m of height and 16.65 m diameter, thickness of wall is 0.35 m. During more than 10 years such parameters as pressure, temperature field in silo wall, parameters of strength of silo wall were measured. 17 gauges were placed in silo cement. Some results of measured pressure and temperature are shown on figure 2 and figure 3.

5. Calculation of coincidence factor $\varphi$.

Some formulas allow to calculate values of coincidental factor $\varphi$. We can name them estimators. Let’s mark

$$k_1 = \overline{A}_s(p_h) + \overline{A}_s(\Delta T)$$

(12)

$$k_2 = \mu^2_{k_s(ph)} + \mu^2_{A_s(\Delta T)}$$

(13)

$$k_3 = \overline{A}_s(p_h) + t_{af} \mu_{A_s(ph)}$$

(14)

$$k_4 = \overline{A}_s(\Delta T) + t_{af} \mu_{A_s(\Delta T)}$$

(15)

$$k_5 = \overline{A}_s(p_h, \Delta T) + t_{af} \mu_{A_s(p_h, \Delta T)}$$

(16)
\[ k_6 = \max(A_s(p_h)) \]  
\[ k_7 = \frac{A_s(p_h, \Delta T) - k_6}{A_s(\Delta T)} \]  
\[ k_8 = \frac{A_s(p_h, \Delta T) - k_3}{k_4} \]  

where:

\( \bar{A}_s, \mu_{A_s} \) - mean value and standard deviation,

\( t_\omega \) – coefficient of quantile of assumed probability, e.g. \( t_\omega = 1.64 \) for Gauss 0.95 level probability.

Based on the calculated coefficients \( k_i \) from equations (2) ÷ (19), coincidence coefficients can be calculated based on the recommendations given, among others in [2], [14]:

\[ \varphi_1 = \max(k_7) \]  
\[ \varphi_2 = \bar{k}_7 + t_\omega \mu_{k_7} \]  
\[ \varphi_3 = \max(k_8) \]  
\[ \varphi_4 = \bar{k}_8 + t_\omega \mu_{k_8} \]  
\[ \varphi_5 = \frac{\max(A_s(p_h, \Delta T)) - \max(A_s(p_h))}{\max(A_s(\Delta T))} \]  
\[ \varphi_6 = \frac{k_5 - k_3}{k_4} \]  
\[ \varphi_7 = \frac{k_1 + t_\omega k_2}{k_3 + k_4} \]  
\[ \varphi_8 = \frac{\varphi_6 \cdot (k_1 + k_3) - k_3}{k_4} \]  

6. Results of calculations

Values of factors \( \varphi \) are computed for cement silo. For solutions presented in tables 2 – 5 used several combinations of pressure and temperature as follows. Left side of equation (11) can be treated as simultaneously acting loads, measured during experiment. Right side of this equation is a typical summation of design loads, for example characteristic loads proposed by silo standards. Coefficients \( \varphi_i \) can allow to decrease this value make them more realistic. In each of presented tables 2 – 5 used combination of loads. In table 5 acting loads of left side are maximum pressure and accompanying temperature. Right side creates maximum pressure, maximum temperature which can appear during whole time of exploitation of silo. Two cases are considered, silo without and with cracks. Rows in tables represent levels in silo walls where gauges measuring pressure and temperature were placed. In this case this rows represents levels where pressure and temperature have maximum value, that is the middle and lower parts of silo cells. To compare the results of many years of researches, proposals for the coefficient of coincidence in Eurocode 1 [1] is shown in the table 6.
Table 2. Maximum values of pressure and accompanying temperature

| \( \varphi_1 \) | \( \varphi_2 \) | \( \varphi_3 \) | \( \varphi_4 \) | \( \varphi_5 \) | \( \varphi_6 \) | \( \varphi_7 \) | \( \varphi_8 \) |
|---|---|---|---|---|---|---|---|
| 0.913 | 0.764 | 0.802 | 0.232 | 0.913 | 0.679 | 0.946 | 0.851 |
| 0.495 | 0.403 | 0.436 | 0.074 | 0.495 | 0.492 | 0.933 | 0.849 |
| 0.767 | 0.990 | 0.553 | 0.712 | 0.767 | 0.845 | 0.948 | 0.872 |
| 0.688 | 0.981 | 0.485 | 0.691 | 0.688 | 0.934 | 0.929 | 0.864 |
| 0.750 | 0.833 | 0.253 | 0.281 | 0.750 | 0.680 | 0.980 | 0.921 |

Table 3. Maximum values of pressure and suspected maximum temperature

| \( \varphi_1 \) | \( \varphi_2 \) | \( \varphi_3 \) | \( \varphi_4 \) | \( \varphi_5 \) | \( \varphi_6 \) | \( \varphi_7 \) | \( \varphi_8 \) |
|---|---|---|---|---|---|---|---|
| 0.772 | 0.798 | 0.797 | 0.360 | 0.772 | 0.808 | 0.945 | 0.851 |
| 0.959 | 0.883 | 0.879 | 0.234 | 0.959 | 0.939 | 0.949 | 0.864 |
| 0.981 | 0.993 | 0.919 | 0.940 | 0.943 | 0.940 | 0.957 | 0.880 |
| 0.802 | 0.987 | 0.612 | 0.787 | 0.802 | 0.945 | 0.919 | 0.855 |
| 0.982 | 0.897 | 0.327 | 0.363 | 0.834 | 0.959 | 0.988 | 0.929 |

Table 4. Left side of equation (11) maximum values of pressure and accompanying temperature, right side is maximum values of pressure and suspected maximum temperature

| \( \varphi_1 \) | \( \varphi_2 \) | \( \varphi_3 \) | \( \varphi_4 \) | \( \varphi_5 \) | \( \varphi_6 \) | \( \varphi_7 \) | \( \varphi_8 \) |
|---|---|---|---|---|---|---|---|
| 0.333 | 0.294 | 0.278 | 0.051 | 0.222 | 0.322 | 0.798 | 0.718 |
| 0.643 | 0.648 | 0.557 | 0.184 | 0.429 | 0.638 | 0.823 | 0.749 |
| 0.600 | 0.595 | 0.518 | 0.151 | 0.400 | 0.593 | 0.801 | 0.737 |
| 0.226 | 0.251 | 0.182 | 0.202 | 0.400 | 0.539 | 0.882 | 0.820 |
| 0.041 | 0.000 | 0.058 | 0.098 | 0.222 | 0.300 | 0.951 | 0.894 |

Table 5. Left side of equation (11) is max. values of pressure and accompanying temperature, which is reduced 50%, and right side is max. values of pressure and suspected maximum temperature.

| \( \varphi_1 \) | \( \varphi_2 \) | \( \varphi_3 \) | \( \varphi_4 \) | \( \varphi_5 \) | \( \varphi_6 \) | \( \varphi_7 \) | \( \varphi_8 \) |
|---|---|---|---|---|---|---|---|
| 0.271 | 0.111 | 0.098 | 0.093 | 0.017 | 0.074 | 0.751 | 0.676 |
| 0.514 | 0.413 | 0.417 | 0.358 | 0.118 | 0.276 | 0.765 | 0.696 |
| 0.499 | 0.360 | 0.357 | 0.311 | 0.091 | 0.240 | 0.698 | 0.642 |
| 0.180 | 0.051 | 0.057 | 0.041 | 0.046 | 0.090 | 0.749 | 0.697 |
| 0.194 | 0.058 | 0.056 | 0.038 | 0.048 | 0.054 | 0.810 | 0.793 |

Table 6. Combination of loads according to [1] and simultaneity coefficients

| Design situations combination to be considered |
|---|---|---|---|---|
| \( \psi_{0.2} \) | \( \psi_{0.3} \) |
| D | Solids discharge | Self-weight | Foundation settlement | 1.0 | Snow or wind thermal | 0.6 |
| I | Imposed loads or deformation | Self-weight | Solids filling | 1.0 | Snow or wind thermal | 0.6 |
| S | Snow | Self-weight | Solids filling | 1.0 |
| F | Foundation settlement | Self-weight | Solids discharge | 1.0 | Snow or wind thermal | 0.6 |

7. Conclusions

Special combinations of loads [6] still constitute one of the most challenging issue in design of reinforced concrete structures. The case becomes even more complicated when thermal [5] and thermodynamic influences [8] are considered in combination with static, dynamic and geodynamic [10] impact of loads.

8
Statistical description of the pressure using the "k" quantile probability method is very simple and useful in calculating the pressure whose values are obtained from the experiment. This simple description is used e.g. in the so-called “Borcz equation” [3, 4].

This description was also used in the standard description, in which the well-known and "eternal" Jansen formula was used to calculate the pressure. The proposed value of the k coefficient for the Borcz (7) description is k=3.0 and for standard [1] applications k=1.5 to k=2.0, because the standard description uses the variability of pressure described by local pressure.

Presented methods of computing the coincidence factor show that his value strongly depends of these methods. Authors [4] propose use the factor equals 0.5 for cracked silos and 0.8 for silos without cracks. In the standard [1], a coefficient of coincidence equal 0.6 was used.

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