Investigation on Steady State Unbalance Response of Rotor with Elastic Ring Squeeze Film Damper

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Abstract. In this paper, the method for calculating steady state response of a rotor system with elastic ring squeeze film damper(s) is studied. The finite difference method is used to calculate the oil film pressure field which excludes the influence of rotating speed and considers the action of oil orifices starting with the governing equations of oil film. The mechanical characteristic curves of ERSFD are obtained by integrating the film pressure field, based on which the steady state response of the rotor system with ERSFD(s) is calculated by the finite element method and iteration method. The calculating response is compared with the experimental data, and it is found that the calculating response is in good agreement with the actual one, with the relative error of no more than 15%, which verifies the correctness of the above algorithm, and conclusion that the elastic ring extrusion oil film damper can be regarded as a linear element when the eccentricity journal of the journal is relatively small is obtained.

1. Introduction
Complicated structure and variable working conditions restrict the performance of aero-engines. Squeeze film damper (SFD) is widely used as a kind of passive isolator in bearings of aero-engines for simple structure and excellent performance. However, the film stiffness of conventional SFDs is highly nonlinear, which may result in bistable state or uncoordinated procession and limits its application [1]. Elastic ring squeeze film damper (ERSFD) is used on Ал-31Ф engine in Russia, for it can not only improve nonlinearity of SFD but also adjust frequency of rotor system. Elastic ring is combined with squeezed film chamber in ERSFD, which divides the oil film into inner and outer parts. Each part is separated by evenly distributing bosses and is connected by radial orifices with each other. The vibration reduction mechanism of ERSFD is analysed and the experiments are carried out by M Zhou [2-4] to prove the good damping effect. The equivalent stiffness and damping of ERSFD with different eccentricity is calculated by L Cao [5-8], and conclusion that mechanical characteristics of ERSFD is more linear than SFD is got. The transformation law between structure parameters and dynamic characteristics of ERSFD is studied by J Hong [9] using finite element method.

However, investigation on dynamics characteristics of a rotor system with ERSFDs can hardly be seen. In this paper, mechanism characteristics of ERSFD is analysed using finite difference method, results of which are used to calculate steady state unbalance response of rotor system with ERSFDs. The comparison with experimental data indicates the correctness of algorithm. Thus the method established in this paper can play a guiding role for the design of ERSFD on rotor system.

2. Mechanism characteristics of ERSFD
The steady pressure field governing equation of inner and outer film in cylindrical coordinates, as equation (1) and (2) shows, can be deduced from generalized Reynolds equation using bounding conditions of oil film [2]:

\[
\frac{1}{R_1^2} \frac{\partial}{\partial \theta} \left( h_1 \frac{\partial p_1}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( h_1 \frac{\partial p_1}{\partial z} \right) = -12 \mu \Omega \left( \frac{\partial h_1}{\partial \theta} - \frac{\partial r}{\partial \theta} \right) + 12 \mu v_d
\]

\[
\frac{1}{R_2^2} \frac{\partial}{\partial \theta} \left( h_2 \frac{\partial p_2}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( h_2 \frac{\partial p_2}{\partial z} \right) = 12 \mu v_d
\]

where \( R_1 \) and \( R_2 \) are radius of the journal and the housing, \( h_1 \) and \( h_2 \) are the film thickness, \( \mu \) is the fluid viscosity, \( \Omega \) is the rotating speed, \( r \) is the radial deformation of ERSFD, \( v_d \) is the net velocity through orifice, and \( z \) and \( \theta \) are the axial and circumferential coordinates. In addition, \( L \) is the axial width of ERSFD and \( n \) is the number of bosses. Figure 1 is the sketch map of ERSFD components. The parameters of ERSFD are illustrated in figure 2.

2.1. Clearance functions of ERSFD

According to equation (1) and (2), clearance functions \( h_1 \) and \( h_2 \) have effect on pressure field of ERSFD, and can be calculated from elastic deformation of ERSFD. Finite element method is accepted to solve the deformation of ERSFD with a bearing force acting on the journal. For example, a specific ERSFD with parameters in table 1 is used to explain the process of calculation.

| \( R_1 (\text{mm}) \) | \( R_2 (\text{mm}) \) | \( c_1 \) and \( c_2 \) (\( \text{mm} \)) | \( h_0 \) (\( \text{mm} \)) | \( L \) (\( \text{mm} \)) | \( b_1 \) and \( b_2 \) (\( \text{mm} \)) | \( d_1 \) and \( d_2 \) (\( \text{mm} \)) | \( n \) |
|---|---|---|---|---|---|---|---|
| 65.5 | 67.5 | 0.3 | 1.4 | 16 | 5 | 10 | 10 |

First of all, three-dimensional models of ERSFD with its inner and outer bushing, just as figure 3 displays, are established.
After importing the material parameters into ANSYS, meshing and setting bounding conditions, problem can be solved. Here the elasticity modulus equals 2e11Pa, and the Poisson’s ratio equals 0.3. Solid 186 element is used to mesh the body. The contact type between boss and lining on the direction of bearing load is bonded, while that of others is frictionless. Because boss and lining directly facing the load are bonded with each other and can hardly move relatively, but others may appear a little radial separation and circumferential slip with each other. Circumferential deformation restriction is imposed on the outer face of outer lining, and a 100N bearing load along X axis is applied to the inner face of inner lining. The deformation of ERSFD shows in figure 4, from which conclusion that the inner lining has translational motion and the eccentric side of ERSFD has gross distortion facing when the radial bearing load exists can be drawn.

The stiffness of ERSFD is calculated through equation (3), and that of example ERSFD is 7.18e6N/m.

\[ K = \frac{F}{s} \]  

(3)

Clearance functions can be got by exporting and disposing deformation data of ERSFD and its inner and outer lining. Relative distance between inner face of ERSFD and outer face of inner lining means the inner clearance function \( h_1 \), while that between outer face of ERSFD and inner face of outer lining means the outer one \( h_2 \). Figure 5 shows the result with 100N applied on inner lining along X axis.

![Clearance functions](image.png)

**Figure 5.** Clearance functions of example ERSFD with 100N along X axis.
Because the deformation of ERSFD and clearance functions vary with the bearing load, different clearance functions under different bearing loads on the same direction are calculated in figure 6, using which to linearly interpolate and get clearance at certain angle, and calculate the pressure field of oil film.

2.2. Pressure field of oil film

Finite difference method is accepted to get the pressure field of oil film, for it is difficult to solve equation (1) and (2) directly. Equation (5) is derived from the geometric equation of inner clearance, namely, equation (4) [2]:

$$h_1(\theta) = c_1 + r(\theta) + e\cos\theta$$  \hspace{1cm} (4)

$$\frac{\partial h_1}{\partial \theta} = \frac{\partial r}{\partial \theta} - e\sin\theta$$  \hspace{1cm} (5)

where $e$ is the eccentricity of the journal. In this way, equation (1) can be written as:

$$\frac{1}{R^2} \frac{\partial}{\partial \theta} \left( h_1^3 \frac{\partial P_1}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( h_1^3 \frac{\partial P_1}{\partial z} \right) = 12\mu \Omega e \sin\theta + 12\mu v_d$$  \hspace{1cm} (6)

There is no $r$ in equation (6), which simplifies the calculation of pressure field. It is obvious that the pressure of film varies with $\Omega$. In order to eliminate the effect of $\Omega$ and improve the versatility of the result, dimensionless parameters in equation (7) are introduced.

$$\begin{align*}
H &= h/c \\
\lambda &= 2z/l \\
P &= pc^2/2\mu R^2 \\
\varepsilon &= e/c
\end{align*}$$  \hspace{1cm} (7)

where $\varepsilon$ is called the eccentricity ratio of ERSFD. Therefore, equation (6) and (2) can be written as:

$$\frac{\partial}{\partial \theta} \left( H_1^3 \frac{\partial P_1}{\partial \theta} \right) + \left( \frac{2R_1}{l_1} \right)^2 \frac{\partial}{\partial \lambda} \left( H_1^3 \frac{\partial P_1}{\partial \lambda} \right) = 6\varepsilon \sin\theta + \frac{6v_d}{\Omega c_1}$$  \hspace{1cm} (8)

$$\frac{\partial}{\partial \theta} \left( H_2^3 \frac{\partial P_2}{\partial \theta} \right) + \left( \frac{2R_2}{l_2} \right)^2 \frac{\partial}{\partial \lambda} \left( H_2^3 \frac{\partial P_2}{\partial \lambda} \right) = \frac{6v_d}{\Omega c_2}$$  \hspace{1cm} (9)
Just as figure 7 shows, inner and outer oil film can be divided into meshes and the derivative terms in equation (8) and (9) are expressed as the difference quotient form like:

\[
\begin{align*}
\frac{\partial}{\partial \theta} \left( \frac{\partial P}{\partial \theta} \right)_{i,j} &\approx \frac{H^3_{i+1/2,j} P_{i+1,j} + H^3_{i-1/2,j} P_{i-1,j} - \left( H^3_{i+1/2,j} + H^3_{i-1/2,j} \right) P_{i,j}}{(\Delta \theta)^2} \\
\frac{\partial}{\partial \lambda} \left( \frac{\partial P}{\partial \lambda} \right)_{i,j} &\approx \frac{H^3_{i+1/2,j} P_{i,j+1} + H^3_{i-1/2,j} P_{i,j-1} - \left( H^3_{i+1/2,j} + H^3_{i-1/2,j} \right) P_{i,j}}{(\Delta \lambda)^2}
\end{align*}
\]

(10)

where \( m \) and \( n \) are numbers of circumferential and axial nodes, \( \Delta \theta = (\theta_{n+1} - \theta)/m, \Delta \lambda = 2/n \).

The oil orifices are seen as capillary holes, so according to the flow equation (11) [2]:

\[
Q = \frac{\pi d_h^4 \Delta p_h}{128 \mu l_h}
\]

(11)

the expression equation of \( v_d \) can be got:

\[
v_d = \frac{d_h^2 \rho c^2 \Delta p_h \Omega}{16 c^2 l_h^3}
\]

(12)

where \( d_h \) and \( l_h \) are the diameter and depth of orifice, and \( \Delta p_h \) is the pressure difference between inner and outer film at the hole. So the pressure at a certain node is expressed as equation (13) by combining equation (8) ~ (12) and adopting successive over-relaxation iteration (SOR):

\[
P^{(k+1)}_{i,j} = \omega \left( A_{i,j} P^{(k)}_{i+1,j} + B_{i,j} P^{(k)}_{i-1,j} + C_{i,j} P^{(k)}_{i,j+1} + D_{i,j} P^{(k)}_{i,j-1} - F_{i,j} \right) / E_{i,j} + (1-\omega)P^{(k)}_{i,j}
\]

(13)

where \( A_{i,j} = H^3_{i+1/2,j} + H^3_{i-1/2,j}; B_{i,j} = H^3_{i+1/2,j}; C_{i,j} = \left[ (2 R \Delta \theta)/(1 \Delta \lambda) \right]^2 H^3_{i,j+1/2}; D_{i,j} = \left[ (2 R \Delta \theta)/(1 \Delta \lambda) \right]^2 H^3_{i,j-1/2}; E_{i,j} = A_{i,j} + B_{i,j} + C_{i,j} + D_{i,j} \), \( F_{i,j} = 6 \epsilon \sin \theta - 3 d_h^2 R^2 \Delta p_h / (8 l_h^3 c^3) \), \( F_{i+1,j} = 3 d_h^2 R^2 \Delta p_h / (8 l_h^3 c^3) \), \( \omega \) is relaxation factor. \( A_{i,j}, B_{i,j}, C_{i,j}, D_{i,j} \) and \( E_{i,j} \) are related to clearance functions, so after initializing the pressure field and getting clearance functions of ERSFD, final pressure field can be got through SOR method. The detailed process is showed in figure 8, and the final results are displayed in figure 9.

**Figure 7.** Meshing of oil film.
Initialization of pressure field and $v_d$

Calculation of $v_d$ at orifices

Calculation of new pressure field

Converge?

Yes

Calculation of $H$ at important nodes

No

Figure 8. Calculation schedule of pressure field.

Figure 9. Pressure field of oil film.

2.3. Equivalent stiffness and damping of ERSFD

Relative pressure fields are integrated by equation (14) to get relative radial and circumferential force.

$$
\begin{bmatrix}
\Delta F_r \\
\Delta F_t
\end{bmatrix} = \sum_{i=1}^{m} \sum_{j=1}^{n} R \Delta \theta_{i,j} \Delta z_{i,j} P_{i,j} \begin{bmatrix}
\cos \theta_{i,j} \\
\sin \theta_{i,j}
\end{bmatrix}
$$

Taking equation (7) into consideration, absolute film forces can be written as:

$$
\begin{bmatrix}
F_{r,\Omega} \\
F_{t,\Omega}
\end{bmatrix} = \frac{2 \mu R^2}{c^2} \begin{bmatrix}
\Delta F_r \\
\Delta F_t
\end{bmatrix}
$$

(15)

where $F_{r,\Omega} = F_r / \Omega$, $F_{t,\Omega} = F_t / \Omega$, $F_r$ and $F_t$ are absolute radial and circumferential force.

Equivalent stiffness and damping can be obtained by the following equation [10]:

$$
K_a = -\frac{F_r}{e}, \quad C_a = -\frac{F_t}{e \Omega}
$$

(16)

By acting different bearing loads on ERSFD, its mechanical characteristics can be got like figure 10, which shows the nonlinearity of ERSFD and is necessarily used to calculate steady state unbalance response of a rotor system with ERSFD(s). It can be seen that ERSFD is approximately a linear element when the eccentricity ratio of journal is less than 0.7.
3. Dynamic characteristics of rotor system with ERSFD(s)

Finite element method is chosen to calculate dynamic characteristics using equation (17) [11]:

\[ M\ddot{q} + (C - \Omega G)\dot{q} + Kq = Q \]  \hspace{1cm} (17)

where \( M \) is mass matrix of the rotor system, \( C \) is damping matrix, \( G \) is gyroscopic matrix, \( K \) is stiffness matrix, \( q \) is generalized displacement vector, and \( Q \) is external force vector.

When ERSFD is installed on a rotor, it brings nonlinearity to the system as well. Nonlinear stiffness \( K_n \) is in parallel with linear stiffness of bearing \( K_0 \), and so is the damping. Therefore, equivalent stiffness and damping at bearing nodes can be written as:

\[ \begin{align*}
\epsilon' &= C_n + C_0 \\
K' &= K_n + K_0
\end{align*} \hspace{1cm} (18)\]

Consequently, the dynamic response of a rotor can be calculated using equation (17) if the nonlinear stiffness and damping at a certain rotating speed are known. However, just as figure 10 shows, eccentricity of journal has influence on \( K_n \) and \( C_n \), and different eccentricity appears at different rotating speed. So the key part is to calculate eccentricity \( \epsilon \).

Interactive method is adopted to get eccentricity and system response. When rotating speed staying at \( \Omega_i \), directly interaction method that using unbalance response with weak damping as initial value is given priority because of its simplicity and quick computation. Using \( q_0 \) to get equivalent stiffness and damping to form new matrix \( K \) and \( C \), then get new interactive response \( q \) by solving equation (17). When error between \( q \) and \( q_0 \) is less than its margin, process terminates. However, ERSFD may come into its bistable area, which makes aforesaid method dis-convergent and out of operation. Then comes to the method that calculating eccentricity of journal interactively at \( \Omega_i \) firstly and using \( \epsilon \) to get unbalance response. Give \( \epsilon_0 \) a certain value bigger than 0 and smaller than 1, get matrix \( K \) and \( C \) to form equation (17), and examine whether \( \epsilon_0 \) is the root of motion equation. If not, give \( \epsilon_0 \) a new value to interact, otherwise, save the result of \( \epsilon \) or \( \epsilon_0 \) and get the response of the system. Detailed steps are showed in figure 11. By traversing all rotating speeds in solution of internal, dynamic response of the rotor with ERSFD(s) can be calculated.
Calculation of unbalance response with weak damping \( q_0 \)

Start

Rotating speed of calculation \( \Omega_i \)

Calculation of unbalance response with weak damping \( q_0 \)

Calculation of equivalent stiffness \( K \) and damping \( C \)

Calculation of unbalance response with nonlinear damping \( q \)

Yes

\[ ||q-q_0|| < \delta ? \]

No

Converge?

Yes

\( q_0 = q \)

No

Initialization of eccentricity \( e_0 \)

Calculation of unbalance response \( q \) and eccentricity \( e \)

Yes

\[ |e-e_0| < \xi ? \]

No

Save the result of unbalance response \( q \) and eccentricity \( e \)

No

\( \Omega_i \geq \Omega_{max} \)

Yes

End

No

\( \Omega_i \neq \Omega_{i+1} \)

Figure 11. Calculation steps of eccentricity and system response.

4. Experiments on rotor with ERSFDs

In order to evaluate the validity of the arithmetic mentioned in chapter 2 and 3, an experiment system in figure 12 has been established, which is divided into 21 beam elements, 5 disk elements, 2 bearing elements and 1 bifurcate element. There are 2 ERSFDs at bearing 1 and bearing 2 relatively. Use finite element method to calculate dynamic characteristics of rotor system and get results in figure 13 and 14.

(a) Objects of experiment system

(b) Finite element model of experiment system

Figure 12. Experiment system.
**Figure 13.** Campbell map of experiment system.

(a) The first translational mode of vibration  
(b) The second pitching mode of vibration

**Figure 14.** Mode shape of experiment system.

In figure 14, $A_x$ and $A_y$ are dimensionless displacement along X and Y axis. It can be seen that there is first one order critical in operating speed range 0 ~ 4000rpm. Vibration of disk 1 is larger than other disks, which provides obvious view to monitor and analyze the system, so displacement of disk 1 is chosen to verify the algorithm mentioned above. The parameters of ERSFDs used in experiment are listed in table 2, and actual components of ERSFDs are shown in figure 15.

**Table 2.** Structural parameters of experimental ERSFDs.

|        | $R_1$(mm) | $R_2$(mm) | $c_1$ and $c_2$(mm) | $h$(mm) | $L$(mm) | $b_1$ and $b_2$(mm) | $d_1$ and $d_2$(mm) | $n$ | $K_0$(N/m) |
|--------|--------|--------|----------------|------|------|----------------|----------------|----|--------|
| ERSFD 1| 65.5   | 67.5   | 0.3           | 1.4  | 16   | 5              | 0.5              | 10 | 7.18e6 |
| ERSFD 2| 75.5   | 77.7   | 0.3           | 1.6  | 22   | 5              | 0.5              | 10 | 8.62e6 |

**Figure 15.** Components of ERSFDs.

(a) ERSFDs  
(b) Inner lining  
(c) Outer lining

**Figure 16.** Mechanical characteristics of ERSFD 2.

(a) Radial force  
(b) Circumferential force
Mechanism characteristics of ERSFD 1 is illustrated in figure 10, and that of ERSFD 2 in figure 16. Experiments about rotor system with ERSFDs are conducted at around 30°C. There is 77.4g·cm unbalance on disk 1, 54g·cm on disk 4 and 63g·cm on disk 5, and unbalance on disk 4 has opposite phase with the others. The results of experiment and calculation are drawn in figure 17. It can be seen from figure 17 and table 3 that calculation response agrees well with the experimental one, with the biggest relative error on damping ratio $D$ less than 15%. The errors come from several resources. Firstly, the assembling relationships of system components when calculating are regarded to be tight, while they can be loose in practical and lead to errors. Secondly, cooperative relationship of ERSFD components will influent practical film stiffness and damping and result in errors. Thirdly, equation (1) and (2) are deduced under several assumptions, which may exists differences with factual condition.

![Figure 17. Comparison diagram of experiment and calculation.](image1)

Table 3. Parameters comparison of experiment and calculation.

|                      | Peak of vibration $A$ (μm) | Relative error of $A$ (%) | Critical speed $\omega_{cr}$ (rpm) | Relative error of $\omega_{cr}$ (%) | Damping ratio $D$ (%) | Relative error of $D$ (%) |
|----------------------|-----------------------------|---------------------------|-------------------------------------|-------------------------------------|------------------------|--------------------------|
| Experiment           | 379                         | 1.58                      | 2761                                | 4.93                                | 3.6                    | 13.8                     |
| Calculation          | 384.9                       | 2.15                      | 2897                                | 5.6                                 | 3.1                    | 13.5                     |

Figure 18 shows the journal eccentricity ratios calculating result of two ERSFDs. It can be seen that the biggest $\varepsilon$ of ERSFD 1 and ERSFD 2 are around 0.6 and 0.4, where they can still be regarded as linear elements. Figure 19 shows that there is not much difference between two kind of response, which means ERSFDs on the rotor system perform well and can be seen as linear elements at this operating condition.

![Figure 18. Results of eccentricity ratios.](image2)

![Figure 19. Response of linear and nonlinear damping.](image3)

5. Conclusion
In this paper, Theoretical research and experimental verification on steady state unbalance response of rotor system with ERSFD(s) has been carried out. Major conclusions are listed as follows:

1) A method for calculating mechanical characteristics of ERSFD with the influence of orifices taken into consideration is established using finite difference method. The variation of equivalent stiffness and damping with eccentricity ratio with the influence of rotating speed eliminated is obtained, which improves the universality of the results.

2) A method for calculating steady state unbalance response of a rotor system with ERSFD(s) is established using finite element method based on the characteristics curves of ERSFD(s). The correctness of the method is verified by comparing the calculation result and the experimental one.
3) ERSFD has good mechanical characteristics, and can be regarded as linear element with big eccentricity ratio.

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