Peculiarities of sub-barrier reactions with heavy ions

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Abstract. With the quantum diffusion approach the behavior of capture cross sections and mean-square angular momenta of captured systems are revealed in the reactions with deformed and spherical nuclei at sub-barrier energies. The calculated results are in a good agreement with existing experimental data. With decreasing bombarding energy under the barrier the external turning point of the nucleus–nucleus potential leaves the region of short-range nuclear interaction and action of friction. Because of this change of the regime of interaction, an unexpected enhancement of the capture cross section is expected at bombarding energies far below the Coulomb barrier. This effect is shown its worth in the dependence of mean-square angular momentum of captured system on the bombarding energy. From the comparison of calculated capture cross sections and experimental capture (fusion) cross sections, the importance of quasifission near the entrance channel is shown for the actinide-based reactions and reactions with medium-heavy nuclei at sub-barrier energies.

1. Introduction

The measurement of excitation functions down to the extreme sub-barrier energy region is important for studying the long range behavior of nucleus-nucleus interaction, and very little data exist on the fusion, fission and capture cross sections at extreme sub-barrier energies [1–27]. The experimental data obtained are of interest for solving astrophysical problems related to nuclear synthesis. Indications for an enhancement of the $S$-factor, $S = E_{c.m.}\sigma \exp(2\pi\eta)$ [28, 29], where $\eta(E_{c.m.}) = Z_1Z_2e^2\sqrt{\mu/(2\hbar^2E_{c.m.)}}$ is the Sommerfeld parameter, at energies $E_{c.m.}$ below the Coulomb barrier have been found in Refs. [10, 15, 18]. Its origin is still under discussion.

From the comparison of capture cross sections and fusion cross sections one can show a significant role of the quasifission channel in the reactions with various medium-light and heavy nuclei at sub-barrier energies. The competition between the complete fusion and quasifission can strongly reduce the value of the fusion cross section and, respectively, the value of the evaporation residue cross section [30–32]. This effect is especially crucial in the production of superheavy nuclei. It worth remembering that first evidences of hindrance for compound nucleus formation in the reactions with massive nuclei at low energies near the Coulomb barrier were observed at GSI already long time ago [33].

To clarify the behavior of capture cross sections at sub-barrier energies, a further development of the theoretical methods is required [32, 34–36]. The conventional coupled-channel approach with realistic set of parameters is not able to describe the capture cross sections either below
or above the Coulomb barrier [18]. The use of a quite shallow nucleus-nucleus potential [37] with an adjusted repulsive core considerably improves the agreement between the calculated and experimental data. Besides the coupling with collective excitations, the dissipation, which is simulated by an imaginary potential in Ref. [37] or by damping in each channel in Ref. [38], seems to be important. The quantum diffusion approach based on the quantum master-equation for the reduced density matrix has been suggested in Ref. [39]. This model takes into consideration the fluctuation and dissipation effects in collisions of heavy ions which model the coupling with various channels. In the present paper the model [39] is applied.

2. Model

2.1. The nucleus-nucleus potential

In the model [39] the potential describing the interaction of two nuclei is taken in the form [40]

\[ V(R, Z_i, A_i, \theta_i, J) = V_C(R, Z_i, A_i, \theta_i, J) + V_N(R, Z_i, A_i, \theta_i, J) + \frac{h^2 J(J + 1)}{2 \mu R^2}, \]

where \( V_N, V_C \), and the last summand stand for the nuclear, the Coulomb, and the centrifugal potentials, respectively. The nuclei are proposed to be spherical or deformed. The potential depends on the distance \( R \) between the center of mass of two interacting nuclei, mass \( A_i \) and charge \( Z_i \) of nuclei \( (i = 1, 2) \), the orientation angles \( \theta_i \) of the deformed (with the quadrupole deformation parameters \( \beta_i \)) nuclei and the angular momentum \( J \). The static quadrupole deformation parameters are taken from Ref. [41] for the even-even deformed nuclei. For the nuclear part of the nucleus-nucleus potential, we use the double-folding formalism, in the form

\[ V_N = \int \rho_1(r_1)\rho_2(r_2)f(r_1 - r_2)dr_1dr_2, \]

where \( f(r_1 - r_2) = C_0[F_{in}(r_{10}) + F_{ex}(1 - r_{10})]d(r_1 - r_2) \) is the density-dependent effective nucleon-nucleon interaction and \( \rho_0(r) = \rho_1(r) + \rho_2(r) \), \( F_{in} = f_{in} + f_{in}'(\beta_{12}) \), \( F_{ex} = f_{ex} + f_{ex}'(\beta_{12}) \), \( \rho_{10} \), and \( N_i \) are the nucleon densities and neutron numbers of the light and the heavy nuclei of the dinuclear system, respectively. Our calculations are performed with the following set of parameters: \( C_0 = 300 \text{ MeV fm}^3 \), \( f_{in} = 0.09 \), \( f_{ex} = -2.59 \), \( f_{in}' = 0.42 \), \( f_{ex}' = 0.54 \), and \( \rho_{10} = 0.17 \text{ fm}^{-3} \) [40]. The densities of the nuclei are taken in the two-parameter symmetrized Woods-Saxon form with the nuclear radius parameter \( r_0 = 1.15 \text{ fm} \) (for the nuclei with \( A_i \geq 16 \) and the diffuseness parameter \( a \) depending on the charge and mass numbers of the nucleus [40]. We use \( a = 0.53 \text{ fm} \) for the light magic nucleus \( ^{16}\text{O} \), \( a = 0.55 \text{ fm} \) for the intermediate nuclei \( ^{30}\text{Si}, ^{32,34,36}\text{S}, ^{48}\text{Ca}, ^{50}\text{Ti}, ^{64}\text{Ni}, ^{208}\text{Pb} \), and \( a = 0.56 \text{ fm} \) for the actinides. For the \( ^4\text{He} \), nucleus \( r_0 = 1.02 \text{ fm} \) and \( a = 0.48 \text{ fm} \).

The Coulomb interaction of two deformed nuclei has the following form:

\[ V_C(R, Z_i, A_i, \theta_i, J) = \frac{Z_1Z_2e^2}{R} + \left( \frac{9}{20\pi} \right)^{1/2} \frac{Z_1Z_2e^2}{R^3} \sum_{i=1,2} R_i^2 \beta_i \left[ 1 + \frac{2}{7} \left( \frac{5}{\pi} \right)^{1/2} \beta_i \right] P_2(\cos \theta_i), \]

where \( P_2(\cos \theta_i) \) is the Legendre polynomial.

In Fig. 1 there is shown the nucleus-nucleus potential \( V \) for the \( ^{16}\text{O} + ^{238}\text{U} \) reaction (for simplicity, \( ^{258}\text{U} \) is assumed to be spherical) which has a pocket. With increasing centrifugal part of the potential the pocket depth becomes smaller, while the position of the pocket minimum moves towards the barrier at \( R = R_b \approx R_1 + R_2 + 2 \text{ fm} \), where \( R_b = 1.15A_1^{1/3} \) are the radii of colliding nuclei. This pocket is washed out at large angular momenta \( J > 65 \). Thus, only a limited part of angular momenta contributes to the capture process.
2.2. Capture cross section

The capture cross section is a sum of partial capture cross sections $\sigma_{\text{cap}}(E_{\text{c.m.}}, J)$ [39]

$$
\sigma_{\text{cap}}(E_{\text{c.m.}}) = \frac{\pi h^2}{2\mu E_{\text{c.m.}}} \sum_J (2J+1) \int_0^{\pi/2} d\theta_1 \sin \theta_1 \int_0^{\pi/2} d\theta_2 \sin \theta_2 P_{\text{cap}}(E_{\text{c.m.}}, J, \theta_1, \theta_2),
$$

where $\mu = m_0 A_1 A_2/(A_1 + A_2)$ is the reduced mass ($m_0$ is the nucleon mass), and the summation is over the possible values of angular momentum $J$ at a given bombarding energy $E_{\text{c.m.}}$. Knowing the potential of the interacting nuclei for each orientation, one can obtain the partial capture probability $P_{\text{cap}}$ which is defined by the passing probability of the potential barrier in the relative distance $R$ at a given $J$. The value of $P_{\text{cap}}$ is obtained by integrating the propagator $G$ from the

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**Figure 1.** (Upper part) The nucleus-nucleus potentials calculated at $J = 0$ (solid curve), 30 (dashed curve), 50 (dotted curve), and 65 (dash-dotted curve) for the $^{16}\text{O} + ^{238}\text{U}$ reaction. The interacting nuclei are assumed to be spherical in the calculations. (Lower part) The position $R_b$ of the Coulomb barrier, radius of interaction $R_{\text{int}}$, and external $r_{\text{ex}}$ and internal $r_{\text{in}}$ turning points for some values of $E_{\text{c.m.}}$ are indicated at the nucleus-nucleus potential ($J=0$) for the same reaction.
\[ \beta \text{ and dotted lines, respectively. The static quadrupole deformation parameters are: } \beta_{2}(^{238}U)=0.286 \text{ and } \beta_{1}(^{16}O)=\beta_{1}(^{36}S)=0. \]

**Figure 2.** (Upper part) The nucleus-nucleus potentials calculated at \( J=0 \) for the reactions \(^{36}S+^{238}U\) and \(^{16}O+^{238}U\). (Lower part) The dependence of the capture cross section for nuclei colliding with a fixed orientation on \( E_{\text{c.m.}} - V^{\text{orient}} \) where \( V^{\text{orient}} \) is the height of the Coulomb barrier for certain orientations. The results of calculations for the sphere-sphere (the interacting nuclei are spherical), sphere-pole and sphere-side configurations are shown by solid, dashed and dotted lines, respectively. The static quadrupole deformation parameters are: \( \beta_{2}(^{238}U)=0.286 \text{ and } \beta_{1}(^{16}O)=\beta_{1}(^{36}S)=0. \)

The initial state \((R_0, P_0)\) at time \( t = 0 \) to the final state \((R, P)\) at time \( t \) \((P\text{ is a momentum})\) [39]:

\[ P_{\text{cap}} = \lim_{t \to \infty} \int_{-\infty}^{\infty} dR \int_{-\infty}^{\infty} dP G(R, P, t|R_0, P_0, 0) = \lim_{t \to \infty} \frac{1}{2} \text{erfc} \left[ \frac{-r_{\text{in}} + R(t)}{\sqrt{\Sigma_{RR}(t)}} \right]. \] (5)

The second line in (5) is obtained by using the propagator \( G = \pi^{-1} |\det \Sigma|^{-1/2} \exp(-q^T \Sigma^{-1} q) \)

\( q^{T} = [q_{R}, q_{P}], \quad q_{R}(t) = R - R(t), \quad q_{P}(t) = P - P(t), \quad R(t=0) = R_{0}, \quad P(t=0) = P_{0}, \quad \Sigma_{kk'}(t) = 2q_{k}(t)q_{k'}(t), \quad \Sigma_{kk'}(t = 0) = 0, \quad k, k' = R, P \)

calculated for an inverted oscillator which approximates the nucleus-nucleus potential \( V \) in the variable \( R \). The frequency \( \omega \) of this oscillator with an internal turning point \( r_{\text{in}} \) (Fig. 1) is defined from the condition of equality of the classical actions of approximated and realistic potential barriers of the same height at given \( J \). It should be noted that the passage through the Coulomb barrier approximated by a parabola has been previously studied in Refs. [35, 42]. This approximation is well justified for the reactions and energy range, which are considered here. Finally, one can find the expression for the capture probability:

\[ P_{\text{cap}} = \frac{1}{2} \text{erfc} \left[ \left( \frac{\pi s_{1}(\gamma - s_{1})}{2\mu(\omega_{0}^{2} - s_{1}^{2})} \right)^{1/2} \frac{\mu \omega_{0}^{2} R_{0}/s_{1} + P_{0}}{\gamma \ln(s_{1}/s_{2})^{1/2}} \right], \] (6)

where \( \gamma \) is the internal-excitation width, \( \omega_{0}^{2} = \omega^{2}\{1 - h\lambda\gamma/\mu(s_{1} + \gamma)(s_{2} + \gamma)\} \) is the renormalized frequency in the Markovian limit, the value of \( \lambda \) is related to the strength of linear coupling in coordinates between collective and internal subsystems. The \( s_{i} \) are the real roots \((s_{1} \geq s_{2} \geq s_{3})\) of the following equation: \((s + \gamma)(s^{2} - \omega_{0}^{2}) + h\lambda\gamma s/\mu = 0\). The details
Figure 3. The calculated (lines) capture cross section (upper part), average angular momenta of captured system (middle part) versus $E_{c.m.}$, and partial capture cross sections (lower part) versus $J$ at $E_{c.m.}$ = 65 (dash-dot-dotted line), 69.5 (dash-dotted line), 73 (dotted line), 83 (dashed line), and 100 (solid line) MeV for the $^{16}$O+$^{208}$Pb reaction with the spherical nuclei. The experimental cross sections marked by closed squares, circles, rhombus, stars are from Refs. [5, 8, 11, 18], respectively. The experimental values of $\langle J^2 \rangle$ (solid squares) are taken from Ref. [45]. The value of the Coulomb barrier $V_B$ is indicated by arrow.

of the used formalism are presented in [39]. We have to mention that most of the quantum-mechanical, dissipative effects and non-Markovian effects accompanying the passage through the potential barrier are taken into consideration in our formalism [35, 39]. For example, the non-Markovian effects appear in the calculations through the internal-excitation width $\gamma$.

As shown in Fig. 1, the nuclear forces start to play a role at $R_{int} = R_b + 1.1$ fm where the nucleon density of colliding nuclei approximately reaches 10% of the saturation density. If the value of $r_{ex}$ corresponding to the external turning point is larger than the interaction radius $R_{int}$, we take $R_0 = r_{ex}$ and $P_0 = 0$ in Eq. (6). For $r_{ex} < R_{int}$, it is naturally to start our treatment
Figure 4. The calculated (solid lines) capture cross section versus \(E_{\text{c.m.}}\) for the reactions \(^{48}\text{Ca} + ^{208}\text{Pb}\) (upper part) and \(^{34}\text{S} + ^{238}\text{U}\) (lower part). The experimental cross sections are taken from Refs. [46] (closed squares and triangle), [19] (open squares), and [26] (open circles). The value of the Coulomb barrier \(V_b\) is indicated by arrow. The static quadrupole deformation parameters are: \(\beta_1(^{48}\text{Ca})=\beta_2(^{208}\text{Pb})=0, \beta_1(^{34}\text{S})=0.125,\) and \(\beta_2(^{238}\text{U})=0.286.\)

with \(R_0 = R_{\text{int}}\) and \(P_0\) defined by the kinetic energy at \(R = R_0\). In this case the friction hinders the classical motion to proceed towards smaller values of \(R\). If \(P_0 = 0\) at \(R_0 > R_{\text{int}}\), the friction almost does not play a role in the transition through the barrier. So, at \(R < R_{\text{int}}\) the relative motion may be more coupled with other degrees of freedom. At \(R > R_{\text{int}}\) the relative motion is almost independent of the internal degrees of freedom. Thus, two regimes of interaction at sub-barrier energies differ by the action of the nuclear forces and the role of friction at \(R = r_{\text{ex}}\).

3. Calculated results
   Besides the parameters related to the nucleus-nucleus potential, two parameters \(h\gamma=32\) MeV (\(h\gamma=15\) MeV) and the friction coefficient \(h\lambda = -h(s_1 + s_2)=2\) MeV are used for calculating the capture probability in reactions with deformed actinides (spherical lead nucleus). The most
Figure 5. The same as in Fig. 4, but for the reactions for the reactions $^{16}\text{O} + ^{232}\text{Th}$ and $^4\text{He} + ^{238}\text{U}$. The experimental data in the upper part are taken from Refs. [47] (open triangles), [2] (closed triangles), [48] (open squares), [49] (closed squares), [6] (open stars) and [7] (closed stars). The fission cross sections from Refs. [9] and [50] are shown in the lower part by open circles and solid squares, respectively. The value of the Coulomb barrier $V_b$ for the spherical nuclei is indicated by arrow. The dashed curve represents the calculation with the Wong formula. The static quadrupole deformation parameters are: $\beta_2(^{232}\text{Th})=0.261$, $\beta_2(^{238}\text{U})=0.286$ and $\beta_1(^4\text{He})=\beta_1(^{16}\text{O})=0$.

Realistic friction coefficients in the range of $\hbar\lambda \approx 1-2$ MeV are suggested from the study of deep inelastic and fusion reactions [43]. These values are close to those calculated within the mean field approach [44]. All calculated results presented are obtained with the same set of parameters and are rather insensitive to a reasonable variation of them [39].
Figure 6. The same as in Fig. 4, but for the reactions $^{16}\text{O} + ^{238}\text{U}$ and $^{36}\text{S} + ^{238}\text{U}$. The experimental cross sections are taken from Refs. [13] (open triangles), [51] (closed triangles), [6] (open squares), [4] (closed squares), [50] (open stars), [24] (closed stars), and [20] (rhombuses). The dashed curve represents the calculation with the Wong formula. The static quadrupole deformation parameters are: $\beta_2(^{238}\text{U})=0.286$ and $\beta_1(^{16}\text{O})=\beta_1(^{36}\text{S})=0$.

3.1. Effect of orientation
The influence of orientation of the deformed heavy nucleus on the capture process in the reactions $^{36}\text{S} + ^{238}\text{U}$ and $^{16}\text{O} + ^{238}\text{U}$ is studied in Fig. 2. We demonstrate that the capture cross section $\sigma_{\text{cap}}$ at fixed orientation as a function of $E_{\text{c.m.}} - V_{\text{orient}}$, where $V_{\text{orient}}$ is the Coulomb barrier for this orientation, is almost independent of the orientation angle $\theta_2$.

3.2. Comparison with experimental data and predictions
In Figs. 3 - 7 the calculated capture cross sections for the reactions $^{16}\text{O}, ^{48}\text{Ca} + ^{208}\text{Pb}$, $^{16}\text{O}, ^{32}\text{S} + ^{232}\text{Th}$, $^4\text{He}, ^{16}\text{O}, ^{30}\text{Si}$, $^{34,36}\text{S} + ^{238}\text{U}$ are in a rather good agreement with the available experimental data. Because of the uncertainties in the definition of the deformation of the light
Figure 7. The same as in Fig. 4, but for the reactions \( ^{32}\text{S} + ^{232}\text{Th} \) and \( ^{30}\text{Si} + ^{238}\text{U} \). The experimental data are taken from Refs. [22] (solid squares), [16] (solid circles), and [26] (open squares). The static quadrupole deformation parameters are: \( \beta_2(^{238}\text{U})=0.286 \), \( \beta_2(^{232}\text{Th})=0.261 \), \( \beta_1(^{32}\text{S})=0.312 \), and \( \beta_1(^{30}\text{Si})=0.315 \). For the \( ^{30}\text{Si} + ^{238}\text{U} \) reaction, the results of calculations with \( \beta_1(^{30}\text{Si})=0 \) (the predictions of the mean-field and macroscopic-microscopic models) are presented by dashed line in the lower part of the figure.

As seen from Figs. 5 and 6 (dashed lines) the Wong formula [52] does not reproduce the capture cross section at \( E_{\text{c.m.}} < V_b \) even taking into consideration the static quadrupole deformation of target-nucleus.
One can see in Figs. 3 - 10 that there is a sharp fall-off of the cross sections just under the Coulomb barrier corresponding to spherical nuclei. With decreasing $E_{c.m.}$ up to about 3 - 10 MeV (when the projectile is spherical and the target is deformed) and 15 - 20 MeV (when both projectile and target are deformed nuclei) below the Coulomb barrier the regime of interaction is changed because at the external turning point the colliding nuclei do not reach the region of nuclear interaction where the friction plays a role. As result, at smaller $E_{c.m.}$ the cross sections fall with a smaller rate. With larger values of $R_{int}$ the change of fall rate occurs at smaller $E_{c.m.}$. However, the uncertainty in the definition of $R_{int}$ is rather small. Therefore, an effect of the change of fall rate of sub-barrier capture cross section should be in the data if we assume that the friction starts to act only when the colliding nuclei approach the barrier. Note that at energies of 10 - 20 MeV below the barrier the experimental data have still large uncertainties to make a firm experimental conclusion about this effect. The effect seems to be more pronounced in collisions of spherical nuclei, where the regime of interaction is changed at $E_{c.m.}$ up to about 1 - 5 MeV below the Coulomb barrier [39]. The collisions of deformed nuclei occur at various mutual orientations affecting the value of $R_{int}$.

The calculated mean-square angular momenta

$$\langle J^2 \rangle = \frac{\pi \hbar^2}{2 \mu E_{c.m.} \sigma_{cap}} \sum_J J(J + 1)(2J + 1) \int_0^{\pi/2} d\theta_1 \sin \theta_1 \int_0^{\pi/2} d\theta_2 \sin \theta_2 P_{cap}(E_{c.m.}, J, \theta_1, \theta_2)$$

of captured systems versus $E_{c.m.}$ are presented in Figs. 3, 10, and 11 for the reactions $^{16}$O,$^{4}$He+$^{208}$Pb and $^{16}$O + $^{232}$Th,$^{238}$U. At energies below the barrier $\langle J^2 \rangle$ has a minimum. This minimum depends on the deformations of nuclei and on the factor $Z_1 \times Z_2$. For the reactions $^{16}$O + $^{232}$Th,$^{238}$U, these minima are about 7 - 8 MeV below the corresponding Coulomb barriers, respectively. The experimental data [45] indicate the presence of the minimum as well. On the left-hand side of this minimum the dependence of $\langle J^2 \rangle$ on $E_{c.m.}$ is rather weak. A similar weak dependence has been found in Refs. [53] in the extreme sub-barrier region. Note that the found behavior of $\langle J^2 \rangle$, which is related to the change of the regime of interaction between the colliding nuclei, would affect the angular anisotropy of the products of fission-like fragments.
Figure 9. The predicted capture cross sections for the reactions $^{34,36}S + ^{244}\text{Pu}$, $^{248}\text{Cm}$ (dashed lines) and $^{36}S + ^{244}\text{Pu}$, $^{248}\text{Cm}$ (solid lines). The values of the Coulomb barriers for the reactions $^{34,36}S + ^{244}\text{Pu}$, $^{248}\text{Cm}$ are indicated by dotted and solid arrows, respectively. The static quadrupole deformation parameters are: $\beta_2^{(244}\text{Pu})=0.293$, $\beta_2^{(248}\text{Cm})=0.297$, $\beta_1^{(34}\text{S})=0.125$, and $\beta_1^{(36}\text{S})=0$.

following capture. Indeed, the values of $\langle J^2 \rangle$ are extracted from data on angular distribution of fission-like fragments [25].

The agreement between calculated with Wong-type formula and experimental $\langle J^2 \rangle$ is not good. At energies below the barrier $\langle J^2 \rangle$ has no a minimum (see Fig. 11). However, for the considered reactions the saturation values of $\langle J^2 \rangle$ are close to those obtained in our formalism.

3.3. Astrophysical factor, L-factor and barrier distribution

In Figs. 10 and 12 the calculated astrophysical $S$–factors versus $E_{c.m}$. are shown for the reactions $^4\text{He}+^{208}\text{Pb}$ and $^{16}\text{O}+^{238}\text{U}$. The $S$-factor has a maximum which is seen in experiments [10,15,37]. After this maximum $S$-factor slightly decreases with decreasing $E_{c.m}$. and then starts to increase. This effect seems to be more pronounced in collisions of spherical nuclei. The same behavior has been revealed in Refs. [54] by extracting the $S$-factor from the experimental data.
In Fig. 12, the so-called logarithmic derivative, \( L(E_{c.m.}) = d(\ln(E_{c.m.}\sigma_{cap})) / dE_{c.m.} \), and the barrier distribution \( d^2(E_{c.m.}\sigma_{cap}) / dE_{c.m}^2 \) are presented for the \(^{16}\text{O} + ^{238}\text{U}\) reaction. The logarithmic derivative strongly increases below the barrier and then has a maximum at \( E_{c.m.} \approx V_b^{\text{orient}}(\text{sphere-pole}) - 3\text{ MeV} \) (at \( E_{c.m.} \approx V_b - 3\text{ MeV} \) for the case of spherical nuclei). The maximum of \( L \) corresponds to the minimum of the \( S \)-factor.

In Fig. 12, the barrier distributions calculated with an energy increment 0.2 MeV have only one maximum at \( E_{c.m.} \approx V_b^{\text{orient}}(\text{sphere-sphere}) = V_b \) as in the experiment [55]. With an increasing increment the barrier distribution is shifted to lower energies. Assuming a spherical target nucleus in the calculations, we obtain a more narrow barrier distribution.

**Figure 10.** The calculated capture cross section (upper part), average angular momenta of compound nucleus (middle part) and astrophysical \( S \)-factor (lower part) with \( \eta_0 = \eta(E_{c.m.} = V_b) \) versus \( E_{c.m.} \) for the \(^4\text{He} + ^{208}\text{Pb}\) reaction.
Figure 11. The calculated mean-square angular momenta versus $E_{\text{c.m.}}$ for the reactions $^{16}\text{O} + ^{232}\text{Th}, ^{238}\text{U}$ are compared with experimental data [6]. The dashed curve represents the calculation by the Wong-type formula.

3.4. Capture cross sections in reactions with large fraction of quasifission

In the case of large values of $Z_1 \times Z_2$ the quasifission process competes with complete fusion at energies near barrier and can lead to a large hindrance for fusion, thus ruling the probability for producing superheavy elements in the actinide-based reactions [31, 56]. Since the sum of the fusion cross section $\sigma_{\text{fus}}$, and the quasifission cross section $\sigma_{\text{qf}}$ gives the capture cross section,

$$\sigma_{\text{cap}} = \sigma_{\text{fus}} + \sigma_{\text{qf}},$$

and $\sigma_{\text{fus}} \ll \sigma_{\text{qf}}$ for actinide-based reactions $^{48}\text{Ca} + ^{232}\text{Th}, ^{238}\text{U}, ^{244}\text{Pu}, ^{246,248}\text{Cm}$ and $^{50}\text{Ti} + ^{244}\text{Pu}$ [31], we have

$$\sigma_{\text{cap}} \approx \sigma_{\text{qf}}.$$

In a wide mass-range near the entrance channel, the quasifission events overlap with the products of deep-inelastic collisions and can not be firmly distinguished. Because of this the
Figure 12. The calculated values of the astrophysical S-factor with $\eta_0 = \eta(E_{c.m.} = V_b)$ (middle part), the logarithmic derivative $L$ (upper part) and the fusion barrier distribution $d^2(E_{c.m.}\sigma_{cap})/dE_{c.m.}^2$ (lower part) for the $^{16}\text{O}+^{238}\text{U}$ reaction. The value of $L$ calculated with the assumption of $\beta_{1}(^{16}\text{O})=\beta_{2}(^{238}\text{U})=0$ is shown by a dashed line. The solid and dotted lines show the values of $d^2(E_{c.m.}\sigma_{cap})/dE_{c.m.}^2$ calculated with the increments 0.2 and 1.2 MeV, respectively. The closed squares are the experimental data of Ref. [55].

mass region near the entrance channel is taken out in the experimental analyses of Refs. [57,58]. Thus, by comparing the calculated and experimental capture cross sections one can study the importance of quasifission near the entrance channel for the actinide-based reactions leading to superheavy nuclei.

The capture cross sections for the quasifission reactions [57–59] are shown in Figs. 13-15. One can observe a large deviations of the experimental data of Refs. [57,58] from the calculated results. The possible reason is an underestimation of the quasifission yields measured in these reactions. Thus, the quasifission yields near the entrance channel are important. Note that
Figure 13. The same as in Fig. 4, but for the $^{48}\text{Ca} + ^{232}\text{Th}, ^{238}\text{U}$ reactions. The excitation energies $E_{CN}^*$ of the corresponding nuclei are indicated. The experimental data are taken from Refs. [57] (marked by squares) and [59] (marked by circles). The static quadrupole deformation parameters are: $\beta_2(^{238}\text{U}) = 0.286$, $\beta_2(^{232}\text{Th}) = 0.261$, and $\beta_1(^{48}\text{Ca}) = 0$.

There are the experimental uncertainties in the bombarding energies.

One can see in Fig. 16 that the experimental and the theoretical cross sections become closer with increasing bombarding energy. This means that with increasing bombarding energy the quasifission yields near the entrance channel mass-region decrease with respect to the quasifission yields in other mass-regions. As seen in Fig. 16, the quasifission yields near the entrance channel mass-region increase with $Z_1 \times Z_2$. 
Figure 14. The same as in Fig. 13, but for the indicated $^{48}\text{Ca},^{50}\text{Ti} + ^{244}\text{Pu}$ reactions. The experimental data are from Refs. [58] (squares) and [57] (circles). The static quadrupole deformation parameters are: $\beta_2(^{244}\text{Pu})=0.293$, and $\beta_1(^{48}\text{Ca})=\beta_1(^{50}\text{Ti})=0$.

4. Origin of fusion hindrance in reactions with medium-mass nuclei at sub-barrier energies

In Figs. 17 and 18 the calculated capture cross section are presented for the reactions $^{36}\text{S} + ^{48}\text{Ca}, ^{64}\text{Ni}$ and $^{64}\text{Ni} + ^{64}\text{Ni}$. The values of $V_b$ are adjusted to the experimental data for the fusion cross sections shown as well. For the systems mentioned above, the difference between the sub-barrier capture and fusion cross sections becomes larger with decreasing bombarding energy $E_{c.m.}$. Assuming that the estimated capture and the measured fusion cross sections are correct, the small fusion cross section at energies well below the Coulomb barrier may indicate that other reaction channel is open and the system evolves by other reaction mechanism after the capture. The observed hindrance factor may be understood in term of quasifission whose cross section should be added to the one of fusion to obtain a meaningful comparison with
Figure 15. The same as in Fig. 13, but for the reactions $^{48}$Ca + $^{246,248}$Cm. The experimental data are from Refs. [57] (squares) and [58] (circles). The static quadrupole deformation parameters are: $\beta_2^{(246}\text{Cm})=0.298$, $\beta_2^{(248}\text{Cm})=0.297$, and $\beta_1^{(48}\text{Ca})=0$.

the calculated capture cross section. The quasifission event corresponds to the formation of a nuclear-molecular state or dinuclear system with small excitation energy that separates (in the competition with the compound nucleus formation process) by the quantal tunneling through the Coulomb barrier in a binary event with mass and charge close to the entrance channel. In this sense the quasifission is the general phenomenon which takes place in the reactions with the massive [30–33], medium-mass and, probably, light nuclei.

From the present analysis an unambiguous signature of the role of quasifission (binary decay) channel could not be inferred. These reaction mode has to be studied in the future experiments: from the mass (charge) distribution measurements one can show the distinct components due to quasifission. The low-energy experimental data would probably provide straight information since the high-energy data may be shaded by competing reaction processes such as quasifission.
Figure 16. The ratio of theoretical and experimental capture cross sections versus the excitation energy $E_{\text{c.m.}}$ of the compound nucleus for the reactions $^{48}\text{Ca} + ^{238}\text{U}$ (closed stars), $^{48}\text{Ca} + ^{244}\text{Pu}$ (closed triangles), $^{48}\text{Ca} + ^{246}\text{Cm}$ (closed squares), $^{48}\text{Ca} + ^{248}\text{Cm}$ (closed circles), and $^{50}\text{Ti} + ^{244}\text{Pu}$ (closed rhombuses).

and deep inelastic collisions. More experimental and theoretical studies of sub-barrier fusion hindrance are needed to improve our understanding of quasifission process, which may be especially important in astrophysical fusion reactions. Note that the binary decay events were already observed experimentally in [61] for the $^{58}\text{Ni} + ^{124}\text{Sn}$ reaction at energies below the Coulomb barrier but assumed to be related to deep-inelastic scattering.

5. Summary

The quantum diffusion approach is applied to study the capture process in the reactions with deformed nuclei at sub-barrier energies. The available experimental data at energies above and below the Coulomb barrier are well described, showing that the static quadrupole deformations of the interacting nuclei are the main reasons for the capture cross section enhancement at sub-barrier energies. Since the deformations of the interacting nuclei mainly influence the slope of curve at $E_{\text{c.m.}} < V_b$ and one can extract the ground state deformation of projectile or target from the experimental capture cross section data.

Due to a change of the regime of interaction (the turning-off of the nuclear forces and friction) at sub-barrier energies, the curve related to the capture cross section as a function of bombarding energy has smaller slope. This change is also reflected in the functions $\langle J^2 \rangle$, $L(E_{\text{c.m.}})$, and $S(E_{\text{c.m.}})$. The mean-square angular momentum of captured system versus $E_{\text{c.m.}}$ has a minimum and then saturates at sub-barrier energies. This behavior of $\langle J^2 \rangle$ would increase the expected anisotropy of the angular distribution of the products of fission and quasifission following capture. The astrophysical factor has a maximum and a minimum at energies below the barrier. The maximum of $L$-factor corresponds to the minimum of the $S$-factor. One can suggest the experiments to check these predictions.

The importance of quasifission near the entrance channel is shown for the actinide-based reactions and reactions with medium-mass nuclei at sub-barrier energies.
Figure 17. The calculated capture cross sections versus $E_{c.m.}$ for the indicated reactions. The experimental fusion cross sections marked by closed and open squares are taken from Refs. [21, 27], respectively. The values of the Coulomb barrier are indicated by arrows. The static quadrupole deformation parameters are: $\beta_1^{(36\text{S})}=\beta_2^{(48\text{Ca})}=0$ and $\beta_2^{(64\text{Ni})}=0.087$ [60].

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Figure 18. The calculated capture cross sections versus $E_{\text{c.m.}}$ for the indicated reaction. The experimental fusion cross sections marked by closed squares and circles are taken from Refs. [1, 14], respectively. Here, $\beta_{1,2}(^{64}\text{Ni})=0.087.$

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