Magnetic Moment Measurement Method Based on Magnetic Sensor Array

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Abstract. The magnetic moment measurement method based on magnetic sensor array is proposed to solve the problems of cumbersome calculation process and low inversion accuracy in the current magnetic moment calculation methods. Based on the concepts of magnetic field intensity, magnetic gradient tensor and Frobenius norm, this method can use the magnetic field information collected by magnetic sensor array to calculate the magnetic moment vector. The factors that may affect the measurement effect, such as sensor array structure, sensor precision and baseline size, are analyzed by simulation, and the advantages and disadvantages of three magnetic moment measurement methods based on magnetic sensor array are compared. Simulation results show that the proposed method is flexible in use, convenient in measurement, accurate in results, and has strong feasibility and reliability in geological survey and other applications.

Keywords: Magnetic anomaly detection, magnetic moment, sensor array.

1. Introduction

Magnetic anomaly detection technology is widely used in geological survey, deformation monitoring, unexploded ordnance detection and other scenarios, [1] which has extremely high civilian and military significance. The obvious magnetic field anomaly of a magnetic target is the premise of magnetic anomaly detection, and the most intuitive physical quantity to describe the magnetic characteristics of a target is magnetic moment. When magnetic anomaly detection technology is used in geological survey or mine clearance, a series of deviations caused by magnetic moment measurement are likely to cause serious and irreversible accidents.

This paper proposes a magnetic moment measurement method based on magnetic sensor array. This method is based on the magnetic dipole model and uses the concepts of magnetic field strength, magnetic gradient tensor and Frobenius-norm to calculate the magnetic moment vector. The proposed method is flexible in use, convenient in measurement, and accurate in results. It provides a theoretical basis and empirical reference for the deeper application of magnetic anomaly data such as inversion and recognition of magnetic targets and 3D reconstruction. [2]
2. Magnetic moment measurement method based on magnetic sensor array

Magnetic moment is an important indicator of ferromagnetic materials, and the magnetic moment of each ferromagnetic material is different. The magnetic moment can be written as the product of the magnetic moment modulus $m$ and the magnetic moment direction vector $\mathbf{m}$, or it can be decomposed into orthogonal three components $(m_x, m_y, m_z)$ according to the coordinate system direction.

When the measuring distance is more than 2.5 times the size of the measured object, the magnetic dipole model can be used to replace magnetic objects with a relatively regular shape. [3]

Since it is difficult to directly measure the magnetic field gradient, we combine multiple vector magnetic sensors into a specific structure, and use the difference of the sensor's short-distance baseline magnetic vector to approximate the second partial differential of the magnetic scalar potential to achieve magnetic gradient tensor component measurement. Therefore, the magnetic sensor array can not only measure the magnetic field intensity of the measuring point, but also measure the magnetic gradient tensor value at the measurement center point. The following will introduce three methods to calculate the magnetic moment using the data measured by the magnetic sensor array.

2.1. Calculation of magnetic moment based on magnetic field strength

The distribution of the dipole magnetic field is related to the magnetic moment and position, so the magnetic field distribution of the dipole with a known position is only related to the magnetic moment. However, due to the existence of the earth's magnetic field, the measured magnetic field information also contains the value of the earth's magnetic field. If it is directly calculated, the error value will be large.

Assuming that the geomagnetic field at the measurement point is $\mathbf{B}_0 = (B_{0x}, B_{0y}, B_{0z})$, the dipole magnetic field at the measurement point is $\mathbf{B}_d$, and the measurement result is $\mathbf{B}_m$, then

$$\mathbf{B}_m = \mathbf{B}_d + \mathbf{B}_0 .$$

So, the following formula can be obtained:

$$\begin{align*}
B_{mx} &= \frac{\mu_0}{4\pi} \cdot \frac{(3x^2 - r^2)m_x + 3xy m_y + 3xz m_z}{r^5} + B_{0x} \\
B_{my} &= \frac{\mu_0}{4\pi} \cdot \frac{3xy m_x + (3y^2 - r^2)m_y + 3yz m_z}{r^5} + B_{0y} \\
B_{mz} &= \frac{\mu_0}{4\pi} \cdot \frac{3xz m_x + 3yz m_y + (3z^2 - r^2)m_z}{r^5} + B_{0z}
\end{align*}$$

Using the measurement data of two or more sensors in the array, the magnetic moment and the geomagnetic field vector value can be solved.

2.2. Calculation of magnetic moment based on magnetic gradient tensor

The magnetic gradient tensor is a second-order tensor composed of the rate of change of the three components of the magnetic field vector in the three directions of the coordinate system. [3] It contains nine components in total and is a real symmetric and traceless matrix.

The geomagnetic field intensity is very large, but the gradient value is small, which can be ignored in a small measurement area.

Assuming that the magnetic dipole with the magnetic moment $\mathbf{m} = (m_x, m_y, m_z)$ is located at the origin of the coordinate system, the five independent elements of the magnetic gradient tensor of the measurement point $\mathbf{P}$ at $\mathbf{r} = (x, y, z)$ can be expressed as:
\[
B_{xx} = \frac{\mu_0}{4\pi} \cdot \frac{9r^2-15x^2}{r^7} m_x + \frac{3y^2-15x^2y}{r^7} m_y + \frac{3z^2-15x^2z}{r^7} m_z \\
B_{xy} = \frac{\mu_0}{4\pi} \cdot \frac{3y^2-15x^2y}{r^7} m_x + \frac{3x^2-15xy^2}{r^7} m_y + \frac{(-15xyz)}{r^7} m_z \\
B_{yy} = \frac{\mu_0}{4\pi} \cdot \frac{3x^2-15xy^2}{r^7} m_x + \frac{3y^2-15y^2z}{r^7} m_y + \frac{3z^2-15y^2z}{r^7} m_z \\
B_{zx} = \frac{\mu_0}{4\pi} \cdot \frac{-15xyz}{r^7} m_x + \frac{3z^2-15yz^2}{r^7} m_y + \frac{(3x^2-15y^2z)}{r^7} m_z \\
B_{zy} = \frac{\mu_0}{4\pi} \cdot \frac{3x^2-15y^2z}{r^7} m_x + \frac{(3z^2-15yz^2)}{r^7} m_y + \frac{(-15xyz)}{r^7} m_z
\]

Substituting the magnetic gradient tensor value measured by the magnetic sensor array into the above formula can solve the magnetic moment.

2.3. Calculation of magnetic moment based on Frobenius norm

The Frobenius norm \( C_T \) of the magnetic gradient tensor matrix is:

\[
C_T = \sqrt{\sum B_{ij}^2}
\]

According to the properties of the matrix, the Frobenius-norm of the matrix \( G \) is an invariant. The \( B_{ij} \) in the above formula represents the change rate of \( B_i \) in the \( j \) direction. Therefore, the Frobenius-norm value can be obtained through the measurement data of the magnetic sensor array. In addition, the relationship between \( C_T \) and magnetic moment is as follows:

\[
C_T = \frac{3\mu_0 m}{4\pi r^7} \sqrt{4(\cos \phi)^2 + 2}.
\]

\( \phi \) is the angle between the magnetic moment vector and the position vector. The above formula and the following formula can be combined to obtain the magnetic moment value:

\[
\begin{cases}
\mathbf{r}_o \cdot \mathbf{m}_o = \cos \phi \\
\mathbf{V}_2 \cdot \mathbf{m}_o = 0 \\
\sqrt{m_{ox}^2 + m_{oy}^2 + m_{oz}^2} = 1
\end{cases}
\]

In the above formula, \( m_{ox}, m_{oy}, m_{oz} \) are the three-axis components of the magnetic moment direction vector \( \mathbf{m}_o \) in the coordinate system.

3. Simulation and analysis

In order to compare the advantages and disadvantages of the above magnetic moment calculation methods, the following simulation analysis is carried out for factors that may affect the accuracy of magnetic moment measurement, such as the magnetic sensor array structure, the accuracy of the magnetic sensor, and the baseline length of the magnetic sensor array. Assuming that the magnetic target is at the origin of the coordinate system, the magnetic moment modulus \( m \) is 1000 A·m², and the magnetic moment direction vector is \( \mathbf{m}_o = (0.35, 0.35, 0.87) \).

3.1. Magnetic sensor array structure

Common magnetic sensor arrays have rectangular tetrahedral, rectangular, and cross structures. The magnetic sensor array with the above structure is used for magnetic field measurement, assuming that the measurement system has been calibrated, and there is no sensor measurement error and sensor installation error. In this section, different types of magnetic sensor array structures are used to solve the magnetic moment using the three methods in the previous chapter. Set the sensor accuracy to 0.1nT and the array baseline distance to 0.2m. The measurement range of the center position of the magnetic sensor array is from (-10m, 0m, 0m) to (10m, 0m, 0m). The attitude is kept unchanged during the movement,
and the sampling interval is 0.3m. Compare the effect of different sensor array structures on the magnetic moment solution.

![Figure 1. Simulation of magnetic sensor array structure](image)

Figure 1. Simulation of magnetic sensor array structure

It can be seen from the results that all three sensor array structures can achieve full tensor field measurement of magnetic anomalies within a certain accuracy range. Under the same field source, sensor accuracy and baseline size, the plane cross-shaped and rectangular structures have higher accuracy. In addition, the plane cross-shaped structure is simple, easy to install, and convenient to calibrate sensor system errors and non-alignment errors between arrays, which is more practical.

Comparing three magnetic moment calculation methods based on magnetic sensor array data. the method based on magnetic field strength depends on sensor data. In the near field, the calculation effect is better, but as the distance increases and the quality of the magnetic measurement data decreases, the calculation effect will become worse. The method based on the magnetic gradient tensor is directly related to the structure of the magnetic sensor array, and an ideal magnetic moment calculation effect can be obtained in the cross and rectangular structures. The method based on Frobenius-norm is widely used in the scene of measuring system attitude transformation, because of its invariant.

In summary, the processing method for the magnetic sensor array data can be selected according to actual specific needs.

3.2. Magnetic sensor accuracy and array baseline
This section takes the cross-shaped magnetic sensor array as an example. In the simulation, the baseline of the structure is 0.02 m to 3.5 m, and the sampling interval is 0.05 m. Simulate the calculation of the magnetic moment when the sensor accuracy is 1 nT, 0.1 nT.
It can be seen from the results that the accuracy of the magnetic sensor and the array baseline have an impact on the results of the three methods. Under the same field source, sensor array structure and baseline size, the higher the sensor accuracy, the better the inversion effect. Under the conditions of the same field source, sensor array structure and sensor accuracy baseline size, the larger the baseline size, the greater the error of the result. But it does not mean that a small baseline size must be good. In fact, the smaller the baseline size, the more susceptible to other factors, so the most suitable baseline size should be selected according to the actual application.

4. Conclusions
This paper proposes a magnetic moment measurement method based on magnetic sensor array, and optimizes the problems of inflexible use of the existing methods, low accuracy, and large influence by the earth's magnetic field. Based on the concepts of magnetic field intensity, magnetic gradient tensor and Frobenius norm, this method can use the magnetic field information collected by magnetic sensor array to calculate the magnetic moment vector. Simulation results show that the proposed method is flexible in use, convenient in measurement, accurate in results, and has strong feasibility and reliability in geological survey and other applications. It provides a theoretical basis and empirical reference for the deeper application of magnetic anomaly data such as inversion and recognition of magnetic targets and 3D reconstruction.

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