Erosion of globular cluster systems: the influence of radial anisotropy, central black holes and dynamical friction

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ABSTRACT
We present the adaptable MUESLI code for investigating dynamics and erosion processes of globular clusters (GCs) in galaxies. MUESLI follows the orbits of individual clusters and applies internal and external dissolution processes to them. Orbit integration is based on the self-consistent field method in combination with a time-transformed leapfrog scheme, allowing us to handle velocity-dependent forces like triaxial dynamical friction. In a first application, the erosion of GC systems (GCSs) in elliptical galaxies is investigated. Observations show that massive ellipticals have rich, radially extended GCSs, while some compact dwarf ellipticals contain no GCs at all. For several representative examples, spanning the full mass scale of observed elliptical galaxies, we quantify the influence of radial anisotropy, galactic density profiles, supermassive black holes and dynamical friction on the GC erosion rate. We find that GC number density profiles are centrally flattened in less than a Hubble time, naturally explaining observed cored GC distributions. The erosion rate depends primarily on a galaxy’s mass, half-mass radius and radial anisotropy. The fraction of eroded GCs is nearly 100 per cent in compact, M32-like galaxies and lowest in extended and massive galaxies. Finally, we uncover the existence of a violent tidal-disruption-dominated phase which is important for the rapid build-up of halo stars.

Key words: methods: numerical – galaxies: elliptical and lenticular, cD – galaxies: nuclei – galaxies: star clusters: general.

1 INTRODUCTION
Globular clusters (GCs) are among the oldest objects in galaxies. They can provide a wealth of information on the formation and evolution histories of their host galaxies as well as on cosmological structure formation (Searle & Zinn 1978; Marks & Kroupa 2010; Harris, Harris & Alessi 2013). This paper is a first step in connecting and understanding properties of GC systems, their host galaxies and their central supermassive black holes (SMBHS).

Surveys of elliptical galaxies show radial GC profiles to be less concentrated when compared to the galactic stellar light profiles (Harris & Racine 1979; Forbes et al. 1996; McLaughlin 1999; Capuzzo-Dolcetta & mastrobuno-Battisti 2009 and references therein). An impressive example is the 10 kpc core in the spatial distribution of GCs in NGC 4874, one of the two dominant elliptical galaxies inside the Coma cluster (Peng et al. 2011). Two competing scenarios attempt to explain these cored distributions. In one scenario, the core originated from processes operating at the onset of galaxy formation in the very early Universe (Harris 1986, 1993). The other scenario assumes that GCs were formed co-evally with the field stars with a cuspy distribution, i.e. comparable to the galactic stellar light profile. In this second scenario, the observed cores in the GC distributions are caused by the subsequent erosion and destruction of GCs in the nucleus of the galaxy itself (Capuzzo-Dolcetta 1993; Baumgardt 1998; Vesperini et al. 2003). It is this scenario we would like to shed light on with this study.

However, taking all the relevant processes that affect the GC erosion rates in elliptical galaxies into account is numerically challenging. This is due to the fact that there are several internal and external processes acting simultaneously on the dissolution of GCs, such as two-body relaxation, stellar mass-loss and tidal shocks (Gnedin & Ostriker 1997; Vesperini & Heggie 1997; Fall & Zhang 2001; Baumgardt & Makino 2003; Gieles et al. 2006). In this study, we

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present a new code named MUESLI to investigate several processes that dominate cluster erosion: (i) tidal shocks on eccentric GC orbits and relaxation-driven dissolution, and their dependence on the anisotropy profile of the GC population, (ii) tidal destruction of GCs due to a central SMBH, (iii) stellar evolution (SEV) and (iv) orbital decay through dynamical friction (DF). That is:

(i) GCs lose mass when stars get beyond the limiting Jacobi radius, $r_J$, and become unbound to the cluster. Two-body relaxation will cause any GC to dissolve with time. The dissolution time depends on the mass and extent of the GC as well as the strength of the tidal field (Baumgardt & Makino 2003). GCs on very eccentric orbits are particularly susceptible for disintegration within a few orbits owing to the strong tidal forces near the galactic centre. In radially biased velocity distributions, large fractions of orbits are occupied by such eccentric, i.e. low angular momentum orbits, and the overall destruction rate of GCs is strongly enhanced over the isotropic case. The same holds for triaxial galaxies when GCs move on box orbits (Ostriker, Binney & Saha 1989; Capuzzo-Dolcetta & Tesserì 1997; Capuzzo-Dolcetta & Vicari 2005).

(ii) The gradient of the potential which is relevant for the destruction of GCs is increased by the presence of SMBHs. SMBHs are commonly found in the cores of luminous galaxies (Mogliari et al. 1998; Lauer et al. 2007a) and the connection between SMBHs and GCs is of particular interest. Burkert & Tremaine (2010) and Harris & Harris (2011) found empirical relations between the total number of GCs and the mass of the central black hole. The origin of this linear $M_\star - N_{GC}$ relation is under debate. See Harris, Poole & Harris (2014) for a most recent version of the $M_\star - N_{GC}$ relation and comparison to other GC/host galaxy relations. There is some evidence that $M_\star$ and $N_{GC}$ are indirectly coupled over the properties of their host galaxies (Rhode 2012); however, a direct causal link cannot be ruled out owing to the difficulty of studying the growth of SMBHs from accreted cluster debris.

(iii) Another effect is mass-loss by SEV. SEV decreases the GC mass most significantly during an initial phase of roughly 100 Myr. In this period, O and B stars lose most of their mass through stellar winds and supernovae (e.g. de Boer & Seggewiss 2008). Over a Hubble time, a stellar population loses about 30–40 per cent of its mass due to SEV (Baumgardt & Makino 2003).

(iv) Finally, massive objects like GCs lose energy and angular momentum due to DF when migrating through an entity of background particles. GCs will gradually approach the centre of the galaxy where they are destroyed efficiently as described above. In low-luminosity spheroids ($L \sim 10^{10} L_\odot$), decaying GCs might also merge together and contribute to the growth of nuclear star clusters (Tremaine, Ostriker & Spitzer 1975; Agarwal & Milosavljević 2011; Antonini 2013; Gnedin, Ostriker & Tremaine 2014). Among other quantities, the efficiency of DF depends on the departure of the host galaxy from spherical symmetry (Peñarrubia, Just & Kroupa 2004), and becomes largest for low angular momentum orbits (Pesce, Capuzzo-Dolcetta & Vietri 1992; Capuzzo-Dolcetta & Vicari 2005).

Like a real muesli, our Multi-Purpose Elliptical Galaxy SCF + Time-Transformed Leapfrog Integrator (MUESLI) consists of several well-chosen ingredients. MUESLI has a high flexibility and is designed for computing GC orbits and erosion rates in live galaxies. It can handle spherical, axisymmetric and triaxial galaxies with arbitrary density profiles, velocity distributions and central SMBH masses for which no analytical distribution functions exist. Since the potential of the galaxy is computed self-consistently, the code can handle time evolving potentials due to e.g. the interaction of the galaxy and a central black hole (Merritt & Quinlan 1998) or even non-virialized structures.

MUESLI is designed to constrain the field star and GC formation efficiencies in the early Universe. This can be done by relating the computational outcomes with observations of the GC-specific frequency, $S_N$, which is the number of observed GCs normalized to total mass/luminosity of the host galaxy (Georgiev et al. 2010; Harris et al. 2013; Wu & Kroupa 2013). The U-shaped $S_N$ distribution, being highest for the least massive and most massive galaxies, traces the impact of feedback processes operating in different galactic environments. However, the quantitative examination of these processes requires knowledge about the total fraction of GCs eroded over time.

In this first paper, we provide detailed information about the code and about $N$-body model generation, and we show results from the code testing. We apply our code to erosion processes of GCs inside spherical galaxies with Hernquist and Sérics profiles, isotropic and radially biased velocity distributions and central SMBHs. This is done for four representative galaxies. These galaxies cover a wide range of masses ($M_{\text{gal}} \approx 10^7–10^{12} M_\odot$), sizes ($R_e \approx 10^2–10^3$pc) and central SMBH masses ($M_* \approx 10^6–10^9 M_\odot$). Erosion rates in axisymmetric and triaxial galaxies, as well as nuclear star cluster and SMBH growth processes by cluster debris are reserved for later publications.

This paper is organized as follows. The MUESLI code and the dynamics governing GC dissolution and disruption processes are specified in Section 2. At the end of this section, we introduce the initial conditions of the GCs and discuss the generation of the underlying galaxy models. Results are presented in Section 3, followed by a critical discussion (Section 4). The main findings are summarized in Section 5. Extensive tests of the code are carried out in Appendix A.

### 2 METHOD

In the following, we briefly describe the main ingredients of MUESLI. These ingredients can easily be modified, exchanged or upgraded, making MUESLI a versatile platform for the study of GC dynamics in elliptical galaxies and related problems.

#### 2.1 SCF integration method and scaling issues

The computations are performed with the self-consistent field (SCF) method (Hernquist & Ostriker 1992). The SCF algorithm uses a basis-function approach to evaluate an expression for the potential $\phi$ from the underlying matter configuration. The orders $n$, $l$ of the radial and angular expansion terms can be adjusted to match the type of galaxy. In the underlying study, we restrict ourselves to spherically symmetric galaxies. The usage of $l > 0$ in spherical galaxy models can result in inhomogeneities and unphysical drifts of the angular momentum vectors. Therefore, $n = 30$, $l = 0$ is adopted for the main computation of the spherical galaxies of this study, while $n = l \geq 10$ is chosen for axisymmetric and triaxial galaxies. Tests are performed in Appendices A1 and A2.

The particle trajectories are integrated forward in time with the time-transformed leapfrog (TTL) scheme (Mikkola & Aarseth 2002) combined with an iteration method to account for the inclusion of external (velocity-dependent) forces. These forces are the DF force (Section 2.2.3) and post-Newtonian forces allowing us to mimic generic relativistic effects arising from the central SMBH. Post-Newtonian terms are not relevant for GC destruction processes as they occur on distances much larger than event horizon scales.
where GR effects are negligible and so we do not consider them for this study.

While the code uses conventional model units, $M_{\text{GAL}} = R_{\text{HI}} = G = 1$, the relevant GC quantities (Section 2.3.1) as well as the DF force (Section 2.2.3) are defined in physical dimensions. The scaling of time, mass and size is performed during computations.

### 2.2 GC dissolution mechanisms

We aim at quantifying the relevance of internal and external effects like SEV (Section 2.2.1), relaxation driven evolution of GCs in tidal fields (Section 2.2.1), tidal disruption through shocks (Section 2.2.2) and DF (Section 2.2.3) for shaping a cored GC distribution.

#### 2.2.1 SEV and two-body relaxation

SEV reduces the cluster mass most significantly during an initial phase of roughly 100 Myr. During this period, the most massive stars lose mass through stellar winds and supernovae (e.g. de Boer & Seggewiss 2008).

We implemented the combined scheme for SEV and energy equipartition-driven evaporation in tidal fields from Baumgardt & Makino (2003).

Compared to the long-term dynamical evolution, SEV decreases the initial cluster mass nearly instantaneously by 30 per cent (Baumgardt & Makino 2003, their fig. 1). Therefore, initial SEV can be taken into account by using a constant GC mass correction factor of 0.70 (Baumgardt & Makino 2003, their equation 12).

On the other hand, two-body relaxation causes a more continuous mass-loss (Hénon 1961; Baumgardt & Makino 2003; Heggie & Hut 2003). Due to two-body encounters, stars gain enough energy so that they can leave the cluster. In the long term, this process leads to the dissolution of any star cluster. The process of relaxation-driven mass-loss is accelerated when GCs are embedded in the external tidal field of a host galaxy as the potential barrier for escape is lowered (Chernoff & Weinberg 1990; Baumgardt & Makino 2003). In this case, a star may separate from the GC when passing beyond a characteristic radius, commonly known as the Jacobi radius, $r_J$ (e.g. King 1962; Spitzer 1987). When the cluster moves on a non-circular orbit the tidal field strength varies with time, and so does the Jacobi radius. With growing eccentricity of the cluster’s orbit, the extremal distances are increasing and the Jacobi radius is increasing (see e.g. Küpper et al. 2010b; Webb et al. 2013).

Baumgardt & Makino (2003) suggests that the time-dependent mass-loss rate of GCs in tidal fields can be approximated by a linear function. A few modifications were added to account for arbitrary galactic density profiles and changing GC orbits caused by DF. This is done by first computing the galactocentric distance, $r_G$, and velocity, $v_G = \sqrt{a \cdot r_G}$, where $a = a(r)$ is the acceleration at the position of the GC. Then, the dissolution time is calculated using equation 7 of Baumgardt & Makino (2003),

$$t_{\text{DISS}} = \frac{1}{[\text{Myr}]} \beta \left( \frac{N_0}{\ln(0.02 N_0)} \right)^\gamma \left( \frac{r_G}{\text{kpc}} \right) \left( \frac{v_G}{220 \text{ km s}^{-1}} \right) \left( \frac{G m}{3} \right),$$

for every cluster individually. The two parameters $\beta$ and $\gamma$ depend on the concentration of the GCs. We chose values of $\beta = 1.91$ and $\gamma = 0.75$ for our main computations (as well as $\beta = 1.21$ and $\gamma = 0.79$ for a particular model), which have been found to reproduce the mass evolution of clusters with a King density profile and a $W_0$ of 5.0. This is typical density profile among Milky Way GC (e.g. McLaughlin & van der Marel 2005). Depending on the density profile of the respective GC, $\beta$ and $\gamma$ can change. The initial number of GC stars, $N_0$, is approximated by $N_0 = m_{\text{GC}}(t = 0)/0.547$ (Baumgardt & Makino 2003). We assume that mass is lost linearly with time, so after each timestep, $\Delta t$, the GC mass is reduced by an amount $\Delta m = \Delta t \cdot m_{\text{GC}}(t = 0)/t_{\text{DISS}}$. The galactocentric distance used in equation (1) gets updated at each peri- or apocentre passage and we use the last pericentre or apocentre distance for $r_G$. In this way, our method reproduces the $(1 - \epsilon)$ scaling of the lifetimes found by Baumgardt & Makino (2003) without having to calculate orbital parameters.

In galaxies where the circular velocity $v_G$ varies with radius, we use an average of the pericentre and apocentre velocity $v = (v_{\text{peri}} + v_{\text{ap}})/2$ to account for the varying circular velocity. Once the mass of a cluster is less than $m_{\text{GC}} = 10^2 M_\odot$, it is assumed to be dissolved.

#### 2.2.2 Tidal shocks

The variation of the tidal field does not only enhance the overall mass-loss rate but also increases the cluster’s internal energy (Gnedin & Ostriker 1997; Gnedin, Hernquist & Ostriker 1999). If the pericentre distance to the galactic centre is small, the energy input through tidal variation acts like a tidal shock and the energy gain of the cluster is significant (Gnedin & Ostriker 1997; Küpper et al. 2010a; Smith et al. 2013; Webb et al. 2013). In these cases, mass-loss of the cluster gets driven by tidal shocks leading to quick cluster dissolution (Gnedin & Ostriker 1997; Vesperini & Heggie 1997; Gnedin, Hernquist & Ostriker 1999; Peñarrubia, Walker & Gilmore 2009; Küpper et al. 2010b). We found that by using a second disruption criterion in addition to Section 2.2.1, we can compensate for the underestimated mass-loss rate for clusters on very eccentric orbits within strong tidal fields and obtain better fits to direct $\text{NBody6}$ integrations (Fig. 2). This criterion is derived below. For quantifying the strength of tidal shocks, we calculate the Jacobi radius, $r_J$, following King (1962) as

$$r_J \approx \left( \frac{G m_{\text{GC}}}{\Omega^2 \cdot \frac{\partial \phi}{\partial r^2}} \right)^{1/3}. \quad (2)$$

Here, $\Omega = \sqrt{\frac{G m}{r^3}}$ is the cluster’s orbital galactocentric angular velocity. We approximate the second spatial derivative of the galactic potential in equation (2) by

$$\frac{\partial^2 \phi}{\partial r^2} \approx \frac{a(r + dr) - a(r - dr)}{2 dr}, \quad (3)$$

where $a(r)$ is the acceleration at galactocentric radius $r$ and $dr << r$ is a sufficiently small distance away from the cluster centre. We choose $dr = r_J$ to be the 3D half-mass radius $^1$ of the respective GC. This approximation works well even for the quickly declining 1/r potential close to the central SMBH (see Fig. 1). If a GCs falls below the galactic centre distance $r = 2r_J$, $r_J$ is taken to be the $r_J$ at $r = 2r_J$.

Equation (2) is valid for star clusters on eccentric orbits in arbitrary spherical potentials (see also, e.g., Spitzer 1987; Read et al. 2006; Just et al. 2009; Küpper et al. 2010a; Renaud, Gieles & Boily 2011; Ernst & Just 2013). It has been compared to $N$-body simulations of dissolving star clusters and found to well reproduce the

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1 For galaxy-specific quantities like the half-mass radius, $R_H$, we use capital letters.
To quantify the dependence of the overall GC system erosion rate on 
$\text{radius at which stars escape from the star cluster into the tidal tails}$ (Küpper, Lane & Heggie 2012). We assume that a cluster is disrupted if its ratio $x = \frac{a}{r_H}$ is larger than $x = 0.5$ at any point during its orbit\(^2\). This approach yields a safe lower limit on the disruption rate as some cluster would be confronted with tidal fields even in excess of $x = 0.5$ at perigalacticon. The motivation behind using the limit $x = 0.5$ is subject to (i) observations of GCs in the Milky Way and (ii) direct N-body computations. The majority of GCs in the Milky Way are on eccentric orbits (Dinescu, Girard & van Altena 1999). Most of them have ratios, $x$, well below 0.2, while only one GC has $x > 0.5$ (Baumgardt et al. 2010; Ernst & Just 2013). The one cluster with $x \approx 0.55$ is the low-mass GC Pal 5, which is thought to be in the very final stages of dissolution due to its pronounced tidal tails (Odenkirchen et al. 2003; Dehnen et al. 2004). Observations therefore suggest a limit of $x = 0.5$ to be reasonable.

In addition to that, we also performed direct N-body computations with the NBODY6 code (Aarseth 1999, 2003) on the GPU computers of the SPODYR group at the AIFIA, Bonn. We ran 32 simulations of compact and massive star clusters ($m_{\text{GC}} > 10^5 \, M_\odot$) on a range of orbits within a galactic tidal field. We followed their dynamical evolution for 10 billion years or until total dissolution, skipping the first 1 Gyr in which the clusters’ evolution is dominated by the SEV processes and expansion as a consequence of rapid mass-loss. See Fig. 2 for a representative sample. Also shown in Fig. 2 is the theoretically predicted mass evolution using equation (1) and the disruption criterion $x = 0.5$. Our model clusters lie in between a

\[^2\text{Furthermore, we assume that GCs passing an SMBH within their 3D half-mass radii, } r_H, \text{ are disrupted as well.}\]

\[^3\text{To quantify the dependence of the overall GC system erosion rate on internal cluster profiles, computations with } \beta = 1.91 \text{ and } \gamma = 0.79 \text{ (i.e. } W_0 = 5.0 \text{) and } \beta = 1.21 \text{ and } \gamma = 0.79 \text{ (green line in Fig. 6) were performed.}\]
2.2.3 Dynamical friction

Massive objects moving through a background of particles will decelerate and lose orbital energy by DF (Chandrasekhar 1943). This effect may have profound implications for the orbital evolution and, hence, the fate of GCs. Our MUESLI code is designed to obtain the impact of DF on the destruction of GCs in galaxies with isotropic and anisotropic velocity distributions as well as axisymmetric and triaxial galaxies.

The equation of motion of a massive object like a GC in a galaxy with DF is

$$\mathbf{a}_{GC}(\mathbf{r}) = -\nabla \phi(\mathbf{r}) + \mathbf{a}_{GC, DF}(\mathbf{r}).$$

Here, $\mathbf{a}_{GC}$ is the total acceleration of the GC, $-\nabla \phi(\mathbf{r})$ is the acceleration due to the combined galactic and SMBH potential, while $\mathbf{a}_{GC, DF}$ describes the deacceleration due to DF. Chandrasekhar’s DF formula has been extended to account for ellipsoidal velocity distributions by Pesce et al. (1992) and $\mathbf{a}_{GC, DF}(\mathbf{r})$ has the form

$$\mathbf{a}_{GC, DF}(\mathbf{r}) = -\gamma_1(\mathbf{r}) \tilde{v}_1(\mathbf{r}) \mathbf{e}_1 - \gamma_2(\mathbf{r}) \tilde{v}_2(\mathbf{r}) \mathbf{e}_2 - \gamma_3(\mathbf{r}) \tilde{v}_3(\mathbf{r}) \mathbf{e}_3,$$

where the DF coefficients $\gamma_i(\mathbf{r})$ can be written as

$$\gamma_i(\mathbf{r}) = \frac{2\sqrt{2} \pi \rho(\mathbf{r}) G m_{GC} \ln \Lambda}{\sigma_{\tilde{v}_i}^2(\mathbf{r})} \times \int_0^\infty \exp \left( -\frac{\tilde{v}_i^2(\mathbf{r})}{1 + u} - \frac{\tilde{v}_j^2(\mathbf{r})}{1 + u} - \frac{\tilde{v}_k^2(\mathbf{r})}{1 + u} \right) \frac{1}{\left( \tilde{v}_i^2 + u \right) \left( \tilde{v}_j^2 + u \right) \left( \tilde{v}_k^2 + u \right)} du. \tag{6}$$

The function $\Lambda$ that appears in the Coulomb logarithm, $\ln \Lambda$, can be obtained for bodies with a finite size (Binney & Tremaine 2008, their equation 8.2) as

$$\Lambda = \frac{b_{max}}{\max (r_0, G M_{GC}/v_{\text{typ}}^2)}. \tag{7}$$

The maximum impact parameter $b_{max}$ is approximated by the galactocentric distance $r_0$. This approach yields a more realistic treatment than by assigning a constant value for the Coulomb logarithm (Hashimoto, Funato & Makino 2003; Spinnato, Fellhauer & Portegies Zwart 2003). The validity of equation (7) is restricted to $\Lambda > 1$ in order to prevent unphysical acceleration by DF. The characteristic velocity is $v_{\text{typ}} \approx G M_{\text{MGAL}}/R_{\text{HI}}$. Here, $M_{\text{MGAL}}$ and $R_{\text{HI}}$ correspond to the total mass and half-mass radius of the galaxy. The parameter $\epsilon^2$ which appears in equation (6) is given by the ratio of the eigenvalues $\sigma_i^2(\mathbf{r})/\sigma_j^2(\mathbf{r})$ of the velocity dispersion tensor $\sigma_i^2 = \nabla \tilde{v}_i \cdot \nabla \tilde{v}_i$. For convenience, the velocity dispersion component with the largest eigenvalue of $\sigma_i^2$ is defined to be $\sigma_1^2(\mathbf{r})$. The integral is evaluated numerically for each integration timestep by using the Gauß–Legendre integration method in combination with logarithmic mapping. The density $\rho(\mathbf{r})$ is obtained directly from the SCF algorithm. The velocity components $\tilde{v}_i(\mathbf{r}) = \cos \theta_i v_{GC}(\mathbf{r})$ with $\cos \theta_i = \frac{\tilde{v}_i(\mathbf{r})}{v_{GC}(\mathbf{r})}$ are obtained by the projection of the GC velocity vector $v_{GC}$ on to the normalized eigenvectors $\mathbf{e}_i$ of the velocity dispersion tensor $\sigma_i^2$. The position-dependent eigenvalues $\sigma_i$ and eigenvectors $\mathbf{e}_i$ are calculated in hundreds of cubic segments which are part of a $5 \times 5 \times 5$ mesh with logarithmically increasing resolution towards the centre (see Fig. 5 for illustration). This is achieved by replacing the inner 27 out of 125 cubes by a second $5 \times 5 \times 5$ grid. The procedure is repeated $G_{\text{depth}} \in \mathbb{N}$ times. The innermost resolution scale is $R_{\text{res}} = 0.2 R_{\text{max}} 0.6^{G_{\text{depth}}}$. The size $R_{\text{max}}$ of the outermost grid is
chosen to encompass the whole galaxy. In this way, a variable DF force acting on GCs in an elliptical galaxy is handled. The underlying galaxy models are specified in Section 2.3.2.

Grid-based calculations are always affected by discontinuities/jumps in combination with discreteness noise subject to finite number of cells and particles. In order to counterbalance these systems, we apply the inverse distance weighting (IDW) method (Shepard 1968). Irregularities are smoothed out by first calculating the centre of mass of the particles in a box (which is used as the position of the box), local eigenvalues and eigenvectors of \( \sigma^2 \). Boxes containing only few particles are left out of consideration. For the spatial interpolation, only cells within a radius corresponding to the galactocentric distance of an orbiting GC are taken into account.

To guarantee that the contribution of the nearest box dominates, the weighting parameter \( p \) (also known as the power parameter) is calibrated in many N-body experiments to be \( p = 64 \). For testing issues, we refer to Appendix A3. The eigenvalues and eigenvectors are calculated at the beginning of the computations and for each time-scale the potential becomes updated by the SCF algorithm.

2.3 Initial conditions

2.3.1 GC mass and size distribution

The present-day GC mass spectrum, \( dN/dm \propto m^{-\beta} \), can be characterized as a power-law distribution with different exponents for characteristic mass scales (McLaughlin 1994). Usually, it is well approximated by the exponents \( \beta = 0.2 \) below and \( \beta = 2 \) above a threshold mass of \( m_{TH} = (1-2) \times 10^6 \, M_\odot \). This two-component power-law distribution resembles a bell-shaped function when expressed in terms of the number of GCs, \( dN \), per constant logarithmic cluster mass interval, \( d\log_{10}m \). For the initial cluster mass function, we are using the single power-law distribution,

\[
\frac{dN}{dm} \propto m^{-2}.
\]  

It finds support by observations of young, luminous clusters in starburst galaxies where the mass spectrum monotonically follows a (single) power-law profile with slope \( \beta \approx 2 \) (Battinelli, Brandimarti & Capuzzo-Dolcetta 1994; Zhang & Fall 1999).4

It is our aim to investigate whether dissolution of low-mass clusters is responsible for turning a power-law mass function into a bell-shaped mass function (see also Baumgardt 1998; Fall & Zhang 2001; Vesperini et al. 2003; McLaughlin & Fall 2008; Elmegreen 2010) by cluster disruption processes, relaxation-driven mass-loss in tidal fields and DF. Scenarios involving gas expulsion (Kroupa & Boily 2002; Parmentier & Gilmore 2007; Baumgardt, Kroupa & Parmentier 2008) are not considered in our main computations (with the exception of one model) and will be added in later publications. The overall GC mass range is chosen to be \( m_{GC} = 10^5 - 10^6 \, M_\odot \). Clusters below \( 10^5 \, M_\odot \) are not considered, because in galaxies with an age of several Gyr, they would have lost most (if not all) of their initial mass by energy-equipartition-driven evaporation (Baumgardt & Makino 2003; Lamers, Baumgardt & Gieles 2010).

Observations find no strong correlation between half-mass radius and mass for GCs which are less massive than \( 10^6 \, M_\odot \) (Eggen et al. 2005; Dabringhausen, Hilker & Kroupa 2008). The median 3D half-mass radius of GCs in typical early-type galaxies centres around \( r_H = 4 \, pc \) (Eggen et al. 2005; Jordán et al. 2005).5 The situation changes when the clusters become more massive than a particular mass scale which is of the order of \( 10^6 \, M_\odot \) (Dabringhausen et al. 2008). Hence, we assume that they follow a trend given by

\[
r_H = \begin{cases} 
4 \, pc & : \quad m_{GC} \lesssim 1.0 \times 10^6 \, M_\odot \\
4 \left( \frac{m_{GC}}{10^6 \, M_\odot} \right)^{0.6} \, pc & : \quad m_{GC} \gtrsim 1.0 \times 10^6 \, M_\odot 
\end{cases}
\]  

The influence of other primordial size relations for GC erosion processes is not considered in this paper.

2.3.2 Spatial distribution of GCs

Having defined cluster masses and sizes, the GC space and velocity vectors have to be distributed within the galaxies by making five underlying assumptions:

(i) the initial GC phase space distribution equals the one of the underlying galaxy model,

(ii) initial GC masses and sizes do not depend on the distance to the galactic centre,

(iii) accumulation of GCs through mergers or subsequent formation in star-forming events is neglected.

4 Recent investigations (Larsen 2009) found that the initial mass distribution is also compatible with a Schechter-type mass function with a particular turn-down mass in the high-GC-mass regime. However, for simplicity, we use a single power-law mass function here, as the differences will be limited to the high-mass end where only relatively few clusters are found.

5 For the conversion \( r_e = 0.75 r_H \) (Spitzer 1987) of the projected half-light radius, \( r_e \), to the 3D half-mass radius, \( r_H \), the mass-to-light ratio, \( \Upsilon(r) \), is assumed to be constant.

6 The code allows for individual adjustment of these aspects.
is the anisotropy radius. The particle positions were distributed $T \propto M_{\text{gal}}$ for 0


e_{\text{mod}} = \frac{1}{2} \left(1 + \frac{3}{2} \right)^{-1}

\text{for Hernquist and McGlynn (1990) and Jaffe (Jaffe 1983) models. Scalefactors $a = (1 + \sqrt{2})^{-1}$ for Hernquist and $a = 1$ for Jaffe models were used in order to fix the half-mass radius to one. Finally, we generated an additional triaxially shaped model (required for testing issues of the DF routine) with a central core and outer Sersic $n = 4$ profile from cold collapse computations (Lynden-Bell 1967; Aarseth & Binney 1978; van Albada 1982; McGlynn 1984; Merritt & Quinlan 1998). A spherical distribution with a $\rho \propto r^{-1.5}$ density profile and virial ratio $2T/|W| = 0$ was set for $0 < r < 2$. It collapsed and settled down into a strongly triaxial configuration with $T = (a^2 - b^2)/(a^2 - c^2) = 0.53$ within its half-mass radius. Here, $T$ is the triaxiality parameter and $a, b, c$ are the three main axes of the ellipsoidal configuration. It was evolved forward in time with the NBODY6 (Aarseth 1999, 2003) code until virialization. The density centre was shifted to the centre of origin and the model was rescaled to $R_{\text{HI}} = 1$. Models generated from collapse simulations are isotropic in their centres and radially biased at large galactocentric distances.

In the scenario of hierarchical structure formation (but see also Samland 2004), where smaller structures merge to build up larger objects such as elliptical galaxies (Toomre 1977; White & Rees 1978; Kauffmann, White & Guiderdoni 1993; Steinmetz & Navarro 2002), violent relaxation causes the merger products to be centrally isotropic and radially biased at large radii (Lynden-Bell 1967). Our models agree with these cosmological predictions.

Table 1. Adopted parameters for the simulated galaxies. $R_{\text{HI}} = 1.35R_e$ is used to calculate the 3D half-mass radius $R_{\text{HI}}$ from the effective radius $R_e$. References: (1) Magorrian et al. (1998); (2) Rose et al. (2005); (3) Karachentsev et al. (2004); (4) Lauer et al. (2007b) for $R_e$ and $M_{\text{gal}}$ by assuming $\gamma = 3$; (5) Bender, Saglia & Gerhard (1994); (6) McConnell et al. (2011) black hole mass from $M_* - \sigma$ relation; (7) Häring & Rix (2004), (8) Cappellari et al. (2002), (9) McConnell et al. (2012), (10) McConnell et al. (2011)

| Model | Galaxy example | $M_{\text{gal}} (10^9 M_\odot$) | $R_e (pc)$ | $R_{\text{HI}} (pc)$ | $M_*(10^6 M_\odot$) | Ref. |
|-------|----------------|------------------------------|-------------|---------------------|---------------------|-----|
| MOD1  | M32            | 0.8                          | 125         | 170                 | 0.0025              | 1.23 |
| MOD2  | NGC 4494       | 100                          | 3715        | 5000                | 0.065               | 4.56 |
| MOD3  | IC 1459        | 300                          | 6000        | 8050                | 2.6                 | 7.8  |
| MOD4  | NGC 4889       | 2000                         | 25000       | 34000               | 20                  | 9.10 |

(iv) the overall dynamics of the host galaxy are not influenced by GC evolution processes,
(v) all galaxy models are virialized and remain in isolation.

For this study, we created several realistic base models. We assume that the stars follow a Sérsic model (Sérsic 1968) with concentration $n = 4$ and constant mass-to-light ratio, $\gamma$. They were generated by the deprojection of 2D Sérsic profiles into 3D density profiles. Afterwards, the density, potential and distribution function was calculated on a logarithmically spaced grid configuration of size $r \in [10^{-4}, 10^2]$ in model units, $G = R_{\text{HI}} = M_{\text{gal}} = 1$. The distribution function for an anisotropic Osipkov–Merritt velocity profile (Osipkov 1979; Merritt 1985) was calculated by making use of equation 4.78a from Binney & Tremaine (2008). Here, the velocity anisotropy parameter has the form $\beta(r) = (1 + R_s^2/r^2)^{-1}$ and $R_s$ is the anisotropy radius. The particle positions were distributed according to the density profile, while the normalized cumulative distribution function was used to allocate particle velocities. It was evaluated by the transformation of the double integral into a single integral according to the substitution described in Merritt (1985) $\gamma$. Central SMBHs of mass $M_*$ were implemented by adding the term $\phi_{\text{BH}} = -M_* r$ to the potential of the underlying mass distribution. Afterwards, all particles were inverted (and doubled) through the origin. In this way, the centre of mass and density centre were located at the point of origin and the model stays at rest during computations. We also created galaxies following Hernquist (Hernquist 1990 and Jaffe (Jaffe 1983) models. Scalefactors $a = (1 + \sqrt{2})^{-1}$ for Hernquist and $a = 1$ for Jaffe models were used in order to fix the half-mass radius to one. Finally, we generated an additional triaxially shaped model (required for testing issues of the DF routine) with a central core and outer Sersic $n = 4$ profile from cold collapse computations (Lynden-Bell 1967; Aarseth & Binney 1978; van Albada 1982; McGlynn 1984; Merritt & Quinlan 1998). A spherical distribution with a $\rho \propto r^{-1.5}$ density profile and virial ratio $2T/|W| = 0$ was set for $0 < r < 2$. It collapsed and settled down into a strongly triaxial configuration with $T = (a^2 - b^2)/(a^2 - c^2) = 0.53$ within its half-mass radius. Here, $T$ is the triaxiality parameter and $a, b, c$ are the three main axes of the ellipsoidal configuration. It was evolved forward in time with the NBODY6 (Aarseth 1999, 2003) code until virialization. The density centre was shifted to the centre of origin and the model was rescaled to $R_{\text{HI}} = 1$. Models generated from collapse simulations are isotropic in their centres and radially biased at large galactocentric distances.

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Related to the fact that a systematic scan over the Fundamental Plane of elliptical galaxies is beyond the scope of this paper, we scaled our models to four representative elliptical galaxies. These are M32, NGC 4494, IC 1459 and NGC 4889. While M32 is a compact dwarf galaxy which is gravitationally bound to M31, NGC 4889 is the most massive and extended galaxy in our sample. It is a brightest cluster galaxy (BCG) and defines together with NGC 4874 the gravitational centre of the Coma cluster. The four galaxies were chosen because they cover the full mass range of elliptical galaxies from small compact dwarf ellipticals (dEs) to giant BCGs. They lie (within 10–45 per cent scatter) on the $R_e - M_{\text{gal}}$ relation (Dabringhausen et al. 2008, their equation 4) for low-redshift bright elliptical galaxies, bulges and very compact dE galaxies. We note that our results concerning GC erosion processes in M32-like compact galaxies should not be extrapolated to much more extended dwarf spheroidal (dSph) galaxies with weaker tidal fields (see fig. 2 in Dabringhausen et al. 2008 and Section 3.1.1 in this paper). Complementary to the galactic mass range, our representative galaxies host central SMBH with masses in the range of a few $10^6 M_\odot$ (MOD1, i.e M32) up to $10^{10} M_\odot$ (MOD4, i.e NGC 4889). The number of observed GCs ranges from 0 (M32; Harris et al. 2013) to about 11 000 GCs (NGC 4889; Harris et al. 2009). The physical properties of all four galaxy models are summarized in Table 1.

### 3 RESULTS

They are divided into three major parts. In Section 3.1, we discuss general aspects of the GC erosion rate in various galaxies. We present evidence for a new phase in the evolution of GC systems. In the following section (Section 3.2), we discuss the formation of cores in GC systems. Finally, in Section 3.3, we investigate the evolution of the cluster mass function.

#### 3.1 GC erosion rate

In order to evaluate the importance of the various processes for the dissolution of GCs, we performed 48 main computations plus additional 18 models which are required to uncover more systematical effects. Each of these models consists of 2 $\times 10^7$ stellar particles and 20 000 GCs, distributed according to a power-law mass distribution as given by equation (8) and the present-day half-mass radius relation (equation 9). We chose to model 20 000 GCs in order to obtain a good statistical significance of our results. The GCs in all computed
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Figure 6. The temporal evolution of the fraction of destroyed GCs. Most of the destruction occurs at early times. Hence, we call this period the tidal-disruption-dominated phase (TDDP). It is followed by a relaxation-driven dissolution phase. The destruction rate rises up to 100 per cent in models representing the least massive and compact galaxies (MOD1), independent of their velocity distribution. The rate decreases to 50 per cent in the most luminous and extended galaxies (MOD4) with an isotropic velocity structure. The different colours (i.e. black solid, blue long and red short dashed lines) represent the degree of radial anisotropy characterized by the anisotropy radius $R_A$ of the Osipkov–Merritt models. Purple lines represent computations with more stringent criteria for GC destruction through tidal shocks (Section 2.2.2). Even by using $x = 0.8$ or $x = \infty$ there is a TDDP in the most compact models (MOD1). With the exception of one model (green solid line, upper-right panel), GCs were assumed to have King profiles with a concentration parameter $W_0 = 5.0$. The green solid line illustrates the difference to a cluster population having parameters between $W_0 = 3.0$ and 5.0, or more precisely by using $\beta = 1.21$ and $\gamma = 0.79$ in equation (1) (Section 2.2.1).

While at large radii, both models agree well with each other, Sersic $n = 4$ models are centrally more concentrated. The models with $R_A/R_H = \infty$ represent the isotropic case. The central SMBHs masses were adopted from Table 1.

We evolved all models for 10 Gyr under the influence of the generalized DF force (Section 2.2.3). As described in Sections 2.2.2 and 2.2.1, clusters were assumed to be destroyed if: (i) the strength of the tidal field, $x = r_H/r_J$, exceeded $x = 0.5$ and (ii) relaxation-driven mass-loss in tidal fields (and SEV) decreases their masses below the limit $m_{GC} = 100 M_\odot$. The temporal evolution of the GC destruction rate is plotted in Fig. 6 and absolute numbers of destroyed GCs after 10 Gyr evolution are summarized in Table 2.

3.1.1 Tidal-disruption-dominated phase

We want to emphasize the strong chronological aspect in the evolution of whole GC systems which can be observed in our
computations (Fig. 6). Significant numbers of GCs are being torn apart early on, i.e. within a few crossing time-scales of the galaxy at its half-mass radius, $T_{\text{cross}} = 42.26(R_0/M_{\text{GAL}})^{0.5}$ Myr $<< T_{\text{Hubble}}$, which is $T_{\text{cross}} = 3.3$ Myr for M32 and $T_{\text{cross}} = 187$ Myr for NGC 4889. This can be seen in form of the steeply rising slope of the fraction of destroyed clusters at very early times. Hence, we characterize it as the tidal-disruption-dominated phase (TDDP). In isotropic galaxy models (MOD2–MOD4) with central SMBHs and S´ersic $n = 4$ density profiles, approx. 10 per cent (MOD4) to 40 per cent (MOD2) of all GCs are destroyed within the TDDP. During the TDDP, tidal shocks dominate cluster dissolution processes. It is subsequently followed by a long-term relaxation-driven dissolution phase in which surviving clusters lose mass more gently. The TDDP can be explained as follows. By assuming the initial GC phase space distribution to be equal to that of the stellar component of the host galaxy, significant numbers of clusters pass close to the galactic centre within their first orbit. Here, tidal shocks cause rapid mass-loss and destruction of GCs. The creation of central cores in the radial GC distribution proceeds rapidly (Section 3.2). Evidently, the fraction of destroyed GCs depends on the mass and size of the galaxy, i.e. the tidal field. The TDDP is most pronounced in very compact and not so massive galaxies and less efficient in very extended galaxies. However, we note that M32 (MOD1) is an extremely compact galaxy and should not be regarded as representative for common dSph galaxies. We note that there are dE galaxies with much larger spatial scales than that of M32. See also fig. 2 in Dabringhausen et al. (2008). In such dEs, the tidal field is much weaker and the fraction of destroyed GCs is strongly reduced. For comparison, we computed a dSph galaxy model with a S´ersic $n = 1$ density profile and isotropic velocity distribution (Fig. 7). In this model, we found no indication for a pronounced TDDP but a strong contribution from DF. It drives large amounts of GCs to the centre where they would merge together and form a nuclear star cluster.

To get a closer insight into the dynamics of the TDDP, we performed eight additional computations with more stringent criteria ($x = 0.8$ or $x = \infty$) for GC disintegration processes by tidal shocking. See the solid and dotted purple lines in Fig. 6). Evidently, the number of destroyed clusters during the TDDP decreases and the overall slope rises less steeply. However, even by using $x = \infty$, there is evidence for the occurrence of a TDDP in the most compact galaxy models MOD1. Interestingly, the TDDP does not change the total fraction of destroyed GCs after 10 Gyr but affects the temporal evolution/slope of disintegration processes. In Section 4, we also critically review our assumptions of initial cluster sizes and galaxy models which affect the strength of the TDDP. In Section 3.1.3, we discuss the influence of secondary aspects like the central SMBH and galactic density profile on the TDDP.

The influence of the host galaxy on the cluster disruption/dissolution rate becomes also evident if we calculate the normalized arithmetic mean radius, $R_0/R_\text{H}$. $R_0/R_\text{H}$ is defined to be the averaged radius at which GCs in our computations were assumed to be destroyed, either by tidal shocks or relaxation-driven destruction. $R_0/R_\text{H}$ anticorrelates with the mass and size of the host galaxy and is largest in the compact M32-like galaxy ($R_0/R_\text{H} \approx 1.1$) and lowest in the most massive and extended galaxy, NGC 4889 ($R_0/R_\text{H} \approx 0.15$).

The existence of a rapid phase in the evolution of GCs might be of strong relevance for the fast build-up of a galaxy’s field star population from eroding clusters. Furthermore, the existence of the TDDP might be relevant for SMBH growth processes in the very early Universe as some fraction of the debris might enter loss cone trajectories and contribute to the feeding of the central black hole.
holes. Especially, as the majority of cluster debris is gravitationally unbound with respect to the black hole, potential and gravitational focusing would enlarge its geometric cross-section. Interestingly, the phase space distribution of the field stars originating from such a TDDP should be complementary to the phase space distribution of the surviving GC which is discussed in Section 3.2.

3.1.2 Radial anisotropy

The overall fraction of destroyed GCs in spherical galaxies with an isotropic velocity distribution (and no central SMBH) depends on the mass and scale of a galaxy (Table 2). While up to 100 per cent of all GCs are destroyed in compact dwarf galaxies like M32 and 75–80 per cent in mid-size galaxies, no more than 50 per cent are eroded over the course of 10 Gyr in the most massive and extended galaxies like NGC 4889 (MOD4). In Fig. 7, the total fraction of dissolved GCs is plotted as a function of the mass of the galaxy. As can be seen, the initial orbital anisotropy has a considerable impact on the overall GC erosion rate in massive elliptical galaxies. Compared to the specific isotropic galaxy model MOD4 with a Sérsic $n = 4$ and central SMBH, orbital anisotropy increases the fraction of destroyed clusters from 50 per cent ($R_A/\nu = 1$) to 70 per cent ($R_A/\nu = \infty$) and 95 per cent ($R_A/\nu = 0.25$). Different formation or merger histories, and thus different degrees of radial anisotropy, may therefore be a reason for considerable scatter in the total number of surviving GCs in observed elliptical galaxies of similar size and mass. The fraction of eroded GCs in compact dE galaxies like M32 (MOD1) centres around 100 per cent. This number is insensitive to the initial velocity distribution. Our computations naturally explain the absence of GCs around M32. However, early GC stripping by M31 might have occurred as well.

3.1.3 Density profile and SMBHs

Secondary aspects like the density profile or central SMBH exert their action only in very massive galaxies. On average the absolute erosion rate is 1–4 per cent higher in the centrally more peaked Sérsic $n = 4$ models. The strongest impact is observed in the galaxy models MOD4 (Table 2). These differences can be explained by a higher initial number density of GCs inside the centrally more concentrated Sérsic $n = 4$ models and a steeper gradient of the tidal field.

The impact of SMBHs on the overall GC erosion rate after 10 billion years of evolution is insignificant. The increase of the total destruction rate compared to models without central SMBHs does not exceed the one per cent level. This is of the same order as the assumed Poisson error related to statistics. However, this does not mean that SMBHs do not contribute to the exact sequence of GC dissolution processes inside galactic nuclei. It is irrelevant for the overall GC erosion rate (after 10 Gyr) if clusters were eroded continuously or disrupted by the central SMBH during a single close passage. In order to isolate the impact of SMBHs during the TDDP, computations without relaxation-driven mass-loss and SEV were performed (lower part of Table 2). Evidently, the $M_H = 2 \times 10^{10} M_\odot$ ultramassive black hole inside the reference galaxy MOD4, which has similar physical properties like the BCG NGC 4889, contributed significantly to the number of tidal disruptions.

Secondary aspects like the density profile or SMBH might also become relevant in low-density dwarf or irregular galaxies in which the overall gradient of the potential and the fraction of tidally disrupted GCs are small.

3.1.4 Dynamical friction

The influence of DF on the overall destruction rate in very massive elliptical galaxies is almost negligible. The reasons behind the subdominant impact of DF on the GC destruction rate are three-fold.

(i) GC masses are distributed according to a single power-law GC mass function (Section 2.3.1). Most initial clusters masses are located at the low-mass end of this distribution. Initial SEV further decreases their masses. However, the strength of de-acceleration by DF is proportional to cluster masses (equations 5 and 6). Fig. 8 compares the influence of DF in models with a single and a double power-law initial cluster mass distribution but otherwise identical physical parameters. In the latter case, DF has a stronger influence on the overall GC erosion rate due to GCs being preferentially more massive. We chose the threshold mass, $m_{TH} = 2 \times 10^5 M_\odot$, with slopes $\beta = 0.2$ below and $\beta = 2$ above $m_{TH}$.

(ii) Low-mass clusters are particularly susceptible for relaxation-driven mass-loss. In this way, their masses are continuously decreased so that DF gets less important.

(iii) Finally, the strength of DF is proportional to the distance-dependent Coulomb logarithm (equation 7) which becomes zero at small galactocentric distances.
The fraction of destroyed clusters depends on the galaxy model, the initial cluster mass distribution [i.e. single (SPD) versus double power-law distribution (DPD)] and DF. The parameter $\Delta_{\text{del}}$ quantifies the difference in the fraction of destroyed GCs after 10 Gyr between models including DF and those where it was ignored.

The influence of DF on the overall destruction rate in models with a single power-law cluster mass function is only evident (up to the percentage level) in computations representing the dwarf compact elliptical galaxy M32 (MOD1). But even in this galaxy, the tidal field strongly dominates GC erosion processes. The only exception where DF contributed significantly to GC erosion processes was observed in the dSph galaxy model (Fig. 7).

### 3.2 GC core formation in giant elliptical galaxies

Galaxy observations reveal the spatial GC distribution to be centrally less peaked than that of the stellar light profile (Harris & Racine 1979; Forbes et al. 1996; McLaughlin 1999; Capuzzo-Dolcetta & Mastrobuono-Battisti 2009). In this section, the formation of core profiles as a consequence of GC disintegration processes in tidal fields will be investigated. The initial and final (after 10 Gyr) 2D number density profiles are compared relative to each other. This is done for the representative galaxies MOD2 (NGC 4494), MOD3 (IC 1459) and MOD4 (NGC 4889) in our sample (Section 2.3.2). The initial population of 20 000 GC was distributed according to the phase space distribution of Sérsic $n = 4$ models with an isotropic and a radially biased $R_{1}/R_{\text{Hil}} = 1$ velocity distribution and a central SMBH. The results are shown in Fig. 9 from which the following conclusions can be drawn.

(i) In all galaxies, the central GC distribution becomes flattened by erosion processes. The outer GC number density profiles in isotropic distributions remain intact and it follows that GC distributions around the most massive and largest galaxies should have preserved their initial conditions. These results are in agreement with findings by Vesperini (2000). Cores are more extended in isotropic velocity distributions despite reduced numbers of destroyed clusters. The reason behind this apparent contradiction is related to the existence of the larger number of GCs on eccentric orbits in radially biased velocity configurations. GCs with large galactocentric distances also get close to the galactic centre, where (at least) the less massive GCs are efficiently eroded. In this way, clusters all along the radial distribution become affected over time while the overall shape (i.e. slope) of the number density distribution is conserved. This observation is also consistent with earlier studies (e.g. Vesperini et al. 2003, their fig. 5). However, in velocity distributions of Osipkov–Merritt type with the most extreme value $R_{1}/R_{\text{Hil}} = 0.25$, mean pericentric distances of GC orbits decrease with increasing galactocentric distance, resulting in a steepening of the outer slope of the GC distribution. This effect is discussed in more detail in Appendix B.

These observations have a profound impact for the study of GC systems. Efforts to compute the number of eroded GCs by simply integrating the central number deficit of clusters by comparison to the stellar light component might be biased. The inferred values should be corrected for the influence of radial anisotropy, mass and scale of the host galaxy.

A systematic study in which core sizes are obtained for comparison issues with actual data of real galaxies must also include a threshold mass scale in order to mimic observational limitations. We found that core sizes depend on the threshold mass, $M_{\text{th}}$. This is related to the fact that massive GCs are less affected by tidal fields. The effect is shown in Fig. 9 for the case of MOD4. Here, the fraction of surviving GCs is largest and the effect is most pronounced. The purple dashed lines represent the unfiltered number density profiles ($M_{\text{th}} = 10^{5} M_{\odot}$) whereas the blue lines correspond to GCs in excess of $M_{\text{th}} = 2 \times 10^{5} M_{\odot}$. This corresponds to the detection limit at the distance of NGC 4889. However, slope differences between unfiltered and those with $M_{\text{th}} = 10^{5} M_{\odot}$ are small in MOD2 and MOD3, owing to a negligible fraction of GCs below $10^{4} M_{\odot}$ (Fig. 12). The initial profiles (black solid lines) were rescaled to match the final profiles at large galactocentric radii. We take from this figure that the observation of a mass-dependent core size of a GC system might be proof of cluster dissolution as the origin of the core (Section 1).

(ii) Core formation occurs on short cosmological time-scales. Shortly after the TDDP, the GC number density profiles are centrally flattened (Fig. 10).

(iii) Despite the influence of radial anisotropy, cores are pronounced in less massive galaxies, as here the percentage of disrupted clusters as well as the ratio $R_{60}/R_{\text{Hil}}$ is highest. However, spatially extended cores are also found in the most luminous galaxies in the Universe, and our computational results indicate that observed GC profiles with central cores (Forbes et al. 1996) are created by disruption and dissolution processes. To see if our computations are in agreement with observations, we plotted GC number density profiles for NGC 4889 (MOD4, blue lines) by taking observational limitations into account. For a typical mass-to-V-band-light ratio $V_{r} = 1.5$ (McLaughlin & van der Marel 2005), $m_{\text{GC}} = 2 \times 10^{5} M_{\odot}$ corresponds to the threshold mass below which a GC would be undetected at the distance of $D \approx 100$ Mpc (Harris et al. 2009). As can be seen in Fig. 9, central flattening is compatible with our models within the central few kiloparsecs. This is in agreement with the inner parts of the galaxy $r \approx 3.5$ kpc (fig. 6 in Harris et al. 2009) where the GC number density profile becomes more shallow than the stellar light profile of NGC 4889. However, in a more realistic scenario, erosion is only partly responsible for the observed central shallow profiles, as in these galaxies’ major merger events will also contribute to the spatial flattening of the radial GC density profiles (Bekki & Forbes 2006).
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Figure 9. Projected 2D number density profiles of GC systems with isotropic and radially biased \( \frac{R_A}{R_H} = 1 \) velocity distributions. The slope of the initial GC configuration (solid black curve) also corresponds to the stellar light profile of its host galaxy. In all cases, surviving GC systems turn into centrally shallower configurations. Interestingly, central core profiles seem to be less pronounced in radially biased configurations provided they are compared to initial profiles (black lines). In anisotropic velocity configurations, GCs at large spatial scales are eroded more efficiently and the shape of the number density profile is less affected. Observational limitations are handled by using a GCs threshold mass \( M_{\text{th}} = 2 \times 10^5 M_\odot \) in MOD4, \( M_{\text{th}} = 10^4 M_\odot \) in MOD3 and MOD2). Core sizes become more pronounced when all clusters are considered (purple curves with \( M_{\text{th}} = 10^2 M_\odot \)). However, core sizes in MOD3 and MOD2 are close to the unfiltered values as the number of GCs below \( 10^4 M_\odot \) is negligible (see the cluster mass distribution in Fig. 12). We note that for comparison issues the blue lines are rescaled by constant factors.

(iv) Our computations also reflect the preferential destruction of GCs on elongated orbits and the consequences for the dynamics of the surviving GC system. After 10 Gyr of evolution, the central regions of the plotted models (Fig. 11) show strong signs of a tangential bias subject to the preferential survival of GCs on circular orbits. In models with initial radial anisotropy, a tangentially biased region develops within the 3D half-mass radius. At large distances, the radial anisotropy is reduced but still persists at significant levels.

Our computations demonstrate a relation between core sizes of GC systems and the host galaxies mass and velocity distributions. A quantitative evaluation of this correlation will be an interesting task for a follow-up investigation.

3.3 Final GC mass distribution

Fig. 12 shows the evolution of the GC mass function (left-hand panels). Results are plotted for three galaxies with Sérsic \( n = 4 \) density profiles, central SMBHs and radially biased velocity distributions. Evidently, a moderate degree of radial anisotropy \( \frac{R_A}{R_H} = 1 \) transforms initial power-law distributions into bell-shaped curves...
(upper and middle panels) with a peak at approximately $10^5 \, M_\odot$. However, the GC destruction rate in the most massive and extended galaxies (MOD4) is reduced owing to weak tidal fields. Here, the total fraction of dissolved GCs is not high enough to turn a power-law distribution into a bell-shaped curve peaking at $2 \times 10^5 \, M_\odot$. Stronger initial anisotropy or mass-loss related to gas expulsion (Kroupa & Boily 2002; Baumgardt et al. 2008) might represent one solution to this discrepancy. Indeed, including gas expulsion during the gas-rich cluster phase results in a bell-shaped mass function after 10 Gyr of evolution. This is shown in Fig. 12 in form of green shaded histograms. In order to mimic the effects of gas expulsion on the embedded cluster mass function, we applied equations 6, 8 and 9 from Kroupa & Boily (2002). However, even by considering gas expulsion, the distribution peaks at a few $10^4 \, M_\odot$ instead of $2 \times 10^5 \, M_\odot$. An additional effect which might naturally explain this discrepancy in the most massive and extended galaxies is related to the idea that BCGs are partly grown from galaxy mergers of smaller constituents at high redshift. Furthermore, the progenitor galaxies of the most massive ones today were initially much more compact (see e.g. Trujillo et al. 2007, van Dokkum et al. 2008). Within these progenitor galaxies, the cluster mass functions quickly transformed into a bell-shaped form before they merged together to form the BCG. Note the fast temporal evolution in Fig. 6 in the less massive but more compact galaxies.

The relations between GC mean masses and galactocentric distances are plotted in the right-hand panels of Fig. 12. Despite a large degree of scatter due to low number statistics of surviving clusters, models MOD2 and MOD3 are in agreement with the hypothesis of having a constant GC mean mass over a broad range of galactocentric distances. The residual slope $b$ which is obtained from a linear regression in the interval $r \in [1-200] \, \text{kpc}$ is of the order of the 1σ error bar ($b/\Delta b = 0.98$, MOD2) and ($b/\Delta b = 1.1$, MOD3), respectively. However, the hypothesis of a constant GC mean mass over a large galactocentric distance is rejected for MOD4 ($b/\Delta b = 2.8$).

Our results imply that in order to reproduce observed GC properties of BCGs, their merger history from more compact progenitors should be considered. Although our computations were not designed to reproduce GC characteristics of particular galaxies but instead to illustrate systematics, they already reproduce a lot of observed features: a bell-shaped mass distribution, a nearly constant GC mean mass over large galactocentric distances and shallow central number density profiles.

### 4 CRITICAL DISCUSSION AND OUTLOOK FOR FUTURE WORK

In this Section, we will critically review our assumptions and results. This will ease the efforts to identify potential weaknesses and help to improve follow-up studies.

(i) The specific implementation of tidal shock and their relevance for disruption processes (Section 2.2.2) require the orbital angular momentum to be conserved or decreasing due to DF. This is because a GC does not necessarily become completely unbound by tidal shocks once the ratio of half-mass radius to Jacobi radius exceeds $r_{1/2}/r_J = 0.5$. If the angular momentum is conserved or monotonically decreasing (which is the case in spherical galaxies), such a cluster will pass the same or an even stronger tidal field within the next crossing time-scale until it would become eroded. In all computed galaxies, the vast majority of crossing time-scales is significantly below the total duration of the integrations, thus yielding safe lower limits on the number of disruptions. A more complicated situation emerges in galaxies deviating from spherical symmetry due to the existence of trajectories where the directional components of the angular momentum vector change in time. In such galaxies, a GC passing a region in which $r_{1/2}/r_J = 0.5$ might not repeat doing so for a long time. The criterion for disruption processes by tidal shocks in non-spherical galaxies represents a much more challenging task and will be part of future studies. We also note that our disruption criterion was adjusted by means of direct NBODY6 integrations in one particular galaxy model as well as by using one particular cluster model. However, in order to compensate these shortcomings, we changed the parameter $x$ to even higher values than $x = 0.5$ and discussed the systematics. We found no quantitative differences in the outcomes. In addition to that, our SEV and relaxation-driven dissolution implementation (Section 2.2.1) was calibrated in direct $N$-body computations (Baumgardt & Makino 2003) which are based on a Kroupa initial (stellar) mass function (IMF) with lower and upper mass limits 0.1 and 15 $M_\odot$. The (initial) mass-loss through SEV would increase by using a higher upper mass limit. However, this would mostly affect the initial correction factor which has no influence on the shape of the single power-law GC mass distribution (Section 2.3).

(ii) In our computations, GC half-mass radii were distributed according to relation (9) and were then integrated by leaving their sizes unchanged. The strength of tidal shocks and hence the efficiency of tidal disruption processes depends on the compactness (i.e size) of GCs. Therefore, the percentage of disrupted GCs during the TDDP depends on initial cluster sizes. If GCs would be much more compact after their rapid gas expulsion phase, the impact of the TDDP on the overall cluster erosion rate would be reduced. In future studies, more realistic initial conditions as well as cluster size evolution should be included. However, the same criticism also applies to the used galaxy models which in this study were assumed to be non-evolving. van Dokkum et al. (2008) show that massive elliptical galaxies at high redshifts were more compact than today. In more compact progenitor galaxies, the TDDP on the other hand would be very pronounced and might quickly transform a single power-law cluster mass function into a bell-shaped form. After (dry) merging processes, these galaxies will inflate their sizes.
but the bell-shaped cluster mass function should remain unaffected.
In conclusion, the efficiency of the TDDP depends on cluster and
galaxy size evolution.

(iii) Massive (elliptical) galaxies display a bimodal colour dis-
tribution of GCs which have different metallicities, kinematics and
number density profiles (Zepf & Ashman 1993; Forbes, Brodie
& Grillmair 1997; Brodie & Strader 2006; Forbes, Ponman &
O’Sullivan 2012). These cluster populations are leftovers of dif-
ferent star formation events. The red and metal-rich GC population
is centrally more concentrated and follows the stellar light profile
of its host galaxy, whereas the number density profile of metal-poor
GCs is flatter and dominates the GC system at large distances. Our
computations address the evolution of the GC distribution which
traces the galaxy light and we neglect GC populations which were
formed (or accreted) later on.

(iv) The compact dwarf galaxy M32 does not contain any GCs.
Our computations indicate that they might have been eroded in
the strong tidal field of this galaxy. The real stellar density profile
of M32 deviates at distances below 15 arcsec (≈55 pc) and above
100 arcsec (≈370 pc) from a Sérsic $n = 4$ profile (Kent 1987)
which we used in our computations. The central density inside M32
is even higher than the corresponding density of an $n = 4$ profile
(see fig. 4 in Kent 1987). This would result in an even stronger
tidal field and an increase of the actual disruption rate. Therefore,
our results concerning the erosion rate in compact M32-like dwarf
galaxies with Hernquist or Sérsic $n = 4$ density profiles should

Figure 11. Final (3D) anisotropy profiles of the surviving GC population, $\beta = 1 - (\sigma_\theta^2 + \sigma_\phi^2)/2\sigma_r^2$, as a function of the galactocentric radius. The error values are obtained by the bootstrapping method. All GCs are weighted equally without discrimination of their mass/luminosity. In all computed models, the central velocity distribution becomes tangentially biased. In configurations with initial radial anisotropy, the tangentially biased region develops within the 3D half-mass radius. For MOD2 and MOD3, only clusters above the mass threshold $m_{GC} = 10^4 M_\odot$ are taken into account (blue dashed curves). Unfiltered values are plotted in MOD4.
Figure 12. Left-hand panels: initial (black) and final (blue histograms) GC mass functions. Error bars show Poisson uncertainties. A moderate degree of radial anisotropy ($R_A/R_H = 1$) transforms initial power-law distributions into bell-shaped curves (MOD2 and MOD3). The inclusion of a primordial gas expulsion phase (green histograms and curve, MOD4) contributes to the shaping of a bell-shaped mass function even in the most massive galaxies. Right-hand panels: the GC mean mass versus galactocentric distance obtained for 2D radial binning. Uncertainties were derived via bootstrapping. Despite a large degree of scatter MOD2 and MOD3 are compatible with a constant mean mass over a broad range of galactocentric distances (see text for more details). The mean mass centres around $(3-4) \times 10^5 M_\odot$ and is strongly affected by a small number of GCs more massive than a few $10^6 M_\odot$. By assuming a Schechter-type function for initial cluster masses with a particular turn-down mass in the high-GC-mass regime, the final mean mass might naturally lower to $2 \times 10^5 M_\odot$. At least in spiral galaxies in which the initial cluster mass distribution can be observed directly, it is compatible with such a Schechter mass function (Larsen 2009).

Hold for M32 itself. However, these results should not be applied to ‘more common’ dSph which are less massive, less dense and have shallower density profiles (e.g. $n = 1$). The GC erosion rate in dSphs is reduced as indicated by the one computation with a Sérsic $n = 1$ density profile (Fig. 7). (v) As already mentioned, our computations are governed by the stellar density profiles specified in Section 2.3.2. The next logical extension would be to use the cumulative density profiles from the stellar, dark matter and gas component. It has to be investigated whether the extended isothermal density profiles of DM haloes would significantly alter the GC erosion rate which is dominated by tidal effects deep within the galaxy where the stellar density dominates.

(vi) DF (Section 2.2.3) is implemented as an external routine in the MUESLI code. While this is a commonly used strategy in numerical investigations, care has to be taken. By assuming GCs to be immune to dissolution processes, all of them would accumulate within given time periods near the centre of the galaxy, driving the mass density upwards. In reality, DF is an energy conserving process and while compact objects spiral inwards, stellar mass is driven outwards. These back-reaction effects are not considered in this study. However, due to the sub-dominant role of DF in our computations, back-reaction processes will have a minor impact on the inferred results. (vii) The main focus of this paper is about destruction rates of GCs by tidal shocks and relaxation-driven mass-loss in tidal fields of spherically symmetric galaxies. We kept it simple and neglected the fate of dissolving GCs and how their debris might affect internal dynamics of galaxies, e.g., by forming a nuclear star cluster (Tremaine et al. 1975; Agarwal & Milosavljević 2011; Antonini 2013; Gnedin et al. 2014). These issues as well as direct SMBH loss cone studies will be part of later studies. To handle them with our MUESLI code requires detailed understanding of GC dissolution mechanisms in evolving galaxies. Nevertheless, our computations already indicate a chronological aspect in the erosion of GC systems which might be of relevance for the fast build-up of massive black holes in the early Universe.

5 CONCLUSIONS

We developed a versatile code, named MUESLI, designed to investigate the dynamics and evolution of GC systems in elliptical galaxies. It uses the SCF method with a TTL scheme to integrate orbits of field stars and GCs. In this way, velocity-dependent forces like DF and
post-Newtonian effects of central massive black holes can be handled accurately. In order to be able to treat spherical galaxies with anisotropic velocity distributions (as well as non-spherical galaxies), the code uses the ellipsoidal generalization of Chandrasekhar’s DF formula (Pesce et al. 1992). The advantage of MUESLI lies in its flexibility to evaluate the impact of complex physical processes on the erosion rates of GCs in evolving galaxies.

In a first application, we have investigated if flat central cores in GC distributions around massive elliptical galaxies result from tidal disruption events and cluster dissolution processes through relaxation. Furthermore, we explored the question if the strong tidal field within the compact dwarf galaxy M32 is responsible for lack of GCs in this galaxy.

We used a power-law distribution for the GC masses, and set the initial phase space distribution of the GCs equal to the stellar phase space distribution of the host galaxy. The rapid phase of gas expulsion was ignored with the exception of one model. We assumed two cluster dissolution channels: (i) a slightly modified version of relaxation-driven mass-loss in tidal fields (which also handles SEV) from Baumgardt & Makino (2003) was implemented. Once a cluster mass becomes less than \( m_{\text{GC}} = 100 M_{\odot} \), it is assumed to be dissolved by relaxation. Additionally (ii), we identified a tidal disruption criterion in terms of the ratio of cluster half-mass radius, \( r_{1/2} \), to Jacobi radius, \( r_J \), in that no cluster was able to survive for a significant amount of time, when the ratio \( x = r_{1/2}/r_J \) passed a threshold of \( x = 0.5 \). The condition for GC disruption in tidal fields was calibrated by means of direct N-body experiments. For this purpose, we used the star cluster code NBODY6 to compute the evolution of massive clusters on various orbits within the tidal field of a host galaxy.

We found that, after 10 Gyr of evolution, all computed GC systems show signs of central flattening with the central core size depending in a non-trivial way on the mass, scale and anisotropy profile of the host galaxy and threshold GC mass. Galaxies with highly radially biased velocity distributions lose a significant fraction of clusters also at large galactocentric radii. As a result, the cores, in their central density profiles, are less pronounced than in galaxies with isotropic distributions. The primary factors which determine the disruption rate of GCs are the half-mass radius and mass of the galaxy and the initial degree of radial anisotropy of the GC system. For host galaxies with an isotropic velocity distribution, the fraction of disrupted GCs is nearly 100 per cent in very compact, M32-like dwarf galaxies.

The rate is lowest in the most massive and extended galaxies (50 per cent) like NGC 4889. The arithmetic mean radius, \( R_D \), where most GC destruction occurred during the last 10 billion years, is roughly equal to the (3D) half-light radius \( R_{1/2} \) in compact dEs and drops to 0.15\( R_{1/2} \) in massive elliptical like NGC 4889. An isotropic initial velocity distribution is mostly preserved at large radius \( R > R_{1/2} \), while the GC velocity profile close to the galactic centre become less radial or even tangentially biased. Different degrees of initial radial anisotropy may be the reason for a considerable scatter in the total number of GCs around more massive elliptical galaxies (see Table 2). In compact M32-like galaxy models with radial anisotropy no single GC survived.

The influence of DF on the overall GC erosion rate in massive elliptical galaxies is insignificant as long as the initial cluster mass function follows a power-law distribution with slope \( \beta = 2 \). However, DF yields a small contribution in compact dEs like M32. Secondary effects like the density profile or the presence of a central massive black hole manifest their influence only in the most massive and extended galaxies. An ultramassive black hole with a mass above 10 billion solar masses inside a galaxy like NGC 4889 has a considerable impact on tidal disruption processes. Its presence increases the total fraction of destroyed GCs during the violent phase of tidal disruptions by 2–5 per cent in absolute terms.

We also found that GC erosion processes result in a bell-shaped GC mass function and a nearly constant relation between GC mean mass and galactocentric distances as long as the galaxies are not too extended and radially biased. Observations of bell-shaped GC mass functions in extended galaxies may indicate that their GC populations were formed in more compact building blocks of these galaxies, which later merged to form the present-day host.

Finally, our results show a strong chronological aspect in the evolution of GC systems. That is, most tidal disruptions occur at early times, on dynamical time-scales of the host galaxy. Hence, we call this a TDDP in the evolution of GC systems. Our simulations strongly suggest that the number of GCs in most galaxies was much higher at their formation. Therefore, depending on the fraction of stars in a galaxy which were born in GCs, the debris of the disrupted clusters should constitute a significant amount of a galaxy’s field population. In the extreme case that all stars in galaxies were born in GCs, our study would imply that larger galaxies like NGC 4889 have to be the merger product of many smaller galaxies and/or that the progenitor galaxies were initially much more compact because otherwise 10–50 per cent of its stellar mass would still have to be locked up in GCs (Fig. 6). Given the fact that only about 0.1 per cent of all stars seem to be locked up in GCs nowadays, our study prefers building blocks of galaxies in the early Universe to either have a small fraction of stars being born in very massive GCs, or being relatively compact like M32, or having highly radially biased GC distributions.

Interestingly, we predict the field population coming from disrupted GCs to have complementary orbital properties to the phase space distribution of the surviving clusters. Moreover, we predict the centrally cored GC distributions around SMBHs to be tangentially biased, and thus parts of the field star population to have a pronounced radially biased component from cluster debris. The diffusion of this cluster debris in phase space (in combination with gravitational focusing relevant for unbounded matter) might therefore contribute to the rapid growth of SMBHs in the early Universe through the refilling of the black hole loss cone. To which degree will be subject to a future study.

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APPENDIX A: TESTING

A1 Discreteness noise

When the potential becomes updated in time intervals $\Delta t_{\text{up}}$ by the SCF algorithm, fluctuations in the overall particle distribution give rise to irregular oscillations of the virial ratio (Hernquist & Ostriker 1992). Additionally, improper selection of the expansion coefficients, especially the angular expansion order leads to multipole-induced precession.

We inferred the magnitude of these fluctuations, which inversely ($\propto N^{-0.5}$) depends on the particle number, by the computation of spherical and axisymmetric Sérsic $n = 4$ and Hernquist profiles. All models were evolved forward up to 100 $N$-body time units. We varied the particle number ($N = 2 \times 10^5$ and $N = 4 \times 10^6$), the time-scales of potential evolution ($\Delta t_{\text{up}} = 1$ and 100) as well as the radial and angular expansion order (between $l = 0$ and 20). In this way, we were able to estimate the fluctuations and their relevance for the accuracy of orbit integrations of elliptical galaxy models. For testing purposes (only), the axisymmetric models were generated by simply reducing the $z$-components by a factor of 2. Without adjusting particle velocities properly, axisymmetric models generated in such a way are dynamically unstable. The only reason for using them instead of existing virialized axisymmetric models generated from cold collapse computations was subject to more controlled conditions required for the evaluation of discreteness noise effects. Orbits in axisymmetric models were therefore computed in fixed potentials, i.e. by using $\Delta t_{\text{up}} = 100$.

Ideally, a test particle with zero velocity in $z$-direction has to orbit the galaxy without changing its $z$-component. Hence, the maximal drift of the angular momentum vector $L$, i.e. the maximal angle $\alpha_{\text{m}}$ between $L_{\text{ini}}$ and $L_{\text{fin}}$, was used as one criterion for the accuracy of orbit integration. As an additional criterion we have used the standard deviation $\sigma_r = \left( \frac{1}{N} \sum_i^{N} \left( r_i - \bar{r} \right)^2 \right)^{1/2}$ from circular motion at different galactocentric distances $r = 0.1, 1, 10$. This quantity reveals the magnitude of potential fluctuations.

Apparently, relaxation arising from discreteness noise is subdominant when performing integrations with high particle numbers ($N \geq 4 \times 10^6$) and by using low angular expansion terms, i.e. $l = 0$ for the computation of spherical galaxies. In axisymmetric or triaxial galaxies, torques from the overall matter configuration are orders of magnitudes larger than local anisotropies or discreteness noise. The integration inaccuracies are listed in the Table A1. On the basis of tabulated data, several trends can be obtained. The amplitude of discreteness noise effects are anticorrelated with the total number of particles. Torques induced by angular multipole expansion affect mostly trajectories with short orbital periods while the precession diminishes altogether by using lowest order (spherical) terms. Converging solutions of the underlying density profile are obtained with high radial order terms. For the parameter space of our main computation, radial orbit fluctuations correspond to only a few tens of parsec over time-scales of several billion years when scaled to the proportions of giant elliptical galaxies like NGC 4889 (Section 2.3.2) and sub-parsec scales for the smallest galaxies.

A2 The conservation of the space phase distribution

The order of the radial expansion coefficient, $n$, required to guarantee the conservation of the initial space phase distribution is investigated in more detail in this section. For that purpose, we evolved spherical Hernquist, Jaffe and Sérsic models forward in time up to 100 $N$-body time-scales ($\Delta t_{\text{up}} = 1$). Several different values for the radial expansion coefficients, $n$, were chosen. Afterwards, we compared the numerical outcomes to the initial models in terms of density profiles, axis ratios at several Lagrange radii ($1, 2, 5, 10, 25, 50, 60, 70, 80, 90$ per cent), radial and tangential velocity dispersion as well as velocity anisotropy parameter, $\beta(r) = \left( 1 + R^2_{\text{fric}} / r^2 \right)^{-1}$. All profiles were found to be perfectly stable when represented by high radial $n = 30$ and lowest angular order ($l = 0$). This is in agreement with the results obtained in Appendix A1. These are the coefficients adopted for the main computations. For illustration, the velocity dispersion profile of an anisotropic Hernquist model with scalelength $a = (1 + \sqrt{2})^{-1}$ is compared to the analytical profile in Fig. A1.

The expansion coefficient evaluation of axisymmetric and triaxial models is postponed. Mesh effects and the DF routine (Section 2.2.3) are investigated and tested in the next section (Appendix A3).

A3 DF and grid effects

The purpose of this section is two-fold. The DF routine itself has to be tested and compared with analytical predictions of idealized DF problems. Systematic effects caused by discreteness noise and grid selection effects have to be evaluated in isotropic and anisotropic velocity distributions as well. In principle, they can affect the computations and the calculus of the velocity dispersion tensor which is required for the generalized DF force (equation 5). We numerically evaluated the inspiral time $t_{\text{GC}}$ of a GC on a circular orbit ($r = 5$ kpc) by using equation (5) as well as Chandrasekhar’s (standard) DF formula for a Maxwellian velocity distribution as

$$a_{\text{GC}} = -\frac{4\pi G^2 m_{\text{GC}} \rho(r) \ln(A)}{v_{\text{GC}}^2} \times \left[ \text{erf}(X) - \frac{2X}{\sqrt{\pi}} \exp(-X^2) \right] v_{\text{GC}}.$$  \hspace{1cm} (A1)

Here, $X = v_{\text{GC}} / (\sqrt{2} \sigma)$ and $\sigma = \sqrt{\sigma_r^2 + \sigma_z^2 + \sigma_\theta^2}$ is the one-dimensional velocity dispersion obtained from the eigenvalues of the local velocity dispersion tensor. The numerical outcomes were then compared with analytical predictions of a decaying

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The radial velocity dispersion, $\sigma_r^2$, is a sensible indicator for model stability by tracing minuscule instabilities along $r$.

The Hernquist base function of the SCF algorithm uses a different scale-length $a$, hence, a high radial order is required for these models as well.
Table A1. Parameters of the discreteness noise evaluation. $N$ specifies the total number of particles and $\Delta t_{\text{imp}}$ is the characteristic time-scale at which the overall potential becomes re-evaluated by the SCF algorithm. The parameters $n, l$ correspond to the order of the radial and angular expansion terms. $\sigma_i$ measures the standard deviation from circular motion at three different radii ($r = 0.1, 1, 10$) and $\alpha_{\text{im}}$ traces the maximal angular deviation caused by multipole-induced precession at the same radial distances. It is given in radians. There is no dipole moment induced precession ($l=1$) since all particles were initially inverted (and doubled) at the origin. The main computations are performed with five times higher particle numbers than the largest test models.

| Hernquist | $N$ | $\Delta t_{\text{imp}}$ | $n$ | $l$ | $\sigma_i(r = 0.1, 1, 10)$ | $\alpha_{\text{im}}(r = 0.1, 1, 10)$ |
|-----------|-----|--------------------------|-----|-----|--------------------------|-----------------------------|
| Spherical | $2 \times 10^3$ | 1 | 30 | 0 | 0.002/0.006/0.003 | 0/0/0 |
| Spherical | $2 \times 10^3$ | 100 | 30 | 0 | 0.0001/0.0001/0.0004 | 0/0/0 |
| Spherical | $2 \times 10^5$ | 1 | 10 | 10 | 0.008/0.01/0.003 | 1.8/0.1/0.001 |
| Spherical | $2 \times 10^5$ | 100 | 10 | 10 | 0.003/0.002/0.007 | 1.8/0.08/0.002 |
| Spherical | $2 \times 10^5$ | 100 | 10 | 5 | 0.003/0.002/0.006 | 1.9/0.07/0.0006 |
| Axis-sym. | $2 \times 10^5$ | 100 | 12 | 1 | - | 0/0/0 |
| Axis-sym. | $2 \times 10^5$ | 100 | 12 | 2 | - | 0.06/0.008/0.0001 |
| Axis-sym. | $2 \times 10^5$ | 100 | 12 | 5 | - | 0.3/0.008/0.0007 |
| Axis-sym. | $2 \times 10^5$ | 100 | 12 | 10 | - | 0.1/0.02/0.001 |
| Axis-sym. | $2 \times 10^5$ | 100 | 20 | 20 | - | 0.09/0.03/0.003 |
| Spherical | $4 \times 10^6$ | 1 | 30 | 0 | 0.0009/0.002/0.0005 | 0/0/0 |
| Spherical | $4 \times 10^6$ | 100 | 30 | 0 | 0.0001/0.0002/0.0009 | 0/0/0 |
| Spherical | $4 \times 10^6$ | 1 | 10 | 10 | -0.005/0.0003 | -0.01/0.0001 |
| Spherical | $4 \times 10^6$ | 100 | 10 | 10 | 0.0001/0.0003/0.0003 | 1.4/0.02/0.0003 |
| Spherical | $4 \times 10^6$ | 100 | 10 | 5 | 0.0001/0.0002/0.0003 | 1.4/0.01/0.0005 |
| Axis-sym. | $4 \times 10^6$ | 100 | 12 | 1 | - | 0/0/0 |
| Axis-sym. | $4 \times 10^6$ | 100 | 12 | 2 | - | 0.01/0.001/0.00002 |
| Axis-sym. | $4 \times 10^6$ | 100 | 12 | 5 | - | 0.03/0.001/0.0001 |
| Axis-sym. | $4 \times 10^6$ | 100 | 12 | 10 | - | 0.3/0.002/0.0001 |
| Axis-sym. | $4 \times 10^6$ | 100 | 20 | 20 | - | 0.3/0.003/0.0003 |
| Sérsic $n = 4$ | $N$ | $\Delta t_{\text{imp}}$ | $n$ | $l$ | $\sigma_i(r = 0.1, 1, 10)$ | $\alpha_{\text{im}}(r = 0.1, 1, 10)$ |
| Spherical | $2 \times 10^6$ | 1 | 30 | 0 | 0.001/0.0004/0.002 | 0/0/0 |
| Spherical | $2 \times 10^6$ | 100 | 30 | 0 | 0.0003/0.0005/0.0002 | 0/0/0 |
| Spherical | $2 \times 10^6$ | 1 | 10 | 10 | 0.005/0.02/0.003 | 1.7/0.03/0.002 |
| Spherical | $2 \times 10^6$ | 100 | 10 | 5 | 0.001/0.002/0.01 | 0.4/0.04/0.001 |
| Spherical | $2 \times 10^6$ | 100 | 12 | 1 | - | 0/0/0 |
| Spherical | $2 \times 10^6$ | 100 | 12 | 2 | - | 0.03/0.02/0.0004 |
| Spherical | $2 \times 10^6$ | 100 | 12 | 5 | - | 0.01/0.02/0.0004 |
| Spherical | $2 \times 10^6$ | 100 | 12 | 10 | - | 0.02/0.02/0.0005 |
| Spherical | $2 \times 10^6$ | 100 | 20 | 20 | - | 0.02/0.02/0.0002 |
| Spherical | $4 \times 10^6$ | 1 | 30 | 0 | 0.0003/0.0009/0.0009 | 0/0/0 |
| Spherical | $4 \times 10^6$ | 100 | 30 | 0 | 0.0004/0.0004/0.0004 | 0/0/0 |
| Spherical | $4 \times 10^6$ | 1 | 10 | 10 | 0.002/0.002/0.002 | 0.6/0.009/0.0001 |
| Spherical | $4 \times 10^6$ | 100 | 10 | 10 | 0.0007/0.0005/0.002 | 0.5/0.02/0.0003 |
| Spherical | $4 \times 10^6$ | 100 | 10 | 5 | 0.0007/0.0005/0.002 | 0.5/0.02/0.0003 |
| Axis-sym. | $4 \times 10^6$ | 100 | 12 | 1 | - | 0/0/0 |
| Axis-sym. | $4 \times 10^6$ | 100 | 12 | 2 | - | 0.004/0.001/0.0001 |
| Axis-sym. | $4 \times 10^6$ | 100 | 12 | 5 | - | 0.007/0.002/0.0004 |
| Axis-sym. | $4 \times 10^6$ | 100 | 12 | 10 | - | 0.008/0.004/0.0005 |
| Axis-sym. | $4 \times 10^6$ | 100 | 20 | 20 | - | 0.01/0.005/0.0005 |

$(m_{\text{GC}} = 10^7 \, M_\odot)$ GC orbit and plotted in Fig. A2. The spherical galaxy had an isotropic velocity distribution, Jaffe density profile (Jaffe 1983), total mass $M_{\text{gal}} = 10^4 \, M_\odot$ and half-mass radius $R_{\text{H}} = 5 \, \text{kpc}$, similar to the properties of NGC 4494 (Section 2.3.2). We selected a Jaffe profile because of its analytical simplicity. Furthermore, it deviates from the underlying base model of the SCF algorithm. In this way, the reliance on higher order base functions was automatically tested as well. We fixed the Coulomb logarithm to $\ln \Lambda = 6$ in order to ease the analytical calculation. Grid selection effects were investigated by carrying out integrations with $N = 4 \times 10^6$ particles and several realizations of the $5 \times 5 \times 5$ grid (Section 2.2.3). Evidently, Chandrasekhar’s (standard) DF formula, its triaxial generalization and the analytical prediction yield (nearly) equal inspiral times $t_{\text{imp}}$ despite the highly non-linear character of DF (Fig. A2). Owing to the IDW interpolation method (Section 2.2.3), the chosen grid configuration does not have any visible systematic effect on $t_{\text{imp}}$. And indeed, the differences in $t_{\text{imp}}$ caused by grid selection effects are only of the order of 200 Myr, compared to $\approx 20 \, \text{Gyr}$ absolute integration time, or 1 per cent in relative terms. Without the IDW interpolation method, they are much bigger (of the order of 25 per cent). The generalized triaxial DF routine (equation 5) yields slightly larger ($\approx 4\%$) inspiral times than the standard DF formula. The reason for this are discrete-ness noise fluctuations in the evaluation of the eigenvalues $\sigma_i$ in
Erosion of globular cluster systems

Figure A1. Radial and tangential velocity dispersion profile of an anisotropic Hernquist model ($R_A = Rh = 1, N = 4 \times 10^6$) which was dynamically evolved over 100 N-body time-scales. Afterwards, we rescaled the model to $Rh = 1 + \sqrt{2}$ in order to compare it with the analytical velocity dispersion profile for anisotropic systems (Baes & Dejonghe 2002). The usage of high radial order terms ($n = 30$) in combination with lowest order (spherical) angular terms ($l = 0$) yields accurate results.

To further explore mesh and discreteness noise effects in anisotropic velocity distributions, we performed a second test. The orbits of 10 000 globular clusters in a strongly triaxial galaxy with a cored density profile and a radially biased velocity distribution at large distances ($\beta \approx 1$) were evolved forward in time under the influence of the generalized DF force (equation 5). The generation of this particular model is described in Section 2.3.2. Total mass and scale of the galaxy were assumed to be identical to the test before. For this test, we restrict to tidal disruption processes from equation (2) and neglect the long-term energy-equipartition-driven evaporation. The cluster masses and sizes were distributed according to the single power-law mass spectrum (equation 8) and present-day half-mass relation (equation 9). Discreteness noise and mesh effects were artificially enhanced by computing a small number $N = 5 \times 10^5$ model over 200 N-body time-scales ($\approx 3.5$ Gyr). Moreover, the overall potential was frequently updated by the SCF algorithm in $\Delta t_{\text{scf}} = 1$ intervals. We then compared the time-scales until given fractions of GCs were destroyed in computations with identical physical properties apart from different grid size realizations. In this way, mesh effects were isolated and analysed. The numerical outcomes are shown in Fig. A3. Evidently, the IDW interpolation method narrows grid effects down to insignificant values. The implemented DF routines yield credible results with insignificant errors.

A4 Potential fluctuations and their relevance for globular cluster disruption

Finally, the performance of the disruption routine (Section 2.2.2) has to be evaluated. Notwithstanding that the base functions as well as their derivatives are continuous, summing up these functions might lead to wiggles along the radial direction. It may therefore be possible that the radial acceleration $a_r(r)$ as well as the evaluation of the Jacobi radius (equation 2) are affected and a cluster is incorrectly assumed to be destroyed by tidal forces. To guarantee computational outcomes free of biased GC disruption rates, we performed
several test integrations by considering tidal disruption processes only, i.e. by neglecting the long-term relaxation-driven mass-loss. Isotropic Hernquist models ($N = 10^6, n = 30, l = 0, \Delta t_{\text{up}} = 1$) with 5000 randomly distributed clusters were evolved forward in time. They were scaled to the physical properties of MOD1 (M32) and MOD4 (NGC 4889, Table 1), the two galaxies in our sample with the most extreme GC-to-galaxy half-mass ratios, $r_H/R_H$. For each of these models, the Jacobian radius was evaluated directly from the SCF algorithm as well as by using an analytical expression for the radial acceleration. The differences in the total number of disrupted clusters $N_{\text{dis}}$ were below 0.05 per cent (M32) and 0.3 per cent (NGC 4889). This corresponds to absolute discrepancies of two and one globular cluster(s), respectively. Being so small in magnitude, these fluctuations can be neglected, particularly because the main computations are performed with twenty times higher particle numbers and without re-evaluation of the potential.

**Figure A3.** Owing to the IDW interpolation method, different grid configurations no longer have any discernible effects on the cluster tidal disruption rate. $R_{\text{max}}$ is the outermost mesh size which affects the positioning of the inner grid cells.

**APPENDIX B: EXTREME RADIAL ANISOTROPY**

The final number density profile of a radially biased Sérsic $n = 4$ model with extreme anisotropy, $R_A/R_H = 0.25$, is plotted in Fig. B1. The profile shows a less pronounced core than the corresponding isotropic configuration (Fig. 9). The reason for this is that clusters all along the spatial extent of the galaxy are eroded efficiently. However, at very large galactocentric distances, the slope becomes steeper. This is due to the fact that pericentre distances at a given galactocentric distance start to decrease again. This effect is illustrated by means of independent Monte Carlo computations in Fig. B2. Although there are reports on galaxies in which the globular cluster system profiles at large radii are indeed steeper than the surface brightness profile of their host galaxies, e.g. NGC 4406 (M86; Rhode & Zepf 2004; Capuzzo-Dolcetta & Mastrobuono-Battisti 2009), such a highly radially biased (initial) GC configuration does not represent a plausible explanation, especially as the degree of radial anisotropy remains still very large at huge galactocentric distances. In the case of M86, it was suggested by Rhode & Zepf (2004) that tidal truncation induced by M84 or even the potential well of the Virgo cluster is responsible for the steep profile. The much flatter outer GCS profile of its companion M84 might support this scenario (Capuzzo-Dolcetta & Mastrobuono-Battisti 2009).
Figure B2. Monte Carlo realizations of pericentre versus galactocentric distances of reference model MOD4 with a Sérsic $n = 4$ density profile, central SMBH and a strongly radially biased (left-hand panels) and isotropic (right-hand panels) velocity distribution. The central (black) data points represent the arithmetic mean. A truncation in the strongly radially biased configuration is evident. It explains why the outer GC profiles in the $R_A/R_H = 0.25$ models are more strongly influenced beyond a characteristic radius being of the order of $R_A = 8.5$ kpc.