Development of a dynamic pressure standard of low amplitudes and frequencies

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Abstract
The height of the gravitational wave is an influential magnitude in the estimation of its energy content, a very important parameter in the design of maritime structures such as piers and breakwaters; however, there are reasonable doubts among researchers regarding the quality of its measurement. The objective of the present work was to demonstrate that a stationary calibrated pressure transmitter allows the indirect calculation of the wave height with satisfactory accuracy despite the fact that it is a dynamic event as long as its frequency range is low, as is the case with the gravitational waves of the Costa Rican central Pacific Ocean: between 0.05 and 0.39 Hz. In the absence of a primary standard of periodic pressure disturbances, an alternate path was developed based on the characterization of the parameters of the differential equation characteristic of a pressure measurement system in a shock tube prototype from normal shock wave theory and subsequent verification in a periodic disturbance generator that its attenuation and delay are practically negligible at the frequency of 0.1 Hz. The effect of the different geometry of the chamber of the pressure measurement system used in the sea was evaluated pneumatically by comparison with the dynamic pressure standard in the prototype of the periodic disturbance generator, while the effect of the compressibility of the fluid was evaluated in water in the wave channel of the Laboratory of Maritime Engineering, Rivers and Estuaries of the University of Costa Rica. The results show that the pressure measured from the least-squared adjustment coefficients of an electric current transmitter obtained by stationary calibration is acceptable to estimate the climatology of the gravitational wave characteristic of the Costa Rican central Pacific Ocean with maximum errors of 136 mm in waves of height up to 1.4 m.

Keywords: dynamic calibration, pressure transducers, shock tube, periodic pressure generator

(Some figures may appear in colour only in the online journal)

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1. Introduction

Currently, the applications of measurements in dynamic processes are increasingly relevant, covering different fields in the technological, medical, and academic sectors \[1\].

This work shows a very specific application related to the measurement of the height of the gravitational wave, a dynamic event characterized by waves with periods from 1 to 30 s and heights up to 15 m \[2\]. The energy content of the gravitational wave is transferred by the wind, and its estimation from the height of the wave defines the design of maritime structures such as marinas and breakwaters.

Fortunately, the gravitational wave on the coasts of the Costa Rican central Pacific Ocean is a dynamic event characterized by waves of low height and frequency: between 0.29 and 4.22 m and between 0.39 and 0.05 Hz respectively \[3\], by the “shield cone” effect that Cocos Island produces to gravitational wave trains formed in the far south Pacific Ocean near New Zealand.

The wave height is calculated indirectly by the pressure of the seawater column according to Bernoulli’s linearized equation \[2\], which for the case of interest represents between 2.81 and 42.31 kPa.

To measure the equivalent pressure of the wave height in the project range, it is possible to use the measurement system \[4\] GW WL16U-030-050 \[5\], composed of a measurement chamber and data acquisition (DAQ) system with low sampling frequency \(f_m \leq 10 \text{ Hz}\).

The measurement chamber is divided into two parts: one is a metallic resistive sensor on a silicon diaphragm exposed to the hydrostatic pressure of the seawater column, electrically connected to a conditioning circuit of the analog electrical signal with a transmitter of 4–20 mA DC electric current located in the other hermetic part of the measuring chamber.

The transmitter cables are connected to the DAQ by a flexible hollow tube of variable length depending on the model, which also allows compensation of the measurements by atmospheric pressure.

Two batteries allow for autonomy of the measurement system until the programmed download of the information through the communication port and the software to the personal computer (PC).

Figure 1 shows a simplified representation of both the gravitational wave (oscillating line in blue) of constant period \(T_p\) as well as the installation of the GW WL16U-030-050 measurement system on the seabed to measure the wave height:

Although the GW WL16U-030-050 \[5\] measurement system is used in a dynamic environment, its analog electrical output is converted into gauge pressure by means of a line of coefficients adjusted by the ordinary least-squares (OLS) method in a direct comparison with a standard pressure in an isobaric environment.

This contradictory situation between the stationary calibration of the measurement system and its application in a dynamic event caused a reasonable doubt among the researchers of the Laboratory of Maritime Engineering, Rivers and Estuaries (IMARES) of the UCR, for which it was proposed to demonstrate that the stationary calibration is suitable for measuring the height of the gravitational wave as a dynamic event in the range of the gravitational wave climatology of the central Costa Rican Ocean \[3\].

For this purpose, their corrected response under stationary conditions was compared with the response of a dynamic pressure standard previously characterized in a shock tube, when both were exposed to periodic disturbances reproduced experimentally in a pneumatic generator.

At the end of the process, its corrected response under stationary conditions was compared with the response of the dynamic pressure standard installed in the IMARES wave channel.

2. Materials and methods

2.1. Measurement of fluid pressure

Considering that the response \(p_t\) (hPa) of the pressure measurement system in a fluid at rest is linear:

\[ p_t = S_0 + S_1 i_s \]  

where \(i_s\) (mA) is the pressure transmitter output DC electric current, \(S_0\) (hPa) is the adjusted intercept, and \(S_1\) (hPa mA\(^{-1}\)) is the static sensitivity coefficient.

Then, the coefficients of equation (1) are estimated by the OLS method by direct comparison with a stationary pressure standard that defines its metrological traceability to SI \[4\]:

\[ (S_0, S_1) \rightarrow \left( \hat{S}_0, \hat{S}_1 \right). \]

The comparison is made in an isobaric comparator with adjustable pressure in the range of interest: up to 42.31 kPa, taking care that the standard and calibrating sensors are in the same plane and depth of the working volume.

If the quality of the calibrating response is linearly acceptable, then the fluid pressure is indirectly measured according to the following statistical model:

\[ \hat{p}_t = \hat{S}_0 + \hat{S}_1 i_m \] (2)
where \( \tilde{p}_t \) (hPa) is the measured pressure of the fluid and \( \tilde{e}_m \) (mA) is the output of the measured pressure transmitter. In this case, the measurement system behaves like a zero-order system and is the simplest dynamically.

However, when the pressure measurement system is used to measure dynamic events such as sea waves, then the parameters set in steady state are not in principle adequate to estimate the pressure.

### 2.2. Response to a periodic disturbance

If that same pressure measurement system is disturbed by a perfect sinusoidal signal of amplitude \( A_p \):

\[
p(t) = A_p \sin(\omega t)
\]

where \( \omega \) (rad s\(^{-1}\)) is the frequency of the periodic disturbance and \( t \) is the time (s), then its response in the time domain is also a sinusoidal periodic function but modified in amplitude and phase with respect to the disturbance [6, 7]:

\[
p_t(t) = A_p M \sin(\omega t + \phi)
\]

where \( M \) (dimensionless) is the amplification and \( \phi \) (rad) the phase shift of the response of the pressure measurement system.

The amplification of the response of the pressure measurement system is calculated according to the following equation [6, 7]:

\[
M = \left( (1 - a^2)^2 + (2\zeta a)^2 \right)^{-0.5}
\]

where \( \zeta \) (dimensionless) is the relative damping factor and \( a \) is an auxiliary variable:

\[
a = \omega \times \omega_n^{-1}
\]

where \( \omega_n \) (rad s\(^{-1}\)) is the natural frequency of the pressure measurement system.

The offset of the response of the pressure measurement system is calculated according to the following equation [6, 7]:

\[
\phi = -\tan^{-1}\left[ (2\zeta a) \left( 1 - a^2 \right)^{-1} \right].
\]

The phase shift between the periodic disturbance and the response of the pressure measurement system (\( \Delta \tau \)) can be expressed in the time domain according to the following equation:

\[
\Delta \tau = 1000 - \frac{\text{ms}}{s} \left( \phi \omega^{-1} \right).
\]

Therefore, the challenge is to determine the parameters \( \zeta \) and \( \omega_n \).

### 2.3. Response to an aperiodic shock

If the sensor is exposed to an aperiodic, step-type disturbance of amplitude \( A_p \):

\[
p(t) = \begin{cases} 
0 & \forall \ t < 0 \\
A_p & \forall \ t \geq 0 
\end{cases}
\]

the response in time (\( \forall \ t \geq 0 \)) corresponds to an underdamped second-order measurement system (\( \zeta < 1 \)) [6, 7]:

\[
p_t(t) = A_p \left[ 1 - \exp(-\omega dt) \sin(\omega dt + \cos^{-1} \zeta) \right]
\]

where \( \omega_d \) (rad s\(^{-1}\)) is the damped frequency

\[
\omega_d = \sqrt{1 - \zeta^2 \omega_n^2}.
\]

Once the parameters of the characteristic equation \( \omega_n \) and \( \zeta \) of the dynamic pressure standard have been calculated with the established experimental accuracy, then in principle it is possible to predict their behavior in the face of a periodic disturbance.

If the disturbance to which it is subjected is sinusoidal, its response will have the same shape, although the amplitude and phase may be remarkably different with respect to those of the disturbance.

### 2.4. Reconstruction of the response according to Fourier

However, a periodic signal does not necessarily have to be sinusoidal to evaluate the dynamic behavior of a pressure transducer, and in experimentation, it is practically impossible to reproduce it.

In general, a periodic half-period function \( L_p \) can be “reconstructed” from its corresponding Fourier series if the Dirichlet condition [7] is fulfilled:

\[
x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n \omega t) + b_n \sin(n \omega t)
\]

where \( n = 1, 2, \cdots, \infty \).

\[
a_0 = \frac{1}{L_p} \int_{-L_p}^{L_p} f(t) \, dt
\]

\[
a_n = \frac{1}{L_p} \int_{-L_p}^{L_p} f(t) \cos(n \omega t) \, dt
\]

\[
b_n = \frac{1}{L_p} \int_{-L_p}^{L_p} f(t) \sin(n \omega t) \, dt
\]

\( L_p = 0.5 T_p \).

The amplitude and frequency of the harmonics of the reconstructed signal are calculated according to the following equations:

\[
A_n = \left[ a_n^2 + b_n^2 \right]^{0.5}
\]
where $\omega_1 = \omega_0$ (rad $s^{-1}$) is the fundamental frequency.

Since the mean is not an is-biased estimator for a periodic function, it is preferable to use the root mean square value:

$$\text{RMS} = \left[ \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( \frac{A_n}{\sqrt{2}} \right)^2 \right]^{0.5}$$

2.5. **The shock tube**

The shock tube [6–8] is an aperiodic step-type pressure disturbance generator, suitable for estimating the parameters $\zeta$ and $\omega_n$ of the pressure measurement system.

The prototype developed (figure 2) is made up of two schedule 80 [9] poly(vinyl chloride) (PVC) chambers of constant cross section (76.2 mm): the compression chamber of length $L_{cc} = 423.00$ (0.30) mm and the chamber expansion of length $L_{ce} = 2233.40$ (0.51) mm, between which an aluminum diaphragm (6) of defined thickness is installed.

The two PVC tubular chambers are supported by supports (1) to a horizontal table (2) that ensures the stability of the diaphragm (6) of defined thickness is installed. The inlet of humid air from a compressor is connected by hose and quick coupling (4) to the compression chamber and is controlled by the regulation valve (5) which, when it abruptly opens, causes the necessary overpressure for the diaphragm (6) to burst.

If the air in the expansion chamber behaves like an ideal gas, it is possible to theoretically predict [6–8] the absolute pressure of the air disturbed by the first reflection of the shock wave $P_{S2}$ (hPa):

$$P_{S2} = P_{21} \frac{2^{\omega_1+1}}{\omega_1+1} \frac{P_{21}}{P_{1e}^{-1}} - 1$$

where $P_{21}$ (hPa) is the theoretical absolute pressure of the disturbed gas at position TP2-1, $\gamma_1$ (dimensionless) is the isentropic coefficient of air, and $P_{1e}$ (hPa) is the absolute pressure of the gas at rest.

The absolute pressure $P_{21}$ is calculated according to [6–8]:

$$P_{21} = P_{1e} \left[ 1 + \frac{2\gamma_1}{\gamma_1+1} \left( M_k^2 - 1 \right) \right]$$

where $M_k$ is the Mach number (dimensionless):

$$M_k = a_s a_1^{-1}.$$
2.6. Measurement of pressure transmitter output

The DC electric current from the output of the pressure transmitters ($i_{m}$ in equation (2)) is measured independently, simultaneously, and synchronized by each of the four input channels of the milliammeter of the HBM QuantumX MX440B DAQ system [14] s/n 0095E008567. The milliammeter calibration of the DAQ system is calibrated in steady state by direct comparison with DC electrical current signals from a FLUKE 753 [15] reference source, calibrated in a laboratory authorized by the manufacturer.

2.7. Drying the transmission gas

The means of displacement of the shock wave is the ambient air trapped in the expansion chamber during the assembly of the diaphragm but dried with dry air for medicinal use (purity $\approx 98\%$), which is supplied from its respective container to the expansion chamber through valve V1 and extracted by overpressure, with respect to atmospherics, with valve V2 (figure 2).

The reduction of air humidity is demonstrated by introducing the datalogger HOBO U12-012 [16] 160 mm into the expansion chamber with a sampling frequency of 10 Hz (figure 3).

The drying of the atmospheric air trapped in the expansion chamber is a continuous process of $n_c = 30$ fill–empty cycles with the medicinal dry air.

The drying starts at point P1, and point P2 ends with an effective reduction of the relative humidity up to 25% in 30 min before performing the shot at point P3.

Although the pressure of the gas disturbed by the first reflection of the shock wave remains constant, the duration of the event is so short that it could not be detected by the DAQ system of the GW WLI6U-030-050 [5].

For this reason, the pressure transmitter EXTECH PT30-SD [17] s/n TA87982 was dynamically characterized with a sensor of the same nature, and although the measurement chamber is smaller, it was considered that the resonant effect is similar.

The gauge pressure measurement system consisting of the EXTECH PT30-SD pressure transmitter [17] and HBM QuantumX MX440B DAQ system [14] was considered the dynamic pressure standard of this project.

3. Results and discussion

3.1. Validation of the shock tube prototype

The validation [4] of the TC-02-(0.6/2) shock tube prototype (figure 4) was based on the measurement of its performance parameters when a 10.2 $\mu$m thick aluminum diaphragm bursts and its comparison with those previously obtained in the TC10 shock tube of the Laboratory of Dynamic Measurements of the École Nationale Supérieure d’Arts et Métiers (LMD-ENSAM) [18, 19], with the expectation of 5% of difference (figure 5).

The first performance parameter is the rise time of the air pressure at rest from its perturbation by the first reflection of the shock wave ($\Delta T_{15}$), and is defined by the milestones P1 (shock wave front impact) and P2 (response to disturbance of the sensor) of the EXTECH PT30-SD pressure transmitter installed in the measurement port TP5 (see figure 2).

In principle $\Delta T_{15} \to 0$ but the imperfection of its reproduction, due to the nature of the dynamic pressure standard, prevents its achievement.
The second performance parameter is the pressure stabilization time ($\Delta t_5$) defined by milestones P3 (the arrival of the rarefaction wave coming from the compression chamber of the shock tube causes the pressure drop) and P2.

In principle $\Delta t_5 \to \infty$ but the effect of the successive reflections of the expansion and rarefaction shock waves [7] shortens its constancy.

Table 1 shows the performance of the TC-02-(0.6/2) prototype with respect to the French TC10, measured with the dynamic pressure standard for a sampling frequency equal to 20 kHz, where $\nu = n_d - 1$ is the degrees of freedom and $\nu$ is the number of ‘shots fired’.

The deviation from the supposed isentropic behavior of the shock wave expansion is estimated with the statistical criterion ‘trueness’ [20]. Table 2 shows the results of the comparison of the ‘trueness’ of the pressures measured in the measurement ports TP2-1 and TP5 of the shock tube prototype, where $u_r(y)$ is the relative uncertainty of the parameter $y$ estimated according to the law of propagation of uncertainties [21] and $\nu_{id}$ the effective degrees of freedom according to Welch–Satterwaite [21].

The responses of the corrected dynamic standard in both shock tubes, TC-02-(0.6/2) (in blue) and TC10 (in green), correspond to those of an underdamped second-order measurement system ($\zeta < 1$) according to equation (10), and although there are different methods to adjust the coefficients, the Levenberg–Marquardt numerical method was used (table 3), based on an iterative algorithm of approximations that minimizes the residual of the adjustment.

The maximum relative error of the adjusted coefficients by the Levenberg–Marquardt numerical method in both shock tubes (table 3) did not exceed the 4.82% < 5% established as a goal.

### 3.2. The generator of periodic pressure disturbances

In 1972, Hilton [7] built a periodic pressure generator that was essentially a tube partially filled with liquid, with one end open and the other closed, mounted on an electrodynamic stirrer, to obtain a sinusoidal disturbance.

Based on the results obtained, Hilton [7] recommended abandoning the primary nature of the method and installing a reference pressure transducer alongside the calibrator.

In 1956 Hermann and Stiefelmeyer [7] built a periodic pressure generator based on a rotary valve that generated square signals slightly distorted by the effect of its sequential opening–closing.

More recently, in 2000, Kobata and Ooiwa [22, 23] developed a generator of periodic pressure disturbances based on a rotary valve of a novel design compared to the one built by Hermann and Stiefelmeyer in 1956 [7] to calibrate transmitters of pressure in amplitudes up to 10 kPa and fundamental frequency between 10 and 50 Hz.

Like their predecessors, Kobata and Ooiwa [22, 23] considered that a reference pressure transmitter is required for the dynamic calibration of another pressure transmitter.

### 3.3. The construction of the prototype

In the same way that Hermann and Stiefelmeyer [7] and later Kobata and Ooiwa [22, 23] predicted that a rotary valve would generate a train of square pulses, it was considered that the alternating and synchronized control of the inlet and outlet of the flow of air through valves could generate a train of triangular pulses, with a period and amplitude similar to those of the gravitational waves of the Costa Rican central Pacific.

To this end, the prototype GPP-02A (figure 6) was built between June and October 2018, consisting of a schedule 40 [9] PVC measuring chamber of constant cross section (76.2 mm) and length $L_c = 423.00 \pm 0.30$ mm.

In the measurement chamber, there are two measurement ports: TPe and TPc displaced 90° in the same plane, where the dynamic pressure standard, previously characterized in the shock tube prototype, and the calibrating one are installed.

To experimentally reproduce the periodic pressure signal, the GPP-02A prototype consists of two control systems, the first consisting of the fine adjustment valve (8) and the second consisting of a programmable logic controller (PLC) that controls the solenoids of the ON-OFF valves normally open, installed at the inlet and outlet (positions 5e and 5s) of the measurement chamber.

To reproduce the pneumatic periodic signal with the desired parameters, the following procedure is carried out:
3.4. Validation of the prototype

The characteristic parameters of the periodic pneumatic signals reproduced in the GPP-02A prototype are peak-peak period $T_p$ (s) and amplitude $A_p$ (hPa):

$$A_p = (p_{c,\text{max}} - p_{c,\text{min}})$$

where $p_{c,\text{max}}$ (hPa) and $p_{c,\text{min}}$ (hPa) are the maximum and minimum corrected gauge pressures of the cycle.

The results of a run composed of five reproduced cycles are shown in the figure 7.

The average results obtained (figure 8) were as follows:

$$T_p = (10.030 \pm 0.080 - 0.070) \text{ s}$$

$$A_p = (71.550 \pm 1.163 - 2.262) \text{ hPa}.$$  

The initial assumption of explaining the nature of the signal generated by a triangular pulse was not fulfilled, but considering that the signal is periodic and satisfies the Dirichlet condition, it is ‘reconstructed’ with a Fourier series:

- The experimental distribution of the signal on each side of the half-period was adjusted by linear regression using a polynomial of degree 3, in this case $L_p = 5.015$ s,

- $F_n(t)$ is calculated according to equation (10). In this case, only harmonics were taken into account up to $n = 13$, that is, $F_{13}(t)$.

The terms and parameters of the Fourier series (table 5) are calculated according to the equations in section 2.4.

The superposition of the measurements (red line) together with the curves fitted by the OLS method (blue line) and reconstructed by Fourier series (green line) demonstrate the agreement of the two approaches to explain the periodic event, where $A_p$ (dimensionless) is the amplitude of the signal but refers to the maximum pressure of the cycle.

where $R^2(A_p)$ is the multiple correlation coefficient equal to 1,000.00 for both ranges (table 4).

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The frequency and amplitude spectra obtained from the processing of the average run are shown in table 6.

The spectra obtained show that the effect of frequency on the amplitude of the harmonics of the reproduced periodic signal is practically zero from the second harmonic. In this sense, the observed distortion estimated by the total harmonic distortion (THD = 34.16%) is attributable to the non-linear behavior of the opening–closing valves in the GPP-02A prototype.

In this regard, Kobata and Ooiwa [22, 23] reported that in their periodic generator the assignable cause was the resonant effect of the chamber on the rotary valve.

### 3.5. Applications

Considering that the geometry of the measurement chamber and the low sampling frequency of the GW WL16U-030-050 [5] measurement system prevent its dynamic calibration in the prototype shock tube TC-02-(0.6/2) verified the hypothesis: ‘the least-squared coefficients estimated in a stationary calibration allow the indirect calculation of the wave height with adequate accuracy in the range of low amplitude and frequency typical of the Costa Rican central Pacific Ocean.’

### 3.6. Stationary calibration of the calibrant

The stationary calibration of the calibrant was based on the estimation of its least-squares coefficients by direct comparison with the stationary pressure standard OMEGA PX4200-030G [10] s/n 56901 calibrated in the Pressure Laboratory of the INM of Costa Rica, adjusting the operation of the prototype GPP-02A for its operation as an isobaric comparison medium (table 7).

### 3.7. Dynamic verification of the calibrant

The first verification of the metrological performance of the calibrant was carried out with humid air in the GPP-02A prototype (figure 9), using as references the stationary and dynamically corrected responses of the dynamic pressure standard.

Considering that the period of the disturbance reproduced in the GPP-02A prototype is \( T_p = 10.015 \) s the response of the dynamic pressure standard is defined by the parameters referred in table 8.

From table 8 it is concluded that the response of the dynamic pressure standard is practically not affected by the disturbance: there is no attenuation \( (M = 1) \) and the delay is negligible \((206 \mu\text{m})\), and therefore, the pneumatic signal is a reference reliable to evaluate the response of the calibrant.

Considering that local acceleration of gravity \( g_1 = 9.7818 \text{ m s}^{-2} \) and the density of water \( \rho_w = 997.52 \text{ kg m}^{-3} \), then the comparative results in equivalent water column height \( (H) \), calculated from Bernoulli’s linearized equation is shown in figure 10.
Table 8. Parameters of the response to periodic disturbance.

| Parameters                  | Symbols | MU | Values |
|-----------------------------|---------|----|--------|
| Auxiliary variable          | $a$     | dm | 0.0001 |
| Attenuation                 | $M$     | dm | 1.0000 |
| Phase shift                 | $\phi$  | rad| $-0.0001$ |
| Phase shift in time         | $\Delta \tau$ (average) | $\mu$s | $-206$ |

Figure 10. Comparative responses to a pneumatic disturbance.

Figure 11. IMARES wave channel.

3.8. IMARES wave channel

The second verification of the metrological performance of the calibrant was carried out with water in the IMARES wave channel (figure 11), using as a reference the installed pressure measurement system based on the electrical conductivity of the fluid.

In this case, the calibrant was submerged 1 m from the free surface to avoid the effect of the water column when the wave passed [2] and it was subjected to regular periodic disturbances of period $T_p = 1.8$ s.

Given the same density of the water in the wave channel, the comparative results in equivalent water column height calculated from Bernoulli is shown in figure 12.

From the swarm of measurements, those corresponding to the four cycles were averaged between 30.5 and 37.9 s (figure 13).

The comparative results of the measurements in column of air and equivalents in water according to Bernoulli are shown in table 9.

4. Conclusions

From table 9 it can be seen that the relative errors of the measurements of the wave heights reproduced in air in the generator of periodic disturbances GPP-02A and in water in the wave channel of the CIMAR-UCR are of the same order: $-9.3\%$ vs
−10.5% despite the fact that the pressure measurement systems and the frequencies of the events were different, demonstrating the robustness of the approach.

Therefore, the height of the gravitational wave as a periodic low-frequency event can be measured with a measurement system of the type GW WL16U-030-050 [5] calibrated in steady state with maximum errors of 136 mm in waves of height up to 1.4 m.

As the frequency of the periodic disturbance is not relevant for the established scope, it is possible to use the conventional theoretical framework of the GUM to calculate the uncertainty of the wave height, but taking into account that it is an oscillating event therefore, it cannot apply parametric statistics [24] to estimate the pooled variability under intermediate precision conditions but the arcsine distribution function [21].

The metrological traceability of the indirect water column height measurement systems is managed through the least-squared estimation of the adjustment coefficients in the stationary calibrations.

This property is of great importance when it is not possible to experimentally determine the coefficients of the characteristic equation of the pressure transmitter, either because it cannot be installed in the shock tube prototype or when the DAQ system does not allow an increase in the sampling frequency to detect the milestones of the isentropic expansion of the gas.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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