Non-equilibrium transport through quantum dots with Dzyaloshinsky–Moriya–Kondo interaction

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We study non-equilibrium transport through a single-orbital Anderson model in a magnetic field with spin-dependent hopping amplitudes. In the cotunneling regime it is described by an effective spin-1/2 dot with a Dzyaloshinsky–Moriya–Kondo (DMK) interaction between the spin on the dot and the electron spins in the leads. Using a real-time renormalization group technique we show that at low temperatures (i) the DMK interaction is strongly renormalized, (ii) the renormalized magnetic field acquires a linear voltage dependence, and (iii) the differential conductance exhibits a voltage asymmetry which is strongly enhanced by logarithmic corrections. We propose transport measurements in which these signatures can be observed.

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Introduction.—Over the past decade it has been established that electronic transport measurements through quantum dots and single molecules can be used to probe various coherent spin phenomena\textsuperscript{1,2} One of the simplest models for the theoretical description of quantum dots is the single-orbital Anderson model, which consists of an energy level that can be occupied by up to two electrons. This energy level is coupled to electronic reservoirs with hopping amplitudes \( t^\alpha \) where \( \alpha = L, R \). In the regime of suppressed charge fluctuations the system can be described by the Kondo model, in which the effective spin \( S \) on the dot is coupled to the electron spins \( s \) in the leads via a Heisenberg exchange interaction \( \propto S \cdot s \) resulting in rich Kondo physics. When the attached reservoirs are held at different chemical potentials, electrons are transported through the Kondo dot by cotunneling processes. The complicated interplay between Kondo physics and the non-equilibrium processes has led to the development of various renormalization group (RG) approaches\textsuperscript{1-3}.

An interesting extension of the Anderson model is obtained by allowing spin-dependent hopping amplitudes \( t^\alpha_{\sigma\sigma'} \) where \( \sigma, \sigma' = \uparrow, \downarrow \). As it was shown by Paaske \textit{et al.}\textsuperscript{2} spin-orbit interaction in materials like SiGe may lead to a mixing of different orbital states of the quantum dot, thus resulting in an effective spin dependence of the hopping amplitudes. Observable manifestations of this effect are voltage asymmetries of the differential conductance\textsuperscript{4,5} and the suppression of Kondo ridges at finite magnetic fields\textsuperscript{6}.

However, spin-orbit interaction is not the only possibility to generate a spin dependence of the hopping amplitudes. Pustilnik \textit{et al.}\textsuperscript{5} have considered quantum dots with an even number of electrons in a finite magnetic field. Tuning the Zeeman energy to the value of an orbital splitting creates a pair of degenerate levels, which originate from different orbital states and thus naturally possess different hopping amplitudes. They further derived the effective Kondo model for the case of symmetric hoppings to the leads, which possesses an anisotropic exchange interaction \( \propto S^x s^x + S^y s^y + \Delta S^z s^z \).

In this Rapid Communication we study a single-orbital Anderson model coupled to electronic leads via arbitrary spin- and lead-dependent hopping amplitudes \( t^\alpha_{\sigma\sigma'} \) (see Fig. 1). We demonstrate that with these two prerequisites the regime of single electron occupancy of the dot is generally described by an effective Kondo model with anisotropic exchange interaction \textit{and a Dzyaloshinsky–Moriya–Kondo (DMK) interaction} \( \propto \mathbf{d} \cdot (S \times s) \) between the effective spin on the dot and the electron spins in the leads.

Dzyaloshinsky–Moriya (DM) interactions arise in most elementary form in magnetic molecules.\textsuperscript{7} For example, dimer molecules can be modeled by two local spins coupled by a DM term, which can give rise to a deviation of the orientation of the magnetization from the direction of the external magnetic field. This situation is of particular relevance\textsuperscript{8} for single-molecule magnets like Mn\textsubscript{12} and Mn\textsubscript{10}. The DM interaction results in a canting of the spins that is also predicted to have characteristic signatures in the transport properties.\textsuperscript{9}

In sharp contrast to these previous works, the DMK interaction studied here does not act within the quantum dot (or molecule) but rather between the spin (or pseudospin) on the dot and the electron spins in the leads. As the DMK interaction is antisymmetric under a left-right inversion of the system, it has drastic consequences for its non-linear transport properties, which we study using a

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.pdf}
\caption{(Color online) Single-orbital Anderson model in a magnetic field \( h_0 \). The energies of the singly occupied levels are \( \epsilon_d \pm h_0 \). The dot levels are coupled to the leads via spin-dependent hopping amplitudes \( t^\alpha_{\sigma\sigma'} = t^1_{\sigma\sigma'} - t^2_{\sigma\sigma'} (\mathbf{d} \cdot \mathbf{s})_{\sigma\sigma'} \). In the Kondo regime of strong Coulomb repulsions \( U \) with \( \epsilon_d = -U/2 \) the system is mapped onto the effective model Eq. (\ref{eq:1}).}
\end{figure}
real-time renormalization group (RTRG) method. We find that (i) the DMK interaction is strongly renormalized during the RG flow, (ii) the renormalized magnetic field (RMF) $h$ acquires a component along the DMK vector $\mathbf{d}$ that grows linearly with the bias voltage $V$, and (iii) the DMK interaction results in an asymmetry of the differential conductance $G(V) \neq G(-V)$, which is strongly enhanced by logarithmic corrections. We further discuss how the RMF manifests itself in the correlation functions and the real-time dynamics of observables, and propose the measurement of the differential conductance in two- and three-terminal set-ups for its experimental detection.

**Model.**—We consider a quantum dot coupled to two electronic reservoirs held at chemical potentials $\mu_{L,R} = \pm V/2$ (Fig. 1). Electronic transport through the dot is mediated by two levels with level splitting $h_0$, which are assumed to be well separated from all other energy levels of the dot. The hopping amplitudes $t_{\alpha\sigma}$ depend on the spin states as well as the attached leads. In the case of normal Fermi-liquid leads the matrices $t_{\alpha\sigma}^\dagger$ can be chosen hermitian and thus parameterized as $t_{\alpha\sigma}^\dagger = t_{\alpha\sigma}^\dagger \delta_{\alpha\sigma} - t_{\alpha\sigma}^\dagger (d_{\alpha} \cdot \sigma)_{\sigma\sigma}^\tau$ in terms of four real numbers $t_{\alpha\sigma}^\dagger$ (one can always choose $t_{\alpha\sigma} > 0$) and two unit vectors $d_{\alpha}; \sigma$ denotes the vector of Pauli matrices. We focus on the case $d_{\alpha} \equiv d = (d^x, d^y)$ where $i = x, y, z$.

In the Kondo regime one can eliminate the empty and doubly occupied states on the dot via Schrieffer-Wolff transformation. This yields an effective Kondo-type Hamiltonian

$$H = \sum_{\alpha\sigma} \epsilon_{\alpha} (\mathbf{c}_{\alpha\sigma}^\dagger \mathbf{c}_{\alpha\sigma}) + h_0 \sum_{i} S^i + \sum_{\alpha\beta\gamma\delta} J_{\alpha\beta\gamma\delta} S^i_{\alpha\beta} S^j_{\gamma\delta} - \frac{1}{2} \sum_{\alpha\beta\gamma} J_{\alpha\beta\gamma} d^i \cdot S^i n_{\alpha\beta},$$

where $J_{\alpha\beta\gamma} = 8 (t_{\alpha\beta}^1 t_{\beta\gamma}^2 + t_{\alpha\beta}^2 t_{\beta\gamma}^1) / U$ is the coupling between the spin-1/2 on the dot and the charge density $n_{\alpha\beta} = \sum_{kk\sigma\sigma'} c_{\alpha k\sigma}^\dagger c_{\beta k\sigma'}$, in the leads, and $U$ denotes the on-site Coulomb repulsion. The exchange coupling between $S^i$ and the spin densities $s_{\alpha\beta}^i = \frac{1}{2} \sum_{kk\sigma\sigma'} c_{\alpha k\sigma}^\dagger \sigma_{\sigma\sigma'} \gamma_{\beta k\sigma'}$ in the leads is given by

$$J_{\alpha\beta}^{ij} = (\delta_{ij} - d^i d^j) J_{\alpha\beta}^{i\dagger} + d^i d^j J_{\alpha\beta}^{d} + \sum_{k} \epsilon_{ij} k J_{\alpha\beta}^{DMK}. \tag{2}$$

The couplings $J_{\alpha\beta}^{i\dagger} = 8 (t_{\alpha\beta}^1 t_{\beta\gamma}^2 + t_{\alpha\beta}^2 t_{\beta\gamma}^1) / U$ constitute the anisotropic Kondo model while $J_{\alpha\beta}^{DMK} = 8 (t_{\alpha\beta}^1 t_{\beta\gamma}^2 - t_{\alpha\beta}^2 t_{\beta\gamma}^1) / U$ is the Dzyaloshinsky–Moriya–Kondo (DMK) coupling between the spin on the dot and the electron spins in the leads.

An important parameter in the model (1) is the angle between the DMK vector $\mathbf{d}$ and the applied magnetic field $h_0$. A parallel alignment $\mathbf{d} \parallel h_0$ describes, for example, the Kondo effect in quantum dots with an even number of electrons. On the other hand, the perpendicular case $\mathbf{d} \perp h_0$ is found in the effective cotunneling model for a Kramers doublet in quantum dots with spin-orbit interactions, provided the coupling between the orbital angular momentum and the magnetic field is neglected.

We stress that a non-vanishing DMK term requires a left-right asymmetry of the hopping amplitudes, which is quite naturally present in nanoscale junctions, and contains the non-local reservoir spin density $s_{LR}$. The corresponding coupling matrix $J_{\alpha\beta}^{DMK}$ is antisymmetric in reservoir space, i.e. $J_{\alpha\beta}^{DMK} \propto i\tau^y$. The other couplings $J_{\alpha\beta}^{i\dagger}$ and $J_{\alpha\beta}$ are symmetric and thus spanned by $1, \tau^x$, and $\tau^z$, where $\tau^i$ denote the Pauli matrices in reservoir space. Hence in total the model (1) contains ten couplings. Although their bare values are expressed just through the four parameters $t_{\alpha\beta}^1, t_{\alpha\beta}^2$, the relations between renormalized couplings appear to be more complex, and all ten of them have to be taken into account in the RG treatment.

**Scaling analysis.**—The renormalization of the couplings $J$ is governed to leading order by the poor-man’s scaling (PMS) equations. For the model (1) they have the form (in the following all matrix operations are performed in the reservoir space)

$$\frac{d}{d\Lambda} J^\parallel = \frac{1}{2} \{ J^\parallel, J^\dagger \} + \frac{1}{2} \{ J_{\alpha\beta}^{DMK}, J \},$$

$$\frac{d}{d\Lambda} J^\dagger = (J^\parallel)^2 - (J_{\alpha\beta}^{DMK})^2, \frac{d}{d\Lambda} J = [J^\parallel, J_{\alpha\beta}^{DMK}], \tag{3}$$

$$\frac{d}{d\Lambda} J_{\alpha\beta}^{DMK} = \frac{1}{2} \{ J_{\alpha\beta}^{DMK}, J^\dagger \} + \frac{1}{2} \{ J^\parallel, J \},$$

where $\Lambda = \ln(D/\Lambda)$ with $\Lambda$ being the flow parameter. Its initial value is given by the band width $D$. The parametrization (2) is preserved under the RG flow governed by (3), i.e. no new terms are generated. The PMS equations (3) also leave invariant $\text{tr} J = \text{tr} J_0 \equiv 2c$ and $\text{tr} [J^\parallel^2 - (J^\parallel)^2 - J^\dagger^2 - (J_{\alpha\beta}^{DMK})^2] = 0$. The latter relation implies that when both $J$ and $J_{\alpha\beta}^{DMK}$ are initially zero one recovers the isotropic Kondo model with $J^\parallel = J^\dagger$.

We emphasize that despite $J^\parallel$ and $J_{\alpha\beta}^{DMK}$ formally satisfy the same RG equation, they have different symmetry properties in the reservoir space.

It is worthwhile to note that the magnetic field $h_0$ does not explicitly enter the PMS equations (3). Its magnitude $h_0$ may appear as a cut-off scale for the RG flow. The Eqs. (3) with the initial (bare) values $J_0 = J(\Lambda = D)$ stated after Eqs. (1) and (2) can be solved exactly. Here we focus on the most significant features of the RG flow and the role of the spin-charge and DMK terms.

The parameters $\text{tr} J^\dagger$ and $\text{tr} J^\parallel$, which essentially determine the flow diagram of the model (1), obey

$$\text{tr} J^\dagger = 2c + \frac{(T_K/\Lambda)^{4c}}{1 - (T_K/\Lambda)^{4c}}, \quad \text{tr} J^\parallel = \frac{4c}{K} \frac{(T_K/\Lambda)^{2c}}{1 - (T_K/\Lambda)^{4c}}, \tag{4}$$

where the invariant Kondo temperature is defined by $T_K = D \sqrt{(\text{tr} J^\dagger - 2c) / (\text{tr} J^\parallel + 2c)}$, and $c$ was defined above. In comparison with the conventional anisotropic
Kondo model, the solution \( \mathcal{H} \) additionally depends on the asymmetry parameter \( K = \sqrt{(\text{tr} J_0^d)^2 - 4 d^2 / (\text{tr} J_0^d)} \), which is in general different from unity unless both equalities \( t^L_1 = t^R_1 \) and \( t^L_2 = t^R_2 \) hold. In the latter case we have \( J^{\text{DMK}} = 0 \) and \( J = \text{const.} \) during the RG flow, thus we recover the scaling equations of Ref. 8.

In contrast, in the generic case of broken left-right symmetry \( J^{\text{DMK}} \) and \( J \) are renormalized under the RG flow and acquire non-zero values even if one of them is absent initially. For example, we find \( \text{tr}[J^{\text{DMK}}] \) recover the scaling equations of Ref. 8. In the following we show that precisely \( J^{\text{DMK}} \) and \( J \) cause interesting observable effects.

Renormalized magnetic field (RMF).—One of these effects is the renormalization of the magnetic field. For its derivation we have to go beyond the PMS analysis. We use the RTRG method, as it is particularly suited to analyze non-equilibrium effects. The derived flow equation for the RMF can be integrated with the result

\[
\hbar = \left[ 1 - \frac{1}{2} \text{tr}(J^d - J^d_0) \right] h_0 - \text{tr}(J^d_0) V / d. \tag{5}
\]

The subscript \( c \) denotes the exchange couplings cut off at the energy scale \( \Lambda_c \equiv \max \{ h_0, V \} < T_K \). The first term in (3) is analogous to the first-order result for the RMF in the anisotropic Kondo model. The second term originates from the renormalization of the coupling between the dot spin and the electron density in the leads, which is generated by the DMK interaction [see Eq. (2)]. Hence the linear voltage dependence of (5) is a consequence of the left-right asymmetry and spin dependence of the hopping amplitudes. We stress that the voltage dependence already appears in first order in the renormalized exchange couplings \( J_0 \), and that the effect on the magnitude \( h \) of the RMF is maximal in the parallel case \( d \parallel h_0 \). The voltage dependence implies that a RMF will be generated even in the absence of an external field \( h_0 \) and that \( h(V) \neq h(-V) \). This is in sharp contrast to the anisotropic Kondo model, where the voltage dependence only enters through logarithmic corrections in \( O(J^2) \) and the RMF remains symmetric under inversion of the bias.

The result (3) has an immediate consequence for the dynamics \( \langle S^z(t) \rangle \), which can be used to probe the RMF. The experimental observation of the precession frequency as well as the spin susceptibility does, however, require time-resolved measurements on the quantum dot. Although time-resolved experiments have been previously performed, it would be rather challenging to extract quantitative results for the RMF. Thus we propose two alternative ways to observe the effect of the RMF in stationary quantities.

Differential conductance.—As the DMK interaction breaks the inversion symmetry of the system, it has drastic consequences on its non-equilibrium transport properties. In Fig. 2 we plot the differential conductance \( G(V) = dI / dV \) through the dot for a parallel alignment of \( d \) and \( h_0 \). For \( |V| < h \) the conductance is dominated by elastic cotunneling processes. At \( V \approx \pm h \) the conductance possesses jumps due to the onset of inelastic cotunneling processes. The magnitude of these jumps is asymmetric with respect to \( V \rightarrow -V \) already in order \( J^2 \) (dashed line in Fig. 2), which qualitatively agrees with previous perturbative results. In leading order this asymmetry is proportional to \( \text{tr}[J^{\text{DMK}}(d \cdot h_0)] / h_0 \). In addition, the RTRG analysis shows that the zero-temperature broadening of these jumps is given by the transverse spin relaxation rates \( \Gamma_2(V = h) \) and \( \Gamma_2(V = -h) \), respectively, which differ from each other by \( \propto h_0 \). This explicitly shows that the asymmetry of \( G(V) \) is a direct consequence of the DMK interaction and it can be quantified in terms of the corresponding renormalized coupling \( J^{\text{DMK}} \). The asymmetry further depends on the angle between \( d \) and \( h_0 \), taking its maximal value in the parallel case and vanishing in the perpendicular case.

A more thorough RTRG analysis in the parallel case including the leading logarithmic corrections in \( O(J^2) \) reveals that the magnitudes of the jumps at \( V = \pm h \) acquire logarithmic terms \( \propto \ln(\Lambda_c / |V \mp h \mp i\Gamma_2(V \mp h)|) \) broadened by the corresponding transverse spin relaxation rates. Due to the difference of the respective prefactors the asymmetry of \( G(V) \) acquires a strong enhancement (see the solid line in Fig. 2), which even influences the elastic cotunneling regime \( |V| < h \).

The logarithmic resonances at the onset of inelastic cotunneling at \( V = \pm h \) correspond to the extrema of the derivative of the differential conductance \( dG/dV = d^2I / dV^2 \). Thus the RMF \( h(\mp V) \) can be read off from the positions of its dip and peak respectively (see insets [Fig. 2]).

FIG. 2. Differential conductance with (solid line) and without (dashed line) logarithmic corrections for \( t^L_1 = 0.08, t^R_1 = 0.04, t^L_2 = 0.05, t^R_2 = 0.03, d \parallel h_0 \), and \( h_0 = 100 T_K \). We observe an asymmetry under \( V \rightarrow -V \), which gets strongly enhanced by the logarithmic corrections. Insets: The asymmetry of the RMF with respect to \( V \rightarrow -V \) can be read off from the resonances in \( dG/dV = d^2I / dV^2 \).
Conductance leads while the probe lead is used to measure the RMF. b) FIG. 3. (Color online) a) Sketch of the three-terminal set-up. The stationary current $I$ can be read off from the resonances in $dG/dV$. All other parameters are as in Fig. 2. c) The RMF can be read off from the resonances in $dG/dV(G_p, \mu_p + V/2)$. The curves shown correspond (from top to bottom) to $V = 20 T_K, 40 T_K, \ldots, 160 T_K$. The dashed line is a guide to the eye indicating the linear voltage dependence of $h(V)$.

in Fig. 2. The second term in (5) is given by the sum of the two positions. We note that due to the restriction of our quantitative analysis to the weak-coupling regime the asymmetry $h(V) \neq h(-V)$ observed in Fig. 2 is relatively small. However, as it turns out to be a generic consequence of asymmetric and spin-dependent hopping amplitudes we expect it to be present in the strong-coupling regime as well.

Three-terminal set-up. — Another way to experimentally observe the linear voltage dependence of the RMF is provided by the use of the three-terminal set-up sketched in Fig. 3a. A similar set-up has been used by Leturcq et al. to measure the splitting of the Kondo resonance in the spectral density. Specifically, the three-terminal set-up consists of the system (1) with an additional, weakly coupled probe lead modeled by $\sum_{\alpha \beta} (\epsilon_k - \mu_p) c^\dagger_{\alpha k} c_{\beta k} \rho$ with exchange interaction $J^p \sum_{\alpha=L,R} S \cdot (s_{\alpha p} + s_{p \alpha}) + J^p S \cdot s_{pp}$, where $|J^p| \ll |J_{\alpha \beta}^F|, |J_{\alpha \beta}^H|$. We assume symmetric and isotropic exchange couplings to the probe lead and note in passing that the calculation can easily be generalized to more complicated situations. As the probe lead is only coupled weakly, its influence on the PMS equations and the RMF is negligible, and the conductance $G_p = dI_p/d\mu_p$ can be calculated using perturbation theory in $J^p$. The result is shown in Fig. 3b. G_p possesses characteristic steps at $\mu_p = \pm (h - V/2), \pm (h + V/2)$ which are broadened by the transverse spin relaxation rate. When plotting $dG_p/dV$ as a function of $\mu_p + V/2$ one can directly read off the RMF from the position of the first resonance. The results shown in Fig. 3c clearly reveal the linear voltage dependence of the resonance position and thus the RMF.

Conclusion. — We have studied a Kondo dot with additional DMK interaction, which is amplified during the RG flow. The DMK term results from the broken inversion symmetry of the system and manifests itself in the non-equilibrium transport properties. In particular, we showed that the RMF acquires a component along the DMK vector depending linearly on the bias voltage, which can be experimentally detected in two- and three-terminal set-ups. In addition, the differential conductance becomes asymmetric with respect to the bias. As these fingerprints of the DMK interaction rely on broken spin and spatial symmetry, they are also expected beyond the weak-coupling regime. A complete description of the strong-coupling regime remains, however, an open problem.

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