Epistemological Obstacles: An Overview of Thinking Process on Derivative Concepts by APOS Theory and Clinical Interview

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Abstract. This study aims to identify the epistemological obstacle of pre-service mathematics teachers on the basic concept of derivatives. The research used descriptive qualitative. Data of the research was obtained from the results of tests and interviews given to six participants as the pre-service mathematics teachers from two different institutions in West Java. The test is given consists of three questions related to the basic concept of the derivative. The answers to the questions were confirmed again by clinical interviews. These data are then reviewed by the framework of APOS theory (Action-Process-Object-Schema) to overview the thinking of processes experienced by participants in answering the questions given. The results of the study show that there is still some learning obstacles experienced by participants in understanding the basic concepts of derivatives. Constraints experienced include the lack of mathematical connection skills regarding the basic concept of derivatives, the operation of a function that is still wrong, and the lack of a meaningful process for the concepts learned so that problem-solving processes are still limited to procedural problems. Based on this research, it is recommended that pre-service mathematics teachers can internalize derivative concepts learned so that the knowledge scheme formed can be better.

1. Introduction

Mathematics is a science that is formed empirically from human experiences, then the experience is processed and analyzed by reasoning which finally comes to concepts that apply in general through justification that is the process of searching for truth based on proof. According to Ernest [1], one of the conceptions of mathematics is based on a problem-solving view that views mathematics as dynamic, namely the space of creation and human discovery that develops continuously where patterns are raised and then filtered into knowledge. The knowledge that has been formed has distinctive characteristics, which have hierarchical properties that are interrelated between one concept with another concept or one term with another term so that good connection skills are needed in connecting mental objects in the form of concepts, terms, definitions, hypotheses, theorems, and others that exist in cognitive structures to construct a full understanding in learning of mathematics [2-4]. The derivative concept is an example of knowledge generated from generally valid experiences through the process of verification and institutionalization. Referring to the details of the education curriculum, derivative concepts become part of mathematical objects that are important to be studied by students in secondary schools [5].
However, some of the results of research conducted indicate that the implementation of derivative material learning still leads to memorizing formulas and then uses them in procedural contexts so that students do not understand and interpret the concepts learned [6-8]. Besides, the views of students who assess those mathematics is a difficult and tedious lesson also influence the learning outcomes that are less than optimal. Such things certainly become obstacles in the learning process. Brousseau [9] defines obstacles in learning as a collection of errors that are closely related to prior knowledge. This happens because these errors arise in the students' initial understanding of a matter, then the error continues to be repeated so that it is embedded in the system's long-term memory as knowledge. In line with the results of these studies, Ernest [1] states that students must understand deeply and systematically from a mathematical concept. The urgency of the statement is further emphasized by the previous discussion that the process of forming knowledge in mathematics is hierarchical, which is interrelated between one concept and another concept.

The process of forming new knowledge, especially in mathematics is believed to be the result of a series of grooves as APOS Theory with abbreviations of Action - Process - Object - Schema [10]. Objects that have been stored in someone's memory as knowledge will be processed when an action occurs due to a certain stimulus on the memory sensor. Dubinsky [10] as an APOS theory developer based his theory on the view that one's knowledge and understanding of mathematics is a tendency to be able to respond to a mathematical situation and reflect it on a social context. Furthermore, the individual constructs or reconstructs mathematical ideas through actions, processes and mathematical objects which are then organized into a scheme to be used in solving the problems at hand. Asiala [11] stated that the goal to be achieved from the APOS theory is the formation of knowledge through the mental construction of students. Suryadi [12] interprets the deep construction of the context of the APOS theory is the formation of actions that are contemplated in a process. Furthermore, the process is encapsulated into objects. The summarized objects can be re-encapsulated into a process. Based on the action, process, and object can then be formed into a knowledge scheme which is then shortened to APOS. There are following the flow of mental construction in Figure 1:

![Figure 1. Flow of Mental Construction](image)

Looking from the flow of mental construction in Figure 1, understanding the mathematical concept needs to begin by carrying out an activity that stimulates the mental construction to form actions. The action is then contemplated or reflected in a process which is then formed into mental objects which will be decomposed back into the process if needed. Action, process, and object will be formed into a scheme to solve the problems.

Based on previous discussions, researchers felt the need to carry out case studies in universities to investigate whether pre-service mathematics teachers who would later teach mathematics to students in schools still had obstacles in understanding the basic concepts of derivatives. But in this study, the barriers studied were more concentrated on epistemological obstacles to review the factors that influence the difficulty of pre-service mathematics teachers in understanding the basic concepts of derivatives.
2. Methods
Based on the previous discussion, the researcher wanted to review the knowledge that had been formed regarding the concept of derivation through case studies without giving effect in the form of learning carried out in higher education. Participants involved were pre-service mathematics teachers from two different universities, each of which consisted of three students. Participants were given a test instrument about the basic concept of derivation consisting of three questions and given clinical interviews consisting of questions from the order of the easy to the difficult with regard to the basic concept of derivatives, the purpose of clinical interviews is to confirm answers to questions that have been given naturally are based on the knowledge that participants have [13].

The stages of data analysis that can be done are as follows [14]: 1) identify participants' answer errors in answering questions; 2) connecting errors with conceptions that arise in cognitive structure participant; 3) explain why the conception identified is an obstacle epistemology in the given context; 3) find the context in the conception identified, and carry out the discussion using the theories discussed in the theoretical framework. Furthermore, the data is reviewed using the framework of APOS (Action-Process-Object-Schema) theory to see participant thinking processes more intact and systematic. The review consists of three steps [15] including 1) interiorization or contemplation of action into a process; 2) the construction of a coordinated process; 3) and encapsulation of processes into objects or de-encapsulation of actions, processes, and objects to form knowledge schemes to fit the situation. APOS theory is relevant for reviewing data related to submitting questions because the theory specifically addresses the nature of learning topics such as the concept of limits [15]. Moru [14] states that the APOS theory is used to investigate the epistemological obstacle of mathematics students in understanding the concept of limits. The epistemological obstacle is an obstacle in learning that arises because of the limitations of students' understanding of a concept [9]. If students are faced with a different concept, the knowledge they have is not can be used in different contexts or have difficulties when using previously owned knowledge.

3. Result and Discussion
The results of the study show that there are still epistemological obstacles experienced by participants. The obstacles found in the study include the lack of understanding of the basic concepts of derivatives, algebraic operations on a function that is still wrong, and the lack of a meaningful process for the concepts learned so that problem-solving abilities are still limited to procedural problems. The following are the tests given to participants regarding the derivative basic concepts:

1) A function is given \( f(x) = 4x^3 + 3x^2 - 2x + 1 \). How many times this function must be differentiation so that the derivative of \( f(x) = 0 \)? Show the differentiation process!

From the question, it can be seen whether anyone discusses the basic concepts of general derivatives with examples to find derivatives of functions \( f(x) = ax^n \) is \( f'(x) = nax^{n-1} \). Based on the results of the answers, most participants know how to find derivation functions from Eq.1.

\[
f(x) = 4x^3 + 3x^2 - 2x + 1 \tag{1}
\]

until the function is worth \( f'(x) = 0 \). They did not seem to have difficulties in using the formula \( nax^{n-1} \) when finding derivation of such polynomial or constants so that the epistemological obstacle was rarely found in the process in question number one.

2) Explain how the relationship between the concept of the gradient of lines and the concepts of limit results in derivative concepts \( f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \! \) !

From the question, it can be seen how far the subject interpreted the derivative concept by knowing the relationship between the concept of the gradient of lines and the concept of limit to produce equations

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \tag{2}
\]
Based on the results of the answer to question number two, none of the participants can explain the interrelationship between the concepts correctly. They forget about the concept of straight line gradients and limit concepts. Besides, when given the interpretation of derivative concepts geometrically, most participants also cannot understand the intent of the interpretations given.

3) If \( f(x) = x^n \), then prove that \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = n \cdot x^{n-1} ! \)

From the question, it will be seen whether the participant knows the general formula
\[
f'(x) = n \cdot x^{n-1}
\] (3)
commonly used to derivative function
\[
f(x) = x^n
\] (4)
obtained from the derivative equation namely
\[
\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\] (5)

If the participant knows the binomial newton formula for the function \((x + h)^n\) then they will be able to use the derivative equation to show that
\[
f'(x) = n \cdot x^{n-1}
\] (6)

On other that, participants can also work by determining the derivative of \((x + h)^1\), \((x + h)^2\), \((x + h)^3\), \((x + h)^4\), and so on using equations derivative equation so that from the derivative results these functions will get a general pattern that shows
\[
f'(x) = n \cdot x^{n-1}
\] (7)

Based on the results of the answer to question number three, none of the participants could show evidence that
\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = n \cdot x^{n-1}
\] (8)
so there is an epistemological obstacle in understanding the interrelationship of general formulas with basic derivative concepts.

Next, an example of the clinical interview is displayed from one of the participants to confirm the results of the answers from the tests previously given. Questions in clinical interviews related to the derivative basic concepts are given in sequence starting from the easy questions to the difficult questions to review the extent to which participants understand the basic concepts of derivatives. There are following the results of the clinical interview from one of the participants in Table 1.

| Table 1. The Results of Clinical Interview |
|-------------------------------------------|
| **Interview**                             | **Answer from Participant**                             |
| Explain how the derivative concepts who you know and also explain your answers to questions number 1! | Suppose there is a function \( f(x) = x^n \), then the derivative will be \( n x^{n-1} \). For question number 1, only four times have to be lowered so that the function will be 0. |
| How does the concept of the gradient of lines relate to \( \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \) do you know? How do you think the meaning is \( \lim_{h \to 0} \) in the equation? | I do not know sir. Later value from \( h \) replaced with 0 |
If there is a function \( f(x) = x^2 + 2 \) then can you show the process and results of substitution on \((x + 2)\)? From the same function, can you show derivative processes using \( \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \)? Yes sir, the value of \((x + 2)\) later substituted to \( x^2 + 2 \). The result is \( x^2 + 4x + 4 + 2 = x^2 + 4x + 6 \). I think, I can do it, \( \lim_{h \to 0} \frac{(x+h)^2 + x^2 + 2x^2 + 2 - x^2}{h} = 2x \).

How do you interpret the operation of departure at \((x + h)^2\) and \((x + h)^3\)? Next, can you show the operating process of the departure at \((x + h)^n\)? That's from the squared rank sir, so the result is \( x^2 + 2x + h^2 \). For \((x + h)^3\) just multiply the result of the squared by \((x + h)\). For the result of \((x + h)^n\) is \( x^n + h^n \).

Can you prove that \( \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = n \)? No, I can not prove that \( \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = n \).

Referring to the results of the test and the clinical interviews, it can be seen briefly that the participant's understanding of the basic concept of derivatives is still limited to procedural understanding. Some of these examples include: 1) participants can generally derivative a function well using the formula they normally use namely \( f'(x) = nx^{n-1} \) for \( f(x) = x^n \) but when asked to explain where the formula is obtained, participants cannot explain it; 2) participants do not know the relevance of mathematical ideas or concepts that build derivative concepts; 3) participants are not used to analyzing the use of derivative concepts in non-routine problems.

These conditions will then be reviewed with the framework of the APOS theory to see the process of participant thinking systematically and see the extent to which participants in constructing knowledge related to the basic concept of derivatives. The review consists of three steps [14] including 1) interiorization or contemplation of action into a process; 2) the construction of a coordinated process; 3) and encapsulation of processes into objects or de-encapsulation of actions, processes, and objects to form knowledge schemes to fit the situation. In the derivative concept, the action phase begins when participants are given a test and interview as a stimulus to carry out mental processes in answering questions. In question number one, after interacting with the action, participants need a little process to derive the function given. Based on the concept image that has been built, participants can only show that the derivative concept is a procedural concept involving mental objects that they already have. In questions number two and number three, after interiorizing the action in the form of attention and connecting ideas contained in their memory randomly, participants try to construct processes that are appropriate to the situation faced by the participants making the connection process of mental objects what they remember from the concept of slope lines and limits to produce derivative concepts and prove that general derivative formula are generalizations of Leibniz notation. Based on these conditions, mental construction does not lead to the formation of derivative schemes because participants cannot verify the answers they put forward. According to Clark [16] to construct the scheme of derivatives at least mental objects are needed including the conception of the process of decline and coordination between mental objects that form the schema of the function and function of composition. From the statement, participants have not been able to show the conception meaning of the process of decline so that the construction of knowledge about basic derivative concepts has not yet arrived at organizing actions, processes and mental objects to derivative schemes to trigger the epistemological obstacle in the process of understanding derivative concepts.

4. Conclusion
Based on the results and discussion, it can be concluded that there are still some learning barriers experienced by pre-service mathematics teachers in understanding the basic concepts of derivatives. Constraints experienced include the lack of mathematical connection skills regarding the basic concept of derivatives, the operation of a function that is still wrong, and the lack of a meaningful process for the concepts learned so that problem-solving processes are still limited to procedural problems. Based
on this, it is recommended that students internalize derivative concepts learned so that the knowledge scheme formed can develop better.

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Acknowledgments
Thanks to the two leaders of higher education institutions in Kuningan Regency, West Java, who were willing to permit research so that the writing articles of the results of this study can be resolved.