A mechanism for FIMPy baryogenesis

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Abstract: We present a simple mechanism which allows the simultaneous generation of the baryon asymmetry of the Universe along with its dark matter content. To this goal, we employ the out-of-equilibrium decays of heavy bath states into a feebly coupled dark matter particle and Standard Model charged fermions. These decays lead to dark matter production via the freeze-in mechanism and, assuming that they further violate \(CP\) can generate a viable matter-antimatter asymmetry in the resonant regime. We illustrate this mechanism by studying a particular realisation of this general scenario, where the role of the heavy bath particles is played by \(SU(3)_{c} \times SU(2)_{L}\)-singlet vector-like fermions with a non-zero hypercharge and dark matter is identified with a gauge-singlet real scalar field. We show that in the context of this simple model the cosmological constraints for the dark matter abundance and the baryon asymmetry are satisfied for masses of heavy vector-like fermion states of a few TeV, potentially within reach of the High-Luminosity Run of the Large Hadron Collider. Dark matter, in turn, is predicted to be rather light, with a mass of a few keV.
1 Introduction

Among the numerous open questions in contemporary high-energy physics, the origin of cosmic dark matter and that of the baryon asymmetry in the Universe occupy a pivotal place. Not only do they constitute two of the most striking and fundamental pieces of evidence for the existence of physics beyond the Standard Model (SM) of particle physics, but they do so only once the latter is placed within a cosmological framework. Resolving either (let alone, both) of these questions most likely requires particle physics to be viewed from a cosmological standpoint and, conversely, cosmology to be analysed in terms of the behaviour of the fundamental constituents of matter and their interactions. And, indeed, in doing so during the past decades model-builders have not been short of ideas concerning the nature of dark matter and the mechanism through which matter came to dominate over anti-matter in the Universe.

Among the various dark matter candidates that have been proposed we can mention axions [1–3], primordial black holes [4–6], a vast number of incarnations of Weakly or Feebly Interacting Massive Particles (WIMPs/FIMPs, for reviews cf e.g. [7, 8]), gravitationally produced dark matter [9] and asymmetric dark matter (for reviews cf e.g. [10, 11]), just to name a few. Similarly, the matter-antimatter asymmetry of the Universe has been explained in terms of different mechanisms of baryogenesis, like GUT baryogenesis [12],
electroweak baryogenesis [13, 14], Affleck-Dine mechanism [15]) and leptogenesis [16]. Interestingly, during the last decade there have also been several attempts to actually link the two questions, most notably in the contexts of WIMP baryogenesis [17–19] or asymmetric dark matter [20, 21].

Recently, the main idea behind ARS leptogenesis [22], namely that of a lepton number asymmetry being generated through $CP$-violating sterile neutrino oscillations was exploited in [23], where the authors proposed that a baryon asymmetry could, instead, be also generated by augmenting the SM with exotic scalars and fermions directly coupling to quarks. The fermions, which were taken to be singlets under the SM gauge group can, moreover, play the role of viable dark matter candidates through the freeze-in mechanism [24, 25], whereas their $CP$-violating oscillations, in the presence of electroweak sphaleron transitions, can generate the observed baryon asymmetry of the Universe.

In this paper we propose a mechanism which borrows ideas both from Dirac leptogenesis [26–29] and from this scenario of “freeze-in baryogenesis”. As we will describe in detail in the following, we consider a heavy particle species which is charged under (parts of) the SM gauge group, and which can decay through feeble interactions into SM fermions along with a neutral particle. The latter is our dark matter candidate, produced upon the decays of the heavy particle through the freeze-in mechanism, along the lines presented in [23]. However, in our case the decays themselves violate $CP$, in a similar manner as in leptogenesis models. Then, in the presence of electroweak sphalerons, we will see that an asymmetry can be generated between SM fermions and antifermions. The relevant processes proceed through feeble couplings, preventing them from ever reaching equilibrium and, thus, satisfying the third Sakharov condition [30]. The first condition, namely baryon number violation, is satisfied due to the active sphaleron processes in a way that resembles neither GUT baryogenesis (explicit $B$ violation in decays) nor leptogenesis (explicit $L$ violation in decays), although we will also comment on the possibility of direct $B/L$ violation as well.

The paper is structured as follows: in Section 2 we discuss the general features of our mechanism, namely the way through which the required dark matter abundance and the baryon/lepton asymmetries can be generated, without adopting any concrete microscopic model. In Section 3 we propose a simple model as a proof-of-concept that concrete incarnations of our mechanism can, indeed, be constructed. We compute the predicted dark matter abundance and baryon asymmetry, quantify the effects of processes that wash out the latter and briefly comment on the phenomenological perspectives of our model, notably in relation to searches for long-lived particles (LLPs) at the Large Hadron Collider (LHC). Finally, in Section 4 we summarize our main findings and conclude. Some more technical aspects are left for the Appendix.

2 Dark matter and baryogenesis from freeze-in: General framework

Before presenting a concrete realisation of our take on the freeze-in baryogenesis idea, let us briefly summarise a few key notions that will be useful for the discussion that follows: frozen-in dark matter and how the freeze-in framework can enable us to satisfy the three
Sakharov conditions that are necessary for successful baryogenesis. A concrete implementation of these ideas will be presented in detail in Section 3.

2.1 Freeze-in DM abundance

The freeze-in mechanism for dark matter production relies on two basic premises:

- The initial DM abundance is zero.
- Dark matter only interacts extremely weakly (“feebly”) with the Standard Model particles (along with any other particles that are in thermal equilibrium with them).

Under these assumptions, and further assuming that dark matter production takes place in a radiation-dominated Universe, dark matter never reaches thermal equilibrium with the plasma. Instead, it is produced through the out-of-equilibrium decays or annihilations of bath particles and all dark matter depletion processes, the rate of which typically scales as \( \langle \sigma v \rangle \times n_{DM}^2 \) (where \( \sigma \) is the dark matter annihilation cross-section, \( v \) its velocity and \( n_{DM} \) its number density), can be ignored.

2.2 Baryon Asymmetry abundance \( Y_B \)

In general, the decays and/or annihilations that are responsible for dark matter production can also violate both the baryon number \( B \) and \( C/CP \). Then, as long as we stick to the freeze-in framework these processes occur out-of-equilibrium with the thermal plasma, thus fulfilling all three Sakharov conditions.

Intuitively, and ignoring all wash-out processes, if we denote the measure of \( CP \) violation by \( \epsilon_{CP} \), we would expect the generated asymmetry in the SM fermion \( \Delta_f \) to be connected to the dark matter abundance \( Y_{DM} \) through a relation of the type

\[
Y_{\Delta_f}(x) \sim \epsilon_{CP} Y_{DM}(x)
\]  

In reality, this limit cannot be attained given that some amount of washout is almost inevitable, whereas concrete realisations typically require the introduction of additional particles and decay channels. In this respect this relation may be viewed as an upper limit to the asymmetry that can be generated through decays that simultaneously produce dark matter.

In fact in the following, when studying a concrete incarnation of our decay-induced freeze-in baryogenesis idea, we will see that

- Since we will be starting with non-self-conjugate initial states \( F_i \) (Dirac fermions), \( CPT \) conservation and unitarity impose the existence of multiple decay channels of the \( F_i \)’s for a non-vanishing \( CP \) asymmetry to be generated. To this goal, we will exploit possible decays of the heavy fermions into different Standard Model fermions (leptons), i.e. flavour effects.

\[1\]Very similar remarks can be made if the decay violates, instead, lepton number. This is also the option that we will adopt later in this work.
• The freeze-in framework will necessitate extremely small values for the couplings involved in the decay process. The predicted \( CP \) violation, being an effect that arises from the interference of tree-level and one-loop processes is, then, even further suppressed, which will lead us to consider resonantly enhanced configurations in self-energy-type diagrams \([31–34]\). Therefore, at least two heavy fermions \( F_i \) must be added.

• Baryon and/or lepton number need not be violated by the decay processes. As proposed, e.g. in \([26]\), a \( CP \) asymmetry can be translated to a baryon asymmetry by the electroweak sphalerons. If, additionally, the BSM heavy fermions are immune to the action of sphalerons, in the end a net asymmetry will be generated in the baryon and lepton sectors.

3 A concrete realisation

Let us now elaborate the previous considerations through a concrete, simple model. We extend the SM by two heavy vector-like leptons \( F_i \) which are singlets under \( SU(3)_c \times SU(2)_L \) but carry hypercharge and a real gauge-singlet scalar \( S \) which is our freeze-in DM candidate. We moreover impose a discrete \( Z_2 \) symmetry on the Lagrangian, under which all exotic states are taken to be odd while the SM particles are even. Under these assumptions, the Lagrangian reads

\[
\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_S + \mathcal{L}_{SF}
\]

(3.1)

where \( \mathcal{L}_{\text{SM}} \) is the SM Lagrangian,

\[
\mathcal{L}_S = \partial_\mu S \partial^\mu S - \frac{\mu_S^2}{2} S^2 + \frac{\lambda_S}{4!} S^4 + \lambda_{sh} S H^\dagger H
\]

(3.2)

describes interactions of dark matter with itself and with the Standard Model Higgs doublet and

\[
\mathcal{L}_{SF} = \sum_i \left( \tilde{F}_i \right) (i\hat{D}) F_i - M_i \tilde{F}_i F_i - \sum_{\alpha, i} \left( \lambda_{\alpha i} S \tilde{F}_i P_R e_\alpha + \lambda^*_{\alpha i} S e_\alpha P_L F_i \right)
\]

(3.3)

where \( e_\alpha \) are the right-handed SM charged leptons of flavour \( \alpha = \{e, \mu, \tau\} \) and \( \lambda_{\alpha i} \) denote the feeble couplings. Note that, without loss of generality, we have neglected potential off-diagonal couplings among the heavy fermions \( F_i \). For simplicity in what follows we will also set the Higgs portal coupling, \( \lambda_{sh} \), to zero.

The tree-level proper decay width \( \Gamma_{F_i \rightarrow e_\alpha S} \) in the limit \( M_i \gg m_{e_\alpha} + m_S \) is given by

\[
\Gamma_{F_i \rightarrow e_\alpha S} \approx \frac{|\lambda_{\alpha i}|^2}{16\pi g_{F_i}^2} M_i
\]

(3.4)

where \( g_{F_i} = 2 \) are the internal degrees of freedom of species \( F_i \). The equilibrium decay rate density \( \gamma_{F_i \rightarrow e_\alpha S} \) evaluated in the decaying particle rest frame is \([25]\)
\[ \gamma_{F_i \rightarrow \alpha S} \equiv g_{F_i} \int d\Pi_{F_i} d\Pi_{\alpha S} (2\pi)^4 \delta^4(p_{F_i} - p_{\alpha S} - p_S) f_{F_i}^\text{eq} |M|_{F_i \rightarrow \alpha S}^2 \]

\[ = \frac{g_{F_i}}{2\pi^2} M_i^2 \Gamma_{F_i \rightarrow \alpha S} T K_1 \left( \frac{M_i}{T} \right) \tag{3.5} \]

where \( d\Pi_k = d^3p_k / (2\pi)^3 2E_k \) is the elementary Lorentz invariant phase space volume of species \( k \), \( |M|^2 \) denotes the squared matrix element summed, but not averaged, over the internal degrees of freedom of the initial states, \( K_1 \) is the modified Bessel function of the second kind of order one and the distribution function of \( F_i \) approximately follows the Maxwell-Boltzmann distribution, \( f_{F_i}^\text{eq} = e^{-E_{F_i}/T} \). The corresponding thermally averaged equilibrium decay rate density \( \langle \gamma_{F_i \rightarrow \alpha S} \rangle \) reads

\[ \langle \gamma_{F_i \rightarrow \alpha S} \rangle \equiv \frac{\gamma_{F_i \rightarrow \alpha S}}{n_{F_i}^\text{eq}} = \frac{K_1 (M_i/T)}{K_2 (M_i/T)} \Gamma_{F_i \rightarrow \alpha S} \approx \begin{cases} \frac{M_i}{\pi T} \Gamma_{F_i \rightarrow \alpha S}, & T \gg M_i \\ \Gamma_{F_i \rightarrow \alpha S}, & T \ll M_i \end{cases} \tag{3.6} \]

where \( n_{F_i}^\text{eq} \) is the equilibrium number density of \( F_i \) assuming zero chemical potential

\[ n_{F_i}^\text{eq} \equiv g_{F_i} \int \frac{d^3P}{(2\pi)^3} f_{F_i}^\text{eq} = \frac{g_{F_i}}{2\pi^2} M_i^2 T K_2 \left( \frac{M_i}{T} \right) \tag{3.7} \]

The time dilation factor \( K_1 (M_i/T) / K_2 (M_i/T) \) implies that decays are inhibited at temperatures higher than the decaying state mass \( M_i \).

The dominant \( 2 \leftrightarrow 2 \) scattering processes modifying the abundance of \( S \) are those which involve the \( U(1)_Y \) gauge boson as an external state, \( i.e. F_i B \leftrightarrow e_\alpha S, F_i S \leftrightarrow e_\alpha B \) and \( F_i \bar{e}_\alpha \leftrightarrow SB \), which will be henceforth referred to as gauge scatterings. The corresponding matrix elements depend on the product of a feeble and a gauge coupling, whereas all other scattering processes involve higher powers of feeble couplings and are therefore subleading. All relevant Feynman diagrams are shown in Fig. 1.
Figure 2: Tree-level decay and gauge scattering rate densities involving $F_1$ and $e_\alpha$ as external states, normalized to the Hubble parameter $H$ and the equilibrium number density of photons $n_\gamma$. The feeble coupling of the interactions is $\lambda_{e1} \approx 2.15 \times 10^{-8}$ and the densities have been evaluated at tree-level and in the limit $M_1 \gg m_{e_\alpha} + m_S$.

The equilibrium interaction rate density of a generic scattering process $ab \rightarrow cd$ is [35]

$$\gamma_{ab\rightarrow cd} \equiv \int d\Pi_a d\Pi_b d\Pi_c d\Pi_d \frac{(2\pi)^4}{\sqrt{\xi}} \delta^{(4)}(p_a + p_b - p_c - p_d) f^e_a f^e_b |M|_{ab\rightarrow cd}^2$$

$$= \frac{T}{512\pi^6} \int_{\tilde{\xi}_{\text{min}}}^{\infty} \frac{d\tilde{\xi}}{\sqrt{\tilde{\xi}}} |P_{\text{in}}| |P_{\text{fin}}| K_1 \left( \frac{\sqrt{\tilde{\xi}}}{T} \right) \int d\Omega |M|_{ab\rightarrow cd}^2$$

(3.8)

where $P_{\text{in}}$, $P_{\text{fin}}$ and $\tilde{\xi}$ are the initial and final momenta and the energy in the centre-of-momentum frame respectively, with $\tilde{\xi}_{\text{min}} = \max \{ (m_a + m_b)^2, (m_c + m_d)^2 \}$. In Fig. 2 we present the decay and gauge scattering rate densities involving $F_1$ and the right-handed electron $e_\alpha$ as external states in terms of the dimensionless parameter $z \equiv M_1/T$. For the feeble coupling we use the value $\lambda_{e1} = 2.145 \times 10^{-8}$ and work at lowest order in perturbation theory and in the limit $M_1 \gg m_{e_\alpha} + m_S$. In order to study their effect on the Boltzmann equations, it is convenient to normalize them to the Hubble parameter $H$ and to the number density of photons $n_\gamma = 2\zeta(3)T^3/\pi^2$, where $\zeta$ is the Riemann zeta function [36]. The integrals appearing in the various scattering rate densities have been computed with the Cuba numerical library [37].

The s-channel resonance that appears in the scattering $F_i B \leftrightarrow e_\alpha S$ has been regularised by the finite decay width of $F_i$. Also the IR-induced resonances due to the exchange of massless SM leptons that appear in the u-channel of $F_i B \leftrightarrow e_\alpha S$ and in the t-channel of $F_i \bar{e}_\alpha \leftrightarrow SB$ have both been regulated by the thermal mass of the right-handed SM leptons $m_\xi^2(T) = g_\xi^2 T^2/8$ (see [38] and references therein). Note that as thermal effects are irrelevant at the temperature of a few TeV that is of interest to us, we include the thermal
mass only when it acts as a regulator of the IR-divergences. For a few examples in which finite temperature effects can become important cf e.g. [39–41].

3.1 Out-of-Equilibrium Decays

As we already mentioned, one of the crucial ingredients of our setup is that all processes involving dark matter (and, for that matter, CP violation) must never attain chemical equilibrium. In order to fulfil this condition, an upper bound on the magnitude of the freeze-in couplings can be obtained by requiring the total thermally averaged decay rate to be smaller than the Hubble expansion rate at the characteristic temperature \( T = M_i \). Due to the feeble nature of the \( \lambda_{\alpha i} \) couplings, successful baryogenesis requires a resonant enhancement of the asymmetry, which in turn implies that the masses of the two heavy fermions have to be very close to each other. Then, for \( M_1 \simeq M_2 \) the out-of-equilibrium condition reads

\[
\left( \sum_{\alpha,i} \langle \gamma_{F_i \rightarrow e_{\alpha} S} \rangle \right) \lesssim H \bigg|_{T = M_1} \tag{3.9}
\]

The Hubble parameter is given by

\[
H(T) \simeq \frac{1.66 \sqrt{g_{*\rho}}}{M_{Pl}} T^2 \tag{3.10}
\]

where \( M_{Pl} \simeq 1.22 \times 10^{19} \) GeV is the Planck mass and \( g_{*\rho} \simeq 106.75 \) are the effective degrees of freedom related to the energy density. At leading order and in the limit \( M_i \gg m_{e_{\alpha}} + m_S \) the out-of-equilibrium condition (3.9) reads (\( g_{F_i} = 2 \))

\[
\sum_{\alpha,i} \Gamma_{F_i \rightarrow e_{\alpha} S} \lesssim 2H \bigg|_{T = M_1} = \sum_{\alpha,i} |\lambda_{\alpha i}|^2 \lesssim 2.83 \times 10^{-13} \left( \frac{M_1}{\text{TeV}} \right) \tag{3.11}
\]

Thus for heavy leptons at the TeV range, the couplings have to be smaller than \( \sim 10^{-7} \) for the decays to proceed out-of-equilibrium.

3.2 Freeze-in DM abundance

The freeze-in DM abundance \( Y_S \) that is produced from decays and \( 2 \rightarrow 2 \) scattering processes in our model follows the Boltzmann equation

\[
s \frac{dY_S}{dt} = \sum_{\alpha,i} \{ F_i \leftrightarrow e_{\alpha} S \} + \sum_{\alpha,i} \{ F_i B \leftrightarrow e_{\alpha} S \} - \sum_{\alpha,i} \{ F_i S \leftrightarrow e_{\alpha} B \} + \sum_{\alpha,i} \{ F_i \bar{e}_{\alpha} \leftrightarrow SB \} + 2 \sum_{i,j} \{ F_i \bar{F}_j \leftrightarrow SS \} + 2 \sum_{\alpha,\beta} \{ \bar{e}_{\alpha} e_{\beta} \leftrightarrow SS \} \tag{3.12}
\]
In writing (3.12) we have used the notations

\[
\begin{align*}
\{ab \leftrightarrow cd\} &\equiv (ab \leftrightarrow cd) + (\bar{a} \bar{b} \leftrightarrow \bar{c} \bar{d}) \\
[a b \leftrightarrow c d] &\equiv (a b \leftrightarrow c d) - (\bar{a} \bar{b} \leftrightarrow \bar{c} \bar{d}) \\
(a b \leftrightarrow c d) &\equiv \int d\Pi_a d\Pi_b d\Pi_c d\Pi_d (2\pi)^4 \delta^{(4)} \left[ |M|_{ab \to cd}^2 f_a f_b (1 \pm f_c) (1 \pm f_d) \\ &\quad - |M|_{cd \to ab}^2 f_c f_d (1 \pm f_a) (1 \pm f_b) \right] 
\end{align*}
\] (3.13)

where \(\delta^{(4)}\) is an abbreviation for \(\delta^{(4)} (p_a + p_b - p_c - p_d)\), \(f_k\) is the distribution function of species \(k\) and \(s\) denotes the entropy density. We will make the following assumptions:

- The initial DM abundance is zero. Combined with the feeble couplings, this allows us to ignore the inverse decays \(e_\alpha S \to F_i\), i.e. \(f_S \simeq 0\).

- The DM production takes place during the radiation dominated era. At this epoch time and temperature are related by \(\dot{\mathcal{T}} \simeq -H \mathcal{T}\), which is valid for \(\partial g^{*} \rho / \partial \mathcal{T} \simeq 0\). Using this relation we may switch variables and write

\[
\frac{d}{dt} S Y_S = s H z \frac{d}{dz} Y_S \tag{3.14}
\]

The entropy density during the radiation dominated era is given by

\[
s = \frac{2\pi^2}{45} g_{*s} T^3 \tag{3.15}
\]

where \(g_{*s}\) are the effective degrees of freedom with respect to the entropy density.

- The distribution functions of the visible sector species obey the Maxwell-Boltzmann statistics, i.e. we neglect Bose-enhancement and Pauli-blocking factors. Hence, we can write in general

\[
(a b \leftrightarrow c d) = \gamma_{ab \to cd} \frac{Y_a}{Y_a^{eq}} \frac{Y_b}{Y_b^{eq}} - \gamma_{cd \to ab} \frac{Y_c}{Y_c^{eq}} \frac{Y_d}{Y_d^{eq}} \tag{3.16}
\]

Let us first focus on the heavy lepton decays \(F_i \to e_\alpha S\) and the corresponding \(CP\)-conjugate processes, which provide the dominant contribution to DM production for \(z \gtrsim 1\) [25]. Under the aforementioned assumptions the Boltzmann equation for the DM abundance can be written as
\( sHz \frac{d}{dz} Y_S \simeq \sum_{\alpha,i} \{ F_i \leftrightarrow e_\alpha S \} \)

\[
= 2 \sum_{\alpha,i} \gamma_{e_\alpha S} \frac{Y_{F_i+\bar{F}_i}}{Y_{e_\alpha S}} + \mathcal{O}(\epsilon^2) \\
= \frac{g F_i}{\pi^2} M_i^2 T K_1 \left( \frac{M_i}{T} \right) \sum_{\alpha} \Gamma_{F_i \rightarrow e_\alpha S} + \frac{g F_2}{\pi^2} M_2^2 T K_1 \left( \frac{M_2}{T} \right) \sum_{\alpha} \Gamma_{F_2 \rightarrow e_\alpha S}
\]

(3.17)

where \( \gamma_{e_\alpha S} \) is the equilibrium decay rate density at tree-level, \( \epsilon \) denotes the CP asymmetry and we have used \( Y_{F_i+\bar{F}_i} \simeq Y_{e_\alpha S}^\text{eq} \). If we consider \( M_1 \simeq M_2 \) (resonant case) and substitute the decay width (3.4), the Hubble parameter (3.10) and the entropy density (3.15) in Eq. (3.17), then at tree-level and in the limit \( M_i \gg m_{e_\alpha} + m_S \) the DM abundance simplifies to

\[
Y_S (z) = \frac{45 M_{Pl}}{32 \times 1.66 \pi^3 g_{*S} \sqrt{g_{*\rho}}} \sum_{\alpha,i} |\lambda_{\alpha i}|^2 M_1 \int_z^{z_{RH}} dz' \frac{z'^3}{z_{RH}^2} K_1 (z')
\]

(3.18)

where \( z_{RH} \equiv M_1 / T_{RH} \) and we have considered for simplicity that \( \partial g_{*S} / \partial T \simeq 0 \). In our analysis \( T_{RH} \) will be set to \( 10^{12} \text{ GeV} \). At the present day \( T = T_0 \simeq 2.73 \text{ K} \), so \( T_0 \ll M_1 \ll T_{RH} \) and the integral contributes a factor of \( 3\pi/2 \), yielding

\[
Y_S (z_0) = \frac{135 M_{Pl}}{64 \times 1.66 \pi^4 g_{*S} \sqrt{g_{*\rho}}} \sum_{\alpha,i} |\lambda_{\alpha i}|^2 M_1
\]

(3.19)

The experimentally observed DM abundance is

\[
Y_S (z_0) = \frac{\Omega_{DM} \rho_c}{m_S s_0}
\]

(3.20)

where \( \Omega_{DM} h^2 = 0.1200 \pm 0.0012 \) \cite{42}, \( \rho_c \equiv 3 H_0^2 / 8\pi G \simeq 10.537 h^2 \text{ GeV m}^{-3} \) is the critical density and \( s_0 \simeq 2.9 \times 10^9 \text{ m}^{-3} \) is the entropy density at the present day \cite{43}. If we ignore dark matter production through scattering processes, the DM mass \( m_S \) can be related to the heavy lepton mass \( M_1 \) and to the feeble couplings as

\[
m_S = \frac{64 \times 1.66 \pi^4 g_{*S} \sqrt{g_{*\rho}} \Omega_{DM} \rho_c M_1}{135 M_{Pl} s_0} \sum_{\alpha,i} |\lambda_{\alpha i}|^2
\]

(3.21)

Lyman-\( \alpha \) forest observations can be used in order to extract a lower bound on the DM mass, the exact value of which depends on the underlying mechanism of DM genesis. For DM candidates that freeze-out the current bound is \( m_{DM} \gtrsim (1.9 - 5.3) \text{ keV} \) at 95% confidence level.

\(^2\)Gauge scatterings keep \( F_i, (\bar{F}_i) \) close to thermal equilibrium down to \( z \sim 25 \), when they eventually freeze-out. Since the baryon asymmetry is generated prior to their freeze-out (when sphalerons become inactive) this is a valid approximation.
Figure 3: DM abundance generated solely by decays, as well as by decays and scatterings for heavy leptons masses $M_1 \simeq M_2 = 1.2$ TeV and couplings $\sum_{\alpha,i} |\lambda_{\alpha i}|^2 \simeq 6.25 \times 10^{-16}$.

C.L. [44–46]. This limit can be mapped onto the case of freeze-in produced DM yielding $m_S \gtrsim (4 - 16)$ keV [47, 48]. This, in turn, imposes an upper limit on the feeble couplings

$$\sum_{\alpha,i} |\lambda_{\alpha i}|^2 \lesssim 7.55 \times 10^{-16} \left( \frac{M_1}{\text{TeV}} \right)$$

in order not to overclose the Universe, where we have used the more conservative bound $m_S \gtrsim 4$ keV. Thus we see that the Lyman-α forest sets a more severe constraint than the out-of-equilibrium condition of Eq. (3.9), forcing the feeble couplings to lie in the $10^{-8}$ range and below for heavy lepton masses at the TeV scale.

For a more rigorous treatment one should take into account the impact of scattering processes, which can modify the predicted DM abundance at $z \lesssim 1$, an epoch when scatterings are dominant. Including such scattering processes will be essential for the calculation of the baryon asymmetry presented in the next section. We focus only on scattering processes which involve gauge bosons as external states and ignore the subleading ones (second row of Eq. (3.12)). In this case the Boltzmann equation takes the form

$$sHz \frac{dY_S}{dz} = \sum_{\alpha,i} \{F_i \leftrightarrow e_\alpha S\} + \sum_{\alpha,i} \{F_i B \leftrightarrow e_\alpha S\} - \sum_{\alpha,i} \{F_i S \leftrightarrow e_\alpha B\} + \sum_{\alpha,i} \{F_i \bar{e}_\alpha \leftrightarrow SB\}$$

$$= 2 \sum_{\alpha,i} \left( \gamma_{e_\alpha S}^F + \gamma_{e_\alpha S}^F + \gamma_{e_\alpha S}^F + \gamma_{e_\alpha B}^F \right) + O(\epsilon^2)$$

(3.23)

where we have made use of the freeze-in approximation $f_S \simeq 0$ and considered $Y_{F_i + F_i} \simeq Y_{F_i + F_i}^{eq}$. In Figure 3 we fix the masses of the heavy leptons $M_1 \simeq M_2 = 1.2$ TeV and the couplings $\sum_{\alpha,i} |\lambda_{\alpha i}|^2 \simeq 6.25 \times 10^{-16}$ and we compare the predicted DM abundance as estimated if we only take into account decays of the heavy leptons (lower, green line) with
the result obtained through a full-blown numerical solution of Eq. (3.23) (upper, yellow line). We observe that, as expected, the inclusion of the scattering processes does not drastically modify the predicted amount of dark matter in the Universe. Note that we have also cross-checked all of our results by implementing our model in FeynRules [49] and computing the predicted freeze-in DM abundance with CalcHEP/micrOMEGAs 5 [50, 51]. Given our findings, we conclude that the analytic estimate of the DM mass given by Eq. (3.21) constitutes a reliable approximation. For the values of the physical parameters used in Figure 3, the value of the dark matter mass in order to reproduce the observed DM abundance in the Universe is \( m_S \simeq 5.81 \text{ keV} \), which is compatible with the Lyman-\( \alpha \) forest bounds discussed previously.

### 3.3 \( CP \) Asymmetry

The \( CP \) asymmetry generated through the decays of \( F_{1,2} \) arises, at lowest order, due to the interference of the tree-level and 1-loop self-energy Feynman diagrams (wave part contribution) as shown in Figures 1a and 1b. It can be defined in terms of the decay widths as

\[
e_{\alpha i} = \frac{\Gamma (F_i \to e_\alpha S) - \Gamma (\bar{F}_i \to \bar{e}_\alpha S)}{\sum_\alpha \Gamma (F_i \to e_\alpha S) + \Gamma (\bar{F}_i \to \bar{e}_\alpha S)} = \frac{\Gamma (F_i \to e_\alpha S) - \Gamma (\bar{F}_i \to \bar{e}_\alpha S)}{2 \Gamma_i} \tag{3.24}
\]

where \( \Gamma_i \) is the total decay width of \( F_i \). If we separate the matrix element \( M|_k \), with \( k \) being the loop order \( (k = 0, 1, \ldots) \), into a coupling constant part \( c_k \) (denoting a collection of coupling constants) and an amplitude part \( A_k \), i.e. \( M|_k = \sum_{i=0}^k c_i A_i \), we can rewrite the \( CP \) asymmetry as [52]

\[
e_{\alpha i} = -2 \frac{\text{Im} \left\{ c_0 c_i^* \right\} \int d\Pi_{e\alpha} d\Pi_S (2\pi)^4 \delta^{(4)} \text{Im} \left\{ A_0 A_i^* \right\}}{\sum_{\alpha} |c_0|^2 \int d\Pi_{e\alpha} d\Pi_S (2\pi)^4 \delta^{(4)} |A_0|^2} \tag{3.25}
\]

The imaginary part of the amplitudes is related to the discontinuity of the corresponding graph as \( 2 \text{Im} \left\{ A_0 A_i^* \right\} = \text{Disc} \left\{ A_0 A_i^* \right\} \), which can be computed using the Cutkosky cutting rules. For a non-degenerate \( F_i \) spectrum, \( M_j - M_i \gg \Gamma_j \), we obtain [53]

\[
e_{ii} = \frac{1}{16\pi} \frac{1}{1 - x_j} \frac{\text{Im} \left\{ \lambda_{ai}^* \lambda_{aj} [\lambda^\dagger \lambda]_{ji} \right\}}{[\lambda^\dagger \lambda]_{ii}} \tag{3.26}
\]

where \( x_j \equiv M_j^2 / M_i^2 \) and we have used the standard notation \( [\lambda^\dagger \lambda]_{ij} \equiv |\lambda_{ei}|^2 + |\lambda_{ji}|^2 + |\lambda_{ri}|^2 \) and \( [\lambda^\dagger \lambda]_{ji} \equiv \lambda_{ej}^* \lambda_{ei} + \lambda_{mj}^* \lambda_{mi} + \lambda_{rt}^* \lambda_{rt} \). This is half the result obtained in standard leptogenesis (see e.g. Eq. (5.13) in [52]), because we deal with Dirac fermions where only a charged lepton propagates in the loop, whereas Majorana neutrinos can decay to both a charged lepton or a light neutrino accompanied by the Higgs field.

From \( CPT \) invariance and unitarity we know that the total decay width of a state and its \( CP \)-conjugate are equal.
\[ \sum_\alpha \Gamma (F_i \to e_\alpha S) = \sum_\alpha \Gamma (\bar{F}_i \to \bar{e}_\alpha S) \equiv \Gamma_i \]  

(3.27)

Hence the CP asymmetry vanishes when summed over the flavours, \( \epsilon_i \equiv \sum_\alpha \epsilon_{\alpha i} = 0 \). This can be seen by summing Eq. (3.26) over flavours, a case in which the argument of the imaginary part becomes real and it vanishes identically,

\[ \sum_\alpha \text{Im}\left\{ \lambda^*_\alpha \lambda_{\alpha j} [\lambda^{\dagger} \lambda]_{ji} \right\} = \text{Im}\left\{ [\lambda^{\dagger} \lambda]_{ij} [\lambda^{\dagger} \lambda]_{ji} \right\} = 0 \]  

(3.28)

However, flavour effects can lead to a non-vanishing baryon/lepton asymmetry, since washout processes are flavour-dependent and, therefore, the lepton asymmetries in each flavour are washed out in a different way [54–57].

Due to the feeble nature of the \( \lambda_{\alpha i} \) couplings, a resonant enhancement of the CP asymmetry is needed in order to generate a sufficiently large baryon asymmetry. This occurs when the mass difference between the heavy leptons is of the order of their decay widths and is related to the wave-part contribution to the CP asymmetry (Figure 1b). The resulting CP asymmetry is given by

\[ \epsilon_{\alpha i} = \frac{1}{16\pi} \frac{1 - x_j}{(1 - x_j)^2 + g_j^2} \frac{\text{Im}\left\{ \lambda_{\alpha i}^* \lambda_{\alpha j} [\lambda^{\dagger} \lambda]_{ji} \right\}}{[\lambda^{\dagger} \lambda]_{ii}} \]  

(3.29)

where \( g_j \equiv \Gamma_j/M_i \). In deriving (3.29) we have regularized the divergence for small \( x_j \) by applying the resummation procedure presented in [33], with the regulator given by \( M_1 \Gamma_2 \). If we also express the coupling constants in terms of their magnitude and phase, \( \lambda_{\alpha i} = |\lambda_{\alpha i}| e^{i\phi_{\alpha i}} \), Eq. (3.29) can be rewritten as

\[ \epsilon_{\alpha i} = \frac{1}{16\pi} \frac{1 - x_j}{(1 - x_j)^2 + g_j^2} \frac{|\lambda_{\alpha i}| |\lambda_{\alpha j}|}{[\lambda^{\dagger} \lambda]_{ii}} \sum_{\beta \neq \alpha} |\lambda_{\beta i}| |\lambda_{\beta j}| \sin (-\phi_{\alpha i} + \phi_{\alpha j} - \phi_{\beta j} + \phi_{\beta i}) \]

\[ \equiv \frac{1}{16\pi} \frac{1 - x_j}{(1 - x_j)^2 + g_j^2} \frac{|\lambda_{\alpha i}| |\lambda_{\alpha j}|}{[\lambda^{\dagger} \lambda]_{ii}} \sum_{\beta \neq \alpha} |\lambda_{\beta i}| |\lambda_{\beta j}| p^{ij}_{\alpha \beta} \]  

(3.30)

where \( p^{ij}_{\alpha \beta} = -p^{ji}_{\alpha \beta} = -p^{ji}_{\beta \alpha} \). The resonance condition reads \( M_2 - M_1 \sim \Gamma_2/2 \) [33] and in this case the CP asymmetry can be maximally enhanced to

\[ \epsilon_{\alpha 1}^{\text{res}} = \frac{g_{F_1}}{2} \frac{|\lambda_{\alpha 1}| |\lambda_{\alpha 2}|}{[\lambda^{\dagger} \lambda]^2_{11} + [\lambda^{\dagger} \lambda]^2_{22}} \sum_{\beta \neq \alpha} |\lambda_{\beta 1}| |\lambda_{\beta 2}| p^{12}_{\alpha \beta} \]  

(3.31a)

\[ \epsilon_{\alpha 2}^{\text{res}} = g_{F_2} \frac{|\lambda_{\alpha 1}| |\lambda_{\alpha 2}|}{[\lambda^{\dagger} \lambda]^2_{11} + [\lambda^{\dagger} \lambda]^2_{22}} \sum_{\beta \neq \alpha} |\lambda_{\beta 1}| |\lambda_{\beta 2}| p^{12}_{\alpha \beta} \]  

(3.31b)

These are the expressions that we will be using in the numerical analysis that follows in order to compute \( \epsilon_{\alpha i} \).
3.4 Baryon Asymmetry

In the previous sections we described how our model can satisfy two out of the three Sakharov conditions, namely C/CP violation and out-of-equilibrium dynamics. The last condition to be fulfilled in order to generate a baryon asymmetry is the violation of the baryon/lepton number. In the case of Majorana heavy states a conserved lepton number cannot be consistently assigned in the presence of interaction and mass terms in the Lagrangian and therefore $L$ is violated. This is not the case in our model where the heavy states are of Dirac nature. In this case, we may rely on sphaleron departure from equilibrium during the epoch that the lepton asymmetry is generated, along the lines described in [29].

In the model we propose the heavy leptons $F_i$ carry the same lepton number as the SM leptons and the total lepton asymmetry is $Y_L = Y_{LSM} + Y_{LF}$, where $Y_{LSM} = \sum_\alpha Y_{L\alpha}$ and $Y_{LF} = \sum_i Y_{LFi}$. All processes conserve the combination $Y_{B-L} = Y_B - Y_{LSM} - Y_{LF}$, i.e. $dY_{B-L}/dz = 0$. We also assume that the Universe is initially totally symmetric, $Y_{B-L,SM}\mid_{z_{RH}} = Y_{LF}\mid_{z_{RH}} = 0$ and therefore at any $z$ it holds $Y_{B-L,SM} = Y_{LF}$.

Since sphalerons are insensitive to the lepton asymmetry $Y_{LF}$, as $F_i$ are $SU(3)_c \times SU(2)_L$-singlets, they only affect the non-zero lepton asymmetry stored in the SM lepton sector $Y_{LSM}$. In particular they convert it to a baryon asymmetry by imposing certain relations among the chemical potentials of the various species (see Appendix A). Once sphalerons depart from equilibrium, which occurs at $T_{sph} = 131.7 \pm 2.3$ GeV [58], the baryon and lepton numbers are separately conserved. When the heavy leptons decay away the total baryon asymmetry, being proportional to $Y_{B-L,SM}$, vanishes. However, if sphalerons become inactive during the decay epoch of the heavy leptons then the baryon asymmetry freezes at $Y_B \propto Y_{B-L,SM}\mid_{T_{sph}}$, which in general is not null.

Taking into consideration all the decay and $2 \rightarrow 2$ scattering processes the full Boltzmann equations of the asymmetries read

$$-sHz \frac{dY_{\Delta F}}{dz} = \sum_\alpha [F_i \leftrightarrow e_\alpha S] + \sum_\alpha [F_i B \leftrightarrow e_\alpha S] + \sum_\alpha [F_i S \leftrightarrow e_\alpha B] + \sum_\alpha [F_i \bar{e}_\alpha \leftrightarrow SB]$$

$$+ \sum_{\alpha,\beta,j} [F_i \bar{e}_\alpha \leftrightarrow \bar{F}_j \bar{e}_\beta] + \sum_{\alpha,\beta} [F_i \bar{e}_\alpha \leftrightarrow \bar{F}_j \bar{e}_\beta] + \sum_{\alpha,\beta} [F_i e_\beta \leftrightarrow F_j e_\alpha] + \sum_{\alpha,\beta} [F_i \bar{F}_j \leftrightarrow e_\alpha \bar{e}_\beta] + \sum_{\alpha,\beta} [F_i \bar{F}_j \leftrightarrow S S] + \sum_{\alpha,\beta} [F_i F_j \leftrightarrow e_\alpha e_\beta]$$

$$+ [F_i S \leftrightarrow F_j \bar{S}]$$

(3.32)
where $Y_{\Delta F_i} \equiv Y_{F_i} - Y_{\bar{F}_i}$ and $Y_{\Delta \alpha} \equiv Y_{B}/3 - Y_{L_{SM\alpha}}$. The primed term indicates that one has to subtract the contribution due to the on-shell propagation of $F_i$, usually referred to as real intermediate state subtraction (RISS), which is already taken into account by the successive decays $e_\alpha S \leftrightarrow F_i \leftrightarrow e_\alpha S$.

The various terms in the Boltzmann equations can be expressed in terms of the $CP$ asymmetry $\epsilon_{\alpha i}$, the tree-level rate densities and the asymmetric abundances. As is typically done, we linearize in the SM chemical potentials $[59]$}

$$
\frac{Y_{e_\alpha (e_\alpha)}}{Y_{e_\alpha}^{eq}} \equiv e^{\mp \mu_{e_\alpha}/T} \simeq 1 \pm \frac{\mu_{e_\alpha}}{T} = 1 \pm \frac{Y_{\Delta e_\alpha}}{2Y_{\gamma}} \tag{3.34}
$$

All non-gauge interactions are subleading in comparison to the gauge interactions and can be safely discarded. We only include the $CP$-violating part of the RISS term, which ensures that the source term (proportional to $\epsilon_{\alpha i}$) takes the correct form, i.e. it vanishes when all species are in chemical equilibrium. The Boltzmann equations can be rewritten as$^3$

$$
-sH \frac{dY_{\Delta F_i}}{dz} = \sum_\alpha \left( y_{F_i} - \frac{Y_S}{Y_S^{eq}} y_{e_\alpha} \right) \left( \gamma_{F_i S}^{e_\alpha} + \gamma_{e_\alpha S}^{F_i S} \right) + \sum_\alpha \left( \frac{Y_S}{Y_S^{eq}} y_{F_i} - y_{e_\alpha} \right) \gamma_{F_i S}^{e_\alpha} + \sum_\alpha \left( y_{F_i} - y_{e_\alpha} \right) \gamma_{F_i S}^{e_\alpha} \tag{3.35}
$$

$$
-sH \frac{dY_{\Delta \alpha}}{dz} = 2 \sum_i \epsilon_{\alpha i} \left[ \left( 1 - \frac{Y_S}{Y_S^{eq}} \right) \sum_\rho \left( \gamma_{\rho S}^{F_i} + \gamma_{S B}^{F_i} + \gamma_{F_i S}^{\rho e_\alpha} - \gamma_{\rho S}^{F_i} e_\alpha S \right) \right] \left( y_{F_i} - \frac{Y_S}{Y_S^{eq}} y_{e_\alpha} \right) \gamma_{F_i S}^{e_\alpha} + \sum_\alpha \left( y_{F_i} - y_{e_\alpha} \right) \gamma_{F_i S}^{e_\alpha} \tag{3.36}
$$

$^3$In deriving Eq. (3.23) we dropped dark matter annihilation processes altogether. In baryo/leptogenesis, inverse reactions can be crucial and need to be taken into account. In order to do so in an efficient manner, in Eqs. (3.35) and (3.36) we will approximate $f_S \simeq -\frac{f_{e_\alpha}}{f_{Y_S}} \gamma_{e_\alpha S}$ - a relation which is rigorously applicable to bath particles. We have checked that – as expected, since we have restricted ourselves to regions of the parameter space in which the condition of Eq. (3.9) is satisfied – this is, indeed, a good approximation which only leads to a small ($\sim 2\%$) reduction of the predicted dark matter abundance for large values of $z$. 

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where \( y_{F_i} \equiv \frac{Y_{\Delta F_i}}{Y_{F_i}} \), \( y_{e\alpha} \equiv \frac{Y_{\Delta e\alpha}}{Y_{e\alpha}} \), \( B_{e\mu}^{F_i} \) denotes the branching ratio of the decay \( F_i \rightarrow e\beta S \) and we have used \( Y_{F_i + \bar{F}_i} \approx y_{F_i}^{eq} Y_{F_i + \bar{F}_i} \). In deriving the equations above we have taken into account that the \( CP \) asymmetry in scattering processes stemming from self-energy diagrams (Figure 1b) is always equal to the \( CP \) asymmetry from decays [60]. As described in [52] one must also include the contributions from the RIS-subtracted \( 2 \rightarrow 3 \) scatterings involving gauge bosons, in order to obtain the correct form for the scattering source terms. These processes can, however, be neglected as subleading with regards to their impact on washout. Note that the source term in the Boltzmann equation of the asymmetry in \( F_i \) vanishes as a consequence of \( CPT \) and unitarity, as explained in Section 3.3. Lastly, these equations are not totally independent, as they are related through the conservation of \( Y_{B - L} \), resulting to \( \sum_i Y_{\Delta F_i} = \sum_{\alpha} Y_{\Delta \alpha} \).

One can express the asymmetries in each SM flavour \( y_{e\alpha} \) in terms of \( y_{\Delta \alpha} \), solving a system of algebraic equations which relate the chemical potentials and abundances in equilibrium of the various species (see Appendix A). For the temperatures of interest we find

\[
\begin{pmatrix}
y_{e} \\
y_{\mu} \\
y_{\tau}
\end{pmatrix} = - \frac{1}{711} \begin{pmatrix}
230 & -7 & -7 \\
-7 & 230 & -7 \\
-7 & -7 & 230
\end{pmatrix} \begin{pmatrix}
Y_{\Delta e} \\
Y_{\Delta \mu} \\
Y_{\Delta \tau}
\end{pmatrix} \frac{1}{Y_\gamma} \tag{3.37}
\]

In the same way we have calculated the amount of the SM lepton asymmetry converted to baryon asymmetry by the sphaleron transitions, to be

\[
Y_B = \frac{22}{79} \sum_\alpha Y_{\Delta \alpha} \tag{3.38}
\]

The system of the five coupled Boltzmann equations of the asymmetries (3.35) and (3.36) has been solved numerically, using the DM abundance generated by decays and scattering processes of Eq. (3.23). We confirm that the proposed model can indeed generate the observed baryon asymmetry \( Y_B = 8.71 \pm 0.06 \times 10^{-11} \) [43] in the resonant regime. As an illustration we present in Figure 4 an explicit example, using the following values of the parameters:

\[
\begin{align*}
M_1 &= 1.2 \text{ TeV}, & M_2 &= M_1 + \Gamma_2 / 2 \\
|\lambda_{e1}| &= 2.145 \times 10^{-8}, & |\lambda_{e2}| &= |\lambda_{\tau 2}| = 9 \times 10^{-10} \\
|\lambda_{\mu 1}| &= |\lambda_{\tau 1}| = 9 \times 10^{-9}, & |\lambda_{\mu 2}| &= 8 \times 10^{-10} \\
p_{e\mu}^{12} &= p_{e\tau}^{12} = p_{\mu\tau}^{12} = 1
\end{align*} \tag{3.39}
\]

The corresponding resonant \( CP \) asymmetries \( \epsilon_{\alpha i} \) turn out to be: \( \epsilon_{e1} \approx 2.1 \times 10^{-1} \), \( \epsilon_{\mu 1} \approx -5.7 \times 10^{-2} \), \( \epsilon_{\tau 1} \approx -1.53 \times 10^{-1} \) and \( \epsilon_{e2} \approx 1.53 \times 10^{-3} \), \( \epsilon_{\mu 2} \approx -4.2 \times 10^{-4} \), \( \epsilon_{\tau 2} \approx -1.11 \times 10^{-3} \). We have explicitly verified that \( Y_{B - L} \) is conserved for all values of \( z \), or equivalently that the relation \( \sum_i Y_{\Delta F_i} = \sum_{\alpha} Y_{\Delta \alpha} \) holds.
Figure 4: The generated DM abundance $Y_S$, the asymmetries $Y_{\Delta \alpha}$, $Y_{\Delta F_i}$ and the baryon asymmetry $Y_B$ as a function of the dimensionless parameter $z = M_1/T$, for the heavy leptons masses and feeble couplings given in Eq. (3.39). The dashed line illustrates the evolution that $Y_B$ would follow in the absence of sphaleron decoupling, while the vertical line denotes the temperature at which sphalerons depart from equilibrium.

Note that, contrary to the standard leptogenesis scenarios, the SM flavour asymmetries $Y_{\Delta \alpha}$ are constantly generated since the $CP$-violating processes always occur out-of-equilibrium. They eventually attain their final value as soon as the DM state freezes-in at $z \sim 3 - 5$. On the other hand, as the heavy leptons decay their asymmetries $Y_{\Delta F_i}$ decrease and eventually vanish at high $z$. The asymmetry in $F_2$ is many orders of magnitude smaller than the one in $F_1$, due to its smaller couplings $\lambda_{\alpha 2} \ll \lambda_{\alpha 1}$, and is not shown in Figure 4. Once the $F_i$ have decayed away, conservation of $Y_{B-L}$ implies that also $Y_{B-L,SM} \rightarrow 0$. Thus the model predicts an equal amount of baryon $Y_B$ and SM lepton $Y_{L,SM}$ asymmetries left in the Universe [29].

Our numerical analysis reveals that at least one of the feeble couplings must have a magnitude larger than $\sim 10^{-8}$ in order to account for the observed baryon asymmetry. Our main results are summarised in Figure 5, where we show the combinations of the feeble couplings and the heavy lepton mass for which a viable baryon asymmetry can be generated, for the two illustrative scenarios depicted in Table 1.

| Scenario 1: | Scenario 2: |
|------------|------------|
| $|\lambda_{\mu 1}|$ | $9 \times 10^{-9}$ |
| $|\lambda_{\tau 1}|$ | $9 \times 10^{-9}$ |
| $|\lambda_{\mu 2}|$ | $9 \times 10^{-10}$ |
| $|\lambda_{\tau 2}|$ | $9 \times 10^{-10}$ |

Table 1: Values of the parameters used for the two scenarios shown in Figure 5.

In both cases the $CP$ asymmetry is resonantly enhanced, \textit{i.e.} the mass of $F_2$ is $M_2 = M_1 + \Gamma_2/2$ and we consider $p_{\mu \mu}^{12} = p_{\tau \tau}^{12} = p_{\mu \tau}^{12} = 1$ for the phases of the feeble couplings. The $\lambda_{e1}$ coupling is being treated as a free parameter and lies in the $10^{-8}$ ballpark.
Figure 5: Combinations of the feeble couplings and the heavy lepton mass which can generate the observed baryon asymmetry, for the set of parameters depicted in Table 1. The dashed lines depict representative DM masses $m_S = \{2, 4, 6, 16\}$ keV for which the observed DM abundance of the Universe can be reproduced. Masses below 4 keV are excluded from current Lyman-α forest observations (grey shaded area).

The dark matter relic density constraint can be satisfied throughout the parameter ranges depicted in the Figure, for an appropriate choice of the mass $m_S$. However, in the shaded region the required dark matter mass turns out to be in conflict with the Lyman-α bound of Eq. (3.22).

We observe that the first scenario (upper, blue line) is excluded from Lyman-α constraints for all values of the heavy lepton mass $M_1$. On the contrary, the second scenario (lower, orange line) is able to account for both the baryon asymmetry and the DM abundance for a wide range of $M_1$, $550 \text{ GeV} \lesssim M_1 \lesssim 2800 \text{ GeV}$. Note that interestingly, we find that for any combination of parameter values the relic density constraint is satisfied for DM masses that do not exceed $\sim 6 \text{ keV}$. This implies that if Lyman-α constraints become stronger in the future, they may be able to fully exclude the model’s viable parameter space.

3.5 Phenomenological aspects

In the previous paragraphs we saw that our simple model, described by the Lagrangian of Eq. (3.1), can provide a common explanation for the observed dark matter content and baryon asymmetry of the Universe. As we showed, this can be achieved by assuming highly degenerate heavy fermions $F_i$ with masses $M_i \sim 1 \text{ TeV}$ and Yukawa-like couplings of the order of $\lambda_{\alpha_1} \sim 10^{-8} - 10^{-7}$ and $\lambda_{\alpha_2} \sim 10^{-10}$ for $F_1$ and $F_2$ respectively.

Our model is intended to serve mainly as a proof-of-concept for the freeze-in baryogenesis mechanism that we propose. In this spirit, a full-blown analysis of its phenomenological predictions goes well beyond the scope of the present work, especially since alternative constructions assuming, e.g., different quantum numbers for the various particles involved, can lead to wildly different phenomenological signatures. Still, despite its simplicity this model does exhibit an interesting phenomenology, which we will briefly comment upon. Note that
a simpler variant of this model involving a single vector-like heavy fermion $F$ was already studied, e.g., in [61, 62], mainly from the viewpoint of its predicted dark matter abundance as well as its signatures in New Physics searches at the LHC.

First, the heavy fermions can mediate lepton flavour-violating decays of the type $\ell_\beta \rightarrow \ell_\alpha \gamma$. In particular, given the constraint $Br(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}$ [63], and assuming that (as was the case throughout the analysis presented in the previous Sections) $\lambda_1 \sim \lambda_{\mu 1}$, the analysis presented in [62] restricts the product of the Yukawa-like couplings of $F_1$ to the first two generation leptons as

$$\frac{\sqrt{\lambda_e \lambda_{\mu 1}}}{M_1} \lesssim 3.6 \times 10^{-5} \text{ GeV}^{-1}$$

(3.40)

which is easily satisfied in the cosmologically relevant part of our parameter space. In deriving this bound we have neglected the interference between Feynman diagrams involving different species of heavy fermions running in the loop, since the couplings $\lambda_{\alpha 2} \ll \lambda_{\alpha 1}$. Besides, constraints stemming from measurements of the muon lifetime turn out to be subleading [62].

On the side of collider searches, let us first point out the fact that given the feeble nature of the $\lambda_{\alpha i}$ couplings, the mean proper decay length of $F_1$ is of the order of a few cm, whereas that of $F_2$ is of the order of several meters. Given these lifetime ranges, both $F_1$ and $F_2$ can be looked for in searches for long-lived particles (LLPs) at the LHC. In particular, as shown in [61, 62], searches for displaced leptons accompanied by missing energy can target the production and decay of $F_1$ and, for the values of $\lambda_{\alpha 1}$ that we assume, already exclude $F_1$ masses up to $\sim 400 - 500$ GeV. $F_2$, on the other hand, survives long enough to escape the detector and can be probed by searches for Heavy Stable Charged Particles, with masses smaller than $\sim 500 - 600$ GeV being already excluded. The corresponding searches at the High-Luminosity Run of the LHC will probe $F_{1,2}$ masses reaching up to $\sim 800$ GeV and $\sim 1.5$ TeV, respectively [61].

In summary, if our model is to simultaneously explain the dark matter abundance and the baryon asymmetry of the Universe while remaining consistent with Lyman-$\alpha$ bounds, then it predicts interesting signatures for LLP searches at the LHC. In conjunction with the fact that, as we showed, explaining the matter-antimatter asymmetry of the Universe tends to favour $F_{1,2}$ masses lying in the range of a few TeV, we can hope that a substantial part of the cosmologically favoured part of the parameter space will be probed by the LHC within the next few years.

4 Conclusions and Outlook

In this paper we discussed a mechanism for the simultaneous generation of the dark matter density and the baryon asymmetry of the Universe. To this goal we relied on the out-of-equilibrium decays of heavy bath particles into a (feebly coupled) dark matter state along with Standard Model charged fermions, which leads to dark matter production via the freeze-in mechanism. If, moreover, $CP$ is violated by these same decay processes, a viable matter-antimatter asymmetry can also be generated either directly (if the decays
also lead to baryon/lepton number violation) or through the interplay of the generated $CP$ asymmetry with the SM electroweak sphalerons.

As a proof-of-concept, we employed a simple model in which the role of the heavy bath particles was played by two $SU(3)_c \times SU(2)_L$-singlet vector-like fermions with a non-zero hypercharge and dark matter was identified with a gauge singlet real scalar field. We showed that, indeed, such a simple construction can lead to both dark matter production and successful baryogenesis and we briefly discussed the phenomenological prospects of such a construction, particularly in relation to searches for long-lived particles at the LHC.

Our proposal draws inspiration from the idea presented in [23]: first, the dark matter content and the baryon asymmetry of the Universe are generated simultaneously through the freeze-in mechanism. Secondly, at least within the framework of our concrete incarnation of this proposal, in both cases the electroweak sphalerons are used in order to convert a $CP$ asymmetry into a baryonic one. In our case, however, instead of considering oscillations – as in ARS leptogenesis – we rather relied on decays in order to generate this initial $CP$ asymmetry – a process which is somewhat reminiscent of GUT-baryogenesis. Moreover, we presented a general thermodynamical treatment that can also be applied in cases in which baryon/lepton number is violated already at the decay level.

There are several topics which we did not expand upon in this paper, and which would merit much more detailed investigation. First, more elaborate models can certainly be developed in this context of freeze-in baryogenesis, potentially establishing connections with theoretically well-motivated UV completions of the Standard Model. As an example, we could mention the fact that we assumed rather ad-hoc values for the couplings of the heavy bath particles to dark matter and to the SM fermions. More realistic flavour structures can be envisaged in quite a straightforward manner, whereas nothing forbids dark matter itself to be asymmetric, along the lines described in [20]. Secondly, albeit related to the previous point, our discussion of more phenomenological aspects has been fairly elementary. Concrete, well-motivated constructions can have important implications for flavour physics (e.g. flavour-violating decays of or contributions to electric/magnetic moments of SM fermions), collider physics (e.g. new resonances and/or final states in LHC searches, potentially involving long-lived particles [64]) or cosmological measurements (e.g. direct detection experiments or CMB observations). Such considerations, which are typically fairly model-dependent, are left for future work.

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A Spectator Processes

During the baryon and/or lepton number violation era there are various processes which can modify the number densities of the particles species. They are called 'spectator processes' because they affect the baryon/lepton number indirectly by distributing the asymmetry among all SM fermions and Higgs fields. These processes include the gauge interactions, the Yukawa interactions and the electroweak and QCD non-perturbative sphaleron transitions. If these processes are in thermal equilibrium with the cosmic plasma, then their effect is to impose certain relations among the chemical potentials of the various particle species.

Recall that the asymmetry in the particle and antiparticle equilibrium number densities of species \( k \), denoted as \( n_{\Delta k} \equiv n_k - n_{\bar{k}} \), is given in the ultra-relativistic limit \( m_k \ll T \) and \( \mu_k \ll T \) by

\[
  n_{\Delta k} = \begin{cases} 
    g_k \mu_k \frac{T^2}{\pi^2}, & k \text{: boson} \\
    g_k \mu_k \frac{T^2}{\pi^2}, & k \text{: fermion}
  \end{cases}
\]  

(A.1)

where \( \mu_k \) is the chemical potential and \( g_k \) are the independent degrees of freedom of species \( k \). The total baryon \( n_B \) and lepton \( n_L \) asymmetries can be written as

\[
  n_B = \sum_k B_k n_{\Delta k}, \quad n_L = \sum_k L_k n_{\Delta k}
\]  

(A.2)

where \( B_k, L_k \) are the baryon and lepton numbers of species \( k \) respectively.

In our case the lepton asymmetry is mainly generated during \( T_{EW} \leq T \ll 10^8 \text{ GeV} \) when all spectator processes are active, including the sphaleron processes. These violate the \( n_B + n_L \) asymmetry but conserve the orthogonal combination \( n_{B-L} \equiv n_B - n_L \). A rough estimation is that sphalerons will erase the \( n_{B+L} \) asymmetry resulting to

\[
  n_B = \frac{n_{B+L}}{2} + \frac{n_{B-L}}{2} \quad \text{sphalerons} \quad n_B = \frac{n_{B-L}}{2}
\]  

(A.3a)

\[
  n_L = \frac{n_{B+L}}{2} - \frac{n_{B-L}}{2} \quad \text{sphalerons} \quad n_L = -n_B = -\frac{n_{B-L}}{2}
\]  

(A.3b)

A more careful treatment shows that in spite the fact that sphalerons violate \( n_{B+L} \), thermodynamic equilibrium requires a non-zero value, i.e. \( n_{B+L} \neq 0 \). To quantify the asymmetry distribution, one can always relate the \( n_{B-L} \) number density asymmetry with \( n_B \) and \( n_L \) as follows
\[ n_{B-L} \equiv n_B - n_L = c n_{B-L} - (c-1) n_{B-L} \Rightarrow \begin{cases} n_B = c n_{B-L} \\ n_L = (c-1) n_{B-L} \end{cases} \quad (A.4) \]

where \( c \lesssim 1 \) is the constant of proportionality. To evaluate it we assign chemical potentials to the particle spectrum of the model. This consists of all the SM species with three families and one Higgs doublet, together with the two BSM vector-like fermions \( F_i \).

### A.0.1 SM

Let us first consider only the SM spectrum and assign the following non-vanishing chemical potentials \([65, 66]\):

\[
\begin{align*}
\mu_{q_\alpha} & \quad \text{(LH quark doublets)} \\
\mu_{u_\alpha} & \quad \text{(RH up-quark singlets)} \\
\mu_{d_\alpha} & \quad \text{(RH down-quark singlets)} \\
\mu_{\ell_\alpha} & \quad \text{(LH lepton doublets)} \\
\mu_{e_\alpha} & \quad \text{(RH charged lepton singlets)} \\
\mu_H & \quad \text{(Higgs doublet, } H = (h^+, h^0)^T) \quad (A.5f)
\end{align*}
\]

where \( \alpha = \{1, 2, 3\} \) is a SM flavour index\(^4\). The corresponding antiparticle states have opposite chemical potentials, e.g. \( \mu_{\tilde{H}} = -\mu_H \) where \( \tilde{H} = (h^{0*}, -h^-)^T \). Note that the electrically chargeless gauge bosons have vanishing chemical potentials, as they carry no conserved quantum number, while the chemical potential of the \( W^\pm \) gauge bosons vanish because the third component of the weak isospin is zero at \( T > T_{EW} \) [65]. Hence the number density asymmetries of the various components can be written, according to Eq. (A.1), as

\[
\begin{align*}
n_{\Delta q_\alpha} & \equiv n_{q_\alpha} - n_{\bar{q}_\alpha} = 6 \mu_{q_\alpha} \frac{T^2}{6} \quad (A.6a) \\
n_{\Delta u_\alpha} & \equiv n_{u_\alpha} - n_{\bar{u}_\alpha} = 3 \mu_{u_\alpha} \frac{T^2}{6} \quad (A.6b) \\
n_{\Delta d_\alpha} & \equiv n_{d_\alpha} - n_{\bar{d}_\alpha} = 3 \mu_{d_\alpha} \frac{T^2}{6} \quad (A.6c) \\
n_{\Delta \ell_\alpha} & \equiv n_{\ell_\alpha} - n_{\bar{\ell}_\alpha} = 2 \mu_{\ell_\alpha} \frac{T^2}{6} \quad (A.6d) \\
n_{\Delta e_\alpha} & \equiv n_{e_\alpha} - n_{\bar{e}_\alpha} = \mu_{e_\alpha} \frac{T^2}{6} \quad (A.6e) \\
n_{\Delta H} & \equiv n_H - n_{\tilde{H}} = 2 \mu_H \frac{T^2}{3} = 4 \mu_H \frac{T^2}{6} \quad (A.6f)
\end{align*}
\]

\(^4\)Note that in this Appendix we are introducing a slightly different notation with respect to the rest of the paper, denoting the SM right-handed leptons by \( \{e_1, e_2, e_3\} \) instead of \( \{e, \mu, \tau\} \).
The baryon and lepton number density asymmetries for each flavour ($n_{B\alpha}$, $n_{L\alpha}$), as well as the corresponding total ones ($n_B$, $n_L$), can be written in terms of the chemical potentials in Eq. (A.5) as follows

\[
n_{B\alpha} \equiv \sum_k B_{k\alpha} \Delta n_k = \frac{1}{3} (n_{\Delta q\alpha} + n_{\Delta u\alpha} + n_{\Delta d\alpha}) = \frac{(2\mu_{q\alpha} + \mu_{u\alpha} + \mu_{d\alpha})}{6} T^2 \quad (A.7a)
\]

\[
n_B \equiv \sum_{\alpha=1}^3 n_{B\alpha} = \sum_{\alpha=1}^3 (2\mu_{q\alpha} + \mu_{u\alpha} + \mu_{d\alpha}) \frac{T^2}{6} \quad (A.7b)
\]

\[
n_{L\alpha} \equiv \sum_k L_{k\alpha} \Delta n_k = n_{\Delta l\alpha} + n_{\Delta e\alpha} = (2\mu_{l\alpha} + \mu_{e\alpha}) \frac{T^2}{6} \quad (A.7c)
\]

\[
n_L \equiv \sum_{\alpha=1}^3 n_{L\alpha} = \sum_{\alpha=1}^3 (2\mu_{l\alpha} + \mu_{e\alpha}) \frac{T^2}{6} \quad (A.7d)
\]

These sixteen SM chemical potentials are not totally independent; they are related through the processes that attain chemical equilibrium during the era of the asymmetry generation. At $T \ll 10^8$ these are:

- $SU(2)_L$ sphaleron transitions which induce an effective 12-fermion effective operator $O_{(B-L)_{SM}} = \sum_{i=1}^{N_f} (q_i q_i \ell_i)$, which implies

  \[
  \sum_{\alpha=1}^3 (3\mu_{q\alpha} + \mu_{\ell\alpha}) = 0 \quad (A.8)
  \]

- $SU(3)_c$ sphaleron transitions which give rise to

  \[
  \sum_{\alpha=1}^3 (2\mu_{q\alpha} - \mu_{u\alpha} - \mu_{d\alpha}) = 0 \quad (A.9)
  \]

- Yukawa interactions which imply

  \[
  \mu_{q\alpha} - \mu_H - \mu_{d\alpha} = 0, \quad (q_{\alpha} \tilde{H} \tilde{d}_{\alpha} + \text{h.c.}) \quad (A.10a)
  \]

  \[
  \mu_{q\alpha} + \mu_H - \mu_{u\alpha} = 0, \quad (q_{\alpha} H \tilde{u}_{\alpha} + \text{h.c.}) \quad (A.10b)
  \]

  \[
  \mu_{l\alpha} - \mu_H - \mu_{e\alpha} = 0, \quad (\ell_{\alpha} \tilde{H} \tilde{e}_{\alpha} + \text{h.c.}) \quad (A.10c)
  \]

Out of the eleven constraints of Eqs. (A.8) - (A.10) only ten of them are linearly independent, as the QCD sphaleron induced relation (A.9) can be obtained by adding Eq. (A.10a) and Eq. (A.10b). An additional independent constraint can be obtained from the hypercharge ($\mathcal{Y} = Q + t_3$) neutrality condition, which implies

- \[
  \mu_{q\alpha} - \mu_{H} - \mu_{d\alpha} = 0, \quad (q_{\alpha} \tilde{H} \tilde{d}_{\alpha} + \text{h.c.})
  \]

- \[
  \mu_{q\alpha} + \mu_{H} - \mu_{u\alpha} = 0, \quad (q_{\alpha} H \tilde{u}_{\alpha} + \text{h.c.})
  \]

- \[
  \mu_{l\alpha} - \mu_{H} - \mu_{e\alpha} = 0, \quad (\ell_{\alpha} \tilde{H} \tilde{e}_{\alpha} + \text{h.c.})
  \]
• Hypercharge constraint

\[ n_Y = \sum_k \Upsilon_k n_{\Delta k} = 0 \]

\[ \Rightarrow \sum_{\alpha=1}^{3} \left( \frac{1}{6} n_{\Delta q_\alpha} + \frac{2}{3} n_{\Delta u_\alpha} - \frac{1}{3} n_{\Delta d_\alpha} - \frac{1}{2} n_{\Delta \ell_\alpha} - n_{\Delta e_\alpha} \right) + \frac{1}{2} n_{\Delta H} = 0 \]

\[ \Rightarrow \sum_{\alpha=1}^{3} (\mu_{q_\alpha} + 2\mu_{u_\alpha} - \mu_{d_\alpha} - \mu_{\ell_\alpha} - \mu_{e_\alpha}) + 2\mu_H = 0 \]  \hspace{1cm} (A.11)

where \( \Upsilon_k \) is the hypercharge of species \( k \). Two more constraints are imposed by the equality of the baryon flavour asymmetries \[52\]

• Baryon flavour asymmetry equality

\[ n_{B_1} = n_{B_2} = n_{B_3} \]

\[ \Rightarrow 2\mu_{q_3} + \mu_{u_3} + \mu_{d_3} = 2\mu_{q_2} + \mu_{u_2} + \mu_{d_2} = 2\mu_{q_1} + \mu_{u_1} + \mu_{d_1} \]  \hspace{1cm} (A.12)

Hence there are in total thirteen independent constraints (Eqs. (A.8), (A.10), (A.11) and (A.12)) and therefore one can express the sixteen SM chemical potentials (A.5) in terms of three chemical potentials, that we chose to be \( \{ \mu_{\ell_\alpha} \} \). Solving the system of equations one obtains

\[ \mu_{q_\alpha} = \frac{1}{9} (\mu_{\ell_1} + \mu_{\ell_2} + \mu_{\ell_3}) \]  \hspace{1cm} (A.13a)

\[ \mu_{u_\alpha} = \frac{5}{63} (\mu_{\ell_1} + \mu_{\ell_2} + \mu_{\ell_3}) \]  \hspace{1cm} (A.13b)

\[ \mu_{d_\alpha} = -\frac{19}{63} (\mu_{\ell_1} + \mu_{\ell_2} + \mu_{\ell_3}) \]  \hspace{1cm} (A.13c)

\[ \mu_{\ell_1} = \frac{1}{21} (17\mu_{\ell_1} - 4\mu_{\ell_2} - 4\mu_{\ell_3}) \]  \hspace{1cm} (A.13d)

\[ \mu_{\ell_2} = \frac{1}{21} (-4\mu_{\ell_1} + 17\mu_{\ell_2} - 4\mu_{\ell_3}) \]  \hspace{1cm} (A.13e)

\[ \mu_{\ell_3} = \frac{1}{21} (-4\mu_{\ell_1} - 4\mu_{\ell_2} + 17\mu_{\ell_3}) \]  \hspace{1cm} (A.13f)

\[ \mu_H = \frac{4}{21} (\mu_{\ell_1} + \mu_{\ell_2} + \mu_{\ell_3}) \]  \hspace{1cm} (A.13g)

Now the baryon and lepton number density asymmetries can be written in terms of the
The flavour asymmetry combination

\[ n_{\Delta \alpha} \equiv n_{B\alpha} - n_{L\alpha} = n_B/3 - n_{L\alpha} \]
can be written as

\[ n_{\Delta \alpha} = (2\mu_{q\alpha} + \mu_{u\alpha} + \mu_{d\alpha} - 2\mu_{\ell\alpha} - \mu_{e\alpha}) \frac{T^2}{6}, \]  

\[ \Rightarrow \begin{pmatrix} n_{\Delta 1} \\ n_{\Delta 2} \\ n_{\Delta 3} \end{pmatrix} = -\frac{1}{63} \begin{pmatrix} 205 & 16 & 16 \\ 16 & 205 & 16 \\ 16 & 16 & 205 \end{pmatrix} \begin{pmatrix} \mu_{\ell 1} \\ \mu_{\ell 2} \\ \mu_{\ell 3} \end{pmatrix} \frac{T^2}{6}. \]

Inverting the expression above one finds

\[ \begin{pmatrix} \mu_{\ell 1} \\ \mu_{\ell 2} \\ \mu_{\ell 3} \end{pmatrix} = -\frac{1}{711} \begin{pmatrix} 221 & -16 & -16 \\ -16 & 221 & -16 \\ -16 & -16 & 221 \end{pmatrix} \begin{pmatrix} n_{\Delta 1} \\ n_{\Delta 2} \\ n_{\Delta 3} \end{pmatrix} \frac{6}{T^2}. \]  

Using Eq. (A.6) we can express the chemical potentials \( \mu_{\ell\alpha} \) in terms of the flavour asymmetries \( n_{\Delta \ell\alpha} \) and relate them to \( n_{\Delta \alpha} \) as

\[ \begin{pmatrix} n_{\Delta \ell 1} \\ n_{\Delta \ell 2} \\ n_{\Delta \ell 3} \end{pmatrix} = -\frac{2}{711} \begin{pmatrix} 221 & -16 & -16 \\ -16 & 221 & -16 \\ -16 & -16 & 221 \end{pmatrix} \begin{pmatrix} n_{\Delta 1} \\ n_{\Delta 2} \\ n_{\Delta 3} \end{pmatrix}. \]  

The total \( B - L \) asymmetry is

\[ n_{B-L} \equiv n_B - n_L = -\frac{79}{21} (\mu_{\ell 1} + \mu_{\ell 2} + \mu_{\ell 3}) \frac{T^2}{6}. \]  

Hence in the SM and for \( T_{sph} \ll T \ll 10^8 \) GeV the total baryon \( n_B \) and the \( n_L \) asymmetries are related to \( n_{B-L} \) by

\[ n_{B-L} \equiv n_B - n_L = -\frac{79}{21} (\mu_{\ell 1} + \mu_{\ell 2} + \mu_{\ell 3}) \frac{T^2}{6}. \]  

Note that our result agrees with reference [55], while the authors of reference [29] define \( n_{\Delta \ell\alpha} \) per gauge degree of freedom and therefore their result is smaller by a factor of two.
\[ n_B = \frac{28}{79} n_{B-L} = \frac{28}{79} \sum_{\alpha=1}^{3} n_{\Delta \alpha} \tag{A.18} \]

\[ n_L = -\frac{51}{79} n_{B-L} = -\frac{51}{79} \sum_{\alpha=1}^{3} n_{\Delta \alpha} \tag{A.19} \]

**A.0.2 FIBG Model**

We extend the SM to include also two heavy vector-like leptons \( F_1 \) and \( F_2 \) with chemical potentials \( \mu_{F_1} \) and \( \mu_{F_2} \) respectively and asymmetry

\[ n_{\Delta F_i} = \mu_{F_i} \frac{T^2}{6} \tag{A.20} \]

Since the BSM fermions are gauged under \( U(1)_{\Upsilon} \), with \( \Upsilon_{F_1} = \Upsilon_e = -1 \), the hypercharge constraint (A.11) is modified to

\[
\begin{align*}
    n_{\Upsilon} &\equiv \sum_{k} \Upsilon_k n_{\Delta k} = 0 \\
    \Rightarrow &\sum_{\alpha=1}^{3} \left( \frac{1}{6} n_{\Delta q_\alpha} + \frac{2}{3} n_{\Delta u_\alpha} - \frac{1}{3} n_{\Delta d_\alpha} - \frac{1}{2} n_{\Delta \ell_\alpha} - n_{\Delta e_\alpha} \right) + \frac{1}{2} n_{\Delta H} - \sum_{i=1}^{2} n_{\Delta F_i} = 0 \\
    \Rightarrow &\sum_{\alpha=1}^{3} \left( \mu_{q_\alpha} + 2\mu_{u_\alpha} - \mu_{d_\alpha} - \mu_{\ell_\alpha} - \mu_{e_\alpha} \right) + 2\mu_H - (\mu_{F_1} + \mu_{F_2}) = 0 \tag{A.21} \end{align*}
\]
The theory conserves the total $n_{B-L} \equiv n_B - n_L = n_B - n_{LSM} - n_{LF}$ asymmetry, where

$$n_{B\alpha} \equiv \sum_k B_{k\alpha} n_{\Delta k\alpha} = \frac{1}{3} (n_{\Delta q\alpha} + n_{\Delta u\alpha} + n_{\Delta d\alpha}) = (2\mu_{q\alpha} + \mu_{u\alpha} + \mu_{d\alpha}) \frac{T^2}{6}$$  \hspace{1cm} (A.22a)

$$n_B \equiv \sum_{\alpha=1}^{3} n_{B\alpha} = \sum_{\alpha=1}^{3} (2\mu_{q\alpha} + \mu_{u\alpha} + \mu_{d\alpha}) \frac{T^2}{6} = 3 \left( 2\mu_{q\alpha} + \mu_{u\alpha} + \mu_{d\alpha} \right) \frac{T^2}{6}$$  \hspace{1cm} (A.22b)

$$n_{L\alpha} \equiv \sum_{k:SM} L_{k\alpha} n_{\Delta k\alpha} = n_{\Delta q\alpha} + n_{\Delta u\alpha} = (2\mu_{q\alpha} + \mu_{e\alpha}) \frac{T^2}{6}$$  \hspace{1cm} (A.22c)

$$n_{LSM} \equiv \sum_{\alpha=1}^{3} n_{L\alpha} = \sum_{\alpha=1}^{3} (2\mu_{q\alpha} + \mu_{e\alpha}) \frac{T^2}{6}$$  \hspace{1cm} (A.22d)

$$n_{LF} \equiv \sum_{i=1}^{2} n_{\Delta F_i} = \sum_{i=1}^{2} \mu_{F_i} \frac{T^2}{6}$$  \hspace{1cm} (A.22e)

$$n_L \equiv n_{LSM} + n_{LF} = \left( \sum_{\alpha=1}^{3} (2\mu_{q\alpha} + \mu_{e\alpha}) + \sum_{i=1}^{2} \mu_{F_i} \right) \frac{T^2}{6}$$  \hspace{1cm} (A.22f)

If we also assume that the early Universe is totally symmetric, $n_{B-L|0} = 0$ and $n_F|0 = 0$, then we obtain an additional constraint to $\mu_{F_1} + \mu_{F_2}$, that is

$$n_B - n_{LSM} - n_{LF} = \sum_{\alpha=1}^{3} n_{\Delta q\alpha} - \sum_{i=1}^{2} n_{\Delta F_i} = 0$$

$$\Rightarrow \sum_{\alpha=1}^{3} (2\mu_{q\alpha} + \mu_{u\alpha} + \mu_{d\alpha} - 2\mu_{\ell\alpha} - \mu_{e\alpha}) - (\mu_{F_1} + \mu_{F_2}) = 0$$  \hspace{1cm} (A.23)

Replacing this into Eq. (A.21) we obtain the modified hypercharge constraint expressed in terms of the SM chemical potentials.

- Modified hypercharge constraint

$$\sum_{\alpha=1}^{3} (-\mu_{q\alpha} + \mu_{u\alpha} - 2\mu_{d\alpha} + \mu_{\ell\alpha}) + 2\mu_H = 0$$  \hspace{1cm} (A.24)

Now the sixteen SM chemical potentials are constrained under the $SU(2)$ sphaleron (A.8) and Yukawa (A.10) processes (ten constraints), as well as under the baryon flavour asymmetry equality (A.12) and the modified hypercharge (A.24) conditions (three constraints). Solving the system of equations we may express all chemical potentials in terms of $\mu_{e\alpha}$.
\[
\begin{align*}
\mu_{q_\alpha} &= -\frac{11}{144}(\mu_{e_1} + \mu_{e_2} + \mu_{e_3}) \\
\mu_{u_\alpha} &= -\frac{13}{72}(\mu_{e_1} + \mu_{e_2} + \mu_{e_3}) \\
\mu_{d_\alpha} &= \frac{1}{36}(\mu_{e_1} + \mu_{e_2} + \mu_{e_3}) \\
\mu_{\ell_1} &= \frac{1}{48}(43\mu_{e_1} - 5\mu_{e_2} - 5\mu_{e_3}) \\
\mu_{\ell_2} &= \frac{1}{48}(-5\mu_{e_1} + 43\mu_{e_2} - 5\mu_{e_3}) \\
\mu_{\ell_3} &= \frac{1}{48}(-5\mu_{e_1} - 5\mu_{e_2} + 43\mu_{e_3}) \\
\mu_H &= -\frac{5}{48}(\mu_{e_1} + \mu_{e_2} + \mu_{e_3})
\end{align*}
\]

Now the baryon and lepton number density asymmetries can be written in terms of the chemical potentials \(\mu_{e_\alpha}\) as

\[
\begin{align*}
n_{B_\alpha} &= \left(2\mu_{q_\alpha} + \mu_{u_\alpha} + \mu_{d_\alpha}\right) \frac{T^2}{6} = -\frac{11}{36}(\mu_{e_1} + \mu_{e_2} + \mu_{e_3}) \frac{T^2}{6} \\
n_B &= 3n_{B_\alpha} = -\frac{11}{12}(\mu_{e_1} + \mu_{e_2} + \mu_{e_3}) \frac{T^2}{6} \\
n_{L_\alpha} &= (2\mu_{\ell_\alpha} + \mu_{e_\alpha}) \frac{T^2}{6} \\
n_{LSM} &= \sum_{\alpha=1}^{3} \left(2\mu_{\ell_\alpha} + \mu_{e_\alpha}\right) \frac{T^2}{6} = \frac{19}{8}(\mu_{e_1} + \mu_{e_2} + \mu_{e_3}) \frac{T^2}{6} \\
n_{LF} &= (\mu_{F_1} + \mu_{F_2}) \frac{T^2}{6} = \sum_{\alpha=1}^{3} \left(2\mu_{q_\alpha} + \mu_{u_\alpha} + \mu_{d_\alpha} - 2\mu_{e_\alpha} - \mu_{e_\alpha}\right) \frac{T^2}{6} \\
&= -\frac{79}{24}(\mu_{e_1} + \mu_{e_2} + \mu_{e_3}) \frac{T^2}{6} = n_B - n_{LSM} \\
n_L &= n_{LSM} + n_{LF} = -\frac{11}{12}(\mu_{e_1} + \mu_{e_2} + \mu_{e_3}) \frac{T^2}{6} = n_B
\end{align*}
\]

The flavour asymmetry combination \(n_{\Delta_\alpha} \equiv n_{B_\alpha} - n_{L_\alpha} = n_B/3 - n_{L_\alpha}\) can be written as

\[
n_{\Delta_\alpha} = (2\mu_{q_\alpha} + \mu_{u_\alpha} + \mu_{d_\alpha} - 2\mu_{e_\alpha} - \mu_{e_\alpha}) \frac{T^2}{6}
\]

\[
\Rightarrow \begin{pmatrix} n_{\Delta_1} \\ n_{\Delta_2} \\ n_{\Delta_3} \end{pmatrix} = -\frac{1}{72} \begin{pmatrix} 223 & 7 & 7 \\ 7 & 223 & 7 \\ 7 & 7 & 223 \end{pmatrix} \begin{pmatrix} \mu_{e_1} \\ \mu_{e_2} \\ \mu_{e_3} \end{pmatrix} \frac{T^2}{6}
\]
Inverting the expression above one finds

\[
\begin{pmatrix}
\mu_{e_1} \\
\mu_{e_2} \\
\mu_{e_3}
\end{pmatrix} = -\frac{1}{711} \begin{bmatrix}
230 & -7 & -7 \\
-7 & 230 & -7 \\
-7 & -7 & 230
\end{bmatrix} \begin{pmatrix}
n_{\Delta 1} \\
n_{\Delta 2} \\
n_{\Delta 3}
\end{pmatrix} \frac{6}{T^2}
\]  

(A.27)

Using Eq. (A.6) we can express the chemical potentials \(\mu_{e_\alpha}\) in terms of the flavour asymmetries \(n_{\Delta e_\alpha}\) and relate them to \(n_{\Delta \alpha}\) as

\[
\begin{pmatrix}
n_{\Delta e_1} \\
n_{\Delta e_2} \\
n_{\Delta e_3}
\end{pmatrix} = -\frac{1}{711} \begin{bmatrix}
230 & -7 & -7 \\
-7 & 230 & -7 \\
-7 & -7 & 230
\end{bmatrix} \begin{pmatrix}
n_{\Delta 1} \\
n_{\Delta 2} \\
n_{\Delta 3}
\end{pmatrix}
\]  

(A.28)

Hence in this model and for \(T_{sph} \ll T \ll 10^8 \) GeV the total baryon \(n_B\) and the SM lepton \(n_{L_{SM}}\) asymmetries are related to \(n_{B-L_{SM}}\) by

\[
n_B = \frac{22}{79} n_{B-L_{SM}} = \frac{22}{79} \sum_{\alpha=1}^{3} n_{\Delta \alpha}
\]  

(A.29)

\[
n_{L_{SM}} = -\frac{57}{79} n_{B-L_{SM}} = -\frac{57}{79} \sum_{\alpha=1}^{3} n_{\Delta \alpha}
\]  

(A.30)

This result agrees with that obtained in reference [23].

References

[1] J. Preskill, M. B. Wise, and F. Wilczek, *Cosmology of the Invisible Axion*, Phys. Lett. B 120 (1983) 127–132.

[2] L. F. Abbott and P. Sikivie, *A Cosmological Bound on the Invisible Axion*, Phys. Lett. B 120 (1983) 133–136.

[3] M. Dine and W. Fischler, *The Not So Harmless Axion*, Phys. Lett. B 120 (1983) 137–141.

[4] S. Hawking, *Gravitationally collapsed objects of very low mass*, Mon. Not. Roy. Astron. Soc. 152 (1971) 75.

[5] G. F. Chapline, *Cosmological effects of primordial black holes*, Nature 253 (1975), no. 5489 251–252.

[6] A. M. Green and B. J. Kavanagh, *Primordial Black Holes as a dark matter candidate*, J. Phys. G 48 (2021), no. 4 043001, [arXiv:2007.10722].

[7] G. Arcadi, M. Dutra, P. Ghosh, M. Lindner, Y. Mambrini, M. Pierre, S. Profumo, and F. S. Queiroz, *The Waning of the WIMP? A Review of Models, Searches, and Constraints*, Eur. Phys. J. C 78 (2018), no. 3 203, [arXiv:1703.07364].

– 28 –
N. Bernal, M. Heikinheimo, T. Tenkanen, K. Tuominen, and V. Vaskonen, *The Dawn of FIMP Dark Matter: A Review of Models and Constraints*, Int. J. Mod. Phys. A 32 (2017), no. 27 1730023, [arXiv:1706.07442].

E. W. Kolb, D. J. H. Chung, and A. Riotto, *WIMPzillas*, AIP Conf. Proc. 484 (1999), no. 1 91–105, [hep-ph/9810361].

H. Davoudiasl and R. N. Mohapatra, *On Relating the Genesis of Cosmic Baryons and Dark Matter*, New J. Phys. 14 (2012) 095011, [arXiv:1203.1247].

K. Petraki and R. R. Volkas, *Review of asymmetric dark matter*, Int. J. Mod. Phys. A 28 (2013) 1330028, [arXiv:1305.4939].

M. Yoshimura, *Unified Gauge Theories and the Baryon Number of the Universe*, Phys. Rev. Lett. 41 (1978) 281–284. [Erratum: Phys.Rev.Lett. 42, 746 (1979)].

V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, *On the Anomalous Electroweak Baryon Number Nonconservation in the Early Universe*, Phys. Lett. B 155 (1985) 36.

A. G. Cohen, D. B. Kaplan, and A. E. Nelson, *Progress in electroweak baryogenesis*, Ann. Rev. Nucl. Part. Sci. 43 (1993) 27–70, [hep-ph/9302210].

I. Affleck and M. Dine, *A New Mechanism for Baryogenesis*, Nucl. Phys. B 249 (1985) 361–380.

M. Fukugita and T. Yanagida, *Baryogenesis Without Grand Unification*, Phys. Lett. B 174 (1986) 45–47.

J. McDonald, *Simultaneous Generation of WIMP Miracle-like Densities of Baryons and Dark Matter*, Phys. Rev. D 84 (2011) 103514, [arXiv:1108.4653].

Y. Cui, L. Randall, and B. Shuve, *A WIMPy Baryogenesis Miracle*, JHEP 04 (2012) 075, [arXiv:1112.2704].

Y. Cui and R. Sundrum, *Baryogenesis for weakly interacting massive particles*, Phys. Rev. D 87 (2013), no. 11 116013, [arXiv:1212.2973].

L. J. Hall, J. March-Russell, and S. M. West, *A Unified Theory of Matter Genesis: Asymmetric Freeze-In*, [arXiv:1010.0245].

J. Unwin, *Towards Cogenesis via Asymmetric Freeze-In: The \( \chi \) Who Came-in from the Cold*, JHEP 10 (2014) 190, [arXiv:1406.3027].

E. K. Akhmedov, V. A. Rubakov, and A. Y. Smirnov, *Baryogenesis via neutrino oscillations*, Phys. Rev. Lett. 81 (1998) 1359–1362, [hep-ph/9803255].

B. Shuve and D. Tucker-Smith, *Baryogenesis and Dark Matter from Freeze-In*, Phys. Rev. D 101 (2020), no. 11 115023, [arXiv:2004.00636].

J. McDonald, *Thermally Generated Gauge Singlet Scalars as Self-Interacting Dark Matter*, Phys. Rev. Lett. 88 (2002) 091304, [hep-ph/0106249].

L. J. Hall, K. Jedamzik, J. March-Russell, and S. M. West, *Freeze-In Production of FIMP Dark Matter*, JHEP 03 (2010) 080, [arXiv:0911.1120].

K. Dick, M. Lindner, M. Ratz, and D. Wright, *Leptogenesis with Dirac Neutrinos*, Phys. Rev. Lett. 84 (2000) 4039–4042, [hep-ph/9907562].

H. Murayama and A. Pierce, *Realistic Dirac leptogenesis*, Phys. Rev. Lett. 89 (2002) 271601, [hep-ph/0206177].
D. G. Cerdeno, A. Dedes, and T. E. J. Underwood, *The Minimal Phantom Sector of the Standard Model: Higgs Phenomenology and Dirac Leptogenesis*, JHEP 09 (2006) 067, [hep-ph/0607157].

M. C. Gonzalez-Garcia, J. Racker, and N. Rius, *Leptogenesis without violation of B-L*, JHEP 11 (2009) 079, [arXiv:0909.3518].

A. D. Sakharov, *Violation of CP Invariance, C asymmetry, and baryon asymmetry of the universe*, Sov. Phys. Usp. 34 (1991), no. 5 392–393.

A. Pilaftsis and T. E. J. Underwood, *Resonant leptogenesis*, Nucl. Phys. B 692 (2004) 303–345, [hep-ph/0309342].

A. Pilaftsis and T. E. J. Underwood, *Electroweak-Scale Resonant Leptogenesis*, Phys. Rev. D 72 (2005) 113001, [hep-ph/0506107].

A. Pilaftsis, *Heavy Majorana neutrinos and baryogenesis*, Int. J. Mod. Phys. A 14 (1999) 1811–1858, [hep-ph/9812256].

A. Anisimov, A. Broncano, and M. Plumacher, *The CP-asymmetry in resonant leptogenesis*, Nucl. Phys. B 737 (2006) 176–189, [hep-ph/0511248].

P. Gondolo and G. Gelmini, *Cosmic abundances of stable particles: Improved analysis*, Nucl. Phys. B 360 (1991) 145–179.

E. W. Kolb and M. S. Turner, *The Early Universe*, Nature 294 (1981) 521.

T. Hahn, *CUBA: A Library for multidimensional numerical integration*, Comput. Phys. Commun. 168 (2005) 78–95, [hep-ph/0404043].

J. M. Cline, K. Kainulainen, and K. A. Olive, *Protecting the primordial baryon asymmetry from erasure by sphalerons*, Phys. Rev. D 49 (1994) 6394–6409, [hep-ph/9401208].

M. J. Baker, M. Breitbach, J. Kopp, and L. Mittnacht, *Dynamic Freeze-In: Impact of Thermal Masses and Cosmological Phase Transitions on Dark Matter Production*, JHEP 03 (2018) 114, [arXiv:1712.03962].

C. Dvorkin, T. Lin, and K. Schutz, *Making dark matter out of light: freeze-in from plasma effects*, Phys. Rev. D 99 (2019), no. 11 115009, [arXiv:1902.08623].

L. Darmé, A. Hryczuk, D. Karamitros, and L. Roszkowski, *Forbidden frozen-in dark matter*, JHEP 11 (2019) 159, [arXiv:1908.05685].

**Planck** Collaboration, N. Aghanim et al., *Planck 2018 results. VI. Cosmological parameters*, Astron. Astrophys. 641 (2020) A6, [arXiv:1807.06209].

**Planck** Collaboration, N. Aghanim et al., *Planck 2018 results. VI. Cosmological parameters*, Astron. Astrophys. 641 (2020) A6, [arXiv:1807.06209]. [Erratum: Astron.Astrophys. 652, C4 (2021)].

A. Garzilli, O. Ruchayskiy, A. Magalich, and A. Boyarsky, *How warm is too warm? Towards robust Lyman-α forest bounds on warm dark matter*, arXiv:1912.09397.

N. Palanque-Delabrouille, C. Yèche, N. Schöneberg, J. Lesgourgues, M. Walther, S. Chabanier, and E. Armengaud, *Hints, neutrino bounds and WDM constraints from SDSS DR14 Lyman-α and Planck full-survey data*, JCAP 04 (2020) 038, [arXiv:1911.09073].

V. Iršič et al., *New Constraints on the free-streaming of warm dark matter from intermediate and small scale Lyman-α forest data*, Phys. Rev. D 96 (2017), no. 2 023522, [arXiv:1702.01764].
[66] W. Buchmuller, R. Peccei, and T. Yanagida, Leptogenesis as the origin of matter, Ann. Rev. Nucl. Part. Sci. 55 (2005) 311–355, [hep-ph/0502169].