Tests of Predictions of the Algebraic Cluster Model:
The Triangular $D_{3h}$ Symmetry of $^{12}C$

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Abstract. A new theoretical approach to clustering in the frame of the Algebraic Cluster Model (ACM) has been developed. It predicts rotation-vibration structure with rotational band of an oblate equilateral triangular symmetric spinning top with a $D_{3h}$ symmetry characterized by the sequence of states: $0^+, 2^+, 3^-, 4^\pm, 5^-$ with a degenerate $4^+$ and $4^-$ (parity doublet) states. Our measured new $2^+$ member in $^{12}C$ allows the first study of rotation-vibration structure in $^{12}C$. The newly measured $5^-$ state and $4^-$ states fit very well the predicted ground state rotational band structure with the predicted sequence of states: $0^+, 2^+, 3^-, 4^\pm, 5^-$ with almost degenerate $4^+$ and $4^-$ (parity doublet) states. Such a $D_{3h}$ symmetry is characteristic of triatomic molecules, but it is observed in the ground state rotational band of $^{12}C$ for the first time in a nucleus. We discuss predictions of the ACM of other rotation-vibration bands in $^{12}C$ such as the $(0^+)$ Hoyle band and the $(1^-)$ bending mode with prediction of (“missing 3$^-$ and 4$^-$”) states that may shed new light on clustering in $^{12}C$ and light nuclei. In particular, the observation (or non observation) of the predicted (“missing”) states in the Hoyle band will allow us to conclude the geometrical arrangement of the three alpha particles composing the Hoyle state at 7.6542 MeV in $^{12}C$. We discuss proposed research programs at the Darmstadt S-DALINAC and at the newly constructed ELI-NP facility near Bucharest to test the predictions of the ACM in isotopes of carbon.

1. Introduction
A recent experiment performed at the HI\gamma S gamma-ray facility [1] using the optical readout TPC (O-TPC) [2] shown in Fig. 1, provided unambiguous identification of the $2^+$ member of the Hoyle rotational band [3]. This observation allows for the first time the study of the predicted rotation-vibration structure of $^{12}C$.

The structure of light nuclei, and specifically of clustering received new interest with major developments of ab-initio calculations of light nuclei and in particular of $^{12}C$. Theoretical ab-initio shell model [4] and symmetry inspired shell model [5] calculations as well as ab-initio Effective Field Theory (EFT) calculations on the lattice [6, 7], Fermionic Molecular Dynamics (FMD) model [8] and Anti-symmetrized Molecular Dynamics (AMD) model [9], are employed to yield a microscopic foundation of the clustering phenomena that naturally occurs in cluster models [10, 11]. For example, one issue of current concern [12] is the geometrical structure of the three alpha-particles in the Hoyle state at 7.6542 MeV in $^{12}C$ (linear chain, obtuse triangle or equilateral triangle) and the Hoyle rotational band built on top of the Hoyle state [13, 14, 15, 16, 17].

Cluster states are best described as molecular states which are characterized by the separation (Jacobi) vector(s) connecting the constituent objects. For diatomic like object one single
Figure 1. A schematic diagram of the Optical Readout Time Projection Chamber (O-TPC) [2]. The gas mixture and pressure as well as the operating voltages are indicated. The drift electron(s) that provide the third (time projection) upward dimension is shown schematically.

(separation) vector is required leading to the predicted U(4) symmetry [18] that was observed in \(^{18}\text{O}\) via the characteristic enhanced E1 decays [19]. Triatomic symmetric spinning tops are characterized by the two perpendicular Jacobi vectors (\(\lambda\) and \(\rho\)) leading to the predicted U(7) symmetry with the geometrical \(D_{3h}\) symmetry [20, 21].

The observation of the \(2^+\) Hoyle rotational excitation in \(^{12}\text{C}\) [3, 14] together with the recently discovered \(4^-\) [22, 23] and \(4^+\) [24] states in \(^{12}\text{C}\) are in agreement with the predicted spectrum of an oblate spinning top with a \(D_{3h}\) symmetry [20, 21]. It was predicted [20, 21] that the three alpha-particle system of \(^{12}\text{C}\) leads to the ground state rotational band including the most unusual sequence of states: \(0^+, 2^+, 3^-, 4^\pm, 5^-\). The new high spin \(5^-\) state [25] as well the previously published \(4^-\) state [22, 23] lead to a \(J(J+1)\) trajectory as predicted by this U(7) model [25] including the nearly degenerate \(4^-\) and \(4^+\) states as shown in Fig. 2.

2. The Algebraic Cluster Model

The spectrum of \(^{12}\text{C}\) predicted by the Algebraic Cluster Model (ACM) [20, 21] is shown in Fig. 3 where it is also compared to the measured spectrum of \(^{12}\text{C}\) [25]. In addition to the ground state rotational band this U(7) model [20, 21] predicts the Hoyle state at 7.6542 MeV in \(^{12}\text{C}\) to be the first vibrational breathing mode of the three alpha-particle equilateral configuration leading to the same rotational structure albeit with a larger moment of inertia (by a factor of 2). Recent measurements revealed the \(2^+\) [14] and \(4^+\) [24] members of the predicted Hoyle rotational band and we are currently searching [26, 27] for the \(4^-\) predicted by the ACM to be nearly degenerated with the \(4^+\) state and the \(3^-\) (broad) state that was suggested to lie between 11 and 14 MeV [22, 28]. The observation (or lack there) of these “missing” states will allow us to determine whether the Hoyle state is composed of three alpha-particles in an equilateral triangle arrangement [25, 8, 9] or an obtuse triangle [7] or whether it is better described as a
vibrational excitation of a “diffuse gas” of three alpha-particles [11].

The U(7) model also predicts the $1^-$ state at 10.84 MeV to be the vibrational bending mode with a rotational band including the $1^-$ and a degenerate $2^\pm$ states. We are searching [26, 27] for the third $2^+$ of $^{12}$C that is predicted by the U(7) model to lie near the observed $2^-$ state at 11.8 MeV.

3. Future Tests of the ACM

We propose a new research program in Nuclear Structure studies to be performed at the ELI-NP facility near Bucharest, Romania, with a newly proposed electronic readout TPC (eTPC) and a Silicon Strip Detector (SSD) [27]. We propose to measure of the multi-alpha decay of $^{12}$C and $^{16}$O with a TPC detector. Such measurements of multi-alpha decay of $^{12}$C were also performed in the past with Silicon Strip Detectors (SSD) [22, 23, 24] hence similar measurements with SSD are also considered for the ELI-NP. We emphasize that the measurements we discuss in $^{12}$C and $^{16}$O are considered as typical examples of studies in nuclear structure that can be performed with our proposed detectors at the ELI-NP and such measurements in other light nuclei will lead to new perspective on the clustering phenomena in light nuclei.

4. Conclusions

In conclusion the ACM appears to open a new chapter in cluster physics of light nuclei and it presents an opportunity for further experimental investigation of light nuclei. We refer the
Figure 3. Comparison between the low-lying experimental spectrum of $^{12}\text{C}$ [25] and the energies of the oblate symmetric top. The levels are organized in columns corresponding to the ground state band and the vibrational bands with $A$ and $E$ symmetry [20, 21] of an oblate top with triangular symmetry. The last column on the left-hand side, shows the lowest observed non-cluster ($1^+$) levels.

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