Tuning the magic angle of twisted bilayer graphene at the exit of a waveguide

Michael Vogl,1 Martin Rodriguez-Vega,1,2 and Gregory A. Fiete2,3

1Department of Physics, The University of Texas at Austin, Austin, TX 78712, USA
2Department of Physics, Northeastern University, Boston, MA 02115, USA
3Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

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We introduce a new method that allows reducing the strength of the interlayer couplings in few-layer van der Waals heterostructures by irradiating them with longitudinal waves of light. As a specific application, we consider twisted bilayer graphene and show that we can tune the magic angles in a controlled manner. According to Maxwell’s equations, the electromagnetic fields associated with light in free space do not possess longitudinal components. However, this can be circumvented imposing suitable boundary conditions in space as in metallic waveguides, which can generate electric fields with longitudinal components (transverse magnetic modes (TM)). We propose to place twisted bilayer graphene at a specific location at the exit of a waveguide, such that it is subjected to purely longitudinal components of a TM wave. We derive an analytic model to predict the magic angle as a function of the drive and waveguide parameters. This approach could be employed in experiments to tune the Fermi velocity in experiments and explore the strongly correlated physics in-situ.

Introduction. Van der Waals heterostructures are flexible platforms to engineer systems with designer properties because they are not constrained by the chemistry of the two-dimensional materials composing the system[11]. The properties of such heterostructures can be tuned with the knobs provided by the layer material choice, temperature, pressure, applied fields, time-dependent drives, etc. Recent experiential advances made possible the introduction of a new knob: a relative twist angle between the layers. In particular, twisted bilayer graphene (TBG) has emerged as a controllable platforms for the study of strongly correlated physics[24,25]. The interplay of the interlayer tunneling amplitude, and the induced moire pattern lead to the formation of isolated flat bands at specific magic angles, where interaction effects can dominate[24,25]. Specifically, TBG exhibits an insulating state at half-filling driven by electron-electron interaction[24,25], which can transition into superconducting[1] and ferromagnetic[25] states at specific filling factors.

One of the main parameters that controls the location of the magic angle is the interlayer tunneling amplitude. Fine tuning of the rotation angle represents one of the main challenges for the experimental realization of strongly correlated states, since the band structure is very sensitive to small changes in this angle. Recent theoretical studies suggested the possibility to overcome small misalignments away from the magic angle by applying uniaxial pressure[28,29]. Furthermore, experiments have already shown that hydrostatic pressure can increase the tunneling amplitude between the layers and lead to strongly correlated states in samples otherwise dominated by the kinetic energy[30,31]. This procedure addresses the layer misalignment issue for samples with angles larger than the magic angle. However, no controlled scheme has been proposed to date to reduce the tunneling amplitude between the layers and decrease the magic angle.

Traditional Floquet engineering in free space using circularly-polarized light allows to break time reversal symmetry, induce non-trivial gaps in the quasienergy spectrum and leads to rich topological phase diagrams by modifying the intralayer hopping amplitude[24,25]. Recently, this traditional Floquet protocol was applied to twisted bilayer graphene. It allows to induce topological transitions at large twist angles using high-frequency drives[32,33]. Thereby, the band structure near charge neutrality and induced induce a transition to a Chern insulating phase with Chern number $C = 1$ in the high-frequency regime, and induce flat bands using near-infrared light in a wide range of twist angles[33]. Despite the high degree of tunability induced by the Floquet protocols, the interlayer tunneling amplitude is not easily affected.

In this work, we demonstrate that in a waveguide a transverse magnetic (TM) mode can directly couple to the inter-layer tunneling in few-layer van der Waals systems. The intensity and frequency of the drive can then be used to modify the tunneling strength without additional not desired effects. Therefore, in the context of twisted bilayer graphene, Floquet drives using light con-
defined inside a waveguide allows to decrease the magic angle, providing the knob in this missing direction. Although this work focuses on twisted bilayer graphene, the drive protocol we propose is general and can be applied to other low-dimensional systems to tune the strength of interlayer couplings. This is useful because a change in strength of interlayer couplings can have many interesting consequences. For example, a theoretical work suggests that MoS$_2$ bilayers can host Moire quantum wells with an energy level structure sensitive to interlayer coupling strengths\textsuperscript{[47]}. In general, our approach increases the degree in which the parameters of low-dimensional systems can be modified in out-of-equilibrium settings. It allows to directly tune the Fermi velocity of the quasienergy spectrum and it increases the degree in which the parameters of low-dimensional systems can be modified in a controlled way. In the following, we discuss the consequences of Floquet drives in waveguides on twisted bilayer graphene. However, we stress that this protocol can be implemented in other low-dimensional van der Waals heterostructures.

Static model. When two graphene layers are misaligned with respect to each other by a rotation, a moire pattern emerges with a tunneling amplitude between the layers that is dominated by processes at lattice sites located directly on top of each other. The low-energy effective Hamiltonian for static twisted bilayer graphene (TBG)\textsuperscript{[34,37–40]}

\[
H = \begin{pmatrix}
  h(-\theta/2, k - \kappa_-) & T(x) \\
  T^\dagger(x) & h(\theta/2, k - \kappa_+)
\end{pmatrix},
\]

which describes two stacked graphene layers that are twisted with respect to each other at an angle $\theta$. Each graphene layer is described by the Hamiltonian

\[
h(\theta, k) = \gamma \begin{pmatrix}
  0 & f(R(\theta)|k|) \\
  f^*(R(\theta)|k|) & 0
\end{pmatrix},
\]

where $k$ is the in-plane momentum. $R(\theta)$ corresponds to a rotation around the axis perpendicular to the graphene sheets by an angle $\theta$, and $f(k) = e^{-2iak_{\perp}^3/3} + 2e^{iak_{\perp}^3/3} \sin(a_{0}k_{\perp}/\sqrt{3} - \pi/6)$ is the intra-layer hopping matrix expressed in momentum space. In each layer, we shift the momenta by the $K$-points $\kappa_{\pm} = k_0(-\sqrt{3}/2, \pm 1/2)$. Here $k_0 = k_D \sin(\theta/2)$ and $k_D = 3\pi/3a_0$ defines the angle dependent momentum scale of the bilayer system. The tunneling processes between the layers are approximated by $T(x) = \sum_{i=-1}^{1} e^{-i\mathbf{b}_i \cdot \mathbf{x}} T_i$, where the matrix structure is given by

\[
T_n = w_0 \mathbf{1}_2 + w_1 \left( \cos\left(\frac{2\pi n}{3}\right) \sigma_1 + \sin\left(\frac{2\pi n}{3}\right) \sigma_2 \right),
\]

where $\mathbf{b}_{\pm} = k_D(\pm \sqrt{3}, 3)/2$ are the reciprocal moire lattice vectors and $\mathbf{b}_0 = (0, 0)$ is introduced to write $T(x)$ compactly.

In bilayer graphene, some stacking configurations are energetically more favorable over others. Particularly, AB/BA stacking, where half the atoms of the upper graphene layer lie on top of the empty hexagon centers of the lower layer, is favored over AA stacked regions where the upper layer is directly on top the lower layer. This energy-dependent stacking configuration leads to relaxation effects in twisted bilayer graphene, where AB/BA regions are favored over AA regions\textsuperscript{[37,38]}. We take this effect and corrugation into account through the additional parameter $w_1$ in the tunneling amplitude\textsuperscript{[34,40]}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.pdf}
\caption{(a) Twisted bilayer graphene Moire Brillouin zone (mBZ). Large orange and black hexagons represent the graphene layer BZs. (b) TBG quasienergy spectrum in a waveguide along a high-symmetry path in the mBZ (orange). The gray curve corresponds to the static case. The specific parameters are $w_0 = 0$, and $\theta = 1.3^\circ$, $\Omega/W = 1.3$, and $\sigma_{AB}A = 0.4$.}
\end{figure}

This can be understood recognizing that the parameter $w_0$ enters the Hamiltonian in the form of a AA stacking hopping. On the other hand, $w_1$ enters as hopping in bilayer graphene AB/BA configuration. Therefore, $w_1/w_0 > 1$ corresponds to systems where AB-stacked regions are favored. Then, the ratio $w_1/w_0$ can be used to model systems with larger or smaller AB/BA regions\textsuperscript{[34,40]}. In appendix B, we describe the numerical band structure implementation. Throughout this work, we use the parameters $\gamma = h v_F/a_0 = 2.36$ eV, $a_0 = 2.46$ Å and $w_1 = 110$ meV.

Driving protocol. We now place the twisted bilayer graphene sample at the exit of a metallic waveguide.
uide, as sketched in Fig. 1. The transverse magnetic modes (TM) are derived from the vector potential \( \mathbf{A} = \hat{z}A \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \Re(e^{-ik_zz-i\Omega t}) \), where \( k_z = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \) is the wavenumber in the \( z \)-direction, \( k^2 = \Omega^2 \mu \varepsilon \), \( \mu \) is the permeability constant of the insulator inside the waveguide, and \( \varepsilon \) is the dielectric constant. The waveguide cut-off frequency is given by \( \Omega_c = \frac{1}{\sqrt{\mu \varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \). A full derivation and the full electric and magnetic fields inside the waveguide are listed in the appendix A.

The maxima of the sine functions are ideal locations to place the TBG sheet because the field has a maximum amplitude \( A \) at these locations and because the vector potential is constant to second order in the \( x, y \) direction. Taking \( \mathbf{A} = A \Re(e^{-ik_zz-i\Omega t})\hat{z} \) therefore is a good approximation if the sample is small compared to the waveguide dimensions \( a, b \). At the tight binding level, \( \mathbf{A} \) enters the Hamiltonian through the hopping elements via the Peierls substitution \( t_{ij} \rightarrow t_{ij} \exp\left( -i \int_{r_i}^{r_j} \mathbf{A} \cdot d\mathbf{l} \right) \), where the line integral is associated with the interlayer hopping direction in real space and \( r_i \) labels site \( i \). The vector potential we consider only has a component in \( z \)-direction and therefore only influences interlayer hopping amplitudes \( w_i \). For computational simplicity, we will assume that the exit of the cavity is at \( z = 0 \) and that the twisted bilayer graphene sheet is placed at such a location. We note that the coupling of the longitudinal vector potential is not particular of twisted bilayer graphene and applies to other van der Waals heterostructures.

In an idealized scenario, the additional phase factor in the tunneling amplitude is uniform across the sample. However, lattice relaxation effects and surface roughness introduce a position dependence to the phase due to different layer separation. An exact treatment of such a complicated scenario is not the purpose of this work. In order to simplify the picture and make analytic progress, let us recall that relaxation effects lead to two different hopping amplitudes: \( w_1 \), the hopping on \( AB \) stacking and \( w_0 \), the hopping of \( AA \) stacking. Therefore, to leading order, the two hopping amplitudes acquire the phases

\[
\begin{align*}
  w_1 &\rightarrow w_1 e^{-i\alpha AA A \cos(\Omega t)}, \\
  w_0 &\rightarrow w_0 e^{-i\alpha AA A \cos(\Omega t)},
\end{align*}
\]

where \( \alpha_{AA} = 3.6 \) Å is the distance of the graphene layers in \( AA \)-stacked regions and \( \alpha_{AB} = 3.4 \) Å the corresponding quantity for \( AB \) stacking. For other types of low-dimensional materials, an analogous replacements is valid for their interlayer coupling elements.

To lowest order, the high frequency approximation of a Floquet Hamiltonian (in \( 1/\Omega \)) is the time average over one period \( 2\pi/\Omega \), \( \tilde{H}_F = \frac{1}{\Omega} \int_0^{2\pi/\Omega} \Omega ds/(2\pi) \hat{H}(s) \). In the present case, the only time dependence of \( \tilde{H} \) is through the hopping amplitudes in Eq. (5). We find that the effective Floquet Hamiltonian has the same structure of the static Hamiltonian Eq. (2), with the hopping amplitudes renormalized as

\[
\begin{align*}
  w_1 &\rightarrow \tilde{w}_1 = J_0(|a_{AB}A|) w_1, \\
  w_0 &\rightarrow \tilde{w}_0 = J_0(|a_{AA}A|) w_0,
\end{align*}
\]

where \( J_0 \) is the zeroth Bessel function of the first kind. This light-induced direct renormalization of the interlayer hopping amplitude is the main result of our work. We stress that the result is only valid in the large frequency limit. The same driving protocol can be used to selectively weaken the inter-layer couplings of other few-layer materials such as multilayer graphenes, transition metal dichalcogenide multilayers, phosphorene multilayers and others that have been of recent interest.

In Fig. 2, we plot the quasienergies along a high-symmetry path in the mBZ. The quasi-energies in the driven case were computed using an extended space Floquet Hamiltonian and they agree well with our effective Hamiltonian. We find that the effect of the drive on the low quasi-energy band structure is an increased Fermi velocity near \( \kappa_+ \). Unlike the case of Floquet drives in free space with circularly polarized light, our protocol does not open gaps in the quasienergy spectrum because time reversal symmetry is not broken.

Now we discuss the effect of the TM modes on the magic angles. To simplify the discussion, we consider the chiral case, where \( w_0 = 0 \). Furthermore, we assume small energies such that the Hamiltonian (2) near one of the \( K \) points of graphene becomes linear in momenta i.e. \( f(k) \approx a_0(k_x - ik_y) \). In this case there is only one dimensionless parameter that enters the Schrödinger equation, \( \alpha = w_1/(2\sqrt{\epsilon} k_0 \sin(\theta/2)) \). In Ref. 50 Tarnopolsky et al. considered this limit in equilibrium, and found perfectly flat bands appearing for \( \alpha_1 \approx 0.586 \) and \( \theta_1 \approx 1.09^\circ \). They found that further flat bands exist for \( \alpha_0 = \alpha_1 + n\Delta \alpha \) with \( \Delta \alpha = 3/2 \) and \( n \in \mathbb{N} \). With our modified values for \( w_1 \) we therefore find that the magic angles are to good approximation \( \sin(\theta/2) \approx \theta/2 \) given as

\[
\begin{align*}
  \alpha_0 &\rightarrow \tilde{\alpha}_0 = J_0(|a_{AB}A|) \alpha_0, \\
  \alpha_1 &\rightarrow \tilde{\alpha}_1 = J_0(|a_{AA}A|) \alpha_1,
\end{align*}
\]
\[ \theta_n = \frac{w_1 J_0 \langle a_{AB} a \rangle}{g \kappa D \alpha_n}. \] (8)

Therefore, the degree of tunability of the hopping amplitude depends on the range of values \( \eta \equiv (e/\hbar) a_{AB} a \) can take in experiments, where we reintroduced \( \hbar \) and \( e \). In terms of the electric field amplitude, we have \( \eta \equiv \alpha a_{AB} E / (\hbar \Omega) \). Then, in a pump-probe setup, with a pump drive frequency \( \Omega = 725 \text{ THz} \) (\( \sim 3 \text{ eV}, \text{ in the range of violet light} \)), laser fluence of 4 mJ/cm², and typical pulse duration \( \tau = 0.1 \text{ ps} \), we obtain \( \eta \approx 0.4 \), which leads to \( \theta_F / \theta = 0.96 \). The pump frequency was chosen to be larger than the bandwidth, which is approximately the point where a high-frequency approximation starts breaking down. \[, 9, 10, 11, 12 \] This value can be further tuned by changing the permittivity inside the cavity. A waveguide of size of the order of a = 1 mm can support frequencies as small as 1 THz when the filling insulator is air. Therefore, the high-frequency regime necessary for the validity of Eq. (8) is allowed in a typical waveguide.

In figure 3, we plot the Fermi-velocity of the lowest band \( \epsilon_k^{(1)} \)

\[ v = \sqrt{\left( \theta_{k_x} \epsilon_k^{(1)} \right)^2 + \left( \theta_{k_y} \epsilon_k^{(1)} \right)^2} |_{\kappa+}. \] (9)

near \( \kappa_+ \) as a function of parameter \( \alpha \). The quasi-energies were calculated using the extended space representation of the Floquet Hamiltonian. \[, 13 \] We find that the points where the Fermi velocity vanishes, and where we have flat bands is shifted to larger values of \( \alpha \), or smaller angles \( \theta \). In the inset, we show that the position of the flat bands for the first magic angle has shifted to \( \tilde{\alpha} = 0.608 \). This shift is a 4% change in the magic angle, and in good agreement with our analytical estimates \[. \] The lowest magic angle as function of the driving strength is plotted in figure 4 for a maximum value of \( \eta = 0.8 \) which could be reached with a short pump pulse of \( \tau = 0.05 \text{ ps} \) and drive frequency \( \Omega = 620 \text{ THz} \).

**Conclusions.** We introduced a new mechanism to dynamically tune the effective interlayer tunneling amplitude in few-layer quantum materials using longitudinal electric fields available in a waveguide. This allows us to simulate some of the effects of uniaxial negative pressure in a controlled manner. We demonstrated an application of this novel drive protocol in twisted bilayer graphene, where we can effectively decrease the magic angle. The use of waveguides in the design of Floquet engineering protocols will increase the range of parameters that can be modified in out-of-equilibrium settings.

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Sujay Ray, Jeil Jung, and Tanmoy Das. Wannier pairs in twisted bilayer graphene. *Science*, 363(6431):1059–1064, 2019. ISSN 0036-8075. doi:10.1126/science.aav1910. URL: https://science.sciencemag.org/content/363/6431/1059

Dmitry V. Chichinadze, Laura Classen, and Andrey V. Chubukov. Nematic superconductivity in twisted bilayer graphene, 2019.

Francisco Guinea and Niels R. Wael. Electrostatic effects, band distortions, and superconductivity in twisted graphene bilayers. *Proceedings of the National Academy of Sciences*, 115(52):13174–13179, 2018. ISSN 0027-8424. doi:10.1073/pnas.1810947115. URL: https://www.pnas.org/content/115/52/13174

Biao Lian, Zhijun Wang, and B. Andrei Bernevig. Francisco Guinea and Niels R. Wael. Electrostatic effects, band distortions, and superconductivity in twisted graphene bilayers. *Proceedings of the National Academy of Sciences*, 115(52):13174–13179, 2018. ISSN 0027-8424. doi:10.1073/pnas.1810947115. URL: https://www.pnas.org/content/115/52/13174

Biao Lian, Zhijun Wang, and B. Andrei Bernevig. Twisted bilayer graphene: A phonon-driven superconductor. *Phys. Rev. Lett.*, 122:257002, Jun 2019. doi:10.1103/PhysRevLett.122.257002. URL: https://link.aps.org/doi/10.1103/PhysRevLett.122.257002

Sujay Ray, Jeil Jung, and Tanmoy Das. Wannier pairs in superconducting twisted bilayer graphene and related systems. *Phys. Rev. B*, 99:134515, Apr 2019. doi:10.1103/PhysRevB.99.134515. URL: https://link.aps.org/doi/10.1103/PhysRevB.99.134515

M. J. Calderon and E. Bascones. Correlated states in magic angle twisted bilayer graphene under the optical conductivity scrutiny, 2019.

Yu Saito, Jingyuan Ge, Kenji Watanabe, Takashi Taniguchi, and Andrea F. Young. Decoupling superconductivity and correlated insulators in twisted bilayer graphene, 2019.

Petr Stepanov, Ipsita Das, Xiaobo Lu, Ali Fahimi, Kenji Watanabe, Takashi Taniguchi, Frank H. L. Koppens, Johannes Lischner, Leonid Levitov, and Dmitri K. Efetov. The interplay of insulating and superconducting orders in magic-angle graphene bilayers, 2019.

Jian Kang and Oskar Vafek. Strong coupling phases of partially filled twisted bilayer graphene narrow bands. *Physical Review Letters*, 122(24), Jun 2019. ISSN 1079-7114. doi:10.1103/physrevlett.122.246401. URL: http://dx.doi.org/10.1103/PhysRevLett.122.246401

G. E. Volovik. Graphite, graphene, and the flat band superconductivity. *JETP Letters*, 107(8):516–517, Apr 2018. ISSN 1090-6487. doi:10.1134/s0021364018080052. URL: http://dx.doi.org/10.1134/s0021364018080052

Hoi Chun Po, Liljune Zou, Ashvin Vishwanath, and T. Senthil. Origin of mott insulating behavior and superconductivity in twisted bilayer graphene. *Physical Review X*, 8(3), Sep 2018. ISSN 2160-3308. doi:10.1103/physrevx.8.031089. URL: http://dx.doi.org/10.1103/PhysRevX.8.031089

Masayuki Ochi, Mikito Koshino, and Kazuhiko Kuroki. Possible correlated insulating states in magic-angle twisted bilayer graphene under strongly competing interactions. *Physical Review B*, 98(8), Aug 2018. ISSN 2469-9969. doi:10.1103/physrevb.98.081102. URL: http://dx.doi.org/10.1103/PhysRevB.98.081102

J. Gonzalez and T. Stauber. Kohn-luttinger superconductivity in twisted bilayer graphene. *Physical Review Letters*, 122(2), Jan 2019. ISSN 1079-7114. doi:10.1103/physrevlett.122.026801. URL: http://dx.doi.org/10.1103/PhysRevLett.122.026801

Yu Ry Shermanov and Joseph J. Betoures. Electronic phases in twisted bilayer graphene at magic angles as a result of van hove singularities and interactions. *Physical Review B*, 98(20), Nov 2018. ISSN 2469-9969. doi:10.1103/physrevb.98.205151. URL: http://dx.doi.org/10.1103/PhysRevB.98.205151

Evan Laksono, Jia Ning Leaw, Alexander Reaves, Manraj Singh, Xinyu Wang, Shaffique Adam, and Xingyu Gu. Singlet superconductivity enhanced by charge order in nested twisted bilayer graphene fermi surfaces. *Solid State Communications*, 282:38–44, Oct 2018. ISSN 0038-1098. doi:10.1016/j.ssc.2018.07.013. URL: http://dx.doi.org/10.1016/j.ssc.2018.07.013

Jorn W. F. Venderbos and Rafael M. Fernandez. Correlations and electronic order in a two-orbital honeycomb lattice model for twisted bilayer graphene. *Physical Review B*, 98(24), Dec 2018. ISSN 2469-9969. doi:10.1103/PhysRevB.98.245103. URL: http://dx.doi.org/10.1103/PhysRevB.98.245103

S. Shallcross, S. Sharma, E. Kandelaki, and O. A. Pankratov. Electronic structure of turbostratic graphene. *Phys. Rev. B*, 81:165105, Apr 2010. doi:10.1103/PhysRevB.81.165105. URL: https://link.aps.org/doi/10.1103/PhysRevB.81.165105

Rafi Bistritzer and Allan H. MacDonald. Moiré bands in twisted double-layer graphene. *Proceedings of the National Academy of Sciences*, 108(30):12233–12237, Jul 2011. ISSN 0027-8424. doi:10.1073/pnas.1108174108. URL: https://www.pnas.org/content/108/30/12233

D. Weckbecker, S. Shallcross, M. Fleischmann, N. Ray, S. Sharma, and O. Pankratov. Low-energy theory for the graphene twist bilayer. *Phys. Rev. B*, 93:035452, Jan 2016. doi:10.1103/PhysRevB.93.035452. URL: https://link.aps.org/doi/10.1103/PhysRevB.93.035452

Yuan Cao, Valla Fatemi, Ahmet Demir, Shiang Fang, Spencer L. Tomarken, Jason Y. Luo, Javier D. Sanchez-Yamagishi, Kenji Watanabe, Takashi Taniguchi, Efthimios Kaxiras, Ray C. Ashoori, and Pablo Jarillo-Herrero. Correlated insulator behaviour at half-filling in magic-angle graphene superlattices. *Nature*, 556(7699):80–84, 2018. ISSN 1476-4687. doi:10.1038/nature26154. URL: https://doi.org/10.1038/nature26154

Kyounghwan Kim, Ashley DaSilva, Shengqiang Huang, Babak Fallahazar, Stefano Larentis, Takashi Taniguchi, Kenji Watanabe, Brian J. LeRoy, Allan H. MacDonald, and Emanuel Tutuc. Tunable moiré bands and strong correlations in small-twist-angle bilayer graphene. *Proceedings of the National Academy of Sciences*, 114(13):3364–3369, 2017. doi:10.1073/pnas.1620140114. URL: https://www.pnas.org/content/114/13/3364

Aaron L. Sharpe, Eli J. Fox, Arthur W. Barnard, Joe Finney, Kenji Watanabe, Takashi Taniguchi, M. A. Kastner, and David Goldhaber-Gordon. Emergent ferromagnetism near three-quarters filling in twisted bilayer graphene. *Science*, 365(6453):605–608, 2019. ISSN 0036-8075. doi:10.1126/science.aaw3780. URL: https://science.sciencemag.org/content/365/6453/605

Stephen Carr, Shiang Fang, Pablo Jarillo-Herrero, and
Efthimios Kaxiras. Pressure dependence of the magic twist angle in graphene superlattices. *Phys. Rev. B*, 98:085414, Aug 2018. doi:10.1103/PhysRevB.98.085414 URL https://link.aps.org/doi/10.1103/PhysRevB.98.085414

Bheema Lingam Chittari, Nicolas Leconte, Srivani Javvaji, and Jeil Jung. Pressure induced compression of flatbands in twisted bilayer graphene. *Electronic Structure*, 1(1):015001, nov 2018. doi:10.1088/2516-1075/aaead3 URL https://doi.org/10.1088%2F2516-1075%2Faead3

Matthew Yankowitz, Jeil Jung, Evan Laksono, Nicolas Leconte, Bheema L. Chittari, K. Watanabe, T. Taniguchi, Shafiique Adam, David Graf, and Cory R. Dean. Dynamic band-structure tuning of graphene moiré superlattices with pressure. *Nature*, 557(7705):404–408, 2018. URL https://doi.org/10.1038/s41586-018-0107-1

Takahsi Oka and Hideo Aoki. Photovoltaic hall effect in graphene. *Phys. Rev. B*, 79:081406, Feb 2009. doi:10.1103/PhysRevB.79.081406 URL https://link.aps.org/doi/10.1103/PhysRevB.79.081406

Netanel H. Lindner, Gil Refael, and Victor Galitski. Floquet topological insulator in semiconductor quantum wells. *Nature Physics*, 7(6):490–495, 2011. URL https://doi.org/10.1038/nphys1926

Gabriel E. Topf, Gregor Jotzu, James W. McIver, Lede Xian, Angel Rubio, and Michael A. Sentef. Topological Floquet engineering of twisted bilayer graphene. *Phys. Rev. Research*, 1:023031, Sep 2019. doi:10.1103/PhysRevResearch.1.023031 URL https://link.aps.org/doi/10.1103/PhysRevResearch.1.023031

Yantao Li, H. A. Fertig, and Babak Seradjeh. Floquet-engineered topological flat bands in irradiated twisted bilayer graphene, 2019.

Or Katz, Gil Refael, and Netanel H. Lindner. Floquet flat-band engineering of twisted bilayer graphene, 2019.

M. Fleischmann, R. Gupta, S. Sharma, and S. Shallcross. Moire quantum well states in tiny angle two dimensional semi-conductors, 2019.

F. Rost, R. Gupta, M. Fleischmann, D. Weckbecker, N. Ray, J. Olivesara, M. Vogl, S. Sharma, O. Pankratov, and S. Shallcross. Nonperturbative theory of effective hamiltonians for deformations in two-dimensional materials: Moire systems and dislocations. *Physical Review B*, 100(3), Jul 2019. ISSN 2469-9969. doi:10.1103/physrevb.100.035101 URL http://dx.doi.org/10.1103/PhysRevB.100.035101

Maximilian Fleischmann, Reena Gupta, Florian Wallschler, Dominik Weckbecker, Velimir Meded, Sangeeta Sharma, Bernd Meyer, and Sam Shallcross. Perfect and controllable nesting in the small angle twist bilayer graphene, 2019.

Ming Xie and Allan H. MacDonald. On the nature of the correlated insulator states in twisted bilayer graphene, 2018.

Nguyen N. T. Nam and Mikito Koshino. Lattice relaxation and energy band modulation in twisted bilayer graphene. *Phys. Rev. B*, 96:075311, Aug 2017. doi:10.1103/PhysRevB.96.075311 URL https://link.aps.org/doi/10.1103/PhysRevB.96.075311

Francisco Guinea and Niels R. Walet. Continuum models for twisted bilayer graphene: Effect of lattice deformation and hopping parameters. *Phys. Rev. B*, 99:205134, May 2019. doi:10.1103/PhysRevB.99.205134 URL https://link.aps.org/doi/10.1103/PhysRevB.99.205134

Yang Cheng, Chen Huang, Hao Hong, Zixun Zhao, and Kaihui Liu. Emerging properties of two-dimensional twisted bilayer materials. *Chinese Physics B*, 28(10):107304, sep 2019. doi:10.1088/1674-1056/ab3e46 URL https://doi.org/10.1088%2F1674-1056%2Fab3e46

Kaihui Liu, Liming Zhang, Ting Cao, Chenhao Jin, Diana Qiu, Qin Zhou, Alex Zettl, Peidong Yang, Steve G. Louie, and Feng Wang. Evolution of interlayer coupling in twisted molybdenum disulfide bilayers. *Nature Communications*, 5(1), Sep 2014. ISSN 2041-1723. doi:10.1038/ncomms5966 URL http://dx.doi.org/10.1038/ncomms5966

Jhao-Ying Wu, Wu-Pei Su, and Godfrey Gumbs. Anomalous magneto-transport properties of bilayer phosphorene, 2019.

M. Vogl, O. Pankratov, and S. Shallcross. Semiclasis for matrix hamiltonians: The gutzwiller trace formula with applications to graphene-type systems. *Phys. Rev. B*, 96:035442, Jul 2017. doi:10.1103/PhysRevB.96.035442 URL https://link.aps.org/doi/10.1103/PhysRevB.96.035442

Ce Shang, Adel Abbout, Xiaoming Zang, Udo Schwinger-schlogl, and Aurelien Manchon. Artificial gauge fields and topological insulators in moire superlattices, 2019.

H M Abdulliah, B Van Duppen, M Zarenia, B Hahloui, and F M Peeters. Quantum transport across van der Waals domain walls in bilayer graphene. *Journal of Physics: Condensed Matter*, 20(42):425303, sep 2017. doi:10.1088/1361-646x/aa81a8 URL https://doi.org/10.1088%2F1361-646x%2Fa81a8

André Eckardt and Egidijs Anisimovas. High-frequency approximation for periodically driven quantum systems from a floquet-space perspective. *New Journal of Physics*, 17(9):093039, sep 2015. doi:10.1088/1367-2630/17/9/093039 URL https://doi.org/10.1088%2F1367-2630/17/9/093039

Grigory Tarnopolsky, Alex Jura Kruchkov, and Ashvin Vishwanath. Origin of magic angles in twisted bilayer graphene. *Phys. Rev. Lett.*, 122:106405, Mar 2019. doi:10.1103/PhysRevLett.122.106405 URL https://link.aps.org/doi/10.1103/PhysRevLett.122.106405

Dmitry A. Abanin, Wojciech De Roeck, Wen Wei Ho, and Francois Huyenc. Effective hamiltonians, prethermalization, and slow energy absorption in periodically driven many-body systems. *Physical Review B*, 95(1), Jan 2017. ISSN 2469-9969. doi:10.1103/physrevb.95.014112 URL http://dx.doi.org/10.1088%2F1361-648x%2Faa81a8

Takahiro Mikami, Sota Kitamura, Kenji Yasuda, Naoto Tsuji, Takashi Oka, and Hideo Aoki. Brillouin-wigner theory for high-frequency expansion in periodically driven systems: Application to floquet topological insulators. *Physical Review B*, 93(14), Apr 2016. ISSN 2469-9969. doi:10.1103/PhysRevB.93.144307 URL http://dx.doi.org/10.1103/PhysRevB.93.144307

Marin Bukov, Luca D Alessio, and Anatoli Polkovnikov. Universal high-frequency behavior of periodically driven systems: from dynamical stabilization to floquet engineering. *Advances in Physics*, 64(2):139226, Mar 2015. ISSN 1460-6976. doi:10.1080/00018732.2015.1055918 URL http://dx.doi.org/10.1080/00018732.2015.1055918

Michael Vogl, Pontus Laurell, Aaron D. Barr, and Gregory A. Fiete. Flow equation approach to periodically driven quantum systems. *Phys. Rev. X*, 9:021037, May 2019. doi:10.1103/PhysRevX.9.021037 URL https://link.aps.org/doi/10.1103/PhysRevX.9.021037

Michael Vogl, Martin Rodriguez-Vega, and Gregory A. Fi-
et al. Effective floquet hamiltonian in the low-frequency regime. Phys. Rev. B, 101:024303, Jan 2020. doi:10.1103/PhysRevB.101.024303 URL https://link.aps.org/doi/10.1103/PhysRevB.101.024303

Appendix A: Electric and magnetic transverse fields.

The wave equation for the vector potential \( \mathbf{A} \) in free space according to Maxwell’s equations is given as

\[
\nabla^2 \mathbf{A} = \mu c \partial_t^2 \mathbf{A} \tag{A1}
\]

A solution to this equation is

\[
\mathbf{A} = \hat{\mathbf{z}} \sin(k_x x) \sin(k_y y) \Re\{e^{-ik_z z - i\Omega t}\}, \tag{A2}
\]

where \( k_z = \sqrt{k^2 - k_x^2 - k_y^2} \) and \( k = \Omega/\sqrt{\mu c} \). We recall that

\[
\mathbf{H} = \nabla \times \mathbf{A}; \quad \partial_t \mathbf{E} = \frac{1}{\epsilon} \nabla \times \mathbf{H} \tag{A3}
\]

From here one may see that the solution for \( \mathbf{A} \) we found is enough to accommodate the boundary conditions

\[
E_y(x = a) = E_z(x = a) = E_x(y = b) = E_z(y = b) = 0 \tag{A4}
\]

that a rectangular metallic waveguide introduces if we choose \( k_x = \frac{m\pi}{a} \) and \( k_y = \frac{n\pi}{b} \). This solution is called a transverse magnetic mode (TM) of the cavity and the fields are given as

\[
E_x \propto \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \Re\{e^{-ik_z z - i\Omega t}\},
\]

\[
E_y \propto \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \Re\{e^{-ik_z z - i\Omega t}\},
\]

\[
E_z \propto \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \Re\{e^{-ik_z z - i\Omega t}\},
\]

\[
H_x \propto \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \Re\{e^{-ik_z z - i\Omega t}\},
\]

\[
H_y \propto \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \Re\{e^{-ik_z z - i\Omega t}\},
\]

\[
H_z = 0
\]

At the points we want to place the twisted bilayer graphene sheets we have \( \sin = 1 \) and \( \cos = 0 \) and therefore only \( E_z \) is non-zero. This coincides with the physical understanding that the effects we discuss in the paper are actually caused by the physical fields in the correct direction.

A more complete description can be found in [58].

Appendix B: Numerical solution of the Hamiltonian

In this appendix we detailed the numerical implementation of the static Hamiltonian for twisted bilayer graphene. We employ the cellular method, that was first introduced in 1933 by Wigner and Seitz [59]. Below we give a short summary of the steps needed.

Our starting point is the Schrödinger equation for a periodic Hamiltonian

\[
E\psi = H(p, x)\psi, \tag{B1}
\]

where \( \hat{p} = -i\nabla \). Since \( H(p, x) \) is periodic, with the same periodicity of the lattice, it commutes with the translation operators \( T_X = e^{ipX} \) for lattice vectors \( X \). That is, it also fulfills the eigenvalue equation

\[
T_X\psi(x) = C\psi(x), \tag{B2}
\]

which has the eigenvalues \( C = e^{ikX} \) and therefore one may rearrange the equation as \( e^{-ikX}\psi(x + X) = \psi(x) \). If we multiply both sides by \( e^{-ikx} \) we see that \( u(x) = e^{ikx}\psi(x) \) is periodic, which is known a Bloch’s theorem. Therefore setting \( \psi(x) = e^{ikx}u(x) \) in the Schrödinger equation gives us

\[
E_k u(x) = H(p + k, x)u(x), \tag{B3}
\]

which describes bands \( E_k \) and has coordinates restricted to one lattice unit cell. The scalar product is restricted in a similar fashion. Calculating bands therefore merely amounts to writing the Hamiltonian in a plane wave basis

\[
H_{nm}(k) = \frac{1}{V_{UZ}} \int_{A_{UZ}} e^{iQ \cdot \mathbf{x}} H(k, p, x) e^{-iQ \cdot \mathbf{x} + p \cdot \mathbf{x}}, \tag{B4}
\]

where \( A_{UZ} \) is a unit cell, \( V_{UZ} \) the volume of the unit cell and \( Q_m \) are reciprocal lattice vectors. The integral over the \( A_{UZ} \) can be performed analytically, leading to efficient implementations of the band structure calculation.