On the first Townsend coefficient at high electric field

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Abstract—Based on the simplified approach it is shown and experimentally confirmed that gas gain in wire chambers at very low pressure becomes higher on thicker wires at the same applied high voltage. This is a consequence of the fact that the first Townsend coefficient at high reduced electric field depends almost entirely on the mean free path of the electrons.

I. INTRODUCTION

An electron drifting between two points \( r_1 \) and \( r_2 \) under the influence of an electric field gains energy and produces secondary electrons due to inelastic collisions. The energy distribution of electrons changes in shape from the Maxwellian (at the absence of an applied field) to a wider distribution under the electric field. When attachment, photoproduction and space (at the absence of an applied field) to a wider distribution under distribution of electrons changes in shape from the Maxwellian

| Townsend coefficient \[1\]. Most of them satisfactorily describe correctly determined parameters. The generalized form of the experimental data on some ranges of the electric field with reduced first Townsend coefficient is given by the expression

\[
S = \frac{Z}{r_n} \int_{r_1}^{r_2} f \, dr;
\]

(1)

where \( f \) is the Townsend coefficient. The Townsend coefficient is a function of reduced electric field strength \( S = E/p \), i.e. \( E = f (E = p) \). There are several forms of the first Townsend coefficient \[1\]. Most of them satisfactorily describe experimental data on some ranges of the electric field with correctly determined parameters. The generalized form of the reduced first Townsend coefficient is given by the expression

\[
P = A \exp\left(\frac{B \rho}{E}\right);
\]

(2)

where \( A \) and \( B \) are parameters depending on the gas type and electric field range.

Let represent an electron free path, i.e. the path between two consecutive collisions of electrons with atoms, \( r \) is a projection of the free path on the field direction, \( m \) is a mean free path and \( 1 \) is an ionization path, i.e. the distance in the field direction required for the electron to travel between two successive ionizations. It is obvious that for each gas \( m \) and \( 1 \) depend on pressure, \( 1 \) is a function of the local electric field as well.

In gases, distances between two consecutive collisions of electrons and atoms have an exponential distribution. In general, the mean free path in gases is defined as \[2\]

\[
m = \frac{1}{n};
\]

(3)

* The study have in the most been done during author’s stay at TRIUMF, Vancouver, BC, Canada.

where \( n \) is the number of atoms per unit volume and \( E \) is the total cross section for electron collision with atoms. Generally, the cross section is a function of electron velocity (energy) \( E = \langle v \rangle \).

It has been shown \[3\] that the generalized form of the first Townsend coefficient expressed by Eq. (2) is a satisfactory description of experimental data in a wide range of electric field. This form of the first Townsend coefficient will be used hereafter as well.

II. TOWNSEND COEFFICIENT AT HIGH REDUCED ELECTRIC FIELD

A. Simplified case

Consider a hypothetical simplified case when free paths between two collisions of electrons with atoms are constant at a given pressure and the probability for the electron with energy above the atom ionization level to produce an ionization at each collision is \( 1 \). Let all electrons have an energy equal to the average energy of a real energy distribution. All quantitative estimations included in this subsection are made under these simplified assumptions. In this simplified case the free paths are equal to \( m = 1 \).

Consider a hypothetical electric field when the field strength \( E \) gradually increases from 0 to some value. Let electrons at each point be in equilibrium with an electric field. The higher the electric field, the higher the average electron energy. At low energies electrons scatter isotropically. However, at a high electric field electrons gain large energy and already at energy of 20-30 eV scattering takes place mostly in forward direction \[4\], \[5\], \[6\]. As a result, \( E \) tends to \( m \). In this case at each collision electron loses energy equal to the gas ionization potential. When distances between two collisions in the field direction become greater than the electron path required to gain enough energy to ionize atoms, i.e. \( E \) \( 1 \), electrons restore their energy and the average energy continues to increase. At this conditions the first Townsend coefficient is expressed very simply as

\[
\frac{1}{E} = \frac{1}{m};
\]

(4)

Therefore the first Townsend coefficient can be expressed as a combination of two components:

\[
1 = A \exp\left(\frac{B \rho}{E}\right); \quad E < 1
\]

\[
2 = 1 = \langle v \rangle \quad 1
\]

(5)
with the general gas multiplication factor of Eq. (11) expressed now as

$$Z = \frac{\ln M}{\ln (r/a)} = \int_1^{(1 + S c^2) (1 + S c^2)} (\ln (r/a)) dx$$

(6)

The first term in Eq. (5) is a standard form for the first Townsend coefficient and describes events when there are several collisions between two consecutive ionizations. The second one takes into account cases when there are no elastic collisions between two consecutive ionizations.

An electron drifting at the electric field $E$ over the distance $z$ gains energy $eEz$, where $e$ is a projection of $m$ onto the field direction. As was mentioned above, at the high electric field electrons have a large average energy and scatter mostly in the forward direction, and $E$ tends to $m$. At these conditions for any gas at a high field there exists an electric field strength $E_{m}$ when electron gains enough energy to ionize the gas over the path $m$, i.e.

$$eE_{m} = I_0;$$

(7)

where $I_0$ is the gas ionization potential. For this case Eq. (5) could be re-written as

$$1 = Ap_{exp} \left( \frac{E}{E_{m}} \right) \text{ when } E < E_{m}$$

$$2 = n \text{ when } E = E_{m}$$

(8)

There are very important consequences from the last equations both for parallel plate avalanche counters (PPAC) and wire chambers.

In case of the PPAC the electric field is homogenous and the multiplication factor over the cathode-anode distance is expressed, according to Eq. (5), as

$$Z = \int_{a}^{b} (\ln (r/a)) dx$$

(9)

when $E < E_{m}$, or

$$Z_{c} = \int_{a}^{b} (\ln (r/a)) dx$$

(10)

when $E = E_{m}$.

This means that in the simplified case of constant free paths when there is a limit of the electric field strength $E_{m}$ where further voltage increase does not increase gas multiplication. This field $E_{m}$ is given by Eq. (7) with $m$ defined by Eq. (5).

At standard conditions for an ideal gas the number of atoms in a unit volume is $n = 2.687 \times 10^16$ cm$^{-3}$ (Lohschmidt number). Generally, the cross section is a function of electron energy and the typical value is of the order of $10^{-15}$ cm$^2$ [7], [8], [9]. Typical ionization potentials for different gases are in the range 10-15 eV, Substituting values for $n$ and $I_0$ into Eq. (7) gives $E_{m}$ of 270-400 kV/cm, which is far beyond a reachable value for PPACs. However, the situation changes at low pressure. At a gas pressure of 20 Torr, $n \approx 7 \times 10^6$ cm$^{-3}$, and Eq. (7) gives an electric field value in the range $E_{m} \approx 7-10.5$ kV/cm, or $S_m = E_{c} = 350-520$ V/cm Torr in terms of reduced electric field. These values are in the range where PPACs are used at low pressure [10].

More important consequences from the consideration of Eq. (5) appear for wire chambers. The electric field in a cylindrical wire chamber is defined as

$$E = \frac{V}{r \ln (r/a)}$$

(11)

where $a$ and $b$ are the wire and cathode radii, and $r$ is the distance from the wire center. The electric field strength has a maximum value on the wire surface and sharply drops off away from the wire. As a result, at the atmospheric pressure the gas gain mainly takes place within 3-5 wire radii.

Electrons drifting toward the wire gradually gain energy. For the simplified case mentioned above, each collision of the electron with energy above the ionization potential results in ionization. When electrons gain energy about few tens of eV, the scattering mostly takes place in the forward direction.

Real gas amplification starts when the electric field becomes greater than some critical value $E_c$. This critical value is a characteristic of the gas and for most gases is in the range 30-70 kV/cm at the atmospheric pressure, or $S_c' > 10$ Torr in terms of reduced electric field. The reduced critical electric field $S_c = E_{c} = p$ is a constant for a given gas and independent of pressure.

In wire chambers with the same geometry and different anode wire diameters at the atmospheric pressure one needs to apply much lower voltage to chambers with thin wires in order to get the same gas gain. At lower pressure the voltage difference required to get the same gas gain on two different anode wire diameters becomes smaller.

Now recall that there exists an electric field $E_{m}$ where an electron gains energy equal to the gas ionization potential between two consecutive collisions. Although the field near the wire is non-uniform, for simplicity one can consider an average electric field over the path between two consecutive collisions. We found that for gases at standard conditions this field was in the range $E_{m} \approx 270-400$ kV/cm or $S_{m} \approx 350-520$ V/cm Torr in terms of reduced electric field. Like the critical reduced electric field strength $S_{c}$, $S_{m}$ is a constant for a given gas and is independent of pressure.

Consider as an example two single wire chambers with cathode diameters 10 mm and wire diameters 10 mm and 50 mm. Figure 1 presents the reduced electric field value near the 10 mm and 50 mm wires as a function of the distance from the wire surface. 1200 V applied to both wires at the pressure 100 Torr and 900 V at 20 Torr. We will use typical values $S_c = 65$ V/cm Torr and $S_{m} = 500$ V/cm Torr in further consideration. These values $S_c$ and $S_{m}$ are shown in the figure as well. For each wire let $r_0$ be the point where $E = E_c$, i.e. the point where the gas avalanche starts, and $r_m$ be the point where $E = E_{m}$. Figure 1 shows that at 100 Torr the avalanche starts about 0.2 mm away from the 50 mm wire surface and about 0.18 mm away from the 10 mm wire. At 20 Torr these values are 1.6 mm and 1.3 mm respectively.

With the same chamber geometry and applied voltages the electric field strength is much higher on the surface of the thin wire. It drops off faster on the thin wire and eventually the field...
strength becomes higher near the thick wire as can be seen in Fig. 1. Figure 2 shows details of the same field strengths with a linear vertical scale.

The gas gain on each wire is defined by Eq. (12) by integrating the first Townsend coefficient from the avalanche start point \( r_c \) to the wire surface \( r_a \). In the simplified case one can divide this integral into two parts. One part includes the path from \( r_c \) to \( r_m \) where \( S < S_m \), and the second one is from \( r_m \) to the wire surface \( r_a \). Thus, the gas gain is defined as

\[
\text{ln} M = \frac{Z}{r_c} \int_{r_c}^{r_m} 1(x)dx + \frac{Z}{r_a} \int_{r_m}^{r_a} 2(x)dx
\]  

(12)

where \( 1(x) \) and \( 2(x) \) are defined by Eq. (8).

For the simplified case at a reduced electric field strength \( S > S_m \) any electron collision with the atom results in ionization. However, due to mainly forward scattering, in the vicinity of a wire surface the electrons experience very few collisions with atoms. The number of collisions depends on the free path and in the simplified case is independent of the electric field, i.e. the first Townsend coefficient becomes independent of the electric field strength.

Let us consider in detail the electric field strengths at a pressure of 100 Torr and applied voltage of 1200 V. The gas gain starts earlier on the 50 m wire. However, the field is very weak here and the contribution to the total gain is insignificant. Eventually, the electric field strength becomes higher on the 10 m wire. Everywhere after the field crossing point the first Townsend coefficient for the 10 m wire is higher than or equal (after the electric field strength on the 50 m wire exceeds the \( E_m \) value) to that on the 50 m wire. As a result, the gas gain on the 10 m wire is higher than on the 50 m wire at this pressure.

At a pressure of 20 Torr and 900 V applied to both chambers the situation is different. As usual, at distances far from the wire surfaces the electric field strength is higher for the thick wire and gas gain starts earlier on that wire. However, in contrast to the previous case the electric field strength lines cross above the \( S_m = 500 \text{ V/cm} \) Torr line. This means that the first Townsend coefficient is higher on the 50 m wire everywhere until the electric field strength near the 10 m wire reaches the value \( S = S_m = 500 \text{ V/cm} \) Torr. After that point both wires have the same first Townsend coefficients. As a result, at a pressure of 20 Torr and applied high voltage of 900 V the total gas gain on the 50 m wire is higher than that on the 10 m wire.

At the atmospheric pressure an amplification in the vicinity of a wire surface, typically within 3-5 wire radii, is the main contribution to the gas gain. The higher electric field near the thinner wire surface results in a higher gas gain. At a very low pressure the situation changes. The thin wire has almost the same gas gain in the vicinity of the wire surface as the thick wire with the same applied voltage. Differences in the total gas gain between thin and thick wires are defined in most by the gain far from the wire surface, where the thick wire has a higher electric field and first ionization coefficient.

Values \( S_c \) and \( S_m \) are constant for each gas and vary from gas to gas. As a result, different gases will have different pressures and applied high voltages at which gas gain becomes higher on the thick wire for the same chamber geometry and applied voltages.
B. Real gas case

In the previous subsection we considered the simplified case with constant electron free path lengths and electron energies equal to the average value of the real energy distribution. In real gases electron free paths have an exponential distribution with a mean value expressed by Eq.\(^{5}\). The electrons energy distribution is very wide with a long tail. Also, the ionization cross section \(i\) is only a fraction of the total cross section at electron energies above the ionization level. At a high reduced electric field the first Townsend coefficient at each point can be defined through the mean free path \([2]\) multiplying it by the probability that the path is longer than the local ionization path \(i\). Multiplying this number by \(n\) takes into account the ratio of the ionization cross section to the total cross section. In this case \(z\) in Eq.\(^{5}\) becomes

\[
 z(t) = -\frac{1}{m} \exp\left(\frac{i(t)}{m}\right); \tag{13}\]

The first term in Eq.\(^{5}\) and Eq.\(^{8}\) should be multiplied by the probability that path is shorter than the local ionization path \(i\) = \(L_0 = eE(t)\). Finally, Eq.\(^{8}\) transforms to

\[
 (r) = A \exp\left(\frac{B}{E}\right) \left(1 - \exp\left(\frac{-i(t)}{m}\right) + \frac{1}{m} \exp\left(\frac{i(t)}{m}\right)\right) \tag{14}\]

or, using definitions of \(i\) and \(m\)

\[
 (r) = A \exp\left(\frac{B}{E}\right) \left(1 - \exp\left(\frac{L_0 n}{eE(t)}\right) + n \exp\left(\frac{L_0 n}{eE(t)}\right)\right) \tag{15}\]

The first term in Eq.\(^{15}\) will dominate at a relatively low reduced electric field. This term vanishes at a high reduced electric field and the Townsend coefficient will almost entirely depend on the second term.

The reduced first Townsend coefficient defined by a standard form of Eq.\(^{4}\) is normally parametrized for some range of a reduced electric field \(E/p\). It tends to a constant value \(A\) with further increase of \(E/p\).

The first Townsend coefficient expressed by Eq.\(^{15}\) coincides with that expressed by Eq.\(^{2}\) at a lower electric field. It reaches the peak value as \(E/p\) increases and drops with a further increase of the electric field, when the average electron energy increases. This happens because the second term in Eq.\(^{15}\) drops due to decrease of the ionization cross section \(i\) with increasing average electron energy.

Experimental results [11] and the numerical solution of the Boltzmann equation along with Monte Carlo calculations [12] demonstrated that the first Townsend coefficient \(A\) is not a function of \(E/p\) only, but a gradient of an electric field as well. Authors of Ref. [12] showed that at low pressure there are two processes which affect the first Townsend coefficient behaviour at a high electric field. First, at high \(E/p\) electrons approaching the wire could miss the wire surface and rotate around it. The probability for this is higher on a thinner wire, which, as the authors noted, results in a higher first Townsend coefficient on the thinner wire at the same value of \(E/p\). However, in this case one can talk about increase of an effective electron drift path rather than increase of the first Townsend coefficient. Such an increase of the electron path due to rotation around the wire will increase the total gas gain. Another process which results in decrease of the first Townsend coefficient is due to the so-called delayed electrons [12]. In a high electric field gradient electrons are not in equilibrium with the electric field and their energy is lower than that in the same constant field. This results, as the authors noticed, in decrease of \(E/p\) with further increase of the electric field.

The authors of Ref. [13] made a Monte Carlo simulation of electron avalanches in proportional counters. They demonstrated that rotation of electrons around the wire did increase the gas gain, but made a small contribution to the total gain. It was shown that the main gain increase happened due to extension of the avalanche region further from the wire. These conclusions are generally in agreement with the results shown in the present paper for the simplified case.

Microscopic calculation of the first Townsend coefficient and gas gain as a function of pressure and a wire diameter made by the authors of Ref. [14] demonstrated that decrease of a gas pressure eventually resulted in a higher gain on the thicker wire at the same applied voltage.

A set of measurements of gas gain in single wire chambers with \(12\) \(12\) mm cell cross sections with different wire diameters (15, 25, 50 and 100 m) have been made in order to check the gas gain behaviour at a high reduced electric field \(E/p\). Chambers were filled with pure iso-C\(_{4}\)H\(_{10}\) at pressures 92, 52, 32 or 12 Torr and irradiated with \(^{55}\)Fe x-ray particles. Most of the electrons released by the photoabsorption process in the gas volume leave the chamber cell. A small fraction of them lose their entire energy inside the cell and give full signals. These photoabsorption peaks were used to calculate the gas gain. The intensity of these full photoabsorption peaks drops with decreasing gas pressure due to the increasing electron range in the gas. The results clearly demonstrated that a decrease of a gas pressure from 92 to 12 Torr led to a higher gas gain on thicker wires compared to that on thinner ones. Detailed results of these measurements are published separately [15].

Figure 3 presents one result from these measurements, namely the charge spectra taken from single wire chambers at a pressure of 12 Torr, with 800 V applied to all chambers. Electrons released by the photoabsorption of \(^{55}\)Fe x-ray particles have a range of about 45 mm in pure iso-C\(_{4}\)H\(_{10}\) at 12 Torr. Almost all of them leave the chamber cell before losing all of their energy and there is no evidence of photoabsorption peaks. The resulting charge spectra from the chambers have continuous distributions. However, all chambers have the same initial ionization distribution and the edges of the spectra do indicate the gas gain on each wire. The figure clearly demonstrates that in pure iso-C\(_{4}\)H\(_{10}\) at 12 Torr and applied 800 V the thicker wires have a higher gas gain. Taking a collected charge on each wire at the half maximum on the edge of the spectra as a reference gives the ratio of gas gains on all these wires M\(_{15}\) : M\(_{25}\) : M\(_{50}\) : M\(_{100}\) = 1 : 1.35 : 1.85 : 3.15.

It should be noted that the chamber simulation program Garfield [16] with the Magboltz [17] interface shows a similar tendency on the gas gain at low pressure. Although Garfield
have large energy and scatter mostly in the forward direction, the function on the electron mean free path. This reflects the well-known fact that the function remains almost the same as in the vicinity of the wire surface where the electric field strength and the first Townsend coefficient are higher for the thicker wire. Thicker wires should be used in wire chambers operating at very low pressure where scattering on the wires is not critical.

II. Gas Amplification

In PPACs at some value of the reduced electric field strength the gas gain, one can compare the gas gain ratios for wires of different diameter. The Garfield estimation of avalanche sizes due to single electrons in the same wire chambers under the same conditions as in the above-mentioned example gives the gas gain ratio $M_{15}:M_{25}:M_{50}:M_{100} = 1:2:6:9:27$.

III. Conclusion

At a high reduced electric field, where drifting electrons have large energy and scatter mostly in the forward direction, the first Townsend coefficient should almost entirely depend on the electron mean free path. This reflects the well-known fact that the function $\frac{1}{\lambda} = P = \frac{1}{\lambda} (1 + \epsilon) = P$ is saturated at a high reduced electric field. The generalized formula for the first Townsend coefficient at a high reduced electric field should include the measured gas data as parameters.

It is experimentally demonstrated that gas gain in wire chambers at very low pressure becomes higher on thicker wires at the same applied high voltage. Thinner wires have a much higher electric field in the vicinity of the wire surface than the same applied high voltage. However, the first Townsend coefficient remains almost the same as in the vicinity of the thicker wires. Very few ionizations occur in the vicinity of the wire surface where the electric field strength and the first Townsend coefficient are higher for the thicker wire. Thicker wires should be used in wire chambers operating at very low pressure where scattering on the wires is not critical.

In PPACs at some value of the reduced electric field strength in the simplified case of constant free paths there is a limit for gas gain, which is defined by gas density, i.e. electron’s free paths. These values vary from gas to gas and are in the range $S_m = 340-500$ V/cm Torr. In real gases free paths have an exponential distribution, and there will be no sharp transition when the electric field reaches the value $S_m$. It should asymptotically approach its limit instead.

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