Off-Fermi Shell Nucleons in Superdense Asymmetric Nuclear Matter

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Recent observations of the strong dominance of proton-neutron (\textit{pn}) relative to \textit{pp} and \textit{nn} short-range correlations (SRCs) in nuclei indicate on possibility of unique new condition for asymmetric high density nuclear matter, in which the \textit{pp} and \textit{nn} interactions are suppressed while the \textit{pn} interactions are enhanced due to tensor interaction. We demonstrate that for sufficiently asymmetric case and high densities the momentum distribution of the smaller \textit{p}-component is strongly deformed with protons increasingly populating the momentum states beyond the Fermi surface. This result is obtained by extracting the probabilities of two-nucleon (2N) SRCs from the analysis of the experimental data on high momentum transfer inclusive electro-nuclear reactions. We fitted the extracted probabilities as a function of nuclear density and asymmetry and used the fit to estimate the fractions of the off-Fermi shell nucleons in the superdense nuclear matter relevant to neutron stars. Our results indicate that starting at three nuclear saturation densities the protons with fractional densities \(\frac{1}{3}\) will populate mostly the high momentum tail of the momentum distribution while only 2\% of the neutrons will do so. We discuss the implications of this condition for neutron stars and emphasize that it may be characteristic to any asymmetric two component Fermi system with suppressed central and enhanced short-range tensor interactions between the two components.

The recent experiments on high-momentum transfer semiexclusive reactions\textsuperscript{[1, 2]} in which the struck nucleon from the nucleus is detected in coincidence with the recoil nucleon from SRC, found striking disbalance between \textit{pn} and \textit{pp}/\textit{nn} correlations. They found that protons struck from the nucleus with initial momenta of \(k_F < p \leq 600\) MeV/c, in the 92\% of the time emerge from the \textit{pn} SRC, while the \textit{pp} and \textit{nn} SRCs are significantly suppressed, contributing only \(\approx 4\%\) to the high momentum part of the nucleon momentum distribution in nuclei. This disbalance was understood based on the dominance of the \textit{NN} tensor interaction in the 300 < \(p < 600\) MeV/c momentum range relevant to 2N SRCs in nuclei\textsuperscript{[1, 3, 4]}. The tensor-interaction dominance is due to the fact that in this momentum range corresponding to inter-nucleon distances of \(\sim 1\) Fm the NN central potential is crossing the zero due to transition from attractive to the repulsive interactions. As a result at these distances overall NN potential is dominated by \textit{pp} and \textit{nn} interactions and enhancement of the interaction in isosinglet, \(L=2\), \textit{pn} channel. The resulting picture for the nuclear matter consisting of protons and neutrons at densities in which inter-nucleon distances are \(\sim 1\) Fm is rather unique: it represents a system with suppressed \textit{pp} and \textit{nn} but enhanced \textit{pn} interactions.

The goal of the present study is to understand how the high momentum part of the momentum distributions of protons and neutrons are defined in the high density nuclear matter under the above described conditions.

**New Relation between High Momentum \textit{p}- and \textit{n}-distributions in Nuclei:** Due to short range nature of \(NN\) interaction the nuclear momentum distribution, \(n^A(p)\), for momenta, \(p\), exceeding the characteristic nuclear Fermi momentum \(k_F\) is predominantly defined by the momentum distribution in the SRCs. There is a rather large experimental body of information indicating that for the range of \(k_F < p < 600\) MeV/c the SRCs are dominated by 2N correlations, which consist of mainly the \textit{pn} pairs (for recent reviews see \textsuperscript{[11, 12]}).

In recent work\textsuperscript{[13]} based on the dominance of the \textit{pn} SRCs we predicted two new properties for the nuclear momentum distributions at \(k_F < p < 600\): (i) There is an approximate equality of \textit{p}- and \textit{n}-momentum distributions weighted by their relative fractions in the nucleus \[ x_p = \frac{2}{3}, \quad x_n = \frac{A-2}{A}. \]

with \(\int n^A_{p/n}(p)dp = 1\). (ii) The probability of proton or neutron being in high momentum NN SRC is inverse proportional to their relative fractions and can be related to the momentum distribution in the deuteron \(n_d(p)\) as:

\[ n^A_{p/n}(p) = \frac{1}{2x_p/n} a_2(A, y) \cdot n_d(p), \]

where \(a_2(A, y)\) is interpreted as a per nucleon probability of finding 2N SRC in the given \(A\) nucleus\textsuperscript{[5, 7]} and the nuclear asymmetry parameter is defined as \(y = |1 - 2x_p|\).

The above two properties are obtained assuming no contributions from \textit{pp}, \textit{nn} as well as higher order SRCs. They follow from the assumption that the whole strength of nuclear high momentum distribution as well as per nucleon probability of proton and neutron to be in the SRC is defined by the same \textit{pn} correlation. In Ref.\textsuperscript{[13]} we demonstrated that these properties are seen in the direct calculations using realistic \(^3\text{He}\) wave function.

Since the SRC is defined by local properties of nuclei, one expects that the \(A\) dependence of \(a_2\), in Eq.\textsuperscript{(2)} is related to the nuclear density, i.e. \(a_2(A, y) = a_2(p, y)\). This could allow us to estimate the high momentum part of the nucleon momentum distribution not only for finite\textsuperscript{[13]} but also for infinite nuclear matter.
The Parameter $a_2$ and $A(e,e')X$ Processes: In principle, any nuclear process which probes high momentum nucleons in nuclei should allow an extraction of $a_2(A, y)$. One of such processes are high momentum transfer inclusive $A(e,e')X$ reactions measured in special kinematics in which electron scatters off a deeply bound nucleon having large momentum in the nucleus. Two parameters, 4-momentum transfer square $-Q^2$ and Bjorken $x_{Bj} = \frac{Q^2}{2m_N q_0}$ allow us to select these kinematics. Introducing the parameter $\alpha$, which defines ($A$ times) the light cone momentum fraction of nucleus carried by the interacting nucleon, within impulse approximation:

$$x_{Bj} = \frac{Q^2}{2m_N q_0} = \frac{q_+}{2q_0} + \frac{q_-}{2q_0} + \frac{m^2 - m^2_N}{2q_0 m_N}$$

(3)

where 4-momentum of the virtual photon is defined as $(q_0, q_3)$ with $q_3 = q_0 \pm q_3$. Also $p_{i+}$ and $\bar{m}$ represent the $^+$-component and mass of the bound initial nucleon. From Eq. (3) in the limit of $Q^2 \gg m_N^2$ such that $\frac{x_{Bj}}{x} \ll 1$ the condition: $\alpha \ll x_{Bj}$ is satisfied and choosing $x_{Bj} > 1$ will select a nucleon in the nucleus that carries momentum fraction more than that of the stationary nucleus. This observation is the basis of the 2N SRC model, according to which at $Q^2 \gg m_N^2$ and $1.4 - 1.5 < x_{Bj} < 2$ the interacting nuclear needs to acquire a substantial momentum fraction from the nucleon with which it is in a short-range space-time correlation. The expectation that the $\gamma N$ interaction in SRC will be weakly influenced by the long-range mean-field of A-2 residual nucleus resulted to the prediction of the onset of plateau in the ratios of $A(e,e')X$ and $d(e,e')X$ cross sections at $x_{Bj} > 1.4 - 1.5$ and $Q^2 \gg m_N^2$. First, such plateau was observed in Ref. and later was confirmed in new experiments for wide range of nuclei. The measurements also confirmed that the onset of the plateau depends on $Q^2$ and sets in at $Q^2 \geq 1.5$ GeV$^2$ as it was predicted in the 2N SRC model (see e.g. 6, 11, 14). It is worth noting that models in which the $x > 1$ cross section is attributed mainly to the final state interaction of the struck nucleon with the residual nucleons is in disagreement with the observed plateau and its onset being a function of $Q^2$ (for detailed discussion of FSI effects see Ref. 12).

Based on the expectation that $A(e,e')X$ probes 2N SRCs, from Eq. (2) one observes that:

$$a_2(A, y) = \frac{2\sigma_{eA}}{A\sigma_{ed}} \text{ with } \sigma_{eA} = \frac{d\sigma}{dE_{e'}/d\Omega_{e'}}$$

(4)

for those values of $x_{Bj}$ and $Q^2$ that the measured ratio of the cross sections exhibits the plateau.

**Extraction of $a_2(A)$:** We analyzed the compilation of the world data on inclusive $A(e,e')X$ reactions from Ref. 10, 12. Only the data for $d, ^{3}$He, $^{4}$He, $^{9}$Be, $^{12}$C, $^{27}$Al, $^{56}$Fe, $^{64}$Cu and $^{197}$Au nuclei satisfied the criteria of $x_{Bj} \geq 1.5$ and $Q^2 \geq 1.5$GeV$^2$. We first constructed the data matrix for central $Q^2$ and $x$ spanning the following values: $Q^2 = 1.75, 2.25, 2.75, 3.25$ and $x_{Bj} = 1.55, 1.65, 1.75$. For each pairs of $Q^2, x_{Bj}$ we averaged the $\sigma_{eA}$ cross sections and their errors with $\Delta Q^2 = \pm 0.25$ GeV$^2$ and $\Delta x_{Bj} = \pm 0.05$. Along with the averaged cross sections we estimated the average values of relevant kinematic variables, $\gamma$ for each bin according to:

$$\gamma = \frac{\sum_{i} \gamma_i \sigma_{eA}^{i} / \sum \sigma_{eA}^{i}}{\sigma_{eA}}$$

(5)

The $a_2$ is estimated for each $Q^2, x_{Bj}$ bin as:

$$a_2(A, y) = \frac{\sigma_{eA}(A, Q^2, \gamma)}{\sigma_{ed}(x_d, Q^2_{d}, \theta_{e,d})} \cdot R,$$

(6)

where $x_{A,d}$, $Q^2_{A(d)}$ and $\theta_{e,A(d)}$ are average values for nuclei and $d$ defined according to Ref. 10. The factor $R$ uses the theoretical calculation of $d(e,e')X$ reaction 16, 17 to correct for the misalignment of averaged $x, Q^2$ and $\theta_e$ for nuclei $A$ and $d$ in the following form:

$$R = \frac{\sigma_{ed}^{th}(Q^2_{d}, x_d, \theta_{e,d})}{\sigma_{ed}(Q^2_{A}, x_A, \theta_{e,A})}$$

(7)

where $\sigma^{th}$ is a model calculation of the cross sections.

The results of $a_2$ for the above mentioned nuclei are given in Fig. 1 and in Table I together with the previous Ref. 7, 9 and recent 10, 11, 12 estimates. Our results are somewhat lower than that of Refs. 7, 9 and together with Ref. 10, they agree with the earlier indication 7 that $a_2$ decreases for heaviest nuclei due to larger asymmetry $y$ which is in agreement with the observation of the suppression of $nn/\bar{pp}$ vs $pn$ SRCs.

**Fitting of $a_2(A, y)$:** We now use the extracted values of $a_2$ to fit them in the parametric form:

$$a_2(A, y) = a_2(A, 0) f(y).$$

(8)

The justification for the factorization of $A$ and $y$ dependence follows from the fact that the asymmetry dependence of $a_2$ is due to its proportionality to the number of the $pn$ pairs per nucleon. Thus one expects same function $f(y)$ for nuclei with different $A$.

First, we fit $a_2(A, 0)$s for symmetric nuclei. However we have only two data points for $a_2(A, 0)$: $^{4}$He and $^{12}$C. To be able to fit the $a_2(A, 0)$s for the range of $A \geq 12$ we use the approximation 5, 6, 11:

$$a_2(A, 0) = C \int \rho^2_{A}(r)d^3r$$

(9)
Extrapolation to Infinite and Superdense Nuclear Matter: The obtained fit in Eq. (5) allows us to estimate $a_2(A, y)$ for infinite nuclear matter since Eq. (5) converges at $A \to \infty$ and $f(y)$ is finite by definition. The estimate for the symmetric nuclear matter at saturation densities $\rho_0$ can be obtained using the relation between the nuclear radius and $A$, $R = r_0 \cdot A^{1/3}$, which yields

$$\langle \rho^2 \rangle_{INM} = \frac{1}{A} \int_0^{\rho^2_{A, sym}(r)} d^3r = \frac{4\pi}{3} \rho_0^2 r_0^3 \approx 1.4 \text{ fm}^{-3},$$

where we use $\rho_0 = 1.6 \text{ fm}^{-3}$ and $r_0 = 1.1 \text{ fm}$. From Eqs. (8) and (10) we obtain for symmetric nuclear matter at saturation density:

$$a_2(\rho_0, 0) \approx 7.03 \pm 0.41,$$

which is quantitatively in agreement with the $a_2$ estimated from the $y$ scaling analysis of the $A(e,e')X$ data extrapolated to infinite nuclear matter [20] which yields [21] $a_2 \approx 8.0 \pm 1.24$. Note that our estimate gives the lower limit for $a_2$ due to the neglect of the $nn$ and $pp$ SRCs.

Next we consider asymmetric nuclear matter. We combine Eqs. (12) and (11) into Eq. (8) to estimate $a_2(\rho, y)$ for given values of nuclear density and asymmetry $y$. As an example of the application of $a_2(\rho, y)$, we estimate the fraction $F_{p/n}$ of off-Fermi-shell nucleons in the $\beta$ equilibrium $e - p - n$ superdense asymmetric nuclear matter using the relation (see Eq. (2)):

$$F_{p/n}(A, y) = \frac{1}{2x_{p/n}} a_2(A, y) \int_{k_F}^{\infty} n_d(p) d^3p.$$

For the asymmetry $y$, in our estimates we use the threshold value of $x_p = \frac{1}{6}$ (corresponding to $y = \frac{2}{3}$) below of which the direct URCA processes:

$$n \to p + e^- + \nu_e, \quad p + e^- \to n + \nu_e$$

will stop in the standard model of superdense nuclear matter consisting of degenerate protons and neutrons [22].

Estimating the Fermi momenta of protons and neutrons in Eq. (13) with $k_{F,N} = (3\pi^2 n_N)^{1/3}$, we present the off-Fermi-shell fractions of protons and neutrons as a
function of nuclear density. The most interesting result of these estimates is that in equilibrium \( pn \) SRCs move the large fraction of protons above the Fermi-shell: at \( 3\rho_0 \) densities half of the protons will be off-Fermi-shell while at \( \rho \gtrapprox 4.5\rho_0 \) all the protons will populate the high momentum tail of the momentum distribution. The situation however is not as dramatic for neutrons, with only \( few\% \) of neutrons populating the high momentum part of the momentum distribution.

**Possible Implications for Nuclei and Neutron Stars:** Our main observation is that with an increase of nuclear asymmetry the lesser component become more energetic. This is confirmed \[13\] by direct calculation of the average kinetic energies of proton and neutron using realistic wave function of \(^3\)He, in which case one expects neutrons to be more energetic than protons: \( \langle T_n \rangle = 18.4 \text{ MeV} \) and \( \langle T_p \rangle = 13.7 \text{ MeV} \). For nuclei with large \( A(\gtrapprox 40) \) one expects protons to be more energetic than neutrons, with larger fraction of protons occupying high momentum tail of the momentum distributions. This may have several verifiable implications for large \( A \) nuclear phenomena\[13\].

Our observation may have more dramatic implications for the dynamics of neutron stars. Some of them are:

- **Cooling of a Neutron Star:** Large concentration of protons above the Fermi momentum will allow the condition for Direct URCA processes \( p_p + p_e \rightarrow p_n \)[22] to be satisfied even if \( x_p < 1/3 \). This will allow a situation in which intensive cooling of the neutron stars continues well beyond the critical point \( x_p = 1/3 \) (see also Ref.\[11\]).

- **Superfluidity of Protons:** Transition of protons to the high momentum tail will smear out the energy gap which will remove the superfluidity condition for the protons.

- **Protons in the Neutron Star Cores:** The concentration of protons in the high momentum tail will result in proton densities \( p_p \sim p_p^0 \gg k_F^2 p \). This will favor an equilibrium condition with "neutron skin" effect in which large concentration of protons populates the core rather than the crust of the neutron star. This and the proton superfluidity condition violation may provide different dynamical picture for generation of magnetic fields in the stars.

- **Isospin locking and the stiff equation of state of the neutron stars:** With an increase in density more and more protons move to the high momentum tail where they are in short range tensor correlations with neutrons. In this case one would expect that high density nuclear matter to be dominated by configurations with quantum numbers of tensor correlations \( (S = 1, I = 0) \). In such a scenario protons and neutrons at large densities will be locked in the NN iso-singlet state. This will double the threshold of inelastic excitation from \( NN \rightarrow N\Delta \) to \( NN \rightarrow \Delta\Delta(NN^*) \) transition thereby stiffening the equation of state which is favored by the recent large neutron star mass observation\[23\].

**Possible Universality of the Obtained Result:** Our observation is relevant to any asymmetric two-component Fermi system in which the interaction within each component is suppressed while the mutual interaction between two components is enhanced. It is interesting that the similar situation is realized for two-fermi-component ultra-cold atomic systems\[24\], but with the mutual s-state interaction\[27\]. One of the most intriguing aspects of such systems is that in the asymmetric limit they exhibit very rich phase structure with indication of the strong modification of the small component of the mixture\[25, 26\]. In this respect our case is similar to that of ultra-cold atomic systems with the difference that the interaction between components has a tensor nature.

**Limitations and Outlook:** Our analysis has several limitations: One is that we neglected the contributions from isotriplet \( 2N \) as well as \( 3N \) SRCs. Even though the statistical errors in the extraction of \( a_2 \) are small (Table I), an additional errors are accumulated due to the fitting procedure, especially for the asymmetry function \( f(y) \). We estimate the overall error in the extrapolation procedure at \( \sim 30\% \).

Finally, the procedure of extraction and fitting of \( a_2(\rho, y) \) can be significantly improved with the new high \( Q^2 \) and \( x_{Bj} > 1 \) experiments covering widest possible range of \( A \) and \( y \). The seminclusine \( A(e, e'NN)X \) will allow an inclusion into the analysis a contribution from \( pp \) and \( nn \) SRCs. Measurements at \( x_{Bj} > 2 \) domain will allow also to obtain similar estimates for \( 3N \) SRCs. Inclusion of all these effects into the analysis will further increase the magnitude of high momentum fraction of the protons. Thus our present results represent most probably the lower limit of the fraction of off-Fermi shell protons in high density nuclear matter.

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