Total-Amount Coordinated Finite-Time Control of Multi-Motors With Saturation Constraints

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ABSTRACT For the multi-motor traction finite-time control (FTC) system with input saturation constraints, considering parameter simplification of “adding a power integrator” (AAPI) and anti-windup (AW), a total-amount coordinated tracking control (TACTC) strategy based on nonsingular terminal sliding mode (NTSM) is proposed to ensure the global finite-time stability and effectively reduce energy consumption. First, a traction model of multiple permanent magnet synchronous motors (PMSMs) with uncertain parameter perturbations and external disturbances was established. Next, a finite-time auxiliary AW system was designed and the auxiliary state was fed back to TACTC. Then, the strong constraint of parameters of AAPI was relaxed, the setting rules of the corresponding parameters were given and the proof of finite-time Lyapunov stability was completed. Finally, the excellent performance of parameter simplification of AAPI and the controller was verified by the Matlab/Simuink simulation and RT-Lab semi-physical experiment of the multi-motor traction system, which shows that the control strategy can suppress the input saturation effect in the multi-motor traction system effectively.

INDEX TERMS Input saturation, finite-time control, adding a power integrator, total-amount coordinated.

I. INTRODUCTION Considering the development of locomotives with heavy-load and high-speed, the energy consumption of locomotive is increasing. Therefore, in the relevant studies on producer energy management [1]–[3], AW strategy is introduced and the control parameters are simplified to reduce the control input, thereby effectively reducing energy consumption and improving the economic benefits of the railway.

The common driving mode of electric locomotives is that multiple PMSMs jointly provide power [4]. The complex and changeable running environment of locomotives often causes the loss of traction performance of the motors. For example, when the rail surface changes from dry to wet or covered with snow, some wheels slip, resulting in motor idling and loss of traction especially in the case of ramp, which easily causes the backward failure [5]. The classical control refers to the multi-motor, which is regarded as a multi-agent system, and the consistency algorithm is introduced to realize the high-performance synchronization of individual states in the system (e.g., speed and location) [6], [7]. In recent years, the theory of total-amount consistency has been proposed in the multi-motor traction system to ensure the overall traction performance of the locomotive, prevent the motor from idling and energy consumption, and achieve the consistency between the sum of the output torques of all motors and the desired traction characteristic curve [8], [9]. However, most of the control rates proposed above can only be asymptotically stable, whereas FTC has fast convergence, high accuracy, and strong robustness [10], [11]. By introducing the terminal sliding mode to design the FTC, the dynamic response performance of the system can be effectively improved [12]. Furthermore, the combination of the terminal sliding mode technique with the AAPI ensures the continuity and the nonsingularity of the controller, and the finite-time (FT) upper bound can be obtained strictly, which improves the precision of the paper [13]. However, in terms of the complex multi-motor traction system, the strong constraints of the

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sliding mode switching gain and the AAPI parameters easily cause a control input saturation problem especially in the theory based on total-amount consistency, and the problem on input saturation of multi-motor is more prominent. Therefore, a TACTC based on the NTSM is proposed in consideration of AAPI parameter simplification and AW strategy to ensure the global FT stability.

The AAPI technique can be regarded as a development of the backstepping control, which is widely used in the design of high-order nonlinear system stability [14], [15]. However, the large use of inequality expansion and contraction in the process of proof causes the strong parameter constraint, which is also the key factor in the difficult application of the AAPI technique to engineering practice [16]. Among them, the combination of the symbolic and the barrier Lyapunov functions can effectively improve the AAPI technique [17], [18]. At the same time, the parameter of AAPI can be simplified by sacrificing the global FT stability [19]. However, the research on the parameter simplification of the AAPI remains lacking. Therefore, this paper relaxes the strong parameter constraint of AAPI, gives the rules of parameter setting, and realizes the global FT stability by using the FT Lyapunov stability theorem.

At the same time, in consideration of the parameter perturbation and the load torque disturbance of each motor in engineering, the traction torque control based on the sliding mode variable structure is constructed to improve the dynamic response performance of each motor. However, its large sliding mode switching gain and the parameter constraints in the AAPI may still lead to the control input saturation problem [20]. At present, studies on input saturation are extensive. The mathematical correlation function is used to deal with saturation [21]. A static or dynamic AW compensator is designed to weaken the saturation effect [22], [23]. The saturation constraint is transformed into the optimization problem with linear matrix inequality constraint [24]. However, in view of the complexity of the input saturation in the multi-motor TACTC, a FT convergent auxiliary AW system, which can improve the research on the relaxation of strong parameter constraints with the AAPI and effectively weaken the influence of input saturation on the traction performance of multiple motors, is further designed in this paper.

The basic idea of total-amount consistency was presented in [8], [9]. This paper is a great improvement over those papers: considering the AAPI parameter simplification and the AW strategy, a multi-motor coordinated control algorithm based on the NTSM is proposed to ensure the global FT stability of the system and weaken the influence of input saturation on the overall traction performance. The specific innovation points are as follows. (1) In the FTC combined with the AAPI technique, this paper relaxes the condition of strong parameter constraint and gives the basis for selecting the AAPI parameter. The global FT convergence is realized using the FT Lyapunov stability theorem. (2) A FT convergent auxiliary AW system, which improves the method of parameter simplification and provides a theoretical basis for engineering application, is designed to prevent the constraints of relevant control parameters that may still cause input saturation in the actual multi-motor TACTC.

The rest of this paper is structured as follows. In the second section, a traction model of the multiple PMSMs is established. In the third section, a TACTC strategy is designed by combining with the FT convergent auxiliary AW system. In the fourth section, the proof of the FT Lyapunov stability is completed using the AAPI technique. In the fifth section, the effectiveness of AAPI parameter simplification and the TACTC strategy is verified by simulation and experiment. The sixth section presents the conclusions.

II. MATHEMATICAL MODEL

In the electric locomotive with development of permanent magnet direct drive system, the mathematical model of PMSM is adopted in the multi-motor traction system [25], [26]. Considering the $j^{th}$ Motor as an example, the correlation equation in the $d-q$ axis reference frame is as follows.

\[ \begin{align*}
    u_{dj} &= R_{ij}i_{dj} + L_{ij}(di_{dj}/dt) - \omega_{j}L_{qj}i_{qj} \\
    u_{aq} &= R_{ij}i_{aq} + L_{aq}(di_{aq}/dt) - \omega_{j}(L_{qj}i_{dj} + \psi_{fj}) \\
    T_{ej} &= 1.5n_{pj}[\psi_{fj}i_{aq} - (L_{dj} - L_{aq})i_{aq}i_{dj}] \\
    J_{j}(d\Omega_{j}/dt) &= -R_{\Omega j}\Omega_{j} + T_{ej} - T_{Lj} \\
    \dot{\Omega}_{j} &= \omega_{j}/n_{pj}
\end{align*} \]

where $u_{dj}, u_{aq}$ are the $d$ and $q$ axis voltage, respectively; $i_{dj}, i_{aq}$ are the $d$ and $q$ axis currents, respectively; $L_{dj}, L_{aq}$ are the $d$ and $q$ axis inductances, respectively; $R_{ij}$ is stator resistance, $\omega_{j}$ is the electrical angular velocity, $\Omega_{j}$ is the mechanical angular velocity, $\psi_{fj}$ is permanent magnet flux linkage, $T_{ej}$ is the electromagnetic torque, $T_{Lj}$ is the load torque, $n_{pj}$ is the number of pole pairs, $J_{j}$ is the moment of inertia, $R_{\Omega j}$ is the friction coefficient.

This paper adopts the no-salient pole PMSM, i.e. $L_{dj} = L_{aq} = L_{j}$. At the same time, the $i_{dj} = 0$ vector control method is adopted to facilitate the control of multi-motor. The correlation equation is as follows.

\[ \begin{align*}
    \dot{L}_{j}(di_{aq}/dt) &= -R_{ij}i_{aq} - \psi_{fj}\omega_{j} + u_{aq} \\
    (J_{j}/n_{pj}) \cdot (d\omega_{j}/dt) &= (R_{\Omega j}/n_{pj})\omega_{j} + T_{ej} - T_{Lj} \\
    T_{ej} &= 1.5n_{pj}\psi_{fj}i_{aq}
\end{align*} \]

Let $\omega_{j} = x_{ij}, x_{2j} = \dot{x}_{ij}, T_{ej} = x_{3j}$. Given that the parameters of the motor are time-varying during operation (e.g., the resistance and inductance), parameter perturbation and load torque disturbance are uniformly attributed to an unknown compound disturbances. Then, the state equation can be expressed as follows.

\[ \dot{x}_{j} = \tilde{A}_{j}x_{j} + \tilde{B}_{j}u_{aq} + d_{j} \]
In this section, the FT convergent auxiliary AW system is designed, and the TACTC is transformed into the sliding mode tracking control. As a result, the FT-AWC is designed, and the basis of parameter setting is given.

The basic idea of auxiliary AW system was presented in [22], [23]. This paper is a good improvement over those papers: to improve the speed of AW, the FT convergent auxiliary AW system is designed for the \( j \)th motor as follows:

\[
\dot{x}_{aj} = \begin{cases}
  -A_{aj}x_{aj} - k_{aj}^{1/q}/q + \Delta u_{aj} \\
  -|[a_{1jq}^{-2}s^{2-1/q}b_{j}\Delta u_{aj}]| + 0.5\Delta u_{aj}^2/|x_{aj}|, & |x_{aj}| \geq \tau \\
  0, & |x_{aj}| < \tau
\end{cases}
\]

(5)

where \( x_{aj} \) refers to the auxiliary state; \( A_{aj} \) refers to the positive constant to be designed, \( A_{aj} - 0.5 - a_{1jq}^{-2}(0.5c_{2jq}/q) > 0; \) \( \tau \) is a small positive constant; and the constant \( k_{aj} > 0. \)

The deviation between the total torque of \( n \) motors and the desired traction characteristic curve is defined as follows.

\[
\sigma_2 = \sum_{j=1}^{n} (x_{3j}) - T^*
\]

(6)

Let \( \dot{\sigma}_1 = \sigma_2 \), the error equation can be obtained as follows.

\[
\dot{\sigma}_2 = \sum_{j=1}^{n} (-\tilde{a}_{ij}x_{1j} - \tilde{a}_{2j}x_{2j} + \tilde{b}_j u_{aj} + d_{2j}) - \dot{T}^*
\]

(7)

The NTSM surface is selected as follows.

\[
s = \sigma_1 + \sigma_2^q
\]

(8)

where \( \sigma_1 \) is the positive constant to be designed. \( 1 < q < 2. \)

Therefore, considering the FT auxiliary AW system, the TACTC based on NTSM combined with the AAPI technique is designed as follows.

\[
v_{qj} = \frac{1}{b_j} \left[ \tilde{a}_{ij}x_{1j} + \tilde{a}_{2j}x_{2j} - K_j sgn(s^{2-1/q}) + \frac{1}{n}T^* - c_{2jq}(s^{1/q} - x_{1jq}^{1/q}) - \frac{1}{n}a_{2jq}^{-2}s^{2-1/q} - \frac{1}{n}a_{2jq}^{-2}s^{2-1/q} - 1 \right]
\]

(9)

where AW coefficient \( c_{2jq} > 0 \) and parameter \( K_j \geq a_{1jq}^{-2}D_j. \)

**Theorem:** For the multi-motor traction system (3), the FT auxiliary AW system (5) is designed, and the TACTC (9) is designed based on NTSM combined with the AAPI technique. When the AAPI parameters meet \( \alpha_1 > 0, \alpha_2 > 0, \) and \( k > 0, \) the total output traction torque can track the desired traction characteristic curve \( T^* \) within \( t_s \leq 2q/[k_{aj}(q-1)] \cdot V(t_0)^{(q-1)/(2q)} \).

**Remark 1:** The classic designs \( \alpha_1 = \frac{1}{2}^{1-1/q} + l_2(1 + q)q^{-1}, \alpha_2 = \frac{1}{2}^{1-1/q}(1 + q)^{-1}[\frac{1}{2}^{1-1/q}\alpha_1(2 - q^{-1})(1 + q) + (2 - q^{-1})\alpha_1^{1/q} + 1] + l_1 \) are required for the AAPI parameters \( \alpha_1 \) and \( \alpha_2 \) in the FTC [27]. However, this paper has relaxed this constraint. Only the AAPI parameters

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**FIGURE 1.** The frame diagram of the TAC-FT-AWC system.

In the multi-motor traction system of locomotive, the frame diagram of the TAC-FT-AWC system is constructed, as shown in Fig. 1. The frame diagram comprises the control module, the drive module and the motor module. The main aim of this study was to design the TAC-FT-AWC strategy that combines the parameter simplification of AAPI and FT auxiliary AW system, as shown in the control module.
α₁ > 0 and α₂ > 0 are required in Eq. (9), and the advantage of global FT stability remains.

Remark 2: To suppress chattering phenomenon caused by the discontinuity of the sign function, a continuous function \( F = s^{2-1/q}/(|s^{2-1/q}| + \sigma) \) is used instead of the sign function, where \( \sigma \) is a small positive constant.

IV. STABILITY ANALYSIS

In this section, the relevant lemmas needed for the stability proof are firstly given in this paper. The FT Lyapunov stability theorem is the basis to complete the stability proof, and the FT upper bound is obtained.

A. THEORETICAL BASIS

Lemma 1 [28]: For the following system \( \dot{x} = f(x), f(0) = 0, \) and \( x \in \mathbb{R}^n \). If there is continuous and positive definite function \( V(x) \) such that

\[
\dot{V}(x) + \alpha V(x)^{p} \leq 0, \quad x \in \mathbb{R}^n
\]

where if \( \alpha > 0 \) and \( 0 < p < 1 \), the system converges in finite time \( t_s \), and the FT upper bound meets

\[
t_s \leq \frac{1}{\alpha(1 - p)} V(x_0)^{1-p}
\]

Lemma 2 [29]: If \( 0 < q < 1 \) and satisfying the division of two coprime odd numbers, the following inequality is satisfied for any \( x, y \in \mathbb{R} \):

\[
|x^q - y^q| \leq 2^{-q} |x - y|^q
\]

Lemma 3 [30]: If \( c, d, \) and \( \lambda \) are positive real numbers, the following inequality is satisfied for any \( x, y \in \mathbb{R} \):

\[
|x|^c |y|^d \leq \frac{c}{c + d} \lambda |x|^{c+d} + \frac{d}{c + d} \lambda^{-c/d} |y|^{c+d}
\]

Lemma 4 [29]: If \( 0 < p < 1 \), the following inequality is satisfied for any \( x_j \in \mathbb{R}, j = 1, 2, \ldots, n \):

\[
|x_1 + \ldots + x_n|^p \leq |x_1|^p + \ldots + |x_n|^p
\]

Lemma 5 [30]: If \( a \) and \( b \) are non-negative real numbers, the following inequality is satisfied for any \( x, y \in \mathbb{R} \):

\[
|x|^a |y|^b \leq \frac{a}{a + b} |x|^{a+b} + \frac{b}{a + b} |y|^{a+b}
\]

B. STABILITY PROOF

In this section, the theorem of the Lyapunov stability is adopted to prove the theorem of this paper.

Proof: The positive definite Lyapunov function is constructed as follows:

\[
V = V_1 + V_2 + V_3
\]

where

\[
V_1 = \frac{1}{2} \alpha_1^2, \quad V_2 = \int_0^{\alpha_2} (\tau q - \alpha_2^q)^{2-1} \sigma d \tau, \quad V_3 = \sum_{j=1}^{n} \left( \frac{1}{2} \lambda_j^2 \right).
\]

The virtual control law is designed to be \( \alpha_2^q = -\alpha_1^{-1/q} |\sigma|^{1/q} \). After the \( V_2 \) is simplified on the basis of the Lemma 2, \( V_2 \leq 2^{-1/q} \alpha_1^2 |\sigma|^2 \) can be obtained.

Step 1: Taking the derivative of \( V_1 \),

\[
\dot{V}_1 = \sigma_1 \dot{\sigma}_1 = \sigma_1 \sigma_2^q + \sigma_1 (\sigma_2 - \sigma_2^q) \leq |\sigma_1| |\sigma_2 - \sigma_2^q| - \alpha_1^{-1/q} |\sigma_1|^{1+1/q}
\]

The following can be obtained to simplify on the basis of Lemmas 2 and 3:

\[
\dot{V}_1 \leq -\alpha_1^{-1/q}(1 - \gamma_1) |\sigma_1|^{1+1/q} + b_1 |\sigma|^{1+1/q}
\]

where \( \lambda_1 = \frac{2^{-1/q} \gamma_1 (1 + q)}{\gamma_1} > 0 \), \( b_1 = (2^{-1/q} \alpha_1^{-1/q} \lambda_1) / (1 + q) \).

Step 2: Taking the derivative of \( V_2 \),

\[
\dot{V}_2 = \frac{\partial V_2}{\partial \sigma_1} \dot{\sigma}_1 + \frac{\partial V_2}{\partial \sigma_2} \dot{\sigma}_2
\]

where \( s = \alpha_1 (\sigma_2 - \sigma_2^q) \) and \( \sigma_1 = -\alpha_1 \sigma_2^q \) are plugged in the first item on the right of Eq. (19). The following can be obtained through the simplification on the basis of Lemmas 2 and 3:

\[
\frac{\partial V_2}{\partial \sigma_1} \dot{\sigma}_1 \leq \alpha_1^{-1/q} \gamma_2 |\sigma_1|^{1+1/q} + 2^{1-1/q} (2-1/q) \alpha_1^{-2} b_2 |\sigma|^{1+1/q}
\]

Then, \( K_2 \geq (1/4) K_2 \) and Eq. (9) are plugged in the second item on the right of Eq. (19). The following is obtained on the basis of Lemmas 3, 4, and 5:

\[
\frac{\partial V_2}{\partial \sigma_2} \dot{\sigma}_2 = (\sigma_2^q - \sigma_2^q) |\sigma_2|^{1+1/q} - \alpha_1^{-1/q} \gamma_3 |\sigma_1|^{1+1/q} + b_3 |\sigma|^{1+1/q} - \alpha_1^{-1/q} \sigma_2 |\sigma|^{1+1/q}
\]

where \( \lambda_3 = \frac{2^{-1/q} \gamma_3 (1 + q)}{(2 - q)} > 0 \), \( b_3 = \alpha_1^{-1/q - 2} q^{-1} \lambda_3^{-1/2} q^{-1}(2 - q) / (1 + q) \).

Eq. (20) is combined with Eq. (21), then:

\[
\dot{V}_2 \leq \left[ 2^{-1/q} (2 - 1/q) \alpha_1^{-2} b_2 + b_3 \right] |\sigma|^{1+1/q}
\]

\[
+ \alpha_1^{-1/q} \gamma_3 |\sigma_1|^{1+1/q} - \alpha_1^{-1/q} \sigma_2 |\sigma|^{1+1/q}
\]

\[
+ \sum_{j=1}^{n} \left[ \alpha_1^{-1/q} (2c_2/2q) |x_j|^2 - \alpha_1^{-1/q} (2c_2/2q) s^2 \right]
\]

Step 3: Taking the derivative of \( V_3 \).

The inequality \( x_j \Delta u_j \leq 0, x_j^2 + 0.5 \Delta u_j^2 \) is plugged in the auxiliary AW system (5), and the following can be obtained:

\[
\dot{V}_3 \leq \sum_{j=1}^{n} \left[ -(A_{ij} - 0.5) x_j^2 + k_4 x_j^{1+q} \right.
\]

\[
\left. - \alpha_1^{-1/q} 2^{-1/q} b_j \Delta u_j \right]
\]
To sum up, the derivative of $V$ is obtained:

$$
\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3
$$

$$
\leq -k_1 |\sigma|^{1+1/q} - k |s|^{1+1/q} - \sum_{j=1}^{n} \left(k_3 x_{aj}^{1+1/q} + (A_{aj} - 0.5) \right) ^{1/q-2} (0.5 c_{j}^q/q) |x_{aj}|^2
$$

(24)

where $k = k_2 - k_3$, $k_1 = k_4 + 1/q - 2 (1 - \gamma_1, \gamma_2, \gamma_3, k_2 = \alpha \gamma_1 - \alpha_2$, and $k_3 = b_1 + 2 (1 - 1/q) \gamma_2 - 2 b_2 + b_3$.

If parameters satisfy $A_{aj} - 0.5 - \alpha_1^{1/q-2} (0.5 c_{j}^q/q) > 0, k_1 > 0, k_2 > k_3, k_4 > 0$, and $\gamma_1 + \gamma_2 + \gamma_3 < 1$, then:

$$
\dot{V} \leq -k_1 |\sigma|^{1+1/q} - k |s|^{1+1/q} - \sum_{j=1}^{n} \left[k_4 x_{aj}^{1+1/q} \right] < 0
$$

(25)

After simplifying and analyzing $V_2$ on the basis of Lemma 2, $|s| \geq 2^{(1-q)/(2q)} \alpha_1 V_1^{1/2}$ is obtained. At the same time, considering $|\sigma| = 2^{1/2} V_1^{1/2}$ and $\sum_{j=1}^{n} |x_{aj}| = 2^{1/2} V_3^{1/2}$, the following can be obtained:

$$
\dot{V} \leq -k_5 V_1^{(1+q)/(2q)} - k_6 V_2^{(1+q)/(2q)} - k_7 V_3^{(1+q)/(2q)}
$$

$$
\leq -k_8 V^{(1+q)/(2q)}
$$

(26)

where $k_5 = 2^{(q+1)/(2q)} k_1$, $k_6 = 2^{(1-q)/(2q)} k_2$, $k_7 = 2^{(q+1)/(2q)} k_4$, and $k_8 = \min \{k_5, k_6, k_7\}$.

Eq. (26) satisfies Lemma 1. Thus, the FT upper bound can be obtained.

$$
t_s \leq 2 q / [k_8(q - 1)] \cdot V(x_0)^{(q-1)/(2q)}
$$

(27)

Therefore, $\sigma_1, s$, and $x_{0j}$ can converge to 0 in a FT. The $\sigma_2$ can converge to 0 in a FT due to $s = \sigma_1 + \alpha_1 \sigma_2^q$. The theory of total-amount consistency is realized.

V. THE SIMULATION AND EXPERIMENTAL ANALYSIS
A. THE SIMULATION ANALYSIS
A multi-motor traction system composed of four PMSMs with different parameters is used as the simulation object to verify the effect of relaxing the strong constraint of the AAPI parameters and the superiority of the TAC-FT-AWC. The specific parameters of each motor are shown in Table 1. To facilitate comparison with the previous results of our team, the control input saturation limit and the initial torque value of each motor are set to $\pm 220$ and 0.25, respectively.

In consideration of the uncertainty of unknown compound disturbances in practice, four different disturbance signals (i.e., fast-varying disturbance, slowly-varying disturbance, high-frequency noise, and uniform noise) are applied to four motors. At the same time, the desired traction characteristic curve is composed of piece-wise functions, as shown in Eq. (28): simulating the acceleration phase of motor in $0 \rightarrow 5s$, the constant-speed operation phase of motor in $5 \rightarrow 10s$, and the deceleration phase of motor in $10 \rightarrow 15s$.

$$
T^* = \begin{cases} 
0.2r, & t < 5 \\
1, & 5 \leq t \leq 10 \\
0.2r + 3, & 10 < t < 15 
\end{cases}
$$

(28)

1) SIMPLIFICATION EFFECT OF AAPI PARAMETERS
In this section, the changes in the control input curve of each motor is analyzed through different AAPI parameters $\alpha_1$ and $\alpha_2$ to verify the effectiveness of the simplified AAPI parameters in reducing the control input saturation.

![FIGURE 2. The control input curve with $\alpha_1 = 10, \alpha_2 = 100$.](image-url)

Fig. 2-Fig. 4 show that the AAPI parameter constraint seriously influences the change in the control input curve. The absolute values of the control input peak values were corresponded by different AAPI parameters (Table 2) to facilitate the analysis.

The red font in Table 2 indicates that the control output exceeds the saturation limit 220. Table 2 shows that the traditional AAPI method has strong constraint on parameters $\alpha_1$ and $\alpha_2$. Columns 1 and 2) and easily causes a saturation

| TABLE 1. The nominal value of the PMSM parameters. |
|-----------------------------------------------|
| Parameters | Motor1 | Motor2 | Motor3 | Motor4 |
|-------------|--------|--------|--------|--------|
| $R_s$ (Ω)   | 2.873  | 2.856  | 2.867  | 2.861  |
| $L$ (mH)    | 8.7    | 8.6    | 8.9    | 8.8    |
| $n_p$       | 2      | 2      | 2      | 2      |
| $J$ (kg-m$^2$) | 4.51*10$^4$ | 4.47*10$^4$ | 4.51*10$^4$ | 4.49*10$^4$ |
| $R_t$ (Ω)   | 4.831*10$^3$ | 4.846*10$^3$ | 4.827*10$^3$ | 4.838*10$^3$ |
| $N_m$ (s$^{-1}$) | 1.017 | 1.017 | 1.017 | 1.017 |
problem in the multi-motor system. In this paper, the constraint on the AAPI parameter is relaxed. When \( \alpha_1 = 1 \), \( \alpha_2 > 5.35 \) is obtained from parameter setting basis \( k > 0 \), the FT stability can be guaranteed, as shown in Column 3 in Table 2. This method can evidently reduce the saturation problem in the multi-motor. However, a small overrun may occur due to the parameter setting basis. Therefore, the effect of input saturation can be effectively suppressed by combining the auxiliary AW system.

### Table 2. The control input peak with different \( \alpha_1, \alpha_2 \).

| Parameters | The absolute values of the control input peak values |
|------------|-----------------------------------------------------|
| \( \alpha_1 \) | \( \alpha_2 \) | Motor1 | Motor2 | Motor3 | Motor4 |
| 10 | 100 | 455 | 78 | 725 | 376 |
| 5 | 50 | 120 | 180 | 308 | 243 |
| 1 | 6 | 231 | 217 | 191 | 205 |

2) SIMULATION OF THE ANTI-WINDUP CONTROL

In this paper, based on the previous research of our team, the superiority of the proposed control strategy is shown through the comparative simulation of no AW, asymptotically stable AW, and FT-stable AW in this paper. At the same time, the AAPI parameters in the TAC-FT-AWC are set on the basis of \( \alpha_1 = 1 \) and \( \alpha_2 = 6 \), which further reflects the superiority of this paper to solve the input saturation problem from two aspects. Fig. 5-Fig. 7 show the tracking curve and error of no AW, asymptotically stable AW, and FT-AW, respectively.

From the analysis of the above three groups of comparison curves, when the motor input saturation occurs in the TACTC, the no AW strategy in Fig. 5 causes about 5 times of fluctuation, which seriously affects the tracking performance. The asymptotically stable AW strategy in Fig. 6 causes about 0.2 times of fluctuation. Fig. 7 shows that under the control strategy of this paper, the fluctuation caused by the input saturation is about half of 1%, which is almost negligible.
Therefore, the TAC-FT-AWC proposed in this paper can effectively suppress the effect of the input saturation on the overall traction performance. At the same time, the AWC parameters $A_{aj} = 50$ and $c_{2j} = 100$ in Fig. 7 are more practical compared with those in the AWC parameters $A_{aj} = 1000$ and $c_{2j} = 1900$ in Fig. 6. Finally, according to the control input curve after the FT-AWC in Fig. 8, each motor can be effectively controlled within the input limit of 220. Careful analysis shows that after Motor 1 is saturated in 11.22s, Motor 2 is saturated again after about 11.69s under the action of the auxiliary AW system and the TACTC. Therefore, the complexity of the input saturation is reflected in the multi-motor TACTC. The control strategy proposed in this paper can effectively suppress the effect of the multi-motor input saturation on the overall traction performance.

**B. THE EXPERIMENTAL ANALYSIS**

The RT-Lab semi-physical experiment platform is used to verify the effectiveness of the control strategy of this paper and reflect the actual engineering environment, as shown in Fig. 9, and the configuration of RT-Lab system is shown in Fig. 10. The RT-Lab experimental
platform is composed of TMS320F2812 digital signal processor (DSP), RT-Lab OP5600 simulation motor, personal computer (PC), and oscilloscope, etc. During the experiment, the experimental parameters and conditions are the same as the simulation. The code of the TAC-FT-AWC is downloaded to the DSP using a PC. Then, the traction model is compiled into OP5600, and the results are obtained. The experimental results are shown in Fig. 11 and Fig.12.

Fig. 11 is shown the experimental results of total-amount coordinated finite-time anti-windup control strategy. Fig. 11 (a) is the tracking effect curve. Fig. 11 (b) is the tracking error curve. As shown in the Fig. 11, half of 1% of the fluctuation caused by input saturation has almost no effect on the overall traction performance, which is consistent with the Matlab/simulink simulation results. The control input curve of Fig. 12 shows that the control inputs of each motor are controlled below the input limit of 220 effectively. To sum up, Fig. 11 and Fig. 12 show that the experimental results are consistent with the simulation results in Simulink.

VI. CONCLUSION
In this paper, facing the traction system of electric locomotive, the multi-motor coordinated control is extended from the
individual state consistency to the total traction torque consistency, which ensures the overall traction performance of the locomotive in the actual operation environment. Against the seriousness and the complexity of the input saturation problem in the multi-motor TAC-FTC, the AAPI parameter simplification and AW are considered. The TAC-FTC is designed on the basis of the NTSM, which achieves the global FT stability and suppresses the effect of the input saturation on the overall traction performance. Based on the simulation and experimental analysis, the effectiveness of the simplified AAPI parameters in reducing the control input saturation is verified. The superiority of the TACTC strategy based on the FT auxiliary JW system is verified through comparative simulation, which provides a theoretical basis for solving the control input saturation problem easily observed in practical engineering. In the future, energy minimization with an optimal dynamic adjustment mode in the multi-motors traction torque coordination will be considered.

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