Parametric uncertainty quantification in natural frequency of sandwich plates using polynomial neural network

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Abstract. In this paper, uncertainty quantification in natural frequencies for laminated soft core sandwich plates is presented by employing finite element (FE) coupled polynomial neural network (PNN) approach. The computational efficiency and accuracy is achieved by using PNN as surrogate model. Latin hypercube sampling method is employed for training of data in PNN model. The stochastic first three natural frequencies of sandwich plates are studied for individual variation in input parameters. The stochasticity in individual input parameters are considered in order to assess their influence on global response of the structure. The algorithm discussed in this article is observed to be converging with the previously published literature (for deterministic case) and validated with full scale Monte Carlo simulation (MCS) i.e. original finite element approach (for stochastic case). The computational time and cost reduced significantly by employing the present surrogate based FE approach compared to that of conventional Monte Carlo simulation approach.

Keywords: Sandwich plate, Finite element, Monte Carlo simulation, Polynomial neural network, Uncertainty quantification.

1. Introduction

In present scenario, speed and efficiency are the most important parameter for any system (either machine or structure). It results in continuously increasing demand of better material. Scientists and researchers are continuously trying for a better and efficient material according to its applications such as aerospace, automobile, submarines and other task defined functions. Therefore, the material developed needs to be highly task specific and withstand in any service condition. That is why lightweight robust structures are highly preferred, predominantly in structural applications where high amount of vibration is suspected to produce due to dynamic loads. Sandwich plates are one of them which is widely used in design and construction of aerospace structures. A sandwich plate consists of face-sheets (both upper and lower) separated by core in between them [1]. Face-sheets consist of laminate of different orientation stacked one over another to get specific property. All the laminates and core are bonded together to serve the purpose. It also shows superior properties like weight sensitiveness, good resistance for environmental degradation, economic, ecofriendly. All these merits make sandwich plate suitable for enormous structural application such as aircrafts, submarine, auto-industries, civil construction, orthopedic devices [2-3]. Sandwich plates are more widely used in cost sensitive than that of weight sensitive applications with respect to monolithic plates. The cost effective sandwich plates requires sandwich core of low cost material which exhibits
better weight sensitiveness as well. The development and automation in production processes make possible the production of low cost sandwich plates. In general, a sandwich plate is made up of facesheet at outer layers and the core inserted in between them wherein the facesheets are fabricated with stiff and strong materials but of relatively lower thickness than that of light and thicker core. It is not necessary for the core as well as facesheet material of a sandwich plate to be homogeneous. In a sandwich plate, a core is having less strength and more energy absorption capability while face sheet is having more strength and ductility. Recently, interest in the sandwich plate was observed to be considered on the core of the lattice structure. Stretching causes the deformation in members of a truss, presented in lattice structured core [4]. Due to its inherent material anisotropy and manufacturing inaccuracies, such structures are always subjected to spatial variabilities. In real-life applications, it is difficult to get the accurate design specifications of complex structures like sandwich plate with laminated facesheet because of the implicit anisotropy and complexity during manufacturing process leading to inexorable uncertainties. Due to material and geometric uncertainties, overall performance of sandwich plates significantly deviate from deterministic results. Here, it is aimed to address the effect of source-uncertainty on the free vibration behaviour of sandwich plates. So, it is required to understand the fundamental principles of stochastic mechanics. The variability in material and geometric properties affect the stiffness of the plate leading to deviation in the performance from deterministic to probabilistic [5]. Thus, it is imperative to consider such variations during design and analysis to ensure better safety, reliability and serviceability of the structures. The bodies having finite mass and elasticity is prone to vibration. The oscillatory motion of the system about an equilibrium point is named as vibration. Vibration can be categorised into linear and nonlinear, damped and undamped, forced and free vibration. When the mechanical system is subjected to external force, it is named as forced vibration, while in case of free vibration only initial displacement is given to the system. In free vibration, no external force is applied to the system after the initial disturbance. The resonance is the phenomenon in which the natural frequency of the system coincides with the forcing frequency leading to catastrophic failure of the structure. Hence, it is required to avoid the specified frequency range for the safe design of the mechanical system and to know that range it is necessary to determine the natural frequencies of the system. The accurate material and geometrical properties are very difficult to obtain due to the unavoidable source-uncertainties. Hence, it becomes mandatory to consider the variabilities in material and geometric properties for safe design of the structure [6-7]. Stochastic analysis in engineering fields attained global attentions for the researchers. In stochastic analysis, the main focus is drawn to determine the variability in responses due to variabilities in material properties, geometric properties, operational conditions and environment variations. Uncertainties or variabilities are associated with the every phase of the modelling and simulation. There are two types of uncertainty namely aleatory uncertainty and epistemic uncertainty. The aleatory uncertainty is random in nature and it cannot be reduced. The range or value of aleatory uncertainty is known, but the exact value is varying according to the time and unit. Epistemic uncertainties occur in the system due to lack of knowledge or information of the modelling [8]. Sandwich plates are widely used in design and construction of Aerospace craft. In such application these materials are subjected to wide environmental changes such as pressure, temperature, density, humidity. This inevitable change in surrounding affects the vibrational response of the structure. Therefore it is essential to include the actual operating condition in order to get the changes in vibration characteristic of sandwich plates. The vibration response of the Aerospace craft is usually carried out in atmospheric condition rather than actual unevenly varying condition for the cause of convenience. Thus, it is essential to consider the material and geometric uncertainty in order to accommodate above mentioned environmental changes as well as other inaccuracies occurring during design and fabrication of the sandwich plate. In most of the scientific literatures, researchers have concentrated on deterministic natural frequency analysis [9-13], however some researchers [14-23] have considered the stochastic behaviour in their study. No study has been carried out for uncertainty quantification of natural frequency for laminated sandwich plate by employing FE coupled PNN surrogate model approach.
2. Mathematical Formulation

The strain in structure occurs as per following strain-displacement relation [10]

\[
\{\varepsilon(\sigma)\} = \{\nabla(\sigma)\} = \{\frac{\partial U(\sigma)}{\partial x} \frac{\partial V(\sigma)}{\partial y} \frac{\partial W(\sigma)}{\partial z} + \frac{\partial U(\sigma)}{\partial y} \frac{\partial V(\sigma)}{\partial z} + \frac{\partial U(\sigma)}{\partial z} \frac{\partial V(\sigma)}{\partial x} + \frac{\partial W(\sigma)}{\partial x} \frac{\partial V(\sigma)}{\partial y} + \frac{\partial W(\sigma)}{\partial y} \frac{\partial V(\sigma)}{\partial x}\} = \{\varepsilon(\sigma)\}
\]

(1)

i.e. \(\{\varepsilon(\sigma)\} = [H(\sigma)]\{\varepsilon(\sigma)\}\). where, \([H]\) represents the unit step function. The displacement vector in generalized form may be written as

\[
\{\varepsilon(\sigma)\} = \sum_{i=1}^{N} N_i(\sigma) S_i(\sigma)
\]

(2)

where, \(s = \{u_0 v_0 w_0 \partial_x \partial_y \partial_z u_a v_a w_a u_b v_b w_b\}^T\), and \(N\) represents the shape function. Equation (1) can be used to write strain vector as

\[
\{\varepsilon(\sigma)\} = \{A(\sigma)\}\{S(\sigma)\}
\]

(3)

where, \([A]\) is strain-displacement matrix. As per Hamilton’s principle, the dynamic equilibrium equation may be written as

\[
\{R(\sigma)\}\{\ddot{S}\} = \lambda^2\{M(\sigma)\}\{\dddot{S}\}
\]

(4)

where, \([R(\sigma)\] is the stochastic natural frequency and \([M(\sigma)\] is global mass matrix which can be expressed as

\[
[M(\sigma)] = \sum_{i=1}^{\infty} \int \int \rho_i(\sigma) [N]^T[J]^T[N]dxdy + \int \int [N]^T[K(\sigma)][N]dxdy
\]

(5)

where, \(\rho_i(\sigma)\) represents the stochastic mass density. The stiffness matrix \([K(\sigma)\] may be written as

\[
[K(\sigma)] = \sum_{i=1}^{\infty} \int \int \rho_i(\sigma) [J]^T[J]dxdy
\]

(6)

2.1 Polynomial neural network model

An input-output dataset in optimal PNN structure [24] can be expressed as

\[(X_i, Y_i) = (x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in}, y_i)\]

(7)

where, \(i = 1, 2, 3, \ldots, n\). Polynomial regression equation is determined for each pair of input variable \(x_i\) and \(x_j\) model output \(Y\)

\[Y = A + B_i x + C_{ij} x_i x_j + D_{ik} x^2_i + E_{kj} x^2_j + F_{ikj} x_i x_j^2\]

(8)

where, \(i, j, k = 1, 2, 3, \ldots, n\) and co-efficient of polynomial equation is represented by \(A, B, C, D, E, F\). The estimated output \(\hat{Y}\) is determined as

\[\hat{Y} = A_0 + \sum_{i=1}^{n} B_i x_i + \sum_{i=1}^{n} C_{ij} x_i x_j + \sum_{i=1}^{n} D_{ik} x_i^2 + \sum_{j=1}^{n} F_{ij} x_j^2 + \sum_{i=1}^{n} E_{ikj} x_i x_j^2\]

(9)

where, \(i, j, k = 1, 2, 3, \ldots, n\). \(X(x_{1i}, x_{2i}, \ldots, x_{ni}) = \) input variable vector

\[P(A_0, B_i, C_{ij}, D_{ik}, E_{ij}, F_{ikj}, \ldots) = \) vector of coefficients. The basic architecture of PNN is represented in figure 1 while the methodology of stochastic natural frequency analysis of sandwich plate by using surrogate model is presented in figure 2.

3. Results and Discussion
In this study, source uncertainty is quantified for natural frequencies analysis of simply supported skewed (30°) sandwich plate considering individual variation of input parameters. The dimensions of the sandwich plate under consideration are as: length = 100 mm, width = 100 mm and thickness = 10 mm. The core and face-sheet thickness is considered as 0.9 cm and 0.1 cm, respectively. Both upper and lower face-sheet is having 8 laminate of 90°/0°/90°/0°/90°/0°/90°/0° orientation angle. Stochasticity in skew angle is taken as Δ =5% whereas Δ =10% is taken for other parameters, unless otherwise mentioned. The material properties considered in this analysis are:

![Figure 1. Taxonomy for architecture of PNN](image1)

![Figure 2. Flow diagram of stochastic free vibration analysis by employing PNN Model](image2)
(a) For core:
\[ E_1 = E_2 = E_3 = 0.5\text{GPa}, \quad G_{12} = G_{13} = 0.4\text{GPa}, \quad G_{23} = 0.2\text{GPa}, \quad \nu_{12} = \nu_{13} = \nu_{23} = \nu_{32} = 0.27, \quad \nu_{21} = \nu_{31} = 0.006, \quad \rho = 1000\text{ kg/m}^3 \text{ and} \]
(b) For face-sheet:
\[ E_1 = 38.6\text{GPa}, \quad E_2 = E_3 = 8.27\text{GPa}, \quad G_{12} = G_{13} = 4.14\text{GPa}, \quad G_{23} = 1.656\text{GPa}, \quad \nu_{12} = \nu_{13} = \nu_{23} = \nu_{32} = 0.26, \quad \nu_{21} = \nu_{31} = 0.006, \quad \rho = 2600\text{ kg/m}^3. \]

3.1 Validation

The deterministic FE code is validated with the published literature [7,10] and it is observed to be in good agreement as presented in Table 1.

Table 1 Non-dimensional deterministic natural frequencies of a four-layered (90°/0°/90°/0°) anti-symmetric composite sandwich plate

| Skew angle | Mode | Present analysis | Kulkarni and Kapuria [10] | Wang et al. [7] |
|------------|------|------------------|---------------------------|----------------|
| 30°        | 1    | 1.8889           | 1.9209                    | 1.9410         |
|            | 2    | 3.4827           | 3.5353                    | 2.9063         |

Two types of validation are required in surrogate assisted probabilistic analysis. First one is FE code validation and second one is surrogate model validation. FE code is already validated and the deterministic result is presented in table 1, whereas PDF and scatter plot is portrayed in figure 3 for surrogate model validation. The least deviation of PDF plot for MCS and PNN validates the surrogate model. For further validation of the surrogate model, scatter plot is shown. Least deviation of scatter plot from diagonal line indicates the predictability and applicability of PNN surrogate model.

3.2 Numerical results

Uncertainty in first three natural frequencies of sandwich plate is quantified and presented in figure 4 to figure 7. The different degree of stochasticity in individual parameter is considered to get the global response of the structure due to local source uncertainties. In Figure 4, the natural frequency response of sandwich plate having variation in only ply-orientation angle with different degree of stochasticity is depicted. The mean natural frequencies for first and second mode are almost same with insignificant variation in response bound whereas uncommon behaviour is observed for third mode. In third natural frequency mean frequency as well as response bound increases with increase in stochasticity. Figure 5 shows the common behaviour of response bound for stochastic natural frequencies, considering variation in only skew angle. The response bound increases with increase in degree of stochasticity with common mean value.
Figure 3. (a) Probability density function (PDF) and (b) Scatter plot for combined variation of all input parameters.
Figure 4. Stochastic natural frequencies (rad/sec) because of variation in ply-orientation angle with different degree of stochasticity.

(a) Fundamental natural frequency

(b) Second natural frequency

(c) Third natural frequency
4. Conclusion

The novelty of this paper lies in uncertainty quantification of natural frequencies by employing PNN approach. PNN is employed in conjunction with conventional FE based Monte Carlo Simulation (MCS) in order to get benefitted from the capability of MCS (probabilistic characterization) and diminishing the lacuna of computational expensiveness. The present algorithm is observed as computationally efficient with adequate accuracy. Here, layer wise bottom up stochastic propagation approach is employed for natural frequency analysis, considering individual variation of random input parameters. The hybrid FE-PNN approach presented in this study is comprehensive in nature i.e. it can be employed to any complex structure. This study can be further extended by incorporating the noise in the system.
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