Constraining modified gravity theories with GSM

Camilo Santa
Instituto de Fisica, Universidad de Antioquia, A.A.1226, Medellin, Colombia

Antonio Enea Romano
Theoretical Physics Department, CERN, CH-1211 Geneva 23, Switzerland
ICRANet, Piazza della Repubblica 10, I-65122 Pescara, Italy
Instituto de Fisica, Universidad de Antioquia, A.A.1226, Medellin, Colombia
(Dated: May 21, 2019)

The measurement of the size of gravitationally bounded structures is an important test of gravity theories. For a given radius different theories can in fact predict a different gravitational stability mass (GSM) necessary to ensure the stability of the structure in presence of dark energy. We compute the GSM of gravitationally bounded structures as a function of the radius for different scalar-tensor theories, including $f(R)$ and generalized Brans-Dicke, and compare the theoretical predictions to observational data.

The results of the analysis show that modified gravity theories (MGT) are compatible with observational data, and in some cases fit the data better than general relativity (GR), but the latter is not in strong tension with the observations. The data presently available do not give a conclusive evidence of the need of a modification of GR, but marginally favor MGT. Future data from galaxy surveys such as the Euclid mission could be important to get stronger constraints.

**Introduction**

Modified gravity theories (MGT) have been extensively investigated as possible solutions of some of the unsolved puzzles of the observed Universe, such as the nature of dark energy or dark matter, and can also provide models of cosmic inflation in very good agreement with observations. It is thereof important to set constraints on MGT using different types of observations, and one important test is provided by the stability of cosmic structure, in particular the turn around radius, i.e. the maximum size of a spherically symmetric gravitationally bounded object in presence of dark energy. The effects of the modification of gravity were considered previously in the case of the Brans-Dicke theory and some classes of Galileon theories, while here we consider a wider class of scalar-tensor theories, including among others $f(R)$, Generalized Brans-Dicke and quintessence theories. For convenience in the comparison with observational data, in this letter we compute closely related quantity, the gravitational stability mass (GSM) necessary to ensure the stability of a gravitationally bounded structure of given radius.

The first calculations of the turn around radius were based on the use of static coordinates, which can be related to cosmological perturbation with respect to the the Friedmann metric via an appropriate background coordinate transformation, allowing to establish a gauge invariant definition of the turn around radius. Here we show that the use of cosmological perturbations theory is more convenient and allows to find general theoretical predictions which can be applied to a wide class of scalar-tensor theory.

The theoretical predictions are compared to observations, finding that MGT are in better agreement with observations for few data points, but without a strong evidence of the incompatibility of GR, which has a mild $2\sigma$ tension with only one data point.

Note that the gravitational stability mass only allows to set lower or upper bounds on the parameters defining the different theories, since the main constraints come from the objects with the lowest observed mass for a given observed radius.

**Effective gravitational constant in scalar-tensor theories**

We will focus on the class of scalar-tensor theories defined by the action:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} f(R, \phi, X) - 2\Lambda + \mathcal{L}_m \right],$$

(1)

where $\Lambda$ is the bare cosmological constant, $\phi$ is a scalar field and $X = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi$ is the scalar field’s kinetic term, and we used a system of units in which $c = 1$. For non-relativistic matter with energy-momentum tensor

$$\delta T^0_0 = \delta \rho_m, \quad \delta T^i_i = -\rho_m v_m, i,$$

(2)

where $v_m$ is the matter velocity potential, and using the metric for scalar perturbations in the Newton gauge

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 - 2\Phi)dx^i dx^j,$$

(3)

the Einstein’s equations give the modified Poisson equation

$$\nabla^2 \Psi = -4\pi G_{eff} a^3 \rho_m \delta_m,$$

(4)

where $k$ is a comoving wavenumber and $\delta_m = \frac{\delta \rho_m}{\rho_m} + 3Hav_m$ is the gauge-invariant matter density contrast.
The quantity $G_{eff}$, normally interpreted as the effective gravitational “constant”, is given by 

$$G_{eff} = \frac{1}{8\pi F} \frac{f_X + 4 \left( f_X \frac{k^2}{\alpha^2} \frac{F_k}{\pi} + \frac{r^2}{\pi} \right)}{f_X + 3 \left( f_X \frac{k^2}{\alpha^2} \frac{F_k}{\pi} + \frac{r^2}{\pi} \right)}, \quad (5)$$

where $F = \frac{\partial f}{\partial \phi}$.

**Gravitational stability mass** According to [19] and [21] the turn around radius can be computed from the gauge invariant Bardeen’s potentials by solving the equation

$$\ddot{a}r - \frac{1}{a} \frac{\partial \Psi}{\partial t} = 0, \quad (6)$$

where the dot and the prime denote derivatives respectively with respect to time and the radial coordinate. Following the approximation of spherical symmetry [18], the gravitational potential around an object of mass $m$ can be obtained from eq.(4)

$$\Psi = -G_{eff} m \frac{1}{r}, \quad (7)$$

which substituted in eq.(6) allows to derive a general expression for the turn around radius for all the scalar-tensor theories defined in eq.(1)

$$r_{TAR} = \sqrt{\frac{3G_{eff} m}{\Lambda}}, \quad (8)$$

It is convenient to define the ratio between the Newton constant $G$ and the effective gravitational constant as $\Delta = G/G_{eff}$ and the gravitational stability mass (GSM) as:

$$m_{gs} = \frac{\Lambda r_{obs}^3}{3G_{eff}} = m_{GR} \Delta, \quad (9)$$

where $m_{GR}(r_{obs}) = \Lambda r_{obs}^3/3G$ is the value of the GSM predicted by GR. Any object of mass $m_{obs}$ should have a radius $r_{obs} < r_{TAR}(m_{obs})$, or vice-versa any gravitational bounded object of radius $r_{obs}$ should have a mass larger than $m_{gs}$

$$m_{obs}(r_{obs}) > m_{gs}(r_{obs}) = \frac{\Lambda r_{obs}^3}{3G_{eff}} = m_{GR}(r_{obs}) \Delta. \quad (10)$$

In fact objects of size $r_{obs}$ with a mass smaller than $m_{gs}(r_{obs})$ would not be gravitationally stable, since the effective force due to dark energy will dominate the attractive gravitational force.

In order to compare theories to experiment, it is important to establish what is the size of gravitationally bounded structures, and for this purpose the caustic method has been developed [22], showing good accuracy when applied to simulated data. In the rest of this letter we will use the results of the application of this method to set constraints on the parameters of the different MGT.

**Galaxy clusters data** [23] can be used to set upper bounds on GSM, and to consequently set constraints on $G_{eff}$, since from eq.(10) we get

$$\Delta < \frac{m_{obs}}{m_{GR}}. \quad (11)$$

Most of the data are consistent with GR, except a couple of data points corresponding to the galaxy clusters A655 and A1413, which give respectively $\Delta < 0.9162 \pm 0.2812$ and $\Delta < 0.9723 \pm 0.0151$, where the error has been obtained from the errors on $m_{obs}$ corresponding to $r_{MAX}$ in [23]. For A1413, the deviation from GR is of order $2\sigma$, so we can conclude that there is not a very strong evidence of the need of a modification of GR, but MGT are marginally favored.

**Generalized Brans-Dicke** These theories [24] [25] are a generalization of Brans-Dicke theory [26], with a more general kinetic term, defined by the action

$$f(R, \phi, X) = \frac{\phi}{8\pi G} R + \frac{g(\phi)}{4\pi G} X. \quad (12)$$

After decomposing the scalar field as the sum of a homogeneous background component and a space dependent perturbative part according to

$$\phi(t, x) = \bar{\phi}(t) + \delta \phi(t, x), \quad (13)$$

at leading order in perturbations the effective gravitational constant is given by

$$G_{eff} = \frac{4 + 2\phi_0 g_0}{3\phi_0 + 2\phi_0^2 g_0} G, \quad (14)$$
FIG. 2: Observed masses and radii of the A655 and A1413 galaxy clusters. These are the objects with the most significant difference with respect to the GR prediction, of order $\sigma$ for A655 and $\sigma$ for A1413 (see inset). The blue lines correspond to scalar-tensor theories with a value of $\Delta$ such that $m_{gs} = m_{obs}$ for the each clusters.

where $\phi_0 = \tilde{\phi}(t_0)$, $g_0 = g(\phi_0)$, and $t_0$ is the cosmic time corresponding to the red-shift of the observed structure.

The corresponding turn around radius and GSM are given by

$$r_{TAR} = \frac{3}{2} \sqrt{\frac{3Gm}{\Lambda} \frac{4 + 2\phi_0 g_0}{3\phi_0 + 2\phi_0^2 g_0}}, \quad (15)$$

$$m_{gs} = \frac{r_{obs}^3 \Lambda}{3G} \frac{4 + 2\phi_0^2 g_0}{3\phi_0 + 2\phi_0^2 g_0}. \quad (16)$$

The regions of the $(\phi_0, g_0)$ parameters space satisfying the condition $m_{obs} > m_{gs}$ are shown in fig.3 for the strongest constraints, which come from the A1413 galaxy cluster.

**Brans-Dicke** The action in Brans-Dicke theory is given by

$$f(R, \phi, X) = \frac{\phi}{8\pi G} R + \frac{\omega}{4\pi G \phi} X, \quad (17)$$

which reduces to general relativity in the $\omega \to \infty$ limit. The effective gravitational constant can be obtained from eq. $[5]$

$$G_{eff} = \frac{G}{\phi_0} \frac{4 + 2\omega}{3 + 2\omega} = 1 + \epsilon \frac{\phi}{\phi_0}, \quad (18)$$

where $\epsilon = 1/(2\omega + 3)$. The turn around radius can then be computed using eq. $(8)$ giving

$$r_{TAR} = \frac{3}{2} \sqrt{\frac{3Gm}{\Lambda_{eff}} (1 + \epsilon)}, \quad (19)$$

which is consistent with previous calculations $[15, 16]$ using both static coordinates or cosmological perturbations in the Newton gauge, assuming $\Lambda_{eff} = \phi_0 \Lambda$ is the observed gravitational constant.

The corresponding GSM is

$$m_{gs} = \frac{\Lambda_{eff} r_{obs}^3 \Lambda}{3G(1 + \epsilon)}. \quad (20)$$

The strongest observational constraint comes from A655 and A1413 which give respectively $\epsilon > 0.0915 \pm 0.3350$ and $\epsilon > 0.0285 \pm 0.0159$. Note that GR is not in strong tension with the observations, since the tightest constraint on $\epsilon$ is less then $2\sigma$ away from the GR limit $\epsilon = 0$.

**$f(R)$ theories** In this case the action is independent of the scalar field, and in the Jordan frame is

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} f(R) + L_m \right], \quad (21)$$

with the effective gravitational constant given by

$$G_{eff} = \frac{1}{8\pi F} \frac{1 + 4\frac{k^2 F_R}{F^2}}{1 + 3\frac{k^2 F_R}{F^2}}. \quad (22)$$

On sub-horizon scales ($\frac{k^2 F_R}{F} \gg 1$) it reduces to $[20]$

$$G_{eff} = \frac{1}{6\pi F}, \quad (23)$$

and the turn around radius is given by

$$r_{TAR} = \frac{3}{2\pi \Lambda F}, \quad (24)$$
which corresponds to this expression for the GSM

$$m_{gs} = 2\pi \Lambda F r_{obs}^3. \quad (25)$$

Observational constraints give $F < (0.0486 \pm 0.0149)G^{-1}$ and $F < (0.0516 \pm 0.0008)G^{-1}$ for A655 and A1413 respectively. It can noted that GR is not incompatible with observations, since the tightest constraint on $F$, corresponding to A1413, is less then $2\sigma$ away from the GR limit $F = 0.0531G^{-1}$.

**$R^n$ theories** For these theories the action is given by

$$f(R, \phi, X) = \frac{1}{8\pi G} R + \frac{\alpha}{8\pi G} R^n, \quad (26)$$

and the corresponding effective gravitational constant is

$$G_{eff} = \frac{4G}{3(1 + n\alpha R^n)} = \frac{4G}{3(1 + \alpha \beta)}, \quad (27)$$

where $\beta = nR^{n-1}$, which gives the following expressions for the turn around radius and GSM

$$r_{TAR} = \sqrt[3]{\frac{4Gm}{\Lambda [1 + \alpha \beta]}}, \quad (28)$$

$$m_{gs} = \frac{\Lambda r_{obs}^3 [1 + \alpha \beta]}{4G}. \quad (29)$$

In fig. (4) we plot the regions of the $(\alpha, \beta)$ parameters space satisfying the condition $m_{obs} > m_{gs}$, with the strongest constraints coming from the A1413 galaxy cluster.

**Quintessence** The action of Quintessence is given by

$$f(R, \phi, X) = \frac{g(\phi)}{8\pi G} R - \frac{1}{4\pi G} X, \quad (30)$$

and in this case the effective gravitational constant is

$$G_{eff} = \frac{G}{2g_0} + \frac{4g_0^2}{g_0 2g_0 + 3g_0^2}, \quad (31)$$

where $g_0 = g(\phi_0), \ g_0' = g'(\phi_0), \ \phi_0 = \bar{\phi}(t_0)$ and the turn around radius and GSM are given by

$$r_{TAR} = \sqrt[3]{\frac{3Gm_{gs} 2g_0 + 4g_0^2}{g_0 \Lambda 2g_0 + 3g_0^2}}, \quad (32)$$

$$m_{gs} = \frac{g_0 \Lambda r_{obs}^3 2g_0 + 3g_0^2}{3G 2g_0 + 4g_0^2}. \quad (33)$$

We plot in fig. (5) the regions of the $(\phi_0, \ g_0)$ parameters space satisfying the condition $m_{obs} > m_{gs}$, with the strongest constraints coming from the A1413 galaxy cluster.

**Conclusions** We have derived the theoretical prediction of the gravitational stability mass for a wide class of scalar-tensor theories including $f(R)$ and generalized Brans-Dicke. Most of observations are consistent with GR except the galaxy clusters A655 and A1413, which have masses smaller than what’s predicted by GR. For A1413 the difference between the estimated mass and the GR prediction is of order $2\sigma$, and we have computed the values of the parameters of different MGT which could provide a better agreement between theoretical prediction and observations for this object. In the future it will be important to increase the size of the data sets used for testing different gravity theories, such as for example the observations of the upcoming Euclid mission. In this letter we have assumed a fixed value of the cosmological constant, but in the future it could be interesting to investigate the interplay between MGT and dark energy by fitting both at the same time, and assess the existence of a possible degeneracy between the two. Another important factor to include in the future analysis could be the effect of non sphericity.

**Acknowledgements** We thank Vasiliki Pavlidou for interesting discussions.
FIG. 5: Allowed regions of the \((g_0, g'_0)\) dimensionless parameters space, for Quintessence. The main constrains come from the galaxy cluster A1413. The dark blue region corresponds to \(m_{\text{obs}} > m_{\text{gs}}\) and the other colours to three different error bands \(m_{\text{obs}} + n\sigma_m > m_{\text{gs}}\).

[1] A. De Felice and S. Tsujikawa, Living Rev. Rel. 13, 3 (2010), arXiv:1002.4928.
[2] A. A. Starobinsky, JETP Lett. 86, 157 (2007), arXiv:0706.2041.
[3] W. Hu and I. Sawicki, Phys. Rev. D76, 064004 (2007), arXiv:0705.1158.
[4] A. Nicolis, R. Rattazzi, and E. Trincherini, Phys. Rev. D79, 064036 (2009), arXiv:0811.2107.
[5] C. Burrage, E. J. Copeland, and P. Millington, Phys. Rev. D95, 064050 (2017), arXiv:1610.07529, [Erratum: Phys. Rev.D95,no.12,129902(2017)].
[6] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, Phys. Rept. 513, 1 (2012), arXiv:1106.2476.
[7] N. Frusciante, R. Kase, N. J. Nunes, and S. Tsujikawa, Phys. Rev. D98, 123517 (2018), arXiv:1810.07957.
[8] R. Kase and S. Tsujikawa, JCAP 1811, 024 (2018), arXiv:1805.11919.
[9] A. A. Starobinsky, Phys. Lett. B91, 99 (1980), [771(1980)].
[10] S. Tsujikawa, Lect. Notes Phys. 892, 97 (2015), arXiv:1404.2684.
[11] J. Ohashi and S. Tsujikawa, JCAP 1210, 035 (2012), arXiv:1207.4879.
[12] A. De Felice, S. Tsujikawa, J. Elliston, and R. Tavakol, JCAP 1108, 021 (2011), arXiv:1105.4685.
[13] K. Bamba and S. D. Odintsov, Symmetry 7, 220 (2015), arXiv:1503.00442.
[14] Planck, Y. Akrami et al., (2018), arXiv:1807.06211.
[15] S. Bhattacharya, K. F. Dialektoopoulos, A. E. Romano, C. Skordis, and T. N. Tomaras, JCAP 1707, 018 (2017), arXiv:1611.05055.
[16] S. Bhattacharya, K. F. Dialektoopoulos, A. E. Romano, and T. N. Tomaras, Phys. Rev. Lett. 115, 181104 (2015), arXiv:1505.02375.
[17] S. Bhattacharya, K. F. Dialektoopoulos, and T. N. Tomaras, JCAP 1605, 036 (2016), arXiv:1512.08856.
[18] V. Pavlidou and T. N. Tomaras, JCAP 1409, 020 (2014), arXiv:1310.1920.
[19] C. S. Velez and A. E. Romano, JCAP 1805, 041 (2018), arXiv:1611.09223.
[20] S. Tsujikawa, Phys. Rev. D76, 023514 (2007), arXiv:0705.1032.
[21] V. Faraoni, Phys. Dark Univ. 11, 11 (2016), arXiv:1508.00475.
[22] H. Yu, A. L. Serra, A. Diaferio, and M. Baldi, Astrophys. J. 810, 37 (2015), arXiv:1503.08823.
[23] K. Rines, M. J. Geller, A. Diaferio, and M. J. Kurtz, Astrophys. J. 767, 15 (2013), arXiv:1209.3786.
[24] A. De Felice and S. Tsujikawa, JCAP 1007, 024 (2010), arXiv:1005.0868.
[25] N. Roy and N. Banerjee, Phys. Rev. D95, 064048 (2017), arXiv:1702.02169.
[26] C. Brans and R. H. Dicke, Phys. Rev. 124, 925 (1961), [,142(1961)].
[27] EUCLID, R. Laureijs et al., (2011), arXiv:1110.3193.
[28] Euclid, I. Tereno et al., IAU Symp. 306, 379 (2014), arXiv:1502.00903.
[29] Euclid, R. Scaramella et al., IAU Symp. 306, 375 (2014), arXiv:1501.04908.
[30] Euclid, J. Amiaux et al., Proc. SPIE Int. Soc. Opt. Eng. 8442, 84420Z (2012), arXiv:1209.2228.
[31] A. Giusti and V. Faraoni, (2019), arXiv:1905.04263.
[32] V. Pavlidou. Private communication