3D Feature of Self-correlation Level Contours at 10^{10} cm Scale in Solar Wind Turbulence

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Abstract

The self-correlation level contours at 10^{10} cm scale reveal a 2D isotropic feature in both the slow solar wind fluctuations and the fast solar wind fluctuations. However, this 2D isotropic feature is obtained based on the assumption of axisymmetry with respect to the mean magnetic field. Whether the self-correlation level contours are still 3D isotropic remains unknown. Here we perform for the first time a 3D self-correlation level contours analysis on the solar wind turbulence. We construct a 3D coordinate system based on the mean magnetic field direction and the maximum fluctuation direction identified by the minimum-variance analysis method. We use data with 1 hr intervals observed by WIND spacecraft from 2005 to 2018. We find, on one hand, in the slow solar wind, the self-correlation level contour surfaces for both the magnetic field and the velocity field are almost spherical, which indicates a 3D isotropic feature. On the other hand, there is a weak elongation in one of the perpendicular directions in the fast solar wind fluctuations. The 3D feature of the self-correlation level contours surfaces cannot be explained by the existing theory.

Key words: magnetic fields – plasmas – solar wind – turbulence

1. Introduction

The magnetohydrodynamic (MHD) turbulence exhibits anisotropic features as a result of the preferred direction that the background magnetic field determines (Shebalin et al. 1983). The solar wind is observed to be in a turbulence state (Tu & Marsch 1995) and various studies proposed that solar wind turbulence is 2D anisotropic based on theories (Oughton et al. 1994; Goldreich & Sridhar 1995), simulations (Cho & Vishniac 2000), and observations related to the power spectral index (Horbury et al. 2008; Podesta 2009; Chen et al. 2010; Wicks et al. 2010; He et al. 2013), structure function (Luo & Wu 2010), and correlation function (Mattheus et al. 1990; Dasso et al. 2005).

The “Maltese cross” is one 2D pioneering work. It consists of two lobes, one elongated along the mean field direction (slab-like fluctuations), and the other elongated along the perpendicular direction to the mean field direction (2D fluctuations) and is obtained by a 2D self-correlation analysis (Mattheus et al. 1990). Dasso et al. (2005) applied the correlation function method by using two-day-long data from Advanced Composition Explorer spacecraft for the slow solar wind and the fast solar wind separately. They find that the fast wind mainly contains slab-like fluctuations and the slow wind 2D fluctuations. However, Wang et al. (2019) find that the self-correlation function level contours of the magnetic field and the velocity field are 2D isotropic for both the slow solar wind and the fast solar wind at the 10^{10} cm scale using the same method as Dasso et al. (2005).

The 2D anisotropic studies have been extended to the 3D scenario, which includes not only the mean magnetic field direction, but also the perpendicular magnetic field fluctuation direction. Boldyrev (2006) predicted theoretically that the solar wind turbulence is 3D anisotropic with \( l_1 > l_{1,2} > l_{1,1} \), where \( l_1, l_{1,2}, l_{1,1} \) are correlation lengths in the mean magnetic field direction, the perpendicular magnetic field fluctuation direction and the direction perpendicular to both, respectively. Chen et al. (2012) used the local structure function method to analyze the 3D structure of turbulence in the fast solar wind in a new local coordinate system from the outer scale to the proton gyroscale. Verdini et al. (2018) used the same local structure function method to analyze the 3D structure taking the wind expansion effect into account.

In the present study, we perform the 3D self-correlation function level contour analysis on the WIND spacecraft measurements. We construct the 3D coordinate system using the mean magnetic field and the maximum variance fluctuation direction \( L \) obtained by minimum-variance analysis (MVA) method. In Section 2, we describe the data and methods used in order to study the 3D anisotropy, including the way to construct the 3D coordinate system and get the 3D contour surfaces. We show our observational results in Section 3. In Section 4, we discuss our results and present our conclusions.

2. Data and Method

We use data from the WIND spacecraft during 14 yr, from 2005 to 2018, when the spacecraft hovered at the Lagrangian point \( L1 \) in the solar wind. The magnetic field investigation (Lepping et al. 1995) provides 3 s resolution magnetic field data, and the three-dimensional plasma analyzer (Lin et al. 1995) measures the plasma data with a cadence of \( \Delta = 3 \) s. We cut the data set into 1 hr intervals with no overlap and require that the data gap accounts for less than 5% in each interval. We remove the intervals with \( \max[|\delta B_x|] < 2 \) nT, \( \max[|\delta V_x|] < 20 \) km, where \( j \) indicates \( x, y, z \) axis in the geocentric-solar- ecliptic coordinate system, and \( \delta \) means the variation between every 3 s, in order to avoid the influence of shear magnetic field and shear flows.

For each interval \( i \), we define the fluctuation as \( \delta U = U - \bar{U} \), where \( U \) is either magnetic field \( B \) or velocity \( V \), and \( \bar{U} \) is obtained by performing a linear fit to \( U \). The two-
time-point self-correlation function of $\delta U$ is calculated as
\[
R_u(i, \tau) = \langle \delta U(t) \cdot \delta U(t + \tau) \rangle,
\]
here, $\tau = 0, \Delta, 2\Delta, ..., 400\Delta$ is the time lag, and $\langle \rangle$ denotes an ensemble time average. In order to easily make a comparison, we use the zero time lag self-correlation $R(i, 0)$ to normalize the self-correlation function and obtain $R_{uu}(i, \tau) = R_u(i, \tau)/R(i, 0)$.

In this way, the $R_{uu}(i, \tau)$ at $\tau = 0$ is always equal to 1. According to the Taylor hypothesis (Taylor 1938), we transfer the time lag to spatial lag using $r = \tau V_{SW}$, where $r$ is the spatial lag and $V_{SW}$ is the mean flow velocity in the corresponding interval $i$.

Wang et al. (2019) has shown the isotropic feature of the self-correlation level contours in a 2D coordinate system. We extend this system to 3D by introducing the maximum variance direction $L$, which is determined by performing the MVA method (Sonnerup & Cahill 1967) to the magnetic field data. This 3D coordinate system uses the mean magnetic field $B_0$ and the projection of maximum variance direction $L$ in the plane perpendicular to the mean field as $r_\parallel$ and $r_{\perp 2}$ components, respectively. $r_{\perp 1}$ components complete this orthogonal coordinate system. Any angles greater than 90° are reflected below 90°. In Figure 1, we show the angle $\theta_{VB}$ between the directions of $V_{SW}$ and $B_0$ and the angle $\phi_L$ between $r_{\perp 2}$ direction and the component of $V_{SW}$ perpendicular to $B_0$ for each interval $i$.

We find 23,083 intervals in the slow solar wind ($V_{SW} < 400$ km s$^{-1}$) and 3347 intervals in the fast solar wind ($V_{SW} > 500$ km s$^{-1}$) and study their 3D self-correlation level contours separately. The probability density function of $\theta_{VB}$ in the left panel of Figure 2 shows that the magnetic field is more oblique to the solar wind velocity in the slow wind than in the fast wind, which is consistent with the Parker Spiral theory. In the right panel, we show in the slow wind, there are more intervals with perpendicular $\phi_L$ than parallel $\phi_L$, while the fast wind group has a roughly even distribution over 0° and 90°.

For each group, we bin $\theta_{VB}$ and $\phi_L$ into 15° bins and calculate the average of the normalized spatial self-correlation functions as follows:
\[
R_{uu}(\theta_{VB}^n, \phi_L^n, r) = \frac{1}{n(\theta_{VB}^n, \phi_L^n)} \times \sum_{\theta_{VB}^n - 7.5 < \theta_{VB}(r, i) < \theta_{VB}^n + 7.5, \phi_L^n - 7.5 < \phi_L(r, i) < \phi_L^n} R_{uu}(i, r)
\]
where $n(\theta_{VB}^n, \phi_L^n)$ is the number of the intervals in the corresponding bin, and, $\theta_{VB}^n = 15^n m + 7.5^n; \phi_L^n = 15^n n + 7.5^n; m, n = 0, 1, 2, ..., 5$.

We obtain 36 averaged self-correlation functions for 36 ($\theta_{VB}, \phi_L = 15^n \times 15^n$ bins. We analyze the contours at level $R_{uu}(\theta_{VB}, \phi_L, r) = 1/e \approx 0.368$, and obtain an $r_{level}$ value by linear interpolation for each ($\theta_{VB}, \phi_L$). In order to plot the contour surface in the 3D coordinate system, we transform ($\theta_{VB}, \phi_L, r_{level}$) into ($r_{\perp 1}, r_{\perp 2}, r_\parallel$) by using $r_{\perp 1} = r_{level} \sin \theta_{VB} \sin \phi_L$, $r_{\perp 2} = r_{level} \sin \theta_{VB} \cos \phi_L$, $r_\parallel = r_{level} \cos \theta_{VB}$. We reflect the surface in the first octant into the other seven octants based on the assumption of reflectional symmetry. The result is presented in the next section.

3. Results

Figure 3 shows the averaged self-correlation functions in $r_{\perp 1}$, $r_{\perp 2}$, and $r_\parallel$ directions, which correspond to the following angular bins:
\[
r_{\perp 1} \rightarrow (75^\circ \leq \theta_{VB} \leq 90^\circ, 75^\circ \leq \phi_L \leq 90^\circ),
\]
\[
r_{\perp 2} \rightarrow (75^\circ \leq \theta_{VB} \leq 90^\circ, 0^\circ \leq \phi_L < 15^\circ),
\]
\[
r_\parallel \rightarrow (0^\circ \leq \theta_{VB} < 15^\circ, 0^\circ \leq \phi_L \leq 90^\circ).
\]
In the left panel of Figure 3, we present the averaged magnetic self-correlation functions with standard error bars for both the
Figure 3. Left panel: averaged normalized self-correlation functions $R_{bb}(r)$ of 1 hr long magnetic field data. The solid and dashed lines are for the slow wind and the fast wind. Red, blue, and yellow colors correspond the $r_P$, $r_\perp 1$, and $r_\perp 2$ directions, respectively. The error bar shows the standard error of $r_\text{level}$ for a given $R_{bb}$. Right panel: averaged normalized self-correlation functions $R_{vv}(r)$ of 1 hr long velocity data, in the same manner as the left panel.

Figure 4. 3D self-correlation level contour surface at level $R_{\text{level}} = 0.368$ of (a) magnetic field in the slow wind; (b) magnetic field in the fast wind; (c) velocity field in the slow wind; (d) velocity field in the fast wind. The color represents $r_\text{level} [10^{10} \text{ cm}]$, which is the distances from the origin. The dashed red and blue lines in the $r_\perp 1 = -1.4$ plane are projections of the intersection lines of the surface with two planes $r_\perp 1 = A_1$ and $r_\perp 1 = A_2$, respectively, where $A_1$ and $A_2$ are shown in the legends with the corresponding colors in the corresponding panel; the dashed red and blue lines in the $r_\perp 2 = -1.4$ plane are projections of the intersection lines of the surface with two plane $r_\perp 2 = A_1$ and $r_\perp 2 = A_2$, respectively; the dashed red and blue lines in $r_P = -1.4$ plane are projections of the intersection lines of the surface with two plane $r_P = A_1$ and $r_P = A_2$, respectively.
slow solar wind (solid lines) and the fast solar wind (dashed lines). It is hard to distinguish the functions of the three directions for the slow wind. The phenomenon of the functions almost overlapping with each other means the 3D isotropic feature of the self-correlation level contours in slow solar wind turbulence. For the fast wind, there is a slight elongation along the \( r_{12} \) direction in the perpendicular plane. Note that the magnetic self-correlation function of the fast wind is larger than that of the slow wind. When we consider self-correlation function with respect to the time lag instead of the spatial lag, the magnetic self-correlation functions for both the slow wind and the fast wind are almost the same (not shown). In the right panel, we show the averaged velocity self-correlation functions. They have almost the same features as the averaged magnetic self-correlation functions except there is no clear elongation along the \( r_{12} \) direction.

We show the 3D self-correlation level contour surfaces at level \( R_{\text{max}} = 0.368 \) in Figure 4. In Figure 4(a), the slow wind magnetic field self-correlation function contour surface is almost a spherical surface. The projection closed curves on the 2D plane are plotted to help visualize the isotropic feature. They are almost round and almost identical to each other, which confirm the isotropic result. In Figure 4(b), the fast wind magnetic field self-correlation level contour surface departs a bit from a spherical surface. \( R_{\text{level}} \) in the \( r_{12} \) direction is slightly longer. The projection closed curves on three planes are not round and have different sizes between each other. In Figure 4(c), the slow wind velocity field self-correlation level contour surface is almost spherical and the projection closed curves on the 2D planes is also round and identical to each other, which shows a clear isotropic feature as the magnetic field. In Figure 4(d), the fast wind velocity field self-correlation level contour surface has a similar shape with that of the magnetic field. The similarity between the magnetic field and velocity field contour shape supports the applicability of the data analysis technique here. We should also note that the \( R_{\text{level}} \) is shorter for the slow wind than for the fast wind and shorter for the velocity field than for the magnetic field.

In order to evaluate the unevenness shown in Figure 4, we reduce the 3D surface into the line trend with \( \theta_{VB} \) and \( \phi_{L} \), as shown in Figure 5. We calculate the averaged \( R_{\text{level}} \) in 6 \( \theta_{VB} \) bins from \( 0^\circ \leq \phi_{L} < 90^\circ \) with a weight of interval number in each \( \phi_{L} \) bin. The result is shown in the left panel. We can clearly see that the variation with \( \theta_{VB} \) is rather small for both the slow wind and the fast wind and for both the magnetic field and the velocity field. We calculate the average \( R_{\text{level}} \) in 6 \( \phi_{L} \) bins from \( 60^\circ \leq \theta_{VB} \leq 90^\circ \) with a weight of interval number in the two \( \theta_{VB} \) bins. The result is shown in the right panel. For the slow wind, the variation with \( \phi_{L} \) is very small; while for the fast wind, there is a weak elongation along \( r_{12} \). Again, it is easily seen that \( R_{\text{level}} \) is shorter for the slow wind and the velocity field.

In Figure 6, we show the variations with \( \theta_{VB} \) and \( \phi_{L} \) for the fast solar wind. The black solid and black dashed lines are the same as those in Figure 5. We check the data intervals in the fast wind (group A) and further rule out the intervals with large gradient by visual inspection. We reserve 2272 cases (group B). The variations with \( \theta_{VB} \) and \( \phi_{L} \) for this new fast wind group B are calculated and shown by blue lines in Figure 6. The anisotropy for the fast group becomes weaker after we remove the structures more strictly.

### 4. Discussion and Conclusions

We present for the first time the 3D self-correlation level contours of the magnetic field and the velocity field at the \( 10^{10} \) cm scale based on \( WIND \) spacecraft measurements from 2005 to 2018. We construct a 3D coordinate system according to the mean magnetic field direction and the maximum variance direction \( L \) of the magnetic field. The self-correlation contour surfaces at level \( R_{\text{max}} \approx 1/e \) in the slow solar wind are 3D isotropic for both the magnetic field and the velocity field. The self-correlation contour surfaces at level \( R_{\text{max}} \approx 1/e \) in the fast solar wind show weak anisotropic feature in the perpendicular plane with an elongation along \( r_{12} \). However, the anisotropy becomes weaker when we exclude the intervals with structures more strictly.

The 3D coordinate system constructed here is consistent with the 3D coordinate system presented by Chen et al. (2012) if we consider the maximum variance direction \( L \) as the \( (B_{1} - B_{2}) \) in their work. Chen et al. (2012) present a structure function analysis in a scale-dependent 3D coordinate system defined as follows: for each pair of points, the local mean field \( B_{\text{local}} = (B_{1} + B_{2})/2 \) was calculated as one axis and the local perpendicular fluctuation direction \( B_{\text{local}} \times [(B_{1} - B_{2}) \times B_{\text{local}}] \) as another axis. Verdin et al.
support Kolmogorov’s theory (Kolmogorov 1941). How to interpret this result in the slow solar wind requires further investigation.

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