A Hydro+Cascade model has been used to describe radial and elliptic flow at the SPS and successfully predicted the radial and elliptic flow measured by the both STAR and PHENIX collaborations. Furthermore, a combined description of the radial and elliptic flow for different particle species, restricts the Equation of State (EoS) and points towards an EoS with a phase transition to the Quark Gluon Plasma (QGP).

1. Introduction

  Relativistic Hydrodynamics provides a link between the Equation of State (EoS) of the excited nuclear matter and collective observables such as elliptic flow ($v_2$-s) and radial flow ($T_{\text{slope}}$-s). At the SPS, pure hydrodynamics calculations can fit the transverse mass spectra for almost any EoS by choosing the freezeout temperature $T_f$. However in non-central collisions, when $v_2$ was calculated for these same EoSs with the same $T_f$-s, $v_2$ was above the data by a factor of two[1]. Bass and Dumitru[2] removed the $T_f$ indeterminacy by injecting the particles into a microscopic transport model at a switching temperature $T_{\text{switch}} \approx T_c \approx 165 \text{ MeV}$, and cascading the particles until they decoupled. With the freezeout parameter removed, the slope parameters for central PbPb collisions were calculated and found to agree with experimental values. The only parameter in this approach is the total multiplicity in the collision.

  Later, elliptic flow was calculated[3] in a similar Hydro+Cascade model and $v_2$ was only 20% above the data. Since at freezeout the viscosity is certainly important, it was not surprising that the introduction of a cascade reduced the elliptic flow. It was surprising that Hydro+Cascade could simultaneously reproduce the elliptic and radial flow for different particle species as a function of impact parameter. Furthermore, the combined analysis of radial and elliptic flow restricted possible EoSs since the freezeout temperature could no longer be adjusted to make any EoS fit any slope parameter. Roughly speaking, a soft EoS produced too little radial flow while a hard EoS produced too much elliptic flow.

  Now, with the EoS roughly fixed from available SPS data, parameter free predictions were made for RHIC. A few of these predictions are: (a) an increase in $v_2$ by approximately 40% over the SPS and (b) a significant increase in the radial flow. (c) curved nucleon $m_T$ spectra. The preliminary data reported in this conference have been in agreement with
Figure 1. (a) A compilation of slope parameters (see e.g. [9]) at the SPS compared to model predictions for different EoSs. The slope parameters are fit from $0 < M_T - m < 0.9$ GeV, corresponding to the WA98 acceptance. (b) Model predictions for slope parameters at RHIC for different EoSs. The slope parameters are fit over the range $0 < M_T - m < 1.6$ GeV and do depend on the fit range used.

Ideally, the cascade should provide a kind of dual description of the hydrodynamics. Although $T_c$ provides a natural place to switch to the cascade, the results (slope parameters and $v_2$-s, lifetimes, etc.) should be insensitive to the switching temperature $T_{\text{switch}}$. Unfortunately, Bass and Dumitru [2] reported that the results were sensitive to the transition surface. By incorporating chemical freezeout into the hydrodynamic calculation, the sensitivity to $T_{\text{switch}}$ was much reduced although the elliptic flow at the SPS remained sensitive to $T_{\text{switch}}$ [3].

If the preliminary data remain unchanged and further predictions are verified, the hydrodynamic description must be taken seriously and the equilibration times and transport cross sections estimated from binary, perturbative, classical parton cascades must be considered only a very coarse guide to the radiating, non-perturbative, quantum glue that makes up the initial state.

2. The EoS, Flow, Predictions and Data

Below a family of EoSs with a first order phase transition are studied and are labeled by the Latent Heat (LH). LH4, LH8, · · · label EoSs with a latent heats of $0.4 \text{ GeV}/\text{fm}^3$, $0.8 \text{ GeV}/\text{fm}^3$, · · ·. LH$\infty$ is studied as a limiting case. A Resonance Gas (RG) EoS (which does not have a phase transition) is also studied.

First in Fig. (a), the measured slope parameters at the SPS are compared to model predictions for different EoSs. LH8 gives the best description of the available spectra. LH4
Figure 2. Slope parameters for $\pi^-$, $K^-$, $p$ and $\bar{p}$ reported in these proceedings by the STAR(a) and PHENIX(b) collaborations. The open(closed) symbols show model predictions for anti-protons(protons). The STAR collaboration fits the $\pi^-$, $K^-$ and $\bar{p}$ spectra over the ranges, $0.12 \text{GeV} < M_T - m < 0.45 \text{GeV}$, $0.04 \text{GeV} < M_T - m < 0.34 \text{GeV}$ and $0.04 \text{GeV} < M_T - m < 0.45 \text{GeV}$ respectively. The PHENIX collaboration fits the $\pi^-$ and $p$ spectra over the ranges, $0.19 \text{GeV} < M_T - m < 0.87 \text{GeV}$ and $0.175 \text{GeV} < M_T - m < 2.2 \text{GeV}$ respectively.

is too stiff (the slope parameters are too high) and LH16 is too soft (the slope parameters are too low). The $T_{\text{slope}}$-s for a RG EoS are comparable to LH4, and the $T_{\text{slope}}$-s for LH\(\infty\) are comparable to LH16.

Although LH8 gives the best fit to SPS spectra, the model-data discrepancy for the other EoSs is not large. As the collision energy is increased from the SPS to RHIC the slope parameters all increase (see e.g. [2,3]). Since at high energies the importance of the QGP phase increases, the differences between these EoSs are magnified during the systems evolution. In Fig.1(b), the model predictions at RHIC collision energies are shown for different EoSs. Note, the spectra are curved and the parameterization in terms of slope parameters is only schematic. LH\(\infty\), with no QGP push, generates only small slope parameters. The differences between the EoSs is clear in the flow of the $\Omega$.

With these predictions, a comparison to the first RHIC data is made in Fig.2. The $M_T$ spectra are curved (see [5]) and therefore the STAR and PHENIX collaborations measure quite different slope parameters. The STAR collaboration fits the observed spectra in a low $M_T$ range and measures large slopes, while the PHENIX collaboration fits in a high $M_T$ range and measures small slopes. The best agreement with the proton and anti-proton slope parameters of both collaborations is found between LH8 and LH16. LH\(\infty\) has too much flow at small $M_T$ and too little flow at high $M_T$ and therefore fails to reproduce the curvature of the $M_T$ spectra seen in the data. The slope parameters reported in this conference implicate a strong transverse expansion.
Figure 3. Elliptic flow for different EoSs as a function of the number of participants relative to the maximum at the SPS(a) and RHIC(b). (a) The data points are for $\pi^-$ and the model points are for all pions. (b) The model points and data points are for all charged particles.

Now, elliptic flow is studied as a function of impact parameter. In Fig. 3(a) and (b), the elliptic flow of pions/charged particles at the SPS and RHIC is shown for different EoSs. Notice that elliptic flow increases by approximately 40% from the SPS to RHIC. This prediction was borne out by the first STAR measurements. At the SPS, EoSs without or only a very weak phase transition (e.g. RG or LH4) produce far too much elliptic flow. At the SPS, LH8 and LH16 give approximately the same elliptic flow since the contribution to $v_2$ of the pure QGP phase is small. At RHIC, the $v_2$-s of LH4, LH8 and LH16 begin to separate as the QGP phase becomes increasingly significant.

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