Development of Nonlinear Flight Mechanical Model of High Aspect Ratio Light Utility Aircraft

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Abstract. The implementation of Flight Control Law (FCL) for Aircraft Electronic Flight Control System (EFCS) aims to reduce pilot workload, while can also enhance the control performance during missions that require long endurance flight and high accuracy maneuver. In the development of FCL, a quantitative representation of the aircraft dynamics is needed for describing the aircraft dynamics characteristic and for becoming the basis of the FCL design. Hence, a 6 Degree of Freedom nonlinear model of a light utility aircraft dynamics, also called the nonlinear Flight Mechanical Model (FMM), is constructed. This paper shows the construction of FMM from mathematical formulation, the architecture design of FMM, the trimming process and simulations. The verification of FMM is done by analysis of aircraft behaviour in selected trimmed conditions.

1. Introduction

The development of Electronic Flight Control System (EFCS) is conducted inside the cooperation between LAPAN and TU Berlin whose purpose is to build a demonstrator aircraft named LAPAN Surveillance Aircraft (LSA-02). The aircraft selected is a light utility single engine aircraft with payload capability up to 350 kg (see Figure 1). The original mechanical flight control system is modified with EFCS to provide full authority to reduce pilot work load and enhance control performance especially during extreme tasks such long endurance flight and high accuracy maneuver. The development of Flight Control Law (FCL) for the Basic EFCS (BEFCS) is conducted to realize the attitude control and flight path control for the LSA-02[8][9]. Other development that is navigation control shall be assigned as the experimental FCL. The development of basic FCL employs a strict development process namely V model development process [1].

The development of basic FCL can be only realized when an accurate model of the aircraft dynamics is available. Assumed as a rigid body, the real aircraft has six degrees of freedom and possess dynamic nonlinearity. Hence, to mimic the real aircraft response and characteristics, the flight mechanical model (FMM) needs to be supported by accurate data such as mechanical properties data (mass, inertias, CG. position, etc.), propulsion system data, and also aerodynamic and stability derivatives coefficients. Since at initial stage some data may be confidential or proprietary which are difficult to obtain, then prediction techniques and assumptions will be used for reconstructing the required data.
The development of FMM considers numerous aspects but it usually starts with the definition of the equations of motion. In addition, the accurate representation of the atmosphere variables should also be considered to provide realistic atmospheric effects to the aircraft response during flight. Furthermore, the contribution of disturbance may be added to the aircraft motion. The formulation of forces and moments coefficients are essential and unique for each aircraft especially for aerodynamic forces and moments coefficients. These may be the most valuable data for an aircraft and often they are confidential. However, some prediction tools may be used to estimate the aerodynamic coefficients including the stability derivatives, which to some degree may help at the early stage of the FMM development. Later in the development process, the data from wind tunnel tests or ultimately the flight tests shall be used for validating the FMM. The propulsion parameters may be easier to acquire from the engine data. However, some parameters can only obtained by means ground tests or flight tests. Other variables like acceleration and load factor shall be included as well in the FMM to provide valuable information especially if the acceleration or load factor exceed the threshold of airframe structural strength, which may be a threat for the safety of human on board.

![Diagram of Aircraft STEMME ES15](image)

**Figure 1.** Aircraft STEMME ES15 which is modified with EFCS to become LSA-02 [1].

The main structure of a FMM may be generic and can be adjusted for some different types of aircraft. However, as the aircraft possesses huge number of parameters which are related to its specific characteristics and the flight regimes it may cover, the development of FMM shall be carried out independently and exclusively for a specific aircraft.

The research objectives of the work presented in this paper are to develop a nonlinear flight mechanical model of LSA-02 aircraft and to verify the flight dynamic characteristics of the aircraft, especially the natural Eigen mode responses from the flight simulation. This paper shows the construction of FMM, including the definition its dynamic variables, the mathematical formulation, and the architecture design of FMM numerical representation using SIMULINK. In addition, a trimming method for longitudinal and lateral motions and the dynamic response of the FMM are also simulated and presented.
2. Formulation of FMM

It is important to firstly define clear terminology used in FMM development [2]. The formulation of FMM involves four main coordinate systems, i.e. geodetic, aerodynamic, kinetic and body-fixed coordinate systems (see Figure 2).

The equations of motion mostly is described in body-fixed coordinate system. The forces equation is the summation of the forces come from the weight of the aircraft, the thrust of the engine and the aerodynamic forces. The thrust is assumed to work coincidently in body-fixed axis $x_f$ which generates no moments. Hence, the moments equation only comes from aerodynamic moments. The airspeed is obtained from the flight path velocity subtracted with the wind velocity. The Euler angles or the attitude angles are calculated from angular rates. The geodetic position is obtained from calculated ground speed.

![Figure 2. Flight mechanical angles with respect to the coordinate systems.](image)

There are six aerodynamic forces and moments coefficients which are firstly calculated before computing the aerodynamic forces and moments. These coefficients derivatives are explained in [3] and can be written to be

\[
C_W = C_{W0} + \frac{C_A^2}{\pi \Lambda e} \\
C_q = C_{Q\beta}\beta + C_{QP}\left(\frac{P_f S}{V}\right) + C_{QR}\left(\frac{V_f S}{V}\right) + C_{Q\xi}\xi + C_{Q\zeta}\zeta \\
C_A = C_{A0} + C_{A\alpha}\alpha + C_{A\eta\xi}\eta_K \\
C_l = C_{l\beta}\beta + C_{l\phi}\left(\frac{P_f S}{V}\right) + C_{l\psi}\left(\frac{V_f S}{V}\right) + C_{l\xi}\xi + C_{l\zeta}\zeta \\
C_m = C_{m0} + C_{mu}\alpha + C_{m\eta}\eta \\
C_n = C_{n\beta}\beta + C_{n\phi}\left(\frac{P_f S}{V}\right) + C_{n\psi}\left(\frac{V_f S}{V}\right) + C_{n\xi}\xi + C_{n\zeta}\zeta
\]

Here, the coefficient index W is drag, Q is side, A is lift. The control surfaces deflection of aileron, elevator, rudder and flaps are denoted respectively with $\xi$, $\eta$, $\zeta$ and $\eta_K$. Other coefficients e.g. $C_{W0},\Lambda,e,C_{Q\beta},C_{QP}$, etc. on Equation (1)-(6) are defined following the description in [6].
The thrust force equation is derived from the engine data [4]. Here, it is assumed that the throttle position is proportional to the fuel consumption. Mapping the engine rotation with the throttle position will result the engine power. The thrust force is calculated by equation (7).

\[ F = \frac{\bar{\eta}_p P_e}{V} \]  

where \( \bar{\eta}_p \) is propeller efficiency, \( P_e \) is engine power and \( V \) is airspeed. The propeller efficiency is assumed to be constant and is given to be 0.85. The construction of relation chart between throttle position and the engine power is displayed in Error! Reference source not found.. The atmospheric variables are assumed to follow the international standard atmosphere (ISA). The variables required are geopotential height, static pressure, temperature and air density. The geometric height or the altitude is obtained from geodetic position which may be computed by solving the navigational parts of the equations of motion.

3. Architecture of FMM
The FMM consists of several subsystems which will be continuously developed. The existing subsystems are namely the command subsystem, trim control subsystem, aircraft subsystem and output subsystem. However, the FMM emphasizes more to the aircraft subsystem or the aircraft model. The original aircraft model receives command signals from the input model and produces responses to the output model.

The environment model is included inside the aircraft model or the aircraft subsystem while the disturbance model is included in the input subsystem. The FMM is realized by means of Simulink / MATLAB applications. The block diagram of the aircraft model and its architecture are depicted in Figure 5.
All sub-systems of the aircraft model are derived based on the formulation of FMM which has been previously developed. Environment sub-system comprises wind speed and ISA atmosphere data computation. Kinematics block calculates airspeed and ground speed. Equations of motion sub-system block consists of forces, moments and states computation. The forces and moments block include the weight, propulsion, aerodynamics and inertial data and computation. The integrator sub-system integrates the state derivatives resulted from the equations of motion. There are four derivatives obtained from the integrator sub-system, i.e. flight path speed, angular rate, attitude and position derivatives.

4. Trimming the Aircraft Model

The next phase after building the aircraft model is to compute the trimmed condition of the aircraft. Trimmed condition is when the resultant of forces and moments are zeros. This means the aircraft is on steady flight condition either in longitudinal or lateral motion. There are many techniques to trim the aircraft, even Simulink MATLAB offers the automatic trim application directly from the developed model. Here, the trimming method is proposed by means of FCL implementation. For this purpose, the FCL for trimming the aircraft model is called by the trim control subsystem and form a closed system. Therefore, the FMM is expanded by introducing the command and trim control subsystems. The previous input subsystem is included inside the trim control subsystem.
The trim control subsystem has purpose of computing the required commands for the control surfaces to trim the aircraft in longitudinal and lateral motion. The trim control in this report is essentially the cascade of the flight-path control and attitude control, which are employed as a means to trim the aircraft, hence the required control surfaces deflection are obtained. Therefore, their implementations only concern the steady state responses of the aircraft. The trim control, based on its function, is divided into two controls, i.e. longitudinal motion control and lateral motion control. The longitudinal control consists of the flight-path velocity control and the altitude control, while the lateral control comprises the angle of sideslip control and heading control. Here, the proportional and integral gains are used as controller. The block diagram of trim control is displayed from Figure 7 to Figure 10.

![Figure 7. The block diagram of flight path velocity control.](image)

![Figure 8. The block diagram of altitude control.](image)

![Figure 9. The block diagram of angle of sideslip control.](image)

![Figure 10. The block diagram of heading control.](image)

The integration between the flight-path velocity control, altitude control, angle of sideslip control, and azimuth control is implemented in trim control subsystem and is displayed in Figure 11.
Initially, the trimming of the aircraft subsystem is carried out for the longitudinal motion, and then is continued for the lateral motion. The longitudinal motion is trimmed by maintaining flight path velocity and altitude. The flight path velocity is realized by the throttle position while the altitude is realized by the elevator. The lateral motion is trimmed by maintaining the angle of sideslip. The angle of sideslip is realized by the rudder while the zero heading angle is realized by the aileron.

In this work, for the longitudinal trimming, the flight path velocity chosen are 130, 160, and 190 km/h while the altitude is fixed at 1000 m. The lateral motion is trimmed by maintaining the angle of sideslip and the heading angle. The angle of sideslip chosen are -10, -5, 0, 5, and 10 degrees, while the heading is fixed at 0 degree. The results of longitudinal and lateral trimming are the steady state values and required control inputs (control surface deflection).

![Diagram](Image)

**Figure 11.** The Simulink model of the trim control.

**Figure 12.** Example of flight simulation. Open loop responses of longitudinal motion (left) and lateral motion (right).
After that, the open loop response is examined to identify the natural behavior of the aircraft model. From trimming values, the disturbance is excited so that the response of the aircraft can be observed. The simulation result for disturbed trimming condition can be seen from Figure 12.

From Figure 12 on the left column, the step input of 1 degree is given to the trimmed elevator deflection. The longitudinal motion consists of two Eigen modes, i.e. short period mode and phugoid mode. The short period can be clearly observed from the pitch rate response around the first two seconds after elevator excitation. The angle of attack although slower response than the pitch rate, but the short period can be distinguished as well within the same time. After two seconds the phugoid appears in all states. From this simulation we can qualitatively estimate that the short period has frequency of 0.5 Hz or around 3 rad/s (because the period is around 2 seconds) while the phugoid has frequency of 0.05 Hz or around 0.3 rad/s (because the period is around 20 seconds). For further examination, the Eigenvalues and Eigenvectors of the linearized system can be computed and compared to the nonlinear simulation results.

Figure 12 on right column shows the state variables response from the step input of 1 degree from the aileron. The aileron excites the rolling motion in the aircraft which is represented by the roll rate and the bank angle responses. The nature of the aircraft is unstable typically because of the spiral mode. The spiral mode is mainly reflected by, from the simulation, the bank angle response. This explains that the bank angle keeps diverging away from zero. The roll mode is depicted from the roll rate but surprisingly it shows slight oscillation which probably is caused by low roll damper $C_{lp}$ value. The Dutch-roll can be clearly seen from the response of the yaw rate around the first two seconds. The weaker Dutch-roll can be observed as well from the angle of sideslip response.

After investigating the open loop responses, the next step is to investigate the trimming curve. The longitudinal and lateral trimming results are observed. The relationships selected for the longitudinal motion are elevator versus flight path velocity and angle of attack versus flight path velocity. On the other hand, the relationships selected for the lateral motion are aileron versus angle of sideslip and rudder versus angle of sideslip. The result of trimming curves are given in Figure 13.

![Figure 13. Trimming curve of the elevator, angle of attack with flight path velocity (left) and Trimming curve of the aileron, rudder with the angle of sideslip (right).](image-url)

From Figure 13 on the left, the relation for elevator and angle of attack with the flight path velocity in longitudinal motion is non-linear. The angle of attack is positive when the flight path velocity is low which explains that the aircraft needs more lift force to fly in low speed. The angle of attack gradually decrease to reduce the lift contribution when the flight path velocity is increased. The elevator has purpose to hold the pitch angle so that the flight path inclination stays at zero. When the flight path velocity is low, the elevator is negative to produce positive pitching moment until the altitude is maintained. If elevator is kept fixed while the flight path velocity is increased, the aircraft will climb...
and the altitude command will fail. Therefore, to maintain the altitude the elevator will increase (from negative to positive) as the flight path velocity is increased.

From Figure 13 on the right, the relation for aileron and rudder with the angle of sideslip in lateral motion seems to be linear. The aileron has purpose to compensate the yawing moment caused rudder deflection. The rudder deflection also causes the rolling moment which is needed to be anticipated by aileron. The aileron has smaller deflection because of long lever arm that is wing. On the other hand, to achieve angle of sideslip command, the rudder is seen to have relatively proportional deflection. In addition, the flight path velocity in longitudinal motion does not affect the relationship in lateral motion. This is because the aircraft is laterally commanded when the longitudinal motion is already trimmed.

5. Conclusions
Some conclusions drawn from this paper are

- The development of the nonlinear flight mechanical model of high aspect ratio light utility aircraft has been done. This is shown by the formulation and architecture design of the nonlinear flight mechanical model
-Trimming points aircraft model have been selected and the flight simulation verifies the natural Eigen mode responses of the real aircraft. The real aircraft has all stable Eigen modes except for spiral mode which is slightly unstable.
- For further investigation, some work can be suggested, i.e. utilization of more comprehensive and accurate stability and control derivatives, computation of trimmed aircraft on more variation of flight condition.

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