TOWARD PLANETESIMALS: DENSE CHONDRULE CLUMPS IN THE PROTOPLANETARY NEBULA

JEFFREY N. CUZZI, ROBERT C. HOGAN, AND KARIM SHARIFF

Received 2008 February 12; accepted 2008 April 11

ABSTRACT

We outline a scenario that traces a direct path from freely floating nebula particles to the first 10–100 km sized bodies in the terrestrial planet region, producing planetesimals that have properties matching those of primitive meteorite parent bodies. We call this "primary accretion." The scenario draws on elements of previous work and introduces a new critical threshold for planetesimal formation. We presume the nebula to be weakly turbulent, which leads to dense concentrations of aerodynamically size-sorted particles that have properties similar to those observed in chondrites. The fractional volume of the nebula occupied by these dense zones or clumps obeys a probability distribution as a function of their density, and the densest concentrations have particle mass densities that are 100 times that of the gas. However, even these densest clumps are prevented by gas pressure from undergoing gravitational instability in the traditional sense (on a dynamical timescale). While in this state of arrested development, they are susceptible to disruption by the ram pressure of the differentially orbiting nebula gas. However, self-gravity can preserve sufficiently large and dense clumps from ram pressure disruption, allowing their entrained particles to sediment gently but inexorably toward their centers, producing 10–100 km "sandpile" planetesimals. Localized radial pressure fluctuations in the nebula, as well as interactions between differentially moving dense clumps, will also play a role that must be accounted for in future studies. The scenario is readily extended from meteorite parent bodies to primary accretion throughout the solar system.

Subject headings: accretion, accretion disks — instabilities — minor planets, asteroids — solar system: formation — turbulence

Online material: color figure, mpeg animations

1. INTRODUCTION

There is no currently accepted scenario for the formation of the parent bodies of primitive meteorites that accounts for the most obvious of their properties. These properties (reviewed by Scott & Krot 2005 and discussed in more detail in § 2.1) include (1) dominance by aerodynamically well-sorted mineral particles of submillimeter size; (2) class-to-class variation in well-defined physical, chemical, and isotopic properties; (3) a spread of 1 Myr or so between the formation times of the oldest and youngest objects found in the same meteorite; (4) a spread of 1–3 Myr in the radiometric ages of different meteorite types; and (5) a dearth of melted asteroids, with model results for even some melted asteroids that imply Myr delays in formation relative to ancient minerals. In recent years, meteoritic evidence has appeared for some planetesimals that were formed very early, which represent a minority of both meteorites and asteroids. This implies that primary accretion started early and continued for several million years; thus, it was fairly inefficient and did not run quickly to completion.

Most current models for this "primary accretion" stage (reviewed by Cuzzi & Weidenschilling 2006; Dominik et al. 2007) can be classified as either (1) "incremental growth," where large particles sweep up smaller ones by inelastic collisions involving porous surfaces, and growth proceeds hierarchically; or (2) "instability," where physical sticking is irrelevant and collective effects drive collapse to sizes of kilometers or larger on very short timescales. Those who favor instability models, most of which rely on gravity and occur in a particle-rich nebula midplane, are concerned by the poorly understood sticking of mineral particle aggregates and the apparent difficulty of growing beyond meter size due to rapid inward migration and collisional disruption. Those who favor incremental growth models have noted that midplane instability models are precluded by even very weak global turbulence and that, in the dense midplane layers that form when turbulence is absent, incremental accretion is at a low relative velocity and the meter-size barrier is not a problem. However, the most sophisticated models of incremental accretion in nonturbulent nebulae find it to be so efficient that large planetesimals grow in only \(10^4\)–\(10^5\) yr throughout the asteroid belt region (Weidenschilling 2000), a short time that is difficult to reconcile with constraints 1–5 above. A third class of scenario suggests that a complex interplay between several nonlinear processes—turbulence, pressure gradients, and gravity—may concentrate appropriately sized particles and lead to planetesimal growth (Cuzzi et al. 2001, 2005, 2007; Johansen et al. 2006, 2007). Scenarios that involve preferentially meter-sized objects encounter concerns about whether meter-sized objects can survive their own high-velocity mutual collisions in this sort of turbulent environment (cf. Sirono 2004; Langkowski et al. 2008; Ormel et al. 2008). However, in principle, such boulder-concentration scenarios can proceed independently in the same environment as that discussed in this paper, where we focus on millimeter-sized particles.

Most of the above scenarios provide no natural explanation for observed meteorite properties 1 and 2 above. In particular, the evidence for the H chondrite class (and perhaps all the ordinary chondrites) suggests that entire asteroids of 10–100 km diameter formed directly from a physically, chemically, and isotopically homogeneous mix of dust-rimmed particles of similar sizes (§ 2.1). In this paper, we outline a possible path by which entire batches
of millimeter-sized, aerodynamically sorted particles might proceed directly in turbulence (even if sporadically) to planetesimals that have the properties outlined above. We find that sufficiently large and dense clumps of millimeter-sized particles can form by turbulent concentration such that, even if classical gravitational instability cannot operate, their self-gravity may still allow them to survive disruptive forces and slowly sediment into a “sandpile” planetesimal. This simple analytical model is backed up by some numerical simulations that support the basic idea. In §§ 2.2 and 2.3, we review the most relevant physics that determines the properties of dense, particle-rich zones or clumps in turbulent nebulae. In § 3, we address the fate of these dense clumps using analytical and numerical models of their evolution. We derive the combination of size and mass density that a clump must have in order to evolve into a “sandpile” that has some degree of internal strength. In subsequent stages not modeled here, we imagine that collisions and thermal sintering transform these sandpiles into the cohesive rocky parent bodies we see today. However, in the outer solar system, lower energy collisions and weaker thermal processing might well allow planetesimals to retain their initial low-strength states, as seen for some cometary objects (e.g., Asphaug & Benz 1996).

2. BACKGROUND

2.1. Meteoritics Background and Evidence for Inefficient Accretion in a Turbulent Environment

One can extract a number of clues from primitive (unmelted) meteorites regarding the primary accretion process by which their parent bodies first formed (Scott & Krot 2005; Taylor 2005). For the best evidence, one must look back through extensive subsequent evolutionary stages. Even unmelted bodies in the 100 km size range have incurred extensive collisional evolution (Bischoff et al. 2006), producing compaction, fragmentation, and physical grinding and mixing on and beneath their surfaces, which may obscure the record of primary accretion. Model studies (e.g., Petit et al. 2001; Kenyon & Bromley 2004, 2006; Bottke et al. 2005; Chambers 2004, 2006; Weidenschilling & Cuzzi 2006) suggest that the collisional stage occurred after dispersal of the nebula gas allowed the orbital eccentricities of primitive bodies to increase. We are concerned with an earlier stage, when the still-abundant nebula gas led to a more benign environment with fewer and gentler collisions. The direct products of primary accretion might be most clearly visible in the rare “primary texture” seen in some CM (Metzler et al. 1992) and CO (Brearley 1993) chondrites. In these objects, or, more specifically, in unbroken fragments within them, the texture consists of similarly sized, dust-rimmed particles that are packed next to each other as if they had been gently brought together and compressed, with no evidence for local fracturing or grinding.

Even after collisional effects associated with subsequent stages of growth have blurred this signal, perhaps even mixing material from different parent bodies, evidence remains in the bulk properties of all chondrite classes. Chondrite classes are defined by their distinctive mineral, chemical, and isotopic properties (e.g., Grossman et al. 1988; Scott & Krot 2005; Weisberg et al. 2006). Large samples of material with a quite well-defined nature were accumulated at one place and/or time, and material of a quite different but equally well-defined nature was accumulated at another place and/or time into a different parent body. Among the most obvious aspects of these class properties is a dominance within chondrites of submillimeter-sized mineral particles (generically “chondrules”; Grossman 1988; Jones et al. 2000, 2005; Connolly et al. 2006), which are aerodynamically well sorted. Aerodynamic sorting of chondrite components was first emphasized by Dodd (1976), Hughes (1978, 1980), and Skinner & Leenhousts (1993), and it has since been discussed by a variety of authors (see Cuzzi & Weidenschilling 2006 for a summary of the observations and arguments supporting aerodynamics). Chondrules have a size that varies from class to class, but is distinctive and narrowly defined within a given class or chondrite.

The evidence suggests that chondrules do not comprise merely a thin surface layer swept up by a large object (Scott 2006). In the case that we suggest as an archetype, the H-type ordinary chondrite parent body is widely believed to be a 100 km radius object (perhaps the asteroid Hebe) that was initially composed entirely of a physically, chemically, and isotopically homogeneous mix of chondrules and associated material. This mix was thermally metamorphosed by accreted $^{26}$Al, to a degree that varied with depth, into an onion-shell structure and was subsequently broken up in several stages (Taylor et al. 1987; Tieloff et al. 2003; McSween et al. 2001; Grimm et al. 2005). Bottke et al. (2005) conclude, from models of collisional evolution in both the primordial (massive) and the current (depleted) asteroid belt, that the primordial asteroid mass distribution was dominated by objects that had diameters of around 100 km, rather than having a power-law size distribution somewhat like that of the current population.

We see the essential challenge as understanding how such a large object can be assembled from such a homogeneous mixture of millimeter-sized constituents, while other objects (arguably the parents of the L- and LL-type ordinary chondrites as well, and logically then the parents of the chondrites of all classes) are assembled from distinct, yet comparably homogeneous, ensembles of qualitatively similar particles. Moreover, this assembly, or primary accretion, phase of nebula evolution must persist for a duration of 1 Myr or more between the formation times of the oldest and youngest meteorites (Wadhwa et al. 2007), and even of objects found in the same meteorite. The $>1$ Myr age spread between ancient refractory inclusions (CAIs) and chondrules is well known (Russell et al. 2006), but there may even be a comparably extended age spread between different chondrules in a given meteorite (Kita et al. 2005).

In recent years, radioisotopic dating of achondrites (melted meteorites) has revealed some planetesimals that formed early (and melted early; Kleine et al. 2005, 2006). One expects early formation and melting to go together, because objects larger than only 10 km or so, which accreted at the same time that refractory inclusions formed (4.567 Gyr ago), would have incorporated their full complement of $^{26}$Al and would have melted (LaTourette & Wasserburg 1998; Woolum & Cassen 1999; Hevey & Sanders 2006). Thus, early accretion implies melting, and lack of melting requires late accretion. Model results imply that, in order to avoid complete melting, 100 km bodies must accrete only after a typical delay of 1.5–2 Myr relative to ancient CAI minerals (reviewed by McSween et al. 2002; Ghosh et al. 2006). While unknown selection and sampling effects may influence the limited meteorite data record, spectral analysis of the entire asteroid database also suggests that melted asteroids are rare. That is, while debate continues as to whether small amounts of melt might be found on the surfaces of many asteroids, which might be caused by collisions (e.g., Gaffey et al. 1993, 2002), only Vesta and a handful of other examples of fully melted basaltic objects have been found in spite of extensive searches (Binzel et al. 2002; Moskovitz et al. 2007).

Put together, this body of information suggests that primary accretion started early but continued for several million years; thus, it was inefficient, or at least sporadic. By comparison, midplane incremental accretion in a nonturbulent nebula accretes numerous Ceres-sized bodies and tens of thousands of 10–100 km
sized objects in only $10^5$ yr (Weidenschilling 2000); thus, it is highly efficient.

Models suggest that the density, temperature, and composition of the nebula evolved significantly over several million years (Bell et al. 1995; D’Alessio et al. 2005; Ciesla & Cuzzi 2006). Thus, one expects the properties of forming planetesimals to change with time, even though the chemical, isotopic, and mineralogical properties of the mixture of solids from which they formed might have been fairly uniform in space at any given time (see Cuzzi et al. 2003 and Ciesla 2007 for more detailed discussions). Reality might have been even more complicated; there is some evidence that chondrites of very distinct chemical and isotopic properties might have formed at roughly the same time (Kita et al. 2005), suggesting a combination of spatial and temporal gradients.

In this paper we present the overview of a scenario by which accumulation of a well-defined particle mix into a large planetesimal—making it a “snapshot” of the local nebula mix—might occur sporadically or inefficiently in weak turbulence over an extended period of time and throughout the terrestrial planet region. The challenge for the future is to show that the accretion processes discussed here can quantitatively account for the mass needed to build planetesimals of the mass needed to create planetary embryos and planets during the age of the nebula (Chambers 2004, 2006; Cuzzi et al. 2007).

2.2. Turbulence

A variety of astronomical and meteoritic evidence supports weak, but widespread and sustained, nebula turbulence. The observed abundance of small particles at high altitudes above the midplane in many visible protoplanetary nebulae for millions of years is most easily explained by ongoing turbulence, both to regenerate the small grains in collisions and to redistribute them to the observed altitudes (Dullemond & Dominik 2005). The survival of ancient, high-temperature mineral inclusions (CAIs) in chondrites, in spite of their tendency to drift inward toward the Sun, can also be explained by turbulent diffusion (Cuzzi et al. 2003, 2005). Moreover, the recent discovery by Stardust of both high- and moderate-temperature crystalline silicates in comet Wild2 (Brownlee et al. 2006; Zolensky et al. 2006) can be easily understood in this way (Ciesla 2007).

It remains a subject of active theoretical debate as to how, or even whether, the nebula can maintain itself in a turbulent state. Ongoing dissipation by molecular viscosity $\nu m$ on the smallest, or Kolmogorov, length scale requires a production mechanism to continually generate turbulent kinetic energy. Although the very existence of ongoing disk accretion into a star, with the ensuing gravitational energy release, provides an ample energy source, the vehicle for transferring that energy into turbulent gas motions is undetermined. Baroclinic instabilities require substantial opacity at thermal wavelengths, but they might be suitable for generating turbulence during the fairly opaque chondrule-CAI epoch in the inner solar system (Klahr & Bodenheimer 2003). The widely accepted magnetorotational instability (MRI; Stone et al. 2000) may or may not be precluded at the high gas densities of the inner solar system (Turner et al. 2007). The role of pure hydrodynamics operating on the radial shear remains unclear, because Keplerian disks are nominally stable to linear (small) perturbations (Balbus et al. 1996; Stone et al. 2000). However, a number of current theoretical studies suggest that turbulence might indeed be present (e.g., Arlt & Urpin 2004; Busse 2004; Umurhan & Regev 2004; Dubrulle et al. 2005; Afshordi et al. 2005; Mukhopadhyay et al. 2005; Mukhopadhyay 2006). Much of this recent work focuses on nonlinear (finite amplitude) instabilities occurring at the very high Reynolds numbers characterizing the nebula, which are well beyond the capability of current numerical and even laboratory models.

In this paper, we simply assume the presence of weak nebula turbulence. The intensity of the turbulence can be characterized by a nondimensional parameter $\alpha = (\nu / c_s)^2$, where $c_s$ is the velocity of the most energetic eddies and $c$ is the sound speed. Values of $\alpha$ in the range $10^{-4} \leq 1$ are consistent with observed nebula lifetimes and can be sustained by only a few percent of the gravitational energy released as the nebula disk flows into the Sun. One then estimates the most energetic (largest) eddy size as $L \sim H \alpha^{1/2}$, where $H$ is the nebula vertical scale height, or the disk thickness. A cascade to smaller scales ensues, where the smallest (Kolmogorov) eddy scale is $\eta = L \alpha^{-3/4} \sim 1$ km and $Re = \alpha c H / \nu m \sim 10^5$ (for $\alpha \sim 10^{-4}$) is the nebula Reynolds number (Cuzzi et al. 2001).

2.3. Turbulent Concentration

It has been found both numerically and experimentally that particles of a well-defined size and density are concentrated into very dense zones in realistic three-dimensional (3D) turbulence (Eaton & Fessler 1994). The optimally concentrated particle has a gas drag stopping time $t_g$ that is equal to the overturn time of the Kolmogorov-scale eddies. In the nebula, $t_g = r_p \rho_p / \rho_g$, where $r$ and $\rho$ are the particle radius and the material density and $c$ and $\rho_g$ are the gas sound speed and density. The Kolmogorov eddy frequency $\omega_\eta$ scales with the orbital frequency $\Omega$ as $\omega_\eta = \Omega \alpha^{-1/2}$, and particles are most effectively concentrated when $t_g \omega_\eta \sim 1$ (Eaton & Fessler 1994). Chondrule-size silicate particles are concentrated for canonical nebula properties if $\alpha \sim 10^{-5} \sim 10^{-3}$; moreover, the shape of the concentrated particle size distribution is parameter-independent and agrees very well with that of chondrules (Paque & Cuzzi 1997; Cuzzi et al. 2001). We define the concentration factor $C = \rho_p / \rho_g$, where we denote the local volume mass density of particles as $\rho_p$ and $\rho_g$ is its nebula average value, and the particle mass loading $\Phi = \rho_p / \rho_g = C \rho_p / \rho_g$. For instance, Figure 1 (Cuzzi et al. 2001) shows the probability distribution function (PDF) for values of $C$ determined from numerical simulations at moderate Reynolds numbers ($\text{curves with triangles}$) and predicted for plausible nebula Reynolds numbers ($\text{curves without triangles}$). Note that the volume fraction that has $C > 100$ is quite substantial, which helps to explain some mineralogical and isotopic properties of chondrules (Cuzzi & Alexander 2006). Not surprisingly, the volume fraction decreases for larger values of $C$. In this work we focus on the less abundant, but higher mass density, concentrations toward the lower right-hand side of the PDF.

The turbulent concentration PDFs of Figure 1 did not account for the feedback of the concentrated particles on the gas. This “mass loading” would be expected to damp gas turbulent motions once the particle mass density significantly exceeded that of the gas, and to lead to a saturation of concentration at some value. Recent work that includes the feedback effects, through drag, of particle density on the damping of turbulence, has shown that values of $\Phi \leq 100$ can indeed be achieved, although less commonly than in the unloaded models such as are shown in Figure 1. This “cascade model” of turbulent concentration (Hogan & Cuzzi 2007) is summarized in Appendix A. On the basis of
1995; Cuzzi et al. 1993, 1994; Johansen et al. 2007). That is, a

Fig. 1.—Cumulative probability distribution, or volume fraction $F_p^\alpha$, of particles that have local concentration factors larger than some value of $C$ (from Cuzzi et al. 2001; see also §2.3). The curves with triangles represent a comparison of the predictive theory described in Cuzzi et al. (2001) with 3D numerical simulations at moderate Reynolds numbers; the curves without triangles are extensions of the same theory to nebula conditions denoted by their values of $\alpha$. Concentration factors of $C \sim 100$ for chondrule precursors (leading to $\Phi = \rho_p/\rho_0 \sim 1$) are fairly common for a range of nebula turbulent intensities (denoted by $\alpha$) and help to explain the lack of chondrule isotopic fractionation (Cuzzi & Alexander 2006). In this paper we focus on the much less common, but much denser, regions to explain the lack of chondrule isotopic fractionation (Cuzzi & Alexander 2006).

these results, we adopt a value of $\Phi = 100$ as an upper bound in all stability and evolution modeling.

3. PRIMARY ACCRETION OF PLANETESIMALS

The ability of gravity to overwhelm opposing forces is a well-known theme in astrophysics, dating back 80 years to early studies of star formation by Jeans. Over 30 years ago, gravitational instability (GI) of solids in a dense layer near the nebula midplane was proposed to lead directly to solid planetesimals on a dynamical timescale (the orbit time; Safronov 1969; Goldreich & Ward 1973). Traditional GI thresholds require the dynamical (collapse) time of a dense region, $t_G = \pi (G \rho_p)^{-1/2}$, where $G$ is the gravitational constant, to be shorter than both the transit time due to random velocity and the local shear (vorticity) timescale ($\tau_v \approx 10^3$). These arguments imply that GI occurs when the local particle density exceeds a few times the Roche density, $\rho_R = (3M_\odot)/(4\pi a^3)$, where $M_\odot$ is the solar mass and $a$ is the distance from the Sun (Safronov 1991; Cuzzi et al. 2001). However, this scenario is frustrated by self-generated midplane turbulence even if the nebula is not globally turbulent (e.g., Weidenschilling 1980, 1995; Cuzzi et al. 1993, 1994; Johansen et al. 2007). That is, a dense layer of centimeter-size and larger particles orbits at a different velocity than the pressure-supported gas, leading to a vertical velocity shear that becomes turbulent and stirs the particle layer. A more recent flavor of GI requires the particles to be small enough that the particle-rich midplane acts as a single fluid, like fog, so that its vertical density stratification stabilizes it against self-generated turbulence (Sekiya 1998; Sekiya & Ishitsu 2001; Youdin & Shu 2002; Sekiya & Takeda 2003; Youdin & Chiang 2004; Garaud & Lin 2004). Because the particles must be very small to satisfy this “one-phase” requirement (millimeter size and smaller), their settling toward the midplane is frustrated by even the faintest breath of nebula turbulence ($\alpha \sim 10^{-3}$). If the particles are prevented from settling into a sufficiently dense layer such that their mass density can stratify the total density (e.g., Dubrulle et al. 1995; Cuzzi & Weidenschilling 2006), the conditions needed to trigger the instability are never achieved.

In the following subsections, we study the evolution of a dense clump such as might form toward the lower right domain of Figure 1, albeit in a simplified way. We treat a clump as an isolated object, when in reality it exists in a context of surrounding material of varying densities. In §4, we discuss the implications of more realistic conditions.

3.1. Gas Pressure Precludes Particle Gravitational Instability

The role of the gas pressure in GI for nebula particles has been almost universally ignored, even though its importance was pointed out decades ago by Sekiya (1983). We encountered it ourselves independently. To test our GI thresholds, we initially ran numerical models of static, but very dense, clumps ($\Phi = 1000$), expecting them to collapse with gravitational free-fall or dynamical collapse times $t_G = \pi (G \rho_p)^{-1/2}$. Here $\rho_p = (l/c)(G \rho_p)^{1/2} = 1/\Phi^2$ is a Mach number. Nebula values of $c$ and $\rho_p$ lie in the top set of nearly linear curves, allowing only slow inward sedimentation to occur on the timescale $t_{\text{sed}}$ (for $\Phi_0 = 100$). That is, under nebula conditions of gas density and temperature, as well as particle loading, the gas behaves as if it were incompressible and prevents the dynamical timescale GI for particles with stopping times of much less than the dynamical collapse time.

Fig. 2.—Results of a 1D compressible numerical model, showing how gas pressure precludes dynamical timescale gravitational instability (Sekiya 1983), as is described in more detail in §3.1. The initial condition was a Gaussian blob with a density distribution of $\rho_p(r) = \rho_{0p} \exp(-r^2/2)$. Here $\rho_p = (l/c)(G \rho_p)^{1/2} = 1/\Phi^2$ is like a Mach number. Nebula values of $c$ and $\rho_p$ lie in the top set of nearly linear curves, allowing only slow inward sedimentation to occur on the timescale $t_{\text{sed}}$ (for $\Phi_0 = 100$). That is, under nebula conditions of gas density and temperature, as well as particle loading, the gas behaves as if it were incompressible and prevents the dynamical timescale GI for particles with stopping times of much less than the dynamical collapse time.

$\Phi_0 = 100$
where nebula parameters lie, well within the flat curves at the top of the plot with $Ma \sim 10^{-4}$ for $l \sim 10^4$ km, $c \sim 10^3$ cm s$^{-1}$, and $\rho_g \sim 10^{-16}$ g cm$^{-3}$. The high value of $Ma$ required for true dynamical timescale collapse (given our chosen value of $\Phi = 100$) would require a smaller gas sound speed or a larger clump size $l$ and gas density $\rho_g$, by orders of magnitude.

The slow shrinkage in the stable regime results from particles sedimenting inward toward their mutual center under their own self-gravity at their terminal velocity $v_t = g t_s$, where $g = 2 \Phi l \rho_g$ is the local gravitational acceleration due to the clump’s mass and $t_s = r_p c / \rho_p$ is the particle stopping time. Thus, $v_t = 2 \Phi l \rho_p / c < V_G$ for $t_s < t_G$, and the “sedimentation” timescale is

$$t_{sed} = \frac{l}{2 v_t} = \frac{1}{4 G \Phi (r_p / c)} = \frac{c}{4 G \Phi l \rho_p} = \frac{1}{4 G \Phi p_b c},$$

roughly 30–300 orbit periods at 2.5 AU for 300 $\mu$m radius chondrules and $\Phi = 1000–100$; that is, much longer than $t_G$. It seems inappropriate to describe this slow ongoing evolution as an instability, especially in midplane scenarios, where it was slow vertical sedimentation by individual particles toward the midplane that produced the “unstable” situation in the first place, and where only further slow sedimentation transpires. The situation is more akin to star formation mediated by ambipolar diffusion than by traditional gravitational instability. Gas pressure even precludes the Safronov-Goldreich-Ward GI in the form in which it was originally proposed, where centimeter-sized particles were envisioned (although $t_{sed}$ would be faster for a literall centimeter-sized particles, and it more closely approaches $t_G$).

The results of Sekiya (1983) have apparently been overlooked by all subsequent workers, who have invariably assumed a Toomre-Safronov-Goldreich-Ward-type particle ensemble that is decoupled from the gas and stabilized (on small scales) only by particle random velocities. This gas pressure constraint is, however, crucial for particles in the chondrule-size regime, where $t_s \sim 1$ hr and $t_G \sim 1$ yr, and must be applied to all GI models that involve chondrule-sized particles. The good news is that for the interesting range of nebula gas density and particle concentration values, the gas does actually behave as if it were incompressible (validating the use of an incompressible code in numerical models such as that shown in § 3.4).

The physics is easily understood in the one-phase regime ($t_s \ll t_G$). The particles feel no direct pressure force, but they are firmly trapped to the gas, which does. The (inward) gravitational force is dominated by particles for $\Phi \gg 1$. The particles drag and compress the gas with them as they begin to collapse under their self-gravity, producing a radial gas density gradient $d \rho_g / dl$, which translates into a large outward pressure gradient $c^2 d \rho_g / dl$ that acts on the gas, and thereby also on the strongly coupled particles.

The gravitational force per unit volume is $f_g = 4GM \rho_b l^2 \sim 2G \Phi l^2 \rho_g$, where $M$ is the clump mass and $R$ is its radius. After some incipient shrinkage and compression occurs, the outward pressure gradient force per unit volume becomes roughly $f_{pg} = c^2 \rho_g l / c^2 > 1$, which gives the criterion for GI as $\Phi > \Phi^*$, where $\Phi^* = c l (G \rho_g)^{1/2}$. The quantity $\Phi^*$ is like an inverse Mach number $Ma^{-1}$, where $Ma = (l/c)(G \rho_g)^{1/2}$. An alternate (perturbation) approach compares the incremental changes $\delta f_g$ and $\delta f_{pg}$ associated with a small gravitationally induced shrinkage characterized by $b l \ll l$. It is straightforward to show (for $t_s \ll t_G$) that $\delta f_g / \delta f_{pg} \approx c^2 \rho_g / GM \Phi \sim 10^3$ under nebula conditions. Thus, the clump is “stiffened” by pressure, and even a small incremental shrinkage is self-limiting. Inspection of Figure 2 shows small oscillations, which suggests that a formal linear stability analysis might lead to an improved stability criterion; this is, however, beyond the scope of the present paper.

For values of $l \sim 10^1$ km and $\rho_g \sim 10^{-16}$ g cm$^{-3}$ (Desch et al. 2005), $\Phi^* \sim 10^4$. This criterion for GI can be compared with the traditional criterion for (marginal, or maximal) instability that is widely used as the criterion for gravitational instability, roughly twice the Roche density $\rho_R = (3 M_p) / 4 \pi a^3$ (Goldreich & Ward 1973; Safronov 1991; Cuzzi et al. 1993). The ratio of the two thresholds is

$$\frac{\Phi^* \rho_g}{2 \rho_R} = \frac{\rho_g}{2 \rho_R Ma} \approx \frac{2c (G \rho_g)^{1/2}}{\Omega^2 l} \sim 10^3 km / l^{1/3}$$

for $\rho_g = 10^{-10} - 10^{-9}$. For the midplane GIs, which have been widely discussed in the past (reviewed by Cuzzi & Weidenschilling 2006), $l \sim 10^3$ km, so the traditional GI criterion falls short by a factor of more than $10^3$ (cf. Sekiya 1983). This degree of mass concentration is unlikely under any plausible circumstances, especially for particles that are already well trapped to the gas. For the ubiquitous dense clumps that we discuss here, which form at all elevations, $l \sim 10^4$ km. Thus, the pressure-supported gas phase prevents the tightly coupled particles from undergoing GI until the particle mass loading is hundreds of times larger than the traditional GI criterion (which is on the order of the Roche density $\rho_R$).

Nevertheless, Sekiya (1983) also noted that when $\rho_p$ is in the range usually cited for GI (a few times $\rho_R$, or $\Phi \sim$ a few hundred), a 3D “incompressible mode” arises, which, while not explicitly stated, we interpret as retaining an identifiable cohort of particles. An “incompressible mode” might resemble a blob of water oscillating in zero gravity. Dense zones that result from turbulent concentration are candidates for such incompressible modes, even at mass loading values that are too low to induce actual GI. However, the fate of such slowly evolving entities in the presence of likely perturbations has never been explored. Below we discuss the least avoidable, most pervasive disruptive perturbation that such clumps would encounter once they form, which we believe to be ram pressure disruption due to systematic velocity differences between the dense zones and the gas, and then derive conditions under which these clumps can survive to become planetesimals.

### 3.2. The Fate of “Incompressible” Clumps

Several kinds of perturbation by the enveloping nebula gas might disrupt a dense clump before the slow sedimentation of its constituent particles toward their mutual center (on the timescale $t_{sed}$) can produce a moderately compact object.

Perhaps the simplest kinds to discuss and dismiss are turbulent pressure, velocity, and/or vorticity fluctuations encountered by the clump. Turbulent pressure fluctuations on the scale $l$ have a typical intensity of $\rho_g$ on $t_G$ $^2$, where $\rho_g$ on $t_G$ is certainly smaller than the fluctuating velocity $v_l$ ($L \sim c \alpha$ the dominate energy-containing eddy). The ratio of even the largest eddy pressure fluctuations to the steady ram pressure from the headwind $\beta \alpha = \beta K$ (a is the semimajor axis, and $K$ is the Keplerian velocity) is $v_l / (\beta K)^2 < \alpha / \beta < 1$ unless $\alpha > 10^{-3}$. Nebula evolution timescales, and parameters leading to turbulent concentrations of chondrule-sized particles in the asteroid belt region, imply that $10^{-6} < \alpha < 10^{-3}$, depending on the gas density and other properties (Cuzzi et al. 2001). This suggests that turbulent velocity fluctuations on clump length scales play a small role. We have run simulations of clumps settling in gravity with and
without enveloping turbulence, and the typical evolutions are qualitatively similar.

We next address local average vorticities on the length scale of a clump. It has long been known that local vorticity is a factor in gravitational instability (Toomre 1964; Goldreich & Ward 1973). Because eddies much smaller than the largest scale of turbulence typically have larger vorticities than the Keplerian shear component (Toomre 1964; Goldreich & Ward 1973). A clump. It has long been known that local vorticity is a factor in gravitational instability (Toomre 1964; Goldreich & Ward 1973).

Qualitatively similar solutions are derived and applied.

Fig. 5 in Appendix A. Below we sketch how such a constraint is derived and applied.

A simplified requirement for gravitational binding of a clump of length scale \( l \) and mass density \( \rho_p \), in the presence of local rotation at angular velocity \( \omega(l) \), is \( 4\pi G \rho_p \sigma_p > \omega^2(l) \). Recalling that the local vorticity \( \omega(l) \) can vary by orders of magnitude from its average value \( \langle \omega(l) \rangle \), we normalize both sides of the relationship between \( \omega_p \) and \( \omega(l) \) by the average inertial range enstrophy \( \langle \omega^2(l) \rangle \) and define the normalized enstrophy: \( S = \omega^2(l)/\langle \omega^2(l) \rangle \). This quantity provides the horizontal axis on Figure 5. Then the simple expression above can be rewritten as \( \Phi > \langle \omega^2(l) \rangle S / 4\pi G \rho_p \). Inertial range relationships can be used to express \( \langle \omega^2(l) \rangle = \Omega^2(L/l)^{4/3} = \Omega^2(L/B)_{(l)}^{4/3} \), where \( \Omega \) is the Kolmogorov scale and \( B \) is some scaling factor. Making use of \( \Delta = \alpha \Omega \Delta \equiv \alpha \Omega (L/B)_{(l)}^{4/3} \) as discussed earlier, and substituting nebula parameters at 2.5 AU, we find that gravitational binding requires a value of \( \Phi > 3 \times 10^3 \alpha S \). This constraint yields diagonal lines on plots of the sort shown in Figure 5 (for an example, see Cuzzi et al. 2007). Interesting regions of parameter space remain accessible to a degree that depends on the quantitative modeling of the PDF \( P(\Phi, S) \).

Obviously, more detailed numerical models of clumps in realistic turbulence are needed to assess these perturbations. We will assume, for the purposes of this study, that turbulent pressure (and associated vorticity) fluctuations are a minor influence and that regions that are stable to their own local vorticity exist (see Cuzzi et al. 2007). This leaves the dominant disruptive process, unavoidably shared by turbulent and laminar nebulae, as ram pressure disruption due to systematic velocity differences between the clump and the gas, as described below. Incidentally, survival against ram pressure gives the horizontal stability boundaries in Figure 3 of Cuzzi et al. (2007). Placing these constraints on PDFs obtained from cascade models is somewhat involved, and a full quantitative description of the situation is deferred to a future paper.

3.3. Ram Pressure Disruption and the Weber Number

Here, for simplicity, we envision the evolution of a single, isolated dense clump with \( \Phi \gg 1 \), merely as an example of one of the many dense clumps formed near the lower right-hand corner of Figure 1. The clump exerts a strong collective influence on the entrained gas, and the mixture experiences solar gravity as an orbit. It attempts to move as an individual large object relative to the nebula gas, dragging the entrained gas along. For instance, a clump forming at some distance above the nebula midplane settles downward under the acceleration of the vertical component of solar gravity, whereas the surrounding gas remains under vertical hydrostatic balance. Also, nebula gas generally orbits at a slower velocity than does the Keplerian orbit velocity \( v_K \) obeyed by massive particles. This is because of its generally outward radial pressure gradient force \( -1/(\rho_0) dP/da \approx 2\beta \alpha a^2 \), where \( a \) is the distance from the Sun, and \( \alpha \approx 10^{-3} \). Thus, particles experience a headwind with magnitude \( w_\phi \sim \beta \alpha a \approx 3 \rho_K \) \( \sim \) a few \( \times 10^3 \) cm s\(^{-1}\). These headwinds result in a ram pressure of \( \rho_0 w_\phi^2/2 \), which can disrupt a strong clump.

For example, a dense drop of ink settling in a glass of water is quickly disrupted by the Rayleigh-Taylor instability and mixes with its surroundings (Thomson & Newhall 1885). However, a dense drop of fluid with surface tension can avoid disruption and settle indefinitely at terminal velocity (as do raindrops, or water in oil). In this situation, the criterion for stability of a fluid droplet, which determines its terminal velocity and thence its size, is given by the Weber number \( We \), where \( We = 2\rho_\phi a^2/\gamma \), and \( \gamma \) is the surface tension coefficient. Drops are disrupted when \( We \) exceeds some critical value \( We^* \sim 1–10 \) (Pruppacher & Klett 1997).

Dense particle clumps in the nebulae have no surface tension, but they do have self-gravity. By direct analogy with the droplet surface tension criterion, we define a gravitational Weber number \( We_G \) as the ratio of the ram pressure force per unit area to the self-gravitational force per unit area for a flattened disk, which is the initial stage of a strengthless spherical clump of diameter \( l \) and particle density \( \rho_p \) upon encountering a headwind \( w_\phi \) (Thomson & Newhall 1885). For a disk of surface mass density \( \sigma_p = \rho_p l \), the gravitational force per unit area is \( G \sigma_p^2 \). Then

\[
We_G = \frac{C_D \rho_p w_\phi^2}{2 G \sigma_p^2} = \frac{C_D w_\phi^2}{2 G \Phi^2 \rho_p l^2},
\]

where \( C_D \) is an effective drag coefficient for the clump, on the order of unity (see Appendix B). The quantity \( We_G \) can be written in other ways as well. The orbit frequency at \( a \) provides the useful relationship \( \Omega^2(a) = (GM_\odot/a^3)^{1/2} \equiv (G \rho^* a^3)^{1/2}, \) or \( G = \Omega^2/a^3 \), where \( \rho^* = M_\odot/a^3 \approx 3.8 \times 10^{-6} \) g cm\(^{-3}\) at 2.5 AU; then

\[
We_G = \frac{C_D \rho^* w_\phi^2}{2 \Phi^2 \rho_p \Omega^2 l^2} = \frac{C_D \beta \rho^* a^2}{2 \Phi^2 \rho_p l^2}.
\]

By analogy with the more familiar surface tension case, there will be some critical gravitational Weber number for stability, \( We_G^* \), which is probably on the order of unity (Pruppacher & Klett 1997); its value must be constrained by numerical experiments as described below. Then we require for stability against headwind disruption that \( We_G < We_G^* \), or

\[
\Phi l > \frac{w_\phi}{(2 G \rho_p We_G^* / C_D)^{1/2}} = \frac{\beta \alpha \Omega}{(2 G \rho_p We_G^* / C_D)^{1/2}}.
\]

These expressions determine the combination of the clump diameter \( l \) and the particle loading \( \Phi \) that will stabilize it against a headwind of magnitude \( \beta \alpha \Omega \). The headwind due to the nebula radial pressure gradient will occur even if a clump is at the midplane and has zero settling velocity; this gives the lower limit on ram pressure that the clump must be stable against. Neglecting vertical settling restricts potentially stable clumps to lying within about 0.01 gas scale heights of the midplane, where the headwind \( \beta \Omega a \) is comparable to the vertical settling velocity. This

\[\text{Note that } \rho^* \text{ is different from Safronov’s (1991) Roche density } \rho_R = (3 M_\odot)/(4\pi a^3), \text{ which we discussed earlier.}\]
restriction must be factored into statistical estimates of planetesimal production (see, e.g., Cuzzi et al. 2007).

For typical nebula parameters, we assume the gas density to be in the range from $\rho_{g} = 10^{-10}$ g cm$^{-3}$ (a nominal minimum mass value) to $10^{-9}$ g cm$^{-3}$ (a more recent value supported by nebula evolution and chondrule formation; cf. Desch et al. 2005), the semimajor axis $a = 3.8 \times 10^{13}$ cm (2.5 AU), the pressure gradient/headwind parameter $\beta \sim 10^{-3}$ (see, e.g., Nakagawa et al. 1986; Cuzzi et al. 1993), and the solar mass $M_\odot = 2 \times 10^{33}$ g. In this regime, $\Phi \gtrsim 1.5 - 5 \times 10^{6} (\beta/10^{-3})$ km. Recall from § 2.3 and Appendix A that mass loading limits the maximum achievable mass loading to $\Phi \sim 100$. A clump satisfying the above constraint, with $l = 1 - 5 \times 10^{4}$ km, would have the mass of a $10 - 100$ km radius body of unit density; thus, these precursors can lead to quite sizeable objects. This characteristic size range is intriguingly close to the roughly 50 km radius primordial building-block size of Bottke et al. (2005). The very existence of any preferred size for the primordial population, if true, is also an intriguing result.

3.4. Numerical Model of Clump Evolution

To obtain a sanity check on the concepts of § 3.3, and to constrain the (unknown) value of $\text{We}_{3}$ for this problem, we developed a very simplified numerical model of a clump experiencing a steady nebula headwind from a more slowly orbiting, pressure-supported gas, using Hill’s approximation, which transforms a cylindrical system into a Cartesian system rotating at some mean rate $\Omega_0$; this is useful if the domain covers a sufficiently narrow radial and angular region that the curvature is negligible. The overall orbital motion of particles (the Keplerian velocity) is thus subtracted out, and the gas exhibits a small differential velocity because of its deviation from Keplerian. A particle clump under the influence of the gas will slowly lag behind in the (orbital) y-direction and slowly drift to smaller (radial) x-values, due to the gas headwind. The model is not intended to be a high-fidelity representation of a realistic nebula situation, but merely to check the basic Weber number model of § 3.3.

The code evolves the velocity field of a two-phase system. The gas is perturbed by a clump of particles, which are mutually attracted by gravitational forces and are themselves dragged by, and exchange momentum with, the gas. The Hill frame, with Cartesian coordinates $(x, y, z)$, represents a radially narrow region corresponding to some range of radius $a$, orbital angle $\theta$, and vertical distance $Z$, respectively. Periodic boundary conditions are used in all three dimensions. The instantaneous Navier-Stokes (Eulerian) equations describing the conservation of mass and momentum for the incompressible gas are expressed in the Hill frame as

$$\nabla \cdot \mathbf{U} = 0, \quad \frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{U} = \frac{\nabla P + \nabla P^{\text{glob}}}{\rho_{g}} + \nu \nabla^2 \mathbf{U} - 2\Omega_0 \times \mathbf{U} + \mathbf{f}^{\text{drag}} + \mathbf{f}^{\text{sp}},$$

where $\mathbf{U}$ is the fluid velocity, $\rho_{g}$ is the gas mass density, $\nu$ is the gas viscosity, $P$ is the local pressure (which fluctuates along with gas velocity and vorticity variations), $\nabla P^{\text{glob}} = (2\rho_{g}w_{g}^{2} \Omega_0, 0, 0)$ represents the (constant) global nebula radial pressure gradient, and $\Omega_0 = (0, 0, \Omega_0)$ is the Keplerian frequency vector. This setup causes the gas to flow toward the clump in the y-direction at a uniform speed $w_{g}$. We are currently neglecting the Keplerian shear term $2\Omega_0 \frac{x}{r} w_{g}^{2} \omega_{g}$, where $\Omega(a) = \Omega_0 (a/a_0)^{-q}$ and $\Omega_0 = \Omega(a_0)$, in the gas and particle equations in order to get a better comparison with our analytical stability model. The Keplerian shear rate associated with this term is $(dw/dx)_{\text{Kep}} = \Omega_{0}/2$, whereas a typical (fluctuating) local shear or strain due only to turbulent motions is $(dw/dx)_{\text{turb}} \sim \omega(t) > \Omega_{0}$. Because the turbulent shear is so widely varying, and likely to be larger than the Keplerian shear, in this study we neglect both of these complications, although they are both suitable avenues for future work.

In our code, particles are followed in the Lagrangian sense. The term $f^{\text{drag}}$ represents the force per unit mass imparted to the gas by the particles. It has the general form

$$f^{\text{drag}} = \frac{\rho_{p}}{\rho_{g} \tau_{sp}} (\mathbf{V} - \mathbf{U}),$$

where $\mathbf{V}$ is the mean weighted particle velocity at a grid point, $t_\ast$ is the particle gas drag stopping time, and $\rho_{p}$ is the particle mass density. To obtain $\mathbf{V}$, a weighted sum is carried out over particles in the eight cell volumes adjacent to each fluid grid point. Weighting functions are used that vary inversely with the distance of the particle from the grid point (see § 5.1 of Squires 1990). Because of the periodic boundary conditions, the wake of a clump can impinge artificially on the clump from the upwind direction. To avoid this, we include the term $f^{\text{sp}}$ in equation (6) as a “sponge force” per unit mass in order to restore the gas velocities downstream of the clump at the outflow boundary plane to their initial values $U_0$; this ensures constant inflow values at the upstream boundary plane.

The sponge force has the form

$$f^{\text{sp}} = \frac{1}{\tau_{sp}} (\mathbf{U} - \mathbf{U}_0),$$

where $\tau_{sp}$ is some timescale. The function $1/\tau_{sp}$ is modeled as a sigmoid in the y-direction very near the outflow boundary. The parameters of the sponge were determined through test runs with the goals of minimizing the sponge’s spatial extent and maximizing its effectiveness without introducing numerical instabilities. Once determined, the same values were used for all production runs.

The (Lagrangian) equation describing the motion of particle $i$ subject to the forces of gas drag and mutual gravity is (again neglecting the Keplerian shear term)

$$\frac{d\mathbf{V}_i}{dt} = 2\Omega_0 \times \mathbf{V}_i + f^{\text{drag}}_i + f^{\text{grav}}_i.$$ 

The term $f^{\text{drag}}_i$ describing the drag force per unit mass on particle $i$ by the gas takes the form

$$f^{\text{drag}}_i = \frac{1}{\tau_{sp}} [\mathbf{U}(\mathbf{X}_i(t)) - \mathbf{V}_i(t)],$$

where $\mathbf{U}(\mathbf{X}_i(t))$ is the gas velocity interpolated to the particle’s position $\mathbf{X}_i(t)$ at time $t$.

Finally, the mutual gravity force per unit mass on each particle is

$$f^{\text{grav}}_i = G \sum_{j \neq i} N_p \frac{m_j}{|\mathbf{X}_j(t) - \mathbf{X}_i(t)|} \left[ |\mathbf{X}_j(t) - \mathbf{X}_i(t)| + \epsilon \right],$$

where $G$ is the gravitational constant and $N_p$ is the number of particles. The quantity $\mathbf{X}_j(t)$ is the unit vector associated with the distance $\mathbf{X}_{ij}(t) = \mathbf{X}_i(t) - \mathbf{X}_j(t)$ between particles $j$ and $i$, and $\epsilon$ is a constant that softens the force at small separations in order to
prevent numerical singularities, chosen to be comparable to a grid cell in extent.

Equations (6) and (9) are solved using pseudospectral methods commonly used to solve Navier-Stokes equations for a turbulent fluid (Canuto 1987). Periodic boundary conditions are assumed, and the number of nodes used in each direction is generally 128, 256, and 128, with a spacing of \( \frac{2\pi}{64} \). A fast Fourier transform (FFT) algorithm is used to evaluate the dynamical variable \( U \) at the computational nodes. A second-order Runge-Kutta scheme is used to time-advance the gas and particle velocities. A third-order Taylor series interpolation scheme is used to determine the gas velocities at the particle positions from values at the eight nearest neighbor nodes. The mutual gravity calculation is done in a brute-force fashion, using an \( N^2 \) algorithm to evaluate the force contributions from all particles. The code is written in Fortran 77 and is parallelized using OpenMP directives. The FFT calculations are done simultaneously on the planes perpendicular to the \( y \)- and \( z \)-axes, and all loops involving particle indices are parallelized. Typically, 3000 “superparticles” are used, but some runs used more than 20,000. Each “superparticle” represents the dynamical effects, and typical response, of a large number of actual chondrules.

Initially the particles are arranged in a uniform spherical clump. The initial velocities for the gas and the particles inside and outside the clump are determined from the expressions in Nakagawa et al. (1986):

\[
U_0 = w_g \frac{\rho_p}{\rho_g + \rho_p} \left( \frac{2D\Omega}{D^2 + \Omega^2}, \frac{D^2}{D^2 + \Omega^2}, 0 \right),
\]

\[
V_0 = -w_g \frac{\rho_g}{\rho_g + \rho_p} \left( \frac{2D\Omega}{D^2 + \Omega^2}, \frac{D^2}{D^2 + \Omega^2}, 0 \right),
\]

where \( D = (\rho_g + \rho_p) \rho_p t_i \). The initial velocity of the gas outside the clump is found by setting \( \rho_p \to 0 \) in equation (12): \( U_0 = (0, -w_g, 0) \); inside the clump, the specific case value of \( \Phi \) is used in equations (12) and (13) to initialize the gas and particle velocities.

Runs using single particles with different values of \( t_i \) were made to verify that the Nakagawa et al. (1986) initial conditions are steady-state solutions for the particle equations. The wall clock time to evolve one integration step is 0.7 s using 256 Intel Itanium 2 processors running at 1.5 GHz. The wall clock time to evolve 1 orbital period is 121 hr.

3.5. Results of the Numerical Model

Figure 3 shows how, as predicted by the simple theory of § 3.3, certain combinations of clump mass loading \( \Phi \), and dimension
remain stable against ram pressure disruption for nebula headwinds produced by a pressure gradient characterized by $\beta$. The Weber number $We$ is dimensionless, and the second expression in equation (2) can be used to obtain the critical value of $We_G$ that separates stable from unstable configurations of the clump, $We_G = We_G^*$, directly from measured values in code units (see Appendix B). We expect that $We_G^*$ will be in the range 1–10. The combination of parameters in the stable case shown (right) gives $We_G \sim 1$; if self-gravity is turned off, the clump is disrupted in $t_{\text{dis}} \sim a^2 / (2\pi)$ orbits, whereas it will sediment into itself on a timescale $t_{\text{sed}} \sim 8$ times as long (see Appendix B). In the stable cases, a dense core is seen to continually shrink and become denser throughout the run, even as material is shed from the periphery of the clump, such that the value of $\Phi/l$ for the core continues to increase (see Appendix C and Fig. 8).

The numerical clump models exhibit considerable erosion over the duration of the runs from a viscously stirred surface layer, and this gradual but inexorable mass loss might lead to the disruption of the clumps if arbitrarily long runs were practical at this time. However, as is discussed in Appendices B and C, this large amount of erosion is a gross overestimate of what would happen in the nebula, because numerical viscosity (and consequently, the shear around the periphery of the clump) plays a far larger role in the current numerical models than it ever would in the actual nebula. That is, the viscously stirred and eroded surface layer in the numerical clumps contains orders of magnitude more mass than would be the case for a nebula clump of the size and density required to survive ram pressure disruption (see Appendix B). In Appendix C, we describe the movies from which Figure 3 is taken and also show how the protected inner regions of the clumps are behaving exactly as predicted, in both the stable and the unstable regimes. For more realistic, larger clump Reynolds numbers ($Re_c$) in the nebula, a far larger fraction of the clump mass will show this behavior, rather than being artificially eroded.

The primary purpose of the numerical models is merely to provide an independent sanity check on the physics of our Weber number model and obtain some idea of $We_G^*$, which gives us an order-of-magnitude estimate for the product $\Phi/l$ that nebula clumps require in order to survive headwinds of a particular value of $\beta\rho_g$. Future coding improvements are needed in order to allow larger (higher $Re_c$) clumps to occur, which would more closely approach nebula conditions in terms of their balance between pressure forces and viscous forces (see Appendices B and C); these improvements will probably include an implicit time advance for the drag terms, and perhaps also a tree or particle-in-mesh code for the particles.

4. DISCUSSION AND SUMMARY

We have shown how self-gravity can stabilize dense clumps of millimeter-sized particles, which form naturally in 3D turbulence, against disruptive gas ram pressure on timescales that are sufficiently long for their constituent mineral grains to sediment toward their mutual centers and form physically cohesive “sandpiles” of order 10–100 km in size. The essence of the result is a critical “gravitational Weber number” on the order of unity, in which self-gravity plays the role of surface tension in more familiar situations such as raindrops. Characteristic mass densities and length scales are determined that meet this requirement. We show numerical results that are in general agreement with the predictions of the simple theory for isolated, spherical initial clumps.

The scenario we have sketched out leads from aerodynamically size-sorted nebula particles, which have the properties of chondrules, to sizeable planetesimals formed entirely from such particles, which contain a snapshot or grab-sample of the local particle mixture. Turbulent concentration first produces the dense zones of size-sorted particles. The more common, less dense of these regions may provide typical chondrule melting environments (Cuzzi & Alexander 2006). Some of the less common, very dense zones, which we have shown elsewhere can achieve mass densities that are 100 times larger than that of the gas, have the potential to become planetesimals, depending on their length scales, nebula locations, and local vorticities. It is intriguing that our characteristic stabilized clump masses are not far from the mass inferred by Bottke et al. (2005) to represent a typical primordial object in the predispersal, pre-erosional asteroid belt.

Ultimately, the sandpiles resulting from completed sedimentation will become compacted further by inevitable collisions with other sandpiles, leading to today’s fairly dense asteroids, while retaining a physical and chemical memory of their parent particle clumps. The mechanism is easily extended to the unmelted aggregate particles of the outer solar system, where, however, if chondrules are absent, the size-sorting fingerprints of the process (Cuzzi et al. 2001) might be less evident.

The conclusions of this paper differ from the suggestions in Cuzzi et al. (2001), who focused on the possible role of ultradense clumps with sizes comparable to a Kolmogorov scale (0.1–1 km). Since that time, Hogan & Cuzzi (2007) have found that particle mass loading saturates the value of $\Phi = \rho_p / \rho_g$, at a value of about 100, which redirected our attention to larger clumps in the $10^{-3}$–$10^{-4}$ km size range. Clumps and fluid structures of these large sizes are much more accessible with numerical codes, both of the standard direct simulation type and the cascade type (Appendix A).

In this scenario, primitive bodies may not primarily represent spatial, but perhaps temporal, samples of the particulate contents of the nebula as its chemical, physical, and isotopic properties evolve over several Myr. The temporal, as well as the spatial, variation in the physical, chemical, and isotopic properties of the concentrated particles, which are being continually altered by thermal events and mineralogical alteration in the nebula gas, can then help account for class-to-class variations between the chondrite groups. An implication of this drawn-out, inefficient process is that younger chondrite types should contain evidence of “leftovers” or “refugees” from earlier times that escaped primary accretion. Several aspects of chondrite makeup are compatible with this implication. Of course, it is well known that ancient CAI minerals and less ancient, less refractory amoeboïd olivine aggregates are found alongside much younger objects in the same chondrites (e.g., Scott & Krot 2005). Also, the oldest chondrites (CVs) contain primarily type I chondrules (which have an age observationally indistinguishable from that of CAIs) and no (more oxidized, heavier O-isotope) type II chondrules, while the younger ordinary chondrites contain a mixture of type I and type II chondrules, some with hints of age variation even within a given chondrite (Kita et al. 2005). The oldest and first-formed objects are all likely to have melted from their abundant $^{26}$Al (Kleine et al. 2005, 2006; Hevey & Sanders 2006), producing differentiated achenotrites and metallic objects, so it is no longer possible to dissect their primordial components.

Of course, the real world is more complicated. Some complications are meteoritic: as only one example, CO chondrites and ordinary chondrites appear to be about the same age (younger than CV chondrites), but they have very different chemical and isotopic properties (Kita et al. 2005), which would, in the context of this scenario, argue for some degree of spatial (radial?) variation in the makeup of nebula particulates at that time anyway.

On the theoretical side, of course, self-consistent numerical models that follow clump evolution in realistic turbulence with headwinds and vertical settling need to be pursued in order to check
these preliminary assessments, which make simplistic assumptions about the local headwind and assume isolated clumps. Several recent studies have found that transient “pressure ridges” arise in the gas in association with spiral density waves or vertical structures (Haghighipour & Boss 2003; Rice et al. 2004; Johansen et al. 2007); in such regions, the local radial pressure gradient and headwind diminish (other regions will have unusually large headwinds). Moreover, clumps do not exist in isolation, but more realistically as a dense core within a larger, less dense envelope, which weakens the headwind felt by the dense clump core relative to the nebula average that we characterize by $\beta$. Both of these conditions would allow some clumps to survive with smaller values of $\Phi l$. Other complications include interactions between different strengthless objects, leading to mergers or disruptions.

Our stability criteria are most likely to be satisfied in a region fairly close to the nebula midplane ($\sim 0.01 H$; see § 3.3), because at higher altitudes the vertical component of solar gravity leads to vertical settling of dense clumps at higher terminal velocities than are easily stabilized against. However, this vertical settling, even if it leads quickly to the disruption of individual clumps, enhances the downward transport rate of small particles and increases the solid/gas ratio closer to the midplane from that generally predicted by simple 1D diffusion models (e.g., Bosse et al. 2006).

All these effects have implications for the occurrence frequency of clumps of the requisite values of $\Phi l$ for stability. Quantitative statistical determinations of the volume fraction of stable clumps that can become planetesimals at any given time will use, for instance, approaches such as the cascade models described in Appendix A. Simplified, preliminary work exploring these factors indicates that while the volume fraction of stable clumps is low at any given time (so the process is clearly not an efficient one), accretion rates are roughly an Earth mass per Myr in the asteroid belt region (Cuzzi et al. 2007). However, considerable work remains in order to establish the statistical formation rate of appropriate clumps that are capable of following this evolutionary path and to study the evolution of clumps in realistic turbulence, with variable headwind velocities and simultaneous vertical settling.

We thank J. Eaton, M. Gaffey, P. Goldreich, W. Hartmann, G. Laughlin, K. Sreenivasan, K. Squires, and A. Wray for very helpful conversations on various aspects of this research, and A. Wray, A. Dobrovolskis, P. Garaud, and S. Weidenschilling for thorough reviews of the original manuscript and a number of useful comments that have been included. N. Turner and A. Carballido also provided useful comments on an earlier version. This work was supported by a grant to J. N. C. from NASA’s Planetary Geology and Geophysics program. Generous grants of CPU time from NASA’s HEC program were essential to the progress of this research; we thank E. Tu, K. Schulbach, C. Niggle, and R. Pesta in particular for their help along these lines. We also thank J. Chang for coding optimization assistance.

**APPENDIX A**

**MASS LOADING AND THE CASCADE MODEL**

This aspect of our work incorporates two related and important concepts: that of intermittency, and that of a statistical cascade process. For instance, it is widely known that dissipation of turbulent kinetic energy occurs at the Kolmogorov scale; it is less widely known that the spatial distribution of dissipation, like that of the particle concentration factor $C$, is highly intermittent. Intermittent quantities are spatially and temporally unpredictable and fluctuate increasingly with increasing Reynolds number $Re$. However, the statistical properties of intermittent quantities such as $C$ (their probability distribution functions, or PDFs) are well determined on any length scale. In what follows, it will be necessary to distinguish between two closely related quantities: the concentration $C$ and the mass loading $\Phi = \rho_p/\rho_g = C\rho_p/\rho_g$. This is because the emergence of particle feedback on the gas due to mass loading ($\Phi$) depends both on $C$ and on the initial value of $\rho_p$. It is ultimately $C$ that is evolved in the cascade, but how it evolves depends on $\Phi$. Thus, both quantities will be alluded to in parallel.

In its inertial range, which is extensive at high $Re$, turbulence is a scale-free process that is often referred to as a cascade and that inspired the approach of cascade modeling (Meneveau & Sreenivasan 1991). The physics of the transportation of kinetic energy, vorticity, and dissipation from their sources at large scales is independent of scale until viscous processes enter at the smallest (Kolmogorov) scale. Scale independence and $Re$ independence are connected because $Re$ determines the depth of the inertial range: $Re^{\alpha/\beta} = L/\eta$ (Tennekes & Lumley 1972). Cascade models simplify this complex, nonlinear, 3D scale-free process into a set of partition rules that are independent of scale, of or level in the cascade. Larger values of $Re$ means a larger number of eddy spatial bifurcations (or levels) between $L$ and $\eta$, as well as stronger fluctuations in intermittent properties (Meneveau & Sreenivasan 1991). Cascade models do not preserve all the information of full 3D models (such as the tubelike spatial structures characterizing vorticity), but they do reproduce the statistical properties (the PDFs), which are of primary importance for our problem. The successful use of such models to describe the intermittency of dissipation, and other quantities, in turbulence (Juneja et al. 1994; Sreenivasan & Stolovitzky 1995) led us to pursue a cascade model for the turbulent concentration of particles.

A cascade model consists of a set of “multipliers”: fractions that define how a quantity of interest is unequally partitioned from a fluid parcel at some level into its equal-sized subdivisions at the next level. The PDF of the particle concentration factor $C$ is determined over all the numerous end-level outcomes of applying a multiplier $m$ [chosen from its own PDF $P(m)$] to each volume element as it bifurcates at each level of the cascade. It has been observed that $P(m)$ is independent of level in the turbulent inertial range (Sreenivasan & Stolovitzky 1995). Hence, multiplier PDFs determined from a direct numerical simulation (DNS) at low $Re$ (with a relatively small number of levels), merely applied repeatedly over additional levels, can predict the properties at higher $Re$ if nothing changes in the physics. Our cascade model (Hogan & Cuzzi 2007) uses a common form for $P(m)$: $p(m) \propto m^{b-1}(1-m)^{b-1}$, where the parameter $b$ determines the width of $p(m)$ (Sreenivasan & Stolovitzky 1995). Both the concentration $C$ and the vorticity $\omega$ are described by their own $b$-distributions. Multiplier PDFs with small values of $b \leq 1$ are broad, and in them multipliers much different from $m = 0.5$ occur with higher probability, producing a grainier, more intermittent spatial distribution. PDFs with large values of $b \gg 1$ are narrow and centered.
on \( m = 0.5 \); because each “eddy bifurcation” then involves a nearly 50–50 partitioning, the resulting spatial distribution is nearly uniform, and the subsequent growth of \( \Phi \) becomes more difficult.

The cascade model can then be used to address the issue of how particle mass loading can affect the cascade; that is, change the physics as the cascade progresses. This dependence is known as “conditioning” of a cascade. We found that the process can be represented quite well as two separate one-phase conditioned cascades for \( C \) and \( \omega \) in which multiplier distributions \( p(m) \) for both \( C \) and \( \omega \) depend only on the local particle mass loading \( \Phi \). We used our full 3D numerical simulations to establish how \( b \) for both these properties depends on local particle or fluid properties.

Figure 4 shows conditioning curves for both \( C \) and \( \omega \) (the latter expressed as enstrophy: \( S = \omega^2 \)), as extracted from multiple 3D DNS simulations at various values of \( \text{Re} \) as large as 2000 (Hogan & Cuzzi 2007). Note how the multiplier PDFs extracted from the DNS results (insets) get narrower (have larger values of \( b \)) as the mass loading increases, choking off intermittency. In these runs, \( \bar{p}_p/\rho_p \approx 1 \), so \( C = \Phi \). These results establish the upper limit \( \Phi \sim 100 \) that can be obtained by turbulent concentration, regardless of the initial value of \( \bar{p}_p/\rho_p \). In the nebula, with a smaller initial value of \( \bar{p}_p/\rho_p \), either higher \( \text{Re} \) or deeper cascades (and larger ensuing \( C \)-values) are needed in order to reach this saturation point.

To determine the joint PDF of the concentration \( C \) and the vorticity \( \omega \), we require not only the conditioned multipliers for \( \omega^2 \) and \( C \) (Fig. 4), but also their spatial correlation. Hogan & Cuzzi (2007) found, on average, a 70–30 preference for anticorrelation at each partitioning, consistent with previous observations that particle concentration zones avoid zones of high fluid vorticity (Squires & Eaton 1990, 1991; Eaton & Fessler 1994; Ahmed & Elghobashi 2000, 2001). Including this partitioning asymmetry factor as a weighted coin gives us a two-phase (particle-gas), conditioned cascade model that shows very good agreement with the joint PDF \( P(\Phi, \omega^2) \) directly determined from our full 3D mass-loaded simulations (see Hogan & Cuzzi 2007). However, to match the full 3D DNS simulations at \( \text{Re} = 2000 \), the cascade model needs only about 15 levels, which would take about 10 CPU hr (for 1024 realizations), compared to the more than 90,000 CPU hr needed in order to converge our full 3D simulation.

Using these conditioned multipliers and the asymmetry factor, entire PDFs of particle concentration (mass loading) and vorticity can be generated for arbitrary numbers of levels (arbitrary \( \text{Re} \)). For example, the 24-level model of Fig. 5 (from Hogan & Cuzzi 2007) is of direct relevance for the \( 10^3 \)–\( 10^4 \) km scale dense zones that are of interest for the nebula. Figure 5 illustrates the saturation of the particle concentration at \( \Phi = 100 \) and the fact that high values of \( \Phi \) occur preferentially at low enstrophy or vorticity. The upper left quadrant is of most interest for determining the numbers of clumps that can survive to become planetesimals (see Cuzzi et al. 2007 for more details).

APPENDIX B

SCALING BETWEEN NUMERICAL CODE AND NEBULA

While the primary result of this paper—the existence of some stability regime determined by a critical gravitational Weber number \( \text{We}_G \)—can be expressed in a nondimensional way, some aspects of the results (specifically, the numerical results and the inferred value of \( \Phi \) for the nebula) require us to pursue the relationship between nebula parameters and code units more deeply. Values in code units are denoted by primes below. The fundamental quantities in the problem are (1) the gas and particle densities \( \rho_g \) and \( \rho_p = \Phi \rho_g \); (2) the clump diameter \( l \); (3) the local orbit frequency \( \Omega(a) \), where \( a \) is the distance from the Sun; (4) the local radial pressure gradient \( dP/da = dP/dx \), where \( x \) is in the Hill frame; (5) the particle stopping time \( \tau_s \); (6) the velocity of material in the Hill frame \( \mathbf{W} \); that is, relative to Keplerian, where \( \mathbf{W} \) represents either the particle or gas velocity; and (7) the gravitational constant \( G \). The principal forces are the Coriolis force due to the rotating frame, \( -2\Omega_b \times \mathbf{W} \), the pressure force, \(( -1/\rho_g )dP/dx \), and the gas drag forces that couple the gas and particles (§ 3.3). The code unit of length is radians (the domain is \( 2\pi \) rad on the short dimension \( x \) or \( z \)), and the grid cell size is therefore \( \Delta = 2\pi/N \), where \( N \) is the number of grid points along that axis). The code time unit (ctu) is \( 1/\Omega' \), where \( \Omega' = 1 \) is the code value of rotation frequency; thus, the orbit period is \( 2\pi \) ctu. We treat as equivalent the values \( a = R = x \).
As noted in § 3.3,
\[ \Omega(a) = \left( \frac{GM_\odot}{a^3} \right)^{1/2} \equiv (G\rho^*)^{1/2}, \quad \text{or} \quad G = \Omega^2 / \rho^*, \]
where \( \rho^* = M_\odot/a^3 \approx 3.8 \times 10^{-8} \text{ g cm}^{-3} \) at 2.5 AU [note that this is different from Safronov’s (1991) Roche density, \( \rho_R = (3M_\odot)/4\pi a^3 \)]. To normalize the mass and mass density (and ultimately establish the code value of the gravitational constant \( G^* \)), we assume a nebula gas mass density of \( \rho_0 \approx 10^{-16} \text{ g cm}^{-3} \) (minimum-mass nebula; Cuzzi et al. 1993) at 2.5 AU; then the code quantities \( \rho^*_g = \rho_0 / \rho_0 = 1 \) and \( \rho^*_g = \rho_0 / \rho_0 = 3 \times 10^{-3} \). Then the dimensionless parameters \( \Phi \) and \( l' \) are set to satisfy various constraints of the code (resolvable clump, adequate particle statistics, reasonable timescale, etc.), and \( \beta' \) is allowed to vary, so that we can empirically determine the critical value of \( \text{We}_{G^*} \), as the largest value of \( \text{We}_{G^*} \) that remains stable. That is, we want the value of \( l' \) to be large enough such that the clump is well resolved, but not so large that it fills the entire cross section of the computational box. Typically, \( l' \) is only 10–20 grid cells so far, and as noted below, this exaggerates the viscous stresses and surficial erosion of material. We want the value of \( \Phi \) to be large enough that the clump is much denser than the gas, and we want a large enough number of particles such that they act like a continuum. The value of \( t_\text{ts} \) should ensure that the clump sedimentation time \( t_\text{sed} \) is larger than its ram pressure disruption time \( t_\text{dis} \) (see below), in order to provide a real test that self-gravity is preserving the clump, rather than simple collapse.

### B1. Important Timescales in the Numerical Model

The dynamical collapse time of the clump under its own self-gravity, in the absence of gas pressure, is \( t_G = \pi(G\rho)^{-1/2} \), or, in code units, \( t_G' = \pi(G^*/\Phi)^{-1/2} \). The mass loading that produces a clump dynamical time comparable to the orbit time is then obtained using the relations \( \pi(G^*/\Phi)^{-1/2} = 2\pi/\Omega' = 2\pi \) or \( \Phi = 1/4G' \sim 100 \), again in fair agreement with prior expectations. However, gas drag prevents clump collapse on this timescale, as described in § 3.1, and actual shrinkage takes a time \( t_\text{sed} \) (eq. [1]). The number of orbits over which we need to follow the clump in order to ensure that it really survives is roughly

\[ t_\text{sed}\Omega \approx \frac{c\Omega}{2\pi} = \frac{H\Omega^2}{2\pi G\Phi r \rho_s} = \frac{\Omega^2}{G\rho_s} \frac{H}{r} \frac{1}{8\pi\Phi} = \frac{\rho^*}{r} \frac{H}{r} \frac{1}{8\pi\Phi}, \]

where we have used \( c = H\Omega \). This expression can be assessed using real quantities, and it comes out to 30–300 orbits for particle radii of 300 \( \mu \text{m} \) and a mass loading parameter of \( \Phi = 1000–100 \). This behavior \( (t_\text{sed} \gg 2\pi/\Omega') \) can be controlled in the code, once the other parameters are established as above, by adjusting the particle stopping time \( t_s \). Because of long code run times, at present we are limited to stipulating only that the sedimentation time significantly exceeds the nominal (self-gravity–free) disruption time; that is, \( t_\text{sed} \gg t_\text{dis} \), where

\[ t_\text{dis} \approx (l/w) \sqrt{2\Phi/C_D}. \]
This expression for the disruption time $t_{\text{dis}}$ is derived from the ram pressure force (§ 2.3) by setting the distance traveled in time $t$ by the windward half of the clump, under acceleration by the ram pressure force, equal to the clump diameter (if we assume that the leeward half of the clump does not get accelerated) and solving for $t$. The value of $C_D$ for our clumps is on the order of unity, as we verify below.

B2. NUMERICAL VISCOITY AND THE CLUMP REYNOLDS NUMBER

The code calls for some input viscosity $\nu'$; with it, the Reynolds number of a clump with a diameter of $l'$ rad, in a headwind of speed $w'_g$ (in code units of rad ctu$^{-1}$), is $Re'_c = l'w'/2\nu'$, where $\nu' = 0.1$ is the defined code viscosity (in code units of rad$^2$ ctu$^{-1}$). We assume an inertial range expression within the wake of the clump in order to determine the wake’s Kolmogorov scale: $\eta'/l' = Re'_c^{-3/4}$, and thus $\eta' = (2\nu' l'^{1/3}/w'_g)^{3/4}$ rad. Our highest resolution runs to date had a gas relative velocity of $w'_g = 38$ rad ctu$^{-1}$ and used a clump size of $l' = 2$ rad. The nominal Kolmogorov scale associated with this flow is $\eta' = [2(0.1)(2)^{13/38}]^{1/4} \sim 0.02$ rad (compared to the grid cell size of 0.05–0.1 rad); the wake turbulence is thus underresolved, and the code’s true viscosity is numerical: $\nu' \sim w'_g \Delta$ (at the boundary of the clump).

While we are not concerned with the fine-scale details of the wake, this marginal resolution introduces some caveats. The ratio of the ram pressure force to the viscous force (both per unit area) is, in general, $(\rho w'_g^2/2)(\rho \nu dw'_g/dr) \sim w'_g l'/4\nu = Re'/2$, where $Re'$ is the Reynolds number of a clump and we have approximated $dw'_g/dr \sim w'_g/(l/2)$. Because numerical viscosity is dominant, $\nu' \sim w'_g \Delta$; thus, in code units, $Re'_c \sim w'_g l'/2w'_g \Delta = l'/2 \Delta \approx 5$ in many cases so far. For comparison, the Reynolds number for nebula clumps of interesting sizes is $> 10^6$. This means that viscous stresses around the periphery of the clump, as well as the fractional depth of the viscous boundary layer, are grossly exaggerated in the numerical model relative to the actual nebula regime of interest. The anomalously large role of (numerical) viscosity leads to an anomalously large “erosion” from the surface of our numerical clumps, and it degrades our results in the sense that potentially stable clumps might appear less stable, due to the erosive mass loss experienced over their sedimentation time. Going to higher resolution cases ($Re'_c \sim 10$) significantly ameliorated this effect, but the problem has certainly not been entirely removed. Another way to characterize the degree of artificial erosion is to estimate the physical Kolmogorov scale for the actual nebula/clump situation: for $\nu = 10^6$ cm s$^{-2}$, $l = 10^4$ km, and $w'_g = 3800$ cm s$^{-1}$, we find a value of $\eta \sim 100$ m, or $10^{-3}$ of the size of a $10^4$ km clump, compared to a fractional size of perhaps $10^{-1}$ or so in the code, as determined by numerical viscosity. Clearly, surface erosion is a much smaller effect in the nebula case than in our crude models. While even at this resolution apparently stable configurations can be found, it would be desirable for future studies to improve on this situation.

We now discuss the clump drag coefficient $C_D$. The drag coefficient of the clump depends on the Reynolds number and must also be scaled to nebula values in order to obtain an estimate of $\Phi$. Weidenschilling (1977) gave expressions for the drag force per unit area (giving the pressure force per unit area) experienced by solid spheres of different $Re_c$ in the Stokes drag regime of interest here as $C_D^2 \rho w_g^2/2$. For $1 < Re_c < 800$, $C_D = 24/Re_c^{0.6}$, or $C_D \sim 8$ for $Re_c \sim 5$, as in our code. The large drag coefficient and ram pressure is partly due to the low-pressure zones set up immediately behind particles in this range of $Re_c$. By comparison, for a solid sphere with $Re_c > 800$, as under nebula conditions, $C_D = 0.44$. However, neither of these values might be entirely applicable to our clumps, which are not rigid spheres. In practice, we have determined $C_D$ empirically from observed values of $t_{\text{dis}}$ with self-gravity turned off. Recalling that $t_{\text{dis}} = (l/w_g)(2\Phi/C_D)^{1/2}$, we simply plot our estimate of $t_{\text{dis}}$, the point at which sizeable vortex-driven internal voids appear in the clump, against the combined parameter $(l/w_g)(2\Phi)^{1/2}$, giving $C_D^{1/2}$ as the slope of the best-fit straight line (Fig. 6). It seems that $C_D \approx 1$ is a good assumption, but this calculation should be redone with higher resolution (lower numerical viscosity) codes. At nebula values of $Re_c$, $C_D$ could reach its theoretical high-Re value of 0.4, and this is what we assume in deriving $\Phi$. 

---

Fig. 6.—Determination of the effective drag coefficient for our clumps. We plotted the estimated disruption times $t_{\text{dis}}$ vs. the quantity $(l/w_g)(2\Phi)^{1/2}$; the slope is then $C_D^{1/2}$. This determination is slightly qualitative as to when we judge the clump to be disrupted; we decided to use the time at which vortex-driven voids first appear in an $a-Z$ planar cross section through the nominal center of the clump.
Finally, the several stable cases we have run (e.g., Fig. 3) can be characterized by \( w'_{ij} = 38 \text{ rad ctu}^{-1}, \rho_{ij}^* = 380 \) (which corresponds to a minimum-mass nebula local density of \( \rho_g = 10^{-10} \) at a distance of 2.5 AU), \( \Phi = 10^3 \), and \( I' \approx 1 \) rad, and although we have not as yet accurately determined the transition value of \( W_{\text{eG}} \), a value of \( W_{\text{eG}} \sim 1 \) is clearly stable. Solving equation (4) of § 3.3 for \( \Phi/l \) under nebula conditions, assuming \( W_{\text{eG}}^* = 1 \), taking \( C_D = 0.44 \) for a high-\( \text{Re}_e \) situation, and assuming a nominal value of \( \beta = 10^{-3} \), we find that \( \Phi/l > 1.5 - 5 \times 10^6 \), independent of \( \rho_g \) and neglecting a questionable coefficient of order unity (because of the contribution of viscous stresses in our low-\( \text{Re}_e \) code). Foreseeable refinements (specifically, lower numerical viscosity) may increase \( W_{\text{eG}}^* \) (the surface tension analog gives \( W_{\text{eG}}^* \sim 10 \)) and thus decrease the value of \( \Phi/l \).

APPENDIX C

TIME EVOLUTIONS AND MOVIES

Movies showing our time evolutions are available in the electronic edition of this article, or in higher resolution versions at http://spacescience.arc.nasa.gov/users/cuzzi/, with file names as given below (for example, http://spacescience.arc.nasa.gov/users/cuzzi/hill26.gif). Figure 7 is a snapshot showing the format of the first frame of each movie. The evolution shown in the three movies is described below.

Each evolution begins with a dense spherical clump in the Hill frame, rotating at local Keplerian velocity, with more slowly orbiting gas impinging on the clump from its leading side. The two left-hand panels show the view from along the radial axis (in the \( Z-\theta \) plane; top) and along the vertical axis (in the \( \alpha-\theta \) plane; bottom). The large right-hand panel shows the view from the orbitally leading direction (in the \( \alpha-Z \) plane). The wake formed by the clump is easily visible in the two left-hand panels. Cases 26 and 29a were sampled at a similar time near their endpoint for Figure 3. The movie of case 26 shows how a clump with no self-gravity is disrupted in about 1.5 ctu for the initial conditions chosen (the orbit period is \( 2\pi \) ctu), in agreement with our simple estimates of \( t_{\text{dis}} \). The movies for cases 29a and 31a are intended to illustrate the same (stable) Weber number case (\( W_{\text{eG}} \sim 1 \)), but we vary two constituent parameters of \( W_{\text{eG}} \) (we decrease both the gas velocity and mass loading by a factor of 3) in order to illustrate similarity; because of the specific parameters chosen, the evolution in case 31a runs for a longer time, but the clump is suffering considerable erosion toward the end of the run. Nevertheless, as is discussed below, the clump retains a dense core that continues to contract. As is discussed in Appendix B, more realistic cases, with higher numerical resolutions, would have smaller numerical viscosities and incur less erosion. Case 29a, while not run for as long as case 31a, appears to be solidly stable, with the clump having a dense core that is continuing to shrink at the end of the run. Note some interesting oscillatory behavior, such as was hinted at in our 1D compressible runs (Fig. 2).

C1. BEHAVIOR OF CENTRAL REGIONS OF DENSE CLUMPS

The simulations that created the movies mentioned above can be used to assess the behavior of the central, dense regions of each clump in a quantitative manner. Figure 8 shows, in three panels, the time variation of some of the properties of the particles that lie within the central regions of the clump (its densest fraction, as sampled out to some cumulative mass threshold). In the top panel, we show how much mass is being eroded from the clump overall (particles moving faster than some comoving velocity threshold are assumed to have left the clump). All three clumps are losing mass, naturally, but the nongravitating clump in case 26 is losing it much more rapidly than the gravitating clumps in cases 29a and 31a. In the middle panel we show the mean mass density as measured over some fraction \( f \) of the clump mass lying at the highest densities; in cases 26 and 29a, we chose \( f = 0.5 \), and in case 31a, we chose \( f = 0.25 \). In the nongravitating clump, the core density never increases, and it quickly blows up, while the central regions of the gravitating clumps (removed from the artificial mass erosion at their peripheries) inexorably get denser at a steady and perhaps even slightly increasing rate (an increasing rate is predicted by the simple model). The bottom panel shows the effective average radius of the region being sampled. The
nongravitating clump never shrinks at all, but both gravitating clumps do. Moreover, as the gravitating clumps shrink, their density increases faster than their linear dimension decreases, so the product $\Phi f$ increases, causing them to become more stable as time goes on. This is an argument for stability even in the presence of erosion. As expected, the density growth timescale (the sedimentation time $t_{\text{sed}}$ of eq. [1]) is 3 times smaller for the denser clump of case 29a (middle). Meanwhile, the apparent agreement of the variation of the fractional radius containing half of the mass seen between cases 29a and 31a in the bottom panel is fortuitous, because the total masses of the clumps are changing (and eroding) at different rates and from different depths in the two cases. Nevertheless, the cores are both shrinking and becoming denser in the two stable cases. The amount of erosion that is incurred from the margins of each clump is related to (proportional to) the thickness of the viscous boundary layer relative to the radius of the clump, which can be expressed in terms of the Reynolds number. As we have argued above in this appendix, actual nebula clumps capable of remaining stable would have Reynolds numbers that were orders of magnitude larger, which would be impossible to study with numerical models. The relevance of all viscous boundary layer effects, including the erosion that inflicts our current numerical runs, will decrease by that factor in the real situation.

Fig. 8.—Properties of the dense central regions in three simulations, as a function of time. All three cases are shown in each panel: the stable, self-gravitating case 31a (solid lines), the stable, self-gravitating case 29a (dashed lines), and the unstable, nongravitating case 26 (dotted lines). Not all cases were run to the same stopping point due to resource constraints. Top, Cumulative total mass retained by the clump; middle, average mass density of the central mass fraction $f$; bottom, effective radius containing a mass fraction $f$. These results show that, away from the artificially perturbed perimeter regions, $\Phi_{c,2}$-stable clumps are behaving as expected by the simple model, and $\Phi_{c,2}$-unstable clumps do not show this behavior.

REFERENCES

Afshordi, N., Mukhopadhyay, B., & Narayan, R. 2005, ApJ, 629, 373
Ahmed, A. M., & Elghobashi, S. 2000, Phys. Fluids, 12, 2906

Arlt, R., & Urpin, V. 2004, A&A, 426, 755
Asphaug, E., & Benz, W. 1996, Icarus, 121, 225

Bell, K. R., Lin, D. N. C., Hartmann, L. W., & Kenyon, S. J. 1995, ApJ, 444, 376
Binzl, R. P., Lupsikko, D., di Martino, M., Whiteley, R. J., & Hahn, G. J. 2002, in Asteroids III, ed. W. F. Bottke, Jr. et al. (Tucson: Univ. Arizona Press), 255

Bischoff, A., Scott, E. R. D., Metzler, K., & Goodrich, C. A. 2006, in Meteorites and the Early Solar System II, ed. D. S. Lauretta & H. Y. McSween, Jr. (Tucson: Univ. Arizona Press), 679

Bosse, T., Kleiser, L., & Meiburg, E. 2006, Phys. Fluids, 18, 027102

Ciesla, F. J., & Cuzzi, J. N. 2006, Icarus, 181, 178
Cuzzi, J. N., & Alexander, C. M. O'D. 2006, Nature, 441, 483

Connolly, H. C., Jr., Desch, S. J., Ash, R. D., & Jones, R. H. 2006, in Meteorites and the Early Solar System II, ed. D. S. Lauretta & H. Y. McSween, Jr. (Tucson: Univ. Arizona Press), 383

Cuzzi, J. N., & Hogan, R. C., Paue, J. M., & Dobrovolskis, A. R. 2001, ApJ, 546, 496

Cuzzi, J. N., Hogan, R. C., Paue, J. M., & Dobrovolskis, A. R. 2001, ApJ, 546, 496

Cuzzi, J. N., Hogan, R. C., Paue, J. M., & Dobrovolskis, A. R. 2001, ApJ, 546, 496

D'Alessio, P., Calvet, N., & Woolfum, D. S. 2005, in ASP Conf. Ser. 341, Chondrites and the Protoplanetary Disk, ed. A. N. Krot, E. R. D. Scott, & B. Reipurth (San Francisco: ASP), 353

Desch, S. J., Ciesla, F. J., Hood, L. L., & Nakamoto, T. 2005, in ASP Conf. Ser. 341, Chondrites and the Protoplanetary Disk, ed. A. N. Krot, E. R. D. Scott, & B. Reipurth (San Francisco: ASP), 849
