Weighting Optimization of Decision Matrix in Fuzzy TOPSIS Using SMARTER Method

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Abstract. Fuzzy TOPSIS is suitable for solving decision-making problems by introducing quantity multiplication operations of triangular fuzzy number (TFN). There is a weakness in determining the weight value in weighting phase of fuzzy TOPSIS. Determination of the weight value calculated by the normalized matrix value requires a relatively better technique to obtain optimal results. In this study to get the optimal value, the weighting stage used the SMARTER (Simple Multi-Attribute Rating Technique Exploiting Ranks) method. SMARTER is able to optimize the weighting value before proceeding to the next stage.

1. Introduction
TOPSIS is one of the multi-criteria decision-making method that using the principle of chosen alternative must have the shortest distance from positive ideal solution and the longest distance from negative ideal solution from a geometric point of view by using the Euclidean distance (the distance between two points) to determine the relative proximity of an alternative with the optimal solution. The positive ideal solution is defined as the sum of all the best attainable values for each attribute, while the negative-ideal solution consists of all the worst values achieved for each attribute. TOPSIS considers both, the distance to a positive ideal solution and the distance to the negative ideal solution by taking the proximity relative to the positive ideal solution. Based on a comparison of the relative distance, alternative priority arrangements can be achieved. This method is widely used to solve practical decision making [3]. But in the weighting phase of fuzzy TOPSIS, there is a weakness in determining the weight value. Determination of the weight value calculated by the normalized matrix value requires a relatively better technique to obtain optimal results. So to get the optimal value, in the weighting stage used the SMARTER method (Simple Multi-Attribute Rating Technique Exploiting Ranks). SMARTER is able to optimize the weighting value before proceeding to the next stage. This method is based on the theory that each alternative consists of a number of criteria that have values and weights with varying degrees of importance. Weighting in the SMARTER method uses a range between 0 and 1, thus facilitating the calculation and comparison of values in each alternative [2].

2. Theoretical Basis
2.1. Fuzzy Logic
Fuzzy logic is used as the linearity and invariant of the control process that cannot be assumed. When the process cannot be assumed and has a model, fuzzy logic provides a methodology and a good way
to assume the process to be applied. This application is for the representation of the application of human knowledge and control of the system, fuzzy logic also requires human knowledge and expertise to provide uncertainty in the process of control [4].

2.2. Fuzzy TOPSIS
Fuzzy TOPSIS requires several steps procedure in making a decision [7]. Establish normalization of the fuzzy decision matrix. Before forming the \( r_{ij} \) element, building \( x_{ij} \) is the first step that must be done. Then calculate \( \Sigma x_{ij} \) function to search for the \( r_{ij} \) element. Having obtained \( \Sigma x_{ij} \), then calculated \((\Sigma x_{ij})^2\) by the formula

\[
\sum x_{ij} = ((x_{11})^2 + (x_{12})^2 + (x_{13})^2 + \ldots + (x_{1n})^2)
\]

(1)

After the \( x_{ij} \) value is obtained, \( r_{ij} \) element resulted from the normalized decision matrix \( R \) calculated by the Euclidean length of a vector.

\[
r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}
\]

(2)

Where :
\( x \) : decision matrix
\( i = 1,2, ..., m \)
\( j = 1,2, ..., n \)

calculate the weighted normalized decision matrix

\[
y_{ij} = w_i r_{ij} \quad \text{with } i = 1,2, ..., m; \text{ and } j = 1,2, ..., n
\]

(3)

where :
\( y_{ij} \) : normalized weight
\( w_i \) : weight

Determine positive ideal solution and negative ideal solution

\[
A^+ = (y_1^+, y_2^+, \ldots, y_n^+);
\]

(4)

\[
A^- = (y_1^-, y_2^-, \ldots, y_n^-);
\]

(5)

Calculate the distance of each alternative from a positive ideal solution and negative ideal solution

\[
D_{i}^+ = \frac{\sum_{j=1}^{n}(y_{ij}^+ - y_{ij})^2}{\sum_{j=1}^{n}(y_{ij}^+)^2};
\]

(6)

\[
D_{i}^- = \frac{\sum_{j=1}^{n}(y_{ij}^- - y_{ij})^2}{\sum_{j=1}^{n}(y_{ij}^-)^2};
\]

(7)

Determine the preference value for each alternative

\[
V_i = \frac{D_{i}^{-}}{D_{i}^{+} - D_{i}^{-}}
\]

(8)

2.3. SMARTER
SMARTER is a decision-making method based on the theory that each alternative consists of a number of criteria that have values and each criterion has a weight that describes how important it is compared to other criteria. The weighting of the SMARTER method uses a range between 0 and 1, thus facilitating the calculation and comparison of values in each alternative [6]. SMARTER, weights are calculated using Rank-Order Centroid (ROC) weighting formula. ROC weighting is obtained by
simple mathematical procedures of priority. This procedure can be formulated as follows (if any K criteria) [2] [1] [5]:

$$W_k = \frac{1}{K} \sum_{i=k}^{K} W_i$$  \hspace{1cm} (9)

Where:
W : Criteria weighted value
K : Number of criteria
i : Alternative value

3. Research Design
3.1 Fuzzy TOPSIS Algorithm
Fuzzy TOPSIS Algorithm flowchart processes are shown on figure 3.1

Flowchart of fuzzy TOPSIS algorithm is a sequence of processes to get a user preference value as the final value to get a ranking of all alternatives and criteria that have been calculated.

3.2 Fuzzy TOPSIS Algorithm with Smarter Method
SMARTER weighted processing flowchart can be seen in figure 3.2.
In this stage, SMARTER weighted processing flowchart is used to get the weight modification value on the fuzzy TOPSIS algorithm in order to obtain more optimal results. The weighted value given is the relative weight average value to be calculated by the normalized matrix value.

4. Result and Discussion

4.1 View of Alternative Data Form

Alternative data is the primary element to calculate determination of scholarship. The data form in the system is looked like figure 4.1.
4.2 View of Fuzzy TOPSIS Testing Form

The distance of positive ideal solution and distance of negative ideal solution result of fuzzy TOPSIS testing form is shown in figure 4.2.

![Figure 4.2 Fuzzy TOPSIS Testing Result](image)

The calculation process consists of student assessment data, alternatives, criteria, Topsis matrix normalization, $R_{ij}$ (result), $Y_{ij}$ ($W_{ij} \times R_{ij}$) result, positive ideal solution, negative ideal solution, the distance of positive ideal solution and the distance of negative ideal solution. So after the calculation is complete it will appear the results of TOPSIS preferences and rankings, based on the calculation is entitled to a scholarship is Alternative 10 (A10). In this fuzzy TOPSIS calculation, the weight used is the weight determined by the decision maker.

4.3 View of Fuzzy TOPSIS with SMARTER Method Testing Form

Result of fuzzy TOPSIS with SMARTER method is shown in figure 4.3.

![Figure 4.3 Fuzzy TOPSIS with SMARTER Result](image)
In the calculation data above, the same as TOPSIS calculation but there is a difference that lies in the value of weight. In this calculation, the weights used are calculated first using SMARTER method. Smarter gets the optimum weight by taking into all account details of all criteria values. This explains that the modified TOPSIS fuzzy weights using SMARTER method also has better preference results.

5. Conclusions
The results of research and discussion can be drawn some conclusions as follows:
1. Weighted modification of fuzzy TOPSIS algorithm with SMARTER method resulted in the same cohesion which Alternative 10 was decided as the eligibility of receiving a scholarship.
2. Results of fuzzy TOPSIS and fuzzy TOPSIS with SMARTER weights, fuzzy TOPSIS receives more alternatives than fuzzy TOPSIS with SMARTER. While TOPSIS with SMARTER accepts fewer or considering alternatives. This is because the use of TOPSIS SMARTER is more selective in choosing and executing alternatives compared to fuzzy TOPSIS.

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