We study the dynamics of a beam of fermions diffracted off a density grating formed by fermionic atoms in the limit of a large grating. An exact description of the system in terms of particle-hole operators is developed. We use a combination of analytical and numerical methods to quantitatively explore the Raman-Nath and the Bragg regimes of diffraction. We discuss the limits in diffraction efficiency resulting from the dephasing of the grating due to the distribution of energy states occupied by the fermions. We propose several methods to overcome these limits, including the novel technique of “atom echoes”.

PACS numbers: 03.75.Fi, 05.30.Fk

I. INTRODUCTION

The availability of quantum-degenerate bosonic atomic systems has recently allowed the extension of atom optics to the nonlinear [8] and the quantum regimes. Matter-wave four-wave mixing [9], coherent matter-wave amplification [10] and superradiance [11], the generation of dark fields [5] and bright [12] atomic solitons and of correlated atomic pairs [13] have been demonstrated, and so has the matter-wave analog of second-harmonic generation, the creation of a molecular condensate component [8, 9].

In contrast, the development of the nonlinear atom optics of fermionic atomic systems is not nearly as far along. While it has been shown theoretically [14, 15] that the four-wave mixing of fermionic matter waves is possible in principle, these predictions have not been verified experimentally so far. Still, the recent achievement of temperatures as low as 0.2$T_F$, where $T_F$ is the Fermi temperature, for the fermions $^{40}$K and $^6$Li [12, 13, 14, 15] is encouraging, and it is hoped that first experiments on fermionic nonlinear atom optics will take place in the near future. In addition to the fundamental goal of achieving a BCS phase transition into pairing and superfluidity [16], research along these lines is also motivated by recent results that hint at the possibility to lower the phase noise in interferometric measurements below the bosonic standard quantum limit by using instead degenerate fermionic beams [17].

The first theoretical discussions of fermionic nonlinear atom optics were presented in Refs. [10] and [11], which treated the case of a single ‘test’ particle scattering off a periodic density grating formed by a degenerate Fermi gas. They showed that for an appropriately prepared grating, the fermionic system can indeed undergo four-wave mixing. In contrast to the fundamental interpretation in terms of “bosonic amplification”, which clearly is not applicable to fermions, this effect was interpreted in terms of the constructive quantum interference between different paths leading to the same final state.

One important aspect of the fermionic case is that, in contrast to bosons, considerable care must be taken in combining two matter waves to form a “grating”, so that their interaction with a third wave can produce a significant four-wave mixing signal. Consider, as we shall do in this paper, counterpropagating matter waves separated in momentum by $q$. In the case of bosons, two obvious possibilities correspond to the states

$$|\psi_{b,1}\rangle = \frac{1}{\sqrt{2(N/2)!}} \left[ (b_{q/2}^\dagger)^{N/2} + (b_{-q/2}^\dagger)^{N/2} \right] |0\rangle,$$

and

$$|\psi_{b,2}\rangle = \frac{1}{\sqrt{2^N N_1}} \left[ b_{q/2}^\dagger + b_{-q/2}^\dagger \right]^N |0\rangle,$$

the $b_{q/2}^\dagger$ being usual bosonic creation operators and $|0\rangle$ the atomic vacuum. The first case describes two counterpropagating beams of $N/2$ atoms each and of momenta $\pm q/2$, while the second state corresponds to a density grating obtained by identically preparing $N$ atoms in a coherent superposition of states of momenta $\pm q/2$. It is known from the study of atomic diffraction by optical fields that these two states lead to different diffraction patterns, because the first one contains “which way” information while the second doesn’t [13]. This difference becomes however insignificant for large gratings.

The situation for fermions is more complicated, since the Pauli exclusion principle precludes one from placing more than one atom per state. One needs instead to consider multimode atomic beams, centered around the mean momenta $\pm q/2$. In this case the states $|\psi_{b,1}\rangle$ and $|\psi_{b,2}\rangle$ are replaced by

$$|\psi_{f,1}\rangle = \frac{1}{\sqrt{2}} \left[ \prod_{\{k\}} a_{k+q/2}^\dagger \prod_{\{k\}} a_{k-q/2}^\dagger \right] |0\rangle,$$

and

$$|\psi_{f,2}\rangle = \prod_{\{k\}} \frac{1}{\sqrt{2}} \left[ a_{k+q/2}^\dagger + a_{k-q/2}^\dagger \right] |0\rangle,$$

where $a_{k+q/2}^\dagger$ are fermionic creation operators for atoms of momenta in the vicinity of $\pm q/2$, the total number of atoms involved being indicated in the appropriate products. From Refs. [10, 11, 13], we know that it is the
quantum coherence apparent in matter-wave states of the form $|\psi_f \rangle$ that is responsible for fermionic four-wave mixing. In order for the required quantum interference to occur, it is essential that every atom be in a coherent superposition of momentum states centered around $q/2$ and $-q/2$.

So far, our discussion has ignored the time dependence of the grating. But since the atoms forming a fermionic grating all have slightly different kinetic energies, their free evolution results in a dephasing that is expected to eventually lead to the disappearance of the four-wave mixing signal. Although the importance of this dephasing was pointed out in Ref. [15], no quantitative discussion of its effect has been presented so far. The present paper addresses this problem quantitatively by a combined analytical and numerical study of the diffraction of a beam of fermionic atoms off a large fermionic grating. We fully include the dynamics of the atomic beam, but neglect its back-action on the grating dynamics, considering only its free evolution. This is the matter-wave analog of the undepleted pump approximation in nonlinear optics.

Section II introduces our model and describes its dynamics in terms of equations of motion for particle-hole operators. The effect of the grating dynamics is characterized in terms of a dephasing time $\tau_d$, whose impact is then illustrated in the simple case of Raman-Nath diffraction. The Bragg regime is analyzed in section III using a combination of an analytical discussion of a simple two-mode system and full numerical simulations. We determine the characteristic time scales governing this process, and conclude that four-wave mixing in degenerate Fermi gases should barely be observable. Noting that the dephasing of the grating is in principle reversible, we turn in Section IV to a discussion of possible ways to achieve such a reversal, based on analogies with the photon echo techniques of quantum optics. Since the physical origin of the dephasing is the difference in kinetic energies for atoms with different momenta, one elegant approach exploits the interaction of the grating with a periodic external potential to achieve a negative effective mass of the atoms. Such methods pave the way for the observation of phenomena that seem out of experimental reach at first glance. Finally, Section V is a summary and outlook.

II. MODEL

A typical atomic four-wave mixing experiment involves three input wave packets that interact, typically via $s$-wave collisions, to produce a fourth packet in a new momentum state. Physically, this can be interpreted as the result of the phase-matched scattering of one of the wave packets off the matter-wave grating produced by the superposition of the other two, as discussed in the introduction.

As is readily apparent from Eq. (4), though, a grating formed by $N$ fermions occupies $2N$ momentum states, with $N$ states around the momentum $q/2$ and $N$ states near $-q/2$ in our case. As a result of the free evolution associated with the spread in kinetic energies of these atoms, the density grating undergoes a dephasing, similar to Doppler broadening in a thermal vapor. The goal of this paper is to investigate the effects of this dephasing and to determine to what extent it can be reduced, or even eliminated.

We consider the situation of a spin-polarized fermionic beam diffracted by an atomic grating of spin-polarized fermions that are either of a different species or in a different spin state, see Fig. 1, the beam and grating atoms interacting coherently via elastic $s$-wave scattering. We neglect $p$-wave scattering, whose cross-section at zero temperature is orders of magnitude smaller than that of $s$-wave scattering.

The diffraction of a fermionic atomic beam by a periodic potential was previously treated for the case of free fermions (or equivalently a one-dimensional bosonic Tonks gas) [21] and of Cooper pairs [22]. These treatments are restricted to the Raman-Nath regime, where the kinetic energies of the diffracted atoms are negligible compared to the potential energy of the grating. The scattering of two fermions by an optical standing wave was also considered in Ref. [22]. Also worth mentioning in the present context is Ref. [23], which investigates the nonlinear mixing of two bosonic and one fermionic wave. In these cases Pauli blocking has a significant effect on the diffraction pattern, but as the gratings are non-fermionic, the dephasing, which is central to our work, is not addressed.

We restrict the quantized description of the system to the dimension along the axis of the grating only, the “transverse” direction. The dynamics of the system in the direction perpendicular to it is treated classically, and parameterized by the time the atoms in the beam interact with the grating. Note that this approximation neglects both the normal reflection and the Bragg reflec-
tion of the incident beam off the grating.

This model is described by the second-quantized Hamiltonian

\[
H = \sum_{k} \left( \hbar \omega_k^{(g)} a_{k\uparrow}^\dagger a_{k\uparrow} + \hbar \omega_k^{(a)} a_{k\downarrow}^\dagger a_{k\downarrow} \right) + \hbar U_0 \sum_{k_1,k_2,q} \left( a_{k_1+q\uparrow}^\dagger a_{k_2-q\downarrow}^\dagger a_{k_2+q\downarrow} a_{k_1\uparrow} \right),
\]

(5)

where \( \omega_k^{(g)} = \hbar k^2/2m_g \) and \( \omega_k^{(a)} = \hbar k^2/2m_a \) is the kinetic energy of a grating (beam) atom of mass \( m_g \) (\( m_a \)) and transverse momentum \( k \),

\[
U_0 = 2\pi \hbar a/(LA\mu),
\]

(6)

\( \mu = m_g m_a/(m_g + m_a) \) is the reduced mass, \( a \) is the s-wave scattering length characterizing the ultracold collisions between grating and beam atoms, \( A \) is an effective transverse cross-sectional area, and \( L \) is the quantization length. The annihilation and creation operators \( a_{k,s} \) and \( a_{k,s}^\dagger \) satisfy the fermionic anticommutation relations \([a_{k,s}, a_{k',s'}]^+ = 0 \) and \([a_{k,s}, a_{k',s'}]^+ = \delta_{kk'} \delta_{ss'} \), the spin components \( s = \{\uparrow, \downarrow\} \) corresponding to the different spin states or atomic species of the grating and the beam, respectively.

Since the elementary process underlying matter-wave diffraction consists in annihilating an atom of momentum \( k \) and creating an atom with momentum \( k+q \), it is useful to describe the dynamics of the atomic beam in terms of “particle-hole” operators

\[
\rho_{k,q}^{(b)} = a_{k+q\uparrow}^\dagger a_{k\downarrow}, \quad \rho_{k,q}^{(g)} = a_{k+q\uparrow}^\dagger a_{k\uparrow},
\]

(7)

where the subscripts “b” and “g” label beam and grating atoms, respectively. The Heisenberg equations of motion for these operators are easily found as

\[
i \frac{d}{dt} \rho_{k,q}^{(b)} = \omega_k^{(a)} \rho_{k,q}^{(b)} + U_0 \sum_{k_1,k_2} \left( \rho_{k+q\uparrow,k_1\downarrow}^{(g)} \rho_{k_2,k_1\uparrow}^{(g)} - \rho_{k,q\uparrow,k_1\downarrow}^{(g)} \rho_{k_2,k_1\uparrow}^{(g)} \right),
\]

(8)

with \( \omega_k^{(a)} = \omega_k^{(g)} - \omega_k^{(b)} \). A similar equation holds for the grating particle-hole operators, with the substitution \( b \leftrightarrow g \).

We consider a beam of \( N_b \) atoms propagating toward the grating with some central transverse momentum \( \overrightarrow{q} \). As discussed in the introduction the grating is taken to be in a superposition of states of transverse momenta \( k-q/2 \) and \( k+q/2 \), with \( -k_{F,g} \leq k \leq k_{F,g} \), where \( k_{F,g} = \pi N_g/L \) is the Fermi momentum (in one dimension) and \( N_g \) is the number of atoms forming the grating. Such a grating can be prepared with two-photon Bragg pulses of bandwidth larger than the Fermi frequency \( \hbar k_{F,g}^2/(2m_g) \) in a two-step process starting from a stationary homogeneous gas, see e.g. Ref. [24]. In that scheme, a \( \pi \)-pulse imparting a transverse momentum \(-\hbar q/2 \) to each atom in the static cloud is followed by a \( \pi/2 \) Bragg pulse that creates the desired superposition state. Thus the initial state of beam-grating system is

\[
|\Psi(t=0)\rangle = |\Psi^{(b)}\rangle |\Psi^{(g)}\rangle,
\]

(9)

where

\[
|\Psi^{(b)}\rangle = \prod_{|k| \leq k_{F,b}} \frac{1}{\sqrt{2}} \left( a_{k-q/2\uparrow}^\dagger + a_{k+q/2\downarrow}^\dagger \right) |0\rangle,
\]

(10)

\[
|\Psi^{(g)}\rangle = \prod_{|k| \leq k_{F,g}} \frac{1}{\sqrt{2}} \left( a_{k-q/2\uparrow} + a_{k+q/2\downarrow} \right) |0\rangle.
\]

(11)

The corresponding initial expectation values of the particle-hole operators are

\[
\langle \hat{\rho}_{k,q}^{(b)} | \hat{\rho}_{k,q'}^{(b)} (0) \rangle = \delta_{q',0} \Theta(k_{F,b} - |k-q|),
\]

\[
\langle \hat{\rho}_{k,q}^{(g)} | \hat{\rho}_{k,q'}^{(g)} (0) \rangle = \frac{1}{2} \left( \delta_{q',0} + \delta_{q',-q} \right) \Theta(k_{F,g} - |k+\frac{q}{2}|) + \frac{1}{2} \left( \delta_{q',0} - \delta_{q',-q} \right) \Theta(k_{F,g} - |k-\frac{q}{2}|).
\]

In the following we assume that the matter-wave grating is sufficiently large that one can safely ignore the back-action of the atomic beam on its dynamics. Mathematically, this amounts to neglecting the term proportional to \( U_0 \) in the grating version of Eq. (8). In this limit, the grating simply undergoes a free evolution fully governed by its transverse kinetic energy.

Assuming, consistently with the assumption of a large grating, that the expectation value of products of beam and grating particle-hole operators can be factorized as

\[
\langle \rho_{k,q}^{(b)} | \rho_{k',q'}^{(b)} \rangle \approx \langle \hat{\rho}_{k,q}^{(g)} | \hat{\rho}_{k',q'}^{(g)} \rangle,
\]

(12)

making explicit use of the grating initial condition (11), and introducing the slowly varying particle-hole operators

\[
\rho_{k,q}^{(b)} (t) = \hat{\rho}_{k,q}^{(b)} (t) \exp(-i\omega_{k,q}^{(b)} t),
\]

and similarly for the grating, yields then the beam particle-hole Heisenberg equations of motion

\[
\frac{d}{dt} \hat{\rho}_{k,p}^{(b)} = -i g(t) \left[ \hat{\rho}_{k,q,p-q}^{(b)} \exp(i\omega_{k,q}^{(b)} t) - \hat{\rho}_{k,p-q}^{(b)} \exp(-i\omega_{k+q,p-q}^{(b)} t) - \hat{\rho}_{k,-q,p+q}^{(b)} \exp(i\omega_{k,-q}^{(b)} t) + \hat{\rho}_{k,p+q}^{(b)} \exp(-i\omega_{k+p,q}^{(b)} t) \right].
\]

(13)
The initial momentum spread of the beam is $\sum_{k} w_k$. We have that the effects of the dephasing grating become apparent. Time in units of the inverse mean-field interaction strength $U_0 N_g$ and $\tau_d = 15$.

where

$$g(t) = \frac{U_0}{2} \sum_{k_2} \Theta(k_{F,g} - |k_2 + q/2|) \exp[-i\omega^{(g)}_{k_2} q t],$$

(14)

reduces in the continuum limit, $\sum_{k} \rightarrow L/2\pi \int dk$, to

$$g(t) = \frac{U_0 N_g}{2} \text{sinc}(\hbar q k_{F,g} t/m_g).$$

(15)

with $\text{sinc}(x) = \sin(x)/x$. For times larger than the dephasing time

$$\tau_d \simeq \frac{\pi m_g}{\hbar q k_{F,g}},$$

(16)

we have that $g(t) \simeq 0$, that is, the diffraction of the beam comes to a stop. This consequence of the dephasing between different momentum states of the atomic grating resulting from their free evolution, represents an essential difference between fermionic and bosonic four-wave mixing: the free evolution of a bosonic grating optically prepared from a Bose-Einstein condensate at zero temperature gives rise to a phase factor that can easily be transformed away. For fermions, though, it results in a dephasing and the effective shutting off of its interaction with the atomic beam.

This impact of the dephasing is particularly easy to see in the Raman-Nath regime of atomic diffraction. We recall that this is the regime where the kinetic energy of the atoms in the beam is small compared to the potential energy of the density grating. However, we retain the kinetic energies of the grating atoms, since they are responsible for the dephasing. As such, this model corresponds to the case where the atoms in the incident beam are much heavier than the atoms in the grating.

Approximating the exponential functions by unity in Eq. (13) consistently with the Raman-Nath approximation gives

$$\frac{d}{dt} \rho^{(b)}_{k,p} = -ig(t) \left( \rho^{(g)}_{k-q,p+q} + \rho^{(g)}_{k+q,p-q} - \rho^{(g)}_{k,p-q} - \rho^{(g)}_{k,p+q} \right).$$

The solution of these operator equations for $g(t) = \text{const}$ are known in terms of Bessel functions of the first kind of order $s$ as

$$\rho^{(b)}_{k,p}(t) = \sum_{s,s'} \int_{-\infty}^{s'} \text{J}_s(U_0 N_g t) \times \text{J}_{s'}(U_0 N_g t) \rho^{(b)}_{k-sq,s'q+p}(0).$$

(17)

Due to conservation of momentum, only states of transverse momenta separated by integer multiples of $q$ are dynamically coupled. For a beam of initial momentum spread $k_{F,b} < q/2$, this implies that a linearly increasing sequence of replica of the initial distribution will be generated in time. Things are more complicated if $k_{F,b} \geq q/2$, since Pauli blocking leads to a situation where only those transverse modes near the edge of the distribution can initially be diffracted, creating holes into which modes deeper into the initial distribution can then subsequently be coupled. This results in an effective broadening of the initial distribution in time, as illustrated in the early stages of Fig. 3.

Since $g(t)$ is actually not a constant, the solution (17) is only valid for times $t \ll \tau_d$. In general, it is necessary to solve the expectation value of the coupled Raman-Nath equations numerically. The effect of the grating diffusion, and the resulting vanishing of $g(t)$ for long enough times, are readily apparent in Fig. 2, which shows that the beam diffraction effectively ceases for $t > \tau_d$. This is seen even more clearly in Fig. 3 which compares the number of significantly occupied transverse modes with and without dephasing of the grating.

III. BRAGG REGIME

We now turn to the analysis of the Bragg regime of fermionic atomic diffraction. We recall that for single atoms and a grating of period $2\pi/q$, this regime corresponds to the situation where only two degenerate modes, of transverse momenta $\pm q/2$, are coupled. The probabilities of occupation of these two states oscillate periodically in time, the so-called Pendellösung [4]. Bragg scattering is widely used in atomic beam splitters, both for single atoms [24] and for Bose-Einstein condensates [27].

Fermi statistics complicate the situation in two ways: First, as we have seen, the distribution of momentum states in the grating around $\pm q/2$ leads to a dephasing resulting in the time-dependent coupling constant $g(t)$. In addition, only those fermions with initial transverse momentum sufficiently close to $q/2$ fulfill the Bragg condition. Hence, it is expected that Bragg diffraction will
burn a spectral hole in the initial Fermi sea of the beam, leaving atoms away from that hole practically untouched.

We can gain some insight into the effects of the dephasing by considering a simple two-state model that retains only the coupling between the transverse modes \( \pm q/2 \), neglecting their coupling to higher-order modes. This approximation is justified if the phase-matching condition is strongly violated for these modes, in other words, if the difference in kinetic energy \( \omega^{(b)}_{q/2} \) is much larger than the mean-field energy \( \hbar U N_g \) responsible for mode coupling, i.e.,

\[
q^2 \gg 4\pi \alpha \rho q \frac{m_b}{\mu},
\]

(18)

\( \rho q \) being the atomic density in the grating.

This situation is described by the closed set of operator equations

\[
\frac{d}{dt} \hat{n}_{q/2}^{(b)} = \pm i g(t) \left[ \hat{\rho}_{q/2}^{(b)} + c.c. \right],
\]

\[
\frac{d}{dt} \hat{\rho}_{q/2}^{(b)} = i g(t) \left[ \hat{n}_{q/2}^{(b)} - \hat{n}_{-q/2}^{(b)} \right],
\]

(19)

where \( g(t) \) is given by Eq. \( (14) \). These equations are reminiscent of the optical Bloch equations that describe the dynamics of a two-level atom driven by a resonant single-mode field of time-dependent amplitude, with the operators \( \hat{n} \) corresponding to the populations of the two levels and the particle-hole operator \( \hat{\rho}_{q/2}^{(b)} \) to the atomic polarization. The evolution of their expectation values can readily be found for the initial conditions \( \langle \hat{n}_{q/2}^{(b)} \rangle = 1 \) and \( \langle \hat{n}_{-q/2}^{(b)} \rangle = 0 \) as

\[
\langle \hat{n}_{q/2}^{(b)} \rangle (t) = \frac{1}{2} (1 \pm \cos \beta(t)),
\]

(20)

\[
\langle \hat{n}_{-q/2}^{(b)} \rangle (t) = \frac{1}{2} (1 \pm \cos \beta(t)),
\]

where

\[
\beta(t) = \int_0^t dt' g(t') = \frac{1}{2} N_g U_0 \int_0^t dt' \text{sinc} \left( \frac{\hbar k_F q t'}{m_g} \right).
\]

(21)

Similarly to the situation that we encountered in the Raman-Nath regime, the dephasing of the grating results in the diffraction effectively coming to an end after a time of the order of \( \tau_d \).

Fig. 3 summarizes the results of the numerical integration of the full equations of motion (8) in the Bragg regime for a \(^6\)Li beam scattering off a grating formed by \(^{40}\)K atoms. It shows the population of the transverse modes of the atomic beam as a function of time for the case where the initial full transverse momentum width of the grating and beam are 1.0q and 0.25q, respectively. The hole burned in the initial Fermi sea is clearly visible, as is the impact of the dephasing of the grating. The first maximum in the Pendelösung oscillations is strongly pronounced for modes near the phase matched mode \( q/2 \), but further maxima cannot be observed due the dephasing of the fermionic grating.

One can characterize the “efficiency” \( \eta \) of the four-wave mixing process by the first maximum of the quantity

\[
\eta(t) = \sum_{k=-q}^0 \langle \hat{n}_k \rangle (t) / N_b,
\]

(22)

which measures the maximum fraction of atoms whose transverse momentum has been flipped about \( k = 0 \). It is plotted in Fig. 4 as a function of the momentum spread of the incident \(^6\)Li beam for three widths of the \(^{40}\)K grating. The behavior of \( \eta \) is as expected, since a wider incident beam contains more atoms that do not fulfill the Bragg condition, and a larger Fermi sea of the grating means a shorter dephasing time.
Experiments are likely to be carried out in a three-dimensional geometry, in which case the time-dependent coupling constant \( g_{3D} \) becomes

\[
g_{3D}(t) = 3U_0N_g \left( \sin \frac{t}{\tau_d} - \cos \frac{t}{\tau_d} \right) \frac{\tau_d^2}{t^2},
\]

where \( \tau_d \) is still given by Eq. (19). It is easily seen that \( g_{3D}(t) \) decays in a manner similar to \( g(t) \), so that the lifetime of the grating is essentially the same as in one-dimension, provided that \( k_{F,g} \) remains the same. Note however that these two times scale differently with the density of the grating, due to the different mode densities in one and three dimensions.

In order to assess the experimental feasibility of a fermionic Bragg diffraction experiment it is necessary to compare the dephasing time \( \tau_d \) to the time \( \tau_B \) that it would take to observe a full Pendellösung oscillation in the absence of dephasing. With the approximation \( \beta(t) \approx N_gU_0t/2 \) we have readily that \( \tau_B \approx \pi/N_gU_0 \). Hence, the observation of Pendellösung oscillations requires

\[
\frac{\tau_d}{\tau_B} = \left( \frac{2\pi}{(6\pi^2)^{1/3}} \right) \left( \frac{m_g}{\mu} \right) \left( \frac{a_{p_g}^2/3}{q} \right) \gg 1,
\]

where we have used the three-dimensional relationship between the density \( \rho_g \) and \( k_{F,g} \), \( k_{F,g} = (6\pi^2\rho_g)^{1/3} \).

Consider for example the diffraction of fermionic potassium \(^{40}\)K off a spin-polarized \(^{40}\)K grating of peak number density \( \rho_g = 10^{15} \text{ cm}^{-3} \) and a scattering length of 85 \( a_0 \). Assuming a grating period of 500 nm \( \tau_d \approx 0.1\tau_B \), so that Pendellösung oscillations will not be observable in that case. The next section discusses possible ways around this difficulty.

**IV. ATOM ECHOES**

The condition (24) shows that one possible way to increase the ratio of \( \tau_d \) to \( \tau_B \) is to increase the grating density \( \rho_g \). This, however, is likely to be impractical. Another solution is to increase the scattering length \( a \) via a Feshbach resonance. Noting that \( m_g/\mu = (m_a + m_b)/m_a \), we can also gain an order of magnitude or so in the ratio (24) by diffracting light fermions such as \(^6\)Li off a potassium grating. Ketterle et al. also proposed a direct way to increase the decay time in three dimensions by reducing the effective Fermi wave vector in the direction of the grating by increasing the size of the Fermi sea in the other directions.

An alternative method that allows one to completely reverse the effects of the dephasing is based on the observation that this dephasing is coherent, simply resulting from the different kinetic energies of the atoms forming the grating. This is reminiscent of Doppler broadening in laser spectroscopy, which leads to a dephasing of the atomic polarization. In that case, the dephasing can easily be reversed by photon echo techniques. Things are slightly more complicated here, because the phase of the individual atoms is just given by their kinetic energy \( h^2k^2/2m_g \). The only way to reverse the sign of this phase is to reverse the mass of the atoms. This can be achieved by placing the atoms forming the grating in an additional periodic potential that gives them a negative effective mass.

Ignoring two-body collisions as well as whatever external fields are required to prepare the initial grating (10) for now, the evolution of the grating atoms in the presence of an optical lattice is governed by the Hamiltonian

\[
H = \frac{\hat{p}^2}{2m} + \frac{V_0}{2} \cos(2k_Lx),
\]

where \( V_0 \) is the lattice depth and \( k_L \) the wave vector of the laser creating the periodic potential. The band struc-
ture associated with this potential is readily obtained numerically, yielding at the center of the first Brillouin zone the effective mass

\[
m_{\text{eff}, n} = \hbar^2 \left( \frac{d^2 E_n}{dk^2} \right)^{-1} \bigg|_{k=0},
\]

where \(n\) is the band index and \(k\) is the quasi-momentum. Figure 6 plots this effective mass of \(^{40}\)K for the first two energy bands. As is well known, \(m_{\text{eff}}\) is alternatively positive and negative as the band index is increased. We remark that since \(U_0\) is a pseudo-potential adjusted so that the scattering problem associated with two-body collisions asymptotically produces the correct collisional cross-section, the appearance of an effective mass does not change the reduced mass \(\mu\) in Eq. (6). Hence Eq. (24) becomes

\[
\frac{\tau_d}{\tau_B} = \left( \frac{2\pi}{(6\pi^2)^{1/3}} \right) \left( \frac{m_{\text{eff}}}{\mu} \right) \left( \frac{a\rho_0^{2/3}}{q} \right).
\]

From Fig. 6, we see that the effective mass can be orders of magnitude larger than \(m_g\), leading to much easier conditions to satisfy to observe fermionic Bragg diffraction.

Even more interesting is the use of the negative effective mass associated with the second energy band, since it permits a full reversal of the dephasing. The excitation of the grating atoms to that band can be achieved e.g. via a two-photon Raman with co-propagating laser beams, or by modulating the phase of one of the lattice beams. Assuming for simplicity that the effective masses corresponding to the \(n = 1\) and \(n = 2\) bands are equal in magnitude, it is easily seen that if the Raman pulse is applied at time \(t_1\), then \(g(t_2 = 2t_1) = g(0)\), and the dephasing has been completely reversed. Clearly, a sequence of pulses alternatively transferring the grating atoms between the two energy bands will keep alternating the sign of \(m_{\text{eff}}\), thereby continuously compensating the Doppler broadening. In case the magnitude of curvatures of the two bands are different, the rephasing time needs to be appropriately adjusted.

Fig. 6 shows the result of this atom echo technique. The second peak of the Pendellö sung oscillations by the atom echoes technique (upper plots) to the case where the dephasing has been arbitrarily turned off (lower plots). In the plots white corresponds to unit probability and black indicates zero probability. All parameters as in Fig. 4.

\[\text{FIG. 7: Comparison between the rephasing of the Pendellö sung oscillations by the atom echoes technique (upper plots) to the case where the dephasing has been arbitrarily turned off (lower plots). In the plots white corresponds to unit probability and black indicates zero probability. All parameters as in Fig. 4.}\]

V. CONCLUSION

In this paper, we have investigated the behavior of purely fermionic scattering, i.e. a beam of fermions diffracted by a fermionic density grating, in the limit of a large grating. We explored different regimes which are determined by the relative magnitude of mean field energy and kinetic energy. Fermi statistic shows its distinct signatures in the limited lifetime of the grating and the inability to satisfy the Bragg condition for large input beams. We proposed novel ways to increase the dephasing time of the fermionic grating which is the most severe limitation for fermionic four-wave mixing. Future work will focus on the complex dynamics arising from the back-action of the scattered atoms on the grating.

This work is supported in part by the US Office of Naval Research, by the National Science Foundation, by the US Army Research Office, by the National Aeronautics and Space Administration, and by the Joint Services Optics Program. H. C. acknowledges the support of the Studienstiftung des deutschen Volkes.
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