Low Temperature Quasi-Particle Transport in Bosonic and Fermionic Superfluids

Rufus Boyack and K. Levin

James Franck Institute, University of Chicago, Chicago, Illinois 60637, USA

In this paper we use a Kubo approach to address low temperature transport associated with the quasi-particles in bosonic and fermionic superfluids. Our analysis for bosonic superfluids utilizes the framework of the one-loop Bogoliubov approximation, whereas for fermionic superfluids we apply strict BCS theory. Interestingly we find that the transport properties of these two different systems have very similar structure, albeit with different quasi-particle dispersions and statistics. Our focus is on thermoelectric transport and the shear viscosity, \( \eta \), which are accessible (and in some instances already measured) in low \( T \) experiments on trapped gases. While the different quasi-particle dispersions lead to rather dramatic contrasts between power law and exponential temperature dependence for \( \eta \) alone, the ratio of shear viscosity to entropy density is more similar. It involves the same linear dependence on the ratio of temperature to inverse quasi-particle lifetime.

Introduction.— Ultracold quantum gases provide rather unique opportunities for addressing non-equilibrium and time-dependent phenomena in superfluids. Central to characterizing these superfluids is the nature of their transport. Prior to the recent focus on trapped atomic gas superfluids are puzzles which originated in contrasting observations of the fermionic and bosonic counterparts of liquid helium. For example, \( ^3\text{He} \) and \( ^4\text{He} \) exhibit a remarkable difference in their shear viscosity, \( \eta \), for low temperatures. At temperatures below the critical temperature the shear viscosity of fermionic \( ^3\text{He} \) has been measured to be a decreasing function of decreasing temperature, whereas the shear viscosity of bosonic \( ^4\text{He} \) is an increasing function of (decreasing) temperature \([1]\).

Another challenge is to address the behavior of the ratio of shear viscosity to entropy density \( s \). This dimensionless parameter has been the focus of recent excitement as it has been conjectured by Kovtun, Son and Starinets (KSS) \([2]\) to satisfy a strict bound \( \eta/s \geq \hbar/(4\pi k_B) \). For atomic Fermi gases there have been experimental \([3]\) and theoretical \([4]\) studies of this ratio, albeit only in the specific unitary regime; no such studies are yet available for the bosonic superfluids. More general investigations of thermoelectric transport in trapped gases have focused on the normal phase of 2d Bose gases \([5]\), and on thermoelectric devices \([6]\). There has been some previous theoretical work by our group above the transition \([7]\) based on the experiments of \([5]\). This is complementary to the present focus on the low temperature superfluid phase.

This background provides a motivation for this paper in which we present a comparison of the dissipative transport properties of bosonic and fermionic superfluids at low temperatures. We focus on the quasi-particle contributions to transport which are the exclusive contributions in the thermal conductivity, \( \kappa \), the \( \omega \neq 0 \) mass conductivity, \( \sigma \), and the off-diagonal thermoelectric coefficients, as well as the shear viscosity. These are to be distinguished from condensate contributions, which dominate the \( \omega = 0 \) mass conductivity. For the latter, a proper theory of transport has to deal with a number of subtle features involving gauge invariance and the important constraint in bosonic transport in which the (two particle) density excitation spectrum or sound modes are intimately coupled to the single particle excitations. For the fermionic superfluids we work at the standard BCS level. Our starting point for bosonic superfluids is due to Wong and Gould \([8]\), and Talbot and Griffin \([9]\), and known as the “one-loop approximation”.

Our calculations show that the one-loop Bogoliubov theory for bosons and the BCS theory for fermions are formally strikingly similar. Nevertheless, primarily as a result of differences in the quasi-particle excitation spectrum, as well as the statistics, there are important differences in superfluid transport. Quite generally, in bosonic systems, because the dispersion relation is gapless, the transport coefficients increase more rapidly as a function of temperature when compared to the gapped fermion systems. This makes transport in the Bose gases more accessible experimentally. In addressing the shear viscosity, we find these differences qualitatively reflect those seen in comparisons between the bosonic and fermionic helium superfluids.

Previous studies of superfluid transport have relied heavily on kinetic theory and a Boltzmann equation ansatz \([10, 11]\). A less widely applied approach has been the use of linear response theory and Kubo formulae, which we will use here. The advantage of the Kubo formulae approach, however, is that by relating directly to Green’s function diagrammatics one has better control over the processes included in transport and the appropriate constraints. This enables a more systematic imposition of perturbation expansions for a weakly interacting bosonic fluid. Also important are the constraints which must be imposed on separating the contributions associated with longitudinal and transverse correlations, since it is only in the former that the condensate will directly enter. Finally, a subtle but important issue here arises in the shear viscosity, for example, where the Kubo formula shows that there are multiple response functions which enter in addition to the simplest stress tensor-stress tensor correlator \([12]\). It is not as apparent how to include these in a Boltzmann based approach.

Since it is likely that dissipation in the ultracold
gases is linked to the details of the experimental set up, we will introduce dissipation via a phenomenological parametrization. The philosophy behind our phenomenological approach to dissipation is rather similar to that articulated by Kadanoff and Martin, who in a series of papers emphasized the importance of the Kubo-based correlation functions and their symmetries [14]. In related work on superfluids [15], they argued for the suitability of introducing a parametrization of the lifetimes associated with transport. In building any phenomenology it is, however, important to stress that inter-particle collisions can not be the sole source of dissipation in mass transport, as in the particle conductivity. This particular transport coefficient reflects the fact that the total momentum would be conserved (in the presence of Galilean invariance) without other sources of momentum relaxation.

The strength of Boltzmann theory is that, when the details of the processes giving rise to dissipation are well established, one can avoid this phenomenology by incorporating specific collision integrals. Within a Boltzmann based theory of bosonic superfluid transport, there is a fairly extensive review by Griffin [11] in which the shear viscosity and the thermal conductivity are addressed. At a qualitative level our Kubo calculations are consistent with this earlier work, but we also include additional transport coefficients. A Kubo formulation of the shear viscosity of a trapped Bose condensed gas was studied in [13] within the second order Beliaev approximation, but this analysis did not incorporate the contribution from anomalous Green’s functions. This paper revisits this earlier work (albeit without a trap) and with the important inclusion of the anomalous Green’s functions which are a crucial component for a consistent treatment of superfluid transport.

Overview.— We begin by summarizing our results, which serve to emphasize the formal similarity of the bosonic one-loop transport theory with the fermionic BCS transport theory. We define the general transport coefficients \( L_{ij} \) via particle \( \mathbf{J}_p \) and heat \( \mathbf{J}_Q \) current densities as follows

\[
\mathbf{J}_p = -L_{11} \nabla \mu - L_{12} \nabla T, \\
\mathbf{J}_Q = -L_{21} \nabla \mu - L_{22} \nabla T,
\]

where \( \nabla \mu \) and \( \nabla T \) represent imposed gradients of the chemical potential (analogous to the electric field for a charged system) and the temperature. (We work in units where \( h = k_B = e = 1 \).) Here the particle or mass conductivity \( \sigma \equiv L_{11} \) and the thermal conductivity \( \kappa \equiv L_{22} \). The off-diagonal transport coefficients appear, for example, in the quasi-particle thermopower.

In the superfluid it should be stressed that the various correlation functions that enter into the \( L_{ij} \) may be distinct for longitudinal and transverse properties. This distinction is most important for the mass conductivity, as the longitudinal contribution reflects the condensate (and diverges at zero momentum and frequency) while the transverse contribution reflects the quasi-particles. The shear viscosity is also represented in terms of this transverse response.

Following the approach of Kadanoff and Martin [15], lifetime effects are phenomenologically incorporated by introducing a parameter \( \gamma \), representing the inverse quasi-particle lifetime. Using the correlation functions which will appear in Eqs. (15–16) below, we find that, for bosons, the normal fluid dissipative transport coefficients (in 3d) are

\[
\eta^B = \int_0^\infty dk \frac{k^6}{30 \pi^2 m^2} \left( \frac{\xi_k}{E_k} \right)^2 \left( -\frac{\partial n(E_k)}{\partial E_k} \right) \frac{1}{\gamma}, \quad (3)
\]

\[
\text{Re} L^B_{ij} = T^{1-j} \int_0^\infty dk \frac{k^4}{6 \pi^2 m^2} e^{i+j-2} \left( -\frac{\partial n(E_k)}{\partial E_k} \right) \frac{1}{\gamma}. \quad (4)
\]

Note we have evaluated \( \eta^B, \text{Re} L^B_{ij} \) in the limit \( \omega \to 0 \).

We introduce \( n_0 \) as the condensate density and \( g \) as the interaction strength. The Hugenholtz-Pines theorem determines the chemical potential, in the Bogoliubov approximation, as \( \mu^B = n_0 g \). The free particle dispersion relation is \( E_k = \frac{k^2}{2m} \) and we define \( \xi_k = \sqrt{E_k^2 - \mu^B} \). The Bogoliubov quasi-particle dispersion relation is then \( E_k^B = \xi_k^2 - (\mu^B)^2 \). We define \( n(x) = [e^{x/T} - 1]^{-1} \) as the Bose-Einstein distribution function.

The same calculations performed above for bosons can be performed for fermions. The only differences that arise are a sign factor due to the different statistics, a degeneracy factor of two due to spin and a redefinition of the dispersion relation. For fermions the transport coefficients are

\[
\eta^F = \int_0^\infty dk \frac{k^6}{15 \pi^2 m^2} \left( \frac{\xi_k}{E_k} \right)^2 \left( -\frac{\partial f(E_k)}{\partial E_k} \right) \frac{1}{\gamma}, \quad (5)
\]

\[
\text{Re} L^F_{ij} = T^{1-j} \int_0^\infty dk \frac{k^4}{3 \pi^2 m^2} e^{i+j-2} \left( -\frac{\partial f(E_k)}{\partial E_k} \right) \frac{1}{\gamma}. \quad (6)
\]

where \( \xi_k = \sqrt{E_k^2 - \mu^F} \), \( E_k^2 = \xi_k^2 + \Delta_k^2 \), and \( f(x) = [e^{x/T} + 1]^{-1} \). (We have again evaluated \( \eta^F, \text{Re} L^F_{ij} \) in the limit \( \omega \to 0 \).) This expression for the shear viscosity has been obtained previously in [4]. Similarly the mass and thermal conductivities \( L_{11}, L_{22} \) are in agreement with strict BCS theory [15]. Comparison between Eqs. (3–4) and Eqs. (5–6) shows the similarities between bosonic and fermionic transport. A key difference between the transport coefficients arises from the soft quasi-particle excitations for bosons as opposed to the gapped excitations for fermions.

General Theory.— We proceed now to derive Eqs. (3–6). In linear response theory the response of a system perturbed slightly from thermal equilibrium is expressed in terms of time independent correlation functions of the unperturbed system [14]. Equations (1–2) lead to four possible correlation functions involving combinations of particle and heat or energy currents. These four correlation functions are

\[
\chi_{ij}(x_1 - x_2, \tau_1 - \tau_2) = -\langle T_r j_i(x_1, \tau_1) j_j(x_2, \tau_2) \rangle, \quad (7)
\]
where \(i, j \in \{1, 2\}\). The particle and heat currents which appear above are defined as [15]

\[
\begin{align*}
\dot{j}_1 &= -\frac{i}{2m} (\nabla_1 - \nabla_1') \psi^+(1')\psi(1)\big|_{\nu=1^+}, \quad (8) \\
\dot{j}_2 &= -\frac{i}{2m} (\partial_{t_2} \nabla_2 + \partial_{t_2}' \nabla_2') \psi^+(2')\psi(2)\big|_{t'2' = 2^+}. \quad (9)
\end{align*}
\]

For a generic superfluid correlation function \(\chi_{ij}\), it is convenient to decompose into longitudinal and transverse components which are given by \(\chi_{ij}^L = \frac{q_\nu \omega \nu \chi_{ij}^L}{\omega}, \chi_{ij}^T = \frac{1}{\pi} (\sum \alpha \chi_{ij}^\alpha - \chi_{ij}^L)\).

We define the FOURIER transform: \(\hat{\chi}_{ij}(x_1 - x_2, \tau_1 - \tau_2) = \frac{1}{\pi} \sum_{\omega} \int \frac{d^3q}{(2\pi)^3} \hat{\chi}_{ij}(q, i\omega) e^{i q \cdot (x_1 - x_2)} e^{-i \omega (\tau_1 - \tau_2)}\).

Then the Kubo formulae for the transport coefficients other than those associated with \(\chi_{11}\) are

\[
\text{Re} \sigma_{ij} = -T^{-1} \text{lim}_{q \to 0} \frac{\text{Im} \chi_{ij}^L(q, \omega)}{\omega}, \quad i, j \neq 1. \quad (10)
\]

Using this definition, one can compute transport coefficients \(\text{Re} \sigma_{ij}^L, i, j \neq 1\) for the bosonic case and \(\text{Re} \sigma_{ij}^F, i, j \neq 1\) for the fermionic case.

The quasi-particle contribution to the mass conductivity and the shear viscosity (for which there is no condensate component) depend only on the transverse component of \(\chi_{11}\) and are given by [14]

\[
\text{Re} \sigma(\omega \neq 0) = -\text{lim}_{q \to 0} \frac{\text{Im} \chi_{11}^T(q, \omega)}{\omega}, \quad (11)
\]

\[
\eta = -m^2 \text{lim}_{\omega \to 0} \text{lim}_{q \to 0} \frac{\omega \text{Im} \chi_{11}^T(q, \omega)}{q^2}. \quad (12)
\]

By limiting consideration in \(\sigma\) as \(\omega \neq 0\), we focus on the quasi-particle transport. The total mass conductivity (which includes the condensate) is \(\text{Re} \sigma(\omega) = \text{Re} \sigma(\omega \neq 0) + \frac{\rho_s}{m} \delta(\omega)\), where \(\rho_s\) is the superfluid density. The mass conductivity of the condensate is infinite but all condensate thermoelectric coefficients vanish. More specifically, the condensate only enters into \(L_{11}\). Finally, we note that the Onsager relation between the associated transport coefficients is \(L_{12} = L_{21}/f\). [?]

**Bosonic One Loop Approximation.**— In order to evaluate these various \(\chi_{ij}\) we introduce the appropriate Green's functions. This is well established for the case of fermionic BCS superfluids. For the bosonic case, the one-loop approximation is based on the Bogoliubov Green's functions and thus involves the Bogoliubov quasi-particle dispersion. Because bosonic superfluid theories involve a controlled perturbation in the interaction strength, they lead to a clear hierarchy of diagrams and we can restrict attention in the dilute fluid limit to those involving one or at most two Green's functions. The latter constitute the “loops” of the transport approximation.

For transverse response functions, the only diagrams that contribute are those that cannot be divided into two parts by removing one line representing a single-particle propagator. Such diagrams are called proper. The condensate contributions to a generic correlation function (dependent on a single-particle Green's function) are not proper, and therefore do not contribute to the transverse response functions. It follows that, for a one-loop theory, the transverse component of a generic correlation function is completely determined by diagrams containing only two single-particle Green's functions. For longitudinal correlation functions, other than \(L_{11}\), there are no condensate contributions and again the leading order contribution involves two single-particle Green's functions. In the superfluid phase there are two such Green's functions (the anomalous and normal Green’s functions.)

**Bosonic Correlation Functions and Kubo Formulae.**— At this bosonic one-loop level we relate these correlation functions to the imaginary time single particle Green’s functions in position space, given by \(G(x, \tau)\) (normal), \(F(x, \tau)\) (anomalous), defined by: \(\langle T_x \psi(x_1)\psi^+(x_2) \rangle = -G(x_1 - x_2) + n_0 \) and \(\langle T_x \psi(x_1)\psi(x_2) \rangle = -F(x_1 - x_2) + n_0\). For convenience, we make the following definitions: the four vector summation \(\sum_k = \frac{1}{2} \sum_{\omega} \int \frac{d^3k}{(2\pi)^3}\), the vertex factors \(v_1 = (\frac{k + \nu q}{m}), v_2 = \frac{q_\nu}{2m}, \text{ and } v_3 = (i\omega_n + \nu \frac{k + q}{2m} + \nu_\omega_n \frac{k + q}{2m})\). The dissipative parameter \(\gamma\) previously introduced also serves to analytically continue the Matsubara frequencies \(i\omega_n\) to real frequencies \(\omega\) via: \(i\omega_n = \omega + i\gamma\).

With these definitions, the four momentum space correlation functions can be computed. The particle current-particle current correlation function is given by:

\[
\begin{align*}
\chi_{11}(q, i\omega) &= n_0 v_1 v_2 [G(Q) + G(-Q) - F(Q) - F(-Q)] \\
&+ \sum_K v_1 v_3 [G(K)G(K + Q) + F(K)F(K + Q)].
\end{align*}
\]

(13)

The particle current-heat current correlation function is:

\[
\begin{align*}
\chi_{12}(q, i\omega) &= \sum_K v_1 v_3 \\
&\times [G(K)G(K + Q) + F(K)F(K + Q)].
\end{align*}
\]

(14)

The heat current-particle current correlation function is:

\[
\begin{align*}
\chi_{21}(q, i\omega) &= \sum_K v_3 v_1 \\
&\times [G(K)G(K + Q) - F(K)F(K + Q)].
\end{align*}
\]

(15)

The heat current-heat current correlation function is:

\[
\begin{align*}
\chi_{22}(q, i\omega) &= \sum_K v_3 v_3 \\
&\times [G(K)G(K + Q) + F(K)F(K + Q)].
\end{align*}
\]

(16)

Our expressions in Eqs. (13–16) contain all possible contributions to the irreducible transverse response functions [16]. As can be seen, the particle current-particle current correlation function \(\chi_{11}\) which appears in Eq. (13),
we define \( \chi \) for bosonic (solid blue) and fermionic (dashed red) superfluids as functions of \( \chi \) unlike all the other \( \chi_{ij} \), contains a term proportional to the condensate density \( n_0 \). This term is purely longitudinal and of no interest here. It is this condensate defined by \( T / T \) to the condensate density \( n \) \( \sim \eta / T \). The normal state for bosonic and fermionic superfluids is defined by \( g_n = 0 \), and \( T = 0 \) respectively. In the Bogoliubov approximation \( n_0 \) is the particle number at \( T = 0 \); thus our calculations are confined to \( T / T_c << 1 \). Here the mass conductivity corresponds to the \( \omega \neq 0 \) contribution.

Finally, from the definitions of the transport coefficients, combined with the correlation functions in Eqs. (13–16) and the Bogoliubov Green’s functions, the resulting transport coefficients \( \eta^B \) and \( \text{Re} \eta^B_{ij} \) are given by the expressions in Eqs. (3–4).

**Calculation of \( \eta / s \).**—Kovtun, Son and Starinets (KSS) [2] have made an interesting conjecture concerning the shear viscosity. They conjecture that any relativistic quantum field theory at finite temperature and zero chemical potential has a ratio of shear viscosity to entropy density satisfying the bound \( \eta / s \sim h/(4\pi k_B) \). Despite the construction of certain systems that violate the KSS bound [17], the KSS conjecture has lead to renewed interest as to what the ‘perfect’ fluids in nature are, i.e., those that come as close as possible to minimizing the conjectured bound. It has been shown by KSS that fluids that saturate this bound are those with a dual gravity description.

An interesting feature of the KSS bound is that it is independent of the speed of light \( c \). Therefore, a non-relativistic quantum system is a possible candidate for the ‘perfect’ fluid. Here we investigate the magnitude of \( \eta / s \) arising from quasi-particle transport in the bosonic one-loop and fermionic BCS superfluids.

A variant of the KSS conjecture extends the applicability of the conjectured bound of \( \eta / s \) to the case of non-zero chemical potential [18]. If we allow \( \mu^B / \mu^F = 0 \), then in the low temperature limits \( (T << \mu^B, T_C) \) we obtain, for bosons and fermions

\[
\eta^B / s^B \rightarrow \frac{1}{5} \times \frac{T}{\gamma},
\]

\[
\eta^F / s^F \rightarrow \frac{p_F^4}{15m^2\Delta_0} \times \frac{T}{\gamma},
\]

where \( p_F \) is the Fermi-momentum. It should be noted that once the entropy density is included, both fermions and bosons exhibit the same \( T/\gamma \) dependence in their viscosity ratios. With the typical temperature dependence used for \( \gamma \) [10], one would predict an upturn in \( \eta / s \) at low \( T \) for both bosonic and fermionic superfluids.

On the other hand, the low temperature limits of the shear viscosity of bosons and fermions are markedly different:

\[
\eta^B \rightarrow \frac{2\pi^2}{225\gamma} \left( \frac{m}{\mu_B} \right)^{3/2} T^4,
\]

\[
\eta^F \rightarrow \frac{2g(E_F)p_F^4}{15m^2\gamma} \left( \frac{2\pi T}{\Delta_0} \right)^{1/2} e^{-\Delta_0/T},
\]

where \( g(E_F) \) is the density of states. Depending on the temperature dependence of the quasi-particle lifetimes \( (\gamma^{-1}) \), the bosonic shear viscosity can exhibit an upturn for low temperatures. However, due to the exponentially suppressed term, the fermionic shear viscosity is not expected to exhibit an upturn, regardless of the parameter \( \gamma \). This is qualitatively consistent with experimental observations in liquid helium.

**Numerical Results.**—In general, the bosonic transport coefficients exhibit power law behavior, whereas the fermionic transport coefficients exhibit an exponentially suppressed response. Explicitly, we find that for bosons

\[
\text{Re} \eta^B_{ij} \rightarrow \frac{2\pi^2}{45\gamma} m^{1/2}(\mu^B)^{i+j-9/2} T^{5-j},
\]
whereas for fermions
\[ \text{Re}L_{11}^F \rightarrow \frac{2p_0^2 g(E_F)}{3m^2 \gamma} \left( \frac{2\pi \Delta_0}{T} \right)^{1/2} e^{-\Delta_0/T}, \]  
\[ \text{Re}L_{22}^F \rightarrow \frac{2p_0^2 g(E_F)}{3m^2 \gamma} \left( \frac{2\pi \Delta_3^0}{T} \right)^{1/2} e^{-\Delta_0/T}. \]  

In BCS theory, assuming the chemical potential is of order \( \mu_F \sim E_F \) and with exact particle-hole symmetry, \( \text{Re}L_{12}^F \rightarrow 0 \).

Figure (1) shows the comparison between the normalized bosonic and fermionic transport coefficients. It is clear from the figures that the quasi-particle transport coefficients at these low \( T \) differ by about 5-7 orders of magnitude. This is, of course, not surprising and due to the differences in the quasi-particle excitation spectrum. From an experimental perspective, it appears rather prohibitive to measure very low \( T \) transport properties of the Fermi gases. By contrast Bose systems lend themselves to these low \( T \) studies. Nevertheless, for Fermi superfluids, away from the ground state there has been considerable progress in mapping out the shear viscosity [3].

Conclusions. We have compared the \( \omega \rightarrow 0 \) mass conductivity, the shear viscosity, and the thermal conductivity in bosonic and fermionic superfluids based on a Kubo formula approach within the one-loop Bogoliubov and closely related BCS approximation. At this level of approximation, our work demonstrates the formal (albeit non-quantitative) similarity between the transport behavior of both superfluid types. The transverse response functions do not contain condensate contributions. Similarly for the longitudinal thermolectric coefficients (aside from the \( \omega \equiv 0 \) mass conductivity) no condensate contributions appear. Thus, it is appropriate to characterize these coefficients entirely in terms of their quasi-particle contributions, as we have here. Of some interest is the fact that even though the shear viscosities \( \eta \) have dramatically different temperature dependence, their ratios in terms of the entropy density \( s \) have precisely the same \( T/\gamma \) (where \( \gamma \) is the inverse quasi-particle lifetime) behavior with very different prefactors. Essentially all reasonable models for the temperature dependence of the transport lifetime will give an upturn in \( \eta/s \) at low \( T \), but not, for the case of fermions, in \( \eta \) itself. This appears consistent with the observed differences between helium-3 and helium-4 superfluids [1].

This work is supported by NSF-MRSEC Grant 0820054. We thank Adam Rançon for many helpful conversations.

[1] H. Guo, D. Wulin, C.-C. Chien, and K. Levin, New J. Phys. 13, 075011 (2011).
[2] P. K. Kovtun, D. T. Son, and A. O. Starinets, Phys. Rev. Lett. 94, 111601 (2005).
[3] A. Turlapov, J. Kinast, B. Clancy, L. Luo, J. Joseph, and J. E. Thomas, J. Low. Temp. Phys. 150, 567 (2008).
[4] H. Guo, D. Wulin, C.-C. Chien, and K. Levin, Phys. Rev. Lett. 107, 020403 (2011).
[5] E. L. Hazlett, L.-C. Ha, and C. Chin, [arxiv:cond-mat.quant-gas/1306.4018].
[6] J. P. Brantut, C. Grenier, J. Meineke, D. Stadler, S. Krimmer, C. Kollath, T. Esslinder, A. Georges, Science 342, 713 (2013).
[7] A. Rançon and K. Levin, [arXiv:cond-mat.quant-gas/1311.0769].
[8] V. K. Wong and H. Gould, Ann. Phys. 83, 252 (1974).
[9] E. Talbot and A. Griffin, Ann. Phys. 151, 71 (1983).
[10] I. M. Khalatnikov, An Introduction to the Theory of Superfluidity (Perseus, New York, 2000).
[11] A. Griffin, T. Nikuni, and E. Zaremba, Bose-Condensed Gases at Finite Temperatures (Cambridge University Press, Cambridge, 2009).
[12] B. Bradlyn, M. Goldstein, and N. Read, Phys. Rev. B 86, 245309 (2012).
[13] M. A. Shahzamanian and H. Yavary, Ann. Phys. 321, 1063 (2006).
[14] L. P. Kadanoff and P. C. Martin, Ann. Phys. 24, 419 (1963).
[15] L. P. Kadanoff and P. C. Martin, Phys. Rev. 124, 670 (1961).
[16] A. Griffin, Excitations in a Bose Condensed Liquid (Cambridge University Press, Cambridge, 1993).
[17] A. Cherman, T. D. Cohen, and P. M. Hohler, JHEP02 026 (2008).
[18] D. T. Son and A. O. Starinets, Ann. Rev. Nucl. Part. Sci. 57, 95 (2007).