Comparison of the contribution of the photon’s vector and scalar Kaluza-Klein partners in the neutron lifetime

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Abstract. A discrete extra dimension of size 11.8 fm would imply the existence of vector $X$ and scalar $H$ bosons as photon’s Kaluza-Klein partners. The massive vector $X$ can be identified with the particle $X_{17}$ suggested by ATOMKI’s experiment. The particle model of extended photon sector coupled to extended nucleon, electron and neutrino also implies the neutron decay to its Kaluza-Klein partner. There are several different channels triggered by $X$ and $H$. In this paper we compare the contribution from them and imply some physical consequences on the parameters of the model.

1. Introduction

1.1. Anomalies
The neutron decay is among the oldest known problems in high energy and elementary particle physics. The current Standard Model allows only the $\beta$-decay channel $n \to p + e^- + \bar{\nu}_e$ and predicts the neutron lifetime with this channel is $\tau_{SM} = 878.7 \pm 0.6$. This value is perfectly compatible with the neutron lifetime measured in the "bottle" measurement $\tau_{bottle} = 879.4 \pm 0.6$. However, the lifetime measured in the beam experiments via counting the produced protons gets 8 seconds longer, $\tau_{beam} = 888 \pm 0.2$ [1]. The discrepancy between the “bottle” and “beam” methods has persisted since these measurements began yielding accurate results in the 1990s. Naturally, the puzzle can be solved by adding some new decay channels, which do not create protons. Thus a smaller number of protons is produced in the neutron decays and the measured value $\tau_{beam}$ could be larger than the true decay time $\tau_{bottle}$. Recently, many physicists are interested in the problem and the main resolutions are suggested with new decay channels, where dark matter can be involved. Fornal and Grinstein [2] have suggested the new neutron decay channels which produce a dark neutron instead of the photon as a possible resolution of the puzzle. Three different channels have been suggested: i) $n \to n_X + \gamma$, ii) $n \to n_X + e^+ + e^-$ and iii) $n \to n_X + \phi$. The authors have also determined that, in the simplest scenario, the hypothetical dark neutron’s mass must fall between 937.9 and 938.8 MeV. However, the recent experiment tests have practically excluded the first two channels [3, 4]. So, only the third channel remains as the possible solution. Later, Ivanov et al [5] have proposed an alternative channel $n \to n_X + \nu_e + \bar{\nu}_e$, where $\nu_e, \bar{\nu}_e$ are neutrino and antineutrino.
Moreover, the scientists in the Xenon collaboration have reported low-energy electronic recoil data recorded by the XENON1T detector, which revealed an excessive recoil of electrons with the confidence of $3\sigma$. One can assume that the recoiled electrons have interacted with a light vector boson particle with a mass in the $MeV$ range. Effortlessly, we can recognize that these anomalies are related with the massive boson $X_{17}$, which was found in the ATOMKI’s Internal Pair Creation (IPC) experiment. In this experiment, Kraszhorkay el al have bombarded the $^7Li$ target with a low-energy proton beam in a Van der Graff accelerator [6]. They have observed an anomalies IPC in the excited $Be$ nucleus transition to its ground state. Thus they have hypothesized the existence of a vector boson with a mass of 17 $MeV$. There are consistently around 5.8 events related to $X_{17}$ in a total of 1 million IPC ones, mostly dominated by photon. The recent observations have also confirmed the hypothesis [7, 8, 9].

In a previous article [10], we have investigated the possibility of having the interaction between the neutron-dark neutron and neutrinos being mediated by $X_{17}$, which lead to the decay channel proposed by Ivanov. The underlying theory to explain the existence of $X_{17}$ together with the dark neutron can be found in Viet’s discrete extra dimension model [11], where $X_{17}$ and dark neutron are interpreted as the Kaluza-Klein (KK) partners of photon and neutron. We have implied the mass splitting between the neutron and its Kaluza-Klein partner being less than 1.102 $MeV$, which suppresses the neutron decay channel into electron-positron pairs in accordance with the recent experimental observation by Tang [12]. Additionally, the masses of Kaluza-Klein partners of proton and electron are assumed to be at least in the TeV range to explain why these particles have not been observed. However, in addition to the massive vector, the photon also has a scalar KK partner. In this article we will discuss the contributions of the new decay channels related top the photon’s scalar KK-partner to the neutron life time problem.

1.2. Particle model with a discrete extra dimension of two points

The discrete extra dimension has been proposed by Viet and Wali as an intuitive interpretation of Connes-Lott’s model noncommutative geometric space-time [13]. Since 1994, Viet and Wali have developed the Discretized Kaluza-Klein theory (DKKT) [14], with the fifth dimension having only two points. Recently, the theory have been applied to some specific cases [15, 16, 17, 10]. In this article we use a particular space-time proposed in [11], which is an extension of the usual one $M^4$ with a discrete extra dimension having the following specific line element and metric

$$ds^2 = G_{MN}dx^Mdx^N = \eta_{\mu\nu}dx^\mu dx^\nu + \lambda^4 dx^5 dx^5,$$

(1)

where $M, N = \mu, 5$ are the indexes of no-normal frame and $\lambda$ is a warping factor similar to the one in the RS1 model [18].

The metric in Eq.(1) corresponds to the extended vierbein

$$E^A = E^A_M DX^M, DX^M = E^M_A E^A,$$

$$G^{AB} = E^A_M G^{MN} E^B_N = diag(-1, 1, 1, 1, 1),$$

$$G^{MN} G_{NL} = \delta^M_L,$$

(2)

where $M, N = a, 5$ are the indexes of locally orthonormal frame.

Thus in this model the vielbeins are in the following particular form

$$E^a_\mu(x) = \delta^a_\mu 1_2, E^5_\mu = 0, E^5_5 = 1/\lambda^2$$

$$E^a_5(x) = \delta^a_5 1_2, E^a_5 = 0, E^5_5 = \lambda^2$$

(3)
In this space-time, the extended fermion field is convenient to represent a given fermion KK pair as a two-column spinor
\[ \Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \]
and the usual Dirac operator is extended to the following matrix
\[ \mathcal{D} = \begin{bmatrix} \not p & -m/\lambda^2 \\ m/\lambda^2 & \not p \end{bmatrix} \]

The Kaluza-Klein partners of photon are obtained by extending the usual vector field’s 1-form \( \not \phi = \gamma^\mu b_\mu(x) \) into the hermitian 2 x 2 matrix containing the photon field \( A_\mu(x) \) and its Kaluza-Klein partners [11].

\[ \mathcal{B} = \Gamma^\mu B_\mu(x) + \Gamma^5 \phi(x) = \begin{bmatrix} \gamma^\mu b_1(x) & \sqrt{2}gH(x) - m/\lambda^2 \\ \sqrt{2}gH(x) - m/\lambda^2 & \gamma^\mu b_2(x) \end{bmatrix} \]

where where \( A_\mu(x) \) is the usual electromagnetic field. Its vector and scalar Kaluza-Klein partners are \( X_\mu(x) \) and \( H(x) \). \( Q \) and \( Q_X \) are, respectively, the electric and dark charge operators. \( g \) and \( g' \) are coupling constants related to photon and \( X_{17} \) respectively. In this model, these coupling constants are not independent, since the gauge Lagrangian constructed from the 1-form \( \mathcal{B} \) must have correct kinetic terms [11].

From the above point of view, we must also extend the particle model of nuclear physics. Neutron, proton, electron and neutrino can also have their own Kaluza-Klein partners. Following [11] we identify \( X_{17} \) with the vector field \( X_\mu(x) \). The interaction of \( X_{17} \) and \( H(x) \) with fermions then can be obtained from a generalization of the Yukawa coupling of photon with fermions as follows

\[ \mathcal{L}_{f-g} = Tr \bar{\psi}(x)(i\not \partial + \mathcal{B})\psi(x), \]

which is a straightforward generalization of the fermion-gauge coupling Lagrangian \( \bar{\psi}(x)(i\not \partial + \not A(x))\psi(x) \).

The Lagrangian in Eq.(7) lead to a non-diagonal mass term for fermion and its K-K partner. Therefore, in order to obtain the mass eigenstates, one must diagonalize the mass matrix by an unitary transformation characterized by a mixing angle [11]. Therefore, the mass splitting between any given fermion \( \psi \) and its Kaluza-Klein partner \( \psi_X \) is given by its mixing angle as follows

\[ m_f - m_{fX} = \frac{2\sqrt{2}m}{\sin 2\theta_f \lambda^2} = \frac{4m_X}{\sin 2\theta_f \lambda^2}, \]

where \( \theta_f \) is the mixing angle of the fermion \( f \) ( \( f \) can be any of \( p, n, e, \nu \) and \( f_X \) is its respective Kaluza-Klein partners \( p_X, n_X, e_X, \nu_X \)).

Therefore, we can have an explicit coupling of neutrino, neutron and their KK partners with the K-K partners of photon as follows [11]

\[ g_{Xnn} = g \sin^2 \theta_n, \quad g_{XnXnX} = g \cos^2 \theta_n, \quad g_{XnnX} = 1/2g \sin 2\theta_n, \]
\[ g_{X\nu\nu} = g \sin^2 \theta_\nu, \quad g_{X\nuX\nuX} = g \cos^2 \theta_\nu, \quad g_{X\nuX\nu} = 1/2g \sin 2\theta_\nu, \]
\[ g_{Hnn} = gHnXn = g \sin^2 \theta_n, \quad g_{HnnX} = g \cos^2 \theta_n, \]
\[ g_{H\nu\nu} = g_{XnX\nuX} = g \sin^2 \theta_\nu, \quad g_{H\nuX} = g \cos^2 \theta_\nu, \]

where \( \theta_n, \theta_\nu \) are respectively the mixing angles of neutron and neutrino with their Kaluza-Klein partners.
Masses of X- and H-photons are contacted with mass parameter as follows

\[ m_X = m/\sqrt{2} = 17 \text{MeV}, \quad m_H = 2m_X/\lambda^2, \quad m = 24 \text{MeV} \]

Now, we are ready to examine the possible decay channels of neutron in the new particle model.

2. Dark decays of neutron

Based on the above model, the decay channels involving \( e^+ - e^- \) pair creation have been excluded by the additional assumption \( m_{n_X} - m_n < 1.102 \text{MeV} \) in accordance with the experiment result of Tang et al [12]. So the remaining possible decay channels of neutron are

(i) \( n \rightarrow p + e^- + \bar{\nu}_e \), (\( \beta - \text{decay} \)),
(ii) \( n \rightarrow n_X + H \), (First order),
(iii) \( n \rightarrow n_X + \nu + \bar{\nu} \),
(iv) \( n \rightarrow n_X + \nu + \bar{\nu}_X \),
(v) \( n \rightarrow n_X + \bar{\nu} + \nu_X \),
(vi) \( n \rightarrow n_X + \nu_X + \bar{\nu}_X \).

The first channel is the usual beta decay via the \( W \) boson exchange in the Standard Model [19]. Its decay width is given as follows

\[ \Gamma_W = \frac{f^R m^5 e}{2\pi^3 Vw^2 G_F} \left( 1 + \frac{3g^2_A}{g^2_V} \right) = \frac{9.504g^4}{64\pi^3 \sin^4 \theta_W m^4_W} m^5_e, \tag{11} \]

In the second channel, the scalar KK partner of photon is involved, while in the remaining neutrino producing ones, the vector boson X17 is exchanged. Now we can calculate the contribution of these decays with the specific coupling constants given in Eq.(9).

2.1. Neutron decay with \( H \) photon production

From the mass splitting formula in Eq.(8), we imply that \( m_H = 2m_X/\lambda^2 < m_n - m_{n_X} < 1.102 \text{MeV} \). So this decay channel is energetically possible.

\[ n(P) \rightarrow n_X(p_1) + H(p_2) \]

[Figure 1. Decay of neutron to the dark one and the H photon]

Applying the golden rule for the two-body decay \( n \rightarrow n_X + H \), in the neutron rest frame, \( P = \left( m_n; 0 \right) \) (see Fig.1), we have

\[ d\Gamma (n \rightarrow n_X + H) = \frac{|\mathcal{M}|^2}{2m_n} (2\pi)^4 \delta^4 (P - p_1 - p_2) \frac{d^3p_1}{2(2\pi)^3 E_1} \frac{d^3p_2}{2(2\pi)^3 E_2} \]

\[ = \frac{|\mathcal{M}|^2 \ |p_1|}{32\pi^2 m^2_n} d\Omega \tag{12} \]
where \( d\Omega = d\phi \ d(\cos\theta) \) is the spherical differential function. The dark neutron momentum magnitude is given as a consequence of the energy conservation in a good approximation, where \( m_H < 1.102 \text{MeV}, \ m_{n_X} > 938.463 \text{MeV} \)

\[
|p_1| = \frac{1}{2m_n} \left[ \left( m_n^2 - (m_{n_X} + m_H)^2 \right) \left( m_n^2 - (m_{n_X} - m_H)^2 \right) \right]^{1/2} \sim \left[ (m_n - m_{n_X})^2 - m_H^2 \right]^{1/2}. \quad (13)
\]

On the other hand, the transition matrix elements are

\[
\mathcal{M}_{\alpha\beta} = g_1 \bar{n}_\alpha (P) n_{X\beta} (p_1), \quad \mathcal{M}_{\alpha\beta}^+ = g_1 \bar{n}_{X\beta} (p_1) n_\alpha (P), \quad (14)
\]

Now we can calculate \(|\mathcal{M}|^2\)

\[
|\mathcal{M}|^2 = g_1^2 \text{Tr}[(p_1 + m_{n_X})(P + m_n)] = 4g_1^2 (p_1.P) + 4g_1^2 m_n m_{n_X} \quad (15)
\]

which can be approximated by

\[
|\mathcal{M}|^2 = 2g_1^2 (m_n + m_{n_X})^2 \quad (16)
\]

So the spherical integral is trivial (\( \int d\Omega = 4\pi \)) and leading to

\[
\Gamma_H = \frac{g_1^2 (m_n + m_{n_X})^3}{8\pi m_n^2} \sqrt{(m_n - m_{n_X})^2 - m_H^2} \sim \frac{g_2^2 \cos^4 \theta_n}{\pi} [(m_n - m_{n_X})^2 - m_H^2]^{1/2} \quad (17)
\]

Finally, using Eqs.(8) and (10), we obtain the decay rate of neutron with the H-photon production channel

\[
\Gamma_H = \frac{2g_2^2 \cos^4 \theta_n m_X}{\pi^2 \lambda^2} \left( \frac{4}{\sin^2 2\theta_n} - 1 \right)^{1/2} \sim \frac{2g_2^2 m_X \cos^3 \theta_n}{\pi \lambda^2 \sin \theta_n}. \quad (18)
\]

In the last step, we have used the condition \( 4/\sin^2 2\theta_n \gg 1 \) as a consequence of the IPC experiment result [11].

2.2. The dark decay of neutron via X17

The neutron decay channels, which produce neutrinos, are triggered by the X17 vector boson exchange. They are depicted by the Feynman diagram in Fig.(2), where \( \psi \) can be either neutrino or its Kaluza-Klein partner.

The generic amplitude for these processes can be represented as follows

\[
\mathcal{M} = \bar{u} (n_X) i g_1 \gamma^\mu u (n) \left\{ \frac{q_{\mu} q_{\nu} - q_{\mu} q_{\nu}/m_X^2}{q^2 - m_X^2} \right\} \bar{u} (\psi) i g_2 \gamma^\nu u (\bar{\psi}), \quad (19)
\]

where \( q_{\mu} \) is the transfer 4-momentum of the virtual X17. \( g_1 \) and \( g_2 \) are coupling constants at the corresponding vertexes to be specified in Eq.(9).
Applying Casimir’s trick and the trace theorems, we find the general formula

\[ |M|^2 = A1 + A2 + A3 \]  \hspace{1cm} (20)

\[ A1 = \frac{32g^2 g^2}{(q^2 - m_X^2)} \left( \frac{(p_1.p_2)(p_3.p_4) + (p_2.p_3)(p_1.p_4) - m_n m_n}{m_n^2} \right) \]

\[ + m_{\nu} m_{\bar{\nu}} (p_1.p_3) + 2 m_{\nu} m_{\bar{\nu}} m_n m_n \]  \hspace{1cm} (21)

\[ A2 = - \frac{32g^2 g^2}{(q^2 - m_X^2)^2 m_X^2} \left[ m_{\bar{\nu}} m_{\nu} (p_1.p_2) - m_{\bar{\nu}} m_{\nu} (p_1.p_4) \right] \]

\[ + m_{\nu} m_n (p_2.p_3) - m_{\bar{\nu}} m_n (p_3.p_4) \]  \hspace{1cm} (22)

\[ A3 = \frac{16g^2 g^2}{(q^2 - m_X^2)^2 m_X^2} \left[ (p_1.p_3)(p_2.p_4) - m_{\nu} m_{\bar{\nu}} (p_1.p_3) \right] \]

\[ + m_n m_n (p_2.p_4) - m_{\nu} m_{\bar{\nu}} m_n m_n \]  \hspace{1cm} (23)

In the neutron decay problem, neutrino can be treated as a massless particle. Therefore we can calculate the decay widths \( \Gamma_i, i = 3, 4, 5, 6 \) of the channels iii)-vi) explicitly.

2.2.1. The decay channel product neutrino and anti-dark neutrino, \( n \rightarrow n_X + \nu + \bar{\nu}_X \). Applying Eqs. (20) with \( m_{\nu} \sim 0, m_{\nu_X} = m \), with condition is \( m < m_n - m_{\nu_X} \), we obtain

\[ |M|^2 = \frac{32g^2 g^2}{(q^2 - m_X^2)^2} \left[ (p_1.p_2)(p_3.p_4) + (p_2.p_3)(p_1.p_4) - m_n m_n \right] \]

\[ - \frac{32g^2 g^2 m^2}{(q^2 - m_X^2)^2 m_X^2} \left[ m_{\nu} (p_1.p_4) + m_{\bar{\nu}} (p_3.p_4) \right] \]

\[ + \frac{16g^2 g^2 m^2}{(q^2 - m_X^2)^2 m_X^2} \left[ (p_1.p_3)(p_2.p_4) + m_n m_n \right] \]

In the neutron rest frame, the Fermi's Golden rule reads

\[ d\Gamma = \frac{\langle |M|^2 \rangle}{(4\pi)^4 \hbar m_n} dE_2 dE_4 \]  \hspace{1cm} (25)

where \( E_2 \) and \( E_4 \) are energies of the outgoing \( \bar{\nu}_X \) and \( \nu \) particles satisfying the conditions

\[ \frac{m^2 - m_{n_X}^2 + m^2}{2 (m_n - E_2 + |p_2|)} < E_4 < \frac{(m_n^2 - m_{n_X}^2 + m^2) - 2m_n E_2}{2 (m_n - E_2 - |p_2|)} \]  \hspace{1cm} (26)

\[ \frac{m^2 - m_{n_X}^2 + m^2}{2 (m_n - E_2 + |p_2|)} < \frac{m^2 - m_{n_X}^2 + m^2}{2 (m_n - E_2 - |p_2|)} \]  \hspace{1cm} (27)
It is convenient to introduce the following small parameters:

\[ \varepsilon = \frac{m_n - m_{nX}}{m_n}, \quad \delta = \frac{m}{m_n}, \quad \eta = \frac{E_2}{m_n}, \quad \phi = \frac{|p_2|}{m_n}, \quad E = \frac{E_4}{m_n}, \quad X = \left( \frac{m_X}{m_n} \right)^2 \]  

These quantities are extremely small in the magnitude order of \(10^{-3}\). Using the lowest order expansions, we obtain

\[
\frac{d\Gamma(E, \eta)}{d\eta} \left( \frac{g_1^2 g_2^2 m_n}{4\pi^3 \hbar} \right)^{-1} = -2E^2 + E \left( 4\varepsilon - 2 \right) + 2\varepsilon - 2\eta - 3\varepsilon^2 - 2\eta^2 + \delta^2 + 4\varepsilon\eta \frac{dE}{(2E + 2\eta - 2\varepsilon + \varepsilon^2 - X)^2} 
\]

When we calculate the above integration over \(E\), we find that the first order terms are eliminated. Therefore, we approximate to the 2nd order. We have

\[
\frac{d\Gamma(\eta)}{d\eta} = \frac{g_1^2 g_2^2 m_n}{4\pi^3 \hbar X^2} \left[ \phi (\eta - \varepsilon) \left( 8\eta^2 - 8\eta \varepsilon - 4\phi \eta + 4\phi \varepsilon - 2\delta^2 \right) \right] 
\]

We continue to put new variables \(x = \eta/\varepsilon; a = \delta/\varepsilon = m/(m_n - m_{nX})\), let

\[
\Gamma_4 = \frac{g_1^2 g_2^2}{2\pi^3 m_X} (m_n - m_{nX})^5 J_4(a)
\]

with

\[
J_4(a) = \int_a^1 \left[ \sqrt{x^2 - a^2} (x - 1) \left( 8x^2 - 8x - 4\sqrt{x^2 - a^2} x + 4\sqrt{x^2 - a^2} - 2a^2 \right) \right] dx 
\]

\[
= -\frac{1}{15} + \frac{2a^3}{3} - \frac{4a^3}{3} + a^4 - \frac{4a^5}{15} + \frac{\sqrt{1-a^2}}{30} (4 - 13a^2 - 6a^4) 
\]

\[
+ \frac{a^4}{2} \ln \left( 1 + \sqrt{1-a^2} \right) - \frac{a^4}{2} \ln a 
\]

where \(0 < a < 1\). Using analytic calculation, we obtain condition \(0.0673 < J_4(a) < 2/15\).

Due to the symmetric properties, the decay width of the channel \(v\) is the same \(\Gamma_5 = \Gamma_4\).

The decay width \(\Gamma_3\) can also be obtained from Eq.(31) by setting \(m = 0\)

\[
\Gamma_3 = \frac{g_1^2 g_2^2}{30\pi^3 m_X} (m_n - m_{nX})^5 
\]

Using the coupling constants in Eq.(9) and replacing the mass difference by the mixing angle in Eq.(8), we have the following bandwidth

\[
\Gamma_3 = \frac{16g_4^4 m_X \sin^4\theta \nu}{15\pi^3 \lambda^{10} \cos \theta_n \sin^5 \theta_n} 
\]

\[
\Gamma_{4,5} = \frac{16g_4^4 m_X \sin^2\theta \nu \cos^2 \theta \nu}{\pi^3 \lambda^{10} \sin^5 \theta_n \cos \theta_n} J_4(a) 
\]
2.2.2. The decay channel product dark neutrino and its antiparticle, \( n \rightarrow n_X + \nu_X + \bar{\nu}_X \). In this case, \( m_{\nu_X} = m_{\bar{\nu}_X} = m \), with condition is \( 2m < m_n - m_{n_X} < 1.1002 \text{ MeV} \), we have

\[
|M|^2 = \frac{32g_1^2g_2^2}{(q^2 - M_X^2)^2} \left[ (p_1 \cdot p_2) (p_3 \cdot p_4) + (p_2 \cdot p_3) (p_1 \cdot p_4) - m_n m_{n_X} (p_2 \cdot p_4) + m^2 (p_1 \cdot p_3) + 2m^2 m_n m_{n_X} \right]
\]

(36)

with the conditions

\[
m < E_2 < \frac{m_n^2 - m_{n_X}^2 - 2m_{n_X} m}{2m_n},
\]

(37)

\[E_- < E_4 < E_+,
\]

(38)

where

\[
E_{\pm} = \frac{m_n - E_2} {2 (m_n^2 + m^2 - 2m_n E_2)} \left( m_n^2 - m_{n_X}^2 + 2m^2 - 2m_n E_2 \right) \pm \sqrt{E_2^2 - m^2} \left[ (m_n^2 - m_{n_X}^2 - 2m_n E_2)^2 - 4m_{n_X}^2 m^2 \right]^{1/2} / 2
\]

(39)

Using the parameters in Eq(28) we have

\[
E_{\pm} \sim \varepsilon - \eta - \frac{\varepsilon^2}{2} - \eta^2 + \delta^2 + \varepsilon \eta \pm \phi \sqrt{(\varepsilon - \eta)^2 - \delta^2}
\]

(40)

We obtain the decay width of this channel as follows

\[
\Gamma_6 = \frac{g_1^2 g_2^2}{2\pi^3 m_X^4} (m_n - m_{n_X})^5 J_6(a)
\]

(41)

with

\[
J_6(a) = \int_a^{1-a} \left\{ -\sqrt{x^2 - a^2} \sqrt{(1-x)^2 - a^2} (4x^2 - 4x - 8a^2) - 2 (x^2 - a^2) \left[ (1-x)^2 - a^2 \right] \right\} dx
\]

(42)

where \( 0 < a < 0.5 \). We can use the following polynomial expansion of \( J_6(a) \) and obtain

\[
\Gamma_6 \sim \frac{g_1^2 g_2^2 m_n \varepsilon^5}{2\pi^3 \hbar X^2} \left( \frac{1}{15} + 1, 4293a^2 - 4, 3946a^3 + 4, 996a^4 + 13, 949a^5 \right)
\]

(43)

\[
= \frac{16g_1^4 m_X}{\pi^3 \lambda^{10}} \cos^4 \theta_{\nu} \sin^5 \theta_n \cos \theta_n J_6(a),
\]

(44)

where \( J_6(a) \leq 1/15 \).

So, we have obtained the decay widths for all the neutrino producing decay channels of neutron via X17. However, these decay widths still depend on the mixing angles \( \theta_n, \theta_{\nu} \). Now, we consider some special cases.

The first, if neutrino mass splitting is small, we can ignore the KK-neutrino mass, \( m = 0 \) and \( a = 0 \). We receive the total decay rate with X17

\[
\Gamma_X = \Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6 = \frac{16g_1^4 m_X}{15\pi^3 \cos \theta_n \sin^4 \theta_{\nu} \lambda^{10}}
\]

(44)
The decay width is independent of $\theta_\nu$.

In the second case, the mass of KK-neutrino is larger than neutron mass splitting. Only the first three channels are possible. So the total rate even $\Gamma_3$

$$\Gamma_X = \Gamma_3 = \frac{16g^4m_X}{15\pi^3\cos\theta_n\sin^2\theta_n\lambda^{10}}\sin^4\theta_\nu \tag{45}$$

In the last case, the dark neutrino mass is in the range $0 < m < m_n - m_X$

$$\Gamma_X = \sum \Gamma_i < \frac{32g^4m_X}{15\pi^3\cos\theta_n\sin^2\theta_n\lambda^{10}} \tag{46}$$

In all the above cases, we have

$$\Gamma_X < \frac{32g^4m_X}{15\pi^3\cos\theta_n\sin^2\theta_n\lambda^{10}} \tag{47}$$

3. Conclusion

The results of Tang’s experiment [12] can be explained by $\delta m_n = 68 \text{ MeV}/\sin 2\theta_n\lambda^2 < 1.102\text{MeV}$. Which implies $\sin 2\theta_n\lambda^2 > 61.7$. On other hand, we have

$$\frac{\Gamma_X}{\Gamma_H} < \frac{256g^2}{15\pi^2\sin^3\theta_n\lambda^8} < \frac{256g^2}{15\pi^2\lambda^{61.74}} < 1.2 \times 10^{-7}g^2 \ll 1, \tag{48}$$

which means that the contributions of the $X_{17}$ mediated and neutrino producing channels is negligible in comparison with the channel with the scalar proton $H$.

In this paper, we have utilized the new dark channels based on the dark sector of the Kaluza-Klein partners of known particles. The specific particle model [11] is based on a further assumption that the extra "dark" dimension can be described (exactly or in a good approximation) by a discrete dimension of size $11.8 \text{ fm}$ to give the hypothetical $X_{17}$ boson, which might explain the ATOMKI anomalies [7]. Although the physical consequences of this "dark" dimension can be found in more details in [11] or elsewhere, here we can make some remarks on the model restricted with photon, electron, neutrino, nucleon and their Kaluza-Klein partners, which are relevant in the MeV energy range.

Since, proton and electron are charged particles, in order to suppress their pair creation under the TeV range, the mixing angles $\theta_p$ and $\theta_e$ must be very close to 0 or $\pi$. In that case, the mass splitting formula 8 will give a large mass for dark proton and electron in the $\text{TeV}$ range. We can consistently assume that $\theta_p \sim 0$, to have proton couplings to $X_{17}$ and $H$ arbitrarily small. It is shown in [11] the electron mixing angle $\theta_e \sim \pi/2$, to have the pair creation $X \rightarrow e^+ + e^-$ nonvanishing according to the ATOMKI observations.

The formula 8 implies that the "warping factor" $\lambda^2$ must be sufficiently large in order to give the small mass splitting of neutron to satisfy the bound $m_n - m_{X_7} < 1.102 \text{ MeV}$. Since neutron and neutrino are neutral, they do not interact with photon directly, so the new channels are mainly with $X_{17}$ and $H$, therefore the phenomenology with photon is not affected.

The only channels which might have consequences on the phenomenology of the standard neutron and electron therefore are related to the dark decays of neutron and neutrino. In addition to the decay channels considered in this paper, one might consider also the phenomenological consequences of the neutrino’s dark decay channels, which can lead to the excessive creation of neutrino, which is not so easy to measure accurately. The matter however is still interesting theoretically and will be considered elsewhere.
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