A global analysis of the experimental data on $\chi_c$ meson hadroproduction

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We carry out a global analysis on the experimental data on $\chi_c$ production rate and the ratio $\sigma(\chi_{c2})/\sigma(\chi_{c1})$ at the LHC and Tevatron. The related long-distance matrix elements (LDMEs) at both leading order (LO) and next-to-leading order (NLO) in QCD coupling constant are renewed. We also present the transverse momentum distribution of the $\chi_c$ production rate and the ratio $\sigma(\chi_{c2})/\sigma(\chi_{c1})$ for several experimental conditions, and find that NLO predictions agree with all sets of experiment measurements except for one. By contrast, at LO, we can not explain all the experiments by using a single LDME. A brief analysis on the non-relativistic QCD scale dependence of the cross sections shows that, for the conditions we concern in this paper, the dependence can almost be totally absorbed into the LDME.

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I. INTRODUCTION

Since the Large Hadron Collider (LHC) started its running, many experimental results have come out, which provided an opportunity to carry out further investigation on phenomenology of QCD-based effective theories. Non-relativistic QCD (NRQCD) is one of the most successful models describing quarkonium productions and decays [1]. Under NRQCD framework, the cross section is factorized into the summation of the products of the short-distance coefficient (SDC), which is independent of the quarkonium state and can be calculated perturbatively, and the long-distance matrix element (LDME), which only depends on the quarkonium state and requires the fit of experimental data to extract its value. The cross section for the process of heavy quarkonium $H$ production or decay can be expressed as

$$d\sigma(H) = \sum_n df_n \langle O^H(n) \rangle$$

where $f_n$ is the SDC for the $q\bar{q}$ state $n$, and $\langle O^H(n) \rangle$ is the LDME of state $n$ for quarkonium $H$.

NRQCD succeeded in many areas where colour-singlet (CS) model [2–6] failed, however, it still faces many challenges. $J/\psi$ polarization at hadron colliders is one among the most puzzling questions NRQCD encounters. Ref. [7, 8] investigated the polarization of direct produced $J/\psi$, while Ref. [9] provided polarization results for prompt $J/\psi$ hadroproduction, which is the first NLO result comparable with experiment. The three references employed three sets of LDMEs, which were obtained from different fit strategy. All of them can describe $J/\psi$ production, yet, despite of this, none of them can explain all the polarization measurements. In addition, the universality of the LDMEs is another challenge. Ref. [10] reconciled experimental data of $J/\psi$ production at the Tevatron and HERA, however, their LDMEs resulted in unphysical cross sections when employed to $J/\psi$ associated with a photon production at hadron colliders [11]. We consider that LO and NLO $3S_1^{[8]}$ inclusive production both behave as $1/p_t^4$ in large $p_t$ region, while for a photon associated case, the cross section of NLO decreases much milder ($1/p_t^8$) than that of LO ($1/p_t^4$) as $p_t$ increases. Since the cross sections for $3S_1^{[8]}$ and $3P_J^{[8]}$ are associated, which is to say, NLO $3P_J^{[8]}$ cross section has one part proportional to minus LO $3S_1^{[8]}$ cross section, the LDMEs obtained from the fit of $J/\psi$ inclusive hadroproduction at NLO can not work in the case of $J/\psi$ hadroproduction associated with a photon at NLO. Only when we push the calculation forward to next-to-next-to-leading order (NNLO), a single set of LDMEs can work in both the processes. However, the case of $J/\psi$ photoproduction is quite similar to that of $J/\psi$ hadroproduction associated with a photon. It is strange that the LDMEs succeeded in explaining HERA data [10] while they caused unphysical results in Ref. [11]. For the above reasons, testing NRQCD is still an important work.

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We say P-wave quarkonia productions provide a very good test on NRQCD factorization scheme for three reasons. Above all, at LO in $v$ (the relative velocity of quark and antiquark in quarkonium), only one LDME is to be obtained from experiment; one does not suffer from the entanglement of too many free parameters in the fit of experimental data, by contrast to the $J/\psi$ case. In addition, at NLO, the transverse momentum ($p_t$) distribution of both $^3P_J$ and $^3S_1$ production rate behaves as $1/p_t^4$ in large $p_t$ region. Higher order corrections can not exceed this behavior, as a result, one can expect NLO predictions to give a good precision in this phase space region. By contrast to the $^3S_1$ case, the significance of NNLO correction is still in the mist. Finally, feeddown from higher excited states to P-wave quarkonia, say $\chi_c$, $h_c$, $\chi_b$, or $h_b$, can almost be neglected (e.g. $\sigma(\psi(2S) \rightarrow \chi_{c1}(1p))/\sigma(\chi_{c1}(1p)) \sim 5\%$ at LHCb [14–16]). One can consider prompt (or inclusive for bottomonium) production rate (the measurement of which is much easier than the measurement of the direct production rate) as the same with the direct one. Notice the advantages stated above, we say the case of $\chi_c$ is "clean"; it is much easier to make definite conclusions than the $J/\psi$ (as well as e.g. $\Upsilon$, $\eta_c$ and $\eta_b$) case.

On the other hand, the study of $\chi_c$ production is not only important itself for phenomenological concerns, but also provides an opportunity of precise study of $J/\psi$ phenomenology (e.g. Ref. [8] indicates that $\chi_c$ feeddown contribution is essential to the study of $J/\psi$ polarization). Many theoretical works on $\chi_c$ production have been published. Ref. [17–19] obtained LDME for $\chi_c$ production at LO, and employed them to the prediction of prompt $J/\psi$ hadroproduction. Ref. [20] presented the first NLO study of $\chi_c$ hadroproduction and gave a favorable choice of the LDME for $\chi_c$ production. Ref. [21] calculated $\chi_c$ production associated with a $c\bar{c}$ pair at hadron colliders. Ref. [22, 23] provide a detailed study on the polarization of hadroproduced $\chi_c$ and $\chi_c$-generated $J/\psi$, at the same time, they compared the theoretical prediction to some of the recent experimental results [24–26].

This paper is devoted to the theoretical predictions of $\chi_c$ hadroproduction, for one thing, as an alternative test of NRQCD. Recently, a number of experiment results have come out from the LHC collaborations, among which, there are many measurements on $\chi_c$ production rate and the ratio $\sigma(\chi_{c2})/\sigma(\chi_{c1})$. This paper will answer the question whether a single LDME can explain all the experimental data. For another thing, the popular values of the LDMEs for $\chi_c$ production, both at LO and NLO, were all given before these experimental results having published; they are out of date. This paper will provide a detailed analysis on the determination of the LDMEs and reasonable values of it at both LO and NLO. Finally, as suggested in Ref. [27], we also observe the NRQCD scale dependence of the cross sections to determine whether NLO prediction stands up.

This paper is organized as follows. Section II gives a brief introduction to NRQCD framework for $\chi_c$ hadroproduction. In Section III, we present the parameter choices in our numerical computation and an analysis to see whether the NRQCD scale dependence is severe. In Section IV, we present the results of the fit and the values of the LDMEs at both LO and NLO and investigate the universality of the LDMEs in detail. Section V is a concluding remark.

### II. $\chi_c$ Production in NRQCD Framework

This section provides quite a brief review of the NRQCD formulas for the calculation of $\chi_c$ production. We do not discuss in detail how the equations are derived. Interested readers can refer to some relative references, e.g. [27, 28].

For $\chi_c$ production at LO of $v$, Eq. (1) can be written as

$$d\sigma(\chi_c J) = df_3^3P_J^{[1]}(O^{\chi_c J}(^3P_J^{[1]})) + df_3^3S_1^{[8]}(2J + 1)(O^{\chi_c a}(^3S_1^{[8]}))$$

The value of CS LDME can be evaluated through [1]

$$\langle O^{\chi_c J}(^3P_J^{[1]}) \rangle = \frac{9}{2\pi}(2J + 1)|R_p'(0)|^2$$

where $R_p'(0)$ is the derivative of the wave function of the related quarkonium with respect to radius at the origin.

The calculation of $f_3^3P_J^{[1]}$ has been described in detail in many of our previous papers, see e.g. [29]. To evaluate $f_3^3P_J^{[1]}$, noticing that it is independent of the long-distance asymptotic states, and replace $\chi_c$ J in Eq. (2) by a $c\bar{c}$ state

$$d\sigma(^3P_J^{[1]}) = df_3^3P_J^{[1]}(O^{3P_J^{[1]}}(3P_J^{[1]})) + df_3^3S_1^{[8]}(O^{3P_J^{[1]}}(3S_1^{[8]}))$$

where we have used the relation $\langle O^{3P_J^{[1]}}(3S_1^{[8]}) \rangle = (2J + 1)\langle O^{3P_J^{[1]}}(3S_1^{[8]}) \rangle$. We should keep in mind that, Eq. (1) is to extract CS SDC, which is expanded in $\alpha_s$. As a result, the quantities $\langle O^{3P_J^{[1]}}(3P_J^{[1]})) \rangle$ and $\langle O^{3P_J^{[1]}}(3S_1^{[8]})) \rangle$ should also
be evaluated perturbatively, and the value of $\alpha_s$ in them should be in accordance with in the SDCs. The left and right side of Eq.(4) should keep those terms up to the same order in perturbative expansion. The evaluation of $d\sigma_{3P_J^{[1]}}^S$ follows the ordinary procedure: writing the squared amplitudes through reading Feynman diagrams and multiplying it by the flux density and the phase space unit. Both $d\sigma_{3P_J^{[1]}}$ and $\langle O^{3P_J^{[1]}}(3S_1^{[8]}) \rangle$ are infrared (IR) divergent, and the IR divergences from the two quantities cancel each other.

We adopt the two-cutoff phase space slicing method [30] and find that the cross section excluding the terms (denoted as $d\sigma_S$) corresponding to the squared real-correction diagrams, in which a gluon connects the quarkonium (as displayed in Fig. 1), integrated over the gluon soft region is free of divergence. We denote this finite part of the cross section as $d\sigma_F$. Neglecting the finite terms proportional to the size of the small region, $d\sigma_S$ can be express as

$$d\sigma_S^{(3P_J^{[1]})} = -\frac{\alpha_s}{3\pi m_c^2} u_c^s \frac{N_c^2 - 1}{N_c^2} d\sigma_{3S_1^{[8]}}^{(3P_J^{[1]})},$$

where $N_c$ is 3 for SU(3) gauge field and

$$u_c^s = \frac{1}{\epsilon_{IR}} + \frac{E}{p} \ln\left(\frac{E + p}{E - p}\right) + \ln\left(\frac{4\pi \mu^2}{s\delta_s^2}\right) - \gamma_E - \frac{1}{3},$$

with $E$ and $p$ being energy and absolute value of momentum of $\chi_c$, respectively, $\gamma_E$ the Euler’s constant, and $\mu$ the scale to complement the dimension. $\delta_s$ is an arbitrary positive number small enough to provide the soft approximation with sufficient accuracy.

Up to the order maintained in our calculation, the transition rate of $c \bar{c}$ state $3S_1^{[8]}$ into $3P_J^{[1]}$ can be calculated in dimensional regularization scheme as

$$\langle O^{3P_J^{[1]}}(3S_1^{[8]}) \rangle_{NLO}^{(3P_J^{[1]})} = -\frac{\alpha_s}{3\pi m_c^2} u_c \frac{N_c^2 - 1}{N_c^2} \langle O^{3P_J^{[1]}}(3P_J^{[1]}) \rangle_{LO},$$

where $u_c$ is defined as

$$u_c^{\epsilon|\mu_{\Lambda}} = \frac{1}{\epsilon_{IR}} - \gamma_E - \frac{1}{3} + \ln\left(\frac{4\pi \mu^2}{\mu_{\Lambda}^2}\right)$$

and

$$u_c^{\epsilon|\overline{MS}} = \frac{1}{\epsilon_{IR}} - \gamma_E + \frac{5}{3} + \ln\left(\frac{\pi \mu^2}{\mu_{\Lambda}^2}\right)$$

in $\mu_{\Lambda}$-cutoff (in which $\mu_{\Lambda}$ is the upper bound of the integrated gluon energy) and $\overline{MS}$ renormalization scheme, respectively. $\mu_{\Lambda}$ is a scale rising from the renormalization of the LDME.

Substituting Eq.(7) and Eq.(5) to Eq.(4), we can solve the SDC for $3P_J^{[1]}$ as

$$df_{3P_J^{[1]}}^{NLO} = df_{3P_J^{[1]}}^{F} - \frac{\alpha_s}{3\pi m_c^2} \frac{N_c^2 - 1}{N_c^2} u_c df_{3S_1^{[8]}}^{LO},$$

where

$$u_c = u_c^s - u_c^c.$$
the expressions of which for $\mu_A$-cutoff and $\overline{MS}$ renormalization scheme are

$$u_{c|\mu_A} = \frac{E}{p} \ln \left( \frac{E + p}{E - p} \right) + \ln \left( \frac{\mu_A^2}{s_0^2} \right) - 2 + 2 \ln(2)$$

(12)

and

$$u_{c|\overline{MS}} = \frac{E}{p} \ln \left( \frac{E + p}{E - p} \right) + \ln \left( \frac{\mu_A^2}{s_0^2} \right),$$

(13)

respectively.

Both of the terms on the right-hand side of Eq. (10) are finite. Now, all the short-distance coefficients are IR divergence free, as a result, the components for calculating the cross section for $\chi_c$ hadroproduction are well defined.

Substituting Eq. (10) into Eq. (2), we obtain the complete expression of cross section for $h_c$ production,

$$d\sigma^{NLO}(\chi_c,J) = d\sigma^F(\chi_c,J) - \frac{\alpha_s}{3\pi m_c^2} \left( \frac{N_c^2 - 1}{N_c^2} \right) u_c \langle O^{c,J} (3P_F^{[1]} S^{[1]}) \rangle df^{LO}_{S^{[1]}} + (2J + 1) \langle O^{\chi_c} S^3 \rangle df^{NLO}_{S^3}. $$

(14)

III. NUMERICAL CALCULATION AND THE ANALYSIS ON $\mu_A$ DEPENDENCE

To calculate $\sigma(3S^1)$ and $\sigma(3P^J_F)$, we apply our Feynman Diagram Calculation package (FDC) [31] to generate all the needed FORTRAN source.

Before we present the numerical results, there are some comments on the obtaining of the CO LDME. Focusing on the last two terms on the right-hand side of Eq. (13), one can notice that, if $\mu_A$ varies its value, in order to fit the cross section $d\sigma^{NLO}(\chi_c)$ to the experiment data, the LDME in the last term should change accordingly, which is to say, the dependence of $\mu_A$ is partly absorbed into the CO LDME. If we proceed our calculation to infinite order of $\alpha_s$, the $\mu_A$ dependence can be totally absorbed into the CO LDME. As a result, this scale actually can be any positive value holding the convergence of $\alpha_s$ expansion. If our results significantly depend on $\mu_A$, the dropped terms in higher orders must contribute significantly, and the calculation up to this order does not reach a sufficient accuracy. Up to NLO, the condition of $\mu_A$ independence requires

$$\frac{\alpha_s}{3\pi m_c^2} \left( \frac{N_c^2 - 1}{N_c^2} \right) \propto \delta_s \langle \rangle,$$

(15)

as well as that the proportional ratio be universal for all the processes. We define a quantity

$$r = \frac{df^{NLO}_{3S^1}}{dp_t} \left/ \left( \frac{\alpha_s}{3\pi} \frac{N_c^2 - 1}{N_c^2} \frac{df^{LO}_{3S^1}}{dp_t} \right) \right.$$

(16)

to determine whether the $\mu_A$ dependence is severe. If $r$ is constrained in a small range through out the whole $p_t$ region for all the processes, we know for sure the dependence of $\mu_A$ can be absorbed into the LDME, vice versa. In this paper, we provide the values of the CO LDME for different renormalization scheme and $\mu_A$ choices.

In the numerical calculation, we have the following common choices of parameters. $|R_p^f(0)|^2 = 0.075 \text{ GeV}^5$ [32] for both LO and NLO calculation, and $m_c = 1.5 \text{ GeV}$. The soft cutoff $\delta_s$ independence is checked in the calculation and $\delta_s = 0.001$ is used. Since the energy scale most of the phase space region exceeds b-quark mass, $\Lambda_{QCD}|n_f=5 = 0.226 \text{ GeV}$ is used. We employ CTEQ6M [33] as the parton distribution function (PDF) and two-loop $\alpha_s$ running for up-to-NLO calculation, and CTEQ6L1 [33] and one-loop $\alpha_s$ running for LO. The renormalization and factorization scales are chosen as $\mu_R = \mu_f = m_t \equiv \sqrt{4m_c^2 + p_t^2}$.

In our fit, we exclude $p_t < 7 \text{ GeV}$ points. That is because, for one thing, below 7 GeV, the relativistic correction contributes a nonlinear part [34], which can not be absorbed into the LDME. For another thing, the $\log(x)$ terms might ruin the perturbative expansion in this region [33, 36], where $x$ denotes Bjorken-$x$.

We find out all the experiment measurements for $\chi_c$ hadroproduction up to now; they are from Ref. [15, 24, 26, 37]. Ref. [37] measured the fraction of prompt $\chi_c \to J/\psi$ production, however, no article provides the production rate of prompt $J/\psi$ for that experiment condition at those $p_t$ points, as a result, we do not include these data in our fit. Notice that Ref. [34] is an updated version of Ref. [24], we exclude Ref. [24] while selecting Ref. [34]. Now, we give all the sets of experimental data used in our fit a number for convenience. We denote the data in Ref. [24, 15, 39] and [26] as E1, E2, E3 and E4, and the data for $\chi_{c1}$ and $\chi_{c2}$ production in Ref. [35] as E5 and E6, respectively.
In E1-4, the values of $p_t$ are given for $\chi_c$ generated $J/\psi$, in accordance to which, we should do the so-called $p_t$ shift as $p_t^{J/\psi} \approx p_t^{\chi_c} m_{J/\psi}/m_{\chi_c}$. Here we choose $m_{J/\psi} = 3.097$ GeV, $m_{\chi_c0} = 3.415$ GeV, $m_{\chi_c1} = 3.510$ GeV and $m_{\chi_c2} = 3.556$ GeV, which are different from Ref. [9], where $m_{J/\psi}$ and $m_{\chi_c}$ are 3.1 GeV and 3.5 GeV, respectively. The branching ratios [40] are 1.27%, 33.9% and 19.2% for $\chi_c0,1,2$ to $J/\psi$.

Before we carry out the fit, we shall first investigate whether $r$ defined in Eq. (10) is universally a constant to hold the $\mu_A$ independence. E2 and E3, and E5 and E6 are in the same experimental condition, respectively, as a result, there are four conditions to present. For these experimental conditions, the values of $r$ are 2.7, 0.054, 0.23, 0.16, 0.058 and 0.37, respectively. We can now expect that, for E1 and E4-E6, the theoretical prediction might agree with the experiment qualitatively.

IV. NUMERICAL RESULTS AND COMPARISON TO THE EXPERIMENTAL DATA

Firstly, we present the values of the CO LDME for the fit of each set of the experimental data at both LO and NLO, and see if they are correspond with each other. In the rest of this paper, we abbreviate $\langle O^{\chi_c}(S_1^0) \rangle$ as $\mathcal{O} \times 10^{-3}$ GeV$^3$.

For LO calculation,

$$
\begin{align*}
\mathcal{O}^{LO}_{E1} &= 0.24 \pm 0.13, \\
\mathcal{O}^{LO}_{E2} &= 1.25 \pm 0.03, \\
\mathcal{O}^{LO}_{E3} &= 1.88 \pm 0.61, \\
\mathcal{O}^{LO}_{E4} &= 0.13 \pm 0.05, \\
\mathcal{O}^{LO}_{E5} &= 1.22 \pm 0.07, \\
\mathcal{O}^{LO}_{E6} &= 0.67 \pm 0.07,
\end{align*}
$$

(17)

and the $\chi^2/d.o.f.$ are 6.4, 0.0075, 0.67, 0.69, 0.11 and 0.43, respectively.

For NLO calculation, as $\mu_A = m_c$ in $\mu_A$-cutoff renormalization scheme,

$$
\begin{align*}
\mathcal{O}^{NLO}_{E1} &= 1.94 \pm 0.16, \\
\mathcal{O}^{NLO}_{E2} &= 2.37 \pm 0.07, \\
\mathcal{O}^{NLO}_{E3} &= 5.21 \pm 0.61, \\
\mathcal{O}^{NLO}_{E4} &= 2.00 \pm 0.07, \\
\mathcal{O}^{NLO}_{E5} &= 2.05 \pm 0.05, \\
\mathcal{O}^{NLO}_{E6} &= 2.03 \pm 0.06,
\end{align*}
$$

(18)

and the $\chi^2/d.o.f.$ are 2.7, 0.054, 0.23, 0.16, 0.058 and 0.37, respectively.

To begin with, we can see that, for all the conditions except E2, the $\chi^2$ for NLO is smaller than that for LO. Moreover, for E1 and E4-6, the obtained values of the CO LDME for NLO are almost the same ($\mathcal{O}$ ranges from 1.94 to 2.05). As we analysed in the previous section, we do not expect theoretical prediction for E2 and E3 agree with the experiment in high precision, however, even for E2, the obtained value of the CO LDME is very close to that for E1 and E4-6. But for E3, the value of the CO LDME obtained from fitting is much larger than those for other sets of experimental data. By contrast, the values of $\mathcal{O}$ obtained for LO range from 0.13 to 1.88; the largest is about 14 times of the smallest, and there is not a common value for any group of the sets; the distribution of the values is dispersive. We can conclude that, there does not exist any universal value for LO LDME, since up to LO, the precision is not

FIG. 2: The value of $r$ defined in Eq. (16) as a function of $p_t^{\chi_c}$.
FIG. 3: The $p_t$ distribution of $\chi_c$ production at the Tevatron and LHC. The blue and black curves are for LO and NLO, respectively.

enough to describe all the experiments. We can also see that the LDME given in Ref. [17] is too large. One might make wrong conclusions if using that value. The LDME given in Ref. [19] is the upper bound of the series of the values presented above. The large value of the LDME might lead to overestimation of the absorption effect (as well as other nuclear matter effects).

Now we carry out a global fit, using all the experimental data in E1-6 for LO, and E1 and E4-6 for NLO, and obtain

$$O^{LO} = 0.47 \pm 0.09, \quad O^{NLO} = 2.00 \pm 0.04.$$  \hspace{1cm} (19)

The $\chi^2/d.o.f.$ are 2.7 and 0.47 for LO and NLO, respectively. The consideration is that we trust the precision of the NLO results for E1 and E4-6, however, for E2 and E3, the situation is not clear, as a result, we fit E1 and E4-6 to obtain NLO CO LDME as a default value to present our results, and employ the LDME to see whether it can explain the experiment E2 and E3.

Theoretical predictions for $\chi_c$ production rate and the ratio $\sigma(\chi_c^2)/\sigma(\chi_c^1)$ are listed in Fig.3 and Fig.4, respectively. Ref. [15, 24, 25, 39] only provide results for $J/\psi p_t$, while Ref. [38] provides results for both $J/\psi$ and $\chi_c p_t$. Since our calculations are carried out at $\chi_c p_t$, as a result, the distribution for E5 and E6 are with respect to $\chi_c p_t$. Since the uncertainty of the LDME for NLO is small, we do not draw the band rising up from this uncertainty. However, the uncertainty of the LDME for LO is quite large, but we do not bother with this matter here, and will provide later more reasonable a band for LO uncertainty.

We can see from Fig.3 and Fig.4 that NLO results are in very good agreement with the experiment except for E3, while LO results can not agree with most of the experiment data. As we mentioned above, for E2 and E3, the NLO
FIG. 4: The ratio $\sigma(\chi_{c2})/\sigma(\chi_{c1})$ as a function of $p_t$ at the Tevatron and LHC. The blue and black curves are for LO and NLO, respectively.

FIG. 5: The LO results for $\chi_c$ production rate and the ratio $\sigma(\chi_{c2})/\sigma(\chi_{c1})$ as a function of $p_t$ at the Tevatron and LHC. The band corresponds to the LO prediction between the results for $O = 0.13$ and $O = 1.88$.

calculation might not be able to provide sufficiently precise results, since as displayed in Fig. 2, that $\mu_\Lambda$ dependence is severe for this experimental condition. This fact might arise from the large rapidity (denoted as $y$), since large $y$ eventually introduces two scales $E_{\chi_c}$ (the energy of $\chi_c$) and $m_t$. When $y = 4.5$, $E_{\chi_c}/m_t \approx 45$, which might ruin the perturbative expansion. One might resum these terms to achieve well converged results.

Since LO results obtained from the default choice of the LDME can not provide good predictions, we shall give a range of the LDME to cover all the experimental data. Here we choose the range between the upper and lower bound of the values given in Eq. (17): $O = 0.13 \sim 1.88$. We can see in Fig. 5 that the large band can cover most of the experimental data, and the upper bound overestimates the significance of $\chi_c$ production rate for some of the conditions. Still, we are not sure whether they are able to explain new experiments, however, to present a band for the range given above might cover the experimental data in the sense of statistics. And we know for sure that, a single value of the CO LDME can not give reasonable results at LO.

At the end of this section, we present the values of the CO LDME at NLO for different choices of $\mu_\Lambda$. For the same
FIG. 6: The NLO results for $\chi_c$ production rate and the ratio $\sigma(\chi_c^2)/\sigma(\chi_c)$ as a function of $p_t$ at the Tevatron and LHC. The band corresponds to the NLO prediction between the upper and lower bounds using the eight LDMEs for different renormalization schemes and the values of $\mu_\Lambda$.

reason, we exclude E2 and E3 data. The LDMEs are listed as follows,

$$O_{m_c}^{NLO} = 2.25 \pm 0.04, \quad \frac{O_{m_c/2}^{NLO}}{O_{m_c}^{NLO}} = 1.68 \pm 0.04, \quad \frac{O_{\Lambda_{QCD}}^{NLO}}{O_{m_c}^{NLO}} = 0.70 \pm 0.04. \quad (20)$$

Here we have used $\overline{MS}$ renormalization scheme (in the calculation of NLO correction to the CO LDME). The $\chi^2/d.o.f.$ are 0.48, 0.46 and 0.42, respectively. We also present here the LDME obtained from including E2 and E3, and see whether the results change much.

$$O_{\mu_\Lambda}^{NLO} = 2.15 \pm 0.09, \quad O_{m_c}^{NLO} = 2.41 \pm 0.09, \quad O_{\Lambda_{QCD}}^{NLO} = 0.84 \pm 0.08. \quad (21)$$

where the subscript $\mu_\Lambda$ stands for $\mu_\Lambda$-cutoff renormalization scheme as well as $\mu_\Lambda = m_c$, and the subscripts $m_c$, $m_c/2$ and $\Lambda_{QCD}$ refer to $\overline{MS}$ renormalization scheme and $\mu_\Lambda$ being the corresponding values. The $\chi^2/d.o.f.$ are 1.8, 1.8, 1.8 and 1.7, respectively. The difference between the LDMEs fitted by including and excluding E2 and E3 ranges from 7% to 20%. The difference increases as $\mu_\Lambda$ gets smaller, which is caused by the different behaviour of $r$ for the four experimental conditions. Including E2 and E3 enhances the $\chi^2/d.o.f.$ significantly, which is to say that theoretical prediction can not fit E2 and E3 as good as it fits E1 and E4-E6. Fig.6 presents the comparison of theoretical predictions for the eight LDMEs to the experimental data. Actually, all the eight LDMEs result in good agreement with experiment except for the E3 case. For E1 and E4-6, the bands hold small as $p_t$ varies, while for E2 and E3, the bands get very large in high $p_t$ regions, which is to say for E1, E4-6 and small $p_t$ region in E2 and E3, the $\mu_\Lambda$ dependence can be absorbed into the LDMEs, while in large $p_t$ regions in E2 and E3, the problem of $\mu_\Lambda$ dependence becomes severe.

V. SUMMARY AND OUTLOOK

In this paper, we calculated $\chi_c$ production rate and the ratio $\sigma(\chi_c^2)/\sigma(\chi_c)$ at hadron colliders, and compared the theoretical predictions to the experiment. We presented a detailed analysis on the CO LDMEs and found that, at LO, there does not exist any universal value of the CO LDME to explain all the experiments, while at NLO, the CO LDME obtained from a global fit is able to explain all the experiments except for E3. At LO, we obtained the value of $O$ ranging from 0.13 to 1.88 when fitting individual experiments E1-6. The upper and lower bounds of $O$ result in
quite a large band, which can cover most of the experimental data, however, the upper bound, corresponding to the value given if Ref. [19], overestimates the significance of $\chi_c$ production rate for some of the experimental conditions. As for NLO case, we carried out a global fit by using eight schemes. Each of them agree well with the experimental data except for E3 case. We also investigated the $\mu_\Lambda$ dependence of the results and found that, for E1, E4-6 and small $p_t$ region in E2 and E3, the dependence on $\mu_\Lambda$ can be absorbed into the LDMEs, while for large $p_t$ region in E2 and E3, the problem of $\mu_\Lambda$ dependence is severe. One should resum the large log terms rising from large rapidity to achieve better results.

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