Two-Phase Stratified Sampling Estimator for Population Mean in the Presence of Nonresponse Using One Auxiliary Variable

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ABSTRACT
In this article, a new estimator for the mean of population in stratified two-phase sampling in the presence of nonresponse using one auxiliary variable is been suggested. The Mean Squared Error (MSE) and the bias of the suggested estimator have been given using large sample approximation. The empirical study shows that the MSE of the suggested estimator is better than existing estimators in terms of efficiency. The optimal values of first and second phase sample have been determined.

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1. Introduction
Double Sampling for stratification is one of those sample survey designs that uses auxiliary information in the process of estimation. It was introduced by Neyman (1938). The efficacy of the procedure of estimation using auxiliary information relies on technique whereby the estimator is been suggested. Research works abound whereby the accuracy of the estimators increases by using auxiliary information. Shabbir and Gupta (2005) and Kadilar and Cingi (2003) extended Singh (1999) estimators in evaluating the mean of the population that has been stratified. Although stratified double sampling is useful, the nonresponse problem is intrinsic in every survey which may lead to incorrect evaluation of the parameters. Hansen and Hurwitz (1946) suggested a method of sub-sampling of non-respondent so as to modify the non-response in their mail surveys. Khoshnevisan et al. (2007), Chaudhary and Singh (2013) and Chaudhary and Kumar (2015) have all suggested different estimators in two-phase stratified sampling under nonresponse.

In this article, an efficient ratio-product estimator in stratified two-phase sampling under nonresponse using one auxiliary variable is been suggested. The characteristics of the estimator suggested have been given.

2. Sampling Design
Consider a population of size N divided into k strata. Let the size of the ith stratum be Ni (i = 1,2,...,k) such that \( \sum_{i=1}^{k} N_i = N \). A large first sample of size ni is drawn from Ni units by (SRSWOR) scheme for the ith stratum and auxiliary variable \( x_i \) is observed to estimate the population mean \( \bar{x} \), which is unknown. Secondly, a smaller second phase sample of size ni is drawn from ni units by SRSWOR. Let Y be the study variable with population mean \( \bar{y} = \sum_{i=1}^{k} p_i \bar{y}_i \) and assume that at the second phase, non-response is examined on the study variable while the auxiliary variable does not undergo nonresponse. Also, assume that at the second phase, there are ni1 respondent units and ni2 non-respondent units in ni units. Using the method of subsampling the non-respondent introduced by Hansen and Hurwitz (1946), \( h_{i2} = \frac{n_{i2}}{L_i} \), \( L_i \geq 1 \) being a sub-sample is drawn from the sample of ni2 non-respondents units and the needed data is obtained from them all.

3. Some Existing Estimators
Some existing estimators for the mean of the population in stratified two-phase sampling with one auxiliary variable under non-response shall be presented in this section.
### 3.1. Rao (1991) Difference-Type Estimator

Rao (1991) gave a difference-type estimator given as

\[ \bar{y}_{p_{i}x_{i}'} = d_{x}^i \bar{x}_{i}^x + d_{y}^i (\bar{x}_{i}^x - \bar{x}_{i}^y) \]  

(1)

Where \( d_{x} \) and \( d_{y} \) are constants.

\[ \bar{x}_{i}^x = \sum_{i}^k p_{i} \bar{x}_{i}^x; \bar{x}_{i}^y = \frac{\sum_{i}^k x_{i} p_{i} \bar{x}_{i}^x}{n_{i}} \]

With Mean Square Error (MSE),

\[ \text{MSE}(\bar{y}_{p_{i}x_{i}'}) = \left[ \sum_{i}^k p_{i} \frac{1}{n_{i}} \frac{1}{n_{i}} S_{i}^x \right] \left( 1 - \rho_{x_{i}y_{i}} \right) \]

(2)

### 3.2. Khare and Srivastava (1993) Estimator

Khare and Srivastava suggested a ratio estimator of the form

\[ \bar{y}_{RBD} = \bar{y}_{p_{i}x_{i}'} \frac{x_{i}'}{x_{i}^y} \]  

(3)

With mean square error,

\[ \text{MSE}(\bar{y}_{RBD}) = \left[ \sum_{i}^k p_{i} \frac{1}{n_{i}} \frac{1}{n_{i}} S_{i}^x \right] \left( 1 - \rho_{x_{i}y_{i}} \right) \]

(4)

### 3.3. Khare and Srivastava (1997) Estimator

Khare and Srivastava (1997) proposed two ratio estimators given as

\[ T_{1} = \bar{y}_{p_{i}x_{i}'} \frac{x_{i}'}{x_{i}^y} \]  

(4)

With mean square error,

\[ \text{MSE}(T_{1}) = \left[ \sum_{i}^k p_{i} \frac{1}{n_{i}} \frac{1}{n_{i}} S_{i}^x \right] \left( 1 - \rho_{x_{i}y_{i}} \right) \]

(5)

### 3.4. Chaudhary et al. (2009) Estimator

Chaudhary et al. (2009) suggested a family of combined-type estimators given as

\[ T_{c} = \bar{y}_{p_{i}x_{i}'} \frac{2 + \rho_{x_{i}y_{i}}}{1 + \rho_{x_{i}y_{i}}} \]  

(6)

With mean square error,

\[ \text{MSE}(T_{c}) = \sum_{i}^k p_{i} \frac{1}{n_{i}} \frac{1}{n_{i}} S_{i}^x \left( 1 - \rho_{x_{i}y_{i}} \right) \]

(7)

Where \( f_{i}' = \left( \frac{1}{n_{i}} - \frac{1}{N_{i}} \right) \) and \( f_{i}' = \left( \frac{1}{n_{i}} - \frac{1}{n_{i}} \right) \)

### 3.5. Chaudhary et al. (2012) Estimator

Chaudhary et al. (2012) gave a combined type estimator given as

\[ T_{c_{1}}(\alpha) = \bar{y}_{p_{i}x_{i}'} \frac{(A + C) \bar{X} + fB\bar{X}_{x}}{(A + B) \bar{X} + C\bar{X}_{x}} \]  

(8)

Where \( \Lambda = (\alpha - 1)(\alpha - 2), B = (\alpha - 1)(\alpha - 4), \)

\[ C = (\alpha - 3)(\alpha - 2)(\alpha - 4), \alpha > 0, \gamma = \frac{n_{i}}{N_{i}} \]

The mean square error as obtained by them is

\[ \text{MSE}(T_{c_{1}}(\alpha)) = \sum_{i}^k \left( \frac{1}{n_{i}} - \frac{1}{N_{i}} \right) \]

(9)

Where \( \phi(\alpha) = \frac{C - fB}{A + fB + C} \)
3.6. Chaudhary And Kumar (2015) Estimator

A combined type estimator was proposed by Chaudhary and Kumar (2015) as follows:

$$T'_c = \bar{y}'_a \left[ \frac{\alpha \bar{X}'_a + b}{\alpha (\bar{X}'_a + b) + (1 - \alpha)(\bar{X}'_a + b)} \right]$$

(12)

With mean square error given as

$$\text{MSE}(T'_c) = \sum_i f'_i p_i^2 \sigma^2_i + \sum_i f'_i p_i^2 (\sigma^2_i + g^2 \lambda^2 R^2 \alpha^2 \sigma^2_i)$$

$$-2g \lambda p_i \rho_i s_i$$

(13)

4. Proposed Estimator and its Properties

The proposed estimator for population mean in two-phase stratified in the presence of nonresponse, the bias of the estimator proposed for the mean of population to derive the expression for the mean squared error and the

$$T_w = \bar{y}'_a \left[ \frac{\bar{X}'_a + \varphi}{\bar{X}'_a - \varphi} \right]$$

(14)

Where $\varphi = \sum_i C_i$, where $C_i$ is the coefficient of variation of the auxiliary variable.

$$\bar{y}'_a = \sum_i p_i \bar{y}'_i, \bar{y}_i = \frac{n_i \bar{y}_i}{n_i}, p_i = N_i / N$$

$\bar{y}_i$ and $\bar{y}_{h_i}$ are the means based on $n_i$ respondent units and $h_i$ subsampled non-respondent units respectively for the study variable and $\bar{X}'_a = \sum_i p_i \bar{X}_i, \bar{X}_i = \sum_i p_i \bar{X}_i$

4.1. The Mean Squared Error (MSE) and the Bias of the Estimator Proposed

To derive the expression for the mean squared error and the bias of the estimator proposed for the mean of population in stratified two-phase sampling under nonresponse, the following symbolization shall be used.

This method is adopted from Chaudhary and Kumar (2015).

$$\bar{y}'_a = 1 + e_a, \bar{X}'_a = \bar{X}(1 + e_1), \bar{X}_i = \bar{X}(1 + e_i), \text{where } e_i, s$$

are the relative error terms and are defined as

$$e_a = \frac{\bar{y}'_a - \bar{y}}{\bar{Y}'}, e_i = \frac{\bar{X} - \bar{X}}{\bar{X}}$$

Such that the following expectations are applied

$$\text{E}(e_a) = (e_i) = (e_1) = 0$$

$$\text{E} = \left( \frac{1}{X} \sum_i \left[ \frac{1 - N_i}{N_i} p_i^2 S_i^2 + \frac{L - 1}{n_i} p_i^2 W_{i2} S_{i2}^2 \right] \right)$$

$$\text{E} = \left( \frac{1}{X} \sum_i \left[ \frac{1}{N_i} p_i^2 S_i^2 \right] \right)$$

$$\text{E} = \left( \frac{1}{X} \sum_i \left[ \frac{1}{N_i} p_i^2 S_i^2 \right] \right)$$

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$$\text{E} = \left( \frac{1}{X} \sum_i \left[ \frac{1}{N_i} p_i^2 S_i^2 \right] \right)$$

Hence, the Mean Squared Error (MSE) of the estimator proposed is stated as

$$\text{MSE}(T_w) = \left[ b^2 R^2 \sum_i \left[ \frac{1 - N_i}{N_i} p_i^2 S_i^2 \right] \right]$$

$$-2ab R^2 \sum_i \left[ \frac{1 - N_i}{N_i} p_i^2 S_i^2 \right]$$

$$+ a^2 R^2 \sum_i \left[ \frac{1 - N_i}{N_i} p_i^2 S_i^2 \right]$$

$$-2b R^2 \sum_i \left[ \frac{1 - N_i}{N_i} p_i^2 S_i^2 \right]$$

$$+ 2a R \sum_i \left[ \frac{1 - N_i}{N_i} p_i^2 S_i^2 \right]$$

$$+ a^2 \left[ \frac{1 - N_i}{N_i} \right] p_i^2 S_i^2 + \frac{(L - 1)}{n_i} p_i^2 W_{i2} S_{i2}^2$$

Where $R = \frac{\bar{Y} - \bar{X}}{\bar{X} + \varphi}$ and $b = \frac{\bar{Y} - \bar{X}}{\bar{X} - \varphi}$.
4.2. The Survey Cost and Obtaining Optimal Values of $n_i, n_i$, and $L_i$ for $T_{sw}$

Let $c_i$ represent the cost in each unit related to the sample size of the first phase $n_i$, also let $c_{0i}$ represent the cost of each unit of the first attempt on the study variable having the sample size of second phase, $n_i$. Consider $c_{i0}$ and $c_{i2}$ to represent the cost in each unit of computing the $n_{i1}$ units that responded and $h_{i2}$ units that did not respond. Hence the overall cost for each stratum is stated as

$$C_i = c_i n_i + c_{0i} n_i + c_{i1} n_{i1} + c_{i2} h_{i2}$$

(15)

The expected cost for each stratum is given by

$$E(C_i) = c_i n_i + n_i \left( c_{0i} + c_{i1} W_{i1} + c_{i2} W_{i2} L_i \right)$$

(16)

Where $W_{i1}$ is the response rate in the $i^{th}$ stratum, $W_{i2}$ represents the rate of non-response rate in the strata and $L_i$ is the inverse sampling rate.

Hence the overall cost over the entire strata is given as

$$C_0 = \sum_i E(C_i)$$

$$= \sum_i \left( c_i n_i + n_i \left( c_{0i} + c_{i1} W_{i1} + c_{i2} W_{i2} L_i \right) \right)$$

(17)

Consider the Lagrange function

$$\phi = MSE(T_{sw}) + \lambda C_0$$

$$= b^2 R^2 \Sigma_i \left[ \frac{1}{n_i - 1} - \frac{1}{N_i} \right] p_i^2 S_{X_i} - 2aR^2 \Sigma_i \left[ \frac{1}{n_i - 1} - \frac{1}{N_i} \right] p_i^2 S_{X_i}$$

$$+ a^2 R^2 \Sigma_i \left[ \frac{1}{n_i - 1} - \frac{1}{N_i} \right] p_i^2 S_{X_i} + 2aR \Sigma_i \left[ \frac{1}{n_i - 1} - \frac{1}{N_i} \right] p_i \rho_S \rho_S S_{X_i}$$

$$+ \Sigma_i \left[ \frac{1}{n_i - 1} - \frac{1}{N_i} \right] p_i^2 S_{X_i} + \frac{(L_i - 1)}{n_i} p_i^2 W_{i2} S_{X_i} \right]$$

$$+ \lambda \Sigma_i \left( c_i n_i + n_i \left( c_{0i} + c_{i1} W_{i1} + c_{i2} W_{i2} L_i \right) \right)$$

(21)

Where $\lambda$ is the multiplier of the Lagrange function.

To derive the optimum values of $n_i, n_i$, and $L_i$, $\phi$ is differentiated with regard to $n_i, n_i$, and $L_i$ individually and the derivatives equated to zero. Hence, for stratum $i$, we get

$$\frac{\partial \phi}{\partial n_i} = \frac{-P_i^2}{n_i^2} \left[ S_{X_i}^2 + b R^2 S_{X_i}^2 - 2b R \rho_S S_{X_i} \right]$$

$$- \frac{(L_i - 1) W_{i2} S_{X_i}^2}{n_i^2} P_i^2$$

$$+ \lambda \left( c_{i0} + c_i W_{i1} + c_{i2} W_{i2} L_i \right) = 0$$

(18)

$$\frac{\partial \phi}{\partial L_i} = \frac{P_i^2}{n_i^2} \left[ 2ab R^2 S_{X_i}^2 - a^2 R^2 S_{X_i}^2 - 2a R \rho_S S_{X_i} \right] + \lambda c_i = 0$$

(19)

From (18)

$$\lambda \left( c_{i0} + c_i W_{i1} + c_{i2} W_{i2} \frac{L_i}{n_i} \right)$$

$$= \frac{P_i^2}{n_i^2} \left[ S_{X_i}^2 + b R^2 S_{X_i}^2 - 2b R \rho_S S_{X_i} \right] + \lambda c_i = 0$$

(22)

From (19)

$$\lambda c_i = \frac{P_i^2}{n_i^2} \left[ a R^2 S_{X_i}^2 + 2a R \rho_S S_{X_i} - 2ab R^2 S_{X_i}^2 \right]$$

(22)

$$n_i = \frac{P_i \sqrt{Q}}{\sqrt{\lambda c_i}}$$

(22)

Where $Q = a R^2 S_{X_i}^2 + 2a R \rho_S S_{X_i} - 2ab R^2 S_{X_i}^2$

From (20)

$$\lambda = \frac{L_i P_i S_{X_i}}{n_i c_{i1}}$$

(23)

putting the value of $\sqrt{\lambda}$ in (23) into (21), we get

$$n_i = \frac{P_i \sqrt{S_{X_i}^2 + b R^2 S_{X_i}^2 - 2b R \rho_S S_{X_i} \right)}{n_i}$$

$$\left( \frac{L_i - 1) W_{i2} S_{X_i}^2}{n_i \sqrt{c_{i2}}} \left( c_{i0} + c_i W_{i1} + c_{i2} W_{i2} \frac{L_i}{n_i} \right) \right)$$

(24)
\[ L_i S_i = \sqrt{c_{i0} + c_{i1} W_{i1} + c_{i2} \frac{W_{i2}}{L_i}} \]

\[ = \sqrt{(S_i^2 + b^2 R^2 S_i^2 - 2b R \rho S_i S_{x_i} + (L_i - 1) W_{i2} S_{y_i}^2)} \cdot c_{i2} \]

Squaring both sides, we have

\[ L_i^2 S_i^2 c_{i0} + L_i^2 S_i^2 c_{i1} W_{i1} + L_i^2 S_i^2 c_{i2} \frac{W_{i2}}{L_i} = \left( S_i^2 + b^2 R^2 S_i^2 - 2b R \rho S_i S_{x_i} - W_{i2} S_{y_i}^2 \right) c_{i2} \]

\[ L_i^2 S_i^2 (c_{i0} + c_{i1} W_{i1}) = \left( S_i^2 + b^2 R^2 S_i^2 - 2b R \rho S_i S_{x_i} - W_{i2} S_{y_i}^2 \right) c_{i2} \]

\[ L_{i\text{(opt)}} = \frac{\sqrt{c_{i2} B_i}}{S_{y_i} A_i} \quad (24) \]

Where \( A_i = \sqrt{c_{i0} + c_{i1} W_{i1}} \cdot B_i = \sqrt{S_{y_i}^2 + b^2 R^2 S_{x_i}^2 - 2b R \rho S_i S_{x_i} - W_{i2} S_{y_i}^2} \)

On substituting the value of \( L_{i\text{(opt)}} \) from (57) into (54), we can express \( n_i \) as

\[ C_0 = \sum_i c_{i} n_i + n_i \left( c_{i0} + c_{i1} W_{i1} + c_{i2} \frac{W_{i2}}{L_i} \right) \]

\[ = \sum_i c_{i} P_i \sqrt{\frac{Q}{\lambda c_i}} + P_i \sqrt{A_i^2 + \frac{\sqrt{c_{i2} B_i W_{i2} S_{y_i}}}{A_i}} c_{i0} + c_{i1} W_{i1} + c_{i2} \frac{W_{i2}}{\sqrt{c_{i2} B_i} S_{y_i} A_i} \]

\[ = \sum_i \sqrt{\frac{c_i}{\lambda}} P_i \sqrt{\frac{Q}{\sqrt{A_i^2 + \frac{\sqrt{c_{i2} A_i W_{i2} S_{y_i}}}{B_i}}}} c_{i0} + c_{i1} W_{i1} + \sqrt{c_{i2} A_i W_{i2} S_{y_i}} \]

\[ \sqrt{\lambda} = \frac{1}{C_0} \sum_i \sqrt{c_i P_i \sqrt{Q} + P_i B_i^2 + \frac{\sqrt{c_{i2} B_i W_{i2} S_{y_i}}}{A_i}} A_i + \frac{\sqrt{c_{i2} A_i W_{i2} S_{y_i}}}{B_i} \]

To obtain the value of \( \sqrt{\lambda} \) with regard to the total cost \( C_i \), the value of \( n_i, n_i', \) and \( L_{i\text{(opt)}} \) are put into (17), hence we have,
\[
\sqrt{\lambda} = \frac{1}{C_u} \sum^2_i \left[ \sqrt{c_i} P_i \sqrt{Q_i} + P_i \left( \frac{B_i A_i + 2 \sqrt{c_i} W_i S_{y_i}}{A_i} \right) \left( \frac{A_i^2 + \sqrt{c_i} A W_i^2 S_{y_i}^2}{B_i} \right) \right]
\]

Substituting the value of \( \sqrt{\lambda} \) from (26) into (25) and (22), the optimum values of \( n_j \) and \( n_i \) are derived as

\[
n_{i_{(w)}} = \frac{c_i \sqrt{Q_i}}{\sqrt{c_i} \sum^i_j \left[ \sqrt{c_i} \sqrt{Q_i} + \left( B_i A_i + \sqrt{c_i} W_i S_{y_i} \right) \right]}
\]

5. Empirical Study

Using the data set used by Chaudhary and Kumar (2015) shown on Table 1 below

| Stratum no | \( N_i \) | \( n_i \) | \( \hat{y}_i \) | \( \bar{y}_i \) | \( S_{y_i}^2 \) | \( S_{r_i}^2 \) | \( \rho_i \) | \( S_{r_i}^2 \) |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1         | 73     | 65     | 26     | 40.85  | 39.56  | 6369.1 | 0.999  | 618.88 |
| 2         | 70     | 25     | 10     | 27.57  | 27.57  | 1051.07| 0.998  | 240.91 |
| 3         | 97     | 48     | 19     | 25.44  | 25.44  | 2014.97| 0.999  | 265.52 |
| 4         | 44     | 11     | 5      | 20.36  | 20.36  | 538.47 | 0.997  | 83.69  |

Table 2: below shows the Mean Squared Error (MSE) and Relative Efficiency percentage (PRE) of different estimators together with the estimator proposed being compared to for different choices of \( W_\alpha \)

| \( W_\alpha \) | \( L_i \) | \( V \) | \( MSE(\hat{T}_i^*) \) | \( MSE(T_i) \) | \( MSE(T_i^\star) \) | \( PRE(\hat{T}_i^*) \) | \( PRE(T_i^\star) \) | \( PRE(T_i) \) |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.1       | 2      | 34.42  | 23.62  | 6.28   | 4.66   | 145.72 | 548.09 | 738.6  |
|           | 2.5    | 34.67  | 24.01  | 6.54   | 4.92   | 144.4  | 530.12 | 704.7  |
|           | 3      | 34.92  | 24.41  | 6.79   | 5.17   | 143.06 | 514.29 | 675.4  |
|           | 3.5    | 35.18  | 24.8   | 7.04   | 5.31   | 141.85 | 499.72 | 662.5  |
| 0.2       | 2      | 34.92  | 24.41  | 6.79   | 5.26   | 143.06 | 514.29 | 663.9  |
|           | 2.5    | 35.43  | 25.2   | 7.3    | 5.66   | 140.6  | 485.34 | 625.97 |
|           | 3      | 35.94  | 25.99  | 7.8    | 6.18   | 138.28 | 460.77 | 581.6  |
|           | 3.5    | 36.44  | 26.78  | 8.31   | 6.70   | 136.07 | 438.51 | 543.9  |
### Conclusion

The optimum values of $n_i^*$, $n_i$ and $L_i$ using the proposed estimator had been ascertained under the cost of the survey. Also, Table 2 shows that the proposed estimator is better than other estimators in terms of efficiency.

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