GM vs biharmonic ocean mixing in the Arctic

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Abstract. As part of the Arctic Ocean Model Intercomparison Project, the LANL ice-ocean modeling team completed two 55-year, global, ice-ocean simulations forced with atmospheric reanalysis data for 1948-2002. These two simulations differ only in the parameterization used for lateral mixing of tracers (potential temperature and salinity) in the ocean, but the resulting circulation and kinetic energy of the simulated oceans are very different, particularly at high latitudes. The differences can be traced to two effects, (1) scale selectivity, in which the Laplacian form of the Gent and McWilliams (GM) parameterization damps wave energy more quickly than the biharmonic mixing formulation, and (2) grid dependence of the diffusion coefficient, which appears in the biharmonic formulation but not in GM, and is particularly important at high latitudes where the grid scale decreases dramatically on the sphere. We conclude that, in order to maintain consistent suppression of numerical noise while allowing for a more energetic circulation in regions of finer grid spacing, future global simulations using the GM parameterization should include a diffusivity scaling factor given by the square root of the grid cell area.

1. Motivation and tools
Diffusion plays an important role in ocean models for two reasons: it dispels grid-scale noise resulting from the cascade of tracer variance to the smallest scales, and it parameterizes important sub-grid-scale fluxes [1]. While the latter role is physically based, the first is purely numerical in nature and modelers try to minimize it by using diffusivities as small as possible (even 0 in some models). In practice, modelers must balance the two effects, allowing enough diffusion to mop up numerical noise, yet not incurring excessive diapycnal mixing.

Biharmonic diffusion has a distinct advantage over Laplacian diffusion in that it primarily acts on the smallest scales, leaving larger scales that are resolved by the grid relatively untouched. Biharmonic diffusion acts in the horizontal plane, however, causing unphysical diapycnal fluxes wherever neutral surfaces are not horizontal. The GM parameterization [2], which is Laplacian in form, was developed for sub-grid-scale mixing on lower resolution, z-coordinate grids and mixes tracers along sloping neutral surfaces, a physical approach that makes it desirable for high resolution modeling also. Our simulations are of moderately high, eddy-admitting resolution (0.4°) and highlight difficulties that may be encountered as modelers move toward eddy-resolving simulations. This study elucidates reasons why our GM and biharmonic simulations are so different at high latitudes, yet similar at lower latitudes.

The simulations were performed with an ocean-sea ice coupled model developed at Los Alamos National Laboratory. Detailed documentation for the ocean and ice models can be found in [3], [4] and other publications referenced therein. The models are configured following the Arctic Ocean Model Intercomparison Project (AOMIP) protocol and integrated using AOMIP-defined
forcing fields and parameterizations, merged across 60–68N with global NCEP reanalysis data. Details regarding the AOMIP protocol can be found at http://fish.cims.nyu.edu/project_aomip/experiments/coordinated_analysis/overview.html.

The Parallel Ocean Program (POP) is a free-surface, z-coordinate ocean model that solves the primitive equations for temperature, salinity and the horizontal velocity components. In the simulations described here, lateral mixing of tracers occurs via the GM mixing parameterization or, alternatively, a biharmonic mixing term. POP is coupled to the Los Alamos Sea Ice Model (CICE), which features an energy conserving thermodynamics model, multiple ice thickness categories, elastic-viscous-plastic ice dynamics, and horizontal advection via an incremental remapping scheme. The ice and ocean models are treated as subroutines, coupled through a driver that also reads pre-interpolated atmospheric data from files and prepares the data for use by the other components.

The ice and ocean models are discretized for nonuniform, general curvilinear grids in which the north pole has been moved smoothly into a nearby land mass to avoid problems associated with converging meridians. For the experiments described here, we use a 900 × 600 global mesh (resulting in 0.3° longitudinal spacing), whose north pole is in North America. The horizontal grid is mercator in the southern hemisphere with latitudinal spacing of 0.4° cos φ. In the northern hemisphere the grid size also decreases with latitude, resulting in a grid spacing that ranges from 9 km (at high latitudes) to 44 km (at the equator). The vertical grid consists of 40 levels which vary in thickness from 10 m at the surface to 250 m at the bottom.

The model is fully parallelized and runs efficiently at high resolution. We utilized 60 multi-streaming vector processors of the Cray X1 in the Center for Computational Studies, Oak Ridge National Laboratory, spending approximately 36,000 CPU-hours for each 55-year simulation.

2. Simplified theory for tracer diffusion
The GM or biharmonic tracer mixing parameterization appears as a diffusion term \( D_H \) in the transport equation

\[
\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T + w \frac{\partial T}{\partial z} = D_H(T) + D_V(T)
\]

for tracer \( T \) (potential temperature or salinity), which also includes advection by the horizontal and vertical velocities, \( \mathbf{u} \) and \( w \), and vertical diffusion \( D_V \). In these simulations \( D_V \) is given by the K-profile parameterization (KPP) [5]. Lateral diffusion terms in the biharmonic and GM cases are

\[
D_{Bi}^H = \nabla^2 (\kappa \Delta^2 \nabla^4 T) \quad \text{and} \quad D_{GM}^H = \nabla_3 \cdot \mathbf{K} \cdot \nabla T,
\]

respectively. Biharmonic diffusion is given by a fourth-order, two-dimensional term where \( \kappa = 2 \times 10^{11} \text{ m}^4\text{s}^{-1} \) and \( A \) is a normalized grid cell area. GM takes a Laplacian form; \( \mathbf{K} \) represents a combined diffusivity and viscosity associated with advection perturbations in layer thickness by the eddy-induced transport velocity [6], whose details are not important here. In the GM case the differential operators are three-dimensional, since the mixing occurs on sloping, neutral surfaces.

Following [7], we neglect advection for now, simplify the equations to one dimension \((T_t = D_H(T))\) and look for time scales associated with the diffusion, substituting a wavenumber \( k \) solution, \( T = \gamma(t) e^{ikx_n} \), where \( x_n = n \Delta \), \( \Delta \) is a grid length and \( n \) is an integer. The ratio of time scales for the biharmonic \( (\tau_B) \) and Laplacian \( (\tau_C) \) operators is

\[
\frac{\tau_B}{\tau_C} = \frac{C}{B} \left( \frac{2}{\Delta} \sin \frac{k \Delta}{2} \right)^{-2} \sim \frac{C}{Bk^2}
\]

if \( k \Delta / 2 \) is small (here, \( B \) and \( C \) are constant biharmonic and Laplacian diffusivities). This ratio illustrates the scale selectivity of the operators. For large \( k \) associated with sub-grid-scale waves,
\(\tau_B\) becomes small compared with \(\tau_C\) (depending on the values of the coefficients), indicating that the biharmonic operator damps those waves more quickly than the Laplacian operator. For smaller wavenumbers, however, corresponding to structures that are resolvable on the grid, the ratio becomes large, indicating that the Laplacian operator damps those scales more quickly. That is, the biharmonic operator admits shorter wavelength structures on the mesh that the Laplacian damps out. This is a fundamental difference between the two types of operators, and it is the primary reason that modelers often use biharmonic diffusion terms.

Besides the form of the operators, the standard diffusivities used for biharmonic and GM mixing terms also differ in the grid scaling factor. Following [8], the biharmonic diffusivity scales with the grid cell area, \(A\), in a manner designed to balance advective and diffusive terms at the grid scale: \(A^{3/2}\). The corresponding grid scaling for the Laplacian case, \(A^{1/2}\), is not included in the standard GM configuration.

The forms of the operators, and resulting differences in scale selectivity, represent a fundamental difference in the two approaches independent of the grid configuration. The scaling factor, however, is much more important at high latitudes where the grid cell size becomes small.

3. Modeled Sensitivities
We ran several short simulations to test the model’s sensitivity to the mixing parameterization. Although differences in basin-average tracer fields are relatively small and therefore difficult to distinguish based on comparison with observations, the currents differ drastically after just one year, driven by the subtle differences in the potential temperature and salinity fields through geostrophy. The sensitivity runs each started from a snapshot taken from the biharmonic AOMIP simulation for Jan 1, 1982, and were run for one year using the standard AOMIP forcing for 1982.

Spatial plots of the kinetic energy and velocity vectors at 466 m are shown in figure 1, averaged over the last month of the 1-year sensitivity simulations. (The simulations reach a quasi-equilibrium level after about 6 months.) The biharmonic case generally maintains its energy level, while the GM run spins down. A run using a plain Laplacian diffusion term with the same diffusivity parameter as in GM spins down even more (not shown). With the grid scaling included in the GM coefficients, however, the energy does not spin down as much as in the standard GM case. The modified GM case also shows faster boundary currents and more eddy-like activity in the interior than the standard GM run.

Naturally, the tracer mixing parameterization has repercussions for water masses in the Arctic, such as the warm Atlantic Layer whose top can be defined by the 0°C isotherm (figure 2). The biharmonic and modified GM cases both show more structure after 1 year than the original GM run does. Gradients are much tighter, especially at the highest latitudes, although differences appear at lower latitudes too, such as in the Beaufort Sea and off the east coast of Greenland.

Preliminary examination of the current structure in the Arctic indicates that the simulations with area-weighted diffusivities are more similar to observations than the standard GM simulation. For example, boundary currents along the continental shelf break turn and follow the Lomonosov Ridge in the biharmonic and modified GM runs, but in the standard GM run the large-scale flow is perpendicular to the ridge (not shown).

4. Discussion
The Laplacian form of the GM parameterization derives naturally from conservation and continuity of layer thickness in isopycnal coordinates [2]. GM mixes tracers along sloping neutral surfaces, a physical approach that makes it desirable for all z-coordinate models, regardless of resolution. The benefits of using GM are well documented for low resolutions as are typically used in ocean climate models. Early studies (including this one) indicate that GM is also useful
Figure 1. Mean kinetic energy (g cm$^{-1}$s$^{-1}$) and velocity vectors for December 1982 from the (a) biharmonic, (b) GM and (c) modified GM sensitivity simulations.

Figure 2. Depth (m) of the 0°C isotherm for December 1982 from the (a) biharmonic, (b) GM and (c) modified GM sensitivity runs.
for eddy-admitting and even eddy-resolving simulations. However, these parameterizations have ramifications for scales resolved by the grid, issues that become more prominent with higher resolution grids.

For the Fourth Assessment of the Intergovernmental Panel on Climate Change (in progress), the GM parameters used in the NCAR Community Climate System Model were tuned such that the strength of the Antarctic Circumpolar Current (ACC) transport through Drake Passage approximately matches observations (Gokhan Danabasoglu, pers. comm., 2005). Generally speaking, the GM parameters are adjusted to produce desirable features in one region, such as the ACC, while balancing the effect on other regions and on the model’s ability to take advantage of the grid resolution available for admitting eddies. Based on our model results, we suggest that including the appropriate grid scaling in the GM diffusivity would mitigate the parameterization’s highly diffusive effects in the Arctic, when tuned for important features at lower latitudes on global grids.

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