The theory and phenomenology of jets in nuclear collisions

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Abstract. We review selected results from a recent in-depth study of jet shapes and jet cross sections in ultra-relativistic reactions with heavy nuclei at the LHC \textsuperscript{1}. We demonstrate that at the highest collider energies these observables become feasible as a new, differential and accurate test of the underlying QCD theory. Our approach allows for detailed simulations of the experimental acceptance/cuts that help isolate jets emerging from a dense QGP. We show for the first time that the pattern of stimulated gluon emission can be correlated with a variable quenching of the jet rates and provide an approximately model-independent approach to determining the characteristics of the medium-induced bremsstrahlung spectrum. The connection between such cross section attenuation and the in-medium jet shapes is elucidated.

PACS. 13.87.-a Jets in large-\(Q^2\) scattering – 24.85.+p Quarks, gluons, and QCD in nuclear reactions – 12.38.Mb Quark gluon plasma

1 Introduction

At present, most measurements of hard processes in heavy ion collisions are limited to single particles and particle correlations, which are only the leading fragments of a jet. There is general agreement that final-state parton energy loss in the QGP controls inclusive particle suppression, the experimental methodology of determining \(R_{AA}(p_T)\) or \(I_{AA}(p_{T1},p_{T2})\) is well-established, and the theory of jet quenching has been very successful \textsuperscript{2}. The data, however, cannot resolve the staggering order of magnitude "systematic" discrepancy in the medium properties extracted via competing phenomenological models \textsuperscript{3}. It is, therefore, critical to find alternatives that accurately reflect the energy flow in strongly-interacting systems and have a more direct connection to the underlying QCD theory.

The intra-jet energy distribution and the related cross section for jets in the case of heavy ion reaction closely match the criteria outlined above. In this paper we study the magnitude of in-medium modification for these observables in Pb+Pb collisions at \(\sqrt{s} = 5.5\) TeV at LHC. We demonstrate that a natural generalization of leading particle suppression to jets,

\[
R_{AA}^{\text{jet}}(E_T; R_{\text{max}}, \omega_{\text{min}}) = \frac{d\sigma^{AA}(E_T; R_{\text{max}}, \omega_{\text{min}})}{d^2E_T} = \frac{\langle N_{\text{bin}} \rangle}{\langle N_{\text{part}} \rangle},
\]

is sensitive to the nature of the medium-induced energy loss. The steepness of the final-state differential spectra amplifies the observable effect and the jet radius \(R_{\text{max}}\) and the minimum particle/tower energy \(p_{T\min} \approx \omega_{\min}\) provide, through the evolution of \(R_{AA}^{\text{jet}}(E_T; R_{\text{max}}, \omega_{\text{min}})\) at any fixed centrality, experimental access to the QGP response to quark and gluon propagation.

We refer the reader to \textsuperscript{4} for a summary of the complications in defining a jet and the related topic of jet-finding algorithms. To make our discussion simpler, we will assume that the complications of the different definitions can be subsumed into an \(R_{\text{sep}}\) parameter \textsuperscript{1}. Once a jet axis and all of the jet particles / calorimeter towers “i” have been identified, the “integral jet shape” is defined as:

\[
\Psi_{\text{int}}(r; R) = \frac{\sum_i (E_T)_i \Theta(R - (R_{\text{jet}})_i)}{\sum_i (E_T)_i \Theta(R - (R_{\text{jet}})_i)}
\]

where \(r, R\) are Lorentz-invariant opening angles, \(R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}\), and \(i\) represents a sum over all the particles in this jet. \(\Psi_{\text{int}}(r; R)\) is the fraction of the total energy of a jet of radius \(R\) within a sub-cone of radius \(r\). It is automatically normalized so that \(\Psi_{\text{int}}(R; R) = 1\) and the differential jet shape is defined as follows:

\[
\psi(r; R) = \frac{d\Psi_{\text{int}}(r; R)}{dr}.
\]

Understanding the many-body QCD theory behind jet shape calculations will naturally lead to understanding the attenuation of jets in reactions with heavy nuclei.
2 Theoretical results for vacuum jet shapes

Jet shapes in vacuum will provide the baseline for jet shape studies in nuclear collisions. Here, we follow the methodology outlined in Ref. \[5\] and generalize this approach to include finite detector acceptance.

2.1 Leading order results

A fixed order QCD calculation of jet shapes is based on the splitting functions \( P_{a\to bc}(z) \), the distributions of the large fractional lightcone momenta (or approximately the energy fractions) of the fragments relative to the parent parton, \( z \) and \( 1-z \) respectively. To lowest order, recalling that \( \psi_a(r; R) \) describes the energy flow \( \propto z \), we can write:

\[
\psi_a(r; R) = \sum_b \frac{\alpha_s}{2\pi r} \int_{z_{\text{min}}}^{1-Z} dz z P_{a\to bc}(z),
\]

with \( Z \) is defined as follows:

\[
Z = \max \left\{ \frac{z_{\text{min}}}{1-R}, \frac{r}{1-R} \right\} \quad \text{if} \quad r < (R_{\text{sep}} - 1)R, \\
Z = \max \left\{ \frac{z_{\text{min}}}{R_{\text{sep}}}, \frac{r}{R_{\text{sep}}R} \right\} \quad \text{if} \quad r > (R_{\text{sep}} - 1)R.
\]

In Eq. (4) \( r = (1-z)\rho \) is related to the opening angle \( \rho \) between the final-state partons. In “elementary” \( p+p \) collisions the inclusion of soft particles \( z_{\text{min}} \approx 0 \) in theoretical calculations is not a bad approximation. In heavy ion reactions, especially for the most interesting case of central collisions, there is an enormous background of soft particles related to the bulk QGP properties. Jet studies will likely require minimum particle energy > 1 – 2 GeV at RHIC and even more stringent cuts at the LHC. Furthermore, control over \( z_{\text{min}} \) can provide detailed information about the properties of QGP-induced bremsstrahlung. The kinematic constraints on the values of \( z \) are discussed in detail in \[1\]. We find for the jet shape functions for quarks and gluons:

\[
\psi_q(r) = \frac{C_F\alpha_s}{2\pi} \frac{2}{r} \left( 2 \log \frac{1-z_{\text{min}}}{Z} - \frac{3}{2} (1-Z)^2 \right)
\]

\[
\psi_g(r) = \frac{C_A\alpha_s}{2\pi} \frac{2}{r} \left( 2 \log \frac{1-z_{\text{min}}}{Z} - \frac{11}{6} - \frac{Z}{3} + \frac{Z^2}{2} \right) \times (1-Z)^2 + \left( 2-z_{\text{min}}^2 - \frac{3}{2} z_{\text{min}}^4 + \frac{3}{2} z_{\text{min}}^2 \right)
\]

\[
\times \frac{T_R N_c \alpha_s}{2\pi} \left( \frac{2}{3} - \frac{2Z}{3} + Z^2 \right) (1-Z)^2 - \left( \frac{3}{2} z_{\text{min}}^2 + \frac{3}{2} z_{\text{min}} + \frac{3}{2} z_{\text{min}}^4 \right).
\]

In the \( z_{\text{min}} \to 0 \) limit Eqs. (7) and (8) reduce to previously known results \[5\].

![Fig. 1. Numerical results for the differential jet shape in p+p collisions at \( \sqrt{s} = 5.5 \text{ TeV} \) at the LHC. Solid lines represent jet shapes with \( R = 0.7, \omega_{\text{min}} = 0 \text{ GeV} \), dashed lines stand for jet shapes with \( R = 0.4, \omega_{\text{min}} = 0 \text{ GeV} \), and dashed-dotted lines are for jet shapes with \( R = 0.7, \omega_{\text{min}} = 10 \text{ GeV} \). The inserts show integrated jet shapes \( \Psi_{\text{int}}(r; R) \).](image)

2.2 The full vacuum jet shape and numerical simulations

A number of important effects beyond the fixed order, Eqs. (7) and (8), contribute to the observed jet shape \[1\].\[3\]. Taking all contributions and ensuring that there is no double counting at small \( r/R \) to \( O(\alpha_s) \) we find:

\[
\psi(r) = \psi_{\text{coll}}(r)(P(r) - 1) + \psi_{\text{LO}}(r) + \psi_{\text{PC}}(r) + \psi_{i,\text{PC}}(r),
\]

On the right-hand-side of Eq. (9) the first term comes from Sudakov resummation with subtraction of the leading 1/r, 1/(l/\rho) log(1/r) contribution at small \( r/R \) to avoid double counting with the fixed order component of the differential jet shape. The second and third terms represent the leading-order contributions in the final-state and the initial-state. The last two terms represent the effect of power corrections. In a full calculation the relative quark and gluon fractions \( f_q + f_g = 1 \) are also needed: \( \psi(r; E_T) = f_q(E_T, \sqrt{s})\psi_q(r; E_T) + f_g(E_T, \sqrt{s})\psi_g(r; E_T) \). This theoretical model is validated against CDF II data \[1\].

Figure 1 shows our numerical results for the jet shape for two different energies \( E_T = 50 \text{ GeV} \) and 250 GeV and two cone radii \( R = 0.7, 0.4 \) in \( p+p \) collisions at \( \sqrt{s} = 5.5 \text{ TeV} \) at LHC. When plotted against the relative opening angle \( r/R \), these shapes are self-similar, i.e. approximately independent of the absolute cone radius \( R \). One of the main theoretical developments in Ref \[1\]...
is the analytic approach to studying finite detector acceptance effects or experimentally imposed low momentum cuts. In Fig. 1 this is illustrated via the selection of $z_{\text{min}} = p_T^{\text{min}}/E_T = 0.2$, 0.04 ($p_T^{\text{min}} = \omega^{\text{min}} = 10$ GeV). Eliminating the soft partons naturally leads to a narrower branching pattern. However, for this effect to be readily observable 10–20% of the jet energy, going into soft particles, must be missed. Thus, even with $p_T^{\text{min}} \sim$ few GeV cuts in Pb+Pb collisions at the LHC aimed at reducing or eliminating the background of bulk QGP particles that accidently fall within the jet cone, the alteration of $\psi (r/R)$ is expected to be small. The integral jet shape $\Psi_{\text{int}} (r/R)$, shown in the inserts of Fig. 1, is not optimal as a tool for identifying kinematic and dynamic effects on jets due to its much smaller sensitivity.

3 Medium-induced contribution to the jet shape

The principal medium-induced contribution to a jet shape comes from the radiation pattern of fast quarks or gluons, stimulated by their propagation and interaction in the QGP. There is a simple heuristic argument which allows one to understand how interference and coherence effects in QCD amplify the difference between the energy distribution in a vacuum jet and the in-medium jet shape. Any destructive effect on the integral average parton energy loss $\Delta E^{\text{rad}}$, such as the Landau-Pomeranchuk-Migdal effect, can be traced at a differential level to the attenuation or full cancellation of the collinear, $k_T \ll \omega$, gluon bremsstrahlung:

$$\Delta E^{\text{rad}}_{\text{LPM suppressed}} \Rightarrow \frac{dI^9}{d\omega} (\omega \sim E)_{\text{LPM suppressed}}$$

$$\Rightarrow \frac{dI^9}{d\omega d^2k_T} (k_T \ll \omega)_{\text{LPM suppressed}} , \quad (10)$$

and we indicate the parts of phase space where the modification of the incoherent $dI^9/d\omega d^2k_T$ is most effective.

In our calculation we use the GLV formalism of expanding the medium-induced radiation in the correlations between multiple scattering centers. Theoretical interest in the angular bremsstrahlung distribution was stimulated by experimental measurements of enhanced triggered opposite-side particle correlations away from the naive $\Delta \phi = \pi$. It has been know that the leading $n = 1$ contribution to final-state medium-induced radiation does not have a collinear with the jet axis component. We now show that this result is general and holds to any order in the opacity expansion:

$$\lim_{r \to 0} \frac{\omega dN^g_{\text{med}}}{d\omega dr} = 0 . \quad (11)$$

Numerical simulations, using Monte-Carlo techniques, confirm independently that $dI^9/d\omega d^2k$ vanishes as $k \to 0$.

The implication of our finding is that there is very little overlap between the techniques used to compute the "vacuum" and medium-induced contributions to the jet shape. Also, selecting different jet radii $R$ and $p_T^{\text{min}}$ of the particles will significantly alter both the jet shape and the amount of energy lost by the hard parton which can be recovered in the experimental measurement. A clear strategy will be to use the measure arms provided by $R (= R^{\text{max}}$ in the evaluation of the $\Delta E^{\text{rad}}$) and $p_T^{\text{min}} (= \omega^{\text{min}}$ in the evaluation of the $\Delta E^{\text{rad}}$) to determine the distribution of the lost energy. This is illustrated schematically in Fig. 2. Theoretically, the first quantity to be calculated is:

$$\frac{\Delta E^{\text{in}}}{E} (R^{\text{max}}, \omega^{\text{min}}) = \frac{1}{E} \int_{\omega^{\text{min}}}^{E} d\omega \int_{0}^{R^{\text{max}}} dr \frac{dI^9}{d\omega dr} (\omega, r) . \quad (12)$$

The separate dependence of $\Delta E^{\text{in}} (R^{\text{max}}, \omega^{\text{min}})/E$ on the cone radius and the momentum acceptance cut is more clearly illustrated in Fig. 3. We show central, mid-central and peripheral collisions, impact parameters $b = 3$, 8, 13 fm, respectively, in $Pb + Pb$ reactions at LHC at nominal $\sqrt{s}$. We notice that, not surprisingly, the ratio $\Delta E^{\text{in}} (R^{\text{max}}, \omega^{\text{min}})/E$ goes down at larger impact parameters because the energy loss of the jet decreases in peripheral collisions. More importantly, at each impact parameter there is a variation of the amount of the bremsstrahlung energy, recovered in the cone. This is precisely the variation that will map on the $R^{\text{jet}}_{AA} (E_T, R^{\text{max}}, \omega^{\text{min}})$ observable.

4 Tomography of jets in heavy ion collisions

The purpose of this Section is to relate the theory of jet propagation in the QGP to experimentally measurable quantities.
sections is:

\[ \left( \frac{d^2 \sigma_{\text{med}}}{d^2 E} \right)(0, R; \omega, E) = \frac{1}{(1 - (1 - f_{q,g}) \cdot \epsilon)} \times \frac{\sigma_{q,g}^{NN}(R, \omega_{\text{min}})}{d^2 E_T dy} \tag{14} \]

where \( E_T' = E_T/(1 - (1 - f_{q,g}) \cdot \epsilon) \). The \((1 - f_{q,g}) \cdot \epsilon\) factor accounts for the total "missed" energy in a jet cone measurement, which necessitates \( E_T' > E_T \). Next, we obtain the full jet shape, including the contributions from the vacuum and the medium-induced bremsstrahlung:

\[ \psi_{\text{tot.}}(r/R) = \frac{1}{\text{Norm}} \int_{\epsilon=0}^1 d\epsilon \sum_{q,g} P_{q,g}(\epsilon) \frac{1}{(1 - (1 - f_{q,g}) \cdot \epsilon)^3} \times \frac{\sigma_{q,g}^{NN}(R, \omega_{\text{min}})}{d^2 E_T' dy} \left[ (1 - \epsilon) \psi_{\text{vac.}}^{q,g}(r/R) + f_{q,g} \cdot \epsilon \psi_{\text{med.}}^{q,g}(r/R) \right]. \tag{15} \]

We recall that, by definition, the area under any differential jet shape, \( \psi_{\text{tot.}}(r/R) \), \( \psi_{\text{vac.}}(r/R) \) and \( \psi_{\text{med.}}(r/R) \), is normalized to unity. Integrating over \( r \) in Eq. \((15)\), we can easily see that the correct “Norm” is the quenched cross section, Eq. \((14)\). Discussion of simple limiting cases for Eqs. \((14)\) and \((15)\) which provide insight into the contributions to the full jet shape and the jet cross section is given in \([1]\).

### 4.2 Numerical results

Clearly, the centrality dependence of \( R_{AA}^{\text{jet}}(R_{\text{max}}, \omega_{\text{min}}) \) at different impact parameters will remain an important handle in understanding the QCD medium. But the major development, that we report on, is the emerging understanding on how to get directly at the mechanisms of jet interaction in matter. In Fig. 4 we show the evolution of the nuclear modification factor for the jet cross section by varying the size of the cone radius \( R_{\text{max}} \) \((\omega_{\text{min}}=0)\). We see that with increasing cone radius \( R_{\text{max}} \) the quenching of the jet cross section disappears at all \( E_T \) and finally reaches unity when \( R_{\text{max}} = 2.0 \) and all of the lost energy is re-captured. It is important to note the factor of 5 to 10 variation in the quenching of the jet cross section. Such variation will pinpoint the angular distribution of the bremsstrahlung gluons. We have also studied in detail the dependence of \( R_{AA}^{\text{jet}}(R_{\text{max}}, \omega_{\text{min}}) \) at different selection of cone radii and different transverse energies \( E_T \) in central \( Pb+Pb \) collisions at \( \sqrt{s} = 5.5 \text{ TeV} \) are shown together for comparison. An interesting conclusion is that there is no significant distinction.

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**Fig. 3.** 2D projections of Eq. \((12)\). The left panel shows the fractional energy loss dependence on the jet radius \( R_{\text{max}} \) \((\omega_{\text{min}}=0)\) and the right panel shows this dependence versus the acceptance cut \( \omega_{\text{min}} \) \( (R_{\text{max}} = R_{\infty}) \). A gluon jet of \( E_{\text{jet}} = 20 \text{ GeV} \) in \( b = 3, 8, 13 \text{ fm} \) \( Pb+Pb \) collisions at LHC was used as an example.

**Fig. 4.** \( E_T \)-dependent nuclear modification factor \( R_{AA}^{\text{jet}}(R_{\text{max}}, \omega_{\text{min}}) \) for different jet cone radii \( R_{\text{max}} \) at \( b = 3 \text{ fm} \) in central \( Pb+Pb \) collisions with \( \sqrt{s} = 5500 \text{ GeV} \).
between the jet shape in vacuum and the total in-medium $\psi(r/R)$. The underlining reason for this surprising result is that although medium-induced gluon radiation produces a broader $\psi_{\text{med.}}(r/R)$, this effect is offset by the fact that the jets lose a finite amount of their energy. Furthermore, when part of the lost energy is missed due to finite acceptance, the required higher initial virtuality jets are inherently narrower.

In Table 1 we present the mean relative jet radii $\langle r/R \rangle$ in the vacuum and in the QGP medium created at the LHC for two different cone selections $R = 0.4$ and $R = 0.7$ and two jet energies $E_T = 20, 100$ GeV. We see that in the realistic numerical simulation of quark and gluon propagation in the QGP there is very little $< +10\%$ variation in this observable. This characteristic difference is slightly larger for a smaller cone, since it emphasizes the large-angle character of the medium-induced radiation. It is important to stress that the QGP is rather “gray” than “black” and only a fraction of the energy of the leading jet is lost in the medium. The effect of even a moderate $\Delta E^\text{jet}(R_{\text{max}}, \omega_{\text{min}})$ can be amplified by the steeply falling cross sections for the $R_{AA}^\text{jet}(E_T; R_{\text{max}}, \omega_{\text{min}})$ observable but this is not the case for $\langle r/R \rangle$.

Finally, we point out where the effect of the QGP medium can be more readily observed in the differential jet shapes. These are the “tails” of the energy flow distribution in $\psi_{\text{tot.}}(r/R)$, near the periphery of the jet cone. We can see that, with cone radius $R = 0.4$, the ratio $\psi_{\text{tot.}}(r/R)/\psi_{\text{vac.}}(r/R)$ could reach about 1.75 when $r/R \sim 1$. However, for experiments to reliably detect and quantify this enhancement of the ratio of the total jet shape in the QCD medium to the jet shape in the vacuum in the $r/R > 0.5$ region, high statistics measurements will be necessary. The required precision can be achieved with the large acceptance experiments at the LHC beyond the proof-of-principle jet identification at RHIC [14].

5 Conclusions

The unprecedentedly high center of mass energy at the LHC will open a new frontier for jet physics in dense QCD matter: tomography of jets that can help pinpoint energy flow in strongly-interacting systems and significantly reduce uncertainties in the QGP properties extracted from different phenomenological models. In this summary we highlighted several important results from a recent comprehensive study of jet shapes and jet cross sections in $\sqrt{s} = 5.5$ TeV $Pb+Pb$ collisions at the LHC [1]. A theory of jet shapes in the vacuum, generalized to deal with experimental acceptance cuts, was discussed and then applied to study jet shapes in heavy-ion collisions by including the contribution from the medium-induced gluon bremsstrahlung. Realistic numerical simulations of jet cross sections and jet shapes were used to illustrate new signatures of quark and gluon propagation in the QGP. It was shown that a suitable generalization of the nuclear modification factor for jets, $R_{AA}^\text{jet}(R_{\text{max}}, \omega_{\text{min}})$, provides a sensitive tool to probe the energy and angular dependence of the stimulated gluon emission pattern. The largest difference between the jet shape in vacuum and the total in-medium jet shape was manifest in the periphery of the

![Fig. 5. Comparisons of the jet shape in vacuum, medium-induced jet shape, and the total jet shape for cone radii $R = 0.7$, 0.4 and for different jet energies $E_T = 20, 50, 100, 200$ GeV, respectively, in $Pb+Pb$ collisions at LHC.](image1)

![Fig. 6. The ratios of the total jet shape in heavy-ion collisions to the jet shape in vacuum for jet energies $E_T = 20, 50, 100, 200$ GeV and cone radius $R = 0.4$ in $b = 3$ fm $Pb+Pb$ collision at $\sqrt{s} = 5.5$ TeV.](image2)
cone, \( r/R \rightarrow 1 \), and for smaller radii, e.g. \( R^{\text{max}} = 0.4 \). The enhancement of the mean relative jet radius \( \langle r/R \rangle \) due to QGP effects was rather modest.

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