High Inclination Planets and Asteroids in Multistellar Systems

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ABSTRACT

The Kozai mechanism often destabilises high inclination orbits. It couples changes in the eccentricity and inclination, and drives high inclination, circular orbits to low inclination, eccentric orbits. In a recent study of the dynamics of planetesimals in the quadruple star system HD98800 (Verrier & Evans 2008), there were significant numbers of stable particles in circumbinary polar orbits about the inner binary pair which are apparently able to evade the Kozai instability.

Here, we isolate this feature and investigate the dynamics through numerical and analytical models. The results show that the Kozai mechanism of the outer star is disrupted by a nodal libration induced by the inner binary pair on a shorter timescale. By empirically modelling the period of the libration, a criteria for determining the high inclination stability limits in general triple systems is derived. The nodal libration feature is interesting and, although affecting inclination and node only, shows many parallels to the Kozai mechanism. This raises the possibility that high inclination planets and asteroids may be able to survive in multistellar systems.

Key words: celestial mechanics – planetary systems – methods: N-body simulations

1 INTRODUCTION

The Kozai instability is a well-known destabilizing mechanism of high inclination satellites. Arnold (1990), in his book Huygens & Barrow, Newton & Hooke, reports a neat illustration due to Lidov (1963). Lidov discovered that if the orbit of the Moon is turned through 90°, its eccentricity increases so rapidly under the action of the tidal forces of the Sun that it collides with the Earth in four years! The effect was described and analysed by Kozai (1962, 1980) in his studies of the survival of high inclination comets and asteroids.

The Kozai mechanism is a secular effect whereby, above a critical inclination i_{crit} of around 40° (depending on the system), the inclination i is coupled to eccentricity e though the Kozai constant (see e.g. Thomas & Morbidelli 1996)

\[ H_{Koz} = \sqrt{a(1-e^2)\cos i} \]  

and is driven to vary in cycles between its initial value and the critical value. The argument of pericentre also librates about ±90° (Kozai 1962, Thomas & Morbidelli 1999, Takeda et al. 2008). In other words, nearly circular, high inclination orbits are driven to high eccentricities in exchange for lower inclination, so that in the Solar system this produces the population of Sun-grazing comets (e.g., Stagg & Bailey 1989).

At first glance, this suggests that planets or asteroids in high inclination orbits in multistellar systems cannot survive for long times. A planet in a circumstellar orbit in a binary star system is known to be subject to Kozai cycles on a timescale (Kiseleva, Ezeleto & Mikkola 1998, Takeda et al. 2008)

\[ \tau_{Koz} \approx \frac{2}{3\pi} \frac{P_{bin}^2}{P_p} (1-e_{bin}^2)^{3/2} \frac{m_A + m_B + m_p}{m_B} \]  

where \( P_{bin} \) is the period of the binary, \( P_p \) that of the planet, \( e_{bin} \) is the binary’s eccentricity, \( m_A \) and \( m_B \) the stellar masses and \( m_p \) the planet’s mass. Surprisingly, then, a recent study of planetesimals in the stellar system of HD 98800 (Verrier & Evans 2008) found a long-lived, high inclination circumbinary population.

HD 98800 has four stars, in two close binaries A and B, in highly inclined and eccentric orbits and hosts a circumbinary debris disc around the B binary pair. The system is well approximated as a hierarchical triple star system, as the secondary star in the A binary is fairly small and in a close orbit. High inclination particles are found to be in long-term stable orbits inclined by 55° to 135° to the inner binary B. This is well above the critical inclination relative to both stellar orbits, yet the orbits are not undergoing Kozai cycles. It is known that the Kozai effect can be disrupted if another mechanism, such as general relativity, tidal forces or other interactions between planets, causes orbital precession on a shorter timescale (Kinoshita & Nakai 1991, Wu & Murray 2003, Takeda & Rasid 2005, Takeda et al. 2008). However, the simulations in Verrier & Evans (2008) deal with the Newtonian dynamics of test particles, so the only additional factor present is the mutual gravitational perturbations of the stars.

This paper explains the origin of the surprising stability. The inner binary causes a nodal precession, instead of Kozai cycles.

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2 NUMERICAL EXPERIMENTS

In Verrier & Evans (2008), numerical studies of the stability of planetesimals in the circumbinary disc of HD9800 were carried using the MOIRA code. This is a fast symplectic code adapted for hierarchical systems (Verrier & Evans 2007). We have checked that our results are not an artefact of the numerical integration method, as similar results are obtained with a standard (Press et al. 1989) Bulirsch-Stoer integration scheme (see Verrier 2008).

The stellar parameters used for the simulations are shown in Table 1. We investigate three possible configurations, labelled orbits I, II and III (see Verrier & Evans 2008 for more details on the rationale for the orbital parameters). High inclination particles are unexpectedly found to be in long-term stable orbits inclined by 55° to 135° to the inner binary, and restricted to a narrow range of semimajor axis. These particles are shown in Figure 1 for the case of the orbit II parameters. Using \( \Omega \) to denote the longitude of the ascending node, we plot the particles in the \( (i \cos \Omega, i \sin \Omega) \) plot. This reveals distinct dynamical structures – in particular, two populations of stable particles. Interestingly, though, there is little structure seen in the \( (e \cos \omega, e \sin \omega) \), where \( \omega \) is the argument of pericentre.

This stabilises the test particles against any Kozai instability driven by the outer star. We first present the evidence in favour of this explanation in Section 2, and then provide a more detailed study of the nodal precession, together with the dynamics behind it, in Section 3. Finally, we summarize our conclusions in Section 4.

Table 1. The initial orbital parameters for the simulations of system HD 98800 used in Verrier & Evans (2008). These are the orbital elements used to evolve the system from 1 Myr in the past to the present state, so the starting parameters for the inner binary are also different for each wide orbit case. The longitudes given are relative to the arbitrary reference direction of the simulations.

| Orbital Parameter | Wide orbit A-B | Inner orbit Ba-Bb |
|-------------------|---------------|------------------|
| \( m_1 \) (\( M_\odot \)) | 1.28 | 0.699 |
| \( m_2 \) (\( M_\odot \)) | 1.3 | 0.582 |
| \( a \) (au) | 61.9 | 67.6 | 78.6 | 0.983 | 0.983 | 0.983 |
| \( e \) | 0.3 | 0.5 | 0.6 | 0.640 | 0.790 | 0.570 |
| \( i \) (°) | 130.7 | 144.7 | 131.7 | 0.0 | 0.0 | 0.0 |
| \( \omega \) (°) | 214.1 | 323.4 | 65.9 | 82.0 | 11.3 | 34.89 |
| \( \Omega \) (°) | 224.5 | 309.2 | 66.6 | 0.0 | 0.0 | 0.0 |
| \( M \) (°) | 166.3 | 90.8 | 16.2 | 225.9 | 32.6 | 259.7 |

2 NUMERICAL EXPERIMENTS

The orbits of all the stable particles librate about 90° inner (i.e. relative to the inner B binary’s orbital plane) inclination. Their nodes also librate about ±90° relative to the inner binary’s pericentre, defining the two populations seen in Figure 1. These areas also defined by their initial node: the upper population in the \( (i \cos \Omega, i \sin \Omega) \) plot starts at 120° and the lower at 240° (relative to the simulations reference direction). All particles starting with \( \Omega = 0° \) are not stable. Unlike the choice of initial node, there appears to be no difference in the evolution of particles with differing pericentres or times of pericentre passage. Interestingly, the stability of the high inclination particles is greater for the simulations with a lower eccentricity for the outer star. This is shown in the far right panels of Figure 1 in which there are more surviving test particles in the case of orbit I as opposed to orbit III. A typical orbit of one of the stable high inclination particles is shown in Figure 2 and illustrates the libration of inclination and node. There also appears...
to be a slight modulation of this variation with the stellar orbital evolution, as indicated in the Figure.

A brief investigation of the variation of stability with the stellar parameters was achieved by integrating the stable particles for a variety of different eccentricities and mutual inclinations of the two stellar orbits. This revealed that the outer stellar eccentricity has little effect on the stability, while the inner eccentricity is crucial! As the inner orbit becomes circular, the high inclination stability, somewhat surprisingly, decreases dramatically. The stellar mutual inclination is only important insofar as it may cause variations in the resulting inner eccentricity. This would appear to be in contradiction to the results shown in Figure 1 where the stable region increases as the outer stars eccentricity decreases. However, in these simulations the test particles do not start similarly aligned with the inner star case (as the original simulations start at 1 Myr in the past, where the inner star has a different longitude for each outer eccentricity case), meaning that in the Orbit I case the particles are closer to the libration island, while in the Orbit III case they are further away and hence less stable. This effect is discussed in more detail in Section 3.1.

The orbital evolution of particles due to each stellar orbit is also of interest. Replacing the inner binary as a single star results in the stable particles now undergoing Kozai cycles and becoming rapidly unstable at very high inclinations. However, removing the outer star instead does not have similar results. In this case, the particles remain stable and still librate about 90° inclination and longitude of the ascending node, as shown in Figure 3. The librations have a period of the order of Kyrs which increases with particle semimajor axis and an amplitude that decreases with semimajor axis.

This nodal libration could be suppressing the Kozai cycles caused by the outer star, and could explain the distribution of the stable particles, if the period is shorter than that of the cycles. At 3 au, $\tau_{Koz} \sim 3$ Kyr and at 8 au $\tau_{Koz} \sim 1.5$ Kyr, while the nodal libration period at 3 au is about 1 Kyr, indeed shorter than $\tau_{Koz}$, and at 8au is about 2 Kyr, longer than $\tau_{Koz}$, supporting the idea.

In summary, the numerical evidence suggests that the inner binary star causes the nodes and inclinations of particles within about 7 au to librate on a shorter timescale than the Kozai cycles induced by the outer star, thereby suppressing the mechanism and stabilising their orbits. The simulations also indicate the period of the librations increase as the inner binary’s eccentricity decreases, explaining the lack of stability of the particles in the triple system with a circular inner binary. This nodal libration is itself a very interesting phenomenon, as is the lack of Kozai cycles.

3 THE NODAL LIBRATION MECHANISM

3.1 The Elliptic Restricted Three-Body Problem

The stellar system is now reduced to that of the inner binary only. The outer binary can be neglected, as we have just demonstrated that the nodal libration takes place in its absence. This arrangement is now identical to the elliptic restricted three-body problem. The circular restricted three-body problem is the limit $e_{bin} = 0$. It is most commonly met under the guise of the Copenhagen problem, in which the two bodies have equal mass. There has been substantial effort on the classification of the orbits in the Copenhagen problem (see e.g., Contopoulos 1967), but the eccentric case has been much less well studied.

Simulations are run for 10 Kyrs, as the short term behaviour is of interest. Since the initial node of the test particles appears to be an important factor in the particle’s orbital evolution, a set of simulations were run looking at a much wider range of this parameter. The underlying stellar parameters are those corresponding to orbit II, whilst the initial test particle grid is shown in Table 1.

The resulting distributions in inclination and the ($i, \cos \Omega$) plane are shown in Figure 4. Once again the two libration islands are clear and are at about $i = 90°$ and $\Omega = \pm 90°$ relative to the pericentre line of the binary star. Figure 5 shows these islands in more detail for a similar simulation that now has particles spaced exactly about these libration centre (i.e. the binary’s pericentre has been set to lie along the simulation reference direction) and from 0° to 180° inclination. Particles in the centre of the islands remain there, and the initially low inclination particles also remain in low inclination orbits, librating about the centre of the ($i, \cos \Omega, i, \sin \Omega$) plane while the retrograde particles circulate. This libration and critical angle is in striking similarity to Kozai mechanism, where similar libration islands are seen, but for $\omega$. Here, however, the argument of pericentre still rapidly circulates, and no structure is apparent in the ($i, \cos \omega, i, \sin \omega$) plane.

A particle’s proximity to the centre of the libration island is
3.2 An Approximate Integral of Motion

The surfaces of section (Figures 4 and 5) suggest that there is an approximate integral of the motion for the circumbinary test particles. From the simulation data, we can verify that this is the case. The constant is the test particle’s component of angular momentum along the direction of the line of apses of the binary star’s orbit, which is

\[ L_{\omega_*} = h \sin i_{tp} \sin \Omega_{\omega_*} \quad (3) \]

where \( i_{tp} \) is the particle’s inclination relative to the plane of the binary’s orbit, and \( \Omega_{\omega_*} \) is the particle’s node in the same frame of reference and relative to the pericentre of the binary’s orbit. An example of the conservation of this quantity is given in Figure 6 for the test particle from Figure 2. The libration seen in the inclinations and nodes of the test particles corresponds to a simple constant precession of their orbital planes about this axis.

This suggests that the physical reason for the test particle’s orbital evolution is a torque on the orbit due to the component of the gravitational force from the binary parallel to the axis of the line of apses, labelled as the direction \( x_{\omega_*} \). This offers an explanation for the increased stability with eccentricity. As the binary’s orbit becomes more eccentric it approaches a straight line along the direction \( x_{\omega_*} \), resulting in a greater torque and hence faster precession. As the binary’s eccentricity decreases, the torque due to forces along the star’s minor axis (i.e. parallel to the \( y_\star \) direction, using the previous notation) will increase, which eventually acts to suppress the precession about the \( x_{\omega_*} \) axis.

3.3 Stability Boundaries

The period and amplitude of the librations vary with initial inclination, node and semimajor axis and the shortest period and minimum amplitude occur for particles that start close to the libration centre. The point at which the period of these variations equals the Kozai timescale of the outer star defines the outer stability boundary. This can be modelled empirically by fitting the period of the inclination variations.

The nodal librations are best defined for initial nodes of \( \Omega = 90^\circ \). To fit the period, sets of simulations are run with a similar grid of test particles as shown in Table 2 but with \( \Omega \) and \( M \) fixed at 90° and 0° respectively and the semimajor axis varied from 3 to 15 au now in steps of 0.5 au. The period of the inclination variations are shown in Figure 7 as a function of initial semimajor axis for the different inclinations. A power law fits each inclination case very well. This gives the following empirical law

\[ P = Na_{tp}^n \text{ Kyr} \quad (4) \]

where \( a_{tp} \) is the test particle semimajor axis and the associated best fit parameters and uncertainties are given in Table 3.

The Kozai timescale for HD 98800 is also shown in Figure 7 and meets the 80° inclination line at 7.4 au. This matches up very well with the outer edge of the high inclination stable test particles for HD 98800 (see the left panels of Figure 1). Therefore, this accurately describes the stability boundary for HD 98800 and can be used as a method of finding a general high inclination stability limit. It should also be noted that the minimum eccentricity of 0.45 obtained by the inner binary in the three star case results in longer period variations, over plotted on Figure 7 which probably causes the slightly diffuse outer edge of the halo seen in Figure 1.

We now carry out the same calculation for sets of test particles in general binary systems. This empirical model can then be used in conjunction with the outer star’s Kozai timescale to place limits on the high inclination stability in a general hierarchical triple system.

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### Table 2. The test particle grid used in the simulations shown in Figures 4 and 5.

| Orbital Elements | Min   | Max   | Step Size |
|------------------|-------|-------|-----------|
| \( a_{tp} \) (au) | 3.0   | 10.0  | 0.2       |
| \( e_{tp} \)     | 0.000 | 0.006 | 0.0002    |
| \( i_{tp} \) (°) | 50    | 130   | 10        |
| \( \omega_{tp} \) | 0     | 300   | 30        |
| \( \Omega_{tp} \) | 0     | 240   | 120       |

### Table 3. Fitted parameters for the period law given in eqn (4).

| Inclination | \( N \) | \( n \) | \( \chi^2 \) |
|-------------|---------|---------|-------------|
| 50°         | 0.0029 ± 0.0008 | 3.33 ± 0.14 | 0.32 |
| 60°         | 0.0027 ± 0.0007 | 3.32 ± 0.14 | 0.26 |
| 70°         | 0.0023 ± 0.0006 | 3.33 ± 0.14 | 0.34 |
| 80°         | 0.0024 ± 0.0006 | 3.31 ± 0.14 | 0.45 |
| 130°        | 0.0026 ± 0.0007 | 3.40 ± 0.14 | 0.26 |
| 120°        | 0.0024 ± 0.0006 | 3.36 ± 0.14 | 0.15 |
| 110°        | 0.0022 ± 0.0006 | 3.36 ± 0.14 | 0.23 |
| 100°        | 0.0022 ± 0.0006 | 3.35 ± 0.14 | 0.08 |

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![Figure 6.](image-url) The conservation of the angular momentum component, \( L_{\omega_*} \), along the inner binary's line of apses for the same particle as Figure 2.
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Figure 4. The inclination variations and surface of section for test particles around the inner binary only. The orbital elements are all relative to the orbit of this binary (i.e. the node is from the pericentre of the binary’s orbit). Colour indicates initial longitude of ascending node, with black through blue through green through yellow through red representing $0^\circ$ to $360^\circ$. The libration islands about $\Omega = \pm 90^\circ$ and $i = 90^\circ$ are clear on the left-hand plot of $i \cos \Omega$ versus $i \sin \Omega$.

Figure 5. The inclination variations and surface of section for particles around the inner binary of HD 98800, but now spaced evenly about $\Omega = 90^\circ$ and for inclinations in the range $0^\circ$ to $180^\circ$. Colour now indicates initial inclination, as shown in the left-hand plot. The initially low inclinations remain in the range $0^\circ$ to $30^\circ$ and are seen in the small libration island about the centre of the right-hand plot.

Simulations show that the libration islands decrease in size as the eccentricity decreases, whilst the critical inclination for the librations to occur increases. The mass ratio of the binary, $\mu = m_B/m_B + m_B$, also has an effect, but not on the geometry of the libration islands. Instead, as this ratio decreases from 0.5, the particles start to escape from the simple libration patterns previously seen. To derive our empirical fit, sets of simulations are again run with similar test particle grids, but now with only one inclination of $85^\circ$ to find the longest possible period of variations. The stellar mass ratio is varied from 0.1 to 0.5 in steps of 0.1 and the eccentricity from 0.1 to 0.8 in steps of 0.1. The binary was generalised to a separation of 1 au, total mass $1 \, M_\odot$ and with all longitudes equal to zero. The parameters to the power law were found to fit

$$P(\text{Kyr})/P_{\text{bin}}(\text{yr}) \approx 0.001 e^{-1.1} \mu^{-0.8} \left( \frac{a}{a_{\text{bin}}} \right)^{(3.37 \pm 0.06)}. \quad (5)$$
Figure 7. The period of the inclination variations plotted as a function of semimajor axis, for test particles around the inner binary of HD 98800. The prograde inclinations are shown only, as the retrograde cases are almost identical. A $50^\circ$ initial inclination is in green, $60^\circ$ in blue, $70^\circ$ in purple and $80^\circ$ in black and the fits to the data are shown as solid lines. The Kozai timescale of the orbit II outer binary star, shown in red, intersects the period variation lines near the outer edge of the orbit II stable halo. The period of the inclination variations is also shown in orange for an inner binary with eccentricity $e = 0.45$, the minimum value obtained by the stars in the HD 98800 orbit II triple system, and can be seen to be much longer than the higher eccentricity case.

Figure 8. The variation of the amplitude and exponent of the fitted power law with mass ratio and eccentricity for the general binary case. In the top two panels different mass ratios are indicated by the colours purple (0.1), blue (0.2), green (0.3), orange (0.4) and red (0.5). In the bottom two panels, different eccentricities are indicated by the colours black (0.1), purple (0.2), blue (0.3), light blue (0.4), green (0.5), yellow (0.6), orange (0.7) and red (0.8). The fitted lines are also shown.

This is expected to scale with semimajor axis, as seen in the coplanar limits in Verrier & Evans (2007). To derive this empirical formula, the powers of $e$ and $\mu$ have been fixed at the values shown in the fitting. The amplitude has been fitted (excluding the $e = 0.1$ case, which seems poorly modelled by this formula, most likely as it is approaching the constant inclination regime), but has an associated error of 0.01. Despite this the equation provides a good fit to the data and a low $\chi^2$ value, as shown in Figure 8.

4 CONCLUSIONS

This paper presents a foray into an area of dynamics that has not been extensively studied, namely small particles around a highly eccentric binary star in a hierarchical triple system. At outset, such particles might be expected to be disrupted by the Kozai instability.

Here, we have demonstrated the existence of stable, high inclination circumbinary test particles. They owe their stability to the high eccentricity of the inner binary. This is somewhat surprising, as it goes against every expectation of planetary dynamics. The inner binary, instead of inducing Kozai cycles, causes smooth inclination variations and nodal precession for certain initial longitudes. This suppresses Kozai cycles that would otherwise occur due to the outer star in the hierarchical triple.

An analytical theory to explain these variations is desirable. However, the stars are very eccentric, their mass ratio is high, and the test particle’s inclination is also large, leaving no obvious small parameters for the standard perturbation expansions of celestial mechanics. The semimajor axis ratios are not very small, but do not explain why existing secular theories (e.g. Kozai 1962; Ford et al. 2000) are unable to describe the dynamics. The libration islands, inclination variations and critical initial inclination are surprisingly familiar to, but contrasting with, the dynamics of the Kozai mechanism. The Kozai mechanism occurs if $\dot{\omega} \approx 0$, whereas the nodal libration appears to be a similar process occurring when $\dot{\Omega} \approx 0$. 

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There exists an approximate integral of motion – namely, the test particle’s component of angular momentum along the direction of the line of apses of the binary star’s orbit. This also is analogous to the Kozai mechanism, which has an approximate integral of motion.

The high inclination libration feature may have important consequences for planetary stability in circumbinary orbits. Many stellar members of triples have exchanged into the system, so high mutual planet-star inclinations are very likely. If there are regions of stability then the outlook for planetary systems in these environments is more promising than previously thought.

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