Spatial correlation description of deformation object based on fuzzy clustering and geological analysis

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KEY WORDS fuzzy clustering; geological investigation; correlation description; deformation object

ABSTRACT The methods of deformation analysis and modeling at single point are realized easily now, but available approaches do not make full use of the information from monitoring points and can not reveal integrated deformation regularity of a deformable body. This paper presents a fuzzy clustering method to analyze the correlative relations of multiple points in space, and then the spatial model for a practical dangerous rockmass in the area of Three Gorges, Yangtze River is established, in which the correlation of six points in space is analyzed by geological investigation and fuzzy set theory.

1. Introduction

In scientific and technological fields, there are many complex things associated with the concept of modeling. Deformation analysis in geoscience is one example of those. Geodesists have tried for many years to establish the decision-making equation for a deformable body by deterministic method, which utilizes information on the loads, properties of the materials, geometry of the body and physical laws governing the stress-strain relationship, or to give a clear picture of the correlation between observed deformations (e.g., displacements) and observed loads (external and internal causes producing the deformation). Nevertheless, due to the complexity and uncertainty of the deformable body, we cannot but notice that the classical approaches to decision-making and cause-reason have been declining while innovative concepts based upon the new mathematics, such as fuzzy set theory and grey system theory, are continually emerging, while it is difficult to make a clear physical concept to all factors.

The author has taken into consideration the correlation among the deformation points in space, the fuzzy set theory is used to analyze the correlation and interaction among the points. Geological investigation and mathematical analysis are used in the analysis of a real dangerous rockmass in the area of Three Gorges and the spatial dynamic deformation model has been established for the rockmass.

2. Fuzzy clustering analysis for multi-point correlation

2.1 The principle of analysis for multi-point correlation

The primary objective of clustering techniques is to partition a given data set into so-called homogeneous clusters. The term homogeneous means that all points in the same group are close to each other and are not close to the points in the other groups. Clustering algorithms may be used to build pattern classes or to reduce the size of a data set while retaining relevant information. In classical algorithms, it is implicitly assumed that disjoint clusters exist in the data set. However, the separation of clusters is a fuzzy notion, and representation of clusters by fuzzy sets seems more appropriate in certain situations, such as the separation of the
monitoring points in the crust deformation. Therefore, the fuzzy clustering analysis is applied to the correlation analysis of multiple points.

Deformation of the crust or engineering structure is continuous, but we can only acquire the measurements at a series of discrete epochs and even very poor measurements sometimes. In order to get the potential information among the observed measurements, the correlation of monitoring points in space and time domain should be taken into consideration. In fact, deformations of the crust or engineering structure are observed by the monitoring network. Strong correlation will be achieved when the points are in the same deformable body or are very close to each other. As we know, the crust is usually taken as the elastic body with isotropism in the study of the crust deformation model, but in fact it is just similar isotropic for the reason of the non-integration, non-uniform, non-elastic, etc. Deformation of a deformable body under the loads is estimated by the displacements of the points in three dimensions, therefore we can consider the deformation points in similar deformation situation in three dimensions as a deformable body or a cluster of points.

2.2 A fuzzy clustering for multi-point correlation in space

According to the fuzzy set theory, the analysis procedures using fuzzy clustering can be summarized as follows:

1) Determination of the sample sets to cluster
Supposing \( X = \{ x_1, x_2, \ldots, x_n \} \) as a whole to cluster, every \( x_i \) is defined as:
\[
x_i = \{ x_i(k) | k = 1, 2, \ldots, n \}
\]

2) Establishment of the fuzzy similarity matrix
To calculate the fuzzy similarity matrix \( R \), the first step is to calculate the similar index \( r_{ij} = R(x_i, x_j) \) of \( x_i \) and \( x_j \). Since there are 14 methods\(^3\) to calculate the similar index and a little different results will be drawn by using different methods, so we have used 14 methods to calculate the similar index \( r_{ij} = R(x_i, x_j) \). One of them is absolute value subtraction which is shown as follows:
\[
r_{ij} = \begin{cases} 
1, & i = j \\
1 - C \sum_{k=1}^{n} | x_i(k) - x_j(k) | & i \neq j
\end{cases}
\]

3) Clustering
With \( r_{ij} \), we obtain fuzzy similarity matrix:
\[
R = \begin{pmatrix} 
    r_{11} & \cdots & r_{1m} \\
    \vdots & \ddots & \vdots \\
    r_{m1} & \cdots & r_{mm}
\end{pmatrix}
\]

Obviously, in \( R = (r_{ij})_{m \times m} \), we have
\[
\begin{align*}
    r_{ii} &= 1, i = j \\
    r_{ij} &= r_{ji}, i \neq j
\end{align*}
\]

That means \( R \) is reflexive and symmetric. In this case \( R \) is called fuzzy similarity matrix. But if \( R \) is also satisfied for \( R \cdot R \subseteq R \), then \( R \) is strongly transitive and is called fuzzy equivalent matrix.

For clustering, a fuzzy similarity matrix can be turned into a fuzzy equivalent matrix by transitive closure method, but it is a trouble. An easy way is that the fuzzy ordering can be pictured using a graph whose edges are labelled by the membership degrees of the couples \( (x_i, x_j) \) in the relation \( R \). Given \( \lambda \in [0, 1] \), if \( r_{ij} \geq \lambda \), then we can take \( i, j \) as a homogeneous cluster; while \( r_{ij} < \lambda \), we can not do this.

3 Analysis of a real dangerous rockmass

3.1 Geological investigation
The Lianziya is a large dangerous rockmass located at the south bank of the Three Gorges. Rockfall and landslide happened here often in history. There was a successful forecasting for the landslide in the opposite bank in 1985. Long term monitoring and investigation has denoted that this area is in an unstable situation.

The lithological association of the Lianziya rockmass are the hard and blocky limestone as the host rock at the upper part and the flaggy carbonaceous shale with multilayer between. There is a weak coal measures strata at the lower part. The rockmass with such a structure causes very easily plastic deformation at the lower part and leads the rock to cleaving and deforming at the upper part. The upper part of hard rock layer causes easily again brittle fracture and brings the whole rockmass into a dangerous situation. Here we apply the principles of analysis for multi-point correlation to \( T_0 \sim T_8 \).
cracks (see Fig. 1). \(T_0 \sim T_6\) cracks are located at the south of the Lianziya rockmass, which is the one of three dangerous areas in Lianziya. The strata tilts to the western mountain and is an inner tilt slope. All cracks are towards the north by west or northwest by west except \(T_6\) which is in the north and south direction. The \(T_0 \sim T_6\) cracks form a dendrograph on the top of cliff and can be taken as a body which is in a quasi critical situation.

\(A_2, A_3, A_4, A_5\) and \(A_6\) are deformation points on the northeast top of \(T_2 \sim T_6\) cracks and in the area of stress concentration. They are the key positions for deformation monitoring. The displacement curves (see Fig. 2) show that the progressive tendency of \(A_2, A_3, A_4, A_5\) and \(A_6\) except \(A_1\) are coincidently towards the north by east.

3.2 Fuzzy clustering analysis

Now we use fuzzy clustering to analyze the correlation of \(A_1 \sim A_6\) points at \(T_0 \sim T_6\) cracks. Table 1 shows the results of deformation similarity matrix of \(A_1 \sim A_6\) points by the method of absolute value subtraction. Given \(\lambda = 0.7\), with the direct clustering method, we consider that \(A_2 \sim A_6\) points in direction \(X\) and \(Z\) are of the same deformation regularity and can be taken as a homogeneous cluster, but \(A_1 \sim A_6\) points are homogeneous in direction \(Y\). The integration results of 14 fuzzy clustering methods are listed in Table 2.

There are separately 3,5 and 4 methods in direction \(X, Y\) and \(Z\) to take \(A_1 \sim A_6\) points as a homogeneous cluster, 9 and 8 methods in direction \(X\) and \(Z\) to take \(A_2 \sim A_6\) points as a homogeneous cluster. The combination of qualitative geological investigation and fuzzy clustering analysis shows that \(A_2 \sim A_6\) points are in a same deformable body.

4 Spatial model for \(A_2 \sim A_6\) points

According to above analysis results, the spatial model for \(A_2 \sim A_6\) points has been established by using the vertical displacements with 18 epochs collected in Lianziya in 1994 and 1995. In modeling, we have kept the original displacements of last 6 epochs as a test of model prediction, so the length of data series of modeling is \(n = 13\).

| Direction | Point \(A_1\) | \(A_2\) | \(A_3\) | \(A_4\) | \(A_5\) | \(A_6\) |
|-----------|--------------|--------|--------|--------|--------|--------|
| \(X\)     | 1            | 0.32   | 0.54   | 0.35   | 0.05   | 0.14   |
|           | 1            | 0.78   | 0.93   | 0.73   | 0.80   |        |
|           | 1            | 0.81   | 0.51   | 0.60   |        |        |
|           | 1            | 0.70   | 0.79   |        |        |        |
|           | 1            | 0.88   |        |        |        |        |
|           | 1            |        |        |        |        |        |
| \(Y\)     | 1            | 0.90   | 0.98   | 0.84   | 0.73   | 0.74   |
|           | 1            | 0.90   | 0.94   | 0.83   | 0.84   |        |
|           | 1            | 0.84   | 0.73   | 0.74   |        |        |
|           | 1            | 0.90   | 0.90   | 0.90   |        |        |
|           | 1            | 0.98   |        |        |        |        |
|           | 1            |        |        |        |        |        |
| \(Z\)     | 1            | 0.22   | 0.27   | 0.04   | 0.25   | 0.50   |
|           | 1            | 0.95   | 0.81   | 0.96   | 0.72   |        |
|           | 1            | 0.77   | 0.97   | 0.77   |        |        |
|           | 1            | 0.78   | 0.54   |        |        |        |
|           | 1            |        | 0.75   |        |        |        |
|           | 1            |        |        |        |        |        |
Assume that $|x_{i}^{(0)}(k)| (k = 1, 2, \cdots, n; i = 1, 2, \cdots, 5)$ is the original time series of 5 points. Corresponding with this, we can get mean value sequence $\{x_{i}^{(0)}(k)\}$ and mean value accumulated-adding generated time series $\{x_{i}^{(1)}(k)\} (k = 1, 2, \cdots, n; i = 1, 2, \cdots, 5)$ the spatial forecasting model[4] is as below

$$\ddot{x}(0)(k) = 2e^{A(k-1)}(I + e^{-A})^{-1}(I - e^{-A}) \dddot{B}$$

The simulative precision of model is:

$$\sigma^2 = \frac{\sum_{i=1}^{12} \sum_{k=1}^{13} [\ddot{x}_{i}^{(0)}(k) - \ddot{x}_{i}^{(0)}(k)]^2}{5 \times 13} = 0.46 \text{ mm}^2$$

The original displacements of 5 points and their forecasting values are listed in Table 3.

Table 3 Original displacements of 5 points and their forecasting values

|      | A2       | A3       | A4       | A5       | A6       |
|------|----------|----------|----------|----------|----------|
|      | original | forecasting | original | forecasting | original | forecasting | original | forecasting | original | forecasting |
| A1   | -26.0    | -26.7    | -24.0    | -25.0    | -29.9    | -31.6    | -22.8    | -24.6     | -15.7    | -17.4      |
| A2   | -25.6    | -26.4    | -24.0    | -25.1    | -30.4    | -32.1    | -23.4    | -24.8     | -15.5    | -17.4      |
| A3   | -26.0    | -26.6    | -25.2    | -25.5    | -33.2    | -32.9    | -25.4    | -25.5     | -17.6    | -17.7      |
| A4   | -26.0    | -27.3    | -24.7    | -26.2    | -32.9    | -34.0    | -25.5    | -26.6     | -17.5    | -18.6      |
| A5   | -27.0    | -28.2    | -26.1    | -27.1    | -33.4    | -35.1    | -26.3    | -28.0     | -18.1    | -19.8      |
| A6   | -27.9    | -29.3    | -27.5    | -28.0    | -35.2    | -36.2    | -28.3    | -29.2     | -20.3    | -21.0      |

5 Conclusions

The spatial analysis of deformable body has taken into account the correlation among the deformation points in space and time domain, the distinction of the monitoring points in the crust deformation is made by fuzzy clustering analysis and geological investigation. An innovative spatial modeling based on fuzzy set theory and grey system theory provides a good approach for extraction of potential information in observed data series and description of integrated deformation regularity.

The study on $T_0 \sim T_6$ cracks demonstrates the flexibility of the spatial analysis, in which an integrated consideration is taken simultaneously on the correlation of six points in space, the geological investigation and fuzzy clustering analysis are used to classify the monitoring points based on the similar deformation regularity in three dimensions. Five points are considered as a deformable body and the spatial model is established.

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