Fretting fatigue investigation on Al 7075-T651 alloy: experimental, analytical and numerical analysis

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ABSTRACT

In this work, a fretting fatigue experimental campaign on Al 7075-T651 alloy specimens is simulated by using both analytical and numerical approaches. According to one of the classifications available in the literature, the theoretical approaches for fretting fatigue assessment may be divided into stress-based models or Stress-Intensity Factor (SIF) models. The analytical approach here applied falls in the group of the stress-based models, whereas two numerical methods fall in the group of the SIF-based models. The fatigue assessment is performed by evaluating both crack path and lifetime.
KEYWORDS: Al 7075-T651, crack path, fretting fatigue, lifetime

NOMENCLATURE

- $a$: theoretical Hertzian contact semi-width
- $C$: shear stress vector lying on the critical plane
- $l$: crack length
- $l_{inc}$: crack increment between two consecutive steps
- $l_{init}$: initial crack length
- $l_0$: distance between the surface and the first microstructural barrier
- $L$: El-Haddad intrinsic crack length
- $N$: constant normal load
- $N_{eq}$: equivalent normal stress
- $N_{exp}$: fretting fatigue life (number of loading cycles to failure)
- $N$: normal stress vector to the critical plane
- $N_{155\mu m}(\Delta K^*_I)$: estimated total fatigue life computed by means of the $\Delta K^*_I$ approach up to a crack length of $155\mu m$
- $N_{155\mu m}(SWT)$: estimated total fatigue life computed by means of the SWT approach up to a crack length of $155\mu m$
- $N_{prop-155\mu m}(\Delta K^*_I)$: estimated fatigue propagation life computed by means of the $\Delta K^*_I$ approach
- $N_{prop-155\mu m}(SWT)$: estimated fatigue propagation life computed by means of the SWT approach
- $N_{3.5mm}(\Delta K^*_I)$: estimated total fatigue life computed by means of the $\Delta K^*_I$ approach up to a crack length of $3.5mm$
- $N_{3.5mm}(SWT)$: estimated total fatigue life computed by means of the SWT approach up to a crack length of $3.5mm$
- $P$: cyclic (harmonic) axial load applied to the specimen
- $Q$: cyclic tangential load
- $R$: loading ratio
- SWT: Smith-Watson-Topper
- $T$: period of the cyclic loading
1, 2, 3  instantaneous principal stress directions
P123  instantaneous principal reference frame
\( \hat{1}, \hat{2}, \hat{3} \)  averaged principal stress directions
\( \hat{p}1\hat{2}\hat{3} \)  averaged principal reference frame

\( \delta \)  off-angle defining the normal, \( w \), to the critical plane
\( \sigma \)  cyclic axial stress applied to the specimen
\( \sigma_{af,-1} \)  material fatigue strength under fully reversed normal stress at \( N_0 \) loading cycles
\( \Delta \sigma_{af} \)  fatigue limit range
\( \sigma_{1,\sigma_2,\sigma_3} \)  instantaneous principal stresses
\( \sigma_{1,\text{max}} \)  maximum value of \( \sigma_1 \) during \( T \)
\( \phi, \theta, \psi \)  instantaneous principal Euler angles
\( \hat{\phi}, \hat{\theta}, \hat{\psi} \)  averaged principal Euler angles
\( \theta \)  kinking angle
\( \theta_{\text{max}} \)  kinking angle that maximises \( \Delta K_I^* \)
\( \tau_{af,-1} \)  material fatigue strength under fully reversed shear stress at \( N_0^* \) loading cycles
\( \Delta K_I \)  range of the Mode I SIF of the actual crack
\( \Delta K_{I,th} \)  threshold range of the stress intensity factor for long cracks
\( \Delta K_I^* \)  range of Mode I SIF for an infinitesimal kinked crack emanating from the actual crack
\( \Delta K_{II} \)  range of Mode II SIF of the actual crack

**SUBSCRIPTS**

\( a \)  Amplitude
\( m \)  Mean
1. INTRODUCTION

For structural components under contact loading, fretting damage can be induced by vibrations in the contact region. Contact loading generates stress concentration at the contact surface, and severe stress gradient inside the component. Vibrations result in an oscillatory relative micro-slip between contacting surfaces. These relative displacements and the friction between surfaces make the fretting damage phenomenon consist in short-crack nucleation at the contact surfaces. At the same time, other mechanisms take place, such as oxidation, forming of wear debris, and material microstructural transformations.

In real conditions, bulk fatigue loading can be applied on one or both structural components in contact. Therefore, the above cracks may propagate causing a catastrophic collapse, called fretting fatigue failure [1-3]. Fretting fatigue is responsible of failure in a great number of in-service applications [4]: railway axels (wheel seats and gear seats [5-7]), aircraft engines (dovetail connections between blades and disk [8-10]), nuclear power plants (steam generator tubes and fuel channel pressure tubes [11-13]).

Over the last 50 years, extensive experimental work has been carried out on fretting fatigue [14-17], and many theoretical approaches have been proposed in the literature. According to the classification presented in Ref.[18], such approaches may be classified into two main groups: (i) stress-based models, where a multiaxial fatigue criterion, generally proposed for plain fatigue,
is applied to a non-local representative stress tensor; (ii) Stress-Intensity Factor (SIF) models, which aim to check whether an assumed initial short crack propagates or arrests, by using Linear Elastic Fracture Mechanics concepts. Note that more detailed classifications are available in the literature. For example, Bhatti et al. [19] classified the theoretical approaches as follows: Critical Plane, Stress Invariant, Fretting Specific parameters and Continuum damage mechanism approaches. Some approaches cannot be included in any classification [20].

Generally, the approaches of the above (i) group require numerical analyses involving finite elements with very small sizes to take into account the stress gradient nearby the contacting surfaces. As a matter of fact, analytical solutions are available for contact pairs involving very simple geometries only. Araujo et al. [21] proposed to use the Modified Wöhler Curve Method in conjunction with the Theory of Critical Distance. Vantadori et al. [22–24] proposed to use the Carpinteri et al. criterion in conjunction with different non-local approaches. Fouvry et al [25,26] presented an approach with the critical distance depending on the stress-gradient.

On the other hand, the approaches of the above (ii) group are time consuming for crack path evaluation. Araujo and Nowell [27], and more recently Dini [28], proposed approaches to estimate the fretting fatigue threshold. Fouvry et al. [29] evaluated the crack propagation based on a short-crack methodology. Navarro et al. [30–32] estimated both fatigue lifetime and crack path, by considering both crack initiation and crack propagation phases.
The experimental observations have highlighted that the main factors influencing the fretting fatigue phenomenon may be summarised in the following eight parameters [33,34]: relative slip amplitude, contact pressure, local stress state, loading cycles number, material and boundary conditions, frequency, temperature, and environment around the contacting surfaces.

In the present paper, the fretting fatigue experimental campaign performed on Al 7075-T651 alloy specimens described in Section 2 is simulated in order to estimate both crack path and fatigue lifetime. Firstly, a novel analytical approach falling in the (i) group (stress-based models) is applied (Section 3). Then two numerical approaches of the (ii) group (SIF-based models) are employed (Section 4). Conclusions are summarised in Section 5.

2. EXPERIMENTAL CAMPAIGN

2.1 Fretting fatigue test set-up

The scheme of the fretting fatigue test device employed is shown in Figure 1(a). At the beginning of each test, a constant normal load $N$ presses two cylindrical pads against a dog-bone specimen (Figure 1(b)). Then, dog-bone specimen is subjected to a cyclic (harmonic) axial load $P$. Due to (1) the friction of the contact surfaces and (2) the compliance of the fretting device, an in-phase cyclic tangential load $Q$ is produced. In such a case, both a constant normal stress distribution and a time-varying shear stress distribution are generated by $N$ and $Q$ at the contact surfaces.
In addition, the axial load $P$ leads to a global stress $\sigma$ in the test specimen.

The device was constructed with two adjustable supports in order to modify the stiffness of the assembly, and thus producing different values for the tangential load $Q$ by applying the same axial load $P$. With this configuration, the fretting fatigue tests have been carried out using multiple independent loading combinations ($N, Q, P$). In order to monitoring in real time the above fretting loads, the test device has been instrumented with a series of load cells, as is shown in Figure 1(a).

![Figure 1](image1.png)

The relevant geometry and sizes for the pads and test specimens are displayed in Figure 1(b). The raw material used for the above components is Al 7075-T651 aluminium alloy. The chemical composition is shown in Table 1 [35], and the mechanical properties are listed in Table 2 [36].

| Table 1 |
|---|

| Table 2 |
|---|

2.2 Results

Two fretting fatigue test configurations are here analysed. One test configuration, including two tested specimens, is characterised by $N=5800N$ (constant normal load), $Q$ with amplitude $Q_a=850N$ (cyclic tangential load, loading ratio $R=-1$), and $\sigma$ with
amplitude $\sigma_a = 50\text{MPa}$ (cyclic axial stress, $R = -1$), resulting in a mean fretting fatigue life equal to $N_{exp} = 627122$ cycles (676704 and 577540 loading cycles to failure for these two tests). The other test configuration, including two tested specimens, is characterised by $N = 5800N$, $Q_a = 1350N$, and $\sigma_a = 70\text{MPa}$, resulting in a mean fretting fatigue life equal to $N_{exp} = 166373$ loading cycles (one test having 167324 cycles and the other 165421 cycles).

Fracture surfaces can be analysed by means of a confocal microscope. Through this technique, it is possible to measure the different cracks appearing in a fracture surface, and thus their corresponding profiles and initiation angles can be obtained. For each crack detected in a fractured surface, a different analysis (i.e. a measure using the confocal microscope) is performed. Each reconstructed crack surface has a resolution of 0.65 $\mu$m in the Oyz-plane, and 2-3 $\mu$m in the x-direction.

Note that, for each of the above test configurations, only one specimen has shown a crack surface suitable to be analysed by confocal microscope. That can be due to the fact that fracture surfaces are damaged by contact between themselves, or the initial part of the crack path is deformed by the pressure produced by the contact pad. The tests which have shown fracture surfaces suitable for confocal microscope analysis are named test No.1 ($N = 5800N$, $Q_a = 850N$, and $\sigma_a = 50\text{MPa}$) and test No.2 ($N = 5800N$, $Q_a = 1350N$, and $\sigma_a = 70\text{MPa}$) in the following.
In Figure 2(a), two different images for the fracture surface corresponding to test No.1 are displayed. In the upper part of this figure, a SEM microscope image clearly shows three fretting cracks (numbered as 1, 2 and 3), and their corresponding surface crack initiation points (marked with a red arrow). Below that, the confocal microscope images belonging to these cracks are plotted, where the colour code represents the distance of the failure surface from a given reference plane parallel to the yz-plane, and the crack initiation point is marked with a white solid circle. The confocal microscope gives us the data (x,y,z cartesian coordinates) of the analysed crack surface. In the present work, these data are processed to create a (3D) mathematical surface represented by a 3D line approximation (spline) that resembles the analysed crack in the best way. In all cases, the experimental (2D) crack paths are obtained from these mathematical surfaces. In order to determine a certain crack path, its corresponding mathematical surface (a 3D spline) is intersected with several planes (see the sectioning planes in Figure 2b), thus producing a set of crack profiles. All these intersection planes (Figure 2(b)) are both perpendicular to the test specimen’s section and passing through the surface point from which the crack initiated (as is observed via confocal microscope). Finally, the resulting crack profiles are averaged, thus producing a mean 2D crack path.

Note that three semi-elliptical cracks have been experimentally observed on the fracture surface for test No.1, where each point of crack initiation is assumed to be coincident with the centre of
each ellipse, whereas only one semi-elliptical crack has been detected for test No.2.

Figure 2.

Now the averaged crack paths obtained for test No.1 and test No.2 are plotted in Figure 3(a) and Figure 3(b), respectively. Such paths are normalised with respect to the theoretical Hertzian contact semi-width, $a = 1.627\text{mm}$. Because the contact stress/strain fields are significant up to a depth equal to about $a$, such crack paths are strongly influenced by the contact load values, $N$ and $Q$.

Note that the crack path measurements are performed up to a depth equal to about 49µm (that is, $y/a \approx 0.03$) because measurements of crack surfaces for a deeper depth would be extremely difficult from a computational point of view.

For two of the three crack paths shown in Figure 3(a) related to test No.1, the crack grows (for the first microns in depth) quite perpendicular to the contact surface. Then the crack profile rotates towards the left-hand side, thus situating below the contact, with an angle (measured from the vertical axis) that ranges between 20° and 28°. A similar fracture behaviour is observed for the crack path related to test No.2 (Figure 3(b)).

A different trend is observed on the third crack path plotted in Figure 3(a): initially growing outside the contact zone, and then (after a few microns) turning towards the opposite direction.
to follow a crack path very similar to that described above, that is to say, turning towards the contact zone with an angle equal to about 22°.

**Figure 3.**

### 3. FRETTING FATIGUE ASSESSMENT: AN ANALYTICAL APPROACH

The approach here presented is suitable for estimating both lifetime and crack path related to metallic components under fretting fatigue loading characterised by a constant amplitude. It is based on the joint application of the multiaxial fatigue criterion by Carpinteri et al. [37,38] and the Point Method (PM) by Taylor [39]. The novelty of the approach here proposed consists in the determination of the critical point $P$ of the specimen, where to apply the Carpinteri et al. criterion in order to perform the fretting fatigue assessment. The position of point $P$ is evaluated by both exploiting the PM (to compute the length of the path) and using a curved path which starts from the hot spot and is normal to an averaged maximum principal stress direction in each of its points, as is detailed in the following Sections.

#### 3.1 Analytical stress field calculation

The experimental test set-up examined (represented by a cylindrical contact characterised by a constant normal load and in-phase tangential and bulk load), the relative sizes between tested specimen and contact pads, and the assumed linear elastic behaviour
of the material ensure that the stress fields in the vicinity of the contact zone can be accurately evaluated through a closed-form analytical solution, by exploiting both the solution by Hertz [40] and that by Mindlin [41], as is detailed in Refs [42,23].

A plane strain condition is assumed. For the fretting fatigue problem being examined, such a condition is verified, from a theoretical point of view, only in the xy-plane shown in Figure 1(b), but it can also be assumed for planes parallel to this, according to the strain behaviour observed for the tested specimens during the experimental campaign.

3.2 Brief description of the Carpinteri et al. criterion

Let us consider the critical point \( P \), lying in the above xy-plane. At a given instant, the principal stress directions 1, 2 and 3 (being \( \sigma_1 \geq \sigma_2 \geq \sigma_3 \)) at point \( P \) can be computed by exploiting the stress tensor expressed with respect to the xyz reference frame (Figure 1(b) and Figure 4). The principal Euler angles \( \phi, \theta \) and \( \psi \), identify the instantaneous reference frame \( P'123 \). Being such angles time-varying, the averaged values \( \hat{\phi}, \hat{\theta} \) and \( \hat{\psi} \) can be computed as follows:

\[
\hat{\phi} = \frac{1}{T} \int_{0}^{T} \phi(t) W(t) dt \\
\hat{\theta} = \frac{1}{T} \int_{0}^{T} \theta(t) W(t) dt \\
\hat{\psi} = \frac{1}{T} \int_{0}^{T} \psi(t) W(t) dt
\]  

(1)

with

\[
W(t) = \begin{cases} 
0, & \sigma_1(t) < \sigma_{1,\text{max}} \\
1, & \sigma_1(t) \geq \sigma_{1,\text{max}}
\end{cases}
\]  

(2)
where $T$ is the period of the cyclic loading, and $\sigma_{1,\text{max}}$ is the maximum value of $\sigma_1$ during $T$. The averaged Euler angles $\hat{\phi}, \hat{\theta}$ and $\hat{\psi}$ identify the averaged reference frame $\hat{\text{P}}\hat{\text{i}}\hat{\text{2}}\hat{\text{3}}$.

The correlation between the above $\hat{1}$-direction and the orientation of the critical plane is expressed by the off-angle $\delta$:

$$\delta = \frac{3\pi}{8} \left[ 1 - \left( \frac{\tau_{af,-1}}{\sigma_{af,-1}} \right)^2 \right]$$

(3)

where $\sigma_{af,-1}$ is the material fatigue strength under fully reversed normal stress at $N_0$ loading cycles, whereas $\tau_{af,-1}$ is the material fatigue strength under fully reversed shear stress at $N_0^*$ loading cycles. In more detail, $\delta$ defines the normal $w$ to the critical plane (Figure 4).

**Figure 4.**

By examining the experimental results presented in Section 2.2, the following assumption can be made: $w$-axis has to lay in the xy-plane. Therefore, the rotation $\delta$ is performed from $\hat{1}$-axis to $\hat{3}$-axis when $\hat{\sigma}_2 = 0$, whereas it is performed from $\hat{1}$-axis to $\hat{2}$-axis when $\hat{\sigma}_3 = 0$.

Let us consider the stress vector $\mathbf{N}$, normal to the critical plane, and the stress vector $\mathbf{C}$, lying on the critical plane. Being $\mathbf{N}$ a vector with a periodic modulus and a fixed direction, the mean value $N_m$ and the amplitude $N_a$ can be easily computed. Instead, since $\mathbf{C}$ is a vector with a time-varying direction, the amplitude
$C_a$ is computed according to the method proposed by Araujo et al. [43].

The number of loading cycles to failure, $N_f$, can be calculated by solving, in an iterative way, the expression here reported:

$$
\sqrt{N_{eq,a}^2 + \left(\frac{\sigma_{af,-1}}{\tau_{af,-1}}\right)^2 \left(\frac{N_j}{N_0}\right)^{2/k} \left(\frac{N_0^*/N_f}{N_0\times100}\right)^{2/k} C_a^2} = \sigma_{af,-1} \left(\frac{N_f}{N_0}\right)^{-1/k}
$$

(4)

where $k$ is the inverse slope of the S-N curve under fully reversed normal stress, $k^*$ is the inverse slope of the S-N curve under fully reversed shear stress, and $N_{eq,a}$ is given by

$$
N_{eq,a} = N_a + \sigma_{af,-1} \left(\frac{N_m}{\sigma_u}\right)
$$

(5)

being $\sigma_u$ the ultimate tensile strength of the material.

3.3 Definition of crack path and critical point $P$

Let us consider the hot spot $H$, lying on the contact surface (i.e. $-a \leq x \leq +a$ in Figure 4). Such a hot spot is assumed as the point where $\sigma_1$ attains its maximum value during the period. For the contact configuration being examined, $H$ coincides with the trailing edge (i.e. $x=+a$ in Figure 4).

The path proposed to compute the position of the critical point $P$ is a curve lying in the $xy$-plane, starting from the above hot spot and normal to the $\hat{1}$-direction in each of its points (Figure 4).

According to the Point Method by Taylor [39], such a path has a length equal to half of the El-Haddad intrinsic crack length $L$:

$$
L = \frac{1}{\pi} \left(\frac{\Delta K_{I,th}}{\Delta \sigma_{af}}\right)^2
$$

(6)
where $\Delta K_{I,th}$ is the threshold range of the stress intensity factor for long cracks, and $\Delta \sigma_{af}$ is the fatigue limit range.

The point $P$ coincides with the end of the above path, which is also assumed as the crack path.

3.4 Results in terms of crack path

The above approach is used to estimate the crack paths for the experimental tests No.1 and No.2, respectively.

The following parameters have to be set. The coefficient of friction is equal to $\mu = 0.72$ according to experimental tests. The fatigue materials parameters, needed to apply such an approach, have not been measured during the experimental campaign, and therefore they are taken from the literature: in more detail, the parameters $\sigma_{af,-1}$ and $k$ are taken from Ref.[44], where they are equal to $169.15 \text{MPa} \ (N_0 = 1(10)^6)$ and $-0.1553$, respectively. To the best knowledge of the authors, experimental data related to S–N curve for torsion are not available in the literature for the material being examined, and hence we assume $\tau_{af,-1} = \sigma_{af,-1} / \sqrt{3} \ (N_0^* = 1(10)^6)$ and $k^* = k = -0.1553$.

The threshold of the stress intensity factor range for long cracks and loading ratio $R$ quite close to zero is taken from Ref.[45], i.e. $\Delta K_{I,th} = 2.1 \text{MPa}\sqrt{m}$. The fatigue limit corresponding to such a loading ratio is assumed equal to half of $\sigma_{af,-1}$ and,
therefore, \( \Delta \sigma_{\text{ef}} = 169.15 \text{MPa} \). According to Eq.(6), the calculated El-Haddad intrinsic crack length is equal to \( L = 0.0491 \text{mm} \).

Figure 5(a) shows the estimated crack path for test No.1 together with three experimental crack paths. It can be observed that the estimated crack path is initially perpendicular to the contacting surfaces (for \( y/a \leq 0.005 \)); then it turns towards the contact zone with an inclination angle equal to about 3° (angle measured with respect to line perpendicular to the contact surface).

Analogously, Figure 5(b) shows the estimated crack path for test No.2 together with the experimental crack path. It can be observed that such an estimated path is very similar to the previous one.

Note that further investigation of the crack path beyond \( L/2 = 0.02455 \text{mm} \) would not provide additional information useful to determine the critical point location. As a matter of fact, a crack path with a length greater than \( L/2 \) would violate one of the assumptions of the proposed approach, that is, the PM assumption.

3.5 Results in terms of fatigue life

For both tests (No.1 and No.2), point \( P \) has the same coordinates in the \( xy \) reference frame, equal to \( (0.9995; -0.0151) \) expressed in mm.
The fatigue life is equal to 410447 and 82892 loading cycles for test No.1 and 2, respectively. By comparing such results with the experimental ones, it can be observed that the theoretical results are conservative and falling in scatter band 2.

4. FRETting FATigUE AsSESSMent: T wo nUMERICAL aPPROACHES

4.1 SIF calculation by means the FEM

In order to compute both the contact stress/strain fields and the Mode I and Mode II stress intensity factors, a finite element method (FEM) model has been used. This model is able to take into account the mechanical contact between crack faces. With this capability, it is possible to model more accurately the crack opening/shear stress distribution and, thus, to evaluate more accurate values of the Mode I and II SIFs.

The FEM software ABAQUS 2017 is employed to construct the model. It is experimentally found that fretting cracks initiate along the slip zones, especially close the contact trailing edge, [32], i.e. at surface points with \( x = a \) and \(-3.5\,\text{mm} \leq z \leq 3.5\,\text{mm}\) (Figure 1(b)). In such a situation, a plane (bi-dimensional) FEM model (plane strain behaviour) is appropriate for the present analysis. Figure 6(a) depicts a scheme of the FEM model and its boundary conditions. Due to symmetry conditions, only a half of the system is modelled. Symmetry conditions are imposed on the xz-plane. Moreover, the points of the pad lying on the yz-plane have their x-displacement suppressed. This is possible because there is no external restriction upon the x-displacement of the specimen, being
such a displacement restricted in this direction only by the friction with the pad. In this model, the test specimen is subjected to three different actions: a constant normal load, \( N \), a cyclic shear load, \( Q \), and a cyclic bulk stress, \( \sigma \). The normal load is applied on the upper face of the pad as a pressure. On the other hand, the extreme sections of the test specimen are subjected to two different pressures, so that the difference between their resultants is the shear load \( Q \): a pressure equal to \( \sigma \) is applied to one end of the specimen, and pressure on the other end is equal to \( \sigma - Q/W \), being \( W \) the specimen cross-section area.

Regarding to the FEM meshes, in all cases they have about 500,000 degrees of freedom. Further, the element size is small enough (less than 2 \( \mu \)m) especially at the contact zone and crack tip, to precisely capture the high strain/stress gradients in those zones. An image of the global mesh and a zoom of the cracked zone are shown in Figure 6(b). Previous analyses performed on uncracked models showed that the maximum von Mises equivalent stress was lower than the material yield stress (503 MPa), thus endorsing the use of a linear-elastic material behaviour. In Figure 6(c), a contourplot for the von Mises stress computed through an uncracked FEM model is depicted. CPE4R finite elements of the ABAQUS code are used in the contact zones, while CPE8R finite elements are employed as far as the crack growth is concerned. Linear elements are used to model the mechanical contact because, as is well-known, quadratic elements give poor results in contact problems. Between the linear element zone and the quadratic element zone, within the
examined specimen, a tie contact constraint has been imposed. The element sides that touch the crack tip have their middle nodes at a distance equal to a quarter of the side length from the tip so that the solution be able to reproduce the singularity. Because of the huge number of elements and nodes, reduced integration elements have been used. The Lagrange multipliers algorithm with a friction coefficient $\mu=0.72$ is employed to model the mechanical contact, according to authors previous experimental work [46]. Regarding the reliability of the results, especially those concerning the mechanical contact, the accuracy of the mesh and the convergence of results are guaranteed by the good reproduction of some theoretical contact elastic solutions. The Mode I and II SIFs are evaluated computing the J-integral. As is remarked above, the convergence of the SIF evaluation is obtained adopting a small enough size for the finite elements surrounding the crack tip. Further, 10 J-integral contour elements have been used to make the SIF evaluation less sensitive to mesh issues, and averaged values for mode I and mode II SIFs have been obtained.

**Figure 6.**

Finally, in order to simulate the crack growth process, the model is re-meshed after each new crack increment $l_{inc}$ by employing a user subroutine. In all FEM crack growth simulations, and between consecutive steps, the crack grows by a fixed length, $l_{inc}$. This crack length increment has the minimum value for which no significant difference has been found in the estimated crack path.
4.2 Description of the maximum $\Delta K_I^*$ approach

The first numerical approach herein used to estimate the crack path is based on the range of the Mode I SIF, $\Delta K_{I}^*$, for an infinitesimal kinked crack emanating from the actual crack (Figure 7(a)). According to Ref.[47], $\Delta K_{I}^*$ is obtained as follows:

$$
\begin{bmatrix}
\Delta K_{I}^* \\
\Delta K_{II}^*
\end{bmatrix} =
\begin{bmatrix}
\frac{180-\theta}{180+\theta}^{\frac{\theta}{360}} \cos \theta - \frac{1}{2\pi} \sin \alpha & -\frac{3}{2} \frac{180-\theta}{180+\theta}^{\frac{\theta}{360}} \sin \theta \\
\frac{180-\theta}{180+\theta}^{\frac{\theta}{360}} \sin \theta & \frac{180-\theta}{180+\theta}^{\frac{\theta}{360}} \cos \theta + \frac{1}{2\pi} \sin \alpha
\end{bmatrix}
\begin{bmatrix}
\Delta K_{I} \\
\Delta K_{II}
\end{bmatrix}
$$

(7)

where

$$
\alpha = \theta \cdot \ln \left( \frac{180-\theta}{180+\theta} \right) - \frac{360\theta^2}{180^2 - \theta^2}
$$

(8)

Further, $\Delta K_{I}$ and $\Delta K_{II}$ are the Mode I and Mode II SIF ranges of the actual crack, respectively, and $\theta$ is the kinking angle.

Figure 7.

In all simulations, the initial crack has length $l_{init}=8\,\mu m$, is perpendicular to surface, and emanates from the contact trailing edge. The values of $\Delta K_{I}$ and $\Delta K_{II}$ for this initial crack are computed and then, by using Eqs. (7) and (8), the kinking angle that maximises $\Delta K_{I}^*$ is found: this angle is named $\theta_{\text{max}}$ (Figure 7(b)).

In next step, the FEM model is re-meshed in order to take into account that the crack length is grown of the increment $l_{inc}$ in the
direction dictated by $\theta_{\text{max}}$. New values for $\Delta K_I$ and $\Delta K_{II}$ are computed (Figure 7(c)), and then a new search for the kinking angle which maximises $\Delta K_I'$ is done. The actual crack is made to grow of $l_{\text{inc}}$ in $\theta_{\text{max}}$ direction (Figure 7(d)). Repeating this procedure again and again, the crack path is obtained.

4.3 Description of the maximum SWT approach

The second numerical approach to predict the crack path is similar to that described in the previous Sub-section but, in the present procedure, the angle of the crack increment between consecutive steps is provided by that direction that, in front of the crack tip, maximises the SWT (Smith-Watson-Topper) parameter. Such a parameter is expressed as follows:

$$SWT = \max \left( \sigma_{\text{max}} \frac{\Delta \varepsilon}{2} \right),$$

where $\sigma_{\text{max}}$ is the maximum normal stress and $\Delta \varepsilon/2$ is the range of the normal strain, for a given material plane and point. Among all the possible values of SWT (one per each material plane considered), only the maximum value is taken into account.

It is noteworthy to mention that, when using a FEM-based approach for examining fractured components, two drawbacks can arise. Firstly, at points located very close to the crack tip, the computed stress and strains fields are extremely dependent on the mesh quality (shape and sizes of the finite elements), and thus the SWT values computed at those points. Secondly, fixing a certain direction in front of the crack tip, it is found that the
stress/strain values vary along that direction, and thus the computed SWT values.

The first drawback can be significantly minimised if the very near crack tip fields are calculated by means of the stress intensity factor ranges \( \Delta K_I \) and \( \Delta K_{II} \) computed through the FEM. Nevertheless, note that the FEM computed stress intensity factor ranges are also affected by the mesh quality, although in a much less degree than for the stress-strain fields. By assuming a plane strain behaviour, the asymptotic stress field ahead of the crack tip is defined by:

\[
\sigma_{rr}(r,\alpha) = \frac{K_I}{\sqrt{2\pi r}} \left( \frac{5}{4} \cos \frac{\alpha}{2} - \frac{1}{4} \cos \frac{3\alpha}{2} \right) + \frac{K_{II}}{\sqrt{2\pi r}} \left( -\frac{5}{4} \sin \frac{\alpha}{2} + \frac{3}{4} \sin \frac{3\alpha}{2} \right) \tag{10a}
\]

\[
\sigma_{\alpha\alpha}(r,\alpha) = \frac{K_I}{\sqrt{2\pi r}} \left( \frac{3}{4} \cos \frac{\alpha}{2} + \frac{1}{4} \cos \frac{3\alpha}{2} \right) + \frac{K_{II}}{\sqrt{2\pi r}} \left( -\frac{3}{4} \sin \frac{\alpha}{2} - \frac{3}{4} \sin \frac{3\alpha}{2} \right) \tag{10b}
\]

\[
\sigma_{r\alpha}(r,\alpha) = \frac{K_I}{\sqrt{2\pi r}} \left( \frac{1}{4} \cos \frac{\alpha}{2} + \frac{1}{4} \cos \frac{3\alpha}{2} \right) + \frac{K_{II}}{\sqrt{2\pi r}} \left( \frac{1}{4} \sin \frac{\alpha}{2} + \frac{3}{4} \sin \frac{3\alpha}{2} \right) \tag{10c}
\]

\[
\sigma_{zz}(r,\alpha) = \frac{2\nu(K_I - K_{II})}{\sqrt{2\pi r}} \cos \frac{\alpha}{2} \tag{10d}
\]

where \( \alpha \) and \( r \) are the relative polar coordinates at crack tip. Using the Hooke’s law and Eqs 10(a)-(d), the asymptotic strain field can be obtained (not shown here for the sake of brevity).

It is straightforward to see that, for a given orientation (defined by a certain value of \( \alpha \)), the stress field defined by Eqs 10(a)-(d) only varies according to \( 1/\sqrt{r} \) and, thus, any value of \( r \) can be considered in order to search that material plane (defined by \( \alpha \)) which maximises the SWT parameter. However, to fulfil the asymptotic behaviour according to Eqs 10(a)-(d), the crack length \( l \)
must satisfy the condition \( r \ll l \). The possibility to use any value of \( r \) eliminates the second drawback exposed above. Analogous to the previous numerical approach, the crack path can be obtained repeating the procedure again and again.

### 4.4 Results in terms of crack path

Now, by applying the above numerical approaches up to a crack depth equal to 155\( \mu \)m, the crack initiation paths for tests No.1 and No.2 are evaluated.

Such a crack depth value is dictated by the following two reasons: (i) the CPU time required for one simulation with final crack depth equal to 155\( \mu \)m is about 4 days; (ii) for too long cracks, the contact algorithm used causes some instabilities in the solution process, and a non-convergence of the FEM model occurs. In other words, the crack depth of 155\( \mu \)m does not correspond to the unstable crack propagation condition given by \( K_I = K_{IC} \) (the material toughness being equal to \( 20 \text{MPa}\sqrt{m} \) according to Ref.[48]), but it is only due to the two reasons above.

*Figure* 8(a) shows the estimated paths for test No.1. In addition, the experimental crack paths are also shown for comparison purposes. On the right-hand side of *Figure* 8(a), a zoom of the area involving the experimental crack paths is displayed. This zoom shows that both approaches produce very similar crack paths that, in some manner, tend to reproduce the experimental ones: initially (that is, \( y \geq 32 \mu m \)) the estimated crack paths are more or less perpendicular to the surface, and then rotate towards
the left, thus situating below the contact. Far from the zoomed area (that is, \( y < -50\mu m \)), both estimated crack paths follow the same trend, although the SWT approach evaluates a less inclined path. In more detail, the inclination angle (measured from the vertical axis) is about 16° according to the \( \Delta K_i^* \) approach, whereas it is equal to about 9° according to the SWT approach. Finally, it is noteworthy that the predicted crack paths are almost straight lines in their final part.

Analogously, the estimated crack paths for test No.2 are plotted in Figure 8(b). The paths show trends very similar to those for test No.1, that is to say: they are quite perpendicular to the contact surface for the first microns; then they turn towards the contact zone following a straight path, with inclination angles of 16° and 11° for the \( \Delta K_i^* \) and SWT approach, respectively. It can be observed that the estimated crack paths reproduce the experimental ones with a good agreement.

4.5 Results in terms of fatigue life

By using the results coming from the above numerical simulations, fatigue life can be evaluated, but unfortunately the crack propagation phase can only be estimated up to a crack depth of 155\( \mu m \), as is discussed above. Due to this limitation, the predicted fretting fatigue life is computed in the following manner: first of all, the crack initiation life \( N_{\text{init}} \) is calculated according to the procedure proposed in Ref.[49], by taking into account a fixed crack initiation length equal to 8\( \mu m \). The Fatemi-
Socie multiaxial fatigue parameter is adopted to compute the crack initiation life \([50]\). Then, the crack is made to grow up to a depth of \(155 \mu m\).

When the crack grows again, a further propagation phase has to be analysed. Therefore, by considering that the initial crack length is of the material microstructure order and by recalling estimations showing that cracks tend to follow paths controlled by the crack opening stress, the following crack growth law is here applied to model the short-crack growth phase \([51]\):

\[
\frac{da}{dN} = C \left( \Delta K_I^m - \left( \Delta K_{I,th} \left( \frac{l_f}{l_f + L_f - l_0} \right) \right)^{m_f} \right)
\] (11)

where \(l\) is the crack length, \(\Delta K_I\) is the range of the Mode I stress intensity factor for a crack having length \(l\), \(f\) is a parameter equal to 2.5 \([52]\), \(\Delta K_{I,th}\) is the Mode I threshold SIF range for long cracks (\(\Delta K_{I,th} = 2.1 MPa\sqrt{m}\) for \(R=0.0\), as has been mentioned previously), \(l_0\) is the distance between the surface and the first microstructural barrier, i.e. the first grain boundary (\(l_0\) is here assumed to be equal to half of the average material grain size reported in Ref.\([45]\) for the examined material, i.e. \(l_0 = 25 \mu m\)), \(C = 8.83 (10)^{-11}\) and \(m = 0.332\) are the Paris law parameters (loading ratio \(R\) equal to 0.1), and \(L\) is the El-Haddad intrinsic crack length, equal to 0.0491mm according to Eq.\((6)\) for \(\Delta \sigma_{ij} = 169.15 MPa\) and \(\Delta K_{I,th} = 2.1 MPa\sqrt{m}\) (loading ratio \(R\) equal to 0.0). Note that the threshold SIF range in Eq.\((11)\) is modified taking into account both
the actual crack length, \( l \), and the microstructural material length, \( l_0 \). According to Eq.(11), a greater crack growth rate is obtained especially for short cracks if, maintaining constant all the other parameters of the equation, the material grain size decreases.

The experimental fretting fatigue lives are listed in **Table 3** for both tests. Further, \( N_{ \text{prop-155\mu m}}(\Delta K^*_I) \) and \( N_{ \text{prop-155\mu m}}(\text{SWT}) \) are the estimated fatigue propagation lives computed through the \( \Delta K^*_I \) approach and the SWT approach, respectively, whereas the total fatigue lives for a crack depth of 155\( \mu \)m are \( N_{155\mu m}(\Delta K^*_I) = N_{\text{init}} + N_{ \text{prop-155\mu m}}(\Delta K^*_I) \) and \( N_{155\mu m}(\text{SWT}) = N_{\text{init}} + N_{ \text{prop-155\mu m}}(\text{SWT}) \).

As was expected, the estimated fatigue lives \( N_{155\mu m}(\Delta K^*_I) \) and \( N_{155\mu m}(\text{SWT}) \) are far from the experimental ones, since the estimated lives are evaluated up to a crack depth equal to 155\( \mu \)m and hence are conservative. As a matter of fact, the unstable crack propagation condition \( K_i = K_{IC} \) is reached for a crack depth equal to 3.5mm.

Therefore, a further propagation up to the final crack length equal to 3.5mm has to be examined. In order to perform such an analysis by avoiding too long computational times and convergence problems, the Mode I stress intensity factor is computed by using the weight function method. In more detail, the Bueckner's unidimensional weight function [53] is employed:

\[
\Delta K_I(l) = \int_0^l w(s) \Delta \sigma(s) ds \tag{12}
\]
where $w(s)$ is the weight function, $\Delta \sigma(s)$ is the range of the crack opening stress distribution along the prospective crack faces, and $l$ is the crack length.

For the case being examined, $\Delta \sigma(s)$ is the stress range during a fretting fatigue cycle (only the positive portion of the stress range is taken into account here), and $l$ is made to vary from $155 \mu m$ to $3.5 mm$. In such a propagation phase, the crack is assumed to grow perpendicular to the contact surface, that is, in the $y$-direction.

The estimated total fretting fatigue lives for a crack depth equal to $3.5 mm$, named $N_{3.5 mm}(\Delta K^*_I)$ and $N_{3.5 mm}(SWT)$, are listed in Table 3. Such values, very close to those previously determined considering a 2D model [49], are falling in scatter bands 3 and 5 for test No.1 and test No.2, respectively.

Note that the computed Mode I stress intensity factors are related to a through-thickness crack (2D model), while the crack has a semi-elliptical shape according to the experimental observations, at least for the first hundreds of microns. Therefore, lower values of the Mode I SIFs would be determined by taking into account semi-elliptical cracks, and thus longer crack propagation lives with respect to those obtained here.

The reasons because a semi-elliptical crack is not taken into account in the present work are twofold. From one hand, a 2D FEM model is examined and, therefore, such a crack geometry cannot be physically introduced in the simulations. From the other hand, although it would be possible to introduce some geometric factor to
the 2D through-the-thickness crack SIF in order to take into account the actual semi-elliptical crack shape, it would not be clear how to apply the above geometric factor to a crack with a not-straight path.

5. CONCLUSIONS

In the present paper, three different approaches (one analytical and two numerical approaches) have been proposed to estimate both the initial crack path and fatigue life under fretting fatigue loading. The analytical approach proposed has the advantage to be able to estimate both the crack path and fatigue life without physically including the crack in the model, and thus is computationally efficient, but it has the drawback that both the fretting fatigue life and crack path estimations are referred to a verification point located at a fixed distance from the contact surface. The two numerical approaches physically include the crack in their steps, that is extremely expensive in terms of computation times, but both the crack path and fatigue life can be evaluated for cracks longer than those in the case of the above analytical approach.

Damage mechanisms of the analytical approach are different from those of the numerical approach, although nearly similar results in terms of crack initiation path have been obtained. More precisely, according to the analytical approach the fatigue limit condition is assumed to occur in the specimen, subjected to far-field multiaxial
loading, when the amplitude of the equivalent normal stress in a point at a certain distance from the hot spot attains the normal stress fatigue limit for plain specimen. On the other hand, the maximum $\Delta K^*$ approach uses a classical elastic fracture mechanics parameter (the mode I stress intensity factor) to estimate the crack path. Although the maximum SWT approach does not employ the SIF directly as a predictor, it uses the SIF to compute the stress field ahead of the crack tip and, by exploiting such a stress field, the direction with the highest value of the SWT parameter is computed.

As far as the experimental tests here discussed are concerned, the estimated crack paths are in quite satisfactory agreement with the experimental ones, especially in the material zone close to the contact surface. The estimated fretting fatigue lives are conservative in all cases examined, falling in the scatter band 2 when applying the analytical approach and in the scatter band 5 when applying the numerical approaches.

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REFERENCES

[1] T. Lindley, Fretting fatigue in engineering alloys, Int. J. Fatigue 19 (1997), pp.39-49.

[2] C. Ruiz, P. Boddington, K. Chen, An investigation of fatigue and fretting in a dovetail joint, Exp. Mech. 24 (1984), pp.208-217.
[3] D. Hills, D. Nowell, Mechanics of fretting-fatigue, Kluwer Academic Publishers, Dordrecht (1994).
[4] D. Nowell, D. Dini, D.A. Hills, Recent developments in the understanding of fretting fatigue, Eng. Fract. Mech. 73 (2006), pp.207-222.
[5] M. Kubota, K. Hirakawa, The effect of rubber contact on the fretting fatigue strength of railway wheel tire, Tribology International 42 (2009), pp. 1389-1398.
[6] S.J. Choi, Y.T. Cho, Fretting fatigue behavior in railway axle materials, J. of Mechanical Science and Technology 29 (2015) 23-31.
[7] D. Zenga, Y. Zhang, L. Lu, L. Zou, S. Zhua, Fretting wear and fatigue in press-fitted railway axle: A simulation study of the influence of stress relief groove, Int. J. Fatigue 118 (2019), pp.225-236.
[8] C. Ruiz, P.H.B. Boddington, K.C. Chen, An investigation of fatigue and fretting in a dovetail joint, Experimental Mechanics 24 (1984), pp.208-217.
[9] J. Murugesan, Y. Mutoh, Fretting fatigue strength prediction of dovetail joint and bolted joint by using the generalized tangential stress range-compressive stress range diagram, Tribology International 76 (2014), pp.116-121.
[10] Y. Su, Q.-N. Han, C.-C. Zhang, H.-J. Shi, L.-S. Niu, G.-J. Deng, S.-S. Rui, Effects of secondary orientation and temperature on the fretting fatigue behaviors of Ni-based single crystal superalloys, Tribology International 130 (2019), pp.9-18.
[11] M.H. Attia, Fretting fatigue and wear damage of structural components in nuclear power stations—Fitness for service and life management perspective, Tribology International 39 (2006), pp.1294-1304.
[12] F. Xue, Z.-X. Wang, W.-S. Zhao, X.-L. Zhang, Bao-Ping Qu, L. Wei, Fretting fatigue crack analysis of the turbine blade from nuclear power plant, Engng Failure Analysis 44 (2014), pp.299-305.
[13] H.-K. Jeung, J.-D. Kwon, C.Y. Lee, Crack initiation and propagation under fretting fatigue of inconel 600 alloy, Journal of Mechanical Science and Technology 29 (2015), pp.5241-5244.
[14] M.H. Attia, R.B. Waterhouse (editors). Standardization of fretting fatigue test methods and equipment. Philadelphia, PA: ASTM STP 1159, American Society of Testing and Materials (1992).

[15] D.B. Rayaprolu, R. Cook. A critical review of fretting fatigue investigations at the Royal Aerospace Establishment. In: Standardization of fretting fatigue test methods and equipment. Edited by M. Helmi, M.H. Attia, B. Waterhouse. Philadelphia, USA: ASTM STP 1159, American Society for Testing and Materials, (1992), pp. 129-152.

[16] M.P. Szolwinski, T.N. Farris, Observation, analysis and prediction of fretting fatigue in 2024-T351 alluminum alloy, Wear 221 (1998), pp.24-36.

[17] A.L. Hutson, T. Nicholas, R. Goodman, Fretting fatigue of Ti-6Al-4V under flat-on-flat contact, Int. J. Fatigue 21 (1999), pp.663-669.

[18] B. Ferrya, J.A. Araújoa, S. Pommierb, K. Demmou, Life of a Ti-6Al-4V alloy under fretting fatigue: Study of new nonlocal parameters, Tribology International 108 (2017), pp.23-31.

[19] N.A. Bhatti, M. Abdel Wahab, Fretting fatigue crack nucleation: A review, Tribology International 121 (2018) pp.121-138.

[20] Q.-N. Han, W. Qiu, Z. He, Y. Su, X. Ma, H.-J.Shi, The effect of crystal orientation on fretting fatigue crack formation in Ni-based single-crystal superalloys: In-situ SEM observation and crystal plasticity finite element simulation, Tribology International 125 (2018), pp. 209-219.

[21] J.A. Araújo, L. Susmel, D. Taylor, J. Ferro, J. Ferreira, On the prediction of high cycle fretting fatigue strength: theory of critical distances vs. hot-spot approach, Eng. Fract. Mech. 75 (2008), pp.1763-1778.

[22] C. Ronchei, A. Carpinteri, G. Fortese, D. Scorza, S. Vantadori, Fretting high-cycle fatigue assessment through a multiaxial critical plane-based criterion in conjunction with the Taylor’s point method. Solid State Phenomena 258 (2016), pp 217-220.
[23] S. Vantadori, G. Fortese, C. Ronchei, D. Scorza, A stress gradient approach for fretting fatigue assessment of metallic structural components, Int. J. Fatigue 101 (2017), pp.1-8.
[24] S. Vantadori, J.A. Araújo, G.M. Juvenal Almeida, G. Fortese, G.C. Veras Pessoa, Early fretting crack orientation by using the critical plane approach, Int. J. Fatigue 114 (2018), pp. 282-288.
[25] R. Ferré, S. Fouvry, B. Berthel, J.-A. Ruiz-Sabariego, Stress gradient effect on the crack nucleation process of a Ti-6Al-4V titanium alloy under fretting loading: comparison between nonlocal fatigue approaches, Int. J. Fatigue 54 (2013), pp.56-67.
[26] S. Fouvry, H. Gallien, B. Berthel, From uni- to multi-axial fretting fatigue crack nucleation: development of a stress-gradient dependent critical distance approach, Int. J. Fatigue 62 (2014), pp.194-209.
[27] J.A. Araújo, D. Nowell, Analysis of pad size effects in fretting fatigue using short crack arrest methodologies, Int. J. Fatigue 21 (1999), pp.947-956.
[28] D. Dini, D. Nowell, I.N. Dyson, The use of notch and short crack approaches to fretting fatigue threshold prediction: theory and experimental validation, Tribology International 39 (2006), pp.1158-1165.
[29] S. Fouvry, D. Nowell, K. Kubiak, D. Hills, Prediction of fretting crack propagation based on a short crack methodology. Eng. Fract. Mech. 75 (2008), pp.1605-1622.
[30] C. Navarro, J. Vázquez, J. Domínguez, A general model to estimate life in notches and fretting fatigue, Int. J. Fatigue 78 (2011), pp.1590-1601.
[31] C. Navarro, J. Vázquez, J. Domínguez, Nucleation and early crack path in fretting fatigue, Int. J. Fatigue 100 (2017), pp.602-610.
[32] J. Vázquez, C. Navarro, J. Domínguez, Analysis of fretting fatigue initial crack path in Al7075-T651 using cylindrical contact, Tribology International 108 (2017), pp.87-94.
[33] J.A. Collins, Fretting-fatigue damage-factor determination, J Eng Indust. 87 (1965), pp.298-302.
[34] S.S. Loen, Berto F., B. Haugen, Predicting fretting fatigue in engineering design, Int. J. Fatigue 117 (2018), pp.314-326.

[35] Alloy 7075 Plate and Sheet: Alcoa Mill Products: SPD-10-037 (2001).

[36] C. Boller, T. Seeger, Materials data for cyclic loading, Elsevier Publishing Company (1998).

[37] A. Carpinteri, A. Spagnoli, S. Vantadori, Multiaxial fatigue assessment using a simplified critical plane-based criterion. Int. J. Fatigue 33 (2011), pp.969-976.

[38] A. Carpinteri, C. Ronchei, D. Scorza, S. Vantadori, Critical plane orientation influence on multiaxial high-cycle fatigue assessment. Phys Mesomech 18 (2015), pp.348-354.

[39] D. Taylor, The theory of critical distances: a new perspective in fracture mechanics, Elsevier, Oxford, UK (2007).

[40] K.L. Johnson, Contact mechanics, Cambridge University Press, Cambridge, UK (1985).

[41] R.D. Mindlin, Compliance of elastic bodies in contact. J. Appl. Mech. 16 (1949), pp.259-268.

[42] G. Fortese, F. Berto, A. Carpinteri, C. Ronchei, D. Scorza, S. Vantadori, Analysis of crack initiation under fretting fatigue loading. Proceedings of the International Symposium on Notch Fracture (ISNF), Santander, Cantabria, Spain (2017).

[43] J. Araújo, A. Carpinteri, C. Ronchei, A. Spagnoli, S. Vantadori, An alternative definition of the shear stress amplitude based on the maximum rectangular hull method and application to the C-S (Carpinteri-Spagnoli) criterion. Fatigue Fract. Eng. Mater. Struct. 37 (2014), pp.764-771.

[44] C. Navarro, S. Muñoz, J. Domínguez, Fracture mechanics approach to fretting fatigue behaviour of coated aluminium alloy components. The Journal of Strain Analysis for Engineering Design 49 (2014), pp.66-75.

[45] C. Navarro, J. Vázquez, J. Domínguez, 3D vs. 2D fatigue crack initiation and propagation in notched plates, Int. J. Fat. 58 (2014), pp.40-46.
[46] Tam R et al. Caracterización de una máquina para realizar ensayos de fretting fatiga con contacto cilíndrico. Anales de la mecánica de la fractura 28 (2011), pp.323-328.

[47] R. Ribeaucourt, M.C. Baietto-Dubourg, A. Gravouil, A new fatigue frictional contact crack propagation model with the coupled X-FEM/LATIN method, Comput. Methods Appl. Mech. Engrg. 196 (2007), pp.3230-3247.

[48] Metals Handbook, Vol.2 - Properties and Selection: Nonferrous Alloys and Special-Purpose Materials, ASM International 10th Ed. 1990.

[49] J. Vázquez, C. Navarro, J. Domínguez, Two dimensional versus three dimensional modelling in fretting fatigue life prediction. J. Strain Anal. 51 (2016), pp.109-117.

[50] A. Fatemi, D.A. Socie, Critical plane approach to multiaxial fatigue damage including out-of-phase loading. Fatigue Fract. Eng. Mater. Struct. 11 (1998), pp.145-165.

[51] S. Muñoz, C. Navarro, J. Domínguez, Application of fracture mechanics to estimate fretting fatigue endurance curves. Eng. Fract. Mech. 74 (2007), pp.2168-2186.

[52] C. Vallellano, J. Domínguez, A. Navarro, On the estimation of fatigue failure under fretting conditions using notch methodologies. Fatigue Fract. Eng. Mater. Struct. 26 (2003), pp.469-478.

[53] H.J. Bueckner, G.C. Sih (eds), Methods of analysis and solutions of crack problems, Noordhoff International Publishing, Leyden (1973).
Experimental, analytical and numerical fretting fatigue investigation on Al 7075-T651 alloy

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FIGURES AND TABLES
Figure 1. Experimental setup of the fretting fatigue tests: (a) fretting fatigue test device, (b) test specimen (sizes in mm).
### Table 1. Chemical composition for the Al 7075-T651 alloy [35].

| %   | Zn | Mg | Cu | Fe | Si | Mn | Cr | Ti | Others |
|-----|----|----|----|----|----|----|----|----|--------|
| Max | 6.1| 2.9| 2.0| 0.5| 0.4| 0.3| 0.28| 0.2| 0.05   |
| Min | 5.1| 2.1| 1.2|--|--|--| 0.18|--|--      |

### Table 2. Mechanical properties for the Al 7075-T651 alloy [36].

| PROPERTY            | $E$    | 71·$10^3$ MPa |
|---------------------|--------|---------------|
| Young’s modulus     | $E$    | 71·$10^3$ MPa |
| Poisson’s ratio     | $\nu$  | 0.33          |
| Yield strength      | $\sigma_y$ | 503 MPa      |
| Tensile strength    | $\sigma_u$ | 572 MPa      |
Figure 2. Test No.1: (a) SEM image of the fracture surface for test No. 1 and the contour maps, obtained via confocal microscope, for the three cracks analysed, (b) schematisation of the procedure employed in order to obtain the experimental crack path.
Figure 3. Experimentally observed fretting fatigue crack paths: (a) test No.1, (b) test No.2.
Figure 4. Schematisation of the procedure employed in order to obtain the analytical crack path.
Figure 5. Estimated crack path by using the analytical approach for (a) test No. 1, (b) test No. 2. Experimental crack paths are also shown.
Figure 6. FEM model used: (a) scheme of the FEM model showing the boundary and loading conditions, (b) global view of the mesh and a zoom of the crack zone, (c) contour plot of the von Mises stress using an uncracked FEM model (stresses in MPa).
Figure 7. Steps in the maximum $\Delta K^*$ crack growth procedure.
Figure 8. Estimated crack path by using the numerical approaches for: (a) test No. 1, (b) test No. 2.
Table 3. Fretting fatigue life predictions (loading cycles to failure) according to two numerical approaches. The averaged experimental fatigue life values $N_{exp}$ are also reported.

| No. of CYCLES          | TEST No. 1 | TEST No. 2 |
|------------------------|------------|------------|
| $N_{prop-155\mu m} (\Delta K_I^*)$ | 15030      | 6008       |
| $N_{prop-155\mu m} (SWT)$       | 22805      | 7650       |
| $N_{155\mu m} (\Delta K_I^*)$   | 91040      | 7204       |
| $N_{155\mu m} (SWT)$           | 98815      | 8846       |
| $N_{3.5\mu m} (\Delta K_I^*)$  | 224866     | 30797      |
| $N_{3.5\mu m} (SWT)$           | 232641     | 32439      |
| $N_{exp}$ (Averaged)           | 627122     | 166373     |