Wavefront propagation in two-dimensional optical bistable device and its application to maze exploration

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Abstract: A bistable system with a spatial expanse can support traveling wavefront which is an interface between different stable states such as “on” and “off”. In this work, a two-dimensional thermo-optical bistable device is investigated in terms of wavefront propagation property and its application to maze exploration. It is shown that wavefront velocity is controllable by light intensity and can take even a negative value, i.e. reduction or retreat of the “on”-state area. Utilizing this feature, an application of this device to maze exploration is demonstrated, where extension of the “on” state area explores the whole paths of the maze, and then the device is switched to reduction mode in which the “on” state area retreats from dead-end paths, resulting in the collect path of the maze.

Key Words: nonlinear optics, optical bistability, thermal diffusion, temperature-dependent optical absorption, wavefront propagation, maze exploration

1. Introduction

Natural computing is a novel information processing scheme utilizing dynamics of various natural phenomena [1]. It is attracting attentions as an alternative to silicon-based digital computing in some categories of problems including combinatorial exploding ones. Maze exploration is one of popular applications of natural computing. For example, Belousov-Zhabotinsky (BZ) excitable reaction wave [2], self-propelling oil droplet [3], movement of true slime mold [4], and many other natural phenomena have been reported to solve mazes. What is necessary for those natural maze explorers is some propagating or expanding property of the medium [5, 6]. BZ wave propagates through any path of the maze, and true slime mold tends to expand until it finds food, exploring the whole paths of the maze. In the case of self-propelling oil droplet, diffusion of acid placed at the goal of maze is the explorer providing gradation of pH, and the oil droplet is just a tracer of the path shown by the pH gradation. The true slime mold presents another useful and important feature: once it finds a food, it shrinks back from unnecessary dead-end paths. Eventually, the true slime mold given foods at the start and goal points shows connection between the start and goal, solving the maze. We considered to realize these functions - extension and (if possible) reduction - using an optical device.
A bistable system with a spatial expanse can keep two stable states (here we assign the terms “on” and “off” to these states) simultaneously at different locations [7]. The interface between “on” and “off” area, here we call it “wavefront”, can spatially propagate. Therefore in this work, we introduce a two-dimensional optical bistable device (2DOBD) as a system for wavefront propagation, to realize maze exploration. We will show controllability of wavefront propagation in 2DOBD, and demonstrate maze exploration using 2DOBD by numerical simulation considering a real optical device structure.

2. Principle of operation and device structure

In our device, optical bistability is realized by positive feedback between heat generated by photoabsorption and temperature-dependent optical absorption. The device structure we have designed is shown in Fig. 1. It is composed of a top cover, a liquid crystal layer, a black light-absorbing layer, and a bottom substrate. The light is irradiated from the top side of the device. The black layer absorbs light transmitted through the liquid crystal layer, converting light to heat. Temperature-dependent optical transmission change is realized by phase transition of the liquid crystal. In this work, we employ 4-cyano-4’-pentylbiphenyl (5CB) showing nematic-isotropic phase transition at about $T_{PT} = 35^\circ C$. At a temperature lower than $T_{PT}$, 5CB is opaque in the nematic phase, and optical transmission is low due to light scattering. If the temperature raises above $T_{PT}$ the liquid crystal transit to isotropic phase and becomes transparent. When the device is irradiated with a weak light, transmitted small fraction of light is absorbed by the black layer and the temperature of the device increases slightly. Increase of the light intensity causes rise of the temperature, and at $T_{PT}$ a positive feedback starts: increase of transmission by phase transition results in raise of heat generation and temperature, which causes further increase of transmission. Finally the device transit to the high transmission “on” state (turn-on transition). When the light intensity is reduced, transition to the low transmission “off” state occurs at a lower light intensity than in the turn-on transition, showing a hysteresis character.

![Fig. 1. Schematic cross-sectional diagram of an opto-thermal bistable device based on nematic-isotropic phase transition of a liquid crystal. The light irradiated from top transmits through the liquid crystal layer and is absorbed in the black substrate to be converted to heat. The liquid crystal presents lower optical transmission due to optical scattering at low temperature in nematic phase (shown in gray), while it shows higher transmission at higher temperature in isotropic phase (shown in white). The wavefront, interface between gray and white area, can propagate laterally.](image)

With a spatial expanse, the device can provide wavefront propagation. Under irradiation of bias light at an intensity in the bistable (hysteresis) region, the whole area of the device can stay at low transmission “off” state, if no perturbation is applied. Once the light intensity is increased above the turn-on threshold at one location, the medium is locally “triggered” and transit to high transmission “on” state. The on-state and off-state regions can coexist simultaneously at different location in the device, and wavefront between those regions can propagate two-dimensionally through thermal diffusion.

If the device is irradiated with a linearly patterned light, one-dimensional wavefront propagation can be realized through the irradiated line. Therefore, a maze pattern can be easily defined by patterned light irradiation, and exploration can be started by irradiating strong trigger light at the start point of the maze.

3. Method of numerical simulation

The opto-thermal bistable device with a spatial expanse can be modeled by a set of equations with thermal diffusion term as follows [8–10]:
\[
\frac{\partial T(x, y, z, t)}{\partial t} = \kappa \Delta T(x, y, z, t) + \sigma(x, y, z, t) - \rho(x, y, z, t)
\]

where \( t \) is the time, \( T \) the temperature, \( \kappa \) thermal diffusion constant, \( \Delta \) Laplacian for the spacial coordinates \((x, y, z)\), \( \sigma \) the generated heat, and \( \rho \) the heat dissipation at the surface of the device to outside. \( A(T) \) is the temperature-dependent optical absorption in the black layer in Fig. 1 as a function of the temperature, and \( I \) the incident light intensity. \( \alpha \) is the heat dissipation constant (heat resistance) at the surface. \( A \) and \( \sigma \) is non-zero only in the black layer in Fig. 1, and \( \rho \) is non-zero only at the surface of the device [9, 10]. Nonlinearity necessary for bistability is included in the term \( A(T) \).

Time-dependent 3-D numerical simulation was performed by the finite element method using FreeFEM++ Ver.3.41 software [11]. Device structure shown in Fig. 1 was simulated. Acrylic polymer was assumed as the material of the substrate and the top cover. Bottom of the substrate contacted to a hot plate at constant temperature of 30°C, and the top cover and side wall of the device were exposed to air at 25°C. Thicknesses of the top cover, 5CB layer, black layer, and substrate were 0.2, 0.1, 0.02, and 1.0 mm, respectively. Optical absorption of the black layer was 100%, so that \( A(T) \) is solely determined by the transmission of 5CB layer. Thermal diffusion constant \( \kappa \) of acrylic polymer and 5CB is 0.92 and 1.0 \( \text{cm}^2 \text{s}^{-1} \), respectively. Heat dissipation constant (heat resistance) \( \alpha \) at the boundary of the devices is \( 9.2 \times 10^{-2} \text{W} \text{°C}^{-1} \). Division number in vertical direction was 140, and in lateral direction, division number was selected so that the element size is 0.05 mm. Time step was 2 sec for investigation on wavefront velocity and 5 sec for maze exploration simulation. Temperature-dependent transmission of 5CB layer was assumed to be 0.5 at lower than 35°C, 1.0 at higher than 37°C, and linearly increasing from 0.5 to 1.0 in the temperature range of 35 to 37°C.

Light intensity used in maze exploration was 0.22 and 0.15 W/cm² at the path and the wall area of the maze, respectively, if not noted otherwise. Trigger light intensity was 0.4 W/cm². These light intensities were determined by preparatory numerical simulation, and found to be within a range of experimental realization using a shrink-focused image of ordinary light projector. For investigation of wavefront propagation velocity, a test maze pattern was designed as shown in Fig. 2(a1)(a2). The dead-end path at the center was the test path. Width of paths and walls was 3 mm, and total size of the maze was 21 × 36 mm. Time-dependent finite element method calculation was performed with a time step of 2 sec. For investigation of the extension mode, an initial maze pattern shown in Fig. 2(a1) was irradiated to achieve stationary “on” area for the first 200 sec, then a test maze pattern shown in Fig. 2(a2) was irradiated for up to 3600 sec. For investigation of the reduction mode, the pattern shown in Fig. 2(a2) was used. At the initial stage, the paths were irradiated at 0.4 [W/cm²] to trigger them on. Then the light intensity was reduced to the test value to observe reduction of the on area in the dead-end path. Position of the wavefront at the end of the

![Fig. 2.](image-url) (a1)(a2) Patterns for test maze to investigate wavefront velocity. For extension mode, (a1) is the initial pattern and (a2) is the test pattern. For reduction mode, (a2) pattern is used both in the initial and test stages (b) Pattern for Steinbock maze. Black area stands for wall irradiated with low intensity light, and white area for path irradiated with high intensity light. Gray area in the test maze shows Gray areas in Steinbock maze (b) stand for start (left) and goal (bottom), and the start area is irradiated at further higher intensity for trigger to turn-on. All path and wall widths are 3 mm.
dead-end path was obtained as the position at the phase transition temperature $T_{PT}$, and was plotted as a function of time. Wavefront propagation velocity was obtained as the slope of the plot.

For the demonstration of maze exploration, the Steinbock’s maze pattern shown in Fig. 2(b) [2] was used, with a width of paths and walls of 3 mm. Total size of the maze was $57 \times 51$ mm. Time-dependent finite element method calculation was performed with a time step of 5 sec.

4. Results and discussions

4.1 Path width for maze exploration

To design a pattern of light irradiation for a maze, a suitable path width was examined. Generally, narrower path width is preferable because the device size becomes smaller. If the device size is same, larger and more complex maze can be realized with a narrower path. Moreover, smaller device size results in shorter time for wavefront propagation from start to goal of the maze, if the wavefront velocity is same. However, thermal diffusion prevents high contrast in temperature distribution, if the path is too narrow. To optimize the path width, cross-sectional finite field element calculation was performed to obtain the stationary temperature distribution at various path width shown in Fig. 3 (left). In Fig. 3 (right), peak temperature of each distribution is plotted as a function of width. At narrower path widths of 1 and 2 mm, peak temperature is significantly reduced from that in wider paths. Considering these results, we decided to use the path width of 3 mm for the following simulation.

![Fig. 3.](image)

4.2 Wavefront propagation property

Wavefront propagation velocity was calculated at various light intensity. Figure 4 (right) shows wavefront velocity as a function of $I_{path}$, the light intensity at the path area. Light intensity at the wall $I_{wall}$ was 0.15 W/cm². If $I_{wall}$ is 0 or small value, the intensity at the path area $I_{path}$ must be increased to be in bistable region, but such a high intensity causes spontaneous turn-on without trigger or wavefront propagation at a three-paths connection (branch).

A positive value of wavefront velocity stands for extension of the “on” area (forward propagation of wavefront), while a negative value stands for retreat (backward propagation of wavefront). The wavefront velocity increases almost linearly with $I_{path}$, from negative value (reduction mode) to positive (extension mode). Therefore, control of wavefront propagation velocity is easily achieved by control of $I_{path}$, although we also have to consider possibility of spontaneous turn-on at three- or four-paths connection area at too high $I_{path}$, and of spontaneous turn-off at the connected path in the reduction mode at too low $I_{path}$.
Fig. 4. (left) An example of temperature distribution of the test maze pattern for wavefront velocity calculation. Irradiated path width is 3 mm. The dead-end path at the center is the test path. Green area shows about 30°C, and blue-purple area about 38–39°C. (right) Wavefront velocity calculated at various light intensity. Positive value stands for extension and negative value reduction of wavefront. The light intensity at the wall area \( I_{\text{wall}} \) is 0.15 W/cm\(^2\) (the green area in the left figure). \( I_{\text{path}} \) stands for the light intensity at the path area (the blue-purple area in the left figure).

4.3 Maze exploration

Figure 5 shows temporal evolution of temperature distribution at the liquid crystal layer of the device, in exploration of Steinbock’s maze pattern. From \( t = 1 \) to 30 sec, trigger light at intensity of 0.4 W/cm\(^2\) was irradiated at the bottom-left start point of the maze. The snapshot at \( t = 20 \) sec shows temperature distribution during the trigger, showing significant increase of temperature at the start point. Temperature of the paths of the maze, shown in light blue in the snapshots, was about 34°C, higher than that of the walls (about 30°C) but lower than the phase transition temperature, thus staying in “off” state. The “turn-on” wavefront propagates through the paths even after the trigger was turned off at \( t = 30 \) sec, exploring the maze \( (t = 5400 \text{ sec}) \). At \( t = 9000 \text{ sec} \), wavefronts reach all path ends, and all the path is in “on” state.

At the moment of full extension at \( t = 9000 \text{ sec} \), intensity of light irradiated to the paths \( I_{\text{path}} \) was reduced from 0.220 to 0.205 W/cm\(^2\) to enter the reduction mode, except for the start and goal points where light intensity was increased to 0.4 W/cm\(^2\) to keep these points “on”. In the reduction mode, the dead end point of each path turns “off” while the connected points remained in “on” state, because a dead end point faces more of cooler surrounding area than a connected point. Simply considering four neighbor pixels, a dead end pixel is surrounded by three low-temperature pixels.

Fig. 5. Numerical simulation of exploration of Steinbock’s maze pattern. Snapshots at \( t = 20 \) (start), 5400, and 9000 sec present temporal evolution of “on” state area (blue) in the extension mode. At \( t = 9000 \text{ sec} \), all maze paths are explored, and the light intensity at the paths is reduced to enter the reduction mode. Snapshot at \( t = 10800 \text{ sec} \) shows “on” state area retreating from the dead-end paths, and finally at \( t = 14400 \text{ sec} \), only the connected paths (solution of the maze) remains in “on” state, showing completion of maze exploration. In the snapshot images, color shows temperature: green stands for 30°C, light blue 34°C where is the path region in “off” state, dark blue 38°C where is the path region in “on” state, red-purple 45°C for trigger, white 45°C in the reduction mode corresponding start and goal points where high light intensity same as the trigger is applied to keep the dead-end start and goal points in “on” state. Color coding for temperature is same as shown in Fig. 4 (left).
and one high-temperature “on” pixel, while a connected pixel is surrounded by two low-temperature pixels (walls) and two high-temperature “on” pixels. This results in a difference of heat flow balance, causing “turn-off” only in a dead-end pixel. Therefore, if the light intensity is too low, not only the dead-end pixels but also all connected paths can be turned “off”. The intensity above was determined after some trial and errors. In the simulation, retreat of the “on” area was observed in the snapshot at $t = 10800$ sec., and finally retreated from all dead-end paths at $t = 14400$ sec. At this moment, only the paths connecting the start and goal points remained in “on” state, thus completion of the maze exploration. The solution was visualized as the “on” area of the device.

In place of the extension mode, the whole paths can be irradiated at a high light intensity, e.g. $0.4 \text{ W/cm}^2$, to trigger simultaneously. This “instantaneous turn-on” mode would reduce the exploration time significantly. However, if the maze has any isolated path that is not connected to the start and goal points, instantaneous turn-on will also turn it on. And if an isolated path has a loop structure, the reduction mode cannot turn it off. Therefore, the instantaneous turn-on mode is available only if the maze has no isolated path. On the contrary, exploration using the extension mode will not turn such an isolated path on, and therefore is more advantageous for general mazes without restriction.

This maze exploration method can find all connected paths even if there exist more than one, but cannot find the shortest path in principle. In general, maze exploration methods in natural computing can be categorized in two types: one is exploration with some kind of potential gradient, and the other without potential gradient [6]. Our method belongs to the latter, as well as the method using BZ reaction wave [2]. Self-propelling oil droplet method [3] belongs to the former category, since the oil droplet traces pH gradient toward the goal with acid. The former can find the shortest path by tracing the most steep gradient of the potential, but finding all solutions (if more than one exist) is not guaranteed. Another possible problem in the former is scalability: if the maze becomes large and complex, the potential gradient gets smaller, and trace will be harder. The latter has opposite advantages and disadvantages: all solutions can be found, but finding the shortest one is not guaranteed (some post-processing is necessary); no problem in scalability and solutions can be found even for a larger and complex maze. True slime mold [4] is a special case: in extension mode it extends without food, so there is no potential gradient; in reduction mode, foods are placed at start and end points, and therefore some nutrient concentration gradient can be formed within the true slime mold, resulting in finding the shortest path. However, this mechanism is not well understood yet. Our 2DOBD can mimic this behavior of the true slime mold, and can also explore maze although slight difference in the function of the reduction mode.

5. Concluding remarks

In this paper, we have demonstrated by the finite element method calculation that a maze can be explored by an experimentally feasible opto-thermal bistable device with a spatial expanse. In this device, wavefront propagation property can be easily controlled by light intensity. Direction of wavefront propagation is also controllable: in extension mode, the “on” state area extends, and in reduction mode, it retreats from the dead-end path if the light intensity is suitably reduced. Temperature distribution was calculated for various path width and $3 \text{ mm}$ is found to be practical minimum width for this device structure and materials. Wavefront propagation velocity was examined as a function of light intensity, showing almost linear dependence.

Exploration of the Steinbock’s maze pattern has been demonstrated utilizing the extension and reduction modes: firstly “on” state area extends to all the paths in extension mode, then turned to reduction mode where the “on” state area retreated only from the dead-end paths, and finally all paths connecting the start and stop points remains in “on” state. Such a reduction mode has been reported only for the true slime mold [4], but not for other natural computing methods.

Controllable wavefront propagation in our two-dimensional optical bistable device can be a powerful tool in natural computing. Experimental investigation is in progress, and will be reported separately.
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