On Chaplygin Gas Braneworld Inflation with Monomial Potential

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Abstract

In this paper we study the Chaplygin gas model as a candidate for inflation in the framework of the Randall Sundrum type-II braneworld model. We consider the original and generalized Chaplygin gas model in the presence of monomial potential. The inflationary spectrum perturbation parameters are reformulated and evaluated in the high-energy limit and we found that they depend on several parameters. We also showed that these perturbation parameters are widely compatible with the recent Planck data for a particular choice of the parameters space of the model. A suitable observational central value of $n_s \simeq 0.965$ is also obtained in the case of original and generalized Chaplygin gas.

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1 Introduction

It is widely believed that the early universe underwent a period of accelerated expansion called inflation \([1]\), which has become the standard paradigm of modern cosmology. Inflation model has been proposed as an attempt to solve the shortcoming of the standard Big-Bang model of cosmology, known usually as the flatness and the horizon problem \([2]\). Inflation is the candidate for understanding the physics of the very early universe, typifying the evolution of the universe to the properties of one or more scalar inflaton fields, responsible for creating an accelerating expanding universe \([3]\). Recently, a great amount of work has been invested in studying the inflationary model with several candidate. Among these candidates, the Chaplygin gas model \([4]\). This model was also used to describe an mysterious dark sector so-called dark energy and dark matter \([5]\) and also used to describe the early universe \([6]\). In cosmology, dark energy is a form of unknown energy dominating the universe with a hugely negative pressure \([7]\). It is demonstrated by various astrophysical observations, including the accelerating universe \([8]\).

Recall that the dark energy is defined by an exotic equation of state of the form \(P_{DE} = \omega \rho_{DE}\), where \(P_{DE}\) and \(\rho_{DE}\) are the pressure and energy density of dark energy, while \(\omega\) is the equation of state parameter of the dark energy \([9]\). There are several candidates to describe the dark matter and dark energy in the cosmology \([10]\). Among them K-essence model \([11]\), the ΛCDM model \([12]\), cosmological constant \([13]\), tachyon model \([14]\) and quintessence model \([15]\). There are different kinds of the Chaplygin gas model which have been proposed in the literature. For example the general model named extended Chaplygin gas which is defined by an exotic equation of state of the form \([16]\)

\[
p = \sum B_m \rho_m - \frac{A}{\rho}
\]

where \(B_m, A, m\) are universal positive constants, and \(0 \lesssim \alpha \lesssim 1\). In the recent years the extended Chaplygin gas was the subject of several cosmological and phenomenological studies \([17]\). Note that, when \(m = 1\) we obtain the case of modified Chaplygin gas \([18]\). The original Chaplygin gas corresponds to the case \(B_m = 0\) and \(\alpha = 1\), this type was studied in \([19]\). In the case of \(B_m = 0\) and \(\alpha \neq 1\) it is known as the generalized Chaplygin gas (GCG) \([20]\) and finally for \(B_m = 0\) and \(\alpha = 0\) corresponds to the ΛCDM model \([12]\). Further, the Chaplygin gas models has been studied in different paper. In the Ref. \([21]\) the authors have studied the interaction of the dark energy with some fluids specially Chaplygin gas in the context of \(f(T)\) theory. In \([22]\), the authors have proposed an attempt for emergent universe scenario with modified Chaplygin gas, and have shown that it is not possible to have emergent scenario with model. In another work \([23]\), B. Pourhassan et all have examined extended model of Chaplygin gas equation of state for which it recovers barotropic fluid with quadratic equation of state, and have found that extended Chaplygin gas may be a more appropriate model than generalized and modified Chaplygin gas and give a best fit with the observational data.

On the other hand, the Chaplygin gas inspired inflation model \([24]\) was the subject of several cosmological and phenomenological studies. In this context, the scalar field is usually
the standard inflaton field, where the energy density can be extrapolated to obtain a successful inflationary period with a Chaplygin gas model. In the same context, a work has been done by R. Herrera [25] where the brane-Chaplygin inflationary model was studied in great details and considered as a viable alternative model that can provide an accelerated expansion of the early universe. In extension of the Ref. [25], the similar work was performed for the case of the tachyon-Chaplygin inflationary model by using an exponential potential in the high-energy regime [26].

In the present paper, we are going to use the Randall-Sundrum II braneworld model to study the original and generalized Chaplygin gas as a candidate for the primordial inflation by assuming that the matter source on the brane consist of a Chaplygin gas. The Chaplygin gas emerges as a effective fluid of a generalized d-brane in a \((d+1, 1)\) spacetime, where the action can be written as a generalized Born–Infeld action [27]. These models have been extensively studied in the literature [6]. The motivation for introducing Chaplygin-brane scenarios is the increasing interest in higher-dimensional cosmological models, motivated by superstring theory, where the matter fields are confined to a lower-dimensional brane while gravity can propagate in the bulk. On the other hand, the Chaplygin gas model seems to be a viable alternative to models that provide an accelerated expansion of the early universe. Our aim is to quantify the modifications of the Chaplygin inspired inflation in the Braneworld scenario. We use the monomial potential to study various perturbation spectrum parameters such as the scalar spectral index \(n_s\) and the ratio \(r\) and the running of the scalar spectral index \(\frac{dn_s}{d\ln(k)}\) in the high-energy limit, particularly for a suitable choice of the different parameters. We show that the inflation parameters are in good agreement with recent Planck 2015 data [28].

An outline of the remainder of this paper is as follows: We first begin in section 2, by recalling the standard inflation and Chaplygin gas Braneworld inflation formalism, in particular the modified Friedmann equation. In section 3, we study different perturbation spectrum concerning monomial potential in the High-energy limit, and we present our results for original and generalalized Chaplygin gas on the brane. The last section is devoted to conclusion.

## 2 Generalized Chaplygin gas braneworld inflation

### 2.1 Inflationary Universe

In this section we propose a short description on standard inflation formalism to enable readers to understand the meanings of terms here involved. Inflation is the period of the early universe that undergoes an accelerating phase [1]. This period is equivalent to \(\ddot{a} > 0\). The candidate that can yield this acceleration phase and responsible for driving inflation is a scalar field named inflaton field. The pressure and energy of the inflaton field are given by \(p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi)\) and \(\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi)\), where \(V(\phi)\) is the scalar potential. The Friedmann equation reads \(H^2 = \frac{8\pi G}{3} \left[ \frac{1}{2} \dot{\phi}^2 + V(\phi) \right] \). During inflation, the potential \(V(\phi) = V\) depends only on the inflaton
field $\phi$. It is supposed that the field equation $\ddot{\phi} + 3H\dot{\phi} = -V'(\phi)$, is well-approximated by $3H\dot{\phi} = -V'(\phi)$. This is called the slow-roll approximation. With very small $\dot{\phi}^2$, the Friedmann equation is approximately $3H^2 \simeq V$.

The condition for acceleration requires that $\dot{\phi}^2 \ll V$, This is called the slow-roll approximation. With very small $\dot{\phi}^2$, the Friedmann equation is approximately $3H^2 \simeq V$, and the flatness conditions $\varepsilon \ll 1$ and $|\eta| \ll 1$, where $\varepsilon = \frac{1}{2}M_p^2 \left(\frac{\dot{\phi}}{\phi}\right)^2$ and $\eta = M_p^2 \frac{\dddot{\phi}}{\phi}$. The amount of inflation can be measured in term of the e-folding number $N$ given by equation $N = M_p^{-2} \int_{\phi_{end}}^{\phi_{end}} \frac{V'}{V} d\phi$, where $\phi_s$ and $\phi_{end}$ are the values of the scalar field at the epoch when the cosmological scales exit the horizon and at the end of inflation, respectively.

The small quantum fluctuations in the scalar field lead to fluctuations in the energy density which was studied in a perturbative theory [29]. As discussed in [30] quantum fluctuations effect of the inflaton are generally negligibles, since the coupling of the scalar field to bulk gravitational fluctuations only modifies the usual 4D predictions at the next order in the slow-roll expansion.

So, one can define the power spectrum of the curvature perturbations as $P_R(k) = \left(\frac{H^2}{2\pi}\right)^2$. On the other hand, the quantum fluctuations in the scalar field lead also to fluctuations in the metric [31]. In this way, one can define the amplitude of tensor perturbations as $P_g(k) = 8M_p^2 \left(\frac{H^2}{2\pi}\right)^2 F^2(x)$. These results lead to the ratio of tensor to scalar perturbations $r = \frac{P_g(k)}{P_R(k)}$.

In relation to $P_R(k)$, the scalar spectral index is defined as $n_s = 1 + d\ln P_R(k) \over d\ln(k)$. Refs. [3] are recommended for further reading on inflation.

2.2 Genaralized Chaplygin gas on the brane

Braneworld inflation [32] is a particular kind of inflation models. It is based primarily on the cosmological model Randall-Sundrum type II which describes the universe in five dimensions with the presence of a brane that includes all ordinary matter. The generalized Chaplygin gas is a perfect fluid characterised by the following equation of state [33]:

$$p = -\frac{A}{\rho^\alpha}$$

where $\rho$ and $p$ are the energy density and pressure of the generalized Chaplygin gas, respectively, $\alpha$ is a constant satisfying $0 < \alpha \leq 1$, and $A$ is a positive constant.

Inserting the equation (1) in the equation of conservation of energy $\dot{\rho} + 3H(\rho + p) = 0$, we obtain the following expression for the energy density

$$\rho_{ch} = \left[A + \left(\rho_{ch0} - A\right) \left(\frac{a_0}{a}\right)^{3(\alpha+1)}\right]^{1\over\alpha+1}$$

where $a_0$ and $\rho_{ch0}$ are the current values of the scale factor and the generalized Chaplygin gas energy density, respectively.

The modification of the equation (1) is realized from an extrapolation of equation (2), where the density matter $\rho_m \sim a^{-3}$ is replaced by the scalar field as $\rho = \left[A + \rho_m^{(\alpha+1)}\right]^{1\over\alpha+1} \rightarrow \rho = \left[A + \rho_{\phi}^{(\alpha+1)}\right]^{1\over\alpha+1}$.
In this section, we will recall briefly some basic facts of Randall-Sundrum type II braneworld model [34]. We suppose that the universe is filled with a perfect fluid with energy density $\rho(t)$ and pressure $p(t)$ in which the Friedmann equation is modified from its usual form [25].

\[ H^2 = k \rho_\phi \left[ 1 + \frac{\rho_\phi}{2\lambda} \right] + \frac{\Lambda_4}{3} + \frac{\xi}{a^4} \]  

(3)

where $H = \frac{\dot{a}}{a}$ defines the Hubble parameter, $\rho_\phi$ represents the matter confined to the brane, $k = \frac{8\pi G}{3} = \frac{8\pi}{3M_p^2}$, $\Lambda_4$ the current cosmological constant $\xi$ is an integration constant and thus transmitting bulk graviton influence onto the brane. This term appears as a form of “dark radiation” and may be fixed by observation [35]. However, during inflation this term is rapidly diluted, so we will neglect it. Where $\lambda$ is the brane tension, $M_p$ is the four-dimensional Planck mass, which is related to the five-dimensional $M_5$ by $M_p = \sqrt{\frac{3M_5^2}{4\pi \lambda}}$. Note that the crucial correction to standard inflation is given by the density quadratic term $\rho_\phi^{(\alpha+1)}$. Note also that in the limit $\lambda \to \infty$, we recover standard four-dimensional general relativistic results.

The Friedmann equation will become [26]

\[ H^2 = \frac{8\pi}{3M_p^2} (A + \rho_\phi^{(\alpha+1)}) \frac{1}{\alpha+1} \left[ 1 + \frac{(A + \rho_\phi^{(\alpha+1)})}{2\lambda} \right]^\frac{1}{\alpha+1}, \]  

(4)

In four-dimensional general relativity, the condition for inflation is $\dot{\phi}^2 \ll V(\phi)$, i.e $p_\phi \ll -\frac{1}{\lambda} \rho_\phi$, where $p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi)$ and $\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi)$, $V(\phi) = V$ is the scalar potential and $\phi$ is the inflaton field the scalar field satisfies the Klein-Gordon equation:

\[ \ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0. \]  

(5)

Note that $\dot{\phi} = \frac{\partial \phi}{\partial t}$, $\ddot{\phi} = \frac{\partial^2 \phi}{\partial t^2}$, $V' = \frac{\partial V}{\partial \phi}$. During inflation, the relation between the energy density and the scalar potential is $\rho_\phi \simeq V$, we consider the slow-roll approximation $\dot{\phi}^2 \ll V(\phi)$ and $\dot{\phi} \ll 3H \dot{\phi}$ the Friedmann equation reduces to:

\[ H^2 = \frac{8\pi}{3M_p^2} (A + V^{(\alpha+1)}) \frac{1}{\alpha+1} \left[ 1 + \frac{(A + V^{(\alpha+1)})}{2\lambda} \right]. \]  

(6)

The term in square brackets is the brane-modification to the standard slow-roll expression for the Hubble rate.

We consider the slow-roll parameters, in the Randall-Sundrum type II Braneworld model [30]. The two first parameters are given for generalized Chaplygin gas model by:

\[ \xi = \frac{M_p^2}{16\pi} \frac{V^\alpha V'^2}{(A + V^{(\alpha+1)})^{\frac{\alpha+1}{\alpha+1}}} \left[ 1 + \frac{(A + V^{(\alpha+1)})}{\lambda \frac{\alpha+1}{\alpha+1}} \right]^\frac{1}{\alpha+1}, \]  

(7)

and

\[ \epsilon = \frac{M_p^2}{16\pi} \frac{V^\alpha V'^2}{(A + V^{(\alpha+1)})^{\frac{\alpha+1}{\alpha+1}}} \left[ 1 + \frac{(A + V^{(\alpha+1)})}{\lambda \frac{\alpha+1}{\alpha+1}} \right]^\frac{1}{\alpha+1}, \]  

(8)
\[ \eta = \frac{M_p^2}{8\pi} \frac{V''}{(A + V^{(\alpha+1)})^{\alpha+1}} \left[ \frac{1}{1 + \frac{(A + V^{(\alpha+1)})^{\alpha+1}}{2\lambda}} \right]. \quad (8) \]

The inflationary phase ends when \( \varepsilon \) or \( |\eta| \) are equal to one, during inflation, the conditions \( \varepsilon \ll 1 \) and \( |\eta| \ll 1 \) are satisfied. Note that, At low energies, \( V \ll \lambda \), the slow-roll parameters reduce to the standard inflation.

In addition, the number of e-folding is given by

\[ N = \frac{-8\pi}{M_p^2} \int_{V_e}^{V_{\text{end}}} \left( A + V^{(\alpha+1)} \right)^{\frac{1}{\alpha+1}} \frac{V'}{V'} \left[ \frac{1}{1 + \frac{(A + V^{(\alpha+1)})^{\alpha+1}}{2\lambda}} \right] dV, \quad (9) \]

where \( V_e \) and \( V_{\text{end}} \) are the values of the potentials at the horizon exit and the end of inflation, respectively.

The inflationary spectrum perturbation is produced by quantum fluctuations of fields around their homogeneous background values. The small quantum fluctuations in the scalar field lead to fluctuations in the energy density and in the metric, for that, we define the power spectrum of the curvature perturbations by [30]:

\[ P_r(k) = \left( \frac{H^2}{2\pi} \right)^2 \quad (10) \]

by using the equation (6) and (10), we find the expression for power spectrum of the curvature perturbations:

\[ P_r(k) = 128\pi \frac{(A + V^{(\alpha+1)})^{\frac{3}{\alpha+1}}}{M_p^6 V'^2} \left[ 1 + \frac{(A + V^{(\alpha+1)})^{\frac{1}{\alpha+1}}}{2\lambda} \right]^3. \quad (11) \]

Another important inflationary spectrum parameter is the amplitude of the tensorial perturbations \( P_g(k) \), describing the primordial gravitational wave perturbations produced by a period of extreme slow-roll inflation, which is defined by [31]:

\[ P_g(k) = \frac{64\pi}{M_p^2} \left( \frac{H}{2\pi} \right)^2 F^2(x), \quad (12) \]

where \( x = H M_p \sqrt{\frac{3}{4\pi}} \) and \( F^2(x) = \left( \sqrt{1 + x^2} - x^2 \sinh^{-1}(\frac{1}{x}) \right)^{-1} \). Note that in the low-energy limit \( (A + V^{(\alpha+1)})^{\frac{1}{\alpha+1}} \ll \lambda \), we have \( F^2(x) \simeq 1 \), and in the high-energy limit \( (A + V^{(\alpha+1)})^{\frac{1}{\alpha+1}} \gg \lambda \), \( F^2(x) \simeq \frac{3}{2} x = \frac{3}{2} \frac{(A + V^{(\alpha+1)})^{\frac{1}{\alpha+1}}}{\lambda} \).

We define the ratio \( r \) of tensor to scalar as:

\[ r = \left( \frac{P_g(k)}{P_r(k)} \right)_{k=k^*}. \quad (13) \]

Here \( k^* \) correspond to the case \( k = H_a \), the value when the universe scale crosses the Hubble horizon during inflation. From equations (11, 12), the tensor to scalar is giving by:
The scalar spectral index is presented by [29]:

\[
n_s - 1 = \frac{d \ln P_R(k)}{d \ln(k)} = M_p^2 \pi \frac{V'(A + V(\alpha + 1))^{\frac{2}{\alpha + 1}}}{1 + (\frac{A + V(\alpha + 1)}{2})^{\frac{1}{\alpha + 1}}} \left( -3 \frac{V''}{V'} \left( \frac{1 + (A + V(\alpha + 1))^{\frac{1}{\alpha + 1}}}{2} \right) + 2V'' \right).
\]

The running of the scalar index is also defined as:

\[
\frac{dn_s}{d\ln(k)} = \frac{M_p^2}{4\pi} \frac{V'}{(A + V(\alpha + 1))^{\frac{1}{\alpha + 1}}} \frac{1}{1 + (\frac{A + V(\alpha + 1)}{2\lambda})^{\frac{1}{\alpha + 1}} \left( 3 \frac{\partial \epsilon}{\partial \phi} - \frac{\partial \eta}{\partial \phi} \right)}.
\]

Note that in the limit \( A \to 0 \), the perturbation spectrum parameters coincides with branoinflation [30] in particular for \( \alpha = 1 \). Also, in the low-energy limit, \((A + V(\alpha + 1))^{\frac{1}{\alpha + 1}} \ll \lambda\), the slow-parameters reduce to the standard form [24]. In what follows, we shall apply the above Braneworld formalism with a monomial potential in the high-energy limit; i.e. \((A + V(\alpha + 1))^{\frac{1}{\alpha + 1}} \gg \lambda\), in relation to recent Planck data.

3 Chaplygin gas with monomial potential

3.1 Original Chaplygin gas

In the following, we will concentrate on the original Chaplygin gas, it is reached as a special case of the general Chaplygin gas, it is proposed as possible explanations of the acceleration of the current univers. The original Chaplygin gas model has been extensively studied. For exemple, in [36] study the behaviour of density perturbations in an Universe dominate by the Chaplygin gas, and found that in spite of presenting negative pressure of Chaplygin gas, is stable at small scale, which opposite to in general what happens with perfect fluids with negative pressure. In [19], the authors have focused to study a Chaplygin gas model in Braneworld inflation with an exponential potential, and have obtained for negligible and small running of the scalar spectral index, the inflationary parameters are in good agreement with observation data. Another example [37], the authors have analysed a phase space of the evolution for a Friedmann Robertson Walker universe driven by an interacting of Chaplygin gas and dark matter, their results are derived from continuity equations, which means that they are independent of any theories of gravity. The original Chaplygin gas model is characterized by an exotic equation of state of the form

\[
p = -\frac{A}{\rho},
\]
where $A$ is a positive constant.

In this section we will propose to investigate monomial potential in braneworld context with generalized Chaplygin gas. This potential used in very recent model in different work. In the paper [38] the authors study the attractors solutions of the dynamical system of a scalar field endowed with monomial potentials, and shown that the behaviour found for monomial potentials is typical in realistic inflationary models. Furthermore, R.Zarrouki et all have studied various inflationary spectrum perturbation parameters with three types of potentials they shown that the monomial potential provides the best fit results to observations data [39]. This potential is given by

$$V = M \phi^n,$$

where $n$ is an positive intege and $M$ is a parameter of dimension $[E]^{4-n}$. In order to derive the inflationary parameters $n_s$, $r$ and $\frac{d n_s}{d\ln(k)}$, let us consider the monomial potential. In this case, the scalar spectral index $n_s$, the ratio $r$ and the running of the scalar index $\frac{d n_s}{d\ln(k)}$ becomes:

$$n_s = \frac{M^2 \lambda}{2 \pi (A + M^2 \phi^2)} \left( - \frac{3n^2 M^2 \phi^{3n-2}}{A + M^2 \phi^{2n}} + n (n-1) M \phi^{n-2} \right) + 1,$$

$$r = \frac{6M^2 \lambda M^2 n^2 \phi^{2n-2}}{\pi (A + M^2 \phi^{2n})^2},$$

and

$$\frac{d n_s}{d\ln(k)} = - \frac{M^2 \lambda^2 n^2 (2n+1)(n+2)}{8\pi^2 \phi^4 (A + (M^n)^2)}.$$

Note that in the limit $A \to 0$, the scalar spectral index $n_s$, the ratio $r$ and $\frac{d n_s}{d\ln(k)}$ coincides with Ref [39].

Although we have analytic results for slow-roll parameters and the number of e-folding, it is not easy to solve them to obtain $\phi_*$ at which the observables $n_s$, $r$ and $\frac{d n_s}{d\ln(k)}$ should be evaluated. For it, we proceed numerically by finding $\phi_{end}$ and using the Eq (9) to obtain $\phi_*$ while making sure that the slow-roll parameters remain small in this range of $\phi$.

To complete our study with original chaplygin gas, we analyse the variations of the perturbation spectrum parameters with respect to $N$ for various values of $n = 1, 2, 3, 4$. We can see that these observables depends on several parameters. For this purpose, we take the inflationary scale $M \sim O(10^{15} GeV)$, the brane tension value $\lambda \sim O\left(10^{68}GeV^4\right)$ and $A \sim 10^{-13} M_p^8$ [25], in order to obtain consistent perturbation spectrum parameters with recent Planck data.

Figure 1 present the variations of scalar spectral index $n_s$ as a function of e-folding number $N$. The scalar spectral index $n_s$ have a increasing behaviour as we increase the e-folding number $N$. We also note that the values of $n_s$ are found to be consistent with the Planck data for large domain of $N$ and the central value of $n_s \simeq 0.965$ is obtained in particular for $n = 1; 2$.

In figure 2 we have plotted $r$ as a function of e-folding number $N$ for different values of $n$. General, the ratio $r$ has a decreasing behaviour with respect to $N$. We remarque also that, in order to confront $r$ with planck data we must have a large values of $N$ for the four values of $n$. 
Figure 1: $n_s$ versus $N$ for various values of $n$. We take $M \sim O(10^{15} \text{ GeV})$, $\lambda \sim O(10^{68} \text{GeV}^4)$ and $A \sim 10^{-13}M_p^8$.

Figure 2: $r$ versus $N$ for various values of $n$. We take $M \sim O(10^{15} \text{ GeV})$, $\lambda \sim O(10^{68} \text{GeV}^4)$ and $A \sim 10^{-13}M_p^8$. 
Figure 3 shows that the observable $\frac{dn_s}{d\ln(k)}$ is an increasing function with respect to $N$. We have obtained extremely weak values which are consistent with Planck data and that it gets smaller as we increase the values of $n$.

To summarize this subsection, we have found that the results reviewed in the context of the original Chaplygin gas model are compatible with the latest observational measurements for a particular choice of e-folding number $N$ and constant values of $n$.

In the following, we will study the case of generalized Chaplygin gas with $\alpha \neq 1$. We will discuss the effect of introducing the constant $\alpha$ on the perturbation spectrum of the model. Our results will be compared to observations and we will show that the inflation can occur successfully in relation to recent observations.

### 3.2 Generalized Chaplygin gas

It is well known that generalized Chaplygin gas is one of the most natural candidates dark energy models to explain the accelerated expansion of the universe. In this context, these models have been extensively studied in the literature. Moreover, in the work [40] the authors have studied an inflationary scenario in the presence of Generalized Chaplygin Gas in the light of the Planck and BICEP2 experiments, and have obtained the constraints on the $n_s$ and $r$. In other work [41] the authors have examined the effect of anisotropy on generalized Chaplygin gas scalar field and its interaction with other dark energy models, they concluded that the increase in anisotropy leads to more correspondence between the dark energy scalar field model and observational data. In the other hand R. Herrera et all have considered an intermediate inflationary universe...
model in the context of a generalized Chaplygin gas in the slow-roll approximation, and have shown that the intermediate generalized Chaplygin gas inflationary models are less restricted than analogous ones standard intermediate inflationary models due to the introduction of $\alpha$ and $\beta$ parameters \[42\]. In the context of brane-inflationary background, the generalized cosmic Chaplygin model was studied by A. Jawad et all \[43\] in the presence of chaotic potential in the high-energy limit, various inflationary parameters was evaluated and compared by planck data.

In this part, we study the generalized Chaplygin gas model in the presence of monomial potential, using the basic formalism obtained in the section 2 in the context of high-energy limit. In this case, the scalar spectral index $n_s$, the ratio $r$ and the running of the scalar index $\frac{dn_s}{d\ln(k)}$ becomes:

\[
 n_s = \frac{M_p^2 \lambda}{2\pi(A + (M\phi^m)^{\alpha+1})} \left( -\frac{3n^2M^3\phi^3n^{-2}}{(A + (M\phi^m)^{\alpha+1})^{\pi+1}} + n(n - 1)M\phi^m - 2 \right) + 1 \tag{22}
\]

The ratio $r$ of tensor to scalar will be given by

\[
r = \frac{6M_p^2\lambda M^2n^2\phi^2n^{-2}}{\pi(A + (M\phi^m)^{\alpha+1})^{\pi+1}} \tag{23}
\]

The running of the scalar spectral index will be in the following form

\[
\frac{dn_s}{d\ln(k)} = -\frac{M_p^4\lambda^2n^2(2n + 1)(n + 2)}{8\pi^2\phi^4(A + (M\phi^m)^{\alpha+1})^{\pi+1}} \tag{24}
\]

We note that, as in the previous case, the inflaton value before the end of inflation $\phi^*$, can be obtained numerically from Eq (9).

Based on the above formulas and to confront simultaneously the observables $n_s$, $r$, and $\frac{dn_s}{d\ln(k)}$ with observations, we study the relative variation of these parameters. We can see that these observables depends on several parameters. Therefore, as the previous case we take the inflationary scale $M \sim O(10^{15} \text{ GeV})$, the brane tension value $\lambda \sim O(10^{68} \text{GeV}^4)$ and $A \sim 10^{-13} M_p^8 \tag{25}$, and $n = 2$ which corresponds to chaotic case.

We will discuss some values of the above inflationary parameters in relation with the e-folds number $N$ and the parameter $\alpha$. The central region, given by the Planck data, of the spectral index $n_s \in [0.959; 0.969]$ gives $45 < N < 75$ for the values of $\alpha$ in order of $\alpha \sim O(10^{-3}) - O(10^{-1})$. Regarding the ratio of scalar to tensor curvature perturbation, the Planck constraint $r < 0.11$, which requires large values of the folding number, that is, $N > 80$, and for $\frac{dn_s}{d\ln(k)} \in [-0.0166; -0.0039]$, we get $56 < N < 150$ for large domain of value of $\alpha$ between $0 < \alpha < 1$. This shows that parameter $\alpha$ has a small influence on the inflationary observables compared to the original Chaplygin gas. But we can have the normal value which is most commonly used for the e-folding number, required for solving the horizon and flatness problems.

Figure 3 shows the variation of $n_s$ with respect to ratio $r$, we remark that $n_s$ is an decreasing linear function with $r$. The ratio $r$ is compatible with Planck data where $r < 0.11$ corresponds to $N > 80$ and $n_s \geq 0.975$. The central value $n_s \simeq 0.965$, where $N \sim 65$ corresponds to the ratio $r \sim 0.17$. 

\[11\]
Figure 4: Evolution of $r$ versus $n_s$ for various values of $N$. We take $M \sim O(10^{15} \text{ GeV})$, $\lambda \sim O(10^{68} \text{GeV}^4)$, $A \sim 10^{-13} M_p^8$ and $n = 2$

Figure 5 presents the running of the scalar spectral index $\frac{dn_s}{d\ln(k)}$ as function of $n_s$. We see that the $\frac{dn_s}{d\ln(k)}$ increases with respect to $n_s$. The central value of the scalar spectral index $n_s \simeq 0.965$ corresponds to $N = 65$, gives $\frac{dn_s}{d\ln(k)} \simeq -0.0005$, which is consistent with Planck data.

In figure 6, we have plotted the ratio $r$ according to the running of the scalar spectral index $\frac{dn_s}{d\ln(k)}$, which is a decreasing function with the variation of $N$. For the ratio tensor-to-scalar given by Planck corresponds to $N > 80$ which gives a domain of the running $\frac{dn_s}{d\ln(k)} \lesssim -0.00024$.

To summarize this subsection, our numerical calculation shows that, for a particular choice of the parameters of the model, the obtained results are compatible with the Planck data, particularly for a suitable choice of the e-folds number $N$. Also, the introduction of parameter $\alpha$ has a small influence on the inflationary parameters.

4 Conclusion

In this paper, we have examined the Chaplygin gas model as a candidate for inflation, which showed some important properties. We have considered the original and generalized Chaplygin gas model in the Randall-Sundrum type II braneworld using an monomial potential in the high-energy limit to study the behaviors of inflationary spectrum perturbation parameters. We have found that the inflationary parameters depend on a parameters space of the model. In the original Chaplygin gas case, $\alpha = 1$, we have shown that for $37 < N < 60$, the central value of the spectral index is reproduced especially for $n = 1; 2$ and the running of the spectral index $\frac{dn_s}{d\ln(k)}$ is consistent with observations for large interval of $N$. A confrontation with recent Planck
Figure 5: Evolution of $\frac{dn}{d\ln(k)}$ versus $n_s$ for various values of $N$. We take $M \sim O(10^{15} \text{GeV})$, $\lambda \sim O(10^{68} \text{GeV}^4)$, $A \sim 10^{-13} M_p^8$ and $n = 2$

Figure 6: Evolution of $\frac{dn}{d\ln(k)}$ versus $r$ for various values of $N$. We take $M \sim O(10^{15} \text{GeV})$, $\lambda \sim O(10^{68} \text{GeV}^4)$, $A \sim 10^{-13} M_p^8$ and $n = 2$
data shows that the best fit is achieved for large values of $N$, in particular for the ratio $r$. The case of generalized Chaplygin gas predicts also a desirable values of $(n_s; r; \frac{dn_s}{d\ln(k)})$ with recent observational data. In particular, the central value, $n_s \simeq 0.965$, where $N = 65$ corresponds to the value of $\alpha$ in order of $O\left(10^{-3}\right) - O\left(10^{-1}\right)$ and for the ratio $r$ is consistent with Planck 2015 $r < 0.11$, is compatible in the case where $N \gtrsim 80$. The values of the tensor to scalar ratio $r$ and running of the spectral index $\frac{dn_s}{d\ln(k)}$ are in excellent agreement with the latest observations of the Planck satellite for a particular choice of the parameter space of the model.

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We declares that there is no conflict of interest regarding the publication of this paper