Scaling of velocity fluctuations in off-wall boundary conditions for turbulent flows

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Abstract.
A model for off-wall boundary conditions for turbulent flow is investigated. The purpose of such a model is to circumvent the need to resolve the buffer layer near the wall, by providing conditions in the logarithmic layer for the overlying flow. The model is based on the self-similarity of the flow at different heights in the logarithmic layer. It was first proposed by Mizuno and Jiménez (2013), imposing at the boundary plane a velocity field obtained on-the-fly from an overlying region. The key feature of the model was that the length scales of the field were rescaled to account for the self-similarity law. The model was successful at sustaining a turbulent logarithmic layer, but resulted in some disagreements in the flow statistics, compared to fully-resolved flows. These disagreements needed to be addressed for the model to be of practical application. In the present paper, a more refined, wavelength-dependent rescaling law is proposed, based on the wavelength-dependent dynamics in fully-resolved flows. Results for channel flow show that the new model eliminates the large artificial pressure fluctuations found in the previous one, and a better agreement is obtained in the bulk properties, the flow fluctuations, and their spectral distribution across the whole domain.

1. Introduction
Wall-bounded turbulent flows are ubiquitous both in nature and in industry, and their simulation is often necessary to predict their behaviour and/or performance. As the Reynolds number of the flow increases, so does the range of scales at play, and the resolution of all of them becomes inaccessible with the computing power available at present – as well as that predicted in the next few decades. To overcome this difficulty, large-eddy simulation (LES) solves only the largest scales, modelling the effect on them of the smaller ones. This approach can be applied with satisfactory results, provided some conditions are met. One is that the largest and smallest scales must be separated by a broad range of scales, called the ‘inertial’ range. Another is that the smallest, dissipative scales must be reasonably isotropic, so that some degree of universality across different flows can be assumed for them. Both assumptions fail in turbulence near walls. The distance $y$ to the wall imposes a constraint on all the turbulent eddies, which could in principle be of any length in the wall-parallel plane, but whose wall-normal extent must vanish as it approaches the wall, and cannot exceed $O(y)$. This constraint not only imposes anisotropy on the eddies, but also a $y$-dependent distribution of sizes. Figure 1, taken from Jiménez (2012), shows that while the size of the smaller, dissipative eddies varies slowly with $y$, the size of large,

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energy-carrying eddies varies linearly as they separate from the wall. As a consequence, the range of separation between the two increases with $y$. Close to the wall, they essentially overlap, in the so-called buffer region, which on these grounds can be defined as $y^+ \lesssim 100$. If we define the logarithmic layer as that in which the larger eddies scale linearly with $y$, it would span from the upper limit of the buffer layer to $y \approx 0.4\delta$, where $\delta$ is the flow thickness (the half-height in the case of channels). Far enough from the wall, e.g. $y \approx 0.1\delta$, and as long as $0.1y^+ \gtrsim 100$, the large and small scales are sufficiently separated, and the distance to the wall is large enough for the small ones to be reasonably isotropic. It is at this height where the assumptions for LES begin to hold.

Below that height, as the larger scales become increasingly small, the resolution required to resolve them becomes comparable to that for the direct numerical simulation (DNS) of all the scales. Resolving all of them is the most straightforward approach to LES of wall-bounded flows, and is called wall-resolved LES (Piomelli, 1999). However, the computational cost of such approach makes it impractical (Piomelli and Balaras, 2002; Choi and Moin, 2012), and models for the near-wall flow have to be used instead. The flow in the wall layer is often represented only in the average sense, using the Reynolds-averaged Navier–Stokes (RANS) equations, or approximated by the thin-turbulent-boundary-layer equations. Many of these wall models attempt, in various ways, to relate the wall stress, which LES cannot obtain accurately because of its insufficient grid resolution, to the outer flow, in order to obtain boundary conditions for the computation. This subclass of wall models are called wall stress models (Cabot and Moin, 1999).

An alternative also described by Cabot and Moin (1999) is to use off-wall boundary conditions. In this case, the outer flow is computed through LES, with a grid providing sufficient resolution only down to a chosen distance from the wall, where approximate boundary conditions are provided. In their review, Cabot and Moin (1999) cite attempts of this type by Bagwell et al. (1993), Balaras et al. (1996) and Nicoud et al. (1998), which they characterize as being largely unsuccessful, due to a lack of accurate representation of the relative phases and time scales of the flow at the artificial boundary. They also cite Jiménez and Vasco (1998), who observed that the wall layer flow is quite sensitive to the transpiration of the vertical velocity across the off-wall plane in order to maintain continuity.

Figure 1. Spectral densities in a channel at $Re_\tau \approx 2000$, as functions of the streamwise wavelength $\lambda_x^+$ and of the wall distance $y^+$ in viscous units, from Jiménez (2012). The shaded contours correspond to the premultiplied spectrum of the velocity fluctuations, $k_x E_{uu}(k_x)$, and the line contours to the surrogate dissipation, $\nu k_x E_{\omega\omega}(k_x)$, where $\omega$ is the vorticity magnitude. The lowest contour is 0.86 times the local maximum at each $y$. The horizontal lines, $y^+ = 100$ and $y/\delta = 0.4$, are the approximate limits in which the energy length scale grows linearly with $y$. The diagonal line through $k_x E_{uu}(k_x)$ is $\lambda_x = 5y$, and the one through $\nu k_x E_{\omega\omega}(k_x)$ is $\lambda_x = 40\eta$. 
A recent attempt to develop off-wall models is that of Chung and Pullin (2009), in an LES of a turbulent channel flow up to very high Reynolds numbers. They determine the slip velocity at an off-wall plane in the logarithmic region, with the Kármán constant calculated dynamically. This is done by relating the slip velocity to the shear stress at each location on the wall which, in turn, is calculated from an ODE obtained by wall-parallel filtering and wall-normal averaging of the streamwise momentum equation. The model is used to calculate a logarithmic relation at the off-wall location, and with it the slip velocity. The original work has been further extended by Inoue and Pullin (2011) and Inoue et al. (2012).

Pascarelli et al. (2000) addressed the need for greater resolution in the wall layer by using a multi-block LES. The outer flow was computed with a lower-resolution grid and the near-wall flow was computed with a higher-resolution one, in a domain periodic in the wall-parallel directions, but smaller than the outer-flow domain. This near-wall block was repeated in a lattice to cover the full streamwise and spanwise lengths of the overlying, low-resolution domain. The length and width of the wall-layer blocks was somewhat larger than those determined to be necessary to sustain turbulence (Jiménez and Moin, 1991). Although the flow at the interface had an artificial wavelength set by the inner-flow grid, the authors found that longer wavelengths occurred within a few grid points from the interface. At much higher Reynolds numbers, where the lengthscale separation within the inner and outer flows becomes much larger, many repeated wall-layer blocks would need to be used in this periodically repeated fashion. When the wall layer block had its upper surface at \( y^+ \approx 100 \), reaching into the logarithmic layer, first- and second-order statistics showed good agreement with single-block calculations. However, when the top of their near-wall blocks was placed at \( y^+ \approx 30 \), within the buffer layer, the Reynolds stresses were underpredicted, and spurious pressure fluctuations occurred.

The idea of simulating the overlying flow separately from the buffer layer, suggested by the work of Pascarelli et al. (2000) and some of the investigations cited above, can be pushed further by removing the buffer layer completely and replacing it by a dynamic boundary condition at its interface with the logarithmic layer, or well within the logarithmic layer itself. This approach has been tested in three recent studies. Podvin and Fraigneau (2011) generated synthetic boundary conditions from proper-orthogonal-decomposition eigenfunctions, which needed to be obtained a priori. Mizuno and Jiménez (2013) constructed boundary conditions dynamically from information in the overlying flow, by capitalizing on the self-similarity of the turbulent fluctuations across the logarithmic layer, and the independence, to a large extent, of this layer from the dynamics beneath it. García-Mayoral et al. (2012) used the idea of ‘minimal’ units of buffer-layer dynamics (Jiménez and Moin, 1991), and the universality of these dynamics across different flows, to construct a boundary condition at \( y^+ \approx 100 \), which could be imposed in a lattice in the spirit of Pascarelli et al. (2000). They synthesized conditions from different flows, including early stages of transition in boundary layers and channel flow, and tested them successfully on turbulent channels.

Compared to Mizuno and Jiménez (2013), the strategy of implementing a boundary condition just above the buffer layer, as proposed by García-Mayoral et al. (2012), has the advantage of not requiring a logarithmic layer to draw information from, so it can be used at low Reynolds numbers, when the logarithmic region is relatively thin. Conversely, it is ill-suited for large Reynolds numbers, in which resolving the flow at \( y^+ \approx 100 \), or any other \( y/\delta \ll 1 \), would be very expensive. In that case, the approach of Mizuno and Jiménez (2013), which can set the boundary condition at a fixed \( y/\delta \) within the logarithmic layer, say \( y/\delta = 0.1 \), would be much more efficient. In that sense, both approaches can be viewed as complementary. For the full simulation of a boundary layer, the strategy of García-Mayoral et al. (2012) could be used in the transitional and early-turbulence regions, while that of Mizuno and Jiménez (2013) could be implemented further downstream.

The present work takes up from that of Mizuno and Jiménez (2013), and attempts to address
Figure 2. Comparison of the spectral energy densities of the streamwise velocity $k_x k_z E_{uu}$ (top) and the spanwise one $k_x k_z E_{vv}$ (bottom), at $y^+ \approx 150$ and $y^+ \approx 300$, from the channel at $Re_\tau \approx 2000$ of Hoyas and Jiménez (2006). The shaded contours represent the spectra at $y^+ \approx 150$, and the solid ones the spectra at $y^+ \approx 300$. In (a) and (e), the stream- and span-wise wavelengths, $\lambda_x$ and $\lambda_z$, have been normalised with the channel half-height $\delta$. In (b) and (f), they are normalised with the distance to the wall, $y$. In (c) and (g), the wavelengths are normalised with the distance $\tilde{y} = y - y_0$ to a plane $y_0^+ = -12$, following Mizuno and Jiménez (2011), and in (d) and (h) using $y_0^+ = -100$, as suggested by García-Mayoral et al. (2012).

some of the caveats encountered in that first approach. Mizuno and Jiménez (2013) based their off-wall model in the fact that the length-scales of the fluctuations in the logarithmic layer are proportional to the distance to the wall (Millikan, 1938; Townsend, 1961; Jiménez, 2012). They constructed a boundary condition by rescaling the length-scales of the velocity fluctuations at a reference plane, and imposed the resulting velocities in a plane closer to the wall, with the ratio of heights of both planes being the rescaling factor. The condition was shifted in the streamwise direction with respect to the reference plane, to enforce a mean advection equal to the mean velocity at the boundary plane. The resulting simulations successfully proved that a logarithmic layer can exist independently of a supporting buffer layer, or even a physical wall, so long as the self-similarity of the lengthscales of the fluctuations is enforced. However, a few shortcomings needed to be addressed for such a method to be accurate enough for general LES applications.

From a physical perspective, the brute-force rescaling of all the flow structures by the same factor is crude. In full wall-bounded flows, the smaller eddies scale linearly with the height, but the larger ones extend all the way to the wall with their stream- and spanwise scales unmodified, except in the the viscous sublayer, $y^+ \leq 5$ where they are finally dissipated before reaching the wall (Jiménez, 2012). This is particularly evident for the structures formed by the streamwise $u$-velocity fluctuations, which can have very large length-scales (Hutchins and Marusic, 2007), as portrayed in Figure 2. The figure shows statistically how the smaller eddies scale linearly with the distance to the wall, while the larger scales remain roughly the same at different heights. A further refinement was suggested in García-Mayoral et al. (2012), who analysed the spectral densities of the fluctuations observed in DNSs, and reported that their linear scaling with $y$ does not have its origin exactly at the wall, but roughly at $y_0^+ = -100$ below it, independently of the Reynolds number of the flow. That scaling is also shown in Figure 2. In contrast, Mizuno and
Jiménez (2011) analysed the scaling with $y$ of the mixing length, defined as

$$
\ell(y) = \left( \frac{1}{u_\tau \, dy} \right)^{-1},
$$

and found that it also has an origin $y_0^+$ below the wall, although it is $Re_\tau$-dependent, tending to vanish with increasing $Re_\tau$, and already of order $\sim -10$ for $Re_\tau \approx 2000$. When a similar $y_0^+$ was calculated a posteriori for the wall-less channels of Mizuno and Jiménez (2013), values of $y_0^+ \approx +30$ for $Re_\tau \approx 850$ and $\approx +80$ for $Re_\tau \approx 2000$ were obtained. Mizuno and Jiménez used those values to rescale their results with $\tilde{y} = y - y_0$ and $\tilde{u}_\tau = u_\tau(\delta/\delta)^{1/2}$, obtaining a better agreement with the full channel data. The scaling using the virtual origin obtained by Mizuno and Jiménez for the channel of Hoyas and Jiménez (2011) is also shown in Figure 2.

Another important problem in Mizuno and Jiménez (2013) was the appearance of very large pressure fluctuations near the boundary plane. The authors traced this problem to the vanishing correlation between the flow field imposed at the boundary and that immediately above. The pressure fluctuations near the boundary plane. The authors traced this problem to the vanishing correlation between the flow field imposed at the boundary and that immediately above. The large pressure gradients would arise in order to enforce the incompressibility of the flow. Finally, the resulting Reynolds stresses $-uv$ were somewhat below those of the full channel, and for the flow to be in equilibrium the defect needed to be compensated by a larger mean shear stress. As a result, the mean profile had a higher slope than for a full DNS, and showed a slightly rounder shape.

In this paper we present some modifications to the method of Mizuno and Jiménez (2013), aimed at improving both its accuracy and dynamical resemblance to fully-resolved wall-turbulence. In Section 2, we briefly outline the method of Mizuno and Jiménez (2013), and describe the modifications we have introduced. In Section 3, we present and discuss the results of our simulations. Conclusions and directions for future work are summarised in the final Section 4.

2. Numerical method

We conduct our simulations using the numerics of Mizuno and Jiménez (2013), modifying only the boundary conditions. The code solves the time-dependent, incompressible flow in a doubly-periodic channel of height $2\delta$ and streamwise and spanwise dimensions $L_x$ and $L_z$. We denote the streamwise, wall-normal and spanwise coordinates by $x$, $y$, and $z$, and the corresponding velocities by $u$, $v$, and $w$. The density is assumed to be $\rho = 1$, and the viscosity is set to $\nu = 1/86000$ to obtain the target friction Reynolds number, $Re_\tau = u_\tau \delta/\nu \approx 2000$. Wall units, scaled with $\nu$ and the friction velocity $u_\tau$, are used throughout the paper, and indicated by the conventional ‘+’ superscript. The code is pseudo-spectral in the two wall-parallel directions, with $2/3$ dealiasing, and uses fourth-order compact finite differences on a non-uniform, staggered grid along $y$ (Lele, 1992). The integration in time uses a fractional-step, third-order RungeKutta (Spalart et al., 1991). The simulation is run at constant, zero flow rate, with the velocity scale set by the difference between the mean $u$ at the centre plane and the mean $u$ at the boundary planes, $\Delta U = 0.3$. We have applied these method on a simulation with DNS-like resolution (DNS for brevity from here on, even if the boundary conditions are artificial, and in no way ‘direct’), and on a coarse-mesh simulation with the same LES treatment of Mizuno and Jiménez (2013). The parameters of the simulations are summarised in Table 1, together with those of the original DNS-like simulation of Mizuno and Jiménez (2013) and the DNS of Hoyas and Jiménez (2006), used for comparison.

In Mizuno and Jiménez (2013), the boundary conditions for the velocities at $y_{bc}^+ \approx 100$ were constructed from the velocity fields at a reference plane $y_{ref}^+ \approx 200$. The fields were ‘shrunk’ by a lengthscale ratio $r = y_{ref}/y_{bc}$, maintaining the original amplitudes, and shifting the result in the streamwise direction to account for the different mean advection velocities at both planes.
which were assumed to be the local means. This procedure is relatively simple in Fourier space, so long as $1/r$ is an integer, and is thoroughly described in Mizuno and Jiménez (2013). Here we have refined the procedure to include the effect of an origin $y_0 = -100$ for the self-similarity law, and to restrict the ‘shrinking’ process to all but the largest scales. For the former refinement, we have chosen $y_{bc}^+ \approx 100$ and $y_{ref}^+ \approx 300$, so that $r = (y_{ref} - y_0)/(y_{bc} - y_0) = 1/2$. For the latter we have applied a wavelength-selective rescaling. Take for instance the streamwise velocity field at $y^+_{ref}$ in the Fourier plane, $\hat{u}_{ref}(k_x, k_z)$, where $k_x$ and $k_z$ are the wavenumbers associated to the wavelengths $\lambda_x$ and $\lambda_z$. In Mizuno and Jiménez (2013), a rescaled field was obtained through a rescaling function $\mathcal{R}$, so that $\hat{u}_{bc} = \mathcal{R}(\hat{u}_{ref})$. Here, we obtain the same field, but multiply it by a filter that leaves out the resulting large scales, and replace them by the original ones from $\hat{u}_{ref}$. The filter is constructed from two one-dimensional ones, $\Phi = \Phi_x(\lambda_x) \times \Phi_z(\lambda_z)$, where $\Phi_x$ is zero for $\lambda_x > 3h$ and unity for $\lambda_x < 17.5 y_{ref}$, and $\Phi_z$ is zero for $\lambda_z > 1.2 h$ and unity for $\lambda_z < 7 y_{ref}$, with smooth fifth-order-spline transitions in-between. The inverse filter $(1 - \Phi)$ is applied to the unrescaled $\hat{u}_{ref}$, to obtain the large scales to be added. In summary, we have

$$\hat{u}_{bc} = \Phi \times \mathcal{R}(\hat{u}_{ref}) + (1 - \Phi) \times \hat{u}_{ref}. \tag{2}$$

The thresholds given above are selected so that the ‘local’ scales at $y_{ref}$ are not filtered by $\Phi$, while those larger—which would be ‘local’ when viewed at some plane higher than $y_{ref}$, are filtered out. According to Flores and Jiménez (2010), ‘healthy’ turbulence can be obtained at a given height $y$ when the largest wavelengths considered are no less than $\lambda_x \approx 5y$, $\lambda_z \approx 2y$, giving a suitable estimate for the size of the ‘local’ turbulence at each height $y$. The criterion that we have imposed is that anything larger than $\approx 6$ times the ‘local’ minimum unit at $y = y_{ref}$ is filtered out completely, and anything smaller than $\approx 3.5$ times that minimum unit is left completely unfiltered.

The filtering process is illustrated in Figure 3. Note that according to Eq. (2), $\Phi$ is applied on the rescaled field and $(1 - \Phi)$ on the unrescaled one, so that some intermediate wavelengths are included in both contributions, and they are in a sense added twice. This is done intentionally to energize the intermediate scales. If both filters were applied on the original, unrescaled signal, a zero-energy gap would appear in the spectral region separating the rescaled and un-rescaled wavelengths. In real wall-turbulence, that gap is filled by a continuous cascade of scales, which are ‘local’ to the planes between $y_{ref}$ and $y_{bc}$. Since information on those scales is missing at $y_{ref}$, it is actually convenient to artificially synthesize a signal to fill that spectral gap. Note also that the streamwise shifting, which accounts for the different advection velocities at $y_{ref}$ and $y_{bc}$, is applied through the operator $\mathcal{R}$, so it does not affect the large scales. This is again intentional, since in real flows the small scales are advected with the local mean velocity, but the large ones travel with a velocity roughly uniform along their wall-normal span, and approximately equal to the bulk velocity (del Álamo and Jiménez, 2009). By making copies of the larger structures

| Case       | $L_x/\delta$ | $L_z/\delta$ | $N_x$ | $N_z$ | $N_y$ | $Re_x$ |
|------------|--------------|--------------|-------|-------|-------|--------|
| C2000      | 8$\pi$       | 3$\pi$       | 6144  | 4608  | 633   | 2003   |
| WL2600     | 4$\pi$       | 2$\pi$       | 3072  | 3072  | 569   | 2570   |
| DNS        | 4$\pi$       | 2$\pi$       | 3072  | 3072  | 569   | 1980   |
| LES        | 4$\pi$       | 2$\pi$       | 384   | 384   | 307   | 2040   |
Figure 3. Sketch of the filtering method in the streamwise direction. ——, filter \( \Phi_x \) applied on the rescaled signal, \( \Re(\hat{u}_{ref}) \); ———, equivalent filter \( \hat{\Phi}_x \) for the un-rescaled \( u_{ref} \), \( \Re(\hat{\Phi}_x \times \hat{u}_{ref}) = \Phi_x \times \Re(\hat{u}_{ref}) \); ————, filter \( 1 - \Phi_x \) applied on the unrescaled signal, to extract the large scales; ———, amplification that would be obtained on each mode of a white-noise signal by applying eq. (2).

exactly below them, we force them to be advected at the same velocity in both planes.

The direct copy of the large scales works well for the wall-parallel velocities, but fails for \( v \). Simulations which tested an implementation of eq. (2) for the three velocities soon developed large pressures near the boundaries and became unstable. However, the large scales of real flows have little \( v \), since they reach the wall and cannot pass beyond it. That impermeability is transmitted through continuity to the regions farther from the wall, where the large scales therefore experience only very weak wall-normal motions. This suggested that we could simply remove all the large scales from \( v \), so that

\[
\hat{v}_{bc} = \Phi \times \Re(\hat{v}_{ref}).
\]  

(3)

The condition on large scales derived from continuity is actually more subtle, and this direct approach has negative implications, particularly on the Reynolds stress, which will be discussed in the following section. A finer, more precise implementation has been left for future work.

3. Results

In this section we present and discuss the results of our simulations. Before that, a word of caution must be said on the DNS case. This simulation is very demanding in computational resources, both in terms of machine allocation and actual running time. Because of that, we have been unable to run the case for a time longer than \( u_\tau t/\delta \approx 1 \). That implies that the results are not statistically converged, and should be taken with caution. On the other hand, the LES results are fully converged, but lack information on how the boundary conditions affect the smaller scales, which are in this case modelled instead of resolved. Fortunately, the characteristic time for those scales is of order \( y/u_\tau \), so the DNS results are somewhat more significant, when restricted to the smaller scales.

The refinements to the boundary condition of Mizuno and Jiménez (2013) proposed here produce significant improvements on the statistics of the resulting simulations. Bulk and fluctuation results are summarised in Figure 4. The most dramatic improvement is the substantial reduction of the spike in the pressure fluctuation \( p^{+} \) as the boundary is approached, which is now essentially confined to the first plane. Besides that, the new simulations show slightly larger kinks in the velocity fluctuations near the boundary, especially for \( u \). These kinks are ubiquitous in off-wall models (Mizuno and Jiménez, 2013; Podvin and Fraigneau, 2011; García-Mayoral et al., 2012). In contrast, the new results are more accurate away from the wall, and match almost exactly the real fluctuations for \( y \gtrsim 0.2\delta \). It was argued in Mizuno and Jiménez (2013) that the effect of the large pressures near the boundary acted on relatively large length scales, and its influence reached well into the channel. Now that those have been
Figure 4. Flow statistics from the present channel DNS and LES, compared to those of Mizuno and Jiménez (2013) and Hoyas and Jiménez (2006). (a) mean velocity profile in defect form; (b) rms velocity fluctuations; (c) Reynolds shear stress; (d) pressure fluctuations. ——, present DNS; ——, present LES; —— DNS of Mizuno and Jiménez (2013); ——, DNS of Hoyas and Jiménez (2006).

eliminated, the perturbations induced by the boundary conditions vanish soon away from their plane.

The better agreement away from the boundary also translates in a better behaviour of the mean profile. To obtain an accurate profile, it is essential that the Reynolds stresses are well represented across the whole channel. A thorough discussion on the delicate equilibrium between the local turbulence and the shear of the mean profile can be found in Tuerke and Jiménez (2013). In essence, any defect in $-uv$ needs to be compensated by a larger mean shear $\nu dU/dy$. Since most of the total stress away from the walls is Reynolds stress, small relative errors in it may lead to large relative errors in $dU/dy$. Those errors accumulate along the $U$-profile, and can lead to substantial errors in $U$ itself. The effect is particularly grievous if it is not confined to a region near the boundary, where the simulation can always be expected to be ‘wrong,’ and the region discarded (Kawai and Larsson, 2012). Even if it shows an improved mean profile compared to Mizuno and Jiménez (2013), the Reynolds stress of the present DNS departs significantly from that of the full-channel DNS of Hoyas and Jiménez (2006). Furthermore, the total stress, not shown, but of which the dominant contribution away from the wall is $-uv$, also departs significantly from the linear trend, even in the core region of the channel. This implies that the mean flow is, in the time span considered, not in equilibrium, and is a clear indicator of the failure to reach statistical convergence, mentioned above. The LES, which is in contrast fully converged, exhibits a sharp dip in $-uv$ at the boundary (as does the DNS), but it nevertheless follows better the results of the full direct simulation C2000 above $y \gtrsim 0.2\delta$, and thus results in a somewhat improved $U$-profile for most of the channel, although the error is still comparable to that of Mizuno and Jiménez (2013).

The fluctuations provide information on the flow in the broad picture, but to analyse whether those fluctuations are correctly distributed among small and large scales, we need to inspect their
Figure 5. Two-dimensional spectral energy densities, from left to right, at $y^+ \approx 100, 105, 150$ and 200. Shaded contours represent the present DNS simulation, and solid lines the reference full-channel DNS (Hoyas and Jiménez, 2006). Premultiplied spectra of the (a) streamwise, (b) wall-normal, and (c) spanwise velocities, $k_xk_zE_{uu}$, $k_xk_zE_{vv}$, and $k_xk_zE_{ww}$, with contours every 0.025 $u'^+$. 0.025 $v'^+$. $w'^+$. (d) Cospectrum of the Reynolds stress, $-k_xk_zE_{uv}$, with contours every 0.035 $uv^+$. (e) Spectrum of the pressure fluctuations, $k_xk_zE_{pp}$, with contours every 0.025 $p'^+$. 
Figure 6. Two-dimensional spectral energy densities, from left to right, at $y^+ \approx 100, 150, 200$ and $300$. Shaded contours represent the present LES simulation, and solid lines the reference full-channel DNS (Hoyas and Jiménez, 2006). Premultiplied spectra of the (a) streamwise, (b) wall-normal, and (c) spanwise velocities, $k_x k_z E_{uu}$, $k_z k_z E_{vv}$, and $k_z k_z E_{ww}$, with contours every $0.025 u'^+2$, $0.025 v'^+2$ and $0.025 w'^+2$. (d) Cospectrum of the Reynolds stress, $-k_x k_z E_{uv}$, with contours every $0.035 u'^+2$.

spectral energy distributions, both along the stream- and spanwise wavelengths, and along the wall-normal coordinate $y$. Energy densities are portrayed in Figure 5 for our DNS, and in Figure 6 for our LES, and compared with the true densities of Hoyas and Jiménez (2006). In Figure 5, they are shown at the boundary plane, one plane immediately above it, and 50 and 100 wall units above. At that height, $y^+ = 200$ or $y = 0.18$, the flow is already beginning to relax to a state that in general terms resembles that of the full, reference simulation C2000 quite closely. The most significant differences occur below that height. Figure 6 includes results at the boundary $y^+ \approx 100$ and at $y^+ = 150, 200$ and $300$, already showing that the flow resembles quite closely the full DNS at that height. Of the three velocities, the spanwise $w$ exhibits a quite accurate behaviour through the full collection of $y$ planes, and the energy in $v$ is essentially distributed
among the correct length scales, although its values are somewhat low immediately away from the boundary. That translates in the dip in \( v' \) observed in Figure 4(b) for \( y \) values just above the boundary. The removal of the large \( v \)-scales, very clear in the absence of the tail for \( \lambda^+_k \gtrsim 5000 \) in Figure 5(b4), and to a lesser extent in Figure 6(b4), is likely responsible of the slight shift of the full \( v \)-spectrum toward smaller scales.

On the other hand, the modifications in the distribution of \( u \) are more profound. The boundary condition is distributing the energy correctly in a statistical sense, as shown in Figures 5(a1) and 6(a1). However, there is a sharp transition between the behaviour at the boundary and one plane immediately above, shown in Figure 5(a2). In the second plane, the intensity of \( E_{uu} \) is unusually large, and spreads into regions to the sides of the true high-density spectral region. As a result, the distance \( y \) taken by \( u \) to relax is somewhat larger than for the other two velocities. The only possible source for the sharp change in \( u \) at the wall is large pressure fluctuations. We have already discussed the existence of a residual spike in \( p' \), compared to Mizuno and Jiménez (2013), which now exists only at the boundary plane. The spectral distribution of that peak is revealed in Figure 5(e1) for the DNS (the computation of the pressure spectra for the LES simulation was under way while preparing this manuscript but, given the similarity in all the other results, there is no reason to believe it should be substantially different from that of the DNS).

At that first plane, the pressure spectrum increases in magnitude, and is slightly shifted towards smaller wavelengths. The shape of the spectral distribution is somewhat reminiscent of that of the \( u \) spectrum in the second plane, particularly in the amplified region with \( \lambda^+_k \approx 1000-2000 \) (figure 5a2), supporting the connection of the sudden increase in \( u \) fluctuations immediately above the boundary with the large pressure fluctuations at the boundary itself. Nevertheless, those large pressures have already vanished at the second plane, and the spectral distribution of \( p \) closely resembles that of Hoyas and Jiménez (2006) from that second panel on. This strongly suggests that, save for the boundary plane, our modifications to Mizuno and Jiménez (2013) have solved the deviations experienced by the pressure.

There is, however, a price to be paid for removing completely the large scales of \( v \), which is made clear in the spectral distribution of the Reynolds stresses in Figures 5(d) and 6(d). While the large-scale tail of \( E_{vv} \) is negligible, its contribution, together with \( E_{uu} \), to populating the cospectral region of Reynolds stresses with \( \lambda^+_k \gtrsim 5000 \) is not. In real flows at moderate or high Reynolds numbers, that tail actually contributes a significant amount to the total \( -uv \), as shown in the figures. That is because, even if the corresponding values of \( v \) are quite low, those of \( u \) can be fairly large. Unlike the rest of the variables, the spectral distribution of the Reynolds stress takes longer wall-normal distances to relax into a ’healthy’ turbulence state. In fact, the tail for the long streamwise wavelengths has not yet recovered at \( y^+ \approx 300 \). Solving this defect in \( -uv \), which is now identified as a large-scale issue, should be the next step.

By introducing a variable rescaling between the flow velocities at \( y = y_{ref} \) and \( y = y_{he} \), we have identified a potential problem in leaving the large scales of the three velocities unrescaled, which in fact led to the destabilization of the simulations. Setting the large scales of the wall-normal velocity to zero has solved that issue, but it is probably not the most accurate solution, even from a physics perspective, and it is easy to see that it should only strictly hold for infinitely long and infinitely wide structures, \( k_x = k_z = 0 \). More accurate procedures are currently under study.

4. Conclusions and future work

The direct simulation of turbulent flows at moderate to high Reynolds numbers is inaccessible for any existing computing facility. Alternatively, LES can successfully reproduce physical flows, but it requires a sufficiently large separation between the scales at which energy is generated and those at which it is dissipated. Unfortunately, that assumption does not hold near walls, since the production scale approaches linearly the viscous one as the distance to the wall decreases.
This forces the resolution requirements of LES to approach those of DNS near walls, so the advantage of the latter is lost, unless the near-wall layer is substituted by a model.

In the present work, we have reassessed the modelling strategy of Mizuno and Jiménez (2013), which constructs boundary conditions at an off-wall plane. This is done in real time during the simulation, by rescaling the length scales of the flow at some reference, overlying plane. The rescaling assumes, and imposes that the length scales of the fluctuations are proportional to the wall-normal distance. We have modified this model to introduce several concepts, based mainly on our physical understanding of the logarithmic layer. From the analysis of the spectral densities of the fluctuations in full-channel DNSs, a virtual origin can be derived for the length scales, which is essentially independent of the Reynolds number, and 100 wall-units below the wall. Other virtual origins have also been proposed (Mizuno and Jiménez, 2011), but the effect is in any case small, and negligible at large Reynolds numbers, once the distances from the wall to the reference and boundary planes are much larger than the distance from the wall to the virtual origin. More importantly, we have introduced a method to incorporate the dependence with wavelength of the rescaling factor. While the length scales of smaller eddies are proportional to the height—as in the original model of Mizuno and Jiménez (2013), larger structures maintain their size constant at different distances from the wall. In our new model, we have proposed to leave the large scales unrescaled. If that is done simultaneously on the three velocities, the code becomes unstable, but this can be sorted by setting the large-scale $v$ to zero. This procedure is actually reasonable from a physical perspective, since very large eddies extend all the way to the wall, where $v$ is always zero. Because of mass conservation, their $v$ should also be very small at any other height.

We have presented results obtained with the model thus refined, showing significant improvement over the original model. In particular, the large artificial pressure fluctuations that appeared at the boundary, and extended well into the flow, have now been removed. Only a much smaller residual peak remains, now affecting only the first plane of the simulation. Nevertheless, that peak seems to perturb intensely the fluctuations of the streamwise velocity immediately above, but those perturbations extend, with milder intensity, not farther than $y \approx 0.1 \delta$. A more important problem is that the slope of the velocity profile is overpredicted, leading to errors in the profile itself. This occurs because of a defect in the Reynolds stress, which needs to be compensated by the viscous one. The defect is mainly located in the large scales, $\lambda^+ \gtrsim 5000$, for which the contribution to the Reynolds stress has been artificially forced to vanish when setting the wall-normal velocity to zero. Strategies to mitigate this effect, while keeping the large-scale $v$ small, are currently under investigation.

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**References**

Bagwell T, Adrian R, Moser R and Kim J 1993 Improved approximation of wall shear stress boundary conditions for large eddy simulation *Near-Wall Turbulent Flows* ed R So et al (New York: Elsevier) pp. 265–76.

Balaras E, Benocci C and Piomelli U 1996 Two-layer approximate boundary conditions for large-eddy simulations *AIAA J.* 34, 1111–19.

Cabot W and Moin P 1999 Approximate wall boundary conditions in the large eddy simulation of high Reynolds number flow. *Flow, Turb. and Comb.* 63, 269–91.

Choi H and Moin P 2012 Grid-point requirements for large-eddy simulation: Chapman’s estimates revisited *Phys. Fluids* 24, 011702.

Chung D and Pullin D 2009 Large-eddy simulation and wall modelling of turbulent channel flow *J. Fluid Mech.* 631, 281–309.
del Álamo J C and Jiménez J 2009 Estimation of turbulent convection velocities and corrections to Taylor’s approximation J. Fluid Mech. 640, 5–26.
Flores O and Jiménez J 2010 Hierarchy of minimal flow units in the logarithmic layer Phys. Fluids 22, 071704.
García-Mayoral R, Pierce B and Wallace J 2012 Off-wall boundary conditions for turbulent simulations Annu. Research Briefs ed P Moin and J Nichols (Stanford: Center for Turbulence Research) pp. 275–88.
Hoyas S and Jiménez J 2006 Scaling of the velocity fluctuations in turbulent channels up to Reₜ = 2003 Phys. Fluids 18, 011702.
Hutchins N and Marusic I 2007 Evidence of very long meandering streamwise structures in the logarithmic region of turbulent boundary layers J. Fluid Mech. 579, 1–28.
Inoue M, Mathis R, Marusic I and Pullin D 2012 Inner-layer intensities for the flat-plate turbulent boundary layer combining a predictive wall-model with large-eddy simulations Phys. Fluids 24, 075102.
Inoue M and Pullin D 2011 Large-eddy simulation of the zero-pressure-gradient turbulent boundary layer up to Reₜ = O(10¹²) J. Fluid Mech. 686, 507–33.
Jiménez J 2012 Cascades in wall-bounded turbulence Ann. Rev. Fluid Mech. 44, 27–45.
Jiménez J and Moin P 1991 The minimal flow unit in near-wall turbulence J. Fluid Mech. 225, 213–40.
Jiménez J and Vasco C 1998 Approximate lateral boundary conditions for turbulent simulations Studying turbulence using numerical simulation ed P Moin and W Reynolds (Stanford: Center for Turbulence Research) pp. 399–412.
Kawai S and Larsson J 2012 Wall-modeling in large eddy simulation: length scales, grid resolution and accuracy Phys. Fluids 24, 015105.
Lele S 1992 Compact finite difference schemes with spectral-like resolution J. Comput. Phys. 103, 16–42.
Millikan C B 1938 A critical discussion of turbulent flows in channels and circular tubes Proc. 5th Int. Conf. on Applied Mechanics (New York: Wiley) pp. 386–92.
Mizuno Y and Jiménez J 2011 Mean velocity and length-scales in the overlap region of wall-bounded turbulent flows Phys. Fluids 23, 085112.
Mizuno Y and Jiménez J 2013 Wall turbulence without walls J. Fluid Mech. 723, 429–55.
Nicoud F, Winkelmanns G, Carati D, Baggett J and Cabot W 1998 Boundary conditions for LES away from the wall Studying turbulence using numerical simulation ed P Moin and W Reynolds (Stanford: Center for Turbulence Research) pp. 413–22.
Pascarelli A, Piomelli U and Candler G 2000 Multi-block large-eddy simulations of turbulent boundary layers J. Comp. Phys. 157, 256–79.
Piomelli U 1999 Large-eddy simulations: achievements and challenges Prog. Aerosp. Sci. 35, 335–62.
Piomelli U and Balaras E 2002 Wall-layer models for large-eddy simulations Annu. Rev. Fluid Mech. 34, 349–74.
Podvin B and Fraigneau Y 2011 Synthetic wall boundary conditions for the direct numerical simulation of wall-bounded turbulence J. Fluid Mech. 12, 1–26.
Spalart P R, Moser R D and Rogers M M 1991 Spectral methods for the Navier-Stokes equations with one infinite and two periodic directions J. Comput. Phys. 96, 297–324.
Townsend A A 1961 Equilibrium layers and wall turbulence J. Fluid Mech. 11, 97–120.
Tuerke F and Jiménez J 2013 Simulations of turbulent channels with prescribed velocity profiles J. Fluid Mech. 723, 587–603.