Robust Fixed-Time Inverse Dynamic Control for Uncertain Robot Manipulator System

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This paper proposes a novel robust fixed-time control for the robot manipulator system with uncertainties. Based on the uniform robust exact differentiator (URED) algorithm, a robust control term is constructed. Then, a robust fixed-time inverse dynamics control (IDC) is proposed. For the proposed control method, the fixed-time stability of a closed-loop system with uncertainties is strictly proved. The newly proposed method exhibits the following two attractive features. First, the proposed control scheme extends the existing fixed-time IDC for the robot manipulator system to the robust control scheme. Second, the proposed method is strictly nonsingular rather than the commonly used approximate approach. Simulation result demonstrates the effectiveness of the proposed control scheme.

1. Introduction

Based on the development of control theory, measurement technology, and material science, the performance of robot manipulator has been greatly improved, and it has been widely used in the engineering areas, such as undersea exploration, space exploration, repetitive labor, and emergency rescue [1–6]. Most of the robot manipulators adopted the trajectory tracking control to complete the desired mission. Thus, the study on the high-precision trajectory tracking control has become one of the most representative control problems in the research field of robot manipulator. As a classical control method, proportional-integral-derivative control (PID) [7] has the advantage of easy implementation in engineering. Many PID-based methods have been applied to design the tracking controller of the robot manipulator, such as fuzzy PID [8], neural PID [9], robust nonlinear PID, [10] and adaptive PID [11]. However, the conventional PID and these PID-based methods in [8–11] are asymptotically stable, which means that the convergence time of tracking error is infinite.

Actually, the fast convergence rate is a very important desired control performance of the robot manipulator. Thus, it is necessary to design a finite-time controller for the robot manipulator. Since the 1990s, based on the development of finite-time stability theories [12–15], the study on finite-time convergence method has become a hotspot. And, to guarantee the robustness of system in the presence of uncertainties, the sliding-mode control (SMC) is a good candidate [16]. Combining the finite-time convergence and SMC, many terminal sliding-mode control (TSMC) schemes have been developed. In [17], for the first-order chaotic system, the authors adopted the FTSMC (fast terminal sliding-mode control) and the composite nonlinear feedback approach to guarantee the finite-time and high-performance synchronization of the chaotic systems in the presence of the external disturbances, parametric uncertainties, Lipschitz nonlinearities, and time delays. For the high-order system, there is a singular problem in the conventional TSMC method [18]. To avoid the singular problem and guarantee the finite-time stability, in [19], a novel recursive singularity-free FTSMC for finite-time tracking control of nonholonomic systems was proposed. In [20], a nonsingular FTSMC was developed, and the uncertainties were compensated by the estimation of disturbance observer. Recently, the development of TSMC also continuously changed the control design of a robot manipulator in engineering [21]. In [22], the authors adopted conventional TSMC to
design the robust MIMO finite-time controller. However, an undesirable singular problem can be caused by the conventional TSMC. To avoid the singular problem, in [23–25], the nonsingular terminal sliding-mode surface was applied to design the nonsingular TSMC scheme. In [26, 27], the authors adopted the integral terminal sliding mode to design controller, and then the singular problem also was eliminated. Besides the TSMC-based methods in [22–27], based on finite-time convergence theory of the homogeneous system, the results in [28, 29] have been developed and also can guarantee the finite-time convergence. These results in [28, 29] not only guarantee the finite-time stability but also eliminate the above singular problem in the conventional TSMC. Although these FTC methods in [22–29] can achieve the finite-time convergence of tracking error, the finite convergence times in [22–29] are heavily dependent on the initial system conditions. Thus, the convergence time may be an unacceptable long time for the large initial system condition cases.

In recent years, an advanced stability notion, that is, fixed-time stability, has been developed [30, 31]. The fixed-time stability control method can not only derive the system trajectories to equilibrium in finite time but also guarantee that the convergence time is independent on the initial system conditions. Thus, the above initial system condition problem in finite-time control can be eliminated. So far, for the robot manipulator, some fixed-time control methods have been developed in [32–34]. In [32, 33], the fixed-time controllers were developed based on the fixed-time sliding-mode control. However, to avoid the singular problem, the schemes in [32, 33] adopted a nonlinear function to approximate the singular control term. Thus, the schemes in [32, 33] can only guarantee that the tracking error converges to a neighborhood of zero. Recently, in [34], a fixed-time inverse dynamics control (IDC) scheme was proposed by using the bilimit homogeneity technique. Unlike the approximate fixed-time schemes in [32, 33], the method in [34] is nonsingular and can strictly guarantee that the tracking error converges to zero. In addition, for the control scheme in [34], since the control design is based on the commonly used IDC, the error dynamics are completely decoupled and the tracking performance is very easy to quantify [34]. However, the fixed-time IDC in [34] does not consider uncertainties.

For the robot manipulator system, the fixed-time control can eliminate the initial system condition problem of conventional finite-time control. However, existing fixed-time control results in [32, 33] which cannot drive the tracking error to zero, which implies that the manipulator cannot accomplish some high-precision operation tasks. The IDC results in [34] can guarantee that the tracking error converges to zero, but the uncertainties are not considered in [34]. Since the actual robot manipulator is a typical uncertain system, the fixed-time IDC in [34] may not achieve the desired fixed-time convergence performance in practical situations. Thus, it is necessary to design a new fixed-time control scheme which can not only strictly drive the tracking error to zero but also compensate the uncertainties in fixed time. The main motivation of this paper is to propose a new scheme to eliminate the above problems of existing fixed-time control for the robot manipulator in [32–34].

The main difficulties to avoid the above two problems of existing results is how to design a scheme which can not only strictly drive the tracking error to zero in fixed time but also fully compensate the uncertainties in fixed time. In this paper, firstly, a robust control term based on the uniform robust exact differentiator (URED) algorithm is designed to compensate the uncertainties in fixed time. Then, a robust control term is integrated into the sliding-mode surface design. In this way, the robust fixed-time IDC is developed by using the IDC, the bilimit homogeneity technique in [34], and the proposed robust control term. In contrast to the aforementioned results, the contributions of this paper are as follows:

(1) The main theoretical contributions of this paper are that the proposed method can not only extend the commonly used IDC for trajectory tracking of robot manipulators to fixed-time stable scheme but also suppress the bounded uncertainties of the robot manipulator system in a fixed time. Thus, the proposed method extends the recent fixed-time IDC result in [34] to a robust scheme.

(2) Since the proposed scheme is designed based on the bilimit homogeneity technique in [34] and the additional URED is nonsingular, the proposed scheme does not require the commonly used approximate approach used in existing fixed-time control schemes in [32, 33]. Thus, the proposed method can drive the tracking error to zero rather than a neighborhood of zero in [32, 33].

The remaining parts of this paper are as follows. In Section 2, the dynamic model, the fundamental facts, and the motivation of this paper are expounded. The main results are presented in Section 3. In Section 3.1, a novel robust fixed-time IDC is developed for an uncertain robot manipulator system. Then, in Section 3.2, fixed-time stability of the proposed controller is presented. In Section 4, a simulation process verifies the effectiveness of the proposed scheme. In Section 5, the conclusion of the whole paper is presented.

Notations: the following notations will be used in this paper: $t$ denotes the time and the initial time is 0. Let $\| \cdot \|$ denote the Euclidean norm of a vector and its induced norm of a matrix.

2. Preliminaries

2.1. Dynamic Model of Robotic Manipulator with Uncertainties. The dynamics of a rigid robotic manipulator with uncertainties and $n$-degree of freedom are given as [32, 33]

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau + d, \]

(1)

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ denote the vector of joint position, vector of joint velocity, and vector of joint acceleration, respectively. $M(q) \in \mathbb{R}^{n \times n}$ is the positive definite symmetric inertia matrix. $C(q, \dot{q}) \in \mathbb{R}^{n \times n}$ is the centrifugal-Coriolis forces matrix. $G(q) \in \mathbb{R}^n$ is the vector of gravitational torque. $\tau = [\tau_1, \ldots, \tau_n]^T \in \mathbb{R}^n$ is the input torque. $d \in \mathbb{R}^n$ denotes the
uncertainties, which can be caused by the unmodeled dynamics, external environment, and parameter uncertainties. The desired joint position is defined as \( \mathbf{q}_d \in \mathbb{R}^n \). The tracking error is defined as

\[
e_1 = q - q_d,
\]

where \( e_1 = [e_{11}, \ldots, e_{1n}]^T \in \mathbb{R}^n \).

### 2.1.1. Control Objective
The objective is to design the input torque \( \tau \) in such a way that the tracking error \( e_1 = 0 \) in fixed time despite the presence uncertainties. And the convergence time is always bounded regardless of system initial conditions.

Let \( \dot{e}_1 = e_2 \), where \( e_2 = [e_{21}, \ldots, e_{2n}]^T \in \mathbb{R}^n \). Then, calculating the time derivative of along the trajectories (1), we have

\[
\begin{align*}
\dot{e}_1 & = e_2, \\
\dot{e}_2 & = f(q, \dot{q}) + b(q)\tau + \Delta,
\end{align*}
\]

where

\[
\begin{align*}
f(q, \dot{q}) & = -(M(q))^{-1}(C(q, \dot{q})\dot{q} + G(q)) - \ddot{q}_d, \\
b(q) & = (M(q))^{-1}, \\
\Delta & = (M(q))^{-1}d = [\delta_1, \ldots, \delta_n]^T.
\end{align*}
\]

Assumption 1 is assumed to be valid throughout this paper.

**Assumption 1.** The uncertainties \( \delta_j (j = 1, \ldots, n) \) is differentiable and satisfies \( |\delta_j| \leq D_{\text{max}} \) and \( |\delta_j| \leq D_{\text{max}}^\delta \), where \( D_{\text{max}} \) and \( D_{\text{max}}^\delta \) are positive constants.

### 2.2. Fundamental Facts
Before giving the control scheme, some useful definitions are recalled for convenience.

**Definition 1** (finite-time stability [35]). For the following uncertain system,

\[
\dot{x} = p(x, \psi),
\]

where \( x \in \mathbb{R}^n \) is the system state, \( \psi \in \mathbb{R}^m \) is the system uncertainty, and \( p(x, \psi): \mathbb{R}^n \rightarrow \mathbb{R}^n \) is a nonlinear function, the origin of (5) is globally finite-time stable if it is globally asymptotically stable and any solution \( x(t, x(0)) \) of (5) reaches the equilibrium at some finite-time moment, i.e., \( \forall t \geq T(x(0)): x(t, x(0)) = 0 \), where \( T(x(0)): \mathbb{R}^n \rightarrow R, 0 \cup [0] \) is the convergence time function.

**Definition 2** (finite-time stability [35]). For uncertain system (5), the origin of (5) is finite-time stable if it is globally finite-time stable and the convergence time function \( T(x(0)) \) is bounded, i.e., \( \exists T_{\text{max}} \geq 0: \forall x(0) \in \mathbb{R}^n, T(x(0)) \leq T_{\text{max}} \), where \( T_{\text{max}} \) is a positive constant and independent on the initial system condition \( \dot{x}(0) \).

**Lemma 1** (see [36], URED algorithm). Consider a dynamic system in the presence of uncertainty \( f_0(t) \):

\[
\begin{align*}
\dot{\sigma}_0 & = -\bar{\Phi}_1(\sigma_0) + \sigma_1, \\
\dot{\sigma}_1 & = -\bar{\Phi}_2(\sigma_0) - f_0(t),
\end{align*}
\]

where \( \sigma_j (j = 0, 1) \) is the system state and \( f_0(t) \) is bounded as \( |f_0(t)| \leq L \). The functions \( \bar{\Phi}_1(\sigma_0) \) and \( \bar{\Phi}_2(\sigma_0) \) are given as

\[
\begin{align*}
\bar{\Phi}_1(\sigma_0) & = |\sigma_0|^{(1/2)} \cdot \text{sign}(\sigma_0) + \frac{\mu}{2} |\sigma_0|^{(2/3)} \cdot \text{sign}(\sigma_0), \\
\bar{\Phi}_2(\sigma_0) & = \frac{1}{2} |\sigma_0|^{(2/3)} \cdot \text{sign}(\sigma_0) + 2\mu |\sigma_0|^{(2/3)} \cdot \text{sign}(\sigma_0),
\end{align*}
\]

where \( \mu > 0 \) is a scalar. If \( \bar{\Phi}_1 \) and \( \bar{\Phi}_2 \) are in the set

\[
\mathcal{R} = \left\{ (\bar{\Phi}_1, \bar{\Phi}_2) \in \mathbb{R}^2 | 0 < \bar{\Phi}_1 < 2\sqrt{L}, \bar{\Phi}_2 > \frac{K_1^2}{4} + \frac{4L^2}{K_1} \right\}
\]

\( \cup \{ (\bar{\Phi}_1, \bar{\Phi}_2) \in \mathbb{R}^2 | \bar{\Phi}_1 > 2\sqrt{L}, \bar{\Phi}_2 > 2L \} \),

then we have the following two propositions.

**Proposition 1.** The Lyapunov function is defined as

\[
V_1 = \sigma^TP_\sigma \sigma,
\]

where \( \sigma = [\bar{\Phi}_1(\sigma_0), \sigma_1]^T \). \( P_\sigma \) is a symmetric and positive definite matrix. For some \( T > 0 \), the derivative of the Lyapunov function \( V_1 \) satisfies the inequality

\[
\dot{V}_1 \leq -k_1(P_\sigma, \epsilon)V_1^{(1/2)} - k_2(P_\sigma, \epsilon)\|\sigma_0\|^{(1/2)}V_1,
\]

where \( k_j(P_\sigma, \epsilon) (j = 1, 2) \) is a positive scalar.

**Proposition 2.** The states \( \sigma_0 \) and \( \sigma_1 \) will converge to the origin uniformly in time \( t_f \). And the convergence time \( t_f \) is bounded as \( t_f \leq T_{\text{max}} \), where \( T_{\text{max}} \) is a positive constant and independent on the initial conditions \( \sigma_0(0) \) and \( \sigma_1(0) \).

**Lemma 2** (see [34]). Consider a dynamic system as follows:

\[
\begin{align*}
\dot{\epsilon}_0 & = -K_p |\epsilon_0|^{a_0} \cdot \text{sign}(\epsilon_0) - L_p |\epsilon_0|^{b_0} \cdot \text{sign}(\epsilon_0), \\
- L_d |\epsilon_0|^{\beta_0} \cdot \text{sign}(\epsilon_0),
\end{align*}
\]

where \( \epsilon_0 \in \mathbb{R} \) and \( \dot{\epsilon}_0 \in \mathbb{R} \) are the system state. If the constants \( K_p > 0, K_d > 0, L_p > 0, L_d > 0, 0 < a_0 < 1, a_0 = (2a_1/(a_1 + 1)), \) \( \beta_0 = 2a_1 + 1 \), and \( \beta_0 = ((2a_1)/((a_1 + 1))) \) and the initial system condition \( \epsilon_0(0) \) is bounded, then the system state is fixed-time stable, i.e.,

\[
\begin{align*}
e_0(t) & = 0, \\
\dot{\epsilon}_0(t) & = 0, \text{ if } t \geq t_c,
\end{align*}
\]

where \( T_c \) is a positive constant and independent on the initial system conditions.

**Remark 1.** The proof of Lemma 1 can be referred to Appendices A and B of [36]. And the expression of \( P_\sigma, \bar{\Phi}_1(P_\sigma, \epsilon) \) and \( \bar{\Phi}_2(P_\sigma, \epsilon) \) are given in Appendix A of [36].
expression of $T_\phi$ is given in Appendix B of [36]. The proof of Lemma 2 can be referred to the proof of Theorem 3.1 in [34].

2.3. Problem Description and Purposes of this Paper

2.3.1. Problem Description. For the robotic manipulator system (1), the conventional finite-time control scheme can achieve a finite convergence time. However, the convergence time is affected by the initial system conditions $e_1(0)$ and $e_2(0)$. Thus, the convergence time may be large in the case that $e_1(0)$ and $e_2(0)$ are large. Recently, based on the fixed-time control, the control schemes proposed in [32–34] can eliminate the effect brought by initial system conditions. However, to avoid the singular problem, the schemes in [32, 33] adopted the approximate method. Thus, these schemes in [32, 33] only guaranteed the tracking error $e_1(0)$ converges to a neighborhood of zero. The results in [34] adopted the IDC method and bilimit homogeneity technique to design a scheme which can solve the singular problem without using approximate method in [32, 33]. However, the method in [34] assumed that the robotic manipulator system is certain, i.e., the uncertainties vector $d = 0$. Obviously, $d$ is not equal to zero in practical case. The uncertainties $d$ may be caused by the unmodeled dynamics, external environment, and parameter uncertainties.

2.3.2. Purpose of This Paper. To further improve the performance of fixed-time control for the robotic manipulator system, the research topic of this paper is to designing a new robust fixed-time IDC which not only can strictly derive the tracking error $e_1(0)$ to zero but also compensate the uncertainties $\tilde{d} = 0$ in fixed time.

3. Main Result

3.1. Control Design. To facilitate the design and analysis, for the vector $\xi = [\xi_1, \ldots, \xi_n]^T \in R^n$ and constant $\alpha > 0$, the vector $S(\xi)^T \in R^n, \phi_1(\xi) \in R^n, \phi_2(\xi) \in R^n$ is defined as

$$S(\xi)^T = [\xi_1 \text{ sign}(\xi_1), \ldots, \xi_n \text{ sign}(\xi_n)]^T,$$

$$\phi_1(\xi) = [\phi_1(\xi_1), \ldots, \phi_1(\xi_n)]^T,$$

$$\phi_2(\xi) = [\phi_2(\xi_1), \ldots, \phi_2(\xi_n)]^T,$$

where $\phi_1(\xi_i)$ and $\phi_2(\xi_i)$ $i = 1, 2, \ldots, n$ are defined as

$$\phi_1(\xi_i) = \sqrt{\xi_i^2 + 1} \text{ sign}(\xi_i),$$

$$\phi_2(\xi_i) = \frac{1}{2} \text{ sign}(\xi_i) + \mu \xi_i \sqrt{\xi_i^2 + 1} \text{ sign}(\xi_i),$$

where $\mu > 0$ is a scalar.

Then, the robust fixed-time IDC for the uncertain robot manipulator system (3) is design as

$$T = (b(q))^{-1} (-f(q, q) + H(e_1, e_2) + R(e_1, e_2)).$$

where $H(e_1, e_2) \in R^n$ is the fixed-time convergence term and designed as in [33]:

$$H(e_1, e_2) = -K_1S(e_1)^{\alpha_1} - K_2S(e_2)^{\alpha_2} - L_1S(e_1)^{\gamma_1} - L_2S(e_2)^{\gamma_2}.$$  \hspace{1cm} (16)

The parameters satisfy the following conditions:

$$\begin{align*}
K_1 &> 0, \\
K_2 &> 0, \\
L_1 &> 0, \\
L_2 &> 0, \\
0 &< \omega_1 < 1, \\
\omega_2 &> \frac{2\omega_1}{(\omega_1 + 1)}, \\
y_1 &< 2\omega_1 + 1, \\
y_2 &> \frac{2\omega_1 + 1}{(\omega_1 + 1)}
\end{align*}$$ \hspace{1cm} (17)

where $R(e_1, e_2)$ is the robust term and designed as

$$R(e_1, e_2) = -k_1\phi_1(\sqrt{H(e_1, e_2)}) + \int_0^t (-k_2\phi_2(\sqrt{H(e_1, e_2)}) \, dv,$$

$$H(e_1, e_2) = e_2 - \int_0^t H(e_1, e_2) \, dv.$$ \hspace{1cm} (18)

The parameters satisfy the following conditions:

$$\{k_1, k_2 \in R^2 | 0 < k_1 \leq 2\sqrt{D_{max}}, k_2 > k_1^2/4 + 4(D_{max})^2\}$$

$$\cup \{k_1, k_2 \in R^2 | k_1 > 2\sqrt{D_{max}}, k_2 > 2D_{max}\}.$$ \hspace{1cm} (19)

3.2. Stability Analysis

Theorem 1. Consider the uncertain robot manipulator system (3) adopts the controller (15). If Assumption 1 is valid and the control parameters are chosen as in (17) and (19), then system (3) is fixed-time stable.

Proof. Substituting the proposed controller (15) into (3), we have
Complementary error satisfaction of Assumption 1, then the fixed-time convergence of $\xi_k$ is fixed-time stable.

\begin{equation}
\bar{e}_i = H_i + R_i + \delta_i,
\end{equation}

where

\begin{equation}
H_i = -K_1|\xi_i|^{\psi_1}\text{sign}(\xi_i) - K_2|\xi_i|^{\psi_2}\text{sign}(\xi_i) - L_1|\xi_i|^{\psi_1}\text{sign}(\xi_i) - L_2|\xi_i|^{\psi_2}\text{sign}(\xi_i),
\end{equation}

\begin{equation}
R_i = -k_1\phi_1(\xi_i - \int_0^{t_0} H_i \, dt) + \int_0^{t_0} (-k_2\phi_2(\xi_i - \int_0^{t_0} H_i \, dt)) \, dt.
\end{equation}

\begin{equation}
\eta_0 = e_{2i} - \int_0^{t_0} H_i \, dt \quad \text{and} \quad \eta_{1i} = \int_0^{t_0} (-k_2\xi_i(\xi_i - \int_0^{t_0} H_i \, dt)) \, dt + \delta_i.
\end{equation}

Let $V_3 = e_{1i}e_{2i} + e_{2i}(-K_1|\xi_i|^{\psi_1}\text{sign}(\xi_i) - K_2|\xi_i|^{\psi_2}\text{sign}(\xi_i) - L_1|\xi_i|^{\psi_1}\text{sign}(\xi_i) - L_2|\xi_i|^{\psi_2}\text{sign}(\xi_i)) + e_{2i}(-k_1\phi_1(\xi_i - \int_0^{t_0} H_i \, dt) + \int_0^{t_0} (-k_2\phi_2(\xi_i - \int_0^{t_0} H_i \, dt)) \, dt + \delta_i).
\end{equation}

Considering $\eta_0 = e_{2i} - \int_0^{t_0} H_i \, dt$ and $\eta_{1i} = \int_0^{t_0} (-k_2\phi_2(\xi_i - \int_0^{t_0} H_i \, dt)) \, dt + \delta_i$, we have

\begin{equation}
V_3 = e_{1i}e_{2i} + e_{2i}(-K_1|\xi_i|^{\psi_1}\text{sign}(\xi_i) - K_2|\xi_i|^{\psi_2}\text{sign}(\xi_i) - L_1|\xi_i|^{\psi_1}\text{sign}(\xi_i) - L_2|\xi_i|^{\psi_2}\text{sign}(\xi_i)) + e_{2i}(-k_1\phi_1(\xi_i - \int_0^{t_0} H_i \, dt) + \int_0^{t_0} (-k_2\phi_2(\xi_i - \int_0^{t_0} H_i \, dt)) \, dt + \delta_i).
\end{equation}

Step 1. A Lyapunov function $V_2$ is defined as (9)

\begin{equation}
V_2 = \eta^T P \eta,
\end{equation}

where $\eta = [\phi_1(\eta_{0i}), \eta_{1i}]^T$. $P_{ij}$ is a symmetric and positive definite matrix. Then, according to Proposition 1 of Lemma 1, if $k_1$ and $k_2$ are in the set (19), for some $\varepsilon > 0$, the derivative of the Lyapunov function $V_2$ satisfies the inequality

\begin{equation}
\dot{V}_2 \leq -k_1(P_{ij}, \varepsilon)|\eta_{0i}|^{1/2}V_2 - k_2(P_{ij}, \varepsilon)|\eta_{1i}|^{1/2}V_2,
\end{equation}

where $k_j(P_{ij}, \varepsilon)$ ($j = 1, 2$) is a positive scalar. And the expression of $P_{ij}, k_1(P_{ij}, \varepsilon)$ and $k_2(P_{ij}, \varepsilon)$ is given in Appendix A in [34]. From (26), it is clear that $V_2 \leq 0$ for $t \geq 0$; thus, $|\phi_1(\eta_{0i})|$ and $|\eta_{1i}|$ are always bounded:

\begin{equation}
|\phi_1(\eta_{0i})| \leq \phi_{1\text{max}},
\end{equation}

\begin{equation}
|\eta_{1i}| \leq \phi_{2\text{max}}.
\end{equation}

where $\phi_{1\text{max}}$ and $\phi_{2\text{max}}$ are positive constants.

Then, a Lyapunov function $V_3$ is defined as

\begin{equation}
V_3 = \frac{1}{2}(e_{1i}^2 + e_{2i}^2).
\end{equation}

Calculating the time derivative of $V_3$ along the trajectories of (21), we get

\begin{equation}
\dot{V}_3 = e_{1i}e_{2i} + e_{2i}(-K_1|\xi_i|^{\psi_1}\text{sign}(\xi_i) - K_2|\xi_i|^{\psi_2}\text{sign}(\xi_i) - L_1|\xi_i|^{\psi_1}\text{sign}(\xi_i) - L_2|\xi_i|^{\psi_2}\text{sign}(\xi_i)) + e_{2i}(-k_1\phi_1(\xi_i - \int_0^{t_0} H_i \, dt) + \int_0^{t_0} (-k_2\phi_2(\xi_i - \int_0^{t_0} H_i \, dt)) \, dt + \delta_i).
\end{equation}

Considering the upper bound of $\phi_1(\eta_{0i})$ and $\eta_{1i}$ given in (27) and (28), (33) can be rewritten as

\begin{equation}
\dot{V}_3 \leq e_{1i}e_{2i} - K_1|\xi_i|^{\psi_1}\text{sign}(\xi_i) - L_1|\xi_i|^{\psi_1}\text{sign}(\xi_i)e_{2i} - L_2|\xi_i|^{\psi_2}\text{sign}(\xi_i)e_{2i} + e_{2i}(-k_1\phi_1(\xi_{0i}) + \eta_{1i}).
\end{equation}

We consider the following two cases:

Case 1: if $e_{1i}e_{2i} < 0$, we have $e_{1i}e_{2i} < 0$. It is clear that $e_{1i}$ is bounded in this case.

Case 2: if $e_{1i}e_{2i} \geq 0$. From (34), we have

\begin{equation}
\dot{V}_3 \leq e_{1i}e_{2i} + |e_{2i}| (k_1\phi_{1\text{max}} + \phi_{2\text{max}}).
\end{equation}
Then, according to Young’s inequality [37], (35) can be rewritten as
\[
V_3 \leq \frac{e_{li}^2 + e_{li}^2}{2} + \frac{(k_1\phi_{1,max} + \phi_{2,max})^2}{2}.
\]
(36)

Let \( C_{\max} = ((k_1\phi_{1,max} + \phi_{2,max})^2/2) \), we have
\[
V_3 \leq 2V_3 + C_{\max}. \tag{37}
\]
From (37), for any initial time \( t = t_g \), we have
\[
V_3 \leq V_3(t_g)e^{2t} + C_{\max}\left(1 - e^{2t}\right), \quad \text{if } t \geq t_g, \tag{38}
\]
where \( e \) is the natural constant. From (38), it is clear that \( V_3 \) and \( e_{li} \) are bounded in arbitrary finite time. From the discussions in case 1 and case 2, it is clear that the control error \( e_{li} \) is always bounded in arbitrary finite time.

Step 2. According to Proposition 2 of Lemma 1, it can be known from (24) that the following equation can be satisfied in a fixed-time \( T_1 \):
\[
\eta_0 = e_{li} - \int_0^t H_i du = 0, \quad \text{if } t \geq T_1,
\]
\[
\eta_{li} = \dot{e}_{li} - H_i = 0, \quad \text{if } t \geq T_1,
\]
where \( T_1 \) is independent on the initial system conditions. Then, we have
\[
e_{li} = 0, \quad \text{if } t \geq T_2 + T_1, \tag{41}
\]
where \( T_2 + T_1 \) is also independent on the initial system conditions.

The proof is finished.

4. Simulation Results

In this section, to illustrate the effectiveness of the proposed method, the mathematical simulation is presented. The dynamic parameters in system (1) are given as a two-DOFs robot manipulator in [34]:
\[
M(q) = \begin{bmatrix}
p_1 + 2p_2 \cos(q_2) & p_3 + p_4 \cos(q_2) \\
p_3 + p_4 \cos(q_2) & p_4
\end{bmatrix},
\]
\[
C(q, \dot{q}) = \begin{bmatrix}
-p_2 \sin(q_2) \dot{q}_1 & -2p_2 \sin(q_2) \dot{q}_1 \\
0 & p_2 \sin(q_2) \dot{q}_2
\end{bmatrix},
\]
\[
G(q) = \begin{bmatrix}
p_5 \cos(q_1) + p_6 \cos(q_1 + q_2), p_6 \cos(q_1 + q_2)
\end{bmatrix}^T,
\]
\[
\begin{aligned}
p_1 &= (m_1 + m_2)r_1^2 + m_2r_2^2 + J_1, \\
p_2 &= m_2r_1r_2, \\
p_3 &= m_2r_1^2, \\
p_4 &= p_3 + J_2, \\
p_5 &= (m_1 + m_2)r_1g_1, \\
p_6 &= m_2r_2g_1,
\end{aligned}
\]
(42)
where \( m_1 = 0.5 \), \( m_2 = 1.5 \), \( r_1 = 1 \), \( r_2 = 0.8 \), \( J_1 = J_2 = 5 \), and \( g_1 = 9.8 \). The desired joint position vector \( q_d \) is selected as
\[
q_d = \begin{bmatrix}-1 - \sin\left(\frac{t}{3}\right), -0.6 - \cos\left(\frac{t}{2}\right)\end{bmatrix}^T \quad \text{rad.} \tag{43}
\]

The parameters of the proposed fixed-time IDC method are selected as
\[
\begin{aligned}
&\omega_1 = 0.3, \\
&K_1 = \frac{8}{3}, \\
&K_2 = \frac{5}{3}, \\
&L_1 = \frac{8}{3}, \\
&L_2 = \frac{5}{3}, \\
&k_1 = 3, \\
&\mu = 1, \\
&k_2 = 2.2.
\end{aligned} \tag{44}
\]

In this section, MATLAB 2016 is chosen as simulation software. The simulation method is chosen as the fixed-step dormand-prince method. The step size of simulation is set as 0.001 s.

For the comparison, the FTSMC scheme in [21], fixed-time IDC scheme in [34], and the proposed robust fixed-time IDC (15) are considered in this section.

According to [21], for system (1), the FTSMC scheme can be designed as
\[
\tau = r_0 + r_{eq} + r_{\tau}, \tag{45}
\]
where the sliding-mode surface \( s_\tau \), the control component \( r_0 \), \( r_{eq} \), and \( r_{\tau} \) are given as
According to [33], for system (1), the fixed-time IDC can be designed by removing the robust term $R(e_1, e_2)$ in (15):

$$
\tau = (b(q)^{-1}(-f(q, \dot{q}) - H(e_1, e_2))
$$

(47)

The parameters of (47) are chosen as in (44).

For verification, we consider the following five cases:

Case 1 (system with none uncertainties, comparison of small and large initial condition): this case shows the performance comparison of finite-time and fixed-time schemes for different initial conditions. We consider there are no uncertainties $d$ in the manipulator system (1). We consider two kinds of initial system conditions:

- small initial condition: $q(0) = [0, 0]^T$ rad,
- large initial condition: $q(0) = [6, 2]^T$ rad,

The simulation results are shown in Figures 1 and 2. From Figure 1(a), for the small initial conditions (48), the three methods can guarantee that the position tracking errors converge to zero in 1.5 s. However, from Figure 2(a), with the same parameters, it can be known that the convergence time of FTSMC is increased to 2.3 s for the large initial condition (49). As stated in the Introduction section and Section 2.3, the reason for the simulation results is that the FTSMC is designed based on finite-time stability, and then the convergence time of FTSMC is affected by different initial system conditions. And, the fixed-time IDC and proposed robust fixed-time IDC can eliminate the negative effect brought by different initial conditions. The input torques of the three methods are given in Figures 1(b) and 2(b), respectively.

Then, the following cases 2 and 5 will show the comparison of the fixed-time IDC and the proposed robust fixed-time IDC for the robot manipulator with uncertainties. We consider four kinds of uncertainties:

1. case 2: uncertainties have small amplitude and low frequency;
2. case 3: uncertainties have large amplitude and low frequency;
3. case 4: uncertainties have small amplitude and high frequency;
4. case 5: uncertainties have large amplitude and high frequency.

And in the following four cases, the initial conditions are chosen as in (48).

Case 2 (uncertainties have small amplitude and low frequency): in this case, the uncertainties are chosen as

$$
d = [30 \cos(0.3t), 40 \cos(0.3t)]^T \text{Nm}.
$$

(50)

The simulation results are shown in Figure 3. From Figure 3(a), it is clear that the convergence precision and convergence rate of the robot manipulator with the fixed-time IDC are affected by the uncertainties (50). And, it also can be observed that the proposed robust fixed-time IDC still can guarantee a fast convergence rate and high convergence precision like case 1. As stated in Section 2.2, the robust term of the proposed method can effectively suppress the uncertainties.

Case 3 (uncertainties have large amplitude and low frequency): in this case, we increase the amplitude of uncertainties in case 1. The uncertainties are chosen as

$$
d = [90 \cos(0.3t), 120 \cos(0.3t)]^T \text{Nm}.
$$

(51)

The simulation results are shown in Figure 4. From Figure 4(a), for the uncertainties with larger amplitude, it is clear that the convergence performance of fixed-time IDC is affected more seriously than that in Case 1. And, the proposed robust fixed-time IDC still can achieve a similar excellent control performance like Case 1.

Case 4 (uncertainties have small amplitude and high frequency): in this case, we increase the frequency of uncertainties in case 1. The uncertainties are chosen as

$$
d = [30 \cos(2t), 40 \cos(2t)]^T \text{Nm}.
$$

(52)

The simulation results are shown in Figure 5. From Figure 5(a), for the uncertainties with these kind of uncertainties, it is clear that the convergence performance of the proposed robust fixed-time IDC can achieve a better convergence performance than fixed-time IDC in this case.

Case 5 (uncertainties have large amplitude and high frequency): in this case, we simultaneously increase the amplitude and frequency of uncertainties in case 1. The uncertainties are chosen as

$$
d = [90 \cos(2t), 120 \cos(2t)]^T \text{Nm}.
$$

(53)

The simulation results are shown in Figure 6. From Figure 6(a), with this kind of uncertainties, it clear that
### Figure 1: Responses of small initial condition in case 1 \( (q(0) = [0,0]\text{ rad}) \).
(a) Position tracking errors. (b) Input torques.

### Figure 2: Responses of large initial condition in case 1 \( (q(0) = [6,2]^T\text{ rad}) \).
(a) Position tracking errors. (b) Input torques.
Figure 3: Responses in case 2. (a) Position tracking errors. (b) Input torques.

Figure 4: Responses in case 3. (a) Position tracking errors. (b) Input torques.
Figure 5: Responses in case 4. (a) Position tracking errors. (b) Input torques.

Figure 6: Responses in case 5. (a) Position tracking errors. (b) Input torques.
the convergence performance of fixed-time IDC is greatly affected. From Figure 6(a), for the fixed-time IDC, an undesirable large oscillation of the position tracking errors can be observed. This undesirable oscillation may lead to the failure of robot manipulator’s control mission. Figure 6(a) also shows that the proposed robust fixed-time IDC still can achieve a fast convergence rate and high control precision despite the presence of this kind of large amplitude and fast-varying uncertainties.

According to the simulation results, the following can be concluded:

(1) The convergence rate of finite-time IDC can be lowered gradually with the increase of initial system conditions (Figures 1 and 2). The convergence rate of fixed-time IDC and proposed robust fixed-time IDC is independent of the initial system conditions, which means that the fixed-time schemes can provide a fast convergence rate without changing the control parameters for the different initial system conditions (Figures 1 and 2).

(2) The fixed-time IDC only can guarantee the desired excellent control performance when the robotic manipulator system does not have uncertainties (Figures 1 and 2). For the different kinds of uncertainties, the convergence rate and control precision of fixed-time IDC are significantly affected (Figures 3–6). Since the actual robot manipulator system is a typical uncertain system, the fixed-time IDC may not achieve the desired convergence performance. By designing a robust term to improve the fixed-time IDC, the proposed scheme can not only maintain the desired convergence performance of fixed-time control but also eliminate the affected by different uncertainties (Figures 3–6).

5. Conclusion

In this paper, a novel robust fixed-time control scheme has been proposed for the robotic manipulator system with uncertainties. By designing a robust control term based on the uniform robust exact differentiator (URED) algorithm, a robust fixed-time IDC extended the existing fixed-time IDC to the robust scheme. Moreover, the proposed method eliminated the singular problem without using approximate approach. It has been strictly proved that the convergence time of the closed-loop system with the proposed control method is fixed-time and independent of the initial conditions. Finally, the simulation results have demonstrated the effectiveness of the proposed method. In the future work, we will focus on the extension of the proposed robust fixed-time control method to more complex manipulator system, such as the flexible joint manipulator. Unlike the rigid manipulator considered in this paper has two orders and is matched uncertain, the flexible joint manipulator system is a high-order system (four orders) and is a typically mismatched uncertain system.

Data Availability

All data, models, and code generated or used during the study appear in the submitted article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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