CP Violation in B Decays and Strategies for Extracting CKM Phases

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Abstract
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CP VIOLATION IN B DECAYS AND STRATEGIES FOR EXTRACTING CKM PHASES

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A brief review of CP violation in the B-meson system and of strategies to determine the angles $\alpha$, $\beta$ and $\gamma$ of the unitarity triangle of the CKM matrix is given. Both general aspects and some recent developments are discussed, including a critical look at the “benchmark” modes, CP violation in penguin decays, $B_s$ decays in the light of the width difference $\Delta \Gamma_s$, charged $B$ decays, and strategies to probe the CKM angle $\gamma$ with $B \to \pi K$ modes.

1 Setting the Scene

At present, CP violation is one of the least experimentally explored phenomena of the Standard Model, and is very promising in the search for indications of “new physics” at future experiments. In order to accomplish this task, it is crucial to have CP-violating processes available that can be analysed in a reliable way within the framework of the Standard Model, where CP violation is closely related to the Cabibbo–Kobayashi–Maskawa matrix (CKM matrix), connecting the electroweak eigenstates of the $d$, $s$ and $b$ quarks with their mass eigenstates. As far as CP violation is concerned, the central feature is that – in addition to three generalized Cabibbo-type angles – also a complex phase is needed in the three-generation case to parametrize the CKM matrix. This complex phase is the origin of CP violation within the Standard Model.

A closer look shows that CP-violating observables are proportional to the following combination of CKM matrix elements:

$$J_{CP} = \pm \text{Im} \left( V_{i\alpha} V_{j\beta} V_{k\gamma}^* V_{l\delta}^* \right) \quad (i \neq j, \alpha \neq \beta),$$

representing a measure of the “strength” of CP violation in the Standard Model. Since $J_{CP} = O(10^{-5})$, CP violation is a small effect. In many scenarios of new physics, several additional complex couplings are present, leading to new sources of CP violation.

Concerning phenomenological applications, the parametrization

$$V_{CKM} = \left( \begin{array}{ccc}
1 - \frac{1}{2} \lambda^2 & \frac{\lambda}{\sqrt{2}} & A \lambda^3 R_0 e^{-i\gamma} \\
-\frac{\lambda}{\sqrt{2}} & 1 - \frac{1}{2} \lambda^2 & A \lambda^2 \\
A \lambda^3 R_0 e^{-i\beta} & -A \lambda^2 & 1
\end{array} \right) + O(\lambda^4)$$

1
with \( \lambda = 0.22 \), \( A \equiv |V_{cb}|/\lambda^2 = 0.81 \pm 0.06 \), \( R_b \equiv |V_{ub}|/(\lambda V_{cb})| = \sqrt{\rho^2 + \eta^2} = 0.36 \pm 0.08 \), and \( R_t \equiv |V_{td}|/(\lambda V_{cb})| = \sqrt{(1 - \rho)^2 + \eta^2} = O(1) \) turns out to be very useful. It is a modification of the Wolfenstein parametrization\(^4\) exhibiting not only the hierarchy of the CKM elements, but also the dependence on the angles \( \beta = \beta(\rho, \eta) \) and \( \gamma = \gamma(\rho, \eta) \) of the usual “non-squashed” unitarity triangle of the CKM matrix\(^5\).

Although the discovery of CP violation\(^6\) goes back to 1964, so far this phenomenon has been observed only within the neutral \( K \)-meson system, where it is described by two complex quantities, called \( \varepsilon \) and \( \varepsilon' \), which are defined by the following ratios of decay amplitudes:

\[
\frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \varepsilon + \varepsilon', \quad \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \varepsilon - 2\varepsilon'.
\]

While \( \varepsilon = (2.280 \pm 0.013) \times 10^{-3} \) parametrizes “indirect” CP violation, originating from the fact that the mass eigenstates of the neutral kaon system are not CP eigenstates, the quantity \( \text{Re}(\varepsilon' / \varepsilon) \) measures “direct” CP violation in \( K \to \pi\pi \) transitions. The CP-violating observable \( \varepsilon \) plays an important role to constrain the unitarity triangle\(^7\) and implies – using reasonable assumptions about certain hadronic parameters – in particular a positive value of the Wolfenstein parameter \( \eta \). Despite enormous experimental efforts, the question of whether \( \text{Re}(\varepsilon' / \varepsilon) \neq 0 \) could not yet be answered. However, in the near future, this issue should be clarified by improved measurements at CERN and Fermilab, as well as by the KLOE experiment at DAΦNE. Unfortunately, the calculations of \( \text{Re}(\varepsilon' / \varepsilon) \) are very involved and suffer at present from large hadronic uncertainties\(^8\) Consequently, this observable will not allow a powerful test of the CP-violating sector of the Standard Model, unless the hadronic matrix elements of the relevant operators can be brought under better control. Probably the major goal of a possible future observation of \( \text{Re}(\varepsilon' / \varepsilon) \neq 0 \) would hence be the unambiguous exclusion of “superweak” models of CP violation\(^9\).

In order to test the Standard Model description of CP violation, the rare decays \( K_L \to \pi^0\nu\bar{\nu} \) and \( K^+ \to \pi^+\nu\bar{\nu} \) are more promising and may allow a determination of \( \sin(2\beta) \) with respectable accuracy\(^9\). Yet the kaon system by itself cannot provide the whole picture of CP violation. Consequently, it is essential to study CP violation outside this system. In this respect, the \( B \)-meson system appears to be most promising, which is also reflected by the tremendous experimental efforts at the future \( B \)-factories. There are of course also other interesting systems to explore CP violation and to search for signals of new physics, for instance the \( D \)-meson system, where sizeable mixing or CP-violating effects would signal new physics because of the tiny Standard Model “background”. In the following, we shall focus on \( B \) decays.
2 The Central Target: CP Violation in the B System

With respect to testing the Standard Model description of CP violation, the major role is played by non-leptonic B decays, which can be divided into three decay classes: decays receiving both tree and penguin contributions, pure tree decays, and pure penguin decays. There are two types of penguin topologies: gluonic (QCD) and electroweak (EW) penguins related to strong and electroweak interactions, respectively. Because of the large top-quark mass, also the latter operators play an important role in several processes.

To analyse non-leptonic B decays theoretically, one uses low-energy effective Hamiltonians, which are calculated by making use of the operator product expansion, yielding transition matrix elements of the following structure:

\[ \langle f | H_{\text{eff}} | i \rangle \propto \sum_{k} C_{k}(\mu) \langle f | Q_{k}(\mu) | i \rangle. \]  

(4)

The operator product expansion allows us to separate the short-distance contributions to this transition amplitude from the long-distance ones, which are described by perturbative Wilson coefficient functions \( C_{k}(\mu) \) and non-perturbative hadronic matrix elements \( \langle f | Q_{k}(\mu) | i \rangle \), respectively. As usual, \( \mu \) denotes an appropriate renormalization scale.

In the case of \( |\Delta B| = 1, \Delta C = \Delta U = 0 \) transitions, which will be of particular interest for the following discussion, we have

\[ H_{\text{eff}} = H_{\text{eff}}(\Delta B = -1) + H_{\text{eff}}(\Delta B = -1)^{\dagger}, \]  

(5)

where

\[ H_{\text{eff}}(\Delta B = -1) = \frac{G_{F}}{\sqrt{2}} \left[ \sum_{j=u,c} V_{j}^{\ast} V_{j} \left\{ \sum_{k=1}^{2} Q_{j}^{q_{k}} C_{k}(\mu) + \sum_{k=3}^{10} Q_{k}^{q} C_{k}(\mu) \right\} \right]. \]  

(6)

Here \( \mu = O(m_{b}) \), \( Q_{j}^{q_{k}} \) are four-quark operators, the label \( q \in \{d, s\} \) corresponds to \( b \to d \) and \( b \to s \) transitions, and \( k \) distinguishes between current–current \((k \in \{1, 2\})\), QCD \((k \in \{3, \ldots, 6\})\) and EW \((k \in \{7, \ldots, 10\})\) penguin operators. The evaluation of such low-energy effective Hamiltonians has been reviewed in Ref. 11, where the four-quark operators are given explicitly and numerical values for their Wilson coefficient functions can be found.

2.1 CP Asymmetries in Decays of Neutral B Mesons

A particularly simple and interesting situation arises, if we restrict ourselves to decays of neutral \( B_{q} \) mesons \((q \in \{d, s\})\) into CP self-conjugate final states...
(|f⟩, satisfying the relation ⟨CP|f⟩ = ± |f⟩). In this case, the corresponding time-dependent CP asymmetry can be expressed as

\[ a_{CP}(t) \equiv \frac{\Gamma(B^0_q(t) \to f) - \Gamma(B^0_{\bar{q}}(t) \to f)}{\Gamma(B^0_q(t) \to f) + \Gamma(B^0_{\bar{q}}(t) \to f)} = \pm \frac{|\langle f |} \]

\[ \mathcal{A}_{CP}^{dir}(B_q \to f) \cos(\Delta M_q t) + \mathcal{A}_{CP}^{mix-ind}(B_q \to f) \sin(\Delta M_q t), \]

where the direct CP-violating contributions have been separated from the mixing-induced CP-violating contributions, which are characterized by

\[ \mathcal{A}_{CP}^{dir}(B_q \to f) \equiv \frac{1 - |\xi_f^{(q)}|^2}{1 + |\xi_f^{(q)}|^2} \quad \text{and} \quad \mathcal{A}_{CP}^{mix-ind}(B_q \to f) \equiv \frac{2 \text{Im} \xi_f^{(q)}}{1 + |\xi_f^{(q)}|^2}, \]

respectively. Here direct CP violation refers to CP-violating effects arising directly in the corresponding decay amplitudes, whereas mixing-induced CP violation is due to interference between \(B^0_q - B^0_{\bar{q}}\) mixing and decay processes.

Note that Eq. (7) has to be modified in the \(B_s\) case for \(t \sim 1/|\Delta \Gamma_s|\) because of the expected sizeable width difference \(\Delta \Gamma_s\). In general, the observable

\[ \xi_f^{(q)} = \mp e^{-i\phi_M^{(q)}} \sum_{j=u,c} V_{jr}^* V_{jb} \langle f | Q^j | \bar{B}^0_q \rangle \sum_{j=u,c} V_{jr} V_{jb} \langle f | Q^j | \bar{B}^0_q \rangle, \]

where \(Q^j \equiv \sum_{k=1}^2 Q_k \tilde{C}_k(\mu) + \sum_{k=3}^{10} Q_k^{(\mu)} \tilde{C}_k(\mu)\), and where

\[ \phi_M^{(q)} = \begin{cases} 2\beta & \text{for } q = d \\ 0 & \text{for } q = s \end{cases} \]

denotes the weak \(B^0_q - B^0_{\bar{q}}\) mixing phase, suffers from large uncertainties, which are introduced by the hadronic matrix elements in Eq. (9). There is, however, a very important special case. If the decay \(B_q \to f\) is dominated by a single CKM amplitude, these matrix elements cancel, and \(\xi_f^{(q)}\) takes the simple form

\[ \xi_f^{(q)} = \mp \exp \left[ -i \left( \phi_M^{(q)} - \phi_D^{(f)} \right) \right], \]

where \(\phi_D^{(f)}\) is a weak decay phase, which is given as follows (\(r \in \{d, s\}\)):

\[ \phi_D^{(f)} = \begin{cases} -2\gamma & \text{for dominant } \bar{b} \to \bar{u} u \bar{r} \text{ CKM amplitudes in } B_q \to f \\ 0 & \text{for dominant } \bar{b} \to \bar{c} c \bar{r} \text{ CKM amplitudes in } B_q \to f. \end{cases} \]
Probably the most important applications of this well-known formalism are the decays $B_d \to J/\psi K_S$ and $B_d \to \pi^+\pi^-$. If one goes through the relevant Feynman diagrams contributing to the former channel (for a detailed discussion, see Ref. 10), one finds that it is dominated by the $\bar{b} \to \bar{c}c\bar{s}$ CKM amplitude. Consequently, the decay phase vanishes, and we have

$$A_{\text{mix-ind}}^{\text{CP}}(B_d \to J/\psi K_S) = + \sin[-(2\beta - 0)]. \quad (13)$$

Since Eq. (11) applies with excellent accuracy to the decay $B_d \to J/\psi K_S$ – the point is that penguins enter essentially with the same weak phase as the leading tree contribution – it is usually referred to as the “gold-plated” mode to determine the CKM angle $\beta$. First attempts to measure $\sin(2\beta)$ through the CP-violating asymmetry [13] have recently been performed by the OPAL and CDF collaborations. Their results are as follows:

$$\sin(2\beta) = \begin{cases} 3.2^{+1.8}_{-2.0} \pm 0.5 & \text{OPAL Collaboration} \\ 1.8 \pm 1.1 \pm 0.3 & \text{CDF Collaboration} \end{cases} \quad (14)$$

they favour the Standard Model expectation of a positive value of this quantity. The presently allowed range arising from the usual fits of the unitarity triangle is given by $0.36 \lesssim \sin(2\beta) \lesssim 0.80$. In the $B$-factory era, an experimental uncertainty of $\Delta \sin(2\beta)|_{\text{exp}} = 0.08$ seems to be achievable.

In the case of the decay $B_d \to \pi^+\pi^-$, mixing-induced CP violation would measure $-\sin(2\alpha)$ through

$$A_{\text{CP}}^{\text{mix-ind}}(B_d \to \pi^+\pi^-) = -\sin[-(2\beta + 2\gamma)] = -\sin(2\alpha), \quad (15)$$

if there were no penguin contributions present. However, we have to deal with such topologies, leading to hadronic corrections to Eq. (15) that were analyzed by many authors [16-18]. Last year, the CLEO collaboration reported the observation of several exclusive $B$-meson decays into two light pseudoscalar mesons [19]. However, $B \to \pi\pi$ modes have not yet been seen and the upper limits for the corresponding branching ratios are not “favourable”. The recent CLEO results indicate moreover that we have in fact to worry about the penguin corrections to Eq. (15).

There are various methods on the market to control the penguin uncertainties in a quantitative way. Unfortunately, these strategies are usually rather challenging in practice. The best known approach was proposed by Gronau and London [20]. It makes use of the $SU(2)$ isospin relations

$$\sqrt{2} A(B^+ \to \pi^+\pi^0) = A(B^0_d \to \pi^+\pi^-) + \sqrt{2} A(B^0_d \to \pi^0\pi^0) \quad (16)$$

$$\sqrt{2} A(B^- \to \pi^-\pi^0) = A(B^0_d \to \pi^+\pi^-) + \sqrt{2} A(B^0_d \to \pi^0\pi^0), \quad (17)$$

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which can be represented in the complex plane as two triangles. The sides of these triangles are determined through the corresponding branching ratios, while their relative orientation can be fixed by measuring the CP-violating observable $A_{\text{mix-ind}}^{\text{CP}}(B_d \to \pi^+\pi^-)$. Following these lines, it is in principle possible to extract a value of $\alpha$ taking into account the QCD penguin contributions. It should be noted that EW penguins cannot be controlled using this isospin strategy. Their effect is, however, expected to be rather small in this case, leading to $|\Delta \alpha_{\text{EW}}/\sin \alpha| \lesssim 4^\circ$. An attempt to analyse other isospin-breaking effects, which may affect the isospin relations (16) and (17) through $\pi^0 - \eta, \eta'$ mixing, was recently performed in Ref. 22. Unfortunately, the Gronau–London approach suffers from an experimental problem, since the measurement of $B_d \to \pi^0\pi^0$ decays is very difficult. Theoretical estimates based on the “factorization” hypothesis (for a critical look at this concept, see Ref. 23) give branching ratios at the $10^{-6}$ level. However, upper bounds on the combined branching ratio $\text{BR}(B_d \to \pi^0\pi^0)$, i.e. averaged over the decay and its charge conjugate, may already lead to interesting upper bounds on the QCD penguin uncertainty affecting the determination of $\alpha$.\[18\]

An alternative to the Gronau–London strategy to extract the CKM angle $\alpha$ is provided by $B \to \rho \pi$ modes.\[26\] Here the isospin triangle relations (16) and (17) are replaced by pentagonal relations, and the corresponding approach is rather complicated. The $SU(3)$ flavour symmetry offers also ways to determine $\alpha$. For example, it is possible to extract this angle with the help of a triangle construction by measuring in addition to the $B_d \to \pi^+\pi^-$ observables those of $B_d \to K^0\bar{K}^0$ decays.\[27\] The latter would also be interesting to obtain insights into certain final-state interaction processes.\[28\] A measurement of both the direct and the mixing-induced CP asymmetries in $B_d \to \pi^+\pi^-$, together with the $B^+ \to \pi^+K^0$ branching ratio, would provide another step towards the control of the QCD penguin uncertainties.\[29\] Several strategies to constrain and determine $\alpha$ along these lines were recently proposed in Ref. 18. As sketched above, a solid extraction of this CKM angle is unfortunately quite difficult and could be out of reach for the first generation of $B$-factory experiments.

A decay appearing frequently in the literature as a tool to determine the CKM angle $\gamma$ is $B_s \to \rho^0K_S$. In this case, however, penguins are expected to lead to serious problems – even more serious than in $B_d \to \pi^+\pi^-$ – so that this mode appears to be the “wrong” way to extract $\gamma$.\[4\] Moreover, the rapid $B_s^0 - \bar{B}_s^0$ oscillations, as well as the small expected branching ratio at the $10^{-7}$ level, make experimental studies of $B_s \to \rho^0K_S$ very difficult. It should be kept in mind, however, that this channel may be in better shape to probe $\gamma$, if the concept of “colour suppression” should not work in this case. A recent model calculation within a perturbative framework can be found in Ref. 30.
2.2 CP Violation in Penguin Modes as a Probe of New Physics

In order to test the Standard Model description of CP violation, penguin-induced modes play an important role. Because of the loop suppression of these “rare” processes, it is possible – and indeed it is the case in several specific model calculations – that new-physics contributions to these decays are of similar magnitude as those of the Standard Model.

An important example is the \( b \to s \) penguin mode \( B_d \to \phi K_S \). The corresponding branching ratio is expected to be of \( \mathcal{O}(10^{-5}) \) and may be large enough to investigate this channel at the future \( B \)-factories. In contrast to the \( b \to d \) penguin case, the corresponding decay amplitude does not contain a sizeable CP-violating weak phase within the Standard Model. Consequently, direct CP violation in \( B_d \to \phi K_S \) is tiny, and mixing-induced CP violation measures simply the weak \( B_d^0 - B_d^- \) mixing phase, implying the relation

\[
\mathcal{A}_{\text{CP}}^{\text{mix-ind}}(B_d \to J/\psi K_S) = \mathcal{A}_{\text{CP}}^{\text{mix-ind}}(B_d \to \phi K_S) = -\sin(2\beta),
\]

which represents an interesting probe of new-physics contributions to \( b \to s \) decay processes. The theoretical accuracy of this relation is limited by certain neglected terms that are CKM-suppressed by \( \mathcal{O}(\lambda^2 R_b) \), and may lead to direct CP violation in \( B_d \to \phi K_S \). Simple model calculations performed at the perturbative quark level indicate asymmetries of at most \( \mathcal{O}(1\%) \). However, the impact of long-distance effects is hard to quantify (for a recent attempt, see Ref. 32). The importance of \( B_d \to \phi K_S \) and similar modes, such as \( B_d \to \eta' K_S \), to search for new-physics effects in \( b \to s \) flavour-changing neutral current processes was emphasized by several authors.

Studies of CP-violating effects in inclusive \( B \to X_s \gamma \) decays, which can be analysed reliably in QCD by means of the operator product expansion, also play an important role in the search for new physics. Within the Standard Model, direct CP violation in \( B \to X_s \gamma \) is very small, i.e. below the 1\% level, whereas it may well be as large as 50\% in new-physics scenarios with enhanced chromomagnetic dipole operators.

2.3 The \( B_s \) System in the Light of \( \Delta \Gamma_s \)

In the \( B_s \) system, very rapid \( B_s^0 - \bar{B}_s^0 \) oscillations are expected, requiring an excellent vertex resolution system. Studies of CP violation in \( B_s \) decays are therefore regarded as being very difficult. An alternative route to investigate CP-violating effects may be provided by the width difference \( \Delta \Gamma_s \). Because of this width difference, already untagged data samples of \( B_s \) decays may exhibit CP-violating effects. Several “untagged strategies” to extract...
the CKM angle $\gamma$ were proposed, using for example angular distributions in $B_s \to K^{*+}K^{*-}$, $K^{*0}\overline{K}^{*0}$ or $B_s \to D^*\phi$, $D_s^{*\pm}K^{*\mp}$ decays.

The $B_s$ system provides interesting probes also for physics beyond the Standard Model. Important examples are the decays $B_s \to D_s^+D_s^-$ and $B_s \to J/\psiK_s$. The latter is the counterpart of the “gold-plated” mode $B_d \to J/\psi K_S$ to measure $\beta$ and is very promising for experiments performed at future hadron machines. These transitions are dominated by a single CKM amplitude and allow – in principle even from their untagged data samples – the extraction of a CP-violating weak phase $\phi_{\text{CKM}} \equiv 2\lambda^2\eta$, which is expected to be of $O(0.03)$ within the Standard Model. Consequently, an extracted value of $\phi_{\text{CKM}}$ that is much larger than this Standard Model expectation would signal new-physics contributions to $B_0^s - \overline{B}_0^s$ mixing.

Suggestions for efficient determinations of the observables of the $B_s \to D_s^+D_s^-$ and $B_s \to J/\psi \phi$ angular distributions, as well as of $\Delta\Gamma_s$ and of the $B_s$ mass difference $\Delta M_s$, were given in Ref. 40.

A time-dependent study of $B_s \to J/\psi \phi$ decays is also interesting to resolve a discrete ambiguity in the determination of the CKM angle $\beta$. Such ambiguities are a typical feature of the strategies to extract CKM phases sketched above; they could complicate the search for new physics considerably. Several strategies were recently proposed to deal with these problems.

\section{2.4 CP Asymmetries in Decays of Charged B Mesons}

Since mixing effects are not present in the charged $B$-meson system, the measurement of a non-vanishing CP asymmetry in a charged $B$ decay would give us unambiguous evidence for direct CP violation, thereby ruling out “superweak” models. Such CP asymmetries arise from interference between decay amplitudes with both different CP-violating weak and CP-conserving strong phases. Whereas the weak phases are related to the CKM matrix, the strong phases are induced by strong final-state interaction effects and introduce in general severe theoretical uncertainties into the calculation. An interesting situation arises, however, in $B^\pm \to \pi^\mp \rho^0(\omega) \to \rho^\pm\pi^+\pi^-$ decays, where $\rho^0(\omega)$ denotes the $\rho^0 - \omega$ interference region. In this case, experimental data on $e^+e^- \to \pi^+\pi^-$ processes can be used to constrain the hadronic uncertainties affecting the corresponding CP asymmetry, which is related to $\sin\alpha$ and may well be as large as $O(20\%)$ at the $\omega$ invariant mass. Direct CP violation in three-body decays such as $B^\pm \to K^\pm\pi^+\pi^-$, which involve various intermediate resonances, was recently considered in Ref. 44. Here the Dalitz plot distributions may provide information on the CKM angle $\gamma$ (see also Ref. 45).

In order to extract angles of the unitarity triangle from decays of charged $B$ mesons, amplitude relations – either exact or approximate ones based on
flavour symmetries – play an important role. A review of these methods can be found in Ref. 10. The “prototype” is the approach to determine $\gamma$ with the help of triangle relations between the $B^\pm \to DK^\pm$ decay amplitudes proposed by Gronau and Wyler. Unfortunately, the corresponding triangles are expected to be very “squashed”. Moreover, one has to deal with additional experimental problems, so that this approach is very difficult from a practical point of view. More refined variants and different ways to combine the information provided by $B \to DK$ decays to probe the CKM angle $\gamma$ were proposed by several authors.\textsuperscript{47, 48} The CLEO collaboration has recently observed the colour-allowed decay $B^- \to D^0 K^-$ and its charge conjugate, which is the first detection of a $B$ decay originating from $b \to c \bar{u} s$ quark subprocesses.\textsuperscript{49}

2.5 Strategies to Probe the CKM Angle $\gamma$ with $B \to \pi K$ Modes

The decays $B^+ \to \pi^+ K^0$, $B^0_d \to \pi^- K^+$ and their charge conjugates, which were observed by the CLEO collaboration last year,\textsuperscript{19} play an important role to probe the CKM angle $\gamma$ at future $B$-factories\textsuperscript{21, 50}. So far, only results for the combined branching ratios $\text{BR}(B^\pm \to \pi^\pm K^\mp)$ and $\text{BR}(B^d \to \pi^\pm K^\mp)$ have been published at the $10^{-5}$ level with large experimental uncertainties. In order to obtain information on $\gamma$, the ratio

$$ R \equiv \frac{\text{BR}(B_d \to \pi^\mp K^\pm)}{\text{BR}(B^\pm \to \pi^\pm K^\mp)} \quad (19) $$

of the combined $B \to \pi K$ branching ratios, and the “pseudo-asymmetry”

$$ A_0 \equiv \frac{\text{BR}(B^0_d \to \pi^- K^+) - \text{BR}(\bar{B}^0_d \to \pi^+ K^-)}{\text{BR}(B^+ \to \pi^+ K^0) + \text{BR}(B^- \to \pi^- K^0)} \quad (20) $$

play a key role. Making use of the $SU(2)$ isospin symmetry of strong interactions, the following amplitude relations can be derived (for a detailed discussion, see Ref. 28):

$$ A(B^+ \to \pi^+ K^0) \equiv P, \quad A(B^0_d \to \pi^- K^+) = -[P + T + P_{\text{ew}}], \quad (21) $$

where the “penguin” amplitude $P$ is defined by the $B^+ \to \pi^+ K^0$ decay amplitude, $P_{\text{ew}} \equiv -|P_{\text{ew}}|e^{i\delta_{\text{ew}}}$ is essentially due to electroweak penguins, and $T \equiv |T|e^{i\delta_T}e^{i\gamma}$ is usually referred to as a “tree” amplitude. However, because of a subtlety in the implementation of the isospin symmetry, the amplitude $T$ does not only receive contributions from colour-allowed tree-diagram-like topologies, but also from penguin and annihilation topologies.\textsuperscript{51, 52} The general expressions for the amplitudes $P$, $T$ and $P_{\text{ew}}$ are given in Ref. 51. Let us
just note here that we have

\[ P \propto \left[ 1 + \rho e^{i\theta} e^{i\gamma} \right], \tag{22} \]

where \( \rho e^{i\theta} \) is a measure of the strength of certain rescattering effects (\( \theta \) is a CP-conserving strong phase). The quantities \( r \equiv |T|/\sqrt{\langle |P|^2 \rangle} \) and \( \epsilon \equiv |P_{ew}|/\sqrt{\langle |P|^2 \rangle} \) with \( \langle |P|^2 \rangle \equiv (|P|^2 + |\overline{P}|^2)/2 \), as well as the strong phase differences \( \delta \equiv \delta_T - \delta_{tc} \) and \( \Delta \equiv \delta_{ew} - \delta_{tc} \), where \( \delta_{tc} \) measures the strong phase of the difference of the penguin topologies with internal top and charm quarks, turn out to be very useful to parametrize the observables \( R \) and \( A_0 \).

If both \( R \) and \( A_0 \) are measured, contours in the \( \gamma - r \) plane can be fixed, corresponding to a mathematical implementation of a simple triangle construction\[\footnote{21}\]. In order to determine the CKM angle \( \gamma \), the quantity \( r \), i.e. the magnitude of the “tree” amplitude \( T \), has to be fixed. At this step, a certain model dependence enters. Using arguments based on “factorization”, one comes to the conclusion that a future theoretical uncertainty of \( r \) as small as \( \mathcal{O}(10\%) \) may be achievable.\[\footnote{50, 52}\] However, since the properly defined amplitude \( T \) does not only receive contributions from colour-allowed “tree” topologies, but also from penguin and annihilation processes,\[\footnote{28, 51}\] it may be shifted sizably from its “factorized” value so that \( \Delta r = \mathcal{O}(10\%) \) may be too optimistic.

Interestingly, it is possible to derive bounds on \( \gamma \) that do not depend on \( r \) at all\[\footnote{23}\] (for a discussion of \( B_s \to K \bar{K} \) decays, see Ref. 39). If we eliminate the strong phase \( \delta \) in the ratio \( R \) of combined \( B \to \pi K \) branching ratios with the help of the pseudo-asymmetry \( A_0 \) and treat \( r \) as a “free” variable, while keeping \( \rho \), \( \theta \) and \( \epsilon \), \( \Delta \) fixed, we find that \( R \) takes the following minimal value:\[\footnote{23}\]

\[ R_{\text{min}} = \kappa \sin^2 \gamma + \frac{1}{\kappa} \left( \frac{A_0}{2 \sin \gamma} \right)^2. \tag{23} \]

In this expression, rescattering and EW penguin effects are described by

\[ \kappa = \frac{1}{w^2} \left[ 1 + 2(\epsilon w) \cos \Delta + (\epsilon w)^2 \right] \quad \text{with} \quad w = \sqrt{1 + 2 \rho \cos \theta \cos \gamma + \rho^2}. \tag{24} \]

An allowed range for \( \gamma \) is related to \( R_{\text{min}} \), since values of \( \gamma \) implying \( R_{\text{exp}} < R_{\text{min}} \), where \( R_{\text{exp}} \) denotes the experimentally determined value of \( R \), are excluded. In the “original” bounds on \( \gamma \) derived in Ref. 53, no information provided by \( A_0 \) has been used, i.e. both \( r \) and \( \delta \) were kept as “free” variables, and the special case \( \rho = \epsilon = 0 \), i.e. \( \kappa = 1 \), has been assumed, implying \( \sin^2 \gamma < R_{\text{exp}} \). A particularly interesting situation arises, if \( R \) is measured to be smaller than 1. In this case, the information on \( \gamma \) provided by the \( B \to \pi K \) decays is complementary to the presently allowed range of \( 41^\circ \lesssim \gamma \lesssim 134^\circ \),
which arises from the usual fits of the unitarity triangle since a certain interval around 90° can be excluded. A measurement of $A_0 \neq 0$ allows us to exclude in addition a range around $0°$ and $180°$. Unfortunately, the present data do not yet provide an answer to the question of whether $R < 1$. The results reported by the CLEO collaboration last year give $R = 0.65 \pm 0.40$, whereas a very recent, updated analysis yields $R = 1.0 \pm 0.4$.

The theoretical accuracy of these bounds is limited both by rescattering processes of the kind $B^+ \rightarrow \{\pi^0 K^+, \pi^0 K^{0*}, \ldots\} \rightarrow \pi^+ K^0$ and by EW penguin effects. An important implication of the rescattering effects may be a sizeable CP asymmetry, $A_+^\pm$, in the decay $B^+ \rightarrow \pi^+ K^0$. The rescattering effects can be controlled in the bounds on $\gamma$ through experimental data. A first step towards this goal is provided by the CP asymmetry $A_+$. In order to go beyond it, $B^\pm \rightarrow K^\pm K$ decays – the SU(3) counterparts of $B^\pm \rightarrow \pi^\pm K$ – play a key role, allowing us to include the rescattering processes in the contours in the $\gamma - r$ plane and the associated constraints on $\gamma$ completely (for alternative strategies, see Refs. 28 and 57). Since the “short-distance” expectation for the combined branching ratio $BR(B^\pm \rightarrow K^\pm K)$ is $O(10^{-6})$, experimental studies of $B^\pm \rightarrow K^\pm K$ appear to be difficult. These modes have not yet been observed, and only upper limits for $BR(B^\pm \rightarrow K^\pm K)$ are available.

However, rescattering effects may enhance this quantity significantly, and could thereby make $B^\pm \rightarrow K^\pm K$ measurable at future $B$-factories. Another important indicator of large FSI effects is provided by $B^\pm \rightarrow K^+ K^-$ decays, for which stronger experimental bounds already exist.

In the case of the decays $B^+ \rightarrow \pi^+ K^0$ and $B^0 \rightarrow \pi^- K^+$, EW penguins contribute only through “colour-suppressed” topologies; estimates based on the “factorization” hypothesis typically give values for $\epsilon$ at the 1% level. These crude estimates may, however, underestimate the role of the EW penguins. An improved description is possible by using the general expressions for the EW penguin operators and performing appropriate Fierz transformations; on the way to controlling the corresponding uncertainties in the bounds on $\gamma$ with the help of experimental data, the decay $B^+ \rightarrow \pi^+ \pi^0$ provides an important first step.

3 Conclusions and Outlook

In conclusion, we have seen that certain non-leptonic $B$ decays offer a fertile ground to test the Standard Model description of CP violation. Detailed experimental studies of these modes at $B$-factories are just ahead of us and may bring unexpected results, which could guide us to the physics lying beyond the Standard Model. The coming years will certainly be very exciting!
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