Integrated Information Theory and Isomorphic Feed-Forward Philosophical Zombies

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Abstract: Any theory amenable to scientific inquiry must have testable consequences. This minimal criterion is uniquely challenging for the study of consciousness, as we do not know if it is possible to confirm via observation from the outside whether or not a physical system knows what it feels like to have an inside - a challenge referred to as the "hard problem" of consciousness. To arrive at a theory of consciousness, the hard problem has motivated theorists to develop phenomenological approaches that adopt assumptions of what properties consciousness has based on first-hand experience and, from these, derive the physical processes that give rise to these properties. A leading theory adopting this approach is Integrated Information Theory (IIT), which assumes our subjective experience is a "unified whole", subsequently yielding a requirement for physical feedback between components within a system as a necessary condition for consciousness. Here, we develop a mathematical framework to assess the validity of this assumption by testing it in the context of isomorphic physical systems with and without feedback. We demonstrate the existence of strictly feed-forward "philosophical zombies" capable of emulating not only the input-output behavior of their integrated counterparts but also exhibiting an isomorphic sequence of internal state transitions. Indeed, the only allowable difference between the integrated system ($\Phi > 0$) and its feed-forward zombie counterpart ($\Phi = 0$) is a permutation of the binary labels used to instantiate the functional states of the system. This proves that isomorphic logical architectures can exist that realize the exact same mathematical computation using different internal representations. For any quantitative theory of consciousness, including IIT, to be robust against isomorphic philosophical zombies, the theory must either justify a connection between consciousness and a particular internal representation or be invariant under isomorphisms.

Keywords: Consciousness; Integrated Information Theory; Krohn-Rhodes Decomposition

1. Introduction

The scientific study of consciousness walks a fine line between physics and metaphysics. On the one hand, there are observable consequences to what we intuitively describe as consciousness. Sleep, for example, is an outward behavior that is uncontroversially associated with a lower overall level of consciousness. Similarly, scientists can decipher "what" is intrinsically experienced when humans are conscious via verbal reports or other outward signs of awareness. By studying the physiology of the brain during these specific behaviors, scientists can build "neuronal correlates of consciousness" (NCCs), which specify where in the brain conscious experience is generated and what physiological processes correlate with it [1]. On the other hand, NCCs cannot be used to explain why we are conscious or to predict whether or not another system demonstrating similar properties to NCCs is conscious. Indeed, NCCs can only tell us what physiological processes correlate with what are assumed to be the functional consequences of consciousness and, in principle, may not actually correspond to the subjective experience of "what it is like" to be conscious [2]. In other words, we can objectively measure behavior we assume accurately reflects consciousness but, currently, there exist no scientific tools...
permitting one to test this assumption. As a result, we struggle to differentiate whether a system is truly "conscious" or is instead simply going through the motions and giving outward signs of, or even actively reporting, an internal experience that does not exist (e.g. [3]).

This is the "hard problem" of consciousness [2] and is what differentiates the study of consciousness from all other scientific endeavors. Since consciousness is subjective (by definition), there is no known objective way to prove when a system experiences it. Addressing the hard problem, therefore, necessitates an inversion of the approach underlying NCCs: rather than starting with observables and deducing consciousness, one must start with consciousness and deduce observables. This has motivated theorists to develop phenomenological approaches that adopt rigorous assumptions of what properties consciousness must include based on human experience, and from these derive the physical processes that give rise to these properties. Support for such a theory, therefore, must ultimately come from how well the deductions of the theory match our intuitive understanding of what consciousness is, as well as the logical consistency and believability of the underlying assumptions [4].

The phenomenological approach to addressing the hard problem is exemplified in Integrated Information Theory (IIT) [5,6], a leading theory of consciousness. Indeed, IIT is a leading contender in modern neuroscience precisely because it takes a phenomenological approach and offers a well-motivated solution to the hard problem of consciousness [7]. Three phenomenological axioms form the backbone of IIT: information, integration, and exclusion. The first, information, states that by taking on only one of the many possibilities a conscious experience generates information (in the Shannon sense, e.g. via a reduction in uncertainty [8]). The second, integration, states each conscious experience is a single unified "whole". And the third, exclusion, states conscious experience is exclusive in that each component in a system can take part in at most one conscious experience at a time (simultaneous experiences are forbidden). Given these three phenomenological axioms, IIT invents a mathematical measure of integrated information called $\Phi$ which is designed to quantify the extent to which a system is conscious based on the logical architecture (i.e. the "wiring") underlying its internal dynamics.

In constructing $\Phi$ as a phenomenologically-derived measure of consciousness, IIT must make a logical leap from its three axioms to the physical processes that embody these axioms. This is what provides a concrete path to solving the hard problem of consciousness - a connection must be assumed between subjective experience (axiomatized as integration, information, and exclusion) and objective (measurable) properties of a physical system. Naturally, this mapping from axioms to physical instantiation is the most important step in the construction of the theory, as everything else follows by deduction. Assessing the justification for how IIT translates each of its three phenomenological axioms into mathematics and the implications for what systems can be conscious is therefore the only way to test the foundations of the theory.

According to IIT, the subjective experience of an "integrated whole" (the integration axiom) translates into the observable property of feedback between components in a system. Consequently, physical feedback is assumed to be a necessary condition for consciousness and is built into $\Phi$ such that any system that lacks feedback lacks integrated information ($\Phi = 0$) and is unconscious by definition. However, it is well known that any "brain" or "circuit" that is wired with internal feedback between components (e.g. neurons) such that $\Phi > 0$ can, in principle, be rewired without feedback in a manner that perfectly preserves the underlying computation being performed but has $\Phi = 0$ [9,10]. This implies IIT admits the existence of feed-forward "philosophical zombies", defined as physical systems that are behaviorally indistinguishable from conscious systems, but nonetheless, lack subjective experience as a consequence of their internal architecture. The existence of philosophical zombies is epistemologically controversial [11,12] and therefore poses a challenge to any candidate theory of consciousness that admits them, including IIT.

In response, philosophical zombies have been explicitly addressed in the IIT literature, primarily in the context of feed-forward neural networks (NN). Because feed-forward NNs can act as universal approximators, they can emulate the behavior of any system with feedback in a way that qualifies as a
philosophical zombie \[6,13\]. The current theoretical framework in IIT has evolved to accommodate the existence of such systems, however, as the difference in subjective experience between feed-forward NNs and their conscious counterparts is not justified solely in terms of the difference in \( \Phi \) values (which would be a circular argument), but instead in terms of important attributes of consciousness assumed to be lacking in feed-forward logical architectures. In particular, it has been argued that feed-forward systems lack the \textit{efficiency} and \textit{autonomy} of their integrated counterparts \[6\]. Thus, the reason feedback is a necessary condition for consciousness, according to IIT, is ultimately justified in terms of the ability for feedback to generate \textit{efficient, autonomous} computations.

Here, we introduce the existence of a fundamentally new type of philosophical zombie within IIT. Namely, a feed-forward system that is \textit{isomorphic} to its conscious counterpart. The isomorphism we exploit produces a philosophical zombie that not only emulates the input-output behavior of its conscious counterpart but also executes an isomorphic sequence of \textit{internal} state transitions. This implies a one-to-one correspondence between internal states in the two systems that makes it clear that the feed-forward architecture is in no way less autonomous or efficient than its counterpart with feedback. Indeed, the only mathematical difference between the integrated system with \( \Phi > 0 \) and the feed-forward system with \( \Phi = 0 \) is a permutation of the binary labels used to instantiate the computational states in the system. This implies \( \Phi \) depends on the specific internal representation of a computation rather than the computation itself. Given a physical system with a specific binary labeling scheme, we show how it is possible to construct an isomorphic physical system that performs the exact same computation using a different internal representation (i.e. one that does not require a logical architecture with feedback). This suggests IIT must either (a) justify a relationship between consciousness and a specific internal representation or (b) modify \( \Phi \) so that it is invariant under isomorphisms. If (a), then the privileged role of feedback has justification beyond simply assuming feed-forward systems lack consciousness and, if (b), only functional differences can result in different subjective experiences.

2. Methods

2.1. Finite-State Automata

Finite-state automata are abstract computing devices, or "machines", designed to model a systems as they transitions between discrete states. Automata theory was invented to address biological and psychological problems \[14,15\] and it remains an extremely intuitive choice for modeling neuronal systems. This is because one can define an automata in terms of how specific abstract inputs lead to changes within a system. Namely, if we have a set of potential inputs \( \Sigma \) and a set of internal states \( Q \), we define an automata \( A \) in terms of the tuple \( A = (\Sigma, Q, \delta, q_0) \) where \( \delta : \Sigma \times Q \rightarrow Q \) is a map from the current state and input symbol to the next state, and \( q_0 \in Q \) is the starting state of the system. To simplify notation, we write \( \delta(s, q) = q' \) to denote the transition from \( q \) to \( q' \) upon receiving the input symbol \( s \in \Sigma \).

For example, consider the "right-shift automaton" \( A \) shown in Figure 1. This automaton is designed to model a system with a two bit internal register that processes new elements from the input alphabet \( \Sigma = \{0, 1\} \) by shifting the bits in the register to the right and appending the new element on the left \[16\]. The global state of the machine is the combined state of the left and right register, so \( Q = \{00, 01, 10, 11\} \) and the transition function \( \delta \) specifies how this global state changes in response to each input, as shown in Figure 1b. In addition to the global state transitions, each individual bit in the register of the right-shift automaton is itself an automaton. In other words, the global functionality of the system is nothing more than the combined output from a system of interconnected automata, each specifying the state of a single component or "coordinate" of the system. Specifically, the right-shift automaton is comprised of an automaton \( A_{Q_1} \) responsible for the left bit of register and an automaton \( A_{Q_2} \) responsible for the right bit of the register. By definition, \( A_{Q_1} \) copies the input from the environment and \( A_{Q_2} \) copies the state of \( A_{Q_1} \). Thus, \( \Sigma_{Q_1} = \{0, 1\} \) and \( \Sigma_{Q_2} = \{ \} \) and the
transition functions for the coordinates are \( \delta_{Q_1} = \delta_{Q_2} = \{ \delta(0,0) = 0; \delta(0,1) = 0; \delta(1,0) = 1; \delta(1,1) = 1 \} \). This fine-grained view of the right-shift automaton specifies its logical architecture and is shown in Figure 1c.

Not all automata require multiple input symbols, however, and it is common to find examples of automata with a single letter input alphabet. In fact, any deterministic state transition diagram can be represented in this way, with a single input letter signaling the passage of time. In this case, the states of the automaton are the states of the system, the input alphabet is the passage of time, and the transition function \( \delta \) is given by the transition probability matrix (TPM) for the system. This provides a concrete link between IIT and automata theory, as \( \Phi \) is a mathematical measure that operates on TPMs. Non-deterministic TPMs can also be described in terms of automata [16,17], but this generalization is not necessary for our purposes.

2.2. Cascade Decomposition

The idea of decomposability is central to both IIT and automata theory. As Tegmark [18] points out, mathematical measures of integrated information, including \( \Phi \), quantify the inability to decompose a transition probability matrix \( M \) into two independent processes \( M_A \) and \( M_B \). Given a distribution over initial states \( p \), if we approximate \( M \) by the tensor factorization \( \hat{M} \approx M_A \otimes M_B \), then \( \Phi \) quantifies an information theoretic distance \( D \) between the regular dynamics \( Mp \) and the dynamics under the partitioned approximation \( \hat{M}p \). If \( \Phi = 0 \), then it is possible to unidirectionally decompose the logical architecture of a system into independent processes without affecting the global dynamics.

Decomposition in automata theory, on the other hand, has historically been an engineering task. The goal is to decompose an automaton \( A \) into an automaton \( A' \) which is made of simpler physical components than \( A \) and maps homomorphically onto \( A \). Here, we define a homomorphism \( h \) as a map from the states, stimuli, and transitions of \( A' \) onto the states, stimuli, and transitions of \( A \) such that for every state and stimulus in \( A' \) the results obtained by the following two methods are equivalent [14]:

1. Use the stimulus of \( A' \) to update the state of \( A' \) then map the resulting state onto \( A \).
2. Map the stimulus of \( A' \) and the state of \( A' \) to the corresponding stimulus/state in \( A \) then update the state of \( A \) using the stimulus of \( A \).

In other words, the map \( h \) is a homomorphism if it commutes with the dynamics of the system. The two operations (listed above) that must commute are shown schematically in Figure 2. If the homomorphism \( h \) is bijective then it is also an isomorphism and the two automata necessarily have
the the same number of states. Isomorphic/homomorphic systems realize the exact same finite-state automaton, modulo the labels used to instantiate computational states in the system. Consequently, homomorphic systems are capable of generating indistinguishable outward behavior using different internal representations. For example, a machine can be programmed to represent "sleep" using an internal state of all zeros (00...0) or an internal state of all ones (11...1). Provided the logical architecture is consistent with a given internal representation, such differences will not have functional consequences beyond the appearance of the internal architecture.

Because we are interested in the role of feedback, the specific type of decomposition we seek is a feed-forward or cascade decomposition of the automata representing the relationship between coordinates in a system. Cascade decomposition takes the automaton $A$ and decomposes it into a homomorphic automaton $A'$ comprised of several elementary automata "cascaded together". By this, what is meant is that the output from one automaton serves as the input to another such that the flow of information in the system is unidirectional (Figure 3). The resulting system of automata is said to be in "cascade" or "hierarchical" form and is functionally identical to the original system but lacks feedback.

At this point, the connection between IIT and cascade decomposition is readily apparent: if an automaton allows a homomorphic cascade decomposition, then the behavior of the resulting system is indistinguishable from the original but utilizes only feed-forward connections. Therefore, there exists a unidirectional partition of the system that leaves the dynamics of the system (i.e. the transition probability matrix) unchanged and $\Phi = 0$. In the language of Oizumi et al. [6], let $C_{\rightarrow}$ be the constellation that is generated as a result of this partition and $C$ be the original constellation. Because $C_{\rightarrow}$ has no effect on the TPM, we are guaranteed that $C_{\rightarrow} = C$ and $\Phi^{MIP} = D(C|C_{\rightarrow}) = 0$. We can repeat this process for every possible subsystem within a given system and, since the flow of information is always unidirectional, $\Phi^{MIP} = 0$ for all subsets so $\Phi^{Max} = 0$. Thus, $\Phi = 0$ for all states and subsystems of a cascade automaton.
Pertinently, the Krohn-Rhodes theorem proves that every automaton can be decomposed into cascade form [10,19]. Because every TPM can be represented as an automaton, this implies every system for which we can measure \( \Phi \) allows a feed-forward decomposition with \( \Phi = 0 \). These feed-forward systems are “philosophical zombies” in the sense that they lack subjective experience according to IIT (\( \Phi = 0 \)), but they nonetheless perfectly emulate the behavior of conscious systems. Yet, the Krohn-Rhodes theorem does not tell us how to construct such systems. Furthermore, the map between systems is only guaranteed to be homomorphic (many-to-one) which fails to address concerns described at the outset regarding the efficiency and autonomy of feed-forward philosophical zombies relative to their conscious counterparts (Section 1). Therefore we must go one step further and insist that the feed-forward decomposition is isomorphic (one-to-one) to the original system, which limits the generality of our approach but allows us to study unique cases which control for all possible confounding variables. As we will show, isomorphic cascade decomposition is possible if and only if the original system can be decomposed into a nested sequence of preserved partitions.

2.3. Feed-forward Isomorphisms via Preserved Partitions

A preserved partition is a way of partitioning the state space of a system into blocks of states (macrostates) that transition together. Namely, a partition \( P \) is preserved if it breaks the state space \( S \) into a set of blocks \( \{B_1, B_2, ..., B_N\} \) such that every state within each block transitions to a state within the same block [14,20]. If we denote the state transition function \( f : S \rightarrow S \), then a block \( B_i \) is preserved when:

\[
\exists j \in \{1, 2, ..., N\} \text{ such that } f(x) \in B_j \forall x \in B_i
\]

In other words, for \( B_i \) to be preserved, every state \( x \) in \( B_i \) must transition to some state in a single block \( B_j \) (where \( i = j \) is allowed). Conversely, \( B_i \) is not preserved if there exist two or more elements in \( B_i \) that transition to different blocks (i.e. \( \exists x_1, x_2 \in B_i \) such that \( f(x_1) = B_j \) and \( f(x_2) = B_k \) with \( j \neq k \)). In order for the entire partition \( P_i \) to be preserved, each block within the partition must be preserved.

For an isomorphic cascade decomposition to exist, we must be able to iteratively construct a hierarchy or “nested sequence” of preserved partitions such that each partition \( P_i \) evenly splits the partition \( P_{i-1} \) above it in half, leading to a more and more refined description of the system. For a system with \( 2^n \) states where \( n \) is the number of binary components in the original system, this implies we need to find exactly \( n \) nested preserved partitions, each of which then maps onto a unique component of the cascade automaton (Section 2.3.1). If one cannot find a preserved partition made of disjoint blocks or the blocks of a given partition do not evenly split the blocks of the partition above it in half, then the dynamics in question cannot be represented isomorphically. In this case, the thing to look for would be a nested sequence of preserved *covers*, which forms the basis for standard (homomorphic) Krohn-Rhodes decompositions [14,21,22]. Our interest, however, lies solely in demonstrating the existence of systems that allow a nested sequence of preserved partitions.

2.3.1. Example: \( AND/OR \equiv COPY/OR \)

As an example, we will isomorphically decompose the system \( X \) comprised of an \( AND \) gate and an \( OR \) gate as shown in Figure 4a. As it stands, \( X \) is not in cascade form because information flows bidirectionally between the components \( Q_1 \) and \( Q_2 \) (i.e. there is feedback). Our goal is to construct an isomorphic system \( X' \) such that information flows strictly unidirectionally (feed-forward) from \( Q'_1 \) to \( Q'_2 \), as shown in Figure 4b. The behavior of \( X \) and \( X' \) must be identical modulo the internal representation used to instantiate functional states. Thus, the isomorphism will result in a one-to-one map between internal representations in the two systems that realizes the finite-state automaton (state transition diagram) shown in Figure 4c.

Given the state transition diagram shown in Figure 4c, our first preserved partition is \( P_1 = \{B_0, B_1\} \) with \( B_0 = \{A, D\} \) and \( B_1 = \{B, C\} \). It is easy to check that this partition is preserved, as one can readily verify that every element in \( B_0 \) transitions to an element in \( B_0 \) and every element in \( B_1 \).
Figure 4. The goal of an isomorphic cascade decomposition is to decompose the integrated logical architecture of the system $X$ (4a) so that it is in cascade form $X'$ (4b) without affecting the state transition topology of the original system (4c).

transitions to an element in $B_1$ (shown topologically in Figure 5a). The next step is to assign the states in $B_0$ a first coordinate value of 0 and the states in $B_1$ a first coordinate value of 1. As required, this guarantees the movement of the first coordinate is independent of later coordinates: if the value of the first coordinate is 0 it will remain 0 and if the value of the first coordinate is 1 it will remain one. This implies the first coordinate in the feed-forward system ($Q_1'$) is a COPY gate taking input from itself, as this results in the correct movement for the blocks of $P_1$.

The second preserved partition $P_2$ must evenly split each block within $P_1$ in half. Letting $P_2 = \{\{B_{00}, B_{01}\}, \{B_{10}, B_{11}\}\}$ we have $B_{00} = \{A\}$, $B_{01} = \{B\}$, $B_{10} = \{C\}$, and $B_{11} = \{D\}$. At this stage, it is trivial to verify that the partition is preserved because each block is comprised of only one element which is guaranteed to transition to a single block. The logic gate for the second coordinate ($Q_2'$) is specified by the way the labeled blocks of $P_2$ transition. Namely, we have $B_{00} \rightarrow B_{00}$, $B_{01} \rightarrow B_{01}$, $B_{10} \rightarrow B_{01}$, and $B_{11} \rightarrow B_{11}$. Note, the transition function $\delta_{Q_2}$ is completely deterministic given input from the first two coordinates (as required) and is given by $\delta_{Q_2} = \{00 \rightarrow 0; 01 \rightarrow 1; 10 \rightarrow 1; 11 \rightarrow 1\}$ which is an OR gate taking input from both $Q_1'$ and $Q_2'$.

Figure 5. The nested sequence of preserved partitions in 5a yields the isomorphism 5b between $X$ and $X'$ which can be translated into the strictly feed-forward logical architecture with $\Phi = 0$ shown in 5c.

At this point, the isomorphic cascade decomposition is complete. We have constructed an automaton for $Q_1'$ that takes input from only itself and an automaton for $Q_2'$ that takes input only from itself and earlier coordinates (i.e. $Q_1'$). The mapping between the states of $X$ and the states of $X'$, shown in Figure 5b, is specified by identifying the binary labels (internal representations) each system uses to instantiate the functional states $A, B, C, D$ of the global finite-state automaton. Because $X$ and $X'$ operate on the same support (the same four computational states) the fact that they are isomorphic
implies the difference between representations is nothing more than a permutation of the labels used to instantiate functional states. By choosing a specific internal representation based on isomorphic cascade decomposition, we can induce a logical architecture that is feedback free and has $\Phi = 0$ while preserving the functionality of the original system.

3. Results/Discussion

We are now prepared to demonstrate the existence of isomorphic feed-forward philosophical zombies in a slightly larger system, similar to those found in Oizumi et al. [6]. To do so, we will decompose the integrated system $Y$ shown in Figure 6 into an isomorphic feed-forward philosophical zombie $Y'$ of the form shown in Figure 3. The system $Y$, comprised of two XNOR gates and one XOR gate, clearly contains feedback between components and, consequently, has $\Phi > 0$ for all states [23]. As in Section 2.3, the goal of the decomposition is an isomorphic labeling of the finite-state automata representing the behavior of the system (6c) such that the induced logical architecture is strictly feed-forward and has the exact same number of components as the original system.

![Figure 6](image)

(a) The transition probability matrix (6a), logical architecture (6b), and state transition topology (6c) of the example system $Y$.

We first evenly partition the state space of $Y$ into two blocks $B_0 = \{A, C, G, H\}$ and $B_1 = \{B, D, E, F\}$. Under this partition, $B_0$ transitions to $B_1$ and $B_1$ transitions to $B_0$, which implies the automaton representing the first coordinate in the new labeling scheme is a NOT gate. Note, we could just as easily have chosen a different preserved partition such as $B_0 = \{A, D, E, H\}$ and $B_1 = \{B, C, F, G\}$, in which case the first coordinate would be a COPY gate; as long as the partition is preserved, the choice is arbitrary and amounts to selecting one of several possible feed-forward logical architectures in cascade form. For the second preserved partition, we let $P_2 = \{\{B_{00}, B_{01}\}, \{B_{10}, B_{11}\}\}$ with $B_{00} = \{C, G\}$, $B_{01} = \{A, H\}$, $B_{10} = \{B, F\}$ and $B_{11} = \{D, E\}$. The transition function for the automaton representing the second coordinate, given by the movement of these blocks, is: $\delta_{Q^2} = \{00 \rightarrow 0; 01 \rightarrow 1; 10 \rightarrow 0; 11 \rightarrow 1\}$, which is again a COPY gate receiving input from itself. The third and final partition $P_3$ assigns each state to its own unique block. As is always the case, this last partition is trivially preserved because individual states are guaranteed to transition to a single block. The transition function for this coordinate, read off the bottom row of Figure 7, is given by:

$\delta_{Q^3^'} = \{000 \rightarrow 0; 001 \rightarrow 0; 010 \rightarrow 1; 011 \rightarrow 1; 100 \rightarrow 0; 101 \rightarrow 0; 110 \rightarrow 1; 111 \rightarrow 1\}$

Using Karnaugh maps [24], it is straightforward to identify $\delta_{Q^3}$ as a COPY gate copying the input from $Q^2$. With the specification of the logic for the third coordinate, the cascade decomposition is complete and the new labeling scheme can be read off the bottom row of Figure 7. A side-by-side comparison of the original system $Y$ and the feed-forward system $Y'$ is shown in Figure 8. As required, the feed-forward system has $\Phi = 0$ but executes the exact same sequence of state transitions as the original system, modulo a permutation of the labels used to internally represent the functional states.
3.1. Behavior is Representation Independent

Isomorphic cascade decomposition results in a permutation of the binary labels assigned to each state in the state transition diagram of the original system. Given the abstract nature of this formalism, it is important to ask whether the feed-forward isomorphic system is actually behaviorally indistinguishable from the original (conscious) system. To address this, we first must recognize that a given internal labeling scheme induces a specific logical architecture in the sense that once binary labels are assigned to states in the state transition diagram the logical architecture of the system is fixed. This is a consequence of the fact that the labeled states must transition correctly, which results in a label-specific truth table specifying the logic for the update of each coordinate. The same argument runs in reverse as well: if one fixes the logical architecture of a system the result is a fixed labeling scheme. The right-shift automaton, for example, is defined in terms of a specific logical architecture (Q₁ copies the state of the environment and Q₂ copies the state of Q₁), which results in a labeled state transition diagram. Because of this, it does not make sense to talk about an "isomorphic right-shift automaton" because an isomorphic system will have a different labeling scheme (and induced logical architecture) and therefore be unrecognizable as a right shift automaton. In this case, it is fair to say that an isomorphism does not result in indistinguishable behavior.

The right-shift automaton is unique, however, in that its behavior is its logical architecture. In general, what we intuitively think of as "behavior" is defined without reference to any particular logical architecture and/or internal representation. Indeed, Krohn-Rhodes decomposition is specifically designed to yield behaviorally indistinguishable machines utilizing different logical architectures - so how is this possible? The answer is that a map always exists between the internal states of the system and what we consider its behavior. That is, there exists a map \( \Gamma : S \to B \) where \( S \) is the set of internal states of the system and \( B \) is the set of behavior. If every element in \( B \) corresponds to a unique element in \( S \), then the behavior of the system is said to "supervene" on its internal state, meaning that one cannot have a difference in behavior without a difference in the internal state. Given this, every internal state in \( S \) encodes a behavior in \( B \) and the map \( \Gamma \) specifies this encoding. The right-shift automaton is unique in that the behavioral states are intimately connected to a specific internal representation (i.e. the behavior specifies the map \( \Gamma \) a priori). In general, this is not the case. Typical behaviors such as walking, talking, sleeping, etc. do not have a natural representation in terms of a unique internal representation. In other words, we do not know a priori what the internal representation scheme for a set of behaviors should look like, nor should we necessarily expect the same internal representation scheme across systems, even if they are behaviorally indistinguishable. Consistency within a representation scheme is all that matters from a behavioral perspective. The system must be able to correctly translate internal states into behavior in a way that realizes the finite-state automaton
for the system. A homomorphic system, therefore, is fully capable of producing indistinguishable behavior using a different internal representation than the original system, provided one is free to create a new map $\Gamma'$ from internal states onto external behavior (i.e., the hardware of the system can be reprogrammed to "recognize" the new labeling scheme).

In IIT, the map $\Gamma$ from internal states to behavior is left unspecified. Because $\Phi$ acts on the transition probability matrix describing the internal dynamics of a system, it operates without reference to any particular set of behavior. Thus, when we isomorphically decompose a system with $\Phi > 0$ onto a system with $\Phi = 0$ we are free to imagine a new map $\Gamma'$ for the feed-forward system that realizes the behavior of the original system using a different internal representation, in which case, the feed-forward system is indeed a philosophical zombie. Granted, in certain situations the physical hardware of a system may place constraints on $\Gamma'$, but such constraints are not present in the calculation of $\Phi$. In other words, IIT treats the internal dynamics of a system as fundamentally distinct from any particular outward behavior, such that we are free to choose $\Gamma$ and $\Gamma'$ in a way that results in a philosophical zombie. In reality, this freedom corresponds to arbitrarily programmable hardware which is easy to realize in some systems, such as computers and digital electronics, and difficult to realize in others, such as the wet chemical hardware of biology.

### 3.2. Efficiency and Autonomy

With this, we can now address whether isomorphic feed-forward philosophical zombies lack the efficiency and autonomy of their integrated counterparts. Recall, these attributes are used to justify the difference in subjective experience between conscious systems ($\Phi > 0$) and feed-forward philosophical zombies ($\Phi = 0$). The one-to-one correspondence defined by the isomorphism we present here immediately implies a feed-forward zombie system is neither less efficient nor less autonomous than its integrated counterpart. Indeed, the isomorphic zombie system shown in Figure 8 is the exact same size as its conscious counterpart, which implies it is equally efficient under the current definition (thermodynamic costs such as those found in [25] are not considered). Similarly, there is no sense in which the system with $\Phi > 0$ is more autonomous than the isomorphic system with $\Phi = 0$, given they

![Figure 8](image-url)
have the same internal dynamics: both act and react (under a relabeling of states) in the exact same way. Specifically, there is no justification for the interpretation that the state transitions of an integrated logical architecture are attributable to internal states and goals while the behavior of feed-forward counterpart is not, as any internal state or goal that the integrated system is either useless to the transitions in states of the system (its actions) or successfully reproduced by the feed-forward system.

4. Moving Forward

Moving forward there are two ways to interpret our results: either all systems with the same function should be prescribed the same level of consciousness (the functionalist view) or consciousness is representation-dependent (IIT’s point of view). Epistemologically, many have argued that only the former interpretation is valid scientifically, as it is difficult, if not logically impossible, to justify a difference in subjective experience without functional consequences [2,11,12]. So how does IIT justify the privileged role of feedback, relative to feed-forward, logical architectures?

To date, IIT has cited only functional differences between systems with and without feedback in explaining why the latter lacks consciousness. This is problematic, given that the Krohn-Rhodes theorem proves the presence or absence of feedback alone has nothing to do with the functionality of a system. For example, IIT cites the unique conscious experience of split-brain patients [26,27] as evidence in support of the privileged role played by feedback in generating a unified subjective experience [5]. And, indeed, it is virtually impossible to imagine that a unified sensory experience (e.g. a unified visual field) can result from disconnected sensory organs that operate independent from one another. Yet, this says nothing about whether or not uni-directional (feed-forward) information exchange, as opposed to bi-directional (feedback) information exchange, is sufficient to “integrate” an experience.

If we consider functionally identical systems with and without feedback, as is the case with the feed-forward isomorphisms presented above, it becomes very difficult to justify a difference in subjective experience. The Krohn-Rhodes theorem guarantees that the behavior of a split-brain patient is perfectly reproducible using a feed-forward architecture, such that it is possible to imagine a second (feed-forward) patient with \( \Phi = 0 \) that perfectly reproduces the behavior of the split-brain patient - they describe the same experience, perform the same in every experiment, etc. In this case, IIT uses the outward behavior of the split-brain patient to justify a particular subjective experience but denies this subjective experience to the feed-forward patient despite the exact same outward behavior. The reason for doing so rests solely on the assumed connection between feedback and subjective experience, which is what IIT was seeking to justify using this experiment in the first place.

As long as functionally equivalent systems can be prescribed different internal experiences, such contradictions are perhaps inevitable [12]. Given our results, the way to avoid this issue is to first and foremost insist that a measure of consciousness be invariant under isomorphisms. This implies that what is being probed is not any particular logical architecture, but rather, the abstract computation that gives rise to a given set of behavior. This results in the interpretation that consciousness is representation-independent and instead probes only the abstract relationship between computational states. This relationship can be quantified topologically using a measure that acts on a finite-state automaton or algebraically using a measure that acts on the corresponding transformation semigroup [14,16]. In either case, what is being quantified is the nature of a given computation. Permuting labels, therefore, will have no measurable effect on conscious experience and isomorphic physical systems will be prescribed the same intrinsic experience because they realize the same (abstract) computation using different internal representations.

Alternatively, IIT must provide proper justification as to why consciousness is tied to a particular logical architecture (e.g. those with feedback). To do this requires an explanation of what differs between functionally identical systems with different logical architectures. Given the strong mathematical equivalence present in our methodology, this difference must ultimately be justified in terms of a specific internal labeling scheme, as \( \Phi \) is a mathematical measure and this is the only
mathematical difference between isomorphic physical systems. This would imply there is something special about representation that leads to conscious experience.

**Author Contributions:** Conceptualization, J.H. and S.W.; formal analysis, J.H.; funding acquisition, S.W.; investigation, J.H.; methodology, J.H.; project administration, S.W.; resources, S.W.; supervision, S.W.; visualization, J.H.; writing-original draft preparation, J.H. and S.W.; writing-review and editing, J.H. and S.W.

**Acknowledgments:** The authors would like to thank Doug Moore for his assistance with the Krohn-Rhodes theorem and semigroup theory, as well as Dylan Gagler and the rest of the emergence@asu lab for thoughtful feedback and discussions. SIW also acknowledges funding support from the Foundational Questions in Science Institute.

**Conflicts of Interest:** The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

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