How the HBT-Puzzle at RHIC might dissipate
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I compute the first-order corrections to two-particle Bose-Einstein correlations due to deviations from equilibrium. Based on that result I argue that for nearly perfect fluids, the HBT radii “freeze out” much later than the single-inclusive distribution. Therefore, to prevent a big longitudinal homogeneity length, the QGP presumably should hadronize not into a nearly perfect fluid of hadrons but rather into a very dissipative one. This could be achieved by a hadronization phase transition away from equilibrium.

Pion interferometry has become a powerful tool for studying the size and duration of particle production in high-energy collisions, from $e^+e^-$ over $pp$ or $p\bar{p}$ to heavy ions like $Au+Au$ or $Pb+Pb$ [1,2]. For the case of nuclear collisions, the interest mainly focuses on the possible transient formation of a deconfined state of matter. This could affect the size of the region from where the pions are emitted as well as the time for particle production. In particular, it was hoped that a strong first-order QCD phase transition would lead to long lifetimes of the particle source [2,3].

Data [4] from BNL-RHIC, and also from the CERN-SPS, represents a big “puzzle” [5] in that the duration of particle emission, as well as the longitudinal homogeneity length, appear rather short. In fact, within the framework of relativistic perfect fluid dynamics, they seem incompatible with even a mere cross-over [3]. Moreover, studies of hadronic rescattering following hadronization of a QGP fluid indicated that one should always observe a long emission duration for hadrons if they are formed at the confinement temperature $T_c \simeq 180 \pm 20$ MeV in equilibrium [7]. This is because the lifetime of the particle source is enhanced by dissipative effects.

In these notes, I compute the corrections to the single-inclusive pion distribution and to the two-pion correlation function due to deviations from perfect equilibrium. I use a simple schematic model which allows for an analytical treatment: one-dimensional (longitudinal) boostinvariant expansion; of course, I must focus on the longitudinal “radius” $R_{\parallel}$ then. The analytical expression for the longitudinal homogeneity length with first-order non-equilibrium corrections is a new result (to my knowledge), generalizing the expression of Makhlin-Sinyukov [8] which applies to perfect local equilibrium. Based on those results, I discuss why HBT correlation functions are much more sensitive to the late, dissipative stages of the evolution than the single-inclusive distributions (in agreement with quantitative kinetic theory solutions [7]): the first-order theory applied here predicts that the correlation function at small relative momentum of the pions can not in fact “freeze out”. This supports the expectation that one should observe a long duration of emission if the post-QGP evolution starts with a hot ($T \simeq T_c$) and dense hadron fluid close to equilibrium. Hence, the way to go might be: start low. A phase transition away from equilibrium can produce cool hadrons with $T < T_c$, and induce large gradients of the temperature and velocity fields. See the last section.

I do not review here the extensive literature on HBT interferometry for nuclear collisions. Recent studies devoted specifically to effects from a confinement transition in $Au + Au$ at RHIC include computations using

1. ideal hydrodynamics [2,3,11],
2. ideal hydrodynamics up to hadronization followed by microscopic hadron kinetic theory [5],
3. parton kinetic theory with [12] or without [13] hadronic rescattering,
4. a QGP evaporation model without interactions among hadrons [14].

Those papers can also be used to trace earlier literature.

I. FIRST-ORDER DEVIATION FROM EQUILIBRIUM

A collection of many particles in a large, static volume approaches a steady distribution in momentum space at times much larger than the microscopic relaxation time for collisions $\tau_c$. This is the equilibrium distribution $f_{eq}(E)$, which is either a Bose or a Fermi distribution, depending on the nature of the particles. The temperature $T$ characterizes how rapidly the probability to find a particle with energy between $E$ and $E + dE$ falls off as $E$ increases. In equilibrium, $T$ is constant throughout the entire volume.
More generally, one can think of situations where $T$ varies in space. That is, on scales much larger than the interparticle distance but much smaller than the total volume, the distribution of particle energies is still described approximately by the equilibrium distribution. If this is not the same distribution everywhere in space, one expects hydrodynamic flow to develop between those regions.

In the absence of a uniform temperature and flow velocity field (global equilibrium), there must also be local corrections to the equilibrium distribution. They can be obtained from the Boltzmann equation \[1.1\]

$$p \cdot \partial f(x^\mu, p^\mu) = C[f] \, .$$

The collision kernel on the right-hand-side is a functional of the distribution function $f(x^\mu, p^\mu)$. It vanishes for $f(x^\mu, p^\mu) = f_{eq}(E; T(x^\mu))$. However, the left-hand-side does not vanish if $f(x^\mu, p^\mu) = f_{eq}(E; T(x^\mu))$ with a non-uniform temperature field $T(x)$ because of the presence of gradients. Therefore, $f_{eq}(E; T(x^\mu))$ is not a solution of the Boltzmann equation \[1.1\]. Expanding the collision kernel to first order in deviations from equilibrium, one can define the relaxation time for collisions via

$$\frac{E}{\tau_c} = \left. \frac{\delta C}{\delta f} \right|_{f_{eq}} \, .$$

Note that $\tau_c$ depends on both momentum and space-time, since $\delta C/\delta f$ is evaluated at $f_{eq}(x^\mu, p^\mu)$. For simplicity, in what follows I neglect the dependence on energy-momentum (more precisely, that on rapidity; $p_t$ dependence is allowed).

The local distribution function is then given by

$$f(x^\mu, p^\mu) = f_{eq}(E(x^\mu); T(x^\mu)) + \delta f(E(x^\mu); T(x^\mu)) \equiv f_{eq}(E(x^\mu); T(x^\mu)) \left[ 1 + \tilde{\delta} f(E(x^\mu); T(x^\mu)) \right] \, ,$$

where $E(x^\mu) \equiv p \cdot u(x^\mu)$ is the energy of the particles measured in the local rest frame. The Boltzmann equation then reads

$$p \cdot \partial (f_{eq} + \delta f) = C[f_{eq} + \delta f] \simeq C[f_{eq}] + \delta f \frac{\delta C}{\delta f} |_{f_{eq}} \, .$$

The first term on the right-hand-side vanishes by definition of $f_{eq}$, while the second term on the left-hand-side can be dropped if considering only corrections to first order in gradients \[15\]. Thus, using \[1.2\],

$$\delta f = \frac{\tau_c}{p \cdot u} p \cdot \partial f_{eq} \, .$$

In what follows, I focus on the limit $m^2 = p_t^2 + m^2 \gg T^2$, i.e. heavy and/or high-momentum particles, such that the quantum mechanical distribution functions can be approximated by the classical Boltzmann distribution. I also assume that no conserved charges such as baryon number are present; in this case viscous corrections to the energy-momentum tensor obtained from $\delta f$ vanish in the frame where $u^\mu = (1, 0)$, which therefore corresponds to the Landau-Lifshitz definition of the local rest frame. Then,

$$\tilde{\delta} f(x^\mu) = \frac{\tau_c}{p \cdot u(x^\mu)} p \cdot \partial \frac{p \cdot u(x^\mu)}{T(x^\mu)} \, .$$

Below, I will apply \[1.4\] to compute corrections to the two-particle correlation function. The Israel-Stewart second order theory of imperfect fluids in principle represents a more satisfactory, yet much more involved approach \[16\].

**II. SINGLE-PARTICLE DISTRIBUTION**

The single-inclusive distribution of particles in a fluid, measured on some space-time hypersurface $\sigma^\mu$ is given by \[17\]

$$\tilde{N}(p) \equiv \frac{dN}{d^2p_t dy} = \int d\sigma \cdot p \cdot f(p \cdot u) \, .$$

(2.1)
Consider a fluid with infinite extent in the transverse directions, and with space-like hypersurfaces of homogeneity. In particular, for longitudinal scaling flow\(^1\) and vanishing transverse flow velocity, those are surfaces of fixed proper time \(\tau = \sqrt{t^2 - x^2}\), which is invariant under Lorentz boosts. Then, 

\[
\sigma \cdot p = d^2r \, d\eta \tau m_t \cosh(y - \eta) . 
\]  

(2.2)

Thus,

\[
\tilde{N}(p) = \int d^2r \, d\eta \tau m_t \cosh(y - \eta) \{ f_{eq}(p \cdot u) + \delta f (p \cdot u) \} 
= \int d^2r \, d\eta \tau m_t \cosh(y - \eta) f_{eq}(p \cdot u) \left\{ 1 + \tilde{\delta f} (p \cdot u) \right\} . 
\]

(2.3)

At large \(m_t/T\) the Bose or Fermi distributions approach a Boltzmann distribution. Here \(T\) denotes the temperature on the hypersurface specified by \(\tau\); for simplicity, let it be independent of the rapidity of the flow. Moreover, in this limit the single-inclusive distribution can be evaluated by a saddle-point integration,

\[
f_{eq}(p \cdot u) \simeq e^{-p \cdot u/T} = e^{-m_t \cosh(y - \eta)/T} \rightarrow \sqrt{\frac{2\pi T}{m_t}} \delta(y - \eta) e^{-m_t/T} . 
\]

(2.4)

In other words, evaluate the integral over the flow rapidity \(\eta\) by expanding (in the exponential) \(p \cdot u = m_t \cosh \theta \simeq m_t (1 + \theta^2/2)\) up to first nontrivial order in \(\theta = y - \eta\), and set \(y = \eta\) throughout the rest of the integrand in (2.3); this reduces the integral over \(\eta\) to a Gaussian integral.

The saddle-point approximation also greatly simplifies \(\tilde{\delta f}\):

\[
p \cdot \partial_p \frac{p \cdot u(\sigma)}{T(\sigma)} = \frac{1}{T(\sigma)} p \cdot \partial_p \cdot u(\sigma) + p \cdot u(\sigma) p \cdot \partial_1 T = \frac{m_t^2}{\tau T} \sinh^2 \theta + m_t^2 \cosh \theta \left[ \cosh \theta \partial_\tau + \frac{1}{\tau} \sinh \theta \partial_\eta \right] \frac{1}{T(\sigma)} 
\rightarrow m_t^2 \partial_\tau \frac{1}{T} , 
\]

(2.5)

and so

\[
\tilde{\delta f}(p \cdot u) \rightarrow \tau c m_t \partial_\tau \frac{1}{T} . 
\]

(2.6)

The integral over \(d^2r\) is trivial, since we assumed that neither the temperature \(T(\sigma)\) nor the flow velocity \(u^\mu(\sigma)\) depend on \(r\); thus, \(\int d^2r\) equals the transverse area \(S_t = \pi R^2\). We obtain

\[
\tilde{N}(p) = S_t \tau \sqrt{2\pi m_t T} e^{-m_t/T} \left\{ 1 - \frac{\tau c m_t}{\tau T} \frac{\partial \log T}{\partial \log \tau} \right\} . 
\]

(2.7)

To be more explicit, we may for example assume that the temperature prior to freeze-out decreases like a power-law in \(1/\tau\):

\[
T \sim \frac{1}{\tau^\gamma} . 
\]

(2.8)

For ideal isentropic expansion the entropy in a comoving volume element is conserved, which for ultrarelativistic particles \((m \ll T)\) leads to \(\gamma = 1/3\). On the other hand, isoergic expansion is perhaps more realistic near freeze-out; in that case the energy per comoving volume element is conserved, which gives \(\gamma = 1/4\). Then,

\[
\tilde{N}(p) = S_t \tau \sqrt{2\pi T m_t} e^{-m_t/T} \left\{ 1 + \frac{\tau c m_t}{\tau T} \right\} . 
\]

(2.9)

The leading term in eq. (2.4), corresponding to \(\tau c \rightarrow 0\), is the result for an equilibrium distribution of the particles. The second term in the curly brackets is due to corrections from local equilibrium, proportional to the microscopic

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\(^1\)This simply means that the flow rapidity \(\eta_f = \text{Artanh} \, v_x\) equals the space-time rapidity \(\eta = \text{Artanh} \, (x/t)\) everywhere in the forward light-cone, where \(x\) denotes the longitudinal direction.
relaxation time $\tau_c$ relative to the expansion time $\tau$. It gives us a hint about the meaning of “freeze-out”. The hadrons measured in the detector are cold $T = 0$ hadrons; they do not stop interacting suddenly at some temperature. However, as $T$ decreases in the course of the evolution, corrections to $f_{\text{eq}}$ become more and more important. The more of them I include in my computation, the lower I can go in $T$. With the full collision kernel I could go down smoothly to $T = 0$ without introducing an abrupt “decoupling”. The important point to realize though is that as corrections to equilibrium grow, they tend to “stabilize” the single-inclusive distribution: from eq. (2.9) one sees that the ideal $\tilde{N}(p)$ at any fixed $m_t$ decreases with $T$, but the correction term acts to slow down that evolution. Eventually, at some time $\tau$ or temperature $T$, the correction terms “freeze” $\tilde{N}(p)$, i.e. it stops evolving any further despite the ongoing approach of $T$ to zero. I can view that as the “freeze-out” temperature.

What this discussion points at, as well, is that “freeze-out” is not universal but depends on the observable under consideration. This is because corrections from deviation from perfect equilibrium are not the same for all observables. We shall return to that below.

The correction to first order in the relaxation time to equilibrium, $\tau_c$, does not make for a quantitative description of the freeze-out process. Nevertheless, to get a rough idea, one might ask when $d\tilde{N}(p)/dT = 0$. This leads to

$$\dot{\tau}_c = 1 + \frac{\gamma m_t \tau_c}{T} - \frac{T}{m_t} \frac{1 - \gamma/2}{\gamma} \frac{\tau_c}{2 \tau} + \frac{\gamma m_t}{T_0} \left( \frac{\tau_c}{\tau_0} \right)^\gamma \frac{\tau_c}{\tau} - \frac{T_0}{m_t} \frac{1 - \gamma/2}{\gamma} \left( \frac{\tau}{\tau_0} \right)^{-\gamma} - \frac{\gamma \tau_c}{2 \tau} . \tag{2.10}$$

Here, $\tau_0$ is the time when corrections to perfect equilibrium set in. This could be as early as hadronization, or any time after that. $T_0$ denotes the temperature at that time ($m_t/T_0$ must be large enough so that $\dot{\tau}_c > 0$).

At $\tau_0$ we may, for example, be dealing with a perfect fluid, i.e. $\tau_c(\tau_0) = 0$. As long as $\tau_c/\tau$ remains negligibly small, and $\tau/\tau_0$ is not too big, the solution of (2.10) is approximately given by

$$\tau_c = \tau - \tau_0 - \frac{T_0 \tau_0}{m_t} \frac{1 - \gamma/2}{\gamma (1 - \gamma)} \left[ \left( \frac{\tau}{\tau_0} \right)^{1-\gamma} - 1 \right] . \tag{2.11}$$

Thus, $\tau_c$ should grow approximately linearly with the expansion time. As $\tau_c$ becomes larger and comparable to $\tau$, its growth must speed up in order that $\tilde{N}(p)$ remains “frozen” (also, if it is non-negligible already at hadronization, i.e. if the hadron fluid is rather viscous from the start). To see this, keep only terms proportional to $\tau_c$ on the right-hand side of (2.10), which leads to

$$\frac{\tau_c(\tau)}{\tau_c(\tau_0)} = \left( \frac{\tau}{\tau_0} \right)^{-\gamma/2} \exp \left( m_t \frac{T_0}{T_0 - m_t} \left( \frac{\tau}{\tau_0} \right)^\gamma - 1 \right) = \sqrt{\frac{T}{T_0}} \gamma m_t / T_0 . \tag{2.12}$$

Clearly, when the rapid growth kicks in, this signals the break-down of the first-order theory and that corrections of higher order in $\tau_c$ also become important.

So, to summarize, the basic picture is as follows. A hadron “fluid” is produced by the decay of the deconfined state. In theory, it may evolve as an ideal fluid for some time, if the relaxation time to equilibrium is extremely short. Eventually, as the fluid becomes more dilute, the relaxation time must grow: at first rather slowly, then turning to a rapid non-linear growth. Alternatively, the relaxation time may be non-negligible already at hadronization, and continue to grow rapidly as the fluid expands. In either case, as soon as the growth rate of $\tau_c$ satisfies eq. (2.10), the single-particle distribution doesn’t change any more, it “freezes”. Smoothly, the fluid cools to $T = 0$.

### III. TWO-PARTICLE CORRELATION FUNCTION

The two-particle correlation function for identical particles is given by

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2For example, for a chiral symmetry which is broken spontaneously (by choice of a vacuum) but not explicitly, at low $T$ corrections to the pressure of non-interacting pions are determined by chiral perturbation theory: $\sim (T/f_\pi)^4 p_{\text{id}}$.

3At some point number-changing reactions must also freeze. One then has to introduce a continuity equation for the number-current, which will lead to a chemical potential $\tilde{N}$. 

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\[ R(p_1, p_2) \equiv \tilde{N}(p_1)\tilde{N}(p_2)[C_2(p_1, p_2) - 1] \]
\[ = \text{Re} \int d\sigma_1 \cdot K d\sigma_2 \cdot K \exp i(\sigma_1 - \sigma_2) \cdot (p_1 - p_2) \]
\[ \times \{ f_{eq}(u_1 \cdot K) + \delta f(u_1 \cdot K) \} \{ f_{eq}(u_2 \cdot K) + \delta f(u_2 \cdot K) \} , \]

(3.1)

with \( K^\alpha = (p_1^\mu + p_2^\mu)/2 \). I consider particle pairs emitted at the same azimuthal angle and with the same transverse momentum, \( \vec{\tilde{p}}_{\perp,1} = \vec{\tilde{p}}_{\perp,2} \). This is no severe restriction as for 1+1d longitudinal expansion only differences in the longitudinal momenta of the emitted particles matter. Introducing the variables \( \alpha = y_2 - y_1 \) for the relative rapidity of the emitted particles and \( \theta_1 = y_1 - \eta_1 \), \( \theta_2 = y_2 - \eta_2 \) for their rapidities relative to the emitting fluid elements, the phase factor becomes

\[ e^{i(\sigma_1 - \sigma_2) \cdot (p_1 - p_2)} = e^{im_\perp \tau [(\cosh \theta_1 - \cosh(y_2 - \eta_1) + \cosh \theta_2 - \cosh(y_1 - \eta_2))} = e^{im_\perp \tau [(1 - \cosh \alpha)(\cosh \theta_1 + \cosh \theta_2) - \sinh \alpha (\sinh \theta_1 - \sinh \theta_2)]} \]
\[ = e^{im_\perp \tau (\theta_2 - \theta_1 - \alpha(\theta_1^2 + \theta_2^2)/4) + \mathcal{O}(\alpha^3, \theta_1^2, \theta_2^2)} . \]

(3.2)

In anticipation of the saddle-point integration over the flow rapidities to be performed below, I expanded the exponent up to second order in \( \theta_1 \) and \( \theta_2 \). Also, I shall be interested mainly in the behavior of the correlation function at small relative rapidity and so expanded up to second order in \( \alpha \).

The same manipulations lead to

\[ d\sigma_1 \cdot K = \frac{\tau m_\perp}{2} [(1 + \cosh \alpha) \cosh \theta_1 + \sinh \alpha \sinh \theta_1] d\theta_1 d^2 r_1 \]
\[ = \frac{\tau m_\perp}{2} \left[ \left( 1 + \frac{\alpha^2}{4} + \frac{\alpha}{2} \right) e^{\theta_1} + \left( 1 + \frac{\alpha^2}{4} - \frac{\alpha}{2} \right) e^{-\theta_1} \right] d\theta_1 d^2 r_1 , \]

(3.3)

\[ f_{eq}(u_1 \cdot K) = e^{-\frac{m_\perp}{2\tau}(\cosh \theta_1 + \cosh(y_2 - \eta_1))} = e^{-\frac{m_\perp}{2\tau}(1 + \cosh \alpha) \cosh \theta_1 + \sinh \alpha \sinh \theta_1} \]
\[ = e^{-\frac{m_\perp}{2\tau}(2 + \alpha^2/2)(1 + \theta_1^2/2) + \mathcal{O}(\alpha, \theta_1^3)} . \]

(3.4)

The expressions for \( d\sigma_2 \cdot K \) and \( f_{eq}(u_2 \cdot K) \) can be obtained by substituting \( \theta_1 \to \theta_2 \) and \( \alpha \to -\alpha \). Integrating over the rapidity \( \theta_1 \) gives

\[ e^{-\alpha^2 m_\perp \tau /2} \int_{-\infty}^{\infty} d\theta_1 e^{-im_\perp \tau (\theta_1 + \theta_1^2/4)} e^{-\frac{m_\perp}{2\tau}(\alpha \theta_1 + (1 + \alpha^2/4)\theta_1^2)} \left[ \left( 1 + \frac{\alpha^2}{4} + \frac{\alpha}{2} \right) e^{\theta_1} + \left( 1 + \frac{\alpha^2}{4} - \frac{\alpha}{2} \right) e^{-\theta_1} \right] \]
\[ \approx \sqrt{\frac{2\pi T}{m_\perp}} \left( 1 - \frac{\alpha^2}{4} (1 + 2i\tau T) \right) e^{-\frac{m_\perp}{2\tau} \tau^2 \theta_1^2} \left( 2 + \frac{\alpha^2}{4} \right) \cos(\alpha \tau T) . \]

(3.5)

On the right-hand side, I performed the following approximations. I expanded the arguments of the exponentials assuming \( \alpha \ll 1 \) but \( \alpha m_\perp / T \) and \( \alpha \tau T \) of order one. I kept only terms up to \( \mathcal{O}(1) \) in this approximation scheme. Nevertheless, I dropped a term \((i/4)\alpha^4(m_\perp / T)(\tau \partial^2 T) = \mathcal{O}(1) \) since I will mainly be interested in the curvature of the correlation function at \( \alpha = 0 \). Since (3.5) is symmetric in \( \alpha \to -\alpha \), the integral over \( \theta_1 \) gives the same result.

Next, I need \( \delta f(u_1 \cdot K) \) in the limit \( \theta_1 \to 0 \), up to second order in \( \alpha \). The result is

\[ \tilde{\delta} f(u_1 \cdot K) = \tau_c m_\perp \left\{ \left[ \left( 1 + \frac{\alpha^2}{4} \right) \partial_\tau + \frac{\alpha}{2\tau} \partial_{y_1} \right] \frac{1}{T} - \frac{\alpha^2}{4 T \tau} \right\} . \]

(3.6)

The result for \( \tilde{\delta} f(u_2 \cdot K) \) is the same, except that the derivative of \( 1/T \) with respect to rapidity is evaluated at \( y_2 \), respectively, and \( \alpha \to -\alpha \). Then,

\[ R(m_\perp, \alpha) \equiv \int d^2 r_1 d^2 r_2 \tau^2 2\pi T m_\perp e^{-2m_\perp / T \cos^2(\alpha \tau T)} e^{-m_\perp \alpha^2 \tau^2 T} \]
\[ \times \left\{ 1 + \tau_c m_\perp \left\{ \left[ \left( 1 + \frac{\alpha^2}{4} \right) \partial_\tau + \frac{\alpha}{2\tau} \partial_{y_1} \right] \frac{1}{T} - \frac{\alpha^2}{4 T \tau} \right\} \right\} \left\{ 1 + \tau_c m_\perp \left\{ \left[ \left( 1 + \frac{\alpha^2}{4} \right) \partial_\tau - \frac{\alpha}{2\tau} \partial_{y_2} \right] \frac{1}{T} - \frac{\alpha^2}{4 T \tau} \right\} \right\} . \]

(3.7)

For any function \( f(y) \), to leading order in \( \alpha = y_2 - y_1 \)

\[ f'(y_2) = f'(y_1) + \alpha f''(y_1) . \]

(3.8)
For \( f(y) = 1/T(y) \), then,
\[
\partial_y \frac{1}{T} = -\frac{1}{T^2} \partial_y T + \alpha \left( -\frac{1}{\tau T} \partial_T T + \frac{2}{\tau^2} (\partial_y T)^2 \right) = \frac{2\alpha}{T} (\partial_y \log T)^2 - \frac{1}{\tau^2} \partial_y \log T - \frac{\alpha}{\tau^2} \partial_T^2 T.
\] (3.9)

On the right-hand-side, \( T \) is evaluated at \( y_1 \). Integrating over the transverse area, and dividing by the product of the single-particle distributions leads to
\[
C_2(m_t, \alpha) - 1 = \cos^2(\alpha T) e^{-m_t \alpha^2 T}
\]
\[
\times \left\{ 1 - \frac{\tau_c m_t \alpha^2}{\tau T} \left[ \frac{1}{2} \left( 1 + \left( \frac{\partial \log T}{\partial \log \tau} \right)^2 - \frac{1}{\tau} \left( \frac{\partial^2 T}{\partial y_1^2} \right) + 2 \left( \frac{\partial \log T}{\partial y_1} \right)^2 \right) \right] \right\}.
\] (3.10)

Here, \( \langle \cdot \rangle \) refer to averages over the transverse plane as well as over events. For consistency, only terms to first order in \( \tau_c \) have been kept.

We can define a longitudinal homogeneity length as the curvature of the correlation function at \( \alpha = 0 \):
\[
R^2_\parallel = -\frac{1}{m_t^2} \frac{1}{2} \frac{\partial \partial^2 C_2(\alpha)}{\partial \alpha^2} \bigg|_{\alpha=0} = \frac{\tau^2 T^2}{m_t^2} \left( \frac{\tau}{m_t} \right) \left( 1 - \frac{\tau_c}{\tau} \frac{1 - \gamma}{\tau} \right) \left( \frac{T}{m_t} \right) - \frac{\alpha}{\tau^2 T^2} \left( 2 \right) \left( \frac{\partial \log T}{\partial y_1} \right)^2.
\] (3.11)

The \( 1/m_t^2 \) prefactor arises because one should in fact take the derivative with respect to the longitudinal momentum difference; for small relative rapidity \( \alpha \), and in the logitudinally comoving frame, \( 1/m_t \tau = \tanh \gamma \alpha \). The first term in (3.11) is subleading in \( T/m_t \ll 1 \) and will be dropped. Then,
\[
R_\parallel = \tau \sqrt{\frac{T}{m_t} \left( \frac{\tau}{m_t} \right) \left( 1 - \frac{\tau_c}{\tau} \frac{1 - \gamma}{\tau} + \frac{2\Delta_T^2}{2\tau^2 T^2} \right)},
\] (3.12)

with
\[
\Delta_T^2 = \left\langle \left( \frac{\partial \log T}{\partial y_1} \right)^2 \right\rangle, \quad \zeta_T = \frac{1}{\tau} \left\langle \partial^2 T \right\rangle.
\] (3.13)

Deviations from equilibrium manifest themselves in the second term under the square root. They do not distort the scaling of \( R_\parallel \) with \( 1/\sqrt{m_t} \) predicted from a local equilibrium distribution \( \[8\] \) (regarding scaling of the out and side radii see \( \[20\] \)). Note, however, the “wrong sign” of the correction relative to the leading term, to which we shall return below in eq. (3.14).

\( \Delta_T^2 \) is the mean-square fluctuation of the temperature \( T \) divided by the typical rapidity scale \( \Delta y \) on which it occurs; it measures fluctuations about the average homogeneous particle source\( \[4\] \). Only fluctuations on scales \( \Delta y \) smaller than the temperature \( T \) at which the correlation function \( C_2(\alpha) \) is being probed matter. Fluctuations on larger scales do not matter as in that case both particles are emitted from the same rapidity “element” and at the same \( T \), so \( \Delta y \) is of order unity. In any case, such rapidity fluctuations of \( T \) can only give a significant correction to \( R_\parallel \) if \( \Delta y \ll 1 \) and if \( \Delta T/T \) is of order unity. In other words, after the transition, there should be regions of longitudinal extension \( \sim 1 \) fm (which at time \( \tau \approx 10 \) fm corresponds to a rapidity interval of 0.1) where \( T \) is about equal the ordinary hydrodynamic freeze-out temperature for a homogeneous fluid, separated by cold regions of similar size with \( T \sim 0 \) (see also \( \[21\] \)). Nevertheless, temperature fluctuations of order \( \Delta T/T \sim 1 \) appear unrealistically large. Moreover, experimentally

\footnote{The average temperature field was of course assumed to be rapidity independent, \( (\partial T/\partial y) = 0 \).}

\footnote{These are analogous to the temperature fluctuations of the cosmic microwave background measured by COBE \( \[21\] \), of order \( \Delta T/T \approx 10^{-5} \).}

\footnote{The single-inclusive distribution \( \[20\] \) is not sensitive to \( \Delta_T \). That is, of course, due to the fact that it measures the temperature at only one rapidity but not correlations of \( T \) between two rapidities \( y_2 \) and \( y_1 \). One might be tempted to measure the single-particle distribution in very narrow bins in rapidity, and in individual events, such as to trace any local disturbance of a boost invariant temperature distribution. This is not possible, however, because the saddle-point \( \[20\] \) of the distribution function has a width \( \sqrt{2\pi T/m_t} \) in rapidity, which can not be less than \( \sim 1 \) for values of \( m_t \) where the application of the collective hydrodynamic theory makes sense. Thus, temperature fluctuations on scales \( \Delta y \ll 1 \) get washed out.}
one usually measures correlations of identical pions, which are the most abundant hadron species. A significant part of
the pion yield is commonly believed to originate from decays of resonances (ρ, η, Δ, ...) after freeze-out. Those
resonance decays smear out any temperature fluctuations on scales Δy ≪ 1, even if those were present before the
resonance decays. Therefore, realistically it seems that the correction proportional to ΔT \^2 in eq. (3.12), which would
tend to enhance R∥, is rather small.

There is also an increase (1 − γ > 0) of R∥ by an amount depending on the temperature gradient in time direction.
If that gradient γ is small, which corresponds to a rather viscous fluid, the increase is largest. If the temperature drops
steeply, γ is larger and so the correction is smaller. In any case, by considering either isentropic or isoergic expansion,
i.e. conservation of either the entropy or the energy in a comoving volume element, one obtains γ ≃ 1/4 − 1/3, and
so 1 − γ = O(1).

The only term leading to a reduction of R∥ is that proportional to ζT, the curvature of the temperature field in
rapidity. This is also obvious intuitively. Note that this curvature term arises from the first-order correction to local
equilibrium (it is proportional to τc). A nonvanishing curvature of T does not contradict the initial assumption that
(∂T/∂y) = 0 at midrapidity. Moreover, one could still argue that T is rapidity independent only on scales probed by
the single-inclusive distribution, say Δy ≃ 1, but does exhibit some local curvature on scales Δy ≪ 1. Even so, it
doesn’t seem likely that the average local curvature, averaged over the transverse plane and events, is big.

Therefore, in summary, all of the corrections from eq. (3.12) proportional to τc should be rather small for the
case of high-energy nuclear collisions, in particular since they are down by a factor 1/(τT)2 relative to perfect local
equilibrium: for nuclear collisions, typically τ is on the order of 10 fm and T on the order of 100 MeV, so τT ≃ 5.

This brings us to the main point. From kinetic solutions for the evolution of hadrons produced from a QGP fluid it
was observed that the two-particle correlation functions are sensitive to a “cloud” of late soft hadronic interactions,
which only minorly affect the single-inclusive distribution [18,7]. We can understand qualitatively why this is so.
Above, I argued that deviations from local equilibrium grow as the temperature T decreases and eventually “freeze”
the single-inclusive distribution (2.9), despite the smoothly ongoing cooling down to T = 0.

The correlation function (3.10) also exhibits a correction term proportional to τc/τ. Note, however, that the
correction vanishes at small relative rapidity (or longitudinal momentum) of the pions, α → 0! The characteristic
relative rapidity over which C2 falls off is set by the exponential in eq. (3.10), α2 ≃ 1/(mπτ2T). Thus, the first-order
correction in the curly brackets in (3.10) is of order τc/(τ3T2), and can only become large when τc/τ ≃ τ2T2. This is a
large number for the case of nuclear collisions, and it grows with time. This means that if we are dealing with a nearly
perfect hadron fluid (τc/τ ≪ 1) with only linear growth of τc in time, as in eq. (2.11), then non-ideal corrections to R∥
actually decrease with time. This is despite the fact that they are sufficient to actually “freeze” the single-inclusive
distribution!

For R∥ to “freeze”, i.e. dR∥/dτ = 0, the relaxation time must evolve as

\[ \dot{\tau}_c = (1 - \gamma) \frac{\tau_c}{\tau} - 2 \frac{2 - \gamma}{1 - \gamma} T^2 \tau^2, \]  \hspace{1cm} (3.14)

where I dropped ζT and ΔT for simplicity. Thus, close to equilibrium, i.e. for small τc/τ, R∥ can not “freeze” since
the relaxation time would have to decrease with time, which seems unphysical. Eq. (3.14) shows that there is no
physical solution (τc ≥ 0 for τ > τ0) which leads to “freezing” of R∥ in a fluid near equilibrium, contrary to \( \hat{N}(p) \).
That is because for \( \dot{\tau}_c \) to stay positive, the first term on the right-hand-side of eq. (3.14) must overwhelm the second
one. However, the first term by itself asks for a rather slow, nearly linear increase \( \tau_c \sim \tau^{1-\gamma} \). In turn, the negative
term in (3.14) grows almost quadratically with τ, and so will eventually take over. The only way out is to stabilize
R∥ through corrections of higher order in τc, which are not taken into account in (3.11). This motivates why the HBT
radii are much more sensitive to the dissipative evolution of the hadron “fluid” than the single-inclusive distribution
or quantities derived from it (flow (2.4)).

The “HBT puzzle” can therefore be formulated as follows: if hadronization leads to a hot and dense hadron fluid,
\( \tau_c/\tau \) is small initially. It will take a long time until the two-particle correlation function “freezes out” because \( \tau_c \)
first has to evolve slowly into the regime where corrections of higher order in \( \tau_c \) become important. Consequently, \( R∥ \)
will be large. In fact, assuming that the initial temperature of the hadrons is equal to the confinement temperature
\( T_c \simeq 180 \pm 20 \text{ MeV} \) already leads to significant deviations of the computed HBT radii [23] to the data [3]. The

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7 At least, if one has in mind typical kinetic estimates of relaxation times, see for example [23]. Near a critical point estimates
of \( \tau_c \) might be altered.

8 Here, “initial” refers to the initial condition for the hadrons, which is set by the hadronization.
theoretical calculations do not fail very badly, but in a clear systematic fashion. While the computed ratio of the out and side radii, \( R_o / R_s \), increases with the pair transverse mass \( m_t \), the data shows it rather flat. Also, the experimental \( R_o(m_t) \) does seem to roughly follow the \( 1/\sqrt{m_t} \) behavior but is systematically lower than the theory at all \( m_t \), even for the most “optimistic” assumption \( T_c = 160 \text{ MeV} \).

**IV. HOW IT MIGHT WORK...**

If a slow growth of the relaxation time prevents \( R_0 \) from freezing quickly, how could one then obtain a small homogeneity length?

It clearly helps to start the final-state hadron kinetic evolution at low temperature, and with big deviations from equilibrium. The closer the initial condition for the hadronic evolution is to the “freeze-out” condition, the better. The results of \( \text{[7]} \) already showed that the picture improves if the initial temperature of the hadron fluid is lowered, but the use of perfect-fluid dynamics as a hadronization scheme implies that on the hadronization surface \( \tau_c/\tau \) was small in those studies. As discussed above, the QGP shouldn’t hadronize into an ideal fluid of hadrons, but into a very viscous one.

Employing ideal hydrodynamics to model the phase transition, the fluid of hadrons starts out essentially at \( T_c \). With \( T_c \geq 160 \text{ MeV} \), one can not describe the HBT data \( \text{[6]} \) very well \( \text{[22]} \); the hadronic fluid is too dense and \( \tau_c \) too small. One should probably go lower in temperature, which implies a non-equilibrium phase transition. One way to obtain it is from parton cascade models \( \text{[14,22]} \), where one can tune the parton→hadron transition such as to yield a cool hadron fluid with initial temperature \( T_0 \) below \( T_c \) and with large velocity and temperature gradients. (Increasing the parton-parton cross section relative to lowest-order pQCD estimates presumably does the job by delaying the transition into hadrons.) This might be the reason why ref. \( \text{[12]} \) apparently is able to fit the radii at RHIC (in fact, \( R_0 \) even comes out too small). In field-theory language, in turn, one would have to consider spinodal decomposition \( \text{[24]} \) if the transition is first-order. If it is a cross over or weak first-order transition, the decay of a condensate which saturates the free energy of the deconfined state at \( T_c \) \( \text{[27]} \) could be a successful approach: while the deconfinement temperature sets the scale for the effective potential, the reheating temperature of the hadrons is determined by the decay process of the condensate and by the overall expansion, and so is quite likely less than \( T_c \) itself.

In summary, the pion correlation function requires much smaller relaxation rate than the single-particle distribution in order to “freeze”. Therefore, HBT interferometry of the QGP “ashes” \( \text{[2]} \) might reveal some of the bulk properties of the QGP hadronization and its non-equilibrium nature. The first-order theory applied here is not capable of quantitative predictions but it clearly points at the fact that “freeze-out” of two-particle correlations at small relative momentum is tightly related to the *dynamics* of the confinement transition.

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