Testing Neutrino Properties at Long Baseline Experiments and Neutrino Factories*

Sandip Pakvasa

Department of Physics & Astronomy
University of Hawaii
Honolulu, HI 96822

Abstract. It is shown that explanations of atmospheric neutrino anomaly other than $\nu_\mu - \nu_\tau$ oscillations (e.g. decay, decoherence and $\nu_\mu - \nu_\tau - \nu_{KK}$ mixing) can be tested at future facilities. Stringent tests of CPT invariance in neutrino oscillations can also be performed.

I INTRODUCTION

In this talk I would like to (a) review some non-oscillatory explanations for the atmospheric neutrino data, and how they can be distinguished from the conventional oscillatory explanations in future neutrino experiments; and (b) describe briefly the strong limits that can be placed on CPT violation both from existing data as well as future experiments especially at neutrino factories.

II NEUTRINO DECAY, DECOHERENCE AND EXTRA DIMENSIONS

Decay

If neutrinos do have masses and mixings; then in general, in addition to oscillating, the heavier neutrinos will decay to lighter ones via flavor changing processes. The only questions are (a) whether the lifetimes are short enough to be interesting and (b) what are the dominant decay modes. To be specific, let us assume that neutrino masses are at most of order eV[1].

For eV neutrinos, the only radiative mode possible is $\nu_i \to \nu_j + \gamma$. From the existing bounds on neutrino magnetic moments, indirect (but model independent) bounds on the decay rates for this mode can be derived: $10^{-11}s^{-1}, 10^{-17}s^{-1}$ and $10^{-19}s^{-1}$ for $\nu_\tau, \nu_\mu$ and $\nu_e$ respectively[1].

* Presented at the NuFACT’00, Monterey, May 22-26, 2000, to be published in the proceedings.
The decay rate for the invisible three body decay $\nu_i \to \nu_j \bar{\nu}$ can be written (for $m_i \gg m_j$) as

$$\Gamma = \frac{\varepsilon_i^2 G_F^2 m_i^5}{192\pi^3}$$  \hspace{1cm} (1)$$

The current experimental bound on $\varepsilon_\mu$ is of $0(100)[2]$ (the one loop result in SM is $\varepsilon^2 \sim 3\times10^{-12}$) and thus $\Gamma$ for $m_i \sim 0(eV)$ has to be less than $10^{-30}\text{s}^{-1}$.

The possible existence of a $I_w = 0$, $J = 0$ and $L = 0$ massless particle, $\chi$ (such as a Nambu-Goldstone boson of broken family symmetry) leads to a flavor changing decay mode:

$$\nu_{\alpha L} \to \nu_{\beta L} + \chi$$ \hspace{1cm} (2)$$

By $SU(2)_L$ symmetry decays $\ell_\alpha \to \ell_\beta + \chi$ will occur with the same strength. Current bounds on $BR(\mu \to e\chi)$ and on $BR(\tau \to \mu\chi)$ of $2\times10^{-6}$ and $7.1\times10^{-6}[3]$ respectively are sufficient to constrain $\nu_\mu$ and $\nu_\tau$ lifetimes to be longer than $10^{29}\text{s}$ and $10^{20}\text{s}$.

The only possibility for fast invisible decays of neutrinos seems to lie with majoron models. There are two classes of models; the $I=1$ Gelmini-Roncadelli[4] majoron and the $I=0$ Chikasige-Mohapatra-Peccei[5] majoron. In general, one can choose the majoron to be a mixture of the two; furthermore the coupling can be to flavor as well as sterile neutrinos. The effective interaction is of the form:

$$g_\alpha \bar{\nu}_\beta \nu_\alpha \ J$$ \hspace{1cm} (3)$$
giving rise to decay:

$$\nu_\alpha \to \bar{\nu}_\beta + J$$ \hspace{1cm} (4)$$

where $J$ is a massless $J = 0$ $L = 2$ particle; $\nu_\alpha$ and $\nu_\beta$ are mass eigenstates which may be mixtures of flavor and sterile neutrinos. Models of this kind which can give rise to fast neutrino decays and satisfy the bounds below have been discussed by Valle, Joshipura and others[6]. These models are unconstrained by $\mu$ and $\tau$ decays which do not arise due to the $\Delta L = 2$ nature of the coupling. The $I=I$ coupling is constrained by the bound on the invisible $Z$ width; and requires that the Majoron be a mixture of $I=1$ and $I=0$. The couplings of $\nu_\mu$ and $\nu_e$ ($g_\mu$ and $g_e$) are constrained by the limits on multi-body $\pi, K$ decays $\pi \to \mu\nu\nu\nu$ and $K \to \mu\nu\nu\nu$ and on $\mu - e$ universality violation in $\pi$ and $K$ decays[7].

Granting that models with fast, invisible decays of neutrinos can be constructed, can such decay modes be responsible for any observed neutrino anomaly?

We assume a component of $\nu_\alpha$, i.e., $\nu_2$, to be the only unstable state, with a rest-frame lifetime $\tau_0$, and we assume two flavor mixing, for simplicity:

$$\nu_\alpha = \cos \theta \nu_2 + \sin \theta \nu_1$$ \hspace{1cm} (5)$$

with $m_2 > m_1$. From Eq. (2) with an unstable $\nu_2$, the $\nu_\alpha$ survival probability is
\[ P_{\alpha\alpha} = \sin^4 \theta + \cos^4 \theta \exp(-\alpha L/E) \]
\[ + 2\sin^2 \theta \cos^2 \theta \exp(-\alpha L/2E) \cos(\delta m^2 L/2E), \] (6)

where \( \delta m^2 = m_2^2 - m_1^2 \) and \( \alpha = m_2/\tau_0 \). Since we are attempting to explain neutrino data without oscillations there are two appropriate limits of interest. One is when the \( \delta m^2 \) is so large that the cosine term averages to 0. Then the survival probability becomes

\[ P_{\mu\mu} = \sin^4 \theta + \cos^4 \theta \exp(-\alpha L/E) \] (7)

Let this be called decay scenario A. The other possibility is when \( \delta m^2 \) is so small that the cosine term is 1, leading to a survival probability of

\[ P_{\mu\mu} = (\sin^2 \theta + \cos^2 \theta \exp(-\alpha L/2E))^2 \] (8)

We note in passing that scenario A does not provide an acceptable fit to atmospheric neutrino data [8,9]. Turning to decay scenario B, consider the following possibility[10]. The three states \( \nu_\mu, \nu_\tau, \nu_s \) (where \( \nu_s \) is a sterile neutrino) are related to the mass eigenstates \( \nu_2, \nu_3, \nu_4 \) by the approximate mixing matrix

\[
\begin{pmatrix}
\nu_\mu \\
\nu_\tau \\
\nu_s
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta & 0 \\
-\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\nu_2 \\
\nu_3 \\
\nu_4
\end{pmatrix}
\] (9)

and the decay is \( \nu_2 \to \bar{\nu}_4 + J \). The electron neutrino, which we identify with \( \nu_1 \), cannot mix very much with the other three because of the more stringent bounds on its couplings [7], and thus our preferred solution for solar neutrinos would be small angle matter oscillations.

In this case the \( \delta m^2_{23} \) in Eq. (6) is not related to the \( \delta m^2_{24} \) in the decay, and can be very small, say \(< 10^{-4} \text{eV}^2\) (to ensure that oscillations play no role in the atmospheric neutrinos). Then the oscillating term is 1 and \( P(\nu_\mu \to \nu_\mu) \) is given by Eq. (8).

The decay model of Equation (8) above gives a very good fit to the Super-K data [11] with a minimum \( \chi^2 = 33.7 \) (32 d.o.f.) for the choice of parameters

\[ \tau_\nu/m_\nu = 63 \text{ km/GeV}, \quad \cos^2 \theta = 0.30 \] (10)

and normalization \( \beta = 1.17 \).

The fits (as shown in Fig. 1 in Ref. 10) show the ratios between the Super-K data and the Monte Carlo predictions calculated in the absence of oscillations or other form of ‘new physics’ beyond the standard model. The best fits of the two models (viz. \( \nu_\mu - \nu_\tau \) oscillations and decay) are of comparable quality. The reason for the similarity of the results obtained in the two models can be understood by looking at Fig. 1, where I show the survival probability \( P(\nu_\mu \to \nu_\mu) \) of muon neutrinos as a function of \( L/E_\nu \) for the two models using the best fit parameters.
In the case of the neutrino decay model (thick curve) the probability \( P(\nu_\mu \rightarrow \nu_\mu) \) monotonically decreases from unity to an asymptotic value \( \sin^2 \theta \simeq 0.49 \). In the case of oscillations the probability has a sinusoidal behaviour in \( L/E_\nu \). The two functional forms seem very different; however, taking into account the resolution in \( L/E_\nu \), the two forms are hardly distinguishable. In fact, in the large \( L/E_\nu \) region, the oscillations are averaged out and the survival probability there can be well approximated with 0.5 (for maximal mixing). In the region of small \( L/E_\nu \) both probabilities approach unity. In the region \( L/E_\nu \approx 400 \text{ km/GeV} \), where the probability for the neutrino oscillation model has the first minimum, the two curves are most easily distinguishable, at least in principle.

For the atmospheric neutrinos in Super-K, two kinds of tests have been proposed to distinguish between \( \nu_\mu - \nu_\tau \) oscillations and \( \nu_\mu - \nu_s \) oscillations. One is based on the fact that matter effects are present for \( \nu_\mu - \nu_s \) oscillations [12] but are nearly absent for \( \nu_\mu - \nu_\tau \) oscillations [13] leading to differences in the zenith angle distributions due to matter effects on upgoing neutrinos [14]. In our case since the mixing is \( \nu_\mu - \nu_\tau \) no matter effect is expected; and hence the recent Super-K results[15] are in accord with expectations of this decay model. The other test is based on the neutral current rate (as measured via production or multi-ring events) which is unaffected in \( \nu_\mu - \nu_\tau \) oscillations but reduced in \( \nu_\mu - \nu_s \) oscillations[16]. In our case of the decay model, the neutral current rate is affected and the expectation is closer to \( \nu_\mu - \nu_s \) mixing.

### Long-Baseline Experiments

The survival probability of \( \nu_\mu \) as a function of \( L/E \) is given in Eq. (1). The conversion probability into \( \nu_\tau \) is given by

\[
P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 \theta \cos^2 \theta (1 - e^{-\alpha L/2E})^2.
\]

(11)

This result differs from \( 1 - P(\nu_\mu \rightarrow \nu_\mu) \) and hence is different from \( \nu_\mu - \nu_\tau \) oscillations. Furthermore, \( P(\nu_\mu \rightarrow \nu_\mu) + P(\nu_\mu \rightarrow \nu_\tau) \) is not 1 but is given by

\[
P(\nu_\mu \rightarrow \nu_\mu) + P(\nu_\mu \rightarrow \nu_\tau) = 1 - \cos^2 \theta (1 - e^{-\alpha L/E})
\]

(12)

and determines the amount by which the predicted neutral-current rates are affected compared to the no oscillations (or the \( \nu_\mu - \nu_\tau \) oscillations) case. Fig. 2 shows the results for \( P(\nu_\mu \rightarrow \nu_\mu), P(\nu_\mu \rightarrow \nu_\tau) \) and \( P(\nu_\mu \rightarrow \nu_\mu) + P(\nu_\mu \rightarrow \nu_\tau) \) for the decay model and compare them to the \( \nu_\mu - \nu_\tau \) oscillations, for both the K2K[16] and MINOS[17](or the corresponding European project[18]) long-baseline experiments, with the oscillation and decay parameters as determined in the fits above.

The K2K experiment, already underway, has a low energy beam \( E_\nu \approx 1-2 \text{ GeV} \) and a baseline \( L = 250 \text{ km} \). The MINOS experiment will have 3 different beams, with average energies \( E_\nu = 3, 6 \) and 12 GeV and a baseline \( L = 732 \text{ km} \). The approximate \( L/E_\nu \) ranges are thus 125–250 km/GeV for K2K and 50–250 km/GeV for MINOS. The comparisons in Figure 2 show that the energy dependence of \( \nu_\mu \) survival probability and the neutral current rate can both distinguish between the
decay and the oscillation models. ICANOE and especially MONOLITH can also test for the oscillation dip[19].

Decoherence[20]

There are several different possibilities that can give rise to decoherence of the neutrino beam. An obvious one is violation of quantum mechanics, others are unknown (flavor specific) new interactions with environment etc[21]. Quantum gravity effects are also expected to lead to effective decoherence[22,23].

The density matrix describing the neutrinos no longer satisfies the usual equation of motion:

\[ \dot{\rho} = -i[H, \rho] \]  

but rather is modified to

\[ \dot{\rho} = -i[H, \rho] + D(\rho) \]  

Imposing reasonable conditions on \( D(\rho) \)[24] it was shown by Lisi et al.[20] that the \( \nu_\mu \) survival probability \( P_{\mu\mu} \) has the form:

\[ P_{\mu\mu} = \cos^2 2\theta + \sin^2 2\theta \ e^{-\gamma L} \cos \left( \frac{\delta m^2 L}{2E} \right). \]  

where \( \gamma \) is the decoherence parameter. If \( \delta m^2 \) is very small (\( \delta m^2 L/2E \ll 1 \)), this reduces to

\[ P_{\mu\mu} = \cos^2 2\theta + \sin^2 2\theta \ e^{-\gamma L} \]  

If \( \gamma = \alpha/E \) with \( \alpha \) constant, then an excellent fit to the Super-K data can be obtained with \( \theta = \pi/4 \) and \( \alpha \sim 7.10^{-3} \) GeV/Km. (If gamma is a constant, no fit is possible and gamma can be bounded by \( 10^{-22} \) GeV). The fits to Super-K data are shown in ref. 20. and they are as good as the decay or \( \nu_\mu - \nu_\tau \) oscillations[25]. The shape of \( P_{\mu\mu} \) as a function of \( L/E \) is very similar to the decay case as shown in Fig. 1.

Large Extra Dimensions

Recently the possibility that SM singlets propagate in extra dimensions with relatively large radii has received some attention[26]. In addition to the graviton, right handed neutrino is an obvious candidate to propagate in some extra dimensions. The smallness of neutrino mass (for a Dirac neutrino) can be linked to this property of the right handed singlet neutrino[27]. The implications for neutrino masses and oscillations in various scenarios have been discussed extensively [28,29]. I focus on one particularly interesting possibility for atmospheric neutrinos raised by Barbieri et al [30]. The survival probability \( P_{\mu\mu} \) is given by

\[ P_{\mu\mu}(L) = | \sum_{i=1}^{3} V_{\alpha i} V_{\alpha i}^* A_i(L) |^2. \]  

where
A_i(L) = \sum_{n=0}^{\infty} U_{on}^{(i)2} \exp(i\lambda_n^{(i)} L/2ER^2) \quad (18)

where \( n \) runs over the tower of Kaluza-Klein states, \( \lambda_n^{(i)}/R^2 \) are the eigenvalues of the mass-squared matrix and \( U_{on}^{(i)}(\approx 1/\pi^2\xi^2) \) are the matrix elements of the diagonalizing unitary matrix.

An excellent fit to the atmospheric neutrino data can be obtained with the following choice of parameters:

\[ \xi_3 = m_3R \sim 3, 1/R \sim 10^{-3} \text{eV}, V_{\mu\beta}^2 \approx 0.4. \quad (19) \]

The fit to Super-K data is shown in Ref.27 and obviously it is as good as oscillations. This case corresponds to \( \nu_\mu \) oscillating into \( \nu_\tau \) and a large number (about 25) of Kaluza-Klein states. Because of the mixing with a large number of closely spaced states, the dip in oscillations gets washed out and \( P_{\mu\mu} \) looks very much like the decay model as shown in the Fig. 1.

### III CPT VIOLATION IN NEUTRINO OSCILLATIONS [31]

Consequences of \( CP, T \) and \( CPT \) violation for neutrino oscillations have been written down before [32]. We summarize them briefly for the \( \nu_\alpha \to \nu_\beta \) flavor oscillation probabilities \( P_{\alpha\beta} \) at a distance \( L \) from the source. If

\[ P_{\alpha\beta}(L) \neq P_{\bar{\alpha}\bar{\beta}}(L), \quad \beta \neq \alpha, \quad (20) \]

then \( CP \) is not conserved. If

\[ P_{\alpha\beta}(L) \neq P_{\beta\alpha}(L), \quad \beta \neq \alpha, \quad (21) \]

then \( T \)-invariance is violated. If

\[ P_{\alpha\beta}(L) \neq P_{\beta\alpha}(L), \quad \beta \neq \alpha, \quad (22) \]

or

\[ P_{\alpha\alpha}(L) \neq P_{\bar{\alpha}\bar{\alpha}}(L), \quad (23) \]

then \( CPT \) is violated. When neutrinos propagate in matter, matter effects give rise to apparent \( CP \) and \( CPT \) violation even if the mass matrix is \( CP \) conserving.

The \( CPT \) violating terms can be Lorentz-invariance violating (LV) or Lorentz invariant. The Lorentz-invariance violating, \( CPT \) violating case has been discussed by Colladay and Kostelecky [33] and by Coleman and Glashow [34].

The effective LV \( CPT \) violating interaction for neutrinos is of the form

\[ \bar{\nu}_L^{\alpha\beta} \gamma_\mu \nu_L^\beta, \quad (24) \]
where $\alpha$ and $\beta$ are flavor indices. We assume rotational invariance in the “preferred” frame, in which the cosmic microwave background radiation is isotropic (following Coleman and Glashow [34]).

$$m^2/2\rho + b_0,$$

where $b_0$ is a hermitian matrix, hereafter labeled $b$.

In the two-flavor case the neutrino phases may be chosen such that $b$ is real, in which case the interaction in Eq. (24) is CPT odd. The survival probabilities for flavors $\alpha$ and $\bar{\alpha}$ produced at $t = 0$ are given by [34]

$$P_{\alpha\alpha}(L) = 1 - \sin^2 2\Theta \sin^2 (\Delta L/4),$$

and

$$P_{\bar{\alpha}\bar{\alpha}}(L) = 1 - \sin^2 2\bar{\Theta} \sin^2 (\bar{\Delta} L/4),$$

where

$$\Delta \sin 2\Theta = \left| (\delta m^2/E) \sin 2\theta_m + 2\delta b e^{i\eta} \sin 2\theta_b \right|,$$

$$\Delta \cos 2\Theta = (\delta m^2/E) \cos 2\theta_m + 2\delta b \cos 2\theta_b.$$  

$\bar{\Delta}$ and $\bar{\Theta}$ are defined by similar equations with $\delta b \rightarrow -\delta b$. Here $\theta_m$ and $\theta_b$ define the rotation angles that diagonalize $m^2$ and $b$, respectively, $\delta m^2 = m_2^2 - m_1^2$ and $\delta b = b_2 - b_1$, where $m_i^2$ and $b_i$ are the respective eigenvalues. We use the convention that $\cos 2\theta_m$ and $\cos 2\theta_b$ are positive and that $\delta m^2$ and $\delta b$ can have either sign. The phase $\eta$ in Eq. (28) is the difference of the phases in the unitary matrices that diagonalize $\delta m^2$ and $\delta b$; only one of these two phases can be absorbed by a redefinition of the neutrino states.

Observable CPT-violation in the two-flavor case is a consequence of the interference of the $\delta m^2$ terms (which are CPT-even) and the LV terms in Eq. (24) (which are CPT-odd); if $\delta m^2 = 0$ or $\delta b = 0$, then there is no observable CPT-violating effect in neutrino oscillations. If $\delta m^2/E \gg 2\delta b$ then $\Theta \simeq \theta_m$ and $\Delta \simeq \delta m^2/E$, whereas if $\delta m^2/E \ll 2\delta b$ then $\Theta \simeq \theta_b$ and $\Delta \simeq 2\delta b$. Hence the effective mixing angle and oscillation wavelength can vary dramatically with $E$ for appropriate values of $\delta b$.

We note that a CPT-odd resonance for neutrinos ($\sin^2 2\Theta = 1$) occurs whenever $\cos 2\Theta = 0$ or

$$(\delta m^2/E) \cos 2\theta_m + 2\delta b \cos 2\theta_b = 0;$$

similar to the resonance due to matter effects [35,36]. The condition for antineutrinos is the same except $\delta b$ is replaced by $-\delta b$. The resonance occurs for neutrinos if $\delta m^2$ and $\delta b$ have the opposite sign, and for antineutrinos if they have the same sign. A resonance can occur even when $\theta_m$ and $\theta_b$ are both small, and for all values of $\eta$; if $\theta_m = \theta_b$, a resonance can occur only if $\eta \neq 0$. If one of $\nu_\alpha$ or $\nu_\beta$ is $\nu_e$, then matter effects have to be included.

If $\eta = 0$, then
$$\Theta = \theta,$$  \hspace{1cm} (31)

$$\Delta = (\delta m^2 / E) + 2 \delta b.$$ \hspace{1cm} (32)

In this case a resonance is not possible. The oscillation probabilities become

$$P_{\alpha \alpha}(L) = 1 - \sin^2 2\theta \sin^2 \left\{ \left( \frac{\delta m^2}{4E} + \frac{\delta b}{2} \right) L \right\},$$ \hspace{1cm} (33)

$$P_{\bar{\alpha} \bar{\alpha}}(L) = 1 - \sin^2 2\theta \sin^2 \left\{ \left( \frac{\delta m^2}{4E} - \frac{\delta b}{2} \right) L \right\}. \hspace{1cm} (34)$$

For fixed $E$, the $\delta b$ terms act as a phase shift in the oscillation argument; for fixed $L$, the $\delta b$ terms act as a modification of the oscillation wavelength.

An approximate direct limit on $\delta b$ when $\alpha = \mu$ can be obtained by noting that in atmospheric neutrino data the flux of downward going $\nu_\mu$ is not depleted whereas that of upward going $\nu_\mu$ is\cite{11}. Hence, the oscillation arguments in Eqs. (33) and (34) cannot have fully developed for downward neutrinos. Taking $|\delta bL/2| < \pi/2$ with $L \sim 20$ km for downward events leads to the upper bound $|\delta b| < 3 \times 10^{-20}$ GeV; upward going events could in principle test $|\delta b|$ as low as $5 \times 10^{-23}$ GeV. Since the $CPT$-odd oscillation argument depends on $L$ and the ordinary oscillation argument on $L/E$, improved direct limits could be obtained by a dedicated study of the energy and zenith angle dependence of the atmospheric neutrino data.

The difference between $P_{\alpha \alpha}$ and $P_{\bar{\alpha} \bar{\alpha}}$

$$P_{\alpha \alpha}(L) - P_{\bar{\alpha} \bar{\alpha}}(L) = -2 \sin^2 2\theta \sin \left( \frac{\delta m^2 L}{2E} \right) \sin(\delta b L),$$ \hspace{1cm} (35)

can be used to test for $CPT$-violation. In a neutrino factory, the ratio of $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$ to $\nu_\mu \rightarrow \nu_\mu$ events will differ from the standard model (or any local quantum field theory model) value if $CPT$ is violated. Fig. 3 shows the event ratios $N(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu) / N(\nu_\mu \rightarrow \nu_\mu)$ versus $\delta b$ for a neutrino factory with $10^{19}$ stored muons and a 10 kt detector at several values of stored muon energy, assuming $\delta m^2 = 3.5 \times 10^{-3}$ eV$^2$ and $\sin^2 2\theta = 1.0$, as indicated by the atmospheric neutrino data\cite{11}. The error bars in Fig. 3 are representative statistical uncertainties. The node near $\delta b = 8 \times 10^{-22}$ GeV is a consequence of the fact that $P_{\alpha \alpha} = P_{\bar{\alpha} \bar{\alpha}}$, independent of $E$, whenever $\delta bL = n\pi$, where $n$ is any integer; the node in Fig. 3 is for $n = 1$. A $3\sigma$ $CPT$ violation effect is possible in such an experiment for $\delta b$ as low as $3 \times 10^{-23}$ GeV for stored muon energies of 20 GeV. Although matter effects also induce an apparent $CPT$-violating effect, the dominant oscillation here is $\nu_\mu \rightarrow \nu_\tau$, which has no matter corrections in the two-neutrino limit; in any event, the matter effect is in general small for distances much shorter than the Earth’s radius.

We have also checked the observability of $CPT$ violation at other distances, assuming the same neutrino factory parameters used above. For $L = 250$ km, the $\delta bL$ oscillation argument in Eq. (35) has not fully developed and the ratio of $\bar{\nu}$ to $\nu$ events is still relatively close to the standard model value. For $L = 2900$ km, a $\delta b$ as low as $10^{-23}$ GeV may be observable at the $3\sigma$ level. However, longer distances may also have matter effects that simulate $CPT$ violation.
IV SUMMARY

At Long Baseline Experiments and Neutrino Factories true signatures of oscillations (dips) can be established and decay like scenarios can be excluded with confidence. Furthermore these facilities can test CPT conservation at levels better than $10^{23}$ GeV.

Acknowledgments
I thank Vernon Barger, John Learned, Eligio Lisi, Paolo Lipari, Maurizio Lusignoli, Tom Weiler and Kerry Whisnant for extensive discussions and collaboration.

This work was supported in part by U.S.D.O.E under grant DE-FG-03-94ER40833.

REFERENCES

1. S. Pakvasa, hep-ph/0004077.
2. M. Bilenky and A. Santamaria, hep-ph/9908272.
3. A. Jodidio et al, Phys. Rev. D34, 1967 (1986).
4. G. Gelmini and M. Roncadelli, Phys. Lett. B99, 411 (1981).
5. Y. Chikasige, R. Mohapatra and R. Peccei, Phys. Rev. Lett. 45, 1926 (1980).
6. J. Valle, Phys. Lett. B131, 87 (1983); G. Gelmini and J. Valle, ibid B142, 181 (1983), A. Choi and A. Santamaria, Phys. Lett. B267, 504 (1991); A. Joshipura and S. Rindani, Phys. Rev. D4, 300 (1992).
7. V. Barger, W-Y. Keung and S. Pakvasa, Phys. Rev. D25, 907 (1982).
8. V. Barger, J. G. Learned, S. Pakvasa and T.J. Weiler, Phys. Rev. Lett. 82, 2640 (1999).
9. P. Lipari and M. Lusignoli, Phys. Rev. D60, 013003 (1999); G. L. Fogli, E. Lisi and A. Marrone, Phys. Rev. D59, 117303 (1999); S. Choubey and S. Goswami, hep-ph/9904257.
10. V. Barger, J. G. Learned, P. Lipari, M. Lusignoli, S. Pakvasa and T. J. Weiler, Phys. Lett. B462, 109 (1999).
11. Y. Fukuda et al. (the Super-Kamiokande Collaboration), Phys. Rev. Lett. 81, 1562 (1998); ibid 82, 2644 (1998).
12. V. Barger, N. Deshpande, P. Pal, R.J.N. Phillips and K. Whisnant, Phys. Rev. D43, 1759 (1991); E. Akhmedov, P. Lipari and M. Lusignoli, Phys. Lett. B300, 128 (1993).
13. J. Pantaleone, Phys. Rev. D49, 2152 (1994).
14. Q. Liu and A. Smirnov, Nucl. Phys. B524, 505 (1998); P. Lipari and M. Lusignoli, Phys. Rev. D58, 073005 (1998).
15. F. Vissani and A. Smirnov, Phys. Lett. B432, 376 (1998); J. Learned, S. Pakvasa, and J. Stone, Phys. Lett. B435, 131 (1998); L. Hall and H. Murayama, Phys. Lett. B436, 323 (1998).
16. KEK-PS E362, INS-924 report (1992).
17. MINOS Collaboration, NuMI-L-375 report (1998).
18. ICANOE, F. Emeudo et al., INFN-AE-99-07; OPERA,CERN/SPSC 2000-028.
19. MONOLITH, M. Aglietta et al., CERN/SPSC 98-20.
20. E. Lisi, A. Marrone and D. Montanino, hep-ph/0002053.
21. R. Alicki and K. Lendi, *Quantum Dynamical Semigroups and Applications*, Lect. Notes Phys. **286** (Springer-Verlag, Berlin, 1987).
22. S.B. Giddings and A. Strominger, *Nucl. Phys. B**307**, 854 (1988); W. H. Zurek, *Physics Today* **44**, No. 10, p.36 (1991); G. Amelino-Camelia, J. Ellis, N.E. Mavromatos, and D.V. Nanopoulos, *Int. J. Mod. Phys. A**12**, 607 (1997); L.J. Garay, *Int. J. Mod. Phys. A**14**, 4079 (1999).
23. J. Ellis, J.S. Hagelin, D.V. Nanopoulos, and M. Srednicki, *Nucl. Phys. B**241**, 381 (1984).
24. F. Benatti and R. Floreanini, *Phys. Lett. B**468**, 287 (1999), and references therein.
25. Decoherence as an explanation for atmospheric neutrinos was also suggested in: Y. Grossman and M. Worah, hep-ph/9867511, C.H. Chang et al., *Phys. Rev. D**60**,033006 (1999).
26. I. Antoniadis, *Phys. Lett. B**246**,377 (1990); J.D. Lykken, *Phys. Rev. D**54**, 3693 (1996); N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett. B**429**, 263(1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett. B**436**, 263 (1998).
27. K.R. Dienes, E. Dudas and T. Gherghetta, *Nucl. Phys. B**557**,25(1999); N. Arkani-Hamed et al, hep-ph/9811448.
28. R.N. Mohapatra, S. Nandi and A. Perez-Lorenzana, *Phys. Lett. B**466**, 115 (1999); R. N. Mohapatra and A. Perez-Lorenzana, hep-ph/9910474; Y. Grossman and M. Neubert, hep-ph/9912408.
29. G. Dvali and A. Yu. Smirnov, *Nucl. Phys. B**563**,63 (1999).
30. R. Barbieri, P-Creminelli and A. Strumia, hep-ph/0002199.
31. V. Barger, S. Pakvasa, T. Weiler and K. Whisnant, hep-ph/0005197.
32. N. Cabibbo, *Phys. Lett. B**72**, 333 (1978); V. Barger, K. Whisnant, R.J.N. Phillips, *Phys. Rev. Lett. 45*, 2084 (1980); S. Pakvasa, in *Proc. of the XXth International Conference on High Energy Physics*, ed. by L. Durand and L.G. Pondrom, AIP Conf. Proc. No. 68 (AIP, New York, 1981), Vol. 2, p. 1164.
33. D. Colladay and V.A. Kostelecky, *Phys. Rev. D**55**, 6760 (1997).
34. S. Coleman and S.L. Glashow, *Phys. Rev. D**59**, 116008 (1999).
35. V. Barger, K. Whisnant, S. Pakvasa, and R.J.N. Phillips, *Phys. Rev. D**22**, 2718 (1980).
36. L. Wolfenstein, *Phys. Rev. D**17**, 2369 (1978); S.P. Mikheyev and A. Smirnov, *Yad. Fiz. 42*, 1441 (1985) [Sov. J. Nucl. Phys. 42, 913 (1986)].
FIGURE 1. Survival probability for $\nu_\mu$ versus $\log_{10} (L/E)$ for the decay model, decoherence, extra dimensions and oscillation.
FIGURE 2. Long-baseline expectations for the K2K and MINOS long-baseline experiments from the decay model and the $\nu_\mu - \nu_\tau$ oscillation model. The upper panel gives the neutral current predictions compared to no oscillations (or $\nu_\mu - \nu_\tau$ oscillations).
FIGURE 3. The ratio of $\bar{\nu}_\mu \to \bar{\nu}_\mu$ to $\nu_\mu \to \nu_\mu$ event rates in a 10 kt detector for a neutrino factory with $10^{19}$ stored muon with energies $E_\mu = 10, 20, 30, 50$ GeV for baseline $L = 732$ km versus the CPT-odd parameter $\delta b$ with $\theta_m = \theta_b \equiv \theta$ and phase $\eta = 0$. The neutrino mass and mixing parameters are $\delta m^2 = 3.5 \times 10^{-3}$ eV$^2$ and $\sin^2 2\theta = 1.0$. The dotted line indicates the result for $\delta b = 0$, which is given by the ratio of the $\bar{\nu}$ and $\nu$ charge-current cross sections. The error bars are representative statistical uncertainties.