Strong electron-photon coupling in one-dimensional quantum dot chain: Rabi waves and Rabi wavepackets

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We predict and theoretically investigate the new coherent effect of nonlinear quantum optics — spatial propagation of Rabi oscillations (Rabi waves) in one-dimensional quantum dot (QD) chain. QD-chain is modeled by the set of two-level quantum systems with tunnel coupling between neighboring QDs. The space propagation of Rabi waves in the form of traveling waves and wave packets is considered. It is shown, that traveling Rabi waves are quantum states of QD-chain dressed by radiation. The dispersion characteristics of traveling Rabi waves are investigated and their dependence on average number of photons in wave is demonstrated. The propagation of Rabi wave packets is accompanied by the transfer of the inversion and quantum correlations along the QD-chain and by the transformation of quantum light statistics. The conditions of experimental observability are analyzed. The effect can find practical use in quantum computing and quantum informatics.

PACS numbers: 32.80.Xx, 42.65.Sf, 71.10.Li, 71.36.+c, 73.21.La, 78.67.Lt

I. INTRODUCTION

Rabi oscillations are periodical transitions of a two-level quantum system between its stationary states under the action of an oscillatory driving field, see e.g. 14, 15. The phenomenon was theoretically predicted by Rabi on nuclear spins in radio-frequency magnetic field16 and was firstly observed by Torrey17. Afterwards, Rabi oscillations were discovered in various physical systems, such as electromagnetically driven atoms18 (including the case of Rydberg atomic states19), semiconductor quantum dots (QDs)20, and different types of solid-state qubits (superconducting charge qubits based on Josephson junctions21,22,23,24,25,26,27, spin-qubits28, semiconductor charge-qubits29). In real physical systems the ideal picture of Rabi effect can be essentially modified by additional features, such as the time-domain modulation of the field-matter coupling constant30,31, the phonon-induced dephasing32 and the local-field effects33,34. New phenomena appear in Rabi oscillators with broken inversion symmetry35 and in systems of two coupled Rabi oscillators36,37,38,39,40.

In spatially extensive samples with a large number of oscillators the propagation effects come into play. As a result, the mechanism responsible for Rabi oscillations causes also a number of nonstationary coherent optical phenomena, such as optical nutation, photon echo, self-induced transparency, etc.41,42. In low-dimensional systems the propagation effects also take place. For example, the numerical modeling of the coherent intersubband Rabi oscillations in a sample comprising 80 AlGaAs/GaAs quantum wells43 shows that the population dynamics depends on the quantum well position in the series. This result demonstrates strong radiative coupling between wells and, more generally, significant difference in the picture of Rabi effect for single and multiple oscillators. Another aspects of such a difference, namely, the effects of quantum interference and correlations between photons in multiatom fluorescence, are demonstrated in44,45 (on the example of atomic chains in nanofibers). From practical point of view, the effect of Rabi oscillations is a key ingredient for realization of binary logic and optical control in quantum informatics and quantum computing.

The theoretical analysis of Rabi oscillations is highly diversified both in form and content. The common feature is the impossibility of consideration of electromagnetic field influence as a small perturbation. Different ways of description are used in the analysis of the model problem of Rabi oscillations in a single two-level atom. One such a way, the probability amplitude method, consists in solving of the Schrödinger equation for wavefunction |Ψ⟩, which is the superposition of various atom-photon states. The second one is the Heisenberg operator method which is based on the analysis of the photonic and atomic operators time evolution. And finally, the third way is the unitary time-evolution operator technique. As is demonstrated in1, all these methods lead to identical solutions; the choice of the concrete one is determined by the convenience considerations.

Taking into account the quantum nature of the electromagnetic field has the special significance in the analysis of Rabi oscillations in complex systems. Two different cases can be marked out. The first could be called quasiclassical one46. In this case external field has classical nature. However, the true field is supposed to be concordant with the quantum motions of the particles. So, the field should contain the contribution of the induced polarization, which has a quantum nature. Another case is really quantum and takes into consideration the photon structure of the electromagnetic field47. During the process of energy-level transitions in the atom the photon structure of the field is also transformed, therefore the atom-photon dynamics should be considered self-consistently.

In this paper we build for the first time a theoretical...
model of a distributed system of coupled Rabi oscillators and predict a new physical effect: propagation of Rabi oscillations in space in the form of traveling waves and wave packets. Quantum oscillators in the system that exhibit Rabi-wave propagation interact strongly enough to stare and exchange the e-h pair excitation between light emission and light absorption acts. Often in quantum optics an "all-matter" picture is employed, where the dynamics of electromagnetic field is integrated out, for example in the optical Bloch equations\cite{32,33}. In\cite{13-15} the light-matter interaction is treated in an "all-light" picture (Lippmann-Schwinger equation approach).

In our model the quantum nature of light is also fully accounted. As the theoretical approach the probability amplitude method, generalized for the case of 1D-chain, is convenient.

The paper is organized as follows. In Sec.II the model is formulated and equations of electron-photon dynamics both for discrete chain and continuous limit are obtained. In Sec.III the plane Rabi waves are considered (their dispersion equations, eigenmode structure, dispersion curves properties). Sec.IV is devoted to the Rabi frequency\cite{34,35}, generalized for the case of 1D-chain, accounted. As the theoretical approach the picture (Lippmann-Schwinger equation approach).

Consider an infinite periodical one-dimensional chain of identical QDs, containing an electron in the size-quantized conduction band. It is assumed that in each p-th QD there are at least two one-electron orbital states, ground $|b_p\rangle$ and excited $|a_p\rangle$, with transition frequency $\omega_0$. Neighboring QDs are coupled via electron tunneling\cite{32}, i.e. the electron can go from state $|a_p\rangle$ to state $|a_{p+1}\rangle$ and from $|b_p\rangle$ to state $|b_{p+1}\rangle$. Transitions between ground state and excited state belonging to different QDs are neglected: $\langle a_{p\pm 1}| b_p \rangle \approx 0$. Let the QD chain be exposed to a plane wave of quantum light, which is incident on the chain at an angle $\alpha$, $\mathbf{E}(x) = \mathbf{E}(\hat{a} e^{ikx} + \hat{a}^* e^{-ikx})$, where $\hat{a}$, $\hat{a}^*$ are photon creation and annihilation operators, $\mathbf{E} = \sqrt{2\pi \hbar^2 / V_0} \hat{a}$ is the normalizing volume, $\hat{a}$ is the unit polarization vector, $\omega = \omega_0 \cos \alpha / c$. The dependance of the field on transverse coordinates is negligibly small because of electrical smallness of QDs. The case of oblique propagation is of interest due to the possibility of wavenumber variation with variation of incidence angle $\alpha$ at constant frequency $\omega$. Then the Hamiltonian of the system in the rotating wave approximation\cite{33} reads

$$\hat{H} = \hat{H}_d + \hat{H}_f + \hat{H}_{df} + \hat{H}_T + \Delta \hat{H},$$

where $\hat{H}_d = (\hbar \omega_0 / 2) \sum_p \sigma_{2p}$ is Hamiltonian of QD-chain in the absence of electron tunneling and QD-field interaction, $\sigma_{2p} = |a_p\rangle \langle a_p| - |b_p\rangle \langle b_p|$, $\hat{H}_f = \hbar \omega \hat{a}^* \hat{a}$ is the Hamiltonian of the free electromagnetic field\cite{32}. The component

$$\hat{H}_{df} = \hbar g \sum_p (\hat{\sigma}_p^+ \hat{a} e^{ikpa} + \hat{\sigma}_p^- \hat{a}^* e^{-ikpa})$$

describes QD-field interaction, where $g = -\mu \mathbf{E} / \hbar$ is interaction constant, $\mu$ is the QD dipole moment. Note that all transition dipole moments in the chain are assumed to be real values and have fixed orientation. The operators $\hat{\sigma}_p^+ = |a_p\rangle \langle b_p|$ and $\hat{\sigma}_p^- = |b_p\rangle \langle a_p|$ are the transition operators from ground to excited states and vice versa for p-th QD. The term

$$\hat{H}_T = -\hbar \xi_1 \sum_p \langle |a_p\rangle \langle a_{p+1}| + |a_p\rangle \langle a_{p-1}| \rangle$$

$$-\hbar \xi_2 \sum_p \langle |b_p\rangle \langle b_{p+1}| + |b_p\rangle \langle b_{p-1}| \rangle$$

(3)

takes into account interdot electron tunneling; $\xi_{1,2}$ are the electron tunneling frequencies for the excited ($\xi_1$) and ground ($\xi_2$) states of the QDs. The term $\Delta \hat{H}$ corresponds to the local-field effects originated from the dipole-dipole electron-hole intradot interaction\cite{17,18,19}; in the mean-field approximation this term is given by

$$\Delta \hat{H} = \frac{4\pi}{V} \mu \tilde{\mathbf{N}} \mu \sum_p \langle \hat{\sigma}_p^- \langle \hat{\sigma}_p^+ \rangle + \hat{\sigma}_p^+ \langle \hat{\sigma}_p^- \rangle \rangle.$$  

(4)

Here $\tilde{\mathbf{N}}$ is the depolarization tensor (see Eq. (18) in Ref.\cite{20}), $V$ is the volume of QD.

B. Equations of motion

The state vector of the "QD-chain+light" system may be represented in terms of the eigenstates of isolated QDs.
as

$$
|\Psi(t)\rangle = \sum_n \sum_p (A_{p,n}(t) |a_p,n\rangle + B_{p,n}(t) |b_p,n\rangle).
$$ (5)

Here $|b_p,n\rangle = |b_p\rangle \otimes |n\rangle$, $|a_p,n\rangle = |a_p\rangle \otimes |n\rangle$, where $|n\rangle$ is the light Fock state with $n$ photons, $B_{p,n}$, $A_{p,n}$ are the probability amplitudes. In the interaction picture the evolution of the system is described by the nonstationary Schrödinger equation $i\hbar \partial_t |\Psi\rangle = \hat{V} |\Psi\rangle$, where the interaction Hamiltonian has the form $H = \sum_p A_{p,n}B_{p,n+1}e^{-i(kpa_\omega t)} - i\Delta \omega B_{p,n} \sum_m A_{p,m}B_{p,m}^*$, where

$$
\frac{\partial B_{p,n+1}}{\partial t} = \frac{i\omega_0}{2} B_{p,n+1} - i\xi A_{p,n}(B_{p,n+1} + B_{p+1,n+1}) - i\Delta \omega A_{p,n+1} \sum_m A_{p,m} B_{p,m},
$$ (6)

$$
\frac{\partial A_{p,n}}{\partial t} = \frac{i\omega_0}{2} A_{p,n} + i\xi A_{p,n}(A_{p,n} + A_{p+1,n}) - i\Delta \omega A_{p,n} \sum_m A_{p,m} B_{p,m}^*,
$$ (7)

where

$$
\Delta \omega = \frac{4\pi}{h}\n\$$ (8)

is the local-field induced depolarization shift. Obtaining equations (5) and (6) we have taken into account that the interaction (2) can cause the transitions between the states $(a_p,n)$, $|b_p,n+1\rangle$ only. It is seen from these equations, two competitive mechanisms manifest themselves in the light-QD-chain coupling: the local-field induced nonlinearity and the dispersion spreading due to the tunneling.

To this point we have no account of the decay processes inside the QDs. To take these processes into account one should account for processes of interaction of e-h pair in QD with phonon bath. Such interaction can be described by conception of quantum trajectories of e-h pair, which are the superpositions of deterministic evolutions and random jumps under the action of Lindblad operators. In this case the Hamiltonian (1) should be replaced by the effective non-Hermitian Hamiltonian $\hat{H}_{eff} = \hat{H} - i\hbar \sum_p \hat{X}_p^+ \hat{X}_p$, where $\hat{X}_p$ is Lindblad operator for $p$-th QD. Suppose that the density of phonon states of the bath has the form of Lorentz line with frequency $\omega_0$ and width $\lambda$. If the bath is weakly coupled with QD then $\hat{X}_p = \sqrt{N/2}\hat{\sigma}^+_p$ (see Refs.37,38,39). It means that the decay, can be taken into account by substitution $\omega_0 \rightarrow \omega_0 - i\lambda$ into Eq(6) and $\omega_0 \rightarrow \omega_0 + i\lambda$ into Eq(7). Let us restrict here consideration to linear regime of the carrier motion and omit the terms $O(\Delta \omega)$ in (6)–(7). In the Sec.VI we shall analyze the limitations imposed by omission of nonlinear terms. By this means, omitting the terms $O(\Delta \omega)$, we arrive at the set of difference-differential equations for vectors $\Psi_{p,n}(t) = \begin{pmatrix} A_{p,n}(t) \\ B_{p,n+1}(t) \end{pmatrix}$, as follows:

$$
\partial_t \Psi_{p,n} = \begin{pmatrix} i\omega_0/2 - i\Delta \omega/2 + \hat{I} - ig\sqrt{n+1}\kappa \xi_p/2 \end{pmatrix} \Psi_{p,n} + i\xi (\Psi_{p-1,n} + \Psi_{p+1,n}),
$$ (9)

where $\hat{I}$ denotes 2D unit operator, $\kappa_p(t) = \hat{\sigma}_x \exp[i(\omega t - kp\sigma_3)/2]$. Coefficients in (9) are expressed in terms of Pauli matrices

$$
\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
$$ (10)

acting on vectors $\Psi_{p,n}(t)$.

C. Continuous limit

Equations (9) are the recurrent ordinary differential equations. In some cases it is more convenient to transform them into the system of partial differential equations. To do this we should turn to the continuous limit, making the standard substitutions (see, for example,40) $\sum \rightarrow \int$, $\Psi_{p,n} \rightarrow \Psi_{p,n}(x)$, $\Psi_{p-1,n} + \Psi_{p+1,n} - 2\Psi_{p,n} \rightarrow a^2 \partial_x^2 \Psi_{p,n}$, $\kappa_p \rightarrow \kappa(x)$. Then the system (9) leads to

$$
\partial_t \Psi_n = \begin{pmatrix} 2i\xi/2 - i\omega_0/2 - \hat{I} - ig\sqrt{n+1}\kappa(\omega t - kx) \end{pmatrix} \Psi_n + i\xi a^2 \hat{\partial}_x^2 \Psi_n.
$$ (11)

Formula (11) represents the system of partial differential equations with the variable coefficient $\kappa = \kappa(\omega t - kx)$. This is the basic system for most of our further calculations.

III. TRAVELING RABI WAVES

A. Eigenmodes

Consider elementary solution of system (11) in the form of damping traveling waves: $A_{n} = u_n e^{i(h-k/2)x} e^{-i(\nu+\omega/2)t} e^{-\lambda t/2}$, $B_{n+1} = v_{n+1} e^{i(h-k/2)x} e^{-i(\nu-\omega/2)t} e^{-\lambda t/2}$, where $h$ is a given wave number and $\nu$ is the eigenfrequency to be found, $u_n$, $v_n$ are the unknown constant coefficients. Substituting them into equations (11) and omitting common factors, we obtain

$$
\{\nu - \Delta/2 + \vartheta_1(h)\} u_n - g\sqrt{n+1} v_{n+1} = 0, \quad (12)
$$

$$
\nu + \Delta/2 + \vartheta_2(h) v_{n+1} = 0, \quad (13)
$$

where $\Delta = \omega_0 - \omega$,

$$
\vartheta_1,2(h) = \xi_{1,2} [2 - a^2 (h \pm k/2)^2],
$$ (14)

System (12)–(13) has a nontrivial solution if its determinant is equal to zero, i.e. we obtain quadratic equation
which connects \(\nu\) and \(h\). Solving the equation with respect to \(\nu\), we determine the eigenfrequencies of system as

\[
\nu_{1,2}(n, h) = -\frac{1}{2} [\vartheta_1(h) + \vartheta_2(h) \mp \Omega(h)] .
\]  

(15)

Here

\[
\Omega(h) = \sqrt{\Delta_{	ext{eff}}^2 + 4g^2(n + 1)},
\]

(16)

\[
\Delta_{	ext{eff}}(h) = \Delta - \vartheta_1(h) + \vartheta_2(h).
\]

(17)

It follows from (15) that there are two eigenmodes for each value of photon number \(n\), namely

\[
|\Psi_{i,n}(t)\rangle = \sum_p (A_{i,n}(pa, t)|a_p, n\rangle + B_{i,n}(pa, t)|b_p, n + 1\rangle ,
\]

(18)

where \(i = 1, 2\) is the number of the eigenmode, and coefficients \(A_{i,n}(x, t)\) and \(B_{i,n}(x, t)\) are defined by the expressions

\[
A_{1,n}(x, t) = \frac{C_1g\sqrt{n + 1}e^{i(h+k/2)x}e^{-i(\nu_1 + \omega/2 - i\lambda/2)t}}{\nu_1 - \Delta/2 + \vartheta_1} ,
\]

(19)

\[
B_{1,n+1}(x, t) = C_1e^{i(h-k/2)x}e^{-i(\nu_1 - \omega/2 - i\lambda/2)t} ,
\]

\[
A_{2,n}(x, t) = C_2e^{i(h+k/2)x}e^{-i(\nu_2 + \omega/2 - i\lambda/2)t} ,
\]

(20)

\[
B_{2,n+1}(x, t) = C_2g\sqrt{n + 1}e^{i(h-k/2)x}e^{-i(\nu_2 - \omega/2 - i\lambda/2)t}}{\nu_2 + \Delta/2 + \vartheta_2} ,
\]

where \(C_{1,2} \equiv C_{1,2}(n, h)\) are normalizing constants, \(\vartheta_{1,2} \equiv \vartheta_{1,2}(h)\), the values \(\nu_{1,2} \equiv \nu_{1,2}(n, h)\) are expressed by the Eqs. (13) and (15), respectively. Each mode corresponds to the electron-photon entangled state, which partial amplitudes oscillate both in time and space. The expression (15) with (13) or (14) describes the state of radiation-dressed QD-chain since the entangled state includes field states with different photon numbers \((n, n + 1)\). This states are the generalization of the single atom dressed states\(^2\) for the case of distributed system. The qualitative distinction of this case is the space-time modulation of dressing parameter: it propagates along the QD-chain according to the traveling wave law \(\exp[i(kx - \omega t)]\). Entanglement and dressing are caused by the interaction of light with QD chain and vanish in the limit of \(g \to 0\). In this case, (11) describes QD-chain electron-hole pairs in equilibrium and inverse states, respectively.

Spatial oscillations of the partial amplitudes are due to QD-coupling. They vanish in the limit of \(\xi_{1,2} \to 0\). Generally, both the modes are excited simultaneously. However, any of them can be excited separately by a proper choice of initial conditions. Similarly to the other coherent excitations in condensed matter, Rabi waves\(^{14}\) and\(^{20}\) introduce a new family of quasi-particles (we name them rabitions). One can apply to them the standard secondary quantization technique. Similarly to the effect of the self-induced transparency, the Rabi wave propagation can be interpreted as the motion of a precessing pseudo dipole\(^{21}\). However, the coherence mechanisms in these two cases are principally different: in the Rabi wave the coherence is settled by the dispersion relation\(^{15}\) while in the case of the self-induced transparency it has solitonic character.

Note that the eigenmodes (19) and (20) each comprise traveling waves with different wave numbers \(h \pm k/2\). Physically, it means that the Rabi wave propagates in an effective periodically inhomogeneous medium formed by spatially oscillating (with period \(2\pi/k\)) electric field. Therefore, the diffraction appears in the system. In the limit \(k \to 0\), the medium turns homogeneous and the diffraction effect vanishes. Reflections of Rabi waves and their mutual transformations at the field inhomogeneities become possible. Thus one obtain a unique possibility to control the processes of the reflection and transmission of Rabi waves by varying the spatial structure of the light.

### B. Dispersion characteristics

Note, that for the waves under consideration the reciprocity conditions break down: \(\nu_{1,2}(n, h) \neq \nu_{1,2}(n, -h)\). This is due to the presence of preferential direction, which is determined by the direction of photon mode propagation along the QD-chain (sign of the value \(k\)). The form of the dispersion characteristics for typical values of parameters is presented on Fig(2). To interpret them one should note that in the limit \(g \to 0\) both of branches have cross points, two in general case and one at \(\xi_1 = \xi_2\). Modes under examination are characterized by continuous spectrum (the value \(h\) varies continuously).

The values \(\nu_{1,2}\) are real for all real \(h\). Therefore, the system being investigated is stable\(^{11}\). Dispersion equation for (12)-(14) can be solved with respect to wave number \(h(\nu)\) for given real-valued frequency \(\nu\). In this case for every mode there are such critical values \(\nu^{cr}_1\), that for \(\nu < \nu^{cr}_1\) the values \(h_{1,2}(\nu)\) become complex. Physically, it means opacity of the QD-chain for Rabi waves with such frequencies\(^{22}\). For \(\nu^{cr}_2 < \nu < \nu^{cr}_1\) the QD-chain is opaque only for one of the modes and for \(\nu < \nu^{cr}_2\) it is non-transparent for both of them. One can choose the system parameters in such a way that in some range of \(\nu\) the all real solutions \(h(\nu)\) are either negative or positive (as an example see Fig(3)). This implies one-way opacity of the system (Rabi waves could propagate in one direction only), which is caused by nonreciprocity of the QD-chain. The critical frequencies are determined from the condition \(\nu^{cr}_{1,2}(n) = \nu_{1,2}(n, h^{(0)})\), where

\[
\frac{\partial \nu_{1,2}(n, h)}{\partial h} \bigg|_{h = h^{(0)}_{1,2}} = 0 .
\]

(21)

It is significant, that critical values \(\nu^{cr}_{1,2}\) depend on photon number \(n\) (see Fig(3)). This gives an important result, namely: the conditions of transparency of the QD-chain
are different for Rabi waves with different photon numbers. Note, that $\nu_{1\sigma}^{\nu_1}$ increases and $\nu_{2\sigma}^{\nu_2}$ decreases with increasing of $n$.

Usually both Rabi waves demonstrate the normal dispersion and propagate in the same direction. But for a special choice of the parameters the abnormal dispersion region for one of the modes appears. For this mode phase and group velocities are oppositely directed (see Fig. 2a). Thus, in current case for a wide range of $h$ one of the modes practically does not transfer the energy along the QD-chain.

\section*{IV. RABI WAVEPACKETS}

\subsection*{A. Electron-photon dynamics}

To find the general solution of the system (11) let us introduce the new variable $\tilde{\Phi}_n(x,t) = e^{i(\omega_0 t - kx)} e^{\lambda t/2} \Phi_n(x,t)$. It leads to the system of partial differential equations with coefficients independent of QD-position $x$:

\begin{equation}
\partial_t \tilde{\Phi}_n - i \left( 2 - a^2 k^2 / 4 \right) \chi(t) \tilde{\Phi}_n + a^2 k \tilde{\sigma}_z \tilde{\xi} \partial_x \tilde{\Phi}_n - a^2 \tilde{z} \partial_x^2 \tilde{\Phi}_n = 0,
\end{equation}

where $\chi(t) = \tilde{\sigma}_z \exp(-i \tilde{\sigma}_z \Delta t)$. This system can be solved by using the Fourier transform with respect to $x$:

\begin{equation}
\tilde{\Phi}_n(x,t) = \int_{-\infty}^{\infty} \tilde{\Phi}_n(h,t) e^{ihx} dh.
\end{equation}

Substituting Eq. (23) into Eq. (22), we obtain the system of ordinary differential equations

\begin{equation}
\frac{d}{dt} \tilde{\Phi}_n = i \left( \hat{\partial} - g \sqrt{n + 1} \tilde{\chi}(t) \right) \tilde{\Phi}_n,
\end{equation}

where $\hat{\partial} = \hat{\partial}(h) = \left( [\hat{\partial}_1(h) + \hat{\partial}_2(h)] \hat{T} + (\hat{\partial}_1(h) - \hat{\partial}_2(h)) \hat{\sigma}_z \right) / 2$. The system (24) can be easily integrated, the solution is given by

\begin{equation}
\tilde{\Phi}_n(h,t) = \hat{\rho}_n(h,t) \tilde{\Phi}_n(h,0),
\end{equation}

where

\begin{equation}
\hat{\rho}_n(h,t) = e^{i[\hat{\partial}_1(h) + \hat{\partial}_2(h)] T} e^{i[k \hat{\sigma}_z \Delta t \hat{\Omega}_n(h)] T}.
\end{equation}
\[ \hat{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z), \quad \mathbf{m} \] is the unit vector of the form 
\[ (-2g \sqrt{n} + 1 / \Omega_n, 0, \Delta_{eff} / \Omega_n). \] The expression can be written in matrix form as follows:

\[ \hat{\rho}_n(h, t) = \begin{bmatrix} \varphi_n^-(h, t) e^{i \rho_n^+(h, t)} & \psi_n(h, t) e^{i \rho_n^+(h, t)} \\ \overline{\psi_n(h, t)} e^{-i \rho_n^+(h, t)} & \varphi_n^+(h, t) e^{-i \rho_n^+(h, t)} \end{bmatrix}, \tag{27} \]

\[ \varphi_n^\pm(h, t) = \cos \frac{\Omega_n(h) t}{2} \pm i \Delta_{eff}(h) \sin \frac{\Omega_n(h) t}{2}, \tag{28} \]

\[ \psi_n(h, t) = -i \frac{2g \sqrt{n} + 1}{\Omega_n(h)} \sin \frac{\Omega_n(h) t}{2}, \tag{29} \]

\[ \delta_n^\pm(h) = \frac{1}{2} [\partial_1(h) + \partial_2(h) \pm \Delta], \tag{30} \]

\[ \Omega_n(h) \text{ and } \Delta_{eff}(h) \text{ are determined by (119) and (177), correspondingly, and } \Phi(h, 0) \text{ can be obtained from initial conditions by the inverse Fourier transform:} \]

\[ \Phi(h, 0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i(h^2 + \delta_n^2 k^2 / 2)} \Psi_n(x, 0) dx. \tag{31} \]

Thus, for the vector of probability amplitudes \( \Psi_n(x, t) \) we have the following expression:

\[ \Psi_n(x, t) = e^{i(kx - \omega t - \delta_n^2 / 2)} e^{-\lambda t / 2} \mathbf{A}_{\rho_n} e^{i(h x + \partial_1(h) + \partial_2(h) t / 2)} dh. \tag{32} \]

Expression (32) is the central result of this section. It allows one to characterize such observables as inversion, as well as the exciton-exciton and exciton-photon correlators for different initial states of light and QDs.

**B. Dispersionless approximation**

In this section we will analyze the propagation of Rabi waves with spatially localized initial state. The QD-chain is supposed to be initially either in the ground state \( (A_n(x, 0) = 0 \text{ for all } n) \), or in the excited state \( (B_n(x, 0) = 0 \text{ for all } n) \), or in the mixed state (arbitrary superposition of ground and excited states).

We describe the spatial-temporal dynamics of Rabi oscillations by the spatial density of the inversion (the inversion per single QD)

\[ w(x, t) = \sum_n (|A_n(x, t)|^2 - |B_{n+1}(x, t)|^2). \tag{33} \]

Space-averaged temporal evolution of Rabi oscillations is characterized by the integral inversion

\[ \bar{w}(t) = \int_{-\infty}^{\infty} w(x, t) dx. \tag{34} \]

Let us approximate the initial spatial distributions by the Gaussian beams \( C_{A,B} \exp[-(x - d_{A,B})^2 / 2 \sigma_{A,B}^2] \), where \( C_{A,B} \) are normalization constants, \( d_{A,B}, \sigma_{A,B} \) are the positions of the beams and their widths, respectively (the indexes \( A,B \) refer to excited and ground state of the electron in the QD-chain, respectively).

The Rabi wavepacket evolution can be qualitatively illustrated with the help of the dispersionless approximation for Eqs. (32). According to (31) we obtain for \( \Phi_n(h, 0) \)

\[ a_n(h) \sim (n+1)e^{-(h+k/2)^2 \sigma_n^2/4}e^{-i \Phi_n(h, 0)}, \tag{35} \]

\[ b_{n+1}(h) \sim n+1e^{-(h-k/2)^2 \sigma_n^2/4}e^{-i \Phi_n(h, 0)}, \tag{36} \]

where \( c(n) \) is an arbitrary photonic distribution. One can see from (35–36) that the spatial spectrum of \( \Psi_n(x, 0) \) is given by two Gaussian peaks with peak widths \( 1/\sigma_{A,B} \) and positions of the peak centers \( h_1, h_2 \) are \( \pm k/2 \), respectively. If \( \sigma_{A,B} \) are sufficiently large, the largest contribution to the integration in (32) comes from the narrow vicinities of points \( h_1, h_2 \). Assuming \( \partial_1(h) \equiv \partial_1(h_1, 0) + (\partial_1 \partial_2 / \partial h)|_{h=h_1,2}(h-h_1,0) \) and \( \rho_n(h, t) \equiv \rho_n(h_1,0, t) \) (the dispersion effects neglecting) and doing standard calculations, we obtain from Eq. (32):

\[ A_n(x, t) = e^{(ikx-\omega t)/2}e^{-\lambda t} \left\{ A_n(x + v_1^0 t, 0) \zeta_2 e^{i \phi_2^0 t} + A_n(x + v_2^0 t, 0) \zeta_1 e^{i \phi_1^0 t} \right\} e^{-ikx/2} + \right\} \right\} e^{-ikx/2}, \tag{37} \]

\[ B_{n+1}(x, t) = e^{-(ikx-\omega t)/2}e^{-\lambda t} \left\{ A_n(x + v_2^0 t, 0) \zeta_2 e^{i \phi_2^0 t} + A_n(x + v_2^0 t, 0) \zeta_1 e^{i \phi_1^0 t} \right\} e^{-ikx/2} + \right\} \right\} e^{-ikx/2}, \tag{38} \]

where the velocities \( v_1^\pm, v_2^\pm \) are defined as

\[ v_1^\pm = v^\pm(h_1^0) = -\xi_1 a^2 k \frac{\Omega_n(k/2) \pm \Delta_{eff}(k/2)}{\Omega_n(k/2)}, \tag{39} \]
and $\theta_{1,2}^{\pm}$ and $\zeta_{1,2}^{\pm}$, $\eta_{1,2}^{\pm}$ are introduced to denote the frequency shifts and amplitude factors, respectively:

$$
\theta_{1,2}^{\pm} = \frac{1}{2} \{ \theta_{1}(h_{1,2}^{(0)}) + \nu_{1}(h_{1,2}^{(0)}) + \Omega_{n}(h_{1,2}^{(0)}) \} - v_{1,2}^{\pm}(h_{1,2}^{(0)}),
$$

$$
\zeta_{1,2}^{\pm} = \frac{\Omega_{n}(h_{1,2}^{(0)}) - \Delta_{\text{eff}}(h_{1,2}^{(0)})}{2\Omega_{n}(h_{1,2}^{(0)})},
$$

$$
\eta_{1,2}^{\pm} = \pm \frac{g\sqrt{n + 1}}{\Omega_{n}(h_{1,2}^{(0)})}.
$$

As is apparent from (37)–(38), generally, any probability amplitude in the Rabi-wave packet is the superposition of four subpackets. Two of them (first and second terms in expressions (37)–(38)) correspond to the excited initial state and two another (third and fourth terms in (37)–(38)) correspond to the ground initial state. The partial subpackets in Eqs. (37)–(38) are characterized by the different frequency shifts $\theta_{1,2}^{\pm}$ and different velocities of motion $v_{1,2}^{\pm}$, given by (29), (10). This velocities coincide with the group velocities of traveling Rabi waves (19)–(20) when $k = \pm k/2$: one can elementary verify, that $v_{1,2}^{\pm} = \text{d}n_{1,2}/\text{d}t|_{h = k/2} = v_{2}^{gr}(k/2)$,

$$
v_{1,2}^{\pm} = \text{d}n_{1,2}/\text{d}t|_{h = -k/2} = v_{2}^{gr}(-k/2).
$$

It is essential that velocities $v_{1,2}^{\pm}$ as well as the frequency shifts $\theta_{1,2}^{\pm}$ depend on photon number $n$. It means that the spatial propagation of wavepacket is accompanied by the change of quantum light statistics (for example, in initially coherent light incoherent component appears).

Under the some specific conditions the number of subpackets could decrease. Two mechanisms of such decrease are possible: the first one is the tending to zero the subpacket amplitude and the second one is the confluence of the subpackets velocities.

As is seen from (39)–(40), if

$$
\Delta_{\text{eff}}(h_{1,2}^{(0)}) = 0,
$$

for one pair of the subpackets the velocity synchronism condition $v_{1,2}^{\pm} = v_{1,2}^{\pm}$ is fulfilled. Notice that Eq. (11) is the analogue of the ordinary synchronism condition in single two-level system $\Delta = 0$. The velocity synchronism condition can be fulfilled by fitting the value of the detuning $\Delta$. If this is the case, there are three subpackets instead of four them due to the confluence mechanism acting. Note, that $\Delta_{\text{eff}}(k/2)$ and $\Delta_{\text{eff}}(-k/2)$ can not be equal to zero together at $k \neq 0$.

If the system is initially prepared in the stationary state, and the synchronism condition is fulfilled in the corresponding point (for example $B_{n+1}(x, 0) = 0$ for any $n$ and $\Delta_{\text{eff}}(-k/2) = 0$), only one subpacket is preserved. In this case the expression for the inversion density becomes rather simple:

$$
w(x, t) = a \sum_{n} A_{n}^{2}(x + \xi_{2}a^{2}kt, 0)[1 - 2\sin^{2}(g\sqrt{n + 1}t)].
$$

Expression (45) sums up contributions of different photonic states (terms with different $n$). Each contribution is the product of two multipliers describing different optical processes. The second multiplier describes Rabi oscillations with the frequencies $v_{n} = 2g\sqrt{n + 1}$, occurring in the QD-chain, while the first one shows that the region of Rabi oscillations moves therewith in space with velocity $v = \xi_{2}a^{2}k$. One should note, that in the case of the exact synchronism $v$ does not depend on $n$. That is why in this partial case the quantum statistics of light is not distorted with propagation of the wavepacket. In particular, the initial coherency of quantum light persists in time. It is immediately follows from Eq. (34) and normalization condition that the integral inversion $w(t)$ corresponding to the inversion density (45) oscillates in time similarly to the Rabi-oscillations in the single two-level system (see Eq.(6.2.21) in [1] at $\Delta = 0$).

Generally, each pair of the subpackets propagates due to the tunneling through its own potential barrier. The velocities of the subpackets motion are proportional to the barrier transparency ($v_{1,2}^{\pm} \sim \xi_{1,2}$). If one of the barriers becomes absolutely opaque, the corresponding pair of subpackets does not move: $v_{1,2}^{\pm} \rightarrow 0$ at $\xi_{1,2} \rightarrow 0$.

The light propagation along the QD-chain is another necessary condition of Rabi subpackets motion: $v_{1,2}^{\pm} \rightarrow 0$ at $k \neq 0$.

For more detail analysis of Rabi wavepackets dynamics (in particular for taking into account the diffraction spreading) the integrals (37)–(38) need to be calculated numerically. The results of calculations is analysed in the following section.

C. Coherent state: collapses and revivals picture

Consider the case of an excited initial state with space distribution in form of single Gaussian beam: $A_{n}(x, 0) = c(n) \exp(-x^{2}/2\sigma^{2})/\sqrt{\pi\sigma^{2}}, B_{n+1}(x, 0) = 0$. Assume that initially light is prepared in coherent state, so photon distribution is given by the Poisson law: $c(n) = (n)! e^{-(n)/2}/\sqrt{n!}$, where $(n)$ defines the average photon number. The spatial-temporal dynamics of inversion density for this case is depicted on Fig 4. As is seen from it, original Gaussian packet oscillates in time and moves along the chain. Oscillations collapse to zero quickly, but after a while they revive in another area of space. The phenomenon of Rabi oscillations collapses and revivals, caused by discreteness of photon distribution, is well studied, but the spacing the collapse and subsequent revival is qualitatively new effect.

Although a variation of the inversion density, depicted in Fig 4, in arbitrary point of the space occupied by the Rabi wavepacket is not too large, the integral inversion, presented in Fig 4, oscillates between -1 and 1, thus indicating presence of strong light-QD coupling.

Let us consider now the case of a mixed initial state (both $A_{n}(x, 0)$ and $B_{n+1}(x, 0)$ are nonzero). As it has been noted above, the synchronism condition can not
be fulfilled for both pair of subpackets (except the case when one of the pairs is motionless, see below). As a consequence, we have four subpackets (or three ones if $\Delta_{eff} = 0$ for one of the pairs). If $\xi_1 = \xi_2 = \xi$ (potential barriers with equal transparency) and $\Delta = 0$, then both packet pairs move with pairwise equal velocities in opposite directions ($v_1^+ = -v_2^-$). If the initial spatial distributions of probability amplitudes for ground and excited states are spaced, the subpackets will collide (see Fig. 5). Initial Gaussian profiles of packets deform with time due to the difference in subpacket velocities.

One should note that for the case of quantum external field in coherent state QD-chain cannot be saturated, i.e. the integral inversion could not be a constant (see Fig. 4). This is due to the fact that $A_n(x,0) = c(n) \exp(-x^2/2\sigma^2)/\sqrt{\pi\sigma^2}$, $B_{n+1}(x,0) = 0$, $\xi_1 = 10g$, $\xi_2 = 7g$, $\Delta = 2(\xi_1 - \xi_2) + \xi_2 a^2 k^2$, $ka = 0.5$, $\sigma = 20a$, $\lambda = 0.05g$.

D. Fock qubit state and vacuum Rabi waves

Let us now consider the interaction of the QD-chain with another important state of initial electromagnetic field, namely, the Fock qubit state, i.e. a superposition of two Fock states: $|\psi_f(0)\rangle = C_1 |N\rangle + C_2 |N + 1\rangle$, where $N$ is an arbitrary fixed number, $C_{1,2}$ satisfy the normal-

---

**FIG. 4:** Space-time distribution of the inversion density (a) and temporal dependence of the integral inversion (b) in the QD chain for a coherent initial state of light ($\langle n \rangle = 4$). The initial state of QD-chain is a single Gaussian wavepacket. $A_n(x,0) = c(n) \exp(-x^2/2\sigma^2)/\sqrt{\pi\sigma^2}$, $B_{n+1}(x,0) = 0$, $\xi_1 = 10g$, $\xi_2 = 7g$, $\Delta = 2(\xi_1 - \xi_2) + \xi_2 a^2 k^2$, $ka = 0.5$, $\sigma = 20a$, $\lambda = 0.05g$.

**FIG. 5:** Space-time distribution of the inversion in the QD chain for the case of two counterpropagating wavepackets. $A_n(x,0) = c(n) \exp[-(x - 3\sigma)^2/2\sigma^2]/\sqrt{\pi\sigma^2}$, $B_{n+1}(x,0) = c(n + 1) \exp[-(x + 3\sigma)^2/2\sigma^2]/\sqrt{\pi\sigma^2}$. The light is initially in coherent state $\langle n \rangle = 5$, $\Delta = 0$, $\xi_1 = \xi_2 = 10g$, $ka = 0.33$. a) $\sigma = 10a$, $\lambda = 0$; b) $\sigma = 20a$, $\lambda = 0.05g$.

**FIG. 6:** Temporal dependence of the integral inversion for a coherent initial state of light at the input parameters as follows: $\langle n \rangle = 5$, $\sigma = 20a$, $\lambda = 0.05g$ (solid line); $\langle n \rangle = 5$, $\sigma = 10a$, $\lambda = 0$ (dotted line) $\langle n \rangle = 0.5$, $\sigma = 20a$, $\lambda = 0$ (dashed line). In all cases, $\xi_1 = \xi_2 = 10g$, $\Delta = 0$, $ka = 0.33$. 

The spatial-temporal dynamics of the inversion density spatial distributions the Gaussian beams will be used. As in the previous case, for initial excitation conditions. In doing this we neglect any changes of the field state. Integrating for us. In this case one can neglect the quantum nature of the light and consider electromagnetic field as a classical one replacing the electric field operator with its expectation value $E(x,t) = Re\{\mathcal{E} \exp[i(kx - \omega t)]\}$. In doing this we neglect any changes of the field state. Omitting the local-field effects as before, we can write the Hamiltonian of the system $\hat{\mathcal{H}}_{\text{QD-chain}}$ - electromagnetic

In the end of this section we consider the vacuum initial state of external electromagnetic field. It is well known that for this case the vacuum harmonic Rabi oscillations occurring due to spontaneous emission can take place. The corresponding Rabi-wave packet in the initially excited QD-chain is illustrated at Fig. 9. This excitation couples single mode electromagnetic vacuum state with one-photon state only and can be imagined as a wave beam which is described by the monochromatic Rabi-frequency spectrum and continuous spatial spectrum at the same time. However, it should be noted that one need to consider an interaction of the QD-chain with a multi-mode field for full treatment of such effects. This problem is a subject for future considerations.

V. CLASSICAL LIGHT LIMIT

The case of a coherent state of electromagnetic field with the large photonic number $n$ is especially interesting for us. In this case one can neglect the quantum nature of the light and consider electromagnetic field as a classical one replacing the electric field operator with its expectation value $E(x,t) = Re\{\mathcal{E} \exp[i(kx - \omega t)]\}$. Omitting the local-field effects as before, we can write the Hamiltonian of the system $\hat{\mathcal{H}}$ QD-chain - electromagnetic

In the end of this section we consider the vacuum initial state of external electromagnetic field. It is well known that for this case the vacuum harmonic Rabi oscillations occurring due to spontaneous emission can take place. The corresponding Rabi-wave packet in the initially excited quantum state $\hat{\psi}(0)$ of the QD-chain is illustrated at Fig. 9. This excitation couples single mode electromagnetic vacuum state with one-photon state only and can be imagined as a wave beam which is described by the monochromatic Rabi-frequency spectrum and continuous spatial spectrum at the same time. However, it should be noted that one need to consider an interaction of the QD-chain with a multi-mode field for full treatment of such effects. This problem is a subject for future considerations.

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The solution reads 

\[ \Psi \text{ where } \] 

and can be solved in the same way. 

Eq. (47) will be the sum of two partial constants. The field in the form \( \hat{\Phi} \) in the form \( \hat{\Phi} = \hat{\Phi}_0 + \hat{\Phi}_T \), where the term 

\[ \hat{H}_0 = \frac{\hbar \omega_0}{2} \sum_n \hat{\sigma}_n - \frac{\hbar \Omega_R}{2} \sum_n \left[ \hat{\sigma}_n^+ e^{i(nk\alpha - \omega t)} + \text{H.c.} \right] \] 

 describes Rabi oscillations in non-interacting QDs and \( \Omega_R = \mu \mathcal{E}/\hbar \) is the Rabi frequency. The interdot interaction mechanism is independent on light properties, so the term \( \hat{H}_T \) is defined as before by equation (3). 

The state vector of the system has the form: 

\[ |\Psi(t)\rangle = \sum_p \left( A_p(t)|a_p\rangle + B_p(t)|b_p\rangle \right), \] 

where \( A_p(t), B_p(t) \) are unknown functions. Now the equations of motion for them can now be obtained using approach of Sec. II B Introducing the decay factor \( \lambda \) into the nonstationary Schrödinger equation and making continuous limit transition in the QD-chain similar to the Sec. II C we obtain the system of equations as follows: 

\[ \dot{\Psi} = \left[ 2i\hat{\xi} - \frac{i\omega_0}{2} \hat{\sigma}_z - \frac{\lambda}{2} f + \frac{i\Omega_R}{2} \hat{\kappa}(kx - \omega t) \right] \Psi + ia^2\hat{\xi}\partial_x^2 \Psi, \] 

where \( \Psi(x, t) = \begin{bmatrix} A(x, t) \\ B(x, t) \end{bmatrix} \). Note, that spontaneous emission can be taken into account in the same way as in II B i.e. by describing it as interaction of QDs with photon bath. In this case the damping constant \( \lambda \) in Eq. (47) will be the sum of two partial constants. The system (47) can be considered as a particular case of the general system (11) and can be solved in the same way. The solution reads 

\[ \Psi(x, t) = e^{i(kx - \omega t)\hat{\sigma}_z/2} e^{-\lambda t/2} \] 

\[ \times \int_{-\infty}^{\infty} \hat{\rho}(h, t)\hat{\Phi}(h, 0) e^{i(hx + |\varphi_1(h) + \varphi_2(h) + t/2|)} dh. \] 

where the values \( \hat{\rho}(h, t), \varphi^\pm(h, t), \psi_h(t), \delta_\pm(h), \Omega(h), \Delta_{e,f}(h) \) are determined, respectively, by equalities (26), (28), (29), (30), (16), (17) if one makes the substitution \( 2g\sqrt{n + 1} \rightarrow \Omega_R \) (not doing that the photonic number dependence in \( \hat{\rho}(h, t), \varphi^\pm, \psi_h, \Omega \) disappears). The vector function \( \hat{\Phi}(h, 0) \) is determined by the initial conditions similar to Eq. (41). 

A typical space-time distribution of the inversion density \( w(x, t) = a||A(x, t)||^2 - ||B(x, t)||^2 \) is shown in fig. 11. As is seen from the figure, the space-time dynamics of Rabi wavepacket is similar to the one for the case of vacuum oscillations (but it is of substantially different physical nature: electromagnetic field is in coherent state with photon number \( n \rightarrow \infty \) and this state does not change when the system oscillates between ground and excited states in contradiction to the case of vacuum Rabi oscilla-
tion). Since the photon distribution does not change, collapses and revivals are absent. However there is another phenomenon of similar nature. One can see in fig. 12 where the plots of the integral inversion are presented, that the oscillations of this quantity at $k \neq 0$, $\xi_1 \neq \xi_2$ decrease with time even at $\lambda = 0$, whereas such a damping is absent at $k = 0$, $\xi_1 = \xi_2$ and integral inversion oscillates harmonically in the range from -1 to 1 (dotted curve in fig. 12). Such a behaviour is the consequence of the classical light for the input parameters as follows: $\Delta = 0$, $\xi_1 = \xi_2 = 3\Omega R$ (dotted line); $\Delta = \xi_2 a^2 k^2$, $ka = 0.33$, $\xi_1 = \xi_2 = 3\Omega R$ (solid line); $\Delta = 2(\xi_1 - \xi_2) + \xi_2 a^2 k^2$, $ka = 0.33$, $\xi_1 = 3\Omega R$, $\xi_2 = 1.5\Omega R$ (dashed line). In all cases $\sigma = 5a$, $\lambda = 0$.

The dependence of the effective detuning $\Delta_{eff}(h)$ (and therefore the Rabi frequency $\Omega(h)$) on $h$. But if $k = 0$ and $\xi_1 = \xi_2$, then $\Delta_{eff} = \Delta$ and $\Omega$ does not depend on $h$. The damping rate is controlled by values of wavenumber $k$ and coupling constants $\xi_{1,2}$. If the initial state of the system is the excited state ($B(x,0) = 0$), the damping rate is predominantly determined by product $\xi_2 k$ (and damping is practically absent if $\xi_2 k = 0$), for the ground initial state ($A(x,0) = 0$) it is defined by $\xi_1 k$. In the case of a mixed initial state all three quantity are important.

Note, that more narrow wavepacket spreads faster, than the wider one (compare fig. 11 and fig. 11).

In the weak coupling limit the indicated dephasing mechanism is analogous to the Landau damping in plasma.

Interaction of two counterpropagating identical Gaussian Rabi wavepackets colliding at $x = 0$ is shown in fig. 13. Unlike the case of quantum field, the space-time distribution of the inversion density is fully symmetrical with respect to the line $x = 0$ and integral inversion does not oscillate: this quantity equals zero for all $t \geq 0$ and arbitrary values of $\Omega_R$.

VI. INFLUENCE OF THE LOCAL-FIELD EFFECTS

Local fields are depolarization fields arising inside QDs due to the dipole-dipole electron-hole interaction. As a result, the field acting on the e-h pair differs from mean field. The local-field action is described by Eq. (4) and leads to nonlinear terms in the equations of motion (last terms in Eqs. (6)–(7)). The presence of nonlinearity results in the qualitatively new features of Rabi oscillations in single QDs. The influence of the local-fields on Rabi waves is the subject for future investigations. In this section we consider some aspects of this influence only and discuss some sufficient conditions of the local-field negligibility.

Let us consider finite (of the length $L$) QD-chain interacting with classical light. The relaxation processes are neglected (i.e. $\lambda = 0$). The equations of motion follow from Eqs. (6)–(7) in the same way as Eq. (11) reduce to Eq. (18) and have the form

$$\partial_t A = -\frac{i}{2}(\omega_0 - 4\xi)A + \frac{i\Omega R}{2}Be^{-i(kx-\omega t)} + i\xi a^2 \partial_x^2 A - i\Delta \omega |B|^2 A, \quad (50)$$

$$\partial_t B = \frac{i}{2}(\omega_0 + 4\xi)B + \frac{i\Omega R}{2}Ae^{-i(kx-\omega t)} + i\xi a^2 \partial_x^2 B - i\Delta \omega |A|^2 B. \quad (51)$$

We use the periodic boundary conditions (Born-von Karman conditions):

$$A(L/2,t) = A(-L/2,t), \quad B(L/2,t) = B(-L/2,t). \quad (52)$$

Suppose that electromagnetic field are also satisfy periodic conditions, i.e. $kL = 2\pi m$, where $m$ is integer number. Then the problem under consideration has a clear physical interpretation: we are dealing with ringed QD-chain of the radius $R = L/2\pi$. QD-chain interacts with electromagnetic mode of whispering gallery type, which is excited in microcavity with rotation axis ($e^{ikx} \rightarrow e^{im\varphi}$, $x \rightarrow R\varphi$, $\varphi$ is the azimuthal coordinate).

Let us seek partial solution of the system (50)–(51) in the form of traveling wave:
where \( \phi_{1,2}(h) = 2\xi_{1,2} + \Delta/2 - \xi_{1,2}a^2(h \pm k/2)^2 \). From boundary conditions one can obtain the quantization condition for \( h = \pi n/L, n \) is integer. Expressing \( v_0 \) from Eq. (54) and substituting into Eq. (53), we obtain

\[
\nu + \phi_1(h) - \frac{\Omega_R^2(\nu + \phi_2(h))}{4(\nu + \phi_2(h) - \Delta\omega|u_0|^2)^2} = 0.
\]

The wavefunction normalization condition results in

\[
\left(1 + \frac{\Omega_R^2}{4(\nu + \phi_2(h) - \Delta\omega|u_0|^2)^2}\right)|u_0|^2 = \frac{1}{L}.
\]

Eqs. (53)-(56) form the closed system of equations with respect to \( \nu \) and \( |u_0|^2 \), which defines the dispersion law of Rabi waves \( \nu = \nu(h) = \nu(n) \) with regard to the local-field effects. It should be noted that only solutions with real-valued \( \nu \) have physical meaning (otherwise the system contradicts to the initial equations). For \( L \to \infty, \Delta\omega \to 0 \) the system reduces to the dispersion equation for Rabi waves in the infinite QD-chain interacting with classical light.

The dispersion characteristics of Rabi waves for two essentially different values of \( L \) but identical \( \Delta\omega \) and other parameters are shown in Fig.14. The curves depicted in Fig.14a corresponds to the QD-chain of large radius. The nonlinear effects in this case are negligibly small. Dispersion curves do not feel nonlinear effects and are practically identical to obtained in the infinite QD-chain interacting with classical light.

The detailed structure of closed loop is shown in Fig.14. The curves depicted in Fig.14a corresponds to the QD-chain of large radius. The nonlinear effects in this case are negligibly small. Dispersion curves do not feel nonlinear effects and are practically identical to obtained in the infinite QD-chain interacting with classical light.

The detailed structure of closed loop is shown in Fig.14. When \( h \) changes adiabatically slow the motion along the loop takes place (it is shown by arrows in Fig.14). The trajectories of motion in different directions prove to be nonidentical. Thus, the loop has an hysteresis character. Interaction of the local-fields and Rabi waves leads to effect of bistability of quantum states in the QD-chain. The detailed analysis of different dispersion branches stability is the subject for future investigations.

Let us now discuss the conditions of the local field effects neglecting in equations (6)–(7). Note, that for \( \xi_1,2 \to 0 \) the system transforms to equations (24), where Rabi oscillations in single QD are described with account of local field effects. An analysis of equations (24) has shown that local field effects are negligibly small for the case of sufficiently large Rabi frequencies, namely, \( \Omega_{(n)} \gg \Delta\omega \), where \( \Omega_{(n)} = \sqrt{\Delta^2 + 4g^2\langle n \rangle} \), \( \langle n \rangle \) is average photon number. If this condition is fulfilled for coherent state, the conventional picture of Rabi oscillations (the alternation of collapses and revivals) takes place. In the opposite case qualitatively new regime of oscillations arises. This regime is due to the local field effects and is described in detail. In this work we are interested only in first regime. In our case the role of \( \Omega_{(n)} \) is played by frequencies \( \nu_1,2(n,h) \), which are determined by Eq. (10). Therefore it is reasonable to suggest that for traveling Rabi waves the local fields effects are negligible under

FIG. 14: Dispersion curves for Rabi waves subject to local-field effects. a) \( L = 200a \), b) \( L = 20a \). In all cases \( \xi_1 = \xi_2 = 3\Omega_R, ka = 0.25\pi, \Delta = 0, \Delta\omega = 100\Omega_R \).
\[ \nu_{1,2}(n, h) \gtrsim \Delta \omega, \]  
(57)

where \( \Delta \omega \) is determined by Eq. (57).

In the case of Rabi-wavepackets general line of reasoning is the same as for Eq. (57) and leads to the following condition of the local-field negligibility: \( \nu_{1,2}(n, h^{(0)}) \gtrsim \Delta \omega. \)

VII. CORRELATION FUNCTIONS

![FIG. 15: The space-time dependence of normalized correlation function \( g^{(1)}(x, 0, t, 0) \) for light in Fock state with \( n = 5 \) and the input parameters as follows: \( \xi = 10 g, \Delta = \xi_2 k^2 a^2, \sigma = 5 a, A_n(x, 0) = \exp(-x^2/2\sigma^2)/\sqrt{\pi\sigma^2}, B_{n+1}(x, 0) = 0. \)]

\[ G_{p,q}^{(1)}(t, t') = \frac{\alpha^4}{16\pi^4} \sum_{l,r} \int_{-\pi/\alpha}^{\pi/\alpha} \int_{-\pi/\alpha}^{\pi/\alpha} e^{ih/(p-q)a} e^{iq/(q-p)h} e^{i(h-r)g} e^{-i\Delta(t-t')} \left[ \varphi^+_{n+1}(h', t') \varphi^-_{n+1}(h', t') - \psi_{n+1}(h', t') \psi_n(h', t') \right] \]

\[ + \varphi_{n+1}^+(h, t) \psi_{n+1}^- (g, t') u_{l,n+1}^+(0) v_{r,n+2}(0) - \psi_{n+1}(h, t) \varphi_{n+1}^-(g, t') u_{l,n+1}^+(0) v_{r,n+2}(0) \]

where \( u_{p,n}(t) = A_{p,n}(t) e^{-i(k_p a - \omega t)/2}, v_{p,n+1}(t) = B_{p,n+1}(t) e^{i(k_p a - \omega t)/2}. \)

The continuous limit transition can be made by means of the following substitutions:

\[ pa \rightarrow x, qa \rightarrow x', \sum \rightarrow \frac{1}{2} \int \... dz, \sum \rightarrow \frac{1}{2} \int \... dz'. \]

One can see, that in general case \( G_{p,q}^{(1)}(t, t') \) can not be represented as \( G_{p-q}^{(1)}(t, t') \) or \( G_{p-q}^{(1)}(t - t'). \) It indicates the absence both spatial and time homogeneity.

Let us consider case of \( B_{n+1}(0) = 0 \) and \( t' = 0. \) Then \( \psi_n(h, t') = 0, \varphi^\pm_{n+1}(h, t') = 1 \) and taking into account that \( \int_{-\pi/\alpha}^{\pi/\alpha} e^{i(q-r)g} dg = 2\pi \delta_{qr}/\alpha, \) we obtain rather simple expression for correlation function:

\[ G_{p,q}^{(1)}(t, 0) = \frac{\alpha^4 e^{-i\Delta t}}{8\pi^4} \sum_{l} \int_{-\pi/\alpha}^{\pi/\alpha} e^{ih/(p-q)a} e^{i(h-l-p)a} \]

\[ \varphi^+_{n+1}(h, t') \varphi^-_{n+1}(h, t) u_{l,n+1}^+(0) v_{q,n+1}(0) dh' dh, \]

(60)

In this section we will calculate exciton-exciton and exciton-photon multi-time correlation functions. For this purpose it is convenient to pass to the Heisenberg picture and to use the discrete model of QD-chain (transition to the continuous limit will be made on the final stage). Thus, we should look for a time dependence of operators \( \hat{\sigma}_p^\pm, \) i.e. we shall seen to find the evolution operator. It can be obtained from [19] (for details see Appendix A) and has the form:

\[ \hat{U}(t, 0) = \frac{a}{2\pi} \sum_{p} \sum_{l} \int_{-\pi/\alpha}^{\pi/\alpha} \left\{ \varphi^+_{n+1}(h, t) e^{i\Delta(t-t')} |a_p, n\rangle \langle a_q, n| + \psi_n(h, t) \left[ e^{i\Delta(t-t')} |a_p, n\rangle \langle b_q, n + 1| + e^{i\Delta(t-t')} |b_p, n + 1\rangle \langle a_q, n| \right] \right. \]

\[ + \left. \varphi^+_{n}(h, t) e^{i\Delta(t-t')} |b_p, n + 1\rangle \langle b_q, n + 1| \right\} e^{ih(p-q)a} dh. \]

(58)

Here \( \varphi^\pm_{n}(h, t), \psi_n(h, t), \Omega_n(h) \) are given by formulas [23], [29], [10]. Quantities \( \delta_{n+1}^\pm, \Delta_{eфф}(h) \) are defined by expressions [30], [17] if one substitutes \( \theta_{1,2} \rightarrow 2\xi_{1,2}\cos[(h \pm k/2)a]. \) Note, that if one makes approximation \( \cos[(h \pm k/2)a] = 1 - (h \pm k/2)^2/2 \) (this is the usability condition for continuous limit), the quantities \( \delta_{n+1}^\pm, \Delta_{eфф}(h) \) coincide with expressions [30], [17].

From expression (58) we obtain by using the formula \( \hat{\sigma}_p^\pm(t) = \hat{U}^+(t, 0) \hat{\sigma}_p^\pm(t) \hat{U}(t, 0) \) the exciton-exciton correlation functions \( G_{p,q}^{(1)}(t, t') \equiv \langle \hat{\sigma}_p^+(t) \hat{\sigma}_q^-(t') \rangle \) as follows:
or, in continuous limit,
\[
G^{(1)}(x, x', t, 0) = \frac{a^2 e^{-i\Delta t}}{8\pi^3} \sum_n \int_{-\infty}^{\infty} \int_{-\pi/a}^{\pi/a} e^{ih'(x-x')} e^{ih(z-x)}
\]
\[
\phi_n^+(h', t) \phi_{n+1}^+(h, t) u_{n+1}^*(z, 0) u_{n+1}(x', 0) dh'dhz. \quad (61)
\]

Plot of normalized correlation function
\[
g^{(1)}(x, x', t, t') = \frac{G^{(1)}(x, x', t, t')}{\sqrt{G^{(1)}(x, x, 0)G^{(1)}(x', x', 0)}} \quad (62)
\]

\[
\langle \hat{E}_q^{-}(t') \hat{a}_p^+(t) \rangle = \frac{a^2 \epsilon}{4\pi^2} \sum_n \sum_{l,r} \int_{-\pi/a}^{\pi/a} dh dh' \left[ \phi_{n+1}^+(h', t) \sqrt{n+1} \left[ \psi_n(h, t) u_{n,l,n}(0) v_{r,n,0}(0) + \phi_n^+(h, t) u_{n,l,n}(0) v_{r,n,l}(0) \right] \right] (63)
\]
\[- \psi_{n+1}(h', t) \sqrt{n+2} \left[ \psi_n(h, t) v_{r,n+1}(0) u_{l,n}(0) + \phi_n^+(h, t) v_{r,n+1}(0) u_{l,n+1}(0) \right] \}
\]
\[
e^{-i\omega t (t-t')} e^{i(kpa-\omega t)} \epsilon e^{i(kpa-\omega t)} e^{i\omega (t-t')}.
\]

VIII. CONCLUSION

The main results, obtained in this work, can be summarized as follows:

i) In QD-chain with tunneling coupling in strong electron-photon coupling regime the space propagation of Rabi oscillations (Rabi waves) takes place. For propagation of Rabi waves the wave vector of the photon mode must have a nonzero component along the chain. Characteristics of the Rabi waves depend strongly on relations between parameter of electron-photon coupling, frequency deviation and transparency factors of tunneling barriers for both of levels.

ii) Traveling Rabi waves are quantum states of the QD-chain dressed by radiation. The qualitative distinction of this states from states of single dressed atom is the space-time modulation of dressing parameter by traveling wave law. Traveling Rabi waves can be interpreted as entangled states of e-h pair and photons. These states are characterized by the dependence of energy on quasi-momentum and can be treated as new type of quasiparticles (in this paper they are called rabitons).

iii) There are two traveling Rabi modes with different frequencies of Rabi oscillations at a given value of wavenumber. The range of Rabi oscillations frequencies is limited by the critical value, which is different for both modes. The QD-chain is opaque for Rabi modes with frequencies lesser than critical one. The critical frequencies as well as dispersion characteristics depend on number of photons in the mode.

iv) In general case the propagation of Rabi wavepackets occurs in the form of four partial packets, which are characterized by different amplitudes and velocities of motion. Rabi wavepackets transfer energy, inversion, quasi-momentum, electron-electron and electron-photon quantum correlations along the chain. Particularly, in

for \( x' = 0, \ t' = 0 \) is presented on Fig[15]. The external field is supposed to be in a single-photon initial state and the electron in QD-chain is in the excited initial state (\( B_{n+1}(x, 0) = 0 \)).

The correlation function of the polarization operator \( \hat{a}_p^+(t) \) with the operator of the external field \( \hat{E}_q^{-}(t) = \epsilon \hat{a} e^{i(kpa-\omega t)} \) (exciton-photon correlators) appears as

\[ \psi_n(h, t) u_{n,l,n}(0) v_{r,n,0}(0) + \phi_n^+(h, t) u_{n,l,n}(0) v_{r,n,l}(0) \]

\[ \psi_n(h, t) v_{r,n+1}(0) u_{l,n}(0) + \phi_n^+(h, t) v_{r,n+1}(0) u_{l,n+1}(0) \]

\[ e^{-i\omega t (t-t')} e^{i(kpa-\omega t)} e^{i\omega (t-t')} \]

the case of QD-chain interacting with quantum light in the state the known collapses-revivals picture is drastically modified due to propagation effect: collapses and revivals take place in different points of space.

v) One pair of Rabi wavepackets exists due to the ground state contribution in initial state of the system, another one appears due to the contribution of excited state (generally, initial state is arbitrary superposition of ground and excited states). Each pair of packets moves through its own potential barrier. If one of the barriers is completely opaque, then corresponding pair of wavepackets does not move and does not spread.

vi) Rabi wavepackets propagation along the QD-chain is followed by the transformation of quantum light statistics (for example, in initially coherent light incoherent component appears). If specific conditions are fulfilled then incoherent part can be decreased to zero.

vii) In particular cases a number of Rabi wavepackets can be diminished. There are two mechanisms of such diminution: tending to zero of partial wavepacket amplitude and merging of the packets due to their velocities equality.

Rabi waves can take place in a number of other distributed systems strongly coupled with electromagnetic field.

The example are superconducting circuits based on Josephson junctions, which are currently the most experimentally advanced solid-state qubits. It is evident, for example, that the chain of qubits placed inside a high-Q transmission-line resonator due to the qubit-qubit capacitance coupling will support the Rabi waves propagation similar to described in this article.

Rabi waves effect can be practically used in nanoelectronics, quantum computing, quantum informatics.
**APPENDIX A: EVOLUTION OPERATOR**

In this appendix, the evolution operator for a discrete QD-chain (Eq. (58)) is obtained from Eq. (9). Let us introduce new variables $u_{p,n} = A_{p,n} e^{-i(k_p a - \omega_0 t)/2}$, $v_{p,n+1} = B_{p,n+1} e^{i(k_p a - \omega_0 t)/2}$. It leads to the system

$$\partial_t u_{p,n} = i\xi_1 \left( u_{p-1,n} e^{-i\kappa a/2} + u_{p+1,n} e^{i\kappa a/2} \right) - i g\sqrt{n+1} v_{p,n+1} e^{i\Delta t}, \quad (A1)$$

$$\partial_t v_{p,n+1} = i\xi_2 \left( v_{p-1,n+1} e^{-i\kappa a/2} + v_{p+1,n+1} e^{i\kappa a/2} \right) - i g\sqrt{n+1} u_{p,n} e^{-i\Delta t}. \quad (A2)$$

Coefficients in this system do not depend from QD-number $p$. It allows us to seek the solution in the form $u_{p\pm 1,n} = u_{p,n} e^{\pm i\kappa a} v_{p+1,n+1} = v_{p,n+1} e^{\pm i\kappa a}, \; h\xi \in [-\pi, \pi]$. Then the equations for different QDs become independent and may be written as

$$\partial_t u_{p,n} = 2i\xi_1 \cos((h + k/2)a) u_{p,n} - ig\sqrt{n+1} v_{p,n+1} e^{i\Delta t}, \quad (A3)$$

$$\partial_t v_{p,n+1} = 2i\xi_2 \cos((h - k/2)a) v_{p,n+1} - ig\sqrt{n+1} u_{p,n} e^{-i\Delta t}. \quad (A4)$$

Expressing $v_{p,n+1}$ from (A3) and substituting into (A4), one obtains the set of independent second-order ordinary differential equations for the variables $u_{p,n}$

$$\frac{d^2 u_{p,n}}{dt^2} - i\delta_h \frac{du_{p,n}}{dt} + \frac{\Omega_n(h)^2 - \delta^2}{4} u_{p,n} = 0, \quad (A5)$$

where $\delta_h = \Delta + 2\xi_1 \cos((h + k/2)a) + 2\xi_2 \cos((h - k/2)a)$. The solution of (A5) reads

$$u_{p,n}(t) = C_1^p(0) e^{i[\delta_h + \Omega_n(h)]t/2} + C_2^p(0) e^{i[\delta_h - \Omega_n(h)]t/2}, \quad (A6)$$

with the constants $C_1^p(0)$ to be determined from initial conditions. Writing the latter in the form $u_{p,n}(0) = a_n e^{ihpa}$, $v_{p,n+1}(0) = b_{n+1} e^{ihpa}$, we rewrite (A6) as

$$u_{p,n}(t) = \left[ \alpha_n e^{i[\delta_h - \Omega_n(h)/2]t} + \beta_n e^{i[\delta_h + \Omega_n(h)/2]t} \right] e^{ihpa}, \quad (A7)$$

and

$$v_{p,n+1}(t) = \left[ \gamma_n e^{-i[\delta_n + \Omega_n(h)/2]t} + \delta_n e^{-i[\delta_n - \Omega_n(h)/2]t} \right] e^{ihpa}. \quad (A8)$$

where

$$\alpha_n = \frac{\Omega_n(h) + \Delta_h}{2\Omega_n(h)} a_n + \frac{g\sqrt{n+1}}{\Omega_n(h)} b_{n+1}, \quad (A9)$$

$$\beta_n = \frac{\Omega_n(h) - \Delta_h}{2\Omega_n(h)} a_n - \frac{g\sqrt{n+1}}{\Omega_n(h)} b_{n+1}, \quad (A10)$$

$$\gamma_n = \frac{g\sqrt{n+1}}{\Omega_n(h)} a_n + \frac{\Omega_n(h) - \Delta_h b_{n+1}}{2\Omega_n(h)}, \quad (A11)$$

$$\delta_n = -\frac{g\sqrt{n+1}}{\Omega_n(h)} a_n + \frac{\Omega_n(h) + \Delta_h b_{n+1}}{2\Omega_n(h)}. \quad (A12)$$

Basis wavefunction has the form

$$|\Psi_h(t)\rangle = \sum_{n,p} \left( u_{p,n}(t) |a_p, n\rangle + v_{p,n+1}(t) |b_p, n+1\rangle \right). \quad (A13)$$

We now can represent the required wavefunction as Fourier integral with basis functions $\Psi_h(t)$:

$$|\Psi(t)\rangle = \int_{-\pi/a}^{\pi/a} M(h) |\Psi_h(t)\rangle dh, \quad (A14)$$

where $M(h)$ is unknown weighting function. To determine it let us take into account, that

$$|\Psi(0)\rangle = \sum_{n,p} \int_{-\pi/a}^{\pi/a} M(h) |a_p, n\rangle + b_{n+1}|b_p, n+1\rangle |\Psi(0)\rangle e^{ihpa} dh, \quad (A15)$$

and the same for $|b_p, n+1\rangle |\Psi(0)\rangle$. Then employing inverse Fourier transform we have

$$a_n M(h) = \frac{1}{2\pi} \sum_{q=-\infty}^{\infty} \langle a_q, n |\Psi(0)\rangle e^{-iha} \quad (A16)$$

and analogously for $b_{n+1}$. Substituting (A17) into (A14), we obtain $|\Psi(t)\rangle = \hat{U}(t,0) |\Psi(0)\rangle$, with $\hat{U}$ given by the Eq. (A5). These calculations allow one to pass from Schrödinger picture to the Heisenberg one. They are used in the calculations of different types of correlators.
