QUARK MATTER DROPLET FORMATION IN NEUTRON STARS

H. HEISELBERG
NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

ABSTRACT

The formation rate of quark matter droplets in neutron stars is calculated from a combination of bubble formation rates in cold degenerate and high temperature matter. Nuclear matter calculations of the viscosity and thermal conductivity are applied. Results show that droplets form only in the core of neutron stars shortly after supernova collapse, where pressures and temperatures are high, and for sufficiently small interface tension between nuclear and quark matter. Coulomb energies hinder formation of large droplets whereas the presence of strange hadrons in nuclear matter increase the droplet formation rate.

1. Introduction

Quark matter in neutron star has recently been found to be able to coexist with nuclear matter and have a rich structure in this mixed phase. This is in contrast to standard models where neutron stars have a pure quark matter core with a mantle of nuclear matter outside. The quark and nuclear matter mixed phase appear at much lower densities. A lattice of quark matter droplets appear already around twice nuclear matter saturation densities which increases the probability for the existence of quark matter in neutron stars.

We shall here discuss the formation of quark matter droplets and calculate the rate by which the first quark matter droplets form at the high densities in centers of neutron stars. (If quark matter is the true ground state of hadronic matter in vacuum, such strangelets would probably reside inside the stars as relics from the early universe and would convert the whole star to a strange star.) We combine bubble formation rates of both cold degenerate and hot systems and apply transport coefficients of nuclear matter. To estimate the time scales for the droplet formation the pressure and temperatures are calculated from standard equation of states used in supernova core collapse for nuclear matter and a simple Bag model for quark matter.

2. Droplet Free Energy

In the core of a supernova collapse or the later neutron star the nuclear matter may be dense enough that it is energetically favorable to form a quark matter droplet. The work forming a droplet of radius $R$ is

$$ W = -\frac{4\pi}{3} R^3 \Delta P + 4\pi \sigma R^2 + N_q \Delta \mu + E_C, $$

(1)
where \( \Delta P = P_{QM} - P_{NM} \) is the pressure difference, \( \sigma \) the surface tension, \( \Delta \mu = \mu_{QM} - \mu_{NM} \) the difference in chemical potentials between nuclear and quark matter, \( N_q = \frac{4\pi}{3} R^3 n_{QM} \) the number of quarks in the droplet and \( E_C \) the Coulomb energy of the droplet.

It is usually assumed that the embedded droplet is in chemical equilibrium with its surroundings at nucleation, i.e.,

\[
\mu_n = \mu_u + 2\mu_d, \quad \mu_p = 2\mu_u + \mu_d.
\]

Olesen and Madsen consider the instantaneous collapse of the nuclear matter into a quark matter containing the original flavor composition. The resulting droplet will be out of chemical equilibrium and diffusion of various particles will take place until chemical equilibrium is attained. We shall take the standard approach that the quark matter is in chemical equilibrium with its surroundings since we expect the diffusion of nucleons in and out of the droplet to be at least as fast as the formation of a large droplet. Furthermore, we shall assume the presence of sufficient strangeness in nuclear matter from strange hadrons like \( \Sigma^- \), \( K^- \), \( \Lambda \) ... . With sufficient rapid diffusion of strangeness during droplet formation we can then also assume \( \mu_d = \mu_s \) inside the quark matter droplet. Otherwise, only \( u \), \( d \) quark matter droplets can form since strangeness production only occur slowly on weak decay time scales. Pure \( u,d \)-quark matter will have higher free energies and are thus harder to form.

Let us first discuss small droplets where Coulomb effects can be neglected. Since the maximal temperatures reached in supernovae are much less than the chemical potentials \( T \ll \bar{\mu} \) we can also ignore thermal energies in (1). The nucleation rate contains the usual Boltzmann factor in which the work to form a critical bubble enters. The critical bubble is determined by the maximum of Eq. (1) which gives a critical bubble radius,

\[
R_c = \frac{2\sigma}{\Delta P},
\]

at which the work is

\[
W_c = \frac{16\pi}{3} \frac{\sigma^3}{\Delta P^2}.
\]

The pressure difference across the boundary of the critical bubble is balanced by the surface tension, \( P_{QM} = P_{NM} + 2\sigma/R_c \). The temperatures and chemical potentials are the same in the two phases and therefore the critical droplet is in complete phase equilibrium with the nuclear matter when created. The surface tension is unfortunately a poorly determined parameter and we refer to Ref. 2 for a discussion. It is probably within the range \( \sigma \sim 10 - 200 \text{MeV/fm}^2 \).

The Coulomb energy, \((3/5)Z^2 e^2/R_c\), grows rapidly with size for a droplet with constant charge density. Screening or charge rearrangement reduces the Coulomb energy significantly but still the critical radius and work are found to increase due to Coulomb effects. A more detailed analysis shows that the formation of droplets more than a few \( fm \) in size is severely hindered.
3. Formation Rate

The droplet formation rate was calculated by Langer & Turski\(^7\)

\[ I = \frac{\kappa}{2\pi} \Omega_0 e^{-W_c/T}. \]  

(5)

The “statistical” prefactor

\[ \Omega_0 = \frac{2}{3\sqrt{3}} \left( \frac{\sigma}{T} \right)^{3/2} \left( \frac{R_c}{\xi_q} \right)^4, \]  

(6)

measures the phase-space volume of the saddle point around \( R_c \) that the droplet has to pass on its way to the lower energy state. Here \( \xi_q \) is the quark correlation length which also estimates the surface thickness. We shall assume that it is of order the QCD Debye screening length \( \xi_q \simeq 1/q_D \) where \( q_D^2 = (2/\pi) \alpha_s \sum_q \mu_q^2 = (2/\pi) \alpha_s N_q \mu^2 \).

Here, \( \alpha_s \) is the QCD fine-structure constant and \( N_q \) the number of quark flavors present. With \( \alpha_s \simeq 0.3, N_q = 3 \) and \( \mu \simeq 400 \text{ MeV} \) we have \( \xi_q = 1/q_D = 0.7 \text{ fm} \). Note that Eq. (6) is only valid when \( R_c \gtrsim \xi_q \) which is the case for reasonable estimates of \( \sigma \) and \( \Delta P \). The “dynamical” prefactor determines the droplet growth rate and is

\[ \kappa = \frac{2\sigma}{\Delta w^2 R_c^3} \left( \lambda T + 2\left( \frac{4}{3} \eta + \zeta \right) \right), \]  

(7)

where \( \Delta w = w_{QM} - w_{NM} \) is the enthalpy difference and \( \lambda, \eta \) and \( \zeta \) are the thermal conductivity, shear and bulk viscosities respectively. The dynamical prefactor \(^7\) incorporates both the low temperature result of Langer & Turski, where the viscous term is ignored, and the high temperature result of Csernai & Kapusta\(^8\), where the thermal conductivity is negligible. Since both calculations were performed in the hydrodynamic limit linearized in the transport coefficients, the two results simply add up\(^7\).

The shear and bulk viscosities and the thermal conductivity have been estimated by Danielewicz\(^10\) within the Boltzmann equation with an effective nucleon-nucleon differential scattering cross section. The contributions to the dynamical prefactor from the bulk viscosity and the thermal conductivity are negligible as compared to that from the viscosity which typically is \( \eta \sim 50 \text{ MeV/fm}^2 \). The result of Langer and Turski\(^7\) include only the thermal conduction term and is therefore not applicable to the case of degenerate quark matter droplet formation. For the formation of hadronic bubbles in quark matter one can use the perturbative QCD calculations of transport coefficients\(^11\). Here the viscous damping dominates as well and the shear viscosity is even larger than in nuclear matter.

Still neglecting Coulomb energies we obtain from Eqs. \((4,5,6)\)

\[ I = \frac{\sigma_0^{7/2} \mu_{100}^2 \eta_{50}}{\Delta P_{10} \Delta w_{10}^2 T_{10}^{3/2}} \exp \left[ 185 - 134 \frac{\sigma_{20}^3}{\Delta P_{10}^2 T_{10}} \right] \text{s}^{-1} \text{km}^{-3}, \]  

(8)
assuming $\alpha_s = 0.3$ and $N_q = 3$. Here we have written all the quantities in typical units: the enthalpy and pressure differences in units of 10 MeV/fm$^3$, the quark chemical potential in 400 MeV, the temperature in 10 MeV, the surface tension in units of 20 MeV/fm$^2$ and the viscosity in 50 MeV/fm$^2$. The resulting rate is very sensitive to $\sigma$, $\Delta P$ and $T$. It is rather insensitive to the prefactor and therefore also the viscosity. Using the prefactor $T^4$ in stead reduces the rate by approximately 11 orders of magnitude which does, however, not have much effect in (8). Here the huge exponent 185 arise mainly from conversion of hadronic scales (fm$^{-4}$) to neutron star dimensions and cooling times (km$^{-3}$s$^{-1}$).

The pressure difference depends on the equation of state. It increases with density and so droplet formation is most likely in the center of the neutron star. The maximum temperature achieved in supernova explosions is largest in the core with a value around 10 MeV (depending on the equation of state and details of the supernova collapse) and it cools down to a few MeV in 10-20 seconds. Consequently, the droplets predominantly form in the neutron star center within few seconds after the core collapse when temperatures are highest as already pointed out in (6, 14, 15) and only if the surface tension is sufficiently low.

The total number of droplets formed is the integrated rate over volume of the neutron star and time after the supernova core collapse

$$N = \int_0^{R_{NS}} 4\pi r^2 dr \int_{t_{SN}}^{\infty} dt I(\Delta P(r), T(t)).$$

(9)

The rate depends mainly on the spatial coordinate through the pressure difference and on time through the temperature. The pressure and temperature depends sensitively on the equation of state of nuclear and quark matter and on the details of the supernova collapse.

To convert the core of the neutron star into quark matter, whether it is in a mixed phase with nuclear matter or not, we must form at least one droplet, i.e. $N > 1$. According to (8) and (3) this requires

$$\sigma \lesssim 24 \text{ MeV} \cdot \text{fm}^{-2} \cdot \Delta P_{c,10}^{2/3} T_{c,10}^{1/3},$$

(10)

where $P_c$ and $T_c$ are now the core values. This condition is almost insensitive to the size and cooling time of the hot core.

4. Summary

A formation rate for quark matter droplets has been derived applying transport coefficients from nuclear matter calculations. The resulting rate is very sensitive to the surface tension at the nuclear/quark matter interface and to the pressure difference and temperature. For large droplets it is important to include Coulomb energies which reduce formation rate for droplets larger than a few $\sim fm$ drastically. The presence of strange hadrons and sufficiently fast strange quark diffusion allow the formation of strange quark matter droplets which requires less work than purely
$u,d$-quark droplets and thus improve the production rate. Quark matter droplets form only in the core of neutron stars, where the pressure difference is highest, and shortly after the core collapse in supernovae explosions, where temperatures are highest. Eq. (10) gives the necessary conditions for creating quark matter in neutron stars. The pressure difference depends strongly on the equation of state used for both nuclear and quark matter and with the present uncertainties in both and in the surface tension, the droplet formation rate cannot be reliably estimated. If neutron stars burn into quark stars\textsuperscript{11} it would happen right after the supernova explosion, and so this mechanism is unlikely to be the cause of gamma ray bursters.

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6. References

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