Passive vibration reduction of a squeeze film damper for a rotor system with fit looseness between outer ring and housing

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Abstract
Clearances between bearing outer ring and sleeve can generally be maintained to provide a margin for the thermal expansion of the bearings. However, temperature variation, improper assembly and long-term vibration can enlarge the clearances and accelerate mechanical wear, leading to what is known as the fit looseness fault. Therefore, it is important to study a fit looseness fault model and investigate how to control the vibration coming from the fit looseness fault. In this paper, a Jeffcott rotor system with three disks was modeled as a single unit. A fit looseness model was applied in the whole rotor model to study the contact problems and response characteristics using a numerical integration method. Then, a squeeze film damper model was applied to assess the vibration reduction effects on the whole rotor system with the fit looseness fault. By comparing the results of the fit looseness fault without squeeze film damper and with squeeze film damper, it is found that the squeeze film damper can reduce nonlinear vibration responses effectively generated by the fit looseness fault for the nonlinear contact. This research work contributes to understanding the mechanism of fit looseness fault and controlling strong nonlinear vibration responses due to the fit clearances.

Keywords
Squeeze film damper, dynamic model, fit looseness fault, vibration reduction, modal analysis

Introduction
Looseness fault is a typical fault in the rotating machinery including gas turbine, compressor and aero-engine. Mechanical looseness fault is a common fault due to the looseness clearances, which can be divided into two categories, which are the pedestal looseness and the fit looseness. Modeling on looseness fault is an effective way to study the mechanism of looseness fault.

At present, scholars have studied deeply on the bolt connection looseness fault or known as the pedestal looseness fault. Piecewise nonlinear stiffness models are usually applied to study the bolt stiffer and softer nonlinear properties. Muszynska and Goldman¹ studied the dynamic characteristics of a rotor with the unbalance and the pedestal looseness or rubbing fault from experiments and simulation aspects. Synchronous and sub-synchronous as well as chaotic phenomenon were exhibited in these system. Ehrich² simulated a model with a piecewise stiffness model to represent a rotor with the looseness fault. The results showed sub-harmonic, super-harmonic dynamic properties. Furthermore, Wang and Chen³ applied a piecewise model to represent the
looseness fault to study the nonlinear response characteristics, and they found that the looseness fault would lead to the longitudinal asymmetrical characteristics in time history.

These piecewise looseness models can only be used to study bolt looseness problems, but they cannot be used to investigate the fit looseness between the bearing ring and housing in the presence of temperature variation, improper assembly and long-term vibration. An illustrated fit looseness fault is shown in Figure 1, where fit looseness clearances are resulting from the shock vibration between the journal bearing and the housing. The difference between the pedestal looseness fault and the fit looseness fault is that a pedestal looseness model has its stiffness directivity, i.e., stiffness changing in horizontal or vertical directions, but a fit looseness model has the property of cyclic changing of stiffness, i.e., stiffness will change along the surface of the bearing housing one time per cycle. Recently, many researchers have applied different methods to study the fit looseness mechanism. Mechanical fit looseness fault comes from the larger clearances between the bearing inner surface and the outer surface of the shaft, or the clearances between the bearing outer surface and the bearing housing inner surface, as shown in Figure 1. As the fit clearances are not easy to control properly, the effects of impact and friction of rotor components on the whole system can be serious, as the temperature between two fit components might increase when a rotor system is running at higher speeds. Severe nonlinear vibration can occur due to the periodic impulse interaction between two components. If these shock vibration are sustained for long durations, this inner surface of components will be rubbed intensively, resulting to the fit looseness clearances being enlarged. Therefore, it is necessary to detect, and diagnosis the looseness fault, as well as to control the looseness clearances.

Researchers are trying different methods to detect and establish the physical models for the fit looseness fault. Electrical and vibration monitoring methods are applied to diagnosis the fit looseness fault. Jung and Lee used an electrical monitoring to detect the mechanical looseness. Inspection of a split sleeve bearing is shown in Figure 2. Abrasion mark was seen on the surfaces of bearing and housing (see Figure 2(a) and (b)). Wang and Liao conducted experiments on the fit looseness fault to study the influences of the fit clearances and the tightening torque on the vibration. Sub-harmonic, super-harmonic and nonlinear resonances occur when a bearing outer ring is in a clearance-fit condition and increasing outer ring tightening torque is one of the main factors that can suppress rotor vibration. Recently, a fit looseness model was established and performed tests were done to verify this model by Chen and Qu, Wang and Guan, considering the interaction of fit pieces. The influences of the fit clearance on vibration responses were studied and verified by the bearing fit clearance tests.

In rotating machinery, active and passive control methods are used to reduce the vibration. Squeeze film damper (SFD) as a passive vibration reduction method was commonly used in aero-engine and other rotor systems. Han and Ding established a rotor/ball bearing system and adopted a hybrid numerical method to solve the linear and SFD support system. It was found that the SFDs can suppress the transient vibration. San and Vance studied the effects of the fluid inertia on the synchronous steady-state operation of a centrally preloaded single mass flexible rotor supported in the squeeze film bearing dampers. It showed that the bistable and jump phenomenon will be invisible at the large Reynolds numbers. Holmes and Sykes applied two SFDs to aero-engine to qualitatively validate a suggested theoretical model. Nonlinear jump phenomena and subharmonic resonance were demonstrated under rotor imbalance. Bonello and Brennan compared the nonlinear vibration characteristics of an aero-engine gas turbine by experiments and simulation. Whereas, Chen established a new rotor–ball bearing–stator dynamic model, which considered the coupling effect between rotor, ball bearing, and stator, elastic support and SFD.
Hertzian contact and rubbing faults. This model was verified by experiments. It shows fractional harmonic, superharmonic, subharmonic, quasi-periodic and chaotic behaviors generated by rubbing faults will be greatly reduced with lower support stiffness and smaller SFD film diameter. Furthermore, El-Shafei\textsuperscript{14} developed a control algorithm in the hybrid squeeze film damper to suppress vibration. Zhou and Luo\textsuperscript{15} established a dynamic model of a rotor supported on ball bearing with floating-ring squeeze film damper. San and Den\textsuperscript{16} measured the dynamic forced response of an idealized SFD with feedholes. Also, they studied an active control model of SFD and the dynamic load tests to identify SFD force coefficients. Heidari and Safarpour\textsuperscript{17} used a theoretical model for an active squeeze film damper to change the stiffness and damper in fluid film.

Though SFD models and experiments for aero-engine vibration reduction have been studied deeply to understand bistable response phenomenon, there is little research on SFD for vibration reduction in the condition of strong nonlinearity; in addition, existing researches mainly focused on fit looseness response characteristics, but vibration control was rarely concerned. In reality, Squirrel cage\textsuperscript{18} including SFD is widely used in an aero-engine support structure to adjust stiffness and damping. SFD used as a vibration absorber reduces vibration passively transmitted from rotor components to foundation. Based on the limited research on SFD application in the vibration reduction in a rotor system with fit looseness fault, a fit looseness model was established in current research, where the interaction between an outer ring and a sleeve is taken account for, which is different from classic rubbing fault model for a bearing outer ring which is not rotating, so the direction of friction needs to be judged. This fit looseness fault model was applied to a rotor-bearing system, and a numerical integration method (Newmark-$\beta$ and Zhai method) was applied to analyze the effects of looseness clearances on rotor's responses. Then a SFD model was applied between the sleeve and ground to absorb the energy and reduce the vibration transmitted from the interaction of the outer ring and the sleeve. Reduction effects and nonlinear phenomenon at different rotational speeds were illustrated by simulation. The current research work has important reference value for understanding SFD vibration reduction of fit looseness fault.

The highlight of this paper is to discuss a SFD applied to a rotor system with fit looseness fault, especially the fit looseness between the outer ring and housing. As fit looseness commonly exists in any rotor system with journal bearings or rolling bearings, the kind of bearings is not emphasized in the paper. We simplified the bearing units as a linear spring. The nonlinearity of the rotor system is from the interaction between the outer ring and housing. A coupling dynamic method was firstly applied to solve this complex system with the fit looseness fault and SFD.

**Modeling method of fit looseness fault and SFD**

**A fit looseness fault model**

Fit looseness modeling method is proposed from a Jeffcott rotor rig developed in Nanjing University of aeronautics and astronautics. The experimental rig is described as follows. This Jeffcott rotor with two disks and flange is supported on two pedestals which are fixed on the plane ground, as shown in Figure 3.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Viewing a sleeve bearing of a rotational machine.}
\end{figure}
A sketch diagram of the rotor rig is shown in Figure 4; where $L_1$, $L_2$, $L_3$, $L_4$, $L_5$ are the relative length of disk and pedestals, $P_1$, $P_2$, $P_3$ denote the name of three disks, $S_1$ and $S_2$ are the simplified pedestal’s position, and $F_{yb1}$ and $F_{yb2}$ are the acting forces of the rotor on the ground. The coordinate system is shown as $xyz$, and bending angles along $x$ and $y$ axes are $\Phi$ and $\Psi$.

At the left pedestal ($S_1$), a fit looseness exists between the ring and the sleeve, which is fixed to the pedestal. The fit looseness clearances were achieved by changing the sleeves with different fit clearances, and the tightening torque was used to tighten or loosen the sleeves, which can be changed to control the nonlinear levels of the stiffness and damping between the outer ring and the sleeve, shown in Figure 5.

The physical parameters measured from the testing rig in Figure 3 are listed in Table 1, where the mass of three disks and sleeve, the positions of three disks and the diameter of the shaft were measured, but polar moment of the disk’s inertia and equator moment of the disk’s inertia were estimated from modal test. Other contact stiffness and viscosity damping coefficient were obtained in the literature by Chen and Qu, Wang and Chen.8,9

In order to understand the interaction mechanism between housing and outer ring, some assumptions are made as follows.

1. The bearing housing is assumed as a mass unit, and the deformation of bearing housing is not considered.
2. The outer ring is harder than bearing housing, so outer ring can be modeled as a mass unit.
3. The nonlinearity resulting from the rolling bearing is neglect to highlight the nonlinear impacts from fit looseness fault, and linear stiffness and damping are used to represent bearing units.
4. Rubbing and friction effects between the outer ring and the sleeve are equivalent to rubbing between the rotor and the stator, but the rubbing directions need to be identified.

According to the conventional rubbing fault models, fit looseness model is established in this paper, referring to literature.8,9

Figure 6 shows the sketch diagram of the rubbing fault between the outer ring and the sleeve, where the center of the sleeve is $O_1$, the center of the bearing outer ring is $O_2$, $P$ is the contact point, the contact angle position is $\theta$,
the normal force is $P_N$, and tangential force is $P_T$. The interactive forces in the horizontal and the vertical directions can be obtained from Han and Ding and are described as 

$$
\begin{align*}
    P_x &= k_r(1 - c/r)[(x_I - x_o) + f \times \text{sign}(\nu_T)(y_I - y_o)] \\
    P_y &= k_r(1 - c/r)[-(y_I - y_o) - f \times \text{sign}(\nu_T)(x_I - x_o)]
\end{align*}
$$

where $k_r$ is the stiffness between two contact surfaces, $f$ is the friction coefficient, $x$ and $y$ are the displacements in two directions, lower scripts $I$ and $O$ represent the inner and outer fit components and $r$ is the relative displacement of the bearing outer ring and the sleeve. $c$ is the fit clearance, and $\nu_T$ is the relative velocity of the sleeve and the bearing outer ring in the tangent direction.
Mathematical model of SFD

In order to reduce the impact vibration transmitted from the interaction of the bearing outer ring and the sleeve, a conventional passive SFD with a squirrel cage is introduced, as shown in Figure 7. The squirrel cage can be used to adjust stiffness, and SFD can be applied to reduce the vibration. A schematic model of a rotor system (Figure 3) with SFD is illustrated in Figure 8. The film forces are acting on the sleeve from the relative motion of the sleeve and ground. In order to simplify this model, the bearing element is considered as linear spring. The film pressure distribution can be deduced based on the generalized Reynolds equation.

Theoretical film forces are got by integrating the pressure distribution over the entire damper surfaces, which are obtained from the Reynolds equation by San and Den. The generalized Reynolds equation can be expressed in the cylindrical coordinates as

\[
\frac{1}{R^2} \frac{\partial}{\partial \theta} \left( R^3 \frac{\partial p}{\partial z} \right) = -12 \mu \psi \frac{\partial h}{\partial \theta} + 12 \mu \frac{\partial h}{\partial t}
\]

where \( R \) is the radius of oil film surface, \( p \) is the pressure, \( h \) is the oil film thickness, \( \mu \) is the fluid viscosity, \( z \) and \( \theta \) are the axial and circumferential coordinates, \( \psi \) is the attitude angle of the journal at rotating speed \( \Omega \), is its first derivatives, with respect to time.

![Figure 7. Passive SFD with squirrel cage.](image1)

![Figure 8. A schematic model of a rotor rig with SFD.](image2)
In order to solve equation (2), based on short bearing assumption and half Sommerfeld boundary condition, the squeezed oil film forces can be expressed as follows

\[
\begin{align*}
F_r &= \frac{\mu R L^3}{c^2} [I_1 \dot{c} + I_2 \dot{\phi}] \\
F_t &= \frac{\mu R L^3}{c^2} [I_2 \dot{c} + I_3 \dot{\phi}]
\end{align*}
\]  

(3)

where \(F_r\) and \(F_t\) respectively, are the radial and tangential oil film forces, \(\dot{c}\) and \(\dot{\phi}\), are the displacement and velocity of the sleeve, and \(\phi\) is the angular coordinate measured from the position of the maximum film thickness in the direction of rotor angular speed. \(L\) is the length of the damper, \(c\) is the damper’s radial clearance and \(\mu\) is the dynamic viscosity of lubricant.

In equation (3), \(I_1, I_2\) and \(I_3\) can be expressed as follows

\[
\begin{align*}
I_1 = &- \frac{\varepsilon \sin \theta_1 (3 + (2 - 5 \varepsilon^2) \cos^2 \theta_1)}{(1 - \varepsilon^2)^2 (1 - \varepsilon^2 \cos^2 \theta_1)^2} + \frac{\sigma (1 + 2 \varepsilon^2)}{(1 - \varepsilon^2)^2} \\
I_2 = &- \frac{2 \varepsilon \cos \theta_1^2}{2 \cos^2 \theta_1} \\
I_3 = &- \frac{\varepsilon \sin \theta_1 (1 - 2 \cos^2 \theta_1 + \varepsilon^2 \cos^2 \theta_1)}{(1 - \varepsilon^2)^2 (1 - \varepsilon^2 \cos^2 \theta_1)^2} + \frac{\sigma (1 + 2 \varepsilon^2)}{(1 - \varepsilon^2)^2}
\end{align*}
\]  

(4)

where \(\theta_1 = \tan^{-1} \left( -\frac{\dot{c}}{\dot{\phi}} \right)\), \(\sigma = 0.5 \pi \tan^{-1} \left( \frac{\sin \theta_1}{(1 - \varepsilon^2)^2} \right)\).

Then the oil film forces in polar coordinate system are translated to the stationary system, so the oil film forces in \(x\) and \(y\) directions can be expressed as follows

\[
\begin{align*}
F_x &= -F_r \frac{x}{\varepsilon} + F_t \frac{y}{\varepsilon} \\
F_y &= -F_r \frac{y}{\varepsilon} - F_t \frac{x}{\varepsilon}
\end{align*}
\]  

(5)

where \(\varepsilon = \sqrt{x^2 + y^2}\), \(\tan \phi = x/y\).

**Solving method**

A classical numerical method by Zhai\(^{20}\) was used to calculate the dynamic responses of rotor–pedestal coupling dynamic model in previous research by Chen and Qu, Wang and Guan.\(^7,8\)The basic calculation procedures are described as follows.

1. Using FE modeling methods to establish the rotor system, which is expressed as

\[
M_s \ddot{q}_s + (C_s - \Omega G_s) \dot{q}_s + K_s q_s = Q_s
\]  

(6)

where \(Q_s\) is the external force vector, \(M_s\) is the mass matrix of the rotor system, \(G_s\) is the gyroscopic matrix of the rotor system, \(K_s\) is the stiffness matrix, \(C_s\) is the damping matrix and \(\Omega\) is the rotational speed.

The sleeve can be modeled as a lumped mass, whose movement equations can be referred in Zhai.\(^{20}\) The rotor is divided into 38 elements, which is enough to converge for the simulated model and real model. The model verification was done by modal test. The left support position is at the 6th node, and the right support position is at the right position. The disks are distributed at 11th, 26th and 39th nodes with the eccentricities 0.03 mm and mass 2.35 kg, 2.35 kg and 1.05 kg, respectively. Furthermore, the hybrid model of finite element(FE) and lumped mass model is used to calculate time–domain responses.
1. Nonlinear rubbing forces in x and y directions which are calculated by the displacements and velocities of the outer ring and the sleeve in equation (1), can be applied in the forces in $Q_s$.

2. Oil-film forces in x and y directions which are calculated by the displacements of the sleeve in equation (5), can be applied in the forces in $Q_s$.

The hybrid calculation procedures are described as follows. Explicit Newmark-$\beta$ method (Zhai method) and Newmark-$\beta$ method were combined to solve the fit looseness fault of a testing rig with SFD. This rotor–pedestal coupling dynamic solution procedure is shown in Figure 9. Implicit Newmark-$\beta$ method is applied to obtain the response of finite element rotor, explicit Newmark-$\beta$ method is applied to get the lumped mass’s response, the fit looseness forces between the outer ring and the sleeve are solved by equation (1) and the SFD forces are solved by equation (5). All forces are obtained from the response of the rotor, sleeve and outer ring at the last computational step. Responses are obtained until the solution is stable and transient solution is neglected. The merit of this method is that it is beneficial to calculate structural coupling interaction, especially for finite element model and lumped mass model.

In order to apply the above numerical computational methods to the test rig (Figure 1), accurate rotor parameters are necessary as an input for the rotor–pedestal model. All the parameters are referred in Chen and Qu, Wang and Guan. This model verification work was done in literature by Wang and Guan. The mode shapes and frequencies from the modal tests and simulation are shown briefly in Figure 10 and Table 2. The first three mode shapes, i.e., the bending modes in $xz$ plane, and natural frequencies from experiments and simulation are close. There is slightly different of the third mode shape from experiments and simulation for the different measuring methods. We used hammer knocking multiple points and outputted a channel signal. We apply time simulation to see the mode shape for different rotational speed, where the gyroscopic effects are considered. To some extent, the geometry of the rig and stiffness of rotor–pedestal system are correct. Based on this valid model, simulations on different conditions with fit looseness fault and SFD were conducted.

![Figure 9. Solving flow for rotor–pedestal model with fit looseness fault and SFD. SFD: squeeze film damper.](image)

![Figure 10. First three modes from experiments and simulation.](image)
Simulation results and discussion

Applying the numerical methods for the rotor system with fit looseness fault and SFD models, this kind of piecewise nonlinear behaviors resulting from the interaction between the bearing outer ring and the sleeve were investigated to illustrate the fit looseness fault mechanism, and vibration reduction effects of SFD are assessed.

Simulation of fit looseness fault without SFD

In order to assess the passive vibration reduction effects of SFD, the fit looseness fault model without SFD was applied to simulate piecewise phenomenon caused by the fit clearance (50 μm). The contact stiffness and viscosity damping between the outer ring and the sleeve is $1 \times 10^6$ N/m and 500 N/(m/s), the contact stiffness and viscosity damping between the sleeve and ground and is $1 \times 10^6$ N/m and 1000 N/(m/s). The stiffness and damping values were estimated to be large enough to keep system stable. The stability of hybrid complex system is not the highlight of this paper. However, the rotor’s dynamic behavior is focused, i.e., the frequency characteristics and contact styles. The nonlinear response phenomenon was displayed as follows.

Figure 11(a) to (d) shows the waterfall of vertical acceleration of the bearing outer ring, the vertical displacement of the rotor and the vertical contact forces at the left position $S_1$ of the rotor system. The contact forces denote the fit looseness contact forces. $1\times$, $2\times$ and $3\times$ represent the harmonic frequency components of the rotational frequency. In Figure 11(a) to (d), when the rotational speeds are around the first critical speed, $1\times$, $2\times$ and $3\times$ components are prominent due to the effects of imbalance; when the rotational speeds are around the second

|                        | The first mode | The second mode | The third mode |
|------------------------|----------------|----------------|----------------|
| Experimental frequency (Hz) | 44.53          | 114.37         | 222.54         |
| Frequency from simulation (Hz) | 40.15          | 111.87         | 214.81         |

Table 2. Results of frequency identification.
and the third critical speeds, $1 \times$, $2 \times$, $3 \times$ components are prominent shown in Figure 11(a), $1 \times$, $2 \times$ components are prominent shown in Figure 11(b) to (d). The intense of nonlinearity of contact is sensitive to the unbalanced force, which depends on the power of rotational speeds, i.e., $\Omega^2$ term. In addition, the contact states are contributed to offset of unbalanced force and weight, i.e., the contact states can be divided into three types, non-contact, contact and co-existence of non-contact and contact. The third contact type will cause the periodic/cyclic impacts between two contact surfaces and more harmonic frequencies can occur.

To better show the nonlinear effects on the response of the rotor, bode diagram were produced for multiple frequency components. Figure 12(a) to (d) are bode diagrams of multiple frequencies of vertical acceleration, vertical displacement and horizontal and vertical forces. As shown in these figures, $1 \times$ is prominent comparing with multiple-frequency components, especially when the rotating speeds are around the third critical speed. The fundamental reason is $1 \times$ is caused by unbalanced forces, and the higher rotational speeds and the unbalanced forces will be; when the rotational speeds are at the second and third critical speeds, the magnitudes of $2 \times$, $3 \times$ and $4 \times$ are decreased, i.e., the nonlinearity is less stronger than that when the rotational speeds are around these critical speeds. This phenomenon is caused by the changing contact styles from discontinuous contact to continuous contact between the outer ring and the sleeve under a larger imbalance. This explanation of contact style will be explored at specific rotational speeds in next section.

**Simulation of fit looseness fault with SFD**

In order to reduce discontinuous contact style, i.e., periodic/cyclic impact between sleeve and bearing outer ring, simulation of rotor system with SFD can be usually conducted to show the vibration alleviation of contact nonlinearity resulted from fit looseness fault. Meanwhile, fit looseness fault characteristics, that is, prominent multiple frequency components and continuous or intermittent impact, are illustrated to verify the effectiveness of SFD.

![Bode diagram of vertical acceleration](image1)

![Bode diagram of vertical displacement](image2)

![Bode diagram of horizontal contact force](image3)

![Bode diagram of vertical contact force](image4)

**Figure 12.** Bode diagrams of acceleration, displacement and forces with respect to rotational speeds and harmonic frequency components.
The main SFD parameters used for equation (4) are set as follows: \( \nu = 0.7 \times 10^{-3} \text{Pa-s} \), \( R = 0.3 \times 10^{-4} \text{m} \), \( c = 0.75 \times 10^{-6} \text{m} \). Other fit looseness parameters are same as the above section. The stable time history of the rotor’s responses with fit looseness fault and SFD is recorded. The effects of SFD on vibration reduction of nonlinear frequencies are illustrated. Figure 13(a) to (f) shows the waterfall plots of vertical acceleration of the bearing outer ring, vertical displacement of the rotor, the vertical contact forces and the film forces at the fit looseness position. The contact forces denote the fit looseness contact forces. The film forces denote the squeeze film forces in horizontal and vertical directions. Comparing with Figure 11(a) to (d), the maximum sleeve’s acceleration amplitude after applying SFD is decreased by 3/4, as shown in Figure 13(a), the rotor’s displacement after applying SFD is close to that before applying SFD, as shown in Figure 13(b), and the fit looseness forces after adding SFD are increased by nearly 20 N at the second and the third critical speeds, as shown in Figure 13(c) to (d). These increased fit looseness forces are balanced by film forces. Higher multiple frequency components are shown in the acceleration frequency components, as shown in Figure 13(a), but \( 1 \times \), \( 2 \times \) are prominent in the waterfall diagrams of displacement, the fit looseness forces and the film forces (see Figure 13(b) to (f)).

Figure 14(a) to (f) are the bode diagrams of multiple frequencies of the vertical acceleration, the vertical displacement and the horizontal and vertical forces. As shown in these figures, \( 1 \times \) are prominent comparing with the multiple-frequency components, especially when the rotating speeds are around the third critical speed. When the rotational speeds are at the second and third critical speeds, the magnitudes of \( 2 \times \), \( 3 \times \) and \( 4 \times \) are decreased. Nonlinearity is less stronger than that when the rotational speeds are around the second and third critical speeds. This phenomenon is caused by continuous contact between the outer ring and the sleeve. Bode

Figure 13. Waterfall of acceleration, displacement and forces.
diagrams of film forces show similar phenomenon under larger imbalance. This explanation of the contact style will be explored at specific rotational speeds in next paragraph.

To clearly show the effectiveness of SFD on vibration reduction of the multiple frequencies and the influence of the contact states, the waveform and the spectrum at typical critical rotational speeds are compared to uncover the mechanism of the vibration reduction of SFD. At the first three critical speeds, i.e., 2200 r/min, 5800 r/min and 9400 r/min, Figures 15 to 17 show comparisons of the waveforms and the spectrums of the vertical acceleration of the sleeve, the vertical displacement of the rotor, the vertical contact forces and the film forces at the fit looseness position, respectively.

In Figure 15(a) to (d), the magnitude of vertical acceleration of sleeve is decreased by 2/3 after applying SFD and the amplitudes of $3x$, $4x$, $5x$ \ldots are decreased prominently, and the magnitude of the vertical displacement of the rotor is decreased by 1/2 after applying SFD and the amplitudes of $1x$ and $2x$ are decreased prominently; in Figure 15(e) to (h), the contact time is decreased and the magnitudes of the contact forces are decreased by 1/2, and the magnitudes of all frequency components are decreased except that of $2x$; In Figure 15(i) to (j), the waveform of the film forces shows the vibration reduction effects in horizontal direction are better than that in vertical direction for the larger film forces in the horizontal direction. Discontinuous contact makes the response
Figure 15. Waveforms and spectrums of acceleration, displacement and forces at 2200 r/min.
Figure 16. Waveforms and spectrums of acceleration, displacement and forces at 5800 r/min. (a) Waveform of vertical acceleration, (b) Spectrum of vertical acceleration, (c) Waveform of vertical displacement, (d) Spectrum of vertical displacement, (e) Waveform of horizontal contact force, (f) Spectrum of horizontal contact force, (g) Waveform of vertical contact force, (h) Spectrum of vertical contact force, (i) Waveform of film force, (j) Spectrum of film force.

Figure 17. Waveforms and spectrums of acceleration, displacement and forces at 9400 r/min. (a) Waveform of vertical acceleration, (b) Spectrum of vertical acceleration, (c) Waveform of vertical displacement, (d) Spectrum of vertical displacement, (e) Waveform of horizontal contact force, (f) Spectrum of horizontal contact force, (g) Waveform of vertical contact force, (h) Spectrum of vertical contact force, (i) Waveform of film force, (j) Spectrum of film force.
Figure 16 shows the vibration reduction effects of SFD on the rotor system with the fit looseness fault at 5800 r/min. In Figure 16(a) to (d), the amplitudes of acceleration and displacement are decreased by 2/3 and 1/3 after SFD. Figures 17(a) to (j) display the waveforms and spectrums of acceleration, displacement, and forces at 9400 r/min. These figures illustrate the capability of SFD to reduce vibration, with the amplitudes decreasing to 1/3 and 1/3, respectively, after SFD.
applying SFD and $1 \times$ is prominent in their spectrums; in Figure 16(e) to (j), the contact forces are decreased by 1/4 after applying SFD and the vibration reduction effects of SFD in vertical direction are same as that in horizontal direction. The nonlinear response of less frequency components is determined by continuous contact and the vibration reduction of SFD does not have its directivity. The vibration reduction effects of SFD are better than that at 2200 r/min.

Figure 18. Waveforms and spectrums of acceleration, displacement and forces at 4200 r/min.
Figure 17 displays vibration reduction effects of SFD on the rotor system with the fit looseness fault at 9400 r/min. In Figure 17(a) to (d), the amplitude of acceleration is decreased by $2/3$, the displacement is not changed after applying SFD and $1/C^2$ is prominent; in Figure 17(e) to (j), the contact forces are increased slightly and the vibration reduction effects of SFD in vertical direction are same as that in horizontal direction. The film forces do not have its directivity due to continuous contact. Due to higher rotational speeds, the vibration reduction...

**Figure 19.** Waveforms and spectrums of acceleration, displacement and forces at 6800 r/min.
effects of SFD on rotor’s displacement are not obvious (see Figure 17(c)). Contact forces between the outer ring and the sleeve are increased for the relative increasing displacements between the outer ring and the sleeve. Hence, the relative displacements between the sleeve and ground are increased, so the vibration reduction effects of SFD are prominent.

Figure 20. Waveforms and spectrums of acceleration, displacement and forces at 12000 r/min.
Besides the response characteristics at the first three critical speeds are analyzed, the nonlinear characteristics at typical rotational speeds (between two critical speeds) are explained to explore the nonlinear contact states and frequency characteristics. Figures 18 to 20 are the waveforms of acceleration, displacement, and contact forces and their spectrums when the rotational speeds are 4200 r/min, 6800 r/min and 12000 r/min. In Figure 18(a) to (d), the acceleration and the displacement are decreased by 2/3 and 1/3. In Figure 18(e) to (h), the periodic impacts disappear and the multiple frequencies disappear. The rubbing and friction between the outer ring and the sleeve disappear after applying SFD and the vibration of the sleeve and the rotor decreased greatly. The magnitudes of contact forces in the horizontal and the vertical axes are 0.01 N and 0.4 N at this rotational speed before adding SFD, which are lower than that at other rotational speeds. Hence, the contact forces decreased to 0 N after adding SFD, which is equivalent to increase the damping in the system.

In Figure 19(a) to (d), the acceleration is decreased by 2/3 and the displacement is decreased slightly. In Figure 19(e) to (h), the magnitude of the periodic impacts is stronger than that before applying SFD and the multiple frequencies are larger than that before applying SFD. The contact strength between the outer ring and the sleeve is increased for the relative increasing displacement between the outer ring and the sleeve. Hence, the relative displacement between the sleeve and ground is increased, so the vibration reduction effects of SFD are prominent.

In Figure 20(a) to (d), the acceleration is decreased by 2/3 and the displacement is increased prominently. In Figure 20(e) to (h), the magnitude of periodic impacts is stronger than that before applying SFD, the contact time of the outer ring and the sleeve is longer and multiple frequencies are larger than that before applying SFD. The energy consumed by SFD should be balanced by the energy from the fit looseness faults between the outer ring and the sleeve. The relative displacements of the outer ring and the sleeve can be increased to achieve the vibration reduction.

**Conclusion**

In order to assess the reduction of the vibration resulting from fit looseness fault, a conventional SFD model was applied to study the effectiveness of vibration reduction in a rotor system with and without SFD. The results of this study can be summarized as follows:

1. Based on the novel hybrid model, Explicit Newmark-β and Implicit Newmark-β were used to establish a rotor dynamic model with fit looseness fault and SFD, which can consider the impacts between the outer ring and the sleeve, and the film forces acting on the sleeve and ground. This method was first used to calculate the vibration responses of a rotor system with fit looseness fault and a conventional SFD.
2. It is found that when the rotational speeds are around the first critical speed, first, second and third harmonics of response are prominent, especially first harmonic response is prominent; when the rotational speeds are around the second and the third critical speeds, 1, 2 are prominent from fit looseness fault without SFD. Nonlinear contact is weak when the rotational speeds are around the second and third critical speeds. This phenomenon is caused by continuous contact between the outer ring and the sleeve. Discontinuous contacts determined periodic/cyclic impacts between the outer ring and the sleeve, resulting in stronger nonlinearity.
3. Applying SFD to assess the vibration reduction of a rotor–pedestal system with fit looseness fault, the magnitudes of acceleration, contact forces are decreased. When the rotational speeds are around the first critical speed and between any two critical speeds, there appears periodic discontinuous contact. When the rotational speeds are around the second and third critical speeds, there appears periodic continuous contact.
4. For the discontinuous contact style between the outer ring and the sleeve, the rubbing and friction effects between the outer ring and the sleeve can be weaken or strengthened by applying SFD. Specially, when the rubbing and friction effects are strengthened, the film forces will be increased as the increment of the rotor’s displacements to balance the contact forces between the outer ring and the sleeve.

**Declaration of Conflicting Interests**

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