Chiral phase transition and color superconductivity in an extended NJL model with higher-order multi-quark interactions

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Abstract

The chiral phase transition and color superconductivity in an extended NJL model with eight-quark interactions are studied. The scalar-type nonlinear term hastens the chiral phase transition, the scalar-vector mixing term suppresses effects of the vector-type linear term and the scalar-diquark mixing term makes the coexisting phase wider.

Quantum Chromodynamics (QCD) has non-perturbative properties. First-principle lattice QCD simulations are useful to study thermal systems at zero or small density [1, 2]. At high density, however, lattice QCD is still not feasible due to the sign problem. Therefore, effective models are used in finite density region. One of the models is the Nambu–Jona-Lasinio (NJL) model [3].

This model has the mechanism of spontaneous chiral symmetry breaking, but it has not the confinement mechanism. However, this model has been widely used [4, 5] with the mean field approximation (MFA), for example, for analyses of the critical endpoint of chiral phase transition [6, 7, 8, 9, 10, 11].

As for the NJL model, only a few studies were done so far on roles of higher-order multi-quark interactions [12, 13], except for the case of the six-quark interaction coming from the
't Hooft determinant interaction [14]. The NJL model is an effective theory of QCD, so there is no reason, in principle, why higher-order multi-quark interactions are excluded.

In this paper, we consider an extended NJL model that newly includes eight-quark interactions and analyze roles of such higher-order interactions on the chiral phase transition and color superconductivity. It is well known that the original NJL model predicts a critical endpoint to appear at a lower temperature ($T$) and a higher chemical potential ($\mu$) than the lattice QCD [2] and the QCD-like theory [15, 16] do.

As for the repulsive vector-type four-quark interaction ($\bar{q}\gamma^\mu q$)$^2$, it is well-known that it makes the chiral phase transition weaker in the low $T$ and high $\mu$ region and makes it a crossover when the vector type interaction is strong enough [7, 11]. In this point of view, an absence of the vector-type four-quark interaction may be preferable in the high density region. On the contrary, a strong vector-type interaction is necessary to reproduce the saturation property of nuclear matter in the relativistic meson-nucleon theory [17]. Thus, it is expected that the vector-type interaction is sizable in the normal density region but suppressed in the higher density region. In the relativistic meson-nucleon theory, it is known that nonlinear meson terms can suppress the effective coupling between mesons and nucleons in the higher density region. Therefore, we consider higher-order multi-quark interactions in the NJL model.

We start with the following chiral-invariant Lagrangian density with two flavor quarks

$$\mathcal{L} = \bar{q}(i\not\partial - m_0)q + \left[ g_{2.0} \left( (\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2 \right) + g_{4.0} \left( (\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2 \right)^2 - g_{0.2} (\bar{q}\gamma^\mu q)^2 - g_{2.2} (\bar{q}\gamma^\mu q)^2 (\bar{q}\gamma^\nu q)^2 + d_{0.2} (i\bar{q}^c \varepsilon \gamma_5 q)(i\bar{q}\varepsilon \gamma_5 q) + d_{2.2} (\bar{q}q)^2 (i\bar{q}^c \varepsilon \gamma_5 q)(i\bar{q}\varepsilon \gamma_5 q^c) + \cdots \right],$$

where $q = q_\alpha$ is the quark field, $q^c = C\bar{q}^T$ and $\bar{q}^c = q^T C$ are the charge-conjugation spinor, $C = i\gamma^2\gamma^0$ is the charge-conjugation matrix, $g_{i,j}$ and $d_{n,m}$ are coupling constants, $m_0$ is the current quark mass, the Latin and Greek indices mean the flavor and color, $\bar{\tau} = (\tau^1 \tau^2 \tau^3)$ are Pauli matrices and $\varepsilon$ and $\varepsilon^b$ are the antisymmetric in each of the flavor and color space.

In this paper, we use the MFA. At the numerical calculation, we ignore higher-order terms represented by dots. Furthermore, we disregard interactions including isovector-vector current not important in symmetric quark matter.

We determine the parameters, $g_{2.0}$, $g_{4.0}$ and $\Lambda$, so as to reproduce the pion mass (138 MeV), the sigma meson mass (650 MeV) and the pion decay constant (93.3 MeV).
The other parameters $g_{0,2}$, $g_{2,2}$, $d_{0,2}$ and $d_{2,2}$ are free parameters. For the typical case, we assume that $g_{0,2} = A g_{2,0}$, $g_{2,2} = B g_{2,0}/\sigma_0^2$, $d_{0,2} = 0.6 g_{2,0}$, $d_{2,2} = C d_{0,2}/\sigma_0^2$, $G_\sigma = 2g_{2,0} + 12g_{4,0}\sigma_0^2$, where $\sigma_0$ stands for the scalar density at $T = \mu = 0$. We take parameters as $(A, B, C) = (1.0, 0, 0)$, $(A, B, C) = (0.8, 0.2, 0)$ and $(A, B, C) = (0, 0, 0.2)$. The current quark masses of up and down quarks are assumed to be 5.5 MeV.

Figure 1 shows the chiral phase transition line in the $\mu$-$T$ plane [18].

![Figure 1: Phase diagram in the $\mu$-$T$ plane. Each curve denotes the location of the first-order phase transition.](image)

It is understood that the $\omega^2$ interaction tends to change the chiral phase transition from a first order to a crossover, but this effect is partially canceled out by the $\sigma^4$ and $\sigma^2\omega^2$ interaction. Consequently, as shown in Fig. 1, there exists a critical endpoint also in the NJL+$\sigma^4+\omega^2+\sigma^2\omega^2$ model.

Figure 2 shows the chiral phase transition line and color superconductivity region in the $\mu$-$T$ plane. Compare the left panel with the right one, the $\sigma^2\Delta^2$ interaction makes the coexisting phase wider.

In conclusion, we have studied effects of eight-quark interactions on the chiral phase transition and color superconductivity. The scalar-type nonlinear term $\sigma^4$ hastens the chiral phase transition and makes the critical endpoint move to a higher temperature and a lower chemical potential than original NJL model calculations. Both the scalar-type nonlinear
FIG. 2: Phase diagram in the $\mu$-$T$ plane. Solid lines represent the first order phase transition and dotted lines represent the second order phase transition. The symbol “$\chi$SB” stands for the chiral symmetry broken phase, “Wigner” stands for the chiral symmetry restored phase, “CSC” stands for color superconductor phase and “coex.” stands for the coexisting phase in which both the chiral condensation and the diquark condensation have finite vacuum expectation values.

The term $\sigma^4$ and the scalar-vector mixing term $\sigma^2\omega^2$ cancel effects of the vector-type linear term $\omega^2$. The scalar-diquark mixing term $\sigma^2\Delta^2$ enlarges the coexisting region of the chiral condensate and the diquark condensate. Therefore, higher-order multi-quark interactions are important for both the chiral phase transition and color superconductivity.

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