Bandwidth-control vs. doping-control Mott transition in the Hubbard model

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We reinvestigate the bandwidth-control and doping-control Mott transitions (BCMT and DCMT) from a spin liquid Mott insulator to a Fermi liquid metal based on the slave-rotor representation of the Hubbard model, where the Mott transitions are described by softening of bosonic collective excitations. We find that the nature of the insulating phase away from half filling is different from that of half filling in the respect that a charge density wave coexists with a topological order (spin liquid) away from half filling because the condensation of vortices generically breaks translational symmetry in the presence of “dual magnetic fields” resulting from hole doping while the topological order remains stable owing to gapless excitations near the Fermi surface. Performing a renormalization group analysis, we discuss the role of dissipative gauge fluctuations due to the Fermi surface in both the BCMT and the DCMT.

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I. INTRODUCTION

Landau-Ginzburg-Wilson (LGW) paradigm has been our unique theoretical framework for classical phase transitions. Starting from an electron Hamiltonian, one can derive an effective LGW free energy functional in terms of order parameters associated with some symmetry breaking. The LGW framework has been also applied to quantum phase transitions by taking temporal fluctuations of order parameters into account, usually called the Hertz-Millis theory.

There are models of quantum phase transitions, on the other hand, which may defy interpretation in the LGW paradigm. Consider the superfluid-insulator transition of a boson Hubbard-type model. The boson density \( \rho \) and its phase \( \phi \) are canonically conjugate, satisfying the uncertainty relation \( \Delta \rho \Delta \phi \gtrsim 1 \), and the competing nature of the two variables results in the condensation of one variable or the other, depending on the ratio of phase stiffness and the compressibility. The quantum conjugate nature of the variables, satisfying an uncertainty relation, lies at the heart of the quantum phase transition in this particular case. It is not obvious how LGW theory, written solely in terms of an order parameter, captures the inherent competing nature of the conjugate variables driving the quantum phase transition.

As another example of a quantum phase transition where an order parameter description is likely to fail, we mention the metal to paramagnetic insulator transition (Mott transition) found in the study of the two dimensional Hubbard model. As recent dynamical mean-field theory (DMFT) studies show, the transition is associated with the vanishing of the spectral weight of the quasiparticle peak, but not with any symmetry breaking field, hence the order parameter approach in the LGW paradigm is not clear to be applicable.

Recently, Florens and Georges (FG) reexamined a bandwidth-control Mott transition (BCMT) from a paramagnetic Mott insulator of a spin liquid to a correlated metal of a Fermi liquid at half filling in the Hubbard model. In order to describe the BCMT they introduced an elegant formulation based on the slave-rotor representation, and investigated properties of metallic and insulating phases of the model. Many of the properties obtained at the mean-field level matched well with the more sophisticated DMFT calculations. In this formulation the competing nature of canonically quantum conjugate variables naturally appears. Within this theoretical framework the Mott transition is understood by softening of bosonic collective excitations, physically associated with zero-sound modes in a Fermi liquid. When these bosonic excitations are gapped, a paramagnetic Mott insulator with charge gap but no spin gap results, thus called a spin liquid. On the other hand, condensation of the boson excitations causes a coherent quasiparticle peak at zero energy, resulting in a Fermi liquid in the low energy limit.

In the present paper we investigate a doping-control Mott transition (DCMT) from the spin liquid to the Fermi liquid based on the slave-rotor representation of the Hubbard model. We find that the DCMT differs from the BCMT in the respect that the nature of the Mott insulator and the mechanism of the Mott transition are different from each other. Hole doping results in a nontrivial Berry phase term to the boson field, leading to an effective magnetic field for its vortex field in the dual formulation. It is shown that this effective magnetic field induces a crystalline phase of doped holes, coexisting with the spin liquid. On the other hand, the paramagnetic Mott insulator at half filling is the same spin liquid, but without any charge orders. We argue that the doped spin liquid with charge order evolves into the Fermi liquid via a continuous phase transition.

The present scenario for the DCMT was discussed before, but based on the boson-only (Hubbard) model, where fermionic excitations are ignored or decoupled to the bosonic excitations in the renormalization group (RG) sense. In this paper we start from the electron Hubbard model, and derive an effective bosonic field theory. It should be noted that this effective field theory...
is totally different from that in the boson Hubbard model owing to the presence of damped gauge fluctuations resulting from gapless fermion excitations. Although the Berry phase plays the same role in both doped (boson and fermion) Mott insulators, nature of the Mott transitions would be different owing to the presence of dissipative gauge fluctuations in the slave-rotor representation of the electron Hubbard model. Performing an RG analysis, we show that the dissipative dynamics of gauge excitations makes both the BCMT and the DCMT in the electron Hubbard model differ from those in the boson Hubbard model. There also exists a previous study considering fermion excitations coupled to bosonic fields. However, this study starts from the quantum dimer model, and considers a valance bond solid instead of the spin liquid with a Fermi surface. Thus, the fermion excitations in the model are gapped (for the s-wave pairing case), thus ignored in the low energy limit. We would like to emphasize that our crystalline phase is nothing to do with Cooper pairs, instead associated with doped holes.

II. EFFECTIVE FIELD THEORY FOR THE MOTT TRANSITION

We derive the slave-rotor representation of the Hubbard model in the path-integral formulation. We note that FG derived it based on not only the canonical quantization method but also the path integral formulation. However, we argue that our path integral derivation more clearly shows the connection between Hubbard-Stratonovich (HS) fields and rotor variables.

We consider the Hubbard model in two dimensions

\[ H = -t \sum_{ij \sigma} c_{i \sigma} c_{j \sigma} + U \sum_i n_i^2. \]  

(1)

Here \( t \) is a hopping integral of electrons, and \( U \) the strength of local interactions. \( n_i = \sum_\sigma c_{i \sigma}^\dagger c_{i \sigma} \) is an electron density.

A usual methodology treating the Hubbard \( U \) term is a HS transformation. Using the coherent state representation and performing the HS transformation, we obtain the partition function

\[
Z = \int D[c_{i \sigma}, \varphi_i] \exp \left[ -\int d\tau \left( \sum_{i \sigma} c_{i \sigma}^\dagger (\partial_\tau - \mu) c_{i \sigma} - t \sum_{ij \sigma} c_{x \sigma}^\dagger c_{y \sigma} \frac{1}{4U} \varphi_i^2 - i \varphi_i \sum_\sigma c_{i \sigma}^\dagger c_{i \sigma} \right) \right],
\]

(2)

where \( \varphi_i \) is an order parameter associated with a charge density wave (CDW), and \( \mu \), the chemical potential of electrons. Physically, the \( \varphi_i \) field corresponds to an effective electric potential. In the usual mean-field manner the CDW order parameter is given by \(-i\varphi_i = 2U \langle \sum_\sigma c_{i \sigma}^\dagger c_{i \sigma} \rangle\).

Integrating over electronic excitations in Eq. (2) and expanding the resulting logarithmic term for the effective potential \( \varphi_i \), one can obtain an effective LGW free energy functional in terms of the CDW order parameter \( \varphi_i \). As mentioned in the introduction, it is not clear that this LGW theoretical framework has the competing nature of quantum conjugate variables because there exists only one CDW order parameter. One can say that the formulation Eq. (2) is exact, and thus the LGW framework may be a good starting point. However, an important point is how to expand the resulting logarithmic term. The expansion should be approximately performed, and thus one cannot say validity of the LGW framework for quantum phase transitions.

It is clear that the metal-insulator transition is associated with charge fluctuations. One way controlling charge fluctuations is to introduce the canonical conjugate variable of the charge density. Unfortunately, \( \varphi_i \) is not the canonically conjugate variable of the charge density because it is an effective electric potential.

We consider the gauge transformation for an electron field

\[ c_{i \sigma} = e^{-i \theta_i} f_{i \sigma}. \]  

(3)

Here \( e^{-i \theta_i} \) is assigned to be an annihilation operator of an electron charge, and \( f_{i \sigma} \) an annihilation operator of an electron spin. In this paper we call \( e^{-i \theta_i} \) and \( f_{i \sigma} \) chargon and spinon, respectively.

Inserting Eq. (3) into Eq. (2), we obtain

\[
Z = \int D[f_{i \sigma}, \theta_i, \varphi_i] \exp \left[ -\int d\tau \left( \sum_{i \sigma} f_{i \sigma}^\dagger (\partial_\tau - \mu - i \partial_\tau \theta_i) f_{i \sigma} - t \sum_{ij \sigma} f_{i \sigma}^\dagger e^{i(\theta_j - \theta_i)} f_{j \sigma}^\dagger \right) \right. \\
\left. + \sum_i \left( \frac{1}{4U} \varphi_i^2 - i \varphi_i \sum_\sigma f_{i \sigma}^\dagger f_{i \sigma} \right) \right].
\]

(4)

Performing the HS transformation \((1/4U)\varphi_i^2 \rightarrow U L_i^2 + i L_i \varphi_i \), and shifting \( \varphi_i \) into \( \varphi_i \rightarrow \varphi_i - \partial_\tau \theta_i \), we obtain the following expression for the partition function

\[
Z = \int D[f_{i \sigma}, \theta_i, \varphi_i, L_i] \exp \left[ -\int d\tau \left( \sum_{i \sigma} f_{i \sigma}^\dagger (\partial_\tau - \mu) f_{i \sigma} - t \sum_{ij \sigma} f_{i \sigma}^\dagger e^{i(\theta_j - \theta_i)} f_{j \sigma}^\dagger \right) \right. \\
\left. + \sum_i \left( U L_i^2 + i L_i (\varphi_i - \partial_\tau \theta_i) - i \varphi_i \sum_\sigma f_{i \sigma}^\dagger f_{i \sigma} \right) \right].
\]

(5)

Integrating over the \( \varphi_i \) field, one finds \( L_i = \sum_\sigma \bar{f}_{i \sigma}^\dagger f_{i \sigma} \). In this respect \( L_i \) corresponds to the density variable of FG.

Eq. (5) has an interesting structure for the quantum phase transition. First of all, there is the competing nature of canonically conjugate quantum variables. The \( \theta_i \) field is canonically conjugate to the charge density \( L_i = \sum_\sigma \bar{f}_{i \sigma}^\dagger f_{i \sigma} \), as one can see from the coupling term \(-i L_i \partial_\tau \theta_i \) of the Lagrangian derived above. These two operators satisfy the commutation relation \([\theta_i, L_j] = i \delta_{ij}\).
and thus the uncertainty relation $\Delta L_i \Delta \theta_i \gtrsim 1$ works. Fluctuations of the $\theta_i$ field correspond to bosonic collective excitations, here associated with zero sound modes of a Fermi liquid when it becomes condensed. This can be justified from the fact that the dispersion of the $\theta_i$ field in its condensed phase is given by that of sound waves.

The quantity $\varphi_i$ is the CDW order parameter in Eq. (2). In the formulation presented in Eq. (5), however, it transforms as the time component of a U(1) gauge field. Under the (U(1) gauge transformation for the matter fields, $f_{i\sigma} \to e^{i\alpha} f_{i\sigma}$ and $\theta_i \to \theta_i + \phi_i$, the effective potential should be transformed into $\varphi_i \to \varphi_i + \partial_t \phi_i$. This gauge-field aspect of the order parameter is introduced due to the mapping of Eq. (3), which involved the new phase degree of freedom.

Integrating over the $L_i$ field, Eq. (5) reads

$$ Z = \int D[f_{i\sigma}, \theta_i, \varphi_i] e^{-\int d\tau L}, $$

$$ L = \sum_{i\sigma} f_{i\sigma}^* (\partial_\tau - \mu - i\varphi_i) f_{i\sigma} - t \sum_{i\sigma,j\sigma} f_{i\sigma}^* e^{i(\theta_i - \theta_j')} f_{j\sigma} + \frac{1}{4U} \sum_i (\partial_\tau \theta_i - \varphi_i)^2. $$

(6)

This expression is nothing but the slave-rotor representation of the Hubbard model, obtained by FG in a different fashion. It is clear that the CDW order parameter appears to be the time component of a U(1) gauge field. This can be understood by the fact that physics of the CDW order parameter is an effective potential.

This effective Lagrangian should be considered to generalize the LGW theoretical framework. If fluctuations of the $\theta_i$ fields are ignored, the resulting effective field theory belongs to the LGW framework. However, as clearly demonstrated by FG, $\theta_i$ fluctuations are mainly responsible for the metal-insulator transition occurring in the Hubbard model at half-filling. Keeping the $\theta_i$ fluctuations, the effective field theory for the Mott transition is naturally given by a gauge theory. In this respect the Mott transition should be viewed beyond the LGW paradigm.

A standard treatment of the hopping term in Eq. (6) yields the effective Lagrangian

$$ L_{\text{eff}} = t \sum_{<ij>} (\alpha_{ij} \beta_{ij}^* + \beta_{ij} \alpha_{ij}^*) $$

$$ + \sum_{i\sigma} f_{i\sigma}^* (\partial_\tau - \mu - i\varphi_i) f_{i\sigma} $$

$$ - t \sum_{<ij>\sigma} (f_{i\sigma}^* \beta_{ij}^* f_{j\sigma} + f_{j\sigma}^* \beta_{ij} f_{i\sigma}) $$

$$ + \frac{1}{4U} \sum_i (\partial_\tau \theta_i - \varphi_i)^2 $$

$$ - t \sum_{<ij>} (e^{i\theta_j} \alpha_{ij} e^{-i\theta_j} + e^{i\theta_j} \alpha_{ij}^* e^{-i\theta_j}), $$

(7)

where $\alpha_{ij}$ and $\beta_{ij}$ are spinon and chargon hopping order parameters, respectively.

A saddle point analysis results in the self-consistent equations

$$ -i\varphi_i = -i(\partial_\tau \theta_i) + 2U \langle \sum_\sigma f_{i\sigma}^* f_{i\sigma} \rangle, $$

$$ \alpha_{ij} = \langle \sum_\sigma f_{i\sigma}^* f_{j\sigma} \rangle, \quad \beta_{ij} = \langle e^{i\theta_j} e^{-i\theta_i} \rangle, $$

(8)

$$ \langle \sum_\sigma f_{\sigma}^* f_{\sigma} \rangle = 1 - \delta, $$

where $\delta$ is hole concentration.

Considering low energy fluctuations around this saddle point, one can set $\alpha_{ij} = \alpha e^{i\alpha_{ij}}$, $\beta_{ij} = \beta e^{i\beta_{ij}}$ and $\varphi_i = \overline{\varphi}_i + a_i$, where $\alpha = |\langle \sum_\sigma f_{\sigma}^* f_{\sigma} \rangle|$ and $\beta = |\langle e^{i\theta_j} e^{-i\theta_i} \rangle|$ are amplitudes of the hopping order parameters, and $\alpha_{ij}$ and $a_i$ are spatial and time components of U(1) gauge fields. Inserting these into Eq. (7), we find an effective U(1) gauge theory for the Mott transition

$$ L_{\text{eff}} = L_0 + L_f + L_\theta, $$

$$ L_0 = 2tN \alpha \beta, $$

$$ L_f = \sum_{i\sigma} f_{i\sigma}^* (\partial_\tau - \mu - i\overline{\varphi}_i - ia_i) f_{i\sigma} $$

$$ - t \beta \sum_{<ij>\sigma} (f_{i\sigma}^* e^{-i\alpha_{ij}} f_{j\sigma} + h.c.), $$

$$ L_\theta = \frac{1}{4U} \sum_i (\partial_\tau \theta_i - \overline{\varphi}_i - a_i)^2 $$

$$ - 2t \alpha \sum_{<ij>} \cos(\theta_j - \theta_i - a_{ij}), $$

(9)

where $N$ is a total number of lattice sites. Eq. (9) is our starting point for the metal-insulator transition.

In this effective gauge theory two important facts should be taken into account since they discriminate the DCMT from the BCMT. One is an effective chemical potential $\mu_{\text{eff}} = \mu + \langle \overline{\varphi}_i \rangle$ in the spinon Lagrangian $L_f$. Particle-hole symmetry at half filling causes the effective chemical potential to vanish. On the other hand, away from half filling the particle-hole symmetry is broken, resulting in a nonzero chemical potential.

The other important feature is a Berry phase term arising from the phase-fluctuation term in the chargon Lagrangian $L_0$

$$ S_B = - \sum_i \int_0^\beta d\tau \frac{1}{2U} \overline{\varphi}_i \partial_\tau \theta_i $$

$$ = - \sum_i \int_0^\beta d\tau \left( \frac{1}{2U} \langle \partial_\tau \theta_i \rangle \partial_\tau \theta_i + i \langle \sum_\sigma f_{\sigma}^* f_{\sigma} \partial_\tau \theta_i \rangle \partial_\tau \theta_i \right) $$

At half filling the Berry phase does not play any roles because time reversal symmetry considered in this paper leads to $\langle \partial_\tau \theta_i \rangle = 0$, and the average occupation number of spinons is given by $\langle \sum_\sigma f_{\sigma}^* f_{\sigma} \rangle = 1$. Inserting this into the expression of Berry phase, one obtains

$$ S_B = - \sum_i \int_0^\beta d\tau i \partial_\tau \theta_i = -2\pi i \sum_i q_i, $$

where $q_i$ is an integer representing an instanton number, here a vortex
charge. Thus, the contribution of Berry phase to the partition function is nothing because of $e^{-\beta n} = 1$. Away from half filling the Berry phase action is obtained to be $S_B = -2\pi i\delta \sum_q q_i$ with modular $2\pi$. This results in a complex phase factor to the partition function, given by $Z = \sum_Q e^{2\pi iQ}Z_Q$, where $Q = \sum_i q_i$ is a total instanton number, and $Z_Q$, the partition function for a fixed $Q$. The observation of Berry phase gives the motivation for this paper. In this paper we investigate how the effect of Berry phase makes the DCMT differ from the BCMT.

III. BANDWIDTH-CONTROL MOTT TRANSITION

First, we discuss the BCMT. Zero effective chemical potential and no Berry phase effect result in the following effective field theory

$$L_f = \sum_{i\sigma} f_{i\sigma}^* (\partial_\tau - i a_{i\sigma}) f_{i\sigma} - t\beta \sum_{<ij>,\sigma} (f_{ij,\sigma} e^{-i a_{ij}} f_{ji,\sigma} + \text{c.c.}),$$

$$L_\theta = \frac{1}{4U} \sum_i (\partial_\tau, \theta_i - a_{i\sigma})^2 - 2t\alpha \sum_{<ij>} \cos(\theta_j - \theta_i - a_{ij}).$$

In the absence of $U(1)$ gauge fluctuations this effective action was intensively studied by FG. For the mean field treatment FG utilized large $N$ generalization of the chargon field, and derived the saddle point equations in Eq. (8) at half filling. They found that there exists a critical $U/t$ for chargon condensation. In the case of $U/t > (U/t)_c$ chargons are gapped, but spinons are massless. Existence of charge gap but no spin gap corresponds to a spin liquid Mott insulator. In the spin liquid there is no coherent quasiparticle peak at zero energy, and only incoherent hump was found near the energy $\pm U$. In the case of $U/t < (U/t)_c$ condensation of chargons occurs, causing a coherent quasiparticle peak at zero energy in the presence of incoherent hump near the energy $\pm U$. As a result a correlated paramagnetic metal appears. Furthermore, FG analyzed the saddle point equations near the Mott critical point $(U/t)_c$, and obtained mean field critical exponents for the charge gap and the quasiparticle weight. They also found that the effective mass of quasiparticles does not diverge near the Mott critical point owing to the spinon dispersion.

However, the mean field analysis of FG should be checked in the presence of $U(1)$ gauge fluctuations since instanton excitations of $U(1)$ gauge fields can cause confinement of spinons and chargons, completely spoiling the mean field picture. In two space and one time dimensions $(2 + 1)D$ it is well known that static charged matter fields are always confined owing to instanton condensation. For the mean field picture of the spin liquid and the Mott transition to be physically meaningful beyond the mean field level, the stability of Eq. (11) should be guaranteed against instanton excitations in the RG sense.

Recently, the present author examined deconfinement of fermions in the presence of a Fermi surface. It has been argued that the fermion Lagrangian $L_f$ in Eq. (11) has a nontrivial charged fixed point as the quantum electrodynamics in $(2 + 1)D (QED_3)$ without a Fermi surface. The present author investigated the stability of the charged critical point against instanton excitations following the strategy in Ref. [16]. In the presence of a Fermi surface the conductivity $\sigma_f$ of fermions is shown to play the similar role as the flavor number $N$ of Dirac fermions in the $QED_3$. Since the flavor number of Dirac fermions is proportional to screening channels for the gauge propagator, large flavors weaken gauge fluctuations in the $QED_3$. In the same way the conductivity of fermions near the Fermi surface determines strength of gauge fluctuations. Remarkably, the charged fixed point is found to be stable against instanton excitations when the fermion conductivity is sufficiently large. This implies that the $U(1)$ gauge field can be considered to be noncompact. Eq. (11) can be a stable theory against instanton excitations. In this respect the spin liquid state can survive beyond the mean field level. But, the spinons are not free particles any more owing to long range gauge interactions, resulting in an algebraic behavior of the spin-spin correlation function with an anomalous critical exponent. The Mott transition beyond the mean field description is more complex owing to the dissipative nature of gauge fluctuations. Integrating over the spinons, one can obtain the effective action for the chargon and gauge fields in the continuum limit

$$S_{eff} = \int d\tau d^2 \theta \left[ \frac{1}{4U} (\partial_\tau, \theta - a_\tau)^2 - 2t\alpha \cos(\theta - a) \right]$$

$$+ \frac{1}{\beta} \sum_{\omega_n} \int d\theta, \omega \left[ \frac{1}{2} a_\mu(q, \omega) D^{-1}_{\mu\nu}(q, \omega) a_{\nu}(-q, -i\omega), \right]$$

where $D^{-1}_{\mu\nu}(q, \omega)$ is the renormalized gauge propagator, given by

$$D_{\mu\nu}(q, \omega) = \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) D(q, \omega),$$

$$D^{-1}(q, \omega) = D_0^{-1}(q, \omega) + \Pi(q, \omega).$$

Here $D_0^{-1}(q, \omega) = (q^2 + \omega^2)/g^2$ is the bare propagator of the gauge field given by the Maxwell gauge action, resulting from integration of high energy fluctuations of spinons and chargons. $g$ is an internal gauge charge of the spinon and chargon. $\Pi(q, \omega)$ is the self-energy of the gauge field, given by a correlation function of spinon charge (number) currents. Since the current-current correlation function is calculated in the noninteracting fermion ensemble, its structure is well known.

$$\Pi(q, \omega) = \sigma(q) |\omega| + \chi q^2.$$
the dirty limit, where $k_0$ is of order $k_F$ (Fermi momentum), and $t$ the spinon mean free path determined by disorder scattering. The diamagnetic susceptibility $\chi$ is given by $\chi \sim m_f^{-1}$, where $m_f \sim (t_0)^{-1}$ is the band mass of spinons. The frequency part of the kernel $\Pi(q, i\omega_n)$ shows the dissipative propagation of the gauge field owing to particle-hole excitations of spinons near the Fermi surface.

Eq. (12) should be a starting point for the BCMT. In the study of FG,[1] U(1) gauge fluctuations are ignored, and thus the physical picture of the Mott transition should be modified. In the absence of U(1) gauge fluctuations the transition falls into the XY universality class. However, long range gauge interactions alter the XY universality nature into the inverted-XY (IXY) universality class. However, long range gauge interactions alter the XY universality nature into the inverted-XY (IXY) universality class if the Landau damping term in Eq. (14) is ignored, and only the Maxwell kinetic energy of the gauge field is taken into account.[9] This means that if one considers a critical exponent $\nu$ associated with the charge gap $\Delta_g \sim |U - U_c|^\nu$ with the critical value $U_c$, the critical exponent changes from $\nu_{\text{XY}}$ of the XY transition to $\nu_{\text{IXY}}$ of the IXY transition. Damped gauge interactions are expected to modify the IXY Mott transition.[9]

Performing the duality transformation for the phase field in Eq. (12), we obtain the dual vortex action

$$S_v = \int d\tau d^2 r \left[\left((\partial_\mu - ic_\mu)\Phi\right)^2 + m_v^2 |\Phi|^2 + \frac{u_v}{2} |\Phi|^4 + \frac{U}{4\alpha} (\partial \times c)^2 - ia_\mu (\partial \times c)_\mu\right] + \frac{1}{\beta} \sum_{\omega_n} dq r \left[\frac{1}{2} a_\mu(q_r, i\omega_n) D^{-1}_{\mu\nu}(q_r, i\omega_n) a_\nu(-q_r, -i\omega_n)\right].$$

(15)

Here $\Phi$ is a vortex field, and $c_\mu$ a vortex gauge field. $m_v$ is a vortex mass, given by $m_v^2 \sim (U/t)_c - U/t$, and $u_v$ a phenomenologically introduced parameter for local interactions between vortices. $(U/t)_c$ is the critical strength of local interactions, associated with the Mott transition in the mean field level.

In the dual vortex formulation the BCMT arises from controlling the vortex mass as a function of the parameter $U/t$. In the case of $m_v^2 < 0 (U/t > (U/t)_c)$ condensation of vortices occurs, resulting in a Mott insulator of chargons. In the case of $m_v^2 > 0 (U/t < (U/t)_c)$ vortices are gapped, implying condensation of chargons, and a paramagnetic metal results.

Performing the Gaussian integration for the gauge field $a_\mu$, we obtain the effective vortex action

$$S_v = \int D[\Phi, c_\mu] e^{-S_v},$$

$$S_v = \int d\tau d^2 r \left[\left((\partial_\mu - ic_\mu)\Phi\right)^2 + m_v^2 |\Phi|^2 + \frac{u_v}{2} |\Phi|^4 + \frac{U}{4\alpha} (\partial \times c)^2 + \frac{1}{\beta} \sum_{\omega_n} dq r \left[\frac{1}{2} a_\mu(q_r, i\omega_n) D^{-1}_{\mu\nu}(q_r, i\omega_n) a_\nu(-q_r, -i\omega_n)\right]\right].$$

where $\gamma$ is a redefined variable including the susceptibility. In the following we consider dirty cases characterized by $\sigma(q_r) = \sigma_0$.

Before we analyze Eq. (16) by using an RG method, we consider two physical limits; one is $\sigma_0 \to 0$ corresponding to an insulator of spinons, and the other, $\sigma_0 \to \infty$ identified with a perfect metal of spinons. In the spinon insulator the kernel $K(q, i\omega_n)$ becomes a constant value, making vortex gauge fluctuations $(c_\mu)$ gapped, thus ignored in the low energy limit. This is because long range gauge interactions $(a_\mu)$ make it massive the low energy mode (Goldstone mode) represented by the vortex gauge field, appearing at high energies. The usual $\Phi^4$ action for the vortex field is obtained. On the other hand, in the perfect spinon metal gauge fluctuations $a_\mu$ are completely screened by spinon excitations, causing the kernel to vanish, and the Maxwell gauge action for the vortex gauge field results. The resulting vortex action is reduced to the standard scalar QED3. Varying the spinon conductivity $\sigma_0$, these two limits would be connected.

We perform an RG analysis for Eq. (16). Anisotropy in the Maxwell gauge action for the vortex gauge field is assumed to be irrelevant, and only the isotropic Maxwell gauge action is considered by replacing $U, 1/4\alpha t$ with $1/(2c_v^2)$. Here $c_v$ is a vortex charge. In the limit of small anisotropy the anisotropy was shown to be irrelevant at one loop level.[9] To address the quantum critical behavior at the Mott transition, we introduce the scaling $r = \rho', r' = \epsilon' r'$ and consider the renormalized theory at the transition point $m_v^2 = 0$

$$S_v = \int d\tau' d^2 - 1 r' \left[Z_v(\partial'_\mu - ic_v c_\mu)\Phi] + \frac{u_v}{2} |\Phi|^4 + \frac{Z_c}{2} (\partial' \times c')^2\right].$$

(18)
where $Z_\Phi$, $Z_u$ and $Z_c$ are the renormalization factors defined by
\[
\Phi = e^{-\frac{D-1}{2}Z_\Phi^2} \phi_{r}, \quad c_\mu = e^{-\frac{D-1}{2}Z_c^2} c_{\mu r}, \\
e_v^2 = e^{-(4-D)/Z_c^2} e_{v r}, \quad u_v = e^{-(4-D)/Z_u Z_c^2} u_{v r}.
\]
In the renormalized action Eq. (18) the subscript $v$ implying "renormalized" is omitted for simple notation.

Evaluating the renormalization factors at one loop level, the RG equations are obtained to be
\[
\frac{d e_v^2}{d \ell} = (4 - D) e_v^2 - \left( \lambda N_v + \frac{\zeta}{\sigma_0} \right) e_v^4, \\
\frac{d u_v}{d \ell} = (4 - D) u_v + h(\sigma_0, e_v^2) e_v^2 u_v - \rho(\sigma_0 + 4) u_v^2 - g(\sigma_0, e_v^2) e_v^4.
\]
(20)

Here $\lambda, \zeta, \rho$ are positive numerical constants, and $h(\sigma_0, e_v^2), g(\sigma_0, e_v^2)$ are analytic and monotonically increasing functions of $\sigma_0$. $N_v$ is the flavor number of the vortex field, here given by $N_v = 1$.

The first RG equation for the vortex charge can be understood in the following way. Integrating out critical vortex fluctuations near the critical point $m_v^2 \sim 0$, we obtain the singular contribution for the effective gauge action
\[
S_c = \frac{1}{\beta} \sum_{q, q', \omega} \int d^2 q \frac{1}{2} c_\mu(q, i \omega) \Xi_{\mu \nu}(q, i \omega) c_\nu(-q, -i \omega), \\
\Xi_{\mu \nu}(q, i \omega) = \left( \delta_{\mu \nu} - \frac{q_\mu q_\nu}{q^2} \right) \Xi(q, i \omega), \\
\Xi(q, i \omega) = \frac{N_v}{8} \sqrt{q^2 + \omega_n^2} + K(q, i \omega) \\
\approx \frac{N_v}{8} \sqrt{q^2 + \omega_n^2} + \frac{q^2 + \omega_n^2}{\sigma_0 |\omega_n|}.
\]
The first term in the kernel $\Xi(q, i \omega_n)$ results from the screening effect of the vortex charge via vortex polarization, causing the $-\lambda N_v e_v^2$ term in the RG equation while the second originates from that via spinon excitations, yielding the $-(\zeta/\sigma_0) e_v^4$ term. The first $(4 - D) e_v^2$ term denotes the bare scaling dimension of the vortex charge.

For the second RG equation, unfortunately, we do not know the exact functional forms of $h(\sigma_0, e_v^2)$ and $g(\sigma_0, e_v^2)$ owing to the complexity of the gauge kernel. Owing to the spinon contribution $K(q, i \omega)$ [Eq. (17)] the kernel of the gauge propagator ($c_\mu$)
\[
D_c(q, i \omega_n) = \frac{1}{q^2 + \omega_n^2 + e_v^4 K(q, i \omega_n)} \\
\approx \frac{\sigma_0 |\omega_n|}{(q^2 + \omega_n^2)(e_v^2 + \sigma_0 |\omega_n|)}
\]
should be utilized instead of the Maxwell propagator in calculating one loop diagrams.\[21\]\[22\]\[23\]\[24\] Note the dependence of the vortex charge $e_v^2$ in the effective gauge propagator. This gives the dependence of the vortex charge to the analytic functions $h(\sigma_0, e_v^2)$ and $g(\sigma_0, e_v^2)$. Although the exact functional forms are not known, the limiting values of these functions are clearly revealed. In the limit of $\sigma_0 \to 0$ the gauge kernel vanishes, thus causing $h(\sigma_0 \to 0, e_v^2) \to 0$ and $g(\sigma_0 \to 0, e_v^2) \to 0$. In the small $\sigma_0$ limit the gauge kernel is given by
\[
D_c(q, i \omega_n) \approx \frac{\sigma_0 |\omega_n|}{e_v^2 q^2 + \omega_n^2},
\]
thus resulting in $h(\sigma_0, e_v^2) = c_h \sigma_0 / e_v^2$ and $g(\sigma_0, e_v^2) = c_g \sigma_0^2 / e_v^4$, where $c_h$ and $c_g$ are positive numerical constants. On the other hand, in the limit of $\sigma_0 \to \infty$ the gauge kernel is reduced to the Maxwell one $D_c(q, i \omega_n) = 1/(q^2 + \omega_n^2)$. Thus, $h(\sigma_0 \to \infty, e_v^2) \to c_1$ and $g(\sigma_0 \to \infty, e_v^2) \to c_2$ are obtained, where $c_1$ and $c_2$ are positive numerical constants.\[21\]\[22\] As a result, Eq. (20) is reduced to the RG equation of the $\Phi^4$ theory\[27\] in the limit of $\sigma_0 \to 0$
\[
\frac{d u_v}{d \ell} = (4 - D) u_v - \rho(N_v + 4) u_v^2,
\]
and that of the scalar QED3\[21\]\[22\] in the limit of $\sigma_0 \to \infty$,
\[
\frac{d e_v^2}{d \ell} = (4 - D) e_v^2 - \lambda N_v e_v^4, \\
\frac{d u_v}{d \ell} = (4 - D) u_v + c_1 e_v^2 u_v - \rho(N_v + 4) u_v^2 - c_2 e_v^4.
\]
(20)

In the small $\sigma_0$ limit the RG equations (20) result in
\[
\frac{d e_v^2}{d \ell} = (4 - D) e_v^2 - \left( \lambda N_v + \frac{\zeta}{\sigma_0} \right) e_v^4, \\
\frac{d u_v}{d \ell} = (4 - D) u_v + c_h \sigma_0 u_v - \rho(N_v + 4) u_v^2 - c_g \sigma_0^2.
\]

In the scalar QED3 there is a delicate issue about the existence of the charged fixed point ($e_0^2 \neq 0$).\[21\]\[22\]\[23\]\[24\] In this paper we do not touch this issue. Instead we assume the existence of the charged critical point in the scalar QED3 by controlling the $\lambda$ value. Then, the charged critical point ($e^c_z = \sigma_0, u^c_v[\sigma^c_z(\sigma_0)]$) in Eq. (20) is expected to vary as a function of the spinon conductivity in the range of
\[
e_v^2(\sigma_0 \to 0) = 0 < e_v^2(\sigma_0) < e_v^2(\sigma_0 \to \infty) = 1/(\lambda N_v), \\
u_v^c[\sigma^c_z(\sigma_0 \to \infty)] < u_v^c[\sigma^c_z(\sigma_0)] < u_v^c[\sigma^c_z(\sigma_0 \to 0)],
\]
where the fixed point ($e^c_z(\sigma_0 \to 0), u^c_v[\sigma^c_z(\sigma_0)]$) corresponds to the IXY one in the original boson model [Eq. (12)], and the fixed point ($e^c_z(\sigma_0 \to \infty), u^c_v[\sigma^c_z(\sigma_0 \to \infty)]$) coincides with the XY one in Eq. (12). The spinon contribution ($\sigma_0$) connects the XY fixed point to the IXY one smoothly in the charge action Eq. (12).\[28\] This implies that the critical exponents near the Mott transition change continuously, depending on the spinon conductivity. This would be measured in some experiments. Because the spinon conductivity depends on disorder, we
would have some different critical points by controlling density of disorder, resulting in various critical exponents between the exponents of the XY and IXY transitions. However, one interesting possibility should be taken into account that the glassy behavior of the chargon field can originate from random potentials. This important subject is under current investigation.

IV. DOPING-CONTROL MOTT TRANSITION

Next, we investigate the DCMT, described by the effective field theory

\[ L_f = \sum_{i\sigma} f_i^\dagger (\partial_\tau - \mu_{eff} - i a_{i\tau}) f_i^\sigma \]

\[ -t\beta \sum_{<ij>\sigma} (f_i^\dagger e^{-ia_{ij}} f_j^\sigma + c.c.), \]

\[ L_\theta = \frac{1}{4U} \sum_i (\partial_\tau \theta_i - a_{i\tau})^2 - 2t\alpha \sum_{<ij>} \cos(\theta_j - \theta_i - a_{ij}) \]

\[ +i\delta(\theta_i a_{i\tau} - a_{i\tau}). \]  \( (21) \)

Note the presence of the effective chemical potential and Berry phase. This Lagrangian is analyzed by employing a duality transformation. In the dual formulation the effect of Berry phase is represented as effective magnetic fields for dual vortex variables.

Following the previous section, the duality transformation of the chargon Lagrangian results in

\[ L_v = \left| (\partial_\mu - ic_\mu) \Phi \right|^2 + m_c^2 |\Phi|^2 + \frac{u_w}{2} |\Phi|^4 - \hbar (\partial \times c)_r \]

\[ +U(\partial \times c)_r^2 + \frac{1}{4\alpha} (\partial \times c)_r^2, \]  \( (22) \)

where the U(1) gauge field \( a_\mu \) was ignored in the mean field level. The Berry phase effect is reflected as an effective magnetic field \( \vec{B} = -2U\delta \) for the vortex field in the term \( -\hbar(\partial \times c)_r \). Remember the expression of the vortex mass \( m_c^2 \sim (U/t) e - U/t \). A cautious reader may suspect that the vortex mass should depend on hole concentration. From the discussion below Eq. (9) it is important to note that the effect of hole doping appears only in the chemical potential and Berry phase terms. Furthermore, at half filling Eq. (22) should be dual to the chargon Lagrangian \( L_\theta \) in Eq. (11). Thus, the vortex mass should depend on only the parameter \( U/t \).

In the dual vortex formulation the BCMT is driven by controlling the vortex mass, as shown in the previous section. On the other hand, the DCMT is nothing to do with the vortex mass. Instead, controlling the effective magnetic field causes the Mott transition. This leads us to consider that the nature of the DCMT differs from the BCMT.

The presence of the effective magnetic field reminds us of a well known Hofstadter problem for vortex fields. In the context of a superfluid-insulator transition this was extensively studied in Refs. Here we briefly sketch the procedure and key results. We first investigate the nature of a doped Mott insulating state in a mean field fashion, i.e., the absence of U(1) gauge fluctuations \( a_\mu \), and discuss the DCMT beyond the mean field level.

Following Ref. 3, we consider commensurate hole concentration \( \delta = p/q \), where \( p \) and \( q \) are relatively prime integers. Under this effective magnetic field \( \delta \), the vortex Lagrangian Eq. (22) has \( q \)-fold degenerate minima in the magnetic Brillouin zone. Low energy fluctuations near the \( q \)-fold degenerate vacua are assigned to be \( \Psi_l \) with \( l = 0, ..., q - 1 \). A key question is how to construct a LGW free energy functional in terms of the \( \Psi_l \) fields. Constraints for an effective potential of \( \Psi_l \) are symmetry properties associated with lattice translations and rotations in the presence of the effective magnetic fields. Based on the symmetry properties one can construct a LGW free energy functional of \( \Psi_l \), and perform a standard mean field analysis. In this free energy a superfluid of original bosons (chargons) is given by \( \langle \Psi_l \rangle = 0 \) for all \( l = 0, ..., q - 1 \) while a Mott insulator is characterized by \( \langle \Psi_l \rangle \neq 0 \) for at least one \( l \). Although the free energy functional has all symmetries, the ground state can be symmetry-broken. In other words, the Mott insulator can have broken translational symmetries.

To see this, one can construct a density wave order parameter by considering bilinear and gauge-invariant combinations of the low energy vortices \( \Psi_l \). Condensation of \( \Psi_l \) leads to a nonzero value of the density wave order parameter, causing a vortex density wave. A density wave of vortices can be interpreted as a crystalline phase of doped holes in the original language. In appendix we review the simple \( q = 2 \) case. Combining this chargon physics with the spinon physics, we can conclude that a doped Mott insulator consists of a density wave of chargons and a spin liquid of spinons with a Fermi surface. It should be noted that this doped Mott insulator is different from the Mott insulator at half filling because there is no charge order in the undoped Mott insulator.

Remember that the crystalline phase of doped holes is nothing to do with the spin liquid in the mean field level. Integrating out the gapped chargon degrees of freedom, we obtain the same spinon-gauge action for the doped spin liquid with that for the undoped one beyond the mean field level. It has been argued that the spinon-gauge action is a critical field theory at the nontrivial charged fixed point, as discussed in the previous section. Thus, the spin-spin correlation function shows a power law behavior with respect to frequency and temperature. On the other hand, no infrared response for charge fluctuations is expected owing to the Mott gap. Instead, the charge order would be reflected in the electron density of states as a spatially modulated pattern because of the translational symmetry breaking. The electron density of states is proportional to the imaginary part of the electron green function, given by convolution of the spinon and chargon propagators in the slave-rotor formulation. Thus, the spatial inhomogeneity of the chargon distribution re-
rults in the spatially modulated pattern in the density of states. Because there is excitation gap in the charogn spectrum, only incoherent hump would be shown in the electron spectral function. This is another different point from the usual density wave.

One cautious person may suspect the coexistence of the spin liquid and charge density wave (CDW) because such a commensurate CDW can destroy the spinon Fermi surface through a space-dependent effective chemical potential, causing the spin liquid to be unstable. However, we argue that the spinon Fermi surface can be preserved even in the commensurate CDW when the Fermi surface nesting is not perfect due to interaction or frustration effects. Considering the low energy vortex excitations $\Psi_l$ near the $q$-fold degenerate vacua, one can find the effective dual action

$$S_f = \int d\tau \left[ \sum_\sigma f^*_\sigma \left( \partial_\tau - i \mathbf{v}_i - i a_{i\tau} \right) f_\sigma \right] - t_\beta \sum_{<ij>|\sigma} \left( f^*_e e^{-i a_{ij}} f_{\mathbf{j}+} + h.c. \right) - \frac{1}{g^2} \sum_\mu \cos(\partial_\tau + a_\mu),$$

$$S_v = -t_v \sum_{<nm>|l} \Psi(l^*)_n e^{i c_{nm} \Psi(l)_m} + V(\{\Psi(l)_n\})$$

$$- \frac{1}{v^2} \sum_\mu \cos(\partial_\tau + c_\mu) + i \sum_{<\mu\nu>} a_{\mu\nu} (\partial_\tau + c_\mu). \quad (23)$$

Here $l = 1, \ldots, q$ corresponds to a color index of low energy vortex fields, and $V(\{\Psi(l)_n\})$ is an effective vortex potential determined by symmetry properties, where the coefficients are effectively doping dependent (see appendix). The last gauge action in the spinon sector originates from high energy contributions of matter fields, where $g$ is an internal gauge charge of spinons.

The question is what happens in the Fermi surface when vortex condensation occurs, resulting in translational symmetry breaking. Ignoring spinon-gauge fluctuations $a_{\mu\nu}$ as the mean field approximation, the spinon-gauge action is completely decoupled from the vortex-gauge action, as discussed before. This indicates that the Fermi surface is not affected by the CDW formation in the vortex sector. Now, we allow spinon-gauge excitations. Integrating out $a_{i\tau}$ in the limit of $g \to \infty$, one obtains the constraint

$$(\nabla \times c)_i = \sum_\sigma f^\dagger_{i\sigma} f_{i\sigma}.$$
the following effective field theory

\[ S_{\text{eff}} = \int d\tau d^2r \sum_{\mu=0}^{q-1} (\partial_\mu - ic_\mu)\Psi_i^\dagger \Psi_i + V(\Psi_i) \\
-\bar{h} - \bar{h}_q(\partial \times c)_x + U(\partial \times c)_r^2 + \frac{1}{4\alpha}(\partial \times c)^2_x \\
+ \int d\tau d\tau_1 d^2r d^2r_1 \frac{1}{2} c_\mu(r, \tau) K_{\mu\nu}(r - r_1, \tau - \tau_1) c_\nu(r_1, \tau_1). \]

(24)

\( K_{\mu\nu} \) results from the anomalous contribution of spinon-gauge fluctuations to vortex-gauge excitations, given by Eq. (17) with a different \( \sigma_0 \) owing to the chemical potential \( \mu_{\text{eff}} \). \( \bar{h} = -2U \delta \) is an applied effective magnetic field, and \( \bar{h}_q = -2U \delta_q \) a nearby one with commensurate hole concentration \( \delta_q = p/q \). One can estimate a critical effective magnetic field \( \bar{h}_c \), with a given \( U/t > (U/t)_c \) by calculating the condensation energy. The critical hole concentration \( \delta_c \) corresponding to the critical magnetic field \( \bar{h}_c \) would be different from \( \delta_q \) generally. In this case one may determine a moderate value of \( q \) near the critical doping \( \delta_c \). Then, there remain residual effective magnetic fields \( \bar{h} - \bar{h}_q \), corresponding to the incommensurability \( \delta - \delta_q \).

We propose that Eq. (24) is a starting point for the DCMT. If the vortex “superconductor” falls into the type-I class, the residual magnetic field would be expelled owing to the dual ”Meissner” effect. A critical field theory for this Mott transition is expected to be without the residual magnetic field

\[ S_{\text{eff}} = \int d\tau d^2r \sum_{\mu=0}^{q-1} (\partial_\mu - ic_\mu)\Psi_i^\dagger \Psi_i + V(\Psi_i) \\
+U(\partial \times c)_r^2 + \frac{1}{4\alpha}(\partial \times c)^2_x \\
+ \int d\tau d\tau_1 d^2r d^2r_1 \frac{1}{2} c_\mu(r, \tau) K_{\mu\nu}(r - r_1, \tau - \tau_1) c_\nu(r_1, \tau_1). \]

(25)

Because the effective vortex action depends on \( q \) and \( V(\Psi_i) \), it is difficult to predict critical vortex dynamics for general \( q \) values. The \( q = 1 \) case corresponds to the undoped spin liquid, already discussed in the previous section. In the \( q = 2 \) case the effective vortex potential is obtained to be

\[ V(\psi_i) = m^2(\psi_0^2 + |\psi_1|^2) + u_4(|\psi_0|^2 + |\psi_1|^2)^2 \\
+ u_4|\psi_1|^2 |\psi_2|^2 - u_8 |\psi_2|^4 + H.c., \]

well discussed in appendix. At the critical point \( m^2 = 0 \) the last eighth-order term is certainly irrelevant owing to its high order. Furthermore, the cubic anisotropy term \((v_4)\) is well known to be irrelevant in the case of \( q < q_c = 4 \).\[27\] As a result, the Heisenberg fixed point \((v_4 = 0 \text{ and } v_4^* \neq 0)\) appears in the absence of vortex gauge fluctuations.\[27\] Introducing the vortex gauge fields at the Heisenberg fixed point, we have qualitatively the same fixed point with Eq. (16) except the \( q = 2 \) vortices. Since the charged critical point depends on the spinon conductivity, the critical exponents vary as a function of the spinon conductivity. At higher \( q \) values we do not understand the nature of the Mott transition owing to the complexity of the vortex potential. Generally speaking, a continuous Mott transition from the U(1) spin liquid with a commensurate density wave order to the Fermi liquid is possible.

On the other hand, if the vortex superconductor belongs to the type-II class, the residual magnetic field can penetrate the superconductor, forming a dual Abrikosov “vortex” lattice. This corresponds to an incommensurate Mott insulator, where hole density is \( \delta \neq \delta_q \). In this case the nature of the Mott transition from the U(1) spin liquid with an incommensurate density wave order to the Fermi liquid is not clear owing to the Berry phase effect. Furthermore, the Landau damping term should be taken into account in the critical field theory as the case of the BCMT because it changes the nature of the Mott transition. A continuous transition to the Fermi liquid may be possible in this case. A detailed analysis of this DCMT is beyond the scope of this paper.

We propose a phase diagram Fig. 1 in the slave-rotor representation of the Hubbard model on two dimensional square lattice. Here U1SL, FL, and CDW represent U(1) spin liquid, Fermi liquid, and charge density wave, respectively.

![FIG. 1: A schematic phase diagram in the slave-rotor representation of the Hubbard model](image-url)

The route 1 is the BCMT from U1SL to FL at half filling while the route 2, that from U1SL + CDW to FL at commensurate hole concentration. In these cases we showed critical field theories, and discussed nature of the continuous phase transitions.

The route 3 is the DCMT at a finite \( U/t \). In this case the critical field theory depends on the nature of the vortex superconductor. Nature of the quantum phase transition from the U1SL with an incommensurate CDW to the FL is not clear owing to incommensurability.
V. DISCUSSION AND SUMMARY

So far, we considered a zero flux state, and thus obtained the U(1) spin liquid with a Fermi surface. The uniform spin liquid phase turns out to be stable (with respect to the flux phase) in the triangular lattice at the mean-field level near the undoped Mott transition point. However, it is important to consider a π flux phase since this phase is usually obtained as a stable mean field state in the square lattice without frustration. In the π flux phase low energy spinon excitations are given by Dirac fermions near four nodal points. As a result the effective spinon Lagrangian is obtained to be $QED_3$. For the BCMT at half filling there is no dissipation in gauge fluctuations. The role of massless Dirac fermions is to weaken gauge interactions, resulting from screening of gauge charges owing to particle-hole polarizability. Thus, increasing the flavor number $N$ of Dirac fermions in the $1/N$ approximation, the IXY transition is expected to turn into the XY one. On the other hand, for the DCMT there remains dissipation in gauge fluctuations owing to a nonzero effective chemical potential. Thus, damped gauge fluctuations would still play some special roles in the DCMT.

In this paper we discussed how the doping-control Mott transition differs from the bandwidth-control one based on the slave-rotor representation of the Hubbard model. We found that the doped Mott insulator consists of a crystalline phase of doped holes and a U(1) spin liquid with a Fermi surface while the Mott insulator at half filling does not destroy the spinon Fermi surface when there is strong frustration, causing the Fermi nesting to disappear. As a result, the spin liquid phase can remain stable to coexist with the density wave. Furthermore, we pointed out that damped U(1) gauge fluctuations resulting from spinon excitations should be taken into account for both Mott transitions because the nature of the Mott transitions is modified by the dissipative gauge excitations. Performing a renormalization group analysis, we showed that the Mott critical point depends on the spinon conductivity characterizing the strength of dissipation. This interesting result leads us to predict that varying the density of disorder would cause different critical exponents because disorder determines the conductivity of spinons.

VI. ACKNOWLEDGEMENT

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APPENDIX: MEAN FIELD ANALYSIS

In this appendix we review a mean field analysis for the $q = 2$ case following Refs. [34, 35]. Ignoring vortex gauge fluctuations in Eq. (22), we can write down the vortex action with the effective magnetic field $\delta = 1/2$ in a lattice version

$$S_M = \int dt \left[ \sum_n |\partial_n \Phi_n|^2 - \sum_{nm} \Phi_n^\dagger \tau^v_{nm} \Phi_m + V(\Phi) \right],$$

(A.1)

where $n, m$ label sites in the dual lattice, and the sign of the hopping integral $\tau^v_{nm}$ around a plaquette is $-1$ owing to the π flux background.

The vortex hopping term can be easily diagonalized in the eigenvalues $\chi^0_n = (1 + \sqrt{2}) - e^{i\pi n}$ and $\chi^1_n = e^{i\pi n}[(1 + \sqrt{2}) + e^{i\pi n}]$, resulting in two low energy modes near the momentum $k = (0, 0)$ and $k = (\pi, 0)$. Then, the low energy dynamics of this system can be described by the low energy vortex fields $\Psi_0$ and $\Psi_1$ in $\Phi_n = \Psi_0 \chi^0_n + \Psi_1 \chi^1_n$.

In order to construct the effective vortex potential one can introduce the following two complex fields

$$\psi_0 = \Psi_0 + i\Psi_1, \quad \psi_1 = \Psi_0 - i\Psi_1. \quad \text{(A.2)}$$

Then, the symmetry transformations are given by

$$T_x : \psi_0 \rightarrow \psi_1, \quad \psi_1 \rightarrow \psi_0,$$

$$T_y : \psi_0 \rightarrow i\psi_1, \quad \psi_1 \rightarrow -i\psi_0,$$

$$R_{\pi/2} : \psi_0 \rightarrow e^{i\pi/4}\psi_0, \quad \psi_1 \rightarrow e^{-i\pi/4}\psi_1, \quad \text{(A.3)}$$

where $T_x$ and $R_{\pi/2}$ are associated with lattice translations and rotations. The LGW effective potential obtained by these symmetry operations is obtained to be

$$V(\psi_1) = m^2(|\psi_0|^2 + |\psi_1|^2) + u_4(|\psi_0|^2 + |\psi_1|^2)^2$$

$$+ v_4|\psi_0|^2|\psi_1|^2 - v_8(|\psi_0^*\psi_1|^4 + H.c.), \quad \text{(A.4)}$$

where $m^2$ is an effective vortex mass, $u_4$ a local interaction, $v_4$ the cubic anisotropy, and $v_8$ breaking the U(1) phase transformation $\psi_{0(1)} \rightarrow e^{i\phi_{0(1)}}\psi_{0(1)}$.

One cautious reader may ask how the coefficients in the LGW free energy functional can be determined. Although the symmetry constraints restrict the functional form of the effective potential, they cannot determine the remaining parameters in the free energy. Our question is whether these parameters are doping dependent or not. Remember that there is no doping dependence in the original vortex mass $m^2_v$ in Eq. (15). It depends on only the parameter $U/t$. However, the effective parameters for
the low energy vortex fields should be considered to be doping dependent. Consider a vortex vacuum resulting from large effective magnetic fields \( \delta \) in spite of \( m_2^2 < 0 \) \((U/t > (U/t)_c))\), corresponding to chargon condensation. Decreasing hole concentration, the flavor number \( q \) of low energy vortices would increase. Decreasing hole concentration further, some components of the low energy vortices are expected to be condensed. In this respect the coefficients in the LGW free energy of \( \Psi \) would increase. Decreasing hole concentration effectively.

Based on the effective vortex potential Eq. (A4), one can perform a mean field analysis. Condensation of vortices occurs in the case of \( m_2^2 < 0 \) and \( \nu_4 > 0 \). The signs of \( \nu_4 \) and \( \nu_8 \) then determine the ground state. For \( \nu_4 < 0 \), both vortices have a nonzero vacuum expectation value \(|\langle \psi_0 \rangle| \neq |\langle \psi_1 \rangle| \neq 0\), and their relative phase is determined by the sign of \( \nu_8 \). In the case of \( \nu_8 > 0 \) the resulting vortex state corresponds to the columnar dimer order, breaking the rotational and translational symmetries. In the case of \( \nu_8 < 0 \) the resulting phase exhibits the plaquette pattern, breaking the rotational symmetries. On the other hand, if \( \nu_4 > 0 \), the ground states are given by either \(|\langle \psi_0 \rangle| \neq 0, |\langle \psi_1 \rangle| = 0 \) or \(|\langle \psi_0 \rangle| = 0, |\langle \psi_1 \rangle| \neq 0\), and the sign of \( \nu_8 \) is irrelevant. In this case an ordinary charge density wave order at wave vector \((\pi, \pi)\) is obtained, breaking the translational symmetries.

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\[
S_g = \frac{1}{\beta} \sum_{\omega_n} \int d^2q_r \frac{1}{2} a^T(q_r, i\omega_n) \Pi(q_r, i\omega_n) a^T(-q_r, -i\omega_n),
\]

where \( a^T(q_r, i\omega_n) \) is the transverse vector of the gauge field. The gauge kernel is given by

\[
\Pi(q_r, i\omega_n) = N_0 \sqrt{q_r^2 + \omega_n^2} + \sigma_0|\omega_n|,
\]

where \( N_0 \) is the flavor number of the chargon field. The first term results from critical chargon fluctuations while
the second one originates from gapless spinon excitations near the Fermi surface. This gauge action can be easily checked to be scale-invariant at the tree level, giving the IXY fixed point in the $\sigma_0 \to 0$ limit and the XY one in the $\sigma_0 \to \infty$ limit. Thus, the spinon contribution characterized by the spinon conductivity $\sigma_0$ connects these two fixed points smoothly.

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