Detecting Polysemy in Hard and Soft Cluster Analyses of German Preposition Vector Spaces

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Abstract

This paper presents a methodology to identify polysemous German prepositions by exploring their vector spatial properties. We apply two cluster evaluation metrics (the Silhouette Value (Kaufman and Rousseeuw, 1990) and a fuzzy version of the V-Measure (Rosenberg and Hirschberg, 2007)) as well as various correlations, to exploit hard vs. soft cluster analyses based on Self-Organising Maps. Our main hypothesis is that polysemous prepositions are outliers, and thus represent either (i) singletons or (ii) marginals of the clusters within a cluster analysis. Our analyses demonstrate that (a) in a subset of the clusterings, singletons have a tendency to contain polysemous prepositions; and (b) misclassification and cluster membership rate exhibit a moderate correlation with ambiguity rate.

1 Introduction

Vector space models have become a steadily increasing, integral part of data-intensive lexical semantics over the past 20 years (cf. Turney and Pantel (2010) and Erk (2012) for two recent surveys). They have been exploited in psycholinguistic (Lund and Burgess, 1996) and computational linguistic research (Schütze, 1998), to explore distributional properties of target objects and the notion of “similarity” within a geometric setting.

While individual vector space approaches have been concerned with sense discrimination, it is still largely unknown how to identify polysemous objects within a vector space model, and which geometric properties characterise the polysemous objects. For example, Schütze (1998) performed sense discrimination of ambiguous word tokens, based on their second-order co-occurrence distributions; Erk (2009) presented two variants of defining regions of word meaning in vector spaces; Erk and Padó (2010) defined a model where polysemous words activated several word vectors; Boleda et al. (2012b) compared two models of representing regular polysemy, one with multiple class assignments for multiple senses, and one incorporating classes with polysemy properties. Our work is different from all these approaches, since we aim to investigate prototypical spatial properties of polysemous objects.

More specifically, this paper is part of a larger framework that systematically explores the vector spatial properties of German prepositions, a notoriously polysemous closed word class. Relying on Self-Organising Maps (SOMs, cf. Kohonen (2001)) and preposition-dependent nouns as vector-space features, we present a methodology to identify the degree of polysemy of the prepositions. For this task, the methodology applies two cluster evaluation metrics, the Silhouette Value (Kaufman and Rousseeuw, 1990) and the V-Measure (Rosenberg and Hirschberg, 2007), to hard vs. soft cluster analyses based on the Self-Organising Maps. Since we start out with a hard clustering, a sub-task is concerned with transferring the SOM hard clusters to soft clusters. Similarly, the original V-Measure applies to hard clusters only, so a second sub-task is concerned with defining a Fuzzy V-Measure that applies to soft clusters. Our main hypothesis is that polysemous prepositions are outliers, and thus represent either (i) singletons or (ii) marginals of the clusters within a cluster analysis.

The paper is organised as follows. After introducing our preposition data in Section 2, Section 3 describes the preposition vector-space features, and the hard and soft clusterings. Section 4 is devoted to the evaluations, and Section 5 relies on the cluster analyses and the evaluations, to detect and discriminate polysemous prepositions.
2 Preposition Data

Although prepositions contribute a considerable portion to the meaning of texts, comparably little effort in computational semantics has gone beyond a specific choice of prepositions (such as spatial prepositions), towards a systematic classification of preposition senses. In recent years, computational research on prepositions has been enforced, mainly driven by the ACL Special Interest Group on Semantics (ACL-SIGSEM). The SIG has organised a series of workshops on prepositions, and a special issue in the Computational Linguistics journal (Baldwin et al., 2009).

Related work across languages includes The Preposition Project for English prepositions (Litkowski and Hargraves, 2005), PrepNet for French prepositions (Saint-Dizier, 2006), and a German project on the role of preposition senses in determiner omission in prepositional phrases (Kiss et al., 2010). The latter is most closely related to the present work, as it is also aimed at German. Their focus however is on manual classifications and corpus annotation, in contrast to our automatic classification approach.

As in many other languages, German prepositions are notoriously ambiguous, e.g. note the quite distinct senses of the German preposition nach in nach drei Stunden/Berlin/Meinung ’after three hours/to Berlin/according to’, referring to a temporal, directional, and accordance meaning. Our gold standard in terms of preposition senses is the German grammar book by Helbig and Buscha (1998). Starting with their class hierarchy, we selected the classes of prepositions that contained more than one preposition. We deleted those prepositions from the classes that appeared less often than 10,000 times in our web corpus containing 880 million words (cf. Section 3.1). This selection process resulted in 12 semantic classes covering between 2 and 27 prepositions each (cf. Table 1). The included prepositions exhibit ambiguity rates of 1 (monosemous) up to 6 (cf. Table 2). Out of the 47 prepositions, 24 are polysemous (51%).

3 Cluster Analyses

The pipeline in our framework is as follows.

1. The prepositions are associated with a distributional feature set.
2. The vector space of prepositions is hard-clustered using Self-Organising Maps.
3. The hard clustering is transferred to a soft clustering.
4. The cluster analyses are evaluated.

The following subsections describe these steps in more detail. While the larger framework plans to perform this pipeline for various cluster algorithms and many feature sets, the current setup of experiments focuses rather on the methodology towards polysemy detection, and is thus restricted to one algorithm (SOMs) and one feature set (nouns).

3.1 Preposition Corpus Features

The distributional features for the German prepositions were induced from the sdeWaC corpus (Faaß and Eckart, 2013), a cleaned version of the German web corpus deWaC created by the WaCky group. The corpus cleaning had focused mainly on removing duplicates from the deWaC, and on disregarding sentences that were syntactically ill-formed (relying on a parsability index provided by a standard dependency parser (Schiehlen, 2003)). The sdeWaC contains approx. 880 million words.

In this paper, we focus on one specific feature set that is expected to provide salient properties towards preposition meaning, i.e., the nouns that are subcategorised by the prepositions. This dependency information was extracted from a parsed version of the sdeWaC using Bohnet’s MATE dependency parser (Bohnet, 2010). So each preposition was associated with a feature vector over its
subcategorised nouns. The overall set of noun features was restricted to the 10,000 nouns from the corpus which co-occurred with the largest number of prepositions.

3.2 Hard Clustering

For hard-clustering the German prepositions, we relied on the Self-Organising Maps (SOMs) artificial neural networks provided by the kohonen library of the R Project for Statistical Computing\(^1\). We expected SOMs to be especially useful for this task, as they create typology-preserving maps, and should thus provide a suitable model to look into the spatial properties of polysemous vectors. Furthermore, SOMs have successfully been applied to semantic classification before (Ontrup and Ritter, 2001; Kanzaki et al., 2002; Guida, 2007).

We created SOM maps with \(k\) clusters, for \(2 \leq k \leq 47\), where 47 represents the total number of prepositions. For each \(k\), we initiated two-dimensional spacings for all possible hexagonal grids. For example, we trained four SOM maps with 30 clusters, using a \(30 \times 1\) grid, a \(15 \times 2\) grid, a \(10 \times 3\) grid, and a \(6 \times 5\) grid. The distance measure used in the maps was Euclidean Distance, which is the only option for SOMs in R.

3.3 Soft Clustering

The soft clustering of the German prepositions was based on the various hard cluster analyses. We performed the hard \(\rightarrow\) soft clustering transfer in two alternative ways, providing two different types of soft cluster analyses.

(1) Centroid-based softening: For each cluster \(c\) within a hard cluster analysis \(C\), we calculated the mean distance \(\text{prep}2\text{cluster}(c)\) over all prepositions \(p\) to the cluster centroid \(z_c\), ignoring any hard assignments in the hard clustering, cf. Equation 1. The individual distances between a preposition \(p\) and a cluster centroid \(z_c\) are denoted as \(d(p, z_c)\).

\[
\text{prep}2\text{cluster}(c) = \frac{\sum_p d(p, z_c)}{|p|}
\]  

For the corresponding soft cluster analysis \(S_t(C)\) of a hard cluster analysis \(C\), a preposition \(p\) was assigned to a cluster \(c\) if the distance \(d(p, z_c)\) was below a threshold \(t \times \text{prep}2\text{cluster}(c)\), with \(t = 0.05, 0.1, 0.15, \ldots, 0.95\). For example, if a distance of a preposition \(p\) to a cluster \(c\) was 5, and the mean distance \(\text{prep}2\text{cluster}(c)\) was 10, then \(p\) would not be assigned to \(c\) for \(t = 0.05, 0.1, \ldots, 0.5\) but for \(t = 0.6, \ldots, 0.95\). In this way, we created 19 different soft cluster analyses \(S_t(C)\) for each hard clustering \(C\), one for each \(t\).

With low values of \(t\), few prepositions (i.e., only those that were very close to the respective cluster centroids) were assigned to the clusters, and the resulting cluster analyses were likely to contain not all of our prepositions, and a low ambiguity rate; with high values of \(t\), more prepositions were assigned to each of the clusters, and the resulting cluster analyses were likely to contain many of the 47 prepositions, and a high ambiguity rate.

(2) Preposition-based softening: For each preposition \(p\) within a hard cluster analysis \(C\), we calculated the mean distance \(\text{cluster}2\text{prep}(p)\) over all cluster centroids \(z_c\) to the preposition \(p\), ignoring any hard assignments in the hard clustering, cf. Equation 2. Again, the individual distances between a preposition \(p\) and a cluster centroid \(z_c\) are denoted as \(d(p, z_c)\).

\[
\text{cluster}2\text{prep}(p) = \frac{\sum_c d(p, z_c)}{|c|}
\]  

Similarly to the centroid-based softening, for the corresponding soft cluster analysis \(S_t(C)\) of a hard cluster analysis \(C\), a preposition \(p\) was assigned to a cluster \(c\) if the distance \(d(p, z_c)\) was below a threshold \(t \times \text{cluster}2\text{prep}(p)\), with \(t = 0.05, 0.1, 0.15, \ldots, 0.95\). By relying on the threshold, we again created 19 different soft cluster analyses \(S_t(C)\) for each hard clustering \(C\), one for each \(t\). In this case, however, we compared the mean distances of an individual preposition to all cluster centroids, and only performed soft cluster assignments if the preposition was close to a cluster centroid in comparison to its distance to other cluster centroids. With low values of \(t\), the prepositions were assigned to none or few clusters, and the resulting cluster analyses were likely to contain not all of our prepositions, and a low ambiguity rate; with high values of \(t\), the prepositions were assigned to many clusters, and the resulting cluster analyses were likely to contain many of the 47 prepositions, and a high ambiguity rate.

4 Evaluation

The evaluation metrics play an important role in our work. On the one hand, we created a large number of hard clustering SOMs (i.e., 96 cluster
analyses since we took all possible grids for each
2 \leq k \leq 47 into account), and for each hard clus-
ter analysis we created 38 soft cluster analyses (19
centroid-based versions, and 19 preposition-based
versions). We thus needed evaluation measures to
decide about the quality of a cluster analysis. On
the other hand, our methodology relies on evaluation
metrics to identify polysemous prepositions, so the
measures are crucial to perform this work.

There is a large body of research regarding the
question of how to compare and evaluate two cluster
analyses. For example, with respect to the specific
task of semantic classification, Schulte im Walde (2003),
compared a range of evaluation measures. Related work in
this area partly adopted the suggested measures, and in addition relied on
Purity or Accuracy (Korhonen et al., 2003; Stevenson and Joannis, 2003). In
more general terms, there is an ongoing discussion about cluster compari-
ton, mainly in the field of Machine Learn-
ing, but also elsewhere. Recent examples include
Meila (2007), Rosenberg and Hirschberg (2007),
and Vinh and Bailey (2010). These approaches all
concentrate on evaluations relying on the entropy
between two cluster analyses, in order to compare
them. Entropy is an information-theoretic mea-
sure of uncertainty; in our context, entropy mea-
sures how uncertain a clustering is, given the in-
formation provided by a gold standard, and vice
versa.

We decided to make use of two evaluation mea-
sures, in order to (i) evaluate and compare our hard
and soft cluster analyses, and (ii) detect polysemy.
The two measures were expected to provide com-
plementary perspectives on the properties of our
cluster analyses, and on the properties of ambigu-
ous prepositions. The following paragraphs de-
scribe these measures, and how they were applied.

(1) With the Silhouette Value (Kaufman and
Rousseeuw, 1990), each cluster is represented by a
silhouette displaying which objects lie well within
a cluster and which objects are marginal to a clus-
ter. The evaluation appeared specifically suited
to our task, as according to our hypotheses, am-
biguous prepositions were expected to represent
marginals in a cluster analysis, i.e., to be com-
parably far away from all cluster centroids.

To obtain the silhouette value \( s_i \) for an object \( o_i \)
within a cluster \( c_A \), we compared the average dis-
tance \( a \) between \( o_i \) and all other objects in \( c_A \) with
the average distance \( b \) between \( o_i \) and all objects
in the neighbouring cluster \( c_B \), cf. Equations 3
to 5. For each object \( o_i \), \(-1 \leq s_i(o_i) \leq 1 \). If
\( s_i(o_i) \) is large, the average object distance within
the cluster is smaller than the average distance to
the objects in the neighbour cluster, so \( o_i \) is well
classified. If \( s_i(o_i) \) is small, the average object
distance within the cluster is larger than the aver-
age distance to the objects in the neighbour cluster,
so \( o_i \) has been misclassified. The silhouette value
was only calculated if cluster \( C_A \) has at least two
members, i.e. if it is not a singleton.

\[
a(o_i) = \frac{1}{|c_A| - 1} \sum_{o_j \in c_A, o_j \neq o_i} d(o_i, o_j) \tag{3}
\]

\[
b(o_i) = \min_{c_B \neq c_A} \frac{1}{|c_B|} \sum_{o_j \in c_B} d(o_i, o_j) \tag{4}
\]

\[
s_i(o_i) = \frac{b(o_i) - a(o_i)}{\max\{a(o_i), b(o_i)\}} \tag{5}
\]

In addition to providing information about the
quality of classification of a single object, the sil-
houette value can be extended to evaluate the in-
dividual clusters and the entire clustering. The av-

erage silhouette width \( s_i(c) \) of a cluster \( c \) is de-
defined as the average silhouette value for all objects
within cluster \( c \), cf. Equation 6, and the average
silhouette width for the clustering \( C \) with \( k \) clus-
ters \( s_i(C_k) \) is defined as the average silhouette
value for the individual clusters, cf. Equation 7.

\[
s_i(c) = \frac{1}{|c|} \sum_{o_i \in c} s_i(o_i) \tag{6}
\]

\[
s_i(C_k) = \frac{1}{k} \sum_{i=1}^{k} s_i(c) \tag{7}
\]

(2) The V-Measure (Rosenberg and Hirschberg,
2007) is an entropy-based cluster evaluation mea-
sure. We chose this measure over other entropy-
based measures (e.g., Variance of Information (VI)
(Meila, 2007), and variants suggested by Vinh and
Bailey (2010)) because the V-Measure \( v(C) \) bal-
ances two desirable properties for a clustering \( C \)
of a given dataset: homogeneity (\( hom \)) and com-
pleteness (\( com \)), cf. Equations 8 to 10.\footnote{Note that Equations 8 and 9 differ from those in
Rosenberg and Hirschberg (2007) in the denominators of the else
condition because there were typos in the definitions (per-
sonal communication with Andrew Rosenberg).}
Homogeneity is similar to purity, and measures how well the clusters within a cluster analysis map to the classes within a gold standard. If each cluster contains only objects from one gold-standard class, then the entropy is at its minimum, \( H(C|G) = 0 \). This represents a maximally homogeneous clustering. Completeness measures how well the classes within a gold-standard map to the clusters within a cluster analysis. If each gold-standard class contains only objects from one cluster, then the entropy is at its minimum, \( H(G|C) \). This represents a maximally complete clustering, because each gold-standard class is completely contained in a cluster.

\[
\begin{align*}
\text{hom}(C) &= 1 \text{ if } H(C, G) = 0; \text{ else } 1 - \frac{H(C|G)}{H(C, G)} \quad (8) \\
\text{com}(C) &= 1 \text{ if } H(C, G) = 0; \text{ else } 1 - \frac{H(G|C)}{H(C, G)} \quad (9) \\
v(C) &= \frac{2 \times \text{hom}(C) \times \text{com}(C)}{\text{hom}(C) + \text{com}(C)} \quad (10)
\end{align*}
\]

There is however a limitation to the V-Measure because it can only be applied to hard classifications which represent an \( N:1 \) relationship between data points and gold-standard classes. This means a given object only belongs to a single class. In our data, this is clearly not the case due to the inherent ambiguity of the prepositions. We thus extended the V-Measure to a fuzzy version Fuzzy V-Measure (fuzzy \( v \)) that applies to \( N:M \) classifications, where a data point can belong to any number of classes.\(^3\)

As for the original calculation of the entropy values, we must define the joint and conditional probabilities across clusters and gold-standard classes. In Rosenberg and Hirschberg (2007), the joint probability of a cluster \( c \) and a gold-standard class \( g \) was estimated as

\[
\hat{p}(c, g) = \frac{a_{cg}}{N}, \quad (11)
\]

where \( a_{cg} \) is the number of prepositions shared by \( c \) and \( g \) and \( N \) is the total number of prepositions. Due to the polysemy of prepositions, we must assume that a preposition occurs in multiple classes. Calculating the probability as above would however give too much weight to highly ambiguous prepositions. Our approach is to give each preposition a total mass of 1 and then equally divide its mass across the classes of which it is a member. Thus, Equation 11 becomes:

\[
\hat{p}(c, g) = \frac{\mu(c \cap g)}{M}, \quad (12)
\]

where \( \mu(c \cap g) \) is the total mass of the prepositions shared by \( c \) and \( g \), and \( M \) is the total mass of the clustering. Note that \( M \) will only be equal to \( N \) if each preposition belongs to exactly as many clusters as classes.

Example: The prepositions \( p_1, p_3 \), and \( p_4 \) each belong to two classes, while preposition \( p_2 \) belongs to three classes (cf. Table 3). Assuming cluster \( c_1 \) contains \( p_1 \) and \( p_2 \), and \( c_2 \) contains \( p_3 \) and \( p_4 \), the contingency table for the clusters \( c_1 \) and \( c_2 \) is given as in Table 4. Thus, while both \( c_1 \) and \( c_2 \) each share two prepositions with the gold-standard classes \( g_1 \) and \( g_2 \) respectively, the higher ambiguity of \( p_2 \) in the first case means there is less evidence for \( c_1 \) given \( g_1 \) than \( c_2 \) given \( g_2 \), namely: \( \hat{p}(c_1|g_1) = 0.83/2 < 1/2 = \hat{p}(c_2|g_2) \).

In addition to being applicable to ambiguous data on the side of the classes themselves, our adaptation of the V-Measure also allows for the application to soft clusterings. In this case, the data points may be present in multiple clusters and simply add their respective mass to the cells in the contingency table.

### Table 3: Prepositions in gold standard.

|     | \( g_1 \) | \( g_2 \) | \( g_3 \) | \( g_4 \) |
|-----|----------|----------|----------|----------|
| \( p_1 \) | 0.5      | 0.5      | 0        | 0        |
| \( p_2 \) | 0.33     | 0        | 0.33     | 0.33     |
| \( p_3 \) | 0        | 0.5      | 0.5      | 0        |
| \( p_4 \) | 0        | 0.5      | 0        | 0.5      |

### Table 4: Evidence for clusters.

|     | \( g_1 \) | \( g_2 \) | \( g_3 \) | \( g_4 \) | \( \sum \) |
|-----|----------|----------|----------|----------|----------|
| \( c_1 \) | 0.83     | 0.5      | 0.33     | 0.33     | 2        |
| \( c_2 \) | 0        | 1        | 0.5      | 0.5      | 2        |

5 Detecting Polysemy

This section applies the evaluation measures to our cluster analyses, in order to detect polysemous prepositions, and to identify their spatial properties. Our hypothesis is that polysemous prepositions are outliers, and thus represent either (i) singletons or (ii) marginals of the clusters within a

\(^3\)Thanks to Andrew Rosenberg for valuable discussions.
cluster analysis. We present a series of assumptions regarding this main hypothesis, and check them according to our hard and soft clusterings.

Singletons represent polysemy. Our first analysis applies to the hard cluster analyses. The assumption here is that clusters that represent singletons contain polysemous prepositions, because singletons contain objects that do not belong to any of the other clusters. Figure 1 plots the number of polysemous singletons (i.e., those singletons whose only cluster member is a polysemous preposition) against the total number of singletons, for each SOM map. The baseline is provided by 51% of the total number of singletons, as 24 out of our 47 preposition types (51%) are polysemous, so the baseline corresponds to a random assignment of preposition types to singletons.

For SOM maps with up to \( k = 13 \) clusters, there is maximally one singleton in the cluster analyses (except for \( k = 4 \) and a grid of \( 2 \times 2 \), which contains two singletons), so it is difficult to judge about the correctness of our prediction. For \( 14 \leq k \leq 26 \), in most cases the number of polysemous singletons clearly outperforms the baseline. For \( k = 22 \) with a grid of \( 22 \times 1 \) and \( k = 26 \) with a grid of \( 13 \times 2 \), the difference to the baseline is even significant (\( \chi^2, p<0.1 \)). For \( k > 27 \), the number of polysemous singletons outperforms the baseline in fewer cases than for smaller \( k \). In sum, our prediction that singletons represent polysemy holds for a restricted subset of our SOM maps, most strongly for \( 22 \leq k \leq 26 \).

![Figure 1: Number of (ambiguous) singletons.](image)

Polysemous prepositions are misclassified. Our second analysis also applies to the hard cluster analyses. Figure 2 exploits the Silhouette Value to predict polysemous prepositions. Since prepositions with several senses are expected to represent marginals in a cluster analysis, they should be comparably far away from all cluster centroids, and thus their silhouette value \( sil \) should be low, i.e., misclassify them. Figure 2 plots the correlation values of Kendall’s tau-b\(^4\) between the silhouette value \( sil(p) \) and the ambiguity rate \( amb(p) \) as defined by the gold standard, across all hard cluster analyses. According to our hypothesis, \( tau \) should be negative: the higher the ambiguity rate, the lower the silhouette value.

The plot demonstrates that our assumption is only partly correct: There are cluster analyses where we find a weak negative correlation, but most clusterings do not exhibit a noticeable correlation, and some clusterings even have a moderate positive correlation. For \( k = 24 \) with a grid of \( 24 \times 1 \) and \( k = 27 \) with a grid of \( 27 \times 1 \), we however find cluster analyses with a moderate negative correlation, \( tau = -0.30 \) and \( tau = -0.32 \).

![Figure 2: Correlation between \( sil(p) \) and \( amb(p) \).](image)

General evaluation of soft clusterings. Before we move on to exploring a further hypothesis regarding polysemous prepositions, we present a general evaluation of our two types of softening approaches. Figures 3 and 4 plot the homogeneity, completeness and fuzzy \( v \) scores after applying centroid-based and preposition-based softening to \( k \) hard clusters, respectively. The soft cluster analyses depend on the threshold \( t \) that controls the assignment of prepositions to clusters. We chose \( t = 0.7 \) as a medium threshold for the two figures. Since the various \( k \) cause strong differences in the coverage of the preposition types in the soft cluster analyses, we also plot the coverage, and the harmonic mean of fuzzy \( v \) and coverage.

The best fuzzy \( v \) scores for the centroid-based soft clusters were obtained with \( k = 16 \) and a \( 8 \times 2 \) grid (0.380), \( k = 12 \) with a \( 6 \times 2 \) grid (0.379) and \( k = 10 \) with a \( 10 \times 1 \) grid (0.377).

\(^4\)Kendall’s tau-b is a measure of association based on concordant and discordant pairs, adjusted for the number of ties.
If we take the coverage into account, the best results were obtained with $k = 20$ with a $20 \times 1$ grid (0.534), $k = 22$ with a $11 \times 2$ grid (0.530) and $k = 25$ with a $5 \times 5$ grid (0.521). For the preposition-based soft clusters the respective fuzzy $v$ scores were $k = 12$ with a $6 \times 2$ grid (0.396), $k = 16$ with a $8 \times 2$ grid (0.376) and $k = 29$ with a $29 \times 1$ grid (0.372); taking coverage into account, the respective scores were $k = 20$ with a $20 \times 1$ grid (0.547), $k = 29$ with a $29 \times 1$ grid (0.536) and $k = 25$ with a $5 \times 5$ grid (0.530). In sum, the best fuzzy $v$ scores for both types of soft cluster analyses were in most cases obtained for $k$ being similar to the number of gold standard classes. Taking coverage into account, the best results were obtained for cluster analyses with $20 \leq k \leq 29$.

A threshold of $t = 0.7$ seemed appropriate for our descriptions, since lower and also higher values of $t$ resulted in less clear preferences for $k$, and the threshold appeared like a useful compromise between low coverage in assigning prepositions to clusters, and highly ambiguous clusters.

Correlation of cluster membership rate with ambiguity rate. This final analysis investigates the relationship between the cluster membership rate of a preposition and its ambiguity rate. Our assumption is that the more clusters a specific preposition is assigned to, the more ambiguous it is. As basis for this analysis we used both the centroid-based and the preposition-based soft clusters, with varying $t$. Figures 5 and 6 present the correlation results, again relying on Kendall’s tau-$b$. For presentation reasons, we restrict the plots to $10 \leq k \leq 30$ with grid shapes $k \times 1$ only, and $t = 0.6, 0.7, 0.8, 0.9$.

Both plots demonstrate that the highest threshold $t = 0.9$ corresponding to highly ambiguous cluster analyses exhibits the best correlations with the ambiguity rates of the prepositions. For the centroid-based softening, this is true for $12 \leq k \leq 20$, for the preposition-based softening, this is true for all but two values of $k$. For lower thresholds, it seems that $t = 0.8 > t = 0.7 > t = 0.6$, but the differences are not at all clear but rather vary depending on $k$. Overall, we reached moderate correlation values, the best correlation being $tau = 0.45$. Interestingly, the best correlation values in the two types of softening approaches were obtained for similar values of $k$, and with $k$ being very similar to the number of gold standard classes (12): the prediction of the centroid-based softening was best with $k = 13$ and $k = 12$ ($tau = 0.453$ and $tau = 0.449$, respectively), and the prediction of the preposition-based softening was best with $k = 12$ and $k = 14$ ($tau = 0.439$ and $tau = 0.368$).
6 Discussion

In the previous section, we performed a series of analyses to investigate the spatial properties of polysemous prepositions in vector space models. Our main hypothesis was that polysemous prepositions are outliers, and thus represent either (i) singletons or (ii) marginals of the clusters within a cluster analysis. Concerning option (i), we showed that for specific values of $k$, there were significantly more polysemous prepositions in the singletons of the hard clusterings than there would be by chance. The relationship did not hold across $k$, however. Concerning option (ii), we performed two analyses. First, we checked whether the silhouette value of a preposition in a hard clustering correlated with its ambiguity rate, based on the assumption that the silhouette value identifies cluster marginals. Again, we found a strong correlation for specific values of $k$, but not across $k$. Second, relying on the soft clusterings we checked whether the cluster membership rate of a preposition correlated with its ambiguity rate. Especially in highly ambiguous cluster analyses there were strong correlations in both types of soft clusterings, for $k$ similar to the number of gold standard classes.

In sum, our analyses confirmed our hypothesis, but (a) with regard to specific $k$ only, and (b) the $k$ varied across the analyses. This might partly be due to our clustering approaches (SOMs for hard clustering, and our two versions of softening approaches), so we are currently experimenting with alternatives. Furthermore, the fuzzy $\nu$ measure that we developed in order to evaluate soft clusterings still seems to provide sub-optimal evidence of clustering quality: The magnitude of the score depends on the threshold, so it is difficult to decide which threshold performed best.

On the other hand, several of our analyses pointed towards similar numbers for an optimal $k$, and these optimal $k$ values were reasonable, as they were close to the number of gold standard classes. Last but not least, we looked into a range of clusterings that performed well according to our fuzzy $\nu$, and it turned out that within a certain magnitude of $k$, the clusterings were very similar to each other, with similar strengths and weaknesses. We thus conclude this paper with a qualitative analysis of the centroid-based soft clustering with $k = 16$ and a $8 \times 2$ grid, the best clustering according to the general evaluation.

The clustering actually contained only 15 clusters (so one cluster was an empty cluster). Three of the clusters were singletons, one with a 3-way ambiguous preposition (nach: local, modal, temporal), one with a 2-way ambiguous preposition (unter: local, modal), and one with a monosemous preposition (samt: modal). From the remaining 12 clusters, 8 could unambiguously be assigned a major sense according to the gold standard classes, and 4 clusters contained prepositions from various gold standard classes.

Overall, we found 27 local preposition senses, 24 modal senses, 21 temporal senses, 5 causal and 3 replacement senses. The minor senses (according to the sizes of the gold standard classes), i.e., final, creator, distributive, partitive, conditional, copulative and restrictive, were not found in the clustering. So there was a clear bias towards the assignment of majority senses. This bias might well be due to the very different sizes of the gold standard classes, so in future work we will experiment with sub-classifications of the large classes.

7 Conclusion

In this paper, we presented a methodology to identify polysemous German prepositions by exploring their vector spatial properties in hard and soft clusterings. The analyses demonstrated that – when looking at clusterings with a similar or slightly larger number of clusters than the gold standard – (a) singletons have a tendency to contain polysemous prepositions; and (b) misclassification and cluster membership rate exhibit a moderate correlation with ambiguity rate.

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