Critical exponents of the degenerate Hubbard model

Holger Frahm*

Institut für Theoretische Physik, Universität Hannover
D-3000 Hannover 1, F. R. Germany

Andreas Schadschneider† ‡

Institut für Theoretische Physik, Universität zu Köln
D-5000 Köln 41, F. R. Germany

July 15, 1992

Submitted to J. Phys. A

Abstract

We study the critical behaviour of the SU($N$) generalization of the one-dimensional Hubbard model with arbitrary degeneracy $N$. Using the integrability of this model by Bethe Ansatz we are able to compute the spectrum of the low-lying excitations in a large but finite box for arbitrary values of the electron density and of the Coulomb interaction. This information is used to determine the asymptotic behaviour of correlation functions at zero temperature in the presence of external fields lifting the degeneracy. The critical exponents depend on the system parameters through a $N \times N$ dressed charge matrix implying the relevance of the interaction of charge- and spin-density waves.

*e-mail: frahm@kastor.itp.uni-hannover.de
†e-mail: as@thp.uni-koeln.de
‡Work performed within the research program of the Sonderforschungsbereich 341 (Köln-Aachen-Jülich)
1 Introduction

The physics of highly correlated electron systems has long been the subject of extensive studies in condensed matter physics. Recently, the non-Fermi liquid character of low-dimensional systems has attracted renewed interest in one-dimensional realizations of these systems where large quantum fluctuations lead to Luttinger liquid behaviour [1]: the correlation functions decay as power-laws at zero temperature, the exponents depending on the system parameters such as electron density, magnetization (or applied magnetic field) and strength of the interaction.

In this context exactly soluble models can provide a variety of new insights, in particular when used together with the general results on quantum critical behaviour in one spatial dimension as provided by the theory of conformal invariance [2]–[4]. Here the universality class of the quantum system is completely determined by a single dimensionless number—the so-called central charge $c$ of the underlying Virasoro algebra. This number $c$ as well as the dimensions of the operators present in the theory can be extracted from analytical results for the spectrum of low-lying states in finite geometries. In this language, Luttinger liquids correspond to a central charge $c = 1$, the dependence of the anomalous dimensions on the system parameters is through a single dimensionless number—the coupling constant of the corresponding Gaussian model.

The situation described above is the generic behaviour of 1D quantum systems with a single critical degree of freedom, as realized in spin-$\frac{1}{2}$ chains or systems of spinless fermions. The situation becomes more complicated for systems where two or more massless excitations are possible: the lack of Lorentz invariance (the corresponding Fermi velocities differ in general) prevents the direct application of the predictions of conformal field theory and the interacting nature of the system complicates the factorization of the problem into independent ones for each critical quasi-particle mode. On the other hand the exact results on the finite-size scaling of the low-lying energies available for Bethe Ansatz soluble models suggests a resolution of this problem: the spectrum is that of a multi-component Gaussian model. In analogy to the analysis of conformal invariant theories the universality class of a system with $N$ massless collective excitation modes is determined by $N$ dimensionless numbers $c_i$—reducing to the central charge in the scalar case. The anomalous dimensions are functions of the system parameters not through a single coupling constant but through a $N \times N$ matrix of dimensionless numbers—the so-called dressed charge matrix. This behaviour has been found in a large number of 1D quantum systems, including certain integrable spin-chains with $S > \frac{1}{2}$ [5, 6], and the Hubbard [7, 8, 9] and $t-J$-models of correlated electrons, the latter both at the integrable supersymmetric point $J/t = 2$ [10] and away from integrability [11].

Most of the integrable models in this list are solved by a hierarchy of Bethe Ansätze [12]: The first one introduces a set of wavenumbers describing the phase of the wave function and determining the spectrum, the others are necessary for the wavefunctions to show the symmetry corresponding to a particular representation of the Permutation group. This allows for the solution of certain systems with various choices of internal degrees of freedom, e.g. an SU(2)-spin in the Hubbard model.

In the present paper we study the critical properties of a generalization of the Hubbard model, describing electrons carrying an SU($N$)-spin index on the lattice. Unlike the case of the regular ($N = 2$) Hubbard model this model allows to study a Mott transition at a finite value of the Coulomb interaction [13, 14, 15]. Our paper is organized as follows: In the following section we shall introduce the model and present a brief review of its Bethe Ansatz solution and a qualitative discussion of its excitation spectrum. In Section 3 the results for the finite-size corrections to the energies of low-lying states as well as their relation to the critical exponents
are given in terms of the $N \times N$ dressed charge matrix. In Section 4 the integral equations for this matrix are solved in the zero-field case and the operator dimensions are computed as a function of the electron density and the strength of the interaction. They are shown to reflect the full $\text{SU}(N)$-spin symmetry present in this case. These results are applied to the computation of the critical exponents for some correlation functions of interest. Finally, in Section 5 we consider states where the $\text{SU}(N)$-symmetry of the ground state is broken by magnetic fields coupling to the various flavours of the internal degree of freedom. In the limiting case of strong coupling we discuss the dependence of the dressed charge matrix and the critical exponents on these external fields.

## 2 The Bethe-Ansatz solution of the degenerate Hubbard model

The Hamiltonian of the degenerate Hubbard model on a chain of length $L$ is given by the following expression:

$$
\mathcal{H}_N = - \sum_{j=1}^{L} \sum_{s=1}^{N} \mathcal{P} \left( c_{j+1,s}^\dagger c_{js} + c_{js}^\dagger c_{j+1,s} \right) \mathcal{P} + 4u \sum_{j=1}^{L} \sum_{s,s' \neq s'} n_{js} n_{js'}^\prime 
+ \mu \sum_{j=1}^{L} \sum_{s=1}^{N} n_{js} + \sum_{j=1}^{L} \sum_{s=1}^{N} h_s n_{ja} 
$$

(1)

The Fermi operator $c_{js}^\dagger$ ($c_{js}$) creates (annihilates) an electron at site $j$ with spin index $s \in \{1, \ldots, N\}$ and $n_{js} = c_{js}^\dagger c_{js}$ is the corresponding number operator. The real parameters $h_s$ may be considered as generalized magnetic fields. Note that these fields do not destroy the integrability of the model since the numbers $N_s = \sum_{j=1}^{L} n_{js}$ of particles with spin index $s$ are conserved. $\mathcal{P}$ projects onto the subspace of states having at most two electrons at each site. This projection is necessary in order to render the Hamiltonian (1) Bethe-Ansatz solvable [13, 16, 17, 19] and has no effect for $N = 2$. The Bethe-Ansatz wave function which solves the Schrödinger equation for (1) for a total number $N_c$ of electrons is characterized by the momenta $k_j$ ($j = 1, \ldots, N_c$) and $N-1$ sets of rapidities $\lambda^{(s)}_{\alpha}$ ($s = 1, \ldots, N-1$; $\alpha = 1, \ldots, M_s$). Imposing periodic boundary conditions on the wave function leads to the Bethe-Ansatz equations

$$
L k_j = 2\pi I_j - \sum_{\beta=1}^{M_1} 2 \text{arctan} \left( \frac{\sin k_j - \lambda^{(1)}_{\beta}}{u} \right),
$$

$$
\sum_{\beta=1}^{M_{s-1}} 2 \text{arctan} \left( \frac{\lambda^{(s)}_{\alpha} - \lambda^{(s-1)}_{\beta}}{u} \right) + \sum_{\beta=1}^{M_{s+1}} 2 \text{arctan} \left( \frac{\lambda^{(s)}_{\alpha} - \lambda^{(s+1)}_{\beta}}{u} \right) + \sum_{\beta=1}^{M_s} 2 \text{arctan} \left( \frac{\lambda^{(s)}_{\alpha} - \lambda^{(s)}_{\beta}}{2u} \right) = 2\pi J_{\alpha}^{(s)} + \sum_{\beta=1}^{M_{s+1}} 2 \text{arctan} \left( \frac{\lambda^{(s)}_{\alpha} - \lambda^{(s+1)}_{\beta}}{2u} \right)
$$

(2)

Here we have set $M_0 = N_c$, $M_N = 0$ and $\lambda^{(0)}_j = \sin k_j$.

The quantum numbers $I_j$ and $J^{(s)}_{\alpha}$ are integer or half-integer depending on the parity of the numbers $N_c$, $M^{(s)}_{\alpha}$:

$$
I_j = \frac{M_j}{2} \mod 1, \quad J^{(s)}_{\alpha} = \frac{M_s - M_{s-1} - M_{s+1} + 1}{2} \mod 1.
$$

(3)
Energy and momentum of the model in a state corresponding to a solution of (2) are completely determined by the momenta $k_j$:

$$
E = -2 \sum_{j=1}^{N_c} \cos k_j + \mu N_c - \sum_{s=1}^{N} h_s N_s,
$$

$$
P = \sum_{j=1}^{N_c} k_j = \frac{2\pi}{L} \left( \sum_{j=1}^{N_c} I_j + \sum_{s=1}^{N-1} \sum_{\alpha=1}^{M_s} J_{\alpha}^{(s)} \right),
$$

where $N_s$ denotes the total number of electrons with orbital index $s$. In the thermodynamic limit ($L \to \infty$, with $N_c/L$, $M_s/L$ kept constant) the eqs. (2) corresponding to the ground state of (1) can be transformed into a set of coupled integral equations for the densities $\rho_c(k)$ and $\rho_s(\lambda)$ of the parameters $k_j$ and $\lambda^{(s)}$, respectively:

$$
\rho_c(k) = \frac{1}{2\pi} + \frac{\cos k}{2\pi} \int_{-\Lambda_1}^{\Lambda_1} d\lambda K_1(\sin k - \lambda) \rho_1(\lambda),
$$

$$
\rho_1(\lambda) = \frac{1}{2\pi} \int_{-k_0}^{k_0} dk \frac{K_1(\lambda - \sin k) \rho_c(k)}{2\pi} - \frac{1}{2\pi} \int_{-\Lambda_1}^{\Lambda_1} d\mu K_2(\lambda - \mu) \rho_1(\mu)
+ \frac{1}{2\pi} \int_{-\Lambda_2}^{\Lambda_2} d\mu K_1(\lambda - \mu) \rho_2(\mu),
$$

$$
\rho_s(\lambda) = \frac{1}{2\pi} \int_{-\Lambda_{s-1}}^{\Lambda_s} d\mu K_1(\lambda - \mu) \rho_{s-1}(\mu) - \frac{1}{2\pi} \int_{-\Lambda_s}^{\Lambda_s} d\mu K_2(\lambda - \mu) \rho_s(\mu)
+ \int_{-\Lambda_{s+1}}^{\Lambda_{s+1}} d\mu K_1(\lambda - \mu) \rho_{s+1}(\mu),
$$

(s = 2, \ldots, N - 1)

with $\Lambda_N = 0$. The kernels $K_{1,2}(x)$ of these eqs. (4) are given by

$$
K_1(x) = \frac{2u}{x^2 + u^2}, \quad K_2(x) = \frac{4u}{x^2 + (2u)^2}.
$$

The values of the parameters $k_0$ and $\Lambda_1, \ldots, \Lambda_{N-1}$ are determined through the normalizations

$$
n_c = \int_{-k_0}^{k_0} dk \rho_c(k),
$$

$$
n_s = \int_{-\Lambda_{s-1}}^{\Lambda_s} d\lambda \rho_{s-1}(\lambda) - \int_{-\Lambda_s}^{\Lambda_s} d\lambda \rho_s(\lambda),
$$

(s = 1, \ldots, N - 1)

where $n_c = N_c/L$ is the total density of electrons and $n_s = N_s/L = (M_{s-1} - M_s)/L$ is the density of electrons with index $s$. Furthermore we have set $\rho_0 \equiv \rho_c$ and $\Lambda_0 = k_0$.

The ground state energy per lattice site is

$$
\epsilon_\infty = \int_{-k_0}^{k_0} dk (\mu + h_1 - 2 \cos k) \rho_c(k) + \sum_{s=1}^{N-1} (h_{s+1} - h_s) \int_{-\Lambda_s}^{\Lambda_s} d\lambda \rho_s(\lambda)
$$

which may alternatively be expressed in terms of the dressed energy

$$
\epsilon_\infty = \frac{1}{2\pi} \int_{-k_0}^{k_0} dk \tilde{\epsilon}_c(k).
$$

\(^\text{1}\text{In the following we will adopt the convention that an index '0' stands for 'c'}.\)
Here $\varepsilon_c(k)$ is the solution of the system of coupled integral equations

$$
\varepsilon_c(k) = \varepsilon_c^{(0)}(k) + \frac{1}{2\pi} \int_{-\Lambda_1}^{\Lambda_1} d\lambda K_1(\sin k - \lambda)\varepsilon_1(\lambda),
$$

$$
\varepsilon_1(\lambda) = \varepsilon_1^{(0)}(\lambda) + \frac{1}{2\pi} \int_{-\Lambda_0}^{\Lambda_0} dk \cos k K_1(\lambda - \sin k)\varepsilon_c(k) - \frac{1}{2\pi} \int_{-\Lambda_1}^{\Lambda_1} d\mu K_2(\lambda - \mu)\varepsilon_1(\mu) + \frac{1}{2\pi} \int_{-\Lambda_2}^{\Lambda_2} d\mu K_1(\lambda - \mu)\varepsilon_2(\mu),
$$

$$
\varepsilon_s(\lambda) = \varepsilon_s^{(0)}(\lambda) + \frac{1}{2\pi} \int_{-\Lambda_{s-1}}^{\Lambda_{s-1}} d\mu K_1(\lambda - \mu)\varepsilon_{s-1}(\mu) - \frac{1}{2\pi} \int_{-\Lambda_s}^{\Lambda_s} d\mu K_2(\lambda - \mu)\varepsilon_s(\mu) + \frac{1}{2\pi} \int_{-\Lambda_{s+1}}^{\Lambda_{s+1}} d\mu K_1(\lambda - \mu)\varepsilon_{s+1}(\mu) \quad (s = 2, \ldots, N - 1).
$$

The bare energies are from \([1]\)

$$
\varepsilon_c^{(0)}(k) = \mu + h_1 - 2\cos k; \quad \varepsilon_s^{(0)}(\lambda) = h_{s+1} - h_s.
$$

The dressed energies \([10]\) obey the conditions

$$
\varepsilon_c(k_0) = 0, \quad \varepsilon_s(\Lambda_s) = 0.
$$

The ground state at half-filling ($n_c = 1$) and with vanishing fields $h_s$ shows for $N > 2$ an interesting behavior which has not been found in the case of the standard Hubbard model ($N = 2$). This has been noticed independently by Schlottmann \([12]\) and one of the authors \([13]\). For $u > u_c$ one finds $k_0 = \pi$ and for $u < u_c$ one has $k_0 < \pi$. Here the critical value $u_c$ is determined through the implicit equation

$$
\int_{-\pi}^{\pi} dk G_N(\sin k; u_c) = 2\pi
$$

where $G_N(x; u)$ in terms of the Digamma-function $\psi(x)$ is given by

$$
G_N(x; u) = \frac{1}{Nu} \text{Re} \left[ \psi \left( 1 + i \frac{x}{2Nu} \right) - \psi \left( \frac{1}{N} + i \frac{x}{2Nu} \right) \right].
$$

For $N = 2$ we have $u_c = 0$ (see Fig. 1) as already shown by Lieb and Wu \([21]\).

The excitation spectrum in zero-fields has also been studied in \([13]\) using an extension of the method developed in \([22, 23]\) and in \([14]\). One finds $N - 1$ gapless spin excitations with soft modes with wave numbers $2sP_F$ ($s = 1, \ldots, N - 1$, $P_F = \frac{\pi}{N} n_c$) and so-called particle-hole excitations.\(^2\) These also are gapless and do exist only for $k_0 < \pi$, i.e. for $n_c < 1$ or $n_c = 1$ and $u < u_c$. In \([13]\) also excitations corresponding to doubly occupied sites have been studied. These are described by complex momenta $k^\pm$ satisfying $\sin k^\pm = \lambda \pm i\mu$ and have a finite gap (at least for $n_c = 1$ and $u > u_c$).

The special structure of the ground state for $n_c = 1$ leads to interesting properties of the model. For $u > u_c$ the only possible charge carrying excitations are those involving complex momenta. As these excitations have a gap the system is in an insulating state. For $u < u_c$ particle-hole excitations become possible. These excitations may carry a current and so the system is in a metallic phase. This shows the existence of Mott-transition at the critical value

\(^2\)For a discussion of the zero-field excitation spectrum for $N = 2$, see e.g. \([22]\) and references therein.
$u_c$ of the Coulomb repulsion $u$. The transition is also reflected in the behavior of other physical quantities, e.g. the charge susceptibility $\chi_c$ and the Fermi velocity $v_c$ [14,15]. As the gap to the excitations with double occupations does not vanish in the limit $u \gtrsim u_c$ the value of the gap to the charge carrying shows a discontinuity at $u = u_c$. We may thus say that the transition is of 'first order'.

In the limit $u \to \infty$ at $n_c = 1$ the model [1] becomes equivalent to the SU($N$) Heisenberg chain [24]. This equivalence generalizes the well-known relation between the regular ($N = 2$) Hubbard model and the Heisenberg antiferromagnet.

Recently, Schlottmann [20] showed that in the continuum limit the particles interact via a potential of the form $1/\sinh^2 r$ where $r$ is the distance between the particles involved in some properly chosen units. This reflects the nonlocal character of the interaction as introduced by the projectors $P$ in the kinetic terms of [1] for $N > 2$.

## 3 Finite-size corrections and conformal properties

As shown in the preceding section the degenerate Hubbard model supports gapless excitations in general. Thus we may apply the concepts of conformal field theory to determine the asymptotic behavior of the correlation functions, e.g. the critical exponents.

First we calculate exactly the finite-size corrections to the ground state energy and the energies of the excited states. This can be done by a straightforward extension of the calculation for the case $N = 2$ [4]. The results can be expressed in terms of the $N \times N$-dressed charge matrix

$$Z = \begin{pmatrix}
\xi_{cc}(k_0) & \xi_{c1}(\Lambda_1) & \cdots & \xi_{c,N-1}(\Lambda_{N-1}) \\
\xi_{1c}(k_0) & \xi_{11}(\Lambda_1) & \cdots & \xi_{1,N-1}(\Lambda_{N-1}) \\
\vdots & \vdots & \ddots & \vdots \\
\xi_{N-1,c}(k_0) & \xi_{N-1,1}(\Lambda_1) & \cdots & \xi_{N-1,N-1}(\Lambda_{N-1})
\end{pmatrix}. \quad (15)$$

The elements of $Z$ can be obtained from the dressed-charge functions $\xi_{rs}(\lambda)$ which obey a system of coupled integral equations similar to (10). For $r = c, 1, \ldots, N - 1$ we have

$$\xi_{rc}(k) = \delta_{rc} + \frac{1}{2\pi} \int_{-\Lambda_1}^{\Lambda_1} d\lambda \cos k K_1(\sin k - \lambda) \xi_{r1}(\lambda),$$

$$\xi_{r1}(\lambda) = \delta_{r1} + \frac{1}{2\pi} \int_{-\Lambda_0}^{\Lambda_0} dk \cos k K_1(\lambda - \sin k) \xi_{rc}(k) - \frac{1}{2\pi} \int_{-\Lambda_1}^{\Lambda_1} d\mu K_2(\lambda - \mu) \xi_{r1}(\mu) + \frac{1}{2\pi} \int_{-\Lambda_2}^{\Lambda_2} d\mu K_1(\lambda - \mu) \xi_{r2}(\mu),$$

$$\xi_{rs}(\lambda) = \delta_{rs} + \frac{1}{2\pi} \int_{-\Lambda_{s-1}}^{\Lambda_{s-1}} d\mu K_1(\lambda - \mu) \xi_{rs-1}(\mu) - \frac{1}{2\pi} \int_{-\Lambda_s}^{\Lambda_s} d\mu K_2(\lambda - \mu) \xi_{rs}(\mu) + \frac{1}{2\pi} \int_{-\Lambda_{s+1}}^{\Lambda_{s+1}} d\mu K_1(\lambda - \mu) \xi_{rs+1}(\mu) \quad (s = 2, \ldots, N - 1). \quad (16)$$

The finite-size scaling behavior of the ground-state energy is found to be

$$E_0 - L\epsilon_\infty = -\frac{\pi}{6L} \sum_{s=0}^{N-1} v_s$$

with the Fermi velocities of charge and spin excitations

$$v_0 \equiv v_c = \frac{1}{2\pi p_c(k_0)} \xi'_c(k_0), \quad v_s = \frac{1}{2\pi p_s(\Lambda_s)} \xi'_s(\Lambda_s) \quad (s = 1, \ldots, N - 1). \quad (18)$$
Energies and momenta of the excitations scale as
\[
E(\Delta \mathbf{M}, \mathbf{D}) - E_0 = \frac{2\pi}{L} \left[ \frac{1}{4} \Delta \mathbf{M}^T (Z^{-1})^T V Z^{-1} \Delta \mathbf{M} + \mathbf{D}^T Z V Z^T \mathbf{D} + \sum_{s=0}^{N-1} v_s (N_s^+ + N_s^-) \right],
\]
\[
P(\Delta \mathbf{M}, \mathbf{D}) - P_0 = \frac{2\pi}{L} \left[ \Delta \mathbf{M}^T \cdot \mathbf{D} + \sum_{s=0}^{N-1} (N_s^+ - N_s^-) \right] + 2 \sum_{s=0}^{N-1} \sum_{r=0}^{s} D_r P_{F,s+1}, \tag{19}
\]
\[V = \text{diag}(v_c, v_1, \ldots, v_{N-1}).\]

Here \(N_c^+, N_s^+\) are positive integers and \(\Delta \mathbf{M}\) and \(\mathbf{D}\) are vectors characterizing the excited state under consideration. \(P_{F,s}\) are the Fermi momenta for electrons with spin index \(s\). For the ground state in the thermodynamic limit we have \(\Delta M_s = 0, D_s = 0\) \((s = c, 1, \ldots, N - 1)\). For an excited state \(\Delta \mathbf{M}\) has integer components denoting the change of the total number of electrons and the number of electrons with index \(s\) with respect to the ground state. \(D_s\) are integer or half-odd integer depending on the parities of the \(\Delta M_s\). Due to (3) we have
\[
D_c = \frac{\Delta N_c + \Delta M_1}{2} \mod 1,
\]
\[
D_s = \frac{\Delta M_{s-1} + \Delta M_{s+1}}{2} \mod 1 \quad (s = 1, \ldots, N - 1) \tag{20}
\]

with \(\Delta M_0 = \Delta N_c\) and \(\Delta M_N = 0\).

In general, all velocities \(v_s\) are different. In this case the results (17) and (19) may be interpreted in terms of a semidirect product of \(N\) independent Virasoro algebras\(^3\). All these Virasoro algebras have central charge \(c_s = 1\). For vanishing fields \(h_s\) all magnon velocities \(v_1, \ldots, v_{N-1}\) are equal \(\text{[13], [15]}\) and we have a semidirect product of a \(c = 1\) Gaussian theory – reflecting the \(U(1)\) symmetry of the charge sector – and a \(c = N - 1\) Wess-Zumino-Witten theory – reflecting the \(SU(N)\)-symmetry of the spin sector.

Comparing (19) with the predictions of the conformal field theory [3, 4]
\[
E(\Delta \mathbf{M}, \mathbf{D}) - E_0 = \frac{2\pi}{L} \sum_{s=0}^{N-1} v_s (\Delta^+_s + \Delta^-_s),
\]
\[
P(\Delta \mathbf{M}, \mathbf{D}) - P_0 = \frac{2\pi}{L} \sum_{s=0}^{N-1} (\Delta^+_s - \Delta^-_s) + 2 \sum_{s=0}^{N-1} \sum_{r=0}^{s} D_r P_{F,s+1}, \tag{21}
\]

one obtains expressions for the conformal dimensions \(\Delta^\pm_s\) of the primary fields in terms of the dressed charge matrix. Requiring that all dimensions are positive we find
\[
2\Delta^\pm_s = \left( (Z^T \mathbf{D})_s \pm \frac{1}{2} (Z^{-1} \Delta \mathbf{M})_s \right)^2 + 2N^\pm_s \tag{22}
\]
which, in general, depend on the system parameters. In the following section we will show that for vanishing fields the \(\Delta^\pm_s\) \((s = 1, \ldots, N - 1)\) are functions of the components of \(\Delta \mathbf{M}\) and \(\mathbf{D}\) only, whereas \(\Delta^\pm_c\) depends on the strength \(u\) of the Coulomb repulsion and the density \(n_c\) of electrons.

We now make further use of the results of conformal field theory to write down the correlation functions for primary fields as
\[
\langle \phi_{\Delta^\pm}(x,t) \phi_{\Delta^\pm}(0,0) \rangle = \prod_{s=0}^{N-1} \frac{\exp(-2i \sum_{r=0}^{s} D_r P_{F,s+1} x)}{(x - iv_s t)^{2\Delta^+_s} (x + iv_s t)^{2\Delta^-_s}}. \tag{23}
\]
\(^3\)See [3] and references therein for a more detailed discussion of this point.
The correlation functions of the physical fields consist of a sum of terms \((23)\). In the following we will study correlators of the form \(\langle O_j(t)O_0^\dagger(0) \rangle\) where \(O\) is given in terms of \(c\) and \(c^\dagger\). To find the asymptotic behavior of the correlator one has to expand \(O\) in terms of the conformal fields. This is not possible in general, but the explicit form of \(O\) allows for an identification of the quantum numbers \(M_{c,M_{1,\ldots,M_{N-1}}\ldots}\) of the intermediate states. Therefore the leading term in the asymptotic expansion of \(\langle O_j(t)O_0^\dagger(0) \rangle\) can be obtained from \((22)\) through minimizing with respect to the \(D_s\) satisfying \((20)\).

4 Critical exponents for vanishing fields

In the absence of magnetic fields it is easily seen that \(\Lambda_r = \infty\) for all \(r\). This allows to eliminate the \(\lambda\)-dependent quantities from the Bethe Ansatz integral equations by Fourier transformation. From Eq. \((16)\) we obtain for the \(k\)-dependent entries of the dressed charge matrix

\[
\xi_{rc}(z) = \frac{N-r}{N} + \frac{1}{2\pi} \int_{-z_0}^{z_0} dy \xi_{rc}(y) G_N(z-y; 1).
\]

where we have introduced a new variable \(z = \sin k/u\).

The solution for \(z_c = \xi_{cc}(z_0)\) is obtained by iteration for small values of \(z_0\) where we obtain

\[
z_c \simeq 1 + \frac{G_N(0; 1)}{\pi} z_0 = 1 - \frac{\gamma + \psi(\frac{1}{N})}{N\pi} z_0 \quad \text{for } z_0 \ll 1.
\]

For \(z_0 \gg N\) a perturbative scheme \([23]\) based on the Wiener-Hopf method can be applied, giving

\[
z_c \simeq \sqrt{N} \left(1 - \frac{N-1}{2\pi z_0}\right) \quad \text{for } z_0 \gg N.
\]

For intermediate values of \(z_0\) the integral equation \((24)\) is easily solved numerically. The dependence of \(z_c\) on the density and the strength of the Coulomb interaction is shown in Fig. 2 for some values of \(N\). In addition to the interpretation of the \(z_c\) as measure of the reordering of the Fermi-sea due to the interaction when an electron is added there exists a direct relation to physical observables: It can be expressed as

\[
z_c^2 = \pi v_c n_c^\ast \kappa \quad \text{(27)}
\]

in terms of the compressibility \(\kappa = -(1/L)\partial L/\partial p\) of the electron gas \((p\) being the pressure).

As in the regular Hubbard model \((N = 2)\) \([7]\) one can employ the Wiener-Hopf method to compute the remaining elements of the dressed charge matrix \((15)\) yielding:

\[
Z = \begin{pmatrix}
z_c & 0 & 0 & \cdots & 0 \\
\frac{N-1}{N} z_c & z_c & 0 & \cdots & 0 \\
\frac{N-2}{N} z_c & 0 & z_c & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
\frac{1}{N} z_c & 0 & 0 & \cdots & z_N
\end{pmatrix}
\]

Note that the symmetric \((N-1) \times (N-1)\)-block \(Z_N\) of this matrix is completely determined by the SU\((N)\)-spin-symmetry of the system: this symmetry is manifest in the kernel of the integral
equations and allows for the reduction of the corresponding matrix Wiener-Hopf problem to \( N - 1 \) scalar ones \[26, 27\]. The latter are soluble by quadratures and one obtains closed expressions for the matrix elements of \( Z_N^{-1} \):

\[
\left( Z_N^{-1} \right)_{rs} = -\frac{2}{N} \sin \frac{\pi}{2N} \frac{\sqrt{1 - z_r^2} \sqrt{1 - z_s^2}}{z_r^2 + z_s^2 - 2 \cos \frac{\pi}{2N} z_r z_s - \sin^2 \frac{\pi}{2N}}, \quad z_n = \cos \frac{\pi n}{N}.
\] (29)

The square of \( Z_N^{-1} \) is the Cartan matrix for the Lie algebra SU(\( N \)):

\[
\left( Z_N^{-2} \right)_{rs} = (C_N)_{rs} = 2 \delta_{r,s} - \delta_{r,s+1} - \delta_{r+1,s}.
\] (30)

Using the properties of the dressed charge matrix we find for the critical exponents

\[
2 \Delta_c^\pm = \left( z_c \sum_{s=0}^{N-1} \frac{N-s}{N} D_s \pm \frac{1}{2z_c} \Delta N_c \right)^2 + 2N_c^\pm
\]

\[
2 \Delta_r^\pm = \left( \sum_{s=1}^{N-1} (Z_N)_{sr} D_s \pm \frac{1}{2} \sum_{s=1}^{N-1} \left( Z_N^{-1} \right)_{rs} \left( \Delta M_s - \frac{N-s}{N} \Delta N_c \right) \right)^2 + 2N_r^\pm
\] (30)

For vanishing magnetic field the magnon velocities \( v_r \) \((r = 1, \ldots, N-1)\) are identical, hence only the quantities

\[
2 \Delta_d^\pm = 2 \sum_{r=1}^{N-1} \Delta_d^\pm = \frac{1}{4} \Delta N_c^T \Delta N_c + D_d^T C_N^{-1} D_d \pm \Delta N_c^T \cdot D_d + 2 \sum_{r=1}^{N-1} N_r^\pm,
\]

where \( (\Delta N_c)_r = \Delta M_r - \frac{N-r}{N} \Delta N_c, \quad (D_d)_r = D_r, \quad r = 1, \ldots, N-1 \) (31)

appear in the exponents describing the asymptotic behaviour of the correlation functions. Note that they are independent of the quantity \( z_c \) incorporating the dependence of the anomalous dimensions on electron density and strength of the interaction. They are completely determined by the SU(\( N \))-symmetry of the zero field ground state and, in fact, of the same form as the exponents characterizing the SU(\( N \))-symmetric critical vertex models and spin-chains \[26, 27\] (the difference being the possibility of fractional values for the elements of \( \Delta N_c \)).

Now we are able to study the asymptotic behaviour of correlation functions of interest: For the field-field correlation function we have \( \Delta N_c = 1 \) and \( \Delta M = (1, \ldots, 1_k, 0, \ldots) \). The corresponding values of \( D \) can be read off from Eq. (20), the contribution at wavenumber \( k = P_F = (\pi/N)n_c \) arises from the choice \( D_r = \frac{1}{2}(\delta_{rs} - \delta_{r,s+1}) \) for any \( s \) giving

\[
2 \Delta_c^\pm = \left( \frac{z_c}{2N} \pm \frac{1}{2z_c} \right)^2, \quad 2 \Delta_d^\pm = \left( 1 - \frac{1}{N} \right), \quad 2 \Delta_d^- = 0.
\] (32)

Hence, the singularity of the momentum distribution function at the Fermi-point

\[
\langle c_{k,s} c_{k,s}^\dagger \rangle \sim |k - P_F|^\alpha
\] (33)

is characterized by the exponent

\[
\alpha = \frac{\theta}{(2N)^2} + \frac{1}{\theta} - \frac{1}{N}
\] (34)
where \( \theta = 2z_c^2 \) varies between 2 and \( 2N \) as \( u \) decreases from \( \infty \) to 0. Hence,

\[
0 = \alpha(u \to 0) < \alpha < \alpha(u \to \infty) = \frac{1}{2} - \frac{1}{N} + \frac{1}{2N^2}
\]

(35)

showing the Luttinger liquid character of the degenerate Hubbard model at nonzero interaction. Numerical data for the exponent \( \alpha \) are presented in Fig. 3.

For the density-density correlation function \( \langle n_s(x,t)n_s(0) \rangle - \langle n \rangle^2 \) we find contributions at wavenumbers \( k = 2mP_F \) \((m = 1, \ldots, N)\) with exponents

\[
2\Delta_c^\pm = 2\left( \frac{m}{2N} \right)^2 \theta, \quad 2\Delta_r^\pm = \frac{m(N-m)}{N}
\]

(36)

arising from the choice \( \Delta N_c = \Delta M_r = 0 \) and \( D_r = \delta_{r,N-m} \). In addition, there are \( k = 0 \)-terms decaying as \( x^{-2} \) asymptotically. They are generated by the marginally relevant secondary operators in the conformal family of the unit operator \( i.e. \Delta N_c = \Delta M_r = 0 \) and \( D_c = D_r = 0 \) but \( N_c^\pm \) or one of the \( N_r^\pm \) in Eq. (30) equal to 1).

From the discussion in the preceding section it is clear that the above statements are valid for any value of the coupling constant and for any filling with \( n_c < 1 \). For \( n_c = 1 \) two cases have to be discussed separately:

(i) For \( n_c = 1 \) and \( u \leq u_c \) the system is in a metallic phase \( described \ by \( k_0 = k_0(u) \leq \pi \) in the Bethe Ansatz equations). The critical exponents of the system are given by (30) with \( z_c \) being a function of the Coulomb interaction through (24). The number \( z_c \) decreases from \( \sqrt{N} \) to 1 as the strength of the Coulomb interaction is varied from 0 to \( u_c \). It is possible to expand the integral equations for the density \( \rho_c(k) \) in the neighbourhood of the Mott transition \( u \leq u_c \) to determine \( k_0(u) \) from (7). For \( N \to \infty \) the resulting expressions simplify, giving \( u_c = \frac{1}{2} \sqrt{3} \) and

\[
z_c \approx 1 + \frac{4\sqrt{14}}{7\pi} \sqrt{1 - \frac{u}{u_c}} + \ldots
\]

(37)

The same square-root singularity in the Coulomb coupling \( u \)—but with a different numerical prefactor—is found near \( u = u_c \) for finite \( N \). Through (30) it also appears in the critical exponents near the Mott transition, e.g. the exponent \( \alpha \) of the momentum distribution (34) varies like

\[
\alpha \approx \frac{1}{2} - \frac{4\sqrt{14}}{7\pi} \sqrt{1 - \frac{u}{u_c}} + \ldots
\]

(38)

as \( u \) approaches \( u_c \) from below \( (N \to \infty) \).

(ii) For \( u > u_c \) with one particle per site the system is in an insulating state, hence charge carrying excitations develop a gap. Excitations in the spin degrees of freedom, however, continue to be massless at zero temperature. The critical properties of this state can be described along the lines of the discussion above: For \( n_c = 1 \) and \( u > u_c \) we have \( k_0 = \pi \). Hence, the \( k \)-dependent quantities disappear from Eq. (16) leaving a system of \( N - 1 \) coupled integral equations for the spin components of the dressed charge matrix. The expression for the conformal dimensions are of the form (22) and depend on the applied magnetic fields. The correlation functions for states corresponding to critical excitations are given by (23). These are exactly those found in the SU\((N)\)-generalization of the Heisenberg spin chain \( see \ e.g. \ [5] \). Note, that (20) implies that the momentum of the intermediate state is shifted by \( \pi \) for states with odd \( \Delta M_1 \).
\section{Magnetic field effects in the strong coupling regime}

Nonzero magnetic fields \( h_s \) in (\ref{eq:1}) lead to finite values of the parameters \( \Lambda_s \) through (\ref{eq:2}). This effect in turn leads to a general dependence of the elements of the dressed charge matrix on the system parameters \( u, n_c \) and all of the fields. Only at and beyond certain critical values of the fields where one or more bands are completely depleted the integral equations simplify to some extent so that analytical results may become available (for a discussion of the \( N = 2 \) case see \ref{eq:3}). A more detailed study of the magnetic field dependence of the dressed charge matrix becomes possible in the strong coupling limit \ref{eq:4}. As in the SU(2) Hubbard model the Bethe Ansatz integral equations (\ref{eq:5}), (\ref{eq:10}) and (\ref{eq:16}) describing the system simplify in the limit \( u \to \infty \) (see also \ref{eq:18}). In this limit the \( k \)-dependent quantities can be eliminated which allows to study the effect of magnetic fields on the critical exponents in more detail. Upon rescaling of the variables \( \lambda/u \to \lambda \equiv \lambda \) one obtains to leading order

\begin{equation}
\varepsilon_r(\lambda) = \varepsilon_r(0) - \frac{1}{2\pi} \int_{-\Lambda_r}^{\Lambda_r} d\mu \tilde{K}_2(\lambda - \mu)\varepsilon_r(\mu) + \frac{1}{2\pi} \int_{-\Lambda_r-1}^{\Lambda_r-1} d\mu \tilde{K}_1(\lambda - \mu)\varepsilon_{r-1}(\mu) + \frac{1}{2\pi} \int_{-\Lambda_{r+1}}^{\Lambda_{r+1}} d\mu \tilde{K}_1(\lambda - \mu)\varepsilon_{r+1}(\mu) \tag{39}\end{equation}

\begin{equation}(r = 1, \ldots, N - 1) \) where \( \Lambda_0 = \Lambda_N \equiv 0 \) and

\begin{equation}e_r(0) = \varepsilon_r(0) - \frac{\sin 2k_0 - 2k_0}{\pi u(1 + \lambda^2)} \delta_{r,1} \equiv \varepsilon_r(0) - \frac{\hbar c}{1 + \lambda^2} \delta_{r,1}. \tag{40}\end{equation}

The integration kernels \( \tilde{K}_i \) are obtained from Eq. (\ref{eq:5}) by setting \( u = 1 \). The dressed energy of the charged excitations is

\begin{equation}\varepsilon_c(k) = -2 (\cos k - \cos k_0). \tag{41}\end{equation}

Similarly, one finds reduced integral equations for the spin-components of the density

\begin{equation}\rho_r(\lambda) = \frac{n_c}{\pi(1 + \lambda^2)} \delta_{r,1} - \frac{1}{2\pi} \int_{-\Lambda_r}^{\Lambda_r} d\mu \tilde{K}_2(\lambda - \mu)\rho_r(\mu) + \frac{1}{2\pi} \int_{-\Lambda_{r-1}}^{\Lambda_{r-1}} d\mu \tilde{K}_1(\lambda - \mu)\rho_{r-1}(\mu) + \frac{1}{2\pi} \int_{-\Lambda_{r+1}}^{\Lambda_{r+1}} d\mu \tilde{K}_1(\lambda - \mu)\rho_{r+1}(\mu), \tag{42}\end{equation}

and of the dressed charge matrix \( (r, s = 1, \ldots, N - 1) \):

\begin{equation}\xi_{sr}(\lambda) = \delta_{sr} - \frac{1}{2\pi} \int_{-\Lambda_s}^{\Lambda_s} d\mu \tilde{K}_2(\lambda - \mu)\xi_{sr}(\mu) + \frac{1}{2\pi} \int_{-\Lambda_{s-1}}^{\Lambda_{s-1}} d\mu \tilde{K}_1(\lambda - \mu)\xi_{s,r-1}(\mu) + \frac{1}{2\pi} \int_{-\Lambda_{s+1}}^{\Lambda_{s+1}} d\mu \tilde{K}_1(\lambda - \mu)\xi_{s,r+1}(\mu) \tag{43}\end{equation}

The other elements of the dressed charge matrix are found to be

\begin{equation}Z_{cc} = 1, \quad Z_{cr} = 0, \quad r = 1, \ldots, N - 1 \tag{44}\end{equation}

\begin{equation}Z_{rc} = \frac{1}{2\pi} \int_{-\Lambda_1}^{\Lambda_1} d\lambda \tilde{K}_1(\lambda)\xi_{r1}(\lambda). \tag{44}\end{equation}

The expression for \( Z_{rc} \) can be rewritten using the symmetry of the kernel in (\ref{eq:13}) to obtain a simple relation to the densities of electrons with SU(\( N \)) index \( s \):

\begin{equation}Z_{rc} = \frac{1}{n_c} \int_{-\Lambda_r}^{\Lambda_r} d\lambda \rho_r(\lambda) = 1 - \sum_{s=1}^{r} \frac{n_s}{n_c} \tag{45}\end{equation}
(since $n_s = n_c/N$ for $h_s \equiv 0$ this reproduces the corresponding entries in Eq. (28) in the $u \to \infty$ limit).

For small magnetic fields (corresponding to large but finite values of the $\Lambda_r$) one can employ the Wiener-Hopf method to the integral equations (39) for the dressed energies together with condition (12) to compute the field dependence of the $\Lambda_r$ ($g_s^{-}(\omega)$ is given in Eq. (62)): \[
\sin \frac{\pi r}{N} \exp \left( \frac{-\pi \Lambda_r}{N} \right) = \sum_{s,t=1}^{N-1} \left( \frac{\sin \pi s/N \sin \pi st/N}{\sin \pi s/2N g_s^{-}(-i\pi/N)} \right) \frac{\varepsilon^{(0)}_t}{\pi h_c} \]  
(46)

An analogous computation yields the actual field dependence of the $Z_{rc}$ in Eq. (45). For $h_s \ll h_c$ we find \[
Z_{rc} = \frac{N - r}{N} - \frac{N}{\pi^2} \sum_{s=1}^{N-1} s_{<} (N - s_{>}) \frac{h_s}{h_c} \]  
(47)

where $s_{<} (s_{>})$ is the smaller (greater) of the integers $r, s$.

From (11) the bare energies $\varepsilon^{(0)}_r$ are known to be proportional to the applied magnetic fields, hence the $\Lambda_r$ in (46) show the logarithmic dependence on the fields found previously in the isotropic Heisenberg spin chain [28] and the SU(2) Hubbard model [9]: \[
\Lambda_r \sim \ln \left( \frac{h_c}{h} \right) \]  
(48)

where $h$ is the typical strength of the fields $h_r$. As is known from the SU(2) Hubbard model this strong dependence on small applied fields shows up in the field dependence of the anomalous dimensions since they contain terms $\propto 1/\Lambda_s$ as the leading corrections in their spin-components $\Delta^\pm_r$ ($r = 1, \ldots, N - 1$) (see Eq. (30)) and is related to a nonanalytic field dependence of the magnetic susceptibility in this system [18] and the SU($N$) Heisenberg model [29]. This is in contrast to the charge-components $\Delta^\pm_c$ of the conformal dimensions where (47) implies a linear dependence on the applied fields.

For sufficiently large fields ($h \simeq o(h_c)$) the system saturates in a state with all electrons occupying states in the band(s) with the lowest magnetic energy. This final state depends on the particular choice of the magnetic fields $h_s$ in (1). In the following we shall consider two possible cases explicitly, generalization to others is straightforward.

—One natural interpretation of the SU($N$)-index is that of a orbital quantum number, i.e. taking the electrons in the $r$-th band as having spin $S + 1 - r$ where $N = 2S + 1$. In this picture the coupling to the magnetic fields should be through Zeeman terms giving \[
h_r = -(S + 1 - r) h, \quad 1 \leq r \leq N - 2S + 1 \]  
(49)

for the fields in (1) or \[
e^{(0)}_r = h - \frac{h_c}{1 + \lambda^2} \delta_{r,1}. \]  
(50)

for the bare energies (10). In this interpretation it is straightforward to see that the quantity $h_c$ introduced in Eq. (10) is simply the large-$u$ limit of the critical magnetic field beyond which the ground state of the system is ferromagnetically ordered (i.e. all electrons are in the band with spin-$S$). For $h > h_c$ only excitations with $\Delta M_r = 0$ ($r = 1 \ldots, N - 1$) are gapless. By construction, the corresponding correlation functions are those of free spinless electrons.

—Another possible interpretation of the $N$ bands in the degenerate Hubbard model is that of degenerate bands of spin-$\pm \frac{1}{2}$ electrons. To be specific let us choose the electrons in the first
$N_+$ bands as having spin $\uparrow$ and the ones in the remaining $N_- = N - N_+$ bands as carrying spin $\downarrow$. This choice gives

$$h_r = -\frac{h}{2} \quad \text{for } 0 \leq r \leq N_+, \quad h_r = \frac{h}{2} \quad \text{for } N_+ < r \leq N,$$

for the coupling of a physical magnetic field to the system corresponding to

$$e^{(0)}_r = h\delta_{r,N_+} - \frac{h_c}{1 + \lambda^2}\delta_{r,1}.$$  

(51)

for the bare energies (40). Again the system saturates at large fields: for

$$h \geq h_x = \frac{h_c}{2N_+} \left( \psi \left( \frac{1}{2} + \frac{1}{N_+} \right) - \psi \left( \frac{1}{2} \right) \right)$$

(52)

the groundstate of the system is determined by filled bands for the $\uparrow$-electrons with densities $n_s = n_c/N_+$ while the $N_-$ bands of $\downarrow$-electrons are empty. This state shows SU($N_+$) spin-symmetry, excitations involving creation of electrons in one of the $\downarrow$-bands are massive. The gapless excitations and corresponding critical exponents are given by the $u \to \infty$-limits of the expressions in the preceding section: the dressed charge matrix for the critical degrees of freedom is of the form (28) with $N$ replaced by $N_+$ and $z_c = 1$.

A. Appendix

In this appendix we briefly list some mathematical results which are helpful in the solution of the Wiener-Hopf equations in Sections 4 and 5. The Wiener-Hopf method itself has been reviewed in the appendix of [9].

In the reduction of the matrix Wiener-Hopf problem to scalar ones one has to diagonalize a tridiagonal $(N-1) \times (N-1)$ Toeplitz-matrix of the type

$$T = \begin{pmatrix}
2y & -x & 0 & \cdots & 0 \\
-x & 2y & -x & \ddots & 0 \\
0 & -x & 2y & \ddots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & -x & 2y & \end{pmatrix}$$

(54)

where $x$ and $y$ are real numbers. In terms of $\lambda_{\pm} = y \pm \sqrt{y^2 - x^2}$ one finds for the determinant of $T$

$$\det T = \frac{\lambda_N^N - \lambda_-^N}{\lambda_+ - \lambda_-},$$

(55)

where we assumed $|y| \neq |x|$. Using this result the eigenvalues $t_j$ of $T$ are easily found to be

$$t_j = 2 \left( y - x \cos \left( \frac{j\pi}{N} \right) \right)$$

(56)

with corresponding eigenvectors

$$v^{(j)} = \begin{pmatrix} v_1^{(j)} \\ \vdots \\ v_N^{(j)} \end{pmatrix}, \quad v_l^{(j)} = \sqrt{\frac{2}{N}} \sin \left( \frac{j l \pi}{N} \right).$$

(57)
Thus $T$ is diagonalized by the matrix $U \in O(N - 1)$ with elements

$$U_{jl} = \sqrt{\frac{2}{N}} \sin \left( \frac{jl\pi}{N} \right).$$

(58)

The inverse of $T$ can be written as follows:

$$T^{-1} = \frac{1}{\det T} S$$

(59)

where the elements of the matrix $S$ are given by

$$S_{jl} = a_{j<} a_{N-j>} x^{j>-j<}, \quad a_j = \frac{\lambda_j^+ - \lambda_j^-}{\lambda_+ - \lambda_-}. \quad (60)$$

Here again $j_\ell = \min (j, l)$ and $j_\triangledown = \max (j, l)$.

The main step of the Wiener-Hopf procedure is the factorization of the Fourier-transformed kernel into a product of two functions $g^\pm$ which are analytic in the upper and lower complex $\omega$-plane, respectively. In the derivation of (46) we have used

$$2e^{-|\omega|} \left( \cosh \omega - \cos \frac{s\pi}{N} \right) = g_+^s(\omega)g_-^s(\omega)$$

(61)

with

$$g_+^s(\omega) = g_-^s(-\omega) = 2\pi \left( -\frac{i\omega}{\pi} \right)^{-i\omega/\pi} \exp \left( \frac{i\omega}{\pi} (1 + \ln 2) \right) \frac{\Gamma \left( \frac{s}{2N} - \frac{i\omega}{2\pi} \right)}{\Gamma \left( 1 - \frac{s}{2N} - \frac{i\omega}{2\pi} \right)}$$

(62)

where $\Gamma$ denotes the Gamma-function. Note that the asymptotic behavior of the functions defined in (62) is simply $\lim_{\omega \to \infty} g_\pm^s(\omega) = 1$.

Finally we want note that in the derivation of (29) we do not need the explicit form of the corresponding kernel $K(\omega)$ into factors $G^\pm(\omega)$. One only needs the value $G^\pm(\omega = 0)$ which is – due to the symmetry property $G^+(\omega) = G^-(-\omega)$ – equal to $\sqrt{G^+(0)G^-(0)} = \sqrt{K(0)}$ (see also [28]).

References

[1] F. D. M. Haldane, Phys. Rev. Lett. 45, 1358 (1980); J. Phys. C14, 2589 (1981).
[2] A. A. Belavin, A. M. Polyakov and A. B. Zamolodchikov, Nucl. Phys. B241, 333 (1984).
[3] J. L. Cardy, Nucl. Phys. B270 [FS16], 186 (1986).
[4] H. W. J. Blöte, J. L. Cardy and M. P. Nightingale, Phys. Rev. Lett. 56, 742 (1986); I. Affleck, ibid. 56, 746 (1986).
[5] A. G. Izergin, V. E. Korepin, and N. Yu. Reshetikhin, J. Phys. A22, 2615 (1989).
[6] H. Frahm and N.-C. Yu, J. Phys. A23, 2115 (1990).
[7] F. Woynarovich, J. Phys. A22, 4243 (1989).
[8] H. Frahm and V. E. Korepin, Phys. Rev. B42, 10553 (1990).
[9] H. Frahm and V. E. Korepin, Phys. Rev. B43, 5653 (1991).
[10] N. Kawakami and S.-K. Yang, J. Phys.: Condens. Matt. 3, 5983 (1991).
[11] M. Ogata, M. U. Luchini, S. Sorella, and F. F. Assaad, Phys. Rev. Lett. 66, 2388 (1991).
[12] C. N. Yang, Phys. Rev. Lett. 19, 1312 (1967).
[13] F. D. M. Haldane, Phys. Lett. 80A, 281 (1980); ibid. 81A, 545 (1981) erratum.
[14] P. Schlottmann, Phys. Rev. B43, 3101 (1991).
[15] A. Schadschneider, Dissertation, Universität zu Köln (1991).
[16] T. C. Choy, Phys. Lett. 80A, 49 (1980).
[17] T. C. Choy and F. D. M. Haldane, Phys. Lett. 90A, 83 (1982).
[18] K. Lee and P. Schlottmann, Phys. Rev. Lett. 63, 2299 (1989).
[19] K. Lee and P. Schlottmann, Physica B163, 398 (1990).
[20] P. Schlottmann, Phys. Rev. B45, 5784 (1992).
[21] E. H. Lieb and F. Y. Wu, Phys. Rev. Lett. 20, 1445 (1968).
[22] A. Klümper, A. Schadschneider and J. Zittartz, Z. Phys. B 78, 99 (1990).
[23] A. Schadschneider and J. Zittartz, Z. Phys. B 82, 387 (1991).
[24] B. Sutherland, Phys. Rev. B12, 3795 (1975).
[25] C. N. Yang and C. P. Yang, Phys. Rev. 150, 327 (1966).
[26] H. J. de Vega, J. Phys. A21, L1089 (1988).
[27] J. Suzuki, J. Phys. A21, L1175 (1988).
[28] N. M. Bogoliubov, A. G. Izergin and V. E. Korepin, Nucl. Phys. B275 [FS17], 687 (1986).
[29] P. Schlottmann, Phys. Rev. B45, 5293 (1992).
Figure Captions

Figure 1:

Dependence of the critical value $u_c$ the degeneracy $N$ as obtained from (13). Note that for $u > u_c$ the system is in an insulating phase whereas for $u < u_c$ it shows metallic behavior (for $n_c = 1$).
(a) Lines of constant $z_c$ for vanishing magnetic fields in the $n_c-u$-plane for $N = 2$. The drawn lines correspond to $z_c = 1.0904, 1.1679, 1.2237, 1.2622, 1.2892, 1.3088, 1.3234$ (corresponding to $z_0 = 0.4, 0.8, \ldots, 2.8$). Note that $z_c \to 1$ for $u \to 0$ and $z_c \to \sqrt{N}$ for $u \to \infty$ with $n_c$ arbitrary, $n_c \to 0$ with $u$ arbitrary, and $n_c \to 1$ with $u > u_c$. (b) As in (a) but for $N = 4$, $z_c = 1.1241, 1.2445, 1.3473, 1.4318, 1.5013, 1.5586, 1.6062$. (c) As in (a) but for $N = 6$, $z_c = 1.1316, 1.2630, 1.3799, 1.4812, 1.5689, 1.6453, 1.7122$. (d) As in (a) but for $N = 8$, $z_c = 1.1344, 1.2701, 1.3928, 1.5012, 1.5973, 1.6830, 1.7601$. 
Figure 3: