Generalization of the DGLAP for the structure function $g_1$ to the region of small $x$

B.I. Ermolaev  
*Ioffe Physico-Technical Institute, 194021 St.Petersburg, Russia*  
M. Greco  
*Dept of Physics and INFN, University Rome III, Rome, Italy*  
S.I. Troyan  
*St.Petersburg Institute of Nuclear Physics, 188300 Gatchina, Russia*

The explicit expressions for the non-singlet and singlet components of the DIS structure function $g_1$ comprising the DGLAP expressions for the coefficient functions and the anomalous dimensions, and accounting for the total resummation of the most singular contributions to those are obtained.

PACS numbers:

I. INTRODUCTION

The standard theoretical instrument for studying the structure function $g_1$ is the DGLAP\[1\]. In this approach, $g_1(x, Q^2)$ can be represented as a convolution of the coefficient functions and the evolved quark distributions calculated with NLO approximation. Those results, having been completed with appropriate fits for the initial quark distributions, provide a good agreement with the available experimental data.

On the other hand, the DGLAP evolution was originally obtained for operating in the range of rather large $x$ where higher-loop contributions to the coefficient functions and the anomalous dimensions are small. It is clear that when $x$ is decreasing, such corrections are becoming more and more essential and the DGLAP should stop to work well at certain values of small $x$.

In the present paper we use the results of our previous papers [2, 3, 4] to demonstrate that the impact of the high-order corrections on the $Q^2$ and $x$-evolutions of the non-singlet structure functions is quite sizable for $x \leq 10^{-2}$.

II. DIFFERENCE BETWEEN DGLAP AND OUR APPROACH

As the DGLAP -expressions for the non-singlet structure functions are well-known. In order to make the all-order resummation of the double-logarithmic contributions to $g_1$, in Refs. [3] was used an alternative approach, composing and solving the Infrared Evolution Equations. This approach was improved in Refs. [2] where the single-logarithmic contributions were accounted for and the QCD coupling was running while in Refs. [5] it was considered as fixed. In contrast to the DGLAP parametrization $\alpha_s = \alpha_s(k^2)$, Refs. [2] used the other parametrization. The argumentation in favor of such a parametrization was given in Ref. [3]. In particular, it was shown there that this new parametrization coincides with the DGLAP -parametrization when $x$ is not far from 1 but those parameterizations differ a lot when $x \ll 1$. Refs. [2] suggest the following formula for the non-singlet component of $g_1(\equiv g_1^{NS})$:  

$$g_1^{NS}(x, Q^2) = \langle e^2/2 \rangle \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} (1/x)^\omega C_{NS}(\omega) \delta q(\omega) \exp \left(H_{NS}(\omega)y\right),$$  

(1)

with $y = \ln(Q^2/\mu^2)$ so that $\mu^2$ is the starting point of the $Q^2$ -evolution. The new coefficient function $C_{NS}$ is expressed in terms of new anomalous dimension $H_{NS}$:  

$$C_{NS} = \omega/(\omega - H_{NS}(\omega))$$  

(2)

and the new anomalous dimension $H_{NS}(\omega)$ accounting for the total resummation of the double- and single-logarithmic contributions is  

$$H_{NS} = (1/2) \left[ \omega - \sqrt{\omega^2 - B(\omega)} \right]$$  

(3)

where Where $B$ and related to it the Mellin transform of $\alpha_s$, $A(\omega)$ are expressed in terms of $l = \ln(1/x), b = (33 - 2n_f)/12\pi$ and the color factors (see Ref. [2] for details).
III. COMPARISON OF THE $x$ EVOLUTION FOR $g_1^{NS}$ BY DGLAP TO THE ONE BY EQ. (1)

Let us compare our results (1) to the expressions for $g_1^{NS}$ obtained with the NLO DGLAP. In order to be independent of fits for $\delta q$, we use the simplest input, the bare quark one. In other words, we compare pure evolutions for the non-singlets. We compare them when $x$ is changing while $Q^2$ is fixed. We define $R_{NLO}$ as follows:

$$R_{NLO}(x) = g_1^{NS}/g_1^{NS}_{NLO}$$

when $Q^2$ is fixed and the initial quark distribution corresponds to the bare quark. The results for $R_{NLO}(x)$ and are presented in Fig. 1 taken from Ref. [4]. It shows that for $x \geq 0.005$ the NLO DGLAP evolution predicts the values for the non-singlets similar to the ones predicted by our evolution. However, for $x \leq 0.005$ the situation is opposite. Therefore, we gather that $x \approx 10^{-2}$ is the point where the impact of higher-loop contributions becomes sizable. However, it is known that DGLAP actually works successfully at lower values of $x$. We suggest the following explanation to this fact: it is the impact of the standard fits for the initial quark density used in DGLAP. Indeed, all such fits contain the terms singular when $x \to 0$ and therefore they are able to mimic the impact of the higher-loop contributions absent in DGLAP.

IV. COMBINING DGLAP WITH OUR HIGHER-LOOP CONTRIBUTIONS

Eq. (1) accounts for the total resummation of the double- and single logarithmic contributions to the non-singlet anomalous dimension and the coefficient function. They are leading when $x$ is small but the method we have used does not allow to account for the other contributions, e.g. constants. One can neglect them when $x$ is small but they become important when $x$ is not far from 1. On the other hand, such contributions are accounted in DGLAP where the non-singlet coefficient function $C_{DGLAP}$ and anomalous dimension $\gamma_{DGLAP}$ are known with the two-loop accuracy:

$$C_{DGLAP} = 1 + \frac{\alpha_s(Q^2)}{2\pi} C^{(1)}, \quad \gamma_{DGLAP} = \frac{\alpha_s(Q^2)}{4\pi} \gamma^{(0)} + \left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2 \gamma^{(1)}$$

Therefore, we can borrow from DGLAP formulae the contributions missing in Eq. (1) by adding $C_{DGLAP}$ and $\gamma_{DGLAP}$ to the coefficient functions and anomalous dimensions of Eq. (1). However, both $C_{DGLAP}$ and $\gamma_{DGLAP}$ contain also the terms already accounted for by Eqs. (1):

$$\tilde{C}_{NS} = 1 + B/(4\omega^2), \quad \tilde{H}_{NS} = B/(2\omega) + B^2/(16\omega^3).$$

Now let us make the new coefficient functions $\tilde{C}_{NS}$ and new anomalous dimensions $\tilde{H}_{NS}$ (see Ref. [4] for more details):

$$\tilde{H}_{NS} = \left[H_{NS} - \tilde{H}_{NS}\right] + \frac{A(\omega)}{16\pi^2 C_F} \gamma^{(0)} + \left(\frac{A(\omega)}{16\pi^2 C_F}\right)^2 \gamma^{(1)},$$

$$\tilde{C}_{NS} = \left[C_{NS} - \tilde{C}_{NS}\right] + 1 + \frac{A(\omega)}{8\pi^2 C_F} C^{(1)}.$$

The new coefficient function and anomalous dimension of Eq. (1) comprise the total resummation of the leading contributions from higher loops and the DGLAP expressions in which $\alpha_s(Q^2)$ is replaced by $A(\omega)$ and therefore can be used in the regions of small and large $x$. However, the initial quark densities should be replaced by the new ones, without singularities when $x \to 0$.

V. CONCLUSION

The total resummation of the most singular logarithmic contributions to the anomalous dimensions and coefficient functions leads to the sizable deviation of $g_1$ from the conventional NLO DGLAP predictions at $x < 10^{-2}$ when the bare quark input is used for the initial parton density. It also leads to the Regge asymptotics $\sim x^{-\omega_0}$, with the value of the intercepts $\omega_0 \approx 0.4$ for the non-singlet $g_1$ and $\omega_0 \approx 0.8$ for the singlet $g_1$. DGLAP compensates lack of the
total resummation by using different kinds of singular in $x$ fits for the initial parton distributions and thereby can successfully work at $x < 10^{-2}$. So, our results can be used for specifying the DGLAP fits, fixing their singular part.

On the other hand, the DGLAP anomalous dimensions and coefficient functions contain the terms quite essential at $x$ not far from 1 whereas our approach cannot account for them. In order to get an approach universally good for large and small $x$, we suggest combining the NLO DGLAP anomalous dimensions and coefficient functions and our formulæ.

VI. FIGURE CAPTIONS

Fig. 2: $R_{NLO}(x)$ for $g_1^{NS}$ for $Q^2 = 20 \text{ GeV}^2$.

VII. ACKNOWLEDGEMENT

Work supported in part by Grant RSGSS-1124.2003.2.

[1] G. Altarelli and G. Parisi, Nucl.Phys. B126 (1977) 297; V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 438; L.N.Lipatov, Sov. J. Nucl. Phys. 20 (1972) 95; Yu.L. Dokshitzer, Sov. Phys. JETP 46 (1977) 641.
[2] B.I. Ermolaev, M. Greco and S.I. Troyan. Nucl.Phys. B594 71 (2001); ibid B571 137(2000); Phys.Lett. B579 321 (2004).
[3] B.I. Ermolaev, M. Greco and S.I. Troyan. Phys.Lett. B522 57 (2001).
[4] B.I. Ermolaev, M. Greco and S.I. Troyan. hep-ph/0509019.
[5] B.I. Ermolaev, S.I. Manaenkov and M.G. Ryskin. Z.Phys.C69 259 (1996); J. Bartels, B.I. Ermolaev and M.G. Ryskin. Z.Phys.C70 273 (1996); J. Bartels, B.I. Ermolaev and M.G. Ryskin. Z.Phys.C72 627 (1996).
