Dynamics of chaotic inflation with variable space dimension

Forough Nasseri

Department of Physics, Shahrood University,
P.O.Box 36155-316, Shahrood, Iran
Institute for Studies in Theoretical Physics and Mathematics,
P.O.Box 19395-5531, Tehran, Iran

(March 24, 2022)

Abstract

Within the framework of a model Universe with variable space dimension, we study chaotic inflation with the potential $m^2 \varphi^2/2$, and calculate the dynamical solutions of the inflaton field, variable space dimension, scale factor, and their interdependence during the inflationary epoch. We show that the characteristic of the variability of the space dimension causes the inflationary epoch in the variable space dimension to last longer than the inflationary epoch in the constant space dimension.

PACS number(s): 98.80.Cq, 04.50.+h, 98.80.Bp, 98.70.Vc
Preprint: IPM/P-2001/048

1 INTRODUCTION

Interest in a speculative model in which the number of space dimension decreases continuously as the Universe expands has increased during the past few years [1-5].

A model Universe with variable space dimension proposed in [6] seems to be singularity free, having two turning points for the space dimension. The way of generalizing the standard cosmological model to the variable space dimension used in [6] is questioned, and another way of writing the field equations is proposed [3]. It has been pointed out that the model discussed in [6] has no upper bound for the space dimension.

Authors of Ref.[3] studied critically previous works in [6], and derived new Lagrangians and field equations. They also discussed the model Universe with variable space dimension from the viewpoint of quantum cosmology,
and obtained a general wave function. In the limit of constant space dimension, the wave function of the model approaches the tunneling Vilenkin wave function or the modified Linde wave function [3].

Ref. [2] considers chaotic inflation in the context of a model with time-varying spatial dimension. The dimensionality of the space is defined by constructing the space out of “cells” with characteristic dimension $\delta$ crumpled in such a way as to exhibit an effective dimension $D$. The authors generalized chaotic inflation with the potential $m^2\phi^2/2$ from the three-space to the constant $D$-space, and then to the variable $D$-space. They also generalized the slow-roll approximation. After doing some necessary treatments, step by step, they obtained dynamical equations of the inflaton field, the scale factor and the variable space dimension.

In Ref. [2], an upper limit for the space dimension at the Planck length has been obtained. Based on the radius of the Universe at the onset of inflation to be larger than the Planck length $l_P$, and than the minimum size of the Universe $\delta$, the space dimension at the Planck length must be smaller than or equal to 4. This result corresponds to the value of the universal constant of the model $C$ to be larger than or equal to $\sim 1678.8$.

In this article, we emphasize on a currently active area of research from the point of view of quantum field theory. In such a research, length scales of order the current horizon size could very easily have started out during inflation smaller than the Planck length. The trans-Planckian problem of inflationary cosmology with variable space dimension is interesting and worthy of study. We will address this problem in [7].

Using the effective time variation of $G$, one can conclude an upper limit, to be equal to 4, for the space dimension at the Planck epoch. This result corresponds to a lower limit on the value of the universal constant, $C$, of the model, see Ref. [4].

A short review of the cosmological model with variable space dimension is given in Sec.2. Using the previous work in [4], we obtain in Sec.3.1 the dynamical solutions of the inflaton field, the variable space dimension, and the scale factor. We also calculate in Sec.3.2 how long the inflationary epoch lasts, and show that the characteristic of the variability of the space dimension causes the inflationary epoch in the variable space dimension to last longer than the inflationary epoch in the constant space dimension. Sec.4 contains our conclusions. We will use a natural unit system in which $c = \hbar = 1$ and $l_P = M_P^{-1} = \sqrt{G} = 1.6160 \times 10^{-33}$ cm.
2 REVIEW OF MODEL UNIVERSE WITH VARIABLE SPACE DIMENSION

In [2, 3], there are some clarifications about the idea of variable space dimension, and some motivations for choosing a model Universe with variable space dimension. There are also some reasonably raised questions: what causes the number of spatial dimension to decrease as the Universe expands? Is there any physical process causes or necessitates such a decrease? And how does this “disappearance” of spatial dimensions take place? Is it that gravity gradually stops “feeling” some dimensions or that the size of some of the dimensions shrinks to zero? The reader can find the answers of these questions in detail in Refs. [2, 3]. It is worth mentioning that for generalization of Friedmann equations from three-space to variable space dimension, we cannot define a metric approach. Rather, we use a Lagrangian approach. There is a constraint in this model which can be written as

\[
\left(\frac{a}{\delta}\right)^D = \left(\frac{a_0}{\delta}\right)^{D_0} = e^C, \tag{1}
\]

or

\[
\frac{1}{D} = \frac{1}{C} \ln \left(\frac{a}{a_0}\right) + \frac{1}{D_0}. \tag{2}
\]

Here \(a\) is the scale factor of Friedmann Universe, \(D\) the variable space dimension, \(\delta\) the characteristic minimum length of the model, \(C\) the universal constant of the model. The zero subscript in any quantity, e.g. in \(a_0\) and \(D_0\), denotes its present value. In our formulation, we consider the comoving length of the Hubble radius to be equal to one. So the scale factor means the physical length in cosmology which is the Hubble radius. Note that in Eqs. (1) and (2), the space dimension is a function of cosmic time \(t\). Time derivative of Eq. (1) leads to

\[
\dot{D} = \frac{-D^2 \dot{a}}{Ca}. \tag{3}
\]

It is easily seen that the case of constant space dimension corresponds to the limit of \(C \to +\infty\). Substituting the Ricci scalar for a \(D\)-manifold with constant space dimension, and constant curvature \(k = -1, 0, +1\)

\[
R = \frac{D}{N^2} \left\{ \frac{2\ddot{a}}{a} + (D - 1) \left[ \left(\frac{\dot{a}}{a}\right)^2 + \frac{N^2 k}{a^2} \right] - \frac{2\dot{a} \dot{N}}{aN} \right\}, \tag{4}
\]
in the Einstein-Hilbert action for pure gravity

\[ S_G = \frac{1}{2\kappa} \int d^{(1+D)}x \sqrt{-g} R, \]  

and using Hawking-Ellis action of a perfect fluid, for the model Universe with variable space dimension, the following Lagrangians have been obtained \[ \mathcal{L}_I :=- \frac{V_D}{2\kappa N} \left( \frac{a}{a_0} \right)^D \left[ \frac{(\dot{a})^2}{a} - \frac{N^2 k}{a^2} \right] \]

\[ - \rho N V_D \left( \frac{a}{a_0} \right)^D, \tag{6} \]

and

\[ \mathcal{L}_{II} := - \frac{V_D}{2\kappa N} \left( \frac{a}{a_0} \right)^D \]

\[ \times \left( \frac{2\dot{D} \dot{a}}{a} + \frac{2D \dot{a} \dot{D}}{a} \ln \frac{a}{a_0} + D(D-1) \left[ \left( \frac{\dot{a}}{a} \right)^2 - \frac{N^2 k}{a^2} \right] \right) \]

\[ - \frac{1}{\kappa N} \frac{D \dot{a} \dot{D} dV_D}{a} \frac{a}{a_0} - \rho V_D N \left( \frac{a}{a_0} \right)^D, \tag{7} \]

where \( \kappa = 8\pi M_P^2 \), \( N \) the lapse function, \( \rho \) the energy density, and \( V_D \) the volume of the space-like sections

\[ V_D = \begin{cases} 
\frac{2\pi^{(D+1)/2}}{\Gamma((D+1)/2)}, & \text{if } k = +1, \\
\frac{\pi^{(D/2)}}{2\Gamma(D/2)} \chi_c^D, & \text{if } k = 0, \\
\frac{2\pi^{(D/2)}}{\Gamma(D/2)} f(\chi_c), & \text{if } k = -1.
\end{cases} \tag{8} \]

Here \( \chi_c \) is a cut-off, and \( f(\chi_c) \) is a function thereof, see Ref. \[ 3 \]. In Eq. (4), Ricci scalar has been given for an arbitrary constant space dimension. To get the action of the model, one can substitute Eq. (4) in Eq. (5), and consider the space dimension as a function of cosmic time. Depending on the way of integration in Einstein-Hilbert action, Lagrangian \( \mathcal{L}_I \) and \( \mathcal{L}_{II} \) can be obtained. In the limit of constant space dimensions, or \( D = D_0 \) and \( C \to +\infty \), \( \mathcal{L}_I \) and \( \mathcal{L}_{II} \) approach to Einstein-Hilbert Lagrangian which is

\[ \mathcal{L}^{0}_{I,II} := - \frac{V_{D_0}}{2\kappa_0 N} \left( \frac{a}{a_0} \right)^{D_0} D_0(D_0 - 1) \left[ \left( \frac{\dot{a}}{a} \right)^2 - \frac{N^2 k}{a^2} \right] \]

\[ - \rho N V_{D_0} \left( \frac{a}{a_0} \right)^{D_0}. \tag{9} \]
So, Lagrangian $\mathcal{L}_I$ and $\mathcal{L}_{II}$ cannot abandon Einstein’s gravity. We have introduced $\kappa_0$ for the value of the gravitational coupling constant in the case of the constant space dimension, $D = D_0$ [3]. It is worth mentioning that the model presented here is consistent in the limit of a flat spacetime. This is very important when considering the overall viability of the model. In the limit $k = 0$, the Lagrangian $\mathcal{L}^0_{I,II}$ leads to the flat FRW cosmology. Varying the Lagrangian $\mathcal{L}_I$ with respect to $N$ and $a$, we find the following equations of motion in the gauge $N = 1$, respectively

$$\left(\frac{\ddot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{2\kappa_0 \rho}{D(D - 1)};$$

$$\left(D - 1\right) \left\{ \frac{\ddot{a}}{a} + \left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right] \left(-\frac{D^2}{2C} \frac{d \ln V_D}{dD} - 1 - \frac{D(2D - 1)}{2C(D - 1)} + \frac{D^2}{2D_0}\right)\right\} + \kappa_0 \left(-\frac{d \ln V_D}{dD} - \frac{D}{C} \ln \frac{a}{a_0} + 1\right) = 0.$$  

(11)

The continuity equation of the model Universe with variable space dimension can be obtained by (10) and (11)

$$\frac{d}{dt} \left[ \rho \left(\frac{a}{a_0}\right)^D V_D \right] + p \frac{d}{dt} \left[ \left(\frac{a}{a_0}\right)^D V_D \right] = 0.$$  

(12)

In the limit of constant space dimension, Eqs. (10), (11) and (12) approach the corresponding equations in the constant $D_0$-space [3, 4]. A complete discussion of the field equations of $\mathcal{L}_{II}$ has been given in [3]. Using (1), (3) and (10), one can get the evolution equation of the space dimension:

$$\dot{D}^2 = \frac{D^4}{C^2} \left[ \frac{2 \kappa_0 \rho}{D(D - 1)} - k \delta^2 e^{-2C/D}\right].$$  

(13)

To study chaotic inflation with the potential $m^2 \phi^2/2$ in the framework of the model Universe with variable space dimension, the crucial equations are obtained by substituting

$$\rho \equiv \frac{1}{2} \left(\dot{\phi}^2 + m^2 \phi^2\right),$$

$$p \equiv \frac{1}{2} \left(\dot{\phi}^2 - m^2 \phi^2\right).$$  

(14)
\( \dot{\phi}^2 \ll m^2 \phi^2, \quad \ddot{\phi} \ll DH \dot{\phi}, \quad -\dot{H} \ll H^2, \quad \) (16)

we are led to

\[
\left( \frac{\dot{a}}{a} \right)^2 \simeq \frac{8\pi m^2 \phi^2}{D(D-1)M_P^2},
\]

(17)

\[
\frac{D^2 \dot{\phi}}{a} \left[ \frac{1}{D_0} - \frac{1}{C} \left\{ \ln \chi_c + \frac{1}{2} \psi \left( \frac{D}{2} + 1 \right) \right\} \right] \simeq -m^2 \phi,
\]

(18)

\[
\dot{D}^2 \simeq \frac{8\pi D^3 m^2 \phi^2}{C^2(D-1)M_P^2}.
\]

(19)

Note that in (16), \( D \) is a function of time, and \( H \equiv \dot{a}/a \). One can rewrite Eq. (18) by neglecting the term of the order \( O(C^{-1}) \). So, we have

\[
\frac{D^2 \dot{\phi}}{a D_0} \simeq -m^2 \phi.
\]

(20)

Using Eqs. (8), (17) and (20), it is easily shown that

\[
\left( \frac{\phi}{M_P} \right)^2 = \frac{C D_0}{4\pi} \int_{D_i}^D \frac{dx(x-1)}{x^3} + \left( \frac{\phi_i}{M_P} \right)^2
\]

\[
= \frac{C D_0}{4\pi} \left[ \frac{1}{D} \left( \frac{1}{2D} - 1 \right) - \frac{1}{D_i} \left( \frac{1}{2D_i} - 1 \right) \right] + \left( \frac{\phi_i}{M_P} \right)^2,
\]

(21)

where subscript \( i \), for example in \( D_i \) and \( \phi_i \), denotes its value at the onset of inflation, when \( t = t_i \).

As shown in [2], for different values of \( C \), corresponding to the space dimension at the Planck length \( D_P = 4, 10, 25 \) and \( +\infty \), we calculated the e-folding number \( N \sim 69, 98, 116 \) and 132, respectively. Using the number of e-foldings, we have also obtained the size of the Universe at the onset of inflation, \( a_i \), see Table 1.

In a standard picture of chaotic inflation, the inflationary period naturally lasts for many more than 60 or so e-folds, and the Universe is driven exponentially toward flatness, so that \( \Omega_0 \) is expected to be unity to high accuracy [8].
Table 1: Values of $D_P$, $C$, $\delta$, $N$, $D_i$, and $a_i$ \cite{2}.

| $D_P$ | $C$     | $\delta$ (cm) | $N$  | $D_i$  | $a_i$ (cm) |
|-------|---------|---------------|------|--------|------------|
| 3     | $+\infty$ | 0             | 58.32| 3.00   | $1.02 \times 10^{-25}$ |
| 4     | 1678.8  | $8.6 \times 10^{-216}$ | 69.80| 3.94   | $1.06 \times 10^{-30}$ |
| 10    | 599.57  | $1.5 \times 10^{-59}$ | 98.97| 16.08  | $2.28 \times 10^{-43}$ |
| 25    | 476.93  | $8.4 \times 10^{-42}$ | 116.42| $-22.65$ | $6.02 \times 10^{-51}$ |
| $\infty$ | 419.70  | $l_P$         | 132.25| $-7.50$ | $8.03 \times 10^{-58}$ |

As discussed in \cite{2}, constraints on the dimensionality of the space are derived from requiring that the radius of the space at the onset of inflation, $a_i$, be larger than the Planck length, and than the minimum size of the model, which is $\delta$. Particularly, for $D_P = 25$ and $+\infty$, we have $a_i < \delta < l_P$, and for $D_P = 10$, $\delta < a_i < l_P$. For $D_P = 4$, we have $\delta < l_P < a_i$, see Table 1. The value of the space dimension at the end of inflationary epoch, $D_f$, and the value of the inflaton field at the onset and end of inflation, $\phi_i$ and $\phi_f$, are given in Table 2. It should be noticed that the values of Table 2 are given for $D_P = 3, 4, 10$ and not for $D_P = 25$ or so. Since for $D_P \geq 25$ we have $a_i < \delta < l_P$, we rule out $D_P \geq 25$ \cite{2}.

We should here emphasize on a currently active area of research from the point of view of quantum field theory. In such a research, length scales of order the current horizon size could very easily have started out during inflation smaller than the Planck length, with no inconsistency from the standpoint of classical physics [10-22]. Based on the issue of the trans-Planckian problem some arguments in the paper \cite{2} no longer apply, particularly the constraints from requiring the initial radius of curvature to be larger than the Planck.
length $l_P$. More clearly, for $D_P = 10$ the value of $a_i < l_P$, and in \[2\], we ruled out $D_P = 10$. This result of Ref. \[2\] could not be correct because for $D_P = 10$ we should study in the trans-Planckian physics. This point will be further discussed in \[3\].

We should here emphasize on the meaning of the scale factor in the model. This is necessary because the analysis depends on an interpretation scale factor of the Universe as an effective radius of the space, so that in $D$ dimension, the volume of the Universe is given by $a^D$, resulting in a constraint on the radius compared to a fundamental length $\delta$ of the form Eq.(1). Constraints on the dimensionality of the space are derived from requiring that the radius of the space at the onset of inflation, $a_i$, be larger than the Planck length. This is a reasonable procedure in a non-flat ($k = \pm 1$), since the scale factor defines a radius of curvature for such spacetimes, and thus has a straightforward interpretation as a physical length. This however becomes problematic in flat ($k = 0$) cosmology, since the radius of curvature of the space becomes infinite, and the scale factor is arbitrary. The proper or physical length are obtained from the comoving length by multiplication of the scale factor \[3\]

$$\ell_{\text{physical}} = a(t)\ell_{\text{comoving}}.$$  \hspace{1cm} (22)

While the comoving length does not change with time, the proper length changes with time because of $a(t)$. We take the scale factor having the dimension of length, and the comoving length is a dimensionless quantity. The comoving length is measured by a set of constant rulers, while the proper length is measured by a set of expanding or contracting rulers. The flat model is unbounded with infinite volume, and with infinite radius. For the flat model the scale factor does not represent any physical radius as in the closed case, or an imaginary radius as in the open case, but merely represents how the physical distance between comoving points scales as the space expands or contracts. Based on Friedmann Universe, for the flat Universe with radiation dominated

$$a(t) = a_0 \left( \frac{t}{t_0} \right)^{1/2},$$  \hspace{1cm} (23)

and with dust dominated

$$a(t) = a_0 \left( \frac{t}{t_0} \right)^{2/3}.$$  \hspace{1cm} (24)
So the physical length to the particle or causal horizon $d_H(t)$, at time $t$ is simply obtained by \[d_H(t) = a(t) \int_0^t \frac{dt'}{a(t')} = \begin{cases} 2t, & \text{radiation dominated}, \\ 3t, & \text{dust dominated}. \end{cases}\] (25)

In this paper, we take the comoving length of the particle horizon to be equal to one

\[\ell_{\text{comoving}} = \int_0^t \frac{dt'}{a(t')} = 1.\] (26)

This means that $d_H(t) = a(t)$. In this case, the scale factor of the flat Universe is the physical length, the horizon.

3 DYNAMICS OF THE MODEL

Let us now study the dynamical behavior of the inflationary cosmology with variable space dimension. In Sec. 3.1, we obtain approximately the dynamical solution of $\phi(t)$, $a(t)$, and $D(t)$. In Sec. 3.2, we then study how long the inflationary epoch lasts. We show that the inflationary epoch of the model with variable space dimension is longer than that of the constant space dimension.

3.1 Dynamical solutions

Let us now obtain the evolution equations of $\phi(t)$, $a(t)$ and $D(t)$. Solving Eq. (21), one can get $D$ in terms of $\phi$. This relation is given by

\[D(\phi) = \frac{1 + \sqrt{1 - \frac{2}{D_i} \left(1 - \frac{1}{2D_i}\right) - \frac{8\pi}{CD_0} \left[\left(\frac{\phi_i}{M_P}\right)^2 - \left(\phi/M_P\right)^2\right]}}{\frac{2}{D_i} \left(1 - \frac{1}{2D_i}\right) + \frac{8\pi}{CD_0} \left[\left(\phi_i/M_P\right)^2 - \left(\phi/M_P\right)^2\right]}.\] (27)

It is worth mentioning that Eq.(27) approaches $D = D_i$ in the limit of constant space dimension or $C \to +\infty$. Using Eqs.(17) and (20), the classical equation of motion for $\phi(t)$ is given by

\[\frac{\dot{\phi}(t)}{M_P} = -\frac{mD_0}{2D} \sqrt{\frac{D - 1}{2\pi D}}.\] (28)
This just tells us that the inflaton field is like a ball rolling down a hill, $\dot{\phi} < 0$. Substituting Eq. (27) in (28), we are led to the non-linear differential equation $\dot{\phi} \equiv f(\phi)$. We approximate this differential equation by expanding $f(\phi)$ in inverse powers of $C$. We are therefore led to

$$\frac{\dot{\phi}(t)}{M_P} = \alpha + \frac{\beta}{C} \left[ \left( \frac{\phi}{M_P} \right)^2 - \left( \frac{\phi_i}{M_P} \right)^2 \right] + \mathcal{O} \left( \frac{1}{C^2} \right) + ..., \quad (29)$$

where

$$\alpha \equiv -\frac{mD_0}{2D_i} \sqrt{\frac{D_i - 1}{2\pi D_i}}, \quad (30)$$

$$\beta \equiv \frac{m(2D_i - 3)}{(D_i - 1)} \sqrt{\frac{\pi D_i}{2(D_i - 1)}}. \quad (31)$$

The above differential equation is the first order Riccati equation [23]. It can be easily shown that the solution of (29) is given by

$$\frac{\phi(t)}{M_P} = \sqrt{\frac{\gamma C}{\beta}} \tanh \left[ -\sqrt{\frac{\beta \gamma}{C}}(t - t_i) + \arctanh \left( \frac{\phi_i}{M_P} \sqrt{\frac{\beta}{C \gamma}} \right) \right] + ... \quad (32)$$

where

$$\gamma \equiv -\alpha + \frac{\beta}{C} \left( \frac{\phi_i}{M_P} \right)^2. \quad (33)$$

Using expression of Eq. (32) in terms of the inverse powers of $C$, one can obtain

$$\frac{\phi(t)}{M_P} = \frac{\phi_i}{M_P} + \alpha(t - t_i) + \frac{\alpha \beta}{C} (t - t_i)^2 \left[ \frac{\alpha(t - t_i)}{3} + \frac{\phi_i}{M_P} \right] + \mathcal{O} \left( \frac{1}{C^2} \right) + ... \quad (34)$$

In the limit of constant $D$-space dimension, i.e. $C \to +\infty$, we have

$$\frac{\phi(t)}{M_P} = \frac{\phi_i}{M_P} - \frac{mD_0}{2D_i} \sqrt{\frac{D_i - 1}{2\pi D_i}} (t - t_i). \quad (35)$$

In this limit, we have $D = D_i = D_0$, and Eq. (35) can be rewritten

$$\frac{\phi(t)}{M_P} = \frac{\phi_i}{M_P} - \frac{m}{2} \sqrt{\frac{D - 1}{2\pi D}} (t - t_i). \quad (36)$$
Also, the dynamical solution of the scale factor in the constant $D$-space is given by [4]

$$a(t) = a_i \exp \left( \frac{4\pi M_P^2 (D - 1)}{M_P^2 (D - 1)} [\phi_i^2 - \phi^2(t)] \right).$$  \hspace{1cm} (37)$$

In the variable space dimension, to obtain the dynamical solution of the scale factor, we rewrite Eq.(17)

$$\frac{\dot{a}}{a} = 2m\phi \sqrt{2\pi D(D - 1)}. \hspace{1cm} (38)$$

We suggest that the solution of Eq.(38) is analogous to that of in the constant $D$-space. This means that we have an exponential behavior for the scale factor

$$a(t) = a_i \exp \left( \mathcal{F}[\phi_i^2 - \phi^2(t)] \right), \hspace{1cm} (39)$$

where $\mathcal{F}$ is a function of $D, D_i,$ and $D_0$. To obtain $\mathcal{F}$, we differentiate Eq.(39), and use Eqs.(28) and (38). Therefore, one can get

$$\mathcal{F} = \frac{8\pi D}{D_0 M_P^2 \left[ (2 - \frac{1}{D_i}) D - 1 \right]}.$$ \hspace{1cm} (40)$$

So, the solution of the scale factor in the chaotic inflation with variable space dimension is given by

$$a(t) = a_i \exp \left( \frac{8\pi D [\phi_i^2 - \phi^2(t)]}{D_0 M_P^2 \left[ (2 - \frac{1}{D_i}) D - 1 \right]} \right). \hspace{1cm} (41)$$

During the inflationary epoch with constant space dimension, $D$ is equal to $D_i$, and Eq.(11) approaches

$$a(t) = a_i \exp \left( \frac{4\pi D_i}{M_P^2 D_0 (D_i - 1)} [\phi_i^2 - \phi^2(t)] \right). \hspace{1cm} (42)$$

More carefully, in the limit of constant $D$-space, we have $D = D_i = D_0$, and Eq. (12) gives Eq. (37).

Let us now calculate the dynamical behavior of the space dimension. Substituting Eq.(32) in Eq.(27), one can obtain

$$D(t) = \left( 1 + \left\{ 1 - \frac{2}{D_i} \left( 1 - \frac{1}{2D_i} \right) - \frac{8\pi}{C D_0} \left( \frac{\phi_i}{M_P} \right)^2 \right\} \right).$$

11
\[- \frac{\gamma C}{\beta} \tanh^2 \left[ - \sqrt{\frac{\beta \gamma}{C}} (t - t_i) + \text{arctanh} \left( \frac{\phi_i}{M_P \sqrt{C \gamma}} \right) \right] \] 
\times \left\{ \frac{2}{D_i} \left( 1 - \frac{1}{2D_i} \right) + \frac{8\pi}{CD_0} \left( \frac{\phi_i}{M_P} \right)^2 \right\}^{-1}. \] 

Expanding Eq. (43) in inverse powers of \( C \), one can easily obtain

\[ D = D_i + \frac{2D_0 m(t - t_i)}{C} \left( \frac{m(t - t_i)D_0}{4D_i} - \frac{\phi_i}{M_P} \sqrt{\frac{2\pi D_i}{D_i - 1}} \right) + \mathcal{O} \left( \frac{1}{C^2} \right) + ... \] 

This equation gives \( D = D_i \) in the limit \( C \to +\infty \), corresponding to the constant space dimension, \( D = 3 \).

### 3.2 How long does the inflationary epoch last?

Let us now obtain how long the inflationary epoch lasts. Using Eqs. (19) and (21), one can write

\[ t - t_i = - \frac{\sqrt{C}}{m} \int_{D_i}^{D} \frac{dD}{\sqrt{\frac{D_0 D (1 - 2D)}{D - 1}} + \left[ \frac{8\pi}{C} \left( \frac{\phi_i}{M_P} \right)^2 - \frac{D_0 (1 - 2D_i)}{D_i^2} \right] \frac{D^3}{(D - 1)^3}}, \] 

where the inflaton mass is \( m \simeq 1.21 \times 10^{-6} M_P \). Inflation begins when \( t = t_i \), and ends when \( t = t_f \). Integrating from \( D_i \) to \( D_f \), and using Table 1 and Table 2, one can easily calculate how long the inflationary epoch lasts. This means

\[ t_f - t_i = \begin{cases} 
1.003 \times 10^{-36} \text{ sec}, & \text{if } D_P = 4, \\
1.159 \times 10^{-36} \text{ sec}, & \text{if } D_P = 10.
\end{cases} \] 

As given in [2], using Eq. (36) for \( D = 3 \) one can obtain \( t_f - t_i \) in the case of constant space dimension

\[ t_f - t_i = \frac{2\sqrt{2\pi D}(\phi_i - \phi_f)}{m M_P \sqrt{D - 1}} \simeq 7.598 \times 10^{-37} \text{ sec}. \]
Comparing these values, one can conclude that the variability of the space dimension causes the time duration of the inflationary epoch to be longer than that of in the constant space dimension. More carefully, when \( D_P = 4 \) the value of \( t_f - t_i \) is about 1.320 times when the space dimension is constant

\[
\frac{1.003 \times 10^{-36}}{7.598 \times 10^{-37}} \approx 1.320. \tag{48}
\]

Also, when \( D_P = 10 \) the value of \( t_f - t_i \) is about 1.525 times when the space dimension is constant

\[
\frac{1.159 \times 10^{-36}}{7.598 \times 10^{-37}} \approx 1.525. \tag{49}
\]

So, the characteristic of the variability of the space dimension causes the inflationary epoch in the variable space dimension to be longer than the inflationary epoch in the constant space dimension.

As mentioned before, we do not consider \( D_P = 25 \) or \(+\infty\) in the model, because for these values of \( D_P \), the radius of the Universe at the onset of inflation is smaller than the minimum length of the model, i.e. \( a_i < \delta \). For this reason, in Eq.(16) we have not calculated the value of \( t_f - t_i \) for \( D_P = 25 \) or so. Contrary to the result of Ref.[2], we do not rule out the case of \( D_P = 10 \) for which \( a_i < l_P \). For this value of \( D_P \), we should consider the trans-Planckian problem in inflationary cosmology with variable space dimension [7].

4 CONCLUSIONS

In this paper, we study the dynamical behavior of the inflaton field, variable space dimension, and scale factor, in the framework of a model Universe with variable space dimension proposed in [4]. Expanding the dynamical equations in terms of the inverse powers of the universal constant of the model, we calculated the dynamical solutions. Our study is based on the inflationary potential to be equal to \( m^2 \phi^2 / 2 \).

We also show that chaotic inflation with variable space dimension lasts longer than chaotic inflation with constant space dimension. More carefully, the characteristic of the variability of the space dimension causes the inflationary epoch of the model to last longer than that of the standard cosmology in 3-space. In cosmology in 3-space, chaotic inflation with the potential
\( m^2 \phi^2/2 \) lasts about \( \sim 10^{-37} \) sec, while chaotic inflation with variable space dimension lasts about \( \sim 10^{-36} \) sec.

In this paper, we clear the meaning of the scale factor in the flat Universe. Taking the comoving length of the horizon to be equal to one, we consider the scale factor to be equal to the physical length of the horizon.

Our treatments are based on the space dimension at the Planck length to be equal to 4 or 10. As mentioned in [2, 4], we rule out the cases for which the space dimension at the Planck length to be equal to 25 or so, because in these cases the radius of the Universe at the onset of inflation is smaller than the minimum size of the model.

Contrary to the result of Refs. [2, 4], we do not rule out the case for which the space dimension at the Planck length to be equal to 10. For this case, the radius of the Universe at the onset of inflation is smaller than the Planck length, and one may consider the inflationary cosmology with the variable space dimension in the trans-Planckian problem, which is a currently active research [10-22]. We will address in detail this problem in [7].

Acknowledgments

The author is grateful to an anonymous referee for useful comments.

References

[1] F. Nasseri, in Proc. Fourth Int. Con. on Gravitation and Cosmology, edited by S. Bharadwaj, N.K. Dadhich and S. Kar, Pramana, J. Phys. 55, 605 (2000).

[2] F. Nasseri and S. Rahvar, Int. J. Mod. Phys. D 11, in press (2002), gr-qc/0008044 v2.

[3] R. Mansouri and F. Nasseri, Phys. Rev. D 60, 123512 (1999).

[4] R. Mansouri, F. Nasseri and M. Khorrami, Phys. Lett. A 259, 194 (1999).
[5] F. Nasseri and S. Rahvar, in *Proc. of Cosmology and Particle Physics 2000*, edited by R. Durrer, J. Garcia-Bellido and M. Shaposhnikov (AIP, New York, 2001), astro-ph/0012157 v2.

[6] J.A.S. Lima and M. Mohazzab, Int. J. Mod. Phys. D 7, 657 (1998); M. Khorrami, R. Mansouri and M. Mohazzab, Helv. Phys. Acta 69, 237 (1996).

[7] F. Nasseri, in preparation.

[8] A. R. Liddle, *An Introduction to Modern Cosmology*, (John Wiley & Sons, England, 1999); E. W. Kolb and M. S. Turner, *The Early Universe*, (John Wiley & Sons, New York, 1990); J. V. Narlikar, *Introduction to Cosmology* (Cambridge University Press, England, 1983).

[9] S. Sarkar, in *Large Scale Structure Formation*, edited by R. Mansouri and R. H. Brandenberger (Kluwer, Dordrecht, 2000).

[10] J. C. Niemeyer and R. Parentani, Phys. Rev. D 64, 101301 (2001).

[11] J. C. Niemeyer, Phys. Rev. D 63, 123502 (2001).

[12] J. Martin and R. H. Brandenberger, Phys. Rev. D 63, 123501 (2001).

[13] R. H. Brandenberger and J. Martin, Mod. Phys. Lett. A 16, 999 (2001).

[14] M. Lemoine, M. Lubo, J. Martin and J.-P. Uzan, Phys. Rev. D 65, 023510 (2002).

[15] L. Mersini, hep-ph/0106134; M. Bastero-Gil, hep-ph/0106133.

[16] B. Reznik, Phys. Rev. D 55, 2152 (1997).

[17] H. Saida and M. Sakagami, Phys. Rev. D 61, 084023 (2000).

[18] A. Kempf, Phys. Rev. D 63, 083514 (2001).

[19] J. Martin and R. H. Brandenberger, astro-ph/0012031.

[20] T. Tanaka, astro-ph/0012431.
[21] L. Mersini, M. Bastero-Gil and P. Kanti, Phys. Rev. D 64, 043508 (2001).

[22] T. Jacobson, Prog. Theor. Phys. Suppl. 136, 1 (2000).

[23] D. R. Shier, K. T. Wallenius, *Applied Mathematical Modeling: A Multidisciplinary Approach* (Chapman & Hall/CRC, U.S., 1999); W. Walter, *Ordering Differential Equations* (Springer-Verlag, New York, 1998).