Double inflation in supergravity and the primordial black hole formation

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Abstract

We study a double inflation model (a hybrid inflation + a new inflation) in supergravity and discuss the formation of primordial black holes (PBHs) with mass $\sim 10^{-20} - 10^5 M_\odot$. We find that in a wide range of parameter space, we obtain PBHs which amount to $\Omega \simeq 1$, i.e., PBH dark matter. Also, we find a set of inflation parameters which produces PBHs evaporating now. Those PBHs may be responsible for antiproton fluxes observed by the BESS experiment.

I. INTRODUCTION

In the framework of supergravity the reheating temperature of inflation should be low enough to avoid overproduction of gravitinos [1,2]. The new inflation model [3] generally predicts a very low reheating temperature and hence it is the most attractive among many inflation models [4]. However, the new inflation suffers from a fine-tuning problem about the initial condition; i.e., for a successful new inflation, the initial value of the inflaton should be very close to the local maximum of the potential in a large region whose size is much longer than the horizon of the universe.

A framework of a double inflation [5] was proposed to solve the initial value problem of the new inflation model [3]. It was shown that the above serious problem is naturally solved by supergravity effects if there exists a preinflation (e.g., hybrid inflation [6]) with a sufficiently large Hubble parameter before the new inflation [5].

\footnote{Different models of double inflation were studied by various authors [7].}
In this double inflation model, density fluctuations produced by both inflations are cosmologically relevant if the $e$-fold number of the new inflation is smaller than $\sim 60$ (the total $e$-fold number $\sim 60$ is required to solve flatness and horizon problems in the standard big bang cosmology [3]). In this case, the preinflation should account for the density fluctuations on large cosmological scales [including the Cosmic Background Explorer (COBE) scales] while the new inflation produces density fluctuations on smaller scales. Although the amplitude of the fluctuations on large scales should be normalized to the COBE data [4], fluctuations on small scales are free from the COBE normalization and can have arbitrary power matched to observations. In Ref. [5], a cosmological implication of the double inflation for the large-scale structure formation was discussed. In this paper, we study primordial black hole (PBH) formation in the double inflation model. In Refs. [11,12], the production of black hole MACHO was investigated in the double inflation model for a special case. Here, we consider a wide range of parameter space where PBHs are formed. In particular, we show that the double inflation creates small PBHs evaporating now if those PBHs are produced during matter-dominated (MD) era, i.e., before the end of reheating process after the new inflation. We stress that these evaporating PBHs may account for antiproton fluxes observed by the BESS experiment [14].

Throughout this paper the gravitational scale ($\sim 2.4 \times 10^{18}$ GeV) is taken to be unity.

**II. BLACK HOLE FORMATION**

In a radiation-dominated (RD) universe, PBHs are formed if the density fluctuations $\delta$ at horizon crossing satisfy a condition $1/3 \leq \delta \leq 1$ [15,16], where $\delta$ is the over density at the horizon scale. Masses of the black holes $M$ are roughly equal to the horizon mass,

$$M \simeq 4\pi \sqrt{\frac{3}{\rho}} \simeq 0.066 M_\odot \left(\frac{T}{\text{GeV}}\right)^{-2} \left(\frac{g_*}{50}\right)^{-1/2},$$

where $\rho$, $T$, and $g_*$ are the total cosmic density, temperature, and statistical degrees of freedom at the horizon crossing, respectively.

The horizon length at the black hole formation epoch ($T = T_*$) corresponds to the scale $L_*$ in the present universe given by

$$L_* \simeq \frac{a(T_0)}{a(T_*)} H^{-1}(T_*) \simeq 0.064 \text{ pc} \left(\frac{T_*}{\text{GeV}}\right)^{-1} \left(\frac{g_*}{50}\right)^{-1/6},$$

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$^2$Different models for the PBH formation have been studied in Ref. [13].

$^3$In this paper we investigate the PBHs with mass $\sim 10^{-20} - 10^5 M_\odot$. The upper bound of $10^5 M_\odot$ is not rigorous but much heavier black holes can be produced in the present model if we take the appropriate model parameters. However, the mass of the black holes should be less than the galactic mass $\sim 10^{12} M_\odot$, otherwise the power spectrum conflicts the observations (e.g., distribution of galaxies). The lower bound $\sim 10^{-20} M_\odot$ comes from the requirement that $e$-fold number of the new inflation should be larger than 0.
where $T_0$ is the temperature of the present universe. The comoving wave number corresponding to this length scale, $k_* \equiv 2\pi/L_*$, is

$$k_* \simeq 1.0 \times 10^8 \text{Mpc}^{-1} \left( \frac{g_*}{50} \right)^{1/6} \left( \frac{T_*}{\text{GeV}} \right).$$

Thus, we can write the PBH mass as a function of comoving wave number as

$$M_* \simeq 6.4 \times 10^{14} M_\odot \left( \frac{g_*}{50} \right)^{-1/6} \left( \frac{k_*}{\text{Mpc}^{-1}} \right)^{-2}.$$  

The mass fraction $\beta_*(= \rho_{BH}/\rho)$ of PBHs of mass $M_*$ is given by [16]

$$\beta_*(M_*) = \int_{1/3}^{1} \frac{d\delta}{\sqrt{2\pi}\sigma(M_*)} \exp \left( -\frac{\delta^2}{2\sigma^2(M_*)} \right) \simeq \sigma(M_*) \exp \left( -\frac{1}{18\sigma^2(M_*)} \right),$$

where $\sigma(M_*)$ is the mass variance at the horizon crossing. Notice that the mass fraction $\beta_*(M_*)$ drops off sharply as $\sigma(M_*)$ decreases. The density of the black holes of mass $M_*$, $\rho_{BH}(M_*)$, is given by

$$\rho_{BH}(M_*) \simeq 3.4 \beta_*(M_*) T_*,$$

where $s$ is the entropy density. Since $\rho_{BH}/s$ is constant at $T < T_*$, we estimate the density parameter $\Omega_{BH}(M_*)$ of the black holes in the present universe as

$$\Omega_{BH}(M_*) h^2 \simeq 2.1 \times 10^8 \beta_*(M_*) \left( \frac{T_0}{\text{GeV}} \right),$$

where $h$ is the present Hubble constant in units of 100 km/sec/Mpc. We write it as a function of PBH mass or PBH scale as

$$\Omega_{BH}(M_*) h^2 \simeq 5.4 \times 10^7 \beta_*(M_*) \left( \frac{g_*}{50} \right)^{-1/4} \left( \frac{M_*}{M_\odot} \right)^{-1/2},$$

or

$$\Omega_{BH}(M_*) h^2 \simeq 2.1 \beta_*(M_*) \left( \frac{g_*}{50} \right)^{-1/6} \left( \frac{k_*}{\text{Mpc}^{-1}} \right).$$

As for the mass of the PBHs produced during the RD era, we have a lower limit $M_R$. The mass of PBH produced at the reheating epoch is given by [see Eq.(1)]

$$M_R \simeq 0.066 M_\odot \left( \frac{T_R}{\text{GeV}} \right)^{-2} \left( \frac{g_*}{50} \right)^{-1/2},$$

where $T_R$ is the reheating temperature. As seen later $T_R$ is less than $10^6\text{GeV}$ in our inflation model, and hence $M_R$ is larger than $\sim 10^{-13} M_\odot$. Thus, the PBHs lighter than $M_R$ should be produced during the MD era.

In a MD universe, a relation between the comoving scale $L_*$ and the horizon mass $M_*$ is
\[ L_s = L_R \left( \frac{M_s}{M_R} \right)^{1/3}, \]  
\[ (11) \]

where \( L_R \) is the comoving scale of the horizon at the reheating epoch. Thus, the mass \( M_s \) of the PBH produced during the MD epoch is given by

\[ M_s \simeq 6.3 \times 10^{22} M_\odot \left( \frac{T_R}{\text{GeV}} \right) \left( \frac{k_s}{\text{Mpc}^{-1}} \right)^{-3}. \]
\[ (12) \]

We see that small PBHs of mass, \( M_s \sim 10^{-19} M_\odot \) for example, would be produced for \( T_R \sim 10^6 \text{GeV} \) and \( k_s \sim 10^{16} \text{Mpc}^{-1} \). The condition for the PBH formation in a MD universe is discussed in Ref. [17], where the mass fraction of PBHs of mass \( M_s \) is estimated as

\[ \beta_s(M_s) \simeq 2 \times 10^{-2} \sigma(M_s)^{13/2}. \]
\[ (13) \]

Notice that the mass fraction has a weaker dependence on \( \sigma \) than in the RD case [see Eq.(10)].

During the MD era, the mass fraction \( \beta_s \) stays constant and hence the density of the black holes of mass \( M_s \), \( \rho_{BH}(M_s) \), is given by

\[ \frac{\rho_{BH}(M_s)}{s} \simeq \frac{3}{4} \beta_s(M_s) T_R. \]
\[ (14) \]

We can write the present density parameter \( \Omega_{BH}(M_s) \) for the black holes of mass \( M_s \) as

\[ \Omega_{BH}(M_s) h^2 \simeq 2.1 \times 10^8 \beta_s(M_s) \left( \frac{T_R}{\text{GeV}} \right). \]
\[ (15) \]

### III. DOUBLE INFLATION MODEL

We adopt a double inflation model proposed in Ref. [5]. The model consists of two inflationary stages; the first one is called preinflation and we take a hybrid inflation [18] (see also Ref. [19]) as the preinflation. We also assume that the second inflationary stage is realized by a new inflation model [20] and its e-fold number is smaller than \( \sim 60 \). Thus, the density fluctuations on large scales are produced during the preinflation and their amplitude should be normalized to the COBE data [9]. On the other hand, the new inflation produces fluctuations on small scales. Since the amplitude of small scale fluctuations is free from the COBE normalization, we expect that the new inflation can produce large density fluctuations enough to form PBHs. We choose the predicted power spectrum to be almost scale invariant \( (n_s \simeq 1) \) on large cosmological scales which is favored for the structure formation of the universe [21]. On the other hand, the new inflation gives the power spectrum which has large amplitude and shallow slope \( (n_s < 1) \) on small scales. Thus, this power spectrum has a large and sharp peak on the scale corresponding to a turning epoch from the preinflation to the new inflation, and we expect that PBHs are produced at that scale.

As for the detailed argument of the dynamics of our model, see Refs. [10, 12].
A. Preinflation

First, let us briefly discuss a hybrid inflation model [18]. The hybrid inflation model contains two kinds of superfields: one is $S(x, \theta)$ and the others are a pair of $\Psi(x, \theta)$ and $\bar{\Psi}(x, \theta)$. Here $\theta$ is the Grassmann number denoting superspace. The model is based on the $U(1)_R$ symmetry under which $S(\theta) \rightarrow e^{2i\alpha}S(\theta e^{-i\alpha})$ and $\Psi(\theta)\bar{\Psi}(\theta) \rightarrow \Psi(\theta e^{-i\alpha})\bar{\Psi}(\theta e^{-i\alpha})$. The superpotential is given by [18]

$$W(S, \Psi, \bar{\Psi}) = -\mu^2 S + \lambda S\bar{\Psi}\Psi.$$  \hfill (16)

The $R$-invariant Kähler potential is given by

$$K(S, \Psi, \bar{\Psi}) = |S|^2 + |\Psi|^2 + |\bar{\Psi}|^2 + \cdots,$$  \hfill (17)

where the ellipsis denotes higher-order terms which we neglect in the present analysis for simplicity. We gauge the $U(1)$ phase rotation: $\Psi \rightarrow e^{i\delta}\Psi$ and $\bar{\Psi} \rightarrow e^{-i\delta}\bar{\Psi}$. To satisfy the $D$-term flatness condition we take always $\Psi = \bar{\Psi}$ in our analysis.

We define $N_{\text{COBE}}$ as the $e$-fold number corresponding to the COBE scale and the COBE normalization leads to a condition for the inflaton potential,

$$\left| \frac{V^{3/2}}{V'} \right|_{N_{\text{COBE}}} \simeq 5.3 \times 10^{-4},$$  \hfill (18)

where $V$ is the inflaton potential obtained from Eqs.(16) and (17). In the hybrid inflation model, density fluctuations are almost scale invariant;

$$n_{\text{pre}} \simeq 1 + 2 \left( \frac{V''}{V} \right) - 3 \left( \frac{V'}{V} \right)^2 \big|_{N_{\text{COBE}}} \simeq 1 - \frac{1}{N_{\text{COBE}}} \simeq 1,$$  \hfill (19)

where $n_{\text{pre}}$ is a spectral index for a power spectrum of density fluctuations.

B. New inflation

Now, we consider a new inflation model. We adopt a new inflation model proposed in Ref. [20]. The inflaton superfield $\phi(x, \theta)$ is assumed to have an $R$ charge $2/(n + 1)$ and $U(1)_{R}$ is dynamically broken down to a discrete $Z_{2nR}$ at a scale $v$, which generates an effective superpotential [20, 21],

$$W(\phi) = v^2 \phi - \frac{g}{n + 1} \phi^{n+1}.$$  \hfill (20)

The $R$-invariant effective Kähler potential is given by

$$K(\phi, \chi) = |\phi|^2 + \frac{\kappa}{4} |\phi|^4 + \cdots,$$  \hfill (21)

where $\kappa$ is a constant of order 1. We require that supersymmetry breaking effects make the potential energy at a vacuum vanish, and we have a relation between $v$ and the gravitino mass $m_{3/2}$ as (for details, see Ref. [20])
The inflaton $\phi(x)$ (the scalar component of $\phi(x, \theta)$) has a mass $m_\phi$ in the vacuum with

$$m_\phi \simeq n |g|^{1/n} |v|^{2 - 2/n}.$$  \hfill (23)

The inflaton $\phi$ may decay into ordinary particles through gravitationally suppressed interactions, which yields reheating temperature $T_R$ given by

$$T_R \simeq 0.1 m_\phi^{3/2} \simeq 2.4 \times 10^{17} \text{GeV} n^{3/2} |g|^{3/2n} |v|^{3(1 - 1/n)}$$

$$\lesssim 10^6 \text{GeV} \quad \text{for} \quad m_{3/2} \lesssim 1 \text{TeV}, n \geq 3.$$  \hfill (24)

An important point on the above density fluctuations is that it results in a tilted spectrum with spectral index $n_{\text{new}}$ given by (see Refs. 5, 20)

$$n_{\text{new}} \simeq 1 - 2 \kappa.$$  \hfill (26)

### C. Initial value and fluctuations of the inflaton $\varphi$

The crucial point observed in Ref. 5 is that the preinflation sets dynamically the initial condition for the new inflation. We identify the inflaton field $\varphi(x)/\sqrt{2}$ with the real part of the field $\phi(x)$. It gets an effective mass $m_{\text{eff}} \sim \mu^2$ during the preinflation 5. Thus, this inflaton $\varphi$ tends to the potential minimum,

$$\varphi_{\text{min}} \simeq -\frac{\sqrt{2}}{\sqrt{\lambda}} v \left( \frac{v}{\mu} \right).$$  \hfill (27)

Notice that $\varphi_{\text{min}}$ deviates from zero due to the presence of a linear term $v^2 \mu^2 S \varphi$ (see Ref. 5). Thus, at the end of the preinflation the $\varphi$ settles down to this $\varphi_{\text{min}}$.

After the preinflation, the universe becomes MD because of the oscillation of the inflaton for preinflation. During the MD era between the two inflations, the energy density scales as $\propto a^{-3}$, and the new inflaton oscillates around $\varphi = 0$ with its amplitude decreasing as $\propto a^{-3/4}$. Since the scale factor increases by a factor $(\mu/v)^{4/3}$ during this era, the mean initial value $\varphi_b$ of $\varphi$ at the beginning of the new inflation is written as

$$\varphi_b \simeq \frac{\sqrt{2}}{\sqrt{\lambda}} v \left( \frac{v}{\mu} \right)^2.$$  \hfill (28)

Therefore, the amplitude of fluctuations with comoving wavelength corresponding to the horizon scale at the beginning of the new inflation is given by

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4 The decay rate of the inflaton $\phi$ is discussed in Ref. 5.
\[ \delta \varphi \simeq \frac{H_{\text{pre}}}{2\pi} \left( \frac{H_{\text{pre}}}{m_{\text{eff}}} \right)^{1/2} \left[ \left( \frac{\mu}{v} \right)^{2/3} \right]^{-3/2} \left[ \left( \frac{\mu}{v} \right)^{4/3} \right]^{-3/4} \simeq \frac{H_{\text{pre}}}{3^{1/2}2\pi} \left( \frac{v}{\mu} \right)^{2}, \]  

(29)

where \( H_{\text{pre}} \) is the Hubble parameter during the hybrid inflation, \( H_{\text{pre}}^2 \simeq \mu^4/3 \). The fluctuations given by Eq. (29) are a little less than newly induced fluctuations at the beginning of the new inflation \( \delta \varphi_{\text{new}} \simeq v^2/(2\pi \sqrt{3}) \). Moreover, the fluctuations produced during the preinflation are more suppressed for smaller wavelength. Thus, we assume that the fluctuations of \( \varphi \) induced in the preinflation are negligible compared with fluctuations produced by the new inflation. As mentioned before, the new inflation gives the tilted spectrum on small scales [see Eq. (26)] and hence the fluctuations at the scale corresponding to the beginning of the new inflation is dominant.

Now let us estimate \( e \)-fold number which corresponds to our current horizon. The \( e \)-fold number is given by [22]

\[ N_{\text{tot}} = 62 - \ln \frac{k}{a_0 H_0} - \ln \frac{10^{16} \text{GeV}}{V_k^{1/4}} + \ln \frac{V_k^{1/4}}{V_{\text{end}}^{1/4}} = \frac{1}{3} \ln \frac{V_{\text{end}}^{1/4}}{\rho_{\text{reh}}^{1/4}}, \]

(30)

where \( V_k \) is a potential energy when a given scale \( k \) leaves the horizon, \( V_{\text{end}} \) that when the inflation ends, and \( \rho_{\text{reh}} \) energy density at the time of reheating.

We take \( V_k \simeq V_{\text{end}} \), and \( \rho_{\text{reh}}^{1/4} \simeq \text{a few} \times T_{\text{reh}} \). For \( k = a_0 H_0 \) (i.e., the present horizon scale), we have

\[ N_{\text{tot}} \simeq 67.1 + \left( \frac{5}{3} - \frac{1}{n} \right) \ln v + \frac{1}{2} \ln n + \frac{1}{2n} \ln g. \]

(31)

In estimating \( N_{\text{COBE}} \) we must take into account the fact that the fluctuations induced at \( e \)-fold number less than \( (2/3) \ln(\mu/v) \) before the end of the hybrid inflation reenter the horizon before the new inflation starts. Such fluctuations are cosmologically irrelevant since the new inflation produce much larger fluctuations [10]. Thus, \( N_{\text{COBE}} \) is given by

\[ N_{\text{COBE}} = N_{\text{tot}} - N_{\text{new}} + \frac{2}{3} \ln \frac{\mu}{v}. \]

(32)

\[ \simeq 67.1 + \left( \frac{5}{3} - \frac{1}{n} \right) \ln v + \frac{1}{2} \ln n + \frac{1}{2n} \ln g - N_{\text{new}} + \frac{2}{3} \ln \frac{\mu}{v}. \]

The COBE normalization in Eq. (18) should be imposed by using this \( N_{\text{COBE}} \).

**D. Numerical Results**

We estimate density fluctuations in the double inflation by calculating evolution of \( \varphi \) and \( \sigma \) numerically. For given parameters \( \kappa \) and \( \lambda \), we obtain the break scale \( k_b \) and the amplitude of density fluctuations produced at the beginning of new inflation \( \delta_b \). Here, \( k_b^{-1} \) is the comoving scale corresponding to the Hubble radius at the beginning of the new inflation (a turning epoch). We can understand the qualitative dependence of \( (k_b, \delta_b) \) on \( \kappa, \lambda \) as follows: When \( \kappa \) is large, the slope of the potential for the new inflation is too steep, and the new inflation cannot last for a long time. Therefore, the break occurs at smaller scales.
As for \( \delta_b \), we can see from Eq.(18) that as \( \lambda \) gets larger, \( \mu \) also gets large. In addition, we can show that

\[
\delta_b \equiv \left( \frac{\delta \rho}{\rho} \right)_{\text{new},k_b} \propto \frac{\sqrt{\lambda \mu^2}}{\kappa} \sim \frac{\lambda^{3/2}}{\kappa},
\]

for a given \( \nu \) (see Ref. [10]). Thus, we have larger \( \delta_b \) for larger \( \lambda \).

**IV. MASS VARIANCE AND DENSITY FLUCTUATIONS**

Our double inflation model predicts the amplitude of density fluctuations \( \delta_b \) as a function of inflation parameters. On the other hand, the black hole abundance is expressed as a function of mass variance \( \sigma \) at the time when the corresponding scale enters the horizon. Therefore, when we compare the observations with the prediction of our model, we need a relation between the mass variance \( \sigma \) and the fluctuations \( \delta_b \).

For the power spectrum with the break scale \( k_b^{-1} \) which enters the horizon during the RD epoch, we have a relation between the mass variance and the amplitude of fluctuations as:

\[
\delta_b \simeq \sigma_b / 0.65,
\]

(the numerical factor depends on the tilted spectral index \( n_s \), and within the parameter range we consider, this factor lies between 0.62 ~ 0.67.). For the power spectrum with the break scale \( k_b^{-1} \) which enters the horizon during the MD epoch, we have a relation between the mass variance and the amplitude of fluctuations as

\[
\delta_b \simeq \sigma_b / 2.3,
\]

(again, the numerical factor depends on the tilted spectral index \( n_s \), and within the parameter range we consider, this factor lies between 2.1 ~ 2.8.).

First, let us consider PBH dark matter with \( \Omega_{BH} \sim 1 \) which are produced during the RD epoch. Since the density fluctuations at the break scale is dominant, and the mass fraction \( \beta_* \) has a sharp peak at that scale, only the PBHs of mass corresponding to the break scale are formed. For PBHs produced during the RD epoch (after reheating process), we have, from Eq. (8),

\[
\Omega_{BH} h^2 \simeq 5.4 \times 10^7 \beta_* \left( \frac{g_*}{50} \right)^{-1/4} \left( \frac{M_*}{M_\odot} \right)^{-1/2}.
\]

For example, if we require that the black holes with mass \( \sim M_\odot \) (=MACHOs) be dark matter in the present universe, i.e. \( \Omega_{BH} h^2 \sim 0.25 \), we obtain \( \beta_* \sim 5 \times 10^{-9} \), and from Eq.(5) we obtain

\[\text{[Equation]}\]

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\[ \sigma(M_\odot) \simeq 0.06. \] (37)

From Eq.(34), we see that the break amplitude is \( \delta_b \simeq 0.06/0.65 \simeq 0.092 \), and we find from Eq.(3) that the break scale is \( 2.5 \times 10^7 \text{Mpc}^{-1} \). In Fig.4, we plot the numerical results of our double inflation model for \( n = 4, g = 1, \) and \( v = 10^{-7} \). We see a wide range of parameter space which may account for DM (\( \Omega_{BH} \simeq 0.1 - 1 \)).

Next, let us consider another interesting mass range of PBHs which are evaporating now (\( M_{\text{evap}} \sim 3 \times 10^{-19} M_\odot \)). The PBHs of such light mass are produced during the MD epoch and from Eq.(15) we find

\[ \Omega_{BH}(M_*) h^2 \simeq 4.2 \times 10^6 \sigma(M_*)^{13/2} \left( \frac{T_R}{\text{GeV}} \right). \] (38)

It has been reported, recently, that the BESS experiment \(^{14}\) has observed antiproton fluxes, which may be explained by the evaporation of PBHs if \( \Omega_{BH} h^2 \simeq 2 \times 10^{-9} \) \(^{14}\). In order to explain the BESS result by evaporating PBHs we need

\[ \sigma(M_{\text{evap}}) \simeq 4.4 \times 10^{-3} \left( \frac{T_R}{\text{GeV}} \right)^{-2/13}. \] (39)

From Eq.(24), we estimate the required fraction of the evaporating PBHs as

\[ \sigma(M_{\text{evap}}) \simeq 9.3 \times 10^{-6} n^{-3/13} |g|^{-3/13} v^{-6(1-1/n)/13}. \] (40)

Since the mass variance \( \sigma(M) \) scales as \( \sigma(M) \propto M^{(1-n_s)/6} \) \(^{15}\) during the MD epoch, we obtain the mass variance at the break scale as

\[ \sigma_b \simeq 9.3 \times 10^{-6} n^{-3/13} |g|^{-3/13} v^{-6(1-1/n)/13} \left( \frac{M_b}{M_{\text{evap}}} \right)^{(1-n_s)/6}, \] (41)

and the amplitude \( \delta_b \) is \( \delta_b \simeq \sigma_b/2.3 \) and \( n_s \simeq 1 - 2\kappa \). In Fig.3, we plot an example of the numerical results of our double inflation model\(^*\) for \( n = 3, g = 10^{-4}, \) and \( v = 10^{-6} \). As shown in the figure, we have a set of inflation parameters (\( \kappa, \lambda \)) which may account for the BESS experiment.

V. CONCLUSIONS AND DISCUSSIONS

In this paper we have studied the formation of PBHs by taking a double inflation model in supergravity. We have shown that in a wide range of parameter space PBHs are produced of various masses. These PBHs are interesting since, for example, they may be identified with MACHOs (\( M \sim M_\odot \)) in the halo of our galaxy. Or, they may be PBHs which are just evaporating now (\( M \sim 10^{-19} M_\odot \)). Such black holes are one of the interesting candidates for the sources of antiproton fluxes recently observed in the BESS detector \(^{14}\).

\(^{6}\)For the case of \( n = 4 \), we do not find a consistent parameter region with the BESS experiment.
The dark matter PBHs play a role of the cold dark matter on the large scale structure formation. The scales of the fluctuations for PBH formation themselves are much smaller than the galactic scale and thus we cannot see any signals for such fluctuations in $\delta T/T$ measurements. However, the PBHs may be a source of gravitational waves. If the PBHs dominate dark matter of the present universe, some of them likely form binaries. Such binary black holes coalesce and produce significant gravitational waves [23] which may be observable in future detectors.

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FIG. 1. The amplitude of density fluctuations ($\delta_b$) and PBH masses. Here we take $n = 4, g = 1, v = 10^{-7}$. The region on left hand side is irrelevant since a break scale exceeds our current horizon. The thick solid lines correspond to $M_{BH}/M_\odot = 10^5, 1, 10^{-5}$, from left to right. The short dash - long dash lines show $\Omega_{BH} \simeq 1$ (top) and 0.1 (bottom).
FIG. 2. The amplitude of density fluctuations ($\delta_b$) and PBH masses. Here we take $n = 3, g = 10^{-4}, v = 10^{-6}$. The region on down right corner is irrelevant since a break scale is too small and does not cross horizon during inflation. The thick solid lines correspond to the break masses $M_b/M_\odot = 10^{-15}, 10^{-17}, 3 \times 10^{-19}, \text{ and } 10^{-20}$, from left to right. The short dash-long dash line shows $\Omega(M_{\text{evap}}) h^2 \approx 2 \times 10^{-9}$. The two thin solid lines show the region $\Omega(M_b) h^2 \approx 1$ (top) and $\approx 2 \times 10^{-9}$ (bottom), respectively.