Data-driven MPC of descriptor systems:  
A case study for power networks

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Abstract: Recently, data-driven predictive control of linear systems has received wide-spread 
research attention. It hinges on the fundamental lemma by Willems et al. In a previous paper, 
we have shown how this framework can be applied to predictive control of linear time-invariant 
descriptor systems. In the present paper, we present a case study wherein we apply data-driven 
predictive control to a discrete-time descriptor model obtained by discretization of the power-
swing equations for a nine-bus system. Our results shows the efficacy of the proposed control 
scheme and they underpin the prospect of the data-driven framework for control of descriptor 
systems.

Keywords: Data-driven control, descriptor systems, MPC, Willems’ fundamental lemma, 
optimal control, power-swing equations, power systems

1. INTRODUCTION

Rotor-angle and frequency stability are crucial for a safe 
power-system operation. They describe the ability of the 
power system to preserve synchrony of generators and to 
bound frequency oscillations after load changes or failures 
of subsystems (Kundur et al., 2004).

A classical approach for achieving these goals is hierar-
chical Automatic Generation Control (AGC). AGC typical-
ly consist of a fast inner droop control, where propor-
tional controllers counteract the disturbance and an 
outer secondary control loop, which ensures asymptotic 
convergence to the nominal frequency as well as load 
sharing among generators (Ibraheem et al., 2005; Li et al., 
2016). However, AGC schemes need a model of the gen-
erator dynamics and the transmission lines for controller 
design. In practice, model parameters are often unknown 
or may change over time, e.g., due to temperature drifts. 
Moreover, AGC schemes often neglect input constraints, 
which might lead to difficulties especially in case of large 
disturbances in small grids with limited power generation.

Model Predictive Control (MPC) has the potential to 
include technical limits such as voltage bounds (Liu et al., 
2019). Moreover, data-driven MPC schemes are able to use 
measured data directly instead of a system model, which 
avoids a potentially costly system identification step. Fur-
thermore, they are able to adapt to slow parameter drifts (Berberich et al., 2021; Bilgic et al., 2022).

Data-driven MPC schemes for frequency control have been 
proposed by Wang et al. (2019); Huang et al. (2021). How-
ever, for these schemes rigorous stability properties have—
to the best of our knowledge—so far not been established 
either due to nonlinearities in the system or due to the 
fact that the system is described by a descriptor system 
rather than a discrete- or continuous-time linear system, 
e.g., governed by an ordinary differential equations.

In the present paper, we use rely on results from our pre-
vious work (Schmitz et al., 2022), wherein we tailor Willem’s 
fundamental lemma to Linear Descriptor Systems (LDSs). 
Therein, we established closed-loop stability guarantees 
for LDSs controlled by a data-driven MPC scheme. In 
the present work, we apply these results for frequency 
control in power systems. Specifically, we illustrate the 
performance of data-driven predictive control on a 9-bus 
system with three generators.

This work is organized as follows: Section 2 describes the 
dynamic power system model as a descriptor system with 
the underlying assumptions. Section 3 continues with the 
fundamental lemma for descriptor systems, followed by 
Section 4 focusing on data-driven predictive control. In 
Section 5 this data-driven control scheme is applied to 
frequency control for a 9-bus system.

Notation: N_0, N denote the natural numbers with and 
without zero, respectively. Moreover, for two numbers 
\(a, b \in \mathbb{N}_0\) with \(a \leq b\), the non-empty interval \([a,b] \cap \mathbb{N}_0\) 
is denoted by \([a:b]\).

For a matrix \(A\) the entry of the \(i\)th column and the \(j\)th 
row is denoted by \(A_{i,j}\). The identity \(\mathbb{R}^{n \times n}\) is denoted by \(I_n\).

For a function \(u : \Omega \rightarrow \Gamma\), we denote the restriction of \(u\) 
to \(\Omega_0 \subset \Omega\) by \(u|_{\Omega_0}\). Considering a map \(u : [t : T - 1] \rightarrow \mathbb{R}^k\)
with \( t < T \), we denote the vectorization of \( u \) by
\[
\mathbf{u}(t, T-1) = \left[ f(t)^\top \ldots f(T-1)^\top \right]^\top \in \mathbb{R}^{(T-t)}
\]
and, for \( N \subseteq \mathbb{N} \) with \( L \leq T - t \), the corresponding Hankel matrix \( H_L(\mathbf{u}(t, T-1)) \in \mathbb{R}^{kL \times (T-t-L+1)} \) is defined by
\[
H_L(\mathbf{u}(t, T-1)) = \begin{bmatrix}
    f(u) & \ldots & u(T - L) \\
    \vdots & \ddots & \vdots \\
    u(t + L - 1) & \ldots & u(T - 1)
\end{bmatrix}.
\]
Given a symmetric positive-definite matrix \( Q \) we define the norm \( \|x\|_Q = (x^\top Q x)^{1/2} \).

2. PROBLEM FORMULATION

Consider a power system with \( \mathcal{N} = \{1, \ldots, n\} \) buses and \( \mathcal{G} \subseteq \mathcal{N} \) generators. Without loss of generality we assume \( \mathcal{G} = \{1, \ldots, g\} \). The grid parameters are described by the bus admittance matrix \( Y = G + iB \in \mathbb{C}^{n \times n} \), where \( G \in \mathbb{R}^{n \times n} \) describes the conductances and \( B \in \mathbb{R}^{n \times n} \) describes the susceptances of all transmission lines. We make the following assumptions:

1. a) lossless system, i.e., \( G = 0 \)

2. b) second-order synchronous generator model

3. c1) constant power loads for all nodes

4. c2) constant bus voltages

5. d) small voltage angle differences

6. e) connected system topology

These assumptions are quite specific but also common in the context of frequency control, cf. (Simpson-Porco et al., 2013; Schiffer et al., 2016; Song et al., 2015).

Following Bergen and Hill (1981), the dynamics of a synchronous generator \( i \in \mathcal{G} \) are described by
\[
M_i \dot{\theta}_i + D_i \dot{\omega}_i = p_i - p_i^d - \sum_{j \in \mathcal{N}} B_{i,j} \sin(\theta_i - \theta_j). \tag{1}
\]
Here, \( M_i > 0 \) is the inertia of the generator, \( D_i > 0 \) is the damping, \( p_i \geq 0 \) is the mechanical power provided to the generator, \( p_i^d \geq 0 \) is the active power demand, and \( \theta_i \in \mathbb{R} \) is the phase angle. We rewrite (1) as first-order system
\[
\begin{bmatrix}
    \dot{\theta}_i \\
    \dot{\omega}_i
\end{bmatrix} = \begin{bmatrix}
    M_i^{-1} \left( p_i - p_i^d - \sum_{j \in \mathcal{N}} B_{i,j} \sin(\theta_i - \theta_j) - D_i \omega_i \right)
\end{bmatrix}.
\]
For all nodes without a generator, we assume constant power loads, cf. assumption c1). This leads to the algebraic equations
\[
-p_i^d - \sum_{j \in \mathcal{N}} B_{i,j} \sin(\theta_i - \theta_j) = 0 \quad \text{for all } i \in \mathcal{N} \setminus \mathcal{G}.
\]
Next, we derive an approximation of this nonlinear system involving assumption d). To this end, we first linearize \( \sin(\theta_i - \theta_j) \approx \theta_i - \theta_j \) (Liu et al., 2021; Zhao et al., 2014). This leads, together with Euler-forward discretization, to the discrete-time LDS
\[
\begin{align*}
    E x(t + 1) &= A x(t) + B u(t) + F w(t), \\
    y(t) &= C x(t)
\end{align*}
\tag{2a}
\tag{2b}
\]
with matrices \( E, A \in \mathbb{R}^{(n + g) \times (n + g)}, B \in \mathbb{R}^{(n + g) \times g} \) and \( C \in \mathbb{R}^{m \times (n + g)} \). Let \( L \) be the Laplacian of the admittance matrix \( B \), i.e.
\[
L = \text{diag} \left( \sum_{j=1} B_{1,j}, \ldots, \sum_{j=n} B_{n,j} \right) - B.
\]
Then
\[
E = \begin{bmatrix}
    0 & I_g \\
    0 & 0
\end{bmatrix}, \quad A = E - \tau \begin{bmatrix}
    I_g & 0 & 0 \\
    0 & M & 0
\end{bmatrix}, \quad D = \begin{bmatrix}
    I_g & 0 & 0 \\
    0 & 0 & I_{n-g}
\end{bmatrix},
\]
\[
B = \tau \begin{bmatrix}
    I_g \\
    0
\end{bmatrix}, \quad F = \tau \begin{bmatrix}
    -I_g & 0 & 0 \\
    0 & -I_{n-g}
\end{bmatrix},
\]
where \( \tau > 0 \) is the discretization step size and \( \bar{M} = \text{diag}(M_1, \ldots, M_g)^{-1}, \bar{D} = \text{diag}(D_1, \ldots, D_g) \).

Further, we identify
\[
x(t) = \begin{bmatrix}
    \omega(t) \\
    \theta(t)
\end{bmatrix} \in \mathbb{R}^{n+g}, \quad u(t) = p(t) \in \mathbb{R}^g
\]
\[
w(t) = p^d(t) \in \mathbb{R}^n.
\]
We consider the input \( w \) as an exogenous variable which cannot be controlled and is predetermined by the actual power demand at the nodes.

Lemma 1. (Regularity of the descriptor system).

If the power network described by the susceptance matrix \( B \) is connected, then the LDS (2a) is regular, i.e., there exists \( \lambda \in \mathbb{C} \) such that \( \det(\lambda E - A) \neq 0 \).

Proof: For \( \lambda \in \mathbb{C} \) let
\[
\bar{L}(\lambda) = \lambda^2 \bar{M}^{-1} + \lambda \bar{D} \begin{bmatrix}
    0 & 0 \\
    0 & 0
\end{bmatrix}.
\]
Then with the determinant formula for block matrices
\[
\det \left( \lambda \begin{bmatrix}
    0 & I_g \\
    0 & 0
\end{bmatrix} + \begin{bmatrix}
    I_g & 0 & 0 \\
    0 & M & 0
\end{bmatrix} \begin{bmatrix}
    I_g & 0 \\
    0 & 0 & I_{n-g}
\end{bmatrix} \right)
= \det(\bar{M}) \det \left( \lambda \begin{bmatrix}
    0 & I_g \\
    0 & 0
\end{bmatrix} + \begin{bmatrix}
    M^{-1} & 0 \\
    0 & 0
\end{bmatrix} \begin{bmatrix}
    I_g & 0 \\
    0 & 0
\end{bmatrix} \right)
= \det(\bar{M}) \det(\bar{L}(\lambda)).
\]
The Laplacian \( L \) is a weakly diagonally dominant matrix, that is \( |L_{i,i}| \geq \sum_{j \neq i} |L_{i,j}| \) for all \( i \in \mathcal{N} \). Since the power system is connected, for every \( i, j \in \mathcal{N} \) there exists a sequence of nonzero elements \( L_{i,i}, L_{i,j}, \ldots, L_{i,n} \). For sufficiently large \( \lambda \) the diagonally perturbed matrix \( \bar{L}(\lambda) \) has these two properties as well. Moreover, there exists at least one index \( i \in \mathcal{N} \) such that \( |L(\lambda_{0,i})| > \sum_{j \neq i} |L(\lambda_{0,i})| \).

This implies the invertibility of \( \bar{L}(\lambda_0) \), cf. Shivakumar and Chew (1974) and, therefore, \( \det(\lambda E - A) \neq 0 \) for \( \lambda = -\tau \lambda_0 + 1 \).

3. Fundamental Lemma for Linear Descriptor Systems

Henceforth, we consider the regular descriptor system (2a), i.e. \( \det(\lambda E - A) \neq 0 \) for some \( \lambda \in \mathbb{C} \). Regularity of (2a) guarantees the existence of invertible matrices \( P, S \in \mathbb{R}^{n \times n} \) which transform the system (2a) into quasi-Weierstraß form (see Berger et al. (2012) and (Daal, 1989, Section 8.2))
\[
SEP = \begin{bmatrix}
    I_g & 0 \\
    0 & N
\end{bmatrix}, \quad SAP = \begin{bmatrix}
    A_1 & 0 \\
    0 & I_r
\end{bmatrix}.
\tag{3}
\]
where $A_1 \in \mathbb{R}^{q \times q}$ and $N \in \mathbb{R}^{(n-q) \times (n-q)}$ is a nilpotent matrix with nilpotency index $s$, i.e. $N^{s-1} \neq 0$ and $N^s = 0$. Although the quasi-Weierstraß form is not unique, the dimension $q$ and the nilpotency index $s$ do not depend on the particular transformation matrices $P, S$ and are, therefore, invariants of the system (2a), cf. (Kunkel and Mehrmann, 2006, Lemma 2.10). For more details on the properties of LDSs we refer to Dai (1989); Stykel (2002).

The input-output trajectories of system (2) are collected in the manifest behavior (cf. Polderman and Willems (1997))

$$
\mathcal{B}_m = \{ (u, w, y) : \mathbb{N}_0 \rightarrow \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p \mid x, u, w, y \text{ satisfy } (2) \},
$$

where $\mathcal{B}_m[t, T] = \{ b[t, T] \mid b \in \mathcal{B}_m \}$ contains the restrictions of the input-output trajectories of (2) to the finite time interval $[t : T]$. We say $(u^*, w^*, y^*) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$ is an stationary setpoint of the system (2) if $(u, w, y) \in \mathcal{B}_m$ with $u(t) = u^*$, $w(t) = w^*$ and $y(t) = y^*$ for all $t \in \mathbb{N}_0$.

Next we recall the concepts of R-controllability and R-observability established by Dai (1989), see also Belov et al. (2018); Stykel (2002).

Definition 2. (R-controllability and R-observability). The descriptor system (2) is called R-controllable if

$$
\text{rank}(\begin{bmatrix} \lambda \mathcal{E} - A \end{bmatrix}) = n
$$

holds for all $\lambda \in \mathbb{C}$. System (2) is called R-observable if

$$
\text{rank}(\begin{bmatrix} \lambda \mathcal{E} - A & C \end{bmatrix}) = n
$$

holds for all $\lambda \in \mathbb{C}$. □

We recall the notion of persistency of excitation.

Definition 3. (Persistency of excitation). A function $w: [0 : T - 1] \rightarrow \mathbb{R}^m$ is said to be persistently exciting of order $L$ if the Hankel matrix $H_L(u_{[0,T-1]})$ has rank $mL$, i.e., full row rank.

The following result for LDSs from (Schmitz et al., 2022, Lemma 5) lays the foundation of the Lemma 5.

Lemma 4. (Fundamental lemma for LDSs). Suppose that the system (2a) is R-controllable and regular. Let $(\hat{u}, \hat{w}, \hat{y}) \in \mathcal{B}_m[0, T - 1]$ be such that $(\hat{u}, \hat{w})^T$ is persistently exciting of order $L + q + s - 1$ and $T, L \in \mathbb{N}$ satisfy $(q + n + 1)(L + q + s) - 1 \leq T$. Then $(u, w, y) \in \mathcal{B}_m[0, L - 1]$ if and only if there is $\alpha \in \mathbb{R}^{(q+n+p)L \times (T-s-L+2)}$ such that

$$
\begin{bmatrix}
H_L(u_{[0,T-s-1]}) \\
H_L(w_{[0,T-s-1]}) \\
H_L(y_{[0,T-s-1]})
\end{bmatrix} \alpha =
\begin{bmatrix}
u_{[0,L-1]} \\
w_{[0,L-1]} \\
y_{[0,L-1]}
\end{bmatrix}.
$$

□

Lemma 4 gives rise to a non-parametric description of system (2), where no explicit knowledge of the matrices $E, A, B, F, C$ is needed. Every element of the manifest behavior $\mathcal{B}_m[0, L - 1]$, that is every input-output trajectory of system (2), corresponds to some vector $\alpha$ and vice versa. In particular, an identification step for the system matrices $E, A, B, F, C$ is not necessary to reconstruct the trajectories of system (2).

4. DATA-DRIVEN PREDICTIVE CONTROL

Next we apply Lemma 4 to an Optimal Control Problem (OCP) and propose a data-driven MPC scheme. Suppose that the system (2a) is regular and (2) is R-controllable and R-observable.

Given an observed trajectory $(u, w, y) \in \mathcal{B}_m[t - q - s + 1, t - 1]$ up to time $t - 1$ and a (predicted) constant future power demand $p^f \in \mathbb{R}^n$, we consider the OCP

$$
\text{minimize}_{u, w, y, \alpha(t)} \sum_{k=0}^{L-1} \left\| \hat{y}(t + k) - y^* \right\|^2_Q + \left\| \hat{u}(t + k) - u^* \right\|^2_R
$$

subject to $(\hat{u}, \hat{w}, \hat{y}) \in \mathcal{B}_m[t - q - s + 1, t - 1]$ and

$$
\begin{bmatrix}
u_{[t-q-s+1,t-1]} \\
w_{[t-q-s+1,t-1]} \\
y_{[t-q-s+1,t-1]}
\end{bmatrix} =
\begin{bmatrix}
u^* \\
w^* \\
y^*
\end{bmatrix},
$$

$$
\begin{bmatrix}
u_{[t+L-q-s+1,t+L-1]} \\
w_{[t+L-q-s+1,t+L-1]} \\
y_{[t+L-q-s+1,t+L-1]}
\end{bmatrix} =
\begin{bmatrix}
u^{p^f} \\
w^{p^f} \\
y^{p^f}
\end{bmatrix},
$$

where $(u^*, p^f, y^*)$ is an stationary setpoint. The matrices $Q \in \mathbb{R}^{p \times p}$ and $R \in \mathbb{R}^{m \times m}$ in the stage cost are assumed to be symmetric and positive-definite. The stage cost penalizes deviations from $y^*$, i.e. the desired output, and the control effort. The consistency constraint (6b) guarantees the lateral state of the predicted and true trajectory are aligned up to time $t - 1$, cf. Schmitz et al. (2022). The constraint (6d) ensures that the power demand $p^f$ is met. The nonnegativeness established by the inequality constraint (6c) serves to match the physical interpretation of the variable $\hat{u}$ as mechanical power provided to the generators.

For the prediction horizon $L$ we assume that $L \geq \hat{L} + q + s - 2$, where $\hat{L} = 2s + q$. This together with the terminal constraint (6c) implies initial as well as recursive feasibility of OCP (6) and that the stationary setpoint $(u^*, p^f, y^*)$ is asymptotically stable with respect to the optimal control in closed loop, at least when the constraints (6d) and (6e) are neglected, cf. (Schmitz et al., 2022, Proposition 10). Imposing the latter constraints, the initial feasibility has to be ensured explicitly, while in this case recursive feasibility and stability follow automatically.

Lemma 4 implies that all trajectories contained in the manifest behavior $\mathcal{B}_m[t - q - s + 1, t - 1]$ can be parameterised by a Hankel matrix. Hence, given an input-output trajectory $(\hat{u}, \hat{w}, \hat{y}) \in \mathcal{B}_m[0, T - 1]$ with persistently exciting input $[\hat{u}, \hat{w}]^T$ of order $L + 2(q + s - 1)$, OCP (6) is equivalent (Schmitz et al., 2022) to

$$
\text{minimize}_{u, w, y, \alpha(t)} \sum_{k=0}^{L-1} \left\| \hat{y}(t + k) \right\|^2_Q + \left\| \hat{u}(t + k) \right\|^2_R
$$

(7a)
with \( (\hat{u}, \hat{w}, \hat{y}) : [t - q - s + 1 : t + L - 1] \rightarrow \mathbb{R}^q \times \mathbb{R}^n \times \mathbb{R}^p \) and \( \alpha(t) \in \mathbb{R}^{T - L - 2q + 3} \) subject to
\[
\begin{align*}
\begin{bmatrix}
\dot{u}_{t-q-s+1, t+L-1} \\
\dot{w}_{t-q-s+1, t+L-1} \\
\dot{y}_{t-q-s+1, t+L-1}
\end{bmatrix} &= 
\begin{bmatrix}
H_{L+q+s-1}(u_{[0, T-s]}) \\
H_{L+q+s-1}(\bar{w}_{[0, T-s]}) \\
H_{L+q+s-1}(\bar{y}_{[0, T-s]})
\end{bmatrix} \alpha(t), \\
\begin{bmatrix}
\bar{u}_{t-q-s+1, t+L-1} \\
\bar{w}_{t-q-s+1, t+L-1} \\
\bar{y}_{t-q-s+1, t+L-1}
\end{bmatrix} &= 
\begin{bmatrix}
u^c \\
y^c \\
\vdots
\end{bmatrix} , \\
\begin{bmatrix}
\bar{w}_{t, t+L-1}
\end{bmatrix} &= 
\begin{bmatrix} \bar{p}^c \\
\bar{y}^c \\
\vdots
\end{bmatrix} , \\
\bar{u}_{t, t+L-1} \geq 0 .
\end{align*}
\]

In our data-driven MPC strategy, OCP (7) is solved at each time step and, for the solution \((u^*, w^*, y^*, \alpha^*(t))\), the value \(u^*(t)\) is applied as new input \(u(t)\) to the system (2). The MPC scheme based on the OCP (7) we propose for the descriptor system (2) is summarized in Algorithm 1.

**Algorithm 1: Data-enabled predictive control**

**Input**: prediction horizon \(L\), (pers. exciting) input/output data \((\bar{u}, \bar{w}, \bar{y})\)

1: Set \(t = 0\)
2: Measure \((u, w, y) \in \mathcal{B}_m[t - q - s + 1, t - 1]\) and predict the future power demand \(\bar{p}^d\)
3: Compute \((u^*, w^*, y^*, \alpha^*(t))\) to (7)
4: Apply \(u(t) = u^*(t)\)
5: \(t \leftarrow t + 1\) and goto Step 2

5. NUMERICAL EXAMPLE

We consider a nine-bus system (Schulz and Turner, 1977), with parameters from Cole and Belmans (2011). The system comprises three generators \((g = |\mathcal{G}| = 3)\) and three consumers, see Fig. 1.

Fig. 1. Nine-bus power system with three generators and three consumers.

The corresponding linearized and temporal discretized descriptor system (2a) is regular \((q = 6, s = 1)\) as well as R-controllable. For the output we assume that only data of the phase angles at the generator nodes is accessible, i.e.
\[
C = [0_{3 \times 3} \, I_3 \, 0_{3 \times 6}] , \quad y = [\theta_1 \, \theta_2 \, \theta_3]^T .
\]  

With this choice of \(C\) the system (2) is R-observable.

![Fig. 2. A trajectory emerging from the data-driven predictive control scheme presented in Algorithm 1. The power demand changes at times \(t = 50\) and \(t = 250\).](image)

We compare the performance of the data-driven control approach described by Algorithm 1 and a classical droop controller realized via a feedback loop
\[
p = \tilde{p} - K\omega_g ,
\]

where \(\tilde{p} \in \mathbb{R}^3\) is some constant offset.

Given a certain power demand \(\bar{p}^d = [p_1^d \ldots p_9^d]\), the aim is to steer the system into a corresponding stationary set-point \((u^*, p^d, y^*)\) of the system (2). We apply the predictive control Algorithm 1 with prediction horizon \(L = 20\). As a persistently exciting input signal of order \(L + 2(q + s - 1) = 32\) we choose a function \(\hat{u} : [0 : 399] \rightarrow \mathbb{R}^{12}\) with values drawn independently from a uniform distribution over the interval \([-1, 1]\). The matrices in the stage cost function are chosen as \(Q = 10 \cdot I_3\) and \(R = I_3\). Fig. 2 illustrates a trajectory generated by Algorithm 1, where two changes in the demand of power occur. Fig. 3 shows the closed-loop trajectory corresponding to a droop controller. Observe that compared to the classical droop controller, the data-driven MPC controller yields a faster frequency stabilization with smaller oscillations. This comes at the cost of a large alteration of the mechanical power on the generators down to their lower bounds, which might not be desirable for synchronous generators due to the risk of mechanical stress. However, in case of mainly inverter-based future power grids, such a control action can be safely applied leading to a better overall control performance.
6. CONCLUSION

In the present work we have proposed an approach for data-driven MPC for frequency stabilization with stability guarantees. Our simulation results show a promising performance of the used controller compared to conventional droop control.

Our current model is linear and relies on assumptions such as constant bus voltages and small voltage angle differences, which can be restrictive in practice. Hence, future work aims at relaxing these assumptions. Moreover, the consideration of stochastic uncertainties is subject to future work.

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