Photon-photon interactions can be a source of CMB circular polarization

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Photon-photon interactions, as described with the standard Heisenberg-Euler interaction, can transform plane polarization of the CMB into circular polarization, in the period right after last scattering. We estimate the standard deviation of the resulting circular polarization parameter, as constrained by confining observations to angular regions of large plane polarization, and find results of the order of $10^{-9}$ for the Stokes parameter $V$.

INTRODUCTION

Plane polarization of the cosmic background radiation (CMB) is induced by Compton scattering in an anisotropic photon bath during the recombination period at a temperature $\approx 25$ eV. But Compton scattering by itself cannot induce circular polarization. Therefore the potential for the detection of a tiny amount of circular polarization, should it exist, might provide evidence for some non-standard ingredients in the early universe. For example it can result from anomalously large magnetic fields in the plasma prior to recombination, or from the use of an exotic metric. It might result from photon-axion mixing effects during the course of propagation from the big-bang to the earth or from beyond-the-standard model effects. It might also result from the propagation through magnetized plasmas in galactic clusters.

Photon-photon interactions over a short period immediately after recombination provide an alternative mechanism by which circular polarization of photons can be produced starting from plane polarization, even in the absence of magnetic fields or exotic physics. Beginning from the vacuum photon-photon interaction coming from QED (Heisenberg-Euler interaction), we propose to observe. We take this photon to move with momentum $q$ in the $\hat{z}$ direction, with basis vectors for polarization given by $(\xi_1 = \hat{x}, \xi_2 = \hat{y})$ and define the Stokes parameters $Q, U, V$, by,

$$P = \frac{1}{2} [\sigma_0 + Q \sigma_3 + U \sigma_1 + V \sigma_2],$$

where $\sigma_i$ are Pauli matrices in the space with basis $\xi_1, \xi_2$, augmented with the identity matrix $\sigma_0$.

Now we want to consider the interactions of the polarization of this photon with those of the entire assemblage of photons in the region in which it was born, which we refer to as “the cloud”. A photon in the cloud, moving in the $\theta, \phi$ direction, is taken to have the density matrix in polarization space,

$$P' = \frac{1}{2} [b_0(\theta, \phi) \tau_0 + q(\theta, \phi) \tau_3 + u(\theta, \phi) \tau_1 + v(\theta, \phi) \tau_2],$$

where the $\tau_i$'s are Pauli matrices acting on the polarization states of the cloud photon, in a basis now chosen as,

$$\vec{\eta}_1 = [\hat{x} \cos \phi + \hat{y} \sin \phi] \cos \theta - \hat{z} \sin \theta,$$

$$\vec{\eta}_2 = -\hat{x} \sin \phi + \hat{y} \cos \phi.$$

There are four steps to what follows:
a). Taking all 16 matrix elements of \( L_{11-E} \) of (1) between initial photon polarizations \( \xi, \eta \) and final polarizations \( \xi', \eta' \) for the dead-forward kinematics \( \gamma_{q_1} + \gamma_{q_2} \rightarrow \gamma_{q_1} + \gamma_{q_2} \); then expressing the result as a bilinear form in the matrices, \( \sigma_i, \tau_j \). Since for our kinematics no photon changes its energy, the resulting form serves as the complete effective Hamiltonian, \( H_{\text{eff}} \), for the transformation process. This leads to the evolution equation \( i d\sigma_2/dt = [\sigma_2, H_{\text{eff}}] \) for the operator \( \sigma_2 \) that measures the circular polarization of the beam photon.

b). Determining the initial polarization density matrices, \( P \) for the beam photon and \( P'(\theta, \phi) \) for the cloud photons. These are set by the final Compton scatterings of an anisotropic photon distribution with a prescribed set of modes indexed by wavenumber \( k \), with intensities \( d_k \).

c). Integrating the evolution equation to get \( V \), the circular polarization parameter for the beam photon, taking into account both the progressive weakening of \( H_{\text{eff}} \) from expansion of the universe, and the environmental change for the beam photon due to the finite wavelength of the acoustic perturbations. Here the integration in time begins at last scattering. The evolution equation is effectively quadratic in the density perturbations, so that the usual spherical harmonic expansion is not applicable.

d). Determining the variances of the elements from which \( V \) was constructed in c), directly from that of \( (U^2 + Q^2)^{1/2} \), the latter both given by standard theory and confirmed by observation; then constructing the variance of \( V \).

A. Evolution equation.

From direct evaluation of the matrix elements of (1) for interaction of a beam photon with a single cloud photon we obtain the polarization dependent part of the effective Hamiltonian,

\[
H_{q_1, q_2}^{\text{eff}} = [\text{vol.}]^{-1} W \sum_{i,j=0}^{i,j=3} h_{i,j}(q_1, q_2, \theta, \phi) \sigma_i \tau_j ,
\]

where

\[
W = \frac{2\alpha^2 \omega_1 \omega_2}{9 m_e^4} .
\]

The non-vanishing elements of \( h_{i,j} \) in (4) are,

\[
h_{3,3} = -h_{1,1} = \cos(2\phi)(1 - \cos \theta)^2 , \\
h_{3,1} = h_{1,3} = \sin(2\phi)(1 - \cos \theta)^2 , \\
h_{2,2} = (1 - \cos \theta)^2 .
\]

To get the approximate \( H_{\text{eff}} \) for the interaction of the beam photon with the cloud, we integrate over a thermal distribution for the cloud, \( |q_2| \), but keep a multiplicative angular dependence. The factor of vol.\(^{-1} \) in (5) is then replaced by the cloud photon density, \( n_n \). Defining \( \langle \tau_j \rangle \) as the cloud expectation values of the operators introduced in \( \Box \), \( \langle \tau_3 \rangle = q(\theta, \phi), \langle \tau_1 \rangle = u(\theta, \phi) \), we obtain from \( \Box \) an \( H_{\text{eff}} \) for the polarization evolution of the beam,

\[
H_{\text{eff}}(t) = \sum_{i=0}^{3} \sigma_i c_i ,
\]

where,

\[
c_i = w(t) \int d\Omega \sum_{j=0}^{3} h_{i,j}(\theta, \phi) \langle \tau_j \rangle ,
\]

and,

\[
w(t) = 0.12 \times 10^{-3} \left( \frac{\omega_1}{\omega_1'} \right) \left( \frac{a(t)}{a(t)} \right) \Gamma_C(t) .
\]

Here \( a(t) \) is the scale factor, \( t \), the time at last scattering, and \( \Gamma_C(t) = 8 n_n(t) \pi a^2 m_e^2/3 \) is what the Compton scattering rate would have been, had there always been total ionization of the medium, i.e. no recombination. In calculating the numerical coefficient in (10), we used \( n_n/n_e = 2 \times 10^9 \). \( \Gamma_C \) serves here only as a convenient unit of rate. The factor \( a(t)/a(t) \) in (10) gives the effect of the red-shift on a cloud photon’s energy after last scattering. The thermal average of \( \omega_2 \) at last scattering has been absorbed in the coefficient in (10). We normalized \( \omega_1 \) with its thermal average at last scattering, \( \omega_1' \), since this will be the same ratio as will be observed on earth.

Once the \( \langle \tau_j \rangle(\theta, \phi) \) is known, and the angular integrals performed, the effective interaction \( H_{\text{eff}} \) acts in the polarization space of the beam photon as \( H_{\text{eff}} = w(t)(c_1 \sigma_1 + c_3 \sigma_3) \) and the Heisenberg equation, \( dP/dt = -i[\mathbf{P}, H_{\text{eff}}] \), for evolution of the circular polarization \( V \) is simply,

\[
\frac{1}{2} \frac{dV}{dt} = w(t)(c_1 Q - c_3 U)
\]

B. Initial conditions.

Turning to the calculation of \( c_1, c_3 \) we first need the dependence of the cloud polarization on photon direction. Beginning from the perturbation of the photon distribution in a direction \( \hat{n} \), with angular coordinates \( \Omega \), produced by an assemblage of sound waves in directions \( \{k\} \), and then calculating the polarization density matrix, \( \rho_{i,j} \) subsequent to a Compton scattering, we express the result in terms of the \( \eta \) basis of (1), obtaining,

\[
\rho_{i,j}(\Omega) = \int d\Omega' \delta f(\Omega') \\
\times \left[ \eta_i(\Omega) \cdot \sum_{m=1,2} \eta_m(\Omega') \eta_m(\Omega') \cdot \eta_j(\Omega) \right] .
\]
where \( \delta f(\Omega') \) describes quadrupole photon density perturbations with wave numbers \( \{k\} \). We take,

\[
\delta f(\Omega') = \sum_{\{k\}} \frac{15}{16} d_k (\hat{n} \cdot \hat{k})^2 ,
\]

(13)

where we have dropped some spherically symmetric contributions of the modes, and normalization is arbitrary. Then \( H_{\text{eff}}(t) \) of (5) is constructed using,

\[
\langle \tau_3(\theta, \phi) \rangle = (\rho_{1,1} - \rho_{2,2})/2
\]

\[
= \frac{1}{2} \sum_k d_k \sin^2 \theta_k (1 + \cos^2 \theta) \cos[2(\phi - \phi_k)] ,
\]

(14)

and

\[
\langle \tau_1(\theta, \phi) \rangle = (\rho_{1,2} + \rho_{2,1})/2
\]

\[
= \sum_k d_k \sin^2 \theta_k \cos \theta \sin[2(\phi - \phi_k)] .
\]

(15)

In (14) and (15) we set \( \theta = 0, \phi = 0 \) to determine \( Q \) and \( U \), as induced by the perturbation (12),

\[
Q = \sum_k d_k \sin^2 \theta_k \cos 2\phi_k ,
\]

\[
U = - \sum_k d_k \sin^2 \theta_k \sin 2\phi_k .
\]

(16)

From (15), (14) and (9) we have,

\[
c_1(t_*) = \frac{9\pi}{8} U ,
\]

\[
c_3(t_*) = \frac{9\pi}{8} Q ,
\]

(17)

for every configuration \( \{k\} \). Here \( t_* \) is the time at recombination, defined as the temperature at which the “visibility function” peaks. Now substituting (15) and (17) into (11) we find \( dV/dt = 0 \), at the time of last scattering.

\( Q \) and \( U \) for the beam photons are frozen in time thereafter, as are the polarizations of individual photons in the cloud as well. But \( c_1(t) \) and \( c_3(t) \), relating to the polarizations of the cloud, evaluated at the position of the beam photon, change over the relevant time region after \( t_* \) for the reasons: a.) the beam direction \( \hat{z} \) may point in a direction of rapidly decreasing polarization of the cloud; b) the weakening of cloud polarization densities from free-streaming of the cloud photons. The situation can be quantified in terms of \( \Delta a/a_* \approx 2a_*^{1/2} \Delta \eta/\eta_0 \) where \( \Delta a \) is a change of scale factor over some short period after recombination, \( \Delta \eta \) the corresponding interval of conformal time and \( \eta_0 \) the conformal time at present. The dilution effects for a particular sound mode with multipolarity \( \ell \), as seen from earth, will become important when \( \ell > \eta_0/\Delta \eta \).

To the extent that it is (usually) rather high values of \( \ell \) (e.g., 700-1000) that lead to a particularly bright spot in \((Q^2 + U^2)^{1/2}\), then even a 20% change in scale, \( a \), can take these fluctuations largely out of the picture, leaving behind fluctuations of smaller \( \ell \) that determine, from then on, the coefficients \( c_1(t) \) and \( c_3(t) \) which then no longer are related to \( Q, U \) by (12), freeing us from the cancellation. In our final results we shall restrict to those directions that have a value of linear polarization \( |Q| \) greater than, say, 2.5 standard deviations of \( \delta Q \) (where we have chosen the transverse axes such that \( U = 0 \)), which will comprise a few percent of the directions surveyed, and for this set of directions we estimate the standard deviation of \( V \).

To estimate numbers, we calculate at the \( V \) that is generated from, a system with \( Q \neq 0, U = 0 \), where \( c_1(t_*) = 0 \), as demanded by \( U = 0 \),

\[
V = 2Q \int_{t_*}^{\infty} dt w(t) c_1(t) 
\]

\[
\approx 226 Q w(t_*) \int_{1}^{\infty} dx x^{-7/2} c_1(x) .
\]

(18)

To get the second line we changed the time variable to \( x = a(t)/a_* \) where \( a(t) \) is the scale factor and \( a_* \) is its value at recombination, used \( dt = dx/(xH) \), where \( H \) is the Hubble rate at time \( t \), and substituted \( \Gamma_C \approx 113(x)^{-3/2}H \) (ref. [13] eq. 3.46.). We shall make a rough estimate by dividing the the integration region in (18) into two ranges; the first, \( 1 < x < 1.2 \), in which \( c_1 = 0 \), and the second \( x > 1.2 \), in which we take \( c_1(x) = (9\pi/8)\bar{U} \), but estimate the variance of \( \bar{U} \), from contributions \( \ell < 400 \). That is to say, in the latter segment we effectively drop all of the earlier-dominant short wave-length fluctuations and treat the longer wave-length ones as statistically independent.

**Distribution functions**

We treat \( d_k \) as a variable with a gaussian distribution, as are therefore the derived quantities \( Q, U, c_1, c_3 \). However, as input data we shall use the present theoretical plus observational results for \((Q^2 + U^2)^{1/2}\) rather than going back to the basic power spectra derived from inflationary models. In a more fundamentally based treatment beginning with the \( d_k \) variables, the magnitudes of the sonic wave-number modes \( k \) would have entered, and the \(|k|\) dependence of the power spectrum as well. But in our estimates, this has all gone into our input values of \( Q \).

The quantities \( Q, U \) here are defined for a beam with a particular direction, \( \hat{z} \), in space, but observational and theoretical results are reported in terms of spherical harmonic expansions indexed with \( \ell \). From results in the literature [12, 14] we find \( \langle Q_i Q_j \rangle \approx 0.06 \pi^{-1}[\mu K]^2 \), after smoothing out the oscillations, up to \( \ell = 1000 \) or so, after which \( \langle Q_i Q_j \rangle \) levels off and then starts to decrease with increasing \( \ell \). Also, in the conventional language per-
turbations are defined as pseudo-variations in temperature, but we take dimensionless \(Q,U\) etc. which are instead: (polarized number-density)/(uniform background number-density), with factors of background density already having been subsumed in the function \(w(t)\). All that is required to change to this dimensionless unit is the replacement \(^{\pm}\mu K\) by the factor \((3/2.7) \times 10^{-6}\). Inverting the expansion in spherical harmonics we have,

\[
\langle \tau_3(\theta, \phi)\rangle = \tau_3(\theta', \phi') + \tau_1(\theta, \phi)\tau_1(\theta', \phi')
\]

\[
+ \sum_{\ell,m} \sum_{\ell',m'} \left[ \left( r_3^{\ell,m} \right) r_3^{\ell',m'} + r_1^{\ell,m} r_1^{\ell',m'} \right] Y_{\ell,m}(\theta, \phi)Y_{\ell',m'}(\theta', \phi').
\]

(19)

The correlation function in (19) vanishes except when \((\ell, m) = (\ell' m')\). Taking the limit in which \(\theta, \phi, \theta', \phi'\) all approach zero we get,

\[
\langle QQ + UU \rangle = \sum_{\ell,|m| \leq \ell} \langle |Q_\ell Q_\ell + U_\ell U_\ell | \rangle \times Y_{\ell, m}(0,0)Y_{\ell, -m}(0,0) \approx 370[\mu^2 K^2].
\]

(20)

where in performing the sum in the last line we have taken

\[
\sigma_\varphi^2 = \langle |Q_\ell Q_\ell + U_\ell U_\ell | \rangle \exp(-\ell/1000)[\mu^2 K^2],
\]

(21)

in accord with the above remarks.

We estimate the variance \(\langle UU \rangle \approx 124[\mu^2 K^2]\) for \(x > \tilde{x}\) by cutting off the sum in (20) at \(\ell = 400\). Then using (17) to obtain \(\langle c_1 c_1 \rangle\) in our later era, and substituting into (18) we obtain,

\[
\sigma_V = 2.3[\langle QQ \rangle \langle UU \rangle]^{1/2} 10^{-12} \approx 24 \times 10^{-9}.
\]

(22)

Given this standard deviation we would expect to find \(V > .6 \times 10^{-9}\) around 5% of the time. The degree of polarization can also be enhanced by making a cut in the beam photon energies; in the above we have taken the thermal average, but since the rate factor (10) contains one power of the beam energy, we could gain. e.g., another factor of 2 by taking photons only from the upper 20% of the distribution.

**DISCUSSION**

There are distinctive features of the \(V\) signal that is produced by the mechanism. The proportionality to photon energy is one that can be tested, in principle, in measurements on a single spot in the sky. Separately, the preference for large \(\ell\), or spots of smaller angular dimension, is even stronger in the \(V\) variance than in the variance of the plane polarization. These features appear to provide a way of distinguishing between the present mechanism and one based on linear into circular transformation in a magnetized plasma living inside a galactic cluster along the transit path (10), or one in which the magnetized plasma is taken to exist before and at the time of recombination (4).

We have not attempted calculation of the effects of our mechanism in the case of tensor perturbations. In the presence of photon-photon interactions they may be more efficient at producing \(V\) than are scalar perturbations, for case in which amplitudes are comparable. Indeed if the numbers reported in the recently published Bicep2 results (17) are sustained, they could be the dominant contributor to circular polarization.

Our use of the basic interaction (11) was perturbative, in the sense that the quantity \(V\) (and \(v\)) remained very small, and had insignificant back-reaction at later times through the evolution equations on the quantities \(Q,U\) that were specified in the initial condition. In some other astrophysical venues in which basic interaction (11) can be expected to play a role, for example in the core of \(\gamma\)-ray bursts, the back reactions will matter. The effective interaction for polarization exchange will still be given by (5). In this connection, we note the close correspondence of the formalism used here with that used in predicting effects of the neutral current \(\nu - \nu\) interaction on the neutrino flavor evolution near the core of the supernova (18). In the latter problem there is a large literature devoted to sudden flavor transformations due to the nonlinear equations (19). We might expect the same with respect to photon polarizations in, e.g., gamma ray burst analysis.

To summarize: we have calculated some ingredients for a respectable calculation of the variance of \(V\). Up through eq. (17) our results are authoritative, but the estimates that follow are very crude. The numbers perhaps justify a more complete approach. While our results are orders of magnitude smaller than presently planned observations could detect (17), we hope that they might encourage the development of observational technology that could make detection possible.

This work was supported in part by NSF grant PHY-0455918.

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