Robust design optimization of a nonlinear monostable energy harvester with uncertainties

Yi Li · Shengxi Zhou · Grzegorz Litak

Received: 12 April 2020 / Accepted: 18 July 2020 / Published online: 24 July 2020
© The Author(s) 2020

Abstract Based on the improved interval extension, a robust optimization method for nonlinear monostable energy harvesters with uncertainties is developed. In this method, the 2nd order terms in the interval extension formula of the objective function (output voltage) are kept so this approach is suitable for a nonlinear energy harvesting system. To illustrate this method, uncertain mass, uncertain capacitance and uncertain electromechanical coupling coefficient are optimized to maximize the central point of output voltage whose deviation of which is simultaneously minimized. Then, an optimal design with different robustness is obtained. The results also show that the robustness of the optimal design of nonlinear monostable energy harvesters is increased, but the cost of performance has to be paid for. Overall, the framework provides the optimal design for nonlinear energy harvesters (monostable, bistable, tristable, multistable harvesters).

Keywords Nonlinear · Energy harvesting · Uncertainty · Robust design · Optimization

1 Introduction
In the last decade, vibration energy harvesting was often researched for its great potential application and market in powering low-power wireless sensor networks, portable electronic mobile devices and embedded devices applied to monitor structural health and vibration control [1–6]. Different kinds of energy harvesters were designed to efficiently harvest energy from flow induced vibration, base vibration, and human bodies, etc. [7–12]. Especially, nonlinear vibration energy harvesters were designed and experimentally tested to enhance broadband energy harvesting [13–20].

Erturk and Inman [21, 22] designed a Duffing-typed bistable energy harvester to efficiently harvest energy from base harmonic excitations, and they found the high-energy interwell orbits. Cottone et al. [23] and later Litak et al. [24, 25] respectively designed nonlinear piezoelectric energy harvesters through exploring the dynamical characteristics of nonlinear oscillators under random excitations. Zhou et al. [26, 27] numerically and experimentally investigated the energy harvesting performance of tristable energy harvesters under harmonic excitations, and then they deduced the analytical solutions. It is found that nonlinear energy harvesters are very sensitive to the system parameters and external excitation conditions [28, 29]. In addition, during the designing and production of nonlinear energy...
harvesters, their structural and electrical parameters are uncertain due to the errors in the manufacturing process. These uncertain parameters have a significant impact on the output voltage. To quantify this influence, two kinds of uncertain analysis methods are developed: non-probabilistic and probabilistic approaches [30].

For the probabilistic method, many samples and data are required to properly determine the probability density functions of uncertainties (for example, Monte Carlo Simulation) [31]. In fact, it is difficult to get enough samples to satisfy statistical requirements. Therefore, the non-probabilistic approaches were developed, in which we just need to know the bounds of uncertain parameters and can use the interval number to define these uncertainties. Researchers developed several interval analysis method, such as the general interval analysis [31–33], the Parameterized Interval Analysis (PIA) [34, 35] which has the optimization and anti-optimization problems [33]. Among interval analysis methods, the Taylor Expansion [36–40] is popular since it is not required to alter the governing functions of system dynamics. Especially, there is a nonlinear relation between the output response of the system and uncertainties. Recently, the uncertainties in the nonlinear vibrational energy harvester have been investigated. Nanda et al. [41] used the method of Quadratures and the Maximum Entropy principle to study the influence of uncertainties on the mean power output of piezoelectric based energy harvesters. Liu et al. [42] carried out the probabilistic response analysis of a nonlinear energy harvester under Gaussian colored noise excitations. Li et al. [43] studied the influence of the uncertain external excitation on the output voltage of nonlinear monostable energy harvesters using an improved interval extension approach based on the 2nd order Taylor series. Meanwhile, some optimization researches are conducted to improve the performance of piezoelectric energy harvesters [44].

In above works, uncertain parameters are not taken into consideration, but these uncertainties can make the objective function apart from the optimal value and harm the gain of the optimization. In this case, a stable design in which the influence of uncertainties on the objective function is small, is urgently needed. This means that the robust design optimization (RDO) [45] should be conducted. The RDO can simulate different uncertain parameters in engineering optimization and reduce the sensitivity of the objective function to uncertain variables. Therefore, such an approach can minimize the mean and variation of the objective function simultaneously.

This paper presents a robust design optimization method based on the improved interval extension to obtain a robust design of the nonlinear monostable energy harvester (NMEH) with uncertain design variables (mass, capacitance, electromechanical coupling coefficient). These uncertain design variables are described by interval number. The central point value and deviation of output voltage are calculated by the improved interval extension based on the 2nd order Taylor series. Then, a robust design optimization is constructed by maximizing the central point value of output voltage and minimizing the deviation of output voltage. This double objective optimization issue is converted into a single objective optimization issue by the penalty function approach, and finally this unconstrained optimization problem is solved.

2 Theoretical solutions of the NMEH

Figure 1 describes the sketch diagram of the magnetic coupled NMEH. As shown in Fig. 1, the magnetic force can be produced by the endmost magnet and two external magnets. Under the excitation \(x_b(t)\), the NMEH can be governed by the following equations [43, 46]:

![Fig. 1 The NMEH with magnetic coupling.](image-url)
\[ M\ddot{x}(t) + C\dot{x}(t) + F_r - \theta V(t) = -nM\ddot{x}_p(t) \] (1)

\[ C_p\ddot{V}(t) + V(t)(R)^{-1} + \theta\dot{V}(t) = 0 \] (2)

where \( x(t) \) presents the tip displacement. \( M, C \) and \( F_r = K_1x(t) + K_3x^3(t) \) are respectively the equivalent mass, damping, nonlinear restoring force. \( C_p, \theta \) and \( R \) are respectively the equivalent capacitance, electromechanical coupling coefficient, load resistance. \( V(t) \) is the output voltage. \( n \) is the amplitude-wise correction factor [46].

Under the base acceleration excitation \( A\cos(\omega t) \) \((A\) stands for the base acceleration excitation amplitude), the governing equations of the NMEH can be rewritten as follows [43, 47]:

\[
\ddot{x}(t) + \frac{\ddot{x}(t)}{C} - x(t) + x^3(t) - \frac{V(t)}{\theta} = nA\cos(\omega t)
\] (3)

\[ C_p\ddot{V}(t) + V(t)(R)^{-1} + \theta\dot{V}(t) = 0 \] (4)

where \( \overline{C} = C/M, \overline{K_1} = K_1/M, \overline{K_3} = K_3/M \) and \( \overline{\theta} = \theta/M \).

The steady-state displacement and voltage solutions of the harvester can be obtained via the lower order harmonic balance method [43, 46]:

\[
\left( \frac{3}{4K_3} - \omega^2 + \frac{SC_p\omega}{K_1} \right) r^2 + \left( \frac{\omega^2}{C} + \frac{S}{R} \right) r^2 = (nA)^2
\] (5)

\[
V = r \theta \omega \left| \frac{1}{\sqrt{1 + (C_p\omega)^2}} \right|
\] (6)

where \( r \) is the amplitude of the response displacement, \( V \) is the amplitude of the response voltage, and \( S = \frac{\theta \omega}{\sqrt{1 + (C_p\omega)^2}} \).

### 3 Uncertain analysis and robust optimization

To reduce this effect, the sensitivities of objective function to these uncertainties should be limited during maximizing the output voltage, and robust optimization should be conducted.

In this paper, \( M, C_p \) and \( \theta \) are assumed to be uncertain parameters. Here, we use interval numbers to define them, and their corresponding central values and deviations are listed in Table 1. Other parameters of the NMEH are given in Table 2. Acceleration and amplitude of excitation are set as 0.5 g and 15 Hz, respectively.

Firstly, the improved interval extension based on the 2nd order Taylor series is employed to take effect of uncertainties listed in Table 1 and achieve the deviation of the objective function (output voltage). For the interval analysis, the uncertainties are defined by the interval variables [43]:

\[
x' = \left[ \frac{x - \bar{x}}{2} \right]
\] (7)

where \( x \) and \( \bar{x} \) are respectively the lower and upper bounds of interval. The central value \( (x_c) \) and deviation \( (\Delta x) \) are shown as:

\[
x_c = \frac{x + \bar{x}}{2}
\] (8)

\[
\Delta x = \frac{\bar{x} - x}{2}
\] (9)

Alternatively, an interval \( x' \) can be rewritten by using the PIA [43]:

\[
x' = x_c + \Delta x\sin x
\] (10)

\[ x \in \left[ \frac{-\pi}{2}, \frac{\pi}{2} \right] \] (11)

The 2nd order Taylor expansion of the output voltage \( V(X') \) around the central value \( X_c \) is [43]:

| Parameters                       | Uncertainty       | Deviation       |
|----------------------------------|-------------------|-----------------|
| **Central value**                |                   |                 |
| \( M \text{ (kg)} \)             | 0.001744162       | 0.000087208     |
| \( C_p \text{ (F)} \)            | \( 8.30150 \times 10^{-9} \) | \( 4.15075 \times 10^{-10} \) |
| \( \theta \text{ (N V}^{-1} \) ) | \( -3.65422 \times 10^{-5} \) | \( 1.8271 \times 10^{-6} \) |
The finite difference approach is used to calculate the sensitivities, and the equations are given as:

\[ V'(x) = \frac{V(x + h) - V(x - h)}{2h} \]

\[ V''(x) = \frac{V(x + h) - 2V(x) + V(x - h)}{h^2} \]

where \( h \) is the finite-difference interval.

According to Eqs. 9, 16 and 17, the deviation of the output voltage is computed as:

\[ \Delta V(X') = \frac{V(X') - V(X)}{2} = \frac{\text{maximum}(G(z_k)) - \text{minimum}(G(z_k))}{2} \]

Secondly, the improved interval extension method is employed to build the robust design optimization issue. Compared with the deterministic optimization problem of the NMEH given in Table 3, uncertain parameters can be simulated and the robust design of the NMEH can be obtained by maximizing the central value of output voltage \((V(X_c))\) and minimizing the deviation of output voltage \((\Delta V(X'))\) in Eq. 20 to limit the effect of these uncertainties on the objective function. As a result, the deterministic optimization with a single objective function is transformed into the robust design optimization with the double objective function (Table 4).

The double objective optimization model listed in Table 4 is further transformed into the following
unconstrained optimization model \( \min f(X^i) \) based on the penalty function method \([45, 48]\):

\[
f(X^i) = \frac{e_1(-V(X_c) + e_2)}{e_3} + \frac{(1 - e_1)(\Delta V(X^i) + e_2)}{e_4}
\]

(21)

where the negative sign in front of \( V(X_c) \) converts finding the maximum of objective function into searching for the minimum. \( e_1 \in [0, 1] \) is the weighting factor whose different values illustrate the different minimization of the central value of minus output voltage \( (-V(X_c)) \) and the deviation of output voltage \( (\Delta V(X^i)) \). This means, a stronger robust design can be obtained by reducing \( e_1 \). The parameter \( e_2 \) is used to keep the non-negativity of \( (-V(X_c) + e_2) \) and \( (\Delta V(X^i) + e_2) \). The parameters \( e_3 \) and \( e_4 \) are normalization factors to guarantee that the polynomial with a central value pertain to the same order of magnitude as the one with deviation. The optimization problem (Eq. 21) with uncertain design variables and constraint listed in Table 4 can be solved by the nonlinear programming method \([49]\).

The above works can be summarized as one process shown in Fig. 2 according to which three cases are optimized; the first is the deterministic optimization given in Table 3, the other two are both the robust design optimizations listed in Table 4. In the robust design optimizations, we alter the parameter \( e_1 \) to obtain optimal designs with different robustness \( (e_1 = 0.9 \text{ and } 0.1) \). The results will be given in the following section.

### 4 Results and discussion

The design variables in the three cases are mass, capacitance and electromechanical coupling coefficient, bounds of which are given in Table 5. These design variables are uncertain especially in the last two cases (robust optimization), and their deviations are listed in Table 1.

In the first case, the deterministic optimization problem defined in Table 3 is carried out, and a stopping criteria is set to determine the convergence of optimization. In this algorithm, if the change in the value of the objective function is smaller than tolerance, \( |f(x_i) - f(x_{i+1})| < \delta \), the iterations end. Here, the tolerance \( (\delta) \) is set as \( 1 \times 10^{-16} \). Starting from nominal design values after 12 interactive calculations, the output voltage converges to 16.7940 V, as shown in Fig. 3. At the same time, \( M \) increases and the other two design variables \( (C_p, \theta) \) drop, shown in Figs. 4, 5 and 6. In the second case, the robust design optimization problem defined in Table 4 is calculated, and \( e_1 \) equals 0.9. It means that we look forward to obtaining as high output voltage as possible, even if the robustness decreases. After interactive computing, the alteration of the objective function is smaller than the tolerance. The central value of output voltage converges to 16.7942 V, and its deviation is 2.3 V, as shown in Figs. 7 and 8. The optimization processes of design variables are given in Figs. 9, 10 and 11.

In the last case, parameter \( e_1 \) is set as 0.1 to obtain optimal design with better robustness, and the robust design optimization problem defined in Table 4 is calculated again. After interactive computing, the objective function tolerance of change is satisfied, and the central value of output voltage converges to a smaller value (14.3232 V) than the one in the second case. However, the deviation decreases from 2.3 to 1.8572 V, as shown in Figs. 12 and 13. Finally, the

| Table 3 Deterministic optimization of the NMEH |

| Content | Uncertainties |
|---------|---------------|
| Objective function | Max: \( V(X) \) | No |
| Design variables | \( X = [M, C_p, \theta] \) | No |
| Constraint function | No | No |

| Table 4 Robust optimization of the NMEH |

| Content | Uncertainties |
|---------|---------------|
| Objective function | (1) Max: \( V(X_c) \) Yes |
| (2) Min: \( \Delta V(X^i) \) |
| Design variables | \( X^i = [M^i, C_p^i, \theta^i] \) Yes, listed in Table 1 |
| Constraint function | No |

---

123
deviation of output voltage is smaller. This means that the optimal design has better robustness than that obtained in the second case. The histories of design variables iterations are shown in Figs. 14, 15 and 16.

5 Conclusion

This paper presents a robust design optimization method for nonlinear monostable energy harvesters with structural uncertainties from the improved
interval extension. Due to the 2nd order items in the extension formula of the objective function (output voltage), this robust design optimization method is suitable for nonlinear systems. Having made a robust design of the NMEH, it is illustrated that the high
central value and the small deviation of output voltage can be obtained by this robust design optimization method. More importantly, the robustness of the NMEH can be increased based on this method. It can be also used for bistable/tristable/multistable energy harvesters thanks to the non-intrusive features of the improved interval extension in this paper.

Acknowledgements This work was supported by the National Natural Science Foundation of China (Grant no.11802237), the Fundamental Research Funds for the Central Universities (Grant no. G2018KY0306), and the 111 Project (No. BP0719007). In addition, GL was supported by the program of the Polish Ministry of Science and Higher Education under the project DIALOG 0019/DLG/2019/10 in the years 2019–2021.

Compliance with ethical standards

Conflict of interest The authors declare that there is no conflict of interest.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use,
sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

References

1. Park G, Rosing T, Todd MD, Farrar CR, Hodgkiss W (2008) Energy harvesting for structural health monitoring sensor networks. J Infrastuct Syst 14:64–79
2. Li Y, Wang X, Liu Z, Liang X, Si S (2018) The entropy algorithm and its variants in the fault diagnosis of rotating machinery: a review. IEEE Access 6:66723–66741
3. Li Y, Wang X, Si S, Huang S (2020) Entropy based fault classification using the Case Western Reserve University data: a benchmark study. IEEE Trans Reliab 69:754–767
4. Du Y, Zhou S, Jing X, Peng Y, Wu H, Kwo N (2020) Damage detection techniques for wind turbine blades: a review. Mech Syst Signal Proc 141:106445
5. Kansal A, Hsu J, Zahedi S, Srivastava MB (2007) Power management in energy harvesting sensor networks. ACM Trans Embed Comput Syst 6:32
6. Wang J, Geng L, Ding L, Zhu H, Yurchenko D (2020) The state-of-the-art review on energy harvesting of flow-induced vibrations. Appl Energy 267:114902
7. Wang J, Tang L, Zhao L, Zhang Z (2019) Efficiency investigation on energy harvesting from airflow in HVAC system based on galloping of isosceles triangle sectioned bluff bodies. Energy 172:1066–1078
8. Zhu H, Li G, Wang J (2020) Flow-induced vibration of a circular cylinder with splitter plates placed upstream and downstream individually and simultaneously. Appl Ocean Res 97:102084
9. Chen N, Wei T, Jung HJ, Lee S (2017) Quick self-start and minimum power-loss management circuit for impact-type micro wind piezoelectric energy harvester. Sens Actuators A Phys 263:23–29
10. Fang S, Wang S, Zhou S, Yang Z (2020) W. H. Liao. Exploiting the advantages of the centrifugal softening effect in rotational impact energy harvesting. Appl Phys Lett 116:063903
11. Chen Z, He J, Liu J, Xiong Y (2019) Switching delay in self-powered nonlinear piezoelectric vibration energy harvesting circuit: mechanisms, effects, and solutions. IEEE Trans Power Electron 34:2427–2440
12. Coccolo M, Litak G, Scoane JM, Sanjuán MA (2015) Optimizing the electrical power in an energy harvesting system. Int J Bifurc Chaos 25:1550171
13. Zhou S, Cao J, Erturk A, Lin J (2013) Enhanced broadband piezoelectric energy harvesting using rotatable magnets. Appl Phys Lett 102:173901
14. Zareporo M, Bilgen O (2018) Constrained-energy dynamic cross-well motion of bistable structures subjected to noise disturbance. Int J Struct Stab Dyn 18:1850047
15. Huang D, Zhou S, Yang Z, Resonance mechanism of nonlinear vibrational multistable energy harvesters under narrow-band stochastic parametric excitations. Complexity (2019) 2019:1050143
16. Chen L, Jiang W (2015) Internal resonance energy harvesting. J Appl Mech Trans ASME 82:031004
17. Lu Z, Ding H, Chen L (2019) Resonance response interaction without internal resonance in vibratory energy harvesting. Mech Syst Signal Proc 121:767–776
18. Lu Z, Chen J, Ding H, Chen L (2020) Two-span piezoelectric beam energy harvesting. Int J Mech Sci 175:105532
19. Adhikari S, Friswell MI, Inman DJ (2009) Piezoelectric energy harvesting from broadband random vibrations. Smart Mater Struct 18:115005
20. Petromichelakis I, Apostolos FP, Kougioumtzoglou IA (2018) Stochastic response determination and optimization of a class of nonlinear electromechanical energy harvesters: a Wiener path integral approach. Probab Eng Mech 53:116–125
21. Erturk A, Hoffmann J, Inman DJ (2009) A piezomagnetoelastic structure for broadband vibration energy harvesting. Appl Phys Lett 94:254102
22. Erturk A, Inman DJ (2011) Broadband piezoelectric power generation on high-energy orbits of the bistable Duffing oscillator with electromechanical coupling. J Sound Vib 330:2339–2353
23. Cottone F, Voca H, Gamaioni L (2009) Nonlinear energy harvesting. Phys Rev Lett 102:080601
24. Litak G, Friswell MI, Adhikari S (2010) Magnetopiezoelectric energy harvesting driven by random excitations. Appl Phys Lett 96:214103
25. Litak G, Borowiec M, Friswell MI, Adhikari S (2011) Energy harvesting in a magnetopiezoelectric system driven by random excitations with uniform and Gaussian distributions. J Theor Appl Mech Pol 49:757–764
26. Zhou S, Cao J, Inman DJ, Lin J, Liu S, Wang Z (2014) Broadband tristable energy harvester: modeling and experiment verification. Appl Energy 133:33–39
27. Zhou S, Cao J, Inman DJ, Lin J, Li D (2016) Harmonic balance analysis of nonlinear tristable energy harvesters for performance enhancement. J Sound Vib 373:223–235
28. Wei C, Jing X (2017) A comprehensive review on vibration energy harvesting: Modelling and realization. Renew Sustain Energy Rev 74:1–18
29. Lu Y, Zhou S (2018) Probability analysis of asymmetric tristable energy harvesters. AIP Adv 8:125212
30. Georgiou G, Manan A, Cooper JE (2012) Modeling composite wing aeroelastic behavior with uncertain damage sensitivity and material properties. Mech Syst Signal Proc 32:32–43
31. Li Y, Yang Z (2010) Uncertainty quantification in flutter sensitivity and material properties. Mech Syst Signal Proc 32:32–43
32. Hansen ER (1975) A generalized interval arithmetic. In: Nickel (ed) Interval mathematics, pp 7–18
33. Muhanna RL, Mullen RL (2001) Uncertainty in mechanics problems interval-based approach. J Eng Mech ASCE 127:557–566
34. Elishakoff I, Miglis Y (2012) Novel parameterized intervals may lead to sharp bounds. Mech Res Commun 44:1–8
35. Elishakoff I, Thakkar K (2014) Overcoming overestimation characteristic to classical interval analysis. AIAA J 52:2093–2097
36. Elishakoff I, Miglis Y (2012) Overestimation-free computational version of interval analysis. Int J Comput Meth Eng Sci Mech 13:319–328
37. Chen S, Wu J, Chen Y (2004) Interval optimization for uncertain structures. Finite Elem Anal Des 40:1379–1398
38. Pownuk A (2004) Efficient method of solution of large scale engineering problems with interval parameters based on sensitivity analysis. In: Proceeding of NSF workshop on reliable engineering computing, Savannah, Georgia, USA, pp 305–306
39. Wang X, Qiu Z (2008) Interval finite element analysis of wing flutter. Chin J Aeronaut 21:134–140
40. Wang X, Qiu Z (2009) Nonprobabilistic interval reliability analysis of wing flutter. AIAA J 47:743–748
41. Nanda A, Karami M, Singla P (2015) Uncertainty quantification of energy harvesting systems using method of quadratures and maximum entropy principle. In: ASME 2015 conference on smart materials, adaptive structures and intelligent systems, Colorado Springs, USA Sept, pp 2015–9026
42. Liu D, Xu Y, Li J (2017) Probabilistic response analysis of nonlinear vibration energy harvesting system driven by Gaussian colored noise. Chaos Soliton Fract 104:806–812
43. Li Y, Zhou S, Litak G (2019) Uncertainty analysis of excitation conditions on performance of nonlinear monostable energy harvesters. Int J Struct Stab Dyn 19:1950052
44. Park J, Lee S, Kwak BM (2012) Design optimization of piezoelectric energy harvester subject to tip excitation. J Mech Sci Technol 26:137–143
45. Wu J, Gao J, Luo Z, Brown T (2016) Robust topology optimization for structures under interval uncertainty. Adv Eng Softw 99:36–48
46. Erturk A, Inman DJ (2011) Piezoelectric energy harvesting. Wiley, Chichester
47. Zhou S, Cao J, Lin J (2016) Theoretical analysis and experimental verification for improving energy harvesting performance of nonlinear monostable energy harvesters. Nonlinear Dyn 86:1599–1611
48. Cheng J, Duan G, Liu Z, Li X, Feng Y, Chen X (2014) Interval multiobjective optimization of structures based on radial basis function, interval analysis, and NSGA-II. J Zhejiang Univ Sci A 15:774–788
49. Beck A (2015) Introduction to nonlinear optimization: theory, algorithms, and applications with MATLAB. Society for Industrial and Applied Mathematics, Philadelphia

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.