Joint influence of the wheel group on vehicle motion parameters

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Abstract. In order to reduce the complexity of researching the dynamics of vehicles with various structural schemes of the "wheel-road" system, the authors recommend a conditional replacement of several wheels on the same axle or on different axles with one equivalent wheel having the dynamic properties of the wheels being replaced. The article considers the case of an arbitrary wheel arrangement and the possibility of making the design scheme more compact. The calculation is carried out relative to the moving coordinate system, rigidly connected with the link frame and having the ability to move parallel to the supporting surface. Based on the analysis of the obtained dependencies, methods are proposed and recommendations are developed for bringing various structural patterns of wheel arrangement to one equivalent wheel.

When developing new vehicle designs, various schemes of support wheels interaction with the road surface and the car are used. Single, dual, steer, non-steerable, drive, non-drive, spring, non-spring, on dependent and independent suspension etc. wheels are used. In this case, even while retaining the wheel design, the interaction with the road will occur in different ways.

This creates significant difficulties for studying the dynamics of vehicles when compiling a system of differential equations, since it requires changing it whenever a constructive variant of the interaction of the wheel-road system is changed.

With a significant variety of design schemes of such a system this creates certain difficulties in research.

To reduce the complexity of research, a number of authors [1-4] conditionally replace several wheels on one axle with one equivalent wheel, with the dynamic properties of the wheels being replaced. So, the slip resistance coefficient of the equivalent wheel will be equal to the sum of the slip resistance coefficients of the wheels on this axis.

However, in many cases the interactions between the wheels are more complex. Consider the case of an arbitrary wheel arrangement relative to the axes of rotation of one link and the possibility of making the design scheme more compact, which provides an equivalent effect on the link and on the road surface during movement.

Figure 1 shows the movement diagram of a road train link, for example a tractor truck or trailer. A common frame connects wheels arbitrarily rotated relative to the frame and interacting with the supporting surface. It is considered that the moving coordinate system XOY is rigidly connected with the link frame and can move parallel to the supporting surface.
Consider the kinematics of the wheel movement that are located in each of the four quadrants of the established coordinate system. Take the notation for the $i$-th wheel:

- $A$ – wheel center;
- $a_i$ – $x$ coordinate of wheel center;
- $b_i$ – $y$ coordinate of wheel center;
- $V_0$ – vector of absolute velocity of wheel center point $O$;
- $V_i$ – vector of absolute velocity of wheel center point $A$;
- $\Theta$ – angle between the axis $OX$ and velocity vector $V_0$;
- $\alpha_i$ – current wheel alignment in relation to the axis $OX$;
- $V_{\Delta i}$ – wheel tire lateral deformation speed;
- $V_{ni}$ – wheel rolling speed;
- $\Psi_i$ – relative speed of point $A$ in rotation around point $O$;
- $\varphi$ – the angle between the fixed coordinate system $O_1X_1$ and the moving axis $OX$ associated with the frame;
- $\omega$ – the angular velocity of link rotation in the horizontal plane.

The sum of the absolute velocity $V_0$ of the point $O$ and the relative speed $V_{\Delta i}$ of the corresponding point $A$ in its rotation around the point $O$ determines the absolute speeds of the wheel centers points:

$$V_i = V_0 + V_{\Delta i}; \quad i = 1 - 4$$

On the other hand, the speed $V_i$ of the point $A_i$ is equal to the sum of the movement speed across the supporting surface of the contact area center $V_{ni}$ and the rate of change of tire lateral under the action of lateral force in contact with the supporting surface $V_{\Delta i}$:

**Figure 1.** Velocity diagram of wheel movement of an articulated vehicle link.
Based on the last two equations:
\[
\overrightarrow{V_i} = \overrightarrow{V_{ni}} + \overrightarrow{V_{di}}; \quad i = 1-4
\]

The sums of the components \(\overrightarrow{V_{0ix}}\) and \(\overrightarrow{V_{0iy}}\) parallel to the axes \(OX\) and \(OY\) respectively, determine the relative velocities \(\overrightarrow{V_{0i}}\). Then the previous vectors equality can be written:
\[
\overrightarrow{V_0} + \overrightarrow{V_{0i}} = \overrightarrow{V_{ni}} + \overrightarrow{V_{di}}; \quad i = 1-4
\]

To take into account the lateral deformation of pneumatic tires, frame of axes \(\xi A, \eta i\) are arranged in the horizontal contact plane of the tires with the supporting surface in the horizontal plane (Figure 1). The equalities obtained above will take the following form in the projections on the axes of this coordinate system:
\[
\begin{align*}
V_0 \cos(\alpha_i - \theta) - b_i \omega \cos(\alpha_i) + \alpha_i \omega \sin(\alpha_i) &= V_{ni} \cos(\psi_i); \\
(V_0 \sin(\alpha_i - \theta) - b_i \omega \sin(\alpha_i) + \alpha_i \omega \cos(\alpha_i)) + \dot{\Delta}_i &= V_{ni} \sin(\psi_i);
\end{align*}
\]

where \(\dot{\Delta}_i\) – the lateral deformation of the tire of the \(i\)-th wheel.

Based on the last equations it is obtained:
\[
tg(\psi_i) = (V_0 \sin(\alpha_i - \theta) - b_i \omega \sin(\alpha_i) - \alpha_i \omega \cos(\alpha_i)) + \dot{\Delta}_i / (V_0 \cos(\alpha_i - \theta) - b_i \omega \cos(\alpha_i)) + \alpha_i \omega \sin(\alpha_i))
\]

For small deformations \(\psi_i\), characteristic of the operating modes of pneumatic tires, the deformation angles are determined as:
\[
\psi_i = \left( (V_0 \cos(\alpha_i - \theta) - b_i \omega \sin(\alpha_i) - (V_0 \sin(\alpha_i + \alpha_i \omega \cos(\alpha_i)) + \dot{\Delta}_i) / (V_0 \cos(\alpha_i + (V_0 \sin(\alpha_i + \alpha_i \omega) \sin(2 \alpha_i))
\]

Considering the process of tire slip in the study of stability and controllability of the steady movement, consider the values \(\Delta_i\) and \(\dot{\Delta}_i\) equal to zero. In this case, the ratio of the angular deformations of the wheel tire before and after the transfer is defined as
\[
\psi_i / \psi_i' = 1 + (2b_i \omega / 2) (V_0 \cos(\theta) - b_i \omega) \cos^2(\alpha_i) + (V_0 \sin(\theta) + \alpha_i \omega) \sin(2 \alpha_i))
\]

For ease of use, this equation can be represented as
\[
\psi_i' = \Phi_i + \psi_i'
\]

where \(\Phi_i = 1 + (2b_i \omega / 2) (V_0 \cos(\theta) - b_i \omega) \cos^2(\alpha_i) + (V_0 \sin(\theta) + \alpha_i \omega) \sin(2 \alpha_i))

It follows that when the wheel moves along its axis of rotation to the longitudinal axis of symmetry of the reduction circuit, the angular deformation changes by \(\Phi_i\) in the process of reduction to the design diagram.

In this case, the transfer results in a change in the angle of the tire \(\psi_i\) of the center of which lies on the OX during axis slide OX. In this case, the initial coordinate of the wheel center changes:
\[
a_i = a_i + b_i \omega g(\alpha_i)\]

The angles of the wheel slip will be determined before and after the second transfer from the following ratio:
\[
\frac{\psi_i''}{\psi_i'} = 1 - (\Delta_i \omega / V_0 \cos(\theta) \omega g(\alpha) - (V_0 \sin(\theta) + \alpha_i \omega)) / (1 + (\Delta_i \omega g(\alpha_i)) / V_0 \cos(\theta) \omega g(\alpha_i))
\]

\[
/ (V_0 \cos(\theta) + (V_0 \sin(\theta) + \alpha_i \omega) \omega g(\alpha_i))
\]
After the first transfer of the wheel, the steering angle is determined as
\[ \psi_i = \chi_i \psi''_i \]
where \( \chi_i = 1 + (\delta_i \omega \tan(a_i))/\left((V_0 \cos(\theta) + V_0 \sin(\theta)) + (a_i + b_i \tan(a_i))\omega\right)/(V_0 \cos(\theta) \tan(a_i) - (V_0 \sin(\theta) + (a_i + b_i \tan(a_i))\omega). \)

Using the last two equations, obtain the functional connection:
\[ \psi_i = \Phi \chi_i \psi''_i \]

This dependence determines the change in the angle of the wheel slip when moving the center of the wheel in accordance with Figure 1 from a point having coordinates \( x = a, y = b \) to an arbitrary point having coordinates \( x = a + b \tan a + \delta; y = 0 \). In this case, the kinematics of the movement of the vehicle link remains.

A necessary condition for equivalent transfer is dynamic motion preservation. The parameters of the force interaction between the wheel and the supporting surface should not change the dynamics. The total interaction can be reduced to the main vector \( P \) and the main moment \( M \) equal to:
\[ P = \psi L_y C; \quad M = f \psi ; \]
where \( \psi \) – angle of the wheel slip; \( C \) – lateral stiffness of a tire; \( f \) – roll stiffness of a tire; \( L_y \) – tangent projection length [5]. Take that the main vector is directed perpendicular to the plane of the wheel.

As is known, two systems of forces are equivalent in the case when their principal vectors and principal moments are equivalent when both systems are brought to the same point. In this case, the center of reduction will take the center of the contact area of the wheel in the initial position with coordinates \( x = a; y = b \). After moving, the coordinates will have the following coordinate values:
\[ x = a + b \tan a + \delta; y = b. \]

The equivalence condition is satisfied when the angle of wheel rotation before and after reduction is the angle \( \alpha \).

For the main vectors, the condition must be met:
\[ \frac{L_{ye} C_e}{L_y C} = \frac{\psi}{\psi_e}; \]

where \( L_{ye}, C_e \) – equivalent wheel characteristics; \( L_y, C \) – initial characteristics of the wheel and after the second transfer.

As a result, it is obtained:
\[ L_{ye} C_e = \Phi \chi \psi L_y C. \]

The magnitude of the main moment of the equivalent wheel is determined by bringing to the center of the contact area for the initial position of the wheel:
\[ M_e = (f_e + L_{ye} C_e (\delta \cos(\alpha) - b \sin(\alpha)))\psi. \]

For the initial position of the wheel, the main moment is:
\[ M = f \psi. \]

Since these moments are equal, then:
\[ \psi/\psi_e = (f_e + L_{ye} C_e (\delta \cos(\alpha) - b \sin(\alpha)))/f. \]

As a result, it can be written:
\[ f = \Phi \chi (f - L_y C e (\delta \cos(\alpha) - b \sin(\alpha))). \]
As a result, dependencies are obtained that determine the parameters of the wheel slip after its transfer.

If there are several arbitrarily located wheels of a link in a vehicle, it is possible to evaluate the dynamic properties of the link in various versions of the layout schemes.

For an equivalent wheel, the orientation of the main vector with respect to the middle plane of the wheel determines the angle \( \alpha_e \):

\[
\alpha_e = \arctg \left( \frac{\sum_{i=1}^{n} P_{ix}}{\sum_{i=1}^{n} P_{iy}} \right),
\]

where \( P_{ix} \) and \( P_{iy} \) — projection of the main vector of the \( i \)-th wheel on the axis \( OX \) and \( OY \) under the condition:

\[
P_i = (L_y C_e (V_0 \cos(\theta) \sin(\alpha_i) - (V_0 \sin(\theta) + \alpha_i \omega) \cos(\alpha_i)) / (V_0 \cos(\theta) \cos(\alpha_i) + V_0 \sin(\theta) + a \omega \sin(\alpha_i))).
\]

In this case, the installation angle of the equivalent wheel will be equal to \( \alpha_e = \arctg((\sum_{i=1}^{n} P_{ix} \sin(\alpha_i)) / (\sum_{i=1}^{n} P_{iy} \cos(\alpha_i))) \).

The main vector \( \vec{P}_e \) for the equivalent wheel is equal to the sum of the projections of the main vectors of all the joined wheels on the direction of the vector \( \vec{P}_e \):

\[
\vec{P}_e = \sum_{i=1}^{n} (L_y C_e (V_0 \cos(\theta) \sin(\alpha_i) - (V_0 \sin(\theta) + \alpha_i \omega) \cos(\alpha_i)) / (V_0 \cos(\theta) \cos(\alpha_i) + V_0 \sin(\theta) + a \omega \sin(\alpha_i))) \cos(\alpha_e - \alpha_i)
\]

When finding the center of the equivalent wheel at a point having coordinates \( x = \alpha; y = \theta \), the slip angle is equal to

\[
\psi_e = (V_0 \cos(\theta) \sin(\alpha_e) - (V_0 \sin(\theta) - \alpha_i \omega) \cos(\alpha_e) / (V_0 \cos(\theta) \cos(\alpha_i) + V_0 \sin(\theta) + a \omega \sin(\alpha_e))
\]

The parameter \( L_{yeC_e} \) characterizing the coefficient of resistance to the slip of the equivalent wheel is defined as:

\[
L_{yeC_e} = P_e / \psi_e.
\]

To determine the angular stiffness of an equivalent wheel, the following functional connection can be used:

\[
f_e = M_e / \psi_e = (\sum_{i=1}^{n} f_{ei}(V_0 \cos(\theta) \sin(\alpha_i) - (V_0 \sin(\theta) - \alpha_i \omega) \cos(\alpha_i)) / (V_0 \cos(\theta) \cos(\alpha_i) + V_0 \sin(\theta) - \alpha_i \omega \sin(\alpha_i)))) / \psi_e
\]

where \( M_e \) — the stabilization torque on an equivalent wheel.

The relationship between normal and lateral loads and moments, lateral and angular deformations acting on the tire during rolling, was considered in detail in [5]. As a result, it becomes possible to obtain a reduction scheme to some unified configuration, which allows one to explore various options for layouts and structural schemes of motor vehicles.

Based on the obtained dependencies, the options for reducing various options for structural schemes to simpler ones are considered:

1. Coaxial and symmetrical arrangement of identical wheels in the original design (Figure 2). It is supposed to bring to a single wheel located in the middle of the axis.

The main characteristics of the equivalent wheel are determined by the following functional connections:
\[ C_{eL_{ye}} = (N + 1)CL_{y} + CL_{y}\sum_{i=1}^{n/2} \left(2l_{i}^{2}\omega^{2}/V_{0}^{2}\cos^{2}\theta - l_{i}^{2}\omega^{2}\right); \]

\[ f_{e} = (N + 1)f + f\sum_{i=1}^{n/2} \left(2l_{i}^{2}\omega^{2}/V_{0}^{2}\cos^{2}\theta - l_{i}^{2}\omega^{2}\right); \]

where \( l_{i} \) - distance from wheel center to the reduction center; \( C, L_{y}, f \) - characteristics of wheels mounted on the axle.

As the obtained dependences show, the replacement of several individual wheels with one equivalent in terms of drive characteristics (slip coefficient, angular stiffness) is possible only with small values of \( l_{i} \), small angular speeds and high speeds. This fully corresponds to the conditions of the study of the stability and controllability of vehicles.

2. The longitudinal arrangement of the wheels in the same plane on different axles (Figure 3) is symmetrical with respect to the central wheel. The reduction to the wheel located inside the base of the car occurs. Equivalent wheel performance is determined by the following equations:

\[ C_{eL_{ye}} = CL_{y} + 1 + \left(\sum_{i=1}^{(k-1)/2} \left(2(1 - d_{i}^{2}\omega^{2}/V_{0}^{2}\sin^{2}(\theta))\right)\right); \]

\[ f_{e} = f\left(1 + \sum_{i=1}^{(k-1)/2} \left(2(1 - d_{i}^{2}\omega^{2}/V_{0}^{2}\sin^{2}(\theta))\right)\right) - CL_{y}\sum_{i=1}^{(k-1)/2} \left((2V_{0}d_{i}^{2}\omega\sin(\theta))/V_{0}^{2}\sin^{2}(\theta) - d_{i}^{2}\omega^{2}\right) \]

where \( k \) – number of wheels.

![Figure 2. The scheme of reduction of coaxial wheels.](image2.png)

![Figure 3. The scheme of reduction of wheels located in the same plane.](image3.png)

The values of the obtained characteristics to a large extent depend on the distance between the wheels, increasing with increasing these distances.

References
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