Reconstructing ATLAS SU3 in the CMSSM and relaxed phenomenological supersymmetry models

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Assuming that the LHC makes a positive end-point measurement indicative of low-energy supersymmetry, we examine the prospects of reconstructing the parameter values of a typical low-mass point in the framework of the Constrained MSSM and in several other supersymmetry models that have more free parameters and fewer assumptions than the CMSSM. As a case study, we consider the ATLAS SU3 benchmark point with a Bayesian approach and with a Gaussian approximation to the likelihood for the measured masses and mass differences. First we investigate the impact of the hypothetical ATLAS measurement alone and show that it significantly narrows the confidence intervals of relevant, otherwise fairly unrestricted, model parameters. Next we add information about the relic density of neutralino dark matter to the likelihood and show that this further narrows the confidence intervals. We confirm that the CMSSM has the best prospects for parameter reconstruction; its results had little dependence on our choice of prior, in contrast to the other models that we study where the situation is less clear. By comparing parameter reconstruction we find that sometimes gaugino mass unification, rather than having fewer free parameters, is critical. We also study prospects for evaluating the relic density of neutralino dark matter from LHC data alone and for direct detection searches with information from both LHC data and the relic density of dark matter. Finally we perform a brief comparison of parameter and relevant mass reconstruction in the models that we study.

I. INTRODUCTION

The search for supersymmetry (SUSY) at the LHC is now in full swing.[1–3] Assuming that the LHC makes a specific positive measurement, we investigate whether the measurement could be explained by various SUSY models, and whether the measurement could be used to distinguish among them and to determine their parameters.[4, 5] As a case study, we consider the ATLAS SU3 benchmark point,[6] a typical low-mass scenario. If a mass spectrum similar to the one of the SU3 point were realised in nature, LHC measurements of dilepton edges would allow one to determine one mass and three mass differences with correlated uncertainties. Taking this as input, within a Bayesian statistical approach we will attempt to reconstruct the ATLAS SU3 benchmark point in five supersymmetry models. For each model, we will find the regions of the model’s parameter space that are in best agreement with the one mass and three mass differences that the LHC would measure. Next, we will examine the extent to which providing additional information from the relic density of the lightest neutralino as the lightest supersymmetric particle (LSP) will help further improve the determination of the parameters. We will find these regions with an adapted version of the publicly available SuperBayeS computer program.[7–10] Then, we will see whether we have reconstructed the ATLAS SU3 benchmark point, that is, whether the regions that we have found closely envelop the ATLAS SU3 benchmark point.

We consider five phenomenological models of supersymmetry. The Constrained Minimal Supersymmetric Standard Model (Constrained MSSM, or CMSSM)[11] is the most economical model; it has only four free continuous parameters: the common gaugino mass $m_{1/2}$, the common scalar mass $m_0$, the common trilinear term $A_0$, all defined at the GUT scale, plus the ratio of the Higgs vacuum expectation values $\tan \beta$, defined at the electroweak (EW) scale. Because of its simplicity, the CMSSM is a tractable model that is well studied, and despite its restrictive boundary conditions, the CMSSM allows a wide range of phenomenology.

There are, however, theoretical reasons for considering “relaxed” models; models in which some parameters are free at the GUT scale. The Non-Universal Higgs Model (NUHM) can be viewed as a relaxed version of the CMSSM. In
the NUHM, the Higgs soft masses are free parameters at the GUT scale, unlike in the CMSSM, where they unify with the common scalar mass. Assuming Higgs mass unification with other scalar masses at the GUT scale is not well motivated, because the Higgs and the scalar fermions belong to different supermultiplets. The NUHM has two more parameters than the CMSSM, namely the Higgs soft masses $m_{H_u}$ and $m_{H_d}$, and it has a richer phenomenology involving Higgs bosons.

Another relaxed phenomenological alternative to the CMSSM that we will consider is the CMSSM with non-universal gaugino masses (CMSSM-NUG). In contrast to the CMSSM’s gaugino soft masses, the CMSSM-NUG’s gaugino soft masses will not unify at the GUT scale; they are free parameters. The CMSSM-NUG has more freedom than the CMSSM, for instance, unlike in the CMSSM, in the CMSSM-NUG, LEP bounds for neutralino masses cannot be derived from LEP bounds for chargino masses. The CMSSM-NUG has two additional parameters; the common gaugino mass is replaced by three independent gaugino soft mass parameters for the bino ($M_1$), the wino ($M_2$) and the gluino ($M_3$).

Furthermore, we consider the MSSM, with all the parameters defined at the EW scale. The general MSSM has over one hundred free parameters. We, however, reduce the MSSM’s number of free parameters, first by imposing the condition of minimal flavour violation in order to satisfy strong bounds on CP-violation and flavour changing neutral currents, and second by assuming some unification at the GUT scale and some degeneracy at the EW scale, so that the resulting model is tractable. Our MSSM has twelve free parameters, which are defined at the EW scale, and are listed below.

Finally, we relax our MSSM by lifting the assumption of gaugino mass unification. We study the MSSM where the gaugino soft masses are free parameters that are defined at the EW scale, and call the model the Non-Universal Gaugino MSSM or MSSM-NUG. In contrast to the MSSM mentioned above, the MSSM-NUG has fourteen free parameters, which are defined at the EW scale, and are listed below.

Our work is an extension of Ref. [15] where the SU3 parameter reconstruction was performed within the framework of the CMSSM only and in which more optimistic uncertainties in computing the neutralino’s relic density were assumed. Otherwise we will follow a similar methodology and apply it to the models listed above. Additionally, here we will make an important preliminary step, which will be to delineate the most likely regions of the parameter space (for every model) by imposing physicality constraints alone, before applying information from an assumed ATLAS measurement or dark matter density. Our main aim in this paper will be to assess the feasibility of reconstructing the SU3 point in the CMSSM and the other SUSY models that we consider here.

The paper is organised as follows. In section II, we discuss the ATLAS SU3 benchmark point and the positive measurements that would be possible at the LHC were Nature described by ATLAS SU3. In section III, we detail our methodology, showing how we construct our likelihood function and describing our Bayesian statistical approach. In section IV, we present the results of our scans of the CMSSM, NUHM, CMSSM-NUG, MSSM, and MSSM-NUG for the reconstruction of the model parameters. We also compare each model’s Bayesian evidence, and present properties of each model relevant to dark matter density and searches. Lastly, we summarise our study in section V.

II. THE ATLAS SU3 BENCHMARK POINT

In preparation for conducting SUSY searches, LHC Collaborations adopted a set of “benchmark points” in the CMSSM and some other models. At each benchmark point, the CMSSM has a distinct phenomenological set of signals. In particular, the ATLAS Collaboration defined the SU3 benchmark point [6] which is specified in Table II. We calculated the CMSSM’s sparticle mass spectrum at ATLAS SU3 with SOFTSUSY [10]. It is shown in Table II and Figure I.

| CMSSM parameter | ATLAS SU3 point | SM parameter | Input value |
|-----------------|----------------|--------------|-------------|
| $m_{1/2}$       | 300 GeV        | $M_t$        | 172.6 GeV   |
| $m_0$           | 100 GeV        | $m_h(m_t)^{\text{MS}}$ | 4.20 GeV   |
| $\tan \beta$   | 6.0            | $\alpha_s(M_Z)^{\text{MS}}$ | 0.1176     |
| $A_0$           | $-300$ GeV     | $1/\alpha_{em}(M_Z)^{\text{MS}}$ | 127.955    |
| $\text{sgn} \mu$ |                |              |             |

TABLE I: Left side: input CMSSM parameter values for the ATLAS SU3 benchmark point. Right side: input values of relevant SM parameters used in our numerical analysis.

Whilst the SU3 point was originally defined in the CMSSM, we extend its definition to the other models considered here as follows. In the NUHM and the CMSSM-NUG, ATLAS SU3 is defined as it was in the CMSSM in Table II but with $m_{H_d} = m_{H_u} = m_0$ and $M_1 = M_2 = M_3 = m_{1/2}$, respectively. In the MSSM and MSSM-NUG, ATLAS SU3
TABLE II: The sparticle mass spectrum in the CMSSM at the ATLAS SU3 benchmark point, calculated with SOFTSUSY.

| Sparticle Mass (GeV) |
|----------------------|
| $\chi^0$ = $\chi_1$ | 117.9 $\tilde{e}_L$, $\tilde{\mu}_L$ | 230.8 $\tilde{d}_L$ | 666.2 |
| $\chi^0$ | 223.4 $\tilde{e}_R$, $\tilde{\mu}_R$ | 157.5 $\tilde{d}_R$ | 639.0 |
| $\chi^0$ | 463.8 $\tilde{\nu}_e$, $\tilde{\nu}_\mu$ | 217.5 $\tilde{u}_L$ | 660.3 |
| $\chi^0$ | 479.9 $\tilde{\tau}_1$ | 152.2 $\tilde{u}_R$ | 644.3 |
| $\chi^0$ | 224.4 $\tilde{\tau}_2$ | 234.2 $\tilde{b}_1$ | 599.0 |
| $\chi^0$ | 476.4 $\tilde{\nu}_e$ | 216.9 $\tilde{b}_2$ | 636.6 |
| $\tilde{g}$ | 717.5 $\tilde{\ell}_1$ | 446.9 |
| $\tilde{g}$ | 670.9 |

FIG. 1: The mass spectrum for the ATLAS SU3 benchmark point.

is found by running the CMSSM parameters to the EW scale with SOFTSUSY to obtain ATLAS SU3 for a general MSSM, and then averaging the parameters that are degenerate in our MSSM. The resulting parameters for ATLAS SU3 in the MSSM are shown in Table III. Because the MSSM parameters at the SUSY mass scale will be the same in all the models, the physical sparticle mass spectrum corresponding to the ATLAS SU3 point will also be the same in all the models.

| Parameter Description | ATLAS SU3 value (masses in GeV) |
|-----------------------|---------------------------------|
| MSSM:                |                                 |
| $M_2$                | Wino mass                        | 231 |
| $m_{\tilde{L}}$      | Left-handed slepton mass         | 224 |
| $m_{\tilde{E}}$      | Right-handed slepton mass        | 149 |
| $m_{\tilde{Q}}$      | Left-handed squark mass          | 622 |
| $m_{\tilde{U}}$      | Right-handed up-type-squark mass | 574 |
| $m_{\tilde{D}}$      | Right-handed down-type-squark mass| 618 |
| $A_U$                | Up-type-quark trilinear coupling | $-824$ |
| $A_D$                | Down-type-quark trilinear coupling| $-1164$ |
| $A_L$                | Lepton trilinear coupling        | $-486$ |
| $m_A$                | Pseudoscalar Higgs mass          | 462 |
| $\mu$                | Higgs parameter                  | 521 |
| $\tan \beta$        | Ratio of Higgs vevs              | 6 |
| MSSM-NUG: MSSM parameters plus: |                                 |
| $M_1$                | Bino mass                        | 123 |
| $M_3$                | Gluino mass                      | 696 |

TABLE III: Input MSSM and MSSM-NUG parameter values for the ATLAS SU3 benchmark point.

One promising way of discovering supersymmetry at the LHC is to study a squark decay chain [6]. A squark can
decay, via intermediate states, to the lightest neutralino, a pair of leptons and a quark,
\[ \tilde{q}_L \rightarrow \chi^0 q \rightarrow \ell^+ \ell^- q \rightarrow \chi^0 \ell^+ \ell^- q, \]
where \( \tilde{q}_L \) denotes the first or second generation left squark, \( \chi^0 = \chi \) and \( \chi^0 \) the first and second neutralino and \( \ell \) an intermediate lepton.

The experimental signature for this chain will be two leptons with opposite signs, a hard jet and missing energy. By measuring the invariant masses of outgoing particles,
\[ M^2 = \left( \sum E \right)^2 - \sum p_T^2, \]
one can determine relationships between the masses of the particles in the decay chain in Eq. (1).

For the ATLAS SU3 benchmark point, the decay chain in Eq. (1) will swamp other processes, because sleptons are lighter than \( \chi^0 \). The invariant mass distribution for the lepton pair will be triangular, like a single sawtooth. The position of the edge of the invariant mass distribution is dependent on the sparticle mass hierarchy; for ATLAS SU3 it will be
\[ M^2_{\ell,\text{edge}} = m_{\chi^0}^2 \left[ 1 - \left( \frac{m_\chi}{m_{\chi^0}} \right)^2 \right] \left[ 1 - \left( \frac{m_\chi}{m_{\ell}} \right)^2 \right]. \]

Measuring \( M^2_{\ell,\text{edge}} \) gives a relationship between \( m_{\chi^0}, m_\chi \) and \( m_\chi \). Measuring other invariant mass edges allows one to determine all the masses involved in the decay chain in Eq. (1). The quark flavours, however, will be indistinguishable, and the lightest slepton will dominate the decay chain. The set of measurable masses is, therefore,
\[ \theta = \left( m_\chi, m_{\chi^0} - m_\chi, m_\tilde{q} - m_\chi, m_\tilde{t} - m_\chi \right), \]
where
\[ m_\tilde{q} = \frac{1}{4} \left( m_\tilde{u}_R + m_\tilde{u}_L + m_\tilde{d}_R + m_\tilde{d}_L \right), \]
\[ m_\tilde{t} = \min \left( m_\tilde{e}_R, m_\tilde{e}_L, m_\tilde{\mu}_R, m_\tilde{\mu}_L, m_\tilde{\tau}_R, m_\tilde{\tau}_L \right). \]

The mass measurements will be correlated; the covariance matrix (which is the inverse of the Fisher matrix) that describes their correlation for the ATLAS SU3 benchmark point is shown in Table IV. It was obtained by ATLAS assuming an integrated luminosity of 1 fb\(^{-1}\) and a positive measurement of the decay chain given by Eq. (1) or the high-\( p_T \) and large missing energy decay chain \( \tilde{q}_R \rightarrow \chi q \), where \( \tilde{q}_R \) denotes the first or second generation right squark.

\[
\begin{array}{cccc}
    m_\chi & m_{\chi^0} - m_\chi & m_\tilde{q} - m_\chi & m_\tilde{t} - m_\chi \\
    3.72 \times 10^3 & 53.4 & 1.92 \times 10^3 & 1.075 \times 10^3 \\
    m_\chi - m_\chi & 3.6 & 29.0 & -1.3 \\
    m_\tilde{q} - m_\chi & 1.12 \times 10^3 & 4.65 & \\
    m_\tilde{t} - m_\chi & 14.1 & \\
\end{array}
\]

**TABLE IV:** The ATLAS SU3 covariance matrix. All entries have dimension GeV\(^2\).

The central values for the mass measurements for the ATLAS SU3 benchmark point are shown in Table V. They were calculated with SOFTSUSY. Table V also shows the simulated relic density for ATLAS SU3, with an error from WMAP5 [17]. The ATLAS SU3 benchmark point is in the “bulk region” of the CMSSM’s parameter space. In this region, neutralino co-annihilation is dominated by slepton exchange, and the neutralino’s relic density is typically of the correct order of magnitude, without excessive fine-tuning.

We calculated the neutralino’s relic density in the CMSSM at ATLAS SU3 with micrOMEGAs [18, 19]. The calculated relic density, \( \Omega h^2_{\chi}^{\text{SU3}} = 0.2332 \), exceeds, and is in statistically significant disagreement with, WMAP5’s measurement, \( \Omega h^2_{\text{WMAP}} = 0.1099 \pm 0.0062 \) [17]. The ATLAS SU3 benchmark point is, however, representative of similar points that result in neutralino’s with relic densities that are in agreement with WMAP’s measurement. By increasing the value of \( \tan \beta \), one can increase the neutralino’s higgsino component. The neutralino’s higgsino component enhances \( \chi\chi \rightarrow WW \) annihilation, and, consequently, suppresses the neutralino’s relic density. Clearly, then, one can adjust the value of \( \tan \beta \) so that the neutralino’s relic density is in agreement with WMAP’s measurement.

\[1\] We have corrected some typos in the covariance matrix given in Ref. [13].
which computes the neutralino’s relic density and direct detection cross section, and a routine that employs nested sampling (NS).

is proportional to the number of points in each bin. The profile likelihood are binned by dividing the parameter space into “bins” and counting the number of points in each bin. The posterior

posterior is maximised).

SuperBayeS to find the regions of parameter space that are in best agreement with experimental data (the regions where the model’s free parameters to experimental data using Bayesian statistical techniques. It uses Monte Carlo methods

is found by summing all the bin widths multiplied by the parameter of interest and the posterior. The evidence is found by summing the likelihood of each closed contour multiplied by the prior volume in each closed contour. The marginalised posteriors and profile likelihoods are normalised by setting their maximum values to unity.

In evaluating prospects for reconstructing the SU3 point in different SUSY models we will proceed in three steps. Firstly, we will delineate the regions of parameters which are physically allowed, as described below. Next, we will add information that the supposed ATLAS edge measurement would provide. Finally, we will examine the impact of additionally assuming that the neutralino’s relic density at the SU3 point is correct.

We perform our numerical analysis with the help of SuperBayeS [7–10] — a sophisticated computer program that fits a model’s free parameters to experimental data using Bayesian statistical techniques. It uses Monte Carlo methods to find the regions of parameter space that are in best agreement with experimental data (the regions where the posterior is maximised). SuperBayeS includes the programs SOFTSUSY, which computes mass spectra, micrOMEGAs, which computes the neutralino’s relic density and direct detection cross section, and MultiNest [20], a scanning routine that employs nested sampling (NS).

The pertinent statistical quantities are calculated numerically, after the list of points has been “binned.” The points are binned by dividing the parameter space into “bins” and counting the number of points in each bin. The posterior is proportional to the number of points in each bin. The profile likelihood is found by finding the bin with the highest likelihood, within the set of bins where the parameter of interest is constant. The marginalised posterior is found by summing the bin widths multiplied by the posterior for that bin, within the set of bins where the parameter of interest is constant. The best-fit point is found by finding the bin with the greatest likelihood. The posterior mean is found by summing all the bin widths multiplied by the parameter of interest and the posterior. The evidence is found by summing the likelihood of each closed contour multiplied by the prior volume in each closed contour. The marginalised posteriors and profile likelihoods are normalised by setting their maximum values to unity.

We adapted the SuperBayeS computer program so that it scanned over the five phenomenological supersymmetry models discussed in section II so that it included a simulated value for the neutralino’s relic density, and so that it included simulated LHC constraints in its likelihood function. The likelihood in our adapted codes is the product of contributions from the LHC edge measurements and the WMAP dark matter density; \( \mathcal{L} = \mathcal{L}_{\text{LHC}} \times \mathcal{L}_{\text{DM}}, \) where

\[
-2 \ln \mathcal{L}_{\text{LHC}} = \left( \theta - \theta_{\text{SU3}} \right) C^{-1} \left( \theta - \theta_{\text{SU3}} \right)^T, \tag{7}
\]

\[
-2 \ln \mathcal{L}_{\text{DM}} = \frac{\left( \Omega_{\chi} h^2 - \Omega_{\chi}^\text{SU3} h_{\text{SU3}}^2 \right)^2}{\sigma^2_{\text{CALC}} + \sigma^2_{\text{WMAP}}}, \tag{8}
\]

where \( \theta \) is the parameter set in Eq. 4, \( \theta_{\text{SU3}} \) is given in Table V, and \( C \) is the covariance matrix in Table IV. \( \Omega_{\chi} h^2 \) is the neutralino’s relic density calculated at a scanned point, \( \Omega_{\chi}^\text{SU3} h_{\text{SU3}}^2 \) is the neutralino’s relic density at the ATLAS SU3 point, given in Table V. \( \sigma^2_{\text{CALC}} \) is the WMAP5 error, given in Table V. \( \sigma^2_{\text{WMAP}} \) is the theoretical error in the neutralino’s relic density. Evidently, we have assumed that the likelihood functions are Gaussians, that is, that the measured values (the ATLAS SU3 values) are described by Gaussians centred on the calculated values.

The theoretical error for the neutralino’s relic density in Eq. 5, \( \sigma_{\text{CALC}} \), results from errors in the particle mass spectrum, rather than from numerical errors or approximations within the relic density calculation itself. These errors originate from approximations that are made when adding radiative corrections to the particle masses. It

\[ \text{TABLE V: The five ATLAS SU3 measurable quantities used in this analysis.} \]

| Quantity | Mean (masses in GeV) | Error |
|----------|----------------------|-------|
| Simulated LHC measurement | | |
| \( m_{\tilde{\chi}} \) | 117.9 | See covariance matrix |
| \( m_{\tilde{\chi}_1} - m_{\chi} \) | 105.5 | See covariance matrix |
| \( m_{\tilde{q}} - m_{\chi} \) | 534.5 | See covariance matrix |
| \( m_{\tilde{\ell}} - m_{\chi} \) | 34.3 | See covariance matrix |
| Simulated WMAP measurement | | |
| \( \Omega_{\chi} h^2 \) | 0.2332 | 6.2 \times 10^{-3} |

In evaluating the profile likelihood for a given parameter (or set of parameters) one maximises the likelihood along the other parameters of the model, in contrast to the marginalised posterior, where one integrates over them; see Ref. [15] for more details.

The MSSM-NUG code was written by Roberto Ruiz de Austri.

2 In evaluating the profile likelihood for a given parameter (or set of parameters) one maximises the likelihood along the other parameters of the model, in contrast to the marginalised posterior, where one integrates over them; see Ref. [15] for more details.

3 The MSSM-NUG code was written by Roberto Ruiz de Austri.

4 These errors in the particle mass spectrum are negligible in Eq. 4 because they are much smaller than the entries in the covariance matrix.
was asserted in a previous study, Ref. [15], that $\sigma_{\text{CALC}}$ will be negligible in the CMSSM’s bulk region. We, however, haven’t studied the theoretical error in the NUHM, CMSSM-NUG, MSSM or MSSM-NUG. Whilst the theoretical error at ATLAS SU3 will be the same in all the models (because the MSSM parameters at the SUSY mass scale will be the same), we don’t know how the theoretical error will behave around ATLAS SU3 in each model. Furthermore, we used an on-line comparison tool\(^5\) to estimate $\sigma_{\text{CALC}}$ at the ATLAS SU3 benchmark point. The comparison tool calculated the sparticle mass spectrum at ATLAS SU3 with three different RGE codes, then calculated the relic density resulting from each sparticle mass spectrum with \texttt{micrOMEGAs}. The differences between the relic densities were of order 5-10%. We, therefore, cautiously take $\sigma_{\text{CALC}} = 0.1 \Omega_\chi_2^{0.1}$ (that is, 10%), which is its default value for the CMSSM in \texttt{SuperBayeS}.

\texttt{SuperBayeS} rejects points in a model’s parameter space that are unphysical; they are assigned zero prior probability. The potential physicality problems are listed in Table VI. Because unphysical points are rejected in this way, a model’s priors are correlated in a complicated way. The resulting marginalised priors will not be sharp, step-function distributions — priors that evenly weight orders of magnitude (log priors) will have a $1/x$ shape and regions of parameter space with more physical points will have a greater prior probability.

| Physicality problems                                      |
|----------------------------------------------------------|
| No radiative electroweak symmetry breaking; $b$ has the wrong sign |
| No radiative electroweak symmetry breaking; $\mu^2$ has the wrong sign |
| Tachyonic Higgs                                          |
| Tachyonic sfermion                                        |
| Landau pole in renormalisation group evolution            |
| No electroweak symmetry breaking; $\mu$ cannot be calculated |
| No electroweak symmetry breaking minimum                  |
| Landau pole; infra-red quasi-fixed point breached         |
| Desired accuracy cannot be reached                        |
| Lightest neutralino is not the LSP                        |

TABLE VI: The potential physicality problems. If a point has one or more of these problems, it is unphysical.

We ran our modified \texttt{SuperBayeS} programs with the NS algorithm on a high-performance computer three times. First, we ran them with no constraints other than the physicality conditions. Second, we did the same but this time applied simulated LHC constraints. Third, we ran them with simulated LHC and simulated WMAP constraints. For mass parameters, we used prior ranges that equally weighted orders of magnitude (log priors). The priors that we used are listed in Table VII. Because of their experimental uncertainties, the nuisance parameters are varied around their mean experimental values. We, however, did not include a likelihood from measurements of the nuisance parameters in our likelihood function.

Our chosen NS parameters were such that the NS algorithm would accurately map the posterior (but not the profile likelihood) and accurately calculate the evidence, in a reasonable CPU time. Accurately mapping the profile likelihood with NS requires more CPU time. The technical details of our runs are listed in Table VIII.

IV. RESULTS

A. The CMSSM

The CMSSM is the most economical model that we study. It has only four continuous free parameters, and, consequently, it ought to be the easiest model in which to recover the ATLAS SU3 parameters.

We show the two-dimensional (2D) marginalised posterior probability density functions (pdfs) for the CMSSM’s parameters for our three scans in Figure 2. Figure 2(a) shows in effect the pdfs for the CMSSM’s parameters’ priors subject to physicality constraints only, because the likelihood is set equal to one. The distributions show regions where it is easier to satisfy all the physicality constraints. The log priors result in residual $1/x$ dependencies, that a priori should favour smaller parameter values. Smaller values of $m_{1/2}$ and smaller absolute values of $A_0$ are favoured. There is no preference for the values of $\tan \beta$ and $m_0$.

\(^5\) Available at \url{http://kraml.web.cern.ch/kraml/comparison/} documented in Refs. [21–24].
### Table VII: The parameters that we scanned over and their prior ranges.

| Parameter | Description | Prior Range | Scale |
|-----------|-------------|-------------|-------|
| $m_0$     | Unified scalar mass | 50, 1000 | Log   |
| $m_{1/2}$ | Unified gaugino mass | 50, 1000 | Log   |
| $A_0$     | Unified trilinear coupling | $-7000, 7000$ | Linear |
| $\tan \beta$ | Ratio of Higgs vevs | 2, 65 | Linear |

**NUHM: CMSSM parameters plus:**

| Parameter | Description | Prior Range | Scale |
|-----------|-------------|-------------|-------|
| $m_{H_d}$ | Higgs down-type doublet mass | 50, 1000 | Log   |
| $m_{H_u}$ | Higgs up-type doublet mass | 50, 1000 | Log   |

**CMSSM-NUG: CMSSM parameters plus:**

| Parameter | Description | Prior Range | Scale |
|-----------|-------------|-------------|-------|
| $M_1$     | Bino mass   | 50, 1000 | Log   |
| $M_2$     | Wino mass   | 50, 1000 | Log   |
| $M_3$     | Gluino mass | 50, 1000 | Log   |

**MSSM**

| Parameter | Description | Prior Range | Scale |
|-----------|-------------|-------------|-------|
| $m_L$     | Left-handed slepton mass | 100, 1000 | Log   |
| $m_E$     | Right-handed slepton mass | 100, 1000 | Log   |
| $m_Q$     | Left-handed squark mass | 100, 1000 | Log   |
| $m_U$     | Right-handed up-type-squark mass | 100, 1000 | Log   |
| $m_D$     | Right-handed down-type-squark mass | 100, 1000 | Log   |
| $A_U$     | Up-type-quark trilinear coupling | 1, 1000 | Log   |
| $A_D$     | Down-type-quark trilinear coupling | 1, 1000 | Log   |
| $A_L$     | Lepton trilinear coupling | 1, 1000 | Log   |
| $m_A$     | Scalar Higgs mass | 100, 1000 | Log   |
| $\mu$     | Higgs parameter | 1, 1000 | Log   |
| $\tan \beta$ | Ratio of Higgs vevs | 2, 60 | Linear |

**MSSM-NUG: MSSM parameters plus:**

| Parameter | Description | Prior Range | Scale |
|-----------|-------------|-------------|-------|
| $M_1$     | Bino mass   | 1, 1000 | Log   |
| $M_3$     | Gluino mass | 1, 1000 | Log   |

**Nuisance**

| Parameter | Description | Prior Range | Scale |
|-----------|-------------|-------------|-------|
| $M_t$     | Top quark pole mass | 163.7, 178.1 | Linear |
| $m_b(M_Z)$ | Bottom quark mass | 3.92, 4.48 | Linear |
| $\alpha(M_Z)$ | Strong coupling | 0.1096, 0.1256 | Linear |
| $1/\alpha_{em}(M_Z)$ | Reciprocal of electromagnetic coupling | 127.846, 127.99 | Linear |

Note that a log scale equally weights orders of magnitude, whereas a linear scale equally weights intervals. Mass parameters and trilinear couplings are in GeV.
little effect on the other two CMSSM parameters.

Figure 4 shows the 1D marginalised posterior pdfs for the ATLAS SU3 measurable masses and mass differences for our three scans of the CMSSM. Whilst the initial ranges (Figure 4(a)) were quite wide, Figure 4(b) shows that the mass constraints were well recovered when the CMSSM was scanned with simulated ATLAS constraints. Like in Ref. [15], assuming the CMSSM improves the determination of the measurable masses and mass differences, that is, the derived confidence intervals are narrower than the experimental confidence intervals. This is because the geometry of the CMSSM’s parameter space disfavours some combinations of masses. The simulated WMAP constraints (Figure 4(c)), however, does not improve the recovery of the mass constraints much further. The biggest effect can be seen in the measurements of $m_1 - m_\chi$ and $m_0 - m_\chi$, because the masses of $\tilde q$ and $\tilde l$ depend on $m_0$ whose reconstruction improves by adding the information on the LSP relic density.

In summary, reconstruction of the ATLAS SU3 parameters is reasonable in the CMSSM. The parameters $m_0$ and $m_{1/2}$ are well reconstructed, but the reconstruction of $A_0$ and $\tan \beta$ is much poorer. Adding the simulated dark matter information squeezes $m_0$, but does not improve the recovery of the other parameters. Assuming the CMSSM narrows the confidence interval for $m_\chi$.

### B. The NUHM

Compared with the CMSSM, the NUHM has two additional parameters, $m_{H_u}$ and $m_{H_d}$, which do not directly enter the assumed ATLAS positive end-point measurement. It is therefore interesting to investigate how their presence will affect mass reconstruction compared with the CMSSM.

In Figure 5 we show the 2D marginalised posterior planes for the NUHM’s parameters for our three scans, in a similar fashion to the the CMSSM results shown above. Figure 5(a) shows the pdfs for the NUHM’s parameters’ priors, because the likelihood is always set equal to one. Like the plots for the CMSSM in Figure 2(a) with physicality constraints only, smaller values of $m_{1/2}$ and smaller absolute values of $m_0$ are favoured. Unlike $m_0$ in the CMSSM, smaller values of $m_0$ in the NUHM are disfavoured. There is no preference for the values of $\tan \beta$ and $m_{H_u}$; however, smaller values of $m_{H_d}$ are favoured.

Figure 5(b) shows the 2D marginalised posterior planes for the NUHM’s parameters, when we scanned the NUHM with simulated LHC constraints. The ATLAS SU3 point is reasonably well reconstructed for $m_{1/2}$, like in the CMSSM. Also, fairly large values of $m_0$ are permitted, but unlike in the CMSSM, small values of $m_0$ are also permitted. Like in the CMSSM, the other parameters, $\tan \beta$, $A_0$, $m_{H_u}$ and $m_{H_d}$ are poorly reconstructed and the sign of $A_0$ is ambiguous.

We can see that the reconstruction of $m_0$ in the NUHM is significantly poorer than it was in the CMSSM, with small values of $m_0$ being permitted in the NUHM. This is because the extra NUHM parameters, $m_{H_d}$ and $m_{H_u}$, contribute (positively) to sfermion masses. Large values of $m_{H_d}$ and $m_{H_u}$ can, therefore, compensate for a small value of $m_0$. This results in small values of $m_0$ being permitted. The reconstruction of the additional NUHM parameters is also poor, especially $m_{H_u}$, in a way resembling the results of a global scan performed in Ref. [14].

Figure 5(c) shows the 2D marginalised posterior planes for the NUHM’s parameters, when we scanned the NUHM with simulated LHC and simulated WMAP constraints. Like in the CMSSM, there is a significant squeeze on $m_0$, though its reconstruction remains poor. The $\tan \beta$, $A_0$, $m_{H_u}$ and $m_{H_d}$ parameters show little improvement over Figure 5(b).

| Technical details                          | SuperBayeS version          | Adapted version 1.5 |
|---------------------------------------------|------------------------------|---------------------|
| Sampling algorithm                          | Nested sampling             | False               |
| Number of processors                        | 1                            | True                |
| Constant efficiency mode                    | False                        | True                |
| Multiple modes expected                      | 2 (usually $m_0$ and $m_{1/2}$) | True                |
| Maximum number of modes expected            | 2                            | True                |
| Maximum number of live points               | 4000                         | True                |
| Evidence tolerance factor                   | 0.5                          | True                |
| Sampling efficiency                         | 2                            | True                |
| Compilers                                   | icfort and icc              | True                |
| Run time                                    | Typically less than 3 days   | True                |

TABLE VIII: The technical details of our SuperBayeS runs.
FIG. 2: Marginalised posterior probability density functions, when we scanned the CMSSM with no constraints (other than those of physicality) [a] simulated LHC constraints [b] and simulated LHC and simulated WMAP constraints [c]. The dark blue and light blue regions are the one-sigma and two-sigma regions respectively. The crosses on the planes are the best-fit values: the values for which $\chi^2$ was minimised. The dots on the planes are the posterior means. The red diamonds on the planes are the ATLAS SU3 benchmark values.
FIG. 3: Marginalised posterior probability (dash blue) and profile likelihood (red) against CMSSM’s parameters, when we scanned the CMSSM with no constraints (other than those of physicality) (a), simulated LHC constraints (b), and simulated LHC and simulated WMAP constraints (c). The crosses on the abscissas are the best-fit values: the values for which \( \chi^2 \) was minimised. The vertical lines are the posterior means. The top and bottom blocks of colour are the one-sigma and two-sigma (68% and 95%) intervals for profile likelihood and for marginalised posterior probability respectively. The red dots on the abscissas are the ATLAS SU3 benchmark values.
FIG. 4: Marginalised posterior probability (dash blue) and profile likelihood (red) against mass constraints, when we scanned the CMSSM with no constraints (a), simulated LHC constraints (b), and simulated LHC and simulated WMAP constraints (c). The crosses on the abscissas are the best-fit values: the values for which $\chi^2$ was minimised. The vertical lines are the posterior means. The top and bottom blocks of colour are the one-sigma and two-sigma (68% and 95%) intervals for profile likelihood and for marginalised posterior probability respectively. The red dots on the abscissas are the ATLAS SU3 benchmark values.
The marginalised pdfs for the ATLAS SU3 measurable masses and mass differences for our three scans of the NUHM are very similar to those for the CMSSM (Figure 4) and for this reason they are not shown here. Like the CMSSM’s parameter space, the geometry of the NUHM’s parameter space also narrows the confidence interval for $m_\chi$.

In summary, compared with the CMSSM, parameter reconstruction in the NUHM is significantly poorer for $m_0$ (with much lower values now allowed), and comparable for $m_{1/2}$, $A_0$ and $\tan \beta$. The reconstruction of $m_{H_d}$ and $m_{H_u}$ is poor because the assumed ATLAS measurements basically do not depend on the Higgs sector.

C. The CMSSM-NUG

Compared with the CMSSM, the CMSSM-NUG has two additional parameters; the common gaugino mass is replaced by three independent gaugino soft mass parameters for the bino ($M_1$), the wino ($M_2$) and the gluino ($M_3$). Unlike the NUHM’s additional parameters, the CMSSM-NUG’s additional parameters do directly enter the assumed ATLAS positive end-point measurement.

In Figure 7 we show the 2D marginalised posterior planes for the CMSSM-NUG’s parameters for our three scans. Figure 7(a) shows the CMSSM-NUG’s parameters’ priors, because the likelihood is always set equal to one. Smaller values of $M_1$, $M_2$ and $M_3$ are favoured. Like in the CMSSM, smaller absolute values of $A_0$ are favoured and there is no preference for the values of $\tan \beta$ and $m_0$.

Figure 7(b) shows the 2D marginalised posterior planes for the CMSSM-NUG’s parameters, when we scanned the CMSSM-NUG with simulated LHC constraints. The reconstruction of $m_0$ in the CMSSM-NUG is significantly different from its reconstruction in the CMSSM. Unlike in the CMSSM, small values of $m_0$ are permitted but large values of $m_0$ are not permitted. The shape of the $m_0$ and $M_3$ plot is different from the $m_0$ and $m_{1/2}$ plot in the CMSSM in Figure 2(b). In the CMSSM-NUG, it favours smaller values of the gaugino masses. Their reconstruction is worse than the reconstruction of $m_{1/2}$ in the CMSSM in Figure 2(b). In the CMSSM-NUG, wider ranges of $M_1$, $M_2$ and $M_3$ are permitted. Like in the CMSSM, $\tan \beta$ is poorly reconstructed and the sign of $A_0$ is ambiguous.

Unfortunately, an egregious prior effect is hampering reconstruction in the CMSSM-NUG. Figure 8(a) shows that the CMSSM-NUG’s priors favour smaller values of $M_1$, $M_2$ and $M_3$, and, consequently, Figure 9(a) shows that the CMSSM-NUG’s priors favour very small neutralino masses. Because of the small neutralino masses, the squark and slepton masses must be small so that the mass differences are correct. This pushes the allowed $m_0$ values down, so that large values are not permitted and so that small values are favoured.

The effect of adding the simulated WMAP constraint marginalised posterior planes for the CMSSM-NUG’s parameters is shown in Figure 7(c). The reconstruction of $M_1$, $M_2$ and $M_3$ is greatly improved over Figure 7(b). Small values of $m_0$ are no longer permitted. Larger values of $m_0$ are preferred over Figure 7(b) and the reconstruction of $\tan \beta$ is a bit worse than in Figure 7(b). It appears that the stronger likelihood removes the egregious prior effect, as the plots bear similarity with Figure 2(c). A similar conclusion can be drawn by comparing Figure 8(b) and 8(c) with Figure 8(a).

Figure 9 shows the 1D marginalised posterior plots for the ATLAS SU3 measurable masses and mass differences for our three scans of the CMSSM-NUG. The plots are different from those for the CMSSM (Figure 4). Figure 9(a) shows that the CMSSM-NUG’s priors favours small lightest neutralino masses. This affects the recovery of $m_\chi$ in Figure 9(b). The simulated WMAP constraints (Figure 9(c)) improved the recovery of the mass constraints, though the recovery was still worse than it was in the CMSSM and the NUHM.

In contrast to the CMSSM’s parameter space, the geometry of the CMSSM-NUG’s parameter space does not narrow the confidence interval for $m_\chi$. The relaxing of gaugino mass unification in the CMSSM-NUG permits combinations of masses that were previously impossible in the CMSSM.

The a priori CMSSM-NUG’s prior preference for small neutralino masses (shown in Figure 9(a)) leads, remarkably, to some tension between the LHC constraint and the WMAP constraint in the model. Generally, relative to the CMSSM’s priors, the NUHM’s priors favour larger relic densities and the CMSSM-NUG’s priors favour smaller relic densities (though in all models small values of the relic density are favoured because of a fairly light sparticle mass spectrum involved). In the CMSSM-NUG scanned with simulated LHC constraints, because the relic density’s prior favours small values, the posterior for the relic density is not centred on its ATLAS SU3 value, but below it. This creates a conflict between the simulated LHC and simulated WMAP constraints.

In summary, parameter reconstruction in the CMSSM-NUG is poorer than it was in the CMSSM, and the results are dependent on the choice of prior. Wide ranges of $M_1$, $M_2$ and $M_3$ are permitted in the CMSSM-NUG, but in the CMSSM, $m_{1/2}$ was tightly squeezed. Furthermore, small values of $m_0$, that were excluded in the CMSSM, are permitted in the CMSSM-NUG. The recovery of $\tan \beta$ and $A_0$ is comparable in the CMSSM-NUG and CMSSM.
FIG. 5: Marginalised posterior probability planes, when we scanned the NUHM with no constraints (a), simulated LHC constraints (b), and simulated LHC and simulated WMAP constraints (c). The dark blue and light blue regions are the one-sigma and two-sigma regions respectively. The crosses on the planes are the best-fit values: the values for which $\chi^2$ was minimised. The dots on the planes are the posterior means. The red diamonds on the planes are the ATLAS SU3 benchmark values.
FIG. 6: Marginalised posterior probability (dash blue) and profile likelihood (red) against NUHM’s parameters, when we scanned the NUHM with no constraints [(a)], simulated LHC constraints [(b)], and simulated LHC and simulated WMAP constraints [(c)]. The crosses on the abscissas are the best-fit values: the values for which $\chi^2$ was minimised. The vertical lines are the posterior means. The top and bottom blocks of colour are the one-sigma and two-sigma (68% and 95%) intervals for profile likelihood and for marginalised posterior probability respectively. The red dots on the abscissas are the ATLAS SU3 benchmark values.
FIG. 7: Marginalised posterior probability planes, when we scanned the CMSSM-NUG with no constraints (a), simulated LHC constraints (b), and simulated LHC and simulated WMAP constraints (c). The dark blue and light blue regions are the one-sigma and two-sigma regions respectively. The crosses on the planes are the best-fit values: the values for which $\chi^2$ was minimised. The dots on the planes are the posterior means. The red diamonds on the planes are the ATLAS SU3 benchmark values.
FIG. 8: Marginalised posterior probability (dash blue) and profile likelihood (red) against the CMSSM-NUG’s parameters, when we scanned the CMSSM-NUG with no constraints (a), simulated LHC constraints (b), and simulated LHC and simulated WMAP constraints (c). The crosses on the abscissas are the best-fit values: the values for which $\chi^2$ was minimised. The vertical lines are the posterior means. The top and bottom blocks of colour are the one-sigma and two-sigma (68% and 95%) intervals for profile likelihood and for marginalised posterior probability respectively. The red dots on the abscissas are the ATLAS SU3 benchmark values.
FIG. 9: Marginalised posterior probability (dash blue) and profile likelihood (red) against mass constraints, when we scanned the CMSSM-NUG with no constraints (a), simulated LHC constraints (b), and simulated LHC and simulated WMAP constraints (c). The crosses on the abscissas are the best-fit values: the values for which $\chi^2$ was minimised. The vertical lines are the posterior means. The top and bottom blocks of colour are the one-sigma and two-sigma (68% and 95%) intervals for profile likelihood and for marginalised posterior probability respectively. The red dots on the abscissas are the ATLAS SU3 benchmark values.
D. The MSSM

The MSSM that we consider is a phenomenological model defined at the EW scale with twelve parameters. Because it has much more freedom than the CMSSM, it is interesting to investigate whether its parameters can be reconstructed.

Unsurprisingly, only a few parameters were reasonably well recovered, namely $M_2$, $\mu$, and $m_Q$. The other parameters, however, were poorly recovered in all cases. We present results for $M_2$, $\mu$ and $m_Q$, the parameters for which some reconstruction was possible, and for $\tan \beta$, to facilitate a comparison with its reconstruction in the other models.

Figure 10 shows the one-dimensional (1D) marginalised posterior pdfs (dash blue), as well as the profile likelihood (solid red) for the selection of MSSM parameters chosen above. Figure 10(a) shows the pdfs for the MSSM’s parameters’ priors, because the likelihood is always set equal to one. The priors for the mass parameters show a 1/$x$ shape. Smaller values of $\tan \beta$ are favoured.

Figure 10(b) shows the one-dimensional (1D) marginalised posterior pdfs (dash blue) when we added the simulated LHC constraints. The parameters $M_2$, $\mu$ and $m_Q$ are well determined, because they feature directly in the ATLAS constraints in the likelihood function. For $M_2$, which to a large extent determines $m_{\chi^0}^2$ and $\mu$ the posterior means are very close to the ATLAS SU3 values. Compared to its reconstruction in the CMSSM, the reconstruction of $\tan \beta$ is poor. Large values of $\tan \beta$ are allowed, and the two-sigma confidence interval only just includes the ATLAS SU3 value. Adding the WMAP dark matter constraints (Figure 10(c)) slightly improves the reconstruction.

Figure 11 shows the 1D marginalised posterior plots for the ATLAS SU3 measurable masses and mass differences for our three scans of the MSSM. Figures 11(b) and 11(c) show that the recovery of the masses in the MSSM is only slightly worse than their recovery in the CMSSM (Figure 4). The geometry of the MSSM’s parameter space narrows the confidence intervals for the measurable masses and mass differences, though less so than the CMSSM’s parameter space.

In summary, it seems that, because of its many free parameters, parameter reconstruction in the MSSM is difficult, though the parameters $M_2$, $\mu$ and $m_Q$ were reasonably well recovered, because they feature prominently in the ATLAS SU3 mass constraints. Like the CMSSM’s parameter space, the geometry of the MSSM’s parameter space prohibits certain mass combinations, and, consequently, narrows the the confidence interval for $m_\chi$.

E. The MSSM-NUG

The MSSM-NUG is a phenomenological model defined at the EW scale with fourteen parameters. Because it has much more freedom than the CMSSM and even the MSSM, it is interesting to investigate whether its parameters can be reconstructed.

Like in the MSSM, only a few parameters were reasonably well recovered, namely $M_2$, $\mu$, and $m_Q$. Surprisingly, unlike in the MSSM, however, $m_L$ and $m_E$ were also reasonably well recovered. This result was not robust; it was dependent on our choice of log priors. When we repeated our scans identically, except with linear priors, the reconstruction for $m_L$ and $m_E$ was significantly worse. We, therefore, only present results for $M_2$, $\mu$, and $m_Q$, the parameters for which reconstruction was robust, and for $\tan \beta$, to facilitate a comparison with its reconstruction in the other models.

Figure 12 shows the 1D marginalised posterior pdfs (dash blue), as well as the profile likelihood (solid red) for the selection of MSSM-NUG parameters chosen above. Figure 12(a) shows the pdfs for the MSSM-NUG’s parameters’ priors, because the likelihood is always set equal to one. The priors show a 1/$x$ shape. The is no strong preference for the value of $\tan \beta$.

Compared to the MSSM, the ATLAS constraints (Figure 12(b)) poorly recover the parameters in the MSSM-NUG, particularly $\mu$ and $\tan \beta$. The two-sigma confidence intervals for $\mu$ and $\tan \beta$ wrongly exclude the ATLAS SU3 values. Adding the WMAP dark matter constraints (Figure 12(c)) improves the reconstruction. Smaller values of $\tan \beta$, that are closer to the ATLAS SU3 value, are now favoured. The two-sigma confidence intervals for $\mu$ and $\tan \beta$ now include the ATLAS SU3 values. The reconstruction for $M_2$ is excellent; the confidence interval is narrow and the posterior mean is very close to the ATLAS SU3 value. It is likely that a prior effect, similar to the problem in the CMSSM-NUG, is removed by the simulated WMAP constraints, and, consequently, the results are greatly improved.

Figure 13 shows the 1D marginalised posterior plots for the ATLAS SU3 measurable masses and mass differences for our three scans of the MSSM. Figures 13(b) and 13(c) show that the recovery of the masses in the MSSM-NUG is worse than their recovery in the CMSSM and MSSM (Figures 4 and 11), but better than their reconstruction in the CMSSM-NUG (Figure 9). Like the CMSSM-NUG’s priors, the MSSM-NUG’s priors favour light neutralino masses. The geometry of the MSSM-NUG’s parameter space slightly narrows the confidence interval for $m_\chi$.

In summary, it seems that parameter reconstruction in the MSSM-NUG is challenging and that the results have a significant dependence on the choice of prior. Only $m_Q$, $M_2$ and $\mu$ could be reasonably well recovered with linear and
Fowlie and Roszkowski (2011) MSSM No constraints

\[ m_Q \, (\text{GeV}) \quad \mu \, (\text{GeV}) \quad \tan \beta \quad M_2 \, (\text{GeV}) \]

(a) MSSM with no constraints

Fowlie and Roszkowski (2011) MSSM ATLAS only

\[ m_Q \, (\text{GeV}) \quad \mu \, (\text{GeV}) \quad \tan \beta \quad M_2 \, (\text{GeV}) \]

(b) MSSM with simulated LHC constraints

Fowlie and Roszkowski (2011) MSSM ATLAS + WMAP

\[ m_Q \, (\text{GeV}) \quad \mu \, (\text{GeV}) \quad \tan \beta \quad M_2 \, (\text{GeV}) \]

(c) MSSM with simulated LHC and simulated WMAP constraints

FIG. 10: Marginalised posterior probability (dash blue) and profile likelihood (red) against the MSSM’s parameters, when we scanned the MSSM with no constraints (a), simulated LHC constraints (b), and simulated LHC and simulated WMAP constraints (c). The crosses on the abscissas are the best-fit values: the values for which $\chi^2$ was minimised. The vertical lines are the posterior means. The top and bottom blocks of colour are the one-sigma and two-sigma (68% and 95%) intervals for profile likelihood and for marginalised posterior probability respectively. The red dots on the abscissas are the ATLAS SU3 benchmark values.
FIG. 11: Marginalised posterior probability (dash blue) and profile likelihood (red) against mass constraints, when we scanned the MSSM with no constraints [a], simulated LHC constraints [b], and simulated LHC and simulated WMAP constraints [c]. The crosses on the abscissas are the best-fit values: the values for which $\chi^2$ was minimised. The vertical lines are the posterior means. The top and bottom blocks of colour are the one-sigma and two-sigma (68% and 95%) intervals for profile likelihood and for marginalised posterior probability respectively. The red dots on the abscissas are the ATLAS SU3 benchmark values.
log priors. Unlike the CMSSM-NUG’s parameter space, the geometry of the MSSM-NUG’s parameter space slightly narrows the confidence interval for $m_\chi$.

F. Dark Matter

The lightest neutralino plays the role of dark matter (DM). We can calculate its relic density and the rate at which we expect DM particles to be detected.

First, Figure 14 shows the 1D marginalised posterior (blue dash) for the relic density $\Omega_\chi h^2$, when we scanned the CMSSM, the NUHM, the CMSSM-NUG, and the MSSM with simulated LHC constraints. The distribution is reasonably well peaked close to the assumed correct value of 0.2332 (red dot) in the CMSSM, the NUHM and even the MSSM, although in the last case the confidence interval is much wider. In the CMSSM-NUG the reconstruction of $\Omega_\chi h^2$ is very poor because of the tension between the LHC constraint and the WMAP constraint in the model mentioned above. The case of the MSSM-NUG (not shown) looks similar to the MSSM plot. For comparison, we show (solid red) the profile likelihood curves for each case.

Turning next to direct detection, Figure 15 shows the 2D marginalised posterior planes for the neutralino mass and the spin-independent cross section on a proton $\sigma_{SI}^p$, when we scanned the CMSSM, the NUHM, the CMSSM-NUG, and the MSSM with simulated LHC and simulated WMAP constraints. The ATLAS SU3 values are well recovered in the CMSSM (Figure 15(a)) and the NUHM (Figure 15(b)). The ATLAS SU3 values are poorly recovered in the CMSSM-NUG (Figure 15(c)). It is likely that this results from the poor recovery of $m_\chi$ in the CMSSM-NUG. The recovery in the MSSM (Figure 15(d)) is worse than it is in the CMSSM and the NUHM, but better than it is in the CMSSM-NUG. The case of the MSSM-NUG (not shown) looks similar to the MSSM plot. In particular, the constraint on $m_\chi$ is only slightly worse than in the MSSM and is much better than in the CMSSM-NUG.

G. Comparison

In Figures 16 and 17 we compare the reconstruction in all five models, by plotting the various parameters’ posterior means and two-sigma (95%) confidence intervals, when we scanned the models with simulated LHC and simulated WMAP constraints. We see that the two-sigma confidence intervals almost always include, and that the posterior means are always close to, the ATLAS SU3 value. The exception is that the two-sigma region for $\tan \beta$ in the MSSM (Figure 16(d)) basically excludes the ATLAS SU3 value, whilst in the MSSM-NUG it barely allows it. Generally, the ranges of $\tan \beta$ are on the high side and not well reconstructed. The parameter $A_0$ is poorly reconstructed in all of the models.

The reconstruction of the ATLAS measurable masses and mass differences (Figure 17) is similar in all five models, apart from the reconstruction of $m_\chi$, which varies between the models. The two-sigma confidence intervals for $m_\chi$ in the CMSSM and in the NUHM are similar, but the two-sigma confidence interval for $m_\chi$ in the CMSSM-NUG is much wider than it is in the other models. In the MSSM and MSSM-NUG, the two-sigma confidence interval for $m_\chi$ is slightly wider than it is in CMSSM, but narrower than it is in the CMSSM-NUG. The geometry of the CMSSM-NUG’s parameter space does not forbid certain combinations of neutralino masses, resulting in a large confidence interval for $m_\chi$.

H. Evidence

We consider whether the simulated LHC constraints or the combination of simulated LHC and simulated WMAP constraints can distinguish between the CMSSM, the NUHM and the CMSSM-NUG. The evidence\footnote{The evidence is a measure of how well and how economically a model agrees with some data. It is given by $Z(D) = \int L(D|H)\pi(H)dm_1 dm_2 \ldots dm_N$; see Ref. 15 for more details.} for each model is listed in Table 1X which also shows odds ratios; ratios of the evidence for different models. The Jefferey’s scale is a subjective scale for assessing whether an odds ratio is significant. The CMSSM and the NUHM are statistically indistinguishable in all cases, the slight preference for the NUHM is not significant according to Jefferey’s scale. The NUHM’s extra parameters improve the fit enough to negate the penalty for having a greater prior volume.
FIG. 12: Marginalised posterior probability (dash blue) and profile likelihood (red) against the MSSM-NUG’s parameters, when we scanned the MSSM-NUG with no constraints (a), simulated LHC constraints (b), and simulated LHC and simulated WMAP constraints (c). The crosses on the abscissas are the best-fit values: the values for which $\chi^2$ was minimised. The vertical lines are the posterior means. The top and bottom blocks of colour are the one-sigma and two-sigma (68% and 95%) intervals for profile likelihood and for marginalised posterior probability respectively. The red dots on the abscissas are the ATLAS SU3 benchmark values.
FIG. 13: Marginalised posterior probability (dash blue) and profile likelihood (red) against mass constraints, when we scanned the MSSM-NUG with no constraints [a] simulated LHC constraints [b] and simulated LHC and simulated WMAP constraints [c]. The crosses on the abscissas are the best-fit values: the values for which $\chi^2$ was minimised. The vertical lines are the posterior means. The top and bottom blocks of colour are the one-sigma and two-sigma (68% and 95%) intervals for profile likelihood and for marginalised posterior probability respectively. The red dots on the abscissas are the ATLAS SU3 benchmark values.
FIG. 14: Marginalised posterior probability (dash blue) and profile likelihood (red) of the neutralino relic density, when we scanned the CMSSM (a), the NUHM (b), the CMSSM-NUG (c) and the MSSM (d). The crosses on the abscissas are the best-fit values: the values for which $\chi^2$ was minimised. The vertical lines are the posterior means. The top and bottom blocks of colour are the one-sigma and two-sigma (68% and 95%) intervals for profile likelihood and for marginalised posterior probability respectively. The red dots on the abscissas are the ATLAS SU3 benchmark values.

The CMSSM and the NUHM are both significantly preferred to the CMSSM-NUG in all cases. The simulated LHC constraints and our models’ priors show a significant preference for the CMSSM and the NUHM over the CMSSM-NUG. The CMSSM-NUG’s extra parameters are not economical; they dilute the regions of parameter space that agree with the constraints, without improving the fit.

A caveat is in order: these results are not entirely robust. We repeated our calculations, identically, except that we used linear priors, rather than log priors. A substantial preference for the CMSSM over the NUHM emerged when the evidence was calculated with simulated LHC and simulated WMAP data, due to the larger number of free parameters in the NUHM and consequently larger volume effect. The CMSSM-NUG was still significantly disfavoured in all cases.
I. Choice of priors

We investigated whether our results were robust, that is, whether they were dependent on our choice of log priors, rather than linear priors. The results for the CMSSM-NUG, MSSM-NUG and for the evidences were not entirely robust. In particular, the models without gaugino mass unification (the CMSSM-NUG and MSSM-NUG) were dependent on our choice of prior.

The prior problems in the CMSSM-NUG and MSSM-NUG could have been caused by a volume effect in the log priors. Log priors equally weighted orders of magnitude. In the CMSSM's log priors, before physicality conditions were imposed, there was a 0.625 chance that the \( m_{1/2} \) parameter would lie between 100 GeV and 1000 GeV. In the CMSSM-NUG's log priors, the chance that \( M_1 \) and \( M_2 \) both lay between 100 GeV and 1000 GeV was 0.625 \( ^2 = 0.391 \). In other words, the fraction of log prior space in which either \( M_1 \) or \( M_2 \) is small in the CMSSM-NUG is greater than the fraction of log prior space in which \( m_{1/2} \) is small in the CMSSM. Because the neutralinos' masses increase with the gaugino masses, this would lead to a prior preference for light neutralinos in the CMSSM-NUG. An identical argument can be applied to the MSSM-NUG.

The CMSSM had very little prior dependence; the results for log and linear priors were statistically indistinguishable. The NUHM's additional parameters, \( m_{H_u} \) and \( m_{H_d} \), showed a strong prior dependence, though the results for the other parameters were indistinguishable. Of the MSSM's parameters, only the trilinear terms had a strong prior dependence. Though this dependence was removed by the simulated WMAP data. Unsurprisingly, because it has so much freedom and is so poorly constrained by the likelihood function, the MSSM-NUG had the strongest prior dependence. It seems that the assumption of gaugino mass unification is critical; without this assumption, CMSSM and MSSM have too much freedom to be constrained by the simulated data.

The prior is information that is known before the inference. We, unfortunately, had an inarticulated prior preference for very small neutralino masses in the CMSSM-NUG and MSSM-NUG, to which we were oblivious, that were in conflict with the ATLAS SU3 sparticle mass spectrum. This is a pitfall of Bayesian statistics; the priors that you choose for the model parameters can result in unexpected and unwanted preferences for some parameters, especially when the likelihood is weakly constraining.

V. CONCLUSIONS

We have analysed the prospects of reconstructing the ATLAS SU3 benchmark point in the framework of several low-energy SUSY models with a Bayesian approach, assuming numerical information about masses and mass differences from a hypothetical positive end-point measurement by ATLAS but otherwise making a Gaussian approximation, following the method developed in Ref. [15].

As a preliminary step, in each model we delineated the ranges of the parameters that were favoured by requiring physical solutions (and that the neutralino was the LSP) alone. Next, we showed that a hypothetical ATLAS measurement would allow one to significantly constrain the common gaugino mass parameter \( m_{1/2} \) and, to a lesser degree, the scalar mass parameter \( m_0 \) in models with gaugino mass unification (the CMSSM and the NUHM), though
FIG. 15: Marginalised posterior probability planes relevant to direct detection experiments, when we scanned the CMSSM\textsuperscript{(a)}, the NUHM\textsuperscript{(b)}, the CMSSM-NUG\textsuperscript{(c)}, and the MSSM\textsuperscript{(d)}. The dark blue and light blue regions are the one-sigma and two-sigma regions respectively. The crosses on the planes are the best-fit values: the values for which $\chi^2$ was minimised. The dots on the planes are the posterior means. The red diamonds on the planes are the ATLAS SU3 benchmark values.
in the other models the situation is much more mixed. In the future, reconstructing $\tan \beta$ will be challenging and reconstructing $A_0$ will be almost impossible, with even its sign remaining ambiguous in our analysis. Adding the relic density of neutralino dark matter to the likelihood improves parameter reconstruction in the unified models, especially for $m_0$, but there is much less improvement in the relaxed models. The assumption of gaugino mass unification seems critical to parameter reconstruction; relaxing this assumption leads to a much poorer determination of the model parameters, the “directly” measured masses and mass differences, the relic density of the neutralino and the elastic spin-independent cross section $\sigma_{SI}^p$.

In our analysis we used a log prior for all parameters other than $\tan \beta$ and $A_0$ (and the nuisance parameters). We did, however, test the prior dependence of our results, by repeating our analysis with a linear (flat) prior for all the parameters. We confirmed that the CMSSM has very little prior dependence [15] but we found that generally the other models have a rather strong prior dependence.

We conclude that with the experimental information provided by the assumed ATLAS measurement it will be challenging to perform a completely reliable parameter reconstruction and model comparison outside of the simplest CMSSM.

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FIG. 17: The two-sigma (95%) confidence intervals and posterior means for $m_{\chi}$ [a], $m_{\chi_2} - m_{\chi}$ [b], $m_{\tilde{q}} - m_{\chi}$ [c] and $m_{\tilde{l}} - m_{\chi}$ [d].
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