The influence of positronium photoionization rate on the heating of J0250+5854 polar cap

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Abstract. The influence of positronium photoionization rate on the heating of PSR J0250+5854 polar cap is considered. It is assumed that the polar cap is heated only by reverse positrons accelerated in pulsar diode. It is supposed that pulsar diode is located near the star surface (polar cap model) and operates in the steady state space charge-limited flow regime. To calculate the production rate of electron-positron pairs we take into account only the curvature radiation of primary electrons and its absorption in magnetic field. It is assumed that some fraction of electron-positron pairs may be created in bound state that can later be photoionized by thermal photons from star surface.

1. Introduction
Radio pulsar J0250+5854 rotates with period $P = 23.54$ s \cite{1}. It is the slowest pulsar among rotation powered pulsars \cite{2}. It is an old pulsar with spin-down age $\tau = 13.7 \times 10^6$ years, its time derivative of period is $\dot{P} = 2.71 \times 10^{-14}$, its spin-down energy loss rate is equal to $\dot{E} = 8.2 \times 10^{28}$erg/s, the strength of dipolar magnetic field at pole estimated by pulsar spin-down rate is $B_{\text{dip}} = 5.1 \times 10^{13}$ G, the distance to the pulsar estimated by dispersion measure is $D_{DM} = 1.56$ kpc \cite{2}. Such pulsars lie beyond conventional pulsar “death line” (see, for example, \cite{3, 4}). The existence of radio pulsar emission in such pulsars is usually explained by the presence of small-scale surface magnetic field \cite{5, 6, 7, 8}. The other explanation is presented by \cite{9}. In this paper, we consider the influence of positronium generation and its photoionization on the J0250+5854 polar cap heating and the corresponding polar cap X-ray luminosity. For other pulsars these processes have been thoroughly considered in, for example,\cite{10, 11}. It is assumed that electron-positron plasma is generated in the inner polar cap bended by the small-scale magnetic field. The calculations are performed in the model of free electron emission with two extreme assumption about value of reverse positron current.

2. Model
Let the neutron star have a radius $r_{ns}$ and a dipolar magnetic moment $\vec{m}$, where $B_{\text{dip}} = 2m/r_{ns}^3$. We also assume that a small-scale magnetic field with strength $B_{sc}$ and characteristic scale $\ell \sim r_{ns}/20$ presents on the polar cap. For simplicity, we suppose that $B_{sc}$ is parallel to surface and lies in the plane of vectors $\vec{\Omega}$ and $\vec{m}$, see figure 1. We also assume that inclination angle $\chi$
Figure 1. A sketch of the vicinity of an inner gap. Neutron star is shown by gray area, pulsar tube boundaries are shown by black lines, the inner gap is shown by brown area.

Figure 2. The dependence of polar cap luminosity $L_{pc}$ due to the reverse positron heating on the small-scale field strength $B_{sc}$ in case of rapid screening model for different parameters is shown. The case of $W_0 = 6 \cdot 10^5 \text{s}^{-1}$ is shown on left panel and the case of $W_0 = 1.2 \cdot 10^8 \text{s}^{-1}$ is shown on the right panel. Black solid line corresponds to $W_0 = +\infty$ (all positroniums are photoionized immediately) on both panels.

(angle between vectors $\vec{\Omega}$ and $\vec{m}$, see figure 1) is equal to $60^\circ$. The model of small-scale magnetic field is described in details in [12, 13, 14]. We consider only the case of inner gap [15] and assume that the inner gap occupies all the pulsar tube cross section and is residing as low as possible. Let us denote the altitudes of inner gap lower plate (cathode) and upper plate (anode) by $z_{lo}$ and $z_{hi}$, respectively (see figure 1). In most cases the inner gap resides exactly on neutron star surface $z_{lo} = 0$ (see [16] for details). We suppose that the inner gap operates in the steady-state space charge-limited flow regime. For simplicity, we take into account the generation of electron-positron pairs only by curvature radiation emitted by primary electrons in magnetic field. It was shown by Usov and Melrose [11] that, for $B < B_{low}$ almost all electron-positron pairs are created in unbound state, i.e. the probability $P_b$ that a pair is created in bound state is equal to zero $P_b = 0$, while for $B > B_{high}$ all pairs are created in bound state, i.e. $P_b = 1$, where $B$ is magnetic strength at point of pair creation, $B_{low} = 0.04 B_{cr}$, $B_{high} = 0.15 B_{cr}$, $B_{cr} \approx 4.41 \cdot 10^{13} \text{G}$ [11]. We approximate the probability $P_b$ at $B_{low} \leq B \leq B_{high}$ by linear
Figure 3. The same in figure 2, for the case of gradually screening model is shown.

Figure 4. The dependence of anode altitude $z_{hi}$ (in units $r_{ns}$) on the small-scale field strength $B_{sc}$ for different parameters are shown by color lines. The colors mean the same as in figure 2, except the brown solid line which shows cathode altitude $z_{lo}$.

function: $P_b = (B - B_{low})/(B_{high} - B_{low})$. The dependence of pair generation properties on the photon polarisation is neglected in the calculations. In order to take into account the positronium photoionization by thermal photons emitted from the star surface we use the following approximate formula for the photoionization rate [11]:

$$\frac{dN}{dt} = W_0 \left( \frac{10^2}{\Gamma} \right)^3 \left( \frac{T}{10^6 K} \right)^2 (1 - \cos \theta_{star}), \tag{1}$$

where $\Gamma$ is positronium Lorentz factor, $T$ is star surface temperature, $\theta_{star}$ is star angular radius, $W_0 = 6 \cdot 10^5 s^{-1}$. Due to small polar cap size, we neglect positronium photoionization by thermal
Figure 5. The dependence of polar cap luminosity $L_{pc}$ on photoionization rate coefficient $W_0$ in case of $B_{sc} = \frac{1}{2} B_{dip}$ for different parameters are shown by color lines. The colors mean the same as in figure 2. Horizontal black solid line corresponds to the case of $W_0 = +\infty$. Left panel corresponds to rapid screening model and right panel corresponds to gradually screening model.

Figure 6. The dependence of the anode altitude $z_{hi}$ (in units $r_{ns}$) on photoionization rate coefficient $W_0$ in the case of $B_{sc} = \frac{1}{2} B_{dip}$ for different parameters are shown by color lines. The colors mean the same as in figure 2. Horizontal black solid line corresponds to the case of $W_0 = +\infty$.

In this paper, we do not take into account photon splitting and positronium decay. In order to roughly estimate the effect of these processes we assume that $(1-f)$ fraction of positronium decays immediately after creation while $f$ fraction does not decay at all. It is worth to note that accordingly [17], the positroniums are quickly photoionized at $T \geq 10^5$ K [18] and, hence, positronium formation can be neglected [10]. In order to consider a such possibility we also perform the calculations for the case of $W_0 = 1.2 \cdot 10^8$s$^{-1}$, which gives a rough estimation of photoionization rate calculated in [17]. Reverse positron current is calculated by using two models: the model of rapid screening [19] according to which the electron-positron plasma screens parallel electric field $E_{||} = (\vec{E} \cdot \vec{B})/B$ almost immediately and the model of gradually screening [20, 21], which allows the parallel electric field to penetrate...
3. Results

The dependence of the polar cap luminosity $L_{pc}$ caused by the reverse positron current heating on strength of small-scale magnetic field $B_{sc}$ in the case of rapid and gradually screening models is shown in figures 2 and 3, correspondingly. The dependence of anode altitude $z_{hi}$ on strength of small-scale magnetic field $B_{sc}$ is shown in figure 4. At $B_{sc} \lesssim 0.5B_{dip}$, altitude $z_{hi}$ decreases with increasing $B_{sc}$ due to increasing total magnetic field strength and, more importantly, due to increasing field line curvature. At larger $B_{sc}$ according to considered field model [16] cathode altitude $z_{lo}$ begins to increase. Hence, primary electrons are accelerated at larger altitude where field strength and its curvature are less. It leads to less effective pair production which, in turn, increases anode altitude $z_{hi}$ at $B_{sc} \gtrsim 0.6B_{dip}$. Due to discrepancies in photoionization rate coefficient $W_0$ [18] we consider the dependence of the polar cap heating on $W_0$ value. The dependence of polar cap luminosity $L_{pc}$ on coefficient $W_0$ is shown in figure 5 and the dependence of anode altitude $z_{hi}$ on $W_0$ is shown in figure 6. It is easy to see that the luminosity $L_{pc}$ at $W_0 \approx 10^7$ sec$^{-1}$ is almost the same as at $W_0 = +\infty$ (all pairs are produced in unbound state). Hence, we confirm result [10] that positronium formation may be neglected but only in case of photoionization rate calculated by [17].

4. Discussion

In the paper we use two assumptions about J0250+5854 pulsar: the presence of surface magnetic field with very small characteristic scale $\ell \approx 500$ m and neutron star surface temperature $T \approx (1-3) \cdot 10^5$ K. The main problem with using these assumptions is that the pulsar is very old $\tau = 13.7 \cdot 10^5$ years. Hence it is difficult to explain why field with so small scale has survived and why star is so hot. But it is worth to note that radio pulsar B0950+08 has spin down age $\tau = 17.5 \cdot 10^6$ years and star surface temperature $T \sim (1-3) \cdot 10^5$ K [23]. A such temperature may be related to internal heating mechanisms like rotochemical heating and heating due to vortex friction [24]. We also may speculate that magnetic field decay event with Hall cascade has occur not so long ago in this pulsar [25]. Hence small-scale magnetic field may be generated during Hall cascades and accompanying field decay may heat up the star.

In the paper only curvature radiation of primary electrons is taken into account. It is worth to note that resonant compton scattering may give a similar quantity of pairs [26] that leads to decreasing anode altitude $z_{hi}$ and hence decreasing polar cap heating $L_{pc}$ by the reverse positron current. Our choice of inclination angle $\chi = 60^\circ$ does not motivated by anything. Although we find that in considered field configuration the pulsar lies below “pulsar death” line in the case of $\chi = 0^\circ$ and $\chi = 30^\circ$. But we guess that it is only artifact of used small-scale field model.

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