Kaon decays shedding light on massless dark photons

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Abstract

We explore kaon decays with missing energy carried away by a massless dark photon, $\gamma$, assumed to have flavor-changing dipole-type couplings to the $d$ and $s$ quarks. We consider in particular the neutral-kaon modes $K_L \to \gamma \gamma$ and $K_L \to \pi^0 \gamma \gamma$ and their $K_S$ counterparts, as well as the charged-kaon channel $K^+ \to \pi^+ \gamma \gamma$, each of which also has an ordinary photon, $\gamma$, in the final state. In addition, we look at $K_{L,S} \to \pi^+ \pi^- \gamma$ and $K^+ \to \pi^+ \pi^0 \gamma$. Interestingly, the same $ds \gamma$ interactions give rise to the flavor-changing two-body decays of hyperons with missing energy and are subject to model-independent constraints that can be inferred from the existing hyperon data. Taking this into account, we obtain branching fractions $B(K_L \to \gamma \gamma)$ and $B(K_L \to \pi^0 \gamma \gamma)$ which can be as high as $10^{-3}$ and $10^{-6}$, respectively, one or both of which may be within the sensitivity reach of the KOTO experiment. Furthermore, we find that $B(K^+ \to \pi^+ \gamma \gamma)$ and $B(K^+ \to \pi^+ \pi^0 \gamma \gamma)$ are allowed to be maximally of order $10^{-6}$ as well, which may be probed by NA62. Complementarily, the hyperon modes can have rates which are potentially accessible by BESIII. Thus, these ongoing experiments could soon be able to offer significant tests on the existence of the massless dark photon.
I. INTRODUCTION

Over the past few decades various phenomenological considerations have motivated the introduction of the so-called dark photon, a spin-one boson associated with a new Abelian gauge symmetry, $U(1)_D$, under which all the fields of the standard model (SM) are singlets [1–20]. The dark photon may be massive or massless, depending on whether $U(1)_D$ is spontaneously broken or stays unbroken, respectively. The massive one, often symbolized by $A'$, can interact directly with SM fermions through a renormalizable operator $\epsilon eA'_\mu J_{\rm EM}^\mu$ containing the electromagnetic current $eJ_{\rm EM}^\mu$ and a small parameter $\epsilon$ generated by the kinetic mixing between the dark and SM Abelian gauge fields [1–8]. It follows that $A'$ could be produced in the decays or collisions of SM fermions and hadrons and it might decay into electrically charged fermions or mesons. In general, it could also decay invisibly into other dark particles. These possibilities have stimulated numerous dedicated quests for it, but with negative results so far, leading to bounds on $\epsilon$ over various ranges of the $A'$ mass [1–7, 21–26].

The massless dark photon, here denoted by $\gamma$, is very dissimilar from the massive one because they differ substantially with regard to both their theoretical underpinnings and experimental signatures [7–16]. If $U(1)_D$ remains unbroken, one can always arrange a linear combination of the dark and SM $U(1)$ gauge bosons so that it has no renormalizable connection to the SM and can be defined as the massless dark photon [8, 9]. Since it therefore has no direct interactions with SM fermions, the limitations implied by the aforementioned hunts for $A'$ are not applicable to $\gamma$. Nevertheless, the latter could still have consequential impact on the SM sector via higher-dimensional operators [9, 10] which may translate into detectable effects. This suggests that potentially promising avenues to seek $\gamma$ may be available and hence should be explored. Some of them will be proposed below, which may be feasible at ongoing or near-future experiments. Given that the viable parameter space of the massive dark photon continues to shrink with accumulating null outcomes of its searches, it is important pay increasing attention to the alternate possibility that the dark photon is massless.

In this paper we concern ourselves with flavor-changing neutral current (FCNC) transitions induced by the massless dark photon, $\gamma$, undergoing nonrenormalizable interactions with the $d$ and $s$ quarks via the dimension-five operators in the Lagrangian

$$
\mathcal{L}_{ds\gamma} = -\overline{d}(C + \gamma_5 C_5)\sigma^{\mu\nu}s \tilde{F}_{\mu\nu} + \text{H.c.},
$$

where $C$ and $C_5$ are constants which have the dimension of inverse mass and can in general be complex, $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$, and $\tilde{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu$ is field-strength tensor of $\gamma$. In the absence of other particles beyond the SM lighter than the electroweak scale, $\mathcal{L}_{ds\gamma}$ could originate from dimension-six operators which respect the SM gauge group and the unbroken $U(1)_D$. One can express operators in the form $\mathcal{L}_{\text{NP}} = -\Lambda_{\text{NP}}^2 \left( C_{12} \overline{Q}_2 \sigma^{\mu\nu} d_2 + C_{21} \overline{Q}_2 \sigma^{\mu\nu} d_1 \right) H \tilde{F}_{\mu\nu} + \text{H.c.}$, where $\Lambda_{\text{NP}}$ represents an effective heavy mass scale, the dimensionless coefficients $C_{12,21}$ are generally complex, $Q_{1,2}$ ($d_{1,2}$) stand for left-handed quark doublets (right-handed down-type quark singlets) from the
first two families, and \( H \) designates the SM Higgs doublet \([9]\). Accordingly \( C = \Lambda_{np}^2 (C_{12} + C_{21}) v/\sqrt{8} \) and \( C_5 = \Lambda_{np}^2 (C_{12} - C_{21}) v/\sqrt{8} \), with \( v \approx 246 \) GeV being the Higgs vacuum expectation value. Both \( \Lambda_{np} \) and \( C_{12,21} \) depend on the details of the underlying new physics (NP).

The interactions in \( \mathcal{L}_{ds\bar{\gamma}} \) bring about the FCNC decays of hyperons into a lighter baryon plus missing energy carried away by the massless dark photon. In Ref. \([14]\) we have studied such two-body processes and demonstrated that their rates are allowed by present constraints to reach values that are within the sensitivity reach of the ongoing BESIII experiment \([27,28]\). Analogous transitions can take place in the kaon sector. In the case of massive dark photon, \( K \to \pi A' \) and \( K^+ \to \mu^+ \nu A' \) might be useful in the quests for it \([6,19,21]\). On the other hand, since \( \gamma \) is massless and has no renormalizable couplings to SM members, \( K \to \pi \gamma \) are forbidden by angular-momentum conservation and gauge invariance and \( K^+ \to \mu^+ \nu \gamma \) would be highly suppressed. Instead, as suggested in Ref. \([12]\), one kaon mode that could probe \( \mathcal{L}_{ds\bar{\gamma}} \) is \( K^+ \to \pi^+ \pi^0 \gamma \), which might be accessible in the NA62 experiment \([29]\).

It turns out that there are other kaon modes which might provide additional and competitive windows into the same \( ds\bar{\gamma} \) couplings. Specifically, here we propose to pursue the neutral-kaon channels \( K_L \to \gamma \gamma \) and \( K_L \to \pi^0 \gamma \gamma \) and the charged one \( K^+ \to \pi^+ \gamma \gamma \), all of which have an ordinary photon, \( \gamma \), among the daughter particles. As our analysis will indicate, the two \( K_L \) modes can have rates which may be big enough to be observable in the currently running KOTO experiment \([30]\). We will also examine \( K_S \to \gamma \gamma, \pi^0 \gamma \gamma \) and \( K_{L,S} \to \pi^+ \pi^- \gamma \) and take another look at \( K^+ \to \pi^+ \pi^0 \gamma \).

The remainder of the paper is organized as follows. In Sec. II with the aid of chiral perturbation theory, we first derive the matrix elements of the quark bilinears in Eq. (1) which are needed to write down the amplitudes for the kaon decays of interest. Then we deal with the corresponding decay rates. In Sec. III we evaluate the maximal branching fractions of the kaon modes, taking into account model-independent restrictions on the \( ds\bar{\gamma} \) couplings deduced from the available hyperon data. We draw our conclusions in Sec. IV.

II. KAON DECAY AMPLITUDES AND RATES

To investigate the influence of \( \mathcal{L}_{ds\bar{\gamma}} \) on our processes of interest, we adopt the framework of chiral perturbation theory \([31]\). In this context, one can obtain the correspondences between operators containing the quark fields \((q_1, q_2, q_3) = (u, d, s)\) and their hadronic counterparts involving the lightest pseudoscalar-meson fields, which constitute a flavor-SU(3) octet and are collected into

\[
\Sigma = e^{i\varphi/f}, \quad \varphi = \sqrt{2} \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 \\
\frac{1}{\sqrt{2}} \pi^- + \frac{1}{\sqrt{6}} \eta_8 \\
K^-
\end{pmatrix}
\begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 \\
\frac{1}{\sqrt{6}} \eta_8 \\
-\frac{2}{\sqrt{6}} \eta_8
\end{pmatrix},
\]

(2)
where $f$ denotes the pion decay constant. At leading chiral order, the bosonization of the quark tensor currents in Eq. (1) has been addressed before [32–36]. Explicitly, it is most generally given by [35]

$$
\overline{q}_I \sigma_{\mu \nu} P_L q_J \Leftrightarrow -\frac{ia_T}{2} f^2 \left[ \left( D_\mu \Sigma \Sigma^\dagger D_\nu \Sigma - D_\nu \Sigma \Sigma^\dagger D_\mu \Sigma + i \epsilon_{\mu \nu \rho \omega} D^\rho \Sigma \Sigma^\dagger \right) \right]_{JI}
$$

$$
\Leftrightarrow \frac{a_T'}{2} f^2 \left[ \left( F_\mu^R + i \tilde{F}_\mu^R \right) + \left( F_\mu^L + i \tilde{F}_\mu^L \right) \right]_{JI}.
$$

where $a_T'$ is a constant having the dimension of inverse mass, $P_{L,R} = (1 \mp \gamma_5)/2$, and $I, J = 1, 2, 3$, implying that $JI = 32 (23)$ for $s \to d$ ($d \to s$) transitions. Electromagnetic effects are included in Eq. (3) via $^1$

$$
D_\mu \Sigma = \partial_\mu \Sigma - i F_\mu^L \Sigma + i \Sigma F_\mu^R,
$$
$$
F_\mu^L = F_\mu^R = -e A_\mu Q_q,
$$
$$
F_\mu^L = \tilde{F}_\mu^R = -e \epsilon_{\mu \nu \rho \omega} \partial^\rho A^\omega Q_q,
$$
$$
F_\mu^L = \partial_\mu A_\nu - \partial_\nu A_\mu,
$$
$$
Q_q = \frac{1}{3} \text{diag}(2, -1, -1),
$$

where $A_\mu$ and $F_{\mu \nu}$ stand for the ordinary photon field and its field-strength tensor, respectively, and $Q_q$ represents the electric-charge matrix of the three lightest quarks. In our numerical treatment later on, we will employ $f = f_\pi = 92.07(85)$ MeV [37] as well as the lattice QCD estimates $a_T = 0.658(23)/\text{GeV}$ [38] and $a_T' = 3.3(1.1)/\text{GeV}$ [36, 39].

From Eq. (3) we can determine the matrix elements required to write down the amplitudes for $K \to \gamma \gamma$ and $K \to \pi \gamma \gamma$ arising from $\mathcal{L}_{ds\gamma}$ which have both an ordinary photon, $\gamma$, and a massless dark photon in the final states. Thus, for $K \to \gamma \gamma$ we arrive at

$$
\langle \gamma | \bar{d} \sigma_{\alpha \omega} s | K^0 \rangle = \langle \gamma | \bar{s} \sigma_{\alpha \omega} d | K^0 \rangle = \frac{i \sqrt{8}}{3} a_T' e f \epsilon_{\alpha \omega \mu \nu} \varepsilon^\mu k^\nu,
$$

$$
\langle \gamma | \bar{d} \sigma_{\alpha \omega} \gamma_5 s | K^0 \rangle = \langle \gamma | \bar{s} \sigma_{\alpha \omega} \gamma_5 d | K^0 \rangle = \frac{\sqrt{8}}{3} a_T' e f \left( \varepsilon^\alpha s_k - \varepsilon_k^s \right),
$$

where $\varepsilon$ and $k$ are the ordinary photon’s polarization vector and momentum, respectively. With the approximation $\sqrt{2} K_{L,S} - K^0 = K^0 \pm K^0$, we then derive the amplitudes

$$
\mathcal{M}_{K_L \to \gamma \gamma} = \frac{4 a_T' e f}{3} \left[ -\epsilon_{\mu \nu \rho \omega} \text{Re} C + \left( g^{\mu \nu} g^{\rho \omega} - g^{\mu \rho} g^{\nu \omega} \right) \text{Im} \mathcal{C}_5 \right] \varepsilon^\mu \varepsilon^\rho \varepsilon^s \varepsilon_k q_k,
$$

$$
\mathcal{M}_{K_S \to \gamma \gamma} = \frac{4 a_T' e f}{3} \left[ \epsilon_{\mu \nu \rho \omega} \text{Im} C + \left( g^{\mu \nu} g^{\rho \omega} - g^{\mu \rho} g^{\nu \omega} \right) \text{Re} \mathcal{C}_5 \right] \varepsilon^\mu \varepsilon^\rho \varepsilon^s \varepsilon_k q_k.
$$

$^1$ Under chiral $\text{SU}(3)_L \times \text{SU}(3)_R$ transformations $\Sigma \to V_L \Sigma V_R^\dagger$, $D_\mu \Sigma \to V_L D_\mu \Sigma V_R^\dagger$, $F_\mu^\nu \to V_\chi F_\mu^\nu V_\chi^\dagger$, and $\tilde{F}_\mu^\nu \to V_\chi \tilde{F}_\mu^\nu V_\chi^\dagger$ for $\chi = L, R$, where $V_\chi \in \text{SU}(3)_\chi$. 

\[^1\] Under chiral $\text{SU}(3)_L \times \text{SU}(3)_R$ transformations $\Sigma \to V_L \Sigma V_R^\dagger$, $D_\mu \Sigma \to V_L D_\mu \Sigma V_R^\dagger$, $F_\mu^\nu \to V_\chi F_\mu^\nu V_\chi^\dagger$, and $\tilde{F}_\mu^\nu \to V_\chi \tilde{F}_\mu^\nu V_\chi^\dagger$ for $\chi = L, R$, where $V_\chi \in \text{SU}(3)_\chi$. 

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where \( \varepsilon \) and \( \bar{q} \) designate the dark photon’s polarization vector and momentum, respectively. These lead to the decay rates

\[
\Gamma_{K_L \rightarrow \gamma\gamma} = \frac{8\alpha_e}{9}(a'_T f)^2 m_K^3 \left[ (\text{Re } C)^2 + (\text{Im } C_5)^2 \right], \\
\Gamma_{K_S \rightarrow \gamma\gamma} = \frac{8\alpha_e}{9}(a'_T f)^2 m_K^3 \left[ (\text{Im } C)^2 + (\text{Re } C_5)^2 \right],
\]

where \( \alpha_e = e^2/(4\pi) = 1/137 \).

Similarly, for \( K \rightarrow \pi\gamma\bar{\gamma} \) we find

\[
\langle \pi^0 | \bar{d} \sigma_{\omega\gamma} | K^0 \rangle = \frac{i\sqrt{2} a'_T e}{3} \left( \varepsilon^*_\omega k_\alpha - \varepsilon^*_\alpha k_\omega \right), \\
\langle \pi^0 | \bar{d} \sigma_{\omega\gamma} | K^0 \rangle = \frac{i\sqrt{2} a'_T e}{3} \varepsilon^*_{\omega\mu\nu} \varepsilon^*_{\kappa\mu} k^\kappa, \\
\langle \pi^- | \bar{d} \sigma_{\omega\gamma} | K^- \rangle = 2ia_T e \left[ \varepsilon^*_\omega (p_K - p_\pi)_\omega - \varepsilon^*_\omega (p_K - p_\pi)_\alpha \right] + \frac{2ia_T e}{3} \left( \varepsilon^*_\omega k_\omega - \varepsilon^*_\omega k_\alpha \right), \\
\langle \pi^- | \bar{d} \sigma_{\omega\gamma} | K^- \rangle = 2 \varepsilon^*_{\omega\mu\nu} \varepsilon^*_{\kappa\mu} \left[ a_T (p'_K - p'_\pi) + \frac{a'_T}{3} k^\kappa \right],
\]

where \( p_K \) and \( p_\pi \) denote the momenta of the kaon and pion, respectively. Accordingly, the decay amplitudes are

\[
\mathcal{M}_{K_L \rightarrow \pi^0\gamma\bar{\gamma}} = \frac{4a'_T e}{3} \left[ - (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}) \right] \text{Re } C + \varepsilon^{\mu\nu\rho\sigma} \text{Im } C_5 \varepsilon^*_{\sigma\nu} k_\mu \bar{q}, \\
\mathcal{M}_{K_S \rightarrow \pi^0\gamma\bar{\gamma}} = \frac{4i a'_T e}{3} \left[ (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}) \right] \text{Im } C + \varepsilon^{\mu\nu\rho\sigma} \text{Re } C_5 \varepsilon^*_{\sigma\nu} k_\mu \bar{q}, \\
\mathcal{M}_{K^- \rightarrow \pi^-\gamma\bar{\gamma}} = 4 \left( a_T + \frac{a'_T}{3} \right) \varepsilon [(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}) \text{C} + i \varepsilon^{\mu\nu\rho\sigma} \text{C}_5 \varepsilon^*_{\sigma\nu} k_\mu \bar{q}].
\]

They translate into the differential rates

\[
\frac{d\Gamma_{K_L \rightarrow \pi^0\gamma\bar{\gamma}}}{ds_{\gamma\bar{\gamma}}} = \frac{\alpha_e a'_T^2 s_{\gamma\bar{\gamma}}^2 K_{K^0}^2 (m_{K^0}^2, m_{\pi^0}^2, s_{\gamma\bar{\gamma}})}{72\pi^2 m_K^4} \left[ (\text{Re } C)^2 + (\text{Im } C_5)^2 \right], \\
\frac{d\Gamma_{K_S \rightarrow \pi^0\gamma\bar{\gamma}}}{ds_{\gamma\bar{\gamma}}} = \frac{\alpha_e a'_T^2 s_{\gamma\bar{\gamma}}^2 K_{K^0}^2 (m_{K^0}^2, m_{\pi^0}^2, s_{\gamma\bar{\gamma}})}{72\pi^2 m_K^4} \left[ (\text{Im } C)^2 + (\text{Re } C_5)^2 \right], \\
\frac{d\Gamma_{K^- \rightarrow \pi^-\gamma\bar{\gamma}}}{ds_{\gamma\bar{\gamma}}} = \frac{\alpha_e (3a_T + a'_T)^2 s_{\gamma\bar{\gamma}}^2 K_{K^-}^2 (m_{K^-}^2, m_{\pi^-}^2, s_{\gamma\bar{\gamma}})}{72\pi^2 m_K^4} \left( |C|^2 + |C_5|^2 \right),
\]

where \( s_{\gamma\bar{\gamma}} \) is the invariant mass squared of the \( \gamma\bar{\gamma} \) pair and \( K(x, y, z) = (x - y - z)^2 - 4yz \). To get each of the corresponding decay rates, the integration range is \( 0 \leq s_{\gamma\bar{\gamma}} \leq (m_K - m_\pi)^2 \).

From the \( a_T \) terms in Eq. (3), we can additionally extract mesonic matrix elements pertaining to processes induced by \( \mathcal{L}_{d\epsilon\gamma} \) without the ordinary photon. Particularly, for the three-body channels
\( K \rightarrow \pi\pi\gamma \) we obtain

\[
\langle \pi^+(p) \pi^-(\bar{p}) | \bar{d} \sigma_{\omega \gamma} s | K^0 \rangle = \frac{i}{2} \frac{a_T}{f} \epsilon_{\alpha \omega \mu \nu} (2\bar{p}^\mu + \bar{q}^\mu) \bar{q}^\nu,
\]

\[
\langle \pi^+(p) \pi^-(\bar{p}) | \bar{\pi} \sigma_{\omega \gamma_5} s | K^0 \rangle = \frac{\sqrt{2} a_T}{f} [p_\alpha (2\bar{p} + \bar{q}) - \omega (2\bar{p} + \bar{q})],
\]

\[
\langle \pi^+(p) \pi^-(\bar{p}) | \bar{\pi} \sigma_{\omega \gamma} s | K^0 \rangle = \frac{\sqrt{2} a_T}{f} [p_\alpha (2\bar{p} + \bar{q}) - \omega (2\bar{p} + \bar{q})],
\]

\[
\langle 0 | \bar{d} \sigma_{\omega \gamma} s | K^0 \rangle = \frac{a_T}{f} \epsilon_{\alpha \omega \mu \nu} [4\bar{p}^\mu p^\nu + (\bar{p}^\mu - p^\mu) \bar{q}^\nu],
\]

\[
\langle 0 | \bar{\pi} \sigma_{\omega \gamma} s | K^0 \rangle = \frac{a_T}{f} [4p_\alpha \bar{p}_\omega - 4\omega p_\omega + (p - \bar{p})_\omega \bar{q}_\alpha],
\]

where we have applied the relation \( p_K = p + \bar{p} + \bar{q} \). These allow us to derive the decay amplitudes, which can be expressed as

\[
M_{K_L \rightarrow \pi^+\pi^-\gamma} = \frac{8a_T}{f} \left[ \epsilon_{\alpha \omega \mu \nu} \bar{\epsilon}^{\alpha \nu} p^-_\mu \bar{p}^\nu \mathrm{Re} C + (p^\mu_- p^\nu_- - p^\nu_+ p^\mu_+) \bar{\epsilon}^{\alpha \nu} \bar{q}_\mu \mathrm{Im} C_5 \right],
\]

\[
M_{K_S \rightarrow \pi^+\pi^-\gamma} = \frac{8a_T}{f} \left[ -\epsilon_{\alpha \omega \mu \nu} \bar{\epsilon}^{\alpha \nu} p^-_\mu \bar{p}^\nu \mathrm{Im} C + (p^\mu_- p^\nu_- - p^\nu_+ p^\mu_+) \bar{\epsilon}^{\alpha \nu} \bar{q}_\mu \mathrm{Re} C_5 \right],
\]

\[
M_{K^- \rightarrow \pi^+\pi^-\gamma} = \frac{8a_T}{f} \left[ \epsilon_{\alpha \omega \mu \nu} \bar{\epsilon}^{\alpha \nu} p^-_\mu \bar{p}^\nu \mathrm{C} + i(p^\mu_- p^\nu_- - p^\nu_+ p^\mu_+) \bar{\epsilon}^{\alpha \nu} \bar{q}_\mu \mathrm{C}_5 \right],
\]

where \( p_{+,-,0} \) represent the momenta of \( \pi^{+,-,0} \), respectively.\(^2\) Subsequently, we arrive at the differential rates

\[
\frac{d\Gamma_{K^- \rightarrow \pi^+\pi^-\gamma}}{ds} = \frac{a_T^2 (m_{K^-}^2 - s)^3}{96\pi^2 f^2 m_{K^-}^3 s} \left( m_{\pi^-}^2, m_{\pi^0}^2, s \right) \left( |C|^2 + |C_5|^2 \right),
\]

\[
\frac{d\Gamma_{K_L \rightarrow \pi^+\pi^-\gamma}}{ds} = \frac{a_T^2 (m_{K^0}^2 - s)^3}{96\pi^2 f^2 m_{K^0}^3 s} \left( s - 4m_{\pi^-}^2 \right)^{3/2} \left( |\mathrm{Re} C|^2 + |\mathrm{Im} C_5|^2 \right),
\]

\[
\frac{d\Gamma_{K_S \rightarrow \pi^+\pi^-\gamma}}{ds} = \frac{a_T^2 (m_{K^0}^2 - s)^3}{96\pi^2 f^2 m_{K^0}^3 s} \left( s - 4m_{\pi^-}^2 \right)^{3/2} \left( |\mathrm{Im} C|^2 + |\mathrm{Re} C_5|^2 \right),
\]

where \( s \) stands for the invariant mass squared of the pion pair. They are to be integrated over \( (m_{\pi^+} + m_{\pi^-})^2 \leq s \leq m_K^2 \) to yield the decay rates.

\(^2\) Although \( \langle \pi^0\pi^0 | \bar{d} \sigma_{\omega \gamma} s | K^0 \rangle \) and \( \langle \pi^0\pi^0 | \bar{\pi} \sigma_{\omega \gamma_5} s | K^0 \rangle \) from Eq. \( 8 \) are not zero, their contributions to the \( K_{L,S} \rightarrow \pi^0\pi^0\gamma \) amplitudes vanish. This is consistent with angular momentum conservation and gauge invariance (Bose symmetry) forbidding the pion pair in these decays from having an angular momentum \( J_{\pi\pi} = 0 \) \( (1) \), similarly to the \( K_{L,S} \rightarrow \pi^0\pi^0\gamma \) case with the ordinary photon \( \gamma \). We can therefore neglect \( K_{L,S} \rightarrow \pi^0\pi^0\gamma \), which are chirally suppressed compared to the \( K \rightarrow \pi\pi\gamma \) modes we consider in Eq. \( 14 \).
We remark that in Eqs. (5), (8), and (11)-(13) each matrix element of $d\sigma_{\omega s}$ and its counterpart are related due to the identity $2i\sigma_{\omega s} = \epsilon_{\omega\mu\nu}\sigma^{\mu\nu}$ for $\epsilon_{0123} = 1$. Furthermore, the amplitudes in Eqs. (6), (9), and (14) respect electromagnetic and U(1)$_D$ gauge invariance.

### III. KAON DECAY PREDICTIONS

From Eqs. (7), (10), and (15) we can evaluate the branching fractions in terms of the coefficients $C$ and $C_5$. Thus, with the central values of $a_T$, $a'_T$, and $f$ quoted above and of the measured kaon lifetimes and masses and the pion masses from Ref. [37], we get

$$B(K_L \to \gamma\gamma) = 5.74 \times 10^{12} \left[ (\text{Re} C)^2 + (\text{Im} C_5)^2 \right] \text{GeV}^2,$$

$$B(K_S \to \gamma\gamma) = 1.00 \times 10^{10} \left[ (\text{Im} C)^2 + (\text{Re} C_5)^2 \right] \text{GeV}^2,$$

$$B(K_L \to \pi^0\gamma\gamma) = 4.95 \times 10^9 \left[ (\text{Re} C)^2 + (\text{Im} C_5)^2 \right] \text{GeV}^2,$$

$$B(K_S \to \pi^0\gamma\gamma) = 8.67 \times 10^6 \left[ (\text{Im} C)^2 + (\text{Re} C_5)^2 \right] \text{GeV}^2,$$

$$B(K^- \to \pi^-\gamma\gamma) = 2.67 \times 10^9 \left[ |C|^2 + |C_5|^2 \right] \text{GeV}^2,$$

$$B(K_L \to \pi^+\pi^-\gamma) = 4.67 \times 10^{10} \left[ (\text{Re} C)^2 + (\text{Im} C_5)^2 \right] \text{GeV}^2,$$

$$B(K_S \to \pi^+\pi^-\gamma) = 8.18 \times 10^7 \left[ (\text{Im} C)^2 + (\text{Re} C_5)^2 \right] \text{GeV}^2,$$

$$B(K^- \to \pi^0\pi^0\gamma) = 1.12 \times 10^{10} \left[ |C|^2 + |C_5|^2 \right] \text{GeV}^2.$$

(16)

Clearly, predictions for their upper values would depend on how large $C$ and $C_5$ might be, subject the pertinent constraints.

The allowed ranges of $C$ have recently been explored in the contexts of a couple of new-physics models in Refs. [12, 13]. They pointed out that in these scenarios the restrictions were mainly from the data on kaon mixing, which receives contributions from the same new particles that participate in the loop diagrams responsible for the $ds\gamma$ couplings. Subsequently, it was shown in Ref. [14] that these interactions also gave rise to the FCNC decays of hyperons into a lighter baryon plus $\gamma$ emitted invisibly and that the less restrained of the models could saturate the limits on the couplings inferred from the existing data on hyperon decays [37]. This implies that the current hyperon data can already translate into model-independent restrictions on the $ds\gamma$ interactions. The resulting bounds on $C$ and $C_5$ can then be used to estimate the maximal values of the kaon branching fractions in Eqs. (16)-(18).

To discuss the impact of the hyperon data more quantitatively, we reproduce here the branching fractions of the aforementioned FCNC hyperon modes calculated in Ref. [14] and expressed in terms
\[ \mathcal{B} (\Lambda \rightarrow n\bar{\gamma}) = 2.75 \times 10^{12} (|C|^2 + |C_5|^2) \text{ GeV}^2, \]
\[ \mathcal{B} (\Sigma^+ \rightarrow p\bar{\gamma}) = 1.54 \times 10^{11} (|C|^2 + |C_5|^2) \text{ GeV}^2, \]
\[ \mathcal{B} (\Xi^0 \rightarrow \Lambda\bar{\gamma}, \Sigma^0\bar{\gamma}) = 1.61 \times 10^{12} (|C|^2 + |C_5|^2) \text{ GeV}^2, \]
\[ \mathcal{B} (\Xi^- \rightarrow \Sigma^-\bar{\gamma}) = 1.32 \times 10^{12} (|C|^2 + |C_5|^2) \text{ GeV}^2, \]
\[ \mathcal{B} (\Omega^- \rightarrow \Xi^-\bar{\gamma}) = 5.18 \times 10^{12} (|C|^2 + |C_5|^2) \text{ GeV}^2. \]
(19)

These transitions, if occur, would be among the yet-unobserved decays of the hyperons. The branching fractions of the latter have approximate upper-limits which can be determined indirectly from the data on the observed channels quoted by the Particle Data Group \[37\]. To do so, for each of the parent hyperons, we subtract from unity the sum of the PDG branching-fraction numbers with their errors (increased to 2 sigmas) combined in quadrature. In the third column of Table I we list the results, the second column displaying the sums of the branching-fraction values.

Comparing the hyperon entries in the last column of this table with Eq. (19), we see that the \( \Xi^0 \) bound is the most stringent and leads to

\[ |C|^2 + |C_5|^2 < \frac{2.1 \times 10^{-16}}{\text{GeV}^2}. \]
(20)

Combining this with Eqs. (16)-(18) and assuming that for the \( K_L \) (\( K_S \)) cases \( \text{Im} \mathcal{C} = \mathcal{C}_5 = 0 \)

| Hadron | Branching-fraction sum of observed modes | Upper limit on total branching fraction of yet unobserved modes |
|--------|----------------------------------------|-----------------------------------------------|
| \( \Lambda \) | 1.0006 ± 0.0071 | 1.4 \times 10^{-2} |
| \( \Sigma^+ \) | 1.0005 ± 0.0042 | 8.0 \times 10^{-3} |
| \( \Xi^0 \) | 1.00000 ± 0.00017 | 3.4 \times 10^{-4} |
| \( \Xi^- \) | 1.00000 ± 0.00042 | 8.3 \times 10^{-4} |
| \( \Omega^- \) | 1.006 ± 0.011 | 1.6 \times 10^{-2} |
| \( K_L \) | 1.0044 ± 0.0018 | 1.8 \times 10^{-3} |
| \( K_S \) | 1.00191 ± 0.00071 | 7.1 \times 10^{-4} |

TABLE I: The second column exhibits the sums of branching fractions of all the observed decays \[37\] of the \( \Lambda, \Sigma^+, \Xi^0, \Xi^-, \) and \( \Omega^- \) hyperons and of the \( K_L \) and \( K_S \) mesons. The last column contains the upper limits on the branching fractions of yet-unobserved decays of these hadrons inferred from the numbers in the second column, as explained in the text.
(C = Im C₅ = 0), we then obtain

\[
\begin{align*}
\mathcal{B}(K_L \rightarrow \gamma \bar{\gamma}) &< 1.2 \times 10^{-3}, & \mathcal{B}(K_S \rightarrow \gamma \bar{\gamma}) &< 2.1 \times 10^{-6}, \\
\mathcal{B}(K_L \rightarrow \pi^0 \gamma \bar{\gamma}) &< 1.0 \times 10^{-6}, & \mathcal{B}(K_S \rightarrow \pi^0 \gamma \bar{\gamma}) &< 1.8 \times 10^{-9}, \\
\mathcal{B}(K_L \rightarrow \pi^+ \pi^- \bar{\gamma}) &< 9.8 \times 10^{-6}, & \mathcal{B}(K_S \rightarrow \pi^+ \pi^- \bar{\gamma}) &< 1.7 \times 10^{-8}, \\
\mathcal{B}(K^- \rightarrow \pi^- \gamma \bar{\gamma}) &< 5.6 \times 10^{-7}, & \mathcal{B}(K^- \rightarrow \pi^- \pi^0 \bar{\gamma}) &< 2.4 \times 10^{-6}.
\end{align*}
\]

(21)

If both C and C₅ are real, the predictions for the \(K^+\) modes are equal to those for their \(K^-\) counterparts.

The second column of Table I also collects the sums of the branching fractions of the observed \(K_{L,S}\) decay channels. Since the central values of these numbers exceed unity by more than 2 sigmas, we may require that the upper limits on the branching fractions of yet-unobserved \(K_{L,S}\) modes be less than the errors shown in the second column. Evidently, these requisites, which are quoted in the last two rows of the third column of the table, are satisfied by the respective \(K_{L,S}\) predictions in Eq. (21).

IV. CONCLUSIONS

To date there have been numerous dedicated hunts for the massive dark photon, but they still have come up empty. If the dark photon exists and turns out to be massless, it would have eluded those quests for the massive one. Therefore, it is essential that future searches for dark photons accommodate the possibility that they are massless, in which case they may have nonnegligible FCNC interactions with SM fermions via higher-dimensional operators.

In this study, we have entertained the latter possibility, specifically in which the massless dark photon has dipole-type flavor-changing couplings to the \(d\) and \(s\) quarks. Concentrating on the implications for the kaon sector, and taking into account indirect model-independent constraints on the \(d\bar{s}\bar{\gamma}\) interactions from the available hyperon data, we examine especially \(K_L \rightarrow \gamma \bar{\gamma}\) and \(K_L \rightarrow \pi^0 \gamma \bar{\gamma}\), both of which have an ordinary photon in the final states, and demonstrate that their rates may reach levels which are potentially testable by KOTO. Moreover, \(K^+ \rightarrow \pi^+ \gamma \bar{\gamma}\) and \(K^+ \rightarrow \pi^+ \pi^0 \bar{\gamma}\) might have big enough rates to be accessible by NA62. We have previously pointed out that the corresponding hyperon decays with missing energy could be probed by BESIII. The results of our analysis will hopefully help stimulate efforts to seek massless dark photons in ongoing and near-future kaon and hyperon experiments.

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