Optimum performance-based design of unsymmetrical 2D steel moment frame

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Abstract
The most commonly used analysis method in performance-based design (PBD) is nonlinear static analysis (NSA). In unsymmetrical 2D frames, unlike for the symmetrical state, NSA should be performed in two lateral loading directions. This complicates the process of achieving a feasible optimal design in addition to increasing the volume of calculations. In this study, to overcoming this issue, a two-step approach is proposed for the optimal PBD of unsymmetrical 2D steel moment-resisting frames (SMRFs). In this approach, in two independent steps, the structure is analyzed with a lateral loading pattern based on the first mode shape in the positive and negative directions. The implementation of the second step is conditional on the satisfactory completion of the first step. The objective function takes into account the differences between successful and unsuccessful steps. The constraints considered are based on the acceptance criteria for SMRF according to FEMA-356 at each performance level. The effectiveness of the proposed approach has been investigated by employing four meta-heuristic optimization algorithms to determine the optimum design for case studies of SMRF structures having three and nine stories.

Keywords Optimisation · Metaheuristic algorithms · Performance-based seismic design · Unsymmetrical 2D steel moment frame

1 Introduction
With the introduction of performance-based design (PBD) to structural and seismic engineering, researchers have focused on improving the reliability of and advancements in this method. The aim of PBD is to enable engineers to design structures that offer predictable performance. The goal is to involve the employer in the selection of the level of building vulnerability at various hazard levels.

In conventional structural design methods, a significant aspect of structural performance within the nonlinear limits that entail damage has not been considered. In addition, expectations about structures vary in terms of their importance and use. It is impossible to design a structure that incurs no damage in a disaster and, in many cases, this is not necessary or justifiable. Using PBD, structures can be reviewed to determine their expected responses under earthquakes and modifications involving engineering judgement can be implemented to improve these responses.

PBD was developed and implemented within the framework of advanced and reliable structural design methods (Gholizadeh and Poorhoseini 2016a). Grierson et al. (2006) employed PBD on steel moment-resisting frames (SMRFs) using modal nonlinear static analysis (NSA) on the dominant modes to assess to the effect of higher modes on a nine-story planar steel frame. Kaveh et al. (2012) used non-dominated sorting genetic algorithm (NSGA-II) for multi-objective optimization of large performance-based steel structures and were able to reduce the time required for NSA using a simple numerical method. Reduction of the initial and lifecycle costs of large-scale structures were the objective functions for their research.

Kaveh and Nasrollahi (2014) assessed the PBD of SMRFs for all performance levels by considering inter-story drift as a design criterion. In their study, NSA was carried out using semi-rigid connections and base shear was the criterion used to determine performance levels.

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Gholizadeh (2015) used target displacement for determining performance levels that were calculated by the equation recommended in FEMA-356. In their study, the structure initially was controlled against the gravity load. If the member resistance was sufficient, NSA was conducted.

Gholizadeh and Milani (2016) used nonlinear time-history analysis to determine structural responses. They used a combination of a radial basis function neural network and optimization algorithms for their design. Gholizadeh and Baghechevan (2017) employed PBD for a SMRF using a multi-objective optimization algorithm. Initially, geometric and strength constraints were controlled under gravity loads and, if the geometric and strength controls were satisfactory, NSA was carried out to assess the maximum inter-story drift as another constraint.

Karimi and Hoseini Vaez (2019) proposed a two-stage optimal seismic design of SMRFs in which the requirements of the load and resistance factor design (LRFD) method initially were controlled and then the PBD method was applied based on the target roof displacement. Fathollahi and Hoseini Vaez (2020) studied the optimum PBD problem of eccentrically braced frames. The constraints were defined according to the acceptance criteria of the PBD method. They proposed a method of evaluating the performance of the link beams by modeling those of eccentrically braced frames.

Gholizadeh et al. (2020) proposed a metaheuristic algorithm for the discrete optimal PBD of SMRFs using a dynamic method to provide a balance between exploration and exploitation. The optimal PBD of the SMRFs was then compared to the performance of the proposed algorithm and other algorithms.

Moghaddam et al. (2021) proposed a method for optimal cross-sectional distribution of steel moment frames. They used a novel adaptive algorithm to improve the computational efficiency of the optimization process. The strength-based demand to capacity ratio and maximum plastic rotation, respectively, were selected as the main performance parameters for force-controlled and deformation-controlled members. Additional studies have been conducted on the use of PBD for steel frames (Degertekin et al. 2021; Gholizadeh and Ebadijalal 2018; Gholizadeh and Poorhoseini 2016b; Mansouri and Maheri 2019; Mergos 2018; Shoiebi et al. 2018; Wang et al. 2020).

Optimization of this type of design adds to the important economic aspect of PBD. Engineering optimization issues can be too complex to solve using conventional optimization methods; thus, metaheuristic algorithms have been developed to solve them. These algorithms are capable of solving several optimization problems (Aljarah et al. 2018; Arora and Singh 2019; Bairathi and Gopalani 2021; Braik et al. 2021; Fathollahi-Fard et al. 2020; Ghasemi-Marzbali 2020; Heidari et al. 2019; Kaveh et al. 2017; Mirjalili et al. 2014; Mohammad Rezapour Tabari and Hashempour 2019; Ozbasaran and Eryilmaz Yildirim 2020).

In some PBD studies, base shear has been used to determine the performance levels. The current study used target displacement to determine the performance levels. Because the most common analytical method for assessing seismic performance is NSA, this method has been used as the basis of structural analysis. Previous studies have assessed symmetrical 2D frames and displacement of the control node in NSA in one direction. However, in 2D unsymmetrical frames, it is necessary to conduct NSA in both directions. The consideration of both directions increases the optimization process; thus, less attention has been paid to optimization of unsymmetrical SMRFs using PBD.

In the present study, a two-step approach is proposed for decreasing the volume of calculations and the complexity of the process of achieving a feasible optimal solution to the PBD of unsymmetrical 2D SMRFs. In this approach, a penalty coefficient is introduced to the optimization process to avoid unnecessary nonlinear static analysis. The aim of optimization is to reduce the structural weight and uniform inter-story drift distribution while satisfying the acceptance criteria for each performance level.

In some studies, constraints such as limits on plastic rotation of plastic hinges, inter-story drift and strength have not been considered; however, the present study has defined the constraints based on SMRF acceptance criteria as defined in FEMA-356 (2000). The enhanced vibrating particles system algorithm (EVPS), enhanced colliding bodies optimization algorithm (ECBO), salp swarm algorithm (SSA) and enhanced whale optimization algorithm (EWOA) were used to optimize three- and nine-story SMRFs to assess the optimal PBD.

The rest of this article is organized as follows: Part 2 provides a brief overview of the concept of optimum performance-based design. Part 3 briefly presents the metaheuristic algorithms. Two numerical examples of SMRFs of three and nine stories are presented in Part 4. Part 5 presents the conclusions of the study.

# 2 Optimum performance-based design

## 2.1 PBD concept

PBD includes the design of the structural members based for the expected performance of the structure under specific types of earthquakes. To determine the design objectives,
the structural performance level and the seismic hazard level should be determined. The current study used the immediate occupancy (IO), life safety (LS), and collapse prevention (CP) performance levels from FEMA-356 based on the enhanced rehabilitation concept at 20%, 10%, and 2% earthquake probability over 50 years, respectively.

Because NSA was used, the structure was subjected to a specific lateral load distribution (first mode of structural vibration) until the lateral displacement of the control point reached the target displacement. The amount of target displacement at each performance level based on FEMA-356 can be calculated as:

\[ \delta_t = C_0 C_1 C_2 C_3 S_a \frac{g T^2_p}{4 \pi^2 \rho}, \]  

where \( C_0 \) is the spectral displacement of an equivalent single-degree of freedom (SDOF) system related to the multi-degree of freedom (MDOF) roof displacement system of the building, \( C_1 \) is the expected maximum inelastic displacement related to displacement for the linearly elastic response, \( C_2 \) is the effect of strength and stiffness degradation and the pinched hysteretic shape on the maximum displacement response, and \( C_3 \) is the increased displacement due to dynamic \( P–\Delta \) effects.

In Eq. (1), \( S_a \) is the response spectrum acceleration corresponding to effective fundamental period \( T_e \) and \( g \) is the gravitational acceleration. The spectral acceleration is the elastic design spectrum of the site at a specific period for a specific damping ratio. The spectral acceleration \( (S_a) \) for all three performance levels at 5\% effective damping can be calculated as:

\[
S_a = \begin{cases} 
F_a S_a (0.4 + 3T_e/T_0) & 0 < T_e \leq 0.2T_0^i \\
F_a S_a & 0.2T_0^i < T_e \leq T_0^i \\
F_a S_a / T_e & T_e > T_0^i 
\end{cases}
\]

where \( S_1 \) and \( S_2 \) are the response acceleration parameters for a 1 – s period and for a short-period (0.1 s), respectively, and \( S_a \) and \( F_a \) are the site class coefficients based on FEMA-356.

In PBD, each element action is classified as either deformation-controlled or force-controlled and the acceptance criteria are defined in accordance with this. The acceptance criteria should be controlled in the target displacement of each performance level for all members.

\[ T_0^i = (F_a S_a) / (F_a S_a), \quad i = \text{IO, LS, CP}, \]

2.2 Optimization problem formulation

Each optimization problem has three parts: the optimization variables (design variables), an objective function, and constraints. In the current study, the design variables were selected for W-shaped steel sections according to the AISC design manual (LRFD-AISC 2001).

2.2.1 Objective function formulation

In addition to minimizing the building weight, the uniform inter-story drift distribution also was assessed. In Eq. (3), the objective function consists of two terms (Grierson et al. 2006):

\[ F(X) = F_1(X) + F_2(X), \]

where \( F_1 \) is the normalized building weight and is formulated as:

\[ F_1(X) = \frac{1}{W_{\text{max}}} \sum_{i=1}^{ng} \rho_i A_i \sum_{j=1}^{nm} L_j \]

where \( W_{\text{max}} \) is the weight of the building and includes the heaviest sections of the final section list of element groups, \( nm \) is the number of structural elements that are collected in \( ng \) design groups, \( \rho_i \) and \( A_i \) are the weight of the unit volume and cross-sectional area of the \( i \)th group section, respectively, and \( L_j \) is the length of the \( j \)th element in the \( i \)th group.

In Eq. (3), \( F_2 \) is the second term of the objective function for considering the uniform inter-story drift distribution and is defined as:

\[ F_2(X) = \left[ \frac{1}{n_s} \sum_{s=1}^{n_s} \left( \frac{v_s^{\text{CP}}(X)/H_s}{\Delta^{\text{CP}}(X)/H} - 1 \right)^2 \right]^{1/2} \]

where \( n_s \) is the number of building stories, \( v_s^{\text{CP}} \) and \( \Delta^{\text{CP}} \) are the drift of story \( s \) and the roof drift at the CP performance level, respectively, \( H \) is the height of the building, and \( H_s \) is the vertical distance from the base of the building to story \( s \). The optimization problem then can be formulated as:

Find : \( X = \{x_1, x_2, x_3, \ldots, x_{ng}\}^T \)

To minimize : \( F(X) = F_1(X) + F_2(X) \)

Subject to : \( g_j(X) \leq 0, \quad j = 0, 1, 2, \ldots, nc, \)

where \( X \) is the vector of the design variables, \( F(X) \) is the objective function, \( g_j(X) \) is the \( j \)th design constraint, and \( nc \) is the number of constraints. The constraints of the optimization problem have been applied to the objective
Select the section by maximum moment of inertia $I_{\text{max}}$ and minimum moment of inertia $I_{\text{min}}$

Calculate $EI_{\text{max}}/L$ for the beams connected to each connection of the $i^{\text{th}}$ column.

Calculate $EI/L$ for the $i^{\text{th}}$ column.

Are columns connected to $i^{\text{th}}$ column from the same group?

Calculate $EI_{\text{min}}/L$ for the columns connected to $i^{\text{th}}$ column

Calculate $K_{\text{min}}$ for $i^{\text{th}}$ column by Eq. (13)

Calculate $(KL/r)_{\text{min}}$ for $z^{\text{th}}$ section of $i^{\text{th}}$ column

Do $(KL/r)_{\text{min}} \leq 200$

Store the $z^{\text{th}}$ section in the list of sections for $i^{\text{th}}$ column.

Is $z \geq N_{CS}$?

Is $z \geq n_f$?

End

Fig. 1 Flowchart for reducing the search space
function based on the exterior penalty function method, which can be expressed as:

$$\varphi(X, r) = F(X) \left( 1 + r \sum_{j=1}^{n_c} V_j^2 \right)$$  \hspace{1cm} (7)

where $\varphi$ and $r$ are the pseudo-objective function and the positive penalty parameter, respectively, and $V_j$ is the violation of the $j$th constraint.

### 2.2.2 Constraints

According to FEMA-356 for SMRFs, the flexural behavior of the beams should be deformation controlled. In steel columns, whenever the axial force is less than 50% of the lower-bound axial compression strength of the column ($P_{CL}$), the flexural and axial behavior of the column should be considered to be deformation controlled and force controlled, respectively.

The constraint corresponding to the plastic rotation of the plastic hinges is defined as:

$$g_{i}^{h} = \left( \frac{\theta_{P,i}^{h}}{\theta_{all,i}^{h}} \right) - 1 \leq 0 \hspace{1cm} (8)$$

where $\theta_{P,i}^{h}$ and $\theta_{all,i}^{h}$ are the plastic rotation of the $i$th plastic hinge and its allowable value at the $i$th performance level based on FEMA-356, respectively, and $nh$ is the number of plastic hinges.

In steel columns with axial compressive forces that are more than 50% of the $P_{CL}$, both the axial loads and flexure shall be considered to be force controlled. According to FEMA-356, the constraint for such columns at the CP performance level is:

$$g_{j}^{\prime} = \left( \frac{P_{Uj}^{\prime} / P_{CL}}{M_{Uj}^{\prime} / M_{CL,j}} \right) - 1 \leq 0 \hspace{1cm} (9)$$

for $P_{Uj}^{\prime} / P_{CL} > 0.5$, $j = 1, 2, ..., nf$

where $g_{j}^{\prime}$ is the strength constraint of the $j$th column, $nf$ is the number of columns, $P_{Uj}$ and $M_{Uj}$ are the axial force and bending moment of the $j$th column derived from analysis, respectively, and $M_{CL,j}$ is the lower-bound flexural strength of the $j$th column.

To satisfy the criteria for the design of the column-column and beam-column joints in steel structures, the geometric constraints should be controlled as Gholizadeh and Baghchevan (2017):

$$g_{G,k} = \left\{ \left( \frac{b_{B}/b_{B}^{bot}}{h_{C}^{top}/h_{C}^{bot}} \right)_k - 1 \leq 0 \right\}, \hspace{1cm} k = 1, ..., nk \hspace{1cm} (10)$$

where $g_{G,k}$ is the geometric constraint of the $k$th connection, $b_{B}$ and $h_{C}^{bot}$ are the flange widths of the beam and bottom column for the $k$th connection, respectively, $h_{C}^{top}$ are the depth of the bottom and top columns for the $k$th connection, respectively, and $nk$ is the number of joints.

The constraint for the slenderness ratio of the column ($\lambda$) is:

$$g_{j}^{\lambda} = \left( \frac{K_l/r_j}{200} \right) - 1 \leq 0 \hspace{1cm} (11)$$

where $K_l$, $l_j$ and $r_j$ are the effective length factor, unsupported length, and cross-section gyration radius of the $j$th column, respectively. The inter-story drift constraint at each performance level is:

$$g_{j}^{D} = \left( \frac{K_d/l_j/r_j}{200} \right) - 1 \leq 0 \hspace{1cm} (12)$$

where $\Delta_l^i$ and $\Delta_{all}^i$ are the inter-story drift of the $i$th story and the allowable inter-story drift at $i$th performance level, respectively. The allowable inter-story drift ($\Delta_{all}$) was considered to be 0.012, 0.031, and 0.061 of the story height for the IO, LS and CP performance levels, respectively (Gong 2003; Karimi and Hoseini Vaez 2019).

### 2.2.3 Reducing the search space

To improve the optimization process before the beginning of optimization, an approach was used to reduce the search space as much as possible and increase the ability of the algorithm to achieve a solution. In this approach, a number of column sections with minimum effective length coefficients greater than the allowable value were removed from the sections list of the columns group and the all number of
Step 1: the pushover analysis with lateral loading pattern in positive direction

- Compute $F_1$ according to Eq. (4)
- Compute $V_G$ according to Eqs. (7) and (10)
- Modeling of unymmetrical steel frame in OpenSees

$\Phi(X, r) = NS \times (F_1(X) + F_2\text{-total}(X)) (1+ r V_\text{total}(X))$

End

Step 2: the pushover analysis with lateral loading pattern in negative direction

- Implement the pushover analysis with lateral loading pattern based on the first mode shape in negative direction
- Compute the target displacement for each performance level
- Compute the constraints violation ($V_{\text{step2}}$) for each performance level
- Compute $F_2\text{-step2}$ According to Eq. (5)
- $V_{\text{total}} = V_{\text{step1}} + V_G + F_2\text{-total} = F_2\text{-step1}$
- $V_{\text{step2}} = 0$

$NS = 1$
$NS = 10$
$NS = 20$
column sections \((N_{CS})\) was decreased. Actually, the search space was limited. The proposed approach for reducing the search space is defined according to the flowchart shown in Fig. 1. The equations in Fig. 1 are as follows:

\[
K = \sqrt{\frac{1.6 G_A G_B + 4.0 (G_A + G_B) + 7.5}{G_A + G_B + 7.5}}, \tag{13}
\]

where \(K\) is the column effective length factor. In Eq. (13), the value of \(G\) for the joints at both ends of each column \((G_A\) and \(G_B)\) can be calculated using Eq. (14). If the
column end is rigidly attached to a properly designed footing, this value is considered to be unity as:

$$G = \frac{\sum \left( \frac{E_c I_c}{L_c} \right)}{\sum \left( \frac{E_b I_b}{L_b} \right)},$$  \hspace{1cm} (14)

where $I_c$, $E_c$, and $L_c$ are the moment of inertia, modulus of elasticity, and unsupported length of the column, respectively, and $I_b$, $E_b$, and $L_b$ are the same parameters for the beam under consideration.

### 2.2.4 Proposed approach

The structure was analyzed using NSA under gravity and lateral loads, which are considered to be the first modes for the lateral loading pattern. Figure 2 shows how to apply lateral loading pattern based on the first mode shape in the positive and negative directions to the structures. When the constraints were satisfied in the positive direction under NSA, the structure then underwent NSA with a lateral loading pattern in the negative direction and the constraints were controlled in this direction. When the constraints were satisfied in the second direction, the solution was deemed to be acceptable; however, if the constraints were not satisfied in the first direction, the second direction was not assessed.

The proposed method can also be used for other structural systems, especially in problems with high computational volume, where this two-step approach can be used to simplify and reduce computational volume. OpenSees software (Mazzoni et al. 2016) was used for the NSA,
Determine the value of maximum number of iterations \( t_{\text{max}} \), population size \( N \) and colliding memory size \( CM \).

Define the \( Pr \) in the \([0, 1]\) range.

Initialize the position of CBs randomly in the search space.

While \( t < t_{\text{max}} \)

- Calculate the coefficient of restitution \( \varepsilon \) by Eq. (T1)*.
- Evaluate the objective function of CBs \( f \).
- Calculate the value of mass matrix \( m \) for the CBs using Eq. (T2).
- Update colliding memory and population.
- Divide CBs into stationary and moving groups and calculate their velocities before collision \( v \) by Eqs. (T3) and (T4).
- Calculate CBs velocities after the collision \( v^r \) by Eqs. (T5) and (T6).
- Update the position of each CB by Eqs. (T7) and (T8).

Select a random number \( rni \) that is distributed in the \([0, 1]\) range.

for \( i=1:N \)

- if \( rni<Pr \)
- Select one dimension of the \( i \)th CB randomly and recalculate its value by Eq. (T9).

end

end

Report the best solution found by the algorithm.

* Eqs. (T1) to (T9) defined in Table 1

**Fig. 6** Pseudo-code for ECBO algorithm

| Equation tag | Equations |
|--------------|-----------|
| (T1)         | \( \varepsilon = 1 - (t/t_{\text{max}}) \) |
| (T2)         | \( m_k = (1/f(k)) \sum_{i=1}^{N} (1/f(X_i)) \) |
| (T3)         | \( v_i = 0, \ i = 1, 2, \ldots, N/2 \) |
| (T4)         | \( v_i = x_i - x_{i-(N/2)} - N/2 + 1, N/2 + 2, \ldots, N \) |
| (T5)         | \( v^r_i = [(1 + \varepsilon)(m_i + m_{i-(N/2)})/\langle m_i + m_{i-(N/2)} \rangle]v_i, i = 1, 2, \ldots, N/2 \) |
| (T6)         | \( v^r_i = [(m_i - \varepsilon m_{i-(N/2)})/\langle m_i + m_{i-(N/2)} \rangle]v_i, i = N/2 + 1, N/2 + 2, \ldots, N \) |
| (T7)         | \( x_{i_{\text{new}}} = x_i + \text{rand} \circ v_i, i = 1, 2, \ldots, N/2 \) |
| (T8)         | \( x_{i_{\text{new}}} = x_{i-(N/2)} + \text{rand} \circ v_i, i = N/2 + 1, N/2 + 2, \ldots, N \) |
| (T9)         | \( x_{i_{\text{new}}} = x_{i_{\text{min}}} + \text{rand} \circ (x_{i_{\text{max}}} - x_{i_{\text{min}}}) \) |

\( t \) is the current iteration number.
\( \text{rand} \) is a random vector with a dimension of the number of problem variables, consisting of random numbers in the interval of \([-1, 1]\).

\( x_{i_{\text{new}}} \) is the value of the \( j \)th variable of the \( i \)th CB.

\( x_{i_{\text{max}}} \) and \( x_{i_{\text{min}}} \) are the upper and lower bounds of the \( j \)th variable, respectively.

\( \text{rand} \) is a random vector in the \([0, 1]\) range.

which is an open-source software framework for finite element analysis (Mazzoni et al. 2005).

Figure 3 shows the flowchart for each stage in which parameter NS is an additional penalty coefficient, the value of which depends on the satisfactory fulfillment of each step and the value of the positive penalty parameter \( r \) is considered as 10. Three target roof displacements (0.007, 0.025, and 0.05 of the total height of the structure) which
corresponded to the performance levels for IO, LS, and CP, respectively, were selected as the initial assumptions for bilinearization.

To evaluate the efficiency of the proposed approach in reducing the computational volume of various problems, the process of a one-step approach has been presented in the flowchart of Fig. 4. Here, nonlinear static analysis has been performed in both directions for any possible solution.

3 Metaheuristic algorithms

The use of metaheuristic algorithms to solve optimization problems is a common approach in research. The ability of these algorithms to cover a search space and escape from local optima lead to suitable results.

3.1 Salp swarm algorithm

Mirjalili et al. (2017) presented an SSA based on the swarming behavior of salps (Anderson and Bone 1980;
To form a mathematical model for a salp chain, the population initially should be divided into two groups: a leader and the followers. An \( n \)-dimensional search space should be considered for salp position \( x \) in which \( n \) represents the number of problem variables and food source \( F \) is the swarm target. The SSA algorithm stores the best solution and allocates it to the food source position. The leader always explores and exploits the space around the food source. SSA updates the position of followers based on their positions and that of the leader. The gradual movement of the followers prevents them from exiting the search space. Figure 5 shows the flowchart of the SSA. The equations mentioned in the flowchart are as follows:

\[
    x^i_j = \begin{cases} 
    F_j + c_1 \left( (u b_j - l b_j) c_2 + l b_j \right) \\
    F_j - c_1 \left( (u b_j - l b_j) c_2 + l b_j \right)
    \end{cases} \quad c_3 \geq 0.5 \\
    \left( 1 - c_3 \right) \frac{x^i_j + x^{i-1}_j}{2} \quad c_3 < 0.5 
\]  

(15)

where \( x^i_j \) is the position of the leader, \( F_j \) is the position of the food source in the \( j \)th dimension, \( u b_j \) and \( l b_j \) are the upper and lower bounds in the \( j \)th dimension, respectively, \( c_2 \) and \( c_3 \) are random numbers that are generated uniformly in the \([0,1]\) interval, and \( c_1 \) is:

\[
c_1 = 2e^{-\left(4t/t_{\text{max}}\right)^2},
\]

(16)

where \( t \) is the number of current iterations, \( t_{\text{max}} \) is the maximum iteration, and:

\[
x^i_j = 0.5 \left(x^i_j + x^{i-1}_j\right) 
\]

(17)

where, for \( i \geq 2 \), \( x^i_j \) is the position of the \( i \)th follower in the \( j \)th dimension. The salp chain can be modelled using Eqs. (15) and (17).

### 3.2 Enhanced colliding bodies optimization algorithm

Kaveh and Ilchi Ghazaan (2014) proposed the ECBO algorithm to improve the convergence rate of the CBO by adding a memory to store some of the best solutions during optimization along with the objective function values and their related mass. The solution vectors stored in the memory are added to the population and an equivalent number of existing worst states colliding bodies (CBs) are deleted. The CBs are sorted in descending order based on their mass. Figure 6 shows the pseudo-code of the ECBO. The equations in the pseudo-code are listed in Table 1.

### 3.3 Enhanced vibrating particles system algorithm

The EVPS algorithm is an alternative to the VPS algorithm that features an increased convergence rate and improved VPS algorithm efficiency (Kaveh et al. 2018). The VPS is a metaheuristic algorithm based on the free vibration of a
SDOF system (Kaveh and Ghazaan 2017). The parameters of this metaheuristic algorithm are:

- **OHB** One of the best positions generated in the entire population until the current iteration. It is a row of memory that is randomly selected.
- **GP** An appropriate particle that is randomly selected from the best solutions in each iteration.
- **BP** An inappropriate particle that is randomly selected from the worst solutions in each iteration.

The memory parameter (MS) stores the best positions of the entire population with the same memory matrix dimensions that have been generated prior to the current iteration. The positions of particles in the EVPS algorithm can be updated using Eq. (18) in which one of the (a)–(c) equations is selected based on the probability of \( \omega_1, \omega_2, \omega_3 \), respectively. Parameters BP, GP, and OHB are determined independently for each particle and \( \omega_1, \omega_2 \) and \( \omega_3 \) are the relative importance of OHB, GP and BP, respectively.

\[
X_t^i = \begin{cases} 
[D.A. \text{ rand}1 + \text{OHB}^i] & \text{if rand} \leq \text{HMCR} \\
[D.A. \text{ rand}2 + \text{GP}^i] & \text{if rand1} \leq \text{HMCR} \\
[D.A. \text{ rand}3 + \text{BP}^i] & \text{if rand2} \leq \text{HMCR} \\
(-1)^{\text{rand}(\text{rand})} (\text{OHB}^i - x_t^i) & \text{if rand3} \leq \text{HMCR} \\
(-1)^{\text{rand}(\text{rand})} (\text{GP}^i - x_t^i) & \text{(a)} \\
(-1)^{\text{rand}(\text{rand})} (\text{BP}^i - x_t^i) & \text{(b)} \\
\omega_1 + \omega_2 + \omega_3 & \text{(c)}
\end{cases}
\]

Table 4: Optimal sections and weights for 3-story frame

| Element group | Proposed two-step approach | One-step approach |
|---------------|----------------------------|------------------|
|               | ECBO | EVPS | EWOA | SSA | ECBO | EVPS | EWOA | SSA |
| 1             | W12 × 120 | W12 × 120 | W14 × 233 | W14 × 159 | W12 × 152 | W12 × 120 | W14 × 370 | W14 × 398 |
| 2             | W14 × 233 | W14 × 193 | W12 × 136 | W14 × 193 | W14 × 233 | W14 × 311 | W14 × 159 | W12 × 210 |
| 3             | W27 × 102 | W27 × 114 | W33 × 130 | W30 × 116 | W21 × 111 | W27 × 102 | W21 × 93 | W18 × 97 |
| 4             | W30 × 99 | W30 × 99 | W30 × 118 | W27 × 102 | W30 × 124 | W30 × 99 | W36 × 160 | W33 × 169 |
| 5             | W14 × 61 | W12 × 58 | W10 × 77 | W10 × 100 | W8 × 58 | W24 × 62 | W12 × 53 | W12 × 53 |
| Best weight (kN) | 297.13 | 280.94 | 318.08 | 318.79 | 324.43 | 338.19 | 368.00 | 410.63 |
| Worst weight (kN) | 649.67 | 457.38 | 546.49 | 486.90 | 705.26 | 562.04 | 612.41 | 640.32 |
| Average weight (kN) | 427.14 | 372.05 | 411.25 | 397.62 | 512.14 | 453.36 | 485.26 | 543.75 |
| COV (%) | 22.61 | 15.96 | 17.22 | 19.92 |

where, rand, rand1, rand2, and rand3 are random numbers that are uniformly distributed in the [0, 1] interval, and \( D \) is the damping surface modeling parameter in the vibration which is defined as:

\[
D = (t/t_{\text{max}})^{-z},
\]

where \( t \) is the current number of iterations, \( t_{\text{max}} \) is the total number of iterations, and \( z \) is a constant value. The HMCR parameter determines whether a violating component should change in response to value changes in OHB or be selected from the allowable search space. If the OHB value changes, another parameter (PAR) will be used to determine whether or not the value should change with the neighboring value. Additionally, parameter P in this algorithm is compared with rand.

### 3.4 Enhanced whale optimization algorithm

The EWOA (Kaveh and Ilchi Ghazaan 2017) has also been used in the present study. This algorithm was developed to enhance the rate of convergence and reliability of the WOA, which was inspired by the hunting of humpback whales in what is known as a bubble-net hunting strategy. The flowchart for the EWOA is shown in Fig. 7. The equations of this flowchart are as follows:

\[
X(t + 1) = X(t) - A \circ D^\omega,
\]

\[
D^\omega = r \circ |X(t)|, \quad a = 2a \circ r - a
\]

where \( r \) is a random vector containing arrays that are uniformly distributed in interval [0,1], \( a \) is a vector of numbers which decrease in linear order in interval [0,2] during the iterations, \( X \) is a vector indicating the position of the whale, \( t \) is the number of the current iterations, and:

\[
X(t + 1) = X(t) - A \circ D^\omega,
\]

\[
D^\omega = r \circ |X(t)|, \quad a = 2a \circ r - a
\]

where \( r \) is a random vector containing arrays that are uniformly distributed in interval [0,1], \( a \) is a vector of numbers which decrease in linear order in interval [0,2] during the iterations, \( X \) is a vector indicating the position of the whale, \( t \) is the number of the current iterations, and:
$X(t + 1) = e^{bk} \cdot \cos(2\pi k) \cdot D' + X'(t)$

$D' = |X'(t) - X(t)|$

(21)

where $b$ is a constant number used to define the logarithmic spiral shape, $k$ is a random number which is uniformly distributed in interval $[-1, 1]$, and $X'$ is the current best position of the whale, and:

$p = 0.3(1 - t/t_{\text{max}})$.
where $t$ and $t_{\text{max}}$ denote the current iteration and number of total iterations in the optimization process, respectively.

4 Numerical examples

Two numerical examples were considered to investigate the optimum PBD of unsymmetrical SMRFs using the SSA, ECBO, EVPS and EWOA. As metaheuristic algorithms are based on a stochastic search to find the optimal answer, different random seeds were generated in each run, which scattered the results. For this reason and to assess the scattering of the results, optimization of each example was performed in 30 independent runs.

Hasan et al. (2002) used a symmetrical shape for these frames to conduct NSA. Kaveh et al. (2010) used them for optimal design and Kaveh and Nasrollahi (2014) assessed the performance of the CSS algorithm for the optimal design of these frames. The frames were considered to be unsymmetrical for the optimal PBD. The unsymmetrical nature of the frames caused the end bay length to be different from the other bays. The length of each bay was 9.15 m and the height of each story, except for the end bay, was 3.96 m. The length of the end bay was 7.62 m.
The expected yield strength ($F_{ye}$) of the steel for the columns and beams was 397 and 339 MPa, respectively. The modulus of elasticity ($E$) was 200 GPa. The column section list was limited to wide-flange sections W8 to W14 and all beam sections were W-shaped sections. The parameter values used in the algorithms for both examples are summarized in Table 2. The values for parameters $S_1$, $S_S$, $F_a$ and $F_v$ are listed in Table 3.

### 4.1 Three-story, four-bay steel frame

Figure 8 shows the grouping of elements and applied loads for the three-story steel frame. Figure 9 shows the number of hinges. A constant gravity load of $W_1 = 32$ kN/m was applied to the first and second stories and a constant gravity load of $W_2 = 28.7$ kN/m was applied to the beams of the roof. The seismic weight for the first and second stories was 4688 kN and for the third story was 5071 kN.

For each optimization algorithm in this problem, a population of 60 and a maximum of 300 iterations were considered. Table 4 presents the optimum sections and the best, worst, average, and standard deviation ($\sigma$) and coefficient of variation (COV) of weights of the structure obtained from each of the optimization algorithms for the one-step and the proposed two-step approaches.
Fig. 17 Number of NSA in each iteration of algorithms for 3-story steel frame
Figure 10 compares the ratio of the best solution to those obtained from the different algorithms in 30 independent runs. As the ratio increased, the difference between the best solution and the solutions achieved from the different runs decreased and a better solution was obtained. It can be seen that EVPS was more capable of finding the optimal solution. The optimum weight of the structure and the objective function value obtained by this algorithm were 280.94 kN and 0.147, respectively.

As the scattering decreased in each chart, the performance of the algorithms became more uniform for the optimal solution from each independent run. Figure 10 and Table 4 indicate that EVPS exhibited less scattering caused by random seeds in the results of each independent run. This indicates its more uniform performance in comparison with the other algorithms.

Figures 11 and 12 show the formation patterns of the plastic hinges in the optimal solution for the three-story frame using NSA with lateral loading pattern based on the first mode shape in the positive and negative directions, respectively, at each performance level. Figure 13 shows the ratio of plastic hinge rotation to the allowable values according to FEMA 356 for nonlinear analysis of a three-story frame at the mentioned load patterns. As seen, this ratio was smaller than unity for all plastic hinges, which indicates an acceptable amount of plastic hinge rotation (plastic hinge rotations which were negligible are not shown). The formation pattern and rotation of the plastic hinges in the unsymmetrical three-story structure at different performance levels confirm that NSA should be performed in both directions to obtain accurate and reliable results.

Figure 14 shows the displacement ($D_{\text{disp}}$) of the stories, the displacement-to-height ratio, and the inter-story drift ratio of the structure for both load patterns. Figure 15 compares the inter-story drift ratios and their permitted values for the best solution. Figure 16 shows the optimization convergence of the best answer for each algorithm. Figure 17 shows the number of NSA in each iteration of algorithms for the three-story steel frame that indicates the reduction of computational volume in analysis.

4.2 Nine-story, five-bay steel frame

Figure 18 shows the geometry, grouping of elements, and applied loads for a nine-story, five-bay steel frame. Figure 19 shows the number of plastic hinges. A constant gravity load of $W_1 = 32$ kN/m was applied to the first to eighth stories and a constant gravity load of $W_2 = 28.7$ kN/m was applied to the beams of the roof. The seismic weight for the first story and roof was 1111 kN and 1176 kN, respectively, and for the second to eighth stories was 1092 kN.

To find the optimal solution for the nine-story structure, a population of 80 and a maximum of 300 iterations were considered for the EWOA, ECBO and EVPS algorithms. The optimum sections and the best, worst, average,
standard deviation and COV of weights for the nine-story frame obtained from each of the algorithms for the one-step and the proposed two-step approaches are presented in Table 5.

The ratio of the best solution to those obtained from different optimization algorithms is shown in Fig. 20 for 30 independent runs. The difference between the best solution and those achieved by ECBO was less than for the two other algorithms, which demonstrates the ability of this algorithm to find the optimal solution. This figure and Table 5 indicate that EWOA showed more uniform performance in this example.

The lowest weight for the nine-story structure was 1326.49 kN and the optimum objective function obtained by ECBO algorithm was 0.247. Figure 21 shows the plastic hinge formation patterns for the optimal solution of the nine-story frame using NSA with lateral loading pattern based on the first mode shape in the positive and negative directions at each performance level. The ratio of plastic hinge rotation to the allowable values in FEMA 356 for a nine-story frame using the NSA with these load patterns is shown in Fig. 22. This ratio was less than unity for all plastic hinges, which indicates an acceptable amount of rotation (rotations of plastic hinges which were negligible are not shown).
The results of the formation pattern and plastic hinge rotation in the unsymmetrical nine-story structure at each performance level indicate that the use of NSA in only one direction was not sufficient and confirmed the need for NSA in both directions. Figure 23 compares the optimization convergence process of algorithms using the results of optimization convergence of the best answer for each algorithm. Figure 24 shows the displacement of the stories, displacement-to-height ratio, and inter-story drift ratio of the structure for both load patterns. Figure 25 compares the inter-story drift ratios and their permitted values for the best solution. Figure 26 shows the number of NSA in each iteration of algorithms for the nine-story steel frame that indicates the reduction of computational volume in analysis.

5 Conclusions

A two-step approach to the performance of PBD optimization of unsymmetrical 2D SMRFs was used that considers the constraints based on the acceptance criteria...
Fig. 23 Results of story drift for a 9-story steel frame with lateral loading pattern based on the first mode shape in: (a) positive direction; (b) negative direction.

Fig. 24 Comparison of inter-story drift ratios (%) and their permitted values for the best solution for a 9-story steel frame with lateral loading pattern based on the first mode shape in: (a) positive direction; (b) negative direction.
for SMRFs according FEMA-356. In this approach, the structure was analyzed with a lateral loading pattern based on the first mode shape in positive and negative directions. The results indicate that the NSA should be performed in both directions.

The objective function was formulated such that the difference between the successful and unsuccessful steps was taken into account and the problem constraints were controlled for each performance level. The constraints in the second step depended on satisfying the constraints in the first step. The optimum designs for two case studies with unsymmetrical SMRFs having three and nine stories were performed using the metaheuristic EWOA, ECBO, EVPS, and SSA algorithms to evaluate the proposed approach. The conclusions are summarized as follows:

- The EVPS was more capable of finding the optimal solution and exhibited less scattering caused by random seeds in each independent run for the three-story steel frame. This indicates its more uniform performance in comparison with the other algorithms.
The results for the nine-story steel frame showed that ECBO performed uniformly when finding optimum solution and led to the most desirable results.

The pattern of plastic hinge formation and their rotation values for the unsymmetrical frames at each performance level confirmed that NSA should be performed in both directions.

The proposed two-step approach for solving such problems can appropriately reduce the computational volume by removing unnecessary calculations of nonlinear static analysis in the second step.

In general, this approach can be efficient and appropriate in problems with high computational volume, due to simplification and reduction of computational volume.

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Declarations

Conflicts of interest/Competing interests The authors declare that they have no conflict of interest.

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