Dynamical transition in a nonlinear two-level system driven by a special hyperbolic-secant external field

Hong Cao · Xi-Jing Liu · Miao Liu

Received: 1 February 2022 / Accepted: 20 May 2022 / Published online: 1 June 2022
© The Author(s), under exclusive licence to Springer Nature B.V. 2022

Abstract We propose an exact solution to a linear two-level system with the existence of a special hyperbolic secant external field and elucidate its transition dynamics. We then extend the model to the nonlinear case and show that the nonlinearity will significantly affect the transition dynamics. As nonlinearity increases, Landau–Zener–Stückelberg–Majorana interference fringes can be constructive and the upper energy level in the adiabatic limit splits into three levels. For fast-sweeping fields, we derive an analytic expression for dynamics transition under the stationary phase approximation and find that transition probability will be blocked as the nonlinearity intensity is much larger than the external field frequency, which agrees with the numerical result.

Keywords The exact solution model · Two-level system · Dynamical transition · Nonlinear effects

1 Introduction

The exactly solvable quantum system with a single-axis driving field and corresponding dynamical behavior has long been a subject of considerable interest in quantum mechanics [1]. Among them, the most famous example is the Landau–Zener–Stückelberg–Majorana (LZSM) model with a linearly sweeping external field [2, 3], which remains a very active area of research due to numerous applications including quantum optics [4], solid-state physics [5], quantum information science [6] and a spin–orbit-coupled Bose–Einstein condensate [7]. Another well-known model is the hyperbolic secant pulse of Rosen and Zener (RZ) [8], which has played an important role in ion–atom collision [9], self-induced transparency [10] and qubit control [11, 12]. In contrast to the LZSM model where the energy bias varies linearly with time for a z-axis control field, in the RZ model, the coupling strength varies with time for an x-axis control field. Both of them belong to the family of analytical exact controls.

Recently, increasing attention has been focused on the application of nonlinear quantum systems, such as the ones involving the ultracold Bose–Einstein condensate (BEC) system and nonlinear optics [13–17]. LZSM and RZ models have also been extended to nonlinear cases [18–22], which proves that nonlinearity will dramatically affect the transition dynamics. For example, in the nonlinear LZSM model, there exists a critical value of the interaction strength beyond which the transition probability becomes nonzero even in the adiabatic limit. For the nonlinear RZ model, the quantum transition can be completely blocked by a strong nonlinearity. Despite these isolated successes, extending exactly solvable single-axis models to nonlinear cases has remained extremely rare. There are two reasons for this, on the one hand, the issues of instabilities and
nonintegrability in the nonlinear quantum system are expected to feature obstructions of control, it is difficult to apply the control principles of a linear quantum system to a nonlinear quantum system. On the other hand, the analytically solvable model is rare, and these exact solutions of them expressed in terms of complicated special functions (such as the Gauss hypergeometric function and the Weber function), the infinite time will be required. To verify the correctness of numerical calculation, the time for the nonlinear case will also be infinite. In real physical systems, the driving field cannot be infinite and the truncation of the pulse is inevitable. However, the transition dynamics of the LZSM and RZ models will change in a finite time [23,24].

It is worth noting that a kind of exactly solvable single-axis driving model based on the invariant method has been proposed, which can produce desired nonadiabatic passages in two-level and three-level systems [25–29]. Differing from the original LZSM and RZ model, these examples of analytically solvable systems are described in terms of simple solutions, rendering the control strategies more transparent. Therefore, numerical results with the nonlinear case will be simply verified through a finite duration. These make us interesting to search for more exactly solvable single-axis driving models and examine what happens when dynamics affected by the nonlinearity.

In this paper, we have constructed an exact solution model for a linear two-level system driven by a hyperbolic-secant pulse. Our model is no redundant parameter and the driving field of the target Hamiltonian here is uniquely determined. Differing from the previous method which aims at finding out the solution to the special function, we resolve the dynamics of the system with a simple solution. Moreover, motivated by the recent increasing research for BEC, where nonlinearity naturally arises from a mean-field treatment of the interaction between particles. We extend the model to the nonlinear case and investigate its transition dynamics in the presence of nonlinearity. Astonishingly, the influence of nonlinearity can be reduced by modulating an external parameter, so we can give a suitable value to get desirable quantum state control.

2 The exact solution to a hyperbolic-secant model in a linear two-level system

We consider the nonlinear two-level system described by the dimensionless Schrödinger equation [17,18]

\[
\frac{d}{dt} \begin{pmatrix} a \\ b \end{pmatrix} = H(t) \begin{pmatrix} a \\ b \end{pmatrix},
\]

with the Hamiltonian given by

\[
H(t) = \frac{\gamma}{2} \sigma_x + \left[ \frac{\Omega_z(t)}{2} + \frac{\delta(t)}{2} \right] \sigma_z,
\]

where \( a \) and \( b \) are the probability amplitudes, the total probability \( |a|^2 + |b|^2 \) is conserved and set to be 1. \( c \) is the nonlinear parameter describing the interparticle interaction. \( \sigma_{x,y,z} \) denote Pauli matrices satisfying \( [\sigma_i, \sigma_j] = i \epsilon_{ijk} \sigma_k \). The coupling strength \( \gamma \) (x-axis field) is constant, and the energy bias between two levels is controlled by a time-dependent \( \Omega_z(t) \) (z-axis field), for example the magnetic field in the z direction for an electron spin [30,31]. When \( c = 0 \), the Hamiltonian of Eq. (3) describes any linear two-level system which is driven along a single z-axis [1].

In the present paper, we consider that the energy bias is time-dependent governed by a hyperbolic secant external field of the form

\[
\Omega_z(t) = 2\gamma \text{sech}(\gamma t).
\]

Differing from LZSM and its nonlinear sweep models [18,32], here the external field \( \Omega_z(t) \) is determined by a unique parameter, and both the amplitude and scanning frequency can be replaced by \( \gamma \). Furthermore, although the pulse form of our model is similar to that of the RZ model, it cannot be regarded as the variants of the RZ model since the energy bias in our model are time-varying, and therefore, it is also not a special case of the Demkov–Kunike model [17].

The above system can be exactly solved at the linear case \( c = 0 \). To resolve the dynamic of this system, we suppose that the present system possesses a dynamical invariant [33,34]

\[
I(t) = \frac{\sin \theta(t) \cos \varphi(t)}{2} \sigma_x + \frac{\sin \theta(t) \sin \varphi(t)}{2} \sigma_y + \frac{\cos \theta(t)}{2} \sigma_z,
\]

\[
= \frac{\sin \theta(t) \cos \varphi(t)}{2} \sigma_x + \frac{\sin \theta(t) \sin \varphi(t)}{2} \sigma_y + \frac{\cos \theta(t)}{2} \sigma_z,
\]

\[
= \frac{\sin \theta(t) \cos \varphi(t)}{2} \sigma_x + \frac{\sin \theta(t) \sin \varphi(t)}{2} \sigma_y + \frac{\cos \theta(t)}{2} \sigma_z,
\]

\[
= \frac{\sin \theta(t) \cos \varphi(t)}{2} \sigma_x + \frac{\sin \theta(t) \sin \varphi(t)}{2} \sigma_y + \frac{\cos \theta(t)}{2} \sigma_z,
\]

\[
= \frac{\sin \theta(t) \cos \varphi(t)}{2} \sigma_x + \frac{\sin \theta(t) \sin \varphi(t)}{2} \sigma_y + \frac{\cos \theta(t)}{2} \sigma_z,
\]

\[
= \frac{\sin \theta(t) \cos \varphi(t)}{2} \sigma_x + \frac{\sin \theta(t) \sin \varphi(t)}{2} \sigma_y + \frac{\cos \theta(t)}{2} \sigma_z,
\]

\[
= \frac{\sin \theta(t) \cos \varphi(t)}{2} \sigma_x + \frac{\sin \theta(t) \sin \varphi(t)}{2} \sigma_y + \frac{\cos \theta(t)}{2} \sigma_z,
\]
which satisfies
\[ \partial_t I(t) = -i[H(t), I(t)]. \] (5)

Here, \( \theta(t) \) and \( \varphi(t) \) are auxiliary time-dependent angles. Equation (5) is readily verified through the following equations of the components:
\[ \gamma = -\frac{\dot{\theta}(t)}{\sin \varphi(t)}, \] (6)
\[ \Omega_{\varphi}(t) = \dot{\psi}(t) - \dot{\theta}(t) \cot \varphi(t). \] (7)

According to Eqs. (6) and (7), we directly calculate the corresponding mixing angles \( \theta(t) \) and \( \varphi(t) \), which can be given by the following equations,
\[ \theta(t) = \frac{\pi}{2} + \arccos(\tanh(\gamma t)), \]
\[ \varphi(t) = \pi - \arccos(\tanh(\gamma t)). \] (8)

We note that for given any \( \theta(t) \) and \( \varphi(t) \), one can construct \( \gamma \) and \( \Omega_{\varphi} \). At this stage, it is direct to verify that this system possesses a dynamical invariant
\[ I(t) = -\tanh(\gamma t) \frac{\sigma_x}{2} + \text{sech}(\gamma t) \tanh(\gamma t) \frac{\sigma_y}{2} \]
\[ -\text{sech}(\gamma t) \frac{\sigma_z}{2}. \] (9)

Obviously, the instantaneous eigenstates of the invariant operator \( I(t) \) in Eq. (9) read
\[ |\phi_{\pm}\rangle = \pm \cos \frac{\theta}{2} |\pm\rangle + \sin \frac{\theta}{2} e^{\pm i\varphi} |\mp\rangle. \] (10)

Here, for two-level system, we have used \( |\pm\rangle = [1, 0]^T, |\mp\rangle = [0, 1]^T \). The dynamical basis of this system can be formulated as
\[ |\Psi(t)\rangle = k_1 e^{i\delta_+(t, t_0)} |\phi_+(t)\rangle + k_2 e^{i\delta_-(t, t_0)} |\phi_-(t)\rangle, \] (11)
where \( k_{1,2} \) are constant, the so-called Lewis–Riesenfeld phase is given by
\[ \delta_{\pm}(t, t_0) = \int_{t_0}^t \langle \phi_{\pm}(t') | i \partial_{t'} - H(t') | \phi_{\pm}(t') \rangle dt'. \]
\[ = \mp \int_{t_0}^t \gamma \frac{\cos \varphi(t')}{\sin \theta(t')} dt'. \] (12)

So far, we have given the exact solution of linear system. As a result, the probability amplitudes can be generically expressed as \( a = \langle + | \Psi(t) \rangle \) and \( b = \langle - | \Psi(t) \rangle \).

In the following study, we assume the quantum state is prepared in one mode initially. As the external field turned on, quantum transition between two modes occurs. Meanwhile, we consider that the driving field is truncated and evolves from 0 to \( \frac{20}{\gamma} \) in a finite time. If the system starts from \( |+\rangle \), then the time-dependent mode superposition coefficients \( (k_{1,2}) \) are determined by \( k_{1,2} = (|+\rangle | \Psi(0)\rangle) \), it yields \( k_1 = 0 \) and \( k_2 = 1 \), and the transition probability is given by
\[ P(t) = | b(t) |^2 = \cos^2 \frac{\theta}{2}. \]

Additionally, if the system starts from \( |-\rangle \), then the superposition coefficients \( (k_{1,2}) \) satisfy \( k_1 = 1 \) and \( k_2 = 0 \), the transition probability can also be expressed as
\[ P(t) = | a(t) |^2 = \cos^2 \frac{\theta}{2}. \]

It is clear that they are symmetrical to each other. This result provides a transparent control in the linear system. We depict the external field format and transition probability in Fig. 1.

Fig. 1 The field pulse \( \Omega_{\varphi}(t) \) and the corresponding transition probability \( (| a(t) |^2 or | b(t) |^2) \) as a function of time. a The field pulse specified by Eq. (3) as time goes from 0 to \( \frac{20}{\gamma} \). b Transition probability between two modes in linear case

3 Nonlinearity effects

3.1 Transition dynamics in nonlinear two-level systems

With the emergence of nonlinearity \( (c \neq 0) \), the transition dynamics will dramatically change [35]. The Schrödinger Eq. (1) is no longer analytically solvable, we therefore exploit a fourth- or fifth-order Runge–Kutta algorithm to trace the quantum state evolution numerically, the contour plots for transition probability with different initial states are shown in Fig. 2. In our calculation, the transition probability is a function of time (evolving from 0 to \( \frac{20}{\gamma} \)) and \( c/\gamma \). As shown in Fig. 2, the blue region stands for low transition probability, and the red area corresponds to high transition probability.

In the linear case \( (c/\gamma = 0) \), it is shown that the transition dynamics is symmetrical when the system starts...
from the state $|+\rangle$ and $|−\rangle$, which is robust and rapidly stable at 0.5, this result agrees with the analytic prediction of Fig. 1. However, the symmetry will be broken as the nonlinear interaction increases. The phenomenon is similar to that of the references [17,21]. Differing from them, in this model, the transition dynamics will approach the linear case when the scanning frequency is fast enough, a situation that can be strongly modified by the scanning frequency. Meanwhile, as the value of $c/\gamma$ increases, LZSM interferometry fringes can be constructed even if the driving field does not vary periodically, and the interference fringes in Fig. 2a, b are asymmetric. Unlike the LZ and RZ interference under an oscillating external field [20,22], the interference phenomenon appeared here is under a non-oscillating external field. These effects provide a good opportunity to precisely measure the interaction between interferometry phenomena and external control fields.

For the weak nonlinear case, i.e., $c/\gamma \leq 1$, the oscillation behavior gradually appears no matter what the initial state $|+\rangle$ or $|−\rangle$ is, and sometimes the maximum value of transition probability is even larger than 0.5. However, the oscillation periods of the two patterns are different. When the system is prepared on state $|+\rangle$ (see Fig. 2a), it will decrease as the nonlinearity strengths. Starting from state $|−\rangle$ exhibits the opposite behavior (see Fig. 2b). For the strong interaction, i.e., $c/\gamma > 1$, quantum transition also exhibits a striking difference between preparing on $|+\rangle$ (see Fig. 2a) and starting from $|−\rangle$. When the system is initially prepared on state $|+\rangle$, the oscillation behavior even appears in $c/\gamma > 2$, and the oscillation pattern is completely broken at $c/\gamma = 4$, both the oscillation period and amplitude have a big change in this case. The quantum transition probability between two modes is blocked and close to zero. When the system is initially prepared on state $|−\rangle$ (see Fig. 2b), the amplitude of the oscillation pattern is dramatically reduced at $c/\gamma = 2$, the transition probability is blocked and tends to zero in this case. Obviously, to form the transition probability blocking, stronger nonlinearity is required when starting from $|+\rangle$.

These results show that there exists a critical value of $c/\gamma$, and the transition probability will be dramatically reduced. The LZSM interference patterns can be destructive or constructive, which are determined by $c/\gamma$. That is to say, if the system is initially prepared on state $|+\rangle$, this point is at $c/\gamma = 4$. Starting from $|−\rangle$ is at $c/\gamma = 1.4$. As a result, we can modulate the value of $c/\gamma$ to get desirable transition probability or LZSM interference patterns. In the following, let us explicitly consider the influence of nonlinearity effects in the case of low or fast scanning frequency.

### 3.2 Adiabatic limit

In the adiabatic limit, the scanning frequency ($\gamma$) is very slow and the characters of transition probabilities should be entirely determined by adiabatic energy levels and eigenstates [36]. These levels as the solution of the time-dependent version of Eq.(1) obtained by replacing $i(\partial/\partial t)$ with energy $E$, after some elaboration, we find the following quartic equation

$$
(2E + c)^2(4E^2 - \gamma^2) - 4\Omega^2 E^2 = 0.
$$

(13)

The adiabatic energy levels of the system are given by the roots of above quartic equation. We illustrate these levels with $\gamma = 0.5$ in Fig. 3 for the case of linearity, weak nonlinearity, and strong nonlinearity, respectively. It is shown that the levels will be dramatically affected by the increasing nonlinearity. In the linear case (see Fig. 3a), two energy levels are symmetric about a horizontal axis, which leads to symmetrical transition dynamics. However, the presence of nonlinearity can break symmetry.

For the case of weak nonlinearity, there exist two situations, one case is that the number of levels is two at $c \leq 0.5$, i.e., $c/\gamma \leq 1$, which is similar to its linear counterpart. Compared with the linearity case, we see that the energy levels lower than the ones for $c = 0$ (see Fig. 3a), which leads to an oscillation behavior of transition probability in Fig. 2 (the case of $c/\gamma < 1$). Another case is that the number of levels is four at

![Fig. 2 Contour plot of transition probability in nonlinear case as the function of time ($\gamma t$) and the ratio $c/\gamma$. Initially, the system starts from $|+\rangle$ (left figure a) or $|−\rangle$ (right figure b).](image-url)
Dynamical transition in a nonlinear two-level system driven

\[ H(t) = \frac{\Omega_2}{2} (2p - 1) - \frac{c}{4} (2p - 1)^2 + \gamma \sqrt{p(1-p)} \cos \eta, \]  

(14)

which can completely describe the dynamical properties of the system (1). Its fixed points correspond to the eigenstates of the nonlinear two-level system, which can be given by the following equations:

\[ \eta^* = 0, \pi, \]  

(15)

\[ \Omega_2 - c (2p^* - 1) + \frac{\gamma (2p^* - 1)}{2 \sqrt{p^*(1 - p^*)}} \cos \eta^* = 0, \]  

(16)

\[ \gamma \sqrt{p^*(1 - p^*)} \sin \eta^* = 0. \]  

(17)

We have depicted the phase space orbits of the corresponding system (14) in Fig. 4 with the time \( t = 20 \), it is shown that there exist two or four fixed points (labeled \( P1, P2, P3, P4 \)). The number of fixed points depends on the nonlinear parameter \( c \). For weak nonlinearity, \( c < 1 \), there exists only two fixed points (\( P1 \) and \( P2 \)) in Fig. 4a, b). They are elliptic points, each being surrounded by closed (elliptic) orbits, which are located on \( \eta^* = 0 \) and \( \eta^* = \pi \), meaning that the two corresponding eigenstates of the two-level system have a relative phase of \( \pi \). The stable elliptic fixed point \( P1 \) corresponds to the lower level in Fig. 3. \( P2 \) is the upper level. For weak nonlinearity, \( c > 1 \), when we increase \( c \), \( P2 \) bifurcates into \( P3 \) and \( P4 \) (see Fig. 4c, d), the corresponding up level split into two unstable level in Fig. 3c, d.

\[ c = 0.7, \text{i.e., } c/\gamma = 1.4 \]  

(see Fig. 3b). In Fig. 3b, the dashed levels are unstable, which can be evaluated by Eq.(13). The eigenvalues \( E \) can be real, complex, or pure imaginary. Only pure imaginary eigenvalues correspond to stable states; others indicate the unstable ones.

We note that it is different from the reference [18,19], there is no looping new feature in our figures. As shown in Fig. 3b, the up-level is divided into three levels, including two unstable levels and one stable level, the breakdown of adiabatic evolution even in the adiabatic limit. However, the lowest level will not split. They are also in good agreement with the situation results in Fig. 2. When the system is prepared initially in state \( |+\rangle \), the system follows the up-level until the level structure destroyed, it only presences the low transition probability and oscillation behavior. As the nonlinearity increases, the up-level splits and the transition probability becomes larger. Since the up-level is not fully converted to the unstable level, the transition probability of starting from \( |+\rangle \) escapes to be blocked. When the system is prepared initially in state \( |\rangle \), the system follows the lowest-level until the up-level structure is destroyed, which leads to no way to jump to the up level, because the middle-level and the lowest-level degenerate. As a result, transition probability block and close to 0 at \( c > 0.7(c/\gamma > 1.4) \).

For the case of strong nonlinearity(\( c > 1 \)), i.e., Fig. 3c, d, there also exists four levels. In Fig. 3c, we note that the up-level gradually turns into an unstable level and the middle two levels close to the right vertical axis at \( c \leq 2 \), i.e., \( c/\gamma = 4 \). The transition probability also exists an oscillatory behavior. However, when the nonlinearity effect is strong enough (see Fig. 3d), i.e., \( c > 2 \), four levels will be completely formed and do not change with increasing nonlinearity parameter \( c \), the transition probability will be blocked. These results perfectly agree with Fig. 2.

To further explore the above peculiar phenomena, we construct the effective classical Hamiltonian introducing the transition probability \( P = |b(t)|^2 \) and \( \eta = \eta_b - \eta_a \) as the relative phase of two modes [36], then we obtain an effective Hamiltonian and satisfy the canonical equation, i.e., \( \dot{P} = -\partial H/\partial \eta, \dot{\eta} = \partial H/\partial P, \)
We begin with the variable transformation,

\[ a = a' \exp \left[ -i \int_0^t \left( \frac{\Omega_x}{2} + \frac{c}{2} \left( |b|^2 - |a|^2 \right) \right) dt \right], \tag{18} \]

\[ b = b' \exp \left[ i \int_0^t \left( \frac{\Omega_x}{2} + \frac{c}{2} \left( |b|^2 - |a|^2 \right) \right) dt \right]. \tag{19} \]

As a result, the diagonal terms in Hamiltonian are transformed away, and we have

\[ b' = \frac{\gamma}{2i} \int_{-\infty}^t dt \exp(-i \int_0^t \right. \nonumber \times \left. \left[ \Omega_z + c \left( |b|^2 - |a|^2 \right) \right] dt \right) \tag{20} \]

Following Ref. [19], we evaluate the self-consistently of Eq. (20). Owing to the large \( \gamma \), the integrand gives a rapid phase oscillation, and it makes the integral small. The dominant contribution comes from the stationary \( t_0 \) of the phase around, and we have

\[ \Omega_z + c(2|b|^2 - 1) = \bar{\alpha}(t - t_0) \tag{21} \]

where

\[ \bar{\alpha} = \left( \frac{d\Omega_z}{dr} + 2c \frac{d |b|^2}{dr} \right)_{t_0} \tag{22} \]

Since \( |b(t)|^2 = |b'(t)|^2 \), then we have

\[ |b|^2 = \left( \frac{\gamma^2}{2} \right)^2 \int_{-\infty}^t dt \exp \left( -\frac{i}{2\bar{\alpha}} (t - t_0)^2 \right) \tag{23} \]

Combining relation (22) with Eq. (23), we come to a closed equation for \( \bar{\alpha} \),

\[ \bar{\alpha} = -2\gamma^2 \text{sech}[\gamma t_0] \text{tanh}[\gamma t_0] + 2c \left( \frac{\gamma^2}{2} \right) \sqrt{\frac{2\pi}{\bar{\alpha}}} \tag{24} \]

Here, we have differentiated Eq. (23) at time \( t_0 \), obtaining a few standard Fresnel integrals with the result \( \Gamma \) of the form

\[ \Gamma = \frac{\pi \gamma^2}{2\bar{\alpha}} \tag{25} \]

Then, the above result yields a closed solution for \( \Gamma \),

\[ \Gamma = -\frac{4}{\pi} \text{sech} \left[ \gamma t_0 \right] \text{tanh} \left[ \gamma t_0 \right] + \frac{2c}{\pi \gamma} \sqrt{\frac{2\pi}{\bar{\alpha}}} \tag{26} \]

Equation (26) gives the approximate solution of transition probability \( |b(t)|^2 \) in the sudden limit. Compared with numerical result, it shows good agreement at \( c/\gamma > 250 \), a clear deviation for \( c/\gamma < 250 \) is observable in Fig. 5, due to a sufficiently fast scanning frequency leading to the invalidity of our assumption \( a \sim 1 \). Consequently, transition probability will be blocked when \( c \gg \gamma \).

---

3.3 Sudden limit

In this section, we will discuss quantum transition in sudden limit, and the scanning frequency is fast enough. We can derive the analytical expression of the transition probabilities using the stationary phase approximation (SPA) [18,19]. Here, we focus on the case of strong nonlinearity \( c/\gamma \gg 1 \). We thus expect the amplitude \( b \) to remain small and \( a \sim 1 \) all the time, and a perturbation treatment of the problem becomes adequate.
Fig. 5 Comparison between our analytic result using SPA and the numerical integration of Schrödinger Eq. (1). The rectangles line demonstrates the analytical results based on Eq. (26); dots line stands for numerical results of transition probability as the function of $c/\gamma$. They are in good agreement at $c/\gamma > 250$. The curves of inset describe comparison between analytic and numerical results at $c/\gamma < 10$

4 Conclusion

In summary, we have constructed a single-axis driving model and given an exact solution to a linear two-level system. For the nonlinear case, we investigate the dynamics transition of this model, which is influenced by the scanning frequency ($\gamma$) and nonlinear parameter ($c$). There exists a critical value for $c/\gamma$, where the LZSM interference fringes will be destructive at this point. Moreover, as nonlinearity increases, the adiabatic energy level will be split and one of the fixed points bifurcates into two points. When the scanning frequency is fast enough, we derive an analytic result for dynamic transition, indicating that large values of nonlinearity are required to bring the transition probability close to zero.

Data Availability All data generated or analyzed during this study are included in this article.

Declarations

Conflict of interest There is no conflict of interest.

References

1. Barnes, E., Sarma, S.D.: Analytically solvable driven time-dependent two-level quantum systems. Phys. Rev. Lett. 109(6), 060401 (2012)
2. Landau, L.: On the theory of transfer of energy at collisions i. Phys. Z. Sowjetunion 2(46), 118 (1932)
3. Zener, C.: Non-adiabatic crossing of energy levels. In: Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character, Vol. 137, No. 833, pp. 696–702 (1932)
4. Bouwmeester, D., Dekker, N., Dorsselaer, F., Schrama, C., Visser, P., Woerdman, J.: Observation of landau-zener dynamics in classical optical systems. Phys. Rev. A 51(1), 646 (1995)
5. Wernsdorfer, W., Sessoli, R., Caneschi, A., Gatteschi, D., Cornia, A.: Nonadiabatic landau-zener tunneling in fe8 molecular nanomagnets. EPL (Europhys. Lett.) 50(4), 552 (2000)
6. Fuchs, G., Burkard, G., Klimov, P.,Awschalom, D.: A quantum memory intrinsic to single nitrogen-vacancy centres in diamond. Nat. Phys. 7(10), 789–793 (2011)
7. Olson, A.J., Wang, S.J., Niffenegger, R.J., Li, C.H., Greene, C.H., Chen, Y.P.: Tunable landau-zener transitions in a spin-orbit-coupled Bose–Einstein condensate. Phys. Rev. A 90(1), 013616 (2014)
8. Rosen, N., Zener, C.: Double Stern–Gerlach experiment and related collision phenomena. Phys. Rev. 40(4), 502 (1932)
9. Olson, R.E.: Charge transfer at large internuclear distances: application to asymmetric alkali-ion-alkali-atom systems. Phys. Rev. A 6(5), 1822 (1972)
10. McCall, S.L., Hahn, E.L.: Self-induced transparency. Phys. Rev. 183(2), 457 (1969)
11. Suominen, K.A., Garraway, B.M., Stenholm, S.: Wavepacket model for excitation by ultrashort pulses. Phys. Rev. A 45(5), 3060 (1992)
12. Poem, E., Kenneth, O., Kodriano, Y., Benny, Y., Khatsevich, S., Avron, J., Gershoni, D.: Optically induced rotation of an exciton spin in a semiconductor quantum dot. Phys. Rev. Lett. 107(8), 087401 (2011)
13. Morsch, O., Oberthaler, M.: Dynamics of Bose–Einstein condensates in optical lattices. Rev. Mod. Phys. 78(1), 179 (2006)
14. Dou, F.Q., Cao, H., Liu, J., Fu, L.B.: High-fidelity composite adiabatic passage in nonlinear two-level systems. Phys. Rev. A 93(4), 043419 (2016)
15. Zhu, J.J., Chen, X., Jauslin, H.R., Guérin, S.: Robust control of unstable nonlinear quantum systems. Phys. Rev. A 102(5), 052203 (2020)
16. Lahini, Y., Pozzi, F., Sorel, M., Morandotti, R., Christodoulides, D.N., Silberberg, Y.: Effect of nonlinearity on adiabatic evolution of light. Phys. Rev. Lett. 101(19), 193901 (2008)
17. Feng, P., Wang, W.Y., Sun, J.A., Dou, F.Q.: Demkov–Kunike transition dynamics in a nonlinear two-level system. Nonlinear Dyn. 91(4), 2477–2484 (2018)
18. Liu, J., Fu, L., Ou, B.Y., Chen, S.G., Choi, D.I., Wu, B., Niu, Q.: Theory of nonlinear Landau–Zener tunneling. Phys. Rev. A 66(2), 023404 (2002)
19. Wang, G.F., Ye, D.F., Fu, L.B., Chen, X.Z., Liu, J.: Landau–Zener tunneling in a nonlinear three-level system. Phys. Rev. A 74(3), 033414 (2006)
20. Li, S.C., Fu, L.B., Liu, J.: Nonlinear Landau–Zener–Stückelberg–Majorana interferometry. Phys. Rev. A 98(1), 013601 (2018)
21. Ye, D.F., Fu, L.B., Liu, J.: Rosen–Zener transition in a nonlinear two-level system. Phys. Rev. A 77(1), 013402 (2008)
22. Li, S.C., Fu, L.B.: Nonlinear Rosen–Zener–Stückelberg interferometry of a Bose–Einstein condensate. Phys. Rev. A 102(3), 033313 (2020)
23. Vitanov, N., Garraway, B.: Landau–Zener model: effects of finite coupling duration. Phys. Rev. A 53(6), 4288 (1996)
24. Militello, B.D., Vitanov, N.V.: Dynamics of a two-state system through a real level crossing. Phys. Rev. A 91(5), 053402 (2015)
25. Li, W., Cen, L.X.: Coherent population transfer in multi-level Allen–Eberly models. Quantum Inf. Process. 17(4), 1–13 (2018)
26. Zhao, P.J., Li, W., Cao, H., Yao, S.W., Cen, L.X.: Exotic dynamical evolution in a secant-pulse-driven quantum system. Phys. Rev. A 98(2), 022136 (2018)
27. Cao, H., Yao, S.W., Cen, L.X.: Explicit construction of nonadiabatic passages for stimulated Raman transitions. Phys. Rev. A 100(5), 053410 (2019)
28. Cao, H., Yao, S.W., Cen, L.X.: Anomalous dynamical evolution and nonadiabatic level crossing in exactly soluble time-dependent quantum systems. Eur. Phys. J. D 73(10), 1–5 (2019)
29. Cao, H., Zhao, R.Q., Chen, H.: Accurate theoretical analysis of light transition in three-waveguide directional coupler based on invariant engineering. Opt. Quant. Electron. 53(6), 1–10 (2021)
30. Martinis, J.M., Geller, M.R.: Fast adiabatic qubit gates using only σ z control. Phys. Rev. A 90(2), 022307 (2014)
31. Stefanatos, D., Paspalakis, E.: Resonant shortcuts for adiabatic rapid passage with only z-field control. Phys. Rev. A 100(1), 012111 (2019)
32. Dou, F.Q., Li, S.C., Cao, H.: Combined effects of particle interaction and nonlinear sweep on Landau–Zener transition. Phys. Lett. A 376(1), 51–55 (2011)
33. Lewis, H.R., Jr.: Classical and quantum systems with time-dependent harmonic-oscillator-type hamiltonians. Phys. Rev. Lett. 18(13), 510 (1967)
34. Lewis, H.R., Jr., Riesenfeld, W.: An exact quantum theory of the time-dependent harmonic oscillator and of a charged particle in a time-dependent electromagnetic field. J. Math. Phys. 10(8), 1458–1473 (1969)
35. Wu, Y., Yang, X.: Strong-coupling theory of periodically driven two-level systems. Phys. Rev. Lett. 98(1), 013601 (2007)
36. Liu, J., Wu, B., Niu, Q.: Nonlinear evolution of quantum states in the adiabatic regime. Phys. Rev. Lett. 90(17), 170404 (2003)

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.