Voltage controllable superconducting state in the multi-terminal superconductor-normal metal bridge

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We study voltage controllable superconducting state in multi-terminal bridge composed of the dirty superconductor/pure normal metal (SN) bilayer and pure normal metal. In the proposed system small control current \(I_{ctrl}\) flows via normal bridge, creates voltage drop \(V\) and modifies distribution function of electrons in connected SN bilayer. In case of long normal bridge the voltage induced nonequilibrium effects could be interpreted in terms of increased local electron temperature. In this limit we experimentally find large sensitivity of critical current \(I_c\) of Cu/MoN/Pt-Cu bridge to \(I_{ctrl}\) and relatively large current gain which originate from steep dependence of \(I_c\) on temperature and large \(I_c\) (comparable with theoretical depairing current of superconducting bridge). In the short normal bridge deviation from equilibrium cannot be described by simple increase of local temperature but we also theoretically find large sensitivity of \(I_c\) to control current/voltage. In this limit we predict existence at finite \(V\) of so called in-plane Fulde-Ferrell state with spontaneous currents in SN bilayer. We argue that its appearance is connected with voltage induced paramagnetic response in N layer.

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INTRODUCTION

The idea to control the superconducting properties of superconductors, which are metals, with help of electric field or voltage is based on their large sensitivity to the form of the electron distribution function \(f(E)\) and ability to modify \(f(E)\) by applied voltage. Origin of the effect could be understood from the equation for the superconducting order parameter \(\Delta\)

\[
\Delta = \lambda_{BCS} \int_0^{\hbar \omega_D} R_2(E) f_L(E) dE, \tag{1}
\]

where \(R_2(E) = Re(\Delta/\sqrt{\omega_D^2 - \Delta^2})\) in the simplest case of spatially homogenous superconductor in absence of superconducting current, \(\lambda_{BCS}\) is a coupling constant in Bardeen-Cooper-Schrieffer theory, \(\omega_D\) is a Debye frequency and \(f_L(E)\) is odd in energy part of \((1 - 2 f(E))\).

With increasing the bath temperature \(T\) the equilibrium Fermi-Dirac distribution \(f(E) = 1/\exp((E/k_B T) + 1)\) changes and \(\Delta(T)\) goes down because more states with \(E > \Delta(T)\) are occupied by electrons and \(f_L(E)\) decreases at low \(E\). At fixed temperature applied voltage \(V\) modifies \(f(E)\) in a similar way, i.e. \(f_L(E)\) decreases with increasing \(V\) (for example see experimental \(f(E,V)\) in Ref. [1]) and one can expect voltage controllable modification of superconducting properties. In some cases effect of \(V\) on \(f(E)\) could be described via introducing the local electron temperature \(T_e(V) \neq T\), for example in the system with strong electron-electron scattering. But sometimes it cannot be done and new effects appear which are connected with nonthermal form of \(f(E,V)\) (thermal form here is Fermi-Dirac distribution with \(T_e \neq T\)).

There are many theoretical and experimental works where voltage controllable superconducting state was studied in metallic superconductors. For example, in Refs. [2] the normal metal-superconductor-normal metal (NSN) voltage biased wire was considered. In Ref. [3] there were found that at \(eV \sim \Delta\) there is jump to the normal state and in finite interval of voltages \(\Delta/2 < eV < \Delta\) several superconducting states could exist in voltage biased 'bulk' superconductor. This result could be related with known transition of the magnetic superconductor to the normal state when magnetic exchange energy \(E_{ex} \sim \Delta\), due to formal analogy between voltage biased and magnetic superconductors as it was discussed in Ref. [3]. In Ref. [3] existence of two stable spatially nonuniform states (symmetric and asymmetric against the center of superconducting part) were predicted for relatively long NSN wire which is consequence of spatially nonuniform nonequilibrium \(f_L(E,V)\). In Ref. [4] the so called bimodal state was found which may be related to enhanced stability of superconductivity near superconductor/normal metal interface [5]. For voltage biased NISIN system there were found several spatially homogeneous states at fixed voltage [4] and in some range of the parameters the Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) state was predicted which develops in lateral direction of NISIN system [3].

Control of critical current of SNS (or SINIS) Josephson junction by applying of the voltage (or, alternatively, current) to the additional N lead, attached to N part of SNS junction was proposed in Refs. [8–10]. In Refs. [11–13] this effect has been experimentally studied. Recently the control of critical current of Ti and Al bridges with help of voltage leads has been observed Refs. [14–15] where the effect is also connected with modification of \(f(E)\) due to applied voltage although in more complicated manner than in previous works.
In our work we study the current/voltage controllable superconducting state in the multi-terminal bridge composed of superconductor/normal metal bilayer and normal metal (see Fig. 1), where superconductor is highly resistive metal (with large resistivity $\rho_S$ in the normal state) and normal metal has $\rho_N \ll \rho_S$. In the proposed system one may control large critical current (about $I_{ep}$ of S layer) flowing along SN bridge instead of much smaller critical current of SNS Josephson junction. In comparison with superconducting bridge in SN hybrid with $\rho_S/\rho_N \gg 1$ and thin S and N layers (order of superconducting coherence length) $I_c(T)$ is much steeper in wide temperature while $I_c$ is much larger at low temperatures \cite{13, 20}. These effects come from the substantial superconducting current flowing along low resistive N layer and small proximity induced minigap $\epsilon_g \sim 1/d_N^2$ \cite{21}. This allows us to expect large sensitivity of $I_c$ even to small deviation from equilibrium caused by applied control current/voltage.

We confirm experimentally these expectations in case of long normal bridge (Cu) and long SN bridge (Cu/MoN/Pt) with lengths $L_N, L_{SN} \gg L_{ee}$, where $L_{ee}$ is an inelastic electron-electron scattering length, when the deviation from the equilibrium can be described in terms of increased local temperature. For our parameters we find current gain about 6 and we discuss how it could be further improved.

We also study theoretically limit of short normal bridge with $L_N \ll L_{ee}$ when nonequilibrium $f_{\ell}(E)$ has nonthermal form in its central part

$$f_{\ell}(E) = \frac{1}{2}(\tanh((E + eV_{ctrl}/2)/(2k_B T)) + \tanh((E - eV_{ctrl}/2)/(2k_B T))).$$  \hspace{1cm} (2)

In this limit we also find large sensitivity of $I_c$ to control current/voltage but in addition there is new effect - appearance of in-plane Fulde-Ferrell (FF) state with spontaneous currents flowing along S and N layers in SN bridge. Previously, in-plane Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state was predicted in similar nonequilibrium SN system in Ref. \cite{23}. In comparison with that work we show that FF state appears at finite voltage $V_{ctrl} \sim k_B T_{cd}$ ($T_{cd}$ is a critical temperature of superconducting layer) and its origin is connected with voltage induced paramagnetic response of N layer which competes with diamagnetic response of S layer. Therefore the situation is similar to FFLO state in equilibrium SF and SFN hybrid structures \cite{21, 22}. And as in case of SFN trilayer one needs large ratio $\rho_S/\rho_N \gg 1$ to realize this state in nonequilibrium SN bilayer.

The structure of the paper is following. In Section II we present our experimental results on current/voltage controllable superconducting state in multi-terminal SN-N bridge with long N bridge. In Section III we theoretically study case of short N bridge and find range of parameters when FF state could be realized in SN bridge and discuss its similarity with FFLO state in equilibrium SFN trilayer. In section IV we conclude our results.

**LONG CONTROL N BRIDGE**

At $L_N \gg L_{ee}$ effect of applied voltage on $f(E)$ could be described via introducing the local electron temperature $T_e$ in the Fermi-Dirac distribution those spatial distribution along N bridge satisfies the one dimensional (when $W_N \ll L_N$) heat conductance equation (see for example Eq. (16) in Ref. \cite{22}). In the limit of short N bridge with $L_N$ smaller than electron-phonon scattering length $L_{ep}$ one may find simple expression for $T_e(y)$

$$T_e = \sqrt{T^2 + \alpha V_{ctrl}^2(1 - y/L_N)y/L_N}$$  \hspace{1cm} (3)

where $\alpha = 3e^2/(\pi^2 k_B^2)$. Eq. (3) illustrates that application of voltage/current to the control bridge changes the electron temperature in SN bridge (which is roughly located at $y = L_N/2$ when $W_{SN} \ll L_N$) and, hence, its critical current.

As we discuss in Introduction we expect relatively good sensitivity (comparable with that for SNS junction) and large current gain in the studied system. To verify it the multi-terminal bridges were made using Cu(30 nm)/MoN(20 nm)/Pt(5 nm) trilayer. The trilayer was grown by magnetron sputtering with a base vacuum level of the order of $1.5 \cdot 10^{-7}$ mbar on standard silicon substrates without removing the oxide layer and at room temperature. At first, Cu is deposited in an argon atmosphere at a pressure of $1 \cdot 10^{-3}$ mbar. Secondly, Mo is deposited in an atmosphere of a gas mixture Ar : $N_2 = 10 : 1$ at a pressure of $1 \cdot 10^{-3}$ mbar, and finally Pt is deposited in an argon atmosphere at a pressure of $1 \cdot 10^{-3}$ mbar (top Pt layer is used for protection purpose). In the next step the multi-terminal Cu/MoN/Pt bridges was made.
with help of mask free optical lithography. At the final stage the MoN/Pt layers were removed (by plasma chemical etching) in the part of the system to create normal banks and bridge. Final configuration is present in Fig. 2(a) where we show image of one of the multi-terminal Cu/MoN/Pt-Cu bridges (nominal width of Cu/MoN/Pt and Cu bridges is 3 µm, length of Cu/MoN/Pt bridge is 19 µm, length of Cu bridge is 5 and 7 µm, $T_{c0} = 7.8K$ of MoN film with thickness 20 nm, coherence length $\xi_0 = \sqrt{hD_{MoN}/1.76k_BT_{c0}} = 4.7 \text{ nm}$).

In Fig. 2(b) we show current-voltage characteristics of Cu/MoN/Pt bridge at different values of the control current $I_{ctrl}$ in Cu bridge measured at $T = 0.8K$. At $I_{ctrl} > 0.35mA$ the critical current of SN bridge goes to zero (see Fig. 2(c)) because the part of normal bridge covered by MoN layer goes to the normal state (it is seen from Fig. 2(d) where the resistance of normal bridge + normal banks as function of control current is present). Similar effect exists at $T = 3K$ (see Fig. 2(b,c)).

Fig. 2(c) demonstrates good sensitivity of studied system at $T = 0.8K$ - even small control current may strongly change $I_c$. Similar sensitivity is typical for SNS junctions [11, 13] which also have steep dependence of $I_c$ on temperature. On contrary, in superconducting bridge only relatively large applied voltage affects $I_c$ [14, 15].

Assuming that the strength of electron-phonon coupling in Au and Cu are close and using electron-phonon scattering time $\tau_{ep}(4.2K) = 1ns$ for Au [11] we find $L_{ep} = \sqrt{D_{Cu}\tau_{ep}} \sim 2.2\mu m$ at $T = 4.2K$ ($D_{Cu} \simeq 50cm^2/s$ according to [1]) which is comparable with the width and length of our Cu and Cu/MoN/Pt bridges. Therefore we are neither in the limit of short (when Eq. (3) is valid) nor long N bridge with $L_N \gg L_{ep}$ (in this case $T_c(L_N/2)$ could be found from the balance between Joule heating and cooling by phonons). Fig. 3 illustrates it where we plot electronic temperature $T_e(I_{ctrl})$ derived from experimental $I_e(T)$, $I_c(I_{ctrl})$ and theoretical $T_e(I_{ctrl})$ in two limits. In calculations we use $V_{ctrl} = I_{ctrl}R_{ctrl}$ where $R_{ctrl} = 6Ω$ is estimated from the geometry of Cu banks, measured $R_{ctrl} + R_{banks}$ - see Fig. 2(d), and known sheet resistance $R_s = 1Ω$ of 30 nm thick Cu layer at 10 K. Electron-phonon coupling strength in Cu is assumed as in Au, leading to above mentioned $\tau_{ep}$.

FIG. 2: (a) Image of one of Cu/MoN/Pt-Cu multi-terminal bridges. Arrows show direction of control ($I_{ctrl}$) and transport ($I$) currents. Photo was taken in four months after transport measurements. (b) Current-voltage characteristics of MoN/Cu/Pt bridge at different values of control current $I_{ctrl}$ in Cu bridge ($T=0.8K$). (c) Dependence of critical current of MoN/Cu/Pt bridge on control current at two temperatures. (d) Dependence of resistance of Cu banks and bridge on control current at two temperatures. Inset in panel (d): temperature dependence of the critical and retrapping currents of MoN/Cu/Pt bridge ($I_{ctrl} = 0$).

From Fig. 2 it follows the current gain $\sim 6$ at $T = 0.8K$ (it is defined as the ratio between $I_c$ at $I_{ctrl} = 0$ and $I_c(I_{ctrl})$ which drives $I_c$ to zero) which is larger than near unity current gain observed in Ref. [11]. Its relatively large value is connected with large critical current of Cu/MoN/Pt bridge (which is about theoretical depairing current $I_{dep}(T=0) = 11.2mA$ of MoN bridge with $d_{MoN} = 20nm$ and larger width $w = 5\mu m$ [20]) while critical current of SNS Josephson junction usually is much smaller.

The current gain could be increased either by going to lower temperatures or by optimizing parameters of the structure. Indeed, in Ref. [11] no signs of phonon emission was found for 5 $\mu m$ long Cu bridge at 25mK. Therefore making SN-N bridge with $L_N = 5\mu m$, $W_N = 200nm$ (and thick normal banks at the ends of N bridge), $L_{SN} = 5\mu m$, $W_{SN} = 1\mu m$ (and thick superconducting banks at the ends of SN bridge) and using parameters of studied Cu/MoN/Pt-Cu system one can obtain current gain $\sim 60$ (expected critical current $I_c(T=0) = 1mA$,...
expected \( I_{\text{ctrl}} = 16\mu A \) and \( V_{\text{ctrl}} = 0.4mV \) which drives SN bridge to normal state at \( T = 100mK \) according to Eq. (3)). Unfortunately such a size and temperature is beyond of our current abilities.

**FULDE-FERREL STATE IN SN-N MULTI-TERMINAL BRIDGE WITH SHORT N BRIDGE**

In this section we theoretically study the limit of short N bridge with length \( L_N < L_{\text{sc}} \) when voltage controlled distribution function in SN bilayer is not thermal and it is described by Eq. (2). As we show below it brings new property, except the possibility to control the critical current as in long N bridge.

In Refs. [8] [10] it was predicted and later experimentally confirmed [12] the sign change of superconducting current flowing via diffusive SNS Josephson junction when distribution function has form of Eq. (2) in N part and applied voltage is large enough. This result could be interpreted as a transition of N part of SNS junction to the paramagnetic state which is consequence of negative spectral current (or current-carrying density of states) in finite energy range in N part and distribution function described by Eq. (2). In voltage driven clean SN system paramagnetic response of N layer has been predicted recently where its connection with so called odd-frequency superconductivity has been discussed [26].

Existence of odd-frequency superconductivity was also predicted in ferromagnetic part of SF bilayer which has a paramagnetic response [27] [28]. At some parameters it may overcome diamagnetic response of S layer and it leads to vanishing of overall magnetic response, signals about instability and appearance of in-plane Fulde-Ferrell-Larkin-Ovchinnikov state [24]. Appearance of FFLO state and vanishing of magnetic susceptibility was also discussed in Ref. [29] for current driven superconductor with Fermi surface nesting. Apparently, these two phenomena are correlated in d-wave superconducting film, where FFLO state with spatially separated paramagnetic and diamagnetic currents flowing in opposite directions across the thickness of the film have been predicted in relatively thin samples [30]. Therefore one may expect that nonequilibrium diffusive SN bilayer also may transit to the FFLO state and our aim is to find the conditions when it could be realized. But first we would like to illustrate the transition to FFLO state in SFN trilayer having in mind to compare it later with nonequilibrium SN bilayer.

In Fig. 4(a,b) we show calculated superconducting sheet current density \( J_z(q_x) = \int j_z(q_x)dx \) flowing along SFN strip and corresponding free energy \( F_S(q_x) \) when temperature driven transition to Fulde-Ferrell state occurs [26] (we do not consider here Larkin-Ovchinnikov state because it has larger energy than FF state in SFN system [31]). Here \( q_x = \nabla \varphi_z + 2(\pi/\Phi_0)A_z \) is gauge invariant gradient of phase of superconducting order parameter along the SFN trilayer and results are obtained using Usadel model (details of calculations are present in Ref. [31]). At temperatures \( T/T_{\text{c0}} = 0.4 \) and 0.5 the ground state is homogenous (minimum of free energy is at \( q_x = 0 \)) and linear magnetic response is again diamagnetic because \( \partial^2 F_S/\partial q_x^2 \big|_{q_x=0} < 0 \). At temperatures \( T/T_{\text{c0}} = 0.1, 0.2 \) and 0.3 the ground state is inhomogenous one (minimum of free energy is at \( q_x = q_{\text{FF}} \)) but the linear magnetic response is again diamagnetic because \( \partial^2 F_S/\partial q_x^2 \big|_{q_x=q_{\text{FF}}} > 0 \) [32]. At temperature \( 0.3 < T_{\text{FF}}/T_{\text{c0}} < 0.4 \) there is transition from homogenous FF state with change of the sign of \( \partial^2 F_S/\partial q_x^2 \big|_{q_x=0} \) (it goes through the zero) and linear magnetic response van-
ishes at \( T = T_{FF} \). Note that in case of relatively large magnetic field (nonlinear regime) transition to FF state may occur in globally paramagnetic state (compare calculated magnetic response of SFN strip at different magnetic fields and temperatures shown in Fig. 7a in Ref. [23]). Physically vanishing of linear magnetic response is connected with compensation of diamagnetic response of S layer by paramagnetic response of FN layers.

In Ref. [23] transition to FFLO state in nonequilibrium SN bilayer was found theoretically using linearized Usadel equations, however its relation with paramagnetic response of N layer was not established. Here we perform Usadel equations, however its relation with paramagnetic fields and temperatures shown in Fig. 7a in Ref. [32]. Physically vanishing of linear magnetic response is connected with compensation of diamagnetic response of S layer by paramagnetic response of FN layers.

![Graph](image_url)

**Fig. 5:** (a,b) Dependence of superconducting sheet current density \( J_\sigma \) flowing along SN strip on \( q_z \) at different \( V_{ctrl} \). At \( eV_{ctrl}/k_B T_{c0} > 0.8 \) the branch with \( J_\sigma > 0 \) and global paramagnetic response at \( q_z = 0 \) appear. In inset of panel (a) we show dependence \( J_\sigma(V_{ctrl}) \) which is qualitatively similar to \( I_c(I_{ctrl}) \) present in Fig. 2(b) at \( T = 0.8 K \). In inset of panel (b) we show dependence of maximal superconducting order parameter (it is located at the boundary of S layer with vacuum) on \( q_z \). At \( eV_{ctrl}/k_B T_{c0} = 1.2 \) and 1.24 there is no homogenous superconducting state with \( q_z = 0 \). Parameters for S and N layers are the same as for SFN trilayer shown in Fig. 4(a), \( T = 0.1 T_{c0} \).

In Fig. 5 we present calculated \( J_z(q_z) \). Voltage drop via N bridge decreases the critical current (it corresponds to maximal \( J_z \) on dependence \( J_z(q_z) \)) qualitatively in the same manner as it does ordinary heating of electrons discussed in section II (compare inset in Fig. 5(a) and \( I_c(I_{ctrl}) \) in Fig. 2(c) at \( T = 0.8 K \)). However at \( V_{ctrl} = V_{FF} \sim 0.8 k_B T_{c0} \) new feature appears: \( J_z \) changes sign at small \( q_z \) and \( J_z \) becomes equal to zero not only at \( q_z = 0 \) but also at \( q_z = q_{FF} \). This points on appearance of the in-plane Fulde-Ferrell state in SN bilayer.

In contrast to SFN trilayer we cannot use free energy to prove directly that FF state is more preferable than homogenous state. Therefore we lean on the qualitative similarity in shape of \( J_z(q_z) \) for equilibrium SFN trilayer (see Fig. 4(a)) and nonequilibrium SN bilayer (see Fig. 5). Indication on advantage of FF state comes from Fig. 4(b) where we show \( J_z(q_z) \) and \( \Delta_{max}(q_z) \) at large voltages (\( \Delta_{max} \) is maximal value of \( \Delta(x) \) in S layer). In FF state superconducting order parameter is larger than in homogenous state - the same effect exists in SFN trilayer. Besides there is an interesting effect - at relatively large \( V_{ctrl} \) homogenous superconducting state with \( q_z = 0 \) does not exist - the same effect was found in Ref. [23].

From Fig. 5(a) one can see that at the transition to FF state \( dJ_z/dq_z \) changes the sign at \( q_z = 0 \). Therefore as in case of SF and SFN hybrids the transition to FF state is accompanied by vanishing of the linear magnetic (Meissner) response. We find that FF appears at finite \( V_{ctrl} = V_{FF} \) with \( q_{FF} = 0 \) and \( q_{FF} \) increases with increasing of \( V_{ctrl} \) as it could be seen from Fig. 5(a,b). With increasing of temperature \( V_{FF} \) increases while \( V_c \) (critical voltage which drives SN bilayer to normal state) decreases which resemble properties of magnetic super-
conductor which hosts FFLO state, where role of \( V \) is played by exchange field, how it was discussed in Ref. [1]. On contrary, in Ref. [23] it was predicted existence of FFLO state with finite \( q_{FF} \) at any voltage. The origin for this discrepancy between our results and results of Ref. [23] is not clear.

In the FF state in absence of transport current or magnetic field there are spontaneous currents flowing in opposite directions across the thickness of SN bilayer (they also exist in equilibrium SFN trilayer [23] and in \( \delta \)-wave thin superconducting film [30]). Their presence is manifestation of locally diamagnetic (in S layer) and paramagnetic (in N layer) magnetic response and finite \( q_{FF} \). In other words coefficient \( \lambda^2 \) (which is inverse square of London penetration depth for ordinary superconductor) in relation \( j_z \sim -\lambda^2 q_z \) has different sign in S and N layers. For SN bilayer with thick S layer there is no transition to FF state because voltage driven paramagnetic response of N layer cannot compensate the diamagnetic response of S layer, as in SFN trilayer with thick S layer [23].

We find that transition to FF state occurs in wide range of parameters similar to one for SFN trilayer [23]. Namely, it occurs at \( T \lesssim 0.37T_c \), it may exist at lower temperature even when \( \rho_S/\rho_N = 20 \) and for S layer as thick as \( 5\xi^c \). The favorite candidates for experimental observation of this state are dirty superconductors like NbN, MoN, WSi etc. with residual resistivity \( \rho_S \gtrsim 100\mu\Omega \cdot cm \), thicknesses \( d_S = 1 - 2\xi^c \) and low resistive metals like Au, Cu, Ag with \( \rho_N = 2 - 5\mu\Omega \cdot cm \) and thicknesses \( 2 - 4\xi^c \).

In Ref. [1] the two-step electron distribution function (Eq. 2) was experimentally observed in the center of Cu bridge with length 1.5 \( \mu m \) at \( T = 25mK \). Transition of Nb/Au/Nb Josephson junction to \( \pi \) state at \( T = 100mK \) with length of Au control bridge 1\( \mu m \) was found in Ref. [12]. Both results give approximate length scales and temperatures when Fulde-Ferrell state could be observed in SN-N multi-terminal bridge. With length of N bridge \( L_N = 1.5\mu m \) and width \( W_N = 100 - 200nm \) the width of SN bridge should be \( W_{SN} \lesssim L_N/5 \approx 200nm \) while its length \( L_{SN} \) is about \( 1 - 1.5\mu m \) to avoid thermalization of electrons along SN bridge.

Spontaneous currents flowing in nonequilibrium SN bilayer being in Fulde-Ferrell state can be checked experimentally by using SQUID magnetometer. Moreover one would expect unusual magnetic properties (global paramagnetic response in Meissner state) and unusual ground states in absence of magnetic field (vortex and onion like ones) connected with finite size (length and width) of SN bridge similar to ones predicted for SFN strip, disk and square [32, 34].

FF state could be also found from transport measurements. In regime of applied current only states with \( \partial J_z/\partial q_z < 0 \) could be realized - these are metastable states (doze with \( J_z \uparrow \uparrow q_z \) and having critical current marked as \( J_{c1} \) in Fig. 5(b)) and ground states (\( J_z \uparrow \downarrow q_z \) with critical current \( J_{c2} \)) [23]. The transition from the metastable state to the ground state with change of the current in the range \( -J_{c2} < J < J_{c2} \) is accompanied by large variation of \( q_z \) (it changes value and sign) when \( |J| \) exceeds \( J_{c1} \) and appearance of moving electric domain [30]. Applying ac current at \( V_{ctrl} < V_{FF} \) with amplitude \( J < J_{c2} = J_c \) (at this control voltage there exist only one critical current) leads to mainly inductive response with the voltage shifted by \( \pi/2 \) from the current. On contrary, at \( V_{ctrl} > V_{FF} \) the resistive response appears, connected with change of \( q_z \) when ac current exceeds \( J_{c1} \).

Another way to detect FF state is to measure current dependent kinetic inductance \( L_k(J) \) of SN bridge. In ordinary superconductor \( L_k(J) = L_k(-J) \) while in FF superconductor \( L_k(J) \neq L_k(-J) \) due to finite \( q_z = q_{FF} \) in the ground state. The last property directly follows from the different slopes of \( J_z(q_z) \) at \( q_z \gtrsim q_{FF} \) and \( q_z \lesssim q_{FF} \) and relation \( L_k^{-1} \sim -\partial J_z/\partial q_z \). For example \( L_k(J = -J_3/2)/L_k(J = J_3/2) \approx 1.4 \) (for \( V_{ctrl} = 1.1k_B T_c \) in Fig. 5(a)) and this ratio increases with further increase of \( J \).

**CONCLUSION**

We demonstrate experimentally the possibility to control critical current of dirty superconductor/low resistive normal metal (SN) hybrid bridge by current/voltage applied to the additional/control normal bridge. We argue that the effect is connected with modification of electron distribution function in SN bilayer. In the experiment for our realization of SN-N multi-terminal bridge we find current gain 6. Its relatively large value is connected with i) large contribution of proximity induced superconductivity in N layer to transport properties of SN bilayer and ii) its large sensitivity to the form of electron distribution function. We argue that the gain could be enhanced by optimization of geometrical parameters of SN-N bridge or going to lower temperatures. Besides we theoretically find that proximity induced superconductivity in N part of SN bilayer may have paramagnetic response at relatively large voltage drop and short N bridge and at some parameters it can be larger than diamagnetic response of host superconductor. It leads to appearance of the in-plane Fulde-Ferrell state with properties similar to ones for hybrid SF or SFN structures, and, apparently, thin \( \delta \)-wave superconducting films and current driven superconductor with Fermi surface nesting.

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Model

To calculate superconducting properties across the thickness of SN bridge being in voltage driven nonequilibrium state we use Usadel equation for anomalous $F = \sin \Theta = N_2 + iR_2$ and normal $G = \cos \Theta = N_1 + iR_1$ Green functions

$$\hbar D \frac{d^2 \Theta}{dx^2} + \left( 2iE - \frac{D}{\hbar} q_z^2 \cos \Theta \right) \sin \Theta + 2\Delta \cos \Theta = 0,$$  \hspace{1cm} \text{(4)}

where $D$ is a diffusion coefficient ($D = D_S$ in superconducting layer and $D = D_N$ in the normal layer), $q_z = \nabla \varphi_z + (2\pi/\Phi_0) A_z$ ($\varphi$ is a phase of the order parameter, $A$ is a vector potential) takes into account nonzero velocity of superconducting electrons $v_s \sim q_z$ in direction parallel to layers ($z$ direction in our case), $\Delta(x)$ is a magnitude of superconducting order parameter which has to be found in the superconducting layer via self-consistency equation

$$\Delta = \lambda_{BCS} \int_0^{\hbar \omega_D} R_2 f_L(E)dE,$$  \hspace{1cm} \text{(5)}

where

$$f_L(E) = \frac{1}{2} \left[ \tanh \left( (E + eV_{crl}/2)/(2k_B T) \right) \right. + \left. \tanh \left( (E - eV_{crl}/2)/(2k_B T) \right) \right].$$  \hspace{1cm} \text{(6)}

To calculate the superconducting sheet current density we use the following expression

$$J_z = \frac{q_z}{e \hbar} \int_0^{d_S + d_N} \frac{1}{\rho} \int_0^\infty 2N_2 R_2 f_L(E)dE dx,$$  \hspace{1cm} \text{(7)}

where $\rho$ is normal state resistivity of S and N layers. We consider thin bilayer with thickness of superconducting layer $d_S \ll \lambda_L$ ($\lambda_L$ is the London penetration depth in S layer) and thickness of normal layer $d_N$ less than characteristic penetration depth of magnetic field in N layer. It allows us to neglect the effect of the current induced magnetic field on the current distribution in SN strip.

At SN interface ($x = d_N$) we follow following boundary condition

$$D_N \frac{d \Theta}{dx} \bigg|_{x = d_N - 0} = D_S \frac{d \Theta}{dx} \bigg|_{x = d_N + 0},$$  \hspace{1cm} \text{(8)}

and continuity of $\Theta$: $\Theta(x = d_S - 0) = \Theta(x = d_S + 0)$ (we assume transparent interface between S and N layers), while at the boundary with vacuum ($x = 0, d_N + d_S$): $d\Theta/dx = 0$.

Equations (A1,A2) are solved numerically by using iteration procedure. For initial distribution $\Delta(x) = \text{const}$ we solve Eq. (A1) in energy interval $0 < E < \hbar \omega_D$ (we take $\hbar \omega_D = 40 k_B T_0$). In numerical procedure we use Newton method combined with tridiagonal matrix algorithm. Found solution $\Theta(x)$ is inserted to Eq. (A2) to find $\Delta(x)$ and than iterations repeat until the relative change in $\Delta(x)$ between two iterations does not exceed $10^{-8}$. Length is normalized in units of $\xi_c = \sqrt{\hbar D_S/k_B T_0}$, energy is in units of $k_B T_0$, current is in units of depairing current of single S layer with the thickness $d_S$. Typical step grid in S and N layers is $\delta x = 0.05 \xi_c$. BSC constant in Eq. (A2) is expressed via $\hbar \omega_D$ and $T_0$ using following expression

$$\lambda_{BCS} = \int_0^{\hbar \omega_D} \frac{\tanh(E/2k_B T_0)}{\bar{E}} dE$$  \hspace{1cm} \text{(9)}

which follows from Eq. (A2) when $\Delta \to 0$, $R_2/\Delta \to 1/E$ and $V_{crl} = 0$.

To decrease the number of free parameters we assume that the density of states in S and N layers is the same and ratio of resistivities is equal to inverse ratio of diffusion constants or mean free paths $\rho_S/\rho_N = D_N/D_S = \ell_N/\ell_S$.

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