Novel phase-locking schemes for the carrier envelope offset frequency of an optical frequency comb

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Optical frequency combs were invented at the end of the last century1,2) and have been benefiting many research fields in physics, including optical frequency metrology,3) ultrafast phenomena,4,5) and astrophysics.6-8) An optical frequency comb links an optical frequency with two microwave frequencies by the following equation:

\[ \nu_n = \nu_n \times f_{\text{rep}} + f_{\text{ceo}}, \quad (1) \]

where \( \nu_n \) is the optical frequency of the \( n \)-th mode of the frequency comb. The repetition rate and carrier envelope offset frequency are presented as \( f_{\text{rep}} \) and \( f_{\text{ceo}} \), respectively. To measure the optical frequencies using the frequency comb, it is generally important to stabilize the two parameters, \( f_{\text{rep}} \) and \( f_{\text{ceo}} \), to a reference frequency standard. An error signal is generated by the phase-locking such that \( f_{\text{ceo}} = (1/2)f_{\text{rep}} \). The Allan deviation and signal-to-noise ratio of the coherent \( \delta \)-function peak for the in-loop beat signal are \( 5.3 \times 10^{-17}/\tau \) and 80–85 dB-Hz, respectively, where \( \tau \) is the averaging time of the frequency measurement. These new locking schemes simplify the sign and mode-number determination in frequency measurements.

In this paper, we propose simple and robust schemes to lock \( f_{\text{ceo}} \) referring to the \( f_{\text{rep}} \) of an optical frequency comb. With the new schemes, we can decrease the parameters of the frequency comb from two to one. In one sample scheme, we use the frequency range where two \( f_{\text{ceo}} \)-related signals overlap around \( f_{\text{rep}}/2 \) [see Fig. 1(b)].

We set our bandpass filter at \( f_{\text{rep}}/2 \) and adjust it so that two signals would overlap inside the passband of the filter. We split the filtered signals into two parts by a simple power splitter and mix them again by a double-balanced mixer (DBM). From the intermediate frequency (IF) output of the DBM, we obtained a signal from the product of the two components of the beat note, mixer output

\[ = \{ \cos(2\pi f_{\text{ceo}}t) + \cos(2\pi (f_{\text{rep}} - f_{\text{ceo}})t) \} \times \{ \cos(2\pi f_{\text{ceo}}t + \varphi) + \cos(2\pi (f_{\text{rep}} - f_{\text{ceo}})t + \varphi) \}
= \cos \varphi \cos(2\pi (f_{\text{rep}} - f_{\text{ceo}})t + \varphi) - \cos(2\pi f_{\text{ceo}}t + \varphi) - \frac{1}{2} \cos(4\pi f_{\text{ceo}}t + \varphi) + \frac{1}{2} \cos(4\pi (f_{\text{rep}} - f_{\text{ceo}})t + \varphi). \quad (2) \]

Here, \( \varphi \) is an additional phase difference between the two signals at the DBM inputs induced by the length difference of the two cables between the splitter and the DBM in Fig. 1(b). We assume that both the phase differences to \( f_{\text{ceo}} \) and \( f_{\text{rep}} - f_{\text{ceo}} \) are \( \varphi \) because the two frequencies are almost the same.

![Conceptual illustration of the \( f_{\text{ceo}} \) locking schemes.](image-url)

(a) Conventional scheme. (b) New scheme for \( f_{\text{ceo}} \) locking such that \( f_{\text{ceo}} = (1/2)f_{\text{rep}} \). (c) New scheme for \( f_{\text{ceo}} \) locking such that \( f_{\text{ceo}} = (1/3)f_{\text{rep}} \).

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mixer output was appropriately time-averaged by a low-pass filter after the DBM to drop the fast varying terms, i.e., 3rd–5th terms in Eq. (2). Consequently, we showed that the time-averaged signal \((\langle \cdot \rangle)\) of the mixer output has the form

\[
\langle \text{mixer output} \rangle_t = \cos \varphi \{\cos[2\pi(f_{\text{rep}} - 2f_{\text{ceo}})t] + 1\} 
\]

where \(\varphi\) is an arbitrary phase obtained when \(2f_{\text{ceo}}\) approaches \(f_{\text{rep}}\). If \(\varphi = \pi/2\), the amplitude of the time-averaged signal drops to zero. Since the time-averaged signal of Eq. (3) has an offset, we need to subtract a stable bias to obtain a direction-sensitive error signal for controlling \(f_{\text{ceo}}\), as shown in Fig. 2. Points A and B reflect the lock points of \(f_{\text{ceo}}\) with an opposite sign in the servo system. The error signal is fed back to the injection current of the pump laser. The error signal was fed back to an injection current of the pump laser.

In this situation, we lock \(f_{\text{ceo}}\) such that the relationship between \(f_{\text{ceo}}\) and \(f_{\text{rep}}\) is \(f_{\text{ceo}} = (2/3)f_{\text{rep}}\).

Figure 3 shows a schematic of the experimental setup. We implemented a fiber-based frequency comb with three output branches.\(^{10,11}\) The \(f_{\text{rep}}\) of the comb was 44 MHz. Every branch was amplified and spectrally broadened by an erbium-doped fiber amplifier and a highly nonlinear fiber. We used one of the branches to observe an \(f_{\text{ceo}}\) interferometric beat note for \(f_{\text{ceo}}\) stabilization, and another to observe a beat note \((f_{\text{beat}})\) between one of the comb components and a 1064 nm Nd:YAG reference laser. The third branch is to be used for any applications.

\(f_{\text{beat}}\) was used for locking one vicinity mode of the comb to the reference laser. The error signal was fed back to an electrooptic modulator and a thermoelctric cooler in the comb oscillator with different time constants to control the effective length of the comb oscillator. The frequency of the reference laser was stabilized to a high-finesse Fabry–Perot cavity.\(^{12,13}\) We note that the frequency of the reference laser varies at a rate of 80 mHz/s owing to the drift of the cavity.

The observed \(f_{\text{beat}}\) beat signal can be monitored by an RF spectrum analyzer, as conceptually shown in Fig. 1(b). The beat signal is filtered and amplified with an appropriate gain. In this branch, \(f_{\text{ceo}}\) and \(f_{\text{rep}}\) were simultaneously measured by frequency counters. In the \(f_{\text{ceo}} = (1/2)f_{\text{rep}}\) locking case, we used a bandpass filter with a 3 dB passband of 17.9–25.3 MHz to filter the beat signals. The filtered signals were split...
To lock controller. It acts as a low-pass oscillator through a proportional-integral-derivative servo injection current of the pump laser diode for the comb was biased by a proper DC voltage and was fed back to the input signal, and the sign of the mixer output described according to Eq. (3). The obtained IF signal was biased by a proper DC voltage and was fed back to the injection current of the pump laser diode for the comb oscillator through a proportional-integral-derivative servo controller. It acts as a low-pass filter because its bandwidth is not so large that the fast varying term in Eq. (2) drops. To lock $f_{\text{CEO}}$, we adjusted $f_{\text{CEO}}$ to be sufficiently close to $(f_{\text{rep}} - f_{\text{CEO}}) \approx (1/2)f_{\text{rep}}$ by varying the bias current to the pump laser diode and then closing the feedback loop. Consequently, we could stabilize $f_{\text{CEO}}$ referring to $f_{\text{rep}}$ such that $f_{\text{CEO}} = (1/2)f_{\text{rep}}$. In this experiment, we need not use an extra RF synthesizer for $f_{\text{CEO}}$ locking.

When $f_{\text{CEO}}$ was locked using the new scheme, we measured the in-loop $f_{\text{CEO}}$ frequency, as shown in Fig. 4(a). To evaluate the locking performance of the scheme without the effect of the slow cavity drift, we calculated $f_{\text{CEO}}/f_{1064}$ and converted it to the Allan deviation [see Fig. 4(b)], where $f_{1064} \approx n \times f_{\text{rep}}$ was approximately considered the optical frequency of the Nd:YAG reference laser. This trace shows the locking performance more quantitatively than the trace in Fig. 4(a). The instability of $5.3 \times 10^{-11}/\sqrt{\tau}$ is much less than that of $f_{\text{rep}}$ and hence does not contribute to that of the frequency comb. The observed instability was limited by the signal-to-noise ratio of the $f_{\text{CEO}}$ signal measured by the frequency counter. If we use a tracking oscillator for determining $f_{\text{CEO}}$, the instability of $f_{\text{CEO}}$ may be further reduced. Figure 5 shows the RF spectrum of the in-loop $f_{\text{CEO}}$ signal. The servo bandwidth was more than 300 kHz, which was estimated on the basis of the bump of the in-loop $f_{\text{CEO}}$ spectrum. We obtained a coherent $\delta$-function peak with a signal-to-noise ratio of 80–85 dB-Hz. The results show that the new locking scheme induces very few additional phase noises to the comb.

In the $f_{\text{CEO}} = (1/3)f_{\text{rep}}$ locking case, it was difficult to arrange the beat measurement as shown in Fig. 1(c) because of the difficulty in finding appropriate bandpass filters due to the relatively small $f_{\text{rep}}$ of 44 MHz in this study. Instead, we compared the doubled frequency of the $(f_{\text{rep}} - f_{\text{CEO}})$ signal with the frequency of the $(f_{\text{rep}} + f_{\text{CEO}})$ signal, as shown in Fig. 6. In this arrangement, we used twice of $f_{\text{rep}}$ and arranged the beat measurements using appropriate bandpass filters experimentally. We actually locked $f_{\text{CEO}}$ such that $f_{\text{CEO}} = (1/3)f_{\text{rep}}$ and verified the locking performance. Similar Allan deviation and phase noise characteristics were observed as in the case of $f_{\text{CEO}} = (1/2)f_{\text{rep}}$.

In the case of $f_{\text{CEO}} = (1/2)f_{\text{rep}}$ locking, we realized a “half-integer comb”. In this comb system, the $f_{\text{CEO}}$-related signals, $f_{\text{CEO}}$ and $(f_{\text{rep}} - f_{\text{CEO}})$, are identical. The frequency of the $n$-th mode is expressed as

$$\nu_n = n f_{\text{rep}} + \frac{1}{2} f_{\text{CEO}} = \frac{n + \frac{1}{2}}{2} f_{\text{rep}}. \quad (7)$$

The $(f_{\text{rep}} - f_{\text{CEO}})$ signal is also recognized as a $-f_{\text{CEO}}$ signal as we can see when we substitute $(f_{\text{rep}} - f_{\text{CEO}})$ into Eq. (1) and shift the integer from $n$ to $n + 1$. Therefore, in the half-integer comb, we need not care about the sign of $f_{\text{CEO}}$. Conventionally, in order to lock $f_{\text{CEO}}$, we need to check and determine the sign of the beat frequency. For simplicity of these procedures, we would like to reduce these sign

![Fig. 4.](image_url) (a) Measured frequency of the in-loop $f_{\text{CEO}}$. (b) Relative Allan deviation of $f_{\text{CEO}}$ to $f_{1064} \approx n \times f_{\text{rep}}$.

![Fig. 5.](image_url) RF spectrum of the in-loop $f_{\text{CEO}}$. The resolution and video bandwidths are 30 kHz and 300 Hz, respectively.

![Fig. 6.](image_url) Actual configuration for locking $f_{\text{CEO}}$ such that $f_{\text{CEO}} = (1/3)f_{\text{rep}}$. 

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\nu_n = n f_{\text{rep}} + \frac{1}{2} f_{\text{CEO}} = \frac{n + \frac{1}{2}}{2} f_{\text{rep}}. \quad (7)
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ambiguities in each comb. If we apply this new method also to $f_{\text{ceo}}$ stabilization, the frequency of the reference laser can be expressed as

$$\nu_{\text{ref}} = \nu_{n} + f_{\text{beat}} = nf_{\text{rep}} + \frac{1}{2}f_{\text{rep}} + \frac{1}{2}f_{\text{rep}} = (n + 1)f_{\text{rep}}. \quad (8)$$

Hence, all the comb modes can be used as an optical frequency ruler referring to the reference laser without any microwave synthesizers. The new locking schemes in this study provide a type of $f_{\text{ceo}}$-free optical frequency comb,\textsuperscript{14–17} which simplifies the sign and mode-number determination in frequency measurements.

In the conventional $f_{\text{ceo}}$ stabilizing scheme [Fig. 1(a)], we can use a frequency divider to widen the capture range of phase locking when the servo bandwidth of $f_{\text{ceo}}$ is not sufficiently large. In the case of the half-integer comb, since we use the square of the sum of the $f_{\text{ceo}}$-related signals, $f_{\text{ceo}}$ and $(f_{\text{rep}} - f_{\text{ceo}})$, as described in Eq. (2), each signal cannot be individually frequency-divided. Therefore, a large servo bandwidth of $f_{\text{ceo}}$ locking is necessary for phase locking. On the other hand, when $f_{\text{ceo}} = (q/p)f_{\text{rep}}$, each $f_{\text{ceo}}$-related signal can be individually frequency-divided. Thus, we can phase-lock $f_{\text{ceo}}$ without a large servo bandwidth by paying the penalty of having a larger residual phase noise than in the case of not using a divider.

In conclusion, we have proposed simple locking schemes for $f_{\text{ceo}}$ referring to $f_{\text{rep}}$ and achieved good locking performance characteristics with our fiber-based optical frequency comb. The performance of the locking schemes has been evaluated by measuring the Allan deviation and phase noise characteristics. With these schemes, we have realized a half-integer comb with $f_{\text{ceo}} = (1/2)f_{\text{rep}}$ and also combs with $f_{\text{ceo}} = (1/3)f_{\text{rep}}$ or $f_{\text{ceo}} = (2/3)f_{\text{rep}}$. Besides this, we have also proposed similar locking schemes that ensure the relationship $f_{\text{ceo}} = (q/p)f_{\text{rep}}$.

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