Research on Coplanar Orbit Maneuvering in the Context of on-Orbit Service

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Abstract. The concept of on-orbit service is introduced, and the development process of on-orbit service is briefly reviewed. The optimal transfer plan of orbit transfer and rendezvous is studied in the context of on-orbit service. The circular orbit motion with perturbation is used to study the near-circle orbit maneuver, and the general equation of coplanar maneuver is established based on the cylindrical coordinate system. The influences of the orbital parameters and the difference initial phase between the coaxial orbits on the optimal transfer orbit are calculated by Matlab simulation.

1. Introduction
The conceptual study of spacecraft on-orbit service began with the Soviet Union replenishment of the space station, followed by special attention in aerospace engineering such as the Soviet spacecraft and the US space shuttle [1]. NASA gave such a description of on-orbit service in the “NASA satellite servicing project report” – In space, people, robots or both work together to complete a type of operation involving the life of various spacecraft and improving the ability to perform tasks[2]. For nearly half a century, the United States, the Soviet Union, Japan, Canada, and ESA have conducted a series of research and experiments in conjunction with specific space missions[3]. In the past 20 years, around the application research of on-orbit service technology, the world's major space powers have carried out a series of exploratory attempts. For example, the US astronauts operated outside the space shuttle and successfully recovered several satellites such as the Palm House B2 and West Star 6, repaired the Hubble telescope, installed giant solar panels for the International Space Station, and installed the space station JEMRMS robotic arm, etc[3-5]. Simultaneously the number of space debris has significantly increased in the near-Earth space as a consequence of continued space activities.[6] Because the positive effect of atmosphere on space debris is very limited[7], active debris removal has become a hot research in on-orbit services.

In this context, the concept of a space-based orbital service system was proposed.[8] The operating system consists of three parts: “Autonomous space transport and orbital robot” (service station), “service vehicle” (active device) and “serviced vehicle” (space vehicle - target). On-orbit service trajectory information as shown in Fig. 1.
In this space-based on-orbit service system, the transfer and rendezvous of spacecraft is a critical step in completing on-orbit service. At present, only the United States, Russia and China independently master space rendezvous and docking technology. Based on the limitations of spacecraft fuels, active spacecraft should minimize fuel consumption during orbital transfer and rendezvous. Therefore, for different initial orbits and service orbits, the design of the optimal transfer orbit has become a research topic for many scholars.[9]

2. Equation of orbital maneuver

When studying the spacecraft maneuver near orbit, the near-circular orbit motion is regarded as the disturbed motion of the circular orbit, and the orbital maneuver equation is established on the cylindrical coordinate system r, u, z (Figure 2.). In this coordinate system, the established coplanar orbit maneuver equation is as in equation (1).[9-15]

\[
\sum_{i=1}^{N} (\Delta V_{ni} \sin \varphi_i + 2\Delta V_n \cos \varphi_i) = \Delta e_x \\
\sum_{i=1}^{N} (\Delta V_{ni} \cos \varphi_i + 2\Delta V_n \sin \varphi_i) = \Delta e_y \\
\sum_{i=1}^{N} 2\Delta V_n = \Delta a \\
\sum_{i=1}^{N} (2\Delta V_{ni}(1-\cos \varphi_i) + \Delta V_n(3\varphi_i - 4\sin \varphi_i)) = \Delta t
\]

where, \( \Delta e_x = e_f \cos \omega_f - e_0 \cos \omega_0 \), \( \Delta e_y = e_f \sin \omega_f - e_0 \sin \omega_0 \), \( \Delta a = (a_f - a_0) / r_0 \), \( \Delta t = \lambda_0(t_f - t_0) \), \( \Delta V_n = \Delta V_n^* / V_0 \), \( \Delta V_n^* = \Delta V_n^* / V_0 \).

It is necessary to clarify that this equation is a non-dimensional equation. Here \( f, 0 \) – indices corresponding to the final and initial orbits, \( e_f, e_0 \) – eccentricity of the orbit; \( a_f, a_0 \) – semi-major axes of the orbits; \( \omega_f, \omega_0 \) – angles between the direction to the pericenter of the corresponding orbit and the direction to the point specified in the final orbit; \( t_f \) – the required time of arrival at a given point, \( t_0 \) – initial maneuver time; \( N \) – number of speed pulses; \( \varphi_i \) – the angle of application of the i-th velocity pulse, measured from the direction to a given point in the direction of the spacecraft; \( \Delta V_n^*, \Delta V_n^* \) – radial and transversal components of the i-th velocity pulse, respectively.
The task of determining the optimal maneuver parameters for coplanar orbits is specified in the following manner: determine \( \Delta V_{r_i}, \Delta V_{t_i}, \varphi_i \) \( (i = 1, \ldots, N) \) to minimize the total feature speed of the orbital maneuver in (2).

\[
\Delta V = \sum_{i=1}^{N} \left( \Delta V_{r_i}^2 + \Delta V_{t_i}^2 \right)^{1/2}
\]  

(2)

3. Impact of parameters on the optimal orbital maneuver

3.1. Impact of perigee and apogee on optimal orbital maneuver

According to formula (1), the velocity impulses of the spacecraft flight from apogee to apogee (T1) and from perigee to perigee (T2) are calculated separately in the coaxial orbit. And compare their size to determine the optimal orbit maneuver and study the influence of orbital parameters on it. Coaxial orbits have intersecting orbits and non-intersecting orbits, as shown in Figure 3.

![Figure 3. Transitions between reverse coaxial orbits: (a) - between intersecting orbits; (b) - between non-intersecting orbits.](image)

3.1.1. According to intersecting coaxial orbits. Change the altitude of the apogee of the orbit, calculate the velocity impulses required for the orbital transfer along the track T1 and the track T2, and obtain the table 1.

| Orbital parameters | \( h_p = 200km \) | \( h_p = 200km \) | \( h_p = 200km \) | \( h_p = 200km \) |
|--------------------|-----------------|-----------------|-----------------|-----------------|
| Eccentricity       | 0.0150          | 0.0187          | 0.0223          | 0.0259          |
| \( \Delta V_{r1}^{T1} / (m/s) \) | 58.8118         | 73.3728         | 87.8769         | 102.3240        |
| \( \Delta V_{t1}^{T1} / (m/s) \) | -57.0746        | -70.6836        | -84.0401        | -97.1494        |
| Difference,\%      | 2.9538          | 3.6652          | 4.3662          | 5.0571          |
| \( \Delta V_{r2}^{T2} / (m/s) \) | -56.6580        | -70.0452        | -83.1384        | -95.9456        |
| \( \Delta V_{t2}^{T2} / (m/s) \) | 59.2643         | 74.0803         | 88.8964         | 103.7125        |
| Difference,\%      | 4.6001          | 5.7608          | 6.9295          | 8.0951          |
| \( \Delta V_{r1} / (m/s) \) | 115.4698        | 143.4180        | 171.0153        | 198.2696        |
| \( \Delta V_{t2} / (m/s) \) | 116.3389        | 144.7640        | 172.9365        | 200.8619        |
| Difference,\%      | 0.7527          | 0.9385          | 1.1234          | 1.3075          |
When the perigee height is constant, the higher the altitude of the apogee, the greater the difference in velocity impulses required for the spacecraft to travel along the track T1 and the track T2. As shown in Table 1, the difference in individual speed pulses reaches several percentage points, while the total speed impulse is not much different, about one percentage point. Therefore, from a mathematical point of view, the optimal transfer orbit for fuel saving should be T1. From the engineering point of view, the T1 and T2 tracks are the best transfer orbit as long as the error is within a reasonable range.

Following the above process, when the apogee is unchanged, change the height of the perigee, and use the same method to get the table 2.

| Orbital parameters | $h_p = 200km$ | $h_p = 250km$ | $h_p = 300km$ | $h_p = 350km$ |
|--------------------|---------------|---------------|---------------|---------------|
| $h_a = 400km$      | 0.0150        | 0.0112        | 0.0074        | 0.0037        |
| $\Delta V_{T1}^1$ (m/s) | 58.8118      | 43.6961       | 28.8596       | 14.2962       |
| $\Delta V_{T2}^1$ (m/s) | -57.0746     | -42.7281      | -28.4334      | -14.1906      |
| Difference,%       | 2.9538        | 2.2153        | 1.4769        | 0.7384        |
| $\Delta V_{T1}^2$ (m/s) | -56.6580     | -42.4935      | -28.3290      | -14.1645      |
| $\Delta V_{T2}^2$ (m/s) | 59.2643      | 43.9457       | 28.9684       | 14.3228       |
| Difference,%       | 4.6001        | 3.4175        | 2.2569        | 1.1180        |
| $\Delta V_{T1}$ (m/s) | 115.4698     | 86.1896       | 57.1886       | 28.4607       |
| $\Delta V_{T2}$ (m/s) | 116.3389     | 86.6738       | 57.4017       | 28.5135       |
| Difference,%       | 0.7527        | 0.5618        | 0.3727        | 0.1855        |

As seen from Table 2, the higher the height of the perigee, the smaller the difference between the individual velocities pulses and the total velocity pulse. At this time, the difference between the individual velocities pulses and the total velocities pulses on the transfer track along T1 and T2 is small, so the track transfer can be achieved on the T1 and T2 tracks.

Based on the comprehensive analysis of Table 1 and Table 2, as the orbital eccentricity increases, the difference between the orbital transfer velocity pulses from T1 and T2 tracks becomes larger and larger. Therefore, when the eccentricity of the track is large enough, the transfer track T1 is selected as the optimum transfer trajectory. When the orbital eccentricity is small, the transfer window should be considered comprehensively.

3.1.2. According to non-intersecting coaxial orbits. Change the orbital parameters of the initial orbit and service orbit respectively, calculate according to the method of 3.1.1 to obtain Table 3. It can be clearly seen from Table 3 that the optimal transfer orbit should be the T2 track.

| Initial orbit parameters | $h_{p1} = 200km$ | $h_{p1} = 200km$ | $h_{p1} = 200km$ | $h_{p1} = 400km$ | $h_{p1} = 600km$ |
|--------------------------|------------------|------------------|------------------|------------------|------------------|
| $h_{a1} = 400km$         | $h_{a1} = 400km$ | $h_{a1} = 400km$ | $h_{a1} = 600km$ | $h_{a1} = 800km$ |                 |
| Service Orbit Parameters | $h_{p2} = 500km$ | $h_{p2} = 600km$ | $h_{p2} = 800km$ | $h_{p2} = 800km$ | $h_{p2} = 800km$ |
| $h_{a2} = 600km$         | $h_{a2} = 800km$ | $h_{a2} = 1000km$| $h_{a2} = 1000km$| $h_{a2} = 1000km$| $h_{a2} = 1000km$|
| $\Delta V_{T1}^1$, (m/s) | 411.6826        | 529.3062        | 646.9298        | 618.6202        | 592.3166        |
| $\Delta V_{T2}^1$, (m/s) | 228.2985        | 285.3731        | 399.5224        | 382.3729        | 366.4162        |
| $\Delta V_{T2}^2$, (m/s) | -227.5388       | -246.0116       | -239.4799       | -318.3964       | -389.3904       |
3.2. The effect of initial phase on optimal orbital maneuver

For spacecraft intersection problems, when the intersection time is given, different initial phases will affect the optimal transfer orbit. Table 4 gives the initial orbit and service orbit information and gives the initial phase and orbit intersection information.

| Parameters                  | Initial orbit | Service orbit |
|-----------------------------|---------------|---------------|
| \( H_{\text{min}} \) (km)  | 180           | 401           |
| \( H_{\text{max}} \) (km)  | 210           | 408           |
| \( \phi_{\text{prg}} \) (degree) | 20           | 102.7         |
| \( U_1 \) (degree)         | 60            | 5             |
| \( U_2 \) (degree)         | 60            | 210           |
| \( U_3 \) (degree)         | 60            | 355           |
| \( N_0 \) — Number of initial turns | 1            | 201           |
| \( N_{\text{ap}} \) — Timing time | 17           | 217           |

The simplest apsidal solution will be used[9], and according to the solution result draw the transfer orbit for different initial phases, as shown in Figure 4.

![Figure 4](image)

In the first case (Figure 4, a), the target spacecraft lags behind the active spacecraft. In order to reduce this lag, it is necessary to significantly increase the semi-major axis of the phase-modulated orbit, which will allow the target spacecraft moving in orbit with a smaller period to "catch up" the active spacecraft. This type of flight is not optimal because of the brake maneuver. On the other hand, due to the number of laps in flight, it is impossible to apply a reverse scheme to make the active spacecraft "catch up" with the target spacecraft.

In the second case (Figure 4, b), the active spacecraft is located behind the target spacecraft. The phase difference is within a reasonable range, and the three velocity pulses obtained are all acceleration pulses. This maneuvering scheme is the optimal maneuver scheme.

In the third case (Figure 4, c), the active spacecraft lags far behind the target spacecraft. In order to catch up with the target spacecraft within a given intersection time, it is necessary to further reduce the orbital height to obtain a track period with a small angle. Therefore, this transfer scheme has a brake pulse, which is not the optimal transfer scheme.
Based on the above analysis, it is found that the difference of the initial phase will cause the occurrence of the brake pulse during the determined intersection time. At this time, the maneuver scheme solved by the simplest apsidal solution is not the optimal maneuver scheme. Observing the calculation process, it is found that the scalar sum is added between the various speed increments in the simplest apsidal solution, which results in the limitation of satisfying the minimum total speed impulse when a certain speed is the braking speed.

4. Conclusions
For the coaxial orbit, the general equations established by the application find that when the initial orbit intersects with the service orbit, the velocities pulses of the active spacecraft from the apogee to the apogee and from the perigee to the perigee is not much different. However, when the orbital eccentricity is large enough, it is recommended to select a transfer orbit from apogee to apogee within a reasonable intersection window. When the initial orbit is not intersected with the service track, the transfer orbit from the perigee to the perigee is the best orbit. When the target spacecraft lags behind the active spacecraft given the constraints of the Rendezvous time, or the target spacecraft is far ahead of the active spacecraft, the transfer orbit solved by the simplest apsidal solution is not the optimal transfer orbit. For the above two cases, the solution of the optimal orbit needs further study.

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