Transportation Cost Effective named Maximum Cost, Corresponding Row and Column minima (MCRCM) Algorithm for Transportation Problem

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Abstract

Transportation model provides a powerful framework to meet the Business challenges. In highly competitive market the pressure is increasing rapidly to the organizations to determine the better ways to deliver goods to the customers with minimum transportation cost. In this paper we proposed a new algorithm based on Least Cost Method (LCM) for finding Initial Basic Feasible Solution (IBFS) to minimize transportation cost. Our proposed algorithm provides a IBFS which is either optimal or near to the optimal value with minimum steps comparatively better than those obtain by traditional algorithm or method. For the validity of this algorithm we considered a numerical transportation problem and comparative study has been made minimum cost with graphically.

Keywords: Transportation Cost, Least Cost Method, Supply, Demand, Initial Basic feasible Solution, Optimum solution.

I. Introduction

The Transportation Problem (T.P.) is a special type of linear programming problem. In the 1920s A.N. Tolstoi was one of the first to study the transportation problem mathematically. In 1930, in the collection Transportation Planning Volume I for the National Commissariat of Transportation of the Soviet Union, he published a paper Methods of Finding the Minimal Kilometrage in Cargo-transportation in space. Tolstoi [XIII] illuminated his approach by applications to the transportation of salt, cement, and other cargo between sources and destinations along the railway network of the Soviet Union. Hitchcock [VIII] worked on the distribution of a production from several sources to numerous localities. Hadley [VI] also included transportation problem in his book: Linear Programming. Lee [XI] and others used goal programming to solve transportation problem. Mackinnon & James developed an algorithm for the generalized transportation problem. Moore et-al performed analysis of a transshipment problem with multiple conflicting objectives. Kwak [X] developed
a goal programming model for improved transportation problem solutions, followed by Kvanli [IX]. It deals with the situation in which a particular commodity shipped from sources to destinations in such a way that the transportation cost is minimum, while satisfying both the supply limits and the demand requirements.

Charnes and Cooper [III] mentioned about transportation in their book – Management Models and Industrial Applications of Linear Programming. Gupta et al. [IV] established a sufficient condition for a paradox in a linear fractional transportation problem with mixed constraints. Adlakha and Kowalski derived a sufficient condition to identify the cases where the paradoxical situation exists. Adlakha et al. proposed a heuristic method for solving TPs with mixed constraints which is based on the theory of shadow price. In the heuristic algorithm for an more-for-less criteria (MFL) solution in Adlakha et al, Vogel Approximation Method (VAM) and Modified Distribution (MODI) method were used.

Goyal[V] worked on improving VAM for unbalanced transportation problem. Kwak & Schniederjans framed goal programming solutions to transportation problem with variable supply and demand requirement. R.K. Ahuja [I] developed an algorithm for minimax transportation problem. In the same Romero has done a survey of generalized goal programming also Currin worked on the transportation problem with inadmissible routes. Transportation problem also solved by Least cost method (LCM), Column Minima (CM), Row Minima (RM) and North West Corner method (NWCM) etc.

In this paper we proposed a new algorithm based on Least Cost Method (LCM) for finding Initial Basic Feasible Solution (IBFS) to minimize transportation cost. Our proposed algorithm provides a IBFS which is either optimal or near to the optimal value with minimum steps comparatively better than those obtain by traditional algorithm or method. For the validity of this algorithm we considered a numerical transportation problem and comparative study has been made with graphically.

II. Transportation Networks and Mathematical Formulation of Transportation Problem as Linear Programming Problem

![General transportation problem](image)

Fig 1: General transportation problem is represented by the network (Operations Research - V K Kapoor, Page-5.1)
There are m sources and n destinations, each represented by a node. The lines linking the sources and destinations represent the routes between the sources and the destinations. The line (i,j) joining source i to destination j carries two information’s

1. The transportation cost per unit, \( c_{ij} \)
2. The amount transported, \( x_{ij} \)

The amount of supply at source \( i \) is \( a_i \) and the amount of demand at destination \( j \) is \( b_j \). The objective of the model is to determine the unknowns \( x_{ij} \) that will minimize the total transportation cost while satisfying all the supply and demand restrictions.

### III. Mathematical form of Transportation Problem as L.P. Problem:

Here the transportation problem can be stated as a linear programming problem as:

Minimize total cost

\[
Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}
\]  

Subject to \( \sum_{j=1}^{n} x_{ij} = a_i \) for \( i = 1, 2, ..., m \)

\( \sum_{i=1}^{m} x_{ij} = b_j \) for \( j = 1, 2, ..., n \)

And \( x_{ij} = 0 \) for \( 1, 2, ..., m \), for all \( j = 1, 2, ..., n \)

The transportation model can also be portrayed in a tabular form by means of a transportation table, shown in Table 1:

|       | \( D_1 \) | \( D_2 \) | ... | \( D_n \) |
|-------|----------|----------|-----|----------|
| \( S_1 \) | \( C_{11} \) | \( C_{12} \) | ... | \( C_{1n} \) |
| \( S_2 \) | \( C_{21} \) | \( C_{22} \) | ... | \( C_{2n} \) |
| \( S_m \) | \( C_{m1} \) | \( C_{m2} \) | ... | \( C_{mn} \) |

**Table 1**: Transportation Cost table

### IV. Proposed algorithm named Maximum cost, corresponding row and column minima (MCRCM) method

This new algorithm for solving transportation problem which is based on Least Cost Method (LCM) and it can be served as an important tool for the decision makers when they are handling various types of logistic problems of transportation problems.
Algorithm:

**Step 1.** Construct the transportation table from given transportation problem.

**Step 2.** Identify a maximum cost value on the transportation table. If there are two or more maximum equal costs, select any one of the maximum cost arbitrarily.

**Step 3.** Find a minimum cost value on that corresponding row and column of maximum cost value. Allocate minimum value of supply/demand on the cell. If there are two or more minimum equal costs, select the row and the column corresponding to the lower numbered row.

**Step 4.** Continue this until the maximum cost value is allocated or crossed.

**Step 5.** Now select next maximum cost value and repeat step (2) to step (6) until and unless all the demand are satisfied and all the supplies are exhausted.

Now for the validity of this algorithm we considered a numerical transportation problem and solved by different existing method and proposed algorithm then comparative study has been made with graphically.

For the validity of this method we considered an example and solved by using our proposed method an existing North-West Corner Method, Least Cost Method and Vogel’s Approximation Method then compared the result with graphically and analysis.

V. Numerical Example

Find an initial basic feasible solution of transportation problem of which transportation cost per unit of production from each warehouse to different stores are given in the following table:

| warehouse | 1  | 2  | 3  | 4  | 5  | Supply |
|-----------|----|----|----|----|----|--------|
| A         | 20 | 18 | 18 | 21 | 19 | 100    |
| B         | 21 | 22 | 23 | 20 | 24 | 125    |
| C         | 18 | 19 | 21 | 18 | 19 | 175    |
| Demands   | 60 | 80 | 85 | 105| 70 | 400(Total) |

Proposed Method: Maximum cost and corresponding row and column minima method Solution:

Identify a maximum cost value on the transportation table. Here maximum number is 24. Then select minimum cost value on that corresponding row and column and that is $x_{35} = \min(175, 70) = 70$ and allocate it in the cell (2,4). Therefore the demand of destination is satisfied completely and hence cross off the other cell of 5th column.
Then the maximum cost value is 23 and here we select 1 at 3rd column and allocate that cell. Set \( x_{13} = (100,85) \) = 85. Therefore the demand of 3 is exhausted and hence cross off the 3rd column.

Applying the same technique in the reduce cost matrix and determine the summation of costs each uncrossed row and column. We will allocate and cross off other cell.

| warehouse | 1 | 2 | 3 | 4 | 5 | Supply |
|-----------|---|---|---|---|---|--------|
| A         | 20| 18| 18| 30| 21| 19 60  |
| B         | 21| 20| 22| 23| 20| 105 24|
| C         | 18| 40| 19| 80| 21| 55  18|
| Demands   | 60| 80| 85| 105| 70| 400(Total) |

Table 2: Maximum cost and corresponding row or column minima method

Total Cost : \((18*30) +(19*70) +(21*20) +(20*105) +(18*40) + (80*19)+(21*55)\) = 7785

VI. Different Existing Methods bellow

North-West Corner Method:

| warehouse | 1 | 2 | 3 | 4 | 5 | Supply |
|-----------|---|---|---|---|---|--------|
| A         | 20| 60| 18| 40| 21| 19 100 |
| B         | 21| 22| 40| 85| 20| 24 125|
| C         | 18| 19| 21| 18| 105| 19 70 175|
| Demands   | 60| 80| 85| 105| 70| 400(Total) |

Table 3: Basic Feasible Solution using North-West Corner Method

Total Cost : \((20*60) +(18*40) +(22*40) +(23*85) +(21*0) + (18*105)+(19*70)\) = 7975
Least Cost Method:

| warehouse | 1 | 2 | 3 | 4 | 5 | Supply |
|-----------|---|---|---|---|---|--------|
| A         | 20| 18| 18| 21| 19| 100    |
| B         | 21| 22| 23| 20| 24| 125    |
| C         | 18| 60| 19| 21| 18| 175    |

Demands: 60 80 85 105 70 400(Total)

Table 4: Basic Feasible Solution using Least Cost Method

Total Cost: \((18 \times 80) + (18 \times 20) + (23 \times 65) + (24 \times 60) + (18 \times 105) + (19 \times 10) = 7895\)

Vogel’s Approximation Method:

| warehouse | 1 | 2 | 3 | 4 | 5 | Supply |
|-----------|---|---|---|---|---|--------|
| A         | 20| 18| 18| 21| 19| 100    |
| B         | 21| 22| 23| 20| 24| 125    |
| C         | 18| 60| 19| 21| 18| 175    |

Demands: 60 80 85 105 70 400(Total)

Table 5: Basic Feasible Solution Using Vogel’s Method

Total Cost: \((18 \times 85) + (19 \times 15) + (22 \times 80) + (18 \times 60) + (24 \times 45) + (18 \times 105) + (19 \times 10) = 7815\)

Optimality Test:

| warehouse | 1 | 2 | 3 | 4 | 5 | Supply | \(u_i\) |
|-----------|---|---|---|---|---|--------|--------|
| A         | 20| 18| 18| 21| 19| 100    | -1     |
| B         | 21| 22| 23| 20| 24| 125    | 3      |
| C         | 18| 60| 19| 21| 18| 175    | 0      |

Demands: 60 80 85 105 70 400(Total)

| \(v_j\) | 18| 19| 17| 19 |

Table 6: Optimal table for optimum Solution
Total Cost : $(18 \times 15) + (18 \times 85) + (22 \times 20) + (20 \times 105) + (19 \times 45) + (18 \times 60) + (19 \times 70) = 7605$

VII. Comparison of Transportation Cost Obtained by Different Methods

Table Shows Comparison of total cost of transportation problem Obtained from various methods.

| Problem | Proposed Algorithm | NWCM | LCM | VAN | Optimal |
|---------|--------------------|------|-----|-----|---------|
| 1       | 7785               | 7975 | 7895| 7815| 7605    |

Graphical Representation for the Cost variation of this Transportation Problem by using Different existing method and our proposed algorithm with optimal cost shown in bellow:
VIII. Conclusion

This article deals an alternate algorithms for which obtaining an IBFS for the minimization of transportation cost is illustrated numerically for all kinds of Transportation Problem. It is observed that proposed i.e Maximum cost, corresponding row and column minima (MCRCM) algorithm is so simple and it provides comparatively a better IBFS solution than those obtained by the traditional algorithms which is either optimal or near to optimal solution with step minima which is helpful for any business organization to reduce their transportation cost. For the further research any one can one can extend the proposed methods in several way.

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