Inclusive spectra of hadrons created by color tube fission 1. Probability of tube fission

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Abstract

The probability of color tube fission that includes the tube surface small oscillation corrections is obtained with pre-exponential factor accuracy on the basis of previously constructed color tube model. Using these expressions the probability of the tube fission in n point is obtained that is the basis for calculation of inclusive spectra of produced hadrons.
1 Introduction

In the previous papers [1-4] the properties of color tubes and their fission are considered on the basis of a semi-phenomenological effective lagrangian of (1+1)-dimensional theory of two scalar fields $r$ and $\phi$ describing the simultaneous change of color tube chromoelectric field and radius [1]. From the point of view of our theory a color tube is a soliton kink $(K)$+antikink $(\bar{K})$-like solutions of field equations. The color charges are arranged in walls $K$ and $\bar{K}$ that are moving apart in opposite directions with large momenta $P_K$ and $P_{\bar{K}}$. The color tube is a metastable state and it is ”hadronized” by fission to primary hadrons, which decay into observable particles.

We are interested in inclusive spectra (IS) of primary hadrons created by fission of a tube with small surface oscillations. Our approach to the tube fragmentation into hadrons is close to Artru- Menessier model [5](as well as other models [6-9] based on Artru - Menessier approach). From the point of view of our effective 1+1 dimensional theory a color tube is a kink and an antikink ($K$ and $\bar{K}$) that are moving apart in opposite directions and between which the color tube is formed. The breaking of the tube is a transition from
the state of a pair \((K, \bar{K})\) to a state\((K, K_1, \bar{K}, \bar{K}_1)\) that is degenerate with it and such that, between extra pair \((K_1, \bar{K}_1)\) that have been formed inside the tube, we have a true vacuum state (the color field is equal to zero).

At each stage of hadronization process the tube splits into pieces with arbitrary masses as far as the distance between the \(K\) and \(\bar{K}\) walls of piece becomes of the order of thickness of the wall. This causes growth of kink mass and rapid decrease of the probability of tube fission and therefore the hadronization process stops. The produced pieces form the primary hadrons that can be attributed to stable hadrons (pions and kaons) as well as hadron resonances.

We limit ourselves by particles with light cone momentum \(P_+ = E + P\) small in comparison with the initial light cone momentum of the tube \(P_{+0}\). Generally the procedure of IS calculation is the following. We must integrate over the probability of tube fission at \(n(n \geq 2)\) space-time points that formed the pieces with given light cone momenta over all allowed positions of fission space-time points [5,9,10].

The basic quantity for calculation is the probability of tube fission as a function of fission point \(P = P(\eta, \tau)\) where \(\eta\) and \(\tau\) are the dimensionless space and time coordinates of fission point. We shell use the probability \(P\)
that includes the tube surface small oscillation corrections.

In our tube model there are two kinds of surface oscillations with large and small quanta masses that play different roles in the tube formation and subsequent evolution [1-4]. The large-mass oscillations play essential role in the formation of color tube [2]: they lead to the breaking of the tube (with the probability $P \approx 1$) that just begins grow. Only small amplitude of a small mass oscillations remain in created pieces [2]. Fission probability of such a tube is small and accordingly its evolution time is large. The tube grows lengthwise and its length become much larger then radius.

Small mass oscillations produce corrections to exponential factor of fission probability of long tube that are proportional to oscillation amplitude squared

$$P(\eta, \tau) \sim \exp[-\pi M^2/\rho + a^2 D(s)].$$  \hspace{1cm} (1)

where $D(s)$ is the function which explicit form depends on initial shape of color tube. Here $s = \tau^2 - \eta^2$ and $M$ and $\rho$ are the dimensionless kink mass

\textsuperscript{1}Perhaps small jets that were observed in high energy hadroproduction experiments are mainly produced by this way. Hadrons in such a jets are created by fission of sufficiently short pieces produced in early stage of tube formation via catastrophic breaking of growing tube.
and tube tension respectively.

For IS calculation we need to more accurately calculate the fission probability including the pre-exponential factor corrections. Therefore before going to calculate IS we discuss (in section 2) the general approach to pre-exponential factor calculation for tunneling transition process from false vacuum + small classical field to true vacuum and calculate a fission probability including the tube surface oscillations corrections with pre-exponential factor accuracy (section 3). In Section 4 we obtain a general formula for tube breaking in $n$ different space-time points that is leading to production of $m$ pieces with lengths $l_1, l_2, \ldots l_m$ respectively. Finally, we summarize the results in Conclusions.

2 The probability of tunneling transition with pre-exponential accuracy

Here we shall explore the pre-exponential factor in the probability $P$ (per unit time and per unit length) of transition false vacuum + classical field $\rightarrow$ true vacuum in Minkowski space.
Following the general approach of Ref.[12] we consider the Lagrangian

\[ L(A) = (1/2)(\partial_\mu A)^2 - U(A), \]

where \( A(x,t) \) is the field variable, \( \mu = 0,1; \partial_0 = \partial/\partial t, \partial_1 = \partial/\partial x; \) the metric is (+,−). We denote the initial state false vacuum and classical field variables respectively by \( A_1 \) and \( A_c \) and final state true vacuum field by \( A_0 \).

The tunneling transition in which we are interested takes place from the state \( A_i = A_1 + A - c \) into such state \( A_f \) that for \( x < x_L \) and \( x > x_R \) we have \( A = A_i \) and for \( x_L < x < x_R \) the field \( A = A_0 \). We assume that \( A_c \) is the field of small classical oscillations and express it in the form

\[ A_c = \sum a_n \Theta_n(x_0,t_0), \]

where \( a_n \gg 1 \) and \( \Theta_n \) are the amplitudes and modes of normal oscillations, respectively.

As it was shown in Ref.[12] the "most probable escape path" (MPEP) in the function space connecting from \( A_i \) to \( A_f \) can be described by \( A(x,t) = f(x,\lambda(t)) \) where \( \lambda(t) \) is the function of time. To obtain the solution of tunneling problem the pair \( (\lambda, \dot{\lambda} = \partial_0 \lambda) \) must be considered as the dynamical variables and the field functional integral should be treated as a path integral over functions \( \lambda \).
Let us consider first the case when all $a_n = 0$. As it was shown in Ref.[2] the proper choice for collective variable $f(x, \lambda)$ is the kink-antikink $(K, \bar{K})$ configuration solution of field equations with $K$ and $\bar{K}$ located at $\Lambda_K$ and $\Lambda_{\bar{K}}$, moving with velocities $\dot{\Lambda}_K$ and $\dot{\Lambda}_{\bar{K}}$ respectively [2]

$$f(x, \lambda) = A_{K, \bar{K}} = K\left(\frac{x - \Lambda_K}{(1 - \Lambda_K^2)^{1/2}}\right) + K\left(\frac{x - \Lambda_{\bar{K}}}{(1 - \Lambda_{\bar{K}}^2)^{1/2}}\right) + C. \quad (4)$$

It must be noted that $A_{K, \bar{K}} = 0$ inside the interval $(\Lambda_K, \Lambda_{\bar{K}})$ and $A_{K, \bar{K}} = 1$ far away from this region. It is obvious that the MPEP parameter $\lambda$ coincide with $\Lambda_{\bar{K}}$ at $\lambda > x_0$ and with $\Lambda_K$ at $\lambda < x_0$ where $x_0$ is the center of the $K, \bar{K}$ pair. Thus the transition amplitude for tunneling process can be written in the form

$$< A_f | A_i > = \int D\lambda(t) \exp[-i \int dt L_{eff}(\lambda(t))] \quad (5)$$

where

$$L_{eff} = \int dx (L[A_{K, \bar{K}}] - L[A_i]) \quad (6)$$

is the effective Lagrangian of dynamical variable $\lambda(t)$ that describes the tunneling process.

It is easy to verify that in the thin wall approximation (i.e. when the sizes of $K$ and $\bar{K}$ are much less then $2M/\rho$) for $L_{eff}$ we obtain

$$L_{eff} = -M[(1 - \dot{\Lambda}_K^2)^{1/2} + (1 - \dot{\Lambda}_{\bar{K}}^2)^{1/2}] + \rho(\dot{\Lambda}_{\bar{K}} - \Lambda_K) \quad (7)$$
where $M$ is the mass of $K$ and $\bar{K}$,

$$M = \int dx (\partial_\lambda A_K) \bar{K}^2 \bigg|_{\lambda=\Lambda_K;\bar{\Lambda}_{\bar{K}}} \tag{8}$$

The first term in (7) is the Lagrangian of the free motion of $K$ and $\bar{K}$ with velocities $\dot{\Lambda}_K$ and $\dot{\Lambda}_{\bar{K}}$, respectively, and the second term is the contribution to the volume energy of the region between $K$ and $\bar{K}$ from $U(A_K,\bar{K}) - U(A_i)$, $\rho$ is the (constant) energy density per unit length. Now we proceed to calculate the $\lambda(t)$ path-integral in the quadratic approximation. According to the semiclassical ideology [12-21] the quadratic approximation for tunneling amplitude path-integral (3) can be obtained by expanding of the effective action around the MPEP that is classical solution of equation of motion for $\lambda_c(t)$ for imaginary time $\tau = -it$ [18]. Thus we have

$$\lambda(t) = \lambda_c(\tau) + \lambda, \tag{9}$$

where $\lambda_c$ obeys the equations

$$- M \frac{d}{d\tau} (\dot{\Lambda}_K/(1 + \dot{\Lambda}_K^2)^{1/2}) = \rho,$$

$$M (\frac{d}{d\tau} (\dot{\Lambda}_{\bar{K}}/(1 + \dot{\Lambda}_{\bar{K}}^2)^{1/2})) = \rho \tag{10}$$

with the initial conditions [2]

$$\Lambda_K = \Lambda_{0K} = M/\rho, \tag{11}$$
\[ \dot{\Lambda}_K = 0, \quad (12) \]
\[ \Lambda_{\bar{K}} = \Lambda_{0\bar{K}} = -M/\rho, \quad (13) \]
\[ \dot{\Lambda}_{\bar{K}} = 0, \quad (14) \]

at \( \tau = 0 \).

Now after simple calculations in quadratic approximation for amplitude

\[ | < A_f | A_i > | \text{ we obtain} \]

\[ | < A_f | A_i > | = |I| \exp(-S_c), \quad (15) \]

where \( S_c \) is the imaginary part of “classical” action for MPEP \( \lambda_c(\tau) \)

\[ S_c = \pi M^2 / 2\rho, \quad (16) \]

and

\[ I = \int \mathcal{D}\sigma \exp\left(-\frac{M}{2}\right) \int_{\rho\tau_K/M}^\rho dz (1 - z^2)^{1/2} \left( \frac{\partial \sigma}{\partial z} \right)^2, \quad (17) \]

with boundary conditions for \( \sigma \)

\[ \sigma(\rho\tau_K/M) = \sigma(\rho\tau_{\bar{K}}/M) = 0. \quad (18) \]

The time of the motion \( K \) and \( \bar{K} \) under barrier \( \tau_K \) and \( \tau_{\bar{K}} \) are given by the relations [2]

\[ \tau_K \approx M/\rho; \quad \tau_{\bar{K}} \approx -M/\rho; \quad (19) \]
and we obtain for $I$ the following simple expression

$$I = \int D\sigma \exp\left(-\int_{-1}^{1} dx (1 - x^2)^{-1/2}\sigma H \sigma\right)$$  \hspace{1cm} (20)$$

where

$$H = (1 - x^2)^{1/2}\left(\frac{d}{dx}\right)(1 - x^2)^{1/2}\left(\frac{d}{dx}\right)$$  \hspace{1cm} (21)$$

The path integral $I$ can be easily calculated by generalized $\zeta$-function method [10,18] and after extracting zero mode contribution [12,13,19] we obtain

$$|I| = \left(\frac{V \rho}{2\pi}\right)^{1/2}$$  \hspace{1cm} (22)$$

where $V$ is the volume of space-time.

Thus for transition probability $P$ we obtain the following expression

$$P = \left(\frac{\rho}{2\pi}\right) \exp(-\pi M^2/2\rho).$$  \hspace{1cm} (23)$$

The above expression is closely related to the result previously obtained by Voloshin [21], who calculated transition probability by using the ”bubble” effective Lagrangian in Euclidean space. As we have seen the same expression can be derived also directly in Minkowski space although the Minkowski $L_{eff}$ differs from Euclidean $L_{eff}$. 

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Now we will show that the $L_{eff}$ have the form (22) for wide class of Lagrangians allowing a true and false vacuum states and $K$ and $\bar{K}$-like instanton solutions of field equations that connect the true vacuum to the false vacuum.

Let us consider the nonlinear lagrangian of fields $A_l$ ($l=1,2,...N$) that has the form

$$L = (1/2)F_{n,m}(A)(\partial_\mu A_n)(\partial_\mu A_m) + U(A), \quad (24)$$

where the matrix $F_{m,n}$ and potential $U(A)$ are the functions of fields $A_l$.

We assume existence of the extrema of $U(A)$ on sets of fields $A^0_n$ and $A^f_n$ that correspond to true and false vacua respectively and $K$ - and $\bar{K}$ -type solutions of field equations.

Without any loss of generality we can assume that $F_{m,n}$ is a diagonal matrix. Then functional integral measure for lagrangian (24) has the form

$$\mathcal{D}[A] = \prod_n \mathcal{D}A_n \Delta_F^{1/2} \quad (25)$$

where $\Delta_F$ is the determinant of matrix $F_{m,n}$.

Let us introduce new field variables $\varphi_n$ as follows

$$\varphi_n = \varphi_n(A) \quad (26)$$

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in such a way that the Jacobian of this variable transformation is proportional to \( \Delta_F^{-1/2} \). In these new field variables the functional integration measure has a very simple form

\[
D[\varphi] = \prod d\varphi
\]  

(27)

and for the Lagrangian we obtain the following expression

\[
L = \sum_{m,n} F_{m,n}(\varphi)(\partial_{\mu}\varphi_m)(\partial_{\mu\nu}\varphi_n) - U(A(\varphi)).
\]  

(28)

where

\[
F_{m,n}(\varphi) = \sum_{l,k} F_{l,k}(A)(\partial A_l/\partial \varphi_m)(\partial A_k/\partial \varphi_n)
\]  

(29)

From our assumption (26) it follows that the MPEP in the \( \varphi \)-function space is the \( K - \bar{K} \)-type solution of the field equations with the same \( \lambda(t) \) for all \( n \). \( \square \)

The "Lorentz invariance" tells us that the \( \varphi_{K\bar{K}}^n \) are dependent only on variable \( (x - \lambda)(1 - \dot{\lambda}^2)^{-1/2} \). Now substituting in (28) the new MPEP

\footnote{Here we are restricted by the case of one parameter \( \lambda(t) \). It is clear that if \( \lambda(t) \) for any \( A_n \) (and, consequently, for \( \varphi_n \)) differs from the others only inside of \( K \) or \( \bar{K} \) walls and in the used thin wall approximation we can restrict ourselves by one path integral variable \( \lambda(t) \). It is quite trivial to see that in the case of two or more different MPEP’s the transition probability is the sum of contributions of all MPEP’s.}
\( \varphi^{K\bar{K}} \) and passing to path integral over \( \lambda \) we obtain for \( L_{\text{eff}} \) in the thin wall approximation the same expression (7) with new definitions of \( M \) and \( \rho \)

\[
M = \int dx \sum_{kl} F_{kl}[\partial_{\varphi}^{K\bar{K}} / \partial x][\partial_{\varphi}^{K\bar{K}} / \partial x],
\]

\[
\rho = (\Lambda_{\bar{K}} - \Lambda_{K})^{-1} \int dx [U(\varphi^{K\bar{K}}) - U(\varphi^{f})]
\]

where \( \varphi^{f} \) is set of \( \varphi \)-fields that correspond to set of \( A_{f} \)-fields.

It is obvious that repeating the above path integral calculation procedure in quadratic approximation we obtain for transition probability the same expression (23).

Up to here we have considered only transition from false vacuum to true vacuum. Now we return to the principal subject of our calculations: the probability of transition false vacuum+classical field \( \rightarrow \) true vacuum. The field configuration \( A_{K\bar{K}} \) which is degenerate with \( A_{i} = A_{1} = A_{c} \) can be approximated by

\[
A_{K\bar{K}}^{c} = A_{K\bar{K}} (1 + A_{c})
\]

where \( A_{K\bar{K}} \) is given by formula (1).

In with the choice of \( A_{K\bar{K}}^{c} \) in this form we must mention two facts. First, this approximation for \( A_{K\bar{K}} \) implies that we have fixed the variations of the form of \( K \) and \( \bar{K} \) beforehand, both in view of the effect of small classical
field and process of motion of $K$ and $\bar{K}$. In the following in the thin wall kink approximation we neglect in general the change of the form of $K$ and $\bar{K}$ (the corresponding corrections are small). Second, the motion of $K$ and $\bar{K}$ now is asymmetric relative to the point $x_0$, since, generally speaking, $\Theta(x, t)$ depends in a nontrivial way both on coordinates and time. Now proceeding as in Ref. [2] in the thin wall limit for the effective Lagrangian we obtain

$$L_{\text{eff}}^c = L_{\text{eff}} - \sum_n a_n^2 \int_{x_0-\Lambda K}^{x_0+\Lambda K} dx F_n(x, t),$$  \hspace{1cm} (33)$$

where $L_{\text{eff}}$ is the effective Lagrangian (7) of with $A_c = 0$, and second term in right hand side is the contribution to the volume energy of the region between $K$ and $\bar{K}$ from small classical oscillations of the field, and

$$F_n(x, t) = (1/2)[(\partial_\mu \Theta_n)^2 - \kappa_n^2 \Theta_n^2]$$  \hspace{1cm} (34)$$
is the Lagrangian of $n$-th normal mode of oscillations.

To calculate the effective action in the quadratic approximation we expand $L_{\text{eff}}^c$ around new MPEP, that is, classical solution of equation of motion for $L_{\text{eff}}^c$ for imaginary time $\tau$. Proceeding as in Ref.[2] after simple but slightly cumbersome calculations we obtain up to terms $\sim a^2$

$$S^c = S_0^c - (\rho/2) \int_0^{\pi} d\theta H,$$  \hspace{1cm} (35)$$
where

\[ H_0(\sigma, \tau) = \left( \frac{\partial \sigma}{\partial \theta} \right)^2; \]  

\[ H_1(\sigma, \theta, x_0, t_0) = \]  

\[-(\frac{\partial \sigma}{\partial \theta})^2 + \sigma^2 \sin \theta (\frac{\partial}{\partial x_0}) \Re F(X_0, T_0), \]  

\[ X_0 = x_0 + \frac{(M/\rho)}{\cos \theta}, \]  

\[ T_0 = t_0 + i(M/\rho)\cos \theta. \]

\[ S_0^c \] is the "classical" action for MPEP [2]:

\[ S_0^c = \frac{\pi M^2}{2 \rho}(1 + \frac{a^2}{\rho}D_1(x_0, t_0)) \]

and the function \( D_1 \) is defined by the integrals

\[ D_1(x_0, t_0) = \]  

\[ \frac{2}{\pi} \Re \left[ \int_0^1 dz (1 - z^2)^{-1/2} \left[ F_n(X_+(z); T(z)) + F_n(X_-(z); T(z)) \right] \right] + \int_0^1 dz \int_{(1-z^2)^{1/2}}^{(1-z^2)^{1/2}} d'z' F_n(Y(z'), T(z)) - \int_{-1}^1 dz F_n(Y(z), t_0) \]

where

\[ X_\pm(z) = x_0 \pm \frac{(M/\rho)}{1 - z^2} \]  

\[ T(z) = i(M/\rho)z + t_0, \]  

\[ Y(z) = x_0 + (M/\rho)z. \]
Thus the probability of tunneling transition has the form

\[ P_c = \frac{\rho}{2\pi \Delta} \exp 2S^c_0 \] (45)

where \( \Delta = \det' H_0 / \det' (H_0 + a^2 H_1 / \rho) \) and \( \det' \) denotes a functional determinant with vanishing eigenvalues removed.

Using generalized \( \zeta \)-function method [18,20] we obtain for \( \Delta \) the expression

\[ \Delta = \exp(a^2 D_2(x_0, t_0) / \rho), \] (46)

where

\[ D_2(x_0, t_0) = \rho a^{-2} (d/ds) \zeta_c(s = 0), \] (47)

\[ \zeta_c(s) = \sum [(n^2 + a^2 U_n(x_0, t_0) / \rho)^{-s} - n^{-2s}] \approx (a^2 / \rho) \sum U_n n^{-2s-1}. \] (48)

\[ U_n(x_0, t_0) = \text{Re} \int_0^\pi d/\theta H_1((2/\pi)^{1/2} \sin n\theta, \theta, x_0, t_0). \] (49)

Substituting (48) in the (15) we obtain

\[ P_c = \frac{\rho}{2\pi} \exp(-\pi M^2 / \rho + a^2 D(x_0, t_0) / \rho), \] (50)

where

\[ D(x_0, t_0) = (\pi M^2 / \rho) D_1(x_0, t_0) + D_2(x_0, t_0). \] (51)

This is the expression for tunneling probability of transition from false vacuum + small classical field into true vacuum. From (51) we see that the
presence of small classical field in initial state does not change the general form of vacuum tunneling probability. The only influence is an appearance of the new factor which depends on the classical field and makes the resulting probability to be a function of point of the transition origin. We do not enter here into a discussion of physical effects arising from small classical field (see discussion in Ref.[2]) and only note that the field appearance can stimulate a ”catastrophic” (with $P_c \approx 1$) transitions.

3 The probability of color tube fission

The main purpose of this section is to obtain the probability $P$ of color tube fission including the small surface oscillation effects with pre-exponential factor accuracy. In other words, we want to calculate the fission probability $P$ per unit length and per unit time of a color tube which has the radius $r \neq r_1$ and color electric field $E = E_1$ and length $\Delta_1$ at the moment of its formation, where $r_1$ and $E_1$ are the stationary color tube radius and electric field respectively.

In order to calculate $P$ we use the previously constructed quasi-phenomenological model [1], based on (1+1)-dimensional theory of two scalar fields $r$ and $E$ de-
scribing the simultaneous change of the radius of a tube and a chromoelectric field.

To study the tube fission it is convenient to choose field variables $\phi = (4\pi\alpha)^{-1/2}E$ and $\chi = 2 \log(r/r_0)$ (where $\alpha = g^2/4\pi$ is the color interaction constant). Then the Lagrangian of the tube has the form [1]

$$L = \lambda_2 [ (\pi/2)(\partial_\nu \phi)^2 + (\alpha/2)\phi^2 (\partial_\nu \chi)^2 - (\epsilon^2)(\exp \chi + \phi^2 \exp -\chi) + \cos 2\pi\phi - 1]$$

where

$$\partial_0 = \partial/\partial \tau, \partial_1 = \partial/\partial \eta,$$

and the metric is $(+, -)$.

It should be emphasized that we are dealing with two scalar fields and, accordingly, the system of vacuum states of a Hamiltonian $H$ consists of pairs of fields $(\phi_n, \chi_n)$ that correspond to one eigenvalue $H_n$. Here we restrict ourselves to the first false vacuum state $(\phi_1, \chi_1)$ that corresponds to a quark tube.

As stated above in our model a color tube is kink and antikink $(K, \bar{K})$ that are moving apart in opposite directions and between which the fields $r$
and $\phi$ are nonzero and equal to values $r = r_1 + \delta r$ and $\phi = \phi_1 + \delta \phi$ where $r_1$ and $\phi_1$ are the fields that correspond to unstable vacuum and $\delta \phi$ and $\delta r$ are the small classical fields. The breaking of the tube is a transition from the state of pair $(K, \bar{K})$ to a state $(K, \bar{K}_1, K_1, \bar{K})$ which is degenerate with it and such that, between extra pair $(\bar{K}_1, K_1)$ that are formed inside the tube we have a true vacuum with $\phi_0 = 0$, $exp(\chi_0) = 0$, $\phi_0 exp(-\chi_0) = 0$ (Ref.[2])

Let us consider first the case of a long tube and pair $(K_1, \bar{K}_1)$ which is formed far from the ends of the tube. Then the effect of the ends can be neglected, the tube can be regarded as infinite, and one can confine oneself to the thin wall approximation for $K_1(\bar{K}_1)$.

As was shown in Ref.[2] the MPEP connecting initial and final states is the $(\bar{K}_1, K_1)$ configuration of fields $(\phi, \chi)$ parameterized by one function $\lambda(t)$. Following the procedure of Ref. [2] after some calculations we obtain (in the thin wall approximation) the effective Lagrangian of $\lambda$ in the form [2]

$$L_{e\!f\!f} = -M[(1 - \dot{\Lambda}_K^2)^{1/2} + (1 - \dot{\Lambda}_{\bar{K}}^2)^{1/2}] + \rho[\Lambda_{\bar{K}} - \Lambda_K]$$

$$- \sum_1^2 a_n^2 \int_{\eta_0 - \Lambda_{\bar{K}}}^{\eta_0 + \Lambda_K} d\eta F^{(n)}(\eta, \tau).$$

(54)

Here $\rho = \eta^2$ is the energy density in the tube,

$$M = 4\pi^{-1/2}[1 + (\alpha/\pi)Z']$$

(55)
is the renormalized (dimensionless) mass of the soliton, and

\[ F^{(n)} = 2^{-1} \left[ (\partial_\nu \Theta_n)^2 - \kappa_n^2 \Theta_n^2 \right] \]  \hspace{1cm} (56)

is the Lagrangian of small surface oscillations of the tube [2],

\[ \dot{\Lambda} = \partial_0 \Lambda. \]  \hspace{1cm} (57)

and \( a_n \) is the amplitude of \( n \)-th oscillation mode.

Note that the presence of two fields \( \chi \) and \( \phi \) in \( L_{\text{eff}} \) manifests itself in
the renormalization of the mass of the soliton and in the two contributions
\( \sim a_n^2 \) corresponding to the two kinds of small oscillations [2].

Now the path integral that define the transition amplitude can be cal-
culated in quadratic approximation just in the same way as it was made in
previous section and one obtains for the tube fission probability

\[ P = \left( \frac{\epsilon^2}{2\pi} \right) \exp\left( -\pi^2 M^2 / \epsilon^2 + \sum \left( \frac{a_n^2}{\epsilon^2} \right) D^{(n)}(\eta_0, \tau_0) \right) \]  \hspace{1cm} (58)

where term \( D(\eta_0, \tau_0) \) accounts for the small oscillations of a tube surface that
are created at the moment of tube formation.

The functions \( D^{(n)} \) contain the contributions of two different origin

\[ D^{(n)} = D_1^{(n)} + D_2^{(n)}. \]  \hspace{1cm} (59)
The part $D_1^{(n)}$ comes from the classical action for MPEP and has the form

$$D_1^{(n)}(x_0, t_0) =$$

$$(2/\pi) Re\left[\int_0^1 dz (1 - z^2)^{-1/2} (F_n(x_0 + (M/\rho)(1 - z^2)^{1/2}; i(M/\rho)z + t_0) +$$

$$F_n(x_0 - (M/\rho)(1 - z^2)^{1/2}; i(M/\rho)z + t_0)) +$$

$$\int_0^1 dz \int_{-(1-z^2)^{1/2}}^{(1-z^2)^{1/2}} dz' F_n(x_0 + (M/\rho)z', i(M/\rho)z + t_0) -$$

$$\int_{-1}^1 dz F_n(x_0 + (M/\rho)z, t_0)\right].$$

The second contribution $D_2^{(n)}$ comes from the determinant of operator that describe the quadratic corrections to MPEP action in path integral. For $D_2^{(n)}$ one obtains (see sec.2)

$$D_2^{(n)} = (d/dq)\zeta^{(n)}_c(q)|_{q=0}$$

(61)

where

$$\zeta^{(n)}_c(q) = \sum_k V_{kk}(\eta_0, \tau_0) k^{-2q-1},$$

(62)

and

$$V_{kk}(\eta_0, \tau_0) =$$

$$Re \int_0^{\pi} d\theta \left[-(2/\pi)k^2 \cos^2 k\theta + \sin^2 k\theta \sin \theta \partial_0\right]$$

(63)

$$F^{(n)}(\tau_0 + (M/\epsilon^2) \sin \theta \cos \theta / |\cos \theta|, i(M/\epsilon^2) \cos \theta + \tau_0).$$
As noted above the large-mass oscillations ($\Theta_1$) cause fission of tube in a very short time. Thus the construction of tube itself terminates after all high frequency oscillations have been ”used” on this fission and a tube has been formed in which only low-frequency interrelated oscillations of a radius and of an electric field $\delta r/r_1\sigma_2 \Theta_2$ and $\delta \phi/\phi_1\sigma_2 \Theta_2$ are kept. This means that on the tube which is formed, only those amplitudes of the oscillations of small mass $\kappa_2$ survive for which the probability of fission stimulated by these oscillations can no longer become $\sim 1$. It can be easily shown [2] that for sufficiently large $\tau_0$ and far from the edges of the tube with $r = r_1 + \delta r$ and $\phi = \phi_1$ at the moment of tube formation $\tau_0 = 0$, the field $\Theta_2$ is the function of $s = \tau_0^2 - \eta_0^2$ only. Accordingly, $D^{(2)}$ is the function of $s$ and one have for probability of a tube fission

$$P = (\epsilon^2/2\pi) \exp(-\pi M^2/\epsilon^2 + (a_2^2/\epsilon^2)D^{(2)}(s)). \quad (64)$$

So far we have considered one MPEP soliton or, equivalently, tube fission via quark-antiquark pair of one type creation inside the tube. In real world the tube fission take place via $q\bar{q}$ pairs of many flavors or various diquark pairs creation. In our tube model to each type of particle-antiparticle pair corresponds its own MPEP or, other words, its own $(K, \bar{K})$-soliton with
mass \( M_i \). This means that in the functional integral contribute many different MPEP’s and the total probability \( P \) is the sum of partial probabilities \( P_i \) of each type MPEP’s. Thus one obtains

\[
P = \sum P_i, \tag{65}
\]

\[
P_i = \left( \frac{\epsilon^2}{2\pi} \right) \exp\left( -\pi M_i^2 / \epsilon^2 + (a_2^2 / \epsilon^2) D_i^{(2)}(s) \right). \tag{66}
\]

It is evident that the sum (65) is dominated by the term of smallest kink mass. However to calculate the inclusive spectra of primary hadrons of different types we need also other partial probabilities.

When the distance of the kink wall from the end of the tube becomes of the order of the thickness of the wall, the influence of the end of the tube becomes important and, strictly speaking, the thin wall approximation for the \( K_1(\bar{K}_1) \) that has been formed near the end of the tube is no longer applicable. It was shown in Ref.[3] that the mass of \( K_1 \) increases with the decrease of the distance between the world point of the center of fission \((\tau_0, \eta_0)\) and the end of the tube, the probability of the fission should decrease near of the tube end. Thus, even if the previous fission occurred at a distance \( l = M/\epsilon^2 \) from the tube end, first the fragment that has been formed ”expands” in such a way that its length become greater than \( 2M/\epsilon^2 \) and only then does the real
chance for the tube to break again.

4 The probability of tube fission at \( n \) points

In our model the tube is formed by the color electric field of the moving (in opposite directions in \( CM \) frame) \( K \) and \( \bar{K} \). The motion of \( K \) and \( \bar{K} \) is decelerated by the tube tension and they will oscillate back and forth just as the yo-yo relativistic string. After some time the tube will break into two parts by production of a pair of \( K \) and \( \bar{K} \) at the space-time point \((\tau_1, \eta_1)\). At later time another pair \( K, \bar{K} \) will be produced at \((\tau_2, \eta_2)\) and so on. The successive breaking of the tube form pieces of small length that are primary hadrons.

For further discussion it is useful to introduce light-cone variables \( u = \tau - \eta \) and \( v = \tau + \eta \) and consider the process of tube breaking in \((u, v)\) coordinates in \( CM \) frame.

We begin with the case of one type of kink. Let us consider first the probability of tube fission into two pieces i.e. let the \( dP(1) \) be the probability of tube breaking in only one world point. Any breaking in the point \((\tau_1, \eta_1)\) means that now we have two independently developing tubes. The
corresponding probability is the product of two factors: one is the probability \( \exp[-W(S_1)] \) that there is not any breaking before the world point \((1) = (u_1, v_1)\) or, in other words, inside the area \(S_1\), limited by the continuation of trajectories of created \(K\) and \(\bar{K}\) \((S_1\) is the area \((0, 1', 1, 1'')\) on Fig.1 or is the strip \(0 \leq u' \leq u_1, 0 \leq v' \leq v_1\) where

\[
W(S_1) = \left(\frac{1}{2}\right) \int_{S_1} du' dv' P((u' + v')/2; (u' - v')/2). \tag{67}
\]

The second factor

\[
dw(1) = \left(\frac{1}{2}\right) P((u_1 + v_1)/2; (u_1 - v_1)/2)du_1dv_1 \tag{68}
\]

is the probability of tube breaking in the point \((1)\).

Thus we have

\[
dP(1) = dw(1)e^{-W(1)} \tag{69}
\]

Now we pass to the fission at the points \((u_1, v_1) = (1)\) and \((u_2, v_2) = (2)\) (see fig.2). There are two different cases. The first one is that there are supplementary breaking between points \((1)\) and \((2)\) (or in the area \((2, D, 1, C)\)). Then the probability \(dP(2,0)\) is the product of:

(i) the probability \(dP(1)\) that the first breaking point is \((1)\);

(ii) the probability \(d\bar{P}(2) = dw(2)e^{W(S_2)}\), \((\text{where } S_2 \text{ is the area } (v_1, C, 2, v_2))\), that the point \((2)\) is just the new breaking point [5,10].
Thus we have

\[ dP(1, 2) = dP(1) dP(2) = dw(1) dw(2) e^{-W(S)}, \] (70)

where \( S \) is the area \((0, u_1, 1, C, 2, v_2)\) or sum of strips \(0 \leq u' \leq u_1, 0 \leq v' \leq v_1\) and \(0 \leq u' \leq u_2, v_1 \leq v' \leq v_2\).

If the piece produced due the breaking \((1)\) and \((2)\) is not broken and develops like the "yo-yo" string then, we have an additional factor \(\exp[-W(S_3)]\) where \(S_3\) is the area \((C, 1, D, 2)\), that is, the probability that there is no any breaking in the \((C, 1, D, 2)\). Finally we have the same expression (70) with the only replacement \(S \rightarrow S = (0, 1, D, 2) = [0 \leq u' \leq u_1, 0 \leq v' \leq v_2]\).

Now it is easy to see that the probability of \(n\) successive breaking can be written in the form

\[ dP(1, 2, \ldots, n) = \prod_{i=1}^{n} dw(i) e^{-W(S_n)}, \] (71)

where the integration area is limited by the continuation of trajectories of created \(K\) and \(\bar{K}\)-s. For small kink mass the integration area \(S_n\) is the sum of successive strips (see fig.3)

\[ [0 \leq u' \leq u_1, 0 \leq v' \leq v_1; 0 \leq u' \leq u_2, v_1 \leq v' \leq v_2; \ldots; 0 \leq u' \leq u_n, v_{n-1} \leq v' \leq v_n] \] (72)
For each stable piece produced by \( i \)-th and \( i+1 \)-th adjacent breaking the supplementary strip \([u_{i=1} \leq u' \leq u_i, v_i \leq v' \leq v_{i+1}]\) must be added.

For many types of kinks repeating the above arguments we obtain for the probability of \( n \) successive breaking when at point (1) is created \( K\bar{K} \) - pair of type \( a_1 \), at point (2) - of type \( a_2 \) etc. the following expression

\[
dP(1, a_1; 2, a_2; \ldots, n, a_n) = \left[ \prod_{i=1}^{n} dw(i, a_i) \right] e^{-W_t},
\]

where \( dw(i, a_i) \) is the partial probability of tube breaking at the point \((i)\) due the creation \( K\bar{K} \) pair of type \( a_i \) given by the expression \((67)\) and \( W_t \) is calculated using the formula \((68)\) with evident replacement \( p \to \) [total probability] = \( p_t \) with \( p_t \) given by Eq. \((65)\).

### 5 Conclusion and outlook

The above expressions enable us to calculate the exclusive and inclusive spectra of primary hadrons, i.e., the spectra of pieces of given length. The procedure of calculation is the following. We must integrate the probability of fission that form the pieces with momenta \( p_1, p_2, \ldots, p_n \) over all allowed positions of fission space-time points. It is convenient to use the the light-cone
coordinates \(u\) and \(v\) and light-cone momenta \(p_{\pm,j} = p_j \pm E_j\) and center-of-mass system of parent tube. Since the light-cone momentum of piece produced by pair \((K_j, \bar{K})\) is given by

\[
p_{+j,l} = \epsilon^2 (u_j - u_l)
\]
\[
p_{-j,l} = \epsilon^2 (v_j - v_l)
\]

the requirement that produced piece has light-cone momentum equal to \(p_{\pm,j,l}\) means that the difference \(\delta u_{j,l}\) must be equal to \(\tilde{p}_{j,l} = p_{+,j,l}/\epsilon^2\). We can take this into account by introducing into integrand the \(\delta(u_j - u_l - \tilde{p}_{j,l})\) and \(\delta(v_j - v_l - m^2_{j,l,t}/\tilde{p}_{j,l})\), where \(m^2_{j,l,t}\) is the piece transversal mass, for each piece.

Then we choose such an ordering of the fission points that corresponds to the momentum space region we are interested in. (It must be noted that each ordering of fission points corresponds to its specific region of momentum space.)

The fission point ordering can be easily taken into account by introducing into the integrand \(\theta\)-functions for \(u\) and \(v\) coordinates. Now the integration area is the \(2n\)-cube \((0 \leq u_1 \leq \tilde{P}, 0 \leq v_1 \leq \tilde{P}_0; \ldots, 0 \leq u_n \leq \tilde{P}_0, 0 \leq v_n \leq \tilde{P}_0)\) where \(n\) is the number of breaking points.

For exclusive spectra the integrand is the probability of \(n-1\) adjacent fission only. This means that all pieces (except two that contain the edges of
parent tube) have the common point of \((K, \bar{K})\) pair creation.

Rather more complicated situation is for an inclusive spectra. In this case we have distinct contributions that correspond to particles (pieces) of different adjacent ranges on the Field-Feynman terminology [22]. The \(k\)-particle inclusive spectra of primary hadrons are the sum of probabilities:

(i) \(P_{2k}\) of the tube breaking in the \(2k\) different points,
(ii) \(P_{2k-1}\) of the tube breaking in the \(2k-1\) different points when one wall of the piece is the edge of parent tube,
(iii) \(P_{2k-2}\) of the tube breaking in the \(2k-2\) different points when there are two pieces the one wall of each is the edge of parent tube. Thus the inclusive spectra are defined by integrals over the probability of at least \(m(m \geq n + 1)\) fission independent on there are or not any other fission. The probability of creation of pieces with common \((K, \bar{K})\) pairs to IS contribute as well as the cases when all or part of \((K, \bar{K})\) of adjacent pieces belong to different \((K, \bar{K})\) pairs (i.e. between these \(K\) and \(\bar{K}\) are any supplementary fission points).

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**FIGURE CAPTIONS** Fig.1. The one breaking point world diagram in $(t, x)$ and $(u, v)$ coordinates.

Fig.2. The two breaking of the tube in world points $(1) = (u_1, v_1)$ and $(2) = (u_2, v_2)$.

Fig.3. The integration area for $n$ breaking of the tube.
