An alternative gauged $U(1)_R$ symmetric model in light of the CDF II $W$ boson mass anomaly

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(Dated: December 15, 2022)

Abstract

We consider an explanation of CDF II $W$ boson mass anomaly by $Z - Z'$ mixing with $U(1)_R$ gauge symmetry under which right-handed fermions are charged. It is found that $U(1)_R$ is preferred to be leptophobic to accommodate the anomaly while avoiding other experimental constraints. In such a case we require extra charged leptons to cancel quantum anomalies and the SM charged leptons get masses via interactions with the extra ones. These interactions also induce muon $g - 2$ and lepton flavor violations. We discuss muon $g - 2$, possible flavor constraints, neutrino mass generation via inverse seesaw mechanism, and collider physics regarding $Z'$ production for parameter space explaining the $W$ boson mass anomaly.
I. INTRODUCTION

Precision measurements of electroweak observable are good test of the standard model (SM) and would provide a hint of beyond the SM. CDFII collaboration recently reported updated result of the SM charged-gauge boson (W boson) mass \[1\]

\[ m_W = (80.433 \pm 0.0064_{\text{stat}} \pm 0.0069_{\text{syst}}) \text{ GeV}, \]  

which deviates from the SM prediction by 7\(\sigma\), where the SM prediction indicates \( m_W = (80.357 \pm 0.006) \text{ GeV} \). The disagreement also appears to the previous global combination of data from LEP, CDF, D0, and ATLAS where they give the mass range of \( m_W = (80.379 \pm 0.012) \text{ GeV} \) \[2\]. This anomaly suggests new physics (NP) beyond the SM \[3–101\], and can be interpreted as the deviation of \( \Delta T \) oblique parameter \[102, 103\], where the oblique parameters are zero in the SM.

One of the straightforward explanations of the anomaly can be realized by introducing extra \( U(1) \) gauge symmetry where the SM Higgs field is charged under it. Then \( \Delta T \) is shifted by effect of mass mixing between the SM \( Z \) and a new neutral gauge boson \( Z' \). In particular one of the minimal scenarios is the explanation by \( Z' \) with right-handed \( U(1)_R \) symmetry \[104\]. The symmetry is originally proposed by chiral anomaly cancellations with three right-handed neutrinos, and it is well testable at the International Linear Collider (ILC) or the Large Hadron Collider (LHC) due to observing the difference of chirality \[105–110\]. However the W boson anomaly cannot be explained if charged-leptons have nonzero \( U(1)_R \) charge because of the constraints from \( W \) and \( Y \) oblique parameters \[7\]. Then, only the possibility to explain the W boson anomaly along this idea is that only the SM Higgs field and quarks have nonzero charge under \( U(1)_R \) symmetry. Note that another possibility is \( U(1)_H \) case where only Higgs doublet is charged under it. In this case we need another Higgs doublet to induce the SM fermion masses and two Higgs doublets can also contribute to \( \Delta T \) parameter at loop level. Remarkably in our setting W boson anomaly is explained by purely \( Z–Z' \) mixing effect.

In this letter, we extend the original \( U(1)_R \) model to make the \( Z' \) to be leptophobic in order to explain this anomaly avoiding other electroweak precision tests. Then extra \( SU(2) \) singlet charged leptons are required to cancel quantum anomalies. As a result the masses of the SM charged leptons are obtained via interactions between the SM lepton and the extra charged leptons. Such interactions also induce lepton anomalous magnetic moments and
II. MODEL SETUP AND CONSTRAINTS

Here, we review our model. We introduce three vector-like neutral fermions $N_{L,R}$ and singly-charged fermions $E_{L,R}$, where only $N_R$ and $E_R$ have nonzero $U(1)_R$ charge with 1 and $-1$, respectively. It suggests that masses of charged-lepton arise not via the SM type but via mixings among singly-charged fermions. One can straightforwardly confirm the typical four patterns of chiral anomaly cancellations per one generation; $[U(1)_R] = [U(1)_R]^3 = [U(1)_R]^2[U(1)_Y] = [U(1)_R][U(1)_Y]^2 = 0$. \(^1\) As for scalar sector, we introduce an isospin singlet fields $\varphi_1$ and $\varphi_2$ with $U(1)_R$ charge $-1$ and $-\frac{1}{2}$ to spontaneous $U(1)_R$ symmetry breaking where SM-like Higgs $H$ also has nonzero charge under $U(1)_R$ in order to induce Yukawa Lagrangian. Each of VEVs is denoted by $\langle H \rangle \equiv [0, v/\sqrt{2}]^T$ and $\langle \varphi_1(2) \rangle \equiv v_1'(2)/\sqrt{2}$. All the field contents and their assignments are summarized in Table I.

\(^1\) In our paper, quark sector is the same as the SM except for the fact that right-handed ones interact with $Z'$ boson.

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TABLE I: Charge assignments of the our fields under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_R$, where its upper index $a$ is the number of family that runs over 1 – 3.

| SU(3)$_C$ | 3 | 3 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|------------|---|---|---|---|---|---|---|---|---|---|---|
| SU(2)$_L$  | 2 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| U(1)$_Y$   | $\frac{1}{6}$ | $\frac{2}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{2}$ | -1 | 0 | 0 | -1 | -1 | $\frac{1}{2}$ | 0 | 0 |
| U(1)$_R$   | 0 | 1 | -1 | 0 | 0 | 1 | 0 | 0 | -1 | 1 | -1 | $-\frac{1}{2}$ |

lepton flavor violations (LFVs) at loop level \[111, 112\]. In addition, we include discussion of active Majorana neutrino mass matrix via inverse seesaw scenario \[107\].

This letter is organized as follows. In Sec. II, we introduce our model and show relevant formulas for phenomenology. In Sec. III, we show our phenomenological analysis of muon $g - 2$, LFVs and collider physics. Finally we devote the summary of our results and the conclusion.
A. Lagrangian and scalar masses

The relevant lepton Yukawa Lagrangian under these symmetries is given by

\[ -L_Y = (y_\ell)_{ab} \bar{\ell}_a L^b \Phi_R + (y_E)_{aa} \bar{E}_a \Phi \bar{E}_a R + (m_E)_{ab} \bar{E}_a L^b R + (y_D)_{ab} \bar{D}_a L^b \tilde{H}_R + (y_N)_{aa} \bar{N}_a \Phi \bar{N}_a R + (M_N)_{ab} \bar{N}_a L^b N^c_L + \text{h.c.}, \]  

(2)

where \( \tilde{H} \equiv i\sigma_2 H \), and upper(lower) indices \((a, b) = 1-3\) for fields(Yukawa or mass matrix) are the number of families, and \( y_E, y_N \) can be diagonal matrix without loss of generality.

The scalar potential is given by

\[ V = \mu_1^2 |\phi_1|^2 - \mu_2^2 |\phi_2|^2 - \mu_H^2 |H|^2 + \lambda_1 |\phi_1|^4 + \lambda_2 |\phi_2|^4 - \mu_3 |\phi_1^* \phi_2^* + \text{h.c.}|^2 + \lambda_3 |\phi_1|^2 |H|^2 + \lambda_4 |\phi_2|^2 |H|^2 + \lambda_5 |\phi_1|^2 |\phi_2|^2. \]

(3)

The scalar fields are parameterized as

\[ H = \begin{pmatrix} w^+ \\ v_1 + i z_1 \\ v_2 + i z_2 \end{pmatrix}, \quad \phi_{1,2} = \begin{pmatrix} v'_{1,2} + r'_{1,2} + i z'_{1,2} \\ v'_1 \\ v'_2 \end{pmatrix}, \]

(4)

where \( w^+ \) and \( z \) are massless Nambu-Goldstone(NG) bosons which are absorbed by the SM gauge bosons \( W^+ \) and \( Z \), and one linear combination of \( z'_{1,2} \) corresponds to NG boson absorbed by an extra \( Z' \) boson from \( U(1)_R \). The VEVs are obtained from tadpole conditions \( \frac{\partial V}{\partial v} = \frac{\partial V}{\partial v_1} = \frac{\partial V}{\partial v_2} = 0 \). We obtain condition for \( v'_1 \) from \( \frac{\partial V}{\partial v'_1} = 0 \) such that

\[ \mu_1^2 v'_1 + \lambda_1 v'_1^3 - \frac{1}{2\sqrt{2}} \mu_3 v'_2^2 + \lambda_5 v'_1 v'_2^2 = 0. \]

(5)

Here the VEV of \( \phi_1 \) is approximately given by

\[ v'_1 \simeq \frac{1}{2\sqrt{2}} \frac{\mu_3 v'_2^2}{\mu_1^2 + \lambda_5 v'_2^2}, \]

(6)

where we assume \( v'_1 \) is much smaller than \( v'_2 \) and \( v'_1^3 \) term is ignored. The hierarchy of VEVs is consistently achieved by choosing \( \mu_3 v'_2 \ll \mu_1^2 \) and/or \( \mu_3 v'_2 \ll \lambda_5 v'_2^2 \). This VEV hierarchy is necessary to explain W boson mass anomaly and to obtain sizable muon \( g - 2 \) at the same time, as we discuss below. In this case, \( z'_2 \) corresponds to the NG boson to be absorbed by \( Z' \) boson.

In our analysis, we assume \( \lambda_1 \) and \( \lambda_5 \) to be negligibly small for simplicity, and only \( r \) and \( r'_1 \) mixes. Then, we obtain the mass matrix for CP even scalar, \( m^2_R \), in the basis of
where the mass eigenstates \{h, H\} is found to be $(r, r'_1)^T = O_R(h, H)^T$, and mass eigenvalues are given by $m^2_{h,H} = O^T_R m^2_R O_R$. $m^2_R$ and $O_R$ are obtained as

$$m^2_R \simeq \begin{bmatrix} 2v^2 \lambda_H & vv' \lambda_3 \\ vv'_1 \lambda_3 & \mu_1^2 \end{bmatrix} , O_R = \begin{bmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{bmatrix} ,$$

where $c_\theta(s_\theta)$ stands for $\cos \theta(\sin \theta)$ with $s_{2\theta} = \frac{2vv'_1 \lambda_3}{m^2_{h} - m^2_{H}}$. The mass eigenvalues are also calculated such that

$$m^2_{h,H} \simeq (v^2 \lambda_H + \mu_1^2) \mp \sqrt{(v^2 \lambda_H - \mu_1^2)^2 + v^2 v'_1 \lambda_3^2} .$$

Here $h \equiv h_{SM}$ is the SM Higgs, therefore, $m_h = 125$ GeV. The mixing effect for CP-even scalar is constrained by the measurements of Higgs production cross section and its decay branching ratio at the LHC, and $s_\theta \lesssim 0.3$ is provided by the current data [120]. The mass of $r'_2$ is approximately given by $m_{r'_2} \simeq \sqrt{2\lambda_2 v'_2}$ which is supposed to be much heavier than $m_H$.

**B. Oblique parameters**

Oblique parameters come from $Z_{SM} - Z'$ mixing. Thus, we firstly discuss this effect. Since $H$ has nonzero $U(1)_R$ charge, there is mixing between $Z_{SM}$ and $Z'$. The resulting mass matrix in basis of $(Z_{SM}, Z')$ is given by

$$m^2_{Z_{SM}Z'} \simeq \frac{1}{4} \begin{bmatrix} (g_1^2 + g_2^2) v^2 & -2\sqrt{g_1^2 + g_2^2} g' v^2 \\ -2\sqrt{g_1^2 + g_2^2} g' v^2 & 4g'^2(v^2 + v'_2) \end{bmatrix} = m^2_{Z'} \begin{bmatrix} \epsilon_1^2 & -\epsilon_1 \epsilon_2 \\ -\epsilon_1 \epsilon_2 & 1 + \epsilon_2^2 \end{bmatrix} ,$$

where $m_{Z_{SM}} \equiv \sqrt{g_1^2 + g_2^2 v^2}$, $m_{Z'} \equiv g' v'_2$, $\epsilon_1 \equiv \frac{m_{Z_{SM}}}{m_{Z'}}$, $\epsilon_2 \equiv \frac{v}{v'_2}$, $g_1$, $g_2$, and $g'$ are gauge coupling of $U(1)_Y$, $SU(2)_L$, and $U(1)_R$, respectively. Note that we ignored $v'_1$ in $m_{Z'}$ formula due to the relation $v'_1 \ll v'_2$. Then its mass matrix is diagonalized by the two by two mixing matrix $V$ as $V m^2_{Z_{SM}Z'} V^T \equiv \text{Diag}(m^2_{Z'}, m^2_{Z_R})$, where we work under $\epsilon_2^2 \ll 1$ and

$$m^2_Z \approx m^2_{Z_{SM}} (1 - \epsilon_2^2) , \quad m^2_{Z_R} \approx m^2_{Z'} (1 + \epsilon_1^2 \epsilon_2^2) , \quad V \approx \begin{bmatrix} c_Z & s_Z \\ -s_Z & c_Z \end{bmatrix} , \quad \theta_Z = \frac{1}{2} \tan^{-1} \left[ \frac{2\epsilon_1 \epsilon_2}{1 + \epsilon_2^2 - \epsilon_1^2} \right] .$$

The $Z_R$ mass can be approximated as $m_{Z_R} \simeq m_{Z'} = g' v'_2$ since $\epsilon_{1,2}$ is small. Thus gauge coupling $g'$ is almost fixed if we choose values of $m_{Z_R}$ and $v'$. 

5
In our case, only $\Delta T$ is nonzero induced via $Z - Z'$ mixing thanks to zero $U(1)_R$ charges of $L_L$, $e_R$ and defined by

$$\Delta T = \frac{1}{\alpha_{em}} \frac{m_{ZSM}^2 - m_Z^2}{m_{ZSM}^2} \approx \frac{\epsilon_2^2}{\alpha_{em}},$$  \hspace{1cm} (12)$$

where we have used Eq. [10] in the last part of the above equation. Thus, $\Delta T$ is straightforwardly given by inserting $\epsilon_2 \equiv v/v'_2$, $v'_2 = m_Z/g'$, $v = 2m_Z \cos \theta_W/g_2$ with $\theta_W$ being the Weinberg angle and simply given by \[7\]

$$\Delta T \approx \frac{v^2 g'^2}{\alpha_{em} m_{Z'}^2} = \frac{4m_Z^2 \cos^2 \theta_W g'^2}{g_2^2 \alpha_{em} m_{Z'}^2}.$$ \hspace{1cm} (13)$$

Note also that contribution to $\Delta T$ at loop level is negligible since our new particles are $SU(2)$ singlet. Although mixing between the SM charged lepton and new charged lepton discussed below can affect the $\Delta T$ at loop level it is highly suppressed by small mixing angle. Thus our relevant parameters for $\Delta T$ are only new gauge coupling $g'$ and $Z'$ mass $m_{Z'}$. From global fit including CDF II $W$ boson mass with $S \sim 0$ we obtain $1\sigma$ range of $T$ as

$$0.09 \leq \Delta T \leq 0.14.$$ \hspace{1cm} (14)$$

Then it is found from Eq. [13]:

$$20 \text{ TeV} \lesssim \frac{m_{Z'}}{g'} (= v'_2) \lesssim 31 \text{ TeV},$$ \hspace{1cm} (15)$$

where we used central value of Eq. [1] for $m_W$. This range is allowed by LEP constraints \[121\] and dijet searches at the LHC \[122-124\]. Here, we emphasize that our model only provides a sizable contribution to $\Delta T$ via $Z - Z'$ mixing. Thus modification of $W$ boson mass is characterized by $\Delta T$ that is estimated by $Z'$ mass, new gauge coupling and other electroweak observables $\{m_Z, \theta_W, \alpha_{em}\}$. It is the advantage of our model that other oblique parameters are not modified.

Here we comment on the situation in the case of original $U(1)_R$ model. In this case, we have contributions to other oblique parameters $W$ and $Y$ due to $Z'$ interactions with charged leptons \[125\]. The oblique parameters, including $W$ and $Y$ are defined from effective Lagrangian

$$\mathcal{L} = -\frac{1}{2} W^3 \Pi_{33}(p^2) W^3 - \frac{1}{2} B_\mu \Pi_{00}(p^2) B^\mu - W^3_\mu \Pi_{30}(p^2) B^\mu - W^+_\mu \Pi_{WW}(p^2) W^- \mu,$$ \hspace{1cm} (16)$$

\[2\] Notice here that the other valid oblique parameters $\Delta S$, $\Delta W$, $\Delta Y$ are zero.
where \( W^3_\mu \), \( B^\mu \) and \( W^\pm \) are gauge fields corresponding to the third component of \( SU(2)_L \), \( U(1)_Y \) and \( W \) boson respectively, and \( p^2 \) is the momentum square carried by the gauge field. The oblique parameters \( W \) and \( Y \) are then given by
\[
W = \frac{m_W^2}{2} \frac{d^2}{d(p^2)^2} \Pi_{33} |_{p^2 = 0}, \quad Y = \frac{m_W^2}{2} \frac{d^2}{d(p^2)^2} \Pi_{00} |_{p^2 = 0}.
\]
\( \Pi \)

It is shown in ref. \[125\] that \( W = Y = 0 \) is obtained when leptons are not charged under new \( U(1) \) symmetry, which is our case. Otherwise, the values of \( W \) and \( Y \) have similar order as \( \hat{T} \equiv \alpha_{em} T \) and the LHC constraints are \( |W| \lesssim 1.8 \times 10^{-4} \) and \( |Y| \lesssim 2.0 \times 10^{-4} \) at 1\( \sigma \) confidence level \[7, 126, 127\]. Thus the parameter region realizing \( \Delta T \) in Eq. \( (14) \) is excluded by the constraints. Therefore we need leptophobic \( U(1)_R \) to explain the \( W \) boson anomaly.

C. Charged-lepton sector

The charged-lepton mass matrix is obtained via mixing among the singly-charged fermions, and the form is give by \[113\]
\[
\begin{pmatrix}
\bar{e}_L \\
\bar{E}_L
\end{pmatrix}^T \mathcal{M}_E \begin{pmatrix}
e_R \\
E_R
\end{pmatrix} = \begin{pmatrix}
\bar{e}_L \\
\bar{E}_L
\end{pmatrix}^T \begin{pmatrix}
0 & m_{eE} \\
M_E & m_{EE}
\end{pmatrix} \begin{pmatrix}
e_R \\
E_R
\end{pmatrix},
\]
\[
\mathcal{M}_E \mathcal{M}_E^\dagger = \begin{pmatrix}
m_{eE} m_{eE} & m_{eE} M_E \\
M_E m_{eE} & M_E^2 + m_{EE} M_{EE}^\dagger
\end{pmatrix}, \quad \mathcal{M}_E^\dagger \mathcal{M}_E = \begin{pmatrix}
m_{eE}^\dagger m_{eE} & m_{eE}^\dagger M_E \\
M_E^\dagger m_{EE} & M_E^2 + m_{EE}^\dagger m_{eE}
\end{pmatrix},
\]
\( \mathcal{M}_E \)

where \( m_{eE} \equiv y_{eL}/\sqrt{2}, \) \( M_E \equiv y_{EE}/\sqrt{2} \). The mass matrix is diagonalized by the transformation \((e_L(R), E_L(R)) \rightarrow V_L(R) \ell_L(R)\). Thus we can obtain diagonalization matrices \( V_L \) and \( V_R \) which respectively diagonalize \( M_E M_E^\dagger \) and \( M_E^\dagger M_E \) as \( V_L^\dagger \mathcal{M}_E \mathcal{M}_E^\dagger V_L = V_R^\dagger \mathcal{M}_E^\dagger \mathcal{M}_E V_R = \text{diag}|D_{Ea}|^2(a=1-6) \), where the first three mass eigenstates correspond to the SM charged-leptons. We then write \( \text{diag}D_{Ea} = \text{diag}(m_e, m_\mu, m_\tau, M_{E1}, M_{E2}, M_{E3}) \).

D. New contribution to \( Z \rightarrow \ell \ell \)

Due to the mixing between the exotic singly-charged fermions and the SM leptons, we have new contribution to \( Z \rightarrow \ell_i \bar{\ell}_j \). Their kinetic Lagrangian in terms of mass eigenvectors
is given by
\[
\frac{g_2}{c_W} \left[ \left( -\frac{1}{2} \sum_{a=1}^{3} V_{L_{L a}}^\dagger V_{L_{a j}} + s_W^2 \delta_{ij} \right) \ell_{L_i} \gamma^\mu \ell_{L_j} + s_W^2 \delta_{ij} \ell_{R_i} \gamma^\mu \ell_{R_j} \right] Z_\mu, \tag{20}
\]
where \( s_W, c_W \) are short-hand notations of Weinberg angles; \( \sin \theta_W, \cos \theta_W \), which are rewritten in terms of \( g_1, g_2 \). Then, the decay rate of \( Z \to \ell_i \bar{\ell}_j \) is given by
\[
\Gamma(Z \to \ell_i \bar{\ell}_j) \simeq \frac{g_2^2}{24\pi c_W^2} m_Z \left[ -\frac{1}{2} \sum_{a=1}^{3} V_{L_{L a}}^\dagger V_{L_{a j}} + s_W^2 \delta_{ij} \right]^2 + s_W^4 \delta_{ij}, \tag{21}
\]
where we assume \( m_Z >> m_\ell \). Here, one confirms that the SM contribution is derived when \( \sum_{a=1}^{3} V_{L_{L a}}^\dagger V_{L_{a j}} = 1; \)
\[
\Gamma(Z \to \ell_i \bar{\ell}_j)_{SM} \simeq \frac{g_2^2}{24\pi c_W^2} m_Z \left[ \frac{1}{4} - s_W^2 + 2s_W^4 \right] \delta_{ij}. \tag{22}
\]
Thus, the new contribution of branching ratios are given by
\[
\Delta BR(Z \to \ell_i \bar{\ell}_j) = \frac{\Gamma(Z \to \ell_i \bar{\ell}_j) - \Gamma(Z \to \ell_i \bar{\ell}_j)_{SM}}{\Gamma_{\text{tot}}^Z} \text{ for } i = j, \tag{23}
\]
\[
\Delta BR(Z \to \ell_i \bar{\ell}_j) = \frac{\Gamma(Z \to \ell_i \bar{\ell}_j)}{\Gamma_{\text{tot}}^Z} \text{ for } i \neq j, \tag{24}
\]
where the total \( Z \) decay width \( \Gamma_{\text{tot}}^Z = 2.4952 \pm 0.0023 \text{ GeV} \)[120].

The current bounds on the lepton-flavor-(conserving)changing \( Z \) boson decay branching ratios\( (BRs) \) at 95 % CL are given by[120]:
\[
\begin{align*}
\Delta BR(Z \to e^\pm e^\mp) &< \pm 4.2 \times 10^{-5}, \\
\Delta BR(Z \to \mu^\pm \mu^\mp) &< \pm 6.6 \times 10^{-5}, \\
\Delta BR(Z \to \tau^\pm \tau^\mp) &< \pm 8.3 \times 10^{-5}, \\
BR(Z \to e^\pm \tau^\mp) &< 9.8 \times 10^{-6}, \\
BR(Z \to \mu^\pm \tau^\mp) &< 1.2 \times 10^{-5}. 
\end{align*} \tag{25}
\]

E. Lepton flavor violations and muon \( g - 2 \)

Due to mixing between the SM charged-lepton and heavier leptons, we have nonzero LFVs and muon \( g - 2 \) via \( y_\ell \). The current experimental upper bounds on LFVs are given by[128, 129]
\[
\begin{align*}
BR(\mu \to e\gamma) &\leq 4.2 \times 10^{-13}, \\
BR(\tau \to \mu\gamma) &\leq 4.4 \times 10^{-8}, \\
BR(\tau \to e\gamma) &\leq 3.3 \times 10^{-8}. 
\end{align*} \tag{26}
\]
FIG. 1: The diagram that gives dominant contributions to $\ell \rightarrow \ell' \gamma$ and muon $g - 2$ in flavor basis. The $\times$ mark indicates mass (mixing) insertion.

On the other hand, new results on the muon ($g - 2$) were recently published by the E989 collaboration at Fermilab [130]:

$$a_{\mu}^{\text{FNAL}} = 116592040(54) \times 10^{-11}. \quad (27)$$

Combined with the previous BNL result, this means that the muon ($g - 2$) deviates from the SM prediction by $4.2\sigma$ level [130–151]:

$$\Delta a_{\mu}^{\text{new}} = (25.1 \pm 5.9) \times 10^{-10}, \quad (28)$$

and it could be a verifiable signature of the physics beyond the SM.

In our scenario, the relevant Lagrangian to induce LFVs and muon $g - 2$ in terms of mass eigenstate is given by

$$\mathcal{L}_{LFV} = (Y_{\alpha\beta}c_\theta - \tilde{Y}_{\alpha\beta}s_\theta) h\bar{\ell}_\alpha P_R \ell_\beta + (Y_{\alpha\beta}s_\theta + \tilde{Y}_{\alpha\beta}c_\theta) H\bar{\ell}_\alpha P_R \ell_\beta + h.c., \quad (29)$$

$$Y_{\alpha\beta} \equiv \frac{1}{\sqrt{2}} \sum_{a=1,2,3} \sum_{b=1,2,3} (V^\dagger_L)_{\alpha,a} (y_\ell)_{ab} (V_R)_{b+3,\beta}, \quad (30)$$

$$\tilde{Y}_{\alpha\beta} \equiv \frac{1}{\sqrt{2}} \sum_{a=1,2,3} \sum_{b=1,2,3} (V^\dagger_L)_{\alpha,a+3} (y_E)_{aa} (V_R)_{a+3,\beta}. \quad (31)$$

\footnote{Even though we have a contribution from a new gauge boson $Z'$ to these phenomenologies, these are subdominant for the value of $g'/m_{Z'}$ in the range of Eq. [15]. We have checked it numerically.}
Then, the dominant contributions to LFVs and muon $g - 2$ at one-loop level are given by

$$\text{BR}(\ell_\beta \to \ell_\alpha \gamma) \simeq \frac{12\pi^2 C_{\beta\alpha}}{(4\pi)^4 m^2_{\ell_\beta} G_F^2} (|a_{L_{\alpha\beta}}|^2 + |a_{R_{\alpha\beta}}|^2),$$

$$\Delta a_\mu \simeq \frac{-m_\mu}{(4\pi)^2} (a_{L_{22}} + a_{R_{22}}),$$

$$a_{L_{\alpha\beta}} \approx -\frac{1}{4} \left( Y_{\alpha\rho c_\theta} - \bar{Y}_{\alpha\rho s_\theta} \right) D_{E_\rho} \left( Y^\dagger_{\rho\beta c_\theta} - \bar{Y}^\dagger_{\rho\beta s_\theta} \right) F(h_1, D_{E_\rho})$$

$$- \frac{1}{4} \left( Y_{\alpha\rho s_\theta} + \bar{Y}_{\alpha\rho c_\theta} \right) D_{E_\rho} \left( Y^\dagger_{\rho\beta s_\theta} + \bar{Y}^\dagger_{\rho\beta c_\theta} \right) F(h_2, D_{E_\rho}),$$

$$a_{R_{\alpha\beta}} \approx -\frac{1}{4} \left( Y_{\alpha\rho c_\theta} - \bar{Y}_{\alpha\rho s_\theta} \right) D_{E_\rho} \left( Y^\dagger_{\rho\beta c_\theta} - \bar{Y}^\dagger_{\rho\beta s_\theta} \right) F(h_1, D_{E_\rho})$$

$$- \frac{1}{4} \left( Y_{\alpha\rho s_\theta} + \bar{Y}_{\alpha\rho c_\theta} \right) D_{E_\rho} \left( Y^\dagger_{\rho\beta s_\theta} + \bar{Y}^\dagger_{\rho\beta c_\theta} \right) F(h_2, D_{E_\rho}),$$

$$F(m_{h_i}, D_{E_\rho}) \approx \frac{m^4_{h_i} - 4m^2_{E_\rho} D^2_{E_\rho} + 3D^4_{E_\rho} - 2m^2_{h_i} (m^2_{h_i} - 2D^2_{E_\rho}) \ln \left[ \frac{m^2_{h_i}}{D^2_{E_\rho}} \right]}{(m^2_{h_i} - D^2_{E_\rho})^3},$$

where $C_{21} \approx 1$, $C_{31} \approx 0.1784$, $C_{32} \approx 0.1736$, $h_1 \equiv h$, $h_2 \equiv H$. The dominant contribution is obtained from the diagram in Fig. [ ] that does not have chiral suppression.

**F. Neutrino mass via inverse seesaw mechanism**

After spontaneous gauge symmetry breaking, we obtain neutral fermion mass matrix in the basis of $(\nu^c_L, N_R, N^c_L)$ as follows

$$M_N = \begin{bmatrix} 0 & m_D & 0 \\
- m^T_D & 0 & M \\
0 & M & M_{NL} \end{bmatrix},$$

where $m_D \equiv v y_D / \sqrt{2}$ and $M \equiv y_N v_1' / \sqrt{2}$. When mass parameters satisfy $M_{NL} \ll m_D \lesssim M$ active neutrino mass can be approximately written by

$$m_\nu \simeq m_D M^{-1} M_{NL} (M^T)^{-1} m_D^T.$$ 

The neutrino mass matrix is diagonalized by a unitary matrix $U_\nu$: $D_\nu = U_\nu^T m_\nu U_\nu$ with $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$. Including charged lepton mixing matrix, the PMNS matrix is defined by $U_{PMNS} \equiv V_L^U U_\nu$.

We discuss constraint from non-unitarity which is described by a matrix $U'_{PMNS}$. This matrix is typically parametrized by the form of

$$U'_{PMNS} \equiv \left( 1 - \frac{1}{2} F F^\dagger \right) U_{PMNS},$$

$$10$$
where $F \equiv (M^T)^{-1}m_D$ is a hermitian matrix. The global constraints on elements of $|FF^\dagger|$ are found combining several experimental results such as the SM $W$ boson mass $M_W$, the effective Weinberg angle $\theta_W$, several ratios of $Z$ boson fermionic decays, invisible decay of $Z$, electroweak universality, measured Cabbibo-Kobayashi-Maskawa, and lepton flavor violations \cite{115}. The result is then given by \cite{116}

$$
|FF^\dagger| \leq \begin{bmatrix}
2.5 \times 10^{-3} & 2.4 \times 10^{-5} & 2.7 \times 10^{-3} \\
2.4 \times 10^{-5} & 4.0 \times 10^{-4} & 1.2 \times 10^{-3} \\
2.7 \times 10^{-3} & 1.2 \times 10^{-3} & 5.6 \times 10^{-3}
\end{bmatrix}.
$$

(40)

If we require $|F| \sim 10^{-5}$ conservatively, $M_{N_L} \sim 1 - 10 \text{ GeV}$ can reproduce active neutrino mass scale. Also we require $y_D \sim 10^{-4}$ for $M = \mathcal{O}(1) \text{ TeV}$. Observed neutrino mixing can be easily obtained since we have sufficient number of free parameters $\{y_D, y_N, M_{N_L}\}$.

### III. Numerical Analysis and Phenomenological Consequences

In this section we carry out numerical study to estimate muon $g - 2$ and LFVs. We also calculate $Z'$ production cross section for parameter space explaining $W$ boson mass anomaly and show phenomenological implications of our scenario at hadron collider experiments.

#### A. Parameter scan for muon $g - 2$ and LFVs

Here we discuss muon $g - 2$ taking into account charged lepton masses and LFV constraints in our model performing numerical analysis. The dominant contribution to muon $g - 2$ comes from the diagram in Fig. \[\text{Fig. 1}\] since it has chirality change inside loop giving heavy charged lepton mass factor. The relevant free parameters are

$$
\{(y_\ell)_{ab}, (y_E)_{aa}, (m_{Ee})_{ab}, \sin \theta, m_H\},
$$

(41)

where $a, b = 1 - 3$. We then scan these free parameters globally to search for best fit value of muon $g - 2$. In Table \[\text{Table II}\] we show our benchmark point(BP) found by numerical analysis. We find that large value of Yukawa coupling $y_E$ is preferred to obtain sizable muon $g - 2$ while exotic heavy charged lepton masses are around 700 GeV. Then values of $v'_1$ and $m_H$ are preferred to be around electroweak scale. This is the reason why we need two singlet scalars $\varphi_1$ and $\varphi_2$ to explain both muon $g - 2$ and $W$ boson mass anomaly as the required
scale of these VEVs are different; \( v'_1 \ll v'_2 \). We also show \( \text{BR}(\ell \rightarrow \ell'\gamma) \) and find that those of \( \mu \rightarrow e\gamma \) and \( \tau \rightarrow \mu\gamma \) are close to the current upper limit. Thus it could be tested in future measurements. Furthermore we show \((\Delta)\text{BR}(Z \rightarrow \ell\ell')\) values that might be also tested in future precision measurements for \( Z \) boson decay.

| Input                  |       |
|------------------------|-------|
| \( v'_1 / \text{GeV} \) | 284   |
| \( m_H / \text{GeV} \)  | 245   |
| \( \sin \theta \)       | 0.250 |
| \( [(y_\ell)_{11}, (y_\ell)_{12}, (y_\ell)_{13}] \) | \([-0.000512, 0.00520, 0.00236]\) |
| \( [(y_\ell)_{21}, (y_\ell)_{22}, (y_\ell)_{23}] \) | \([-0.000105, 0.000624, 0.0643]\) |
| \( [(y_\ell)_{31}, (y_\ell)_{32}, (y_\ell)_{33}] \) | \([0.000148, 0.0145, 0.0647]\) |
| \( [(m_{\ell E})_{11}, (m_{\ell E})_{12}, (m_{\ell E})_{13}] / \text{GeV} \) | \([0.674, 16.8, 5.92]\) |
| \( [(m_{\ell E})_{21}, (m_{\ell E})_{22}, (m_{\ell E})_{23}] / \text{GeV} \) | \([-16.2, -46.3, 22.3]\) |
| \( [(m_{\ell E})_{31}, (m_{\ell E})_{32}, (m_{\ell E})_{33}] / \text{GeV} \) | \([-12.2, 69.5, 19.8]\) |
| \( [(y_E)_{11}, (y_E)_{12}, (y_E)_{13}] \) | \([-3.26, 3.43, -3.41]\) |
| **Output**             |       |
| \( [m_{E_1}, m_{E_2}, m_{E_3}] / \text{GeV} \) | \([655, 690, 694]\) |
| \( \Delta a_\mu \)      | \(2.11 \times 10^{-9}\) |
| \( \text{BR}(\mu \rightarrow e\gamma) \) | \(3.59 \times 10^{-13}\) |
| \( \text{BR}(\tau \rightarrow e\gamma) \) | \(9.91 \times 10^{-11}\) |
| \( \text{BR}(\tau \rightarrow \mu\gamma) \) | \(2.22 \times 10^{-8}\) |
| \( \Delta \text{BR}(Z \rightarrow e^+e^-) \) | \(-1.55 \times 10^{-9}\) |
| \( \Delta \text{BR}(Z \rightarrow \mu^+\mu^-) \) | \(-6.11 \times 10^{-7}\) |
| \( \Delta \text{BR}(Z \rightarrow \tau^+\tau^-) \) | \(-3.82 \times 10^{-5}\) |
| \( \Delta \text{BR}(Z \rightarrow e^+\mu^-) \) | \(6.27 \times 10^{-17}\) |
| \( \Delta \text{BR}(Z \rightarrow e^+\tau^-) \) | \(3.83 \times 10^{-14}\) |
| \( \Delta \text{BR}(Z \rightarrow \mu^+\tau^-) \) | \(3.64 \times 10^{-11}\) |

**TABLE II:** Benchmark point that explains muon \( g - 2 \).
FIG. 2: The 1σ region to explain W boson mass anomaly on \{m_{Z'}, g'\} plane where the red colored line indicates the value providing central value of \(\Delta T\) in Eq. (14).

B. Signature of \(Z'\) production at collider

We consider signature of \(Z'\) production at hadron collider experiments focusing on parameter space that can explain W boson mass anomaly. In Fig. 2, we show the 1σ region to explain W boson mass anomaly on \(\{m_{Z'}, g'\}\) plane where the red colored line indicates the value providing central value of \(\Delta T\) in Eq. (14). For the parameter region we estimate cross section of \(pp \to Z'\) process using CalcHEP \[117\] implementing relevant gauge interactions.

Our \(Z'\) dominantly decays into \(\{q\bar{q}, N_R\bar{N}_R, E_R\bar{E}_R\}\) modes. The decay width is estimated by

\[
\Gamma(Z' \to f\bar{f}) = \frac{g'^2 N_c}{12\pi} m_{Z'} \sqrt{1 - \frac{4m_f^2}{m_{Z'}^2}} \left(1 - \frac{m_f^2}{m_{Z'}^2}\right)
\]

where \(N_c\) is color degrees of freedom. BRs of \(Z'\) decay can be estimated by the width. In Fig. 3, we show products of cross section and the BRs for \(jj\) and \(E\bar{E}(N\bar{N})\) modes where we chose \(m_{E(N)} = 700\) GeV for three generations. We thus obtain sizable cross section that can be tested at future LHC experiments. Extra fermions \(E_a\) and \(N_a\) dominantly decay into the SM lepton with boson as

\[
E_a \to \ell_i h, \quad N_a \to \nu_i h, \nu_i Z, \ell_i W,
\]

where lepton flavor dependence is determined by structure of Yukawa couplings. Therefore
FIG. 3: The products of $Z'$ production cross section and the BRs for $jj$ and $E\bar{E}(N\bar{N})$ modes at the LHC 14 TeV where we chose $m_{E(N)} = 700$ GeV for three generations.

signatures of our $Z'$ are dijet, top quark pair and SM leptons with higgs or gauge bosons. More detailed numerical analysis is beyond the scope of this paper and left as future work.

IV. SUMMARY AND CONCLUSIONS

We have proposed a model with leptophobic $U(1)_R$ gauge symmetry to explain CDF $W$ boson mass anomaly. The shift of $W$ boson mass is realized via neutral gauge boson mass which induces non-zero oblique $\Delta T$ parameter. Also we have shown that $U(1)_R$ should be leptophobic since parameter region to explain $W$ boson mass anomaly is excluded by the LHC constraints on $W$ and $Y$ oblique parameters if $Z'$ is coupled to the SM charged leptons. It is then found that $Z'$ mass and new gauge coupling should satisfy $20 \text{ TeV} \lesssim m_{Z'}/g' \lesssim 31$ TeV to accommodate CDF $W$ boson mass.

In our model charged lepton masses are induced via interactions between the SM leptons and extra charged leptons. These interactions also induce muon $g-2$ and LFVs at loop level. We have investigated muon $g-2$ and LFVs taking into current experimental constraints.
It is found that some LFV BRs, $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$, tends to be close to current upper limits when we explain muon $g - 2$ and the BRs can be tested in future experiments. In addition we have discussed realization of active neutrino mass and mixings via inverse seesaw mechanism. Moreover we have discussed collider physics focusing on $Z'$ production and its decay at the LHC. We have found that sizable cross section can be expected considering parameter region explaining W boson mass anomaly.

Acknowledgments

KIN was supported by JSPS Grant-in-Aid for Scientific Research (A) 18H03699, (C) 21K03562, (C) 21K03583, Okayama Foundation for Science and Technology, and Wesco Scientific Promotion Foundation. This research was supported by an appointment to the JRG Program at the APCTP through the Science and Technology Promotion Fund and Lottery Fund of the Korean Government. This was also supported by the Korean Local Governments - Gyeongsangbuk-do Province and Pohang City (H.O.). H. O. is sincerely grateful for KIAS and all the members. The work was also supported by the Fundamental Research Funds for the Central Universities (T. N.).

[1] T. Aaltonen et al. [CDF], Science 376 (2022) no.6589, 170-176 doi:10.1126/science.abk1781
[2] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98 (2018) no.3, 030001 doi:10.1103/PhysRevD.98.030001
[3] Y. Z. Fan, T. P. Tang, Y. L. S. Tsai and L. Wu, [arXiv:2204.03693 [hep-ph]].
[4] C. T. Lu, L. Wu, Y. Wu and B. Zhu, [arXiv:2204.03796 [hep-ph]].
[5] P. Athron, A. Fowlie, C. T. Lu, L. Wu, Y. Wu and B. Zhu, [arXiv:2204.03996 [hep-ph]].
[6] G. W. Yuan, L. Zu, L. Feng and Y. F. Cai, [arXiv:2204.04183 [hep-ph]].
[7] A. Strumia, [arXiv:2204.04191 [hep-ph]].
[8] J. M. Yang and Y. Zhang, [arXiv:2204.04202 [hep-ph]].
[9] J. de Blas, M. Pierini, L. Reina and L. Silvestrini, [arXiv:2204.04204 [hep-ph]].
[10] G. G. di Cortona and E. Nardi, [arXiv:2204.04227 [hep-ph]].
[11] X. K. Du, Z. Li, F. Wang and Y. K. Zhang, [arXiv:2204.04286 [hep-ph]].
[12] T. P. Tang, M. Abdughani, L. Feng, Y. L. S. Tsai and Y. Z. Fan, [arXiv:2204.04356 [hep-ph]].
[13] G. Cacciapaglia and F. Sannino, [arXiv:2204.04514 [hep-ph]].
[14] M. Blennow, P. Coloma, E. Fernández-Martínez and M. González-López, [arXiv:2204.04559 [hep-ph]].
[15] K. Sakurai, F. Takahashi and W. Yin, [arXiv:2204.04770 [hep-ph]].
[16] J. Fan, L. Li, T. Liu and K. F. Lyu, [arXiv:2204.04805 [hep-ph]].
[17] X. Liu, S. Y. Guo, B. Zhu and Y. Li, [arXiv:2204.04834 [hep-ph]].
[18] H. M. Lee and K. Yamashita, [arXiv:2204.05024 [hep-ph]].
[19] Y. Cheng, X. G. He, Z. L. Huang and M. W. Li, [arXiv:2204.05031 [hep-ph]].
[20] H. Song, W. Su and M. Zhang, [arXiv:2204.05085 [hep-ph]].
[21] E. Bagnaschi, J. Ellis, M. Madigan, K. Mimatsu, V. Sanz and T. You, [arXiv:2204.05260 [hep-ph]].
[22] A. Paul and M. Valli, [arXiv:2204.05267 [hep-ph]].
[23] H. Bahl, J. Braathen and G. Weiglein, [arXiv:2204.05269 [hep-ph]].
[24] P. Asadi, C. Cesarotti, K. Fraser, S. Homiller and A. Parikh, [arXiv:2204.05283 [hep-ph]].
[25] L. Di Luzio, R. Gröber and P. Paradisi, [arXiv:2204.05284 [hep-ph]].
[26] P. Athron, M. Bach, D. H. J. Jacob, W. Kotlarski, D. Stöckinger and A. Voigt, [arXiv:2204.05285 [hep-ph]].
[27] J. Gu, Z. Liu, T. Ma and J. Shu, [arXiv:2204.05296 [hep-ph]].
[28] J. J. Heckman, [arXiv:2204.05302 [hep-ph]].
[29] K. S. Babu, S. Jana and V. P. K., [arXiv:2204.05303 [hep-ph]].
[30] Y. Heo, D. W. Jung and J. S. Lee, [arXiv:2204.05728 [hep-ph]].
[31] X. K. Du, Z. Li, F. Wang and Y. K. Zhang, [arXiv:2204.05760 [hep-ph]].
[32] K. Cheung, W. Y. Keung and P. Y. Tseng, [arXiv:2204.05942 [hep-ph]].
[33] A. Crivellin, M. Kirk, T. Kitahara and F. Mescia, [arXiv:2204.05962 [hep-ph]].
[34] M. Endo and S. Mishima, [arXiv:2204.05965 [hep-ph]].
[35] T. Biekötter, S. Heinemeyer and G. Weiglein, [arXiv:2204.05975 [hep-ph]].
[36] R. Balkin, E. Madge, T. Menzo, G. Perez, Y. Soreq and J. Zupan, [arXiv:2204.05992 [hep-ph]].
[37] N. V. Krasnikov, [arXiv:2204.06327 [hep-ph]].
[38] Y. H. Ahn, S. K. Kang and R. Ramos, [arXiv:2204.06485 [hep-ph]].
[39] X. F. Han, F. Wang, L. Wang, J. M. Yang and Y. Zhang, arXiv:2204.06505 [hep-ph].
[40] M. D. Zheng, F. Z. Chen and H. H. Zhang, arXiv:2204.06541 [hep-ph].
[41] J. Kawamura, S. Okawa and Y. Omura, arXiv:2204.07022 [hep-ph].
[42] A. Ghoshal, N. Okada, S. Okada, D. Raut, Q. Shafi and A. Thapa, arXiv:2204.07138 [hep-ph].
[43] P. Fileviez Perez, H. H. Patel and A. D. Plascencia, arXiv:2204.07144 [hep-ph].
[44] K. I. Nagao, T. Nomura and H. Okada, arXiv:2204.07411 [hep-ph].
[45] S. Kanemura and K. Yagyu, arXiv:2204.07511 [hep-ph].
[46] P. Mondal, arXiv:2204.07844 [hep-ph].
[47] K. Y. Zhang and W. Z. Feng, arXiv:2204.08067 [hep-ph].
[48] D. Borah, S. Mahapatra, D. Nanda and N. Sahu, arXiv:2204.08266 [hep-ph].
[49] T. A. Chowdhury, J. Heeck, S. Saad and A. Thapa, arXiv:2204.08390 [hep-ph].
[50] G. Arcadi and A. Djouadi, arXiv:2204.08406 [hep-ph].
[51] V. Cirigliano, W. Dekens, J. de Vries, E. Mereghetti and T. Tong, arXiv:2204.08440 [hep-ph].
[52] L. M. Carpenter, T. Murphy and M. J. Smylie, arXiv:2204.08546 [hep-ph].
[53] O. Popov and R. Srivastava, arXiv:2204.08568 [hep-ph].
[54] K. Ghorbani and P. Ghorbani, arXiv:2204.09001 [hep-ph].
[55] M. Du, Z. Liu and P. Nath, arXiv:2204.09024 [hep-ph].
[56] A. Bhaskar, A. A. Madathil, T. Mandal and S. Mitra, arXiv:2204.09031 [hep-ph].
[57] A. Batra, S. K.A., S. Mandal and R. Srivastava, arXiv:2204.09376 [hep-ph].
[58] J. Cao, L. Meng, L. Shang, S. Wang and B. Yang, arXiv:2204.09477 [hep-ph].
[59] Y. P. Zeng, C. Cai, Y. H. Su and H. H. Zhang, arXiv:2204.09487 [hep-ph].
[60] S. Baek, arXiv:2204.09585 [hep-ph].
[61] D. Borah, S. Mahapatra and N. Sahu, arXiv:2204.09671 [hep-ph].
[62] E. d. Almeida, A. Alves, O. J. P. Eboli and M. C. Gonzalez-Garcia, arXiv:2204.10130 [hep-ph].
[63] Y. Cheng, X. G. He, F. Huang, J. Sun and Z. P. Xing, arXiv:2204.10156 [hep-ph].
[64] J. Heeck, arXiv:2204.10274 [hep-ph].
[65] A. Addazi, A. Marciano, A. P. Morais, R. Pasechnik and H. Yang, arXiv:2204.10315 [hep-ph].
[66] S. Lee, K. Cheung, J. Kim, C. T. Lu and J. Song, \texttt{arXiv:2204.10338 [hep-ph]}.
[67] C. Cai, D. Qiu, Y. L. Tang, Z. H. Yu and H. H. Zhang, \texttt{arXiv:2204.11570 [hep-ph]}.
[68] R. Benbrik, M. Boukidi and B. Manaut, \texttt{arXiv:2204.11755 [hep-ph]}.
[69] T. Yang, S. Qian, S. Deng, J. Xiao, L. Gao, A. M. Levin, Q. Li, M. Lu and Z. You, \texttt{arXiv:2204.11871 [hep-ph]}.
[70] A. Batra, S. K. A, S. Mandal, H. Prajapati and R. Srivastava, \texttt{arXiv:2204.11945 [hep-ph]}.
[71] H. B. T. Tan and A. Derevianko, \texttt{arXiv:2204.11991 [hep-ph]}.
[72] H. Abouabid, A. Arhrib, R. Benbrik, M. Krab and M. Ouchemhou, \texttt{arXiv:2204.12018 [hep-ph]}.
[73] T. K. Chen, C. W. Chiang and K. Yagyu, \texttt{arXiv:2204.12898 [hep-ph]}.
[74] Q. Zhou and X. F. Han, \texttt{arXiv:2204.13027 [hep-ph]}.
[75] R. S. Gupta, \texttt{arXiv:2204.13690 [hep-ph]}.
[76] V. Basiouris and G. K. Leontaris, \texttt{arXiv:2205.00758 [hep-ph]}.
[77] J. W. Wang, X. J. Bi, P. F. Yin and Z. H. Yu, \texttt{arXiv:2205.00783 [hep-ph]}.
[78] F. J. Botella, F. Cornet-Gomez, C. Miró and M. Nebot, \texttt{arXiv:2205.01115 [hep-ph]}.
[79] J. Kim, Phys. Lett. B 832 (2022), 137220 \texttt{arXiv:2205.01437 [hep-ph]}. Copy to Clipboard-
Download
[80] B. Barman, A. Das and S. Sengupta, \texttt{arXiv:2205.01699 [hep-ph]}.
[81] J. Kim, S. Lee, P. Sanyal and J. Song, \texttt{arXiv:2205.01701 [hep-ph]}.
[82] X. Q. Li, Z. J. Xie, Y. D. Yang and X. B. Yuan, \texttt{arXiv:2205.02205 [hep-ph]}.
[83] J. Isaacson, Y. Fu and C. P. Yuan, \texttt{arXiv:2205.02788 [hep-ph]}.
[84] J. L. Evans, T. T. Yanagida and N. Yokozaki, \texttt{arXiv:2205.03877 [hep-ph]}.
[85] T. A. Chowdhury and S. Saad, \texttt{arXiv:2205.03917 [hep-ph]}.
[86] S. S. Kim, H. M. Lee, A. G. Menkara and K. Yamashita, \texttt{arXiv:2205.04016 [hep-ph]}.
[87] G. Lazarides, R. Maji, R. Roshan and Q. Shafi, \texttt{arXiv:2205.04824 [hep-ph]}.
[88] G. Senjanović and M. Zantedeschi, \texttt{arXiv:2205.05022 [hep-ph]}.
[89] R. Ghosh, B. Mukhopadhyaya and U. Sarkar, \texttt{arXiv:2205.05041 [hep-ph]}.
[90] T. Li, J. Pei, X. Yin and B. Zhu, \texttt{arXiv:2205.08215 [hep-ph]}.
[91] M. C. Rodriguez, \texttt{arXiv:2205.09109 [hep-ph]}.
[92] E. Ramirez and P. Roig, \texttt{arXiv:2205.10420 [hep-ph]}.
[93] J. Kawamura and S. Raby, \texttt{arXiv:2205.10480 [hep-ph]}.

18
[94] S. S. Afonin, [arXiv:2205.12237 [hep-ph]].
[95] B. C. Allanach and J. Davighi, [arXiv:2205.12252 [hep-ph]].
[96] S. S. Xue, [arXiv:2205.14957 [hep-ph]].
[97] T. G. Rizzo, [arXiv:2206.09814 [hep-ph]].
[98] D. Van Loi and P. Van Dong, [arXiv:2206.10100 [hep-ph]].
[99] S. Yaser Ayazi and M. Hosseini, [arXiv:2206.11041 [hep-ph]].
[100] N. Chakrabarty, [arXiv:2206.11771 [hep-ph]].
[101] S. Centelles Chuliá, R. Srivastava and S. Yadav, [arXiv:2206.11903 [hep-ph]].
[102] M. E. Peskin and T. Takeuchi, Phys. Rev. D 46 (1992), 381-409 
doi:10.1103/PhysRevD.46.381
[103] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65 (1990), 964-967 
doi:10.1103/PhysRevLett.65.964
[104] S. Jung, H. Murayama, A. Pierce and J. D. Wells, Phys. Rev. D 81, 015004 (2010) 
[arXiv:0907.4112 [hep-ph]].
[105] T. Nomura and H. Okada, Phys. Rev. D 97 (2018) no.1, 015015 
doi:10.1103/PhysRevD.97.015015 [arXiv:1707.00929 [hep-ph]].
[106] T. Nomura, H. Okada and H. Yokoya, [arXiv:1702.03396 [hep-ph]].
[107] T. Nomura and H. Okada, LHEP 1 (2018) no.2, 10-13 doi:10.31526/LHEP.2.2018.01 
[arXiv:1806.01714 [hep-ph]].
[108] T. Nomura and H. Okada, [arXiv:1704.03382 [hep-ph]].
[109] W. Chao, [arXiv:1707.07858 [hep-ph]].
[110] O. Seto and T. Shimomura, JHEP 04 (2021), 025 [arXiv:2006.05497 [hep-ph]].
[111] H. M. Lee, J. Song and K. Yamashita, J. Korean Phys. Soc. 79 (2021) no.12, 1121-1134 
[arXiv:2110.09942 [hep-ph]].
[112] G. Guedes and P. Olgoso, [arXiv:2205.04480 [hep-ph]].
[113] U. K. Dey, T. Nomura and H. Okada, Phys. Rev. D 100 (2019) no.7, 075013 
doi:10.1103/PhysRevD.100.075013 [arXiv:1902.06205 [hep-ph]].
[114] I. Esteban, M. C. Gonzalez-Garcia, A. Hernandez-Cabezudo, M. Maltoni and T. Schwetz, 
JHEP 01 (2019), 106 [arXiv:1811.05487 [hep-ph]].
[115] E. Fernandez-Martinez, J. Hernandez-Garcia and J. Lopez-Pavon, JHEP 08 (2016), 033 
doi:10.1007/JHEP08(2016)033 [arXiv:1605.08774 [hep-ph]].
[116] N. R. Agostinho, G. C. Branco, P. M. F. Pereira, M. N. Rebelo and J. I. Silva-Marcos, Eur. Phys. J. C **78** (2018) no.11, 895 doi:10.1140/epjc/s10052-018-6347-2 [arXiv:1711.06229 [hep-ph]].

[117] A. Belyaev, N. D. Christensen and A. Pukhov, Comput. Phys. Commun. **184**, 1729 (2013) [arXiv:1207.6082 [hep-ph]].

[118] P. M. Nadolsky, H. L. Lai, Q. H. Cao, J. Huston, J. Pumplin, D. Stump, W. K. Tung and C.-P. Yuan, Phys. Rev. D **78**, 013004 (2008) [arXiv:0802.0007 [hep-ph]].

[119] L. Cerrito, D. Millar, S. Moretti and F. Spano, [arXiv:1609.05540 [hep-ph]].

[120] P. A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).

[121] S. Schael et al. [ALEPH, DELPHI, L3, OPAL and LEP Electroweak], Phys. Rept. **532** (2013), 119-244 [arXiv:1302.3415 [hep-ex]].

[122] G. Aad et al. [ATLAS], JHEP **03** (2020), 145 [arXiv:1910.08447 [hep-ex]].

[123] A. M. Sirunyan et al. [CMS], JHEP **05** (2020), 033 [arXiv:1911.03947 [hep-ex]].

[124] B. A. Dobrescu and F. Yu, [arXiv:2112.05392 [hep-ph]].

[125] G. Cacciapaglia, C. Csaki, G. Marandella and A. Strumia, Phys. Rev. D **74** (2006), 033011 doi:10.1103/PhysRevD.74.033011 [arXiv:hep-ph/0604111 [hep-ph]].

[126] A. Tumasyan et al. [CMS], JHEP **07** (2022), 067 doi:10.1007/JHEP07(2022)067 [arXiv:2202.06075 [hep-ex]].

[127] M. Aaboud et al. [ATLAS], JHEP **10** (2017), 182 doi:10.1007/JHEP10(2017)182 [arXiv:1707.02424 [hep-ex]].

[128] A. M. Baldini et al. [MEG], Eur. Phys. J. C **76** (2016) no.8, 434 doi:10.1140/epjc/s10052-016-4271-x [arXiv:1605.05081 [hep-ex]].

[129] J. Adam et al. [MEG], Phys. Rev. Lett. **110** (2013), 201801 doi:10.1103/PhysRevLett.110.201801 [arXiv:1303.0754 [hep-ex]].

[130] B. Abi et al. [Muon g-2], Phys. Rev. Lett. **126** (2021) no.14, 141801 doi:10.1103/PhysRevLett.126.141801 [arXiv:2104.03281 [hep-ex]].

[131] K. Hagiwara, R. Liao, A. D. Martin, D. Nomura and T. Teubner, J. Phys. G **38**, 085003 (2011) [arXiv:1105.3149 [hep-ph]].

[132] T. Aoyama, M. Hayakawa, T. Kinoshita and M. Nio, Phys. Rev. Lett. **109**, 111808 (2012) doi:10.1103/PhysRevLett.109.111808 [arXiv:1205.5370 [hep-ph]].

[133] T. Aoyama, T. Kinoshita and M. Nio, Atoms **7**, no.1, 28 (2019) doi:10.3390/atoms7010028
[134] A. Czarnecki, W. J. Marciano and A. Vainshtein, Phys. Rev. D 67, 073006 (2003) [erratum: Phys. Rev. D 73, 119901 (2006)] doi:10.1103/PhysRevD.67.073006 [arXiv:hep-ph/0212229 [hep-ph]].

[135] C. Gnendiger, D. Stöckinger and H. Stöckinger-Kim, Phys. Rev. D 88, 053005 (2013) doi:10.1103/PhysRevD.88.053005 [arXiv:1306.5546 [hep-ph]].

[136] A. Keshavarzi, D. Nomura and T. Teubner, Phys. Rev. D 97, no.11, 114025 (2018) doi:10.1103/PhysRevD.97.114025 [arXiv:1802.02995 [hep-ph]].

[137] G. Colangelo, M. Hoferichter and P. Stoffer, JHEP 02, 006 (2019) doi:10.1007/JHEP02(2019)006 [arXiv:1810.00007 [hep-ph]].

[138] M. Hoferichter, B. L. Hoid and B. Kubis, JHEP 08, 137 (2019) doi:10.1007/JHEP08(2019)137 [arXiv:1907.01556 [hep-ph]].

[139] A. Keshavarzi, D. Nomura and T. Teubner, Phys. Rev. D 101, no.1, 014029 (2020) doi:10.1103/PhysRevD.101.014029 [arXiv:1911.00367 [hep-ph]].

[140] A. Kurz, T. Liu, P. Marquard and M. Steinhauser, Phys. Lett. B 734, 144-147 (2014) doi:10.1016/j.physletb.2014.05.043 [arXiv:1403.6400 [hep-ph]].

[141] K. Melnikov and A. Vainshtein, Phys. Rev. D 70, 113006 (2004) doi:10.1103/PhysRevD.70.113006 [arXiv:hep-ph/0312226 [hep-ph]].

[142] P. Masjuan and P. Sanchez-Puertas, Phys. Rev. D 95, no.5, 054026 (2017) doi:10.1103/PhysRevD.95.054026 [arXiv:1701.05829 [hep-ph]].

[143] G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, JHEP 04, 161 (2017) doi:10.1007/JHEP04(2017)161 [arXiv:1702.07347 [hep-ph]].

[144] M. Hoferichter, B. L. Hoid, B. Kubis, S. Leupold and S. P. Schneider, JHEP 10, 141 (2018) doi:10.1007/JHEP10(2018)141 [arXiv:1808.04823 [hep-ph]].

[145] A. Gérardin, H. B. Meyer and A. Nyffeler, Phys. Rev. D 100, no.3, 034520 (2019) doi:10.1103/PhysRevD.100.034520 [arXiv:1903.09471 [hep-lat]].

[146] J. Bijnens, N. Hermansson-Truedsson and A. Rodríguez-Sánchez, Phys. Lett. B 798, 134994 (2019) doi:10.1016/j.physletb.2019.134994 [arXiv:1908.03331 [hep-ph]].

[147] G. Colangelo, F. Hagelstein, M. Hoferichter, L. Laub and P. Stoffer, JHEP 03, 101 (2020) doi:10.1007/JHEP03(2020)101 [arXiv:1910.13432 [hep-ph]].

[148] T. Blum, N. Christ, M. Hayakawa, T. Izubuchi, L. Jin, C. Jung and C. Lehner, Phys. Rev. Lett. 124, no.13, 132002 (2020) doi:10.1103/PhysRevLett.124.132002 [arXiv:1911.08123]
[149] G. Colangelo, M. Hoferichter, A. Nyffeler, M. Passera and P. Stoffer, Phys. Lett. B 735, 90-91 (2014) doi:10.1016/j.physletb.2014.06.012 \texttt{arXiv:1403.7512 [hep-ph]].

[150] M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, Eur. Phys. J. C 77, no.12, 827 (2017) doi:10.1140/epjc/s10052-017-5161-6 \texttt{arXiv:1706.09436 [hep-ph]].

[151] M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, Eur. Phys. J. C 80, no.3, 241 (2020) [erratum: Eur. Phys. J. C 80, no.5, 410 (2020)] doi:10.1140/epjc/s10052-020-7792-2 \texttt{arXiv:1908.00921 [hep-ph]].