Loading of satellite bearing of planetary cycloid gear by forces acting in meshing

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Abstract. The main feature of planetary cycloid gears is the multi-pair contact of the pin-rollers and teeth of the cycloidal disk. To establish liaisons in the gear, mathematical models of the satellite's force interaction with pin-rollers are used. The dependences of the total forces in meshing, which occur under the action of a torque applied to the output shaft, from the gear parameters, clearances in gear and the relative position of gear elements are given. The results of calculations performed in Mathcad using the proposed model show how the forces change in the direction of the inter-axis line of gear with increasing clearances at a constant torque on the output shaft. This solution allows estimating the forces acting on the satellite bearings at the specified manufacturing accuracy at the design stage of the gear.

1. Introduction

The load capacity of the planetary cycloid gear, especially with a long operating time, can be limited by satellite bearings [1]. One of the first was the work of V.N. Kudryavtsev, in which the static loading of gearing without clearance and satellites was studied. Subsequent studies [2-7] indicated that gaps affect a complex of interrelated quantities: backlash and kinematic error of rotation, forces acting in meshing, loss in meshing, torsional stiffness of transmission, etc. The influence of design parameters on forces and contact stresses are presented in [2] by S.K. Malhotra and M.A. Parameswaran. Mathematical models for calculating deformations, gaps and forces between the teeth of the cycloidal disk and the gear teeth are presented by L. Lixing et al. [3]. J.G. Blanche and D.C.H. Yang developed models for the manufacturing errors effects on clearance and torque ripple of the cycloidal drive [5]. In force analysis, forces in contacts are more frequently examined, while much less attention is paid to the study of forces acting on satellite supports. The current work provides a more complete understanding of the work of planetary cycloid gears.
2. Analytical solutions

In a precisely manufactured planetary cycloid gear, all the pin-holes (rollers) are in simultaneous contact with the teeth of the satellite. Due to gaps without load, the pin must have one pair of pin-tooth of the satellite. However, other contact pairs come into operation under load, contact zones are formed and the load is transmitted by all contact pairs of these zones. When gear operating, the position of the satellite changes and with it the distribution of the load along the contact pairs, the angle of elastic rotation of the satellite and the total load transmitted to the support also change.

Consider the case of identical initial clearances in contact pairs. Figure 1(a) shows the initial location of the satellite and the pinwheel, while all the initial clearances in the meshing in the direction of the normals to the tooth profiles are the same and equal to $\Delta_z$. When determining the forces in contact, it is usually assumed that when the wheel is stationary, the satellite rotates by a certain angle $\beta$ under the action of the moment applied to it [1]. In this case, in the $i$-th contact pair, the normal to the contact surfaces approaches and deformation appears in some pairs $\delta_i = \beta l_i - \Delta_z$. The value $l_i = r_{wi} \sin \theta_i$ is the distance from the center of the $O_i$ satellite to the line of action of the force in the contact going from the center of the pin $i$ to the gearing pole $P_z$. Therefore

$$\delta_i = \beta r_{wi} \sin \theta_i - \Delta_z.$$  \hspace{1cm} (1)

The forces $F_z$ acting in the gearing can be determined by the formula [8]:

$$F_z = k_z \frac{4T}{z_2 r_{wi}} (\sin \theta_i - p_w (1 - \sin \theta_i)) \text{при } \sin \theta_i - p_w (1 - \sin \theta_i) \geq 0$$  \hspace{1cm} (2)
where \( i \) — the number of the rollers in the pinwheel, counting from the roller located on the vertical axis of symmetry, it is assigned the number one; \( T_s \) — satellite torque; \( k_z \) — coefficient equal to the ratio of the maximum forces in gearing with and without clearance [9]; \( z_2 \) — the number of rollers in the pinwheel; \( r_{w1} \) — satellite initial radius; \( \lambda \) — coefficient of shortening of the epicycloids, \( \lambda = r_{w2}/r_{w2} \); \( r_{w2} \) — radius of the centrode circle of the pinwheel, \( r_{w2} = z_2a_w \); \( a_w \) — center distance; \( r_{p} \) — pin-roller radius; \( p_{un} \) — a parameter equal to the ratio of the initial clearance in gearing to the maximum strain.

Assuming that \( \forall \) all the forces in gearing with the gap are directed to the pole \( P_z \), then the sum of the projections of the forces on the axis \( OX \) both in the gears with the gap and in the gears without the gap:

\[
\sum_i F_{zix} = T_i/r_{w1} \tag{3}
\]

Force projection \( F_{zj} \) on the axis \( OY \)

\[
F_{zj} = F_{zj} \cos \theta_j \tag{4}
\]

The sum of the projections of the forces acting in meshing on the axis \( OY \):

\[
\sum_i F_{zj} = \sum_i F_{zj} \cos \theta_i = k_z4T_s(z_2r_{w1})^{-1}\sum_i[(1 + p_{un})\sin \theta_i - p_{un}]\cos \theta_i \tag{5}
\]

where \( \sin \theta_i = \sin \tau_{ci}/(1 - 2\lambda \cos \tau_{ci} + \lambda^2)\), \( \cos \theta_j = (\lambda - \cos \tau_{ci})/(1 - 2\lambda \cos \tau_{ci} + \lambda^2)\), \( \tau_{ci} \) — pin angle on the pin wheel.

Calculate the sum \( \sum \sin \theta_i \cos \theta_j \) according to the formula:

\[
\sum \sin \theta_i \cos \theta_j = \frac{z_2}{2\pi} \int_{\tau_{ci}} \frac{\sin \tau_{ci}(\lambda - \cos \tau_{ci})d\tau_{ci}}{1 - 2\lambda \cos \tau_{ci} + \lambda^2} \tag{6}
\]

Integration in formula (6) extends to the contact zone, the position of which can be determined by the values \( \Delta z \), \( \beta \) and \( r_{w1} \). The integral in expression (6):

\[
I_1 = \int_{\tau_{ci}} \frac{\sin \tau_{ci}(\lambda - \cos \tau_{ci})d\tau_{ci}}{1 - 2\lambda \cos \tau_{ci} + \lambda^2} = \frac{1}{2\lambda} \left[ \frac{\cos \tau_{c2} - \frac{1 + \lambda^2}{2\lambda}}{\cos \tau_{c1} - \frac{1 + \lambda^2}{2\lambda}} + \cos \tau_{c1} - \cos \tau_{c2} \right] \tag{7}
\]

Coefficient of radial force:

\[
k_y = \sum_i F_{zij} / \sum_i F_{zix} = k_z \frac{4}{z_2} \sum_i [(1 + p_{un})\sin \theta_i - p_{un}]\cos \theta_i \tag{8}
\]

For gearing without clearance \( k_z = 1 \), \( p_{un} = 0 \), \( \tau_{c1} = 0 \), \( \tau_{c2} = \pi \) and
The sum \( \sum \cos \theta_i = \sum_{i}(\lambda - \cos \tau_{ci})/\left(1 - 2\lambda \cos \tau_{ci} + \lambda^2\right) \) can be calculated by the formula:

\[
\sum \cos \theta_i \approx \frac{z}{2\pi} \int_{\tau_{ci}}^{\tau_{ci}^2} \frac{(\lambda - \cos \tau_c) d\tau_c}{\sqrt{1 - 2\lambda \cos \tau_c + \lambda^2}} = \frac{z}{4\pi\lambda} \left[\left(\frac{\lambda^2}{2}\right) - 1\right] \int_{\tau_{ci}}^{\tau_{ci}^2} \frac{d\tau_c}{\sqrt{1 - 2\lambda \cos \tau_c + \lambda^2}} + \int_{\tau_{ci}}^{\tau_{ci}^2} \sqrt{1 - 2\lambda \cos \tau_c + \lambda^2} d\tau_c
\]

The first integral in (10) can be calculated using the normal elliptic Legendre integral of the first kind \( F(k, \varphi) \) [9]:

\[
I_2 = \int_{\tau_{ci}}^{\tau_{ci}^2} \frac{d\tau_c}{\sqrt{1 - 2\lambda \cos \tau_c + \lambda^2}} = \frac{2}{1 + \lambda} \left[F(k, \varphi_2) - F(k, \varphi_1)\right]
\]

The second integral in (10) can be calculated using the normal elliptic Legendre integral of the second kind \( E(k, \varphi) \):

\[
I_3 = \int_{\tau_{ci}}^{\tau_{ci}^2} \sqrt{1 - 2\lambda \cos \tau_c + \lambda^2} d\tau_c = 2(1 + \lambda) \left[E(k, \varphi_2) - E(k, \varphi_1)\right] + \frac{4\lambda}{\sqrt{1 - 2\lambda \cos \tau_{ci} + \lambda^2}} - \frac{\sin \tau_{ci}}{\sqrt{1 - 2\lambda \cos \tau_{ci} + \lambda^2}} + \frac{\sin \tau_{ci}^2}{\sqrt{1 - 2\lambda \cos \tau_{ci}^2 + \lambda^2}}
\]

where \( k \) = elliptic integral module:

\[
k = 2\sqrt{k}/(1 + \lambda)
\]

\( \varphi_{1,2} \) = amplitude of integral:

\[
\varphi_{1,2} = \arcsin \frac{1 + \lambda}{\sqrt{2}} \sqrt{\frac{1 - \cos \tau_{ci,2}}{1 + \lambda^2 - 2\lambda \cos \tau_{ci,2}}}
\]

Then, the sum of the projections of the forces acting in meshing onto the axis \( Oy \):

\[
\sum F_{xy} = k_z \frac{2T_s}{2\pi R_{sw}} \left(1 + P_{tu}^{-1} \right) Y_1 + \frac{P_{tu}}{2\lambda} \left[1 - \lambda^2 \right] Y_2 + I_3
\]

Coefficient of radial force:
When turning from the initial position of the satellite, the forces in the contact pairs change; the total radial force transmitted to the satellite support also changes.

3. Solution interpretations

Figure 2 shows graphs of the coefficients of the radial gear force for three values of the epicycloid shortening coefficient as a function of the initial gear clearance. Three curves calculated by formula (16) from Figure 2 (a) in Figure 2 (b) are highlighted by thickness and color. Comparison of the calculation results by formula (16) and the complete transmission calculation using a special program shows close agreement.

![Figure 2](image)

**Figure 2.** Dependence of the coefficient of radial force in gearing on the coefficient of shortening of the epicycloid and the initial clearance in the gearing.

The minimum value of the coefficient of radial force in the gearing is zero if one contact pair remains in the gearing at any time and the force in the contact is directed perpendicular to the center line.

Figure 3 shows the change in the forces in the gearing with clearance $\Delta_z = 0.03 \text{ mm}$ along the contact pairs of the satellite tooth - pin on the rotation of the pin wheel by an angle $\gamma$ (in fractions of the angular pitch). In this case, the entire load is perceived only by five contact pairs (with the number of pins $z = 34$).

Figure 4 shows the change in the radial force coefficient $k_y$. When the satellite rotates from its initial position, the forces in the contact pairs change; the total radial force transmitted to the satellite support also changes. When the fore-wheel rotates by an angular pitch, the pattern of power loading is repeated.

$$k_y = k_z \frac{2}{\pi} \left\{ (1 + p_{un}) I_1 + \frac{P_{un}}{2\lambda} \left[ (1 - \lambda^2) I_2 + I_3 \right] \right\}$$

(16)
1 – \( \gamma = 0; \) 2 – \( \gamma = 0,2; \) 3 – \( \gamma = 0,4; \) 4 – \( \gamma = 0,6; \) 5 – \( \gamma = 0,8; \) 6 – \( \gamma = 1,0 \)

Figure 3. The forces acting in meshing at different angles of rotation of the satellite \( \gamma \) relative to the initial position (in fractions of the angular pitch).

4. Conclusions

The results of the analysis show that the radial force in the gearing decreases with a decrease in the shortening coefficient of the epicycloid and is especially noticeable with an increase in the clearance in the gearing. The total radial forces in the gearing and the mechanism \( W \) change in the working gear with periods equal to the angular steps of the location of the rollers.

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