Bethe-Salpeter equation for doubly heavy baryons in the covariant instantaneous approximation

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In the heavy quark limit, a doubly heavy baryon is regarded as composed of a heavy diquark and a light quark. We establish the Bethe-Salpeter (BS) equations for the heavy diquarks and the doubly heavy baryons, respectively, to leading order in a $1/m_Q$ expansion. The BS equations are solved numerically under the covariant instantaneous approximation with the kernels containing scalar confinement and one-gluon-exchange terms. The masses for the heavy diquarks and the doubly heavy baryons are obtained and the non-leptonic decay widths for the doubly heavy baryons emitting a pseudo-scalar meson are calculated within the model.

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I. Introduction

The past few years have seen many important developments concerning hadron colliders, especially the advent of the LHC. Recently, the discovery of and searches for double charm baryons have been reported by various experimental collaborations [1–4]. One is convinced that more and more doubly heavy baryons will be observed in the near future. Consequently, it is an urgent task for theorists to investigate the properties of these states.

On the other hand, the existence of three valence quarks in a baryon makes the theoretical study much more complicated than the case of mesons. People have suggested the presence of diquark structure in a baryon and studied the properties of heavy baryons in such a picture [5–9]. No matter whether the diquark is a real physical object or simply a theoretical approximation, this picture reduces the three body system to a two body problem which is much simpler for investigation.

In recent years, heavy quark effective theory (HQET) has been widely used in the study of doubly heavy baryons [10–17]. Two heavy quarks are reasonably bound into a color-antitriplet heavy diquark whose radius is much smaller than the typical scale ($1/\Lambda_{QCD}$) of the nonperturbative QCD interactions in the heavy quark limit ($m_Q \gg \Lambda_{QCD}$, $m_Q$ denotes the heavy quark mass). The leftover light quark involved in the baryon moves in the color field induced by the heavy diquark. Unlike the heavy quark and light quark system, the internal motion in the heavy diquark cannot be ignored even at leading order in the $1/m_Q$ expansion. This is because the relative momentum in the heavy diquark is not simply $O(\Lambda_{QCD})$, but $\sim \alpha_s^2 m_Q$ as calculated in the Coulomb potential model [11, 18].

As a formally exact equation to describe the relativistic bound system, the Bethe-Salpeter (BS) equation was initially formulated in Minkowski space based on the relativistic quantum theory [19, 20]. However, it is difficult to solve the BS equation in Minkowski space due to its singular behavior. In order to overcome

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this difficulty, with the so-called ”Wick rotation”, the formalism for the BS equation in Euclidean space was developed and investigated in detail [21–24]. With the perturbation theory integral representation, the BS equation was solved in Minkowski space for the scalar and fermion systems [25–30]. Recently, based on the Nakanishi integral representation of the BS amplitude and the projection of the BS equation on the light-front plane, a new method for solving the BS equation in Minkowski space was proposed and was applied to study the electromagnetic form factor [31–36]. In another formalism to solve the BS equation, the covariant instantaneous approximation is adopted in the kernel. In recent decades, this formalism has been successfully used to investigate heavy mesons, heavy baryons, and exotic states [7–9, 12, 37–44].

Based on the diquark picture of the composition of the doubly heavy baryon, we will establish the BS equations for both the heavy diquarks and the doubly heavy baryons in the leading order of a $1/m_Q$ expansion. Motivated by the potential model, the kernel for the BS equation is assumed to be composed of the scalar confinement and one-gluon-exchange terms [45]. We will solve the BS equations numerically under the covariant instantaneous approximation [41–44]. Since the heavy diquark is not really a point object, a few form factors for the effective vertex of the heavy diquark coupling to the gluon are introduced to reflect the inner structure of the heavy diquark. These form factors will be expressed in terms of the BS wave functions obtained for the heavy diquarks. Finally, we will calculate the non-leptonic decay widths for the doubly heavy baryons emitting a pseudo-scalar meson in the BS formalism.

The remainder of this paper is organized as follows. In Sec. II, we establish the BS equation for the heavy diquarks in the leading order of $1/m_Q$ expansion. We also give the normalization conditions for the BS wave functions for the heavy diquarks in this section. In Sec. III, the form factors for the effective vertex of the heavy diquark coupling to the gluon are derived from the BS wave functions obtained for the heavy diquarks. In Sec. IV, we establish the BS equation for the doubly heavy baryons at leading order in the $1/m_Q$ expansion. The normalization conditions for the BS wave functions for the doubly heavy baryons are also given in this section. In Sec. V, the non-leptonic decay widths for the doubly heavy baryons emitting a pseudo-scalar meson are calculated in the BS formalism. Sec. VI is reserved for our summary and some discussions.

II. BS equation for heavy diquarks

In general, the parity of a ground state baryon is positive. Since the parity of quark is supposed to be positive, the parity of the diquark involved in a ground state baryon should be positive. Due to the Pauli principle, two quarks with the same flavor can only constitute an axial-vector diquark. On the other hand, two quarks with different flavors can constitute either a scalar diquark or an axial-vector diquark. It can be easily shown that a heavy diquark which is in the ground state can not be a tensor diquark.

Suppose two heavy quarks $Q_1$ and $Q_2$ (with masses $m_{Q_1}$ and $m_{Q_2}$ respectively) compose a ground state heavy diquark. Define two ratios $\lambda_1 = m_{Q_1}/(m_{Q_1} + m_{Q_2})$ and $\lambda_2 = m_{Q_2}/(m_{Q_1} + m_{Q_2})$. The BS wave function for the heavy diquark is defined as follows:

$$\chi_{PD}(x_1, x_2)_{\alpha\beta} = \epsilon^{ijk} \langle 0 | T \psi_1(x_1)^i_\alpha \psi_2(x_2)^j_\beta | PD, k \rangle$$

$$= e^{-ip_Dx} \int \frac{d^4p}{(2\pi)^4} \chi_{PD}(p)_{\alpha\beta} e^{-ipx}, \quad (1)$$

$$\bar{\chi}_{PD}(x_2, x_1)_{\beta\alpha} = \epsilon^{ijk} \langle PD, k | T \psi_2^*(x_2)^i_\beta \psi_1^*(x_1)^j_\alpha | 0 \rangle$$

$$= e^{ip_Dx} \int \frac{d^4p}{(2\pi)^4} \bar{\chi}_{PD}(p)_{\beta\alpha} e^{ipx}, \quad (2)$$
where $\psi_1$ and $\psi_2$ stand for the field operators of the heavy quarks $Q_1$ and $Q_2$, respectively, $i$, $j$, $k$ represent the color indices, $\alpha$ and $\beta$ represent the spin indices, $X \equiv \lambda_1 x_1 + \lambda_2 x_2$ is the coordinate of the heavy diquark mass center, $x \equiv x_1 - x_2$ is the relative coordinate of the two heavy quarks, $P_D$ is the momentum of the heavy diquark, and $p$ is the relative momentum between the two heavy quarks.

The BS equation for the heavy diquark can be written in the following form (details can be found in Ref. [3]):

$$\chi_{P_D}(p) = S(p_1) \otimes S(p_2) \int \frac{d^4p'}{(2\pi)^4} [\gamma^\mu \otimes \gamma_\mu K^{(1g)}(p - p') + I \otimes IK^{(cf)}(p - p')] \chi_{P_D}(p'),$$

(3)

where $p_1 = \lambda_1 P_D + p$ and $p_2 = \lambda_2 P_D - p$ are the momenta of heavy quarks $Q_1$ and $Q_2$, respectively, $S(p_1)$ and $S(p_2)$ are the propagators of heavy quarks $Q_1$ and $Q_2$, respectively. $K^{(1g)}$ and $K^{(cf)}$ are the one-gluon-exchange and scalar confinement terms of the kernel for the BS equation given by (after imposing the covariant instantaneous approximation [41, 44]):

$$K^{(1g)}(p_i - p'_i) = -\frac{8i\pi}{3} \frac{\alpha_s}{(p_i - p'_i)^2 - \mu^2},$$

(4)

and

$$K^{(cf)}(p_i - p'_i) = \frac{4i\pi\kappa}{[-(p_i - p'_i)^2 + \mu^2]^2} - \frac{\delta^3(p_i - p'_i)}{(2\pi)^3} \frac{4i\pi\kappa}{[-(p_i - k_i)^2 + \mu^2]^2},$$

(5)

where $\alpha_s$ and $\kappa$ are the coupling parameters related to one-gluon-exchange and scalar confinement terms, respectively, $p_i$ is the transverse projection of the relative momentum $(p)$ along the heavy diquark momentum $(P_D)$ (see the definition below Eq. (8)), the second term of $K^{(cf)}$ is introduced to remove the infrared singularity near the point $p_i' = p_i$, and the small parameter $\mu$ is introduced to avoid the divergence in the numerical calculations. This kernel is motivated by the potential model which has been successfully applied in mesons [45]. Furthermore, we assume that the kernel of the heavy diquark is related to the meson by the one-half rule [46, 47].

Eq. (6) can be written in a more usual matrix form as [9]

$$\tilde{\chi}_{P_D}(p) = S(p_2) \int \frac{d^4p'}{(2\pi)^4} [-\gamma^\mu \chi_{P_D}(p')\gamma_\mu K^{(1g)}(p_i - p'_i) + \tilde{\chi}_{P_D}(p')K^{(cf)}(p_i - p'_i)]S(-p_1),$$

(6)

where $\tilde{\chi}_{P_D}(p) = C\chi_{P_D}(p)$ ($C$ is the charge conjugation matrix) and the superscript $T$ represents the transpose of the spinor indices.

In the leading order of a $1/m_Q$ expansion, the heavy quark propagators ($S(p_1)$ and $S(p_2)$) can be written as

$$S(p_1) = \frac{i m_Q (\not\! p_D + 1)}{2\omega_{Q_1} (\lambda_1 m_D + p_l - \omega_{Q_1} + i\epsilon)},$$

(7)

and

$$S(p_2) = -i \frac{m_Q (\not\! p_D + 1)}{2\omega_{Q_2} (-\lambda_2 m_D + p_l + \omega_{Q_2} - i\epsilon)},$$

(8)

where $m_D$ and $v_D$ are the mass and velocity of the heavy diquark, respectively, $p_l = p \cdot v_D$ and $p'_l = p' \cdot v_D$ are the longitudinal and transverse projections of the relative momentum $(p)$ along the heavy diquark momentum $(P_D)$, respectively, the energy $\omega_{Q_{i(j)}} = \sqrt{m_{Q_{i(j)}}^2 - p_l^2}$, and $\epsilon$ is the infinitesimal. As studied in Refs. [10, 11], $|p_i| = O(\alpha_s^2 m_Q)$. Consequently, $|p_i|/m_Q$ terms should not be neglected in calculations carried out to leading order in the $1/m_Q$ expansion.
Substituting Eqs. (7) (8) into Eq. (6), one finds the following two constraints for the BS wave function for the heavy diquarks:

\[
\hat{\gamma}_D \tilde{\xi}^T_{P_0} (p) = \tilde{\xi}^T_{P_0} (p),
\]

are the Lorentz-scalar functions of \( p_1^2, p_1, \) and \( P_D^2 = m_D^2. \)

After some algebra, we find that the BS scalar wave functions for both the scalar heavy diquark \( (f_1) \) and the axial-vector heavy diquark \( (f_2) \) satisfy the same integral equation as follows:

\[
\hat{f}(p_t) = \frac{m_{Q_1} m_{Q_2}}{\omega_{Q_1} \omega_{Q_2} (-m_D + \omega_{Q_1} + \omega_{Q_2})} \int \frac{d^4 p'_t}{(2\pi)^4} [V^{(1g)}(p_t - p'_t) + V^{(cf)}(p_t - p'_t)] \hat{f}(p'_t),
\]

where we define \( \hat{f}(p_t) = \int \frac{d^4 p'_t}{(2\pi)^4} f_{1(2)}(p_t), \) \( V^{(1g)} = -i K^{(1g)}, \) and \( V^{(cf)} = -i K^{(cf)}. \)

In general, the normalization condition for the heavy diquark can be written as (after imposing the covariant instantaneous approximation on the kernel)

\[
\frac{1}{36} \delta^{i_1 j_1} \int \frac{d^4 p d^4 p'}{(2\pi)^8} \tilde{\chi}_{P_0}(p) \frac{\partial}{\partial P_D^2} \left[ I_{P_0}(p, p') \right]^{i_1 j_1} \tilde{\chi}_{P_0}(p') = 1,
\]

where \( i_{1(2)} \) and \( j_{1(2)} \) represent the color indices of the heavy quarks, \( \delta^{i_1 i_2} = \delta^{i_1} \delta^{i_2} - \delta^{i_2} \delta^{i_1} \) and \( I_{P_0}^{i_1 i_2 j_1 j_2}(p, p') \) stands for the inverse of the four point function,

\[
I_{P_0}^{i_1 i_2 j_1 j_2}(p, p') = \delta^{i_1 j_1} \delta^{i_2 j_2} (2\pi)^4 \delta^4(p - p') |S(p_1)\gamma_0|^{-1} |S(p_2)\gamma_0|^{-1}.
\]

Now, it is straightforward to obtain the normalization condition for the BS wave function for the heavy diquark as the following:

\[
-\frac{1}{6} \int \frac{d^4 p}{(2\pi)^4} \left\{ \text{Tr} \left[ \lambda_1 S(-p_1) \tilde{\chi}^{(c)}_{P_0}(p_t) S(p_2) \tilde{\xi}^T_{P_0}(p_t) S(-p_1) \hat{f} \right] + \text{Tr} \left[ \lambda_2 S(-p_1) \tilde{\chi}^{(c)}_{P_0}(p_t) S(p_2) \hat{f} S(p_2) \tilde{\xi}^T_{P_0}(p_t) S(-p_1) \right] \right\} = 1,
\]

where \( \varepsilon = (1, 0) \), \( \tilde{\chi}_{P_0}(p_t) \) and \( \tilde{\chi}^{(c)}_{P_0}(p_t) \) are the transverse projections of the BS wave function given by

\[
\tilde{\chi}_{P_0}(p_t) = -i S(p_2)^{-1} \tilde{\xi}^T_{P_0}(p) S(-p_1)^{-1},
\]

and

\[
\tilde{\chi}^{(c)}_{P_0}(p_t) = C \tilde{\chi}^{(c)}_{P_0}(p) C^{-1}.
\]
respectively.

For the scalar heavy diquark, the transverse projections of the BS wave function \( \tilde{\chi}_{P_0}(p_t) \) and \( \tilde{\chi}^{(c)}_{P_0}(p_t) \) can be obtained from Eqs. (17) (18) as follows respectively:

\[
\tilde{\chi}_{P_0}(p_t) = f_{s_1}(p_t)\gamma^5 + f_{s_2}(p_t)\gamma^5,
\]

(19)

and

\[
\tilde{\chi}^{(c)}_{P_0}(p_t) = \tilde{f}_{s_1}(p_t)\gamma^5 - \tilde{f}_{s_2}(p_t)\gamma^5,
\]

(20)

where

\[
f_{s_1}(p_t) = \int \frac{d^3p_t'}{(2\pi)^3} [V^{(1g)}(p_t - p_t') + 4V^{(cf)}(p_t - p_t')] \tilde{f}(p_t'),
\]

(21)

and

\[
f_{s_2}(p_t) = \int \frac{d^3p_t'}{(2\pi)^3} [V^{(1g)}(p_t - p_t') - 2V^{(cf)}(p_t - p_t')] \tilde{f}(p_t').
\]

(22)

After substituting Eqs. (19) (20) into Eq. (16), carrying out the trace calculation, and integrating out the longitudinal momentum \( p_t \), we obtain the normalization condition for the BS wave function for the scalar heavy diquark as the following:

\[
\int \frac{d^3p_t}{(2\pi)^3} E_{Dm_{Q_1}m_{Q_2}}(\lambda_1 m_{Q_1}\omega_{Q_2} + \lambda_2 m_{Q_2}\omega_{Q_1}) [f_{s_1}(p_t) + f_{s_2}(p_t)]^2 = 1,
\]

(23)

where \( E_D = P_D \cdot \epsilon \).

For the axial-vector heavy diquark, the transverse projections of the BS wave function \( \tilde{\chi}^{(r)}_{P_0}(p_t) \) and \( \tilde{\chi}^{(r)(c)}_{P_0}(p_t) \) can be obtained from Eqs. (17) (18) as follows respectively:

\[
\tilde{\chi}^{(r)}_{P_0}(p_t) = \tilde{f}_{v_1}(p_t)\tilde{g}^{(r)}(p_t) + \tilde{f}_{v_2}(p_t)\tilde{g}\tilde{g}^{(r)},
\]

(24)

and

\[
\tilde{\chi}^{(r)(c)}_{P_0}(p_t) = -\tilde{f}_{v_1}(p_t)\tilde{g}^{(r)}(p_t) + \tilde{f}_{v_2}(p_t)\tilde{g}\tilde{g}^{(r)},
\]

(25)

where

\[
f_{v_1}(p_t) = \int \frac{d^3p_t'}{(2\pi)^3} [V^{(1g)}(p_t - p_t') + 2V^{(cf)}(p_t - p_t')] \tilde{f}(p_t'),
\]

(26)

and

\[
f_{v_2}(p_t) = \int \frac{d^3p_t'}{(2\pi)^3} V^{(1g)}(p_t - p_t') \tilde{f}(p_t').
\]

(27)

Analogously, the normalization condition for the BS wave function for the axial-vector heavy diquark is given by

\[
\int \frac{d^3p_t}{(2\pi)^3} \frac{E_{Dm_{Q_1}m_{Q_2}}(\lambda_1 m_{Q_1}\omega_{Q_2} + \lambda_2 m_{Q_2}\omega_{Q_1})}{3m_D\omega_{Q_1}\omega_{Q_2} (-m_D + \omega_{Q_1} + \omega_{Q_2})} [\tilde{f}_{v_1}(p_t) + \tilde{f}_{v_2}(p_t)]^2 = 1.
\]

(28)

In the numerical calculations, we take the constituent masses of the heavy quarks to be \( m_t = 4.88 \text{ GeV} \) and \( m_c = 1.486 \text{ GeV} \) which were obtained by fitting the real spectra of charmonium and bottomonium in Ref. [43].
The parameters in the kernel $\alpha_s = 0.4$ and $\kappa = 0.18$ are determined by fitting the experimental data for heavy meson spectra \cite{45}. In order to solve the integral equation (13), we discretize the integration region into $n$ pieces (with $n$ sufficiently large). Then Eq. (13) becomes an eigenvalue equation for the $n$ dimensional vector $\tilde{f}$. After solving the eigenvalue equation, the heavy diquark masses are obtained and are displayed in Table I. We find that the heavy diquark masses are independent of the heavy diquark spin and only determined by the flavors of the constituent heavy quarks. In Fig. 1, the normalized BS scalar wave functions for the heavy diquarks are shown. It can be seen that the amplitudes of the BS scalar wave functions do not distinguish the different spins of the heavy diquarks. As discussed in Refs. [10, 11], unlike the heavy quark and light quark system, only the spin symmetry survives (when the diquark mass is below $\sim 10$ GeV) in the leading order $1/m_Q$ expansion for the heavy diquark system. Our results are consistent with this statement.

**TABLE I: Values of the heavy diquark masses used here.**

| $m_{Q_1}$ (GeV) | 4.88 | 4.88 | 1.486 |
|-----------------|------|------|-------|
| $m_{Q_2}$ (GeV) | 4.88 | 1.486| 1.486 |
| $m_D$ (GeV)     | 9.80 | 6.55 | 3.23  |

**FIG. 1:** The normalized BS scalar wave functions ($\tilde{f}(p_t)$) for the heavy diquarks. The solid, dashed, and dotted lines are for the heavy diquarks composed of double $b$ quarks, $b$ and $c$ quarks, and double $c$ quarks, respectively.

### III. Form factor of heavy diquark coupling to gluon

Since the heavy diquark is not really a point object and its radius is enhanced by $\ln^2m_Q$ with respect to $1/m_Q$, we introduce a few form factors for the effective vertex of the heavy diquark coupling to gluon to reflect the inner structure of the heavy diquark.

The effective current for the scalar heavy diquark coupling to a gluon is given as follows \cite{5}:

$$J^\mu = ig s \frac{\lambda^a}{2} (P_{Df}^\mu + P_{Di}^\mu) F_s(Q^2),$$  \hspace{1cm} (29)
where $g_s$ is the coupling constant of the strong interaction, $\lambda^a$ ($a = 1, 2, \cdots, 8$) denote the Gell-Mann color matrices, $F_s(Q^2)$ is the form factor for the effective vertex, $P_{D_i}^\mu$ and $P_{D_f}^\mu$ are the momenta of the initial and final heavy diquarks, respectively, and $Q^2$ is the square of the momentum transfer.

On the other hand, the effective current for the scalar heavy diquark coupling to a gluon can be written as the following in the BS equation formalism (The Feynmann diagram for the vertex is shown in Fig. 2):

$$J^\mu = ig_s\frac{\lambda^a}{2}(M_1^\mu + M_2^\mu),$$

where

$$M_1^\mu = -\frac{1}{6}(2\pi)^4\delta^4(P_{D_i} - P_{D_f} - k)\int \frac{d^4p}{(2\pi)^4}\int \frac{d^4p'}{(2\pi)^4}(2\pi)^4\delta^4(p_2 - p_2')\times\text{Tr}[S(-p_1)\tilde{\chi}_{P_{D_f}}(p_1)\tilde{\chi}_{P_{D_i}}(p_2)\tilde{\chi}_{P_{D_i}}(p_1')\tilde{\chi}_{P_{D_f}}(p_2')v'^\mu],$$

and

$$M_2^\mu = -\frac{1}{6}(2\pi)^4\delta^4(P_{D_i} - P_{D_f} - k)\int \frac{d^4p}{(2\pi)^4}\int \frac{d^4p''}{(2\pi)^4}(2\pi)^4\delta^4(p_1 - p''_1)\times\text{Tr}[S(-p_1)\tilde{\chi}_{P_{D_f}}(p_1)\tilde{\chi}_{P_{D_i}}(p_2)v''^\mu\tilde{\chi}_{P_{D_i}}(p_1')\tilde{\chi}_{P_{D_f}}(p_2')],$$

where $k$ denotes the momentum carried by the gluon, $p$ and $p''^\mu$ in this section denote the relative momenta of final and initial diquarks, respectively.

Then, comparing Eq. (29) with Eq. (30), one can get the form factor ($F_s(Q^2)$) in the leading order of a $1/m_Q$ expansion as follows:

$$F_s(Q^2) = -\frac{(2\pi)^4}{3m_D}\int \frac{d^4p_t}{(2\pi)^4}\left\{\frac{m_{Q_2}^2[m_{Q_2}^2[f_{s1}(p_1) + f_{s2}(p_1)][f_{s1}(p_1') + f_{s2}(p_1')]}{\omega_{Q_1}\omega_{Q_2}\omega_{Q_1}(m_D - \omega_{Q_1} - \omega_{Q_2})(m_D - \omega_{Q_1}\tilde{\omega} - |p_t|\sqrt{\tilde{\omega}^2 - 1cos\theta - \omega_{Q_1}^2})} + \frac{m_{Q_1}m_{Q_2}^2[f_{s1}(p_1) + f_{s2}(p_1)][f_{s1}(p_1') + f_{s2}(p_1')]}{\omega_{Q_1}\omega_{Q_2}\omega_{Q_2}(m_D - \omega_{Q_1} - \omega_{Q_2})(m_D - \omega_{Q_1}\tilde{\omega} + |p_t|\sqrt{\tilde{\omega}^2 - 1cos\theta - \omega_{Q_2}^2})}\right\},$$

where the velocity transfer $\tilde{\omega} = v_{D_i} \cdot v_{D_f}$ ($v_{D_i}$ and $v_{D_f}$ are the velocities of the initial and final heavy diquarks respectively), $p_t^{(in)} = p^{(in)} \cdot v_{D_i}$, and $p_t^{(out)} = p^{(out)} - p_t^{(in)} v_{D_i}$, are the longitudinal and transverse projections of the initial heavy diquark relative momenta ($p^{(in)}$) along their momenta ($P_{D_i}$), respectively, $\omega_{Q_1} = \sqrt{m_{Q_1}^2 - p_t^{(in)}^2}$, $\omega_{Q_2} = \sqrt{m_{Q_2}^2 - p_t^{(out)}^2}$, and $\theta$ is the angle between $p_t$ and $P_{D_i}^{\mu} (= P_{D_i}^{\mu} - v_{D_i}^{\mu}P_{D_i} \cdot v_{D_i}$).

Now, let us turn to the effective current of the axial-vector heavy diquark coupling to gluon [3].

$$J^{\alpha\beta} = ig_s\frac{\lambda^a}{2}\left[g^{\alpha\beta}(P_{D_f}^\mu + P_{D_i}^\mu)F_{v_1}(Q^2) - (P_{D_f}^\beta g^{\alpha\mu} + P_{D_f}^{\mu\beta})F_{v_2}(Q^2) + P_{D_f}^\beta P_{D_i}^\beta (P_{D_f}^\mu + P_{D_i}^\mu)F_{v_3}(Q^2)\right],$$

![FIG. 2: The schematic diagram for the heavy diquark coupling to gluon. $Q_1(p_1^{(i)})$ and $Q_2(p_2^{(i)})$ stand for the heavy quarks $Q_1$ and $Q_2$ with momenta $p_1^{(i)}$ and $p_2^{(i)}$, respectively.](image)
where \( F_1(Q^2) \), \( F_2(Q^2) \), and \( F_3(Q^2) \) are the form factors for the effective vertex.

Analogously, the effective current of the axial-vector heavy diquark coupling to the gluon can be written as follows in the BS equation formalism:

\[
J^{\alpha\mu\beta} = ig_s \frac{\lambda^\alpha}{2} (M_1^{\alpha\mu\beta} + M_2^{\alpha\mu\beta}),
\]

where

\[
M_1^{\alpha\mu\beta} = -\frac{1}{6}(2\pi)^4 \delta^4(P_D - P_{Df} - k) \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4p'}{(2\pi)^4}(2\pi)^4 \delta^4(p_2 - p'_2) \times \text{Tr}[S(-p_1)\tilde{\chi}_{\rho\nu}^\alpha(p_1)S(p_2)\tilde{\chi}_{\rho\nu}^\beta(p'_1)S(-p'_1)\gamma^\mu],
\]

and

\[
M_2^{\alpha\mu\beta} = -\frac{1}{6}(2\pi)^4 \delta^4(P_D - P_{Df} - k) \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4p''}{(2\pi)^4}(2\pi)^4 \delta^4(p_1 - p''_1) \times \text{Tr}[S(-p_1)\tilde{\chi}_{\rho\nu}^\alpha(p_1)S(p_2)\gamma^\mu S(p''_2)\tilde{\chi}_{\rho\nu}^\beta(p''_1)].
\]

As discussed in Ref. [3], the contribution of the \( F_3(Q^2) \) term is suppressed at small and intermediate momentum transfer, \( Q^2 \), since such a term is multiplied by high powers of momenta. Consequently, \( F_3(Q^2) \) is ignored in our calculation. Comparing Eq. (34) with Eq. (35), we derive the other two form factors (\( F_1(Q^2) \) and \( F_2(Q^2) \)) in the leading order of a \( 1/m_Q \) expansion as follows:

\[
F_1(Q^2) = -\frac{(2\pi)^4}{12m_D} \int \frac{d^3p_1}{(2\pi)^3} \left\{ \frac{m_{Q_1} m_{Q_2}}{\omega_{Q_1} \omega_{Q_2} \omega_{Q_2}^\prime} \left[ \tilde{f}_{v_1}(p_1) + \tilde{f}_{v_2}(p_1) \right] \left[ \tilde{f}_{v_1}(p'_1) + \tilde{f}_{v_2}(p'_1) \right] \right. \\
+ \frac{m_{Q_1} m_{Q_2}}{\omega_{Q_1} \omega_{Q_2} \omega_{Q_2}^\prime} \left[ \tilde{f}_{v_1}(p_1) + \tilde{f}_{v_2}(p_1) \right] \left[ \tilde{f}_{v_1}(p''_1) + \tilde{f}_{v_2}(p''_1) \right] \right\},
\]

\[
F_2(Q^2) = 0.
\]

It can be seen that the form factors for both the effective vertex of the scalar heavy diquark coupling to the gluon and the effective vertex of the axial-vector heavy diquark coupling to the gluon are equal to each other in the leading order of a \( 1/m_Q \) expansion. So, we redefine the form factors as \( F(Q^2) \equiv F_1(Q^2) = F_2(Q^2) \) for convenience.

It is well known that when \( Q^2 \to 0 \), the heavy diquark is seen by the gluon as a point particle without the inner structure, and hence the form factor for the effective vertex should be normalized to unity. When \( Q^2 \to \infty \), the gluon can see the individual quarks inside the diquark, and hence the form factor for the effective vertex should approach to zero. We calculate the form factors with the BS wave functions obtained numerically for the heavy diquarks. The dependence of the form factors on the square of the momentum transfer is shown in Fig. 3. We can see that the behavior of our results coincides with the tendency of the above physical picture.

**IV. BS equation for doubly heavy baryons**

As discussed in Sec. I, the doubly heavy baryon can be regarded as a bound state of a heavy diquark and a light quark in the heavy quark limit.

Let us define the ratios \( \eta_1 = m_l/(m_l + m_D) \) and \( \eta_2 = m_D/(m_l + m_D) \) (\( m_l \) is the light quark mass) for convenience. The BS wave function for the doubly heavy baryon composed of a scalar heavy diquark and a
light quark is defined as the following:

$$\chi_P(y_1, y_2) = \langle 0| \psi_l(y_1) \phi_D(y_2)|P \rangle = e^{-iPY} \int \frac{d^4q}{(2\pi)^4} \chi_P(q) e^{-iqy}, \quad (39)$$

where $\psi_l$ and $\phi_D$ stand for the light quark field and the scalar heavy diquark field, respectively, $Y \equiv \eta_1 y_1 + \eta_2 y_2$ is the coordinate of the doubly heavy baryon mass center, $y \equiv y_1 - y_2$ is the relative coordinate, $P$ is the momentum of the doubly heavy baryon, and $q$ is the relative momentum between the heavy diquark and the light quark.

It is straightforward to derive the following BS equation for the doubly heavy baryon containing a scalar heavy diquark and a light quark:

$$\chi_P(q) = S_l(q_1) \int \frac{d^4q'}{(2\pi)^4} G(P, q, q') \chi_P(q') S_D(q_2), \quad (40)$$

where $q_1 = \eta_1 P + q$ and $q_2 = \eta_2 P - q$ are the momenta of the light quark and the heavy diquark, respectively, $S_l(q_1)$ and $S_D(q_2)$ are the propagators of the light quark and the heavy diquark, respectively, $G$ is the kernel which is, motivated by the potential model, given by [45]

$$-iG(P, q, q') = I \otimes IV_1(q, q') + \gamma_\mu \otimes \Gamma^\mu V_2(q, q'), \quad (41)$$

where $\Gamma^\mu = (q_1^\mu - q_2^\mu)F(Q^2)$ is the effective vertex of a gluon with two scalar heavy diquarks, which has been derived in Sec. III, $V_1$ and $V_2$ are the scalar confinement and one-gluon-exchange terms given in the following respectively (after imposing covariant instantaneous approximation [41–44]):

$$V_1(q_t - q'_t) = \frac{8\pi \kappa'}{[-(q_t - q'_t)^2 + \mu^2]^2} - (2\pi)^3 \delta^3(q_t - q'_t) \int \frac{d^3k_t}{(2\pi)^3} \frac{8\pi \kappa'}{[-(q_t - k_t)^2 + \mu^2]^2}, \quad (42)$$

and

$$V_2(q_t - q'_t) = -\frac{16\pi}{3} \frac{\alpha_s}{(q_t - q'_t)^2 - \mu^2}, \quad (43)$$
where \( q^{(i)} \) is the transverse projection of the relative momentum \( q \) along the baryon momentum \( P \), the second term of \( V_1 \) is introduced to remove the infrared singularity near the point \( q_t = q_t' \), and the small parameter \( \mu \) is introduced to avoid the divergence in the numerical calculations. As discussed in Ref. 3, the dimension of \( \kappa' \) is three and that of \( \kappa \) in the meson case is two. This extra dimension in \( \kappa' \) should be caused by nonperturbative diagrams which include the form factor effects at low momentum region. We expect that \( \kappa' \sim \Lambda_{QCD} \kappa \), since \( \Lambda_{QCD} \) is the only parameter related to confinement. In the numerical calculations, we let \( \kappa' \) vary between 0.01 \( GeV^3 \) and 0.06 \( GeV^3 \) [39, 49].

The light quark propagator can be written as the following form [9, 44]:

\[
S_l(q_t) = i\gamma^t \left[ \frac{\Lambda^+}{\eta_1 M + q_t - \omega_t + i\varepsilon} + \frac{\Lambda^-}{\eta_1 M + q_t + \omega_t - i\varepsilon} \right],
\]

where \( M \) and \( v \) are respectively the mass and velocity of the doubly heavy baryon, \( q_t = q \cdot v \) and \( q^\mu_t = q^\mu - q\nu v^\nu \) are the longitudinal and transverse projections of the relative momentum \( (q) \) along the doubly heavy baryon momentum \( P \), respectively, \( \omega_t = \sqrt{m_t^2 - q_t^2} \), and \( \Lambda^\pm_t \) are the projection operators given by

\[
\Lambda^\pm_t = \frac{\omega_t \mp \gamma^t q_0 + m_t}{2\omega_t}.
\]

In the leading order of a \( 1/m_Q \) expansion, the propagator of the scalar heavy diquark can be written as

\[
S_D(q_t) = \frac{i}{2m_D^2 + q_t^2},
\]

where \( u(v) \) is the spinor of the doubly heavy baryon, \( g_{s_1} \) and \( g_{s_2} \) are the Lorentz-scalar functions of \( q_t^2 \), \( q_t \) and \( P^2 = M^2 \).

Defining \( \tilde{g}_{s_1(2)}(q_t) \equiv \int \frac{dq_t}{2\pi} g_{s_1(2)}(q_t) \), one finds that the BS scalar wave functions satisfy the coupled integral equations as follows:

\[
\tilde{g}_{s_1}(q_t) = -\int \frac{d^3q_t'}{(2\pi)^3} \frac{(m_l + \omega_t)}{4\omega_l m_D(M - m_D - \omega_l)} V_1(q_t - q_t') \tilde{g}_{s_1}(q_t')
- \int \frac{d^3q_t'}{(2\pi)^3} \frac{V_1(q_t - q_t') - 2m_D F(Q^2) V_2(q_t - q_t')}{4\omega_l m_D(M - m_D - \omega_l)} q_t \cdot q_r \tilde{g}_{s_2}(q_t'),
\]

\[
\tilde{g}_{s_2}(q_t) = -\int \frac{d^3q_t'}{(2\pi)^3} \frac{V_1(q_t - q_t') + 2m_D F(Q^2) V_2(q_t - q_t')}{2\omega_l m_D(M - m_D - \omega_l)} \tilde{g}_{s_1}(q_t')
- \int \frac{d^3q_t'}{(2\pi)^3} \frac{(m_l - \omega_t)}{4\omega_l m_D(M - m_D - \omega_l)} V_1(q_t - q_t') \tilde{g}_{s_1}(q_t')
- \int \frac{d^3q_t'}{(2\pi)^3} \frac{V_1(q_t - q_t') - 2m_D F(Q^2) V_2(q_t - q_t')}{4\omega_l m_D(M - m_D - \omega_l)} q_t \cdot q_r \tilde{g}_{s_2}(q_t').
\]

The normalization condition for the doubly heavy baryon with a scalar heavy diquark and a light quark is given by (after imposing the covariant instantaneous approximation on the kernel)

\[
i\delta_{j_1 j_2} d_q d_q' \tilde{\chi}_P(q, s) \left[ \frac{\partial}{\partial P_0} I_P(q, q') \right]_{j_1 j_2} \tilde{\chi}_P(q', s') = \delta_{ss'},
\]

where \( q_1^2 = m^2 \), the instanton field \( \chi \) is normalized as

\[
\int \frac{d^4x}{(2\pi)^4} \tilde{\chi}_P(q, s) \tilde{\chi}_P(q', s') = \delta_{ss'}.
\]
where $i_1(2)$ and $j_1(2)$ represent the color indices of the heavy diquark and the light quark, respectively, $s^{(i)}$ is the spin index for the doubly heavy baryon and $I_P^{i_1i_2j_1j_2}$ is the inverse of the four point propagator defined as follows:

$$I_P^{i_1i_2j_1j_2}(q, q') = \delta^{i_1j_1} \delta^{i_2j_2} (2\pi)^4 \delta^4(q - q') S_I^{-1}(q_1) S_D^{-1}(q_2). \quad (51)$$

After substituting Eqs. (55) (56) into Eq. (52) and integrating out the longitudinal momentum $q_t$, the inverse four point propagator can be written in the following form:

$$-\frac{i}{6} \int \frac{d^4q}{(2\pi)^4} \left\{ \text{Tr} [\hat{\chi}_P(q_t) \hat{\chi}_P(q_t) S_I(q_4) S_D(q_2)] \right. \left. + \text{Tr} [\hat{\chi}_P(q_t) \hat{\chi}_P(q_t)(-2i\eta_2) q_2 \cdot \varepsilon S_I(q_1) S_D^2(q_2)] \right\} = 1, \quad (52)$$

where $\varepsilon = (1, 0)$, $\hat{\chi}_P(q_t)$ and $\hat{\chi}_P(q_t)$ are the transverse projections of the BS wave functions given as follows:

$$\hat{\chi}_P(q_t) = -iS_I(q_1)^{-1} \chi_P(q) S_D(q_2)^{-1}, \quad (53)$$

and

$$\hat{\chi}_P(q_t) = -iS_D(q_2)^{-1} \chi_P(q) S_I(q_1)^{-1}, \quad (54)$$

respectively.

Then, one can derive the transverse projections of the BS wave functions from Eqs. (53) (54) into Eq. (52) and integrating out the longitudinal momentum $q_t$, the normalization condition can be written in the following form:

$$\int \frac{d^3q_t}{(2\pi)^3} \frac{1}{24 M m_D \omega^2} \left\{ (m_1 + \omega_l)(2\eta_2 M \omega^2_l + E\eta_1(m_1 m_D - \omega m_D + M m_D - \omega) + 4\omega_l^2 \eta^2_l q_t^2 (M \omega - \omega) q_t^2) \tilde{h}_{s_1}^2(q_t) - \omega m_D + 2m_\omega + M \omega - M m_1 + E\eta_1(-M + m_D + 2\omega) q_t^2 \tilde{h}_{s_2}^2(q_t) + \eta_2 M \eta_1 \tilde{h}_{s_1}(q_t) \tilde{h}_{s_2}(q_t) + (2\eta_2 M (m_1 - \omega_l) q_t^2 \omega_l^2 + E\eta_1(-M + m_D + 2\omega) q_t^4 + E\eta_1(m_1 - \omega_l)(-m_1 m_D - \omega m_D - 2m_\omega + M (m_1 + \omega_l)) q_t^2 \tilde{h}_{s_2}^2(q_t) \right\} = 1, \quad (59)$$

where $E = P \cdot \varepsilon$. 

...
Now let us define the BS wave function for the doubly heavy baryon composed of an axial-vector heavy diquark and a light quark as follows:

\[ \chi^\mu_P(y_1, y_2) = \langle \psi(y_1) A^\mu_P(y_2) | P \rangle = e^{-iP} \int \frac{d^4q}{(2\pi)^4} \chi^\mu_P(q)e^{-iqy}, \]  

(60)

where \( A^\mu_P(y_2) \) stands for the axial-vector heavy diquark field.

The BS equation for the doubly heavy baryon with an axial-vector heavy diquark and a light quark is given by

\[ \chi^\mu_P(q) = S_1(q_1) \int \frac{d^4q'}{(2\pi)^4} G_{\rho\nu}(P, q, q') \chi^\nu_P(q') S^\mu_P(q_2), \]  

(61)

where \( S^\mu_P(q_2) \) is the propagator of the axial-vector heavy diquark and \( G_{\rho\nu} \) is the kernel for the BS equation given by

\[ -iG_{\rho\nu}(P, q, q') = -g_{\rho\nu}I \otimes IV_1(q, q') - \gamma^\mu \otimes \Gamma_{\rho\nu}V_2(q, q'), \]  

(62)

where \( \Gamma_{\mu\nu} = (q_2\mu + q_2\nu)g_{\rho\nu}F(Q^2) \) is the effective vertex for the axial-vector heavy diquark coupling to the gluon, which was derived in Sec. III.

In the leading order of a \( 1/m_Q \) expansion, the propagator of the axial-vector heavy diquark can be written as

\[ S^\mu_P(q_2) = \frac{g_{\mu\nu} - \eta_{\mu\nu}v^{\nu}}{2m_D(q_2\tilde{M} - q_2 - m_D + i\varepsilon)}. \]  

(63)

Similar to the case of the doubly heavy baryon containing a scalar heavy diquark, we can parametrize the BS wave function for the doubly heavy baryon containing an axial-vector heavy quark and a light quark in the following form:

\[ \chi^\mu_P(q) = (g_{v1} + \tilde{g}_{v1}g_{v2})u^\mu(v), \]  

(64)

where \( u^\mu(v) \) is the spinor of the heavy baryon, \( g_{v1} \) and \( g_{v2} \) are the Lorentz-scalar functions of \( q_1^2, q_l \) and \( P^2 = M^2 \). When the spin of the doubly heavy baryon is 1/2, \( u^\mu(v) = \frac{1}{\sqrt{2}}(\gamma^\mu + v^\mu)\gamma^5u(v) \), while the spin of the doubly heavy baryon is 3/2, \( u^\mu(v) \) is the Rarita-Schwinger vector spinor.

After defining \( \tilde{g}_{v1(2)}(q_l) = \int \frac{d^4q}{(2\pi)^4} g_{v1(2)} \), one can find that the BS scalar wave functions satisfy the coupled integral equations in the following:

\[ \tilde{g}_{v1}(q_l) = -\int \frac{d^3q'_l}{(2\pi)^3} \left( m_l + \omega_l \right) \left[ V_1(q_l - q'_l) + 2m_DF(Q^2)\gamma^5V_2(q_l - q'_l) \right] \tilde{g}_{v1}(q'_l), \]  

\[ \tilde{g}_{v2}(q_l) = -\int \frac{d^3q'_l}{(2\pi)^3} \left( m_l + \omega_l \right) \left[ V_1(q_l - q'_l) + 2m_DF(Q^2)\gamma^5V_2(q_l - q'_l) \right] \tilde{g}_{v2}(q'_l). \]  

(65)

(66)

The normalization condition for the BS wave function for the doubly heavy baryon with an axial-vector heavy diquark and a light quark is given by

\[ i\delta_{j_1j_2}^{i_1i_2} \int \frac{d^4q}{(2\pi)^8} \frac{d^4q'}{(2\pi)^8} \chi^\mu_P(q, s) \left[ \frac{\partial}{\partial P_0} I_{\rho\nu}(q, q') \right]_{i_1i_2j_1j_2} \chi^\nu_P(q', s') = \delta_{ss'}, \]  

(67)
where $I_{P_{\mu \nu}}^{i_{1}i_{2}j_{2}j_{1}}$ is the inverse of the four point propagator defined as follows:

$$I_{P_{\mu \nu}}^{i_{1}i_{2}j_{2}j_{1}}(q, q') = \delta^{i_{1}j_{1}} \delta^{i_{2}j_{2}} (2\pi)^{4} \delta^{4}(q - q') S_{i}^{-1}(q_{1}) S_{D_{\mu \nu}}^{-1}(q_{2}).$$

(68)

After some algebra, Eq. (67) can be written in the following form:

$$\frac{i}{6} \int \frac{d^{4}q}{(2\pi)^{4}} \left\{ \text{Tr}[\bar{\chi}_{P_{\sigma}}(q_{t}) \bar{\chi}_{P_{\sigma}}(q_{t}) S_{l}(q_{1})(-i\eta_{1} \bar{\chi} S_{l}(q_{1}) S_{D_{\rho}}^{\rho}(q_{2}))]ight.$$  

$$- \text{Tr}[\bar{\chi}_{P_{\sigma}}(q_{t}) \bar{\chi}_{P_{\sigma}}(q_{t})(i\eta_{2})(2q_{2} \cdot \varepsilon_{\mu \nu} - \varepsilon_{\mu} q_{2 \nu} - q_{2 \mu} \varepsilon_{\nu}) S_{l}(q_{1}) S_{D_{\rho}}^{\rho}(q_{2}) S_{D}^{\mu \nu}(q_{2})] \right\} = 1,$$

(69)

where $\bar{\chi}_{P_{\sigma}}(q_{t})$ and $\bar{\chi}_{P_{\rho}}(q_{t})$ are the transverse projections of the BS wave functions given by

$$\bar{\chi}_{P_{\sigma}}(q_{t}) = -i S_{l}(q_{1})^{-1} \chi_{P_{\sigma}}(q) S_{D_{\alpha \sigma}}(q_{2})^{-1},$$

(70)

and

$$\bar{\chi}_{P_{\rho}}(q_{t}) = -i S_{D_{\rho \beta}}(q_{2})^{-1} \chi_{P_{\rho}}(q) S_{l}(q_{1})^{-1},$$

(71)

respectively.

From Eqs. (70) (71) we get the expressions for the transverse projections of the BS wave functions as follows:

$$\bar{\chi}_{P_{\sigma}}(q_{t}) = \left[ \bar{h}_{v_{1}}(q_{t}) + \bar{g}_{1} \bar{h}_{v_{2}}(q_{t}) \right] u_{\sigma}(v),$$

(72)

and

$$\bar{\chi}_{P_{\rho}}(q_{t}) = \bar{u}_{\rho}(v) \left[ \bar{h}_{v_{1}}(q_{t}) + \bar{g}_{1} \bar{h}_{v_{2}}(q_{t}) \right],$$

(73)

where

$$\bar{h}_{v_{1}}(q_{t}) = \int \frac{d^{3}q_{l}}{(2\pi)^{3}} [V_{1}(q_{t} - q_{l}) + 2m_{D} F(Q^{2}) V_{2}(q_{t} - q_{l})] \bar{g}_{v_{1}}(q_{l}),$$

(74)

and

$$\bar{h}_{v_{2}}(q_{t}) = \int \frac{d^{3}q_{l}}{(2\pi)^{3}} [V_{1}(q_{t} - q_{l}) - 2m_{D} F(Q^{2}) V_{2}(q_{t} - q_{l})] \frac{q_{l} \cdot q_{l}'}{q_{l}^{2}} \bar{g}_{v_{2}}(q_{l}).$$

(75)

After substituting Eqs. (72) (73) into Eq. (68) and integrating out the longitudinal momentum $q_{l}$, the normalization condition can be written in the following form:

$$\int \frac{d^{3}q_{l}}{(2\pi)^{3}} 24 M m_{D} \omega_{l}^{3} \left\{ (m_{l} + \omega_{l})[2\eta_{2} M \omega_{l}^{2} + E \eta_{l}(m_{l} m_{D})ight.$$  

$$- \omega_{l} m_{D} + 2 m_{l} \omega_{l} - M \omega_{l} - M m_{l}) + E \eta_{l}(-M + m_{D} + 2 \omega_{l}) \omega_{l}^{2} \bar{h}_{v_{1}}^{2}(q_{l})]$$  

$$+ 4 \omega_{l}^{2} \bar{q}_{l}^{2} \eta_{l} M + \eta_{l} E \bar{h}_{v_{1}}(q_{l}) \bar{h}_{v_{2}}(q_{l})$$  

$$+(2 \eta_{2} M(m_{l} - \omega_{l} q_{l}^{2} \omega_{l}^{2} + E \eta_{l}(-M + m_{D} + 2 \omega_{l}) q_{l}^{4})$$  

$$+ E \eta_{l}(m_{l} - \omega_{l})(-m_{l} m_{D} - \omega_{l} m_{D} - m_{l} \omega_{l} + M(m_{l} + \omega_{l})) q_{l}^{2} \bar{h}_{v_{2}}^{2}(q_{l}) \right\} = 1.$$  

(76)

In our calculation, we take the constituent masses of the light quarks as $m_{u} = m_{d} = 0.33 \text{ GeV}$, $m_{s} = 0.45 \text{ GeV}$, and the scale of nonperturbative interaction $\Lambda_{QCD} \simeq 0.2 \text{ GeV}$. In order to solve the coupled integral equations (48) (49), we discretize the integration region into $n$ pieces (with $n$ sufficiently large). In this way the integral equations are transformed into coupled matrix equations for the $n$ dimensional vectors $\bar{g}_{s_{1}(2)}$. Then, it
is easy to obtain the eigenvalue equations for \( \tilde{g}_{s(\nu)} \). The same method is applied in dealing with the coupled integral equations [55, 56]. After solving the eigenvalue equations, we obtain masses of the doubly heavy baryons shown in Table II. The mass of the \( \Xi_{cc} \) obtained in our model is consistent with the experimental value, 3518.9 ± 0.9 MeV [58]. The obtained BS scalar wave functions for the doubly heavy baryons composed of a heavy diquark and a light quark are displayed in Fig. 4 and Fig. 5. One finds that the masses and the amplitudes of BS scalar wave functions for the doubly heavy baryons are independent of the spins of both the heavy diquarks and the doubly heavy baryons. In fact, this is just the consequence of the heavy diquark spin symmetry. So, we redefine \( \tilde{g}_1 \equiv \tilde{g}_{s(\nu)_1} \) and \( \tilde{g}_2 \equiv \tilde{g}_{s(\nu)_2} \) in Fig. 4 and Fig. 5 for convenience.

**TABLE II:** Values of the masses of baryons containing two heavy quarks. The lower (upper) masses correspond to \( \kappa' = 0.01 \) (0.06) GeV.

|        | \( \Xi_{bb} \)   | \( \Xi_{bc} \)   | \( \Xi_{cc} \)   | \( \Omega_{bb} \) | \( \Omega_{bc} \) | \( \Omega_{cc} \) |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( M \) (GeV) | 10.08~10.10     | 6.83~6.85       | 3.52~3.56       | 10.18~10.19     | 6.94~6.95       | 3.62~3.65       |

V. Non-leptonic decay of doubly heavy baryons

In this section, we will apply the obtained BS wave functions to calculate the non-leptonic decay widths for the doubly heavy baryons emitting a pseudo-scalar meson in the BS formalism. The Hamiltonian describing such decays reads [51]

\[
H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ub}^* (a_1 O_1 + a_2 O_2),
\]

(77)

where \( V_{cb} \) and \( V_{ub}^* \) are the elements of Cabibbo-Kobayashi-Maskawa matrix, \( U \) and \( D \) stand for the fields of the light quarks involved in the decay, \( O_1 = [\bar{D}\gamma^\sigma(1-\gamma^5)U][\bar{c}\gamma_\sigma(1-\gamma^5)b] \) and \( O_2 = [\bar{c}\gamma^\sigma(1-\gamma^5)U][\bar{D}\gamma_\sigma(1-\gamma^5)b] \).

The first matrix element on the right hand side of Eq. (78) is related to the decay constant of the pseudo-scalar meson composed of a (an) scalar (axial-vector) heavy diquark and a light u (or d) quark.

\[
\langle \Xi^{(s)}_{Qq}(P_f) | P(k) | \Xi^{(s)}_{Qq}(P_i) \rangle = -\frac{i}{\sqrt{2}} G_F V_{cb} V_{ub}^* a_1 \int d^4 z (P(k)) \bar{D}(z)\gamma_\sigma(1-\gamma^5)U(z)|0\rangle e^{-ikz}
\]

(78)

where \( P(k) \) stands for the pseudo-scalar meson with momentum \( k \), \( \Xi^{(s)}_{Qq} \) stands for the doubly heavy baryon composed of a scalar (axial-vector) heavy diquark and a light u (or d) quark.

The first matrix element on the right hand side of Eq. (78) is related to the decay constant of the pseudo-scalar meson, \( f_{P} \), which is defined as

\[
\langle P(k) | \bar{D}(z)\gamma_\sigma(1-\gamma^5)U(z) | 0 \rangle = -i f_{P} k_\sigma.
\]

(79)
FIG. 4: The normalized BS scalar wave functions for the doubly heavy baryons ($\Xi_{QQ'}$) composed of a heavy diquark and a light quark ($u$, $d$). The solid (dotted) lines are for $\kappa' = 0.01$ (0.06) GeV. The upper solid and dotted lines are for $\tilde{g}_2(q_t)$ in unit of GeV$^{-1}$. The lower solid and dotted lines are for $\tilde{g}_1(q_t)$. 
FIG. 5: The normalized BS scalar wave functions for the doubly heavy baryons ($\Omega_{QQ'}$) composed of a heavy diquark and a strange light quark ($s$). The solid (dotted) lines are for $\kappa' = 0.01$ (0.06) GeV. The upper solid and dotted lines are for $\tilde{g}_2(q_t)$ in unit of GeV$^{-1}$. The lower solid and dotted lines are for $\tilde{g}_1(q_t)$. 
The decay amplitude (Eq. (78)) is classified into three cases according to different initial and final doubly heavy baryon states as follows:

\[
\mathcal{M}_1 = \langle \Xi_{Qb}(P_f)P(k)\Xi_{Qb}^*(P_i) \rangle, \quad (80)
\]
\[
\mathcal{M}_2 = \langle \Xi_{Qc}^*(P_f)P(k)\Xi_{Qb}^*(P_i) \rangle, \quad (81)
\]
\[
\mathcal{M}_3 = \langle \Xi_{Qc}(P_f)P(k)\Xi_{Qb}(P_i) \rangle. \quad (82)
\]

Let us first calculate the decay amplitude \(\mathcal{M}_1\). The Feynman diagram for \(\mathcal{M}_1\) is shown in Fig. 6. \(\mathcal{M}_1\) is related to the BS wave functions through the following equation:

\[
\mathcal{M}_1 = \sqrt{2} \frac{G_F V_{cb} V_{ub}^* d_1 f_P k_1}{\sqrt{3}} \int d^4x_1 d^4x_2 d^4y_1 d^4y_2 \epsilon^{ikz} \chi_{P_f(1,2)}(x_1) \chi_{P_i(1,2)}(y_1) \chi_{P_f(1,2)}(x_2) \chi_{P_i(1,2)}(y_2) \langle 0|\phi_{Qc}(z_1)\tilde{b}(z)\gamma^\sigma(1-\gamma^5)c(z)\varphi_{Qb}(z_2)|0 \rangle, \quad (83)
\]

where \(\chi_{P_f}\) and \(\chi_{P_i}\) stand for the BS wave functions for the final and initial doubly heavy baryons, respectively, \(\phi_{Qc}\) and \(\varphi_{Qb}\) are the diquark field operators involved in the final and initial doubly heavy baryons, respectively.

The matrix element in Eq. (83) is related to the BS wave functions for the heavy diquarks through the following equation:

\[
\langle 0|\phi_{Qc}(z_1)\tilde{b}(z)\gamma^\sigma(1-\gamma^5)c(z)\varphi_{Qb}(z_2)|0 \rangle = \sum_\lambda \frac{1}{6} \int \frac{d^4p_{D_f}}{(2\pi)^8} \frac{d^4p_{D_i}}{(2\pi)^8} \delta(p_{D_f,l} - m_{D_f}^2 - P_{D_f,l}^2) \delta(p_{D_i,l} - m_{D_i}^2 - P_{D_i,l}^2) \epsilon^{ikz} \chi_{P_f(1,2)}(p_{D_f}) \chi_{P_i(1,2)}(p_{D_i}) S(-p_{D_f}) S(-p_{D_i}) (1-\gamma^5)\gamma^\sigma \xi^{\nu(\lambda)}_{\mu(\lambda)}
\]

\[
\times e^{-iz_{D_f,l} - P_{D_f,l} + k} e^{-iz_{D_i,l} - P_{D_i,l}} e^{-iz_{D_f,l} - 2z_{D_i,l}}, \quad (84)
\]

where \(\tilde{\chi}_{P_{D_f}}\) and \(\tilde{\chi}_{P_{D_i}}\) are the BS wave functions for the heavy diquarks involved in the final and initial doubly heavy baryons, respectively, \(m_{D_f}\) and \(m_{D_i}\) are the masses of the heavy diquarks involved in the final and initial doubly heavy baryons, respectively, \(P_{D_f,l} = P_{D_f,l} \cdot v\) (\(v\) denotes the velocity of the initial doubly heavy baryon) and \(P_{D_i,l} = P_{D_i,l} \cdot v\). (\(v\) are the longitudinal and transverse projections of the final (initial)
heavy diquark momentum \((P_{D_f})\) along the initial doubly heavy baryon momentum \((P_i)\), respectively, and \(\xi^{\nu(\lambda)}\) is the \(\lambda\)-th polarization vector of the axial-vector heavy diquark involved in the initial doubly heavy baryon.

Substituting Eq. (83) into Eq. (83), the decay amplitude \(\mathcal{M}_1\) can be written in momentum space as follows:

\[
\mathcal{M}_1 = -\frac{1}{\sqrt{2}} G_F V_{cb} V_{UD}^* a_1 f_P k_\sigma (2\pi)^4 \delta^4(P_i - P_f - k) \int \frac{d^4q d^4q'}{(2\pi)^8} (2\pi)^4 \delta^4(q_1 - q_1')
\]

\[
\times (2\pi)^2 \delta(P_{D_f} - \sqrt{m_D^2 + P_{D_f}^2}) \delta(P_{D_i} - \sqrt{m_D^2 + P_{D_i}^2}) \tilde{\chi}_{P_{D_f}'}(q_1') S(q_i) \tilde{\chi}_{P_{D_i}}(q_1) \tilde{\omega}_P(q_i). (85)
\]

Analogously, we can derive the other two decay amplitudes \(\mathcal{M}_2\) and \(\mathcal{M}_3\) in the BS formalism as follows:

\[
\mathcal{M}_2 = -\frac{1}{\sqrt{2}} G_F V_{cb} V_{UD}^* a_1 f_P k_\sigma (2\pi)^4 \delta^4(P_i - P_f - k) \int \frac{d^4q d^4q'}{(2\pi)^8} (2\pi)^4 \delta^4(q_1 - q_1')
\]

\[
\times (2\pi)^2 \delta(P_{D_f} - \sqrt{m_D^2 + P_{D_f}^2}) \delta(P_{D_i} - \sqrt{m_D^2 + P_{D_i}^2}) \tilde{\chi}_{P_{D_f}'}(q_1') S(q_i) \tilde{\chi}_{P_{D_i}}(q_1) \tilde{\omega}_P(q_i). (86)
\]

and

\[
\mathcal{M}_3 = -\frac{1}{\sqrt{2}} G_F V_{cb} V_{UD}^* a_1 f_P k_\sigma (2\pi)^4 \delta^4(P_i - P_f - k) \int \frac{d^4q d^4q'}{(2\pi)^8} (2\pi)^4 \delta^4(q_1 - q_1')
\]

\[
\times (2\pi)^2 \delta(P_{D_f} - \sqrt{m_D^2 + P_{D_f}^2}) \delta(P_{D_i} - \sqrt{m_D^2 + P_{D_i}^2}) \tilde{\chi}_{P_{D_f}'}(q_1') S(q_i) \tilde{\chi}_{P_{D_i}}(q_1) \tilde{\omega}_P(q_i). (87)
\]

where \(\tilde{\xi}^{(\lambda')\lambda}\) is the \(\lambda'\)-th polarization vector of the axial-vector heavy diquark involved in the final doubly heavy baryon.

The differential decay width for the two body decay reads

\[
d\Gamma = \frac{1}{32\pi^2} |\mathcal{M}|^2 \frac{|k|}{M_i} d\Omega,
\]

where \(\tilde{\Omega}\) denotes the solid angle, \(\mathcal{M}\) stands for the decay amplitude, \(M_i\) is the mass of the initial baryon, and \(|k|\) is the absolute value of the three momentum of the particles in the final state in the rest frame of the initial state.

Numerically, the parameters in the Hamiltonian \((G_F, V_{cb}, V_{UD})\), and the masses and the decay constants of the pseudo-scalar mesons are taken to have the following values \([56]\): \(G_F = 1.16637 \times 10^{-5} \text{GeV}^{-2}\), \(V_{cb} = 0.0412, V_{ud} = 0.97418, V_{us} = V_{cd} = 0.2255, V_{cs} = 0.9742, m_\pi = 0.1396 \text{GeV}, m_K = 0.4937 \text{GeV}, m_D = 1.8696 \text{GeV}, m_{D_s} = 1.9685 \text{GeV}, f_\pi = 0.1304 \text{GeV}, f_K = 0.1555 \text{GeV}, f_D = 0.2058 \text{GeV}, f_{D_s} = 0.273 \text{GeV}\).

Our predictions for the non-leptonic decay widths for the doubly heavy baryons emitting a pseudo-scalar meson are shown in Tables \([3][5][6][\text{and}][7]\) where the superscripts \(\frac{3}{2}\) and \(\frac{5}{2}\) denote the spin of the doubly heavy baryons.
diquark and a light quark. We first establish the BS equations for both the heavy diquarks and the doubly heavy for the heavy diquark coupling to the gluon in order to reflect the inner structure of the heavy diquark. It is found that of the doubly heavy baryons are consistent with those from the lattice simulations [54, 55]. Since the size of the heavy 2 used in the cases of mesons and heavy baryons containing a single heavy quark. Since the size of the heavy

\begin{table}[h]
\centering
\caption{Predictions for the non-leptonic decay widths for the doubly heavy baryons emitting $\pi$ meson.}
\begin{tabular}{|c|c|}
\hline
process & decay width ($10^{-14} a_s^2 GeV$) \\
\hline
$\Gamma(\Xi_{bb}^{\frac{1}{2}} \rightarrow \Xi_{cc}^{\frac{1}{2}} \pi)$ & 0.343~0.362 \\
$\Gamma(\Xi_{bb}^{\frac{3}{2}} \rightarrow \Xi_{cc}^{\frac{1}{2}} \pi)$ & 0.205~0.211 \\
$\Gamma(\Xi_{bb}^{3} \rightarrow \Xi_{cc}^{3} \pi)$ & 4.110~4.234 \\
$\Gamma(\Xi_{bc}^{\frac{1}{2}} \rightarrow \Xi_{cc}^{\frac{1}{2}} \pi)$ & 0.848~1.101 \\
$\Gamma(\Xi_{bc}^{\frac{3}{2}} \rightarrow \Xi_{cc}^{\frac{3}{2}} \pi)$ & 0.415~0.587 \\
$\Gamma(\Xi_{bc}^{3} \rightarrow \Xi_{cc}^{3} \pi)$ & 8.626~12.110 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{Predictions for the non-leptonic decay widths for the doubly heavy baryons emitting $K$ meson.}
\begin{tabular}{|c|c|}
\hline
process & decay width ($10^{-15} a_s^2 GeV$) \\
\hline
$\Gamma(\Xi_{bb}^{\frac{1}{2}} \rightarrow \Xi_{cc}^{\frac{1}{2}} K)$ & 0.265~0.267 \\
$\Gamma(\Xi_{bb}^{\frac{3}{2}} \rightarrow \Xi_{cc}^{\frac{3}{2}} K)$ & 0.165~0.171 \\
$\Gamma(\Xi_{bb}^{3} \rightarrow \Xi_{cc}^{3} K)$ & 3.317~3.425 \\
$\Gamma(\Xi_{bc}^{\frac{1}{2}} \rightarrow \Xi_{cc}^{\frac{1}{2}} K)$ & 0.649~0.845 \\
$\Gamma(\Xi_{bc}^{\frac{3}{2}} \rightarrow \Xi_{cc}^{\frac{3}{2}} K)$ & 0.328~0.466 \\
$\Gamma(\Xi_{bc}^{3} \rightarrow \Xi_{cc}^{3} K)$ & 6.810~9.612 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{Predictions for the non-leptonic decay widths for the doubly heavy baryons emitting $D$ meson.}
\begin{tabular}{|c|c|}
\hline
process & decay width ($10^{-15} a_s^2 GeV$) \\
\hline
$\Gamma(\Xi_{bb}^{\frac{1}{2}} \rightarrow \Xi_{cc}^{\frac{1}{2}} D)$ & 0.818~0.832 \\
$\Gamma(\Xi_{bb}^{\frac{3}{2}} \rightarrow \Xi_{cc}^{\frac{3}{2}} D)$ & 0.693~0.741 \\
$\Gamma(\Xi_{bb}^{3} \rightarrow \Xi_{cc}^{3} D)$ & 13.885~14.834 \\
$\Gamma(\Xi_{bc}^{\frac{1}{2}} \rightarrow \Xi_{cc}^{\frac{1}{2}} D)$ & 1.136~1.528 \\
$\Gamma(\Xi_{bc}^{\frac{3}{2}} \rightarrow \Xi_{cc}^{\frac{3}{2}} D)$ & 0.945~1.464 \\
$\Gamma(\Xi_{bc}^{3} \rightarrow \Xi_{cc}^{3} D)$ & 19.525~30.028 \\
\hline
\end{tabular}
\end{table}

VI. Summary and discussion

In the heavy quark limit, a doubly heavy baryon can be regarded as a bound state composed of a heavy diquark and a light quark. We first establish the BS equations for both the heavy diquarks and the doubly heavy baryons, respectively, in the leading order of a $1/m_Q$ expansion. The kernel for the BS equation contains the scalar confinement and one-gluon-exchange terms, which are motivated by the potential model and successfully used in the cases of mesons and heavy baryons containing a single heavy quark. Since the size of the heavy diquark is enhanced by $\ln^2 m_Q$ with respect to $1/m_Q$, we also introduce a few form factors to the effective vertex for the heavy diquark coupling to the gluon in order to reflect the inner structure of the heavy diquark.

The BS equations are solved numerically under the covariant instantaneous approximation, which is suitable for the weakly bound states of both the heavy diquark and the doubly heavy baryon. The obtained masses of the doubly heavy baryons are consistent with those from the lattice simulations [54, 55]. It is found that
the properties of both the heavy diquarks and the doubly heavy baryons are independent of their spin in the leading order of a $1/m_Q$ expansion.

As we know, the superflavor symmetry relates doubly heavy baryons to heavy mesons, and hence the form factors of the transitions of doubly heavy baryons are reduced to the Isgur-Wise function, which is well known for heavy mesons [10, 11, 56]. The calculation of the doubly heavy baryon transitions is greatly simplified under the superflavor symmetry at the cost of ignoring the derivation from non-pointlike spatial dispersion of the heavy diquark. In this work, we directly calculate the decay amplitudes for the doubly heavy baryons using the BS wave functions obtained for both the heavy diquark and the doubly heavy baryons, instead of employing the superflavor symmetry. We give the predictions for the non-leptonic decay widths of doubly heavy baryons emitting a pseudo-scalar meson. Our results will be tested in the future experiments.

In our calculation, for the propagators of the heavy diquarks and the light quarks involved in the bound states, we simply assume the forms of free propagators with the masses of the heavy diquarks and the light quarks taken to be the constituent ones. Actually, the real propagators should be solved using the Dyson-Schwinger equation. In such an approach, one has to guarantee the consistency between the kernel of the BS equation and that of the Dyson-Schwinger equation as required by the axial-vector Ward-Takahashi identity [24]. This is a very complicated procedure and needs more careful investigations in the future.

At the HERA-B and Tevatron facilities more than $10^5$ events involving double charm baryons are expected, while at the LHC one can expect about $10^9$ events [57]. Since the available energy at the LHC is much higher than the masses of $\Xi_{cc}$ and $\Xi_{bb}$, it is believed that their production rates should be comparable. The decay widths of the doubly heavy baryons we present in this work will be tested in the forthcoming experiments.

Since the BS equations are established at the leading order in a $1/m_Q$ expansion, we do not distinguish the different spins of both the heavy diquarks and the doubly heavy baryons. Such differences should happen at $O(1/m_Q)$. $1/m_Q$ corrections will be studied in the future.

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