The QCD Kondo phase in quark stars

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Abstract

We study light (u, d) quark matter with charm impurities. These impurities are added to the Lagrangian density. We derive the equation of state (EOS) of this kind of quark matter, which contains a Kondo phase. We explore this EOS and study the structure of stars, identifying the effects of the Kondo phase. Solving the TOV equations and computing the mass-radius diagram, we find that the presence of a Kondo phase leads to smaller and lighter stars.
I. INTRODUCTION

A long standing question in the theory of compact stars \[1–9\] is: Are there quark stars? This question has been around for decades and it has received a renewed attention after the appearance of new measurements of masses of astrophysical compact objects \[10–12\]. These measurements suggest that stellar objects may have large masses, such as \((1.97 \pm 0.04) M_{\odot} \) \[10\], \((2.01 \pm 0.04) M_{\odot} \) \[11\] or even \((2.4 \pm 0.12) M_{\odot} \) \[12\]. In principle larger masses imply larger baryon densities in the core of the stars and we expect very dense hadronic matter to be in a quark gluon plasma (QGP) phase. On the other hand, most of the equations of state based on quark degrees of freedom are too soft to support heavy stars.

The existence of quark stars depends ultimately on the details of the equation of state of cold quark matter. According to most models, deconfined quark matter should be formed at baryon densities in the range \(\rho_B = 2\rho_0 - 5\rho_0\), where \(\rho_0\) is the ordinary nuclear matter baryon density. Since at low temperatures and high baryon densities we can not rely on lattice QCD calculations, the quark matter equations of state must be derived from models. Many of them are based on the MIT bag model \[13\] or on the Nambu-Jona-Lasinio (NJL) model \[14\]. At very high baryon densities there are constraints derived from perturbative QCD calculations \[7–9, 15\].

The description of cold quark matter is not unique and it may (or may not) contain specific QCD features such as color superconductivity, diquarks, or a Gribov-Zwanziger phase \[16\]. One of these QCD features is the QCD Kondo phase. Recently \[17\] the Kondo effect has been studied in the context of quark matter. In \[17\] it was pointed out that the Kondo effect occurs when a system has i) heavy impurities, ii) a Fermi surface of fermions, iii) quantum fluctuations, and iv) non-Abelian interactions. All these features are present in a dense and cold light quark system with some heavy quarks as impurities. This kind of quark matter was called Kondo phase in \[17\] and its existence in the core of dense stars may change the thermodynamic and transport properties of the stellar medium.

In compact stars the heavy impurities are charm quarks in low concentration. Charm can be produced in strange quark stars by neutrino interactions. A constant neutrino flux traverses the star. During their interactions with quark matter, neutrinos emit a \(W^+ (\nu_e \rightarrow W^+ e^-)\), which is absorbed by an \(s\) quark (or to a lesser extent by a \(d\) quark), which turns into a charm quark \((W^+ s \rightarrow c)\). After being produced the \(c\) quark can decay back to an \(s\) quark but Pauli blocking will reduce the efficiency of this reaction.

The existence of charm in quark stars was first investigated in Ref. \[18\]. At the time the conclusion was that this kind of star would be unstable. Very recently \[19\] this question was addressed again in the context of perturbative QCD (pQCD). The authors investigated the effects of charm quarks in the equation of state for large values of the quark chemical
potential, where pQCD should be reliable. The radial stability analysis suggested that this star would be unstable. Even though charm stars probably do not exist, it is conceivable that some finite amount of charm will always be present in the star and this may be enough to generate the Kondo phase.

In early works the QCD Kondo effect was studied with the perturbative renormalization group equation obtained at the one-loop level \cite{20}. In \cite{17} the ground state of the quark matter with heavy impurities was investigated with a non-perturbative mean field approach. The authors used the following Lagrangian density \cite{17}

\[
\mathcal{L} = \bar{\psi} i \partial \psi + \mu \bar{\psi} \gamma^0 \psi + \bar{\Psi} i \partial \Psi - m_Q \bar{\Psi} \Psi - G_c (\bar{\psi} \gamma^\mu T^a \psi) (\bar{\Psi} \gamma^\mu T^a \Psi)
\]  

(1)

where \(\psi\) and \(\Psi\) represent the light and heavy quark fields respectively. In the heavy quark limit the latter can be replaced by \(\Psi \rightarrow \Psi_v = \frac{1}{2} (1 + \frac{1}{v}) e^{imQv - x} \Psi\), where \(v\) is the velocity of the heavy quark. The coupling strength \(G_c\) in the interaction part of \(\mathcal{L}\) is positive and has dimensions of inverse mass square. The interaction term has the color structure \(\lambda^a \lambda^a\) as prescribed by the one-gluon exchange. The values of the coupling and the cutoff parameter which we have denoted as \(\Lambda\) are taken from the usual NJL model (for \(N_f = 2\)): \(G_c \Lambda^2 = (9/2)2.0\) and \(\Lambda = 0.65\) GeV. These numbers are chosen so as to describe the pion decay constant and the quark condensate density. We have suppressed the flavor index for the fields \(\psi\) and we have assumed that they have the same chemical potential \(\mu\). In \cite{17} it was assumed that the heavy quarks are spatially uniformly distributed within the light quark matter and the density of heavy quarks is large so that the averaged distance between heavy quarks is small when compared to a typical coherence length of the QCD Kondo effect. The above Lagrangian is treated in the mean-field approach and the four-quark term appearing in (1) can be factorized, giving rise to condensates such as, for example, \(\langle \bar{\psi} \Psi_v \rangle\).

In momentum space the (bilinear) mean-field Lagrangian appears as follows \cite{17}

\[
\mathcal{L}^{MF} = \bar{\psi} k \psi + \mu \bar{\psi} \gamma^0 \psi + \bar{\Psi}_v v \cdot k \Psi_v - \lambda (\bar{\Psi}_v \Psi_v - n_Q) + \Delta \bar{\Psi}_v \frac{1 + \gamma^0}{2} (1 + \hat{k} \cdot \vec{\gamma}) \psi
\]

\[
+ \Delta^* \bar{\psi} (1 + \hat{k} \cdot \vec{\gamma}) \frac{1 + \gamma^0}{2} \Psi_v - \frac{8 N_f}{G_c} |\Delta|^2
\]

(2)

where the term weighted by means of the Lagrange multiplier \(\lambda\) was added to impose the constraint of number conservation of the heavy quarks and \(n_Q\) is the averaged heavy quark density. This constraint is required since the heavy quark total number must be fixed, on average. As it was emphasized in Ref. \cite{21}, since \((-\lambda)\) is the coefficient of \(\bar{\Psi}_v \Psi_v\) one might formally interpret it as the chemical potential of heavy quarks. Then, it is the chemical potential for the redefined heavy-quark field \(\Psi_v\) rather than the original heavy-quark field \(\Psi\). In other words, a nonzero \(\lambda\) can be regarded as the energy necessary to put a virtual component of a heavy quark into the system. Even though \(\lambda\) is not the real heavy quark
chemical potential, it is useful to treat it as if it were. As it will be seen, there is a strong correlation between \((-\lambda\)) and the number of heavy quarks \(n_Q\). For a fixed light quark density (fixed \(\mu\)), \(n_Q\) increases when \((-\lambda)\) increases, as we would expect for a chemical potential.

The quantity \(\Delta\) is a complex number associated with the gap function, which is defined as \([17]\):

\[
\Delta_{\delta\alpha} = \frac{G_c}{2} \langle \bar{\psi}_\alpha \psi_\delta \rangle = \Delta \left( \frac{1+\gamma^0}{2} (1 - \hat{k} \cdot \vec{\gamma}) \right)_{\delta\alpha}
\] (3)

II. THE EQUATION OF STATE AND THE KONDO PHASE

From the Lagrangian \([2]\) we can derive the thermodynamic potential, which is given by \([17]\):

\[
\Omega(T, \mu, \lambda) = \frac{N_c}{\pi^2} \int_0^\Lambda k^2 f(T, \mu, \lambda, k) \, dk + \frac{8 N_f}{G_c} |\Delta|^2 - \lambda n_Q(T, \mu, \lambda)
\] (4)

with \(f\) being

\[
f(T, \mu, \lambda, k) = -T \, \log \left( \exp \left( -\frac{E_+(\mu, \lambda, k)}{T} \right) + 1 \right) - T \, \log \left( \exp \left( -\frac{E_-(\mu, \lambda, k)}{T} \right) + 1 \right) - T \, \log \left( \exp \left( -\frac{E(\mu, k)}{T} \right) + 1 \right)
\] (5)

\(E_+(\mu, \lambda, k), E_-(\mu, \lambda, k)\) and \(E(\mu, k)\) are the real parts of the Bogoliubov eigenenergies, which are

\[
E_\pm(\mu, \lambda, k) = \pm \sqrt{(k - \lambda - \mu)^2 + 8 N_f |\Delta|^2 + k + \lambda - \mu}, \quad E(\mu, k) = k - \mu
\] (6)

Minimizing the potential with respect to the Lagrange multiplier \(\lambda\), i.e., taking

\[
\frac{\partial \Omega(T, \mu, \lambda)}{\partial \lambda} = 0
\] (7)

we obtain an expression for the number density of heavy quarks:

\[
n_Q(T, \mu, \lambda) = \frac{N_c}{\pi^2} \int_0^\Lambda k^2 \frac{\partial f(T, \mu, \lambda, k)}{\partial \lambda} \, dk
\] (8)

In the zero-temperature limit, \([4]\) reduces to the following form:

\[
\Omega(\mu, \lambda) = -\lambda \frac{N_c}{\pi^2} \int_0^\Lambda k^2 \frac{\partial f_0(\mu, \lambda, k)}{\partial \lambda} \, dk + \frac{N_c}{\pi^2} \int_0^\Lambda k^2 f_0(\mu, \lambda, k) \, dk + \frac{8 N_f}{G_c} |\Delta|^2
\] (9)

with

\[
f_0(\mu, \lambda, k) = \theta(-k - \lambda + \mu - \sigma) E_+(\mu, \lambda, k) + \theta(-k - \lambda + \mu + \sigma) E_-(\mu, \lambda, k) + \theta(\mu - k) E(\mu, k)
\] (10)
where θ is the unit step function, and where we define \( \sigma = \sqrt{(k - \lambda - \mu)^2 + 8N_f |\Delta|^2} \). In the calculations, the parameters were taken from Ref. [17]. The pressure and energy density are given by

\[
P(\mu, \lambda) = -\Omega(\mu, \lambda) \tag{11}
\]

\[
\varepsilon(\mu, \lambda) = -P(\mu, \lambda) + \mu n_q(\mu, \lambda) - \lambda n_Q(\mu, \lambda) \tag{12}
\]

where \( n_q \) is the number density of light quarks:

\[
n_q(\mu, \lambda) = -\frac{\partial \Omega(\mu, \lambda)}{\partial \mu} = \frac{N_c}{\pi^2} \left( \lambda \int_0^\Lambda k^2 \frac{\partial^2 f_0(\mu, \lambda, k)}{\partial \mu \partial k} \, dk - \int_0^\Lambda k^2 \frac{\partial f_0(\mu, \lambda, k)}{\partial \mu} \, dk \right) \tag{13}
\]

In Fig. [1] we show the quark densities \( n_q \) and \( n_Q \) and the gap \( |\Delta| \) as a function of \( \mu \) and \( \lambda \). From the top picture (Fig. [1a]) we see that the light quark density is independent of \( \lambda \). In contrast, the number of heavy quarks is strongly sensitive to the value of \( \mu \). As can be seen from Fig. [1b], higher light quark densities imply lower heavy quark densities. According to a naive expectation, the Kondo phase is the consequence of a non-vanishing gap \( |\Delta| \), which in turn is a consequence of non-vanishing impurities \( n_Q \). Indeed, there is a correlation between \( n_Q \) and \( |\Delta| \) seen in the top of Figs [1b] and [1c]: \( |\Delta| = 0 \) when \( n_Q = 0 \). However, at lower light quark densities, \( |\Delta| \) is zero when \( n_Q \) is maximal, as can be seen in bottom left corner of Figs. [1b] and [1c].

The equation of state is shown in Fig. [2] where we can see the pressure as a function of the energy density. There is a connection between Figs. [1] and [2]. For example, looking at the line of constant \( \lambda = 0.005 \) GeV both in Fig. [1b] and [1c] we see that, at a certain critical value of \( \mu \) both \( n_Q \) and \( |\Delta| \) suddenly start to grow. This change of behavior corresponds to the appearance of the plateau in Fig. [2c], which marks the onset of the Kondo phase.

For comparison, we also show the equation of state of the MIT bag model. This EOS for a QGP with \( u, d \) and \( s \) quarks (of equal masses \( m_q \)) has pressure given by

\[
P_{MIT} = \sum_{q=u}^{d,s} \frac{\gamma_q}{6\pi^2} \int_0^{k_F} dk \frac{k^4}{\sqrt{m_q^2 + k^2}} - B \tag{14}
\]

and energy density given by

\[
\varepsilon_{MIT} = \sum_{q=u}^{d,s} \frac{\gamma_q}{2\pi^2} \int_0^{k_F} dk \, k^2 \sqrt{m_q^2 + k^2} + B \tag{15}
\]

where \( B \) is the bag constant, \( m_q = 10 \) MeV and \( \gamma_q = 6 \) is the statistical factor for quarks. The quark density is given by

\[
\rho_{MIT} = \sum_{q=u}^{d,s} \frac{\gamma_q}{2\pi^2} \int_0^{k_F} dk \, k^2 \tag{16}
\]
which gives the highest occupied level $k_F$. Using the baryon density $\rho_B = \rho_{MIT}/3$, we get a simple expression for the Fermi momentum

$$k_F = \pi^{2/3} \rho_B^{1/3}$$

which allows the calculation of the pressure and energy density as functions of the baryon density. In Fig. 3 we compare the MIT EOS with our equation of state for $\lambda = 0.01$ GeV. We can reach higher pressures with our model. We emphasize that, as can be seen from Fig. 2 all the relevant values of $\lambda$ will generate EOS curves which will lie above the MIT curves.

III. STELLAR STRUCTURE

As usual, in order to describe the structure of a static (non-rotating) compact star, the Einstein’s field equations are solved for a medium with an isotropic relativistic fluid and in the case of a spherically symmetric metric tensor. Under these conditions, the Einstein equations imply the TOV system which becomes an integro-differential equation for the pressure, $P$, as a function of the radius, $r$. The system reads:

$$P'(r) = -\frac{G M(r) \varepsilon(r) \left( \frac{P(r)}{\varepsilon(r)} + 1 \right) \left( \frac{4\pi r^3 P(r)}{c^2 M(r)} + 1 \right)}{c^2 r^2 \left( 1 - \frac{2G M(r)}{c^2 r} \right)}, \quad M(r) = \frac{4\pi}{c^2} \int_0^r s^2 \varepsilon(s) \, ds + M(0)$$

where $G$ is Newton’s gravitational constant and $c$ is the speed of light. Choosing the dimensionless variables $\tilde{P}(r) = \frac{P(r)}{P_0}$, $\tilde{\varepsilon}(r) = \frac{\varepsilon(r)}{\varepsilon_0}$, and $\tilde{M}(r) = \frac{M(r)}{M_0}$, following [22], we have computed, for several fixed pairs of input values $(P_0, \varepsilon_0)$, the total stellar mass (in solar masses) and then the corresponding stellar radius (in km) for all the relevant values of the parameter $\lambda$. Natural units have been adopted for all calculations.

In Fig. 4 we present some solutions of the TOV system of equations in the mass-radius diagram for both EOS models. In colored thin lines we show the results obtained with the MIT bag model equation of state. In thick black lines those obtained with the model described here. From Fig. 3 we see that the model studied here generates harder equations of state than the MIT. As a consequence it also generates heavier stars, as shown by the thick black lines in Fig. 4. The solid line shows the curve for $\lambda = 0$ GeV which yields the smallest value for $M_{max}$. All the other values of $\lambda$, both positive and negative, lead right-lying curves in Fig. 4. The origin of this non-trivial behavior is in the lower panel of Fig. 1. Indeed, from Fig. 1c we see that the gap $|\Delta|$ goes to zero when $\lambda$ moves away from zero both to positive and negative values. Since non-zero values of $|\Delta|$ are the signature of the Kondo phase, we can conclude that the existence of a Kondo phase softens the equation of state and leads
to lighter and smaller stars. In Fig. 5 we show the dependence of $M_{\text{max}}$ on the light quark density, $n_q$. In line with the results shown in Fig. 4, we see that the smallest values of $M_{\text{max}}$ occur for the $\lambda = 0$, being larger for all other values of $\lambda$. In Fig. 6 we show the dependence of $M_{\text{max}}$ on the heavy quark density, $n_Q$. Here again, the smaller maximal masses occur for $\lambda = 0$. Since heavy quarks are impurities in the present model of quark matter, we expect to have $n_Q << n_q$. This condition will be satisfied for small values of $n_Q$, where, according to Fig. 6, the dependence of $M_{\text{max}}$ on $n_Q$ is weak and $M_{\text{max}} \to 3.7M_\odot$.

IV. CONCLUSIONS

We have evaluated the equation of state derived from the model developed in Ref. [17]. We have applied this equation of state, which contains heavy quark impurities and has a Kondo phase, to the study of quark stars. Solving the TOV equations and computing the mass-radius diagram, we find that the existence of the Kondo phase (when the gap $|\Delta|$ is larger than zero) leads to softer equations of state and hence to lighter and smaller stars.

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FIG. 1: (a) Light quark density as a function of $\mu$ and $\lambda$; (b) the same as (a) for the heavy quark density; (c) the same as (a) for the gap.
FIG. 2: Equation of state obtained from (8), (11), (12) and (13).
FIG. 3: Comparison between the EOS developed in this work with the MIT EOS.

FIG. 4: Solution of the TOV equations (18).
FIG. 5: Maximal star mass as a function of the light quark density $n_q$.

FIG. 6: Maximal star mass as a function of the heavy quark density $n_Q$. 