Research on the improved algorithm of radar signal sorting based on maximum SNR

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Abstract. This paper proposes an improved radar signal sorting algorithm based on the maximum signal-to-noise ratio criterion, aiming at the high computational complexity or poor separation effect of traditional signal sorting algorithms. The algorithm uses the maximum signal-to-noise ratio when the independent signals are completely separated to establish an objective function. The source signal is replaced by the mixed signal processed by the adaptive length moving average. The extreme value problem of the objective function is transformed into a generalized eigenvalue problem. Compared with the traditional method, the improved algorithm not only retains the separation effect of the traditional information theory sorting algorithm, but also has lower computational complexity. Experimental simulation proves that the algorithm can separate linearly aliased radar signals more effectively.

1. Introduction

In modern electronic countermeasures, sorting unknown radar signals in complex scenarios is a key link in radar countermeasures. However, due to the rapid increase in signal density in the electromagnetic environment and the lack of prior information under non-cooperative conditions, traditional sorting algorithms are increasingly difficult to apply. The emergence of blind source separation technology provides ideas for solving this problem. Literature[1] proposed a separation method based on information maximization or entropy maximization; Literature[2] proposed the ICA algorithm of the maximum likelihood criterion; then Hyvärinen et al. used fixed point iteration theory to solve the non-Gaussian maximization objective function, and proposed the famous FastICA algorithm[3], which greatly reduced the amount of calculation and convergence speed.

Although domestic research in the field of blind source separation started relatively late, there are still many results. In 2007, Yanru Yao used the fast independent variable analysis algorithm based on the maximization of negative entropy to successfully sort out the phase-encoded signal, the single pulse signal and the chirp signal[4]; then, Juanfang Chai combined the global optimal blind source separation algorithm and proposed a pseudo-based The blind source separation algorithm for maximizing the signal-to-noise ratio[5], the optimization of the objective function is achieved by solving the generalized eigenvalues, as long as the source is independent, the algorithm can be guaranteed to have a solution; In 2011, Yajun Li used the complex FastICA algorithm based on the kurtosis idea[6] to successfully extract the mixing matrix and the separation matrix to complete the separation of the multi-component radar emitter signal; in 2017, Chao Tong applied the FastICA algorithm based on negative entropy[7] to alias
signal separation. However, the above methods all have corresponding disadvantages. The separation effect of the algorithm based on information theory is ideal, but the amount of calculation is huge; the computational complexity of the algorithm based on maximizing the signal-to-noise ratio is greatly reduced, but the separation effect is deteriorated.

Therefore, this paper proposes an improved radar signal sorting algorithm based on the maximum signal-to-noise ratio criterion. The algorithm uses the maximum signal-to-noise ratio when the independent signals are completely separated to establish an objective function. The source signal is replaced by the mixed signal processed by the adaptive length moving average. The extreme value problem of objective function is transformed into a generalized eigenvalue problem. The improved algorithm has the separation effect of the information theory sorting algorithm and the low computational complexity of the maximum signal-to-noise ratio algorithm. Simulation proves that the algorithm can separate linearly aliased radar signals more effectively.

2. Improved algorithm principle

2.1. Blind source separation model

Suppose \( S \) is \( n \) mutually independent source signals, and \( H \) represents the mixing matrix. Then the mixed signal, that is, the observation signal \( X \) is expressed as:

\[
X = HS
\]

The main task of blind source separation is to find a separation matrix \( W \), so that the separation result is \( Y = X : \)

\[
Y = WX = WHS = S
\]

That is to say, the separation of the signal is realized when \( WH = I \). Therefore, the problem of signal sorting is transformed into finding a suitable matrix \( W \). In the context of this subject, only the mixed signal \( X \) is known, and the others are unknown.

2.2. Signal preprocessing

Because the radar emitter information is unknown, the signal needs to be preprocessed to reduce the difficulty of analysis. There are two most common pretreatment processes, one is de-averaging and the other is whitening. This is an indispensable step in blind source separation. In the FastICA problem, such processing of data can make the model more symbolic of the ICA constraints, simplify its structure, and improve the efficiency of problem-solving.

Zero averaging of the signal. The process of de-averaging can also be called centralization, which can be obtained by simplifying calculations. After completing the process of data de-averaging, it can be assumed that the signal is zero-averaged. The observation signal is \( X \), just replace \( X \) with \( X - E(X) \), where \( E(X) \) is the expectation of \( X \). In actual calculations, the arithmetic mean is generally used instead of expectation, assuming the data sample:

\[
x(n) = \{x_1(n), x_2(n), x_3(n), \ldots, x_N(n)\}, n = 0, 1, 2, \ldots, N - 1.
\]

\( N \) is the length of the sample. Then the arithmetic mean is:

\[
\bar{x}_i(n) = x_i(n) - \frac{1}{N} \sum_{n=0}^{N-1} x_i(n), i = 1, 2, 3, \ldots, N
\]

3. Signal whitening and algorithm principle

Whitening is also called spheroidizing. Its essence is to remove the correlation between signal components, so that the whitened signal components are second-order statistically independent, that is, uncorrelated, which can improve the performance of the algorithm. The so-called whitening of random vectors is to find a whitening matrix \( T \). Replace \( x \) with \( \bar{x} = Tx \) so that the correlation matrix \( R_x \) satisfies \( R_x = E(\bar{x}\bar{x}^T) = I \). There are many commonly used bleaching methods. Among them, the
numerical algorithm of matrix singular value decomposition has become the most common algorithm because of its better stability.

Taking the error between the source signal \( s \) and the estimated signal \( y \) obtained after separation as the noise signal, the noise signal is expressed as: \( e = s - y \), and the signal-to-noise ratio function is expressed as:

\[
\text{SNR} = 10\log_{10} \frac{s \cdot s^T}{e \cdot e^T} = 10\log_{10} \frac{s \cdot s^T}{(s-y) \cdot (s-y)^T}
\]  

Because the source signal \( s \) is an unknown quantity, it is necessary to consider replacing it with a certain signal. This article uses the moving average \( \bar{y} \) of the separated signal \( y \) instead of \( s \) to redefine a new function \( F_i(\bar{y},y) \):

\[
F_i(\bar{y},y) = 10\log_{10} \frac{\bar{y} \cdot \bar{y}^T}{(\bar{y} - y)(\bar{y} - y)^T}
\]  

Among them, \( \bar{y} = \frac{1}{p} \sum_{j=0}^{p} y(n-j) \), \( p \) is the moving average length.

To a certain extent, this function can replace the signal-to-noise ratio function to represent a certain physical meaning and is used to measure the separation effect of the separation system. As long as this function is maximized, it can be approximated that the signal-to-noise ratio is maximized.

Make \( \bar{y} = Wx \), \( y = Wx \), \( \bar{x} \) is the moving average of \( x \). Then formula (5) becomes:

\[
F_i(W,x) = 10\log_{10} \frac{W\bar{x} \cdot x^T W^T}{(W\bar{x} - Wx)(W\bar{x} - Wx)^T}
\]

\[
= 10\log_{10} \frac{W\bar{x} \cdot x^T W^T}{W(\bar{x} - x)(\bar{x} - x)^T W^T}
\]

\[
= 10\log_{10} \frac{WC_1 W^T}{WC_2 W^T}
\]  

Among them, \( C_1 = \bar{x} \cdot x^T \), \( C_2 = (\bar{x} - x)(\bar{x} - x)^T \).

Differentiate the two ends of the formula (6) and set it equal to 0:

\[
\frac{\partial F_i}{\partial W} = \frac{2WC_1}{WC_1 W^T} - \frac{2WC_2}{WC_2 W^T} = 0
\]  

Simplify it as:

\[
WC_2 W^T WC_1 = WC_1 W^T WC_2
\]  

Continue to simplify:

\[
C_1^{-1}C_2 = W^T WC_2 C_1^{-1}(W^T W)^{-1}
\]  

It can be seen that the separation matrix \( W \) is the eigenvector of the matrix \( C_2^{-1}C_1 \). Therefore, to obtain the separation matrix, only the eigenvectors of \( C_2^{-1}C_1 \) are required.

The separated source signal vector is \( y = Wx \), where each row of \( y \) represents a separated signal, that is, \( y \) is an estimate of the source signal \( s \).

The essence of this algorithm is to transform the optimization process into a generalized eigenvalue problem solving without the lengthy iterative steps in other algorithms, which is also the advantage of this algorithm. Literature [6] also proved that the algorithm is solvable as long as the signal obtained by the separation matrix and its derivative are not correlated. Since each radiation source in the space is statistically independent, the signals obtained through the separation matrix are also statistically independent, which ensures that the algorithm has a solution.
The method proposed in literature[6] does not specify how to choose the moving average length; although literature[7] studies the influence of different values on the separation effect, and obtains an optimized value in a specific scenario, the fixed value is not universal. Therefore, this paper proposes an adaptive value moving average algorithm. The value selection process is as follows:

\[ p = n \]

\[ 1/2x_{i+1} < x_i < 3/2x_{i+1}, i = 1, 2, ..., n-1 \]

\[ 1/2x_{i+1} < x_i < 3/2x_{i+1}, i = 1, 2, ..., n-2 \]

\[ 1/2x_{i+1} < x_i < 3/2x_{i+1}, i = 1 \]

\[ p = n \]

\[ p = n-1 \]

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Observing the signal in Figure 3, it can be seen that the two source signals are completely covered by noise and the waveforms are seriously distorted. As a result, it is no longer possible to distinguish the types of source signals.

Figure 2. Source signals

Figure 3. Observed signals
The results using the method of literature[6] are shown in Figure 4.

![Figure 4](image1)

Figure 4. Sorting algorithm based on fixed p-value

![Figure 5](image2)

Figure 5. Sorting algorithm based on adaptive p-value

It can be seen that the single carrier signal is ideal, but the LFM signal amplitude has a certain degree of distortion. Figure 5 uses the improved algorithm of this paper. It can be seen that the effect of the LFM signal in the separation result is improved.

4.2. Performance analysis

In order to evaluate the separation effect of the algorithm, the similarity coefficient $\zeta_{ij}$ between the separated signal and the source signal is taken as the performance index, which is defined as:

$$\zeta_{ij} = \zeta(s_i(t), y_j(t)) = \frac{\sum_{i=1}^{M} y_i(t)s_i(t)}{\sqrt{\sum_{i=1}^{M} y_i^2(t) \sum_{j=1}^{M} s_j^2(t)}}$$

(10)

Among them, $s_i(t)$ is the component of the radar radiator signal. $y_j(t)$ is the component of the separated signal, $i, j = 1, 2, 3, \ldots, N$. The closer the correlation coefficient is to 1, the higher the consistency between the separated signal and the source signal, and the better the separation effect of the algorithm. When the correlation coefficient is 1, the separation effect is obviously the best.

The similarity coefficients of the two separation methods in the previous section are calculated as follows:

1. Sorting algorithm based on fixed $p$ value:

$$\zeta = \begin{bmatrix} 0.0005 & 1.0000 & 0.0405 \\ 0.9665 & 0.0000 & 0.2681 \\ 0.2490 & 0.0000 & 0.9646 \end{bmatrix}$$

(11)

2. Sorting algorithm based on adaptive $p$ value:

$$\zeta = \begin{bmatrix} 0.0005 & 1.0000 & 0.0406 \\ 0.9998 & 0.0000 & 0.0036 \\ 0.0088 & 0.0000 & 0.9992 \end{bmatrix}$$

(12)

Obviously, compared with the method described in literature[6], the similarity of the LFM signal in the improved algorithm has been greatly improved.

Perform 100 Monte Carlo experiments on the method described in literature[6] and the improved method in this paper, and compare the similarity coefficients of the LFM signals as follows:
Figure 6. Algorithmic similarity coefficient comparison

It can be seen that the fixed value sorting algorithm is very unstable, the similarity coefficient is sometimes very good, sometimes particularly bad, and the overall fluctuation is large. The adaptive value sorting algorithm is very stable, and the overall stability is very close to 1. Simulation experiments prove that the improved algorithm is significantly better than the method described in literature[6].

5. Conclusion
Based on the maximum signal-to-noise ratio criterion, this paper proposes an improved radar signal sorting algorithm. Using the observation signal to carry out the adaptive length moving average processing result as the source signal, and establish the objective function based on the signal-to-noise ratio. Then optimize the objective function by solving the generalized eigenvalues. The final feature vector is the optimal solution. This method does not require iteration and solves the problem of high computational complexity of traditional information theory sorting methods, which is not conducive to engineering realization; it also improves the problem of poor separation effect of traditional sorting algorithms based on signal-to-noise ratio due to the fixed sliding length. The computer simulation results show that the improved algorithm can separate mixed radar signals more effectively. However, the algorithm is still in the simulation stage, and how to effectively implement the project remains to be studied.

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