Transverse-momentum dependent parton distributions in large-$N_c$ QCD

P.V. Pobylitsa

Institute for Theoretical Physics II, Ruhr University Bochum, D-44780, Bochum, Germany
and
Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg, 188300, Russia

Abstract

The transverse-momentum dependent quark distributions are studied in the limit of the large number of colors $N_c$. The leading orders of $N_c$ are determined for all twist-2 distributions including the $T$-odd functions. The isospin structure of the dominant contributions is found.

1 Introduction

Transverse-momentum dependent parton distributions appear in the theoretical description of single spin asymmetries in various hard processes including semi-inclusive deep inelastic scattering, Drell-Yan process and inclusive pion production [1, 2, 3, 4]. Although the phenomenological applications of transverse-momentum dependent distributions typically involve QCD-inspired methods, whose status does not reach the standards of solid factorization theorems, certain aspects of this work have been recently clarified by taking a more careful attitude to the role played by the initial- and final-state interactions [5, 6, 7]. It has been emphasized [6] that the choice of the Wilson lines in the definition of the transverse-momentum dependent parton distributions is sensitive to the structure of the initial- and final-state interactions in the considered hard process. In particular, this gives a theoretical argument in favor of nonvanishing $T$-odd distributions.

The nonperturbative nature of the transverse-momentum dependent parton distributions leaves us not too much space for a serious theoretical analysis of their properties. One of the standard methods allowing to study nonperturbative quantities in QCD is the limit of the large number of colors $N_c$ which was introduced in QCD by 't Hooft [8] and analyzed for baryons by Witten [9]. In this paper we study the large-$N_c$ limit of the twist-2 transverse-momentum dependent parton distributions in nucleon, assuming the $SU(2)$ isospin symmetry. In particular, we determine the dominant spin and isospin components of parton
distributions at large $N_c$. Certainly the $1/N_c$-expansion has its limitations. The expansion parameter $1/N_c = 1/3$ is not too small. Even in the leading order of the $1/N_c$-expansion, QCD is not solved, so that usually the work is reduced to the counting of powers of $N_c$ for various quantities and to the identification of dominant and suppressed structures. However, keeping in mind the general success of the phenomenological applications of the $1/N_c$-expansion, we hope that our results obtained for the transverse-momentum dependent distributions at $N_c \to \infty$ may have at least qualitative attitude to the real world.

The naive definition of the transverse-momentum dependent parton distributions is based on the matrix element of the quark fields $\psi$ over the nucleon state $|P,S\rangle$ with momentum $P$ and spin $S$

$$P_\mu P^\mu = M^2, \quad S_\mu S^\mu = -1, \quad P_\mu S^\mu = 0.$$  \hfill (1.1)

Introducing a light-cone vector $n$

$$n_\mu n^\mu = 0,$$  \hfill (1.2)

one can write the following decomposition \cite{3}

$$\begin{align*}
\frac{2(nP)}{(2\pi)^3M} & \int d^4z \delta(nz) e^{ikz} \langle P,S|\bar{\psi}_j(0)\psi_i(z)|P,S\rangle \\
& \begin{cases}
  f_{1T}(P\gamma^\mu) M \\
  + f_{-1T}^\perp \epsilon_{\mu\nu\rho\sigma} \gamma^\rho P^\nu S^\sigma T \\
  \frac{P^\mu k_T^\rho S_T^\rho}{M^2}
\end{cases} \\
+ \left[ \lambda g_{1L} - \frac{(k_T S_T)}{M} g_{1T} \right] \frac{(P\gamma^\mu) \gamma^5 S_T^\mu P^\nu}{M} \\
- \left[ \lambda h_{1L}^\perp - \frac{(k_T S_T)}{M} h_{1T}^\perp \right] i\sigma_{\mu\nu} \frac{P^\mu S_T^\nu}{M^2} + h_{1T}^\perp \frac{\sigma_{\mu\nu} k_T^\mu P^\nu}{M^2} \\
+ \ldots
\end{align*}$$  \hfill (1.3)

where

$$\lambda = M \frac{(nS)}{(nP)}.$$  \hfill (1.4)

The transverse components of vectors ($k_T, S_T$ etc.) are defined as projections on the plane orthogonal to $n$ and $P$. Parton distributions appearing in the RHS of Eq. (1.3) depend on $|k_T|^2$ and

$$x = \frac{(nk)}{(nP)}.$$  \hfill (1.5)

The ellipsis in the RHS of Eq. (1.3) stands for the higher twist contributions.

Functions $f_{1T}, h_{1T}^\perp$ are $T$-odd and vanish in the framework of the naive definition \cite{12}. However, as it was noted in Ref. \cite{5}, the naive definition of transverse-momentum dependent parton distributions (1.3) should be modified by including properly chosen Wilson lines. Let us introduce the compact notation

$$\Psi^{(\pm)}(z,n) = W(\pm \infty, z, n) \psi(z),$$  \hfill (1.6)
where $W(\pm \infty, z, n)$ is the Wilson line directed from the point $z$ to the future (past) infinity along the light-cone vector $n$. By analogy with Eq. (1.3) we can write the following decomposition:

$$
\frac{2(nP)}{(2\pi)^3 M} \int d^4 z \delta(nz) e^{ik_z z} \langle P, S | \bar{\Psi}_j(\pm)(0) \Psi_i(\pm)(z) | P, S \rangle ,
$$

$$
= \left\{ f_1^{(\pm)} \frac{(P \gamma)}{M} + f_{1T}^{(\pm)} e_{\mu\nu\rho\sigma} \gamma_{\mu}^{\rho} \frac{P_{\nu} \Sigma_{\rho\sigma}}{M^2} \right. 
- \left[ \lambda g_{1L}^{(\pm)} - \frac{(k_T S_T)}{M} g_{1T}^{(\pm)} \right] \frac{(P \gamma) \gamma_{5}}{M} - h_{1T}^{(\pm)} \frac{i\sigma_{\mu\nu} g_{1T}^{(\pm)} P_{\nu}}{M} 
- \left[ \lambda h_{1L}^{(\pm)} - \frac{(k_T S_T)}{M} h_{1T}^{(\pm)} \right] \frac{i\sigma_{\mu\nu} g_{1T}^{(\pm)} P_{\nu}}{M^2} + \frac{h_{1\perp}^{(\pm)} \sigma_{\mu\nu} h_{1T}^{(\pm)} P_{\nu}}{M^2} \right\}_{ij} + \ldots (1.7)
$$

which defines functions

$$
F_r^{(\pm)} = \{ f_1^{(\pm)}, g_1^{(\pm)} g_{1L}^{(\pm)}, g_{1T}^{(\pm)}, h_{1L}^{(\pm)}, h_{1T}^{(\pm)}, h_{1\perp}^{(\pm)} \} (1.8)
$$
depending on $x$ and $|k_T|^2$

$$
F_r^{(\pm)} = F_r^{(\pm)}(x, |k_T|^2) . (1.9)
$$

The $T$-invariance leads to the following relations between $F_r^{(+)}$ and $F_r^{(-)}$

$$
F_r^{(+)} = \left\{ \begin{array}{ll}
-F_r^{(-)} & \text{for } f_{1T}^{(\perp)}, h_{1T}^{(\perp)} \\
F_r^{(-)} & \text{otherwise.}
\end{array} \right. (1.10)
$$

which allows nonvanishing functions $f_{1T}^{(\perp)}, h_{1T}^{(\perp)}$.

Multiplying Eq. (1.7) by $(n\gamma)$ both on the left and on the right we can get rid of the higher twist contributions. Taking the nucleon at rest

$$
P^{\mu} = (M, 0) , \quad S^{\mu} = (0, S) , \quad |S| = 1 , (1.11)
$$

$$
S = -\lambda n + S_T , \quad (S_T n) = 0 (1.12)
$$

and normalizing the light-cone vector $n$ so that

$$
n^0 = |n| = 1 , (1.13)
$$

we find

$$
\int \frac{d^3 z}{(2\pi)^3} \exp \left[ izM(nz) - i(k_T z) \right] 
\times \langle P, S \mid [\Pi(n)\Psi^{(\pm)}(0, n)]^T [\Pi(n)\Psi^{(\pm)}(z, n)] \mid P, S \rangle \big|_{z^0 = n^0 z^+ , P^0 = 0}
$$
\[
\Pi(n) = \frac{1}{2} \gamma_0 (n \gamma) = \frac{1 - \gamma_0 (n \gamma)}{2},
\]

(1.15)

\[
[\Pi(n)]^2 = \Pi(n).
\]

(1.16)

The transverse components of vectors are orthogonal to \( n \):

\[
(k_T n) = 0.
\]

(1.17)

The projector \( \Pi(n) \) is defined as follows:

\[
\Pi(n) = \frac{1}{2} \gamma_0 (n \gamma) = \frac{1 - \gamma_0 (n \gamma)}{2},
\]

(1.15)

\[
[\Pi(n)]^2 = \Pi(n).
\]

(1.16)

The transverse components of vectors are orthogonal to \( n \):

\[
(k_T n) = 0.
\]

(1.17)

The structure of the paper is as follows. In Section 2 the main results are described. Section 3 contains a brief summary of the general properties of the large-\( N_c \) baryons. The transverse-momentum dependent quark distributions at large \( N_c \) are studied in Section 4. Several remarks about the \( T \)-odd distributions are made in Section 5.

## 2 Results

In this section the main results of the paper are described. Let us first remind the well known large-\( N_c \) properties of the usual (\( k_\perp \) integrated) parton distributions. At large number of colors \( N_c \), parton distributions are concentrated at values of Bjorken variable \( x \sim N_c^{-1} \). In the naive quark model this can be qualitatively explained as a result of the distribution of the nucleon momentum in the infinite momentum frame between \( N_c \) quarks. Therefore in the large-\( N_c \) limit, the \( x \) dependence \( x \) appears via the product \( x N_c = O(N_c^0) \). For example, the twist-2 parton distributions: unpolarized \( q(x) \), longitudinally polarized \( \Delta_L q(x) \) and transversity \( \Delta_T q(x) \)

\[
q_i = (q, \Delta_L q, \Delta_T q)
\]

have the following large-\( N_c \) behavior

\[
q_i(x) = N_c^2 \left[f_i(x N_c) + O\left(\frac{1}{N_c}\right)\right]
\]

(2.2)

where functions \( f_i \) are \( N_c \) independent. Concerning the isospin properties of parton distributions, we can make the flavor decomposition into the isoscalar \((T = 0)\) and isovector \((T = 1)\) parts:

\[
[q_i(x)]_{T_3} = q_i^{T=0}(x) + \text{sign}(t_3 I_3) q_i^{T=1}(x).
\]

(2.3)
Table 1: The $N^m_r$ behavior and the dominant isospin-$T_r$ structure of transverse-momentum dependent quark distributions $F_r^\pm$.

| $F_r^\pm$ | $m_r$ | $T_r$ |
|-----------|------|------|
| $f_r^\pm$ | 2    | 0    |
| $g_{1T}^\pm$ | 3 | 1 |
| $g_{1L}^\pm$ | 2 | 1 |
| $f_{1T}^\pm$ | 3 | 1 |
| $h_{1T}^\pm$ | 3 | 0 |
| $h_{1L}^\pm$ | 3 | 1 |
| $h_{1T}^\pm$ | 4 | 1 |
| $h_{1T}^\pm$ | 2 | 1 |

Here $t_3, I_3 = \pm 1/2$ are the isospin projections of the quark and nucleon respectively. In the leading order of the $1/N_c$-expansion we have the following isospin structure:

\[
q : \quad T = 0, \\
\Delta_L q, \Delta_T q : \quad T = 1.
\]  

(2.4)

The large-$N_c$ behavior (2.2), (2.4) was first observed in the context of the chiral quark-soliton model \[10\] but can be also derived using the standard methods of large-$N_c$ counting in QCD \[11\].

In the case of the transverse-momentum distributions $F_r^\pm$ \[13\] we find the following large-$N_c$ structure:

\[
F_r^\pm(x,|k_T|^2) = N^m_r \left[ G_r^\pm(x N_c,|k_T|^2) + O\left(\frac{1}{N_c}\right)\right],
\]  

(2.5)

where functions $G_r^\pm$ are $N_c$ independent:

\[
G_r^\pm = O(N_c^0).
\]  

(2.6)

The $N_c$ orders $m_r$ are given in Table 1. Also note that in the leading order of the large-$N_c$ limit every function $F_r$ is either isoscalar ($T_r = 0$) or isovector ($T_r = 1$). This structure is also shown in Table 1. We see from this table that the distributions insensitive to the nucleon spin ($f_1$ and $h_1^\perp$) are isoscalar, whereas other distributions dependent on the nucleon spin are isovector 1.

1This correlation between the spin and isospin structures is a general property of nucleon observables in the large-$N_c$ limit. It is a consequence of the spin-flavor symmetry of the large-$N_c$ baryons \[12\].
Note that the nucleon mass \( M = O(N_c) \) grows with \( N_c \). The definition of the transverse-momentum dependent distributions \( (1.14) \) contains explicit factors of \( M \) introduced as an auxiliary dimensional parameter. These factors of \( M \) contribute to our counting of \( N_c \) orders. This should be kept in mind if one tries to draw phenomenological conclusions about the suppression or enhancement of various distributions to physical processes from the large \( N_c \) counting.

3 Large-\( N_c \) baryons

In the large-\( N_c \) limit the mass of the nucleon \( M \) grows as \( O(N_c) \) \[9\]:

\[
M = O(N_c). \tag{3.1}
\]

In the naive quark model this can be attributed to the fact that nucleon consists of \( N_c \) quarks. According to the standard picture of the large-\( N_c \) baryons in QCD, the lowest baryonic excitations have spin \( S \) equal to isospin \( I \) (in the case of the \( SU(2) \) flavor group) \[12\]

\[
I = S = \frac{1}{2}, \frac{3}{2}, \ldots \tag{3.2}
\]

These states become degenerate at large \( N_c \)

\[
M_B = M \left[ 1 + O(N_c^{-1}) \right] \tag{3.3}
\]

and can be described as “rotational excitations” of the mean field solution of the effective theory describing large-\( N_c \) QCD. We use subscript \( B \)

\[
B = (I, I_3, S_3), \quad I = S \tag{3.4}
\]

for the set of discrete quantum numbers characterizing the baryon state.

It is convenient to work in the frame where this heavy large-\( N_c \) nucleon is nonrelativistic. But we still use the relativistic normalization of nucleon states:

\[
\langle B_1, P_1 | B_2, P_2 \rangle = 2 P_0 (2\pi)^3 \delta_{B_1 B_2} \delta^{(3)}(P_1 - P_2)
\]

\[
= 2 M \left[ 1 + O(N_c^{-1}) \right] \delta_{B_1 B_2} (2\pi)^3 \delta^{(3)}(P_1 - P_2). \tag{3.5}
\]

The exact form of the effective action describing large-\( N_c \) QCD is not known. But certain conclusions can be derived from the symmetry of the mean field: the mean field is assumed to have the spin-flavor invariance: it is invariant under simultaneous space and isospin rotations. In the leading order of the \( 1/N_c \)-expansion, for a certain class of gauge invariant operators \( \hat{A} \) (including bilocal quark operators accompanied by Wilson lines) their matrix elements over baryon states \( (3.2) \) can be represented in the form

\[
\langle B_1, P_1 | \hat{A} | B_2, P_2 \rangle = 2 M \int d^3 X e^{i(P_2 - P_1) \cdot X}
\]
\[ \times \int dR \phi_{B_1}^* (R) \phi_{B_2} (R) A(X, R) \left[ 1 + O \left( \frac{1}{N_c} \right) \right]. \quad (3.6) \]

Here \( A(X, R) \) is the mean field function associated with the operator \( \hat{A} \). The 3-vector \( X \) and the matrix \( R \in SU(2) \) have the meaning of the collective coordinates corresponding to the 3-translations and isotopic rotations respectively. The function \( e^{iPX} \) can be interpreted as the translational wave function of the nucleon with momentum \( P \), whereas \( \phi_{B} (R) \) is its “rotational” wave function.

For the eigenstates of spin and isospin, these rotational wave functions can be expressed in terms of Wigner functions \( D_{jmm'}^j(R) \) (see Appendix A):

\[ \phi_I = S, I_s = S_3 (R) = (-1)^{I+I_s} \sqrt{2I+1} D_{I-I', I'_s, S_3} (R). \quad (3.7) \]

Here the normalization coefficient is fixed by the condition

\[ \int dR \phi_{B_1}^* (R) \phi_{B_2} (R) = \delta_{B_1 B_2} \quad (3.8) \]

and the \( SU(2) \) Haar measure \( dR \) is normalized as follows:

\[ \int dR = 1. \quad (3.9) \]

Usually equations of the type (3.6) appear in various soliton models of nucleon. In these models one can find function \( A(X, R) \) using the the action of the model. In the case of large-\( N_c \) QCD the action of the corresponding effective theory is not known. Therefore for most of operators \( \hat{A} \) the explicit form of the function \( A(X, R) \) is not known. However, it is still possible to derive interesting consequences from the representation (3.6).

As a simple illustration of Eq. (3.6) one can consider the case of the unit operator:

\[ \hat{A} = 1, \quad A(X, R) = 1. \quad (3.10) \]

Then using relation (3.8) and

\[ \int d^3 X e^{i(P_2 - P_1) \cdot X} = (2\pi)^3 \delta(3)(P_1 - P_2) \quad (3.11) \]

we can “derive” the normalization condition (3.5) from Eq. (3.6).

### 4 Parton distributions at large \( N_c \)

Let us apply the general equation (3.6) to the following operator \( \hat{A} \)

\[ \hat{A}^{(\pm)}_{z_1 t_1 \lambda t_1} (z_2, z_1, n) = \left[ \Psi_{z_1 t_1}^{(\pm)} (z_1, n) \right]^\dagger \Psi_{z_2 t_2}^{(\pm)} (z_2, n). \quad (4.1) \]
Here $\Psi^{(\pm)}_{s_2 t_2}$ are quark fields modified by the Wilson lines directed along the light-cone vector $n$. The indices $t_k$ stand for the isospin, and the $s_k$ are Dirac 4-spinor indices.

For this operator $A^{(\pm)}(z_1, z_2, n)$ relation takes the form

$$
(B_1, P_1) \left[ \Psi^{(\pm)}_{s_1 t_1}(z_1, n) \right] \left[ \Psi^{(\pm)}_{s_2 t_2}(z_2, n) \right] (B_2, P_2) = 2M \left[ 1 + O(N_c^{-1}) \right] 
$$

$$
\times \int d^3 x e^{i(p_2 - p_1) \cdot x} \int dR \phi_{B_1}^*(R) \phi_{B_2}(R) A^{(\pm)}_{s_1 t_1, s_2 t_2, z_1, n}(z_2, z_1, n | X, R). \tag{4.2}
$$

Starting from this expression we can compute the matrix element appearing the LHS of Eq. (4.3):

$$
\int \frac{d^3 z}{(2\pi)^3} \exp \left[ i \gamma \cdot M n z - i(k_{T} \cdot z) \right] 
\times \left< B_1, P = 0 \left| \Pi(n) \Psi^{(\pm)}(0, n) \right|_{s_1 t_1} \left[ \Pi(n) \Psi^{(\pm)}(n z, z; n) \right]_{s_2 t_2} | B_2, P = 0 \right> 
= 2MN_c \left[ 1 + O(N_c^{-1}) \right] \int dR \phi_{B_1}^*(R) \phi_{B_2}(R) \left[ R_{s_1 t_1}^T \rho^{(\pm)}_{s_2 t_2, s_1 t_1, k_{T}, n}(x, k_{T}, n)(R^{-1})_{t_1, t_1} \right]. \tag{4.3}
$$

This equation is derived in Appendix B where one can find the explicit expression for the function $\rho^{(\pm)}$ in terms of $A^{(\pm)}$. Actually neither $A^{(\pm)}$ nor $\rho^{(\pm)}$ can be computed from the first principles, since large-$N_c$ QCD is not solved. Therefore we can proceed relying only on the symmetries of $\rho^{(\pm)}$.

Using the $P$-invariance properties of the matrix element appearing in the LHS of Eq. (4.3) we find

$$
\gamma^0 \rho^{(\pm)}(x, k_{T}, n) \gamma^0 = \rho^{(\pm)}(x, -k_{T}, -n). \tag{4.4}
$$

Similarly the $T$-invariance leads to the following relation

$$
\left[ \rho^{(\pm)}(x, k_{T}, n) \right]^T = \tau^2 C \gamma_5 \rho^{(-)}(x, -k_{T}, -n) \gamma_5 C^{-1} \tau^2. \tag{4.5}
$$

Here the superscript $T$ stands for the matrix transposition (both in spin and flavor indices) and $C$ is the matrix with the property

$$
C \gamma_\mu C^{-1} = -\gamma_{\mu}^T. \tag{4.6}
$$

We use notation $\tau^a$ for the isospin Pauli matrices and take into account that

$$
\tau^2 \tau^a \tau^2 = -(\tau^a)^T. \tag{4.7}
$$

Due to the presence of the projector $\Pi(n)$ in the LHS of Eq. (4.3) we have

$$
\Pi(n) \rho^{(\pm)}(x, k_{T}, n) \Pi(n) = \rho^{(\pm)}(x, k_{T}, n). \tag{4.8}
$$
In Appendix C we find the general solution of constraints (1.4) and (3.8):

\[
\rho(\pm)(x, k_T, n) = \left\{ V_1^{(\pm)} + V_2^{(\pm)} \gamma_5 (k_T \tau) + V_3^{(\pm)} \gamma_5 (n \tau) + V_4^{(\pm)} (n \cdot [k_T \times \tau]) + V_5^{(\pm)} i(k_T \gamma) + V_6^{(\pm)} i(n \cdot [\gamma \times k_T]) (n \tau) + V_7^{(\pm)} i(n \cdot [\gamma \times k_T]) (k_T \tau) + V_8^{(\pm)} i(n \cdot [\gamma \times \tau]) \right\} \Pi(n) .
\]

Here

\[
V_r^{(\pm)} = V_r^{(\pm)}(x, |k_T|^2).
\]

According to Eqs. (4.9) and (B.24) we have

\[
V_r = O(N^0_c) .
\]

The \( T \)-invariance (4.10) leads to the property

\[
V_r^{(+)} = \begin{cases} -V_r^{(-)} & \text{for } r = 4, 5, \\ V_r^{(-)} & \text{otherwise} . \end{cases}
\]

In Appendix D using decomposition (4.10), we compute the integral over \( R \) appearing in the RHS of Eq. (4.13):

\[
\int dR \phi^*_l_s S(R) \phi_{l_5,s} \left[ R_{t'_5 t'5} \rho_{l_5}^{(\pm)}(x, k_T, n)(R^{-1})_{t'_5 t'_5} \right]
\]

\[
= \left( \delta_{t_5 t'_1} \left[ V_1^{(\pm)} + V_5^{(\pm)} i(k_T \gamma) \right] \right)_{s_2 s_1} - \frac{2}{3} I_3 (\tau^3)_{t_2 t_5} \left\{ V_2^{(\pm)} \gamma_5 (Sk_T) + V_3^{(\pm)} (nS) \gamma_5 + V_4^{(\pm)} [n \times k_T] \cdot S + V_7^{(\pm)} i(n \cdot [\gamma \times k_T]) (Sk_T) + V_8^{(\pm)} i(n \cdot [\gamma \times \tau]) \right\} \Pi(n) \right)_{s_2 s_1} .
\]

Inserting this expression into Eq. (4.13) and combining the result with Eq. (1.14) we find

\[
\delta_{t_5 t_1} \left[ V_1^{(\pm)} + V_5^{(\pm)} i(k_T \gamma) \right]_{s_2 s_1} - \frac{2}{3} I_3 (\tau^3)_{t_2 t'_5} \left\{ V_2^{(\pm)} \gamma_5 (Sk_T) + V_3^{(\pm)} (nS) \gamma_5 + V_4^{(\pm)} [n \times k_T] \cdot S + V_7^{(\pm)} i(n \cdot [\gamma \times k_T]) (Sk_T) + V_8^{(\pm)} i(n \cdot [\gamma \times \tau]) \right\} \Pi(n) \right)_{s_2 s_1}
\]

\[
= \frac{1}{4 M N_c} \left\{ f_1^{(\pm)} + f_1^{(\pm)} \frac{1}{M} [n \times k_T] \cdot S + \left[ -(nS) g_{1L}^{(\pm)} + \frac{(k_T S)}{M} g_{1T}^{(\pm)} \right] \gamma_5 \right\} .
\]
\[- \left[ - (nS) h_{1L}^{(\pm)} + \left( \frac{k_T S}{M} h_{1T}^{(\pm)} \right) \right] \frac{i}{M} \left[ (n \times \gamma) \cdot k_T \right] \]

\[- h_{1T}^{(\pm)} i [(n \times \gamma) \cdot S] - h_{1L}^{(\pm)} \frac{i}{M} (k_T \gamma) \right]_{s_2 t_2, s_1 t_1} \] \quad (4.14)

Using Eq. (1.12), we see that the LHS of Eq. (4.14) contains the same spin-isospin structures as the RHS so that

\[
\left( f_1^{(\pm)} \right)_{t_1 t_2} = 4 M N_c \delta_{t_2 t_1} V_1^{(\pm)} = O(N_c^2),
\] \quad (4.15)

\[
\left( g_{1T}^{(\pm)} \right)_{t_1 t_2} = - \frac{8}{3} I_3(\tau^3)_{t_2 t_1} M^2 N_c V_2^{(\pm)} = O(N_c^3),
\] \quad (4.16)

\[
\left( g_{1L}^{(\pm)} \right)_{t_1 t_2} = \frac{8}{3} I_3(\tau^3)_{t_2 t_1} M N_c V_3^{(\pm)} = O(N_c^2),
\] \quad (4.17)

\[
\left( f_{1T}^{(\pm)} \right)_{t_1 t_2} = - \frac{8}{3} I_3(\tau^3)_{t_2 t_1} M^2 N_c V_4^{(\pm)} = O(N_c^3),
\] \quad (4.18)

\[
\left( h_1^{(\pm)} \right)_{t_1 t_2} = - 4 M^2 N_c \delta_{t_2 t_1} V_5^{(\pm)} = O(N_c^3),
\] \quad (4.19)

\[
\left( h_{1L}^{(\pm)} \right)_{t_1 t_2} = - \frac{8}{3} I_3(\tau^3)_{t_2 t_1} M^2 N_c V_6^{(\pm)} = O(N_c^3),
\] \quad (4.20)

\[
\left( h_{1T}^{(\pm)} \right)_{t_1 t_2} = \frac{8}{3} I_3(\tau^3)_{t_2 t_1} M^3 N_c V_7^{(\pm)} = O(N_c^4),
\] \quad (4.21)

\[
\left( h_{1T}^{(\pm)} \right)_{t_1 t_2} = \frac{8}{3} I_3(\tau^3)_{t_2 t_1} M N_c V_8^{(\pm)} = O(N_c^2). \] \quad (4.22)

The orders of $N_c$ are determined taking into account Eqs. (4.1) and (4.11). We remind that indices $t_1, t_2$ are isospin projections of the quark fields in the definition of the parton distributions \[1.17\]. We see that distributions $f_1, h_1^\pm$ are isoscalar in the leading order of the $1/N_c$-expansion whereas the leading parts of all other distributions are isovector.

Note that relations (4.15) – (4.22) are compatible with the $T$-parity properties of parton distributions \[1.10\] and functions $V_r \ [1.12]$. Our results (4.15) – (4.22) are summarized in Table 1 of Section 2.

5 \textit{T-odd distributions in large-$N_c$ QCD}

The status of the $T$-odd distributions $f_{1T}^\pm$ and $h_1^\pm$ was clarified in Ref. \[6\] where it was noted that the orientation of the Wilson lines accompanying the quark fields is sensitive to the considered process. Therefore one deals with two sets of
distributions $F_r^{(+)}$ and $F_r^{(-)}$ which are related by the $T$-parity relations \(^{11,10}\). In particular, for the $T$-odd functions one has

$$f_{1T}^{\perp (+)} = -f_{1T}^{\perp (-)}, \quad h_{1}^{\perp (+)} = -h_{1}^{\perp (-)}.$$  

Due the dependence of parton distributions on the direction of Wilson lines functions $f_{1T}^{\perp (\pm)}$, $h_{1}^{\perp (\pm)}$ do not vanish.

In our construction of the $1/N_c$-expansion the sensitivity of parton distributions to the direction of Wilson lines is properly taken into account. Therefore the nonvanishing $T$-odd functions $f_{1T}^{\perp (\pm)}$, $h_{1}^{\perp (\pm)}$ naturally appear via the $T$-odd structures $V_4, V_5^{11,10}$ in our analysis of the large-$N_c$ limit.

As it is well known, various chiral models properly reproduce the large-$N_c$ counting of QCD. For example, the large-$N_c$ structure \(^{2,2}, 241\) of $k_{\perp}$ integrated distributions historically was first established in the context of the chiral quark soliton model. The large-$N_c$ properties of the $T$-even distributions can be also reproduced in chiral models. However, with the $T$-odd distributions the situation is different. In Ref. \(^{13}\) an attempt was made to generate $T$-odd distributions in the chiral sigma model model. But the $T$-odd distributions do not vanish in QCD only due to the special choice of Wilson lines. The absence of Wilson lines (or other effective fields imitating Wilson lines) in the chiral models does not allow to obtain $T$-odd distribution functions in these models \(^{13}\). This failure of the chiral models has nothing to do with our analysis of large-$N_c$ limit of QCD where the $T$-odd functions are clearly seen in Eq. (4.14).

6 Conclusions

In this paper we have performed the large-$N_c$ analysis of the transverse-momentum dependent quark distributions. The leading powers of $N_c$ are determined for all twist-2 distributions. The isospin structure of the leading contributions is also found. We have explicitly checked that the large-$N_c$ limit allows the existence of nonzero $T$-odd distributions. The results are based only on the spin-flavor symmetry of the large-$N_c$ baryons in QCD without any model assumptions.

Acknowledgments. The author appreciates discussions with Ya.I. Azimov, D. Boer, A.V. Efremov, K. Goeke, A. Metz, P.J. Mulders, M.V. Polyakov and P. Schweitzer. This work was supported by DFG and BMBF.

Appendices

A Wigner functions

Wigner functions $D_{mm'}^j(R)$ correspond to the spin-$j$ representation of $SU(2)$ matrices $R$

$$\sum_{m'} D_{mm'}^j(R_1) D_{m'm''}^j(R_2) = D_{mm''}^j(R_1 R_2).$$  

(A.1)
Assuming the unit normalization of the $SU(2)$ Haar measure

$$\int dR = 1, \tag{A.2}$$

we have

$$\int dRD_{m_1m'_1}(R)D_{m_2m'_2}(R) = \frac{1}{2j_1+1}\delta_{j_1j_2}\delta_{m_1m_2}. \tag{A.3}$$

The integrals containing products of three $D$ functions can be expressed in terms of $3j$-symbols:

$$\int dRD_{j_1m_1m'_1}(R)D_{j_2m_2m'_2}(R)D_{j_3m_3m'_3}(R) = (j_1j_2j_3,m_1m_2m_3) (j_1j_2j_3,m'_1m'_2m'_3). \tag{A.4}$$

### B Large-$N_c$ limit and spin-flavor symmetry

In this appendix we derive relation (4.3). As it was mentioned above, we cannot compute functions $A^{(\pm)}$ appearing in the RHS of Eq. (4.2) from the first principles in large-$N_c$ QCD. However, we can study the symmetry properties of $A^{(\pm)}$ following from the symmetries of the mean field describing the baryons in the (unknown) effective theory equivalent to large-$N_c$ QCD.

1) The mean field is “static” (i.e. translationally invariant in time):

$$A_{s_2t_2s_1t_1}^{(\pm)}(z_0^0 + T, z_2; z_1^0 + T, z_1; n|X, R) = A_{s_2t_2s_1t_1}^{(\pm)}(z_0^0, z_2; z_1^0, z_1; n|X, R). \tag{B.1}$$

2) The mean field has the spin-flavor symmetry $[12]$:

$$S_{s_2t_2s_1t_1}(R)R_{t_2t_1}A_{s_2t_2s_1t_1}^{(\pm)}(z_2^0, O(R^{-1})z_2; z_1^0, O(R^{-1})z_1; n^0, O(R^{-1})n|0, 1) \times (R^{-1})_{t_2t_1}S_{s_2t_2s_1t_1}(R^{-1}) = A_{s_2t_2s_1t_1}^{(\pm)}(z_2^0, z_2; z_1^0, z_1; n^0, n|0, 1). \tag{B.2}$$

Here $R$ is an arbitrary $SU(2)$ matrix acting on the isospin indices of $A^{(\pm)}$. Next, $O(R)$ is the space rotation $SO(3)$ matrix associated with the $SU(2)$ matrix $R$:

$$O^{ab}(R) = \frac{1}{2} \text{Tr}(\tau^a R^b R^{-1}). \tag{B.3}$$

The matrix $S(R)$ is the representation of the 3-dimensional rotations for Dirac spinors: if

$$R = \exp(i\omega^a\tau^a) \tag{B.4}$$

then

$$S(R) = \exp(i\omega^a\gamma^5\gamma^0\gamma^a). \tag{B.5}$$
3) The mean field describing the nucleon is not translationally invariant but the collective coordinate $X$ allows us to restore this invariance:

$$A_{s_{2}t_{2}s_{1}t_{1}}^{(\pm)}(z_{2}^{0}, z_{2}; z_{1}^{0}, z_{1}; n|X, R) = A_{s_{2}t_{2}s_{1}t_{1}}^{(\pm)}(z_{2}^{0}, z_{2} - X; z_{1}^{0}, z_{1} - X; n|0, R). \quad (B.6)$$

4) Similarly the isotopic invariance broken by the mean field is restored by using the collective coordinate $R$

$$A_{s_{2}t_{2}s_{1}t_{1}}^{(\pm)}(z_{2}^{0}, z_{2}; z_{1}^{0}, z_{1}; n|X, R) = R_{s_{2}t_{2}'}A_{s_{2}t_{2}s_{1}t_{1}'}^{(\pm)}(z_{2}^{0}, z_{2}; z_{1}, n|X, 1)(R^{-1})_{t_{1}t_{1}'}. \quad (B.7)$$

Combining the properties (B.1), (B.3), (B.7), we conclude that

$$A_{s_{2}t_{2}s_{1}t_{1}}^{(\pm)}(z_{2}^{0}, z_{2}; z_{1}^{0}, z_{1}; n|X, R) = R_{s_{2}t_{2}'}A_{s_{2}t_{2}s_{1}t_{1}'}^{(\pm)}(z_{2}^{0} - z_{1}^{0}, z_{2} - X; 0, z_{1} - X; n|0, 1)(R^{-1})_{t_{1}t_{1}'} \quad (B.8)$$

Let us introduce the following notation:

$$F_{s_{2}t_{2}'}^{(\pm)}(T, z_{2}, z_{1}, n) \equiv N_{c}^{-1}A_{s_{2}t_{2}s_{1}t_{1}'}^{(\pm)}(T, z_{2}; 0, z_{1}; n|0, 1). \quad (B.9)$$

Here we have included the factor of $N_{c}^{-1}$ into the definition of $F^{(\pm)}$ in order to have

$$F_{s_{2}t_{2}'}^{(\pm)} = O(N_{c}^{0}). \quad (B.10)$$

Indeed, the definition of the operator $A_{s_{2}t_{2}s_{1}t_{1}}^{(\pm)}$ implicitly contains the contraction of the color indices of the quark fields $\Psi^{(\pm)}$ which leads to the $O(N_{c})$ behavior of $A_{s_{2}t_{2}s_{1}t_{1}}^{(\pm)}$.

We find from Eq. (B.8)

$$A_{s_{2}t_{2}s_{1}t_{1}}^{(\pm)}(z_{2}^{0}, z_{2}; z_{1}^{0}, z_{1}; n|X, R) = N_{c}R_{s_{2}t_{2}'}F_{s_{2}t_{2}'}^{(\pm)}(z_{2}^{0} - z_{1}^{0}, z_{2} - X, z_{1} - X, n)(R^{-1})_{t_{1}t_{1}'} \quad (B.11)$$

and the spin-flavor symmetry relation (B.2) takes the form

$$S_{s_{2}t_{2}'}(R)R_{s_{2}t_{2}'}F_{s_{2}t_{2}'}^{(\pm)}(z_{2}^{0} - z_{1}^{0}, O(R^{-1})z_{2}, O(R^{-1})z_{1}, n, 0, O(R^{-1})n)$$

$$\times(R^{-1})_{t_{1}t_{1}'}S_{t_{1}'s_{1}}(R^{-1}) = F_{s_{2}t_{2}s_{1}t_{1}}^{(\pm)}(z_{2}^{0} - z_{1}^{0}, z_{2}, z_{1}, n, 0, n). \quad (B.12)$$

Inserting Eqs. (B.1), (B.11) into Eq. (B.2) we find

$$\langle B_{1}, P_{1}\rangle [\Psi_{s_{1}t_{1}}^{(\pm)}(z_{1}^{0}, z_{1}, n)]^\dagger \Psi_{s_{2}t_{2}}^{(\pm)}(z_{2}^{0}, z_{2}, n)|B_{2}, P_{2}\rangle$$

$$= 2MN_{c} \int d^{3}Xe^{i(P_{2} - P_{1} \cdot X)} \int dR\phi_{B_{1}}^{*}(R)\phi_{B_{2}}(R)$$

13
Defining

\[ \times \left[ R_{t_2 t_1}^+ F_{s_2 t_2, s_1 t_1}^{(\pm)} (z^0 - z_1, z_2; z_1 - X, z_2 - X, n^0, n)(R^{-1})_{t_1 t_1} \right] . \]  (B.13)

we arrive at

\[ z_1 = 0, \quad z_2 = z, \quad P_1 = P_2 = 0, \]  (B.14)

we find from Eq. (B.16)

\[ \langle B_1, P = 0 \left| \Psi_{s_1 t_1}^{(\pm)}(0, n) \right| \Psi_{s_2 t_2}^{(\pm)}(z^0, z; n)\rangle B_2, P = 0 \rangle = 2MN_e \int d^3X \]

\[ \times \int dR \phi_{B_1}^* (R) \phi_{B_2}(R) \left[ R_{t_2 t_1}^+ F_{s_2 t_2, s_1 t_1}^{(\pm)} (z^0, z - X, n^0, n)(R^{-1})_{t_1 t_1} \right] . \]  (B.15)

Taking \( z^0 = (nz) \), we obtain

\[ \int \frac{d^3z}{(2\pi)^3} \exp \left[ ixM(nz) - i(k_T z) \right] \times \langle B_1, P = 0 \left| \Psi_{s_1 t_1}^{(\pm)}(0, n) \right| \Psi_{s_2 t_2}^{(\pm)}((nz), z, n)\rangle B_2, P = 0 \rangle \]

\[ = 2MN_e \int \frac{d^3z}{(2\pi)^3} \exp \left[ ixM(nz) - i(k_T z) \right] \int d^3X \]

\[ \times \int dR \phi_{B_1}^* (R) \phi_{B_2}(R) \left[ R_{t_2 t_1}^+ F_{s_2 t_2, s_1 t_1}^{(\pm)} ((nz), z - X, n^0, n)(R^{-1})_{t_1 t_1} \right] . \]  (B.16)

Defining

\[ \tilde{\rho}_{s_2 t_2, s_1 t_1}^{(\pm)} (x, k_T, n) = \int \frac{d^3z}{(2\pi)^3} \exp \left[ ixM(nz) - i(k_T z) \right] \] \[ \times \int d^3X F_{s_2 t_2, s_1 t_1}^{(\pm)} ((nz), z - X, n^0, n) . \]  (B.17)

we find from Eq. (B.14)

\[ \int \frac{d^3z}{(2\pi)^3} \exp \left[ ixM(nz) - i(k_T z) \right] \times \langle B_1, P = 0 \left| \Psi_{s_1 t_1}^{(\pm)}(0, n) \right| \Psi_{s_2 t_2}^{(\pm)}((nz), z; n)\rangle B_2, P = 0 \rangle \]

\[ = 2MN_e \int dR \phi_{B_1}^* (R) \phi_{B_2}(R) \left[ R_{t_2 t_1}^+ \tilde{\rho}_{s_2 t_2, s_1 t_1}^{(\pm)} (x, k_T, n)(R^{-1})_{t_1 t_1} \right] . \]  (B.18)
The $SU(2)$ spin-flavor symmetry (B.12) of $F_{\sigma \tau \sigma_1 \tau_1}$ results in the spin-flavor symmetry of $\tilde{\rho}_{\sigma \tau \sigma_1 \tau_1}$:

$$S_{\sigma \sigma_1'}(R) R_{\tau \tau_1} \tilde{\rho}_{\sigma \tau \sigma_1 \tau_1}^{(\pm)}(x, O(R^{-1})k_T, O(R^{-1})n)(R^{-1})_{\tau_1 \tau_1'} \tilde{\rho}_{\sigma_1' \sigma_1}^{(\pm)}(R^{-1})$$

$$= \tilde{\rho}_{\sigma \tau \sigma_1 \tau_1}^{(\pm)}(x, k_T, n). \quad \text{(B.19)}$$

Next we introduce notation

$$\rho^{(\pm)}(x, k_T, n) = \Pi(n) \tilde{\rho}^{(\pm)}(x, k_T, n) \Pi(n) \quad \text{(B.20)}$$

with $\Pi(n)$ given by (1.15). According to Eq. (B.10) we have

$$\rho^{(\pm)}_{\sigma \tau \sigma_1 \tau_1} = O(A^0_c). \quad \text{(B.21)}$$

Eq. (B.19) yields

$$S_{\sigma \sigma_1'}(R) R_{\tau \tau_1} \rho^{(\pm)}_{\sigma \tau \sigma_1 \tau_1}(x, O(R^{-1})k_T, O(R^{-1})n)(R^{-1})_{\tau_1 \tau_1'} S_{\sigma_1' \sigma_1}(R^{-1})$$

$$= \rho^{(\pm)}_{\sigma \tau \sigma_1 \tau_1}(x, k_T, n). \quad \text{(B.22)}$$

We easily obtain Eq. (4.3) from Eqs. (B.18), (B.20).

### C \hspace{1em} P-parity constraints

In this appendix we show that the matrix function $\rho^{(\pm)}$ satisfying constraints (4.4), (4.8) can be represented in the form (4.9).

We remind that the matrix $\rho^{(\pm)}$ acts on 4-component Dirac spinors and 2-component isospinors. Due to the property (B.20) we effectively deal with 2-component spinors so that there are $2 \times 2 = 4$ independent spin structures in which $\rho^{(\pm)}$ can be decomposed. For example, we can choose

$$1, \gamma_5, i(a_T \gamma) \quad \text{(C.1)}$$

where $a_T$ is an arbitrary 3-vector obeying

$$(a_T n) = 0. \quad \text{(C.2)}$$

Note that all structures (C.1) commute with the projector $\Pi(n)$ (1.15). Taking

$$a_T = k_T, [n \times k_T], \quad \text{(C.3)}$$

we obtain the following basis of spin matrices in which $\rho^{(\pm)}$ can be decomposed:

$$\{B_\alpha\} = \{1, \gamma_5, i(k_T \gamma), i n \cdot [k_T \times \gamma]\}. \quad \text{(C.4)}$$
In the isospin space we can use the following basis:

\[ \{C_\beta\} = \{1, (k_T \tau), (n \cdot [k_T \times \tau]\} . \]  

(C.5)

Now we can build

4_{spin} \times 4_{isospin} = 16 \]  

(C.6)

spin-isospin structures

\[ B_\alpha C_\beta \Pi(n) \]  

(C.7)

which can be used as a basis for the decomposition of \( \rho(\pm) \).

The matrix \( \rho(\pm) \) should satisfy \( P \)-parity property (4.4). Note that among \( B_\alpha \) and \( C_\beta \) only one half of structures are \( P \)-even and others are \( P \)-odd:

\[ \{B_\alpha\} = \{1, i (k_T \gamma)\} p_{-even} \cup \{\gamma_5, i (n \cdot [k_T \times \gamma])\} p_{-odd} , \]  

(C.8)

\[ \{C_\beta\} = \{1, n \cdot [k_T \times \tau]\} p_{-even} \cup \{(k_T \tau), (n \tau)\} p_{-odd} . \]  

(C.9)

Therefore only 8 structures \( B_\alpha C_\beta \) are \( P \)-even:

\[ \{1, i (k_T \gamma)\} \otimes \{1, n \cdot [k_T \times \tau]\} , \]  

(C.10)

\[ \{\gamma_5, i (n \cdot [k_T \times \gamma])\} \otimes \{(k_T \tau), (n \tau)\} . \]  

(C.11)

Thus the matrix \( \rho(\pm) \) obeying the relation (4.4) can be decomposed in these structures (C.10), (C.11). However, for our purposes we prefer to replace one of these structures by a linear combination. To this aim we use the following identity

\[ (n \cdot [\gamma \times k_T]) (k_T \tau) + (k_T \cdot [\tau \times n]) (k_T \gamma) = (\tau \cdot [n \times \gamma]) |k_T|^2 . \]  

(C.12)

Indeed, in the 3-dimensional space the antisymmetrization with respect to four indices always gives zero:

\[ \varepsilon^{ijkl} \delta^n = 0 . \]  

(C.13)

Contracting this equality with

\[ n^j \gamma^k k^l \tau^m k^n , \]  

(C.14)

we obtain Eq. (C.12). Note that both terms entering in the LHS of Eq. (C.12) appear among the structures (C.10), (C.11). The identity (C.12) means that in the basis (C.10), (C.11) we can replace

\[ (n \cdot [k_T \times \tau]) (k_T \gamma) \]  

(C.15)

by the other independent structure

\[ (\tau \cdot [n \times \gamma]) . \]  

(C.16)

With this change of the basis (C.10), (C.11) we arrive at the decomposition (4.9) for \( \rho(\pm) \). Note that due to Eq. (4.10) representation (4.9) for \( \rho(\pm) \) satisfies condition (B.22).
D Calculation of $R$ integrals

In this appendix we derive Eq. (4.13).

We have according to Eq. (4.9)

$$\rho_{s_2t_2s_1t_1}^{(\pm)}(x, k_T, n) = \delta_{t_2t_1} \rho_{s_2s_1}^{(\pm)0}(x, k_T, n) + \tau_{t_2t_1}^{a} \rho_{s_2s_1}^{(\pm)a}(x, k_T, n).$$  \hspace{1cm} (D.1)

where

$$\rho^{(\pm)0}(x, k_T, n) = \left\{ V_{1}^{(\pm)} + V_{5}^{(\pm)i}(k_T \gamma) \right\} \Pi(n),$$  \hspace{1cm} (D.2)

$$\rho^{(\pm)a}(x, k_T, n) = \left\{ V_{2}^{(\pm)} \gamma_5 k_T^a + V_{3}^{(\pm)} \gamma_5 n^a + V_{4}^{(\pm)} [n \times k_T]^a \right\} \Pi(n).$$  \hspace{1cm} (D.3)

Then

$$R_{t_2t_1}^{(\pm)} \rho_{s_2t_2s_1t_1}^{(\pm)}(x, k_T, n)(R^{-1})_{t_1t_1} = \delta_{t_2t_1} \rho_{s_2s_1}^{(\pm)0}(x, k_T, n) + (R \tau^a R^{-1})_{t_2t_1} \rho_{s_2s_1}^{(\pm)a}(x, k_T, n).$$  \hspace{1cm} (D.4)

Here

$$R \tau^a R^{-1} = O^{ba}(R) \tau^b.$$  \hspace{1cm} (D.5)

and the matrix $O^{ba}(R)$ was defined in Eq. (4.3). Now we have

$$R_{t_2t_1}^{(\pm)} \rho_{s_2t_2s_1t_1}^{(\pm)}(x, k_T, n)(R^{-1})_{t_1t_1} = \delta_{t_2t_1} \rho_{s_2s_1}^{(\pm)0}(x, k_T, n) + O^{ba}(R)(\tau^b)_{t_2t_1} \rho_{s_2s_1}^{(\pm)a}(x, k_T, n).$$  \hspace{1cm} (D.6)

For longitudinally polarized states of nucleon described by wave functions (3.7) with $I = S = 1/2$, we find using equations of Appendix A

$$\int dR \left[ \phi_{I_3S_3}^{I}(R) \right]^* \phi_{I_3S_3}^{I}(R) O^{ba}(R) = -\frac{1}{3} (\tau^b)_{I_3I_3} S^a.$$  \hspace{1cm} (D.7)

In the case of an arbitrary polarization $S^\mu$ of the nucleon in the rest frame, we have the following generalization of Eq. (D.7):

$$\int dR \phi_{I_3S_3}^{I}(R) \phi_{I_3S_3}^{I}(R) O^{ba}(R) = -\frac{1}{3} (\tau^b)_{I_3I_3} S^a = -\frac{2}{3} \delta_{I_3I_3} S^a.$$  \hspace{1cm} (D.8)

Now we can compute the integral in the LHS of Eq. (4.9) using Eqs. (3.8), (D.8):

$$\int dR \phi_{I_3S_3}^{I}(R) \phi_{I_3S_3}^{I}(R) \left[ R_{t_2t_1}^{\pm} \rho_{s_2t_2s_1t_1}^{(\pm)}(x, k_T, n)(R^{-1})_{t_1t_1} \right] = \int dR \phi_{I_3S_3}^{I}(R) \phi_{I_3S_3}^{I}(R)$$
\[
\times \left[ \delta t_{2t_1} \rho_{s_{2s_1}}^{(\pm)0}(x, k_T, n) + O^{ba}(R)(\tau^b)_{t_2t_1} \rho_{s_{2s_1}}^{(\pm)0}(x, k_T, n) \right] \\
= \delta t_{2t_1} \rho_{s_{2s_1}}^{(\pm)0}(x, k_T, n) - \frac{2}{3} \delta b_3 I_3 S^a(\tau^b)_{t_2t_1} \rho_{s_{2s_1}}^{(\pm)0}(x, k_T, n) \\
= \delta t_{2t_1} \rho_{s_{2s_1}}^{(\pm)0}(x, k_T, n) - \frac{2}{3} I_3(\tau^3)_{t_2t_1} S^a \rho_{s_{2s_1}}^{(\pm)0}(x, k_T, n).
\]

(D.9)

Taking into account Eqs. (D.2), (D.3), we find

\[
\int dR \phi^{*}_{t_3S}(R) \phi_{t_3S} \left[ R_{t_2t_3} \rho_{s_{2s_1} s_1 t_1}^{(\pm)}(x, k_T, n)(R^{-1})_{t_1 t_2} \right] \\
= \left( \delta_{t_2t_1} \left[ V^1(\pm) + \frac{1}{3} V^3(\pm) i (k_T \gamma) \right] \\
- \frac{2}{3} I_3(\tau^3)_{t_2t_1} S^a \left\{ V^2(\pm) \gamma_5 k_T^a + V^3(\pm) \gamma_5 n^a \right\} \\
+ V_4(\pm) [n \times k_T]^a + V_5(\pm) i (n \cdot [\gamma \times k_T]) n^a \right] \\
+ V_7(\pm) i (n \cdot [\gamma \times k_T]) k_T^a + V_8(\pm) i (n \times \gamma)^a \right\} \Pi(n) \right)_{s_{2s_1}}.
\]

(D.10)

Now we can derive Eq. (4.13) from Eq. (D.10).

References

[1] D. Sivers, Phys. Rev. D41 (1990) 83 [Annals Phys. 198 (1990) 371].
D. Sivers, Phys. Rev. D43 (1991) 261.

[2] M. Anselmino, M. Boglione, and F. Murgia, Phys. Lett. B362 (1995) 164.
M. Anselmino and F. Murgia, Phys. Lett. B442 (1998) 470.

[3] D. Boer and P. J. Mulders, Phys. Rev. D57 (1998) 5780.
D. Boer, Phys. Rev. D60 (1999) 014012.

[4] M. Boglione and P. J. Mulders, Phys. Rev. D60 (1999) 054007.

[5] S. J. Brodsky, D. S. Hwang, and I. Schmidt, Phys. Lett. B530 (2002) 99.

[6] J. Collins, Phys. Lett. B536 (2002) 43.

[7] X. Ji and F. Yuan, Phys. Lett. B543 (2002) 66.
A. V. Belitsky, X. Ji, and F. Yuan, hep-ph/0208038.

[8] G. ’t Hooft, Nucl. Phys. B72 (1974) 461.

[9] E. Witten, Nucl. Phys. B160 (1979) 57.

[10] D.I. Diakonov, V.Yu. Petrov, P.V. Pobylitsa, M.V. Polyakov, and C. Weiss,
Nucl. Phys. B480 (1996) 341.
P.V. Pobylitsa and M.V. Polyakov, Phys. Lett. B389 (1996) 350.
[11] A.V. Efremov, K. Goeke, and P.V. Pobylitsa, Phys. Lett. B448 (2000) 182.
P.V. Pobylitsa and M.V. Polyakov, Phys. Rev. D62 (2000) 097502.

[12] A.P. Balachandran, V.P. Nair, S.G. Rajeev, and A. Stern, Phys. Rev. D27 (1983) 1153.
E. Witten, Nucl. Phys. B223 (1983) 433.
J.-L. Gervais and B. Sakita, Phys. Rev. Lett. 52 (1984) 87.
A.V. Manohar, Nucl. Phys. B248 (1984) 19.
D.I. Diakonov, V.Yu. Petrov, and P.V. Pobylitsa, Nucl. Phys. B306 (1988) 809;
R. Dashen, E. Jenkins and A.V. Manohar, Phys. Rev. D49 (1994) 4713.

[13] M. Anselmino, V. Barone, A. Drago, and F. Murgia, Nucl. Phys. Proc. Suppl. 105 (2002) 132.
M. Anselmino, V. Barone, A. Drago, and F. Murgia, hep-ph/0209073

[14] P.V. Pobylitsa, hep-ph/0212027