Combining Chiral and Heavy Quark Symmetries

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Abstract

The chiral and heavy quark symmetries of QCD are reviewed. These symmetries are used to predict some low-momentum properties of hadrons containing a single heavy quark.

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1. Introduction

The low-momentum properties of the strong interactions cannot be described using perturbation theory in the strong coupling constant. However, there are systematic methods that have proven very useful in this nonperturbative regime.

In the limit where the light up, down and strange quark masses go to zero, QCD possesses an $SU(3)_L \times SU(3)_R$ chiral symmetry that is spontaneously broken to the $SU(3)_V$ subgroup. There are eight Goldstone bosons (associated with the broken symmetry generators), and their interactions are described by an effective chiral Lagrangian that is invariant under $SU(3)_L \times SU(3)_R$. At low momentum the chiral Lagrangian can be expanded in derivatives and at the lowest order of this expansion the self interactions of the Goldstone bosons are described by a single parameter, the pion decay constant. It is possible to treat the quark masses as perturbations and include their effects. This is a good approximation provided the light quark masses are small compared with the chiral symmetry breaking scale. The quark mass terms transform under $SU(3)_L \times SU(3)_R$ as $(\bar{3}_L, 3_R) + (3_L, \bar{3}_R)$, and including in the chiral Lagrangian terms that transform this way explicitly breaks the chiral $SU(3)_L \times SU(3)_R$ symmetry, giving the Goldstone bosons $\pi, K$ and $\eta$ small masses. It is also possible to describe using an effective Lagrangian, the interactions of the Goldstone bosons with other particles whose mass does not go to zero in the chiral limit, e.g., the hyperons.$[^1]$

Symmetry methods are also useful for describing the low momentum properties of hadrons containing a single heavy quark. In this case it is appropriate to take the limit where the heavy quark mass goes to infinity with its four-velocity fixed. In this limit QCD possesses a heavy quark spin-flavor symmetry. Heavy quark symmetry has proven very useful for describing the interactions of a heavy quark with light quarks and gluons in the kinematic regime where the light degrees of freedom typically have a momentum that is small compared with the heavy quark mass.$[^2,3,4]$ For example, heavy quark symmetry implies that all six form factors for $B \to D e\bar{\nu}_e$ and $B \to D^* e\bar{\nu}_e$ semileptonic decays are described by a universal function of velocity transfer and that
this universal function is normalized to unity at zero recoil (where in the rest frame of the $B$ the $D$ (or $D^*$) is also at rest).

The interactions of hadrons containing a single heavy quark with the $\pi$, $K$ and $\eta$ are constrained by both heavy quark and chiral symmetry. Recently there has been a large amount of activity devoted to examining the implications of this combination of symmetries. This paper is meant to provide a pedagogical introduction to chiral perturbation theory for hadrons containing a heavy quark. The general principles of chiral perturbation theory for the Goldstone bosons are developed and applied to a few examples. Heavy quark symmetry will also be introduced. Finally the two symmetries are combined and some predictions that use both symmetries are made.

2. Chiral Symmetries of the Strong Interactions

The part of the Lagrange density for QCD involving the light quark fields is

$$\mathcal{L} = \sum_a \bar{q}_a i \mathcal{D} q_a - \sum_a \bar{q}_a (m_q)_{ab} q_b . \quad (2.1)$$

Here $q_a$ are the light quark fields, $q_1 = u, q_2 = d, q_3 = s$, and $m_q$ is the light quark mass matrix

$$(m_q)_{ab} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} . \quad (2.2)$$

In eq. (2.1) $D_\mu$ denotes a covariant derivative

$$D_\mu = \partial_\mu + ig_s A^A_\mu T^A , \quad (2.3)$$

where $g_s$ is the strong coupling, $A^A_\mu$ denotes the color gauge field, $A = 1, \ldots, 8$, and $T^A$ is an $SU(3)$ color generator.

To make the symmetries of eq. (2.1) explicit, it is convenient to introduce left-
and right-handed fields

\[ q_{aL} = \frac{1}{2}(1 - \gamma_5)q_a , \]  
\[ q_{aR} = \frac{1}{2}(1 + \gamma_5)q_a , \]

and express the Lagrange density in eq. (2.1) in terms of them. Actually \( q_{aL} \) and \( q_{aR} \) are the more fundamental objects. They transform amongst themselves under proper Lorentz transformations. It is the parity invariance of the strong interactions that makes it convenient to combine these two-component fields into a single four-component Dirac spinor field. In terms of the left- and right-handed fields

\[ \mathcal{L} = \sum_a \bar{q}_{aL}i\gamma^\mu\partial_\mu q_{aL} + \sum_a \bar{q}_{aR}i\gamma^\mu\partial_\mu q_{aR} \]

\[ -\sum_{ab} [\bar{q}_{aL}(m_q)_{ab}q_{bR} + \bar{q}_{aR}(m_q)_{ab}q_{bL}] . \]  

(2.5)

Let’s examine eq. (2.5) in the limit \( m_q \to 0 \). This will end up being a good approximation because the up, down, and strange quark masses are small compared with the chiral symmetry breaking scale. In this limit

\[ \mathcal{L} = \sum_a \bar{q}_{aL}i\gamma^\mu\partial_\mu q_{aL} + \sum_a \bar{q}_{aR}i\gamma^\mu\partial_\mu q_{aR} , \]

(2.6)

and \( \mathcal{L} \) possesses the global chiral symmetry \( G = SU(3)_L \times SU(3)_R \), under which

\[ q_{aL} \to L_{ab}q_{bL} , \quad L \in SU(3)_L \]  
\[ q_{aR} \to R_{ab}q_{bR} , \quad R \in SU(3)_R \],  

(2.7a)

and

\[ q_{aL} \to L_{ab}q_{bL} , \quad L \epsilon SU(3)_L \]  
\[ q_{aR} \to R_{ab}q_{bR} , \quad R \epsilon SU(3)_R \],  

(2.7b)

(the repeated index \( b \) is summed over 1, 2, 3).
Eqs. (2.7) represent a symmetry of the Lagrange density, eq. (2.6), but not of the vacuum. In QCD the quark bilinear $\bar{q}_aRq_bL$ has the vacuum expectation value

$$<0|\bar{q}_aRq_bL|0> = v\delta_{ab}. \quad (2.8)$$

Performing a $SU(3)_L \times SU(3)_R$ transformation on the vacuum state, we find

$$<0|\bar{q}_aRq_bL|0> \rightarrow <0|\bar{q}_cR R^*_{ac} R_{bd} L_{bd} |0>$$

$$= R^\dagger_{ca} L_{bd} <0|\bar{q}_cRq_dL|0> = R^\dagger_{ca} L_{bd} v\delta_{cd}$$

$$= v(LR^\dagger)_{ba}. \quad (2.9)$$

So only if the transformation is in the vector subgroup $SU(3)_V$ where $L = R$ is the vacuum invariant. The symmetry $G = SU(3)_L \times SU(3)_R$ is spontaneously broken to the vector subgroup $H = SU(3)_V$. Because $G$ is a symmetry, transformations in the coset space $G/H$ take the vacuum state into another state of the same energy. Consequently the theory must contain massless particles, one for each broken generator of $G$. To ensure that field configurations related by $G/H$ transformations have the same energy, these massless particles are derivatively coupled.

Before proceeding further with QCD, let’s consider the simpler example of a theory with a single complex scalar field $\phi$ and the Lagrange density

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \lambda(\phi^2 - v^2)^2. \quad (2.10)$$

The Lagrange density, eq. (2.10), is invariant under the global $U(1)$ symmetry

$$\phi \rightarrow e^{i\Omega} \phi. \quad (2.11)$$

This symmetry $G = U(1)$ is spontaneously broken by the expectation value

$$<0|\phi|0> = v. \quad (2.12)$$

The resulting Goldstone boson field can be thought of as arising from transforming $\phi$ away from its vacuum expectation value by an element of $G$. For (low energy) long
wavelength configurations of the field $\phi$, only this direction can be excited and we write

$$\phi \simeq ve^{ia/f}, \quad (2.13)$$

where $a$ is the Goldstone boson field and $f$ is a constant. The field $a$ corresponds to excitations that cost no potential energy. Under a $U(1)$ transformation

$$a/f \rightarrow a/f + \Omega. \quad (2.14)$$

Putting (2.13) into eq. (2.10) gives

$$\mathcal{L} = (v/f)^2 \partial_\mu a \partial^\mu a, \quad (2.15)$$

so for a properly normalized kinetic term the constant $f$ is given by

$$f = \sqrt{2}v. \quad (2.16)$$

The symmetry, eq. (2.14), insures that the Lagrange density, eq. (2.15), has no mass term for $a$.

An analysis very similar to the above holds for QCD. The Goldstone bosons are included in a $3 \times 3$ special unitary matrix $\Sigma_{ba}$ that can be thought of as arising from transforming $\bar{q}_a R q_b L$ away from its vacuum expectation value. In analogy with the $U(1)$ case for long-wavelength (low-energy) excitations we write

$$\bar{q}_a R q_b L \simeq v\Sigma_{ba}. \quad (2.17)$$

Under an $SU(3)_L \times SU(3)_R$ transformation the above identification implies that

$$\Sigma \rightarrow L \Sigma R^\dagger. \quad (2.18)$$

The matrix $\Sigma$ is the analog of $e^{ia/f}$ in the $U(1)$ case. To display explicitly the
Goldstone boson fields we write

$$\Sigma = \exp \left(\frac{2iM}{f}\right)$$  \hspace{1cm} (2.19)

where

$$M = \begin{bmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & 0 & K^0 \\ \sqrt{2}\eta & K^0 & -2\sqrt{6}\eta \end{bmatrix}.$$  \hspace{1cm} (2.20)

In eq. (2.19) $f$ is a constant with dimensions of mass. It is straightforward to see using eq. (2.17) that the particle assignments above are correct. Note that under an unbroken $SU(3)_V$ transformation $L = R = V$, eq. (2.18) implies that

$$M \rightarrow VMV^\dagger,$$  \hspace{1cm} (2.21)

so the $\pi, K$, and $\eta$ transform as an $SU(3)_V$ octet.

3. An Effective Lagrangian for the Strong Interactions of the Goldstone Bosons

To describe the strong interactions of the $\pi, K$ and $\eta$ at low momentum an effective chiral Lagrangian is constructed. This effective Lagrangian contains only these fields (heavier degrees of freedom, e.g., the $\rho$-meson, have been integrated out), and because we are interested in low-momentum physics the Lagrangian can be expanded in derivatives. Terms with more derivatives are suppressed since derivatives bring down factors of the small momentum. We want the Lagrangian to respect the symmetries of QCD, i.e., parity and charge conjugation. If for the moment the light quark masses are neglected, the Lagrangian should also be invariant under chiral $SU(3)_L \times SU(3)_R$. Thus

$$\mathcal{L} = \frac{f^2}{8} Tr\partial_\mu \Sigma \partial^\mu \Sigma^\dagger + \ldots$$  \hspace{1cm} (3.1)

where the ellipsis denotes terms with more than two derivatives. Note that there are no terms with zero derivatives because $Tr\Sigma^\dagger \Sigma = 1$. The factor $f^2/8$ is inserted to get properly normalized kinetic terms for the Goldstone bosons.
So far we haven’t included the quark mass terms. Recall

\[ \mathcal{L}_{\text{mass}} = - (\bar{q}_L m_q q_R + \bar{q}_R m_q q_L) , \]  

(3.2)

and under a $SU(3)_L \times SU(3)_R$ transformation

\[ \mathcal{L}_{\text{mass}} \rightarrow - (\bar{q}_L L^\dagger m_q R q_R + \bar{q}_R R^\dagger m_q L q_L) . \]  

(3.3)

Hence the quark mass terms transform as $(\bar{3}_L, 3_R) + (3_L, \bar{3}_R)$. If we add to eq. (3.1) the most general terms that transform this way then we have included, to first order in $m_q$, the effects of the quark masses. There is a very simple way to do the group theory. If we pretend that the quark mass matrix transforms as $m_q \rightarrow L m_q R^\dagger$ and construct invariants under $SU(3)_L \times SU(3)_R$ then the effects of the $(\bar{3}_L, 3_R)$ term in eq. (3.2) are taken into account. Similarly if we pretend that the quark mass matrix transforms as $m_q \rightarrow R m_q L^\dagger$ and construct invariants under $SU(3)_L \times SU(3)_R$ the effects of the $(3_L, \bar{3}_R)$ term in eq. (3.2) are taken into account. Including terms linear in the light quark mass matrix, the chiral Lagrange density becomes

\[ \mathcal{L} = \frac{f^2}{8} Tr \partial_\mu \Sigma \partial^\mu \Sigma^\dagger + \lambda_0 Tr (m_q \Sigma + \Sigma^\dagger m_q) , \]  

(3.4)

where $\lambda_0$ is a constant with dimension $(mass)^3$. The quark mass terms give masses to the Goldstone bosons (i.e., they are only approximate Goldstone bosons because $SU(3)_L \times SU(3)_R$ symmetry is not exact)

\[ m_{\pi^\pm}^2 = \frac{4\lambda_0}{f^2} (m_u + m_d) \]  

(3.5a)

\[ m_{\rho^0}^2 = \frac{4\lambda_0}{f^2} (m_d + m_s) \]  

(3.5b)

\[ m_{K^*}^2 = \frac{4\lambda_0}{f^2} (m_u + m_s) , \]  

(3.5c)
and for the $\pi^0$ and $\eta$ there is a $2 \times 2$ mass matrix

$$m^2_{(\pi^0, \eta)} = \frac{4\lambda_o}{f^2} \begin{bmatrix} (m_u + m_d) & \frac{1}{\sqrt{3}}(m_u - m_d) \\ \frac{1}{\sqrt{3}}(m_u - m_d) & \frac{1}{3}(m_u + m_d) + \frac{4}{3}m_s \end{bmatrix}. \tag{3.6}$$

Since $m^2_K/m^2_{\pi} \simeq m_s/(m_u + m_d)$ is large, the strange quark mass is much larger than the up and down quark masses. The eigenvalues of the matrix in eq. (3.6) are then given approximately by the diagonal elements; corrections to this from the off diagonal elements are of order $((m_u - m_d)/m_s)^2$. Neglecting $m_{u,d}$ compared with $m_s$ gives the relation

$$(3/4)m^2_\eta = m^2_K. \tag{3.7}$$

An examination of isospin splittings leads to the expectation that $m_d$ is about twice as large as $m_u$. Despite the fact that $m_d/m_u$ differs significantly from unity, the neutral and charged pions are almost degenerate because the strange quark mass is much larger than both the up and down quark masses.

The transformation properties of $\Sigma$ under charge conjugation

$$C\Sigma C^{-1} = \Sigma^T, \tag{3.8}$$

and parity

$$P\Sigma(\vec{x},t)P^{-1} = \Sigma^\dagger(-\vec{x},t), \tag{3.9}$$

follow from eq. (2.17).

Under an infinitesimal space-time-dependent left-handed transformation

$$L \simeq 1 + i \epsilon_L^A T^A, \tag{3.10}$$

the QCD Lagrange density (2.6) changes by

$$\delta \mathcal{L} = -L^A_\mu \partial^\mu \epsilon_L^A, \tag{3.11}$$
where

\[ L^A_\mu = \bar{q}_L T^A \gamma_\mu q_L , \]  

(3.12)
is the left-handed current. On the other hand, under the transformation eq. (3.10), the chiral Lagrange density, eq. (3.1), changes by

\[ \delta \mathcal{L} = -\frac{if^2}{4} Tr (\partial_\mu \Sigma \Sigma^\dagger T^A) \partial^\mu \epsilon^A_L . \]  

(3.13)

Comparison of eqs. (3.11) and (3.13) yields an expression for the left-handed current in terms of \( \pi, K \) and \( \eta \) fields

\[ L^A_\mu = \frac{if^2}{4} Tr (\partial_\mu \Sigma \Sigma^\dagger T^A) . \]  

(3.14)

At higher orders in derivatives this Noether procedure for extracting the symmetry currents become ambiguous. Terms in the Lagrange density that are total derivatives contribute to the currents (but not the charges.) An alternate method for deriving the symmetry currents is to gauge the chiral symmetries. Then, for example, the coupling of the external left-handed gauge field, \( A^L_\mu \), is proportional to \( A^C_\mu L^C_\mu \). In this procedure the ambiguity associated with total derivatives corresponds to terms in the gauged chiral Lagrangian that contain a field strength tensor.

Low-momentum matrix elements of the left handed current, eq. (3.12), involving \( \pi, K \) and \( \eta \) fields are given by the matrix elements of eq. (3.14). Expanding \( \Sigma \) in terms of \( M \) gives

\[ L^A_\mu = -\frac{f}{2} Tr \partial_\mu MT^A + ... , \]  

(3.15)

where the ellipsis denote higher powers of \( M \).

The invariant matrix element for \( \pi^-(p) \rightarrow \mu(p_\mu)\bar{\nu}(p_\nu) \) decay is

\[ \mathcal{M} = \frac{G_F}{\sqrt{2} c_1} < 0|\bar{u} \gamma_\mu (1 - \gamma_5) d|\pi^-(p_\pi) > \]

\[ \cdot \bar{u}(p_\mu) \gamma^\mu (1 - \gamma_5) v(p_\nu) , \]  

(3.16)

where \( c_1 \) is the cosine of the Cabibbo angle. But according to eqs. (3.12) and (3.15),
\[ \bar{u} \gamma_{\mu} (1 - \gamma_5) d = -f \partial_{\mu} \pi^- + ..., \] so eq. (3.16) becomes

\[ \mathcal{M} = i G_{F} c_{1} f \frac{p^\mu}{\sqrt{2}} \bar{u}(p_\mu) \gamma_{\mu} (1 - \gamma_5) v(p_\nu) = i G_{F} c_{1} f \frac{m_\mu}{\sqrt{2}} \bar{u}(p_\mu) (1 - \gamma_5) v(p_\nu). \] (3.17)

Comparing the rate that results from eq. (3.17) with experiment gives \( f \simeq 132 \) MeV.

4. Power Counting for \( \pi \pi \) Scattering

Chiral perturbation theory can be used to predict the cross section for \( \pi \pi \) scattering at low momentum. Taking Lagrange density (3.4) and expanding \( \Sigma \) in powers of \( M \) the pieces with four \( M \)'s give the tree level contribution to the invariant matrix element for the \( \pi \pi \to \pi \pi \) process. The lowest four-momentum \( p \) accessible is of order \( m_\pi \). Since \( m_\pi^2 \) is linear in the light quark masses, an insertion of \( m_q \) is of the same order as two derivatives. At tree level the Lagrange density, eq. (3.4), gives an invariant matrix element of order \( p^2/f^2 \).

One-loop Feynman diagrams like that in Figure 1 also contribute to the invariant matrix element for \( \pi \pi \) scattering. To describe their contribution it is convenient to pick dimensional regularization with minimal subtraction as the renormalization scheme. This is particularly convenient because there is no dimensionful cutoff and the subtraction point \( \mu \) only appears in the argument of logarithms (e.g., as \( \log(p^2/\mu^2) \)). The Feynman diagram in Figure 1 contains four factors of \( 1/f \) (two from each vertex) and is of order \( (1/16\pi^2)(p^4/f^4) \ln(p^2/\mu^2) \). Hence, it is subdominant to the tree level contribution of the two derivative term in the chiral Lagrangian, and roughly comparable in importance with the tree level contribution of terms in the chiral Lagrangian containing four derivatives. Such higher-dimension terms in the Lagrangian have coefficients that depend on \( \mu \), and this dependence cancels the logarithmic dependence on \( \mu \) from the one-loop diagrams[5]. For small \( p^2 \), and \( \mu \) of order the chiral symmetry breaking scale \( \sim 1GeV \), the one-loop contribution is enhanced by a large logarithm over the tree level contribution of the terms in the Lagrangian with four derivatives.
Higher loops will contain more powers of \((1/f)\) and are even less important at low momentum. So we have seen that at low momentum the tree level contribution of the terms in the Lagrange density with two derivatives or one factor of the light quark mass matrix dominate the \(\pi\pi \to \pi\pi\) matrix element. This matrix element is predicted just in terms of \(f\) and the pion mass.

Similar power counting applies to other processes. For example, for the matrix element \(<0|L^A_\nu|\pi>\) the contributions from loops and higher-derivative terms in the chiral Lagrangian are suppressed compared to the contribution evaluated in the previous section.

5. Semileptonic Kaon Decay

In this section chiral perturbation theory is applied to the semileptonic decays\(^6\) \(K^0 \to \pi^- e^+ \nu_e\) and \(K^0 \to \pi^0 \pi^- e^+ \nu_e\). At the quark level the effective Hamiltonian density for these decays is

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} f_1 (\bar{s} \gamma_\mu (1 - \gamma_5) u) (\bar{\nu}_e \gamma_\mu (1 - \gamma_5) e) .
\] (5.1)

The amplitude for \(K^0 \to \pi^- e^+ \nu_e\) decay is dependent on the hadronic matrix element

\[
<\pi^- (p_\pi)|\bar{s} \gamma_\mu (1 - \gamma_5) u|K^0(p_K)> = f_+(p_K + p_\pi)_\mu + f_-(p_K - p_\pi)_\mu .
\] (5.2)

where \(f_\pm\) are (Lorentz invariant) functions of the square of the momentum transfer \(q^2 = (p_K - p_\pi)^2\). According to eqs. (3.12) and (3.14), for low energy matrix elements involving the \(\pi, K\) and \(\eta\) we can write

\[
\bar{s} \gamma_\mu (1 - \gamma_5) u = \frac{if^2}{2} Tr (\partial_\mu \Sigma \Sigma^\dagger T) ,
\] (5.3)

with

\[
T = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
1 & 0 & 0
\end{pmatrix} .
\] (5.4)
Expanding $\Sigma$ in terms of $M$ in eq. (5.3), terms with two $M$'s lead to the prediction

\[ f_+ = 1 \ , \quad f_- = 0 \ . \]  

For $K^0 \rightarrow \pi^0\pi^- e^+ \nu_e$ decay the relevant hadronic matrix element is

\[ < \pi^- (q) \pi^0 (k) | \bar{s} \gamma^\mu (1 - \gamma_5) u | K^0 (p) > \]

\[ = iF_1 (q + k)^\mu + iF_2 (q - k)^\mu + iF_3 (p - q - k)^\mu + F_4 \epsilon_{\mu\nu\lambda\sigma} p_\nu q_\lambda k_\sigma \ . \]  

(5.6)

The terms with three $M$'s in eq. (5.3) contribute (Figure 2a) to this matrix element as well as the pole graph in Figure 2b. (Isospin-violating effects are neglected here.) In Figure 2b shaded circle represents a strong interaction vertex while a shaded square denotes an insertion of the left-handed current. The two derivatives in the strong vertex produce two factors of momentum which are cancelled (so far as the power counting is concerned) by the propagator. Consequently both diagrams in Figure 2 contribute to the matrix element (5.6) in the leading order of chiral perturbation theory and give

\[ F_1 = 0, \quad F_2 = -\sqrt{2} \frac{f}{f}, \quad F_3 = \frac{\sqrt{2} (q - k) \cdot (p - q - k)}{f (m_K^2 - (p - q - k)^2)} \ . \]  

(5.7)

The leading contribution to the form factor $F_4$ arises from the Wess–Zumino term. This is discussed in the next section.

6. The Wess–Zumino Term

The currents $L_\mu^A$ and $R_\mu^A$ associated with chiral $SU(3)_L \times SU(3)_R$ symmetry are exactly conserved in the $m_q = 0$ limit. However, quark loops give an anomalous contribution to matrix elements involving these currents (e.g., the contribution to $\frac{\partial}{\partial x} < 0|T(L^A \mu (x)L^B \nu (y)L^C \alpha (z))|0 >$ from the usual triangle diagram). The chiral Lagrangian of Section 3 does not have these anomalous contributions. A simple way
to include the anomalous effects of quark loops on the $\pi, K$ and $\eta$ interactions is to add to the chiral Lagrange density massive left-and right-handed fermion fields $\hat{q}_aL$ and $\hat{q}_aR$ that transform under chiral symmetry the same way the quarks do: $\hat{q}_L \to L\hat{q}_L, \hat{q}_R \to R\hat{q}_R$. The Lagrange density for these fields is

$$\mathcal{L} = i\bar{\hat{q}}_L\gamma_\mu\partial_\mu\hat{q}_L + i\bar{\hat{q}}_R\gamma_\mu\partial_\mu\hat{q}_R - M_q(\bar{\hat{q}}_L\Sigma\hat{q}_R + \bar{\hat{q}}_R\Sigma^\dagger\hat{q}_L) .$$  \hfill (6.1)

Integrating out these massive fermions will reproduce anomalous effects of quark loops on $\pi, K$, and $\eta$ interactions. Note that the $\pi, K$ and $\eta$ have $SU(3)_L \times SU(3)_R$ invariant nonderivative couplings to $\hat{q}_{L,R}$ proportional to $M_q$. Large $M_q$ factors from vertices can cancel those from propagators. It is these nonderivative couplings proportional to $M_q$ that are responsible for the anomalous $\pi, K$ and $\eta$ interactions.

Now suppose we make a field redefinition

$$\hat{q}_L(s) = \Sigma_\dagger(s)\hat{q}_L \quad (6.2a)$$
$$\hat{q}_R(s) = \hat{q}_R \quad (6.2b)$$

where $\Sigma(s)$ is an element of a continuous one-parameter family of special unitary matrices that satisfies

$$\Sigma(0) = 1 \quad \Sigma(1) = \Sigma .$$  \hfill (6.3)

In terms of $\hat{q}_{L,R}(s)$ the action becomes

$$S = \int d^4x[\bar{\hat{q}}_L(s)(i\gamma_\mu - A_\mu)L\hat{q}_L(s) + \bar{\hat{q}}_R(s)i\gamma_\mu\hat{q}_R(s)$$
$$- M_q(\bar{\hat{q}}_L(s)\Sigma_\dagger(s)\hat{q}_R(s) + \bar{\hat{q}}_R(s)\Sigma_\dagger(s)\hat{q}_L(s))] + \Delta(s) ,$$  \hfill (6.4)

where $\Delta(s)$ comes from the anomaly (i.e., from the Jacobian associated with the transformation in eq. (6.2)) and $A_\mu^L$ is

$$A_\mu^L = i\Sigma_\dagger(s)\partial_\mu\Sigma(s) .$$  \hfill (6.5)

At $s = 1$ the full effect of the anomaly on $\pi, K$ and $\eta$ interactions is in $\Delta(1)$
since there are no longer nonderivative couplings of the Goldstone bosons to the heavy fermions that are proportional to $M_q$. It is straightforward using the Noether procedure to compute how $\Delta$ changes with $s$. Under a change from $s$ to $s + \delta s$

$$\delta \hat{q}_L(s) = i\epsilon \hat{q}_L(s), \quad \delta \hat{q}_R(s) = 0,$$

(6.6)

where

$$\epsilon(s) = -i(\delta s)\partial_s \Sigma^\dagger(s)\Sigma.$$

(6.7)

The change in $\Delta(s)$ from the anomaly is

$$\delta \Delta(s) = \int d^4x Tr(\epsilon(D_\mu L^\mu)_{\text{anomalous}}),$$

(6.8)

where the anomalous divergence of the current\* $L^\mu$ is

$$(D_\mu L^\mu)_{\text{anomalous}} = -\frac{1}{24\pi^2} \epsilon_{\mu\nu\rho\sigma} [\partial_\mu A_\nu^L \partial_\rho A_\sigma^L - \frac{i}{2} \partial_\mu (A_\nu^L A_\rho^L A_\sigma^L)],$$

(6.9)

since $A_\nu^L$ appears as an external “gauge field” in eq. (6.4). With $A_\nu^L$ given by eq. (6.5), eq. (6.9) becomes

$$(D_\mu L^\mu)_{\text{anomalous}} = \frac{1}{48\pi^2} \epsilon_{\mu\nu\rho\sigma} \partial_\mu \Sigma^\dagger \partial_\nu \Sigma \partial_\rho \Sigma^\dagger \partial_\sigma \Sigma.$$

(6.10)

So as $s$ changes from $s$ to $s + \delta s$,

$$\delta \Delta(s) = \delta s \left( \frac{i}{48\pi^2} \int d^4x \epsilon_{\mu\nu\rho\sigma} Tr(\Sigma^\dagger \partial_s \Sigma \partial_\mu \Sigma^\dagger \partial_\nu \Sigma \partial_\rho \Sigma^\dagger \partial_\sigma \Sigma) \right).$$

(6.11)

Integrating this from $s = 0$ to $s = 1$ gives the effect of the anomaly on $\pi, K$ and $\eta$ interactions. This is the Wess–Zumino term\[7,8]\]

$$S_{WZ} = 3\Delta(1)$$

$$= \frac{i}{16\pi^2} \int_0^1 ds \int d^4x \epsilon_{\mu\nu\rho\sigma} Tr[\Sigma^\dagger \partial_s \Sigma \partial_\mu \Sigma^\dagger \partial_\nu \Sigma \partial_\rho \Sigma^\dagger \partial_\sigma \Sigma].$$

(6.12)

The factor of three is inserted in eq. (6.12) because $\Delta(1)$ only includes the effects of a single color of up, down and strange quarks.

\* The current $L^\mu$ is the matrix with components $L^\mu_{ab} = \bar{q}_a L^\mu \gamma^\mu q_b L$. 

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The integration region in (6.12) is over a five-dimensional disc of which spacetime is the boundary (see eq. 6.3) at \( s = 1 \). Using \( x^5 = s \) the Wess–Zumino term can be written in the more symmetrical form

\[
S_{WZ} = \frac{i}{80\pi^2} \int d^5x \epsilon^{\mu\nu\rho\sigma\alpha} Tr[\Sigma^\dagger \partial_\mu \Sigma \partial_\nu \Sigma^\dagger \partial_\rho \Sigma \partial_\sigma \Sigma^\dagger \partial_\alpha \Sigma] .
\] (6.13)

When \( \Sigma \) is expanded in terms of \( M \) the various terms can be written as integrals over four-dimensional spacetime. However, the \( SU(3)_L \times SU(3)_R \) invariance of the Wess–Zumino term is only manifest when it is written as an integral over a five-dimensional disk.

The Wess–Zumino term contributes to the left-handed current

\[
I_{WZ}^{\mu A} = \frac{1}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} Tr[\partial_\nu \Sigma \partial_\rho \Sigma^\dagger \partial_\sigma \Sigma \partial_\alpha T] .
\] (6.14)

It is this anomalous piece of the left-handed current that is responsible for the leading contribution to the form factor \( F_4 \) in \( K^0 \rightarrow \pi^- \pi^0 e^+ \nu_e \) decay. This is because other terms in the strong interaction Lagrange density with four derivatives and an anti-symmetric tensor (e.g., \( Tr\epsilon^{\mu\nu\lambda\sigma} \Sigma^\dagger \partial_\mu \Sigma \Sigma^\dagger \partial_\nu \Sigma \Sigma^\dagger \partial_\lambda \Sigma \Sigma^\dagger \partial_\sigma \Sigma \) are forbidden by parity. Expanding eq. (6.14) out in powers of \( M \), the piece with three \( M \)'s gives

\[
F_4 = \frac{\sqrt{2}}{\pi^2 f^3} .
\] (6.15)

7. Heavy Quark Symmetries of the Strong Interactions

The part of the Lagrange density for QCD involving the heavy quark field \( Q \) is

\[
\mathcal{L} = \bar{Q}(i\gamma^\mu \partial_\mu - m_Q)Q .
\] (7.1)

We shall be interested in the kinematic situation where the heavy quark is interacting with light degrees of freedom (i.e., gluons and light quarks and antiquarks) carrying
momenta that are typically much smaller than the heavy quark mass. In this situation it is appropriate to take the limit of QCD where the heavy quark mass $m_Q$ goes to infinity with its four-velocity $v^\mu$ fixed. To do this we write for the heavy quark with velocity $v$

$$Q = e^{-im_Qv \cdot x}[h_v^{(Q)} + \chi_v^{(Q)}], \quad (7.2)$$

where

$$\not h_v^{(Q)} = h_v^{(Q)}, \quad \not \chi_v^{(Q)} = -\chi_v^{(Q)}. \quad (7.3)$$

In the kinematic situation of interest the heavy quark is almost on shell and so $\chi_v^{(Q)}$ can be treated as a small quantity. $h_v^{(Q)}$ and $\chi_v^{(Q)}$ have a much less rapid dependence on spacetime than the phase factor explicitly factored out in eq. (7.2). Neglecting $\chi_v^{(Q)}$ and substituting eq. (7.2) into (7.1) we find

$$\mathcal{L} = \bar{h}_v^{(Q)}[m_Q(\not h - 1) + i\not D]h_v^{(Q)}$$

$$= \bar{h}_v^{(Q)}i\not D h_v^{(Q)}. \quad (7.4)$$

This can be further simplified using eq. (7.3) to

$$\mathcal{L}_v = \bar{h}_v^{(Q)} \left( \frac{\not h + 1}{2} \right) i\not D h_v^{(Q)}$$

$$= \bar{h}_v^{(Q)} \left[ iv \cdot D - i\not D \left( \frac{\not h - 1}{2} \right) \right] h_v^{(Q)}$$

$$= \bar{h}_v^{(Q)} iv \cdot D h_v^{(Q)}. \quad (7.5)$$

The Lagrange density for the effective heavy quark theory in eq. (7.5) has as its Feynman rules: $i/(v \cdot k + i\epsilon)$ for the heavy quark propagator, and $-igT^A v_\mu$ as the vertex for the gluon heavy quark interaction. The full four-momentum of the heavy quark is $p_Q = m_Qv + k$. The momentum $k$ that occurs in the heavy quark propagator
of the effective theory is called the residual momentum; it is a measure of how much
the heavy quark is off-shell. For the heavy quark effective theory to be valid \( k \) must
be much less than \( m_Q \).

Note that in the effective theory the field \( h_v^{(Q)} \) destroys a heavy quark of four-
velocity \( v \); it does not create the corresponding antiquark. Pair creation of heavy
quarks does not occur in the effective theory. The equation of motion for the field
\( h_v^{(Q)} \) is

\[
v \cdot D h_v^{(Q)} = 0 .
\]

The heavy quark effective theory has symmetries that are not manifest in the full
theory of QCD.\(^{[9,10]} \) Since there is no pair creation in the effective theory, there is
a \( U(1) \) symmetry of the Lagrange density in eq. (7.5) associated with heavy quark
conservation. Under an infinitesimal \( U(1) \) transformation of this type

\[
h_v^{(Q)} \rightarrow h_v^{(Q)} + \delta h_v^{(Q)} ,
\]

with

\[
\delta h_v^{(Q)} = i \epsilon_0 h_v^{(Q)} .
\]

Here \( \epsilon_0 \) is an arbitrary (real) infinitesimal parameter. Since gamma matrices no longer
occur in the effective theory, the spin of the heavy quark is conserved by the heavy
quark-gluon interaction. Associated with this is an \( SU(2) \) symmetry group of the
Lagrange density in eq. (7.5). To define the action of the \( SU(2) \) group on the heavy
quark fields, we introduce three orthonormal four-vectors \( e_a^\mu \), \( a = 1, 2, 3 \), that are
orthogonal to the heavy quark’s four-velocity:

\[
e_a^\mu e_b^\mu = -\delta_{ab} ,
\]

\[
v_\mu e^\mu_a = 0 .
\]
Then the three \(4 \times 4\) matrices

\[
S^a = \frac{i}{8} \sum_{b,c} \epsilon^{abc} [\hat{\sigma} b, \hat{\sigma} c],
\tag{7.11}
\]

are the generators of the heavy quark spin symmetry. The Lagrange density in eq. (7.5) is invariant under the infinitesimal transformations

\[
h_v^{(Q)}(Q) \rightarrow h_v^{(Q)} + \delta h_v^{(Q)},
\tag{7.12}
\]

where

\[
\delta h_v^{(Q)} = i \sum_a \epsilon^a S^a h_v^{(Q)}.
\tag{7.13}
\]

Furthermore, \([\hat{\sigma}, S^a] = 0\). So these transformations preserve the constraint \(\hat{\sigma} h_v^{(Q)} = h_v^{(Q)}\). It is easy to see that these transformations correspond to heavy quark spin symmetry in the heavy quark rest frame where \(v^\mu = (1, \vec{0})\). In this frame \(h_v^{(Q)}\) is a two-component Pauli spinor (the lower two components vanish because of the constraint \(\gamma^0 h_v^{(Q)} = h_v^{(Q)}\)). Using the explicit representation

\[
\gamma^j = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix},
\tag{7.14}
\]

and picking the \(e_{a\mu}\) to be unit vectors along the three spatial axes we find that eq. (7.11) implies \(S^a = \frac{1}{2} \begin{pmatrix} \sigma^a & 0 \\ 0 & \sigma^a \end{pmatrix}\).

If there are \(N_h\) heavy quarks \(Q_1...Q_{N_h}\) moving with the same four-velocity \(v\), then, denoting the corresponding fields in the effective theory by \(h_v^{(i)}\), the Lagrange density becomes

\[
\mathcal{L}_v = \sum_{j=1}^{N_h} \bar{h}_v^{(j)} i_v \cdot D h_v^{(j)}.
\tag{7.15}
\]

The Lagrange density in eq. (7.15) is independent of the heavy quark masses, and so the \(SU(2)\) spin symmetry generalizes to a \(SU(2N_h)\) spin-flavor symmetry. Note
that because the heavy quark masses can be very different, this symmetry relates quarks of the same four-velocity but generally different momentum. This is one of the unusual aspects of heavy quark symmetry.

In nature there are three heavy quarks, \( c, b \) and \( t \), and so the heavy quark spin-flavor symmetry is \( SU(6) \). However, because the top quark is so heavy it is likely to decay before it forms a hadron. Ironically, heavy quark symmetry is not a very useful concept for the heaviest of all quarks.

Heavy quark symmetry is not just a nonrelativistic symmetry. In our derivation of the effective theory the spatial components of the four-velocity are not necessarily small. With just a single heavy quark interacting strongly this is not a particularly significant statement, since one can always choose to work in the rest frame where \( v^\mu = (1, \vec{0}) \). However, we will be interested in cases where an external source (e.g., a \( W \)-boson) takes a heavy quark and changes it into a heavy quark with a different four-velocity (and possibly a different flavor as well). Then it is not possible to be in the rest frame of both the initial and final quarks.

8. Heavy Hadron Multiplets

In the \( m_Q \to \infty \) limit the total angular momentum of the light degrees of freedom

\[
\vec{s}_\ell = \vec{S} - \vec{S}_Q ,
\]

commutes with the Hamiltonian. Thus \( s_\ell \), the angular momentum of the light degrees of freedom in the hadron’s rest frame, is a good quantum number.\[^{[11]}\] Consequently in the \( m_Q \to \infty \) limit hadrons containing a single heavy quark come in degenerate doublets of total spin

\[
s_\pm = s_\ell \pm 1/2 ,
\]

unless \( s_\ell = 0 \), in which case the total spin is \( s = 1/2 \). For example when \( s_\ell = 1/2 \), there are spin-zero and spin-one states

\[
|0> = \frac{1}{\sqrt{2}}[|↑↓> - |↓↑>] \quad (8.3a)
\]
\[ |1, 1 > = | \uparrow \uparrow > \]  
\[ |1, 0 > = \frac{1}{\sqrt{2}} [ | \uparrow \downarrow > + | \downarrow \uparrow >] \]  
\[ |1, -1 > = | \downarrow \downarrow > \]

where the first arrow represents the heavy quark spin (along the 3rd axis) while the second arrow denotes the spin of the light degrees of freedom. Since

\[ S_Q^3 |0 > = \frac{1}{2} |1, 0 > , \]  

and \( S_Q \) commutes with the Hamiltonian, these spin-zero and spin-one states are degenerate in mass.

For mesons with \( Q \bar{q}_a \) flavor quantum numbers, the ground state multiplet has \( s_\ell = 1/2 \) and negative parity, corresponding to the pseudoscalar mesons \( P_a \) and vector mesons \( P_a^* \). For \( Q = c \) these are the \( (D^0, D^+, D_s) \) and \( (D^{*0}, D^{*+}, D_s^*) \) mesons while for \( Q = b \) these are the \( (B^-, B^0, B_s) \) and \( (B^{*-}, B^{*0}, B_s^*) \) mesons. We denote the fields that destroy hadrons of this type, with four-velocity \( v \), by \( P_a \) and \( P_a^{*\mu} \). The vector field satisfies the constraint

\[ v_\mu P_a^{*\mu} = 0 . \]  

It is convenient to combine these fields into a \( 4 \times 4 \) matrix \( H_a \) in the following fashion\[^3\]

\[ H_a = \left( \frac{\not{v} + 1}{2} \right) [P_a^{*\mu} \gamma_\mu - P_a \gamma_5] . \]  

This is a shorthand notation. In cases where the flavor of the heavy quark \( Q \) and the value of the four-velocity \( v^\mu \) are important we shall use \( H_a^{(Q)}(v) \). One can think of the relationship between \( H_a \) and the underlying degrees of freedom schematically as

\[ h_v^{(Q)} \bar{\ell}_a \sim H_a^{(Q)}(v) , \]

where \( \bar{\ell}_a \) is a spinor field that destroys the light degrees of freedom. Eq. (8.7) implies that with respect to Lorentz transformations \( H_a(v) \rightarrow D(\Lambda^{-1}) H_a(\Lambda v) D(\Lambda^{-1})^{-1} \).
where \( D(\Lambda) \) is the usual \( 4 \times 4 \) Dirac representation of the Lorentz group (i.e., \( H_a(v) \) transforms as a bispinor). This transformation law gives \( P^* \nu \to \Lambda^\nu_\mu P^* \mu \) because 
\[ D(\Lambda^{-1}) \gamma^\nu D(\Lambda^{-1})^{-1} = \Lambda^\nu_\mu \gamma^\mu. \]
\( H_a \) transforms under heavy quark spin symmetry \( SU(2)_v \) as
\[ H_a \to S H_a, \]
where \( S \) is an element of \( SU(2)_v \). In the heavy meson rest frame \( v^\mu = (1, \vec{0}) \) eq. (8.8) becomes
\[ \delta P_a = \frac{1}{2} i \epsilon^k P^*_a, \]
\[ \delta P^*_a = \frac{1}{2} i \epsilon^k P_a - \frac{1}{2} \epsilon^{ijk} \epsilon^j P^*_a, \]
for infinitesimal transformations \( S = 1 + i \sum \epsilon^j S^j_Q \). Eq. (8.9) corresponds to eq. (8.4) and the analogous equations that result from applying the heavy quark spin raising and lowering operators \( S^\pm_Q \) to the spin-zero state.

The matrix \( H_a^{(Q)}(v) \) satisfies the identities
\[ \gamma^\mu H_a^{(Q)}(v) = H_a^{(Q)}(v), \]
\[ H_a^{(Q)}(v) \gamma^\mu = -H_a^{(Q)}(v). \]

It is convenient to introduce \( \bar{H}_a = \gamma^0 H_a^{1 \gamma^0} \). Under Lorentz transformations \( \bar{H}_a \to D(\Lambda^{-1}) \bar{H}_a D(\Lambda^{-1})^{-1} \), while under heavy quark spin symmetry \( \bar{H}_a \to \bar{H}_a S^{-1}. \)

Some excited mesons with \( Q \bar{q}_a \) flavor quantum numbers have been observed (for the case \( Q = c \)). In the nonrelativistic constituent quark model, the lowest mass excitations arise from giving the light antiquark a unit of orbital angular momentum. This results in two positive parity multiplets of heavy mesons, one with \( s_\ell = 1/2 \) and the other with \( s_\ell = 3/2. \)
For \( s_\ell = 1/2 \) we denote the fields that destroy the spin-zero and spin-one positive parity mesons in this multiplet by \( P_a^* \) and \( P_a' \) respectively \((v_\mu P_a' = 0)\). Again it is convenient to combine them into a \( 4 \times 4 \) matrix

\[
G_a = \frac{(1 + \gamma)}{2}(P_a' \gamma_\mu \gamma_5 - P_a^*) ,
\]

(8.12)

that transforms under heavy quark spin symmetry as

\[
G_a \rightarrow S G_a ,
\]

(8.13)

where \( S \epsilon SU(2)_\nu \). \( G_a \) transforms under Lorentz transformations in the same way as \( H_a \).

For \( s_\ell = 3/2 \) we denote the fields that destroy the spin-one and spin-two mesons in this multiplet by \( P_a^\mu \) and \( P_a^{\mu\nu} \) respectively \((v_\mu P_a^\mu = 0, v_\mu P_a^{\mu\nu} = 0, P_a^{*\mu} = 0 \) and \( P_a^{*\mu\nu} = P_a^{*\nu\mu} \)). It is convenient to combine these fields into the \( 4 \times 4 \) matrix\(^{[12]}\)

\[
F_\mu^a = \frac{(1 + \gamma)}{2} \left\{ P_a^{*\mu\nu} \gamma_\nu - \sqrt{\frac{3}{2}} P_a^\nu \gamma_5 \left[ g_\nu^\mu - \frac{1}{3} \gamma_\nu (\gamma^\mu - v^\mu) \right] \right\} ,
\]

(8.14)

that transforms under heavy quark spin symmetry as

\[
F_\mu^a \rightarrow S F_\mu^a .
\]

(8.15)

One can think of the relationship between the underlying degrees of freedom and \( F_\mu^a \)

as

\[
h_\nu^{(Q)} \tilde{\ell}_a^\mu \sim F_\mu^a ,
\]

(8.16)

where \( \tilde{\ell}_a^\mu \) is a Rarita–Schwinger field that destroys the spin-3/2 light degrees of freedom. Eq. (8.16) implies that under Lorentz transformations \( F_\mu^a \rightarrow \Lambda^\mu_\nu D(\Lambda^{-1}) F_\nu^a D(\Lambda^{-1})^{-1} \) (the four-velocity also transforms as \( v^\mu \rightarrow \Lambda^\mu_\nu v^\nu \)).

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The matrices $G_a$ and $F^\mu_a$ satisfy the relations

\begin{equation}
\not\!G_a = G_a \quad \not\!F^\mu_a = F^\mu_a \tag{8.17a}
\end{equation}

\begin{equation}
G_a \not\!\not\!\not\!v = -G_a \quad F^\mu_a \not\!\not\!\not\!\not\!v = -F^\mu_a \tag{8.17b}
\end{equation}

\begin{equation}
F^\mu_a v_\mu = 0 \quad F^\mu_a \gamma_\mu = 0 . \tag{8.17c}
\end{equation}

It is convenient to introduce $\bar{G}_a = \gamma^0 G_a^\dagger \gamma^0$ and $\bar{F}_a^\mu = \gamma^0 F_a^\dagger \gamma^0$. These barred fields transform under Lorentz transformations in the same way as the unbarred fields. Under heavy quark spin symmetry $\bar{G}_a \to \bar{G}_a S^{-1}$ and $\bar{F}_a^\mu \to \bar{F}_a^\mu S^{-1}$.

The members of the $s_\ell = 3/2$ multiplet have been observed for $Q = c$ (and $a = 1$). They are the $D_1(2420)^0$ and $D_2^*(2460)^0$. The $Q = c$ ($a = 1$ or 2) members of the $s_\ell = 1/2^+$ multiplet are expected to have a mass of about 2360 MeV and to be very broad resonances. They have not yet been detected experimentally.

The 4×4 matrix of fields $H_a$ is an antitriplet with respect to the unbroken $SU(3)_V$ group. We need to assign $H_a$ a transformation rule with respect to the full chiral symmetry group $SU(3)_L \times SU(3)_R$. Here, there is considerable freedom associated with our ability to make field redefinitions. Suppose $H_a$ transforms as $(\bar{3}_L, 1_R)$ under $SU(3)_L \times SU(3)_R$. Then under an $SU(3)_L \times SU(3)_R$ transformation

\begin{equation}
H_a \to H_b L_{ba}^\dagger , \tag{8.18}
\end{equation}

where $L \in SU(3)_L$. While this is an acceptable transformation law, it leads to a definition of parity that is somewhat awkward. Since parity interchanges left- and right-handed quark fields, the parity image of $H$ must transform under $SU(3)_L \times SU(3)_R$ as $(1_L, \bar{3}_R)$. A suitable definition of parity is thus\[13\]

\begin{equation}
PH_a(\vec{x}, t) P^{-1} = \gamma^0 H_b(-\vec{x}, t) \gamma^0 \sum_{ba}(-\vec{x}, t) . \tag{8.19}
\end{equation}

It is possible to redefine fields so that $H_a$ transforms in a simpler way under
parity. Introduce
\[ \xi = \exp \left( \frac{iM}{f} \right), \] (8.20)
which transforms under \( SU(3)_L \times SU(3)_R \) as
\[ \xi \rightarrow L \xi U^\dagger = U \xi R^\dagger, \] (8.21)
since
\[ \xi^2 = \Sigma. \] (8.22)

Under parity
\[ P\xi(\vec{x}, t)P^{-1} = \xi^\dagger(-\vec{x}, t). \] (8.23)

In eq. (8.21) the special unitary matrix \( U \) is (typically) a complicated nonlinear function of \( L, R \) and the meson fields \( M \). Consequently \( U \) depends on spacetime. However, for elements of the unbroken \( SU(3)_V \) subgroup \( L = R = U \). The redefined field
\[ \hat{H}_a = H_b \xi_{ba}, \] (8.24)
transforms under chiral \( SU(3)_L \times SU(3)_R \) as
\[ \hat{H}_a \rightarrow \hat{H}_b U_{ba}^\dagger. \] (8.25)

The advantage of using the hatted fields is that the parity transformation of eq. (8.19) becomes
\[ P\hat{H}_a(\vec{x}, t)P^{-1} = \gamma^0 \hat{H}_a(-\vec{x}, t)\gamma^0. \] (8.26)

For the remainder of this paper we shall use fields in the parity odd \( s_\ell = 1/2 \) multiplet that transform under chiral symmetry as in eq. (8.25) and under parity as in eq. (8.26), although for simplicity we shall not put a hat on these fields. Similar transformation laws hold for \( G_a \) and \( F_a^\mu \).
Baryons with $Qq_αq_β$ flavor quantum numbers transforms according to the $\bar{3}$ and 6 representations of $SU(3)_V$. The $\bar{3}$ contains an isosinglet with zero strangeness ($Λ_Q$) and an isodoublet with strangeness $−1(Ξ_Q)$. The lowest-lying heavy baryons in the $\bar{3}$ representation have $s_ℓ = 0$ and positive parity. We denote the spin-1/2 fields that destroy these baryons by $T_a(T_3 = Λ_Q, T_{1,2} = Ξ_Q)$ they transform under $SU(3)_L × SU(3)_R$ as

$$T_a → T_b U_{ba}^\dagger ,$$ (8.27)

and satisfy

$$\not{\gamma} T_a = T_a .$$ (8.28)

Under $SU(2)_v$ heavy quark spin symmetry $T_a → ST_a$, and under Lorentz transformations $T_a → D(Λ^{-1})T_a$.

The lowest lying baryons in the 6 representation have $s_ℓ = 1$. (The higher spin occurs because of fermi statistics. The ground state baryons in the six have wave functions for the two light quarks that are antisymmetric in color and symmetric in flavor and space. Therefore, they are symmetric in spin.) This angular momentum for the light degrees of freedom gives multiplets with total spin $s_- = 1/2$ and $s_+ = 3/2$. We denote the fields that destroy these baryons by $S^{ab}$ and $S^{*ab}$ where $\nu^\mu S^{*ab} = 0$. It is convenient to combine them into the object

$$S^{ab}_\mu = \sqrt{\frac{1}{3}} (\gamma_\mu + v_\mu) \gamma^5 S^{ab} + S^{*ab}_\mu .$$ (8.29)

Then under heavy quark spin symmetry

$$S^{ab}_\mu → SS^{ab}_\mu ,$$

where $S ∈ SU(2)_v$. Under Lorentz transformations $S^{ab}_\mu → Λ^\mu_ν D(Λ^{-1})S^{ab}_\nu$. The combination of fields $S^{ab}_\mu$ satisfies

$$\not{\gamma} S^{ab}_\mu = S^{ab}_\mu , \ S^{ab}_\mu v^\mu = 0 .$$ (8.30)
9. Semileptonic $B \rightarrow D \bar{e} \nu_e$ and $B \rightarrow D^* \bar{e} \nu_e$ Decay

Heavy quark spin-flavor symmetry determines many properties of hadrons containing a single heavy quark. Perhaps the best illustration of its utility is provided by the semileptonic decays $B \rightarrow D \bar{e} \nu_e$ and $B \rightarrow D^* \bar{e} \nu_e$. In these decays the square of the four-momentum transfer imparted by the virtual $W$-boson to the heavy quarks is large,

$$q^2 = (m_B v - m_D v')^2 = m_B^2 + m_D^2 - 2 m_B m_D v \cdot v' ,$$

and grows with the heavy quark masses. However, as far as the light degrees of freedom are concerned there isn’t a large momentum transfer. When the $B$ with four-velocity $v$ changes to a $D$ (or $D^*$) with four-velocity $v'$ the light degrees of freedom go from a four-momentum of order $\Lambda_{\text{QCD}} v$ to a four-momentum of order $\Lambda_{\text{QCD}} v'$. The square of the momentum transfer felt by the light degrees of freedom is only of order

$$q^2_\ell \simeq (\Lambda_{\text{QCD}} v - \Lambda_{\text{QCD}} v')^2 = -2 \Lambda^2_{\text{QCD}} (v \cdot v' - 1) .$$

Use of the effective heavy quark theory where $m_b$ and $m_c \rightarrow \infty$ is appropriate, because typical momentum transfers felt by the light degrees of freedom are small compared with the heavy bottom and charm quark masses. Of course, there are always virtual gluons with arbitrarily large momentum, and their effects are not adequately taken into account by the effective theory. Fortunately, because of asymptotic freedom the differences between the full theory of QCD and the effective theory arising from high-momentum effects can be taken into account using (renormalization-group improved) QCD perturbation theory. These high-momentum differences change the relationship between the currents $\bar{c} \gamma_\mu \gamma_5 b$ and $\bar{c} \gamma_\mu b$ in QCD and the operators that represent them in the effective theory. In the leading logarithmic approximation$^{[15]}$ (appropriate for $m_b \gg m_c \gg \Lambda_{\text{QCD}}$)

$$\bar{c} \gamma_\mu (1 - \gamma_5) b = \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} \left[ \frac{\alpha_s(m_c)}{\alpha_s(\mu)} \right] a_L \bar{h}_\nu^{(c)} \gamma_\mu (1 - \gamma_5) h_v^{(b)} ,$$

(9.3)
where
\[ a_L(v \cdot v') = \frac{8}{25} [v \cdot v' r(v \cdot v') - 1] \quad (9.4) \]
and
\[ r(v \cdot v') = \frac{1}{\sqrt{(v \cdot v')^2 - 1}} \ln \left( \frac{v \cdot v' + \sqrt{(v \cdot v')^2 - 1}}{v \cdot v' - \sqrt{(v \cdot v')^2 - 1}} \right) \quad (9.5) \]

In the full theory the partially conserved current \( \bar{c} \gamma_\mu (1 - \gamma_5) b \) doesn’t require renormalization. However, its matrix elements contain (for \( v \cdot v' \neq 1 \)) large logarithms of the bottom and charm quark masses that become divergences in the effective theory. Consequently (for \( v \cdot v' \neq 1 \)) \( \bar{h}^{(c)}_v \gamma_\mu (1 - \gamma_5) h^{(b)}_v \) requires renormalization in the effective theory and has dependence on the subtraction point \( \mu \). This dependence cancels that of the coefficient in eq. (9.3)

For \( v = v' \) the current \( \bar{h}^{(c)}_v \gamma_\mu (1 - \gamma_5) h^{(b)}_v \) is not renormalized. (This is consistent with eq. (9.3) since \( a_L(1) = 0. \) This is because \( \bar{h}^{(c)}_v \gamma_\mu h^{(b)}_v \) is the conserved current associated with heavy quark flavor symmetry. Heavy quark spin symmetry ensures that \( \bar{h}^{(c)}_v \gamma_\mu h^{(b)}_v \) is renormalized in the same way as \( \bar{h}^{(c)}_v \Gamma h^{(b)}_v \), where \( \Gamma \) is any collection of gamma matrices.

For \( B_a \to D_a \) and \( B_a \to D^*_a \) matrix elements of \( \bar{h}^{(c)}_v \gamma_\mu (1 - \gamma_5) h^{(b)}_v \), heavy quark spin symmetry and \( SU(3)_V \) symmetry (the heavy quark current is a singlet with respect to \( SU(3)_V \)) imply that\[9,15]\[ \bar{h}^{(c)}_v \gamma_\mu (1 - \gamma_5) h^{(b)}_v = -\eta (v \cdot v') Tr[ H^{(c)}_a (v') \gamma^\mu (1 - \gamma_5) H^{(b)}_a (v)] \quad (9.6) \]
where \( \eta \) is a universal function of \( v \cdot v' \) independent of the heavy quark masses, (such universal functions are commonly referred to as Isgur-Wise functions). \( \eta \) has subtraction point dependence because the current on the left-hand side of eq. (9.6) requires renormalization in the effective heavy quark theory.

Note that heavy quark spin symmetry forces \( \gamma^\mu (1 - \gamma_5) \) to occur between the \( H \)'s in eq. (9.6). On the outside of the \( H \)'s other factors like \( \not{v} \) or \( \not{v'} \) could occur, but because of eq. (8.11) they can be reduced to the form in eq. (9.6).
Taking the traces in eq. (9.6) gives

\[ < D(v')|\bar{h}_v^{(c)}\gamma_\mu h_v^{(b)}|B(v) > \frac{1}{\sqrt{m_Bm_D}} = \eta(v \cdot v')[v + v']_\mu, \] (9.7)

\[ < D^*(v', \epsilon)|\bar{h}_v^{(c)}\gamma_\mu\gamma_5 h_v^{(b)}|B(v) > \frac{1}{\sqrt{m_Bm_D^*}} = \eta(v \cdot v')[(1 + v \cdot v')\epsilon^*_\mu - (\epsilon^* \cdot v)v'_\mu], \] (9.8)

\[ < D^*(v', \epsilon)|\bar{h}_v^{(c)}\gamma_\mu h_v^{(b)}|B(v) > \frac{1}{\sqrt{m_Bm_D^*}} = i\eta(v \cdot v')\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}v'^\alpha v^\beta. \] (9.9)

In eqs. (9.7), (9.8) and (9.9), the factors of $\sqrt{m_Bm_D}$ and $\sqrt{m_Bm_D^*}$ are inserted in the denominator because the heavy meson states $|M(\vec{p}, s) >$ are normalized according to the usual convention

\[ < M(\vec{p}', s')|M(\vec{p}, s) >= 2E\delta_{ss'}(2\pi)^3\delta^3(\vec{p} - \vec{p}'), \]

and the factor of the energy $E$ is proportional to the heavy meson mass.

At zero recoil, $v = v'$, the vector current $\bar{h}_v^{(c)}\gamma_\mu h_v^{(b)}$ is the conserved current associated with heavy quark flavor symmetry. Consequently its matrix element is fixed, implying that\[9,16,17\]

\[ \eta(1) = 1. \] (9.10)

Eqs. (9.7)–(9.10) represent a tremendous amount of predictive power. Lorentz (and parity) invariance imply that $B \rightarrow D$ and $B \rightarrow D^*$ matrix elements of the vector and axial vector current are parametrized by six form factors. We have found that all those form factors are simply related to the single function $\eta(v \cdot v')$ whose value at zero recoil is known.
10. $\Lambda_{\text{QCD}}/m_Q$ Corrections

The Lagrange density for the heavy quark effective theory, given in eq. (7.5), is valid for $m_Q \to \infty$. The corrections that exist at finite $m_Q$ can be found using a systematic expansion in powers of $1/m_Q$. The part of the QCD Lagrange density that involves a heavy quark field $Q$ is

$$L = \bar{Q}(i\not{\partial} - m_Q)Q, \quad (10.1)$$

and it implies the equation of motion

$$(i\not{\partial} - m_Q)Q = 0. \quad (10.2)$$

As shown in Section 7 to go over to the effective theory we write

$$Q = e^{-im_Q v \cdot x} \left[ h_v^{(Q)} + \chi_v^{(Q)} \right], \quad (10.3)$$

where

$$\not{\partial} h_v^{(Q)} = h_v^{(Q)}, \quad \not{\partial} \chi_v^{(Q)} = -\chi_v^{(Q)}. \quad (10.4)$$

The field $\chi_v^{(Q)}$ does not represent the heavy antiquark. It occurs because the heavy quark is not precisely on-shell as it propagates. Substituting (10.3) into (10.2) it is possible to solve for $\chi_v^{(Q)}$ order by order in $1/m_Q$. Eq. (10.2) thus becomes

$$[m_Q(\not{\partial} - 1) + i\not{\partial}] [h_v^{(Q)} + \chi_v^{(Q)}] = 0. \quad (10.5)$$

Treating $\chi_v^{(Q)}$ as a small quantity we find

$$\chi_v^{(Q)} = \frac{1}{2m_Q} i\not{\partial} h_v^{(Q)} + O(1/m_Q^2). \quad (10.6)$$

The derivative on $h_v^{(Q)}$ produces a factor of the residual momentum which is typically of order $\Lambda_{\text{QCD}}$. Hence, the expansion is in powers of $\Lambda_{\text{QCD}}/m_Q$. (By $\Lambda_{\text{QCD}}$ we mean a
typical hadronic scale. We do not distinguish, in such order of magnitude estimates, between the chiral symmetry breaking scale and the confinement scale.) At order $\Lambda_{\text{QCD}}/m_Q$ the relationship between $Q$ and $\bar{h}_v^{(Q)}$ is

$$Q = e^{-im_Q v \cdot x} \left[ 1 + \frac{iD}{2m_Q} \right] \bar{h}_v^{(Q)}.$$  \hfill (10.7)

Putting this into (10.1) yields the Lagrange density

$$\mathcal{L}_v = \bar{h}_v^{(Q)} i v \cdot D h_v^{(Q)} + \frac{1}{2m_Q} \bar{h}_v^{(Q)} \left[ (iD)^2 - g_s \sigma_{\mu\nu} G^{A\mu
u} T^A \right] h_v^{(Q)}.$$  \hfill (10.8)

The terms of order $\Lambda_{\text{QCD}}/m_Q$ are to be treated as a perturbation in the computation of $S$-matrix elements. Hence the equation of motion $v \cdot D h_v^{(Q)} = 0$ can be used to simplify the couplings (e.g., $\bar{h}_v^{(Q)} (iv \cdot D)^2 h_v^{(Q)}$ vanishes using the equation of motion). Also note that the equation of motion insures that the expression for $\chi_v^{(Q)}$ in eq. (10.6) is consistent with the constraint $\bar{\chi}_v^{(Q)} = -\chi_v^{(Q)}$.

In deriving the Lagrange density of eq. (10.8), we treated the gluon field as a fixed background field and used the equation of motion (10.2). This amounts to matching tree graphs in the full theory of QCD with tree graphs in the effective theory. When quantum loop corrections are included in the Lagrange density for the heavy quark effective theory becomes

$$\mathcal{L}_v = \bar{h}_v^{(Q)} i v \cdot D h_v^{(Q)} + \frac{1}{2m_Q} \bar{h}_v^{(Q)} \left[ a_1 (iD)^2 - a_2 g_s \sigma_{\mu\nu} G^{A\mu
u} T^A \right] h_v^{(Q)} + \text{counter terms}.$$  \hfill (10.9)

The couplings $a_1$ and $a_2$ are subtraction-point dependent. The tree level matching in eq. (10.8) determines that

$$a_1(m_Q) = 1 + \mathcal{O}(\alpha_s(m_Q)),$$  \hfill (10.10a)

$$a_2(m_Q) = 1 + \mathcal{O}(\alpha_s(m_Q)).$$  \hfill (10.10b)

The $\mu$ dependence of $a_{1,2}$ follows from the renormalization of the operators $\bar{h}_v^{(Q)} (iD)^2 h_v^{(Q)}$ and $\bar{h}_v^{(Q)} g_s \sigma_{\mu\nu} G^{A\mu\nu} T^A h_v^{(Q)}$. Explicit calculation demonstrates that
(in the leading logarithmic approximation) $a_1$ is independent of $\mu$ and

$$a_2(\mu) = \left[ \frac{\alpha_s(m_Q)}{\alpha_s(\mu)} \right]^{-9/(33-2N)}, \quad (10.11)$$

where $N$ is the number of quark flavors appropriate to the momentum interval between $m_Q$ and $\mu$.

The fact that $a_1(\mu) = 1$ can be understood as a consequence of reparametrization invariance.\[19\] Recall that the heavy quark four-momentum is the sum of a term proportional to $m_Q$ and the residual momentum.

$$p = m_Q v + k. \quad (10.12)$$

However, this decomposition is not unique. The physics must be the same if the small changes

$$v \to v + \epsilon/m_Q, \quad (10.13a)$$

$$k \to k - \epsilon, \quad (10.13b)$$

are made. Since the four-volicity satisfies $v^2 = 1$ the infinitesimal parameter $\epsilon$ satisfies

$$v \cdot \epsilon = 0. \quad (10.14)$$

In addition to the changes in eq. (10.13) to preserve the constraint $\delta h_v^{(Q)} = h_v^{(Q)}$ the heavy quark field changes, $h_v^{(Q)} \rightarrow h_v^{(Q)} + \delta h_v^{(Q)}$, where

$$\delta h_v^{(Q)} = \frac{\dot{f}}{2m_Q} h_v^{(Q)}. \quad (10.15)$$

Note that $\delta h_v^{(Q)} = -h_v^{(Q)}$. Neglecting, for the moment, the gauge fields, the Lagrange density (10.9) should be invariant under (10.13). Since a derivative brings down a factor of the residual momentum, replacing $D_\mu \rightarrow -i k_\mu$ in eq. (10.9) and then demanding invariance under (10.13) gives $a_1(\mu) = 1$. Because it follows from reparametrization invariance this result holds to all orders in perturbation theory.
In eq. (10.9) only the last term violates heavy quark spin symmetry. It is the matrix element of this term that gives rise to the $P^*_Q - P_Q$ mass difference. Since it is the unique operator, $\bar{h}_v^{(Q)} g_s \sigma_{\mu \nu} G^{A \mu \nu} T^A h_v^{(Q)}$, that causes the splitting for two heavy quarks\cite{20} $Q_i$ and $Q_j$, we deduce

$$m_{P^*_Q} - m_{P_Q} = \left(\frac{m_{Q_j}}{m_{Q_i}}\right) \left[\frac{\alpha_s(m_{Q_j})}{\alpha_s(m_{Q_i})}\right]^{-9/(33-2N)} \left(m_{P^*_Q} - m_{P_Q}\right), \quad (10.16)$$

where $N$ denotes the number of quark flavors appropriate to the momentum interval between $m_{Q_i}$ and $m_{Q_j}$. The measured $B^* - B$ and $D^* - D$ mass differences agree well with eq. (10.12).

11. Chiral Lagrangian for Heavy Mesons

The ground state heavy mesons have $s^\pi_\ell = 1/2^-$ for the spin-parity of the light degrees of freedom. The low-momentum strong interactions of these heavy mesons are described by a Lagrange density that is invariant under chiral $SU(3)_L \times SU(3)_R$ symmetry, heavy quark symmetry, parity, and Lorentz transformations. (For invariance under parity and Lorentz transformations explicit factors of $v$ are treated as if the four-velocity transforms are a true four-vector.) The chiral Lagrange density that describes the low momentum strong interactions of heavy $P_a$ and $P^*_a$ mesons is\cite{21,22,23}

$$\mathcal{L} = -iTr \bar{H}_\alpha v_\mu \partial^\mu H_a + \frac{i}{2} Tr \bar{H}_\alpha H_b v^\mu (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)_{ba}$$

$$+ \frac{i}{2} g Tr \bar{H}_\alpha H_b \gamma_\nu \gamma_5 (\xi^\dagger \partial^\nu \xi - \xi \partial^\nu \xi^\dagger)_{ba} + ... \quad (11.1)$$

Here $H_a$ represents the $4 \times 4$ matrix containing the heavy meson fields defined in eq. (8.6), $\xi$ contains the Goldstone boson $\pi, K$ and $\eta$ fields as defined in eq. (8.20), and the ellipsis denotes terms with more derivatives. The traces in eq. (11.1) are over the $4 \times 4$ matrices. The SU(3) indices $a, b$ are explicitly displayed and repeated indices are summed over $1, 2, 3$. Factors of $\sqrt{m_{P_a}}$ and $\sqrt{m_{P^*_a}}$ have been absorbed into the heavy meson $P_a$ and $P^*_a$ fields so that they have dimension 3/2.
Eq. (11.1) has been simplified using eqs. (8.11). For example, (8.11) implies that the term $T r H_a H_b \gamma_5 v_\nu (\xi^\dagger \partial^\nu \xi - \xi \partial^\nu \xi^\dagger)_{ba}$ vanishes. In the terms of eq. (11.1) no gamma matrices can occur between the factors of $\bar{H}$ and $H$ because of heavy quark spin symmetry.

Expanding $\xi$ in powers of the Goldstone boson matrix $M$ and taking the traces yields Feynman rules for the interaction of $\pi, K, \text{ and } \eta$ with the heavy mesons. The $P_a$ and $P_a^*$ propagators that follow from eq. (11.1) are $i \delta_{ab} / 2 v \cdot k$ and $-i \delta_{ab} (g_{\mu\nu} - v_\mu v_\nu) / 2 v \cdot k$ respectively. The $PP^*M$ and $P^*P^*M$ couplings arise from the term proportional to $g$ in the Lagrange density (11.1). The second term does not give rise to couplings of the heavy mesons to a single Goldstone boson field. Heavy quark flavor symmetry implies that the coupling $g$ is independent of the heavy quark mass, $m_Q$.

The light quark mass terms in the QCD Lagrange density transform as $(\bar{3}_L, 3_R) + (3_L, \bar{3}_R)$ under chiral $SU(3)_L \times SU(3)_R$. To incorporate the leading effects of explicit symmetry breaking from light quark masses, we add

$$\delta \mathcal{L}^{(1)} = \lambda_1 T r H_b H_a (\xi m_q \xi + \xi^\dagger m_q \xi^\dagger)_{ab} + \lambda'_1 T r H_a H_a (\xi m_q \xi + \xi^\dagger m_q \xi^\dagger)_{bb} + ...$$

(11.2)

to the chiral Lagrange density. The ellipsis denotes terms with more derivatives and $m_q$ represents the light quark mass matrix (see eq. (2.21)). The first term in eq. (11.2) fixes the mass splitting between heavy mesons containing an anti-strange quark and those containing anti-up or anti-down quarks. The second term contributes an equal amount to the heavy meson masses and does not contribute to SU(3) violating heavy meson mass splittings. The couplings $\lambda_1$ and $\lambda'_1$ are independent of the heavy quark mass (in the $m_Q \rightarrow \infty$ limit).

It is also possible to include deviations from the $m_Q \rightarrow \infty$ limit that violate heavy quark symmetry. At order $\Lambda_{QCD} / m_Q$ the heavy quark spin symmetry is broken only by the color magnetic moment operator $\bar{h}_v^{(Q)} g_s \sigma_{\mu\nu} G^{A\mu\nu} T^A h_v^{(Q)}$. This operator is a
singlet under chiral $SU(3)_L \times SU(3)_R$, and to include its effects

$$\delta \mathcal{L}^{(2)} = \frac{\lambda_2}{m_Q} T_r \bar{H}_a \sigma^{\mu\nu} H_a \sigma_{\mu\nu} + \ldots$$

is added to the Lagrange density. Here the ellipsis denotes terms with derivatives. It follows from eq. (10.11) that the coupling $\lambda_2$ has logarithmic dependence on $m_Q$. The term in eq. (11.3) is independent of the Goldstone boson fields. Its only effect is to shift the $P^*$ and $P$ masses. In terms of the mass difference

$$\Delta = m_{P^*} - m_P = -\frac{8\lambda_2}{m_Q},$$

the propagators for the $P_a$ and $P_a^*$ mesons become $i\delta_{ab}/2(v \cdot k + \frac{3}{4}\Delta)$ and $-i\delta_{ab}(g_{\mu\nu} - v_\mu v_\nu)/2(v \cdot k - \frac{1}{4}\Delta)$. Now in the rest frame $v = (1, \vec{0})$ an on-shell $P_a$ meson has residual energy $-\frac{3}{4}\Delta$ and an on-shell $P_a^*$ meson has residual energy $\frac{1}{4}\Delta$. It is convenient when dealing with situations involving a real $P_a$ meson and a virtual $P_a^*$ meson to redefine the heavy meson fields by $\exp\left(i\frac{3}{4}\Delta v \cdot x\right)$ so the $P_a$ and $P_a^*$ propagators become $i\delta_{ab}/2v \cdot k$ and $-i\delta_{ab}(g_{\mu\nu} - v_\mu v_\nu)/2(v \cdot k - \Delta)$ respectively.

The $\Lambda_{QCD}/m_Q$ corrections due to the operator $\bar{h}_v^{(Q)}(iD)^2 h_v^{(Q)}$ violate heavy quark flavor symmetry and cause the couplings $g, \lambda_1$ and $\lambda'_1$ to depend on the heavy quark mass $m_Q$.

In the next few sections we explore the implications of chiral perturbation theory for the interactions of heavy mesons. The combination of chiral and heavy quark symmetry provides a powerful tool for studying these interactions.

12. The Coupling $g$

In chiral perturbation theory for heavy mesons, the fundamental coupling is $g$. The heavy meson contribution to the light quark axial current is obtained from the Lagrange density in eq. (11.1) using the Noether procedure. Under an infinitesimal axial transformation $\delta M = -f \epsilon^A T^A + \ldots$, while eqs. (8.18) and (8.24) imply that
$H_a \to H_a + \ldots$. Here the ellipsis denotes terms containing the Goldstone boson fields $M$. It follows that

$$\bar{q}_a T^A_{ab} \gamma^5 q_b = -g Tr \tilde{H}_a H_b \gamma^5 \gamma^5 T^A_{ab} + \ldots.$$  \hspace{1cm} (12.1)

Treating the quark fields in eq. (12.1) as constituent quarks and using the nonrelativistic constituent quark model to estimate the $D^*$ matrix element of left hand side of eq. (12.1) gives $g = 1$. A similar estimate of the pion nucleon coupling implies that $g_A = 5/3$. (Recall that experimentally $g_A = 1.25$.) Thus our expectation is that $g$ is around unity.

Expanding the Lagrangian (11.1) in powers of the Goldstone boson fields $M$, we find at linear order that

$$\mathcal{L} = -\frac{g}{f} Tr \tilde{H}_a H_b \gamma^5 \gamma^5 \partial^\nu M_{ba}$$

$$\hspace{1cm} = \left[ \left( -\frac{2g}{f} \right) \partial^\nu M_{ba} P^*_{a \mu} P^*_{b \nu} + h.c. \right] + \left( \frac{2g}{f} \partial^\nu M_{ba} P^{* a \mu} P^*_{b \nu} \epsilon_{\alpha \beta \mu \nu} \sigma^\lambda \right].$$  \hspace{1cm} (12.2)

Eq. (12.2) contains the $P^* P M$ and $P^* P^* M$ couplings. (Note that because of parity invariance there is no $P P M$ coupling.) Using eq. (12.2) for $Q = c$ and $M = \pi$ gives

$$\Gamma(D^{*+} \to D^{0} \pi^+) = \frac{g^2}{6\pi f^2} |\vec{p}_\pi|^3.$$  \hspace{1cm} (12.3)

The decay width for $D^{*+} \to D^{+} \pi^0$ is a factor of two smaller by isospin symmetry. The experimental upper limit$^{[24]}$ of 131 KeV on the $D^*$ width when combined with the $D^{*+} \to D^{+} \pi^0$ and $D^{*+} \to D^{0} \pi^+$ branching ratios$^{[25]}$ of Table I imply that $g^2 \lesssim 0.5$. In evaluating eq. (12.3), $f = f_\pi \simeq 132$ MeV was used.
TABLE I

| Decay Mode       | Branching Ratio % |
|------------------|-------------------|
| $D^* \rightarrow D^0 \pi^0$ | $63.6 \pm 2.3 \pm 3.3$ |
| $D^* \rightarrow D^0 \gamma$  | $36.4 \pm 2.3 \pm 3.3$ |
| $D^* \rightarrow D^0 \pi^+$  | $68.1 \pm 1.0 \pm 1.3$ |
| $D^* \rightarrow D^+ \pi^0$  | $30.8 \pm 0.4 \pm 0.8$ |
| $D^* \rightarrow D^+ \gamma$  | $1.1 \pm 1.4 \pm 1.6$ |

Even if the $D^*$ decay width is too small to measure, radiative $D^*$ decay may provide a (indirect) determination of $g$.[26,27] The $D_a^* \rightarrow D_a \gamma$ matrix element has the form

$$\mathcal{M}(D_a^* \rightarrow D_a \gamma) = e \mu_a e^{\mu \alpha \beta \lambda} \epsilon_\mu^*(\gamma) v_\alpha k_\beta \epsilon_\lambda(D^*) ,$$

(12.4)

where $e \mu_a/2$ is the transition magnetic moment, $k$ is the photon momentum, $\epsilon(\gamma)$ is the polarization vector for the photon, and $\epsilon(D^*)$ is the polarization vector for the $D^*$. The resulting decay rate is

$$\Gamma(D_a^* \rightarrow D_a \gamma) = \frac{\alpha e}{3} |\mu_a|^2 |\vec{k}|^3 .$$

(12.5)

The $D_a^* \rightarrow D_a \gamma$ matrix element gets contributions from the photon coupling to the light quark part of the electromagnetic current, $\frac{2}{3} \bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d - \bar{s} \gamma_\mu s$, and the photon coupling to the heavy charm quark part of the electromagnetic current, $\frac{2}{3} \bar{c} \gamma_\mu c$. The part of $\mu_a$ that comes from the heavy charm quark piece of the electromagnetic current, $\mu^{(h)}$, is determined by heavy quark symmetry. In the effective heavy quark theory the Lagrange density for strong and electromagnetic interactions of the charm quark is

$$\mathcal{L}_v^c = \bar{h}_v^c(i\gamma \cdot \vec{D}) h_v^c + \frac{1}{2m_c} \bar{h}_v^c(iD^c)^2 h_v^c$$

$$- \frac{g_s}{2m_c} \bar{h}_v^c \sigma^{\mu \nu} A^c A_{\mu \nu} - \frac{e}{3m_c} \bar{h}_v^c \sigma^{\mu \nu} h_v^c F_{\mu \nu} + ... ,$$

(12.6)

where the ellipsis denotes terms suppressed by more factors of $1/m_c$, and the sub-
traction point is chosen to be \( \mu = m_c \). This is an extension of the result presented in eq. (10.9) to include electromagnetic interactions,

\[
D_\mu = \partial_\mu + i g_s A_\mu^A T^A + \frac{2}{3} i e A_\mu .
\] (12.7)

Only the last term in eq. (12.6) contributes to the \( D_a^* \rightarrow D^* \gamma \) matrix element. Using the generalization of eq. (9.6)

\[
\bar{h}^{(c)}_\nu \Gamma h^{(c)}_\nu = -\eta (v \cdot v') \text{Tr}[\bar{H}^{(c)}_a (v') \Gamma H^{(c)}_a (v)] ,
\] (12.8)

with \( \Gamma = \sigma^{\mu \nu} \) and \( v' = v \), implies (using \( \eta (1) = 1 \)) that

\[
\mu^{(h)} = \frac{2}{3m_c} .
\] (12.9)

The part of \( \mu_a \) that comes from the photon coupling to the light quark piece of the electromagnetic current, \( \mu^{(\ell)}_a \), is not fixed by heavy quark symmetry. The light quark piece of the electromagnetic current transforms as an octet under the unbroken \( SU(3)_V \) flavor symmetry group. Since there is only one way to combine an 8, 3, and \( \bar{3} \) into a singlet, the \( \mu^{(\ell)}_a \) are expressible in terms of a single reduced matrix element

\[
\mu^{(\ell)}_a = Q_a \beta ,
\] (12.10)

where \( \beta \) is an unknown constant and \( Q_a \) denotes the light quark charges \( Q_1 = 2/3, \ Q_2 = -1/3, \ Q_3 = -1/3. \) (In the nonrelativistic constituent quark model, \( \mu^{(\ell)}_a \) arises from the magnetic moment of a constituent quark. This leads to the expectation that \( \beta \simeq 3 \text{GeV}^{-1} \).) Eq. (12.10) includes effects suppressed by powers of \( 1/m_c \) since it follows only from \( SU(3)_V \) symmetry.

The leading \( SU(3)_V \) violating contribution to \( \mu_a \) has a nonanalytic dependence on the light quark masses \( m_q \) of the form \( m_q^{1/2} \), and arises from the one-loop Feynman
diagrams in Figure 3. It is straightforward to compute these diagrams. Gauging the Goldstone boson chiral Lagrange density in eq. (3.11),

$$\mathcal{L} = \frac{f^2}{8}(D_\mu \Sigma)(D_\mu \Sigma)^\dagger,$$

(12.11)

where

$$D_\mu \Sigma = \partial_\mu \Sigma + ie[Q, \Sigma]A_\mu,$$

(12.12)

and

$$Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}.$$  

(12.13)

Eq. (12.11) gives the electromagnetic interactions of the Goldstone bosons, and it yields the photon vertices in Figure 3. The remaining vertices needed for Figure 3 follow from eq. (12.2). The resulting loop integral has the form

$$I^{\nu \alpha} \equiv \int \frac{d^n q}{(2\pi)^n} \frac{q^\nu q^{\alpha}}{(v \cdot q + i\epsilon)(q^2 - m_K^2 + i\epsilon)^2}$$

$$= 4 \int_0^\infty d\lambda \int \frac{d^n q}{(2\pi)^n} \frac{q^\nu q^{\alpha}}{(q^2 + 2\lambda v \cdot q - m_K^2 + i\epsilon)^3}$$

$$= \left(\frac{4}{n}\right) g^{\nu \alpha} \int_0^\infty d\lambda \int \frac{d^n q}{(2\pi)^n} \frac{q^2}{(q^2 - m_K^2 - \lambda^2 + i\epsilon)^3} + \ldots ,$$

(12.14)

where the ellipsis denotes terms proportional to $v^\nu v^\alpha$ that don’t contribute to the amplitude. Finally the $q$ and $\lambda$ integrations are performed using the formulas

$$\int \frac{d^n q}{(2\pi)^n} \frac{(q^2)^\alpha}{(q^2 - M^2 + i\epsilon)^\beta} = i(-1)^{\alpha+\beta}(M^2)^{\frac{\alpha}{2} + \alpha - \beta} \frac{\Gamma(\alpha + n/2)\Gamma(\beta - \alpha - n/2)}{\Gamma(n/2)\Gamma(\beta)}$$

(12.15)

and

$$\int_0^\infty d\lambda (1 + \lambda^2)^{-p} = \frac{\pi \Gamma(2p - 2)}{2^{2p-2} \Gamma(p-1)\Gamma(p)},$$

(12.16)
This gives as $n \to 4$,
\[
I^{\nu\alpha} = \frac{-i}{16\pi} g^{\nu\alpha} m_K + \ldots
\]  
(12.17)
where the ellipsis denotes terms proportional to $v^\nu v^\alpha$.

Including the $SU(3)_V$ violations that follow from Figure 3 the expression for $\mu_{a}^{(\ell)}$ becomes
\[
\mu_1^{(\ell)} = \frac{2}{3} \beta - \frac{g^2 m_K}{4\pi f^2} - \frac{g^2 m_\pi}{4\pi f^2}
\]  
(12.18a)
\[
\mu_2^{(\ell)} = -\frac{1}{3} \beta + \frac{g^2 m_\pi}{4\pi f^2}
\]  
(12.18b)
\[
\mu_3^{(\ell)} = -\frac{1}{3} \beta + \frac{g^2 m_K}{4\pi f^2}.
\]  
(12.18c)

For $m_K \neq m_\pi$ the one-loop contribution to $\mu_1^{(\ell)}, \mu_2^{(\ell)}$ and $\mu_3^{(\ell)}$ is not in the ratio $2 : -1 : -1$ and hence violates $SU(3)_V$. The most important corrections to eqs. (12.18) come from $SU(3)_V$ violating terms of order $m_s$. These are analytic in the strange quark mass and so are not calculable.

Using
\[
\mu_a = \mu_a^{(\ell)} + \mu^{(h)}
\]  
(12.19)
with $\mu_a^{(\ell)}$ and $\mu^{(h)}$ given by eqs. (12.18) and (12.9), determines the rates for $D_0^{*+} \to D_0^0 \gamma, D_0^{*+} \to D^+ \gamma$ and $D_s^* \to D_s \gamma$ in terms of $\beta$ and $g$. The measured ratio of branching ratios $Br(D_0^{*0} \to D_0^0 \gamma)/Br(D_0^{*0} \to D_0^0 \pi^0)$ thus gives $\beta$ as a function of $g$. The ratio of branching functions $Br(D_0^{*+} \to D^+ \gamma)/Br(D_0^{*+} \to D_0^0 \pi^0)$ can therefore be expressed in terms of $g$. At the present time there is only an upper limit on the value of the $Br(D_0^{*+} \to D^+ \gamma)$, but when it is measured a value for the coupling of $g$ can be extracted.
13. Semileptonic $B \to D\bar{e}e$ and $B \to D^*\bar{e}e$ Decay at Zero Recoil

At zero recoil (i.e., $v = v'$) heavy quark symmetry implies that

$$\frac{< D(v)|\bar{c}\gamma_{\mu}b|B(v)>}{\sqrt{m_Bm_D}} = \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} 2v_{\mu} \left( \Delta^e \right)^2,$$

(13.1a)

$$\frac{< D^*(v,\epsilon)|\bar{c}\gamma_{\mu}\gamma_5b|B(v)>}{\sqrt{m_Bm_{D^*}}} = \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} 2\epsilon^*_{\mu} \left( \Delta^e \right)^2.$$

(13.1b)

Furthermore, it has been shown that there are no order $1/m_c$ or $1/m_b$ corrections to the relations (13.1). Chiral perturbation theory has been used to examine the order $(1/m_c)^{n+2}, n = 0, 1, 2, ...$ corrections to eqs. (13.1). For small up and down quark masses, the leading corrections result from the one-loop diagram in Figure 4 and wavefunction renormalization. In Figure 4 the shaded square denotes an insertion of the weak current vertices (13.1), and the shaded circles denote $P^*P^*\pi$ or $P^*P\pi$ vertices (see eq. (12.2)) from the chiral Lagrangian in eq. (11.1). This gives

$$\frac{< D(v)|\bar{c}\gamma_{\mu}b|B(v)>}{\sqrt{m_Bm_D}} = \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} 2v_{\mu} \left( \Delta^e \right)^2 \left[ \ell n(\mu^2/m^2_\pi) + f(\Delta^e/m_\pi) \right].$$

(13.2a)

$$\frac{< D^*(v,\epsilon)|\bar{c}\gamma_{\mu}\gamma_5b|B(v)>}{\sqrt{m_Bm_{D^*}}} = \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} 2\epsilon^*_{\mu} \left( \Delta^e \right)^2 \left[ \ell n(\mu^2/m^2_\pi) + f(-\Delta^e/m_\pi) \right],$$

(13.2b)

where

$$f(x) = 2\int_0^\infty dq \frac{q^4}{(q^2 + 1)^{3/2}} \left\{ \frac{1}{[(q^2 + 1)^{1/2} + x]^2} - \frac{1}{q^2 + 1} \right\}.$$  

(13.3)

In eqs. (13.2) $C(\mu)$ and $C'(\mu)$ are the contribution of tree level “counter terms” of order $1/m_c^2$, and $\Delta^e = m_{D^*} - m_D$. The dependence of $C$ and $C'$ on the subtraction
point $\mu$ is cancelled by that of the logarithm. For $\mu$ of order the chiral symmetry breaking scale, $\sim 1\text{GeV}$, $C(\mu)$ and $C'(\mu)$ contain no large logarithms and (at least formally) are less important than the terms with a logarithm of the pion mass (which are also of order $1/m_c^2$ since $\Delta^{(c)}$ is order $1/m_c$). The function $f$ takes into account the effects of corrections of order $(1/m_c)^{2+n}$, $n = 1, 2, \ldots$. It is enhanced by powers of $1/m_\pi$ over terms we have neglected and should provide a reliable estimate of the order $(1/m_c)^{2+n}$, $n = 1, 2, \ldots$ effects. Because the pion mass occurs in the denominator, the expansion in powers of $1/m_c$ breaks down in the limit where the pion mass goes to zero. Experimentally, $m_\pi$ is about equal to $\Delta^{(c)}$ and so all the terms of order $(1/m_c)^{2+n}$, $n = 1, 2, 3, \ldots$ are of comparable importance.

Since $\Delta^{(c)}$ is greater than $m_\pi$ the $B \to D^*$ matrix element has an imaginary part. However, experimentally $\Delta^{(c)}$ is very close to $m_\pi$ and it is a good approximation to set $\Delta^{(c)}/m_\pi = 1$. The expression in eq. (13.3) gives $f(1) = 2(\frac{7}{3} - \pi)$ and $f(-1) = 2(\frac{7}{3} + \pi)$. Numerically for $g^2 = 0.5$ and $\mu = 1\text{GeV}$, the correction to the $B \to D$ matrix element from the “large logarithm” is -2.1% and the correction from $f$ is 0.9%. For the $B \to D^*$ matrix element the correction from the large logarithm is -0.7% and from $f$ is -2.0%.

In this section we have only used chiral $SU(2)_L \times SU(2)_R$. The order $1/m_c^2$ effects of kaon and eta loops are absorbed into the constants $C(\mu)$ and $C'(\mu)$.

14. **Semileptonic $B \to \pi e\bar{\nu}_e$ or $D \to \pi e\bar{\nu}_e$ Decay**

For most of the Dalitz plot, chiral perturbation theory cannot be applied to $B \to \pi e\bar{\nu}_e$ and $D \to \pi e\bar{\nu}_e$ decay since (in the $B$ or $D$ rest frame) the pion has a large energy compared with the chiral symmetry breaking scale. In this section we focus on the tiny region of phase space where the pion has an energy small enough that chiral perturbation theory can be applied. For definiteness let’s focus on the decay $B \to \pi e\bar{\nu}_e$. Then the relevant hadronic matrix element is

\[
<\pi(p_\pi)|\bar{u}\gamma_\mu(1 - \gamma_5)b|B(v)> = f_+(p_B + p_\pi)_\mu + f_-(p_B - p_\pi)_\mu ,
\]

where $p_B = m_B v$. 41
The semileptonic decays $B \to \pi e \bar{\nu}_e$ and $B \to \pi \mu \bar{\nu}_\mu$ depend only on $f_+$; the contribution of $f_-$ is proportional to the lepton mass and can be neglected. In the large $b$–quark mass limit (when $v \cdot p_\pi \ll m_b$), the left-hand side goes as $\sqrt{m_b}$ from the normalization of states (there is also a logarithmic dependence on $m_b$ from perturbative QCD effects). Consequently, for $v \cdot p_\pi \ll m_b$,

$$f_+ + f_- \sim O(1/\sqrt{m_b}), \quad (14.2a)$$

$$f_+ - f_- \sim O(\sqrt{m_b}). \quad (14.2b)$$

So, in the limit $m_b \to \infty$, $f_+ = -f_-$. The known dependence of the form factors $f_\pm$ on the heavy quark mass (and isospin symmetry) means that form factors for $B \to \pi e \bar{\nu}_e$ are related to those for $D \to \pi \bar{\nu} e$. Including perturbative QCD effects, we find the relationship\[29\]

$$(f_+^{(B \to \pi)} + f_-^{(B \to \pi)}) = \sqrt{\frac{m_D}{m_B}} \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} (f_+^{(D \to \pi)} + f_-^{(D \to \pi)}), \quad (14.3a)$$

$$(f_+^{(B \to \pi)} - f_-^{(B \to \pi)}) = \sqrt{\frac{m_B}{m_D}} \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} (f_+^{(D \to \pi)} - f_-^{(D \to \pi)}) \quad (14.3b)$$

Since $f_+ = -f_-$ as $m_Q \to \infty$, eq. (14.3b) implies the important relation

$$f_+^{(B \to \pi)} = \sqrt{\frac{m_B}{m_D}} \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} f_+^{(D \to \pi)}. \quad (14.4)$$

Eqs. (14.3) and (14.4) are consequences of heavy quark flavor symmetry. Naively, these formulae hold as long as $v \cdot p_\pi$ is small compared to the heavy quark masses $m_c$ and $m_b$. However, we shall see shortly that, for very small $v \cdot p_\pi$, chiral perturbation theory implies that eqs. (14.3) and (14.4) are not valid.\[30\]
The operator

\[ L_a' = \bar{q}_a \gamma^\nu (1 - \gamma_5) b, \] (14.5)

transforms under chiral \( SU(3)_L \times SU(3)_R \) as \((\bar{3}_L, 1_R)\), and in chiral perturbation theory its hadronic matrix elements are given by those of

\[ L_a' = \left( \frac{i\alpha}{2} \right) Tr\gamma^\nu (1 - \gamma_5) H_{b \bar{b}_a} + \ldots , \] (14.6)

where the ellipsis denotes terms with derivatives, factors of the light quark mass matrix \( m_q \), or factors of \( 1/m_Q \). The constant \( \alpha \) has a logarithmic dependence on the \( b \)-quark mass and is related to the \( B \) meson decay constant \( f_B \). Using equation (14.6) with \( a = 1 \) to calculate the matrix element

\[ < 0 | \bar{u} \gamma^\nu \gamma_5 b | B^- (v) > = i f_B p_B' , \] (14.7)

gives

\[ \alpha = f_B \sqrt{m_B} . \] (14.8)

The form factors \( f_\pm \) are given by the \( B \rightarrow \pi \) matrix element of \( L_1' \) in eq. (14.6). Calculating the Feynman diagrams in Figure 5 and using eq. (14.8) to express \( \alpha \) in terms of \( f_B \) gives\([21,22,23]\) for \( B^0 \rightarrow \pi^+ e \bar{\nu}_e \)

\[ f_+ + f_- = -(f_B / f) \left[ 1 - g v \cdot p_\pi / (v \cdot p_\pi + \Delta^{(b)}) \right] , \] (14.9a)

\[ f_+ - f_- = -g f_B m_B / f (v \cdot p_\pi + \Delta^{(b)}) , \] (14.9b)

where \( \Delta^{(b)} = m_{B^*} - m_B \). For \( B^- \rightarrow \pi^0 e \bar{\nu}_e \) decay there is an additional \( 1/\sqrt{2} \). Eqs. (14.9) are valid for \( v \cdot p_\pi \) much less than the chiral symmetry breaking scale. Note that they don’t depend on heavy quark flavor symmetry but do use the heavy quark spin symmetry. (In the pole graph it is the heavy quark spin symmetry that relates the
$B^*$ decay constant to that of the $B$.) Eqs. (14.9) indicate that $f_+ + f_-$ is negligible compared with $f_+ - f_-$ provided $g$ is not too small. For $g$ around unity

$$f_+ = -gf_B m_B / 2 f(v \cdot p_\pi + \Delta^{(b)}) \ .$$  \hspace{1cm} (14.10)

Eqs. (14.9) and (14.10) also hold for $D \to \pi$ provided one replaces $f_B \to f_D, m_B \to m_D$ and $\Delta^{(b)} \to \Delta^{(c)} = m_{D^*} - m_D$. The pion mass is comparable with $\Delta^{(c)}$ and so the relations in eqs. (14.3) and (14.4) break down for very small $v \cdot p_\pi$. For $v \cdot p_\pi \gg \Delta^{(c)}$ one recovers eq. (14.3) from eq. (14.9) using the heavy quark flavor symmetry prediction for the relation between $B$ and $D$ meson decay constants$^{[31,32]}$

$$f_B = \left[ \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} \sqrt{\frac{m_D}{m_B}} f_D .$$ \hspace{1cm} (14.11)

There are indications from lattice QCD,$^{[33]}$ QCD sum rules$^{[34]}$ and 1 + 1 dimensional QCD$^{[35]}$ in the large $N_c$ limit that the charm quark mass is not large enough for the corrections to eq. (14.11) to be neglected. Even if this is true, eqs. (14.9) may still be a good approximation for both $B \to \pi e\bar{\nu}_e$ and $D \to \pi e\bar{\nu}_e$. If chiral $SU(3)_L \times SU(3)_R$ is used then eqs. (14.9) can be used for $D \to K e \bar{\nu}_e$ decay. However, it is not clear that the kaon mass is small enough for operators with one derivative to be neglected in $L^3_{\nu}$.

15. Concluding Remarks

These notes are meant to provide an introduction to chiral perturbation theory for hadrons containing a single heavy quark. For these hadrons the combination of heavy quark and chiral symmetries is very powerful and it makes a number of interesting predictions.

Much of these notes (Chapters 2—11) consisted of a review of chiral perturbation theory and heavy quark symmetry. Applications of the combination of these methods to properties of $B, B^*$ and $D, D^*$ mesons were made in Chapters 11—14. There has been considerable activity in this area over the last year, and the few applications discussed in these notes do not do justice to the breadth of applicability.
of the methods developed here. These notes provide the general background needed to do research into the properties of heavy hadrons that can be studied with chiral perturbation theory. I encourage the reader to explore some of the other applications of the combination of heavy quark and chiral symmetries that have been discussed in the recently published literature.

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Figure Captions

Fig. 1. One loop Feynman diagram contributing to $\pi\pi \to \pi\pi$ scattering.

Fig. 2. Diagrams for the hadronic matrix element in $K^0 \to \pi^-\pi^0e^+\nu_e$ semileptonic decay.

Fig. 3. Feynman diagrams that give nonanalytic $m_q^{1/2}$ contribution to the $D^* \to D\gamma$ matrix element.

Fig. 4. Feynman diagram that give correction to heavy quark symmetry predictions for $B \to D$ and $B \to D^*$ matrix elements at zero recoil.

Fig. 5. Tree graphs that determine $B \to \pi e\bar{\nu}_e$ near zero recoil.