Test of asymptotic freedom and scaling hypothesis in the 2d O(3) sigma model

J. Balog\textsuperscript{1} and P. Weisz\textsuperscript{2}

\textsuperscript{1}Research Institute for Particle and Nuclear Physics  
H-1525 Budapest 114 Pf. 49, Hungary  
\textsuperscript{2}Max-Planck-Institut für Physik  
D-80805 Munich, Germany

Abstract

The 7–particle form factors of the fundamental spin field of the O(3) nonlinear $\sigma$–model are constructed. We calculate the corresponding contribution to the spin–spin correlation function, and compare with predictions from the spectral density scaling hypothesis. The resulting approximation to the spin–spin correlation function agrees well with that computed in renormalized (asymptotically free) perturbation theory in the expected energy range. Further we observe simple lower and upper bounds for the sum of the absolute square of the form factors which may be of use for analytic estimates.
1 Introduction

1 + 1 dimensional integrable models are theoretical laboratories where one can study selected problems in field theory. These include the question whether the continuum limit of the lattice regularized version of a field theory coincides with the theory constructed by continuum techniques. One can study here structural questions related to asymptotic freedom, topological excitations etc., and, in the case that the theories agree, the nature of lattice artifacts.

The form factor bootstrap method [1, 2] of constructive field theory has been used to construct many integrable models. The method consists of three main steps. The starting point is the exact scattering matrix that is calculated (sometimes conjectured) by the bootstrap method [3]. Next the form factors (matrix elements of local operators between scattering states) are calculated by solving the form factor (FF) axioms [2]. Finally correlation functions are constructed by inserting a complete set of intermediate states between the local operators.

The program has so far only been carried out completely for the Ising model [4]. The S–matrix is known for many integrable models and if there are no internal degrees of freedom also the many–particle form factors are easily constructed as a multiple product of the basic two–particle form factor (which is known again for most models). On the other hand, for models with internal degrees of freedom the solution of the FF axioms, matrix functional difference equations in this case, is not known in general. Nevertheless, the form factors of some models belonging to this class, the Sine–Gordon model, the chiral Gross–Neveu model, and the O(3) and O(4) nonlinear σ–models, are available [2, 5]. Finally (with the exception of the Ising model) the $r$–particle contributions to the correlation functions can only be computed numerically and therefore this sum has to be truncated after the first few terms.

Among models with internal (isospin) degrees of freedom, it is the O(3) model, where one can go furthest. This is because in this special case the many–particle form factors (after removing a simple factor) are polynomials in the particle rapidities which can be determined recursively [6]. Fortunately it is also a model which is of particular interest since it has many properties akin to Yang–Mills theory in 4 dimensions in that it exhibits asymptotic freedom and has instanton solutions.

The question mentioned above concerning the equivalence between the FF construction and that from the lattice regularization has been addressed in ref. [7]. The two constructions are both non–perturbative and in both cases it is the low energy properties that are most easily accessible. However in the bootstrap approach it is simpler to obtain reliable results at higher energies. For example for the Fourier transform of the 2–point correlation function of Noether currents, the contributions of higher $r$–particle states become significant at typical energies rapidly increasing with $r$. Of course, despite the fact that in practice renormalized perturbation
theoretic computations often seem to be quite accurate down to unexpectedly low energies, the property of asymptotic freedom is a statement pertaining to asymptotically high energies, and to establish this property in the bootstrap framework it is necessary to have rigorous knowledge on the contributions of $r$–particle states with arbitrarily large $r$. Although the latter has not yet been achieved, based on the exact form factors up to 6 particles, Max Niedermaier and one of the present authors (J.B.) presented convincing evidence for remarkable scaling properties of the higher intermediate particle contributions \[8, 9\], which are very powerful because they make it possible to include dominant contributions from all states in the form factor expansion of correlation functions. Furthermore the scaling hypothesis is completely compatible with asymptotic freedom.

Although the lattice regularization is an important non–perturbative method (and practically the only one available in 4d QCD), there are many physical phenomena which are inaccessible in this approach for example nuclear structure functions at small Bjorken $x$. In the framework of the bootstrap approach in 2d however, we hope that properties of the structure functions at small $x$ can be extracted, an insight which would hopefully also be relevant for QCD. This hope would be realized if we could find evidence for scaling properties of the structure functions similar to those already observed for the 2–point function spectral densities.

With this goal we have initiated a project to compute the structure functions in the O(3) sigma model, and with the known form factors we have computed contributions from intermediate states up to 5 particles. To establish their scaling properties however it is helpful to extend the list of known form factors to higher particle number. In this paper we discuss the calculation of the 7–particle form factors, which although in principle trivial, is technically challenging because for $r = 7$ one has to deal with quite large polynomials. We will report on the structure functions in a future publication. Here we restrict attention to the contribution of the 7–particle intermediate states to the spin–spin correlation function, which yields an additional test of the scaling hypothesis. We also exhibit simple bounds on the square of the known form factor polynomials, which if they could be generalized to an arbitrary number of particles might be useful for analytic estimates to establish general properties.

2 Some basic definitions

The central object of attention in this paper is the 7–particle form factor of the spin operator of the O(3) model. This is the $r = 7$ case of

$$
\langle 0|S^{a}(0)|a_{1}, \theta_{1}; \ldots; a_{r}, \theta_{r}\rangle = \frac{2}{\sqrt{\pi}} f_{a_{1} \ldots a_{r}}^{a}(\theta_{1}, \ldots, \theta_{r}),
$$

(1)
where the particles are labelled by their isospin indices and rapidities and the spin operator $S^a$ also carries an isospin index. The normalization factor $2/\sqrt{\pi}$ is introduced here for later convenience. The spin operator itself is normalized by

$$\langle 0 | S^a(0) | a_1, \theta_1 \rangle = \delta^{a_1}, \quad \langle b, \chi | a, \theta \rangle = 4\pi \delta^{ab} \delta(\chi - \theta),$$

which means that the Minkowski propagator has unit residue at $p^2 = M^2$, where $M$ is the physical mass of the O(3) particle.

Introducing the squared form factors

$$\delta^{ab} F^{(r)}(u) = \sum_{a_1\ldots a_r} f_{a_1\ldots a_r}^a(\theta_1, \ldots, \theta_r) f_{a_1\ldots a_r}^b(\theta_1, \ldots, \theta_r)$$

the spectral density is given by

$$\rho^{\text{spin}}(\mu) = \frac{4}{\pi} \sum_{k=0}^{\infty} \rho^{(2k+1)}(\mu),$$

where

$$\rho^{(r)}(\mu) = \int_0^\infty \frac{d u_1 \ldots d u_{r-1}}{(4\pi)^{r-1}} F^{(r)}(u) \delta(\mu - M^{(r)}(u)).$$

Here $u_j = \theta_j - \theta_{j+1}$ are rapidity differences and $M^{(r)}(u)$ is the $r$–particle invariant mass. The Fourier transform of the correlation function is represented as the Stieltjes transform of the spectral density:

$$I^{\text{spin}}(p^2) = \int_0^\infty d \mu \frac{\rho^{\text{spin}}(\mu)}{p^2 + \mu^2}. $$

We will compare (the truncation of) (6) to the results of perturbation theory. The 2–loop order perturbative result is [10]

$$p^2 I^{\text{spin}}(p^2) = \lambda_1 \left\{ \frac{1}{\alpha(p)} + (2 + \xi_0) + (2 + \xi_0) \alpha(p) + O(\alpha^2(p)) \right\}. $$

Here the running coupling function $\alpha(p)$ is the solution of

$$\frac{1}{\alpha(p)} + \ln \alpha(p) = \ln \frac{p}{M}$$

and the parameter $\xi_0$ gives the connection between the perturbative mass parameter $\Lambda_{\overline{\text{MS}}}$ and the exact mass gap $M$. In the O(3) model their ratio is known exactly [11]:

$$\xi_0 = \ln \frac{M}{\Lambda_{\overline{\text{MS}}}} = \ln 8 - 1 \approx 1.07944.$$
The overall constant $\lambda_1$ cannot be calculated in perturbation theory, but has been determined exactly \[8\] using the scaling hypothesis (see also Sect. 5):

$$\lambda_1 = \frac{4}{3\pi^2}. \quad (10)$$

Instead of the physical form factor (1) it is convenient to consider the reduced form factors $g_{a_1...a_r}^a$ defined by

$$f_{a_1...a_r}(\theta_1, \ldots, \theta_r) = \frac{\pi^{\frac{3r}{2} - 1}}{2} \Psi(\theta_1, \ldots, \theta_r) g_{a_1...a_r}^a(\theta_1, \ldots, \theta_r). \quad (11)$$

Here

$$\Psi(\theta_1, \ldots, \theta_r) = \prod_{i<j} \psi(\theta_i - \theta_j), \quad (12)$$

with

$$\psi(\theta) = \frac{\theta - i\pi}{\theta(2\pi i - \theta)} \tanh^2 \frac{\theta}{2}. \quad (13)$$

We note that (13) is the basic 2–particle form factor and the product (12) would be the complete $r$–particle form factor if there were no isospin degrees of freedom.

The advantage of using the reduced form factors is that they are polynomials and can (in principle) be recursively calculated \[6\]. The simplest nontrivial case is $r = 3$:

$$g_{a_1a_2a_3}^a(\theta) = \delta^{aa_3} \delta^{a_1a_2} (\theta_2 - \theta_1) + \delta^{aa_2} \delta^{a_1a_3} (\theta_1 - \theta_3 - 2i\pi) + \delta^{aa_1} \delta^{a_2a_3} (\theta_3 - \theta_2). \quad (14)$$

The higher reduced form factors very rapidly become complicated because of the large number of isospin components and rapidity variables.

The quantity entering the spectral densities and two–point functions is the absolute square of the form factors, summed over the internal symmetry indices. For the reduced form factors the corresponding quantities are

$$G^{(r)}(\theta_1, \ldots, \theta_r) = \frac{1}{3} \sum_{a_1 \ldots a_r} |g_{a_1...a_r}^a(\theta_1, \ldots, \theta_r)|^2. \quad (15)$$

For the $r = 3$ example we get

$$G^{(3)}(\theta_1, \theta_2, \theta_3) = 2 \left[ (\theta_1 - \theta_2)^2 + (\theta_1 - \theta_3)^2 + (\theta_2 - \theta_3)^2 \right] + 12\pi^2. \quad (16)$$
3 The 7–particle form factors

The 7–particle reduced form factors
\[ g_{a_1...a_7}^a(\theta_1, \ldots, \theta_7) \] (17) are polynomials of degree 15 in the rapidity differences. To count the independent isospin components we can obviously fix the operator index \( a = 3 \) since the rest of the components are related by simple isospin transformations. Then we get 274 non–vanishing components not counting terms related by the \( 1 \leftrightarrow 2 \) isospin symmetry twice. These components can be calculated from the known 5–particle form factors using the inhomogeneous FF axioms [6]. The resulting polynomials contain typically \( \sim 10^4 \) terms. Actually it is not necessary to do this calculation for all the 274 components since we can find 4 components such that the rest can be obtained from them by using the homogeneous FF axioms (corresponding to permutations of the rapidity variables). After having computed all terms we checked once more that all the FF axioms are satisfied by (17) and also that the coefficients of the leading terms in the last variable (those proportional to \( \theta_5^2 \)) are proportional to the 6–particle reduced form factors as they should [8].

After the calculation of the form factors the next step is to compute the sum of the absolute square of the components since this is needed in the calculation of the 7–particle contribution to the spin–spin correlation function and other physical quantities. This calculation is rather challenging because at intermediate steps one has to handle quite large polynomials (of degree 30 in 7 variables consisting of about \( \sim 4 \cdot 10^5 \) terms). The final result can be simplified enormously because, being a polynomial symmetric under permutations of the rapidity variables, it can be expanded in terms of the basic symmetric polynomials \( \sigma_1, \sigma_2, \ldots, \sigma_7 \) (defined in Eq. (A.3) of ref. [8]). Written in this way, the final result consists of only 3214 terms and is reduced to a manageable size. ¹

To be sure that we have obtained the correct result for the square \( G(7)(\theta_1, \ldots, \theta_7) \) we have performed a number of checks. First we checked that the leading terms in the last variable (the coefficient of \( \theta_7^{10} \)) are twice the analogous 6–particle square \( G(6)(\theta_1, \ldots, \theta_6) \) [8]. Next we checked that the overall leading terms (the terms of total degree 30) agree with those of a simple symmetric polynomial \( P^{(7)}(t) \) [8]. This is defined by (for \( r \) particles)
\[ P^{(r)}(t) = \sum_{\text{perms}} P_0^{(r)}(t), \]
where
\[ P_0^{(r)}(t) = \prod_{i\neq j>1} [(\theta_i - \theta_j)^2 + t \pi^2]. \]

¹Which is however still too large to usefully reproduce here.
In (18) the sum extends over the $r!$ permutations of the rapidity variables $\theta_1, \ldots, \theta_r$ and in (19) the correction terms depending on the parameter $t$ do not affect the overall leading terms, they are included here for later convenience. Finally, we verified that $G^{(7)}(\theta_1, \ldots, \theta_7)$ vanishes at the points

$$\theta_1 - \theta_2 = \theta_2 - \theta_3 = i\pi, \quad \theta_j = \text{arbitrary} \quad j > 3. \quad (20)$$

The fact that $G^{(7)}$ passed all these nontrivial tests gives us some confidence in the correctness of our results.

4 The spin–spin correlation function

Having calculated the square of the 7–particle form factors the corresponding contributions to the spin–spin correlation function and the spectral density can now be computed straightforwardly. The only difficulty is that to get correct numerical results one has to use high precision arithmetic in order to avoid huge rounding errors in calculating $G^{(7)}$. As explained above, to put it into a manageable form, we expressed it in terms of the basic symmetric polynomials $\sigma_1, \ldots, \sigma_7$. Although it is a sum of absolute squares and thus it is obviously positive, it is not manifestly positive in this form. Actually we experienced large cancellations between positive and negative contributions. After rescaling all rapidities by $\pi$ all coefficients of the polynomial become integers and we can illustrate the large degree of cancellation by considering the following integer rapidities:

$$\theta_1 = 12, \quad \theta_2 = 11, \quad \theta_3 = 10, \quad \theta_4 = 9, \quad \theta_5 = 8, \quad \theta_6 = 7, \quad \theta_7 = 0. \quad (21)$$

We can exactly calculate the sum of positive and negative contributions in this case and we get

- positive part : 26512984286926120356867283491794696305689824
- negative part : 26512984286926120356855853186993819121689824
- total : 11430304800877184000000

showing a 21–digit cancellation here!

The 7–particle contribution to the spin–spin correlation function using the VEGAS integration routine, and a subroutine computing $G^{(7)}$ invoking quartic precision (32 digit) arithmetic. The results, compared with the prediction of two–loop perturbation theory, are shown in Figure 1. Note that here we have no free parameters at our disposal in the perturbative calculation as would be the case in most other models. As discussed above, we know the exact relation between the perturbative $\Lambda$ parameter and the particle mass $M$ and also the absolute normalization of the perturbative curve is available in this model. The form factor results
are in very good agreement with perturbation theory in the expected (high) energy range. The small deviation for energies $p/M \sim 10^4$ can be accounted for by the contribution of $r > 7$ intermediate particles. To illustrate the fact that this good agreement is quite nontrivial, in Figure 1 we also show the perturbative result with the overall factor changed arbitrarily by 5%.

5 Scaling

The 7–particle results also corroborate the scaling hypothesis \[\text{[8]}\] for the spectral densities. To study this aspect we introduce the modified $r$–particle spectral density depending on the logarithmic variable $x$ by the definition

$$
\mu \rho^{(r)}(\mu) = R^{(r)}(x), \quad \mu = M e^x.
$$

\[\text{(22)}\]
These are defined for all \( r \geq 2 \), where the cases with \( r \) even refer to the spectral densities of the two–point function of the isospin Noether current. The graph of this function is a bell–shaped curve starting as zero at \( x = \ln r \), reaching its maximum \( M^{(r)} \) at \( x = \xi^{(r)} \) and then slowly decreasing for larger \( x \). Let us introduce the rescaled spectral density \( Y^{(r)} \) by

\[
Y^{(r)}(z) = \frac{1}{M^{(r)}} R^{(r)}(\xi^{(r)} z).
\]

(23)

Figure 2: Illustration of the self-similarity property of the rescaled spectral densities. The plots show \( Y^{(r)}(z) \) (dashed) compared with \( Y^{(r+1)}(z) \) (solid) for \( r = 3, 4, 5, 6 \).

It has been found (based on the study of up to 6 particles) that the shape of the rescaled spectral density and the parameters \( \xi^{(r)}, M^{(r)} \) satisfy self–similarity

\[
\lim_{r \to \infty} Y^{(r)}(z) \to Y(z)
\]

(24)

with universal shape function \( Y(z) \) and asymptotic scaling,

\[
M^{(r)} \sim M_{\ast} r^{-\gamma}, \quad \xi^{(r)} \sim \xi_{\ast} r^{1+\alpha}
\]

(25)

for large \( r \), with some coefficients \( M_{\ast}, \xi_{\ast} \) and exponents \( \gamma, \alpha \). Whereas the properties of the form factors are consistent with \( \gamma = 1 \) only, the other exponent can only be
determined numerically with result $\alpha = 0.27$. For those readers unfamiliar with this model we point out the amusing fact that the self–similarity holds for all $r \geq 2$ and thus interrelates the spectral functions of two different isovector operators, the spin field and the conserved vector current. Similarly (25) is equally valid for even and odd $r$ values, provided we use the normalization introduced in (1).

Self–similarity continues to be satisfied very well as demonstrated in Figure 2. To test asymptotic scaling, we used the fitted numerical values based on results for up to 6 particles to “predict” for the $r = 7$ case

$$M_{sc}^{(7)} = 0.03188 \quad \text{and} \quad \xi_{sc}^{(7)} = 17.73,$$

and compared it to the values directly determined from our 7–particle results

$$M^{(7)} = 0.03189 \quad \text{and} \quad \xi^{(7)} = 17.77.$$  \hspace{1cm} (27)

Similarly for the integrals

$$c^{(r)} = \int_M^\infty d\mu \, \rho^{(r)}(\mu) = \int_0^\infty dx \, R^{(r)}(x) \quad \text{and} \quad h^{(r)} = \int_0^\infty \frac{dx}{x} R^{(r)}(x)$$

we predict

$$c_{sc}^{(7)} = 1.464 \quad \text{and} \quad h_{sc}^{(7)} = 0.04879,$$

and actually get

$$c^{(7)} = 1.46(1) \quad \text{and} \quad h^{(7)} = 0.0488(1).$$  \hspace{1cm} (30)

### 6 Ultra–positivity

Because of the large cancellation between positive and negative terms in the representation of $G^{(r)}$ in terms of symmetric polynomials, it is natural to ask if there exists an alternative representation that is manifestly positive. It is indeed possible to find such representations. If we arrange the rapidities in decreasing order (which is always possible since the polynomial is symmetric under permutations) then all $u_j = \theta_j - \theta_{j+1}$ rapidity differences are positive and we found (for all available particle numbers $3 \leq r \leq 7$) that if we expand $G^{(r)}$ in terms of these differences then all coefficients are positive. Moreover, it is possible to find upper and lower bounds both of which are of the simple form \[18\] such that, expanded in terms of the rapidity differences $u_j$, each and every term in the expansion of $G^{(r)}$ has smaller coefficient than the corresponding one of the upper bound $P^{(r)}(t^{(r)}_u)$ and similarly larger than the coefficient of the corresponding term of the lower bound $P^{(r)}(t^{(r)}_l)$.\[18\]
Table 1: Numerical values of the integrals \( c^{(r)} \), \( c^{(r)}(2) \). Numerical errors are estimated as ±1 on the last digit quoted.

| \( r \) | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|---------|-----|-----|-----|-----|-----|-----|-----|
| \( c^{(r)} \) | 0.785 | 1.009 | 1.140 | 1.242 | 1.327 | 1.400 | 1.46 |
| \( c^{(r)}(2) \) | 1.140 | 1.206 | 1.229 | 1.225 | 1.200 |       |     |

| \( r \) | 8   | 9   | 10  | 11  |
|---------|-----|-----|-----|-----|
| \( c^{(r)}(2) \) | 1.164 | 1.118 | 1.067 | 1.013 |

The value of the parameter characterizing the lower bound is uniformly \( t_{l}^{(r)} = 2 \) for all cases \( 3 \leq r \leq 7 \), whereas the \( t_{u}^{(r)} \) giving the lowest upper bound are

\[
t_{u}^{(3)} = 2, \quad t_{u}^{(4)} = 9/4, \quad t_{u}^{(5)} = 2.434, \quad t_{u}^{(6)} = 2.576, \quad t_{u}^{(7)} = 2.688.
\]  

(31)

The last three values are numerically approximate \(^2\) and come from the requirement that the constant term of \( P^{(r)}(t_{u}^{(r)}) \) be larger than that of \( G^{(r)} \). This is clearly necessary, but at least in the available cases for \( r \geq 5 \) it is also sufficient \(^3\).

The integrals over the spin and current spectral densities are given as sums of the integrals defined in (28):

\[
C^{\text{spin}} = \int_{M}^{\infty} d\mu \rho^{\text{spin}}(\mu) = \frac{4}{\pi} \sum_{k=0}^{\infty} c^{(2k+1)},
\]

\[
C^{\text{curr}} = \int_{M}^{\infty} d\mu \rho^{\text{curr}}(\mu) = \sum_{k=0}^{\infty} c^{(2k+2)}.
\]

(32)

An outstanding question is whether these are finite or infinite in the FF construction. Certainly renormalized perturbation theory predicts that \( C^{\text{spin}} = \infty = C^{\text{curr}} \). Also the validity of the scaling hypothesis requires this because in this scenario \( c^{(r)} \) grows as \( \sim r^\alpha \); the numerical evidence thereof is shown in Figure 3.

If the bounds we found above for \( 3 \leq r \leq 7 \) continue to be true for all \( r \), then the simple structure of (18) allows to study the structure of the correlation function analytically. The existence of an upper limit of simple form may help proving the existence of the correlation function whereas the existence of the lower bound may facilitate the construction of a proof that \( C^{\text{spin}} = \infty = C^{\text{curr}} \) independent of the validity of the scaling hypothesis.

\(^2\)e.g. the exact value for the case \( r = 5 \) is \( t_{u}^{(5)} = 208^{1/6} \)

\(^3\)The case \( r = 4 \) is an exception since the requirement in this case yields a value \( (34/3)^{1/3} = 2.246 \), which is not quite sufficient.
We are not going to discuss these questions further in this paper, but to illustrate the usefulness of the existence of the simple lower bound we have calculated the integrals $c^{(r)}(2)$. These are defined analogously to the ones in (28) using $P^{(r)}(2)$ instead of $G^{(r)}$. The simplicity of the integrand allows us to calculate these integrals numerically quite effortlessly up to $r = 11$. The results of the numerical integration are given in Table 1.

Although the integrals $c^{(r)}(2)$ are (apparently) decreasing with $r$, it is possible that they are decreasing slow enough to make the series (32) diverge. Indeed, as seen in Figure 3, $rc^{(r)}(2)$ is monotonically increasing for all $r$ evaluated up to now.

Acknowledgements
J. B. wishes to thank the Max–Planck–Institut für Physik in Munich where most of this work has been done, for hospitality. This investigation was supported in part by the Hungarian National Science Fund OTKA (under T030099 and T034299).

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4This is because one can represent $c^{(r)}(2)$ as an integral with the integrand just involving the polynomial $P^{(r)}_0(2)$ in Eq. (19) (instead all its permutations), at the cost of extending the limits of integration over the $u_i$ from $-\infty$ to $+\infty$. 

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Figure 3: Values of $\ln c(r)$ (circles) and of the product $rc(r)(2)/(4\pi)$ (squares). The error on $\ln c(r)$ is approximately the radius of the circle.

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