Flow rate measurements of binary gas mixtures through long trapezoidal microchannels

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Abstract. The flow rate of two noble gas mixtures, namely He/Ar and He/Kr, is measured through a microsystem containing 400 long trapezoidal microchannels placed in parallel configuration. Each microchannel has a trapezoidal cross section with long base 5.38 micrometers and height 1.90 micrometers, while its length is 5000 micrometers. The experiment is based on the constant volume method. The flow is driven by pressure gradient. The flow rate measurements refer to downstream pressures of 15.1 kPa and 8.05 kPa. The pressure ratio is in the range of 3-7 and 4-7 for the larger and smaller downstream pressures, respectively. The investigated rarefaction range is in the slip and early transition regions. The concentration of He varies from zero to one. The measured flow rates are compared to the corresponding computational ones obtained by the numerical solution of the McCormack kinetic model. Very good agreement between the experimental and computational results is reached. The difference between the corresponding results is less than the experimental uncertainty. Typical pressure and concentration profiles along the axis and the velocity profiles in the center of the channel obtained from the numerical solution are also presented.

1. Introduction

Over the last years, gaseous micro-flows have captured an increasing attention in the scientific community. This interest is well-justified by the appearance of new technologies at the micro-scale, such as micro-electromechanical systems or vacuum technology. The description of the gaseous flow in small scale devices requires the consideration of the molecular distribution function and the Boltzmann or other kinetic equations. By comparing the predictions of different theoretical models with experiments, the validity of these models can be evaluated.

Previous experimental works on gaseous micro-flows have mainly focused on the measurement of the flow rate of single gases through various micro-channels [1, 4, 5, 7, 22]. For micro-flows, the constant volume or the bubble tracking methods may be used for these purposes. Slip models have been developed or tested for gaseous micro-flows [2, 3, 10]. The slip coefficients have also been determined in several studies. The properties of single gases are generally well-known; however, there are only a few works on gaseous mixtures. Recently, the flow rate of gas mixtures through rectangular micro-channels has been measured, and the results have been compared to the solution of the McCormack kinetic model [13, 17, 18].

Much work has been devoted to the numerical solution of linearized kinetic models for single gas flows through various channels with rectangular [14], elliptical [6], triangular [11, 20] or
Figure 1. Geometry of the trapezoidal channel.

trapezoidal [21] cross sections. The solution of these models has been carried out by the discrete velocity method. The McCormack kinetic model has been used to model the flow of gaseous mixtures in channels [12, 15, 19]. The McCormack kinetic model has the advantageous property that the hydrodynamic behavior of the mixture can be properly captured. In Ref. [19], a flow rate formula has been derived for deducing the flow rate in long channels. It has been used for comparison between computational and experimental flow rates through rectangular channels [17, 18].

The flow of gas mixtures is affected by diffusion effects. At finite rarefactions, the different components of the gas tend to travel with different velocities in the channel due to their different molecular speeds. Hence, the species tend to separate. The separation phenomenon caused by the gas diffusion has been investigated in Refs. [8, 19, 18, 16].

In the present work, the flow rate of the noble gas mixtures He/Ar and He/Kr is measured through long trapezoidal micro-channels and compared to the numerical solution of the McCormack model. The flow in micro-channels with this cross section has not been analyzed earlier for gaseous mixtures. The measurements are carried out for a wide range of concentrations for the He/Ar mixture, while the concentration of 50% is used for the He/Kr mixture. The downstream pressure is 15.1kPa and 8.05kPa. In addition to the comparative study in terms of the flow rates, the representative axial distributions of the pressure and the concentration and the typical velocity profiles in the center of the channel are analyzed.

2. Definition of the problem

The isothermal fully-developed flow of binary gaseous mixtures through long trapezoidal channels is considered. The channel lies along the $x$ coordinate, whereas the cross section is located in the $(y, z)$ coordinate sheet. The longer base and the height of the cross section are along the $y$ and $z$ coordinates. The acute angle of the cross section is $\omega = 54.74^\circ$, while the height of the channel is $h = 1.90 \pm 0.013\mu m$, its longer base is $B = 5.38 \pm 0.019\mu m$ and its length is $L = 5000\mu m$. The geometry of the channel is presented in Figure 1. The origin of the Descartes coordinate system is in $(x, y, z) = (0, 0, 0)$.

The mixture has two components $\alpha = 1, 2$. The molar densities and masses of the species are denoted by $n_\alpha$ and $m_\alpha$. The molar masses of the specific gases are given by $m_{He} = 4.003g/mol$, $m_{Ar} = 39.95g/mol$ and $m_{Kr} = 83.80g/mol$. The concentration of the first component is defined by $C = n_1/(n_1 + n_2)$. The flow is driven by the pressure difference between the inlet and the outlet of the channel. The downstream and upstream pressures and concentrations are denoted by $P_A$, $P_B$ and $C_A$, $C_B$. In the present work, $C_A = C_B$. The temperature of the gas $T$ is
constant. The flow is characterized by the rarefaction parameter
\[
\delta(x) = \frac{P(x)D_h}{\mu(x)v_0(x)},
\]
where \(P\) is the pressure, \(\mu\) is the viscosity, \(D_h\) is the hydraulic diameter of the channel and \(v = (2R_gT/m)^{1/2}\) is the characteristic molecular speed. In addition, \(R_g\) is the global gas constant and \(m = Cm_1 + (1 - C)m_2\) is the average molar mass. The average Knudsen number in the channel is defined as the inverse of the average rarefaction parameter \(Kn_0 = 2/(\delta_A + \delta_B)\), where \(\delta_{A,B}\) is the inlet or outlet rarefaction parameter.

In this study, one of the important quantities is the molar flow rate of the components
\[
J_\alpha = \int_A n_\alpha u_{\alpha x} dA,
\]
where \(u_{\alpha x}\) is the axial velocity of the components and \(A\) is the area of the cross section. The total molar flow rate is given by \(J = J_1 + J_2\).

3. Experimental approach

The measurement of the flow rates is based on the constant volume method. The experimental setup, which was described in detail in Ref. [13], is located in a thermally insulated plastic chamber, Figure 2. The circuit consists of upstream A and downstream B reservoirs and the microsystem \(\mu S\), which is a collection of microchannels etched into a silicon plate, Figure 3. The inlet A and the outlet B of the microchannels are connected to reservoirs A and B via valves VA2 and VB2. The reservoirs can be vacuumed out by the vacuum pump VP or filled up with the gas mixture from the tank G. The pressure in the reservoirs is measured by two Inficon capacitance gauges (CDG025D). The capacitance gauges are denoted as \(P_A\) and \(P_B\) in Figure 2. The accuracy of the capacitance gauges is ±0.2% of reading. The temperature is regulated by Peltier modules and accurately measured by three PT100 temperature sensors. The accuracy of the temperature sensors is 0.15\(K\). Two sensors are attached to the inlet and outlet reservoirs as denoted by \(T_1\) and \(T_2\) in Figure 2, while the third sensor \(T_3\) is connected to the microsystem, Figure 3. These sensors provide the same results within their accuracy. The third sensor is used to define the temperature in the system.

The measurement is carried out in a three-step procedure. First, the whole circuit is filled up with the gas at the downstream pressure. Then, valve VA2 is closed, and the possible leakage or outgassing flow rate \(J_{Bo}\) is measured in circuit B. Secondly, reservoir A is filled up with the mixture at the upstream pressure. After thermal regulation is reached, VA2 is opened,
and the gas flows through the microsystem. In this step, the inlet and outlet flow rates \( J_A \) and \( J_B \) are measured on the basis of the pressure variations in the two containers. Finally, the whole circuit is filled up with the mixture at the actual upstream pressure. Valve VB2 is closed, and the possible leakage or outgassing flow rate \( J_{Ao} \) is measured in circuit A. The inlet and outlet experimental flow rates, \( J_A^e \) and \( J_B^e \), are deduced by correcting \( J_A \) and \( J_B \), with \( J_{Ao} \) and \( J_{Bo} \). The measured flow rate is defined as the average \( J^e = (J_A^e + J_B^e)/2 \). The duration of the outgassing/leakage measurement is between 1.5 and 2 hours, while the typical time of the flow rate measurement is around 40 minutes. The allowed pressure variation in the flow rate measurement is less than 2% in the downstream reservoir. The temperature of the gas is \( T = 299.5K \) and the volumes of the upstream and downstream containers are \( V_A = 1.708e - 4m^3 \) and \( V_B = 1.658e - 4m^3 \). In the experiments, the pressure variation in the reservoirs is less than 2%, which indicates that the amount of the gas flowing through the microsystem is less than 0.02\( \times P_BV_B/(R_gT) = [2e - 4mol, 1.1e - 4mol] \) for \( P_B = 15.1kPa \) and \( P_B = 8.05kPa \), respectively.

The flow rates are determined from the pressure variations in the reservoirs in accordance with the equation of state

\[
J_{A,B} = \mp \frac{dN_{A,B}}{dt} = \mp \frac{V_{A,B}}{R_g} \frac{dP_{A,B}}{dT_{A,B}} ,
\]

where \( N_{A,B} \) is the molar amount of the gas in the upstream and downstream containers. The above equation can be written by

\[
J_{A,B} = \mp \frac{dN_{A,B}}{dt} = \mp \frac{V_{A,B} dP_{A,B}}{R_g T_{A,B}} \left( 1 - \frac{dT_{A,B}/T_{A,B}}{dP_{A,B}/P_{A,B}} \right). \tag{4}
\]

In the experiments, the relative temperature and pressure variations are in the order of \( dT_{A,B}/T_{A,B} = 4\times10^{-4} \) and \( dP_{A,B}/P_{A,B} = 2\times10^{-2} \). As a result, the flow rate can be determined as

\[
J_{A,B} = \mp \frac{V_{A,B}}{R_g T_{A,B}} a_{A,B} c_{A,B}, \tag{5}
\]

where the time derivative of the pressure variation is denoted by \( a_{A,B} \) with \( P_{A,B}(t) = a_{A,B}t + b_{A,B} \), and \( c_{A,B} = (1 - dT_{A,B}/T_{A,B})/(dP_{A,B}/P_{A,B}) = 1 \pm 0.02 \). Eq. (5) is used to calculate the flow rates, \( J_{A,B} \), after the flow rate measurement. The time-averaged value of the temperatures \( T_{A,B} \) is taken from sensor T3, the parameter \( a_{A,B} \) is determined from a least square fit of the pressure profiles and \( c_{A,B} \) is unity. The uncertainty of the measurement can be estimated as follows. The relative standard deviation of \( a_{A,B} \) is less than \( \pm0.5\% \). This value is determined in the way as given in Ref. [13]. The standard deviation defines confidence interval with 68.2% acceptance. The relative standard deviation is used to estimate the uncertainty of \( a_{A,B} \). The total uncertainty of the experiment consists of the uncertainty of the volume \( V_{A,B} \), the temperature \( T_{A,B} \) and the coefficients \( a_{A,B} \) and \( c_{A,B} \). The volume was determined by using the method given in Ref. [13], and its uncertainty was estimated as \( \pm1.3\% \). The overall uncertainty of the temperature is estimated by the deviation of the temperature inside the chamber in a test measurement lasting six hours. The deviation of the temperature is less than \( \pm0.2\% \). The uncertainty of \( c_{A,B} = (1 - dT_{A,B}/T_{A,B})/(dP_{A,B}/P_{A,B}) \) is in the order of \( \pm2\% \). Combining these factors, the total experimental uncertainty is estimated as

\[
\frac{\Delta J_{A,B}}{J_{A,B}} = \frac{\Delta V_{A,B}}{V_{A,B}} + \frac{\Delta T_{A,B}}{T_{A,B}} + \frac{\Delta a_{A,B}}{a_{A,B}} + \frac{\Delta c_{A,B}}{c_{A,B}} = \pm (1.3\% + 0.2\% + 0.5\% + 2\%) = \pm4\%. \tag{6}
\]
4. Computational approach

The McCormack linearized kinetic model is utilized to describe the flow. Since the channel is long, the speed of the gas is small compared to the characteristic molecular speed and the linearized description can be used. The end effects can also be neglected. The calculation is divided into two steps. First, the local flow problem at a particular cross section is solved. Secondly, the global behavior of the flow including the flow rates and the distributions of the pressure and the concentration is deduced.

The flow is described by the $J_P$, $J_C$ thermodynamic fluxes \cite{18, 19} defined by

\[ J_P = -\int_A (n_1 u_{1x} + n_2 u_{2x})dA, \quad J_C = -n_1 \int_A (u_{1x} - u_{2x})dA. \]  

These fluxes are the linear functions of the local pressure and concentration gradients

\[ X_P = \frac{\partial P}{\partial x} D_h \frac{P}{P'}, \quad X_C = \frac{\partial C}{\partial x} D_h \frac{C}{C'}, \]  

such that

\[ J_P = \frac{nv_0 A}{2} (\Lambda_{PP} X_P + \Lambda_{PC} X_C), \]  

\[ J_C = \frac{nv_0 A}{2} (\Lambda_{CP} X_P + \Lambda_{CC} X_C), \]  

where $n = n_1 + n_2$ and $\Lambda_{PP}$, $\Lambda_{PC}$, $\Lambda_{CP}$, $\Lambda_{CC}$ are the so-called kinetic coefficients. These coefficients are obtained by the numerical solution of the McCormack kinetic model, which is achieved by an accelerated discrete velocity method \cite{19, 20}. The realistic potential of Kestin et al. is used in the McCormack collision frequencies \cite{9}. The coefficients are the functions of the local rarefaction parameter and the concentration. By solving the McCormack model in a wide range of the rarefaction and the whole range of the concentration a database is created for the kinetic coefficients. The discrete velocity algorithm has been used with the computational grid having a number of nodes $M \times N = 16 \times 300$ for $\delta < 1$ and $M \times N = 16 \times 72$ for $\delta \geq 1$ in the velocity space, where $M$ and $N$ denote the number of magnitudes and polar angles. In the spatial space, the number of nodes along the large base is 1500 for all $\delta$. The process of iteration for the calculation of the kinetic coefficients is terminated when the relative convergence error is less than $1e^{-6}$.

The flow rates of the components can be expressed by the thermodynamic fluxes according to

\[ J_1 = -C J_P - (1 - C) J_C, \quad J_2 = -(1 - C) (J_P - J_C). \]  

The flow rates of the species are constant along the axis of the channel due to the conservation of mass. By using the ideal gas law, the definition of the rarefaction parameter and Eqs. (7)-(10), Eq. (11) can be written by

\[ J_1 = -\frac{PA D_h}{nv_0 L} \left[ (C \Lambda_{PP} + (1 - C) \Lambda_{CP}) \frac{\partial P}{\partial x'} \frac{1}{P'} + (C \Lambda_{PC} + (1 - C) \Lambda_{CC}) \frac{\partial C}{\partial x'} \frac{1}{C'} \right], \]  

\[ J_2 = -\frac{PA D_h}{nv_0 L} (1 - C) \left[ (\Lambda_{PP} - \Lambda_{CP}) \frac{\partial P}{\partial x'} \frac{1}{P'} + (\Lambda_{PC} - \Lambda_{CC}) \frac{\partial C}{\partial x'} \frac{1}{C'} \right], \]  

where $x' = x/L$ is the dimensionless coordinate along the axis of the channel. These equations are supplemented with the boundary conditions for the pressure and the concentration at the inlet and the outlet of the channel such that

\[ P(0) = P_A, \quad P(1) = P_B, \quad C(0) = C_A, \quad C(1) = C_B. \]
This boundary condition is justified by the facts that relatively small pressure variation is allowed in the two reservoirs during the measurement and the concentration variation due to the gaseous separation is also negligible. Eqs. (12)-(13) constitute a system of two ordinary differential equations for the primitive functions $P(x')$ and $C(x')$. For deducing the kinetic coefficients, the rarefaction parameter is determined from the local value of the pressure and the concentration and the viscosity function of the gaseous mixtures, see Eq. (1). The actual value of the kinetic coefficients are deduced from the pre-calculated database mentioned above. Eqs. (12)-(13) together with Eq. (14) are numerically solved by discretizing the spatial coordinate and using the Euler method to compute the spatial derivatives. The solution yields the flow rates of the components $J_1$, $J_2$ and the distributions of the pressure $P(x')$ and the concentration $C(x')$.

5. Results and discussion

The flow rate of He/Ar and He/Kr gas mixtures are measured and compared to the kinetic calculation at downstream pressures 15.1kPa and 8.05kPa. The microsystem consists of 400 parallel channels. The channels are etched in a circular silicon plate and covered with glass plates via anodic bonding. The parallel channels are connected to the inlet A and the outlet B of the microsystem, Figure 3. The flow rate through an individual channel is considered in the following.

Table 1 shows the flow rates for downstream pressure 15.1kPa. For the He/Ar mixture, the concentrations $C_A = [0.0, 0.1017, 0.3012, 0.5010, 0.7019, 0.9014, 1.0]$, which include the pure single gas cases, is investigated, while the concentration $C_A = 0.5010$ is studied for the He/Kr mixture. The flow rates are measured in the range of the pressure ratio $3 - 7$. In the table, the columns from left to right in order show the concentration of the gas, the upstream pressure, the pressure ratio, the average Knudsen number, the experimental flow rate, the difference between the inlet and outlet experimental flow rates, the kinetic flow rates of the species $J_1$, $J_2$, the total kinetic flow rate $J$ and the discrepancy between the experimental and the total kinetic flow rates, defined by $\Delta = 100(1 - J^e/J)$, respectively. The experimental flow rate $J^e$ is determined as described in section 3. The difference between the inlet and outlet experimental flow rates are defined by $\Delta_e = 100(1 - J_A^e/J_B^e)$. The $J_A^e$ and $J_B^e$ experimental flow rates are determined as defined by Eq. (5) in section 3. The kinetic flow rates $J_1$, $J_2$, $J$ are obtained by the solution of Eqs. (12)-(14) in section 4. The inlet and outlet values of the pressures $P_A$, $P_B$ measured in the experiment and the constant concentrations $C_A = C_B$ in Table 1 are used as input parameters for the calculation. In the table, it can be seen that the flow rates increase with increasing pressure drop. In addition, they also increase with decreasing concentration. This latter behavior is caused by the smaller mass of the first species, which results into a larger molecular speed. This correlation remains valid in the whole range of the gas rarefaction. This has been confirmed by us both numerically and experimentally. The flow rates for the He/Kr mixture are smaller than the corresponding ones for the He/Ar mixture at the same concentration, which is caused by the larger mass of the Kr than that of the Ar. The difference between the inlet and outlet experimental flow rates is always less than the experimental uncertainty, which justifies that possible leakage or outgassing effects are well taken into account. It is found that the discrepancy between the experiment and kinetic flow rates are always less than the experimental uncertainty; hence, a very good agreement is reached between the two different approaches.

Table 2 shows the corresponding results at downstream pressure 8.05kPa. For the He/Ar mixture, the same concentrations excluding the single gas cases are examined as for the larger downstream pressure. For the He/Kr mixture, the flow with the same concentration is studied as in Table 1. The pressure ratio is in the range of $4 - 7$. The columns of Table 2 presents the data in the same scenario as in Table 1. The flow rates are deduced in a similar manner
Table 1. Flow rates through an individual microchannel for He/Ar and He/Kr mixtures at downstream pressure $P_B = 15.1 kPa$.

| $C_A$ | $P_A (Pa)$ | $P_A/P_B$ | $Kn_0$ | $J^e (mol/s)$ | $\Delta e$ | $J_1 (mol/s)$ | $J_2 (mol/s)$ | $J (mol/s)$ | $\Delta$ |
|-------|------------|-----------|--------|---------------|-----------|--------------|--------------|-------------|--------|
| He/Ar |
| 0.0000 | 45355 | 3.00 | 0.11 | 1.02e-11 | 1.22 | 0 | 1.01e-11 | 1.01e-11 | -0.35 |
| 59422 | 3.94 | 0.09 | 1.66e-11 | -1.41 | 0 | 1.64e-11 | 1.64e-11 | -1.35 |
| 74977 | 4.97 | 0.07 | 2.49e-11 | -2.45 | 0 | 2.45e-11 | 2.45e-11 | -1.75 |
| 90110 | 5.97 | 0.06 | 3.39e-11 | -2.63 | 0 | 3.36e-11 | 3.36e-11 | -0.90 |
| 105010 | 6.95 | 0.05 | 4.41e-11 | 1.66 | 0 | 4.38e-11 | 4.38e-11 | -0.67 |
| 0.1017 | 45328 | 3.00 | 0.12 | 1.04e-11 | 1.30 | 1.30e-12 | 9.30e-12 | 1.06e-11 | 2.37 |
| 59944 | 3.97 | 0.09 | 1.76e-11 | 2.00e-12 | 1.54e-11 | 1.74e-11 | -1.58 |
| 74994 | 4.97 | 0.08 | 2.56e-11 | 2.59e-11 | 2.26e-11 | 2.54e-11 | -0.73 |
| 90439 | 5.99 | 0.07 | 3.51e-11 | 3.51e-11 | 3.11e-11 | 3.48e-11 | -0.93 |
| 105938 | 7.02 | 0.06 | 4.69e-11 | -1.50 | 4.80e-12 | 4.05e-11 | 4.53e-11 | -3.47 |
| 0.3012 | 45340 | 3.00 | 0.13 | 1.15e-11 | 4.08e-12 | 7.82e-12 | 1.19e-11 | 2.88 |
| 59885 | 3.97 | 0.11 | 1.86e-11 | 6.28e-12 | 1.27e-11 | 1.90e-11 | 2.05 |
| 74925 | 4.96 | 0.09 | 2.81e-11 | 8.83e-12 | 1.87e-11 | 2.75e-11 | -2.41 |
| 90251 | 5.98 | 0.08 | 3.79e-11 | 3.74e-12 | 3.73e-11 | 3.14e-11 | -1.64 |
| 105761 | 7.02 | 0.07 | 4.91e-11 | 4.91e-11 | 4.85e-11 | 4.85e-11 | -1.27 |
| 0.5010 | 45200 | 2.99 | 0.15 | 1.40e-11 | 7.61e-12 | 5.99e-12 | 1.36e-11 | -2.49 |
| 59840 | 3.96 | 0.12 | 2.11e-11 | 9.88e-12 | 2.15e-11 | 1.89 |
| 74731 | 4.95 | 0.10 | 3.10e-11 | 8.83e-12 | 3.06e-11 | 3.06e-11 | -1.12 |
| 90368 | 5.98 | 0.09 | 4.09e-11 | 1.17e-11 | 2.56e-11 | 3.73e-11 | -1.64 |
| 105126 | 7.00 | 0.08 | 5.21e-11 | 2.72e-11 | 5.26e-11 | 4.85e-11 | 0.78 |
| 0.7019 | 45082 | 2.99 | 0.19 | 1.55e-11 | 1.21e-11 | 4.03e-12 | 1.61e-11 | 3.35 |
| 60203 | 3.96 | 0.15 | 2.51e-11 | 1.16e-11 | 9.88e-12 | 2.15e-11 | 1.89 |
| 75192 | 4.98 | 0.12 | 3.52e-11 | 1.62e-11 | 3.06e-11 | 3.06e-11 | -1.12 |
| 90210 | 5.98 | 0.11 | 4.69e-11 | 3.41e-11 | 3.14e-11 | 4.72e-11 | 0.72 |
| 105449 | 6.98 | 0.09 | 5.99e-11 | 4.29e-11 | 5.99e-11 | 4.85e-11 | 0.78 |
| 0.9014 | 45255 | 3.00 | 0.24 | 1.95e-11 | 1.85e-11 | 2.15e-11 | 2.15e-11 | 3.57e-11 | 1.41 |
| 59883 | 3.97 | 0.19 | 3.02e-11 | 2.86e-11 | 2.15e-11 | 2.15e-11 | 3.57e-11 | 1.41 |
| 74922 | 4.96 | 0.16 | 4.30e-11 | 3.98e-11 | 3.81e-11 | 4.36e-11 | 1.37 |
| 90389 | 5.98 | 0.14 | 5.64e-11 | 5.24e-11 | 5.14e-11 | 5.75e-11 | 1.95 |
| 105850 | 7.00 | 0.12 | 7.17e-11 | 6.61e-11 | 6.61e-11 | 7.28e-11 | 1.50 |
| 1.0000 | 45482 | 3.01 | 0.30 | 2.29e-11 | 2.32e-11 | 2.32e-11 | 2.32e-11 | 1.35 |
| 60343 | 4.00 | 0.24 | 3.57e-11 | 3.63e-11 | 3.63e-11 | 3.63e-11 | 1.70 |
| 75293 | 4.99 | 0.20 | 4.94e-11 | 5.07e-11 | 5.07e-11 | 5.07e-11 | 2.66 |
| 89976 | 5.96 | 0.17 | 6.54e-11 | 6.61e-11 | 6.61e-11 | 6.61e-11 | 1.05 |
| 105450 | 6.98 | 0.15 | 8.23e-11 | 8.37e-11 | 8.37e-11 | 8.37e-11 | 1.77 |

He/Kr

| 0.5010 | 45275 | 3.00 | 0.12 | 1.11e-11 | -3.32 | 6.47e-12 | 4.83e-12 | 1.13e-11 | 1.87 |
| 59875 | 3.97 | 0.10 | 1.80e-11 | -0.89 | 9.83e-12 | 7.97e-12 | 1.78e-11 | -1.24 |
| 74947 | 4.96 | 0.08 | 2.53e-11 | -2.84 | 1.36e-11 | 1.18e-11 | 2.54e-11 | 0.18 |
| 90320 | 5.98 | 0.07 | 3.50e-11 | -3.29 | 1.80e-11 | 1.61e-11 | 3.41e-11 | -2.62 |
| 105832 | 7.01 | 0.06 | 4.43e-11 | 2.29e-11 | 2.11e-11 | 4.40e-11 | 0.56 |
Table 2. Flow rates through an individual microchannel for He/Ar and He/Kr mixtures at downstream pressure $P_B = 8.05\, kPa$.

| $C_A$ | $P_A(Pa)$ | $P_A/P_B$ | $Kn_0$ | $J^*(mol/s)$ | $\Delta_e$ | $J_1(mol/s)$ | $J_2(mol/s)$ | $J(mol/s)$ | $\Delta$ |
|-------|-----------|-----------|---------|-------------|-------------|--------------|--------------|------------|---------|
| **He/Ar** | | | | | | | | | |
| 0.1017 | 31974 | 3.97 | 0.17 | 7.50e-12 | -3.50 | 9.92e-13 | 6.30e-12 | 7.29e-12 | -2.88 |
| 40237 | 5.00 | 0.14 | 1.03e-11 | -1.97 | 1.32e-12 | 9.08e-12 | 1.04e-11 | 1.32 |
| 47965 | 5.96 | 0.12 | 1.33e-11 | -0.77 | 1.62e-12 | 1.18e-11 | 1.34e-11 | 0.74 |
| 56065 | 6.96 | 0.11 | 1.68e-11 | 0.46 | 1.95e-12 | 1.50e-11 | 1.70e-11 | 1.03 |
| 0.3012 | 32038 | 3.98 | 0.20 | 8.58e-12 | 1.48 | 3.12e-12 | 5.31e-12 | 8.43e-12 | -1.82 |
| 40155 | 4.99 | 0.16 | 1.16e-11 | -1.11 | 4.15e-12 | 7.65e-12 | 1.18e-11 | 1.14 |
| 47940 | 5.96 | 0.14 | 1.48e-11 | 1.22 | 5.20e-12 | 1.00e-11 | 1.52e-11 | 2.80 |
| 56086 | 6.97 | 0.12 | 1.92e-11 | -0.66 | 6.39e-12 | 1.27e-11 | 1.91e-11 | -0.56 |
| 0.5010 | 32063 | 3.98 | 0.23 | 9.77e-12 | -1.02 | 5.88e-12 | 1.00e-11 | 2.50 |
| 40143 | 4.99 | 0.19 | 1.35e-11 | 2.62 | 7.85e-12 | 5.95e-12 | 1.38e-11 | 2.14 |
| 47955 | 5.96 | 0.16 | 1.72e-11 | -0.07 | 9.79e-12 | 7.91e-12 | 1.77e-11 | 2.53 |
| 55789 | 6.93 | 0.14 | 2.23e-11 | -1.54 | 1.19e-11 | 9.92e-12 | 2.18e-11 | -1.96 |
| 0.7019 | 31879 | 3.96 | 0.28 | 1.18e-11 | -2.76 | 9.24e-12 | 2.76e-12 | 1.20e-11 | 1.24 |
| 40248 | 5.00 | 0.23 | 1.61e-11 | -0.54 | 1.26e-11 | 4.05e-12 | 1.66e-11 | 2.57 |
| 47842 | 5.94 | 0.20 | 2.05e-11 | -2.04 | 1.57e-11 | 5.32e-12 | 2.10e-11 | 2.03 |
| 55793 | 6.93 | 0.17 | 2.55e-11 | -3.54 | 1.91e-11 | 6.74e-12 | 2.58e-11 | 1.10 |
| 0.9014 | 31945 | 3.97 | 0.36 | 1.48e-11 | 0.26 | 1.40e-11 | 1.02e-12 | 1.50e-11 | 1.26 |
| 40188 | 4.99 | 0.30 | 1.99e-11 | -2.62 | 1.91e-11 | 1.54e-12 | 2.06e-11 | 3.35 |
| 47947 | 5.96 | 0.26 | 2.60e-11 | -0.29 | 2.41e-11 | 2.09e-12 | 2.62e-11 | 0.62 |
| 55821 | 6.93 | 0.23 | 3.18e-11 | 0.91 | 2.95e-11 | 2.61e-12 | 3.21e-11 | 0.85 |
| **He/Kr** | | | | | | | | | |
| 0.5010 | 31971 | 3.97 | 0.19 | 7.99e-12 | 0.64 | 5.10e-12 | 3.20e-12 | 8.30e-12 | 3.70 |
| 40264 | 5.00 | 0.15 | 1.11e-11 | -2.13 | 6.70e-12 | 4.70e-12 | 1.14e-11 | 2.34 |
| 47974 | 5.96 | 0.13 | 1.43e-11 | -3.37 | 8.31e-12 | 6.29e-12 | 1.46e-11 | 2.03 |
| 55979 | 6.95 | 0.12 | 1.76e-11 | -2.14 | 9.98e-12 | 7.52e-12 | 1.75e-11 | -0.21 |

As described in the previous paragraph. It can be seen that the flow rates are always smaller than the corresponding values at the larger downstream pressure because of the smaller value of the density at smaller pressure. In terms of the pressure ratio and the concentration, a similar observation can be made as for Table 1. The difference between the inlet and outlet flow rates is also always smaller than the experimental uncertainty, and the discrepancy between the experimental and kinetic results are also smaller than the experimental uncertainty. A very good agreement is reached between the two approaches again. The agreement between experimental and computational flow rates is similar to the one observed in [17] and much better than the one given in [18]. This improvement has been caused by the improved experimental device, where the plastic connections have been replaced by metal ones, which provides better tightness. The good agreement in both situations justifies that the McCormack model is a viable tool for the description of the present flow problem.

In Table 3, a comparison of single gas, approximated and total kinetic flow rates, $J_s$, $J^*$ and $J$, are shown for $P_B = 8.05\, kPa$ and $C_A = 0.5010$. The single gas results are deduced by assuming a single gas with the average mass and the proper viscosity of the mixture. For the He/Ar and He/Kr mixtures, the average masses and the viscosities are given by $m = [21.9g/mol, 43.8g/mol]$
Table 3. Comparison of single gas, approximated and exact flow rates for the two mixtures at downstream pressure $P_B = 8.05kPa$ and $C_A = 0.5010$.

| $P_A/P_B$ | $J_s (mol/s)$ | $J^* (mol/s)$ | $J (mol/s)$ | $\Delta_s$ | $\Delta^*$ |
|-----------|---------------|---------------|-------------|-------------|-----------|
| He/Ar     |               |               |             |             |           |
| 3.98      | 8.33e-12      | 1.02e-11      | 1.00e-11    | 16.7        | -1.9      |
| 4.99      | 1.17e-11      | 1.41e-11      | 1.38e-11    | 15.0        | -2.0      |
| 5.96      | 1.53e-11      | 1.81e-11      | 1.77e-11    | 13.6        | -2.1      |
| 6.93      | 1.92e-11      | 2.24e-11      | 2.18e-11    | 12.0        | -2.6      |
| He/Kr     |               |               |             |             |           |
| 3.97      | 6.24e-12      | 8.53e-12      | 8.30e-12    | 24.9        | -2.7      |
| 5.00      | 8.95e-12      | 1.18e-11      | 1.14e-11    | 21.5        | -3.7      |
| 5.96      | 1.17e-11      | 1.51e-11      | 1.46e-11    | 19.6        | -3.4      |
| 6.95      | 1.49e-11      | 1.88e-11      | 1.75e-11    | 14.9        | -7.2      |

Figure 4. The distributions of the pressure (a) and the concentration (b) for He/Ar mixture at $C_A = 0.5010$ and $P_B = 15.1kPa$. The symbols, empty and solid triangles, empty and solid squares and empty circles, represent the cases with pressure ratios 2.99, 3.96, 4.95, 5.98, 6.96, respectively.

and $\mu = [2.38c - 5Pas, 2.70c - 5Pas]$. The single gas flow rate is obtained from the solution of Eqs. (12)-(14) by setting $C = 1$ and using the average mass and the viscosity as input quantities to determine the inlet and outlet rarefaction parameters. The approximated flow rates $J^*$ are deduced by using the methodology neglecting the concentration gradient for pressure driven flow and developed in Ref. [19]. The differences $\Delta_s = 100(1 - J_s/J)$ and $\Delta^* = 100(1 - J^*/J)$ are tabulated in the last columns of the table. It can be seen that the single gas assumption does not provide reliable results. The $\Delta_s$ difference can be as large as 16.7% and 24.9% for the He/Ar and He/Kr mixtures. The approximated formula provides an improvement with the largest absolute deviation of 2.6% and 7.2% for the two cases.

In addition to the flow rates, Figures 4 and 5 show the typical pressure and concentration profiles along the axis of the channel as deduced from the numerical calculation for He/Ar and He/Kr mixtures at concentration of 0.5010 and downstream pressure of 15.1kPa. As it is described in section 4, the solution of Eqs. (12)-(14) provide the flow rates $J_1, J_2$ and the distribution of the pressure and the concentration along the axis of the channel. The input parameters for the calculation of the pressure and concentration distributions are exactly the
Figure 5. The distributions of the pressure (a) and the concentration (b) for He/Kr mixture at $C_A = 0.5010$ and $P_B = 15.1kPa$. The symbols, empty and solid triangles, empty and solid squares and empty circles, represent the cases with pressure ratios $3.00, 3.97, 4.96, 5.98, 7.01$, respectively.

The same as those in Table 1 at $C_A = 0.5010$. By examining the pressure profiles, it can be concluded that the distributions are nearly non-linear at this downstream pressure. The non-linearity is caused by the compressibility and reduced by the rarefaction of the gas. The dimensionless flow rate depends on the rarefaction degree at intermediate values of the rarefaction. The concentration profiles exhibit the separation phenomenon. The concentration starts at the upstream value at the entrance of the channel, it starts to decrease, taking its minimum at the second half of the channel, and, finally, it reaches the downstream value at the exit. The non-uniformity of the concentration depends on the pressure ratio. With increasing pressure drop, the gaseous separation becomes larger. The corresponding distributions for the two mixtures are nearly the same, but it can be seen, on the concentration profiles, that the separation is stronger for the He/Kr mixture, which has larger mass ratio compared to the He/Ar mixture.

Figure 6 shows the typical dimensional velocity profiles of the species of the gas along the center line of the trapezoidal cross section, $y = 0$, at the center of the channel, $x = L/2$, for the two types of mixture at $P_B = 8.05kPa$, $C_A = 0.5010$ and $P_B/P_A \sim [4,6]$. The exact pressure ratios are given in Table 2. The dimensional velocity is calculated according to $u_{\alpha} = v(u_{\alpha}^{(P)}X_P + u_{\alpha}^{(C)}X_C)$, where $u_{\alpha}^{(P)}$ and $u_{\alpha}^{(C)}$ are the dimensionless gas velocities for uniform pressure or concentration driven flows and $X_P$ and $X_C$ are the local pressure or concentration gradients. This calculation is based on the methodology of Ref. [19] for the solution of the McCormack model. For the examined cases, the local rarefaction and concentration values and $X_P$, $X_C$ at the center of the channel are deduced from the distribution of the pressure and the concentration. In the figure, the separation of the components is clearly visible. The lighter species have larger velocity than that of the heavier one due to their larger molecular speed. The velocities are larger if the pressure ratio becomes larger. In addition, the velocities of the components of the He/Kr mixture are smaller those of the He/Ar mixture. This can be explained by the fact that the He/Kr gas has a smaller characteristic molecular speed than that of the He/Ar mixture.

6. Conclusions

In the present work, the flow rates of He/Ar and He/Kr mixtures through long trapezoidal micro-channels are measured by using the constant volume method. The downstream pressures $15.1kPa$ and $8.05kPa$ are investigated. The pressure ratio is in the range of $3−7$ and $4−7$ for the larger and smaller outlet pressures. The flow rates are compared to the results of the McCormack
kinetic model. The relative difference between the experimental and kinetic results is less than the experimental uncertainty, which is ±4%. This indicates a complete agreement between the two approaches within the range of the precision (uncertainty) of the experiment. This agreement can be considered as very good. The McCormack model reproduces the experimental results within their uncertainty. It is shown that the single gas assumption does not provide reliable results for this type of flow. Hence, the kinetic modelling of the gas mixture, taking into account the gas diffusion, is necessary. The typical axial distributions of the pressure and the concentration are also analyzed. The typical dimensional velocity profiles are also studied. The gaseous mixture exhibits the separation phenomenon. The good agreement between the theoretical and experimental flow rates suggests that the McCormack model is a viable tool for modeling pressure driven isothermal flows of gas mixtures in micro-channels at the considered parameters.

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