EasyConvPooling: Random Pooling with Easy Convolution for Accelerating Training and Testing

Jianzhong Sheng∗
Huazhong University of Science and Technology
City University of Hong Kong
csjianzhong@gmail.com

Chuanbo Chen
Huazhong University of Science and Technology
chuanboc@163.com

Chenchen Fu
City University of Hong Kong
chenfu2@cityu.edu.hk

Chun Jason Xue†
City University of Hong Kong
jasonxue@cityu.edu.hk

ABSTRACT
Convolutional operations dominate the overall execution time of Convolutional Neural Networks (CNNs). This paper proposes an easy yet efficient technique for both Convolutional Neural Network training and testing. The conventional convolution and pooling operations are replaced by Easy Convolution and Random Pooling (ECP). In ECP, we randomly select one pixel out of four and only conduct convolution operations of the selected pixel. As a result, only a quarter of the conventional convolution computations are needed. Experiments demonstrate that the proposed EasyConvPooling can achieve 1.45x speedup on training time and 1.64x on testing time. What’s more, a speedup of 5.09x on pure Easy Convolution operations is obtained compared to conventional convolution operations.

CCS CONCEPTS
• Computer systems organization → Embedded systems; Redundancy; Robotics; • Networks → Network reliability;

KEYWORDS
Easy Convolution, Random Pooling, Training, Testing

ACM Reference Format:
Jianzhong Sheng, Chuanbo Chen, Chenchen Fu, and Chun Jason Xue. 2018. EasyConvPooling: Random Pooling with Easy Convolution for Accelerating Training and Testing. In Proceedings of Archive. ACM, New York, NY, USA, 9 pages. https://doi.org/10.1145/mnnnnnn.nnnnnnn

1 INTRODUCTION
Convolutional Neural Networks (CNNs) are a promising class of machine learning algorithms that achieve remarkable performance in various computer vision tasks, e.g., image classification [18]. One of the key reason for this success is their deep architecture [23]. It has been proved that deeper architecture makes better performance. As a result, the performance of CNNs over past few years has been improved mainly by designing a deeper architecture. It is not uncommon for a neural network to have massive parameters in its model, costing more time to train and test the network.

In this study, we propose an effective technique called EasyConvPooling (ECP) to accelerate both training and testing. EasyConvPooling is consist of two parts: Easy Convolution and Random Pooling. In Random Pooling, we select one pixel out of four randomly, and then compute convolution of the selected pixel only. This leads to reduction in 75% convolution computation compared to conventional convolution and thus reducing both training time and testing time.

In order to realize the proposed method, we are facing two questions. The first question is how to determine the selected pixel in Random Pooling and how to obtain its index for conducting Easy Convolution in the upper layer. For selecting pixel, we randomly appoint one pixel out of four to be the “lucky” pixel; for its index, we keep the index of the “lucky” pixel which is appointed before pooling. This does not lead to significant loss in accuracy.

The second question is how to conduct Easy Convolution in selected mode and keep the shape of output feature map unchanged. In order to solve this problem, we determine the mode of Easy Convolution according to the index of the selected pixel to assure that they match. Based on the experiments, we find that conducting Random Pooling alone does reduce training time and testing time. The reduction grows to be more significant when combined with Easy Convolution. Experimental results demonstrate that the proposed ECP achieves 1.45x speedup on training time and 1.64x speedup on testing time. In addition, we obtain a speedup of 5.09x on pure Easy Convolution operations compared to conventional convolution operations.

The contributions of this work are as follows:

• Proposes a novel EasyConvPooling technique to conduct convolution and pooling, in which only 25% of convention convolution operations is needed.
• Proposes an universal technique to accelerate both training and testing.
• The proposed novel technique (ECP) can be transfered to any other platform supporting Python.

Remainder of this paper is organized as follows. Section 2 summarizes the related work. Section 3 presents the proposed method.
We propose an easy yet efficient technique called EasyConvPooling (ECP) for Convolutional Neural Networks to conduct convolution and pooling. In the proposed method ECP, only 25% of original convolution operations are done, which reduces 75% multiplications in convolutions with little loss in accuracy. ECP is consist of two parts: Easy Convolution and Random Pooling. Here is how to conduct ECP:

1. Randomly set Mode K.
2. Determine the positions of selected pixels for Random Pooling and Easy Convolution.
3. Conduct Easy Convolution on the selected pixels and pad the neighbor pixels to recover the output shape for pooling layer.
4. Conduct Random Pooling.

In the following subsection, we first present the architecture of the network, convolutional convolution and pooling, then describe Random Pooling and Easy Convolution in details.

### 3.1 Architecture

Figure 1 shows overall architecture of the proposed network for conducting ECP compared to conventional convolution and pooling. In Figure 1, we design a two-convolution neural network with two fully connected layers. Each convolution layer is followed by a pooling layer and a ReLU layer. In the fully connected layers, we add one ReLU layer at the end of the first layer and connect the second fully connected layer to the Softmax layer directly.

The upper part of Figure 1 is the proposed ECP technique and the lower part is conventional way to do convolution and pooling, such as Average Pooling and Max Pooling. In ECP, we replace convolutional convolution and pooling operations by Easy Convolution and Random Pooling. Both Easy Convolution and Random Pooling have a Mode K to control operation mode. In order to assure that they are matched under the same Mode K, each Easy Convolution layer is followed by a Random Pooling layer.

### 3.2 Conventional Convolution and Pooling

Convolution operations occupy the most time of CNNs, and in Figure 2 we look into conventional convolution and pooling to make an overall view of the conventional convolution and pooling. In the following subsection, we will describe and compare conventional convolution and pooling with the proposed Easy Convolution and Random Pooling in details.

In Figure 2, every sliding (convolution) window is consist of four pixels, taking kernel size $2 \times 2$ for easy demonstration, and every time we make convolution of a sliding window (weights) and input pixels Beneath it to form a feature map element. Considering one step stride, sliding windows are overlapped. After computing one feature map element out, we move the sliding window one step right to compute another feature map element and as well as in the second row. Finally, we achieve a feature map for pooling. Here is how we compute convolution:

$$W \ast x(m, n) = \sum_{u} \sum_{v} W(u, v)x(m + u, n + v)$$

where $x$ is an input image and $W$ is a weight matrix of the convolution filter. The operator `$\ast$" means 2D convolution.

In the pooling layer, output is calculated by selecting one pixel out of four to represent the whole four pixels. In Average Pooling, the output is the average value of the four pixels in feature map; in Max Pooling, we select the max value pixel as the output pixel.

In short, we compute four conventional convolutions to form the pooling elements required for pooling window. However, the output of both Average Pooling and Max Pooling are one pixel only, wasting extra 75% convolutions. If we can determine which pixel
to be selected in the pooling layer, we can reduce the extra 75% convolutions in convolution layer. That’s where we benefit in the proposed Easy Convolution and Random Pooling.

3.3 Random Pooling

In conventional convolution, we need to calculate the outputs of every convolution window to make the feature map, however, in the pooling layer, only one pixel out of four (stride = 2) is chosen to represent the output of the pooling window. In Average Pooling, we compute the average of the four pixels to make the output of pooling window and in Max Pooling we make the output by choosing the max value pixel. Random Pooling just randomly select one pixel out of four to represent the output of pooling window reducing 75% extra convolutions.

In Random Pooling, we obtain the index of the selected pixel by setting Random Pooling Mode K. Random Pooling Mode K stands for the position index of the four pixels in the pooling windows. It varies from 0 to 3, from up to down and left to right. The Random Pooling Mode K is randomly set before pooling so that we can figure out how to conduct Easy Convolution in the upper layer.
Figure 3 demonstrates how exactly Random Pooling works. Once the Random Pooling Mode $K$ is set, we can determine which pixel to be selected in the dotted pooling window. Mode 0 means pixel 0 is selected from the dotted pooling window every time. After selecting the first pixel 0 element, we slide the pooling window two step right to obtain the second pixel 0 element. The pooling window slides from left to right, top to down with two strides every time. Finally, the output feature map of Random Pooling is formed by those pixel 0 elements. In Mode 1, 2 and 3, the same operations are done to pixel 1, 2 and 3 elements. Random Pooling actually always select the pixel of the same position in the pooling window to make up the outputs of the pooling windows and thus form the output feature map of the pooling layer. Various Mode $K$ means various pixel position in the pooling window.

In conventional Average Pooling/ Max Pooling, the output of pooling window is always the Averaged/ Maxed value of the pooling window. In the middle of Figure 3 is the proposed Random Pooling, and beside it is conventional Average/ Max Pooling.

### 3.4 Easy Convolution

In order to match and control the Easy Convolution Mode with Random Pooling Mode, we use the same $K$ to control Easy Convolution Mode. Due to the overlapping in convolution sliding window, we need two variables to locate the position of selected convolution window. In programming level, we make the two variables in Easy Convolution to match the Random Pooling Mode $K$ so that we can use the same $K$ in Easy Convolution layer to extract the same position elements matching Random Pooling.

In subsection Random Pooling, we obtain the selected pixel’s index by setting Random Pooling Mode $K$ and then we conduct Easy Convolution using the same Mode $K$. Figure 4 shows how we conduct the Easy Convolution operations, it’s very similar to Random Pooling.

In Figure 4, every convolution window on input image contains several weights. In order to obtain feature map for the next layer, convolution operations are carried out on these input image with sliding convolution window. Window 0 is the selected window for producing selected pixel 0 element for pooling window in pooling layer. Every window 0 is a sliding convolution window over input image under Mode 0. The output of these convolution window is pixels needed in pooling layer for Random Pooling. The first window 0 produces the first pixel 0 element in the pooling window in Figure 3, and the second window 0 produces the second pixel 0 element in the pooling window. The convolution window slides over the input image to produce the output feature of convolution layer. Various Mode $K$ determine various window $K$ to produce various pixel $K$ element needed in the pooling window. After sliding from left to right and up to down, the feature map of convolution layer is formed.

In conventional convolution, sliding convolution window slides over all input image area to produce feature map elements. In Easy Convolution, sliding convolution window slides only to window $K$ position to extract selected data, reducing 75% extra data with a quarter of original shape.

After extracting data we need from input image, we can easily compute convolution as usual, reducing 75% convolutions. The
remaining problem is how we can get the pruned shape back. In some situation, we make use of padding technique to keep the output shape unchanged. We add padding to the output of Easy Convolution to restore the shape of the feature map so that the network can run as usual. For Easy Convolution, we pad the same value to its neighbor empty pixels as shown in Figure 4.

4 EXPERIMENTS AND EVALUATIONS
To demonstrate that the proposed technique ECP is effective and reliable, we perform experiments on various hidden layers compared with Average Pooling and Max Pooling under different Mode K. We coded a one-convolution layer network and a two-convolution layer network to evaluate ECP’s performance under various depth of layers. All the codes are written in Python without any framework, and all the experiments are conducted on CPU, making it universal to all platform supporting Python.

4.1 Experimental Settings
We set batch size to 50 and learning rate to 0.001 in the experiments. For MNIST [20], the input dimension is 28×28 = 784, and the output dimension is 10. The experiments are conducted on Intel(R) Core i7-7700HQ 2.80GHz CPU with Python 3.6.3 installed on Windows 10 operation system.

Table 1 and Table 2 show the parameters of the networks under various hidden layers.

| Layer Name | Parameter |
|------------|-----------|
| Input image | size: 28×28, channel: 1 |
| Convolution | kernel: 5×5, channel: 20 |
| Pooling | kernel: 2×2, stride: 2 |
| ReLU | |
| Fully connected | channel: 100 |
| ReLU | |
| Fully connected | channel: 10 |
| Softmax | |

Table 2: Network Parameters of Two-convolution layer CNN.

| Layer Name | Parameter |
|------------|-----------|
| Input image | size: 28×28, channel: 1 |
| Convolution | kernel: 5×5, channel: 20 |
| Pooling | kernel: 2×2, stride: 2 |
| ReLU | |
| Convolution | kernel: 5×5, channel: 32 |
| Pooling | kernel: 2×2, stride: 2 |
| ReLU | |
| Fully connected | channel: 100 |
| ReLU | |
| Fully connected | channel: 10 |
| Softmax | |
4.2 Experimental Results on Time Performance

In order to verify the time performance of the proposed technique ECP, we evaluate both training time and testing time at the same time. Furthermore, we design a special test on pure convolution with the proposed ECP and conventional convolution method to compare their real operation time on convolution. This special test is conducted on MNIST database for 100 epochs, and we have tested it for several times.

Table 3 shows the overall time performance of the proposed ECP compared with the conventional Max/ Average Pooling. The first column shows the exact epoch when testing accuracy first reaches 98%, and the others indicate time performance. After applying ECP, we achieve 1.45x speedup on training time and 1.64x on testing time compared to Average Pooling. In terms of the pure convolution time, we achieve the speedup of 5.09x. This speedup is even larger than the theoretical speedup value. This is because of the limitation of the memory space. Less convolution data can save the space of memory and thus avoid the content switch operations due to the lack of memory. In addition, based on the result over Iteration, we can be sure that the ECP technique does not lead to more training data.

4.3 Experimental Results on Accuracy

Besides the time performance, accuracy is another critical parameter in both training and testing steps. Considering the randomness of ECP, it may lead to drop in accuracy. In order to figure out this, we first conduct experiments by training the MNIST dataset for 200 times to make sure we can get its best accuracy during experiment. Results indicate that 100 epochs are already enough. For most situation, they achieve their best accuracy within 80 epochs. Sometimes we get worse accuracy while training more due to overfitting. So in the experiments, we decide to evaluate the accuracy within 100 epochs, which is more valuable than training another more 100 epochs to gain accuracy improvement less than 0.5%.

The experiments are conducted on a two-convolution layer network demonstrated in Figure 1. Results in Table 4 indicate the proposed ECP achieves good improvement with little loss in accuracy.

The experiment does not only consider the accuracy performance of ECP compared with Max Pooling and Average Pooling but also takes Mode K into consideration to evaluate the Robustness of the proposed ECP. The results are reliable, and Mode K will be discussed in detail in the following subsection.

4.4 Varying Mode K

Another interesting problem is to check the role of Mode K. In Random Pooling and Easy Convolution, we randomly set a parameter by Mode K, and it determines how to conduct Random Pooling and where to apply the Easy Convolution. In the following experiment, we test what happens if we vary the parameter Mode K.

To find out the role of Mode K plays in the proposed ECP, we design tests on a two-convolution network using Random Pooling technique only and ECP technique respectively. Figure 5 is Random Pooling under different Mode K, and Figure 6 shows results for ECP.

From the figures we can conclude that the randomly set Mode K is not crucial to the results, while it does affect the training process in some aspect. Randomly selected Mode K does not affect the overall convergence of the network no matter it’s conducted alone or together with Easy Convolution, and Mode K has little influence on training time and testing time based on Table 4.

However, in Figure 5 and Figure 6, the randomly selected Mode K seems to have some effect on the convergence in the very beginning of the training process and it seems to have effect on the final accuracy. But recall that the weights in the kernel are initiated randomly by uniform distribution. It can be noted that Mode K has little influence on ECP’s time performance as well as accuracy.
Table 3: Overall Time Performance of ECP vs Max/ Average Pooling.

| Method   | Iter (98%) | Training Time (ms) | Testing Time (ms) | PureConvTime (ms) |
|----------|------------|--------------------|-------------------|-------------------|
| Max Pooling | 3          | 299650.15          | 23814.87          | 4496.77           |
| ECP      | 4          | 243077.47 (1.23x)  | 13084.98 (1.82x)  | 883.65 (5.09x)    |

Table 4: Random Pooling and ECP vs Max /Average Pooling under Mode K.

| Method   | Iter (98%) | Training Time (ms) | Testing Time (ms) | Best Accuracy (%) |
|----------|------------|--------------------|-------------------|-------------------|
| Ave Pooling | 5          | 352544.96          | 21426.24          | 98.69             |
| Random k=0 | 4          | 280866.26 (1.07x)  | 20691.98 (1.15x)  | 98.65 (-0.52)     |
| Random k=1 | 4          | 281443.49 (1.06x)  | 20689.19 (1.15x)  | 98.78 (-0.39)     |
| Random k=2 | 5          | 277848.33 (1.08x)  | 20422.44 (1.17x)  | 98.77 (-0.40)     |
| Random k=3 | 4          | 282693.13 (1.06x)  | 20577.25 (1.16x)  | 98.78 (-0.39)     |
| ECP k=0   | 4          | 243077.47 (1.45x)  | 13084.98 (1.64x)  | 98.65 (-0.52)     |
| ECP k=1   | 5          | 243607.47 (1.45x)  | 13403.70 (1.78x)  | 98.67 (-0.50)     |
| ECP k=2   | 5          | 24106.84 (1.25x)   | 13271.43 (1.80x)  | 98.78 (+0.09)     |
| ECP k=3   | 5          | 243466.94 (1.45x)  | 13303.02 (1.80x)  | 98.77 (+0.08)     |

Figure 7: Random Pooling Convergence vs Average/ Max Pooling under One-convolution Layer.

4.5 Varying Convolution Layers

In this part, we evaluate the proposed ECP under various hidden layers: one-convolution network and two-convolution network. Table 5 is the result of ECP compared with Max Pooling and Table 6 is for Average Pooling.

From the tables, we notice that the performance compared to Average Pooling is better than that of Max Pooling. We gain more performance speedup compared to Average Pooling with a little accuracy improvement rather than drop. What’s more, comparing to Max Pooling, the time performance of the proposed ECP is even better when we make the network deeper, with little accuracy loss.

4.6 Convergence under Various Convolution

The proposed ECP is consist of two parts: Easy Convolution and Random Pooling. In order to compare convergence of Random Pooling alone with Average Pooling/ Max Pooling, we conduct the same conventional convolution in the upper convolution layer of Random Pooling.

Figure 7 and Figure 8 show Random Pooling convergence vs Average/ Max Pooling under one-convolution layer and two-convolution Layer respectively. Figure 9 and Figure 10 show ECP convergence...
Table 5: ECP vs Max Pooling under Various Convolution Layers.

| ConvLayer | Method      | Iter (98%) | Training Time (ms) | Testing Time (ms) | Best Accuracy (%) |
|-----------|-------------|------------|--------------------|-------------------|------------------|
| 1         | Max Pooling | 8          | 215387.59          | 12500.73          | 98.37            |
| 1         | ECP         | 5          | 207065.46 (1.04x)  | 11520.13 (1.09x)  | 98.69 (+0.32)    |
| 2         | Max Pooling | 3          | 299650.15          | 23814.87          | 99.17            |
| 2         | ECP         | 4          | 243077.47 (1.23x)  | 13084.98 (1.82x)  | 98.81 (-0.36)    |

Table 6: ECP vs Average Pooling under Various Convolution Layers.

| ConvLayer | Method      | Iter (98%) | Training Time (ms) | Testing Time (ms) | Best Accuracy (%) |
|-----------|-------------|------------|--------------------|-------------------|------------------|
| 1         | Ave Pooling | 12         | 303580.05          | 18579.89          | 98.29            |
| 1         | ECP         | 5          | 207065.46 (1.47x)  | 11520.13 (1.61x)  | 98.69 (+0.40)    |
| 2         | Ave Pooling | 5          | 352544.96          | 21426.24          | 98.69            |
| 2         | ECP         | 4          | 243077.47 (1.45x)  | 13084.98 (1.64x)  | 98.81 (+0.12)    |

Figure 8: Random Pooling Convergence vs Average/ Max Pooling under Two-convolution Layer.

Figure 9: ECP Convergence vs Average/ Max Pooling under One-convolution Layer.

4.7 Remarks

Based on the experiments above, in the following we summarize the major characteristics of the proposed ECP technique:

- Testing performance is always much better than training.
- ECP has more advantage over Average Pooling than Max Pooling due to the speedup of training.
- We can achieve more performance improvement when conducting ECP on a deeper network, with little loss in accuracy.

5 CONCLUSIONS

Deeper network architecture usually leads to better performance, as a result, it’s getting more and more difficult to train Convolutional Neural Networks. Considering the fact that the overall execution time of Convolutional Neural Networks is dominated by convolution operations, we propose a novel technique named EasyConvPooling (ECP) to solve this problem. In ECP, we conduct convolution operations according to the index from following pooling layer, which reduces 75% of original convolution operations. The experiments demonstrate that we achieve 1.45x speedup on training time and 1.64x on testing time with little loss in accuracy. What’s more, we achieve a speedup of 5.09x on pure Easy Convolution operations compared to conventional convolution operations.

REFERENCES

[1] Carlo Baldassi, Alessandro Ingrosso, Carlo Lucibello, Luca Saglietti, and Riccardo Zecchina. 2015. Subdominant dense clusters allow for simple learning and high computational performance in neural networks with discrete synapses. Physical review letters 115, 12 (2015), 128101.
Figure 10: ECP Convergence vs Average/ Max Pooling under Two-convolution Layer.

(2) Zhiyong Cheng, Daniel Soudry, Zexi Mao, and Zhenzhong Lan. 2015. Training binary multilayer neural networks for image classification using expectation backpropagation. arXiv preprint arXiv:1503.03562 (2015).

(3) Matthieu Courbariaux, Yoshua Bengio, and Jean-Pierre David. 2015. Binaryconnect: Training deep neural networks with binary weights during propagations. In Advances in neural information processing systems. 3125–3131.

(4) Matthieu Courbariaux, Itay Hubara, Daniel Soudry, Ran El-Yaniv, and Yoshua Bengio. 2016. Binaryized neural networks: Training deep neural networks with weights and activations constrained to+ 1 or-1. arXiv preprint arXiv:1602.02830 (2016).

(5) Misha Denil, Babak Shakibi, Laurent Dinh, Nando De Freitas, et al. 2013. Predicting parameters in deep learning. In Advances in neural information processing systems. 2148–2156.

(6) Emily L Denton, Wojciech Zaremba, Joan Bruna, Yann LeCun, and Rob Fergus. 2014. Exploiting linear structure within convolutional networks for efficient evaluation. In Advances in neural information processing systems. 1269–1277.

(7) Jiashi Feng and Trevor Darrell. 2015. Learning the structure of deep convolutional networks. In Proceedings of the IEEE international conference on computer vision. 2749–2757.

(8) Song Han, Junlong Kang, Huizi Mao, Yiming Hu, Xin Li, Yuhin Li, Dongliang Xie, Hong Luo, Song Yao, Yu Wang, et al. 2017. Ese: Efficient speech recognition engine with sparse lstm on fpga. In Proceedings of the 2017 ACM/SIGDA International Symposium on Field-Programmable Gate Arrays. ACM, 75–84.

(9) Song Han, Xingyu Liu, Huizi Mao, Jing Pu, Arvadian Pedram, Mark A Horowitz, and William J Dally. 2016. EIE: efficient inference engine on compressed deep neural network. In Computer Architecture (ISCA), 2016 ACM/IEEE 43rd Annual International Symposium on. IEEE, 243–254.

(10) Song Han, Huizi Mao, and William J Dally. 2015. Deep compression: Compressing deep neural networks with pruning, trained quantization and huffman coding. arXiv preprint arXiv:1510.00149 (2015).

(11) Song Han, Jeff Pool, Sharan Narang, Huizi Mao, Shijian Tang, Erich Elsen, Bryan Catanzaro, John Tran, and William J Dally. 2016. Dnnd: Regularizing deep neural networks with dense-sparse-dense training flow. arXiv preprint arXiv:1607.04381 3, 6 (2016).

(12) Song Han, Jeff Pool, John Tran, and William Dally. 2015. Learning both weights and connections for efficient neural network. In Advances in neural information processing systems. 1135–1143.

(13) Sepp Hochreiter and Jürgen Schmidhuber. 1997. Long short-term memory. Neural computation 9, 8 (1997), 1735–1780.

(14) Kyuyoung Hwang and Wonhyo Sung. 2014. Fixed-point feedforward deep neural network design using weights+ 1, 0, and-1. In Signal Processing Systems (SIPS), 2014 IEEE Workshop on. IEEE, 1–6.

(15) Yani Ioannou, Duncan Robertson, Jamie Shotton, Roberto Cipolla, and Antonio Criminisi. 2015. Training cnns with low-rank filters for efficient image classification. arXiv preprint arXiv:1511.06744 (2015).

(16) Max Jaderberg, Andrea Vedaldi, and Andrew Zisserman. 2014. Speeding up convolutional neural networks with low rank expansions. arXiv preprint arXiv:1405.3886 (2014).

(17) Minje Kim and Paris Smaragdis. 2016. Bitwise neural networks. arXiv preprint arXiv:1601.06071 (2016).

(18) Alex Krizhevsky, Ilya Sutskever, and Geoffrey E Hinton. 2012. Imagenet classification with deep convolutional neural networks. In Advances in neural information processing systems. 1097–1105.

(19) Vadim Lebedev and Victor Lepmitsky. 2016. Fast convnets using group-wise brain damage. In Computer Vision and Pattern Recognition (CVPR), 2016 IEEE Conference on. IEEE, 2554–2564.

(20) Yann LeCun, Léon Bottou, Yoshua Bengio, and Patrick Haffner. 1998. Gradient-based learning applied to document recognition. Proc. IEEE 86, 11 (1998), 2278–2324.

(21) Min Lin, Qiang Chen, and Shuicheng Yan. 2013. Network in network. arXiv preprint arXiv:1311.2420 (2013).

(22) Baoyuan Liu, Min Wang, Hassan Faroosh, Marshall Tappen, and Marianna Pensky. 2015. Sparse convolutional neural networks. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition. 806–814.

(23) Guido F Montufar, Razvan Pascanu, Kyunghyun Cho, and Yoshua Bengio. 2014. On the number of linear regions of deep neural networks. In Advances in neural information processing systems. 2924–2932.

(24) Karen Simonyan and Andrew Zisserman. 2014. Very deep convolutional networks for large-scale image recognition. arXiv preprint arXiv:1409.1556 (2014).

(25) Xu Sun, Xuancheng Ren, Shuming Ma, and Houfeng Wang. 2017. meProp: Sparserified back propagation for accelerated learning with reduced overfitting. arXiv preprint arXiv:1706.06197 (2017).

(26) Cheng Tai, Tong Xiao, Yi Zhang, Xiaogang Wang, et al. 2015. Convolutional neural networks with low-rank regularization. arXiv preprint arXiv:1511.06067 (2015).

(27) Wei Wen, Chunpeng Wu, Yandan Wang, Yiran Chen, and Hai Li. 2016. Learning structured sparsity in deep neural networks. In Advances in Neural Information Processing Systems. 2074–2082.

(28) Jianchao Yang, Kai Yu, Yihong Gong, and Thomas Huang. 2009. Linear spatial pyramid matching using sparse coding for image classification. In Computer Vision and Pattern Recognition, 2009. CVPR 2009. IEEE Conference on. IEEE, 1794–1801.

(29) Ming Yuan and Yi Lin. 2006. Model selection and estimation in regression with grouped variables. Journal of the Royal Statistical Society: Series B (Statistical Methodology) 68, 1 (2006), 49–67.