Active distribution network planning considering linearized system loss

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Abstract. In this paper, various distribution network planning techniques with DGs are reviewed, and a new distribution network planning method is proposed. It assumes that the location of DGs and the topology of the network are fixed. The proposed model optimizes the capacities of DG and the optimal distribution line capacity simultaneously by a cost/benefit analysis and the benefit is quantified by the reduction of the expected interruption cost. Besides, the network loss is explicitly analyzed in the paper. For simplicity, the network loss is appropriately simplified as a quadratic function of difference of voltage phase angle. Then it is further piecewise linearized. In this paper, a piecewise linearization technique with different segment lengths is proposed. To validate its effectiveness and superiority, the proposed distribution network planning model with elaborate linearization technique is tested on the IEEE 33-bus distribution network system.

1. Introduction

Distribution network planning (DNP) is a basic module in power systems, which directly affects the safety, economy and stability of the distribution network. The main purpose of the DNP is to determine the location, capacity of the substation, feeders and DGs at a minimum economic cost in order to meet the load growth demand and the new loads in the modern distribution system, while the reliability and stability of the system should also be ensured [1,2]. The presence of distributed generations (DGs) in distribution network could improve the system efficiency, economy and quality of service [3, 4]. However, the output of some renewable DGs, such as wind turbine generation and photovoltaic, are very volatile. The introduction of these non-dispatchable DGs would cause operation problems in terms of system reliability, power quality, system stability, fault level, and protection coordination, etc [5]. So distribution network planning considering DGs has become a hot research topic.

In traditional distribution networks (TDN), DGs are generally operated with fixed power factors. Distribution system operators (DSOs) traditionally install firm-DGs based on the “fit-and-forget” approach, it means that the load growth forecast is given and there is no control on the output of DG units to determine the installation of future distribution [6]. In traditional network assets, these “fit and forget” approaches will put barrier to the further penetration of renewable energy sources (RES) in the distribution system and they may lead to a higher economic cost [7–9].

The major research contents of ADM power planning include the location and capacity planning of substations, DGs, the unit planning of substations and DGs, and energy storage, electric vehicles, flexible load planning. An exhaustive review of the typical distribution network planning models and
strategies is given in [1, 2, 10]. DNP problem considering DGs can be divided into single planning and comprehensive coordinated planning according to the type of decision variables [11]. Single planning is to determine optimal placement and capacity of DGs without considering the planning of substation configuration and feeders [12], while comprehensive coordinated planning is an overall planning of DGs, substations and distribution feeders. And this paper will focus on the latter one, i.e., optimizing the capacities of DG and the optimal distribution line simultaneously.

The rest of this paper is organized as follows: Section 2 describes the planning model of ADN. A piecewise linearized method is introduced in Section 3 in order to transfer the traditional planning model into the stand mixed integer linear programming (MILP) problem. Then a piecewise linearization technique with different segment length is proposed. The above method is applied to the IEEE 33-bus distribution network and the results are presented in Section 4. Finally, some conclusions are drawn in Section 5.

2. Planning model of ADN

In this section, a formulation of the active distribution network planning problem is presented. The planning model, which assumes that the location of DGs and the topology of the network are fixed, optimizes the capacities of DG and distribution line by a cost/benefit analysis and the benefit is quantified by the reduction of the expected interruption cost. The objective function is to maximize the social welfare, or it can be equally described as minimizing the total cost. The optimization model is set up as follows:

2.1. Objective function

The objective function for ADN planning:

\[
\min TC = C_{DG} + C_L + C_{grid} + C_{loss} + C_{LOL} - C_{fin}
\]

where TC is the abbreviation of total cost, which means the total cost of the ADN planning.

1) DG construction, operation and maintenance costs

\[
C_{DG} = \sum_{i=1}^{N_{DG}} (\beta \cdot C_{equ} + C_{ope} + C_{rep}) \lambda_i \cdot S_{DG_i} + \beta \cdot C_{ins}
\]

where \(N_{DG}\) is the number of total DG nodes containing DGs; \(\beta\) is DG fixed investment annual average cost factor:

\[
\beta = \frac{r(1+r)^t}{(1+r)^t-1}
\]

where \(r\) is annual percentage rate, \(t\) is planning period. And \(C_{equ}\) is the equipment investment cost of DG on node \(i\); \(C_{ope}\) is the operate cost of DG on node \(i\); \(C_{rep}\) is annual maintenance cost of DG on node \(i\); \(\lambda_i\) is power factor of \(i\)th DG unit; \(S_{DG_i}\) is the rated capacity of DG on node \(i\); \(C_{ins}\) is the fixed installation cost of DG unit on node \(i\) [11].

2) feeders investment costs

\[
C_L = \sum_{b=1}^{N_l} k_b \cdot l_b \cdot T_b
\]

where \(k_b\) is line annual investment cost of per unit length; \(l_b\) is line planning length; \(T_b\) is line planning capacity, and \(N_l\) is the number of total distribution lines.

3) Power purchase cost

\[
C_{grid} = C_e (P_L - \sum_{i=1}^{n} \lambda_i \cdot S_{DG_i}) \cdot T_{max}
\]

where \(C_e\) is the electricity price; \(P_L\) is the total load capacity of the distribution system; and \(T_{max}\) is the maximum load equivalent hours.

4) Power loss cost
The details of calculating $P_{loss}$ will be given in part 3.

5) Expected interruption cost

$$C_{LOL} = \sum_{ij} (VOLL \cdot LOL_{ij})$$

where $LOL_{ij}$ is the power expected interrupt loss on node $i$ in time $t$, and $VOLL$ is the value of lost load.

2.2. Constraints

$$\sum_{i} P_{DG} = \sum_{i}^{N_i} (P_i - LOL_i) + \sum_{i} P_{loss}$$

$$S_{ij} \leq S_{ij}^{max}$$

$$U_{ij}^{min} \leq U_i \leq U_{ij}^{max}$$

$$P_i^{G_{min}} \leq P_i \leq P_i^{G_{max}}$$

$$Q_i^{C_{min}} \leq Q_i \leq Q_i^{C_{max}}$$

$$T_k^{min} \leq T_k \leq T_k^{max}$$

$$P_{i,j} = P_i + H_{i,j} \cdot P_{ll}$$

$$-P_i^{max} \leq P_{i,j} \leq P_i^{max}$$

The constraints (8) enforce the total power balance, where $N_i$ is the number of total distribution lines. The constraints (9) enforce line flow limits at every distribution line. The constraints (10) enforce the nodal voltage limits. In this model, the fluctuation range is within 7% of the normal operation voltage. The constraints (11) and (12) are output limits of units, where $Q_f$ means the reactive power absorbed by a reactive compensator. And the constraints (13) is co-ordinated voltage regulation (area voltage control) using OLTC, $T_k$ [13]. The constraints (14) and (15) represent the $N$-I security criterion, where $P_{i,j}$ means the active power flow in the line “I” when the other line “II” in the network fails, and $H_{i,j}$ is a transfer factor.

3. Linearization of power loss

3.1. Traditional linearization method

The network loss is explicitly analyzed in this section. For normal operation, under the flat voltage assumption, the network loss is appropriately simplified as a quadratic function of difference of voltage phase angle [14]. That is, the power injection in the line $(i, j)$ computed at bus $i$, $p_i(\delta, \delta_j)$, and the power injection in the line $(i, j)$ computed at bus $j$, $p_j(\delta, \delta_j)$, are given by:

$$P_i(\delta, \delta_j) = U_i^2 G_{ij} - U_i U_j (G_{ij} \cos \delta_j + B_{ij} \sin \delta_j)$$

$$P_j(\delta, \delta_j) = U_j^2 G_{ij} - U_i U_j (G_{ij} \cos \delta_i + B_{ij} \sin \delta_i)$$

where $y_{ij} = G_{ij} + jB_{ij}$, $y_{ij}$ is the admittance of the line $(i, j)$. Here, we assume that $U_i \approx 1$ in the normal operation of distribution network, then the power loss of the line $(i, j)$ can be obtained as follows:

$$P_{loss} = P_i + P_j = 2G_{ij}(1 - \cos \delta_j) = G_{ij}(\delta - \delta_j)^2$$
Then it is further piecewise linearized by using 2L piecewise linear blocks as shown in Figure.1.

![Figure 1. Piecewise linearization of system loss in a branch.](image)

However, only L piecewise linear blocks are sufficient by using the positive orthant only. In order to achieve this purpose, we need introduce the linearization of absolute sign:

\[
\delta_i = |\delta - \bar{\delta}|
\]

(19)

\[
\delta_{ij} = \sum_{l=1}^{L} \delta_{i}(l)
\]

(20)

\[
P_{ij}^{loss}(\delta, \delta) = G_{ij} \sum_{l=1}^{L} k_{ij}(l) \bar{\delta}_{i}(l)
\]

(21)

where \(k_{ij}(l)\) and \(\delta_{i}(l)\) means, the slope and value of the \(l\)th block of angle, respectively. The quadratic formulation of (18) is piecewise linearized to the above expression with the introduction of absolute sign. While the absolute value is still not a linear expression, a linear expression of the absolute value in (19) is needed, which is obtained by the following math substitution [15]:

\[
\bar{\delta}_{i} = \delta_{ij} + \delta_{ij}
\]

(22)

\[
\delta - \bar{\delta} = \delta_{ij} - \delta_{ij}
\]

(23)

\[
\delta_{ij} \geq 0, \delta_{ij} \geq 0
\]

(24)

Then the power flow can be expressed as follows by using the above piecewise linearization methods:

\[
P_{ij}^{0} = \frac{1}{2} P_{ij}^{loss}(\delta, \delta) - B_{ij} \sin(\delta - \bar{\delta}) \equiv \frac{1}{2} G_{ij} \sum_{l=1}^{L} k_{ij}\bar{\delta}_{i} - B_{ij}(\bar{\delta} - \bar{\delta})
\]

(25)

\[
P_{ij}^{1} = \frac{1}{2} P_{ij}^{loss}(\delta, \delta) + B_{ij} \sin(\bar{\delta} - \bar{\delta}) \equiv \frac{1}{2} G_{ij} \sum_{l=1}^{L} k_{ij}\bar{\delta}_{i} + B_{ij}(\bar{\delta} - \bar{\delta})
\]

(26)

Then the entire model can be transferred into a mixed integer linear model and it can be solved by state-of-art mixed integer linear programming (MILP) commercial solver.

3.2. An advanced piecewise linearization technique

The piecewise linearization strategy has great effect on the approximation accuracy of the network loss. If less linearization segment is used, the approximation error will be considerable although the computation speed will be fast. If more linearization segment is applied, the approximation accuracy will be better at the expense of a heavier computation burden.

In our paper, it is found that with the variation of number of linearization segment, the value of network loss would change significantly. After careful analysis, it is found that the difference of voltage phase angle is usually very small in reality, which means that the operation point of the simplified quadratic loss formulation is usually around the zero point. Under this condition, even a large number of linearization segment would cause a poor approximation error of network loss. Based on this observation, a piecewise linearization technique with different segment length is proposed. When do the piecewise linearization, more segments is introduced when the operating point is near zero. With the deviation from the zero point, less piecewise linearization segments is introduced. The
proposed linearization technique is tested on the IEEE 33-bus distribution network in the following case study.

4. Case study
The proposed model has been applied to IEEE-33 bus distribution network system. The proposed methodology has been developed in GAMS. The IEEE 33-bus system used in this paper has 33 nodes, 37 existing branches, and 32 loads, it is presented in Figure 2.

The optimal planning results of the capacities of DG and distribution line is showed in table 1 and table 2. It should be noted that the node 1 here is a substation, not DGs is connected, this means the capacity showed on node 1 is the power purchased from upstream grid. And it assumes that the installation location of the DG is given in the model, here for the nodes 5,10,20,30, and then we only need to determine their capacities.

| Node number | Capacity(MW) | Node number | Capacity(MW) | Node number | Capacity(MW) |
|-------------|--------------|-------------|--------------|-------------|--------------|
| 1           | 2.7423       | 5           | 0.0000       | 10          | 0.1914       |
| 5           | 0.0000       | 20          | 1.0000       |

The distribution line capacity planning results when the number of linearization segments of calculating line loss is 180 is as follows:

| Line index | Capacity(MW) | Line index | Capacity(MW) | Line index | Capacity(MW) |
|------------|--------------|------------|--------------|------------|--------------|
| 1          | 2.7423       | 14         | 0.0112       | 27         | 0.1194       |
| 2          | 3.4559       | 15         | 0.1637       | 28         | 0.0170       |
| 3          | 1.2517       | 16         | 0.0418       | 29         | 0.4722       |
| 4          | 1.0336       | 17         | 0.0784       | 30         | 1.1196       |
| 5          | 0.9176       | 18         | 1.7355       | 31         | 0.8097       |
| 6          | 0.5238       | 19         | 1.5397       | 32         | 0.3837       |
| 7          | 0.1700       | 20         | 1.7160       | 33         | 0.6428       |
| 8          | 0.4522       | 21         | 0.8209       | 34         | 0.2815       |
| 9          | 0.0457       | 22         | 1.3692       | 35         | 0.6300       |
| 10         | 0.0479       | 23         | 1.2028       | 36         | 0.2593       |
| 11         | 0.1191       | 24         | 0.4940       | 37         | 0.2070       |
| 12         | 0.3065       | 25         | 0.3313       |
| 13         | 0.1987       | 26         | 0.2244       |

Table 3 gives the itemized cost with and without DG units. It is shown that DGs can help reduce the power loss cost, and the total cost is decreased after DG installed.

Table 4 shows the variation of run time and total cost versus the number of linearization segment, and the corresponding figure is shown in Figure.3. It can be seen that if more linearization segment is applied, the approximation accuracy will be better at the expense of a heavier computation burden, and finally the approximation result tends to a stable value with the linearization segment arises.
Table 3. Index comparison.

| Itemized cost                                | Consider DG ($\times 10^4$/day) | Initial network ($\times 10^4$/day) |
|----------------------------------------------|----------------------------------|------------------------------------|
| DG investment and maintenance costs          | 0.1165                           | 0.0000                             |
| Line investment costs                        | 0.0178                           | 0.0274                             |
| Power purchase and DG operation costs        | 4.7536                           | 4.9383                             |
| Power loss costs                             | 0.0109                           | 0.0200                             |
| Expected interruption costs                  | 0.0000                           | 0.0000                             |
| Total costs                                  | 4.8989                           | 4.9857                             |

Table 4. Computation time and total cost versus the number of segments.

| Number of segments | Line loss (MW) | Calculate time (s) | Number of segments | Line loss (MW) | Calculate time (s) |
|--------------------|----------------|--------------------|--------------------|----------------|--------------------|
| 30                 | 0.5302         | 0.695              | 240                | 0.1458         | 1.986              |
| 45                 | 0.3585         | 0.732              | 300                | 0.1443         | 2.243              |
| 60                 | 0.2867         | 0.750              | 360                | 0.1396         | 2.273              |
| 75                 | 0.2293         | 0.912              | 480                | 0.1294         | 4.589              |
| 90                 | 0.2187         | 0.940              | 600                | 0.1310         | 5.512              |
| 105                | 0.1971         | 1.102              | 720                | 0.1335         | 6.223              |
| 120                | 0.1845         | 1.213              | 960                | 0.1285         | 7.293              |
| 150                | 0.1688         | 1.506              | 1200               | 0.1291         | 9.128              |
| 180                | 0.1531         | 1.155              | 1500               | 0.1288         | 16.753             |

Figure 3. Variation of run time and total cost versus the number of segment.

Table 5 shows the line loss under different segment strategies proposed in part 3.2. From table 5 it can be found that the proposed segmenting strategy yield a very good performance. In strategy 3, only 480 segments is applied, and it can get the same approximation accuracy as original piecewise technique with 1200 segments.

Table 5. Variation of line loss with different segmenting strategies.

| Number of segments in $[0, \pi/12]$ | Number of segments in $[\pi/12, \pi/4]$ | Number of segments in $[\pi/4, \pi/2]$ | Line loss |
|------------------------------------|----------------------------------------|--------------------------------------|-----------|
| Strategy 1: 90                     | 45                                     | 15                                   | 0.1331    |
| Strategy 2: 180                    | 90                                     | 30                                   | 0.1302    |
| Strategy 3: 300                    | 150                                    | 30                                   | 0.1289    |

5. Conclusions
The problem of optimal capacity of DGs and distribution lines in ADN planning has been considered in this paper. The proposed model is based on premises that the location of DGs and the topology of the network are fixed. And a piecewise linearization technique to calculate network loss is explicitly analyzed, and then the model can be transformed into a MILP problem so that it can be solved by
state-of-art MILP commercial solver. An advanced piecewise linearization technique with different segment length is proposed. The effectiveness of the proposed ADN planning model with elaborate linearization technique is verified on the IEEE 33-bus distribution network. The results show that the total cost and line loss cost are decreased after DG installed, and the proposed linearization technique can get a good balance between the approximation accuracy and the computation efficiency.

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