Modifying two-body relaxation in N-body systems by gas accretion

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ABSTRACT

We consider the effects that accretion from the interstellar medium onto the particles of an N-body system has on the rate of two-body relaxation. To this end, we derive an accretion-modified relaxation time by adapting Spitzer’s two-component model to include the damping effects of accretion. We consider several different mass-dependencies and efficiency factors for the accretion rate, as well as different mass ratios for the two components of the model.

The net effect of accretion is to accelerate mass segregation by increasing the average mass $\bar{m}$, since the relaxation time is inversely proportional to $\bar{m}$. Under the assumption that the accretion rate increases with the accretor mass, there are two additional effects that accelerate mass segregation. First, accretion acts to increase the range of any initial mass spectrum, quickly driving the heaviest members to even higher masses. Second, accretion acts to reduce the velocities of the accretors due to conservation of momentum, and it is the heaviest members that are affected the most. Using our two-component model, we quantify these effects as a function of the accretion rate, the total cluster mass, and the component masses. We conclude by discussing the implications of our results for the dynamical evolution of primordial globular clusters, primarily in the context of black holes formed from the most massive stellar progenitors.

Key words: globular clusters: general – stellar dynamics – stars: formation – black hole physics.

1 INTRODUCTION

For most of the life of a massive star cluster, two-body relaxation is the dominant physical mechanism driving its evolution (e.g. Henon 1960, 1973; Spitzer 1987; Heggie & Hut 2003; Gieles, Heggie & Zhao 2011). That is, the cumulative effects of long-range gravitational interactions between stars act to alter their orbits within the cluster. These interactions push the cluster toward a state of energy equipartition in which all objects have roughly the same kinetic energy. Consequently, the velocities of the most massive objects decrease, and they accumulate in the central regions of the cluster. Similarly, the velocities of the lowest mass objects increase, and they are subsequently dispersed to wider orbits. This mechanism, called mass segregation, also contributes to the escape of stars from their host cluster across the tidal boundary, with the probability of ejection increasing with decreasing stellar mass. Therefore, two-body relaxation acts to slowly modify the radial distribution of stellar masses within clusters, and can cause very dynamically evolved clusters to be severely depleted of their low-mass stars (e.g. von Hippel & Sarajedini 1998, De Marchi, Paresce & Portegies Zwart 2011, Leigh et al. 2012).

Energy equipartition is an idealized state that should arise after the cumulative effects of many long-range interactions. In a real star cluster with a full spectrum of stellar masses, however, equipartition may not actually be achievable (e.g. Binney & Tremaine 1987; Heggie & Hut 2003). As mentioned, the tendency towards energy equipartition reduces the velocities of the heaviest stars, causing them to sink in to the central cluster regions. Here, they are re-accelerated by the central cluster potential and gain kinetic energy. As this process proceeds, it leads to a contraction of the core and subsequently a shorter central relaxation time (e.g. Spitzer 1987; Heggie & Hut 2003). A shorter relaxation time leads to a faster rate of energy transfer from heavier to lighter stars. Eventually, this makes the heaviest stars evolve away from equipartition.

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This was first demonstrated by Spitzer (1969) using analytic techniques and a number of simplifying assumptions. Spitzer (1969) adopted a two-component system with masses $m_1$ and $m_2$ (where $m_1 > m_2$), forming sub-systems with total masses $M_1$ and $M_2$. Provided that $M_1 < M_2$, Spitzer derived the conditional requirement for a cluster to achieve energy equipartition in equilibrium. Based on this, Spitzer argued that energy equipartition could not be achieved in a cluster with a realistic mass spectrum, since there should always be enough mass in the heavier species for it to form a sub-system in the central cluster regions that decouples dynamically from the lighter species. This is commonly called the Spitzer instability (e.g. Spitzer 1987; Heggie & Hut 2003; Portegies Zwart et al. 2004).

A particularly compelling example of the Spitzer instability involves stellar-mass black holes (BHs) in globular clusters (GCs). Phinney & Sigurdsson (1991) first argued that BHs formed from the most massive stars should rapidly segregate into the core where they decouple dynamically from the rest of the cluster to form a distinct sub-system. Three-body scattering events then lead to the formation of BH-BH binaries, which in turn encourage other BHs and BH-BH binaries. These 3- and 4-body interactions are sufficiently energetic to eject the BHs from the cluster. In the end, most BHs are expected to be ejected, leaving only a handful behind.

This picture has recently been challenged in the literature. In particular, several authors have argued that the Spitzer instability should break down before most BHs are ejected (e.g. Moody & Sigurdsson 2009), and that the timescale for all BHs to be ejected could exceed a Hubble time in some clusters (e.g. Downing et al. 2010). This view is supported by recent claims in the literature that stellar-mass BHs may be present in GCs in surprising numbers. For instance, Strader et al. (2012) recently reported two flat-spectrum radio sources in M22, which appear to be accreting stellar-mass BHs. This suggests that this cluster could contain on the order of $\sim 5\sim 100$ stellar-mass BHs. If BHs were indeed efficiently dynamically ejected, this, in turn, would suggest that a more substantial population of BHs once existed in M22, and likely other GCs as well.

The emerging picture for the formation of massive GCs involves multiple episodes of star formation (e.g. Piotto et al. 2007; Gratton, Carretta & Bragaglia 2012; Conroy & Spergel 2012). In this context, Leigh et al. (2013) recently considered the implications of the mass growth of BHs formed from massive progenitors belonging to the first generation due to accretion from the interstellar medium. The authors argued that, in principle, BHs could deplete a significant fraction of the available gas reservoir within $\lesssim 10^7$ years. If BHs were indeed to accrete efficiently from the ISM, they should not only grow in mass, but their velocities should also decrease due to conservation of momentum. This should preferentially accelerate the process of mass segregation for the BHs, causing them to rapidly accumulate in the central regions of the cluster if they did not form there in the first place. This could accelerate the dynamical decoupling of the BH sub-population from the rest of the system, and hence the phase of dynamical BH ejections due to the Spitzer instability.

In this paper, we consider how accretion from the interstellar medium affects the rate of mass segregation in a star cluster. We are especially interested in the implications for BHs in primordial GCs. Thus, we re-visit Spitzer’s two-component model to derive an accretion-modified relaxation time. We argue that the rate of mass segregation should be affected by accretion in the following way. First, assuming the accretion rate increases with the accretor mass, accretion acts to increase the range of any initial mass spectrum, driving the heaviest members to higher masses the fastest. Second, accretion acts to reduce the velocities of the accretors due to conservation of momentum, and it is the heaviest members whose velocities are reduced the fastest. Both of these effects exacerbate the Spitzer instability, and should accelerate the rate of mass segregation in a primordial star cluster.

In order to better quantify this qualitative picture, we present our adapted version of Spitzer’s two-component model in Section 2. Specifically, we derive an accretion-modified relaxation time, as well as the critical accretion rate at which the rates of mass segregation due to both two-body relaxation and accretion are equal. We present our results in Section 3 for several different assumptions regarding the total cluster mass and accretion rate. In Section 4, we discuss the implications of our results for both star formation and stellar remnants in primordial globular clusters. We summarize our results in Section 5.

2 METHOD

In this section, we present our analytic derivation of an accretion-modified relaxation time for a two-component model star cluster, and derive the critical accretion rate required for the mass segregation timescales due to two-body relaxation and accretion to be equal. We begin by summarizing briefly the relevant background related to both two-body relaxation and accretion.

2.1 Two-body relaxation

Consider a two-component model for a star cluster with component masses $m_1$ and $m_2$, such that $m_1 > m_2$. The populations for these two species have total masses $M_1$ and $M_2$ with $M_1 \ll M_2$. We let $v^2$ denote the initial mean square speed of both species, since at birth the cluster is not in a state of energy equipartition. The e-folding time for the tendency to equipartition bears a striking resemblance to the relaxation time (Heggie & Hut 2003). Thus, to order of magnitude, the relaxation time can be approximated by calculating the time required for the mean square speed of the heavier species to fall from $v^2$ to a value $\sim m_2 v^2 / m_1$.

If the potential well of the lighter species is modelled using a parabolic profile, then equipartition will lead to the heavier species being confined to a region of size $\sim r_h \sqrt{m_2 / m_1}$ (Heggie & Hut 2003). The total mass of the heavier species within this region is $M_1$, whereas that for the lighter species is $M_2 (m_2 / m_1)^{3/2}$. At this point, however, it is not clear whether or not the lighter species remains the dominant mass component in this region. If not, the heavier species becomes increasingly affected by its own self-gravity, and can decouple dynamically from the remainder of the system. Consequently, it may only be possible to achieve
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This is known as Spitzer’s criterion.

In general, theoretical work has shown that a wide range of accretion rates are possible. We will use the Bondi-Hoyle and Eddington limits for the accretion rate (combined with an accretion efficiency parameter) since these provide two different dependences on the accretor mass. However, the derivation presented in the subsequent section can be used to model other mass-dependences for the accretion rate as well.

2.3 Deriving the Relaxation Time

We are interested in the mass segregation timescale due to two-body relaxation for the heavier species in the two-component model described in Section 2.1. To first order, this is approximated by:

\[ \tau_{\text{seg,2body}}(m_1, t) = \frac{\bar{m}(t)}{m_1(t)} \tau_{\text{bh}}(t), \]  

where \( \tau_{\text{bh}}(t) \) denotes the half-mass relaxation time obtained by using the total number of objects \( N = N_1 + N_2 \) and average object mass \( \bar{m}(t) = (m_1(t)N_1 + m_2(t)N_2)/(N_1 + N_2) \) in Equation 2.

We calculate the time-dependence for the mass of an object belonging to species 1 as follows. First, assuming an object of initial mass \( m_1(0) \) accretes at a rate \( \dot{m}_i = dm_i/dt \) for a total time \( t \), we have:

\[ m_i(t) = m_i(0) + \int_0^t \dot{m}_i dt. \]  

For the accretion rate, we assume a mass-dependence of the form:

\[ \dot{m}_i = \lambda \delta m_i^\epsilon, \]  

where \( \lambda, \delta, \) and \( \epsilon \) are all free parameters. The power-law exponent \( \epsilon \) decides the mass-dependence for the accretion rate. The accretion coefficient \( \delta \) is derived according to the physical assumptions that decide the rate of accretion. For example, adopting the Bondi-Hoyle approximation implies \( \epsilon = 2 \), and (e.g. Bondi & Hoyle 1944; Maccarone & Zurek 1999).
from a root-mean-square speed of $v_{rms}$ to change the velocities of the heavier species by $\delta$.

We calculate the time needed for accretion to affect the stellar velocities should $\delta$ be roughly the same amount as $\delta$. Provided the accretion rate increases with increasing accretor mass, this will reduce the kinetic energy of the heavier species. Provided the accretion rate increases with increasing accretor mass, this will reduce the kinetic energy of the heavier species and affect the stellar velocities via conservation of momentum.

Equation 9 or Equation 10 for both species 1 and 2 into the expression for $\tau_{seg, acc}$ yields

$$\tau_{seg, acc}(m_1, t) = \frac{m_1(t)^{1+\epsilon} (m_1(t)/m_2(t))^{(1-\epsilon)/2} - 1}{\lambda \delta (1 - \epsilon)}.$$  

for $\epsilon > 1$. Similarly, for $\epsilon = 1$ or Eddington-limited accretion, we obtain using Equation 10

$$\tau_{seg, acc}(m_1, t) = \frac{\ln(m_1(t)/m_2(t))}{2 \lambda \delta}.$$  

We consider accretion rates with a mass-dependence such that $\epsilon \geq 1$, since this includes both Bondi-Hoyle and Eddington-limited accretion, as well as intermediate and even steeper mass accretion rates.

The total rate (taken to be the inverse of the total mass segregation timescale $\tau_{seg, tot}$) at which the heavier species achieves mass segregation can be written as the sum of the rate of two-body relaxation and the rate at which accretion pushes the heavier species to equipartition. Re-arranging this equation, we arrive at the total accretion-modified mass segregation timescale for the heavier species:

$$\tau_{seg, tot}(m_1, t) = \frac{\tau_{seg, acc}(m_1, t)_{\tau_{seg, 2body}}(m_1, t)}{\tau_{seg, acc}(m_1, t) + \tau_{seg, 2body}(m_1, t)}.$$  

2.4 Deriving the critical accretion rate

To derive the critical accretion rate $\delta_{crit}$ at which two-body relaxation and accretion drive the mass segregation process at the same rate, we set $\tau_{seg, acc} = \tau_{seg, 2body}$ and solve for $\delta$ as a function of $\epsilon$, $\lambda$, $m_1$, and $m_2$. This gives for $\epsilon > 1$:

$$\delta_{crit} = \frac{m_1(t)^{2-\epsilon} (m_1(t)/m_2(t))^{(1-\epsilon)/2} - 1}{\lambda (1 - \epsilon) \delta(t) \bar{\delta}(t)}.$$  

The procedure is similar for $\epsilon = 1$, except we use Equation 13 instead of Equation 12. This gives for Eddington-limited accretion:

$$\delta_{crit} = \frac{m_1(t) \ln(m_1(t)/m_2(t))}{2 \lambda \bar{\delta}(t) \bar{\delta}(t)}.$$  

In Section 5.1 we will use Equation 13 and Equation 16 in order to study the interplay between our assumptions regarding the gas properties, which affect the accretion rate, and our assumption for the total cluster mass, which determines the rate of two-body relaxation.

2.5 Accretion efficiency

Given our limited understanding of the precise physics of accretion onto a BH, it is not possible to reliably define a functional form for the accretion efficiency parameter $\lambda$. In principle, any realistic accretion model should include a time-dependence for $\lambda(t)$. For example, fluctuations in the local gas density due to turbulence, a gradual or even sudden depletion of the available gas reservoir, or dynamical interactions between accreting objects may cause the accretion efficiency to vary over time.

Our analysis is easily modified to treat time-dependent accretion rates by substituting an appropriate choice for $\lambda(t)$.
into either Equation 12 or 13 and then solving for $\tau_{\text{seg,acc}}$. Plausible choices for $\lambda(t)$ may either oscillate or decline (steadily or abruptly) in time. The first case, i.e. an oscillating accretion efficiency, is more easily understood, because under these circumstances, our analysis can simply be interpreted as discussing the time-averaged accretion efficiency parameter. Thus, in the subsequent sections, we assume an oscillating (or constant) accretion efficiency parameter, and discuss only the time-averaged value.

For example, the function $\lambda(t) = (1 + \sin(\pi t / t_0))$ oscillates between 0 and 2 with a frequency of $2 / t_0$. In this case, the time-averaged value for $\lambda(t)$ is equal to 1, so that the time-averaged value for $\tau_{\text{seg,acc}}$ remains the same as for a constant $\lambda = 1$.

Accretion efficiency parameters that oscillate in time should be appropriate to cases where the accretors have alternating “on” and “off” phases. This may well be the case with accreting BHs, since the radiation emitted due to accretion can heat the surrounding gas, which in turn decreases the accretion rate (e.g. Blaes, Warren & Malalay 1992). In this case, the source of energetic photons responsible for heating the gas is turned off, allowing the gas to cool and accretion to re-start in a “feedback regulated” loop (e.g. King & Pounds 2003; Yuan, Xie & Ostriker 2009).

Accretion efficiency parameters that decline in time should be appropriate to cases where the available gas reservoir is depleted over time. This could arise gradually if the gas is used to form stars, or if significant quantities of gas are accreted by BHs. Alternatively, the gas reservoir could be depleted suddenly, e.g. due to, energy injected from supernovae, stellar winds, or winds from accreting compact objects. In either case, Equations 12 and 13 should include the explicit time-dependence for the accretion efficiency parameter. This will contribute to an increase in $\tau_{\text{seg,acc}}$ with time, since the decreasing gas mass should translate into a decreasing gas density, and hence accretion rate. In Section 3 we will assume that the amount of gas lost from the system is negligible over the calculated mass segregation timescales, and our interpretation of $\lambda$ as a time-independent quantity remains valid. This is reasonable provided the mass segregation timescales due to accretion are much less than the timescale for gas depletion. As we will show in Section 3, the current picture for the formation of globular clusters and their multiple populations is consistent with this scenario (e.g. Krause et al. 2012, 2013; Leigh et al. 2013).

3 RESULTS
In this section, we present the results of our analytic two-component model for an accretion-modified two-body relaxation time. Our aim is to quantify the relative rates at which two-body relaxation and gas accretion drive a star cluster towards mass segregation, as a function of our assumptions for the gas properties, component masses, and total system mass. To this end, we present the time evolution of all three mass segregation timescales, namely $\tau_{\text{seg,acc}}$, $\tau_{\text{seg,2body}}$, and $\tau_{\text{seg,tot}}$, and discuss the critical accretion rate required for the mass segregation timescales due to two-body relaxation and accretion to be equal as a function of the mass-dependence for the accretion rate.

3.1 Time evolution of the mass segregation timescales
We begin by quantifying the relative rates of mass segregation due to two-body relaxation and gas accretion for different model assumptions. Specifically, we consider several different mass ratios and total system masses for our two-component model, as well as different mass-dependences for the rate of accretion. This is meant to quantify the sensitivity of the two different mass segregation mechanisms to the cluster and gas properties that decide their rates.

First, we describe our assumptions for the two-component model star cluster, which are needed in order to calculate $\tau_{\text{seg,2body}}$. We adopt $m_2 = 1 M_\odot$ for the lighter species, but consider two different masses for the heavier species, namely $m_1 = 10 M_\odot$ and $m_1 = 50 M_\odot$. We assume a population size of $N_1 = 10^5$ for the heavier species, but vary the population size of the lighter species by considering the values $N_2 = 10^3, 10^4, 10^5$. The component masses and population sizes are chosen to represent reasonable mass ratios between the average stellar and BH masses, and to ensure that the Spitzer criterion (i.e. Equation 1) is initially satisfied. We adopt a half-mass radius for our model cluster of $r_h = 10$ pc, and note that assuming a lower value for the half-mass radius would only shorten the calculated mass segregation timescales.

Next, we describe our assumptions for the properties of the accreted gas, which are needed to calculate $\tau_{\text{seg,acc}}$, and therefore $\tau_{\text{seg,tot}}$. We assume a uniform time-independent gas density throughout the cluster, so that the accretion rate changes only with the stellar mass. We further assume that the gas is always at rest relative to the accretor when calculating the final accretor velocity using conservation of momentum. For the accretors, we adopt a root mean-square-speed of $v = 10$ km s$^{-1}$, which is guided by the relation $v = \sqrt{2GM/r_h}$ (Binney & Tremaine 1987) for a total cluster mass $M \sim 10^6 - 10^8$ M$_\odot$. For the gas, we assume a sound speed of $c_s = 10$ km s$^{-1}$, and a particle number density of $n = 10^6$ cm$^{-3}$. These assumptions are representative of dense giant molecular clouds, and should be reasonable for what is expected in a massive primordial GC for the first $\sim 10^8$ years (e.g. Dehnen et al. 2008; Maccarone & Zurek 2012; Conroy 2012; Krause et al. 2012; Leigh et al. 2013).

We show our results for two different mass-dependencies for the accretion rate. The left panels in Figure 1 show our results assuming $\epsilon = 2$ in Equation 8 which corresponds to Bondi-Hoyle accretion. We use Equation 7 for $\delta$, and $\lambda = 0.1$ for the accretion efficiency parameter. The panels to the right in Figure 1 show our results assuming Eddington-limited accretion, which means that $\epsilon = 1$ in Equation 8, and we use Equation 5 for $\delta$.

The main conclusion to be drawn from Figure 1 is that, for all but the least massive clusters and the lowest accretion rates considered here, the rate of mass segregation due to accretion can actually exceed the rate due to two-body relaxation. The timescale at which this occurs is on the order of $\sim 10^8$ years. Interestingly, this timescale is similar to the total time thought to be required for multiple episodes of star formation to occur in primordial GCs (e.g. Conroy & Spergel 2011; Conroy 2012). Thus, our results suggest that accretion from the ISM could significantly affect
both the spatial and velocity distributions of the heaviest objects in a primordial GC before the gas reservoir is depleted. For a typical primordial GC, this should be the case provided the average accretion rate is greater than $\sim 5 - 10\%$ of the Eddington-limited rate, assuming the mass-dependence for the accretion rate is linear. Similarly, if the accretion rate scales with the square of the accretor mass, then accretion from the ISM is non-negligible as long as the average accretion rate is greater than $1\text{-}10\%$ of the Bondi-Hoyle rate.

### 3.2 The critical accretion rate

In this section, we calculate the critical accretion rate required for the rates of mass segregation due to two-body relaxation and gas accretion to be equal. Our aim is to quantify the relative importance of the different parameters for the cluster and gas properties in establishing a balance between the competing effects of two-body relaxation and accretion.

In Figure 2, we show the critical accretion rate $\delta_{\text{crit}}$ as a function of the mass of the heavier species $m_1$. These results are calculated using Equations (15) and (16) which correspond to Bondi-Hoyle (blue) and Eddington-limited (red) accretion, respectively. In both cases, we assume a constant mass for the lighter species of $m_2 = 1 \text{ M}_\odot$, a constant population size for the heavier species of $N_1 = 10^5$, and a constant accretion efficiency parameter $\lambda = 1.0$. In order to vary the rate of two-body relaxation without affecting the rate of mass segregation due to accretion, we consider three different population sizes for the lighter species, namely $N_2 = 10^5, 10^6, 10^7$.

Figure 2 shows that for the case of Eddington-limited accretion, the critical accretion rate increases with increasing accretor mass. This is because, as the accretor mass in-
creases, the mass segregation timescale due to two-body relaxation decreases faster than the mass segregation timescale due to accretion. In the case of Bondi-Hoyle accretion, however, the critical accretion rate depends only very weakly on the accretor mass, which is due to the fact that the accretion rate scales as the square of the accretor mass. We emphasize that with the exception of the Eddington-limited rate at large accretor masses, the critical accretion rates are comparable to, or even smaller than, those observed in nearby star-forming regions (e.g. McKee & Ostriker 2007).

4 DISCUSSION

In this section, we discuss the implications of our results for mass segregation in primordial globular clusters, in particular with regards to black holes.

4.1 Enhanced mass segregation

One of the key conclusions arising from our analysis is that accretion should accelerate the rate at which a star cluster becomes mass segregated compared to two-body relaxation alone. In fact, accretion can dominate over two-body relaxation in massive clusters for accretion rates that are below the Bondi-Hoyle or Eddington-limited rates by one or even two orders of magnitude. This is because the relaxation time increases with the cluster mass, whereas the mass segregation timescale due to accretion is independent of the cluster mass (assuming that the gas properties are independent of the cluster mass). Our results suggest that two-body relaxation should dominate the mass segregation process in low-mass primordial clusters with global relaxation times $\lesssim 10^7$ – $10^8$ years and hence total cluster masses $\lesssim 10^4$ – $10^5 M_\odot$, provided that our models assumptions are valid. In this regime, accretion should only have a small effect on the rate of mass segregation, and long-range gravitational interactions should alter the accretors’ velocities faster than they are reduced by the accretion process. In more massive clusters, however, the damping effects of accretion could play a significant role in accelerating the rate of mass segregation.

4.2 The effects of a realistic mass spectrum

Our assumption of a two-component model serves to demonstrate the effects of accretion from the ISM on a cluster’s dynamical evolution. The qualitative nature of our results should hold if a realistic mass spectrum is adopted instead. Accretion can modify the distribution of velocities on relatively short timescales in gas-embedded clusters. How exactly the velocities become modified depends on several parameters, in particular the mass spectrum, the total cluster mass, the properties of the gas, and the accretion rate.

In general, we expect accretion to amplify or exacerbate the Spitzer instability. This is due to the mass-dependence of the accretion rate, and the fact that typically $\epsilon > 0$ in Equation 6 (i.e. the accretion rate), which causes the more massive component to grow in mass the fastest. Thus, according to Equation 11 Spitzer’s criterion should typically break down sooner as a result of accretion. However, accretion also acts to reduce the velocities of the accretors due to conservation of momentum, and this should most strongly impact the most massive objects due once again to the mass-dependence of the accretion rate. This can actually serve to combat the effects of the Spitzer instability by inhibiting the most massive objects from decoupling dynamically from the rest of the system once they have segregated to the central regions of the cluster. Clearly, a more sophisticated treatment will be needed in future studies in order to properly quantify these effects and their implications for the Spitzer instability, and a cluster’s ability to achieve energy equipartition.

4.3 Gas properties and the accretion rate

We stress that our results depend sensitively on our assumption for the accretion rate, which is poorly constrained, both theoretically and observationally. Indeed, the accretion efficiency parameter $\lambda$ adopted in Equation 6 is needed to account for the many sources of uncertainty in the gas properties, and hence the accretion rate. For example, our assumption of a uniform, time-independent gas density is an over-simplification. For one, stellar winds and supernovae could create over- and under-densities in the form of sheets and/or filaments, and the efficiency of these processes should fluctuate in time given the presence of a realistic mass function combined with stellar evolution and the cluster dynamics (e.g. Krause et al. 2012, 2013). These effects could contribute to a reduction in the accretion rate by increasing the relative velocity between the gas and the accretors, or by reducing the gas density along the trajectories of the accretors. Realistic hydrodynamical simulations of star cluster formation will be needed in order to properly quantify these effects and their implications for the accretion rate.

Our results can be used to guide the parameter space relevant to these future studies. In particular, we have placed a lower limit on the minimum accretion rate required for accretion to significantly affect the distribution of stellar velocities on timescales shorter than the relaxation time, as a function of the cluster and gas properties. Specifically, the results of our simple model suggest that, for a typical primordial GC, the average accretion rate cannot be much less than $\sim 5 – 10\%$ of the Eddington-limited rate, assuming the mass-dependence for the accretion rate is linear. Similarly, for our model assumptions, the average accretion rate cannot be much less than $1 – 10\%$ of the Bondi-Hoyle rate if the accretion rate scales with the square of the accretor mass.

We have adopted the same root-mean-square speed for all models, independent of the total cluster mass. This is a reasonable assumption since the root-mean-square speed scales as $v \propto (M/r_h)^{0.5}$, and $r_h$ itself depends weakly on the total cluster mass. Thus, in total, the root-mean-square speed depends only very weakly on the total cluster mass. Nevertheless, if the accretion rate scales inversely with the velocity of the accretor, as is the case with the Bondi-Hoyle approximation, then the dependence of the root-mean-square speed on the total cluster mass should contribute to a decrease in the accretion rate with increasing cluster mass. A proper treatment of this effect is beyond the scope of this paper, however, it should certainly be considered in future studies.
4.4 Black hole dynamics

The results presented in this paper are especially relevant for black holes in primordial globular clusters, since they should be the most massive objects in the cluster within a few Myrs of its formation. Recent evidence suggests that there should be a substantial gas reservoir in GCs for the first $\sim 10^5$ years (e.g. Conroy & Spergel 2011), albeit perhaps intermittently, and that nearly all BHs should form from the most massive cluster members within the first few Myrs (e.g. Maeder 2000). It follows that the BHs could have on the order of $10^5$ years to accrete gas from the ISM. Additionally, since any BHs formed from progenitors more massive than $\sim 50 \, M_\odot$ are only slightly less massive than the progenitors themselves and do not experience natal kicks (Fryer et al. 2012), these BHs should have both the shortest mass segregation timescales due to two-body relaxation and the highest accretion rates (ignoring BH winds and/or Compton heating; see below).

The key point is that accretion should act to reduce the mass segregation times of BHs in primordial GCs, and that this effect could be dramatic. Beyond this, more detailed modeling will be needed to determine the fates of the BHs. In particular, should the increased rate of mass segregation contribute to accelerating the onset of the hypothesized phase of dynamical BH ejections? Or could the damping effects of accretion be so dramatic that the BHs are driven to merge (e.g. Davies, Miller & Bellovary 2011)? If so, the formation of an intermediate-mass BH (IMBH) could be the inevitable result. Alternatively, it could be that black hole winds are sufficiently powerful to ejet the bulk of the gas from the cluster. Another possibility is that the gas in the immediate vicinity of the BHs becomes very hot due to, for example, Compton heating (Blaes, Warren & Madau 1994; Yuan, Xie & Ostriker 2009), such that the accretion rate becomes drastically reduced and BH growth is severely limited?

A better understanding of how the presence of significant quantities of gas modifies the black hole dynamics in a primordial GC could help to constrain the initial cluster conditions. For example, if massive BHs should inevitably merge in the presence of gas but no IMBHs are observed in present-day GCs, does this necessarily imply that the BHs never formed in the first place? If so, this would suggest that stars with masses $\geq 50 \, M_\odot$ must have been rare. This could be the case, for instance, if massive primordial GCs were assembled from the mergers of many low-mass sub-clumps, as opposed to a single monolithic collapse. This is because the mass of the most massive cluster member correlates with the total cluster mass (e.g. Kirk & Myers 2011, 2012), and hence the massive stellar progenitors of the most massive BHs are unlikely to form in low-mass clusters.

5 SUMMARY

In this paper, we have considered the effects of accretion from the interstellar medium on the rate of two-body relaxation in a star cluster. To do this, we derived an accretion-modified relaxation time by adapting Spitzer’s two-component model to include the effects of accretion. We considered several different mass-dependencies and efficiency factors for the accretion rate, as well as different mass ratios for the two components of the model.

We have shown that accretion acts to increase the rate of mass segregation. This is because the relaxation time is inversely proportional to the average mass, which increases due to accretion. There are two additional effects that accelerate the mass segregation process, assuming that the accretion rate increases with the accretor mass. First, accretion acts to increase the range of any initial mass spectrum, quickly driving the heaviest members to even higher masses. Second, accretion acts to reduce the velocities of the accretors due to conservation of momentum, and it is the heaviest members that are affected the most. Using our two-component model, these effects have been quantified as a function of the accretion rate, the total cluster mass, and the component masses. We have discussed our results in the context of the dynamical evolution of primordial globular clusters and their black hole sub-populations.

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REFERENCES

Bauzardt H., Makino J. 2003, MNRAS, 340, 227
Bauzardt H., De Marchi G., Kroupa P. 2008, ApJ, 685, 247
Binney J., Tremaine S., 1987, Galactic Dynamics (Princeton: Princeton University Press)
Blaes O., Warren O., Madau P. 1995, ApJ, 454, 370
Bondi H., Hoyle F. 1944, MNRAS, 104, 273
Conroy C., Spergel D. N. 2011, ApJ, 726, 36
Conroy C. 2012, ApJ, 758, 21
Davies M., Miller M. C., Bellovary J. M. 2011, ApJ, 740, 42
De Angeli F., Piotto G., Cassisi S., Busso G., Recio-Blanco A., Salaris M., Aparicio A., Rosenberg A. 2005, AJ, 130, 116
De Marchi G., Paresce F., Pulone L. 2007, ApJ, 656, L65
De Marchi G., Paresce F., Portegies Zwart S. 2010, ApJ, 718, 105
D’Ercole A., Vesperini E., D’Antona F., McMillan S. L. W., Recchi S. 2008, MNRAS, 391, 825
Downing J. M. B., Benacquista M. J., Giersz M., Spurzem R. 2010, MNRAS, 407, 1946
Dopita M. A., Smith G. H. 1986, ApJ, 304, 283
Eddington A. S. 1926, The Internal Constitution of the Stars (Cambridge: Cambridge University Press)
Eddington A. S. 1930, MNRAS, 90, 279
Full S. M., Zhang Q. 2001, ApJ, 561, 751
Foglizzo T., Ruffert M. 1990, A&A, 347, 901
Fryer C. L., Kalogera V. 2001, ApJ, 554, 548
Fryer C. L., Belczynski K., Wiktorowicz G., Dominik M., Kalogera V., Holz D. E. 2012, ApJ, 749, 91
Fryxell B. A., Taam R. E. 1988, ApJ, 335, 862
Gieles M., Heggie D., Zhao H. 2011, MNRAS, accepted
Two-body relaxation modified by gas accretion

Gratton R., Carretta E., Bragaglia A. 2012, Astronomy & Astrophysics Review, in press (arXiv:1201.6526)

Harris, W. E. 1996, AJ, 112, 1487 (2010 update)

Heggie D. C., Hut P. 2003, The Gravitational Million-Body Problem: A Multidisciplinary Approach to Star Cluster Dynamics (Cambridge: Cambridge University Press)

Heggie D. C., Giersz M. 2008, MNRAS, 389, 1858

Heggie D. C., Giersz M. 2009, MNRAS, 397, 46

Henon M. 1960, Annales d’Astrophysique, 23, 668

Henon M. 1973, Dynamical Structure and Evolution of Dense Stellar Systems, ed. L. Martinet & M. Mayor (Geneva Obs.) Pringle J. E. 2006, MNRAS, 373, L90

Hoyle F., Lyttleton R. A. 1939, in Proceedings of the Cambridge Philosophical Society, 35

King A. R., Pounds K. A. 2003, MNRAS, 345, 657

Kirk H., Myers P. C. 2011, ApJ, 727, 64

Kirk H., Myers P. C. 2012, ApJ, 745, 131

Krause M., Charbonnel C., Decressin T., Meynet G., Prantzos N., Diehl R. 2012, A&A, 546, L5

Krause M., Charbonnel C., Decressin T., Meynet G., Prantzos N., 2013, A&A, accepted (arXiv:1302.2494)

Krumholz M. R., McKee C. F., Klein R. I. 2004, ApJ, 611, 399

Krumholz M. R., McKee C. F., Klein R. I. 2005, ApJ, 618, 757

Krumholz M. R., McKee C. F., Klein R. I. 2006, ApJ, 638, 369

Leigh N. W., Umbreit S., Sills A., Knigge C., Glebbeek E., Sarajedini A. 2012, MNRAS, 422, 1592

Leigh N. W., Böker T., Maccarone T. J., Perets H. B. 2013, MNRAS, 429, 2997

Maccarone T. J., Zurek D. R. 2012, MNRAS, 423, 2

Maeder A. 2009, Physics, Formation and Evolution of Rotating Stars. Berlin: Springer-Verlag

Marks M., Kroupa P., Baumgardt H. 2008, MNRAS, 386, 2047

Marks M., Kroupa P. 2010, MNRAS, 406, 2000

McKee C. F., Ostriker E. C. 2007, ARA&A, 45, 565

Moody K., Sigurdsson S. 2009, ApJ, 690, 1370

Paczynsky B., Wiita P. J. 1980, A&A, 88, 23

Park K., Ricotti M. 2013, ApJ, submitted (arXiv:1211.0542)

Phinney S. E., Sigurdsson S. 1991, Nature, 349, 220

Piotto G., Bedin L. R., Anderson J., King I. R., Cassisi S., Milone A. P., Villanova S., Pietrin- ferri A., Renzini A. 2007, ApJ, 661, L53

Portegies Zwart S. F., Baumgardt H., Hut P., Makino J., McMillan S. L. W. 2004, Nature, 428, 724

Ruffert M. 1994, ApJ, 427, 342

Ruffert M. 1997, A&A, 317, 793

Rybicki G. B., Lightman A. P. 1979, Radiative Processes in Astrophysics (New York: Wiley-Interscience)

Spitzer L. Jr. 1969, ApJ, 158, 139

Spitzer L. Jr. 1987, Dynamical Evolution of Globular Clusters (Princeton, NJ: Princeton Univ. Press)

Strader J., Chomiuk L., Maccarone T. J., Miller-Jones J. C. A., Seth A. C. 2012, Nature, 490, 71

Tremaine S. D., Ostriker J. P., Spitzer L. Jr. 1975, ApJ, 196, 407

Tutukov A. V. 1978, A&A, 70, 57

Vesperini E., Heggie D. C. 1997, MNRAS, 289, 898

Vishniac E. T. 1978, ApJ, 223, 986

von Hippel T., Sarajedini A. 1998, AJ, 116, 1789

Webb J. J., Harris W. E., Sills A. 2012, ApJ, 759, 39

Yuan F., Xie F., Ostriker J. P. 2009, ApJ, 691, 98

Zonoozi A. H., Kupper A. H. W., Baumgardt H., Haghi H., Kroupa P., Hilker M. 2011, MNRAS, 411, 1989

Zhang Q., Fall S. M. 1999, ApJ, 527, 81

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