Photon or meson formation in $J/\psi$ decays into $p\bar{p}$

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The measurements of the $J/\psi \rightarrow \gamma p\bar{p}$ decays by the BES Collaboration indicate an enhancement at the $p\bar{p}$ threshold which, however, is not present in the $J/\psi$ decays into $\omega p\bar{p}$ and into $\pi p\bar{p}$. Here, two processes for describing the decays $J/\psi \rightarrow B p\bar{p}$ where $B = \gamma, \omega$ are presented in some detail and the cases $B = \phi, \pi$ are briefly touched on. The first process, applied not only to the radiative decay to reproduce the threshold peak but also to the $\omega p\bar{p}$ decay channel to improve the description of the spectrum, postulates a direct emission of the boson before the baryon pair is formed. The second process assumes that the boson $B$ is emitted from the baryon pair following the $J/\psi$ decay and includes for the decays into $\gamma p\bar{p}$ a final state nucleon-antinucleon interaction based on the Paris $NN$ potential. The reproduction of the $p\bar{p}$ distribution in the $J/\psi \rightarrow \omega p\bar{p}$ decays needs a final state interaction involving a $N(2050)\ 3/2^-$ resonance. The photon- and meson-emission rates are reproduced in a semi-quantitative way.

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I. INTRODUCTION

The $J/\psi$ decays with a proton-antiproton ($p\bar{p}$) pair in the final state are interesting for at least two reasons:

- They are related to the searches for exotic states in the nucleon-antinucleon ($NN$) systems. Such searches have been pursued for a few decades, but significant results have been obtained only recently;
- They are closely related to the $p\bar{p}$ reactions, planned at FAIR [1], aiming at the formation of the $J/\psi$ in atomic nuclei.

The first topic is discussed in this paper but the model developed here may be useful to describe the second one. Indication of exotic states below the $NN$ threshold may be given by scattering lengths for a given spin and isospin state. However, a clear separation of quantum states in scattering experiments is not easy. Equivalent measurements of the X ray transitions in the antiproton hydrogen atoms could also select some partial waves if the fine structure of atomic levels is resolved. So far, only partial selections have been achieved [2]. On the basis of the existing data, the present authors have argued that even averaged fine structure atomic level widths in the lightest atoms indicate the existence of quasi-bound $NN$ states [3]. Full resolution of the hyperfine structures should be the purpose of future experiments.

To reach specific states one can also use formation experiments. For instance, in the radiative $J/\psi$ decay,

$$J/\psi \rightarrow \gamma p\bar{p},$$

an enhancement close to the $p\bar{p}$ threshold has been observed by the BES $^1$ Collaboration [4,5]. We note that both the $J/\psi$ and the photon have $J^{PC} = 1^{--}$. There are three final $p\bar{p}$ states allowed by parity, $P$, and charge-conjugation,

\footnote{1 Beijing Electron Sprectrometer}
$C$, conservations in the $\gamma p\bar{p}$ channel: $^1S_0$, $^3P_1$, and $^3P_0$. Table [I] and Table [II] indicate the allowed $p\bar{p}$ states, denoted by $^2S+1L_J$ or $^{2L+1}S+1L_J$, where $S, L, J$ denote the spin, angular momentum, total momentum of the pair, respectively, and $I$ the isospin. Two isospin states, $I = 0, 1$, enter the $p\bar{p}$ system. A first indication that the system is in an $I = 0$ state was obtained in a simple quark model in Ref. [7]. In Ref. [6] a unified picture and a limited description of the radiative decays has been achieved in a semi-quantitative way. It suggests that the final $\gamma p\bar{p}$ state is dominated by the $^1S_0$ partial wave. In this partial wave the Paris potential generates a 52 MeV broad quasi-bound state at 4.8 MeV below threshold [8]. The conclusion that a near threshold peak is formed in the $^1S_0$ wave has been reached by the Jülich group although the Bonn-Jülich potential does not generate a bound state in this wave [9] and by Chen et al. [11] in the framework of an effective $NN$ interaction model. Another study of the near threshold enhancement performed in Ref. [12] finds a quasibound state to be the explanation. The Bonn-Jülich group found recently a good description of the threshold behaviour in all mesonic channels with a chirally motivated $NN$ potential [13]. The conclusion reached is similar to that obtained with the Paris potential, the near threshold enhancement indicates the presence of a quasi-bound state.

To understand better the nature of the $p\bar{p}$ states involved, one should look directly into the subthreshold energy region. This may be achieved in the antiproton-deuteron or the antiproton-helium reactions at zero or low energies. Another way to look below the threshold is the detection of $NN$ decay products. The specific decay mode

\[ J/\psi \rightarrow \gamma \pi^+\pi^- \eta' \]  

has been studied by the BES Collaboration [14]. This reaction is attributed by BES to an intermediate $p\bar{p}$ configuration in the $J^{PC}(p\bar{p}) = 0^{-+}$ state which corresponds to spin singlet $S$-wave. The peak observed in the invariant mass of the mesons has been interpreted as a new baryon state and named $X(1835)$.

Under the assumption that all mesons are produced in relative $S$-waves, reaction (2), if attributed to an intermediate $p\bar{p}$ state, is even more restrictive than reaction (1). It allows only one intermediate state, the $p\bar{p}$ $^1S_0$, which coincides with the previous findings. The intermediate $p\bar{p}$ state in reaction (2) is possible but not warranted. In Ref. [15] a more consistent interpretation is obtained with the dominance of the $^1S_0$ state which is a mixture of $p\bar{p}$ and $n\bar{n}$ pairs. It has been argued that the peak is due to an interference of a quasi-bound, isospin 0, $NN$ state with a background amplitude. This quasi-bound state was found in Loiseau and Wycech [6] to be responsible for the threshold enhancement in reaction (1). A recent BES III experiment [16] has studied the radiative decay $J/\psi \rightarrow \gamma \gamma \phi$ and observes a broad bump in the $M(\gamma \phi)$ invariant mass distribution. The shape of this bump is consistent with that observed in the absorptive $NN$ amplitude obtained in Ref. [15]. A related strong enhancement of the absorption is observed in the light antiprotonic atoms. The comparison of atomic level widths in a series of atoms ($H, ^2H, ^3H, ^3He, ^4He$) allows to test the absorption of antiprotons on more and more strongly bound protons up to subthreshold energies of $E_{\gamma \pi}$ down to $-40$ MeV [3]. The enhancement of absorption below the $p\bar{p}$ threshold is consistent with both results from Refs. [10] and [8]. This, in our view, provides evidence that the $X(1835)$ meson is due to attraction in the $NN$ system.

A similar decay mode

\[ J/\psi \rightarrow \pi^0 p\bar{p} \]  

displays no near threshold enhancement [4]. Recent BES III experiments [17] [18] have extended these measurements to the reaction

\[ J/\psi \rightarrow \omega p\bar{p}. \]  

No clear near threshold enhancement is found although Haidenbauer et al. [19] claim the existence of a small signal above phase space very close to this threshold. Beyond, a depression at low $p\bar{p}$ energies is seen in the data. These two reactions indicate a strong $P$-wave dominance in reaction (3) and a sizable $P$ wave in reaction (4). Both find a natural explanation in the model developed in the present work. Recent experiments find no $p\bar{p}$ threshold structure in the

\[ \psi' \rightarrow \gamma p\bar{p}. \]  

\(^2\) For completeness we want to mention that earlier in Ref. [10] this group had claimed that within a Watson-Migdal approach they could reproduce the near threshold spectrum with an $I = 1$ state. Also, later in Ref. [13], using for the $NN$ interaction a potential derived within chiral effective field theory fitted to results of a partial-wave analysis of $p\bar{p}$ scattering data, the authors claim that the near-threshold spectrum observed in various decay reactions can be reproduced simultaneously and consistently by their treatment of the $p\bar{p}$ final-state interactions and that the interaction in the isospin-1 $^1S_0$ channel, required to fit the decay $J/\psi \rightarrow \gamma p\bar{p}$, predicts an $NN$ bound state.
Different experimental branching fractions for the $J/\psi$ decay modes implying a $p\bar{p}$ pair based on Fermi Lab [21] and BES experiments [17, 18, 21, 22] are shown in Table I. One notable fact from this Table is that the radiative decay is comparable to the decay into strongly interacting mesons. We will see that this is due to a balance between the phase space (see Appendix A), the coupling constants and the fact of strong $N\bar{N}$ interactions and a direct emission process.

Table I: The states of low energy $p\bar{p}$ pairs allowed in the $J/\psi \rightarrow \gamma p\bar{p}$ and $J/\psi \rightarrow \pi^0 p\bar{p}$ decays. The first column gives decay modes and specifies the internal states of the $p\bar{p}$ pair. Both the $J/\psi$ meson and the photon have $J^{PC} = 1^{--}$. The second column gives $J^{PC}$ for the $p\bar{p}$ system, the last column gives the relative angular momentum of the photon or pion vs. the $p\bar{p}$ pair.

| Decay mode | $J^{PC}(p\bar{p})$ | Relative $l$ |
|------------|---------------------|--------------|
| $\gamma p\bar{p}(^3 S_0)$ | 0$^-$ | 1 |
| $\gamma p\bar{p}(^3 P_0)$ | 0$^+$ | 0 |
| $\gamma p\bar{p}(^3 P_1)$ | 1$^+$ | 0 |
| $\pi^0 p\bar{p}(^3 P_1)$ | 1$^+$ | 0 |
| $\pi^0 p\bar{p}(^3 S_1)$ | 1$^-$ | 1 |

Table II: Experimental branching fractions for some decay modes of the $J/\psi$ meson into channels implying $N\bar{N}$ pairs and the corresponding allowed states of the $N\bar{N}$ pair. All data from Ref. [21] but for the $p\bar{p}\phi$ channel recently measured in Ref. [22].

| Decay mode | Experimental branching fractions | $N\bar{N}$ allowed states |
|------------|---------------------------------|---------------------------|
| $p\bar{p}\pi^0$ | $1.19(0.08) \times 10^{-3}$ | $33 S_1, 31 P_1$ |
| $p\bar{p}\pi^-$ | $2.12(0.09) \times 10^{-3}$ | $33 S_1, 31 P_1$ |
| $p\bar{p}\gamma$ | $3.8(1.0) \times 10^{-4}$ | $3 S_0, 31 P_1, 3 P_0$ |
| $p\bar{p}\omega$ | $9.8(1.0) \times 10^{-4}$ | $11 S_0, 13 P_1, 13 P_0$ |
| $p\bar{p}\phi$ | $5.23(0.34) \times 10^{-5}$ | $11 S_0, 13 P_1, 13 P_0$ |
| $p\bar{p}n\pi^+$ | $2.120(0.029) \times 10^{-3}$ | $13 S_1$ |
| $n\pi^0$ | $2.09(0.16) \times 10^{-3}$ | $13 S_1$ |

The purpose of the present work is to discuss and correlate the physics of $N\bar{N}$ states produced in the $J/\psi$ decays. The main assumption is that the bosons (photon and mesons) are emitted after the $N\bar{N}$ baryons have been produced. In this way one obtains branching ratios $\Gamma(N\bar{N}B)/\Gamma(N\bar{N})$ consistent with experimental data for the $\pi^0, \pi^-$ and $\phi$ mesons formation, listed in Table I. One free parameter $R_0$ - the size of initial $N\bar{N}$ source enters this model and it comes out with a reasonable value of 0.28 fm. On the other hand, to obtain the invariant $p\bar{p}$ mass spectra in the decays and in particular to generate the threshold peak it is necessary to include an additional mechanism for the photon emission before the baryon formation phase. The peak of interest arises as a result of $p\bar{p}$ final state interaction in the way described in Refs. [6, 9]. The rate of this decay enters as another free parameter.

The content of this paper goes as follows. Section II recalls briefly the derivation of the width of the $J/\psi \rightarrow p\bar{p}$ decay mode. Section III develops a model for radiative decay which assumes the photon to be emitted at an early stage of the process. This internal emission model explains the two maxima in the final $p\bar{p}$ spectrum: one is due to baryonium while the other represents a shape resonance in the $p\bar{p}$ interaction. It can be extended to the case of emission of any meson. Section IV discusses the photon or meson ($\omega, \phi, \pi$) emission from the final baryon currents, i.e., once the baryons are formed following the decay of the $J/\psi$. Section V collects the results. In the case of the $J/\psi \rightarrow \omega p\bar{p}$ decay, the description of the $p\bar{p}$ spectrum requires final state interactions with a $N^*(3/2^-)$ resonance while that of the $\omega p$ spectrum requires a contribution of the mechanism of $\omega$ emission before the baryon pair formation occurs. A brief summary together with some outlook are given in Section VI. Finally, appendices tackle a number of technical questions.
II. THE J/ψ → p ¯{\!\!p} AMPLITUDE AND ITS WIDTH

Let the initial J/ψ wave function in momentum space \( \psi_i(\mathbf{P}) \) be normalized as

\[
\psi_i(\mathbf{P}) = \frac{1}{\sqrt{V_0}} (2\pi)^3 \delta^{(3)}(\mathbf{P}),
\]

where \( V_0 \) is the normalization volume. In the rest frame of the J/ψ, the amplitude \( A_{N\overline{N}}(\mathbf{q}_1, \mathbf{q}_2) \) that describes the J/ψ → (N\overline{N})_{I=0} reaction is given by

\[
A_{N\overline{N}}(\mathbf{q}_1, \mathbf{q}_2) = \langle N(\mathbf{q}_1) \overline{N}(\mathbf{q}_2) | A_{N\overline{N}} | \psi_i \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{q}_1 + \mathbf{q}_2) \frac{1}{\sqrt{V_0}} \mathcal{F}_{J/\psi}(\mathbf{q}_r),
\]

where \( \mathcal{F}_{J/\psi} \) denotes the source function associated to the creation of the N\overline{N} pair from the initial J/ψ meson and where \( \mathbf{q}_1 \) and \( \mathbf{q}_2 \) denote the momenta of the nucleon and the antinucleon respectively. This source function is assumed to depend only on the relative N\overline{N} momentum \( \mathbf{q}_r \).

\[
\mathbf{q}_r = \frac{\mathbf{q}_1 - \mathbf{q}_2}{2}.
\]

We postulate furthermore the following smooth phenomenological form for the source function

\[
\mathcal{F}_{J/\psi}(\mathbf{q}_r) = \mathcal{F}_{J/\psi}(q_r) = F_0 \exp(-q_r^2 R_0^2/2),
\]

where \( R_0 \) is the radius of the source for the formation of the N\overline{N} pair and \( F_0 \) is a normalization constant.

The probability for the J/ψ → p ¯{\!\!p} decay channel can be written as

\[
\Gamma(p\overline{p}) = \frac{1}{2} \int \frac{dq_1}{(2\pi)^3} \frac{dq_2}{(2\pi)^3} \frac{\delta(M_{J/\psi} - E(\mathbf{q}_1) - E(\mathbf{q}_2))}{2E(\mathbf{q}_1) 2E(\mathbf{q}_2)} |A_{p\overline{p}}(\mathbf{q}_1, \mathbf{q}_2)|^2,
\]

where we have taken into account the probability to find p\overline{p} in the isospin 0 state, |< I = 0|p\overline{p}>|^2 = 1/2. Using Eq. 7 and the relation \((2\pi)^3 \delta^{(3)}(0) = V_0\), one gets

\[
\Gamma(p\overline{p}) = \frac{1}{2} \frac{\delta^{(3)}(0)}{V_0} \int dq \frac{\delta(M_{J/\psi} - 2E(q))}{[2E(q)]^2} |\mathcal{F}_{J/\psi}(q)|^2 = \frac{1}{4\pi^2} \int q^2 dq \frac{\delta(M_{J/\psi} - 2E(q))}{[2E(q)]^2} |\mathcal{F}_{J/\psi}(q)|^2
\]

\[
= \frac{1}{16\pi^2} \frac{q_m}{M_{J/\psi}} |\mathcal{F}_{J/\psi}(q_m)|^2,
\]

where the delta function has provided \( q = q_m = \frac{1}{2} \sqrt{M_{J/\psi}^2 - 4 m^2} \) where \( M_{J/\psi} \) denotes the mass of the J/ψ meson and \( m \) the nucleon mass. This derivation is recalled here to ascertain that the same factors are used for the particle B formation reactions J/ψ → ppB. The corresponding decay rates will be referred to the prime J/ψ → p\overline{p} rate.

III. DIRECT (INTERNAL) EMISSION AMPLITUDES

The essence of this approach is presented in figure 1 where we illustrate the processes at stake in the case of the photon. The photon is emitted before the p\overline{p} pair is formed. It has been shown in references 6 and 9 that this assumption allows to reproduce the near threshold enhancement in the p\overline{p} invariant mass \( (M_{p\overline{p}}) \) distribution. This enhancement is due to the final state interaction of the two protons. The interactions, Paris potential in 6 and Bonn/Jülich potential in 9, are strongly attractive. In the Paris potential case a quasi-bound state is generated.

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3 The isospin structure of the N\overline{N} states, following the convention used in the Paris potential model, is given by |I = 0⟩ = (|p\overline{p}| + |n\overline{p}|)/√2 and |I = 1⟩ = (|p\overline{p}| − |n\overline{p}|)/√2, so that one has

\[
\langle 0|p\overline{p}⟩ = \langle 1|p\overline{p}⟩ = \langle 1|n\overline{p}⟩ = 1/\sqrt{2} \quad \text{and} \quad \langle 0|n\overline{p}⟩ = −⟨ 0|p\overline{p}⟩.
\]
while none appears in the Bonn potential case. An extension of these calculations to larger values of $M_{\eta\eta}$ is presented below.

In this approach, which will be referred to as the direct emission (DE) model, the direct internal emission process arises either from the charmed $c\bar{c}$ quark pair or from the quark rearrangement stage of the process and its rate is hard to calculate. Here, this rate is fixed by an optimal description of the ratio $\Gamma(\eta\eta\gamma)/\Gamma(\eta\eta)$ and of the magnitude of the threshold peak. The spectrum is generated by a Born operator, $\hat{A}^{B,DE}_{\eta\eta\gamma}$, and final state interactions (FSI) summed in the operator $\hat{A}^{FSI,DE}_{\eta\eta\gamma}$ and collected into the full internal emission operator $\hat{A}^{DE}_{\eta\eta\gamma}$ which can be formally written as

$$\hat{A}^{DE}_{\eta\eta\gamma} = \hat{A}^{B,DE}_{\eta\eta\gamma} + \hat{A}^{FSI,DE}_{\eta\eta\gamma} = \hat{A}^{B,DE}_{\eta\eta\gamma} \left[ 1 + G^{+}_{0,N\gamma}(E_{N\gamma}) \right]$$

where $G^{+}_{0,N\gamma}$ is the free $N\gamma$ propagator at the energy $E_{N\gamma}$ in the presence of the photon of momentum $k$ and $T_{N\gamma}(E_{N\gamma})$ is the $N\gamma$ scattering $T$-operator. This operator can act in both $I = 0$ and $I = 1$ state, which occurs for the $N\gamma$ pair in the diagram representing the final state interactions (right panel in Fig. 1) and will be written when necessary $T_{N\gamma}(E_{N\gamma})$. The Born operator $\hat{A}^{B,DE}_{\eta\eta\gamma}$ is factorized into two contributions: the $N\gamma$ pair creation from the $J/\psi$ meson described through the operator $\hat{A}_{N\eta\gamma}$ and the direct photon emission from the $J/\psi$ meson given by the operator $\hat{V}_{\gamma}^{DE}$

$$\hat{A}^{B,DE}_{\eta\eta\gamma} = \hat{A}_{N\eta\gamma} \hat{V}_{\gamma}^{DE}. \quad (13)$$

The direct photon emission operator $\hat{V}_{\gamma}^{DE}$ has to conserve the charge-conjugation-parity, $CP$, symmetry. In momentum space one has three vectors available: $k, \xi$ - the initial orientation of the $J/\psi$ spin $\gamma$, and the vector product $\xi \wedge k$ to be combined with the polarization vector of the photon of helicity $\lambda$, $\epsilon^{*}(\lambda)$. The matrix element of the operator $\hat{V}_{\gamma}^{DE}$ associated to a transition to reach a $^1S_0$ state should then be of the form

$$V^{DE}_{\gamma}(k) = g_{DE} \epsilon^{*}(\lambda) \cdot (\xi \wedge k), \quad (14)$$

where the constant $g_{DE}$ is a free parameter.

The initial $J/\psi$ meson at rest is described by the momentum space wave function $\psi_{\gamma}$ given in Eq. (6) and the Born amplitude $A^{B,DE}_{\eta\eta\gamma}(q_{\perp}, k)$ is given by the relations

$$\langle p(q_{1}) \mid P(q_{2}) \gamma(k) \mid A^{B,DE}_{\eta\eta\gamma} \mid \psi_{\gamma} \rangle = (2\pi)^{3} \delta^{(3)}(q_{1} + q_{2} + k) \langle q_{\perp} \rangle A^{B,DE}_{\eta\eta\gamma}(q_{\perp}, k), \quad (15)$$

The semi-relativistic three particle free $N\gamma$ propagator matrix elements read here

$$\langle q_{1}, q_{2}, k \mid G^{+}_{0,N\gamma} \mid q_{1}', q_{2}', k \rangle = (2\pi)^{6} \delta^{(3)}(q_{1} - q_{1}') \delta^{(3)}(q_{2} - q_{2}') G^{+}_{0,N\gamma}(q_{1}, q_{2}, k), \quad (16)$$

FIG. 1: Photon emission from the $J/\psi$: the left panel (a) corresponds to the Born term while the right one (b) includes final state corrections. The nucleon (antinucleon) line is denoted $N (\bar{N})$ with respective momenta $q' - k (-q')$ while $p (\bar{p})$ represents a proton (antiproton) propagating with momentum $q - k (-q)$. The wavy line is associated to the photon of momentum $k$. 

In this approach, which will be referred to as the direct emission (DE) model, the direct internal emission process arises either from the charmed $c\bar{c}$ quark pair or from the quark rearrangement stage of the process and its rate is hard to calculate. Here, this rate is fixed by an optimal description of the ratio $\Gamma(\eta\eta\gamma)/\Gamma(\eta\eta)$ and of the magnitude of the threshold peak. The spectrum is generated by a Born operator, $\hat{A}^{B,DE}_{\eta\eta\gamma}$, and final state interactions (FSI) summed in the operator $\hat{A}^{FSI,DE}_{\eta\eta\gamma}$ and collected into the full internal emission operator $\hat{A}^{DE}_{\eta\eta\gamma}$ which can be formally written as

$$\hat{A}^{DE}_{\eta\eta\gamma} = \hat{A}^{B,DE}_{\eta\eta\gamma} + \hat{A}^{FSI,DE}_{\eta\eta\gamma} = \hat{A}^{B,DE}_{\eta\eta\gamma} \left[ 1 + G^{+}_{0,N\gamma}(E_{N\gamma}) \right]$$

where $G^{+}_{0,N\gamma}$ is the free $N\gamma$ propagator at the energy $E_{N\gamma}$ in the presence of the photon of momentum $k$ and $T_{N\gamma}(E_{N\gamma})$ is the $N\gamma$ scattering $T$-operator. This operator can act in both $I = 0$ and $I = 1$ state, which occurs for the $N\gamma$ pair in the diagram representing the final state interactions (right panel in Fig. 1) and will be written when necessary $T_{N\gamma}(E_{N\gamma})$. The Born operator $\hat{A}^{B,DE}_{\eta\eta\gamma}$ is factorized into two contributions: the $N\gamma$ pair creation from the $J/\psi$ meson described through the operator $\hat{A}_{N\eta\gamma}$ and the direct photon emission from the $J/\psi$ meson given by the operator $\hat{V}_{\gamma}^{DE}$

$$\hat{A}^{B,DE}_{\eta\eta\gamma} = \hat{A}_{N\eta\gamma} \hat{V}_{\gamma}^{DE}. \quad (13)$$

The direct photon emission operator $\hat{V}_{\gamma}^{DE}$ has to conserve the charge-conjugation-parity, $CP$, symmetry. In momentum space one has three vectors available: $k, \xi$ - the initial orientation of the $J/\psi$ spin $\gamma$, and the vector product $\xi \wedge k$ to be combined with the polarization vector of the photon of helicity $\lambda$, $\epsilon^{*}(\lambda)$. The matrix element of the operator $\hat{V}_{\gamma}^{DE}$ associated to a transition to reach a $^1S_0$ state should then be of the form

$$V^{DE}_{\gamma}(k) = g_{DE} \epsilon^{*}(\lambda) \cdot (\xi \wedge k), \quad (14)$$

where the constant $g_{DE}$ is a free parameter.

The initial $J/\psi$ meson at rest is described by the momentum space wave function $\psi_{\gamma}$ given in Eq. (6) and the Born amplitude $A^{B,DE}_{\eta\eta\gamma}(q_{\perp}, k)$ is given by the relations

$$\langle p(q_{1}) \mid P(q_{2}) \gamma(k) \mid A^{B,DE}_{\eta\eta\gamma} \mid \psi_{\gamma} \rangle = (2\pi)^{3} \delta^{(3)}(q_{1} + q_{2} + k) \langle q_{\perp} \rangle A^{B,DE}_{\eta\eta\gamma}(q_{\perp}, k), \quad (15)$$

The semi-relativistic three particle free $N\gamma$ propagator matrix elements read here

$$\langle q_{1}, q_{2}, k \mid G^{+}_{0,N\gamma} \mid q_{1}', q_{2}', k \rangle = (2\pi)^{6} \delta^{(3)}(q_{1} - q_{1}') \delta^{(3)}(q_{2} - q_{2}') G^{+}_{0,N\gamma}(q_{1}, q_{2}, k), \quad (16)$$
where
\[ G_{0,N\gamma}^{\pm}(q_1, q_2, k) = \frac{1}{E_{N\gamma} + i\epsilon - \sqrt{q_1^2 + m^2 - \sqrt{q_2^2 + m^2}}}. \] (17)

\( E_{N\gamma} = M_{J/\psi} - k \) being the \( N\gamma \) pair energy and \( k = |\mathbf{k}| \) the emitted photon energy. In the evaluation of the final state interaction contribution, the \( N\gamma \) pair may be in either isospin \( I = 0 \) or \( I = 1 \) state. Thus we may write this contribution as

\[ \langle p(q_1) \bar{p}(q_2) | T_{N\gamma}^I(E_{N\gamma}) | A_{0,N\gamma}^{B,DE}(q_1, q_2)| \psi_i \rangle = \frac{1}{\sqrt{V_0}} \int \langle q_1 | q_2 | T_{N\gamma}^I(E_{N\gamma}) | q_1' q_2' \rangle (2\pi)^3 \delta^{(3)}(q_1' + q_2' + k) \times \frac{d'q_1}{(2\pi)^6} \frac{d'q_2}{(2\pi)^6} \frac{1}{E_{N\gamma} + i\epsilon - \sqrt{q_1^2 + m^2 - \sqrt{q_2^2 + m^2}}} \mathcal{F}_{J/\psi}(q'_r) V_{\gamma}^{DE}(k), \] (18)

where \( q'_r = (q_1' - q_2')/2 = q' - k/2 \) with \( q_1' = q' - k \) and \( q_2' = -q' \). Since

\[ \langle q_1 | q_2 | T_{N\gamma}^I(E_{N\gamma}) | q_1' q_2' \rangle = (2\pi)^3 \delta^{(3)}(q_1' + q_2' - q_1 - q_2) T_I(q_r, q'_r, E_{N\gamma}), \] (19)

we finally arrive at the loop integral that yields the contribution of final state interactions for the direct photon emission process

\[ \langle p(q_1) \bar{p}(q_2) | T_{N\gamma}^I(E_{N\gamma}) | A_{0,N\gamma}^{B,DE}(q_1, q_2)| \psi_i \rangle = (2\pi)^3 \delta^{(3)}(q_1 + q_2 + k) A_{p\bar{p}r}^{FSI,DE}(q_r, k, E_{N\gamma}), \] (20)

where

\[ A_{p\bar{p}r}^{FSI,DE}(q_r, k, E_{N\gamma}) = \frac{1}{\sqrt{V_0}} V_{\gamma}^{DE}(k) \sum_{I=0,1} \int \frac{d'q_2}{(2\pi)^6} \frac{1}{E_{N\gamma} + i\epsilon - \sqrt{q_2^2 + m^2}} \mathcal{F}_{J/\psi}(q'_r) V_{\gamma}^{DE}(k), \] (21)

\( q_r \) being defined in Eq. (5). Then the full amplitude for the direct photon emission reads

\[ \langle p(q_1) \bar{p}(q_2), \gamma(k) | \hat{A}_{p\bar{p}r}^{DE}(\psi_i) \rangle = (2\pi)^3 \delta^{(3)}(q_1 + q_2 + k) A_{p\bar{p}r}^{DE}(q_r, k, E_{N\gamma}) \]

\[ = (2\pi)^3 \delta^{(3)}(q_1 + q_2 + k) \left[ A_{p\bar{p}r}^{B,DE}(q_r, k) + A_{p\bar{p}r}^{FSI,DE}(q_r, k, E_{N\gamma}) \right], \] (22)

where \( A_{p\bar{p}r}^{B,DE}(q_r, k) \), given by Eq. (15), corresponds to the Born amplitude while the effect of final state interactions is given by the loop integral \( A_{p\bar{p}r}^{FSI,DE}(q_r, k, E_{N\gamma}) \) of Eq. (21). These results can be similarly extended for the internal emission of a vector meson \( B \) where one has simply to replace the potential \( V_{\gamma}^{DE}(k) \) by an appropriate potential.

The isospin symmetry is violated by the “internal photon” and as we wrote above in this section, the intermediate state of the baryon pair in Fig. 1(b) is a superposition of \( I = 0 \) and \( I = 1 \) \( N\gamma \) states or of \( p\bar{p} \) and \( n\bar{n} \) ones. However, the \( n\bar{n} \rightarrow p\bar{p} \) transition is weak as the \( n\bar{n} \rightarrow p\bar{p} \) cross section is smaller by a factor of the order of 1/15 as compared to the \( p\bar{p} \rightarrow p\bar{p} \) cross section, see Ref. 8 for comparison. Hence, in our calculation, the small correction due the \( n\bar{n} \) interaction is neglected.

To complete this phenomenological approach, we will assume in addition that, in this process, the source radius has a weak energy dependence on the \( p\bar{p} \) invariant mass that reads

\[ M_{p\bar{p}} = \sqrt{(M_{J/\psi} - k)^2 - (q_1 + q_2)^2} = \sqrt{M_{J/\psi} (M_{J/\psi} - 2k)}. \] (23)

We thus write, with masses expressed in units of \( \text{fm}^{-1} \),

\[ R(M_{p\bar{p}}) = R_0 + \beta \sqrt{M_{p\bar{p}} - 2m} = R_0 + \beta \sqrt{M_{J/\psi} \left( 1 - \frac{2k}{M_{J/\psi}} \right)^{1/2} - 2m}. \] (24)

The values \( R_0 = 0.28 \text{ fm} \) and \( \beta = 0.175 \text{ fm}^{3/2} \) are found to represent the data fairly well. This expression \( 24 \) can also be reinterpreted as a modification of the functional form of the source function \( \mathcal{F}_{J/\psi}(q_r) \) [see Eq. (7)]. We stay
with this parametrization as it indicates a physical effect indicated below.

Calculating the related loop integral and averaging the probability over the phase space (see Appendices B and E), one obtains the $M_{p\bar{p}}$ spectrum plotted in Fig. 2 for different values of $\beta$. This spectrum has several interesting features summarized in Table III and described below.

- It displays two peaks. The narrow peak that arises at the threshold is related to the near $p\bar{p}$ threshold ($E_{NN} = -4.8$ MeV, 52 MeV broad Paris-potential quasi-bound $^{11}S_0$ state [8]). The other, broad, peak is formed at $M_{p\bar{p}} \approx 2130$ MeV. It corresponds to a shape resonance at which the wavelength equals to the size of the $p\bar{p}$ potential well in the Paris potential for the $^{11}S_0$ state [8]. The isospin 0 part of the potential well that generates such structures and the corresponding energy dependent absorptive part are shown in figure 3.

- Figure 2 shows the expansion from the initial radius $R_0 = 0.28$ fm to some radius $R_f$, i.e., when the probability of the photon emission falls to zero and when the $p\bar{p}$ pair is well formed. One sees from the curve on figure 2 that the limiting radius $R_f$ varies from 0.28 fm to $R_f \approx 0.61$ fm with the invariant $M_{p\bar{p}}$ mass varying from 2.90 to 2.60 Gev/$c^2$ when $\beta$ varies from 0.0 to 0.25 fm$^{3/2}$.

- The first minimum moves very slowly to slightly increasing invariant $M(p\bar{p})$ mass but remains below the experimental value at about 1.97 GeV/$c^2$.

- The broad maximum in the spectrum moves to decreasing values of $M_{p\bar{p}}$ as $\beta$ increases, i.e. when $\beta$ goes from 0 to 0.25 fm$^{3/2}$, the maximum moves from 2.15 to 2.01 GeV/$c^2$ when the experiment displays a maximum around 2.13 GeV/$c^2$. Furthermore the ratio of the height of the second maximum over the height of the first minimum decreases and goes to 1 as $\beta$ reaches the value of 0.25 fm$^{3/2}$; for larger values of $\beta$ there is neither a minimum nor a maximum.

![Graph](image_url)

**FIG. 2:** The invariant $M_{p\bar{p}}$ mass distribution calculated within the direct emission DE model for different values of the parameter $\beta$. The data (histogram) is extracted from Fig. 1 in Ref. [5].
TABLE III: Position and values of 1st minimum and second maximum of the direct emission (DE) spectra, all normalized at first maximum value of 2450 events per 0.02 GeV/c^2

| DE / β in fm^{3/2} | 0   | 0.05 | 0.075 | 0.10 | 0.15 | 0.175 | 0.20 | 0.25 | exp   |
|--------------------|-----|------|-------|------|------|-------|------|------|-------|
| 1st min (pos)      | 1.945 | 1.955 | 1.955 | 1.9645 | 1.9645 | 1.974 | 1.9745 | 1.9745 | 1.9745 |
| 1st min (height)   | 628 | 782 | 834 | 869 | 897.7 | 896.4 | 883 | 836 | 1040 |
| 2nd max (pos)      | 2.15 | 2.12 | 2.11 | 2.10 | 2.067 | 2.058 | 2.04 | 2.01 | 2.13 |
| 2nd max (height)   | 1547 | 1457 | 1369 | 1274 | 1097 | 1021 | 953 | 849 | 1350 |
| ratio max/min      | 2.46 | 1.86 | 1.64 | 1.47 | 1.22 | 1.14 | 1.08 | 1.02 | 1.3 |
| becomes negligible | 2.90 | 2.85 | 2.80 | 2.75 | 2.70 | 2.67 | 2.65 | 2.60 |

We will see further on that the contribution of the baryon current in this process shows a maximum at values of \( M_{\bar{p}p} \) slightly above 2.15 GeV.

This internal emission model can be extended to the case of a vector meson emission: the main change comes in the definition of the energy \( E_{NN} \), i.e., \( E_{NN} = M_{J/\psi} - \sqrt{m_B^2 + k^2} \) where \( m_B \) is the mass of the emitted boson.

IV. BARYON CURRENT AMPLITUDES

This calculation is based on a model suggested in Ref. \[6\] (similar ideas have been developed quantitatively by Barnes et al. in Ref. \[23\]). The initial assumption is that the mesons are emitted after the \( N\bar{N} \) pair has been formed. In the decay process, the initial heavy \( cc \) quarks in the \( J/\psi \) state of \( J^{PC} = 1^{-+} \) have to disappear and form another \( qq \) pair. The easiest way to do that is a three gluon intermediate state \[24\] which generates a pair of the same \( J^{PC} \). Next this system generates two extra \( \bar{q}q \) pairs from the vacuum, e.g., by the \( 3^3P_0 \) mechanism. This leads to the formation of a \( 3S_1 \) state. The emission of \( \gamma, \pi, \phi \) or \( \omega \) is assumed to happen after the baryons have been formed. It turns out that this assumption yields a generally consistent description of the mesonic decays. Yet, in the case of the \( \gamma \) or \( \omega \) bosons it represents only a sizable fraction of the decay rate and has to be completed by the contribution of the direct (internal) process just described in the preceding section for the photon case. The mechanism is visualized in Fig. 4 and to quantify it one needs three basic ingredients:

- A wave function to describe the initial \( N\bar{N} \) state. It is generated by the \( cc \) transition to the \( 3S_1 \ N\bar{N} \) state of
The intermediate $N\bar{N}$ pair being in an isospin $I = 0$ state, the lowest order (Born) amplitude in the absence of final state interaction (left panel of Fig. 4) is given, since in that case one has only an intermediate $p\bar{p}$ pair, formally as

$$\langle p|\bar{p},\tilde{B}\rangle (N\bar{N})_{I=0} = \langle p|\bar{p},\tilde{B}\rangle (N\bar{N})_{I=0} = \langle p|\bar{p},\tilde{B}\rangle (N\bar{N})_{I=0}$$

where $G^+_{0,NN}$ denotes the free three-body Green’s function (similar to Eq. (17) in the photon case).

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where $G^+_{0,NN}$ denotes the free three-body Green’s function (similar to Eq. (17) in the photon case).
In these equations, the photon (meson) is emitted from the nucleon of momentum \(q_1\) and the antinucleon is the spectator with momentum \(q_2\) such that \(q_2 = -q, q_1 = -q_2 - k = q - k\) and \(q_r = q - k/2\).

The FSI contribution contains the formal expression

\[
\langle p\bar{p}B|T_{NN}^+ G_{++,NN\bar{B}}^+ \bar{U}_{B}^0 |(N\bar{N})_{I=0}\rangle = \sum_{I} \langle p\bar{p}B|(N\bar{N})_{I} \rangle \langle (N\bar{N})_{I}|T_{NN}^+ G_{++,NN\bar{B}}^+ |(N\bar{N})_{I} \rangle \langle (N\bar{N})_{I}|\bar{U}_{B}^0 |(N\bar{N})_{I=0}\rangle \tag{29}
\]

where the sum over the isospin \(I\) is restricted to \(I = 0, 1\). We may then write explicitly

\[
A^{FSI,BC}_{pp\bar{B}}(q_1, q_2, k) = \langle p(q_1) \bar{p}(q_2) B(k)|T_{NN}^+ G_{++,NN\bar{B}}^+ \bar{U}_{B}^0 |N\bar{N}\rangle_{I=0} G_{++,NN\bar{B}}^+ |(N\bar{N})_{I=0}|\tilde{A}_{N\bar{N}\bar{B}}\psi_i \rangle = \sum_{I=0,1} \int \langle p(q_1) \bar{p}(q_2) B(k)|T_{NN}^+ G_{++,NN\bar{B}}^+ \bar{U}_{B}^0 |N\bar{N}\rangle_{I=0} G_{++,NN\bar{B}}^+ |(N\bar{N})_{I=0}|\tilde{A}_{N\bar{N}\bar{B}}\psi_i \rangle \times \langle [N(q_1) \bar{N}(q_2)]_I B(k')|\tilde{U}_{B}^0 |(N\bar{N})_{I=0} \rangle G_{++,NN\bar{B}}^+ |(N\bar{N})_{I=0}|\tilde{A}_{N\bar{N}\bar{B}}\psi_i \rangle \times \frac{dq'_1}{(2\pi)^3} \frac{dq'_2}{(2\pi)^3} \frac{dk'}{(2\pi)^3} \delta^{(3)}(q'_1 + q'_2 + k) A^{B,BC}_{N\bar{N}B}(q'_2, k), \tag{30}
\]

Then, from Eqs. (19), (27) and (28), we obtain

\[
A^{FSI,BC}_{pp\bar{B}}(q_1, q_2, k) = (2\pi)^3 \delta^{(3)}(q_1 + q_2 + k) \tilde{A}^{FSI,BC}_{pp\bar{B}}(q_r, k),
\]

\[
\tilde{A}^{FSI,BC}_{pp\bar{B}}(q_r, k) = \sum_{I} \int \frac{dq'_2}{(2\pi)^3} \frac{dk}{(2\pi)^3} \frac{\gamma_{\bar{B},2}}{E_{\bar{B}}(k)} G_{++,NN\bar{B}}^+ (q'_2 - \frac{k}{2}, k) A^{B,BC}_{N\bar{N}B}(q'_2, k), \tag{31}
\]

where

\[
G_{++,NN\bar{B}}^+ (q'_2 - \frac{k}{2}, k, q_2, k) = \frac{1}{M_{J/\psi} + i\epsilon - E_{\bar{B}}(k) - \sqrt{(q_2^2 + k^2) + m^2 - \sqrt{q_2^2 + m^2}}} \tag{32}
\]

with \(E_{\bar{B}}(k) = \sqrt{k^2 + m_{\pi}^2}\). The FSI amplitude \(31\) requires integrations over the corresponding loop momenta and its detailed form will be discussed in the appendices. Finally, the amplitude in the BC+FSI model is given by

\[
A^{BC}_{pp\bar{B}}(q_1, q_2, k) = (2\pi)^3 \delta^{(3)}(q_1 + q_2 + k) \tilde{A}^{BC}_{pp\bar{B}}(q_r, k) + \tilde{A}^{FSI,BC}_{pp\bar{B}}(q_r, k), \tag{33}
\]

with \(\tilde{A}^{BC}_{pp\bar{B}}(q_r, k)\) given by Eq. (28) and \(\tilde{A}^{FSI,BC}_{pp\bar{B}}(q_r, k)\) by Eq. (31). One has to add a similar contribution for the emission from the antinucleon. This specific final state interaction correction will be evaluated with the half-off-shell scattering matrix \(\tilde{M}_{N\bar{N}}\) arising from the Paris potential \(\tilde{M}_{N\bar{N}}\). It will be applied in what follows to the case of the photon and of the \(\omega\) meson. The explicit expression of the amplitude for the photon emission is calculated in Appendix C.

The lowest order amplitude is enhanced by the \(N\bar{N}\) final state interactions and the effect is significant in the spin singlet \(S\) wave. As indicated by the summation over the isospin \(I\) states, it involves also radiation of magnetic photons from the intermediate \(N\bar{N}\) pairs and generates a delicate interference pattern. Since the intermediate states involve \(I = 0\) this amplitude is expected to determine, or contribute significantly, to the shape of the threshold peak in the invariant \(p\bar{p}\) mass distribution.

A. The initial \(N\bar{N}\) state

In the two models describing the \(p\bar{p}\) threshold peak \(6, 9\), it was assumed that in the course of radiative process the \(p\bar{p}\) final state is formed in the spin singlet \(^1S_0\) state \(15\). The near threshold enhancement arises as a result of the \(I = 0\) \(p\bar{p}\) final state interaction. However, in order to understand the full energy spectrum and mesonic emission rates one needs a better description of the formation mechanism. We go one step further, in addition to the state...
indicated above, the initial \( N\bar{N} \) state is assumed to inherit the spin and isospin quantum numbers of the \( J/\psi \), \( S = 1, I = 0 \), hence it is a \( ^3S_1 \) state.

Before the emission of the photon, the process is given by the matrix element:

\[
\langle \{N(q_1)\bar{N}(q_2)\} | G^{++}_{0,N\bar{N}} \hat{A}_{N\bar{N}} \psi_i \rangle = (2\pi)^3 \delta^{(3)}(q_1 + q_2) \tilde{G}_{N\bar{N}}(q_2),
\]

where \( \tilde{G}_{N\bar{N}}(q_2) \) is given by Eq. (28) with, in the \( J/\psi \) rest frame, \( q_1 = -q_2 = q_r = q \). This free Green’s function becomes singular when the momentum approaches its on-shell value. This singularity leads, in the case of electric photon emissions, to the well known infrared catastrophe. The effect of infrared enhancement should be seen in the experimental data as a peak at the end of the spectrum, that is, in the soft photon limit. However, it is also clear \[4, 5\] that contributions from the infrared photons (\( k < 50 \) MeV/c) have been effectively cut out from the data. We refer the reader to the discussion at the end of Section V A.

**B. The emission vertices**

The electromagnetic current associated to the photon emission from the nucleon is given by

\[
< q' | J_\nu | q > = e \bar{u}(q') [\gamma_\nu + \frac{\kappa}{2im} \sigma_{\mu\nu}(q' - q)\mu] u(q) = e \bar{u}(q') [\gamma_\nu + i \frac{\kappa}{2m} \sigma_{\mu\nu} k^\mu] u(q),
\]

where \( \sigma_{\mu\nu} = i \frac{1}{2} [\gamma_\mu, \gamma_\nu] \), \( e \) is the unit of charge and \( \kappa \) is the anomalous magnetic moment of the nucleon (\( \kappa_p = 1.793 \) for the proton and \( \kappa_n = -1.913 \) for the neutron). The final nucleon four-momentum \( q' \) is related to the initial four-momentum \( q \) by \( q' = q - k \) where \( k \) is the emitted boson four-momentum. The corresponding current for the photon emission from the antinucleon will be given by the substitution \( e \rightarrow -e \) and \( q \rightarrow -q \).

More generally, the emission of a vector particle by a nucleon is described by the operator

\[
L = g_V \gamma_\nu \epsilon^{\nu*}(\lambda) + i \frac{g_T}{2m} \sigma_{\mu\nu} k^\mu \epsilon^{\nu*}(\lambda),
\]

where the four-vector \( \epsilon^{\nu*}(\lambda) \) denotes the polarization vector of the emitted particle, \( \lambda \) its helicity while \( g_V \) and \( g_T = \kappa g_V \) are the vector and tensor coupling constants, respectively.

The final photon or vector meson may be produced in a magnetic or an electric transition. The relevant formation amplitudes are obtained from the transition matrix elements of the operator \[36\] reducing bispinors \( u \) to spinors \( \chi_S \).

We have

\[
\bar{u}(q') Lu(q) = \chi_S' \tilde{U}_B \chi_S,
\]

where \( S \) and \( S' \) denote the initial and final nucleon spin, and one obtains the vertex coupling in the two dimensional spin space

\[
\tilde{U}_B = g_V \hat{A}_V + i \frac{g_T}{2m} \hat{A}_T.
\]

The explicit expressions for the vector \( \hat{A}_V \) and the tensor \( \hat{A}_T \) parts are derived in Appendix C (see Eqs. \[C11\] and \[C17\], respectively).

**1. The photon case**

The full photon potential operator, \( U_0^\gamma(k,q) \), combining the photon emission from either baryon is given by Eqs. \[C22\] and \[C23\]. The magnetic terms in \( U_0^\gamma(k,q) \), proportional to \( \sigma \cdot [k \wedge \epsilon^{*}(\lambda)] \), change the \( ^3S_1 \) state of
the initial $N\overline{N}$ system to the final $^1S_0$ state. Electric terms in $U_0^0(k, q)$ proportional to $q \cdot \epsilon^*(\lambda)$, change the $^3S_1$ state to final $^3P$ states. Relativistic corrections generate additional terms, most of these cancel in the $p\overline{p}$ system, some spin operators bilinear in $q$ lead to D waves but contribute corrections only on 1% level and are not included in the calculations. As discussed in Appendix [C] the two basic couplings add coherently for the proton and the antiproton. Thus the summary coupling of photons to the $p\overline{p}$ system becomes
\[
U_0^0(q, k) = V_{E,\gamma} + V_{M,\gamma}
\]
with
\[
V_{E,\gamma} = \frac{e}{2m} \left[ C_E q \cdot \epsilon^*(\lambda) \right],
\]
\[
V_{M,\gamma} = \frac{ie}{2m} \left[ C_M \cdot (k \wedge \epsilon^*(\lambda)) \right],
\]
which are still operators in the spin-isospin space and where [see Eqs. (D14) and (D19)]
\[
C_E &= -\left\{ \left( \frac{\zeta + \bar{\zeta}}{2} + \frac{1}{2} \right) + \kappa \frac{k}{2m} \left( \frac{\zeta + \bar{\zeta} - 1}{2} \right) \right\} + \kappa \frac{k^2}{2m\Omega} \left\{ \frac{1}{2} \frac{1}{\zeta + \bar{\zeta}} \right\},
\]
\[
C_M &= r_- (\sigma_1 - \sigma_2) + r_+ (\sigma_1 + \sigma_2),
\]
with
\[
r_- = \frac{1}{2} \left\{ \left( \frac{1}{2} \frac{1}{\zeta + \bar{\zeta}} \right) \left( \frac{1}{2} + \kappa \frac{k}{2m} \right) + \kappa \frac{\Omega}{2m} (\zeta + \bar{\zeta}) \right\},
\]
\[
r_+ = \frac{1}{2} \left\{ \left( \frac{1}{2} \frac{1}{\zeta - \bar{\zeta}} \right) \left( \frac{1}{2} + \kappa \frac{k}{2m} \right) + \kappa \frac{\Omega}{2m} (\zeta - \bar{\zeta}) \right\}.
\]
The energies $\Omega, \Omega'$ and the coefficients $\zeta, \bar{\zeta}$ are defined in Appendix [C] [Eqs. (C6), (C9) and (C13)]. The approximation leading to Eqs. (40-43) may be acceptable close to the central region of the $M_{p\overline{p}}$ distribution (see Fig. 5). It is too crude in the threshold region where $k/m \approx 1$ and at the other extremity where $q/m \approx 1$. Nevertheless, some coefficients in $U_0^0(k, q)$ [Eqs. (C22) and (C23)] display remarkable stability. This, in particular, concerns the terms $\zeta + \bar{\zeta}, \zeta + 1/\zeta, \bar{\zeta} + 1/\zeta$ which are approximately 2 within 1% over all the phase space. On the other hand, there are a number of terms involving more complicated combinations of the spin and momenta which are less stable, but small due to other reasons. The terms
\[
\frac{ie}{2m} \frac{k}{2m\Omega} \left[ \left( \frac{\sigma_1}{\zeta} - \frac{\sigma_2}{\bar{\zeta}} \right) \cdot (k \wedge q) \right] [q \cdot \epsilon^*(\lambda)],
\]
\[
\frac{ie}{2m} \frac{k}{2m\Omega} q \cdot k \left\{ \frac{\sigma_1}{\zeta} - \frac{\sigma_2}{\bar{\zeta}} \right\} \cdot [q \wedge \epsilon^*(\lambda)],
\]
and
\[
\frac{ie}{2m} \frac{k}{2m\Omega} q \cdot [k \wedge \epsilon^*(\lambda)] \left( \frac{\sigma_1}{\zeta} - \frac{\sigma_2}{\bar{\zeta}} \right) \cdot q,
\]
involve a spin flip transition. According to $CP$ conservation (see Table 1) these terms lead predominantly to final spin singlet S-wave state. The resulting contribution would give an average $\langle q_1q_3 \rangle = q^2/3$ and would mainly contribute at large $k$, i.e., in the threshold region where corrections will be of the order of $q^2/12m^2$, i.e., about 2%. The term proportional to $q \wedge \epsilon^*(\lambda)$ in Eq. (C22) reads
\[
[g(q, k) \sigma_1 + \bar{g}(q, k) \sigma_2] = [\tilde{r}_- (\sigma_1 - \sigma_2) + \tilde{r}_+ (\sigma_1 + \sigma_2)] = 2 (\tilde{r}_- \sigma_- + \tilde{r}_+ \sigma_+),
\]
with
\[
\tilde{r}_- = \frac{1}{2} \left\{ \left( \zeta - \bar{\zeta} \right) \left( 1 - \kappa \frac{k_0}{2m} \right) - \left( \frac{1}{2} \frac{1}{\zeta} - \frac{1}{\bar{\zeta}} \right) \left( 1 + \kappa \frac{k_0}{2m} \right) \left( 1 + \kappa \frac{k_0}{\Omega} \right) \right\},
\]
\[
\tilde{r}_+ = \frac{1}{2} \left\{ \left( \zeta + \bar{\zeta} \right) \left( 1 - \kappa \frac{k_0}{2m} \right) - \left( \frac{1}{2} \frac{1}{\zeta} + \frac{1}{\bar{\zeta}} \right) \left( 1 + \kappa \frac{k_0}{2m} \right) \left( 1 + \kappa \frac{k_0}{\Omega} \right) \right\}.
\]
It involves a dominant $\sigma_+ \cdot [q \wedge \epsilon^*(\lambda)]$ combination which generates spin triplet $P$-wave states. It could be contributing as much as 20% of the dominant electric term. However, it is only important close to the threshold region where

$$\tilde{r}_+ \approx \frac{k_0}{\Omega} \left[ 1 + \frac{\kappa}{m} \left( 1 + \frac{k_0}{2\Omega} \right) \right], \quad (46)$$

but where the $P$-wave contributions are strongly suppressed by the phase space.

In practical calculations it is sufficient to neglect small corrections of the order of $k^2/4m^2$ which contribute about 3% to the electric rate, and about one percent to the total rate. On the other hand, a sizable, i.e., of the order of $P^\omega$ but where the

$$\tilde{r}_+ \approx \frac{k_0}{\Omega} \left[ 1 + \frac{\kappa}{m} \left( 1 + \frac{k_0}{2\Omega} \right) \right], \quad (46)$$

term is of the order of 0.

since they both lead to $S$ waves.

The emitted $\omega$ meson has a negative $G$ parity and couplings to the proton or the antiproton differ in sign (this also applies to the case of the $\pi$ meson emission). Again the emission by a baryon or an antibaryon is predominantly coherent as this sign is compensated by the momentum and/or spin involved in the vertices. In the case of $\omega$ meson the tensor coupling is known to be consistent with zero and the main contribution comes from the vector coupling [25]. From Eqs. (C25) one infers that the first $\epsilon_0^\omega(\lambda) = k \cdot \epsilon^*(\lambda)/k_0$ term [Eq. (C3)] almost disappears, by the $G$-parity rule, when the emissions from nucleon and antinucleon are added. Retaining only the dominant pieces in Eq. (C26) that is neglecting terms of the order of $k^2/2\Omega E_q$ or $k^2/4m^2$, one obtains, with $k_0$ defined in Eq. (C24) and $E_q$ in Eq. (C7),

$$U^0_\omega(q, k) \approx -\frac{g_{V\omega}}{2m} \left[ 4 q \cdot \epsilon^*(\lambda) + 2 \frac{k \cdot q}{E_q} \frac{k \cdot \epsilon^*(\lambda)}{k_0} - i(\sigma_1 - \sigma_2) \cdot \{k \wedge \epsilon^*(\lambda)\} \right], \quad (48)$$

which implies that one neglects the following contributions

$$-i \frac{g_{V\omega}}{2m\Omega} \left[ (\sigma_1 - \sigma_2) \cdot \{q \wedge \epsilon^*(\lambda)\} + \frac{k \cdot \epsilon^*(\lambda)}{k_0} \right] \left\{ (\sigma_1 + \sigma_2) \cdot \{k \wedge q\} \right\} \quad (49)$$

in addition to a term that disappears since it contains the expression

$$1 - 2 \frac{E_q}{\Omega} + \frac{q^2}{\Omega^2} = 0.$$

The first term in Eq. (49)

$$\frac{g_{V\omega}}{2m\Omega} \frac{k \cdot q}{E_q} \left[ (\sigma_1 - \sigma_2) \cdot \{q \wedge \epsilon^*(\lambda)\} \right]$$

has to be compared to the basic magnetic contribution

$$i \frac{g_{V\omega}}{2m} \left[ (\sigma_1 - \sigma_2) \cdot \{k \wedge \epsilon^*(\lambda)\} \right],$$

since they both lead to $S$-wave magnetic transitions. For average momenta of the order of 500 MeV/c the neglected term is of the order of 0.1 of the dominant one. Taking into account the interference contribution in the probability, reduces further this contribution, justifying its neglect.

The second term in Eq. (49)

$$i \frac{g_{V\omega}}{2m\Omega} \frac{k \cdot \epsilon^*(\lambda)}{k_0} \left\{ (\sigma_1 + \sigma_2) \cdot \{k \wedge q\} \right\},$$

which gives rise to $3P$ waves has to be compared to the basic electric contribution

$$-4 \frac{g_{V\omega}}{2m} q \cdot \epsilon^*(\lambda),$$
and, in the absence of any interference contribution, leads to a very small contribution, of the order of $10^{-3}$ of the basic decay rate.

Apart from the magnitude of the coupling constant there is one important difference with respect to the photon. The magnetic coupling is weak in comparison to the electric one. The reverse was true in the $\gamma$ case due to the large proton magnetic moment. This is the basic reason making the transition to the final $^1S_0$ state small (about 1/10 of the total).

3. The pion case

For $\pi$ mesons we use the standard $\gamma_5$ coupling

$$U^0_\pi(q,k) \approx g_\pi \sigma \cdot \left[ \frac{q-k}{E_{q-k} + m} - \frac{q}{E_q + m} \right] \tau \cdot \varphi_\pi \simeq -\frac{g_\pi}{2m} (\sigma \cdot k) \left( \tau \cdot \varphi_\pi \right).$$

(50)

In this case, the emission requires a spin flip and a change of nucleon angular momentum leading to the final $p\bar{p}$ in the $^3P_1$ state. This mechanism eliminates the possibility of $^1S_0$ states and does not produce any threshold enhancement as indicated by the BES experiments [1, 5, 17, 18].

C. The $N\bar{N}$ final state interactions

The emission of a magnetic photon from the nucleon in the reaction

$$J/\psi \rightarrow \gamma p\bar{p}$$

(51)
generates, within our model, the final spin 0 state in the $p\bar{p}$ system. The corresponding operator in spin space is denoted, in the small $k/2m$ limit and up to the relativistic correction $\kappa\Omega/2m$, by [see Eq. (47)]

$$V_{M,\gamma}(q,k) \approx \frac{ie}{2m} (\sigma_1 - \sigma_2) \cdot (k \wedge \epsilon^*(\lambda)).$$

(52)

It shows no dependence on the nucleon momentum before emission $q$. The Born amplitude associated to this approximation of the magnetic contribution reads then in spin space, using Eqs. (27) and (28) with $q_2 = -q$.

$$A^{M,(B,BC)}_{p\gamma\bar{p}}(q,k) = (2\pi)^3 \delta^{(3)}(q_1 - q + k) V_{M,\gamma}(q,k) \tilde{G}_{p\bar{p}}(q).$$

(53)

We consider the $^3S_1 \rightarrow ^1S_0$ transition in the $p\bar{p}$ system. As read from Eq. (52) and discussed in appendix C, the radiation from both baryons is described by $\sigma_1 - \sigma_2$. The related transition matrix element may be expressed in terms of the direction of spin in the triplet state, $\xi$. Then, the relation

$$(0 0 \frac{1}{2}(\sigma_1 - \sigma_2) | 1 \frac{1}{2} \xi) \cdot (k \wedge \epsilon^*(\lambda)) = \xi \cdot (k \wedge \epsilon^*(\lambda)),$$

(54)

leads to a formula which may be used in the case of the limit defined by Eq. (52) for a transition from the $^3S_1 \rightarrow ^1S_0$ state. Indeed, from Eqs. (39), (47) and (53), we obtain

$$\langle ^1S_0|A^{M,(B,BC)}_{p\gamma\bar{p}}(q_r,k)^3S_1 \rangle = (2\pi)^3 \delta^{(3)}(q_1 - q + k) \tilde{G}_{p\bar{p}}(q_r) \left< ^1S_0|V_{M,\gamma}|^3S_1 \right>$$

$$= (2\pi)^3 \delta^{(3)}(q_1 - q + k) A^{M,(B,BC)}_{p\gamma\bar{p}}(q_r,k),$$

(55)

with

$$\tilde{A}_{p\gamma\bar{p}}^{M,(B,BC)}(q_r,k) = \frac{ie}{m} \xi \cdot (k \wedge \epsilon^*(\lambda)) \tilde{G}_{p\bar{p}}(q_r).$$

(56)

The electric contribution to the amplitude is

$$A^{E,(B,BC)}_{p\gamma\bar{p}}(q,k) = (2\pi)^3 \delta^{(3)}(q_1 - q + k) V_{E,\gamma}(q) \tilde{G}_{p\bar{p}}(q).$$

(57)
with the electric potential given in Eq. (40). In the same limit \((k/2m << 1)\) and up to the relativistic correction \(\kappa k^2/12m\), the approximate electric potential reads

\[
V_{E,\gamma}(q, k) \approx -\frac{e}{2m} \, 4 \, q \cdot e^*(\lambda).
\]

(58)

It leads to transitions from the \(^1S_0\) state to \(^3P\) states. However, final state interactions in the P wave state are weak, at least in the Paris potential model and they are therefore neglected.

On the other hand, in the magnetic transitions the scalar amplitude \(A_{\gamma p\gamma}^{\mathcal{M},(0,BC)}(q, k)\) has to be corrected and these corrections turn out to be very important. They are described by the loop integral (see Eq. (31))

\[
\tilde{A}_{\gamma p\gamma}^{\mathcal{M},(FSI,BC)}(q, k) = \sum_{I=0,1} \int \frac{d^3q'}{(2\pi)^3} \, T_I(q, q' - k/2, E_{NN}) \, G^{+}_{I0,NN}(q', k) \, A_{\gamma p\gamma}^{\mathcal{M},(B,BC)}(q', k),
\]

(59)

calculated with the recent Paris potential \([8]\), in a way described in Ref. [15]. On shell, this \(T\) matrix element is normalized to the scattering length. The numerical evaluation of the loop integral in the presence of two singular propagators has to be done with considerable care. The procedure is described in Appendix E. The full amplitude for magnetic transitions becomes

\[
A_{\gamma p\gamma}^{\mathcal{M},BC}(q_1, q_2, k) = (2\pi)^3 \, \delta^{(3)}(q_1 + q_2 + k) \left[ A_{\gamma p\gamma}^{\mathcal{M},(B,BC)}(q, k) + \tilde{A}_{\gamma p\gamma}^{\mathcal{M},(FSI,BC)}(q, k) \right],
\]

(60)

with \(q_2\) and \(q_r\) given in Eq. (8).

For the results presented in the following section, the amplitude (60) has been evaluated with the potential \(V_{\mathcal{M},\gamma}(q, k)\) given in Eq. (41) including the \(\kappa/2m\) relativistic correction but dropping the \(t_+\) \((\sigma_1 + \sigma_2)\) since this term generates spin triplet \(P\) waves only important close to the threshold region where they are strongly suppressed by the phase space.

V. THE RESULTS

A. The radiative decays

The \(M_{\gamma p}\) data in the region of high photon energy are dominated by a peak as can be seen in Figs. 5 and 6. The explanation is related to strong nucleon-antinucleon attraction essentially in \(NN\) the isospin-spin singlet \(^{11}S_0\) state but to a certain extent also in the isotriplet spin singlet \(^{31}S_0\). Now, with the radiation due to baryonic currents this peak is strongly suppressed due to interference of the intermediate \(p\bar{p}, n\bar{n}\) channels and the cancellations of the magnetic moments involved.

The various contributions to the \(M_{\gamma p}\) spectrum plotted in Fig. 5 together with the experimental data have been obtained with the following procedure:

- a) \(|F_0|\) is the overall normalization that is fixed by the \(J/\psi \rightarrow p\bar{p}\) decay rate,
- b) magnetic and electric amplitudes are calculated independently for the DE and BC emission modes,
- c) the emission rates are added and the normalizations of the DE and BC rates are fixed to reproduce the experimental ratios shown in Table IV and the invariant mass distribution.

We are not able at present to evaluate the phase difference between the amplitudes associated to the DE and BC mechanisms. However, the interference effects are most likely fairly weak for two reasons:

- a) in the low \(M_{\gamma p}\) region, characterized by magnetic photons the contribution of the DE mechanism dominates largely the contribution of the BC mechanism,
- b) in the high \(M_{\gamma p}\) region a similar, although less striking, dominance is attributed to the electric photons whereas there is practically no contribution any more of the DE mechanism.

The experimental data, displayed in Fig. 5 indicates possible interference effects in the region between 2.3 and 2.6 GeV. However, in view of the quality of the data, we hesitated to include an additional parameter to the already many parameters introduced in our description. Thus we have neglected the possible interferences and have simply added the contributions of the DE and BC mechanisms on the probability level.

The emission from final baryons (BC) - see table IV - generates about half of the experimental rate and misses the full strength of the threshold enhancement. Photons may be emitted also by exchange currents related to charged
mesons exchanged between baryons. Such processes are known and well described in the $NN$ systems, but to our best knowledge, have not been discussed in the $NN$ systems. Calculations have been performed. We found effects of about 10% which do not change the overall picture in any significant way. The BC model has to be supplemented by the other internal emission DE mechanism discussed in Section III. The relative strength of the later is a free parameter which is set to try to reproduce the two peaks in $M_{p\bar{p}}$ spectrum (see Figs 5 and 6). In this way the total branching ratio becomes consistent with the data. The direct emission is characterized by different $NN$ final state interactions, in particular there is no cancelation of $p$ and $n$ magnetic moments during the emission process. Hence the interaction in the $^{11}\Sigma_0$ wave is stronger and the two resonances at threshold and at $M_{p\bar{p}} \approx 2170$ MeV are more distinct. As discussed previously in Section III the first is due to quasi-bound state, the second is a shape resonance. Both are generated by the Paris potential model [8].

Now, a comparison of radiative decays $J/\psi \rightarrow \gamma p\bar{p}$ and $\psi(2S) \rightarrow \gamma p\bar{p}$ could be discussed qualitatively. The baryon current emission does not depend on the internal structure of the $J/\psi \rightarrow \gamma p\bar{p}$ and $\psi(2S) \rightarrow \gamma p\bar{p}$ mesons. What is shown in the present work is that the near threshold peak is suppressed by the difference in the proton and neutron magnetic moments. On the other hand, the probability of internal photon radiation does depend on the internal structure. As a consequence, the relative weight of the two modes depends on the internal wave functions which is nodeless in the $J/\psi$ case and has a node in the relative $c\bar{c}$ coordinate in the $\psi(2S)$ case. We are not able to calculate this wave function. Our qualitative argument is that the internal emission from the $\psi(2S)$ meson (that yields a peak) has to be small. Apparently this node suppresses the magnetic radiative transitions via the DE mode and no peak is generated. We see some, although not fully convincing argument for this suppression mechanism in the experimental $\gamma p\bar{p}$ branching ratios equal $3.8(1.0) \times 10^{-4}$ for $J/\psi$ and much smaller $3.9(0.5) \times 10^{-5}$ for $\psi(2S)$ (Ref. [21]).

- The end of $p\bar{p}$ spectrum
The origin of the experimental peak at the end of spectrum is the sequential decay

$$J/\psi \rightarrow \eta_c \gamma \quad \text{and} \quad \eta_c \rightarrow p\bar{p}$$

(61)
TABLE IV: Ratio $\mathcal{R} = \Gamma(p\bar{p}\gamma)/\Gamma(p\bar{p})$ in % units with consecutive steps of improvement. The experimental ratio $\mathcal{R} = 18(5)$ is evaluated from the experimental branching fractions displayed in Table III. The first column indicates the involved mechanism: BC for the first line and BC+DE for the second. The other columns give the different contributions in these mechanisms (see text). The additional DE radiation arises from the quark phase and is fitted to the magnitude of the near threshold peak.

| Mechanism         | Electric | Magnetic (BC) | Magnetic (BC+FSI) | Total |
|-------------------|----------|---------------|-------------------|-------|
| BC                | 4.38     | 1.65          | 1.97              | 6.35  |
| BC + DE           |          |               | 5                 | 6.51  |

FIG. 6: As in Fig. 5 but for the near threshold region. Note the small $P$-wave renormalized electric contribution.

which generates a peak at the invariant mass $M_{p\bar{p}} = M(\eta_c) = 2983.4$ MeV. The decay rates are known experimentally and $J/\psi \to \eta_c \gamma$ is $1.7 (0.4) \times 10^{-2}$ of the total while $\eta_c \to p\bar{p}$ is $1.50 (0.16) \times 10^{-3}$ of its decay rate [21]. The dotted line in Fig. 5 results from the modulus squared of the following relativistic Breit-Wigner amplitude for the description of the $\eta_c$

$$A_{\eta_c} = -f_{\eta_c} \frac{m_{\eta_c} \Gamma_{el}}{M_{p\bar{p}}^2 - m_{\eta_c}^2 + i m_{\eta_c} \Gamma_{tot}},$$

with $\Gamma_{tot} = 31.8 (0.8)$ MeV and $\Gamma_{el}/\Gamma_{tot} = 1.50 (0.16) \times 10^{-3}$. In the energy range of this $\eta_c$ contribution, the interference of the $A_{\eta_c}$ amplitude with the very small magnetic $S$-wave is neglected and for the curve shown in Fig. 5 the free parameter $f_{\eta_c}$ is fixed at the value $23.2 \times 10^3$ events/ 0.02 (GeV/c$^2$). Together the expected area under the end peak would amount to $2 \times 10^{-5}$ of the total decay rate, i.e., about 5% of the $p\bar{p}\gamma$ decay rate. The first experimental result of reference [4] indicated a 1% effect but more recent measurements yield comparable results [5].

In addition to the $\eta_c$ peak, another peak arises within the BC model. It is related to the infrared enhancement in the intermediate state $p\bar{p}$ propagator. The real infrared catastrophe does not occur since the initial $J/\psi$ has a finite width. This effect produces an enhancement in the region $M_{p\bar{p}} > 2820$ MeV and a narrow bump at the end
The radiative decays

\[ 3090 < M_{\eta \pi} < 3100 \text{ MeV}. \] The area under this enhancement amounts to 3% of the rate \( R_\gamma \), calculated with the baryon emission model. That is about 0.7% of the experimental rate. The experimental check is not easy as the errors in the photon energy determination - \( \sigma_E \) - are large in this region and these two effects overlap. The BES detector offers \( \sigma_E = \frac{21\%}{\sqrt{E/\text{GeV}}} \), and in the region of interest \( \sigma_E \approx E \approx 100 \text{ MeV} \) and thus the position of the peak is not well determined. With a better resolution, the magnitude and shape of the infrared bump would be a check for decay models.

As indicated in Section IV A in the comment below Eq. (34), the BES data, where a \( k < 50 \text{ MeV/c} \) cut is applied, does not display the contribution of the infrared photon contribution. In other terms, a phenomenological final state interaction correction is applied to the \( P \)-wave electric amplitude \( A_{\gamma \eta \pi}^{E, (B,BC)}(q, k) \) given by Eq. (67). Hence, the short-dash line in Fig. 5 is the result of the renormalized electric photon amplitude where the \( M_{\eta \pi} = M_{J/\psi} \) pole has been eliminated

\[
A_{\gamma \eta \pi, E}^{E, (B,BC)}(q, k) = f_r A_{\gamma \eta \pi}^{E, (B,BC)}(q, k),
\]

\[
f_r = |N_E \frac{M_{J/\psi} - M_{\eta \pi}}{M_{J/\psi} - M_{\eta \pi} - i\Gamma_E}|.
\]

The free parameters are fixed at the respective values \( \Gamma_E = 500 \text{ MeV} \) for the width and \( N_E = 3.5 \) for the normalization. Despite the Breit-Wigner form we do not suggest that there is a new resonant mechanism involved. This form is used only for a parametrization that serves two purposes:

1) removing the infrared enhancement since it is removed in the experimental data and

2) enhancing the electric photon emission approximately by a factor 3 to reach consistency with the data.

Although a new resonance is a possibility we are inclined to view \( f_r \) in Eq. (65) as a result of another DE mechanism. This possibility is discussed in the next subsection devoted to the \( \omega \) emission and where, indeed, the broad bump in the spectrum is due to the DE decay mode and not to an intermediate resonance. A quantitative analysis may be performed in the \( \omega \) emission case as more data exist. It is not feasible in the radiative decay mode and we limit the analysis to the phenomenological formula (64).

B. The \( \omega, \phi, \) and \( \pi \) emission rates

1. The \( \omega \) emission

Its rate seems to be easier to understand than those for the radiative decays due to the weak tensorial coupling which favors strongly the electric type transitions. The corresponding branching fraction

\[
R_\omega = \frac{\Gamma(\eta \gamma \omega)}{\Gamma(\eta \pi \gamma)}
\]

obtained with the basic final state emission (BC) model is shown in table 5. The electric type transitions (E) lead to \( P \)-wave \( p \bar{p} \) states with small final state interactions. The magnetic type transitions (M) generate \( S \) wave states strongly affected by the final \( N \bar{N} \) interactions. In comparison to the photon case these interactions are stronger as isospin conservation requires baryons to be in isospin 0 state. On the other hand, due to large meson mass, the final \( p \) and \( \bar{p} \) are less strongly oriented close to threshold and the tensor \( NN\omega \) coupling is weak and consistent with zero. Hence, effects of these final state interactions are not well visible in the emission rate, see figure 7. It is the electric transition (labelled EE in Fig. 5) that dominates. Now in distinction to the photon case the longitudinal component \( e^*(\lambda = 0) \) exists and the corresponding strength of the dominant electric transition is given in Eq. (D30). It yields approximately \((g_{\gamma \omega q}/2m)^2\). One finds that the longitudinal component gives a large contribution to the low part of the \( M_{\eta \pi} \) spectrum which is not supported by the data.
TABLE V: Calculated ratios $\Gamma(p\bar{p} \text{ meson})/\Gamma(p\bar{p})$ of channel widths allowed in the $J/\psi$ decays. The errors correspond to uncertainties of the $p\bar{p}$ meson coupling $g$. Values of $R_{\exp}$ calculated from the experimental branching fractions listed in Table II.

| meson | $R_{\exp}$ | $\mathcal{R}[BC]$ | $\mathcal{R}[BC, FSI]$ | $\mathcal{R}[BC, FSI, DE]$ | $g^2/(4\pi)$ |
|-------|------------|-----------------|-----------------|-----------------|-------------|
| $p\bar{p}\pi^0$ | 0.575 (0.05) | 0.43 | | | 13.8 |
| $p\bar{p}\pi^-$ | 0.966 (0.06) | 0.85 | | | 13.8 |
| $p\bar{p}\omega$ | 0.507 (0.07) | 0.87 (0.16) | 0.67 (0.13) | | 8.1 (1.5) |
| $p\bar{p}\omega$ | 0.507 (0.07) | 0.40 | 0.33 | 0.39 | 4.16 |
| $p\bar{p}\phi$ | 0.0247 (0.0016) | 0.023 | | | 5.5 this work |

The decay rates are given in Table V and these results are obtained with the coupling constant $g^2/4\pi = 8.1 (1.5)$, $g^2/4\pi = 0.16 (0.46)$ obtained with dispersion relations [20], more recent values from $NN$ interactions are $g^2/4\pi = 9.73$, $g^2/4\pi = 0.005$ [27]. In both cases the tensorial coupling is negligible and it was neglected. However, the most significant parameter is the source radius and the rates of $\omega$ (and $\pi$) meson emissions put very strong limits on $R_0$. The final choice is obtained from the best fit of $R_\omega$ and $R_\pi$ is $R_0 = 0.28(1)$. The $\omega$ coupling constants are uncertain, those indicated above are extracted from NN scattering data. On the other hand coupling constants derived from semi-phenomenological meson formation data are smaller. A value $g^2/4\pi = 1.19$ has been obtained in Ref. [28] with a very small tensorial coupling although Ref. [29] reaches a value $g^2/4\pi = 4.16$. In our calculations, we have chosen $g^2/4\pi = 3.8$ and $g_\pi/4\pi = 0.05$.

The omega emission case differs strongly from the photon emission as apparently the BC mode dominates. However, as already shown in the BES Collaboration paper [18], the emission of $\omega$ meson requires the involvement of excited nucleon states $N^*$ and the final state involves three interacting particles. The multiple scattering method presented here requires the leading corrections due to $N^*$ to reach some 25% of the leading order. Roughly the next (missing) order is expected to reach about 10%.

The related mechanism is presented only schematically here. The Rarita-Schwinger particle propagation is given by

$$G^{\mu\nu} = \frac{\gamma^\rho + m_{3/2}}{m_{3/2}^2 - p^2 + im_{3/2}\Gamma} P^{\mu\nu},$$

where $P^{\mu\nu}$ projects on spin 3/2 states. We follow Ref. [30] which underlines some controversies in the formulation. These are of small concern as in the situation discussed here one finds this particle to be non relativistic and [31]

$$P^{\mu\nu} \rightarrow P^{ij} = 2/3 [\delta^{ij} + i/2 \epsilon_{ijk}\sigma^k].$$

2. The $M_{p\bar{p}}$ and $M_{\omega p}$ spectra

The BC model of meson emission from final baryons yields a fair estimate of the decay rate, Table V, unfortunately subject to large uncertainty in the $\omega NN$ coupling constant. On the other hand, the spectra of the invariant masses $M_{p\bar{p}}$ and $M_{\omega p}$ pose a more difficult problem. The $M_{p\bar{p}}$ distribution, plotted in the left panel of Fig. 7, requires strong reduction in the lower mass region which may be generated only by a destructive interference. Such a possibility is offered by final state interactions involving a $N(3/2^-)$ resonant state expected to mediate the $\omega-p$ interaction in the 2 GeV energy region [see Fig. 5(b)]. The bulk of available phase space covers region between $M_{\omega p} = 2052$ MeV at $p\bar{p}$ threshold and $M_{\omega p} = 1805$ MeV at the end of $M_{p\bar{p}}$ spectrum. Hence, interference of the intermediate $N(3/2^-)$ and the basic decay mode may be constructive in part and destructive in another part of the phase space.

The related mechanism is presented only schematically here. The Rarita-Schwinger particle propagation is given by

$$G^{\mu\nu} = \frac{\gamma^\rho + m_{3/2}}{m_{3/2}^2 - p^2 + im_{3/2}\Gamma} P^{\mu\nu},$$

where $P^{\mu\nu}$ projects on spin 3/2 states. We follow Ref. [30] which underlines some controversies in the formulation. These are of small concern as in the situation discussed here one finds this particle to be non relativistic and [31]

$$P^{\mu\nu} \rightarrow P^{ij} = 2/3 [\delta^{ij} + i/2 \epsilon_{ijk}\sigma^k].$$

(67)

(68)
The $\omega, \phi,$ and $\pi$ emission rates

FIG. 7: Left panel (a): the $M_{p\bar{p}}$ spectrum obtained with the BC model. Right panel (b): the $M_{p\bar{p}}$ spectrum obtained with the BC+FSI model. The same arbitrary normalization is used to fit the experimental shape for both graphs. The electric contribution is labelled EE. No FSI contribution in the DE model but the weak energy dependence of the source radius [Eq. (24)] has been kept. Here $R_0 = 0.28$ fm, $g_V^\omega/(4\pi) = 3.8$ and $g_T^\omega = 0$. Data extracted from Figs. 2 in [18].

FIG. 8: The $M_{p\bar{p}}$ spectrum as in Fig. 7 (b) but for the near threshold region.

This formula sets the main simplification of this final state interaction. In addition, we drop the second term which leads to spin flip leading to no interference with the basic Born amplitude. The calculation involves a standard loop integral which follows the procedure of Appendix E. Three uncertain parameters are implied: we use $m_{3/2} = 2050$ MeV, $\Gamma = 300$ MeV for the $N(3/2^-)$ position and width (nonessential), and 3% for the strength of the $N(3/2^-)$ coupling to the $\omega - N$ channel. The effect is shown in Fig. 7(b) and in Fig. 10. The resonance parameters ($m_{3/2} = 2050$ MeV, $\Gamma = 300$ MeV) are close to those of the $N(1875)$ and of the more uncertain $N(2120)$ $3/2^-$ resonances (see Ref. [21]).

On the other hand the $\omega$ spectra are not reproduced and the $M_{\omega p}$ and $M_{\omega \bar{p}}$ distributions miss a bump in the data at large masses. Inspection of figure 10 indicates a broad structure missing around 2 GeV. Such resonance has been already introduced into our description of the final state interaction. Nevertheless, its effect is not seen in the distribution of $M_{\omega \bar{p}}$ [18]. The formalism developed so far indicates a strong correlation of $M_{\omega p}$ and $M_{\omega \bar{p}}$. The phase space region close to the $p\bar{p}$ threshold overlaps with the region of $M_{\omega \bar{p}} \sim 2.05$ GeV. Thus enhancing the high energy tail of $M_{\omega p}$ reduces the low energy end of $M_{\omega \bar{p}}$. Within the BC+FSI approach it is not possible to reproduce both distributions and another mechanism has to be found. Another option tried was $J/\psi \to N(1/2^-)N^*(3/2^-) \to NN\omega$, ...
The \( \omega, \phi, \) and \( \pi \) emission rates

**FIG. 9:** Emission of the \( \omega \) meson from intermediate baryons. The left graph (a) corresponds to the Born term while the right one (b) includes final state corrections involving the \( N(3/2^-) \) nucleon resonance.

but for the reason given above it was not able to explain simultaneously the \( M_{p\bar{p}} \) and the \( M_{\omega p} \) spectra.

**FIG. 10:** The \( M_{\omega p} \) spectrum obtained with the BC+FSI+DE model (see Fig. 7 caption). The dash-dot curve (Basic model) refers to the BC+FSI calculation.

As the introduction of resonances brings no success we resort to another formation mechanism which was found useful in the study of radiative decays. A fraction of \( \omega \) mesons is assumed to be emitted internally i.e. before the baryons are formed in a \( P \) wave state. This emission process is depicted in Fig. 1(a), the photon line being replaced by an \( \omega \) meson line. Instead of the matrix element given by Eq. (14) related to \( S \) states, that associated to \( P \) states should be of the form

\[
V_{DE}^{\omega}(\lambda) = f \epsilon^i(\lambda) \cdot \xi,
\]

where \( f \) is a free coupling constant. In the center-of-mass (c.m.) system of \( J/\psi \) the bilinear form of \( V_{DE}^{\omega} \) averaged over directions of \( \xi \) leads to sum over polarizations

\[
\sum_{\lambda = -1}^1 | < V_{DE}^{\omega}(\lambda) > |^2 = f^2 \sum_\lambda \epsilon_i^* \epsilon_i = f^2 (3 - k^2/m^2_\omega).
\]

Now the essential point is that this coupling does not depend on \( q^2 \) in contrast to the helicity sum of \( |V_E^{\omega}|^2(\lambda) \) in Eq. (D30). Jointly with the assumed expansion of the system during the decay expressed in Eq. (24) one is able to avoid the unrequired correlation of low energy \( M_{p\bar{p}} \) and high energy \( M_{\omega p} \). Here, final state interactions are not introduced in the direct emission DE model but the same weak energy dependance for the source radius [Eq. (24)]
as for the photon case has been kept, i.e., $R_0 = 0.28$ fm and $\beta = 0.175$ fm$^{3/2}$. The strength of coupling to $V_{DE}^\gamma$ is obtained by the best fit to both $M_{p\bar{p}}$ and $M_{p\bar{p}}$ spectra. The results are given in Figs. 7 and 10. It is apparent that the bump in the last figure is not due to a resonance but to a different decay mode. This internal emission mode contributes about 22% of the decay rate.

3. The $\pi$ and $\phi$ emission rates

They are given in Table V. The neutral pion is emitted coherently from the intermediate $p\bar{p}$ system. The negatively charged pion may be emitted from the antiproton only. However, in the intermediate $I = 0$ state one has also $n\bar{n}$ component and the $\pi^-$ may be emitted by the neutron. These processes are coherent. Therefore the ratio $\Gamma(p\pi^-)/\Gamma(p\bar{p}) = 1.78$ (0.22) within error limits equals 2 minus the square of the relevant pion-nucleon coupling constants. This indicates that pions are emitted predominantly in the baryonic phase of decay and that final state interactions are not essential, the $p\bar{p}$ and $p\bar{n}$ interactions being different. The BES data indicate effects of $N^*$ resonances in the invariant mass distribution which, depending on the way of description, amount to some 25% of the total rate.

VI. SUMMARY AND OUTLOOK

A. Summary

In the present work, the $J/\psi \to Bp\bar{p}$ decays where $B = \gamma, \omega, \phi, \pi$ have been studied. Two processes have been introduced to describe the BES Collaboration data on the photon [3, 5] or $\omega$ meson [17, 18] formation in $J/\psi$ decays into $p\bar{p}$. For the radiative decays both processes include final state nucleon-antinucleon interactions with $S$-wave half-off shell functions [13] based on the Paris $NN$ potential [8]. The $J/\psi$ source is described in momentum space by a phenomenological Gaussian function with radius $R_0$ (see Eq. 9). The value $R_0 = 0.28$ fm is found to be the best choice to reproduce the different particle $B$ decay rates as compared to that of the $J/\psi \to p\bar{p}$ decay. Before presenting some outlook, the description of the two mechanisms and of the free parameters for the photon and meson emissions are summarized below.

1. Direct emission process. Here, the emission of photons or $\omega$ mesons occurs before the final baryons are formed. In the radiative decay, the final state interactions generate two distinct final resonant states in the $p\bar{p}$ system. One close to the threshold (very sharp peak in the $p\bar{p}$ spectra) is explained as an effect of baryonium - a broad 52 MeV wide quasi-bound state at 4.8 MeV below threshold generated in the $^{11}S_0$ wave by the Paris potential. Another - a resonant state at 2170 MeV - is formed as a shape resonance in the same partial wave. The Born contribution of the direct emission process allows to achieve a good description of the full $\omega$ spectrum at large $M_{p\bar{p}}$ and $M_{p\bar{p}}$ invariant masses. For the $\gamma$ or $\omega$ meson it is found necessary to introduce, for the source radius, a weak energy dependence on $M_{p\bar{p}}$ [see Eq. (21)], in both case the same dependence is used.

2. Emission from baryonic current. The second mechanism assumes the emission of photons (or mesons) by the baryonic currents in the final state of the $J/\psi \to p\bar{p}$ decay. This emission occurs thus after the initial decay of the $J/\psi$ into an $NN$ pair. In the radiative decay channel this process is not sufficient to provide a fair reproduction of final resonant states. This is the reason why this model has to be completed by the direct emission model just described above. For the $\omega$ meson production case, the Born term of this process is the dominant mode as it is in the $\pi$ or $\phi$ formation case. However, the $\omega$ invariant mass distribution $M_{p\bar{p}}$ requires a strong reduction in the lower mass region. This is obtained by introducing a specific final state interaction involving a $N^*(3/2)$ or $\bar{N}^*(3/2)$ resonance created by an $\omega-p$ ($\omega-\bar{p}$) interaction through an $\omega$ meson exchange between the $p(p)$ and $p(\bar{p})$ pairs [see Fig. 9(b)].
3. **Free parameters for the radiative emission.** For the photon emission case, to reproduce the $M_{p\pi}$ spectra (see Figs. 5 and 10) and the relative decay rate (see Table IV), seven free parameters are used: the initial radius of the source function, $R_0 = 0.28$ fm, the slope parameter associated to the energy dependence of this radius $\beta = 0.175$ fm$^{3/2}$, the normalization of the DE model contribution [Eq. (22)], that of the baryon current contribution [Eq. (33)], the coefficients $\Gamma_E = 500$ MeV and $N_E = 3.5$ entering the renormalized electric photon amplitude [Eq. (44)] and the normalization $f_{\eta_c}$ for the $\eta_c$ Breit-Wigner parametrization in Eq. (62).

4. **Free parameters for the meson emissions.** In the case of the $\omega$ meson emission, to fit the invariant masses $M_{p\pi}$ and $M_{p\omega}$ distributions (see Figs. 7 and 10) and the relative decay rate (see Table V), five free parameters are introduced: the normalization of the BC+FSI model [see Eqs. (33), (48) and (D30), (D33)], the two $N(3/2^-)$ parameters in Eq. (67), viz., $m_{3/2} = 2050$ MeV, $\Gamma = 300$ MeV plus the 0.3% strength of its coupling to $N\omega$ and the direct $\omega$ emission coupling constant $f$ [see Eq. (70)]. Looking at the ratio of the decay rates $R(p\omega)/R(p\eta)$ given in Table V, a $g^2_{\omega}\phi/4\pi$ coupling between 4.16 and 8.1 would bring this ratio closer to the experimental value. This table also shows that the Born amplitude [Eq. (27)] allows to reproduce well the relative decay rates for the cases of the $\pi$ or $\phi$ emission for known values of $g^2(p\omega)$ or $g^2(p\eta\phi)$.

5. **Uncertainties, shortcomings.** The basic mechanism for pion emission from the baryonic currents yields decay rates smaller than the experimental ones (see Table V). A proper description of the proton-pion invariant mass distribution requires at least three pion-nucleon resonant states and a good control over relative phases. The limitation to single dominant final partial wave is not sufficient to describe the rather precise data.

   For radiative decays, the separation of two formation mechanisms is only approximate due to two effects:

   (a) unknown relative phase of both amplitudes may affect the region $M_{p\pi} \approx 2.45 \pm 0.15$ GeV where these mechanisms give comparable rates,

   (b) there might be some presumably weak effect of the isospin symmetry violation in the course of the internal photon emission. Both effects are difficult to calculate.

### B. Outlook

The internal structure of the $J/\psi$ and $\psi(2S)$ being different, the direct emission model is less likely to give a reasonable description of the radiative decay of the $\psi(2S)$ state. This might explain, in a qualitative way, why no resonant states are visible in this process.

The phenomenological part of the $N\bar{N}$ Paris potential [8] has been determined to reproduce the $N\bar{N}$ data up to $E_{N\bar{N}} \approx 200$ MeV, i.e., $M_{p\pi} \approx 2.1$ GeV. It is interesting to observe that, nevertheless, it produces reasonable results beyond the region tested in scattering experiments. The present approach could also be applied with other $N\bar{N}$ scattering matrices, for instance that of Ref. [33]. Furthermore, with more accurate experimental results, effects of weakly populated final $N\bar{N}$ states might enter. In the present study, they do not seem to give sizable contributions.

Complications were found for the $\omega$ emission channel: the $M_{p\pi}$, $M_{p\omega}$ and $M_{p\eta}$ spectra [13] might indicate contributions of two $N^*$ states. The description of these is complementary to that of the mesic and radiative excitations of the nucleon. With increased precision of the BES Collaboration measurements, the extraction of the resonance parameters and nucleon-meson coupling constants should be more accurate.

Spectra of the $J/\psi \rightarrow p\bar{p}\pi^0$ decays (see, e.g., Ref. [34]) albeit not discussed here, indicate at least effects of established $N(1535)$ and $N(1650)$ states. The description of these decays seems to be a pressing question which may yield more information that those arising from the uncertain $\omega$ case.

The infrared catastrophe is approached by the baryon current model. It would thus be interesting to improve the energy resolution at the end of the spectrum to validate or disprove the photon (light meson) emission process from the final baryons.

Finally, the present work should allow to approach the related $\bar{p}p \rightarrow J/\psi + meson$ reaction on nuclei which sooner or later will be studied experimentally [1].

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Appendix A: Phase space

Let $p = (p_0, \mathbf{p})$, 
$$p^2 = p_0^2 - \mathbf{p}^2$$ 
and 
$$p_0 = E(p) = E(|\mathbf{p}|) = E(p) = \sqrt{\mathbf{p}^2 + m^2}.$$ 
Then the restricted two body phase space for the $J/\psi \rightarrow p\bar{p}$ decay at rest reads, with $q_1$ and $q_2$ denoting the four-momenta of the nucleon and the antinucleon with masses $m_1 = m_2 = m$, 
$$L_2 = (2\pi)^4 \int \prod_{i=1,2} \frac{d^4q_i}{(2\pi)^4} \theta(q_i^0) \delta(q_i^2 - m_i^2) \delta^{(3)}(q_1 + q_2) \delta(M_{J/\psi} - E(q_1) - E(q_2))$$ 
$$= \frac{1}{4\pi} \int \frac{q^2 dq}{E^2(q)} \delta(M_{J/\psi} - 2E(q)) = \frac{1}{8\pi} \frac{\sqrt{M_{J/\psi}^2 - 4m^2}}{M_{J/\psi}}. \quad (A1)$$
which is numerically equal to 0.03164.

For a $J/\psi$ at rest decaying into a $p\bar{p}\pi$ channel with respective four-momenta $q_1, q_2$ and $k$, the three-body phase space reads 
$$L_3 = (2\pi)^4 \int \prod_{i=1,2} \frac{d^4q_i}{(2\pi)^4} \theta(q_i^0) \delta(q_i^2 - m_i^2) \delta^{(3)}(q_1 + q_2 + k) \delta(M_{J/\psi} - E(q_1) - E(q_2) - E_B(k))$$ 
$$= \frac{1}{(2\pi)^5} \int \frac{dq_1}{2E(q_1)} \frac{dq_2}{2E(q_2)} \int \frac{dk}{2E_B(k)} \delta^{(3)}(q_1 + q_2 + k) \delta(M_{J/\psi} - E(q_1) - E(q_2) - E_B(k))$$ 
$$= \frac{1}{(2\pi)^5} \int \frac{dq}{2E(q)} \int \frac{dk}{2E_B(k)} \frac{dx}{2E_B(k) 2E(-q-k)} \delta(f(x)) \quad (A2)$$
with 
$$x = \frac{k \cdot q}{k q} \quad (A3)$$
and 
$$f(x) = M_{J/\psi} - E(q) - \sqrt{k^2 + q^2 + 2kq x + m^2 - E_B(k)}. \quad (A4)$$

Thus, energy conservation implies that $f(x)$ cancels for 
$$x = x_0 = \frac{M_{p\bar{p}}^2 - 2E(q)[M_{J/\psi} - E_B(k)]}{2 k q} \quad (A5)$$
and we have introduced the invariant nucleon-antinucleon invariant mass squared 
$$M_{p\bar{p}}^2 = s = (M_{J/\psi} - E_B(k))^2 - k^2. \quad (A6)$$
The invariant $NN$ mass spans the interval $[2m, M_{J/\psi} - m_B]$. Then, we have 
$$L_3 = \frac{1}{(2\pi)^3} \int_0^\infty \frac{k^2 dk}{2E_B(k)} \int_0^\infty \frac{q^2 dq}{2E(q)} \int_{-1}^{1} dx \frac{\delta(f(x))}{\sqrt{k^2 + q^2 + 2kq x + m^2}}$$ 
$$= \frac{1}{(2\pi)^3} \int_0^\infty \frac{k dk}{2E_B(k)} \int_0^\infty \frac{q dq}{2E(q)} \int_{-1}^{1} dx \delta(x - x_0). \quad (A7)$$
From Eq. (A5) where one has $-1 \leq x_0 \leq 1$, one gets $1 - x_0^2 \geq 0$ so that 
$$4k^2 q^2(1-x_0^2) = -4M_{p\bar{p}}^2 E^2(q) + 4[M_{J/\psi} - E_B(k)] E(q) M_{p\bar{p}}^2 - (M_{p\bar{p}}^2 + 4m^2 k^2)$$
must be positive. Hence $E(q)$ will lie between the two positive roots, $E_+$ and $E_-$, of the trinomial 
$$E_{\pm} = \frac{M_{J/\psi} - E_B(k)}{2} \pm \frac{k}{2 M_{p\bar{p}}} \sqrt{M_{p\bar{p}}^2 - 4m^2}, \quad (A8)$$
where $E_- > m$ and, from Eq. (A6), $k$ is a function of the invariant mass $M_{\gamma\bar{p}}$

$$k = k(M_{\gamma\bar{p}}) = \sqrt{[(M_{\gamma\psi} + m_\gamma)^2 - M_{\gamma\bar{p}}^2] \left[ (M_{\gamma\psi} - m_\gamma)^2 - M_{\gamma\bar{p}}^2 \right]} \left[ 2M_{\gamma\psi} \right] = \frac{\sqrt{\lambda(m_\gamma^2, M_{\gamma\psi}^2, M_{\gamma\bar{p}}^2)}}{2M_{\gamma\psi}}, \quad (A9)$$

where we have introduced the standard triangle (Källen) function $\lambda(x, y, z)^4$.

We may now transform the remaining integrations using $q \, dq/E(q) = dE$ and from Eq. (A6) changing the variable $k$ to $M_{\gamma\bar{p}}$ to obtain

$$L_3 = \frac{1}{(2\pi)^3} \int_{2m}^{M_{\gamma\psi} - m_\gamma} \frac{M_{\gamma\bar{p}} \, dM_{\gamma\bar{p}}}{4M_{\gamma\psi}} (E_+ - E_-) = \frac{1}{(2\pi)^3} \frac{1}{4M_{\gamma\psi}} \int_{2m}^{M_{\gamma\psi} - m_\gamma} k(M_{\gamma\bar{p}}) \, \sqrt{M_{\gamma\bar{p}}^2 - 4m^2} \, dM_{\gamma\bar{p}} \quad (A10)$$

and arrive at

$$L_3 = \frac{1}{(2\pi)^3} \frac{1}{8M_{\gamma\psi}} \int_{2m}^{M_{\gamma\psi} - m_\gamma} \sqrt{\lambda(m_\gamma^2, M_{\gamma\psi}^2, M_{\gamma\bar{p}}^2) [M_{\gamma\bar{p}}^2 - 4m^2]} \, dM_{\gamma\bar{p}}$$

$$= \frac{1}{(2\pi)^3} \frac{1}{16M_{\gamma\psi}^2} \int_{4m^2}^{(M_{\gamma\psi} - m_\gamma)^2} \frac{du}{u} \, \sqrt{\lambda(m_\gamma^2, M_{\gamma\psi}^2, u) \lambda(m^2, m^2, u)}. \quad (A11)$$

In the case where the vector particle is a photon, the integral (A11) can be calculated exactly (see, e.g., Ref. [35]) and gives

$$L_{3,\gamma} = \frac{1}{32\pi^3} \left\{ \frac{M_{\gamma\psi}^2}{8M_{\gamma\psi}} \sqrt{M_{\gamma\psi}^2 - 4m^2} - \frac{m^2}{M_{\gamma\psi}^2} (M_{\gamma\psi}^2 - m_\gamma^2) \ln \left[ \frac{M_{\gamma\psi} + \sqrt{M_{\gamma\psi}^2 - 4m^2}}{2m} \right] \right\}, \quad (A12)$$

which is numerically equal to 261.718 (MeV)$^2$.

**Appendix B: The width for $J/\psi \to p\bar{p}B$ decay process**

Let us evaluate the decay amplitude in the Born approximation, the boson being radiated either from the proton or from the antiproton. If we let the proton radiate, left panel on figure 4 indicates that $q_1 = q - k$ denotes the momentum of the proton after the photon emission while $q_2 = -q$ is the spectator antiproton final momentum, $k$ the boson momentum. Then, the potential $U_{p\bar{p}B}(q_1, k)$ can be expressed as [Eq. (68)]

$$U_{p\bar{p}B}^0(q_1, k) = A_{V,B}(q_1, k) + \frac{i\kappa}{2m} A_{T,B}(q_1, k) \quad (B1)$$

with $\kappa$ the anomalous magnetic moment. From this expression one then evaluates the associated amplitude [Eq. (28)] $A_{p\bar{p}B}^{B,BC}(q_1, k)$ which is still an operator in the spin-isospin space. From Eqs. (27) and (28), we obtain the probability for the decay with a boson emission

$$\Gamma(p\bar{p}B) = \frac{1}{(2\pi)^5} \int \frac{dq_1 \, dq_2 \, dk}{2E(q_1)2E(q_2)2E_B(k)} \delta(M_{\gamma\psi} - E(q_1) - E(q_2) - k) \, \delta^{(3)}(q_1 + q_2 + k) |A_{p\bar{p}B}^{B,BC}(q_2, k)|^2. \quad (B2)$$

For a boson emission of mass $m_\gamma$ with an energy $E_B(k) = \sqrt{m_\gamma^2 + k^2}$, we have

$$\Gamma(p\bar{p}B) = \frac{1}{(2\pi)^5} \int \frac{dq \, dk}{2E(q)2E(q + k)2E_B(k)} \delta(M_{\gamma\psi} - E(q) - E(q + k) - E_B(k)) |A_{p\bar{p}B}^{B,BC}(q, k)|^2$$

$$= \frac{1}{(2\pi)^4} \int \frac{dq}{2E(q)} \int \frac{k^2 \, dk}{2E_B(k)} \int dx \frac{\delta(f(x))}{2E(q + k)} |A_{p\bar{p}B}^{B,BC}(q, k, x)|^2, \quad (B3)$$

---

4 One has $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx = (x + y - z)^2 - 4xy = (-x + y + z)^2 - 4yz = (x - y + z)^2 - 4zx$. 
where $x$ has been defined in Eq. (A3) and $f(x)$ is given in Eq. (A4). The $x$ integration, based on the energy conservation relation and assuming, $|A_{ppB}^{B,BC}(q,k,x)|$ to be independent of $x$ and depend mainly on $q$, i.e., $|A_{ppB}^{B,BC}(q_1,k)|$ has been defined in Eq. (A3) and $f(x)$ evaluated using Eq. (A5), the condition $-1 \leq x \leq 1$. Therefore, we have

$$\Gamma(ppB) = \frac{1}{32\pi^3 M_{J/\psi}} \int M_{p\bar{p}} \ dM_{p\bar{p}} \int \frac{q dq}{E(q)} |h(q)|^2,$$

(B5)

The experimental spectrum of the $p\bar{p}$ invariant mass is rather complicated and its description is a check for the theory in question. Since we may write the width for the emission of a vector particle as

$$\Gamma(ppB) = \int dM_{p\bar{p}} \ S(M_{p\bar{p}}),$$

(B6)

where $S(M_{p\bar{p}})$ denotes the spectral function, we have

$$S(M_{p\bar{p}}) = \frac{M_{p\bar{p}}}{32\pi^3 M_{J/\psi}} \int \frac{q dq}{E(q)} |h(q)|^2 = \frac{M_{p\bar{p}}}{32\pi^3 M_{J/\psi}} \int_{E_-}^{E_+} dE \ |h(q)|^2,$$

(B7)

where the integration limits $E_-$ and $E_+$ are given in Eq. (A8) while the invariant mass $M_{p\bar{p}}$ is defined by Eq. (A6) and the emitted particle momentum $k(M_{p\bar{p}})$ is read from Eq. (A9). Numerical calculations at the endpoints require care and the following approximate expression is helpful to check the accuracy

$$S(M_{p\bar{p}}) \approx \frac{M_{p\bar{p}}}{32\pi^3 M_{J/\psi}} (E_+ - E_-) |h|^2,$$

(B8)

assuming further that $|h|^2$ depends only weakly on $E$. From Eq. (A8), the difference $E_+ - E_-$ is simply

$$E_+ - E_- = k(M_{p\bar{p}}) \sqrt{M_{p\bar{p}}^2 - 4m^2}.$$ 

So, finally, the spectral function reads

$$S(M_{p\bar{p}}) = \frac{\sqrt{\lambda(M_{p\bar{p}}^2 - 4m^2)} \sqrt{M_{p\bar{p}}^2 - 4m^2}}{64\pi^3 M_{J/\psi}} |h|^2$$

(B9)

and this formula is useful to understand the end points. Close to the $p\bar{p}$ threshold, $M_{p\bar{p}} = 2m$, one finds $S \sim \sqrt{k_M - k}$ where $k_M = (M_{J/\psi}^2 - 4m^2)/2M_{J/\psi} = 979.14$ MeV is the maximal value of $k$ reached at the threshold. This limit leads to $k = 2q$ and the partners in the $p\bar{p}$ pair run parallel in the direction opposite to the photon direction. Such a configuration enhances final state interactions. This dependence in $k_M - k$ determines the position of the threshold peak in $S$. At the other end of the spectrum, $M_{p\bar{p}} = M_{J/\psi} - m_B$, one has $S \sim k = 0$.

**Appendix C: Explicit expressions for the electromagnetic operators**

Let $q$ be the initial nucleon four momentum and $q' = q - k$ that of the nucleon after emission of the boson with four-momentum $k$. The following Lorentz condition applies

$$\mathbf{k} \cdot \mathbf{e}^*(\lambda) - k_0 e_0^*(\lambda) = 0,$$

(C1)

which in the case of the photon leads to

$$\mathbf{k} \cdot \mathbf{e}^*(\lambda) = 0,$$

(C2)
and for a massive vector particle to

$$
\epsilon_0^*(\lambda) = k \cdot e^*(\lambda)/k_0.
$$

(C3)

The vector part of the current reads

$$
A_V(q', q, e^*(\lambda)) = \bar{u}(q') \gamma_\mu e^{\mu*}(\lambda) \ u(q) = \chi_{S'}^\dagger \ A_V(q, k, e^*(\lambda)) \ \chi_S,
$$

(C4)

where $\chi_S$ and $\chi_{S'}$ denote the standard two-dimensional spin vectors. The four-dimensional spinors read

$$
u(q) = \sqrt{\frac{\Omega}{2m}} \ \left( \frac{\sigma \cdot q'}{\Omega} \chi_S \right),$$

$$\bar{u}(q') = u(q')^\dagger \gamma_0 = \sqrt{\frac{\Omega'}{2m}} \left( \chi_{S'}^\dagger \frac{\sigma \cdot q' - \sigma \cdot q}{\Omega'} \chi_{S'} \right) \ \gamma_0 = \sqrt{\frac{\Omega}{2m}} \left( \chi_{S'}^\dagger - \frac{\sigma \cdot q'}{\Omega'} \chi_{S'} \right),$$

(C5)

where the energies $\Omega$ and $\Omega'$ are

$$
\Omega = m + E_q \quad \text{and} \quad \Omega' = m + \sqrt{(q - k)^2 + m^2},
$$

(C6)

$$E_q = \sqrt{q^2 + m^2},$$

(C7)

with $m$ the nucleon mass. In the following we use the Bjorken and Drell definitions of the Pauli $\sigma$ and $\gamma$ matrices[36]. Hence, from Eq. (C4) for the vector term we can write

$$A_V(q', q, e^*(\lambda)) = N \left[ (1 + \frac{\sigma \cdot q'}{\Omega'} \frac{\sigma \cdot q}{\Omega}) \ \epsilon_0^*(\lambda) - \left( \frac{\sigma \cdot e^*(\lambda)}{\Omega} \frac{\sigma \cdot q}{\Omega} + \frac{\sigma \cdot q' \cdot e^*(\lambda)}{\Omega'} \right) \right],$$

(C8)

with the normalization factor $N$

$$N = \frac{\sqrt{\Omega \Omega'}}{2m} = \frac{\Omega \zeta}{2m} \quad \text{with} \quad \zeta = \sqrt{\frac{\Omega'}{\Omega}}.$$  

(C9)

Upon using the standard relation for any two vectors $v$ and $w$

$$ (\sigma \cdot v) (\sigma \cdot w) = v \cdot w + i \ \sigma \cdot (v \wedge w),$$

(C10)

the spin operator $A_V(k, q, e^*(\lambda))$ becomes

$$A_V(q, k, e^*(\lambda)) = \frac{k \cdot e^*(\lambda)}{2mk_0} \left\{ \Omega \zeta + \frac{q \cdot (q - k)}{\Omega \zeta} - i \ \frac{\sigma \cdot (k \wedge q)}{\Omega \zeta} \right\}$$

$$- \frac{1}{2m} \left\{ \left( \zeta + \frac{1}{\zeta} \right) q \cdot e^*(\lambda) - \frac{1}{\zeta} k \cdot e^*(\lambda) - i \left( \zeta - \frac{1}{\zeta} \right) \sigma \cdot (q \wedge e^*(\lambda)) - i \frac{1}{\zeta} \sigma \cdot (k \wedge e^*(\lambda)) \right\}.$$  

(C11)

For the emission from the antinucleon with momentum $-q$, we simply have to change in Eqs. (C11) and (C18) $q$ into $-q$

$$\bar{A}_V(q, k, e^*(\lambda)) = A_V(-q, k, e^*(\lambda)).$$

(C12)

The substitution $q \rightarrow -q$ induces $\zeta \rightarrow \zeta$ where

$$\zeta = \sqrt{\frac{\Omega}{\Omega'}} \quad \text{with} \quad \Omega = m + \sqrt{(q + k)^2 + m^2}.$$  

(C13)

The tensor piece is more elaborate

$$A_T(k, q, e^*(\lambda)) = \bar{u}(q') \ \sigma_{\mu \nu} k^\mu e^{\nu*}(\lambda) \ u(q) = \chi_{S'}^\dagger \ A_T(k, q, e^*(\lambda)) \ \chi_S,$$

(C14)

with

$$A_T(q, k, e^*(\lambda)) = \frac{\Omega \zeta}{2m} \left( 1 - \frac{\sigma \cdot q'}{\Omega'} \right) \ \sigma_{\mu \nu} k^\mu e^{\nu*}(\lambda) \ \left( \frac{1}{\Omega} \ \frac{\sigma \cdot q}{\Omega} \right).$$

(C15)
Since 

\[ \sigma_{\mu
u} k^\mu e^\nu(\lambda)(k) = -i k_0 \begin{pmatrix} 0 & \sigma \cdot e^\lambda(\lambda) \\ \sigma \cdot e^\lambda(\lambda) & 0 \end{pmatrix} + i \epsilon_0(\lambda) \begin{pmatrix} 0 & \sigma \cdot k \\ \sigma \cdot k & 0 \end{pmatrix} + \begin{pmatrix} \sigma \cdot (k \wedge e^\lambda(\lambda)) & 0 \\ 0 & \sigma \cdot (k \wedge e^\lambda(\lambda)) \end{pmatrix}. \]

we obtain

\[
A_T(k, q, e^\lambda(\lambda)) = \frac{\Omega \zeta}{2m^2} \left\{ -i k_0 \left[ \frac{\sigma}{\Omega} \cdot e^\lambda(\lambda) \frac{\sigma \cdot q}{\Omega} - \frac{\sigma \cdot q'}{\Omega'} \right] \right. 
+ \left. \sigma \cdot [k \wedge e^\lambda(\lambda)] - \sigma \cdot \frac{q'}{\Omega'} \sigma \cdot [k \wedge e^\lambda(\lambda)] \frac{\sigma \cdot q}{\Omega} \right\}. 
\]

(C16)

With the repeated use of Eq. (C10) and of double vectorial product properties, we are led to the following explicit expression

\[
A_T(q, k, e^\lambda(\lambda)) = \frac{i k_0}{2m} \left( \frac{k \cdot e^\lambda(\lambda)}{k_0} \right) \left[ \left( \zeta - \frac{1}{\zeta} + \frac{k_0}{\Omega \zeta} \right) k \cdot q + \frac{1}{\zeta} \left( k^2 - k_0^2 \right) + i \left( \zeta + \frac{1}{\zeta} + \frac{k_0}{\Omega \zeta} \right) \sigma \cdot (k \wedge q) \right] 
- \frac{i k_0}{2m} \left[ q \cdot e^\lambda(\lambda) \left[ \zeta - \frac{1}{\zeta} + \frac{k^2}{k_0 \Omega \zeta} + \frac{i}{k_0 \Omega \zeta} \sigma \cdot (k \wedge q) \right] + \frac{\Omega \zeta}{2m} \sigma \cdot [k \wedge e^\lambda(\lambda)] \left[ 1 + \frac{k_0}{\Omega \zeta^2} \right] \right] 
- \frac{k_0}{2m} \sigma \cdot (q \wedge e^\lambda(\lambda)) \left[ \zeta + \frac{1}{\zeta} - \frac{1}{k_0 \Omega \zeta} (q - k) \cdot k \right] - \frac{1}{2m} \frac{1}{\Omega \zeta} q \cdot [k \wedge e^\lambda(\lambda)] \sigma \cdot q. 
\]

(C17)

where \( k_0 = \sqrt{k^2 + m^2} \). The tensor amplitude for the emission from the antinucleon will be obtained from the replacements \( q \rightarrow -q \) and hence \( \zeta \rightarrow \zeta' \) in Eq. (C17).

1. The specific case of the photon

For the photon, since \( \epsilon_0(\lambda) = 0 \) and thus \( k \cdot e^\lambda(\lambda) = 0 \), the vector amplitude reduces to

\[
A_{V,\gamma}(q, k, e^\lambda(\lambda)) = -\frac{1}{2m} \left\{ \left( \zeta + \frac{1}{\zeta} \right) q \cdot e^\lambda(\lambda) - i \left( \zeta - \frac{1}{\zeta} \right) \sigma \cdot [q \wedge e^\lambda(\lambda)] - i \frac{1}{\zeta} \sigma \cdot [k \wedge e^\lambda(\lambda)] \right\},
\]

(C18)

while the tensor amplitude becomes

\[
A_{T,\gamma}(q, k, e^\lambda(\lambda)) = -\frac{i k_0}{2m} q \cdot e^\lambda(\lambda) \left[ \zeta - \frac{1}{\zeta} + \frac{k^2}{k_0 \Omega \zeta} + \frac{i}{k_0 \Omega \zeta} \sigma \cdot (k \wedge q) \right] + \frac{\Omega \zeta}{2m \kappa} \sigma \cdot [k \wedge e^\lambda(\lambda)] \left[ 1 + \frac{k_0}{\Omega \zeta^2} \right] 
- \frac{k_0}{2m} \sigma \cdot (q \wedge e^\lambda(\lambda)) \left[ \zeta + \frac{1}{\zeta} - \frac{1}{k_0 \Omega \zeta} (q - k) \cdot k \right] - \frac{1}{2m} \frac{1}{\Omega \zeta} q \cdot [k \wedge e^\lambda(\lambda)] \sigma \cdot q.
\]

(C19)

where, here, \( k_0 = |k| = k \). The corresponding photon amplitude for the antinucleon emission, \( A_{T,\gamma} \), will be obtained with the replacement \( q \rightarrow -q \) which induces \( \zeta \rightarrow \zeta' \).

The vertex coupling yields for the photon emission from the nucleon

\[
V_N^\gamma(q, k) = e \left( A_{V,\gamma} + i \frac{\kappa}{2m} A_{T,\gamma} \right) 
= -\frac{e}{2m} \left\{ \left( \zeta + \frac{1}{\zeta} \right) - \frac{\kappa k_0}{2m} \left[ \zeta - \frac{1}{\zeta} + \frac{k_0}{\Omega \zeta} + i \frac{\sigma \cdot (k \wedge q)}{k_0 \Omega \zeta} \right] \right\} q \cdot e^\lambda(\lambda) 
+ \frac{ie}{2m} \left\{ \left( \zeta - \frac{1}{\zeta} \right) - \frac{\kappa k_0}{2m} \left( \zeta + \frac{1}{\zeta} - \frac{q \cdot k - k_0^2}{k_0 \Omega \zeta} \right) \right\} \sigma \cdot (q \wedge e^\lambda(\lambda)) 
+ \frac{ie}{2m} \left\{ \frac{1}{\zeta} + \frac{\kappa}{2m} \left( \Omega \zeta + \frac{k_0}{\Omega \zeta} \right) \right\} \sigma \cdot [k \wedge e^\lambda(\lambda)] 
- \frac{iek}{2m} \frac{q \cdot [k \wedge e^\lambda(\lambda)]}{2m \Omega \zeta} \sigma \cdot q. \]

(C20)
and correspondingly for the emission from the antinucleon

\[ V^\gamma_N(q, k) = -e \left[ \mathcal{A}_{V\gamma}(-q, k) + i \frac{k}{2m} \mathcal{A}_{T\gamma}(-q, k) \right] \]  

(C21)

obtained by making the substitutions, \( e \rightarrow -e, q \rightarrow -q \) and, hence, \( \zeta \rightarrow \bar{\zeta} \).

Labelling the nucleon part by 1 and the antinucleon part by 2 we will get the potential for the photon emission and recombing these expressions

\[
\begin{align*}
U^0_\gamma(q, k) &= \frac{e}{2m} \left\{ \hat{f}(q, k) + \tilde{f}(q, k) + i \frac{k}{2m\Omega} \left( \frac{\sigma_1}{\zeta} - \frac{\sigma_2}{\bar{\zeta}} \right) \cdot (k \wedge q) \right\} \cdot q \cdot \epsilon^*(\lambda) \\
&+ \frac{ie}{2m} \left[ \hat{g}(q, k) \sigma_1 + \tilde{g}(q, k) \sigma_2 + \frac{k}{2m\Omega} q \cdot k \left( \frac{\sigma_1}{\zeta} - \frac{\sigma_2}{\bar{\zeta}} \right) \right] : [q \wedge \epsilon^*(\lambda)] \\
&+ \frac{ie\kappa}{2m} \frac{q \cdot (k \wedge \epsilon^*(\lambda))}{2m\Omega} \left( \frac{\sigma_1}{\zeta} - \frac{\sigma_2}{\bar{\zeta}} \right) \cdot q ,
\end{align*}
\]

(C22)

where we have introduced

\[
\begin{align*}
f(q, k) &= - \left( \zeta + \frac{1}{\zeta} \right) + \frac{k\kappa}{2m} \left( \zeta - \frac{1}{\zeta} + \frac{k}{\Omega\zeta} \right) , \\
g(q, k) &= \left( \zeta - \frac{1}{\zeta} \right) - \frac{k\kappa}{2m} \left( \zeta + \frac{1}{\zeta} + \frac{k}{\Omega\zeta} \right) , \\
h(q, k) &= \frac{1}{\zeta} + \frac{k\kappa}{2m} \left( \frac{\Omega\zeta}{k} + \frac{1}{\zeta} \right) ,
\end{align*}
\]

(C23)

where we have used \( k = k_0 \). The bar functions are identical but with \( \zeta \rightarrow \bar{\zeta} \).

2. The case for the \( \omega \) meson

For completeness we present below the exact expressions for the \( \omega \) meson emission. Only a few terms are really exploited in the present work. With now

\[ k_0 = E_\omega = \sqrt{k^2 + m^2_\omega} , \]

(C24)

where \( m_\omega \) denotes the mass of the \( \omega \) meson, and, taking into account the change of sign of the \( \omega \) coupling at the \( \mathbf{N} \) vertex because of \( \mathbf{G} \) parity, from Eqs. (38) and (C8), the vector piece will read,

\[
\begin{align*}
A_{V,\omega} &= g_{V\omega} \frac{k \cdot \epsilon^*(\lambda)}{2mE_\omega} \left[ \Omega(\zeta - \bar{\zeta}) + \frac{q^2}{\Omega} \left( \frac{1}{\zeta} - \frac{1}{\bar{\zeta}} \right) - \frac{k \cdot q}{\Omega} \left( \frac{1}{\zeta} + \frac{1}{\bar{\zeta}} \right) - i \left( \frac{\sigma_1}{\zeta} + \frac{\sigma_2}{\bar{\zeta}} \right) \cdot (k \wedge q) \right] \\
&- \frac{g_{V\omega}}{2m} \left( \left( \zeta + \frac{1}{\zeta} + \bar{\zeta} + \frac{1}{\bar{\zeta}} \right) q \cdot \epsilon^*(\lambda) - \left( \frac{1}{\zeta} - \frac{1}{\bar{\zeta}} \right) k \cdot \epsilon^*(\lambda) \right) \\
&- i \left( \frac{\sigma_1}{\zeta} - \frac{\sigma_2}{\bar{\zeta}} \right) \cdot (k \wedge \epsilon^*(\lambda)) - i \left\{ \left( \zeta - \frac{1}{\zeta} \right) \sigma_1 + \left( \bar{\zeta} - \frac{1}{\bar{\zeta}} \right) \sigma_2 \right\} \cdot [q \wedge \epsilon^*(\lambda) ,
\end{align*}
\]

(C25)

which to order \( k^2/4m^2 \) reduces to

\[
\begin{align*}
A_{V,\omega} &\simeq g_{V\omega} \frac{k \cdot \epsilon^*(\lambda)}{2m} \left( 2 \left( -1 + \frac{q^2}{2\Omega q} - \frac{\Omega}{2E_\omega q} \right) \frac{k \cdot q}{\Omega} - i \frac{1}{\Omega} \left( \sigma_1 + \sigma_2 \right) \cdot (k \wedge q) \right) \frac{k \cdot \epsilon^*(\lambda)}{E_\omega} \\
&- \frac{g_{V\omega}}{2m} \left\{ 4 q \cdot \epsilon^*(\lambda) - i(\sigma_1 - \sigma_2) \cdot [k \wedge \epsilon^*(\lambda)] + \frac{k \cdot q}{\Omega E_\omega} (\sigma_1 - \sigma_2) \cdot [q \wedge \epsilon^*(\lambda)] \right\} ,
\end{align*}
\]

(C26)
where $E_q$ has been defined in Eq. (C7) and $Ω$ in Eq. (C6). Then we have to add the tensor piece, knowing that the coupling constant for this part is rather ill known but most likely small

$$E_ω A_{T,ω} = i \frac{g_{Tω}}{2m} \left\{ i \left[ \left( \frac{1}{Ω} \right) + \left( \frac{1}{Ω'} \right) \right] k \cdot q + \left( \frac{1}{Ω} - \frac{1}{Ω'} \right) \right\} (k^2 - E_ω^2)$$

$$+ i \left( ζσ_1 + ζ′σ_2 \right) \cdot \left( k \cdot q \right) + i \left( 1 + \frac{E_ω}{Ω} \right) \left( \frac{σ_1}{Ω} + \frac{σ_2}{Ω'} \right) \cdot \left( k \cdot q \right) \frac{k \cdot e^∗(λ)}{2mE_ω}$$

$$- i \frac{E_ω}{2m} \left[ \left( ζ + ζ′ - \left( 1 - \frac{k^2}{E_ωΩ} \right) \frac{1}{Ω} + \frac{1}{Ω'} \right) + i \frac{E_ω}{Ω} \left( \frac{σ_1}{Ω} - \frac{σ_2}{Ω'} \right) \cdot \left( k \cdot q \right) \right] q \cdot e^∗(λ)$$

$$+ \frac{Ω}{2m} \left[ \left( ζ + \frac{E_ω}{Ω} σ_1 \right) - \left( \frac{ζ}{Ω} + \frac{E_ω}{Ω} σ_2 \right) \cdot \left( k \cdot e^∗(λ) \right) \right]$$

$$- \frac{E_ω}{2m} \left[ \left( 1 + \frac{ζ}{Ω} + \frac{E_ω}{Ω} \left( \frac{k \cdot q}{Ω} \right) \right) σ_1 + \left( \frac{ζ}{Ω} + \frac{1}{Ω'} + \frac{k^2}{E_ωΩ} + \frac{k \cdot q}{E_ωΩ} \right) σ_2 \right] \cdot \left( k \cdot q \right) q \cdot e^∗(λ)$$

We have then

$$V_ω = A_{V,ω} + A_{T,ω}.$$  \hspace{1cm} (C28)

### Appendix D: Amplitudes for boson emission

The boson with 3-momentum $k$ is emitted either from the nucleon (labelled 1) or from the antinucleon (labelled 2) lines. In the rest frame of the $J/ψ$, the nucleon momentum prior to the photon emission is denoted $q$ and $-q$ is that of the antinucleon. The mass of the nucleon and antinucleon is denoted by $m$. The various variables that occur in this appendix, $Ω$, $Ω'$, $Ω$, $ζ$ and $ζ'$, are given in Eqs. (C6), (C9) and (C13).

1. Photon emission

a. Vector emission

When the photon is emitted from the nucleon line the contribution of the vector term to the amplitude is given by Eq. (C18) multiplied by the charge $e$ and where the helicity $λ$ takes the values $±1$. To obtain the amplitude corresponding to the photon emission from the antinucleon line one has simply to substitute $e → -e$ and $q → -q$. Thus the full amplitude arising from the vector coupling reads

$$\tilde{A}_V(q, k, e^∗(λ)) = \tilde{A}_{V,a} + \tilde{A}_{V,b} + \tilde{A}_{V,c},$$  \hspace{1cm} (D1)

with

$$\tilde{A}_{V,a}(q, k, e^∗(λ)) = -\frac{e}{2m} \left[ \left( \frac{1}{ζ} + \frac{1}{ζ'} \right) + \left( ζ + \frac{1}{ζ'} \right) \right] q \cdot e^∗(λ),$$  \hspace{1cm} (D2)

$$\tilde{A}_{V,b}(q, k, e^∗(λ)) = \frac{ie}{2m} \left[ \left( ζ - \frac{1}{ζ} \right) σ_1 \cdot q \cdot e^∗(λ) + \left( \frac{1}{ζ} - ζ' \right) σ_2 \cdot q \cdot e^∗(λ) \right],$$  \hspace{1cm} (D3)

$$\tilde{A}_{V,c}(q, k, e^∗(λ)) = \frac{ie}{2m} \left( \frac{σ_1}{ζ} - \frac{σ_2}{ζ'} \right) \cdot k \cdot e^∗(λ).$$  \hspace{1cm} (D4)

In these equations $λ = ±1$ and since, for the photon, $e_0^*(λ) = 0$ and $k$ is chosen to lie along the z-axis, the orthogonality relation $k \cdot e^∗(λ) = 0$ implies

$$e^∗(λ = ±1) = \frac{1}{\sqrt{2}} \left( -λ, -i, 0 \right).$$  \hspace{1cm} (D5)

The $\tilde{A}_{V,a}$ amplitude contributes to the $^{3}S_1 → ^3 P_1$ electric coupling amplitude (D19). The $\tilde{A}_{V,b}$ amplitude leads to a final $^3 P_0$ state not included, so far, in our work. Note that, it vanishes in the approximation $ζ \sim \tilde{ζ}$. The $\tilde{A}_{V,c}$ amplitude contributes to the $^3 S_1 → ^1 S_0$ transition.
b. Tensor coupling

With the same notations as above, the exact amplitude for the tensor coupling when the photon is emitted from the nucleon line reads

$$A_1^T(q, k, e^*(\lambda)) = \frac{e\kappa_N}{4m^2} \left\{ k \cdot q \cdot e^*(\lambda) \left[ \zeta - \frac{1}{\zeta} + \frac{k}{\Omega \zeta} + \frac{i}{\Omega \zeta} \sigma_1 \cdot k \cdot q \right] + i \sigma_1 \cdot k \cdot e^*(\lambda) \left( \Omega \zeta + \frac{k}{\zeta} \right) \right\}, \quad (D6)$$

where $\kappa_N$ is either $\kappa_p$ or $\kappa_n$. In the following we do not keep the terms proportional to $\sigma_1 \cdot q \cdot e^*(\lambda)$ and depending on $q^2$ as they will contribute to final $3P_0$ and $D$ waves, respectively. These $P$ and $D$ wave are absent in our model. Thus, the amplitude is reduced to

$$A_1^T(q, k, e^*(\lambda)) \approx \frac{e\kappa_N}{4m^2} \left\{ k \cdot q \cdot e^*(\lambda) \left[ \zeta - \frac{1}{\zeta} + \frac{k}{\Omega \zeta} \right] + i \sigma_1 \cdot k \cdot e^*(\lambda) \left( \Omega \zeta + \frac{k}{\zeta} \right) \right\}, \quad (D7)$$

to which we have to add the part associated to the emission from the antinucleon $A_2^T(q, k, e^*(\lambda))$. Thus the total tensor amplitude in this approximation $A_1^T(q, k, e^*(\lambda))$ can be split into two contributions (with $\lambda = \pm 1$)

$$A_1^T(q, k, e^*(\lambda)) = A_1^T(q, k, e^*(\lambda)) + A_2^T(q, k, e^*(\lambda)) = A_{1,a}^T(q, k, e^*(\lambda)) + A_{1,b}^T(q, k, e^*(\lambda)), \quad (D8)$$

with

$$A_{1,a}^T(q, k, e^*(\lambda)) = \frac{e\kappa_N}{4m^2} \left\{ k \cdot q \cdot e^*(\lambda) \left[ \zeta + \frac{1}{\zeta} + \frac{k}{\Omega \zeta} \right] \right\}, \quad (D9)$$

and

$$A_{1,b}^T(q, k, e^*(\lambda)) = \frac{e\kappa_N}{4m^2} \left\{ \sigma_1 \cdot k \cdot e^*(\lambda) \left( \Omega \zeta + \frac{k}{\zeta} \right) - i \sigma_2 \cdot k \cdot e^*(\lambda) \left( \Omega \zeta + \frac{k}{\zeta} \right) \right\}. \quad (D10)$$

The amplitude $A_{1,a}^T(q, k, e^*(\lambda))$ term $D9$ will add up to the $A_{1,b}^T(q, k, e^*(\lambda))$ term $D2$ to give the $3S_1 \rightarrow 3P_1$ transition amplitude while the $D10$ together with the $D4$ amplitude will contribute to the $3S_1 \rightarrow 3S_0$ transition.

c. Magnetic and electric transitions

• $3S_1 \rightarrow 3S_0$ transitions

The $3S_1 \rightarrow 3S_0$ magnetic coupling amplitude will be given by the sum of the $D4$ and $D10$ amplitudes. For any vector $a$ the spin matrix elements read

$$\langle 1S_0 | \sigma_1 \cdot a | 3S_1 \rangle = -\langle 1S_0 | \sigma_2 \cdot a | 3S_1 \rangle = \frac{1}{\sqrt{3}} \left( -i\sqrt{2}a_y + a_z \right), \quad (D11)$$

since the spin contents of the $1S_0$ and $3S_1$ states are, with $|+\rangle$ the $1/2$ and $|-\rangle$ the $-1/2$ spin states, respectively,

$$|1S_0\rangle = \frac{1}{\sqrt{2}} \left[ |+\rangle - |\rangle \right], \quad |3S_1\rangle = \frac{1}{\sqrt{3}} \left[ |+\rangle + \frac{1}{\sqrt{2}} \left( |+\rangle + |\rangle \right) + |\rangle \right]. \quad (D12)$$

With $k$ along the $z$ axis, as already defined, we have (see Ref. [37] p. 62)

$$e^*(\lambda = \pm 1) = \left( -\frac{\lambda}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0 \right), \quad k \cdot e^*(\lambda = \pm 1) = \left( \frac{ik}{\sqrt{2}}, -\frac{i\lambda k}{\sqrt{2}}, 0 \right), \quad (D13)$$

and therefore to

$$V_M^T(q, k, e^*(\lambda)) = \langle 1S_0 | A_{1,a}^T(q, k, e^*(\lambda)) + A_{1,b}^T(q, k, e^*(\lambda)) | 3S_1 \rangle = -\frac{e\lambda k}{2m\sqrt{3}} \left\{ \left( 1 + \frac{\kappa_N k}{2m} \right) \left[ \frac{1}{\zeta} + \frac{1}{\zeta} \right] + \frac{\kappa_N \Omega}{2m} \left( \zeta + \zeta \right) \right\}. \quad (D14)$$
Upon summing over $\lambda$ the modulus squared we get
\[
\sum_{\lambda=\pm1} |V_M^\gamma(q, k, e^*(\lambda))|^2 = \frac{e^2}{4m^2} \frac{2k^2}{3} \left\{ \left( 1 + \frac{\kappa_N k}{2m} \right) \left[ \frac{1}{\zeta} + \frac{1}{\bar{\zeta}} \right] + \frac{\kappa_N \Omega}{2m} \left( \zeta + \bar{\zeta} \right) \right\}^2.
\] (D15)

Note that in the small $k/2m$ limit, this reduces to
\[
\sum_{\lambda=\pm1} |V_M^\gamma(q, k, e^*(\lambda))|^2 \approx \frac{8e^2}{3} \frac{k^2}{4m^2} \left[ 1 + \frac{\kappa_N \Omega}{2m} \right]^2,
\] (D16)
a small contribution indeed. Using only the vector piece of the amplitude one would have obtained
\[
\frac{e^2}{4m^2} \sum_{\lambda=\pm1} \langle 1S_0 \rangle \left( \frac{\sigma_1}{\zeta} - \frac{\sigma_2}{\bar{\zeta}} \right) \cdot k \wedge e^*(\lambda) |^{3S_1}^2 \rangle = \frac{2e^2}{3} \frac{k^2}{4m^2} \left[ \frac{1}{\zeta} + \frac{1}{\bar{\zeta}} \right]^2.
\] (D17)

Considering the final state scattering contributions with intermediate $p\bar{p}$ and $n\bar{n}$ states (30) one obtains the following magnetic amplitude
\[
V_M^\gamma(q, k, e^*(\lambda)) = -\frac{\lambda ek}{4m\sqrt{3}} \left\{ \left( \frac{1}{\zeta} + \frac{1}{\bar{\zeta}} \right) (T_0 + T_1) \right\} + \frac{1}{2m} \left[ \Omega (\zeta + \bar{\zeta}) + k \left( \frac{1}{\zeta} + \frac{1}{\bar{\zeta}} \right) \right] T_0 (\kappa_p + \kappa_n) + T_1 (\kappa_p - \kappa_n),
\] (D18)

where $T_{0,1}$ are the $N\bar{N}$ scattering amplitudes in the corresponding $I = 0, 1$ isospin states.

- $^3S_1 \rightarrow ^3P_1$ transitions

The $^3S_1 \rightarrow ^3P_1$ electric coupling amplitude is given by the sum of the amplitudes (D2) and (D9)
\[
A_{V,a}^\gamma(q, k, e^*(\lambda)) = -\frac{e}{2m} q \cdot e^*(\lambda) \left\{ \zeta + \frac{1}{\zeta} + \bar{\zeta} + \frac{1}{\bar{\zeta}} - \frac{k\kappa_p}{2m} \left[ \zeta + \bar{\zeta} + \left( \frac{1}{\zeta} + \frac{1}{\bar{\zeta}} \right) \left( \frac{k}{\Omega} - 1 \right) \right] \right\}.
\] (D19)

Summing the squared $q \cdot e^*(\lambda)$ term over $\lambda$ gives
\[
\sum_{\lambda=\pm1} |q \cdot e^*(\lambda)|^2 = \frac{1}{2} \sum_{\lambda=\pm1} \left| -\lambda q_x + iq_y \right|^2 = q^2 \left[ 1 - \cos^2 \theta_q \right],
\] (D20)

where
\[
\cos \theta_q = \frac{k \cdot q}{kq}.
\] (D21)

Hence
\[
\sum_{\lambda=\pm1} |A_{V,a}^\gamma(q, k, e^*(\lambda)) + A_{T,a}^\gamma(q, k, e^*(\lambda))|^2 = \frac{e^2 q^2}{4m^2} \left[ 1 - \cos^2 \theta_q \right] \left\{ \zeta + \frac{1}{\zeta} + \bar{\zeta} + \frac{1}{\bar{\zeta}} - \frac{k\kappa_p}{2m} \left[ \zeta + \bar{\zeta} + \left( \frac{1}{\zeta} + \frac{1}{\bar{\zeta}} \right) \left( \frac{k}{\Omega} - 1 \right) \right] \right\}.
\] (D22)

which in the small $k/2m$ limit reduces to
\[
\frac{4e^2 q^2}{m^2} \left[ 1 - \cos^2 \theta_q \right].
\]
2. \( \omega \) emission

For the \( \omega \) meson, the polarization vector reads (see Ref. [37] p. 62)

\[
e^*(\lambda = \pm 1) = \frac{1}{\sqrt{2}} (0, -\lambda, -i, 0), \quad e^*(\lambda = 0) = \frac{1}{m_\omega} (k, 0, 0, E_\omega),
\]

As before the momentum of the emitted \( \omega \) meson is assumed to lie along the \( z \)-axis, so that one has

\[
k \cdot e^*(\lambda = \pm 1) = 0, \quad k \cdot e^*(\lambda = 0) = \frac{kE_\omega}{m_\omega}, \quad k \wedge e^*(\lambda = \pm 1) = \frac{k}{\sqrt{2}} (i, -\lambda, 0), \quad \text{and} \quad k \wedge e^*(\lambda = 0) = 0.
\]

The Lorentz condition, \( k^\mu \cdot e^*(\lambda)_\mu = E_\omega e_\omega^*(\lambda) - k \cdot e^*(\lambda) = 0 \), implies

\[
e^*_\omega(\lambda) = \frac{k \cdot e^*(\lambda)}{E_\omega}.
\]

The \( \omega \) tensor coupling being small we shall neglect its contribution [27]. If the \( \omega \) is emitted from the nucleon line, the exact amplitude for the vector coupling is given by Eq. (C11) multiplied by the coupling constant \( g_{V\omega} \). To obtain the full amplitude one has to add the amplitude corresponding to the \( \omega \) emission from the antinucleon given by Eq. (C12). It is given by Eq. (C25). In addition to Eq. (D11), we have, for any vector \( a \), the equality

\[
\langle 3P_1 | \sigma_1 \cdot a | 3S_1 \rangle = \langle 3P_1 | \sigma_2 \cdot a | 3S_1 \rangle.
\]

The dominant electric contribution for \( \lambda = \pm 1 \) will then read

\[
V^E_\omega(\lambda = \pm 1) = -\frac{g_{V\omega}}{2m} q \cdot e^*(\lambda \pm 1) \left( \zeta + \frac{1}{\zeta} + \frac{1}{\bar{\zeta}} \right),
\]

while for \( \lambda = 0 \), since \( e^*_\omega(\lambda = 0) = k/m_\omega \), one has

\[
V^E_\omega(\lambda = 0) = -\frac{g_{V\omega}}{2m} \left\{ q \cdot e^*(\lambda = 0) \left( \zeta + \frac{1}{\zeta} + \frac{1}{\bar{\zeta}} \right) + \frac{k}{m_\omega} \left[ \frac{1}{\zeta} + \frac{1}{\bar{\zeta}} \right] \frac{k \cdot q}{\Omega} \right\},
\]

and, thus,

\[
V^E_\omega(\lambda = 0) = -g_{V\omega} \frac{q \cos \theta_q}{2mm_\omega} \left\{ E_\omega \left( \zeta + \frac{1}{\zeta} + \frac{1}{\bar{\zeta}} \right) + \frac{k^2}{m_\omega} \left[ \frac{1}{\zeta} + \frac{1}{\bar{\zeta}} \right] \right\}.
\]

This contributes to the \( ^3S_1 \rightarrow ^3P_1 \) transition. Summing over the helicity \( \lambda \) the squared amplitudes we obtain

\[
\sum_{\lambda=-1}^{1} |V^E_\omega(\lambda)|^2 = g_{V\omega}^2 \frac{q^2}{4m^2} \left\{ (1 - \cos^2 \theta_q) \left[ \zeta + \frac{1}{\zeta} + \frac{1}{\bar{\zeta}} \right]^2 \right.
\]

\[
+ \left. \cos^2 \theta_q \left[ \frac{E_\omega}{m_\omega} \left( \zeta + \frac{1}{\zeta} + \frac{1}{\bar{\zeta}} \right) + \frac{k^2}{m_\omega} \Omega \left( \frac{1}{\zeta} + \frac{1}{\bar{\zeta}} \right) \right]^2 \right\}.
\]

In the small \( k/2m \) limit, this reduces to

\[
\sum_{\lambda=-1}^{1} |V^E_\omega(\lambda)|^2 \approx g_{V\omega}^2 \frac{4q^2}{m^2} \left\{ (1 - \cos^2 \theta_q) + \left( \frac{E_\omega}{m_\omega} \right)^2 \cos^2 \theta_q \left[ 1 + \frac{k^2}{E_\omega \Omega} \right] \right\},
\]

giving rise again to a very small correction to the dominant piece \( 4g_{V\omega}^2 q^2/m^2 \).

For the magnetic contribution, using Eqs. (D24) and (C25) we retain the dominant piece and get

\[
V^M_\omega(\lambda = \pm 1) = i \frac{g_{V\omega}}{2m} \left\{ \frac{\sigma_1}{\zeta} - \frac{\sigma_2}{\bar{\zeta}} \right\} \cdot [k \wedge e^*(\lambda = \pm 1)].
\]
It is null for \( \lambda = 0 \). Then, let us thus look at the \( ^3S_1 \to ^1S_0 \) magnetic transition using Eqs. (D12)

\[
\langle ^1S_0|V_M^\omega(\lambda = \pm 1)|^3S_1 \rangle = i \frac{g_V}{2m} \langle ^1S_0| \left( \frac{\sigma_1}{\zeta} - \frac{\sigma_2}{\zeta} \right) \cdot [k \wedge e^*(\lambda = \pm 1)]|^3S_1 \rangle \\
= -\frac{\lambda g_v \omega k}{2m \sqrt{3}} \left( \frac{1}{\zeta} + \frac{1}{\zeta} \right). \tag{D32}
\]

Summing over the helicities the amplitude squared, one gets

\[
\sum_{\lambda = \pm 1} |\langle ^1S_0|V_M^\omega(\lambda = \pm 1)|^3S_1 \rangle|^2 = \frac{g_v^2 \omega}{4m^2} \frac{2k^2}{3} \left( \frac{1}{\zeta} + \frac{1}{\zeta} \right)^2, \tag{D33}
\]

and thus about

\[
\frac{8g_v^2 \omega}{3} \frac{k^2}{4m^2}
\]

in the small \( k/2m \) limit, i.e., a very small contribution. The \( J/\psi \) and the \( \omega \) being isospin 0 states only the \( I = 0 \) component of the \( pp \to p\bar{p} \) and \( n\bar{n} \to p\bar{p} \) rescattering terms contributes. In the Paris potential the modulus of these two components being equal, there will be a cancellation either in the convention given by the Eqs. (22) and (23) of Ref. [8] or in that of Eqs. (49) and (50) [there, one will have to change the sign of the Paris \( n\bar{n} \to p\bar{p} \) amplitude].

**Appendix E: Numerical calculation of the loop integrals**

In this appendix we outline the numerical calculation of the loop integrals in Eqs. (31) or (59) with propagators \( \frac{d^\prime}{(2\pi)^3} \) and \( \frac{d}{(2\pi)^3} \). The structure of these equations goes like

\[
I(q, k) = \int \frac{dq'}{(2\pi)^3} T_{N\bar{N}}(q - k/2, q' - k/2, E_{N\bar{N}}) G^{+}_{0,N\bar{N}B}(q', k) \tilde{G}_{p\bar{p}}(q') U^0(q', k), \tag{E1}
\]

where we will use the half off-shell values of the \( N\bar{N} \) scattering matrix evaluated from the Paris potential.

With \( k \) along the \( z \)-axis, \( q \) in the \( x-z \)-plane, \( q = (q \sin \theta_q, 0, q \cos \theta_q) \), we have

\[
q \cdot q' = qq' \sin \theta_q \sin \theta_{q'} \cos \varphi' + \cos \theta_q \cos \theta_{q'} \quad \text{and} \quad k \cdot q' = kq' \cos \theta'. \tag{E2}
\]

Let us assume that the half off-shell dependence of the scattering matrix depends on the momentum transfer

\[
T_{N\bar{N}}(q - k/2, q' - k/2, E_{N\bar{N}}) = T_{N\bar{N}}(\chi', E_{N\bar{N}}), \tag{E3}
\]

with

\[
\chi' = (q - \frac{k}{2}) - (q' - \frac{k}{2}) = \chi - q' \quad \text{so that} \quad \chi' = \sqrt{q^2 + q'^2 - 2qq' \sin \theta_q \sin \theta_{q'} \cos \varphi' + \cos \theta_q \cos \theta_{q'}}. \tag{E4}
\]

Then,

\[
G^{+}_{0,N\bar{N}B}(q', k) = \frac{1}{M_{J/\psi} + i\epsilon - E_B(k) - \sqrt{q'^2 + m^2 - E'}}, \tag{E5}
\]

where \( E_B(k) = \sqrt{k^2 + m_B^2} \), the energy of the emitted boson while \( E \) is the off-shell energy of the nucleon from which the boson \( B \) has been emitted:

\[
E = \sqrt{(q' - k)^2 + m^2}. \tag{E6}
\]

Using \( E \) as a variable, rather than \( \cos \theta' \), we may reexpress Eq. (E1) as

\[
I(q, k) = \frac{1}{(2\pi)^3} \frac{d^\prime}{k \sqrt{V_0}} \int_0^\infty q' dq' \frac{\mathcal{F}(q')}{M_{J/\psi} + i\epsilon - 2\sqrt{q'^2 + m^2}} \int_{E-}^{E+} \frac{E dE}{M_{J/\psi} + i\epsilon - E_B(k) - \sqrt{q'^2 + m^2 - E}} \times \int_0^{2\pi} d\varphi' T_{N\bar{N}}(\chi', E_{N\bar{N}}) U^0(q', k), \tag{E7}
\]

where

\[
\mathcal{F}(q') = \frac{\sqrt{q'^2 + m^2}}{2\pi^2} \frac{(q'^2 - m^2)^2}{2(q^2 + q'^2 - 2qq')}. \tag{E8}
\]
where we have used the relation $E\ dE = -k\ q'\ d\cos\theta'$ and where $F(q')$ denotes the source function as given by Eq. (9). The explicit dependence on $q$ in $I(q,k)$ comes from the $\chi'$ dependence in the half off-shell scattering matrix. The limits of integration for the $E$-integration are

$$E_\pm = \sqrt{m^2 + (k \pm q')^2}.$$  

(E8)

The invariant mass squared of the $NN$ pair, $s = M_{NN}^2$ is

$$s = \left(\sqrt{q'^2 + m^2 + (q' - k)^2 + m^2}\right)^2 - k^2,$$  

(E9)

since the total momentum of this pair is $-k$ and the relative energy

$$E_{NN} = \sqrt{s + k^2} - \sqrt{4m^2 + k^2}.$$  

(E10)

Note that in the non-relativistic limit this expression goes to

$$E_{NN} \approx \frac{s - 4m^2}{4m}.$$  

(E11)

At threshold $E_{NN} = 0$, so that $s = 4m^2$ and the emitted boson reaches its maximum momentum value, i.e., $k = 979.9$ MeV/c for the photon and $k = 742.5$ MeV/c for the omega. At the other end of the spectrum for the emitted boson $k = 0$ and $s = (M_{J/\psi} - M_B)^2$ and we point out that there is no singularity in the integral due to this value of $k = 0$.

The integral over $\phi'$ is performed numerically without difficulty and displays no singularity. In each of the other two integrations, there is the presence of a pole. In the $q'$-integral the pole in $q'$ lies at

$$q_p = \sqrt{M_{J/\psi}^2 - 4m^2} = 1231.82\text{ MeV},$$  

(E12)

For practical calculation of the integral (E7) it is sufficient to integrate the $q'$ variable up to the maximum value $q_{Max} = 12\text{ fm}$

In Eq. (E7) there will be a pole in $E$ at

$$E_p = M_{J/\psi} - E_B(k) - E(q'),$$  

(E13)

if $E_- \leq E_p \leq E_+$, i.e., if

$$\sqrt{(q' - k)^2 + m^2} \leq M_{J/\psi} - \sqrt{k^2 + M_B^2} - \sqrt{q'^2 + m^2} \leq \sqrt{(q' + k)^2 + m^2}.$$  

(E14)

Study of the inequalities (E14) allows to write the integral (E7) as

$$I(q,k) = I_1 + I_2 + I_3,$$  

(E15)

with

$$I_1 = \int_0^{q_1} dq'\ ...\ (no\ pole\ in\ q')\ \int_{E_-}^{E_+} dE\ ...\ (no\ pole\ in\ E),$$  

(E16)

5 In the spirit of the Paris potential and its parametrization, the non-relativistic approximation to this expression would read

$$E_{NN} \approx \frac{T_{lab}}{2} \left\{ 1 - \left( \frac{T_{lab}}{8m} + \frac{k^2}{8m^2} \right) \right\} \quad \text{with} \quad T_{lab} = \frac{s - 4m^2}{2m}.$$
\[
I_2 = \int_{q_1}^{q_2} dq' \text{(no pole in } q') \int_{E_-}^{E_+} dE \text{(pole in } E),
\]
and
\[
I_3 = \int_{q_2}^{q_{\text{max}}} dq' \text{(pole in } q') \int_{E_-}^{E_+} dE \text{(no pole in } E).
\]

In the above integrals, \(I_1\), \(I_2\) and \(I_3\) the ... are to be identified with the corresponding functions given in Eq. (E7). Defining
\[
q^\pm = -ks \pm \sqrt{s^2(k^2 + s - 4m^2) - 4m^2k^2s} = \frac{k}{2} - \frac{1}{2} \sqrt{(s + k^2)(s - 4m^2)},
\]
and
\[
E_{NN}^0 = \frac{M_{J/\psi}^2 - m_B^2}{4(M_{J/\psi} - m)} - m = \frac{(M_{J/\psi} - 2m)^2 - m_B^2}{4(M_{J/\psi} - m)},
\]
one finds, for \(E_{NN} \leq E_{NN}^0\), \(q_1 = q^+, q_2 = q^-\) and for \(E_{NN} \geq E_{NN}^0\), \(q_1 = -q^+, q_2 = q^-\). For \(E_{NN} = E_{NN}^0\), \(q_1 = 0\). The boundary \(E_{NN}^0\) is equal to 172.48 MeV for the photon case and to 101.54 MeV for the omega one. Note that at the \(p\bar{p}\) threshold there is no pole in the integrals over \(dE\), and
\[
I(0, k) = \int_{0}^{q_{\text{max}}} dq' \text{(pole in } q') \int_{E_-}^{E_+} dE \text{(no pole in } E).
\]

In the numerical program for the loop calculation, the principal value integrals in \(E\) and in \(q'\) are calculated using the FORTRAN subroutine dqawce.f download from the quadpack-netlib website (http://www.netlib.org/quadpack/).

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