Gravitational lens system as a potential tool to detect extremely low frequency primordial gravitational wave

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The imprint of extremely low frequency primordial gravitational wave on a gravitational lens system with a non-aligned source-deflector-observer configuration is investigated in work (Liu, 2022, MNRAS, 517, 2769) from which it shows that time delay with perturbation from extremely low frequency primordial gravitational wave could deviate from the one deduced from the theoretical model as much as 100 percent with a series of chosen parameters. However, the frequency of gravitational wave chosen in work (Liu, 2022, MNRAS, 517, 2769) is a little bit confusing. Here, with the suitable parameters chosen in this work, the results show that time delay between different images of the source in the gravitational lens system with perturbation from primordial gravitational wave with extremely low frequency could strongly deviate from the one resulting from the theoretical model as much as about several hundred percent, indicating that time delay from gravitational lens system could be used to detect extremely low frequency primordial gravitational wave.

INTRODUCTION

Primordial gravitational waves (PGWs) with a nearly scale invariant spectrum [1,7] are a robust prediction of inflation which not only leads to a flat, homogeneous, and isotropic Universe but also produces seed perturbations growing to create large scale structure in the Universe [8,11]. In order to confirm inflation and determine the energy scale of it, it is of great significance to detect PGWs. The traditional method of detection of PGWs with extremely low frequency in the range of $10^{-18}\text{Hz}$-$10^{-16}\text{Hz}$ is the B-modes of polarization of cosmic microwave background (CMB) [12,13]. However, detection of PGWs with extremely low frequency using B-mode polarization faces challenges due to the foreground contamination from dust in our Milky Way. Thus, it is worth finding an alternative observational feature induced by such extremely low frequency PGWs.

Recently, work of Liu [14] proposes a new method to detect extremely low frequency PGWs using gravitational lens system with a non-aligned source-detector-observer configuration. It shows from Liu [14] that the time delay from a non-aligned gravitational lens system perturbed by extremely low frequency PGWs could deviate from the time delay deduced from the theoretical model as much as about several hundred percent, indicating that a non-aligned gravitational lens system could be used as a detector of extremely low frequency PGWs.

TIME DELAY WITH PERTURBATION FROM PGW

We adopt the same non-aligned gravitational lens system as that in Liu [14] where the projection of the source on the lens axis and the observer are equidistant from the deflector. The source, the deflector and the observer are at $(x = 2L\beta, y = 0, z = -L), (x = 0, y = 0, z = 0)$ and $(x = 0, y = 0, z = L)$, respectively. The speed of light is set to be $c = 1$. The metric of the gravitational wave is

$$h_{ij} = \begin{bmatrix} -\cos^2 \phi h_+ & -\cos \phi h_x & \sin \phi \cos \phi h_+ \\ -\cos \phi h_x & h_+ & \sin \phi h_x \\ \sin \phi \cos \phi h_+ & \sin \phi h_x & -\sin^2 \phi h_+ \end{bmatrix} \times \cos (\omega t - k \cdot x)$$

where $t = t_e + (z + L)$ in order to approach the level of approximation, $t_e$ is the time the photons were emitted at $(x = 2L\beta, y = 0, z = -L)$ so that $\omega t_e$ acts as the initial phase, $k = \omega (\sin \phi, 0, \cos \phi)$ is the propagation vector, $\omega = 2\pi f$, $f$ is the frequency of gravitational wave, $h_+$ and $h_x$ are the amplitude of the two polarizations of the gravitational wave, respectively.

Same as that in Liu [14], we set $\phi = \frac{\pi}{2}$ and $h_x = 0$ for simplicity. Then, as shown in Liu [14], the angular position of the image of the source with respect to the line that joins the observer and the deflector is

$$\theta_{1,2} = \frac{\beta^*}{2} \pm \sqrt{\frac{\beta^*^2 + 4\eta^2}{2}} + \theta_0$$

where $\eta = \sqrt{\frac{2\omega M}{L}}$ is the Einstein radius, $M$ is the mass of the deflector, $\beta^* = \beta + \beta_0$ and

$$\theta_0 = \frac{1}{2} \frac{h}{L} \cos [\omega t_e + 2L] - \frac{1}{2} \frac{h}{2L} \sin [\omega t_e + 2L]$$
and the magnification deduced from observation is
\[
\beta_0 = \frac{1}{4\omega L} \sin(\omega t_e) - \frac{1}{2\omega L} \sin[\omega(t_e + L)] + \frac{1}{4\omega L} \sin[\omega(t_e + 2L)]
\]  
(4)

With the angular position of the image of the deflector with respect to the line that joins the observer and the deflector shown as
\[
\theta_d = \frac{1}{2} h \cos[\omega(t_e + 2L)] - \frac{1}{2\omega L} \sin[\omega(t_e + 2L)] + \frac{1}{2\omega L} \sin[\omega(t_e + L)]
\]  
(5)

From the point of view of real observation, the image angular position of the source is the image of the source with respect to the image of the deflector
\[
\theta_{1,2}' = \theta_{1,2} - \theta_d = \frac{\beta^*}{2} \pm \frac{\sqrt{\beta^* + 4\eta^2}}{2}
\]  
(6)

The actual magnification is expressed as
\[
\mu = \frac{\theta(\theta - \theta_0)^3 \lambda}{[(\theta - \theta_0)^2 - \beta_0(\theta - \theta_0) - \eta^2][\theta - \theta_0]^2 + \eta^2]}
\]  
(7)

and the magnification deduced from observation is
\[
\mu' = \frac{\theta^4}{\theta^4 - \eta^4}
\]  
(8)

From the theoretical point of view, the misalignment angle \(\beta^*\) is inferred by the observer through \(\beta^* = \theta_1 + \theta_2'\) and \(\eta\) through \(\eta = \sqrt{-\theta_1^2 \theta_2'}\). Then we get \(\nu = \left| \frac{\mu_1}{\mu_2} \right|\) in observation and \(\nu' = \left| \frac{\mu_2'}{\mu_2} \right|\) in theory.

As in Liu [14], \(T_{SAO}/T_{SBO}\) represents the time travel of the light traveling from the source S through the point A/B to the observer O. Then, the actual time delay between different images of the source is
\[
\Delta T_{Observation} = T_{SAO} - T_{SBO} = \frac{(2GM)^2}{L} \left( \frac{1}{\theta_1^2} - \frac{1}{\theta_2^2} \right) - 4GM\ln \left( \frac{\theta_1^2}{\theta_2^2} + 2GMh/\omega \right) \left( \frac{1}{R_A} - \frac{1}{R_B} \right) \times \{ \sin(\omega t_e) - 2\sin[\omega(t_e + L)] + \sin[\omega(t_e + 2L)] \}
\]  
(9)

where \(R_{1,2} = \frac{L^2 + \sqrt{L^2 + 8GM}}{2}\) and the time delay deduced from theoretical model, \(\Delta T_{Theory}\), is
\[
\Delta T_{Theory} = \frac{(2GM)^2}{L} \left( \frac{1}{\theta_1^2} - \frac{1}{\theta_2^2} \right) - 4GM\ln \left( \frac{\theta_1^2}{\theta_2^2} \right)
\]  
(10)

Based on Eq. (9) and Eq. (10), we define
\[
\kappa = \frac{\Delta T_{Theory} - \Delta T_{Observation}}{\Delta T_{Theory}}
\]  
(11)
as the deviation of time delay induced by extremely low frequency PGWs.

In the following calculations, we set \(\eta = 4.8 \times 10^{-6}\) and \(L = 1\) Gpc. According to [15], we set \(h = 10^{-5}, f = 10^{-18}\) Hz in this work. However, in Liu [14], parameters of \(h = 10^{-5}, \omega = 10^{-18}\) Hz are adopted. It shows that the real frequency adopted in Liu [14] is \(f = \frac{\omega}{2\pi} = 10^{-18} / 2\pi \approx 1.59 \times 10^{-19}\), this could underestimate the deviation of time delay.

From the up panel in Figure 1, it shows that the maximum absolute value of time delay deviation increases as \(\beta\) decreases and the deviation of time delay with \(\beta = 2 \times 10^{-6}\) could reach about 600 percent. When \(\beta\) is less than \(2 \times 10^{-6}\), the maximum absolute value of time delay deviation continues to increase and this is not plotted in Figure 1. In real observation, when time delay anomaly which is usually observed in gravitational lensing, is larger than other perturbations (a precise model of the deflector potential, the line-of-sight weak lensing effect, gravitational microlensing, etc), we may expect that we confirm the existence of extremely low frequency PGWs. The bottom panel in Figure 1 presents \(\nu\), this could be interpreted as flux ratio anomaly usually observed in gravitational lensing and gravitational microlensing could induce.

**CONCLUSION AND DISCUSSION**

In order to find the possible hint of extremely low frequency PGWs in the Universe, the effect of extremely low frequency PGWs on time delay between different images of the source in gravitational lens system with a non-aligned source-deflector-observer configuration is investigated in this work where suitable parameters are adopted and the distance from the projection of the source on the lens axis to the deflector is equal to that from the observer to the deflector. It shows from this work that, with a series of suitable parameters, time delay between different images of the source in the non-aligned gravitational lens system could be strongly perturbed by extremely low frequency PGWs and may present obvious deviation from that deduced from theoretical model as much as about several hundred percent in some situations, meaning that gravitational lens system with a non-aligned configuration could serve as a potential long-base-line detector of extremely low frequency PGWs. Thus, in addition to B-mode polarization, obvious deviation of time delay in gravitational lensing could be used as an alternative observational feature generated by extremely low frequency PGWs.
FIG. 1: Left, middle and right show the results along with $t_e$ when $\beta = 2 \times 10^{-6}$, $\beta = 3 \times 10^{-6}$ and $\beta = 4 \times 10^{-6}$, respectively. Up and bottom show $\kappa$ and $\nu_\nu'$ along with $t_e$, respectively.

In addition to the condition that the projection of the source on the lens axis and the observer are equidistant from the deflector, the deflector is a point (or thin axially symmetric) gravitational deflector and the specific direction and polarization of the PGWs is adopted, it should be noted that this conclusion holds with the general condition that the projection of the source on the lens axis and the observer are not equidistant from the deflector, the direction of propagation and polarization of PGWs are arbitrary, the deflector is not limited to a point (or thin axially symmetric) gravitational deflector and can be elliptical lenses and the linear superposition of PGWs is present.

Compared with perturbation (deviation with several hundred percent) from extremely low frequency PGWs, time delay deviation resulting from other perturbations (for example, a precise model of the deflector potential across the images, the line-of-sight weak lensing effect and gravitational microlensing) is small (several tens percent in total) [14]. Time delay deviation generated by extremely low frequency PGWs may have the same order as that resulting from other perturbations described above, in this case, we may extract the time delay induced by other perturbations to test whether there is additional time delay deviation.

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