Design optimization by numerical characterization of fluid flow through the valveless diffuser micropumps

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Abstract. Valveless piezoelectric micropumps are in wide practical use due to their ability to conduct particles with absence of interior moving mechanical parts. In this paper, an extended numerical study on fluid flow through micropump chamber and diffuser valves is conducted to find out the optimum working conditions of micropump. In order to obtain maximum generality of the reported results, an analytical study along with a dimensional analysis is presented primarily, to investigate the main dimensionless groups of parameters affecting the micropump net flux. Consequently, the parameters appeared in the main dimensionless groups have been changed in order to understand how the pump rectification efficiency and optimum diffuser angle depend on these parameters. A set of characteristic curves are constructed which show these dependencies. The application of these curves would have far reaching implications for valveless micropumps design and selection purposes.

1. Introduction

The valveless diffuser micropumps are driven by a piezoelectric element bounded to a flexible membrane with no moving parts. Due to ability to conduct particles and absence of interior moving mechanical parts, valveless micropumps have wide practical applications.

Several researchers have explained the working principle valveless diffuser micropumps [1,2]. They indicated that the when the pump membrane is deflected in pump and supply modes presented in figure1, the flow resistance across the nozzle and diffuser elements become different. Therefore, the diffuser element may act as a passive valve.

Due to model complexity and limited knowledge of the physics of the device, design of micropumps is a difficult task as well as challenging. Since Micropump system analysis is a sophisticated multi-disciplinary problem with various field couplings, numerous scientific and engineering researches have been conducted in the past decade to optimize micropump working parameters. Effect of valve geometry and working conditions on valve rectification has been investigated by several researchers [2-4]. Olsson et al [5] used lumped mass model to study flat walls diffuser micropumps to simulate numerically the pressure limit, piezoelectric excitation level and flow performance of micropump in single and double chamber micropumps. Moreover, variety of experimental works is reported about micropumps shape optimization in the literature [2,6]. Fan et al [7] numerically studied the performance of a piezoelectrically actuated valveless micropump with consideration of the three-way electro–mechanical–fluid coupling. Pan et al. [8,9] have obtained analytical solution for a valveless micropump in order to investigate the dynamic response of coupled...
membrane-fluid system with respect to piezoelectric actuation. Nguyen [10] studied the effect of membrane deflection and actuation frequency on flow rate and found that flow rate increases by increasing these parameters at low frequencies.

In this paper, using finite element method, an extended numerical study on fluid flow through micropump chamber and diffuser valves is conducted to find out the optimum working conditions of micropump. In order to obtain maximum generality of the reported results, an analytical study along with a dimensional analysis is presented primarily, to investigate the main dimensionless groups of parameters affecting the micropump net flux. Consequently, utilizing the concept of parametric programming in the numerical simulation, the parameters appeared in the main dimensionless groups have been changed in order to understand how the pump rectification efficiency and optimum diffuser angle depends on these parameters.

2. Theoretical analysis of micropump rectification

Micropump net flow rate can be evaluated from its valves loss coefficients using continuity equation. Loss coefficients in nozzle/diffuser directions in figure 2 would be stated as:

\[ K_d = \frac{1}{2} \frac{\Delta P_{d}}{\rho_{in,d}} \]  
\[ K_n = \frac{1}{2} \frac{\Delta P_{n}}{\rho_{in,n}} \]  
\[ Q_{in} = \frac{1}{2} \frac{P_{in} - P_{out}}{\rho_{in,n}} \]

where, \( K \) is the pressure loss coefficient, \( \rho \) is the fluid density, \( v \) is the flow velocity and \( P \) is the static pressure. Indices \( n, d \) determine the valve elements are whether nozzle or diffuser and Indices \( in, out \) and \( th \), represent inlet, outlet and throat of the element respectively.

From continuity equation the following equation can be written:

\[ \frac{dV}{dt} = Q_{d} + Q_{n} \]  

where, \( V \) represents the displaced volume of the micropump membrane and \( Q \) is the volume flux of fluid through the valves.

From the simultaneous solution of (1), (2) and (3), the micropump net flux could be obtained as:

\[ Q_{net} = Q_{d} - Q_{n} = \frac{K_{d} - \sqrt{K_{d}}}{K_{d} + \sqrt{K_{d}}} \]  

where, \( Q_{net} \) is representative of the micropump net flux. Writing (4) in dimensionless form results in:

\[ \gamma = \frac{\sqrt{\eta} - 1}{\sqrt{\eta} + 1} \]
where, $\eta$ is Efficiency ratio defined as the ratio of loss coefficients in diffuser and nozzle directions and $\gamma$ is dimensionless net flux which is known as flow rectification characteristic of micropump. This dimensionless parameter may be written as:

$$\gamma = \frac{Q_{net}}{dV/dt}$$  \hspace{1cm} (6)

It is evident that rectification efficiency of typical diffuser elements is small compared to conventional check valves employed in mechanical micropumps. Therefore, an optimization based on design parameters would be necessary in order to approach the maximum available net flow of a diffuser type micropump.

3. Dimensional analysis

In order to obtain a general sense about the effectiveness of main parameters involving in fluidic part of a valve-less micropump on its performance, it is convenient to conduct a dimensional analysis. Although some researches on loss coefficients of passive valves, their rectification [3,4] or the effect of Reynolds number on diffuser flow rate have been made [5,11], no complete dimensionless grouping for micropump design parameters has been observed in the literature. In this regard, the parameters which affect the micropump performance are categorized in three main groups. According to Buckingham II theorem, six dimensionless variables can be established based on these parameters. The performance parameters and their corresponding dimensionless variables are presented in table 1.

The parameter $Q$ in a 2D model can be defined as the pump flux per unit width of diffuser. For unsteady internal flows a multiplication of Reynolds and Strouhal numbers which is called frequency number, $Fn$, may be suggested.

$$Fn = \frac{\rho D^2}{\mu}$$  \hspace{1cm} (7)

Frequency number can be regarded as a frequency based Reynolds number for unsteady internal flows.

Regarding the problem of determining the optimum diffuser angle in which maximum rectification occurs, it can be written that:

$$\gamma = F_1(Re, Eu_s, Eu_p, Fn, AR, \theta)$$  \hspace{1cm} (8)

where, $Eu_s$ and $Eu_p$ are representatives of micropump supply mode and pump mode Euler numbers, respectively. It should be noted that the Euler Number which is derived here is exactly the same as diffuser or nozzle loss coefficients defined in the previous section.

| Performance parameter type | Performance parameter Description | Symbol | Dimensionless variables | Equation |
|----------------------------|----------------------------------|--------|-------------------------|----------|
| Geometric                  | Diffuser length                  | $L$    | Diffuser aspect ratio    | $AR = L/D$ |
|                            | Diffuser throat diameter          | $D$    | Diffuser angle           | $\theta$ |
|                            | Diffuser angle                   | $\theta$ |                        |          |
| Kinematical                | Total flux                       | $Q_t$  | Flow rectification      | $\gamma = Q_{net}/Q_t$ |
|                            | Net flux                         | $Q_{net}$ |                      |          |
|                            | Excitation frequency             | $f$    | Strouhal number         | $St = fQ/D^2$ |
| Dynamic                    | Fluid viscosity                  | $\mu$  | Reynolds number         | $Re = \rho Q/\mu$ |
|                            | Fluid density                    | $\rho$ |                        |          |
|                            | Diffuser upstream pressure        | $P$    | Euler number            | $Eu = 2PD^2/\rho Q^2$ |

From (5) the flow rectification, $\gamma$, can be written as a function of the efficiency ratio, $\eta$:

$$\gamma = F_2(\eta)$$  \hspace{1cm} (9)
while, by definition, the efficiency ratio can be stated as:

$$\eta = \frac{K_d}{K_p} = \frac{Ep}{Ep_s}$$  \hspace{1cm} (10)

Hence, by investigation of the relationship between $\eta$ and other dimensionless variables, along with the application of (9), the optimization task can be completed:

$$\eta = F_3(Re,Fn,AR,\theta)$$  \hspace{1cm} (11)

In this study, the steady state conditions are studied so that the frequency number can be excluded from (11):

$$\eta = F(Re,AR,\theta)$$  \hspace{1cm} (12)

Equation (12) implies that the efficiency ratio (or rectification) of a micropump diffuser depends on its geometry and the Reynolds number. The numerical study presented in Section 4 follows the objective of finding such dependencies.

4. Numerical model and solution algorithm

In equations (1) and (2), the total pressure loss coefficients for the diffuser and the nozzle directions can be divided into three parts: Pressure losses due to sudden contraction at the entrance, gradual contraction or expansion through the length of the nozzle/diffuser and, sudden expansion at the exit. In order to study the diffuser performance under applied pressures, all parts of the pressure loss must be considered.

In this work, a simplified accurate numerical model of micropump is suggested. The model geometry shown in figure 3 consists of source and destination chambers and a diffuser element connecting them. The model is simplified in geometry for simple programming purposes. Planar diffuser elements are preferred to be selected rather than conic ones, because of their popularity in manufacturing and fabrication by simple silicon machining processes [2]. The advantage of symmetry in micropump geometry is taken to minimize the computational costs. It is evident from figures 3 and 4 that only half of the system is modeled.

For fluid flow through the micropump diffusers, a range of Reynolds numbers between 100 and 500 are predicted and parabolic velocity profiles have been observed and simulated [3,4]. Based on the aforementioned assumptions, in the present study, the flow is to be laminar under considered actuation pressures.

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Figure 3. Half of the 2D geometry used in simulation

Figure 4. Boundary conditions used in numerical simulation

The governing equations can be obtained by using conservation laws of mass and momentum in differential form. In this paper, using finite element method, an extended numerical study on fluid flow through micropump chamber and diffuser valves is conducted to find out the optimum working conditions of micropump. Consequently, utilizing the concept of parametric programming in the numerical simulation, the parameters appeared in the main dimensionless groups have been changed in order to understand how the pump rectification efficiency, depends on these parameters.

Initially, an 8×8 mm chamber and a throat diameter of 100μm for diffuser valves are considered. Actuation pressures of 1kPa to 15kPa, diffuser half angles of 1º to 20º and valve aspect ratios (ratio of valve length to its throat diameter) between 5 and 40 are examined. At a specified actuation pressure, the fluid flow through micropump is simulated for pump and supply modes, separately. The pump and supply modes flow rates, the net flow rate, dimensionless variables such as Reynolds number, loss
coefficient and flow rectification for different diffuser angles are calculated. Consequently, the optimized diffuser angle in which maximum flow rate occurs is determined. Overall procedure is repeated for several actuation pressures. Finally, correlation curves for optimized diffuser angle with respect to actuation pressure and dimensionless variables are deduced.

5. Results and discussion

The flow rates for two cases of supply and pump modes are obtained applying the proposed numerical analysis. Figure 5 shows these fluxes with respect to diffuser angle at different pressure actuations. Symbols P, S and N represent pump, supply and net fluxes, respectively; meanwhile, the numbers 1, 5, 10 and 15 used in the figure 5 indicate the applied pressures on the chamber. It is evident that the general trend of all constant pressure curves is identical.

Although the supply mode curves show an increasing scheme with diffuser angle, a gradual increase up to a maximum point followed by a gentle decline is observed for pump mode. These two different trends yield an optimized point at which maximum net flow occurs. Figure 6 presents the Optimized Diffuser Angle, ODA, vs. actuation pressure in log-log scale. As the result of curve fitting the following correlation between ODA and actuation pressure, AP, is obtained:

\[
(ODA) = 8.1 \times (AP)^{-0.3}
\]  

Although the obtained correlation covers wide practical implications for design and selection purposes, it has some limitations. The main restriction is that only planar diffusers have been considered in the simulation. The effect of chamber size on correlated ODA is not studied in the present work and finally, the aspect ratios of diffusers, AR, must be around 10 in order to provide accurate predications of ODA. As will be discussed, AR has substantial effects on ODA.

Since experimental results on ODA could not be found in literature, to check the correlation validity, a particular analysis with actuation pressure of 30kPa has been performed and an ODA of 2.7° is obtained. Using the correlation function, ODA of 2.91° will be predicted which has an error of 7.7%.

Figure 7 shows the efficiency ratio of diffuser (pump mode to supply mode Euler number ratio) with respect to the Reynolds number at different diffuser angles. The results are in agreement with the findings of figure 5. The curves show an optimized Reynolds number at which maximum efficiency ratio can be obtained. Since the applied pressure and the Reynolds number of diffuser are in direct relation, the optimized Reynolds number decreases by increasing the diffuser angle. Hence, according to the findings, if it is desired to set high actuation pressures, smaller diffuser angles should be utilized in order to approach the maximum possible rectification of the diffuser.

The effect of diffuser aspect ratio AR on optimized diffuser angle at 5kPa back pressure is presented in figure 8. The curve shows sharp decline in ODA as AR increases. Hence, in practical point of view, smaller diffuser lengths should be considered while employing large diffuser angles.
6. Conclusions

In this paper, using finite element method, an extended numerical study on fluid flow through micropump chamber and diffuser valves is conducted. Based on the simulation results, a correlation between optimized diffuser angle and actuation pressure is presented. The effects of governing dimensionless design parameters including valve diffuser angle, flow Reynolds number and valve aspect ratio on micropump rectification efficiency is examined and, a set of characteristic curves are constructed which show these effects. The optimum working condition of micropump can be clearly found out through the use of these general characteristic curves.

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