Perturbative Minimal Superconformal Technicolor

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Abstract. We investigate the perturbative regime of the Minimal Supersymmetric Conformal Technicolor. The model allows for a stable vacuum correctly breaking the electroweak symmetry. The particle spectrum features several light particles stemming out from the new $\mathcal{N} = 4$ sector of the theory. We find that the Tevatron and the LHC can rule out a significant portion of the parameter space of this model.

1. Introduction

In technicolor models \cite{1,2} one gives a mass to the electroweak gauge bosons via new strong dynamics. These models must be extended in order to give mass to the standard model (SM) fermions \cite{3,4}. In typical extensions one expects large flavor changing neutral current (FCNC) processes. Using near conformal dynamics alleviates the FCNC problem \cite{5}, but the price for the early models was a seemingly inconsolable tension with the electroweak (EW) precision data \cite{6}.

For over a decade it was hoped that near conformal dynamics might anyways eliminate the tension with precision data even if one had a large number of technidoublets gauged under the electroweak (EW) symmetry. However recently in \cite{7} it has been shown that to be phenomenologically viable the near conformal models should contain the minimal number of flavors gauged under the EW symmetry.

The simplest models of this type which are shown to pass the precision tests, or have the smallest deviation from the precision data, while still providing a (near) conformal behavior were put forward in \cite{8}. Among these, the Minimal Walking Technicolor (MWT) features the most economical particle content. In MWT the gauge group is $SU(2)_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y$ and the field content of the technicolor sector is constituted by two flavors of techni-fermions and one techni-gluon all in the adjoint representation of $SU(2)_{TC}$. The model features also a pair of Dirac leptons, whose left-handed components are assembled in a weak doublet, necessary to cancel the Witten anomaly \cite{9} arising when gauging the new technifermions with respect to the weak interactions.

The model requires, however, still additional ingredients in order to give mass to the SM fermions. For example, one may postulate the existence of an Extended Technicolor (ETC) sector, traditionally featuring new gauge interactions linking the SM fermions to the techniquarks, which can generate mass terms for the SM fermions (as well as for the technimesons and -baryons) via a new dynamical mechanism. Another alternative is to reintroduce
new bosons (bosonic technicolor) [10, 11] able to give masses to the SM fermions using standard
Yukawa interactions. Eventhough these models are phenomenologically viable, they suffer from a
SM-like fine tuning and are therefore unnatural. Supersymmetric technicolor has been considered
[12, 13] as a way to naturalize bosonic technicolor. Another possibility would be to imagine the
new scalars also to be composite of some new strong dynamics.

In [14] we made the observation that the techni-fermions and techni-gluons of the Minimal
Walking Technicolor fit perfectly in an N = 4 supermultiplet, provided that we also include
three scalar superpartners. In fact the SU(4) global symmetry of MWT is nothing but the well
known SU(4)_R R symmetry of the N = 4 Super Yang Mills theory. This is the global quantum
symmetry that does not commute with the supersymmetry transformations.

Supersymmetrizing MWT in this way leads to an approximate N = 4 supersymmetry of the
technicolor sector that is broken to N = 1 only by EW gauge and Yukawa interactions. Due to
approximate N = 4 invariance the beta function of the technicolor gauge coupling is zero at one
loop, i.e. the associated technicolor model is approximately conformal. We called this model
Minimal Supersymmetric Conformal Technicolor (MSCT).

This model can also be viewed as the first extension of the SM exhibiting maximal
supersymmetry in four dimensions. MSCT constitutes an interesting theoretical as well as
phenomenological model to explore since it naturally allows to investigate different regimes
according to how strongly coupled the maximally supersymmetric Yang-Mills theory is taken to
be. Here we will assume the technicolor coupling is perturbative.

2. The Model

The fermionic particle content of the MWT is given explicitly as

\[ Q_L^a = \left( \begin{array}{c} U^a_L \\ D^a_R \end{array} \right), \quad U_R^a, \quad D_R^a, \quad a = 1, 2, 3; \quad L_L = \left( \begin{array}{c} N \\ E \end{array} \right)_L, \quad N_R, \quad E_R, \quad (1) \]

where U and D are techni-fermions in the adjoint representation of SU(2)_TC, whose left-handed
components form a doublet under SU(2)_L, and the chiral leptons required to cancel the Witten
anomaly are denoted by N and E. The following generic hypercharge assignment is free from
gauge anomalies:

\[ Y(Q_L) = \frac{y}{2} \quad Y(U_R, D_R) = \left( \frac{y+1}{2}, \frac{y-1}{2} \right) \]

\[ Y(L_L) = -3\frac{y}{2} \quad (N_R, E_R) = \left( \frac{-3y+1}{2}, \frac{-3y-1}{2} \right) \]

The global symmetry of the technicolor sector is SU(4) which breaks explicitly to SU(2)_L ×
U(1)_Y by the natural choice of the electroweak embedding [8, 15]. Electroweak symmetry
breaking is triggered by a fermion bilinear condensate. The vacuum is stable against the SM
quantum corrections [16].

To build MSCT we set y = 1 in Eqs. (2) so that D_R^a is a singlet under EW symmetry and
can play the role of the techni-gaugino. We define the N = 4 supermultiplet in terms N = 1
superfields, whose scalar and fermionic components are expressed by:

\[ (\tilde{U}_L, \ U_L) \in \Phi_1, \quad (\tilde{D}_L, \ D_L) \in \Phi_2, \quad (\tilde{U}_R, \ \bar{U}_R) \in \Phi_3, \quad (G, \ \bar{D}_R) \in V, \quad (4) \]

where we used a tilde to label the scalar superpartner of each fermion. We indicated with \Phi_i,
i = 1, 2, 3 the three chiral superfields of 4SYM and with V the vector superfield. The superfields
associated with the remaining MWT fermions N and E are given by:

\[ (\bar{N}_L, \ N_L) \in \Lambda_1, \quad (\bar{E}_L, \ E_L) \in \Lambda_2, \quad (\bar{N}_R, \ N_R) \in N, \quad (\bar{E}_R, \ E_R) \in E. \quad (5) \]
Table 1. MSCT $\mathcal{N} = 1$ superfields and transformation properties. Adj denotes adjoint and F fundamental.

| Superfield | SU(2)$_{TC}$ | SU(2)$_{L}$ | U(1)$_{Y}$ |
|------------|--------------|--------------|--------------|
| $\Phi_{1,2}$ | Adj | F | 1/2 |
| $\Phi_{3}$ | Adj | 1 | -1 |
| $V$ | Adj | 1 | 0 |
| $\Lambda_{1,2}$ | 1 | F | -3/2 |
| $N$ | 1 | 1 | 1 |
| $E$ | 1 | 1 | 2 |
| $H$ | 1 | F | 1/2 |
| $H'$ | 1 | F | -1/2 |

The quantum numbers of the superfields in Eqs.(4,5) and of those labeled by $H$ and $H'$, which contain each a Higgs scalar weak doublet, are given in Table 1.

The renormalizable lepton and baryon number conserving superpotential for the MSCT is

$$P = P_{MSSM} + P_{TC},$$

where $P_{MSSM}$ is the minimal supersymmetric standard model (MSSM) superpotential, and

$$P_{TC} = -\frac{g_{TC}}{3\sqrt{2}}\epsilon_{ijk}\epsilon^{abc}\Phi^i_a\Phi^j_b\Phi^k_c + y_U\epsilon_{ij}\Phi^i_3H_0^\alpha\Phi^j_3 + y_N\epsilon_{ij}\Lambda_iH_0^\alpha N + y_E\epsilon_{ij}\Lambda_iH_0^\alpha E + y_R\Phi^i_3\Phi^j_3. \quad (7)$$

In the last equation $\Phi^i_a = Q^i_a$, $i = 1, 2$, with $a$ the technicolor index. Gauge invariance and $\mathcal{N} = 1$ supersymmetry does not ensure the Yukawa coupling of the first term to be equal to $g_{TC}$, however, setting it to this value amounts to the $\mathcal{N} = 4$ limit. We have also investigated the running of a general Yukawa coupling and shown that it tends towards $g_{TC}$ at low energies [17]. This result justifies our choice to set it equal to the technicolor gauge coupling itself.

3. Electroweak Symmetry Breaking

In perturbative MSCT we assume SUSY is broken softly and electroweak symmetry breaking is driven by the negative mass squared of $\tilde{D}_L$, $\tilde{H}$ and $\tilde{H}'$. Electroweak symmetry breaking follows the pattern $SU(2)_L \times U(1)_Y \times SU(2)_{TC} \rightarrow U(1)_{EM} \times U(1)_{TC}$. The phenomenological constraints on a new $U(1)$ massless gauge boson were studied in [18]. The lower limits on the scale of dimension six operators with SM fields and the new massless photon are in the TeV range.

We choose the vacuum expectation value (vev) of the techni scalar to be aligned in the third direction of the $SU(2)_{TC}$ gauge space. We define the vevs:

$$\langle \tilde{D}^\alpha_L \rangle = \frac{v_{TC}}{\sqrt{2}}, \quad \langle \tilde{H}_0 \rangle = s_\beta \frac{v_H}{\sqrt{2}}, \quad \langle \tilde{H}'_0 \rangle = c_\beta \frac{v_H}{\sqrt{2}},$$

where all the vevs are chosen to be real, $s_\beta = \sin \beta$, and $c_\beta = \cos \beta$.

After EWSB the mass terms of gauge bosons are written as a function of the mass eigenstates as:

$$-\mathcal{L}_{g-mass} = g_{TC}v_{TC}^2G_L^\mu G_{L}^{-\mu} + \frac{g_2^2}{2}\left(v_{TC}^2 + v_H^2\right)W^\mu_+ W_{L}^{-\mu} + \frac{g_2^2 + g_Y^2}{4}\left(v_{TC}^2 + v_H^2\right)Z_{\mu}Z^\mu \quad (9)$$

Notice that we indicate the scalar component of each weak doublet superfield with a tilde.
where
\[
G_\mu^\pm = \frac{1}{\sqrt{2}} \left( G_\mu^1 \mp i G_\mu^2 \right), \quad W_\mu^\pm = \frac{1}{\sqrt{2}} \left( W_\mu^1 \mp i W_\mu^2 \right), \quad Z_\mu = c_w W_\mu^3 - s_w B, \quad t_w = \frac{g_Y}{g_L}.
\] (10)

The ± exponent of the techni-gluon refers to the \( U(1)_{TC} \) charge, while the ± exponent on the EW gauge bosons refer to the usual EM charge. We see the electroweak scale is set by
\[
\sqrt{v_H^2 + v_{TC}^2} = 246 \text{ GeV}.
\] (11)

The remaining, massless states are the techni-photon and the EW photon:
\[
G_\mu = G_\mu^3, \quad A_\mu = s_w W_\mu^3 + c_w B
\] (12)

4. Spectrum and Constraints

We can write the gauge boson, fermion, and scalar squared mass matrices in block diagonal form in the basis of EM- and TC-charges and L and B numbers [17]. The mass matrices of all the SM fermions and their superpartners assume the same form, in terms of the Higgs vevs, as of those obtained in the MSSM and can be found for example in [19]. The EW gauginos, Higgs scalar doublets and their superpartners mix with the \( \mathcal{N} = 4 \) technicolor sector. Finally the fields \( N_L, \bar{N}_R \), and their scalar superpartners will not mix at tree level with other SM fields with EM charge \( Q_{EM} = 1 \) (where we defined \( Q_{EM} = T_3^L + Y \)).

We also calculate the one-loop contributions to the CP-even and -odd neutral (both under \( U(1)_{EM} \) and \( U(1)_{TC} \)) scalars by numerically evaluating the derivatives of the mass eigenvalues with respect to the fields evaluated on the vevs [20]. In this first estimate we compute to the neutral Higgs masses neglecting the contributions from top-stop mass splitting. We consider the fields given in Table 1, plus the \( W \) and \( B \) bosons and their superpartners. It is seen that the masses of the lightest physical states, \( h_0^0 \) and \( A_0 \), which are massless at tree level, gain a mass at 1-loop with a strong dependence on the size of the Yukawa couplings.

The lower bounds on the mass of the lightest neutralino and chargino are [21]:
\[
m_{\chi^0_0} > 46 \text{ GeV}, \quad m_{\chi^\pm_0} > 94 \text{ GeV}.
\] (13)

These limits refer to the MSSM, but are rather general, since they are extracted mostly from the \( Z \) decay to neutralino-antineutralino pair the former, and from photo-production of a chargino-antichargino pair at LEP II the latter. We can therefore assume these limits to hold also for the MSCT. Because of their generality and independence from the coupling strength (as long as it is not negligible), we use the lower bound on the chargino mass also for the mass of the doubly-charged chargino \( E \). Note that the presence of the term proportional to \( y_R \) in the superpotential, Eq.(7) allows it to decay into singly charged ordinary particles, thereby escaping cosmological constraints on charged stable particles.

The techni-gaugino \( \bar{D}_R^3 \) is an EW singlet fermion and therefore is a right-handed neutrino, which can be very light. Because of this, and because the mass of the lightest techineutralino is a decreasing function of the mass of \( \bar{D}_R^3 \), we assume the soft SUSY breaking mass of the technigaugino to be small.

Other useful limits on the parameters are obtained by using the fact that the smallest eigenvalue of a semi-positive definite square matrix is smaller or equal to any eigenvalue of the principal submatrices. By using the neutralino, chargino, and doubly charged chargino mass matrices, we get for example
\[
v_{TC} > 2 \frac{46 \text{ GeV}}{\sqrt{g_L^2 + g_Y^2}} = 124 \text{ GeV}, \quad v_H < 213 \text{ GeV}, \quad \frac{y_{EC} v_H}{\sqrt{2}} > 94 \text{ GeV}, \quad m_t = \frac{y_t}{y_E} m_{cc}
\] (14)
where the subscript $t$ refers to the top quark and $m_{cc}$ is the doubly charged lepton mass. One of the most important inequalities is

$$y_t > \frac{173}{213} \sqrt{\frac{1}{\frac{94^2}{y_t^2} - \frac{94}{y_t^2}}}.$$  \hspace{1cm} (15)

This last bound is plotted in Figure 1, where the shaded area shows the values of $y_t$ and $y_E$ excluded by the experiment: it is evident from the plot in Figure 1 that either $y_t$ or $y_E$ is constrained to be larger than about 1.3. Such large Yukawa couplings imply a Landau pole at some intermediate scale and we cannot discuss unification of the gauge couplings.

By using the renormalization group equations [17] we find that a phenomenologically reasonable compromise for the values of $y_t$ and $y_E$ allowing perturbativity at the energy scale of a few TeVs is, respectively, 1.65 and 2.2. The value of the lightest chargino mass is controlled mainly by $y_U$, which we take equal to $y_t$, while $y_{TC} = y_N = 1.1$, since they are less constrained to be large. In Figure 2 are plotted $y_{TC}, y_U, y_t, y_N, y_E$ as a function of the renormalization scale $M$: the couplings are normalized for $M = m_Z$ to $y_t = y_U = 1.65, y_E = 2.2, y_N = y_{TC} = 1.1$.

By maximizing the minimum eigenvalue of $\mathcal{M}_n \cdot \mathcal{M}_n^\dagger$ on the parameter space allowed by the bounds (14,15) with $y_t = 1.65$ and $y_E = 2.2$, we obtain

$$m_{\chi_0}^{\text{max}} = 46 \text{ GeV}, \ m_{\chi_\pm} = 60 \text{ GeV},$$  \hspace{1cm} (16)

where $m_{\chi_\pm}$ is calculated at the point in parameter space that maximizes $m_{\chi_0}$. With these chosen values of the Yukawa couplings at the EW scale, a Landau pole arises around 2.3 TeV and the chargino mass is light compared to the experimental bounds. A scan of the allowed parameter space, with the same values of the Yukawa couplings shows that it is not possible to satisfy both constraints in Eq.(13) simultaneously. However, by further lowering the Landau pole scale we can achieve a heavy enough spectrum:

$$m_{\chi_0}^{\text{max}} = 47 \text{ GeV}, \ m_{\chi_0}^{\text{max}} = 96 \text{ GeV}, \ m_{b_0} = 95 \text{ GeV}, \ m_{A_0} = 32 \text{ GeV}, \ M_{\text{pole}} = 400 \text{ GeV}$$  \hspace{1cm} (17)

The results correspond to having chosen at the EW scale the values $y_N = 1.8, y_t = g_{TC} = y_U = 2.3, y_E = 2.4$. 

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**Figure 1.** Shaded area shows experimentally excluded values of the Yukawa couplings $y_t$ and $y_E$.

**Figure 2.** Plot of $y_{TC}, y_U, y_t, y_N, y_E$ as a function of the renormalization scale $M$: the couplings are normalized for $M = m_Z$ to $y_t = y_U = 1.65, y_E = 2.2, y_N = y_{TC} = 1.1$. 
5. Conclusions
Our preliminary results on the spectrum of the MSCT indicate that the Tevatron and the LHC can rule out a significant portion of the parameter space of this model. Part of the particle spectrum is very similar to the one of the MSSM, however, MSCT also features several new light states, with respect to the EW scale, such as a doubly charged particle. Therefore an interesting experimental signature would be the discovery of a doubly charged particle together with a very light chargino and/or neutralino. Finally, since the Yukawa couplings are larger than the SM ones we expect 100% increase of several production cross sections such as the Higgs scalar ($h_0^0$) one via the gluon-gluon fusion process.

Another characteristic of the model relevant for collider experiments is that due to the presence of one extra Higgs-type particle, coming from the supertechnicolor sector, the spectrum features scalars and pseudoscalars lighter than in the MSSM case. These states will also yield interesting signatures at collider experiments.

Since our model features, at the EW scale, a new $\mathcal{N} = 4$ supertechnicolor sector, collider experiments have the possibility to explore directly string theory. This is so since the new scalars coming from this sector can be directly identified with the extra six space coordinates of ten dimensional supergravity. This link is even more clear when considering the present supertechnicolor sector in the nonperturbative regime which can be investigated using AdS/CFT techniques.

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