Non–Singlet QCD Analysis of the Structure Function $F_2$ in 3-Loops

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First results of a non–singlet QCD analysis of the structure function $F_2(x, Q^2)$ in 3–loop order based on the non–singlet world data are presented. Correlated errors are determined and their propagation through the evolution equations is performed analytically. The value for $\alpha_s(M_Z)$ is determined to be $0.1135 + 0.0023 / −0.0026$, compatible with results from other QCD analyses. Low moments for $u_v(x)$, $d_v(x)$ and $u_v(x) − d_v(x)$ with correlated errors are calculated which may be compared with results from lattice simulations.

1. INTRODUCTION

A consistent 3-loop QCD analysis of the unpolarized structure function $F_2(x, Q^2)$ can be carried out after having the recently completed next-to-next-to-leading order (NNLO) anomalous dimensions available \[1\] in addition to the 2-loop Wilson coefficients \[2\]. Two NNLO QCD analyses have been performed previously based on structure function and other hard scattering data \[3\], and structure function data only \[4\]. In these analyses singlet– and non–singlet evolution was dealt with in parallel. The present analysis concentrates on the non–singlet evolution only in order to firstly obtain an accurate as possible picture for the valence quark distributions and the value of the QCD coupling constant $\alpha_s(M_Z^2)$ with correlated errors in NNLO. Analyzing only the non–singlet data has the advantage that large gluon and sea quark effects remain decoupled. A NNLO QCD analysis of $F_2(x, Q^2)$ allows to reduce the theoretical error in determining $\alpha_s(Q^2)$ to at least the level of the experimental error since the factorization and renormalization scale uncertainties reduce significantly. Comparison of QCD analysis results with results from recent lattice simulations concerning the low order moments has shown astonishing agreement in the polarized case \[5\]. With the steadily improving lattice calculations this might also become feasible in the unpolarized case, where systematic effects have still been large during the last years. In this letter we describe the results of an analysis of the deeply inelastic non–singlet world data for charged lepton–nucleon scattering.

2. QCD FORMALISM

In Mellin–N space the non–singlet (NS) parts of a structure function $F_i(N, Q^2)$ are given by

$$F_i^{\pm, V}(N, Q^2) = \left[ 1 + C_{ki}(N)a + C_{li}(N)a^2 \right] f^{\pm, V}(N, Q^2),$$

where the $C_{ki}(N)$ are the corresponding Wilson coefficients and $f^{\pm, V}(N, Q^2)$ are the non–singlet quark combinations. The symbol $a$ denotes the strong coupling constant normalized to $a(Q^2) = \alpha_s(Q^2)/4\pi$. The quark combinations to be considered are

$$\Delta^\pm = (u \pm \bar{u}) - (d \pm \bar{d}),$$
$$v = (u - \bar{u}) + (d - \bar{d}).$$

The non–singlet parts of $F_2$ are proportional to

$$F_2^{NS} \propto \frac{1}{3} \Delta^+, \quad F_2^{p,v} \propto \frac{5}{18} v + \frac{1}{6} \Delta^-, \quad \text{and} \quad F_2^{d,v} \propto \frac{5}{18} v.$$
\( F_2^{NS} \) stands for the difference of proton and deuteron data in the range \( x < 0.3 \) while the other combinations are used in the valence approximation for \( x > 0.3 \). All these combinations evolve as \( + \)-combinations in \( Q^2 \). The relevant parton densities to be determined in a nonsinglet QCD analysis are \( xu_v(x, Q^2) \), \( xd_v(x, Q^2) \), and \( x(d-\bar{u})(x, Q^2) \).

The solution of the evolution equation to 3-loops reads for the valence distribution as an example

\[
V(Q^2) = V(Q_0^2) \left( \frac{a}{a_0} \right)^{-\hat{P}_0/\beta_0} \\
\left\{ \begin{array}{c}
1 - \frac{1}{\beta_0} (a - a_0) \left[ \hat{P}_1^{\pm} - \frac{\beta_1}{\beta_0} \hat{P}_0 \right] \\
-\frac{1}{2\beta_0} (a^2 - a_0^2) \left[ \hat{P}_2^{\pm,\gamma} - \frac{\beta_1}{\beta_0} \hat{P}_1^{\pm} + \left( \frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0} \right) \hat{P}_0 \right] \\
+ \frac{1}{2\beta_0} (a - a_0)^2 \left[ \hat{P}_1^{\pm} - \frac{\beta_1}{\beta_0} \hat{P}_0 \right] \end{array} \right\} \quad (5)
\]

where the \( \hat{P}_i \) are the splitting functions in Mellin space.

### 3. PARAMETERIZATION

The parameterizations of the above mentioned parton densities at the input scale of \( Q_0^2 = 4.0 \) GeV\(^2\) are determined as

\[
xu_v(x, Q_0^2) = A_{u_v} x^{a_u} (1 - x)^{b_u} \\
\quad (1 - 1.108x^{\frac{4}{2}} + 26.283x), \quad (6)
\]

and

\[
xd_v(x, Q_0^2) = A_{d_v} x^{a_d} (1 - x)^{b_d} \\
\quad (1 + 0.895x^{\frac{4}{2}} + 18.179x). \quad (7)
\]

Here the values for the coefficients in the polynomial given as numbers in the intermediary range of \( x \) are obtained by a fit and are then kept fixed since their respective errors are still large. Furthermore, the analysis requires different \( \pi \) and \( \bar{d} \) quark densities. We adopted the choice \( 6 \) which gives a good description of the \((d-\bar{u})\) data from E866 \( 7 \).

\[
x(d-\bar{u})(x, Q_0^2) = 1.195x^{1.24}(1 - x)^{9.10} \\
\quad (1 + 14.05x - 45.52x^2). \quad (8)
\]

The normalization constants \( A_{u_v} \) and \( A_{d_v} \) are fixed by the conservation of the number of valence quarks: \( \int_0^1 u_v(x)dx = 2 \), \( \int_0^1 d_v(x)dx = 1 \).

The remaining four parameters are determined in the fit for \( u_v \) and \( d_v \), the low–\( x \) and high–\( x \) parameters \( a \) and \( b \), respectively, along with \( \Lambda_{QCD} \).

### 4. DATA

The results presented here are based on 762 data points for the structure function \( F_2(x, Q^2) \) measured on proton and deuteron targets. The experiments contributing to the statistics were:
BCDMS [8], SLAC [9], NCM [10], H1 [11], and ZEUS [12]. The BCDMS data were recalculated replacing $R_{QCD}$ by $R_{1998}$ [13]. All deuteron data were corrected for Fermi motion and offshell effects [14]. The non–singlet structure function $F^N_{2S}$ was constructed according to the relation $F^N_{2S} = 2(F^p_{2S} - F^d_{2S})$ from proton and deuteron data given at the same $x$ and $Q^2$ values. The region $x > 0.3$ which was selected for $F^p_{2S}$ is expected to be dominated by valence quark contributions while for the region $x < 0.3$ $F^N_{2S}$ data were used. In addition, a $Q^2$ region of $4.0 < Q^2 < 30000 \text{ GeV}^2$ was chosen and a cut in the hadronic mass $W^2 > 12.5 \text{ GeV}^2$ was applied in order to diminish higher twist effects.

In the fitting procedure we allowed for a relative normalization shift between the data sets within the overall normalization uncertainties quoted by the experiments or assumed accordingly. These normalization shifts were fitted once and then kept fixed.

5. CORRELATED ERROR CALCULATION

The systematic errors which are known to be partly correlated are not treated separately in this analysis. For all data sets we used the simplest procedure by adding the statistical and the total systematic errors in quadrature being aware of the fact that this will eventually overestimate the errors.

The fully correlated $1\sigma$ experimental error bands are determined via Gaussian error propagation demanding that only fits with a positive definite covariance matrix are accepted. The gradients of the parton distributions with respect to the variable parameters needed here can be calculated analytically at the input scale $Q^2_0$. Their values at $Q^2$ are given by evolution in Mellin–$N$ space.

6. RESULTS

The fit results are summarized in table 1.

|     | $u_v$ | $d_v$ | $\Lambda_{QCD}^{(4)}$ | $\chi^2/ndf$ |
|-----|-------|-------|-----------------------|--------------|
| $a$ | 0.314 ± 0.007 | 0.413 ± 0.047 | 227 ± 30 MeV | 652/757 = 0.86 |
| $b$ | 4.199 ± 0.032 | 6.196 ± 0.332 |                |              |

Table 1: Parameter values of the NNLO QCD fit based on the non–singlet world data on $F_{2}^{em}(x, Q^2)$.

The value of $\Lambda_{QCD}^{(4)}$ is quite stable against a variation of the $Q^2$ cut on the data when varying it between 4.0 GeV$^2$ to 10.0 GeV$^2$. From the fitted value of $\Lambda_{QCD}^{(4)}$ the following value of $\alpha_s$ is extracted:

$$\alpha_s(M_Z^2) = 0.1135^{+0.0023}_{-0.0026} \text{ (expt.).}$$

This value is within the errors in good agreement with results from other NLO/NNLO QCD analyses, see Table 2, and with the latest value for the world average of $0.1182 \pm 0.0027$ [15].

Figure 2. The parton density $xd_v$ at the input scale $Q^2_0$ with the same conditions as in Figure 1a.
The resulting parton densities $xu_v(x)$ and $xd_v(x)$ at the input scale of $Q_0^2 = 4.0 \text{ GeV}^2$ are presented in Figures 1, 2. Comparison with results from global analyses shows satisfactory agreement. While $xu_v$ is rather well determined, the error band for $xd_v$ is a bit broader as also found by the other analyses.

| Parameter | NLO | NNLO |
|-----------|-----|------|
| $\alpha_s(M_Z^2)$ | | |
| CTEQ6 | 0.1165 ± 0.0006 | [17] |
| MRST03 | 0.1165 ± 0.0002 | [3] |
| A02 | 0.1171 ± 0.0015 | [4] |
| ZEUS | 0.1166 ± 0.0049 | [19] |
| H1 | 0.1150 ± 0.0017 | [12] |
| BCDMS | 0.110 ± 0.006 | [9] |
| BB (pol) | 0.113 ± 0.004 | [5] |

Table 2: Comparison of NLO and NNLO results on $\alpha_s(M_Z^2)$ from deep inelastic scattering, including also global fits. The NLO H1 value is subject to an additional error of $+0.0009$/−0.0005 and the NLO ZEUS value of $±0.0018$ due to model dependence.

Low moments for $u_v$, $d_v$ and $u_v − d_v$ with correlated 1σ errors have been calculated. An interesting comparison can be made for the second moment of $u_v − d_v$, with a result from recent lattice simulations. A value of $0.180 ± 0.005$ was derived here and can be compared with results of upcoming lattice simulations, cf. e.g. [10]. This comparison should be performed for higher moments in the future as well as for the moments of the individual distributions $u_v$ and $d_v$.

7. CONCLUSIONS

A non–singlet QCD analysis of the structure function $F_2(x, Q^2)$ based on the non–singlet world data has been performed at 3–loop order. The value determined for $\alpha_s(M_Z^2)$ is compatible within the errors with results from other QCD analyses and with the world average. New parameterizations of the parton densities $u_v$ and $d_v$ including their fully correlated 1σ errors are derived. Low order moments for $u_v$, $d_v$ and $u_v − d_v$ are calculated from this analysis. It will be interesting to compare them with upcoming results from lattice simulations.

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