PERSPECTIVE ON AFTERGLOWS: NUMERICALLY COMPUTED VIEWS, LIGHT CURVES, AND THE ANALYSIS OF HOMOGENEOUS AND STRUCTURED JETS WITH LATERAL EXPANSION

JAY D. SALMONSON
Lawrence Livermore National Laboratory, Livermore, CA 94551
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ABSTRACT

Herein I present numerical calculations of light curves of homogeneous and structured afterglows with various lateral expansion rates as seen from any vantage point. Such calculations allow for direct simulation of observable quantities for complex afterglows with arbitrary energy distributions and lateral expansion paradigms. A simple, causal model is suggested for lateral expansion of the jet as it evolves: namely, that the lateral expansion kinetic energy derives from the forward kinetic energy. As such, the homogeneous jet model shows that lateral expansion is important at all times in the afterglow evolution and that analytical scaling laws do a poor job at describing the afterglow decay before and after the break. In particular, I find that lateral expansion does not cause a break in the light curve as had been predicted. A primary purpose of this paper is to study structured afterglows, which do a good job of reproducing global relationships and correlations in the data and thus suggest the possibility of a universal afterglow model. Simulations of structured jets show a general trend in which jet breaks become more pronounced with increasing viewing angle with respect to the jet axis. In fact, under certain conditions a bump can occur in the light curve at the jet-break time. I derive scaling relations for this bump and suggest that it may be a source of some bumps in observed light curves such as that of GRB 000301C. A couple of lateral expansion models are tested over a range of efficiencies and viewing angles, and it is found that lateral expansion can, in some cases, substantially sharpen the jet break. I show flux surface contour maps and simulated images of the afterglows that give insight into how they evolve and determine their light curves.

Subject headings: gamma rays: bursts — gamma rays: theory
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1. INTRODUCTION

It is currently widely believed that gamma-ray bursts (GRBs) derive from narrow (half-opening angle $\theta_{0,GRB} \sim$ a few degrees) jets of relativistic ejecta pointing toward the observer. One of the basic motivations for this has been to relieve the energy crisis in GRBs by reducing the necessary total energy from the inferred isotropic equivalent energy (up to several $10^{54}$ ergs) by a factor $\theta_{0,GRB}^2/2 \sim 10^{-4}$ to $10^{-6}$ (and thus boosting the total event rate by the reciprocal, $2/\theta_{0,GRB}^2$, to include unseen jets directed away from the observer). If the GRB emission is collimated, it is plausible that the afterglow shock is also collimated into a cone with opening angle $\theta_0$, generally thought to be larger than $\theta_{0,GRB}$.

A rich area of inquiry is then to predict and look for observable consequences of a narrow jet both in the GRB phase (e.g., Salmonson 2000, 2001) and in the afterglow phase (e.g., Rhoads 1999; Sari, Piran, & Halpern 1999). The most fundamental consequences derive from the deceleration of the afterglow as it propagates into the interstellar medium (ISM). The relativistic motion of the emitting shock causes the radiation to be beamed into an angle $1/\Gamma$ in the observer frame. At early times, this relativistic beaming angle is smaller than the physical jet opening angle, $1/\Gamma < \theta_0$; thus, the emission appears to be isotropic. Eventually, as the jet decelerates, it transitions to $1/\Gamma > \theta_0$, and the finite, nonisotropic extent of the jet becomes apparent to both the observer and the jet itself. The observer sees a deficit of flux compared to that expected from an isotropic emitter. The jet, now being entirely causally connected, begins to expand sideways. These two effects combine to cause a break in the light curve (Rhoads 1999; Sari et al. 1999). Roughly 20 such jet breaks have been observed (see Frail et al. 2001 and references therein), and these constitute the most direct evidence for beaming to date.

In addition, since the later, slower emission is beamed into a wider angle than the earlier emission, we expect to see off-axis optical and radio afterglows where no gamma-ray or X-ray burst has been seen (Rhoads 1997; Dalal, Griest, & Pruett 2002). To date, these so-called orphan afterglows have yet to be positively observed.

Thus far, the study of this afterglow jet break has assumed an afterglow pointed directly at the observer with constant, homogeneous, unstructured energy and mass density $\propto H(\theta_0 - \theta)$ across the jet surface, with a hard edge at $\theta = \theta_0$, where $H(\ldots)$ is the Heaviside step function. As such, from scaling laws $t_j \propto \theta_0^{-1/3}$, the observation of a jet-break time $t_j$ gives direct information about the opening angle of the jet. While this model is relatively simple to calculate and is amenable to analytical calculation, it is not necessarily the easiest jet morphology for nature to produce and thus may not accurately represent a physical jet. First, one certainly expects the observer’s viewing angle $\theta_i$ of the jet to vary. Furthermore, a homogeneous jet with a hard edge will be unstable to expansion and rarefaction and is thus unlikely to propagate intact and unlikely to have been
formed in the first place. Recent numerical simulations by Zhang, Woosley, & MacFadyen (2003) of relativistic jets emerging from stars, within the context of the “collapsar” model, show substantial structure, with the most energetic material along the jet axis and decreasing with larger angles from the axis. The first semianalytical calculations of an afterglow jet with structure, i.e., with a decreasing energy density and/or Lorentz factor as a function of angle from the jet axis, were done by Rossi, Lazzati, & Rees (2002), while an analytical treatment was done by Zhang & Mészáros (2002b). Not long before, a qualitative discussion of a structured jet model was put forward by Salmonson & Galama (2002). It was found by Rossi et al. (2002) and Zhang & Mészáros (2002b) that a universal structured jet, viewed at different angles, will yield a range of afterglows with jet-break times relating to the viewing angle by \( t_j \propto \theta^{-5/3} \). By letting the energy per solid angle of the jet decrease with angle from the jet axis as \( \epsilon \propto \theta^{-5/2} \), they were able to effectively reproduce the observed relation \( E \propto t_j^{-1} \) (Frail et al. 2001).

In this paper I present calculations that further refine the work by Rossi et al. (2002) and Zhang & Mészáros (2002b). By discretizing the surface of the afterglow, arbitrary functions of energy density, the Lorentz factor, and the lateral Lorentz factor (defined in the fluid frame) can be simulated and light curves produced for arbitrary viewing angles \( \theta \). In so doing, I corroborate some of the key results of Rossi et al. (2002) and Zhang & Mészáros (2002b), i.e., \( t_j \propto \theta^{-5/3} \) and \( E_{10} \propto t_j^{-1} \) if \( \epsilon \propto \theta^{-2} \). In addition, these more detailed calculations allow for a quantitative discussion of how the light curve breaks; in particular, I find that a flattening, and even a bump, in the light curve is possible just prior to the jet-break time. In addition, I study lateral expansion and find that some of the general scaling laws that are widely used in afterglow work are incorrect.

2. NUMERICAL JET CALCULATIONS

The calculation presented here begins by discretizing the surface of the afterglow mapped with polar coordinates \((\theta, \phi)\) into small elements of solid angle \( d\Omega = \cos \psi \sin \theta \, d\theta \, d\phi \), where herein we assume the afterglow is spherical, i.e., zero inclination \( \psi = 0 \), and where \( \theta = 0 \) corresponds to the jet axis. Each surface element plows into the ISM, sweeping up mass according to its cross section \( dA = R^2 \, d\Omega \), decelerating, shocking, and radiating. The physics of this calculation can be broken up into two parts: (1) the dynamical evolution of each surface element of the afterglow, as dictated by conservation of energy and momentum, and (2) the radiative mechanism, which I take to be the standard synchrotron shock model (Mészáros & Rees 1997).

To calculate the evolution of the afterglow shock, one needs only to invoke conservation of energy and momentum along with an assumption of radiation losses. Herein I assume radiative losses are dynamically insignificant, i.e., the evolution is adiabatic. Define the initial bulk Lorentz factor \( \Gamma_0 = \dot{E}_0 / \dot{M} c^2 \) of a surface element of the afterglow, where \( \dot{E}_0 \) and \( \dot{M} \) are the initial energy and rest mass per solid angle. Thus, following Paczyński & Rhoads (1993) and Rhoads (1999), the energy and radial momentum of a surface element of the afterglow are

\[
\Gamma_0 + f = (1 + f) \xi \Gamma, \quad \Gamma_0 \beta_0 = (1 + f) \xi \Gamma \beta, \quad (1)
\]

where \( \xi = (E + M) / M \) is the internal energy of the expanding shell.

The mass fraction \( f \) of accumulated interstellar mass density, \( \rho_{\text{ISM}} \equiv n_{\text{H}_2} \), where \( n_{\text{H}_2} \) is the mass of the proton, is

\[
f = \frac{\dot{M}}{\dot{M} c^2} = \frac{\Gamma_0 \dot{E}_0}{\dot{M} c^2} = \frac{\Gamma_0 \dot{E}_0}{\dot{M} c^2} \frac{\Delta \Omega}{\Delta \Omega_0} r^2 dr,
\]

\[
= \frac{\Gamma_0 \dot{E}_0}{\dot{M} c^2} \Delta \Omega_0 = 1.9 \times 10^{-54} \frac{\Gamma_0}{\dot{E}_0} \Delta \Omega_{52} \Delta \Omega_0 \Delta N_e, \quad (2)
\]

where the number of electrons swept up into the shock element is

\[
\Delta N_e = \int_0^R \rho_{\text{ISM}} \Delta \Omega_0 r^2 dr, \quad (3)
\]

the energy per solid angle is \( \dot{E}_{52} = \dot{E} / (10^{52} / 4\pi \text{ ergs}) \), and the element solid opening angle is

\[
\Delta \Omega = \sin \theta d\phi d\theta, \quad (4)
\]

where the position of a surface element \( \theta \) will evolve as the shock laterally expands because of internal pressure (\( \xi \leq 3 \)). The velocity magnitude is \( \beta = \left| v / c \right| = (1 - 1/\Gamma^2)^{1/2} \) and the proper time in the fluid frame is

\[
t' = \int_0^R dr / \Gamma \beta c. \quad (5)
\]

Equations (1) can be solved for the Lorentz factor

\[
\Gamma = \frac{\Gamma_0 + f}{\sqrt{1 + 2f}} \cdot (6)
\]

Therefore, by specifying \( \dot{E}_0 \) and \( \Gamma_0 \) and a prescription for lateral expansion \( v_L \) (\( \xi \leq 3 \)), the entire evolution of the afterglow as a function of \( R \) is determined.

In order to calculate the observed flux \( F_v \) at a given frequency \( \nu \), note that the intensity transforms as \( I = I' \delta^2 \), where the Doppler factor is

\[
\delta = \left| \Gamma (1 - \beta \cdot \hat{n}) \right|^{-1}, \quad (7)
\]

1 Eqs. (1) are the correct equations for the evolution of a jet. It has been noted by several authors (e.g., Huang, Dai, & Lu 2000a) that eqs. (1) yield a velocity \( v \propto R^{-1} \) in the nonrelativistic limit that is not consistent with the Sedov-Taylor (S-T) blast wave solution, \( v \propto R^{-1/2} \) (e.g., Shu 1992). Thus, it was assumed that these equations are incorrect for describing afterglow shock dynamics for the entire evolution. There have been alternative formulations of the afterglow shock dynamics. The reason eqs. (1) do not reproduce the S-T solution is because the S-T solution (and the Blandford-McKee [B-M] solution; Blandford & McKee 1976) is a “blast wave,” and such a fireball does work as it expands, and thus radial momentum is not conserved. The S-T solution depends on the assumption of spherical symmetry (the lateral expansion of a fluid element is opposed by the expansion of adjacent elements, thus forcing the fluid radially) and thus does not apply to a jet. The jet, having nothing to push on, expands laterally and, thus lacking a radial force, must conserve radial momentum. Afterglow jet is more akin to a bullet fired from a gun than to a spherically expanding blast wave; the jet and the bullet are isolated, momentum-conserving bodies decelerating because of their interaction with their surroundings, while the blast wave continues to do work as it expands, by virtue of its flatness, and thus is not momentum-conserving. Thus, I argue that eqs. (1) describe the entire evolution of a jet more accurately than do the S-T or B-M solutions in their respective regimes (as long as the jet has not expanded so far as to become spherically symmetric, which I do not find to be the case over the time interval discussed herein). Note that the degeneracy of the energy and momentum equations for \( \Gamma > 1 \) make the behavior of eqs. (1) indistinguishable from the B-M solution in the relativistic regime.
\( \beta = v/c \), and \( \hat{n} \) is the unit vector pointing toward the observer. Following Rossi et al. (2002), here I focus on the power-law branch of the spectrum between the peak, \( \nu_m \), and cooling, \( \nu_c \), frequencies. As such, the proper intensity at the proper peak frequency \( \nu_m \) is

\[
I_{\nu_m} = \frac{P_\nu \Delta N_e}{4\pi R^2 \Delta \Omega},
\]

where the shock element has surface area \( R^2 \Delta \Omega \) and the proper power per electron radiated at \( \nu_m \) is (Wijers & Galama 1999)

\[
P_\nu = 5.4 \times 10^{-24} \times \left( \frac{\phi_{\nu}}{0.59} \right) n_{0^n}^{1/2} \epsilon_{0^n}^{1/2} \beta^2 \ \text{ergs s}^{-1} \ \text{Hz}^{-1} \ \text{electron}^{-1},
\]

where the factor \( \beta^2 \) has been included here to make this equation valid in the nonrelativistic limit (Rybicki & Lightman 1975). The observed intensity at a frequency \( \nu \) is

\[
I_\nu = I_{\nu_m} \left( \frac{\nu}{\nu_m} \right)^{-\alpha} \delta^3,
\]

and integrating over the population of radiating electrons, the flux will scale as

\[
F_\nu = \int \frac{P_\nu \Delta A}{4\pi R^2 \Delta \Omega} \left( \frac{\nu}{\nu_m} \right)^{-\alpha} \delta^3 \left( 1 + z \right) D_L \Delta N_e, (10)
\]

where \( D_L \) is the luminosity distance and \( \Delta N_e \) is the integral. The surface area of an afterglow element as seen by the observer, \( \Delta A \), is calculated self-consistently in the code by projecting the elements onto a surface perpendicular to the observer line of sight (LOS). The minimum electron frequency is

\[
\nu_m = 3.5 \times 10^9 \left( \frac{\phi_{\nu}}{0.64} \right) c_{e-1} n_{0^n}^{1/2} \epsilon_{0^n}^{1/2} \ \text{Hz},
\]

and \( \nu' = \nu/\delta \), where we take \( \nu = 4.4 \times 10^{14} \) Hz for the \( R \) band. Therefore,

\[
\left( \frac{\nu}{\nu_m} \right)^{-\alpha} = \left[ 7.96 \times 10^{-6} \left( \frac{\phi_{\nu}}{0.64} \right) c_{e-1} n_{0^n}^{1/2} \epsilon_{0^n}^{1/2} \delta \right]^{-\alpha}, (13)
\]

and thus equation (10) scales as

\[
I_\nu \propto \Gamma^{3 \alpha + 1 + \delta \alpha + 3 + R \nu^{-\alpha}} \propto \Gamma^{3 \alpha + 2 + \delta \alpha + 4} \mu^\alpha, \quad \text{where} \quad t \propto R/\Gamma/\delta,
\]

thus demonstrating the explicit scalings and consistency with Rossi et al. (2002). For \( \alpha = \frac{1}{2} \) the observed flux is

\[
F_\nu = 6.4 \times 10^{-57} \left( \frac{\phi_{\nu}}{0.59} \right) \left( \frac{\phi_{\nu}}{0.64} \right)^{1/2} c_{e-1} n_{0^n}^{3/4} \epsilon_{0^n}^{3/4} \Delta A \ \text{mJy},
\]

where \( \cosmology \) \((\Omega_\Lambda, \Lambda) = (0.3, 0.7)\) was used with \( H_0 = 65 \text{ km s}^{-1} \ \text{Mpc}^{-1} \) to give \( D_L(1) = 2.2 \times 10^{28} \) cm. It is important to note that by explicitly evolving \( \Gamma \) (eq. [6]) and \( F_\nu \) (eq. [14]) in terms of the number of swept-up electrons, \( N_e \propto \text{volume} \) (eq. [3]), this formulation consistently accommodates sideways expansion of the jet (§ 3.1). Finally, the flux light curve as a function of observer time \( t \) is calculated by

\[
t_{\text{obs}} = (1 + z) \int_0^R (1 - \beta \cdot \hat{n}) \frac{dr}{\beta c}.
\]

To better compare physical timescales, the redshift dependence is removed from the times plotted in this paper; \( t \equiv t_{\text{obs}}/(1 + z) \).

A calculation proceeds as follows: An initial afterglow is specified by \( \delta(\theta, \phi), \Gamma_0(\theta, \phi), \) and a lateral expansion prescription and is allowed to plow into the ISM by incrementing the radius by \( \Delta R/R \sim 0.1 \) percent. The intensity, equation (10), and the observer time, equation (15), are saved at each surface element. Thus, a lattice of \((t_n, t)\)-pairs are evaluated in \((\theta, \phi, R)\)-space. Intensity is then interpolated on data slices of constant observer time \( t \). Finally, the total flux \( F_\nu \) at each observation is derived from equation (14), where surface areas \( \Delta A \) are calculated from a projection of the positions of the observed intensities onto the observer plane of view.

3. REVIEW OF THE HOMOGENEOUS JET MODEL

It is worthwhile to begin this discussion with a brief review of the homogeneous jet model, which is amenable to analytical calculations and thus allows for comparison and validation of numerical results with known results. The apparent surface area of the afterglow scales as \( dA \propto (R \theta_A)^2 \), where \( \theta_A \) is the angular size of the effective viewable aperture onto the afterglow surface:

\[
\theta_A \approx \begin{cases} \frac{1}{\Gamma}, & \theta_e < \frac{1}{\Gamma} + \theta_0 \quad \text{(relativistic-beaming dominated)}, \\ \theta_0, & \theta_e > \frac{1}{\Gamma} + \theta_0 \quad \text{(physical-jet-extent dominated)}. \end{cases}
\]

In addition, one can divide the Doppler factor (eq. [7]) into asymptotic limits:

\[
\delta = 2 \begin{cases} \frac{1}{\Gamma \theta_e}, & \theta_e < \frac{1}{\Gamma} + \theta_0, \\ 1, & \theta_e > \frac{1}{\Gamma} + \theta_0. \end{cases}
\]

Using equation (14) the flux at a given frequency is

\[
F_\nu \propto \Gamma^{3 \alpha + 2 + \delta \alpha + 4} R^3 \theta_A^2 \quad \text{mJy},
\]

and from equation (15) the observer time scales as

\[
t \approx \frac{R}{\beta c}. \quad (19)
\]

Before the afterglow shock has reached its deceleration radius \( R_d \), it coasts freely, \( \Gamma \approx \Gamma_0 \), and so \( \delta \approx \text{const} \); thus, \( F_\nu \sim \Gamma \sim t^\gamma \), where \( t \sim R \). After the shock passes the deceleration radius, i.e., \( R > R_d \), the radius and Lorentz factor are related by \( R \propto \Gamma^{-2/3} \) (eqs. [2] and [6]). In this regime there exist asymptotic power-law slopes for \( F(t) \) only in the simple cases in which one of the three angular scales, \( \theta_0, \theta_e \), and \( 1/\Gamma \), dominates over the other two. These three cases are summarized in Table 1 and can be seen in Figure 1 and 2.
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The Three Asymptotic Limits at Which One of the Three Angular Scales, \( \theta_{0}, \theta_{0}', \) and \( 1/\Gamma \), Dominates the Other Two

| Limit | \( \theta_{0} \) | \( \delta \) | General \( \alpha = 1/2 \) |
|-------|--------------|--------|-----------------|
| \( \theta_{0} \gg \theta_{0}' \), \( \Gamma \), \( 1/\Gamma \) | \( \Gamma \) | 2\( \Gamma \) | \( t^{-3/2} \)
| \( 1/\Gamma \gg \theta_{0}, \theta_{0}' \) | \( \theta_{0}' \) | 2\( \theta_{0}' \) | \( t^{-3/2} \)
| \( \theta_{0} \gg \theta_{0}', \Gamma \), \( 1/\Gamma \) | \( 2/(\Gamma \theta_{0}') \) | \( \theta^{2} \) | \( \theta^{2} \)

Notes.—Asymptotic expressions for \( \theta_{0} \) and \( \delta \) are given by eqs. (16) and (17), respectively. The last column is for the spectral slope \( \alpha = 1/2 \) (eq.[10]).

One characteristic timescale of the afterglow model is the deceleration time

\[
 t_{d} = \frac{R_{d}}{\Gamma^{2} \delta c} \approx 0.2 \left( \frac{E_{52}}{n} \right)^{1/3} \Gamma^{-8/3} \left( 1 + \frac{\delta^{2}}{\theta_{0}^{2}} \right) \text{ s}, \quad (20)
\]

where I define \( R_{d} = \left( 3E_{0}/4\pi\Gamma_{0}^{2}\rho_{\text{ISM}}c^{2} \right)^{1/3} \) as the radius at which \( f = 1/\Gamma_{0} \) (eq. [2]). For jets viewed well off-axis, \( \theta_{0} \gg 1/\Gamma \), the flux (eq. [18]) varies as \( F_{d} \propto \delta^{7/2} \). Since \( t \propto 1/\delta \), then \( F_{d} \propto t^{-7/2} \).

Another key timescale is the jet-break time \( t_{b} \), which occurs when the shock has decelerated to a Lorentz factor \( \Gamma \sim 1/\theta_{0} \):

\[
 t_{b} = \frac{5}{4} \left( \frac{\Gamma_{0} \theta_{0}}{n} \right)^{8/3} t_{d} \approx 8.5 \left( \frac{E_{52}}{n} \right)^{1/3} \theta_{0,1}^{8/3} \text{ minutes}, \quad (21)
\]

where \( \theta_{0,1} = \theta_{0}/1^{\circ} \). The factor 5/4 derives from equation (15), and the fact that the jet-break time occurs when \( 1/\Gamma \sim \theta_{0} \) is observed at the edge of the jet rather than at the center. This is a factor of 5/2 greater than derived at the jet center (e.g., Sari et al. 1999). For \( \delta \gg \theta_{0} \), the jet break at time \( t_{j} \) is largely washed out; thus, a more pertinent timescale in this regime is that at which the flux is a maximum. This occurs roughly when \( \theta_{j} \sim 1/\Gamma \gg \theta_{0} \), where \( \delta \approx 2\Gamma / \left( 1 + \theta_{0}^{2} \right) \sim \Gamma \), so \( F_{\text{max}} \propto \Gamma^{4} \sim t^{-3/2} \), where \( t \sim \Gamma^{-8/3} \).

The final phase of the afterglow is when the shock motion becomes nonrelativistic, \( \beta \ll 1 \). This regime is beyond the purpose of this paper; however, it is necessary to analytically describe the nonrelativistic light-curve behavior of the present model so as to understand the asymptotic behavior of the simulations. For \( \beta \ll 1 \), equations (1) yield \( \beta \propto R^{-3} \), equation (14) scales as \( F_{\nu} \propto R^{3} \beta^{2} \), and equation (15) becomes \( t \propto R/\beta \). Thus, the nonrelativistic light curve of the present model is \( F_{\nu,\text{nonrel}} \propto t^{-3/4} \). Note that this is independent of the spectral slope. This nonrelativistic model is likely incomplete; for instance, Dai & Gou (2001) suggest modifying the shock amplification of the shock-frame magnetic field from \( B' \propto \gamma B \) to \( B' \propto [\gamma(\gamma - 1)]^{1/2} \). Such modifications can produce substantially different behavior of the afterglow light curve, including a break at the relativistic/ nonrelativistic transition (Huang, Dai, & Lu 2006). Since our understanding of such field generation is still incomplete (for example, Rossi & Rees 2003), here I choose not to make any such modifications. Let it suffice to understand that all postbreak light curves modeled in this paper will asymptotically approach \( F_{\nu} \propto t^{-3/4} \) as they approach the nonrelativistic regime (for example, see the late times of the bottom panels of Figs. 4–7).

In order to validate the results presented here, I compared with existing solutions. Comparing these homogeneous afterglow solutions with those of Granot, Piran, & Sari (1999), I find very good agreement when one accounts for three parameter differences. As mentioned above, equation (10) is valid for observed frequencies above the synchrotron frequency \( \nu_{\text{hr}} \). This simplification is necessary to remove unwanted spectral breaks when attempting to study the cause and quality of dynamical jet breaks. This limit corresponds to the case in which the parameter \( \phi \gg 1 \) in Granot et al. (1999). Second, Granot et al. (1999) take the velocity of the emitting electrons behind the shock to be \( \Gamma/\sqrt{2} \), where I have assumed it to be \( \Gamma \). I have tested both choices (the former can be implemented by effectively replacing \( \Gamma \rightarrow \Gamma/\sqrt{2} \) in eq. [14]) and found a quantitative difference in the flux surface, but the difference is minimal for the integrated flux, so the shape of the light curves is not significantly affected by this choice, as can be seen from the results presented in this section. The last difference is the choice of spectral index; herein the value \( \alpha = 1/4 \) is employed, while Granot et al. (1999) use \( \alpha = 2/3 \). Adopting the values used by Granot et al. (1999), I find very good agreement with their results.
Fig. 2.—Views of the flux contour of the afterglow for various points indicated in Fig. 1. The z-axis is the flux magnitude, and the x-y plane is the view plane of the observer, who would be located in the distance to the top left. The flux (z) and spatial (x and y) axes have been normalized to facilitate viewing. The grid lines demarcate the coordinate surface of the afterglow in \((\theta, \phi)\) at a resolution of \(\Delta\theta = 1'' / 50, \Delta\phi = 360'' / 100\). In (a) the shock, which is poorly resolved at this resolution, has not reached the deceleration time (eq. [20]) and thus exhibits a dome-like morphology. After passing the deceleration time, in (b) the flux becomes concave at the center, as was first described by Granot et al. (1999). This concavity is because the on-axis material is observed to evolve more quickly and thus decelerate and cool faster than off-axis material. In general, larger angles from our viewing axis correspond to earlier times in afterglow evolution and thus higher Lorentz factors \(\Gamma\). Physically, the rim of maximum flux around the bowl corresponds to the angle \(1/\Gamma\) because material at larger angles having higher \(\Gamma\) and thus tighter beaming, \(1/\Gamma\), cannot be seen, and material at smaller angles corresponding to later times has decelerated and thus has reduced flux. The rim thus moves outward as the afterglow evolves. Eventually, (c) this rim approaches the physical edge of the jet. As the rim passes the edge of the jet, (d) the light curve breaks. In (e) we see two views of the afterglow at a large viewing angle, \(\theta_0 = 5^\circ\), at a time when most of the flux is just outside of the rim and thus is basically viewed edge-on by the observer. These calculations do not include the thickness of the shock (\(\sim R/10^2\)), and as such one does not expect to observe such a deep notch in a light curve in (e); however, this suggests very high levels of polarization in orphan afterglows. [See the electronic edition of the Journal for a color version of this figure.]
3.1. Lateral Expansion

Thus far the discussion has concerned jets with no lateral (sideways) expansion, \( v_\perp = 0 \). It has been argued that when \( c_b t \gg \theta_0 c t \), i.e., when the Lorentz factor has decreased such that \( 1/\Gamma > \theta_0 \), where \( c_b \sim c \) is the sound speed, the shock will begin expanding sideways (Rhoads 1999; Sari et al. 1999). This increasing angular size of the shock, sweeping up an ever larger region of ISM, is supposed to impede the forward propagation, and the shock begins to decelerate exponentially with radius. It is argued that this shift from forward expansion to lateral expansion will cause an exponential decay in the Lorentz factor as a function of radius, \( \Gamma(R) \propto \exp(-R) \), and thus a break in the afterglow light curve. Thus, one can derive the predicted postbreak light curve slope by assuming that forward expansion essentially ceases, \( R \approx \text{const} \). In addition, if the jet is narrow, then the jet opening angle will expand as \( \theta \propto t \propto 1/\Gamma \) (e.g., Rhoads 1999), and since this physical extent of the jet is growing, one has \( \theta_A \sim 1/\Gamma \) (eq. [16]). Thus, from equation (18) the flux scales as \( F_\nu \propto \Gamma^{4+\alpha/2} \), where \( \delta \sim \Gamma \) for \( \theta_0 = 0 \). In addition, the observer time varies as \( t \propto \Gamma^{-2} \) (eq. [19]). Thus, \( F_\nu \propto \Gamma^{-2(\alpha+4)/2} \sim t^{-2} \) for \( \alpha = \frac{1}{2} \).

However, the assumption that the radial expansion in \( R \) effectively halts as a result of the lateral expansion of \( \theta_0 \) is an extreme limit characterized by a narrow opening angle, \( \theta_0 \ll 1/\gamma \), and a lateral expansion velocity, assumed constant, that eventually exceeds the decelerating forward velocity. Several authors (Wei & Lu 2000; Moderski, Sikora, & Bulik 2000; Panaitescu & Mészáros 1999) have shown that in practice the lateral expansion of the jet does not reproduce the scaling discussed above and cannot create a prominent “sharp” break and instead tends to smooth and broaden the jet break of equation (21). Certainly, the approximation of a narrow opening angle does not hold in general, and we now consider the behavior of \( v_\perp \).

The lateral expansion is a result of internal pressure in the shocked medium that, in turn, is the result of the forward motion of the shock into the ISM. Thus, I argue that a physically motivated model for lateral expansion is to make the rate of lateral expansion a function of the rate of forward expansion. Thus, one does not expect the lateral expansion velocity to exceed the forward motion of the shock, from which it derives its energy. There are two basic reasons for this: First, as the shock progresses into the ISM, it decelerates; thus, the rate of injection of internal energy into the shock from bulk kinetic energy is monotonically decreasing \( \sim \Gamma \). Furthermore, the shock is sweeping up interstellar gas at an increasing rate \( \sim R^2 \). As was pointed out by Moderski et al. (2000), this mass has no inherent lateral velocity component, and thus new mass must be constantly accelerated. This is a significant source of “drag” on the decreasing internal pressure responsible for the expansion. Thus, the assumption that lateral expansion is at all times constant is unphysical.

In this section calculations are presented for afterglows with a simple lateral expansion model, and it is found that lateral expansion serves primarily to smooth the break if it has any effect at all and, perhaps more importantly, provides very little if any steepening of the postbreak light curve.

Each shock element originally moves radially with a Lorentz factor \( \gamma \). The relativistic jump conditions imply the shocked gas is heated by a factor of \( \sim \gamma - 1 \) and is thus driven to expand laterally via its internal pressure. The actual lateral expansion that results is a complex balance between internal pressure forces of the shocked gas and the drag of constantly sweeping up and laterally accelerating fresh ISM. This balance depends intimately on the hydromagnetic nature of collisionless shocks and is not well understood as of yet. Here I simply prescribe that the lateral kinetic energy of the shock, in its radially comoving frame, is a constant proportion of the radial kinetic energy. Thus, I define this ratio:

\[
R_k = \frac{\gamma_\perp - 1}{\gamma - 1} ,
\]

which is a parameter measuring the shock’s efficiency at converting radial kinetic energy, \( \gamma - 1 \), into lateral kinetic energy, \( \gamma_\perp - 1 \), and thus can vary from zero to unity.

Let us define lateral velocity as the velocity of the shock in the radially comoving frame. This paper assumes axisymmetry of the afterglow, so the proper coordinate velocity, 4-velocity, and Lorentz factor in this frame are respectively denoted \( v'_\perp(\theta) \), \( U'_\perp(\theta) \), and \( \gamma'_\perp = (1 - v'_\perp^2(\theta))^{-1/2} \). Let us assume the shock expands uniformly, i.e., \( v_\perp(\theta) \propto \theta \) (see § 4.3 for a relaxation of this condition); thus, noting that \( R_k \) scales as the energy, given a ratio \( R_{k0} \) at the edge of the jet, \( \theta = \theta_0 \), this ratio at all angles \( \theta \) will be determined by

\[
R_k \equiv \left( \frac{\theta}{\theta_0} \right)^2 R_{k0} .
\]

It is a convenient fact that since the radial velocity \( v \) and lateral velocity \( v'_\perp \) are orthogonal in the shock frame, the Lorentz factor of the shock in the lab frame is

\[
\Gamma = \gamma \gamma'_\perp ,
\]

where \( \gamma = (1 - v^2)^{-1/2} \) and the observer frame lateral expansion velocity is then \( v_\perp = v'_\perp/\gamma \). Thus, using equations (22)-(24), a given Lorentz factor \( \Gamma \), and a lateral expansion efficiency \( R_k \), one can determine \( \gamma \) and \( \gamma'_\perp \). The total Lorentz factor \( \Gamma \) is the quantity that is evolved by the dynamical equation (6). The angle made by the velocity vector with respect to the original radial vector of the shock element, at \( \theta_0 \), is

\[
\alpha_0 \equiv \arctan\left( \frac{v'_\perp}{\gamma} \right) .
\]

In other words, we measure the sideways expansion with respect to the original radial vector of the shock element; the shock momentum does not undergo a torque as \( \theta \) expands. The angle between the shock element velocity and the radial vector at \( \theta \) is

\[
\alpha = \theta_0 - \theta + \alpha_0 ,
\]

and the shock position \( \theta \) is incremented with radius \( R \) as

\[
\theta(R) = \theta_0 + \int \alpha \frac{dR}{R} .
\]

There are many conceivable elaborations of this model. For instance, it is likely that the efficiency of conversion of forward motion to lateral motion is dependent on the Lorentz factor and radius: \( R_k(\gamma, R) \).

This model implies a maximum asymptotic value for the opening angle \( \theta \) that is achieved when the shock has
 decelerated to nonrelativistic velocities, at which equations (22) and (25) imply \( \alpha_0 = \arctan(R_k^{1/2}) \), and since \( \alpha \) will become 0 at large radii, then equation (27) gives

\[
\theta_{\text{max}} = \theta_0 + \arctan \left( R_k^{1/2} \right) .
\]

(28)

Thus, as seen in Figures 3 and 10, one does not see exponential growth of the jet opening angle, i.e., \( \theta \not\sim \exp(R) \), and thus the analytically predicted runaway expansions are physically unrealizable.

3.2. Amending the Homogeneous Jet Scaling Laws

An interesting result of these calculations is that the afterglow decay slopes are altered from the theoretical asymptotes both before and after the jet-break time. In this section I describe why this occurs and give amended analytical asymptotic decay slopes. In Figure 3 is shown the same homogeneous, narrow, \( \theta_0 = 1^\circ \) jet as in Figures 1 and 2, but now with a range of lateral expansions. A key feature is that, for \( \theta_0 = 0^\circ \), greater lateral expansion is seen to steepen the light curve prior to the jet break, while not steepening the light curve after the jet break (in fact, this slope becomes less steep). This can also be seen in the sequence of Figures 4–7 for a more “realistic” jet with \( \theta_0 = 5^\circ \). Thus, we calculate behavior opposite to that predicted by Rhoads (1999) and Sari et al. (1999).

To explain this, begin by ignoring Doppler factors, \( \delta \propto \Gamma \); thus, from equation (18) the flux varies as

\[
F_\nu \propto \Gamma^{4(1+\alpha)} R^3 \theta_A^2 ,
\]

(29)

and the time (eq. [19])

\[
t \propto \frac{R}{\Gamma^2} .
\]

(30)

Now we parameterize the Lorentz factor dependence as

\[
\Gamma \propto R^{3 \alpha/2} ,
\]

(31)

where \( \alpha \) is unity for the case of no lateral expansion and is estimated otherwise. These equations with equation (16) imply

\[
F_\nu \propto \left\{ \begin{array}{ll}
\Gamma^{- (1+2 \alpha-1/\alpha)}/[1+1/(3 \alpha)] , & \text{pre-jet-break : } \theta_A \propto \frac{1}{\Gamma} , \\
\Gamma^{- (2+1+\alpha-1/\alpha)}/[1+1/(3 \alpha)] , & \text{post-jet-break : } \theta_A \propto \theta_0 .
\end{array} \right.
\]

(32)

In order to estimate the parameter \( \alpha \), note from equation (2) that

\[
f(R) \propto \int [\theta R] \, dR
\]

(33)

FIG. 3—Three different lateral expansion rates for an afterglow jet of initial opening angle \( \theta_0 = 1^\circ \), initial Lorentz factor \( \gamma_0 = 1000 \), and isotropic equivalent energy of \( 10^{52} \) ergs. These are \( R_k = 0 \) (no lateral expansion), \( R_k = 0.01 \), which corresponds to \( \gamma'_0 = 3.7 \), and \( R_k = 0.1 \), or \( \gamma'_0 = 10.5 \). Top: Light curves as seen on-axis, \( \theta_0 = 0^\circ \). Middle: Light curves viewed at \( \theta_0 = 2^\circ \). Lateral expansion Doppler boosts the flux into larger angles \( \theta \) from the jet axis, and thus on-axis observers see a reduction in flux (top curve), while off-axis observers see a surplus of flux (middle curve). Bottom: Evolution of the jet opening angle \( \theta_0 \) with observer time. It is important to note that none of these sideways expansion rates show marked steepening of the light curve after the jet-break time \( t_j \), as has been predicted analytically (see §§ 3.1 and 3.2). This is because the growth of \( \theta_0 \) with time (bottom curve), while rapidly accelerating throughout the evolution, does not become exponential, as analytic arguments would indicate. [See the electronic edition of the Journal for a color version of this figure.]

FIG. 4—R-band light curves (top), polarization curves (middle), and power-law index (bottom) for a homogeneous jet with opening angle \( \theta_0 = 5^\circ \), \( \delta = (10^{52} \text{ ergs})/4\pi \), and \( \Gamma_0 = 100 \). The polarization curves closely match those of Ghisellini & Lazzati (1999, Fig. 4). The first peak of the double-peaked polarization curves indicates the “near” edge of the jet coming into view, \( 1/\Gamma > \theta_0 - \theta_* \), and the second peak corresponds to the “far” edge coming into view, \( 1/\Gamma > \theta_0 + \theta_* \). Therefore, comparison of the three panels shows an initial steepening of the light curve for a given viewing angle \( \theta_* \), corresponding to the first polarization peak, and additional steepening occurring at the second peak. [See the electronic edition of the Journal for a color version of this figure.]
and from equations (27) and (25) that

$$\theta(R) \approx \theta_0 + \int \frac{dR}{\gamma R}.$$  \hspace{1cm} (34)

A crucial point here is that the lab-frame angle between the velocity and radial vectors (eq. [25]) $\alpha_0 \propto 1/\gamma$ and not $1/\Gamma$. The reason is that maximal sideways expansion occurs when the shock-frame lateral expansion energy $\gamma'_{\perp}$ is roughly the same as the radial expansion energy $\gamma$. From equation (24) this implies that the total specific energy of the shock $\Gamma \sim \gamma'_{\perp}$. Therefore, if an expansion angle of $\alpha_0 \sim 1/\Gamma$ is desired, this implies $\gamma'_{\perp} \sim \Gamma$, and the total specific energy of the shock is $\sim \Gamma^2$, which is energetically untenable. The latter, erroneous choice of $\alpha_0$ scaling as $1/\Gamma$ results in the standard exponential decay of $\Gamma$ with $R$ that was discussed in the first paragraph of $\S$ 3.1. From equation (6)

$$\Gamma \approx \frac{\Gamma_0}{2f},$$ \hspace{1cm} (35)

so $1/\gamma \propto 1/\sqrt{\Gamma} \propto f^{1/4}$. Thus, one can solve equations (33) and (34) for the asymptotic behaviors. In the limit of no lateral expansion, $\theta(R) \rightarrow \theta_0$, and so equation (33) becomes $f \propto R^2$; thus $\Gamma \propto R^{-3/2}$, and so $\alpha = 1$, as expected. For the limiting case in which lateral expansion dominates, equation (34) gives $\theta(R) \rightarrow \int \frac{dR}{\gamma R} / R \propto \int f^{1/4} dR / R$, and equation (33) becomes $f \propto R^2$; thus $\Gamma \propto R^{-3}$, and so $\alpha = 2$. Therefore, from equation (32), for $\alpha = 1/2$ the prebreak light curve is steepened from $F_\nu \propto t^{-5/4}$ for negligible lateral expansion to $F_\nu \propto t^{-4/7}$ for maximal lateral expansion. This range nicely brackets the prebreak decays of
simulated light curves over a range of lateral expansion rates as seen in the bottom panels of Figures 4–7. It is also worth noting that inspection of Figures 7–9 indicates that, not surprisingly, Doppler effects become important for large lateral expansions. Such effects were neglected in the derivation of these scaling laws.

The postbreak afterglow decays less steeply than analytical models predicted. This is because, as discussed in § 3.1, a realistic afterglow jet will expand laterally only in proportion to its forward expansion; thus, the lateral expansion rate decreases at late times (see Fig. 10), and the postbreak decay curve will asymptotically move toward the limit of no lateral expansion: $A = 1$.

To summarize, I have implemented a simple model for jet dynamics in which the lateral expansion of the jet is an effect of forward expansion. I argue that this basic cause/effect relationship is a necessary component of any model for jet dynamics. Resulting jet simulations show that lateral expansion (1) smooths the break in the jet light curve, (2) steepens the prebreak light curve, but does not significantly steepen the postbreak slope of the light curve, and (3) can significantly alter the shape of a light curve. Thus, I suggest that the oft-cited scaling laws described at the beginning of this section do not accurately represent the evolution of a laterally expanding jet. To properly diagnose light-curve slopes and breaks, simulations such as those shown here are necessary. A preliminary conclusion that can be drawn from the simulations shown in this section is that large efficiencies in conversion of forward kinetic energy to lateral kinetic energy, i.e., $R_{ke} \gtrsim 0.1$, are probably not physical in the

Fig. 8.—Flux contours for selected points of Fig. 7 for $\theta_0/\theta_b = 0.67$. See Fig. 3 for a description of the axes. Panels (a) and (b) show the flux rim (e.g., Fig. 3b) to be heavily skewed and distorted because of the Doppler factor of the sideways expansion; material near the core of the jet is seen to be brighter. In (b) the rim expands beyond the pole of the jet. Panel (c) shows the rim becoming less distorted as the jet decelerates, but still sloped, and the edge of the jet is visible. Panel (d) shows the rim to be very flat and symmetrical, since the jet has decelerated substantially; thus, sideways expansion and resultant aberration has subsided. Grid resolutions are $\Delta \theta = 5'' / 150$ and $5'' / 50$ for panels (a) and (b), and (c) and (d), respectively, with $\Delta \phi = 360'' / 90$. [See the electronic edition of the Journal for a color version of this figure.]
context of the homogeneous jet model, because the light curves they produce bear little resemblance to observed light curves. In this way, one can begin to constrain the physics of afterglow shocks with observations.

4. THE STRUCTURED, UNIVERSAL JET

The primary purpose of the numerical framework discussed thus far is to quantitatively study the afterglow light curves from structured jets, i.e., with a nonuniform energy or velocity distribution. As suggested by Rossi et al. (2002) and Zhang & Mészáros (2002b), an asymptotic decrease in energy per solid angle as $E/\theta^2$ can reproduce the observed relation $E/\theta$ (Frail et al. 2001). Furthermore, allowing $\delta \sim \text{const}$ for angles within a core $\theta < \theta_c$ provides a natural explanation for the dearth of small-angle jets, less than $3^\circ$, also reported by Frail et al. (2001). As such, I define an Ansatz energy profile

$$\delta(\theta) = \frac{\delta_{52}}{1 + (\theta/\theta_c)^2} \frac{10^{52}}{4\pi} \text{ ergs sr}^{-1}.$$  

How does one choose an initial $\Gamma_0(\theta)$? Based on simulations by Zhang et al. (2003), $\Gamma_0 \sim \text{const}$, and so this choice is used for the structured jet runs in this paper. However, it is important to realize, and the following simulations confirm this, that the initial $\Gamma_0$ determines only the deceleration time of the shock, but subsequent evolution is independent of it. To see this, noting that the global energy has dependence $\delta \propto \Gamma^2 R$ from equations (1), then for any radius greater than the deceleration radius, $R > R_d$, the Lorentz factor is determined by $\Gamma \propto \delta^{1/2}$, independent of the initial $\Gamma_0$.

In Figure 11 is shown a series of light curves from a universal structured jet with isotropic equivalent energy.
Fig. 10.—For the homogeneous jet model, the evolution of the jet specific kinetic energy $\Gamma - 1$ and opening angle $\theta_0$ for four jet expansion parameters $R_0$ (eq. [22]). The kinetic energy (top) does decrease more rapidly with radius $R$ for large lateral expansion parameters but does not become exponential in $R$ as argued by Rhoads (1999). Instead, it asymptotically steepens to the nonrelativistic limit, $\Gamma - 1 \propto R^{-\delta}$. For this reason, the sequence of Figs. 4–7 does not exhibit a progressively more pronounced break because of lateral broadening of the jet. The opening angle (bottom) is seen to expand rapidly once the jet decelerates to roughly $\theta_0 \sim 1/\Gamma_0$, as expected, but because lateral expansion depends on forward expansion, it slows down to a maximum final opening angle $\theta_{\text{max}} = \theta_0 + \arctan(R_0^{1.8})$. Since these simulations commence at $R = 1$ lt-day (prior to deceleration; $\Gamma \approx \text{const}$), one can see some early spreading, $\theta_0 \sim \int (v_{\parallel}/R) dt \sim \log(R)$. While this particular behavior is an artifact of starting the simulation at $R = 1$ lt-day, it does indicate that strong lateral spreading can significantly rearrange the initial jet morphology before it begins to decelerate, thus hinting at the possibility of a universal jet shape independent of initial conditions. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 11.—Light curves for a structured jet with isotropic equivalent energy $10^{53}$ ergs, $\delta_{E} = 10$, using eq. (36), and initial Lorentz factor $\Gamma_0 = 100$ everywhere. One sees corroboration of the basic thesis of Rossi et al. (2002), with some flattening at larger viewing angles. [See the electronic edition of the Journal for a color version of this figure.]

There can be seen a gradual flattening, and eventual appearance of a bump, in the light curves of Figure 11 with increasing viewing angle $\theta_v$. This can be understood when one considers that the early phase of the light curve, i.e., $\Gamma > 1/\theta_v$, is dominated by emission primarily along the LOS with energy $\delta \propto (\theta_v/\theta_i)^{-2}$, while the flux at the break is dominated by the energetic core moving at angle $\theta_i$ with respect to the observer. Here I show that these two components have distinct laws for their breaks, and this difference between these two laws makes each component distinct for large viewing angles $\theta_v$, thus creating a bump.

The LOS component can be modeled as a homogeneous jet with opening angle $\theta_i$. As such, one expects a break when $\Gamma \sim \delta \sim 1/\theta_A \sim 1/\theta_i$, so $F_v,\text{LOS} \propto \Gamma^{4q/3} \delta^{-1} \theta_i^{-(4q+7)/3}$. Furthermore, $t_{\text{LOS}} \propto \delta^{1/3} \Gamma^{-8/3} \propto \theta_i^{3/2}$. Therefore,

$$F_v,\text{LOS} \propto t_{\text{LOS}}^{1/2(4q+7)} \propto \theta_i^{3/2},$$

(40)

where $\alpha = \frac{1}{2}$ and $q = 2$. The jet core component can be modeled as a narrow jet seen far off-axis; $\theta_i \gg \theta_A$, where $\theta_A = \theta_i$. One expects a maximum flux when $\Gamma \sim 1/\theta_i$; thus, $F_v,\text{core} \propto \Gamma^{4q+7} \delta \propto \theta_i^{-(4q+7)}$, where $\delta = 1$ for the core. It follows that $t_{\text{core}} \propto \delta^{1/3} \Gamma^{-8/3} \propto \theta_i^{3/2}$. Thus, the core is brightest at

$$F_v,\text{core} \propto t_{\text{core}}^{3/8(4q+7)} \propto \theta_i^{3/2},$$

(41)

for $\alpha = \frac{1}{2}$ and $q = 2$. This relation traces the jet break in the light curve $F_v(t)$ over a range of viewing angles. The different decay laws for equations (40) and (41) demonstrate why both components of a light curve from a structured jet will
dive for large break times. Seen another way,

$$
\frac{t_{\text{core}}}{t_{\text{LOS}}} \propto \theta_c^{\gamma/3};
$$

thus, the contribution to the light curve from the core happens progressively later than that of the LOS material.

### 4.1. The “Bump”

As discussed above, under certain conditions a “bump” can appear in the light curve at the jet-break time. This typically happens at large viewing angles and small jet cores. For example, Figure 12 shows three light curves from identical afterglows, except for variation in the size of the jet core, \( \theta_c \), demonstrating that a narrower core produces a more prominent bump. This bump can be explained by considering how the flux varies as the observer sees an ever-increasing \((\sim 1/\gamma)\) angular region of a surface with varying energy density. It can be shown that the observed flux varies as \( F \propto \gamma^p \theta_c^q \), where \( p = 2 \) for \( 1/\gamma < \theta_0 \) (isotropic) and \( p = 4 \) otherwise (jetlike). In addition, the energy per steradian of the afterglow scales as \( E \propto (\theta_c/\theta)^q \), where in this case \( q = 2 \). Thus, the flux will vary as

$$
F(\frac{1}{\gamma}) \propto (\frac{1}{\gamma})^{-p} \left( \frac{\theta_c}{\theta} \right) \left( \frac{\theta_c}{\theta - 1/\gamma} \right)^{q-1},
$$

so that early in the evolution, when \( 1/\gamma \ll \theta \), the flux is down by \( (\theta_c/\theta)^q \), as expected, but when the beaming angle has expanded to reach the core, i.e., \( 1/\gamma = \theta - \theta_c \), the flux averaged over the surface will be \( \propto (\theta_c/\theta)^q \). The flux of equation (43) decreases as \( \gamma \) decreases until \( 1/\gamma = p/(p+q-1)\theta \), so the smaller \( q/p \) is, the farther out the bump will occur. The condition for no bump to be observed is when the jet core becomes visible, i.e., \( 1/\gamma = \theta - \theta_c \), and the flux has not yet begun to increase; thus,

$$
\theta_c > \frac{q-1}{p+q-1} \theta.
$$

Therefore, for \( p = 2 \) and \( q = 2 \) we have \( \theta_c > \theta/3 \), and so a bump will be visible if the jet is viewed at angles \( \theta \) much in excess of \( 3\theta_c \). This analysis is born out in the simulations and gives constraints on the range of allowed viewing angles \( \theta \) with respect to the size of the jet core \( \theta_c \) and the steepness of the jet decay structure \( q \).

There are several examples of bumps in light curves, including GRB 970228, GRB 970508, and GRB 980326 (for a discussion, see Zhang & Mészáros 2002a). While in general GRB afterglows do not exhibit bumps in their light curves just prior to the break, GRB 000301C exhibited a prominent, achromatic bump at the jet-break time that has been interpreted as a gravitational lensing event (Garnavich, Loeb, & Stanek 2000) and alternatively as continuous energy injection by a millisecond pulsar (Zhang & Mészáros 2001). In light of the present calculations, it is possible that the bump in the light curve of GRB 000301C was due to a simple perspective effect onto a jet with a narrow core or a steep decay curve. This explanation is appealing in that it does not require external mediators (i.e., a lensing body or a pulsar) to create it. If the bump in GRB 000301C and possibly those of other burst light curves could be positively attributed to perspective onto a structured afterglow, key information could be determined about the size (i.e., the jet core \( \theta_c \)) and shape (the decay structure \( q \)) of the afterglow.
Fig. 14.—Flux contours for selected points of Fig. 13 for $\theta_0 = 12^\circ$. At early times (a), the flux surface is distorted from a symmetric bowl (see Fig. 1, point b) by both lateral expansion, as in Fig. 8a, and the intrinsic structure of the jet. Near the jet-break time, panels (b1)-(b3) show different views of the flux surface. Panel (b3) shows the physical pole of the afterglow nearly coincident with the flux peak. This demonstrates the origin of the break in the light curve in the structured jet model: the coincidence of the peak flux rim and the actual pole of the jet. As such, the edge of the jet does not play a role in the jet break as it does in the homogeneous jets. Panels (c) and (d) show this coincidence clearly. Grid resolutions are $\Delta \theta = 30^\circ/300$, $\Delta \phi = 360^\circ/360$ for panel (a) and $\Delta \theta = 30^\circ/100$, $\Delta \phi = 360^\circ/60$ for panels (b), (c), and (d). [See the electronic edition of the Journal for a color version of this figure.]
4.2. Uniform Lateral Expansion

Having constructed a structured jet model, we are interested in the effects of lateral expansion on said model. Since the evolution of the afterglow shock is uncertain, I choose to study two basic paradigms: First, in this section I simply apply uniform expansion to the afterglow, as was done for the homogeneous jet in § 4.1. Thus, expansion is governed by equation (23). The second model, discussed in § 4.3, employs nonuniform expansion, in which the hotter core expands faster than the cooler wings of the afterglow.

Examination of the progression of increasing lateral expansions in Figures 11 and 13–15 shows that lateral expansion suppresses the bump at the jet-break time viewed at large $\theta_c$. However, this also makes the breaks at small $\theta_c$ less pronounced. This behavior, in which the jet break is more pronounced at high viewing angles $\theta_c$ than at small ones, is quite general. Even a nonuniform expansion paradigm discussed in § 4.3 demonstrates this.

4.3. Nonuniform Lateral Expansion

Uniform expansion of the structured jet is likely oversimplified. A more accurate sideways expansion prescription should encapsulate one’s hydrodynamic intuition that a fluid element will be accelerated proportionally to the gradient of its internal energy density. A simple model with this characteristic is

$$R_k(\theta) = R_{k0} \left[1 - \frac{\delta(\theta)}{\delta_{S2}}\right] = R_{k0} \left(\frac{\theta_c}{\theta_c}\right)^2 ,$$

using equation (36) and where $\theta_c$ is not constant but allowed to laterally expand with the jet by staying assigned to a particular fluid element. Note that equation (45) replicates the uniform expansion of equation (23) for small angles $\theta \ll \theta_c$ but transitions to a constant, rigid-body expansion for large angles. Figures 16–18 demonstrate that nonuniform...
Fig. 18.—Selected flux surfaces for a structured jet with a nonuniform expansion paradigm at $\theta_e = 12^\circ$. Grid resolutions are $\Delta \theta = 30^\circ/100$ and $\Delta \phi = 360^\circ/120$. Prior to the break (a), the morphology is similar to the uniform expansion (Fig. 14). The hot core of the jet expands and decelerates more rapidly than the wings. Thus, a crater appears as the core comes into view in panels (b)–(e). This sharp flux deficit makes a sharp jet break (Fig. 17). [See the electronic edition of the Journal for a color version of this figure.]
expansion can be very effective at creating a very sharp break in the light curve. A key approximation of equation (45) is that it assumes a fixed functional form for \( R_k \), even as the shock surface evolves. This represents a model in which the proper hydrodynamic timescale, which increases with time, becomes longer than the deceleration timescale; thus, the early-time expansion morphology is “frozen in” and predominantly determines the subsequent evolution. Further work is required to determine the hydrodynamic evolution of the afterglow.

5. DISCUSSION

I have implemented a simple model for afterglow jet spreading in which lateral expansion depends on energy from the forward expansion. This model demonstrates that a homogeneous jet does not exhibit a lateral-expansion-dominated phase, as described by analytical arguments (Rhoads 1999; Sari et al. 1999). In particular, dynamically

\[
\gamma \sim \exp(-R)
\]

(Fig. 10), and observationally

\[
F_\nu \sim t^{-p}
\]

after the break (Figs. 5–7). The only source of a break in this model is the observation of the physical edge of the jet. As shown in Figures 5–7, this break is “sharpened” by lateral expansion in that it becomes less dependent on viewing angle.

In addition, I have studied structured jets as seen at various viewing angles and with various sideways expansions. This study confirms the key results of Rossi et al. (2002) and Zhang & Mészáros (2002b): \( t_j \sim \theta_{\text{core}}^{1/3} \) and \( E_{\text{iso}} \sim t_j^{-1} \). Furthermore, the jet break is seen to be sharp and can exhibit some flattening and even a bump prior to the break at large viewing angles compared to the core size, \( \theta_\text{v} \gtrsim \theta_\text{core} \). It appears to be a general feature of the universal jet model that late-time jet breaks (i.e., \( \theta_\text{v} \)) are more pronounced than early-time breaks. That is, if early-time breaks are sharp, then late-time breaks will tend to have a flattening or a bump (e.g., Fig. 13), but if this bump is quenched, perhaps by nonuniform spreading of the jet, the early-time breaks become weak or are washed out altogether (e.g., Fig. 17). In general, the sharpness of the structured jet break is determined by the size of the core, \( t \propto \theta_\text{core}^{1/3} \), and is thus intrinsically sharper than the break in the homogeneous jet model, which depends on the outside edge \( t \propto \theta_0^{1/3} \). As such, the structured jet can explain sharper breaks than the homogeneous jet model.

Future work with E. Rossi et al. will focus on polarization (e.g., Fig. 4) as a tool to discriminate between the homogeneous and structured jet paradigms. In addition, improved understanding of the evolution of the jet, whether it be hydrodynamic or otherwise, will allow for more quantitative expansion models and more predictive simulations.

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