Axions and the Graceful Exit Problem in String Cosmology

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Abstract

We reexamine the graceful exit problem in the Pre-Big-Bang inflationary scenario. The dilaton-gravity action is generalized by adding the axion and a general axion/dilaton potential. We provide a phase space analysis of the dynamics which leads us to extend the previous no-go theorem and rule out the branch change necessary for graceful exit in this context.

Submitted to Physics Letters B
1 Introduction

The standard cosmological model provides a convincing and consistent picture of the universe back to the period of primordial nucleosynthesis. When extrapolated further into the past, however, it reveals a need for a very special set of initial conditions, both finely tuned (the flatness problem) and coherent over acausal scales (the horizon problem). Together with other well known problems and gaps, not the least of which is that the model predicts its own demise by necessitating a past singularity \cite{1}, these problems justify the search for mechanisms in the early universe which can erase deviations from special initial conditions, providing a dynamical solution to the fine tuning. The inflationary paradigm \cite{2} does just this despite the fact that there is no consensus on a concrete mechanism.

Since string theory \cite{3} is the only current candidate for a theory capable of uniting gravity with the other forces of nature, there have been many investigations of the effects of stringy dynamics on cosmological evolution \cite{4}-\cite{23}. In particular, the massless modes (dilaton, axion and graviton) of the string can develop classical fields whose couplings are fixed by the requirement of conformal invariance, leading to the low energy effective action for these fields.

In addition to providing new evolutionary possibilities, solutions to these equations have a larger symmetry group \cite{21}-\cite{23} including scale-factor duality \((r(t) \rightarrow 1/r(t))\), where \(r(t)\) is the scale factor). Combining this with time reversal symmetry, one can find that corresponding to an era of decelerated expansion defined for positive time there is another solution representing accelerated expansion (inflation) defined for negative times. Assuming these could be connected, one would obtain a cosmological solution describing a slowly expanding universe which accelerates into a period of rapid expansion, identified as the ‘big bang’, and then settles into a standard Friedmann-Robertson-Walker cosmology. This is the ‘pre-big-bang’ scenario \cite{12,17}.

To promote this idea into a realistic model we are still lacking two essential components. First, the stringy fields must be tamed in the post-big bang universe. In particular, the dilaton must be decoupled since variations in the dilaton field correspond to changes in masses and coupling constants, which are strongly constrained by observation \cite{15,24}. This can be accomplished by including dilaton self-interaction potentials and trapping the dilaton in a potential minimum. While the introduction of a general potential will destroy exact
scale factor duality symmetry we can expect the asymptotic forms of the solutions to remain similar in regions where the potential tends to a constant value. Second, we need to check if the symmetry related branches can be linked. Previously we followed a suggestion [17] that the potential itself could catalyze this branch changing, though in [17] the authors in fact noted the difficulty of achieving such a branch change and claimed a hard-to-go theorem obstructing the change. In [19], we confirmed their negative conclusions by proving an exact no-go theorem for the correct form of branch changing by a dilaton potential, even with a stringy fluid or higher genus terms as suggested in [15].

Recently there have been some speculations about the fate of the no-go theorem when axions, with or without self interactions, are present. Namely, it is known that the axion terms could affect dynamics of collapsing universes, and actually overturn collapse into expansion [18], eventually asymptotically linking separate solutions of the axionless theory. Thus there may have been hope that, with the inclusion of the axion interactions, a similar link-up between expanding solutions belonging to different branches might occur. This turns out not to be the case. In this letter we extend the dilatonic no-go result by including the axion and an arbitrary dilaton/axion potential and showing that one cannot connect solutions well-behaved in the past with those well-behaved in the future. The proof is a direct analog of the proof for the case of a potential depending only on the dilaton.

2 The Gravitational Action

There is a considerable amount of literature concerning the tree level gravitational action in string theory and its expansion in the string tension $\alpha' [25]$. In what follows we will ignore the corrections of the derivative expansion of order $O(\alpha')$ and higher. Though these terms would be expected to be important near the singularity, we will neglect them for the reasons of simplicity and consistency (these terms would involve higher derivative corrections in the equations of motion, including spurious solutions which must be non-perturbative in $\alpha'$ by dimensional analysis [26]). Retaining the conventions of [19], in the string world-sheet frame the tree-level action becomes:

$$S = \int d^4x \sqrt{g} e^{-2\phi} \left\{ R + 4(\nabla \phi)^2 - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} \right\}$$

(1)

where the 3-form $H = dB$ is the field strength of the Kalb-Ramond 2-form $B_{\mu\nu}$. In four dimensions, this 3-form is dynamically dual to a pseudoscalar axion field. The correspondence
is given through the Hodge duality to a vector $Y$

$$H_{\mu\nu\lambda} = \sqrt{g} \epsilon_{\mu\nu\lambda\rho} Y^\rho$$  \hspace{1cm} (2)

which has to be constrained further in order to satisfy the Bianchi identity $dH = 0$. This is ensured by adding a Lagrange multiplier term of the form $a(dH)$ to the action, and integrating out the 3-form $H$ and the 1-form $Y$, in favor of the pseudoscalar degree of freedom $a$. The final correspondence is given by

$$H_{\mu\nu\lambda} = \sqrt{2} e^{2\phi} \sqrt{g} \epsilon_{\mu\nu\lambda\rho} \partial^\rho a$$  \hspace{1cm} (3)

resulting in the replacement of the 3-form kinetic term coupled to the inverse string coupling $e^{-2\phi}$ by the pseudoscalar kinetic term coupled to the dual string coupling.

Finally, motivated by the idea that a potential could simulate generic properties of branch changing, we can introduce a potential for the dilaton and axion fields, coming from the loop expansion of the effective action (we will not speculate as to the origin of this potential and leave it completely general). This yields the following action:

$$S = \int d^4x \sqrt{g} e^{-2\phi} \left\{ R + 4(\nabla \phi)^2 - e^{4\phi} (\nabla a)^2 - 2\Lambda(\phi, a) \right\}$$  \hspace{1cm} (4)

We shall be working with the FRW metric in the string frame,

$$ds^2 = -n(t)^2 dt^2 + r(t)^2 d\bar{x}^2$$  \hspace{1cm} (5)

where $d\bar{x}^2$ is the three dimensional volume element for a space of constant curvature $k$. We will concentrate on the case of $k = 0$. To be consistent with this form of the metric we will also assume that the dilaton and axion field are functions only of time. To obtain the equations of motion we can work directly with the action. Expressing the action in terms of the functions $r(t)$ and $n(t)$ we can Kaluza-Klein reduce it to a one-dimensional problem in constrained mechanics, and look at the variations of the resulting action with respect to the functions $n(t)$, $a(t)$, $\phi(t)$ and $r(t)$, finally setting $n(t) = 1$. The equations of motion which arise from these variations are respectively

$$\Lambda + 6\dot{\phi}h - 3h^2 - 2\dot{\phi}^2 + \frac{1}{2} e^{4\phi} \dot{a}^2 = 0$$  \hspace{1cm} (6)

$$\ddot{a} + 3h \dot{a} + 2\dot{\phi} \dot{a} + e^{-4\phi} \frac{\partial \Lambda}{\partial a} = 0$$  \hspace{1cm} (7)
\[ 4(\ddot{\phi} - \dot{\phi}^2 + 3h\dot{\phi}) - 6(\dot{h} + 2h^2) + 2\Lambda - \frac{\partial \Lambda}{\partial \phi} + e^{4\phi} \dot{a}^2 = 0 \] (8)

\[ 4(\ddot{\phi} - \dot{\phi}^2 + 2h\dot{\phi}) - 4\dot{h} - 6h^2 - e^{4\phi} \dot{a}^2 + 2\Lambda = 0 \] (9)

where \( h = \dot{r}/r \). Multiplying the axion equation of motion (7) by \( \dot{a} \) we can recast it in terms of the variable \( \rho = e^{4\phi} \dot{a}^2 \). By further noting that

\[ \frac{\partial \Lambda}{\partial a} \dot{\phi} = \dot{\Lambda} - \frac{\partial \Lambda}{\partial \phi} \dot{\phi} \] (10)

we can rewrite this equation as:

\[ \dot{\rho} + 6h\rho + 2\dot{\Lambda} - 2\frac{\partial \Lambda}{\partial \phi} \dot{\phi} = 0 \] (11)

Manipulating (8) and (9) to eliminate \( \ddot{\phi} \) and \( \dot{\phi}^2 \) and retaining the constraint equation (6) we finally obtain

\[ \dot{h} = 2h\dot{\phi} - 3h^2 + \rho - \frac{(\partial \Lambda/\partial \phi)^2}{2} \] (12)

\[ 0 = \dot{\rho} + 6h\rho + 2\dot{\Lambda} - 2\frac{\partial \Lambda}{\partial \phi} \dot{\phi} \] (13)

\[ 0 = 2\dot{\phi}^2 + 3h^2 - 6h\dot{\phi} - \Lambda - \rho/2 \] (14)

which is the set of equations of motion we wish to analyze.

### 3 Branch Changing and the Graceful Exit

To investigate the dynamics of the trajectories we consider motion in the four dimensional phase space of the variables \( h, \phi, a \) and \( \rho \). The constraint equation (14) can be solved for \( \dot{\phi} \)

\[ \dot{\phi} = \frac{3h \pm \sqrt{3h^2 + 2\Lambda + \rho}}{2} \] (15)

To be consistent with previous work [17, 19] we should designate the trajectories having the plus/minus sign in the above quadratic as (+)/(−) branch trajectories. We will also need the egg function

\[ e = \sqrt{3h^2 + 2\Lambda + \rho} \] (16)
where we note that trajectories can switch branches only when $e = 0$, because only here $\dot{\phi}$ is identical on both branches. (In fact, branch changing must generally occur here to keep higher derivatives continuous). The region where $e \leq 0$ is referred to as the “egg”. Clearly an egg can occur only where $\Lambda \leq 0$.

By substituting (13) into (12) we get the evolution equation for $h$:

$$\dot{h} = \pm h\sqrt{3h^2 + 2\Lambda + \rho + \rho - (\partial \Lambda / \partial \phi) / 2}$$

(17)

We can now see that the (+) branch solutions are susceptible to runaway expansion when $h$ dominates this equation whereas the (−) branches are stable against this type of future behavior. This in turn indicates that a (+) will tend to evolve out of a well-behaved past into a future singularity and the reverse for a (−) branch. Thus a (+) into (−) transition, if dynamically allowed, could be a globally nonsingular cosmology as indicated in the introduction, having a region of very large curvature mimicking the Big Bang, and possibly an inflationary phase, in the neighborhood of this transition.

Now, as we have shown before, in the pure dilaton-metric system such transitions are impossible. However, referring to (13), we see that in the simple case of an axion-independent potential, $\rho \propto r^{-6}$. This in fact means that a non-zero axion can prevent the universe from a singular collapse, as shown in [18]. However, the free axion (note the tentative use of the label “free” here, meaning axion-independent potential; the axion still couples exponentially to the dilaton) redshifts away rapidly in an expanding phase, as can be seen from comparing the $\rho$ v.s. $r$ dependence to other sources. More detailed studies of the cosmological effects of the free axion have been carried out [4, 4, 10, 18]. Very recent work [20] studied the special case of a constant potential (central charge) and included the dilaton, the axion and the spatial curvature. These authors find that generic solutions remain singular; while there are some special solutions having an infinite lifetime, they observe that these evolve into the strong coupling region ($\phi \to \infty$) where the theory is expected to break down, concluding that no graceful exit can arise as a result of a free axion. In order to complete the check of the axion’s (in)ability to produce graceful exit, it is of interest to include an axion-dependent potential and see if it can yield to interpolation between well behaved branches, evolving to and from the weak coupling regime.

Let us therefore consider the possibility that a (+) branch solution can evolve toward the region of the egg, bounce off the egg becoming a (−), and evolve away from the egg.
Consider the quantity
\[ \dot{e} = \frac{6h\dot{h} + 2\dot{\Lambda} + \rho}{2e} \] (18)
Substituting the equations of motion, we can rewrite this equation as
\[ \pm \dot{e} = 3h^2 + \frac{1}{2} \frac{\partial \Lambda}{\partial \phi} = 2h\dot{\phi} - \dot{h} + \rho \] (19)
with the sign chosen according to the branch. Integrating this between two times \( t_0 < t_1 \) along a trajectory remaining on a single branch yields
\[ \pm (e(t_1) - e(t_0)) + h(t_1) - h(t_0) = 2 \int_{\phi(t_0)}^{\phi(t_1)} h d\phi + \int_{t_0}^{t_1} \rho dt \] (20)
This equation represents one of the trajectory equations in implicit (integral) form. It can be thought of as a constraint on trajectories, and will be the key to the proof of the no-go theorem for (+) to (−) branch changing using the egg, much like in the axionless case [19].

Next we need to determine the direction of \( \phi \) flow around the egg. To visualize the phase space, one can think of the \( h \) axis as a ‘vertical’, all the other variables lying in a ‘horizontal’ hyperplane. Then for a point in the phase space to be ‘above’ or ‘below’ the egg (lying on a line parallel to the \( h \) axis and intersecting the egg), we need \( 2\Lambda + \rho \leq 0 \), as we can see from (16). But solving (15) for the condition \( \dot{\phi} = 0 \) shows that \( 6h^2 = 2\Lambda + \rho \), implying that \( 2\Lambda + \rho \geq 0 \). Combining these two statements, we see that trajectories cannot reverse the direction of \( \phi \) flow above or below the egg. Thus the value of the dilaton \( \phi \) is increasing for the trajectories above the egg (where \( h > 0 \)) and decreasing for the trajectories below it (where \( h < 0 \)). This leads to the conclusion that the contribution of the first integral on the rhs of equation (20) is positive for such trajectories, as it represents the area of the projection of a trajectory on the \( h - \phi \) plane. The second integral is also positive by the definition of \( \rho \).

Finally, to complete the no-go theorem, we establish the following two lemmas. First, we show that no (−) branch trajectory leaving the top surface of the egg can pass the boundary of the region vertically above and below the egg (defined by the condition that \( 2\Lambda + \rho = 0 \)). To see this, let \( t_0 \) be the instant when the trajectory leaves the egg and \( t_1 \) the instant when it reaches the vertical boundary. Clearly \( e(t_0) = 0 \), and since the vertical boundary of the egg region is defined by \( 2\Lambda + \rho = 0 \), we have \( e(t_1) = \sqrt{3}h(t_1) \). Inserting this into the lhs of
the (-) branch version of (20), we find

\[ \text{lhs} = -\sqrt{3}h(t_1) + h(t_1) - h(t_0) \]  

(21)

This is clearly nonpositive (recall \( h \geq 0 \) since we are above the egg), contradicting our conclusion that the rhs is positive. An analogous argument shows that a (+) trajectory passing the boundary and entering the region below the egg cannot hit it. The result is in fact just the time-reversal of the previous statement.

From these two lemmas we can conclude that egg is *generically* incapable of changing (+) branch solutions to (-). A (+) trajectory entering above the egg must leave the region of the egg as a (+), from the first lemma, which prohibits the conversion to a (-). Further, a (+) trajectory entering below the egg must also leave as (+), since it cannot hit the egg at all. We qualify this incapability as generic since there is still a finely tuned set of trajectories that may evade the proof, namely those that touch the egg neither on the top surface nor the bottom, but at the egg boundary in the \( h = 0 \) hyperplane. Thus there remains the possibility that a (+) trajectory could hit one of these boundary points, convert to a (-) and then either leave the egg region or pass underneath the egg.

To see that this cannot happen, consider the quantity (19) at a point touching the egg in the \( h = 0 \) hyperplane, \( \pm \dot{e} = \frac{1}{2} \frac{\partial \Lambda}{\partial \phi} \). Because \( e \) provides a measure of distance to the egg (\( e \) is negative inside the egg and positive outside), we see that points where \( \frac{\partial \Lambda}{\partial \phi} < 0 \) repel the (-) solutions and attract the (+) ones, and conversely for \( \frac{\partial \Lambda}{\partial \phi} > 0 \). Thus the only case that could produce an exception to our no-go theorem is \( \frac{\partial \Lambda}{\partial \phi} \leq 0 \).

To consider this case, we define \( f = 2\Lambda + \rho \). As we pointed out earlier, we have \( f < 0 \) at points on trajectories which are vertically above or below the egg. At a point where a trajectory could hit the egg in the \( h = 0 \) plane we must have \( f = 0 \). Now, we compute the time derivative of this function

\[ \dot{f} = 2\ddot{\Lambda} + \dot{\rho} = 2 \frac{\partial \Lambda}{\partial \phi} \dot{\phi} + 2 \frac{\partial \Lambda}{\partial a} \dot{a} + \dot{\rho} = 2 \frac{\partial \Lambda}{\partial \phi} \dot{\phi} - 6\rho \]  

(22)

using (13). From (14) we see that \( \dot{\phi} = 0 \) at this point, thus \( \dot{f} = 0 \) also. Differentiating again and making further use of the equations of motion shows that at this point

\[ \ddot{f} = 2 \frac{\partial \Lambda}{\partial \phi} \ddot{\phi} - 6\dot{\rho}^2 + 6 \rho \frac{\partial \Lambda}{\partial \phi} - \left( \frac{\partial \Lambda}{\partial \phi} \right)^2 \]  

(23)
Considering $f$ as a function of time along the trajectory, we notice that if $\ddot{f} < 0$, then $f$ is concave and must have been negative immediately before and after the egg hit at $f = 0$. As we have shown above that we need only worry about the case where $\partial \Lambda \over \partial a \leq 0$, we see that if either $\partial \Lambda \over \partial \phi$ or $\rho$ are non-zero the trajectory cannot escape the region directly above or below the egg. Referring to (17) we see that $\dot{h} > 0$. Thus these trajectories are (+) branch trajectories coming from under the egg and emerging above it as (−). However, because we have already shown that (−) trajectories can not exit to the right of the egg, we see that this case is in fact ruled out.

This argument breaks down in the special case $\dot{a} = 0$ ($\rho = 0$) and $\partial \Lambda \over \partial a = 0$. To examine the behavior here we need to find the lowest non-vanishing derivatives of $h$, $e$ and $f$. After some calculation we find

$$h^{(3)} = e^{-4\phi}(2(\partial \Lambda \over \partial a)^2 + \frac{1}{2} \partial \Lambda \over \partial a \partial \phi \partial \dot{\phi})$$

$$\pm e^{(3)} = -\frac{1}{2} e^{-4\phi} \partial \Lambda \over \partial a \partial \phi \partial \dot{\phi}$$

$$f^{(6)} = -10(\partial \Lambda \over \partial a)^2(24(\partial \Lambda \over \partial a)^2 + 12 \partial \Lambda \over \partial a \partial \phi \partial \dot{\phi} + (\partial \phi \partial \phi)^2)$$

where ($n$) indicates the $n$th derivative with respect to time. To have the (+) attracted to the egg we need $e^{(3)} \leq 0$. This forces $h^{(3)} \geq 0$ and $f^{(6)} \leq 0$ with equality holding only in the case where $\partial \Lambda \over \partial a = 0$. If $\partial \Lambda \over \partial a = 0$ then the trajectory passing through the point discussed above with $h = e = f = 0$ is just a constant and so does not represent a branch change. This point is a generalization of the exceptional fixed points discussed in [19]. Otherwise we again have a (−) branch entering the region above the egg from which cannot escape without touching the egg again.

This completes the proof that a (+) branch, originating outside of the egg region, cannot use it to convert to a (−) branch solution outside of the egg region. As a consequence, we see that the axions cannot catalyze branch changing and graceful exit even when endowed with a potential.

4 Conclusion

We have investigated the graceful exit problem in Pre-Big-Bang inflation with the dynamics governed by stringy action including the axion field, as well as a potential dependent on
both the axion and the dilaton. Although the non-interacting axion can asymptotically link up distinct solutions of the dilaton-metric system, in such cases the related solutions still belong to the same branch, one past-collapsing and the other future-expanding \cite{18}. One could have hoped that with the addition of an axion potential, actual branch changes could have been induced, leading to the resolution of the graceful exit problem. Specifically, one could have expected the violations of the positivity arguments employed in the integral formula (20), or the creation of qualitatively new pathologies leading to at least isolated nonsingular perturbative solutions. This behavior could not be ruled out solely on the grounds of singularity theorems, as we have shown previously. Our analysis demonstrates that the axion in fact cannot facilitate branch changing, and lead to graceful exit. We thus extend our earlier result, stating that no (+) branch solution (generically regular in the past) can evolve into a (−) branch solution (generically regular in the future), to hold even in the presence of axions with arbitrary interactions. Thus the axion can be safely ignored in all expanding universes, as it does not change their qualitative behavior.

In light of this result, we see that the only hope to resurrect the Pre-Big-Bang inflation is to show that higher order $\alpha'$ corrections asymptotically link dual solutions, and lead to nonsingular cosmologies. This conjecture is partly supported by the nonsingular solutions presented in \cite{14}, but the evidence is hardly conclusive because the solutions are non-perturbative in $\alpha'$, indicating their sensitivity to even higher order corrections, present in string theory. A more comprehensive approach as advocated in \cite{27} is in order, where one should construct an exact conformal field theory and study all the higher order corrections systematically. Indeed, these authors present certain models which can be interpreted as anisotropic universes and which possess the desired duality-related asymptotia. However, the final answer should be based on an isotropic and homogeneous model, which is still lacking.

**Acknowledgements**

We would like to thank M. Gasperini, R. Khuri, E. Kiritsis, and G. Veneziano for helpful conversations. This work was supported in part by DOE grant DE-FG02-94ER40823, and in part by NSERC of Canada. NK was also supported in part by an NSERC postdoctoral fellowship.
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