SECONDARY LS CATEGORY OF MEASURED LAMINATIONS

BY

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ABSTRACT

A new version of the tangential LS category [5] for foliated spaces depending on a transverse invariant measure, called the measured category, was introduced in [12, 13, 14]. This measured category vanishes easily for interesting dynamics. When it is zero, by taking arbitrarily large homotopies, the rate of convergence to zero of the quantity involved in the definition gives a new invariant, called the secondary measured category. Several versions of classical results are proved for the secondary measured category. It is also shown that the secondary measured category is a transverse invariant and it has a deep relation with the growth of pseudogroups.

1. Introduction

The LS category is a homotopy invariant given by the minimum number of open subsets contractible within a topological space needed to cover it. It was introduced by L. Lusternik and L. Schnirelmann in the 1930’s in the setting of variational problems. Many versions of this invariant have been given. In

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particular, H. Colman and E. Macías have introduced a tangential version for foliations, where they use leafwise contractions to transversals [4, 5]. In [12, 13, 14], another version of the tangential category, called the measured LS category, is defined. We show that this invariant vanishes in many dynamically interesting examples. So we are going to introduce dynamics in the definition of the measured category giving rise to the secondary category.

The paper begins with a brief introduction to laminations, tangential category and measured category. Also, we illustrate the reason underlying the vanishing of the measured category in interesting dynamics.

We continue introducing the secondary version of the measured LS category for laminations on compact spaces. For each positive integer $n$, the $n$-measured LS category is defined like the measured LS category by considering only deformations of (leafwise) length $\leq n$, where the length of a deformation is the maximum length of the induced paths at each point. Length is defined by using chains of plaques of a fixed foliated atlas, or by using a Riemannian metric in the case of smooth foliations. The measured LS category vanishes just when the $n$-measured LS category converges to zero as $n \to \infty$. In this case, the secondary category is defined as the growth type of the sequence of inverses of $n$-measured LS categories. The secondary category can be defined also at the level of pseudogroups and it is in fact a transverse invariant (considering good generators of the pseudogroup in the sense of J. A. Álvarez and A. Candel [1]).

Finally, we study its relation with the growth of groups and pseudogroups (see, e.g., [18]), obtaining a link between these growth types and an equality in the case of free suspensions by Rohlin groups [16]. This final result allows to compute our invariant in simple examples and gives a criterion to find groups that do not satisfy the Rohlin condition.

2. Definition of the $\Lambda$-category

We refer to [3] for the basic notion and definitions about laminations, foliated chart, foliated atlas, holonomy pseudogroup and transverse invariant measure. They are recalled here in order to fix notation.

A **Polish space** is a separable and completely metrizable topological space. Let $X$ be a Polish space. A **foliated chart** in $X$ is a pair $(U, \varphi)$ such that $U$ is an open subset of $X$ and $\varphi : U \to B^n \times S$ is a homeomorphism, where $B^n$ is an open ball of $\mathbb{R}^n$ and $S$ is a Polish space. The sets $B^n \times \{\ast\}$ are called the **plaques**