Comparative investigation of the energetic ion spectra comprising the magnetospheric ring currents of the solar system

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Abstract Investigated here are factors that control the intensities and shapes of energetic ion spectra that make up the ring current populations of the strongly magnetized planets of the solar system, specifically those of Earth, Jupiter, Saturn, Uranus, and Neptune. Following a previous and similar comparative investigation of radiation belt electrons, we here turn our attention to ions. Specifically, we examine the possible role of the differential ion Kennel-Petschek limit, as moderated by Electromagnetic Ion Cyclotron (EMIC) waves, as a standard for comparing the most intense ion spectra within the strongly magnetized planetary magnetospheres. In carrying out this investigation, the substantial complexities engendered by the very different ion composition distributions of these diverse magnetospheres must be addressed, given that the dispersion properties of the EMIC waves are strongly determined by the ion composition of the plasmas within which the waves propagate. Chosen for comparison are the ion spectra within these systems that are the most intense observed, specifically at 100 keV and 1 MeV. We find that Earth and Jupiter are unique in having their most intense ion spectra likely limited and sculpted by the Kennel-Petschek process. The ion spectra of Saturn, Uranus, and Neptune reside far below their respective limits and are likely limited by interactions with gas and dust (Saturn) and by the absence of robust ion acceleration processes (Uranus and Neptune). Suggestions are provided for further testing the efficacy of the differential Kennel-Petschek limit for ions using the Van Allen Probes.

1. Introduction

What criterion or standard do we use when comparing the particle populations within the highly diverse magnetospheres of the solar system? Should a planet that is 10 times bigger or that has a magnetic field that is 10 times stronger have energetic particle populations that are 10 times more intense or 10 times more energetic? For relativistic electron spectra of the strongly magnetized planets of the solar system, one answer has been suggested and supported. Specifically, following the pioneering work of Kennel and Petschek [1966], Schulz and Davidson [1988], Davidson et al. [1988], and Summers et al. [2009], Mauk and Fox [2010] developed a general approach to evaluating a relativistic differential Kennel-Petschek (KP) limit for energetic magnetospheric electrons that successfully provides a standard for comparing the relativistic electron radiation belts of all of the strongly magnetized planets of the solar system. The electron KP limit makes use of the expected runaway growth of whistler waves under certain conditions and the associated strong scattering of electrons into the magnetic loss cone. This work also confirmed for the relativistic regime the nonrelativistic Schulz and Davidson [1988] prediction of a characteristic $E^{-1}$ shape of the spectra for energies $<1$ MeV. It was found specifically that for Earth, Jupiter, and Uranus, the electron spectra are pegged close to the KP limit between 0.1 and 1 MeV and that the spectra are characterized by the $\sim E^{-1}$ shape within the same range of energy. At higher energies the spectra are below the corresponding KP limits. Neptune and Saturn were shown to be different. Neptune achieves the limit only close to the 0.1 MeV energy, and it is suggested that the energization processes are not robust enough to challenge the KP limit at energies as high as 1 MeV. At Saturn the spectra fall well below the limit for all energies, at least during the Cassini mission epoch, and it is suggested that the Saturnian spectra are strongly degraded by the unique gas and dust populations within that system.

The present work is a companion to that earlier work on electrons. Here we consider the question of the limiting factors for the energetic ion spectra that comprise the ring current populations of the five planetary magnetospheres. Primarily by means of the mechanism of the diamagnetic current that arises from particle pressure gradients, these populations carry major fractions of the electric currents that encircle these planets in...
the inner to middle regions and distort the magnetic field shapes somewhat away from their dipolar configurations. Here we develop for ions the differential Kennel-Petschek limit as a standard for comparing the intensities and spectral shapes. For ions, the electromagnetic ion cyclotron (EMIC) waves are thought to play the role that whistler waves play for electrons. The analysis provided here is performed in a fashion that accommodates the complexities of the multispecies composition of the plasmas that determine the properties of the EMIC waves. This examination has already been carried out for the case of Earth in Mauk [2013] where it was found that the proton spectra reside somewhat above the classical KP limit for energies < 100 keV by a roughly consistent amount. The observed flat spectral shapes also may correspond to the spectral sculpting that is expected with the KP process. Here we expand that examination to all of the other strongly magnetized planets of the solar system. We note here that there are other processes that likely play a role in the general loss of energetic ions, but under appropriate conditions the EMIC moderated scattering losses have the potential to be much faster than other processes that have been invoked [Daglis et al., 1999; Erlandson and Ukhorskiy, 2001]. At Earth, for example, collisional processes such as charge exchange are not fast enough to explain the initial rapid storm recovery phase [e.g., Keika et al., 2006].

2. Planetary Ion Spectra

The ion spectra that we will be examining are shown in Figure 1. Here are fits to spectra that are the most intense recorded in the literature for the ring current regions of our five target planets. Specifically, they are the most intense at one of two energies, 100 keV and 1 MeV, or both. Solid line spectra are protons, and dashed line spectra are heavy ions (O, S, or Water Group [W⁺]). For Earth and Saturn the spectra are the most intense published (Earth) or observed (Saturn) over at least several years of observation. Despite the use of an orbital mission (Galileo), the Jupiter Spectra have been much more sparsely observed in their entirety, but we have spectra from both the Galileo mission and a spectrum from the Voyager 1 flyby. For Uranus and Neptune we have only the Voyager 2 flybys, and so the spectra represent the most intense spectra for just one brief period of time. The Saturn W⁺ spectrum is a spectrum sampled at the same time as the most intense proton spectrum observed (a separate search was not made for the most intense W⁺ spectrum). The literature sources for these various spectra are provided in Table 1, along with the fitting parameters for all of the spectra shown. The Saturn spectra were captured with the Cassini MIMI instrument [Krimigis et al., 2004] at the following times: 2005, day 266, 0900–1000 UT (L = 8.4) and 2005, day 332, 0100–0200 (L = 9.5).

Figure 1. Energetic ion spectra sampled within the ring current regions of the five strongly magnetized planets of the solar system and fitted with equation (3). The spectra are the most intense observed within these systems specifically at 100 keV and 1 MeV, or both. Solid line spectra are protons, and dashed line spectra are heavy ions (O, S, or Water Group [W⁺]). For Earth and Saturn the spectra are the most intense published (Earth) or observed (Saturn) over at least several years of observation. Despite the use of an orbital mission (Galileo), the Jupiter Spectra have been much more sparsely observed in their entirety, but we have spectra from both the Galileo mission and a spectrum from the Voyager 1 flyby. For Uranus and Neptune we have only the Voyager 2 flybys, and so the spectra represent the most intense spectra for just one brief period of time. The Saturn W⁺ spectrum is a spectrum sampled at the same time as the most intense proton spectrum observed (a separate search was not made for the most intense W⁺ spectrum). The literature sources for these various spectra are provided in Table 1, along with the fitting parameters for all of the spectra shown. The Saturn spectra were captured with the Cassini MIMI instrument [Krimigis et al., 2004] at the following times: 2005, day 266, 0900–1000 UT (L = 8.4) and 2005, day 332, 0100–0200 (L = 9.5).
Table 1. Spectral and Other Selected Parameters for the Most Intense Planetary Energetic Ion Spectra in the Solar System

| Planet | $E_{\text{Low}}$ (keV) | C | $kT$ | $\gamma_1$ | $E_0$ | $\gamma_2a$ | $\gamma_2b$ | BD (nT)$^\dagger$ | Beta$^\ddagger$ | Eo | $\gamma_2$ | 2 | $a$ | Dipole | Beta$^\ddagger$ | CRM$^\ddagger$ | ERM keV | Comment |
|--------|------------------------|---|------|---------|-------|---------|---------|-----------|---------|-----|---------|---|-----|----------|---------|----------|----------|--------|
| E(H$^+$) | 3.0 | 5 | 6.5E6 | $-0.308$ | 776 | 1 | 1.69 | 1152 | 0.123 | 1100 | 0.263 | 45 | 3.12 | 45 | ERM | 38 | Exp45; [Lyons and Williams, 1980] |
| E(H$^+$) | 3.5 | 10 | 8.0E5 | $-0.244$ | 890 | 1 | 0.41 | 486 | 0.350 | 350 | 0.44 | 45 | 4.4 | 45 | ERM | 44 | Exp45; [Lyons and Williams, 1980] |
| E(H$^+$) | 4.0 | 5 | 7.6E6 | $-0.428$ | 4944 | 4 | 41.25 | 486 | 0.419 | 350 | 0.36 | 45 | 3.6 | 45 | ERM | 45 | Exp45; [Lyons and Williams, 1980] |
| E(H$^+$) | 5.0 | 5 | 2.6E7 | $-0.269$ | 87.3 | 5 | 5.74 | 249 | 0.752 | 350 | 0.31 | 45 | 3.1 | 45 | ERM | 57 | Exp45; [Williams and Lyons, 1974b] |
| E(H$^+$) | 6.0 | 10 | 4.0E7 | $-0.286$ | 245 | 1 | 2.13 | 486 | 0.355 | 350 | 0.30 | 45 | 3.0 | 45 | ERM | 57 | Exp45; [Williams and Lyons, 1974b] |

$^a$This is a poor fit. It uses only the peaks of very structured proton spectra.

$^b$This spectrum also has some unmodeled structure, but the fit is accurate to a factor of 2 everywhere.

$^c$Ratio of the integrated particle pressure for the designated spectrum, normalized by the dipole magnetic pressure.

$^d$CRM is the maximum value of the $C_R/C_p$ versus $E_r$ profile.

$^e$ERM is the value of the resonant energy $E_r$ at which CRM occurs.

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3. Kennel-Petschek Theory

Considering only the case of ions, the idea of the Kennel-Petschek (KP) limit is as follows: (1) magnetic flux tubes within a planetary inner magnetosphere may be robustly populated by energetic ions by some acceleration processes, perhaps during magnetic bottle configurations like those of the inner magnetosphere (i.e., by the Petschek (FP) limit). (2) accelerated ions trapped in magnetic bottle configurations like those of the inner magnetosphere are intrinsically unstable through gyroresonant interactions to the generated waves near the magnetic equator; (3) a small fraction of the generated waves propagating along the magnetic field lines of the generated waves propagating along the magnetic field lines may be reflected back toward the equatorial regions (i.e., $\sim 5\%$ in amplitude, or $\sim 2.5\%$ in power flux), and (4) accelerated ions trapped in magnetic bottle configurations like those of the inner magnetosphere may be intrinsically unstable through gyroresonant interactions to the generated waves near the magnetic equator. Considered here only the case of ions, the idea of the Kennel-Petschek (KP) limit is as follows: (1) magnetic flux tubes within a planetary inner magnetosphere may be robustly populated by energetic ions by some acceleration processes, perhaps during magnetic bottle configurations like those of the inner magnetosphere (i.e., by the Petschek (FP) limit). (2) accelerated ions trapped in magnetic bottle configurations like those of the inner magnetosphere are intrinsically unstable through gyroresonant interactions to the generated waves near the magnetic equator; (3) a small fraction of the generated waves propagating along the magnetic field lines of the generated waves propagating along the magnetic field lines may be reflected back toward the equatorial regions (i.e., $\sim 5\%$ in amplitude, or $\sim 2.5\%$ in power flux), and (4) accelerated ions trapped in magnetic bottle configurations like those of the inner magnetosphere may be intrinsically unstable through gyroresonant interactions to the generated waves near the magnetic equator.

For Saturn, the water group (W$^+$) ion spectrum presented in Figure 1 is the more intense of the W$^+$ spectra sampled during the two time periods represented by the Saturn proton spectra. A separate search has been made for the most intense proton spectra observed during the Galileo mission epoch [Mauck et al., 2004]. For Saturn, the water group (W$^+$) spectrum sampled during the two time periods represented by the Saturn proton spectra. A separate search has been made for the most intense proton spectra observed during the Galileo mission epoch [Mauck et al., 2004].

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medium but any mechanism of feedback would suffice); and (4) if \( \mathbf{G} \cdot \mathbf{R} \geq 1 \), there occurs runaway growth of the waves and, as a consequence, ions are lost at the very fast strong diffusion limit, whereby the loss cone of the magnetic field configuration is filled by wave-particle scattering as fast as the loss cone particles can be precipitated into the atmosphere. And so, for robust acceleration processes, it is predicted that the particle distributions will adjust themselves so that \( \mathbf{G} \cdot \mathbf{R} \leq 1 \). The result of the initial work by Kennel and Petschek [1966] was an expressed upper limit on the integral intensity of the particle distribution of the form \( \int_{\mathbf{E}} \mathbf{E} \cdot \mathbf{D} \leq \text{KPL} \), where \( \mathbf{E} \) is particle intensity as a function of energy, \( \mathbf{E}^* \) is the energy above which the particles contribute to the positive growth of the waves, and \( \text{“KPL”} \) is short for “Kennel-Petschek Limit,” a number that is dependent on the magnetospheric radial position parameter “\( \mathbf{L} \).”

The differential KP limit examines the maximum allowed intensities not just at a single minimum resonant energy but as a function of the minimum resonant energy; this process constrains not only integral intensities but spectral shapes as well [Schulz and Davidson, 1988]. We approximate the net EMIC wave gain \( \mathbf{G} \) with the following expression:

\[
\mathbf{G} \approx \exp \left[ \frac{\gamma_{\mathbf{T}}}{V_{\mathbf{g}}} \cdot \mathbf{D} \cdot \mathbf{R}_{\mathbf{p}} \right] \tag{1}
\]

where \( \mathbf{G} \) is the ratio of final wave amplitude to initial wave amplitude as the waves propagate through the equatorial regions, \( \gamma_{\mathbf{T}} \) is wave temporal growth rate \((1/\mathbf{s})\), \( V_{\mathbf{g}} \) is the wave group velocity \((\mathbf{cm}/\mathbf{s})\), \( \mathbf{D} \) is the distance in planetary radii along the magnetic field direction and centered on the magnetic equator where the wave growth rate remains positive and large, and \( \mathbf{R}_{\mathbf{p}} \) is the planetary radius in cm. Manipulating the condition that \( \mathbf{G} \cdot \mathbf{R} \leq 1 \) using (1) results in the following:

\[
\mathbf{D} \cdot \mathbf{R}_{\mathbf{p}} \cdot \gamma_{\mathbf{T}} [\omega_r (E)] \leq \ln \left[ 1/\mathbf{R} \right] \cdot V_{\mathbf{g}} [\omega_r (E)] \tag{2}
\]

For the initial calculations to come we will assume, as did Kennel and Petschek [1966], that \( \mathbf{R} \sim 0.05 \), so that \( \ln [1/\mathbf{R}] \sim 3 \). Here both \( \gamma_{\mathbf{T}} \) and \( V_{\mathbf{g}} \) are shown as functions of \( \omega_r \) which is the EMIC mode wave frequency that is in gyroresonance with a proton that has a user specified minimum resonant energy \( \text{E}_r \). Equation (2) is evaluated using analytic fits to measured energetic ion distributions characterized with the highly flexible form:

\[
\mathbf{I} \left[ \frac{1}{\mathbf{cm}^2 \mathbf{ s \ sr \ keV}} \right] = C \cdot E_k \mathbf{w} \cdot \left[ kT \cdot \gamma_{\mathbf{T}} (1 + 1 + E_{\mathbf{keV}})^{\gamma_{\mathbf{T}} (1 + 1)} \right] \left( \frac{E_{\mathbf{keV}}}{E_{\mathbf{w}}} \right)^{\gamma_{\mathbf{T}} (2b)} \cdot \sin 2\gamma_{\mathbf{T}} (\omega) \tag{3}
\]

containing five free fitting parameters: \( C, kT, \gamma_{\mathbf{T}}, \gamma_{\mathbf{2a}}, \gamma_{\mathbf{2b}} \), and \( E_{\mathbf{w}} \) and with the anisotropy parameter “\( \gamma_{\mathbf{T}} \)” simply set to a guessed value. For this study we adopt the choice made by Kennel and Petschek [1966] and set \( S = 1/6 \) (we discuss this choice in a later section). We choose to use the combinations \( (\gamma_{\mathbf{2a}}, \gamma_{\mathbf{2b}}) = (1, 1/2) \) or \( (1/2, 1) \) depending on how sharply the spectrum breaks at higher energies. The choice of a spectral fitting function that separates energy from angular variations was discussed extensively and justified in Mauk and Fox [2010]. The parameters described here are the parameters shown in Table 1.

For the spectral fitting, we define two different values of \( C, C_m \) is the value that comes out of the spectral fitting process, and \( C_K \) is the value of \( C \) that is needed in order for the intensity to be exactly at the Kennel-Petschek limit for a given \( E_r \). Using these definitions and using the fact that \( \gamma_{\omega_r (E_r)} \) is linearly proportional to the normalization parameter \( C \), we can change (2) from an inequality to an equality by replacing \( \gamma_{\omega_r (E)} \) with \( (C_K/C_m) \cdot \gamma_{\omega_r (E_r)} \) and then rearranging to yield

\[
\frac{C_m}{C_K} = \frac{\mathbf{D} \cdot \mathbf{R}_{\mathbf{p}} \cdot \gamma_{\omega_r (E)} [\omega_r (E_r)]}{\ln \left[ 1/\mathbf{R} \right] \cdot V_{\mathbf{g}} [\omega_r (E_r)]} \tag{4}
\]

When this ratio is greater than 1, equal to 1, or less than 1 for a given specified resonant energy \( E_r \), that means that the proton integral intensity for that given minimum resonant energy is greater than, equal to, or less than the differential Kennel-Petschek limit. Our prediction would be that \( C_m/C_K \) will be \( \leq 1 \) for all values of \( E_r \).

It is worth commenting that in equation (4) the value of \( R \) is highly uncertain. What makes the KP limit and equation (4) useful is that the results vary only as the logarithm of that highly uncertain number.
The spatial wave growth rates \( \gamma_z = \gamma / V_p \) for parallel propagation are evaluated with the following expressions (derived from Kennel and Petschek [1966] and Kroll and Trivelpiece [1973]; see also Mauk and McPherron [1980] for the formulation of \( \gamma_z \) that eliminates \( V_p \) for parallel propagation):

\[
\frac{\gamma}{V_p} = \frac{2\pi^2 e^2 V_\phi}{m_e c^2 \omega} \eta_{p_x} \left( A_{p_x}^+ - \frac{1}{\Omega_{h}^+} - 1 \right)
\]

where

\[
\eta_{p_x} = 2\pi m_h V_\phi \left( \Omega_{h}^+ - 1 \right) \int_0^{P_i} dP_z f_h(P_z) |P_i - P_z|
\]

and

\[
P_R = m_h V_\phi \left( 1 - \frac{\Omega_{h}^+}{\Omega_{h}^+} \right)
\]

where \( m_h \) is hot ion mass, \( e \) is unit charge, \( c \) is speed of light, \( V_\phi \) is the EMIC wave phase velocity with a wave assumed to be propagating parallel to \( B, \omega \) is wave frequency (radians/s), \( \Omega_{h} \) is hot ion gyrofrequency (\( |\omega| B/(m_h c) \)), \( B \) is magnetic field strength, \( f_h(P) \) is the hot ion phase space density (normalized to hot ion density) as a function of hot ion momentum (assumed to be gyrotrropic), \( P_{i} \) is hot ion momentum perpendicular to \( B, P_{h} \) is hot ion momentum parallel to \( B, \) and \( P_R \) is the parallel momentum that a hot ion must have to be in gyroresonance with the EMIC wave. The anisotropy parameter \( A^+ \) is a complicated integral of various differentiations of the \( f_h(P) \) function, but for a distribution function with the form of (3) it can be shown that in the nonrelativistic limit (assumed to be valid for our ion calculations here) \( A^+ \) reduces simply to the “S” parameter in (3). It is assumed for the equations (5)–(7) that the hot ions are singly charged. Note that in subsequent discussions we will be referring to the minimum resonant energy \( (E_i) \), which for these nonrelativistic calculations is just \( P_{i}^2/m_h \). For a given wave frequency, the EMIC wave instability involves integrations along a cut in phase space in equation (6) from that minimum energy to infinite energy. Note finally that all of the complexity engendered by the presence of multiple ion species is contained within the specification of \( V_\phi \).

We assume here that the propagation properties of the waves are dominated by cold, multispecies plasmas as specified with the cold plasma dispersion relations [Stix, 1992] (see Appendix A). Several authors have considered the effects of warm plasmas on the dispersion properties of EMIC waves at Earth [e.g., Silin et al., 2011; Chen et al., 2011]. The results are that the dispersion properties can shift somewhat quantitatively and sometimes qualitatively for some parameter states. For example, under appropriate conditions, waves can be generated within the so-called frequency “stop bands” of the dispersion curves, and the true resonances at ion cyclotron frequencies can be turned into “quasi-resonances.” However, at Earth there is a long history of statistical analyses observations as compared with theory that show qualitative and semiquantitative agreement with the cold plasma results at the geosynchronous orbit (e.g., beginning with Mauk and McPherron [1980], Mauk [1982, 1983], and Roux et al. [1982]), and as one moves closer to the plasmasphere-dominated regions as we are moving here, the multicomponent plasma species are likely to be even colder.

The complexity of the propagation properties of EMIC waves in cold, multispecies plasmas is shown for Earth in the example plotted in Figure 2. While we will be assuming in this study that the waves propagate parallel...
parameter that the distributions develop. They argued for something close to we found in our study of the KP limit at Earth that it is the LT(O+) mode that sets the limit, not the LT(He+) or

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during storm conditions [Ukhorskiy et al., 2010]. However, for distributions that challenge the KP limit, the LT(H+) modes [Mauk, 2013]. For general magnetospheric conditions, with more general anisotropy parameters (e.g., $S \sim 1$), it is the LT(He+) and the LT(H+) modes that are expected to, and are observed to, dominate the structure of observed EMIC wave distributions, although the LT(O+) mode is certainly observed during storm conditions [Ukhorskiy et al., 2010]. However, for distributions that challenge the KP limit, the LT(O+) mode is expected to play the most significant role because of the low anisotropy values. EMIC wave observations have not been ordered in the literature in a fashion that allows this claim to be yet tested. Most recently, for example, the LT(O+) has largely been ignored because automated selection routines are unable to distinguish the LT(O+) mode from other phenomena that occur within similar ranges of frequency [Usanova et al., 2012; Min et al., 2012].

At other planets, we report here very similar results. Specifically, it is the lowest-frequency mode that invariably provides the limiting condition. And so at Earth, Jupiter, Saturn, Uranus, and Neptune, the modes demonstrated that the Kennel-Petschek results that we report here are quite insensitive to the exact numerical fractions used and to the addition of very minor species with contribution fractions <1%. How is it that such a diverse set of EMIC mode structures can be expected to yield some kind of unifying influence on planetary ion spectra? The answer resides in one final characteristic of the Kennel-Petschek limit theory having to do with angular anisotropies. There is a wide diversity of particle pitch angle distribution characteristics within any one magnetosphere. However, as the acceleration of the population begins to challenge the KP limit, the very first thing that is thought to happen is that wave-particle scattering pushes the distributions much closer to isotropy; that is the anisotropy parameter ($A^+$ or $S$) becomes very small. This fact is apparent from the storm-time proton distributions at Earth [Williams and Lyons, 1974a] and is also quite apparent from the electron pitch angle study reported in Mauk and Fox [2010]. Mauk and Fox [2010] proposed that for a population that is challenging the KP limit, there is an approximate “universal” anisotropy parameter that the distributions develop. They argued for something close to $S \sim 0.3$ for electrons. Kennel and Petschek [1966] themselves chose a very low value of the anisotropy parameter ($\sim 1/6$) based on a simple saturated pitch angle diffusion formulation. That is the anisotropy parameter that we start with in the present work.

When the anisotropy gets small the wave growth strongly favors the lowest frequency modes. Specifically, we found in our study of the KP limit at Earth that it is the LT(O+) mode that sets the limit, not the LT(He+) or the LT(H+) modes [Mauk, 2013]. For general magnetospheric conditions, with more general anisotropy parameters (e.g., $S \sim 1$), it is the LT(He+) and the LT(H+) modes that are expected to, and are observed to, dominate the structure of observed EMIC wave distributions, although the LT(O+) mode is certainly observed to Figure 2 to better display the mode structure. We will focus here on the left-handed “transverse” modes (LT; the word “transverse” referring to the orientation of the major axis of the magnetic perturbation ellipse with respect to $k \cdot B$

plane; the probable very minor role played by the LC “compressional” modes is discussed by Mauk [1982]). Even more complexity is introduced when one considers the diverse ion compositions of our five target planets. Figure 3 shows a comparison of the normalized EMIC wave phase velocities for these planets. Again, the propagation angle for these waves was assumed to be 30° to make the mode structure clearer, although again, for the Kennel-Petschek analyses, we will assume propagation parallel to the magnetic field. Table 2 shows the compositions that were assumed for the five planets. Much experimentation has
that set the limiting conditions are LT(O⁺), LT(S⁺), LT(H₂O⁺), LT(H₂⁺), and LT(N⁺). However, at Saturn and Uranus, respectively, the H₂O⁺ and H₂⁺ ions are relatively minor species, and we find in our analyses that if we simply eliminate those species, the limiting modes become, respectively, LT(H₂O⁺) and LT(H⁺), with only imperceptible changes in the calculated KP limits. And so, the reason that such complicated and diverse EMIC wave mode structures can perhaps provide a unifying influence on ion spectra is because of the following: (a) the mechanism picks out just one controlling mode out of the multiplicity of modes that may be available, and (b) the mechanism is not too particular about the exact quantitative details of that mode.

It is of interest for this study whether or not the EMIC waves of interest have been observed within the various magnetospheres addressed here. While the distribution of the different EMIC wave branches during the times of occurrence of the most intense ion spectra has not been established for Earth's inner magnetosphere, it is nonetheless well known that EMIC waves occur commonly within that region of space [Ukhorskiy et al., 2010; Usanova et al., 2012; Min et al., 2012]. At Jupiter, the literature is sparse about the kinds of EMIC waves of interest here; however, definitive measurements have been reported with the Ulysses mission by Lin et al. [1993], where a key element was the ability to measure the latitudinal profile of the wave spectra.

Mass-species-engendered banded frequency structures were observed in the L = 8 to 10 RJ regions, including the LT(S⁺) mode of greatest interest here (although puzzles remain about the ratio of wave electric to wave magnetic field magnitudes). EMIC waves have been observed in Saturn’s magnetosphere, but they have been interpreted as being generated by the neutral atom ionization pickup process and might not be relevant to the processes being discussed here [Barbosa, 1993; Leisner et al., 2011; Russell et al., 2006]. Whether or not some EMIC waves of interest here are hidden within the ubiquitous pickup spectra is unclear. A search for reports of EMIC waves at Uranus and Neptune has resulted in no definitive findings. It is noted that the extensive studies of plasma waves at Uranus and Neptune were performed using the plasma wave instrument on Voyager 2 [Kurth and Gurnett, 1991], and with its low-frequency limit of 10 Hz, that instrument was not able to report on the presence or absence of the EMIC waves of interest here. A search for definitive reports of EMIC wave measurements using the Voyager 2 magnetometer instrument at Uranus or Neptune was unsuccessful, but it is unclear whether or not there has been a definitive search. It will be of interest to later discussions that definitive measurements of the EMIC waves of interest here have been reported for Earth and Jupiter but have not been reported for Saturn, Uranus, and Neptune.

**Figure 3.** Comparisons of the theoretical dispersion properties of linear, Electromagnetic Ion Cyclotron (EMIC) waves propagating in the cold, multispecies magnetospheric plasmas of Earth, Jupiter, Saturn, Uranus and Neptune. Shown is the phase speed normalized with the Alfvén speed for waves propagating with an angle 30° from the magnetic field direction. The composition assumed for each planetary magnetosphere is shown in Table 2 along with the literature sources for those compositions.
4. Summary of Electron Results

For several reasons, some time is spent here reviewing the results of Mauk and Fox [2010] regarding energetic electrons. First and foremost, the electron results are somewhat more definitive in supporting the efficacy of the Kennel-Petschek limit than are the results reported in the present paper for ions. In discussing the limitations of the findings reported here, it is important to have the more definitive case available for immediate comparison. Second, comparisons between the electron results and ion results are important elements in section 8; Uranus is particularly interesting in this respect. Third, the figure that is the focus of the discussions here on electrons (Figure 4) was plotted in a misleading fashion in the original version [Mauk and Fox, 2010, Figure 16], and its publication here represents a correction.

Figure 4 is a summary of the electron results of Mauk and Fox [2010]. Figure 4 (top) shows the most intense electron spectra (most intense specifically at 1 MeV) sampled from our five target planets. Figure 4 (bottom) shows the relativistic electron version of equation (4), the profile of \( \frac{C_m}{C_K} \) as a function of the minimum resonant energy \( (E_r) \). Note that the energy scales on the two panels represent different things: specific energy values in the Figure 4 (top) and the minimum energy that goes into integration over a broad range of energies in Figure 4 (bottom) (this is the aspect that was plotted in misleading fashion in the original publication). \( C_m/C_K \) profiles in Figure 4 (bottom) that reside within the horizontal blue-shaded region (± a factor of 3) are judged to be at the KP limit. As a distribution of electrons is accelerated up toward the KP limit, the \( C_m/C_K \) profile is theoretically expected to flatten itself along the \( C_m/C_K = 1 \) line. That behavior is roughly what we see for the Earth, Jupiter, and Uranus profiles. What we also find is that profiles that do indeed flatten themselves along the \( C_m/C_K = 1 \) profile also end up having flat spectral shapes that roughly conform to \( I \sim E^{-1} \) (see Figure 4, top). This result using a relativistic treatment confirms the prediction of Schultz and Davidson [1988] on the basis of a nonrelativistic treatment. That expectation for relativistic electron spectra has recently been theoretically confirmed by Summers and Shi [2014]. Several features in Figure 4 are of interest to later discussions here. First is the fact that the Uranian electron spectrum appears to have been robustly accelerated during the measurement period. In the present work we will find that condition not to be true for the ions. Second, the variation from planet to planet of these most intense electron intensity spectra is about 2 orders of magnitude, as compared to the roughly 4 orders of magnitude apparent for the ions in Figure 1.

A caveat to the results discussed here is that, in the regions where intensities at relativistic energies (~1 MeV) are not high, and at radial distance greater than that used for Figure 4, the particle intensities for \( E_r \leq 0.2 \) MeV can sometimes substantially exceed the Kennel-Petschek limit at Earth (see discussion in the work of Mauk and Fox [2010]). Tang and Summers [2012] discovered that this same condition prevails at Saturn for \( L \geq 7 R_S \). Dynamic injections may be sufficiently active there to overwhelm losses, or alternatively, the feedback process may be disrupted in the more distant regions.

### Table 2. Ion Composition Near the Positions of the Most Intense Planetary Energetic Ion Spectra

| Planet   | Earth | Jupiter | Saturn | Uranus | Neptune |
|----------|-------|---------|--------|--------|---------|
| \( L \)  | 3 to 5| 7.1 to 9.5| 8.4 to 9.5| 6.5 | 9.4     |
| Elec. Den. | 130 to 1100| 120 to 900| 6 to 10| ~1 | ~0.1 |
| Species   | Fractions | Fractions | Fractions | Fractions | Fractions |
| \( H^+ \) | 0.78 | 0.025 | 0.16 | 0.995 | 0.6  |
| \( H_2^+ \) | 0.2 | 0.068 | 0.17 | 0.025 | 0.005 |
| \( He^+ \) | 0.17 | 0.025 | 0.02 | 0.005 | 0.005 |
| \( O^+ \) | 0.02 | 0.27 | 0.38 | 0.395 | 0.395 |
| \( S^+ \) | 0.18 | 0.17 | 0.2 | 0.2 |
| \( O_2^+ \) | 0.17 | 0.27 | 0.38 | 0.395 |
| \( S_2^+ \) | 0.04 | 0.28 | 0.28 | 0.28 |

References: Horwitz et al. [1986], Belcher [1983], Thomasen et al. [2010]; D. C. Hamilton contribution to Mauk et al. [2009], Krimigis et al. [1986]; Belcher et al. [1991], Richardson et al. [1995]
5. Earth Ions

A Kennel-Petschek analysis of the most intense ring current proton spectra within Earth’s magnetosphere was performed by Mauk [2013]. A sample of that analysis is shown in Figure 5 (top) for one proton spectrum measured at $L = 4$, the first of the two $L = 4$ Earth spectra documented in Table 1. Shown is the Kennel-Petschek analysis of this spectrum in the usual form of $C_m/C_K$ profiles (equation (4)) as a function of the minimum resonant energy. The red profiles show the limits imposed by the LT(O$^+$) mode, and the black profiles show the limits imposed by the LT(He$^+$) mode. For the parameters chosen (our standard $S = 1/6$, $D = L$, $R = 0.05$; and cold plasma composition fractions from Horwitz et al. [1986]: 78% H$^+$, 20% He$^+$, and 2% O$^+$) the LT(H$^+$) mode makes no contributions whatsoever. For each of the two modes that do appear (LT(O$^+$) and LT(He$^+$)), three different profiles are shown for three different assumed total electron densities representing a range of densities that might be encountered at a the given $L$ value (see the discussion by Mauk [2013]; the lower two values are from Sheeley et al. [2001]). The results for the three densities are shown with dotted, dashed, and solid lines, and labeled “a,” “b,” and “c.” It is of interest that the limits imposed by the mode that dominates (LT(O$^+$)) follow essentially the same curve irrespective of density; a higher density simply pushes the solution farther and farther to the left, that is to lower minimum resonant energies. Figure 5 (bottom) shows that as the fraction of O$^+$ increases, the LT(He$^+$) contribution disappears, but the LT(O$^+$) solution does not change very much at all; mostly just pushing the solution to lower values of the minimum resonant energy.

What we see in Figure 5 (top) is that the chosen spectrum is more than a factor of 4 higher than the classical Kennel-Petschek limit over a range of resonant energies. This result occurs relatively consistently for a broad range of intense proton spectra observed in Earth’s inner magnetosphere [Mauk, 2013]. Based on that observation, the blue horizontal line in Figure 5 is viewed as representing an empirically derived Kennel-Petschek limit. That limit can be achieved by modest alterations of the standard parameters. There are three key parameters in the simple theory: $D$ (the distance along the magnetic field where the wave growth remains positive and high), $R$ (the wave reflective or feedback coefficient), and $S$ (the assumed anisotropy parameter). The “classical” parameters are: $D = L$, $R = 0.05$, and $S = 1/6$ (the $S$ parameter is somewhat $L$ dependent). The blue horizontal line in the panels of Figure 5 represents the relative level that would be achieved if we were to alter the parameters to the following: $D = L/2$, $R = 0.005$, and $S = 1/6$. Alternatively, one could choose to modify $S$ to a smaller value, given the extremely flat pitch angle distributions reported by Williams and Lyons [1974a]; a numerical characterization is very difficult from the highly compressed plots provided. None of the “new” parameter possibilities ($R$, $D$, and $S$) for achieving quantitative concurrence with the KP expectations are unreasonable, but it is acknowledged that the need for manipulating the model...
parameters weakens arguments in favor of the efficacy of the KP limit for ions. Other details of the analyses in Figure 5 and in Mauk [2013], however, argue more in favor of a role for the KP limit. For example, the spectral shapes for energies less than 100 keV are relatively flat for the most intense spectra, with low spectral index values ($\gamma$; see Table 1) similar to what is observed for the electrons. The results in Figure 5 and other results from Mauk [2013] are further summarized in Table 1. In that table, the peaks of the $C_m/C_K$ values observed from the KP analysis are shown in the column labeled “CRM,” which is short for C-Ratio Maximum. The minimum resonant energy where that peak value was observed is provided in the column labeled “ErM.”

Mauk [2013] concluded the analysis summarized in Figure 5 by saying that: “—the evidence discussed here provides indications that an energetic ion, differential KP limit is active in helping to control the maximum intensities of ring current ion intensities within Earth’s inner magnetosphere, but the evidence is not definitive.” In the present paper we seek additional evidence by considering the ion spectra sampled at other planets throughout the solar system.

In the section that follows, we will be addressing the ion spectra of the outer planets Jupiter, Saturn, Uranus, and Neptune. Much background information was provided about these planetary magnetospheres in the early companion to the present work [Mauk and Fox, 2010]. Here we choose not to repeat that information and instead refer the reader to that earlier work.

6. Outer Planet Ions

Five spectra from Jupiter are displayed in Figure 1. Heavy ions, in the form of $\text{Sn}^{+}$ and $\text{O}^{+}$ are important constituents of Jupiter’s magnetosphere, with $\text{S}^{++}$ dominating over $\text{O}^{++}$ for all of the energetic ion moments [Mauk et al., 2004]. Hence, we have chosen to display $\text{H}^{+}$ and $\text{Sn}^{+}$ spectra from the Galileo mission that are most intense at 100 keV and at 1 MeV (hence, four spectra). Figure 1 also shows one Jupiter spectrum from the Voyager epoch because Jupiter has proven to be a dynamic place even though the major source of energy for magnetospheric processes in Jupiter’s inner to middle magnetosphere is thought to be planetary rotation. Figure 6 shows a comparison between a ring current moment sampled during the Voyager epoch of 1979 and several samples of that same moment during the Galileo epoch (1995 and 1999). The moment displayed is the current collected by a solid state detector utilized by the Voyager instrument and modeled for both the Voyager and Galileo epochs based on the spectral inputs [Mauk et al., 2004]. That moment is roughly proportional to energy flux (but somewhat modified according to some detector characteristics).
What was found, and discussed by Mauk et al. [2004] is that the ring current population during the Voyager epoch was much inflated as compared to that observed during the Galileo epoch. The suggested reason for the variation, supported by other unrelated observations, was that the volcanic action of the moon Io emitted more copious amounts of neutral gas during the Galileo epoch than it did during the Voyager epoch. Such gases strongly degrade the energetic ion populations by charge exchange in the regions close to Io. The Voyager spectrum displayed in Figure 1 was measured by a single, magnetic-field-shielded solid state detector which was not able, in design, to discriminate between heavy ions and light ions. However, detailed analyses by Mauk et al. [1996] demonstrated that the detector current shown in Figure 6 could only be reproduced under the assumption that the measured ions are mostly heavy (O\(^{16+}\) or S\(^{6+}\)).

Figure 7 (bottom) shows the Kennel-Petschek analyses for the five Jupiter spectra shown in Figure 1 (Figure 7 (top) is a replot of the Jupiter spectra from Figure 1). It would appear from these analyses that Jupiter’s energetic ion populations are constrained by the classical Kennel-Petschek limit. Assuming that the Kennel-Petschek limit is playing the role ascribed to it here, the reason why Jupiter’s ion populations would be constrained by the classical limit whereas Earth’s ion populations are constrained by a somewhat higher level (factor of ~3) is not known. However, these systems are very different and it would not be a surprise if one or more of the fundamental parameters (D, R, and S) were also different. It is of interest that the C\(_{m}/C_{K}\) profiles from three out of four of the spectra obtained from the Galileo mission reach up to “contact” the C\(_{m}/C_{K}\) = 1 line at various ranges of energy. It is only the Voyager spectrum, observed during the period of the inflated ring current epoch, that has a C\(_{m}/C_{K}\) profile that lies flat against the C\(_{m}/C_{K}\) = 1 line over an extended range of energies. That is also the Jupiter spectrum in Figure 1 that is the flattest for energies below a break point near ~2 MeV.

The KP analyses for the Saturn spectra shown in Figure 1 and replotted in Figure 8 (bottom). Saturn is clearly a very different place in that its spectra do not challenge the KP limit at any energy. This result for ions is consistent with the results for electrons. As with the electrons, we assume that the copious presence of neutral gas and dust degrades the ion spectra faster than the ions can be accelerated to the highest levels [Paranicas et al., 2008].

The results of our KP analyses for Uranus and Neptune are shown in Figure 9, as a part of a five planet summary of the results of this paper. The most intense ion spectra from Uranus and Neptune join Saturn in being far below their respective KP limits. Figure 9 makes it doubly clear how special both Earth and Jupiter are with respect to the robustness of the acceleration processes that are happening. The question naturally arises as to whether simple modifications of the R, D, and/or S parameters might bring Uranus and Neptune (and Saturn) into line with Earth and Jupiter as magnetospheres with KP-limited ion spectra, in the same way that the nominal factor of 3 difference between Earth and Jupiter’s has been reconciled. That question is discussed in the section 8.

7. Other Limiting Factors

The Kennel-Petschek limit is a highly nonlinear limit representing a relatively sharply defined upper threshold or demarcation. For magnetospheres that do not challenge the KP limit, there is no reason that a...
magnetospheric population must be limited by such a sharply defined limit. For such magnetospheres it is a reasonable assumption that there exists a quasi-linear balance between source and loss processes. At Saturn, for example, ion populations that are energized in the middle to outer magnetosphere are transported (by diffusion arising from multiplicities of small-scale injections) into the increasingly dense neutral gas and dust populations generated by the plumes of Saturn’s moon Enceladus [Dougherty et al., 2009]. If the transport and energization rates increase, the populations are transported more deeply into the gas and dust populations before they are overcome by charge exchange and scattering losses [Paranicas et al., 2008]. Other magnetospheres may establish a quasi-linear source versus loss balance using the very same wave-particle losses considered here for the Kennel-Petschek limit. As intensities increase the wave growth, the corresponding wave intensities increase, causing increasing proportions of scattered losses. The difference there would be that the threshold would not have been crossed, whereby the feedback of wave energies would lead to runaway growth, giving rise to a sharp threshold.

It is worth considering whether there might be other threshold-inducing mechanisms. One possibility is that particle pressures might increase to the extent that the magnetic field cannot mechanically contain the populations. Table 1 shows two additional columns to address this point. “BD” is the dipole magnetic field strength at the position where the spectrum was measured (as distinct from the locally measured magnetic field strength) and “betaD” is the ratio of the integrated particle pressure (for the particular spectrum in question) normalized by the dipole magnetic field pressure. We have chosen to parameterize with the dipole field because the local measured magnetic field strength depends much on the configuration of electric currents over large-scale regions. This approach is commensurate with some characterizations of laboratory plasmas where the “beta” parameter is not the local particle pressure normalized by the local magnetic field strength, but rather the internal particle pressure normalized by the external magnetic field that is confining the plasma. Note that for the configuration of a single plasma-populated flux tube within an otherwise vacuum field, a “betaD” of 0.5 would correspond roughly to a traditional local plasma beta parameter equal to 1, since inside the flux tube plasma and magnetic field pressures would be roughly equal. Clearly if “betaD” approaches the value of 1, as it does for the Earth L = 5 spectrum in Table 1, it is reasonable to assume that the system might struggle to contain such a population. Whether or not the system would struggle to contain lower values of betaD is a question that we are unable to answer here. But it is of substantial interest that for the two magnetospheres that seem to challenge the KP limit, Earth and Jupiter, the betaD values for the most intense spectra are very different for the two planets; Earth’s are relatively high whereas Jupiter’s are all very low. It is clear that the KP limit comes closest to ordering the vast differences between the Jupiter and Earth spectra than do pressure balance considerations.

Figure 7. (top) A replotting of the Jupiter energetic ion spectra shown in Figure 1. (bottom) The Kennel-Petschek analysis of the Jupiter spectra, comprising the Cm/Ck profiles derived using equation (4).
8. Summary and Discussion

We have compared the most intense energetic ion spectra that comprise the magnetospheric ring current populations in all five of the strongly magnetized planets of the solar system, specifically at Earth, Jupiter, Saturn, Uranus, and Neptune. The chosen spectra are most intense at 100 keV, 1 MeV, or both. Acknowledging the limitations of comparing spectra obtained from orbital missions (Earth, Jupiter, and Saturn) with those measured during single flybys (Uranus and Neptune), we note that the spectra are diverse in intensity at selected energies by as much a 4 orders of magnitude, substantially greater than the mere 2 orders of magnitude diversity of “most intense” electron spectra within these very same systems. We find for energies <0.5 MeV that Earth’s energetic ion spectra are by far more intense than are the spectra of any other planet. For energies >1 MeV, Jupiter has by far the most intense ion spectra.

We have tested the degree to which the spectral diversity might be ordered by an updated version of the classical Kennel-Petschek limit, taking into account the diversity of ion species within these various systems. We find that, given some flexibility in specifying the parameters that go into the classic theory, all of these most intense ion spectra have intensities that are comparable to, below, or far below the Kennel-Petschek limit; and that therefore the ion Kennel-Petschek limit does represent a true threshold for the intensities of energetic ions within planetary magnetospheres. However, only the ring current populations of Earth and Jupiter have acceleration processes sufficiently robust to clearly challenge the Kennel-Petschek limit. The observed “most intense” spectra of Saturn, Uranus, and Neptune reside far below this limit at all energies. Earth and Jupiter spectra that challenge the KP limit over extended ranges of resonant energies also have the characteristic flat energy distributions (low spectral indices) within the region of minimum resonant energies where the limits are challenged. These observations suggest that these spectra are sculpted by the KP process. The fact that only Earth and Jupiter have reports in the literature of definitive measurements of EMIC waves thought to be generated by hot anisotropic ions (and not by the ion pickup process), as discussed at the end of section 3, is consistent with the findings reported here that only at Earth and Jupiter do the most intense ion spectra seem to challenge the KP limit.

The question arises as to whether the Cm/Ck profiles calculated for Neptune, Uranus, or Saturn might be raised and brought into line with those of Earth and Jupiter by modifying one or more of the relevant parameters: R, D, or S; just as was done for Earth to reconcile the roughly factor of 3–4 (average) discrepancy when the Earth Cm/Ck profile peaks were consistently found to be somewhat higher than expected. The factors that must be made up are ~11 for Saturn, ~30 for Uranus, and ~160 for Neptune (Figure 9). For the parameter D it is hard to imagine that it can be much greater than L given the great variation in plasma and magnetic field parameters along the field lines; one could possibly stretch credibility by increasing D by a factor of 2, thereby increasing Cm/Ck by a corresponding factor of 2. It is also unlikely that the R factor can help us very much since it enters into the Cm/Ck logarithmically. If we modify R to its theoretical limit, that is, change it from 0.05 to 1, representing the highly unrealistic 100% reflective feedback into the system,
\( C_m/C_K \) can be increased by only \( \ln \left( 1/0.05 \right) = \ln 20 \) = a factor of 3, a highly unlikely additional factor that even still does not solve the problem. It might be reasonable to allow an overall combined factor of 2 to be achieved by some manipulations of the \( R \) and \( D \) parameters. That leaves us with the \( S \) factor. While we can certainly contemplate modest increases in the \( S \) parameter (e.g., factor of 2), large increases would violate the fundamental premise of the Kennel-Petschek process. It is assumed with that process that once the EMIC wave turbulence becomes strong in association with re\textit{flective} feedback, the pitch angle distributions are strongly \textit{frat}tened such that the scattering losses approach the so-called strong diffusion limit. And since such strong flattening is certainly observed at Earth [Williams and Lyons, 1974a], there is no reason why such flattening would not also accompany strong EMIC wave turbulence at other planets. For completeness we show in Figure 10 that a substantial violation of our assumption about the flattening of the pitch angle distributions can indeed move the \( C_m/C_K \) profiles for Saturn, Uranus, and Neptune substantially closer to the limit line. Here a rather typical quiet time or storm recovery phase pitch angle anisotropy of \( S = 1 \) is assumed [e.g., Williams and Lyons, 1974b; Lui et al., 1990; Chen et al., 1998]. However, again, Figure 10 violates the fundamental premise of the Kennel-Petschek limit and therefore cannot be used to alter our general conclusions about Saturn, Neptune or Uranus. In the paragraphs that follow, we address the individual issues raised with the results at these three planets.

Although Saturn is a highly active magnetosphere [Mitchell et al., 2009], the results for Saturn are not surprising given the dense clouds of neutral gas and dust which engender very fast loss rates for energetic ions via the charge exchange process [Paranicas et al., 2008].

Uranus is an interesting case because (a) its electron populations show evidence of very robust acceleration processes that challenge the electron KP limit over an extended range of energies (Figure 4), (b) observations suggest that Uranus’ magnetosphere was very dynamic during the Voyager 2 encounter [Mauk et al., 1987, 1995; Sittler et al., 1987; Belcher et al., 1991], probably as a result of solar wind interactions given the Sun-Uranus alignment of the Uranian spin axis [Selesnick and McNeely, 1987], and (c) the whistler mode emissions, thought to be an important agent of electron acceleration were particularly intense [Kurth and Gurnett, 1991]. Thus, it is fair to say that the Uranian magnetosphere was highly active during the Voyager 2 flyby, but nonetheless the level of ion acceleration appears not to have been commensurate with the level of electron acceleration.

Neptune’s magnetosphere was quiet during the Voyager 2 encounter, and there is the question as to whether it is ever particularly active. Neptune's interaction with the solar wind is expected to be weak, and there is no strong internal source of plasma, like that at Saturn and Jupiter, that can be energized by the rapid rotation of the planet. No residual signatures of dynamic injections were evident in the energetic particle data, the radial profiles of energetic particles were the most symmetric observed within the

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**Figure 9.** (top) A five-planet comparison of sample intense ion spectra from Earth (E), Jupiter (J), Saturn (S), Uranus (U), and Neptune (N). The Earth spectrum is the second \( L = 4 \) spectrum in Table 1 (different than the one used for Figure 5). The numbers following the planet designation is the approximate \( L \) value where the spectrum was sampled. The symbols H, O, and S refer to hydrogen, oxygen, and sulfur ions. (bottom) The Kennel-Petschek analyses of those same five spectra comprising the \( C_m/C_K \) profiles derived using equation (4).
magnetospheres of the outer planets, and the energetic ion spectra quantitatively formed nearly perfect Maxwellians or Kappa distributions, a condition that is actually quite rare within planetary magnetospheres [Mauk et al., 1995]. It is a fair guess, although not certain, that the lack of robust ion (and electron) acceleration at Neptune is a general characteristic of that magnetosphere.

Regarding the use of the differential Kennel-Petschek as a standard for comparing spectra and acceleration processes at different planets, again, as acknowledged in Mauk [2013], it is well understood that the generation and propagation of EMIC waves in planetary magnetospheres can be far more complicated than the simple model represented here [e.g., Omidi et al., 2013; Silin et al., 2011]. The question is can a simplified recipe be found that provides an approximate standard for comparing different space environments and conditions? Such a simple recipe seems to work fairly well for the electrons (Figure 4) despite the well-known complexities of whistler-electron interactions. The proof resides in the extent to which we can obtain consistent results over a broad range of events and environments.

The fact that Jupiter’s energetic ion spectra appear to be well ordered by the Kennel-Petschek limit adds considerable new evidence in favor of the efficacy of the Kennel-Petschek limit as an ordering mechanism for energetic ions. However, the variations of the results between Earth and Jupiter (the two planets needing different KP parameters) make the evidence somewhat less convincing than that for the electron results. Again, the most definitive evidence may in the future come from detailed studies needed using the comprehensively instrumented Van Allen Storm Probes Mission launched 30 August 2012, now that instrument responses are well understood and once a diversity of storms is encountered.

To date the geomagnetic storms occurring since the launch of the Van Allen Probes mission and the full commissioning of the instruments have been modest; the storm of 17 March 2013 was the most intense at $\text{Dst} \approx -130$ nT, with “betaD” values roughly 1/3 of the higher values shown in Table 1 when compared at each value of $L$ (using pressures provided by Gioudou et al. [2014]). Questions that can be addressed now are as follows: to what extent do modest storms challenge the Kennel-Petschek limit, or is this limit only challenged during the strongest storms? How does the distribution of EMIC wave modes (among LT(H$^+$), LT(He$^+$), and LT(O$^+$)) change with activity level, and does the LT(O$^+$) mode become more prominent with respect to the other modes during the strongest or most active storms? Is any change in that distribution correlated with the flattening of the energetic ion pitch angle distributions during modest and strong storms?

Appendix A

The dispersion relationships for the propagation of electromagnetic ion cyclotron (EMIC) waves plotted in Figures 2 and 3, and used in equations (5)–(7) for determining $V_\phi$ are derived directly from equations in Stix [1992]. The source equations are reproduced here for convenience. The vector and scalar of the so-called index of refraction ($n$ and $n$) are defined as follows:

$$ n = \frac{k c}{\omega} \quad (A1) $$

$$ n = \frac{c}{V_\phi} \quad (A2) $$

where $k$ is the wave vector (magnitude $2\pi/\lambda$ where $\lambda$ is the wavelength), $c$ is the speed of light, $\omega$ is the wave frequency ($2\pi/T$ where $T$ is the wave period), and $V_\phi$ is the wave phase speed. Most critical for the present
purposes is the determination of the wave phase speed $V_\omega$. That function is determined by solving that following quadratic (in $n^2$) equation:

$$An^4 - Br^2 + C = 0$$  \hspace{1cm} (A3)

where:

$$A = S \sin^2(\theta) + P \cos^2(\theta)$$  \hspace{1cm} (A4)

$$B = RL \sin^2(\theta) + PS \left(1 + \cos^2(\theta)\right)$$  \hspace{1cm} (A5)

$$C = PRL$$  \hspace{1cm} (A6)

where $\theta$ is the angle of the $k$ vector with respect to the background magnetic field $B_0$, and where

$$S = (R + L)/2$$  \hspace{1cm} (A7)

$$R = 1 - \sum_i \frac{\alpha_{pi}^2}{\omega (\alpha + \Omega_i)}$$  \hspace{1cm} (A8)

$$L = 1 - \sum_i \frac{\alpha_{pi}^2}{\omega (\alpha - \Omega_i)}$$  \hspace{1cm} (A9)

$$P = 1 - \sum_i \frac{\alpha_{pi}^2}{\omega^2}$$  \hspace{1cm} (A10)

$$\alpha_{pi}^2 = 4\pi n_i q_i^2/m_i$$  \hspace{1cm} (A11)

$$\Omega_i = q_i B_0/(m_i c)$$  \hspace{1cm} (A12)

and where $n_i$ is the density of species $i$, $q_i$ is the charge of species $i$, and $m_i$ is the mass of species $i$. Note that electrons are one of the species that must be included, and the summations are over electrons plus all charged ion species. The final solutions for $n^2$ (and then for $V_\omega$) using a multispecies plasma, with up to six ion species, as considered in the present work, is exceedingly messy. Modern technology in the form of symbolic operation routines like Mathematica® aids greatly in the manipulation of these solutions.

The wave ellipticity equation used to create Figure 2 (bottom) and to determine the ellipticity-dependent coloration of the lines in Figures 3 and 2 (top) is

$$\frac{i \delta E_x}{\delta y} = \frac{i \delta B_y}{\delta x} = \frac{n^2 - S}{D}$$  \hspace{1cm} (A13)

where

$$D = (R - L)/2$$  \hspace{1cm} (A14)

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