REALIZATION OF GHZ STATES AND THE GHZ TEST VIA CAVITY QED

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Abstract
In this article we discuss the realization of atomic GHZ states involving three-level atoms and we show explicitly how to use this state to perform the GHZ test in which it is possible to decide between local realism theories and quantum mechanics. The experimental realizations proposed makes use the interaction of Rydberg atoms with a cavity prepared in a coherent state.

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1 INTRODUCTION
Quantum mechanics has given rise to many questions of philosophical nature, for instance, about the concept of reality \[1 \ 2 \ 3 \ 4 \ 5\]. "Objective reality" (the reality of objects outside ourselves) seems, as Heisemberg put, to have "evaporated" as a result of quantum physics. Based on the intuitive notion that the world outside the self is real and has at least some properties that exists independent of the human consciousness we would expect that "objective reality" would maintain. The counterintuitive concepts and notions in quantum mechanics are not easy to accept without any questioning and spent of time thinking about so strange behavior which nature reveals to us theoretically and experimentally, which gives us a completely new view of the reality. Even nowadays there are many interpretations of the quantum theory \[1 \ 2 \ 3 \ 4\] and there is no consensus among which of the several interpretations is the correct one and there are still some points to be clarified. This conterintuitive concepts and their consequences lead some outstanding scientists as, for instance Einstein, to question the theory.

Entanglement is a feature of quantum mechanics which is fascinating but, on the other hand, yields as a consequence non-locality which contradicts completely our common sense. Einstein, Podolsky and Rosen, noticed the implications of quantum entanglement and proposed a *gedanken* experiment, the EPR experiment \[6 \ 2 \ 3 \ 4 \ 7\] in a fine article which deserves some comments. Although the EPR experiment was imagined to show that quantum theory was not a complete theory to describe
reality ironically it turned out to be a very important contribution to the theory. The EPR strategy was to describe an experimental arrangement involving correlated pair of particles (EPR states or Bell states). These particles interact and then are separated. Therefore, measurements made on one particle can be used via correlation to generate predictions about the other particle. In the EPR thought experiment the incompatible observables in question are position and momentum. In their article EPR state that the physical concepts with which a theory operates are intended to correspond with the objective reality. EPR look at the mathematical model supplied by quantum mechanics and give us a sufficient condition for an element of that model to represent an element of reality. They state: "If, without in any way disturbing a system we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity". As a necessary condition for the completeness of the theory, EPR state that "every element of the physical reality must have a counter part in the physical theory". According to the EPR claim, for instance, if the non-commuting spin-$\frac{1}{2}$ observables $S_z$ and $S_x$ had both simultaneous reality, and thus definite values, these values would enter into the complete description, according to the condition of completeness. If then the wave function provided a complete description of reality, it would contain these values. However, according to quantum mechanics the spin state vector cannot "contain" the values of $S_z$ and $S_x$ simultaneously. For instance, in the eigenstate $|z, +\rangle$ (z component of the spin up) of $S_z$ the particle has the property $(S_z, +)$ and the value of $S_z$ is predictable with certainty, and so there is an element of reality corresponding to it. However, we could also say that, in this state, the particle has neither the property $(S_x, +)$ nor the property $(S_x, -)$ since the particle state could be projected in the eigenstates $|x, +\rangle$ and $|x, -\rangle$ with the same probability $1/2$. That is, neither of these properties constitute an element of reality. EPR saw that the fact that quantum mechanics admits no dispersion-free states does not, on its own, tell us whether the theory is complete or not. In their words: "From [the dispersion principle] it follows (1) the quantum mechanics description of reality given by the wave function is not complete or (2) when operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality". Shortly after the EPR paper Bohr published a reply in which he defends the completeness of the quantum-mechanical description of nature which could not be refuted properly by EPR and which came to be known as the Copenhagen interpretation [8]. A modified version of the EPR experiment, however analogous, was suggested by Bohm [9, 2, 3]. In Bohm’s experiment the observables are different components of spin of the spin-$\frac{1}{2}$ particle. For instance, the EPR pair could be an electron-positron pair or twin photons emitted by excited atoms in a two-photon transition in which the photons are emitted with orthogonal polarizations. There are nowadays several experimental proposals of such experiment and some have been performed in laboratory (see for instance [10]). A proposal of generating EPR states via cavity QED is presented, for instance, in [11, 12]. Hidden variable theories [2, 3] were proposed in which one would recover local realism and one of its proponents was Bohm [13, 2, 3]. Bell developed a clever theoretical tool which could be used to perform experiments based on it which could decide between local realism theories and quantum mechanics, the Bell’s inequality [14, 2, 3, 7]. In the 1980s it was realized experiments by Aspect and collaborators [15, 7] based on the Bell’s inequality which although strongly favored quantum mechanics, there remained some possibility for which a local reality view could still be maintained. Concerning the experimental works developed to test Bell’s inequality, in 1981 Bell stated: "It is hard for me to believe that quantum mechanics works so nicely for inefficient practical set-ups, and is yet going to fail when sufficient refinements are made". We should point out that the main difficulty in the test of the Bell’s inequality is related to the fact that there is no perfect particle detectors. For a review of Bell’s inequalities and some of its variants see [16].

As we have mentioned Bell’s inequality has been a clever tool in order to test quantum mechanics.
However, in 1989, Greenberger, Horne and Zeilinger found out an ingenious theoretical tool to test quantum mechanics confronted with local hidden variable theories, the Bell theorem without inequalities [17, 18, 16], which can demonstrate the spookiness of quantum mechanics even more dramatically than the Bell’s analysis. The GHZ scheme is based on a clever choice of three commuting operators which are each formed by the product of three particle spin-$\frac{1}{2}$ operators, two for the $y$ component and one for the $x$ component, and a fourth operator formed by the product of three particle spin-$\frac{1}{2}$ operators all of them for the $x$ component. The GHZ test is performed applying these operators to a three particle entangled state, the GHZ state. Contrary to experimental tests involving the Bell’s inequalities, the GHZ test is to be performed in just one run of the experiment. The decision between quantum mechanics and local hidden variable theories is taken observing the eigenvalue of the GHZ state for the operator involving the three particle spin-$\frac{1}{2}$ operators all of them for the $x$ component. Such eigenvalue can be $\pm 1$ since there are two possibilities for defining the GHZ state. One involving a plus sign and other involving a minus sign. That is, these states are constructed with the product of the three spin up states for the three particles $\pm$ the product of the three spin down states for the three particles. Depending on which GHZ state we are using the eigenvalue will be $+1$ or $-1$. If the sign in the GHZ state is $+1$, according to quantum mechanics, the eigenvalue of the GHZ state for the operator involving the three particle operators all of them for the $x$ spin component will be $+1$, and if the sign in the GHZ state is $-1$, according to quantum mechanics, this eigenvalue of the GHZ state will be $-1$. However, for instance, for the first case (eigenvalue $+1$) if we assume that there are elements of reality to be revealed according to a local hidden variable theory the eigenvalue should be $-1$. As we see, the decision between quantum mechanics and local realism theories is a binary simple one showing the power of the GHZ formalism. There are several proposals of preparation of GHZ states. For instance, proposals involving cavity QED are presented in [19, 11, 20, 12, 21, 22]. A scheme to prepare a two mode cavity EPR state and a three cavity mode GHZ state is presented in [23]. A non-maximally entangled GHZ state is used in a scheme of teleportation in [24]. A scheme to produce GHZ atomic states and for teleportation of atomic states is discussed in [25]. The realization of a spin-type GHZ state via an atomic interference method is discussed in [20]. An interesting proposition of generating EPR states and realization of teleportation using a dispersive atom-field interaction where two atoms interact simultaneously with a cavity is discussed in [27]. A scheme of generating GHZ states also using a dispersive atom-field interaction where three atoms interact simultaneously with a cavity is presented in [28]. A method for preparation of an entangled field state is proposed in [29]. For a concrete experimental realization of GHZ states see [30, 12]. As a last remark about GHZ states and their many particle generalizations we mention that they are essential ingredients in many quantum communication and quantum computation schemes [10, 30, 31].

In reference [19] we have proposed a preparation of atomic GHZ states and a GHZ test similar the one discussed in this article. However, in reference [19] we have used three-level lambda atoms in which the lower states are degenerate states. The advantage of using lambda atoms is that it is not necessary to perform many atomic state rotations as in the case of the non-degenerate system used here. However, the atomic state rotation and atomic state detection are simpler in the non-degenerate case used in the present article. This is the main reason we developed the study reported in this article.

In the sections which follow we assume that the atoms we are going to use are Rydberg atoms of relatively long radiative lifetimes [32]. We also assume perfect microwave cavities, that is, we neglect effects due to decoherence. Concerning this point, it is worth to mention that nowadays it is possible to build up niobium superconducting cavities with high quality factors $Q$. It is possible to construct cavities with quality factors $Q \sim 10^8$ [33]. Even cavities with quality factors as high as
\( Q \sim 10^{12} \) have been reported \[34\], which, for frequencies \( \nu \sim 50 \) GHz gives us a cavity field lifetime of the order of a few seconds. Maybe future technological achievements will allow us to build up cavities with very high quality factors in which the fields can be stored for a very long time. This perhaps is one of the main goal of the cavity QED experimental physicists.

2 THE EPR EXPERIMENT

Let us consider a three-level cascade atom \( Ak \) with \( |e_k\rangle, |f_k\rangle \) and \( |g_k\rangle \) being the upper, intermediate and lower atomic states (see Fig. 1). We assume that the transition \( |f_k\rangle \leftrightarrow |e_k\rangle \) is far enough from resonance with the cavity central frequency such that only virtual transitions occur between these states. In addition we assume that the transition \( |f_k\rangle \leftrightarrow |g_k\rangle \) is highly detuned from the cavity frequency so that there will be no coupling with the cavity field. Here we are going to consider the effect of the atom-field interaction taking into account only levels \( |f_k\rangle \) and \( |g_k\rangle \). We do not consider level \( |e_k\rangle \) since it will not play any role in our scheme. Therefore, we have effectively a two-level system involving states \( |f_k\rangle \) and \( |g_k\rangle \).

Considering levels \( |f_k\rangle \) and \( |g_k\rangle \) we can write an effective time evolution operator (see Appendix)

\[
U_k(t) = e^{i\varphi a^\dagger a} | f_k\rangle \langle f_k | + | g_k\rangle \langle g_k |,
\]

where the second term above was put by hand just in order to take into account the effect of level \( |g_k\rangle \).

Now, consider first an atom A1 is prepared in the Ramsey cavity \( R1 \) in a superposition. If atom A1 is initially in the state \( |g_1\rangle \), according to the rotation matrix

\[
R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix},
\]

we have

\[
| \psi \rangle_{A1} = \frac{1}{\sqrt{2}} ( | f_1\rangle + | g_1\rangle ).
\]

Now, let us assume that we have a cavity \( C \) prepared in coherent state \( |\alpha\rangle \). A coherent state \( |\beta\rangle \) is obtained applying the displacement operator \( D(\beta) = e^{(\beta a^\dagger - \beta^* a)} \) to the vacuum, that is, \( |\beta\rangle = D(\beta)|0\rangle \), and is given by

\[
|\beta\rangle = e^{-\frac{1}{2}|\beta|^2} \sum_{n=0}^{\infty} \frac{(\beta)^n}{\sqrt{n!}} |n\rangle.
\]

Experimentally, it is obtained with a classical oscillating current in an antenna coupled to the cavity. Let us take \( \varphi = \pi \). Then, according to (2.1), the system \( A1 - C \) evolves to

\[
| \psi \rangle_{A1-C} = \frac{1}{\sqrt{2}} ( | f_1\rangle - \alpha \rangle + | g_1\rangle |\alpha\rangle ),
\]

where we have used \( e^{\varphi a^\dagger a} |\alpha\rangle = |e^{\varphi} \alpha\rangle \)[35]. Now, if atom A1 enters a second Ramsey cavity \( R2 \) where the atomic states are rotated according to the rotation matrix (2.2), we have

\[
| f_1\rangle \rightarrow \frac{1}{\sqrt{2}} ( | f_1\rangle - | g_1\rangle ),
\]

\[
| g_1\rangle \rightarrow \frac{1}{\sqrt{2}} ( | f_1\rangle + | g_1\rangle ).
\]
and, therefore,
\[ |\psi\rangle_{A1-C} = \frac{1}{2} [ | f_1\rangle (|\alpha\rangle + | - \alpha\rangle) + | g_1\rangle (|\alpha\rangle - | - \alpha\rangle)]. \tag{2.7} \]

It is worth to mention at this point that if we define the non-normalized even and odd coherent states
\[ | \pm \rangle = |\alpha\rangle \pm | - \alpha\rangle, \]
\[ | \pm \rangle = |\alpha\rangle \pm | - \alpha\rangle, \tag{2.8} \]

with \( N^\pm = \langle \pm | \pm \rangle = 2 \left( 1 \pm e^{-2|\alpha|^2} \right) \) and \( \langle + | - \rangle = 0 \), we have already an EPR state involving the atomic states of A1 and the cavity field state, that is we have
\[ |\psi\rangle_{A1-C} = \frac{1}{2} [ | f_1\rangle (|+\rangle + | g_1\rangle | - \rangle)] \tag{2.9} \]

Now, let us prepare a two-level atom A2 in the Ramsey cavity R3. If atom A2 is initially in the state \( | g_2\rangle \), according to the rotation matrix \( \text{2.22} \), we have
\[ |\psi\rangle_{A2} = \frac{1}{\sqrt{2}} (| f_2\rangle + | g_2\rangle), \tag{2.10} \]

and let us send this atom through cavity C. Taking into account \( \text{2.21} \) with \( \varphi = \pi \), after the atom has passed through the cavity we get
\[ |\psi\rangle_{A1-A2-C} = \frac{1}{2\sqrt{2}} [ | f_1\rangle (| f_2\rangle + | g_2\rangle)(|\alpha\rangle + | - \alpha\rangle) - | g_1\rangle (| f_2\rangle - | g_2\rangle)(|\alpha\rangle - | - \alpha\rangle)]. \tag{2.11} \]

Then, atom A2 enters a Ramsey cavity R4 where the atomic states are rotated according to the rotation matrix \( \text{2.22} \), that is,
\[ \frac{1}{\sqrt{2}} (| f_2\rangle + | g_2\rangle) \rightarrow | f_2\rangle, \]
\[ \frac{1}{\sqrt{2}} (| f_2\rangle - | g_2\rangle) \rightarrow - | g_2\rangle, \tag{2.12} \]

and we get
\[ |\psi\rangle_{A1-A2-C} = \frac{1}{2} [ | f_1\rangle | f_2\rangle(|\alpha\rangle + | - \alpha\rangle) + | g_1\rangle | g_2\rangle(|\alpha\rangle - | - \alpha\rangle)]. \tag{2.13} \]

If we inject \( |\alpha\rangle \) in cavity C which mathematically is represented by the operation \( D(\beta)|\alpha\rangle = |\alpha + \beta\rangle \)
and, taking \( \beta = \alpha \), this gives us
\[ |\psi\rangle_{A1-A2-C} = \frac{1}{2} [ | f_1\rangle | f_2\rangle(2\alpha\rangle + | 0\rangle) + | g_1\rangle | g_2\rangle(2\alpha\rangle - | 0\rangle)]. \tag{2.14} \]

In order to disentangle the atomic states of the cavity field state we now send a two-level atom A3 resonant with the cavity, with \( | b_3\rangle \) and \( | a_3\rangle \) being the lower and upper levels respectively, through C. If A3 is sent in the lower state \( | b_3\rangle \), under the Jaynes-Cummings dynamics \( \text{3.8} \) (see \( \text{A.71} \) with \( \Delta = 0 \)) we know that the state \( | b_3\rangle | 0\rangle \) does not evolve, however, the state \( | b_3\rangle | 2\alpha\rangle \) evolves to \( | a_3\rangle | \chi_a\rangle + | b_3\rangle | \chi_b\rangle \), where \( | \chi_a\rangle = \sum_n C_n \cos(\sqrt{n}/\sqrt{n}) | n\rangle \) and \( | \chi_b\rangle = -i \sum_n C_{n+1} \sin(\sqrt{n}/\sqrt{n}) | n\rangle \) and
\[ C_n = e^{\frac{1}{2}|2\alpha|^2}(2\alpha)^n/\sqrt{n}!. \] Then we get
\[ |\psi\rangle_{A1-A2-C} = \frac{1}{2} [ | f_1\rangle | f_2\rangle(| a_3\rangle | \chi_a\rangle + | b_3\rangle | \chi_b\rangle + | b_3\rangle | 0\rangle) + | g_1\rangle | g_2\rangle(| a_3\rangle | \chi_a\rangle + | b_3\rangle | \chi_b\rangle - | b_3\rangle | 0\rangle)]. \tag{2.15} \]
and if we detect atom A3 in state \(|a_3\rangle\) finally we get the EPR (or Bell) state

\[
| \Phi^+ \rangle_{A_1-A_2} = \frac{1}{\sqrt{2}} (|f_1\rangle |f_2\rangle + |g_1\rangle |g_2\rangle),
\]

which is an entangled state of atoms A1 and A2, which in principle may be far apart from each other.

In the above disentanglement process we can choose a coherent field with a photon-number distribution with a sharp peak at average photon number \(\langle n \rangle = |\alpha|^2\) so that, to a good approximation, \(|\chi_b\rangle \approx C_n \cos(\sqrt{n}g \tau) |n\rangle\) and \(|\chi_a\rangle \approx C_n \sin(\sqrt{n}g \tau) |n\rangle\), where \(n\) is the integer nearest \(\langle n \rangle\), and we could choose, for instance \(\sqrt{n}g \tau = \pi/2\), so that we would have \(|\chi_a\rangle \approx C_n |n\rangle\) and \(|\chi_b\rangle \approx 0\). In this case, atom A3 would be detected in state \(|a_3\rangle\) with almost 100% of probability. Therefore, proceeding this way, we can guarantee that the atomic and field states will be disentangled successfully as we would like.

Notice that starting from (2.13) if we had injected \(|-\alpha\rangle\) in the cavity and detected \(|a_3\rangle\) we would get the EPR state

\[
| \Phi^- \rangle_{A_1-A_2} = \frac{1}{\sqrt{2}} (|f_1\rangle |f_2\rangle - |g_1\rangle |g_2\rangle).
\]

Now, if we apply an extra rotation on the states of atom A2 in (2.16) in a Ramsey cavity \(R5\), according to the rotation matrix

\[
R = |g_2\rangle \langle f_2| - |f_2\rangle \langle g_2|,
\]

we get

\[
| \Psi^- \rangle_{A_1-A_2} = \frac{1}{\sqrt{2}} (|f_1\rangle |g_2\rangle - |g_1\rangle |f_2\rangle),
\]

and applying (2.18) on (2.17) we get

\[
| \Psi^+ \rangle_{A_1-A_2} = \frac{1}{\sqrt{2}} (|f_1\rangle |g_2\rangle + |g_1\rangle |f_2\rangle).
\]

The states (2.16), (2.17), (2.19) and (2.20) form a Bell basis [39, 40] which is a complete orthonormal basis for atoms A1 and A2. These states show that quantum entanglement implies non-locality. The manifestation of non-locality shows up when we perform a measurement on one of the atoms. For instance, from (2.16) it is clear that if we detect atom A1 in state \(|f_1\rangle\) then atom A2 collapses instantaneously to the state \(|f_2\rangle\) and if we detect atom A1 in state \(|g_1\rangle\) then atom A2 collapses instantaneously to the state \(|g_2\rangle\), no matter how distant they are from each other. The same applies to the other states (2.17), (2.19) and (2.20).

3 THE GHZ EXPERIMENT

In this section we are first going to show how to prepare an atomic GHZ state of the Mermin kind [18] and then show in detail how to perform the GHZ test once we have a GHZ state. Considering the states (2.8) we can write the state (2.13) as

\[
| \psi, + \rangle_{A_1-A_2-C} = \frac{1}{2} (|f_1\rangle |f_2\rangle |+\rangle + |g_1\rangle |g_2\rangle |-\rangle),
\]

which is a GHZ state involving atomic states of A1 and A2 and the cavity field state. It is easy also to obtain the GHZ state

\[
| \psi, - \rangle_{A_1-A_2-C} = \frac{1}{2} (|f_1\rangle |f_2\rangle |+\rangle - |g_1\rangle |g_2\rangle |-\rangle),
\]
Let us prepare a two-level atom $A_3$ in the Ramsey cavity $R5$. If atom $A3$ is initially in the state $|g_3\rangle$, according to the rotation matrix (2.2) we get

$$|\psi_{A3}\rangle = \frac{1}{\sqrt{2}}(|f_3\rangle + |g_3\rangle).$$

(3.23)

We take into account (2.1) with $\phi = \pi$. Starting from (3.21), after the atom has passed through the cavity we get

$$|\psi_{A1-A2-A3-C}\rangle = \frac{1}{2\sqrt{2}}[|f_1\rangle |f_2\rangle (|f_3\rangle + |g_3\rangle)(|\alpha\rangle + |\beta\rangle) - |g_1\rangle |g_2\rangle (|f_3\rangle - |g_3\rangle)(|\alpha\rangle - |\beta\rangle)].$$

(3.24)

Then, we let atom $A3$ to enter a Ramsey cavity $R6$ where the atomic states are rotated according the rotation matrix (2.2), which gives us

$$\frac{1}{\sqrt{2}}(|f_3\rangle + |g_3\rangle) \rightarrow |f_3\rangle,$$

$$\frac{1}{\sqrt{2}}(|f_3\rangle - |g_3\rangle) \rightarrow -|g_3\rangle,$$

(3.25)

and we get

$$|\psi_{A1-A2-A3-C}\rangle = \frac{1}{2}[|f_1\rangle |f_2\rangle |f_3\rangle (|\alpha\rangle + |\beta\rangle) + |g_1\rangle |g_2\rangle |g_3\rangle (|\alpha\rangle - |\beta\rangle)].$$

(3.26)

Injecting $|\alpha\rangle$ in cavity $C$ we have

$$|\psi_{A1-A2-A3-C}\rangle = \frac{1}{2}[|f_1\rangle |f_2\rangle |f_3\rangle (|2\alpha\rangle + |0\rangle) + |g_1\rangle |g_2\rangle |g_3\rangle (|2\alpha\rangle - |0\rangle)].$$

(3.27)

Now, we follow the same prescription used in the previous section in order to disentangle the atomic states of the cavity field state. That is, we send a two-level atom $A4$ resonant with the cavity through $C$ in the lower state $|b_4\rangle$ and we detect atom $A4$ in the upper state $|a_4\rangle$. Then finally we get the GHZ state

$$|\psi_+,\rangle_{A1-A2-A3} = \frac{1}{\sqrt{2}}(|f_1\rangle |f_2\rangle |f_3\rangle + |g_1\rangle |g_2\rangle |g_3\rangle).$$

(3.28)

Notice that, if starting from (3.26), we had injected $|\beta\rangle$ and detected $|a_4\rangle$, we would obtain the other GHZ state

$$|\psi_-,\rangle_{A1-A2-A3} = \frac{1}{\sqrt{2}}(|f_1\rangle |f_2\rangle |f_3\rangle - |g_1\rangle |g_2\rangle |g_3\rangle).$$

(3.29)

Let us now first discuss a summary of the GHZ test prescription. We will follow closely the discussion presented in the very clear article by Mermin [18]. First we define the atomic operators

$$A = \sigma_x^1\sigma_x^2\sigma_x^3,$$

$$B = \sigma_y^1\sigma_x^2\sigma_y^3,$$

$$C = \sigma_y^1\sigma_y^2\sigma_x^3,$$

$$D = \sigma_x^1\sigma_y^2\sigma_x^3,$$

(3.30)
where
\[
\begin{align*}
\sigma^k_x &= | f_k \rangle\langle g_k | + | g_k \rangle\langle f_k |, \\
\sigma^k_y &= -i( | f_k \rangle\langle g_k | - | g_k \rangle\langle f_k |),
\end{align*}
\]  
\tag{3.31}

\(k = 1, 2\) and \(3\). It is easy to show that
\[
[A, b] = [A, C] = [B, C] = 0,
\tag{3.32}
\]
and that
\[
A | \psi, \pm \rangle_{A1-A2-A3} = B | \psi, \pm \rangle_{A1-A2-A3} = C | \psi, \pm \rangle_{A1-A2-A3} = \mp 1 | \psi, \pm \rangle_{A1-A2-A3},
\tag{3.33}
\]
and
\[
D | \psi, \pm \rangle_{A1-A2-A3} = \pm 1 | \psi, \pm \rangle_{A1-A2-A3}.
\tag{3.34}
\]

If we assume that there are six elements of reality \(m^k_x\) and \(m^k_y\) \((k = 1, 2\) and \(3\)) each having value \(+1\) or \(-1\) waiting to be revealed according to a local realism theory, then we can write
\[
\begin{align*}
    a_\pm &= m^1_x m^2_y m^3_y = \mp 1, \\
    b_\pm &= m^1_y m^2_x m^3_y = \mp 1, \\
    c_\pm &= m^1_y m^2_y m^3_x = \mp 1,
\end{align*}
\tag{3.35}
\]
and we have
\[
d_\pm = a_\pm b_\pm c_\pm = m^1_x m^2_x m^3_x = \mp 1,
\tag{3.36}
\]
where the upper and lower subindexes refer to the GHZ state \(3.28\) and \(3.29\) respectively and we have used \((m^k_y)^2 = 1\). So, the existence of elements of reality implies that if we measure the value of the observables \(\sigma^k_x\) \((k = 1, 2\) and \(3\)) (that is the elements of reality associated with them) in the state \(| \psi, \pm \rangle_{A1-A2-A3}\) the product of the three resulting values must be \(d_\pm = \mp 1\). But according to \(3.34\), the eigenvalue of the operator \(D\) applied the state \(| \psi, \pm \rangle_{A1-A2-A3}\) is \(\pm 1\). Therefore, measuring this eigenvalue we can decide between theories based on local realism and quantum mechanics.

Now, let us see how we proceed to perform the GHZ test (we follow closely the scheme presented in \[19\]). We start letting atoms \(A_k\) to pass through the Ramsey zones \(K_k\) \((k = 1, 2\) and \(3\)) where the atomic states are rotated according to the rotation matrix
\[
K_k = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix},
\tag{3.37}
\]
or
\[
K_k = \frac{1}{\sqrt{2}}(| f_k \rangle\langle f_k | - | f_k \rangle\langle g_k | + | g_k \rangle\langle f_k | + | g_k \rangle\langle g_k |).
\tag{3.38}
\]
These rotation matrixes are the key ingredient which allow us to perform the GHZ test. The method of the test we are going to describe is based on a gradual unraveling of the GHZ state being considered.

The eigenvalues of the operators \(\sigma^k_x\) are
\[
| \psi^k_x, \pm \rangle = \frac{1}{\sqrt{2}}(| f_k \rangle \pm | g_k \rangle).
\tag{3.39}
\]
Let us take the state \(3.28\). Writing the states \(| f_1 \rangle\) and \(| g_1 \rangle\) in terms of the states \(3.39\) for \(k = 1\) and substituting in the GHZ state \(3.28\) we get
\( \ket{\psi, +} \rangle_{A1-A2-A3} = \frac{1}{2} (\ket{\psi_2^1, +} (\ket{f_2} \ket{f_3} + \ket{g_2} \ket{g_3}) + \ket{\psi_2^1, -} (\ket{f_2} \ket{f_3} - \ket{g_2} \ket{g_3})). \) (3.40)

Applying (3.38) to the state (3.28) for \( k = 1 \), we have

\[ K_1 \ket{\psi, +} \rangle_{A1-A2-A3} = \frac{1}{2} (\ket{f_1} (\ket{f_2} \ket{f_3} - \ket{g_2} \ket{g_3}) + \ket{g_1} (\ket{f_2} \ket{f_3} + \ket{g_2} \ket{g_3})). \] (3.41)

Now, we compare (3.41) and (3.40). We see that the rotation by \( K_1 \) followed by the detection of \( \ket{g_1} \rangle \) corresponds to the detection of the the state \( \ket{\psi_2^1, +} \rangle \) whose eigenvalue of \( \sigma_x^1 \) is +1. After we detect \( \ket{g_1} \rangle \), we get

\[ \ket{\psi} \rangle_{A2-A3} = \frac{1}{\sqrt{2}} (\ket{f_2} \ket{f_3} + \ket{g_2} \ket{g_3}). \] (3.42)

As before, we rewrite the states \( \ket{f_2} \rangle \) and \( \ket{g_2} \rangle \) in terms of these states (3.39) for \( k = 2 \) and substitute the above state and we have

\[ \ket{\psi} \rangle_{A2-A3} = \frac{1}{2} (\ket{\psi_3^1} + \ket{g_3} \rangle) + \ket{\psi_2^2, -} (\ket{f_3} - \ket{g_3}). \] (3.43)

If we apply (3.38) for \( k = 2 \) to the state (3.42) we get

\[ K_2 \ket{\psi} \rangle_{A2-A3} = \frac{1}{2} (\ket{f_2} (\ket{f_3} - \ket{g_3}) + \ket{g_2} (\ket{f_3} + \ket{g_3})). \] (3.44)

Again, if we compare (3.44) with (3.43), we see that the rotation by \( K_2 \) followed by the detection of \( \ket{g_2} \rangle \) corresponds to the detection of the the state \( \ket{\psi_2^2, +} \rangle \) whose eigenvalue of \( \sigma_x^2 \) is +1. After we detect \( \ket{g_2} \rangle \), we get

\[ \ket{\psi} \rangle_{A3} = \frac{1}{\sqrt{2}} (\ket{f_3} + \ket{g_3} \rangle) = \ket{\psi_3^2, +}. \] (3.45)

which is the eigenvector of \( \sigma_x^3 \) with eigenvalue +1. Now, we finally apply (3.38) for \( k = 3 \) to the above state and we have

\[ K_3 \ket{\psi} \rangle_{A3} = \ket{g_3}, \] (3.46)

and the only possibility is to detect atom A3 in the state \( \ket{g_3} \rangle \).

We can repeat the above procedure and see that we have only four possibilities which are presented schematically below, where on the left, we present the possible sequences of atomic state rotations through \( K_k \) and detections of \( \ket{f_k} \rangle \) or \( \ket{g_k} \rangle \) and on the right, we present the sequences of the corresponding states \( \ket{\psi_k^k, \pm} \rangle \) where \( k = 1, 2 \), and 3 which corresponds to the measurement of the eigenvalue of the operator \( D \) given by (3.30),

\[ (K_1, \ket{g_1}) (K_2, \ket{g_2}) (K_3, \ket{g_3}) \leftrightarrow \ket{\psi_2^1, +} \ket{\psi_2^2, +} \ket{\psi_3^3, +}, \]
\[ (K_1, \ket{g_1}) (K_2, \ket{f_2}) (K_3, \ket{f_3}) \leftrightarrow \ket{\psi_2^1, +} \ket{\psi_2^2, -} \ket{\psi_3^3, -}, \]
\[ (K_1, \ket{f_1}) (K_2, \ket{f_2}) (K_3, \ket{g_3}) \leftrightarrow \ket{\psi_2^1, -} \ket{\psi_2^2, -} \ket{\psi_3^3, +}, \]
\[ (K_1, \ket{f_1}) (K_2, \ket{g_2}) (K_3, \ket{f_3}) \leftrightarrow \ket{\psi_2^1, -} \ket{\psi_2^2, +} \ket{\psi_3^3, -}. \] (3.47)

Notice that all these results give us the eigenvalue +1 of the operator \( D \) (see (3.31)). Therefore, in one run, if we succeed to perform the above rotations of the atomic states according to (3.38), each one followed by the detection of the respective state \( \ket{f_k} \rangle \) or \( \ket{g_k} \rangle \), we get the result of the experiment in favor of quantum mechanics.
If we perform the test for the state \( \psi^\alpha \) (see (3.24)), following the above procedure, it is easy to see that we have the four possible outcomes,

\[
(K_1, | g_1 \rangle)(K_2, | g_2 \rangle)(K_3, | f_3 \rangle) \leftrightarrow |\psi^1_x, +\rangle|\psi^2_x, +\rangle|\psi^3_x, -\rangle,
\]

\[
(K_1, | g_1 \rangle)(K_2, | f_2 \rangle)(K_3, | g_3 \rangle) \leftrightarrow |\psi^1_x, +\rangle|\psi^2_x, -\rangle|\psi^3_x, +\rangle,
\]

\[
(K_1, | f_1 \rangle)(K_2, | f_2 \rangle)(K_3, | f_3 \rangle) \leftrightarrow |\psi^1_x, -\rangle|\psi^2_x, -\rangle|\psi^3_x, -\rangle,
\]

\[
(K_1, | f_1 \rangle)(K_2, | g_2 \rangle)(K_3, | g_3 \rangle) \leftrightarrow |\psi^1_x, -\rangle|\psi^2_x, +\rangle|\psi^3_x, +\rangle.
\]

(3.48)

Notice that all these results give us the eigenvalue \(-1\) of the operator \( D \) (see (3.34)), as it should be. See Fig. 2 where we present a scheme of the GHZ experiment.

Notice that if we start from (3.21), following the prescription (3.47) and (3.48), we do not have to apply \( K_3 \) and detect \( | f_3 \rangle \) or \( | g_3 \rangle \) but instead, after applying \( K_2 \) followed by the corresponding detection of the state of \( A_2 \), we would get the cavity in the state \(|\psi^3_x, +\rangle = |\alpha\rangle \) or \(|\psi^3_x, -\rangle = |\alpha\rangle \). Putting in another way, if we define

\[
D = \sigma^C_x \sigma^2_x \sigma^C_x,
\]

(3.49)

where

\[
\sigma^C_x = |+\rangle\langle-| + |\rangle\langle+|
\]

(3.50)

and \(|+\rangle \) and \(|-\rangle \) are given by (2.3), we have for the eigenvectors of \( \sigma^C_x \)

\[
|\psi^2_x, +\rangle = |\psi^C_x, +\rangle = \frac{1}{2}(|+\rangle + |\rangle - |\rangle \rangle = |\alpha\rangle
\]

\[
|\psi^3_x, -\rangle = |\psi^C_x, -\rangle = \frac{1}{2}(|+\rangle - |\rangle - |\rangle \rangle = |\alpha\rangle
\]

(3.51)

Then, in order to conclude the GHZ test, in the case in which the cavity is left in the state \(|\psi^C_x, +\rangle = |\alpha\rangle \) we could send an atom \( A_3 \) in the state

\[
|\psi\rangle_{A3} = \frac{1}{\sqrt{2}}(| f_3 \rangle + | g_3 \rangle),
\]

(3.52)

through \( C \). After atom \( A_3 \) interacts with the cavity we would have

\[
|\psi\rangle_{A3-C} = \frac{1}{\sqrt{2}}(| f_3 \rangle - |\alpha\rangle + | g_3 \rangle |\alpha\rangle),
\]

(3.53)

Now, we follow the same prescription used in the previous section in order to disentangle the atomic states of the cavity field state. That is, we inject \( |\alpha\rangle \) in the cavity and we get

\[
|\psi\rangle_{A3-C} = \frac{1}{\sqrt{2}}(| f_3 \rangle | 0 \rangle + | g_3 \rangle | 2\alpha \rangle),
\]

(3.54)

and we send a two-level atom \( A_4 \) resonant with the cavity through \( C \) in the lower state \(|b_4\rangle \) and if we detect atom \( A_4 \) in the upper state \(|a_4\rangle \) we have

\[
|\psi\rangle_{A3-C} = | g_3 \rangle |\alpha\rangle,
\]

(3.55)

and the detection of \(| g_3 \rangle \) is the signature that the cavity was previously in the state \(|\alpha\rangle \). If the cavity was in the state \(|\alpha\rangle \) after atom \( A_3 \) has interacted with the cavity we have

\[
|\psi\rangle_{A3-C} = \frac{1}{\sqrt{2}}(| f_3 \rangle |\alpha\rangle + | g_3 \rangle |\alpha\rangle).
\]

(3.56)
Now, we inject $|\alpha\rangle$ and we have

$$|\psi\rangle_{A3-C} = \frac{1}{\sqrt{2}}(|f_3\rangle \ 2\alpha\rangle + |g_3\rangle \ |0\rangle).$$

(3.57)

and we send a two-level atom $A4$ resonant with the cavity through $C$ in the lower state $|b_4\rangle$ and if we detect atom $A4$ in the upper state $|a_4\rangle$ we have

$$|\psi\rangle_{A3-C} = |f_3\rangle |\chi_a\rangle,$$

(3.58)

and the detection of $|f_3\rangle$ is the signature that the cavity was previously in the state $|-\alpha\rangle$. With the detection of $|a_4\rangle$ and $|g_3\rangle$ or $|a_4\rangle$ and $|f_3\rangle$ we conclude the GHZ test based on the GHZ state (3.21) or state (3.22).

We summarize below the results of the GHZ test: if we perform this test for the state (3.21), following the above procedure, it is easy to see that we have the four possible outcomes,

$$(K_1, \ |g_1\rangle)(K_2, |g_2\rangle)(|a_4\rangle \ (|g_3\rangle) \leftrightarrow |\psi^1_x, +\rangle |\psi^2_x, +\rangle |\psi^3_x, +\rangle),$$

$$(K_1, \ |g_1\rangle)(K_2, |f_2\rangle)(|a_4\rangle \ (|g_3\rangle) \leftrightarrow |\psi^1_x, +\rangle |\psi^2_x, -\rangle |\psi^3_x, -\rangle),$$

$$(K_1, \ |f_1\rangle)(K_2, |f_2\rangle)(|a_4\rangle \ (|g_3\rangle) \leftrightarrow |\psi^1_x, +\rangle |\psi^2_x, -\rangle |\psi^3_x, +\rangle),$$

$$(K_1, \ |f_1\rangle)(K_2, |g_2\rangle)(|a_4\rangle \ (|g_3\rangle) \leftrightarrow |\psi^1_x, +\rangle |\psi^2_x, +\rangle |\psi^3_x, -\rangle).$$

(3.59)

Notice that all these results give us the eigenvalue +1 of the operator $D$ (see (3.34)).

If we perform the test for the state (3.22), following the above procedure, it is easy to see that we have the four possible outcomes,

$$(K_1, \ |g_1\rangle)(K_2, |g_2\rangle)(|a_4\rangle \ (|g_3\rangle) \leftrightarrow |\psi^1_x, +\rangle |\psi^2_x, -\rangle |\psi^3_x, +\rangle),$$

$$(K_1, \ |g_1\rangle)(K_2, |f_2\rangle)(|a_4\rangle \ (|g_3\rangle) \leftrightarrow |\psi^1_x, +\rangle |\psi^2_x, -\rangle |\psi^3_x, -\rangle),$$

$$(K_1, \ |f_1\rangle)(K_2, |f_2\rangle)(|a_4\rangle \ (|g_3\rangle) \leftrightarrow |\psi^1_x, -\rangle |\psi^2_x, -\rangle |\psi^3_x, +\rangle),$$

$$(K_1, \ |f_1\rangle)(K_2, |g_2\rangle)(|a_4\rangle \ (|g_3\rangle) \leftrightarrow |\psi^1_x, -\rangle |\psi^2_x, +\rangle |\psi^3_x, -\rangle).$$

(3.60)

Notice that all these results give us the eigenvalue −1 of the operator $D$ (see (3.34)), as it should be. See Fig. 3 where we present a scheme of this GHZ experiment.

### 4 CONCLUSION

Concluding, we have presented a scheme of realization of atomic GHZ state and the GHZ test making use of cavity QED. In the scheme presented here we use atoms interacting with a superconducting cavity prepared in a coherent state which is a state relatively easy to be prepared and handled. In our scheme we make use of atoms in a cascade configuration and in the scheme presented in [19] we make use of atoms in a lambda configuration. The advantage of using a cascade atomic configuration is that the atomic state rotation and detection process is simpler than in the lambda configuration where we have states which are degenerated. On the other hand, for the cascade configuration we have to perform more rotations of the atomic states using Ramsey cavities than in the case of the lambda configuration.
A Time evolution operator for two-level atoms

Let us consider a two-level atom interacting with a cavity field, where $|e\rangle$ and $|f\rangle$ are the upper and lower states respectively, with $\omega_e$ and $\omega_f$ being the two atomic frequencies associated to these two states and $\omega$ the cavity field frequency (see Fig. 1). The Jaynes-Cummings Hamiltonian, under the rotating-wave approximation, is given by

$$H = \hbar a^\dagger a + \hbar \omega_e |e\rangle\langle e| + \hbar \omega_f |f\rangle\langle f| + \hbar g (a|e\rangle\langle f| + a^\dagger |f\rangle\langle e|),$$

(A.61)

where $a^\dagger$ and $a$ are the creation and annihilation operators respectively for the cavity field, $g$ is the coupling constant and we write

$$H = H_0 + H_I,$$

(A.62)

where we have settled

$$H_0 = \hbar a^\dagger a + \hbar \omega_e |e\rangle\langle e| + \hbar \omega_f |f\rangle\langle f|,$$

$$H_I = \hbar g (a|e\rangle\langle f| + a^\dagger |f\rangle\langle e|).$$

(A.63)

Let us define the interaction picture

$$|\psi_I\rangle = e^{\frac{i \hbar t}{\hbar} H_0} |\psi_S\rangle.$$  

(A.64)

Taking into account

$$i\hbar \frac{d}{dt} |\psi_S\rangle = H |\psi_S\rangle,$$

(A.65)

we get

$$i\hbar \frac{d}{dt} |\psi_I\rangle = V_I |\psi_I\rangle,$$

(A.66)

where

$$V_I = e^{\frac{i \hbar t}{\hbar} H_I} e^{-\frac{i \hbar t}{\hbar} H_0} = \hbar \begin{bmatrix} 0 & ge^{i \Delta t} a \\ ge^{-i \Delta t} a^\dagger & 0 \end{bmatrix},$$

(A.67)

and

$$\Delta = (\omega_e - \omega_f) - \omega.$$  

(A.68)

Considering

$$|\psi_I(t)\rangle = U_I(t) |\psi_I(0)\rangle = U_I(t) |\psi_S(0)\rangle,$$

(A.69)

we have to solve the Schrödinger’s equation for the time evolution operator

$$i\hbar \frac{d U_I}{dt} = V_I U_I,$$

(A.70)

where

$$U_I(t) = \begin{bmatrix} u_{ee}(t) & u_{ef}(t) \\ u_{fe}(t) & u_{ff}(t) \end{bmatrix}$$

(A.71)

and

$$U_I(0) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$  

(A.72)
That is,

\[
\begin{align*}
  i \frac{d}{dt} u_{ee}(t) &= g e^{i \Delta t} u_{ef}(t), \\
  i \frac{d}{dt} u_{ef}(t) &= g e^{i \Delta t} u_{ff}(t), \\
  i \frac{d}{dt} u_{fe}(t) &= g e^{-i \Delta t} a^\dagger u_{ee}(t), \\
  i \frac{d}{dt} u_{ff}(t) &= g e^{-i \Delta t} a^\dagger u_{ef}(t).
\end{align*}
\]  

(A.73)

Now, we are going to use the abbreviation

\[\alpha = i \frac{\Delta}{2}\]  

(A.74)

and we have

\[
\begin{align*}
  i e^{-\alpha t} \frac{d}{dt} u_{ee}(t) &= g e^{\alpha t} u_{ef}(t), \\
  i e^{-\alpha t} \frac{d}{dt} u_{ef}(t) &= g e^{\alpha t} u_{ff}(t), \\
  i e^{\alpha t} \frac{d}{dt} u_{fe}(t) &= g e^{-\alpha t} a^\dagger u_{ee}(t), \\
  i e^{\alpha t} \frac{d}{dt} u_{ff}(t) &= g e^{-\alpha t} a^\dagger u_{ef}(t),
\end{align*}
\]

(A.75)

which can be solved easily using, for instance, Laplace transformation \(L\{u(t)\} = \tilde{u}(s)\) and we have

\[
\begin{align*}
  i[(s + \alpha) \tilde{u}_{ee}(s + \alpha) - 1] &= g a \tilde{u}_{ef}(s - \alpha), \\
  i(s + \alpha) \tilde{u}_{ef}(s + \alpha) &= g a \tilde{u}_{ff}(s - \alpha), \\
  i(s - \alpha) \tilde{u}_{fe}(s - \alpha) &= g a^\dagger \tilde{u}_{ee}(s + \alpha), \\
  i[(s - \alpha) \tilde{u}_{ff}(s - \alpha) - 1] &= g a^\dagger \tilde{u}_{ef}(s + \alpha),
\end{align*}
\]

(A.76)

and solving these algebraic equations and taking the inverse Laplace transformation we get

\[
U_I(t) = \begin{bmatrix}
  e^{i \frac{\Delta}{2} t} (\cos \mu t - i \frac{\Delta}{2 \mu} \sin \mu t) & -i g e^{i \frac{\Delta}{2} t} \frac{\mu}{\mu} (\sin \mu t) a \\
  -i g a^\dagger e^{-i \frac{\Delta}{2} t} \frac{\mu}{\mu} (\sin \mu t) & e^{-i \frac{\Delta}{2} t} (\cos \nu t + i \frac{\Delta}{2 \nu} \sin \nu t)
\end{bmatrix},
\]  

(A.77)

where we have defined

\[
\begin{align*}
  \mu &= \sqrt{g^2 a a^\dagger + \frac{\Delta^2}{4}}, \\
  \nu &= \sqrt{g^2 a^\dagger a + \frac{\Delta^2}{4}}.
\end{align*}
\]  

(A.78)

In the large detuning limit \((\Delta \gg g)\) we have

\[
\begin{align*}
  \mu &= \sqrt{g^2 a a^\dagger + \frac{\Delta^2}{4}} \approx \frac{\Delta}{2} + \frac{g^2 a a^\dagger}{\Delta}, \\
  \nu &= \sqrt{g^2 a^\dagger a + \frac{\Delta^2}{4}} \approx \frac{\Delta}{2} + \frac{g^2 a a^\dagger}{\Delta}.
\end{align*}
\]  

(A.79)
and we get easily
\[ U_d(t) = e^{-i\varphi(a^\dagger a+1)} |e\rangle\langle e| + e^{i\varphi a^\dagger a} |f\rangle\langle f|, \] (A.80)
where \( \varphi = g^2t/\Delta \). If we neglect spontaneous emission, the time evolution operator can also be obtained from the effective Hamiltonian
\[ H_d = \hbar g^2 a^\dagger a (|e\rangle\langle e| - |f\rangle\langle f|), \] (A.81)
which agrees with the expression presented in [20, 41, 42, 7, 36]. The subindexes in (A.80) and (A.81) are related to the atom-field interaction described by them, that is, a dispersive interaction.

**Figure Captions**

**Fig. 1**- Energy states scheme of a three-level atom where \(|e\rangle\) is the upper state with atomic frequency \(\omega_e\), \(|f\rangle\) is the intermediate state with atomic frequency \(\omega_f\), and \(|g\rangle\) is the lower state with atomic frequency \(\omega_g\) and \(\omega\) is the cavity field frequency and \(\Delta = (\omega_e - \omega_f) - \omega\) is the detuning. The transition \(|f\rangle \leftrightarrow |e\rangle\) is far enough of resonance with the cavity central frequency such that only virtual transitions occur between these levels (only these states interact with field in cavity \(C\)). In addition we assume that the transition \(|e\rangle \leftrightarrow |g\rangle\) is highly detuned from the cavity frequency so that there will be no coupling with the cavity field in \(C\).

**Fig. 2**- Set-up for the GHZ experiment. Atom A1 passes through the Ramsey cavity \(R1\) where it is prepared in a coherent superposition, cavity \(C\) and through the Ramsey cavity \(R2\). Atom A2 passes through the Ramsey cavity \(R3\) where it is prepared in a coherent superposition, cavity \(C\) and through the Ramsey cavity \(R4\). Atom A3 passes through the Ramsey cavity \(R5\) where it is prepared in a coherent superposition, and cavity \(C\) through the Ramsey cavity \(R6\). Then we inject a coherent state \(|\alpha\rangle\) or \(|-\alpha\rangle\) in the cavity and send a two-level atom A4, initially in the lower state \(|b_4\rangle\), resonant with cavity \(C\) through \(C\) and then we detect A4 in detector \(D4\) in the upper state \(|a_4\rangle\). Once the GHZ state has been obtained, the GHZ test is performed making use of the Ramsey cavities \(K1, K2\) and \(K3\) and detectors \(D1, D2\) and \(D3\) as described in the text.

**Fig. 3**- Set-up for the GHZ experiment. Atom A1 passes through the Ramsey cavity \(R1\) where it is prepared in a coherent superposition, cavity \(C\) and through the Ramsey cavity \(R2\). Atom A2 passes through the Ramsey cavity \(R3\) where it is prepared in a coherent superposition, cavity \(C\) and through the Ramsey cavity \(R4\). Atom A3 passes through the Ramsey cavity \(R5\) where it is prepared in a coherent superposition, and cavity \(C\). Then we inject a coherent state \(|\alpha\rangle\) or \(|-\alpha\rangle\) in the cavity and send a two-level atom A4, initially in the lower state \(|b_4\rangle\), resonant with cavity \(C\) through \(C\) and then we detect A4 in detector \(D4\) in the upper state \(|a_4\rangle\). Once the GHZ state has been obtained, the GHZ test is performed making use of the Ramsey cavities \(K1\) and \(K2\) and detectors \(D1, D2\) and \(D3\) as described in the text.

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Fig. 2 - E. S. Guerra

D3(\langle g_3 \rangle)

D3(\langle f_3 \rangle)

D2(\langle g_2 \rangle)

D2(\langle f_2 \rangle)

D1(\langle g_1 \rangle)

D1(\langle f_1 \rangle)

D4(\langle a_4 \rangle)
Fig. 3 - E. S. Guerra

D3(|g_3\rangle)

D3(|f_3\rangle)

D2(|g_2\rangle)

D2(|f_2\rangle)

D1(|g_1\rangle)

D1(|f_1\rangle)

D4(|a_4\rangle)

A_4

R_4

K_2

R_2

K_1

A_3

R_5

A_2

R_3

A_1

R_1