Influence of the conical seamount on underwater sound propagation in a 3D shallow sea environment

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Abstract. The sound field of a 3D shallow sea environment with a conical submarine mountain is simulated and calculated using the finite element method. To verify the accuracy of the finite element calculation results, the effects of submarine mountain height and sound source depth on sound-propagation characteristics are evaluated. The simulation results indicate that owing to the reflection and concentration of acoustic energy due to the submarine mountain, before the sound wave reaches the subsea mountain, the impact of subsea mountain on the acoustic energy can reach 10 dB. In addition, the sound energy is hindered by the seamount. Therefore, the sound energy loss after the sound wave reaches the subsea mountain is higher than the sound energy loss in the absence of the seamount under the same circumstance, and the maximum is up to 20 dB. The larger the seamount height and sound source depth, the more significant the blocking and accumulation of the sound energy.

1. Introduction
Research on the underwater-propagation characteristics of sound waves has been a popular topic worldwide. In classical sound-field modeling, it is typically assumed that the marine environment is a horizontally layered structure and the $N \times 2D$ method is used to determine the approximate solution in 3D to reduce the calculation difficulty, and ignore the influence of transverse waves of sound propagation. However, in oceans, the changes in seafloor topography are complex, and there are widespread abrupt changes in the terrain including slopes and seamounts, and low frequency sound waves can penetrate into the elastic bottom of the sea is deep and reflection back to the sea, at the same time, the horizontal refraction effect due to the undulation of the sea floor cannot be ignored. It is necessary to improve the sound-field model for further research. Although many research results have proven that the existence of a submarine mountain significantly affects the underwater acoustic wave-propagation characteristics [1-3], there is a lack of systematic research and analysis on this effect. However, due to limited space, mainly conical seamounts were selected for systematic analysis in this study.

After several decades of development, a variety of sound field calculation methods have been established in the field of underwater acoustic propagation [4], but the existing methods make different degrees of ideal approximations of wave equations and environmental parameters. Approximations have certain limitations [4], specifically when dealing with the problem of underwater acoustic
propagation in complex seafloor terrain. The finite element method [4-6] (FEM) is a general numerical method for solving partial differential equation problems with complex boundary conditions. However, FEM has not been widely used because it requires a large amount of computer memory for calculating sound fields, especially for 3D sound fields. With the rapid improvement in computer performance in recent years, the conditions for applications of FEM in 3D sound field calculation have gradually evolved. However, there is still a lack of published results on the calculation of 3D ocean sound fields using FEM, and related research is urgently needed.

Given the above reasons, in our study, the underwater sound propagation characteristics of a conical seamount in a 3D shallow sea environment were calculated using FEM. This study focuses on the influence of the conical seamount on underwater sound propagation and hopes to provide a reference for underwater sonar detection.

2. Finite element form of the sound field
Considering the problem of low-frequency sound propagation in shallow water analyzed in this study, we establish a waveguide model in a 3D xyz rectangular coordinate system, as shown in Figure 1(A). The harmonic point sound source is located on the 3D coordinate z axis, the seawater layer is a fluid medium, and the seabed layer is an elastic medium. The constructed sound field calculation model is shown in Figure 1(B).

\[ W^{\text{i}} = W^{\text{f}} + \mathbf{F}_{\text{M},i} + \mathbf{C}_{\text{S},i} \alpha_{\text{p}}, \]  
\[ (K_y + j\omega C_y - \omega^2 M_y) \mathbf{p}_i = \mathbf{F}_{\text{E},i}, \]  
\[ M_S = M_y, K_S, C_S \]  
\[ p(x, y, z) = 0 \quad \text{on } S \]  

(A) Schematic of a submarine mountain model in a 3D Cartesian coordinate system. (B) Schematic of a sound field model for the finite element method (FEM) calculations.

**Figure 1.** Sound field model of the submarine mountain.

In Figure 1(B), \( V_w \) is defined as the seawater layer, and \( V_s \) is the bottom of the sea and the S-plane is the boundary between the seawater layer and the shallow sea surface.

From the weighted integration of the Helmholtz equation combined with Gaussian theory, the acoustic finite element equation in the \( V_w \) layer can be expressed as [7]

\[ (K_y + j\omega C_y - \omega^2 M_y) \mathbf{p}_i = \mathbf{F}_{\text{E},i}, \]  
where \( p_i \) is the sound pressure at the seawater layer and the bottom node \( i \); \( M_y, K_y, \) and \( C_y \) are the mass matrix, stiffness matrix, and damping matrix, respectively, and \( \mathbf{F}_{\text{E},i} \) denotes the acoustic excitation.

For elastic structures, the finite element vibration equation in the \( V_s \) layer is [7]

\[ (K_s + j\omega C_s - \omega^2 M_s) \mathbf{u}_i = \mathbf{F}_{\text{S},i}, \]  
where \( M_s, K_s, \) and \( C_s \) are the stiffness matrix, mass matrix, and damping matrix of the structural grid without constraints (displacement \( u_i \)), respectively, and \( \mathbf{F}_{\text{S},i} \) is the excitation load on the structure.

In this calculation, the Dirichlet boundary condition is set to the S-boundary surface of the sea surface, and the boundary condition is given by

\[ p(x, y, z) = 0 \quad \text{on } S \]  

\[ 0 < z \leq 50 \]
To simulate the propagation of acoustic signals in the infinite ocean environment, a perfectly matched layer (PML) \[8\] was added to the model and converted into a PML equation by increasing the absorption coefficient in the finite element equation.

By coupling the physical field equations, flow-elastic coupling equations, and PML equations established by the aforementioned FEM, the sound pressure field \( p \) in the seawater layer and the sea floor in a shallow ocean environment can be modeled. The model can also determine the relationships between the velocity field \( v \) and other physical field values \[9\]. In this study, the above calculation process shall be implemented on the COMSOL platform. Moreover, sound energy flux density is used to discuss the acoustic propagation characteristics of a low-frequency three-dimensional shallow sea environment \[10\].

3. Verification of 3D sound field calculation

In this study, in order to verify the accuracy of the three-dimensional shallow sea calculation model established by the finite element method, the wedge-shaped ocean waveguide, which is also known as ASA wedge-shaped waveguide \[9\] or standard wedge-shaped waveguide, is selected in this study to carry out tests; the waveguide is shown in as shown in Figure 2. It is commonly used for the test of ocean acoustic field calculation software. The sound source position is set in a 3D rectangular coordinate system (0 m, 0 m, 100 m), the sound source frequency is 25 Hz, the receiving depth is 30 m, and the wedge angle is 2.68°. The seawater layer parameters are the sound speed \( c_w = 1500 \text{ m/s} \) and water density \( \rho_w = 1000 \text{ kg/m}^3 \); the seafloor parameters are the zonal sound speed \( c_p = 1700 \text{ m/s} \), shear wave sound speed \( c_s = 800 \text{ m/s} \), seabed density \( \rho_b = 1500 \text{ kg/m}^3 \), and seabed sound speed attenuation coefficient \( \alpha_p = \alpha_s = 0.5 \text{ dB/λ} \). Figure 3 presents the results of the sound propagation loss curve at a reception depth of 30 m, calculated using the FEM and parabolic equation (PE) \[11-13\] under the above simulation conditions.

![Figure 2. ASA wedge-shaped uphill model.](image)

![Figure 3. Numerical simulation results of the 3D wedge-shaped subsea sound field. (Transmission loss)](image)

A comparison between the acoustic propagation loss curves calculated by the FEM and the PE method is presented in Figure 3. It suggests that the propagation loss calculated by the FEM is consistent with that calculated by the PE method, which validates the FEM sound field calculation method developed in this study. Furthermore, we shall use FEM to calculate the established 3D sound field model and analyze the influence of submarine mountains on the propagation characteristics of low-frequency sound signals in shallow water by considering varying heights of the subsea mountain and depths of the sound source.
4. Effect of conical seamount on sound propagation

4.1 Analysis of the influence of the height of the seabed on the propagation of low-frequency sound

This section first discusses the influence of submarine mountain height on low-frequency sound propagation in a 3D shallow ocean environment and then analyzes the distribution of acoustic energy flow in the x-z and x-y planes as the height of the subsea mountain changes. The 3D submarine mountain model is presented in Figure 4, where the sound source position is set in the 3D rectangular coordinate system at (0 m, 0 m, 100 m); the sound source frequency is 25 Hz, and the reception depth is 30 m. The parameters of the seawater and seabed are consistent with the model shown in Figure 2, the center coordinates of the bottom circle of the bottom mountain (2000 m, 0 m, 200 m), the radius of the bottom circle is \( r = 800 \) m, and the heights of the bottom mountain are \( H = 0 \) m, 50 m, 100 m and 150 m.

![Figure 4. Schematic of the 3D seamount model.](image)

The height \( H \) of the subsea mountain is changed, and the sound-propagation characteristics are compared with those when there is no subsea mountain. Figure 5 presents the propagation loss curve at a depth of 30 m when there is no seabed and when the heights of the seabed are 50 m, 100 m, and 150 m. Figures 6(A), 7(A), 8(A) and 9(A) present the pseudo-color pictures of the sound energy flow at the \( y = 0 \) m plane for different seamount heights, whereas Figures 6(B), 7(B), 8(B), and 9(B) present those at the \( z = 150 \) m plane for different seamount heights.

![Figure 5. Curves of acoustic propagation loss at different altitudes.](image)
Figure 6. The height of the submarine mountain – $H = 0$ m.

Figure 7. The height of the submarine mountain - $H = 50$ m.
Figure 8. The height of the submarine mountain - $H=100$ m.

Figure 9. The height of the submarine mountain - $H=150$ m.

The propagation loss curve presented in Figure 5 indicates that before the sound waves reach the bottom of the mountain, the sound propagation loss at the bottomless mountain and that at the bottom of the mountain are nearly the same. Before sound waves reach the top of the mountain, the acoustic propagation loss in the presence of a seamount is less than that in the absence of the seamount under the same conditions, and the propagation loss difference can reach up to 10 dB. However, behind the seamount, the propagation loss is larger than in the absence of a seamount under the same conditions. The propagation loss difference of the subsea mountain can reach up to 20 dB; the larger the height of the subsea mountain, the larger the difference in propagation loss. Figures 6(A), 7(A), 8(A), and 9(A) indicate that when there is no submarine mountain, the sound energy gradually decreases with increasing distance, and when there is a submarine mountain, there is a sound energy concentration above it caused by the obstruction. The concentration and attenuation of the sound energy behind the seamount are higher compared with the case where there is no seamount; moreover, the larger the
height of the seamount, the more significant are the blocking and concentration effects. Figures 6(B), 7(B), 8(B), and 9(B) indicate that the obstruction of the submarine mountain is in front of it, and that the front of the wave is behind the submarine mountain. The acoustic energy gathers above the submarine mountain, whereas the acoustic energy attenuation is more significant behind the submarine mountain. The larger the height of the submarine mountain, the higher is the obstruction. A significant aggregation effect is caused by the sound wave being in the uphill area before reaching the top of the mountain. The sound energy is concentrated above the sea mountain due to the partial reflection of the sound wave by the sea mountain, such that the propagation loss of the tapered sea mountain is smaller than the transmission loss when there is no sea mountain. The blocking effect of the seamount [14] on the sound wave is in the downhill area, and the sound energy spreads, such that the propagation loss of the cone-shaped seamount is greater than the propagation loss when there is no seamount.

4.2 Influence of sound source depth on low-frequency sound propagation
This section discusses the influence of the submarine mountain on the sound energy when the height of the submarine mountain is constant. The 3D subsea mountain model is shown in Figure (4), where the sound source position is set on the z-axis, and the sound source depths $SD = 50 \text{ m}, 100 \text{ m}, \text{ and } 150 \text{ m}$ are considered, the sound source frequency is 25 Hz, and the reception depth is 30 m. The parameters of the seawater and seabed are consistent with the model shown in Figure 2. The center coordinates of the bottom circle of the bottom mountain are $(2000 \text{ m}, 0 \text{ m}, 200 \text{ m})$; the radius of the bottom circle is $r = 800 \text{ m}$, and the height of the bottom mountain is $H = 100 \text{ m}$.

![Figure 10. Acoustic propagation loss curves for different sound source depths.](image)

![Figure 11. Sound source depth $SD = 50 \text{ m}$](image)
Figure 12. Sound source depth $SD = 100$ m.

Figure 13. Sound source depth $SD = 150$ m.

By comparing the propagation loss curves shown in Figure 10, it can be seen that when the receiving depth and the geometric parameters of the seamount remain unchanged during the simulation. For a sound source depth of 50 m, 100 m and 150 m, respectively, before the sound wave reaches the seamount, the attenuation speed of the sound energy decreases with an increase of the sound source depth. Behind the seamount, the attenuation rate of the sound energy increases with an increase of sound source depth. By comparing the sound energy flow diagram shown in Figures 11-13, and looking at the front of submarine mountains, it is obvious that the deeper the sound source, the slower the sound energy attenuation. However, due to the reflection and aggregation of sound energy on the seamount, the sound energy attenuation on the seamount is very small, and the deeper the sound source is, the more the sound energy accumulates. Behind the seamount the sound energy decays quickly due to the barrier effect the seamount has on the sound energy; the deeper the sound source, the more obvious the attenuation.

5. Conclusions
In this study, the sound-propagation characteristics of a cone-shaped seamount in a 3D shallow ocean environment were calculated using FEM. By comparing the effects of different seamount heights and
sound source depths on the sound-propagation characteristics, the following conclusions have been reached.

1) For the area in front of the submarine mountains at a fixed sound source depth, the sound energy propagation characteristics are basically the same, regardless of whether the influence of the submarine mountains is taken into consideration or not. Before the sound wave reaches the top of the mountain, the sound energy in the model with the seamount is higher than that without the seamount owing to the reflection and concentration of acoustic energy caused by the seamount. Behind the bottom mountain, the sound energy of the seamount is lower than when there is no seamount; this is due to the blocking effect the mountain has on the sound energy. The larger the height of the submarine mountain, the more significant the aggregation and blocking effects.

2) When the height of the seamount is fixed, before the sound wave reaches the top of the mountain, the attenuation speed of the sound energy decreases with an increase of sound source depth. Behind the seamount, the attenuation rate of sound energy increases with an increase of sound source depth.

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