The Strong Magnetic Field Decay and Evolution of Radio Pulsars on the P–$\dot{P}$ Diagram

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Abstract

In this work we have analysed various data on radio pulsars and we have shown that magnetic field decay of a factor about 10-20 is necessary to explain their evolution, in particular to remove the discrepancy between the characteristic and the real ages. The character of the field decay is exponential with a characteristic time of about $3 \times 10^6$ yr. Observational data on single X-ray pulsars which radiate due to cooling also support this result.

Key words: pulsar, magnetic field, evolution

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1 Introduction

Spin period (P) and time derivative of the spin period (Ṗ) of radio pulsars (PSRs) are known highly precisely and using these quantities one can precisely estimate: 1) rate of rotational energy loss (Ė), 2) effective (overestimated) or real value of the perpendicular component of the magnetic field (B) depending on the presence or absence of additional torques other than the magnetodipole radiation torque, 3) the value of characteristic time (τ) which is the characteristic age if the values of the braking index n are known along the evolutionary tracks. Since the change in n with respect to time is not known, τ has no physical meaning and it is not related to the age of the PSR especially for the case of older PSRs. Such definitions are necessary under the existence of strong magnetic field decay throughout the evolution of PSRs. Please note that as we can estimate only the component of the magnetic field (but not the magnetic field itself) perpendicular to the spin axis using P and Ṗ values, by saying ‘magnetic field decay’ we mean a decrease in the value of the perpendicular component.

Although the evolution of PSRs on the P-Ṗ diagram has been studied since 1970s, there are still some uncertainties and open questions about the evolution of these objects. 1–3 Below, widely used definitions of τ and B (i.e. for n=3) will be used unless another definition is given.

On the P-Ṗ diagram, PSRs must move along constant magnetic field lines if no additional torque other than the magnetodipole radiation torque is present and/or no decay in the magnetic field occurs and/or the rotation and the magnetic axes do not align in time. Actually, these processes may take place with different degrees for different PSRs and this has been discussed since seventies. 4–10 On the other hand, radio luminosity of PSRs may decrease several times during the evolution. When PSRs approach ‘the death belt’ (of which the place on the P-Ṗ diagram is not known well) the effect of nulling (in small degree), narrowing of the radiation beam, and turning-off of PSRs begin to take place. All of these create difficulties in determining the evolutionary tracks of PSRs on the P-Ṗ diagram. Note that the origin and evolution of radio PSRs seem to be known better and investigating this type of pulsar is easier compared to other types of pulsar and in particular to other types of neutron star. In this work, we try to improve and advance in this very important problem.
2 What do we see on the P-\dot{P} diagram?

Today, one can estimate PSR distances with errors \( \sim 30\% \) if they are located up to about 5-6 kpc from the Sun. Most of the far away PSRs have been discovered in recent surveys (southern sky close to the Galactic plane). So, we will limit our statistical investigation by examining only the PSRs located at \( d<4 \) kpc from the Sun. In this region, there are about 600 single PSRs out of about 1400 PSRs detected up to date in the Galaxy.

The rate of rotational energy loss of PSRs is

\[
\dot{E} = \frac{32\pi^4 R^6 B^2}{3c^4 P^4}
\]

where \( R \) is the radius and \( c \) is the speed of light. As seen from expression (1), positions of the PSRs with small values of \( B \) and large values of \( P \) must change very slowly on the P-\dot{P} diagram if the component of the magnetic field perpendicular to the spin axis does not diminish. On the other hand, parameters of young PSRs considerably rapidly change compared to PSRs with large characteristic time \( (\tau=P/(n-1)\dot{P}) \). Therefore, we begin to investigate the single-born PSRs with the values \( \dot{P}>10^{-17} \) s/s.

All the known PSRs with \( \dot{P}>10^{-17} \) s/s are represented in Figure 1. As seen from this figure, the direction of the increase in the number density of PSRs on the P-\dot{P} diagram does not coincide with the directions of the lines of constant magnetic field. For PSRs with \( 10^{12}<B<10^{13} \) G and \( \tau>10^5-10^6 \) yr, this may be related to PSRs entering the 'death-belt'. But these lines also do not coincide in the region where \( \tau<10^6 \) yr. For the PSRs with \( \tau<10^5 \) yr the observed values of the braking index \( (n) \) are less than or approximately equal to 3. The value of \( n \) must be greater than 3 during and immediately after the magnetic field decay when influence of the activity processes of the PSRs begin to come to an end. Therefore, we may say that there are also some observational data which confirm the magnetic field decay for the youngest PSRs, a natural decay as a result of high temperatures.

On the other hand, such an effect can also be due to a considerable decrease in the beaming fraction of PSRs as the value of \( P \) increases, e.g. a decrease proportional to \( P^{-1/2} \). As the number of the PSR data is very large, we have constructed a dependence between the ratio of the pulse width to the period and the period and also a dependence between this ratio and \( \tau \).
We see that the known dependence between the beaming fraction and the period remains valid.

As the PSR searches have mainly taken place in the Galactic plane (especially in the last few years) and as the old PSRs can go far away from the Galactic plane, the number of young PSRs among the observed ones in the surveys must be relatively large. So, the number density of PSRs on the P-\(\dot{P}\) diagram must be shifted to large values of \(\tau\) if we consider only the PSRs close to the Sun. In Figure 2 we have plotted only the PSRs with values of projected distances on the plane of the Galaxy \(d \times \cos b \leq 3\) kpc. The distance values were taken from Guseinov et al.\(^\text{11}\) and these values are similar to the ones given in ATNF catalogue\(^\text{12}\). As seen from a comparison of this figure with Figure 1, there is an increase in the number density distribution of PSRs with large values of \(\tau\). (In Figure 2, the number density of PSRs with \(\tau < 10^6\) yr and with large values of \(B\) is considerably less.) But the increase is not so much as must be under linear dependence between the number density and \(\tau\). And this must not only connected to low sensitivity of the surveys for large Galactic latitude values at which most of the PSRs with large values of \(\tau\) must be located.

Is this mainly related to a considerable decrease in the luminosity with the increase in characteristic time? It is true that this can not be a strong effect, but it is necessary to consider the influence of it. In Figure 3, the 30 PSRs which have the largest luminosities at 1400 MHz \((L_{1400} \geq 9.0\) mJy kpc\(^2\)) are denoted with the symbol 'X' and the 50 PSRs with the smallest values of luminosity \((L_{1400} \leq 1.58\) mJy kpc\(^2\)) are represented with 'circles'. As seen from the figure, the average luminosity of PSRs may decrease a factor of only a few times during the evolution while the differences between the initial luminosity values of PSRs are about 4-5 orders of magnitude.\(^\text{10,19}\) We have constructed also a similar figure for PSRs with \(d \times \cos b \leq 3\) kpc to check this result. Again, there is no considerable difference in the average luminosity values of young and old PSRs. Therefore, diminishing of the luminosity can not have an important role in the 'death' of PSRs with \(\dot{E} > 10^{31}\) erg/s.

Above, we see that the number of PSRs with \(10^{11} \leq B \leq 10^{13}\) G does not increase proportionally to the value of \(\tau\). In order to make a better analysis, let us consider the P-\(\dot{P}\) diagram of the PSRs with \(d \times \cos b \leq 3\) kpc displayed in Figure 2. The numbers of PSRs with \(10^{11} \leq B \leq 10^{13}\) G and with characteristic times \(\leq 10^5\), \(\leq 10^6\), \(\leq 10^7\) and \(\leq 10^8\) yr are written in the caption of Figure 2.
We have tested the change of the number of PSRs with respect to $\tau$ also by adopting a PSR sample in a cylindrical volume with $d \times \cos b \leq 2$ kpc, because we want to see the influence of PSRs going out from and coming in to the volumes under consideration. We see that the ratio of the number of PSRs with respect to $\tau$ changes with $\tau$ practically similarly in both cases. This ratio is about 4.5-4.6 when $\tau$ changes from $10^5$ to $10^6$ yr and from $10^6$ to $10^7$ yr. But when the value of $\tau$ increases from $10^7$ yr up to $10^8$ yr, the number ratio of PSRs is only about 1.7 (in both cases). This slow increase in the number of PSRs under the increasing value of $\tau$ is a result of the PSRs going deeper through the death-belt where it is not certainly known what processes lead to the turn-off of PSRs and the decay of the magnetic field. We only know that this effect strongly depends on the values of $B$ and $P$ and hence on $\dot{E}$. The rate of PSRs’ turn-off increases with the increase in the values of $B$ and $P$. This can directly be seen from Figure 2. The decrease in the number of some of these PSRs up to $\tau \sim 10^7$ yr may be related to the decreasing of the beaming fraction, but the fact that the PSRs with $\tau > 10^7$ yr are deficient is weakly dependent on the beaming fraction. If we consider only the PSRs with $10^{11} \leq B \leq 10^{12}$ G, then the number ratio with respect to $\tau$ is $\sim 7.9$ and $\sim 2.4$ under the increase of $\tau$ from $10^6$ to $10^7$ yr and from $10^7$ to $10^8$ yr, respectively, for both considered volumes. The considerably large number ratio in this $B$ interval may be the result of strong magnetic field decay under which PSRs in the interval $10^{12} \leq B \leq 10^{13}$ G penetrate into the interval $10^{11} \leq B \leq 10^{12}$ G. In the latter magnetic field interval PSRs can not reach to large values of period as they do under large values of $B$, because the death-belt is parallel to the constant $\dot{E}$ lines. As a result of the diminishing of the perpendicular component of the magnetic field, PSRs penetrate through the death-belt and turn-off in considerably small time intervals. Thus, the increase in the difference between the real age and the characteristic time under the increasing of the value of $\tau$ can be understood.

3 A direct evidence for the strong magnetic field decay

In the previous section, we see that the number density of PSRs increases in the direction of increasing value of $\tau$, but not as a linear dependence. On
the other hand, this increase depends on the depth of penetration through the death-belt, the value and the decay of the magnetic field, and the period values of PSRs. Let us now discuss this problem using another approach.

The average distance of PSRs from the plane of the Galaxy is

$$|z| = |V_z| \times t$$

(2)

where $V_z$ is the value of the z-component of the space velocity and $t$ is the real age of PSR. As

$$t = \frac{P}{(n-1)P} (1 - \left(\frac{P_0}{P}\right)^{n-1})$$

(3)

value of $\tau$ is equal to or linearly depends on $t$ for small values of initial spin periods ($P_0<<P$) and

$$|z| \sim \tau.$$  

(4)

In Figure 4, $|z|$ versus $\tau$ distribution of the PSRs with $d \times \cos b \leq 3$ kpc and $P>10^{-17}$ s/s is represented. Effect of the radio luminosity and the sensitivity of the latest PSR surveys may have some influence on this dependence. In order to exclude (or diminish) this effect, we have used only the PSRs which were observed at 400 MHz and which have $\log L_{400} = \log (F_{400} \times d_{kpc}^2) \geq 1$. As seen from Figure 4, the average value of $|z|$ increases about a factor of 2 if we compare the PSRs in the intervals $10^6 < \tau < 10^7$ yr and $10^7 < \tau < 10^8$ yr (this increase may be up to a factor of 3 if we take into account the decrease in PSR luminosity and comparably worse search of PSRs at large latitude values). When we compare the PSRs in the intervals $10^7 < \tau < 10^8$ yr and $10^8 < \tau < 10^9$ yr we see that the average value of $|z|$ practically does not increase beyond $\tau \sim 10^7$ yr. This strongly contradicts the existence of a linear dependence between the real and the characteristic ages for PSRs with $\tau > 10^6$ yr and the evolution of PSRs along B=const lines if the component of the magnetic field perpendicular to the rotation axis does not decrease. In principle this was known many years ago but on the basis of small statistical data and with larger uncertainties in the distance values of PSRs. The differences between the characteristic times and the kinematic ages do not depend on the beaming fraction nor on the effects which may influence the character of the number density change under the increasing of $\tau$.

We have also constructed the dependence of $|z|$ on $\tau$ for the same volume as in Figure 4 but for the PSRs with $10^{11} < B < 10^{12}$ G. The average values
of $|z|$ in different $\tau$ intervals for this sample are practically the same as in Figure 4. It is especially necessary to note that kinematic ages of the PSRs with $\log \tau \geq 8.5$ are about 2 orders of magnitude smaller than the $\tau$ values as seen from Figure 4. As real ages of the PSRs with $\log \tau \geq 7.5$ are considerably smaller than the $\tau$ values their location on this part of the P-\dot{P} diagram shows that the component of the magnetic field perpendicular to the rotation axis diminishes more than one order of magnitude (see Fig. 2).

From Figure 4, not only the absence of a linear dependence between the real ages and the characteristic times but also the absence of a linear dependence of the number density of PSRs on $\tau$ is seen, similar to Figure 2. These effects may strongly be the result of also the influence of the decrease in the beaming angles (and in accordance the beaming fraction which is related to it) as $t$ and $\tau$ increase. Of course it is in more degree a result of the decrease in the magnetic field and the magnetic polar areas.

In Figure 2, the PSRs which have also been detected in X-rays are displayed with sign ‘X’. The X-ray luminosity strongly depends on the value of $\dot{E}^{20-22}$ and the $\dot{E}$ values of the PSRs in Figure 2 are in a wide interval from $\sim 10^{37}$ erg/s down to $\sim 10^{29}$ erg/s. Therefore, the number density of the PSRs with small $L_x$ values is very high, though this is not seen directly from Figure 2 which represents a considerably large volume. If we take into account these facts, then from the locations of these objects on the P-\dot{P} diagram we can be sure about the decay in the magnetic field (by diminishing of the magnetic field component which is perpendicular to the rotational axis) of a factor about 3-4 until $\tau \sim 10^6$ yr.

The large birth rate of PSRs with $B > 10^{13}$ G, which later drop down to $10^{11} \leq B \leq 10^{12}$ G interval, supports their considerably large number density at large values of $\tau$. Although the magnetic field of these PSRs decreases, in general they remain in the considered interval of the magnetic field. The braking index can be also less than 3 at some points during the evolution up to $\tau \sim 10^6$ yr, but the dominant factor must be the field decay and so the braking index must be greater than 3 throughout the whole evolutionary track.

Let us now discuss the character of the magnetic field decay in relation to the observational data. Above, from the analysis of the observational data, we have seen that the component of the magnetic field perpendicular to the rotational axis must decrease and the character of this decrease must roughly be a simple exponential law. In this case, the characteristic time ($\tau = P/2\dot{P}$
for n=3) of PSRs depends on both the characteristic time of the field decay ($\tau_d$) and the real age ($t$):

$$\tau = \frac{\tau_d}{2} e^{2t/\tau_d}$$

(5)

if $t >> \tau_d$. If we adopt $\tau_d = 3 \times 10^6$ yr, then the evolutionary tracks of PSRs approximately lead to the actual distribution of PSRs on the P-\(\dot{P}\) diagram. In order to estimate the values of $P$ and $\dot{P}$ we can use the equation

$$P \dot{P} = \frac{B_0^2}{10^{39}} e^{-2t/\tau_d}.$$ 

(6)

In Table 1, $t$, $\tau$, $P$ and $\dot{P}$ values (found from eqn.(4) and eqn.(5)) for 2 different values of $B_0$ are represented. The positions of PSRs (corresponding to $t=6 \times 10^6$ yr and $9 \times 10^6$ yr) with initial magnetic field values $10^{12}$ G and $10^{13}$ G under the exponential magnetic field decay with a characteristic time of $3 \times 10^6$ yr are displayed in Figure 2.

4 Discussion and Conclusions

As known, the X-ray radiation depend very strongly on the value of rate of rotational energy loss ($\dot{E}$) of PSRs. Moreover, the acceleration of particles mainly take place in the field of magnetodipole radiation wave. Therefore, the 'death-belt', as commonly adopted, passes parallel to the constant $\dot{E}$ lines. PSR radio radiation may begin to turn off practically for $\tau \sim 10^6$-$10^8$ yr depending on the magnetic field value (correspondingly $10^{13}$-$10^{11}$ G), in other words, beginning from $\dot{E} \sim 10^{32}$-$10^{31}$ erg/s. Processes for the turn-off become very strong as $\dot{E}$ decreases. In spite of these processes, the radio radiation of PSRs practically do not decrease till the turn-off or we can say that the decrease in their radio radiation is not important in our investigation. The evolutionary tracks of PSRs on the P-\(\dot{P}\) diagram and the places of the turn-off strongly depend on the magnetic field decay which has roughly an exponential character. As follows from our analysis of the data, the characteristic time of this decay is about $3 \times 10^6$ yr. Following this discussion we understand that the main and dominant reason for the considerable deviation of the direction of the increase in the number density of PSRs (direction of evolution) from the constant B lines is the magnetic field decay and the turning off of PSRs under the same values of $\dot{E}$. 

8
The problem under discussion is the oldest one in PSR astronomy. As we have seen in the sections above, the main reason to suggest a magnetic field decay more than one order of magnitude up to $\sim 10^7$ yr is that the kinematic ages of PSRs practically do not increase for $\tau \geq 3 \times 10^7$ yr (see Figure 4). The strong decay of the magnetic field also comes from the distribution of single X-ray PSRs including the PSRs which radiate due to cooling. In principle, we can explain the direction of the increase in the number density of PSRs on the P-P diagram and the absence of a correlation between the number of PSRs and $\tau$ based on decreasing of the luminosity and the turn-off. But these ways are too artificial compared to explaining the direction of the increase in the number density by magnetic field decay.

The idea of exponential decay of the magnetic field of PSRs was more popular in the past, before the observation of millisecond PSRs with weak magnetic fields (i.e. $B < 10^{10}$ G). After estimating the magnetic field of neutron stars in X-ray binary systems, the idea of magnetic field decay without accretion was rejected. In this work we see from the analysis of the observational data that the component of the magnetic field perpendicular to the spin axis decreases significantly. The decay must weaken after $t \sim 10^7$ yr. We think that Ruderman \(^3\) may be right not only about the character of the magnetic field decay but also about the increasing or the invariability of the magnetic field when PSRs are very young. There exist many complex problems about the evolution of magnetic field of neutron stars, which date back to the seventies. \(^24,25\) The magnetic field decay is necessary to explain the evolution of PSRs based on the actual/available observational data and especially to get rid of the discrepancy between $\tau$ and $t$. 
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Table 1 - The values of $t$, $\tau$, $P$ and $\dot{P}$ found from equations (4) and (5) for 2 different values of $B_0$ ($\tau_d=3 \times 10^6$ yr).

| $B_0$ (10$^{12}$ G) | $t$ (10$^6$ yr) | $\tau$ (10$^6$ yr) | $P$ (s) | $\dot{P}$ (10$^{-18}$ s/s) |
|----------------------|-----------------|---------------------|--------|---------------------------|
| 10                   | 6               | 80                  | 3.0    | 610                       |
|                      | 9               | 583                 | 3.1    | 84                        |
| 1                    | 6               | 80                  | 0.3    | 60                        |
|                      | 9               | 583                 | 0.31   | 8.38                      |
Figure Captions

Figure 1: P-\dot{P} diagram of all PSRs with \dot{P}>10^{-17} s/s in the ATNF Pulsar Catalogue.

Figure 2: P-\dot{P} diagram of all PSRs with d×\cos b \leq 3 kpc. The PSRs which have also been detected in X-rays are denoted with 'X'. There are 8 PSRs with \tau \leq 10^5 yr, 41 PSRs with \tau \leq 10^6 yr, 181 PSRs with \tau \leq 10^7 yr, and 299 PSRs with \tau \leq 10^8 yr. The 'black squares' show the positions of PSRs under exponential decay of the perpendicular component of the magnetic field: the two squares on the right are for the case of B_0=10^{13} G (for the upper one t=6×10^6 yr and for the lower one t=9×10^6 yr) and the ones on the left are for B_0=10^{12} G (again for the upper one t=6×10^6 yr and for the lower one t=9×10^6 yr).

Figure 3: P-\dot{P} diagram of PSRs with d×\cos b \leq 2 kpc. The 30 PSRs which have the highest L_{1400} values are displayed with 'X'. The 50 PSRs which have the lowest L_{1400} values are shown as 'circles'. The other PSRs are represented with 'plus' signs.

Figure 4: |z| versus log \tau diagram of the PSRs which have \dot{P}>10^{-17} s/s, d×\cos b \leq 3 kpc and log L_{400} \geq 1.
