Looking from the East at an Elephant Trotting West 

Direct $CP$ Violation in $B^0$ Decays

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Abstract

With the time dependant CP asymmetry in $B_d(t) \rightarrow \psi K_S$ well measured, the most powerful method to search for direct CP violation in $B_d$ decays is to analyze whether the CP asymmetry of other transitions like $B_d(t) \rightarrow \pi^+\pi^-, \eta'K_S$ etc. can be described by a $\sin$ term taken from $B_d(t) \rightarrow \psi K_S$ and without a $\cos$ term. The failure of such a constrained fit would establish direct CP violation (even if no CP asymmetry were observed in these other modes).

Prologos

The occurrence of large $CP$ violation in $B_d \rightarrow \psi K_S$ predicted in 1981 [1] has been established in 2001 [2, 3]. BABAR and BELLE have just now presented updates:

\[
\sin(2\phi_1) = 0.82 \pm 0.12 \pm 0.05 \quad \text{BELLE [5]}
\]

\[
\sin(2\beta) = 0.75 \pm 0.09 \pm 0.04 \quad \text{BABAR [4]}
\]

These findings are of a paradigmatic nature in several respects even beyond the obvious one that they constitute the first observation of CP violation outside the $K_L$ system: (i) The effect being truly large ‘de-mystifies’ CP violation: for it tells us that if CP invariance can be broken, that violation is not intrinsically small [6]. (ii) While the observed asymmetry could still contain sizeable contributions from New Physics, it conforms to the CKM prediction in a completely unforced way. It

\[1\text{With due apologies to Sir Charles Laughton.}\]
behooves us now to refer to the CKM description of CP violation as a *theory* rather than giving it the somewhat patronizing label *ansatz*.

**Parabasis**

Despite the success of the CKM theory so far, it would be premature to accept even the overall pattern of its predictions and consider only deviations from it as noteworthy. In particular there is another paradigmatic issue to be raised concerning the classification of the underlying dynamics: is there also *direct* CP violation, i.e. CP violation in $\Delta B = 1$ transitions? The CKM theory definitely does not produce a superweak scenario for $B$ decays – alas, one still has to verify it.

Observing a CP asymmetry in charged $B$ decays would establish it unequivocally. The situation concerning neutral $B$ decays is more subtle. Consider

$$ e^+ e^- \rightarrow \Upsilon(4S) \rightarrow B_d \bar{B}_d \rightarrow f_{tag} f_{CP} $$

(3)

where $f_{tag}$ denotes a final state tagging the flavour of one beauty meson and $f_{CP}$ a CP eigenstate, into which the other beauty meson decays and which is reconstructed. Then one finds for the decay rate $R_+ [R_-]$ where the tagging decay is that of a $B_d$ $[\bar{B}_d]$:

$$ R_\pm(\Delta t) \propto e^{-|\Delta t|\Gamma(B_d)} \times [1 \pm S\sin(\Delta m_d \Delta t) \pm C\cos(\Delta m_d \Delta t)] $$

(4)

with $\Delta t$ denoting the difference in (proper) time of the two decays $[\Delta t \equiv t_{CP} - t_{tag}]$, $\bar{\rho}(f_{CP})$ the ratio between the two decay amplitudes $[\bar{\rho}(f_{CP}) \equiv T(\bar{B}_d \rightarrow f_{CP})/T(B_d \rightarrow f_{CP})]$ and $q/p$ coming from the diagonalization of the $B_d - \bar{B}_d$ mass matrix. The asymmetry then reads:

$$ \frac{R_+(\Delta t) - R_-(\Delta t)}{R_+(\Delta t) + R_-(\Delta t)} = S\sin(\Delta m_d \Delta t) + C\cos(\Delta m_d \Delta t) $$

(5)

Observing a *cos* term in the time evolution establishes direct CP violation, since in that case $|T(\bar{B} \rightarrow f_{CP})|^2 \neq |T(B \rightarrow f_{CP})|^2$ – an obvious and well-known statement. For this to happen, two transition amplitudes with different weak and strong phases have to contribute; this situation is realized due to the intervention of Penguin operators.

However – as is known as well [7], though less appreciated – direct CP violation can manifest itself also in a different way. The quantity $(q/p)\bar{\rho}(f_{CP})$ reflects $\Delta B = 2$ and $\Delta B = 1$ dynamics in the first and second factors, respectively; $(q/p)$ and $\bar{\rho}(f_{CP})$ separately depend on the phase convention $\xi$ adopted for the definition of antiparticles: $|\bar{B}_d \rangle \equiv e^{i\xi\text{CP}}|B_d \rangle$. Their product, however, is phase convention independent and thus an observable. As long as CP violation is studied in a single channel $B_d(t) \rightarrow f_{CP}$, one cannot draw an empirical distinction between classifying a signal as ‘superweak’ or ‘direct’ from measuring the *sin* term; for a change in the
phase convention $\xi \rightarrow \xi + \Delta \xi$ can make either $q/p$ or $\bar{\rho}(f_{CP})$ real. A superweak scenario of CP violation is thus best defined as one where there exists a convention for the phase $\xi$ s.t. all $\Delta B = 1$ decay amplitudes are real. However once one studies the time evolution in two different channels

$$B_d(t) \rightarrow f_{CP} \text{ vs. } B_d(t) \rightarrow \tilde{f}_{CP}$$

an empirical distinction can be drawn: in a superweak scenario the cos term is effectively absent since $|\langle q/p \rangle \bar{\rho}(f_{CP})|^2 = |\langle q/p \rangle \bar{\rho}(\tilde{f}_{CP})|^2 = |q/p|^2 \simeq 1$ with $|q/p| = 0.998 \pm 0.006 \pm 0.007$ experimentally [5]; likewise the sin term is then of equal size for both modes:

$$S(f_{CP}) = \frac{2}{1 + |\frac{q}{p}|^2} \Im \bar{\rho}(f_{CP}) = \eta \bar{\eta} S(\tilde{f}_{CP}),$$

where $\eta_1$ and $\bar{\eta}_1$ denote the CP parities of $f_{CP}$ and $\tilde{f}_{CP}$, respectively. Any deviation from this simple pattern reveals direct CP violation! One should also note that if the two transitions $B_d \rightarrow f_{CP}$ and $B_d \rightarrow \tilde{f}_{CP}$ are each described by a single amplitude or if there are no significant final state interactions in those two modes, then one has $|\bar{\rho}(f_{CP})|^2 = 1 = |\bar{\rho}(\tilde{f}_{CP})|^2$. Yet $\Im \langle q/p \rangle \bar{\rho}(f_{CP}) \neq \Im \langle q/p \rangle \bar{\rho}(\tilde{f}_{CP})$ can hold due to different weak phases in the $\Delta B = 1$ amplitudes for $B_d \rightarrow f_{CP}$ and $B_d \rightarrow \tilde{f}_{CP}$. I.e., such direct CP violation could not manifest itself through a cos term, only through a difference in the sin terms.

Such a method becomes quite powerful now, since the well measured asymmetry in $B \rightarrow \psi K_S$, Eqs.(4,5), provides an excellent calibrator: the sin term has been well measured and a fairly tight bound placed on the cos term. I.e., one considers other $B_d \rightarrow \tilde{f}_{CP}$ decays into CP eigenstates and analyzes whether the time evolution of the rate can adequately be described without a cos term and with a sin term that has the same [opposite] coefficient as in $B_d \rightarrow \psi K_S$ if $\tilde{f}_{CP}$ is CP odd [even].

BELLE and BABAR have presented data on two such additional channels:

- **BELLE** describes the time evolution of the asymmetry in the decay $B_d \rightarrow \eta' K_S$ with just a sin term [3]:

$$\Im \frac{q}{p} \bar{\rho}(\eta' K_S) = 0.29^{+0.53}_{-0.54} \pm 0.07$$

This result is consistent with no asymmetry, which would imply large direct CP violation to make it consistent with the findings in $B_d \rightarrow \psi K_S$. At the same time it falls within one sigma of a superweak CP violation (both $\eta' K_S$ and $\psi K_S$ are CP odd).

- **BELLE** fits the $B_d(t) \rightarrow \pi^+ \pi^-$ evolution using a sin as well as cos term with coefficients [3]

$$S = -1.21^{+0.38+0.16}_{-0.27-0.13} \quad (9)$$
$$C = +0.94^{+0.25}_{-0.33} \pm 0.09 \quad (10)$$

\[ \text{2Theoretically one expects } |1 - |q/p|^2| \sim O(10^{-3}). \]
\textit{BABAR} on the other hand finds \[9\] \[11\]:

\[
S = -0.01 \pm 0.37 \pm 0.07 \\
C = +0.02 \pm 0.29 \pm 0.07 .
\]

Those two sets of numbers convey the sense of two different messages: the BELLE analysis presents tantalizing evidence for a CP asymmetry in a second class of $B$ decays that furthermore has a large direct component, whereas the \textit{BABAR} study shows no evidence for a CP asymmetry. Future measurements will clarify the experimental picture.

The first step towards properly interpreting the data is to analyze whether the time evolution of $B_d(t) \rightarrow \pi^+ \pi^-$ contains a \textit{qualitatively new} feature beyond what is seen in $B_d(t) \rightarrow \psi K_S$, i.e. whether there is direct CP violation. This question has to be answered experimentally irrespective of the KM theory predicting such an effect! This is most efficiently achieved by studying whether the CP asymmetry in $B_d(t) \rightarrow \pi^+ \pi^-$ can be described with $C = 0$ and $S_{\pi \pi} = -S_{\psi K_S}$ ($\pi \pi$ and $\psi K_S$ carry opposite CP parity), see Eqs.\((11)\), i.e. \textit{without} any free fit parameter! What drives $B_d(t) \rightarrow \psi K_S$ has to contribute to $B_d(t) \rightarrow \pi^+ \pi^-$ as well. Before such a ‘minimal’ scenario has not been ruled out, it is \textit{premature to undertake fits where $S$ and $C$ are allowed to float}! Only after direct CP violation has been established due to the failure of a superweak fit (or quantitative theoretical control has been established over the impact of Penguin operators) should one concentrate on fitting the data with $S$ and $C$ not a priori fixed. The goal there is to separate the two variants of direct CP violation, namely Im$^4 \rho(\psi K_S) \neq -\text{Im}^2 \rho(\pi \pi)$ and $|\rho(\pi \pi)|^2 \neq 1$, and ultimately also of isolating the Penguin contribution.

One should note that a fit to $B_d(t) \rightarrow \pi^+ \pi^-$ yielding $S \simeq 0 \simeq C$ as suggested by the \textit{BABAR} findings – viewed together with the data on $B_d(t) \rightarrow \psi K_S$ – would reveal the intervention of large \textit{direct} CP violation \textit{without} exhibiting any CP asymmetry!

\textit{In summary:} The main point of this short note is to emphasize that one should and can empirically establish the categorical feature whether direct CP violation occurs in $B_d$ decays. This is best achieved by analyzing whether the time evolution of a CP asymmetry in $B_d(t) \rightarrow \tilde{f}_{CP}, \tilde{f}_{CP} \neq \psi K_S$ can be described \textit{without} a \textit{cos} term and with a \textit{sin} term of equal size as in $B_d(t) \rightarrow \psi K_S$. Once it has been decided that such a description without a free fit parameter fails, one can address the more challenging task to extract the coefficients of the \textit{sin} and \textit{cos} terms in an unconstrained fit with the ultimate goal of determining the size of the Penguin

\footnote{With the definition for $C$ given in Eq.\((11)\) $C$ has the opposite sign from the \textit{BABAR} convention.}
contribution, phases etc.

Epilogos

The great actor Sir Charles Laughton once referred to his face as looking like the east side of an elephant walking west. The point of analogy here differs from this imagery, though: Evaluating the beauty of an elephant is a rather complex task, yet spotting him from any direction should be quite straightforward.

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