Mathematical model of interaction between forest machine mover and consolidating soil

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Abstract. The paper aims at development of mathematical model of interaction process between forest machine rover and soil, the properties of which change during the interaction process. The study bases on the off-road operation theory and methods of mathematical analysis. The basis of the mathematical model includes equation of interaction between a mover die and deformable soil mass of specified thickness placed on a rigid non-deformable base. The model uses power dependence of the mass deformation modulus on its density, which is changed under the impact of the mover die following the relative compression of the soil. The equation solution for sinkage is obtained in form of Taylor polynomials, this allows obtaining the rut depth calculated value without iterations, which simplifies development of automated workplaces designed to evaluate forestry machines and cutting sites soils interaction at timber harvesting planning stage. Using an example of interaction between forest machine mover and weak forest soils, the paper shows the difference between rut depth calculated values, obtained using proposed mathematical model, and the values, obtained with a constant value of the deformation modulus. With a contact pressure in range of 0.06-0.1 MPa, rut depth predicted values are lower by 30-60%. In conclusion the paper notes prospective areas of further research, which include studying the changes of soil physical and mechanical properties, characterizing its shear resistance, and development of mathematical models that take into account the soil bearing capacity, the value of which also changes during compaction.

1. Introduction.

Studying of interaction between forest machine movers and soils is an important research area of logging production science. Movers of wheeled and tracked vehicles cause the soils compaction and rutting, which is one of the indicators for assessing ecological compatibility of wood harvesting technology [1]. In addition, rutting study results present a base for forest machine tractive performance evaluation. When the soil is being compacted, its mechanical properties increase, this leads to soil mass solidification. This phenomenon has not received a complete scientific description so far.

Aim of the study is to present a mathematical model of the interaction process between forest machine mover and the ground, the properties of which change in the course of interaction process.

Research methods - off-road operation theory, methods of mathematical analysis.
Results of the study. When developing the mathematical model, we consider machine mover as a die affecting a deformable soil mass of specified thickness laying on a rigid non-deformable base, by analogy with [4], [5], [6]. Figure 1 presents the scheme.

![Figure 1. Scheme of the interaction process: 1 – machine mover, 2 - deformable soil mass, 3 - rigid (non-deformable) base](image)

Following equation determines deformation of elementary layer of the mass [7], [8]:

\[ dh = \frac{\varepsilon}{1-\varepsilon} dz \]  (1)

where \( \varepsilon \) is the relative deformation of elementary layer of the soil, \( z \) is the coordinate reckoned from the site of contact of the rover and the ground.

Total compressive strain of the soil mass is determined by the formula [7], [8]:

\[ h = \frac{H-h}{1-\varepsilon} \]  (2)

where \( H \) is deformable soil layer thickness.

Traditionally, the solution of the problem is obtained using a linear expression [7], [8]:

\[ E = \frac{\sigma}{\varepsilon} \]  (3)

where \( E \) is deformation modulus of the soil, \( \sigma \) is normal stress in the soil mass, decreasing with depth [7], [8]:

\[ \sigma = \frac{Jp}{1+\left(\frac{z}{aB}\right)^2} \]  (4)

where \( J \) is the coefficient of geometrical parameters of the contact patch, \( p \) is the average pressure of the mover on the surface, \( a \) is the deformable soil mass thickness coefficient, \( B \) is width of the contact patch.

The deformation modulus in formula (3), as a rule, is considered to be a constant, independent of \( \sigma, \varepsilon \) [4], [5], [6].

We take the change in \( E \) during compaction into account. We should clarify that in geometric interpretation of the stress-strain dependence, \( E \) is tangent of the angle of the tangent to the stress \( \sigma \) plotted on strain \( \varepsilon \). For a nonlinear function \( \sigma (\varepsilon) \), we define the “instantaneous” modulus \( E \) as:

\[ E = \frac{d\sigma}{d\varepsilon} \]  (5)
The relative deformation $\varepsilon$ is related to the density of the soil $\rho$ by the formula [9]:

$$\rho = \frac{\rho_0}{1 - \varepsilon}$$  \hspace{1cm} (6)

where $\rho_0$ is the initial density of the soil before the stress $\sigma$.

Research results show that the strain modulus $E$ is expressed in terms of the density of the soil by a power function [9], [10]:

$$E = a_E \rho^b_E$$  \hspace{1cm} (7)

where $a_E$, $b_E$ are numerical coefficients depending on the properties of the soil, its type and state.

For example, forest soils are usually classified by categories. Table 1 presents characteristic values of $\rho$, $E$, $H$ for various categories of forest soils.

| Soil category | $\rho$, g/cm$^3$ | $E$, MPa | $H$, m |
|---------------|-----------------|---------|-------|
| Weak (III)    | 0.75            | 0.4     | 0.8   |
| Normal (II)   | 1               | 1       | 0.4   |
| Stiffish (I)  | 1.25            | 3       | 0.3   |

According to the data from Table 1, using the least squares method, we establish that for forest soil the coefficients in formula (7) are $a_E = 1.16$, $b_E = 3.91$.

Equation (7) with the expression (6) appears as follows:

$$E = a_E \left( \frac{\rho_0}{1 - \varepsilon} \right)^b_E = E_0 \left( \frac{1}{1 - \varepsilon} \right)^b_E$$  \hspace{1cm} (8)

where $E_0$ is deformation modulus of the soil by the formula (7) with $\rho = \rho_0$ (before the interaction).

Basing on the formulae (5), (8), we obtain:

$$\frac{d\varepsilon}{d\sigma} = \frac{1}{E_0} \left( 1 - \varepsilon \right)^{-b_E}$$  \hspace{1cm} (9)

Solution of the differential equation (9) with the initial condition $\varepsilon (0) = 0$ has the form:

$$\varepsilon = 1 - \left( \frac{E_0}{\sigma (b_E - 1) + E_0} \right)^{-\frac{1}{b_E - 1}}$$  \hspace{1cm} (10)

Immediate attempt to take the definite integral in (2) with (4), (10) leads to mathematical difficulties. To facilitate the solution and simplify the subsequent calculations, we represent the integrand in equation (2) in form of a Taylor polynomial:

$$\varepsilon = \left( \frac{b_E - 1}{E_0} \right) \sigma + \left( \frac{b_E^2 - 7b_E + 6}{3E_0} \right) \sigma^2 + \left( -\frac{6b_E^3 - 29b_E^2 + 46b_E - 24}{4E_0^4} \right) \sigma^3$$  \hspace{1cm} (11)

where $f_1$, is the derivative of the function $f$ of order $n$ with respect to $\sigma$ at the point 0, the coefficients $\varsigma_n$ with $\sigma_0$:

$$\varsigma_1 = \frac{1}{E_0}, \quad \varsigma_2 = -\frac{b_E}{2E_0^2}, \quad \varsigma_3 = \frac{2b_E^2 - 7b_E + 6}{3E_0^3}, \quad \varsigma_4 = -\frac{6b_E^3 - 29b_E^2 + 46b_E - 24}{4E_0^4}$$  \hspace{1cm} (12)

The coefficients $\varsigma_n$ do not depend on $z$, thus equation (2) appears as follows:

$$h \approx \int_{0}^{z} \frac{1}{\sigma} d\zeta = \sum_{n=1}^{k} \frac{H^{-\rho}}{\varsigma_n}$$  \hspace{1cm} (13)

Using formula for indefinite integral of a rational fraction:
Taking into account expressions (13), (15) the equation for determining soil mass compression (2) presents as follows:

\[ h \approx \sum_{n=1}^{k} \xi_{n} \lambda_{n} \]  

(16)

where \( L \) is the length of the contact patch.

Calculations when varying \( B = 0.4 \ldots 0.8 \text{ m}, L = 0.8 \ldots 5 \text{ m}, E, H \) according to table 1, \( h = 0 \ldots 0.8H \), show that in equation (16) it is possible to limit \( k = 2 \) practically without loss of the accuracy, then:

\[ h = \xi_{1} \lambda_{1} + \xi_{2} \lambda_{2} = \frac{J \rho a B}{E_{0}} \arctg \left( \frac{H - h}{aB} \right) + \frac{J^{2} p^{2} aB}{4E_{0}^{2}} \left( \frac{aB(H - h)}{a^{2}B^{2} + (H - h)^{2}} + \arctg \left( \frac{H - h}{aB} \right) \right) \]  

(17)

Moreover, in the same ranges of variation:

\[ \frac{aB(H - h)}{a^{2}B^{2} + (H - h)^{2}} \approx \frac{aB}{H} \arctg \left( \frac{H - h}{aB} \right) \]  

(18)

\[ \arctg \left( \frac{H - h}{aB} \right) \approx \frac{3}{4} \left( \frac{H - h}{aB} \right) \]  

(19)

Taking into account (12), (15), (18), (19), we obtain from (17):

\[ h = \frac{3}{4} J \rho \frac{(H - h)}{E_{0}} - \frac{3}{16} \frac{J^{2} p^{2} (b_{k} - 2)(aB + H)}{E_{0}^{2} H} (H - h) \]  

(20)

Solving equation (20) for \( h \), we get:

\[ h = \frac{pC \xi}{1 + pC \xi} H \]  

(21)

where \( C \) is the stiffness of the soil massif, \( \xi \) is the dimensionless function of soil massif strengthening:

\[ C = \frac{3}{4} \frac{J}{E_{0}} \]  

(22)

\[ \xi = 1 - \chi p \]  

(23)

where \( \chi \) is the coefficient of soil massif strengthening during compaction:

\[ \chi = \frac{1}{4} \frac{J}{E_{0}} \frac{aB + H}{H} (b_{k} - 2) \]  

(24)

The parameters \( a, J \) in expressions (22), (24) are calculated by the formulae [7], [8], [10]:

\[ a = 0.64 \frac{B + H}{H} \]  

(25)
2. Example of calculation results and conclusions.

Comparison of the results of the computation of compression \( h \) by the formula (21) with the solution of equation (2) with \( E = \text{const} \) is presented in graphs in Figure 2 \((B = 0.7 \text{ m}, L = 1.3 \text{ m}, \text{III category of forest soil})\).

\[
J = \frac{0.03B + L}{0.43B + 0.6L}
\]  

(26)

![Graph showing forest soil compression](image)

**Figure 2.** Forest soil category III compression depending on mover’s pressure

As the graphs show, the calculated values of the rut depth \( h \), obtained using the proposed mathematical model, differ markedly from the values obtained by calculation with a constant value of the deformation modulus. With the pressure in range of 0.06-0.1 MPa, which is characteristic of the interaction of forest machines wheeled movers with the soil, the \( h \) values predicted by the proposed model are lower by 30-60%.

Figure 3 illustrates graphically the obtained results of the calculation of the change in \( E, \rho \) effected by the mover.

![Graph showing deformation modulus and density](image)

**Figure 3.** Deformation modulus and density of forest soil category III, depending on the mover pressure

Calculations show that with the pressure in range of 0.06-0.1 MPa, the density of the soil under the mover influence increases approximately by 10-15%, as a result, the deformation modulus increases by 50-90%.
In our opinion, the proposed mathematical model allows us to take stiffening of the soil under the mover’s pressure into account, thereby to develop the basis for theoretical studies of the mover and soil interaction. Equation (21), in contrast to the previously known ones, allows obtaining the calculated value of the rut depth $h$ without resorting to iterations, this simplifies the development of automated workplaces designed to evaluate forestry machines and cutting sites soils interaction at timber harvesting planning stage.

Further prospective research, in our opinion, should be focused on the following problems:
1. Studying changes in physical and mechanical properties of the soil, characterizing soil shear resistance.
2. Development of mathematical models that take into account bearing capacity of the soil, the estimated value of which also changes during compaction.

On the basis of the obtained results, in the future, it is possible to develop mathematical models that predict rovers’ tractive performance, which will be useful in assessing mobility of forest machines.

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