Low-lying mixed symmetry states in the N=84 isotones \(^{140}\)Ba and \(^{142}\)Ce within interacting boson model-2

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Abstract: The characteristics of the low-lying mixed-symmetry states for \(^{140}\)Ba and \(^{142}\)Ce in the even-even N=84 isotones are investigated within the framework of the IBM-2 model. Electromagnetic transitions, B(E2) and B(M1) were calculated. Results showed that the lowest mixed-symmetry state is \(2^+_5\) for \(^{140}\)Ba and \(^{142}\)Ce nuclei. A good agreement is found between experiment and predictions made using the U (5) limit of IBM-2.

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1. Introduction

The neutron-proton version of the interacting boson model (IBM-2) [1-4] is distinguish between proton boson and neutron boson, and contains states called mixed symmetry states. The N = 84 isotones have been the subject of many theoretical [5] and experimental [6] investigations into the collective and particle nature of low-lying excitations in nuclei near the closed shell regions (N = 82). Experimentally, J. Vanhoy et al. [7] have studied excited levels and electromagnetic transitions between these levels in \(^{152}\)Ce up to 3.3 MeV using the \((n, n'\gamma)\) reaction. W. Kim [8] investigated properties of low-lying states in \(^{142}\)Ce via high resolution electron scattering, and measured differential cross sections for inelastic scattering from \(^{142}\)Ce. Theoretically, J. Copnell et al. [9] studied a collective model using the interacting boson model-2 for the N = 84 isotones \(^{130}\)Xe to \(^{146}\)Sm, as well as an approach explicitly treating both the shell-model and collective-model degrees of freedom. A.R. Subber et al. [10] studied the level structure of even–even neutron-rich \(^{140-148}\)Ba isotopes in the framework of the interacting boson model, and calculated the reduced transition probabilities B(E2) of these nuclei. In this study, the calculations of positive-parity state energies of \(^{140}\)Ba and \(^{142}\)Ce nuclei have been done using IBM-2, and amplitudes squared of \(F = F_{\text{max}}\) component of \(2^+_m\) and \(2^+_l\) states as a function of Majorana parameter \(\xi_2\) of these nuclei are plotted.

2. The interacting boson model

The IBM-2 model has very successful in describe the collective properties in nuclei. when there are distinctions between proton and neutron bosons, the Hamiltonian can be written as [11]

\[
H = H_r + H_\nu + H_{rv}
\]  

(1)

The Hamiltonian generally used in phenomenological calculations can be written as[2]
The first term represents the single-boson energies for neutron and protons, \( \varepsilon_d \) is the energy difference between s- and d- boson and \( n_{d\rho} \) is the number of d-bosons, where \( \rho \) correspond to \( \pi \) (proton) or \( \nu \) (neutron) bosons. The second term denotes the main part of the boson-boson interaction, i.e. the quadrupole-quadrupole interaction between neutron and proton bosons with the strength \( \kappa \). In the IBM-2 model, the quadrupole moment operator is given by

\[
Q_\rho = (d_\rho^s s_\rho + s_\rho^d d_\rho^-)^{(2)} + \chi_\rho (d_\rho^s d_\rho^-)^{(2)}
\]

Where \( \chi_\rho \) determines the structure of the quadrupole operator and is determined empirically. The terms \( V_{\pi\pi}^\wedge \) and \( V_{\nu\nu}^\wedge \), which correspond to interaction between like-boson, are sometimes included in order to improve the fit to experimental energy spectra. They are of the form

\[
V_{\rho\rho}^\wedge = \frac{1}{2} \sum_{L=0,2,4} C_\rho^{(L)} [d_\rho^s d_\rho^+ \rho^{-}]^{(L)} [d_\rho^- d_\rho^+ \rho^{-}]^{(L)}
\]

\( V_{\rho\rho}^\wedge \) shows from microscopic considerations represents the interaction between same bosons. The Majorana term \( M_{\nu\nu} \) which contains three parameters \( \xi_1, \xi_2 \) and \( \xi_3 \)

\[
M_{\nu\nu} = \frac{1}{2} \xi_1 \{s_\pi^d d_\pi^+ \rho^{-} - d_\pi^s s_\pi^+ \rho^{-}\}^{(2)} + \xi_2 \{s_\nu^d d_\nu^+ \rho^{-} - d_\nu^s s_\nu^+ \rho^{-}\}^{(2)} - \sum_{k=1,3} \xi_k \{d_\nu^s d_\nu^+ \rho^{-}\}^{(k)} [d_\nu^s d_\nu^+ \rho^{-}]^{(k)}
\]

is responsible for the location of mixed symmetry states with respect to fully symmetric ones.

### Table 1: IBM-2 parameters (in MeV unit) used for the description of positive parity states of \(^{140}\)Ba and \(^{142}\)Ce nuclei, except \( \chi_\pi \) and \( \chi_\nu \) which are dimensionless.

| Nuclei  | N  | N_\pi | N_\nu | \varepsilon_d | \kappa_\pi | \kappa_\nu | \chi_\pi | \chi_\nu | \xi_1 | \xi_2 | \xi_3 | C_\pi^0 | C_\pi^2 | C_\nu^0 | C_\nu^2 | C_\nu^4 |
|---------|----|-------|-------|---------------|------------|------------|---------|---------|-------|-------|-------|--------|--------|--------|--------|--------|
| \(^{140}\)Ba | 4  | 3     | 1     | 0.80          | -0.29      | -0.15      | 0.93    | 0.18    | 0.24  | 0.18  | 0.08  | -0.40  | 0.00   | 0.00   | 0.00   |
| \(^{142}\)Ce | 5  | 4     | 1     | 0.86          | -0.31      | -0.15      | 0.10    | 0.06    | 0.35  | 0.07  | -0.27 | 0.00   | 0.00   | 0.00   |

3. Results and discussion

In \(^{140}\)Ba and \(^{142}\)Ce isotopes (N = 84), both valence protons and neutrons are in the same major-shell between shell-closures 28 and 50. For these isotopes the parameters \( \chi_\pi \) and \( \chi_\nu \) have the values -0.10 and -0.15. The values deduced for the IBM-2 Hamiltonian parameters are reported in Table 1. The parameters \( C_\rho^L \) (L = 0,2,4) is set to zero. A comparison between experimental [13] and calculated, positive parity states, below 3 MeV is shown in Figs. 1-2. It can be seen from Fig.1 that the experimental energy value of \( 6^+ \) level is equal to 1.660 MeV, which agrees well with the calculated one 1.660 MeV. In \(^{140}\)Ba, the identification of the \( 1^+_2 \) state as the mixed symmetry state at 2.814 MeV is close to experimental ones at 2.873 MeV. For \( \beta \)-band energy levels in \(^{140}\)Ba isotope, its can see that the \( 0^+_2 \) and \( 2^+_2 \) states are predicted at 1.513 and 2.201 MeV and therefore their values are close to the experimental ones, which are 1.510 and 2.320 MeV, respectively. For \( \gamma \)-band energy levels, it can be seen that the \( 2^+_\gamma \) and \( 4^+_\gamma \) states are predicted at 1.513 and 2.201 MeV and therefore their values are close to the experimental ones, which are 1.510 and 2.320 MeV, respectively. For \(^{140}\)Ba the \( 1^+_2, 4^+_1, 3^+_2 \) and \( 5^+_2 \) states are dominated by the \( F = F_{\max} = \) configurations. The \(^{142}\)Ce has \( N_\pi = 4 \) and \( N_\nu = 1 \). As shown in Fig. 2, the ground state band is well reproduced for \(^{142}\)Ce isotope. In this figure, one can see that the calculated energy values of \( 6^+_1, 8^+_1 \) and \( 10^+_1 \) states are equal to 1.780, 2.337 and 3.501 MeV, these values are close to experimental ones which happen to be 1.743, 2.624 and 3.536 MeV, respectively.
The experimental value of $1^+_2$ state locates at 2.398 MeV, which has been identified experimentally as the known mixed-symmetry state from its decay properties, is perfectly reproduced by the calculated result at 2.631 MeV. Hamilton et al. [5] have suggested that the $2^+_3$ level at 2.004 MeV is a mixed-symmetry state, and equal to 2.009 MeV in present study. It is found that the calculated energy of $0^+_2 = 2.030$ MeV is very good agreement of the experimental value is equal to 2.030 MeV. The $2^+_3$ and $3^+_3$ states are also dominated by the $F = F_{\text{max}} - 1$ configurations. The $R$-values of low-lying states in IBM-2 for $^{140}$Ba and $^{142}$Ce nuclei are shown in Fig. 3. We have varied of the Majorana parameter $\xi_2$ around the best-value while keeping all other parameters at their best-fit values, the variations of the energy of these states with this parameter are shown in Fig. 4. As it is clear from this figure that illustrates that the $\xi_2$ strongly affects the energies of the levels considered to have mixed symmetry character.

In the U (5) dynamical limit of the IBM-2, the characteristic ratio $R_{4/2} = E_{4/2}/E_{2/2}$ is 2.00 for spherical nuclei. The characteristic ratio $R_{4/2}$ is 2.50 for the O (6) symmetry, and $R_{4/2} = 3.33$ for rigid rotors [12]. For $^{140}$Ba and $^{142}$Ce the experimental values of $R_{4/2}$ are 1.87, 1.90 the calculation gives $R_{4/2} = 1.93, 2.00$ respectively, both of them indicate that it are a near-harmonic. In table 2, we have introduced experimental and theoretical values of energy ratios in $^{140}$Ba and $^{142}$Ce together with the values of IBM limits.

![Figure 1: Calculated and experimental [13] energy level schemes for $^{140}$Ba isotope.](image1)

![Figure 2: Calculated and experimental [13] energy level schemes for $^{142}$Ce isotope.](image2)
4. Mixed Symmetry Stat

The full symmetry (FS) states are characterized by \( F = F_{\text{max}} \), while mixed symmetry (MS) states are characterized by \( F = F_{\text{max}} - 1, F_{\text{max}} - 2, \) etc. In the vibrational U(5) limit, the lowest mixed-symmetry state has \( j^n = 2^+ \). MS states occur when the motions of the proton and neutrons are not in same phase [14]. An important quantity indicating the F-spin nature of each state \( |J> \) is the ratio \( R \) defined by

\[
R = \frac{< J | F^2 | J >}{F_{\text{max}}(F_{\text{max}} + 1)},
\]

as a measure of the mixing F-spin states. Since we are interested mainly in the \( F = F_{\text{max}} \) and \( F = F_{\text{max}} - 1 \) states, we can assume a state has the following form

\[
|J> = \alpha |F_{\text{max}}> + \beta |F_{\text{max}} - 1>, \quad \alpha^2 + \beta^2 = 1 \quad (7)
\]

It is simple to calculate

\[
< J | F^2 | J > = \alpha^2 F_{\text{max}}(F_{\text{max}} + 1) + \beta^2(F_{\text{max}} - 1)F_{\text{max}}. \quad (8)
\]
where $\alpha$ and $\beta$ are important to measure the amount of mixed symmetry in each state. For $^{140}$Ba and $^{142}$Ce nuclei, the amplitudes squared of $F = F_{\text{max}}$ component of $2_{\text{ms}}^+$ and $2^+$ states as a function of Majorana parameter $\xi_2$ are plotted as shown in Fig. 5, and the $\xi_2$ varies around the best fitted value.

![Figure 5: Amplitudes squared of $F = F_{\text{max}}$ component of $2_{\text{ms}}^+$ and $2^+$ states as a function of Majorana parameter $\xi_2$ for $^{140}$Ba and $^{142}$Ce nuclei.](image)

### 5. Electromagnetic Transitions

We have investigated the electromagnetic transition properties in $^{140}$Ba and $^{142}$Ce using the IBM-2. For the calculation of reduced transition probability and moments the following one-body electromagnetic operators were considered

$$T^{(E2)} = e_\pi Q_\pi + e_\nu Q_\nu$$

$$T^{M1} = \frac{3}{4\pi} \left( g_\pi L^{(1)}_\pi + g_\nu L^{(1)}_\nu \right)$$

where $L^{(1)}_\rho = \sqrt{10} \left[ d^+ d^- \right]^{(1)}$$

$$T^{M1} = 0.77 \left[ (d^+ d^-)^{(1)}_\pi - (d^+ d^-)^{(1)}_\nu \right] (g_\pi - g_\nu)$$

However, direct measurement of B(M1) matrix elements is normally difficult, so the M1 strength of gamma transition may be expressed in terms of the multipole mixing ratio, which can be written as [15]

$$\delta \left( \frac{E^2}{M1} \right) = 0.835 \ E_\gamma \ (MeV) \Delta$$
where
\[
\Delta = \frac{\langle j_f \parallel T^{E2} \parallel j_i \rangle}{\langle j_f \parallel T^{M1} \parallel j_i \rangle}
\]
(13)

where \(e_p\) and \(e_n\) are the proton and neutron boson charges, \(g_p\) and \(g_n\) are g-factors for proton and neutron boson, \(L_\rho\) is the angular momentum operator. For a gamma ray decaying from \(j_i\) state to \(j_f\) state carried by a photon of multipole order \(L\), the reduced transition probabilities, for electric \(B(EL)\) and magnetic transition \(B(ML)\) are defined as follows
\[
B(E2; j_i \rightarrow j_f) = \frac{1}{2J_f+1} |\langle j_f \parallel T^{E2} \parallel j_i \rangle|^2
\]
(14)
\[
B(M1; j_i \rightarrow j_f) = \frac{1}{2J_i+1} |\langle j_f \parallel T^{M1} \parallel j_i \rangle|^2
\]
(15)

From Table 3, one can see that the computed \(B(E2)\) transition probabilities are in good agreement with the experimental data. The parameters \(e_\rho(p = \pi, \nu)\) in the \(T(E2)\) operator are estimated, by normalizing the calculated \(B(E2)\) value to the experimental value for the \(B(E2; 2^+_1 \rightarrow 0^+_1)\). In both \(^{140}\)Ba and \(^{142}\)Ce the effective charges \(e_\mathcal{R}\) are 0.14 \(e\) while \(e_\mathcal{V}\) is equal to 0.04 \(e\) and 0.02 \(e\) respectively. For the magnetic transitions, we take \(g_\mathcal{V}=0.05\ \mu_N\), \(g_\pi = 0.76\ \mu_N\) fitting to the experimental result of \(B(M1; 2^+_1 \rightarrow 2^+_2) = 0.13^{+0.01}_{-0.004} \mu^2_N\) for \(^{142}\)Ce nucleus. The calculated transition \(B(E2; 6^+_1 \rightarrow 4^+_2) = 0.0762 \text{e}^2\text{b}^2\) is consistent with the experimental one 0.081(4) \text{e}^2\text{b}^2\) for \(^{140}\)Ba. For \(^{142}\)Ce, the calculated value of \(B(E2; 2^+_2 \rightarrow 2^+_1)\) is 0.1308 \text{e}^2\text{b}^2\) a little lower than the experimental value of 0.162(0.037) \text{e}^2\text{b}^2\) and also, the calculated value of \(B(E2; 2^+_2 \rightarrow 2^+_1) = 0.0003 \text{e}^2\text{b}^2\) a little lower than the experimental value of 0.033(0.011) \text{e}^2\text{b}^2\). In the present calculation of the \(B(E2; 4^+_1 \rightarrow 2^+_1)\) is equal to (0.1331, 0.117(0.01)) for the \(^{142}\)Ce isotope in the (IBM, EXP) results, respectively.

The M1 operator in this model is given in Eq.(11). From Table 4, for \(^{142}\)Ce, it is seen that the theoretical values of \(B(M1; 1^+_1 \rightarrow 2^+_1) = 0.02032\ \mu^2_N\) very close to the experimental value is 0.0022\(\pm0.0006\) \mu^2_N\. In the meantime, the \(2^+_2 \rightarrow 2^+_1\) M1 transition strength is small, the experimental value is \(0.046\(\pm0.0025\) \mu^2_N\), the calculated value is \(0.000260\ \mu^2_N\. We observed a large M1 strength for \(2^+_2 \rightarrow 2^+_1\) transition, \(B(M1) = 0.082970\ \mu^2_N\) in \(^{140}\)Ba. The M1 decay of \(3^+_2\) to \(2^+_1\) and \(2^+_2\) states are dominant for both nuclei.

**Table 2:** Experimental and theoretical values of energy ratios in \(^{140}\)Ba and \(^{142}\)Ce nuclei together with the values of IBM limits [10].

| Nuclei | \(E4^+_2/E2^+_1\) | \(E2^+_1/E2^+_1\) | \(E0^+_1/E4^+_1\) | \(E0^+_2/E4^+_1\) |
|--------|----------------|----------------|----------------|----------------|
| \(^{140}\)Ba | EXP | IBM-2 | EXP | IBM-2 | EXP | IBM-2 |
| \(^{142}\)Ce | 1.90 | 2.00 | 2.36 | 2.44 | 1.66 | 1.63 | 3.16 | 3.27 |
| U(5) | 2.00 | 2.00 | \(\geq2.00\) | \(-1.00\) | 4.50 |
| O(6) | 3.30 | 3.00 | \(\geq1.00\) | \(\geq2.00\) |
| SU(3) | 1.90 | 2.00 | 2.36 | 2.44 | 1.66 | 1.63 | 3.16 | 3.27 |

**Table 3:** The absolute \(B(E2)\) values calculated in \(\text{e}^2\text{b}^2\), compared with the available experimental [6,13] data.

| \(j_f \rightarrow j_f\) | \(j_f \rightarrow j_f\) | \(j_f \rightarrow j_f\) |
|----------------|----------------|----------------|
| \(^{140}\)Ba | EXP | IBM | EXP | IBM | EXP | IBM |
| \(^{142}\)Ce | 0.073(34) | 0.0730 | 0.093(21) | 0.0937 |
| \(1^+_1 \rightarrow 0^+_1\) | \(0.0951\) | \(0.117(0.01)\) | \(0.1331\) |
Table 4: The absolute B(M1) values calculated in $\mu_N^2$ unite, compared with the available experimental[13,16,17] data.

| $^{140}$Ba | $^{142}$Ce |
|------------|-----------|
| $I^J \rightarrow I^J_f$ | EXP | IBM | EXP | IBM |
| $2_2 \rightarrow 2_1$ | 0.000229 | > 0.012 | 0.000309 |
| $2_2 \rightarrow 2_1$ | 0.013164 | 0.13$^{+0.01}_{-0.00}$ | 0.132353 |
| $2_4 \rightarrow 2_2$ | 0.016538 | 0.20$^{+0.03}_{-0.03}$ | 0.007490 |
| $2_2 \rightarrow 2_1$ | 0.000001 | 0.0046$^{+0.0028}_{-0.00025}$ | 0.000260 |
| $3_2 \rightarrow 2_1$ | 0.0001477 | 0.001148 |
| $3_2 \rightarrow 2_1$ | 0.000656 | 0.000766 |
| $3_2 \rightarrow 2_1$ | 0.082970 | 0.060463 |
| $3_2 \rightarrow 2_1$ | 0.000013 | 0.000028 |
| $1_2 \rightarrow 2_2$ | 0.001531 | 0.0022$^{+0.0008}_{-0.0006}$ | 0.002032 |
| $1_2 \rightarrow 2_2$ | 0.000003 | 0.000019 |
| $1_2 \rightarrow 0_2$ | 0.05022 | 0.059992 |
| $1_2 \rightarrow 0_2$ | 0.039541 | 0.023145 |
| $3_1 \rightarrow 4_1$ | 0.001445 | 0.002668 |

6. Conclusion

The nuclear structure of $^{140}$Ba and $^{142}$Ce were studied in the framework of the IBA-2 model. We have calculated the properties low-lying mixed symmetry states only of $2^+$, $3^+$ and $1^+$ states. The influence of model parameter ($\xi_2$) on level schemes has been investigated, the results shown that the $\xi_2$ strongly affects the energies of the levels considered to have mixed symmetry character. The comparison between the recent experimental data and calculated ones shows that they are in good agreement.

7. References

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