Implications of a light radion on the RG evolution of higgs self coupling in the Randall-Sundrum model

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Abstract

In this paper we determine how the beta function of the higgs self coupling $\lambda$ at one loop order is modified by a light stabilized radion in the Randall-Sundrum model. We then use the modified beta function to derive a lower bound on the radion vev $\langle \phi \rangle$, both for perturbative and non-perturbative values of $\lambda$ at the ultra violet cut off $\Lambda$. The lower bound on $\langle \phi \rangle$ is obtained by demanding that the renormalized coupling $\lambda(\mu)$ at $\mu = 100$ GeV be consistent with the present experimental bound of 110 GeV on the higgs mass from LEP II searches. We also show that if $\lambda(\Lambda)$ is sufficiently small then an upper bound on $\langle \phi \rangle$ can be determined by requiring that $\beta(\lambda)$ be positive over the relevant momentum range.

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Introduction

Recently several attractive proposals[1,2] based on theories in extra dimensions have been put forward to explain the hierarchy problem. Among them the Randall-Sundrum model is particularly interesting because it considers a five dimensional world based on the following non-factorizable metric

\[ ds^2 = e^{-2kr_c\theta} \eta_{\mu \nu} dx^\mu dx^\nu - r_c^2 d\theta^2. \]  

(1)

Here \( r_c \) measures the size of the extra dimensions which is an \( S^1/Z_2 \) orbifold. \( x^\mu \) are the coordinates of the four dimensional space-time. \( -\pi \leq \theta \leq \pi \) is the coordinate of the extra dimension with \( \theta \) and \(-\theta\) identified. \( k \) is a mass parameter of the order of the fundamental five dimensional Planck mass \( M \). Two 3 branes are placed at the orbifold fixed points \( \theta = 0 \) (hidden brane) and \( \theta = \pi \) (visible brane). Randall and Sundrum showed that any field on the visible brane with a fundamental mass parameter \( m_0 \) gets an effective mass \( m = m_0 e^{-kr_c\pi} \) due to the exponential warp factor. Therefore for \( kr_c \approx 14 \) the electroweak scale is generated from the Planck scale by the warp factor.

In the Randall-Sundrum model \( r_c \) is the vacuum expectation value (vev) of a massless scalar field \( T(x) \). The modulus was therefore not stabilized by some dynamics. In order to stabilize the modulus Goldberger and Wise[3] introduced a scalar field \( \chi(x, \theta) \) in the bulk with interaction potentials localised on the branes. This they showed could generate a potential for \( T(x) \) and stabilize the modulus at the right value \( (kr_c \approx 14) \) needed for the hierarchy without any excessive fine tuning of the parameters.

In the Randall-Sundrum model the SM fields are assumed to be localized.
on the visible brane at $\theta = \pi$. However the SM action is modified due to the exponential warp factor. Small fluctuations of the modulus field from its vev gives rise to non-trivial couplings of the modulus field with the SM fields. In this report we shall derive the couplings of the radion to the higgs field up to quadratic order in $\hat{\phi}$. Here $\hat{\phi}$ is a small fluctuation of the radion field from its vev and is given by $\phi = f e^{-k\pi T(x)} = \langle \phi \rangle + \hat{\phi}$. $\langle \phi \rangle = f e^{-k\pi v_c}$ is the vev of $\phi$ and $f$ is a mass parameter of the order of $M$. We shall then determine the modification in the beta function for $\lambda$ to one loop due to a light stabilized radion. The phenomenological implications of the Randall-Sundrum model depends on two unknown parameters, the radion mass $m_\phi$ and its vev $\langle \phi \rangle$. The requirement that the interbrane seperation in the Randall-Sundrum model be such so as to solve the hierarchy problem implies that $\langle \phi \rangle$ must be of the order of a TeV. Since the radion coupling to the SM fields is inversely proportional to $\langle \phi \rangle$ the phenomenology of the RS model is expected to depend quite sensitively on $\langle \phi \rangle$. In fact studies of radion phenomenology in the context of the RS model show that in order to be consistent with the collider data $\langle \phi \rangle$ must be of the order of $v$ (higgs vev) or greater[4]. In this paper we shall use the RG equation for $\lambda$ in the RS model to derive a lower bound on $\langle \phi \rangle$ for both perturbative and non-perturbative values of $\lambda$ at the cut off scale $\Lambda$. The lower bound on $\langle \phi \rangle$ will be derived by demanding that the renormalized coupling $\lambda(\mu)$ at $\mu \approx 100$ GeV should be consistent with the present experimental bound of 110 GeV on the higgs mass. We also show that if $\lambda(\Lambda)$ is sufficiently small then it is possible to derive an upper bound on $\langle \phi \rangle$ by requiring that $\beta(\lambda(\mu))$ must be positive for all $\mu \leq \Lambda$.

**Radion contribution to the RG equation for $\lambda$**

The radion couplings to the higgs scalar is completely determined by general covariance. The action for the higgs scalar in the Randall-Sundrum
model can be written as

\[ S = \int d^4 x \sqrt{-g_v} [g_v^{\mu\nu} \frac{1}{2} \partial_\mu h \partial_\nu h - V(h)]. \] (2)

where \( V(h) = \frac{1}{2} \mu^2 h^2 + \frac{\lambda}{4} h^4 \). \( h \) is a small fluctuation of the higgs field from its classical vacuum \( v \). In absence of graviton fluctuations we have

\[ g_v^{\mu\nu} = e^{2k\pi T(x)} \eta^{\mu\nu} = \left( \frac{\phi}{T} \right)^2 \eta^{\mu\nu} \]
\[ \sqrt{-g_v} = \left( \frac{\phi}{T} \right)^4 \]

where
\[ \phi = f e^{-k\pi T(x)} \]

Rescaling \( h \) and \( v \) as \( h \to \frac{f}{\langle \phi \rangle} h \) and \( v \to \frac{f}{\langle \phi \rangle} v \) we get

\[ S = \int d^4 x [\left( \frac{\phi}{\langle \phi \rangle} \right)^2 \frac{1}{2} \eta^{\mu\nu} \partial_\mu h \partial_\nu h - \left( \frac{\phi}{\langle \phi \rangle} \right)^4 V(h)]. \] (3)

where
\[ V(h) = \frac{\lambda}{4} (h^4 + 4h^3 v + 4h^2 v^2). \] (4)

The Feynman diagrams that give rise to the radion contribution to the renormalization of the four higgs vertex in the RS model are shown in Fig 1.

\[ \text{Figure. 1.} \quad \text{Feynman diagrams that give rise to the radion contribution to the vertex renormalization.} \]
It is clear from these diagrams that to evaluate them we need the couplings of one and two radions to the higgs sector. Note first that the radion coupling to the kinetic energy term of the higgs boson will not contribute to the renormalization of the vertex associated with the operator $h^4$. The reason being such couplings will give rise to operators involving derivatives of higgs field. Second the radion couplings to the SM fields can be expressed as a power series expansion in $\frac{1}{\langle \phi \rangle}$. Hence naive dimensional analysis(NDA)[6] can be used to estimate the ultraviolet (UV) cut off $\Lambda$. Following the usual prescription of NDA we shall equate the cut-off $\Lambda$ to $4\pi \langle \phi \rangle$. In general the ratio $\frac{\Lambda}{\langle \phi \rangle}$ is expected to lie between 1 and $4\pi$. However the estimates presented in this paper will not change much as long as $\frac{\Lambda}{\langle \phi \rangle}$ lies in this range. Further since perturbation theory is defined only about a stable minimum we shall expand both $h$ and $\phi$ about their respective vevs. Evaluating the vertex renormalization diagrams explicitly with a cut off $\Lambda$ we find that the leading log terms of these diagrams are given by

$\Gamma_1 = 6\lambda \frac{288 \lambda v^2}{16 \pi^2 \langle \phi \rangle^2} \ln \frac{\Lambda^2}{\mu^2}$. \hspace{1cm} (5a)$

$\Gamma_2 = 6\lambda \frac{144 \lambda v^4}{16 \pi^2 \langle \phi \rangle^4} \ln \frac{\Lambda^2}{\mu^2}$. \hspace{1cm} (5b)$

$\Gamma_3 = 6\lambda \frac{128 \lambda v^2}{16 \pi^2 \langle \phi \rangle^2} \ln \frac{\Lambda^2}{\mu^2}$. \hspace{1cm} (5c)$

and
Here $\mu$ is the renormalization mass scale. In the SM model the wavefunction renormalization constant of the higgs boson $Z_h$ is equal to one at one loop order even after the higgs field is shifted by its vev. However the radion coupling to the KE term of the higgs boson gives rise to a non-trivial wavefunction renormalization of the higgs boson. Evaluating the radion mediated self energy diagram (Fig.2) of the higgs boson,

\[
\Gamma_4 = -6\lambda \frac{6}{16\pi^2 \langle \phi \rangle^2} [\Lambda^2 - m^2_\phi \ln \frac{\Lambda^2}{\mu^2}],
\]  

(5d)

we find that $Z_h = 1 + \frac{1}{32\pi^2} \frac{7m^2_\phi - m^2_\phi}{\langle \phi \rangle^2} \ln \frac{\Lambda^2}{\mu^2}$.

Using the above vertex and wavefunction renormalizations induced by a light radion it can be shown that the complete one loop beta function for $\lambda$ in the RS model is given by

\[
\beta(\lambda) = \mu \frac{d\lambda}{d\mu} = \frac{1}{8\pi^2} [9\lambda^2 + \frac{402\lambda^2 v^2}{\langle \phi \rangle^2} + \frac{144\lambda^2 v^4}{\langle \phi \rangle^4} + \frac{7\lambda m^2_\phi}{\langle \phi \rangle^2} + \lambda(6g^2_y - \frac{9}{2}g^2 - \frac{3}{2}g'^2)]
\]
\[
+ \frac{1}{8\pi^2} [-6g^4_y + \frac{3}{16}(g^4 + \frac{1}{2}(g^2 + g'^2)^2)]
\]

(6)

The purely SM contribution to $\beta(\lambda)[7]$ can be obtained by letting the expansion parameter $\langle \phi \rangle$ approach infinity.

**Lower bound on radion vev**

For simplicity we shall first consider the higgs-radion system in isolation from the remaining fields. The beta function corresponding to this idealized
situations can be obtained by setting $g_y = g = g' = 0$. Such an approximation would be meaningful provided $\lambda(\mu)$ is much greater than the remaining couplings over the entire momentum interval of interest. Further for a light radion ($m_\phi \ll \langle \phi \rangle$) we can drop the term proportional to $m_\phi^2$ from the expression of $\beta(\lambda)$. The beta function for $\lambda$ then contains only quadratic terms in $\lambda$. Solving the RG equation for $\lambda$ under the above approximation we get

$$\lambda(\mu) = \frac{\lambda(\Lambda)}{1 + \frac{\lambda(\Lambda)}{8\pi^2} (9 + 402 \frac{v^2}{\langle \phi \rangle^2} + 144 \frac{v^4}{\langle \phi \rangle^4}) \ln \frac{\Lambda}{\mu}}$$ (7)

In fig. 3 we have plotted the renormalized coupling $\lambda(\mu)$ at $\mu = 100$ GeV against the radion vev $\langle \phi \rangle$ for $\lambda(\Lambda) = \infty$ and $\lambda(\Lambda) = e$ under the quadratic approximation to $\beta(\lambda)$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Showing the variation of $\lambda(\mu)$ at $\mu = 100$ GeV with $\langle \phi \rangle$ when only the $O(\lambda^2)$ terms of $\beta(\lambda)$ are kept.}
\end{figure}
We find that for $\lambda(\Lambda) = \infty$, in order that $\lambda(\mu)$ at $\mu = 100$ GeV be greater than 0.099 (the value corresponding to the present experimental bound on $m_h$) the radion vev $\langle \phi \rangle$ must be greater than 378.2 GeV (solid curve). The lower bound on $\langle \phi \rangle$ does not change much from this value as long as the value of $\lambda(\Lambda)$ remains non-perturbative i.e. $\lambda(\Lambda) \geq \sqrt{4\pi}$. On the other hand if $\lambda$ lies in the perturbative regime e.g. if $\lambda(\Lambda) = e$ then in order that $\lambda(\mu)$ at $\mu = 100$ GeV be consistent with the LEP II bound on the higgs mass, the radion vev $\langle \phi \rangle$ must be greater than 468.4 GeV (dotted curve). The above results were obtained by keeping only the $O(\lambda^2)$ terms in the beta function for $\lambda$. If $\lambda(\Lambda)$ is much greater than the remaining couplings then clearly the evolution of $\lambda(\mu)$ towards low energies will be determined mainly by the $O(\lambda^2)$ terms of $\beta(\lambda)$. However if $\lambda(\Lambda)$ is small then the $O(\lambda^2)$ terms of $\beta(\lambda)$ become smaller than the $O(\lambda)$ and $O(\lambda^0)$ terms and the above approximation breaks down. We have therefore considered the full expression for $\beta(\lambda)$ and determined the lower bound on $\langle \phi \rangle$ by demanding that $\lambda (100$ GeV $)$ be consistent with the present experimental bound on $m_h$. By considering the full beta function for $\lambda$ and assuming for simplicity that $g_y$, $g$ and $g'$ do not scale with $\mu$ it can be shown that the solution for $\lambda(\mu)$ is given by

$$\lambda(\mu) = \lambda_1 + \frac{\lambda_1 - \lambda_2}{\lambda(\Lambda) - \lambda_1} \frac{\lambda(\Lambda) - \lambda_2}{\lambda(\mu) - \lambda_2} \frac{a(\lambda_1 - \lambda_2)}{a - 1}.$$  \hspace{1cm} (8a)

where $\lambda_1 = \frac{-b+\sqrt{(b^2-4ac)}}{2a}$ and $\lambda_2 = \frac{-b-\sqrt{(b^2-4ac)}}{2a}$

$$a = \frac{1}{8\pi^2} \left[ 9 + \frac{402}{\langle \phi \rangle^2} + \frac{144}{\langle \phi \rangle^4} \right].$$  \hspace{1cm} (8b)

$$b = \frac{1}{8\pi^2} \left[ \frac{7m_{\phi}^2}{\langle \phi \rangle^2} + \left( 6g_y^2 - \frac{9}{2}g^2 - \frac{3}{2}g'^2 \right) \right].$$  \hspace{1cm} (8c)

and

$$c = \frac{1}{8\pi^2} \left[ -6g_y^4 + \frac{3}{16} \left( g^4 + \frac{1}{2} \left( g^2 + g'^2 \right)^2 \right) \right].$$  \hspace{1cm} (8d)
Figure 4. Showing the variation of \( \lambda(\mu) \) at \( \mu = 100 \) GeV with \( \langle \phi \rangle \) using the full expression for \( \beta(\lambda) \).

In Figure 4 we have plotted \( \lambda(\mu) \) at \( \mu = 100 \) GeV against different values of \( \langle \phi \rangle \). The solid curve corresponds to the UV boundary condition \( \lambda(\Lambda) = \infty \) and the dotted curve to \( \lambda(\Lambda) = e \). Both plots were obtained with the following values of \( g_y, g \) and \( g' \): \( g_y = \sqrt{2m_t/v} = 1.001 \), \( g = \frac{e}{\sin \theta_w} = 0.644 \) and \( g' = \frac{e}{\cos \theta_w} = 0.356 \). Further the radion mass \( m_\phi \) was assumed to be 50 GeV. From these two plots we find that \( \langle \phi \rangle \) must be greater than about 243 GeV so that \( \lambda(\mu) \) at \( \mu = 100 \) GeV is greater than 0.099 (\( \approx 0.1 \)). This estimate of lower bound on \( \langle \phi \rangle \) will not change much with \( m_\phi \) as long as the radion is light and \( m_\phi \) lies in the few tens of GeV range. We find that the lower bound on \( \langle \phi \rangle \) obtained by using the complete expression for \( \beta(\lambda) \) does not depend at all on the UV boundary condition. In fact fig. 4 shows that for \( \langle \phi \rangle \) less than 250 Gev the renormalized value of \( \lambda(\mu) \) at low energies is governed by the infrared properties of the theory and not the ultraviolet.

**Upper bound on radion vev**

From the full expression of \( \beta(\lambda) \) it is clear that if \( \lambda \) is sufficiently small and \( \langle \phi \rangle \) is large then \( \beta(\lambda) \) can become negative due to the dominance of the \( g_y^4 \) term which is negative. Hence for sufficiently small values of \( \lambda(\Lambda) \) a reasonable upper bound on \( \langle \phi \rangle \) can be obtained by demanding that \( \beta(\lambda(\mu)) \)
be positive for all \( \mu \leq \Lambda \). This criterion will ensure that the RG evolution of \( \lambda(\mu) \) from \( \Lambda \) towards low energies exhibits infrared free behaviour. In particular \( \beta(\lambda(\Lambda)) \) must be positive. At the crossover from infrared free behaviour to asymptotically free behaviour the beta function vanishes and we get

\[ A(\lambda)x^2 + B(\lambda)x + C(\lambda) = 0. \]  

(9a)

where

\[ A(\lambda) = [9\lambda^2(\Lambda) + \lambda(\Lambda)(6g^2_v - \frac{9}{2}g^2 - \frac{3}{2}g'^2) - 6g_v^4 + \frac{3}{16}(g^4 + \frac{1}{2}(g^2 + g'^2)^2)]. \]  

(9b)

\[ B(\lambda) = 402\lambda^2(\Lambda)v^2 + 7\lambda(\Lambda)m^2. \]  

(9c)

\[ C(\lambda) = 144\lambda^2(\Lambda)v^4. \]  

(9d)

and \( x^2 = \langle \phi \rangle^2 \).

Since \( \langle \phi \rangle \) is real, \( x \) must be positive. Using this condition it can be shown that the physical root of the above equation is

\[ x = \frac{-B(\lambda) - \sqrt{B^2(\lambda) - 4A(\lambda)C(\lambda)}}{2A(\lambda)}. \]

The upper bound on \( \langle \phi \rangle \) for any given small value of \( \lambda(\Lambda) \) can be determined from the above root. For example for \( \lambda(\Lambda) \approx e \) we find that the radion vev must be greater than 806 Gev so that \( \beta(\lambda(\Lambda)) \) is positive. In Figure 5 we have plotted the upper bound on \( \langle \phi \rangle \) against \( \lambda(\Lambda) \). As expected the upper bound on \( \langle \phi \rangle \) increases with increasing \( \lambda(\Lambda) \). For \( \lambda(\Lambda) \) slightly greater than 0.6 both roots become unphysical and no bound on \( \langle \phi \rangle \) is obtained. The reason being once \( \lambda(\Lambda) \) becomes sufficiently large \( \beta(\lambda) \) remains positive irrespective of the value of \( \langle \phi \rangle \). Note that the upper bound on \( \langle \phi \rangle \) rises very sharply in the vicinity of this region. In fact our estimate for the upper bound becomes somewhat unreliable here.
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