Lorentz symmetry breaking in Abelian vector-field models with Wess-Zumino interaction

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Abstract

We consider the abelian vector-field models in the presence of the Wess-Zumino interaction with the pseudoscalar matter. The occurrence of the dynamic breaking of Lorentz symmetry at classical and one-loop level is described for massless and massive vector fields. This phenomenon appears to be the non-perturbative counterpart of the perturbative renormalizability and/or unitarity breaking in the chiral gauge theories.
1. Introduction

The abelian vector-field models we consider are given by the lagrangian which contains the Wess-Zumino interaction (in the Minkowski space-time):

\[
\mathcal{L}_{\text{WZ}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu - b \partial_\mu A^\mu
+ \frac{\beta^2}{2} \partial_\mu \theta \partial^\mu \theta + \frac{\kappa}{4M} \theta F_{\mu\nu} \tilde{F}^{\mu\nu},
\]

(1)

where \( \tilde{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \), the universal dimensional scale \( M \) is introduced, \( m \equiv \beta \sqrt{V} M \) is the mass of the vector field and the coupling parameters \( \beta \) and \( \kappa \) take particular values depending on their origin as we explain below.

The Wess-Zumino interaction (the last term in (1)) can be equivalently represented in the following form,

\[
\int d^4x \frac{\kappa}{4M} \theta F_{\mu\nu} \tilde{F}^{\mu\nu} = -\int d^4x \frac{\kappa}{2M} \partial_\mu \theta A_\nu \tilde{F}^{\mu\nu},
\]

(2)

when it is treated in the action. Therefore the pseudoscalar field is involved into the dynamics only through its gradient \( \partial_\mu \theta(x) \) due to topological triviality of abelian vector fields.

These models have different roots.

1. They may represent the anomalous part of the chiral abelian theory when the lagrangian is prepared in the gauge invariant form by means of integration over gauge group and after the Landau gauge-fixing,

\[
\mathcal{L}_{\text{ch}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 (A_\mu + \partial_\mu \vartheta)(A^\mu + \partial^\mu \vartheta)
- b \partial_\mu A^\mu + \bar{\psi}(\vartheta + ie(\vartheta + \partial \vartheta)P_L)\psi,
\]

(3)

where \( P_L = (1 + \gamma_5)/2 \). In this case \( \kappa \equiv e^3/12\pi^2 \), \( \theta = M \vartheta \), and \( \beta = m/M \). The latter relation provides the cancellation of the ghost pole in the vector-field propagator coupled to the chiral fermion current or, in other words, it leads to the Proca propagator for vector field. As it is known this is just the anomalous vertex,

\[
\theta \partial_\mu J_\mu^L = i\frac{e^3}{48\pi^2} \theta F_{\mu\nu} \tilde{F}^{\mu\nu}
\]

(4)

which yields the violation of perturbative unitarity at high energies. Therefore the nonperturbative properties of the model (1) might be crucial in understanding of what happens with the chiral abelian model due to breakdown of perturbative unitarity and of power-counting renormalizability.

Indeed as it follows from canonical rescaling arguments

\[
\begin{align*}
x & \to \lambda x; \quad \partial_\mu \to \partial_\mu/\lambda; \quad A_\mu \to A_\mu/\lambda; \quad \theta \to \theta/\lambda; \quad b \to b/\lambda^2; \\
\psi & \to \psi/\lambda^{3/2}; \quad S_{\text{kin}} + S_{\text{int}}^\perp \to S_{\text{kin}} + S_{\text{int}}^\perp; \quad S_{\text{int}}^\| \to S_{\text{int}}^\|/\lambda,
\end{align*}
\]

(5)
for $\lambda << 1$ the notion of high energies is roughly related to the limit $M \to 0$ or $m = \beta V M \to 0; \kappa/M \to \infty$. Thus one expects that the effective coupling is rapidly increasing with energies and therefore approaching to the strong-coupling regime.

2. On the other hand the model (I) with $\beta^2 \to -\beta^2$ can shed light on the breaking of unitarity in the chiral abelian vector models (CAVM) with the anomaly compensating ghost field

$$L^\perp_{ch} = L_{ch} - \frac{1}{2}m^2 \partial_\mu \eta \partial^\mu \eta - \eta \partial_\mu J^\mu_L$$ (6)

This model is renormalizable by power counting, exhibits extended gauge invariance and it is equivalent to the CAVM (2) with additional constraint $\Box \theta = 0$ that leads to the non-local lagrangian with purely transversal gauge fields. However the presence of triangle anomaly is still troublesome since it eventually gives rise to the lack of decoupling of the massless ghost pole in the transversal projector, in spite of the extended gauge or BRST invariance.

Thus the model (I) with the negative "ghost" sign of kinetic term for scalar fields might be helpful to unravel the infrared region of transversal CAVM.

3. The models family (I) can be obtained as a result of dimensional reduction of the dim-5 abelian vector-field model with the Chern-Simons interaction,

$$L_{CS} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa_5}{10} \epsilon^{\mu\nu\rho\sigma\lambda} A_\mu \partial_\nu A_\rho \partial_\sigma A_\lambda - b \partial_\mu A^\mu$$ (7)

where now $\mu, \ldots = 0, \ldots, 4; \text{dim}[\kappa_5] = -3/2$ and the metric is $\text{diag}(+,-,-,-,+)$. This choice of metric provides the correct sign for kinetic terms of $A_\mu$ and $\theta$ fields in the four-dimensional space-time and corresponds to the reduction of $O(3,2)$-symmetry to $O(3,1)$-symmetry. Other choices, $O(4,1) \to O(3,1)$ or $O(4,1) \to O(4)$, lead to the ghost field, either the scalar one or the vector one respectively.

Then on the stationary $x_4$-independent solutions of the field equations one can perform the dimensional reduction of this space dimension $|x_4| \leq \tau/2 \to 0$ so that

$$\partial_4 A_\mu \approx 0; \quad A_4 \equiv \tau^{-1/2} \beta \theta; \quad A_\mu|_{\mu \neq 4} \to \tau^{-1/2} A_\mu.$$

The reduced dim-4 lagrangian,

$$L_{WZ} = \int_{-\tau/2}^{\tau/2} dx_4 L_{CS} \simeq \tau L_{CS}|_{\tau \to 0}$$

takes the form (1) with $m^2 = 0$ and the coupling parameter depending on the reduction scale $\kappa_{WZ}/M = 3\kappa_5 \beta/5\sqrt{\tau}$. Thereby we have two options either to impose $\beta \sim \sqrt{\tau M}$ or to assume that the CS coupling is vanishing $\kappa_5 \sim \sqrt{\tau}/M$. In the first case one obtains the model (I) without kinetic term for $\theta$-field while in the second one the kinetic term survives.

Thus the properties of dim-5 CS and dim-4 WZ models are intimately connected as within the perturbative expansion as well as beyond it.

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"This connection is manifested in Eqs.of motion, their classical solutions and furthermore in the comparison of one-loop effective potentials."
Now let us take the model \( (1) \) on its own right and proceed to the evaluation of tree-level ”classical” solutions, which may be considered as a starting point to develop the loop ”quasiclassical” approximation.

2. Equations of motion and ”vacuum” solutions

The Euler-Lagrange equations for the model \( (1) \) read:

\[
\partial_{\mu} F^{\mu\nu} + m^2 A^\nu + \partial^\nu b - \frac{\kappa}{M} \partial_{\mu} \vartheta \tilde{F}^{\mu\nu} = 0;
\]

\[
\partial_{\mu} A^\mu = 0 \Rightarrow \Box b = 0;
\]

\[
\pm \beta^2 \Box \theta = \frac{\kappa}{4M} F_{\mu\nu} \tilde{F}^{\mu\nu}.
\]

We search for the constant solutions for \( \partial_{\mu} \theta \) and related field-strength solutions. Let us classify four distinguished sectors:

**Case A.** \( m = 0, \beta = 0 \), i.e. the vector field is massless and there is no kinetic term for scalar field which becomes a pure WZ field.

**Case B.** \( m \neq 0, \beta = 0 \), the vector field is massive but the scalar one represents a pure WZ field.

**Case C.** \( m = 0, \beta \neq 0 \), vector and scalar fields are massless and the scalar field is a D’Alembert field or a ”ghost” massless scalar for \( \beta^2 \rightarrow -\beta^2 \).

**Case D.** \( m \neq 0; \beta \neq 0 \), the vector field is massive but the scalar one is still massless.

We shall see that the classical ”vacuum” solutions and the one-loop effective potentials are different in above cases.

It is evident from Eq.(8) that the constant solutions

\[
\partial_{\mu} \theta = M\eta_{\mu} = \text{const}; \quad F_{\mu\nu} = 0
\]

are there for all the cases. In A, B the classical action vanishes for these Lorentz symmetry breaking solutions and therefore the Lorentz symmetric vacuum configuration \( \eta_{\mu} = 0 \) is degenerate. The quantum effective potential (in the Coleman-Weinberg spirit) is needed to select out the true vacuum configuration (see below). In C, D for \( \eta_{\mu}\eta^{\mu} \neq 0 \) the above solution shifts the potential energy by a constant which might lead to an inequivalent quantum theory. Again the quantum effective potential is helpful for the resolution of the true vacuum configuration.

Let us also analyze constant solutions with \( F_{\mu\nu} \neq 0 \) which correspond to \( A_{\nu} = 1/2 F_{\mu\nu} x^\mu \). From Eqs.(8) it immediately follows that possible solutions obey the condition \( F_{\mu\nu} \tilde{F}^{\mu\nu} = 0 \). In A, C the constant field-strength configuration and the constant value of \( \partial_{\mu} b \equiv \zeta M^2 n^{(1)}_{\nu} \) are allowed. The first Eq.(8) now reads,

\[
\zeta M^2 n^{(1)}_{\nu} = \kappa \eta^{\mu} \tilde{F}_{\mu\nu}
\]
and therefore \( \eta^\mu n^{(1)}_\mu = 0 \). The arbitrary field-strength \( F_{\mu\nu} \) can be conveniently represented in terms of linear-independent vectors

\[
\begin{align*}
\eta &\equiv n^{(0)}_\mu; n^{(1)}_\mu; n^{(2)}_\mu; n^{(3)}_\mu \equiv n^i_\mu; \\
\epsilon^{\mu\nu\rho\sigma} n^{(1)}_\mu n^{(2)}_\nu n^{(3)}_\rho n^{(4)}_\sigma &\neq 0; \\
F_{\mu\nu} &= \sum_{i<j} a_{ij} n^i_\mu n^j_\nu; \\
n^i_\mu n^j_\nu &\equiv n^i_\mu n^j_\nu - n^j_\mu n^i_\nu.
\end{align*}
\]

(11)

The above conditions mean that only three independent vectors are available in building solutions, namely \( \eta_\mu; n^{(2)}_\mu; n^{(3)}_\mu \).

Let us impose the complete decoupling of \( b \)-field, i.e. set \( \zeta \to 0 \). Then the solution is parametrized as follows:

\[
F_{\mu\nu} = a \eta n^{(2)}_{[\mu} n^{(3)}_{\nu]}.
\]

(12)

If \( \eta_\mu n^\mu \neq 0 \) the vector \( n^{(2)} \) can be chosen orthogonal to \( \eta^\mu \) yielding

\[
\tilde{F}_{\mu\nu} = \tilde{a} n^{(1)}_{[\mu} n^{(3)}_{\nu]}.
\]

(13)

The coefficients \( a, \tilde{a} \) are not fixed by Eqs. of motion as well as the vector \( \eta_\mu \).

In \( \mathbf{B}, \mathbf{D} \) the constant solution for \( F_{\mu\nu} \) is incompatible with constant \( \eta_\mu \) but rather needs the configuration \( \theta(x) = M (\alpha x^2/2 + (\eta \cdot x)) \), \( \partial_\mu \theta = M (\alpha x^2 + \eta_\mu) \). However from Eq.(8) one obtains the relations

\[
\begin{align*}
\frac{m^2}{2} F_{\mu\nu} &= \kappa \alpha \tilde{F}_{\mu\nu}; \\
\left( \frac{m^4}{2} + \kappa^2 \alpha^2 \right) F_{\mu\nu} F^{\mu\nu} &= 0; \\
F_{\mu\nu} \tilde{F}_{\mu\nu} &= 0 = \pm \frac{8 M^2 \beta^2 \alpha}{\kappa}
\end{align*}
\]

(14)

which lead to \( F_{\mu\nu} = 0, \alpha = 0 \) in \( \mathbf{D} \), whereas in \( \mathbf{B} \) we are lead to \( F_{\mu\nu} F^{\mu\nu} = F_{\mu\nu} \tilde{F}_{\mu\nu} = 0 \) which corresponds to the radiation-like constant field \( \vec{E} \perp \vec{H}, |\vec{E}| = |\vec{H}| \).

We conclude that, if there is a smooth massless limit for vector fields and the kinetic term for \( \theta \) field is present, then the Lorentz symmetry breaking (LSB) may be induced by the non-trivial average value for \( \partial_\mu \theta(x) \), but unlikely due to constant field-strength configurations. Therefore let us examine the LSB induced by a non-zero pseudoscalar background \( \partial_\mu \theta = M \eta_\mu \neq 0 \) when the quantum effects are taken into account.

3. Spectrum of vector fields in \( \eta \)-background

It is evident from Eqs.(3) that the longitudinal components decouple \((\partial_\mu A^\mu = 0)\) and the auxiliary field \( b(x) \) is free. In the momentum representation the Eqs.(8) read (in the constant \( \eta \)-background),

\[
\left\{ (p^2 - m^2) g^{\mu\nu} + i \kappa \epsilon^{\mu\nu\rho\sigma} \eta_\rho p_\sigma \right\} A^\perp_{\nu} \equiv K \cdot A^\perp = 0.
\]

(15)

Let us denote

\[
\mathcal{E}^{\mu\nu}(p) = \kappa \epsilon^{\mu\nu\rho\sigma} \eta_\rho p_\sigma = -\mathcal{E}^{\mu\nu}(-p),
\]

(16)
and multiply Eq.(13) by a transposed matrix $K^t = K^* = K(-p)$ which have the same eigenvalues as $K$ due to its hermiticity,

$$K^t K \cdot A^\perp = (p^2 - m^2)^2 \mathbf{1} + \hat{\mathcal{E}}^2 A^\perp = 0. \quad (17)$$

The matrix $\hat{\mathcal{E}}^2$ represents in fact the projector on a two-dimensional plane (for $\eta_\mu, p_\mu$ not collinear),

$$\mathcal{E}_{\mu\nu}\mathcal{E}^{\nu\lambda} = \kappa^2 \left\{ \delta^\lambda_\mu \left( \eta^2 p^2 - (\eta \cdot p)^2 \right) - (\eta^\lambda \eta_\mu p^2 + p^\lambda p_\mu \eta^2) + (\eta^\lambda p_\mu + p^\lambda \eta_\mu)(\eta \cdot p) \right\}$$

$$\equiv \kappa^2 \left( \eta^2 p^2 - (\eta \cdot p)^2 \right) [P_2]^\lambda_\mu, \quad P_2^2 = P_2; \quad \text{tr} P_2 = 2. \quad (18)$$

Evidently,

$$\hat{\mathcal{E}} P_2 = \hat{\mathcal{E}}, \quad P_\perp P_2 = P_2, \quad P_2 \cdot p = P_2 \cdot \eta = 0. \quad (19)$$

Therefore in the $\eta_\mu$-direction one finds the free massive field,

$$(p^2 - m^2) A^\mu_\eta = 0; \quad A^\mu_\eta = \frac{\eta^\mu}{\eta^2}(\eta \cdot A^\perp), \quad (20)$$

whereas in the two-dimensional plane selected by $P_2$ the dispersion law is different from a free-particle one,

$$\left( (p^2 - m^2)^2 \mathbf{1} + \hat{\mathcal{E}}^2 \right) P_2 \cdot A = \left( (p^2 - m^2)^2 + \kappa^2 (\eta^2 p^2 - (\eta \cdot p)^2) \right) P_2 \cdot A = 0. \quad (21)$$

Let us restrict ourselves to space-like $\eta_\mu$ and choose the coordinate frame where $\eta = (0, \eta)$. Then the energy spectrum is defined by the following dispersion law,

$$p_0^2 = p^2 + m^2 + \frac{\kappa^2 \eta^2}{2} \pm \sqrt{\frac{\kappa^4 (\eta^2)^2}{4} + m^2 \kappa^2 \eta^2 + \kappa^2 (\eta p)^2} \geq 0. \quad (22)$$

Thus in the plane orthogonal to both $\eta_\mu$ and $p_\mu$ there appear two types of waves (for two different polarizations). In the soft-momentum region ($p^2 << m^2 + \frac{\kappa^2 \eta^2}{4}$) one reveals two massive excitations with masses,

$$m^2_\pm \approx m^2 + \frac{\kappa^2 \eta^2}{2} \pm \sqrt{\frac{\kappa^4 (\eta^2)^2}{4} + m^2 \kappa^2 \eta^2}. \quad (23)$$

In particular, when the bare vector particle is massless, then after the interaction with LSB background the mass splitting arises and in the soft-momentum region we find only one polarization for nearly massless excitations but a complementary polarization behaves as a massive one.
4. Effective potential for pseudoscalar field

Let us examine the role of quantum corrections in the formation of v.e.v. for \( \partial_\mu \theta \) and derive the one-loop effective potential for the gradient of WZ field induced by the virtual creation of vector particles. We follow the recipe of the background-field method\(^7,8\) to obtain the effective potential and consider the second variation of the action \( S_{WZ} \) around constant space-like \( \partial_\mu \theta = M \eta_\mu, \ \eta^2 < 0 \) and zero vector-field configurations.

For space-like \( \eta_\mu \) the energy spectrum of vector fields is real (see Eq. (22)). Therefore one can employ the causal prescription for vector-field propagators and furthermore perform the Wick rotation in computing the effective action. Then the transversal part of the (euclidean) vector-field action reads

\[
A_\mu^\perp \left[ (-\Box + m^2) \delta_{\mu \nu} - \mathcal{E}_{\mu \nu}(\hat{p}) \right] A_\nu^\perp = \left( A_\mu^\perp \cdot \mathbf{K} A_\mu^\perp \right).
\]

(24)
The one-loop effective potential is then obtained from the functional determinant of the above operator \( \text{Det}\mathbf{K} = (\text{Det}\mathbf{K}^t \mathbf{K})^{1/2} \) in the conventional way\(^8\),

\[
V_{\text{eff}} \equiv V^{(0)} + V^{(1)}; \quad V^{(0)} = \pm \frac{\beta_{\text{bare}}^2 M^2}{2} \eta^2;
\]

\[
V^{(1)} = \frac{1}{V o l} \left\{ \frac{1}{4} \text{Tr} \left[ \mathbf{P}_2 \log \mathbf{K}^t(\eta) \mathbf{K}(\eta) \right] \right\} - \{ \eta_\mu = 0 \}
\]

\[
= \frac{1}{2} \int_{|p|<\Lambda} \frac{d^4p}{(2\pi)^4} \ln \left( 1 + \frac{\kappa^2 (\eta^2 p^2 - (\eta \cdot p)^2)}{(p^2 + m^2)^2} \right)
\]

(25)
where the relations corresponding to Eqs. (13), - (21) are used for the euclidean space metric (\( \eta^2 > 0 \) from now on). One can evaluate the effective potential, for instance, by means of perturbative expansion,

\[
V^{(1)}(\eta) = \frac{1}{2} \int_{|p|<\Lambda} \frac{d^4p}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \frac{(\kappa^2 (\eta^2 p^2 - (\eta \cdot p)^2))^n}{(p^2 + m^2)^{2n}} \equiv \sum_{n=1}^{\infty} V_n^{(1)}
\]

(26)
The integration over angular variables is performed with the help of the following identity,

\[
\int_{S^{(3)}} d\Omega (\eta^2 p^2 - (\eta \cdot p)^2)^n = 2\pi^2 \frac{(2n+1)!}{2^{2n} n! (n+1)! \eta^{2n} p^{2n}}.
\]

(27)
In four dimensions the first two terms in (26) are divergent and in the finite-cutoff regularization they have the following cutoff dependence,

\[
V_1^{(1)} = \frac{3\kappa^2}{2^7\pi^2} \left( \Lambda^2 - 2m^2 \ln \frac{\Lambda^2}{m^2} + m^2 \right) \eta^2 + O \left( \frac{m^2}{\Lambda^2} \right);
\]

\[
V_2^{(1)} = -\frac{5\kappa^4}{2^9\pi^2} \ln \frac{\Lambda^2}{m^2} (\eta^2)^2 + O \left( \frac{m^2}{\Lambda^2} \right).
\]

(28)
\(^d\)Insofar as the field \( \theta \) is massless the action (1), (2) is invariant under field translations \( \theta \rightarrow \theta + \text{const} \) and the conventional effective potential for this field is irrelevant.
Evidently, the renormalization is required with two counterterms,

$$\Delta V(\theta) = \frac{\Delta \beta^2}{2} \partial_\mu \theta \partial^\mu \theta + \frac{\Delta g}{4M^4} (\partial_\mu \theta \partial^\mu \theta)^2. \quad (29)$$

We see that the second divergence cannot be generally cured in the minimal model (1) and implies the inclusion of the dim-8 vertex into the bare lagrangian (1). The appearance of higher-dimensional vertices in the effective lagrangian is not surprising since the model is not perturbatively renormalizable. Thus we should specify the boundary conditions for the effective potential which provide the minimal form of the lagrangian (1) at a particular scale (by means of the fine-tuning of all higher-dimensional vertices to zero value). Then the form of the effective lagrangian for other scales will be governed by the effective renormalization-group flow.

The remaining part of \( V^{(1)} \) for \( n \geq 3 \) is finite and contains the following terms,

$$V^{(1)}_{n \geq 3} = (-1)^{n+1} \left( \frac{\kappa^2 \eta^2}{4m^2} \right)^n \frac{m^4(2n+1)}{16\pi^2 n(n-1)(n-2)}. \quad (30)$$

It seems that for massive vector fields there exists a weak coupling and soft-momentum regime \( \kappa^2 \eta^2 < 4m^2 \) which is perturbatively safe, in the sense that the boundary condition reproducing (1) can be imposed at \( \eta_\mu = 0 \).

The related effective potential is found from (26) after summation and reads

$$V_{\text{ren}} = \pm \mu_1^2 \eta^2 + \frac{g}{4}(\eta^2)^2 + \frac{1}{32\pi^2} \ln \left( \frac{m^2 + z}{\mu_2^2} \right) (5z^2 + 6m^2 z + m^4), \quad (31)$$

where we set \( z \equiv \kappa^2 \eta^2/4 \). The constants \( \mu_1, \mu_2, g \) are fixed by boundary conditions. For instance, the soft momentum and weak coupling normalization (at \( \eta_\mu = 0 \)) is given by

$$\pm \mu_1^2 = \pm \beta^2 M^2 - \frac{\kappa^2 m^2}{64\pi^2}; \quad \mu_2^2 = m^2; \quad g = -\frac{\kappa^4}{256\pi^2}. \quad (32)$$

However such boundary conditions do not allow the massless limit or the strong-coupling regime which is expected to take place at high energies (see Sec.1). In the latter case we will use the normalization at the fixed scale \( M \) which has the meaning of a scale for our measurements.

5. Dynamic breaking of the Lorentz symmetry

Let us search for the Coleman-Weinberg\(^6\) instabilities in the effective potential of pseudoscalar field that cause the dynamic symmetry breaking of the Lorentz symmetry (for other

\( ^6 \)We omit for a moment the "dipole-ghost" term with four derivatives \((\partial^2 \theta)^2\) which however is important in the RG-flow for such models.

\( ^7 \)Thereby we simplify the RG analysis which should support our consideration.
scenarios of LSB, see \(^{10}\)). We examine separately the models with massless and massive vector fields.

In the first case the one-loop effective potential cannot be normalized at zero momenta, i.e. at \(\eta^2 = 0\). Instead one has to provide the basic lagrangian at the main scale of the model \(\eta^2 = M^2\)

\[
V_{\text{ren}}(\eta, \mu = \frac{\kappa M^2}{2}) = \pm \frac{\beta^2 M^2}{2} \eta^2 + \frac{5 \kappa^4}{2^9 \pi^2} \eta^2 \ln \frac{\eta^2}{M^2}.
\]  

(33)

The minimum is obtained from the conditions,

\[
\frac{\partial V}{\partial \eta} = 2 \eta V'(\eta^2) = 0; \quad V'(\eta^2) \equiv \frac{dV(\eta^2)}{d(\eta^2)} = \pm \frac{\beta^2 M^2}{2} \eta^2 + \frac{5 \kappa^4}{2^9 \pi^2} 2 \ln \frac{\eta^2}{M^2} + 1,
\]

(34)

and

\[
\frac{\partial^2 V}{\partial \eta_{\mu} \partial \eta_{\nu}} = 2 \delta_{\mu \nu} V'(\eta^2) + 4 \eta_{\mu} \eta_{\nu} V''(\eta^2) \geq 0,
\]

\[
V''(\eta^2) \equiv \frac{d^2V(\eta^2)}{(d(\eta^2))^2} = \frac{5 \kappa^4}{2^9 \pi^2} \left(2 \ln \frac{\eta^2}{M^2} + 3\right).
\]

(35)

The symmetric solution \(\eta_{\mu} = 0\) leads to the minimum if \(V'(0) > 0\) which corresponds to the positive sign in the first term (33). In the latter case other solutions may arise for a strong coupling \(\kappa\) when \(V'(\eta^2) = 0\).

In order to find the critical value of \(\kappa\) let us substitute this equation (34) into (33) to provide \(V''(\eta^2) \geq 0\). Then one obtains that for \(\kappa^4 \geq 128 \pi^2 e^{3/2} \beta^2 \beta^2 / 5\) the second minimum appears. However it lies higher than the symmetric one as compared with the value of the effective potential at \(\eta_{\mu} = 0\). They are degenerate when \(\kappa^4_{\text{cr}} = 256 \pi^2 e \beta^2 / 5\) and for higher values of \(\kappa\) the LSB vacuum is favourable. By its character the corresponding phase transition is of the first order since at \(\kappa_{\text{cr}}\) the v.e.v. of \(\eta^2\) jumps to \(\eta_{\text{cr}}^2 = M^2/e\). This v.e.v. entails the LSB due to creation of space-like constant gradient of pseudoscalar field \(\partial_\mu \theta \sim M^2 e^{-1/2}(0, n); \quad n^2 = 1\).

When going back to Sec.1 one can conclude that the plausible scenario of what happens in the chiral gauge model (3) with Proca vector fields at high energies is the LSB at strong coupling. This is what might be behind the breaking of perturbative unitarity in such models.

If \(\beta = 0\) (Sec.2, Case C, the pure WZ interaction without a kinetic term for \(\theta\)-field) the LSB still occurs and the Lorentz symmetric extremum becomes a maximum. For the negative sign of the first term in Eq. (34) the LSB minimum always exists and a normalization scale is not there to prevent the Lorentz symmetric vacuum from decay.

Let us extend our analysis to the massive vector-field models. We pay the special attention to the power-counting renormalizable chiral model (1) with transversal vector fields. This model is suitably described in the ”ghost” sector by the effective lagrangian (11), with a ”ghost” sign of the kinetic term for \(\theta\)-field (\(\beta^2 \rightarrow - \beta^2; \quad m = \beta M\)) and can be consistently normalized by the choice (32) at the infrared point. The LSB conditions (34), (35) are fulfilled both in the strong and the weak coupling regimes and the LSB minimum is unique.
since $V''(\eta^2) > 0$ for positive $\eta^2$. In particular,

$$\eta_{\min}^2 \simeq \frac{m^2}{5\kappa^4 \ln(32\pi^2/\kappa^2)} \quad \text{for} \quad \kappa << 1;$$
$$\eta_{\min}^2 \simeq \frac{m^2}{32\pi \kappa^3 \sqrt{7}} \quad \text{for} \quad \kappa \gg 1.$$  

(36)

Thus the symmetric vacuum in such a model is always unstable.

Once the Lorentz symmetry is spontaneously broken one should expect the occurrence of Goldstone modes in the spectrum of fluctuations, $\partial_\mu \tilde{\theta} = \partial_\mu \theta - M\eta_\mu$ around the minimum. In our case it gives rise to the degeneracy in the kinetic term of pseudoscalar fluctuations. As it follows from (35), the second variation contains the projector on the $\eta_\mu$ direction. Consequently, after Wick rotating to the Minkowski space-time, the kinetic term $-1/2 \left( (\eta \cdot \partial) \tilde{\theta}(x) \right)^2$ describes the dynamics of a massless free mode whose support, in the momentum space, lies on the space-like hyperplane $\eta_\mu n^\mu$ (this feature typically arises in the quantization of Yang-Mills fields in algebraic non-covariant gauges\(^{11}\)). In other directions the dynamics is generated by higher-derivative ”ghost-dipole” terms in the effective action, $\sim (\partial^2 \theta)^2$.

For a different sign of kinetic term and for $\beta = 0$ there is no LSB solutions with space-like $\eta_\mu$ but there are time-like LSB configurations which yield the extremum for the one-loop action. However the very notion of vacuum energy should be carefully revised since those configurations are involved in the construction of the Hamiltonian by means of Legendre transformation. We postpone the analysis of this problem as well as the description of energy spectrum of $\theta$-field fluctuations to a more detailed paper.

6. Conclusions

As a result of the present investigation one can argue that the cancellation of anomalies strongly prevents the chiral theories from the Lorentz symmetry breaking.

On the other hand in the presence of Wess-Zumino interaction the occurrence of LSB, at least in small domains, seems to be natural. Indeed, the WZ action is invariant under general coordinate transformations, $x_\mu \to n_a(x_\mu)$. This mapping is a local diffeomorphism when

$$\det [\partial^\mu n_a] \equiv \epsilon^{\mu\nu\rho\sigma} \partial^\mu n_0 \partial^\nu n_1 \partial^\rho n_2 \partial^\sigma n_3 \neq 0.$$ 

Under these transformations the fields and derivatives behave as vectors,

$$A^\mu = \tilde{A}^a(n) \frac{\partial n_a}{\partial x_\mu}.$$ 

Therefore the pseudoscalar Chern-Pontryagin density in the WZ action turns out to be multiplied by $\det [\partial^\mu n_a]$, just compensating the change in the integration measure. Let us select out the curvilinear coordinate system with $\theta(x)$ as one of the coordinate vector, i.e. $\theta \equiv \tilde{\theta}(n) = M\eta_a n^a$, as it is always possible inside the domains where $\partial_\mu \theta \neq 0$. Then the WZ
action (2) in new coordinates takes the form of the Chern-Simons action in the hyperplane orthogonal to the constant vector $\eta_a$,

$$S_{WZ} = -\frac{\kappa}{2} \int d^n \eta_a \int d^3 n^\perp \epsilon^{abcd} A_b \partial_c A_d,$$

(37)

which provides the reduction of dynamics from four to three dimensions. Thus locally, in domains of smooth $\partial_\mu \theta$, one actually deals with the dynamics described in our paper.

We see some similarities of LSB between the (2 +1)-dimensional case, as it is connected with the dynamics in two dimensions, and the (4 + 1)-dimensional case in its reduction to our model. These similarities will be discussed elsewhere.

Apart from the benefit for understanding the chiral model, the spectral properties of the light in a pseudoscalar medium (“pseudoscalar optics”) could happen to be applicable in two situations: first, to explore the axion matter if it exists in the Universe and, second, to check the possibility of strong-coupling LSB in the neutral-pion matter under extreme conditions. If the latter one is conceivably characterized by a vanishing pion decay constant $F_\pi \to 0$ at the phase transition of chiral symmetry restoration then the WZ interaction of pions and photons may be enhanced considerably to invoke LSB.

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