Enhancement of tunnel conductivity by Cooper pair fluctuations in electron-hole bilayer

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Abstract. Influence of Cooper pair fluctuations that are precursor of pairing of electrons and holes located on opposite surfaces of topological insulator film on tunnel conductivity between the surfaces is investigated. Due to restrictions caused by momentum and energy conservation dependence of tunnel conductivity on external bias voltage has peak that becomes more prominent with decreasing of disorder and temperature. We have shown that Cooper pair fluctuations considerably enhance tunneling and height of the peak diverges in vicinity of critical temperature with critical index $\nu = 2$. Width of the peak tends to zero in proximity of critical temperature. Pairing of electrons and holes can be suppressed by disorder and in vicinity of quantum critical point height of the peak also diverges as function of Cooper pair damping with critical index $\mu = 2$.

1. Introduction

Recently pairing of spatially separated electrons and holes located on opposite surfaces of thin film of topological insulator was predicted [1, 2, 3]. Pairing can lead to a number of interesting effects including superfluidity [4], anomalous drag effect [5] and anomalous Andreev reflection [6]. Condensate formed by electron-hole Cooper pairs is topological one [1] and can host Majorana fermions on topological defects. But the most prominent manifestation of pairing is internal Josephson effect.

Internal Josephson effect leads to colossal enhancement of tunnel conductance between electron and hole layers. It was observed [7] in a system of composite electrons and composite holes that was realized in semiconductor GaAs structures in strong magnetic field. Its quantitative description is complicated one due to disorder, strong phase fluctuations that take place in two-dimensional superfluids and topological defects ([8] and references therein).

Here we propose new physical effect that can be observed in normal state in vicinity of critical temperature. In the normal state pairing areas can appear as thermodynamical fluctuations and can transfer coherent Josephson current. We have shown that pairing fluctuations lead to considerable enhancement of tunneling in normal state in vicinity of critical temperature. In spite of the fact that described effect is universal and can be realized in different realizations of electron-hole bilayers, for example in graphene bilayer [9], we considered it for definiteness in thin film of topological insulator.

The rest part of the article is organized in following way. In section 2 we briefly discuss electronic structure of topological insulator film, microscopical approach for description of
Cooper pair fluctuations and calculation of tunnel conductivity. In section 3 we discuss results of calculation and conclude. Section 4 is devoted to brief summary.

2. Calculation of tunneling conductivity

The considered system consists of topological insulator thin film. Surface states of its opposite surfaces are populated by electrons and holes controlled by external gates. In weak coupling regime [3] Hamiltonian of the system is given by

$$H = \sum_k (\xi_{kT} a_{kT}^+ a_{kT} + \xi_{kB} a_{kB}^+ a_{kB}) + \frac{\lambda}{\nu_F} \sum_{k' q} a_{k+qT}^+ a_{k' -qB} a_{k' B} a_{kT}$$

where $\lambda$ is dimensionless coupling constant in Bardeen-Cooper-Schrieffer (BCS) approximation; $a_{kT}$ and $a_{kB}$ are annihilation operators for electrons on top and bottom surfaces of topological insulator film; $\xi_{kT} = v_F k - \mu$ and $\xi_{kB} = -v_F k + \mu$ are their dispersion law (electrons from bottom layer have negative velocity indicating that spectrum belong hole type); $v_F$ is Fermi velocity and $\nu_F$ is density of states on Fermi level. Pairing is very sensitive to mismatch of concentrations of electrons and holes so here we consider only balanced case. Intralayer Coulomb interaction leads only to renormalization of quasiparticle spectrum and can be omitted. Definition of dimensionless coupling constant $\lambda$ in BCS approximation including interaction screening and specifics of topological insulator surface states can be found in [3]. Also theory of electron-hole pairing in more complicated realistic model that includes screening, disorder and hybridization caused by tunneling is developed in [3].

In microscopic theory of fluctuations in superconductors [10] Cooper pair propagator is usually introduced. It corresponds to vertex function that is sum of infinite series of scattering diagrams between electrons in Cooper channel. On Fig. 1a introduction of propagator for Cooper pairs $L^R_c(\omega)$ formed by electrons and holes in film of TI is represented. Its analytical value is given by

$$L^R_c(\omega) = \frac{\lambda}{1 - \lambda/\nu_F \Pi^R_c(\omega)},$$

Figure 1. a) Introduction of Cooper propagator $L^R_c(\omega)$ for electron-hole bilayer b) Diagrams for tunnel conductivity including contribution of Cooper pair fluctuations

Figure 2. Dependence of tunnel conductivity on external bias voltage for $T = 0.2K$ and $\gamma = 0.2K$. Inset: Phase diagram of the system in presence of disorder.
where $\Pi^R_c(\omega)$ correspond to bubble diagram on Fig. 1 b in and is given by

$$
\Pi^R_c(\omega) = \nu_F \left( \frac{1}{\lambda} - \left[ \ln \frac{T}{T_0} + \Psi(\frac{1}{2}) + \frac{2\pi T}{\omega} - i \frac{\omega}{4\pi T} \right] - \Psi(\frac{1}{2}) \right). \quad (3)
$$

Here $T_0$ is critical temperature of pairing in absence of disorder, $\gamma$ is Cooper pair damping that is average value of damping of electron and hole and $\Psi(x)$ is logarithmic derivative of Gamma function. At critical temperature denominator of (2) become zero indicating Cooper instability of normal state of the system against electron-hole pairing.

For calculation of tunnel conductivity we used linear response approach [11]. We showed that if momentum of electron is conserved in tunneling process then in absence of interactions between electrons and holes tunneling conductivity is given by

$$
\sigma_T(V) = \frac{e^2}{h} \frac{4\pi |t|^2}{eV} \text{Im}[\Pi^R_c(eV)], \quad (4)
$$

where $V$ is external bias voltage, $t$ is amplitude of tunneling process. This term corresponds to first term on diagram on Fig 1b. Contribution of Cooper pair fluctuation correspond second term on diagram on Fig 1b. These contribution can be interpreted as fluctuation internal Josephson effect. The value of tunneling conductivity in presence of Cooper pair fluctuations is given by

$$
\sigma_T(V) = \frac{e^2}{h} \frac{4\pi |t|^2}{eV} \text{Im}[\frac{\Pi^R_c(eV)}{1 - \lambda/\nu_F \Pi^R_c(eV)}]. \quad (5)
$$

In vicinity of critical temperature denominator of expression tends to zero hence Cooper pair fluctuations leads to critical behavior of tunnel conductivity.

3. Conclusions

For numerical calculation we used the following set of parameters [3] $\mu = 0.02\text{eV}$, $T_0 = 0.1\text{K}$, $\lambda = 0.13$, $t = 0.01\text{K}$. In BCS regime disorder can easily suppress pairing since it acts differently on components of Cooper pair if its damping $\gamma$ exceed critical value $\gamma_0 = 0.89T_0$. The phase diagram of the system in presence of disorder is presented on inset of Fig 2.

The dependence of tunnel conductivity on external bias is plotted for $T = 0.2\text{K}$ and $\gamma = 0.2\text{K}$ on Fig 2. For other values of these parameters the dependence is qualitatively the same. Peak is caused by conservation laws for momentum and energy that should be satisfied for each tunneling process. Coulomb interaction considerably enhances tunneling between layers.

Dependence of tunnel conductivity peak height and peak width as function of temperature is presented on Fig. 3 and Fig. 4 for two values of Cooper pair damping. At $\gamma_1 = 0.06\text{K}$ disorder only reduce critical temperature and at $\gamma_2 = 0.1\text{K}$ pairing is suppressed. In vicinity of critical temperature peak height diverges and its width tends to zero. In critical region corresponding analytical dependence is given by

$$
\sigma_T(V = 0) \sim (T/T_{0d} - 1)^{-\nu},
$$

where $\nu = 2$ is critical index and $T_{0d}$ is critical temperature in presence of disorder. Dependence of height on temperature is universal — it depends only on proximity to critical temperature $T_{0d}$ and does not depend on disorder explicitly. We found that even if pairing of electrons and holes is suppressed by disorder the tunneling is strongly enhanced in region $\gamma < \gamma_0/\lambda$. Width of peak is decreasing with decreasing of temperature and its minimal value depends on Cooper pair damping. In vicinity of quantum phase transition tunneling conductivity also diverges and is given by

$$
\sigma_T(V = 0) \sim (\gamma/\gamma_0 - 1)^{-\mu},
$$

3
where $\mu = 2$ is corresponding critical index.

Interlayer Coulomb interaction considerably influence tunneling in electron-hole bilayer even if Cooper pairing is suppressed by disorder. The situation is completely different from one in electron-electron bilayer [12] where interlayer interaction in ladder approximation does not influence height of the peak and insignificantly decreases its width.

4. Summary
Influence of fluctuation of Cooper pairs formed by electron and hole in normal state in thin film of topological insulator on tunneling between its surfaces is considered. We find critical behavior of tunneling conductivity in vicinity of classical phase transition with critical index $\nu = 2$. In vicinity of quantum phase transition dependence of tunnel conductivity on damping of Cooper pairs is also critical with critical index $\mu = 2$.

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