Sliding mode control of continuous time systems with reaching law based on exponential function

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Abstract. In this paper a pseudo-sliding mode control is proposed by introducing a continuous and smooth input signal in order to guarantee both chattering elimination and boundedness of sliding variable derivative in the presence of non-zero external disturbance. For this purpose, having fixed a suitable sliding manifold, a homogeneous differential equation describing the sliding variable evolution is considered. It is discussed later in this paper that the input signal formed on the basis of this equation provides asymptotic convergence of the sliding variable and its derivative to zero as well as the asymptotic stability of the non-linear system in the absence of external disturbance. The dynamics of the system affected by non-zero external disturbance make the state vector converge to domains in a vicinity of the origin at the exponential rate, as the norm of arbitrary trajectory is limited to decreasing exponential function. In order to expand the variety of controllers based on a reaching law and providing the above-mentioned properties, a certain class of functions is presented.

1. Introduction
In many practical applications it is very difficult to obtain an accurate model reflecting a physical phenomenon that needs to be controlled. Discrepancy between the actual dynamics and its mathematical model induced by complexity and non-linearity of the system, time-varying or unknown parameters as well as unmodelled structures, forced engineers to introduce a new control technique combating aforementioned modelling difficulties. One widely adopted approach to both robust and computationally effective control of high-order non-linear plants is the sliding mode control (SMC). The idea of SMC is based on the application of a control signal always moving the state vector toward a region adjacent to a differentiable manifold [10, 11] and allowing the representative point to remain on the sliding surface in its close vicinity thereafter. Once the system state hits a manifold, the dynamics of the system becomes insensitive to some disturbances and uncertainties of the model. The development of these concepts began in the late 1950s in the Soviet Union - a significant contribution in the progress of SMC is owed to Utkin [1, 2] and Itkis [3]. Since the sliding mode control law is usually not a continuous function of time, thus one needs to take into consideration control chattering problem impeding SMC implementations. Although chattering attenuation received a major boost in the early 1990s due to second-order concepts [6] and then in 2000s, when higher-order ideas appeared [7], it is still one of the most important aspects in practical applications of SMC.

In this paper we consider an alternative control strategy based on a reaching law [8, 9]. The designed controller is both continuous and smooth in order to provide complete chattering elimination in the presence of non-zero disturbance. Furthermore, proposed algorithm
guarantees boundedness of the sliding mode derivative. The price we pay for obtaining such an input signal is a loss of accuracy, as the system trajectories converge to a vicinity of the origin (instead of the origin) in the presence of non-zero external disturbance.

2. Problem statement
We consider the time-varying, nonlinear n-th order dynamic system

\[ \dot{x}_1 = x_2, \dot{x}_2 = x_3, \ldots, \dot{x}_n = f(t, x_1, \ldots, x_n) + b \cdot u + d(t) \quad (\dot{x} = dx/dt, t \geq 0), \]

where \( x_1, x_2, \ldots, x_n \) are the state variables of the system and \( x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \) is the state vector, \( t \) denotes time, \( u \in \mathbb{R} \) is the input signal. Function \( f \) is a priori known and maps \( [t_0; \infty) \times \mathbb{R}^n \) into \( \mathbb{R} \). Besides, \( f = f(t, x) \) is assumed to be bounded and Lipschitz continuous with respect to the second coordinate. Moreover, \( b \) is a non-zero constant, while scalar-valued function \( d \) defined on \( [t_0; \infty) \) is unknown and represents the system external disturbance. Further in this paper, it is assumed that \( d \) is continuous almost everywhere (discontinuity set is a measure-zero set) [16] and there exists a positive constant \( D \) such that

\[ |d(t)| \leq D, \text{ for all } t \in [t_0; \infty). \]

Switching function \( \sigma : \mathbb{R}^n \to \mathbb{R} \) is introduced as follows

\[ \sigma(x) := c_1 x_1 + \ldots + c_n x_n, \]

where \( c_1, \ldots, c_n \) are real constants. Furthermore, it is assumed that the constants \( c_1, \ldots, c_n \) are chosen so that the eigenvalues of the homogeneous differential equation

\[ c_1 x_1 + \ldots + c_n x_n = 0 \]

are identical, real and less than zero.

Until the end of the next chapter we will be analyzing dynamic system (1), wherein \( d \equiv 0 \) \((t_0 \leq t < \infty)\). The differential equation describing the sliding variable evolution is proposed in this brief in the following form

\[ \dot{\sigma} = -k \left( 1 - e^{-|\sigma|/\sigma_0} \right) \text{sign}(\sigma), \]

where \( k \) and \( \sigma_0 \) are certain constants greater than zero. Note that the latter equation is satisfied when the following controller is applied

\[ u(t, x) = -\frac{k}{b} \left( 1 - e^{-|\sigma|/\sigma_0} \right) \text{sign}(\sigma) - \frac{1}{b} (c_1 x_2 + \ldots + c_{n-1} x_n) - \frac{1}{b} f(t, x). \]

The solution of Eq. 5 is determined by the equations

\[ \sigma(t) = \sigma_0 \ln \left( 1 + e^{-k(t+C)/\sigma_0} \right) \text{sign}(\sigma), \quad C = -\frac{\sigma_0}{k} \ln \left( e^{\sigma(t_0)/\sigma_0} - 1 \right) \text{sign}[\sigma(t_0)]. \]

Taking \( \sigma \) as a measure of the distance between a representative point \( x \) and a hyperplane \( H = \{ x \in \mathbb{R}^n : \sigma(x) = 0 \} \) we obtain, that while representative point is reaching the hyperplane, its velocity of convergence \( \dot{\sigma} \) is decreasing exponentially. Since the function \( \dot{\sigma} \) is bounded upper and below by \( \pm k \), thus the absolute value of \( \dot{\sigma} \) never exceeds \( k \).

Note that the control function (6) is both continuous and smooth despite discontinuity of \( \text{sign} \) function. Although simple calculation shows that

\[ \lim_{t \to \infty} \sigma(t) = 0, \quad \lim_{t \to \infty} \dot{\sigma}(t) = 0, \]

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the smooth feedback controller (6) cannot provide finite-time convergence of the sliding variable to zero. Convergence of the system trajectories is discussed later in this paper.

The use of the controller (6) for the system carries some mathematical advantage. Indeed, since one can prove that function (6) has Lipschitz property in the absence of external disturbance \( d \), thus there is no need to understand the solutions of the system in the Filippov sense [5, 12] any longer. Thanks to Picard - Lindelöf theorem [13] the existence and uniqueness of the solution is guaranteed.

3. Asymptotic stability of the system in the absence of external disturbance

In this section asymptotic stability of the system including feedback controller (6) is proven. Let us consider once more dynamic system (1) in the absence of external disturbance \( d \). The idea is to show that every solution of non-homogeneous differential equation

\[
c_1x_1 + ... + c_nx_n = \sigma(t),
\]

where \( \sigma \) is determined by (7), is asymptotically stable. To do so, one can select a proper Lyapunov - candidate function, however in this very situation it is not an easy task. Therefore, a general law in the theory of ordinary differential equations is proven in this chapter.

**Definition 1:** Consider differential equation

\[
\dot{x} = A(t) \cdot x + r(t),
\]

where \( t \in (a;b) \), \( A : (a;b) \to L(\mathbb{R}^n) \) is a continuous function, \( L(\mathbb{R}^n) \) denotes a space of bounded linear operators \( \mathbb{R}^n \to \mathbb{R}^n \) and \( r : (a,b) \to \mathbb{R}^n \) is continuous almost everywhere. Function \( U : (a;b)^2 \to L(\mathbb{R}^n) \) defined by the formula

\[
U(t;t_0) := \sum_{m=0}^{\infty} K_m(t;t_0),
\]

where \( K_0(t,t_0) = I \) and

\[
K_{m+1}(t;t_0) = \int_{t_0}^{t} \int_{t_0}^{s_m} \cdots \int_{t_0}^{s_1} A(s_m) \cdots A(s_1)A(s)ds_m \cdots ds_1ds = \int_{t_0}^{t} K_m(s;t_0)ds
\]

is called a resolvent of Eq. 10 [14, 15].

Let \( \phi = \phi(t) \) be a solution of Eq. 10 determined by initial condition \( \phi_0 = \phi(t_0) \). Note that the function \( g(t,x) = A(t) \cdot x \) is Lipschitz continuous with respect to the second coordinate, thus every solution can be extended to infinity. Taking into account that \( U(t_0;t_0) = I \) and \( \dot{U}(t;t_0) = A(t)U(t;t_0) \), we obtain that the resolvent is a normalized fundamental matrix of Eq. 10, hence

\[
\phi(t) = U(t;t_0) \cdot \phi_0.
\]

Furthermore, differentiating the following equation

\[
\phi(t) = U(t;t_0)\phi_0 + \int_{t_0}^{t} U(t;s)r(s)ds,
\]

one can easily prove that (14) represents a unique solution of (10).

**Lemma 1:** If the trivial solution \( \phi \equiv 0 \) of homogeneous equation associated with Eq. 10 is asymptotically stable, then every solution of Eq. 10 is asymptotically stable.
Proof: Let \( \psi = \psi(t) \) and \( \eta = \eta(t) \) be arbitrary solutions of (10) determined respectively by initial conditions \( \psi_0 = \psi(t_0), \eta_0 = \eta(t_0) \). From (14) we get
\[
\psi(t) - \eta(t) = U(t; t_0)\psi_0 + \int_{t_0}^{t} U(t; s)r(s)ds +
\]
\[
= -U(t; t_0)\eta_0 - \int_{t_0}^{t} U(t; s)r(s)ds =
\]
\[
= U(t; t_0)(\psi_0 - \eta_0).
\]

The latter equation shows that the difference between any two non-homogeneous system solutions is a solution of a homogeneous system, thus taking into account that the trivial solution of homogeneous equation is asymptotically stable, we obtain the following implication
\[
\forall \varepsilon > 0 \exists \delta > 0 \ ||\psi(t_0) - \eta(t_0)|| < \delta \Rightarrow ||\psi(t) - \eta(t)|| < \varepsilon, \text{ for all } t_0 \leq t \leq \infty. \tag{18}
\]

Obviously \( \lim_{t \to \infty} ||\psi(t) - \eta(t)|| = 0 \). This concludes the proof. Now we are in a position to formulate the main result of this chapter.

Theorem 1: If \( d \equiv 0 \ (t_0 \leq t < \infty) \), then system (1) including control signal given by (6) is asymptotically stable.

Proof: Simple calculation shows that Eq. 9 might be represented in the following form
\[
\dot{x} = A \cdot x + r(t), \tag{19}
\]
where \( x, r(t) \) are vectors of dimension \((n-1) \times 1\), wherein \( x = [x_1, ..., x_{n-1}]^T \) and \( r(t) = [0, ..., \sigma(t)]^T \). \( A \) is in turn matrix of dimension \((n-1) \times (n-1)\) given by the formula
\[
A = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 & 0 \\
0 & 0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & 1 \\
-c_1 & -c_2 & -c_3 & \cdots & -c_{n-2} & -c_{n-1}
\end{bmatrix} \tag{20}
\]

Since homogeneous equation corresponding to (9) is asymptotically stable, therefore in particular trivial solution is asymptotically stable. Relying on lemma 1, we obtain asymptotic stability of (9). The theorem is proven.

4. Uniform and asymptotic boundedness of the system in the presence of non-zero external disturbance

Until now, we have been considering system (1) assuming the absence of disturbance function. Suppose now that \( d \neq 0, k > D \) and control law is designed as in (6). Under above assumptions, one can easily notice that sliding variable satisfies the following differential equation
\[
\dot{\sigma} = -k \left( 1 - e^{-|\sigma|/\sigma_0} \right) \text{sign}(\sigma) + d(t). \tag{21}
\]

Define a set
\[
\Sigma_D := \{ \sigma : |\sigma| \leq -\sigma_0 \ln(1 - D/k) \}. \tag{22}
\]

Lemma 2: If there exists time \( t_1 \in [t_0; \infty) \) such that \(|\sigma(t_1)| = -\sigma_0 \ln(1 - D/k)\), then \( \sigma(t) \in \Sigma_D \) for all \( t \geq t_1 \).

Proof: Suppose that there exists time \( t_2 > t_1 \) such that \( \sigma(t_2) > -\sigma_0 \ln(1 - D/k) \). Since disturbance function \( d \) is continuous almost everywhere, thus there must exist an interval
(a;b) ⊂ (t1;t2) and for all t ∈ (a;b) the switching function σ is increasing. Hence, ˙σ > 0 if t ∈ (a;b). On the other hand, Eq. 21 yields ˙σ < 0 for all t and σ(t) > −σ0 ln (1 − D/k), which ultimately leads to a contradiction. Due to the symmetry of the problem, it is enough to consider the case of σ > 0.

The latter lemma means that if sliding variable reaches boundary set ΣD, then it remains there thereafter. The following Lyapunov - candidate function

\[ V(σ) = \frac{1}{2} [σ + \text{sign}(σ)σ0 \ln (1 − D/k)]^2 \]  

(23)
guarantees the asymptotic convergence of σ to ΣD. Considering V(σ) = 1/2σ2 as a Lyapunov - candidate function, one can easily prove finite-time convergence. Now we are in a position to formulate the main result of the paper.

**Theorem 2:** Let f = φ(t) be an arbitrary solution of system (1) in the presence of non-zero external disturbance, determined by initial condition φ0 = φ(t0). Let λ denotes the eigenvalue of A. Then

\[ ||φ(t)|| ≤ \frac{σ0}{λ} \ln (1 − D/k), \text{ if } t \to ∞. \]  

(24)

**Proof:** The idea is to show, that every solution of non-homogeneous differential equation

\[ c_1 x_1 + ... + c_n x_n = σ(t), \]  

(25)

where σ is is related to Eq. 21, satisfies condition (24). As we have already seen, Eq. 25 is is equivalent to (19). Taking into account the Ważewski’s inequality [15, 17]

\[ ||U(t; t_0)|| = ||e^{A(t−t_0)}|| ≤ e^{λ(t−t_0)} \text{ for all } t ∈ [t_0; ∞), \]  

(26)

we obtain that

\[ ||φ(t)|| ≤ ||U(t; t_0)|| \cdot ||φ(t_0)|| + \int_{t_0}^{t} ||U(t; s)|| \cdot ||r(s)|| ds ≤ \]  

(27)

\[ ≤ e^{λ(t−t_0)} ||φ(t_0)|| + \int_{t_0}^{t} e^{λ(t−s)} ||σ(s)|| ds = \]  

(28)

\[ = e^{λ(t−t_0)} ||φ(t_0)|| + e^{λt} \int_{t_0}^{t} e^{−λs} ||σ(s)|| ds. \]  

(29)

Obviously, \( \lim_{t \to ∞} e^{λ(t−t_0)} ||φ(t_0)|| = 0 \). Relying on the generalized de l’Hospital theorem in the form of Stolz [15], one can easily prove that

\[ \lim_{t \to ∞} e^{λt} \int_{t_0}^{t} e^{−λs} ||σ(s)|| ds = \frac{σ0}{λ} \ln (1 − D/k). \]  

(30)

Note that the existence of the latter limit is guaranteed only when σ ∉ ΣD. Suppose that there exists time t_1 ∈ [t_0; ∞) and ||σ(t_1)|| = −σ0 ln (1 − D/k). Simple calculation shows that if t → ∞, then

\[ ||φ(t)|| ≤ \lim_{t \to ∞} e^{λ(t−t_0)} ||φ(t_0)|| + \lim_{t \to ∞} e^{λt} \int_{t_0}^{t_1} e^{−λs} ||σ(s)|| ds + \]  

(31)

\[ + \lim_{t \to ∞} e^{λt} \int_{t_1}^{t} e^{−λs} ||σ(s)|| ds = \lim_{t \to ∞} e^{λt} \int_{t_1}^{t} e^{−λs} ||σ(s)|| ds ≤ \]  

(32)

\[ ≤ \lim_{t \to ∞} e^{λt} \int_{t_1}^{t} e^{−λs} [−σ0 ln (1 − D/k)] ds = \frac{σ0}{λ} \ln (1 − D/k). \]  

(33)
The last equality follows from the generalized de l’Hospital theorem in the form of Stolz. The proof of the theorem is complete.

It is worth noting that the record \( \lim_{t \to \infty} ||\phi(t)|| \) is formally incorrect as the considered limit does not need to exist, though the statement (26) remains valid.

**Corollary 1**: Let \( \phi = \phi(t) \) be an arbitrary solution of system (1) in the presence of non-zero external disturbance, determined by initial condition \( \phi_0 = \phi(t_0) \). Let \( \lambda \) denotes the eigenvalue of \( A \). Then

\[
||\phi(t)|| \leq ||\phi(t_0)||e^{\lambda(t-t_0)} + \frac{\sigma_0}{\lambda} \ln(1-D/k) \quad \text{for all } t \in [t_0; \infty).
\]

**Proof**: Since

\[
\frac{d}{dt} \int_{t_0}^{t} e^{\lambda(t-s)} |\sigma(s)| ds = |\sigma(t)| \geq 0
\]

and the function of the upper limit is increasing, hence

\[
||\phi(t)|| \leq ||\phi(t_0)||e^{\lambda(t-t_0)} + \lim_{t \to \infty} \int_{t_0}^{t} e^{\lambda(t-s)} |\sigma(s)| ds.
\]

Computing the latter limit, we obtain the thesis.

**Corollary 2**: Let \( \phi = \phi(t) \) be an arbitrary solution of system (1) in the presence of non-zero external disturbance, determined by initial condition \( \phi_0 = \phi(t_0) \). Let \( \lambda \) denotes the eigenvalue of \( A \). If \( \text{det}(A) \neq 0 \), then

\[
||\phi(t)|| \leq ||\phi(t_0)||e^{\lambda(t-t_0)} + |\sigma(t_0)| \cdot ||A^{-1}|| \cdot \sup_{t \in [t_0; \infty)} (||U(t; t_0) - I||) \leq \sum_{k=0}^{n} ||A^{-1}|| \cdot n,
\]

for all \( t \in [t_0; \infty) \).

**Proof**: Note that

\[
||\phi(t)|| \leq ||\phi(t_0)||e^{\lambda(t-t_0)} + || \int_{t_0}^{t} e^{A(t-s)} |\sigma(t_0)| ds || = \sum_{k=0}^{n} ||A^{-1}|| \cdot n,
\]

As the assumptions of the Fubini’s theorem [16] are satisfied, thus

\[
|| \sum_{k=0}^{\infty} \frac{A^{k+1}(t-t_0)^{k+1}}{(k+1)!} || \leq \sup_{t \in [t_0; \infty)} (||U(t; t_0) - I||) \leq \sup_{t \in [t_0; \infty)} (e^{\lambda(t-t_0)} + ||I||) \leq n,
\]

which concludes the proof.

5. **Generalization on the control law**

In this section a generalization on the differential equation (5) is proposed. Consider function \( z : \mathbb{R} \to \mathbb{R} \) of variable \( \sigma \) such that

- \( z \geq 0 \), \( z \) is continuous and even,
- \( z \) is differentiable or differentiable except 0,
\[ u(t, x) = -\frac{k}{b} \left( 1 - a^{-z(\sigma)/\sigma_0} \right) \text{sign}(\sigma) - \frac{1}{b} (c_1 x_2 + \ldots + c_{n-1} x_n) - \frac{1}{b} f(t, x). \] (43)

It is easy to observe that appliance of controller (43) to the system makes the sliding variable satisfy the following differential equation
\[ \dot{\sigma} = -k \left( 1 - a^{-z(\sigma)/\sigma_0} \right) \text{sign}(\sigma) + d(t). \] (44)

Depending on the choice of \( z \), homogeneous equation associated with the Eq. 44 may be impossible to solve analytically, yet the existence and uniqueness is guaranteed by Picard - Lindelöf theorem. Indeed, if \( \sigma > 0 \), then
\[ P'_\sigma = \left[ -k \left( 1 - a^{-z(\sigma)/\sigma_0} \right) \text{sign}(\sigma) \right]' = -\frac{k \cdot \ln(a)}{\sigma_0} \cdot a^{-z(\sigma)/\sigma_0} \cdot z'_\sigma \leq 0 \] (45)
and \( z \in C_{a,L}^0 \), thus \( P'_\sigma \) is continuous on \([0; \infty)\), which implies that \( \lim_{\sigma \to 0} P'_\sigma = 0 \), \( \lim_{\sigma \to \infty} P'_\sigma = -k \cdot \ln(a) \cdot g/\sigma_0 \). Hence \( P'_\sigma \) is bounded and as a result Lipschitz continuous. The same is true for \( \sigma < 0 \).

Selecting a proper Lyapunov - candidate function, one can easily prove that in the absence of external disturbance
\[ \lim_{t \to \infty} \sigma(t) = 0, \quad \lim_{t \to \infty} \dot{\sigma}(t) = 0. \] (46)

From the foregoing considerations, we obtain that controller (43) makes the sliding variable and its derivative possess the characteristics and properties of the variable that satisfies (5). Therefore, all the lemmas and theorems discussed so far may be prescribed to an accuracy of some constants. Indeed, consider a set
\[ \sigma_{\min} := \inf \left\{ \sigma \in (0; \infty) : z(\sigma) > -\sigma_0 \log_a (1 - D/k) \right\}, \] (47)
where \( k > D \). Note that \( z \in C_{a,L}^{0,1} \) implies the existence of \( \sigma_1 \in [0, \infty) \) such that function \( z_{\min} = z|_{[\sigma_1; \sigma_{\min}]} \) is strictly increasing and as a result injective. Hence,
\[ \Sigma^D_{\sigma_1} := \{ \sigma \in \mathbb{R} : |\sigma| \leq z_{\min}^{-1} (\sigma_0) \} \] (48)
is used instead of \( \Sigma_D \). Obviously, if the function \( z \) is injection on \([0; \infty)\), then it is enough to consider \( z^{-1} \) instead of \( z_{\min}^{-1} \).

### 5.1. Example

For the mass-spring-damper system with friction [18] modeled by
\[ m\ddot{x} + b_0 \dot{x} + b_1 \text{sign}(\dot{x}) + k_1 x = u + d(t), \] (49)
where \( x \) is the displacement from equilibrium, \( u \) is the external force applied to system, \( m \) is the mass of the block, \( b_0 \) is damping constant, \( b_1 \) is friction, \( k_1 \) is force constant of spring, \( d \) denotes unknown external disturbance, the control law

\[
u(x) = -km \left( 1 - e^{-\sigma^2} \right) \text{sign}(\sigma) - m\dot{x} + b_0\dot{x} + b_1\text{sign}(\dot{x}) - k_1x
\]  

(50)

applied to (49) makes the sliding variable satisfy (44) (for the purposes of this example, we will call (50) non-conventional control law), if the switching function is given by \( \sigma(x) = x + \dot{x} \) and \( a = \exp, z(\sigma) = \sigma^2 \). The control system is simulated with \( m = 1 \) [kg], \( b_0 = 0.1 \) [kg/s], \( b_1 = 0.05 \) [kg/s²], \( k_1 = 0.5 \) [kg/s], \( k = 2 \), \( \sigma_0 = 1 \), \( d(t) = \sin(t) \), \( D = 1 \), \( x(0) = 7 \) [m] and \( \dot{x}(0) = 10 \) [m/s]. Furthermore, we demonstrate the results of the simulation with the same control system simulated with conventional sliding mode control law given by

\[
u(x) = -x - 3 \cdot \text{sign}(x + \dot{x}).
\]  

(51)

Lines located above and below the hyperplane \( \sigma(x) = 0 \) (see Fig. 1) are given respectively by

\[
x_2 = -x_1 \mp \sigma_0 \ln \left( 1 - \frac{D}{k} \right) = -x_1 \mp \ln \left( 0.5 \right),
\]  

(52)

wherein \( x_1 = x \) and \( x_2 = \dot{x} \). Lines located in Fig. 3 are determined respectively by

\[
\sigma = \mp \ln \left( 1 - \frac{D}{k} \right) = \mp \ln \left( 0.5 \right).
\]  

(53)
6. Conclusions
In this paper an alternative sliding mode control for continuous time systems has been proposed. The method employs a reaching law - the strategy has been designed so that the rate of change of the sliding variable is always bounded and decreases exponentially with the sliding variable
decrease. On the one hand, presented algorithm enforces the convergence of the representative point to the origin while considering unperturbed systems. On the other hand, for systems affected by external disturbance it has been proven that sliding variable enters in finite time a specified set and never leaves it. As a result, every trajectory starting from any initial position converges to a vicinity of the origin and the system becomes uniformly and asymptotically bounded. Furthermore, application of the proposed reaching law has resulted in chattering elimination. Techniques demonstrated in this article can be adapted for discrete time systems.

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