Two-Higgs-Doublet Models with a Flavored $Z_2$

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Two Higgs-doublet models usually consider an ad-hoc $Z_2$ discrete symmetry to avoid flavor changing neutral currents. We consider a new class of two Higgs-doublet models where $Z_2$ is enlarged to the symmetry group $F \ltimes Z_2$, i.e. an inner semi-direct product of a discrete symmetry group $F$ and $Z_2$. In such a scenario the symmetry constrains the Yukawa interactions but goes unnoticed by the scalar sector. In the most minimal scenario, $Z_3 \times Z_2 = D_4$, flavor changing neutral currents mediated by scalars are absent at tree and one-loop level, while at the same time predictions to quark and lepton mixing are obtained, namely a trivial CKM matrix and a PMNS matrix (upon introduction of three heavy right-handed neutrinos) containing maximal atmospheric mixing. Small extensions allow to fully reproduce mixing parameters, including cobimaximal mixing in the lepton sector (maximal atmospheric mixing and a maximal $CP$ phase).

I. INTRODUCTION

The discovery of a Higgs boson with a mass of $m_H \simeq 125$ GeV has opened the door to the possibility of having in Nature multiple fundamental scalars. In principle, nothing forbids their proliferation. Nonetheless, the amount of parameters dramatically increases, both in the Yukawa and scalar sector. Here we consider a simple extension to the standard model (SM) by only introducing a second Higgs doublet (2HDM) with quantum numbers identical to the SM Higgs, and three right-handed neutrinos to generate active neutrino masses. Furthermore, we mainly focus on the problem of fermion mixing by first adopting the common 2HDM framework with natural flavor conservation (NFC) [1, 2], achieved through a $Z_2$ reflection symmetry. Then, we add flavor to it via the enlargement of the symmetry group in a very particular manner, $F \ltimes Z_2$. This denotes an inner semi-direct product of a discrete symmetry group $F$ and a $Z_2$ symmetry. The non-Abelian nature of the enlarged symmetry group then strongly reduces the number of Yukawa couplings, thus providing a more predictive theory. Moreover, the ad-hoc nature of the $Z_2$ is explained as a part of a larger group.

To understand the need for the $Z_2$ symmetry we briefly sketch its impact. In a general setup, one may immediately write the Yukawa Lagrangian for a given fermion

$$-\mathcal{L}_Y \supset \bar{\psi}_L \left( Y_1^{\psi} \Phi_1 + Y_2^{\psi} \Phi_2 \right) \psi_R + \text{H.c.,}$$

where $\psi_R$ or $\psi_L$ are three-dimensional vectors in flavor space each denoting a weak singlet or a weak doublet, respectively, and $\psi$ representing any of the four fermion types, $\psi = q_u, q_d, \ell, \nu$. Notice that the Higgs doublets must be replaced by their charge-conjugate fields, $\Phi^*_k = i \sigma_2 \Phi_k^*$, for the up-type quark and neutrino cases. If the neutral components of both scalar doublets acquire a vacuum expectation value (VEV), $\langle \Phi_1^0 \rangle = v_1$ and $\langle \Phi_2^0 \rangle = v_2$, both Yukawa matrices contribute to the fermion masses and mixing. It is clear that diagonalization of the mass matrix,

$$M = v_1 Y_1 + v_2 Y_2,$$

cannot mean, in general, diagonalization of the individual Yukawa matrices. This brings about dangerous tree-level flavor-changing-neutral-currents (FCNC). To avoid them it is sufficient to assume NFC by introducing a $Z_2$ symmetry and by assigning a single scalar doublet for a given fermion species such that only one of the two Yukawa matrices contributes to the mass matrix. This is, the scalar fields transform under the discrete symmetry such that

$$\Phi_1 \rightarrow -\Phi_1, \quad \Phi_2 \rightarrow +\Phi_2,$$

while the left-handed quarks and leptons transform trivially and the right-handed parts transform appropriately. The different assignment possibilities lead to four non-equivalent types of 2HDMs:

- Type I: All charged fermions couple to $\Phi_2$.
- Type II: $q_d$ and $\ell$ couple to $\Phi_1$ and $q_u$ to $\Phi_2$.

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1 There are other possibilities to explain the ad-hoc $Z_2$, for instance by linking it to the remnant symmetry of a spontaneously broken $U(1)$, see e.g. [3–5].

2 For brevity, we left aside the Majorana option. However, we return to it later.

3 The type X and Y are also called the lepton-specific and flipped scenarios, respectively.
• Type X: $q_u$ and $q_d$ couple to $\Phi_2$ and $\ell$ to $\Phi_1$.

• Type Y: $q_u$ and $\ell$ couple to $\Phi_2$ and $q_d$ to $\Phi_1$.

Other possibilities are the Type III which is the general 2HDM with all couplings permitted and the inert doublet model where $\Phi_2$ couples to all fermions while $\Phi_1$ has no VEV thus leaving unbroken the $Z_2$ symmetry and providing a viable dark matter candidate. Although other approaches, such as Yukawa-alignment [6] or singular-alignment [7], may also avoid tree-level FCNC, here we only focus on those 2HDM employing the discrete symmetry $Z_2$. Note that the non-equivalence nature between the four types comes from the fact that each framework ends up having different effective Yukawa couplings of the fermions to the various scalar particles; for a thorough discussion on various phenomenological and theoretical aspects of 2HDMs see Ref. [8].

On the other hand, a general feature shared by the four different types (I, II, X, and Y) is the $Z_2$-invariant scalar potential given by

$$ V_{2\text{HDM}}^{Z_2} = \sum_{i=1}^2 \left[ m_{ix}^2 (\Phi_i^d \Phi_x) + \frac{\lambda_5}{2} (\Phi_i^d \Phi_x)^2 \right] + \frac{3}{2} \left[ \lambda_5 (\Phi_1^d \Phi_2) + \lambda_4 (\Phi_1^d \Phi_2) (\Phi_2^d \Phi_1) \right] + \frac{1}{2} \left[ \lambda_5 (\Phi_1^d \Phi_2) \right]^2 $$

The hermiticity condition of the potential leaves $\lambda_5$ as the only complex coefficient while the rest, $m_{11}^2$, $m_{22}^2$, and $\lambda_{1,2,3,4}$, are real. There are in total eight real parameters. However, not all of them are physical. A phase redefinition can make $\lambda_5$ real and only seven parameters are physical. Note that our potential has explicitly become $CP$-symmetric.

No matter the amount of Higgs doublets one employs, the full mass matrix for any given fermion is parametrized by nine complex parameters. The initial arbitrariness may then be reduced via weak-basis transformations (unitary transformations leaving invariant the kinetic terms), but not enough to claim predictivity. In the mass basis, for either quarks or leptons, one has six fermion masses and four (six for Majorana neutrinos) mixing parameters, plus arbitrary Yukawa couplings. The flavor sector thus gives to the SM and its extensions (without symmetries) the highest amount of arbitrariness. It is only when symmetries are introduced that the initial arbitrariness can be drastically reduced. Here we intend to explore the effect of symmetries in the flavor sector such that we find correlations among the quark and lepton mixing parameters.

The paper is organized as follows: in Sec. II, we discuss the meaning of adding flavor to $Z_2$. Next, in Sec. III, we provide the most minimal scenario realizing the features of our approach. Also we highlight the main differences when compared to the four types of 2HDMs. Thereafter, in Sec. IV, we take the incompleteness of fermion mixing in our simple model as a hint to the presence of additional new physics and introduce a flavor doublet of real scalar gauge singlets. Finally, in Sec. V, we conclude. Some technical details are delegated to appendices.

II. ADDING FLAVOR TO $Z_2$

We are interested in those finite symmetry groups, $G$, which can be written as an inner semi-direct product of an arbitrary group $F$ and $Z_2$,

$$ G = F \rtimes Z_2. $$

There are in fact many examples of such groups (see Ref. [9] for more details): $D_N = \Delta(2N) \simeq Z_N \rtimes Z_2$, $\Sigma(2N^2) \simeq (Z_N \times Z_N) \rtimes Z_2$, $\Sigma(24) \simeq Z_2 \times Z_6 \times Z_2$, etc.

The main property of this kind of groups is that they contain two one-dimensional irreducible representations (denoted singlets), which behave exactly as if we only had a $Z_2$ symmetry. Thus, by assigning each Higgs doublet to one of these singlets, we are mimicking in the scalar sector any of the NFC models with a $Z_2$ symmetry. On the other hand, the non-Abelian nature of the symmetry only impacts the Yukawa interactions, thus providing a way to approach the problem of mixing while simultaneously tackling minimal scalar extensions to the SM.

An additional feature of this approach is the following. Since the number of Higgs doublets in a theory restricts the maximum group order of allowed symmetries (‘realizable symmetries’) that would otherwise imply massless Goldstone bosons [10], then by implementing symmetry groups as here proposed we avoid these constrictions.

Let us take as a first example the Klein group given by $Z_2 \times Z_2$. It is the smallest possibility within this approach. It has four elements and four irreducible representations (irreps): $1_{++}$, $1_{+-}$, $1_{-+}$, and $1_{--}$. However, as it is still an Abelian group its effect on the Yukawa couplings is only of reduction but not of relation. For example, we could assign the Higgs doublets as $1_{-+}$ and $\Phi_2 \sim 1_{++}$ while the third, second, and first fermion families as $1_{-+}$, $1_{-+}$, and $1_{++}$, respectively. In return the mass matrix for Dirac fermions would take the generic form

$$ M = \begin{pmatrix}
  y_1 v_{++} & y_2 v_{--} & 0 \\
  y_5 v_{--} & y_2 v_{++} & 0 \\
  0 & 0 & y_3 v_{++}
\end{pmatrix}, $$

where $\langle \Phi_1 \rangle = v_{--}$ and $\langle \Phi_2 \rangle = v_{++}$. Therefore, although we have reduced the number of complex parameters from nine to five, we yet have no predictions except for the fact that we only expect mixing between the first two generations. Nevertheless, it demonstrates that the combination of the flavor-safe $Z_2$ with an additional group will simplify the Yukawa sector. Going to the non-Abelian case will result in predictive scenarios, and we will study a very minimal approach in what follows.
III. THE MINIMAL CASE: $Z_3 \times Z_2$

The smallest non-Abelian finite group has six elements and is denoted by $D_3 \equiv Z_3 \times Z_2$. This dihedral group describes the symmetrical properties of an equilateral triangle.\(^4\) It has three irreducible representations: two singlets $1_+ , 1_-$, and one doublet $2$. The product rules can be found in Appendix A.

Although different assignments between the $D_3$ irreps and the fermion fields could be done, here we opt to consider

$$Q_{L,3} \sim 1_+ , \quad Q_{L,D} = \left( \begin{array}{c} Q_{L,1} \\ Q_{L,2} \end{array} \right) \sim 2 ,$$

$$u_{R,3} \sim 1_+ , \quad u_{R,D} = \left( \begin{array}{c} u_{R,1} \\ u_{R,2} \end{array} \right) \sim 2 , \quad (7)$$

$$d_{R,3} \sim 1_-, \quad d_{R,D} = \left( \begin{array}{c} d_{R,1} \\ d_{R,2} \end{array} \right) \sim 2 ,$$

whereas in the lepton sector,

$$E_{L,1} \sim 1_+ , \quad E_{L,D} = \left( \begin{array}{c} E_{L,2} \\ E_{L,3} \end{array} \right) \sim 2 ,$$

$$e_{R,1} \sim 1_- , \quad e_{R,D} = \left( \begin{array}{c} e_{R,2} \\ e_{R,3} \end{array} \right) \sim 2 , \quad (8)$$

$$N_{R,1} \sim 1_-, \quad N_{R,D} = \left( \begin{array}{c} N_{R,2} \\ N_{R,3} \end{array} \right) \sim 2 .$$

We are motivated to this choice, as we will see, because the dominant contributions to quark and lepton mixing are the Cabibbo and atmospheric angle, correspondingly.

Recall that the scalar sector should be assigned to $\Phi_1 \sim 1_-$ and $\Phi_2 \sim 1_+ . \quad (9)$

The neutral component of both Higgs doublets acquires a VEV, spontaneously breaking the electroweak symmetry; we denote them as

$$v_1 \equiv \langle \Phi_1 \rangle \quad \text{and} \quad v_2 \equiv \langle \Phi_2 \rangle . \quad (10)$$

Note we are using the convention $v^2 = v_1^2 + v_2^2 = (174 \text{ GeV})^2$.

The $Z_3 \times Z_2$-symmetric Yukawa Lagrangian is

$$-L_Y = L_Y^Q + L_Y^E \quad (11)$$

with

$$L_Y^Q = y_L \bar{Q}_{L,3} \tilde{\Phi}_2 u_{R,3} + y_R \bar{Q}_{L,3} \Phi_1 d_{R,3} + \frac{1}{2} \left[ y_L^u \left( \bar{Q}_{L,D} u_{R,D} \right) + \tilde{\Phi}_2 \right] + \frac{1}{2} \left[ y_R^d \left( \bar{Q}_{L,D} d_{R,D} \right) + \Phi_2 \right] + \text{H.c.} , \quad (12)$$

$$L_Y^E = y_{E_L} \bar{E}_{L,1} \tilde{\Phi}_1 N_{R,1} + y_{E_L} \bar{E}_{L,1} \Phi_1 e_{R,1} + \frac{1}{2} M_1 \bar{N}_{R,1} N_{R,1} + \frac{1}{2} M_2 \left[ \bar{N}_{R,D} N_{R,D} \right] + \text{H.c.}, \quad (13)$$

where $\{ 1_+, 1_-, 2 \}$ represents one of the three possible outputs from the $D_3$ tensorial product. Also notice that we are now assuming Majorana neutrinos by virtue of a standard seesaw.

In the quark sector, the resulting Yukawa matrices take the form

$$\Delta_1 = \left( \begin{array}{ccc} 0 & y_u & 0 \\ -y_u & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) , \quad \Delta_2 = \left( \begin{array}{ccc} y_d & 0 & 0 \\ 0 & y_d & 0 \\ 0 & 0 & 0 \end{array} \right) , \quad (14)$$

while in the lepton sector we have

$$\Pi_1 = \left( \begin{array}{ccc} y_e & 0 & 0 \\ 0 & 0 & y_l \\ 0 & -y_l & 0 \end{array} \right) , \quad \Pi_2 = \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & y_e & 0 \\ 0 & 0 & y_e \end{array} \right) , \quad (15)$$

where all the parameters are real and positive and where we have taken $\{ y_u, y_d, y_l, y_e \} \in \mathbb{R}^+$ without loss of generality. All Dirac mass matrices satisfy

$$M = v_1 \Xi_1 + v_2 \Xi_2 , \quad (16)$$

where $\Xi = \Gamma, \Delta, \Pi, \Omega$. Each mass matrix has three complex parameters and possesses the feature of being diagonalisable by the same transformation that brings to diagonal form its individual Yukawa matrices. It is this property that guarantee the absence of FCNC at tree level and it represents an explicit realization of the singular alignment ansatz [7].

Note how we end up, in the quark sector, with only eight real parameters, six of which correspond to the six

\(^4\) $D_3$ is isomorphic to $S_3$, the group describing the permutations of three indistinguishable objects.
quark masses while the other two, being complex phases, are forced to be nearly $\pm \pi/2$ due to the phenomenological observation of hierarchical fermion masses. We return to this point later.

The effective Majorana neutrino mass matrix can be computed from the standard seesaw formula, $\mathcal{M}_\nu = -M_e M_R^{-1} M_e^T$, and is found to be diagonal:

$$
\mathcal{M}_\nu = - \begin{pmatrix}
\frac{(y_{e1} v_1)^2}{M_1^2} & 0 & 0 \\
0 & \frac{(y_{e2} v_2)^2 - (y_{e1} v_1)^2}{M_2^2} & 0 \\
0 & 0 & \frac{(y_{e3} v_3)^2 - (y_{e1} v_1)^2}{M_3^2}
\end{pmatrix}.
$$

(17)

Here $M_R = \text{diag}(M_1, M_2, M_3)$, which is a consequence of the $D_3$ flavor symmetry. The mass matrix has a mass degeneracy between the two neutrino states, $\nu_{\ell,2}$ and $\nu_{\ell,3}$, while, since it is diagonal, it does not contribute to the mixing.

Towards studying the phenomenology of this scenario we note that complex matrices of the form

$$
m = a\mathbb{1} + \begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix},
$$

are brought to diagonal shape via a maximal bi-unitary transformation

$$
u_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \pm i \\ \pm i & 1 \end{pmatrix}, \quad \nu_R = \begin{pmatrix} e^{i\gamma_1} & 0 \\ 0 & e^{i\gamma_2} \end{pmatrix} \cdot \nu_L,
$$

(19)

that is,

$$
\nu_L \cdot m \cdot \nu_R^\dagger = \begin{pmatrix} |a \mp ib| & 0 \\ 0 & |a \pm ib| \end{pmatrix},
$$

(20)

with $\gamma_1 = \text{arg}(a \mp ib)$ and $\gamma_2 = \text{arg}(a \pm ib)$ implying real and positive masses. The choice of the signs will depend on the ordering of the masses. The singular values of such a matrix $m$ are given by

$$
m_{1,2} = |a \pm ib| = \sqrt{|a|^2 + |b|^2 \pm 2|a||b| \sin \rho},
$$

(21)

where $\rho = \text{arg}(a) - \text{arg}(b)$. Moreover, note that if the parameters $a$ and $b$ are taken to be real ($\rho = 0$) then the masses would be degenerate. In particular, if $\rho$ is in the first quadrant then $\rho \in \text{ArcSin}\left(\frac{m_3 - m_1}{m_2 + m_1}\right), \pi/2].$ The transformations $\rho \rightarrow \pm \rho + \pi$ and $\rho \rightarrow - \rho$ will lead to the same masses as $\rho$. Additionally, when the masses are hierarchical, $m_2 \gg m_1$, the allowed interval for $\rho$ shrinks to $\rho \in [\pi/2 - 2m_1/m_2 + O(m_1^2/m_2^3), \pi/2]$, essentially implying that $\rho \simeq \pm \pi/2$. We have chosen the off-diagonal Yukawas to be real and positive without loss of generality. Therefore the complex phase of the diagonal Yukawas is found to be $\gamma_1 \simeq \pm \pi/2$.

With these results in mind and looking at the form of the mass matrices of the charged fermions shown in Eqs. (14)-(16) we can extract the masses and mixing parameters:

$$
m_i = y_{e1} v_1, \quad m_b = y_{b1} v_1, \quad m_c = y_{c1} v_1, \quad m_{c,u} = |y_{e1}^u v_2 \pm y_{e1}^d v_1|, \quad m_{s,d} = |y_{e1}^d v_2 \pm y_{e1}^d v_1|, \quad m_{\tau,\mu} = |y_{e1}^\mu v_2 \pm y_{e1}^\mu v_1|,
$$

(22)

with the Majorana neutrino masses as given in Eq. (17). The quark Yukawa couplings can now be generically fixed to (defining $\tan \beta = v_2/v_1$)

$$
y_1^{(u,d)} = \frac{m_{c(s)} - m_{u(d)}}{2\sqrt{2} \sin \beta}\quad \text{and} \quad y_2^{(u,d)} = \frac{m_{c(s)} + m_{u(d)}}{2\sqrt{2} \sin \beta}.
$$

(23)

An alternative solution exists when one exchanges $y_1^{\nu} \leftrightarrow y_2^{\nu}$. Similarly for the charged leptons,

$$
y_1^{\ell} = \frac{m_{\tau} - m_{\mu}}{2\sqrt{2} \cos \beta}\quad \text{and} \quad y_2^{\ell} = \frac{m_{\tau} + m_{\mu}}{2\sqrt{2} \cos \beta},
$$

(24)

and again it is possible to exchange $y_1^{\nu} \leftrightarrow y_2^{\nu}$.

Turning to fermion mixing, we can parametrize the relevant diagonalization matrices in terms of the complex rotation matrices $U_{ij}(\theta, \phi)$, which are defined as

$$
U_{12}(\theta, \phi) = \begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} & 0 \\ -\sin \theta e^{i\phi} & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix},
$$

(25)

and similarly for $U_{13}$ and $U_{23}$. Then, the mixing matrices for the up and down quarks and for the charged leptons are simply given by

$$
U^{(q)}_L = U_{12}(\pi/4, \pm \pi/2),
$$

(26)

$$
U^{(q)}_d = U_{12}(\pi/4, \pm \pi/2),
$$

(27)

$$
U^{(q)}_e = U_{23}(\pm \pi/2, \pm \pi/2).
$$

(28)

We obtain for the Cabibbo-Kobayashi-Maskawa (CKM) and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrices

$$
V^{(0)}_{\text{CKM}} = U^{(q)}_L U^{(d)}_L^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},
$$

(29)

$$
V^{(0)}_{\text{PMNS}} = U^{(q)}_L U^{(\nu)}_L^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & \pm 1/\sqrt{2} \\ 0 & \pm 1/\sqrt{2} & \pm 1/\sqrt{2} \end{pmatrix},
$$

where one of the signs of the PMNS matrix is realized when Eq. (24) applies and the opposite when $y_1^{\nu} \leftrightarrow y_2^{\nu}$.

This is, by enlarging $\mathbb{Z}_2$ to $\mathbb{Z}_3 \times \mathbb{Z}_2$, we are now able to predict trivial mixing in the quark sector and a maximal atmospheric mixing angle in the lepton sector. There is also a maximal $CP$ violation phase, which is unphysical if the angles $\theta_{12}$ and $\theta_{13}$ remain 0, but it will become important later. These features have to be understood as the dominant characteristics of this model at leading
order. Its incompleteness points to further investigation on how the model should be extended, see Sec. IV.

A. FCNC

There are no tree level FCNC since all Yukawa matrices are simultaneously diagonalisable. However, at the one-loop level quantum corrections could induce misalignment in the different Yukawa matrices and generate FCNC. To check this effect we employ the formulas obtained for a theory with $N$--Higgs doublets [11] and given in Appendix B. It is straightforward to see that for our particular model in all cases the one-loop renormalization-group-equations may only give place to flavor-conserving terms\(^5\)

\[
16\pi^2 \mu \frac{d}{d\mu} \Xi_a \propto \Xi_a \quad (a = 1, 2),
\]

where $\mu$ is the renormalization scale and $\Xi = \Gamma, \Delta, \Pi, \Omega$. More details can be found in Appendix B\(^6\).

B. Nonuniversal charged fermion-scalar couplings

In order to find the couplings between the charged fermions and the Higgs scalars we need to move both of them to their mass basis. In our case, only the latter are still in the symmetry adapted basis. We first introduce their notation

\[
\Phi_1 = \left( \begin{array}{c} \Phi_1^+ \\ v_1 + \frac{i\phi_0^0 + i\phi_0^0}{\sqrt{2}} \end{array} \right), \quad \Phi_2 = \left( \begin{array}{c} \Phi_2^+ \\ v_2 + \frac{i\phi_0^0 + i\phi_0^0}{\sqrt{2}} \end{array} \right).
\]

Since the scalar potential is $CP$-symmetric there are states with definite $CP$-odd and $CP$-even quantum numbers. This allows one to write two independent mass matrices

\[
M_{CP, even}^2 = \begin{pmatrix} 2v_2^2\lambda_2 & 2v_1 v_2 \lambda_{345} \\ 2v_1 v_2 \lambda_{345} & 2v_1^2\lambda_1 \end{pmatrix},
\]

where $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$, and

\[
M_{CP, odd}^2 = \begin{pmatrix} 2v_2^2\lambda_5 & -2v_1 v_2 \lambda_5 \\ -2v_1 v_2 \lambda_5 & 2v_1^2\lambda_5 \end{pmatrix}.
\]

The first case can be brought to diagonal form by means of the orthogonal transformation

\[
\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \phi_0^0 \\ \phi_0^0 \end{pmatrix},
\]

with $\tan 2\alpha = 2v_1 v_2 \lambda_{345}/(v_1^2\lambda_1 - v_2^2\lambda_2)$, while the second one by

\[
\begin{pmatrix} G^0 \\ A \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \phi_0^0 \\ \phi_0^0 \end{pmatrix}.
\]

Here the latter angle of rotation satisfies $\tan \beta = v_2/v_1$ and $G^0$ is the neutral pseudo-Goldstone boson to be ‘eaten’ by the $Z$ mass. Similarly, one has for the charged scalars a mass matrix

\[
M_{charged}^2 = \begin{pmatrix} 2v_2^2(\lambda_4 + \lambda_5) & -2v_1 v_2(\lambda_4 + \lambda_5) \\ -2v_1 v_2(\lambda_4 + \lambda_5) & 2v_1^2(\lambda_4 + \lambda_5) \end{pmatrix},
\]

diagonalised by the same rotation as for the $CP$-odd neutral scalars,

\[
\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \phi_0^0 \\ \phi_0^0 \end{pmatrix}.
\]

The Yukawa Lagrangian related to the interactions to the neutral scalars is

\[
-L_Y \supset \sum_{f} \frac{m_f}{(246 \text{ GeV})^2} \left( \xi_f^T \tilde{T} f h + \xi_f^T \tilde{T} H - i \xi_f^T \gamma_5 f A \right),
\]

while the one related to the interactions to the charged scalar is

\[
-L_Y \supset - H^+ \frac{\mathbf{V}_{CKM}^{(0)} \mathbf{V}_{CKM}^{(0) \dagger}}{(246 \text{ GeV})^2} \sum_{i} \left( m_{i\xi_{f\alpha}} P_L + m_{j\xi_{f\beta}} P_R \right) d_j - H^+ \frac{m_f}{(246 \text{ GeV})^2} \mathbf{V}_{CKM}^{(0)} \mathbf{V}_{CKM}^{(0) \dagger} \sum_{r} \epsilon^{H^+} \mathbf{P}_{L,R} \xi_{r} + \text{H.c.,}
\]
masses, \( m_3 \gg m_2 \gg m_1 \), may be applied to Eq. (40) to obtain the approximate relations

\[
\begin{align*}
    f_{\pm} &\approx \frac{m_2}{2m_{2,1}} \left( \frac{c_\beta}{s_\beta} + \frac{s_\alpha}{c_\beta} \right), \\
    g_{\pm} &\approx \frac{m_2}{2m_{2,1}} \left( \frac{s_\beta}{s_\beta} + \frac{c_\alpha}{c_\beta} \right).
\end{align*}
\]

(41)

For small or large \( \tan \beta \), both relations reduce to \( f_- \approx \frac{m_2}{m_1} f_+ \) and \( g_- \approx \frac{m_2}{m_1} g_+ \), meaning that the fermion with a lighter mass (\( m_1 < m_2 \)) has an \( \mathcal{O}(100) \) enhancement in its coupling to the scalars compared to the heavier one. Moreover, for \( \alpha \to \beta - \pi/2 \) all couplings to the 125 GeV scalar state, \( h \), including the new functions \( f_{\pm} \), are automatically made SM-like, i.e. \( g_{\pm} \to 1 \), while the other couplings end up only depending on \( \tan \beta \). A further implication of the alignment limit is that the coupling of the \( CP \)-even state \( H \) with the \( W \) and \( Z \) bosons becomes null.

The resulting couplings have been grouped into different sets corresponding to similar characteristics in Table II. This also holds for couplings which depend on the Yukawa parameters (and therefore, to the different fermion masses), like \( \xi_{\tau,s,c}^{H^+} \equiv g_+ (\alpha, \beta, y_1, y_2) \) for \( f = \tau, s, c \). As they have the same functional dependence they are grouped under the category \( \xi_{\tau,s,c}^{H^+} \).

In general, conventional 2HDMs with NFC have a moderate behaviour for moderate values of \( \tan \beta \). Their main differences appear in the small (or large) \( \tan \beta \) limits. For example, take the couplings to the charged scalar, \( H^+ \). In the type-II scenario, its coupling to \( \bar{t}b \) is large (philic) at large \( \tan \beta \), whereas in the same limit, it is always small (phobic) for the type-I case. In contrast to this typical situation, the \( Z_3 \times Z_2 \) model shows already at moderate values of \( \tan \beta \) either phobic or philic behaviour, see Figs. 1–3. Also it can be seen that for a given value of \( \beta \) a given fermion may completely decouple from one of the four scalars and accidentally become inert to that scalar.

### IV. Completing Mixing as a Guide for New Physics

While possessing attractive features, the minimal \( Z_3 \times Z_2 \) model presented so far does not fully reproduce the fermion mixing and masses. We take this ‘incomplete mixing’ as a hint pointing towards new physics. In the quark sector, the vanishing mixing points to the introduction of a \( dim > 4 \) operator that generates small corrections. In a similar fashion, the Majorana nature of neutrinos could allow \( dim \)-4 operators and therefore large contributions to mixing. The simplest possibility is obtained by introducing a real singlet scalar field, which is assumed to transform under \( D_3 \) as a doublet,

\[
\eta \sim [2].
\]

(42)

This field acquires a VEV

\[
\langle \eta \rangle \equiv \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}.
\]

(43)

Note that by introducing \( \eta \) and its non-renormalizable interactions we have allowed at tree level the appear-
FIG. 1. Effective couplings to the 125 GeV scalar ($h$), as a function of $\beta$, in any of the 2HDM types and the $Z_3 \rtimes Z_2$ model. We considered $\alpha = 0.7$. The blue and red continuous lines represent any of the conventional 2HDMs couplings with NFC whereas the black (dotted) and gray (dashed) lines the new couplings of our model. The left and right (magenta) shaded regions depict the small and large tan $\beta$ limits while the upper (cyan) and lower (yellow) regions the philic ($|\xi_{h}^{f}| > 1.5$) and phobic ($|\xi_{h}^{f}| < 0.5$) limits, respectively. The middle (green) line the SM limit. Note that the funnel is a consequence of plotting the absolute values of the coupling $\xi_{h}^{f}$.

FIG. 2. Effective couplings to the heavy $CP$-even scalar ($H$), as a function of $\beta$, in any of the 2HDM types and the $Z_3 \rtimes Z_2$ model. We considered $\alpha = 0.7$. The blue and red continuous lines represent any of the conventional 2HDMs couplings with NFC whereas the black (dotted) and gray (dashed) lines the new couplings of our model. The left and right (magenta) shaded regions depict the small and large tan $\beta$ limits while the upper (cyan) and lower (yellow) regions the philic ($|\xi_{H}^{f}| > 1.5$) and phobic ($|\xi_{H}^{f}| < 0.5$) limits, respectively. Note that the funnel is a consequence of plotting the absolute values of the coupling $\xi_{H}^{f}$.

FIG. 3. Effective couplings to the neutral $CP$-odd ($A$) and charged ($H^+$) scalars, as a function of $\beta$, in any of the 2HDM types and the $Z_3 \rtimes Z_2$ model. The blue and red continuous lines represent any of the conventional 2HDMs couplings with NFC whereas the black (dotted) and gray (dashed) lines the new couplings of our model. The left and right (magenta) shaded regions depict the small and large tan $\beta$ limits while the upper (cyan) and lower (yellow) regions the philic ($|\xi_{A,H}^{f}| > 1.5$) and phobic ($|\xi_{A,H}^{f}| < 0.5$) limits, respectively. Note that the funnel is a consequence of plotting the absolute values of the coupling $\xi_{A,H}^{f}$.

Yukawa matrices to be added. At last, notice that integrating out the singlet scalar means that our theory has become a 2HDM of Type III. We will later demonstrate that the model can be easily made flavor-safe. An explicit numerical example will be provided in Sec. IV D.

A. Quark mixing

In the quark sector, the non-renormalizable dim-5 operators leading to a correct CKM matrix requires a complete UV formulation to be realized. As a simple example that serves as a plausibility argument, consider the following dim-5 effective interactions, invariant under the SM gauge group and the flavor symmetry:

$$\Delta L_{Y}^{q} = \frac{g_{d}^{q}}{\Lambda} [\bar{Q}_{L,D}\eta]_{-} d_{R,3} \Phi_{2} + \frac{g_{d}^{q}}{\Lambda} [\bar{Q}_{L,D}\eta]_{+} d_{R,3} \Phi_{1}$$

$$+ \frac{g_{d}^{q}}{\Lambda} [\bar{Q}_{L,3}]_{+} d_{R,D} \eta_{-} \Phi_{2} + \frac{g_{d}^{q}}{\Lambda} [\bar{Q}_{L,3}]_{-} d_{R,D} \eta_{-} \Phi_{1}$$

$$+ \frac{g_{d}^{q}}{\Lambda} [\bar{Q}_{L,D}]_{+} d_{R,D} \eta_{+} \Phi_{2} + \frac{g_{d}^{q}}{\Lambda} [\bar{Q}_{L,D}]_{-} d_{R,D} \eta_{-} \Phi_{1} + H.c.$$  \hspace{1cm} (44)

These contributions give rise to small corrections in quark mixing through perturbations to the down quark mass
matrix of the form

\[
\Delta M_d = \begin{pmatrix}
\epsilon_1 & \epsilon_2 & \epsilon_3 \\
\epsilon_2 & -\epsilon_1 & \epsilon_3' \\
\epsilon_3 & \epsilon_3' & 0
\end{pmatrix},
\]

which should be enough to perturb our initial identity matrix and reproduce the CKM mixing. These effective operators can be realized in an UV-complete model just by adding a vector-like pair of coloured particles with the same gauge quantum numbers as the right-handed down quarks.

In the basis where \(M_u\) and \(M_d\) are diagonal, the perturbation matrix becomes

\[
\Delta \tilde{M}_d = \begin{pmatrix}
0 & -\epsilon_1 + i\epsilon_2 & \frac{\epsilon_3 - i\epsilon_4}{\sqrt{2}} \\
\epsilon_1 + i\epsilon_2 & 0 & \frac{\epsilon_4 - i\epsilon_3}{\sqrt{2}} \\
\frac{i(\epsilon_3 + i\epsilon_4)}{\sqrt{2}} & \frac{-i(\epsilon_3 + i\epsilon_4)}{\sqrt{2}} & 0
\end{pmatrix}.
\]

Recall that here we still have trivial quark mixing. In order to obtain a realistic mixing scenario, the perturbations need to be sufficiently small compared to the bottom quark mass but large enough compared to the down and strange quark masses. This implies that the Yukawa parameters \(y_{1,2}\) are no longer completely satisfying Eq. (23). Through a qualitative analysis we find that for

\[
\tilde{M}_d = \text{diag}(|y^d_2|v_2 - y^d_1 v_1, |y^d_2|v_2 + y^d_1 v_1, y_6 v_1) + \Delta \tilde{M}_d
\]

\[
\sim \begin{pmatrix}
m_d & \lambda m_s & \lambda m_b \\
\lambda m_s & m_s & \lambda^2 m_b \\
0 & - & m_b
\end{pmatrix},
\]

where \(\lambda \simeq 0.225\), it is possible to fully reproduce quark mixing without introducing unacceptably large amounts of flavor violation at tree level. The (3, 1) and (3, 2) matrix elements could be taken as zero or of the same order that their transpose counterparts. On the other hand, all entries are given up to \(O(1)\) complex factors. It is interesting to note that Eq. (47) shows an approximate \(U(2)\) flavor symmetry for the first two generations, \(m_b \gg m_{d,s}\) (analogously for the up-type quarks). The above resulting mass matrix is a similar realization of the 'flavorful' 2HDMs investigated in Ref. [14] wherein Yukawa couplings, for all charged fermions, are chosen as to approximately preserve a \(U(2)^5\) flavor symmetry acting on the first two generations.

Alternatively, we could have introduced perturbations through the up-type quarks; however, to reproduce the CKM mixing would have required a larger modification of the initial Yukawa parameters, \(|y^u_2|^2\) and \(y^u_4\), by at least one order of magnitude. This may be easily appreciated by considering that a perturbation to the \(1 - 2\) sector of the size \(m_1 m_2\) is enough in the down quark sector, \(\sqrt{m_d m_s} \sim 10\) MeV, to generate Cabibbo mixing, while for the up-type quarks it would still require an additional order of magnitude, \(O(10)\sqrt{m_u m_s} \sim 100\) MeV, plus some extra tuning in the Yukawa parameters to get the correct light quark masses, \(m_u\) and \(m_c\).

B. Lepton mixing

In the lepton sector, the dominant perturbation contributions come through the right-handed neutrinos,

\[
-\Delta \mathcal{L}_Y^N = \frac{1}{2} g_1^N \sqrt{\rho_{R,1}^N [\rho_{N,1}]} + \text{H.c.},
\]

producing

\[
\Delta M_R = \begin{pmatrix}
0 & \delta_{N1} & r\delta_{N1} \\
\delta_{N1} & 0 & \delta_{N2} \\
r\delta_{N1} & \delta_{N2} & 0
\end{pmatrix},
\]

where we have defined \(r = \omega_2/\omega_1\) and \(\delta_{N1} = g_1^N \omega_1\).

The charged lepton contribution remains untouched by the addition of the scalar \(\eta\) and is given by Eq. (28).

Once we consider the contributions to the mass matrix of the right-handed neutrinos shown in Eq. (49) the initial lepton mixing given by Eq. (29) gets modified. If the Yukawa couplings appearing in the neutrino mass matrix are taken real then we have

\[
U_L^* = U_{23}(\pi/4, \pm\pi/2),
\]

\[
U_R^* = O_{23}(\theta_{23}',\theta_{13}, \phi_{12}) O_{12}(\theta_{12}', \phi_{23}),
\]

where \(O_{ij}(\theta)\) is the usual rotation matrix in the \((i, j)\) plane. It can be shown that

\[
U_{23}(\pi/4, \pm\pi/2) O_{23}(\theta_{23}') = P \cdot U_{23}(\pi/4, \mp\pi/2),
\]

where \(P\) is a diagonal unitary matrix which is unphysical. Therefore, if the neutrino sector is real we obtain cobimaximal mixing [15] with \(\theta_{23} = \pi/4\) and \(\delta_{CP} = \pm\pi/2\) in the lepton sector. While the sign of \(\delta_{CP}\) is not fixed, data seems to favor the negative option [16]. Note that this is a particular case of the general theorem derived in Ref. [17], i.e. if cobimaximal mixing is present in the charged lepton sector and the neutrino sector is real, then the full PMNS matrix is also cobimaximal. In particular, the full lepton mixing parameters are given by

\[
\theta_{12} = \theta_{12}', \quad \theta_{13} = \theta_{13}', \quad \theta_{23} = \pi/4 , \quad \delta_{CP} = \pm\pi/2, \quad \phi_{12} = 0, \pi/2 = \phi_{13} ,
\]

irrespective of \(\theta'_{23}\). That is, the large hierarchy between the charged lepton masses coupled with the assumption that the neutrino Yukawas are real leads to cobimaximal mixing i.e. maximal atmospheric mixing angle and \(\delta_{CP} = \pm\pi/2\). For the other two mixing angles \(\theta_{12}\) and \(\theta_{13}\) no predictions can be made, but the parameters can be chosen in such a way that they lie inside the experimental constraints. Moreover, the Majorana phases rel-
evant for neutrinoless double beta decay maintain their CP conserving values.

We remark that of course there is no need to assume the neutrino sector to remain real, in the most general scenario with complex parameters there is enough freedom to fit all the mixing parameters. The assumption that the neutrino Yukawas are real, while the charged lepton Yukawas are forced to be complex due to hierarchical masses, may seem ad-hoc but can actually be justified in many different scenarios. For example in Ref. [18] the author derives a general loop mechanism in which the neutrino mass matrix is complex but diagonalized by a real orthogonal matrix. This same mechanism could be applied here by changing the type I seesaw neutrino mass generation by an inverted loop seesaw mediated by three real scalars. Then, the cobimaximal nature of the PMNS would remain. Another option would be to explicitly impose a remnant CP symmetry in the neutrino sector.

It is worth to note that our scenario is minimal and quite simple, yet, it leads to such a restricted scenario. The SM symmetry group is extended by just $D_3$ while the particle content is enlarged by an extra Higgs doublet and an $D_3$ doublet $\eta$ which is a gauge singlet.

### C. The scalar potential

The most general scalar potential invariant under $Z_3 \times Z_2$ is

$$V = V_{2\text{HDM}} + V_\eta^Z + V_{\Phi \eta}^Z,$$

with the first term given in Eq. (4) and

$$V_\eta^Z = \frac{\mu^2}{2} \left[ |\eta|^2 + \frac{\lambda_1}{2} |\eta|^2 + \frac{\lambda_2}{2} |\eta|^4 \right] + \frac{\mu_\eta^2}{2} \left[ |\eta|_2^2 |\eta|_2^2 \right],$$

$$V_{\Phi \eta}^Z = \left[ \zeta_1 (\Phi_1 \Phi_1^* + \zeta_2 (\Phi_2 \Phi_2^*) \right] (|\eta|^2),$$

where $|\eta|^2$ represents one of the three possible choices $(1_+, 1_-, 2)$, all couplings are real (due to hermiticity) ensuring a CP conserving potential and we have omitted those terms involving $|\eta|^4$ as it is zero. This potential has an extra Goldstone boson due to the fact that $|\eta|^2 = n_1^2 + n_2^2$ is equivalent to $(n_1 - i n_2)(n_1 + i n_2)$ and $|\eta|^2 |\eta|^2 = (|\eta|^2)^2$. Then, Eq. (54) is accidentally invariant under a global $U(1)$ symmetry originated from the two components of the flavor doublet scalar, $\eta$. To avoid its appearance we softly-break the accidental symmetry by introducing

$$V_{\text{soft}} = \frac{\mu^2}{2} |\eta|^2 + \frac{\mu_\eta^2}{2} |\eta|^4 - \mu_1 n_1 n_2 .$$

Additionally, the fact that the heavy quark masses are simply given by $m_t \simeq y_t v_2$ and $m_b \simeq y_b v_4$ naturally points to having order one Yukawas and hierarchical VEVs in the range

$$v_2 \simeq v \quad \text{and} \quad v_1 \sim [1, 10] \text{ GeV},$$

meaning that $\tan \beta \in (10, 100)$. To create such a hierarchy while maintaining all scalar masses around the electroweak scale we need to consider $m_{12}^2 < 0$, $m_{11}^2 > 0$, and introduce the soft-breaking term

$$-m_{12}^2 (\Phi_1^* \Phi_2 + \text{H.c.}),$$

where $m_{12} \sim \mathcal{O}(10) \text{ GeV}$. By assuming $|m_{11}|, |m_{22}| \sim 100 \text{ GeV}$, a straightforward calculation then leads to

$$v_2 \simeq \sqrt{-\frac{m_{12}^2}{\lambda_2}} \quad \text{and} \quad v_1 \simeq \frac{m_{12}^2 v_2}{m_{11}^2 + \lambda_3 v_2^2} .$$

The smallness of $v_1$ is thus natural as one recovers a larger symmetry when setting it to zero.

The minimization conditions read

$$-m_{11} = \lambda_1 v_1^2 + \lambda_3 v_2^2 + (w_1^2 + w_2^2) \cos \beta ,$$

$$-m_{22} = \lambda_2 v_2^2 + \lambda_3 v_2^2 + 2(w_1^2 + w_2^2) \cos \beta ,$$

$$-\mu_1^2 = 2(w_1^2 + w_2^2) \lambda_1 + 2v_2^2 \lambda_2 + 2v_1^2 \lambda_3 + \mu_2 - \frac{w_1 v_1}{w_2} ,$$

$$-\mu_2^2 = 2(v_1^2 + v_2^2) \lambda_1 + 2v_2^2 \lambda_2 + 2v_1^2 \lambda_3 + \mu_1^2 - \frac{w_2 v_1}{w_1} \mu_2^2 ,$$

where $\lambda = \lambda_1 + \lambda_3$. The latter two conditions can only be met if

$$\mu_1^2 = -\frac{w_1 w_2}{w_1^2 - w_2^2} (\mu_2^2 - \mu_2^2) .$$

The general expressions for the squared mass matrices are given in Appendix C.

In order to decouple $\eta$ from the 2HDM we assume its mass (or VEV) to be large enough and $\zeta_{12} \rightarrow 0$. Then, for the full potential, $V + V_{\text{soft}}$, to be bounded from below we require the well-known relations

$$\lambda_1 \geq 0 , \quad \lambda_2 \geq 0 , \quad \lambda_3 \geq -\sqrt{\lambda_1 \lambda_2} ,$$

$$\lambda_3 + \lambda_4 - |\lambda_5| \geq -\sqrt{\lambda_1 \lambda_2} ,$$

while for the new contributions

$$\lambda_{11} \geq 0 \quad \text{and} \quad \lambda_{12} \geq 0 ,$$

which all are sufficient and necessary conditions.

### D. Numerical example

In the following, we give a numerical example of how the perturbations brought by the addition of $\eta$ modify our initial 2HDM setup. We assign a best-fit value to our initial 2HDM setup. We assign a best-fit value to our initial 2HDM setup. We assign a best-fit value to our initial 2HDM setup.
mixing parameters

\[ \chi^2 = \sum_{f=d,s,b} \frac{(m_f^{\text{th}} - m_f^{\text{exp}})^2}{\sigma_f^2} + \frac{|V_{12}^\text{th} - |V_{12}^\text{ckm}||^2}{\sigma_{12}^2} \]

\[ + \frac{|V_{23}^\text{th} - |V_{23}^\text{ckm}||^2}{\sigma_{23}^2} + \frac{|V_{13}^\text{th} - |V_{13}^\text{ckm}||^2}{\sigma_{13}^2} \]

\[ + \frac{(J_q^\text{th} - J_q^{\text{exp}})^2}{\sigma_J^2} \]

(62)

where the value of the masses is taken at the Z boson mass scale, \( M_Z \), using the RunDec package [19]

\[ m_d^{\text{exp}}(M_Z) = 0.0027^{+0.0003}_{-0.0002} \text{ GeV} , \]

\[ m_s^{\text{exp}}(M_Z) = 0.055^{+0.004}_{-0.002} \text{ GeV} , \]

\[ m_b^{\text{exp}}(M_Z) = 2.86 \pm 0.02 \text{ GeV} , \]

(63)

as shown in the most recent global fit from the PDG [20].

As a proof of principle, we consider a minimal scenario with the least number of parameters. We assume all of them real except for \( \epsilon_4 \) which we consider it as purely imaginary and set \( \epsilon_5, \epsilon_6 = 0 \). Also we allow for small variations in the initial down quark Yukawa couplings appearing in Eq. (22).

The following best-fit values,

\[ \epsilon_1 = 4.4634 \text{ MeV} , \quad \epsilon_2 = 12.1428 \text{ MeV} , \]

\[ \epsilon_3 = 103.6520 \text{ MeV} , \quad \epsilon_4 = -i \times 60.9786 \text{ MeV} , \]

(65)

reproduce the down quark masses and the observed CKM mixing at the 1\( \sigma \) level with a quality of fit of \( \chi^2_{\text{d.o.f.}} = 0.49 \).

Besides their role in mixing, the introduction of perturbations has also brought FCNC at tree level. We now show how the size of the contributions is still sufficiently small. Through the best-fit values we calculate the unitary transformations for the left- and right-handed fields. With them the corresponding down quark Yukawa matrices, in the mass basis, are

\[ \Gamma_1 \lesssim \begin{pmatrix} 10^{-4} & 10^{-4} & 10^{-7} \\ 10^{-4} & 10^{-3} & 10^{-5} \\ 10^{-5} & 10^{-5} & 10^{-1} \end{pmatrix} + i \begin{pmatrix} 10^{-9} & 10^{-7} & 10^{-7} \\ 10^{-7} & 10^{-8} & 10^{-6} \\ 10^{-6} & 10^{-5} & 10^{-9} \end{pmatrix} , \]

\[ \Gamma_2 \lesssim \begin{pmatrix} 10^{-4} & 10^{-5} & 10^{-8} \\ 10^{-5} & 10^{-4} & 10^{-7} \\ 10^{-6} & 10^{-6} & 10^{-9} \end{pmatrix} + i \begin{pmatrix} 10^{-10} & 10^{-8} & 10^{-8} \\ 10^{-8} & 10^{-8} & 10^{-8} \\ 10^{-7} & 10^{-6} & 10^{-10} \end{pmatrix} , \]

(66)

where we have assumed \( v_1 \sim 10 m_b \) and \( v_2 \sim m_t \) to estimate the upper bounds and which are all consistent with those presented in Refs. [14, 21]. There are in fact three different scenarios from which Eq. (66) represents one of them. As all the independent perturbations defined in Eq. (45) originate from both Higgs doublets, \( \Phi_1 \) and \( \Phi_2 \), we can define three different benchmark scenarios as follows: all the perturbations come from i) \( \Phi_1 \), ii) \( \Phi_2 \), or iii) both. Our choice in Eq. (66) depicts the first case. We left for future work a detailed study of the differences between this approach and the conventional 2HDMs.

V. CONCLUSIONS

We have considered a new class of 2HDM where the conventional \( \mathbb{Z}_2 \) symmetry, by which FCNC can be naturally avoided, has been enlarged to \( \mathcal{F} \times \mathbb{Z}_2 \) such that symmetry constrains the Yukawa sector but goes unnoticed by the scalar sector. In particular, we have shown that the minimal case with \( \mathbb{Z}_3 \times \mathbb{Z}_2 \) is able to provide trivial quark and maximal atmospheric mixing at leading order. A further implication to this class of models is that couplings between the fermions to the scalars are nonuniversal, compared to the conventional types where couplings are universal. At last, we have taken the incompleteness of fermion mixing as a hint pointing towards new physics. To this end we have included two real scalar gauge singlets which transform as a flavor doublet, and are later integrated out by assuming them to be properly heavy. We have shown that quark mixing can be set in agreement with the latest global fits while the lepton mixing can become cobimaximal, i.e. maximal atmospheric mixing and maximal CP violation. We have treated the introduction of the real scalars as a new way of adding perturbations to 2HDMs in a systematic manner by demanding them to be invariant under the flavor symmetry. In general, these additions have the effect of breaking flavor conservation and tree level FCNC, mediated by the neutral scalars, are induced. However, the size of the contributions remains sufficiently small thanks to the approximate presence of a \( U(2)^3 \) global flavor symmetry in the light quark sector.

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Appendix A: Product rules of $D_3 \simeq \mathbb{Z}_3 \times \mathbb{Z}_2$

$D_3$ is the smallest non-Abelian discrete symmetry group. It describes the symmetrical properties of an equilateral triangle. It has three irreducible representations: two singlets, $1_+$ and $1_-$, and one doublet, $2$. Their product rules are

\[ 1_+ \times 1_+ = 1_+ , \quad 1_- \times 1_- = 1_+ , \quad 1_- \times 1_+ = 1_- \, , \quad 1_+ \times 2 = 2 , \quad 1_- \times 2 = 2 , \quad 2 \times 2 = 1_+ + 1_- + 2 . \]  

(A1)

In particular, the tensor product of two doublets, $a = (a_1, a_2)^T$ and $b = (b_1, b_2)^T$, is explicitly given as

\[ a \times b = (a_1 b_1 + a_2 b_2) 1_+ + (a_1 b_2 - a_2 b_1) 1_- + \left( \begin{array}{c} a_1 b_2 + a_2 b_1 \cr a_1 b_1 - a_2 b_2 \end{array} \right) 2 . \]  

(A2)

Appendix B: Renormalization group equations

In a model with $N$ Higgs doublets, wherein all Higgses couple to all fermions, the fermion mass matrices are expressed as a linear combination of $N$ Yukawa matrices times a VEV. Given an initial setup, the one-loop renormalization group equations (RGE) tell us how stable are the initial mass matrices at higher scales and if new flavor structures may appear, giving rise to misalignments. The one-loop RGE has been calculated in Ref. [11] and reads

\[ 16\pi^2 \mu \frac{d}{d\mu} \Delta_k = a_\Delta \Delta_k \]  

\[ + \sum_{l=1}^{N} \left[ 3 \text{Tr}(\Delta_k \Delta^l) + \Gamma_k l \text{Tr}(\Pi_k \Pi_l) \right] \Delta_l \]  

\[ + \sum_{l=1}^{N} \left( -2\Delta_l \Delta^l \Gamma_l + \Gamma_k l \Delta^l \Gamma_l \right) \]  

\[ + \frac{1}{2} \sum_{l=1}^{N} \left( \Delta_l \Delta^l \Gamma_k + \Gamma_l \Delta^l \Gamma_k \right) \]  

\[ + \frac{1}{2} \sum_{l=1}^{N} \left( \Gamma_k \Delta^l \Gamma_l + \Gamma_l \Delta^l \Gamma_k \right) , \]  

(B2)

where we have followed the notation introduced in [11]. Here $\mu$ denotes the renormalization scale and

\[ a_\Gamma = -8g^2 - \frac{9}{4} g^2 - \frac{5}{12} g'^2 , \]  

\[ a_\Delta = -8g^2 - \frac{9}{4} g^2 - \frac{17}{12} g'^2 , \]  

\[ a_\Pi = -\frac{9}{4} g'^2 - \frac{15}{4} g'^2 , \]  

(B3)

where $g$, $g'$ are the gauge couplings of the SM gauge group, $SU(3)_C \times SU(2)_L \times U(1)_Y$, respectively.

Appendix C: Squared mass matrices

The squared mass matrix for the $CP$-even scalars reads

\[ M^2_{CP-even} = \begin{pmatrix} m^2_{\phi \phi} & m^2_{\phi \eta} \\ m^2_{\eta \phi} & m^2_{\eta \eta} \end{pmatrix} , \]  

(C1)

where

\[ m^2_{\phi \phi} = \begin{pmatrix} 2v^2 \lambda_2 + m^2_{\eta 2} \cot \beta & 2v_1 v_2 \lambda_{345} - m^2_{\eta 2} v_1 v_2 \\ 2v_1 v_2 \lambda_{345} - m^2_{\eta 2} & 2v_1 v_2 \lambda_1 + m^2_{\eta 2} \tan \beta \end{pmatrix} , \]  

(C2)

\[ m^2_{\phi \eta} = \begin{pmatrix} 2\sqrt{2} \xi_1 v_1 v_2 & 2\sqrt{2} \xi_2 v_1 v_2 \\ 2\sqrt{2} \xi_1 v_2 v_2 & 2\sqrt{2} \xi_2 v_1 v_2 \end{pmatrix} , \]  

(C3)
\[
m^2_{\eta\eta} = \begin{pmatrix} 4\lambda w_1 w_2 - \frac{w_1 w_2 (\mu_1^2 - \mu_2^2)}{w_2 - w_1} & 4\lambda w_2^2 + \frac{w_1^2 (\mu_1^2 - \mu_2^2)}{w_2 - w_1} \\ 4\lambda w_1^2 + \frac{w_2^2 (\mu_1^2 - \mu_2^2)}{w_2 - w_1} & 4\lambda w_1 w_2 - \frac{w_1 w_2 (\mu_1^2 - \mu_2^2)}{w_2 - w_1} \end{pmatrix},
\]

\(\text{(C4)}\)

whereas for the \(CP\)-odd and charged scalars we have

\[
M^{2}_{CP\text{-odd}} = \begin{pmatrix} 2v_2^2 \lambda_5 - m_{12}^2 \tan \beta & -2v_1 v_2 \lambda_5 + m_{12}^2 \\ -2v_1 v_2 \lambda_5 + m_{12}^2 & 2v_1^2 \lambda_5 - m_{12}^2 \cot \beta \end{pmatrix}.
\]

\(\text{(C5)}\)

and

\[
M^{2}_{\pm} = \begin{pmatrix} 2v_2^2 (\lambda_4 + \lambda_5) - m_{12}^2 \tan \beta & -2v_1 v_2 (\lambda_4 + \lambda_5) + m_{12}^2 \\ -2v_1 v_2 (\lambda_4 + \lambda_5) + m_{12}^2 & 2v_1^2 (\lambda_4 + \lambda_5) - m_{12}^2 \cot \beta \end{pmatrix}.
\]

\(\text{(C6)}\)

Note that the mass matrices for the \(CP\)-odd and charged scalars are rank one while rank four for the \(CP\)-even case.

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