Acoustic waves in bubbly liquids for two kinds of gas bubbles with phase transitions

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Abstract. The propagation of acoustic waves in a mixture of liquid with vapor-gas bubbles is studied. The dispersed phase consists of two fractions of the bubbles differing on gas structure and radii (a two-fractional mixture). The phase transitions take place at each of the fractions. The volume content of the bubbles is the low (about 1%). The dispersion relation, unified for plane, cylindrical and spherical waves is obtained. It is shown that a presence of the second fraction in the structure of a disperse phase of the bubbles changes significantly a dispersion and a dissipation of acoustic waves.

1. Introduction

A propagation of acoustic waves in a liquid with gas bubbles is studied intensively both theoretically and experimentally. Nigmatulin [1], Nakoryakov et al. [2], Temkin [3] considered various basic aspects of acoustics of multiphase flows. Hao and Prosperetti [4] obtained the results of study of the vapor bubble dynamics in liquid. Commander and Prosperetti [5] presented a model of the propagation of plane waves with small amplitude in a mixture of liquid and of gas bubbles. This model is adequate for the frequencies far from resonance at the volume content of a disperse phase of 1-2%. Nigmatulin et al. [6] showed the need of allowing for the effect of liquid compressibility for problems of acoustics of bubbly liquids. Shagapov [7] considered a problem of propagation of small disturbances in liquids with polydisperse bubbles. Nigmatulin et al. [8] studied the small radial oscillations of vapor-gas bubbles in liquid under the influence of an acoustic field. They showed that capillary effects and phase transitions lead to the new resonant frequency of small bubbles. This frequency is different from the Minnaert frequency. Vakhitova and Shagapov [9] investigated the propagation of small disturbances in two-component two-phase bubbling media with a three-temperature model. They found that heat and mass transfer rather than an interphase friction mainly determine dispersion. Gubaidullin and Nikiforov [10] obtained a dispersion relation describing a propagation of harmonic spherical and cylindrical disturbances in monodisperse mixture of a liquid with the vapor-gas bubbles. They showed strong dependence of an attenuation of pulse waves on a vapor concentration in bubbles. Gubaidullin, Nikiforov and Gafiyatov [11] investigated a propagation of acoustic waves in a mixture of liquid with vapor-gas bubbles and with insoluble gas bubbles taking into account the phase transitions only in the one fraction.
In the present paper, a propagation of a sound in the mixture of liquid with two kinds of vapor-gas bubbles with the phase transitions on the bubble surface is studied.

2. Dispersion relation

Linearized equations for one-dimensional disturbances in a two-fractional mixture of liquid with vapor-gas bubbles are obtained from the general equations of motion for two-phase mixtures [1]. Solving this system this dispersion relation is obtained

\[
\left( \frac{K}{\omega} \right)^2 = \frac{\sigma_{\omega}^2}{C_i^2} + \frac{\sigma_{\omega} p_0}{p_0 N_{Rb}} \left( 1 + \frac{H_{3b} + H_{3a} + H_{2a} \chi_1}{M_{3a} \chi_2 / B_{2a}} \right) + \frac{\sigma_{\omega} p_0}{p_0 N_{Rb}} \left( 1 + \frac{H_{3a} + H_{3b} + H_{2a} \chi_3}{M_{3b} \chi_4 / B_{1b}} \right)
\]

(1)

Here

\[
\chi_1 = \frac{M_{4b} - B_{2a} M_{4a}}{M_{3b}}, \quad \chi_2 = \frac{H_{3a} B_{4a}}{M_{3a} \chi_1} + \frac{H_{3b} B_{4b}}{M_{3b}} \chi_3 = \frac{M_{4a} - B_{2a} M_{4b}}{B_{1b} M_{3a}},
\]

\[
\chi_4 = \frac{H_{3a} B_{4a}}{M_{3a}} + \frac{H_{3b} B_{4b}}{M_{3b}},
\]

\[
\delta = \frac{m_j - 1}{1 - k_{yj}}, \quad H_{ij} = \frac{m_j}{B_{1j} \tau_{T_{ij}}} \left( B_{1j} M_{2j} + N_{i1} M_{4j} \right), \quad H_{2j} = \frac{m_j}{B_{1j} \tau_{T_{ij}}} \left( B_{1j} M_{1j} - N_{i1} M_{3j} \right),
\]

\[
M_{1j} = G_j - M_{2j} - \frac{L_{2j} N_{3j}}{N_{rj}} - \frac{L_{2j} N_{3j}}{N_{rj} \chi_1} - \frac{1}{N_{rj} \chi_1}, \quad M_{2j} = \frac{N_{2j} L_{2j}}{N_{rj} - L_{2j} - \delta N_{2j}},
\]

\[
M_{3j} = \frac{N_{3j} \delta - L_{2j} N_{3j}}{L_{2j} - \delta N_{2j}} + L_{2j} + M_{4j}, \quad M_{4j} = \frac{L_{2j} - L_{2j} N_{2j}}{N_{rj} \chi_3},
\]

\[
B_{ij} = \frac{c_1 \tau_{T_{2j}}}{c_2 \tau_{T_{1j}}}, \quad N_{rj} = \frac{\rho_0^c (i \omega (j_0)^2 G_{rj})}{p_0 (3G_{rj} t_{rj} + 1)}, \quad G_{rj} = \frac{1}{t_{rj}},
\]

\[
t_{rj} = \left( \frac{j_0}{4v_1} \right)^2, \quad t_{rj} = \frac{j_0}{C_1 (j_0)^{1/3}},
\]

\[
N_{i1} = \frac{i \omega \tau_{T_{1j}}}{2m_j} - 1, \quad N_{i2} = \frac{i \omega \tau_{T_{2j}}}{2} - 1, \quad N_{3j} = k_{3j} (c_{2j} - R_j) - 1 + G_j,
\]

\[
L_{1j} = E_j (i \omega (j_0)^2 - 1), \quad L_{2j} = \frac{L_{2j} k_{2j}}{1 - k_{yj} T_0} + \Delta R_j - L_{2j} (1 + B_{1j}),
\]

\[
L_{3j} = 1 - G_j (1 + B_{1j}), \quad L_{4j} = L_{1j} + \Delta R_j N_{3j}, \quad k_{3j} = \frac{i \omega \tau_{T_{2j}}}{c_{2j}},
\]

\[
m_j = \frac{\rho_j}{\rho_0}, \quad m_j = \frac{\rho_j^2}{\rho_0}, \quad j = a, b.
\]

Dispersion relation (1) (i.e. the function of the complex wave number $K$, on the frequency $\omega$) determines a propagation of acoustic plane, cylindrical and spherical waves in two-fractional mixtures of liquid with vapor-gas bubbles (different initial radii $a_j$, $b_j$, different initial volume contents $\alpha_{a\omega}$, $\alpha_{b\omega}$, and different thermal properties of fractions) with the interphase diffusion mass transfer in each fraction.
3. Results
Figure 1 shows the phase velocity and attenuation coefficient in water for the volume contents of the fraction of $\alpha_{2a} = \alpha_{2b} = 0.005$, for the radius of air bubbles of $a_0 = 2 \cdot 10^{-3}$ m, for the radius of helium bubbles (solid lines) of $b_0 = 10^{-3}$ m, calculated by dispersion relation (1). In unperturbed state a pressure of a two-fractional mixture is $p_0 = 0.1$ MPa, a temperature is $T_0 = 327$ K, a mass concentration of water vapor in bubbles is $k_{v_{0a}} = k_{v_{0b}} = 0.1$. This also presents the results of calculations for the monodisperse mixtures of water with air bubbles ($a_0 = 2 \cdot 10^{-3}$ m, $\alpha_2 = 0.01$, dashed lines) and helium bubbles ($a_0 = 10^{-3}$ m, $\alpha_2 = 0.01$, dotted lines).

![Figure 1](image_url)

**Figure 1.** Frequency dependences of the phase velocity (a) and of the attenuation coefficient (b) for the mixture of water with: vapor–air and vapor–helium bubbles (solid line), vapor–air bubbles (dashed line), vapor–helium bubbles (dotted line).

The difference of gases in bubbles have small effect on the phase velocity value (figure 1(a)). However, the presence of two different bubble sizes in the mixture is important significantly. This leads to the occurrence of the two minimum points of the phase velocity curve in the near of the resonance frequencies of the oscillations of bubbles and to a characteristic inflection of a phase velocity curve (figure 1(a), solid line). In case of propagation of the harmonic waves in a liquid with bubbles of one radius, only one minimum of the phase velocity is observed (figure 1(a), dashed and dotted lines). The phase velocity values (figure 1(a), solid line) in low- and high-frequency areas are
equal to the values for the monodisperse mixtures (figure 1(a), dashed and dotted lines), because the volume content of bubbles is the same in both cases ($\alpha_2 = 0.01$, $\alpha_{2a} = \alpha_{2b} = 0.005$).

The phase velocity and the attenuation coefficient have maximum points, corresponding to the two resonance frequencies (figure 1(b), solid lines). By equal volume content of bubbles ($\alpha_2 = 0.01$) the value of the attenuation coefficient in low-frequency area in two-fractional mixture of water with vapor–air and vapor–helium bubbles (figure 1(b), solid line) is more, than the value for mixture of water with vapor–air bubbles only (figure 1(b), dotted line).

4. Conclusions
The propagation of acoustic waves in two-fractional mixture of liquid with vapor-gas bubbles taking into account the phase transitions is studied. The dispersed phase consists of two fractions of the bubbles with different size and gas structure. The presence of two fractions of bubbles leads to occurrence of two maxima in a frequency dependence of attenuation coefficient, and this causes a characteristic inflection of a frequency dependence of phase velocity. It is shown that dispersion and dissipation of acoustic waves depends significantly on presence of the second fraction in a part of bubbles.

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