Mesoscopic conductance fluctuations in InAs nanowire-based SNS junctions

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\textbf{Abstract.} We report a systematic experimental study of mesoscopic conductance fluctuations in superconductor/normal/superconductor (SNS) devices Nb/InAs-nanowire/Nb. These fluctuations far exceed their value in the normal state and strongly depend on temperature even in the low-temperature regime. This dependence is attributed to high sensitivity of perfectly conducting channels to dephasing and the SNS fluctuations thus provide a sensitive probe of dephasing in a regime where normal transport fails to detect it. Further, the conductance fluctuations are strongly nonlinear in bias voltage and reveal subgap structure. The experimental findings are qualitatively explained in terms of multiple Andreev reflections in chaotic quantum dots with imperfect contacts.
1. Introduction

As a consequence of the quantum mechanical interference of electron wavefunctions the low-temperature conductance $G$ of mesoscopic samples fluctuates when varying the chemical potential or an applied magnetic field. These conductance fluctuations were demonstrated more than 20 years ago as one of the first examples of mesoscopic quantum phenomena in submicron samples [1, 2]. Through the Landauer formula $G = (2e^2/h) \sum T_i$, the conductance can be expressed in terms of a sample-specific set of transmission eigenvalues $\{T_i\}$, and the variance of the conductance fluctuations, $\text{Var} G$, provides important information about the statistical properties of the transmissions, such as the distribution $\rho(T)$ and correlations. An important energy scale for electron interference in random systems is the so-called Thouless energy $E_{\text{Th}}$, being the shift in chemical potential $\mu_c \sim E_{\text{Th}}$ sufficient to uncorrelate the transport properties. At high temperatures $T$, strong dephasing due to inelastic scattering with rate $\gamma_\phi \gg E_{\text{Th}}$ subdivides the sample into many uncorrelated parts and the conductance fluctuations are suppressed by self-averaging. As the temperature is lowered $\text{Var} G$ increases and it is a remarkable result that when $T, \gamma_\phi \ll E_{\text{Th}}$ (usually $\gamma_\phi \ll T$ at low $T$), $\text{Var} G$ saturates to a value of the order of $(e^2/h)^2$, independent of the sample size and degree of disorder. For this reason, the phenomenon is denoted universal conductance fluctuations (UCF) [1, 2] and in this regime transport remains practically insensitive to dephasing.

A fundamentally different situation occurs if the leads to the normal (N) sample turn superconducting (S). In this case, a gap $\Delta$ opens at the Fermi level, and a subgap energy...
electron incident on the S interface cannot penetrate into the lead, but is instead coherently Andreev reflected (AR) as a hole upon injection of a Cooper pair. Instead of being a sum of \( \{ T_i \} \), the transport properties now depend on Andreev states modified by finite-voltage \( V \) in a way highly dependent on the transmissions \( \{ T_i \} \) [3]–[11] and Landau–Zener transitions between the states lead to quasi-particle current [4]–[6]. Transitions are most probable when levels come close for \( T \approx 1 \) and \( \phi \approx \pi \) as schematically illustrated in figure 1(a). We will show that this has important consequences for the statistical properties of the differential conductance, because its fluctuations develop extreme sensitivity to the statistics of the almost perfect channels, \( T \approx 1 \).

This paper focuses on this intriguing interplay of interference and Andreev processes and its consequences for the statistical properties of mesoscopic junctions. Enabled by recent progress in nanoscale device fabrication [12]–[16] we measure the low-temperature fluctuations of differential conductance in short mesoscopic superconductor/normal/superconductor (SNS) devices based on semiconducting nanowires contacted by niobium (Nb) leads. We systematically study the temperature and bias dependence of the fluctuation amplitude, the correlation potential \( \mu_c \), and the average differential conductance, and find that the normal-lead universal limit for the fluctuations is broken in SNS devices as was also recently pointed out by Doh et al [16] and Ojeda-Aristizabal et al [17]. In addition, we here show that unexpectedly, the fluctuations maintain a strong dependence on \( T \) even at low temperatures \( T \ll E_{\text{Th}} \) where the normal-state fluctuations are saturated. To explain the data we theoretically analyze how dephasing modifies the statistics of the almost perfect channels \( T \approx 1 \) and find that transmissions \( T \approx 1 \) are suppressed. This mechanism explains the strong temperature dependence of the SNS fluctuations and shows that they provide a much more sensitive probe of dephasing than normal UCF. Furthermore, varying the bias, we find that the fluctuation amplitude diverges as a power law as \( V \to 0 \) and we observe, for the first time, that

**Figure 1.** (a) Energy of Andreev bound states versus phase difference \( \phi \) for various \( T \) (schematic). Arrows indicate Landau–Zener transitions induced by the time dependence of \( \phi \) at finite bias. (b) \( dI/dV \) versus \( V_g \) at various temperatures (for clarity the 6.5, 17, 180 and 290 K traces have been offset by 1, . . . , 4e^2/h, respectively). Inset: scanning electron micrograph (SEM) of a typical device. (c) Device schematic.
multiple Andreev reflections (MAR) lead to subgap structure (SGS) in the fluctuation amplitude and in $\mu_c$. The finite bias results are compared with computations based on MAR-theory in chaotic quantum dots with imperfect contacts with good qualitative agreement between theory and experiment.

The organization of the paper is as follows. In section 2, we introduce the sample fabrication and the theoretical framework. In sections 3 and 4, we analyze the temperature dependence and the bias dependence of the fluctuations, respectively. Finally, conclusions are given in section 5 and the appendices are organized as follows: appendix A contains supporting data from additional devices; appendix B describes the extraction of device parameters; appendix C discusses the properties of the superconducting niobium leads and appendix D and appendix E discuss the properties of the circuit and possible effects of Nyquist noise. Further, in appendix F we present the theoretical model of the quantum dot and in appendix G and appendix H, we discuss the role of imperfect contacts and dephasing on the transmission distribution $\rho(T)$, respectively.

2. Experimental and theoretical setup

The InAs nanowires used in this study are grown by molecular beam epitaxy and transferred to a doped Si substrate capped with 200 nm SiO$_2$. Contacts to individual wires are defined by e-beam lithography and dc sputtering of 70 nm Nb following a brief etch in BHF (see [14, 15] for details). The leads have a critical temperature $T_c \approx 1.7$ K resulting in a gap $\Delta = 1.76T_c \approx 0.25$ meV at low temperature. The inset to figure 1(a) shows a SEM of a typical device; the wires have diameters $d \sim 80–100$ nm and the distance between the contacts is $L \sim 100$ nm. The nanowires are $n$-type and the device discussed here has mobility $\mu \sim 10^3$ cm$^2$/Vs$^{-1}$, carrier density $n \sim 4 \times 10^{17}$ cm$^{-3}$, mean free path $l_e \sim 18$ nm, diffusion constant $D \sim 60$ cm$^2$ s$^{-1}$, and Thouless energy $E_{Th} \sim 0.4$ meV estimated from the transfer characteristics $G(V_g)$ (see appendix B). Due to the design of the outer circuit, the measurable supercurrent is strongly suppressed allowing a study of the quasi-particle current alone (within the RCSJ/’tilted washboard’ model of Josephson junctions the device constitute a strongly underdamped junction—see appendix D). We measure the two-terminal differential conductance $G \equiv dI/dV$ using standard lock-in techniques ($V_{ac} = 12$ $\mu$V, 77 Hz) while varying the bias $V$, back-gate potential $V_g$ (applied to the doped substrate), and temperatures from 300 K to 300 mK. In the following, data from one device is presented, but similar results have been obtained on two additional Nb-based and one Al-based device demonstrating the generality of the phenomena. These results are presented in appendix A.

Disorder in the InAs crystal together with a multifaceted wire surface [18] presumably make the system chaotic and the barriers formed in the NS interface dominate the resistance. Therefore, we compare data with predictions from theory of MAR [4–6], [10] and energy independent scattering random matrix theory (RMT) for multi-channel chaotic dots with imperfect contacts, see figure 1(c) [19]. This RMT is valid for both diffusive and ballistic dots if $E_{Th}$ of dot + contacts is large, $eV \Delta, T \ll E_{Th}$. Thus, we ignore the energy dependence of $T_i$ (relaxing this assumption makes our numerics impractical and discussion more involved [9]). For the instructive case of perfect contacts we analytically find the effect of weak dephasing ($T \ll E_{Th}$) on the distribution $\rho(T)$ and of small bias $eV \ll \Delta$ on $\text{Var}G$ at $T = 0$. For the general case of imperfect contacts ($N = 16$ channels, transparencies $\Gamma_L = \Gamma_R$ chosen to match the experiment) the bias dependence of transport statistics is computed at $T = 0$. For

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details of the theory and a discussion of the role of contact asymmetry see appendix F and appendix G.

Figure 1(b) shows examples of the measured $G(V_g)$ for $V = 0$ V for various temperatures. For $T \lesssim 100$ K $G(V_g)$ exhibits a large number of reproducible, aperiodic fluctuations allowing a statistical analysis of the data. To characterize the fluctuations, we extract for each trace the average value of the differential conductance $\langle G \rangle \equiv \langle G \rangle_{V_g}$, the variance of the fluctuations $\text{Var} G = \langle G^2 \rangle - \langle G \rangle^2$, and from the correlation function $\tilde{F}(\delta V_g) = (\langle G(V_g) - \langle G \rangle \cdot \langle G(V_g + \delta V_g) - \langle G \rangle \rangle)$ the typical $V_g$-scale of the fluctuations $V_c$ (proportional to $\mu_c$) as $F(V_c) = \frac{1}{2} F(0) = \frac{1}{2} \text{Var} G$ [20]. The normal state behavior at temperatures below $T_c$ is measured by applying a magnetic field $B = 0.5$ T to suppress the superconductivity of the leads.

3. Effect of temperature

3.1. Experimental findings

Let us first consider the role of temperature. Figure 2 shows the temperature dependence of the extracted parameters at zero bias. For $T > T_c = 1.7$ K, $\langle G \rangle$ is almost constant $3 e^2/h$ showing that the current is not carried by thermally excited carriers. At $T = 1.7$ K when the leads turn superconducting $\langle G \rangle$ increases as a consequence of AR. The increase occurs over a range $1 \text{ K} \lesssim T \lesssim T_c$ corresponding to the $T$-dependence of the superconducting gap $\Delta(T)$ (included in the figure) which, below 1 K, is very weak and $\langle G \rangle$ is effectively saturated.

The fluctuation amplitude displays a different dependence on $T$: upon lowering $T$ from room temperature, $\text{Var} G$ increases as $T^{-1.7}$ (solid line). This reflects the self-averaging discussed above and the saturation at $T \sim 5$ K agrees with $E_{\text{Th}} \sim 5$ K estimated from the transfer characteristics. The transition to superconducting leads at $T = T_c$ is accompanied by a sudden increase of $\text{Var} G$, but unexpectedly it keeps increasing all the way to the lowest $T$; the upper inset to figure 2 emphasizes the low-$T$ behavior of $\text{Var} G$. Thus, the $T$ dependence of $\text{Var} G$ is not governed by $\Delta(T)$. Interestingly for $T \lesssim 1$ K $\text{Var} G$ seems to rejoin the $T^{-1.7}$ relationship that was followed above 5 K. We note that the normal-state saturation value $0.09(e^2/h)^2$ (measured with $B = 0.5$ T) is of the order of the theoretical normal-state universal value [21]. In the superconducting state, however, $\text{Var} G$ reaches $2.5(e^2/h)^2$ at 300 mK, $\sim 30$ times larger than the normal-state value. The upper bound for our measurement of $\text{Var} G$ due to finite $V_{ac}$ and Nyquist noise can be estimated from figure 3 and we conclude that neither can contribute to the observed power-law $T$-dependence (see appendix E).

3.2. Theoretical analysis

To describe this behavior we consider the Andreev states that are formed when the leads turn superconducting. These appear at energies sensitive to the phase difference of the leads $\phi$ and the transparency of the channels, $\epsilon_{i,\pm} = \pm \Delta(1 - T_i \sin^2 \phi/2)^{1/2}$ [3] as illustrated in figure 1(a). A finite bias $V \ll \Delta/e$ leads to a quasi-particle current $\propto \exp(-\pi \Delta(1 - T_i)/eV)$ since the resulting time evolution of the phase difference, $\phi = 2 eV t/h$, induces Landau–Zener transitions between low-energy pairs $\epsilon_{i,\pm} \approx 0$ [4]–[6]. Such transitions are most probable for $T_i \rightarrow 1$ and $\phi \approx \pi$, and in contrast to the normal case, transport is therefore dominated exponentially by the almost perfect channels. To analyze the temperature dependence we therefore study

\[ V_c \] has been transformed into the corresponding change in chemical potential using the gate coupling factor $\alpha \approx 2.5 \text{ meV V}^{-1}$ found from gate-dependent resonances in figure 3(b).
Figure 2. Temperature dependence of $\langle G \rangle$ (□), $\text{Var} G$ (○) and $\mu_c$ (△). For $\text{Var} G$ and $\mu_c$ solid lines are fits ($aT^b$). For $\text{Var} G$ only the data points with $T > 5$ K were included in the fit and the dashed extension is the extrapolation to lower temperatures. Solid symbols show parameters measured in the normal state. Upper inset: the $T$-dependence of $\text{Var} G$ on a log-linear scale. All data are for zero dc bias. Lower inset: schematic illustration of $\rho(T)$ for no (dashed), and weak dephasing, $T, \gamma_\phi \ll E_{\text{Th}}$ (shaded).

the role of dephasing on the statistics of $T \to 1$. Using the dephasing-probe model[22], Brouwer and Beenakker [23] numerically demonstrated that in a single-channel dot dephasing suppresses $\rho(T)$ for $T \to 1$. Using this approach, we consider the limit $\gamma_\phi \ll \delta$, ($\delta$ is the level spacing) and for $1 - T \ll \gamma_\phi/\delta$, we derive in appendix H an exponential suppression of the transmission density, $\rho(T) \propto \exp[-\beta(\gamma_\phi/2\delta)/(1-T)]$ (here $\beta$ is the usual Dyson parameter). Extending to the multichannel limit $N \gg 1$, we find that dephasing leads to the appearance of a temperature-dependent upper bound $T_* = 1 - \gamma_\phi/2\pi E_{\text{Th}}$ such that $\rho(T) = 0$ for $T > T_*$ (see lower inset to figure 2). For normal transport this is accompanied by a practically undetectable correction $-\gamma_\phi/\pi E_{\text{Th}}$ to the UCF [23]. However, for SNS transport due to the exponential sensitivity of the current to $T$’s near 1 the appearance of $T_*$ makes transport strongly temperature-dependent and unlike normal UCF theoretically $\text{Var} G$ diverges, $\text{Var} G \to \infty$, $T, V \to 0$ (the $V$ dependence is discussed in section 4). The details of our calculations are given in appendix H. In conclusion, lowering $T$ decreases $\gamma_\phi(T)$ and increases $T_*$ and thus allows the exponential contributions from progressively more transparent channels to play a role in the transport and its fluctuations thus increasing $\text{Var} G$. We are, at present, not able to predict the functional form of the increase and the power-law relationship $\text{Var} G \propto T^{-1.7}$ suggested by
the experiment remains unexplained. Also, the combined inclusion of dephasing and imperfect contacts remains a challenging theoretical problem.

Consider now the $T$ dependence of $\mu_c$ presented in figure 2. Generally, $\mu_c$ reflects the dependence of $\mathcal{T}$ on $V_g$ and provides information about the statistics of $\{T_i\}$ complimentary to $\text{Var}\mathcal{G}$. For $T > E_{Th}$, the correlations depend on dephasing and thermal averaging and $\mu_c$ decreases with $T$ and saturates for $T \ll E_{Th}$ to a value $\sim 0.35$ meV in agreement with the previous estimates of $E_{Th}$. As $T$ is lowered from $T_c$ we observe a further dramatic increase in the sensitivity to $V_g$ (see figure 1) and $\mu_c$ decreases significantly below its normal-state saturation value ($\mu_c \propto T^{0.7}$). In the S-state $\mu_c$ probes correlations of the nearly perfect channels as discussed above. However, the functional form of the $T$ dependence (and, in particular, the $T = 0$ value of $\mu_c$) is a complicated and fully open theoretical problem, which needs further work.

Figure 3. (a) $\mathcal{G}$ versus $V$ measured at 300 mK for constant $V_g$ as indicated by arrows in panel (b) (bottom curve off-set by $-e^2/h$ for clarity). (b) Gray-scale representation $\mathcal{G}$ versus $V$ and $V_g$ (brighter: more conductive). (c) Experimental $\langle \mathcal{G} \rangle$ ($\square$), $\text{Var}\mathcal{G}$ ($\circ$) and $\mu_c$ ($\triangle$, in units of $E_{Th} = 5$ K), respectively, as a function of $|V|$. The dashed lines show the result when measured at 2 K with the contacts in the normal state. (d) Measured and calculated $\text{Var}\mathcal{G}$ displayed on logarithmic scales; dashed line show $V^{-1}$. The bottom curves show the residual of fitting the experiment to a power law $\propto V^{-0.8}$ to enhance the SGS. (e) As (c) computed at $T = 0$ K.
4. Effect of bias

4.1. Experimental findings

We now discuss the bias-dependence of the transport and its fluctuations. The nature of the important AR-processes depends strongly on $V$, because a sequence of $n$ AR-processes that transfer a quasi-particle across the junction, is energetically possible only when $eVn \geq 2\Delta$. At the same time, a large number of AR requires a high transparency. Hence, as the bias voltage is decreased an enhanced sensitivity to the tail $\rho(T \to 1)$ is indeed expected, and it is interesting to study the characteristics of the fluctuation pattern as a function of bias voltage. Figure 3(a) shows measurements of $\mathcal{G}(V)$ for two different $V_g$ illustrating the strong dependence on $V_g$. Upon lowering $|V|$ the differential conductance shows an increase at $V \approx \pm 0.5\text{meV}$ corresponding to enhanced quasi-particle transport when the peaks in the density of state (DOS) of the leads line-up at $V = \pm 2\Delta/e$. The SGS at lower bias is the consequence of the bias thresholds for MAR as described above.

Figure 3(b) shows a gray-scale representation of $\sim 5000$ such traces covering $0 \leq V_g \leq 9.5\text{V}$. The enhanced quasi-particle transport for $|V| \leq 2\Delta/e$ is seen throughout the plot and the pattern of tilted bands of high differential conductance observed for $|V| > 2\Delta/e$ is characteristic of conventional gate and bias-dependent fluctuations in disordered mesoscopic samples [24]. Extracting again $\langle \mathcal{G} \rangle$, $\text{Var} \mathcal{G}$ and $\mu_c$ for each bias value leads to the result of figure 3(c) (left) which also includes normal-state data measured at 2 K. The average $\langle \mathcal{G} \rangle$ increases at $|V| < 2\Delta/e$ and the SGS clearly survives the averaging. The correlation potential $\mu_c$ decreases slowly as $V$ is lowered from $\Delta/e$, whereas $\text{Var} \mathcal{G}$ displays a pronounced peak for $V \to 0$. Both contain structure resembling MAR.

4.2. Theoretical analysis

To understand these results we theoretically consider multichannel samples at $T = 0$ first for $eV \ll \Delta$, when the generic distribution tail $\rho(T \to 1) \propto 1/\sqrt{T - T}$ gives a nonlinear dependence, $I \propto \sqrt{eV/\Delta}$ [4]–[6]. To find the fluctuations around this value in fully coherent wires or quantum dots with ideal contacts we use the known correlators of $T$, $T'$ [25] leading to $\text{Var} \mathcal{G} \propto 1/V^2$. The appearance of a divergence agrees with the experiment; however, as seen in figure 3(d) the experiment finds $V^{-0.8}$. Therefore, a more realistic model of the experiment should take into account the barriers formed in the NS interfaces. Figure 3(c) (right) shows $\langle \mathcal{G} \rangle$, $\text{Var} \mathcal{G}$ and $\mu_c$ computed for $T = 0$ (see appendix F). Importantly, we now find $\text{Var} \mathcal{G}$ close to $V^{-1}$ qualitatively similar to the experimental trend. Interestingly, we see that the SGS for $V = 2\Delta/ne$, $n = 1, 2, \ldots$ appears not only in the average current, but also in the fluctuations $\text{Var} \mathcal{G}$ and $\mu_c$. The SGS peaks in $\mu_c$ do not merge to form a divergence as for $\text{Var} \mathcal{G}$, but rather decrease slowly for $V \to 0$ in qualitative agreement with the experiment. No prior theoretical results exists for $\mu_c(V)$ and therefore the found agreement with the experiment is quite satisfactory. We note that, at lowest bias, the computed values of $\text{Var} \mathcal{G}$ are considerably larger than the measured values (factor 20), and that computed MAR peaks appear considerably sharper and higher than observed in the experiment. We attribute this to our simplifying assumptions of symmetric contacts and energy-independent elastic scattering, but most importantly; however, the strong suppression of $\text{Var} \mathcal{G}$ induced by dephasing is absent in the $T = 0$ model. A quantitative agreement with the experiment is therefore not expected. Future
studies could investigate this by repeating the measurement of figure 3 at various temperatures in devices with individually tunable barriers.

5. Conclusion

In conclusion, we present the first systematic study of the fluctuations of SNS transport. We find a very large enhancement of the fluctuation amplitude compared to normal-state UCF and an extreme temperature-sensitivity $\text{Var } G \propto T^{-1.7}$ even for temperatures where the normal-state fluctuations are saturated. We argue theoretically that this can be understood as the combined effect of the almost perfectly transmitting channels dominating the transport and the cut-off of transmissions close to one with increasing dephasing. Thus, SNS fluctuations provide a sensitive probe of quantum interference, which might be used for measuring weak dephasing, unavailable from normal UCF. Moreover, we reveal that the statistical properties of SNS fluctuations exhibit SGS as a function of bias. Good qualitative agreement is found with numerical calculations based on scattering RMT and MAR theory.

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Appendix A. Data from additional devices

Above, results were presented from measurements of one device (S0); an InAs nanowire contacted by superconducting niobium leads. Figures A.1(a)–(f) show data from one device (S1) with superconducting leads based on a Ti/Al/Ti trilayer [15] with $\Delta \approx 115 \mu eV$ and two additional niobium-based devices (S2 and S3). The qualitative behavior analyzed in relation to figure 3 is again observed for all three devices with the key features being: (i) The enhancement of $\langle G \rangle$ at the quasi-particle onset $|V| = 2\Delta/e$, and SGS at lower bias. (ii) The strongly peaked fluctuation amplitude as $|V| \to 0$ V (see below) and structure in $\text{Var } G$ at lower bias, and (iii) The slow decrease of the correlation potential $V_c$ for $|V| \to 0$ V and SGS also in $V_c$. Furthermore, for comparison, figure A.1(g)–(h) shows the corresponding measurements for a similar nanowire device contacted by normal-metal titanium/gold leads [14]. The low-bias behavior of $\langle G \rangle$, $\text{Var } G$ and $V_c$ remains featureless, thus confirming the significant role of the superconductors. The increased noise of the results in figure A.1 in comparison with the results of figure 3 is due to poorer statistics for S1–S3. Nevertheless the results supports the generic nature of the discussed phenomena.

In figure A.2 the bias dependence of the fluctuation amplitude for samples S0–S4 is displayed on logarithmic scales where each data point represents an average of positive and negative bias values. For the samples with S-leads (S0-S3) the $\text{Var } G \propto V^p$ dependence is again observed with best-fit powers $-1.13 \leq p \leq -0.79$ (inset) in good agreement with our numerical results for the fluctuations of multichannel quantum dots with imperfections contacts. We ascribe the spread in $p$ to sample-specific transparency and asymmetry of the contacts which have great influence on the fluctuation amplitude and its bias dependence. We note that the gap
Figure A.1. Measurements from additional devices: (a,b) Results from a nanowire device (S1) with a Ti/Al/Ti tri-layer contact turning superconducting below 750mK ($\Delta \approx 115 \mu$eV). (c)–(f) Results from nanowire devices (S2 and S3) with Nb contacts similar to those of the S0. (a,c,e) Conductance as a function of bias and gate voltages showing the increased conductance below the superconducting gap and the characteristic pattern for gate- and bias-dependent CF. (b,d,f) Average conductance, variance and critical voltage as a function of bias voltage, displaying similar phenomenology as displayed in figure 2 for device S0. (g,h) Results from a nanowire device (S4) with normal Ti/Au contacts to allow a comparison with the corresponding measurement in the absence of superconductivity. All results are measured at $T = 300$ mK.

Figure A.2. Variance versus source–drain bias $V$ in units of $\Delta/e$ for samples S0–S4 on logarithmic scales. Positive and negative bias values have been averaged. A power-law dependence $\text{Var } \mathcal{G} \propto V^p$ is observed for devices S0–S3, which have superconducting leads and the best-fit exponents are collected in the table. In contrast, the results of S4, which has normal-state (Ti/Au) leads display a strong saturation (for S4, $\Delta$ was set to $0.25$ meV to allow easy comparison with the Nb-based devices S0, S2 and S3). All results are measured at $T = 300$ mK.
Table B.1. Results of the analysis based on the transfer characteristics of the nanowire device. The analysis is based on the charge control model and the quoted values are the mean values over the relevant gate voltage interval.

| Sample id. | S0 | S1 | S2 | S3 | S4 |
|------------|----|----|----|----|----|
| Lead material | Nb | Al | Nb | Nb | Au |
| Device length (nm) | L | 100 | 300 | 100 | 100 | 300 |
| Device diameter (nm) | d | 80 | 70 | 80 | 80 | |
| Gate capacitance (aF) | C<sub>g</sub> | 2.5 | 1.2 | 2.5 | 1.6 | 1.8 |
| Carrier density (10<sup>17</sup> cm<sup>−3</sup>) | n | 4.1 | 1.9 | 4.7 | 2.0 | 1.2 |
| Mobility (10<sup>3</sup> cm<sup>2</sup> Vs<sup>−1</sup>) | µ | 1.0 | 4.1 | 0.9 | 1.0 | 4.5 |
| Mean free path (nm) | l<sub>e</sub> | 18 | 60 | 18 | 15 | 56 |
| Fermi wavevector (10<sup>6</sup> cm<sup>−1</sup>) | k<sub>F</sub> | 2.9 | 2.2 | 3.0 | 2.3 | 1.8 |
| Fermi velocity (10<sup>8</sup> cm s<sup>−1</sup>) | v<sub>F</sub> | 1.3 | 1.0 | 1.3 | 1.0 | 0.8 |
| Fermi wavelength (nm) | λ<sub>F</sub> | 22 | 28 | 21 | 27 | 33 |
| Diffusion constant (cm<sup>2</sup> s<sup>−1</sup>) | D | 60 | 180 | 73 | 45 | 120 |
| Thouless energy (meV) | E<sub>Th</sub> | 0.4 | 0.1 | 0.5 | 0.3 | 0.1 |

Δ<sub>S1</sub> ∼ 115 µeV of the leads for sample S1 is significantly smaller than the Nb-based devices. Thus, at low bias the results will be more sensitive to thermal effects and noise as well as influence from by the ac-bias (V<sub>ac</sub> ∼ 12 µV ∼ 0.1Δ<sub>Al</sub>/e) required by the lock-in technique. Therefore, Var G saturates at a larger bias ∼ 0.2Δ for S1 than for the other devices.

The figure also shows the corresponding data for sample S4 having normal leads. As compared to S0–S3, the fluctuation amplitude for S4 is effectively independent of bias and has a magnitude similar to that of the superconducting samples when these are biased outside the gap (|V| > 2Δ/e).

Appendix B. Device parameters

The characteristic parameters of the devices (carrier density n, Fermi wave vector k<sub>F</sub>, Fermi velocity v<sub>F</sub>, mean free path l<sub>e</sub>, number of channels N, diffusion constant D, and Thouless energy E<sub>Th</sub>) were estimated from the measurements of the transfer characteristics and bias spectroscopy in the Coulomb blockade regime. In the following, we describe the analysis for sample S0 but the parameters of S1–S4 are obtained similarly and the values are collected in table B.1. Figure B.1(a) shows the linear conductance G versus V<sub>g</sub> at a temperature of 17 K where conductance fluctuations are not yet dominant (The analysis is insensitive to the temperature as the G versus V<sub>g</sub> traces for T ≲ 150 K are very similar. Below ∼10 K, however, UCF makes an accurate determination of the transconductance problematic). As seen in figure B.1(a) the wire is depleted from carriers at low gate voltages. The threshold appears at V<sub>g</sub><sub>T</sub> ∼ −8 V from which on G increases linearly with V<sub>g</sub> until V<sub>g</sub> ∼ −5 V. Within the charge control model [26], which is widely used for analysis of nanowire field effect transistors (FETs) [27], the transconductance g<sub>m</sub> = ∂G/∂V<sub>g</sub> ≈ 0.6 e<sup>2</sup>/hV is given by g<sub>m</sub> = µC<sub>g</sub>/L<sup>2</sup>, where µ is the mobility, C<sub>g</sub> the capacitance to the back-gate and L ∼ 100 nm the length of the device. The capacitance is found from the V<sub>g</sub>-separation δV<sub>g</sub> ∼ e/C<sub>g</sub> of Coulomb blockade conductance peaks which appear close to pinch-off [14] (this slightly underestimates C<sub>g</sub> due to a finite level spacing; however, this correction is not significant for the analysis). Figure B.1(b) shows
Figure B.1. (a) The linear conductance as a function of $V_g$ measured at 17 K for device S0. For $V_g$ above the threshold at $V_{G,T} \sim -8$ V the conductance increases linearly with a transconductance $g_m \approx 0.6e^2/hV$, which allows a determination of the mobility. (b) as (a) but measured at $T = 300$ mK and for $V_g$ close to pinch off where Coulomb blockade dominates the transport.

A measurement of $dI/dV$ versus bias and gate for $V_g \sim -7$ V exhibiting the characteristic Coulomb-blockade diamonds with $\delta V_g \sim 65$ meV yielding $C_g \approx 2.5$ aF in good agreement with similar studies in other nanowire devices [14]. This $C_g$-value is somewhat lower than the result of an electrostatic cylinder-over-plane model where $C_g = \frac{2\pi \epsilon_0 \epsilon_r L}{(ln(2h/r))}$ ($\epsilon_0$, $\epsilon_r$, $L$, $r$ and $h$ are the free-space permittivity, relative dielectric constant of SiO$_2$, length of wire segment between the leads, the wire radius and center-to-plane distance, respectively). However, this is expected since the electric field from the back gate in our device is effectively screened by the large metal leads (see SEM-image on figure 1). As $C_g$ depends mainly on the geometry of the sample we do not expect it to depend significantly on $V_g$. Using $C_g = 2.5$ aF we get a mobility $\mu \approx 960 \text{cm}^2\text{V}^{-1}\text{s}^{-1}$.

The carrier density in the wire at a gate potential $V_g$ can be estimated from the charge induced by the gate-voltage with respect to the pinch-off, $n = C_g(V_g - V_{G,T})/\pi r^2 L e$ (here $r$ is the wire radius and $L$ the length of the wire segment between the leads). This gives $n \approx 2.5-5.7 \times 10^{17} \text{cm}^{-3}$ for $V_g = 0-10$ V. We note that these estimates of mobility and density are similar to other studies of InAs nanowire devices [12, 28].

The Fermi wave vector $k_F$ is calculated using the three-dimensional (3D) expression for the Fermi energy $k_F = (6n\pi^2)^{1/3}$ and using the bulk value for the effective electron mass in InAs $m^* = 0.026m_e$ ($m_e$ being the electron mass) we find also the Fermi velocity $v_F = h_F/m^*$. Finally, since $\mu = el_e/v_Fm^*$ we find the mean free path $l_e$, the diffusion constant $D = \frac{1}{3}v_Fl_e$, and the Thouless energy $E_{Th} = hD/L^2$. In order to transform changes in $V_g$ into the corresponding change in chemical potential the parameter $\alpha = C_g/C_{\text{total}}$ is needed. In analogy with the standard procedure for finding $\alpha$ in the Coulomb blockade regime it is determined as the typical slope of the high-conductance ridges observed in figure 3 giving $\alpha = 2.5 \text{meV V}^{-1}$ for S0. The results are summarized in table B.1, which includes also the results of a similar analysis for devices S1–S4.

Appendix C. Properties of the Nb film

Devices S0, S2 and S3 have the InAs nanowires contacted by niobium leads deposited by dc sputtering. Figure C.1(c) shows traces of $dI/dV$ versus $V$ at constant gate $V_g = 2.5$ V as
the temperature is lowered from 1.8 to 0.3 K for device S0. Below $T_c \sim 1.7$ K the low-bias conductance increases and at lower temperatures discernable SGS develops. The dashed curve shows the temperature dependence of (twice) the gap $2\Delta_{BCS}(T)$ saturating to $2 \times 1.75T_C = 0.5$ meV at low temperature. This agrees with the observed increase of the conductance for $|V| \sim 0.5$ mV as the peaks in the DOS of the leads line up at $2\Delta/e$.

The critical temperature measured in the device is considerably lower than that of bulk Nb, which has a critical temperature of 9.2 K. Such differences between bulk properties of the lead material and the actual properties of the nanoscale devices is often observed. For example, in [15, 29] aluminum was used for contacting carbon nanotubes and InAs nanowires, respectively, with an observed transition temperature of 750 mK considerably lower than $T_c$ of bulk A.1 (1.2 K). In the case of the present device the dramatic decrease of $T_c$ may be due an impure Nb film in the interface between contact and nanowire due to a reaction of the sputtered Nb with outgassing from the electron–beam resist (PMMA) on the substrate or traces of oxygen during sputtering [30, 31].

The critical magnetic field of niobium can exceed several tesla depending on the quality and geometry of the film, making niobium a good candidate for nanostructure-based superconducting quantum interference devices (SQUIDs) [32] where robustness to an external magnetic field is desired. In our case, however, the critical field turns out to be relatively small which is beneficial as we can then measure the normal-state behavior for temperatures below $T_c$.

Figures C.1(a) and (b) show $dI/dV$ versus $V$ and $V_g$ with $B = 0$ and $B = 0.5$ T, respectively. For $B = 0$, the conductance exhibits pronounced subgap peaks due to AR (similar to figure 3(b)). The SGS disappears upon the application of $\sim 350$ mT, however, at $B = 0.5$ T, a small conductance decrease remains for $|V| \leq 0.1$ mV as also seen in figure C.1(d). This feature repeats for all gate-voltages and indicates that some reminiscence of superconductivity may still exist. From the measurements for $T > T_c$ in figure 3 and of sample S4 in figure A.1 it is known

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure.png}
\caption{(a), (b) Grey-scale representation of $dI/dV$ versus $V$ and $V_g$ for applied magnetic field $B = 0$ T and $B = 0.5$ T, respectively (brighter, more conductive). (c) $dI/dV$ versus $V$ and temperature at fixed gate potential. The BCS-like evolution of $2\Delta$ is indicated by the dashed line. (d) Traces of $dI/dV$ versus $V$ for various combinations of temperature and magnetic field as indicated on the figure.}
\end{figure}
that in the normal state the statistical properties (\( \text{Var} \mathcal{G}, \langle \mathcal{G} \rangle \) and \( \mu_c \)) are bias-independent for small \( V \). Therefore, to ensure that the normal-state data reported for \( T < T_c \) in figure 2 (solid points) are free from superconducting correlations they are measured with \( B = 0.5 \, \text{T} \) and a small bias \( V = 0.2 \, \text{mV} \).

Appendix D. Considering the suppressed supercurrent

The theoretical supercurrent through the wire is given by \[ I_c = \pi \Delta_{\text{BCS}}/2eR_n \] where \( R_n \) is the normal-state resistance of the device. In our case \( R_n \sim (3e^2/h)^{-1} \), which yields a theoretical supercurrent of \( I_c \sim 30 \, \text{nA} \). The measurable supercurrent, however, depends on the external circuit and in the extended RCSJ/’tilted-washboard’ model \[ Q^{-1} = \omega_p(RC + \frac{h}{2eI_cR_n}) \] and \( \omega_p = \sqrt{2eI_c/h(C(1 + R/R_n) + C_j)} \). Here \( C_j \) is the junction capacitance and \( I_j = I_c \sin(\phi) \) is the phase-dependent super-current through the junction. \( C \) is the relatively large area bonding-pad capacitance and \( R \) is the resistance of the on-chip wiring connecting the bonding pads to the device. The circuit is schematically shown in figure E.1, where \( R_W \sim 50 \, \Omega \) is the resistance of the wiring of the cryostat. In [29] efforts were made to optimize these parameters for a large measurable supercurrent in carbon nanotube based junctions (essentially by making \( R \) large). In the present case of the Nb–NW–Nb junctions we have \( C_j \sim 2 \, \text{fF}, R_n \sim 10 \, \text{kΩ}, C \sim 1 - 2 \, \text{pf}, \) and \( R \lesssim 10 \, \text{Ω} \), and with these parameters we obtain \( Q \sim 10 \) showing that the measurable supercurrent is strongly suppressed. This is consistent with the experimental results: at all gate voltages we observe the reminiscence of the highly suppressed supercurrent as a narrow weak peak in \( dI/dV \) at zero bias (see figure 3(a)). The peak is, however, negligible compared to the quasi-particle conductance, which allows us to analyze the results in terms of the quasi-particle transport and AR alone.

Appendix E. Possible effects of thermal noise and \( V_{ac} \)

In principle, the temperature dependence presented in figure 2 could be affected by thermal noise and the finite excitation voltage of the lock-in amplifier \( V_{ac} = 12 \, \mu eV \). Due to down-mixing by the nonlinear SNS device, thermal noise may result in a dc voltage \( V_N(T) \propto \sqrt{T} \). Since a bias dependence of \( \text{Var} \mathcal{G} \) is observed in figure 3 at \( T = 0.3 \, \text{K} \) down to \( V \sim V_{ac} \), we conclude that \( V_N(0.3 \, \text{K}) \lesssim 12 \, \mu eV \), so the Nyquist contribution must satisfy \( V_N(T) \lesssim \sqrt{T[K]} \times 22 \, \mu eV \).
The approximate form of $\text{Var}\ G$ as a function of voltage, $\text{Var}\ G/(e^2/h)^2 \approx 0.23(\Delta/eV)^{0.8}$ for $V > V_{ac}$, $V_N(0.3)$, allows us to evaluate the upper boundaries for $\text{Var}\ G$ allowed in our experiment at $V = 0$. Using $V = V_{ac}$ and $\Delta = 250\ \mu\text{eV}$ gives $\approx 1.6\ T^{-0.4}$, $T > 0.3\ \text{K}$. Only at lowest temperature do our data reach 2.6, and at higher $T$ values significantly lower than the Nyquist upper bound are observed. Thus, we conclude that the power-law extracted from our data cannot be governed by a thermal noise. Further, the values of $\text{Var}\ G$ are affected by $V_{ac}$ only at the lowest temperature.

Appendix F. Construction of the $S$-matrix

Since we expect that the fluctuational phenomena we consider are generic, the particular geometry of the sample is not expected to make a qualitative difference. On the one hand, the barriers on the NS boundaries are the regions where the main voltage drop occurs. The mesoscopic sample itself can then be considered point-like. On the other hand, the samples are disordered due to impurities. These two features allow us to consider a chaotic quantum dot as a realistic geometry for the experiment. Importantly, the quantum dot model allows us to study how the fluctuations depend on the voltage $V_g$ on a nearby electrostatic gate and on the imperfect contacts. Such information, unavailable from other models, is important for the comparison to the experiment:

1. Effect of the gate voltage $V_g$. Conventionally, a closed chaotic dot is characterized by its large $M \times M$ Hamiltonian matrix $H$ from a Gaussian ensemble of random matrices with relevant symmetry. This symmetry is characterized by Dyson parameter $\beta$ for pure ensembles: orthogonal, $\beta = 1$, if time- and spin-reversal symmetry are present (GOE), Unitary, $\beta = 2$, if time-reversal symmetry is broken (GUE), or Symplectic, $\beta = 4$, if spin-symmetry is broken (GSE). The ensemble average (denoted by $\langle \ldots \rangle$) of any Hamiltonian element vanishes, $\langle H_{\alpha\gamma} \rangle = 0$. The pair correlator of the elements is defined by $\beta$, the matrix size $M$, and mean level spacing $\delta$, \[ \langle H_{\alpha\gamma} H_{\alpha'\gamma'} \rangle = \frac{M^2\delta^2}{\pi^2} \left( \delta_{\alpha\gamma'} \delta_{\gamma\alpha'} + \delta_{\alpha\alpha'} \delta_{\gamma\gamma'} \delta_{\beta 1} \right), \quad \beta = 1, 2. \]

An open dot with $N$ ballistic channels is fully characterized by its $N \times N$ scattering matrix $U$. Usually $U$ is assumed to be uniformly distributed over the ensemble of unitary matrices of relevant symmetry (Dyson circular ensembles with $\beta = 1, 2, 4$). However, a gate voltage $V_g$ coupled to the dot via capacitor $C_g$, see figure F.1, shifts the chemical potential (the bottom of the band) and thus affects the matrix $U$. Variations in $V_g$ lead to a shift of the dot’s Hamiltonian and therefore

$$ U = \mathbb{1}_N - 2\pi i W^\dagger W \frac{1}{\mathbb{1}_M \cdot \mu - H + i\pi W W^\dagger W}. $$

(F.1)

The unit matrix of $N \times N$ size is denoted by $\mathbb{1}_N$ and the coupling $M \times N$ matrix $W$ consists of the matrix $(\sqrt{M}/\pi)\mathbb{1}_N$ and zeros in the lower $M-N$ rows [35]. For $N \gg 1$ universal results (independent of $M$) are reached only when $M \rightarrow \infty$. In the limit $N \ll M$ the Hamiltonian Gaussian ensemble was shown to give the same uniform distribution for $U$ as the circular ensembles [36]. Equation (F.1) is essential for finding the role of $V_g$ in numerical mesoscopic averaging.
2. Effect of imperfect contacts. For ballistic contacts, the scattering matrix $U$ is taken from the Circular ensemble directly, or by using equation (F.1). For a dot with imperfect channels with transparency $\Gamma_i$, $0 \leq \Gamma_i \leq 1$, the scattering matrix $S$ is distributed according to the Poissonian ensemble [19]. A representative of this ensemble can be obtained from the matrix $U$ of an open dot after including possible multiple reflections from the contacts, see figure F.1:

$$S = \sqrt{1_N - \Gamma - \sqrt{1_N - \Gamma} \frac{1}{1_N - \sqrt{1_N - \Gamma} U \sqrt{1_N - \Gamma}}. \quad (F.2)$$

Indeed, expanding the last term to $n$th order in $U$ accounts for $n - 1$ internal reflections before the electron exits through one of the contacts.

We use quantum dots to model the SNS samples and find the fluctuations of conductance $G = dI/dV$. Each sample is specified by its $S$ matrix, and we find its set of transmission eigenvalues $\{\tau_i\}$. Using the scattering theory, which includes MAR theory in SNS structures, developed by Averin and Bardas [4], we compute the current $I$ as a sum of currents in each channel for the slightly shifted voltages to find the sample-specific $G$. Repeating this calculation for many different $S$ matrices allows to find statistical properties of $G$.

In general, the mesoscopic averaging can be performed after measurements in many samples, or by using a single sample and varying the gate voltage $V_g$. Justification for this widely used procedure comes from the hypothesis that energy averaging equals ensemble averaging. Therefore, the mesoscopic averaging denoted by $\langle \ldots \rangle$ should be understood as $\langle \ldots \rangle_{V_g}$.

Experimentally, correlations for traces taken at the same bias voltage $V$ and gate voltages shifted by $\delta V_g$ are quantified by the correlator $F$ [20],

$$F(V, \delta V_g) = \langle G(V, V_g)G(V, V_g + \delta V_g) \rangle_{V_g} - \langle G(V, V_g) \rangle \langle G(V, V_g + \delta V_g) \rangle_{V_g}. \quad (F.3)$$

The variance of the conductance is then given by $\text{Var} G = F(V, 0)$. For large $\delta V_g$ the conductances become completely uncorrelated, $F(V, \delta V_g) \to 0$, and the correlation potential $\mu_c$ is defined as the shift in chemical potential $\mu_c$ which diminishes the correlator twice,

$$F(V, \delta V_g^c) = \frac{1}{2} F(V, 0), \quad \mu_c = \alpha \delta V_g^c. \quad (F.4)$$
In normal transport through a dot with ballistic contacts \( \mu_c \) is naturally measured in units of its escape rate, or the Thouless energy of an open dot, \( E_{\text{Th}} = N \delta/2\pi = h/\tau_d \). For example, for normal linear conductance \( \mu_c = E_{\text{Th}}, \beta = 1, 2 \) and it reaches its maximum [(\( 2^{1/2} + 3^{1/2} \))\( (2^{1/2} - 1) \)]\( 1/2 \)\( E_{\text{Th}} \approx 1.14E_{\text{Th}} \) in the crossover between \( \beta = 1, 2 \). If the contacts are imperfect with equal transmission \( \Gamma \), the Thouless energy (and thus \( \mu_c \)) diminishes according to the time an electron typically spends inside the dot, \( E_{\text{Th}} = N \Gamma \delta/2\pi \).

In SNS transport, to find \( \mu_c(V) \) we have to numerically solve equation (F.4) (iteratively by the Newtonian method). We substitute the gate voltage averaging in equation (F.3) by averaging over several hundred (100–400) samples at each iteration, their \( S \) matrices being found combining the procedure in 1) and 2). While the correlator \( F \) is mathematically well defined in equation (F.3), in reality both sides of equation (F.4) fluctuate, and our procedure may converge slowly. Convergence after few iterations results in noise in \( \text{Var} \, G \), see figures 3(d) and (e). Indeed, averaging the rhs of equation (F.4) in just few hundred samples may differ from the true value of \( \text{Var} \, G \). Similar fluctuations in the lhs result in noise for the \( \mu_c(V) \) plots.

The RMT described here uses energy-independent \( S \) matrices for electrons with (generally) different kinetic energies. This assumption is valid if the electrons are close to the Fermi level. A typical energy of an electron is limited by the largest energy among the temperature scale \( k_B T \), the superconducting gap in the contacts, \( \Delta(T) \), and the bias \( eV \). The scattering matrix \( S \) can be taken energy-independent if \( T, \Delta, eV < E_{\text{Th}} = N \Gamma \delta/2\pi \) due to large level spacing \( \delta \) and good conduction of the sample, \( N \Gamma \gg 1 \).

In addition, for validity of our analytical results we must ensure that for small bias voltages, \( eV \ll \Delta \), the band \( \sim (eV/\Delta)^{1/2} \) still contains many transmission eigenvalues \( T \). This is fulfilled, if \( 1/N \ll (eV/\Delta)^{1/2} < 1 \). Numerics for \( N \gg 1 \) are performed for \( \beta = 2 \), but the results are easily generalized on \( \beta = 1, 4 \), since \( \rho(T) \) is insensitive to \( \beta \) for \( N \gg 1 \) and the correlator \( K \) is simply rescaled with \( 1/\beta \) [25].

Appendix G. Effect of imperfect contacts

Analytical results for \( \text{Var} \, G \) in SNS transport can be obtained after combining transmission correlators \( K(T, T') \) [25] with the MAR theory of Averin and Bardas [4], who showed that for \( eV \ll \Delta \) only channels with high transmissions, \( T \approx 1 \), are important. Landau–Zener transitions between Andreev bound states lead to nonlinear current \( I \propto (eV/\Delta)^{1/2} \) in diffusive wires [5]. We point out that \( \rho(T) \propto 1/\sqrt{1-T} \) is generic for mesoscopic samples with perfect contact leads, such as diffusive wires or quantum dots with ballistic contacts, and obtain

\[
\langle I(V)I(V') \rangle \propto \frac{\sqrt{V'V}}{\beta(V+V')}. \tag{G.1}
\]

\( \text{Var} \, G \) is obtained from equation (G.1) after differentiating both sides of equation \( \partial_V \partial_{V'} \) and setting \( V' = V \). The final result is \( \text{Var} \, G \propto 1/V^2, eV \ll \Delta \). Numerical results for multichannel dots with ballistic contacts, \( N = 16 \), indeed show this behavior. However, this instructive example does not take the contacts into account, which are important for the current experiment.

The opposite limit is a random mode mixer, or the Fabry–Perot interferometer, where electrons gain random phases traversing between the contacts. The internal reflections are absent and the main resistance comes from the contacts (‘opaque mirrors’) [37, 38]. This model is thus relevant for almost perfect conductors and its results are also universal. The distribution \( \rho(T) \)
of such a random Fabry–Perot interferometer depends on the transparency \( \Gamma_{1,2} \) of the contacts, and is bound by \( \mathcal{T}_- < \mathcal{T} < \mathcal{T}_+ \) [37],

\[
\rho(\mathcal{T}) = \frac{\sqrt{\mathcal{T}_+ \mathcal{T}_-}}{\mathcal{T} \sqrt{(\mathcal{T} - \mathcal{T}_-)(\mathcal{T}_+ - \mathcal{T})}}. 
\]  

}\text{(G.2)}

\[
\mathcal{T}_\pm = \frac{\Gamma_1 \Gamma_2}{(1 \mp \sqrt{1 - \Gamma_1})(1 - \Gamma_2)^2}. 
\]  

}\text{(G.3)}

The transmission can be parameterized by a uniformly distributed phase \( \varphi \in [0, 2\pi] \) as \( \mathcal{T} = \mathcal{T}_+ \mathcal{T}_- / (\mathcal{T}_+ \cos^2 \varphi/2 + \mathcal{T}_- \sin^2 \varphi/2) \). Even though the correlator \( \mathcal{K}(\mathcal{T}, \mathcal{T}') \propto 1/((\beta \sin^2 (\varphi - \varphi')/2) \) is formally independent of the cut-offs \( \mathcal{T}_\pm \), they do affect fluctuations for \( eV \ll \Delta \):

\[
(I(V)I(V')) \rightarrow \frac{8T_+^2 \sqrt{VV'}}{\pi \beta (V + V')} \exp \left( \frac{\pi \Delta (\mathcal{T}_+ - 1)(V + V')}{eVV'} \right). 
\]  

}\text{(G.4)}

Asymmetry in transmissions of the contacts \( \Gamma_1 \neq \Gamma_2 \) modulates \( \mathcal{T}_\pm \) in equation (G.3) and suppresses currents and their fluctuations. Importantly, the appearance of a fixed cut-off for perfect channels affects the current fluctuations exponentially. One reason for the appearance of \( \mathcal{T}_- < 1 \), the contact asymmetry, is obvious from the last example, and below we consider it more quantitatively for chaotic quantum dots. The universality of the results (G.1), (G.4) does not hold in a general situation: the shape of \( \rho(\mathcal{T}) \) and the cut-off values \( \mathcal{T}_\pm \), found by the methods of [21, 39] depend on conductance \( g_N \) and the contact asymmetry, \( N_1 \neq N_2 \) or \( \Gamma_1 \neq \Gamma_2 \). Symmetric contacts with \( N_1 = N_2 \) still yield \( \rho(\mathcal{T} \rightarrow 1) \propto 1/\sqrt{1 - \mathcal{T}} \), even for \( \Gamma_1 = \Gamma_2 \ll 1 \), but for asymmetric contacts we can have \( \rho(\mathcal{T} \rightarrow \mathcal{T}_+ - 0) \propto \sqrt{\mathcal{T}_+ - \mathcal{T}} \). For transmission distribution \( \rho(\mathcal{T}) \) we can take into account the contact asymmetry following the method of Brouwer and Beenakker [21]. We find that the perfect channels do not vanish, \( \rho(1) \neq 0 \), if the contacts are not very asymmetric, and \( \rho(\mathcal{T}) \) near \( \mathcal{T} = 1 \) is suppressed compared to the universal distribution as

\[
\rho(\mathcal{T})|_{\mathcal{T} \rightarrow 1} \rightarrow \frac{\sqrt{(N\Gamma_1 - \tilde{\Gamma} N_2)(N\Gamma_2 - \tilde{\Gamma} N_1)}}{\pi \tilde{\Gamma} \sqrt{1 - \mathcal{T}}}, 
\]  

}\text{(G.5)}

where \( \tilde{\Gamma} \equiv \Gamma_1 + \Gamma_2 - \Gamma_1 \Gamma_2 \). Indeed, for \( N_{1,2} = N/2 \) perfect channels exist when the contact transparencies are close, \( |\Gamma_1 - \Gamma_2| < \Gamma_1 \Gamma_2 \). In the limit \( \Gamma_{1,2} \ll 1 \) this condition can be easily violated by a relatively small difference in \( \Gamma \). We cannot analytically predict the behavior of the current fluctuations, since \( \mathcal{K}(\mathcal{T}, \mathcal{T}') \) is unknown and most probably non-universal. However, we assume that our contacts are not very asymmetric and \( \rho(1) \neq 0 \). Even if asymmetry in contacts does affect \( \text{Var} \mathcal{G} \) in our experiment, it cannot account for a strong temperature dependence of our data. To explain the strong \( T \) dependence of our results, we consider the effect of dephasing on the transmission statistics.

**Appendix H. Effect of dephasing**

As discusses above, in the low-temperature limit, \( \gamma_\phi \ll E_{1h} \), and small bias voltages, \( eV \ll \Delta \), the statistics of \( \mathcal{T} \) close to perfect transmission become extremely important for SNS transport. As seen in equation (G.4), the appearance of a cut-off \( \mathcal{T}_+ < 1 \) strongly suppresses \( \text{Var} \mathcal{G} \). Our samples are expected to combine effects of imperfect contacts and dephasing. For simplicity we now consider the role of dephasing alone, assuming ballistic contacts to the
reservoirs. Temperature is assumed to be sufficiently low to make the dephasing rate small, \( \gamma_\phi = \hbar/\tau_\phi \ll E_{Th} \). The dephasing probe model, proposed by Büttiker [22], has been extensively used in the literature. In this model, the quantum coherence in the dot is destroyed by attaching a probe, where the voltage can be either externally controlled or remain floating. This probe exchanges electrons with the dot, and one distinguishes non-uniform (or localized) versus uniform dephasing by such a probe depending on its coupling to the dot, see figure F.1.

Uniform dephasing denotes a probe having large number of poorly coupled channels, \( \Gamma_\phi \to 0, N_\phi \to \infty \). However, the dephasing rate \( \gamma_\phi = N_\phi \Gamma_\phi \delta \) remains fixed. Known results for uniform dephasing correspond to the results of the imaginary potential model in the Hamiltonian approach [23]. Non-uniform dephasing, on the other hand, usually takes perfect coupling to the dot, \( \Gamma_\phi = 1 \), and finite \( N_\phi \). The advantage of this approach is the technical simplicity due to the lack of back reflection from the probe, but on the other hand \( \gamma_\phi \) should be strictly quantized. For dots that are not artificially dephased by a local probe, we assume the uniform dephasing model to be more realistic.

For small dephasing in a dot with arbitrary \( N \) only the uniform dephasing model can be used. To illustrate the role of \( \gamma_\phi \) in the tails of \( \rho(\mathcal{T}) \) we first take a single-channel quantum dot, \( N = 2 \), and later consider the limit \( N \gg 1 \). Coherent quantum dots with \( N = 2 \) have only one transmission eigenvalue \( \mathcal{T} \) and \( \rho(\mathcal{T}) = (\beta/2)^{\mathcal{T}^2-1} \), \( \beta = 1, 2 \) [25]. For weakly dephased dots we use the intermediate results of [23] and express the dimensionless conductance \( g = hG/2e^2 \) close to \( g = 1 \) as a sum

\[
g \approx \sum_{i,j=1}^{N} \left( 1 - \frac{N_\phi \Gamma_\phi(x_i + x_j)}{2x_i x_j} \right) u_1^i u_2^i u_1^j u_2^j + \sum_{i,j=1}^{N} |u_1|^2 |u_2'|^2 \frac{N_\phi \Gamma_\phi}{x_i + x_j}, \tag{H.1}\]

where \( u, u' \) are 2 \times 2 unitary random matrices and \( u' = u^\dagger \), \( \beta = 1 \). If the particles were absorbed by the probe, only the first term in the sum (H.1) would have been present. The second term is due to reinjection of particles by the probe and results from the requirement of particle conservation. The parameters \( N_\phi \Gamma_\phi/x_{1,2} \) characterize the coupling strength of the probe to the dot, \( x \to \infty \) corresponds to weak coupling. The distribution \( \rho(\mathcal{T}) \) for \( N_\phi \Gamma_\phi \ll 1 \) is found after integration over the uniform distribution of \( u, u' \) in the unitary group and the distribution \( P(x_1, x_2) \) (see [23] for its general form),

\[
\rho(\mathcal{T}) = \int_{N_\phi \Gamma_\phi}^{\infty} dx_1 dx_2 \int du du' \delta(\mathcal{T} - g) P(x_1, x_2), \tag{H.2}\]

\[
P(x_1, x_2) \approx \frac{\beta(x_1 x_2 |x_1 - x_2|)^\beta}{48} \frac{e^{-\beta(x_1 + x_2)/2}}{1 - \mathcal{T} \ll N_\phi \Gamma_\phi}. \]

For \( 1 - \mathcal{T} \ll N_\phi \Gamma_\phi \), we find an exponential suppression, \( \rho(\mathcal{T}) \propto \exp[-\beta N_\phi \Gamma_\phi / 2(1 - \mathcal{T})] \). This result \( \rho(\mathcal{T}) \propto \exp[-\beta W(\mathcal{T})] \) can be understood as the density of classical particles at a point \( \mathcal{T} \in [1, 1] \) and temperature \( 1/\beta \) in an external potential

\[
W(\mathcal{T}) = \frac{N_\phi \Gamma_\phi}{1 - \mathcal{T}}. \tag{H.3}\]

Even though this potential is weak, it repels the transmission \( \mathcal{T} \) from the perfect \( \mathcal{T} = 1 \). Only at \( N_\phi \Gamma_\phi \ll 1 \) it is found that \( \rho(1) = \beta/2 \) at \( 1 - \mathcal{T} \approx 0.17 N_\phi \Gamma_\phi, \beta = 1 \) and \( 0.20 N_\phi \Gamma_\phi, \beta = 2 \). This result can be interpreted as an effective cut-off \( \mathcal{T}_c \) in transmissions due to finite dephasing, \( 1 - \mathcal{T}_c \approx N_\phi \Gamma_\phi = \gamma_\phi/\delta \).

New Journal of Physics 11 (2009) 113025 (http://www.njp.org/)
Figure H.1. The tail of the distribution $\rho(T)$ in a weakly dephased quantum dot, $N_\phi \Gamma_\phi \ll 1$. Distributions in a single-channel dot, $N = 2$, plotted for $\beta = 1$ (long-dashed) and $\beta = 2$ (solid) are exponentially suppressed at $1 - T \ll N_\phi \Gamma_\phi$ and return to $\rho \approx \beta/2$ at $1 - T \sim N_\phi \Gamma_\phi$. They should be contrasted with qualitatively plotted $\rho(T) \propto (T - T)^{1/2}$, $T \approx T_*$ (short-dashed) for a multichannel dot, $N \gg 1$.

Can we expect that in the multichannel limit, $N \gg 1$, the density $\rho(T \to 1)$ is enhanced in dephased quantum dots? This question is natural from comparison between $\rho(1) = \beta/2$, $N = 2$ and $\rho(T) \propto 1/\sqrt{1 - T} \to \infty$, $N \gg 1$ for $T \to 1$ in coherent dots. It turns out to be more convenient to consider not $T \in [0, 1]$, but $\lambda \gg 0$, such that $T = 1/(\lambda + 1)$. Following our result for $N = 2$, we assume that some potential $W(\lambda)$ induced by dephasing acts on each eigenvalue $\lambda$. The exact form of this potential and its dependence on dephasing strength are unknown. Dephasing potential $W(\lambda)$ should be distinguished from a many-body potential $W_0(\lambda)$ and repulsion $u(\lambda, \lambda')$ from another eigenvalue $\lambda'$. The former, $W_0 = (N/2 \ln(\lambda + 1)$, weakly repels $\lambda$ from $\infty$ and appears due to smaller phase space available to large $\lambda$ in the limit $N \gg 1$ (we neglect with corrections $O(1)$ to $W_0(\lambda)$, leading, e.g. to weak localization correction). The latter, the repulsion $u(\lambda, \lambda') = -\ln|\lambda - \lambda'|$ between the eigenvalues $\lambda, \lambda'$, origins in the chaotic dynamics in the sample, and we further assume for simplicity that it maintains its universal form [25] (see discussion below).

If $W(\lambda) + W_0(\lambda)$ and/or asymmetry in the contacts, $N_1 \neq N_2$, lead to cut-off values $\lambda = a, b$ such that $\rho(\lambda) = 0, \lambda = a, b$, we find [40]

$$\rho(\lambda) = \frac{1}{\pi \sqrt{(\lambda - a)(b - \lambda)}} \left( \frac{N \pi}{2} \sqrt{(a + 1)(b + 1)} + \pi z - \int_a^b d\lambda' \frac{dW(\lambda')}{d\lambda'} \sqrt{(\lambda' - a)(b - \lambda')} \right),$$

(H.4)

where $z = N_{\min} - N/2 \leq 0$ is defined by the contact asymmetry. The distribution $\rho(\lambda)$ depends on the exact shape of $W(\lambda)$. Integration of (H.4) over $\lambda$ satisfies the normalization condition, $\int d\lambda \rho(\lambda) = N_{\min}$, since the $W(\lambda')$ contribution is canceled after integration over $\lambda$. When $W = 0$, the boundary is given by $\sqrt{b_0 + 1} = -N/2z$, and the minimal possible value of $\lambda, a_0 = 0$. For symmetric dots, $z = 0$, one reproduces bimodal distribution $\rho(T) = N/(2\sqrt{T(1 - T)})$.

However, if the potential $W(\lambda)$ is unknown, the resulting distribution $\rho(\lambda)$ must reproduce results obtained in the uniform dephasing model, Var $\mathcal{G}$ in particular [23]. The transmission
repulsion of coherent dots allows us to add another equation relating \( \text{Var} \, \mathcal{G} \) and \( a, b \):

\[
\frac{\text{Var} \, \mathcal{G}}{(2e^2/h)^2} = (T_+ - T_-)^2 = \left( \frac{1}{a + 1} - \frac{1}{b + 1} \right)^2 = \left( \frac{N_1 N_2}{N(N + N\varphi\Gamma\varphi)} \right)^2.
\]

Together with boundary conditions on \( \rho(\lambda) \) the total system of equations reads

\[
\left\{ \begin{array}{l}
\int_a^b \frac{d\lambda}{\sqrt{(\lambda - a)(b - \lambda)}} \frac{dW(\lambda)}{d\lambda} = -\frac{\pi N}{2\sqrt{(1+a)(1+b)}}, \\
\int_a^b \frac{d\lambda(\lambda + 1)}{\sqrt{(\lambda - a)(b - \lambda)}} \frac{dW(\lambda)}{d\lambda} = \pi z, \\
1 - \frac{1}{1 + a} - \frac{1}{1 + b} = \frac{N^2 - 4z^2}{N(N + N\varphi\Gamma\varphi)}.
\end{array} \right.
\]

If the potential only repels \( \lambda \) from 0, the second and third equations in (H.6) for symmetric dots, \( z = 0 \), give \( a = N\varphi\Gamma\varphi / N, b = \infty \). If we additionally assume that \( W(\lambda) \propto N\varphi\Gamma\varphi \), the first equation in (H.6) results in \( W(\lambda) = N\varphi\Gamma\varphi \arctan(1/\sqrt{\lambda})/\sqrt{\lambda} \). However, a different assumption about functional dependence of \( W(\lambda) \) on \( N\varphi\Gamma\varphi \) leads to different dependence on \( \lambda \). Therefore, additional information about \( W(\lambda) \) is needed.

On the other hand, we could assume that the potential \( W(\lambda) \) maintains the functional form given by (H.3), \( W(\lambda) = (N\varphi\Gamma\varphi / 2)(1 + 1/\lambda) \). This potential only repels \( \lambda \) from the perfect transmission, \( \lambda = 0 \), and for a symmetric dot, \( z = 0 \), the second equation in (H.6) gives \( b \to \infty \). First equation in (H.6) allows us to express \( a \) as a function of dephasing strength and find the total distribution:

\[
\rho(\lambda) = \frac{N\sqrt{\lambda - a}}{2}\left[ (1 + 2a)\lambda + 2a \right],
\]

\[
a^3 = \frac{\left( \frac{N\varphi\Gamma\varphi}{2N} \right)^2}{a + 1}.
\]

Instead of generic \( \rho(\mathcal{T}) \propto 1/\sqrt{1 - \mathcal{T}}, \mathcal{T} \to 1 \) the distribution behaves as \( \rho(\mathcal{T}) \propto (\mathcal{T}_\ast - \mathcal{T})^{1/2}, \mathcal{T} \approx \mathcal{T}_\ast \), see figure H.1. However, the cut-off value \( a \) in equation (H.8) does not satisfy the third equation in (H.6). In fact, equation (H.3) is only an asymptote \( \lambda \to 0 \) and the part which repels \( \lambda \) from \( \infty \) is absent. This part might become important for strong dephasing, see numerical results of [23]. The potential (H.3) leads to formation of the gap in transmissions, \( \rho(\mathcal{T}) = 0, \mathcal{T}_\ast < \mathcal{T} < 1 \), but it probably overestimates the size of this gap.

The system of equations (H.6) defines \( a, b \) through the unknown potential \( W(\lambda) \). The function \( W(\lambda) \) cannot be universal, since in the strong dephasing limit \( N\varphi\Gamma\varphi \gg N \) the last equation gives \( a \approx b \), but then the first two equations cannot be simultaneously satisfied. This shows that, in general, the phenomenological treatment of dephasing in form of a wisely chosen potential \( W(\lambda) \) is not complete. The repulsion between transmissions should itself change from its universal form \( -\ln|\lambda - \lambda'| \) as a result of dephasing and a more accurate treatment should be taken.

However, from the analytical results for \( N = 2 \) we expect that the potential \( W(\lambda) \) is the main effect of weak dephasing, \( N\varphi\Gamma\varphi \ll N \) and modification of transmission repulsion would affect the size of the gap only perturbatively with small parameter \( N\varphi\Gamma\varphi / N \ll 1 \). Without
finding the exact form of $W(\lambda)$, we find from the third equation in (H.6) perturbations to the coherent values of the boundaries $a_0$ and $b_0$. If the contact asymmetry affects only $\mathcal{T}_-$, we find

$$T_+ \approx \frac{N}{N + N_\phi \Gamma_\phi}, \quad T_- \approx \frac{4\varepsilon^2}{N(N + N_\phi \Gamma_\phi)}.$$ (H.9)

Comparing equations (H.8) and (H.9) we conclude that the potential equation (H.3) overestimates the dephasing effect and gives too large a value of $a$ for $N_\phi \Gamma_\phi \ll N$. Close to the cut-off $\rho(\lambda) \propto \sqrt{\lambda - a}$, $\lambda \ll a$ independently of the exact form of $W(\lambda)$ and we conclude that for $\rho(T) \approx \sqrt{T_+ - T}$, $T \approx T_+$ effects of dephasing and asymmetric contacts are similar.

Even though $\rho(\lambda)$ does not behave as the coherent distribution $\rho_0(\lambda) = N/(2\pi \sqrt{\lambda})$, it should converge to $\rho_0(\lambda)$ as $a \to 0$. While its exact form depends on $W(\lambda)$, we can factorize it using the unknown functions $f$ and $g$, which are smooth on a small scale $<< 1$, as $\rho(\lambda) = (N/2\pi)\sqrt{\lambda - a} f(a/\lambda) g(\lambda)$. Keeping $\lambda/a \gg 1$ fixed, from the asymptotic $\rho_0(\lambda)$ we find $g(\lambda) = 1/\lambda$ and $f(0) = 1$. For $a \neq 0$ the distributions equalize, $\rho(\lambda) = \rho_0(\lambda)$, when $\sqrt{T - a/\lambda} f(a/\lambda) = 1$. Obviously, the second solution to this equation should exist at $a/\lambda < 1$ (for example, for $\rho(\lambda)$ given by equation (H.7) the root is $a/\lambda = 3\sqrt{2}/ \lambda$) and as $a \to 0$, this solution moves, $\lambda \to 0$. In $\mathcal{T}$-variables not a divergent $\rho_0(\mathcal{T}) \propto 1/\sqrt{1 - \mathcal{T}}$, but rather a peak of $\rho(\mathcal{T})$ is then expected, $\rho(\mathcal{T}) \sim 1/\sqrt{\gamma_\phi/E_{Th}}, 1 - \mathcal{T} \sim \gamma_\phi/E_{Th}$ and the bulk of $\rho(\mathcal{T})$ is only slightly perturbed compared to $\rho_0(\mathcal{T})$. Schematically, this distribution is presented in the inset to figure 2.

The important result (H.9) shows that the dephasing-induced gap for transmissions close to $\mathcal{T} = 1$ survives the limit $N \gg 1$, $1 - \mathcal{T}_+ \approx \gamma_\phi/2\pi E_{Th} = \tau_d/\tau_\phi$. The shape of $\rho(\mathcal{T}) \propto (\mathcal{T}_+ - \mathcal{T})^{1/2}, \mathcal{T}_+ \approx \mathcal{T}_-$ for $N \gg 1$ should be contrasted with $\rho(\mathcal{T}) \propto \exp[-\beta N_\phi \Gamma_\phi/2(1 - \mathcal{T})]/\mathcal{T}_+, \mathcal{T}_+ \approx \mathcal{T}_-$ for $N = 2$. We conclude that lowering temperature $\mathcal{T}$, $\gamma_\phi$ diminishes and thus opens conducting channels. In a normal system the widening of $\rho(\mathcal{T})$ and the growth in $\text{Var} \mathcal{G}$ usually come together. In some models, like the Fabry–Perot interferometer with $\mathcal{T}_\pm$ given in equation (G.3), or a weakly dephased quantum dot with $\mathcal{T}_\pm$ presented in equation (H.9), $\text{Var} \mathcal{G}$ is defined by $(\mathcal{T}_+ - \mathcal{T}_-)^2$ only, see equation (H.5). Even though the connection of $\text{Var} \mathcal{G}$ to $\mathcal{T}_\pm$ is generally much more complicated [21], we suggest that a wider distribution is a signature of enhanced $\text{Var} \mathcal{G}$. Since the normal current is only proportional to $\mathcal{T}$, $\mathcal{T}(\mathcal{T}) \propto V \mathcal{T}$, this enhancement is hardly noticeable in a dephased dot at small $\gamma_\phi/E_{Th} \ll 1$, see equation (H.5). However, the SNS current is exponentially dominated by perfect channels, $\mathcal{I}(\mathcal{T}) \propto \exp(-\pi \Delta(1 - \mathcal{T})/eV)$, and a weak dephasing $\gamma_\phi/E_{Th} \ll 1$ remains important for SNS transport fluctuations. In a realistic model of our SNS experiment both dephasing and imperfect contacts should be taken into account.

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