WHAT IS THE EVANS-VIGIER FIELD?

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Abstract. We explain connections of the Evans-Vigier model with theories proposed previously. The Comay’s criticism is proved to be irrelevant.

The content of the present talk is the following:

− Evans-Vigier definitions of the $B^{(3)}$ field [1];
− Lorentz transformation properties of the $B^{(3)}$ field and the B-Cyclic Theorem[2, 3];
− Clarifications of the Ogievetskiĭ-Polubarinov, Hayashi and Kalb-Ramond papers [4, 5, 6];
− Connections between various formulations of massive/massless $J = 1$ field;
− Conclusions of relativistic covariance and relevance of the Evans-Vigier postulates.

In 1994-2000 I presented a set of papers [7] devoted to clarifications of the Weinberg (and Weinberg-like [8, 9]) theories and the concept of Ogievetskiĭ-Polubarinov notoph. In 1995-96 I received numerous e-mail communications from Dr. M. Evans, who promoted a new concept of the longitudinal phaseless magnetic field associated with plane waves, the $B^{(3)}$ field (which is later obtained the name of M. Evans and J.-P. Vigier). Reasons for continuing the discussion during 2-3 years were: 1) the problem of massless limits of all relativistic equations does indeed exist; 2) the dynamical Maxwell equations have indeed additional solutions with energy $\mathcal{E} = 0$ (apart of those
with $\mathcal{E} = \pm |\kappa|$, see [10, 11, 12, 13, 14];

1) the $\mathbf{B}^{(3)}$ concept met strong non-positive criticism (e. g., ref [15, 16, 17]) and the situation became even more controversial in the last years (partially, due to the Evans' illness).

What are misunderstandings of both the authors of the $\mathbf{B}^{(3)}$ model and their critics? In *Enigmatic Photon* (1994), ref. [1], the following definitions of the longitudinal Evans-Vigier $\mathbf{B}^{(3)}$ field have been given:

\[ \mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = iB^{(0)} \mathbf{B}^{(3)*} \]  
\[ \mathbf{B}^{(3)} = \mathbf{B}^{(3)*} = -\frac{ik^2}{B^{(0)}} \mathbf{A}^{(1)} \times \mathbf{A}^{(2)} \]

Definition 3. [p.16, formula (41)]

\[ \mathbf{B}^{(3)} = B^{(0)} \hat{k} \]

The following notation was used: $\kappa$ is the wave number; $\phi = \omega t - \kappa \cdot \mathbf{r}$ is the phase; $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$ are usual transverse modes of the magnetic field; $\mathbf{A}^{(1)}$ and $\mathbf{A}^{(2)}$ are usual transverse modes of the vector potential.

The main experimental prediction of Evans [1a,b] that the magnetization induced during light-matter interaction (for instance, in the IFE)

\[ \mathcal{M} = \alpha I^{1/2} + \beta I + \gamma I^{3/2} \]

where $I = \frac{1}{2} \epsilon_0 E_0^2$, $E_0 = c|\mathbf{B}_\pi|$ has not been confirmed by the North Caroline group [18]. As one can see from Figure 4 of [18] “the behaviour of the experimental curve does not match with Evans calculations”.

Nevertheless, let us try to deepen understanding of the theoretical content of the Evans-Vigier model. In their papers and books [1] Evans and

\[ \nabla \times [\mathbf{E} - i\mathbf{B}] + i(\partial/\partial t)[\mathbf{E} - i\mathbf{B}] = 0, \]
\[ \nabla \times [\mathbf{E} + i\mathbf{B}] - i(\partial/\partial t)[\mathbf{E} + i\mathbf{B}] = 0. \]

we come to $\nabla \times \mathbf{E} = 0$ and $\nabla \times \mathbf{B} = 0$, i. e. to the conditions of longitudinality. The method of deriving this conclusion has been given in [19].

1 If we put energy to be equal to zero in the dynamical Maxwell equations

2 I apologize for not citing all numerous papers of Evans et al and papers of their critics due to page restrictions on the papers of this volume.
Vigier used the following definition for the transverse antisymmetric tensor field:

\[
\begin{pmatrix}
B_{\perp} \\
E_{\perp}
\end{pmatrix} = \begin{pmatrix}
\frac{B^{(0)}}{\sqrt{2}} \\
\frac{E^{(0)}}{\sqrt{2}}
\end{pmatrix} + \begin{pmatrix}
\frac{B^{(0)}}{\sqrt{2}} \\
\frac{E^{(0)}}{\sqrt{2}}
\end{pmatrix} e^{+i\phi} + \begin{pmatrix}
\frac{B^{(0)}}{\sqrt{2}} \\
\frac{E^{(0)}}{\sqrt{2}}
\end{pmatrix} e^{-i\phi},
\]

If \(B^{(0)} = E^{(0)}\) this formula describes the right-polarized radiation. Of course, a similar formula can be written for the left-polarized radiation. These transverse solutions can be rewritten to the real fields. For instance, Comay presented them in the following way [16c] in the reference frame \(\Sigma\):

\[
\begin{align*}
E_{\perp} &= \cos[\omega(z - t)]\hat{i} - \sin[\omega(z - t)]\hat{j}, \\
B_{\perp} &= \sin[\omega(z - t)]\hat{i} + \cos[\omega(z - t)]\hat{j},
\end{align*}
\]

and analyzed the addition of \(B_{||} = \sqrt{2}\mathbf{k}\) to (9). Making boost to other frame of reference \(\Sigma'\) he claimed that a) \(B^{(3)}\) is not parallel to the Poynting vector; b) with the Evans postulates \(E^{(3)}\) has a real part; c) transverse fields change, whereas \(B^{(3)}\) is left unchanged when the boost is done to the frame moving in the \(z\) direction. Comay concludes that these observations disprove the Evans claims on these particular questions. Furthermore, he claimed that the \(B^{(3)}\) model is inconsistent with the Relativity Theory.

According to [20, Eq.(11.149)] the Lorentz transformation rules for electric and magnetic fields are the following:

\[
\begin{align*}
E' &= \gamma(E + c\beta \times B) - \frac{\gamma^2}{\gamma + 1} \beta (\beta \cdot E), \\
B' &= \gamma(B - \beta \times E/c) - \frac{\gamma^2}{\gamma + 1} \beta (\beta \cdot B),
\end{align*}
\]

where \(\beta = \mathbf{v}/c\), \(\beta = |\beta| = \tanh \phi\), \(\gamma = \frac{1}{\sqrt{1 - \beta^2}} = \cosh \phi\), with \(\phi\) being the parameter of the Lorentz boost. We shall further use the natural unit system \(\hbar = 1\). After introducing the spin matrices \((S')_{jk} = -ie^{ijk}\) and deriving relevant relations:

\[
(S \cdot \beta)_{jk} a^k \equiv i[\beta \times a]^j,
\]

\[
\beta^j \beta^k \equiv [\beta^2 11 - (S \cdot \beta)^2]_{jk},
\]
one can rewrite Eqs. (10,11) to the form

\[
\mathbf{E}^{ij'} = \left( \frac{\gamma^2}{\gamma + 1} \left[ (\mathbf{S} \cdot \beta)^2 - \beta^2 \right] \right)_{ij} \mathbf{E}^j - i\gamma (\mathbf{S} \cdot \beta)_{ij} \mathbf{B}^j, \quad (12)
\]

\[
\mathbf{B}^{ij'} = \left( \frac{\gamma^2}{\gamma + 1} \left[ (\mathbf{S} \cdot \beta)^2 - \beta^2 \right] \right)_{ij} \mathbf{B}^j + i\gamma (\mathbf{S} \cdot \beta)_{ij} \mathbf{E}^j. \quad (13)
\]

Pure Lorentz transformations (without inversions) do not change signs of the phase of the field functions, so we should consider separately properties of the set of \( \mathbf{B}^{(1)} \) and \( \mathbf{E}^{(1)} \), which can be regarded as the negative-energy solutions in QFT and of another set of \( \mathbf{B}^{(2)} \) and \( \mathbf{E}^{(2)} \), the positive-energy solutions. Thus, in this framework one can deduce from Eqs. (12,13)

\[
\mathbf{B}^{(1)'}_{ij} = \left( 1 + \frac{\gamma^2}{\gamma + 1} (\mathbf{S} \cdot \beta)^2 \right)_{ij} \mathbf{B}^{(1)}_{ij} + i\gamma (\mathbf{S} \cdot \beta)_{ij} \mathbf{E}^{(1)}_{ij}, \quad (14)
\]

\[
\mathbf{B}^{(2)'}_{ij} = \left( 1 + \frac{\gamma^2}{\gamma + 1} (\mathbf{S} \cdot \beta)^2 \right)_{ij} \mathbf{B}^{(2)}_{ij} + i\gamma (\mathbf{S} \cdot \beta)_{ij} \mathbf{E}^{(2)}_{ij}, \quad (15)
\]

\[
\mathbf{E}^{(1)'}_{ij} = \left( 1 + \frac{\gamma^2}{\gamma + 1} (\mathbf{S} \cdot \beta)^2 \right)_{ij} \mathbf{E}^{(1)}_{ij} - i\gamma (\mathbf{S} \cdot \beta)_{ij} \mathbf{B}^{(1)}_{ij}, \quad (16)
\]

\[
\mathbf{E}^{(2)'}_{ij} = \left( 1 + \frac{\gamma^2}{\gamma + 1} (\mathbf{S} \cdot \beta)^2 \right)_{ij} \mathbf{E}^{(2)}_{ij} - i\gamma (\mathbf{S} \cdot \beta)_{ij} \mathbf{B}^{(2)}_{ij}, \quad (17)
\]

and

\[
\mathbf{B}^{(1)'}_{ij} = \left( 1 + \gamma (\mathbf{S} \cdot \beta) + \frac{\gamma^2}{\gamma + 1} (\mathbf{S} \cdot \beta)^2 \right)_{ij} \mathbf{B}^{(1)}_{ij}, \quad (18)
\]

\[
\mathbf{B}^{(2)'}_{ij} = \left( 1 - \gamma (\mathbf{S} \cdot \beta) + \frac{\gamma^2}{\gamma + 1} (\mathbf{S} \cdot \beta)^2 \right)_{ij} \mathbf{B}^{(2)}_{ij}, \quad (19)
\]

\[
\mathbf{E}^{(1)'}_{ij} = \left( 1 + \gamma (\mathbf{S} \cdot \beta) + \frac{\gamma^2}{\gamma + 1} (\mathbf{S} \cdot \beta)^2 \right)_{ij} \mathbf{E}^{(1)}_{ij}, \quad (20)
\]

\[
\mathbf{E}^{(2)'}_{ij} = \left( 1 - \gamma (\mathbf{S} \cdot \beta) + \frac{\gamma^2}{\gamma + 1} (\mathbf{S} \cdot \beta)^2 \right)_{ij} \mathbf{E}^{(2)}_{ij}, \quad (21)
\]

(when the definitions (7) are used). To find the transformed 3-vector \( \mathbf{B}^{(3)'} \) is just an algebraic exercise. Here it is

\[
\mathbf{B}^{(1)'} \times \mathbf{B}^{(2)'} = \mathbf{E}^{(1)'} \times \mathbf{E}^{(2)'} = i\gamma (B^{(0)})^2 (1 - \beta \cdot \hat{k}) \left[ \hat{k} - \gamma \beta + \frac{\gamma^2 (\beta \cdot \hat{k}) \beta}{\gamma + 1} \right]. \quad (22)
\]
We know that the longitudinal mode in the Evans-Vigier theory can be considered as obtained from Definition 3. Thus, considering that $B^{(0)}$ transforms as zero-component of a four-vector and $B^{(3)}$ as space components of a four-vector: [20, Eq.(11.19)]

$$B^{(0)'} = \gamma (B^{(0)} - \beta \cdot B^{(3)}), \quad (23)$$

$$B^{(3)'} = B^{(3)} + \frac{\gamma - 1}{\beta^2}(\beta \cdot B^{(3)}) \beta - \gamma \beta B^{(0)}, \quad (24)$$

we find from (22) that the relation between transverse and longitudinal modes preserves its form:

$$B^{(1)'} \times B^{(2)'} = iB^{(0)'}B^{(3)*'}, \quad (25)$$

that may be considered as a proof of the relativistic covariance of the $B^{(3)}$ model.

Moreover, we used that the phase factors in the formula (7) are fixed between the vector and axial-vector parts of the antisymmetric tensor field for both positive- and negative- frequency solutions if one wants to have pure real fields. Namely, $B^{(1)} = +iE^{(1)}$ and $B^{(2)} = -iE^{(2)}$. As we have just seen the $B^{(3)}$ field in this case may be regarded as a part of a 4-vector with respect to the pure Lorentz transformations. We are now going to take off the abovementioned requirement and to consider the general case:

$$\left( \begin{array}{c} B_{\perp} \\ E_{\perp} \end{array} \right)' = \Lambda \left\{ \left( \begin{array}{c} \tilde{B}^{(1)} \\ \tilde{E}^{(1)} \end{array} \right) e^{+i\phi} + \left( \begin{array}{c} \tilde{B}^{(2)} \\ \tilde{E}^{(2)} \end{array} \right) e^{-i\phi} \right\} = \Lambda \left\{ \left( \begin{array}{c} e^{i\alpha(x^\mu)} \tilde{B}^{(1)} \\ -e^{i\beta(x^\mu)} \tilde{B}^{(2)} \end{array} \right) e^{i\phi} + \left( \begin{array}{c} \tilde{B}^{(1)} \\ \tilde{B}^{(2)} \end{array} \right) e^{-i\phi} \right\}. \quad (26)$$

Our formula (26) can be re-written to the formulas generalizing (6a) and (6b) of ref. [2] (see also above (18,19)):

$$B^{(1)'} = \left( 1 + i e^{i\alpha} \gamma (S \cdot \beta) + \frac{\gamma^2}{\gamma + 1} (S \cdot \beta)^2 \right)_{ij} B^{(1)} \quad (27)$$

$$B^{(2)'} = \left( 1 - i e^{i\beta} \gamma (S \cdot \beta) + \frac{\gamma^2}{\gamma + 1} (S \cdot \beta)^2 \right)_{ij} B^{(2)} \quad (28)$$

One can then repeat the procedure of ref. [2] (see the short presentation above) and find out that the $B^{(3)}$ field may have various transformation laws when the transverse fields transform with the matrix $\Lambda$ which can be extracted from (12,13). Since the Evans-Vigier field is defined by the
formula (3) we again search the transformation law for the cross product of the transverse modes \[ [B^{(1)} \times B^{(2)}]' =? \] with taking into account (27,28).

\[
[B^{(1)} \times B^{(2)}]'^j = i\gamma B^{(0)} \left\{ \left[ 1 - \frac{e^{i\alpha} + e^{i\beta}}{2} (i\beta \cdot \hat{k}) \right] (1 + \frac{\gamma^2 (\beta^2 - (S \cdot \beta)^2)}{\gamma + 1})ij B^{(3)} + \\
+ \frac{i}{2} (S \cdot \beta)ij B^{(3)} - \gamma B^{(0)} \left[ \frac{e^{i\alpha} + e^{i\beta}}{2} + e^{i(\alpha + \beta)} (\beta \cdot \hat{k}) \right] \beta^j \right\}. 
\]

We used again the Definition 3 that \( B^{(3)} = B^{(0)} \hat{k} \).

One can see that we recover the formula (8) of ref. [2] (see (22) above) when the phase factors are equal to \( \alpha = -\pi/2, \beta = -\pi/2 \). In the case \( \alpha = +\pi/2 \) and \( \beta = +\pi/2 \), the sign of \( \beta \) is changed to the opposite one.\(^3\)

We are able to obtain the transformation law as for antisymmetric tensor field, for instance when \( \alpha = -\pi/2, \beta = +\pi/2 \).\(^4\) Namely, since under this choice of the phases

\[
B^{(1)'} \times B^{(2)'} = i\gamma \left[ B^{(0)} \right]^2 \left( \hat{k} - \frac{\gamma \beta (\beta \cdot \hat{k})}{\gamma + 1} + (i\beta_y - i\beta_x) \right),
\]

the formula (30) and the formula for opposite choice of phases lead precisely to the transformation laws of the antisymmetric tensor fields:

\[
[B^{(3)}']^j = \left( 1 \pm \gamma (S \cdot \beta) + \frac{\gamma^2 (S \cdot \beta)^2}{\gamma + 1} \right) ij B^{(3)}. 
\]

\( B^{(0)} \) is a true scalar in such a case.

What are reasons that we introduced additional phase factors in Helmholtz bivectors? In [21] a similar problem has been considered in the \((1/2,0) \oplus (0,1/2)\) (cf. also [7, 22]). Ahluwalia identified additional phase factor(s) with Higgs-like fields and proposed some relations with a gravitational potential. However, the \( E \) field under definitions \( \alpha = -\pi/2, \beta = +\pi/2 \) becomes to be pure imaginary. One can also propose a model with the corresponding introduction of phase factors in such a way that

\(^3\)By the way, in all his papers Evans used the choice of phase factors incompatible with the \( B \)-Cyclic Theorem in the sense that not all the components are entries of antisymmetric tensor fields therein. This is the main one but not the sole error of the Evans papers and books.

\(^4\)In the case \( \alpha = +\pi/2 \) and \( \beta = -\pi/2 \), the sign in the third term in parentheses (formula (30) is changed to the opposite one.
$B_\perp$ to be pure imaginary. Can these transverse fields be observable? Can the phase factors be observable? A question of experimental possibility of detection of this class of antisymmetric tensor fields (in fact, of the anti-hermitian modes on using the terminology of quantum optics) is still open. One should still note that several authors discussed recently unusual configurations of electromagnetic fields [23, 24].

Let us now look for relations with old formalisms. The equations (10) of [4] is read

$$f_{\mu\nu}(p) \sim [\epsilon_{\mu}^{(1)}(p)\epsilon_{\nu}^{(2)}(p) - \epsilon_{\nu}^{(1)}(p)\epsilon_{\mu}^{(2)}(p)]$$

(32)

for antisymmetric tensor $f_{\mu\nu}$ expressed through cross product of polarization vectors in the momentum space. This is a generalized case comparing with the Evans-Vigier Definition 2 which is obtained if one restricts oneself by space indices.

The dynamical equations in the Ogievetski˘ı-Polubarinov approach are

$$\Box f_{\mu\nu} - \partial_\mu \partial^\lambda f_{\lambda\nu} + \partial_\nu \partial^\lambda f_{\lambda\mu} = J_{\mu\nu},$$

(33)

and the new Kalb-Ramond gauge invariance is defined with respect to transformations

$$\delta f_{\mu\nu} = \partial_\mu \lambda_\nu - \partial_\nu \lambda_\mu.$$  

(34)

It was proven that the Ogievetski˘ı-Polubarinov equations are related to the Weinberg $2(2j+1)$ formalism [25, 26] and [7b-f,i].

Furthermore, they [4] also claimed "In the limit $m \to 0$ (or $v \to c$) the helicity becomes a relativistic invariant, and the concept of spin loses its meaning. The system of $2s + 1$ states is no longer irreducible; it decomposes and describes a set of different particles with zero mass and helicities $\pm s, \pm(s-1), \ldots, 0$ (for integer spin and if parity is conserved; the situation is analogous for half-integer spins)." In fact, this hints that actually the Proca-Duffin-Kemmer $j = 1$ theory has two massless limit, a) the well-known Maxwell theory and b) the notoph theory ($h = 0$). The notoph theory has been further developed by Hayashi [5] in the context of dilaton gravity, by Kalb and Ramond [6] in the string context. Hundreds (if not thousands) papers exist on the so-called Kalb-Ramond field (which is actually the notoph), including some speculations on its connection with Yang-Mills fields.

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Footnotes:

5Cf. with [27]. I am grateful to an anonymous referee of Physics Essays who suggested to look for possible connections. However, the work [27] does not cite the previous Ogievetski˘ı-Polubarinov statement.
In [28] I tried to use the Ogievetskiı-Polubarinov definitions of \( f_{\mu\nu} \) (see (32)) to construct the “potentials” \( f_{\mu\nu} \). We can obtained for a massive field

\[
f_{\mu\nu}(p) = \frac{iN^2}{m} \begin{pmatrix}
0 & -p_2 & 0 & 0 \\
p_2 & 0 & p_{\rho\rho} & \frac{p_{\rho\rho}}{p_{0+m}} \\
-p_1 & -m - \frac{p_{\rho\rho}}{p_{0+m}} & 0 & \frac{p_{\rho\rho}}{p_{0+m}} \\
0 & -\frac{p_{\rho\rho}}{p_{0+m}} & \frac{p_{\rho\rho}}{p_{0+m}} & 0
\end{pmatrix}
\]

This tensor coincides with the longitudinal components of the antisymmetric tensor obtained in refs. [9a, Eqs.(2.14,2.17)] (see also below and [7i, Eqs.(16b,17b)]) within normalizations and different forms of the spin basis. The longitudinal states reduce to zero in the massless case under appropriate choice of the normalization and only if a \( j = 1 \) particle moves along with the third axis \( OZ \). Finally, it is also useful to compare Eq. (35) with the formula (B2) in ref. [29] in order to realize the correct procedure for taking the massless limit.

Thus, the results (at least in a mathematical sense) surprisingly depend on a) the normalization; b) the choice of the frame of reference.

In the Lagrangian approach we have

\[
\mathcal{L}^{Proca} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu \Rightarrow \mathcal{L}^{Maxwell}(m \to 0)
\]

and

\[
\mathcal{L} = -\frac{1}{2} F_{\mu} F^{\mu} + \frac{m^2}{4} f_{\mu\nu} f^{\mu\nu} \Rightarrow \mathcal{L}^{Notoph} = -\frac{1}{2} F_{\mu} F^{\mu}(m \to 0)
\]

where

\[
F^\mu = \frac{i}{2} \epsilon^{\mu\nu\alpha\beta} \partial_\nu f_{\alpha\beta} = \partial_\beta \tilde{f}^{\mu\beta}
\]

(if one applies the duality relations). Thus, we observe that a) it is important to consider the parity matters (the dual tensor has different parity properties); b) we may look for connections with the dual electrodynamics [30].

The above surprising conclusions induced me to start form the basic group-theoretical postulates in order to understand the origins of the Ogievetskiı-Polubarinov-Evans-Vigier results. The set of Bargmann-Wigner equations, ref [31] for \( j = 1 \) is written, e.g., ref. [32]

\[
[i\gamma^\mu \partial_\mu - m]_{\alpha\beta} \Psi_{\beta}(x) = 0,
\]

\[
[i\gamma^\mu \partial_\mu - m]_{\gamma\beta} \Psi_{\alpha\beta}(x) = 0,
\]

\footnote{There is also another way of thinking: namely, to consider “unappropriate” normalization \( N = 1 \) and to remove divergent part (in \( m \to 0 \)) by a new gauge transformation.}
where one usually uses
\[ \Psi_{\{\alpha\beta\}} = m\gamma^\mu_\alpha R\delta_\beta A_\mu + \frac{1}{2}\sigma^{\mu\nu}_{\alpha\delta} R\delta_\beta F_{\mu\nu}, \] (41)

In order to facilitate an analysis of parity properties of the corresponding fields one should introduce also the term \( \sim (\gamma^5 \sigma^{\mu\nu} R)_{\alpha\beta} \tilde{f}_{\mu\nu} \). In order to understand normalization matters one should put arbitrary (dimensional, in general) coefficients in this expansion or in definitions of the fields and 4-potentials [28]. The \( R \) matrix is
\[ R = \begin{pmatrix} i\Theta & 0 \\ 0 & -i\Theta \end{pmatrix}, \quad \Theta = -i\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \] (42)

Matrices \( \gamma^\mu \) are chosen in the Weyl representation, i.e., \( \gamma^5 \) is assumed to be diagonal. The reflection operator \( R \) has the properties
\[ R^T = -R, \quad R^\dagger = R = R^{-1}, \] (43)
\[ R^{-1}\gamma^5 R = (\gamma^5)^T, \] (44)
\[ R^{-1}\gamma^\mu R = -\gamma^\mu, \] (45)
\[ R^{-1}\sigma^{\mu\nu} R = -\sigma^{\mu\nu}. \] (46)

They are necessary for the expansion (41) to be possible in such a form, i.e., in order the \( \gamma^\mu R, \sigma^{\mu\nu} R \) and (if considered) \( \gamma^5 \sigma^{\mu\nu} R \) to be symmetrical matrices.

I used the expansion which is similar to (41)
\[ \Psi_{\{\alpha\beta\}} = \gamma^\mu_\alpha R\delta_\beta F_\mu + \sigma^{\mu\nu}_{\alpha\delta} R\delta_\beta F_{\mu\nu}, \] (47)
and obtained
\[ \partial_\alpha F^{\alpha\mu} + \frac{m}{2} F^\mu = 0, \] (48)
\[ 2m F^{\mu\nu} = \partial_\mu F^{\nu} - \partial_\nu F^{\mu}. \] (49)

If one renormalizes \( F^\mu \to 2m A^\mu \) or \( F_{\mu\nu} \to \frac{1}{2m} F_{\mu\nu} \) one obtains “textbooks” Proca equations. But, physical contents of the massless limits of these equations may be different.

Let us track origins of this conclusion in detail. If one advocates the following definitions [33, p.209]
\[ \epsilon^\mu(0,+1) = -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix}, \quad \epsilon^\mu(0,0) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \epsilon^\mu(0,-1) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix}. \] (50)
and \((\vec{p} = p^i / |p|, \gamma = E_p/m)\), ref. [33, p.68] or ref. [34, p.108],

\[
\epsilon^\mu(p, h) = L^\mu \nu(p) \epsilon^\nu(0, h),
\]

\[
L^0_0(p) = \gamma, \quad L^i_0(p) = L^0_i(p) = \hat{p}_i \sqrt{\gamma^2 - 1},
\]

\[
L^i_k(p) = \delta_{ik} + (\gamma - 1) \hat{p}_i \hat{p}_k
\]

(51)

for the field operator of the 4-vector potential, ref. [34, p.109] or ref. [35, p.129]²,³

\[
A^\mu(x) = \sum_{h = 0, \pm 1} \int \frac{d^3p}{(2\pi)^3 2E_p} \left[ \epsilon^\mu(p, h)a(p, h)e^{-ip \cdot x} + (\epsilon^\mu(p, h))^* b^\dagger(p, h)e^{ip \cdot x} \right],
\]

(54)

the normalization of the wave functions in the momentum representation is thus chosen to the unit, \(\epsilon^\mu(p, h)\epsilon^\mu(p, h) = -1\).⁸ We observe that in the massless limit all defined polarization vectors of the momentum space do not have good behaviour; the functions describing spin-1 particles tend to infinity. This is not satisfactory, in my opinion, even though one can still claim that singularities may be removed by rotation and/or choice of a gauge parameter. After renormalizing the potentials, e.g., \(\epsilon^\mu \rightarrow u^\mu \equiv m \epsilon^\mu\) we come to the field functions in the momentum representation:

\[
u^\mu(p, +1) = -\frac{N}{\sqrt{2m}} \left( \begin{array}{c} p_r \\
\frac{p_1}{E_p + m} \\
\frac{p_2}{E_p + m} \\
\frac{p_3}{E_p + m} \end{array} \right), \quad \nu^\mu(p, -1) = \frac{N}{\sqrt{2m}} \left( \begin{array}{c} m + \frac{p_1}{E_p + m} \\
-im + \frac{p_2}{E_p + m} \\
-ip_3 \frac{p_2}{E_p + m} \end{array} \right),
\]

(55)

⁷Remember that the invariant integral measure over the Minkowski space for physical particles is

\[
\int d^4p \delta(p^2 - m^2) \equiv \int \frac{d^3p}{2E_p}, \quad E_p = \sqrt{p^2 + m^2}.
\]

Therefore, we use the field operator as in (54). The coefficient \((2\pi)^3\) can be considered at this stage as chosen for convenience. In ref. [33] the factor \(1/(2E_p)\) was absorbed in creation/annihilation operators and instead of the field operator (54) the operator was used in which the \(\epsilon^\mu(p, h)\) functions for a massive spin-1 particle were substituted by \(u^\mu(p, h) = (2E_p)^{-1/2} \epsilon^\mu(p, h)\), which may lead to confusions in searching massless limits \(m \rightarrow 0\) for classical polarization vectors.

³In general, it may be useful to consider front-form helicities (or “time-like” polarizations) too. But, we leave a presentation of a rigorous theory of this type for subsequent publications.

⁸The metric used in this paper \(g^{\mu\nu} = \text{diag}(1, -1, -1, -1)\) is different from that of ref. [33].
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\[ u^\mu(p, 0) = N \frac{p_3}{m} \begin{pmatrix} \frac{p_3}{E_p + m} \\ \frac{p_1 p_3}{E_p^2 + m^2} \\ \frac{p_2 p_3}{E_p^2 + m^2} \\ m + \frac{p_3^2}{E_p} \end{pmatrix}, \]

\( (N = m \text{ and } p_{r,l} = p_1 \pm ip_2) \) which do not diverge in the massless limit.

Two of the massless functions (with \( h = \pm 1 \)) are equal to zero when the particle, described by this field, is moving along the third axis (\( p_1 = p_2 = 0, p_3 \neq 0 \)). The third one (\( h = 0 \)) is

\[ u^\mu(p_3, 0) \mid_{m \to 0} = \begin{pmatrix} p_3 \\ 0 \\ 0 \\ \frac{p_3^2}{E_p} \end{pmatrix} \equiv \begin{pmatrix} E_p \\ 0 \\ 0 \\ E_p \end{pmatrix}, \]

and at the rest (\( E_p = p_3 \to 0 \)) also vanishes. Thus, such a field operator describes the “longitudinal photons” which is in complete accordance with the Weinberg theorem \( B - A = h \) for massless particles (let us remind that we use the \( D(1/2, 1/2) \) representation). Thus, the change of the normalization can lead to the change of physical content described by the classical field (at least, comparing with the well-accepted one). Of course, in the quantum case one should somehow fix the form of commutation relations by some physical principles.\(^1\)

If one uses the dynamical relations on the basis of the consideration of polarization vectors one can find fields:

\[
B^{(+)}(p, +1) = -\frac{iN}{2\sqrt{2}m} \begin{pmatrix} -ip_3 \\ p_3 \\ ip_r \\ 0 \end{pmatrix} = e^{-i\alpha_{-1}}B^{(-)}(p, -1),
\]

\[
B^{(+)}(p, 0) = \frac{iN}{2m} \begin{pmatrix} p_2 \\ -p_1 \\ 0 \end{pmatrix} = -e^{-i\alpha_0}B^{(-)}(p, 0),
\]

\[
B^{(+)}(p, -1) = \frac{iN}{2\sqrt{2}m} \begin{pmatrix} ip_3 \\ p_3 \\ -ip_l \\ 0 \end{pmatrix} = e^{-i\alpha_{+1}}B^{(-)}(p, +1),
\]

and

\[
E^{(+)}(p, +1) = -\frac{iN}{2\sqrt{2}m} \begin{pmatrix} E_p - \frac{p_1 p_r}{E_p + m} \\ iE_p - \frac{p_2 p_r}{E_p + m} \\ -\frac{p_3 p_r}{E_p + m} \end{pmatrix} = e^{-i\alpha_{-1}}E^{(-)}(p, -1).\]

\(^1\)I am very grateful to the anonymous referee of my previous papers (“Foundation of Physics”) who suggested to fix them by requirements of the dimensionless nature of the action (apart from the requirements of the translational and rotational invariances).
\[ E^{(+)}(p, 0) = \frac{iN}{2m} \begin{pmatrix} -\frac{p_3 p_1}{E_p + m} \\ \frac{p_2 p_3}{E_p + m} \\ -p_2 \end{pmatrix} = -e^{-i\alpha'_0} E^{(-)}(p, 0), \] (62)

\[ E^{(+)}(p, -1) = \frac{iN}{2\sqrt{2}m} \begin{pmatrix} E_p - \frac{p_2 p_1}{E_p + m} \\ -iE_p - \frac{p_2 p_3}{E_p + m} \\ -p_2 \end{pmatrix} = +e^{-i\alpha'_{+1}} E^{(-)}(p, +1), \] (63)

where we denoted, as previously, a normalization factor appearing in the definitions of the potentials (and/or in the definitions of the physical fields through potentials) as \( N \). \( E^{(+)}(p, 0) \) and \( B^{(+)}(p, 0) \) coincide with the strengths obtained before by different method [9a,28], see also (35). \( B^{\pm}(p, 0) = E^{\pm}(p, 0) = 0 \) identically. So, we again see a third component of antisymmetric tensor fields in the massless limit which is dependent on the normalization and rotation of the frame of reference.

However, the claim of the pure “longitudinal nature” of the antisymmetric tensor field and/or “Kalb-Ramond” fields after quantization still requires further explanations. As one can see in [5] for a theory with \( L = -\frac{1}{8} F_{\mu} F^{\mu} \) the application of the condition \( (A_{ij}^{(+)}(x,j))|\Psi >= 0 \) (in our notation \( \partial_{\mu} f^{\mu\nu} = 0 \)), see the formula (18a) therein, leads to the above conclusion. Transverse modes are eliminated by a new “gauge” transformations. Indeed, the expanded lagrangian is

\[ L^H = \frac{1}{4} (\partial_{\mu} f^{\mu\nu}) (\partial_{\nu} f^{\mu\alpha}) - \frac{1}{2} (\partial_{\mu} f^{\mu\alpha}) (\partial_{\nu} f^{\mu\alpha}) = -\frac{1}{4} L^{(2j+1)} + \frac{1}{2} (\partial_{\mu} f^{\mu\nu}) (\partial_{\nu} f^{\alpha\nu}). \] (64)

Thus, the Ogievetski˘ı-Polubarinov-Hayashi Lagrangian is equivalent to the Weinberg’s Lagrangian of the \( 2(2j+1) \) theory [36] and [7a-e], which is constructed as a generalization of the Dirac Lagrangian for spin 1 (instead of bispinors it contains bivectors). In order to consider a massive theory (we insist on making the massless limit in the end of calculations, for physical quantities) one should add \( +\frac{1}{4} m^2 f_{\mu\nu} f^{\mu\nu} \) as in (37).

The spin operator of the massive theory, which can be found on the basis of the Noether formalism, is

\[ J^k = \frac{1}{2} \epsilon^{ijk} f^{ij} = \] (65)

\[ = \epsilon^{ijk} \int d^3x \left[ f^{0i} (\partial_{\mu} f^{mn}) + f^{mj} (\partial^0 f^{\mu i} + \partial^{\mu} f^{0i} + \partial^{i} f^{0\mu}) \right] = \frac{m}{2} \int d^3x \tilde{E} \times \tilde{A}. \]

The formal difference in Lagrangians does not lead to physical difference. Hayashi said that this is due to the possiblity of applying the Fermi method *mutatis mutandis*. 
In the above equations we applied dynamical equations as usual. Thus, it becomes obvious, why previous authors claimed the pure longitudinal nature of massless antisymmetric tensor field after quantization, and why the application of the generalized Lorentz condition leads to equating the spin operator to zero.\(^3\) But, one should take into account the normalization issues. An additional mass factor in the denominator may appear a) after “re-normalization” \(\mathcal{L} \rightarrow \mathcal{L}/m^2\) (if we want to describe long-range forces an antisymmetric tensor field must have dimension \([\text{energy}]^2\) in the \(c = \hbar = 1\) unit system, and potentials, \([\text{energy}]^1\) in order the corresponding action would be dimensionless; b) due to appropriate change of the commutation relations for creation/annihilation operators of the higher-spin fields (including \(\sim 1/m\)); c) due to divergent terms in \(E, B, A\) in \(m \rightarrow 0\) under certain choice of \(N\). Thus, one can recover usual quantum electrodynamics even if we use fields (not potentials) as dynamical variables.

The conclusions are:

- While first experimental verifications gave negative results, the \(B^{(3)}\) construct is theoretically possible, if one develops it in a mathematically correct way;
- The \(B^{(3)}\) model is a relativistic covariant model. It is compatible with the Relativity Theory. The \(B^{(3)}\) field may be a part of the 4-potential vector, or (if we change connections between parts of Helmholtz bivector) may be even a part of antisymmetric tensor field;
- The \(B^{(3)}\) model is based on definitions which are particular cases of the previous considerations of Ogievetskii and Polubarinov, Hayashi and Kalb and Ramond;
- The Duffin-Kemmer-Proca theory has two massless limits that seems to be in contradictions with the Weinberg theorem \((B - A = \hbar)\);
- Antisymmetric tensor fields after quantization may describe particles of both helicity \(h = 0\) and \(h = \pm 1\) in the massless limit. Surprisingly, the physical content depends on the normalization issues and on the choice of the frame of reference (in fact, on rotations).

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\(^3\)It is still interesting to note that division of total angular momentum into orbital part and spin part is not gauge invariant.
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