Experience in using a synthetic turbulence generator for eddy-resolving simulation of the free convection boundary layer on a vertical plate

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Abstract. The paper presents results of eddy-resolving numerical simulation of the turbulent free convection boundary layer that develops along a vertical isothermally heated plate. The problem of definition of boundary conditions at the computational domain inlet, where a time-varying flow field has to be specified, is considered. Mean characteristics of inflow are defined from results of RANS simulation of a 2D boundary layer, and a synthetic turbulence generator is used to introduce unsteady fluctuations at the inlet. Calculations are performed with an in-house unstructured finite-volume code; results of comparison with known experimental data are presented. The influence of the choice of the turbulence model used for RANS-based calculations of the mean inlet flow profiles is considered.

1. Introduction
Turbulent transfer in free convection boundary layers developing on heated vertical or inclined surfaces are of apparent interest in many practical applications. Numerical simulation of such phenomena is one of the powerful tools that allow obtaining detailed information about flow structure, as well as for heat transfer characteristics. It is known that the RANS approach tends to underestimate the heat transfer rate in the turbulent free boundary layer [1]. Thus, the most preferable way to get reliable numerical solutions for such cases is using the eddy-resolving approach, which shows a relatively good agreement with experiments (e.g. see [2]). However, such simulation techniques take a lot of computational resources, especially for accurate prediction of laminar-turbulent transition in the boundary layer on a vertical plate [3, 4]. When the only turbulent regime of free convection is under consideration, there is no need to simulate the flow over the whole plate; the most effective way is to impose proper inlet boundary conditions that correspond to a fully developed turbulent flow. With this approach, a time-varying flow field has to be specified for the resolved length scales. The imposed inflow fluctuations should emulate, as close as possible, the real turbulence peculiar to the problem being studied, since they affect the downstream flow dynamics. The simplest way for introducing unsteadiness at the inlet is to superimpose proper mean profiles of flow characteristics with a synthetic turbulent content. Mean flow characteristics can be obtained from a preliminary RANS calculation. Artificially generated synthetic fluctuations have to endure a transition region (adaptation distance) in which they are molded into realistic turbulence by solving the full or filtered Navier–Stokes equations.
Such techniques are widely used in different applications dealing with forced convection flows, and employment of a synthetic turbulence generator allows significantly shortening the adaptation distance (see, for instance, [5] and references there). Contrary to that, there is insufficient published information concerning usage of such techniques for the simulation of turbulent free convection shear flows.

The aim of the present study is (i) to adjust the methodology of generating inflow boundary conditions with a synthetic turbulence generator for turbulent free convection boundary layer simulation based on the Implicit LES (ILES) approach; (ii) to perform computations of the turbulent free convection boundary layer on a vertical heated plate, and to evaluate the adaptation distance; (iii) to compare ILES results obtained with inlet profiles of mean flow characteristics from RANS-based calculations using different turbulence models.

2. Mathematical model and numerical aspects

2.1. Description of the case

The present numerical study has been carried out for the physical conditions that correspond to the known experiments by Tsuji&Nagano [6] dealing with the air free convection boundary layer on a vertical heated isothermal plate. The plate surface temperature, $T_w$, is set to 60°C. An ambient fluid temperature, $T_a$, is set to 16°C.

The mathematical model is based on the three-dimensional incompressible-fluid Navier-Stokes and energy equations using the Boussinesq approximation for describing the buoyancy effects. Physical properties of air are assumed constant and evaluated at the mean temperature $T_f = (T_w + T_a)/2$, except for the thermal expansion coefficient $\beta$ that is evaluated at $T = T_a$. The Prandtl number, Pr, is equal to 0.7.

Figure 1a illustrates the used 3D rectangular computational domain. The vertical size of the heated plate (1) is $L = 2.56$ m. The width of the computational domain is 0.48 m. The external boundary of the domain, parallel to the plate, is at a distance of 0.4 m from the plate. “Synthetic” boundary conditions are imposed at the domain inlet (2), as described below. The no-slip condition is specified at the plate. For the outlet section (3), a generalized inflow/outflow (“pressure-outlet”) condition is used with normal-to-boundary inflow (if any). The “pressure-inlet” condition is applied at the external permeable boundary (4), also with normal-to-boundary inflow. The simulation is performed with periodicity boundary conditions specified at planes (5) and (6).

![Figure 1. Computational domain for (a) 3D and (b) 2D calculations.](image)

Let us introduce the local Rayleigh number based on a characteristic boundary layer thickness as

$$Ra_\delta = \frac{\beta(T_w - T_a)\delta^3}{\nu^2 \cdot Pr}.$$  

Here $g$ is the gravity acceleration, and $\nu$ is the fluid kinematic viscosity. The used local integral thickness of the layer, $\delta$, is defined as integral of the normalized mean streamwise velocity, $u/u_{max}$, across the boundary layer (over the $y$-direction). The integration is
performed up to \( y = \delta_T \), where \( \delta_T \) is the thermal boundary layer thickness associated with the point where the fluid temperature differs from the ambient temperature by 1% of \((T_w - T_a)\).

Profiles of mean velocity components, temperature and two turbulence parameters provided by a RANS solution for the 2D turbulent free-convection boundary layer (figure 1b) at Rayleigh number \( \text{Ra}_\delta = 1.3\times10^6 \) are specified at the inlet section (2). The RANS computations were performed with three turbulence models, known as \( k-\varepsilon \) RNG, \( k-\omega \) SST and \( k-\varepsilon \) Lien-Leschziner (L-L). Among these models, the last one, suggested in [7], gives the best results when applied to prediction of 2D free convection boundary layers [8]. Velocity and temperature fluctuations necessary for the inlet boundary condition for ILES simulation are generated applying the method described in Section 2.3.

2.2. Numerical aspects
To perform calculations, we use the in-house unstructured finite-volume code SINF/Flag-S, which is under development at the Department of Fluid Dynamics, Combustion and Heat Transfer of SPbPU. For time advancing, an original semi-implicit fractional step method based on second-order Crank-Nicolson scheme is used [9]. The QUICK scheme was applied for evaluation of convective fluxes, both for the momentum and energy equations. The time step was 0.001 seconds. Samples with duration of about 40 s have been calculated for getting flow statistics after a transient period.

The used computational grid consisted of about 18 millions of hexahedral cells. The grid points were clustered near the plate to provide the value of the average normalized distance from the center of the first computational cell to the wall, \( y^+ \), of about 0.35, and non-dimensional cell sizes in the \( x \)- and \( z \)-direction were \( \Delta x^+ \approx \Delta z^+ \approx 14 \).

2.3. Synthetic turbulence generator
Various synthetic turbulence generators are described in the literature [10-12]. They have received much attention during the last two decades, e.g. in reviews [5, 13]. The vast majority of generators are intended for the forced convection flow with free and/or near-wall turbulence. In this paper, we consider the generator proposed in [10, 11] and denoted below as GST. This generator can realistically reproduce the anisotropy of vortex structures, which is an essential feature of near-wall turbulence. This involves the superposition of harmonic functions with random amplitudes and phases to generate a velocity field with preset turbulent length and time scales and a required energy spectrum. The original version of the generator provides velocity fluctuations only. Using the approach [14], we also introduced the temperature fluctuations based on the assumption that RMS value of temperature fluctuations is proportional to the square root of the turbulent kinetic energy \( k \) (this modification of the original generator is denoted below as GST/T). Both variants (GST and GST/T) have been implemented into SINF/Flag-S.

For the modified version of the generator [11], velocity and temperature fluctuations are evaluated using the superposition of amplitude-modulated Fourier modes, where amplitude modulation is achieved through the modified dimensionless spatial von Karman energy spectrum of the turbulence kinetic energy (see [11] for details). Profiles of \( k \) and the specific dissipation \( \omega \) should be provided also as input data for the generator. An additional parameter for the generator is the integral time scale \( \tau = a\delta/\overset{\text{max}}{u} \), where \( \overset{\text{max}}{u} \) is the maximal value in the RANS velocity profile and \( \delta \) is the (defined above) integral thickness of the boundary layer at the computational domain inlet section; constant \( a \) is set equal to 2.

3. Results and discussion
As an example, figure 2 shows results obtained in case of using the \( k-\varepsilon \) RNG model for getting inflow profiles of mean characteristics and the GST/T generator. Instantaneous \( x \)-velocity and temperature fields, and maps of root mean square (RMS) values of \( x \)-velocity and temperature fluctuations are presented (both the time- and the span-averaging procedures were applied). One can see that the whole boundary layer thickness increases downstream with an increase in RMS values of velocity and temperature fluctuations. It is also evident that for further calculations the distance between the wall...
and the opposite computation domain boundary should be increased since in the presented case the turbulence vortices almost reach the boundary closer to the outlet.

A comparison of results obtained with the GST and the GST/T has shown that the only significant difference is in the level of temperature fluctuations at a relatively short section near the inlet boundary.

Figure 2. Flow fields: (a) instantaneous velocity $x$-component, (b) RMS of $x$-velocity fluctuations, (c) instantaneous temperature and (d) RMS of temperature fluctuations.

To illustrate the streamwise transformation of the mean flow, time- and span-averaged profiles of velocity and temperature at two sections are presented in figure 3 in conjunction with the measurement data [6]. Here $\theta = (T - T_a)/(T_w - T_a)$ is dimensionless temperature, and $\zeta = -y(\partial \theta/\partial y)_{y=0}$ is dimensionless normal coordinate. The first section is positioned at $x = 0.73$ m and corresponds to $Gr_{\delta} = 2.3 \times 10^6$, the second section, $x = 1.71$ m, corresponds to $Gr_{\delta} = 7.0 \times 10^6$. The two given values of $Gr_{\delta}$ were evaluated from the mean velocity profiles presented in [6] for the positions of $Gr_{x^*} = 3.624 \times 10^{10}$ and $Gr_{x^*} = 8.441 \times 10^{10}$ (here the coordinate $x^*$ is counted from the experimental plate leading edge). A good agreement between the predicted and the experimental profiles can be observed for the second section, whereas the first-section profiles differ noticeably from the experimental data, especially in case of temperature profiles.

For the second section, figure 4 presents a comparison of RMS values of velocity and temperature fluctuations, obtained with different turbulent models, with experimental data [6]. It is seen that the temperature fluctuation profile in this section is weakly sensitive to variations of the inlet RANS-generated profiles, whereas the RMS values of velocity fluctuations are more sensitive. Notably, a considerable underestimation of peak RMS values of velocity fluctuations is observed in case of the inlet profiles obtained with the $k-\omega$ SST model, while results obtained with two $k-\varepsilon$ models are almost identical and much closer to the experimental data.
To estimate the distance needed for adaptation of the artificially generated inlet turbulent flow to a natural state, computed streamwise distributions of the time- and span-averaged wall heat flux, \( q \), were analysed. Figure 5a shows distributions obtained for three variants of the inlet profiles. One can see that in all the cases the considered behaviour of the calculated curves in the flow adaptation region is similar: the heat flux firstly decreases to a minimum value, and then grows again, up to achieving a nearly constant value corresponding to the natural state of the turbulent free-convection boundary layer. For all the cases considered, the adaptation distance for the mean flow characteristics can be estimated as \( 14\delta \), where \( \delta \) is the integral boundary layer thickness introduced in Section 2.1.

The calculated data for the local Nusselt number, \( \text{Nu}_\delta = q_\text{w}\delta/\lambda\Delta T \), versus \( Ra_\delta \) is presented in figure 5b. Non-monotonic behaviour of the prediction curves at lower values of \( Ra_\delta \) is attributed to some decrease of the boundary layer thickness in the initial region of the flow development, where fluctuations, artificially generated at the inlet, first slightly decay. The "asymptotic" dependence \( \text{Nu}_\delta(Ra_\delta) \) that is observed at \( Ra_\delta > (3...4)\times10^6 \) for all the computational cases, predicts the local Nusselt numbers that are approximately 15% lower than those given by the correlation \( \text{Nu} = 0.12\cdot Ra^{1/3} \) established in experiments [6]. This is apparently due to some numerical aspects, such as limited grid resolution and/or domain size. So, further investigations are required.

**Figure 3.** Comparison of the computed profiles of the mean (a) velocity and (b) temperature with experimental data for two sections of the boundary layer.

**Figure 4.** Comparison of the computed profiles of RMS values of (a) \( x \)-velocity and (b) temperature fluctuations with experimental data for the second section (\( Gr_\delta = 7.0\times10^6 \)).
Figure 5. (a) Computed distributions of the averaged wall heat flux along the plate and (b) the local Nusselt number vs. the local Rayleigh number.

4. Conclusions
A valuable experience has been gained from multi-variant Implicit LES computations of the vertical-plate turbulent free-convection boundary layer that was developing from artificially-generated inlet boundary conditions, including a turbulent content, to its natural state. It has been established that the choice of the RANS model used for getting the mean inlet flow profiles weakly affects the length of the transition region.

The adaptation distance has been estimated as $14\delta$, where $\delta$ is the integral thickness of the boundary layer at the inlet of the computational domain. The calculations performed with the implemented generator of synthetic turbulence have given almost identical results in variants with and without inlet temperature fluctuations, except for a short section near the inlet. The wall heat flux calculated for the flow region positioned downstream of the adaptation section was underestimated by about 15% as compared with experimental data. Further calculations are required to evaluate, in particular, grid-dependence issues and effects of increasing the normal-to-plate and span sizes of the computational domain.

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