Fano resonances in optical spectra of semiconductor quantum wells driven by an oscillating field

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Optical spectra of semiconductor quantum wells driven by an off-resonant oscillating field are studied theoretically. Due to the dynamical stabilization effect, the field induces the quasi-stationary electron states confined at repulsive scatterers and immersed into the continuum of states of conduction electrons. As a result, the Fano resonances in the spectra of interband optical transitions appear near the energies of the quasi-stationary states.

I. INTRODUCTION

The engineering of various quantum systems by an off-resonant electromagnetic field (Floquet engineering) became the established research area which resulted in many fundamental effects (see, e.g., Refs. \textsuperscript{1–9}). Since the frequency of the off-resonant field lies far from the optical absorption range, the field cannot be absorbed and only “dresses” electrons (dressing field), changing their physical characteristics. The effects induced by the dressing field were actively studied during last years both experimentally and theoretically in various nanostructures, including quantum rings\textsuperscript{10–13}, quantum wells\textsuperscript{14–18}, topological insulators\textsuperscript{19–23}, graphene and related 2D materials\textsuperscript{24–28}, etc. Among these effects, the field-induced stabilization of unstable systems (the dynamical stabilization) should be noted especially (see, e.g., Ref. \textsuperscript{2}). Recently, we demonstrated that the dynamical stabilization results in quasi-stationary electron states confined at repulsive scatterers in 2D electron systems irradiated by a circularly polarized dressing field\textsuperscript{29}. The present article is aimed to develop the theory describing impact of these quasi-stationary states on optical properties of semiconductor quantum wells (QWs).

The article is organized as follows. In Sec. II, we discuss the model describing quasi-stationary electron states induced in QW by a circularly polarized dressing field. In Sec. III, the theory of optical properties modified by these states is developed. The last sections of the article contain conclusion and acknowledgements.

II. MODEL

Let us consider a semiconductor quantum well (QW) confining electrons in the $x,y$ plane within the area $S$, where the conduction band is empty, while the valence band is filled by electrons. The QW is irradiated by the two-mode electromagnetic wave (EMW) propagating along $z$ axis perpendicularly to the QW plane (see Fig. 1). The first mode is a strong off-resonant circularly polarized EMW (dressing field) with the vector potential $\mathbf{A}(t) = (A_x, A_y) = [E_0/\omega_0](\sin \omega_0 t, \cos \omega_0 t, 0)$, where $E_0$ is the electric field amplitude, and $\omega_0$ is the frequency which lies far from characteristic resonant frequencies of the QW (particularly, $\hbar \omega_0 < \varepsilon_g$, where $\varepsilon_g$ is the band gap of the QW). Since the dressing field is off-resonant, it cannot be absorbed by the QW and only mixes electron states within the conduction band. The second mode is a weak resonant EMW (probing field) with the electric field amplitude $E$ and the frequency $\omega$, which is linearly polarized along the $x$ axis ($\hbar \omega > \varepsilon_g$). The resonant probing field induces electron transitions between the conduction and valence bands of the QW and serves to detect the features of optical spectra originated from the dressing field.

![FIG. 1: Sketch of the system under consideration: Semiconductor quantum well (QW) irradiated by a two-mode electromagnetic wave consisting of a strong circularly polarized electromagnetic wave with the electric field amplitude $E_0$ and the frequency $\omega_0$ (dressing field) which induces quasi-stationary electron states bound at repulsive scatterers and a weak linearly polarized electromagnetic wave with the electric field amplitude $E$ and the frequency $\omega$ (probing field) which serves to detect these states.](image)

The behavior of a conduction electron near a scatterer with the repulsive potential $U(\mathbf{r})$ in the presence of the dressing field is defined by the Hamiltonian $\hat{H} = [\hat{\mathbf{p}} - e\mathbf{A}(t)/c]^2/2m_e + U(\mathbf{r})$, where $\hat{\mathbf{p}} = (\hat{p}_x, \hat{p}_y)$ is the plane momentum operator, $m_e$ is the effective electron mass, $e$ is the electron charge, $\mathbf{r} = (x,y) = (r \cos \varphi, r \sin \varphi)$ is the plane radius vector of an electron, and $\varphi$ is the azimuth angle in the plane. In the frame of classical electrodynamics\textsuperscript{30}, the circularly polarized field induces the rotational movement of an electron along the circular trajectory described by the radius vector $r_0(t) = (-r_0 \cos \omega_0 t, r_0 \sin \omega_0 t)$, where

$$r_0 = \frac{|e|E_0}{m_e\omega_0^2}$$

is the radius of the trajectory. The Hamiltonian rewritten in the rest frame of the rotating electron reads\textsuperscript{31}.

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\[ \hat{H} = \frac{\hat{p}^2}{2m_e} + U(r - r_0(t)). \] 

Thus, the field dependence of the Hamiltonian in the new reference frame is transferred from the operator of kinetic energy to the potential energy. Expanding the oscillating potential, \( U(r - r_0(t)) \), into a Fourier series, one can take into account only the zero harmonic of the expansion,

\[ U_0(r) = \frac{1}{2\pi} \int_{-\pi}^{\pi} U(r - r_0(t)) \, d(\omega t), \quad (2) \]

which describes the smooth dynamics of an electron. As a consequence, the electron dynamics near a repulsive potential can be described by the Hamiltonian

\[ \hat{H}_0 = \frac{\hat{p}^2}{2m_e} + U_0(r), \quad (3) \]

where the stationary potential \( U_0(r) \) should be treated as a repulsive potential dressed by an oscillating field \( \delta \). The general feature of the dressed repulsive potentials of kind \( \delta \) is the field-induced local minimum in the core of the repulsive potentials \( \delta \). As a consequence, the electron states confined near the local minimum appear. To proceed, let us restrict the following analysis by the case of short-range scatterers which are conventionally modeled in QWs by the delta potential, \( U(r) = u_0 \delta(r) \), where \( u_0 > 0 \) is the strength of the repulsive potential (see, e.g., Ref. 10). This model of scatterers corresponds, particularly, to a semiconductor QW doped by neutral atoms. Substituting the delta potential into Eq. (2), we arrive at the dressed delta potential,

\[ U_0(r) = \frac{u_0 \delta(r - r_0)}{2\pi \tau_0}. \quad (4) \]

Thus, the circularly polarized dressing field turns the repulsive delta potential \( U(r) = u_0 \delta(r) \) into the delta potential barrier of ring shape \( \delta \) pictured in Fig. 2a.

As a consequence, the bound electron states which are confined inside the area fenced by the ring-shape barrier \( 0 < r < r_0 \), appear. The delta potential barrier \( \delta \) serves as a boundary between the bound electron states and the continuum of delocalized wave functions of conduction electrons. Certainly, the bound electron states are quasi-stationary since they can decay via the tunnel transition through the potential barrier into the continuum of conduction electrons. As a consequence, the energy broadening of the bound states appears. To avoid the destructive impact of scattering processes on the bound states, we will assume that the field frequency, \( \omega_0 \), is high enough to satisfy the condition \( \omega_0 \tau_e \gg 1 \), where \( \tau_e \) is the mean free time of conduction electrons.

It should be noted also that the effect of the oscillating terms — which were neglected by us in the Hamiltonian \( \delta \) — on the bound states is negligible small if the field frequency, \( \omega_0 \), lies far from resonant frequencies of the bound states \( \delta \). The Schrödinger problem with the Hamiltonian \( \delta \) and the dressed potential \( \delta \) can be easily solved \( \delta \) in the limiting case of strong delta potential \( \alpha = 2\hbar^2/m_e u_0 \ll 1 \) and results in the energy spectrum of the bound quasi-stationary states,

\[ \varepsilon_{nm} = \frac{\hbar^2 \varepsilon_{nm}^2}{2m_e r_0^2} + O(\alpha), \]

energy broadening of the states,

\[ \Gamma_{nm} = \frac{4\varepsilon_{nm}^2 \alpha^2}{N_{m+1}^2(\xi_{nm}) - N_{m-1}^2(\xi_{nm})} + O(\alpha^3), \]

and their wave functions

\[ \psi_{nm} = \frac{e^{im\varepsilon\omega t}}{\sqrt{(\pi r_0)} J_{m+1}(\xi_{nm})} \left\{ \begin{array}{ll} J_m(\xi_{nm} \frac{r}{r_0}), & 0 < r \leq r_0 \\ 0, & r \geq r_0 \end{array} \right. + O(\alpha), \]

where \( J_m(\xi) \) and \( N_m(\xi) \) are the Bessel functions of the first and second kind, respectively, \( \xi_{nm} \) is the \( n \)th zero of the Bessel function \( J_m(\xi) \), \( n = 1, 2, 3, ... \) is the principal quantum number which numerates zeros of the Bessel function \( J_m(\xi) \), \( m = 0, \pm 1, \pm 2, ... \) is the angular momentum. The ground bound quasi-stationary state with the energy \( \varepsilon_{10} \) is pictured schematically in Fig. 2a, where the tunnel transition from this state to the continuum of free conduction electrons is marked by the arrow.

![FIG. 2: Scheme of electron transitions: (a) tunnel transition (the red solid arrow) from the ground quasi-stationary state with the energy \( \varepsilon_{10} \) (the horizontal yellow strip) to the state of free conduction electron (the green wave arrow) through the ring-shape delta potential barrier \( U_0 = \omega_0 \delta(r - r_0)/2\pi \tau_0 \) (the vertical blue line); (b) optical transitions from the valence band \( \varepsilon_v \) to the conduction band \( \varepsilon_c \) induced by the probing field with the frequency \( \omega \): the direct transition (the solid arrow 1) and the transition through the quasi-stationary state with the energy \( \varepsilon_{10} \) (the dashed arrow 2).](image)
masses of electrons (holes) in the subbands. In the following, the basic electron states corresponding to these energies will be denoted as \( |k_{c(v)}\rangle = |\sqrt{S} e^{ikr}\phi_{c(v)}\rangle\), where \(\phi_{c(v)}\) are the wave functions corresponding to the subband edges (they include both the Bloch functions of the semiconductor material and the subband wave function arisen from the size quantization in the QW). The quasi-stationary electron states at bound places of the \(\phi\) will be denoted as \(|s_{nm}\rangle = |\psi_{nm}\phi_c\rangle\), where the index \(j = 1, 2, \ldots N\) numerates scatterers located in different places of the \(x, y\) plane and \(N\) is the total number of scatterers in the QW.

The considered two-band electron system interacting with the two-mode electromagnetic field can be described by the effective Hamiltonian

\[
\hat{H}_{\text{eff}} = \hat{H}_e + \hat{H}_T + E \cos \omega t \hat{H}_D, \tag{8}
\]

where

\[
\hat{H}_e = \sum_{j=1}^{N} \sum_{n,m} |s_{nm}\rangle \langle s_{nm}| + \sum_{k_c} \hat{H}_e \xi_{ek} (k_c) + \sum_{k_v} \xi_{vk} (k_v) \tag{9}
\]

is the electron Hamiltonian describing energies of the basic electron states,

\[
\hat{H}_T = \sum_{j=1}^{N} \sum_{n,m,k_c} |k_c\rangle \langle k_c|H_T|s_{nm}\rangle \langle s_{nm}| + \text{H.c.} \tag{10}
\]

is the tunnel Hamiltonian describing the tunnel transitions from the quasi-stationary bound states to the states of the conduction electrons through the radial potential barrier (see Fig. 2a), and

\[
\hat{H}_D = \sum_{k_v} \left[ \sum_{k_c} |k_c\rangle \langle k_c|\hat{H}_D|s_{nm}\rangle \langle s_{nm}| + \text{H.c.} \right] \tag{11}
\]

is the Hamiltonian of the dipole interaction between electron states in the valence band and the conduction band induced by the probing field.

### III. RESULTS AND DISCUSSION

Assuming the tunneling to be weak and the probing field amplitude, \(E\), to be small, the last two terms of the Hamiltonian \(\hat{H}_D\) can be considered as a perturbation. Then the probability of optical interband electron transition from the state \(|k_{c}'\rangle\) to the state \(|k_c\rangle\) per unit time reads

\[
w_{k_c, k_{c}'} = \frac{\pi E^2}{2\hbar} \delta \varepsilon_{ck} + \varepsilon_{ck'} + \varepsilon_g - \hbar \omega \times \left| \frac{4\pi^2 \delta (k - k')}{} \right|
\]

where \(\varepsilon_{ck} = \varepsilon_{ck'}\) (the state of conduction electron with the bound states energy \(\varepsilon_{nm} = \hbar^2 k_{nm}^2 / 2m_c\), and \(|s_{nm}\rangle\) is the quasi-stationary electron state with the energy \(\varepsilon_{nm}\) confined at a scatterer positioned in the zero point of the coordinate system. In what follows, the matrix elements in Eq. (12) will be rewritten as

\[
\langle k_c|H_T|s_{nm}\rangle \langle s_{nm}|H_D|k_{c}'\rangle = e^{i(k - k')\omega} r_j \langle k_c|H_T|s_{nm}\rangle \langle s_{nm}|H_D|k_{c}'\rangle, \quad \text{where } r_j \text{ is the radius vector of } j\text{th scatterer position.}
\]

The first term under the modulus in the probability \(\langle k_c|H_T|s_{nm}\rangle\) describes the usual direct interband transition (see the vertical solid arrow 1 in Fig. 2b), whereas the second term corresponds to the transitions through the intermediate quasi-stationary states (see the dashed arrow 2 in Fig. 2b, which marks such a transition through the ground quasi-stationary state \(\varepsilon_{10}\)). It follows from the energy conservation law that the transitions through the quasi-stationary states are possible only within the narrow energy range of conduction electrons, \(\varepsilon_{ck} \approx \varepsilon_{nm} \pm \Gamma_{nm}/2\). If the broadening \(\Gamma_{nm}\) is small, the matrix elements \(\langle k_c|H_T|s_{nm}\rangle\) in Eq. (12) varies little around the energy \(\varepsilon_{ck} = \varepsilon_{nm}\) for which the probability of the transition is not negligible. Therefore, one can replace the tunnel matrix elements \(\langle k_c|H_T|s_{nm}\rangle\) in Eq. (12) with the resonant matrix elements, \(\langle k_{nm}|H_T|s_{nm}\rangle\).

Taking into account Eq. (13) and the solutions of the Schrödinger problem \(\hat{H}_{\text{eff}}\), the resonant matrix element of the conventional tunnel Hamiltonian \(\hat{H}_D\) can be written as \(\langle k_{nm}|H_T|s_{nm}\rangle = -\hbar \sqrt{\Gamma_{nm}/S} m_c\). Then the probability \(\langle k_c|H_T|s_{nm}\rangle\) can be rewritten as

\[
w_{k_c, k_{c}'} = \pi E^2 \delta \varepsilon_{ck} + \varepsilon_{ck'} + \varepsilon_g - \hbar \omega \times \left| \frac{4\pi^2 \delta (k - k')}{} \right|^2 \frac{\pi |D_{cv}|^2 E^2}{2\hbar S^2}, \tag{14}
\]

where \(\Phi_{nm}(k) = \langle \psi_{nm}|e^{ikr}\rangle\) is the Fourier transform of the bound state wave functions \(\langle \psi_{nm}|\rangle\), \(D_{cv} = (\hbar m_0 / S \varepsilon_g) \langle \phi_{c}||p_x||\phi_{c}\rangle\) is the interband matrix element of electric dipole moment, and \(m_0\) is the electron mass.
in vacuum. Next, we have to average the probability \([14]\) over coordinates of all \(N\) scatterers. Assuming the scatterers to be randomly arranged in the QW plane, the averaging procedure is defined by the operator \(\hat{A} = (1/S)^N \prod_{j=1}^{N} [\int_S d^2\mathbf{R}_j]\). Then the intensity of absorption of the probing field, \(I = (\hbar \omega / S) \sum_{\mathbf{k}, \mathbf{k}'} \hat{A} \omega \epsilon_{\mathbf{k}, \mathbf{k}'}\), reads

\[
I = \int_{\mathbf{k}} d^2k' \int_{\mathbf{k}'} d^2k \left( \frac{\hbar^2 n_s [1 + 4\pi^2 n_s \delta(k-k')]}{m_e} \right) \times \sum_{n,m} \frac{\Gamma_{nm}^{1/2} \Phi_{nm}(k')}{\varepsilon_{e0} + \varepsilon_{ck} - \varepsilon_{nm} + i\Gamma_{nm}/2}^2 + 4\pi^2 \delta(k-k')
\]

\[-\frac{8n_s \pi^2 \hbar}{\sqrt{m_e}} \text{Re} \left[ \sum_{n,m} \frac{\Gamma_{nm}^{1/2} \Phi_{nm}(k')}{\varepsilon_{e0} + \varepsilon_{ck} - \varepsilon_{nm} + i\Gamma_{nm}/2} \right] \times \delta(k-k') \left( \frac{\omega D_{e\Gamma}^2 E^2}{16\pi^3} \right) \delta \varepsilon_{ck} + \delta \varepsilon_{v\mathbf{k}'} + \varepsilon_g - \hbar \omega, \tag{15}\]

where \(n_s = N/S\) is the density of quasi-stationary states (density of scatterers). Restricting the consideration by the ground quasi-stationary state \((n = 1, m = 0)\) which defines the low-energy optical properties, the absorption intensity \([15]\) can be rewritten as

\[
I = \left( \frac{\omega m_e |D_{e\Gamma}^2 E^2|}{2\hbar^2} \right) \left[ 1 + \frac{\hbar^2 n_s^2 \Gamma_{10}|\Phi_{10}(k_\omega)|^2 + n_s \Phi_{10}}{m_e [(\varepsilon_\omega - \varepsilon_{10})^2 + (\Gamma_{10}/2)^2]} \right]
\]

\[-\frac{2n_s \hbar \Gamma_{10}^{1/2} \Phi_{10}(k_\omega)(\varepsilon_\omega - \varepsilon_{10})}{\sqrt{m_e [(\varepsilon_\omega - \varepsilon_{10})^2 + (\Gamma_{10}/2)^2]}}, \tag{16}\]

where \(k_\omega = \sqrt{2(h\omega - \varepsilon_g)m_e m_h/(m_e + m_h)} / \hbar\) is the resonant electron wave vector corresponding to the direct interband optical transition (see Fig. 2b), \(\varepsilon_\omega = \varepsilon_{e0} + \hbar^2 k_\omega^2 / 2m_e\) is the resonant energy in the conduction band (see Fig. 2b), and \(\Phi_{10} = \int_0^\infty |\Phi_{10}(k)|^2 dk\). Substituting Eqs. (1) and (5)–(7) into Eq. (16), one can find the dependence of the absorption spectrum of QW on the dressing field intensity, \(I_0 = cE_0/2\pi\), which is plotted in Fig. 3 for GaAs-based quantum well (the effective mass of electrons is \(m_e \approx 0.0067 m_0\) and the effective mass of holes in the first subband is \(m_h \approx 0.11 m_0\)).

The first term in the square brackets of Eq. (16) arises from the direct optical transition (see the solid arrow 1 in Fig. 2b), which does not depend on the probing field frequency, \(\omega\), since the density of electron states near edges of 2D subbands does not depend on the electron energy. Just this term describes the intensity of optical absorption in the absence of the dressing field, \(I = \omega m_e |D_{e\Gamma}^2 E^2| / 2\hbar^2\). The second term there arises from optical absorption through the quasi-stationary state \(\varepsilon_{10}\) (see the dashed arrow 2 in Fig. 2b) and is described by the Lorentzian centered at the resonant energy \(\varepsilon_\omega = \varepsilon_{10}\), whereas the third term arises from the quantum interference of the absorption ways 1 and 2 in Fig. 2b and depends on the broadening of quasi-stationary state \(\Gamma_{10}\).

Since the interference term changes its sign at the resonant energy \(\varepsilon_\omega = \varepsilon_{10}\), we arrive at the asymmetrical structure of the total absorption spectrum plotted in Fig. 3, which is typical for the Fano resonances\([21]\). In the present plots, we restricted the consideration by the resonance arisen from the ground quasi-stationary state \(\varepsilon_{10}\). Certainly, analogous Fano resonances will appear from other quasi-stationary states (overlying in energy) in the high-frequency area of the absorption spectrum.
IV. CONCLUSION

An off-resonant circularly polarized electromagnetic field (dressing field) can induce the quasi-stationary electron states bound at repulsive scatterers in semiconductor quantum wells (QWs). These states manifest themselves in optical spectra of the QWs as Fano resonances arisen from the quantum interference of direct interband optical transitions and the transitions through the quasi-stationary states. It is demonstrated that resonance peaks are positioned at energies of the states, whereas the Fano asymmetry of the peaks depends on the energy broadening of the quasi-stationary states. Therefore, the developed theory allows to use optical measurements as a direct experimental method to detect energy structure of the quasi-stationary states induced by the dressing field.

Acknowledgments

The reported study was funded by the Russian Science Foundation (project 20-12-00001).
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