Investigation of noscale supersymmetry breaking models with a gauged $U(1)_{B-L}$ symmetry

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**Abstract**

Noscale supersymmetry (SUSY) breaking model is investigated in the minimal extension of the minimal supersymmetric standard model (MSSM) with a gauged $U(1)_{B-L}$ symmetry. We specifically consider a unification-inspired model with the gauge groups $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \subset SU(5) \times U(1)_5$ for illustration. While the noscale boundary condition at the grand unification scale ($M_G \simeq 2 \times 10^{16}$ GeV) in the MSSM is not consistent with phenomenological constraints, we show that it is if the gaugino of the $U(1)_5$ multiplet is several times heavier than the gauginos of the MSSM. However, if $SU(5) \times U(1)_5$ is further embedded in a larger simple group, e.g. $SO(10)$, the noscale boundary condition at $M_G$ is inconsistent with phenomenological constraints. If we relax the noscale boundary condition and allow non-zero soft scalar masses for the Higgs fields which spontaneously break the $U(1)_5$ symmetry, the resultant spectrum of SUSY particles becomes consistent with all the phenomenological constraints, even if we impose the GUT relation on the gauge coupling and the gaugino mass of the $U(1)_5$. In this case, the SUSY CP problem is also solved, since the condition $B\mu = 0$ at the boundary can be imposed consistently with the electroweak symmetry breaking.
1 Introduction

Low energy supersymmetry (SUSY) is expected to serve as a basis for physics beyond the Standard Model (SM). If we just consider the minimal supersymmetric standard model (MSSM) with generic soft SUSY breaking terms as an effective low energy theory, then we must face with over a hundred of additional parameters. However, although the SUSY particles are not discovered at this moment, we already know from some low energy experiments, such as detecting flavor changing neutral currents (FCNC) and CP violation [1], that these parameters cannot be generic. These experiments give us a hint what kind of pattern the soft SUSY breaking terms should have.

In this paper, we concentrate on models which have the so called “noscale boundary condition” [2]. In such a model, all soft breaking terms except the gaugino masses are assumed to vanish at some high energy scale $M_X$, which is usually taken as the GUT scale $M_G \simeq 2 \times 10^{16}$ GeV. Soft breaking terms except gaugino masses at the weak scale are generated by renormalization group effects dominated by the gaugino loops, which are automatically flavor blind and naturally suppress the FCNC interactions [3]. Furthermore, if we can set the $B\mu$ term to be zero at the boundary scale $M_X$, consistently with the radiative electroweak symmetry breaking, the SUSY CP violation problem is also solved. Therefore, under the assumption of the noscale boundary condition, we can naturally avoid the SUSY FCNC and CP violation problems and get a phenomenologically desirable, highly predictive mass spectrum of SUSY particles. Recently, models with the noscale boundary condition begin to attract much attention, since a natural and a simple geometrical realization was proposed, i.e. the gaugino-mediated SUSY breaking models [4].

However, it was shown recently that the minimal noscale model with $M_X = M_G$ is actually not consistent with phenomenological bounds [5, 6, 7], mainly due to the lower bound on the Higgs boson mass and the cosmological requirement that a charged particle (in particular stau) is not the lightest SUSY particle (LSP). There might be several ways to reconcile the noscale boundary condition with these phenomenological bounds, e.g. by imposing non-universal gaugino masses [6], or by imposing the noscale boundary condition above the GUT scale [8, 7].

In this paper, we propose another way, which is to change the mass spectrum of SUSY particles by gauging some symmetry. We consider the minimal extension of the MSSM by adding a gauged $U(1)_{B-L}$ symmetry to the MSSM gauge groups. Actually, the $U(1)_{B-L}$ symmetry is the unique global symmetry which can be gauged without introducing any particles charged under the MSSM gauge groups. Furthermore, the existence of three right-handed Majorana neutrinos is automatically required by the anomaly cancellation condition of this symmetry. They naturally get large masses of the order of the $B - L$ breaking scale, which allows us to have a realistic mass spectrum of the lighter neutrinos via the “seesaw” mechanism [9]. Gauging the $U(1)_{B-L}$ symmetry is also motivated from obtaining an exact R parity [10].
In this work, we consider the minimal extension of the MSSM with a gauged \( U(1)_{B-L} \) symmetry and analyze whether the noscale boundary condition at the GUT scale is consistent with phenomenological bounds or not.

The organization of this paper is as follows. In section 2, we explain the setup of our model. We concentrate on the \( SU(5) \times U(1)_5 \) unification-inspired model. Here the \( U(1)_5 \) is the so called “fiveness”, which is a linear combination of the weak hypercharge \( U(1)_Y \) and the \( U(1)_{B-L} \), and we assume that this \( U(1)_5 \) symmetry is spontaneously broken at an intermediate scale. We also discuss the subtlety of the mixing between the \( U(1)_Y \) and the \( U(1)_5 \), which arises due to the decoupling of the colored Higgs fields. In section 3, we show the results of the analyses, and compare the differences between noscale models with and without a gauged \( U(1)_{B-L} \) symmetry. In section 4, we consider some variations which relax the noscale boundary condition, and analyze whether they are consistent with phenomenological constraints. Section 5 contains the summary and concluding remarks.

2 Models with a gauged \( U(1)_{B-L} \) symmetry

We consider the minimal extension of the MSSM with gauge groups \( SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_5 \subset SU(5) \times U(1)_5 \) below the GUT scale. The superpotential is given by the following simplest form,

\[
W = (y_\nu)_{ij} \bar{N}_i L_j H_u + \lambda_1 X (S \bar{S} - v^2) + \frac{1}{2} (\lambda_2)_{ij} S \bar{N}_i \bar{N}_j + W_{\text{MSSM}},
\]

where \( \lambda_1, \lambda_2, \ y_\nu \) are dimensionless coupling constants, \( v \) is the vacuum expectation value for the \( S \) and \( \bar{S} \) fields, which roughly corresponds to the \( B-L \) breaking scale, and \( W_{\text{MSSM}} \) is the superpotential of the MSSM,

\[
W_{\text{MSSM}} = (y_u)_{ij} \bar{u}_i Q_j H_u - (y_d)_{ij} \bar{d}_i Q_j H_d - (y_e)_{ij} \bar{e}_i L_j H_d + \mu H_u H_d.
\]

Here, \( \bar{N}_i \) is the chiral superfield of the right-handed Majorana neutrino, and \( X, S, \bar{S} \) are those which are responsible for the \( B-L \) symmetry breaking. All of these extra chiral superfields are singlets under the MSSM gauge groups. The complete list of the matter content of our model and the \( U(1) \) charge assignment are given in Table 1. In that table, the charge \( Q_5 \) of the \( U(1)_5 \) is given by the normalization consistent with the unification into \( E_6 \) and any of its subgroups. The \( U(1)_{B-L} \) charge \( B-L \) is given by a linear combination of the weak hypercharge \( Y \) and \( Q_5 \) as

\[
B-L = -\frac{1}{5} (2\sqrt{10} \ Q_5) + \frac{4}{5} Y.
\]

Now, we are at the point to discuss the renormalization group equations (RGEs). There are some subtleties caused by the kinetic term mixing between the two \( U(1) \) gauge multiplets. This is because \( \text{Tr}[Y Q_5] \neq 0 \) below the GUT scale due to the decoupling of colored Higgs fields. (Here,
Table 1: The list of the matter content and the U(1) charge assignment of our model. Here, the subscript “i” denotes the generation and runs 1, 2, 3. The charge of the U(1)$_5$, $Q_5$, is given by the normalization consistent with the unification into $E_6$ and any of its subgroups.

Tr is taken with all the chiral superfields.) After we perform a rotation on the two U(1) gauge multiplets to diagonalize their kinetic terms, there appear mixings in the couplings between the matter fields $\phi_i$ and the two U(1) gauge fields. We parameterize these couplings as follows:

\[
D_\mu \phi_i = \left( \partial_\mu + \frac{1}{i} \left[ \bar{g}_Y^i A_\mu^Y + \bar{g}_5^i A_\mu^5 \right] \right) \phi_i , \\
\bar{g}_Y^i = g_Y Y^i + g_{5,Y} Q_5^i , \\
\bar{g}_5^i = g_{5,Y} Y^i + g_5 Q_5^i.
\]  

Here, the index “i” runs through all the chiral superfields. Note that, in the following discussion, we choose the GUT normalization also for the weak hypercharge; for example, $Y^Q = \sqrt{3/5} (1/6)$.

The one-loop RGEs of the gauge couplings can be obtained by following the methods given in Ref. [11] as:

\[
\frac{d}{dt} \begin{pmatrix} g_Y & g_{5,Y} \\ g_{5,Y} & g_5 \end{pmatrix} = \frac{1}{16\pi^2} \begin{pmatrix} g_Y & g_{5,Y} \\ g_{5,Y} & g_5 \end{pmatrix} \times \begin{pmatrix} b_Y & b_{5,Y} \\ b_{5,Y} & b_5 \end{pmatrix},
\]  

where

\[
b_Y = \text{Tr} [\bar{g}_Y g_Y] = \frac{33}{5} g_Y^2 + \frac{57}{5} g_{5,Y}^2 + \frac{2\sqrt{6}}{5} g_Y g_{5,Y}, \\
b_5 = \text{Tr} [\bar{g}_5 g_5] = \frac{33}{5} g_5^2 + \frac{57}{5} g_{5,Y}^2 + \frac{2\sqrt{6}}{5} g_{5,Y} g_5 , \\
b_{5,Y} = b_{5,Y} = \text{Tr} [\bar{g}_5 \bar{g}_Y] = \frac{33}{5} g_Y g_5 + \frac{\sqrt{6}}{5} (g_Y g_5 + g_{5,Y} g_5) + \frac{57}{5} g_Y g_{5,Y},
\]

and $t \equiv \ln(\mu/\mu_0)$.

Note that, because of the kinetic term mixing, $g_Y$ does not follow the same RGE as in the MSSM. Even if we set the off diagonal gauge couplings to be zero at some scale, they develop nonzero values because of the non-vanishing $\text{Tr}[Y Q_5]$. Then, how can we discuss the gauge coupling unification? Actually, the gauge field $A_\mu^Y$ also couples with the fields which have nonzero U(1)$_5$ charges. Hence, the basis given in Eq. (2.4) is inadequate to discuss the running of the unbroken U(1)$_Y$ gauge coupling $g_1$. We move to the on-shell basis in which one of the U(1) gauge
fields couples with only the fields which have zero U(1)_5 charges by rotating the gauge multiplets. We can extract the unbroken U(1)_Y gauge coupling g_1 from interactions of matter fields with this massless gauge field as

\[ g_1 = \frac{g_Y g_5 - g_{Y,5} g_{5,Y}}{\sqrt{g_5^2 + g_{5,Y}^2}}. \tag{2.7} \]

Using Eqs. (2.5) and (2.6), one finds that g_1 defined by Eq. (2.7) satisfies the one-loop RGE

\[ \frac{d}{dt} g_1 = \frac{1}{16\pi^2} \frac{33}{5} g_1^3, \tag{2.8} \]

which is the same as in the MSSM, both above and below the B – L breaking scale v. Therefore, the condition for the unification of g_1 with SU(3)_C and SU(2)_L gauge couplings g_3 and g_2 is unaffected by mixing, up to two-loop and threshold effects. Thus, the gauge coupling unification is not spoiled in this model, and it is sensible to impose the gauge coupling unification at the GUT scale:

\[ g_3 = g_2 = g_1 \equiv g_U, \quad g_{Y,5} = g_{5,Y} = 0. \tag{2.9} \]

Note that this condition indicates g_Y = g_1, while g_5 is still undetermined at the boundary. We will assume this boundary condition throughout this paper.

The RGEs for the gaugino masses are given by

\[ \frac{d}{dt} \begin{pmatrix} M_Y & M_{Y,5} \\ M_{5,Y} & M_5 \end{pmatrix} = \frac{2}{16\pi^2} \begin{pmatrix} M_Y & M_{Y,5} \\ M_{5,Y} & M_5 \end{pmatrix} \times \begin{pmatrix} b_Y & b_{5,Y} \\ b_{Y,5} & b_5 \end{pmatrix}, \tag{2.10} \]

where b_Y, b_{5,Y}, b_{Y,5}, b_5 are identical to those in Eq. (2.6). These mass terms of the two U(1) gauginos and their interactions between the matter fields are given by

\[ V \supset \sum_i \left[ \sqrt{2}(g_Y^i \phi_Y^i \lambda_Y \psi_{\phi_i} + \bar{g}_5^i \phi_Y^i \lambda_5 \psi_{\phi_i}) + \text{h.c.} \right] \]

\[ - \frac{1}{2} \left[ (\lambda_Y, \lambda_5) \begin{pmatrix} M_Y & M_{Y,5} \\ M_{5,Y} & M_5 \end{pmatrix} \begin{pmatrix} \lambda_Y \\ \lambda_5 \end{pmatrix} + \text{h.c.} \right], \tag{2.11} \]

where the index “ i ” runs through all the chiral superfields as before, and ψ_{\phi_i} is the fermion superpartners of \phi_i. λ_Y and λ_5 are the gauginos which are the superpartner of the gauge fields given in Eq. (2.4). One can show that the well-known gaugino mass relation

\[ \left( \frac{M_3}{g_5^2}, \frac{M_2}{g_2^2}, \frac{M_1}{g_1^2} \right) = \text{const.}, \tag{2.12} \]

is precisely satisfied by the one-loop RGEs of this model. Here, the mass of the gaugino which is the superpartner of the massless U(1)_Y gauge field is given by

\[ M_1 = \frac{g_5^2 M_Y - g_{5,5,Y} (M_{5,Y} + M_{Y,5}) + g_5^2 M_5}{g_5^2 + g_{5,Y}^2}, \tag{2.13} \]
which is obtained by performing the same rotation on the gaugino fields as the one performed on
the two U(1) gauge fields. Thus, it is also natural to impose the GUT relation among the gaugino
masses with vanishing off-diagonal elements,

\[ M_3 = M_2 = M_1 = M_{1/2}, \quad M_{Y,5} = M_{5,Y} = 0 , \]  

at the GUT scale. Note that, together with Eq. (2.3), \( M_Y \) is equal to \( M_1 \), while \( M_5 \) is still
undetermined at the boundary. We will also assume this boundary condition in the remaining
part of this paper.

If we set the off-diagonal elements of the U(1) gauge couplings and gaugino mass terms to be
zero at the GUT scale, they remain fairly small even at the intermediate scale\(^1\) and hence many
authors neglect small effects caused by these mixings in their analyses of the soft SUSY breaking
parameters. However, in our analyses, we take into account the dominant mixing effects from the
gauginos on the soft scalar mass terms to determine them accurately. The contribution of the two
U(1) gauginos to the RGE for soft scalar mass terms is calculated as

\[
\frac{d}{dt} m_i^2 \supset -\frac{1}{16\pi^2} \left[ 8\bar{g}_Y^2 |M_Y|^2 + 8\bar{g}_5^2 |M_5|^2 + 8(\bar{g}_Y^2 + \bar{g}_5^2) |M_{\text{off D}}|^2 \right] ,
\]  

(2.15)

where \( M_{\text{off D}} \equiv (M_{Y,5} + M_{5,Y})/2 \). In the following analyses, when we assume that \( g_Y \neq g_5 \) and/or
\( M_Y \neq M_5 \) at the GUT scale, we use Eq. (2.15) and neglect small effects of the U(1) mixing on
the other soft breaking terms, e.g. the SUSY breaking trilinear terms (A terms).

On the other hand, a dramatic simplification occurs when we further impose the GUT relation
also on the U(1)\(_5\) gauge coupling and gaugino mass as

\[ g_Y = g_5 = g_U, \quad M_Y = M_5 = M_{1/2} \]  

(2.16)
at the GUT scale with vanishing off-diagonal elements. In this case, we can go to the basis where
the two U(1) gauge couplings and the corresponding two gaugino masses do not mix at arbitrary
scales below \( M_G \). The gauge interaction of a matter field \( \phi_i \) is

\[
D_\mu \phi_i \supset i(Y^i, Q^i_5) \left( \begin{array}{cc} g_Y & 0 \\ 0 & g_5 \end{array} \right) \left( \begin{array}{c} A^Y_\mu \\ A^5_\mu \end{array} \right) \phi_i \]  

(2.17)
at \( M_G \). The existence of the U(1) mixing can be seen from the traces of the U(1) charges,

\[
Q \equiv \begin{pmatrix} \text{Tr}[Y Y] & \text{Tr}[Y Q_5] \\ \text{Tr}[Q_5 Y] & \text{Tr}[Q_5 Q_5] \end{pmatrix} = \begin{pmatrix} 33/5 & \sqrt{6}/5 \\ \sqrt{6}/5 & 57/5 \end{pmatrix} . \]  

(2.18)

\(^1\)If we assume the relations given in Eq. (2.16), \( \frac{\sqrt{g_Y^2 + g_5^2}}{\sqrt{g_Y^2 + g_5^2}} \sim 1\% \) and \( \frac{M_{Y,5}^2 + M_{5,Y}^2}{\sqrt{M_{Y,5}^2 + M_{5,Y}^2}} \sim 2\% \) at the intermediate
scale \( \mu \simeq 10^{10}\text{GeV} \). Even if we assume \( g_5 = 5g_Y \) and \( M_5 = 5M_Y \) at the GUT scale \( M_G \), these quantities only
slightly increase to be \( \sim 2\%, 4\% \), respectively.
We can go to the basis where $Q$ is diagonal by rotating the basis in the following way,

\[
(g'_Y, g'_5) = (g_Y, g_5) R, \\
(A'_Y, A'_5) = (A_Y, A_5) R, \\
(\lambda'_Y, \lambda'_5) = (\lambda_Y, \lambda_5) R,
\]

\[
R^{-1}Q R = \begin{pmatrix} 9 + \sqrt{6} & 0 \\ 0 & 9 - \sqrt{6} \end{pmatrix}, \tag{2.19}
\]

where $R$ is a $2 \times 2$ rotational matrix. In this basis, the off-diagonal elements of the $\beta$ functions are zero, $b_{Y', 5'} = b_{5', Y'} = 0$, and by virtue of Eq. (2.16), the off-diagonal elements of the gauge couplings and gaugino masses are also zero at the GUT scale. Therefore, we need not worry about the mixings of the $U(1)$ gauge couplings and gaugino mass terms at any scale, and hence the calculation of the RGEs can be carried out straightforwardly. The price for the choice of this basis is that the $U(1)$ charges of the fields are now complicated. The new charges $Y'$ and $Q'_5$ are given by

\[
Y' = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{10}} Y + \frac{\sqrt{3} + \sqrt{2}}{\sqrt{10}} Q_5, \\
Q'_5 = -\frac{\sqrt{3} + \sqrt{2}}{\sqrt{10}} Y + \frac{\sqrt{3} - \sqrt{2}}{\sqrt{10}} Q_5 \tag{2.20}
\]

in this basis. In the remainder of this paper, we use this basis when we assume the extended GUT relation given by Eq. (2.16).

3 Noscale boundary conditions in models with a gauged $U(1)_{B-L}$ symmetry

In this section we show the results of the analyses and their implications. We work on the $SU(5) \times U(1)_5$ unification-inspired model. The soft scalar masses are assumed to vanish at the GUT scale $M_G$, $m_0 = 0$, and they are generated by the RG effects at lower energies. We also assume the SUSY breaking trilinear terms $A_0$ to vanish at $M_G$, but leave the SUSY breaking Higgs mass term ($B\mu$ term) to be generic for a while, since we do not have a reliable explanation for the origin of the Higgsino mass term ($\mu$ term), and hence also for the $B\mu$ term. The condition for the radiative electroweak symmetry breaking (EWSB) relates the absolute value of the $\mu$ term, $|\mu|$, and $B\mu$ (both at the weak scale) with the $Z$ boson pole mass $m_Z$ and the ratio of the two VEVs of the Higgs doublets, $\tan \beta \equiv v_u/v_d$. We choose $\tan \beta$ to be a free parameter, and then $|\mu|$ and $B\mu$ are predicted. We assume the three standard model gaugino masses $M_1, M_2, M_3$ to be universal, but remain the $U(1)_5$ gaugino mass $M_5$ to be free:

\[
M_3 = M_2 = M_1 = M_{1/2}, \quad M_5 = \text{free}. \tag{3.1}
\]

Actually, under the assumption of this relation, one can show that this model is equivalent to the SO(10)-inspired $SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ model by performing a similar rotation on the basis of the two $U(1)$ charges.
Thus, the parameters of the model are the gaugino masses $M_{1/2}$ and $M_5$, $\tan \beta$, and $\text{sgn}(\mu)$ (the sign of $\mu$). The set of the parameters which corresponds to $B\mu = 0$ at $M_G$ will be shown as a hypersurface in the parameter space in the remaining analyses. We calculate the spectrum of the model and search for the parameter regions which are consistent with all the phenomenological bounds.

3.1 Analytical procedure and phenomenological bounds

Let us see how our analyses are done. We first evolve the boundary conditions at $M_G$, i.e.,

$$\{m_0 = A_0 = 0, \quad M_1 = M_2 = M_3 = M_{1/2}, \quad M_5\},$$

(3.2)
down to the U(1)$_{B-L}$ breaking scale $g_5v$ by the one-loop RGEs. In addition to the MSSM parameters, these include the additional U(1) gauge coupling, the soft mass terms for the fields $S, \overline{S}, X, \overline{N}_i$, the Yukawa couplings $y_\nu, \lambda_1, \lambda_2$, and the corresponding $A$ terms (see Eq.(2.1)). The one-loop RGEs are presented in the Appendix.

In our analyses, however, we neglect the Dirac neutrino Yukawa coupling $y_\nu$ and the effects induced by this coupling. This is justified if we use a relatively low $B - L$ breaking scale, which allows us to obtain a conservative bound for the slepton masses. Small $y_\nu$ also allows us to neglect the threshold effects at the $B - L$ breaking scale. Actually this assumption is preferable to avoid the large lepton-flavor-violating-interaction (LFVI) rates. The off-diagonal elements in the slepton masses at the scale $\mu$ are roughly given by

$$(m_L^2)_{ij} \sim \frac{1}{16\pi^2} m_0^2(y_\nu^i y_\nu^j)_{ij} \log \left( \frac{M_G}{\mu} \right)$$

(3.3)
in the minimal supergravity (mSUGRA) scenario. In the models with the noscale boundary condition, the universal soft scalar masses vanish at the GUT scale, hence the rate of the LFVIs are expected to be suppressed. However, in the presence of a gauged U(1)$_{B-L}$ symmetry, there exists an additional contribution to the slepton masses from the gaugino of this extra U(1) gauge multiplet. This contribution is mainly induced at high energy scales near the GUT scale, because of the non-asymptotic freedom of the U(1) gauge symmetry. Therefore, in our model, it is expected that the rate of the LFVIs are not so much suppressed compared with those of the mSUGRA scenario [12]. In the following analyses, we set the $B - L$ breaking scale, which is roughly equal to the right-handed Majorana neutrino masses, to be $10^{10}\text{GeV}$. This is low enough to satisfy the constraints coming from the LFVIs.

Secondly, we use the SOFTSUSY code [13] to evolve the gauge, Yukawa couplings and the soft SUSY breaking parameters down to the weak scale. The Dirac neutrino Yukawa coupling $y_\nu$ is not included in this code, but this is not a problem; the effects of the neutrino Yukawa coupling
on the MSSM parameters disappear below the $B-L$ breaking scale, at least at the one-loop level. The two loop effects are expected to be negligibly small.

This code does not only solve the RGEs, but calculates the one-loop self energies of all the particles and determines the physical pole masses by identifying the pole of the propagator [14]. It determines the physical mass spectrum which is consistent with the boundary conditions at the high energy scale (the $U(1)_{B-L}$ breaking scale in this case) and the low energy scale. The low energy boundary conditions are: $\overline{\text{MS}}$ masses of the quarks and leptons at energy scale $Q = 91.19$ GeV, top quark pole mass, $\overline{\text{MS}}$ and $\overline{\text{MS}}$ gauge couplings of SU(3) and $U(1)_{\text{em}}$. This is done by performing the following iteration procedure: (step 1) evolve boundary conditions at $M_G$ to the $U(1)_{B-L}$ breaking scale by one-loop RGEs → (step 2) input the running (DR) soft parameters to the SOFTSUSY code → (step 3) SOFTSUSY finds a physical spectrum consistent with high and low energy boundary conditions → (step 4) run the DR gauge and Yukawa couplings up to the $U(1)_{B-L}$ breaking scale by SOFTSUSY, which are in general different from those obtained at (step 1) → (step 5) run the DR gauge and Yukawa couplings to high energies and determine $M_G$, then run them back to the $U(1)_{B-L}$ breaking scale by SOFTSUSY, which are in general different from those obtained at (step 1) → (step 2) → (step 3) ... This iteration procedure is performed until the gauge and Yukawa couplings match at the $U(1)_{B-L}$ breaking scale, that is, at (step 2) and (step 4). This procedure amounts to taking into account the loop effects and the weak scale SUSY threshold corrections, and it determines the mass spectrum precisely.

Determining the mass spectrum and the running DR parameters (the gauge couplings, the Yukawa couplings, the $\mu$ term, the $B\mu$ term, and the $A$ terms) allows us to identify the region of the parameter space of the model which is excluded by particle search experiments and cosmological requirements. We specifically consider the lower bounds on the masses of the Higgs boson (114.1 GeV [14]) and the selectron (99 GeV [16]) from the LEP experiments. We also consider the ratio of the stau mass to the neutralino mass, $m_{\tilde{\tau}_1}/m_{\tilde{\chi}}$, which should be larger than 1 to avoid charged LSP. In addition, bounds from experiments of rare processes can also be applied to restrict the parameter space. We consider the branching ratio (BR) of $b \rightarrow s\gamma$ in particular, since it provides a powerful experimental testing ground for physics beyond the SM, because of its sensitivity to virtual effects of new particles. In this work, we perform a calculation of $\text{BR}(b \rightarrow s\gamma)$ based on Ref. [17], which includes the dominant next-to-leading-order (NLO) corrections enhanced by large $\tan\beta$ factors. For the conservative bound, we adopt $2.0 \times 10^{-4} < \text{BR}(b \rightarrow s\gamma) < 4.5 \times 10^{-4}$ [18].

3 The pole mass of the Higgs boson $m_h$ is quite sensitive to the top quark pole mass $m_t$ (with its dependence $\partial m_h/\partial m_t > 0$). We set $m_t = 175.0$ GeV in our analyses.

4 Actually, the lower bound on the Higgs boson mass in the MSSM is somewhat weaker in large $\tan\beta$ region. However, in such a region, the most severe bound comes from the BR($b \rightarrow s\gamma$) or the requirement of neutral LSP, and hence the allowed region of the parameter space is not altered by adopting the Higgs boson mass bound ($m_h = 114.1\text{GeV}$).
from CLEO experiments. The bounds we adopted cast the most stringent constraints on the model.

We provide some comments on the Higgs boson mass. As is explained in Ref. [13], the SOFTSUSY code predicts the Higgs boson mass to be systematically 2–4 GeV heavier than the combination of the codes SSARD [19] and FeynHiggs [20]. We adopt SOFTSUSY, since this code performs calculations of full one-loop self energies or accurate approximations of them based on Ref. [14] to determine the pole masses of SUSY particles, and since it predicts the larger Higgs boson mass which gives us a more conservative bound.

3.2 Results and their implications
3.2.1 The minimal noscale model

Before the results for the SU(5) × U(1) model, we show the result for the minimal noscale model in Figs. 1. The figure on the left corresponds to the case \( \mu > 0 \), and the one on the right corresponds to the case \( \mu < 0 \). This model is the conventional MSSM with the noscale boundary condition, i.e., \( m_0 = 0 \) for all scalar soft masses and nonzero universal gaugino masses \( M_{1/2} \neq 0 \) at \( M_G \). The red (solid) line is the contour of the Higgs boson mass \( m_h = 114.1 \) GeV. The region below this line is currently excluded. The blue (dashed) line is the upper bound on \( M_{1/2} \) from the cosmological requirement that the stau is not the LSP. The green (dotted) line is the lower bound on \( M_{1/2} \) from the \( b \to s\gamma \) experiments. The orange (dot-dashed) line is the contour of the right-handed selectron mass \( m_{\tilde{e}_R} = 99 \) GeV. The purple (solid) line in the case \( \mu > 0 \) denotes the predicted \( \tan \beta \) when we set the condition \( B\mu = 0 \) at the GUT scale.

The black shaded region on the upper right side is the part where the radiative electroweak symmetry breaking (EWSB) cannot be implemented. The constraint from \( b \to s\gamma \) is much stringent in the case \( \mu < 0 \), since the SUSY contribution to the BR(\( b \to s\gamma \)) interferes constructively with the SM contribution, which is opposite to the case \( \mu > 0 \). However, as is already shown in Refs. [3, 4], we can see that there is almost no region which simultaneously satisfies these constraints even in the case \( \mu > 0 \). The obstacle is mainly due to the fact that in noscale models, the lightest neutralino (mostly bino) is nearly degenerate with the right-handed stau and, unfortunately, is slightly heavier than the stau.

If we can alter the particle spectrum and make the stau heavier, the requirement of neutral LSP becomes less restrictive and a wider parameter region compatible with the constraints may arise. In this work, we work on the minimal extension of the MSSM with a gauged U(1)\(_{B-L}\) symmetry. As we will see in the remaining sections, the extra positive contribution to the stau mass from the U(1)\(_{B-L}\) gaugino loops makes the noscale boundary condition consistent with all

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5As for the case \( \mu < 0 \), the RG effects of the gauginos on the \( B\mu \) term is of the opposite sign compared with the case \( \mu > 0 \), and hence \( B\mu = 0 \) at the GUT scale cannot be consistent with the EWSB.
the constraints. In addition, if we relax the noscale boundary condition and allow the non-zero soft scalar masses of the order of the gaugino mass for the fields \(S, \bar{S}\) at the GUT scale, the \(U(1)_{B-L}\) \(D\)-term contribution gives us another solution for the charged LSP problem.

### 3.2.2 \(SU(5)\times U(1)_5\) model

Now we go to the \(SU(5)\times U(1)_5\) model. First of all, consider the case where the extended gauge coupling unification and the gaugino mass relation are imposed:

\[
\begin{align*}
  g_1 &= g_2 = g_3 = g_5, \\
  \frac{M_1}{g_1^2} &= \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2} = \frac{M_5}{g_5^2}.
\end{align*}
\]

They are natural assumptions for the case in which the gauge groups \(SU(5)\times U(1)_5\) are embedded in a larger gauge group, such as \(SO(10)\). The result for the case \(\mu > 0\) is shown in Fig.2. The conventions are the same as those in the Figs.1. The parameter region consistent with \(B\mu = 0\) at the GUT scale is not shown in this figure, since it is almost the same as in the minimal noscale model. The black shaded regions in the lower and upper right side are the parameter spaces where either the EWSB does not occur, or tachyonic scalars arise. We can see that there is almost no region which is compatible with the constraints. The result for the case \(\mu < 0\) is not presented, but it is also almost the same as in the minimal noscale model with \(\mu < 0\) shown in the Figs.1.

To understand this result, we explain the phenomenology of the \(SU(5)\times U(1)_5\) model.

First of all, consider the ratio of the stau mass to the neutralino mass, \(m_{\tilde{\tau}_i}/m_{\tilde{\chi}}\). The RGEs of the three standard model gaugino masses are unaffected by gauging the \(U(1)_{B-L}\) symmetry (as is explained in section 2). On the other hand, new sources for the squark and slepton masses exist. One is the additional positive RGE effects from the \(U(1)_5\) gaugino mass, and another is the \(D\)-term contributions due to the spontaneous breaking of the \(U(1)_5\). The \(D\)-term contribution to the mass squared of a scalar field \(\phi_i\) is approximately given by

\[
(\Delta m_i^2)_{D\text{-term}} = \frac{1}{\sqrt{10}}(m_S^2 - m_{\bar{S}}^2)Q_5^i,
\]

where \(Q_5^i\) is the \(U(1)_5\) charge of \(\phi_i\) and \(m_S^2, m_{\bar{S}}^2\) are the soft masses for the fields \(S, \bar{S}\). Here, small mixing effects are neglected. In our numerical calculations, all of these effects are included by using the diagonal basis given in Eq. (2.20). This contribution is added at the \(B - L\) breaking scale \(g_5v\), and is renormalized down to lower energy scales. As for the detailed discussions about the \(D\)-term contributions to soft scalar masses, see Refs. [21, 22]. Eq.(3.6) is zero at \(M_G\) if we impose the noscale boundary condition, but it is nonzero (and negative for particles with
negative $U(1)_{5}$ charges) at the breaking scale of the $U(1)_{5}$. This is because $m_{S}^{2}$ receives a negative contribution from the renormalization by the Yukawa coupling $\lambda_{2}$ which is absent for $m_{S}^{2}$, and hence $m_{S}^{2} - m_{S}^{2} > 0$. Due to these effects, the mass spectrum is shifted from that of the minimal noscale model. In particular, the mass squared of the right-handed slepton at the weak scale is approximately shifted by an amount (neglecting the mixing effects)

$$\Delta m_{\tilde{e}_{R}}^{2} = \frac{2(Q_{5}^{\tilde{e}_{R}})^{2}}{b_{5}} M_{5}^{2} \left[ 1 - \frac{1}{1 + \frac{b_{5}}{2\pi \alpha_{5} \log \left( \frac{M_{G}}{\mu_{B-L}} \right)}} \right] - \frac{1}{20} (m_{S}^{2} - m_{S}^{2}),$$

(3.7)

where $M_{5}$ is the $U(1)_{5}$ gaugino mass at $M_{G}$ and $m_{S}^{2}$, $m_{S}^{2}$ are evaluated at the $B - L$ breaking scale. The constants are defined as $b_{5} \equiv 57/5$ and $Q_{5}^{\tilde{e}_{R}} (= -1/2\sqrt{10})$ is the $U(1)_{5}$ charge of the right-handed selectron, and $\mu_{B-L}$ is the $B - L$ breaking scale. In the second line, we use $\alpha_{5}^{-1} (= \alpha_{\text{GUT}}^{-1}) = 24$ and $\mu_{B-L} = 10^{10}\text{GeV}$.\[7\]

This shows that the right-handed sleptons acquire positive soft masses from the $U(1)_{5}$ gaugino, and negative contributions from the $U(1)_{5}$ $D$ term. The $D$-term contribution depends on the size of the Yukawa coupling $\lambda_{2}$. If we assume $\lambda_{2}$ has a comparable size as the $U(1)_{5}$ gauge coupling $g_{5}$ at the $U(1)_{5}$ breaking scale, this $D$-term contribution is relatively small.\[8\] Because of the small coefficients in Eq. (3.7), the resultant selectron mass is almost the same as in the minimal noscale model. The Higgs boson mass also remains the same as that in the minimal noscale model since the contribution to the stop masses from the $U(1)_{5}$ gaugino is also very small and the stop masses at the weak scale are dominated by the gluino-loop contribution. This is why the lower bound on the Higgs boson mass and the requirement of neutral LSP still conflict with each other. Therefore, if we assume that the gauge groups $SU(5) \times U(1)_{5}$ are embedded in a larger gauge group such as SO(10), we cannot set the noscale boundary condition at the GUT scale without additional assumptions.\[9\] If the SO(10) gauge group breaks down into the gauge groups $SU(5)_{\text{GUT}} \times U(1)_{5}$ above the GUT scale, the conditions given in Eqs. (3.4) and (3.5) do not hold. However, the above conclusion is not altered, because the gauge coupling $g_{5}$ is smaller than those of the MSSM gauge groups at the scale $M_{G}$ due to the non-asymptotic freedom of the $U(1)_{5}$.

\[8\] After taking the mixing effects into account, soft breaking masses increase by a small amount. For example, if we assume the extended GUT relations given in Eqs. (3.4) and (3.5), the right-handed selectron mass squared $m_{\tilde{e}_{R}}^{2}$ at the weak scale becomes about 1% larger by including the mixing effects.

\[9\] If we take $\lambda_{2} \simeq 0.1g_{5}$ at the $U(1)_{5}$ breaking scale ($\simeq 10^{10}\text{GeV}$), the $D$-term contribution is about 10% of the gaugino contribution, which is the first term of Eq. (3.7). It reaches 40% of the gaugino contribution, if we take $\lambda_{2} \simeq 0.8g_{5}$ at the breaking scale. Through out this paper, we assume $\lambda_{2} \simeq 0.1g_{5}$ at the $U(1)_{5}$ breaking scale ($\simeq 10^{10}\text{GeV}$) to obtain a conservative bound on the slepton masses.

\[9\] Because of the reason mentioned in footnote 2, this is also true for the $SU(3)_{C} \times SU(2)_{L} \times U(1)_{R} \times U(1)_{B-L}$ model embedded in SO(10).
Now, let us relax the extended GUT relations given in Eqs. (3.4) and (3.5). In this case, we have no reason to expect that the gauge coupling $g_5$ and the gaugino mass $M_5$ of the $U(1)_5$ are the same as those of the MSSM gauge groups, since the vector multiplet of the $U(1)_5$ does not belong to the $SU(5)_{GUT}$ vector multiplet above the GUT scale. In fact, such a $SU(5)$ unification model (rather than a $SO(10)$ GUT) is also desirable for obtaining bimaximal mixings among the lighter neutrinos [23, 24], since the leptons and quarks reside in the different multiplets. In this case, we can easily obtain much larger slepton masses and satisfy all the phenomenological bounds by increasing $M_5$. (One can easily see from Eq. (3.7) that the resultant spectrum can be altered only slightly even if we significantly increase the gauge coupling $g_5$.) In the following representative examples of numerical calculations, we set the gaugino mass relation as

\[ M_1 = M_2 = M_3 = M_{1/2}, \]
\[ M_5 = 5 M_{1/2}, \]  

(3.8)

at the GUT scale. As for the relation between the gauge couplings, we use Eq. (3.4) because of the reason mentioned above. The results for the cases $\mu > 0$ and $\mu < 0$ are shown in Figs. 3 and 4, respectively. The conventions are the same as those in Figs. 1. We can see that there exists a wide parameter region consistent with all the constraints, even for the case $\mu < 0$.

4 Relaxing the noscale boundary condition

So far we have considered a model with the noscale boundary condition and a gauged $U(1)_{B-L}$ symmetry. As we have seen, we can easily obtain a spectrum of SUSY particles consistent with experimental and cosmological constraints by imposing the boundary conditions, e.g., Eqs. (3.4) and (3.8). These conditions are expected to be quite plausible in a $SU(5)$ (not a $SO(10)$) unification model, which is preferable to explain the bimaximal mixings among the lighter neutrinos. Unfortunately however, we can set the condition $B\mu = 0$ at the GUT scale only in a small restricted parameter region. Therefore, in this model, although the FCNC interactions are naturally suppressed by virtue of vanishing soft scalar masses at the GUT scale, the SUSY CP problem requires an accidental phase cancellation between the $B\mu$ term and the $\mu$ term, which are expected to have independent phases, in most of the parameter space.

However, there is a quite natural solution to this problem by considering a variation of the model we are working on. The gauged $U(1)_{B-L}$ symmetry inevitably requires a set of new Higgs fields which are singlets under the MSSM gauge groups to spontaneously break the $B-L$ symmetry at some high energy scale. They correspond to the fields $S, \bar{S}$ in our model. By assigning even $B-L$ charges to these fields, as is done in our model, there is no allowed coupling to the MSSM

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10 This is true even if we assume a much larger mass for the gaugino of the $U(1)_5$, $M_5$. 

12
fields at the renormalizable level. (Exact R-parity conservation is also automatically guaranteed.) Therefore, there appears no FCNC problem even if we allow non-zero soft scalar masses for the fields $S$, $\bar{S}$ to be of the order of the gaugino masses at the GUT scale.

These soft scalar masses arise from the nonrenormalizable couplings with the SUSY breaking fields,

$$
\mathcal{L} \supset \int d^4 \theta \left( \lambda_S \frac{Z^\dagger Z}{M_{pl}^2} S + \lambda_{\bar{S}} \frac{Z^\dagger Z}{M_{pl}^2} \bar{S} \right),
$$

where $Z$ stands for the SUSY breaking fields and $\lambda_S$, $\lambda_{\bar{S}}$ are unknown dimensionless couplings. In this case, it is natural to expect that $m_S^2$ and $m_{\bar{S}}^2$ differ by a factor of order one, since there is no reason to believe that the $\lambda_S$ and $\lambda_{\bar{S}}$ are degenerate. Thus, the $D$-term contributions to the soft scalar masses due to the breaking of $U(1)_{B-L}$ do not in general vanish at tree level (see eq.(3.6)). If $m_S^2 - m_{\bar{S}}^2 > 0$, the right-handed slepton masses acquire positive $D$-term contributions (cf. $(2\sqrt{10} Q^e_{\tilde{N}}) = -1$) while the neutralino mass almost remains the same, and the requirement of neutral LSP might be extremely relaxed. Note that this non-vanishing $D$ term also induces relatively large RG effects through the tadpole diagram, which is proportional to $dm_i^2/dt \propto g_5^2 Q^e_i S$ in the limit of vanishing $U(1)$ mixing effects. (See the Appendix for notations.)

Such a situation can be easily realized, for example, in gaugino-mediated SUSY breaking models by allowing the $S$ and $\bar{S}$ fields to propagate in the bulk and to have contact interactions with SUSY breaking fields which reside in the hidden sector brane.

In Fig.5, we show the result when we vary $m_S^2$ and $m_{\bar{S}}^2$. In this analysis, we take $m_S^2 = -m_{\bar{S}}^2$ at the GUT scale for simplicity, and assume the relations in Eqs. (3.4) and (3.5), i.e. the extended unification conditions. We choose $\{M_{1/2}, m_S^2\}$ as free parameters, and $B\mu$ is fixed to be zero at the GUT scale (tan $\beta$ is a prediction rather than a parameter). The two red (solid) lines are the contours of the Higgs boson mass, 114.1 GeV and 120 GeV. The left side of the contour $m_h = 114.1$GeV is excluded by current experiments. The blue (dashed) line denotes the lower bound on $m_S^2 = -m_{\bar{S}}^2$ from the neutral LSP condition $m_{\tilde{\tau}_1}/m_{\tilde{\chi}} \geq 1$. As we can see, the $D$-term contribution is large enough to make the stau heavier than the neutralino, although relatively large tan $\beta(\gtrsim 20)$ is predicted by the condition $B\mu = 0$ at the GUT scale. The green (dotted) lines are the contours of $BR(b \to s\gamma) \times 10^4 = 2.0$ and 2.5, respectively. The left side of the contour $BR(b \to s\gamma) \times 10^4 = 2.0$ is currently excluded. The orange (dot-dashed) line is the contour of the selectron mass $m_{\tilde{e}_R} = 99$GeV, and the inner region surrounded by this line is excluded by the LEP experiment. The thin black lines are the contours for tan $\beta = 20, 22, 24, 26$, respectively. The black shaded region in the upper left is where the EWSB does not occur or tachyonic scalars arise. From Fig. 5, we can see that a parameter region consistent with phenomenological bounds exist for $M_{1/2} \gtrsim 350$ GeV and $m_S^2 = -m_{\bar{S}}^2 \gtrsim (500\text{GeV})^2$. There is still such a region even if the lower bound on the Higgs boson mass is pushed up to 120 GeV.
Before we close this section, we present some comments. Even within the MSSM, we can obtain a SUSY spectrum consistent with the Higgs boson mass bound and the requirement of neutralino LSP by allowing non-zero soft scalar masses for the Higgs fields at the GUT scale. Such a situation can also be easily realized in gaugino-mediation models by allowing the Higgs multiplets to propagate in the bulk [4]. Such a setting may provide a simple solution for generating the $\mu$ term with the correct size by the Giudice-Masiero mechanism [25]. In this case however, the $B\mu$ term is generally expected to be non-zero at the boundary and to have an independent phase from the $\mu$ term. To solve the SUSY CP problem, this requires an accidental phase cancellation between the $\mu$ and $B\mu$ terms with $O(1\%)$.

5 Conclusions and Discussions

Models with the noscale boundary condition naturally solve the SUSY FCNC problem and possibly also the SUSY CP problem. Unfortunately, the minimal noscale model was shown to be not consistent with phenomenological bounds, mainly due to the lower bound on the Higgs boson mass and the cosmological requirement that the charged particle is not the LSP. In this paper, we investigate the minimal extension of the MSSM with a gauged $U(1)_{B-L}$ symmetry, especially the SU(5) $\times U(1)_5$ unification-inspired model, and consider whether the noscale boundary condition at the GUT scale is consistent with phenomenological constraints or not.

First, we consider the case with the extended GUT relations given in Eqs. (3.4) and (3.5), which are quite natural if the MSSM gauge groups and the $U(1)_5$ are unified into a single group, such as SO(10). In this case, we find that the particle spectrum is almost the same as the minimal noscale model and that there exists almost no parameter region consistent with the experimental and cosmological constraints.

Next, we relax the extended GUT relations and assume that the $U(1)_5$ is not unified into a single group. In this case, it is very natural that the gaugino mass for the $U(1)_5$ is different from those of the MSSM gauge groups, which are assumed to be universal, by a factor of order one. As a result, we find that the stau is heavy enough not to be the LSP when the gaugino of the $U(1)_5$ is somewhat heavier than those of the MSSM gauge groups, and that a wide parameter region is consistent with all the constraints. This may imply a SU(5) unification, rather than a SO(10), in models with the noscale boundary condition.

Finally, we consider the case in which the $S$ and $\bar{S}$ fields, which are the Higgs fields to break the $U(1)_5$ spontaneously at an intermediate scale, have non-vanishing soft scalar masses at the GUT scale. This does not introduce any dangerous flavor-violating interaction, but provides a large $D$-term contribution which easily solves the charged LSP problem, even if we impose the extended GUT relations given in Eqs. (3.4) and (3.5). In this case, we can also impose the condition $B\mu = 0$.
at the GUT scale consistently with the EWSB in a wide parameter region, and hence this case is free also from the SUSY CP problem.

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A RGEs for the Yukawa couplings and the soft SUSY breaking terms

In this appendix, we show the list of the RGEs for the Yukawa couplings and soft SUSY breaking terms for the SU(5) × U(1)\textsubscript{5} model. Here, we include the Dirac neutrino Yukawa couplings and their effects to the soft SUSY breaking terms for completeness, which we have neglected in the numerical analyses. As for the kinetic term mixings between the two U(1) gauge multiplets, we neglect them and only show the terms induced from the diagonal parts. When the extended GUT relation given in Eq. (2.16) is imposed, one can easily include all the mixing effects at one-loop level by working in the diagonal basis denoted by Eq. (2.20). Even when that relation is not imposed, one can include the dominant mixing effects on the soft scalar mass terms by replacing the appropriate terms with the terms given in Eq. (2.15) as we have done in this work. Our conventions for soft breaking terms except the gaugino masses are given by

\begin{equation}
V_{\text{soft}} = \tilde{u} A_u \tilde{Q} H_u - \tilde{d} A_d \tilde{Q} H_d - \tilde{e} A_e \tilde{L} H_d + B \mu H_u H_d + \tilde{N} A_\nu \tilde{L} H_u + A_1 \tilde{X} \tilde{S} \tilde{S} + \frac{1}{2} \tilde{S} \tilde{N} A_2 \tilde{N} \\
+ \tilde{Q}^i \tilde{m}_Q^2 \tilde{Q} + \tilde{L}^i \tilde{m}_L^2 \tilde{L} + \tilde{u}_w^2 \tilde{u}_w^i + \tilde{d}_w^2 \tilde{d}_w^i + \tilde{e}_w^2 \tilde{e}_w^i + m^2_{H_u} H_u^2 H_u + m^2_{H_d} H_d^2 H_d \\
+ \tilde{N}_X^2 \tilde{N}_X^i + m^2_{\tilde{X}} \tilde{X}^* \tilde{X} + m^2_{\tilde{S}} \tilde{S}^* \tilde{S} + m^2_{\tilde{S}} \tilde{\tilde{S}}^* \tilde{\tilde{S}}.
\end{equation}

(A.1)

As for the gaugino masses, see Eq. (2.11).

A.1 RGEs for the Yukawa couplings and the \( \mu \) term

\begin{equation}
\frac{d}{dt} (y_\nu)_{ij} = \frac{1}{16 \pi^2} \left[ \begin{array}{c}
-3 g_2^2 - \frac{3}{5} g_1^2 - \frac{19}{10} g_5^2 + 3 \text{Tr}(y_u^\dagger y_u) + \text{Tr}(y_\nu^\dagger y_\nu) \\
+ 3 (y_\nu y_\nu^\dagger)_{ij} + (y_\nu y_\nu^\dagger y_\nu)_{ij} + (\lambda_2 \lambda_2^\dagger)_{ij}
\end{array} \right] (y_\nu)_{ij}
\end{equation}

(A.2)

\begin{equation}
\frac{d}{dt} (y_e)_{ij} = \frac{1}{16 \pi^2} \left[ \begin{array}{c}
-3 g_2^2 - \frac{9}{5} g_1^2 - \frac{7}{10} g_5^2 + 3 \text{Tr}(y_d^\dagger y_d) + \text{Tr}(y_e^\dagger y_e) \\
+ 3 (y_e y_e^\dagger)_{ij} + (y_e y_e^\dagger y_e)_{ij}
\end{array} \right] (y_e)_{ij}
\end{equation}

(A.3)
\[
\frac{d}{dt}(y_{ij}) = \frac{1}{16\pi^2} \left[ \left\{ -\frac{13}{15}g_2^2 - 3g_2^2 - \frac{16}{3}g_3^2 - \frac{3}{10}g_5^2 + 3\text{Tr}(y^\dagger y) + \text{Tr}(y^\dagger y) \right\} (y_{ij}) \right] + 3\text{Tr}(y_{ij}y_{ij}^\dagger) + (y_{ij}y_{ij}^\dagger)_{ij} \\
\frac{d}{dt}(y_{ij}) = \frac{1}{16\pi^2} \left[ \left\{ -\frac{7}{15}g_2^2 - 3g_2^2 - \frac{16}{3}g_3^2 - \frac{7}{10}g_5^2 + 3\text{Tr}(y^\dagger y) + \text{Tr}(y^\dagger y) \right\} (y_{ij}) \right] + 3(y_{ij}y_{ij}^\dagger) + (y_{ij}y_{ij}^\dagger)_{ij} \\
\frac{d}{dt}(\lambda_{ij}) = \frac{1}{16\pi^2} \left[ \left\{ -\frac{15}{2}g_5^2 + \text{Tr}(\lambda_2^\dagger \lambda_2) + \lambda_1^\dagger \lambda_1 \right\} (\lambda_{ij}) \right] + 2(\lambda_2^\dagger \lambda_2)_{ij} + 2(\lambda_2^\dagger \lambda_2)_{ij} + 2(\lambda_2^\dagger \lambda_2)_{ij} \\
\frac{d}{dt}(\lambda_1) = \frac{1}{16\pi^2} \left[ \left\{ -10g_5^2 + \text{Tr}(\lambda_2^\dagger \lambda_2) \right\} \lambda_1 + 3(\lambda_1^\dagger \lambda_1) \right] \\
\frac{d}{dt}(\mu) = \frac{1}{16\pi^2} \left[ \text{Tr} \left\{ 3y^\dagger y + 3y_d^\dagger y_d + y^\dagger y + y^\dagger y \right\} - 3g_2^2 - \frac{3}{5}g_1^2 - \frac{2}{5}g_5^2 \right] \\
\] A.2 RGEs for the soft SUSY breaking terms

\[
S \equiv m_H^2 - m_H^2 + \text{Tr} \left[ m_Q^2 + m_u^2 - 2m_n^2 - m_L^2 + m_e^2 \right] \\
S_5 \equiv 4m_H^2 - 4m_H^2 + 10m_S^2 - 10m_S^2 + \text{Tr} \left[ -6m_Q^2 - 3m_u^2 + 9m_n^2 + 6m_L^2 - m_e^2 - 5m_N^2 \right] \\
\frac{d}{dt}(m_e^2) = \frac{1}{16\pi^2} \left[ 2(m_e^2 y_e y_e^\dagger + y_e y_e^\dagger m_e^2)_{ij} + 4(y_e^\dagger m_l^2 y_e^\dagger + m_e^2 y_e y_e^\dagger + A_e A_e^\dagger)_{ij} \right] + \left\{ -\frac{24}{5}g_1^2 |M_1|^2 - \frac{1}{5}g_5^2 |M_5|^2 + \frac{6}{5}g_2^1 S - \frac{1}{20}g_5^2 S_5 \right\} \delta_{ij} \\
\frac{d}{dt}(m_L^2) = \frac{1}{16\pi^2} \left[ (m_L^2 y_e y_e^\dagger + y_e y_e^\dagger m_L^2)_{ij} + (m_L^2 y_e y_e^\dagger + y_e y_e^\dagger m_L^2)_{ij} + 2(y_e^\dagger m_L^2 y_e^\dagger + m_e^2 y_e y_e^\dagger + A_e A_e^\dagger)_{ij} \right] + \left\{ -6g_2 |M_2|^2 - \frac{6}{5}g_1^2 |M_1|^2 - \frac{9}{5}g_5^2 |M_5|^2 - \frac{3}{5}g_1^2 S + \frac{3}{20}g_5^2 S_5 \right\} \delta_{ij} \\
\frac{d}{dt}(m_N^2) = \frac{1}{16\pi^2} \left[ 2(m_N^2 y_e y_e^\dagger + y_e y_e^\dagger m_N^2)_{ij} + 4(y_e^\dagger m_N^2 y_e^\dagger + m_N^2 y_e y_e^\dagger + A_e A_e^\dagger)_{ij} \right] + \left\{ -5g_2^2 |M_5|^2 - \frac{1}{4}g_5^2 S_5 \right\} \delta_{ij} \\
(A.4) \\
(A.5) \\
(A.6) \\
(A.7) \\
(A.8) \\
(A.9) \\
(A.10) \\
(A.11) \\
(A.12) \\
(A.13)
\[
\frac{d}{dt}(m_Q^2)_{ij} = \frac{1}{16\pi^2} \left[ (m_Q^2 y_u^i y_u + y_u^i y_u m_Q^2)_{ij} + (m_Q^2 y_d^i y_d + y_d^i y_d m_Q^2)_{ij} 
\right. \\
+ 2(y_u^i m_u^2 y_u + y_u^i y_u m_H^2 + A_u^i A_u)_{ij} + 2(y_d^i m_d^2 y_d + y_d^i y_d m_H^2 + A_d^i A_d)_{ij} \\
+ \left\{ -\frac{2}{15} g_1^2 |M_1|^2 - \frac{32}{3} g_3^2 |M_3|^2 - \frac{1}{5} g_5^2 |M_5|^2 + \frac{1}{2} g_5^2 S - \frac{1}{20} g_5^2 S_5 \right\} \delta_{ij} \right]
\]  
(A.14)

\[
\frac{d}{dt}(m_d^2)_{ij} = \frac{1}{16\pi^2} \left[ 2(m_d^2 y_d y_d^i + y_d y_d^i m_d^2)_{ij} + 4(y_d m_d^2 y_d^i + m_d^2 y_d y_d^i + A_d A_d^i)_{ij} 
\right. \\
+ \left\{ -\frac{8}{15} g_1^2 |M_1|^2 - \frac{32}{3} g_3^2 |M_3|^2 - \frac{9}{5} g_5^2 |M_5|^2 + \frac{2}{5} g_5^2 S + \frac{3}{20} g_5^2 S_5 \right\} \delta_{ij} \right]
\]  
(A.15)

\[
\frac{d}{dt}(m_u^2)_{ij} = \frac{1}{16\pi^2} \left[ 2(m_u^2 y_u y_u^i + y_u y_u^i m_u^2)_{ij} + 4(y_u m_u^2 y_u^i + m_u^2 y_u y_u^i + A_u A_u^i)_{ij} 
\right. \\
+ \left\{ -\frac{32}{15} g_1^2 |M_1|^2 - \frac{32}{3} g_3^2 |M_3|^2 - \frac{1}{5} g_5^2 |M_5|^2 - \frac{4}{5} g_5^2 S - \frac{1}{20} g_5^2 S_5 \right\} \delta_{ij} \right]
\]  
(A.16)

\[
\frac{d}{dt} m_{H_d} = \frac{1}{16\pi^2} \left[ 6 \text{Tr} \left[ m_{H_d}^2 y_d y_d^i + y_d m_{H_d}^2 y_d + A_d A_d^i \right] 
\right. \\
+ 2 \text{Tr} \left[ m_{H_d}^2 y_e y_e^i + y_e m_{H_d}^2 y_e + A_e A_e^i \right] \\
+ \left\{ -6 g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 - \frac{4}{5} g_3^2 |M_3|^2 - \frac{3}{5} g_5^2 S - \frac{1}{10} g_5^2 S_5 \right\} \right]
\]  
(A.17)

\[
\frac{d}{dt} m_{H_u} = \frac{1}{16\pi^2} \left[ 6 \text{Tr} \left[ m_{H_u}^2 y_u y_u^i + y_u m_{H_u}^2 y_u + A_u A_u^i \right] 
\right. \\
+ 2 \text{Tr} \left[ m_{H_u}^2 y_v y_v^i + y_v m_{H_u}^2 y_v + A_v A_v^i \right] \\
+ \left\{ -6 g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2 - \frac{4}{5} g_3^2 |M_3|^2 + \frac{3}{5} g_5^2 S + \frac{1}{10} g_5^2 S_5 \right\} \right]
\]  
(A.18)

\[
\frac{d}{dt} m_S^2 = \frac{1}{16\pi^2} \left[ 2 m_S^2 \text{Tr}(\lambda_2^\dagger \lambda_2) + 2 \text{Tr} \left[ \lambda_2^\dagger m_X^2 \lambda_2 + \lambda_2 m_X^2 \lambda_2^* + A_2^\dagger A_2 \right] 
\right. \\
+ \left\{ -20 g_5^2 |M_5|^2 + \frac{1}{2} g_5^2 S_5 \right\} \right]
\]  
(A.19)

\[
\frac{d}{dt} m_S^2 = \frac{1}{16\pi^2} \left[ 2(m_S^2 + m_X^2 + m_S^2)^\dagger \lambda_1 + 2 A_1^\dagger A_1 
\right. \\
+ \left\{ -20 g_5^2 |M_5|^2 - \frac{1}{2} g_5^2 S_5 \right\} \right]
\]  
(A.20)
\[
\frac{d}{dt} m_X = \frac{1}{16\pi^2} \left[ 2(m_X^2 + m_S^2 + m_{\tilde{S}}^2)\lambda_1^\dagger\lambda_1 + 2A_1^\dagger A_1 \right]
\] (A.21)

\[
\frac{d}{dt} (A_e)_{ij} = \frac{1}{16\pi^2} \left[ 3\text{Tr}(\bar{y}_d y_d) + \text{Tr}(\bar{y}_e y_e) \right] (A_e)_{ij} + 2 \left[ 3\text{Tr}(\bar{y}_d A_d) + \text{Tr}(\bar{y}_e A_e) \right] (y_e)_{ij} \\
+4(y_e y_e^\dagger A_e)_{ij} + 5(A_e y_e y_e)_{ij} + 2(y_e y_e^\dagger A_e)_{ij} + (A_e y_e y_e)_{ij} \\
+ \left\{ -3g_2^2 - \frac{9}{5}g_1^2 - \frac{7}{10}g_5^2 \right\} (A_e)_{ij} + 2 \left\{ 3g_2^2 M_2 + \frac{9}{5}g_1^2 M_1 + \frac{7}{10}g_5^2 M_5 \right\} (y_e)_{ij}
\] (A.22)

\[
\frac{d}{dt} (A_\nu)_{ij} = \frac{1}{16\pi^2} \left[ 3\text{Tr}(\bar{y}_d y_d) + \text{Tr}(\bar{y}_e y_e) \right] (A_\nu)_{ij} + 2 \left[ 3\text{Tr}(\bar{y}_d A_d) + \text{Tr}(\bar{y}_e A_e) \right] (y_\nu)_{ij} \\
+4(y_\nu y_\nu^\dagger A_\nu)_{ij} + 5(A_\nu y_\nu y_\nu)_{ij} + 2(y_\nu y_\nu^\dagger A_\nu)_{ij} + (A_\nu y_\nu y_\nu)_{ij} \\
+ (\lambda_2 \lambda_2^\dagger A_\nu)_{ij} + 2(A_2 A_2^\dagger y_\nu)_{ij} + \left\{ -3g_2^2 - \frac{3}{5}g_1^2 - \frac{19}{10}g_5^2 \right\} (A_\nu)_{ij} \\
+ 2 \left\{ 3g_2^2 M_2 + \frac{3}{5}g_1 M_1 + \frac{19}{10}g_5^2 M_5 \right\} (y_\nu)_{ij}
\] (A.23)

\[
\frac{d}{dt} (A_u)_{ij} = \frac{1}{16\pi^2} \left[ 3\text{Tr}(\bar{y}_d y_d) + \text{Tr}(\bar{y}_e y_e) \right] (A_u)_{ij} + 5(A_u y_d y_d)_{ij} \\
+ 6\text{Tr}(A_u y_u^\dagger)(y_u)_{ij} + 4(y_u y_u^\dagger A_u)_{ij} + 2(y_u y_u^\dagger A_d)_{ij} \\
+ \left\{ -\frac{16}{3}g_3^2 - g_2^2 - \frac{13}{15}g_1^2 - \frac{3}{10}g_5^2 \right\} (A_u)_{ij} \\
+ 2 \left\{ \frac{16}{3}g_3^2 M_3 + 3g_2^2 M_2 + \frac{13}{15}g_1^2 M_1 + \frac{3}{10}g_5^2 M_5 \right\} (y_u)_{ij}
\] (A.24)

\[
\frac{d}{dt} (A_d)_{ij} = \frac{1}{16\pi^2} \left[ \text{Tr} \left[ 3y_d y_d + y_e y_d^\dagger \right] \right] (A_d)_{ij} + 5(A_d y_d y_d)_{ij} + (A_d y_d^\dagger y_u)_{ij} \\
+ \text{Tr} \left[ 6A_d y_d + 2A_d y_e^\dagger \right] (y_d)_{ij} + 4(y_d y_d^\dagger A_d)_{ij} + 2(y_d y_u^\dagger A_u)_{ij} \\
+ \left\{ -\frac{16}{3}g_3^2 - g_2^2 - \frac{7}{15}g_1^2 - \frac{7}{10}g_5^2 \right\} (A_d)_{ij} \\
+ 2 \left\{ \frac{16}{3}g_3^2 M_3 + 3g_2^2 M_2 + \frac{7}{15}g_1^2 M_1 + \frac{7}{10}g_5^2 M_5 \right\} (y_d)_{ij}
\] (A.25)
\[
\frac{d}{dt} A_2 = \frac{1}{16\pi^2} \left[ 3(\lambda_1^2\lambda_2)(A_2)_{ij} + 6(\lambda_1\lambda_2^2)(A_2)_{ij} + 3(A_2\lambda_1^2\lambda_2)_{ij} \\
+ 2Tr(\lambda_2^2 A_2)(\lambda_2)_{ij} + (\lambda_1^2\lambda_1)(A_2)_{ij} + 2(\lambda_1^2 A_1)(\lambda_2)_{ij} \\
+ 2(y_\nu y_\nu^T A_2)_{ij} + 2(y_\nu y_\nu^T A_2)_{ji} + 4(A_\nu y_\nu^T \lambda_2)_{ij} + 4(A_\nu y_\nu^T \lambda_2)_{ji} \\
+ \left\{ -\frac{15}{2} g_5^2 \right\} (A_2)_{ij} + 2 \left\{ \frac{2}{15} g_5^2 M_5 \right\} (\lambda_2)_{ij} \\
\right] \\
\frac{d}{dt} A_1 = \frac{1}{16\pi^2} \left[ 3(\lambda_1^2\lambda_1^2)A_1 + 6(\lambda_1\lambda_1^2 A_1) + Tr(\lambda_1^2\lambda_2^2)A_1 + 2Tr(\lambda_1^2 A_2)\lambda_1 \\
+ \left\{ -10g_5^2 \right\} A_1 + 2 \left\{ 10g_5^2 M_5 \right\} \lambda_1 \\
\right] \\
\frac{d}{dt} B\mu = \frac{1}{16\pi^2} \left[ B\mu \left\{ Tr \left[ 3y_u^T y_u + 3y_d^T y_d + y_e^T y_e + y_\nu^T y_\nu \right] - 3g_2 - \frac{3}{5} g_1 - \frac{2}{5} g_5^2 \right\} \\
+ \mu \left\{ Tr \left[ 6y_u^T A_u + 6y_d^T A_d + 2y_e^T A_e + 2y_\nu^T A_\nu \right] + 6g_2^2 M_2 + \frac{6}{5} g_1^2 M_1 + \frac{4}{5} g_5^2 M_5 \right\} \\
\right]
\]

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Figure 1: Constraints on the tan$\beta - M_{1/2}$ plane for the minimal noscale model with $\mu > 0$ (left) and $\mu < 0$ (right), respectively. The blue (dashed) line denotes the upper bound on $M_{1/2}$ for the neutralino to be the LSP. The purple (solid) line for the case $\mu > 0$ denotes the parameter region predicted from the condition $B\mu = 0$ at the GUT scale. The other lines correspond to the lower bounds on $M_{1/2}$ from various constraints, which are those from the Higgs boson mass (red solid), the selectron mass (orange dot-dashed) and the BR($b \to s\gamma$) (green dotted), respectively. In the black shaded region, the EWSB does not occur or tachyonic scalars emerge. The light shaded region is allowed (present only for the case $\mu > 0$).

Figure 2: Constraints on the tan$\beta - M_{1/2}$ plane for the SU(5)×U(1)$_5$ inspired model with $\mu > 0$ and the extended GUT relations given in Eqs. (3.4) and (3.5). The conventions are the same as those in Figs. 1. The light shaded region is allowed.
Figure 3: The same as Fig. 2 but with the condition $M_5 = 5M_{1/2}$ at the GUT scale. Now there is a wide allowed region.

Figure 4: The same as Fig. 3 but for $\mu < 0$. Even in this case, a wide region is allowed.
Figure 5: Constraints on the model with $m_S^2 = -m_{\tilde{S}}^2 (\neq 0)$, $B\mu = 0$ and with the conditions given in Eqs. (3.4), (3.5) at the GUT scale. The thin black (solid) lines are the contours of the predicted $\tan\beta$, which are 20, 22, 24, and 26, respectively. Other conventions are the same as those in Figs. [4].