Lorentz Force Correction and Radiation Frequency Property 
of Charged Particles in Magnetic Dipole

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(Dated: October 29, 2018)

By concern of compression of charge density field, the corrected Lorentz force formula and consequent inference is presented. And further radiation frequency property of an individual charge density field in magnetic dipole is analyzed respectively for radiant property of the charged particle and the emitted electromagnetic wave transfer property between the moving radiant source and observer. As results, the behavior and radiation frequency property of the electron beam in magnetic dipole is interpreted upon the individual’s behavior and property. At final, the potential application is put forward for wider interest.

I. INTRODUCTION

In view point of electrical charge density field [1], Lorentz interaction of individual charge and magnetic field can happen only in a specified space — overlapped volume of the charge density field and magnetic field. When the volume is in compressed status or varies with a change of the charge internal potential energy due to energy exchange, the Lorentz force will be affected since the magnetic action amount is proportion to the volume; consequently the centrifugal force’s variation will result in the angular velocity’s variation and radiant power’s of the charged particle. By considering the charge’s internal structure and compressed status, Lorentz force correcting factor is studied. In addition, for the Lorentz force to work for causing electromagnetic radiation, there will be a corresponding property on the radiation frequency of a single particle during a single pass; however on the radiation property involving the particle’s radiant frequency, observer detecting frequency and radiant power, currently relevant study is not complete and explicit yet in some points for neglecting the charge’s micro-structure and undistinguishing the particle charge itself’s radiation in magnetic field from detecting radiation at observer location [2, 3, 4], while it is one of our studying subjects following. Further based on the new results of Lorentz force correction and radiation frequency property for single charged particle in magnetic dipole, we give a qualitative interpretation on the relevant behavior or property of collectively charged particles or macro beam element during it passes through a magnetic dipole, and discuss verifying method on our theorem as well as important application.

II. DERIVATION OF THE CORRECT RELATION AND INFERENCE

A. Corrected Formula of Lorentz Force

For a micro charge element within a charge density field, where \( dq_i = \rho_i(t) dV_i(t) = \rho_i(r_{aic}) dV_0(r_{aic}) \) and \( \int \rho |r_{nic}(t)| dV = \int \rho_0(r_{nic}) dV_0 = q \) (Ref. [1]), there exists a corrected Lorentz force exerted on the \( dq_i \) as below:

\[
d\vec{f}[r_{nic}(t)] = dq_i \vec{v}[r_{nic}(t)] \times \vec{B}[r_{nic}(t)] k_i[r_{nic}(t)] \quad (1)
\]

\[
k_i(t) = \frac{dV_i}{dV_0} = e^{-\eta_i(t)} \quad \eta_i \in [0, \infty) \quad k_i \in (0, 1]
\]

* Author thanks Fang-an Wang for valuable discussion.
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Integrating Eq. (1)

$$\int [r_{cc}(t)] = \int \int \int \rho(r_{ic}(t))[\bar{v}[r_{ic}(t)] \times \vec{B}[r_{ic}(t)]dV[r_{ic}(t)]$$

$$= \bar{v}_{cc}(t) \times \vec{B}(t)k(t)q$$  \hspace{1cm} (2)

where $k(t)$ is average compression ratio and $\bar{k}(t) = e^{-\eta(t)}$.

**B. Consequent Inferences**

**A. Angular velocity relation of a charged particle with mass $m_q$ in uniform magnetic dipole**

From

$$\rho(t)\frac{d\bar{n}_\rho(t)}{dt} = \rho(t)\omega(t)\bar{n}_\perp \rho(t) = \rho(t)\omega_\rho(t)\bar{n}_\rho(t) = \bar{v}(t)\bar{n}_\rho(t)$$  \hspace{1cm} (3)

Take derivative Eq. (3) with respect to $t$

$$[\dot{\rho}(t)\omega_\rho(t) + \rho(t)\dot{\omega}_\rho(t)]\bar{n}_\rho - \rho\omega_\rho^2(t)\bar{n}_\rho = \bar{v}(t)\bar{n}_\rho + v(t)\omega_\rho(t)\bar{n}_\perp \rho$$  \hspace{1cm} (4)

From Eq. (2) and Eq. (4) there

$$m_q\rho(t)\omega_\rho^2(t) = \bar{k}(t)qB(t)$$  \hspace{1cm} (5)

where $\rho(t)$ is curvature radius of the charge center of a charged particle in magnetic dipole, $\bar{n}_\rho(t)$, $\bar{n}_\rho(t)$ is unit vector of $\vec{\rho}(t)$, $\vec{\tau}(t)$ respectively, $\vec{\rho} = \rho\bar{n}_\rho$, $\vec{\tau} = v\bar{n}_\rho$; $m_q$ — rest mass of charged particle.

There

$$\omega_\rho(t) = \frac{\bar{k}(t)qB}{m_q} = \frac{qB}{m_q} \frac{1}{e^{\eta(t)}}$$  \hspace{1cm} (6)

That is, the particle’s angular or revolution frequency is inversely proportional to the internal potential energy of the particle or to the compression status of the charge density field.

And

$$\frac{d\omega_\rho(t)}{dt} = \frac{qB}{m_q} \frac{d\bar{k}(t)}{dt} = -\frac{qB}{m_q}e^{-\eta(t)} \frac{d\bar{n}(t)}{dt} = -\omega_\rho(t)\frac{d\bar{n}(t)}{dt}$$  \hspace{1cm} (7)

**B. Resultant force exerted on the charged particle and derivation of the rate of curvature radius of charge center**

$$\vec{f}_q(t) = m_q\bar{v}(t)\bar{n}_\rho + m_qv(t)\omega_\rho(t)\bar{n}_\perp \rho$$

$$= m_q(\bar{\rho}\omega_\rho + \rho\dot{\omega}_\rho)\bar{n}_\rho + m_q\rho\omega_\rho^2\bar{n}_\perp ca$$

$$= m_q(\bar{\rho}\omega_\rho + \rho\dot{\omega}_\rho)\bar{n}_\rho + \bar{k}(t)q\bar{\tau}(t) \times \vec{B}$$

$$= f_v(t)\bar{n}_\rho(t) - f_\rho\bar{n}_\rho(t)$$  \hspace{1cm} (6)

In frame fixated on the particle, according to the fact that the rate of line velocity, denoted as $a_/\perp(t)$, results from centrifugal acceleration $a_/\perp(t)$, in other words $a_/\perp(t)$ is determined by $a_/\perp(t)$ and its value is just profile of the centrifugal acceleration on $\bar{n}_\perp \rho$ direction.

So there

$$\frac{a_/\perp(t)}{a_/\perp(t)} = \frac{\dot{\bar{v}}(t)}{a_/\perp(t)} = \pm \gamma\omega_\rho(t)$$  \hspace{1cm} (7)
where $\tau = 1$ second, and as $\dot{v} > 0$, take the symbol as $"+"$; as $\dot{v} < 0$, take the symbol as "$-"$. To substitute $a_\perp(t) = v(t)\omega_\rho(t)$ to Eq. (7), then

$$\dot{v} = \pm^*\tau v(t)\omega_\rho^2(t) = \begin{cases} *\tau v^2 & \dot{v} > 0 \\ -^*\tau v^2 & \dot{v} < 0 \end{cases}$$

Resolve Eq. (4) by using Eq. (7′) thus

$$\dot{\rho}(t) = \dot{v}/\omega_\rho - \rho\dot{k}/k = \pm^*\tau \rho \frac{\dot{k}(t)qBv(t)}{m_q} - \rho \frac{\dot{k}}{k}$$

C. Radiation power formula

(i) Radiant power formula of an electron in magnetic dipole

In case $\dot{v} < 0$, $\dot{k} > 0$, by using Eq. (6) or $P_{kl}(t) = \int f_q(t) \cdot \dot{r}(t) = m_qv\dot{v}$ and $\dot{v} = -^*\tau v^2$, $E_p = \mu e(\ln \bar{k} - 1 + \bar{k} - 1)$ [Ref. [1]]

$$P_{rt}(t) = -P_r(t) = P_{kt}(t) + P_{pt}(t) = m_qv\dot{v} + \frac{dE_p(t)}{dt} = \frac{d}{dt} \left[ \frac{m_qv^2(t) + E_p(t) + E_0}{dt} \right] = -^*\tau m_qv^2\omega_\rho^2 - \mu e\frac{1 - \bar{k}}{\bar{k}} = -^*\tau e^2B^2\bar{k}^2(t)v^2(t) - \mu e\frac{1 - \bar{k}}{\bar{k}}$$

$$P_r = ^*\tau m_qv^2\omega_\rho^2 + \mu e\frac{1 - \bar{k}}{\bar{k}}$$

where $P_r$ — radiant power; $P_{lt}$ — power of electron’s total energy loss; $P_{kt}$ — power of electron’s kinetic energy loss; $P_{pt}$ — power of electron’s potential energy loss. $E_p$ — internal potential energy of electron’s charge density field; $E_0$ — electron’s intrinsic energy, and $E_0 = constant$ Ref. [1].

(ii) Detecting power

Ref. [5] and suppose $J_{exc}(t') = 0$, $J_l(t') = 0$; there

$$P_0(t') = J(t') = \int J(t) \frac{dt}{dt'} = P_r(t) \left[ 1 - \frac{dT(t')}{dt'} \right]$$

III. RADIATION FREQUENCY PROPERTY FOR SINGLE PASS OF SINGLE PARTICLE

A. Charge’s radiant frequency property

From

$$\dot{\rho}(t) = \bar{k}(t)\lambda_0\nu(t) = \pm^*\gamma \frac{\bar{k}(t)qBv}{m_q} + \rho \frac{\dot{k}}{k}$$
\[ \nu(t) = \pm \frac{\tau q B v(t)}{m_q \lambda_0} + \rho \frac{\dot{k}}{k^2 \lambda_0} \]  
(12)

and

\[ \frac{d\nu(t)}{dt} = \pm \frac{\tau q B}{m_q \lambda_0} \dot{v}(t) + d\left(\rho \frac{\dot{k}}{k^2 \lambda_0}\right) \]  
(13)

and

\[ \Delta \nu_r = \left[ \pm \frac{\tau q B v(t)}{m_q \lambda_0} + \frac{1}{\lambda_0} \rho(t) \frac{\dot{k}}{k^2(t)} \right] t_b \]
\[ = \pm \frac{\tau q B}{m_q \lambda_0} |v(t_a) - v(t_b)| + \frac{1}{\lambda_0} \left[ \rho(t_a) \frac{\dot{k}(t_a)}{k^2(t_a)} - \rho(t_b) \frac{\dot{k}(t_b)}{k^2(t_b)} \right] \]  
(14)

where \( \lambda_0 \) — diameter of intrinsic charge density field.

Here there is continuous electromagnetic compression status in horizontal direction, \( \vec{n} \rho(t) \); it is resulted from a resultant electrical field action consisted of two induced electrical field in opposite directions; one in centrifugal direction for Lorentz deflection or Lorentz force to work, the other in opposite direction of the displacement for created radiant magnetic field to oppose against the Lorentz work and convert the work into horizontal polarized electromagnetic radiation in tangential direction of the particle’s trajectory.

**B. Observer detecting frequency property**

Ref. [6] there is a frequency transfer relation between the radiant source and observer as following

\[ \nu(t') = \nu(t) \frac{dt}{dt'} = \nu(t) \left[ 1 - \frac{dT(t')}{dt'} \right] \]  
(15)

here \( \frac{dT(t')}{dt'} < 0 \), so \( 1 - \frac{dT(t')}{dt'} > 1 \) and \( \nu(t') > \nu(t) \)

Specifically in optical non-dispersion medium,

\[ \nu(t') = \nu(t) \left[ 1 + \frac{v(t')}{c} \right] \]  
(16)

here suppose \( v_o(t') = 0 \), where \( v_o(t') \) — speed of observer.

In optical dispersion medium

\[ \nu(t') = \nu(t) \left[ 1 + \frac{v(t') + a_{sis}(t)T(t)}{c - a_{sis}(t)T(t)} \right] \]  
(17)

Furthermore Ref. [7]

\[ a_{sis}(t) = a_{im}(t) = \frac{dv_{im}(t)}{dt} = \frac{dv_{im}}{d\omega} \frac{d\omega_{im}(t)}{dt} + \frac{dv_{im}}{dn} \frac{dn}{dr_s} v(t) \]
\[ = \frac{dv_{im}}{d\omega} \left[ \frac{d\omega_s(t)}{dt} + \rho_{\phi}(t, n) a_{sm}(t) \right] + \frac{dv_{im}}{dn} \frac{dn}{dr_s} v(t) \]
\[ = \frac{dv_{im}}{d\omega} \left[ \frac{2\pi d\nu(t)}{dt} + \rho_{\phi}(t, n) \dot{v}(t) \right] + \frac{dv_{im}}{dn} \frac{dn}{dr_s} v(t) \]  
(18)

where \( n \) — refractive index of the dispersion medium

\( v_{im} \) — velocity of electromagnetic wave with respect to medium

\( \rho_{\phi}(t, n) \) — phase density of radiant source at instant \( t \) and in medium with refractive index \( n \)
As the medium has same refractive index, then from Eq. (17)

\[
a_{sis}(t) = \frac{dv_{im}}{d\omega} \left[ 2\pi \frac{dv(t)}{dt} + \rho_{\phi}(t, n) \dot{v}(t) \right]
\]  

(19)

From Eq. (17) and (19), it is known that in even dispersion medium detected frequency at observer location is related to various information about the charged particle and signal transfer time \(T(t) = T(t')\).

C. Observing frequency width

According to Eq. (15) as for radiant source’s highest frequency \(\nu_h(t_h)\) and lowest frequency \(\nu_l(t_l)\) there exist the corresponding relations

\[
\nu_h(t'_h) = \nu_h(t_h) \left[ 1 - \frac{dT(t')}{dT'} \bigg|_{t'_h} \right]
\]

and

\[
\nu_l(t'_l) = \nu_l(t_l) \left[ 1 - \frac{dT(t')}{dT'} \bigg|_{t'_l} \right]
\]

then

\[
\nu'_h(t'_h) - \nu'_l(t'_l) = \nu_h(t_h) \left[ 1 - \frac{dT(t')}{dT'} \bigg|_{t'_h} \right] - \nu_l(t_l) \left[ 1 - \frac{dT(t')}{dT'} \bigg|_{t'_l} \right]
\]  

(20)

Suppose \(\frac{dT(t')}{dt}|_{t'_h} = \frac{dT(t')}{dt}|_{t'_l}\) and using Eq. (12)

\[
\nu'_h - \nu'_l = (\nu_h - \nu_l) \left[ 1 + \frac{v(t')}{c} \right] = 2\Delta \nu_r = 2 \left\{ \pm \frac{\pi q B}{m_q A_0} |v(t_a) - v(t_b)| + \frac{1}{\lambda_0} \left[ \rho(t_a) \frac{\dot{k}(t_a)}{k^2(t_a)} - \rho(t_b) \frac{\dot{k}(t_b)}{k^2(t_b)} \right] \right\}
\]  

(21)

Therefore for non-dispersion medium

\[

\nu'_h - \nu'_l = (\nu_h - \nu_l) \left[ 1 + \frac{v(t')}{c} \right] = 2\Delta \nu_r \left\{ \pm \frac{\pi q B}{m_q A_0} |v(t_a) - v(t_b)| + \frac{1}{\lambda_0} \left[ \rho(t_a) \frac{\dot{k}(t_a)}{k^2(t_a)} - \rho(t_b) \frac{\dot{k}(t_b)}{k^2(t_b)} \right] \right\}
\]

(22)

for even or uniform dispersion medium, and combine Eq. (13), (19) with Eq. (21)

\[
\nu'_h - \nu'_l = (\nu_h - \nu_l) \left[ 1 + \frac{v(t')}{c - a_{sis}(t) T(t)} \right] = \Delta \nu_r \left\{ \pm \frac{\pi q B}{m_q A_0} |v(t_a) - v(t_b)| + \frac{1}{\lambda_0} \left[ \rho(t_a) \frac{\dot{k}(t_a)}{k^2(t_a)} - \rho(t_b) \frac{\dot{k}(t_b)}{k^2(t_b)} \right] \right\}
\]

(23)

IV. DISCUSSION

A. Lorentz force, in a complete inertial frame, is particle’s total energy related

For a single charged particle, its energy \(E_q(t) = \frac{m_q}{2} v^2(t) + \mu q [\ln \dot{k}^{-1}(t) + \dot{k}(t) - 1] + E_{q0} = \frac{m_q}{2} v^2(t) + \mu q [\ddot{\eta}(t) + e^{-\eta(t)} - 1] + E_{q0}\) (Ref. [3]) relates to corrected Lorentz force as centrifugal force \(f_{\rho}(t) = \dot{k}(t) q\ddot{v}(t) \times \vec{B}(t)\); where \(\ddot{v}(t)\)
reflects that the force is particle’s kinetic energy related, and \( \vec{F}(t) = e^{-\eta(t)} \) shows the corrected force is also particle charge’s internal potential energy related.

For energy loss process of a charged particle in magnetic field, there \( \dot{\vec{v}} = -\tau \omega_\rho(t) \), or \( \dot{\vec{v}} = -\tau \vec{v} \left( \frac{kqB}{m_q} \right)^2 \). While for energy increase process of a charged particle in magnetic field from external electrical field, where its intensity is \( \mathbf{E}(t) \), \( \dot{\vec{a}} = \omega_\rho(t) \), or \( \dot{\vec{a}} = \omega_\rho = \frac{\mathbf{E}(t)}{m_q} \).

**B. Interpretation on behavior and radiation property of electron beam in magnetic dipole**

**A. Beam’s horizontal size decreasing**

This is because individual’s charge density field is compressed by two opposite electrical fields (Ref. III A) along centrifugal direction or \( \vec{n}_r(t) \). The compression contributes to decreasing space charge effect along this direction.

**B. Decreasing of beam’s longitudinal kinetic energy spread**

This is because along \( \vec{n}_\perp r(t) \) there exists \( \frac{\partial \vec{v}(t,z)}{\partial z} < 0 \); thus along the longitudinal direction beam’s internal force increase, that help to improve the kinetic energy spread and increase internal potential energy.

**C. Observer’s frequency width is about two times of source itself’s frequency width**

This is because there is derivative of frequency transfer relation Eq. (15)

\[
\frac{d\nu(t')}{dt'} = \frac{d\nu(t)}{dt} \left( \frac{dt}{dt'} \right)^2 + \frac{d\nu(t)}{dt} \frac{d^2t}{dt'^2} \nonumber
\]

\[
= \frac{d\nu(t)}{dt} \left\{ \left[ 1 - \frac{dT(t')}{dt'} \right]^2 - \frac{d^2T(t')}{dt'^2} \right\}
\]

or

\[
d\nu(t') = d\nu(t) \left\{ 1 - \frac{dT(t')}{dt'} - \left[ 1 + \frac{dT(t)}{dt} \right] \frac{d^2T(t')}{dt'^2} \right\}
\]

For example in non-dispersion medium, \( \frac{d^2T(t')}{dt'^2} = 0 \), \( \frac{dT(t')}{dt'} = -\frac{\nu(t)}{c} \approx -1 \)

So from Eq. (24) there

\[
d\nu(t') \approx 2d\nu(t)
\]

For beam the observer frequency width will be wider than single’s due to energy spread of the electron beam.

**C. Application**

A. Corrected Lorentz force formula can be used to analyze the deflecting angle of the charge particles with different energy in magnetic dipole. Consequent gyral frequency formula can be used to analyze circling frequency of charge particle in storage ring or cyclotron directly, without supposing mass’s increasing or decreasing of particle as particle’s energy varies. (Ref. Eq. (5) (5’))

B. The radiation frequency and power property can be applied to analyze radiant frequency and power property of synchrotron light sources. The property can also be used in analysis of light source of high red-shift astronomic observation.
V. CONCLUSION

Lorentz force in magnetic field is corrected by charge field’s compression factor which manifests charge density field’s internal potential energy status. The radiation frequency property of individual charged particle in magnetic field is consisted of two characteristics, radiant frequency property due to Lorentz deflection radiation and detecting or observing frequency property which is determined by time function of signal transportation between time domains of radiant source and observer. Consequently the behavior and radiation property of electron beam can be interpreted upon the behavior and radiation property of single charged particle. In addition based on the corrected Lorentz force formula, it is inferred that the angular frequency or velocity of a charged particle in magnetic dipole is timely independent to its mass but its total energy status or specifically its internal potential energy status that relates the charge field’s compression status.

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