Open-chain transfer matrices for AdS/CFT

Rajan Murgan\textsuperscript{1,2} and Rafael I. Nepomechie\textsuperscript{3}

Physics Department, P.O. Box 248046, University of Miami
Coral Gables, FL 33124 USA

Abstract

We extend Sklyanin’s construction of commuting open-chain transfer matrices to
the $SU(2|2)$ bulk and boundary $S$-matrices of AdS/CFT. Using the graded version of
the $S$-matrices leads to a transfer matrix of particularly simple form. We also find an
$SU(1|1)$ boundary $S$-matrix which has one free boundary parameter.

\textsuperscript{1}Current address: Physics Department, Gustavus Adolphus College, Olin Hall, 800 West College Avenue,
St. Peter, MN 56082 USA
\textsuperscript{2}rmurgan@gustavus.edu
\textsuperscript{3}nepomechie@physics.miami.edu
1 Introduction

The factorizable $SU(2|2)$-invariant bulk $S$-matrix proposed by Beisert \[1, 2\] plays a central role in understanding integrability in the closed string/spin chain sector of AdS/CFT. Indeed, this $S$-matrix can be used to derive \[1, 3, 4\] the all-loop asymptotic Bethe ansatz equations \[5\] and to compute finite-size effects \[6\]. (For reviews and further references, see for example Ref. \[7\].)

Integrability also extends to the open string/spin chain sector of AdS/CFT. (See for example \[8\]-\[11\] and references therein.) Hofman and Maldacena \[12\] have proposed boundary $S$-matrices corresponding to open strings attached to maximal giant gravitons \[13\] in $AdS_5 \times S^5$. While there has been some subsequent work (see for example \[14\]-\[20\]), the study of integrability in the open string/spin chain sector is considerably less well developed compared with the closed string/spin chain sector. In particular, corresponding all-loop asymptotic Bethe ansatz equations have yet to be derived.

An important prerequisite for deriving such Bethe ansatz equations is to construct a commuting open-chain transfer matrix, which is the main purpose of this note. Sklyanin \[21\] long ago made the key observation that the transfer matrix should be of the “double-row” form. However, because the bulk $S$-matrix is not of the difference form and has a peculiar crossing property \[22, 2\], it is necessary to generalize his construction. Indeed, we argue that the transfer matrix contains an unexpected factor \(2.19\) which is essential for commutativity. This factor can be removed by working instead with graded versions of the $S$-matrices.

The $SU(2|2)$ bulk $S$-matrix has an $SU(1|1)$ submatrix which itself satisfies the Yang-Baxter equation \[5, 23\]. We find here a corresponding boundary $S$-matrix which, unlike those found in \[12\], contains an arbitrary boundary parameter. The simplicity of the $SU(1|1)$ bulk and boundary $S$-matrices suggests that they can serve as useful toy models of the more complicated $SU(2|2)$ case.

The outline of this paper is as follows. In Section \[2\] we construct two different commuting open-chain transfer matrices. The first, constructed with non-graded $S$-matrices, contains an unexpected factor; and the second, constructed with graded versions of the $S$-matrices, does not have this extra factor. In Section \[3\] we present the $SU(1|1)$ boundary $S$-matrix. We conclude in Section \[4\] with a brief discussion of our results. An appendix contains the $SU(2|2)$ bulk $S$-matrix and explains some of our notation.
2 Transfer matrix

Bulk and boundary $S$-matrices are the two main building blocks of the transfer matrix. We assume here that the bulk $S$-matrix is essentially the one found by Beisert [1] based on $SU(2|2)$ symmetry, but in a basis [2] where the standard Yang-Baxter equation (YBE)

$$S_{12}(p_1, p_2) S_{13}(p_1, p_3) S_{23}(p_2, p_3) = S_{23}(p_2, p_3) S_{13}(p_1, p_3) S_{12}(p_1, p_2).$$

is satisfied. We use the standard convention $S_{12} = S \otimes I$, $S_{23} = I \otimes S$, and $S_{13} = \mathcal{P}_{12} S_{23} \mathcal{P}_{12}$, where $\mathcal{P}_{12} = \mathcal{P} \otimes I$, $\mathcal{P}$ is the permutation matrix, and $I$ is the four-dimensional identity matrix. For convenience, this $S$-matrix is given explicitly in the Appendix. For simplicity, we omit the scalar factor. Hence, this matrix has the unitarity property

$$S_{12}(p_1, p_2) S_{21}(p_2, p_1) = I,$$

where $S_{21} = \mathcal{P}_{12} S_{12} \mathcal{P}_{12}$, as well as the crossing property [22, 2]

$$C_{2}(p_2) S_{12}(p_1, \bar{p}_2) C_{2}(p_2)^{-1} S_{12}(p_1, p_2)^{t_2} = I f(p_1, p_2),$$

where $C(p)$ is the matrix

$$C(p) = \begin{pmatrix}
0 & i \text{sign}(p) & 0 & 0 \\
-i \text{sign}(p) & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0
\end{pmatrix},$$

and the scalar function $f(p_1, p_2)$ is given by

$$f(p_1, p_2) = \frac{\left(\frac{1}{x_1^+} - x_2^-\right) (x_1^+ - x_2^+)}{\left(\frac{1}{x_1^-} - x_2^+\right) (x_1^- - x_2^+)}.$$

Moreover, $\bar{p} = -p$ denotes the antiparticle momentum, with

$$x^\pm(\bar{p}) = \frac{1}{x^\pm(p)}.$$

As we shall see, the peculiar dependence of the charge conjugation matrix $C(p)$ on the sign of $p$ gives rise to a nontrivial factor in the transfer matrix.

We assume here that the right boundary $S$-matrix $R^-(p)$ is essentially the one found by Hofman and Maldacena [12] for the so-called $Y = 0$ giant graviton brane, but in a basis [16] where the standard (right) boundary Yang-Baxter equation (BYBE) [24, 25]

$$S_{12}(p_1, p_2) R_1^-(p_1) S_{21}(p_2, -p_1) R_2^-(p_2) = R_2^-(p_2) S_{12}(p_1, -p_2) R_1^-(p_1) S_{21}(-p_2, -p_1)$$

(2.7)
is satisfied. It is a diagonal matrix given by \[16\]
\[
R^{-}(p) = \text{diag} \left( e^{-ip}, -1, 1, 1 \right).
\]
(2.8)

As noted in \[12\],
\[
 x^{\pm}(-p) = -x^{\mp}(p), \quad \eta(-p) = \eta(p),
\]
(2.9)

From the bulk $S$-matrix, we construct a pair of monodromy matrices
\[
T_{a}(p; \{p_{i}\}) = S_{aN}(p, p_{N}) \cdots S_{a1}(p, p_{1}), \quad \hat{T}_{a}(p; \{p_{i}\}) = S_{1a}(p_{1}, -p) \cdots S_{Na}(p_{N}, -p),
\]
(2.10)

where $\{p_{1}, \ldots, p_{N}\}$ are arbitrary “inhomogeneities” associated with each of the $N$ quantum spaces, and the auxiliary space is denoted by $a$. (As usual, the quantum-space “indices” are suppressed from the monodromy matrices.) These matrices obey the relations
\[
S_{ab}(p_{a}, p_{b}) T_{a}(p_{a}; \{p_{i}\}) T_{b}(p_{b}; \{p_{i}\}) = T_{b}(p_{b}; \{p_{i}\}) T_{a}(p_{a}; \{p_{i}\}) S_{ab}(p_{a}, p_{b}),
\]
\[
S_{ba}(-p_{b}, -p_{a}) \hat{T}_{a}(p_{a}; \{p_{i}\}) \hat{T}_{b}(p_{b}; \{p_{i}\}) = \hat{T}_{b}(p_{b}; \{p_{i}\}) \hat{T}_{a}(p_{a}; \{p_{i}\}) S_{ba}(-p_{b}, -p_{a}),
\]
\[
\hat{T}_{a}(p_{a}; \{p_{i}\}) S_{ba}(p_{b}, -p_{a}) T_{b}(p_{b}; \{p_{i}\}) = T_{b}(p_{b}; \{p_{i}\}) S_{ba}(p_{b}, -p_{a}) \hat{T}_{a}(p_{a}; \{p_{i}\})
\]
(2.11)
as a consequence of the YBE. The “decorated” right boundary $S$-matrix given by
\[
T_{a}^{-}(p; \{p_{i}\}) = T_{a}(p; \{p_{i}\}) R_{a}^{-}(p) \hat{T}_{a}(p; \{p_{i}\})
\]
(2.12)
also satisfies the BYBE, i.e.,
\[
S_{ab}(p_{a}, p_{b}) T_{a}^{-}(p_{a}; \{p_{i}\}) S_{ba}(p_{b}, -p_{a}) T_{b}^{-}(p_{b}; \{p_{i}\})
= T_{b}^{-}(p_{b}; \{p_{i}\}) S_{ab}(p_{a}, -p_{b}) T_{a}^{-}(p_{a}; \{p_{i}\}) S_{ba}(-p_{b}, -p_{a}),
\]
(2.13)
by virtue of (2.7) and (2.11).

Following Sklyanin [21], we assume that the open-chain transfer matrix is of the double-row form
\[
t(p; \{p_{i}\}) = \text{tr}_{a} R_{a}^{+}(p) T_{a}^{-}(p; \{p_{i}\})
= \text{tr}_{a} R_{a}^{+}(p) T_{a}(p; \{p_{i}\}) R_{a}^{-}(p) \hat{T}_{a}(p; \{p_{i}\}),
\]
(2.14)
where the trace is over the auxiliary space, and the left boundary $S$-matrix $R^{+}(p)$ is chosen to ensure the essential commutativity property
\[
[t(p; \{p_{i}\}), t(p'; \{p_{i}\})] = 0
\]
(2.15)
for arbitrary values of $p$ and $p'$. By repeating the (not short) computation in \[21\] but now making use of the unitarity and crossing properties \[2.2\] and \[2.3\], we find that the commutativity property is indeed obeyed, provided that $R^+(p)$ satisfies the relation

$$S_{21}(p_2, p_1)^{t_{12}} R_1^+(p_1)^{t_1} C_1(-p_1) S_{21}(p_2, -p_1)^{t_2} C_1(-p_1)^{-1} R_2^+(p_2)^{t_2} = R_2^+(p_2)^{t_2} C_2(-p_2) S_{12}(p_1, -p_2)^{t_1} C_2(-p_2)^{-1} R_1^+(p_1)^{t_1} S_{12}(-p_1, -p_2)^{t_{12}}. \tag{2.16}$$

In obtaining this result, we also make use of the identity

$$f(p_1, p_2) = f(-p_2, -p_1) \tag{2.17}$$

which is satisfied by the function defined in \[2.5\]. The relation \[2.16\] can be simplified using again the crossing property \[2.3\]. Eventually, we arrive at

$$S_{12}(p_1, p_2) M_1 R_1^+(-p_1) S_{21}(p_2, -p_1) M_2 R_2^+(-p_2) = M_2 R_2^+(-p_2) S_{12}(p_1, -p_2) M_1 R_1^+(-p_1) S_{21}(-p_2, -p_1), \tag{2.18}$$

where the matrix $M$ is given by

$$M = C(-p) C(p)^{-1} = \text{diag}(-1, -1, 1, 1) = M^{-1}. \tag{2.19}$$

In obtaining this result, we make use of the identities

$$f(p_1, p_2) = f(-p_2, -p_1) \tag{2.20}$$

and

$$M_1 S_{12}(p_1, p_2) M_2 = M_2 S_{12}(p_1, p_2) M_1. \tag{2.21}$$

Comparing the $R^+(p)$ relation \[2.18\] with the $R^-(p)$ relation \[2.7\], we conclude that the left boundary $S$-matrix is given by

$$R^+(p) = M R^-(p), \tag{2.22}$$

where $M$ is given by \[2.19\]. We emphasize that this matrix $M$, which arises from the peculiar dependence of the charge conjugation matrix on the sign of the momentum, is essential in order for the transfer matrix \[2.14\] to have the commutativity property \[2.15\], which we have verified numerically for small numbers of sites. A formally similar matrix appears in the construction of open-chain transfer matrices for nonsymmetric $R$-matrices \[26\].

The matrix $M$ does not appear if we work instead with corresponding graded quantities. Indeed, let us make the parity assignments

$$p(1) = p(2) = 0, \quad p(3) = p(4) = 1, \tag{2.23}$$

\footnote{For the generalization of Sklyanin’s formalism to graded $S$-matrices, see for example \[27\].}
and define the graded bulk $S$-matrix by (see, e.g., [3])

$$S^g(p_1, p_2) = P g S(p_1, p_2),$$

(2.24)

where $P^g$ is the graded permutation matrix

$$P^g = \sum_{i,j=1}^{4} (-1)^{p(i)p(j)} e_{ij} \otimes e_{ji},$$

(2.25)

and $S(p_1, p_2)$ is given in the Appendix. We consider the transfer matrix given by

$$t(p; \{p_i\}) = \text{str}_a R_a^+(p) T_a(p; \{p_i\}) R_a^-(p) \hat{T}_a(p; \{p_i\}),$$

(2.26)

where str denotes the supertrace, the monodromy matrices are formed as in (2.10) except with the graded $S$-matrix (2.24) using the graded tensor product (instead of the ordinary tensor product), and $R^-(p)$ is again given by (2.8), which also satisfies the graded BYBE. The transfer matrix (2.26) satisfies the commutativity property (2.15) for $R^+(p)$ given by (2.22) with $M = 1$. That is,

$$t(p; \{p_i\}) = \text{str}_a R_a^-(p) T_a(p; \{p_i\}) R_a^-(p) \hat{T}_a(p; \{p_i\}).$$

(2.27)

This transfer matrix evidently has the right structure for formulating the Bethe-Yang equation on an interval with left and right boundaries.

3 \textit{SU}(1|1) boundary $S$-matrix

The $SU(2|2)$ bulk $S$-matrix contains an $SU(1|1)$ submatrix which itself satisfies the graded YBE, namely, [5, 23]

$$S(p_1, p_2) = \begin{pmatrix} x_1^+ - x_2^- & 0 & 0 & 0 \\ 0 & x_1^- - x_2^+ & (x_1^+ - x_1^-) \omega_1 & 0 \\ 0 & (x_2^+ - x_2^-) \omega_2 & x_1^+ - x_2^- & 0 \\ 0 & 0 & 0 & x_1^- - x_2^+ \end{pmatrix},$$

(3.1)

where the parity assignments are $p(1) = 0$, $p(2) = 1$. (We are again not concerned here with overall scalar factors.) Curiously, as already noted by Beisert and Staudacher [5], the YBE holds even without imposing any constraint between $x^+(p)$ and $x^-(p)$, and without specifying $\omega(p)$.

\footnote{In the closed string/spin chain sector, it is necessary to formulate the Bethe-Yang equation using the graded $S$-matrix in order to properly implement periodic boundary conditions [3].}
We find that the corresponding right BYBE has the following diagonal solution
\begin{equation}
R^-(p) = \text{diag} \left( a - x^+(p), a + x^-(p) \right),
\end{equation}
where \( a \) is an arbitrary boundary parameter. While the appearance of boundary parameters is common for boundary \( S \)-matrices associated with affine Lie algebras, we emphasize that no such boundary parameter appears in the \( SU(2|2) \) boundary \( S \)-matrices \[12, 16\]. As is the case for the bulk, the BYBE is satisfied without imposing any constraint between \( x^+(p) \) and \( x^-(-p) \) other than (2.9), and without specifying \( \omega(p) \) other than (3.3).

The corresponding commuting open-chain transfer matrix is given by (2.26), where the left boundary \( S \)-matrix is given by
\begin{equation}
R^+(p) = R^-(p) \bigg|_{a \rightarrow b} = \text{diag} \left( b + x^-(p), b - x^+(p) \right),
\end{equation}
where \( b \) is another arbitrary boundary parameter. The commutativity (2.15) holds for arbitrary \( x^+(p), x^-(-p), \omega(p) \) obeying (2.9), (3.3).

4 Discussion

We have found that the \( SU(2|2) \) bulk and boundary \( S \)-matrices of AdS/CFT can be used to construct a commuting open-chain transfer matrix given by (2.14), where \( T_a(p; \{p_i\}) \) and \( \hat{T}_a(p; \{p_i\}) \) are given by (2.10), and \( R^+(p) \) is given by (2.22), which contains the unexpected factor \( M \) (2.19). Alternatively, using graded versions of the \( S \)-matrices, one can construct the simpler transfer matrix (2.27). Moreover, we have found a new \( SU(1|1) \) boundary \( S \)-matrix (3.2) which, in contrast to the \( SU(2|2) \) case (2.8), contains an arbitrary boundary parameter.

For the \( SU(2|2) \) closed chain, a local Hamiltonian can be obtained from the closed-chain transfer matrix
\begin{equation}
t_{\text{closed}}(p; \{p_i\}) = \text{tr}_a T_a(p; \{p_i\})
\end{equation}
by setting all the inhomogeneities equal \( p_i \equiv p_0 \), and taking the logarithmic derivative,
\begin{equation}
H_{\text{closed}} = \frac{d}{dp} \ln t_{\text{closed}}(p; \{p_i = p_0\}) \bigg|_{p=p_0}.
\end{equation}
As noted by Beisert \[1\], in contrast to the conventional case, this Hamiltonian depends on the value of \( p_0 \), since the bulk \( S \)-matrix does not have the difference property. Nevertheless, this Hamiltonian is local, since the \( S \)-matrix is regular, \( S(p_0, p_0) \propto \mathcal{P} \), and therefore \( t_{\text{closed}}(p_0; \{p_i = p_0\}) \) is the one-site shift operator.
It is not clear whether an analogous local Hamiltonian can be obtained from the open-chain transfer matrix \((2.14)\). Indeed, in contrast to the conventional homogeneous case \([21]\), \(t(p_0; \{p_i = p_0\})\) is not proportional to the identity. This is due to the fact that
\[
\hat{T}_a(p_0; \{p_i = p_0\}) = S_{1a}(p_0, -p_0) \cdots S_{N\alpha}(p_0, -p_0),
\]
which is not a product of permutation operators, and the fact that \(R^-(p_0)\) is not proportional to the identity matrix. (This is true even for the conventional inhomogeneous case.) Hence, the naive guess
\[
\left. \frac{d}{dp} t(p; \{p_i = p_0\}) \right|_{p=p_0}
\]
does not give a local Hamiltonian; and multiplying \((4.4)\) by \(t(p_0; \{p_i = p_0\})^{-1}\) does not help.

It would be interesting to determine the eigenvalues and Bethe ansatz equations of the \(SU(2|2)\) open-chain transfer matrix. We expect that the \(SU(1|1)\) case will serve as a useful warm-up exercise.

**Acknowledgments**

This work was supported in part by the National Science Foundation under Grants PHY-0244261 and PHY-0554821.

**A The \(SU(2|2)\)-invariant bulk \(S\)-matrix**

We arrange the bulk \(S\)-matrix elements into a \(16 \times 16\) matrix \(S\) as follows,
\[
S(p_1, p_2) = \sum_{i,j=1}^{4} S_{i j}^{i' j'}(p_1, p_2) e_{i i'} \otimes e_{j j'},
\]
where \(e_{ij}\) is the usual elementary \(4 \times 4\) matrix whose \((i, j)\) matrix element is 1, and all others are zero. Although \((A.1)\) is the standard convention, Arutyunov et al.
use a different convention (see Eq. (8.4) in \([2]\)), such that our matrix \(S\) is the transpose of theirs. The nonzero matrix elements are \([2]\)
\[
S_{aa}(p_1, p_2) = A, \quad S_{\alpha\alpha}(p_1, p_2) = D,
\]
\[
S_{ab}(p_1, p_2) = \frac{1}{2}(A - B), \quad S_{\beta\alpha}(p_1, p_2) = \frac{1}{2}(A + B),
\]
\[
S_{a\beta}(p_1, p_2) = \frac{1}{2}(D - E), \quad S_{\alpha\beta}(p_1, p_2) = \frac{1}{2}(D + E),
\]

\[ S^\alpha\beta_{ab}(p_1, p_2) = -\frac{1}{2}\epsilon_{ab}\epsilon^{\alpha\beta} C, \quad S^{ab}_{\alpha\beta}(p_1, p_2) = -\frac{1}{2}\epsilon^{ab}\epsilon_{\alpha\beta} F, \]

\[ S^\alpha\alpha_{ab}(p_1, p_2) = G, \quad S^{\alpha\alpha}_{ab}(p_1, p_2) = H, \quad S^{\alpha\alpha}_{aa}(p_1, p_2) = K, \quad S^{\alpha\alpha}_{aa}(p_1, p_2) = L, \quad (A.2) \]

where \( a, b \in \{1, 2\} \) with \( a \neq b \); \( \alpha, \beta \in \{3, 4\} \) with \( \alpha \neq \beta \); and

\[
A = \frac{x_2^- - x_1^+ \eta_1 \eta_2}{x_2^+ - x_1^- \bar{\eta}_1 \bar{\eta}_2}, \quad B = -\left[ \frac{x_2^- - x_1^+}{x_2^- - x_1^-} + 2 \frac{(x_1^- - x_1^+)(x_2^- - x_2^+)(x_2^- + x_1^+)}{(x_1^- - x_2^-)(x_1^- x_2^- - x_1^+ x_2^+)} \right] \eta_1 \eta_2 \bar{\eta}_1 \bar{\eta}_2, \quad C = \frac{2ix_1^- x_2^- (x_1^- - x_2^-) \eta_1 \eta_2}{x_1^+ x_2^+ (x_1^- - x_2^-)(1 - x_1^- x_2^-)}, \quad D = -1, \quad E = \left[ 1 - 2 \frac{(x_1^- - x_1^+)(x_2^- - x_2^+)(x_1^+ + x_2^+)}{(x_1^- - x_2^-)(x_1^- x_2^- - x_1^+ x_2^+)} \right], \quad F = \frac{2i(x_1^- - x_1^+)(x_2^- - x_2^+)(x_1^+ - x_2^+)}{(x_1^- - x_2^-)(1 - x_1^- x_2^-) \eta_1 \eta_2}, \quad G = \frac{(x_2^- - x_1^-) \eta_1}{(x_2^+ - x_1^+) \bar{\eta}_1}, \quad H = \frac{(x_2^- - x_2^-) \eta_1}{(x_1^- - x_2^-) \bar{\eta}_1}, \quad K = \frac{(x_1^- - x_1^-) \eta_2}{(x_1^- - x_2^+) \bar{\eta}_1}, \quad L = \frac{(x_1^- - x_2^+) \eta_2}{(x_1^- - x_2^+) \bar{\eta}_2}, \quad (A.3) \]

where

\[ x_i^\pm = x^\pm(p_i), \quad \eta_1 = \eta(p_1)e^{ip_2/2}, \quad \eta_2 = \eta(p_2), \quad \bar{\eta}_1 = \eta(p_1), \quad \bar{\eta}_2 = \eta(p_2)e^{ip_1/2}, \quad (A.4) \]

and \( \eta(p) = \sqrt{i\left[x^-(p) - x^+(p)\right]} \). Also,

\[ x^+ + \frac{1}{x^+} - x^- - \frac{1}{x^-} = \frac{i}{g}, \quad \frac{x^+}{x^-} = e^{ip}. \quad (A.5) \]

References

[1] N. Beisert, “The su(2|2) dynamic S-matrix,” [arXiv:hep-th/0511082];
N. Beisert, “The Analytic Bethe Ansatz for a Chain with Centrally Extended su(2|2) Symmetry,” J. Stat. Mech. 0701, P017 (2007) [arXiv:nlin/0610017].

[2] G. Arutyunov, S. Frolov and M. Zamaklar, “The Zamolodchikov-Faddeev algebra for AdS5 × S5 superstring,” JHEP 0704, 002 (2007) [arXiv:hep-th/0612229].

[3] M.J. Martins and C.S. Melo, “The Bethe ansatz approach for factorizable centrally extended S-matrices,” Nucl. Phys. B 785, 246 (2007) [arXiv:hep-th/0703086].
[4] M. de Leeuw, “Coordinate Bethe Ansatz for the String S-Matrix,” *J. Phys. A* **40**, 14413 (2007) [arXiv:0705.2369].

[5] N. Beisert and M. Staudacher, “Long-range $PSU(2, 2|4)$ Bethe ansaetze for gauge theory and strings,” *Nucl. Phys. B* **727**, 1 (2005) [arXiv:hep-th/0504190].

[6] Z. Bajnok and R.A. Janik, “Four-loop perturbative Konishi from strings and finite size effects for multiparticle states,” [arXiv:hep-th/0807.0399].

[7] A.A. Tseytlin, “Spinning strings and AdS/CFT duality,” in Ian Kogan Memorial Volume, *From Fields to Strings: Circumnavigating Theoretical Physics*, M. Shifman, A. Vainshtein, and J. Wheater, eds. (World Scientific, 2004) [arXiv:hep-th/0311139]; N. Beisert, “The dilatation operator of $\mathcal{N} = 4$ super Yang-Mills theory and integrability,” *Phys. Rept.* **405**, 1 (2005) [arXiv:hep-th/0407277]; K. Zarembo, “Semiclassical Bethe ansatz and AdS/CFT,” *Comptes Rendus Physique* **5**, 1081 (2004) [Fortsch. Phys. **53**, 647 (2005)] [arXiv:hep-th/0411191]; J. Plefka, “Spinning strings and integrable spin chains in the AdS/CFT correspondence,” *Living Rev. Rel.* **8**, 9 (2005) [arXiv:hep-th/0507136]; J.A. Minahan, “A brief introduction to the Bethe ansatz in $\mathcal{N} = 4$ super-Yang-Mills,” *J. Phys.* **A39**, 12657 (2006); K. Okamura, “Aspects of Integrability in AdS/CFT Duality,” [arXiv:0803.3999].

[8] D. Berenstein and S.E. Vazquez, “Integrable open spin chains from giant gravitons,” *JHEP* **0506**, 059 (2005) [arXiv:hep-th/0501078].

[9] T. McLoughlin and I. Swanson, “Open string integrability and AdS/CFT,” *Nucl. Phys. B* **723**, 132 (2005) [arXiv:hep-th/0504203].

[10] A. Agarwal, “Open spin chains in super Yang-Mills at higher loops: Some potential problems with integrability,” *JHEP* **0608**, 027 (2006) [arXiv:hep-th/0603067]; K. Okamura and K. Yoshida, “Higher loop Bethe ansatz for open spin-chains in AdS/CFT,” *JHEP* **0609**, 081 (2006) [arXiv:hep-th/0604100].

[11] N. Mann and S.E. Vazquez, “Classical open string integrability,” *JHEP* **0704**, 065 (2007) [arXiv:hep-th/0612038].

[12] D.M. Hofman and J.M. Maldacena, “Reflecting magnons,” *JHEP* **0711**, 063 (2007) [arXiv:0708.2272].

[13] J. McGreevy, L. Susskind and N. Toumbas, “Invasion of the giant gravitons from anti-de Sitter space,” *JHEP* **0006**, 008 (2000) [arXiv:hep-th/0003075].
M.T. Grisaru, R.C. Myers and O. Tafjord, “SUSY and Goliath,” *JHEP* **0008**, 040 (2000) [arXiv:hep-th/0008015];
A. Hashimoto, S. Hirano and N. Itzhaki, “Large branes in AdS and their field theory dual,” *JHEP* **0008**, 051 (2000) [arXiv:hep-th/0008016].

[14] H.Y. Chen and D.H. Correa, “Comments on the Boundary Scattering Phase,” *JHEP* **0802**, 028 (2008) [arXiv:0712.1361].

[15] C. Ahn, D. Bak and S.J. Rey, “Reflecting Magnon Bound States,” *JHEP* **0804**, 050 (2008) [arXiv:0712.4144].

[16] C. Ahn and R.I. Nepomechie, “The Zamolodchikov-Faddeev algebra for open strings attached to giant gravitons,” *JHEP* **0805**, 059 (2008) [arXiv:0804.4036].

[17] R. Murgan and R.I. Nepomechie, “$q$-deformed $su(2|2)$ boundary $S$-matrices via the ZF algebra,” *JHEP* **0806**, 096 (2008) [arXiv:0805.3142].

[18] N. Beisert and F. Loebbert, “Open Perturbatively Long-Range Integrable $gl(N)$ Spin Chains,” [arXiv:0805.3200].

[19] L. Palla, “Issues on magnon reflection,” [arXiv:0807.3646].

[20] D.H. Correa and C.A.S. Young, “Reflecting magnons from D7 and D5 branes,” [arXiv:0808.0452].

[21] E.K. Sklyanin, “Boundary conditions for integrable quantum systems,” *J. Phys.* **A21**, 2375 (1988).

[22] R.A. Janik, “The $AdS_5 \times S^5$ superstring worldsheet $S$-matrix and crossing symmetry,” *Phys. Rev.* **D73**, 086006 (2006) [arXiv:hep-th/0603038].

[23] N. Beisert, “An $SU(1|1)$-Invariant $S$-Matrix with Dynamic Representations,” *Bulg. J. Phys.* **33S1** (2006) 371 [arXiv:hep-th/0511013].

[24] I.V. Cherednik, “Factorizing particles on a half line and root systems,” *Theor. Math. Phys.* **61**, 977 (1984).

[25] S. Ghoshal and A.B. Zamolodchikov, “Boundary $S$-Matrix and Boundary State in Two-Dimensional Integrable Quantum Field Theory,” *Int. J. Mod. Phys.* **A9**, 3841 (1994) [arXiv:hep-th/9306002].

[26] L. Mezincescu and R.I. Nepomechie, “Integrable open spin chains with nonsymmetric $R$ matrices,” *J. Phys.* **A24**, L17 (1991).
[27] A. Foerster and M. Karowski, “The supersymmetric t-J model with quantum group invariance,” *Nucl. Phys.* B408, 512 (1993); A. González-Ruiz, “Integrable open-boundary conditions for the supersymmetric t-J model. The quantum group invariant case,” *Nucl. Phys.* B424, 468 (1994) [arXiv:hep-th/9401118]; R.H. Yue, H. Fan and B.Y. Hou, “Exact diagonalization of the quantum supersymmetric $SU_q(n|m)$ model,” *Nucl. Phys.* B462, 167 (1996) [cond-mat/9603022]; M. Shiroishi and M. Wadati, “Integrable Boundary Conditions for the One-Dimensional Hubbard Model,” *J. Phys. Soc. Jpn.* 66, 2288 (1997) [arXiv:cond-mat/9708011]; A.J. Bracken, X.-Y. Ge, Y.-Z. Zhang and H.-Q. Zhou, “Integrable open-boundary conditions for the $q$-deformed supersymmetric $U$ model of strongly correlated electrons,” *Nucl. Phys.* B516, 588 (1998) [arXiv:cond-mat/9710141]; X.-W. Guan, “Algebraic Bethe ansatz for the one-dimensional Hubbard model with open boundaries,” *J. Phys. A* 33, 5391 (2000) [arXiv:cond-mat/9908054]; D. Arnaudon, J. Avan, N. Crampé, A. Doikou, L. Frappat and E. Ragoucy, “General boundary conditions for the $sl(N)$ and $sl(M|N)$ open spin chains,” *J. Stat. Mech.* P08005, 1 (2004) [math-ph/0406021].