On Indirect CP Violation and Implications for $D^0 - \bar{D^0}$ and $B_s - \bar{B_s}$ mixing

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The two kinds of indirect CP violation in neutral meson systems are related, in the absence of new weak phases in decay. The result is a model-independent expression relating CP violation in mixing, CP violation in the interference of decays with and without mixing, and the meson mass and width differences. It relates the semileptonic and time-dependent CP asymmetries, and CP-conjugate pairs of time-dependent $D^0$ CP asymmetries. CP violation in the interference of decays with and without mixing is related to the mixing parameters of relevance to model building: the off-diagonal mixing matrix elements $|M_{12}|$, $|\Gamma_{12}|$, and $\phi_{12} \equiv \arg(M_{12}/\Gamma_{12})$. Incorporating this relation into a fit to the $D^0 - \bar{D^0}$ mixing data implies a level of sensitivity to $|\phi_{12}|$ of 0.10 [rad] at 1\sigma. The formalism is extended to include new weak phases in decay, and in $\Gamma_{12}$. The phases are highly constrained by direct CP violation measurements. Consequently, the bounds on $|\phi_{12}|$ are not significantly altered, and the effects of new weak phases in decay could be difficult to observe at a high luminosity flavor factory ($D^0$) or at the LHC ($B_s$) via violations of the above relations, unlike in direct CP violation.

I. INTRODUCTION

There are two kinds of indirect CP violation in neutral meson decays, CP violation in pure mixing (CPVMIX) and CP violation in the interference of decays with and without mixing (CPVINT) (see, for example, [1, 2]). Let $M^0$ and $\bar{M}^0$ be the interaction eigenstates of a neutral meson system. Indirect CP violation in pure mixing is due to a non-vanishing relative phase, $\phi_{12} = \arg(M_{12}/\Gamma_{12})$, between the dispersive ($M^0$) and absorptive ($\bar{M}^0$) parts of the $M^0 - \bar{M}^0$ transition amplitude. It is responsible for CP asymmetries in semileptonic decays ($\bar{M}^0, M^0 \to \ell^X$). Indirect CP violation in the interference of decays with and without mixing ($M^0 \to \bar{M}^0 \to f$ and $M^0 \to f$) can occur in decays to final states which are common to $M^0$ and $\bar{M}^0$, leading to time-dependent CP asymmetries.

Direct CP violation corresponds to different magnitudes for decay amplitudes related by CP conjugation. It requires at least two amplitude contributions with different CP violating weak phases and different CP conserving strong phases. The weak phases present in the decay amplitudes in addition to the dominant Standard Model (SM) weak phase, subsequently referred to as “new weak phases in decay”, can also lead to unequal CPVINT measurements for different final states, and to $T$-violating triple-product correlations for $VV$ final states $\Psi$, even if strong phase differences are absent.

In general, CPVINT receives contributions from CPVMIX and from new weak phases in decay. However, if the latter is absent, then CPVINT originates solely from the mixing phase $\phi_{12}$, and therefore it must be connected to CPVMIX. Consequently, two related formulae can be derived: (i) an expression for CPVINT in terms of the mixing parameters $\phi_{12}$, $|M_{12}|$, and $|\Gamma_{12}|$, see Eq. (52). Such a relation was first derived in the limit $M_{12} \ll \Gamma_{12}$ $\Psi$; (ii) a model-independent expression relating the four mixing observables, i.e., the two kinds of indirect CP violation and the neutral meson mass and width differences, see Eq. (53). (i) allows a fit of the three mixing parameters to the four observables to be performed. (ii) leads to model-independent correlations between time-dependent and semileptonic CP asymmetries. It also leads to simple relations between CP-conjugate time-dependent CP asymmetries in $D^0$ decays to non-CP eigenstates.

Examples in which the connection between the indirect CP asymmetries can be realized are provided by the tree-level dominated decays, e.g., $K^0 \to \pi \pi$, $D^0 \to K^{\pm} \pi^{\mp}$, $D^0 \to K^+ K^-$, $\pi^+ \pi^-$, and $B_s \to J/\Psi \phi$. Contributions to these decays beyond the SM tree-level charged current interactions can be neglected in the SM itself, as well as in many of its proposed extensions. Thus, the underlying hypothesis of no new weak phases in decay is often valid. Of particular interest are applications to the $D^0$ and $B_s$ systems, where non-vanishing indirect CP asymmetries would constitute a clear signal for new physics. In many SM extensions they could be present at levels which can be measured at ongoing, imminent, or planned experiments.

We review the neutral meson mixing and CP violation formalism in Sections II and III. Attention is paid to the independence of physical observables with respect to the sign convention for the neutral meson mass or width differences. In Section IV we derive the expression for CPVINT in terms of $\phi_{12}$, $|M_{12}|$, and $|\Gamma_{12}|$: the model-independent relation between CPVMIX and CPVINT; and the resulting correlations among the time-dependent and semileptonic CP asymmetries. In the case of the $D^0$, we discuss singly Cabibbo suppressed (SCS), Cabibbo favored (CF), and doubly Cabibbo suppressed decays to CP and non-CP eigenstates. In the case of the $B_s$, we focus on $b \to c \bar{c} s$ mediated transitions, e.g., $B_s \to J/\Psi \phi$. In Section V a fit to the $D^0 - \bar{D^0}$ mixing data is carried out to determine the allowed ranges for $\phi_{12}$, $|M_{12}|$, and $|\Gamma_{12}|$ in models with negligible new weak phases in the tree-level dominated decays.

In Section VI we discuss in detail how the above results would be modified by the appearance of subleading weak phases in the decay amplitudes, and in $\Gamma_{12}$. Order of magnitude bounds on these weak phases, hence on vio-
lations of the relations between CPVMIX and CPVI

can be obtained from existing direct CP violation measures.
It then follows that (i) the bounds on $\phi_{12}^f$ (and $|M_{12}^f|$) do not change significantly, and (ii) it could be difficult to detect violations at currently allowed levels in the $D^0$ and $B_s$ systems, at a super $B$ factory and at the LHC, respectively. In fact, the existence of new weak phases in decay would be much easier to discover directly, via direct CP asymmetry measurements. However, with sufficient statistics it could be possible to isolate and measure shifts in $\arg(\Gamma_{12})$ (due to new physics, or to subleading $O(V_{ub}^*V_{cb}/V_{cs}^*V_{cb})$) SM contributions in $B_s$ mixing) from such violations. We conclude in Section VII.

While this work was in progress, Ref. appeared, which also explores the relation between the two indirect CP asymmetry, in the absence of new weak phases in decay. Our starting point for the derivation of a model-independent relation differs, in that it explicitly removes a discrete ambiguity in $\phi_{12} \rightarrow -\phi_{12}$, and allows us to obtain a simple general expression. The reader is referred to for a discussion of all four neutral meson systems. Also see for a discussion, based on , of correlations between time-dependent and semileptonic CP asymmetries in decays to CP eigenstates. After completion of this work, we discovered that our model-independent relation, Eq. (51), can be found in . We augment their presentation by providing its derivation from the neutral meson mixing formalism, and by discussing its significance for relating CPVMIX and CPVI

Our fit procedure for $D^0 - \overline{D^0}$ mixing differs by removing the discrete ambiguity in $\phi_{12} \rightarrow \phi_{12} + \pi$.

II. FORMALISM

We begin with a summary of the formalism for neutral meson mixing and decays . The neutral meson mass eigenstates are linear combinations of the strong interaction eigenstates $|M^0\rangle$ and $|\overline{M}^0\rangle$,

$$|M_{1,2}\rangle = p|M^0\rangle \pm q|\overline{M}^0\rangle,$$

where $|q|^2 + |p|^2 = 1$. We define the mass and width differences as

$$x \equiv \frac{m_2 - m_1}{\Gamma}, \quad y \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma},$$

where $\Gamma \equiv (\Gamma_1 + \Gamma_2)/2$ is the average width.

The decay amplitudes of the neutral mesons $M$ and $\overline{M}$ to CP conjugate final state $f$ and $\overline{f}$ are denoted as

$$A_f = \langle f | \mathcal{H} | M^0\rangle, \quad \overline{A}_f = \langle f | \mathcal{H} | \overline{M}^0\rangle,$$

$$A_{\overline{f}} = \langle \overline{f} | \mathcal{H} | M^0\rangle, \quad \overline{A}_{\overline{f}} = \langle \overline{f} | \mathcal{H} | \overline{M}^0\rangle,$$

where $\mathcal{H}$ is the weak interaction effective Hamiltonian.

The decay amplitudes for the tree-level dominated decays can, in general, be written as

$$A_f = A_f^T e^{i\phi_f} [1 + r_f e^{i(\delta_f + \phi_f)}],$$

$$\overline{A}_{\overline{f}} = A_{\overline{f}}^T e^{i(\delta_{\overline{f}} + \phi_{\overline{f}})} [1 + r_{\overline{f}} e^{i(\delta_{\overline{f}} - \phi_{\overline{f}})}],$$

$$\overline{A}_f = A_f^T e^{-i\phi_f} [1 + r_f e^{i(\delta_f - \phi_f)}],$$

$$\overline{A}_{\overline{f}} = A_{\overline{f}}^T e^{-i(\delta_{\overline{f}} - \phi_{\overline{f}})} [1 + r_{\overline{f}} e^{i(\delta_{\overline{f}} + \phi_{\overline{f}})}],$$

(4)

where $A_f^T$ and $A_{\overline{f}}^T$ are the magnitudes of the dominant SM tree-level contributions. The ratios $r_f$ and $r_{\overline{f}}$ are the relative magnitudes of subleading contributions containing new weak phases (they could arise from new physics, or from SM amplitudes with suppressed CKM structure). $\phi_f$, $\phi_{\overline{f}}$, $\delta_f$, and $\delta_{\overline{f}}$ are weak (CP violating) phases which appear with opposite signs in CP conjugate amplitudes, and $\delta_f$, $\delta_{\overline{f}}$ are strong (CP conserving) phases which appear with the same signs in CP conjugate amplitudes. All of the quantities entering Eq. (4), except the weak phases, are understood to be phase space independent for 3-body and higher final states.

In the case of decays to CP eigenstates, $\Delta_f = 0(\pi)$ for CP even (odd) final states. Eq. (4) therefore reduces to

$$A_f = A_f^T e^{i\phi_f} [1 + r_f e^{i(\delta_f + \phi_f)}],$$

$$\overline{A}_f = \eta_f^{CP} A_f^T e^{-i\phi_f} [1 + r_f e^{i(\delta_f - \phi_f)}],$$

(5)

where $\eta_f^{CP} = +(-)$ for CP even (odd) final states. The time-dependent CP asymmetries depend on the universal quantity

$$\lambda_f \equiv \frac{q A_f}{p A_f} = -\eta_f^{CP} \frac{q}{p} e^{i\phi},$$

(6)

where $r_f$ of Eq. is neglected in the equality, and $\phi$ is the relative weak phase between the mixing and decay amplitudes. Examples of decays to CP eigenstates include $D^0 \rightarrow K^+ K^-, \pi^+ \pi^-, K_s \pi^0$, and $B_s \rightarrow J/\Psi \phi, D_{s}^{(*)} K_s^{(*)}$. In the “pure-penguin” decay $B_s \rightarrow \phi \phi, A_T^f$ is the magnitude of the Standard Model penguin amplitude. Neglecting $r_f$, the weak phase $\phi$ in Eq. is the same as in the tree-level $B_s$ examples above, up to a small correction $\delta \phi = 2 \text{Im}(V_{ub}^*V_{cb}/V_{cs}^*V_{cb})$.

For final states which are not CP eigenstates, the time-dependent CP asymmetries depend on

$$\lambda_f \equiv \frac{q \overline{A}_f}{p A_f} = -\frac{q}{p} R_f e^{i(\phi + \Delta_f)},$$

$$\lambda_{\overline{f}} \equiv \frac{q \overline{A}_{\overline{f}}}{p A_{\overline{f}}} = -\frac{q}{p} R_{\overline{f}} e^{i(\phi - \Delta_{\overline{f}})},$$

(7)

where $r_f$ and $r_{\overline{f}}$ of Eq. are neglected in the equalities, and thus $R_{\overline{f}}^{-1} = R_f = A_{\overline{f}}^T/A_f^T$. Examples are $D^0 \rightarrow K^+ \pi^\pm, K^+ K$ and $B_s \rightarrow D_{s}^{(*)} D_{s}^{(*)}$. The weak phase $\phi$ is
the same in Eqs. (6) and (7) for the $D^0$ decays (up to negligible corrections $\lesssim |(V_{ub}V_{cb})/(V_{ub}V_{cb})| \sim 10^{-3}$) and tree-level $B_s$ decays listed above. The $M^0$ and $M^0$ transition amplitudes are

$$\langle M^0|H|M^0\rangle = M_{12} \frac{i}{2} \Gamma_{12},$$
$$\langle M^0|H|M^0\rangle = M_{12}^{*} \frac{i}{2} \Gamma_{12}^{*},$$

where $H$ is the $2 \times 2$ effective Hamiltonian governing neutral meson mixing. We define the mixing parameters $x_{12} \equiv 2|\tilde{M}_{12}|/\Gamma$, $y_{12} \equiv |\Gamma_{12}|/\Gamma$, $\phi_{12} \equiv \arg(M_{12}/\Gamma_{12})$. (9)

The notation $x_{12}$, $y_{12}$ is borrowed from [3]. $\phi_{12}$ is a CP violating weak phase which is responsible for CP violation in mixing ($|q/p| \neq 1$). Solving the eigenvalue problem yields

$$(x - iy)^2 = x_2^2 - y_2^2 - i2x_2y_2 \cos \phi_{12}, \quad \text{or}$$
$$x^2 - y^2 = x_2^2 - y_2^2, \quad xy = x_2y_2 \cos \phi_{12},$$

and

$$\frac{q}{p} = -\frac{\Gamma(x - iy)}{2(M_{12}^{*} - \frac{i}{2} \Gamma_{12}^{*})} = -\frac{2(M_{12}^{*} + \frac{1}{2} \Gamma_{12}^{*})}{\Gamma(x - iy)}. \quad (11)$$

The phase transformation $|M^0\rangle \rightarrow e^{i\theta}|M^0\rangle$, $|M^0\rangle \rightarrow e^{-i\theta}|M^0\rangle$ has no physical effects, due to conservation of Strangeness, Charm, or Beauty number by the strong interactions. Indeed, it is easily seen that $\phi_{12}$, $\lambda_f$, $\lambda_T$, $x$, and $y$ (which are related to, or are themselves observables) are invariant under these phase redefinitions [1]. Furthermore, the mass eigenstates are rotated by a common phase factor.

One is free to identify $M_2$ or $M_1$ with either the short-lived meson ($M_S$) or the heavier meson ($M_H$), by redefining $q \rightarrow -q$. This is equivalent to choosing a sign-convention for $y$, which in turn fixes the sign of $x$ via $\text{sign}(\cos \phi_{12})$, or vice-versa. Note that changing the sign-convention for $y$ (or $x$) takes $\lambda_f \rightarrow -\lambda_f$, or equivalently, $\phi \rightarrow \phi + \pi$. However, the combinations $y \lambda_f$, $x \lambda_f$, or $y \cos \phi$, $y \sin \phi$ and $x \cos \phi$, $x \sin \phi$ are sign-convention independent, which is seen explicitly from Eq. (11). Thus, they are candidates for being related to physical observables.

Examples of CP conserving observables are $\text{sign}(y \cos \phi)$ and $\text{sign}(x \cos \phi)$. In the limit of small or no CP violation, respectively: (i) $M_S$ would be approximately or exactly CP-even if and only if $\text{sign}(y \cos \phi) = +1$, and (ii) $M_H$ would be approximately or exactly CP-even if and only if $\text{sign}(x \cos \phi) = +1$. This is seen from Eqs. (11), (6), and (8) by requiring that CP-even ($M_+$) and CP-odd ($M_-$) states decay into CP-even and CP-odd final states, respectively. In fact, in the $D^0$ system in the limit of CP conservation, the observable $y_{CP}$, defined in Eq. (32), is equivalent to [8]

$$y_{CP} \equiv \frac{\Gamma(D^0(t) \rightarrow f)}{\Gamma(D^0(t) \rightarrow f)}.$$ 

The world average is [9],

$$y_{CP} = (1.07 \pm 0.26). \quad (13)$$

Taking into account that $|q/p| \approx 1$ and $|\sin \phi| \ll 1$, see Eq. (33), one finds that $y_{CP} \approx y \cos \phi$ to very good approximation [3], thus explicitly realizing (i) above.

An alternative choice employed by the PDG [10] and HFAG [11] collaborations for the $K^0$ and $D^0$ systems, is to identify $M_2$ with the would-be CP-even state in the limit of no CP violation. This amounts to choosing a convention for $\phi$, i.e., $\phi \approx 0$ rather than $\phi \approx \pi$. Given that in both systems the approximately CP-even state is $M_S$, this choice is equivalent to the sign-convention $y > 0$.

If $M_2$ were identified with $M_S$ ($y > 0$), Eq. (11) would give

$$x = \text{sign}(\cos \phi_{12}) \times$$

$$\left( x_2^2 - y_2^2 + \sqrt{(x_2^2 + y_2^2)^2 - 4x_2y_2x_1y_1 \sin^2 \phi_{12}} \right)^\frac{1}{2},$$

and

$$y =$$

$$\left( y_2^2 - x_2^2 + \sqrt{(x_2^2 + y_2^2)^2 - 4x_2y_2x_1y_1 \sin^2 \phi_{12}} \right)^\frac{1}{2}. \quad (15)$$

If, instead, $M_2$ were identified with $M_H$ ($x > 0$), then the factor $\text{sign}(\cos \phi_{12})$ would be moved to the equation for $y$, with appropriate modifications for the choices $y < 0$ or $x < 0$. These equations relate the the neutral meson mass and width differences to the underlying mixing parameters $x_{12}$, $y_{12}$, and $\phi_{12}$.

III. THE CP ASYMMETRIES

CP violation in pure mixing corresponds to $|q/p| \neq 1$. It can be measured via the “wrong-sign” semileptonic CP asymmetry,

$$a_{SL} \equiv \frac{\Gamma(M^0(t) \rightarrow f^- X) - \Gamma(M^0(t) \rightarrow f^+ X)}{\Gamma(M^0(t) \rightarrow f^- X) + \Gamma(M^0(t) \rightarrow f^+ X)} = \frac{\langle q/p \rangle}{|q/p|^2}.$$ 

In the limit $|q/p| \rightarrow 1 \ll 1$, which holds to good approximation for all four meson systems, $a_{SL} = 2(|q/p| - 1)$.

The $D^0$ time-dependent decay rates into a final state $f$ can be written as (see, for example, [2])

$$\Gamma(D^0(t) \rightarrow f) = \frac{1}{2} e^{-\tau|A_f|^2} \left\{ 1 + |\lambda_f|^2 \cosh(y \tau) \right\} \times$$

$$+(1 - |\lambda_f|^2) \cos(x \tau) + 2\text{Re}(\lambda_f)$$

$$\times \sinh(y \tau) - 2\text{Im}(\lambda_f) \sin(x \tau) \right\}, \quad (17)$$

$$\Gamma(D^0(t) \rightarrow f) = \frac{1}{2} e^{-\tau|\overline{A_f}|^2} \left\{ 1 + |\lambda_f^{-1}|^2 \cosh(y \tau) \right\} \times$$

$$+(1 - |\lambda_f^{-1}|^2) \cos(x \tau) + 2\text{Re}(\lambda_f^{-1})$$

$$\times \sinh(y \tau) - 2\text{Im}(\lambda_f^{-1}) \sin(x \tau) \right\}. \quad (18)$$
where $\tau = \Gamma_D t$.

For $D^0$ decays to CP eigenstates the above expressions yield, to good approximation, purely exponential forms due to the small values of $x$ and $y$,

\[ \Gamma(D^0(t) \to f) \propto \exp[-\hat{\Gamma}_{D^0 \to f} t], \]
\[ \Gamma(D^0(t) \to \bar{f}) \propto \exp[-\hat{\Gamma}_{D^0 \to \bar{f}} t]. \] (19)

The decay rate parameters are \[ [4] \]

\[ \hat{\Gamma}_{D^0 \to f} = \Gamma_D [1 + \eta_f^{CP} |q/p| (y \cos \phi - x \sin \phi)], \]
\[ \hat{\Gamma}_{D^0 \to \bar{f}} = \Gamma_D [1 + \eta_f^{CP} |p/q| (y \cos \phi + x \sin \phi)], \] (20)

where $\phi$ is defined in Eq. (6), and $r_f$ has been neglected. Note that Eq. (20) applies to singly Cabibbo suppressed (SCS) 2-body decays (e.g., $D^0 \to K^+ K^-, \pi^+ \pi^-$), and to 2-body decays in which both Cabibbo favored (CF) and doubly Cabibbo suppressed (DCS) amplitudes contribute (e.g., $D^0 \to K^0 \pi^0$). In the case of decays to CP eigenstates which are resonances or multi-body states, Eq. (20) is valid when ignoring the interference of these amplitudes with other amplitudes in phase space, see below.) One defines the CP violating combination (or lifetime CP asymmetry),

\[ \Delta Y_f = \frac{\hat{\Gamma}_{D^0 \to f} - \hat{\Gamma}_{D^0 \to \bar{f}}}{2}\Gamma_D = a^m + a^i, \] (21)

where

\[ a^m = -\eta_f^{CP} \frac{y^*}{2} \cos \phi \left( \frac{|q|}{|p|} - \frac{|p|}{|q|} \right), \]
\[ a^i = \eta_f^{CP} \frac{x}{2} \sin \phi \left( \frac{|q|}{|p|} + \frac{|p|}{|q|} \right). \] (22)

$a^m$ and $a^i$ are the contributions due to CPVMIX ($|q/p| \neq 1$) and CPVINT ($\sin \phi \neq 0$), respectively, and are universal quantities. Note that they are independent of sign convention for $x$ or $y$. Subleading, non-universal corrections to $\hat{\Gamma}_{D^0 \to f}$ due to $r_f \neq 0$ are discussed in Section VI.

In SCS $D^0$ decays to non-CP eigenstates (e.g., $D^0 \to K^+ K^-$), the final states are essentially resonances or multi-body states. The time-dependence of the decays is again exponential, to good approximation, and is independent of phase space if the interference of these amplitudes with other amplitudes is ignored. In general, in decays to resonances, or multi-body decays, the exponential decay rates depend on phase space (e.g., for 3-body decays, the location in the Dalitz plot) and give two CP violating combinations \[ [11], \]

\[ \Delta Y_f = \frac{\hat{\Gamma}_{D^0 \to f} - \hat{\Gamma}_{\bar{D}^0 \to \bar{f}}}{2\Gamma_D} = a_f^m + a_f^i, \]
\[ \Delta Y_{\bar{f}} = \frac{\hat{\Gamma}_{\bar{D}^0 \to f} - \hat{\Gamma}_{\bar{D}^0 \to \bar{f}}}{2\Gamma_D} = a_{\bar{f}}^m + a_{\bar{f}}^i, \] (23)

where, neglecting $r_f$ and $r_{\bar{f}}$,

\[ a_f^m = -R_f \frac{y_f^*}{2} \cos \phi \left( \frac{|q|}{|p|} - \frac{|p|}{|q|} \right), \]
\[ a_f^i = R_f \frac{x_f}{2} \sin \phi \left( \frac{|q|}{|p|} + \frac{|p|}{|q|} \right), \] (24)

(for $a_{\bar{f}}$, replace $f \to \bar{f}$),

\[ x_f = x \cos \Delta_f + y \sin \Delta_f, \quad y_f = y \cos \Delta_f - x \sin \Delta_f, \]
\[ x_{\bar{f}} = x \cos \Delta_f - y \sin \Delta_f, \quad y_{\bar{f}} = y \cos \Delta_f + x \sin \Delta_f, \] (25)

and $\phi$, $\Delta_f$ are defined in Eq. (7). In SCS decays one expects $R_f = O(1)$, implying that the CP asymmetries for non-CP eigenstates should be of same order as for CP eigenstates.

The quantities $a_f^m$, $a_f^i$, $a_{\bar{f}}^m$, $a_{\bar{f}}^i$ are not universal for non-CP eigenstate final states, due to the presence of strong phases. However, the latter can be determined, e.g., for 3-body decays, from Dalitz plot analyses. For example, in the simple case of a single resonance, $K^+ K^-$, in the Dalitz plot, $\Delta_{K^+ K^-}$ can be determined from the interference region of $K^+ K^-$ with $K^- K^+$. Consequently, $x$, $y$, $|q/p|$, and $\phi$ can be determined (up to discrete ambiguities) in Dalitz plot analyses of final states such as $D^0 \to K_s K^+ K^-$ and $D^0 \to \pi^+ \pi^- \pi^0$ \[ [11]. \]

In the case of CF and DCS decays to non-CP eigenstates, the time-dependence for $D^0$ decays to the “wrong-sign” (WS) final states $D^0(t) \to f$ and $\bar{D}^0(t) \to \bar{f}$ is expanded to quadratic order in $\tau$ inside the curly brackets of Eqs. (17) and (18), due to the small values of $\tan^2 \theta_c$, $x$, and $y$ ($A_{f}^{T}$ is chosen to be the DCS amplitude, e.g., $D^0 \to K^+ \pi^-$ or $\bar{f} = K^+ \pi^{-}$). The result can, in general, be written as (we adopt a notation similar to the one used in the experimental analysis of $D^0 \to K^+ \pi^0$ \[ [13], \]

\[ \Gamma[D^0(t) \to f] = e^{-\tau |A_f|^2} \times \]
\[ \left[ (R_f^+)^2 + R_f^{-} y^+ \tau + \frac{(x^+)^2 + (y^+)^2}{4} \tau^2 \right], \]
\[ \Gamma[\bar{D}^0(t) \to \bar{f}] = e^{-\tau |A_{\bar{f}}|^2} \times \]
\[ \left[ (R_{\bar{f}}^+)^2 + R_{\bar{f}}^{-} y^- \tau + \frac{(x^-)^2 + (y^-)^2}{4} \tau^2 \right], \] (26)

where $R_f^+ = |A_f^+|/|A_f|$ and $R_{\bar{f}}^+ = |A_{\bar{f}}^+|/|A_{\bar{f}}|$ are the magnitudes of the DCS to CF amplitude ratios for $D^0$ and $\bar{D}^0$ decays. Neglecting $r_f$ and $r_{\bar{f}}$ (until Section VI), $R_f^+ = R_f^*= R_f = |A_f^+|/|A_f| = A_{f}^{T}/A_{f}^{T}$, as in Eq. (7), so that

\[ y^\pm = \left( \pm \frac{|q|}{|p|} - \frac{|p|}{|q|} \right) \times (x_{f}^{T} \sin \phi \pm y_{f}^{T} \cos \phi), \]
\[ x^\pm = \left( \pm \frac{|q|}{|p|} - \frac{|p|}{|q|} \right) \times (x_{f}^{T} \cos \phi \pm y_{f}^{T} \sin \phi), \] (27)
where \( x_f^2 \) and \( y_f^2 \) have been defined in Eq. (25). In addition,
\[
|q/p|^2(x^2 + y^2) = (x^\pm)^2 + (y^\pm)^2,
\]
allowing \(|q/p|\) to be expressed solely in terms of \((x^\pm)^2 + (y^\pm)^2\) above. The expressions given in Refs. [4] and [13] for \( y^\pm \) and \( x^\pm \) differ from those in Eq. (28) due to choice of convention, and are recovered by substituting \( \Delta f \rightarrow -\Delta f + \pi \) in \( x_f^2 \) and \( y_f^2 \). The time dependence for \( D^0 \) decays to the “right-sign” (RS) final states is, to good approximation, exponential and given by
\[
\Gamma[D^0(t) \rightarrow f] = e^{-\tau |A_f|^2},
\]
\[
\Gamma[D^f(t) \rightarrow f] = e^{-\tau |A_f|^2}.
\]
Thus, the decay rate parameter is \( \hat{\Gamma}_{D^0 \rightarrow K^-\pi^+} = \Gamma_D \).

A fit to the time-dependence in Eqs. (26) and (27) yields measurements of \( R_{f}^\pm \), \( y_{f}^\pm \), and \( x_{f}^\pm \), which can be used to determine or constrain \( |q/p| \) and \( \phi \), as carried out in [13] for \( D^0 \rightarrow K^\pm \pi^\mp \). Note that the CP violating quantity \((y^+ - y^-)\) satisfies
\[
R_{f}(y^+ - y^-) = \Delta Y_f,
\]
where \( R_{f} = R_{f}^{-1} \) is the magnitude of the CF to DCS amplitude ratio (for \( r_f = r_{f} = 0 \)), see Eqs. (7) and (28). Finally, the contributions of CPVMIX and CPVINT in \( D^0 \) decays to RS final states are relatively suppressed by \( \tan^4 \theta_c \), and are therefore not considered.

An important CP conserving quantity \( y_{CP} \), mentioned in Section II, can be defined in terms of the decay rate parameters \( \hat{\Gamma}_{D^0 \rightarrow f_{CP}} \) (for SCS decays to CP eigenstates) and \( \hat{\Gamma}_{D^0 \rightarrow K^-\pi^+} \),
\[
y_{CP} = \eta_{f} \hat{\Gamma}_{D^0 \rightarrow f_{CP}} + \hat{\Gamma}_{D^0 \rightarrow f_{CP}} - 1.
\]
The expressions for the decay rate parameters given above (in the \( r_f = 0 \) limit) imply \[4\]
\[
y_{CP} = \frac{y}{2} \cos \phi \left( \left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) - \frac{x}{2} \sin \phi \left( \left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right).
\]
The time-integrated CP asymmetry for \( D^0 \) decays to CP eigenstates (SCS and CF/DCS) is defined as
\[
a_f = \frac{\Gamma(D^0 \rightarrow f) - \Gamma(D^f \rightarrow f)}{\Gamma(D^0 \rightarrow f) + \Gamma(D^f \rightarrow f)}. \tag{34}
\]
Expanding to leading order in \( x, y, r_f \) yields \[11\]
\[
a_f = a_f^d + a_f^m + a_f^t,
\]
where \( a_f^m \) and \( a_f^t \) are given in Eq. (22), and
\[
a_f^d = 2r_f \sin \phi_f \sin \delta_f \tag{36}
\]
is the (non-universal) direct CP violation contribution. The time-dependent CP asymmetry (\( \Delta Y_f \)) and the time-integrated CP asymmetry (\( a_f \)) are equal if there is no direct CP violation.

For SCS \( D^0 \) decays to non-CP eigenstates there are two time-integrated CP asymmetries to consider,
\[
a_f = \frac{\Gamma(D^0 \rightarrow f) - \Gamma(D^f \rightarrow f)}{\Gamma(D^0 \rightarrow f) + \Gamma(D^f \rightarrow f)}, \tag{37}
\]
Expanding to leading order in \( x, y, r_f, r_{f} \) yields \[11\]
\[
a_f = a_f^d + a_f^m + a_f^t, \quad a_{f} = a_f^d + a_f^m + a_f^t \tag{38}
\]
where \( a_f^m \), \( a_f^m \), and \( a_f^t \), \( a_f^t \) are given in Eq. (24), and
\[
a_f^d = 2r_f \sin \phi_f \sin \delta_f, \quad a_f^d = 2r_{f} \sin \phi_f \sin \delta_{f} \tag{39}
\]
are the direct CP violation contributions. Again, if there are no new weak phases in decay, the time-dependent and time-integrated CP asymmetries are equal, i.e., \( \Delta Y_f = a_f \) and \( \Delta Y_f = a_{f} \).

In the case of CF/DCS decays to non-CP eigenstates, and in our convention for RS and WS final states, the definitions of \( a_f \) and \( a_{f} \) in Eq. (37) correspond to the RS and WS time-integrated CP asymmetries, respectively (e.g., \( a_{K^-\pi^+} \) and \( a_{K^+\pi^-} \) for \( D^0 \rightarrow K^\pm \pi^\mp \)). To leading order in \( x, y, r_f, r_{f} \) they are given by
\[
a_f = a_f^d, \quad a_{f} = R_{f}(y^+ - y^-) + a_f^d \tag{40}
\]
where the RS \( (a_f^d) \) and WS \( (a_{f}^d) \) direct CP asymmetries are as in Eq. (39).

The time-dependent CP asymmetry for \( B_s \) decay to a CP eigenstate, to leading order in \( r_f \) and for \( |q/p| = 1 \) (the HFAG average is \( |q/p| = 1.002 \pm 0.005 \)), takes the simple form \[1\]
\[
\Gamma(B_s(t) \rightarrow f) - \Gamma(B_s(t) \rightarrow f) \Gamma(B_s(t) \rightarrow f) + \Gamma(B_s(t) \rightarrow f) = S_f \sin(|x| \Gamma t) - C_f \cos(|x| \Gamma t), \tag{41}
\]
where
\[
S_f = \eta_{f}^{CP} \sin(x) \sin \phi, \quad C_f = 2r_f \sin \phi_f \sin \delta_f \tag{42}
\]
are the contributions due to interference between mixing and decay, and direct CP violation, respectively. The factor \( \sin(x) \) in \( S_f \) originates from the time-dependence of the decay rates, via \( \sin(x \Gamma t) = \sin(x \sin(|x| \Gamma t)) \), and insures that \( S_f \) is independent of sign convention.

For \( B_s \) decays to non-CP eigenstates there are two
time-dependent CP asymmetries to consider,
\[
\frac{\Gamma(B_s(t \to f)) - \Gamma(B_s(t \to \bar{f}))}{\Gamma(B_s(t \to f)) + \Gamma(B_s(t \to \bar{f}))} = S_f \sin(|x| \Gamma t) - C_f \cos(|x| \Gamma t),
\]
\[
\frac{\Gamma(B_s(t \to \bar{f})) - \Gamma(B_s(t \to f))}{\Gamma(B_s(t \to f)) + \Gamma(B_s(t \to \bar{f}))} = S_f^\prime \sin(|x| \Gamma t) - C_f^\prime \cos(|x| \Gamma t),
\]
where (again leading order in \(r_f\), and for \(|q/p| = 1\),
\[
S_f = S_f^\prime = 2 \, \text{sign}(x) \, \sin(\phi) \, \sin(\delta_f) \, r_f/(1 + r_f^2), \quad C_f = 2 \, r_f \, \sin(\phi) \, \sin(\delta_f), \quad C_f^\prime = 2 \, r_f \, \sin(\phi) \, \sin(\delta_f).
\]
The equality between \(S_f\) and \(S_f^\prime\) holds, up to negligible corrections of \(O(|q/p| - 1)\).

\section{IV. Relating the Indirect CP Asymmetries}

In general, we are interested in decays to final states common to \(M^0\) and \(M^0\), whose leading contributions to \(\Gamma_{12}\) are proportional to the dominant CKM structure entering this quantity, i.e., \((V_{cb} V_{ub}^*)^2\) for the \(D^0\) and \((V_{cb} V_{us}^*)^2\) for the \(B_s\). All of the examples we have mentioned previously are in this class. In this section we assume that there are no subleading amplitudes with new weak phases in these decays \(|r_f| = r_f = 0\) in Eq. (2), and we neglect CKM suppressed contributions to \(\Gamma_{12}\). The following relations are then satisfied:

\[
\frac{\Gamma_{12}}{\Gamma_{12}} = \frac{A_f A_f^*}{A_f^* A_f} = \left(\frac{A_f}{A_f^*}\right)^2,
\]
and
\[
\frac{\Gamma_{12}}{\Gamma_{12}} = \frac{A_f A_f^* + A_f^* A_f}{A_f^* A_f + A_f A_f} = \frac{A_f}{A_f^*} \frac{A_f^*}{A_f},
\]
for CP-eigenstate and non-CP-eigenstate final states, respectively. CKM suppressed contributions to \(\Gamma_{12}\) and to \(r_f\) within the SM yield corrections to these relations of \(O((V_{cb} V_{ub})/(V_{cb} V_{ub})) \approx 6 \times 10^{-4}\) for \(D^0\) decays, and of \(O((V_{ub} V_{us})/(V_{ub} V_{us})) \approx 0.02\) for \(B_s\) decays [see Eq. (113)].

The following formulae, obtained from Eqs. (11) and (111), will be useful:

\[
|q/p|^2 (x^2 + y^2) = x_{12}^2 + y_{12}^2 + 2x_{12} y_{12} \sin \phi_1,
\]
\[
\frac{|q/p|^4}{p} = \left(\frac{x_{12}^2 + y_{12}^2 + 2x_{12} y_{12} \sin \phi_1}{x_{12}^2 + y_{12}^2 - 2x_{12} y_{12} \sin \phi_1}\right),
\]
\[
y_{12} = \frac{y^2 + A_m^2 x^2}{1 - A_m}, \quad x_{12} = \frac{x^2 + A_m^2 y^2}{1 - A_m},
\]
where
\[
A_m = \frac{(|q/p|^2 - 1)}{|(|q/p|^2 + 1)|}
\]
is related to CP violation in mixing. Note that Eq. (18), which also appears in [6], relates CPVMIX to the underlying mixing parameters \(x_{12}, y_{12}\), and \(\phi_{12}\).

Multiplying (see Eq. (11))
\[
\left(\frac{q}{p}\right)^2 = \frac{M_{12} - \frac{1}{2} \Gamma_{12}}{\Gamma_{12} - \frac{1}{2}},
\]
on the l.h.s. by \((A_f/A_f)^2\) for decays to CP eigenstates to obtain \(\lambda_f^2\) (or by \((A_f^*/A_f)/A_f\) for decays to non-CP eigenstates to obtain \(\lambda_f^2\)), and on the r.h.s. by \(\Gamma_{12}/\Gamma_{12}\) yields
\[
\frac{\Gamma_{12}}{\Gamma_{12}} = \frac{\sin(\phi_1)}{\cos(\phi_1)}, \quad \frac{\sin(\phi_f)}{\cos(\phi_f)} = \frac{2A_m x y}{y^2 + A_m^2 x^2}, \quad \sin(\phi_2) = \frac{2A_m x y}{y^2 + A_m^2 x^2}.
\]

The first relation is incorporated into the fit of \(x_{12}, y_{12}\), and \(\phi_{12}\) using the \(D^0 - \bar{D}^0\) mixing data. The last two relations are obtained by eliminating the dependence of \(\sin(\phi_2)\) and \(\cos(\phi_2)\) on \(x_{12}, y_{12}\), and \(\phi_{12}\), using Eqs. (11). Finally, a trigonometric identity yields
\[
\frac{\Gamma_{12}}{\Gamma_{12}} = \frac{1 - |q/p|^2}{x/y} = \frac{a_{SL} x}{y^2}.
\]
As discussed in [6], this relation gives an excellent description of the data in the neutral kaon system.

It is straightforward to relate \(\Delta Y_f\) and the semileptonic CP asymmetry using Eq. (113), after expanding to first order in \(|q/p|\). In the case of \(D^0\) decays, the same relations also apply to the time-integrated CP asymmetries (for \(r_f = r_f = 0\)). For decays to CP eigenstates, one obtains
\[
\Delta Y_f = a_f = -y \cos(\phi) \eta_f e^{\phi} \frac{a_{SL} x^2 + y^2}{y^2}.
\]
We know from experiment that the level of CP violation in the \(D^0\) system is small and that the short-lived meson is approximately CP-even, implying \(|\cos(\phi)\approx 1\), sign(\(y \cos(\phi)\) = +1 (as in the Standard Model) and, to good approximation,
\[
\Delta Y_f = a_f = -\eta_f e^{\phi} \frac{a_{SL} x^2 + y^2}{y^2}|y|,
\]
which is independent of sign convention for \(x\) or \(y\). Similarly, we obtain
\[
y_{CP} = y \cos(\phi) = |y|.
\]
up to corrections of order $\sin^2 \phi$ or $a^2_{S_L}$.

For SCS $D^0$ decays to non-CP eigenstates, one obtains

$$R_f \Delta Y_T = \Delta Y_f/R_f = R_f \alpha_T = \alpha_f/R_f$$

$$= -\cos \Delta_f \frac{a_{S_L} y^2 + x^2}{|y|}. \quad (59)$$

Confirmation of the relation between the hadronic CP asymmetries in the first line of Eq. (59) does not require knowledge of $\Delta_f$. In terms of the CP-averaged branching ratios for $D^0 \to f$ and $D^0 \to \bar{f}$, it is simply given by

$$\Delta Y_T = \frac{\alpha_T}{\alpha_f} = \frac{\text{Br}(D^0 \to f)}{\text{Br}(D^0 \to \bar{f})}. \quad (60)$$

This relation follows non-trivially from Eq. (59): $R_f \Delta Y_f$ (or $R_f \alpha^C_f$) and $\Delta Y_f/R_f$ (or $\alpha^C_f/R_f$) could, in principle, differ by $O(1)$ given that $y \sim x$, so that $\Delta_f$ could be large, and that $|(q/p) - 1| \sim \sin \phi$ is allowed.

In the case of CF/DCS $D^0$ decays to non-CP eigenstates, Eqs. (53), (50), and (55) imply that the time-dependent, time-integrated, and semileptonic CP asymmetries are related as (recall $f = K^- \pi^+$ for $D \to K \pi$ in our convention)

$$y'^+ - y'^- = R_f \alpha_T = -\cos \Delta_f \frac{a_{S_L}(D^0)_y^2 + x^2}{2 |y|}. \quad (61)$$

The strong phase $\delta_{K\pi}$ for $D^0 \to K^+ \pi^-$ decays can be precisely measured by the BES-III Collaboration at the $\Psi(3770)$ charm threshold.

For $B_d$ decays to CP eigenstates, the time-dependent and semileptonic CP asymmetries are related as

$$2S_f/(1 - S_f^2) = -\eta_{f^C} \text{sign}(y \cos \phi) a_{S_L}|x/y|, \quad (62)$$

which is independent of sign convention for $x$ or $y$ ($|x|$ follows from sign($x$) in $S_f$). At this point, we elaborate on the determination of sign($y \cos \phi$) in $\delta_{f^C}$. Starting with Eq. (11), the last relation in Eq. (10), and taking $y \ll x$ (using the HFAG averages), the central value for $y/x$ is $\approx 0.003$, we obtain

$$\text{sign}(y \cos \phi) = \text{sign}([M_{12}]^2 \text{Re}[\Gamma^*_1 \bar{A}_f/A_f] + \text{Im}[M_{12}^* \text{Im}(M_1 \bar{A}_f/A_f)]. \quad (63)$$

The ratio of second to first terms above is given by $\sin \phi_1 \sin \phi/ \cos(\phi_1 + \phi)$. However, for $y \ll x$, $\phi_1 = \phi \text{mod}(\pi)$, see Eq. (52), implying that the magnitude of the ratio is less than 1. Thus, Eq. (63) simplifies, and

$$\text{sign}(y \cos \phi) = \text{sign}(\text{Re}[\Gamma^*_1 \bar{A}_f/A_f]). \quad (64)$$

Given that the impact of new physics on the r.h.s. would in general be subleading, we conclude that $\text{sign}(y \cos \phi) = \text{sign}(y \cos \phi)_{\text{SM}} = +1$, and that

$$2S_f/(1 - S_f^2)^{1/2} = -\eta_{f^C} |x/y| a_{S_L} \quad (65)$$

$$\begin{array}{|c|c|c|c|}
\hline
x \text{[\%]} & y \text{[\%]} & |q/p| & \phi \text{[rad]} \\
\hline
1.00 \pm 0.25 & 0.77 \pm 0.18 & 0.94 \pm 0.14 & -0.046 \pm 0.093 \\
1.00 \pm 0.25 & 0.76 \pm 0.18 & 0.86 \pm 0.16 & -0.15 \pm 0.13 \\
\hline
\alpha_{K\pi} \text{[rad]} & \delta_{K\pi} \text{[rad]} & R_D \text{[\%]} & A_D \text{[\%]} \\
\hline
0.40 \pm 0.19 & 0.20 \pm 0.37 & 0.336 \pm 0.008 & 0 \\
0.39 \pm 0.18 & 0.20 \pm 0.37 & 0.336 \pm 0.009 & -2.1 \pm 2.4 \\
\hline
\end{array}$$

TABLE I: HFAG outputs for $A_D = 0$ (first row) and $A_D \neq 0$ (second row)

in the absence of new weak phases in decay, as in [3]. For decays to non-CP eigenstates,

$$S_f = S_T = -\kappa |x/y| a_{S_L}/2, \quad (66)$$

where

$$\kappa = (4 R_f^2 \cos^2 \Delta_f - S_f^2 (R_f^2) + 1)^{1/2}/(R_f^2 + 1). \quad (67)$$

and, as usual, $R_f = \text{det}(A_f/A_f) = A_f^T/A_f$. The (near) equality of the two time-dependent CP asymmetries, already noted in Eq. (14), is a trivial consequence of $y \ll x$ and $|q/p| - 1 \ll 1$, unlike in $D^0$ decays.

V. FIT RESULTS FOR $D^0 - \bar{D}^0$ MIXING

The current $D^0 - \bar{D}^0$ mixing and CP violation fit results reported by the Heavy Flavor Averaging Group (HFAG) [6] can be expressed in terms of the four universal parameters ($x$, $y$, $|q/p|$, and $\phi$), two strong phases ($\delta_{K\pi}$ and $\delta_{K\pi}$), the CP averaged ratio of wrong-sign to right-sign $D^0 \to K^+ \pi^-$ decay rates ($R_D$), and the corresponding direct CP violation parameter ($A_D$). In terms of our notation for CF/DCS decays,

$$R_D = \frac{(R_f^+)^2 + (R_f^-)^2}{2},$$

$$A_D = \frac{(R_f^+)^2 - (R_f^-)^2}{(R_f^+)^2 + (R_f^-)^2} = a_f^2 - a_{S_L}^2, \quad (68)$$

with $f = K^- \pi^+$. The four universal parameters are extracted from fits to the time-dependent decay rates for $D^0 \to K^+ \pi^-$, $K^- \pi^+$, $K^- \pi^+$, $K^\pi \pi$, and the semileptonic decay rates [8]. The HFAG fit only allows for new weak phases in the $D^0 \to K^+ \pi^-$ amplitudes, via $A_{D_f} \neq 0$. New weak phases in decay and their impact on the $D^0 - \bar{D}^0$ mixing and CP violation fit are discussed in more detail in Sec. VI.

In general, $x$, $y$, and $|q/p|$ can be expressed in terms of the mixing parameters $x_{12}$, $y_{12}$, $\phi_{12}$, see Eqs. (12), (13), (18). In the absence of new weak phases in decay, the same is true for $\phi$, see Eq. (52). Using these four equations (recall that Eqs. (13) and (14) correspond to the HFAG convention, which identifies $M_2$ with the approximately CP-even state) $x$, $y$, $|q/p|$, and $\phi$
Parameter $A_D = 0$ \hspace{1cm} Eq. (52) removed \hspace{1cm} $A_D \neq 0$

| Parameter | $A_D = 0$ | $A_D = 0$ removed | $A_D \neq 0$ |
|-----------|------------|-------------------|-------------|
| $x_{12} \ [%]$ | $1.00 \pm 0.25$ | $1.00 \pm 0.25$ | $1.02 \pm 0.24$ |
| $y_{12} \ [%]$ | $0.77 \pm 0.18$ | $0.78 \pm 0.18$ | $0.75 \pm 0.18$ |
| $\phi_{12} \ [\text{rad}]$ | $0.02 \pm 0.08$ | $-0.12 \pm 0.22$ | $0.07 \pm 0.08$ |

TABLE II: Results for the mixing parameters at 1σ, see Sec. V (VI) for $A_D = 0$ ($A_D \neq 0$).

are determined by the mixing parameters $x_{12}, y_{12}, \phi_{12}$. Ranges for these underlying parameters can be extracted directly from experimental data under the assumption that $A_D = 0$; where HFAG currently reports seven parameters, only six would be reported.

For this work, we adopt a simpler strategy for extracting values of $x_{12}, y_{12},$ and $\phi_{12}$: we take the HFAG fit results (for the $A_D = 0$ case) for $x, y, |q/p|$, $\phi$, $\delta_{K\pi}$, $\delta_{K\pi\pi}$, $R_D$, shown in Table II and minimize

$$\chi^2 = \epsilon_i W_{ij} \epsilon_j$$

\hspace{1cm} (69)

where $\epsilon_i$ is the difference between the HFAG value for the $i^{th}$ parameter and the fitted value predicted using the equations which relate $x, y, |q/p|,$ and $\phi$ to $x_{12}, y_{12},$ and $\phi_{12}$; the weight matrix $W_{ij}$ is the inverse of the full error matrix for the values reported by HFAG, including the correlation coefficients [14]. The fitted values for $\delta_{K\pi}$, $\delta_{K\pi\pi}$, and $R_D$ are very close to the HFAG values; they change only due to (small) diagonal elements in $W_{ij}$. The HFAG parameters used as input are taken from a fit with $\chi^2 = 24.9$ for 21 degrees of freedom (28 experimental results minus 7 parameters). The value of $\chi^2$ in our fit is 0.2. In effect, the overall $\chi^2$ increases slightly as one degree of freedom is restored to the mix of measurements and the parameters to be extracted.

The fitted values of $x_{12}, y_{12},$ and $\phi_{12}$ are listed in the second column of Table II. In particular, we obtain

$$\phi_{12}^D [\text{rad}] = 0.02 \pm 0.08 \ (1 \sigma) .$$

\hspace{1cm} (70)

Our results for $x_{12}$ and $y_{12}$ are very close to the fitted values for $x$ and $y$ in Table I as would be expected for small $\phi_{12}$, see Eqs. (14), (15). A bound equivalent to a precision on $\phi_{12}^D$ of $0.18 \ (1 \sigma)$, which assumes no correlations between the experimental measurements, has recently been obtained in [13]. The HFAG error matrix corresponds to parabolic errors, and thus our two sigma and higher CL intervals are simple multiples of our 1σ CL interval. However, a preliminary HFAG fit [10] to the data, which uses Eqs. (14), (15), (48), and (52), as discussed above, indicates that the errors on $\phi_{12}$ are non-parabolic (and thus we do not list higher-CL intervals). Therefore, our fit result for $\phi_{12}$ is only approximate. The preliminary HFAG 1σ and 95% CL intervals for non-parabolic errors are

$$\phi_{12}^D [\text{rad}] = 0.02^{+0.06}_{-0.13} \ (1 \sigma) ,$$

\hspace{1cm} (71)

$$\in [-0.30, +0.30] \ (95\% \ CL) .$$

The former is similar to our result using parabolic errors. The HFAG fit results for parabolic errors are in agreement with ours.

The impact of the relation between $\phi$ and $\phi_{12}$ on the precision with which $\phi_{12}$ is constrained is seen by repeating the fit for the $A_D = 0$ case, but with Eq. (52) removed. In this case $\phi$ is treated as an independent parameter which is trivially fit. The result is reported in the third column of Table II. We observe that the error on $\phi_{12}$ increases by roughly a factor of three, and thus conclude that the relationship between CPVmix and CPVINT provides a powerful constraint on the allowed magnitude of CP violation in $D^0 - \bar{D}^0$ mixing.

Finally, to understand the implications of the bound on $\phi_{12}^D$ for model building, we separate $M_{12}$ into its SM and new physics parts,

$$M_{12} = M_{12}^\text{SM} e^{i\phi_{12}^\text{SM}} + M_{12}^\text{NP} e^{i\phi_{12}^\text{NP}} ,$$

\hspace{1cm} (72)

where only the difference of the weak phases $\phi_{12}^\text{NP} - \phi_{12}^\text{SM}$ is physical. We continue to assume that there are no new weak phases in decay, and identify $\Gamma_{12}$ with its SM value. The definition of $\phi_{12}$ then yields

$$\sin \phi_{12}^D = \frac{M_{12}^\text{NP}}{M_{12}^\text{SM}} \sin(\phi_{12}^\text{NP} - \phi_{12}^\text{SM}) ,$$

\hspace{1cm} (73)

where $|M_{12}|$ follows from the fitted value of $x_{12}$. In the usual phase convention in which $M_{12}^\text{SM}$ is real ($\phi_{12}^\text{SM} = 0$), the above bounds on $\phi_{12}^D$ thus imply that

$$\frac{\text{Im}(M_{12}^\text{NP})}{|M_{12}|} \in [-0.06, +0.10] \ (1\sigma) ,$$

\hspace{1cm} (74)

for parabolic errors, and

$$\frac{\text{Im}(M_{12}^\text{NP})}{|M_{12}|} \in [-0.11, +0.08] \ (1\sigma) ,$$

\hspace{1cm} (75)

$$\in [-0.30, +0.30] \ (95\% \ CL) .$$

for the (preliminary) non-parabolic HFAG errors. As shown in the next section, these bounds can not be substantially altered if we allow for new weak phases in decay.

VI. NEW WEAK PHASES IN DECAY

A. General considerations

In this section we discuss how the relations between CPVmix and CPVINT are modified by new weak phases from subleading decay amplitudes (originating from new physics, or CKM suppressed SM amplitudes). We begin with the resulting shifts in $\arg(\lambda_f)$, $\arg(\lambda_T)$, and $\arg(\Gamma^*_\mu/\Gamma_{12})$. Expressions relating $\arg(\lambda_f)$ and $\arg(\lambda_T)$ to $1 - |q/p|$, as well as to $\phi_{12}$, which depend on these shifts, are obtained, replacing the previous expressions...
involving $\phi$. In turn, new relations between the time-dependent and semileptonic CP asymmetries are derived for $D^0$ and $B^0$ decays. Direct CP violation bounds are used to constrain deviations from the $r_f = r_\tau = 0$ case, and the 1σ intervals for $x_{12}$, $y_{12}$, and $\phi_{12}$ from an appropriately modified fit to the $D^0 - \bar{D}^0$ mixing data are presented.

The argument $\phi_{12} \equiv \arg(-\lambda_f)$ for a decay to a CP eigenstate in Eq. (6) is shifted, to first order in $r_f$, as

$$
\phi_{12} = \phi + \delta \phi_{12}, \quad \delta \phi_{12} = -2r_f \cos \delta_f \sin \phi_f.
$$

(76)

For non-CP eigenstates, the arguments $\phi_{12} \equiv \arg(-\lambda_f)$ and $\phi_{12} \equiv \arg(-\lambda_{f'})$ in Eq. (6) are shifted by

$$
\delta \phi_{12} = -r_f \sin(\delta_f + \phi_f) + r_\tau \sin(\delta_f - \phi_f),
\delta \phi_{12} = -2r_f \sin(\delta_f + \phi_f) + r_f \sin(\delta_f - \phi_f).
$$

(77)

The new contribution to Arg$(\Gamma_{12}/\Gamma_{12}^\ast)$ is defined as

$$
\delta \phi_\Gamma \equiv \arg \left( \frac{\Gamma_{12}}{\Gamma_{12}^\ast} \right) - \arg \left( \frac{\Gamma_{0}^{12}}{\Gamma_{0}^{12}} \right) = 2 \ln \left( \frac{\Gamma_{12}^\ast}{\Gamma_{0}^{12}} \right),
$$

(78)

to leading order in $\delta \Gamma_{12} \equiv \Gamma_{12} - \Gamma_{0}^{12}$, where $\Gamma_{0}^{12}$ is the leading SM contribution to $\Gamma_{12}$ proportional to $(V_{cb}V_{ub}^\ast)^2$ for the $D^0$ and $(V_{cb}V_{ub}^\ast)^2$ for the $B_s$. Note that $\delta \phi_\Gamma$ is phase redefinition invariant and is an observable, unlike arg$(\Gamma_{12})$.

$\delta \phi_\Gamma$ receives contributions from CKM suppressed corrections to $\Gamma_{12}$ within the SM, and from subleading decay amplitudes ($r_f, r_\tau \neq 0$). (We note that a recent analysis of $\Gamma_{12}$ in the $D^0$ system indicates that the CKM suppressed corrections to $\delta \phi_\Gamma$ could be enhanced from $O(|V_{cb}V_{ub}^\ast|)$ in the SM to $O(0.01)$ in models with a fourth family.) The contribution to $\delta \phi_\Gamma$ from subleading decay amplitudes ($\delta \phi_{12}'$), expressed as a sum over exclusive final states, and to leading order in $r_f, r_\tau$, is given by

$$
\sum_f \sum_{f'} A_{f}^{f'} A_{f}^{f'} (r_f \cos(\Delta_f - \delta_f) \sin \phi_f + r_\tau \cos(\Delta_f + \delta_f) \sin \phi_\tau) = -2\left( \sum_f \eta_f^{CP} (A_{f}^{f})^2 \sum_{f',f''} 2A_{f'}^{f''} A_{f'}^{f''} \cos \Delta_f \right) =
$$

(79)

$$
\sum_f A_{f}^{f} A_{f}^{f} (r_f \cos(\Delta_f - \delta_f) \sin \phi_f + r_\tau \cos(\Delta_f + \delta_f) \sin \phi_\tau),
$$

where the sums are over CP and non-CP eigenstates. We learn that $\delta \phi_\Gamma$ is of $O(4r_f \sin \phi_f)$, roughly weighted by the fraction of $\Gamma_{12}$ that is attributed to the affected decay amplitudes within the SM. $r_f$ is the “generic” size of $r_f$ and $r_\tau$ in these amplitudes. The same qualitative conclusion can also be reached via the OPE treatment for $\Gamma_{12}$, in the case of the heavier $B_d$ and $B_s$ mesons.

The relation between $\phi$ and $\phi_{12}$ in Eq. (62) is replaced by

$$
\tan(2\phi_{12} - 2\delta \phi_{12} + \delta \phi_\Gamma) = -\frac{\sin 2\phi_{12}}{\cos 2\phi_{12} + y_{12}^2/x_{12}^2}.
$$

(80)

for decay to a CP eigenstate. The argument on the l.h.s. is simply $2\phi + \delta \phi_\Gamma$, which takes into account the shift in arg$(\Gamma_{12}/\Gamma_{12}^\ast)$ in Eq. (15). The relation between CPVMIX and CPVINT for decay to a CP eigenstate is now given, in terms of the observable $\phi_{12}$, by

$$
\tan(\phi_{12} - \delta \phi_{12} + \delta \phi_\Gamma/2) = -A_{12}/x_{12}.
$$

(81)

Expanding to lowest order in $r_f$ and $|q/p| - 1$ yields

$$
\tan(\phi_{12} - \delta \phi_{12} - \phi_\Gamma/2) = -\frac{\sin 2\phi_{12}}{\cos 2\phi_{12} + y_{12}^2/x_{12}^2} = \tan(\phi_{12} + \phi_{12}^\ast + \Delta_f + \delta \phi_\Gamma/2)
$$

(82)

for the dependence of the observables $\phi_{12}$ and $\phi_{12}^\ast$ on $\phi_{12}$, and

$$
\tan(\phi_{12} - \delta \phi_{12} - \Delta_f + \delta \phi_\Gamma/2) = -A_{12}/x_{12}.
$$

(83)

for the modified relations between CPVMIX and CPVINT.

Approximate bounds on $\delta \phi_{12}$, $\delta \phi_{12}^\ast$, and $\delta \phi_\Gamma$ for $D^0$ and $B_s$ decays can be obtained from direct CP violation measurements. It is instructive to compare them to the current experimental sensitivity to $\delta \phi_{12}$, $\delta \phi_{12}^\ast$, and $\delta \phi_\Gamma$ in time-dependent (mixing-related) measurements.

B. $D^0 - \bar{D}^0$ mixing

We need to consider new weak phases in singly Cabibbo suppressed (SCS) decays, and their combined effects in Cabibbo favored (CF) and doubly Cabibbo suppressed (DCS) decays. We begin with a discussion of the former. The HFAG average for $\Delta Y_f$ [9], obtained from the BaBar and Belle $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ measurements [18], is

$$
\Delta Y_f = (-0.123 \pm 0.248)\%.
$$

(85)

The time integrated CP asymmetries for $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ are [9],

$$
a_{K^+K^-} = (-0.16 \pm 0.23)\%,
\ a_{\pi^+\pi^-} = (0.22 \pm 0.37)\%.
$$

(86)
The direct CP asymmetries are obtained by subtracting $\Delta Y_f$ from the time integrated CP asymmetries, see Eq. (85), yielding

$$a_{K+K^0}^d = (-0.04 \pm 0.34)%, \quad a_{\pi^+\pi^-}^d = (0.34 \pm 0.45)%. \quad (87)$$

(Predictions for $a_f^d$ in the Standard Model suffer from large hadronic uncertainties spanning an order of magnitude or more, and could be as large as $\approx 0.1\%$.) Unless the new physics has a very special structure, e.g., parity conserving $K$, the results for $a_{K+K^0}^d$ and $a_{\pi^+\pi^-}^d$ give rough bounds on the direct CP asymmetries for all decays mediated by $c \rightarrow u(s\bar{s},d\bar{d})$ transitions. Models which can easily produce direct CP asymmetries of this size or larger in SCS decays have been discussed in [11].

In general, strong phase differences enter as $\sin \delta$ in the direct CP asymmetries, and as $\cos \delta$ in $\delta\phi_{\lambda_y}, \delta\phi_{\lambda_y}$, and $\delta\phi_T$. However, the relevant new physics ($|\Delta C| = 1$) effective operators for SCS decays differ from the tree-level SM operators in their color and chirality structures (the QCD penguin operators, most notably the chromo-magnetic dipole operator, are relatively unconstrained by $D^0 - \bar{D}^0$ mixing). Thus, strong phase suppression is not expected [11], implying that $\delta\phi_{\lambda_y} \sim a_f^d$. This justifies taking

$$|\delta\phi_{\lambda_y}|, |\delta\phi_T| \lesssim 1\% \quad (88)$$

for SCS decays, and similarly for the last term in Eq. (52).

The SCS decays enter the HFAG $D^0 - \bar{D}^0$ mixing fit via $\Delta Y_f$ (averaged over $\pi^+\pi^-$ and $K^+K^-$) and $y_{CP}$. In the case of decays to CP eigenstates a new weak phase would shift $\Delta Y_f$, to lowest order in $(1 - |q/p|)$ and $r_f$, by

$$\delta(\Delta Y_f) = -\eta_{CP}^f (|y| a_f^d - |x| \delta\phi_{\lambda_y}). \quad (89)$$

This result follows by substituting $\phi \rightarrow \phi_{\lambda_y}$, and $|y/p| \rightarrow |y_{CP}/p_{CP}|$ in Eq. (22), and expanding in small quantities. Note that the impact of $r_f \sin \phi_{\lambda_y}$ is suppressed by mixing ($x, y \sim 10^{-2}$), unlike in $a_f^d$ which enters the time integrated CP asymmetry. With $a_f^d, \delta\phi_{\lambda_y} < 1\%$ and $x, y \sim 1\%$, we find

$$|\delta(\Delta Y_f)| \lesssim 10^{-4}, \quad (90)$$

which is less than a few percent of the experimental uncertainty, see Eq. (53). The shift in $y_{CP}$ due to new weak phases in decay must be even smaller relative to its experimental uncertainty, given in Eq. (13), because its dependence on CP violating quantities must be quadratic (and still suppressed by $x$ or $y$).

The relation between $a_{SL}$ and $\Delta Y_f$ in Eq. (57) for decays to CP eigenstates is modified, to lowest order in $r_f$ and $(|q/p| - 1)$, as

$$\eta_{CP}^f \Delta Y_f = -a_{SL} \frac{y_f^2 + x_f^2}{2} - \frac{|y_f a_f^d}{2} + |x| (\delta\phi_{\lambda_y} - \delta\phi_T/2). \quad (91)$$

Given that the approximately CP-even $D^0$ mass eigenstate is the shorter-lived and heavier one, we have substituted $y \cos \phi \rightarrow |y|$ (as before) and $x \cos \phi \rightarrow |x|$, and similarly below. Applied to SCS decays, the new physics correction is again $\lesssim 10^{-4}$. The modified relations satisfied by $a_{SL}, \Delta Y_f$, and $\Delta Y_T$ for SCS decays to non-CP eigenstates which replace Eq. (57) are

$$\frac{\Delta Y_f/\Delta Y_T}{\cos \Delta_f} = -a_{SL} \frac{y_f^2 + x_f^2}{|y|} - \frac{|y_f a_f^d}{|y|} + |x| (\delta\phi_{\lambda_y} - \delta\phi_T), \quad (92)$$

$$\frac{\Delta Y_f/\Delta Y_T}{\sin \Delta_f} = |y| (\delta\phi_{\lambda_y} - \delta\phi_T) + |x| (a_f^d + a_f^d), \quad (93)$$

for SCS decays, and similarly for the last term in Eq. (52).

The difference between the CF and DCS direct CP asymmetries for $D^0 \rightarrow K^-\pi^+\pi^0$ (CF) and $D^0 \rightarrow K^+\pi^-\pi^0$ (DCS) are

$$a_f = (+0.16 \pm 0.89)%; \quad f = K^-\pi^+\pi^0,$$

$$a_f = (-1.4 \pm 5.2)%; \quad f = K^+\pi^-\pi^0. \quad (94)$$

The modified relations satisfy significant strong phase differences. The direct and time-integrated CP violation bounds therefore imply

$$|\delta\phi_{\lambda_y}|, |\delta\phi_{\lambda_y}|, |\delta\phi_T| \leq O(\text{few percent}) \quad (97)$$

(following our convention, take $r_f$ and $r_T$ in Eqs. (39) and (77) to correspond to CF and DCS new physics amplitudes, respectively). For completeness, we note that for CF and DCS decays to CP eigenstates (e.g., $D^0 \rightarrow K_s\pi^0, D^0\bar{D}^0$), $|\delta\phi_{\lambda_y}| \leq O(1\%)$, which is the approximate bound on CF direct CP violation (DCS contributions are suppressed by $\theta_c^d$).
New weak phases in CF and DCS transitions would enter the HFAG $D^0 \rightarrow \overline{D}^0$ mixing via $D^0 \rightarrow K^+\pi^-$ and $D^0 \rightarrow K_s\pi^+\pi^-$. For illustrative purposes, let’s consider $D \rightarrow K\pi$ in more detail. The general form for the time-dependent amplitudes $D^0(t) \rightarrow K^+\pi^-$ and $\overline{D}^0(t) \rightarrow K^-\pi^+$ is the same as in Eqs. (20) and (27). However, the corrected expressions for $y^\pm$ and $x^\pm$, see Eq. (28), are given by $(f = K^-\pi^+)$

\[
y^\pm = \left( + \frac{A_T}{A_f} q \right) \left( - \frac{A_f p}{A_T} q \right) \times (x_T \sin \phi^\pm + y_T \cos \phi^\pm),
\]

\[
x^\pm = \left( + \frac{A_T}{A_f} q \right) \left( - \frac{A_f p}{A_T} q \right) \times (x_T \cos \phi^\pm \pm y_T \sin \phi^\pm),
\]

(98)

where (in terms of the direct CP asymmetry for the CF decays),

\[
\left| \frac{A_T}{A_f} \right| = 1 + a^d_f = 1 + 2r_f \sin \delta_f \sin \phi_f ,
\]

(99)

and

\[
\phi^+ = \phi_{\lambda T} + \delta_T f = \phi + \delta \phi_{\lambda T},
\]

\[
\phi^- = \phi_{\lambda T} - \delta_T f = \phi + \delta \phi_{\lambda T}.
\]

(100)

Corrections to the relation between the time-dependent CP asymmetry $(y^{+} - y^{-}) = R_f \Delta y_T$ and $a_{SM}$ in Eq. (61) are easily obtained from Eqs. (92) and (93), applied to CF/DCS decays.

Measurements of $y^\pm$, $x^\pm$, and $R_D$, $A_D$ [defined in Eq. (68) for $D^0 \rightarrow K\pi$ have been reported by BaBar and Belle (15, 20), also see (9). Averaging over the two experiments yields

\[
y^+ - y^- = (-0.19 \pm 0.64)\%
\]

(101)

for the time-dependent CP violation. The experimental error is an order of magnitude larger than the maximal allowed shift due to new weak phases in decay, of order $x$ or $y$ times the bound in Eq. (97). In addition, a fit for $x_T^\mp$, $y_T^\mp$ and $\phi$ has been carried out in the Belle analysis (13), yielding

\[
\phi = (0.16 \pm 0.41) \text{ [rad].}
\]

(102)

However, the fit uses the formulae for $x^\pm$ and $y^\pm$ in Eq. (28), thus neglecting the corrections in Eq. (98). In particular, it assumes that $\phi^+ = \phi^- = \phi$. Fortunately, the reported error on $\phi$ is an order of magnitude larger than the upper bounds on $|\delta \phi_{\lambda T}|$ and $|\delta \phi_{\lambda T}|$ of a few percent, in $\phi$. Moreover, $a^d_f$ should be $< 1\%$, hence negligible in Eq. (99). Thus, the use of Eq. (28) turns out to be a good approximation.

The Belle Collaboration also fits for $\phi$ in a time-dependent Dalitz plot analysis for $D^0 \rightarrow K_s\pi^+\pi^-$ (23), obtaining

\[
\phi = (-0.24 \pm 0.32) \text{ [rad].}
\]

(103)

Again, this analysis assumes that $\phi^+ = \phi^- = \phi$ (in general, $\phi^+$ and $\phi^-$ would vary across the Dalitz plot). Again, the error on $\phi$ is about an order of magnitude larger than the allowed shifts in $\phi^\pm$, implying that this is a good approximation.

The outputs of the HFAG fit for $A_D \neq 0$ (new weak phases in decay) listed in Table I have been obtained under the assumption that $\phi^+ = \phi^- = \phi$ in the time-dependent $D^0 \rightarrow K^+\pi^-$ and $\overline{D}^0 \rightarrow K^-\pi^+$ amplitudes. We have just seen that this is a good approximation. In addition, HFAG has not allowed for new weak phases in $D^0 \rightarrow K^+\pi^-$ and $\overline{D}^0 \rightarrow \pi^+\pi^-$. Again, this is a good approximation for SCS decays, given that the impact of new weak phases on $\Delta Y_f$ and $y_{CP}$ would be negligible. Finally, modifications to the relation between $\phi$ and $\phi_{12}$ in Eq. (92) see Eqs. (90) and (83) are smaller than the experimental sensitivity to $\phi$ in CF/DCS decays by an order of magnitude, and in SCS decays by more than an order of magnitude.

In view of the above considerations and in the case of new weak phases in decay, the mixing parameters $x_{12}$, $y_{12}$, and $\phi_{12}$ can be obtained, to good approximation, along the lines of the fit carried out in Sec. V (for $A_D = 0$). In particular, $\phi$ is once again expressed in terms of $x_{12}$, $y_{12}$, and $\phi_{12}$ using Eq. (82) as are $x$, $y$, and $|q/p|$, using Eqs. (14), (15), and (48). However, now we take the HFAG fit results for $A_D \neq 0$, see Table I, and add $A_D$ to the sum over HFAG outputs in Eq. (69) for the $\chi^2$ function. The validity of this approximation reflects the suppression due to mixing ($x$ or $y$) of the effects of new weak phases in decay on CPVINT (continued use of Eq. (62)), and the lack of such suppression in the direct CP asymmetries [use of the $A_D \neq 0$ fit results]. The HFAG parameters used as input in this case are taken from a fit with $\chi^2 = 25.3$ for 20 degrees of freedom (28 experimental results minus 8 parameters). The value of $\chi^2$ in our fit is 1.3. Thus, as in the $A_D = 0$ fit, the overall $\chi^2$ increases by a small amount as the number of degrees of freedom is increased by one.

The fitted values of $x_{12}$, $y_{12}$, and $\phi_{12}$, with 1$s$ parabolic errors, are shown in the last column of Table II. In particular, we obtain

\[
\phi_{12}^D \text{ [rad]} = 0.07 \pm 0.08 \text{ (1s)}
\]

(104)

for parabolic errors. This is fully consistent (within 1$s$) with Eq. (74) for no new weak phases in decay, as expected. To ascertain the impact on models in which new weak phases in decay are possible, we note that the relation between $\phi_{12}^D$ and $M_{12}^{NP}$ in Eq. (73) is modified, to very good approximation, as

\[
\sin \phi_{12}^D = \frac{M_{12}^{NP}}{M_{12}} \sin (\phi_{NP}^M - \phi_{SM}^M) - \frac{\delta \phi_T}{2}.
\]

(105)

Therefore, we obtain the approximate (parabolic) 1$s$ CL interval

\[
\frac{\text{Im}(M_{12}^{NP})}{M_{12}} \in [-0.01, +0.15],
\]

(106)
(for the usual phase convention in which \( M_{12}^{SM} = 0 \) is real), up to small corrections of a few percent or less from \( \delta \phi_T/2 \). This is consistent, within 1\( \sigma \), with Eq. (71) for no new weak phases allowed. Similarly, the corresponding HFAG (non-parabolic error) analysis, i.e., a direct fit to the experimental data which allows \( A_D \neq 0 \) and incorporates Eqs. (52), should be consistent with Eq. (75).

What will be the sensitivity to new weak phases in decay be at a high luminosity flavor factory? We have seen that their impact on the time-dependent CP asymmetries \((\Delta Y_f)\) in SCS decays can be at most a few percent of the current errors (for \( D^0 \to \pi^+\pi^- \) and \( D^0 \to K^+K^- \)). In the case of CP and DCS decays (e.g., \( D^0 \to K_s\pi^+\pi^-, K^+\pi^\mp \)) we saw that their current sensitivity to \( \phi \) is roughly an order of magnitude weaker than the maximal shifts allowed in \( \phi^\pm \), and similarly for the precision with which Eq. (61), relating \( \delta \phi_T \) to the experimental data which allows \( A_D \neq 0 \) and incorporates Eqs. (52), should be consistent with Eq. (75).

The new physics satisfies the inequality \(|\delta \phi_T| < 1\) (unless \( \phi \approx \pi/2 \), which implies that \( \text{sign}(y \cos \phi_{\lambda_f}) = \text{sign}(y \cos \phi) = +1 \) in the first term, and \( \text{sign}(x \cos \phi_{\lambda_f}) = \text{sign}(x \cos \phi) \) in the second term.

C. \( B_s - \bar{B_s} \) mixing

Moving to \( B_s - \bar{B_s} \) mixing, the CDF and D0 collaborations are probing \( S_f \), in \( B_s \to J/\Psi \phi \) and \( a_{SL} \) with combined uncertainties of 0.4 and 0.009, respectively [4]. At LHCb with 2 fb\(^{-1}\) the expected uncertainties are \( \delta S_f \approx 0.02 \) [24] and \( \delta a_{SL} \approx 0.002 \) [22]. If new subleading weak phases appear in \( B_s \) decays to CP eigenstates, then

\[
S_f = \eta_f^CP \text{sign}(x) \sin \phi_{\lambda_f},
\]

with \( \phi_{\lambda_f} \) given in Eq. (79). The modified relation between \( a_{SL} \) and \( S_f \) for decays to CP eigenstates [see Eq. (65)] follows from Eq. (82), and is given in lowest order in \( r_f \) and \( |y/p| < 1 \) by

\[
\eta_f^C P \frac{S_f}{(1 - S_f^2)^{1/2}} = \frac{-\text{sign}(y \cos \phi_{\lambda_f})}{|y|} \frac{a_{SL}}{2} - \frac{\text{sign}(x \cos \phi_{\lambda_f})}{\cos^2 \phi_{\lambda_f}} \frac{\delta \phi_T}{2},
\]

The new physics satisfies the inequality \(|(2r_f \cos \delta \sin \phi_T) \tan \phi| < 1 \) (unless \( \phi \approx \pi/2 \), which implies that \( \text{sign}(y \cos \phi_{\lambda_f}) = \text{sign}(y \cos \phi) = +1 \) in the first term, and \( \text{sign}(x \cos \phi_{\lambda_f}) = \text{sign}(x \cos \phi) \) in the second term.

There are no direct CP asymmetry measurements yet for \( B_s \) decays mediated by \( b \to c\bar{c}s \) transitions. However, their magnitudes should be of same order as those for \( b \to c\bar{c}s \) transitions in \( B_d \) decays. The best bound is \( \approx 2\% \), from the direct CP asymmetry for \( B_d \to J/\Psi K^0 \) [9]. As previously noted, the strong phase differences enter as \( \sin \delta \) in the direct CP asymmetries, and as \( \cos \delta \) in \( \phi_{\lambda_f} \) and \( \delta \phi_T \). However, in the Standard Model the \( B \to J/\Psi \phi \) amplitude is given by a particular color-suppressed combination of two effective operator matrix elements \((Q_1, 2)\). Moreover, the soft gluon contributions to these matrix elements are formally suppressed by \( \Lambda_{QCD}/(m_{s_{\phi}}) \) rather than \( \Lambda_{QCD}/m_b \) [26]. We also note that significant strong phase differences \((\sim 30^\circ - 50^\circ)\) between the different isospin amplitudes in \( B \to D^{(*)}\pi \) and \( B \to D^{(*)}K \) decays are known to exist, due to color-suppressed channels [27]. Thus, significant strong phase differences between the SM and new physics \( B_s \to J/\Psi \phi \) or \( B_d \to J/\Psi K^0 \) amplitudes can be expected. We therefore take

\[
|\delta \phi_{\lambda_f}|, |\delta \phi_T| \lesssim 5\%,
\]

in the case of decay to a CP eigenstate, then we will have obtained a measurement of \( \delta \phi_T \) (since \( \delta \phi_{\lambda_f} \sim \phi_{\lambda_f}^2 \)). For example, this situation could be realized: (i) in SCS decays, if new phases in CF/DCS amplitudes are near the current direct CP violation bounds, or (ii) in CF/DCS decays, in the more likely possibility that new phases only appear in SCS amplitudes. The determination of \( \delta \phi_T \) could be combined with a measurement of \( \phi_{\lambda_f}^2 \) to fix \( |M_{12}^{NP}| \) in Eq. (105).
for new physics in $b \to c\bar{c}s$ transitions. Effects of this size in the second term on the r.h.s. of Eq. (110), applied to $B_s \to J/\Psi \phi$, would be difficult to observe at LHCb, given an order of magnitude larger projected experimental uncertainty on the first term

$$\delta \left( \frac{a_{\text{SL}}}{2} \frac{x}{y} \right) = O(0.4), \quad (112)$$

for 2 fb$^{-1}$ (obtained from $\delta a_{\text{SL}}$ above, and the SM central value for $|x/y|$ in [28]).

New CP violating effects at the 5% level in the tree-amplitudes would be quite exotic. If the new physics enters the $b \to c\bar{c}s$ transitions via gluonic or electroweak penguins, which we believe is a far more likely scenario, then its contributions to $\delta \phi_{\lambda/\phi}$ and $\delta \phi_T$ would be negligible. Recall that new CP violating amplitudes in penguin dominated $B_d$ decays, e.g., $B_d \to \phi K_s$, are constrained to lie below $O(10\%)$.

Finally, $\delta \phi_T$ receives a significant Standard Model contribution (relative to the leading $[\lambda_c/\lambda_2]^2$ CKM structure in $\Gamma_{12}/\Gamma_{12}^{\text{SM}}$)

$$\delta \phi_{\text{SM}}^{T} = 4 \text{Im} \frac{\lambda_c}{\lambda} \frac{\Gamma_{12}^{\text{uc}}}{\Gamma_{12}^{\text{SM}}} \approx 8\% \left( \frac{\Gamma_{12}^{\text{uc}}}{\Gamma_{12}^{\text{SM}}} \right), \quad (113)$$

where $\lambda_i \equiv V_{is}V_{ib}$. $\Gamma_{12}^{\text{uc}}$ and $\Gamma_{12}^{\text{SM}}$ are defined in the Standard Model expression for $\Gamma_{12}$,

$$\Gamma_{12}^{\text{SM}} = -\frac{\lambda_i^2}{2} \Gamma_{12}^{\text{cc}} + 2\lambda_i \lambda_u \Gamma_{12}^{\text{uc}} + \lambda_u^2 \Gamma_{12}^{\text{us}}. \quad (114)$$

In the OPE treatment they differ only with respect to quark content in loops: two charm quarks vs. one charm and one up quark, and satisfy $\Gamma_{12}^{\text{uc}} \equiv \Gamma_{12}^{\text{SM}}$. This is likely to be the dominant contribution to $\delta \phi_T$, certainly if new physics only enters through gluonic or electromagnetic penguins. With sufficient statistics it could be possible to isolate and measure $\delta \phi_T$ via Eq. (110) applied to $B_s \to J/\Psi \phi$. This would require that the hierarchy in Eq. (108), equivalent to $|\delta \phi_T/2| \gg |\delta \phi_{\lambda/\phi}|$, is satisfied. In practice, a substantial improvement of the direct CP asymmetry bounds for $b \to c\bar{c}s$ transitions would also be required.

VII. DISCUSSION AND CONCLUSION

If $\phi_{12} = \text{arg}(M_{12}/\Gamma_{12})$ is the only source of CP violation in neutral meson decays, then CP violation in pure mixing (CPVMIX), i.e., $|q/p| \neq 1$, and CP violation in the interference of decays with and without mixing (CPVINT), i.e., $\phi \neq 0$, are related phenomena. Moreover, $\phi$ would be related to the underlying mixing parameters $|M_{12}|$, $|\Gamma_{12}|$, and $\phi_{12}$ of relevance to model building. New weak phases in the decay amplitudes would enter and modify these relations. However, existing direct CP violation measurements provide stringent constraints on their magnitudes in the (tree-dominated) $D^0$ and $B_s$ decays of interest to us, implying that any modifications to the relations between CPVMIX and CPVINT must be small perturbations. We summarize these results, and their implications below.

The general relation between $\phi$ and $|q/p|$ (CPVINT and CPVMIX) in the limit of no new weak phases in decay is derived in Section IV, see Eq. (43). It leads to correlations between the semileptonic and time-dependent CP asymmetries and additionally, in the $D^0$ system, between the semileptonic and time-integrated CP asymmetries. We remind the reader that in $D^0$ decays the time-dependent ($\Delta \Phi_f$) and time-integrated ($\phi_f$) CP asymmetries must be equal in the limit of no direct CP violation (11) (no new weak phases in decay), see Section III.

Below we will refer to the whole complex of relations obtained via applications of Eq. (51) as the CPVMIX/CPVINT relations. We give them a fairly complete treatment in the case of tree-level dominated $D^0$ decays, covering singly Cabibbo suppressed (SCS) decays to CP $(K^+K^-, \pi^+\pi^-)$ and non-CP $(K^*K)$ eigenstates, as well as Cabibbo favored (CF) and doubly Cabibbo suppressed (DCS) decays to CP eigenstates $(K_s\pi^0)$ and to “wrong-sign” non-CP eigenstates $(K^-\pi^0)$, where examples are included in parentheses. In the case of $B_s$ decays, we confirm the correlation between the semileptonic and time-dependent CP asymmetries obtained in [2] for decays to CP eigenstates, and we also obtain the correlation for decays to non-CP eigenstates, see Eqs. (65) and (66).

For SCS $D^0$ decays to non-CP eigenstates, CP conjugate decay rates are of same order. Therefore, pairs of time-dependent and time-integrated CP asymmetries are accessible to experiment. We find that the relation between $\phi$ and $|q/p|$ implies that the ratio of CP asymmetries within each pair is given by the inverse ratio of the corresponding CP averaged decay rates, see Eq. (60). By contrast, the near equality of CP conjugate pairs of time-dependent CP asymmetries ($S_f$ and $S_{\bar{f}}$) for $B_s$ decays to non-CP eigenstates is a trivial consequence of $y \ll x$ and $|q/p| - 1| \ll 1$.

The general expression derived for $\phi$ in terms of the underlying mixing parameters $|M_{12}|$, $|\Gamma_{12}|$, and $\phi_{12}$, in the limit of no new weak phases in decay, is given in Eq. (52). It can be combined with similar expressions for $x$, $y$, and $|q/p|$, see Eqs. (14), (15), and (48), to extract the underlying $D^0 - \bar{D}^0$ mixing parameters $|M_{12}^D|$, $|\Gamma_{12}^D|$, and $\phi_{12}^D$ from a direct fit to the experimental data. In this work we adopt the simpler strategy of extracting the mixing parameters from a fit to the HFA output, which include $x$, $y$, $|q/p|$, and $\phi$. The (parabolic) HFA output error matrix is used to construct a $\chi^2$ function, see Section V. We find that

- $\phi_{12}^D$ is currently being probed at the level of 0.10 [rad] at 1\sigma, see Eq. (70).

- Incorporating the relation between $\phi$ and $\phi_{12}$, Eq. (72), into the fit reduces the experimental errors on $\phi_{12}^D$ by a factor of three for the current data set.

The preliminary (non-parabolic) HFA fit directly to the data [16], also obtained using Eqs. (14), (15), (48), and
for $x, y, |q/p|$, and $\phi$, yields a sensitivity to $\phi_{12}^D$ of 0.10 [rad] (1$\sigma$); the 95% CL bound is $\phi_{12}^D \leq 0.30$ [rad], see Eq. (71).

Two questions concerning the impact of new weak phases in decay need to be addressed: (i) to what extent can the CPVMIX/CPVINT relations be violated in the $D^0$ and $B_s$ systems, and how well could such violations be measured in the future; (ii) to what extent can $\delta \phi$ be modified. The violations in (i) can be characterized precisely in terms of the CPVINT observables $\phi_{\lambda j} = \arg[q\bar{A}_j/pA_f]$ with respect to $\phi$, the shift $\delta \phi_T$ in $\arg(\Gamma_{12}/\Gamma_{12}^*)$ with respect to the appropriate leading SM contribution, and the direct CP asymmetries $\phi_f^D$ (see Eqs. (89)–(93), Eqs. (98)–(100), and Eq. (110) in Section VI). Thus, we need to know how large these quantities can be.

Direct CP violation bounds provide stringent constraints on subleading amplitudes containing new weak phases. Strong phase differences enter as $\cos \delta$ in the direct CP asymmetries, and as $\sin \delta$ in $\delta \phi_{\lambda j}$ and $\delta \phi_T$. However, we argue that in all cases of interest the new physics amplitudes would have significant strong phase differences with respect to the leading SM amplitudes (due, essentially, to different color and chirality structures for the underlying effective operators). Therefore, the direct and time-integrated CP violation bounds also translate into order of magnitude bounds on $\delta \phi_{\lambda j}$, $\delta \phi_T$, and $\delta \phi_T$ due to new weak phases in decay (see Eqs. (SS)–(97), and (111) for the SCS, CF/DCS, and $b \to c\bar{c}s$ transitions, respectively).

The main implications of these bounds for $D^0 - \bar{D}^0$ and $B_s - \bar{B}_s$ mixing today are:

- In SCS $D^0$ decays the maximal allowed violations of the CPVMIX/CPVINT relations are of $O(\text{a few }%)$ of the current experimental errors on the time-dependent CP asymmetries (CPVINT), see Eqs. (SS)–(92).

- In CF/DCS $D^0$ decays the maximal allowed violations of the CPVMIX/CPVINT relations are of $O(10\%)$ of the current experimental errors on the time-dependent CP asymmetries (CPVINT), see Eqs. (98)–(103).

- Violations of Eq. (52), relating $\phi$ and $\phi_{12}^D$, are similarly bounded relative to the present day experimental sensitivity to $\phi$ in SCS and CF/DCS $D^0$ decay modes, respectively.

- Consequently, the bounds on $\phi_{12}^D$ in Eqs. (70) and (71) can not be significantly modified by new weak phases in decay, see Eq. (114).

- For $b \to c\bar{c}s$ transitions, the maximal allowed violation of the $B_s$ CPVMIX/CPVINT relations is $O(5\%)$ in absolute terms, see Eq. (110).

At a high luminosity flavor factory (with 75 ab$^{-1}$), we assume that there will be an order of magnitude improvement in precision for individual $D^0 - \bar{D}^0$ mixing measurements, and in the global fit to the data (a reduction of $\approx 6$ in the errors on $\phi$ and $|q/p|$ for the global HFAG fit is projected in (29)). The error on the $B_s$ semileptonic CP asymmetry at LHCb (with 2 fb$^{-1}$) is expected to be $\delta \phi_{12} \approx 0.002$ (23). Therefore, our conclusions on the sensitivity of mixing measurements at these facilities to new weak phases in decay are:

- Violations of the $D^0 - \bar{D}^0$ CPVMIX/CPVINT relations will be probed at the same order as the currently allowed maximal violations (obtained from direct and time-integrated CP violation measurements), implying that they could be difficult to observe at a high luminosity flavor factory.

- The “goodness” of a global fit to the $D^0 - \bar{D}^0$ mixing data which assumes no new weak phases in decay would probably be more sensitive to their effects than violations of the CPVMIX/CPVINT relations in individual decay modes.

- The expected error in the $B_s$ semileptonic CP asymmetry at LHCb is prohibitively large for a meaningful probe of the $B_s$ CPVMIX/CPVINT relations to be carried out.

In principle, with sufficient statistics it would be possible to determine $\delta \phi$ in the $D^0$ and $B_s$ systems, if the violations of the CPVMIX/CPVINT relations are much larger than the direct CP asymmetries in the SCS or CF/DCS transitions and the $b \to c\bar{c}s$ transitions, respectively (see the discussions at the ends of Sections VIB and VIC).

We emphasize that the $D^0$ and $D^\pm$ direct CP asymmetry measurements provide much more sensitive probes of new weak phases in decay than the time-dependent CP asymmetries (which correspond to differences between the $D^0$ and $\bar{D}^0$ time of decay profiles). Recall that in $D^0$ decays, the direct CP asymmetries are obtained from comparison of the time-integrated and time-dependent CP asymmetries. The effects of new weak phases in the time-dependent CP asymmetries are necessarily suppressed by mixing ($x$ or $y$). Therefore, the most likely scenario at a high luminosity flavor factory is that improved precision in the time-integrated or direct CP violation measurements will imply that the effects of new weak phases in decay lie beyond the reach of the time-dependent CP asymmetry measurements. It will of course still be possible to probe for new weak phases in $b \to c\bar{c}s$ transitions at a super-B factory ($B$ decays) and at the LHC ($B$ and $B_s$ decays) via direct CP violation measurements.

Finally, and of immediate interest for CP violation in $D^0 - \bar{D}^0$ mixing, the bounds on $\phi_{12}^D$ imply that $\text{Im}(M_{12}^{NP})/|M_{12}|$ is being probed at the 10% level at 1$\sigma$ (for the usual phase convention in which $M_{12}^{SM}$ is real, and where $M_{12}^{NP}$ is the new physics contribution). The preliminary HFAG 95% CL interval for $\phi_{12}^D$ implies that
\[ \text{Im}(M^{NP}_{\phi_{12}})/M_{12} \leq 0.30 \text{ at 95\% CL.} \] These results apply to models without new weak phases in decay, or with new weak phases in decay (up to an additive correction of less than a few percent), see Eqs. (73)–(75), and Eqs. (105), (106). Examples in which CP violation in mixing at such levels is possible have recently been discussed in the context of supersymmetry, little Higgs models, warped extra dimension models, and the minimal flavor violation framework [6, 15, 31, 32].

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