Higgs Decay to Photons at Two Loops*

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The calculation of the two-loop corrections to the partial width of an intermediate-mass Higgs boson decaying into a pair of photons is reviewed. The main focus lies on the electroweak (EW) contributions. The sum of the EW corrections ranges from -4% to 0% for a Higgs mass between 100 GeV and 150 GeV, while the complete correction at two-loop order amounts to less than ±1.5% in this regime.

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1. Introduction

The standard model (SM) predicts the existence of one scalar particle, the Higgs boson (H). The Higgs boson is the only particle of the SM which has not been found until now. Electroweak precision data mainly collected at CERN LEP and SLAC SLC in combination with the direct top-quark mass measurement at the Tevatron would favour a light Higgs boson with a mass below about 200 GeV at the 95% confidence level, while the direct search at LEP leads to a lower bound of 114 GeV at the 95% confidence level [1]. This mass range is compatible with the so-called intermediate-mass range, defined by $M_W \leq M_H \leq 2M_W$, $M_W$ and $M_H$ being the mass of the W boson and the Higgs boson, respectively. In this mass regime the decay of the Higgs boson into a pair of photons is an important detection channel at the Large Hadron Collider (LHC) due to its clear signature, though the branching fraction does not exceed 0.3%. Furthermore, this decay channel is useful in determining the properties of the Higgs boson. At a future International Linear Collider (ILC) precision measurements would be possible. In particular, at the ILC the two photon mode could be made possible, which allows for the production of Higgs bosons via the fusion of two photons. This way a precise measurement of $\Gamma(H \rightarrow \gamma\gamma)$, with a

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precision of 2.3% for $M_H = 120$ GeV [2], would be possible. Also the CP-properties of the Higgs boson could be studied at the ILC operating in the two photon mode. A comprehensive review of SM Higgs boson physics is given in [2]. The Higgs-decay into two photons is furthermore sensitive to new charged, heavy particles of physics beyond the SM. For these reasons a precise prediction of the partial decay width $\Gamma(H \rightarrow \gamma\gamma)$ in the intermediate-mass range is required. To this end the two-loop calculations of the partial decay width have recently been completed.

Here, a short review of these calculations is given focusing on the EW contributions. Firstly, in section 2 the Born level results are given. The individual EW contributions at two-loop order are discussed in the following sections, namely the corrections due to light fermions in section 3, the top-quark-induced corrections in section 4, and the purely bosonic corrections in section 5. The sum of these contributions together with the QCD corrections at two-loop order are presented in section 6, which also contains the conclusion.

2. Born level

At Born level there exist two diagrams for each fermion (Fig. 1). However, only the heavy top-quark contributes sizeably while the other contributions are negligible. In addition one has to consider 26 diagrams with virtual W bosons, Goldstone bosons, and ghosts (Fig. 2). Since heavy particles give sizeable contributions to this loop-induced process, it is sensitive to new charged particles of physics beyond the SM.

The exact results for the decay rate have been known since long [3].
However, it is instructive to have a look at the expansion of these results in the external momenta. Expansions will partly be performed also at the two-loop level, where the exact results are not known. A natural choice for the expansion parameters is \( \tau_t = M_H^2/(4M_t^2) \) and \( \tau_W = M_H^2/(4M_W^2) \), respectively. It turns out that the approximation consisting of the first three terms of the expansion in \( \tau_t \) is practically indistinguishable from the exact result up to \( \tau_t \approx 0.25 \) for the case of the diagrams with virtual top-quarks. In the second case, the convergence is slightly worse, since \( M_H = 2M_W \) corresponds to \( \tau_W = 1 \) and the exact result behaves like \( \sqrt{1 - \tau_W} \) in this limit. Nevertheless, for \( M_H = 120 \) GeV, 140 GeV, and 2\( M_W \), the approximation by five expansion terms deviates from the exact result by as little as 0.3\%, 1.6\%, and 19.9\%, respectively.

In the intermediate-mass regime of the Higgs boson the contribution from virtual bosons and ghosts dominates, while it is partially cancelled by the contribution due to virtual top-quarks.

### 3. Corrections due to light fermions

Light fermions can contribute at the two-loop level since it is possible to avoid the direct coupling of the fermions to the Higgs boson. The relevant diagrams are shown in Fig. 3. Furthermore, one has to sum over all generations of light fermions. Therefore one can expect a non-negligible contribution due to light fermions. The respective calculation was done in [4]. In this case an Asymptotic Expansion is not possible since thresholds occur at \( M_H = M_W \) and at \( M_H = 2M_W \), while we consider a Higgs boson mass just in this mass range. For this reason an analytic calculation was per-
formed. The authors considered the limit of vanishing masses for the light fermions and employed the Background Field Method (BFM) quantisation framework in order to reduce the number of diagrams. Then they projected out the scalar amplitudes and reduced them to a set of linearly independent ones. These were in turn reduced to a set of master integrals by means of the Integration-By-Parts technique. Finally the master integrals were calculated using differential equations and the results were expressed in terms of Generalised Harmonic Polylogarithms. The unphysical singularity at the 2W-threshold was regularised through the introduction of the width of the W boson by performing the replacement $M_W \rightarrow M_W - i\Gamma_W / 2$. It could be shown that the result is independent on the regulator except in the region between 150 GeV and 170 GeV, where the result has to be taken with some caution. The relative correction as a function of the Higgs boson mass is shown in Fig. 4. For comparison also the two-loop QCD result is shown and the sum of these corrections. It turns out that in the intermediate-mass region of the Higgs boson the corrections due to light fermions are small but indeed non-negligible, between 1% and 2%.

4. Top-quark-induced corrections

The top-quark-induced corrections have first been considered in the limit of a large top-quark mass in [5]. Sample diagrams for this class of corrections are depicted in Fig. 5. In order to obtain the correction of order $O(G_F m_t^2)$ as an expansion in $\tau_W$ up to and including the terms of order $O(\tau_W^4)$ the
Asymptotic Expansion technique was applied taking the bottom-quark to be massless. Furthermore, the on-shell-scheme, dimensional regularisation, the anticommuting definition of $\gamma_5$, and a general $R_\xi$-gauge have been employed. Also the Tadpole diagrams had to be included since the $m_t^4$-terms cancel out non-trivially in the sum of contributions from genuine Tadpole diagrams, counterterms and non-trivial terms in the Asymptotic Expansion. In Fig. 5 the normalised amplitude is shown as a function of $\tau_W$. It turns out that the convergence behaviour is similar to the one at Born level for the diagrams with virtual bosons and ghosts. For this reason it was assessed

Fig. 5. Two-loop sample diagrams for top-quark-induced corrections.

![Two-loop sample diagrams for top-quark-induced corrections.](image)

Fig. 6. Normalised amplitude for the top-quark-induced two-loop electroweak corrections proportional to $G_F M_W^2$ as a function of $\tau_W$. The dashed curves represent the sequence of approximations that are obtained by successively including higher powers of $\tau_W$ in the expansion. The dotted vertical line and the right edge of the frame encompass the intermediate-mass range of the Higgs boson; taken from [5].

![Normalised amplitude for the top-quark-induced two-loop electroweak corrections.](image)
that the approximation should be very good for Higgs boson masses up to 140 GeV and still reasonably good up to the right edge of the intermediate-mass regime. By now also the full top-quark-induced corrections have been completed in [6], where also the leading term in the top-quark mass could be recovered. The method of the calculation is the same as in the case of the purely bosonic corrections and will be reviewed in the respective section. In Fig. 7 the result is shown as the relative correction to the Born result as a function of the Higgs boson mass. The correction lies in the range between 2.5% and 3%. Also the result for the leading term is shown in this figure. It is obviously a very good approximation to the full result.

![EW Corrections to $\Gamma(H \rightarrow \gamma\gamma)$](image)

Fig. 7. Individual relative EW corrections as functions of the Higgs boson mass; taken from [6].

5. Purely bosonic corrections

Finally the purely bosonic corrections have to be taken into account, one sample diagram of which is shown in Fig. 8. The calculation of these corrections was also performed in [6]. The authors employed the BFM quantisation framework and used the Tadpole counterterm in order to cancel the diagrams containing a Tadpole. They projected out the relevant form factor and performed a Taylor expansion in the parameter $q_W = q^2/(4M_W^2)$, where $q$ is the external momentum of the Higgs boson, up to and including
terms of order $O(g_W^3)$. The gauge parameter was renormalised in order to obtain finite terms in this expansion which in turn allows for an improvement of the results by means of a Padé approximation. The respective result is also shown in Fig. 7, where it is denoted as YM. Its value is around 2% with a sign opposite to the corrections discussed above.

6. Resulting NLO corrections and conclusion

The purely bosonic corrections have a different sign compared to the other corrections as is the case at Born level. Therefore a partial cancellation takes place between the individual EW contributions as can also be seen in Fig. 7. The QCD corrections at the two-loop level (see [7], for the result at three loops see [8]) are also small and cancel partially against the resulting EW corrections. This is shown in Fig. 9. The reason for the smallness of the QCD result could be due to the fact that only the Born level diagrams containing virtual quarks are affected by QCD corrections. Note, that also in the case of the QCD corrections at two loops a naive expansion in the external momenta is possible, again leading to a rapidly converging series as is the case for the Born level contributions of the diagrams with virtual top-quarks. The sum of the complete EW corrections ranges from $\sim -4\%$ to $\sim 0\%$ for a Higgs boson mass between 100 GeV and 150 GeV. The full two-loop result amounts to less than $\pm 1.5\%$. The NLO calculation therefore already gives a very reliable prediction.

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Fig. 9. Full relative EW and relative QCD corrections as functions of the Higgs boson mass; taken from [6].

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