Microwave-controlled generation of shaped single photons in circuit quantum electrodynamics

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(Dated: May 11, 2014)

Coherent generation of single photons with waveforms of a given shape plays an important role in many protocols for quantum information exchange between distant quantum bits. Here we create shaped microwave photons in a superconducting system consisting of a transmon circuit coupled to a transmission line resonator. Using the third level of the transmon, we exploit a second-order transition induced by a modulated microwave drive to controllably transfer an excitation to the resonator from which it is emitted into a transmission line as a travelling photon. We demonstrate the single-photon nature of the emitted field and the ability to generate photons with a controlled amplitude and phase. In contrast to similar schemes, the presented one requires only a single control line, allowing for a simple implementation with fixed-frequency qubits.

One of the most important challenges in the rapidly developing field of quantum information processing and quantum communication is the efficient implementation of quantum state transfer between spatially separated quantum bits. Even though small systems with only a few qubits successfully use coupling schemes based on atomic vibrational modes [1], discrete electromagnetic modes of optical or microwave cavities [2] or on-chip resonators [3], itinerant rather than localized photons are preferable as information carriers for distribution of entanglement and quantum networking over larger distances [4].

Such a quantum channel between distant qubits can be established in a variety of ways – for example in heralded schemes using interference and subsequent detection of photons radiated [5,6] or scattered [7] by the qubits. A deterministic approach relying on reabsorption of a photon emitted from one qubit by another has been the subject of several theoretical proposals [8,9]. This scheme requires efficient generation of single photons on demand [10,12], their entanglement with the emitting qubit [13,18] and typically a symmetric temporal shape of the photon to allow time-reversal of the emission process. This necessitates techniques for the generation of controllably shaped single photons which have been realized with optical photons [19,21] and shown to enable photon reabsorption [22], albeit with a limited efficiency.

Here we utilize superconducting circuits as one of the promising platforms for quantum information processing, offering good coherence times [23] and strong coupling between qubits and microwave radiation [24] in an easily controllable and compact solid-state system. The field emitted from a superconducting circuit is naturally confined to a one-dimensional transmission line without any need for spatial mode matching and can be easily routed between different elements of a network. For these systems, photon shaping schemes have been proposed [9] and experimentally realized [25], which are based on tunable couplers regulating the emission rate of a photon localized in a resonator into a transmission line. Systems with a fixed resonator emission rate but a tunable coupling between the resonator and the qubit [26] can also be used for photon shaping. Both of these approaches rely on flux tuning of a SQUID loop to achieve variation in the qubit-resonator or resonator-transmission line coupling. However, because of the varying Josephson inductance of the loop the frequency of the resonator changes together with the coupling. Therefore, to control the phase of the emitted photon as well as its amplitude envelope, the frequency shift needs to be compensated by another tunable parameter in the circuit, such as the qubit-resonator detuning [27].

In this letter, we present an alternative, microwave-based approach to photon shaping. Both the amplitude and the phase of the emitted photon are controlled by a single phase- and amplitude-modulated microwave signal which induces an effective tunable qubit-resonator coupling via a second-order process. Since qubit frequency and coupling remain fixed in this scheme, it can be realized in simple circuits without additional tuning elements.

Our system consists of an on-chip asymmetrically coupled transmission line resonator with a resonance frequency \( \omega_r/2\pi = 7.126 \text{GHz} \) and a linewidth \( \kappa/2\pi = 6.6 \text{MHz} \) coupled by a Jaynes-Cummings interaction of strength \( g/2\pi = 65 \text{MHz} \) to a transmon-type superconducting circuit [28] with its transition frequency between the ground state \( |g\rangle \) and the first excited state \( |e\rangle \) set to \( \omega_g/2\pi = 8.126 \text{GHz} \). In the presented photon shaping protocol, we make explicit use of the multi-level structure of the transmon, treating it as an effective three-level ladder-type system with the transition between the first and the second excited state \( |f\rangle \) detuned from \( \omega_q \) by the anharmonicity \( \alpha/2\pi = -380 \text{MHz} \). The direct transition between states \( |g\rangle \) and \( |f\rangle \) is forbidden to first order.

The bare eigenstates of the qubit-resonator system are
coupled to each other by the time-independent Jaynes-Cummings interaction and by an external microwave drive applied to the qubit through its gate line [Fig. 1(a)]. The coupled system is in the dispersive regime, that is, the qubit and the resonator are detuned from each other by $\Delta = \omega_q - \omega_r \gg g$ and no energy is exchanged between them.

To obtain a tunable Jaynes-Cummings type coupling allowing effectively resonant swapping of excitation, we make use of second-order processes [29, 31]. We apply the qubit drive at a frequency $\omega_d$ slightly detuned by $\delta$ from the energy difference $2\omega_q + \alpha - \omega_c$ between the states $|f0\rangle$ and $|g1\rangle$. In a reference frame rotating with the frequency $\omega_d$, the Hamiltonian of the system is

$$H(t) = \delta_q b^\dagger b + \frac{1}{2} \alpha b^\dagger b^\dagger b + \delta_r a^\dagger a + g(ab^\dagger + a^\dagger b) + \frac{1}{2}(\Omega^* (t)b + \Omega(t)b^\dagger),$$

where $\delta_q = \omega_q - \omega_d$ and $\delta_r = \omega_r - \omega_d$ are the transition frequencies of the transmon and the resonator in the rotating frame, $\Omega(t) = \Omega_0(t)e^{i\phi(t)}$ is the complex drive strength corresponding to the physical drive signal $\Omega_0(t)\cos(\omega_d t - \phi(t))$ with slowly varying amplitude $\Omega_0(t)$ and phase $\phi(t)$. The operators $a, a^\dagger$ are the standard annihilation and creation operators of the resonator and $b, b^\dagger$ their analogues for the transmon [28].

The states $|f0\rangle$ and $|g1\rangle$ are nearly resonant in this rotating frame, while the intermediate states $|e0\rangle$ and $|e1\rangle$ are far off-resonant. They can therefore be adiabatically eliminated, giving rise to an effective Jaynes-Cummings type coupling between the resonator and the two-level system consisting of qubit states $|g\rangle$ and $|f\rangle$ [Fig. 1(b)].

This interaction is described by the effective Hamiltonian

$$H_{\text{eff}}(t) = (\Delta f_{0g1}(t) + \delta)|f\rangle\langle f| + \tilde{g}(t)|f0\rangle\langle g1| + \text{h.c.},$$

which we have simplified by absorbing the parts of the AC Stark shifts independent of $\Omega$ into the renormalized frequencies of the qubit and the resonator. The remaining AC Stark shift $\Delta f_{0g1}(t)$ of the transition frequency between $|f0\rangle$ and $|g1\rangle$ is to leading order quadratic in the drive strength $\Omega$. The effective second-order coupling $\tilde{g}(t)$ can be expressed as

$$\tilde{g}(t) = \frac{1}{\sqrt{2}} \frac{g\alpha}{\Delta + \alpha} \Omega(t).$$

Most importantly, it is proportional to the applied drive signal $\Omega(t)$ and therefore fully tunable. Also, its dependence on the anharmonicity implies that stronger effective couplings and therefore shorter photon pulses could be achieved with more anharmonic qubits such as the fluxonium [32].

Similarly to the atomic optics experiment in [20], this tunable effective coupling is used to generate single photons by controllably transferring population of the $|f0\rangle$ state into $|g1\rangle$ which decays into $|g0\rangle$ by photon emission. Since the drive is off-resonant with the $g \leftrightarrow e$ transition, the system remains trapped in the ground state [Fig. 1(b)], ensuring that only a single photon is emitted. By modulating the amplitude and the phase of the drive signal in time using sideband mixing, we control the temporal shape of the output field $a_{\text{out}}(t) = \sqrt{\kappa}a(t)$ resulting in the emission of a single photon state $|1\rangle = \int \psi(t)a_{\text{out}}^\dagger(t)|0\rangle dt$ characterized by its mode function $\psi(t)$.

We initialize the qubit in the state $(|g\rangle + |f\rangle)/\sqrt{2}$, which is then mapped onto an itinerant photon state $(|0\rangle + |1\rangle)/\sqrt{2}$ by using a pulse of the simple form

$$\Omega(t) = \Omega_0 \sin^2(\pi t/T) \exp(i\phi(t)),$$

parametrized by the maximum amplitude $\Omega_0$, the duration $T$ and the phase $\phi(t)$. This process preserves the coherence of the superposition leading to non-zero voltage quadratures of the detected photon signal. To determine the waveform of the photon, we measure both quadratures $I(t)$ and $Q(t)$ of the output field $a_{\text{out}}(t)$ emitted into the detection line in a heterodyne setup [12]. The averaged value of the complex signal $v(t) = I(t) + iQ(t)$ is then proportional to the expectation value $\langle a_{\text{out}}(t) \rangle = \psi(t)/2$ [33]. To detect the weak single-photon signal, a broadband low-noise HEMT amplifier is used. For measurements requiring only small bandwidth, we also employ a phase-preserving Josephson parametric amplifier to further increase the signal-to-noise ratio. A Chebyshev filter is applied to the digitized signal to reduce noise and to eliminate unwanted signals arising in the detection process such as the pump tone of the parametric amplifier or the DC offset of the A/D converter.

FIG. 1. (a) The energy level scheme of the qubit-resonator system. The matrix elements of the drive $\Omega$ and the Jaynes-Cummings coupling $g$ are indicated by solid and dotted lines, respectively, connecting the coupled bare states. The two second order paths connecting the states $|f0\rangle$ and $|g1\rangle$ are indicated by arrows. (b) The second order effective coupling $\tilde{g}$ between $|f0\rangle$ and $|g1\rangle$ after adiabatic elimination of the intermediate states $|e0\rangle$ and $|e1\rangle$ is indicated by the solid blue line. The decay of the state $|g1\rangle$ into $|g0\rangle$ by photon emission is shown by the wavy yellow arrow.
To obtain the symmetric photon shape depicted in Fig. 2, we calibrate the phase $\phi(t)$ of the drive signal to compensate the amplitude-dependent Stark shift $\Delta_{\phi_{01}}(t)$. The Stark shift is approximated by a quadratic function of the drive amplitude $\Omega(t)$ whose coefficients are adjusted to keep the phase of the emitted photon signal constant in time. The parameters $\Omega_0$ and $T$ are chosen empirically to minimize the residual population in the initial $|f0\rangle$ state and maximize time-reversal symmetry of the photon quantified by the normalized overlap between the voltage trace and its time inverse. For the shaped photon generated in our experiment, this symmetry parameter reaches the measured value 0.99. For comparison, we prepare a field with exponential shape [Fig. 2] by applying a strong short 10 ns drive pulse, transferring a fraction of the $|f0\rangle$ state population into $|g1\rangle$ which then undergoes spontaneous emission. The measured value of 0.73 for its symmetry parameter closely matches the theoretical value of $2/e \approx 0.74$. The system dynamics including imperfections of the second-order transition and decoherence effects are modelled by solving the master equation for the full Jaynes-Cummings model including imperfections of the second-order transition and the theoretical value of $\frac{2}{e}$ which lies well below one, its classical limit, and is consistent with the value of zero for a perfectly antibunching single-photon state. The density matrix $\rho$ of the photon state shown in Fig. 3 is extracted from the measured moments by employing a maximum likelihood algorithm. From the numerical simulation of the emission process, we find the normalization condition $\langle A^\dagger A \rangle = 0.36$, corresponding to an emission efficiency of 72% which is limited by the finite lifetime $T_1 = 340$ ns of the $|f\rangle$ state. This accounts for the deviation of the diagonal elements of $\rho$ from the theoretical values of 1/2 while the off-diagonal elements are reduced due to dephasing effects.

Finally, to demonstrate the phase tunability of the scheme, we generate a photon with a double-peaked time trace and vary the phase difference between the two peaks [Fig. 4]. The transmon is first prepared in the state $(|g\rangle + |f\rangle)/\sqrt{2}$. The subsequent photon shaping signal consists of an initial pulse with amplitude and length adjusted to transfer approximately one half of the qubit-resonator system’s population from $|f0\rangle$ to $|g1\rangle$, followed after 150 ns by a saturation pulse inducing emission of the remaining fraction of the photon. Since the overlap between the two peaks is small, their phases can be tuned almost independently by adjusting the phases of the two corresponding drive pulses [Fig. 4(c)]. The lower amplitude of the second peak relative to the first one is caused by dephasing of the qubit. The loss of coherence between the qubit states $|g\rangle$ and $|f\rangle$, which in our sample has a
characteristic time scale of $T_2 \approx 100$ ns, translates into a reduction of the expectation value $\langle a_{\text{out}}(t) \rangle$ and hence the heterodyne voltage. We note that dephasing does not affect the measured averaged power [Fig. 4(b)].

In conclusion, we have presented a photon shaping technique relying fully on a phase- and amplitude-controllable microwave drive acting on three transmon levels. We have shown that this method can be used to generate single photon pulses of nearly symmetric shapes with a controllable amplitude and phase. The shaping method can be refined by using simulations and optimization techniques to find drive pulses needed to generate given photon shapes. With the time-reversed scheme applied to a second distant qubit the emitted photon can be in principle reabsorbed to map its quantum state onto the qubit [8]. The simple nature of this scheme with respect to the required control elements makes it also a prime candidate for use in 3D circuit QED architectures [36], where low-loss microwave waveguides can be exploited for high-fidelity state transfer.

This work was supported by the European Research Council (ERC) through a Starting grant, by the Swiss National Science Foundation through the National Center of Competence in Research “Quantum Science and Technology” and by ETH Zurich.

**Fig. 4.** (a) The amplitude of the drive signal is shown by the thick line. The thin line displays one of the quadratures of the signal to illustrate the frequency variation with the amplitude necessary to compensate the AC Stark shift. (b) The emitted power as a function of time for the double-peaked photon. (c) The $I$ quadrature of the detected voltage as a function of time for different relative phases between the two drive pulses. The voltage along the dashed constant-time line passing through the maximum of the second peak is shown separately in (d).
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