MUON CAPTURE AND THE PSEUDOSCALAR FORM FACTOR OF THE NUCLEON

V. BERNARD
ULP, Physique Théorique, F-67084 Strasbourg

T. R. HEMMERT, U.-G. MEIßNER
FZ Jülich, IKP (Theorie), Jülich, Germany

We summarize recent work on muon capture and the pseudoscalar form factor of the nucleon.

1 Introduction

Ordinary \( \mu^- p \rightarrow \nu \mu n \) and Radiative \( \mu^- p \rightarrow \nu \mu \gamma n \) Muon Capture (OMC, RMC) on a proton are venerable subjects in nuclear physics \( (e.g. \text{ ref.} 1) \). After having served for decades as a testing ground for the symmetries and structure of the weak interaction, today these reactions can also be regarded as unique tests of the axial structure of the nucleon as mandated by the explicitly and spontaneously broken chiral symmetry of QCD at low energies. In particular, they can give us access to the elusive pseudoscalar form factor \( G_P(q^2) \) of the nucleon which has received new attention \( (e.g. \text{ ref.} 2, 3) \) many years after the pioneering analyses in the 1960s \( (e.g. \text{ ref.} 4) \). Due to experimental constraints most of the muon capture work in the single nucleon sector so far has focused on OMC. Therein one is only sensitive to one particular kinematic point in the pseudoscalar form factor \( G_P(q^2 = -0.88 m^2_\mu) \equiv g_P \), which is commonly referred to as the pseudoscalar coupling constant. It took until 1995 that the first measurement of RMC on Hydrogen was reported \( (e.g. \text{ see} 4) \), but quite surprisingly the extracted number for \( g_P \) disagreed by as much as 50% from the very precise theoretical calculations \( (e.g. \text{ see} 4, 2) \). In the following years RMC on the proton was reanalyzed in the framework of Heavy Baryon Chiral Perturbation Theory (HBChPT) by several groups \( (e.g. \text{ ref.} 6) \), but as of September 1998 (i.e. BARYONS 98) no new hadronic structure effect could be identified that would have invalidated Fearing’s calculation \( (e.g. \text{ used in the analysis of the data}) \). In parallel, the chiral structure of \( G_P(q^2) \) was reanalyzed \( (e.g. \text{ ref.} 7) \) and explicitly shown to be unaffected by contributions from the first nucleon resonance \( \Delta(1232) \). For now the discrepancy remains unexplained \( (e.g. \text{ ref.} 9) \), with new theoretical \( (e.g. \text{ ref.} 8) \) and experimental \( (e.g. \text{ ref.} 10) \) investigations under way.

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email: th.hemmert@fz-juelich.de
In the remainder of this brief contribution we want to show that OMC and RMC indeed are low energy hadronic processes where a good convergence behavior of HBChPT can be expected. We then address a remaining open question in the existing RMC calculations. Furthermore, we emphasize the existence of very precise predictions for the momentum dependence of $G_P(q^2)$—which go beyond the usual focus on $g_\alpha$ in the literature—and present the poor state of “world data” for this “black sheep” among the nucleon form factors. Finally, we point out that pion electroproduction is a promising window to improve our knowledge of $G_P(q^2)$.

2 Ordinary Muon Capture

In the Fermi approximation of a static $W^-\mu$ field, the invariant matrix element of ordinary muon capture can be written as

$$\mathcal{M}_{\mu^-p-\nu_\mu n} = \mathcal{M}_{\text{OMC}}^\text{LO} = \langle \nu_\mu | W_\mu^+ | \mu \rangle i \frac{g_{\mu\nu}}{M_W^2} \left[ \langle n | V_\nu^- | p \rangle - \langle n | A_\nu^- | p \rangle \right]. \quad (1)$$

The leptonic matrix element $\langle \nu_\mu | W_\mu^+ | \mu \rangle$ is uniquely fixed by the electroweak vertices of the Standard Model, utilized here as the source of well-understood external fields that probe the hadronic structure of a nucleon of mass $M_N$ and isovector magnetic moment $\mu_v$ at low energies. Now one calculates the charge-changing hadronic vector $\langle n | V^- | p \rangle$ and axial-vector $\langle n | A^- | p \rangle$ currents in HBChPT to the order desired. To $O(p^2)$ one finds

$$\langle n | V^- | p \rangle^{(2)} = -i \frac{g_2 V_{ud}}{\sqrt{8}} \bar{n}(r') \left\{ v_{\alpha} + \frac{(r + r')_\alpha}{2 M_N} + \frac{\mu_v}{M_N} [S_{\alpha}, S \cdot (r' - r)] \right\} p(r),$$

$$\langle n | A^- | p \rangle^{(2)} = -i \frac{g_2 V_{ud}}{\sqrt{8}} \bar{n}(r') \left\{ 2 g_A S_{\alpha} - \frac{g_A}{M_N} S \cdot (r + r') v_{\alpha} - \frac{2 g_A S \cdot (r' - r)}{(r' - r)^2 - m_\pi^2} (r' - r)_{\mu} \right\} p(r) + O(1/M_N^2), \quad (2)$$

where $v_{\alpha}$, $[S_{\alpha}]$ corresponds to the velocity- [spin]-vector of HBChPT and $g_A$, $|g_2|$ denotes the axial vector [weak] coupling constant of the nucleon with CKM matrix element $V_{ud}$. Eq.(2) contains the coupling of the axial source to the nucleon via an intermediate pion of mass $m_\pi$ as required by chiral symmetry, leading to a $O(p)$ effect in the transition current.

Assuming that the initial muon-proton system constitutes the ground-state of a bound system described by a $1s$ Bohr-wavefunction $\Phi(x)_{1s}$ of a muonic atom one finds the spin-averaged capture rate $O(p^3)$

$$\Gamma_{\text{OMC}} = \frac{\alpha^3 G_F^2 V_{ud}^2 m_\mu^5}{2\pi^2 (m_\pi^2 + m_\mu^2)^2} \left\{ (2g_A^2 + 1)m_\mu^4 + (4g_A^2 + 2)m_\mu^2 m_\pi^2 + (3g_A^2 + 1)m_\pi^4 \right\}.$$
\[ + \frac{2m_\mu}{(m_\mu^2 + m_\pi^2)M_N} \left[ (g_{A\mu\nu} - 5g^2_{A\nu} - 2)(m_\mu^6 + 3m_\mu^4m_\pi^2) ight. \\
+ (3g_{A\mu\nu} - 16g^2_{A\nu} - 6)m_\nu^2m_\pi^4 + (g_{A\mu\nu} - 7g^2_{A\nu} - 2)m_\pi^6 \right] + \mathcal{O}(1/M_N^2) \]
\[ = \left( 247 - 59 \right) \times s^{-1} + \mathcal{O}(1/M_N^2), \]

with \( G_F = g_F^2 \sqrt{2}/(8M_W^2) \). Note that the \( \mathcal{O}(p^2) \) contribution amounts to a correction of less than 25\% of the leading term. The expectation that OMC has a well behaved chiral expansion is also supported by the observation that in the case of no explicit chiral symmetry breaking (i.e. \( m_\pi = 0 \)) the spin-averaged capture rate is only changed by 10\%: \( \Gamma_{\text{OMC}}^{\text{OMC}} = (214 - 46) \times s^{-1} + \mathcal{O}(1/M_N^2) \). The physical reason for the nice stability of perturbative calculations for OMC is of course the fact that contributions of order \( n \) are suppressed by \( (m_i/\Lambda_\chi)^{n-1} \), with \( i = \pi, \mu \) and \( \Lambda_\chi \sim M_N \sim 1 \text{GeV} \). Analogous suppression effects are at work for RMC. We therefore note that at \( \mathcal{O}(1/M_N^2) \)—when the calculation becomes sensitive to the internal structure of the nucleon beyond just the isovector magnetic moment \( \mu_v \) and the leading pion pole of Eq.(2)—the new structure effects are strongly suppressed and therefore present a formidable challenge for the required precision of muon capture experiments.

### 3 Some comments on RMC

Several HBChPT calculations of RMC have been performed since BARYONS 95, the most elaborate one by Ando and Min. They found that the \( \mathcal{O}(p^3) \) effects are small, in accordance with the analysis presented in the previous section. No large structure effect that would invalidate the Born-term analysis of Fearing could be identified. However, in our opinion there is one point left to be examined in detail regarding the contribution of \( \Delta(1232) \) in muon capture. In HBChPT these effects are incorporated via \( \mathcal{O}(p^3) \) counterterms, leading only to a small effect—consistent with previous phenomenological analyses. While this result is reassuring it is also surprising from the viewpoint of effective field theories. Introducing \( \Delta(1232) \) into the theory leads to a new scale \( \Delta = M_\Delta - M_N \sim 300 \text{MeV} \), which would suggest that the resulting effects \( (m_i/\Delta) \), \( i = \pi, \mu \) could be of the order of 30\%! We have started to investigate this problem to identify the origin of this strong suppression of \( \Delta(1232) \). First numerical results confirm the smallness of the contributions, but the analytical structure and the physics behind this suppression is hard to pin down. It’s origin lies in the fact that due to the atomic structure of the \( \mu p \) system both OMC and RMC are very sensitive to spin structure of the initial state which seems to act as filter mechanism.
4 The elusive Form Factor

The electroweak structure of a nucleon is typically encoded via 6 form factors (e.g. ref\[^3\]). Muon captures provides us with the opportunity to study the axial-weak structure of a nucleon. In the absence of second class currents the corresponding relativistic matrix element of the hadronic axial current reads

\[ \langle n| A_\alpha^- | p \rangle = \bar{n}(p_2) \left[ G_A(q^2) \gamma_\alpha \gamma_5 + \frac{G_P(q^2)}{2M_N} q_\alpha \gamma_5 \right] p(p_1). \] (4)

Here, \( G_A(q^2) \) and \( G_P(q^2) \) are the axial and the induced pseudoscalar form factor, respectively. While \( G_A(q^2) \) can be extracted from (anti)neutrino–proton scattering or charged pion electroproduction data, \( G_P(q^2) \) is harder to pin down and in fact constitutes the least known nucleon form factor. In Fig.1 we present the “world data” for \( G_P(q^2) \). In OMC one is sensitive to the point

\[ g_P \equiv \frac{m_\mu}{2M_N} G_P(q^2 = -0.88m_\mu^2) = \frac{2m_\mu F_\pi g_{\pi NN}}{m_\pi^2 + 0.88m_\mu^2} - \frac{1}{3} g_A m_\mu M_N r_A^2 \]

\[ = 8.23 \pm 0.46, \] (5)

with \( F_\pi \) denoting the pion-decay constant and \( r_A \) the axial radius of the nucleon extracted from \( G_A(q^2) \). It is this prediction for \( g_P \) with which the RMC result from TRIUMF\[^5\] disagrees, whereas the OMC\[^12\] measurements are consistent with it, within errors (see Fig.1). Note that the (theoretical) error of Eq.(5) is much smaller and comes from the uncertainty in the strong coupling constant \( g_{\pi NN} \). Eq.(5) is obtained via several quite different theoretical analyses\[^4\],\[^2\],\[^3\] and nowadays is considered to rest on firm ground.

There are 2 curves shown in Fig.1 to display the difference between the usual pion-pole parameterization for \( G_P(q^2) \) and analyses that take into ac-
count the full chiral structure of the form factor, yielding
\[ G^\chi_P(q^2) = \frac{4 M_N g_{\pi NN} F_\pi}{m_\pi^2 - q^2} - \frac{2}{3} g_A M_N^2 r_A^2. \] (6)

In the kinematical region of RMC, which mainly lies to the “left” of the OMC point in Fig.1 the structure effect proportional to \( r_A \) is expected to play only a small role. Certainly, the present experimental uncertainties both in OMC and in RMC are too large to distinguish between the 2 curves, but new efforts are under way. Finally we want to emphasize that there exists another window on \( G_P(q^2) \)—pion electroproduction. So far there has only been one experiment that took up the challenge, with the results shown in Fig.1. In this kinematical regime the structure proportional to \( r_A \) produces the biggest effect and a new dedicated experiment should be able to identify it—thereby enhancing our knowledge of this poorly known form factor considerably!

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