Scaling and dynamics of turbulence over sparse canopies

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Turbulent flows within and over sparse canopies are investigated using direct numerical simulation. We study how such canopies affect the surrounding flow through two distinct mechanisms. The first is through the flow induced directly by the presence of the canopy elements as obstacles. When the element-induced flow is filtered out, the remaining background turbulence exhibits a balance of the viscous and the Reynolds shear stresses within the canopy layer similar to that over smooth walls, even if their sum behaves differently. From this, a scaling based on the sum, at each height, of these two stresses is proposed. Using this height-dependent scaling, the background turbulence fluctuations within the canopies show similarities to those over smooth walls. This suggests that the background turbulence within the canopy scales with the local stress at each height, rather than the total drag as in smooth walls. This effect is essentially captured when the canopy is substituted by a drag force that acts on the mean flow alone, aiming to produce the correct local stress without modifying the fluctuations directly. This forcing is shown to produce better estimates for the turbulent fluctuations compared to a conventional, homogeneous-drag model. The effect of the element-induced flow, however, requires the representation of the individual canopy elements.

Key words:

1. Introduction

Canopy flows are ubiquitous in both natural and artificial settings. The most commonly studied examples of such flows are those through crops and forests. In addition, they are relevant to flows over engineered surfaces, such as pin fins for heat transfer or piezoelectric filaments for energy harvesting. The study of turbulent flows over canopies, therefore, has wide ranging applications, including reducing crop loss (de Langre 2008), energy harvesting (McGarry \& Knight 2011, Elahi \textit{et al.} 2018) and improving heat transfer (Fazu \& Schwerdtfeger 1989, Bejan \& Morega 1993). On the basis of the geometry and spacing of the canopy elements, a canopy can be classified as dense, sparse or transitional (Nepf 2012). In the dense limit, the canopy elements are in close proximity to each other and turbulence is essentially not able to penetrate within the canopy layer. In the sparse limit, the spacing between canopy elements is large and the turbulent eddies can penetrate the full depth of the canopy. An intermediate or transitional regime lies between these two limits. Turbulent flows in the dense regime, reviewed by Finnigan (2000) and Nepf (2012), are characterised by the formation of Kelvin–Helmholtz-like,

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or mixing-layer, instabilities, originating from the inflection point at the canopy tips (Raupach et al. 1996). As the sparsity of the canopies is increased, the importance of the Kelvin–Helmholtz-like instability decreases (Poggi et al. 2004; Pietri et al. 2009; Huang et al. 2009). Eventually, the flow would resemble that over a smooth wall, albeit perturbed by the discrete presence of the individual canopy elements (Finnigan 2000). The separation between these regimes is still somewhat unclear. Nepf (2012) proposed an approximate classification of the canopy regime based on the roughness frontal density, $\lambda_f$. Nepf (2012) observed that canopies are dense for $\lambda_f \gg 0.1$, sparse for $\lambda_f \ll 0.1$, and intermediate for $\lambda_f \approx 0.1$. However, in addition to the geometric parameter $\lambda_f$, the lengthscales of the flow should also be considered when determining the regime of the canopy. The lengthscales in a turbulent flow may be much larger than the element spacing at a particular Reynolds number, so that the turbulent eddies are precluded from penetrating within the canopy. As the Reynolds number is increased, however, the turbulent lengthscales will eventually become comparable to the element spacing and allow the turbulent eddies to penetrate within the canopy efficiently.

In the present work, we study flows within and above sparse canopies. The canopies have low roughness densities $\lambda_f \lesssim 0.1$, with element spacings large enough to limit their interaction with the near-wall turbulence dynamics. Owing to the sparse nature of these canopies, we would expect the flow within them to be dominated by the footprint of the canopy elements, rather than by a mixing-layer instability. Conventionally, a homogeneous-drag is used to represent the effect of canopies (Dupont & Brunet 2008; Finnigan et al. 2009; Huang et al. 2009; Bailey & Stoll 2016). This approach would only be strictly valid to represent very closely packed canopies, where the element spacing is much smaller than any lengthscale in the overlying flow, and even small flow structures perceive the canopy elements as acting collectively (Zampogna & Bottaro 2016). Using a homogeneous drag to capture the effects of sparser canopies tends to overdamp turbulent fluctuations within the canopies (Yue et al. 2007; Bailey & Stoll 2013). This is typically attributed to the inability of homogenised models to capture the element-induced flow, and the lack of representation of the gaps between the canopy elements, where the fluctuations would not experience any damping (Bailey & Stoll 2013). In the present work we separate the effect of the element-induced coherent flow from the incoherent background turbulence, and focus mainly on the properties of the latter. We study different element spacings and geometries. We propose a scaling that suggests that the dynamics of the background turbulence within sparse canopies are mainly governed by their effect on the mean velocity, rather than by the direct interaction of the canopy elements with the flow. Based on this scaling, we propose that the effect on the background turbulence is represented better by a drag acting on the mean flow alone than by a homogeneous drag. Partial results from some of the simulations can be found in Sharma & García-Mayoral (2018a,b).

The paper is organised as follows. The numerical methods used and the canopy geometries simulated are described in §2. The results of the canopy-resolving simulations and the scaling of turbulent fluctuations are discussed in §3. The results obtained from simulations that substitute the canopy by a drag force are discussed in §4. The conclusions are presented in §5.

2. Numerical simulations

We conduct direct numerical simulations of an open channel with canopy elements protruding from the wall. The streamwise, wall-normal and spanwise coordinates are $x$, $y$ and $z$, respectively, and the associated velocities are $u$, $v$ and $w$. The size of the
**Figure 1.** Schematic of the numerical domain. An instantaneous field of the fluctuating streamwise velocity from case TP1 is shown in three orthogonal planes.

| Case            | \(N_x \times N_z\) | \(u_\tau\) | \(Re_\tau\) | \(\lambda_f\) | \(\int D^+\) | \(\Delta x^+\) | \(\Delta z^+\) |
|-----------------|---------------------|-------------|-------------|--------------|---------------|---------------|---------------|
| Smooth S        | –                   | 0.052       | 538.8       | –            | –             | 8.8           | 4.4           |
| P0              | 32\times16          | 0.155       | 532.5       | 0.22         | 0.94          | 4.36          | 4.36          |
| P0-H            | –                   | 0.173       | 594.2       | –            | 0.93          | 9.72          | 4.86          |
| P0-H0           | –                   | 0.194       | 549.4       | –            | 0.90          | 8.99          | 4.49          |
| Impermeable     |                     |             |             |              |               |               |               |
| prismatic       |                     |             |             |              |               |               |               |
| elements P1     | 16\times8           | 0.118       | 520.3       | 0.05         | 0.79          | 4.26          | 4.26          |
| P1-H0           | –                   | 0.125       | 553.8       | –            | 0.80          | 9.06          | 4.53          |
| P2              | 8\times4            | 0.079       | 529.4       | 0.01         | 0.57          | 4.33          | 4.33          |
| P2-H0           | –                   | 0.078       | 522.9       | –            | 0.57          | 8.55          | 4.28          |
| Impermeable     |                     |             |             |              |               |               |               |
| T-shaped        |                     |             |             |              |               |               |               |
| elements T1     | 16\times8           | 0.113       | 505.6       | 0.07         | 0.80          | 4.14          | 4.14          |
| T1-H            | –                   | 0.136       | 503.3       | –            | 0.85          | 11.00         | 5.50          |
| T1-H0           | –                   | 0.141       | 519.9       | –            | 0.82          | 11.34         | 5.67          |
| T2              | 8\times4            | 0.080       | 513.3       | 0.02         | 0.58          | 4.20          | 4.20          |
| T2-H0           | –                   | 0.082       | 527.4       | –            | 0.60          | 8.63          | 4.31          |
| Permeable       |                     |             |             |              |               |               |               |
| T-shaped        |                     |             |             |              |               |               |               |
| elements TP1    | 16\times8           | 0.137       | 505.5       | 0.07         | 0.81          | 8.27          | 4.14          |
| TP1-L           | –                   | 0.142       | 527.1       | –            | 0.81          | 8.62          | 4.31          |

Table 1. Simulation parameters. \(N_x\) and \(N_z\) are the number of canopy elements in the streamwise and spanwise directions, respectively, \(u_\tau\) is the friction velocity based on the net drag, \(Re_\tau\) is the friction Reynolds number based on \(u_\tau\) and \(\delta\), and \(\lambda_f\) is the roughness frontal density. \(\int D^+\) is the net canopy drag force scaled with \(u_\tau\), that is, the proportion of the total drag on the fluid exerted by the canopy elements, with the remainder being the friction at the bottom wall. The grid resolutions in the streamwise and spanwise directions are \(\Delta x^+\) and \(\Delta z^+\), respectively.
simulation domain is $2\pi \delta \times \delta \times \pi \delta$, with the channel height $\delta = 1$. This box size has been shown to be adequate to capture one-point statistics up to the channel height for the friction Reynolds numbers used in the present study [Lozano-Durán & Jiménez 2014]. A schematic representation of the numerical domain is shown in figure 1. The domain is periodic in the $x$ and $z$ directions. No-slip and impermeability conditions are applied at the bottom boundary, $y = 0$, and free slip and impermeability at the top, $y = \delta$. It is shown in §3 that the height of the roughness sublayer for the canopies studied here extends to only half of the domain height, so the top boundary of the channel does not interfere with the canopy flow. The flow is incompressible, with the density set to unity. All simulations are run at a constant mass flow rate, with the viscosity adjusted to obtain a friction Reynolds number based on the total stress $\text{Re}_\tau = u_\tau \delta / \nu \approx 520$.

The numerical method used to solve the three-dimensional Navier-Stokes equations is adapted from Fairhall & García-Mayoral (2018). A Fourier spectral discretisation is used in the streamwise and spanwise directions. The wall-normal direction is discretised using a second-order centred difference scheme on a staggered grid. The grid in the wall-normal direction is stretched to give a resolution $\Delta y^+ = 0.2$ at the wall, stretching to $\Delta y_{\max}^+ \approx 2$ at the top of the domain. The wall-parallel resolutions for the different cases are given in table 1. The time advancement is carried out using a three-step Runge-Kutta method with a fractional step, pressure correction method that enforces continuity [Le & Moin 1991]

$$[I - \Delta t \frac{\beta_k L}{\text{Re}}] u^n_k = u^n_{k-1} \Delta t \left[ \frac{\alpha_k}{\text{Re}} L u^n_{k-1} - \gamma_k N(u^n_{k-1}) - \zeta_k N(u^n_{k-2}) - (\alpha_k + \beta_k) G(p^n_k) \right] , \quad k \in [1, 3] ,$$  \quad (2.1)

$$D G(\phi^n_k) = \frac{1}{(\alpha_k + \beta_k) \Delta t} D(u^n_k) ,$$  \quad (2.2)

$$u^n_{k+1} = u^n_k - (\alpha_k + \beta_k) \Delta t G(\phi^n_k) ,$$  \quad (2.3)

$$p^n_{k+1} = p^n_k + \phi^n_k .$$  \quad (2.4)

Where $I$ is the identity matrix and $L$, $D$ and $G$ are the Laplacian, divergence and gradient operators respectively. $N$ is the dealiased advective term. $\alpha_k, \beta_k, \gamma_k$ and $\zeta_k$ are the Runge-Kutta coefficients for substep $k$ from Le & Moin [1991], and $\Delta t$ is the timestep.

### 2.1. Canopy-resolving simulations

In these simulations, the canopy elements are assumed to be solid obstacles, and their geometry is resolved using an immersed-boundary method adapted from García-Mayoral & Jiménez (2011). The simulation parameters for the different cases studied here are summarised in table 1. Case S is an open channel with a smooth-wall floor. The canopy-resolving simulations include two canopy geometries, as portrayed in figure 2, with varying element spacings. The first geometry, denoted by the letter ‘P’, consists of collocated prismatic-posts with a square top-view cross-section with sides $l_x^+ \approx 20$, and height $l_s^+ \approx 110$. The spacing between the canopy elements in the wall-parallel directions for cases P0, P1 and P2 are $L_x^+ \approx 100$, 200 and 400, respectively. The second geometry, denoted by the letter ‘T’, consists of frontally-extruded T-shaped canopies, as portrayed in figures 2(b–c). The head of these canopy elements has dimensions $l_x^+ \approx 40$ in the wall-parallel directions. The base of the canopy elements has $l_s^+ \approx 80$ and $l_h^+ \approx 30$, respectively. The canopy elements are in a collocated arrangement,
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Figure 2. Contours of instantaneous streamwise velocity in planes passing through the centre of a canopy element. (a–c) represent cuts in the $z$–$y$ plane, and (d–f) represent cuts in the $x$–$y$ plane. (a,d) cuboidal canopy elements from case P1; T-shaped canopy elements from (b,e) case T1 and (c,f) case TP1. The white lines mark the positions of the canopy elements. The contours are scaled using the global friction velocity, $u_\tau$, of each case.

and the spacing between the elements for cases T1 and T2 is $L_1^+ = L_2^+ \approx 200$ and 400, respectively. Case TP1 has the same geometry and layout as T1, but the canopy elements are represented by a drag force only applied within them, rather than by immersed boundaries. This method allows some flow to permeate into the canopy elements, as shown in figures (c,f), and has been observed to be a suitable model for certain natural canopies (Yan et al. 2017; Yue et al. 2007). The drag force in case TP1, applied only at the grid points that are within the canopy elements, is of the form $C_{dc}u_i|u_i|$, similar to Yue et al. (2007), Bailey & Stoll (2013) and Yan et al. (2017), where $C_{dc}$ is a drag coefficient and $u_i$ is the instantaneous local velocity in every $i$ direction. $C_{dc}$ is set such that further increasing its magnitude does not significantly increase the net drag force on the mean flow. This forcing provides a local body force opposing the flow inside the canopy elements, and thus results in a small velocity within the canopy elements. The net mean drag force for this canopy is similar to that of the impermeable canopy, T1, as noted in table 1, in spite of the different character of the canopy elements.

The roughness densities of the canopies are given in table 1. All the canopies studied lie within the sparse to transitional regime empirically demarcated by Nepf (2012). The spanwise spacings between the canopy elements are $L_2^+ \geq 100$, which is comparable to, or larger than, the width of near-wall streaks, $\lambda_2^+ \approx 100$ (Kline et al. 1967). This implies that the canopies should be sparse from the point of view of the near-wall turbulent fluctuations as well.
Figure 3. Drag coefficients, $C_{dh} = D/U^2$, obtained from ‘T’ shaped canopies. —, case T1; ——, case TP1; ——, case T2; ---, cases T1-H/H0; and ---, case T2-H0. The inset provides a magnified view of the drag coefficients for cases T2 and T2-H0.

2.2. Drag force representations

In order to explore the canopy-flow dynamics, we also conduct simulations where the canopy is replaced by some drag force, that does not resolve the geometry of the canopy elements. Sparse canopies consisting of bluff elements, such as those in the present work, are generally better characterised by a quadratic drag (Coceal et al. 2008), whereas for dense canopies where viscous effects dominate a drag force proportional to the velocity would be more appropriate (Tanino & Nepf 2008). Therefore, in the present study, the canopy elements are replaced by a quadratic drag force, that is, where the drag is proportional to the square of the velocity.

The drag coefficient, $C_{dh}$, is obtained by approximating the canopy drag force obtained from the canopy-resolving simulations, $D$, to a form $D \approx C_{dh}U|U|$, where $U$ is the mean streamwise velocity. The drag coefficients obtained from cases T1, TP1 and T2 are portrayed in figure 3. This quadratic form provides a reasonable approximation of the drag force for $y^+ > 10$, once viscous effects are small. This is consistent with observations made in previous studies (Coceal et al. 2008; Böhme et al. 2013).

For the simulations labelled with the suffix ‘-H’, the presence of the canopy is replaced by a force $C_{dh}u_i|u_i|$ applied homogeneously below the canopy tips. This is the conventional homogeneous-drag model. It also requires the prescription of drag coefficients in the spanwise and wall-normal directions. We estimate these by rescaling the streamwise drag coefficient based on the relative change in the ‘blockage ratio’ of the canopy elements in the different directions (Luhar et al. 2008), in the spirit of the method proposed by Luhar & Nepf (2013). The blockage ratio in each direction is proportional to the frontal area of the canopy elements in that direction. In the wall-normal and spanwise directions this would be the top-view and the side-view areas respectively. For the wall-normal drag, this assumption is particularly coarse, but Busse & Sandham (2012) have shown that the flow is relatively insensitive to moderate changes in the wall-normal drag coefficient. In the simulations labelled with the suffix ‘-H0’, a forcing $C_{dh}U|U|$ is applied in the region below the canopy tips, where $U(y)$ is the mean velocity profile. The drag is only applied to the mean-streamwise velocity, and has no fluctuating component. While the drag force in cases labelled ‘-H’ varies along any given wall-parallel plane depending on the local velocity, in cases labelled ‘-H0’ the drag force is homogeneous along any given wall-parallel plane, as it depends only on the mean velocity and the drag coefficient at that height. Finally, the simulation labelled with the suffix ‘-L’ applies a drag $C_{dh}U|U|$, as in cases H0, but distributed in a reduced-order representation of the canopy elements, which consists of a 24-mode, $x$-$z$ Fourier-truncation of the canopy geometry.
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3. Canopy-resolving simulations

In this section, we present and discuss the scaling of turbulent fluctuations in sparse canopies, and compare them with those over a smooth wall. Over a smooth wall, the balance of stresses within the channel can be obtained by averaging the momentum equations in the wall-parallel directions and time and integrating in $y$, which yields

$$
\frac{dP}{dx} y + \tau_w = -\overline{u'v'} + \nu \frac{dU}{dy},
$$

(3.1)

where $\tau_w$ is the wall shear stress, $dP/dx$ is the mean streamwise pressure gradient, $-\overline{u'v'}$ is the Reynolds shear stress, $U$ is the mean streamwise velocity, and $\nu$ is the kinematic viscosity. Particularising (3.1) at $y = \delta$, we obtain the expression for the wall shear stress and the friction velocity, $u_\tau$,

$$
u^2 = \tau_w = -\delta \frac{dP}{dx}.
$$

(3.2)

In the presence of a canopy, the stress balance also includes the drag exerted by the canopy elements,

$$
\frac{dP}{dx} y + \tau_w = -\overline{u'v'} + \nu \frac{dU}{dy} - \int_0^y D \, dy,
$$

(3.3)

where the canopy drag averaged in $x, z$ and time, $D$, is zero for $y > h$. Equation (3.3) can be rewritten as

$$
\frac{dP}{dx} y + \tau_w + \int_0^h D \, dy = -\overline{u'v'} + \nu \frac{dU}{dy} + \int_y^h D \, dy,
$$

(3.4)

so that the net drag, $\tau_w + \int_0^h D \, dy$, is on the left-hand side, as in (3.1). From this net drag, a ‘global’ friction velocity can be defined,

$$
u^2 = \tau_w + \int_0^h D \, dy = -\delta \frac{dP}{dx}.
$$

(3.5)

This is the equivalent of the smooth-wall $u_\tau$ of equation (3.2) for canopy flows. Equation (3.4), however, shows that in canopy flows, the total stress is made up of the viscous stress, the Reynolds stress and the canopy drag stress. Their sum, the total stress, is linear in $y$ as in smooth-wall flows. Above the canopy, the canopy drag is zero and
the magnitude of the viscous and Reynolds shear stresses is similar to that over smooth walls. Within the canopy, however, the canopy drag can dominate and the viscous and Reynolds shear stresses are smaller than over smooth walls, as shown in figure 4(a).

As can be observed in figure 1, the presence of the canopy elements induces a coherent flow. Several studies have shown that the flow around the canopy elements and the flow far away from them have significantly different characteristics, and consequently they are typically studied separately [Finnigan 2000, Böhm et al. 2013, Bailey & Stoll 2013]. A commonly used technique to separate the element-induced flow from the background turbulence is through triple decomposition [Reynolds & Hussain 1972]

\[ \mathbf{u} = \mathbf{U} + \mathbf{\bar{u}} + \mathbf{u}', \quad (3.6) \]

where \( \mathbf{u} \) is the full velocity. The mean velocity, \( \mathbf{U} \), is obtained by averaging the flow in time and space. The element-induced velocity, also referred to as the dispersive flow, \( \mathbf{\bar{u}} \), is obtained by averaging the flow in time alone. We refer to the remaining part of the flow, \( \mathbf{u}' \), as the incoherent, background-turbulence velocity. A method analogous to triple decomposition, called ‘double averaging’ [Raupach & Shaw 1982], can also be used to separate the element-induced and background-turbulence flows [Finnigan 2000, Nepf 2012, Bai et al. 2015, Giometto et al. 2016, Yan et al. 2017].

The intensity of the element-induced flow can vary with the shape [Balachandar et al. 1997, Taylor et al. 2011], permeability [Yu et al. 2010, Ledda et al. 2018], and distribution of the canopy elements. It is possible, however, for canopies to have different element-induced flows but similar background turbulence. To illustrate this, we compare two canopy simulations, T1 and TP1, which have similar canopy layouts and roughly the same net drag, as shown in figure 5(h). The difference between the two cases is that in T1 the canopy elements are impermeable, whereas in TP1 some flow penetrates into the elements. The rms fluctuations of the full and background-turbulence velocity components for these cases are shown in figure 5. The magnitude of the full streamwise fluctuations within the canopy is significantly larger for T1 than for TP1. This increase, however, can be attributed essentially to the stronger element-induced fluctuations generated by the impermeable canopy elements. This is evidenced by the fact that the background-turbulence streamwise fluctuations for both cases essentially collapse, as shown in figure 5(b). The cross-flow fluctuations and Reynolds shear stress profiles for both these canopies are also similar. The impermeable canopy, however, has a slightly larger damping effect on the spanwise fluctuations.

The fluctuating velocities portrayed in figure 5 are scaled using the ‘global’ friction velocity defined by equation (3.5), which includes the full contribution of the canopy drag. Tuerke & Jiménez [2013] studied smooth-wall flows with artificially forced mean profiles, and observed that the turbulent fluctuations in such flows scaled with the local sum of the viscous and Reynolds shear stresses, or the local stress \( \tau_f \), at each height. This was the case even when \( \tau_f \) was not linear with \( y \) due to the artificial forcing. Tuerke & Jiménez [2013] defined a ‘local’ friction velocity, \( u^* \), by linearly extrapolating the local stress at each height to the wall,

\[ u^*(y)^2 = \frac{\delta}{\delta - y} \tau_f(y). \quad (3.7) \]

Notice that, for a smooth unforced channel, \( u^* = u_\tau \) at every height. Following Tuerke & Jiménez [2013], we define the sum of the viscous and background-turbulence Reynolds stresses as \( \tau_f \). In the present work, we only discuss the scaling of the background-turbulence fluctuations. Hence, only the contribution of the background-turbulence Reynolds shear stresses to \( \tau_f \) are considered. A similar concept was also proposed by
Figure 5. Rms velocity fluctuations and shear stresses scaled with the global friction velocity, $u_\tau$, from the canopy-resolving simulations. In the left column, the lines represent —, case P0; —, case P1; and —, case P2. In the right column, the lines represent —, case T1; —, case TP1; and —, case T2. Dashed lines represent the full velocity fluctuations and solid lines represent the background-turbulence fluctuations. The thin, solid black line represents the smooth-wall case, S, for reference.
Högström et al. (1982) for flows over urban canopies. They scaled turbulence with a local friction velocity, defined as the square root of the magnitude of the local Reynolds shear stress, but had measurements only at heights where the contribution of the viscous stress to \( \tau_f \) would be small. Using \( u^* \), a local viscous lengthscale can also be defined, \( \nu/u^* \), and from it an effective viscous height, \( y^* = yu^*/\nu \). Both \( u^* \) and \( y^* \) are portrayed, for the prismatic-post canopies, in figure 6. Near the canopy tips, where the element-induced drag is no longer present, the local friction velocity, \( u^* \) becomes equal to the global \( u_\tau \), and \( y^* \) becomes equal to \( y^+ \). Making the canopy sparser reduces the canopy drag, and hence the difference between \( u^* \) and \( u_\tau \) within the canopy reduces with increasing canopy sparsity.

When scaled with \( u_\tau \), as is done conventionally, the viscous and Reynolds shear stresses near the base of the canopy are highly damped compared to smooth walls. However, the balance of these stresses in \( \tau_f \) remains close to that over smooth walls. This is illustrated in figure 4, which portrays the terms in the stress balance within a channel with canopies scaled with \( u_\tau \) and \( u^* \). The similarity of the viscous and Reynolds shear stresses in the canopy-flow and smooth-wall cases suggests that the canopy acts on the background turbulence essentially through changing their local scale, rather than through a direct interaction of the canopy elements with the flow. To explore the scaling further, the background-turbulence rms fluctuations are portrayed scaled with \( u^* \) in figure 7. Scaling the fluctuations with the conventional \( u_\tau \) shows a reduction of the fluctuations within the canopy compared to a smooth wall, as shown in figure 5. With our proposed scaling with \( u^* \), in contrast, the streamwise fluctuations appear similar to those in a smooth channel. The increase in spanwise and wall-normal fluctuations, shown in figures 7(c–f), suggests however, that there is a relative increase in the intensity of the cross flow within the canopy compared to a smooth channel.

Although \( u''^* \) and \( u''v''^* \) within the canopy appear similar to those over smooth walls, there are some differences in the distribution of energy across different lengthscales in the flow, particularly in the region close to the wall. In order to examine this, we compare the spectral energy densities, at \( y^* \approx 15 \), for a smooth-wall and for case P1 in figure 8. This is the height roughly corresponding to the location at which the magnitude of the fluctuations peaks in smooth-wall flows (Jiménez & Pinelli 1999). In global units, the energy is observed to be in larger wavelengths when compared to a smooth channel, especially in \( \lambda_z \). In local scaling, however, there is a greater overlap of the regions with highest intensity, particularly for \( E_{uu} \) and \( E_{uv} \). In addition, the canopy case exhibits
Figure 7. Rms velocity fluctuations and shear stresses for the background turbulence scaled using the local friction velocity, $u^*$, from the canopy-resolving simulations. The lines in the left column represent --, case P0; ---, case P1; and ----, case P2. In the right column, the lines represent --, case T1; ---, case TP1; and ----, case T2. The thin black lines represent the smooth-wall case, S.
a concentration of energy at the canopy wavelengths and its harmonics. Note that the canopy spacing, for case P1, at $y^* \approx 15$ is reduced to $L_+ = L_+ \approx 100$, while in global scaling it is $L^+_x = L^+_z \approx 200$. The increase in the energy in the canopy wavelengths is a reflection of the element-induced flow. The large streamwise scales, in turn, are damped by the presence of the canopy, which results in a reduction of their energy.

The differences in the energy distribution observed within the canopy eventually disappear above it. To illustrate this, figure 9 portrays the spectra near the canopy tips, $y^* \approx 105$. Here, the concentration of energy in the canopy wavelengths and its harmonics is weak, and the smaller scales in the flow are smooth-wall like. There is, however, still a deficit of energy in large streamwise wavelengths, compared to a smooth wall, associated with the damping of these scales by the canopy elements, as discussed in the previous paragraph. This effect diminishes away from the canopy, and the spectra are essentially smooth-wall-like for $y^+ \gtrsim 250$, as shown in figure 10, indicating that outer-layer similarity is recovered and delimiting the height of the roughness sublayer.

So far, we have mainly focussed on the results for case P1, with prismatic canopy elements with spacings $L^+_x = L^+_z \approx 200$. We now discuss the effect of changing the canopy element geometry and spacing on the flow. An increased sparsity results in an increase in the magnitude of both the full and background-turbulent velocity fluctuations, as shown in figure 5. In local scaling, however, the background turbulent fluctuations follow a similar trend to that observed for case P1. We observe that $u'^*$ and $u'v'^*$ appear smooth-wall-like, while there is a relative increase in the magnitude of the cross fluctuations. For the denser canopy of case P0, on the other hand, the fluctuations become less similar to those over smooth-walls. The streamwise fluctuations are damped more intensely within the canopy, and there are additional Reynolds shear stresses near the wall. Figure 11 shows that, compared to the sparser canopies, P0 has an accumulation of energy in streamwise wavelengths corresponding to the canopy harmonics but across a range of spanwise wavelengths. These regions of excess energy have also been noted by Abderrahaman-Elena et al. (2019), who studied densely packed cuboidal roughness. They noted that these regions were an imprint of the large, background-turbulence scales modulating the smaller scale coherent flow generated by the roughness. This effect diminishes as the canopy element spacing is made larger than the energetic scales in the background-turbulence, as evidenced by the lack of these regions in the spectra of the sparser canopies portrayed in figures 8, 9 and 11. For case P2, the spectra are already close to smooth-wall-like at the canopy tips, suggesting that both the element-induced flow and the damping of large scales are already weak at this height.

Previous studies have noted the formation of Kelvin–Helmholtz-like instabilities near the canopy tips in dense canopies (Finnigan 2000; Nepf 2012; Bailey & Stoll 2016). These instabilities originate from an inflection point in the mean-velocity profile over canopies (Raupach et al. 1996). The T-shaped canopies were originally designed as a flow control device, aiming to produce a strong inflection point at the canopy tips in order to generate these instabilities, while leaving the near-wall flow relatively undisturbed (Sharma & García-Mayoral 2018b). When present, these instabilities leave a distinct footprint in $E_{vv}$ and $E_{uv}$, causing an increase in energy in a narrow range of streamwise wavelengths and for large spanwise wavelengths (García-Mayoral & Jiménez 2011; Gómez-de Segura et al. 2018; Abderrahaman-Elena et al. 2019). However, such a footprint is not observed in the canopies studied here. This suggests that, if the instabilities are present over these sparse canopies, they are weak compared to the other fluctuations in the flow, and are therefore masked by them.
Figure 8. Pre-multiplied spectral energy densities for case P1 (filled contours) and for case S (line contours) normalised with their respective rms values at (a–d) $y^+ \approx 15$, and (e–h) $y^* \approx 15$. The contours from the left to right columns are in increments of 0.03, 0.06, 0.05 and 0.06, respectively.

Figure 9. Pre-multiplied spectral energy densities for case P1 (filled contours) and case S (line contours) at $y^* \approx 105$, normalised by their respective $u^*$. The contours in (a–d) are in increments of 0.125, 0.06, 0.075 and 0.06, respectively.

Figure 10. Pre-multiplied spectral energy densities at $y^+ \approx 250$. , case P0; —, case P1; and —, case P2, normalised by their respective $u_r$. Filled contours represent case S. The contours in (a–d) are in increments of 0.075, 0.04, 0.06 and 0.03, respectively.
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**Figure 11.** Pre-multiplied spectral energy densities at (a–d) $y^* \approx 15$; and (e–h) $y^* \approx 105$, normalised by their respective $u^*$. Filled contours represent case P2; , case S; and , case P0. The contours in (a–h) are in increments of 0.3, 0.03, 0.1, 0.06, 0.12, 0.06, 0.075 and 0.05, respectively.

4. Simulations with artificial forcing

The results discussed so far suggest that sparse canopies affect their surrounding flow through two mechanisms, an element-induced flow, and a change in the local scale for the background-turbulence fluctuations. With respect to the second mechanism, the effect of the canopy elements would be indirect, through modifying the mean-velocity profile and thus the local stress, $\tau_f$. The latter would, in turn, set the scale for the fluctuations. If this is the case, applying the mean drag produced by the canopy on the mean flow alone should capture the essential effects of the canopy on the background-turbulence. We test this in the simulations labelled with the suffix '-H0'. For the densest canopies simulated for each geometry, that is, P0 and T1/TP1, we also compare the mean-only-drag simulations with conventional, homogeneous drag simulations labelled with the suffix '-H'. Note that simulations T1-H0 and T1-H correspond to both the permeable and impermeable canopies of T1 and TP1, as they have similar net drags and drag coefficients.

The streamwise fluctuations and the Reynolds shear stresses of the mean-only-drag simulations are in good agreement with the corresponding background-turbulence fluctuations from the resolved canopies, except for the densest canopy studied, case P0, as shown in figure [12]. For the sparsest canopies, cases P2 and T2, the cross-flow fluctuations, particularly in the wall-normal direction, are slightly larger than their mean-only-drag counterparts. A likely reason for this discrepancy is the presence of an unsteady element-induced flow for these canopies, whose contribution cannot be filtered out by the conventional triple-decomposition technique that we have used. The fluctuating velocities scaled by the local friction velocities for the drag-force simulations are provided in appendix [A] for reference.

For case P0, a homogeneous drag provides a better representation of the cross-flow fluctuations than the mean-only drag, and the streamwise fluctuations are not well represented by either forcing method. For this case, there is significant interaction between the element-induced flow and the background turbulence, as discussed in [3]. Thus, it is not surprising that neither the mean-only drag nor the homogeneous drag is able to capture the full effect of this canopy on the background turbulence.
Figure 12. Rms velocity fluctuations and shear stresses scaled with the global friction velocity, $u_\tau$, of the canopy-resolving and mean-only drag and homogeneous drag simulations. In the left column, lines represent $\text{-}$, case P0; $\cdots$, case P0-H0; $\cdots$, case P0-H; $\longrightarrow$, case P1; $\cdots$, case P1-H0; $\longrightarrow$, case P2; and $\cdots$, case P2-H0. In the right column, lines represent $\text{-}$, case T1; $\cdots$, case T1-H0; $\cdots$, case T1-H; $\longrightarrow$, case TP1; $\longrightarrow$, case T2; and $\cdots$, case T2-H0. The thin black line represents the smooth-wall case, S. Note that only the background-turbulence fluctuations are portrayed for the canopy-resolving simulations.
Figure 13. Pre-multiplied spectral energy densities at (a–d) $y^* \approx 15$ and (e–h) $y^* \approx 105$, normalised by their respective $u^*$. Filled contours represent case TP1; — case T1-H0; — case T1-H; and — case TP1-L. The contours in (a–h) are in increments of 0.3, 0.075, 0.175, 0.075, 0.12, 0.06, 0.075 and 0.05, respectively.

Figure 14. Drag force distribution in the streamwise direction in a plane passing through the canopy heads for case TP1-L (blue); — —, distribution of the mean-only drag force, as in case T1-H0. The location of the canopy elements is sketched in grey at the bottom of the figure.

For the sparser canopies of T1 and TP1, compared to a mean-only drag, the homogeneous drag tends to overdamp the fluctuations within the canopy, particularly in the streamwise direction, as can be observed in figure 12(b). The excessive damping of fluctuations by a homogeneous drag, in comparison to a resolved canopy, was also noted by Yue et al. (2007) and Bailey & Stoll (2013). Figure 13 shows that this decrease in the intensity of fluctuations within the canopy is mainly a result of damping of the smaller streamwise wavelengths in the flow, $\lambda^*_x \lesssim 200$. The homogeneous drag simulation, T1-H, reproduces well the larger scales of the resolved canopy simulation, TP1. This suggests that scales much larger than the canopy spacing still perceive the canopy as homogeneous. Using the mean-only drag recovers some of the smaller streamwise scales, but it does not act directly on the larger scales as the actual canopy does. As the element spacing is increased, the range of scales affected in a homogenised fashion is shifted to larger scales, so that the energetic turbulent scales become less damped. This is consistent with the results portrayed in figures 11(e–h), which show that, near the canopy tips, the dense canopy P0 damps the energy at $\lambda^*_x \sim 1000–2000$, compared to smooth walls, while the sparse canopy P2 leaves these scales relatively undisturbed.

The accumulation of energy in the lengthscales of the order of the canopy wavelengths
and its harmonics observed in the canopy-resolving simulations requires a discrete representation of the canopy elements. Hence, it cannot be captured by either the mean-only- or the homogeneous-drag approaches. To introduce information of the canopy layout in the model, we distribute the drag calculated from the mean flow into a reduced-order representation of the canopy elements in case TP1-L. The representation consists of a truncation in Fourier space in $x$ and $z$, of the actual layout. The procedure is illustrated in figure 14 by the streamwise distribution of the drag force used by this model. In addition to capturing the local scaling of the flow, discussed in §4, this model is also able to represent the concentration of energy in the canopy scales for case TP1, as observed in figure 13. This is reflected by the collapse of the rms fluctuations of the full velocity components of TP1-L and TP1, as shown in figure 15. The magnitude of spanwise velocity fluctuations within the canopy of TP1-L is slightly larger than TP1, likely due to the fact that TP1-L does not apply any form of spanwise drag force. The drag force, although only applied in the streamwise direction, is also able to reproduce the canopy harmonics in the spectra of $E_{vv}$ and $E_{ww}$, which are caused by the deflection of the streamwise flow around the canopy elements as a result of continuity. The large scales in the flow are similar to those in the mean-only drag, as the drag in this case also does not act on these scales directly. This method, however, is only able to capture the weak coherent flow generated by the permeable canopy, and still under-predicts the full streamwise velocity fluctuations of the impermeable canopy.
5. Conclusions

In the present work, we have studied turbulent flows within and over sparse canopies. Two different canopy element geometries have been studied, each for various different element spacings. We have also compared canopies with permeable and impermeable elements in the same arrangement. The flow was decomposed into an element-induced component and a background-turbulence component. It was found that although the element-induced flow in the permeable and impermeable canopies studied here differ, the background turbulence was essentially the same. A new scaling for the background-turbulence fluctuations within sparse canopies was proposed. This scaling uses the friction velocity based on the local sum of the viscous and Reynolds shear stresses, $\tau_f$, at each height, rather than the conventional friction velocity, based on the net drag. When scaled with the proposed local friction velocity, the background-turbulence fluctuations and the viscous and Reynolds shear stresses appear more smooth-wall-like, compared to when conventional net-drag scaling is used. This suggests that the sparse canopy acts in a large part on the background turbulence through a change in the local scale, rather than through a direct interaction with the canopy elements. Based on the proposed scaling, we investigated the extent to which a drag force acting on the mean flow captures the effect of the canopy on the background turbulence. The mean-only drag directly modifies the mean flow alone, which in turn sets $\tau_f$ and, hence, the scale for the fluctuations. We show that the mean-only drag is able to capture the background-turbulence fluctuations within the canopies better than a conventional, homogeneous drag. Neither approach is, however, sufficient to capture the element-induced flow. The element-induced flow can be partially recovered by redistributing the mean-only drag in a low-order representation of the canopy.

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Appendix A.

Figure 16 compares the background-turbulence velocity fluctuations from the canopy resolving simulations with their corresponding mean-only or homogeneous drag simulations, in the proposed local scaling.

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Figure 16. Rms velocity fluctuations and shear stresses, scaled with the local friction velocity, $u^*$, of the canopy-resolving and mean-only drag and homogeneous drag simulations. In the first column, lines represent –, case P0; ---, case P0-H0; -----, case P0-H; ----, case P1; ---, case P1-H0; ----, case P2; and -----, case P2-H0. In the second column, lines represent –, case T1; ---, case T1-H0; -----, case T1-H; ----, case TP1; ----, case T2; and ---, case T2-H0. The thin black lines represent case S. Note that only the background-turbulence fluctuations are portrayed for the canopy-resolving simulations.
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