Adaptive traffic control at motorway bottlenecks with time-varying Fundamental Diagram

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Abstract: This paper deals with the problem of controlling traffic at motorways bottlenecks in presence of an unknown, time-varying, Fundamental Diagram (FD). The FD may change over time due to traffic composition or to the presence of Connected and Automated Vehicles (CAVs) with varying driving characteristics and penetration rates. A novel methodology, based on Model Reference Adaptive Control, is presented to robustly estimate the time-varying set-points that maximise the bottleneck throughput. The proposed approach is integrated in a control scheme that includes a linear quadratic integral regulator designed to control traffic which comprises a percentage of CAVs. Simulation experiments, based on a first-order multi-lane macroscopic traffic flow model that also considers for the capacity drop phenomenon, are presented to illustrate the effectiveness of the proposed approach.

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Keywords: Adaptive control, traffic control, connected and automated vehicles, time-varying Fundamental Diagram, robust estimation.

1. INTRODUCTION

During the recent decades, developments in automation have affected many aspects of our society, addressing practical challenges that include planning, designing, evaluating, and employing new technologies, which, in latter years, also involved vehicle automation (Bishop, 2005). In turn, Connected and Automated Vehicles (CAVs) are expected to strongly affect transportation systems; in fact, despite currently CAVs are designed to mainly improve safety and convenience of drivers, some technologies have direct influence on the traffic flow characteristics (Diaikaki et al., 2015; Tajdari et al., 2020b, 2019a; Ghaffari et al., 2015, 2018). In addition, CAVs may allow access to control variables that are not available in existing traffic control systems; for example, lane distribution optimization, via in-vehicle lane changing commands, is expected to have great potential in the context of Automated Highway Systems (AHS) and has been investigated by a number of scholars (Varaiya, 1993; Hall and Lotspeich, 1996; Roncoli et al., 2015b; Zhang and Ioannou, 2017), although there are considerably further potential of improvement in the area.

From a control perspective, many existing works rely on feedback control (Papageorgiou et al., 1991; Diakaki et al., 2002; Huang et al., 2016; Roncoli et al., 2016, 2017; Tajdari et al., 2020a). However, these studies assume predefined constant set-points, typically critical density, which is supposed to be determined based on, e.g., historical data. However, even if a set-point is known, it may not always be optimal due to possible changes in traffic behaviour characteristics, such as, e.g., a different traffic composition. In addition, the presence of CAVs at various penetration rates may have an impact on the traffic characteristics. The most affected factors are road capacity and critical density, due to their varying car-following and lane-changing behaviour, which is dependent on the CAV controller settings (Roncoli, 2019). For example, a higher degree of freedom in control can mitigate the negative impact of lane-changing, leading to higher capacities than the current values obtained with spontaneous lane-changes (Yu et al., 2019). It is therefore expected that the bottleneck capacity, as one of traffic characteristics, changes in time due to various factors and control schemes, highlighting the need to estimate in real-time accurate set-points values to achieve maximum benefits.

With a similar purpose, adaptive algorithms have been proposed and employed for on-line tuning the design parameters within urban control strategies, such as, e.g., Kouvelas et al. (2011); Kutadinata et al. (2016). Roncoli et al. (2016) employed a methodology based on discrete-time extremum seeking applied on off-line traffic data, which is a model-free method for real-time optimisation. Extremum seeking has been broadly investigated and utilized in several applications, such as, e.g., Ariyur and Krstić (2003); Kutadinata et al. (2016). Yu et al. (2019) proposed an extremum seeking control approach to find an optimal density input for freeway traffic online when there is a downstream bottleneck extended from Krstic and Smyslyavaev (2008); Oliveira et al. (2016). However, the method is restricted to a single lane with a single segment network. This paper presents a novel adaptive control approach capable to estimate accurate critical density (constant or time-varying) integrated with the control methodology in Tajdari et al. (2019b, 2020a) as...
Fig. 1. The FD assumed at the bottleneck area.

an integrated lane-changing and ramp metering robust control, considering the presence of partly automated and connected vehicles. Regarding communication, we assume that the equipped vehicles have the capability of bidirectional communication with the infrastructure (V2I); see, e.g., Roncoli et al. (2015a,b).

The paper is structured as follows: The proposed adaptive estimator is introduced in Section 2. Section 3 describes the overall control framework where the estimator is employed. Section 4 introduces the experiment setup, while in Section 5 the obtained simulation results are presented. Section 6 concludes the paper, highlighting its main results and indicating future research directions.

2. ADAPTIVE ESTIMATOR DESIGN

We aim at designing an estimator that produces the critical densities for each lane at a motorway bottleneck. For this purpose, we first introduce some necessary assumptions, then we proceed with the controller design, and finally we demonstrate the convergence of the estimated values and the stability of the method. In the design of our estimator, we assume a parabolic Fundamental Diagram (FD): however, we will show in Section 5 that the method is effective also when other shapes for an FD are used, as long as it is concave and has a unique maximum point, such a triangular-shaped FD.

We employ a \( q - \rho \) relationship, shown in Fig. 1, where

\[
q_{\text{out}} = a\rho^2 + b\rho,
\]

which has a maximum point \((\rho^*, q^*)\) as

\[
\rho^* = \frac{-b}{2a}, \quad q^* = \frac{-b^2}{4a}.
\]

We introduce nominal values \(q^*\) and \(\rho^*\) for variables \(q\) and \(\rho\), respectively, and we rewrite (1) as

\[
q_{\text{out}} - q^* = a(\rho^2 - \rho^{*2}) + b(\rho - \rho^*).
\]

Let us define the integral error states

\[
E_Q = \int (q_{\text{out}} - q^*)dt
\]

\[
E_\rho = \int (\rho - \rho^*)dt,
\]

leading to the integral error system

\[
\dot{E}_Q = q_{\text{out}} - q^*
\]

\[
\dot{E}_\rho = \rho - \rho^*,
\]

which can be rewritten as

\[
\dot{X} = B_eu_e + r_e,
\]

where

\[
X = \begin{bmatrix} \int q \, dt \\ \int \rho \, dt \end{bmatrix}, \quad u_e = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \rho^2 - \rho^{*2} \\ \rho - \rho^* \end{bmatrix},
\]

\[
B_e = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}, \quad r_e = \begin{bmatrix} q^* \\ \rho^* \end{bmatrix}.
\]

We propose to control system (8) via Model Reference Adaptive Control (MRAC) (Slotine et al., 1991), which allows to identify the unknown parameters \(a\) and \(b\) (appearing in \(B_e\)), and minimize the tracking error simultaneously. Therefore, we consider the feedback control law

\[
u_e = \begin{bmatrix} X^T \\ r_e \end{bmatrix}.
\]

(12)

We consider a model reference

\[
\dot{X}_M = -A_MX_M + B_Mr_e,
\]

where \(A_M\) and \(B_M\) are arbitrary matrices that make the dynamic of model reference stable. Assuming the error of the integral states and the model reference \(e = X - X_M\), then

\[
\dot{e} = \dot{X} - \dot{X}_M
\]

\[
= B_e(-\dot{B}X - \dot{C}r_e) + A_MX_M - B_Mr_e + A_MX - A_MX
\]

\[
= -A_M(X - X_M) + B_e(-\dot{B} + A_M)X + B_e(-\dot{C} - \frac{B_M}{B_e})
\]

\[
= -A_Me + B_e(-\dot{B} + A_M)X + B_e(-\dot{C} - \frac{B_M}{B_e}),
\]

(14)

which leads to

\[
e = \frac{B_e}{sI + A_M} \left[ -\dot{B} + A_M \quad \dot{C} - \frac{B_M}{B_e} \right] \begin{bmatrix} X \\ r_e \end{bmatrix},
\]

(15)

where \(s\) is the Laplace variable. Accordingly, the error dynamic of (15) is stable over time, as \(A_M\) was chosen as a stable matrix ((\(sI + A_M\) is stable or in other words \(A_M\) has negative eigenvalues), if \(\dot{\bar{a}}\) is finite (or \(\dot{\bar{a}}\) is convergence). Please note that \(A_M\), and \(B_M\) are finite. By replacing (11) in (8), we obtain

\[
\ddot{X} = B_e(-\dot{B}X + (\frac{1}{B_e} - \dot{C})r_e)
\]

(16)

this system is stable if

\[
\lim_{t\to\infty} \dot{B} \to B_e
\]

\[
\lim_{t\to\infty} \dot{C} \to \frac{1}{B_e}
\]

(17)

Then, to demonstrate the stability of the system, we assume

\[
u_e = \text{sign}(B_e)\bar{a} \begin{bmatrix} X \\ r_e \end{bmatrix}.
\]

(18)

To investigate the convergence of \(\bar{a}\), the following Lyapunov function is used:

\[
V = X^T P X + \bar{a}^T \Gamma^{-1} \bar{a},
\]

(19)

where \(P \geq 0\) and \(\Gamma > 0\) imply that \(V > 0\). For stability it would be enough if \(V \leq 0\), then

\[
\frac{dV}{dt} = X^T P \dot{X} + X^T \dot{P} X + \bar{a}^T \Gamma^{-1} \dot{\bar{a}} + \bar{a}^T \Gamma^{-1} \dot{\bar{a}}.
\]

(20)

By replacing \(V = \begin{bmatrix} X \\ r_e \end{bmatrix} \) in (18), we obtain

\[
u_e = \text{sign}(B_e)\bar{a} u^T \bar{a},
\]

thus

\[
\frac{dV}{dt} = 2X^T P B_e \text{sign}(B_e) u^T \bar{a} + 2\bar{a}^T \Gamma^{-1} \bar{a}.
\]

(21)

If we assume \(PB_e = C^T\), \(e = X^T C^T\), and \(\frac{dV}{dt} = 0\), then from (21) we have

\[-2 \text{sign}(B_e) u e u^T \bar{a} = 2\bar{a} \Gamma^{-1} \bar{a};
\]

(22)
thus, for the stability the changes of the unknown parameters should be
\[
\dot{a} = -\Gamma \text{sign}(B_x) \omega T.
\]
(23)

According to our observations, when $|\dot{a}| > 1$, the estimated values of $q^*$, and $p^*$ are highly fluctuating which results in a deterioration of the controller performance. To avoid this, we design $\Gamma$ as follows in order to keep $\Gamma \leq 1$ and to guarantee a smooth convergence
\[
\Gamma(k) = \begin{cases} \frac{1}{|\dot{a}(k)|}, & \text{if } |\dot{a}(k)| > 1, \\ (|\dot{a}(k)|)^\nu, & \text{if } |\dot{a}(k)| \leq 1. \end{cases}
\]
(24)

3. MOTORWAY TRAFFIC CONTROL FRAMEWORK

The control framework proposed in this work, depicted in Fig. 2, consists of a feedback controller designed to maintain the density at a motorway bottleneck on the estimated critical set-point through the adaptive estimator.

For self-completeness, we present here the formulation of an optimal control problem, originally proposed by Tajdari et al. (2020a), designed to manipulate the lateral flows, depending on the network topology, some terms of (25).

The model is formulated in a motorway network as follows (see also, e.g., Roncoli et al. (2020a), designed to manipulate the lateral flows, with the overall goal of avoiding the creation of congestion and maximising the bottleneck throughput. More specifically, lane-changing control aims at increasing the bottleneck capacity encountered with human-driver lane choices via more efficient lane distribution; while ramp metering aims at avoiding the creation of congestion and maintaining the bottleneck throughput at the increased capacity level.

We consider a discrete-time linear system that describes the conservation law equation for each cell $(i,j)$ of a motorway network as follows (see also, e.g., Roncoli et al. (2015b))
\[
\rho_{i,j}(k+1) = \rho_{i,j}(k) + \frac{T}{L_i} [q_{i-1,j}(k) - q_{i,j}(k)]
\]
\[+ \frac{T}{L_i} [f_{i,j-1}(k) - f_{i,j}(k)] + \frac{T}{L_i} d_{i,j}(k) + \frac{T}{L_i} r_{i,j}(k), \tag{25}
\]
where $\rho_{i,j}(k)$ is the traffic density, defined as the number of vehicles present within the cell at time instant $k$ divided by the cell length; $q_{i,j}(k)$ is the longitudinal flow leaving cell $(i,j)$ and entering cell $(i+1,j)$; $f_{i,j}(k)$ is the net lateral flow moving from cell $(i,j)$ to cell $(i,j+1)$; $d_{i,j}(k)$ is any external (uncontrolled) flow entering the network in cell $(i,j)$; $r_{i,j}(k)$ is the flow allowed to enter the network from the ramp located in $(i,j)$. The model is formulated in discrete time, considering the discrete time step $T$, indexed by $k = 0, 1, \ldots, K$, where the time is $t = kT$. Note that, depending on the network topology, some terms of (25) may not be present. Considering the well-known relation
\[
q_{i,j}(k) = \rho_{i,j}(k) v_{i,j}(k)
\]
and assuming that the controller operates in (and in fact maintains) congestion-free traffic conditions, that is, the speed in all cells is approximately constant (e.g., the critical speed) $v_{i,j}(k) \equiv v_{i,j} \forall i,j,k$, the resulting system is in the form of the following Linear Time Invariant (LTI) system
\[
\bar{x}(k+1) = \bar{A} \bar{x}(k) + \bar{B} u(k) + d(k),
\]
(27)
where $\bar{x}$ contains all the densities (state variables); $u$ includes all the lateral flows $f_{i,j}$ and the ramp flow $r_{i,j}$ that are assumed controllable; while $d$ includes the external flows that are not in $u$. Matrices $A$ and $B$ are derived from (25),(26).

Assuming the availability of nominal (desired) steady-state values and introducing the notation $\Delta \omega(k) = \omega(k) - \omega_\text{d}$, where $\omega$ replaces $\bar{x}$, $u$, and $d$, allows to reformulate the system in terms of error dynamics as
\[
\Delta \bar{x}(k+1) = \bar{A} \Delta \bar{x}(k) + \bar{B} \Delta u(k) + \Delta d(k).
\]
(28)

We then assume $\Delta d(k) = 0$ and, in order to avoid offset at the stationary state, we employ an integral controller to reject constant disturbances (Åström and Hågglund, 1995), thus removing the need for measuring the external inflows, by augmenting the system (28) with integral states, denoted as $z$, characterised by dynamics
\[
z(k+1) = z(k) + C \Delta \bar{x}(k),
\]
(29)
where $C$ extracts the elements of $\Delta \bar{x}$ corresponding to the bottleneck cells, which are the cells in the segment where the on-ramp nose is located, i.e., where congestion is likely to first appear (see Fig. 3). These modifications lead to the following augmented system:
\[
\Delta x(k+1) = A \Delta x(k) + B \Delta u(k),
\]
(30)
where
\[
\Delta x = \begin{bmatrix} \Delta \bar{x} \\ z \end{bmatrix}, A = \begin{bmatrix} \bar{A} & 0_{d \times S} \\ C \bar{F} & \bar{S} \end{bmatrix}, B = \begin{bmatrix} -\bar{B} \\ 0_{S \times (F+1)} \end{bmatrix}.
\]
We consider the following quadratic cost function, over an infinite time horizon, which accounts for minimisation of state and control input errors:
\[
\min J = \sum_{k=0}^{\infty} \begin{bmatrix} \Delta x(k) \end{bmatrix}^T C^T Q C \Delta x(k) + \Delta u(k)^T R \Delta u(k) \bigg|_{k=0}^{\infty} ,
\]
(31)
where
\[
Q = \varphi_1 I_{S \times S}, \quad R = \begin{bmatrix} \varphi_1 & I_{F \times F} & 0_{F \times 1} \\ 0_{1 \times F} & \varphi_2 \end{bmatrix}, \quad C = \begin{bmatrix} 0_{S \times H^T} & I_{S \times S} \end{bmatrix}.
\]
(32)

The matrices $Q$ and $R$ are weighting matrices associated to the magnitude of the integral and control errors, respectively, defined by parameters $\varphi_1 > 0$, $\varphi_1 > 0$, and $\varphi_2 > 0$, where $\varphi_1$ penalises lateral flow errors and $\varphi_2$ penalises ramp-metered flow error.

The resulting optimal control problem (31), (30) can be solved through a Linear Quadratic Regulator (LQR), resulting in the following linear feedback control law
\[
\Delta u(k) = -K \Delta x(k),
\]
(34)
where (see, e.g., Anderson and Moore (1971))
\[
K = (R + B^T P B)^{-1} B^T P A,
\]
(35)
\[
P = C^T Q C + A^T P A - A^T P B (R + B^T P B)^{-1}.
\]
(36)

For practical implementation, gain $K$ is split as
\[
K = [K_{\text{F}}\ K_{1}],
\]
(37)
which allows to rewrite the control law as
\[
\Delta u(k) = -K_{\text{F}} \Delta \bar{x}(k) - K_1 z(k),
\]
(38)
and, consequently,
\[
u(k) = u(k-1) - K_{\text{F}} [\bar{x}(k) - \bar{x}(k-1)] - K_1 [z(k) - z(k-1)].
\]
(39)

Since in practice, it may not be always possible to achieve the desired density set-point at the bottleneck (e.g., due
4. EXPERIMENT SET-UP

4.1 Nonlinear multi-lane traffic flow model

In order to test and evaluate the performance of the proposed control strategy, we present simulation experiments using a first-order traffic flow model based on Roncoli et al. (2015b). The model is used for reproducing the traffic behaviour for a multi-lane motorway and it features: (i) non-linear functions for the lateral flows of manually driven vehicles (which may also act as disturbances for the designed controller); (ii) a Cell Transmission Models (CTM)-like formulation for the longitudinal flows; and (iii) a non-linear formulation to account for the capacity drop phenomenon. Briefly, we consider the conservation law equation (25), where all variables are defined as in Section 3. Lateral flows due to manual lane-changing, \( f_{i,j}^M(k) \), are considered between adjacent lanes of the same segment, and corresponding rules are defined in order to properly assign and bound their values. They are computed as

\[
f_{i,j}^M(k) = l_{i,j+1}(k) - l_{i,j+1}(k),
\]

where

\[
l_{i,j}(k) = \min \left\{ 1, \frac{E_{i,j}(k)}{D_{i,j-1}(k) + D_{i,j+1}(k)} \right\} D_{i,j}(k)
\]

\[
E_{i,j}(k) = \frac{L_i}{T} \left[ \rho_{i,j}^{\text{jam}} - \rho_{i,j}(k) \right]
\]

\[
D_{i,j}(k) = \frac{L_i}{T} \rho_{i,j}(k) A_{i,j}(k)
\]

\[
A_{i,j}(k) = \mu \max \left\{ 0, \frac{G_{i,j}(k) \rho_{i,j}(k) - \rho_{i,j}(k)}{G_{i,j}(k) \rho_{i,j}(k) + \rho_{i,j}(k)} \right\},
\]

and \( j = j \pm 1 \). Variable \( E \) denotes the available space, in terms of flow acceptance, while \( D \) denotes the lateral demand flow, which is computed via definition of the attractiveness rate \( A \). Equation (42) accounts for the potentially limited space that may not be sufficient for accepting the lateral flow entering from both sides of a cell. In (45), the factor \( G \) is mostly equal to 1, which implies the intent of drivers to move towards a faster lane; while \( \mu \) is a constant coefficient in the range \([0, 1]\) reflecting the “aggressiveness” in lane-changing.

Longitudinal flows are the flows going from a cell to the next downstream one, while remaining in the same lane. We employ the Godunov-discretised first-order model proposed in Roncoli et al. (2015b). Moreover, the model accounts also for the capacity drop phenomenon, via a linearly decreasing demand function for over-critical densities (Kontorinaki et al., 2017) and a linear reduction of the maximum flow as a function of the entering lateral flows. The overall formulation for longitudinal flow is

\[
q_{i,j}(k) = \min \{ Q_{i,j}^E(k), Q_{i+1,j}^E(k) - d_{i+1,j}(k) \},
\]

where

\[
Q_{i,j}^E(k) = \begin{cases} 
\nu_{i,j} \rho_{i,j}(k), & \text{if } \rho_{i,j}(k) < \rho_{i,j}^{\text{cr}} \\
(1-\gamma)Q_{i,j}^\text{cap} - \nu_{i,j} \rho_{i,j}^{\text{jam}} + Q_{i,j}^R(k), & \text{otherwise}
\end{cases}
\]

\[
Q_{i+1,j}^E(k) = \begin{cases} 
Q_{i+1,j}^\text{cap} - \nu_{i+1,j} \rho_{i+1,j}(k), & \text{if } \rho_{i+1,j}(k) < \rho_{i+1,j}^{\text{cr}} \\
\nu_{i,j} \rho_{i,j}^{\text{jam}} + Q_{i,j}^R(k), & \text{otherwise}
\end{cases}
\]

Parameter \( \nu^{\text{max}} \) denotes the maximum speed, \( \rho^{\text{cr}} \) is the critical density, and \( Q^{\text{cap}} = \nu^{\text{max}} \rho^{\text{cr}} \) is the capacity flow. Parameter \( \varphi \) influences the extent of capacity drop due to overcritical densities, while \( \nu \) affects the capacity drop due to entering lateral flows. Note that, setting \( \varphi = 1 \) and \( \nu = 0 \), we obtain a conventional first-order model, i.e., no capacity drop appears at the head of congestion.

To model a time-varying FD, we modify (47)–(49) by assuming a time-varying critical density \( \rho^{\text{cr}}(t) \equiv \rho^{\text{cr}}(k) \) and, consequently, a time-varying capacity flow \( Q^{\text{cap}}(k) = \nu^{\text{max}} \rho^{\text{cr}}(k) \).

4.2 Network description and simulation configuration

We consider a two-lane motorway stretch, shown in Fig. 3, to test and evaluate the performance of the proposed strategy. In particular, we consider a network composed of 10 segments of the same length \( L_i = 0.5 \) km, while we employ a time step \( T = 10 \) s. The used traffic demand is depicted in Fig. 4. In addition, different lanes feature
different parameters, namely a different FD, which may reflect different traffic composition (e.g., a high number of heavy vehicles reducing the capacity of a specific lane). The characteristics of each lane vary over time, in particular we consider that there is a step change in the FD at $t = 150$ min, that is

$$\begin{align*}
\rho_{1,i}^c(k) &= \begin{cases} 
18 & \text{if } k < 900 \\
16 & \text{if } k \geq 900 
\end{cases} (50) \\
\rho_{2,i}^c(k) &= \begin{cases} 
20 & \text{if } k < 900 \\
18 & \text{if } k \geq 900. 
\end{cases} (51)
\end{align*}$$

We test our controller by considering only a percentage $\eta$ of vehicles that are connected and automated, i.e., implementing the lane-changes dictated by the controller, while the other vehicles are assumed to behave according to the lane-changing model for manual vehicles described in (41)–(45). The lateral flow implemented in numerical experiments is therefore

$$\bar{f}_{i,j}(k) = \begin{cases} 
M_{i,j}(k), & \text{if } \Phi^c(k) = 0 \\
(1 - \eta)M_{i,j}(k), & \text{if } \Phi^c(k) = 1. 
\end{cases} (52)$$

Since ramp metering actions may create a queue outside the motorway network, we introduce the following dynamics for the queue length $w(k)$ (in veh)

$$w(k + 1) = w(k) + T(d_{10,1}(k) - \bar{r}_{10,1}(k)), (53)$$

where $d_{10,1}(k)$ is the on-ramp external demand during time interval $(k, k + 1)$. The ramp flow implemented in our numerical experiments is

$$\bar{r}_{10,1}(k) = \begin{cases} 
d_{10,1}(k), & \text{if } \Phi^c(k) = 0, \\
\text{sat}(r_{10,1}(k)), & \text{if } \Phi^c(k) = 1. 
\end{cases} (54)$$

We consider the following bounds for the control inputs:

$$\begin{align*}
\text{sat}(f_{i,j}) &= \begin{cases} 
f_{i,j}^{\min} = -\frac{L_i}{T} \rho_{i,j} + 1, & \text{if } f_{i,j} \leq f_{i,j}^{\min} \\
f_{i,j}^{\max} = \frac{L_i}{T} \rho_{i,j}, & \text{if } f_{i,j} \geq f_{i,j}^{\max} \\
 f_{i,j}, & \text{otherwise;}
\end{cases} (55) \\
\text{sat}(r_{10,1}) &= \begin{cases} 
r_{10,1}^{\min} = 0, & \text{if } r_{10,1} \leq r_{10,1}^{\min} \\
r_{10,1}^{\max} = \min \left( \frac{Q}{T} + d_{10,1}, Q_{c}^{\text{cap}} \right), & \text{if } r_{10,1} \geq r_{10,1}^{\max} \\
r_{10,1}, & \text{otherwise.} 
\end{cases} (56)
\end{align*}$$

As performance metric we employ the TTS over a finite time horizon $K$, defined as

$$\begin{align*}
TTS &= T \sum_{k=0}^{K} \sum_{i=1}^{N} \sum_{j=m_i}^{M_i-1} \rho_{i,j}(k) + T w(k). (57)
\end{align*}$$

4.3 Reference model for the estimator

As discussed in Section 2, we should build a reference model with a stable dynamics, where one of the states is the integral of the other state. Thus, we use a two-state system with globally stable dynamic around $r_e$, as follows

$$\dot{X}_r = A_r X_r + B_r r_e (58)$$

where $X_M = X_{r_{11}}$. Moreover, the initial condition for the estimated critical densities and the maximum flows are set as conservative, i.e., the same used in the controlled case with constant set-point. Accordingly, $\hat{\rho}_1^c(0) = 18 \text{veh/km}$ and $\hat{\rho}_1^c(0) = 20 \text{veh/km}$, while $\hat{q}_1^c(0) = 1800 \text{veh/h}$ and $\hat{q}_1^c(0) = 2000 \text{veh/h}$. These values result in $\bar{a}(0) = -10.3$ for lane 1 and $-4.5$ for lane 2, while $b(0)$ is $330$ for lane 1 and $180$ for lane 2.

5. EXPERIMENTAL RESULTS

5.1 No-control scenario

The no-control case is investigated through implementation of the nonlinear traffic model (41)-(49) in the presented motorway stretch. According to Figs. 5(a) and 6(a), one may see that a strong congestion creates at the merge area (segment 10) and spills back reaching segment 2. The congestion occurs due to a) the relatively high inflow entering from the ramp, because the overall demand during the peak period is about 4600 veh/h, while the total capacity is 4200 veh/h; as well as b) the inefficient “natural” lane-changing flow. Capacity drop also happens at the bottleneck cells of the stretch, which intensifies the congestion.

5.2 Controlled scenario with constant set-points

The linear dynamic compensator (38), (40) is used for controlling the non-linear traffic model (41)-(49). Parameters of $\varphi_Q$, $\varphi_{R_1}$ and $\varphi_{R_2}$ are tuned based on a sensitivity analysis (see Tajdari et al. (2020a)) and the following values are chosen: $\varphi_Q = 1$, $\varphi_{R_1} = 1$ and $\varphi_{R_2} = 0.001$. Accordingly, congestion fully disappears; however, the densities at the bottleneck area are maintained at their initial critical values as [18, 20], see Figs. 6(b) and 6(b) and cannot follow the time-varying set-points value results in TTS loss reported in Table 2. In addition, a queue is generated at the on-ramp location during the peak period, which is not saturated to the upper-bound in this experiments (look at Fig. 7). The TTS improvement is about 9% (see also Table 2).
We see that the density converges to a neighbourhood of
Fig. 5. Density for the no-control case (a), controlled case (b) and controlled case with estimator (c).

Table 2. TTS for the different scenarios.

|                       | No control case | Without                         | With adaptive estimator |
|-----------------------|-----------------|---------------------------------|-------------------------|
| TTS [veh.h]           | 964.39          | 872.78                          | 769.95                  |

Improvement (%) 9 20

5.3 Controlled scenario with dynamically estimated set-points

In this section, we report the results of experiments to investigate the performances of the adaptive estimator and the impact of the estimated matrices in (17) on the overall performance. In the experiments, we consider $\zeta = \nu = 2$ in (24). Also, from (10), the estimated critical densities, and maximum out flows are $\hat{\rho}_c^*(k) = \rho(k) - u_2(k)$, and from (10) and (17), we have $\hat{q}^*(k) = \frac{B_2^T}{4B_{1,1}}$. The simulation result of the closed-loop system control with the adaptive estimator is shown in Figs. 5–8. One can observe from Fig. 6(c) that the congestion disappears, similarly as in the previous case; on the other hand, by inspecting Fig. 5(c), we see that the density converges to a neighbourhood of

the time-varying optimal values that are identified by the estimator as shown in Fig. 8(b) and the output flow of the bottleneck in Fig. 8(a) converges to its maximum value. The convergence to optimal values is achieved in about 20 min (equal to 120 time steps).

6. CONCLUSIONS

This paper presents a novel adaptive methodology to robustly estimate the critical set-point values for a traffic controller designed to achieve maximum throughput at a bottleneck area, assuming the FD is unknown and time-varying. Next steps involve more thorough analyses of the stability features of the estimator, as well as performing further simulation experiments to investigate the robustness of the estimator to different type of disturbances.
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