A study of local domination number of $Sn \triangleright H$ graph

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Abstract. All graphs in this paper are undirected, connected and simple graph. Let $G = (V, E)$ be a graph of order $|V|$ and size $|E|$. We define a set $D$ as a dominating set if for every vertex $u \in V - D$ is adjacent to some vertex $v \in D$. The domination number $\gamma(G)$ is the minimum cardinality of dominating set. By a locating dominating set of graph $G = (V, E)$, we define for every two vertices $u, v \in V(G) - D$, $N(v) \cap D \neq \emptyset$. Locating dominating set is a special case of dominating set with an extra constrain above. The minimum cardinality of a locating dominating set is locating dominating number $\gamma_L(G)$. The value of locating dominating number is $\gamma_L(G) \subseteq V(G)$. This paper studies locating dominating set of edge comb product of graphs, denoted by $G \triangleright H$. The graph $G \triangleright H$ is a graph obtained by taking one copy of $G$ and $|E(G)|$ copies of $H$ and grafting the $i$-th copy of $H$ at the edge $e$ to the $i$-th edge of $G$, where $G$ is star graph $S_n$ and $H$ is any special graph.

1. Introduction
Let $G = (V(G), E(G))$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$. A subset $D$ of $V(G)$ is called a vertex dominating set of $G$ if every vertex not in $D$ is adjacent to some vertices in $D$. A graph $G = (V, E)$ is called a locating dominating set if for every two vertices $u, v \in V(G) - D$, $N(v) \cap D \neq \emptyset$. Slater [5], [6] defined the locating-dominating number $\gamma_L(G)$ of a graph $G$ is the minimum cardinality of a locating-dominating set of $G$. The value of locating dominating number is $\gamma_L(G) \subseteq V(G)$. In this paper, we will initiate to analyze locating dominating set of edge comb product of two graphs, denoted by $G \triangleright H$, where $G$ is path graph and $H$ is any special graph.

Saputro et al [7] firstly introduction a comb product of graph. Let $G$ and $H$ be two connected graphs. Let $o$ be a vertex of $H$. The comb product between $G$ and $H$, denoted by $G \triangleright H$, is a graph obtained by taking one copy of $G$ and $|V(G)|$ copies of $H$ and grafting the $i$-th copy of $H$ at the vertex $o$ to the $i$-th vertex of $G$. By the definition of comb product, we can say that $V(G \triangleright H) = \{(a, v) | a \in V(G), v \in V(H)\}$ and $(a, v)(b, w) \in E(G \triangleright H)$ whenever $a = b$ and $vw \in E(G)$ and $v = w = o$.

A natural extension of comb product of graph is an edge comb product of graph. Let $G$ and $H$ be two connected graphs. Let $e$ be an edge of $H$. The edge comb product between $G$ and $H$, denoted by $G \triangleright H$, is a graph obtained by taking one copy of $G$ and $|E(G)|$ copies of $H$ and grafting the $i$-th copy of $H$ at the edge $e$ to the $i$-th edge of $G$. By the definition of comb
product. We can say that \( p = |V(G \supset H)| = q_1(p_2 - 2) + p_1 \) and \( q = |E(G \supset H)| = q_1q_2 \), see [8] for detail.

Let \( G \) be path \( P_n \) with vertex set \( V(P_n) = \{ x_i; 1 \leq i \leq n \} \), and edge set \( E(G) = \{(i, i+1); 1 \leq i \leq n-1 \} \) so \( |V(P_n)| = n, |E(P_n)| = n - 1 \) and \( H \) is helm graph with vertex set and edge set are \( V(H_m) = \{ A \} \} \cup \{ x_i; 1 \leq i \leq m \} \cup \{ y_i; 1 \leq i \leq m \} \), \( E(H_m) = \{ Ax_i; 1 \leq i \leq m \} \cup \{ x_ix_{i+1}; 1 \leq im - 1 \} \cup \{ y_iy_{i+1}; 1 \leq i \leq m \} \). Thus \( |V(H_m)| = 2m + 1, \) \( |E(H_m)| = 3m \). Furthermore, the graph \( P_n \supset H_m \) has a vertex set \( V(P_n \supset H_m) = \{ A_i; 1 \leq i \leq n - 1 \} \cup \{ x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m - 1 \} \cup \{ y_{i,j}; 1 \leq i \leq n - 1; 1 \leq j \leq m \} \) and edge set \( E(P_n \supset H_m) = \{ x_{i,j}x_{i+1,j+1}; 1 \leq i \leq n - 1; 1 \leq j \leq m - 2 \} \cup \{ A_iy_{i,j}; 1 \leq i \leq n - 1; 1 \leq j \leq m - 1 \} \). The order and size of \( (P_n \supset H_m), n \geq 3, m \geq 3 \) are \( |V(P_n \supset H_m)| = 2nm - m - 2 \) and \( |E(P_n \supset K_m)| = 3mn - 3m \).

We can say that \( p = V|G \supset H| = q_1(p_2 - 2) + p_1 \) and \( q = |E(G \supset H)| = q_1q_2 \). See Figure 1 as an example of edge comb product of graphs.

![Figure 1. Locating dominating set of edge comb product \( S_8 \supset P_4 \)](image)

2. Main Results

In this section, we determine the exact values of locating dominating number of some edge comb product of graphs, namely \( S_n \supset H \). In this paper, \( H \) are complete graph \( K_m \), star graph \( S_m \), triangular book \( Bt_m \) and path graph \( P_n \).

**Theorem 2.1.** Let \( x_ix_{i+1} \) be an edge of \( K_m \), as well as be a grafting edge of \( K_m \). The locating domination number of comb product graph \( S_n \supset K_m \) is \( \gamma_L(S_n \supset K_m) = nm - 2n \).

**Proof.** An edge comb product graph \( S_n \supset K_m \), \( n \geq 3 \) and \( m \geq 4 \), is a connected graph with vertex set \( V(S_n \supset K_m) = \{ A \} \cup \{ x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m - 1 \} \) and edge set \( E(S_n \supset K_m) = \{ Ax_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m - 1 \} \cup \{ x_{i,j}x_{i+1,j+1}; 1 \leq i \leq n - 1; 1 \leq j \leq m - 2 \} \cup \{ A_iy_{i,j}; 1 \leq i \leq n - 1; 1 \leq j \leq m - 1 \} \).

The order and size of \( (S_n \supset K_m), n \geq 3, m \geq 4 \) are \( |V(S_n \supset K_m)| = nm - n + 1 \) and \( |E(S_n \supset K_m)| = (n)(\frac{m(m-1)}{2}) \).
First we analysis the lower bound. We claim the lower bound is $\gamma_L(S_n \triangleright K_m) \geq nm - 2n$. To show that is the lower bound, we prove by contradiction. Assume $\gamma_L(P_n \triangleright K_m) < nm - 2n$, we choose locating dominating set of $(S_n \triangleright K_m)$ namely $D = \{x_{i,j}; 2 \leq i \leq n-1; 1 \leq j \leq m-1\} \cup \{x_{1,j}; 1 \leq j \leq m-2\}$ with cardinality of $D$ is $|D| = nm - 2n - 1$. The vertex set without locating dominating set of $(S_n \triangleright K_m)$ is $V - D = \{A\} \cup \{x_{i,j}; 2 \leq i \leq n; j = m - 1\} \cup \{x_{1,j}; 1 \leq j \leq m - 2 \leq j \leq m - 1\}$. The intersection between vertex set $\forall v \in (V - D)$ and $D$ are as follows.

$$
N(A) \cap D = \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m - 2\}
$$

$$
N(x_{i,j}) \cap D = \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m - 2\}
$$

$$
N(x_{1,j}) \cap D = \{x_{1,j}; 1 \leq j \leq m - 2\}
$$

For the vertices $x_{1,m-1}, x_{1,m-2} \in (V - D)$, it can be seen that $N(x_{1,m-1}) \cap D = N(x_{1,m-2}) \cap D$, and $D$ does not comply the properties of locating dominating set. So, we can conclude that $\gamma_L(S_n \triangleright K_m) < nm - 2n$ is a contradiction. Hence, the location domination number of $(S_n \triangleright K_m)$ is $\gamma_L(S_n \triangleright K_m) \geq nm - 2n$.

Furthermore, we show that $\gamma_L(S_n \triangleright K_m) \leq nm - 2n$, by choosing locating dominating set of $(S_n \triangleright K_m)$ is $D = \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m - 2\}$ with $|D| = nm - 2n$. The vertex set without locating dominating set of $(S_n \triangleright K_m)$ is $V - D = \{A\} \cup \{x_{i,j}; 1 \leq i \leq n; j = m - 2\}$. Intersection between vertex set $\forall v \in (V - D)$ and $D$ are as follows.

$$
N(A) \cap D = \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m - 3\}
$$

$$
N(x_{i,j}) \cap D = \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m - 3\}
$$

For the vertices $x_A, x_{i,j} \in (V - D)$ can be seen that $N(x_A) \cap D \neq \emptyset$ and $N(x_{i,j}) \cap D \neq \emptyset$, $D$ is locating dominating set of $(S_n \triangleright K_m)$ because $D$ satisfied the definition 1. In the otherhand $N(x_A) \cap D \neq N(x_{i,j}) \cap D$, it conclude that $D$ satisfied to be locating dominating set, so $D$ satisfied the definition 2. Hence the location domination number of $(S_n \triangleright K_m)$ is $\gamma_L(S_n \triangleright K_m) \leq nm - 2n$. Because $\gamma_L(S_n \triangleright K_m) \geq nm - 2n$ and $\gamma_L(S_n \triangleright K_m) \leq nm - 2n$, then we can say that $\gamma_L(S_n \triangleright K_m) = nm - 2n$. 

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**Figure 2.** Edge Comb Product $S_4 \triangleright K_5$

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**Theorem 2.2.** Given that a comb product graph $S_n \triangleright S_m$. Suppose $Ax_i$ is a grafting edge of $S_m$. Then the locating domination number of $\gamma_L(S_n \triangleright S_m) = nm - n$.

**Proof.** An edge comb product graph $S_n \triangleright S_m$ $n \geq 3$ and $m \geq 3$, is a connected graph with vertex set $V(S_n \triangleright S_m) = \{A\} \cup \{x_i; 1 \leq i \leq n\} \cup \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m - 1\}$ and edge set $E(S_n \triangleright S_m) = \{Ax_i; 1 \leq i \leq n\} \cup \{x_{i}; 1 \leq i \leq n; 1 \leq j \leq m - 1\} \cup \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m - 1\}$. The order and size of $(S_n \triangleright S_m), n \geq 3, m \geq 3$ are $|V(S_n \triangleright S_m)| = nm + 1$ and $|E(S_n \triangleright S_m)| = nm$.

First we analysis the lower bound. We claim the lower bound is $\gamma_L(S_n \triangleright S_m) \leq nm - n$. To show that is the lower bound, we prove by contradiction. Assume $\gamma_L(S_n \triangleright S_m) < nm - n$, we
choose locating dominating set of \( (S_n \supseteq K_m) \) namely \( D = \{x_{i,j}; 2 \leq i \leq n-1; 1 \leq j < m-1\} \cup \{x_{1,j}; 1 \leq j \leq m-2\} \) with cardinality of \( D \) is \(|D| = nm - 2n - 1\). The vertex set without locating dominating set of \( S_n \supseteq S_m \) namely \( D = \{x_i; 1 \leq i \leq n-1\} \cup \{x_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m-2\} \) with cardinality of \( D \) is \(|D| = nm - n - 1\). The vertex set without locating dominating set of \( S_n \supseteq S_m \) is \( V - D = \{A\} \cup \{x_n\} \cup \{x_{i,j}; 1 \leq i \leq n; j = m - 2\} \) Intersection between vertex set \( \forall v \in (V - D) \) and \( D \) are as follows.

\[
N(A) \cap D = \{x_i; 1 \leq i \leq n - 1\} \\
N(x_{i,j}) \cap D = \{x_i; 1 \leq i \leq n - 1\} \\
N(x_n) \cap D = \emptyset
\]

For the vertices \( x_n \in (V - D) \), it can be seen that \( N(x_n) \cap D = \emptyset \), and \( D \) not satisfied properties of locating dominating set. So, we can conclude that \( \gamma_L(S_n \supseteq S_m) < nm - n \) is a contradiction. Hence, the location domination number of \( (S_m \supseteq S_m) \) is \( \gamma_L(S_n \supseteq S_m) \geq nm - n \).

Furthermore, we show that \( \gamma_L(S_n \supseteq S_m) \leq nm - n \), by choosing locating dominating set of \( (S_m \supseteq S_m) \) is \( D = \{x_i; 1 \leq i \leq n\} \cup \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m - 2\} \) with \(|D| = nm - n\). The vertex set without locating dominating set of \( (S_m \supseteq S_m) \) is \( V - D = \{A\} \cup \{x_{i,m-1}; 1 \leq i \leq n\} \) Intersection between vertex set \( \forall v \in (V - D) \) and \( D \) are as follows.

\[
N(A) \cap D = \{x_i; 1 \leq i \leq n\} \\
N(x_{i,m-1}) \cap D = \{x_i; 1 \leq i \leq n\}
\]

For the vertices \( A, x_{i,m-1} \in (V - D) \) can be seen that \( N(A) \cap D \neq \emptyset \) and \( N(x_{i,m-1}) \cap D \neq \emptyset \), \( D \) is locating dominating set of \( S_n \supseteq S_m \) because \( D \) satisfied the definition 1. In the otherhand \( N(A) \cap D \neq N(x_{i,m-1}) \cap D \) it conclude that \( D \) satisfied to be locating dominating set, so \( D \) satisfied the definition 2. Hence the location domination number of \( S_n \supseteq S_m \) is \( \gamma_L(S_n \supseteq S_m) \leq nm - n \). Because \( \gamma_L(S_n \supseteq S_m) \geq nm - n \) and \( \gamma_L(S_n \supseteq S_m) \leq nm - n \), then we can say that \( \gamma_L(S_n \supseteq S_m) = nm - n \).

\[
\text{Figure 3. Edge Comb Product } S_4 \supseteq S_4
\]

**Theorem 2.3.** Given that a comb product graph \( S_n \supseteq B_{t_m} \) and \( AB \) is a graft edge of \( B_{t_m} \), then location domination number of \( \gamma_L(S_n \supseteq B_{t_m}) = nm \).

**Proof.** An edge comb product graph \( S_n \supseteq B_{t_m} \), \( n \geq 3 \) and \( m \geq 3 \), is a connected graph with vertex set \( V(S_n \supseteq B_{t_m}) = \{A\} \cup \{x_i; 1 \leq i \leq n\} \cup \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m\} \) and edge set \( E(S_n \supseteq B_{t_m}) = \{Ax_i; 1 \leq i \leq n\} \cup \{Ax_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m\} \cup \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m\} \). The order and size of \( (S_n \supseteq B_{t_m}) \), \( n \geq 3, m \geq 3 \) are \(|V(S_n \supseteq B_{t_m})| = n + nm + 1\) and \(|E(S_n \supseteq B_{t_m})| = n + 2nm\).
First we analysis the lower bound. We claim the lower bound is $\gamma_L(S_n \supseteq Bt_m) \geq nm$. To show that is the lower bound, we prove by contradiction. Assume $\gamma_L(S_n \supseteq Bt_m) < nm$, we choose locating dominating set of $S_n \supseteq Bt_m$ namely $D = \{x_{i,j}; 2 \leq i \leq n - 1; 1 \leq j \leq m\} \cup \{x_{1,j}; 2 \leq m\}$, with cardinality of $D$ is $|D| = nm - 1$. The vertex set without locating dominating set of $S_n \supseteq Bt_m$ is $V - D = \{A\} \cup \{x_{1,1}\}$. Intersection between vertex set $\forall v \in (V - D)$ and $D$ are as follows.

$$N(A) \cap D = \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m\}$$
$$N(x_{1,1}) \cap D = \emptyset$$

For the vertices $x_{1,1} \in (V - D)$, it can be seen that $N(x_{1,1}) \cap D = \emptyset$, and $D$ does not comply the properties of locating dominating set. So, we can conclude that $\gamma_L(S_n \supseteq Bt_m) < nm$ is a contradiction. Hence, the location domination number of $(S_n \supseteq Bt_m)$ is $\gamma_L(S_n \supseteq Bt_m) \geq nm$.

Furthermore, we show that $\gamma_L(S_n \supseteq Bt_m) \leq nm$, by choosing locating dominating set of $(S_n \supseteq Bt_m)$ is $D = \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m\}$ with $|D| = nm$. The vertex set without locating dominating set of $(S_n \supseteq Bt_m)$ is $V - D = \{A\} \cup \{x_{i,1}; 1 \leq i \leq n\}$. Intersection between vertex set $\forall v \in (V - D)$ and $D$ are as follows.

$$N(A) \cap D = \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m\}$$
$$N(x_{i,1}) \cap D = \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m\}$$

For the vertices $A, x(i) \in (V - D)$ can be seen that $N(A) \cap D \neq \emptyset$ and $N(x_{i}) \cap D \neq \emptyset$, $D$ is locating dominating set of $S_n \supseteq Bt_m$ because $D$ satisfied the definition 1. In the otherhand $N(A) \cap D \neq N(x_{i}) \cap D$ it conclude that $D$ satisfied to be locating dominating set, so $D$ satisfied the definition 2. Hence the location domination number of $S_n \supseteq Bt_m$ is $\gamma_L(S_n \supseteq Bt_m) \leq nm$. Because $\gamma_L(S_n \supseteq Bt_m) \geq nm$ and $\gamma_L(S_n \supseteq Bt_m) \leq nm$, then we can say that $\gamma_L(S_n \supseteq Bt_m) = nm$. $\square$

![Figure 4. Edge Comb Product $S_4 \supseteq Bt_3$](image)

**Theorem 2.4.** Given that a comb product graph $S_n \supseteq P_m$ and $x_1x_2$ is a grafting edge of $P_m$. Then locating domination number of $\gamma_L(S_n \supseteq P_m) = n(\lfloor \frac{2m}{5} \rfloor)$.

**Proof.** An edge comb product graph $S_n \supseteq P_m$, $n \geq 3$ and $m \geq 5$, is a connected graph with vertex set $V(S_n \supseteq P_m) = \{A\} \cup \{x_{i,j}; 1 \leq i \leq n; 1 \leq j \leq m - 1\}$, and edge set $E(S_n \supseteq P_m) = \{Ax_{i,j}; 1 \leq i \leq n; j = 1\} \cup \{x_{i,j}x_{i,j+1}; 1 \leq i \leq n; 1 \leq j \leq m - 2\}$. The order and size of $(S_n \supseteq P_m)$, $n \geq 3, m \geq 5$ are $|V(S_n \supseteq P_m)| = nm - n + 1$ and $|E(S_n \supseteq P_m)| = nm - n$.

First we analysis the lower bound. We claim the lower bound is $\gamma_L(S_n \supseteq P_m) \geq n(\lfloor \frac{2m}{5} \rfloor)$. To show that is the lower bound, we prove by contradiction. Assume $\gamma_L(S_n \supseteq P_m) < n(\lfloor \frac{2m}{5} \rfloor)$, we choose locating dominating set of $S_n \supseteq P_m$ namely $D = \{x_{i,j}; 1 \leq i \leq n - 1; j \equiv 2 + 0 \text{ mod}\}$.
\[ \{x_n, j; j \geq 3; j \equiv 2 + 0 \mod 2\} \] with cardinality of \( D \) is \(|D| = n\left(\frac{2m}{5}\right) - 1\). The vertex set without locating dominating set of \( S_n \geq P_m \) is \( V - D = \{A\} \cup \{x_{i,j}; 1 \leq i \leq n; j \equiv 1 \mod 2\} \cup \{x_{n,j}; 1 \leq j \leq 2\} \) Intersection between vertex set \( \forall v \in (V - D) \) and \( D \) are as follows.

\[
\begin{align*}
N(A) \cap D &= \{x_{i,1}; 1 \leq i \leq n - 1\} \\
N(x_{i,j} \cap D) &= x_{i,j}; 1 \leq i \leq n; j \equiv 2 + 0 \mod 2 \\
N(x_{n,j} \cap D) &= \emptyset
\end{align*}
\]

For the vertices \( x_{n,j} \in (V - D) \), it can be seen that \( N(x_{n,j}) \cap D = \emptyset \), and \( D \) does not comply the properties of locating dominating set. So, we can conclude that \( \gamma_L(S_n \geq P_m) < n\left(\frac{2m}{5}\right) \) is a contradiction. Hence, the locating dominating number of \( (S_n \geq P_m) \) is \( \gamma_L(S_n \geq P_m) \geq n\left(\frac{2m}{5}\right) \).

Furthermore, we show that \( \gamma_L(S_n \geq P_m) \leq n\left(\frac{2m}{5}\right) \), by choosing locating dominating set of \( (S_n \geq P_m) \) is \( D = \{x_{i,j}; 1 \leq i \leq n; j \equiv 2 + 0 \mod 2\} \) with \(|D| = n\left(\frac{2m}{5}\right) \). The vertex set without locating dominating set of \( (S_n \geq P_m) \) is \( V - D = \{A\} \cup \{x_{i,j}; 1 \leq i \leq n; j \equiv 1 \mod 2\} \). Intersection between vertex set \( \forall v \in (V - D) \) and \( D \) are as follows.

\[
\begin{align*}
N(A) \cap D &= \{x_{i,1}; 1 \leq i \leq n\} \\
N(x_{i,j} \cap D) &= \{x_{i,j}; 1 \leq i \leq n; j \equiv 2 + 0 \mod 2\}
\end{align*}
\]

For the vertices \( x_A, x_{i,j} \in (V - D) \) can be seen that \( N(x_A) \cap D \neq \emptyset \) and \( N(x_{i,j}) \cap D \neq \emptyset \), \( D \) is locating dominating set of \( S_n \geq P_m \) because \( D \) satisfied the definition 1. In the otherhand \( N(x_A) \cap D \neq N(x_{i,j}) \cap D \) satisfied to be locating dominating set, so \( D \) satisfied the definition 2. Hence the location domination number of \( S_n \geq P_m \) is \( \gamma_L(S_n \geq P_m) \leq n\left(\frac{2m}{5}\right) \). Because \( \gamma_L(S_n \geq P_m) \geq n\left(\frac{2m}{5}\right) \) and \( \gamma_L(S_n \geq P_m) \leq n\left(\frac{2m}{5}\right) \), then we can say that \( \gamma_L(S_n \geq P_m) = n\left(\frac{2m}{5}\right) \).

Figure 5. Edge Comb Product \( S_8 \geq P_4 \).
3. Concluding Remarks
In this paper, we have obtained the exact values of locating dominating number of some edge comb product graph, namely $S_n \cong H$. In this paper, we studied $H$ as complete graph $K_m$, star graph $S_m$, triangular book $Bt_m$ and path graph $P_n$. We have found the exact values of their locating dominating number. However for $H$ is any graph we have found any result yet. Therefore we proposed the following open problem.

Open Problem 3.1. Determine the sharp lower bound or upper bound of locating dominating number of $S_n \cong H$ for $H$ is any graph.

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References
[1] A. Finbow, B.L. Hartnell 1988 On locating dominating sets and well-covered graphs, Congr. Numer. 65 191-200.
[2] C.J. Colbourn, P.J. Slater, L.K. Stewart 1987 Locating-dominating sets in series-parallel networks, Congr. Numer. 56 135-162.
[3] Chartrand G and Lesniak L 2000 Graphs and digraphs 3rd ed (London: Chapman and Hall).
[4] Foucaud, F., Henning, M. A., Lawenstein, C. and Sasse, T 2016 Locating dominating sets in twin-free graphs, Discrete Applied Mathematics. 200 52-58.
[5] P.J. Slater 1987 Dominating and location in acyclic graphs, Networks 17 55-64.
[6] P.J. Slater 1988 Dominating and reference sets in graphs, J. Math. Phys. Sci. 22 455.
[7] Saputro, S. W. 2013. The Metric Dimension of Comb Product Graphs. Graph Theory Conference in Honor of Egawas 60th Birthday. 1-2.
[8] Wardani, D. A. R., Dafik, A. C. Prihandoko, and A. I. Kristiana 2016 On The Total r-Dynamic Coloring of Graph: A New Graph Coloring Study. International Conference on Mathematics: Education, Theory and Application. ICMETA. Surakarta.