Discrete model of hybrid stepper motor by optimal state space control

P V Lekomtsev

Department of Mechatronic Systems, Kalashnikov Izhevsk State Technical University, Studencheskaya 42, Izhevsk, Russia

lekomtsev@istu.ru

Abstract. The article provides an overview of modern control methods of a stepper motor. Continuous and discrete models of a hybrid stepper motor in the state space have been developed. At each step of the control algorithm, the required torque is set in the state matrix and the values of the optimal voltage in the dq frame used in field oriented control are calculated.

1. Introduction
The electric drive based on stepper motors (SM) is currently widely used in CNC machines, 3d printers, robots and other industrial automation systems. Their widespread use, primarily in systems of positional electric drive of robots, is due to the distinctive features of the SM itself as an electromechanical energy converter, such as the possibility of working out small discrete movements, the absence of a brush-collector assembly, as well as the development of the element base and the capabilities of modern microprocessor program management systems [1–6]. Recently, field oriented control has been used for effective control of SM [7–15]. This approach overcomes the most important drawbacks of SM - such as increased vibration, the existence of resonance zones in the operating speed range, the low dynamic characteristics, the presence of pronounced detent moment at each step, a high operating temperature of the actuator and the low efficiency, low positioning accuracy.

In synchronous and asynchronous motors field oriented control has been used for a long time. The mathematical basis of this control strategy is differential equations that describe an electric motor equally correctly in different operating modes. Field oriented control makes it possible to build highly dynamic and precision electric drives.

The digital closed-loop principle was introduced in SM in 1968 and 1974 by Fredriksen [3] and Kuo [4] to increase positioning accuracy and reduce the sensitivity of the engine to load noise. SM used in machines and robotic manipulators are increasingly controlled by closed loop.

Classical feedback algorithms based on PID controllers are inadequate, since these algorithms are sensitive to mechanical configuration changes [5, 6]. Classical methods that use linear models to design control system of a stepper motor can only be used for small changes around the operating point. This problem can be solved by applying modern feedback control methods, such as self-tuning regulation (STR) [7]. This problem can be solved by applying the modern control techniques with feedback, such as self-tuning control, the controller on the basis of the internal model [8] or nonlinear feedback control [9], where in the controller is forced to adapt to the conditions of engine operation.
When applied to SM, STR provides better performance than the PID controller. However, this type of control is difficult to implement due to the high computational complexity.

Crnosija et al. implemented an optimal feedback control algorithm of SM [10]. Defoort et al. [11] and Shah et al. [12] proposed third-order sliding mode controller (SMC) for SM. A control was proposed based on the Lyapunov function with a nonlinear observer in the microstep for a permanent magnet SM [13]. The application of the method of conditional servo-compensators to control the position of a permanent magnet SM was presented in [14].

2. Hybrid stepper motors model in the state space

A hybrid stepper motor (HSM) model has been developed in which electrical quantities are represented in the \(dq\) frame which rotates at a speed \(\omega_s = p \omega\), where \(\omega\) is the rotor speed, \(p\) is the number of motor pole pairs. The field oriented principle of stepper motor control is based on independent regulation of \(i_q\) and \(i_d\) currents. The model of HSM in the \(dq\) frame is obtained, given by the following equations:

\[
\frac{d}{dt} i_d = \frac{u_d}{L} - \frac{R}{L} i_d + p \omega i_q ,
\]

\[
\frac{d}{dt} i_q = \frac{u_q}{L} - \frac{R}{L} i_q - \frac{K_m}{L} \omega - p \omega i_d ,
\]

\[
\frac{d}{dt} \omega = \frac{K_m i_q - T_{dm} \sin(2p \theta) - F \omega - T_L}{J} ,
\]

\[
\frac{d}{dt} \theta = \omega ,
\]

where \(u_d, u_q\) – voltage in the \(dq\) frame; \(i_d, i_q\) – stator currents in the \(dq\) frame; \(\omega\) – the angular velocity of the rotor; \(\theta\) – angular position of the motor shaft; \(R\) – stator winding resistance; \(L\) – stator winding inductance; \(p\) – number of motor pole pairs; \(F\) – viscous friction coefficient; \(T_L\) – load torque on the shaft; \(T_{dm}\) – maximum detent torque ; \(K_m\) – motor constant; \(J\) – inertia momentum of the motor and the load.

Vector-matrix model of HSM in the state space represented in the classical form

\[ \dot{x} = Ax + Bu , \]

where \(A\) – state matrix, \(B\) – control matrix, \(x\) – state vector, \(u\) – control vector.

Since the drive controller should provide adjustment torque, rotational speed and angular displacement, as the generalized coordinates selected electrical currents, rotor speed \(\omega\), the angular displacement \(\theta\). The control is the voltage \(u_d, u_q\), the disturbance is the load torque on the shaft. Having solved the initial system (1-4) with respect to the first derivatives, the HSM equation in the state space is obtained. A nonlinearity associated with a variable load torque was added to the classical model of HSM with constant parameters.

We represent the vector-matrix model of HSM in the state space in the form:

\[
\dot{x} = \begin{bmatrix} i_d \\ i_q \\ \omega \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & p \omega & 0 & 0 \\ -p \omega & -\frac{R}{L} & -\frac{K_m}{L} & 0 \\ 0 & \frac{K_m}{L} & -\frac{F}{J} - \frac{T_L}{J} & -\frac{T_{dm} \sin(2p \theta)}{J} \\ 0 & \frac{F}{J} & \frac{1}{J} & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ \omega \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} u_d \\ u_q \end{bmatrix} .
\]

In general terms, when at least one of the matrix \(A, B\) is time-dependent, the problem is non-linear and has only particular solutions.
3. Discrete HSM model in the state space
To find the equation of state represent (6) in a discrete form, with the sampling time \( T \) to zero and trajectory at each discrete time intervals is linear.

We write the solution for the nonlinear problem in a discrete form, when the matrices \( A \) and \( B \) are constant at time moments \( k, k=0,1,2,3, \ldots \)

\[
\frac{x_{k+1} - x_k}{T} = A_k x_k + B_k u_k, \tag{7}
\]

or

\[
x_{k+1} = \tilde{A}_k x_k + \tilde{B}_k u_k, \tag{8}
\]

where \( \tilde{A}_k = TA + E, \tilde{B}_k = TB_k \).

This equation relates the system transition from state \( x_k \) to state \( x_{k+1} \). On the time interval \( T \), we take the values of the matrices \( A_k \) and \( B_k \) to be constant. For convenience in the subsequent entries will remove the sign "wavy line".

We write the vector-matrix model of HSM (6) in discrete form:

\[
\left[ \begin{array}{c}
i_d(k+1) \\
i_q(k+1) \\
\omega(k+1) \\
\theta(k+1)
\end{array} \right] = \left[ \begin{array}{cccc}
1 - \frac{T}{L} & Tp_o(k) & 0 & 0 \\
Tpo(k) & 1 - \frac{T}{L} & -\frac{T}{L} & 0 \\
0 & \frac{T}{J} & 1 - \frac{T}{J} & \frac{T}{J} \sin(2\rho \theta(k)) \\
0 & 0 & \frac{T}{J} & 1
\end{array} \right] \left[ \begin{array}{c}
i_d(k) \\
i_q(k) \\
\omega(k) \\
\theta(k)
\end{array} \right] + \left[ \begin{array}{c}
T \\
0 \\
0 \\
0
\end{array} \right] \left[ \begin{array}{c}
u_d(k) \\
u_q(k) \\
\rho(k) \\
\theta(k)
\end{array} \right], \tag{9}
\]

where \( i_d(k), i_q(k), \omega(k), \theta(k) \) – measured values of currents, angular velocity and displacement, respectively;

\( \omega(k+1) \) – planned value of angular velocity;

\( \theta(k+1) \) – planned value of the angular position of the motor shaft.

It is proposed to calculate the load torque by the formula [16-18]:

\[
T_L(k) = \frac{\omega(k+1) - \omega(k)}{T}, \tag{10}
\]

\[
\omega(k+1) = \frac{\theta(k+1) - \theta(k)}{T}. \tag{11}
\]

The simulation of the stepper motor was performed under the condition that there were no measurement errors, which depend on the accuracy class of the sensors.

4. HSM model by optimal control
To implement the optimal control of the HSM, a quadratic quality functional was used that determines the control and displacement energy, which is expressed as follows

\[
I = \frac{1}{2} \int_{t_0}^{t_f} (x^T Q x + u^T G u) dt, \quad Q \succeq 0, \; G > 0, \tag{12}
\]

where \( Q \) and \( G \) – positive-definite matrices.

The solution for the quality criterion (12), minimizing the energy of control and displacement, is determined by the following expression [19]:

\[
u = -\left( G + B^T K B \right)^{-1} B^T K A x, \tag{13}\]

where \( K \) – the Cauchy matrix found by solving the Riccati equation [19]:

\[
-K = Q + A^T K + K^T A - K^T B G^{-1} B^T K, \quad K(t_f) = 0.
\]

The FL86ST94-560A hybrid stepper motor was simulated in the MatLab program to obtain the optimal voltage taking into account (13). The simulation results at angular velocity \( \omega=5 \text{ rad/s} \) are presented in figures 1,2.
Figure 1. Applied to the HSM voltage represented: a – in the rotational (fixed to the rotor) $dq$ frame; b – in the (fixed to the stator) $ab$ frame.

Figure 2. Motor currents: in the rotational (fixed to the rotor) $dq$ frame; b – in the (fixed to the stator) $ab$ frame.

The simulation results shown in Figures 1 and 2 show that for smooth rotation of the HSM shaft, it is necessary to change the phase currents and voltages in the moving coordinate system within the range of movement near each pole.

5. **Conclusion**
This paper provides an overview of modern stepper motor control methods. It has been revealed that in practice is necessary to control the stepper motor in variable load devices with the requirement of smooth rotation, which is achieved by using feedback control algorithms. Moreover, impulse stepping motor control algorithms require increased energy consumption, and the power control characteristic is nonlinear.

The article presents the optimal discrete model of a hybrid stepper motor in the state space. A significant difference of this model is its adaptation to a variable load torque, which minimizes control energy.

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