The Effect of Radiative Feedback on Bondi–Hoyle Flow around a Massive Star

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ABSTRACT

We apply an algorithm for radiative feedback on a dusty flow (detailed in Edgar & Clarke (2003)) to the problem of Bondi–Hoyle accretion. This calculation is potentially relevant to the formation of massive stars in ultradense cores of stellar clusters. We find that radiative feedback is more effective than in the case of previous calculations in spherical symmetry. The Bondi-Hoyle geometry implies that material is flowing nearly tangentially when it experiences the sharp radiative impulse at the dust destruction radius, and consequently it is readily perturbed into outflowing orbits. We find that it is difficult for stellar masses to grow beyond around 10 M\(_\odot\) (for standard interstellar dust abundances). We discuss the possible implications of this result for the formation mechanism of OB stars in cluster cores. We end by proposing a series of conditions which must be fulfilled if Bondi–Hoyle accretion is to continue.

Key words: stars: formation – stars: early-type – hydrodynamics – methods: numerical – radiative transfer

1 INTRODUCTION

Massive stars are immensely important astrophysical objects, far more so than their rarity would imply. They are incredibly bright, being visible at cosmological distances. They manufacture most of the metals in the Universe, both during their brief lives, and in the supernovae which mark their deaths. Massive stars also exert large forces on their surroundings, through winds, ionisation and radiation pressure.

Despite this importance, massive star formation is a poorly understood process. Observational studies are hampered by the distance to massive star forming regions, and the high degree of obscuration in such regions. From a theoretical point of view, the very existence of massive stars presents a challenge. Early studies (in spherical symmetry) suggested that radiation pressure on dust should prevent the assembly of such stars unless the opacity of the dust is arbitrarily reduced by an order of magnitude compared with interstellar values (Wolff & Cassinelli 1986, 1987). More recently, simulations under conditions of axisymmetry have shown that accretion via a disc can ease the problem somewhat (Yorke & Sonnhalter 2002). However, other authors have continued to seek other possible formation mechanisms for massive stars. One suggestion (postulated by Bonnell et al. (1998) and developed as Bonnell & Bate (2002)) is that massive stars are formed in the ultra-dense cores of young stellar clusters. Such cores are associated with OB star formation (Blum et al. 2002), although observed cores do not reach the densities required by Bonnell et al. In the merger models of Bonnell et al., continued accretion onto the stars drives an adiabatic contraction of the cluster core to the high densities required. Bonnell et al. (2001) used hydrodynamical simulations of cluster formation to show that accretion onto the stars in the core is reasonably well described by Bondi–Hoyle accretion (i.e. the accretion cross section is appropriate to a compact object moving supersonically through a gaseous medium). However, such simulations include no feedback, and thus cannot address the question of whether massive stars can continue to accrete by this mechanism in an ultra-dense cluster core.

The interaction of outflowing radiation with a dusty accretion flow may be envisaged in the following terms: Dust sublimes at a radius in the flow, \(r_d\), where its temperature attains a value of 1700 K. Inward of this radius, there is no significant momentum exchange between the radiation field and the gas, until very small radii, where the gas become ionised. Radiation that impacts the dusty flow at \(r_d\) is absorbed within a very narrow region, since the Rosseland mean opacity of the dust at the stellar radiation temperature (\(\sim 30000 \) K) is very high. The absorption of stellar luminosity \(L\) in this region is associated with momentum transfer to the gas at a rate \(L/c\) and the inflow suffers a sharp deceleration at this point. This absorbed radiation is subsequently thermalised, i.e. re-emitted at a temperature equal to the local dust temperature (\(\sim 1700 \) K). The rate of...
momentum transfer to the flow that is associated with this thermalised field is $\sim \tau_0 L/c$, where $\tau_0$ is the optical depth of the flow to the thermalised radiation.

In this paper, we explore the issue of radiative feedback by radiation pressure on dust in the Bondi–Hoyle geometry. In Bondi–Hoyle accretion of isothermal gas, the flow is gravitationally focused behind the massive object (star) and forms a shock in the downstream direction. For gas entering with an impact parameter less than a critical value (the Bondi–Hoyle radius), the loss of tangential momentum renders the shocked gas bound to the star and enables it to be accreted by way of radial infall along the shock. Such accretion column geometry evidently has different implications for the interaction between the mass flow and radiation field than either of the geometries in which radiative feedback has been simulated to date (i.e. spherical or disc accretion). Our study will be a first look at how radiative feedback on dust operates in Bondi–Hoyle geometry. We will also be mindful that in the environment in which such accretion may be important, it is not only the geometry that is different from conventional studies but also the physical parameters. In particular, the hydrodynamical simulations suggest that the mean gas densities in the hypothesised ultra-dense core phase is extremely high ($10^9$–$10^{10}$ cm$^{-3}$) and we will find that this is an important factor in determining the efficacy of radiative feedback.

In the simulations described below, we employ an algorithm for simulating the effect of radiative feedback on a dusty flow that we developed in a previous paper (Edgar & Clarke 2003). The aim of this algorithm is to preserve the sharp deceleration at the point where the dust sublimes, while avoiding the computational cost of a full frequency dependent calculation. Wolfire & Cassinelli (1987) showed that a frequency averaged (grey) approach missed this sharp impulse (which corresponds to $L/c$ worth of momentum). The reader should refer to Edgar & Clarke (2003) for a full description (and test) of our method, but the general algorithm is as follows:

- The radiation field is split into direct and thermalised components
- The direct field is attenuated using wavelength dependent opacities
- The balance of the energy is placed into the thermalised field, which is then solved using the diffusion approximation

The initial location of the dust destruction front is estimated based on the temperature of a bare grain in the stellar radiation field, but the algorithm iterates if the thermalised field implies the dust melts further from the star. The mechanical effect of anisotropic scattering is included in the radiation pressure opacity, but the scattered radiation is not followed further. In this paper, we solve the diffusion approximation along each radius under the assumption of spherical symmetry. This was done for reasons of simplicity, although it should be possible to relax this approximation in future work. Spherical symmetry is obviously not a good approximation for the accretion column, but we shall show that the flow is disrupted in a region where the deviations from spherical symmetry are not large. This gives us some confidence that our conclusions are not merely an artifact of our algorithm.

The structure of this paper is as follows: In section 2, we present a brief mathematical analysis of Bondi–Hoyle flow. Section 3 details our search for suitable simulation parameters. The results from our hydrodynamic simulations follow in section 4. We discuss our results and present out conclusions in sections 5 and 6 respectively.

2 BONDI–HOYLE ACCRETION: ANALYTIC APPROACH

Wolfire & Cassinelli (1987) considered accretion by a star (mass $M$) moving at a steady speed ($v_\infty$) through an infinite gas cloud. The gravity of the star focuses the flow into a wake which it then accretes. The geometry is sketched in figure 1.

By considering ballistic orbits, Wolfire & Cassinelli concluded that all material with an impact parameter satisfying

$$\zeta < \zeta_{HL} = \frac{2GM}{v_\infty^2}$$  \hspace{1cm} (1)

would be accreted. If the cloud has density $\rho_\infty$, the mass flux is

$$\dot{M}_{HL} = \pi \zeta^2 \zeta_{HL} v_\infty \rho_\infty = \frac{4\pi G^2 M^2 \rho_\infty}{v_\infty^2}$$ \hspace{1cm} (2)

which is known as the Hoyle–Lyttleton accretion rate. By considering the stability of the accretion column (the wake following the point mass on the $\theta = 0$ axis), Bondi & Hoyle (1944) concluded that the true accretion rate could be as little as half this value.

Bondi (1952) studied spherically symmetric accretion onto a point mass. The analysis shows (see e.g. Frank et al. 2002) that a Bondi radius may be defined as

$$r_B = \frac{2GM}{c_{SB}}$$ \hspace{1cm} (3)

where $c_{SB}$ is the sound speed at $r_B$. Flow outside this radius is subsonic, and the density is almost uniform. Within it, the gas becomes supersonic and moves towards a freefall solution. The similarities between equations 1 and 3 led Bondi to propose an interpolation formula:

$$\dot{M} = \frac{2\pi G^2 M^2 \rho_\infty}{(c_{SB}^2 + v_\infty^2)^{3/2}}$$ \hspace{1cm} (4)

This is known as the Bondi–Hoyle accretion rate. The corresponding $\zeta_{BH}$ is formed by analogy with equation 1. On the basis of their numerical calculations, Shima et al. (1983) suggest that equation 4 should acquire an extra factor of two, to become

$$\dot{M}_{BH} = \frac{4\pi G^2 M^2 \rho_\infty}{(c_{SB}^2 + v_\infty^2)^{3/2}}$$ \hspace{1cm} (5)
This pair of equations bears a suspicious resemblance to the counters the dust destruction front, and changes density and will be unchanged. The flow is incident from the left, endial, since the radiative impulse is radial. The \( \theta \) velocity. Assuming that the gas remains isothermal, we can where the sin values immediately inside approximation:

\[
v_r = \sqrt{v_\infty^2 + \frac{2GM}{r} - \frac{\zeta^2 v_\infty^2}{r^2}} \tag{6}
\]

\[
v_\theta = \frac{\zeta v_\infty}{r} \tag{7}
\]

\[
r = \frac{\zeta^2 v_\infty^2}{GM(1 + \cos \theta) + \zeta v_\infty^2 \sin \theta} \tag{8}
\]

\[
\rho = \frac{\rho_\infty \zeta^2}{r \sin \theta(2\zeta - r \sin \theta)} \tag{9}
\]

The first three equations are obtained from the Newtonian continuity equation using the previous three.

2.1 Simple Feedback

The simplest way of including radiative feedback is to place \( r/c \) worth of momentum into the flow at the dust destruction radius, \( r_d \). This misses the effect of the thermalised radiation field, but is a useful first approximation.

Consider the flow sketched in figure 2. The ‘d’ subscripts denote quantities immediately outside \( r_d \), ‘i’ subscripts denote values immediately inside \( r_d \). All the velocities are radial, since the radiative impulse is radial. The \( \theta \) velocities will be unchanged. The flow is incident from the left, encounters the dust destruction front, and changes density and velocity. Assuming that the gas remains isothermal, we can conserve mass and momentum to find

\[
\rho_d v_d = \rho_i v_i \tag{10}
\]

\[
2\pi \rho_d v_d^2 r_d \sin \theta d\theta - \frac{L}{2c} \sin \theta d\theta = 2\pi \rho_i v_i^2 r_i \sin \theta d\theta \tag{11}
\]

where the \( \sin \theta d\theta \) terms are taking care of solid angles. This pair of equations bears a suspicious resemblance to the Rankine–Hugoniot conditions for shocks. We can eliminate the value of \( \rho_i \) between these equations, to give

\[
v_i = v_d - \frac{L}{4\pi r_d^2 \rho_d v_d} \tag{12}
\]

At first sight, this appears to be independent of angle. However, it is not - \( v_d \) and \( \rho_d \) are somewhat complex functions of \( \theta \).

In simple physical terms, the flow will slow dramatically in the radial direction as it passes through \( r_d \). This will cause gas streamlines to move onto more circular orbits (since their angular momentum is unaffected). The gas streams could then flow around inside \( r_d \), and still end up on the accretion column trailing the star. Indeed, if \( r_d \) were larger than \( \zeta_{BH} \), then a small radiative impulse could increase the accretion rate. Material originating from large \( \zeta \) would be moved onto orbits closer to circular (since the radial impulse would not affect angular momentum), and hence be more likely to be bound to the star after encountering the accretion column. For a protostar in a protocluster, this would require \( v_\infty > 36 \text{ km s}^{-1} \), which is a little on the high side. The accretion rate (proportional to \( v_\infty^2 \) - cf equation 2) would also be rather low, unless the density were increased to compensate.

There will be a gas streamline which has its closest approach to the star at \( r_d \). Equation 12 is obviously not going to apply to this streamline, since it will have \( v_d = 0 \). If the angle, \( \theta_c \), at which this occurs is close enough to zero, this will not be a problem, since it will be lost in the wake. However, there will be a range of angles with \( \theta < \theta_c \) for which equation 12 will imply a negative value for \( v_i \). Physically, this corresponds to the flow being turned around at \( r_d \). While this is fine for particles, it will be disastrous for the ballistic approximation of the gas. Streamlines will cross, destroying the assumptions necessary to treat the gas using simple Newtonian theory.

Using equation 12 and the fact that the velocity in the \( \theta \) direction is unchanged, it is fairly straightforward to compute the trajectory followed by the gas inside the \( r_d \) (on the assumption of ballistic flow).

Figure 3 plots some sample trajectories for flow past a 10 M_☉ star. In this plot, \( \rho_\infty = 10^{-16} \text{ g cm}^{-3} \) and \( v_\infty = 5 \times 10^3 \text{ cm s}^{-1} \). We shall show in the next section that these parameters are appropriate for the ultra-dense cores postulated by Bonnell & Bate. To make the perturbations to the flow visible, we used an artificially large value for the luminosity of \( 10^6 \text{ L}_\odot \) (approximately twice the ZAMS value for a 10 M_☉ star). We placed the dust destruction radius at \( r_d = 5 \times 10^{14} \text{ cm} \), which is a reasonable value for these conditions.

Our simplified feedback model implies that radiation pressure on dust will give rise to orbit crossing (and hence the formation of a shock cone) in the downstream direction (i.e. for \( \theta < \theta_c \)). For a luminosity appropriate to a 10 M_☉ ZAMS star, \( \theta_c \approx 0.2 \text{ rad} \), which far exceeds the width of the unperturbed Bondi–Hoyle wake.

To proceed further, it is necessary to resort to full hydrodynamic calculations, both because these can follow the flow in the orbit crossing regime and because they can in-
clude additional feedback from the thermalised radiation field. This important effect, omitted in the above analysis, decelerates the flow before it encounters the dust destruction front. In addition, we can use hydrodynamical simulations to explore the case that the equation of state of the gas is not isothermal, since it is well known (e.g. Hunt (1971, 1973) and Ruffert (1994); Ruffert & Arnett (1994)) that Bondi–Hoyle flow is not well described by the equations of Bisnovatyi-Kogan et al in this case.

3 PARAMETERS

Simulating Bondi–Hoyle flow in the context of star formation is rather difficult, due to the large range in length scales which must be resolved. Although we are interested in stars rather more massive than the compact objects typically studied in Bondi–Hoyle simulations, the protostellar velocities in a cloud core are generally much lower as well - typically only a few km s$^{-1}$. This tends to make $\Omega_{\text{HL}}$ rather large (cf equation 1), and many times larger than the protostellar radius. We can reduce the problem, since we are not interested in following the flow right down to the protostar, but only to the dust destruction front, $r_d$. However, the ratio $\Omega_{\text{HL}}/r_d$ can still be painfully large. Another motivation for keeping $\Omega_{\text{HL}}$ small is self-gravity. If the mass enclosed in the simulation is comparable to that of the protostar, consistency requires that the flow is self-gravitating - which would require a great deal of computational effort. The H$\alpha$ region surrounding the protostar must also be kept away from the dust. If ionised gas reaches the dust, then it might be able to destroy the dust before the grains reach their nominal sublimation temperature.

To find suitable simulation parameters ($M$, $v_\infty$, $\rho_\infty$), we make use of the analytic formulae for pressure free Bondi–Hoyle flow derived by Bisnovatyi-Kogan et al (1972). We estimate the volume of ionised gas by balancing the number of ionisations and recombinations along each sight line, in a straightforward generalisation of the Strömgren Sphere (Strömgren 1939). To calculate the number of ionising photons, we make use of the ZAMS formulae of Tout et al (1996) and the Stefan–Boltzmann Law. We seek parameters which satisfy the following conditions:

- The predicted accretion rate will cause a significant change in mass on a timescale of a million years or so
- The volume of ionised gas must lie within $r_d$
- The ratio $r_d/\zeta_{\text{BH}}$ must not be too small

Note that the first two requirements are motivated by physical considerations. The third is a computational practicality. Unfortunately, these requirements tend to conflict with each other - a faster gas flow will help reduce $\zeta_{\text{BH}}$, but it will also reduce the central concentration of the fluid. This will let the ionised region grow.

Based on information supplied by Ian Bonnell (published as Bonnell & Bate (2002)) the reasonable ranges for the relevant parameters are given in Table 1. However, the upper end of each range is beginning to push limits to the extreme. In particular, there is little point seeking Bondi–Hoyle accretion solutions for a 100 $M_\odot$ star, unless we have already established that it can grow to this mass. Having said this, more massive stars make it easier to keep the ionisation problem to a minimum. This is because $r_d$ is an ‘area’ effect (cf the Stefan–Boltzmann Law) whereas ionisation is a ‘volume’ effect (cf the Strömgren Sphere).

From this, the parameters given in Table 2 squeeze through the selection criteria. These give an accretion rate of $3 \times 10^{-4} M_\odot$ yr$^{-1}$ and $\zeta_{\text{BH}} = 1.1 \times 10^{16}$ cm. The dust destruction front due to direct stellar radiation is located at $r_d = 2.1 \times 10^{14}$ cm. The ionised gas should always lie within $1.3 \times 10^{14}$ cm of the star, which is sufficiently inside $r_d$ for our purposes. This is helped by the following considerations:

- The value of $r_d$ is likely to increase, due to the thermalised radiation field
- The escape velocity at the edge of the H$\alpha$ region is on the order of forty kilometres per second, while the sound speed in the ionised gas is only a few kilometres per second

Therefore, ionisation is unlikely to be a problem at the start of the simulations. So long as accretion continues, the H$\alpha$ region is unlikely to grow. However, if the radiative feedback causes significant disruption to the flow, then falling densities within $r_d$ are likely to allow the ionised volume to grow, violating our assumptions.

4 DYNAMICAL SIMULATIONS

The Bondi–Hoyle solution does not occur for gaseous flow. Shocks and instabilities occur, requiring numerical solutions. To simulate the flow, we used the latest version (2.0.3) of the ZEUS2D code of Stone & Norman (1992) in spherical polar mode. Radiative feedback was included using the radiative transfer algorithm developed by Edgar & Clarke (2003), with the radiative feedback appearing as an extra source of momentum. In the present work, we solved the diffusion approximation (and hence $r_d$ as well) along each ray using the assumption of spherical symmetry. This should be a good approximation on the upstream side, but will be poor in the wake, where the density gradient will tend to transport radiation sideways. This would have the effect of expanding the wake, and reducing its ram pressure. Despite the computational efficiency of our new algorithm,  

| Parameter          | Range                     |
|--------------------|---------------------------|
| Gas Density        | $10^{-18}$ g cm$^{-3}$ < $\rho_\infty$ < $10^{-15}$ g cm$^{-3}$ |
| Gas Velocity       | $10^4$ cm s$^{-1}$ < $v_\infty$ < $10^5$ cm s$^{-1}$ |
| Stellar Mass       | $10 M_\odot$ < $M$ < $100 M_\odot$ |

Table 1. Range of Parameters for an ultra–dense core

Table 2. Chosen parameters for Bondi–Hoyle simulations

$1$ $r_d$ is calculated by computing the sublimation temperature of a dust grain in the stellar radiation field. See Edgar & Clarke (2003) for details

$2$ Note that this is still substantially smaller than the Hypercompact H$\alpha$ regions discussed by Kurtz (2002)
it is still expensive to solve on every timestep. Accordingly, the density structure along each ray was stored whenever the radiative feedback was calculated. The solution for a particular ray was only recomputed when this structure changed significantly.

We calculated accretion by depleting the innermost grid cells of material, by a fixed fraction each timestep. On the upstream side, the boundary conditions were those given by equations (1) to (3). This enabled the outer boundary of the simulation to be moved inwards (cf Koshe et al. 1999), easing the computational load. On the downstream side, we imposed outflow conditions. For the equation of state, we tried $\gamma = 1, 4/3, 5/3$. Although the densities suggest isothermality is the best approximation (Low & Lynden-Bell 1976), there will also be radiative heating and possibly other processes. The star was parameterised by the ZAMS formulae of Tout et al. (1996).

In the majority of our calculations, we used 50 grid cells in the $\theta$ direction, and 160 in the radial direction. The angular spacing was uniform, but the radial spacing was not. Only the inner five radial zones (extending to $r = 0$) were uniformly spaced, and formed the accretion cavity. The remaining 155 grid cells were logarithmically spaced, joining smoothly on to the uniform grid. For some test cases, we increased the resolution, but this did not change our results significantly. The resolution we used means that most of the direct stellar impulse is applied in a single grid cell, but Zeus2D does not seem to mind this (cf section 4.2). In the isothermal case, the accretion column is covered by about five azimuthal zones.

In all our calculations, the only source of gravity is the star itself. The self-gravity of the gas is neglected, due to the computational cost. However, the neglect is not without justification. The gas mass contained in our simulations should remain less than the stellar mass. Furthermore, these simulations are meant to be a small sample around a larger cluster core. In this case, the structure of the cluster itself would provide some support against local gravitational collapses of the gas. As we shall see, isothermal gas can give rise to high gas densities in the accretion column, and it is possible that self-gravitating clumps could form. However, such clumps would still be accreted by the star. Finally, any over-densities which do form will tend to absorb more radiation, and the increased temperature will tend to stabilise them against gravitational collapse.

4.1 Flow without Feedback

As mentioned above, the ratio $\zeta_{HL}/r_d$ is rather large for our simulations - larger than most simulations of Bondi–Hoyle flow to date. Therefore, we first simulated Bondi–Hoyle flow without feedback, to provide a benchmark for the runs with feedback included.

Figure 4 shows the resultant accretion rates as a function of time. The simulations with $\gamma$ values of $4/3$ and $5/3$ settle into a quasi-steady state, with the stiffer equation of state giving a slightly lower accretion rate. The isothermal run shows quite strong oscillations in its accretion rate, although the mean value is close to $\dot{M}_{HL}$. These oscillations are likely to be due to the gas suddenly shocking as it encounters the $\theta = 0$ axis. In a very short distance, it has to turn around (the test of accretion in the standard analysis is for material flowing away from the accretor), and it is evidently unable to do this in a steady manner.

We plot sample density and velocity fields for these runs in Figures 5 and 6. Note that the size of these plots is rather smaller than $\zeta_{HL} \approx 10^{16}$ cm. While the isothermal run is obviously qualitatively similar to the flow described by Hoyle & Lyttleton, the stiffer equations of state lead to rather different behaviour. For $\gamma = 5/3$, a bow shock forms, with the downstream flow close to spherical (cf e.g. Hunt 1971). Putting $\gamma = 4/3$ moves the bow shock back, attaching it to the accretor. The isothermal flow is qualitatively similar to that described by equations (1) to (3). We found that the oscillating accretion rate plotted in Figure 4 was due to a slight expansion and contraction in the accretion column, rather than a dramatic change in the flow pattern.
Figure 6. Velocity fields for high Mach number flow around a small accretor. The approximate location of the shock front is shown by the dotted line. Flow is incident from the left.

We can also make a useful comparison between the ram pressure in the radial direction and the sharp impulse at the dust destruction radius. At the most basic level, accretion can only proceed if

\[ \Xi = \frac{4\pi \rho v^2 r^2}{L} > 1 \]  \hfill (13)

(the angular factors are the same for both ram and radiation pressure). For a 10 M\(_\odot\) star, the formulae of Tout et al. (1996) predict \( L = 5552 \, L_\odot \). Results for an expected dust destruction radius of \( 2.1 \times 10^{14} \, \text{cm} \) are plotted in Figure 7. From this, it is obvious that we have happened upon an interesting region of parameter space. The adiabatic runs have \( \Xi \) values which get as low as a few. This means that the direct impulse will not be sufficient to halt the flow. However, the extra momentum from the thermalised field may slow the flow sufficiently to allow the accretion flow to be shut down. The \( \Xi \) values for the isothermal case vary even more dramatically. Just outside the wake, the radiation pressure at the dust destruction front will dominate the ram pressure (unsurprising, given the velocity field shown in Figure 6). In the wake, the ram pressure is orders of magnitude greater than the radiation pressure.

4.2 Direct Impulse Only

To provide comparison with later simulations, we performed three runs with only the direct impulse included. We achieved this by injecting \( L/c \) worth of momentum into the grid at a fixed radius. As may be seen from Figure 8, the effect on the accretion rate is minimal. The greatest differences occur for the isothermal case, and even then, the changes probably have more to do with the intrinsic instability of the flow, rather than any significant changes due to feedback. These results are not too surprising in the light of Figure 4 which showed that the ram pressure should exceed the radiation pressure for most of the flow at the dust destruction radius.
4.3 Feedback with Reduced Dust

Three runs were performed with the depleted dust abundances of Wolfire & Cassinelli (1987). This involves an ad hoc reduction of the dust abundances by a factor of ten, and also a reduction in graphite grain size. Wolfire & Cassinelli found this reduction was necessary to ensure material was always accelerating inwards in their calculations.

Accretion rates for these runs are plotted against time in figure 9. From these, it is apparent that the addition of feedback is not having a dramatic effect on the flow. The run with $\gamma = 5/3$ now shows larger fluctuations in its accretion rate, and the overall rate has dropped too. Smaller changes have occurred for the $\gamma = 4/3$ gas, with the accretion rate only falling slightly when compared to the pure hydrodynamic run. For the isothermal case, the change in accretion rate is barely visible compared to the large fluctuations present.

On examining the density contours, we found that the run with $\gamma = 4/3$ was little changed - unsurprisingly, in view of Figure 9. The isothermal simulation showed a small density enhancement adjacent to the wake, corresponding to the portion of Figure 7 where $\Xi$ was less than unity. Beyond this, there were few changes. There were greater changes for the $\gamma = 5/3$ gas - the flow was far less smooth (as one might expect, given the changing accretion rate). Since the reduced dust abundances made the Rosseland optical depth low, the shape of the dust destruction front was close to spherical in all cases. The only significant deviation was for the accretion column in the isothermal case. Even here, the dust destruction front was only pushed out slightly.

4.4 Feedback with Full Dust

Increasing the dust abundances to their normal galactic values had a dramatic effect on the flow. Soon after the impulse switched on, material ceased penetrating $r_d$, and the grid cells inside drained of material. $\textsc{Zeus2d}$ was then forced to terminate. Consider Figure 10. This plots $\Xi$ and $\tau_R$ values for the isothermal gas shortly after the impulse is switched on. If the gas is outflowing, $\Xi$ is set to zero. It is relevant to compare these two quantities because the amount of momentum imparted to the flow by the thermalised field is $L \tau_R/c$. Since $\Xi$ measures the ram pressure of the flow at $r_d$ in units of $L/c$, if $\tau_R \gg \Xi$, then the thermalised radiation should reduce the ram pressure at $r_d$ enough to allow the direct radiation field to halt inflow. Figure 10 shows that the total momentum imparted to the flow by the luminosity completely dominating the ram pressure in the flow. This is the reason accretion halted.

When we examined the position of the dust destruction front, we found that it was still close to spherical for most of the flow. However, the attached shock in the $\gamma = 4/3$ case, and the accretion column in the isothermal case caused noticeable deviations. In the isothermal run, the dust destruction front in the accretion column was located roughly four times further from the accretor than on the upstream side.

We attempted various numerical ‘fixes’ in order to ensure that numerical problems were not interfering with the physical analysis. In some runs, we spread the effect of the direct stellar radiation field over a number (typically seven) of grid cells. In others, we tried adjusting the CFL condition, to incorporate a term based on $s = 0.5a t^2$. Finally, we tried both methods together. None of these ‘worked’ - accretion was still halted in every case. We therefore concluded that our results were not an artefact of the numerical method. This is consistent with the theoretical arguments discussed above.

5 DISCUSSION

Radiative feedback has a much more dramatic effect on Bondi–Hoyle accretion than spherically symmetric accretion. This is brought about by the lower central concentration and presence of centrifugal support found in Bondi–Hoyle flow.
Although we have only been able to study one set of parameters, there are several implications for the formation of massive stars in clusters. Since $L \sim M^3$ for stars close to $10 \, M_\odot$, the fact that our simulations were obviously on the borderline between accreting and not accreting implies that

- Less massive stars will have few problems accreting in the Bondi–Hoyle geometry
- More massive stars are unlikely to accrete significant amount of mass under these conditions

Unfortunately, it is not easy to determine an exact limit - especially since we have not been able to test a wide range of parameters, and the fact that it is the reprocessed radiation which is of critical importance. Consider equation 12. Since $L/r_d^3$ should be approximately constant (cf the Stefan–Boltzmann equation), if the ‘inside’ radial velocity is positive (indicating accretion) for a protostar at some mass, it should remain positive as the mass increases. The sharp impulse at the dust destruction radius cannot stop the inflow by itself. It can only do so if the flow has been slowed by the thermalised radiation. Of course, if the value of $v_\infty$ rises too much (due to the contraction of the cluster), then the accretion timescale (cf equation 2) will become longer than the main sequence timescale of the star.

Estimating the effect of the thermalised radiation is much more difficult, since the impulse from it is distributed throughout the flow. Based on the findings presented above, requiring that $\Xi > \tau_R^{-1}$ at the dust destruction radius is the most reasonable condition for the purpose of making estimates. However, this should be treated with caution, since it is not rigorously derived.

Despite these problems, we can still suggest regions of parameter space which might permit accretion, and hence be worthy of future investigation. These regions must fulfil several conditions:

- $M_{\text{BH}}$ must be large enough to permit the star to gain appreciable mass before it leaves the main sequence
- They must have $\Xi > 1$ at the dust destruction radius (or the flow will be reversed)
- The Hii region must lie within the dust destruction radius
- They must have $\Xi > \tau_R$ at the dust destruction radius (again, to avoid reversal of the flow)

For a given mass of star, these conditions define a region of $(\rho_\infty, v_\infty)$ space which are likely to permit accretion in the Bondi–Hoyle geometry. In Figure 11, we show the allowed parameters for a variety of stellar masses, evaluated using equations 6 to 9 for the full dust abundances. For the first condition, we required $M_{\text{BH}} > 10^{-7} M_\odot \, \text{yr}$. The last three conditions above are functions of angle, so in these cases we tested the condition on the upstream ($\theta = \pi$) direction. The Hii region will be largest in this direction. Furthermore, the analytic approximation will hold to smaller radii. We also added the condition that $\zeta_{\text{HL}} > 2r_d$, and took the edge of the calculation to lie at $2\zeta_{\text{HL}}$ (for the purposes of calculating $\tau_R$). Both of these conditions are somewhat arbitrary. First, note that the parameters tested above ($\rho_\infty = 10^{-16} \, \text{g cm}^{-3}$ and $v_\infty = 5 \times 10^5 \, \text{cm s}^{-1}$ for $M = 10 \, M_\odot$) lie outside the allowed range for the full dust (this point lies inside the permitted region, as calculated for the reduced dust). Of course, this is not a full test of the validity of these conditions, but it is encouraging. The sloping straight line boundary is caused by the requirement on $M$. The horizontal boundary which appears close to $v_\infty = 2 \times 10^{10} \, \text{cm s}^{-1}$ for the higher masses is set by the constraint on $\zeta_{\text{HL}}$.

For accretion to occur, Figure 11 suggests that the optical depth must be kept low, either by high velocities (so that the distance to the outer boundary is small) or low densities. The permitted region for the higher mass stars at high velocities is almost independent of density because $\rho$ does not vary much between $r_d$ and $2\zeta_{\text{HL}}$. This means that $\rho$ cancels from the ratio $\Xi/\tau_R$, leaving the ratio a function of $v_\infty$ only. At low densities, the rather complicated shape of the allowed region is due to the solution of the thermalised radiation field, and the calculation of $\tau_R$. Note that there is no allowed low density region for the $11 \, M_\odot$ case.

It seems that Bondi–Hoyle accretion is going to run into serious trouble at about $10 \, M_\odot$. Even for masses of only $8 \, M_\odot$, the allowed region is curtailed quite severely. However, these are only estimates, and assume that Bondi–Hoyle accretion will be a good approximation over all the parameter space covered by figure 11. Unfortunately, this is unlikely to be the case. For example, a velocity of $10^4 \, \text{cm s}^{-1}$ for a $10 \, M_\odot$ star gives $\zeta_{\text{HL}} \sim 10^{19} \, \text{cm}$, which is far larger than the interstellar separation in a cluster core. Also, a $10 \, M_\odot$ star moving at $10^5 \, \text{cm s}^{-1}$ through at cloud with $\rho_\infty = 10^{-15} \, \text{g cm}^{-3}$ would have an unreasonably high accretion rate of $M_{\text{BH}} \sim 0.3 \, M_\odot \, \text{yr}^{-1}$. These concerns would restrict the allowed parameter space further.

There are further problems, which our simulations do

$^3$ Recall that $\Xi$ is the ratio of ram to radiation pressure at $r_d$ - see equation 18.

$^4$ Additionally, simulating such a large volume would be computationally painful, due to the enormous difference in length scales. We will still have $r_d \sim 10^{14} \, \text{cm}$. Worse still, a cloud that big is going to be self-gravitating.

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Figure 11. Allowed parameters for Bondi–Hoyle accretion for a variety of stellar masses, as suggested by the conditions in the text. The contours enclose the allowed regions for the marked stellar masses, when the dust abundance is assumed to be Galactic. The star marks the location of the simulations presented in this paper.
not address. We have not included an accretion luminosity, since it will complicate the feedback mechanism. However, the accretion luminosity implied by the parameters of Table 4 is roughly three times the ZAMS luminosity. A short test, on the assumption that the accretion luminosity was also isotropic (this may not be the case), suggested that accretion was not possible even for the reduced dust abundances. Our inability to simulate the ionised region is also a concern. Flow in the Bondi–Hoyle geometry is far less centrally concentrated than in the spherically symmetric case, so the HII region might be able to grow. If it can reach the dust destruction radius, some rather unpleasant chemistry would have to be simulated as well. Our imposition of axisymmetry and low resolution in the inner portions of the grid also prevent us examining the question of accretion discs. If the flow at infinity is non-uniform (rather likely in a protocluster), then there is the potential for angular momentum accretion too (Ruffert 1997, 1998). This would drive the formation of an accretion disc.

The problems radiative feedback presents in the Bondi–Hoyle geometry have potentially serious implications for the merger model of massive star formation proposed by Bonnell et al. (1998). Accretion drove the contraction of their clusters to the high stellar densities required for mergers. Bonnell & Bate (2002) presented a full hydrodynamic simulation of such a cluster, but did not have a cut off mass for accretion (their model was scale free). If the most massive stars in the cluster core stopped accreting, then it is possible that the cluster core would not contract enough to drive a high merger rate. However, this is very uncertain. Bonnell et al. (2001) simulated another cluster, and their results show many low mass stars in the core, in addition to higher mass ones. It is possible that the lower mass stars could accrete enough material to drive the stellar density to that required for mergers. Note also that Bonnell et al. (1998) did prevent stars more massive than 10 M⊙ accreting in their simulations, but still found merging stars.

6 CONCLUSION

We have shown that if stars form in the ultra-dense cores of young clusters, as postulated in the models of Bonnell et al. (1998), Bonnell & Bate (2002), then the feedback from radiation pressure on dust is likely to prevent accretion on to stars more massive than ~10 M⊙. This feedback is more disruptive to the accretion flow than in our previous simulations (Edgar & Clarke 2003) which used the initial conditions employed in the classic work of Wolfire & Cassinelli (1987) (i.e. spherically symmetric accretion and initial gas densities of 10^{-19} g cm^{-3}).

The reasons for this more effective feedback in the present case are two-fold. Firstly, the ultra-dense cluster cores that are invoked by Bonnell et al. are unobservably short lived. In this phase the local gas density of their cores is extremely high, exceeding that seen in regions of massive star formation, or that employed by Wolfire & Cassinelli by many orders of magnitude. Consequently, the optical depth to the thermalised radiation field (τR) is very high. Since the total rate of momentum input into the gas is ~τR L/c, this increases the coupling between the radiation and gas dramatically. Secondly, in the case of spherically symmetric accretion, the gas density is centrally concentrated and the velocity field is radial. Both of these effects enhancing the radial component of the ram pressure in the accretion flow at the dust destruction radius. In the case of cluster cores, by contrast, accretion proceeds in Bondi–Hoyle geometry, and flow is nearly tangential as it approaches the accretion wake (see Figure 4). Consequently the radial component of the ram pressure is relatively small, and this again favours the disruption of the accretion flow.

The above results imply that if massive stars indeed form in these hypothetical ultradense cores, then for masses greater than ~10 M⊙, they must be assembled entirely by stellar collisions and consequent mergers rather than by accretion. The attainment of these ultradense conditions is however driven by accretion by stars in the core and it currently unclear whether accretion on to low mass stars (i.e. those with mass <10 M⊙) will suffice for this purpose. This issue needs to be explored by further simulations that incorporate feedback from radiation pressure on dust into realistic cluster simulations.

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