Cosmic acceleration in brane cosmology

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Cosmic acceleration may be the result of unknown physical processes involving either new fields in high energy physics or modifications of gravitation theory. In the latter case, such modifications are usually related to the existence of extra dimensions (which is also required by unification theories), giving rise to the so-called brane cosmology. In this paper we investigate the phenomenon of the acceleration of the Universe in a particular class of brane scenarios in which a large scale modification of gravity arises due to a gravitational leakage into extra dimensions. By using the most recent supernova observations we study the transition (deceleration/acceleration) epoch as well as the constraints imposed on the parameters characterizing the model. We show that these models provide a good description for the current supernova data, which may be indicating that the existence of extra dimensions play an important role not only in fundamental physics but also in cosmology.

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I. INTRODUCTION

The current idea of a negative-pressure dominated universe seems to be inevitable in light of the impressive convergence of the recent observational results (see, e.g., [1] for a review). This in turn has led cosmologists to hypothesize on the possible existence of an exotic dark component that not only would explain these experimental data but also would reconcile them with the inflationary flatness prediction ($\Omega_{\text{Total}} = 1$). This extra component, or rather, its gravitational effects is thought of as the first observational piece of evidence for new physics beyond the domain of the standard model of particle physics, a conclusion that has given rise to many speculations on its fundamental origin [2].

Alternatively, another possible route to deal with this dark pressure problem could be a modification in gravity instead of any adjustment to the energy content of the Universe. This idea naturally brings to light another important question at the interface of fundamental physics and cosmology: extra dimensions. As is well known the existence of extra dimensions is required in various theories beyond the standard model of particle physics, especially in theories for unifying gravity and the other fundamental forces, such as superstring or M theories. As suggested in Ref. [3], extra dimensions may also provide a possible explanation for the huge difference between the two fundamental energy scales in nature, namely, the electroweak and Planck scales [$M_{\text{Pl}}/m_{\text{EW}} \sim 10^{16}$] (see also [4]).

In the cosmological context, the role of extra spatial dimensions as source of the dark pressure is translated into the so-called brane world (BW) cosmologies [5]. The general principle behind such models is that our 4-dimensional Universe would be a surface or a brane embedded into a higher dimensional bulk space-time on which gravity and only gravity could propagate. Brane world scenarios has been a topic of much interest recently. In [6], for instance, a class of BW models which admit a wider range of possibilities for the dark pressure than do the usual dark energy scenarios was investigated. An interesting feature of this class of models is that the acceleration of the Universe can be a transient phenomena, which could help reconcile the supernova evidence for an accelerating universe with the requirements of string/M-theory [7].

Another particularly interesting scenario is the one proposed by Dvali et al. [8], which we will refer to it as DGP model. It describes a self-accelerating 5-dimensional BW model with a noncompact, infinite-volume extra dimension in which the whole dynamics of gravity is governed by a competition between a 4-dimensional Ricci scalar term, induced on the brane, and an ordinary 5-dimensional Einstein-Hilbert action. For scales below a crossover radius $r_c$ (where the induced 4-dimensional Ricci scalar dominates), the gravitational force experienced by two punctual sources is the usual 4-dimensional $1/r^2$ force whereas for distance scales larger than $r_c$ the gravitational force follows the 5-dimensional $1/r^3$ behavior. The theoretical consistency of the model, and in particular of its self-accelerating solution, is still a matter of debate in the current literature (see, e.g., [9]). From the observational viewpoint, however, DGP models have been successfully tested in many of their predictions, ranging from local gravity to cosmological observations [8, 10, 11, 12].

In this paper we are particularly interested in testing the viability of DGP scenarios in light of the latest supernova (SNe Ia) data, as provided recently by Riess et al. [13]. The sample used, which consists of 157 gold events distributed over the redshift interval $0.01 \lesssim z \lesssim 1.7$, is the compilation of the best observations made so far by the two supernova search teams plus 16 new events observed by HST. In agreement with other independent analyses, it is shown that these models constitute a good
II. BASIC EQUATIONS AND THE TRANSITION EPOCH

In DGP models, the modified Friedmann's equation due to the presence of an infinite-volume extra dimension reads

\[
\left( \frac{\rho}{3M_{pl}^2} + \frac{1}{4r_c^2} + \frac{1}{2r_c} \right)^2 = H^2 + \frac{k}{R(t)^2},
\]

where \( k, R(t), H \) and \( \rho \) are, respectively, the curvature parameter of the spatial section, the cosmological scale factor, the Hubble parameter and the energy density of the cosmic fluid (which we will assume to be composed only of nonrelativistic particles). The crossover scale defining the gravitational interaction among particles located on the brane is expressed as \( r_c = M_{pl}^2/2M_5^3 \), where \( M_{pl} \) is the Planck mass and \( M_5 \) is the 5-dimensional reduced Planck mass. Note that whenever the condition \( \rho/3M_{pl}^2 > 1/r_c^2 \) is valid, DGP and standard models are analogous so that the cosmological evolution for the early stages of the Universe (when the above condition holds) is exactly the same in both scenarios.

Equation (1) also implies that the normalization condition is given by

\[
\Omega_k + \left[ \sqrt{\Omega_{rc} + \sqrt{\Omega_{rc} + \Omega_m}} \right]^2 = 1
\]

(2)

where \( \Omega_m \) and \( \Omega_{rc} \) are, respectively, the matter and curvature density parameters (defined in the usual way) and

\[
\Omega_{rc} = 1/4r_c^2H_o^2,
\]

(3)

is the density parameter associated to the crossover radius \( r_c \). For a flat universe (\( \Omega_k = 0 \)), Eq. (2) reduces to

\[ \Omega_{rc} = (1 - \Omega_m^2)/4. \]

In order to study the acceleration phenomenon in these scenarios we derive the deceleration parameter, defined as

\[ q(z) = \frac{1}{j} \frac{d\ln \mathcal{F}(z; \Omega_j)}{d\ln (1+z)} - 1, \]

(4)

where \( j \) stands for \( m, r_c \) and \( k \) and the dimensionless function \( \mathcal{F}(z; \Omega_j) \) is given by

\[
\mathcal{F} = \left[ \Omega_k (1+z)^2 + \left( \sqrt{\Omega_{rc} + \sqrt{\Omega_{rc} + \Omega_m(1+z)^2}} \right)^2 \right]^{1/2}.
\]

(5)

From the Eqs. (4) and (5), it is possible to obtain the transition redshift \( z_* \), at which the Universe switches from deceleration to acceleration or, equivalently, the redshift at which the deceleration parameter vanishes. Regardless of the geometry adopted, \( z_* \) can always be expressed in a closed analytic form, i.e.,

\[
(1 + z_*)^{3/2} = 2 \left( \frac{\Omega_{rc}}{\Omega_m} \right)^{1/3}.
\]

(6)

If now we restrict our analysis to the flat case, in accordance to CMB measurements [16, 17], and assume \( \Omega_m = 0.3 \ (\Omega_{rc} = 0.1225) \), as suggested by clustering estimates [18], Eq. (6) provides \( z_* \approx 0.48 \), which is surprisingly close to the transition redshift estimated from the current SNe Ia data, i.e., \( z_* = 0.46 \pm 0.13 \) [14] (see also [19] for other recent estimates of \( z_* \)). For the sake of comparison, we also compute \( z_* \) for the so-called concordance model, namely, a flat universe with \( \Omega_m = 0.27 \) and a vacuum energy contribution of \( \Omega_{\Lambda} = 0.73 \) (LCDM).

Such a model yields \( z_* \approx 0.75 \), which is off by \( \sim 2 \sigma \) from the central value estimated in [14].

In Fig. 1a we show the \( \Omega_m - \Omega_{rc} \) plane for the transition redshift lying in the interval \( 0.33 \leq z_* \leq 0.59 \), which corresponds to \( \pm 1 \sigma \) of the value for \( z_* \) given in [14].

Note that a considerable portion of this parameter space is compatible with such a constraint. The vertical hachured rectangle stands for the estimate of matter density parameter as provided by WMAP, i.e., \( \Omega_m = 0.27 \pm 0.04 \) [16]. In particular, for the best-fit values of \( \Omega_m \) and \( \Omega_{rc} \), obtained from the current gravitational lensing data [20] we find \( z_* \approx 0.29 \) while for these esti-
FIG. 2: a) Confidence regions (68.3%, 95.4% and 99.7%) in the \( \Omega_m - \Omega_c \) plane by considering the gold sample of Riess et al. \[14\]. Note that the area corresponding to the confidence intervals is considerably reduced when compared with previous SNe Ia analyses \[14, 22\]. b) The same as in Panel (a) by assuming a Gaussian prior on the matter density parameter, \( \Omega_m = 0.27 \pm 0.04 \). For this analysis the best-fit model occurs at \( \Omega_m = 0.28 \) and \( \Omega_c = 0.21 \) with \( \chi^2_{\text{min}}/\nu \simeq 1.13 \).

In this section we test the viability of DGP scenarios through a statistical analysis involving the most recent SNe Ia data, as provided recently by Riess et al. \[14\]. The total sample presented in \[14\] consists of 186 events distributed over the redshift interval 0.01 \( \lesssim z \lesssim 1.7 \) and constitutes the compilation of the best observations made so far by the two supernova search teams plus 16 new events observed by \textit{HST}. This total data-set was divided into “high-confidence” (gold) and “likely but not certain” (silver) subsets. Here, we will consider only the 157 events that constitute the so-called gold sample. In what follows we briefly outline our main assumptions for this analysis (see \[22\] for some recent SNe Ia analyses).

The predicted distance modulus for a supernova at redshift \( z \), given a set of parameters \( \mathbf{s} \), is

\[
\mu_p(z|\mathbf{s}) = m - M = 5\log d_L + 25, \tag{7}
\]

where \( m \) and \( M \) are, respectively, the apparent and absolute magnitudes, the complete set of parameters is \( \mathbf{s} \equiv (H_o, \Omega_j) \) and \( d_L \) stands for the luminosity distance (in units of megaparsecs),

\[
d_L = \frac{c(1 + z)}{H_o\sqrt{|\Omega_k|}} S_k \left[ \sqrt{|\Omega_k|} \int_{x'}^1 \frac{dx}{x^2 F(x; \Omega_j)} \right], \tag{8}
\]

with \( x' = \frac{R(z)}{H_o} = (1 + z)^{-1} \) being a convenient integration variable, \( F(x; \Omega_j) \) the expression given by Eq. (6) and \( S_k \) a function defined by one of the following forms: \( S_k(r) \equiv \sinh(r), r, \) and \( \sin(r) \), respectively, for open, flat and closed geometries.

We estimated the best fit to the set of parameters \( \mathbf{s} \) by using a \( \chi^2 \) statistics, with

\[
\chi^2 = \sum_{i=1}^{157} \frac{[\mu_p^i(z|\mathbf{s}) - \mu^i_p(z|\mathbf{s})]^2}{\sigma_i^2}, \tag{9}
\]

where \( \mu^i_p(z|\mathbf{s}) \) is given by Eq. (7), \( \mu^i_p(z|\mathbf{s}) \) is the extinction corrected distance modulus for a given SNe Ia at \( z_i \), and \( \sigma_i \) is the uncertainty in the individual distance moduli, which includes uncertainties in galaxy redshift due to a peculiar velocity of 400 km/s. The Hubble parameter \( H_o \) is considered a “nuisance” parameter so that we marginalize over it by using the analytical method of Ref. \[21\].

Figure 2a shows the confidence regions (68.3%, 95.4% and 99.7%) in the \( \Omega_m - \Omega_c \) plane by considering the gold sample of Riess et al. \[14\]. Compared to Fig. 1 of \[22\] and Fig. 2 of \[8\] (which used the then available SNe Ia data), the gold sample studied here reduces considerably the area corresponding to the confidence intervals. The best-fit parameters for this analysis are \( \Omega_m = 0.33 \) and \( \Omega_c = 0.24 \) with \( \chi^2_{\text{min}}/\nu \simeq 1.12 \) (\( \nu \equiv \) degrees of freedom).
freedom. At 95% c.l. we obtain $0.24 \leq \Omega_m \leq 0.43$ and $0.17 \leq \Omega_r \leq 0.29$. In particular, the above relative value of $\chi^2$ is slightly smaller than the one we found for the concordance scenario, $\chi^2_{\text{min}}/\nu \simeq 1.14$, and equal to the value we obtained for the $\Lambda$CDM with arbitrary curvature. By restricting the analysis to the flat case, we note that the data favour a lower value of the matter density parameter, i.e., $\Omega_m = 0.21$ ($\Omega_r = 0.156$) with $\chi^2_{\text{min}}/\nu \simeq 1.13$. Such a value, however, is inside the $2\sigma$ interval of the current WMAP estimates of the quantity of matter in the Universe, $\Omega_m = 0.27 \pm 0.04$ [10], and therefore does not imply any possible conflict between the predictions of the model and the independent measurements of $\Omega_m$ (see [22, 23] for a discussion on this point).

By using SNe Ia-independent constraints in the $\Omega_m - \Omega_r$ plane yields more precise limits on these parameters due to the degeneracy between them for SNe Ia data. Thus, in Fig. 2b we show a similar analysis to Fig. 2a having assumed a Gaussian prior on the matter density parameter, $\Omega_m = 0.27 \pm 0.04$, as provided by WMAP team [10]. The parameter space now is considerably reduced relative to the former analysis, with the best-fit model occurring at $\Omega_m = 0.28$ and $\Omega_r = 0.21$ ($\chi^2_{\text{min}}/\nu \simeq 1.13$). Such a model corresponds to an accelerating universe with $q_0 \simeq -0.65$ and a total expanding age of $t_0 \simeq 9.9h^{-1}$ Gyr. Note that the best-fit value for $\Omega_r$ leads to an estimate of the crossover scale $r_c$ in terms of the Hubble radius $H_0^{-1}$ (see Eq. 3),

$$r_c \simeq 1.09H_0^{-1},$$

(10)

which is slightly smaller than the one obtained in Ref. [9] by using the old SNe Ia data (see also [11] for summary of the current estimates of $r_c$). By fixing $\Omega_k = 0$, we find $\Omega_m = 0.23$ ($\Omega_r = 0.148$) with $\chi^2_{\text{min}}/\nu \simeq 1.15$. This particular value of $\Omega_r$, corresponds to a crossover distance between 4-dimensional and 5-dimensional gravities of the order of $r_c \simeq 1.3H_0^{-1}$. In Table I we summarize the main results of the paper.

## IV. CONCLUSION

The results of observational cosmology in the past years have opened up an unprecedented opportunity to establish a more solid connection between Fundamental Physics and Cosmology. Surely, the most remarkable finding among these results comes from SNe Ia observations which suggest that the cosmic expansion is undergoing a late time acceleration. Such a phenomenon has been often explained from two different ways, i.e., either by considering the presence of a negative-pressure dark component, the so-called dark energy, or by assuming the existence of extra spatial dimensions, as motivated by recent developments in particle physics. In this paper we have followed the second route. We have studied the transition (deceleration/acceleration) epoch in the context of DGP models and shown that these scenarios based on a large scale modification of gravity are in good agreement with the current SNe Ia data for values of $\Omega_m = 0.33^{+0.10}_{-0.09}$ and $\Omega_r = 0.24^{+0.05}_{-0.07}$ (95.4% c.l.). By assuming a SNe Ia-independent constraint on the matter density parameter, we have found $\Omega_r = 0.21$, leading to an estimate of the crossover scale $r_c \simeq 1.09H_0^{-1}$. If flatness is imposed, then we find $\Omega_m = 0.23$ ($\Omega_r = 0.148$), which corresponds to a acceleration universe with $q_0 \simeq -0.65$ and a total expanding age of $t_0 \simeq 9.5h^{-1}$ Gyr.

In summary, what we have shown is that at least at the level of background tests (like tests involving SNe Ia measurements), DGP models constitute a viable alternative for the dark energy or dark pressure problem. This may be understood as an indication that the existence of extra dimensions play an important role not only in fundamental physics but also in cosmology.

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### Table I: Best-fit values for $\Omega_m$, $\Omega_r$ and $r_c$

| Test               | $\Omega_m$ (flat) | $\Omega_r$ (flat) | $r_c$ (flat) |
|--------------------|-------------------|-------------------|--------------|
| SNe Ia             | 0.33 (0.21)       | 0.24 (0.156)      | 1.02 (1.26)  |
| SNe Ia + $\Omega_m$| 0.28 (0.23)       | 0.21 (0.148)      | 1.09 (1.30)  |

*a in units of $H_0^{-1}$

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