Destination Choice Game: A Spatial Interaction Theory on Human Mobility

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With remarkable significance in migration prediction\textsuperscript{1,2}, global disease mitigation\textsuperscript{3,4}, urban plan\textsuperscript{5,6} and many others, an arresting challenge is to predict human mobility fluxes between any two locations\textsuperscript{7}. A number of methods have been proposed against the above challenge, including the intervening opportunity model\textsuperscript{8}, the gravity model\textsuperscript{9}, the radiation model\textsuperscript{10,11}, the population-weighted opportunity model\textsuperscript{12,13}, and so on. Despite their theoretical elegance, all models ignored an intuitive and important ingredient in individual decision about where to go, that is, the possible congestion on the way\textsuperscript{14} and the possible crowding in the destination\textsuperscript{15,16}. Here we propose a microscopic mechanism underlying mobility decisions, named destination choice game (DCG), which takes into account the crowding effects resulted from spatial interactions among individuals. In comparison with the state-of-the-art models, the present one shows more accurate prediction on mobility fluxes across wide scales from intracity trips to intercity travels, and further to internal migrations. The well-known gravity model is happen to be the equilibrium solution of a degenerated DCG neglecting the crowding effects in the destinations.

The number of individuals $T_{ij}$ travelling from the starting location $i$ to the destination $j$ is resulted from the cumulation of destination choices of all individuals at location $i$. We model such decision-making process by a multiplayer game with spatial interactions, where each individual chooses one destination from all candidates to maximize his utility. Specifically speaking, the utility $U_{ij}$ of an arbitrary individual at location $i$ to choose location $j$ as destination consists of the following four parts. (i) The fixed payoff of the destination $h(A_j)$, where $h$ is intuitively assumed to be a monotonically increasing function of $j$’s attractiveness $A_j$ that is usually dependent on $j$’s population, GDP, environment, and so on. (ii) The fixed travelling cost $C_{ij}$. (iii) The congestion effect $g(T_{ij})$ on the way, where $T_{ij}$ is the target quantity and $g$ is a monotonically non-decreasing function. (iv) The crowding effect $f(D_j)$ at the destination, where $f$ is a monotonically non-decreasing function and $D_j = \sum_i T_{ij}$ is the total number of individuals choosing $j$ as their destination. In a word, the utility function $U_{ij}$ reads

$$U_{ij} = h(A_j) - f(D_j) - C_{ij} - g(T_{ij}).$$

In the above destination choice game (DCG), if every individual knows complete information, the equilibrium solution guarantees that all $O_i$ individuals at the same starting location $i$ have exactly the same utility no matter which destinations to be chosen. Strictly speaking, the variable $T_{ij}$ has to be continuous to guarantee the existence of an equilibrium solution, which is a reasonable approximation when there are many individuals in each journey $i \rightarrow j$. Figure 1(a) illustrates a simple game scene. Considering a simple utility function $U_{ij} = A_j - \frac{1}{2}D_j - C_{ij} - T_{ij}$ that takes into account both the congestion effect on the way and the crowding effect in the destination, we can obtain the equilibrium solution based on the equilibrium condition ($U_{i3} = U_{i4}$) and the conservation law ($T_{i3} + T_{i4} = O_i$ and $T_{ij} + T_{2j} = D_j$). The solution is shown in figure 1(b).

Generally speaking, we cannot obtain the analytical expression of the equilibrium solution, instead, we apply the method of successive averages\textsuperscript{18} (MSA, see Methods) to iteratively approach the solution. Inspired by the successful economic applications of the Weber-Fechner law\textsuperscript{19} (see Methods), we apply the following utility function

$$U_{ij} = \alpha \ln A_j - \beta \ln d_{ij} - \gamma \ln D_j - \ln T_{ij},$$

where $\alpha$, $\beta$ and $\gamma$ are nonnegative parameters that can be fitted by real data, subject to the largest Sørensen similarity index\textsuperscript{20} (SSI, see Methods). $A_j$ is the location $j$’s attractiveness, which is approximated by the actual number of attracted individuals in the real data and $d_{ij}$ is the geometric distance between $i$ and $j$.

We use three real data sets, including intracity trips in Abidjan, intercity travels in China and internal migrations in US, to test the predictive ability of the DCG model. The data set of intracity trips in Abidjan is extracted from the anonymous Call Detail Records (CDR) of phone calls and SMS exchanges between Orange Company’s customers in Ivory Coast\textsuperscript{21}. To protect customers’ privacy, the customer identifications have been anonymized. The positions of corresponding base stations are used to approximate the positions of starting points and destinations. The data set of intercity travels in China\textsuperscript{13} is extracted from anonymous users’ check-in records at Sina Weibo, a large-scale social network in China with functions similar to Twitter. Since here we focus on movements between cities, all the check-ins within a prefecture-level city are regarded as the same with a proxy position being the center of the city. The data set of internal migrations in US is downloaded from https://www.irs.gov/statistics/soi-tax-
For both (b) and (c), the equilibrium solutions are shown in Table 1. In all the above three data sets and other data sets presented in the Supplementary Note 1, every location can be chosen as a destination.

We use three different metrics to quantify the proximity of the DCG model to the real data. Firstly, we investigate the travel distance distribution, which is the most representative feature to capture human mobility behaviours [22][24]. As shown in figures 2(a-c), the distributions of travel distances predicted by the DCG model are in good agreement with the real distributions. We next explore the probability $P(D)$ that a randomly selected location has eventually attracted $D$ travels (in the model, for any location $j$, $D_j$ is the total number of individuals choosing $j$ as their destination). $P(D)$ is a key quantity measuring the accuracy of origin-constrained mobility models, because origin-constrained models cannot ensure the agreement between predicted travels and real travels to a location [23]. Figures 2(d-f) demonstrate that the predicted and real $P(D)$ are almost statistically indistinguishable. Thirdly, we directly look at the mobility fluxes between all pairs of locations [10][13]. As shown in figures 2(g-i), the average fluxes predicted by the DCG model are in reasonable agreement with real observations.

We next compare the predicting accuracy on mobility fluxes of DCG with well-known models including the gravity models, the intervening opportunities model, the radiation model and the population-weighted opportunities (PWO) model (see Methods). In terms of SSI, as shown in figure 3, DCG performs best. Specifically speaking, it is remarkably better than parameter-free models like the radiation model and the PWO model and slightly better than the gravity model with two parameters. Supplementary Note 1 shows extensive empirical comparisons between predicted and real statistics as well as accuracies of different methods for more data sets involving travels inside and between cities in Japan, UK, Belgium, US and Norway. Again, DCG outperforms other benchmarks in all cases.

To further understand the advantage of the DCG model in comparison with the well-adopted gravity models, we give a close look at the key mechanism differentiated from all previous models, that is, the extra cost caused by the crowding effect, as inspired by the famous minority game [10][26]. Accordingly, we test a simplified model without the term $f(D_j)$ in Eq. (1). Figure 1(c) illustrates an example with a simple utility function $U_{ij} = A_j - C_{ij} - T_{ij}$ that only takes into account the congestion effect on the way. Similar to the case shown in figure 1(b), the equilibrium solution can be obtained by the equilibrium condition and the conservation law. For a more general and complicated utility function (by removing the term related to the crowding effect in Eq. (2))

$$U_{ij} = \alpha \ln A_j - \beta \ln d_{ij} - \ln T_{ij},$$

based on the potential game theory [27], one can prove

\[ \text{Figure 1: Illustration of a simple example of DCG. (a) The game scene. The nodes 1 and 2 represent two starting locations while the nodes 3 and 4 are two destinations. $O_i$ is the number of individuals located in $i$, $A_j$ is the attractiveness of $j$, and $C_{ij}$ is the fixed travelling cost from $i$ to $j$. (b)} \]
TABLE I. **Fundamental statistics of the data sets.** The second to fifth columns present the number of individuals, the number of recorded movements, the number of locations and how to estimate the geographical positions of these locations. For migration data, we do not know the precise number of individuals, but it should be close to the number of total records since people usually do not migrate frequently.

| data set                  | #individuals | #movements | #locations | positional proxy   |
|---------------------------|--------------|------------|------------|--------------------|
| intracity trips in Abidjan| 154849       | 519710     | 381        | base station       |
| intercity travels in China| 1571056      | 4976255    | 340        | prefecture-level city |
| internal migrations in US | N/A          | 2498464    | 51         | state capital      |

FIG. 2. **Comparing the predictions of DCG model and the empirical data.** (a-c) Predicted and real distributions of travel distances $P(d)$. (d-f) Predicted and real distributions of locations’s attracted travels $P(D)$. (g-i) Predicted and observed fluxes. The gray points are scatter plot for each pair of locations. The blue points represent the average number of predicted travels in different bins. The standard boxplots represent the distribution of predicted travels in different bins. A box is marked in green if the line $y = x$ lies between 10% and 91% in that bin and in red otherwise. The data presented in (d-i) are binned using the logarithmic binning method [17].

that the equilibrium solution is equivalent to the solution of the following optimization problem

$$\max Z(x) = \sum_j \int_0^{T_{ij}} (\alpha \ln A_j - \beta \ln d_{ij} - \ln x) dx,$$

s.t. $\sum_j T_{ij} = O_i, \ T_{ij} \geq 0$. (4)

Since the objective function is strictly convex, the solution is existent and unique. Applying the Lagrange multiplier method, we can obtain the solution of Eq. (4), which is exactly the same to the gravity model with two free parameters (i.e., Gravity 2, Eq. (11)), and if we set $\alpha = 1$ in Eq. (3), the solution degenerates to the gravity model with one free parameter (i.e., Gravity 1, Eq. (10)). The detailed derivation is shown in Supplementary Note 2. The significance of such interesting finding is threefold. Firstly, it provides a theoretical bridge that
connecting the DCG model and the gravity model, which are seemingly two unrelated theories. Indeed, it provides an alternative way to derive the gravity model. Secondly, comparing with the gravity models, the higher accuracy of the prediction from the DCG model suggests the existence of the crowding effect in our decision-making about where to go, which also provides a positive evidence for the validity of the critical hypothesis underlying the minority game. Thirdly, the improvement of accuracy from Gravity 2 to the DCG model can be treated as a measure for the crowding effect, which is, to our knowledge, the first quantitative measure for the crowding effect in human mobility.

In summary, the theoretical advantages of DCG are twofold. First of all, it does not require any prerequisite from God’s perspective, like the constraint on total costs in the maximum entropy approach [28, 29] and the deterministic utility theory [30], or any oversuble assumption, like the independent identical Weibull distribution to generate the hypothetically unobserved utilities associated with travels in the random utility theory [31, 32]. Instead, the two assumptions underlying DCG, namely (i) each individual chooses a destination to maximize his utility and (ii) congestion and crowding will depress utility, are very reasonable. Therefore, in comparison with the above-mentioned theories, DCG shows a more realistic explanation towards the gravity model by neglecting the crowding effect in destinations (see some other derivations to the gravity model in Supplementary Note 3). Secondly, the present game theoretical framework is more universal and extendable. As the travelling costs and crowding effects are naturally included in the utility function, DCG is easy to be extended to deal with more complicated spatial interactions that depend on individuals’ choices about not only destinations, but also departure time, travelling modes, travelling paths, and so on [32, 33].

In addition to theoretical advantages, DCG could better aid government officials in transportation intervention. For example, if the government would like to raise congestion charges in some areas (e.g., in Beijing, the parking fees in central urban areas are surprisingly high), the Radiation model and PWO model cannot predict the quantitative impacts on travelling patterns since the population distribution is not changed, instead, the game theoretical framework could respond to the policy changes by rewriting its utility function. Another example is to forecast and regulate tourism demand [37]. In China, in the vacations of the National Day and the Spring Festival, many people stream in a few most popular tourist spots, leading to unimaginable crowding and great environmental pressure. Recently, Chinese government forecasts tourism demand before those golden holidays based on the booking information about air tickets, train tickets and entrance tickets, and then the visitors are effectively redistributed to more diverse tourist spots with remarkable decreases of visitors to the most noticed a few spots. Such phenomenon can be explained by the crowding effects in the destination choices, but none of other known models. In a word, DCG is more relevant to real practices and thus of potential to be enriched towards an assistance for decision making.

METHODS

Method of Successive Averages

The method of successive averages (MSA) is an iterative algorithm to solve various mathematical problems [18]. For a general fixed point problem $x = F(x)$, the $n$th iteration in the MSA uses the current solution $x^{(n)}$ to find a new solution $y^{(n)} = F(x^{(n)})$. The next current solution is an average of these two solutions $x^{(n+1)} = (1 - \lambda(n))x^{(n)} + \lambda(n)y^{(n)}$, where $0 < \lambda(n) < 1$ is a parameter. For the DCG model, the MSA contains the following steps:

**Step 1**: Initialization. Set the iteration index $n = 1$. Calculate an initial solution

$$T_{ij}^{(n)} = O_i A_{ij}^\alpha d_{ij}^{-\beta}$$

**Step 2**: Calculate a new solution

$$F_{ij}^{(n)} = O_i A_{ij}^\alpha d_{ij}^{-\beta} (D_{ij}^{(n)})^{-\gamma}$$

where $D_{ij}^{(n)} = \sum_k T_{ij}^{(n)}$.

**Step 3**: Calculate the average solution

$$T_{ij}^{(n+1)} = (1 - \lambda(n))T_{ij}^{(n)} + \lambda(n)F_{ij}^{(n)}.$$  

If $|T_{ij}^{(n+1)} - T_{ij}^{(n)}| < \varepsilon$ ($\varepsilon$ is a very small threshold, set as 0.01 in the work), the algorithm stops with current solution being the approximated solution; Otherwise, let $n = n + 1$ and return to **Step 2**.

For simplicity, we use a fixed parameter $\lambda(n) = \lambda = 0.5$. 

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**FIG. 3.** Comparing predicting accuracy of the DCG model and well-known benchmarks in terms of SSI.
Weber-Fechner Law

Weber-Fechner Law (WFL) is a well-known law in behavioural psychology [19], which represents the relationship between human perception and the magnitude of a physical stimulus. WFL assumes the differential change in perception $dp$ to be directly proportional to the relative change $dW/W$ of a physical stimulus with size $W$, namely $dp = \kappa dW/W$, where $\kappa$ is a constant. From this relation, one can derive a logarithmic function $p = \kappa \ln(W/W_0)$, where $p$ equals the magnitude of perception, and the constant $W_0$ can be interpreted as stimulus threshold. This equation means the magnitude of perception is proportional to the logarithm of the magnitude of physical stimulus. The WFL is widely used to determine the explicit quantitative utility function in behavioural economics [19], and thus we adopt it in Eq. (2).

Sørensen Similarity Index

Sørensen similarity index is a similarity measure between two samples [20]. Here we apply a modified version [38] of the index to measure whether real fluxes are correctly reproduced (on average) by theoretical models, defined as

$$SSI \equiv \frac{1}{N(N-1)} \sum_i \sum_{j \neq i}^{N} \frac{2 \min(T'_{ij}, T_{ij})}{T_{ij} + T'_{ij}},$$

where $T'_{ij}$ is the predicted fluxes from location $i$ to $j$ and $T_{ij}$ is the empirical fluxes. Obviously, if each $T'_{ij}$ is equal to $T_{ij}$, the index is 1, while if all $T'_{ij}$ are far from the real values, the index is close to 0.

Benchmark Models

We select two classical models, the gravity model and the intervening opportunities model, and two parameter-free models, the radiation model and the population-weighted opportunities model, as the benchmark models for comparison with the DCG model.

(i) The gravity model is the earliest proposed and the most widely used spatial interaction model [39]. The basic assumption is that the flow $T_{ij}$ between two locations $i$ and $j$ is proportional to the population $m_i$ and $m_j$ of the two locations and inversely proportional to the power function of the distance $d_{ij}$ between the two locations, as

$$T_{ij} = \frac{\alpha m_i m_j}{d_{ij}^\beta},$$

where $\alpha$ and $\beta$ are parameters. To guarantee the predicted flow matrix $T$ satisfies $O_i = \sum_j T_{ij}$, we use two origin-constrained gravity models [24]. The first one is called Gravity 1 as it has only one parameter, namely

$$T_{ij} = O_i \frac{A_j d_{ij}^{-\beta}}{\sum_j A_j d_{ij}^{-\beta}}.$$  (11)

(ii) The intervening opportunities (IO) model [8] argues that the destination choice is not directly related to distance but to the relative accessibility of opportunities to satisfy the traveller. The model’s basic assumption is that for an arbitrary traveller departed from the origin $i$, there is a constant very small probability $\alpha/\beta$ that this traveller is satisfied with a single opportunity. Assume the number of opportunities at the $j$th location (ordered by its distance from $i$) is proportional to its population $m_j$, i.e., the number of opportunities is $\beta m_j$, and thus the probability that this traveller is attracted by the $j$th location is approximated $\alpha m_j$. Let $q_i^{(j)} = q_i^{(j-1)}(1-\alpha m_j)$ be the probability that this traveller has not been satisfied by the first to the $j$th locations ($i$ itself can be treated as the 0th location), we can get the relationship $q_i^{(j)} = e^{-\alpha S_{ij}/(1-e^{-\alpha M})}$ between the probability $q_i^{(j)}$ and the total population $S_{ij}$ in the circle of radius $d_{ij}$ centred at location $i$, where $M$ is the total population of all locations. Furthermore, we can get the expected fluxes from $i$ to $j$ is

$$T_{ij} = O_i(q_i^{(j-1)} - q_i^{(j)}) = O_i \frac{e^{-\alpha (S_{ij} - m_j)} - e^{-\alpha S_{ij}}}{1 - e^{-\alpha M}}.$$  (12)

(iii) The radiation model [10] assumes that an individual at location $i$ will select the nearest location $j$ as destination, whose benefits (randomly selected from an arbitrary continuous probability distribution $p(z)$ where $z$ is proportional to the population $m_j$) are higher than the best offer available at the origin $i$. The fluxes $T_{ij}$ predicted by the radiation model is

$$T_{ij} = O_i \frac{m_j m_i}{(S_{ij} - m_j) S_{ij}}.$$  (13)

(iv) The population-weighted opportunities (PWO) model [12] assumes that the probability of travel from $i$ to $j$ is proportional to the attractiveness of destination $j$, inversely proportional to the population $S_{ij}$ in the circle centred at the destination with radius $d_{ij}$, minus a finite-size correction $1/M$. It results to the analytical solution as

$$T_{ij} = O_i \frac{m_j (\frac{1}{S_{ij}} - \frac{1}{M})}{\sum_j m_j (\frac{1}{S_{ij}} - \frac{1}{M})}.$$  (14)
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