Parameter-Independent Quark Mass Relation
in the U(3)×U(3)' Model

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Abstract

Recently, we have proposed a quark mass matrix model based on U(3)×U(3)' family symmetry, in which up- and down-quark mass matrices $M_u$ and $M_d$ are described only by complex parameters $a_u$ and $a_d$, respectively. When we use charged lepton masses as additional input values, we can successfully obtain predictions for quark masses and Cabibbo-Kobayashi-Maskawa mixing. Since we have only one complex parameter $a_q$ for each mass matrix $M_q$, we can obtain a parameter-independent mass relation by using three equations for $\text{Tr}[H_q]$, $\text{Tr}[H_qH_q]$ and $\det H_q$, where $H_q \equiv M_qM_q^\dagger$ ($q = u, d$). In this paper, we investigate its parameter-independent feature of the quark mass relation in the model.

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1 Introduction

Recently, we have proposed a quark mass matrix model\textsuperscript{1} based on U(3)×U(3)' symmetry, in which mass matrices for up-quarks, down-quarks, charged leptons, and neutrinos, $M_f$ ($f = u, d, e, \nu$), are described respectively with only one complex parameters $a_f$ by

\begin{equation}
(M_f)_{ij} = m_{0f}(\Phi_f)_{ij}^\alpha(S_f^{-1})_{\alpha\beta}(\bar{\Phi}_f)_{\beta j}.
\end{equation}

Here $\Phi_f$ and $S_f$ are vacuum expectation values (VEVs) matrices. The indexes $i, j = 1, 2, 3$ are ones of U(3) family and $\alpha, \beta = 1, 2, 3$ are indexes of U(3)' family. Although $\Phi_f$ and $S_f$ have a dimension of "mass", we put the factor $m_{0f}$ with a dimension of mass in Eq.(1.1), since we treat those as dimensionless quantities as seen in (1.2), (1.3) and (1.5) later.

In (1.1), $M_\nu$ is a Dirac neutrino mass matrix. Although we consider that the observed neutrinos are Majorana neutrinos and the Majorana neutrino mass matrix is given by a similar mechanism \textsuperscript{1, 2} to the so-called neutrino seesaw mechanism \textsuperscript{3}, we do not discuss the structure of $M_\nu$ in the present paper because the purpose of the present paper is to discuss the quark mass relation.

We define structure of the matrix $\Phi_f$ as an dimensionless expression

\begin{equation}
\Phi_f = \Phi_0 P_f,
\end{equation}

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where
\[ \Phi_0 = \text{diag}(z_1, z_2, z_3), \tag{1.3} \]
\[ P_f = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}). \tag{1.4} \]

Since we consider that the U(3)' is broken into a discrete symmetry S_3, the matrix \((S_f^{-1})\) is given by
\[ (S_f)^{-1} = (1 + a_f X) = (1 + b_f X)^{-1}, \tag{1.5} \]
where
\[ 1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{1.6} \]
and \(a_f\) is a complex parameter:
\[ a_f = -\frac{b_f}{1 + b_f}. \tag{1.7} \]

Only for the charged lepton mass matrix \(M_e\), the parameter \(a_e\) is given by \(a_e = 0\), so that the mass matrix \(M_e\) is given by
\[ M_e = m_{0e} \Phi_e \Phi_e^\dagger = m_{0e} \Phi_0 \Phi_0, \tag{1.8} \]
where we can put \(P_e = 1\) without losing a generality. Namely, we take \(S_e = 1\) only for \(f = e\). Therefore, the parameters \(z_i\) (\(i = 1, 2, 3\)) are given by
\[ z_i = \sqrt{\frac{m_{ei}}{m_{e1} + m_{e2} + m_{e3}}}, \tag{1.9} \]
where \((m_{e1}, m_{e2}, m_{e3}) = (m_e, m_\mu, m_\tau)\). Here, as the input values \((m_e, m_\mu, m_\tau)\), the running mass values of the charged leptons at a scale \(\mu = m_Z\), \((m_e, m_\mu, m_\tau) = (0.000486849 \text{ GeV}, 0.102751 \text{ GeV}, 1.7467 \text{ GeV})\), are used, not the pole masses, because the predicted quark mass values are calculated at the scale \(\mu = m_Z\). (The study of the quark mass matrix (1.1) with the form (1.5) has been substantially done in Ref.[4] although the model has been based on U(3)-family symmetry, not U(3)×U(3)'.)

In this model, when we choose suitable values of the complex parameters \(a_q\) (\(q = u, d\)) together with additional input values, \((m_e, m_\mu, m_\tau)\), we can successfully obtain [1] predictions for quark masses and Cabibbo-Kobayashi-Maskawa (CKM) mixing [5]. However, so far, it is not clear whether the successful parameter fitting is unique or not, and that there are another good parameter solutions or not.

In order to settle these questions, parameter-independent mass relations are useful, which can be obtained in this model; We have three independent equations for the each mass matrix.
$M_q$ ($q = u, d$), while we have only one complex parameter $a_q$, therefore we can obtain one mass relation. In this paper, we investigate such the parameter-independent mass relation in the U(3)×U(3)′ model. At present, the observed quark mass values, especially, for the first generation quarks have considerably large error, i.e. $m_u = 1.27^{+0.50}_{-0.42}$ MeV and $m_d = 2.90^{+1.24}_{-1.19}$ MeV at $\mu = m_Z$ \cite{6}. By obtaining such a parameter-independent quark mass relation, we check whether the U(3)×U(3)′ model is reasonable or not and what values of $m_u$ and $m_d$ are acceptable to the U(3)×U(3)′ model.

2 Brief review of the U(3)×U(3)′ model

In our model based on U(3)×U(3)′ symmetry\cite{1,2}, we consider hypothetical fermions $F_\alpha$ ($\alpha = 1, 2, 3$), which belong to (1,3) of U(3)×U(3)′, in addition to quarks and leptons $f_i$ ($i = 1, 2, 3$) which belong to (3,1).

We assume that the VEV form (1.1) originates from the following 6×6 mass matrix model:

\[
\begin{pmatrix}
\bar{f}_L & F_\alpha
\end{pmatrix}
\begin{pmatrix}
(0)_{i}^{j} & (\Phi_f)_{i}^{j}_{\beta} \\
(\Phi_f)_{\alpha}^{j} & -(S_f)_{\alpha}^{j}
\end{pmatrix}
\begin{pmatrix}
\bar{f}_Rj \\
F_{R\beta}
\end{pmatrix}, \tag{2.1}
\]

Here, $F_{L(R)}$ are heavy fermions with (1,1,3) of SU(2)$_L$×SU(3)×SU(3)′. On the other hand, $f_R$ are right-handed quarks and leptons, $f_R = (u, d, \nu, e^-)_R$, while $f_L$ are not physical fields. They are given by the following combinations:

\[
f_L \equiv (f_u, f_d, f_\nu, f_e)_L \equiv \left( \frac{1}{\Lambda_H} H_u^L q_L, \frac{1}{\Lambda_H} H_d^L q_L, \frac{1}{\Lambda_H} H_u^L \ell_L, \frac{1}{\Lambda_H} H_d^L \ell_L \right), \tag{2.2}
\]

where $\Lambda_H$ is a flavon VEV scale, and

\[

t_L = \left( \begin{array}{c}
u_L \
e_L
\end{array} \right), \quad \ell_L = \left( \begin{array}{c}H_u \\H_d
\end{array} \right), \quad H_u = \left( \begin{array}{c}H_u^0 \\H_u^-
\end{array} \right), \quad H_d = \left( \begin{array}{c}H_d^+ \\H_d^0
\end{array} \right). \tag{2.3}
\]

In other words, the matrix given in Eq.(2.1) denotes would-be Yukawa coupling constants.

After the U(3) and U(3)′ have been completely broken, the quarks and leptons are described by the effective Hamiltonian

\[
\mathcal{H}_Y = (\bar{\nu}_L)^i (M_\nu)_{ij} (\nu_R)_j + (\bar{\ell}_L)^i (M_\ell)_{ij} (\ell_R)_j + y_R (\bar{\nu}_R)^i (Y_R)_{ij} (\nu_R^c)^j + (\bar{u}_L)^i (M_u)_{ij} (u_R)_j + (\bar{d}_L)^i (M_d)_{ij} (d_R)_j. \tag{2.4}
\]

Note that the quarks and leptons $f_i$ are not U(3) family triplet any more in the exact meaning, but they are mixing states between the fermions $f$ and $F$. However, for convenience, we will still use the index of U(3) family for these fermion states.
By performing a seesaw-like approximation with \( \Lambda_2 = O(\langle \Phi_f \rangle) \ll \Lambda_1 = O(\langle S_f \rangle) \), the mass matrix (2.1) leads to the following Dirac mass matrix of quarks and leptons:

\[
(M_f)_{ij} \approx \frac{\langle H_u/d \rangle}{\Lambda_H} \langle \Phi_f \rangle^i \langle (S_f)^{-1} \rangle^\alpha \langle \bar{\Phi}_f \rangle^j \langle \Phi_f \rangle^\beta.
\] (2.5)

However, in this paper, since we interest only in the relative ratios of the quark masses in the same sector \((q = u \text{ or } q = d)\), the factor \( \langle H_u/d \rangle/\Lambda_H \) takes a common value, so that the factor \( \langle H_u/d \rangle/\Lambda_H \) does not play any essential role in our study.

As seen in this section, \( \Phi_f \) and \( S_f \) have a dimension of mass. However, for convenience, hereafter, we use a dimensionless expressions (1.2) and (1.5), and define the parameter \( m_{0f} \) with a mass dimension by Eq.(1.1).

### 3 Three equations for mass relations

When we define an Hermitian mass matrix

\[ H_q = M_q M_q^\dagger, \] (3.1)

the explicit form of \( H_q \) is given by

\[
H_q = M_q M_q^\dagger = m_{0q}^2 \Phi_q S_q^{-1} \Phi_q^\dagger \Phi_q (S_q^\dagger)^{-1} \Phi_q^\dagger
\]
\[
= k_q^2 P_q D_q^{1/2} (1 + a_q X) D_q (1 + a_q X) D_q^{1/2} P_q^\dagger,
\] (3.2)

where

\[ k_q = \frac{m_{0q}}{m_{0e}}. \] (3.3)

The Hermitian matrix \( H_q \) is diagonalized as

\[ U_q H_q U_q^\dagger = D_q^2 \equiv \text{diag}(m_{q_1}^2, m_{q_2}^2, m_{q_3}^2). \] (3.4)

Hereafter, for convenience, we define

\[ (m_1, m_2, m_3) = \frac{1}{k_q} (m_{q_1}, m_{q_2}, m_{q_3}), \] (3.5)

and

\[ \tilde{D}_q \equiv \text{diag}(m_1, m_2, m_3) = \frac{1}{k_q} D_q. \] (3.6)

In general, we have the following three equations for the matrix \( H_q \):

\[ c_1 \equiv m_1^2 + m_2^2 + m_3^2 = \text{Tr}[\tilde{D}_q^2] = \frac{1}{k_q^2} \text{Tr}[H_q], \] (3.7)
\[ c_2 \equiv m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_1^2 = \frac{1}{2} \left( \text{Tr}[\tilde{D}_q^2 \tilde{D}_q^2] - (\text{Tr}[\tilde{D}_q^2])^2 \right) \]

\[ = \frac{1}{k_q^2} \frac{1}{2} \left\{ \text{Tr}[H_q H_q] - (\text{Tr}[H_q])^2 \right\}, \quad (3.8) \]

and

\[ c_3 \equiv m_1^2 m_2^2 m_3^2 = \det \tilde{D}_q^2 = \frac{1}{k_q^6} \det H_q. \quad (3.9) \]

For convenience, hereafter, we denote \( \text{Tr}[A] \) as \( [A] \) simply. By using the explicit form (3.2), we obtain \( c_1, c_2 \) and \( c_3 \) as follows:

\[ c_1 = \left| 1 + \frac{1}{3} a_q \right|^2 \left( D_e^2 \right) + \frac{1}{9} |a_q|^2 ([D_e]^2 - [D_e^2]), \quad (3.10) \]

\[ c_2 = \frac{1}{2} \left| 1 + \frac{2}{3} a_q \right|^2 \left( [D_e^2]^2 - [D_e^2^4] \right) + \frac{2}{9} |a_q|^2 [D_e] \det D_e, \quad (3.11) \]

\[ c_3 = |1 + a_q|^2 (\det D_e)^2 = |1 + a_q|^2 \det D_e^2. \quad (3.12) \]

Let us define

\[ Q_1 \equiv \frac{[D_q^2]^2}{[D_e^2]^2}, \quad Q_2 \equiv \frac{[D_q^4]^2 - [D_q^6]}{[D_e^2]^2 - [D_e^4]}, \quad Q_3 \equiv \frac{\det D_q^2}{\det D_e^2}, \quad (3.13) \]

and

\[ L_1 \equiv \frac{[D_e]^2 - [D_e^2]}{[D_e^2]^2}, \quad L_2 \equiv \frac{[D_e] \det D_e}{[D_e^2]^2 - [D_e^4]}. \quad (3.14) \]

then, we obtain the following relations:

\[ Q_1 = \left| 1 + \frac{1}{3} a_q \right|^2 + \frac{1}{9} |a_q|^2 L_1, \quad (3.15) \]

\[ Q_2 = \left| 1 + \frac{2}{3} a_q \right|^2 + \frac{4}{9} |a_q|^2 L_2, \quad (3.16) \]

\[ Q_3 = |1 + a_q|^2. \quad (3.17) \]

Note that \( L_1 \) and \( L_2 \) are given only by the charged lepton masses, and \( Q_1, Q_2 \) and \( Q_3 \) are expressed by quark masses \( (m_1, m_2, m_3) \) after \( (m_e, m_\mu, m_\tau) \) are substituted.
Finally, by eliminating the parameter $a_q$ from Eqs. (3.15) - (3.17), we obtain the mass relation

$$-b_0 + b_1(m_1^2 + m_2^2 + m_3^2) - b_2(m_1^2m_2^2 + m_2^2m_3^2 + m_3^2m_1^2) + b_3m_1^2m_2^2m_3^2 = 0.$$  (3.18)

Here the coefficients $b_0$, $b_1$, $b_2$, and $b_3$ are defined by

$$b_0 = \left(1 + \frac{1}{2}L_1 - 4L_2\right),$$  (3.19)

$$b_1 = 3(1 - 2L_2)\left[\frac{1}{[D_e^2]}\right],$$  (3.20)

$$b_2 = 6\left(1 + \frac{1}{2}L_1\right)\left[\frac{1}{[D_e^2] - [D_e^4]}\right],$$  (3.21)

$$b_3 = (1 - L_1 + 2L_2)\left[\frac{1}{\det D_e^2}\right],$$  (3.20)

which are expressed only by the charged lepton masses.

4 Behavior of $m_1/m_2$ versus $m_2/m_3$

In order to investigate the behavior of $m_1/m_2$ versus $m_2/m_3$, we define parameters

$$x \equiv \frac{m_2}{m_3} = \frac{m_{q2}}{m_{q3}}, \quad y \equiv \frac{m_1}{m_2} = \frac{m_{q1}}{m_{q2}}.$$  (4.1)

Note that the parameters $x$ and $y$ are independent of the value of $k_q$ defined in (3.3). Then, since $(m_1, m_2, m_3)$ are expressed as

$$(m_1, m_2, m_3) = (xy, x, 1)m_3,$$  (4.2)

the relation (3.18) becomes

$$-b_0 + b_1m_3^2(1 + x^2 + x^2y^2) - b_2m_3^4(x^2 + x^2y^2 + x^4y^2) + b_3m_3^6x^4y^2.$$  (4.3)

Therefore, we get a relation $y = f(x)$:

$$y = \frac{1}{x}\sqrt{\frac{(b_1m_3^2 - b_2m_3^4)x^2 - (b_0 - b_1m_3^2)}{(b_2m_3^4 - b_3m_3^6)x^2 - (b_1m_3^2 - b_2m_3^4)}}.$$  (4.4)

As seen in (4.4), the function $y = f(x)$ has poles at

$$x = 0, \quad \text{and} \quad x = \pm\frac{b_1m_3^2 - b_2m_3^4}{b_2m_3^4 - b_3m_3^6},$$  (4.5)
and a zero point at
\[ x = \pm \sqrt{\frac{b_0 - b_1 m_3^2}{b_1 m_3^2 - b_2 m_3}}. \]  

(4.6)

The explicit values \( b_0, b_1, b_2 \) and \( b_3 \) are given by

\begin{align*}
    b_0 &= 1.04888, \\
    b_1 &= 0.97489 \text{ (GeV)}^{-2}, \\
    b_2 &= 87.6457 \text{ (GeV)}^{-4}, \\
    b_3 &= 1.16204 \times 10^8 \text{ (GeV)}^{-6}.
\end{align*}

(4.7)

Here we have used running mass values \( m_e(\mu) = 0.000486847 \text{ GeV} \), \( m_\mu(\mu) = 0.102751 \text{ GeV} \) and \( m_\tau(\mu) = 1.7467 \text{ GeV} \) as the charged lepton mass values at \( \mu = m_Z \) [6].

The behavior of \( y = f(x) \) is illustrated in Fig.1. The behavior depends on the input value of \( m_3 \). Note that since

\[ \frac{b_0}{b_1} = 1.0758 \text{ (GeV)}^2, \]

(4.8)

the factor \((b_0 - b_1 m_3^2)\) in (4.4) changes the sign according as \( m_3 > m_{30} \) or \( m_3 < m_{30} \), where

\[ m_{30} \equiv \sqrt{\frac{b_0}{b_1}} = 1.0373 \text{ GeV}. \]

(4.9)

Hereafter, we call the behavior in the case \( m_3 > m_{30} \) as normal type, and the behavior in the case \( m_3 < m_{30} \) as non-normal type.

Now, let us compare our parameter-independent results with the observed quark mass values in detail. The observed quark mass values at \( \mu = m_Z \) [6] are as follows:

\begin{align*}
    m_u &= 0.00127^{+0.00050}_{-0.00042} \text{ GeV}, \quad m_c = 0.619 \pm 0.084 \text{ GeV}, \quad m_t = 171.7 \pm 3.0 \text{ GeV}, \\
    m_d &= 0.00290^{+0.00124}_{-0.00119} \text{ GeV}, \quad m_s = 0.055^{+0.016}_{-0.013} \text{ GeV}, \quad m_b = 2.89 \pm 0.09 \text{ GeV}.
\end{align*}

(4.10)

Hereafter, we use the following values as the mass ratios \( m_1/m_2 \) and \( m_2/m_3 \):

\begin{align*}
    \frac{m_u}{m_c} &= 0.00205^{+0.00126}_{-0.00084}, \quad \frac{m_c}{m_t} = 0.00361^{+0.00056}_{-0.00054}, \\
    \frac{m_d}{m_s} &= 0.0527^{+0.0508}_{-0.0286}, \quad \frac{m_s}{m_b} = 0.0190^{+0.0063}_{-0.00056}.
\end{align*}

(4.11)
Figure 1: $m_3$ dependence of the behavior of $m_1/m_2$ versus $m_2/m_3$. The curves of the mass relation $y = f(x)$ given in Eq.(4.4) are drawn in the $(m_2/m_3, m_1/m_2)$ plane for the cases (a) $m_3 = 1$ GeV, (b) $m_3 = 30$ GeV, and (c) $m_3 = 100$ GeV. The shaded square regions are correspond to the observed mass ratios in (4.11) and (4.12) for up-quark sector and down-quark sector respectively obtained by Xing et. al.

As seen in Fig.1, the behavior of $m_1/m_2$ in the normal type has a maximum whose value is smaller than $\sim 10^{-2}$. On the other hand, as seen in Eq.(4.12), the observe value of $m_d/m_s$ is $m_d/m_s \simeq 0.05$. Therefore, the mass ratios for down-quark sector cannot be described by the behavior of the normal type. Thus we have the solution for Eqs.(4.11) and (4.12) by the behavior of the normal type for the up-quark sector, and by the behavior of the non-normal type for the down-quark sector.

**Down-quark sector**

First, let us see behaviors of the mass ratios $(m_1/m_2, m_2/m_3)$ in the down-quark sector.

As seen in Fig.2, we can determine a value $m_3$ from the observed center values in (4.12) as $m_3 = 1.03$ GeV. If we take $m_3 > 1.04$ GeV, the behavior of the mass ratios becomes the normal type from non-normal type as seen in the curve (c) of Fig.2. Furthermore, if we take a larger value $m_3 > 1.05$ GeV, then the curve is out of the error region as seen in the curve (d) of Fig.2. Similarly, there is no solution of $m_3$ for $m_3 < 0.91$ GeV as seen in the curve (a) of Fig.2. Thus
Figure 2: The $m_3$ value in the mass relation consistent with the observed values $m_d/m_s$ and $m_s/m_b$ in the down-quark sector. The curves of the mass relation $y = f(x)$ in Eq.(4.4) are drawn in the $(m_2/m_3, m_1/m_2)$ plane for the cases with (a) $m_3 = 0.91$ GeV, (b) $m_3 = 1.03$ GeV, (c) $m_3 = 1.04$ GeV, and (d) $m_3 = 1.05$ GeV. The each curve in the case of (c) and (d) has a left end point, in the left region from which there is no real solutions for $m_1/m_2$. The shaded square region is correspond to the observed mass ratios in (4.12) for down-quark sector obtained by Xing et. al.

we obtain

$$m_3 = 1.03^{+0.02}_{-0.12} \text{ GeV},$$

(4.13)

from the consistency between and the mass relation (4.4) and the observed mass ratios (4.12).

If we take $m_3 = 1.03$ GeV, we obtain

$$k_d = \frac{m_b}{m_3} = 2.89,$$

(4.14)

from the definition (3.5). In this choice of the value of $k_d$, we have

$$m_b = 2.98^{+0.06}_{-0.35} \text{ GeV},$$

(4.15)

which has smaller error bar for upper limit than that of the observed $m_b$ in (4.10).

**Up-quark sector**
Figure 3: The $m_3$ value in the mass relation consistent with the observed values $m_u/m_c$ and $m_c/m_t$ in the up-quark sector. The curves of the mass relation $y = f(x)$ in Eq.(4.4) are drawn in the $(m_2/m_3, m_1/m_2)$ plane for the cases with (a) $m_3 = 56.5$ GeV, (b) $m_3 = 114$ GeV, and (c) $m_3 = 228$ GeV. The each curve has a left end point, in the left region from which there is no real solution for $m_1/m_2$. The shaded square region is correspond to the observed mass ratios in (4.11) for up-quark sector obtained by Xing et. al.

In order to get a reasonable value of $m_3$ in the mass relation, we illustrate curves of the mass relation for several values of $m_3$ in Fig. 3 and Fig 4. We find that there are two solutions of the $m_3$ which are consistent with the observed up-quark mass ratios (4.11) as seen in Figs.3 and 4:

$$m_3 = 30.5^{+8.5}_{-4.8} \text{GeV}, \quad m_3 = 114^{+114}_{-57} \text{GeV.} \quad (4.16)$$

Both solutions can give reasonable quark mass ratios $(m_u/m_c, m_c/m_t) \simeq (2.4, 3.6) \times 10^{-3}$. However, those two center values $m_3 = 30.5$ GeV and $m_3 = 114$ GeV give

$$k_u = 5.63 \quad \text{and} \quad k_u = 1.51, \quad (4.17)$$

respectively. On the other hand, we have obtained $k_d = 2.9$ in the down-quark sector as seen in Eq.(4.14). Therefore, whichever we take the value in (4.17), the value is poor agreement with $k_d = 3$. It is natural to consider that the relations of quark mass matrices $M_u$ and $M_d$ to the charged lepton mass matrix $M_e$ take the same weight between the up- and down-quark sectors, i.e. $k_u = k_d$, except for the parameters $a_u$ and $a_d$. If we want to consider $k_u = k_d = 3$, we have
Figure 4: Another $m_3$ value in the mass relation consistent with the observed values $m_u/m_c$ and $m_c/m_t$ in the up-quark sector. The curves of the mass relation $y = f(x)$ in Eq.(4.4) are drawn in the $(m_2/m_3, m_1/m_2)$ plane for the cases with (a) $m_3 = 25.7$ GeV, (b) $m_3 = 30.5$ GeV, and (c) $m_3 = 39$ GeV. The each curve has a left end point, in the left region from which there is no real solution for $m_1/m_2$. The shaded square region is correspond to the observed mass ratios in (4.11) for up-quark sector obtained by Xing et. al.

First generation quark masses

Next, we see the constraints on the first generation quark masses $m_u$ and $m_d$. From the curves (a) $m_3 = 0.91$ GeV, (b) $m_3 = 1.03$ GeV and (c) $m_3 = 1.04$ GeV in Fig.2, we obtain

$$m_d = 2.9^{+3.7}_{-1.8} \text{ MeV},$$ \hspace{1cm} (4.18)

where we have used the input value $[6] (m_s)^{obs} = 55$ MeV. Our result (4.18) has wide error compared with the observed value $[6] (m_s)^{obs} = 2.90^{+1.24}_{-1.19}$ MeV. Similarly, from Figs.3 and 4, we obtain the following two solutions of $m_u$,

$$m_u = 1.49^{+0.45}_{-0.75} \text{ MeV}, \quad \text{and} \quad m_u = 1.3^{+0.7}_{-0.5} \text{ MeV},$$ \hspace{1cm} (4.19)
respectively, where we have taken \((m_c)^{\text{obs}} = 0.619\) GeV. We also see that our results (4.19) cannot put any severe constraint on the observed value \((m_u)^{\text{obs}} = 1.27^{+0.50}_{-0.42}\) MeV.

5 Concluding remarks

In conclusion, we have investigate a parameter-independent quark mass relation in the \(U(3) \times U(3)\)' model. Considering our results with the observed quark mass values \([6]\), we conclude that the choice \(k_u = k_d = 3\) in the previous work \([1]\) of the explicit parameter fitting of \(a_u\) and \(a_d\) was reasonable. However, we have found that there are two solutions in the up-quark sector as we have shown in Figs.3 and 4. This is not so serious problem when we take the error range of the observed quark mass values into consideration.

We are convinced that our parameter-independent analysis of the mass relation is useful for model checking in future study of the \(U(3) \times U(3)'\) model.

We did not investigate a similar parameter-independent study for the CKM mixing. The similar study of the CKM matrix elements \(V_{ij}\) cannot been obtained unless the results include quark masses. We are interested in the values \(V_{us}, V_{cb}, V_{td}\) and so on, while those values will be disturbed by existence of large elements \(V_{ud}, V_{cs}, V_{tb}\) and so on. It is our future task to obtain a relation without such large contribution terms.

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References

[1] Y. Koide and H. Nishiura, Phys.Rev. D 92, 111301(R) (2015).
[2] Y. Koide and H. Nishiura, IJMPA 32, 1750085 (2017).
[3] P. Minkowski, Phys. Lett. B 67, 421 (1977); M. Gell-Mann, P. Ramond, and R. Slansky, Proceedings of the Supergravity Stony Brook Workshop, New York, 1979, edited by P. Van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, Japan 1979, edited by A. Sawada and A. Sugamoto [KEK Report No. 79-18, Tsukuba]; R. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980); J. Schechter and J. W. F. Valle, Phys. Rev. D 22, 2227 (1980).
[4] Y. Koide and H. Fusaoka, Z. Phys. C 71, 459 (1996).
[5] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
[6] Z.-z. Xing, H. Zhang, and S. Zhou, Phys. Rev. D 77, 113016 (2008). And also see H. Fusaoka and Y. Koide, Phys. Rev. D 57, 3986 (1998).