Promote and Application of Rigid Body Moment of Inertia Experiment on College Students' Humanistic Quality Ability

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Abstract. In the experiment of rotational inertia of rigid body, there is often a big difference between the experimental value and the theoretical value, and the error source cannot be found accurately, which leads to the failure of the experiment. In this paper, the basic principle and formula derivation of the tri-linear pendulum method for measuring the rotational inertia of rigid body are given. The physical meaning of each character in the expression of moment of inertia and its possible influence on the accuracy of experimental results are clarified. Secondly, through the measurement of the original data, calculate the experimental results, guide students to analyze the source of error independently and put forward measures to reduce the error. Thirdly, the influence of different factors on the experimental results is analyzed, and the factors influencing the moment of inertia of rigid body are summarized. Finally, this paper discusses how to excavate physical thought, research methods and humanistic knowledge in the process of physical experiment course, and gives the measures and countermeasures to cultivate and improve the scientific and technological ability and humanistic quality of college students, so as to realize the purpose of mutual penetration of scientific knowledge and humanistic quality.

Keywords: Rotational inertia, tri-linear pendulum method, error analysis, physical thinking, humanistic quality.

1. Introduction
The rotational inertia is a measure of the inertia of a rigid body when it rotates around an axis, which is an important physical quantity to study the law of rigid body rotation [1-3]. It is also an important parameter in engineering technology, scientific experiment, machinery, electric power and other industrial fields [4-5]. In our daily life, objects can be divided into regular and irregular bodies according to their shapes. Uniform rigid bodies with regular shapes can be directly calculated by formula. However, it is difficult to calculate the rotational inertia for irregular shaped non-uniform rigid body, and many of them cannot be solved. Therefore, it is necessary to use experimental means to measure the moment of inertia of the rigid body [6-7].

The experiment of measuring the rotational inertia of rigid body by tri-linear pendulum method is always a classic experiment in college physics because of its clear thinking and simple method [8-9]. According to the law of mechanical energy conversion and conservation, this paper deduces the...
expression of the rotational inertia of the disk measured by the tri-linear pendulum method, gives the detailed experimental data, analyses and discusses the influence of different physical quantities on the accuracy of the experimental results, and puts forward specific measures and suggestions to reduce the error. The research results can provide certain data reference and guidance for experimental teaching.

Physical experiment has developed into an independent course. In the university, this course is mainly opened for freshmen, and it is a compulsory course for science and engineering students. This is because physical experiment itself has a set of systematic experimental knowledge, methods, habits and skills, which is also the necessary basis for them to receive professional study or engage in scientific research in the future. At present, under the background of new engineering, it is required to give full play to the scientific and technological role of physical experiments, as well as the role of physical experiments in cultivating college students' humanistic quality. Therefore, from the perspective of physical experiment teaching, we should fully dig out the humanistic quality elements contained in it, realize the function of humanistic quality education, and improve the humanistic quality of college students through effective training ways and countermeasures [10].

2. Basic theory and formula derivation

As shown in Figure 1, when the suspension line is at $BA$ position, it is the balance position of the tri-linear pendulum. When the maximum angular displacement of the lower disc $\theta_0$ occurs, the suspension line moves to $BA_1$ position, as shown by the dotted line in the figure. At this time, the center of gravity of the lower disk rises $h$. The potential energy at the equilibrium position is zero, the kinetic energy of the lower disk is zero when the maximum angular displacement $\theta_0$ occurs.

If the friction resistance and the translational kinetic energy of the up and down motion of the mass center of the disc are neglected, according to the law of conservation of mechanical energy, it can be obtained that

$$m_0gh = \frac{1}{2}J_0\omega_0^2$$

Where $m_0$ is the mass of the lower disk, $g$ is the acceleration of gravity, $J_0$ and $\omega_0$ are respectively the rotational inertia of the lower disk and the angular velocity passing through the equilibrium position.

![Figure 1. Diagram of geometric analysis of tri-liner pendulum](image-url)
If the maximum angular displacement $\theta_0$ is very small, it can be proved that the lower disk will vibrate simply. According to the law of harmonic vibration, the rotation angle $\theta$ of the lower disk relative to the equilibrium position at any time $t$ is

$$\theta = \theta_0 \cos \left( \frac{2\pi \theta_0}{T_0} t + \varphi \right)$$

(2)

Where $T_0$ is the vibration period of the lower disk and $\varphi$ is the initial phase. The angular velocity of vibration can be obtained by derivation

$$\omega = \frac{d\theta}{dt} = -\frac{2\pi}{T_0} \sin \left( \frac{2\pi \theta_0}{T_0} t + \varphi \right)$$

(3)

When the lower disc passes through the balance position, the maximum angular velocity is $\omega_0 = \frac{2\pi}{T_0} \theta_0$. Substituting Equation (2) and (3) into Equation (1), we can get

$$m_0 g h = \frac{1}{2} I_0 \left( \frac{2\pi}{T_0} \theta_0 \right)^2$$

(4)

After finishing, the rotational inertia of the lower disc can be obtained

$$J_0 = \frac{m_0 g T_0^2}{2\pi^2 \theta_0^2} h$$

(5)

According to the geometric relations of Figure. 1, there are

$$h = O_1 O = BC - BC_1 = \frac{(BC)^2 - (BC_1)^2}{BC + BC_1}$$

(6)

$$BC^2 = (AB)^2 - (AC)^2 = l^2 - (R - r)^2$$

$$BC_1^2 = (A_1 B)^2 - (A_1 C_1)^2 = l^2 - (R^2 + r^2 - 2Rr \cos \theta_0)$$

Where, $l$ is the length of the three-line suspension line, $r$ and $R$ are the radius of the upper and lower disks respectively. After finishing

$$h = \frac{2Rr(1-\cos \theta_0)}{BC + BC_1} = \frac{4Rr \sin^2 \frac{\theta_0}{2}}{BC + BC_1}$$

(7)

When the deflection angle is very small, $\sin \frac{\theta_0}{2} \approx \frac{\theta_0}{2}$. Let $OO' = H$, $H$ is the vertical distance between the upper and lower disks. When $l \gg R$, $BC \approx BC_1 \approx H$, so $h = \frac{Rr \theta_0^2}{2H}$, substituting $h$ into Equation (5), can get

$$J_0 = \frac{m_0 g Rr}{4\pi^2 H} T_0^2$$

(8)

It can be seen from the formula that, for the same object to be measured, besides the mass of rigid body $m_0$, the main factors affecting the rotational inertia of rigid body are $H$ and the vibration period $T_0$ of the disk, and the radius measurement of the upper and lower disks must be accurate.
In order to reduce the experimental error as much as possible, this paper proposes a more accurate and effective method for measuring the radius of upper and lower disks. When the horizontal upper disk is slightly rotated, due to the tension of the three suspension wires placed symmetrically, the lower disc, that is, the central line between the upper disk and the lower disk, acts as the central axis for periodic torsion.

The circumscribed circle of the triangle has a common center with the disk. The circumcircle radius is $R$, which is less than the geometric radius $R_0$ of the disc. If the distance between the catenary points of the lower disk is $b$, then it can be known $R = \frac{\sqrt{3}}{3} b$ from the geometric relationship. In this way, through mathematical transformation, the radius of the disk which is difficult to measure accurately is converted into a convenient and less error method, which improves the accuracy of the experimental results, and enables students to master this method, so as to bypass the analogy, and enhance the students’ ability to find and solve problems.

The experimental instrument used in this experiment is the three-line pendulum experimental device in the Physics Laboratory of Nanchang Institute of Technology (Figure. 3).

### Figure 2. Measuring method of disc radius

### Figure 3. The equipment for measuring rotational inertia of rigid body with tri-linear pendulum method

#### 3. Measurement and calculation of basic parameters

The mass of the lower disk used in this experiment is 241g. The local gravity acceleration is 9.8m/s$^2$ and the rotation angle is 8°. In Table. 1, $a$ is the distance between the connection points of the upper disk, and $\bar{a}$ is the corresponding average value. $b$ is the distance between the connecting points of the lower disc, $\bar{b}$ is the corresponding average value. $l$ is the catenary length.
Table 1. Relevant length measurement data (unit: cm)

| Order number |  |  |  |
|--------------|---|---|---|
| 1 | 7.630 | 11.324 | 43.30 |
| 2 | 7.535 | 11.344 | 43.81 |
| 3 | 7.626 | 11.480 | 43.39 |
| Average value | $\bar{a}=7.594$ | $\bar{b}=11.383$ | $l=43.50$ |

Table 2. Measurement of vibration period of tri-linear pendulum method (unit: s)

| Time required for vibration 10 times | Disk data |
|-------------------------------------|-----------|
|                                    | 1         | 14.4 |
|                                    | 2         | 11.8 |
|                                    | 3         | 16.5 |
| Average value                      |           | 14.2 |

| Time required for vibration once   | $T_0$ |
|-----------------------------------|------|
|                                    | 1.42 |

According to the data in Table. 1 and Table. 2, the calculation is carried out as follows.

Upper- and lower-disc radius, $r = \frac{\sqrt{3}}{3}\bar{a}=4.382\text{cm}$, $R = \frac{\sqrt{3}}{3}\bar{b}=6.572\text{cm}$

Vertical distance between upper and lower disks $H = \sqrt{l^2 - (R - r)^2}=43.445\text{cm}$

Rotational inertia calculated by theoretical value $J_0 = \frac{1}{2}n_0R^2=5.16\times10^{-4}\text{kg}\cdot\text{m}^2$

Rotational inertia measured experimentally $J' = \frac{n_0gR}{4\pi^2H}T_0^2=4.69\times10^{-4}\text{kg}\cdot\text{m}^2$

The relative error of the experiment $\eta = \frac{|J_0-J'|}{J_0}\times100\% = 9.10\%$

4. Analysis and discussion of experimental error

In this paper, the quantitative method is used to explore the influence of the angle and the length of suspension on the experimental error.

![Relative error under different rotational angles $\theta$](image.png)

It can be seen from Figure. 4 that the relative error of the experimental results gradually increases with the increase of the angle $\theta$. When the angle is more than $8^\circ$, the relative error will be more than $9\%$. 


which leads to the difference between the measurement results and the actual value. Therefore, it is suggested that the maximum rotation angle should not exceed 5° in actual measurement, which will reduce the relative error. In the experimental teaching, before the experiment starts, the teacher will directly tell the students that the angle should not exceed 5° as far as possible, so that the students will operate by the mechanic and will not think about the reasons behind it. Therefore, it is suggested that in this experiment and other experiments, such as simple pendulum experiment and gravity acceleration measurement experiment, students should be asked to measure many times, find out the reason of error change, understand its essence, and improve students' ability to find and solve problems.

Figure 5. Relative error under different vertical distance $l$

Figure 5 shows the relative error of the experimental results when the catenary length $l$ changes. Strictly speaking, when the length of suspension wire changes, the vibration period will also change. However, in order to explore the influence of $l$ change on the experimental accuracy, the data in Table. 1 and Table. 2 are used for other physical quantities, which are not rigorous but still feasible. As can be seen from Figure. 5, the relative error gradually decreases with the increase of the catenary, which indicates that in the experiment, the length of the catenary should not be too short, and if it is too short, the experimental error is too large. But it can't be too long, because if it is too long, the disc will not do harmonic motion, which will lead to serious errors in experimental principle, at the same time, if the suspension line is too long, it is easy to break, which will affect the smooth progress of the experiment.

5. Cultivating and improving college students' Humanistic Quality in physics experiment

The concepts and laws of physics contain simple and harmonious natural beauty. For example, the conservation of mechanical energy, energy and momentum reflects the perfect unity of nature. Different physics experiments tell us different philosophic views, which enable students to treat things with scientific and rigorous attitude, and to analyze problems from multiple perspectives, so as to grasp the main contradictions of things. There are rich moral education factors in physics experiment course. Teachers should give full play to the moral education function of physics teaching and put moral education in the process of knowledge teaching. With the charm of the subject, they can infect and motivate students with vivid examples. This will play a positive role in cultivating students' good moral quality and improving their personality. At the same time, physical experiments contain rich aesthetic ideas. For example, the use of physical model building method reflects the beauty of simplicity. The formal symmetry of Coulomb's law and the law of universal gravitation reflects the beauty of symmetry, etc. Therefore, physical experiment is the best material to cultivate students' humanistic quality.
6. Conclusion
This paper takes tri-linear pendulum method to measure the rotational inertia of rigid body as an example. The basic principle of the experiment is introduced, and the mathematical expression of the rotational inertia is derived. Using the actual data in the laboratory, the theoretical results are compared with the experimental results, and the relative error of the experiment is obtained. Moreover, the change trend of the accuracy of the experimental results with the change of the angle and the length of the catenary is studied qualitatively, the method to improve the accuracy of the experiment is given, and the essential reason is analyzed, which lays a theoretical foundation for students to have a good understanding. At the same time, in the experimental principle, method and result analysis, the hidden elements of humanistic quality are explored, which provides a reference for the cultivation of college students' humanistic quality.

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