Quark-Meson Coupling Model for a Nucleon

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ABSTRACT

We considered the quark-meson coupling model for a nucleon. The model describes a nucleon as an MIT bag, in which quarks are coupled to the scalar and the vector mesons. A set of coupled equations for the quark and the meson fields are obtained and are solved in a self-consistent manner. We show that the mass of a dressed MIT bag interacting with \(\sigma\)- and \(\omega\)-meson fields differs considerably from the mass of the free MIT bag. The effects of the density-dependent bag constant are investigated. The results of our calculations imply that the self-energy of the bag in the quark-meson coupling model is significant and needs to be considered in doing the calculations for nuclear matter or finite nuclei.

1 Introduction

For more than a decade the success of quantum hadrodynamics\(^\text{1}\) has been rather impressive in describing the bulk properties of nuclear matter as well as the properties of finite nuclei. The model is rather simple with only a few parameters, and yet it has been successfully applied to a great number of problems for nuclear matter and nuclear structure. In this model the relevant degrees of freedom are nucleons and mesons, but the nucleons having a composite structure are treated as Dirac particles. Several years ago a model to remedy this problem was proposed by Guichon\(^\text{2}\). He proposed a quark-meson coupling (QMC) model, in which quarks and mesons are explicitly dealt with. The model describes nuclear matter as non-overlapping, static, spherical MIT bags\(^\text{3}\) interacting through the self-consistent exchange of scalar (\(\sigma\)) and vector (\(\omega\)) mesons in the mean-field approximation. Using this model, he investigated the direct quark degrees of freedom in nuclear matter. The model was refined
later to include the nucleon Fermi motion and the center-of-mass corrections to the bag energy and was applied to a variety of problems [6-10].

In the MIT bag model the bag constant $B$ and a phenomenological parameter $Z$ are fixed such that the nucleon mass of 939 MeV is reproduced for some bag radius $R$. The bag constant $B$ produces the pressure to make a bubble in the QCD vacuum. $Z$ is to account for various corrections including the zero-point motion. In the QMC model for nuclear matter the quark-$\sigma$ coupling constant ($g_q^\sigma$) and the quark-$\omega$ coupling constant ($g_q^\omega$) are additionally introduced. Usually, $B$ and $Z$ are fixed first so that the mass of the free MIT bag becomes equivalent to the nucleon mass of 939 MeV for a certain bag radius. We denote by $B^{\text{free}}$ and $Z^{\text{free}}$, respectively, the bag constant $B$ and the parameter $Z$ for the free MIT bag. $B^{\text{free}}$ and $Z^{\text{free}}$ thus obtained are used in the QMC model calculations, and then $g_q^\sigma$ and $g_q^\omega$ are determined to reproduce the binding energy per nucleon (B.E. = $-16$ MeV) at the saturation density ($\rho_N^0 = 0.17$ fm$^{-3}$).

However, due to the interaction between the quarks and the $\sigma$- and the $\omega$-mesons the mass of a single dressed MIT bag in free space, when calculated with $B^{\text{free}}$ and $Z^{\text{free}}$ described above, may be different from the nucleon mass of 939 MeV. Such a possible deviation of the mass of a dressed bag from the value of 939 MeV has been neglected in previous QMC model calculations for nuclear matter. If the deviation in mass is significant, it is necessary to modify the parameters $B^{\text{free}}$ and $Z^{\text{free}}$ before implementing them in the nuclear matter calculations. In this paper we investigate this change in the nucleon mass. In Section 2 the QMC model for a nucleon is described. Some numerical results are presented in Section 3. Section 4 contains a summary.

### 2 Quark-meson coupling model for a nucleon

The Lagrangian density for the MIT bag in which quark fields $\psi$ are coupled to the $\sigma$- and $\omega$-fields may be written as

$$
\mathcal{L} = \left[ \frac{i}{2} \left( \bar{\psi} \gamma^\mu \partial_\mu \psi - (\partial_\mu \bar{\psi}) \gamma^\mu \psi \right) - \bar{\psi} \left( g_q^\sigma \gamma_\mu \omega^\mu + \left( m_q - g_q^\omega \sigma \right) \right) \psi - B(\sigma) \right] \theta_v \\
- \frac{1}{2} \bar{\psi} \Delta_s - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2_\omega \omega_{\mu} \omega^\mu + \frac{1}{2} \left[ (\partial_\mu \sigma) (\partial^\mu \sigma) - m^2_\sigma \sigma^2 \right],
$$

(1)

where $F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, $\theta_v$ is the step function for confining the quarks inside the bag, and $\Delta_s$ is the $\delta$-function at the bag surface. Here, we assume the density-dependent bag constant introduced in Ref. [10] and express the
bag constant $B(\sigma)$ as

$$
\frac{B}{B_0} = \left[ 1 - g_\sigma^2 \frac{4}{\delta M_N^2} \right]^\delta
$$

(2)

with $M_N^2 = 939$ MeV. We will neglect the isospin breaking and take $m_q = (m_u + m_d)/2$ hereafter. For actual numerical calculations $m_q$ will be taken to be zero. The mass of $\sigma$ ($\omega$) is taken as 550 (783) MeV. From this Lagrangian density the equations of motion for quark fields $\psi$, sigma fields $\sigma$, and omega fields $\omega_\mu$ follow. For $\psi$ we have

$$
[\gamma^\mu (i\partial_\mu - g_\omega^\omega \omega_\mu) - (m_q - g_\sigma^\sigma \sigma)] \psi_{\theta_v} = \frac{1}{2} (1 - i\gamma^\mu n_\mu) \psi \Delta_s,
$$

(3)

where $\Delta_s = -n \cdot \partial(\theta_v)$ is used. The left hand side of Eq. (3) gives us the equation for the quarks inside the bag, and the right hand side gives us the linear boundary condition at the bag surface. The equations for $\sigma$ and $\omega_\mu$ are, respectively,

$$
\partial_\mu \partial^\mu \sigma^\sigma + m_\sigma^2 \sigma^\sigma = \left[ g_\omega^\omega \bar{\psi}\psi - \frac{\partial B(\sigma)}{\partial \sigma} \right] \theta_v
$$

(4)

and

$$
\partial^\nu F_{\nu\mu} + m_\omega^2 \omega_\mu = g_\omega^\omega \bar{\psi} \gamma_\mu \psi \theta_v.
$$

(5)

If we consider only the ground state quarks in the static spherical MIT bag and keep only the time-component of $\omega_\mu$, Eq. (3) for $r < R$ becomes coupled linear differential equations for $g(r)$ and $f(r);

$$
\frac{df(r)}{dr} = - \left[ \frac{2f(r)}{r} + E^- (r)g(r) \right]
$$

(6)

$$
\frac{dg(r)}{dr} = E^+ (r)f(r),
$$

(7)

where $g(r)$ and $f(r)$ are the radial parts of the upper and the lower components of $\psi$, i.e.,

$$
\psi (t, r) = e^{-i\epsilon_q t/R} \left( \begin{array}{c} g(r) \\ -i\vec{\sigma} \cdot \hat{r} f(r) \end{array} \right) \frac{\chi_q}{\sqrt{4\pi}},
$$

(8)

where $\vec{\sigma}$ is the Pauli spin matrix and $\chi_q$ is the quark spinor. Also,

$$
E^+ (r) = \frac{\epsilon_q}{R} - g_\omega^\omega \omega_0 (r) - (m_q - g_\sigma^\sigma \sigma (r))
$$

(9)

$$
E^- (r) = \frac{\epsilon_q}{R} - g_\omega^\omega \omega_0 (r) - (m_q - g_\sigma^\sigma \sigma (r))
$$
with \( \sigma(\mathbf{r}) \) and \( \omega_0(\mathbf{r}) \) being the \( \sigma \)- and the time component of the \( \omega \)-fields. For the ground state Eqs. (6) are the equations in the radial coordinate \( r \) only. The linear boundary condition from the right hand side of Eq. (3), when rewritten by using Eq. (8), becomes
\[
f\left(\frac{xr}{R}\right) = -g\left(\frac{xr}{R}\right)
\]
(10)
at the bag surface, \( r = R \), and determines the eigenvalue \( x \) of the quarks. \( \epsilon_q \) is then given by \( \sqrt{x^2 + (Rm_q)^2} = x \) for \( m_q = 0 \). \( g(r) \) and \( f(r) \) are normalized such that the quark density \( \rho_q(= \bar{\psi}\gamma_0\psi) \) integrated over the bag is the unity.

In the static spherical approximation Eqs. (4) and (5) are reduced to
\[
(\nabla^2 - m_i^2)\sigma(\mathbf{r}) = \left[ \frac{\partial B(\sigma)}{\partial \sigma} - g^2(3\rho_s) \right] \theta(R - r)
\]
(11)
and
\[
(\nabla^2 - m_i^2)\omega_0(\mathbf{r}) = -g^2(3\rho_q) \theta(R - r)
\]
(12)
respectively, where \( \rho_s(= \bar{\psi}\psi) \) is the scalar density and 3 is explicitly multiplied by the densities \( \rho_q \) and \( \rho_s \) here to account for the sum over 3 quarks. Equations (11) and (12), which are nothing but the Klein-Gordon equations with source terms, may be readily solved using the Green’s function defined by the equation
\[
(\nabla^2 - m_i^2)G_i(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') \quad (i = \sigma, \omega).
\]
(13)
For the s-wave state \( G_i(\mathbf{r}, \mathbf{r}') \) may be written as
\[
G_i(\mathbf{r}, \mathbf{r}') = -\frac{1}{4\pi} \frac{1}{m_i r r'} \sinh(m_i r <) e^{-m_i r >}.
\]
(14)

Equations (4), (5), (11), and (12) form a set of coupled equations for \( \psi \), \( \sigma \), and \( \omega_0 \), which need to be solved self-consistently.

By solving these equations we can obtain the eigenvalue of the quarks and the energy of the nucleon bag. One way to calculate the energy \( E_N \) of the nucleon is computing the energy-momentum tensor \( T^{\mu\nu} \) and using the relation
\[
E_N = \int d^3r \ T_{00}.
\]
(15)
\( T_{00} \) may be written as
\[
T_{00} = \left( \frac{E_q}{R} \bar{\psi}\gamma^0\psi + B(\sigma) \right) \theta(R - r) - \frac{1}{2} \left( (\nabla\omega_0)^2 + (m_\omega\omega_0)^2 \right) + \frac{1}{2} \left( (\nabla\sigma)^2 + (m_\sigma\sigma)^2 \right),
\]
(16)
where \( E_q \) is given by

\[
\frac{E_q}{R} = 3 \frac{e_q}{R} - Z \frac{R}{R} = 3 \frac{x}{R} - Z
\]

with the sum over 3 quarks taken into account. Correcting for spurious center-of-mass motion in the bag, the mass of the bag at rest is taken to be

\[
M_N = \sqrt{E_N^2 - \langle p_{c.m.}^2 \rangle}
\]

where \( \langle p_{c.m.}^2 \rangle = \sum_{k=1}^{3} \langle p_k^2 \rangle = 3(x/R)^2 \). By calculating \( M_N \) for each bag radius and minimizing \( M_N \) with respect to the bag radius \( R \), we can get the nucleon mass and the bag radius.

3 Results

We first present some results of our calculations of the nucleon mass when we take the model parameters from Ref. [10]. There are several sets of parameters used in Ref. [10]. Here we only consider the cases when \( R = 0.6 \) fm and \( \delta = 8 \). In Fig. 1 \( M_N \) is plotted as a function of the bag radius \( R \). The solid curve represents the free MIT bag mass \( M_N^{free} \) without a coupling of the quarks with the mesons: \( g^g_\sigma = g^g_\omega = g^B_\sigma = 0 \). \( B_0^{1/4} \) and \( Z \) are taken to be 188.1 MeV and 2.030, respectively, which produce the minimum of \( M_N^{free} \) at 939 MeV and \( R = 0.6 \) fm. Now we include the couplings between the quarks and the mesons. We take the parameter sets used to plot Fig. 1 of Ref. [10]. When \( g^g_\sigma = 0 \) and \( \delta = 8 \), we have \( g^B_\sigma = 8.45 \) and \( g^B_\omega = 3.00 \) from Table 1 of Ref. [10]. \( M_N \) obtained with these parameters are plotted by the dashed curve in Fig. 1. \( M_N \) is reduced to 848 MeV with \( R = 0.694 \) fm. When \( g^g_\sigma = 1.0 \) and \( \delta = 8 \), and thus \( g^B_\sigma = 7.26 \) and \( g^B_\omega = 3.08 \) from Table 1 of Ref. [10], we get \( M_N \) as plotted by the dotted curve in Fig. 1 with \( M_N = 834 \) MeV at \( R = 0.674 \) fm. Let us consider the case when \( g^g_\sigma = 2.0 \) and \( \delta = 8 \). Then from Table 1 of Ref. [10] we have \( g^B_\sigma = 5.65 \) and \( g^B_\omega = 2.85 \). \( M_N \) obtained with these parameters are plotted by the dash-dotted curve in Fig. 1 with \( M_N = 830 \) MeV and \( R = 0.659 \) fm. Fig. 1 of Ref. [10] shows that for these three different sets of parameters the nuclear matter binding energies calculated by the QMC model are the same particularly near the saturation density. However, the present calculations show that the nucleon masses calculated with these parameter sets are considerably different from each other, all being far from \( M_N^{free} = 939 \) MeV. The couplings induce large attraction and reduce the nucleon mass by about 100 MeV. Fig. 1 also shows that the attraction is not uniform with respect to the bag radius. There is more attraction at larger bag radii, and as a result the bag radius increases significantly.
Fig. 1: Nucleon mass $M_N$ plotted as a function of the bag radius $R$ for different coupling constants, $g_q^a$, $g_q^\omega$ and $g_B^\sigma$. $B_0^{1/4} = 188.1$ MeV and $Z = 2.030$ are used for all calculations. For the dashed curve, $g_q^a = 0$, $g_q^\omega = 3.00$ and $g_B^\sigma = 8.45$. For the dotted curve, $g_q^a = 1$, $g_q^\omega = 3.08$ and $g_B^\sigma = 7.26$. For the dash-dotted curve, $g_q^a = 2$, $g_q^\omega = 2.85$ and $g_B^\sigma = 5.65$.

To see how these results come about we consider some simple cases. Let’s go back to the free MIT bag case; $g_q^a = g_q^\omega = g_B^\sigma = 0$. We still use $B_0^{1/4} = 188.1$ MeV and $Z = 2.030$. The free MIT bag mass is plotted again in Fig. 2 by the solid curve. Let us introduce the coupling between the quarks and the $\omega$-meson by use of $g_q^a = 0$ and $g_q^\omega = 2$ and use a constant bag constant, i.e., $g_B^\sigma = 0$. Then the dashed curve in Fig. 2 is obtained, giving us $M_N = 957$ MeV at $R = 0.595$ fm due to the repulsive interaction. When $g_q^a = 2$ and $g_q^\omega = 0$, i.e., when there is only quark-$\sigma$ coupling, attraction is induced and we get $M_N = 926$ MeV at $R = 0.601$ fm, as shown by the dotted curve. When we take $g_q^a = g_q^\omega = 2$, cancellation between the repulsion and the attraction takes place, and a small repulsion is left as plotted by the dash-dotted curve, resulting in $M_N = 948$ MeV at $R = 0.598$ fm.

Now we consider the density-dependency in the bag constant using Eq. $[2]$. We start with the case that $g_q^a = g_q^\omega = 2$ and $g_B^\sigma = 0$, corresponding to the dash-dotted curve in Fig. 2. This curve is plotted again in Fig. 3 by the dash-dotted curve. ($M_N^{free}$ is also plotted in Fig. 3 by the solid curve.) To see the effects of the density-dependency of the bag constant, we take $g_B^\sigma = 1$, keeping $g_q^a = g_q^\omega = 2$. The calculated $M_N$ is represented by the dashed curve. Reducing the bag constant by introducing the density-dependency...
Fig. 2: Nucleon mass $M_N$ plotted as a function of the bag radius $R$ for different coupling constants, $g_\sigma^q$, $g_\omega^q$ and $g_\sigma^B$. $B_0^{1/4} = 188.1$ MeV and $Z = 2.030$ are used for all calculations. For the dashed curve, $g_\sigma = 0$ and $g_\omega = 2$ are used. For the dotted curve, $g_\sigma^q = 2$ and $g_\omega = 0$. For the dash-dotted curve, $g_\sigma^q = 2$, $g_\omega = 2$. For all the calculations shown here $g_\sigma^B = 0$ are used.

Fig. 3: Nucleon mass $M_N$ plotted as a function of the bag radius with the coupling constant given in the figure. $B_0^{1/4}$ and $Z$ are the same as in Fig. 2.

with $g_\sigma^B = 1$ causes attraction. For $g_\sigma^B = 3$, more attraction is induced, as shown by the dotted curve in Fig. 3. This can be seen from Eqs. (11) and (13). The source function for the $\sigma$ field in Eq. (13) increases (negatively) due to $\partial B(\sigma)/\partial \sigma$, so the $\sigma$ field increases, which results in enhanced attraction in the eigenvalue. In Fig. 4 we show the $\sigma(\mathbf{r})$ field for $R = 0.6$.
fm as a function of the radial coordinate $r$. The dotted, dashed, and dash-dotted curves in Fig. 4 are obtained with the corresponding parameters used for the dotted, dashed, and dash-dotted curves in Fig. 3, respectively. The figure shows that as $g_{\sigma}^B$ becomes larger the $\sigma$ field increases due to the increase of the source function. Hence, the smaller nucleon mass for the larger $g_{\sigma}^B$. Of course, a smaller $B(\sigma)$ in Eq. (16) causes a smaller mass, too.

![Graph showing the $\sigma$ fields for $R = 0.6$ fm. The coupling constants used here for the dotted, dashed, dash-dotted curves are the same as those for the dotted, dashed, dash-dotted curves, respectively, in Fig. 3.](image)

As we mentioned earlier, when $g_{\sigma}^B$ is large enough, not only the nucleon mass becomes smaller, but also the bag radius becomes larger. When the bag constant is kept constant as in Fig. 2, the bag radii are essentially fixed as $R = 0.6$ fm. Even if we change $g_{\sigma}^q$ and $g_{\omega}^q$ to some extent, the nucleon mass curves move more or less uniformly in energy. However, when we introduce the density-dependency in the bag constant, the bag radius increases as shown in Figs. 1 and 3. The density-dependent bag constant brings the $M_N$ curve downward in energy, but not uniformly. There is more attraction at larger bag radii. The reason for this can be seen from Fig. 5, where the source function in Eq. (11) for $R = 0.6$ fm is plotted by the solid curve. $\partial B/\partial \sigma$ and $-g_{\omega}^3\rho_s$ are also shown by the dotted and the dashed curves, respectively. Due to $\partial B/\partial \sigma$ the source function does not vanish even at large radii. Therefore, if the bag radius $R$ is larger, $\sigma(r)$ in Eq. (11) becomes greater for large radii. This is shown in Fig. 6, where $\sigma(r)$ is plotted for $R = 0.6$ and 0.8 fm. $\sigma(r)$ for $R = 0.8$ fm is greater than $\sigma(r)$ for $R = 0.6$ fm at large radial region, which is the significant radial region. As a result, there is more attraction at large bag radii.
Fig. 5: The source function is plotted by the solid curve. $\partial B/\partial \sigma$ and $-g_q^3 \rho_s$ are plotted by the dotted and the dashed curves, respectively. The coupling constants used are $g_q^2 = g_q^3 = 2$ and $g_B^3 = 3$.

Fig. 6: $\sigma(r)$ is plotted with respect to the radial coordinate $r$ by the solid and the dashed curves, respectively, for $R = 0.6$ and 0.8 fm. The coupling constants used are $g_q^2 = g_q^3 = 2$ and $g_B^3 = 3$.

We made similar studies for other bag radii ranging from 0.6 to 1.0 fm and obtained similar results. Our results show that the change in the bag mass due to the coupling of the quarks with the mesons is not negligible. Such self-energy effects on the mass due to the meson coupling need to be taken into account in
choosing the parameters for the calculation of nuclear matter properties. The density-dependent bag constant can not only change the nucleon mass but also shift the bag radius.

4 Summary

We have applied the QMC model to a single nucleon. Recently the model has been often used to investigate explicit quark degrees of freedom in describing the nuclear matter. However, in the previous calculations the change in the mass of the bag due to the self-energy was ignored. Our calculations suggest that this change in the nucleon mass is not negligible, and thus the model parameters need to be modified to take this effect into consideration when they are used in nuclear matter calculations.

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