Evolution of Baryon-Free Matter
Produced in Relativistic Heavy-Ion Collisions

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A 3-fluid hydrodynamic model is introduced for simulating heavy-ion collisions at incident energies between few and about 200 A·GeV. In addition to the two baryon-rich fluids of 2-fluid models, a new model incorporates a third, baryon-free (i.e. with zero net baryonic charge) fluid which is created in the mid-rapidity region. Its evolution is delayed due to a formation time $\tau$, during which the baryon-free fluid neither thermalizes nor interacts with the baryon-rich fluids. After formation it thermalizes and starts to interact with the baryon-rich fluids. It is found that for $\tau = 0$ the interaction strongly affects the baryon-free fluid. However, at reasonable finite formation time, $\tau \simeq 1$ fm/c, the effect of this interaction turns out to be substantially reduced although still noticeable. Baryonic observables are only slightly affected by the interaction with the baryon-free fluid.

I. INTRODUCTION

Nearly fifty years have passed since the paper ”Relativistic kinetic equation” by Spartak T. Belyaev and Gersh I. Budker was published [1]. The proposed relativistic formulation of the distribution function and the kinetic equation with small-angle scattering have been included in many textbooks and found numerous applications in atomic physics and electron-positron plasma. Recently a generalized relativistic kinetic equation of this type was implemented for describing the partonic evolution in very early stages of a heavy-ion collision at ultra-relativistic RHIC energies [2]. In the present paper we address more mod-
erate, but nevertheless highly relativistic, energies, i.e. up to those reached at the CERN SPS. The relativistic kinetic equation is used in a peculiar way, namely to derive a coupling term for three-fluid hydrodynamic equations.

Two-fluid hydrodynamics with free-streaming radiation of pions was advanced first in [3, 4]. The initial stage of heavy-ion collisions definitely is a highly non-equilibrium process. Within the hydrodynamic approach this non-equilibrium is simulated by means of a 2-fluid approximation, which takes care of the finite stopping power of nuclear matter [5, 6], and simultaneously describes the entropy generation at the initial stage. The radiated pions form a baryon-free matter in the mid-rapidity region, while two baryon-rich fluids simulate the propagation of leading particles. The pions are the most abundant species of the baryon-free matter which may contain any hadronic and/or quark-gluon species including baryon-antibaryon pairs.

First applications of the 2-fluid model [7, 8] to the description of heavy-ion collisions in the wide range of incident energies, from those of SIS to SPS, were quite successful. One of the advantages of the hydrodynamic models is that they directly address the equation of state (EoS) of nuclear matter, which is of prime interest for this domain of physics. In particular, we have shown [9] recently that the experimental excitation function of the directed flow is well described by the mixed-phase EoS in contrast to earlier predictions of the two-phase bag-model EoS. In these 3D hydrodynamic simulations we describe the whole process of the reaction, i.e. the evolution from the formation of a hot and dense nuclear system to its subsequent decay. This is in distinction to numerous other simulations, which treat only the expansion stage of a fireball formed in the course of the reaction, while the initial state of this dense and hot nuclear system is constructed from either kinetic simulations or more general albeit model-dependent assumptions (e.g. see [10, 11]).

However, the approximation of free-streaming pions, produced in the mid rapidity region, was still irritating from the theoretical point of view, in particular, because the relative momenta of the produced pions and the leading baryons are in the range of the $\Delta$ resonance for the incident energies considered. This would imply that the interaction between the produced pions and the baryon-rich fluids should be strong. The free-streaming assumption relies on a long formation time of produced pions. Indeed, the proper time for the formation of the produced particles is commonly assumed to be of the order 1 fm/c in the comoving frame. Since the main part of the produced pions is quite relativistic at high incident
energies, their formation time should be long enough in the reference frame of calculation to prevent them from interacting with the baryon-rich fluids. However, this argument is qualitative rather than quantitative, and hence requires further verification. The first attempt to do this was undertaken by the Frankfurt group [12], which started to explore an opposite extreme. They assumed that the produced pions immediately thermalize, forming a baryon-free fluid (or a “fireball” fluid, in terms of [12]), and interact with the baryon-rich fluids. No formation time was allowed, and the strength of the corresponding interaction was guessed rather than microscopically estimated. This opposite extreme, referred to as a (2+1)-fluid model and being not quite justified either, yielded results substantially different from those of the free-streaming approximation. This was one of the reasons why in subsequent applications the Frankfurt group neglected the interaction between baryon-free and baryon-rich fluids while keeping the produced pions thermalized [13], thus effectively restoring the free-streaming approximation. However, the assumed immediate thermalization of the fireball fluid together with the lack of interaction with baryon-rich fluids still was not a consistent approximation.

In this paper we would like to return to the problem of verification of the free-streaming approximation for the produced pions. To do this, we extend the 2-fluid model of refs. [3, 4, 7, 8, 9] to a 3-fluid model, where the created baryon-free fluid (which we call a “fireball” fluid, according to the Frankfurt group) is treated on equal footing with the baryon-rich ones. However, we allow a certain formation time for the fireball fluid, during which the constituents of the fluid propagate without interactions. Furthermore, we estimate the interaction between fireball and baryon-rich fluids by means of relativistic kinetic equation and elementary cross sections.

In this paper we consider incident energies in the range from few to about 200 A·GeV (i.e. from AGS to SPS energies). The interest to this energy was recently revived in connection with the project of the new accelerator facility at GSI SIS200 [14]. The goal of the research program on nucleus-nucleus collisions at this planned facility is the investigation of nuclear matter in the region of incident energies \( E_{\text{lab}} \simeq 10 - 40 \) A·GeV), in which the highest baryon densities and highest relative strangeness at moderate temperatures are expected. Here, the QCD phase diagram is much less explored, both experimentally and theoretically, as compared to the higher energy region characterized by higher temperatures, but lower net baryon densities, where lattice QCD calculations [15] and a large body of
experimental data from SPS (CERN) and RHIC (BNL) are available by now.

II. 3-FLUID HYDRODYNAMIC MODEL

Unlike the one-fluid hydrodynamic model, where local instantaneous stopping of projectile and target matter is assumed, a specific feature of the dynamic 3-fluid description is a finite stopping power resulting in a counter-streaming regime of leading baryon-rich matter. Experimental rapidity distributions in nucleus–nucleus collisions support this counter-streaming behavior, which can be observed for incident energies between few and 200 A·GeV.

The basic idea of a 3-fluid approximation to heavy-ion collisions is that at each space-time point \( x = (t, \mathbf{x}) \) the distribution function of baryon-rich matter, \( f_{br}(x, p) \), can be represented as a sum of two distinct contributions

\[
 f_{br}(x, p) = f_p(x, p) + f_t(x, p),
\]

initially associated with constituent nucleons of the projectile (p) and target (t) nuclei. In addition, newly produced particles, populating the mid-rapidity region, are associated with a fireball (f) fluid described by the distribution function \( f_f(x, p) \). Note that both the baryon-rich and fireball fluids may consist of any type of hadrons and/or partons (quarks and gluons), rather than only nucleons and pions. However, here and below we suppress the species label at the distribution functions for the sake of transparency of the equations.

With the above-introduced distribution functions \( f_\alpha (\alpha = p, t, f) \), the coupled set of relativistic Boltzmann equations looks as follows:

\[
 p_\mu \partial_\nu f_p(x, p) = C_p(f_p, f_t) + C_p(f_p, f_f),
\]

\[
 p_\mu \partial_\nu f_t(x, p) = C_t(f_p, f_t) + C_t(f_t, f_t),
\]

\[
 p_\mu \partial_\nu f_f(x, p) = C_t(f_p, f_t) + C_t(f_p, f_f) + C_t(f_t, f_t),
\]

where \( C_\alpha \) denote collision terms between the constituents of the three fluids. We have omitted intra-fluid collision terms, like \( C_p(f_p, f_p) \), since below they will be canceled any way. The displayed inter-fluid collision terms have a clear physical meaning: \( C_{p/t}(f_p, f_t) \), \( C_{p/t}(f_p/t, f_t) \), and \( C_t(f_p/t, f_t) \) give rise to friction between p-, t- and f-fluids, and \( C_t(f_p, f_t) \) takes care of particle production in the mid-rapidity region. Note that up to now we have done no approximation, except for hiding intra-fluid collision terms.
Let us proceed to approximations which justify the term “fluids” having been used already. We assume that constituents within each fluid are locally equilibrated, both thermodynamically and chemically. In particular, this implies that the intra-fluid collision terms are indeed zero. This assumption relies on the fact that intra-fluid collisions are much more efficient in driving a system to equilibrium than inter-fluid interactions. As applied to the fireball fluid, this assumption requires some additional comments, related to the concept of a finite formation time. During the formation proper time $\tau$ after production, the fireball fluid propagates freely, interacting neither with itself nor with the baryon-rich fluids. After this time period, the fireball matter locally thermalizes and starts to interact with both itself and the baryon-rich fluids. Being heated up, these three fluids may contain not only hadronic but also deconfined quark-gluon species, depending on the EoS used.

The above assumption suggests that interaction between different fluids should be treated dynamically. To obtain the required dynamic equations, we first integrate the kinetic Eqs. (2)–(4) over momentum and sum over particle species with weight of baryon charge. This way we arrive to equations of the baryon charge conservation

$$\partial_\mu J_\alpha^\mu(x) = 0,$$

for $\alpha = p$ and $t$, where $J_\alpha^\mu = n_\alpha u_\alpha^\mu$ is the baryon current defined in terms of baryon density $n_\alpha$ and hydrodynamic 4-velocity $u_\alpha^\mu$ normalized as $u_\alpha^{\alpha\mu} u_\alpha^\mu = 1$. Eq. (5) implies that there is no baryon-charge exchange between $p$- and $t$-fluids, as well as that the baryon current of the fireball fluid is identically zero, $J_f^\mu = 0$. Integrating kinetic Eqs. (2)–(4) over momentum with weight of 4-momentum $p^\nu$ and summing over all particle species, we arrive at equations of the energy–momentum exchange for energy–momentum tensors $T_\alpha^\mu\nu$ of the fluids

$$\partial_\mu T_p^\mu\nu(x) = -F_p^\nu(x) + F_{fp}^\nu(x),$$

$$\partial_\mu T_t^\mu\nu(x) = -F_t^\nu(x) + F_{ft}^\nu(x),$$

$$\partial_\mu T_f^\mu\nu(x) = F_p^\nu(x) + F_t^\nu(x) - F_{fp}^\nu(x) - F_{ft}^\nu(x),$$

where the $F^\nu$ are friction forces originating from inter-fluid collision terms in the kinetic Eqs. (2)–(4). $F_p^\nu$ and $F_t^\nu$ in Eqs. (6)–(7) describe energy–momentum loss of baryon-rich fluids due to their mutual friction. A part of this loss $|F_p^\nu - F_t^\nu|$ is transformed into thermal excitation of these fluids, while another part $(F_p^\nu + F_t^\nu)$ gives rise to particle production into the fireball fluid (see Eq. (8)). $F_{fp}^\nu$ and $F_{ft}^\nu$ are associated with friction of the fireball fluid with
the p- and t-fluids, respectively. Note that Eqs. (6)–(8) satisfy the total energy–momentum conservation
\[ \partial_{\mu}(T_{\mu}^{\rho} + T_{\mu}^{\pi} + T_{\mu}^{f}) = 0. \] (9)

As described above, the energy–momentum tensors of the baryon-rich fluids (\(\alpha = p\) and \(t\)) take the conventional hydrodynamic form
\[ T_{\alpha}^{\mu\nu} = (\varepsilon_{\alpha} + P_{\alpha}) u_{\alpha}^{\mu} u_{\alpha}^{\nu} - g^{\mu\nu} P_{\alpha} \] (10)
in terms of the proper energy density, \(\varepsilon_{\alpha}\), and pressure, \(P_{\alpha}\). For the fireball, however, only the thermalized part of the energy–momentum tensor is described by this hydrodynamic form
\[ T_{f}^{(eq)\mu\nu} = (\varepsilon_{f} + P_{f}) u_{f}^{\mu} u_{f}^{\nu} - g^{\mu\nu} P_{f} \] (11)
Its evolution is defined by a Euler equation with a retarded source term
\[ \partial_{\mu}T_{f}^{(eq)\mu\nu}(x) = \int d^{4}x' \delta^{4}(x - x' - U_{F}(x')\tau) \left[ F_{p}^{\nu}(x') + F_{t}^{\nu}(x') \right] - F_{fp}^{\nu}(x) - F_{ft}^{\nu}(x), \] (12)
where \(\tau\) is the formation time, and
\[ U_{F}^{\mu}(x') = \frac{F_{p}^{\mu}(x') + F_{t}^{\mu}(x')}{|F_{p}(x') + F_{t}(x')|} \] (13)
is a free-streaming 4-velocity of the produced fireball matter. In fact, this is the velocity at the moment of production of the fireball matter. According to Eq. (12), the energy and momentum of this matter appear as a source in the Euler equation only later, at the time \(U_{F}^{\rho}\tau\) after production, and in different space point \(x' = U_{F}(x')\tau\), as compared to the production point \(x\). From the first glance, one can immediately simplify the r.h.s. of Eq. (12) by performing integration with the \(\delta\)-function. However, this integration is not that straightforward, since the expression under the \(\delta\)-function, \(x - x' - U_{F}(x')\tau = 0\), may have more than one solution with respect to \(x'\). The latter would mean that the matter produced in several different space-time points \(x'\) simultaneously thermalizes in the same space-time point \(x\). This is possible due to the nonlinearity of the hydrodynamic equations.

The residual part of \(T_{f}^{\mu\nu}\) (the free-streaming one) is defined as
\[ T_{f}^{(fs)\mu\nu} = T_{f}^{\mu\nu} - T_{f}^{(eq)\mu\nu}. \] (14)
The equation for $T^{(eq)\mu\nu}_f$ can be easily obtained by taking the difference between Eqs. (8) and (12). If all the fireball matter turns out to be formed before freeze-out, than this equation is not needed. Thus, the 3-fluid model introduced here contains both the original 2-fluid model with pion radiation [3, 4, 7, 8, 9] and the (2+1)-fluid model [12, 13] as limiting cases for $\tau \to \infty$ and $\tau = 0$, respectively.

The nucleon–nucleon cross sections at high energies are strongly forward–backward peaked. In this case the Boltzmann collision term can be essentially simplified, since the involved 4-momentum transfer is small. The small-angle-scattering expansion of the collision integral results in the relativistic Fokker–Planck equation, as first derived by Belyaev and Budker [1]. Precisely this equation was used in [5] to estimate the friction forces, $F^\nu_p$ and $F^\nu_t$, proceeding from only $NN$ elastic scattering. Later these friction forces were calculated [19] based on (both elastic and inelastic) experimental inclusive $NN$ cross sections

$$F^\nu_{\alpha} = \rho_p \rho_t \left[\left(u^\nu_{\alpha} - u^\nu_{\bar{\alpha}}\right) D_P + \left(u^\nu_p + u^\nu_t\right) D_E\right],$$

(15)

$\alpha = p$ and $t$, $\bar{p} = t$ and $\bar{t} = p$. Here, $\rho_\alpha$ denotes the scalar densities of the p- and t-fluids,

$$D_{P/E} = m_N \frac{V_{pt}^{\text{rel}}}{\sigma_{P/E}(s_{pt})},$$

(16)

where $m_N$ is the nucleon mass, $s_{pt} = m_N^2 \left(u^\nu_p + u^\nu_t\right)^2$ is the mean invariant energy squared of two colliding nucleons from the p- and t-fluids, $V_{\text{rel}}^{\text{pt}} = [s_{pt}(s_{pt} - 4m_N^2)]^{1/2}/2m_N$ is the mean relative velocity of the p- and t-fluids, and $\sigma_{P/E}(s_{pt})$ are determined in terms of nucleon-nucleon cross sections integrated with certain weights (see [3, 4, 7, 8, 19] for details). It was found in [19] that a part of these friction terms, which is related to the transport cross-section, may be well parameterized by an effective deceleration length $\lambda_{\text{eff}}$ with a constant value $\lambda_{\text{eff}} \approx 5$ fm. However, there are reasons to consider $\lambda_{\text{eff}}$ as a phenomenological parameter, as it was pointed out in [7]. Indeed, as it is seen from Eq. (16), this friction is estimated only in terms of nucleon-nucleon cross sections while the excited matter of baryon-rich fluids certainly consists of great number of hadrons and/or deconfined quarks and gluons. Furthermore, these quantities may be modified by in-medium effects. In this respect, $D_{P/E}$ should be understood as quantities that give a scale of this interaction.

Eqs. (5)–(7) and (12), supplemented by a certain EoS and expressions for friction forces $F^\nu$, form a full set of equations of the relativistic 3-fluid hydrodynamic model. To make this set closed, we still need to define the friction of the fireball fluid with the p- and t-fluids, $F^\nu_{fp}$ and $F^\nu_{ft}$ in terms of hydrodynamic quantities and some cross sections.
III. INTERACTION BETWEEN FIREBALL AND BARYON-RICH FLUIDS

Our aim here is to estimate the scale of the friction force between the fireball and baryon-rich fluids, similar to that done before for baryon-rich fluids \[19\]. To this end, we consider a simplified system, where all baryon-rich fluids consist only of nucleons, as the most the abundant component of these fluids, and the fireball fluid contains only pions.

For incident energies from 10 (AGS) to 200 A·GeV (SPS), the relative nucleon-pion energies are in the resonance range dominated by the \(\Delta\)-resonance. To estimate this relative energy we consider a produced pion, being at rest in the center of mass of the colliding nuclei, \(q_\pi = \{m_\pi, 0, 0, 0\}_{cm}\). Baryon-rich fluids decelerate each other during their interpenetration. This means that the nucleon momentum should be smaller than the incident momentum, \(|p_N| < |\{m_N\gamma_{cm}, p_{cm}\}|\), where \(\gamma_{cm}\) is the gamma factor of the incident nucleon in the cm frame. Calculating the invariant relative energy squared \(s = (p+q)^2\) at \(E_{lab} = 158\) A·GeV, we obtain \(s^{1/2} < 1.8\) GeV. This range of \(s\) precisely covers the resonance region, \(1.1\) GeV \(< s^{1/2} < 1.8\) GeV \[20\]. At \(E_{lab} = 10\) A·GeV we arrive at \(s^{1/2} < 1.3\) GeV, which is also within the resonance region. At even lower incident energies the strength of the fireball fluid becomes so insignificant, as compared with thermal mesons in the p- and t-fluids, that the way of treatment of its interaction with the baryon-rich fluids does not essentially affect the observables. For the same reason we do not apply any special prescription for the unification of the fireball fluid with the baryon-rich fluids, since this may happen only at relatively low incident energies \(E_{lab} < 10\) A·GeV.

The resonance-dominated interaction implies that the essential process is absorption of a fireball pion by a p- or t-fluid nucleon with formation of an \(R\)-resonance (most probably \(\Delta\)). This produced \(R\)-resonance still belongs to the original p- or t-fluid, since its recoil due to absorption of a light pion is small. Subsequently this \(R\)-resonance decays into a nucleon and a pion already belonging to the original p- or t-fluid. Symbolically, this mechanism can be expressed as

\[N^\alpha + \pi^f \rightarrow R^\alpha \rightarrow N^\alpha + \pi^\alpha.\]

As a consequence, only the loss term contributes to the kinetic equation for the fireball fluid.

Proceeding from the above consideration, we write down the collision term between
fireball-fluid pions and α-fluid nucleons (α = p or t) as follows

\[ C_t(f_\alpha, f_t) = - \int \frac{d^3q}{q_0} W^{N\pi\to R}(s) f_t^{(eq)}(x, p) f_\alpha(x, q), \]  

(17)

where \( s = (p + q)^2 \),

\[ W^{N\pi\to R}(s) = \frac{1}{2} \left[ (s - m_N^2 - m_\pi^2)^2 - 4m_N^2m_\pi^2 \right]^{1/2} \sigma_{tot}^{N\pi\to R}(s) \]

is the rate to produce a baryon \( R \)-resonance, and \( \sigma_{tot}^{N\pi\to R}(s) \) is the parameterization of experimental pion–nucleon cross-sections [20]. Here, only the distribution function of formed (and hence thermalized) fireball pions, \( f_t^{(eq)} \), enters the collision term, since the non-formed particles did not participate in the interaction by assumption.

Integrating \( C_t(f_\alpha, f_t) \) weighted with the 4-momentum \( p' \) over momentum, we arrive at

\[ F_{f_\alpha}^{\nu}(x) = \int \frac{d^3q d^3p}{p_0 p_0'} p'^\nu W^{N\pi\to R}(s) f_t^{(eq)}(x, p) f_\alpha(x, q) \]

\[ \simeq \frac{W^{N\pi\to R}(s_{f_\alpha})}{m_\pi u_t^0} \left( \int \frac{d^3q}{q_0} f_\alpha(x, q) \right) \left( \int \frac{d^3p}{p_0} p^0 p'^\nu f_t^{(eq)}(x, p) \right) = D_{f_\alpha} T_t^{(eq)\nu} \rho_\alpha, \]  

(18)

where we substituted \( p^0 \) and \( s \) by their mean values, \( < p^0 > = m_\pi u_t^0 \) and \( s_{f_\alpha} = (m_\pi u_t + m_N u_\alpha)^2 \), and introduced the transport coefficients

\[ D_{f_\alpha} = \frac{W^{N\pi\to R}(s_{f_\alpha})}{(m_N m_\pi)} = V_{rel}^{f_\alpha} \sigma_{tot}^{N\pi\to R}(s_{f_\alpha}). \]  

(19)

Here, \( V_{rel}^{f_\alpha} = [(s_{f_\alpha} - m_N^2 - m_\pi^2)^2 - 4m_N^2m_\pi^2]^{1/2}/(2m_N m_\pi) \) denotes the mean invariant relative velocity between the fireball and the α-fluids. Thus, we have expressed the friction \( F_{f_\alpha}^{\nu} \) in terms of the fireball-fluid energy-momentum density \( T_t^{\nu\nu} \), the scalar density \( \rho_\alpha \) of the α fluid, and a transport coefficient \( D_{f_\alpha} \). Note that this friction is zero until the fireball pions are formed, since \( T_t^{(eq)\nu\nu} = 0 \) during the formation time \( \tau \).

In fact, the above treatment is an estimate of the friction terms rather than their strict derivation. This peculiar way of evaluation is motivated by the form of the final result (18). An advantage of this form is that \( m_\pi \) and any other mass do not appear explicitly, and hence allows a natural extension to any content of the fluid, including deconfined quarks and gluons, assuming that \( D_{f_\alpha} \) represents just a scale of the transport coefficient.

**IV. SIMULATIONS OF NUCLEUS–NUCLEUS COLLISIONS**

The relativistic 3D code for the above described 3-fluid model was constructed by means of modifying the existing 2-fluid 3D code of refs. [3, 4, 7, 8, 9]. In actual calculations
we used the mixed-phase EoS developed in \[21, 22, 23\]. This phenomenological EoS takes into account a possible deconfinement phase transition of nuclear matter. The underlying assumption of this EoS is that unbound quarks and gluons may coexist with hadrons in the nuclear environment. In accordance with lattice QCD data, the statistical mixed-phase model describes the first-order deconfinement phase transition for pure gluon matter and crossover for that with quarks \[21, 22, 23\].

We performed simulations of nucleus–nucleus collisions Pb+Pb at $E_{\text{lab}} = 158 \ A GeV$ and Au+Au at $E_{\text{lab}} = 10.5 \ A GeV$. General dynamics of heavy-ion collisions is illustrated in Fig. 1 by the energy-density evolution of the baryon-rich fluids ($\varepsilon_b = \varepsilon_p + \varepsilon_t$, in the cm frame of colliding nuclei) in the reaction plane of the Pb+Pb collision. Different stages of interaction at relativistic energies are clearly seen in this example: Two Lorentz-contracted nuclei (note the different scales along the $x$- and $z$-axes in Fig.1) start to interpenetrate through each other, reach a maximal energy density by the time $\sim 1.1 \ fm/c$ and then expand predominantly in longitudinal direction forming a “sausage-like” freeze-out system.

At this and lower incident energies the baryon-rich dynamics is not really disturbed by the fireball fluid and hence the cases $\tau = 0$ and 1 fm/c turned to be indistinguishable in terms of $\varepsilon_b$.

In Fig. 2 the dynamic evolution of the fireball energy density ($\varepsilon_f$, in the cm frame of the colliding nuclei) in the reaction plane of the Pb+Pb collision at impact parameter $b = 2 \ fm$ is shown for two values of the formation time, $\tau = 0 \ fm/c$ (the left column of panels) and 1 fm/c (the right column of panels). It starts to form near the time moment, when the maximal energy density $\varepsilon_b$ is reached. The $f$-fluid evolution indeed looks like that for an expanding fireball, its density essentially depends on the formation time.

To quantitively reveal the role of the interaction between fireball and baryon-rich fluids, we followed the evolution of the total energy released into the fireball fluid

$$E_f^{(\text{released})}(t) = \int_0^t dt' \int d^3x' \left( F_p^0(x') + F_t^0(x') \right), \quad (20)$$

cf. Eq. (8), and the total energy kept in the fireball fluid (both thermalized and nonthermalized) after interaction,

$$E_f^{(\text{tot})}(t) = \int d^3x \ T_{ij}^{(0)}(t, x) \right)$$

$$= \int_0^t dt' \int d^3x' \left( F_p^0(x') + F_t^0(x') - F_{fp}^0(x') - F_{ft}^0(x') \right), \quad (21)$$
Figure 1: Time evolution of the energy density, $\varepsilon_b = \varepsilon_p + \varepsilon_t$, for the baryon-rich fluids in the reaction plane ($xz$ plane) for the Pb+Pb collision ($E_{\text{lab}} = 158$ A·GeV) at impact parameter $b = 2$ fm. Shades of gray represent different levels of $\varepsilon_b$ as indicated at the right side of each panel. Numbers at this palette show the $\varepsilon_b$ values (in GeV/fm$^3$) at which the shades change. Arrows indicate the hydrodynamic velocities of the fluids.
Figure 2: The same as in Fig. 1, but for the fireball-energy density ($\varepsilon_f$ in the cm frame of the colliding nuclei) for two formation times, $\tau = 0$ fm/c (the left column of panels) and 1 fm/c (the right column of panels).

cf. Eq. (8), in the cm frame of two colliding nuclei. Results of the calculation are presented in Fig. 3. To provide a common scale, the $E_{\text{f released}}$ quantity calculated with the formation time $\tau = 100$ fm/c is presented in all the panels. The $\tau = 100$ fm/c case practically
implies absence of interaction between the fireball and baryon-rich fluids and the equality
\( E_f^{(\text{tot})} = E_f^{(\text{released})} \), because \( T_f^{(\text{eq})0} = 0 \) and hence \( F_{fp}^0 = F_{ft}^0 = 0 \), cf. Eq. \( \text{[18]} \).

Figure 3: Time evolution of the total energy released into the fireball fluid, \( E_f^{(\text{released})} \) (solid lines), and the total energy kept in the fireball fluid after interaction, \( E_f^{(\text{tot})} \) (dashed-dotted lines). Two nucleus–nucleus collisions Pb+Pb at \( E_{\text{lab}} = 158 \text{ A·GeV} \) (three upper panels) and Au+Au at \( E_{\text{lab}} = 10.5 \text{ A·GeV} \), both at zero impact parameter, were calculated with different formation times \( \tau \) indicated in the panels. The upper dashed curve in all 6 panels represent \( E_f^{(\text{released})} \) calculated with the formation time \( \tau = 100 \text{ fm/c} \).

First, we see that the energy release into in the fireball fluid occurs only during a short time of interpenetrating of colliding nuclei. As it has been expected, at zero formation time \( \tau = 0 \) the interaction with the baryon-rich fluids strongly affects the fireball fluid: It reduces its total energy \( E_f^{(\text{tot})} \) as compared to the case without interaction (i.e. \( \tau = 100 \text{ fm/c} \)). Even the released energy \( E_f^{(\text{released})} \) drops down. This effect results from additional stopping of
baryon-rich fluids associated with friction with the fireball fluid. Because of this additional stopping, the baryon-rich fluids produce less secondary particles (and those produced are less energetic). Naturally, this effect is more pronounced at the energy 158 A·GeV, since the amount of produced secondary particles is much larger in this case than that at lower energies.

At realistic values of the formation times, $\tau = 0.5$ and 1 fm/c, the effect of the interaction is substantially reduced. It happens because the fireball fluid starts to interact only near the end of the interpenetration stage. As a result, by the end of the collision process it looses only 10% of its available energy $E_t^{(\text{released})}$ at $E_{\text{lab}} = 158$ A·GeV and 30%, at $E_{\text{lab}} = 10.5$ A·GeV. Certainly, this effect should be observable in mesonic quantities, in particular, in such fine observables as directed and elliptic flows. The global baryonic quantities stay practically unchanged at finite $\tau$. Indeed, $E_t^{(\text{released})}$ remains almost the same as at $\tau = 100$ fm/c.

The energy content of the baryon-rich fluids exceeds that of the fireball by an order of magnitude at $E_{\text{lab}} = 158$ A·GeV (3440 GeV) and even more at $E_{\text{lab}} = 10.5$ A·GeV (929 GeV). Therefore, the interaction with the fireball fluid does not essentially change the global baryonic quantities. As for refined baryonic observables, our preliminary calculations of the directed nucleon flow show no changes at $E_{\text{lab}} = 10.5$ A·GeV and only slight changes in the mid-rapidity region at $E_{\text{lab}} = 158$ A·GeV. This means that our previous results on the nucleon directed flow and its excitation function, obtained within the 2-fluid model [9], are not affected by the interaction between the baryon-rich and fireball fluids.

V. CONCLUSIONS

In this paper we have developed a 3-fluid model for simulating heavy-ion collisions in the range of incident energies between few to about 200 A·GeV. In addition to two baryon-rich fluids, which constitute the 2-fluid model [3, 4, 7, 8, 9], a delayed evolution of the produced baryon-free (fireball) fluid is incorporated. This delay is governed by a formation time, during which the fireball fluid neither thermalizes nor interacts with the baryon-rich fluids. After the formation, it thermalizes and comes into interaction with the baryon-rich fluids. This interaction is estimated from elementary pion-nucleon cross-sections.

The hydrodynamic treatment of heavy-ion collisions is an alternative to kinetic simu-
lations. The hydrodynamic approach has certain advantages and disadvantages. Lacking the microscopic feature of kinetic simulations, it overcomes their basic assumption, i.e. the assumption of binary collisions, which is quite unrealistic in dense matter. It directly addresses the nuclear EoS that is of prime interest in heavy-ion research. Furthermore, our 3-fluid model uses only friction forces instead of a vast body of differential cross-sections of elementary processes, which are generally unknown experimentally. Naturally, we have to pay for all these pleasant features of hydrodynamics: the treatment assumes that the nonequilibrium stage of the collision can be described by the 3-fluid approximation. However, all the assumptions used are quite transparent and can be tested numerically.

We have simulated relativistic nuclear collisions within the 3D code based on the relativistic 3-fluid hydrodynamics combined with the EoS of the statistical mixed-phase model of the deconfinement phase transition, developed in [21, 22, 23]. We performed calculations of nucleus–nucleus collisions Pb+Pb at $E_{\text{lab}} = 158$ A·GeV and Au+Au at $E_{\text{lab}} = 10.5$ A·GeV. To reveal the role of the interaction between fireball and baryon-rich fluids, we examined the evolution of global quantities of the fireball fluid.

For zero formation time ($\tau = 0$) the interaction strongly affects the fireball fluid: It considerably reduces its total energy as compared to that without interaction. However, for realistic reasonable finite formation time, $\tau \approx 1$ fm/c, the effect of the interaction is substantially reduced. The fireball fluid looses only 10% of its available energy at $E_{\text{lab}} = 158$ A·GeV and 30%, at $E_{\text{lab}} = 10.5$ A·GeV. Certainly, this effect should be observable in mesonic quantities, in particular, in such sensitive observables like directed and elliptic flows. Since the energy content of the baryon-rich fluids is much higher than that of the fireball fluid, global baryonic quantities remain insensitive to this interaction. As our preliminary calculations show, even the directed nucleon flow remains practically unaffected by this interaction. In particular, this fact justifies our previous results on the nucleon directed flow and its excitation function, obtained within the 2-fluid model [9].

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