Superconducting transition induced by columnar disorder in strong magnetic field

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Abstract: The superconducting transition in presence of strong columnar disorder parallel to the magnetic field is considered. A solvable model appropriate for description of the broad crossover regime towards the true "glassy" critical behavior is constructed, and the behavior of the thermodynamical quantities and of the Edwards-Anderson order parameter near transition is obtained. The critical exponents for the correlation lengths orthogonal and parallel to the magnetic field are the same and in agreement with the experimental values. The dynamical critical exponent is $z = 2$, also in agreement with the measured value. Several perturbations to the solvable model are considered and shown to be irrelevant for the critical behavior. It is argued that there exists an optimal density of defects at which the transition temperature at given magnetic field reaches its maximum.
1 Introduction

The problem of superconducting transition in the magnetic field in presence of strong disorder has been a subject of great interest in recent years. From the point of view of applications, there is a need to know how to introduce defects into a superconducting material in a way that would maximize pinning of vortices and therefore increase critical currents [1]. On the theoretical side, understanding of the phase transitions in presence of quenched disorder and description of the low-temperature disordered phases have always been challenging problems, and often required novel physical concepts and mathematical techniques. A prime example of this is the theory of spin glasses. For the superconducting transition in the magnetic field, a variety of novel low-temperature phases have been proposed, differing in the cases of point like [2, 3] and line-like disorder [4, 5, 6]. In the present paper we discuss a theory of superconducting transition at high magnetic fields in presence of columnar (line-like) defects [6]. We have in mind a three-dimensional, anisotropic, strongly type-II superconductor (YBCO, for example) in typical magnetic fields of ∼1T, irradiated by a flux of some heavy ions with energies ∼1GeV and with trajectories parallel to the external magnetic field. If the thickness of the sample in the direction of the beam is ∼10µm, the ions are able to penetrate through the entire material, leaving behind the continuous tracks of damaged superconductor of diameter d ∼ 50Å. In the absence of disorder, the high-field fluctuations of the order parameter, ψ(⃗r), are strongly enhanced by formation of Landau levels (LLs) for Cooper pairs. Such fluctuations lead to $D \rightarrow D - 2$-dimensional reduction in the pairing-susceptibility, $\chi_{sc}(\vec{r}, \vec{r}')$, and, strictly speaking, eliminate the superconducting (Abrikosov) transition for $D = 2, 3$ [7]. The Abrikosov phase is then replaced by a new fluctuation-induced state, the density-wave of Cooper pairs (SCDW), in which the thermal average $\langle |\psi(\vec{r})|^2 \rangle$ has a weak modulation not necessarily accompanied by a long-range phase coherence [8]. The chief effect of disorder is to remove the LL degeneracy and so to restore a possibility of a true superconducting transition. In this sense the superconducting transition that we will consider is induced by the presence of disorder. $\chi_{sc}$ can now diverge at some finite temperature, $T_{sc}(H)$, determined by the strength of disorder. The superconducting transition corresponds to the Bose condensation of Cooper pairs into the lowest energy eigenstate of the random potential, which, we will argue, extends over the whole sample in situations of experimental interest. Furthermore, for experimentally relevant parameters, $T_{sc}(H)$ can be far above the SCDW transition line [8] over much of the $H - T$ phase diagram, allowing us to treat the fluctuations that produce SCDW in an approximate way.

In the following sections we first define a model for superconductor in a magnetic field parallel to the columnar defects, which represents a realistic description of the problem for certain ranges of magnetic fields and densities of columnar defects. It is demonstrated that the model is exactly solvable. The solution exhibits “dimensional transmutation”, i.e. the effective dimensionality of the transition changes continuously as a function of magnetic field. This effect is a direct consequence of analytic properties of LL wavefunctions and is a signature of the high-field limit. We determine the transition line in $H - T$ phase diagram, the Edwards-Anderson order parameter, and the behavior of specific heat and magnetic
susceptibility in vicinity of the transition. There are similarities between the transition considered here and the one in the spherical model for spin-glasses [9]. Using the information on diffusion in strong magnetic field that comes from the studies of the quantum Hall effect, we determine the critical exponents for the correlation lengths and the dynamical critical exponent. The calculated exponents are in good agreement with the experimental values [5]. We then consider perturbations to our exactly solvable model. In particular, we discuss the breakdown of the model for higher density of defects. It is argued that disorder then becomes less efficient in removing LL degeneracy, enabling fluctuations again to suppress superconducting transition temperature, similarly to what happens in the homogeneous case. This naturally suggests the existence of the optimal dose of irradiation which produces the highest transition temperature at given magnetic field. We conclude by reviewing the salient points of our model and by briefly reanalyzing the approximations built into it.

2 Statement of the problem and the solvable model

We are interested in strongly anisotropic layered superconductors described by the Ginzburg-Landau (GL) Lawrence-Doniach model, with magnetic field perpendicular to the layers. Fluctuations in magnetic field are neglected (GL parameter $\kappa \gg 1$). We focus on the high-field limit, where the LL structure of Cooper pairs dominates the fluctuation spectrum: This is the case for fields above $H_b \approx (\theta/16)H_c(0)(T/T_c(0))$, where $\theta$ is the Ginzburg fluctuation parameter [10]. (For instance, in BSCCO 2:2:1:2, $\theta \approx 0.045$ and $H_b \approx 1$ Tesla.) In this regime, the essential features of the physics are captured by retaining only the lowest Landau level (LLL) modes. Generally, the partition function is

$$Z = \int D[\psi^*, \psi] \exp(-S),$$

where

$$S = \frac{b}{T} \sum_{n=1}^{N_k} \int d^2 \vec{r} \eta |\psi_n(\vec{r}) - \psi_{n+1}(\vec{r})|^2 + (\alpha'(T, H) + \lambda \sum_i V(|\vec{r} - \vec{r}_i|))|\psi_n(\vec{r})|^2 + \frac{\beta}{2} |\psi_n(\vec{r})|^4,$$

(1)

where $\alpha'(T, H) = a(T - T_c(H))$, $b$ is the effective layer separation, $n$ is the layer index and $a$, $\beta$ and $\eta$ are phenomenological parameters. The magnetic field is assumed to be parallel to columnar defects, which are modeled by an effective potential $V(\vec{r} - \vec{r}_i) > 0$, peaked at $\vec{r} = \vec{r}_i$ with a width comparable to the diameter of the columns $d$. The parameter $\lambda > 0$ represents the effective strength of disorder, and its positiveness reflects the fact that the superconducting temperature is locally suppressed by damaging the material. Random variables in the problem are two-dimensional coordinates of defects, $\{\vec{r}_i\}$. We assume that the positions of columns of damaged superconductor are uncorrelated, i. e. that they are distributed according to the Poisson distribution $P_N(\vec{r}_1, ... \vec{r}_N) = (e^{-\rho A}/\rho^N)/N!$ where $P_N$ is the probability for finding $N$ impurities at the positions $\vec{r}_1, ... \vec{r}_N$, $A$ is the area of the system and $\rho$ is the concentration of impurities [11].

The partition function in Eq. 1 is quite general, and defines the problem we want to study. To proceed, note that in certain range of parameters one may make two simplifying
assumptions. First, if the magnetic length \( l \) and the average distance between defects are both much larger than the diameter of the columns \( d \), we may assume that \( V(\vec{r}) = \delta(\vec{r}) \). At fields of \( \sim 1 \) T, \( l \approx 200 \text{Å} \), and for moderate doses of \( 10^9 - 10^{11} \text{ion/cm}^2 \) these conditions are reasonably satisfied. This approximation makes the random potential which describes the disorder completely uncorrelated in space, and facilitates the exact treatment. Second, after rescaling the fields and the lengths as \((2b \beta \pi l^2 / T)^{1/4} \psi \to \psi, r / (l \sqrt{2 \pi}) \to r\), the quartic term can be rewritten as

\[
\frac{1}{4} \sum_n \int d^2 \vec{r} |\psi_n|^4 = \frac{1}{4N} \sum_n \beta_A(n) \left( \int d^2 \vec{r} |\psi_n|^2 \right)^2,
\]

where \( \beta_A(n) = (N \int |\psi_n|^4) / (\int |\psi_n|^2)^2 \) is the generalized Abrikosov ratio corresponding to configuration \( \psi_n(\vec{r}) \), and \( N = A / 2 \pi l^2 \) is the degeneracy of the LLL. We now observe that \( \beta_A(n) \) is only weakly dependent on the actual configuration, the well known example being the small difference in \( \beta_A \) between triangular and square lattice of zeroes \[10\]. Thus, we may approximate \( \beta_A(n) \) in the quartic term by a constant of order unity. This replaces the local quartic term in the general theory (1) by an interaction still diagonal in layer indices, but infinitely ranged within a layer. This approximation neglects the lateral fluctuations that produce the SCDW transition \[7\], and without disorder this theory has no finite temperature phase transition below four dimensions. Thus, the disorder is assumed strong on the fine energy scale set by the local quartic term, but weak compared to the energy scale of LL separation, so that the LLL approximation is sensible. The neglect of weak lateral correlations is justified if \( T_{sc}(H) \) is far above the SCDW transition line \[8\]. In that case the SCDW fluctuations start to matter only very close to the glassy transition and can be ignored in most realistic situations. Since superconducting and SCDW transitions arise from two distinct mechanisms, the respective transition lines scale differently in the \( H - T \) phase diagram and, for moderate disorder, we are assured of a wide crossover region near \( H_{c2}(T) \) where the neglect of SCDW fluctuations in our model is justified.

3 Transition line and the order parameter

After the \( \beta_A(n) \to \langle \beta_A \rangle \sim 1 \) substitution the thermodynamics of the model becomes exactly solvable \[3\]. We first introduce variables \( \{x_n\} \) to decouple the quartic term and integrate over the fields \((\psi^*, \psi)\). This leads to \( Z = \int \prod_n dx_n \exp(-NS') \), where

\[
S' = -\sum_n \frac{x_n^2}{\langle \beta_A \rangle} + \int_0^\infty dV \rho_f(V) Tr_{n,m} \ln[g_\eta(2\delta_{n,m} - \delta_{n,m-1} - \delta_{n,m+1}) + (g_\alpha + x_n + g_\lambda V) \delta_{n,n}].
\]

We drop the terms coming from the rescaling of \( \psi_n(\vec{r}) \) and introduce dimensionless combinations of GL parameters \( g_\eta,\alpha,\lambda = \{\eta,\alpha', \lambda / 2 \pi l^2 \} \times \sqrt{(b \pi l^2) / (T \beta)} \). The density of states for an uncorrelated random potential in the LLL can be found exactly by using the supersymmetric
formalism \[12\]. For the Poisson infinitely short-range scatterers it is given by the integral:

\[
\rho_f(V) = \frac{1}{\pi} \Im \frac{d}{dV} \ln \int_0^\infty dt \exp(\imath Vt - f \int_0^t \frac{dy}{y}(1 - e^{-\imath y})) ,
\]

(4)

where \( f = \rho 2\pi l^2 = H_\Phi / H \), and \( H_\Phi \) is the matching field, at which there is precisely one defect per unit of flux. In the thermodynamic limit \( N \to \infty \), the partition function is completely determined by the saddle-point of \( S' \). Assuming that the saddle point is at \( x \) independent of the layer index, we finally write the free energy above the critical temperature \( F_{\text{NN}}(T) = -x^2 \langle \beta A \rangle + \frac{1}{2} \int_{-1}^{1} dk \int_0^\infty dV \rho_f(V) \ln [g_q e(k) + g_\alpha + x + g_\Lambda V] , \)

(5)

where \( e(k) = 1 - \cos(k) \) and \( x \) is determined by the solution of

\[
x = \frac{\langle \beta A \rangle}{4} \int_{-1}^{1} dk \int_0^\infty \frac{\rho_f(V)dV}{g_q e(k) + g_\alpha + x + g_\Lambda V} .
\]

(6)

In Eq. 6 it is important to know the behavior of density of states at low energies. For \( f < 1 \), density of states has a delta-function singularity at \( V = 0 \), while for \( f > 1 \), \( \rho_f(V) \sim V^{-2} \) when \( V \to 0 \) \[12\] \[13\]. The transition line, \( T_{\text{sc}}(H) \), in the \( H - T \) diagram is determined by Eq. 6 and \( x + g_\alpha = 0 \), which corresponds to condensation of Cooper pairs into \( k = 0 \) and \( V = 0 \) eigenstate of the random potential. Without disorder the LLL is completely degenerate and \( \rho(V) = \delta(V) \). The integral on the right hand side of Eq. 6 is then infrared divergent when \( x + g_\alpha = 0 \), and there can be no finite temperature phase transition. With the columnar disorder present, from the behavior of the density of states it is easily seen that there will be a non-zero transition temperature only if concentration of impurities and magnetic field are such that \( f > 3/2 \). Below this value of \( f \) LLL degeneracy is not sufficiently lifted by the random potential and thermal fluctuations prevent a finite temperature phase transition in our model. \( f = 3/2 \) determines the effective lower critical dimension for our model. After introducing dimensionless quantities \( t = T/T_c(0) \), \( h = H/H_{c2}(0) \) and \( \lambda' = \lambda H_{c2}(0)/\phi_0 a T_0 \), where \( \phi_0 \) is the flux quantum, we perform the integration over wave-vector \( k \) in Eq. 6 to obtain the expression for transition temperature:

\[
t_{\text{sc}}(h) = (1 - h) \left[ 1 + \frac{\beta A}{2 \lambda'} \int_0^\infty \frac{\rho_f(V)dV}{\sqrt{V^2 + (2\eta V)/(h \lambda' a T_0)}} \right]^{-1} .
\]

(7)

Notice that when \( \lambda' \to 0 \) we have \( t_{\text{sc}}(h) \to 0 \), while for increasing \( \lambda' \), transition temperature \( t_{\text{sc}}(h) \) increases. Also, with increasing density of columnar defects (increasing parameter \( f \)), \( t_{\text{sc}}(h) \) increases. This is related to the experimental observation \[1\] that the irreversibility line shifts to higher temperatures with increasing dose of irradiation with heavy ions.

As temperature drops below \( t_{\text{sc}}(h) \), \( x \) remains at the value it had at the transition. There is now a macroscopic occupancy of the lowest energy state at \( V = 0 \) and \( k = 0 \). As is well known, condensation into this state is a meaningful concept only if the state is extended.
Condensation into a localized lowest energy state would imply a diverging Abrikosov ratio $\langle \beta_A \rangle$ below the transition, making the transition impossible. It is a special feature of this problem that the lowest lying state must indeed be extended for certain range of impurity concentrations. The density of states, Eq. 4, changes from being infinite at $V = 0$ when $f < 2$, to being zero when $f > 2$. Thus, for fields and impurity concentrations such that parameter $f < 2$, true extended states (which always exist in the LLL [14]) must lay at the bottom of impurity band, since the number of states there diverges. The change of behavior in the density of states at $f = 2$ could be caused by the fact that the mobility edge shifts to positive energies at some $f_0 > 2$, leaving spread-out but localized states at $V = 0$, which now becomes the tail of the distribution. Numerical diagonalization studies indicate that mobility edge is indeed located near the band center for $f > 4$ [15]. Thus, strictly speaking, our model is appropriate for $f < f_0$. However, even for $f$ above but close to $f_0$, which is often the case for fields and concentrations of experimental interest, the states at $V = 0$ are still near mobility edge and will appear extended in a finite size sample. On this basis, we expect that useful information about the transition can still be obtained within our model.

With these cautionary remarks in mind, the natural order parameter is the thermal average of the component of $\psi_n(\vec{r})$ corresponding to the eigenvalue with $V = 0$ and $k = 0$. This is $\langle \psi_{0,0} \rangle = (NN_L(g_{\alpha}|_{t=t_{sc}(h)} - g_{\alpha})/\langle \beta_A \rangle)^{1/2}$. The disorder-averaged value of the field is $\langle \psi_n(\vec{r}) \rangle = 0$, due to random phases of the state $\phi_{V=0}(\vec{r})$. Under the assumption that the lowest state is extended through the sample, $|\phi_{V=0}(\vec{r})|^2 \approx 1/N$; the Edwards-Anderson order parameter [16] $q_{EA} = |\langle \psi_n(\vec{r}) \rangle|^2$ then equals

$$q_{EA} = \frac{2}{\langle \beta_A \rangle} (g_{\alpha}|_{t=t_{sc}(h)} - g_{\alpha})$$

below $t_{sc}(H)$, and is zero above. Thus, $q_{EA} \propto (t_{sc}(h) - t)^{2\beta}$, with the exponent $\beta = 1/2$.

The free energy below $t_{sc}(h)$ is

$$\frac{F}{NN_LT} = -\frac{g_{\alpha}^2}{\langle \beta_A \rangle} + \frac{1}{2} \int_{-1}^1 dk \int_0^\infty \rho_f(V)dV \ln (g_\eta e(k) + g_\lambda V).$$

**4 Correlation lengths and dynamical exponent**

To calculate the exponents that determine the divergence of correlation lengths parallel and perpendicular to the field we first note that from Eq. 6 and the definition of critical line it follows

$$(g_{\alpha} + x)[1 + \frac{\langle \beta_A \rangle}{4} \int_{-1}^1 dk \int_0^\infty \frac{\rho_f(V)dV}{(g_\eta e(k) + g_\lambda V)(g_\eta e(k) + g_\lambda V + g_{\alpha} + x)}]$$

$$= g_{\alpha} - g_{\alpha}|_{t=t_{sc}(h)}.$$

The integral in the last equation diverges for $f < 5/2$ as $(g_{\alpha} + x)^{f-5/2}$ when the transition line is approached from above, and it is finite for $f > 5/2$. Thus, we obtain $(g_{\alpha} + x) \propto$
\((t - t_{sc})^{1/(f-3/2)}\) for \(f < 5/2\) and \((g_\alpha + x) \propto (t - t_{sc})\) for \(f \geq 5/2\). The same behavior follows if the transition line is approached along the line of constant temperature. This determines the value of the exponent for the correlation length parallel to the field \(\xi_{\parallel} \propto \frac{2}{2f - 3}\) :

\[
\nu_{\parallel} = 1/(2f - 3),
\]

(11)

for \(f < 5/2\) and the classical value \(\nu_{\parallel} = 1/2\) for \(f > 5/2\). The concentration corresponding to \(f = 5/2\) determines the effective upper critical dimension in the problem. We now turn to correlation length perpendicular to the field, \(\xi_{\perp} \propto [t - t_{sc}(h)]^{1/(2f - 3)}\) :

\[
\nu_{\perp} = \frac{1}{2(2f - 3)},
\]

(12)

and \(\phi_i(\vec{r})\) are the eigenstates of the random potential. If we now introduce \(V = (V_1 + V_2)/2\) and \(\omega = (V_1 - V_2)/2\), for \(V\) close to mobility edge and small \((q, \omega)\), the Fourier transform of \(F\) has a diffusive form \[17, 18\]

\[
F(\vec{q}, V, \omega) = \frac{\rho_f(V)q^2D(q^2/\omega)}{\pi(\omega^2 + q^4D^2(q^2/\omega))},
\]

(15)

where \(D(q^2/\omega)\) is the generalized “diffusion constant”. Assuming this form for \(F(\vec{q}, V, \omega)\) and rescaling everything by the appropriate power of temperature in Eq. 12, we find:

\[
\nu_{\perp} = \nu_{\parallel}.
\]

(16)

Surprisingly, the scaling turns out to be isotropic, in spite of the strong anisotropy in the model.

The expression for two-particle spectral density yields immediately another important result. Since the energy and the momentum variables appear always in combination \(\omega/q^2\), the dynamical exponent has the value:

\[
z = 2.
\]

(17)

This may also be found by directly calculating the dc conductivity along the field using the time-dependent version of the model, which yields \(\sigma_{zz} \propto \xi_{\parallel}\), the same as Aslamazov-Larkin result in three dimensions.
The dependence of the correlation length exponents on magnetic field through parameter $f$ is the consequence of power-law behavior of the density of states close to the bottom of the band. A closer look at the density of states $\rho_f(V)$ reveals it to be roughly constant except in a narrow region, typically less than 1% of total bandwidth, around $V = 0$, where it either diverges or vanishes [13]. One might expect that taking the effective potential $V(\vec{r})$ to have a finite range would tend to wash away this fine feature of the density of states. This suggests that the experimentally relevant situation corresponds to $f = 2$ in our model, where the density of states is flat down to the lowest energies. Based on this observation one would expect that the observable values of the exponents are $\nu_\perp = \nu_\parallel = 1$. This expectation is met by the experimental results of Ref. 5: $\nu_\perp = 1.0 \pm 0.1$, $\nu_\parallel = 1.1 \pm 0.1$. The value of dynamical exponent is also in agreement with the measured $z = 2.2 \pm 0.2$ [5].

5 Magnetization and specific heat

Magnetization per unit volume is found by differentiating the averaged free energy with respect to the magnetic field:

$$
\frac{M}{ANLd} = -\frac{2T_0 \sqrt{\theta}}{d\phi_0 \sqrt{\theta}} \left( q_{EA} + \frac{2x}{\langle \beta_A \rangle} \right).
$$

(18)

Below the transition line this coincides with the usual mean-field result. Above the transition line $q_{EA} = 0$ and from Eq. 10 it follows that at constant temperature close to the transition $(g + x) \propto [h - h_{sc}(t)]^{1/[(f - 3/2)]}$ when $f < 5/2$ and $(g + x) \propto [h - h_{sc}(t)]$ otherwise. Thus the magnetic susceptibility is a smooth function of the field at the transition for $f < 2$, but has an upward cusp for $2 < f < 5/2$ and a discontinuity for $f > 5/2$. The size of this discontinuity depends on the location of the transition in the $H - T$ diagram. Differentiating the free energy twice with respect to temperature one obtains the specific heat. It is straightforward to show that at the transition it behaves the same way as susceptibility; smooth for $f < 2$, has a cusp for $2 < f < 5/2$ and has the usual discontinuity for $f > 5/2$. More precisely, both magnetic susceptibility and specific heat behave as $[t - t_{sc}(h)]^{-\alpha}$ for $3/2 < f < 5/2$, where $\alpha = (f - 5/2)/(f - 3/2)$. The behavior of the specific heat, order parameter and correlation length in our model is related to the one obtained from $O(2N)$ vector model in the limit $N \to \infty$ and in the effective dimension $D_{eff} = 2f - 1$.

6 Perturbations

Having a solvable version of the general theory (1), we are in position to study small perturbations around it and check their relevance for the critical behavior. First, let us assume that the strength of disorder $\lambda$ is also allowed to fluctuate from one columnar defect to another, i.e., the random potential in Eq. 1 is taken to be

$$
\sum_i \lambda_i \delta(\vec{r} - \vec{r}_i), \quad (19)
$$

\[7\]
with the random variable $\lambda_i$ distributed according to the distribution:

$$P(\lambda_i) = \frac{1}{2w},$$

(20)

for $|\lambda_i - \lambda| < w$, $0 < w < \lambda$, and zero otherwise. This random potential, being still spatially uncorrelated, allows an exact calculation of the density of states \[1\,2\]. One can show however \[13\], that the power-law behavior of the density of states at low energies is not changed by allowing finite width $w$, which implies the same critical exponents as already obtained. This perturbation is therefore irrelevant.

A more complicated extension of the model is to assume a particular form of three-dimensional disorder represented by a random potential in Eq. 1:

$$\lambda_n \sum_i \delta(\vec{r} - \vec{r}_i),$$

(21)

where the effective strength of disorder $\lambda_n$ is the same for all defects within the $n$-th layer, but fluctuates from one layer to another according to a Gaussian distribution

$$P[\{\lambda_n\}] \propto \exp - \sum_n (\lambda_n - \lambda)^2 / w^2,$$

(22)

where the width of distribution $w$ is a small parameter, $w << \lambda$. Since the model is not fully three, but rather $2 + 1$ dimensional, one can still study the phase transition by the method described here in combination with the replica formalism \[16\]. Without going into details, the result is that the critical behavior is not changed by a small $w$, and the only effect of additional disorder along the field is a decrease in transition temperature. We conclude that this perturbation is also irrelevant for the critical behavior.

As emphasized earlier, the replacement of disorder potential representing disorder by a sum of delta-functions is sensible only if the other two length scales in the problem, magnetic length and the average separation between the defects, are much larger than the average diameter of the columns. The latter condition obviously breaks down for higher doses of irradiation, when there is a considerable probability of overlap between the columnar defects. In fact, at higher densities of defects the trend of increase of the transition temperature with defect concentration should be reversed. To see this, consider first a small density of defects. Then our model is applicable, and as we already noted, at fixed magnetic field the transition temperature given by eq. 7 increases with increasing parameter $f$. This is because larger $f$ shifts the density of states towards higher energies, decreasing the integral in denominator in the Eq. 7. For high defect densities the model ceases to be useful, but it is still possible to qualitatively understand what happens by considering the limit of very large concentrations. For very large density of defects, the effect of randomly distributed finite width scatterers is similar to having a potential which is almost constant in space. In such a potential the LLL is again nearly degenerate, only the average energy is shifted by some amount from where it was without any external potential. But this being the case, thermal fluctuations are again able to suppress the transition temperature because of the dimensional reduction! Thus,
for large densities of defects transition temperature must be decreasing with the density. This suggests that there is a field dependent optimal defect concentration at which the transition temperature reaches its maximum value, still below the mean-field $T_{c2}(H)$. This would explain the observation of saturation of transition temperature with increase of dose of irradiation in ref. 16.

7 Summary and conclusions

We have studied the high-field superconducting glassy transition induced by columnar disorder. It was argued that in certain range of magnetic fields and for not too large concentrations of defects the random potential that represents disorder may be assumed to be uncorrelated in space. By assuming the self-interaction of the fluctuating superconducting order-parameter to be infinitely ranged in directions perpendicular to the field we defined a model which describes the crossover regime towards the true glassy critical behavior, and which can be solved exactly. Within this model we have calculated the transition line, Edwards-Anderson order parameter, magnetization, specific heat, critical exponents for the correlation lengths and the dynamical exponent. The critical behavior of the model is shown to be isotropic, and the values of the critical exponents are in agreement with existing measurements. Several extensions of our model are considered, and some are shown to be irrelevant for the critical behavior. We presented a qualitative arguments for the behavior of transition temperature with density of defects, and argued for the existence of optimum concentration of defects at which the transition temperature reaches its maximum.

The crucial simplifications of the general GL theory with columnar disorder in Eq. 1 which facilitated the exact solution of the model were the assumption of delta-function scatterers for representation of disorder and the neglect of weak SCDW correlations, i.e., $|\phi|^4$ interaction term was replaced by interaction infinitely long-ranged within layers. The first approximation is not a drastic one for small densities of defects; allowing for some range of random scatterers is likely to wash away the power-law behavior of density of states at low energies and make the critical behavior analogous to the $f = 2$ case for our model. This picture is in agreement with the measured values of critical indices for correlation lengths \[5\]. Neglect of SCDW correlations represents a more serious approximation, which necessarily breaks down sufficiently close to the critical temperature. The model solved here is thus best understood as a description of the crossover region before the true critical (glassy) behavior is reached. It is a disordered equivalent of spherical model for spin systems. Strong disorder however, guarantees a rather wide crossover region where our model is useful. This is supported by experimental observation of nearly isotropic scaling \[4\], while the general model in Eq. 1 would be most likely to exhibit an anisotropic critical exponents. Also, for calculations of non-universal quantities, such as the transition line in $H-T$ phase diagram, the present model augmented with more realistic potential $V(\mathbf{r})$ should suffice. It is therefore a useful step in quantitative understanding of superconducting transition in presence of correlated disorder in magnetic field.
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References

[1] Civale L. et al. (1991), Vortex confinement by columnar defects in $YBa_2Cu_3O_7$ crystals: enhanced pinning at high fields and temperatures, Phys. Rev. Lett. 67, 648-651.

[2] Fisher M. P. A. (1989) Vortex-glass superconductivity: a possible new-phase of high-$T_c$ oxides, Phys. Rev. Lett. 62, 1415-1418; Dorsey A. T., Huang M., and Fisher M. P. A. (1992), Dynamics of the normal to vortex-glass transition: mean-field theory and fluctuations, Phys. Rev. B 45, 523-526.

[3] Koch R. et al (1989) Experimental evidence for vortex-glass superconductivity in $Y-Ba-Cu-O$, Phys. Rev. Lett. 63, 1511-1514.

[4] Nelson D. R. and Vinokur V. M. (1993) Boson localization and correlated pinning of superconducting vortex-arrays, Phys. Rev. B 48, 13060-13077.

[5] W. Jiang et al (1994), Evidence of a Bose-glass transition in superconducting $YBa_2Cu_3O_7$ single crystals with columnar defects, Phys. Rev. Lett. 72, 550-553.

[6] Tešanović Z. and Herbut I. F. (1994) High-field superconducting transition induced by correlated disorder, Phys. Rev. B 50, 10389-10392.

[7] Tešanović Z. (1994) Critical behavior in type-II superconductors, Physica (Amsterdam) C, 220, 303-309; Herbut I. F. and Tešanović Z. (1994) Density-functional theory of freezing of vortex liquid in quasi-two-dimensional superconductors, Phys. Rev. Lett. 73, 484-487.

[8] Herbut I. F. and Tešanović Z. (1995) First-order melting of vortex lattice in strongly type-II three-dimensional superconductors, Physica (Amsterdam) C, 255, 324-328.

[9] Kosterlitz J., Thouless D. and Jones R. (1976) Spherical model of a spin-glass, Phys. Rev. Lett. 36, 1217-1220.

[10] Tešanović Z. and Andreev A. V. (1994) Thermodynamic scaling functions in the critical region of type-II superconductors, Phys. Rev. B 49, 4064-4075.
[11] Friedberg R. and Luttinger J. M. (1975) Density of energy levels in disordered systems, Phys. Rev. B 12, 4460-4474.

[12] Brezin E., Gross D. and Itzykson C. (1984) Density of states in presence of strong magnetic field and random impurities, Nucl. Phys. B 235, 24-45.

[13] Herbut I. F. (1995) Spectral boundary of a positive random potential in a strong magnetic field, Phys. Rev. B 51, 9820-9824.

[14] Laughlin R. (1981) Quantized Hall conductivity in two dimensions, Phys. Rev. B 23, 5632-5633.

[15] Liu D. and Das Sarma S. (1994) Universal scaling of strong-field localization in an integer quantum Hall liquid, Phys. Rev. B 49, 2667-2690 and references therein.

[16] Edwards S. F. and Anderson P. W. (1975) Theory of spin-glasses, J. Phys. F 5, 965-974.

[17] Abrahams E., Anderson P. W., Lee P. A. and Ramakrishnan T. V. (1981) Quasiparticle lifetime in disordered two-dimensional metals, Phys. Rev. B 24, 6783-6789.

[18] Chalker J. T. and Daniell G. J. (1988) Scaling, diffusion and integer quantum Hall effect, Phys. Rev. Lett. 61, 593-596.

[19] A. V. Samoilov A. V., Feigel’ man M. V., Konczykowski M. and Holtzberg F. (1996) Upper limit of the Bose-glass transition in $YBa_2Cu_3O_7$ at high density of columnar defects, Phys. Rev. Lett. 76, 2798-2801.