From factorizations of noncommutative polynomials to combinatorial topology

ABSTRACT: Factorization of polynomials is one of the most fundamental and most classical problems in mathematics but we do not know that much about factorizations of polynomials over noncommutative rings. Beginning 1995, I. Gelfand and the speaker constructed $n!$ different factorizations of a noncommutative polynomial in one variable with $n$ roots in “generic” position and studied “noncommutative splitting algebras” associated with such factorizations. These algebras can be described in terms of ranked directed graphs. There are surprising connections between homology of such graphs and related splitting algebras. Here I construct a bridge between noncommutative algebra related to factorizations of polynomials and combinatorial topology.