POMERON and AdS/CFT CORRESPONDENCE FOR QCD

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The Maldacena conjecture that QCD is holographically dual to gravity in extra dimensions is briefly reviewed. On the basis of this duality conjecture, the complete glueball spectrum is computed which bears a striking resemblance to the known QCD$_4$ spectrum as determined by lattice simulations. In particular, a strong coupling expansion for the Pomeron intercept is obtained.

1 Introduction

It has been a long held belief that QCD in a non-perturbative setting can be described by a string theory. This thirty-year search for the QCD strings has recently led to a remarkable conjecture: QCD is holographically dual to gravity in extra dimensions. To be more precise, the Maldacena conjecture and its further extensions state that Yang-Mills theory is exactly dual to a critical string theory in a non-trivial gravitational background. In such a framework, Pomeron should emerge as a closed string excitation.

In this talk, we provide a brief review for the Maldecena duality conjecture, and summarize results for the glueball spectrum and the Pomeron intercept in the strong coupling limit. The present discussion is heuristic, disregarding important details in order to provide a picture for the relevant physics.

Let us begin by first recalling that in the early days of string theory, (or the “dual resonance model” to use the nomenclature that predates both string theory and QCD), one observed that it was reasonable to represent the hadronic spectrum beginning with zero width “resonances” on exactly linear Regge trajectories. With the advent of QCD this approach was reformulated as the 1/N expansion at fixed ’t Hooft coupling, $g_{YM}^2 N$. States with vacuum quantum numbers could be assigned to closed-strings, including a massive $2^{++}$ tensor glueball on the leading Pomeron trajectory, $\alpha P(t) = \alpha P(0) + \alpha' P t$. Soon a three-fold crisis appeared:

- zero-mass states,
- extra dimensions,
- supersymmetries.

A careful study of negative norm states (i.e. ghosts), tachyon cancellation and the consistency of the perturbative expansion at the one loop level led to super-
symmetric string theories in 10 space-time dimensions. At the one-loop level
unitarity requires that pair creation of two open strings, each contains "zero-
mass" spin-1 states, is dual to a vacuum exchange with an intercept \( \alpha_P(0) = 2 \).
This leads to a massless \( 2^{++} \) state, the graviton. In fact the low energy theory
was clearly not QCD but rather supergravity in 10 dimensions!

What is the mechanism which allows our 4-d space/time and yet is able
to generate a non-zero mass gap for tensor glueballs? How can one “lower”
the Pomeron intercept so that \( \alpha_P(0) \) takes on its phenomenological value of
\( 1.1 \sim 1.2 \)? The key ingredient turns out to be duality, which allows a dual
description of QCD involving extra dimensions and a nontrivial background
metric which breaks supersymmetries.

On the basis of Maldacena’s duality conjecture, a rich glueball spectrum
can be computed at strong coupling. This recent development has led to the
rebirth of active QCD string studies.

2 AdS/CFT duality for QCD

Indeed, there is a trivial kinematic advantage in having (at least) one extra
dimension. A zero-mass graviton in 5-d has 5 (rather than 2) on-shell states,
[the little group for a lightlike vector \( \mathbf{p} = (p_\mu, p_5) = (p_5, 0, 0, 0, p_5) \) is SO(3)].
Thus a single 5-d graviton can mimic the five spin-components for a 4-d massive
\( 2^{++} \) glueball on the leading trajectory, if it propagates through a non-trivial
“medium”. It is then possible that a graviton in 5-d, when seen from a 4-d
perspective, becomes massive, thus leading to a Pomeron with an intercept
less than 2.

This simple observation is at the heart of the modern approach. Maldacen’s conjecture begins with a full 10-d critical superstring with the background
“medium” provided dynamically as a solution to appropriate set of supergrav-
ity Einstein’s equation. The self-consistency of this approach presumably is
critical to the still poorly understood mathematical framework required to
prove Maldecena’s duality conjecture.

2.1 Mass Generation and Extra Dimensions

Before mentioning some technical details, let us demonstrate how the incorpor-
ation of extra-dimensions with a nontrivial background provides a natural
mechanism for mass-generation. Consider a situation where the addition of a
fifth-dimension, \( r \), leads to a metric, \( ds^2 = u(r)^2 \sum_{i=1,2,3,4} dx_i^2 + w(r)^{-2} dr^2, \)
where \( w(r) \) takes on an appropriate non-trivial form to be specified later.
The wave equation for a minimally coupled massless scalar field is: \( \{ \partial^2_\mu + \)
\[ \sigma(r)^{-1} \partial_r \tau(r) \partial_r \phi(x, r) = 0, \] where \( \tau(r) = u^4 w \) and \( \sigma(r) = u^2 w^{-1} \). If one attempts to send a 4-d plane-wave with 4-momentum \( p_\mu \), i.e., \( \phi(x, r) = e^{ip \cdot x} \phi(r) \), one finds that \( \phi(r) \) satisfies an ordinary differential equation

\[ - \partial_r \tau(r) \partial_r \phi(r) = m^2 \sigma(r) \phi(r), \]

(1)

with \( -p^2 = m^2 \). This equation is of the standard Sturm-Liouville form, and the allowed \( m^2 \) values can be found by treating this as an eigenvalue problem. If \( \tau(r) \) and \( \sigma(r) \) could be chosen so that no massless mode exists, one would have massive propagation from a 4-d perspective.

### 2.2 AdS/BH Background for QCD

One example of this approach begins with type-IIB string theory leading to \( QCD_3 \) strings. We know that there are many extended solitonic objects beyond the perturbative string expansion. In particular in IIB, there are 3 + 1 dimensional objects called Dirichlet 3-branes, (D3-branes), and the low-energy dynamics of a set of \( N \) parallel D3-branes is described by open strings with end-points restricted on these branes. Indeed the effective theory (or Born-Infeld action) reduces exactly to \( SU(N) \) Yang-Mills theory at weak coupling. (To be exact, in the present context, the full target gauge theory is \( \mathcal{N} = 4 \) SUSY YM in 4-d.) Note that one set of zero-mass states, the spin-1 \textbf{gauge bosons}, is no longer an embarrassment as they represent the weakly coupled gluonic modes. Because of branes, YM theories live in dimensions less than 10.

The Maldacena conjecture states that both the open string/Yang-Mills and the closed string/gravity descriptions are simultaneously true or equivalent in the near horizon limit, where the above metric is \( AdS^5 \times S^5 \). In this limit, the theory does not contain gravity, i.e., massless graviton \textbf{decouples}. However this background is so symmetric that the resultant target Yang-Mills theory is conformal with \( \mathcal{N} = 4 \) supersymmetries. Witten has suggested a further modification to remove these unwanted symmetries. One imagines raising the “temperature” by compactifying one spatial co-ordinate on a circle, \( S^1 \), parallel to the brane, with anti-periodic fermionic boundary conditions to break SUSY. In the gravity language, the resultant background metric collapses into an \( AdS^5 \)/black-hole, while on the gauge side the 4-d theory deconfines and lifts both the fermion masses and through quantum corrections the scalar masses to the “cut-off” scale provided by this temperature. At energies well below the cut-off in the weak coupling region, it is conjectured that one is left with a dimensionally reduced pure 3-d Yang-Mills theory or quarkless \( QCD_3 \).
To arrive at $QCD_4$ as the target theory, one begins with the eleven-dimensional M theory on $AdS^7 \times S^4$ or 10-d type-IIA string theory after compactifying the “eleventh” dimension (on a very small circle of radius $R_0$). Again following the suggestion by Witten, the “temperature” is raised with a second compact radius $R_1$ in a direction $\tau$ parallel to the type-IIA D4-branes. On the “thermal” circle, the fermionic modes have anti-periodic boundary conditions breaking conformal and all SUSY symmetries. Therefore, if all goes as conjectured, in the scaling limit $g_s^2 N = g_s N \beta / R_1 \to 0$ there should be a fixed point mapping type-IIA string theory in a background $AdS^7$/black-hole metric,

$$ds^2 = f(r) dr^2 + r^2 \sum_{i=1,2,3,4,11} dx_i^2 + f(r)^{-1} dr^2 + d\Omega_4^2 , \quad (2)$$

$f = r^2 - r^{-4}$, into pure $SU(N)$ Yang-Mills or quarkless $QCD_4$. Therefore, the threefold crisis mentioned earlier has now been resolved.

3 Wave Equations and Glueball Spectra

To compute the glueball excitations for $QCD_4$, in the extreme strong coupling limit, one simply needs to find the spectrum of harmonic fluctuations for the bosonic supergravity fields around these $AdS$/black-hole backgrounds. The “warp factor” in the radial “fifth” coordinate forms a “cavity” so that all modes are discrete and massive.

The goal is to compute all strong coupling states that might survive for $QCD_4$ in the scaling (weak coupling) limit. We therefore ignore throughout any Kaluza-Klein modes in compact manifolds (compactified $S^1$ co-ordinates in $AdS$ or the Sphere $S^4$), which are charge states in their own superselection sector. To illustrate the approach consider metric fluctuations about the fixed $AdS^7$ background, $G_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}(x)$, where

$$h_{\mu\nu}(x) = H_{\mu\nu}(r)e^{i k_4 x_4} . \quad (3)$$

With all other fields set to zero, we look for plane wave solutions with Minkowski time, $t = ix_4$ and discrete mass eigenvalues, $m = -ik_4$.

Because of an accidental $SO(4)$ symmetry in $(x_1, x_2, x_3, x_{11})$ in strong coupling, the spectrum is degenerate when classified by spin under $SO(3)$. Here we exhibit explicitly the fluctuations leading to spin-2 tensor glueballs. There are five independent perturbations, $h_{ij} = q_{ij} r^2 T_4(r) e^{-m x_4}$, which form the spin-2 representations of $SO(3)$, where $i, j = 1, 2, 3$ and $q_{ij}$ is an arbitrary constant traceless-symmetric $3 \times 3$ matrix. $T_4(r)$ satisfies the free wave equation,

$$r(r^6 - 1) T_4''(r) + (7r^6 - 1) T_4'(r) + (m^2 r^3) T_4(r) = 0 . \quad (4)$$
This equation can be expressed covariantly, as if one is dealing with a minimally coupled massless scalar field. In particular, the mass for the lowest tensor glueball can be obtained:

\[ m_T \simeq [9.86 + 0(\frac{1}{g^2N})] \beta^{-1}, \]

where \( \beta = 2\pi R_1 \), with \( R_1 \) being the thermal radius. (We have adopted a simple normalization for the AdS/black-hole metric, e.g., for AdS\(^7\), \( \tilde{g}_{\tau\tau} = f(r) = r^2 - r^{-1} \). This corresponds to fixing the “thermal-radius” \( R_1 = 1/3 \) so that \( \beta = 2\pi R_1 = 2\pi/3 \).) In general, \( \beta \) serves as the mass scale in the strong coupling limit.

4 Pomeron in Strong Coupling

We have found that there is indeed a rather remarkable correspondence of the overall mass and spin structure between the spectrum determined by lattice simulations at weak coupling and that captured by the lowest states for fluctuations associated with the type-IIA supergravity bosonic multiplet. The glueball mass calculations are based on the earlier work of several authors. This supports the belief that the Maldacena conjecture may well be correct and that further efforts to go beyond strong coupling are worthy of sustained effort. We note that for each value of \( PC = (++++, --+++, +++, --) \), the lowest state is present in approximately the right mass range.

At higher masses, the discrepancies increase. One reason is the obvious fact that on the supergravity side all orbital excitations of higher spin states are pushed to infinity in strong coupling by virtue of the divergent closed-string tension,\( \sigma = \frac{16\pi^2 N^2}{3g^2N^2} \left[ 1 + 0(\frac{1}{g^2N^2}) \right] \). For example, a 3++ state is a purely stringy effect outside of the classical limit of supergravity.

Finally, we must emphasize that our comparison is premised on the neglect of many states in the strong coupling limit that are in the wrong superselection sector to survive in the weak coupling limit of QCD. A major challenge is to understand how this could be accomplished, e.g., by modifying or by adopting a different background metric.

We next comment on the intercept of the leading glueball trajectory as a way to estimate the crossover value for the bare coupling, where continuum physics might begin to hold. The Pomeron is the leading Regge trajectory passing through the lightest glueball state with \( J^{PC} = 2^{++} \). In a linear approximation, it can be parameterized by \( \alpha_P(t) = 2 + \alpha'_P(t - m_T^2) \), where our strong-coupling result provides for the lightest tensor mass. The Pomeron slope can be related to the QCD string tension, \( \alpha'_P \simeq \frac{27}{16\pi^2 g^2N} + 0(\frac{1}{g^4N^2}) \beta^2 \). Putting these together, we obtain a strong coupling expansion for the Pomeron intercept

\[ \alpha_P(0) \simeq 2 - 0.66 \left( \frac{4\pi}{g^2N} \right) + 0(\frac{1}{g^4N^2}). \]

(5)
Turning above argument around would allow one to estimate a crossover value between the strong and weak coupling regimes by fixing $\alpha_P(0) \simeq 1.2$ at its phenomenological value. In fact this yields for $N = 3$ QCD a reasonable value for the fine structure constant: $g^2/4\pi \simeq 0.176$ at a characteristic confinement scale, $\Lambda_{QCD}$. Such estimates have proven sensible in the lattice approach to strong coupling QCD. Much more experience with this new approach to strong coupling must be gained before such numerology can be taken seriously. However if it proves to be an accurate guide, one may be able to follow the general strategy used in lattice simulations. Postpone the difficult question of analytically solving the QCD string to find the true UV fixed point. Work at a fixed but physically reasonable cut-off scale and calculate the spectrum of QCD in the cut-off theory. If one is near enough to the fixed point, mass ratios should be reliable. After all, the real benefit of a weak/strong duality is to use each method in the domain where it provides the natural language. On the other hand, clearly from a fundamental point of view, finding analytical tools to understand the renormalized trajectory and prove asymptotic scaling within the context of the gauge invariant QCD string would also be a major achievement — an achievement that should include a proof of confinement itself.

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