Time refraction in expanding plasma bubbles

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Abstract. We consider the elementary processes associated with time refraction in non-stationary plasmas. We describe the frequency shifts of longitudinal and transverse wave propagation in isotropic plasmas, and their corresponding energy variation, which is a direct consequence of non-stationarity. We also show the appearance of reflected waves, propagating with the same (time-varying) frequency, but in the opposite direction, as a direct consequence of time refraction. The case of an expanding (or contracting) plasma bubble is discussed, and will be applied to an expanding universe, and to sonoluminescence.

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1. Introduction

In recent years, there has been an increasing interest in wave propagation in non-stationary optical media, and a number of new concepts such as photon acceleration [1, 2], and time refraction [3] have been defined. Outside plasma physics, other closely related concepts, such as instantaneous optics [4], self-phase modulation and cross-phase modulation [5], have also been explored. Such an interest is mainly motivated by the development of ultra-short laser pulses [6], and can be extended to any time-varying process occurring in a plasma, affecting the propagation properties of electrostatic waves as well. This means that a whole range of phenomena in the temporal domain can be explored. They include time refraction at a sharp temporal boundary [7], temporal beam splitting and temporal interference [8]. The symmetry between space and time occurring in such wave phenomena is obvious, and new physical phenomena are being revealed in the temporal domain. Such new effects associated with wave propagation in a time-varying medium include the existence of a force acting on photons [9], and at a quantum level, the occurrence of photon-pair creation from out of vacuum [10]. As recently discussed, these quantum vacuum effects are closely related to the dynamical Casimir effect and Unruh radiation [11].

In plasma physics, experimental evidence of time refraction can already be found in the observation of laser blue-shift due to flash ionization phenomena [12, 13]. But a clear theoretical description of time refraction in a plasma is still missing, which is the motivation for the present work. Here we ignore quantum effects and concentrate on isotropic plasmas. We retain the influence of wave dispersion, which is undissociated from the plasma state, and consider both electrostatic and electromagnetic wave modes.

Our main target will be the understanding of the spectral changes of radiation inside expanding (or contracting) plasma bubbles. Several examples of expanding and contracting plasmas can be found. Such plasma bubble models can be applied to laser compressed targets for inertial fusion research [14, 15], to pulsed discharges in a gaseous background, such as those related to sonoluminescence [16, 17], to the mini-magnetosphere created around a magnetized space craft for solar sailing purposes [18], or to astrophysical and cosmological objects [19]. In such a large variety of non-stationary plasmas, the electron mean density will vary significantly in time, due to ionization and recombination, or simply to plasma expansion. We can then generically describe the electron mean density as $n_0(t) = n_0 f(t)$, where $f(t)$ is a form function varying between zero and one. Another source of non-stationarity is associated with intense laser fields, such that the electron effective mass is modified by relativistic oscillations. In such case, we have $m_e(t) = m_e \gamma_{\text{eff}}(t)$, where $m_e$ is the electron rest mass, and the following expression for the averaged relativistic gamma factor can be used [20]: $\gamma_{\text{eff}}(t) = \sqrt{1 + \langle a^2(t) \rangle}$, where $a(t)$ is the normalized vector potential of the intense laser radiation, and the average is taken over a laser period. We could also envisage similar temporal variations of the electron and hole effective masses in a semi-conductor plasma.

One of the main characteristics of time refraction is the occurrence of space reflection. Waves counter-propagating with respect to the waves initially excited in the medium result from the existence of field constraints, such as the temporal continuity relations [8]. This unexpected property is, therefore, a direct consequence of the temporal variations of the medium, and can be derived from the time-dependent wave equations, as shown here.

In this work, we consider waves propagating in isotropic plasmas with frequencies above the electron plasma frequency, and we neglect the ion motion. But our results can easily be
extended to ion acoustic waves, and to anisotropic plasmas. In section 2, we consider time refraction of electron plasma waves that has been disregarded in the literature. In section 3, we show that a reflected wave always exists as a direct consequence of the temporal dependence of the medium. In section 4, the case of electromagnetic waves will be discussed. In section 5, we discuss the frequency shifts of radiation inside an expanding or contracting plasma bubble. In the case of expansion, this can be relevant to the expanding universe, considered as a gigantic plasma bubble. It can also be relevant to supernovae explosions and to relativistic fire balls. In the case of contraction, it can be relevant to laser compression of dense plasmas, as used in inertial fusion research, and to the intriguing phenomenon of sonoluminescence. In section 6, we state our conclusions.

2. Electron plasma waves

Let us consider the case of electron plasma waves in a time-varying plasma, where both the plasma density and the electron effective mass can evolve in time. The ions are assumed to be at rest, providing a charge neutralization background, and the electrons are described by the fluid equations

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \vec{v}) = F(t),
\]

(1)

\[
\frac{d}{dt} \left[ m_e(t) \vec{v} \right] + e \vec{E} + \frac{k_B T}{n} \nabla n = 0,
\]

(2)

where \( n \) and \( \vec{v} \) are the electron mean density and electron mean velocity, \( e \) the electron charge, \( F(t) \) the ionization or recombination rate, \( k_B \) the Boltzmann constant and \( T \) the electron plasma temperature, assumed constant. The electric field \( \vec{E} \) is determined by Poisson’s equation

\[
\nabla \cdot \vec{E} = \frac{e}{\epsilon_0} [n_0(t) - n],
\]

(3)

where \( n_0(t) = n_0 f(t) \) is the neutralizing ion density. We now consider the electron mean density of the form \( n = n_0(t) + \tilde{n} \), where the last term describes a wave perturbation. If we linearize the above equations with respect to the perturbation and assume, without loss of generality, that the wave propagation takes place along the \( z \)-axis, we obtain

\[
\frac{\partial \tilde{n}}{\partial t} = -n_0 f(t) \frac{\partial v}{\partial z} + F(t),
\]

(4)

\[
\frac{\partial v}{\partial t} = -\frac{eE}{m_e(t)} - \frac{S_e^2(t)}{n_0 f(t)} \frac{\partial \tilde{n}}{\partial z} - v \frac{d}{dt} \ln m_e(t),
\]

(5)

where \( S_e(t) = \sqrt{k_B T/m_e(t)} \) is the electron thermal velocity, and

\[
\frac{\partial E}{\partial z} = -\frac{e}{\epsilon_0} \tilde{n}.
\]

(6)

From these three equations a wave equation can be established, which takes the following form:

\[
\frac{\partial^2 \tilde{n}}{\partial t^2} - S_e^2(t) \frac{\partial^2 \tilde{n}}{\partial z^2} + \omega_p^2(t) \tilde{n} = \nu(t) \left[ \frac{\partial \tilde{n}}{\partial t} - F(t) \right] + F'(t),
\]

(7)
where \( F'(t) = \frac{dF}{dt} \), and the time-dependent electron plasma frequency \( \omega_p(t) \) is defined by

\[
\omega_p(t) = \left[ \frac{e^2 n_0}{\epsilon_0 m_e(t) f(t)} \right]^{1/2}.
\] (8)

In equation (7), the coefficient \( \nu(t) \) determines the rate of change of the background electron plasma properties

\[
\nu(t) = \frac{d}{dt} \ln \left[ \omega_p^2(t) \right].
\] (9)

At this point, it should be emphasized that the wave equation (7) is exact and that no approximation, apart from linearization with respect to the wave perturbation, was introduced.

In order to understand the physical meaning of this wave equation, and the main differences with respect to the stationary plasma, we first consider the case of a very slow variation in the medium, such that all the terms on the right-hand side can be neglected, \( \nu(t) \approx 0, \) and \( F(t) \approx 0. \) For a given wavenumber \( k, \) we have the simple solution of

\[
\tilde{n}_k(z, t) = A_{\pm k} \exp \left[ \pm ikz - i \int_0^t \omega(t') \, dt' \right],
\] (10)

where \( A_{\pm k} \) are constant wave amplitudes and the time-varying frequency \( \omega(t) \) obeys the time-dependent dispersion relation

\[
\omega^2(t) = \omega_p^2(t) + S_e^2(t) k^2.
\] (11)

Let us now consider a more interesting case, where the rate of change in the plasma medium (expansion or contraction, for \( m_e \) constant) is exactly compensated by the ionization (or recombination) rate, in order to keep the plasma frequency constant. We then have \( \nu(t) = 0, \) and \( F(t) \neq 0. \) It is obvious that, in this case, the term \( F(t) \) can be a source of wave radiation, which is determined, for a given wavenumber \( k, \) by a forced wave solution similar to equation (10), but where the wave amplitudes are now time dependent, and determined by

\[
A_{\pm k}(t) = \frac{1}{2} \int_0^t F(t') \exp \left[ +i \int_0^{t'} \omega(t'') \, dt'' \right] \, dt'.
\] (12)

Notice that the excited waves propagate symmetrically along the positive and negative \( z- \)direction. We finally consider the important case of plasma frequency variations (due for instance to expansion or contraction), in the absence of any ionization (or recombination) process. This corresponds to \( F(t) = 0 \) and \( \nu(t) \neq 0. \) Due to the importance of this case to our subsequent discussion, we explicitly write the corresponding form of the wave equation

\[
\left[ \frac{\partial^2}{\partial t^2} - S_e^2(t) \frac{\partial^2}{\partial z^2} + \omega_p^2(t) \right] \tilde{n} = \nu(t) \frac{\partial \tilde{n}}{\partial t}.
\] (13)

The derivation of physically relevant solutions for this wave equation is discussed in the next section. We have previously shown [3, 8] that, for electromagnetic waves propagating in arbitrary optical medium, a time variation of the medium always implies the excitation of reflected waves, propagating with the same frequency spectrum but in the opposite direction with respect to the initially excited wave modes. This demonstration will be extended here for electrostatic waves propagating in a non-stationary plasma.

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3. Reflected waves

We first consider a simple and ideal situation of a stepwise change in the medium where, at 
$t = 0$ the plasma suddenly changes, following the law:

$$\omega_p(t) = \omega_{p0} \quad (t < 0), \quad \omega_p(t) = \omega_{p1} \quad (t > 0),$$

with the corresponding values, $f(t < 0) = f_0$ and $f(t > 0) = f_1$. A more realistic model will 
be considered later in this section. Because we are restricting our analysis to an expanding 
(or contracting) plasma with $F(t) = 0$, we can exclude any discontinuity associated with wave 

sources.

The continuity and momentum equations (4) and (5) have to stay valid for all times, which 
implies that the time derivatives of $\tilde{n}$ and $\nu$ have to be defined at $t = 0$. This is satisfied by 
imposing the following continuity (or time boundary) conditions:

$$\tilde{n}(z, t = 0^+) = \tilde{n}(z, t = 0^-), \quad \tilde{\nu}(z, t = 0^+) = \tilde{\nu}(z, t = 0^-).$$

Let us then assume that, for $t < 0$, we have a single wave solution of the form

$$\tilde{n}_k(z, t < 0) = A_0 e^{ikz-i\omega_0 t} + c.c.,$$

where $k$ and $\omega_0$ are related by the dispersion $\omega_0^2 = \omega_{p0}^2 + S_e^2 k^2$. Similarly, for $t > 0$, we will expect 
to have a new wave solution

$$\tilde{n}_k(z, t > 0) = A_1 e^{ikz-i\omega_1 t} + A_2 e^{-ikz-i\omega_1 t} + c.c.,$$

where now $\omega_1$ is determined by the new dispersion relation $\omega_1^2 = \omega_{p1}^2 + S_e^2 k^2$. The counter-

propagating term with amplitude $A_2$ is included in order to satisfy the temporal continuity 
conditions (15), as shown next. Using the relation between the electron density perturbation 
$\tilde{n}(z, t)$ and the associated velocity oscillations $\nu(z, t)$, we can easily realize from equations (16) 
and (17) that these continuity conditions lead to the following amplitude relations:

$$A_0 = A_1 + A_2', \quad A_0 = \alpha(A_1 - A_2'),$$

where

$$\alpha = \frac{\omega_1}{\omega_0} \frac{f_0}{f_1}. \quad (19)$$

For simplicity, we have assumed that the wave amplitudes are real, otherwise we would have to 
replace $A_2'$ by its complex conjugate in the relations (18). From here we can then easily obtain 
the following (temporally) transmitted and reflected wave amplitudes:

$$A_1 = \frac{(1+\alpha)}{2\alpha} A_0, \quad A_2' = -\frac{(1-\alpha)}{2\alpha} A_0.$$

This completely determines the electron plasma wave solution for $t > 0$, and shows that, for 
$\alpha \neq 1$, a reflected wave has necessarily to exist in order to satisfy the temporal continuity 
relations (15).

Let us now consider a more general situation, where the electron plasma frequency changes 
continuously in time. We can now assume a wave solution of the form

$$\tilde{n}_k(z, t) = A_k(t) e^{ikz-i\phi(t)} + A_{-k} e^{-ikz-i\phi(t)} + c.c.,$$

where $A_{\pm k}(t)$ are slowly varying amplitudes, such that $|dA_{\pm k}(t)/dt| \ll |\omega(t)A_{\pm k}(t)|$, and

$$\phi(t) = \int_{t'}^t \omega(t') \, dt'.$$
Replacing this solution in the wave equation (13), assuming that the instantaneous dispersion relation is always verified, and that \(|v(t)| \ll \omega(t)|\), we arrive at the following two relations:

\[ \frac{d}{dt} A_k = \frac{1}{2} \left[ \frac{\omega'}{\omega} - v(t) \right] A_k e^{2i\phi(t)} \] (23)

and

\[ \frac{d}{dt} A_{-k} = \frac{1}{2} \left[ \frac{\omega'}{\omega} - v(t) \right] A_k e^{-2i\phi(t)} \] (24)

where \(\omega' \equiv \frac{d\omega(t)}{dt}\). These coupled equations can easily be solved, but here we choose to focus on a situation where the wave propagating in the positive direction is always dominant, \(|A_k(t)| \gg |A_{-k}(t)|\), because it more clearly reveals the nature of the mode coupling process. We, therefore, assume that \(A_k(t) \simeq A_k(0) = \text{const.}\) and \(A_{-k}(0) = 0\), and focus on equation (24). Integration then leads to

\[ A_{-k}(t) = R(t) A_k(0), \] (25)

where the temporal reflection coefficient is determined by

\[ R(t) = \frac{1}{2} \int t' \left[ \left( \frac{\omega'}{\omega} \right)' - v(t') \right] \exp \left[ -2i \int t'' \omega(t'') dt'' \right] dt'. \] (26)

This solution can also be derived by using an alternative approach, where the continuous variation of the medium is decomposed into a succession of infinitesimal steps at times \(t_j = j\tau\), with \(j = 0, 1, 2, 3, \ldots\), such that

\[ \omega_p(t) = \omega_p,j \quad (j - 1 < t/\tau < j), \quad \omega_p,j = \omega_p,j-1 + \frac{d\omega_p}{dt} \tau \quad (\tau \to 0). \] (27)

Each step is similar to that considered at the beginning of this section. Generalization of the continuity conditions (15) is straightforward and takes the form

\[ \tilde{n}(z, t_j + 0) = \tilde{n}(z, t_j - 0), \quad \tilde{v}(z, t_j + 0) = \tilde{v}(z, t_j - 0). \] (28)

By following the procedure of our previous work [8], equations (25) and (26) can then be recovered. We can therefore conclude that the existence of reflected waves is imposed by the continuity of the electron density and velocity perturbations.

### 4. Transverse waves

The case of electromagnetic waves can be treated in a similar form, with Poisson’s equation (3) replaced by the full set of Maxwell’s equations. We can write them in the following form:

\[ \nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}, \quad \nabla \times \vec{H} = \vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}, \] (29)

where the electron current is defined by \(\vec{J} = -en\vec{v}\). Without loss of generality, we consider wave propagation along the \(z\)-direction, and field polarization in the \(x\)-direction. The above equations can then lead to

\[ \left[ \frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial z^2} + \omega_p^2(t) \right] E_x = \frac{en_0}{\epsilon_0} \frac{\partial f}{\partial t} v_x, \] (30)
where the electron velocity perturbation is determined by
\[
\frac{\partial}{\partial t} [m_e(t) v_z] = -e E_x. \tag{31}
\]
Here again, the term on the right-hand side of the wave equation couples wave modes with the same wavenumber and frequency but propagating in opposite directions. We can then use a solution similar to (21), such that
\[
E_x(z, t) = E_k(t) e^{ikz - i\phi(t)} + E_{-k} e^{-ikz - i\phi(t)} + c.c. \tag{32}
\]
Replacing this in the wave equations (30) and (31), and assuming that the time-dependent dispersion relation for electromagnetic waves is always verified
\[
\omega^2(t) = \omega_p^2(t) + k^2 c^2, \tag{33}
\]
we obtain a similar relation for the two counter-propagating wave amplitudes, as given by
\[
\frac{d}{dt} E_{-k} = \frac{1}{2} \left[ \frac{\omega'}{\omega} - \frac{\omega_p^2}{\omega^2} v(t) \right] E_k e^{-2i\phi(t)}. \tag{34}
\]
Again, this is valid for \( |\nu(t)| \ll \omega(t) \), and for slowly varying amplitudes, such that the wave mode propagating in the forward direction is dominant, \( |E_k(t)| \gg |E_{-k}(t)| \), we can easily integrate equation (34) and arrive at the solution
\[
E_{-k}(t) = R(t) E_k(0), \tag{35}
\]
where the temporal reflection coefficient for transverse waves is now defined by
\[
R(t) = \frac{1}{2} \int^t \left( \frac{\omega'}{\omega} - \frac{\omega_p^2}{\omega^2} v(t') \right) \exp \left[ -2i \int^t \omega(t'') dt'' \right] dt'. \tag{36}
\]
We notice that this is very similar to the result obtained above for the electron plasma waves. It is also formally analogous to the reflection coefficients for stationary but inhomogeneous media, where the time variable and time derivatives have to be replaced by a space variable and by space derivatives. It should be mentioned that, just by using the dispersion relation (33), we can always write
\[
\frac{\omega'}{\omega} = \frac{1}{2} \frac{\omega_p^2}{\omega^2} v(t). \tag{37}
\]
Approximate solutions for the reflected wave can also be derived by using the continuity relations for the \( \vec{B} \) and \( \vec{D} \) fields, as shown in our previous work for a generic optical medium [8].

5. Expanding plasma bubble

We now consider a spherical plasma bubble with a varying radius \( a(t) \). The resulting electron plasma density will be varying according to
\[
n_e(t) = n_{e0} \frac{a_0^3}{a(t)^3}, \tag{38}
\]
where \( n_{e0} \) and \( a_0 \) are the initial density and plasma radius. We assume that the plasma radius \( a(t) \) is always much larger than the wavelength \( 2\pi / k \) of any given wave mode, and concentrate
on wave propagation far away from the plasma boundary. The frequency of a generic transverse wave mode will evolve in time according to

\[ \omega^2(t) = k^2 c^2 + \omega_0^2(t) = k^2 c^2 + \omega_{p0}^2 \frac{a_0^3}{a(t)^3}. \]  

(39)

If the plasma expands with a constant radial velocity \( v_R(t) \), we can write

\[ \frac{a_0^3}{a(t)^3} = \frac{a_0^3}{(a_0 + v_0 t)^3} = \frac{1}{(1 + \alpha t)^3}, \]  

(40)

with \( \alpha = v_0/a_0 \). We can apply the expanding plasma bubble to the expansion of the universe, because nearly all the matter in the universe is in the ionized state. Using spherical coordinates, we can use the Robertson–Walker metric \([21]\), and write

\[ ds^2 = dt^2 + a(t)^2 dx^2, \quad dx^2 = \frac{dr^2}{1 - Kr^2} + r^2 (d\theta^2 + \sin^2 \theta \, d\psi^2), \]  

(41)

where \( K = 0 \) holds for flat, \( K = 1 \) for a closed and \( K = -1 \) for open universe solutions, and the expansion rate is determined by the acceleration equation

\[ \frac{1}{a} \frac{d^2 a}{dt^2} = -\frac{4\pi}{3} G (\rho + 3 p), \]  

(42)

where \( G \) is the gravitational constant, \( \rho \) the energy density and \( p \) the pressure. For a flat \((K = 0)\) matter-dominated universe, we have \( p = 0 \) and \( \rho \sim a^{-3} \), which leads to \( a \sim t^{2/3} \), or

\[ \frac{a_0^3}{a(t)^3} = \frac{1}{(1 + \alpha t)^2}. \]  

(43)

In contrast, for a flat \((K = 0)\) radiation-dominated universe, we have \( p = \rho/3 \) and \( \rho \sim a^{-4} \), which implies that \( a \sim \sqrt{t} \), or

\[ \frac{a_0^3}{a(t)^3} = \frac{1}{(1 + \alpha t)^{3/2}}. \]  

(44)

Another interesting expansion law corresponds to the inflation period for which we can use \( p = -\rho \), \( \rho = \text{constant} \) and \( a \sim \exp(HT) \), where \( H \) is the expansion constant. In this case, we get

\[ \frac{a_0^3}{a(t)^3} = \exp(-\alpha/t), \]  

(45)

with \( \alpha = 3H \). We can see from these various plasma expansion laws that the photon frequency shifted spectrum will depend on the type of expansion. It is then useful to illustrate our discussion by using a dimensionless version of equation (39), written as

\[ X(t) = [1 + \Omega^2 Y(t)]^{1/2}, \]  

(46)

where \( X = \omega/kc \), \( \Omega = \omega_{p0}/kc \) and \( Y = a_0^3/a(t)^3 \) for the two different kinds of expansion laws

\[ Y(t) = \frac{1}{(1 + \alpha t)^m}, \quad Y(t) = e^{-\alpha t}, \]  

(47)
where this last expression is pertinent for inflation, and the first law is used for $m = 3/2, 2, 3$ with $\Omega$ and $a$ as arbitrary parameters. It is important to notice that the spectral down-shift induced by time refraction in the expanding background plasma is physically distinct from the well-known cosmological red shift due to the change in the curvature of space–time, which is determined by a completely independent law \[21\]

$$\omega(t) = \omega_0 \frac{a_0}{a(t)}, \quad X(t) = Y(t)^{1/3}. \quad (48)$$

It should also be noticed that according to our laws of time refraction for an expanding plasma bubble, the initial and final frequency values for the electromagnetic wave mode are always the same, and that the various expansion models only correspond to different possible transitions between these extreme values. The initial frequency is always determined by our choice of $k$ and $\omega_p$, and is given by

$$\omega_0^2 = k^2 c^2 + \omega_p^2, \quad X(0) = (1 + \Omega^2)^{1/2}. \quad (49)$$

On the other hand, the asymptotic final value for the mode frequency is just

$$\omega_\infty^2 = k^2 c^2, \quad X(t \to \infty) = 1. \quad (50)$$

The maximum frequency shift, relative to the initial value, is therefore independent of the particular expansion model, and is given by the quantity

$$\frac{\Delta \omega}{\omega_0} = 1 - \frac{\omega_\infty}{\omega_0} = 1 - \frac{1}{\sqrt{1 + \Omega^2}}, \quad (51)$$

which means that it will vary between $\Delta \omega/\omega_0 \simeq 0$, for $\Omega = 0$ or $k \to \infty$, the high-frequency spectral range, and $\Delta \omega/\omega_0 \simeq 1$, for low frequencies near to the initial plasma cut-off. This spectral asymmetry shows that, if we start from an equilibrium spectrum at a given initial temperature $T_0$, the spectrum will be slightly distorted from its equilibrium Planck shape. Finally, another interesting application of time refraction is sonoluminescence, which can be described by a plasma bubble contraction and collapse. In this case, we can use the collapsing model

$$\frac{a_0}{a(t)} = \frac{1}{(1 - t/t_0)^{3/2}}, \quad (52)$$

where $t_0$ is the collapsing time. From this law, arbitrarily large frequency blue shifts can be expected, as determined by (figure 1)

$$\omega(\tau) = \left[k^2 c^2 + \frac{\omega_p^2}{(1 - \tau)^2}\right]^{1/2}, \quad (53)$$

with $\tau = t/t_0$. This leads to $\omega(\tau) \to \infty$, when $\tau \to 1$. This simple classical law will lose its validity for high energies, due to quantum effects and to the limited energy of the contraction process. The strong frequency up-shifts predicted by time refraction inside a contracting plasma bubble seem compatible with the observations of sonoluminescence. They provide a possible alternative explanation for the observations, where the frequency shifts are a direct result of the explosive increase of the plasma density, affecting the entire radiation spectrum.
Figure 1. Relative variation of the transverse wave frequency $\omega(t)$, normalized to the initial frequency $\omega_0$. In blue (full line) $\Omega^2 = 0.1$, and in red (broken line) $\Omega^2 = 10$, for an imploding plasma bubble described by equation (53).

6. Conclusions

We have considered wave propagation in a non-stationary plasma. For simplicity, the medium was assumed isotropic and homogeneous. Approximate solutions for electron plasma waves, and for transverse electromagnetic waves, were derived. These results correspond to the time refraction process, previously discussed for special cases of non-dispersive optical media [3, 4]. In the present work, arbitrarily changing plasmas were considered and electrostatic waves were also discussed. Time-varying dispersion relations for both longitudinal and transverse waves were considered. Our solutions explicitly show that the wave frequency, as well as the wave amplitude, vary in time as a consequence of the temporal variation of the plasma parameters. It was also shown that such temporal changes imply the excitation of reflected waves. This remarkable property is particularly interesting for experimental purposes.

As examples of application of a time-varying plasma, we have considered the special case of expanding or contracting plasma bubbles, and have exemplified our model with the expansion of the universe, considered as a gigantic plasma bubble, and with sonoluminescence. These are quite speculative but interesting examples that are included here for illustration. Another possible application could be the compression of dense plasmas for inertial fusion. In all these cases, the present work shows that time refraction could be used as a non-perturbative optical diagnostic for the evolving plasma properties. Boundary effects were neglected, but could be easily included. Their main influence for waves propagating inside the plasma bubble, would be the conversion of the continuum spectrum considered here into a discrete one. This was previously discussed by us for a non-dispersive medium inside an optical cavity [10]. Boundary phenomena also include radiation leaking and mode conversion. In general, they can also lead to additional frequency shifts, which cannot be simply reduced to the usual Doppler effect [3]. However, pure Doppler effects would occur due to plasma convective motion inside the expanding bubble, or due to bubble motion with respect to an external observer. In the present work these Doppler shifts were avoided, by assuming that the plasma mean velocity is equal to zero and by ignoring wave leaking. Extension to the case of trapped modes in non-uniformly expanding plasmas will be examined in a future work.

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