Research on initial alignment method of vehicle-mounted SINS based on reverse navigation

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Abstract. In order to improve the alignment accuracy and speed of vehicle-mounted SINS, an initial alignment scheme of "inertial solidification alignment + Kalman filter fine alignment based on reverse navigation" is proposed in this paper. Firstly, an inertial solidification alignment algorithm is derived, the initial alignment problem is transformed into Wahba problem, and the optimal attitude matrix at the initial moment is obtained by singular value decomposition algorithm. The inertial solidification alignment algorithm is more concise and intuitive than the traditional alignment algorithm of geocentric inertial coordinate system. Then, we analyzed the effectiveness of rotation modulation technology to improve alignment accuracy theoretically and proved it by simulation. And it is pointed out that the application platform space is limited and the system cost is increased when using this technology. Furthermore, in order to overcome the shortcomings of using rotation modulation technology, a Kalman filter algorithm based on reverse navigation is derived. By reusing the data stored during the inertial solidification alignment, the alignment time is extended equivalently, the initial attitude error is reduced and the attitude calculation accuracy after forward navigation is improved, higher precision alignment is realized. Finally, the effectiveness of the initial alignment scheme is verified by simulation.

1. Introduction
The main purpose of initial alignment of strapdown inertial navigation system (SINS) is to determine the spatial orientation of the carrier coordinate system relative to the reference coordinate system, so as to provide initial attitude for subsequent navigation[1-2]. Therefore, the accuracy and time-consuming of initial alignment directly affect the subsequent navigation accuracy of the carrier, and restrict the survivability and response ability of the carrier. Scholars at home and abroad have made a lot of research results in the field of static base alignment and shaking base alignment[3-5]. However, the location of the carrier is fixed during the initial alignment of the stationary base and the shaking base, the mobility and survivability of the carrier is limited. Therefore, for the initial alignment of vehicle-mounted SINS, the focus is mainly on the in-motion alignment, that is "alignment while driving", the main purpose is to shorten the preparation time for the carrier to enter the navigation state, and improve the viability and response ability of the carrier. Aiming at the problem of in-motion alignment of vehicle-mounted SINS, the coarse alignment algorithm based on attitude estimation and the fine alignment algorithm based on Kalman filter (KF) are designed in [6], and the effectiveness is verified by simulation and experiment, but the coarse alignment and fine alignment are strictly separated in this scheme which takes a little longer time. Aiming at SINS/OD (odometer, OD) alignment with variable velocity and variable acceleration, an improved coarse alignment algorithm is
proposed in [7], the output of inertial sensor and OD is linearized in the sampling interval which no longer regarded as a constant, the accuracy of vector observation and coarse alignment is improved. A method of anti-sloshing initial alignment and zero speed correction for vehicle-mounted single axis rotary SINS is proposed in [8], which can achieve high accurate initial alignment in 5 minutes, however, the rotary SINS is more expensive and bulkier than traditional SINS. An improved initial coarse alignment algorithm based on reverse navigation algorithm is proposed in [9], which has higher initial alignment accuracy and stability than the optimal alignment method.

In order to further improve the accuracy and speed of initial alignment for vehicle-mounted SINS, an initial alignment scheme of "inertial solidification alignment + Kalman filter alignment based on reverse navigation" is proposed in this paper. Firstly, the navigation coordinate system is solidified in the inertial space at the initial moment to form a new inertial coordinate system, based on it, the inertial solidification alignment algorithm is constructed, which makes the alignment algorithm more intuitive and concise than the traditional alignment method of geocentric inertial coordinate system; After that, the improvement effect of single axis continuous rotation modulation technology on the initial alignment accuracy is verified, considering that the rotation modulation technology needs the support of transposition mechanism, it will inevitably increase the system cost and volume, so its application value is limited in the limited platform space environment. Therefore, the precise alignment method of Kalman filter based on reverse navigation is further deduced. This method can effectively improve the initial alignment accuracy without increasing the cost and alignment time. Finally, the effectiveness of the initial alignment scheme is verified by simulation.

2. Fast and high precision alignment algorithm on moving base

2.1. Inertial solidification alignment based on attitude optimal estimation (ISABAOE)

Defining the longitude and latitude of the carrier at the initial time are respectively \( \lambda_0 \) and \( L_0 \), the longitude and latitude of the carrier at time \( t \) are respectively \( \lambda_t \) and \( L_t \), satisfying equation \( \Delta \lambda = \lambda_0 + \Delta \lambda_t \) and \( \Delta L = L_0 + \Delta L_t \), \( \Delta \lambda_t \) and \( \Delta L_t \) are respectively the changes in longitude and latitude generated by the motion of the carrier. According to the chain rule, the strapdown attitude matrix can be decomposed into

\[
C_n^b(t) = (C_n^{b(t)})^T C_n^b(0) C_{b(0)}^{b(t)}
\]

Where

\[
C_n^{b(t)} \approx I - \begin{bmatrix}
0 & -\Delta \lambda \sin L_0 & \Delta \lambda \cos L_0 \\
\Delta \lambda \sin L_0 & 0 & \Delta L \\
-\Delta \lambda \cos L_0 & -\Delta L & 0
\end{bmatrix}
\]

\( C_n^b(t) \) is the conversion matrix between carrier coordinate system (\( b \), right-forward-upward) and navigation coordinate system (\( n \), east-north-up) at time \( t \), that is the strapdown attitude matrix. \( C_n^b(0) \) is the conversion matrix between inertial solidification coordinate system \( b(0) \) and inertial solidification coordinate system \( n(0) \), where \( b(0) \) formed by solidification of \( b \) coordinate system in inertial space at the initial moment, \( n(0) \) formed by solidification of \( n \) coordinate system in inertial space at the initial moment; \( C_{n(t)}^{b(0)} \) represents the attitude change of the coordinate \( n \) from time \( t \) to time 0; \( C_{b(t)}^{b(0)} \) represents the transformation matrix between \( b(0) \) and \( b \) at time \( t \), which satisfies the differential equation \( \dot{C}_{b(t)}^{b(0)} = C_{b(t)}^{b(0)} (\omega_{b(0)b} \times b) \approx C_{b(t)}^{b(0)} (\omega_b^b \times b) \), it can be obtained through attitude update.
according to the gyro measurement value. Here, the equivalent rotation vector method is adopted to update the direction cosine matrix. The process is as follows

\[ C_{b(t)}^{h(0)} = C_{b(t-1)}^{h(t-1)} C_{b(t)}^{h(t-1)} \]  

\[ C_{b(t)}^{h(t-1)} = I_3 + \frac{1}{2} \sin(\|\phi(T)\|)(\phi(T) \times) + \frac{1}{2} \cos(\|\phi(T)\|)(\phi(T))^2 \]  

Where, initial value \( C_{b(0)}^{h(0)} = I \), \( \phi(T) \) is the equivalent rotation vector of the angular velocity vector integral during the sampling period \([0, T]\), which can be obtained by using the "monomeric + previous period" algorithm. Compared with the monomeric algorithm, this algorithm can improve the compensation accuracy of non-commutative error, that is

\[ \phi(T) = \Delta \theta_i + \frac{1}{12} \Delta \theta_0 \times \Delta \theta_i \]  

Where

\[ \Delta \theta_0 = \int_{-T}^{0} \omega_{b}^h dt \]  

\[ \Delta \theta_i = \int_{0}^{T} \omega_{b}^h dt \]  

Now, the key is to solve for \( C_{b}^{a}(0) \) by the specific force equation. First the specific force equation in the navigation coordinate system is

\[ \dot{v}^a = C_{b}^{a} f^b - (2 \omega_{ae}^a + \omega_{en}^a) \times v^a + g^a \]  

Where \( v^a \) is the carrier velocity in the navigation coordinate system, \( f^b \) is the specific force measured by the accelerometer, \( 2 \omega_{ae}^a \times v^a \) is the Coriolis acceleration caused by the motion of the carrier and the rotation of the earth, \( \omega_{en}^a \times v^a \) is the centripetal acceleration against the earth caused by the motion of the carrier, \( g^a \) is the acceleration due to gravity.

As the carrier velocity measured by the odometer is in the carrier coordinate system, it cannot directly provide the carrier velocity in the navigation coordinate system, so it is inconvenient to use \( v^a \) it as input in the algorithm, which needs to be converted to \( v^b \) in the carrier coordinate system. This is also an important difference between the odometer and GNSS during initial alignment.

\[ \dot{v}^b = \frac{d(C_{h}^{a} v^b)}{dt} = (C_{h}^{a} \dot{v}^b + C_{h}^{a} \dot{v}^b) \]  

Where \( v^b \) is measured by the odometer.

Equation (9) is substituted into Equation (8), and then the specific force equation is converted to

\[ \left(C_{b}^{a}(0) C_{h}^{b}(0) \omega_{ab}^h \times v^b + (C_{a}^{0})^T C_{b}^{a}(0) C_{h}^{b}(0) \dot{v}^b \right) \]  

\[ \left(C_{a}^{0})^T C_{b}^{a}(0) f^b - (2 \omega_{ae}^a + \omega_{en}^a) \times v^a + g^a \right) \]
As the Coriolis acceleration and centripetal acceleration of the earth caused by the motion of the carrier has little influence, so it can be ignored and both sides of the specific force equation can be multiplied by \( C_{n(0)}^n \)

\[
C_b^n(0) \left[ C_{b(t)}^{b(0)} \omega_{mb}^b \times \mathbf{v}_b^b + C_{b(t)}^{b(0)} \mathbf{v}_b^b - C_{b(t)}^{b(0)} \mathbf{f}_b^b \right] = C_{n(t)}^n \mathbf{g}_n^n
\]  

(11)

The odometer has some inherent interference in the measurement, and the integral method can reduce its influence to a certain extent. So we can integrate both sides of this equation in time \([0, t]\)

\[
C_b^n(0) \left[ \int_0^t C_{b(t)}^{b(0)} \omega_{mb}^b \times \mathbf{v}_b^b + C_{b(t)}^{b(0)} \mathbf{v}_b^b - C_{b(t)}^{b(0)} \mathbf{f}_b^b \right] dt = \int_0^t C_{n(t)}^n \mathbf{g}_n^n dt
\]  

(12)

Then we can expand both sides of this equation, and ignore this minimal interference term \((\omega_{\omega}^b + \omega_{\omega}^b) \times \mathbf{v}_b^b\),

\[
C_b^n(0) \left( C_{b(t)}^{b(0)} \mathbf{v}_b^b (t) - \mathbf{v}_b^b (0) - \int_0^t C_{b(t)}^{b(0)} \mathbf{f}_b^b dt \right) = \int_0^t C_{n(t)}^n \mathbf{g}_n^n dt
\]  

(13)

And then we can construct two vectors that are

\[
\alpha_v(t) = C_{b(t)}^{b(0)} \mathbf{v}_b^b (t) - \mathbf{v}_b^b (0) - \int_0^t C_{b(t)}^{b(0)} \mathbf{f}_b^b dt
\]  

(14)

\[
\beta_v(t) = \int_0^t C_{n(t)}^n \mathbf{g}_n^n dt
\]  

(15)

Where \(\alpha_v(t)\) and \(\beta_v(t)\) can be solved by discretization, the detailed process is described in [10].

That is to say

\[
C_b^n(0) \alpha_v(t) = \beta_v(t)
\]  

(16)

Here, the initial alignment problem can be turned into a Wahba problem[11]. Then, we use the singular value decomposition (SVD) algorithm of the attitude matrix to solve this problem to obtain the optimal matrix \(C_b^n(0)\), considering the length of the article, there is no derivation here, the algorithm can be seen in [12].

2.2. Rotation modulation technology

Rotational modulation is a constant error self-compensation technique for inertial sensors, the basic principle is as follows.

Assuming that the carrier coordinate system coincides with the IMU rotating coordinate system (s, three axes point to the sensitive axis of the inertial sensor respectively) at the initial moment, the IMU is controlled to rotate continuously around the vertical \(z\) axis at angular velocity \(\omega\), then the relationship between the carrier coordinate system and IMU rotating coordinate system can be expressed as

\[
C_b^* = \begin{bmatrix}
\cos \omega t & \sin \omega t & 0 \\
-\sin \omega t & \cos \omega t & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  

(17)
Then, assuming that the constant gyro drift of the three axes is \( \varepsilon_x^s, \varepsilon_y^s \) and \( \varepsilon_z^s \) respectively, and the constant zero bias of the accelerometer is \( \nabla_x^s, \nabla_y^s \) and \( \nabla_z^s \) respectively. In order to analyze the mechanism of rotation modulation on constant error intuitively, assuming that the navigation coordinate system coincides with the carrier coordinate system, the modulation forms of gyro constant drift and accelerometer constant bias at time \( t \) in navigation coordinate system are as follows

\[
\begin{align*}
\varepsilon^n &= \mathbf{C}_n^s \varepsilon^b_s \\
\mathbf{C}_n^s &= \begin{bmatrix} \varepsilon_x^t \\ \varepsilon_y^t \\ \varepsilon_z^t \end{bmatrix} = \begin{bmatrix} \varepsilon_x^t \cos(\omega t) - \varepsilon_y^t \sin(\omega t) \\ \varepsilon_y^t \cos(\omega t) + \varepsilon_x^t \sin(\omega t) \\ \varepsilon_z^t \end{bmatrix} \\
\nabla^n &= \mathbf{C}_n^s \nabla^b_s \\
\mathbf{C}_n^s &= \begin{bmatrix} \nabla_x^t \\ \nabla_y^t \\ \nabla_z^t \end{bmatrix} = \begin{bmatrix} \nabla_x^t \cos(\omega t) - \nabla_y^t \sin(\omega t) \\ \nabla_y^t \cos(\omega t) + \nabla_x^t \sin(\omega t) \\ \nabla_z^t \end{bmatrix}
\end{align*}
\]

(18)

(19)

We can see that the constant error in the direction perpendicular to the axis of rotation is modulated into a periodic signal, after integration, it can be offset. That is to say, the constant error is compensated. Therefore, the rotation modulation technology will help to improve the initial alignment precision.

### 2.3. Kalman filter fine alignment based on reverse navigation

In strict sense, the inertial solidification alignment is still coarse alignment. Therefore, in order to further improve the initial alignment accuracy, a Kalman filter fine alignment algorithm based on reverse navigation has been discussed in this paper, the IMU and odometer data are stored in the coarse alignment process. After the coarse alignment, the attitude and velocity position at the start time are obtained by reverse navigation, and then the attitude at the end of alignment is obtained by forward navigation. Through the repeated use of data, the initial error is reduced and then the final alignment accuracy is improved.

Forward strapdown inertial navigation algorithm and forward dead reckoning algorithm are relatively common, and will not be repeated here[13]. The discretization algorithm of reverse strapdown inertial navigation is as follows

\[
\mathbf{C}^n_{k-1} \approx \mathbf{C}^n_k \left( \mathbf{I} + T_s \tilde{\mathbf{\Omega}}^h_{k-1} \right)
\]

\[
v^n_{Rk-1} \approx v^n_{Rk} + T_s \left[ \mathbf{C}^n_{bb} f^n_{k-1} - (-2 \omega_n^{Rk} + \omega_m^{Rk}) \times v^n_{Rk} + g^n \right]
\]

\[
\begin{align*}
L_{k-1} &\approx L_k + T_s \frac{v^{n}_{Rk}}{R_M + h_k} \\
\lambda_{k-1} &\approx \lambda_k + T_s \frac{v^{n}_{Rk} \sec \lambda_k}{R_N + h_k} \\
h_{k-1} &\approx h_k + T_s v^{n}_{CRk}
\end{align*}
\]

(20)

(21)

(22)
Where \( \tilde{w}^n_{\text{proj} \rightarrow k} = \left( \hat{w}^n_{\text{proj} \rightarrow k-1} \times \right) \), \( \omega^b_{\text{proj} \rightarrow k-1} = - \omega^b_{\text{proj} \rightarrow k-1} - C^b_n \left( \omega^u_{\text{proj}} + \omega^e_{\text{proj}} \right) \), \( \omega^u_{\text{proj}} = \begin{bmatrix} \omega \cos L \hline \omega \sin L \end{bmatrix} \),

\[
\alpha^n_{\text{en} \rightarrow k} = \begin{bmatrix} -v^{\nu}_{\text{en} \rightarrow k} / (R_M + h_k) \\ v^{\nu\nu}_{\text{en} \rightarrow k} / (R_N + h_k) \\ v^{\nu\nu}_{\text{en} \rightarrow k} \tan L / (R_N + h_k) \end{bmatrix} ;
\]

\( C^b_{k-1}, v^b_{k-1}, L_{k-1}, \hat{\lambda}_{k-1} \) and \( h_{k-1} \) respectively represent the strapdown attitude matrix, velocity, latitude, longitude and altitude of inertial navigation coordinate system at \( (k-1) \); \( f^b_{k-1} \) is the specific force measured by accelerometer and \( \omega^b_{\text{proj} \rightarrow k-1} \) is the measured output of gyroscope; \( \omega^u_{\text{proj}} \) is the rotation angle velocity of the earth, \( g^n \) is the acceleration of gravity; \( R_M \) and \( R_N \) are the principal curvature radius of meridian circle and the principal curvature radius of prime unitary circle respectively.

The algorithm of reverse dead reckoning is as follows

\[
C^n_{bk \rightarrow k} = C^n_{bk} \left( I + T_s \tilde{\omega}^b_{\text{proj} \rightarrow k-1} \right)
\]

\[
v^n_{sk \rightarrow k} = C^n_{bk} v^b_{sk \rightarrow k-1}
\]

\[
L_{\text{DR} \rightarrow k-1} = L_{\text{DR} \rightarrow k} + T_s \frac{v^n_{\text{proj} \rightarrow k}}{R_M + h_{\text{DR} \rightarrow k}}
\]

\[
\hat{\lambda}_{\text{DR} \rightarrow k-1} = \hat{\lambda}_{\text{DR} \rightarrow k} + T_s \frac{v^n_{\text{proj} \rightarrow k} \sec L_{\text{DR} \rightarrow k}}{R_N + h_{\text{DR} \rightarrow k}}
\]

\[
h_{\text{DR} \rightarrow k-1} = h_{\text{DR} \rightarrow k} + T_s v^n_{\text{DR} \rightarrow k-1}
\]

(25)

Where \( L_{\text{DR} \rightarrow k-1}, \hat{\lambda}_{\text{DR} \rightarrow k-1} \) and \( h_{\text{DR} \rightarrow k-1} \) are the latitude, longitude and altitude of the carrier obtained from the dead reckoning.

Then the error equation is established as follows

\[
\begin{aligned}
\dot{\varphi} &= \varphi \times \left( \omega^u_{\text{proj}} + \omega^e_{\text{proj}} \right) + \delta \omega^u_{\text{proj}} + \delta \omega^e_{\text{proj}} - C^n_b \delta v^b \\
\delta \dot{v}^n &= - \varphi \times f^n + \delta \dot{v}^n \times \left( 2 \omega^u_{\text{proj}} + \omega^e_{\text{proj}} \right) + v^n \times \left( 2 \delta \omega^u_{\text{proj}} + \delta \omega^e_{\text{proj}} \right) + \left( C^n_b \right)^\nu \nabla^b \\
\delta \dot{p} &= M_1 \delta v^u + M_2 \delta p^b + M_1 \delta p_{\text{proj}} + M_1 M_\rho \beta \\
\hat{\dot{v}}^b &= \left[ 0 \ 0 \ 0 \right]^T \\
\hat{\dot{\beta}} &= \left[ 0 \ 0 \ 0 \right]^T 
\end{aligned}
\]

(26)

Where, \( \beta = \left[ \alpha_\varphi \ \delta k \ \alpha_\varphi \right]^T, \ M_\rho = v_{\text{OD}} \left[ -C^n_b (\cdot,3) \ C^n_b (\cdot,2) \ C^n_b (\cdot,1) \right], \)
\[
M_1 = \begin{bmatrix}
0 & \frac{1}{R_N + h} & 0 \\
\sec L & R_N + h & 0 \\
0 & 0 & 1
\end{bmatrix},
M_2 = \begin{bmatrix}
0 & 0 & -v_E \\
v_E \tan L \sec L & R_N + h & v_E \sec L \\
0 & 0 & 0
\end{bmatrix};
\]

\(\phi\) represents the attitude error; and \(\delta \omega_e^n\), \(\delta \omega_m^n\) is the calculation error of \(\omega_e^n\), \(\omega_m^n\) respectively; \(\delta v^n\) represents the velocity error; \(\delta p\) represents the position error of inertial navigation solution, and \(\delta p_{DR}\) represents the position error of dead reckoning; \(\alpha_p\), \(\alpha_y\) is pitch installation error and heading installation error respectively; \(v_{OD}\) is the forward velocity of carrier measured by odometer.

Then we can get the 21 dimensions state error equation is
\[
\dot{X} = FX + GW
\]
(27)

Where
\[
X = [\phi_E, \phi_N, \phi_L, \delta v_E, \delta v_N, \delta v_U, \delta L, \delta \lambda, \delta h, \delta L_{DR}, \delta \lambda_{DR}]^T;
\]

\(F\) is the state transition matrix which can be obtained by the above error equation; \(G\) is the system noise matrix; \(W\) is white Gaussian noise which satisfies \(N(0, Q)\).

The difference between the position calculated by sins and the position calculated by dead reckoning is selected as the observation measurement, so the measurement equation can be expressed as
\[
Z = \delta p - \delta p_{DR} = HX + V
\]
(28)

Where
\[
H = \begin{bmatrix} 0_{3x3} & 0_{3x3} & I_{3x3} & -I_{3x3} & 0_{3x3} \end{bmatrix},
\]
represents the measurement matrix; \(V\) is white Gaussian noise which satisfies \(N(0, R)\).

Then, Kalman filter algorithm was used to achieve fine alignment, considering the length of the article, the detailed process is not shown here.

3. Simulation and analysis

3.1. Simulation parameters setting

The initial position of the carrier is set as 116.18°E, 39.58°N and height is set at 24m; when the rotation modulation is introduced, continuous rotation around the azimuth axis is selected, and the rotation angular rate is 15°/s. at the same time, the error parameters of sensor are shown in Table 1, including constant bias, random error of inertial devices and error parameters of odometer.

| Parameters                        | Setting Value |
|-----------------------------------|---------------|
| Gyro constant drift               | 0.01°/h       |
| Gyro random noise                 | 0.01°/h       |
| Accelerometer zero bias           | 100μg         |
| Accelerometer measurement noise   | 50μg          |
| Odometer scale factor error       | 0.01          |
At the same time, a vehicle trajectory including pitching, heading turning and other manoeuvres are set. The total driving distance is about 3340m, the time is 300s, and the maximum speed is 10m/s. The vehicle trajectory is shown in figure 1.

Based on the above simulation conditions, three schemes are set up and verified in this paper. The specific schemes are shown in Table 2.

| Scheme number | Scheme setting                                           |
|---------------|----------------------------------------------------------|
| Scheme 1      | ISABAOE (300s)                                          |
| Scheme 2      | RM + ISABAOE (300s)                                     |
| Scheme 3      | ISABAOE (300s) + KF fine alignment based on reverse navigation |

3.2. Simulation results
Firstly, scheme 1 (ISABAOE) and scheme 2 (RM + ISABAOE) are simulated and verified. After initial alignment for 300s, the attitude error angles were obtained as shown in figure 2.
Figure 2. Attitude error angles estimated by scheme 1 and scheme 2

It can be seen that after 300s of inertial solidification alignment based on the attitude optimal estimation, the three attitude error angles converge. At 300s, the attitude error angles are about (-23.11″, 6.149″, -5.523′), which achieves relatively high precision attitude alignment. On the basis of scheme 1, after the introduction of single axis continuous rotation modulation technology, we can see that the convergence speed of attitude error curve is improved, the curve is more stable, and the accuracy is significantly improved, at 300s, the three attitude error angles are about (5.364″, 2.245″, -2.042′), and the heading alignment accuracy is effectively improved. It can be seen that the theory and simulation analysis show that the rotation modulation technology can effectively improve the alignment accuracy. The main reason is that the constant error of inertial sensor in the vertical direction of the rotating axis is compensated in the initial alignment process, and the equivalent eastern gyro drift which restricts the heading alignment accuracy is eliminated, so the alignment accuracy is effectively improved. However, in order to achieve this effect in practical application, it is necessary to introduce transposition mechanism, which undoubtedly increases the cost and volume of the system.

Then, the paper verifies the Kalman filter fine alignment algorithm based on reverse navigation (scheme 3). After 300s inertial solidification alignment based on the attitude optimal estimation, the stored alignment data are used for reverse navigation and forward navigation, and the attitude error angle is obtained as shown in figure 3.
It can be seen that after inertial solidification alignment, it can extend the alignment time equivalently and improve the estimation accuracy of attitude error effectively by making full use of the same data through reverse navigation and forward navigation. The error estimation of three attitude angles are about (4.025″, 4.253″, 2.282′), compared with the inertial solidification alignment based on attitude optimal estimation, the alignment accuracy is further improved. Therefore, the scheme can achieve fast and accurate initial alignment without increasing the cost, volume and complexity of the system.

4. Conclusion
This paper discusses the problem of further improving the accuracy based on the inertial solidification alignment. Rotation modulation technology can effectively improve the accuracy, but compared with traditional SINS, it increases the cost and volume of the system. The Kalman filter fine alignment method based on the reverse navigation makes the initial attitude more accurate by reusing the data stored during the inertial solidification alignment, thus improving the forward navigation accuracy after the reverse calculation, and improving the alignment accuracy. This scheme is suitable for the traditional strapdown inertial navigation system, and does not need to add transposition mechanism compared with the rotation modulation scheme; at the same time, compared with the traditional Kalman filter fine alignment algorithm, it can shorten the alignment time, so it has important application value in the background of initial alignment in short time.

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