Vacuum Decay Time in Strong External Fields

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We consider dynamics of vacuum decay and particle production in the context of short pulse laser experiments. We identify and evaluate the invariant “materialization time,” $\tau$, the timescale for the conversion of an electromagnetic field energy into particles, and we compare to the laser related time scales.

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In the past decade high intensity short pulse laser technology has advanced rapidly \cite{1}, pulses achieved intensities of $10^{26}$ W/m$^2$ \cite{1,2,3}. With subsequent concentration by coherent harmonic focusing allowing a further gain in intensity of around six orders of magnitude \cite{1}, laser technology is nearing the scale of rapid vacuum instability, \cite{4}. Laser technology has advanced rapidly \cite{1}, pulses achieved intensities of $10^{26}$ W/m$^2$, where $E_0 \equiv m^2c^3/\hbar = 1.32 \times 10^{18}$V/m. The study of vacuum instability with laser pulses involves dynamics on a timescale set by the pulse length, which at optical frequencies implies that the fields are in existence for $\sim 10^{-15}$s, and may reach $\sim 10^{-18}$s when coherent harmonic focusing is used.

The vacuum state of quantum electrodynamics (QED) is metastable in the presence of electrical fields of any strength, but only in proximity of $E_0$ does the effect occur on an observable time scale \cite{1,4}, as we exhibit below. Specifically, we investigate whether the laser pulse timescale allows the vacuum in strong fields to relax, thereby admitting the new vacuum to experimental investigation using pulsed lasers. The dynamics of ‘false’ vacuum decay have been studied in the context of spontaneous positron creation in heavy ion collisions \cite{7,8,9} and cosmological models \cite{10,11,12}. The QED vacuum decay has not been directly observed in heavy ion collision experiments, due to the relatively long time scale of vacuum decay dynamics as compared to competing processes. However, particle production in strong fields has found a fertile field in quantum chromodynamics \cite{13,14}.

Considerable effort went into generalizing the Euler-Heisenberg-Schwinger (EHS) \cite{3,4} pair production mechanism for a variety of large-scale (compared to $\lambda_c = \hbar/m_ec = 3.86 \times 10^{-13}$m) space- and time-dependent field configurations \cite{13,14,15,16,17} and to incorporating back reaction \cite{13,18,19,20,21}. A stable, modified vacuum state has only been obtained when the field fills a finite space-time domain \cite{18}. The perturbative vacuum is also stable for an ideal plane wave (laser) field of arbitrary strength, and thus many investigations have focused on understanding pair production in optimized pulsed laser field configurations \cite{22,23,24,25,26,27,28,29,30,31,32}. More recently it has been also noted that in the interaction of laser pulses with thin foils, the charge separation effect due to a much greater electron mobility helps in achieving longitudinal electrical fields of comparable strength as are present in the laser pulse, a phenomenon used in laser-ion acceleration \cite{33}. We thus address in this work the general circumstance of a spatially homogeneous electric field.

In all laboratory experiments supercritical fields (fields capable of spontaneous particle production) will be strongly time dependent. One must distinguish two profoundly different experimental regimes involving vacuum rearrangement:

a) the field is established on a time scale much faster than the typical vacuum decay time, such as was studied in heavy ion collisions \cite{8,34}, with the spectrum of produced particles (positrons) representative of the single particle states achieved with the ultimate field strength;
b) the field is established on a time scale slower than the decay time of the vacuum (adiabatic switching). In this case a particle will be produced just upon achievement of supercriticality, and thus at zero (longitudinal) momentum. This is the assumption under which the EHS instability is derived.

In both cases the Fourier-frequency spectrum of the field formation assists the process of vacuum emission of particles, in which case we speak of induced (as compared to spontaneous) vacuum decay \cite{35,36,37,38}.

We evaluate the rate per unit of time and volume of field energy materialization in the adiabatic EHS switch-on limit by calculating the tunneling probability in the presence of the local potential generated by a (nearly) constant field \cite{13}. One starts with the action of an electron with transverse energy $\epsilon_{\perp}^2 = p_{\perp}^2 + m^2$ in the inverted (i.e. Euclidean time) potential

$$S = \int_0^{\epsilon_{\perp}/eE} |p|dz = \int_0^{\epsilon_{\perp}/eE} dz \sqrt{\epsilon_{\perp}^2 - (eEz)^2} = \frac{\pi \epsilon_{\perp}^2}{4eE},$$

with the upper bound determined by the turning point in the potential, at which the quasi-longitudinal momentum (and therefore the pair) becomes real. The tunneling probability for the electron-positron pair is then twice the action in Eq. (1)

$$\Gamma(\epsilon_{\perp}^2) = |\exp(-2S)|^2 = e^{-\pi \epsilon_{\perp}^2}.$$  

By integrating $\prod_s \prod_{\epsilon_{\perp}} [1 - \Gamma(\epsilon_{\perp}^2)]$ over all $s, p_{\perp}$ one reproduces the Schwinger series expression for the total
vacuum persistence probability, as was first noted by Nikishov [40]. However, to obtain the rate \( d(N)/dt dV \), we sum the probability Eq. (2) over the volume-normalized density of states for a spin-1/2 particle \( 2 \cdot d^3p/(2\pi)^3 \).

With the assumption of adiabaticity, the longitudinal momentum is determined entirely by the electric field, so that \( dp_3/dt = eE \), and

\[
W = \frac{d(N)}{dt dV} = 2s \int \frac{d^3p}{(2\pi)^3} \frac{d\Gamma}{d\epsilon} (\epsilon_+^2) = \frac{\alpha e^2}{\pi^2} E^2 e^{-\epsilon_0/\pi E}. \quad (3)
\]

We obtain the rate at which energy is transferred to EHS pairs by including in Eq. (3) the transverse energy conversion becomes

\[
\frac{d(u_m)}{dt} = 2s \int \frac{d^3p}{(2\pi)^3} \frac{d\Gamma}{d\epsilon} (\epsilon_+ + \epsilon_-) \Gamma(\epsilon_+^2), \quad (4)
\]

in which \( \epsilon_+ (\epsilon_-) \) is the asymptotic energy of the electron (positron). As we explicitly assume that only one pair is created per event, momentum conservation allows us to put \( \epsilon_+ = \epsilon_- = \sqrt{\rho_0^2 + m^2} \). Thus, the rate of energy conversion becomes

\[
\frac{d(u_m)}{dt} = \frac{cE}{2\pi^2} \int \frac{d\epsilon}{\epsilon} \epsilon^2 e^{-\epsilon/\pi E}. \quad (5)
\]

Integration by parts results in

\[
\frac{d(u_m)}{dt} = \omega_0 E^2 e^{-\epsilon_0/E} \left\{ 1 + h\left(\frac{\pi E_0}{E}\right) \right\}, \quad (6)
\]

in which \( \omega_0 := \alpha e/\pi^2 \lambda e = \alpha mc^2/\pi^2 h = 5.740 \times 10^{17} \text{s}^{-1} \) and \( h(z) := \sqrt{\pi e^2} \text{erfc}(z)/2z \). The asymptotic behavior of the complementary error function implies that \( h(z) \) increases linearly with \( E/E_0 \), specifically, for \( z \gg 1 \), \( h(z) \sim E/4E_0 \).

The relaxation time of the metastable with-field vacuum state via materialization is the ratio of the available supply of field energy \( u_f \) (density) to the rate of electromagnetic field energy conversion Eq. (6):

\[
\tau := u_f \left(\frac{d(u_m)}{dt}\right)^{-1}. \quad (7)
\]

We assume that the pairs decohere rapidly so that the reverse process is impossible, and Eq. (7) then generates the usual kinetic result

\[
u(t) = u(t) e^{-\int_0^t \frac{d\tau}{\tau}}.
\]

\( \tau \) provides the time at which all field energy is converted into mass, and hence we refer to it as the materialization time of the field.

A rough time scale may be obtained by ignoring the second term Eq. (6) and the nonlinear corrections in the field energy, using \( u_f = E^2/2 \):

\[
\tau(0) := \frac{\omega_0^{-1}}{4} e^{-\epsilon_0/\pi E}. \quad (8)
\]

This is the central result of this paper in its simplest qualitative form, shown in figure 1 as the upper (red) line. Materialization time for \( E < E_0 \) is longer than \( 8 \) as.

The lifetime of the field is directly related to the quantity of (usable) energy present, and positive or negative corrections to the energy density translate into increases or decreases in \( \tau \). The nonlinear contributions are governed by the real part of the effective Lagrangian via

\[
u_f = E \cdot B - \text{Re} \mathcal{L}, \quad \text{where } D = \frac{\partial}{\partial E} \text{Re} \mathcal{L}.
\]

Using the resummed expression of Eq. (8), \( u_f \) may be calculated explicitly. Reference [42] displays contour plots for parallel and perpendicular electromagnetic fields up to \( 5E_0 \) using expansions of the integrated expression [43], showing that the correction \( |\Delta u_f| \) never exceeds \( 10^{-2}E_0^2 \). More importantly, the correction is positive for pair producing field configurations (for which no frame exists such that \( E \equiv 0 \)), implying that the field should persist longer than expected based on the linear Maxwellian formula. However, the effect is over-compensated by the correction in Eq. (7) introduced by proper weighting of the phase space integration. In figure 1 the resulting materialization time \( \tau \) is shown (solid lower black line) for a constant, purely electric field.

We next evaluate \( \tau \) adding a constant, homogeneous magnetic field. The energy available in presence of both electric and magnetic fields is evaluated in the local rest frame from the four-vector of energy-momentum \( T^{\mu\nu} \nu \) (see e.g. [44]) i.e. \( u_f = \left| T^{\mu
u} \nu \right| \cdot F \). This “mass density” of the field is expressed in terms of the invariants \( F = (1/4)F^{\mu\nu} F_{\mu\nu} = 1/4 (B^2 - E^2) \) and \( \mathcal{G} = (1/4)F^{\mu\nu} F^{\mu\nu} = E \cdot B \) as

\[
u_f = \sqrt{F^2 + \mathcal{G}^2} f^2 (F, \mathcal{G}) + A^2 (F, \mathcal{G}), \quad (9)
\]

where \( f = \partial \mathcal{L}/\partial F \rightarrow -1 \) (Maxwell) and \( A \) is the conformal anomaly induced by external fields, \( T_0^\mu = 4 A \).
We remark at this point that \( d(u_m)/dt \) is Lorentz invariant. It follows that \( \tau \) as defined is the Lorentz invariant (proper) decay time of the vacuum, and we may choose a suitable frame in which to evaluate \( \tau \). When \( E \cdot B = 0 \), pairs are produced whenever the generalized electric field \( a = \sqrt{\mathcal{F}^2 + \mathcal{G}^2} - \mathcal{F} \) is nonzero, and the rate reduces to that of Eq. (6). For \( E \cdot B \neq 0 \), a reference frame exists where \( E \) and \( B \) are either parallel or antiparallel. Carrying through the tunneling calculation in this frame, we observe the quantization of the transverse momentum of the final states:

\[
\Gamma_{l,r} = e^{-\frac{2 \pi}{\omega} t_{l,r}^2},
\]

with \( t_{l,r}^2 \) the energy eigenvalues in the combined field,

\[
t_{l,r}^2 = \begin{cases} m^2 + 2eBl & r = -1, \\ m^2 + 2eB(l+1) & r = +1, \end{cases} \tag{10}
\]

in which \( l = 0,1,2,... \) The integration over transverse momenta converts into the usual sum over Landau levels,

\[
\frac{d(u_m)}{dt} = e(E)(eB) \sum_{l,r} \frac{1}{(2\pi)^2} \epsilon_{l,r} e^{-\frac{2 \pi}{\omega} t_{l,r}^2}. \tag{11}
\]

Noting the double degeneracy of all but the \( l = 0 \) mode, we sum over \( r \) and apply the Euler-Maclaurin summation formula. This produces the convergent form

\[
\frac{d(u_m)}{dt} = \omega_0 E^2 e^{-\frac{eE_0}{\omega}} \left\{ 1 + h \left( \sqrt{\frac{\pi E_0}{E}} - \chi(B^2, E) \right) \right\} \tag{12}
\]

\[
\chi := \frac{E_0}{E} \sum_{k=1}^{\infty} \frac{\mathcal{B}_{2k}}{(2k)!} \left( \frac{2B^2}{E_0} \right) e^{\frac{eE_0}{\omega}} \frac{d^{2k-1}}{dx^{2k-1}} \left[ \sqrt{xe^{\frac{eE_0}{\omega}}} \right]_{x=1}
\]

with \( \omega_0 \) and \( h(z) \) defined as above and \( \mathcal{B}_{2k} \) the Bernoulli numbers. The "\( x \coth x \)" found in the pair creation rate \( (10, 19, 16) \) has been replaced due to the weighting of the phase space integral, resulting in a dependence on the magnitude \( B^2 \).

Combining Eqs. (11), (12) the materialization time is

\[
\tau = \frac{u_f}{E^2} \frac{\omega_0^{-1} e^{(\pi E_0/E)}}{1 + h \left( \sqrt{\frac{eE_0}{\omega}} \right) - \chi(B^2, E)}. \tag{13}
\]

In figure 2, Eq. (13) is evaluated for \( B/E = 10^{-2}, 1, 10^2 \). For \( B \geq E \) the lifetime of the field is increased over the pure electric case despite the augmented production rate evidenced by the coth factor mentioned above.

In Table I we exhibit a few points of reference for a pure electric field, listing the particle creation rate from Eq. (13) and the expected lifetime of the field given complete materialization from Eq. (17). At \( E/E_0 = 0.2 \) (4% of critical power intensity) and a time scale of \( 10^{-15}s \) for example, the materialization rate shows that 0.036% of the field energy is converted and therefore approximately 280 nC of electrons (positrons) created per \( \mu m^{-3} \), a signature which should be well observable. If the field energy is in kJ range, this implies that 0.15 J materialize, which greatly exceeds the energy converted into particles \( O(10 \text{ erg}) \) in most extreme laboratory particle collision reactions. Thus, for experiments alert to pair production, actually attaining the critical field \( E_0 \) is not necessary for a clear signal of vacuum decay (materialization) processes.

In the context of relativistic focusing, where the characteristic time scale is expected to be as short as a few attoseconds, the critical field strength needs to be reached to achieve spontaneous vacuum decay (as opposed to induced pair creation).

We further note that in the above example the percentage of the field energy converted will remain small, and materialization will not present a significant source of dissipation in practice. However materialization of near-critical fields \( (E/E_0 \sim 1) \) obtained in relativistic focusing can lead to the formation of large \( O(50 \text{ nm}) \) spatial domains of electron-positron-photon plasma with \( T \approx 2 \text{ MeV} \), allowing experiments to test the strongly coupled regime of QED and provide an accessible analogy for the current interest in quark-gluon plasma [17].

In summary, we have studied the materialization time \( \tau \) of the electromagnetic field in view of pair production at high field intensity/energy density. We presented the field dependence of \( \tau \) and found that a field of order \( 0.2E_0 \)}
is sufficient for observable materialization. Our current study relies on an adiabatically changing field configuration, as is appropriate in the EHS context. This is consistent a posteriori for $E \rightarrow E_0$ given the characteristic times for materialization in critical fields, which are 1000 times shorter than the typical intense pulse laser fields operating at 10 femtosecond scale. On the other hand, our results imply that stronger fields $\sim E_0$ and closer investigations are necessary for the much shorter (attosecond range) field pulses generated in relativistic focusing. The here introduced concept of the field materialization time may be generalized to such more intense and shorter lived field configurations, because these calculations can be undertaken within the same semiclassical approach.

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