An integrated structure and control design of a coaxis planar parallel manipulator for pick-and-place applications based on elastic potential energy

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Abstract
During high-speed pick-and-place operations, elastic deformations are quite apparent if the mass of the manipulator is low. These irregular deformations are accompanied by vibrations, then errors. Since elastic potential energy reflects elastic deformation, the vibration of the whole manipulator can be controlled well when the elastic potential energy is decreased. In this article, to design manipulators with flexible links for pick-and-place operations, an integrated structure and control design framework is proposed. The dynamic model of a coaxis planar parallel manipulator is obtained by the finite element method, an effective method. A proportional–derivative controller is utilized for this industrial application. Simultaneously, the optimal structural and control parameters are derived by minimizing the elastic potential energy via integrated design, in which actuated systems and accuracies are regarded as constraints. Finally, simulations show that the performance of the parallel manipulator is improved by this design methodology.

Keywords
Parallel manipulator, dynamics, integrated design, pick-and-place, coaxis

Introduction
Pick-and-place operations are needed in industries of electronics, pharmaceutics, foods, and so on. In these operations, manipulators must be complete the tasks in short-cycle time. Dynamic performance, therefore, takes a critical role in manipulator evaluation and design. An effective way to increase acceleration is to reduce masses of moving parts. Yet it’s noteworthy that mass reduction will eventually cause obvious flexibility effects so that vibrations and errors of manipulators will become severe. Thus, elastic potential energy cannot be neglected in the dynamic analysis. In the conventional mechanical design process, structure design and control design are performed separately. This design process has been unable to achieve the demanding requirement in some modern industries. By optimizing structural parameters to suppress vibration, the natural frequency is a widely used index. To reduce...
vibrations of robot manipulators, many researchers have focused preferentially on control strategies. A popular study object is the single-link flexible manipulator system. Mohamed and Tokhi\textsuperscript{5} apply command shaping techniques based on input shaping, low-pass, and band-stop filtering for vibration control. Hassan et al.\textsuperscript{6} propose a model-based predictive controller for vibration suppression. A neural network controller is designed to eliminate the effects of the input dead zone of the actuators by He et al.\textsuperscript{7} Vibration control has also been studied for two-link flexible manipulators.\textsuperscript{8,9,10} Since the coupled effects of structural and control parameters, Maghami et al.\textsuperscript{11} have demonstrated the first attempt at an integrated structure and control design. The spacecraft design is improved by optimizing both structural and control parameters simultaneously. Then, this design methodology has been used for a four-bar manipulator by employing the simple proportional–derivative (PD) control,\textsuperscript{12} a compliant two-axes mechanism to suppress vibration,\textsuperscript{13} a feed drive system to increase the system bandwidth and decrease Abbe offset,\textsuperscript{14} and a five-bar linkage manipulator to improve settling time,\textsuperscript{15} etc. It is not hard to see structures of the subjects in these studies that are simple, which have only less than three translation motions. And these studies are not about high-speed pick-and-place operations.

Lou et al.\textsuperscript{16} have presented a planar three-degree-of-freedom robot manipulator, which has three RRR (R: revolute joint) subchain. This robot manipulator is called V3 (Figure 1), which has unlimited rotation capability. Similar structures have been presented by Isaksson\textsuperscript{17} in 2011. The kinematics and workspace of V3 robot have been analyzed in detail.\textsuperscript{18} To improve the velocity performance in workspace, the lengths of the links of the V3 robot have been optimized for pick-and-place operations. In this article, the integrated design methodology is utilized to further improve the 3T1 R parallel manipulator. In high-speed operation, the end-effector will be required not only to change the position but also the orientation in a surprisingly short time simultaneously. After reducing masses of moving parts, the horizontal flexibility increases conspicuously. In an early study,\textsuperscript{19} efficiency has been employed as the design objective, since the cycle time is a critical requirement for this industrial application. However, the elastic vibration was not controlled well. The elastic vibrations of links cannot be ignored, which may induce unstable motion of manipulator.\textsuperscript{20,21} Thus, accuracy of manipulator is influenced by elastic deformation. To reduce adverse efforts and insure the whole performance, the integrated design methodology is an effective technique to improve the V3 robot system. For pick-and-place operations, a new general framework of the integrated structure and control design is presented in this article. The elastic potential energies of links are applied to characterize elastic vibrations, which are minimized to decrease the vibrations and also diminish the errors in this methodology. To simplify this problem, we just first consider the horizontal flexibility.

To begin with, a flexible-link robot system must be modeled. The elastodynamic model is required, which is characterized by nonlinear differential equations. Commonly, the assumed modes method (AMM) and finite element method (FEM) are the most frequently used methods to discretize the dynamic model.\textsuperscript{22} With dominant assumed modes for cantilever and pinned–pinned beams, the inverse dynamics model of flexible two-link manipulators has been studied by Green and Sasiadek.\textsuperscript{23} A computationally efficient method based on the AMM for modeling flexible robots has been proposed by Celentano and Coppola.\textsuperscript{24} To derive an appropriate low-order dynamic model for the design of the controller, Zhang et al.\textsuperscript{25} have employed the AMM to discretize elastic motion of a rigid–flexible planar parallel manipulator. The AMM is also applied to achieve an n-dimensional discretized model of the two-link flexible manipulator for dynamic uncertainty analysis and controller design.\textsuperscript{26} The FEM is widely used in modeling and analysis of robot manipulators as well. A relatively complicated case, a three-PRR planar parallel manipulator, has been analyzed by using FEM.\textsuperscript{27} The finite element model is presented for a unique approach for active vibration control of a one-link flexible manipulator.\textsuperscript{28} And the finite element model is also applied to simulate the bending range of the central spring and static properties of a compliant gripper.\textsuperscript{29} A comprehensive stiffness analysis method via finite element analysis has been obtained by Yu et al.\textsuperscript{30} Both methods are effective to study elastodynamic model. However, for a manipulator with multilinks and irregular shapes, the AMM is not easy to achieve the model, while the FEM can be an effective alternative.\textsuperscript{31} With the AMM, it is usual to assume the modes of the links. If the shapes of the links are irregular, it is very hard to get the precise assumed modes. The model based on the FEM is composed of every element. No matter what the shapes of the links are, and they can be discretized into finite and regular elements. The model of the regular element can be precisely obtained. Thus, the manipulator with irregular shapes and multilinks is more easily modeled. In this article, we want to study a manipulator with variable cross-sectional links, so the FEM may be better suited for modeling.

The organization of this article is as follows. In the section “Dynamic modeling,” the V3 parallel manipulator...
system with flexible links is modeled by the FEM. In the section “Integrated design problem formulation,” the Lagrange multiplier is utilized to achieve the system dynamics with the loop-closure conditions. And a PD controller is employed in the integrated design problem which is regarded as an optimal design problem. Beginning with a regular example analysis, two-step integrated designs are put into effect. The computer simulations show the improvements of this system in section “Simulation and discussion.” In section “Comparison,” the results of conventional design are shown to compare with the results of integrated design. Conclusions are provided in the last section.

Dynamic modeling

This study supposes that all the links are only bent in the horizontal plane and other parts are rigid bodies. Referring to Figure 2, frame $OXYZ$ is an inertial coordinate system, and point $O$ is coincident with point $A_1$, which is the center of the actuated joint in the subchain $A_1B_1C_1$.

Kinetic energy and potential energy

In Figure 3, there is a coordinate system to describe the $i$th flexible subchain. To facilitate the description of the robot system, the meanings of mathematical notations are proposed in Table 1. First, kinetic and potential energy of the parallel manipulator should be obtained, since the dynamic model will come from the Lagrangian formulation.

Suppose all the discretized elements on every link are Bernoulli–Euler beam, there are $n_i$ elements on the actuated link $i_1(A_iB_i)$. Let’s consider a point $P$ on the element between node $k$ and $k + 1$ (element $k$). The vector $OP$ is referred to as $p$, which can be obtained as follow.
The inertial coordinate frame $S_{ij}$, the included angle from $\omega_{ijkl}$, and $\phi$ is the included angle from the $+X$ direction to the vector $H_i H_2$.

The actuated angle variable of the $i$th subchain $\gamma_i$ is the angle variable between actuated link and passive link of the $i$th subchain $\gamma_i$.

The angle variable between actuated link and passive link of the $i$th subchain $\gamma_i$.

The angle variable between actuated link and passive link of the $i$th subchain $\gamma_i$.

Flexural displacement and slope of node $k$ of link $ij$.

Flexural displacement and slope at the end of link $ij$.

Cross-sectional area of the element $k$.

Length of element $k$.

Cross-sectional area of the actuated link.

Length of the bar $\bar{a}_{ijkl}$.

Cross-sectional area of the passive link.

Flexural displacement and slope at the end of link $ij$.

Cross-sectional area of the element $k$.

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\[ q = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \\ \sin \theta_i & \cos \theta_i \end{bmatrix} \begin{bmatrix} l_i \\ u_{i,1,e} \end{bmatrix} + \begin{bmatrix} \cos (\gamma_i + v_{i,1,e}) & -\sin (\gamma_i + v_{i,1,e}) \\ \sin (\gamma_i + v_{i,1,e}) & \cos (\gamma_i + v_{i,1,e}) \end{bmatrix} \begin{bmatrix} l_{i,2,k} + x_{i,2,k} \\ y_{i,2,k} \end{bmatrix} \] (9)

where \((x_{i,2,k}, y_{i,2,k})\) are the coordinates of \(Q_i\) in the body frame \(O_{i,2,k}Y_{i,2,k}\). The kinetic energy of the element is derived

\[ T_{i,2,k} = \frac{1}{2} \dot{U}_{i,2,k}^T M_{i,2,k} \dot{U}_{i,2,k} \] (10)

where \(U_{i,2,k} = [\theta_i, u_{i,1,e}, v_{i,1,e}, \gamma_i, u_{i,2,k}, v_{i,2,k}, u_{i,2,k+1}, v_{i,2,k+1}]^T\). The flexural displacement \(y_{i,2,k}\) can be approximated as

\[ y_{i,2,k} = \Psi_{i,2,k} U_{i,2,k} \] (11)

where a similar shape function matrix is defined as follows

\[ \Psi_{i,2,k} = \begin{bmatrix} 0_{4 \times 1} \\ 1 - 3x_{i,2,k}^2/a_{i,2,k}^2 + 2x_{i,2,k}^3/a_{i,2,k}^3 \\ x_{i,2,k} - 2x_{i,2,k}^2/a_{i,2,k} + x_{i,2,k}^3/a_{i,2,k}^2 \\ 3x_{i,2,k}^2/a_{i,2,k}^2 - 3x_{i,2,k}^3/a_{i,2,k}^3 \\ -x_{i,2,k}^2/a_{i,2,k} + x_{i,2,k}^3/a_{i,2,k}^2 \end{bmatrix} \] (12)

The element mass matrix is a 8 x 8 matrix and can be expressed as

\[ M_{i,2,k} = \rho_2 S_{i,2,k} \int_0^{a_{i,2,k}} \left( \frac{\partial q}{\partial U_{i,2,k}} \right)^T \frac{\partial q}{\partial U_{i,2,k}} dx_{i,2,k} \] (13)

The elastic potential energy is described as

\[ V_{i,2,k} = \frac{1}{2} U_{i,2,k}^T K_{i,2,k} U_{i,2,k} \] (14)

where

\[ K_{i,2,k} = \begin{bmatrix} 0_{4 \times 4} & 0_{4 \times 4} \\ 0_{4 \times 4} & 0_{4 \times 4} \end{bmatrix} \]

\[ K_{i,2,k,22} \] has a similar form as \(K_{i,1,k,22}\).

Therefore, the total kinetic energy of the V3 manipulator is the summation of the kinetic energies of all elements on the six links, the kinetic energies of the nine joints \(T_A, T_B, T_C\), and the kinetic energy of the end-effector \(T_e\) and the payload \(T_p\), which is given as follows

\[ T = \sum_{i=1}^{3} \sum_{j=1}^{2} \sum_{k=1}^{n_i} T_{ij,k} + \sum_{i=1}^{3} (T_A + T_B + T_C) + T_e + T_p \] (15)

where

\[ T_A = \frac{1}{2} J_A \theta_i^2 \]

\[ T_B = \frac{1}{2} m_B \dot{p}_B^T \dot{p}_B + \frac{1}{2} J_B (\dot{\theta}_i + \dot{v}_{1,e} + \dot{\gamma}_i)^2 \]

\[ T_C = \frac{1}{2} m_C \dot{q}_C^T \dot{q}_C + \frac{1}{2} J_C (\dot{\theta}_i + \dot{v}_{1,e} + \dot{\gamma}_i + \dot{v}_{2,e})^2 \]

\[ T_e = \frac{1}{2} m_e \dot{q}_e^T \dot{q}_e + \frac{1}{2} J_e \dot{\phi}_e^2 \]

\[ T_p = \frac{1}{2} m_p \dot{q}_p^T \dot{q}_p + \frac{1}{2} J_p \dot{\phi}_p^2 \]

The design for the V3 robot manipulator, which presents the improved lengths of links. Here, the set of parameters

**Boundary and constraint conditions**

In this system model, the head of each link is clamped on the rotation axis of the joint, so that all links are fixed head beams. The flexural displacements and slopes of node 1 of each link are zeros

\[ u_{i,j,1} = 0, v_{i,j,1} = 0, i = 1, 2, 3, j = 1, 2 \]

Thus, the vector of generalized coordinate can be described by

\[ r = [\theta_1 \ \theta_2 \ \theta_3 \ \gamma_1 \ \gamma_2 \ \gamma_3 \ \dot{R}_1^T \ \dot{R}_2^T \ \dot{R}_3^T]^T \]

where

\[ R_i = [u_{i,1,2} \ v_{i,1,2} \ \cdots \ u_{i,n_i,1} \ v_{i,n_i,1} \ u_{i,1,e} \ v_{i,1,e} \ u_{i,2,2} \ v_{i,2,2} \ \cdots \ u_{i,n_i,2} \ v_{i,n_i,2} \ u_{i,2,e} \ v_{i,2,e}]^T \]

are flexural displacements of the links. By using the generalized coordinate vector, the kinetic energy and potential energy in (15) and (16) can be summarized as

\[ T = \frac{1}{2} r^T M r \]

\[ V = \frac{1}{2} r^T K r \] (18)

where \(M\) and \(K\) are the total mass matrix and the total stiffness matrix, obtained from those individual mass matrix and stiffness matrix, respectively.

Liao et al.\(^{18}\) have implemented an optimal kinematic design for the V3 robot manipulator, which presents the improved lengths of links.
will be applied to define the lengths in this study. In the end-effector, the length \( b_2 = 0 \), which means the bar \( H_3H_4 \) is vanished. The upper two subchains, thus, compose a five-bar mechanism. It is also regarded as a positioning device of the V3 manipulator. The orientation is characterized by the vector \( \mathbf{x}_i \). By mathematic operation, the dynamic equation becomes

\[
\begin{bmatrix}
M(r) & H_r^T \\
H_r & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\mathbf{r}} \\
\dot{\lambda}
\end{bmatrix}
= \begin{bmatrix}
\mathbf{r} - C(r, \dot{\mathbf{r}})\dot{\mathbf{r}} - K\mathbf{r} \\
-H_r\dot{\mathbf{r}}
\end{bmatrix}
\]  

(23)

Then, we obtain ordinary differential equations, which can be solved by Runge–Kutta method.

**Integrated design problem formulation**

**Controller**

For high-speed robot control, many efficient control strategies have been introduced. Theoretically, any reasonable control strategy can be used to implement the integrated design. However, to check the performance of this integrated design initially, we expect to utilize a fundamental but effective control strategy. The PD controller is one of the most frequently used controllers in the industry. It is fortunate if a simple controller can realize a good effect. It is undeniable that other advanced controllers, such as active disturbance rejection adaptive controller, variable parameters controller, and fuzzy logic controller, can achieve better results, especially in the electronic industry. For the pick-and-place operations with variable payloads, these control methodologies must be more effective. To further improve the performance of the V3 robot system, these advanced controllers must be investigated for the integrated structure and control design in the future. But in this early study, PD controller is applied first. During the control process, it is very difficult to measure the flexure on line. The actuated angle \( \theta_i \) at joint \( A_i \) (\( i = 1, 2, 3 \)) are the quantities that can be easily measured in real time. The velocity can be achieved by difference technique. By the PD control law, the input torque can be calculated as follows

\[
\tau_i = K_{pi}\theta_i + K_{di}\dot{\theta}_i
\]  

(24)

where \( \theta_i = \theta_i - \theta_{ii}, \dot{\theta}_i = \dot{\theta}_{ii} - \dot{\theta}_i \), and \( \theta_{ii} \) and \( \dot{\theta}_{ii} \) are the desired joint position and the desired joint angular velocity, respectively. \( K_{pi} \) and \( K_{di} \) are nonnegative control parameters to be determined. Thus, a closed-loop control system, which contains structural and control parameters, is obtained by combining (23) and (24). Figure 4 shows the control process of the manipulator.

**Performance indices**

The demanding requirements for automatic equipments can be translated into instantaneous and steady-state performances in a direct way. There are several performance indices, which can be chosen to characterize a system.

- **Settling time.** It shows how long a system response enters a steady state. And it reflects the response speed of a system.
- **Steady-state error.** This performance index is applied to characterize the system accuracy in steady-state.
- **The maximum value of the tracking error.** This maximum value \( e_{tm} \) is a performance index symbolizing the accuracy of the motion trajectory.
- The ending value of the tracking error. The less is the ending value $e_{e,t}$, the closer to the steady-state is the system. It is a performance index reflecting accuracies of the picking or placing position and orientation.

- The maximum value of the elastic potential energy. This performance index ($V_m$) reflects the maximum elastic deformation of the whole robot mechanism in the trajectory. Simultaneously, the elastic deformation can be used to characterize the amplitude of vibration from a profile.

For a high-speed pick-and-place task, the object has usually been picked or placed before the system reaches its steady state. And the motion time is usually given in advance. Consequently, steady-state error and setting time are the common performance indices. They will not be utilized in this article. The accuracy of the operation position, which is at the end of the trajectory, is most significant in a pick-and-place operation. Bounding values may help to complete the picking or placing task with high accuracy. The maximum value of the tracking error is secondary, while the ending value of the tracking error must be taken into account seriously. The tracking error originates from the actuated module, joint clearance, flexible link, and so on. During high-speed motion, if the tracking error is limited, the flexibility effects of the links will be reduced, and the vibration will also be attenuated. The flexibility effect generates elastic deformation, which comes with vibration. The extreme elastic deformation may make link bent largely, even broken off. Since the elastic potential energy reflects the elastic deformation, limiting the maximum value of elastic potential energy in the motion can constrain the elastic deformations and vibrations of the links.

Since the V3 robot manipulator has two translations and one rotation, the tracking errors can be determined as the tracking translation error $e_t = \sqrt{(x_d - x)^2 - (y_d - y)^2}$ and the tracking rotation error $e_r = |\phi_d - \phi|$, where $x_d$, $y_d$, and $\phi_d$ are the desired position and orientation of the end-effector, respectively.

**Design variables**

Both structural and control parameters are optimized simultaneously in integrated design. In the dynamic model, $S_{ij,k}$, $a_{ij,k}$ are structural parameters under determination. Therefore, the indeterminant structural parameters are $S_{ij,k}$ and $a_{ij,k}$. Take $S = [S_{11,1}, \cdots, S_{11,n_1}, S_{12,1}, \cdots, S_{12,n_2}, \cdots]$ and $\mathbf{A} = [a_{11,1}, \cdots, a_{11,n_1}, a_{12,1}, \cdots, a_{12,n_2}, \cdots]^T$ as the vector of cross-sectional area and length parameters, respectively. Take $K_p = [K_{p1}, K_{p2}, K_{p3}]^T$ and $K_d = [K_{d1}, K_{d2}, K_{d3}]^T$ as the vectors of control parameters. In summary, the set of design variables $\mathcal{P}$ are the collection of both control parameters and structural parameters as follows

$$\mathcal{P} = [K_p^T, K_d^T, S^T, \mathbf{A}^T]^T$$

**Problem formulation**

Efficiency is a crucial target in a pick-and-place operation. Hence, the cycle time for each task should be as short as possible. In this study, the ending values of the tracking errors $e_{e,t}$ and $e_{e,r}$, reflecting the achievement, should be minimized to complete the picking or placing task with high accuracy. The maximum values of the tracking errors $e_{e,m}$ and $e_{e,r}$ should be limited to satisfy the motion trajectory. Meanwhile, to decrease the effect of vibration, an effective and efficient way is to minimize the maximum value of elastic potential energy $V_m$. Furthermore, the actuated torque $\tau$ also should be limited since actuated saturation exists for each actuator. Apparently, the above-mentioned problem can be formulated as a multiobjective optimization problem. Considering its complexity, some of the objective functions can be treated as constraints. In practice, it is difficult to given an appropriate range for the elastic potential energy, while the accuracy can be ensured as long as the error is limited in a small range. Those constraints are employed to ensure operational effectiveness. Then, according to the analyses above, the integrated design problem can be formulated as follows.

**Problem 1.** Integrated design problem based on elastic potential energy.

Find a set of optimal design parameters $\mathcal{P}$ to

$$\min_{\mathcal{P}} V_m$$

Subject to $e_{e,t} \leq e_1, e_{e,r} \leq e_2$

$$e_{e,m} \leq e_3, e_{e,m} \leq e_4$$

$$\tau_i \leq \tau_{\text{max}}, i = 1, 2, 3$$

Figure 4. The control block diagram.
where $e_n$, $n = 1, \cdots, 4$, are the prespecified tracking errors bounds and $\tau_{\max}$ is considered as the actuated torque limit.

This formulation is a general framework of the integrated design for pick-and-place operations. For a defined manipulator with a trajectory, these constraints will be determined.

**Simulation and discussion**

Before the simulation, some symbols of lengths, materials, masses, and moments of inertia should be set specified values.

- The lengths of links are obtained from our previous study, the lengths of actuated links $l_1 = 0.306$ m ($i = 1, 2, 3$), the lengths of passive links $l_2 = 0.532$ m, $b_1 = 0.069$ m, and $b_2 = 0$ m.
- In this case, we first consider simple discrete model, while focus on the performance improvement of the manipulator with variable cross-sectional links. The actuated links $i1$ ($A_iB_i$) are discretized into two elements averagely ($a_{i1,k} = l_{i1}/2$) and for passive links $i2$ ($B_iC_i$), $k = 3$ ($a_{i2,k} = l_{i2}/3$). Suppose the links are cylinders, the cross-sectional areas are, thus, the functions of the radii. In the implementation, the structural parameters are only the radii ($R_1$ and $R_2$ for links $i1$, $r_1$, $r_2$, and $r_3$ for links $i2$). Therefore, $\delta = [R_1, R_2, r_1, r_2, r_3]^T$.
- In real-world, the torque of any electric motor cannot be unlimitedly increased. Usually, an actuated system in an industrial equipment is composed of one motor and one gearbox. In this simulation, the limitation of an actuated system is supposed to be $\tau_{\max} = 120$ N·m.
- Aluminium alloy is chosen as the material of actuated links. Its density is $2.7 \times 10^3$ kg/m$^3$ and Young’s modulus is 71 GPa. Carbon fiber is chosen as the material of the passive links. Its density is $1.8 \times 10^3$ kg/m$^3$ and Young’s modulus is 150 GPa.
- $J_{A_i} = 0.2$ kg·m$^2$, $J_{B_i} = 0.005$ kg·m$^2$, $m_{R_i} = 0.5$ kg, $J_{C_i} = 0.005$ kg·m$^2$, $m_{C_i} = 0.5$ kg, $i = 1, 2, 3$, $J_e + J_p = 0.005$ kg·m$^2$, and $m_e = m_p = 0.5$ kg.

Note: The payload at the end-effector is not usually limited to the maximum payload case in this integrated architecture, its characteristics are studied only in radial direction in workspace. For any configuration, there must be a configuration with the same performance at another radius. They are symmetrical configurations. Therefore, any selected trajectory has infinite symmetrical trajectories, which all can satisfy these required indices. To measure the tracking errors, the coordinates of the end-effector can be calculated by the flexible kinematics. The choices of the control parameters are the expectations of the errors of the positions and the velocities. First, beginning from a regular example, $K_{ps} = 1.2 \times 10^5$, $K_{pi} = 800$, $R_j = 0.02$ m, $r_i = 0.01$ m, $i = 1, 2, 3, j = 1, 2$, the variations of the elastic potential energy, and the errors and the actuated torques in the operation can be analyzed. Moreover, the ordinary differential equations are calculated by the MATLAB ode45 solver. The tracking errors depend on the position coordinate of the end node of each subchain. The translation error is described by the desired position and the point $C_1$. The orientation of the end-effector can be calculated by coordinates of the points $C_1$ and $C_2$ (or $C_3$). These points’ coordinates are determined by the actuated and the passive angles, the flexural displaces, and the slopes at the ends of $l_{i1}$ and $l_{i2}$, $i = 1, 2, 3$, which can be described as equation (9). Figures 5 and 6 show the tracking errors in $X$-direction, $Y$-direction, and orientation, respectively. In accelerating stage, the translation error increases rapidly at beginning, while at beginning of the decelerating stage, the error becomes negative quickly, since
deceleration can be considered as acceleration in the opposite direction. The peak value of $\eta_{\text{tt}}$, which is more than 2.5 mm, appears at about 0.088 s. At the end of the deceleration, the translation error reduces, but the value is also more than 1 mm. The primary cause of the irregular change of the orientation error is that the errors of the point $C_1$ and $C_2$ (or $C_3$) cannot change in the same trend. However, the peak value of the orientation error $e_{\varphi}$ is less than $5 \times 10^{-3}$ rad ($5 \times 10^{-3}$ rad $\approx$ 0.29 deg). The elastic energy depends on the flexural displaces and slopes of all the nodes. Simultaneously, it reflects the vibration effect from another perspective. Figure 7 shows the elastic potential energy response in this operation. The peak value 0.27 J appears in the changeover period between acceleration and deceleration. And the vibration is more obvious in the deceleration. Figure 8 shows the actuated torques responses of the system. The $\tau_2$ and $\tau_3$ reach the boundary in this operation. [Please approve the edits made in table citations.]

The choices of the prespecified tracking errors bounds depend on the results of the simulation example and the requirements of pick-and-place operations.
1. The translation tracking errors bounds: $e_1 = 0.001 \text{m}$ and $e_3 = 0.003 \text{m}$.

2. The rotation tracking errors bounds: $e_2 = 5 \times 10^{-4} \text{rad}$ and $e_4 = 5 \times 10^{-2} \text{rad}$.

These ending values of the tracking errors can satisfy the most requirements of pick-and-place operations.

In simulation, the integrated design problem is realized as follows.

**Problem 2.** Integrated design of V3 parallel manipulator based on elastic potential energy

Find a set of optimal design parameters $\mathcal{P} = [K_{p1}, K_{p2}, K_{p3}, K_{d1}, K_{d2}, K_{d3}, R_1, R_2, r_1, r_2, r_3]^T$ to minimize $V_m$

subject to

- $e_{tc} \leq e_1, e_{tc} \leq e_2$
- $e_{tm} \leq e_3, e_{tm} \leq e_4$
- $\tau_1 \leq \tau_{\text{max}}, i = 1, 2, 3$
- $K_{p1}, K_{p2}, K_{p3} \in [10 \times 10^5, 1.5 \times 10^5]$
- $K_{d1}, K_{d2}, K_{d3} \in [500, 1000]$
- $R_1, R_2 \in [0.015 \text{m}, 0.045 \text{m}]$
- $r_1, r_2, r_3 \in [0.005 \text{m}, 0.015 \text{m}]$

This integrated design problem, by analyzing, is a non-linear and nonconvex optimization problem. Thus, the genetic algorithm (GA) will be chosen to solve this optimization problem. To improve the computational efficiency, the GA toolbox and the parallel computing toolbox in MATLAB are utilized. The GA toolbox uses matrix functions to build a set of versatile routines for implementing a wide range of GA methods. The parallel computing toolbox solves computationally and data-intensive problems using multicore processors, GPUs, and computer clusters. Let consumers use the full processing power of multicore computers.

There are 11 parameters to be designed. If the optimum is searched directly, it is time-consuming. Hence the integrated design can be thus in two steps. It is assumed that the cross-sectional areas of the elements on links 1 and 2 ($i = 1, 2, 3$) are united, respectively, in the first step, that is, $R_1 = R_2 := R$ and $r_1 = r_2 = r_3 := r$. It leads to eight independent design variables. For another purpose, it is used to consider the integrated design with the same shapes of the manipulator links. In the first step, the initial set is generated randomly and uniformly in the parameter intervals. Once a set satisfies all the constraints, it is regarded as an effective initial set and is applied to initialize the search. When the optimum is found in this step, it is served as the initial set for the second-step integrated design, which takes all 11 parameters as independent ones.

The initial and the optimal parameters generated by first-step integrated design are shown in Table 2. Comparing with the control parameters, the structural parameters change little. In the optimal design, $K_{p3}$ is smallest in the $K_{pi}, i = 1, 2, 3$, but $K_{d3}$ is biggest in the $K_{di}$. Figures 9 and 10 show the translation and orientation trajectories with the first-step initial and optimal parameters, respectively. There are small errors between the simulated controlled responses and the planned trajectories. In the accelerating stages of the initial and optimal design, the translation tracking errors increase rapidly at the beginning and the

| $K_{p1}$ | $K_{p2}$ | $K_{p3}$ | $K_{d1}$ | $K_{d2}$ | $K_{d3}$ |
|---------|---------|---------|---------|---------|---------|
| Initial | 131336  | 118194  | 134676  | 682     | 601     | 876     |
| Optimal | 124854  | 120571  | 108102  | 812     | 750     | 915     |

**Table 2.** The initial and optimal design generated by the first-step integrated design.

**Figure 9.** The translation trajectories with the first-step initial and optimal parameters.

**Figure 10.** The orientation trajectories with the first-step initial and optimal parameters.
end, as shown in Figure 11. At the end of these operations, the translation errors achieve the peak values, 0.84 mm with the initial design and 0.99 mm with the optimal design. The $e_t$ of the optimal design is not obviously fluctuant. Figure 12 shows that the maximum value of the orientation tracking error of the initial design is $8.31 \times 10^{-4}$ rad and the end value is $8.26 \times 10^{-5}$ rad, and $e_t,m = 1.23 \times 10^{-3}$ rad and $e_t,c = 3.98 \times 10^{-4}$ rad, respectively, with the optimal design, which increase apparently. However, the elastic potential energy is effectively controlled by the first-step optimization, as shown in Figure 13. The maximum value of the elastic potential energy of the initial design is 0.0459 J, which also appears in the acceleration changeover period. In the optimal system, the $V_m$ is 0.0184 J, which is lowered by 59.91%. And at the end of the operation, the plot of the elastic potential energy becomes gentle. It implies the vibration is weakened. Figure 14 shows the actuated torque responses with optimal parameters. At the end of the accelerating and the decelerating stage, the $\tau_2$ reaches the boundary. And the $\tau_3$ reaches the boundary at the beginning of the decelerating
The convergence of the first-step integrated design process is shown in Figure 15.

The second-step integrated design started from the results (the optimal parameters) of the first-step design. Table 3 shows the comparison of the initial and optimal parameters.

|      | $K_{p1}$ | $K_{p2}$ | $K_{p3}$ | $K_{d1}$ | $K_{d2}$ | $K_{d3}$ |
|------|----------|----------|----------|----------|----------|----------|
| Initial | 124854   | 120571   | 108102   | 812      | 750      | 915      |
| Optimal | 141436   | 129506   | 122940   | 840      | 834      | 560      |

For the structural parameters, the radii of the links change obviously. For the link $i_1$, not only the radius of the first element but also the radius of the second element become greater. The radius of the first element is bigger than the second element’s, which increases about 26.92%. The variation of the radius of the second element is less than 1 mm. For the link $i_2$, the radius of the first element increases about 0.2 mm. The radii of the second and third elements reduce 0.18 mm and 0.9 mm, respectively. Figures 16 and 17 show the translation and orientation trajectories with the second-step initial and optimal parameters, respectively. The responses with the optimal parameters are better. From Figure 18, we can know that the variation trend of the translation tracking error of the optimal design is similar to the initial one. The maximum value of the optimal design also appears at the end of the operation, $e_{im} = e_{ic} = 0.98$ mm. In the translation tracking errors in $X$-direction, the optimum response is worse than the initial response after 0.08 s in a short time. It mainly causes by high deceleration. However, this local peak value is significantly smaller than the maximum value. This phenomenon can be accepted. The maximum values and the ending values of the tracking errors in $X$-direction and $Y$-direction are not beyond the constraints. The orientation tracking errors are shown in Figure 19. The peak value of the orientation tracking error of the optimal design is $2.11 \times 10^{-3}$ rad, which does not reduce, and the end value is $1.40 \times 10^{-3}$ rad. After the second-step integrated design, the elastic potential energy is lowered again, see Figure 20. The $V_m$ of the optimal design is 0.0112 J. The vibration of the operation is controlled better. Figure 21 shows the actuated torque responses with the second-step optimal parameters. In the decelerating stage, the actuated torque responses gradually begin to oscillate obviously. The $\tau_1$ response oscillates most at the end of this operation. The $\tau_2$ also reaches the boundary at the end of the accelerating and the decelerating stage, and the $\tau_3$ reaches the

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**Table 3.** The initial and optimal design generated by the second-step integrated design.

|      | $K_{p1}$ | $K_{p2}$ | $K_{p3}$ | $K_{d1}$ | $K_{d2}$ | $K_{d3}$ |
|------|----------|----------|----------|----------|----------|----------|
| Initial | 124854   | 120571   | 108102   | 812      | 750      | 915      |
| Optimal | 141436   | 129506   | 122940   | 840      | 834      | 560      |

| $R_1$(mm) | $R_2$(mm) | $r_1$(mm) | $r_2$(mm) | $r_3$(mm) |
|-----------|-----------|-----------|-----------|-----------|
| ln        | 34.66     | 10.83     | 10.83     | 10.83     |
| op        | 43.99     | 11.03     | 10.65     | 9.93      |

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**Figure 16.** The translation trajectories with the second-step initial and optimal parameters.

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**Figure 17.** The orientation trajectories with the second-step initial and optimal parameters.
boundary at the beginning of the decelerating stage. Figure 22 shows the convergence of the second-step integrated design process.

**Comparison**

In this section, the results of conventional design in which the structural parameters $P_s = [R_1, R_2, r_1, r_2, r_3]^{T}$ are optimized first and the control parameters $P_c = [K_p1, K_p2, K_p3, K_d1, K_d2, K_d3]^{T}$ second is shown to compare with the results of integrated design. The set of regular parameters in “Simulation and Discussion” is chosen as the initial parameters. The design problem is similar to Problem 2 but the design parameters are divided into two sets. In the first step, to optimize the structural parameters, the control parameters are fixed to the initial parameters. When

**Figure 18.** The translation tracking errors with the second-step initial and optimal parameters: (a) in X-direction and (b) in Y-direction.

**Figure 19.** The orientation tracking errors with the second-step initial and optimal parameters.

**Figure 20.** The elastic potential energy responses with the second-step initial and optimal parameters.
the optimal structural parameters are obtained, they remain unchanged to optimize the control parameters in the second step.

Table 4 shows the initial and optimal parameters. After the optimal design, the radii of the elements on the link $i1$ become big. The radii of the first two elements on the link $i2$ are nearly equal. The third element becomes small. For control parameters, $K_{pi}$ ($i = 1, 2, 3$) decreases but $K_{di}$ ($i = 1, 2, 3$) increases. Figures 23–26 show the tracking performances are not much better than the results of the integrated design, which all fulfill requests. However, the maximum value of the elastic potential energy after the conventional design is 0.01699 J, which is bigger than 1.5 times of $V_m$ with the optimal parameters of the integrated design (see Figures 20 and 27). The actuated torque responses with the optimal parameters of conventional design fluctuate greatly in Figure 28. Figure 29 shows the convergence of the conventional design process. Thus, the integrated structure and control design method are more effective to improve the performance on elastic deformation.
Figure 25. The translation tracking errors with initial and optimal parameters of conventional design and optimal parameters of integrated design: (a) in X-direction and (b) in Y-direction.

Figure 26. The orientation tracking errors with initial and optimal parameters of conventional design and optimal parameters of integrated design.

Figure 27. The elastic potential energy responses with initial and optimal parameters of conventional design and optimal parameters of integrated design.

Figure 28. The actuated torque responses with optimal parameters of conventional design.

Figure 29. The convergence of the conventional design process.
Conclusion
A new integrated design framework based on elastic potential energy is presented in this article. And it is applied to design a coaxis planar parallel manipulator, V3. The FEM is utilized to derive the system dynamic model. A PD controller is employed in the closed-loop system to verify its effects. In this study, the vibration is suppressed via minimizing the maximum value of the elastic potential energy. For pick-and-place operation, the limitations of the actuated system, the ending, and maximum values of the tracking errors are also regarded as the major performance indices. Then, this integrated design problem is formulated as a nonlinear, multimodal optimization problem and the GA is applied to locate the global optimum. Last, the performance on elastic deformation for this manipulator system has been obviously improved by this integrated design method in the simulation. The vibration during the moving process can be controlled better. In further study, a prototype will be set up and the experiments will be carried out.

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