HADRONIZATION EFFECTS IN INCLUSIVE $\tau$ DECAY

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Abstract. It is shown that the nonperturbative effects due to hadronization play a crucial role in low–energy strong interaction processes. Specifically, such effects impose a stringent constraint on the infrared behavior of the Adler function and play an essential role in the theoretical analysis of inclusive $\tau$ lepton decay.

1 Introduction

This paper briefly presents the results of the studies of effects due to hadronization in the theoretical description of inclusive $\tau$ lepton decay [1–8]. In this strong interaction process the experimentally measurable quantity is

$$R^\tau = \frac{\Gamma(\tau^\pm \rightarrow \text{hadrons}^\pm \nu_\tau)}{\Gamma(\tau^\pm \rightarrow e^\pm \bar{\nu}_e \nu_\tau)} = R^{J=0}_{\tau,V} + R^{J=1}_{\tau,V} + R^{J=0}_{\tau,A} + R^{J=1}_{\tau,A} + R_{\tau,S}. \quad (1)$$

In what follows we shall restrict ourselves to the consideration of parts $R^{J=1}_{\tau,V}$ and $R^{J=1}_{\tau,A}$. The theoretical prediction for these quantities reads

$$R^{J=1}_{\tau,V/A} = \frac{N_c}{2} |V_{ud}|^2 S_{\text{EW}} (\Delta_{QCD}^{V/A} + \delta_{\text{ew}}^{V/A}), \quad \Delta_{QCD}^{V/A} = 2 \int_{\frac{M^2}{m_{V/A}}} f\left(\frac{s}{M^2}\right) R_{V/A}(s) \frac{ds}{M^2}, \quad (2)$$

where $N_c = 3$ is the number of colors, $|V_{ud}| = 0.9738 \pm 0.0005$ is Cabibbo–Kobayashi–Maskawa matrix element [11], $S_{\text{EW}} = 1.0194 \pm 0.0050$ and $\delta_{\text{ew}}^{V/A} = 0.0010$ stand for the electroweak corrections (see Refs. [12–14]), and $\Delta_{QCD}^{V/A}$ denotes the QCD contribution. In Eq. (2) $M_\tau = 1.777$ GeV is the mass of $\tau$ lepton [11], $m_{V/A}$ stands for the total mass of the lightest allowed hadronic decay mode of $\tau$ lepton in the corresponding channel, $f(x) = (1 - x)^2 (1 + 2x)$, and

$$R_{V/A}(s) = \frac{1}{2\pi i} \lim_{\varepsilon \to 0^+} \left[ \Pi_{V/A}(s + i\varepsilon) - \Pi_{V/A}(s - i\varepsilon) \right] = \frac{1}{\pi} \text{Im} \lim_{\varepsilon \to 0^+} \Pi_{V/A}(s + i\varepsilon), \quad (3)$$

with $\Pi_{V/A}(q^2)$ being the hadronic vacuum polarization function. The superscripts “V” and “A” will only be shown when relevant hereinafter.

In general, it is convenient to perform the theoretical analysis of inclusive $\tau$ lepton hadronic decay in terms of the Adler function [15]

$$D(Q^2) = -\frac{d \Pi(-Q^2)}{d \ln Q^2}, \quad Q^2 = -q^2 = -s. \quad (4)$$

Within perturbative approach the ultraviolet behavior of $D(Q^2)$ (4) can be approximated by power series in the strong running coupling $\alpha_s(Q^2)$

$$D(Q^2) \simeq D_{\text{pert}}^{(1)}(Q^2) = 1 + \sum_{j=1}^\ell d_j \left[ \alpha_s^{(1)}(Q^2) \right]^j, \quad Q^2 \to \infty. \quad (5)$$

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In this paper we shall restrict ourselves to the one–loop level ($\ell = 1$): $\alpha_s^{(1)}(Q^2) = 4\pi/(\beta_0 \ln z)$, $z = Q^2/\Lambda^2$, $\beta_0 = 11 - 2n_f/3$, $\Lambda$ is the QCD scale parameter, $n_f$ denotes the number of active flavors ($n_f = 2$ is assumed in what follows), and $d_1 = 1/\pi$.

2 Perturbative approach

In this Section we shall study the massless limit, that implies that the masses of all final state particles are neglected. In this case, by making use of definitions (3) and (4), and additionally employing Cauchy theorem, the quantity $\Delta_{QCD}^{(2)}$ can be represented as

$$\Delta_{QCD}^{(2)} = \frac{1}{2\pi} \int_{-\pi}^{\pi} D(M^2_\tau e^{i\theta})(1 + 2e^{i\theta} - 2e^{i3\theta} - e^{i4\theta}) d\theta,$$

see, e.g., papers [10, 12, 16] and references therein. In fact, the only available option within perturbative approach is to directly use in the theoretical expression for $\Delta_{QCD}^{(2)}$ the perturbative approximation of Adler function $D_{pert}(Q^2)$ (5), which contains unphysical singularities at low energies. In this case Eq. (6) eventually takes the form (see Refs. [7, 8] for the details)

$$\Delta_{pert} = 1 + \frac{4}{\beta_0} \int_{0}^{\pi} \frac{\lambda A_1(\theta) + \theta A_2(\theta)}{\pi(\lambda^2 + \theta^2)} d\theta, \quad \lambda = \ln \left( \frac{M^2_\tau}{\Lambda^2} \right),$$

where $A_1(\theta) = 1 + 2\cos(\theta) - 2\cos(3\theta) - \cos(4\theta)$, $A_2(\theta) = 2\sin(\theta) - 2\sin(3\theta) - \sin(4\theta)$.

Figure 1: Comparison of perturbative expression $\Delta_{pert}$ (7) (solid curves) with experimental data (8). The left and right plots correspond to vector and axial–vector channels, respectively.

It is worth noting here that perturbative approach gives identical predictions for functions $\Delta_{QCD}^{(2)}$ (2) in vector and axial–vector channels (i.e., $\Delta^{V}_{pert} \equiv \Delta^{A}_{pert}$). However, their experimental values are different, namely [9, 10, 17]

$$\Delta^{V}_{exp} = 1.221 \pm 0.057, \quad \Delta^{A}_{exp} = 0.748 \pm 0.032.$$
For vector channel the comparison of perturbative result (7) with experimental data (8) gives \( \Lambda = (465 \pm 140) \text{ MeV} \) (the second value, \( \Lambda = (1646 \pm 26) \text{ MeV} \), will not be considered herein), see Fig. 1. As for the axial–vector channel, the perturbative approach fails to describe experimental data on \( \tau \) lepton decay.

3 Dispersive approach

It is crucial to emphasize that the analysis presented in Sect. 2 entirely leaves out the effects due to hadronization, which play an important role in the studies of strong interaction processes at low energies. Such effects were properly accounted for in the framework of Dispersive approach to QCD, that has eventually led to the following integral representations for functions (3) and (4) (see Refs. [3, 4, 6, 7] for the details):

\[
R(s) = \left(1 - \frac{m^2}{s}\right)^{3/2} + \theta\left(1 - \frac{m^2}{s}\right) \int_s^\infty \rho(\sigma) \frac{d\sigma}{\sigma},
\]

\[
D(Q^2) = 1 + \frac{3}{\xi} + \frac{3u(\xi)}{2\xi} \ln\left[1 + 2\xi(1 - u(\xi))\right] + \frac{1}{u^2(\xi)} \int_{m^2}^\infty \rho(\sigma) \frac{\sigma - m^2}{\sigma + Q^2} d\sigma.
\]

It is worth noting that Eq. (9) by construction automatically takes into account the effects due to analytic continuation of spacelike theoretical results into timelike domain and Eq. (10) embodies the nonperturbative constraints, which relevant dispersion relation imposes on the Adler function. In Eqs. (9) and (10) \( \theta(x) \) denotes the unit step–function, \( u(\xi) = \sqrt{1 + 1/\xi} \), \( \xi = Q^2/m^2 \), and \( \rho(\sigma) \) is the spectral density. For the latter the following expression will be employed [7]:

\[
\rho(\sigma) = \frac{1}{\beta_0} \ln^2(\sigma/\Lambda^2) + \frac{\Lambda^2}{\sigma},
\]

see also Refs. [1, 2, 6, 18]. In the right–hand side of Eq. (11) the first term is the one–loop perturbative contribution, whereas the second term represents intrinsically nonperturbative part of the spectral density. Within the approach on hand the quantity \( \Delta_{QCD}^{V/A} (2) \) ultimately takes the following form:

\[
\Delta_{QCD}^{V/A} = \sqrt{1 - \zeta_{V/A}} \left(1 + 6\zeta_{V/A} - \frac{5}{8} \zeta_{V/A}^2 + \frac{3}{16} \zeta_{V/A}^3\right) + \int_{m_{V/A}^2}^\infty H\left(\frac{\sigma}{M^2}\right) \rho(\sigma) d\sigma
\]

\[
-3\zeta_{V/A} \left(1 + \frac{1}{8} \zeta_{V/A}^2 - \frac{1}{32} \zeta_{V/A}^3\right) \ln\left[\frac{2}{\zeta_{V/A}}\left(1 + \sqrt{1 - \zeta_{V/A}}\right) - 1\right],
\]

where \( \zeta_{V/A} = m_{V/A}^2/M^2 \), \( H(x) = g(x)\theta(1 - x) + g(1)\theta(x - 1) - g(\zeta_{V/A}) \), and \( g(x) = x(2 - 2x^2 + x^3) \), see papers [7, 8] and references therein for the details.

The comparison of obtained result (12) with experimental data (8) yields nearly identical values of QCD scale parameter \( \Lambda \) in both channels, namely, \( \Lambda = (412 \pm 34) \text{ MeV} \) for vector channel and \( \Lambda = (446 \pm 33) \text{ MeV} \) for axial–vector one, see Fig. 2. Additionally, both these values agree very well with perturbative estimation of QCD scale parameter described in Sect. 2.
Figure 2: Comparison of expression $\Delta_{QCD}$ (12) (solid curves) with experimental data (8). The left and right plots correspond to vector and axial–vector channels, respectively.

4 Conclusions

The significance of effects due to hadronization in the theoretical description of inclusive $\tau$ lepton decay is convincingly demonstrated. The Dispersive approach to QCD has proved to be capable of describing experimental data on $\tau$ lepton hadronic decay in vector and axial–vector channels. The vicinity of values of QCD scale parameter $\Lambda$ obtained in both channels testifies to the self-consistency of employed approach.

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