The elastic neutron form factors $G_{E_n}$ and $G_{M_n}$ are calculated in a GPD framework using GPDs obtained from fits to proton elastic form factors $G_{E_p}$ and $G_{M_p}$, and isospin symmetry, with no further changes in parameters. The results for $G_{E_n}$ are in good agreement with existing data, while those for $G_{M_n}$ are fair. The calculations predict the form factors for future measurements at higher $Q^2$.

In recent years, the development of generalized parton distributions (GPDs) [1, 2, 3] has opened the possibility of describing a great variety of exclusive reactions in the multi GeV range in terms of a common nucleon structure. The constraints imposed by the description of many types of reactions offers the possibility of modeling the longitudinal and transverse parton structure of nucleons.

Among the most direct consequences of the GPD formalism are the sum rules which relate the various GPDs to the hadronic form factors. Thus the proton elastic helicity conserving and helicity-flip form factors may be written, respectively, as:

$$ F_{1p}(t) = \int \frac{dx}{1} \sum_q e_q H^u_p(x, \xi, t) dx $$

$$ F_{2p}(t) = \int \frac{dx}{1} \sum_q e_q E^u_p(x, \xi, t) dx $$

where $t = Q^2$ is the momentum transfer to the proton, $\xi$ is the longitudinal momentum transfer, and $q$ signifies quark flavors. Without loss of generality one may work in a coordinate system in which the momentum transfer $t$ is transverse so that $\xi = 0$, and the GPDs may be written:

$$ H^q_p(x, t) = H^u_p(x, t, \xi = 0) \quad E^q_p(x, t) = E^u_p(x, t, \xi = 0) $$

Several authors [4, 5, 6, 7] have modeled the GPDs by Gaussian functions which embody general expected properties. In particular, $H^q_p(E, t = 0) \rightarrow f^q_p(x)$, the unpolarized quark distribution function and asymptotically $H^q(E, -t \rightarrow \infty)$ narrows toward $x = 1$ (see [7, 8]). In terms of a Gaussian a simple model is,

$$ H^q_p(x, t) = f^q_p(x) e^{-\bar{x}t/4x\lambda^2_H} \quad (1) $$

in which $\bar{x} \equiv 1 - x$. For $E^u_p(x, t)$ the we take the simple ansatz,

$$ E^u_p(x, t) = k^u_p(x)e^{-\bar{x}t/4x\lambda^2_H}. \quad (2) $$

To account for hard components of $F_{1p}$ at $-t > 10$ ref. [8] modified the specific functional form for $H^u_p(x, t)$ and $E^u_p(x, t)$ as a Gaussian plus small power law shape in $-t$.

$$ H^u_p(x, t) = f^u_p(x)exp(\bar{x}t/4x\lambda^2_H) + \cdots \quad (3) $$

$$ E^u_p(x, t) = k^u_p(x)exp(\bar{x}t/4x\lambda^2_H) + \cdots, \quad (4) $$

in which $\cdots$ indicates the addition of small power law components in $-t$.

To obtain $E^u_p$ and $E^d_p$, needed for eqs. 1 to 4, the available data for $G_{M_p}$ and the recent JLab data [10, 11] on $G_{E_p}/G_{M_p}$ were fit, as in ref. [3].

The conditions at $t = 0$ were also required, i.e.

$$ H^q_p(x, 0) = c_u f^q_u(x) + c_d f^q_d(x) \quad E^q_p(x, 0) = k^q_p(x) + k^q_p(x). $$

The valence quark distribution functions $f^q_p(x)$ and $f^q_p(x)$ are measured in DIS, and obtained from refs. [4, 5, 6, 7]. The functions $k^q_p(x)$ and $k^q_p(x)$ are not obtainable from evaluations of DIS. Following ref. [8] the simple phenomenological assumption $k^q_p(x) \propto \sqrt{1 - x f^q_p(x)}$ was used. This results in a satisfactory ratio of $F_{2p}/F_{1p}$, since for large $-t$, the quantity $\sqrt{1 - x} \rightarrow 1/\sqrt{-t} = 1/Q$ with normalization obtained by requiring the proton $F_{2p}(0) = 1.79$.

Adequate fits to the measured $G_{M_p}$ and $G_{E_p}/G_{M_p}$, or equivalently $F_{1p}$ and $F_{2p}/F_{1p}$, were obtained with $\lambda_H = 0.76$ GeV/c and $\lambda_E = 0.67$ GeV/c. The results are shown in figs. [1, 2].

This gives

$$ F^u_{1p}(0) = \int c_u H^u_p(x, 0) dx = \int c_u f^u_p(x) dx = 4/3 $$

$$ F^d_{1p}(0) = \int c_d H^d_p(x, 0) dx = \int c_d f^d_p(x) dx = -1/3 $$

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The Pauli form factor $F_\pi(Q^2)$ relative to the dipole $G_D = 1/(1 + Q^2/0.71)^2$. The data are extracted using the recent JLab data [10, 11] for $G_{Ep}/G_{Mp}$, and a recent reevaluation [12] of SLAC data of $G_{Mp}$ [15, 16]. The curve is the result of the fit as discussed in the text.

![Fig. 1: Dirac form factor $F_\pi(Q^2)$ relative to the dipole $G_D = 1/(1 + Q^2/0.71)^2$. The data are extracted using the recent JLab data [10, 11] for $G_{Ep}/G_{Mp}$, and a recent reevaluation [12] of SLAC data of $G_{Mp}$ [15, 16]. The curve is the result of the simultaneous fit to the $G_{Ep}/G_{Mp}$ and $G_{Mp}$ data as discussed in the text and fig. 1.](image)

and

$$E_n^u(x, t) = E_p^u(x, t)$$

$$E_n^d(x, t) = E_p^d(x, t)$$

with $\kappa_n = -1.91 \mu_N$.

The result for $G_{En}$ is shown in Fig. 3. The calculated form factor is somewhat lower than the existing data in the region $Q^2 = -t < 0.75 \text{ GeV}^2/c^2$, but accounts well for the new JLab Hall C data for $Q^2 > 0.75 \text{ GeV}^2/c^2$. There is excellent agreement with the results of a calculation of ref. [19], which is also shown in the figure. The calculation of ref. [19] uses a completely different framework, employing a relativistic constituent quark model with a pion cloud. However, for $Q^2 > 1 \text{ GeV}^2/c^2$ the quarks become most important, with the role of the pion cloud diminishing. In the present calculation, the contribution of the sea quark pairs, which presumably would mimic the pion cloud, was set to zero. The importance of a rigorously relativistic calculation of both the constituent quarks and pion cloud is stressed in ref. [19]. For example, at high $Q^2$ the lower components of the Dirac spinors, which introduce orbital angular momentum, become important. The calculation of ref. [19] employs several parameters, however the $Q^2$ dependence of the form factor at higher $Q^2$ appears to be governed more by relativistic effects than the specific parameter set used. In particular a large number of sets of these parameters can be found to give similar $Q^2$ dependence.

As seen in Fig. 3 both calculations give results at high $Q^2$ which lie above the Galster parameterization [20], as do the most recent experimental data [18]. This is not surprising since the Galster parameterization is simply an ad hoc fit to low $Q^2$ data.

The result for $G_{Mn}$ are shown in Fig. 4. Here, the fit to the experimental data is somewhat poorer than for $G_{En}$. Also shown is the result of the calculation of ref. [19]. Curves are shown for two of the many parameter sets which fit the data. A possible reason for the better fit
may be that ref. [19] chooses parameters in such a way that requires the fit to be rather good for all four elastic form factors, while in the present case only the proton form factors are fit, and then isospin symmetry is applied to obtain the neutron form factors with the parameters fixed.

This note has pointed out the usefulness of GPDs in describing elastic form factors. Alternatively, the elastic form factors, together with isospin symmetry can be very important for constraining nucleon structure through GPDs. Further constraints of details of nucleon structure will be possible by including other high $-t$ experiments into the fit procedure. These include high $W$ high $-t$ real and virtual Compton scattering, and single meson photo and electroproduction, such as described in refs. [1, 8, 22, 23]. It would be quite interesting if conceptual connections could be made between this technique and those of recent relativistic constituent quark models with pion cloud such as in ref. [19], or recent helicity non-conserving pQCD based approaches ref. [30, 31] which have had some success in explaining the $Q^2$ dependence of $F_{2p}$.

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