Why should you reverse the order when dividing a fraction?  
A study of pre-service mathematics teachers’ pedagogical content knowledge in fractional concept

R A Apsari\textsuperscript{1}, S Sariyasa\textsuperscript{2}, G Indrawan\textsuperscript{3}, M A Maulyda\textsuperscript{4}, Radiusman\textsuperscript{5}
\textsuperscript{1} Mathematics Education Study Program, Universitas Mataram, Mataram, Indonesia
\textsuperscript{2} Mathematics Education Study Program, Universitas Pendidikan Ganesha, Singaraja, Indonesia
\textsuperscript{3} Electrical Engineering Study Program, Universitas Pendidikan Ganesha, Singaraja, Indonesia
\textsuperscript{4,5} Primary School Teacher Education Study Program, Universitas Mataram, Mataram, Indonesia

email: ra.apsari@unram.ac.id

Abstract. This study was a part of larger study in examining pre-service mathematics teachers’ Pedagogical Content Knowledge. The aim of the present study was to elaborate the prospective teachers’ conceptual understanding in fraction and how they will teach it for the students. There were 50 participants in this study, all of them are the final year students of mathematics education study program of a state university in Bali who already took the teaching internship program in the school. The data were collected through written test and continued by interview with the participants. The data were analysed qualitatively using descriptive method. From the result, it is found that the prospective mathematics teachers have a difficulties in understanding the concept of fraction. It leads to the lack comprehension in teaching the concept to the students. Reflect to the result, it is recommended to the teacher training institution to emphasize the big ideas in school mathematics’ topics to ensure the prospective teachers have a sufficient knowledge to teach mathematics meaningfully.

Keywords: Pedagogical Content Knowledge, Fraction, Pre-service mathematics teacher

1. Introduction

The focus of current mathematics classroom is to enable students in mastering procedures instead of gaining conceptual understanding [1]. The tendency contributed to the students’ lack of awareness toward the meaning of mathematical symbols and formulas. As showed in the result of PISA [2], it can inferred that the students in Indonesia is seeing mathematics is seen as the body of rigid knowledge that need to be remembered and mostly has no actual implementation. Hence, when the problem is about general application of mathematics in daily life, the students were performed poorly.

There are a number of causes that affect students’ achievement. According to [3] at least four major factors including social economic status, support from parents, peers’ interaction and school’s climate should be addressed to explain the case. Also, the students’ achievement is highly impacted by their internal factors, family factors and school factors [4]. While the other factors are externally happened outside the classroom, the factor from school, especially their interaction with teacher, play significant role inside the classroom.

The study of [5], [6] and [7] found that the low learning result of students is influenced by the teacher’s ability in conducting the lesson. This ability is called teacher’s Pedagogical Content
Knowledge (PCK), a special type of teacher knowledge which focuses on the strategy used by teacher in conducting a meaningful lesson [8]. PCK includes the representations, examples, and applications of a specific content that can be employed by the teacher to make the subject matter comprehensive and make sense [9]. It also refers to the strategies that teachers use in order to overcome learning difficulties encountered by the students.

Besides as a support for the students in learning, PCK is also used as a guideline to conduct a further study related specific topic in mathematics education [10]. By examining the PCK needed in certain mathematical topic, a researcher can elaborate model, strategy, method and technique that can make the teaching and learning more effective.

Most of the trend on studies in mathematics education usually focus on the type of difficulties in conducting lesson as can be seen in [11] & [12] and factors that influence the teachers’ PCK [13]. Reflect on the problem in the teachers’ conceptual mastery and ability to support the students in understanding mathematics, the trend in mathematics education research are shifting to the PCK related studies.

Even though there are a number of previous studies discussing PCK of prospective mathematics teacher, a deep study related to the skills is still limited. Hence, the present study attempt to answer the research question of how far the pre-service mathematics teacher understand the concept in fraction? Due to limitation of the space, the discussion will merely focus on the case of dividing two fractions. Fraction was chosen as one of the topic in PCK because it is considered as difficult topic to explain in the classroom [14].

2. Method

This is a Survey Study which is aimed to investigate the prospective mathematics teacher’s Pedagogical Content Knowledge in various domain of school mathematics. The present study is discussing the result on the fraction related problem which asking the participants to explain how will they explain the division procedures of two fractions as usually in the classroom the students be taught that it can be done by multiply the first fraction with the inverse of the second fraction \( \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} \).

The participants of the study were 50 students of mathematics education study program of a state university in Bali, Indonesia. The criteria for the participants was the final year students who experienced teaching internship program.

The data were collected from students’ responses in written Pedagogical Content Knowledge test and completed by interview. Descriptive qualitative was employed as data analysis method.

3. Results and Discussion

Classification of Answer

![Figure 1. Classification of Answer](image)

From the Figure 1 it can be observed that even though the question is a basic concept, it is not easy for the prospective mathematics teachers. During the interview, the majority of the participants said they
never think about the reason of reverse the order of the fraction and multiply it when the original aim is to divide two fractions. As they remember, the method was directly given from elementary school level and they just memorized it since then. The following Figure 2 provides the example of explanation of the participant.

**Translation:**

If two fractions be divided, the first fraction is \( \frac{a}{b} \) and the second fraction is \( \frac{c}{d} \). Then, \( \frac{a}{b} \div \frac{c}{d} \) can be solved by reverse the order of second fraction and change into \( \frac{a}{b} \times \frac{d}{c} \).

**Figure 2. Example of Direct Explanation**

Actually, the response in Figure 2 showed the common method in explaining fraction division in the Indonesia’s classroom. Without knowing nor questioning the reason, the students will follow the procedure of reversing order and multiply it.

About 6% of participants also responses the problem by giving incomplete example. It means, they tried to show why the reverse and multiplication procedure applied in the case of dividing two fractions, but the explanation is jumping. The following Figure 3 showed the participant’s attempted to explain the idea.

**Translation:**

First, explain that \( \frac{d}{c} \) is the reverse (of \( \frac{c}{d} \)) or \( \frac{1}{c/d} \).

Hence, \( \frac{a}{b} \times \frac{1}{c/d} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} \).

Therefore, we can tell the students that division of two fractions can be done by multiplying the first number with the reverse of the second fraction.

**Figure 3. Example of Incomplete Explanation**

From Figure 3, it can be observed that there is an effort to explain why the division can be solved using multiplication. However, some steps are insufficiently elaborated. First, when the participant said “explain that \( \frac{d}{c} \) is the reverse (of \( \frac{c}{d} \)) or \( \frac{1}{c/d} \)” will lead a confusion in students’ mind since there is no reason why those numbers become reverse for each other. It followed to partial understanding when coming to the second step of “\( \frac{a}{b} \times \frac{1}{c/d} = \frac{a}{b} \times \frac{d}{c} \)”.

Actually, to illustrate why a number is called as a reverse of another number, some elaboration using example might be beneficial. The strategy was employed by 28% of participants in this study. The example of the responses can be seen in the following Figure 4.
Even though the strategy is a good starting point, a teacher should remember not to let the students generalize a mathematical idea from 1 or 2 examples – not even 100 example; as mathematics is a deductive knowledge. However, since we currently deal with students in elementary school level, we also cannot use a rigor proofing method. In this study, only 4% participants come to the general form. One of them employed negative power in proofing which cannot be implemented in the real classroom as the students did not learn power rule yet.

The possible strategy is by encouraging the students to see the general form from their pattern investigation that it can be used in any fractions. The teachers may start with any number and then asked the students to investigate what will happen if we divide a number with 1. Afterwards, ask the students to think how to convert \( \frac{a}{b} \) into \( \frac{a}{1} \). It will be the time when the students explore the concept of invers of the fraction. The conclusion will be the general procedure for division that usually be taught in the school. Even though the final procedure is similar, but there is an understanding in the application of it. In other words, the students use their common sense in doing mathematics – what was promoted since 1974 [15].

**Classification of Correctness**

We classified the correctness of participants’ responses according to their abilities in mathematically explain the process to the students. The summary can be observed in the Figure 5.

---

**Translation:**

\[
\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}
\]

It is explained by forming it into ordinary division which is \( \frac{a}{b} \) can be changed into multiplication \( \frac{a}{b} \times \frac{d}{c} \). To make it clear, teacher can give illustration, for instance \( 4 \div 2 = 2 \) in the division operation can be written as \( \frac{4}{1} \div \frac{2}{1} = 2 \). The result is same.

---

**Figure 4. Example of Using Example Explanation**

---

**Figure 5. Classification of Correctness**

It was found that most prospective teachers are left their sheet blank or directly stated that the fraction should be reversed. For those two cases, we classified the as incorrect. The participants who
were classified as giving incomplete explanation in Figure 1 were also marked Partially Correct in the present classification with addition some type of answer given from the participant who use pattern exploration (see Figure 7). Some participants who use number pattern investigation but add a further exploration to see the general form is considered as Correct.

It happened because there is an effort to show the pattern of the result of division and multiplication of fractions, but not in enough portion. It makes the conclusion of division of fraction is equal to multiply the first fraction to the inverse of the second fraction emerged too early.

As a prospective mathematics teachers, the participants should consider how their students will come to the conclusion. One number example might not be enough to support students in seeing the pattern. Also, it is mathematically incorrect to generalize form example.

**Classification of Representation**
Besides of correctness and completeness of the prospective teachers’ explanation related to the question, we also curious about the type of representation used. In teaching and learning mathematics, representation play important role to bridge the communication between teacher and students and among the students. In Realistic Mathematics Education (RME), representation become one of important tenets called emerging modelling. It is a useful tool to illustrate the context of problem and to support students’ working mathematically to solve it. In this study, unfortunately, the prospective teachers hardly support their explanation with representation as can be seen in the following Figure 6.

![Figure 6. Classification of Representation](image)

It can be seen that very small portion of prospective mathematics teachers employed mathematical model or representation in their explanation. Surprisingly, this finding is similar with BAL (2014) that the teacher candidates are reported to find an appropriate representation to express their ideas. Further analysis showed that all of the participants who use representation in their work, directing their explanation by pattern investigation using example. One of the type of answer can be seen in the following Figure 7.

![Figure 7. The example of students’ representation of fraction.](image)

**Translation:**
Use example, e.g. $\frac{1}{2} \times \frac{4}{1}$
1 from 2 parts is shaded, it is a $\frac{1}{2}$
This $\frac{1}{2}$ will be divided again to 4 people. Each person will get $\frac{1}{8}$ part. To get $\frac{1}{2} \times \frac{4}{1} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$
So, $\frac{a}{b} \times \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$
Connected to the Classification of Correctness in Figure 7, the participants whose answer as similar as is given in the Figure 7 were classified as “Partially Correct” answer. In term of visual support, the answer in Figure 7 provides a meaningful bridge to enhance students’ understanding in dividing fractions. It provides a connector from previous knowledge in division natural numbers into the present topic of fraction. According to [16] connection from previous knowledge to present study playing essential role to the students’ conceptual understanding.

Furthermore, it also found that only 6% of participants who use daily life illustration to explain the concept of division in the fraction. The context used was dividing bread, watermelon and pizza. The rest of the participants were directly using numbers. This finding is contrary to the finding of [17] that most of the teachers were using daily activities to illustrate the fraction such as dividing cake into equal parts.

Reflect on the findings of this study, it can be inferred that the prospective teachers’ difficulties in conducting meaningful mathematics lesson are including: (1) the lack understanding of certain concept in mathematics, (2) the lack of critical thinking to making sense the concept in mathematics, (3) the lack of consideration of students’ prior knowledge to the present learning topic, (4) the lack of communication skill especially to illustrate ideas using visual representation and (5) the lack of literacy skills to see the application of mathematical concepts in daily life. According to [18] to face the challenge in improving the quality of teacher education institution, it is important to consider the use of experience sharing in the classroom to discuss about mathematical concepts and how to teach it.

4. Conclusion
From the results, it can be concluded that the prospective mathematics teachers’ pedagogical content knowledge is need to be improved as it was found that only 12% were able to correctly solve the problem. Hence, it is necessary to develop a pedagogical-based course focused on how to teach specific mathematical content. By that, the course should enable the pre-service mathematics teacher to notice the big ideas or the core indicators in every mathematical topics. Also, we need to encourage teachers to use supported representation tools such as table, diagram, graphic, or other mathematical models to support the students’ conceptual mastery.

References
[1] Molina C 2014 Teaching Mathematics Conceptually. *SEDL insights* 1 1-8
[2] OECD 2015 *PISA 2015 Results in Focus* (OECD)
[3] Kiwanuka H, Van Damme J, Van Den Noortgate W, Anumendem D and Namusisi S 2015 Factors affecting Mathematics achievement of first-year secondary school students in Central Uganda. *South African Journal of Education* 35 1-16
[4] Al-Zoubi S M and Younes M A (n.d.) Low academic achievement: Causes and results *Theory and Practice in Language Studies* 5 2262-2268
[5] Mehdipour Y and Balaramulu D 2013 The influence of teacher’s behavior on the academic achievement *International Journal of Advancements in Research & Technology* 2 217-224
[6] Kimani G, Kara A and Njagi L 2013 Teacher factors influencing students’ academic achievement in secondary schools in Nyandarua Country Kenya *International Journal of Education and Research* 1 1-14.
[7] Munawaroh 2017 The influence of teaching methods and learning environment to the student’s learning achievement of craft and entrepreneurship subjects at vocational high school *International Journal of Environmental & Science Education* 12 665-678
[8] Fennema E and Franke L M 1992 Teachers’ knowledge and its impact ed. D A Grouws *Handbook of research on mathematics teaching and learning* (New York NY: Macmillan) pp 147-164
[9] Maher N, Muir T and Chick H 2015 Examining PCK in a senior secondary mathematics lesson ed. V G M Marshman *Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia* (Sunshine Coast: MERGA) pp 389-396
[10] Petrou M and Goulding M 2011 Conceptualising Teachers’ Mathematical Knowledge in Teaching ed T R Ruthven Mathematical Knowledge in Teaching (New York: Springer) pp 9-25
[11] Whyte J and Anthony G 2012 Maths anxiety: The fear factor in the mathematics classroom New Zealand Journal of Teachers’ Work 9 6-15
[12] Fuchs L S, Fuchs D and Compton D L 2012 The early prevention of mathematics difficulty: Its power and limitations J Learn Disabil 45 257-269
[13] Ratcliff N, Costner R, Carroll K, Jones C, Sheehan H and Hunt G 2016 Causes of and solutions to the achievement gap: Teachers’ perceptions Teacher Educators’ Journal 9 97-111
[14] Coetzeea J and Mammen K 2017 Science and engineering students’ difficulties with fractions at entry-level to university International Electronic Journal of Mathematics Education 12 281-310
[15] Freudenthal H 1974 Soviet research on teaching algebra at the lower grades of the elementary school Educational Studies in Mathematics 5 391-412
[16] Hiebert J and Grouws D 2007 The Effect of Classroom Mathematics Teaching on Students’ Learning ed F K Lester Second handbook of research on mathematics teaching and learning (Charlotte, NC: Information Age) pp 371–404
[17] Baştürk S 2016 Primary student teachers’ perspectives of the teaching of fractions Acta Didactica Napocensia 9 35-44
[18] Cyrino M C 2016 Mathematics teachers' professional identity development in communities of practice: Reifications of proportional reasoning teaching Bolema Mathematics Education Bulletin 30 165-187