Loop Corrections to Scalar Quintessence Potentials

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The stability of scalar quintessence potentials under quantum fluctuations is investigated both for uncoupled models and models with a coupling to fermions. We find that uncoupled models are usually stable in the late universe. However, a coupling to fermions is severely restricted. We check whether a graviton induced fermion-quintessence coupling is compatible with this restriction.

1 Introduction

Observations indicate that dark energy constitutes a substantial fraction of our Universe. The range of possible candidates includes a cosmological constant and - more flexible - some form of dark energy with time dependent equation of state, called quintessence. Commonly, realizations of quintessence scenarios feature a light scalar field.

The evolution of the scalar field is usually treated at the classical level. However, quantum fluctuations may alter the classical quintessence potential. In this talk which is based mainly on, we have a look at one-loop contributions to the effective potential both from quintessence and fermion fluctuations. We will show that in the late universe, quintessence fluctuations are harmless for most of the potentials used in the literature. For inverse power laws and SUGRA inspired models, this has been demonstrated in. On the other hand, we will find that fermion fluctuations severely restrict the magnitude of a possible coupling of quintessence to fermionic dark matter.

In Euclidean conventions, the action we use for the quintessence field $\Phi$ and a fermionic species $\Psi$ to which it may couple is

$$S = \int d^4x \sqrt{g} \left[ M_P^2 R + \frac{1}{2} \partial_\mu \Phi(x) \partial^\mu \Phi(x) + V(\Phi(x)) + \bar{\Psi}(x) \left[ i \nabla + \gamma^5 m_f(\Phi) \right] \Psi(x) \right], \quad (1)$$

with $m_f(\Phi)$ as a $\Phi$ dependent fermion mass. This $\Phi$ dependence (if existent in a model) determines the coupling of the quintessence field to the fermions. As long as one is not interested in quantum gravitational effects, one may set $\sqrt{g} = 1$, $R = 0$ and replace $\nabla \rightarrow \partial$ in the action.

By means of a saddle point expansion, we arrive at the effective action $\Gamma[\Phi_{cl}]$ to one loop order of the quintessence field. The equation governing the dynamics of the quintessence field is then determined by $\delta \Gamma[\Phi_{cl}]|_{\Phi_{cl} = \Phi_{cl}^*} = 0$. When estimating the magnitude of the loop corrections, we will assume that $\Phi_{cl}^*$ is close to the solution of the classical field equations: $\delta S = 0$. Evaluating $\Gamma$ for constant fields, we can factor out the space-time volume $U$ from $\Gamma = UV$. This gives the
effective potential
\[ V_{1\text{-loop}}(\Phi_{cl}) = V(\Phi_{cl}) + \frac{\Lambda^2}{32\pi^2} V''(\Phi_{cl}) - \frac{\Lambda_{\text{term}}^2}{8\pi^2} [m_f(\Phi_{cl})]^2. \] (2)

Here, primes denote derivatives with respect to \( \Phi \); \( \Phi_{cl} \) is the classical field value and \( \Lambda \) and \( \Lambda_{\text{term}} \) are the ultra violet cutoffs of scalar and fermion fluctuations. The second term in Eq. (2), is the leading order scalar loop. We neglect graphs of the order \( (V''_{cl})^2 \) and higher, because \( V \) and its derivatives are of the order \( 10^{-120} \) (see section 3). We have also ignored \( \Phi \)-independent contributions, as these will not influence the quintessence dynamics. However, the \( \Phi \)-independent contributions add up to a cosmological constant of the order \( \Lambda^4 \approx O(M_P^4) \). This is the old cosmological constant problem, common to most field theories. We hope that some symmetry or a more fundamental theory will force it to vanish. The same symmetries or theories could with the same right remove the loop contribution by some cancelling mechanism. After all, this mechanism must be there, for the observed cosmological constant is far less than the naively calculated \( O(M_P^4) \).

Besides, none of the potentials under investigation can be renormalized in the strict sense.

There is also a loophole for all models that will be ruled out in the following: The potential used in a given model could be the full effective potential including all quantum fluctuations, down to macroscopic scales. For coupled quintessence models, this elegant argument is rather problematic and the loophole shrinks to a point (see section 3).

We use units in which \( M_P = 1 \).

2 Uncoupled quintessence

Let us calculate here only the simplest example, the pure exponential \( V(\Phi_{cl}) = A \exp(\lambda \Phi_{cl}) \) (for other potentials see 12 and references therein). Inserting into Eq. (2) we find the one loop corrected potential:
\[ V_{1\text{-loop}}^{\text{EP}} = V_{\text{cl}}^{\text{EP}} \left\{ 1 + \frac{1}{32\pi^2} \Lambda^2 \lambda^2 \right\}. \] (3)

The potential is simply multiplied by a field independent constant. It is easy to see that a rescaling \( A \rightarrow A/(1 + \frac{\Lambda^4}{32\pi^2} \lambda^2) \) absorbs the loop correction, leading to a stable potential up to order \( V''_{cl} \). The next to leading order \( (V''_{cl})^2 \) is non trivial but very small since
\[ V'' \sim V \sim H^2 \sim 10^{-120} \] (4)
in the late universe. However, it is this term which spoils the strict renormalizability in four dimensions for the pure exponential potential.

Other potentials are usually not completely form invariant even to lowest order. Nevertheless, keeping in mind that \( \frac{1}{32\pi^2} \approx 0.003 \) and using reasonable cutoffs \( \Lambda \lesssim 1 \) (remember we use units where \( M_P = 1 \)) loop corrections are usually small in the late universe. Therefore, most potentials are stable (for a hand picked exception see 12).

3 Coupled quintessence

Various models featuring a coupling of quintessence to some form of dark matter have been proposed 12,13,14,15,16,18,19,17,19,17. From the action Eq. (1), we see that the mass of the fermions could be \( \Phi \) dependent: \( m_f = m_f(\Phi_{cl}) \). Two possible realization of this mass dependence are for instance

\[ \text{This symmetry must be unbroken or only very slightly broken. E.g. SUSY is broken too badly to do this as has been pointed out in 10.} \]
\[ m_f = m_f^0 \exp(-\beta \Phi_{cl}) \] and \[ m_t = m_t^0 + c(\Phi_{cl}), \] where in the second case, we may have a large field independent part together with small couplings to quintessence. For the model discussed in \[ 4 \text{, the coupling is of the first form, whereas in } 5, \text{ the coupling is realized by multiplying the cold dark matter Lagrangian by a factor } f(\Phi). \] Hence, the coupling is \[ m_f(\Phi) = f(\Phi) m_f^0, \] if we assume that dark matter is fermionic. If it were bosonic, the following arguments would be similar. We will only discuss the new effects coming from the coupling and set \[ V_{1\text{-loop}} = V_{cl} - \Delta V, \] where \[ \Delta V = \Lambda_{\text{ferm}}^2 [m_f(\Phi_{cl})]^2 / (8\pi^2). \] Let us consider the case that all of the fermion mass is field dependent, i.e. we consider cases like \[ m_f = m_f^0 \exp(-\beta \Phi_{cl}). \] The ratio of the ‘correction’ to the classical potential is

\[ \frac{\Delta V}{V_{cl}} = \frac{1}{8\pi^2} \frac{\Lambda_{\text{ferm}}^2 [m_f(\Phi_{cl})]^2}{V_{cl}} \approx 10^{80}. \]  

(5)

here we used a fermion cutoff at the GUT scale \( \Lambda_{\text{ferm}} = 10^{-3} \), and a fermion mass of the order of \( 100 \text{GeV} = 10^{-16} \text{M}_P \) for an estimate. Thus, the classical potential is negligible relative to the correction induced by the fermion fluctuations.

Having made this estimate, it is clear that the fermion loop corrections are only harmless, if the square of the coupling takes on exactly the same form as the classical potential itself. If, for example we have an exponential potential \( V_{cl} = A \exp(-\lambda \Phi_{cl}) \) together with a coupling \( m_f(\Phi_{cl}) = m_f^0 \exp(-\beta \Phi_{cl}) \), then this coupling can only be tolerated, if \( 2\beta = \lambda \). Taken at face value, this finding restricts models with these types of coupling. It is however interesting to note that for the exponential coupling, the case \( 2\beta = \lambda \) is not ruled out by cosmological observations.

Turning to the possibility of a fermion mass that consists of a field independent part and a coupling, i.e. \( m_f = m_f^0 + c(\Phi_{cl}) \), we find by similar reasoning that

\[ c(\Phi_{cl}) \ll 3 \times 10^{-97}, \]  

(6)

is needed in order for the potential to be stable. Once again, the bound from Eq. \[ 6 \] only applies if the functional form of the loop correction differs from the classical potential.

The coupled models share one property: the loop contribution from the coupling is by far larger than the classical potential. At first sight, the golden way out of this seems to view the potential as already effective: all fluctuations would be included from the start. However, there is no particular reason, why any coupling of quintessence to dark matter should produce just exactly the effective potential used in a particular model: there is a relation between a coupling and the effective potential generated. Put another way, if the effective potential is of an elegant form and we have a given coupling, then it seems unlikely that the classical potential could itself be elegant or natural.

Finally let us remark that the restrictions on a coupling to fermions are so severe that we checked whether a gravitationally induced coupling could violate these bounds. However the \( c(\Phi) \)-induced by graviton exchange is always proportional to \( V(\Phi) \) and therefore the correction to the potential resulting from this effective coupling is proportional to \( V(\Phi_{cl}) \). Therefore, it does not change the form of the potential and is harmless.

4 Conclusions

We have calculated quantum corrections to the classical potentials of various quintessence models. In the late universe, most potentials are stable with respect to the scalar quintessence fluctuations.

\footnote{The constant \( m_f^0 \) is not the fermion mass today, which would rather be \( m_{\text{today}} = m_f(\Phi_{cl}(\text{today})) \).}

\footnote{Of course, sufficiently small \( \beta \), will lead to a more or less constant contribution, where \( m_f(\Phi_{cl}) \approx m_f^0 - \beta \Phi_{cl} \).}
An explicit coupling of the quintessence field to fermions (or similarly to dark matter bosons) seems to be severely restricted. The effective potential to one loop level would be completely dominated by the contribution from the fermion fluctuations. All models in the literature share this fate. One way around this conclusion could be to view these potentials as already effective. They must, however, not only be effective in the sense of an effective quantum field theory originating as a low-energy limit of an underlying theory, but also include all fluctuations from this effective QFT. In this case, there is a strong connection between coupling and potential and it is rather unlikely that the correct pair can be guessed.

The bound on the coupling is so severe that for consistency, we have calculated an effective coupling due to graviton exchange. To lowest order in $V(\Phi)$, this coupling leads to a fermion contribution which can be absorbed by redefining the pre-factor of the potential.

Surely, the one-loop calculation does not give the true effective potential. Symmetries or more fundamental theories that make the cosmological constant small as it is, could force loop contributions to cancel. In addition, the back-reaction of the changing effective potential on the fluctuations remains unclear in the one loop calculation. A renormalization group treatment would therefore be of great value. We leave this to future work.

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