Non exponential spin relaxation in magnetic field in quantum wells with random spin-orbit coupling

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We investigate the spin dynamics of electrons in quantum wells where the Rashba type of spin-orbit coupling is present in the form of random nanosize domains. We study the effect of magnetic field on the spin relaxation in these systems and show that the spatial randomness of spin-orbit coupling limits the minimum relaxation rate and leads to a Gaussian time-decay of spin polarization due to memory effects. In this case the relaxation becomes faster with increase of the magnetic field in contrast to the well known magnetic field suppression of spin relaxation.

The effect of magnetic field on spin relaxation in low-dimensional structures in the presence of spin-orbit (SO) coupling is an interesting theoretical and experimental problem. The understanding of this effect which allows to a certain extent the engineering of the spin dynamics in these systems can have a crucial impact on the possible spintronics applications\textsuperscript{1}. The SO coupling of electrons in two-dimensional (2D) zincblende-based systems with the structural asymmetry grown along the [001] direction is described by the Rashba Hamiltonian $H_R = \alpha(\sigma_x k_y - \sigma_y k_x)$ (Ref. \textsuperscript{2}), where $\alpha$ is the coupling constant, $\sigma_i$ are the Pauli matrices, and $k$ is the in-plane wavevector of the electron. The random spin precession necessary for the spin relaxation is introduced by electron scattering by impurities, phonons, other electrons, and due to random dynamics in regular systems\textsuperscript{2}. In the "dirty" limit, $\alpha k \tau / \hbar \ll 1$ with $\tau$ being the momentum relaxation time, the motional narrowing leads to the Dyakonov-Perel' mechanism\textsuperscript{3,4} causing the exponential spin relaxation with the rate of the order of $(\alpha k / \hbar)^2 \tau$. The spin relaxation rate decreases when a magnetic field is applied\textsuperscript{5} due to the effect of the Lorenz force on the orbital movement of the electron. From the analysis of the spin relaxation in a magnetic field valuable information both on spin and charge dynamics can be extracted\textsuperscript{6}. In the clean limit, $\alpha k \tau / \hbar \gg 1$ spin relaxation, on a time scale of the order of $\tau$, occurs on the top of the spin precession\textsuperscript{7}. Various mechanisms of spin relaxation in solids are reviewed in Ref. \textsuperscript{10}.

In most quantum wells (QW) the Rashba-type SO coupling is achieved by asymmetric remote doping at the sides of the well; the same doping forms a smooth random potential scattering electrons. A crucially important step in being able to quickly manipulate the spins has been made by demonstrating that, by applying external bias across the quantum well, it is possible to change the magnitude of $\alpha$\textsuperscript{11,12,13,14,15}. Almost all previous studies assumed the $\alpha$ parameter to be constant in space. However, the evidence is growing that the SO coupling both in zincblende and Si$_x$Ge$_{1-x}$/Si-based QWs\textsuperscript{16,17} is a random function of coordinate. The randomness arises due to imperfections in the system, e.g. either due to shot noise accompanying the doping\textsuperscript{18}, or due to random variations of the bonds at Si/Ge interfaces\textsuperscript{19} in the Si$_x$Ge$_{1-x}$/Si systems considered as a hope for spintronics due to a very small SO coupling there\textsuperscript{18,20}. Therefore, the spin of a moving electron interacts with a randomly time-dependent effective SO field. This randomness leads to an irregular precession of spins and contributes to the spin relaxation rate. Here we concentrate on the systems where the Rashba term dominates in the SO-coupling, like InGaAs-based structures\textsuperscript{12}, and demonstrate new qualitative effects of the randomness on the spin relaxation in a magnetic field such as a possibility of the non-exponential spin relaxation, and an increase of the spin relaxation rate in a strong magnetic field. The effect of the Dresselhaus coupling arising due to the unit cell asymmetry\textsuperscript{20} will be briefly discussed in the context of our results also. The importance of the randomness of electric field of dopants causing the SO coupling on the donor electrons was first demonstrated by Mel'nikov and Rashba\textsuperscript{21} for bulk Si. Recently, Vagner et al.\textsuperscript{22} showed that a random SO coupling for electrons in GaAs mesoscopic rings interacting with nuclear spins can lead to an Aharonov-Bohm-like effect.

Since we are interested in the description of random spin and charge dynamics, we consider as an example a QW of the width $w$ extended between $-w/2 < z < w/2$, and surrounded by two $\delta$-doped layers at $z = \pm z_0$, $z_0 > w/2$, with a mean two-dimensional concentration of dopants $N_u$ (upper layer) and $N_d$ (lower layer) with charge $|e|$. The local SO coupling, being a linear response of the system to the symmetry-breaking perturbation, is presented as $\alpha_{R}(\rho) = \alpha_{SO} |e| E_z(\rho)$, where $E_z(\rho)$ is the $z-$component of electric field of the dopant ions, $\rho$ is the 2D coordinate at the $z = 0$ plane, and $\alpha_{SO}$ is a system-dependent phenomenological parameter. The $z$-and in-plane $i = x, y$ components of the Coulomb field of the dopant ions are given by\textsuperscript{23,24}:

$$ E_z(\rho) = -\frac{|e|}{\epsilon} \sum_j \frac{z_j}{[(\rho - r_{ij})^2 + z_j^2]^{3/2}}, $$

$$ E_i(\rho) = -\frac{|e|}{\epsilon} \int_0^\infty q A_q \frac{\partial}{\partial \rho_i} J_0 (q |\rho - r_{ij}|) dq, \quad (1) $$
where \( \mathbf{r}_j = (r_{j||}, z_j) \), \( r_{j||} = (r_x, r_y) \) is the 2D radius-vector of the \( j \)th dopant ion, \( z_j = \pm z_0 \) for the upper and lower layer, respectively, \( \epsilon \) is the dielectric constant, \( J_0(q\rho) \) is the zero-order Bessel function, and \( A_q = \exp(-qz_0)/(q + q_s) \), where \( q_s = (2\pi e^2/\epsilon) dN/d\varepsilon_F \) describes the Thomas-Fermi screening of the in-plane Coulomb field with \( N = N_a + N_d \) being the electron concentration, and \( \varepsilon_F \) being the Fermi energy. As a result, electrons move in a smooth 2D random potential \( U(\rho) \) with \( E_{\parallel}(\rho) = -\nabla_{\parallel} U(\rho) \), with the mean value \( \langle U \rangle = 0 \).

We consider the semiclassical movement of the electron in external magnetic field \( \mathbf{H} \):

\[
\frac{\hbar}{m} \frac{d\mathbf{k}}{dt} = eE_{\parallel}(\rho) + \frac{\hbar e}{mc} \mathbf{k} \times \mathbf{H},
\]

where \( m \) is the electron effective mass. The momentum relaxation time for the electron moving in a smooth random potential is given by

\[
\frac{1}{\tau} = \frac{1}{m^2 v_F^2} \int_0^\infty \frac{dp}{\rho} \frac{dC_{UU}(\rho)}{dp},
\]

where \( C_{UU}(\rho) = \langle U(\rho)U(0) \rangle \) is the correlation function of the random potential, and \( v_F = \hbar k_F/m \) is the Fermi velocity corresponding to the Fermi wavevector \( k_F = \sqrt{2\pi N} \). The relaxation time can be estimated as \( \tau \sim \tau_d \varepsilon_F^2 \langle U^2 \rangle \), the potential fluctuation \( \langle U^2 \rangle \sim N (\varepsilon_F^2/\varepsilon q \varepsilon z_0^2) \), and \( \tau_d \) is the time of passing through one domain of random force and random SO coupling, with typical size \( z_0 \) in this case. In InGaAs based structures with \( m = 0.02m_0 \), where \( m_0 \) is the bare electron mass, \( q_s \) is close to \( 2 \times 10^9 \) cm\(^{-1}\). For numerical estimates we use the parameters from Refs. \cite{13,22} with \( N_a = 10^{11} \) cm\(^{-2}\) and \( N_d = 3 \times 10^{11} \) cm\(^{-2}\). Then the ratio \( \varepsilon_F^2/\langle U^2 \rangle \) is of the order of 100 leading to \( \tau \approx 2 \tau_d \) of magnitude longer than \( \tau_d \approx 0.3 \) ps. In weak fields \( \omega_c \tau \ll 1 \), where \( \omega_c = |e|H/mc \) is the cyclotron frequency, the electron path is random, while at \( \omega_c \tau > 1 \) it becomes close to the regular circular movement in magnetic field with a random path of the orbit center. At \( m = 0.02m_0 \) the crossover from the random to the regular movement occurs at \( H \approx 0.1 \) T. The semiclassical description of the orbital movement is possible in a non-quantizing field up to \( H \approx 1 \) T, when \( h \omega_c/\varepsilon_F \approx 0.1 \).

The Rashba parameter is the sum of the mean value \( \langle \alpha \rangle = 2\pi \sigma_{SO} e^2 (N_d - N_a)/\epsilon \), and a random term with the zero mean contribution: \( \alpha(\rho) = \langle \alpha \rangle + \delta \alpha(\rho) \) and the variation

\[
\langle \delta \alpha \rangle^2 = \langle \alpha \rangle^2 = \frac{N}{8\pi (N_d - N_a)^2} z_0^2.
\]

The random and regular contributions become comparable at \( N z_0^2 \lesssim 0.1 \); the condition satisfied in most of the experimentally investigated QWs. The regular term \( \langle \alpha \rangle \) can be reduced or completely removed by applying an electric bias across the quantum well, but even in this case the random term \( \delta \alpha(\rho) \) remains and causes spin relaxation. Figure 1 presents a pattern of \( \alpha_R(\rho) \) obtained by a Monte-Carlo produced white-noise distribution of dopant ions corresponding to the experimental data of Ref. \cite{12}. As can be seen in Fig. 1, the random variations of \( \alpha_R(\rho) \) are large and correlated on the distances of the order of \( z_0 \). Therefore, the pattern of SO coupling is random nanosize domains rather than a constant.

![FIG. 1: (Color online) A realization of the random Rashba SO coupling (in arbitrary units) due to fluctuations of the dopant concentration, \( N_a = 10^{11} \) cm\(^{-2}\), \( N_d = 3 \times 10^{11} \) cm\(^{-2}\), and \( z_0 = 12 \) nm.](image-url)
In high-mobility structures the smooth random potential of dopant ions causes two main sources of randomness leading to a spin relaxation: (i) random $\alpha(\rho)$ due to fluctuations of the electric field $E_z$ and (ii) random direction of $k$. The former contribution causes the randomness in the amplitude of the SO field and, in turn, the precession rate, on the time scale $\tau_d$, while the latter causes the random orientation of the precession axis on the much longer time scale $\tau$. These effects determine the correlator in Eq. 4.

To take into account different possible mechanisms of randomness of the SO coupling (fluctuations in the dopant concentrations, randomness of the bonds at the interfaces, variations of the directions of the growth axis, random strain, etc.) we consider a simple model with

$$\langle \delta \alpha(\rho) \delta \alpha(0) \rangle = \langle (\delta \alpha)^2 \rangle e^{-\rho/l_d}, \quad (7)$$

where $l_d = \tau_d e$ is the characteristic domain size of the order of 10 nm. The details of the shape of the correlator in Eq. 4 have no qualitative influence on our results.

One can show that the correlator of the effective Larmor frequencies in Eq. 3 is given by

$$C_{\Omega l}(t) = \frac{4k^2}{\hbar^2} C_{kk}(t) \times \left[ \langle \alpha \rangle^2 + \langle (\delta \alpha)^2 \rangle \exp \left( -\frac{2}{\omega_c \tau_d} \sin \frac{\omega_c t}{2} - \frac{\delta r}{l_d} \right) \right] \quad (8)$$

Here $\delta r$ is the displacement of the center of cyclotron orbit due to drift in the random potential $\Omega/k$, and $C_{kk}(t) = \cos \omega_c t e^{-t/\tau}$ is the electron momentum correlator in the magnetic field. The first term in the square brackets gives the contribution to the spin dynamics due to regular SO coupling, the second one stands for the contribution from random fluctuations of $\alpha$. In Eq. 8 the correlations between the fluctuations of SO coupling and momentum are neglected since they have drastically different time scales $\tau_d$ and $\tau$, respectively.

To begin we consider the case of a not very strong field $\omega_c \tau_d \ll 1$, where $\delta r \gg l_d$. In the collision dominated regime $\langle \alpha \rangle kT/\hbar \ll 1$, spin decays exponentially and the relaxation rate obtained with Eqs. 4 and 7 is given by $\Gamma_{zz} = 1(z) + \Gamma(z)$, where

$$\Gamma(z) = \frac{4\langle (\delta \alpha)^2 \rangle k^2}{\hbar^2 \tau_d}, \quad \Gamma(z) = \frac{4\langle \alpha \rangle^2 k^2 \tau}{\hbar^2 (1 + \omega_c^2 \tau^2)}. \quad (9)$$

At $H = 0$ both terms are the product of the typical spin-splitting squared and correlation time corresponding to the D’yakonov-Perel’ relaxation mechanism. Here the electron passes many domains moving along the straight path (line 1 in Fig. 1), the spin splitting in each domain is different and the time scale of the variations of the effective Larmor field $\Omega[k(t), \rho(t)]$ is $\tau_d$. In a magnetic field $\omega_c^2 \tau^2 \gg 1$, in the case of the regular SO coupling, the effective Larmor frequency $\Omega[k(t + T/2)] = -\Omega[k(t)]$ is reversed each half of the cyclotron period on the circular trajectory causing spin precession backwards, and the relaxation slows down. It is not the case when the SO constant fluctuates in space and $\langle \alpha \rangle = 0$: if the cyclotron radius is large enough ($\omega_c \tau_d \ll 1$), the random contributions to $\Omega[k(t), \rho(t)]$ are not correlated, the reversal of precession does not take place making the relaxation rate $H$-independent. We emphasize that the source of randomness here is not the momentum scattering but the random fluctuations of the spin-orbit constant. A numerical estimate at $\sqrt{\langle (\delta \alpha)^2 \rangle} = 1.5 \times 10^{-10}$ eVcm and experimental conditions of Ref. 13 gives $1/\Gamma_{zz} \sim 60$ ps.

To understand the qualitatively different situation when the magnetic field becomes so strong that after a cyclotron period electron returns to the same domain of spin-splitting, we consider the ‘compensated’ case with $\langle \alpha \rangle = 0$. For the particle in a smooth potential the condition for such a ballistic regime is $\langle \rho(t + T) - \rho(t) \rangle = \delta r < l_d$, $T = 2\pi/\omega_c$, which can be achieved at $\omega_c \tau \gg (\tau/\tau_d)^{2/3}$. This condition can be fulfilled in non-quantizing fields at $\tau \gg 10$ ps, corresponding to high mobilities $\mu \gtrsim 10^6$ cm$^2$/Vs. In such a case on each cyclotron period the electron passes exactly the same configuration $\alpha(\rho)$ (see circle (2) in Fig. 1). It means that the effective correlation time of Larmor frequency would be much longer than $\tau_d$ and the spin relaxation becomes faster in a strong magnetic field. This result is in a sharp contrast to the well-known magnetic field suppression of the spin relaxation in Eq. 4. During the first cyclotron revolution the spin dynamics is the same as in the absence of a magnetic field. For subsequent revolutions the spin rotates by the same angle as during the first one because the electron passes the same configuration of $\alpha(\rho)$. This memory effect enhances the spin relaxation rate on each cycle and makes the spin relaxation non-exponential, namely

$$\frac{\langle s_z(t) \rangle}{\langle s_z(0) \rangle} = \exp \left\{ -\Gamma_{zz}[K^2 T + (1 + 2K)(t - KT)] \right\}, \quad (10)$$

with $K = [t/T]$ and $[\ldots]$ standing for the integer part.

It is interesting to note that Cremers et al. found that the effect of non-uniform SO coupling on weak localization in quantum dots can be interpreted as a speeding up of spin relaxation. The cusps in $\langle s_z(t) \rangle$ at $t = nT$ are smoothed out if one takes into account the drift of the cyclotron orbit $\delta r$ in the random potential. At $t \gg T$ we obtain $\langle s_z(t) \rangle \sim \exp \left\{ -\Gamma_{zz} t^2/T \right\}$. Figure 2 shows the calculated time dependence of the $\langle s_z \rangle$ for different values of the external magnetic field.

We note that in the high-mobility structures the collision dominated regime can be violated and spin relaxation occurs on the top of spin precession making Eq. 4 invalid. However in strong magnetic field $\omega_c \gtrsim \langle \alpha k/h \rangle$ the applicability of Eq. 4 can be justified, besides, the regular contribution to the spin relaxation is strongly suppressed by the magnetic field. Therefore the spin relaxation rate drops to its residual value of the order of $\Gamma_{zz}$ solely determined by the fluctuations of ran-
The further increase of magnetic field to $\omega_c \tau \sim (\tau_0/\tau)^{2/3}$ makes the spin relaxation non-exponential (see Eq.(10)) with the characteristic decay time $\tau_s \sim (T/\Gamma_{zz})^{1/2}$. Finally, we mention that the addition of the regular Dresselhaus SO coupling will not change our main results since it is suppressed by the magnetic field in the same way as the Rashba contribution, and, therefore, only the effect of a random Rashba coupling remains in a sufficiently strong magnetic field.

To conclude, we have shown that the inevitable randomness of SO coupling has an important effect on the spin dynamics in 2D structures, e.g. it determines the minimum spin relaxation rate achieved in a magnetic field. In the ballistic regime, achieved in sufficiently strong magnetic fields, the domain pattern of SO coupling leads to the non-exponential decay of spin polarization being a manifestation of a spin memory effect. The spin relaxation in a system with random SO-coupling becomes faster with the increase of the magnetic field. These effects limit possibilities of spin manipulation. The predicted magnetic field dependence of the spin relaxation can be observable in 2D structures with $\mu \gtrsim 10^6$ cm$^2$/Vs, where the "ballistic" regime is achievable in non-quantizing fields.

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