Exploring CP Violation through B Decays

Robert Fleischer

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract

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EXPLORING CP VIOLATION THROUGH B DECAYS

ROBERT FLEISCHER
Theory Division, CERN
CH-1211 Geneva 23, Switzerland
Robert.Fleischer@cern.ch

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The $B$-meson system provides many strategies to perform stringent tests of the Standard-Model description of CP violation. In this brief review, we discuss implications of the currently available $B$-factory data on the angles $\alpha$, $\beta$ and $\gamma$ of the unitarity triangle, emphasize the importance of $B_s$ studies at hadronic $B$ experiments, and discuss new, theoretically clean strategies to determine $\gamma$.

Keywords: CP violation; unitarity triangle; non-leptonic $B$ decays.

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1. Introduction

The discovery of CP violation through the observation of $K_L \to \pi^+\pi^-$ decays in 1964 came as a big surprise. This particular kind of CP violation, which is referred to as “indirect” CP violation and is described by a complex quantity $\varepsilon_K$, originates from the fact that the $K_L$ mass eigenstate is not a pure eigenstate of the CP operator with eigenvalue $-1$, but one that receives a tiny admixture of the CP eigenstate with eigenvalue $+1$. Another milestone in the exploration of CP violation through neutral kaon decays came in 1999, when also “direct” CP violation, i.e. CP-violating effects arising directly at the amplitude level, could be established by the NA48 (CERN) and KTeV (FNAL) collaborations through a measurement of a non-vanishing value of $\text{Re}(\varepsilon'_K/\varepsilon_K)$. Unfortunately, this observable does not provide a stringent test of the Standard-Model description of CP violation, unless significant theoretical progress concerning the relevant hadronic matrix elements can be made.

One of the phenomenologically most exciting topics in this decade is the exploration of decays of $B$ mesons, which offer various powerful tests of the CP-violating sector of the Standard Model (SM) and provide, moreover, valuable insights into hadron dynamics. The experimental stage is now governed by the asymmetric $e^+e^-$ $B$ factories operating at the $\Upsilon(4S)$ resonance, with their detectors BaBar (SLAC) and Belle (KEK). These experiments have already established CP violation in the $B$-meson system in 2001, which is the beginning of a new era in the exploration of
Fig. 1. The two non-squashed unitarity triangles of the CKM matrix, where (a) and (b) correspond to $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ and $V_{ub}^*V_{tb} + V_{us}V_{ts} + V_{ud}^*V_{td} = 0$, respectively.

CP violation. Many interesting strategies can now be confronted with data. In the near future, also run II of the Tevatron is expected to contribute significantly to this programme, providing – among other things – first access to $B_s$-meson decays.

In the era of the LHC, these decay modes can then be fully exploited, in particular at LHCb (CERN) and BTeV (FNAL).

Within the framework of the SM of electroweak interactions, CP violation can be accommodated through a complex phase in the Cabibbo–Kobayashi–Maskawa (CKM) matrix, as pointed out by Kobayashi and Maskawa in 1973. Since the CKM matrix is unitary, we obtain six orthogonality relations, which can be represented in the complex plane as six triangles. Applying the Wolfenstein parametrization, we find that there are only two phenomenologically interesting triangles, where all three sides are of the same order in $\lambda \equiv |V_{us}| = 0.22$. If we divide all sides by $A \equiv |V_{cb}|/\lambda^2$, we obtain the triangles illustrated in Fig. 1 where $\mathfrak{p}$ and $\mathfrak{q}$ are related to $\rho$ and $\eta$ through $\mathfrak{p} \equiv (1 - \lambda^2/2) \rho$ and $\mathfrak{q} \equiv (1 - \lambda^2/2) \eta$, respectively, and $\delta \gamma \equiv \gamma - \gamma' = \lambda^2 \eta$. Whenever we refer to a unitarity triangle (UT) in the following discussion, we mean the one shown in Fig. 1 (a).

The main goal is now to overconstrain the UT as much as possible, with the hope to encounter discrepancies, which may shed light on new physics (NP). To this end, we may, on the one hand, obtain indirect information on the UT angles $\alpha$, $\beta$ and $\gamma$ through measurements of the UT sides $R_b$ and $R_t$: the former can be fixed through exclusive and inclusive semileptonic $B$ decays caused by $b \to u\ell \nu\ell$ quark-level transitions, whereas $R_t$ can be determined, within the SM, with the help of $B_0^q$-$\bar{B}_0^q$ mixing ($q \in \{d, s\}$). Finally, using the SM interpretation of the indirect CP violation in the neutral kaon system, which is measured by $\varepsilon_K$, we may fix a hyperbola in the $\mathfrak{p}$-$\mathfrak{q}$ plane. Many different strategies to deal with the corresponding theoretical and experimental uncertainties can be found in the literature; a detailed discussion of these approaches is beyond the scope of this brief review, and was recently given in Ref. 14. Let us here just list typical ranges for $\alpha$, $\beta$ and $\gamma$ that follow from such “CKM fits”:

$$70^\circ \lesssim \alpha \lesssim 130^\circ, \quad 20^\circ \lesssim \beta \lesssim 30^\circ, \quad 50^\circ \lesssim \gamma \lesssim 70^\circ.$$ (1)
If we measure, on the other hand, CP-violating effects in $B$-meson decays, we may obtain direct information on the UT angles $\alpha$, $\beta$ and $\gamma$, as well as on $\delta\gamma$. This topic will be the focus of the following considerations.

In Section 2, we shall classify the main strategies to explore CP violation through $B$ decays. The implications of the currently available $B$-factory data for various benchmark modes will then be discussed in Section 3, whereas we shall focus on the $B_s$-meson system – the “El Dorado” for $B$-physics studies at hadron colliders – in Section 4. In Section 5, we turn to new, theoretically clean strategies to extract $\gamma$, and finish in Section 6 by summarizing our conclusions and giving a brief outlook.

2. Classification of the Main Strategies

In the exploration of CP violation through $B$ decays, non-leptonic transitions play the key rôle, since CP asymmetries may there be induced through certain interference effects. In particular, if the decay receives contributions from two amplitudes with non-trivial weak and strong phase differences, we obtain direct CP violation. Unfortunately, hadronic matrix elements of local four-quark operators, which are very hard to estimate in a reliable manner, enter the calculation of such CP asymmetries, thereby precluding a clean extraction of the weak phase differences, which are related to the angles of the UT and are usually given by $\gamma$. Nevertheless, interesting progress could recently be made in this very challenging direction through the development of the QCD factorization formalisms and perturbative hard-scattering (PQCD) formalisms, QCD light-cone sum-rule approaches, and soft collinear effective theory (SCET), as reviewed in detail in Ref. 22.

In order to extract solid information on the angles of the UT from CP-violating asymmetries, the theoretical input on hadronic matrix elements should be reduced as much as possible. Such strategies, allowing in particular the determination of $\gamma$, are provided by fortunate cases, where we may eliminate the hadronic matrix elements through relations between different decay amplitudes: we distinguish between exact relations, involving pure tree-diagram-like decays of the kind $B \rightarrow DK$ or $B_c \rightarrow DD_s$, and relations, which follow from the flavour symmetries of strong interactions, involving $B_{(s)} \rightarrow \pi\pi, \pi K, KK$ decays.

If we employ decays of neutral $B_{d}$ or $B_{s}$ mesons, another avenue to deal with the problems arising from hadronic matrix elements emerges. It is offered by a new kind of CP violation, which is referred to as mixing-induced CP violation, and is due to interference effects between $B_{q}^{0} - \overline{B}_{q}^{0}$ ($q \in \{d, s\}$) mixing and decay processes. In the time-dependent rate asymmetry

$$\frac{\Gamma(B_{q}^{0}(t) \rightarrow f) - \overline{\Gamma}(\overline{B}_{q}^{0}(t) \rightarrow f)}{\Gamma(B_{q}^{0}(t) \rightarrow f) + \overline{\Gamma}(\overline{B}_{q}^{0}(t) \rightarrow f)} = \left[ \frac{A_{\text{dir}}(B_{q} \rightarrow f) \cos(\Delta M_{q} t) + A_{\text{mix}}(B_{q} \rightarrow f) \sin(\Delta M_{q} t)}{\cosh(\Delta \Gamma_{q} t/2) - A_{\Delta \Gamma}(B_{q} \rightarrow f) \sinh(\Delta \Gamma_{q} t/2)} \right],$$

where $\Delta M_{q} \equiv M_{H}^{(q)} - M_{L}^{(q)}$ and $\Delta \Gamma_{q} \equiv \Gamma_{H}^{(q)} - \Gamma_{L}^{(q)}$ are the mass and decay widths.
differences of the $B_q$ mass eigenstates ("heavy" and "light"), respectively, and $(CP)|f⟩ = ±|f⟩$, this road shows up in the form of the coefficient of the $\sin(\Delta M_q t)$ term, whereas the one of $\cos(\Delta M_q t)$ measures the direct CP-violating effects discussed above. If the decay $B_q \to f$ is dominated by a single CKM amplitude, the corresponding hadronic matrix element cancels in $A_{\text{mix}}^\text{CP}(B_q \to f)$. This observable is then simply given by $\pm \sin(\phi_q - \phi_f)$, where $\phi_f$ and $\phi_q$ are the weak $B_q \to f$ decay and $B^0_q \to \bar{B}^0_q$ mixing phases, respectively. Within the SM, the former phases are induced by the CKM matrix elements entering the $B_q \to f$ decay amplitude, whereas the latter are related to the famous box diagrams mediating $B^0_q \to \bar{B}^0_q$ mixing and are given as follows:

$$\phi_q = 2\text{arg}(V_{tb}^* V_{tb}) = \begin{cases} +2\beta = \mathcal{O}(50^\circ) & (q = d) \\ -2\delta\gamma = \mathcal{O}(-2^\circ) & (q = s). \end{cases}$$

(3)

Mixing-induced CP violation plays a key rôle for several benchmark modes, and is also a powerful ingredient to complement analyses of $B$ decays, which are related to one another through amplitude relations of the kind discussed above.

3. Status of Benchmark Modes

3.1. $B \to J/\psi K$

One of the most famous $B$ decays, the "gold-plated" mode $B^0_d \to J/\psi K_S$ to extract $\sin 2\beta$, originates from $b \to c\bar{c}s$ quark-level processes. If we look at the corresponding Feynman diagrams arising within the SM, we observe that it receives contributions both from tree and from penguin topologies, and that the decay amplitude takes the following form:

$$A(B^0_d \to J/\psi K_S) \propto \left[1 + \lambda^2 ae^{i\phi} e^{-i\gamma}\right],$$

(4)

where the hadronic parameter $ae^{i\phi}$ measures, sloppily speaking, the ratio of the penguin to tree contributions. Since this parameter enters in a doubly Cabibbo-suppressed way, and is na"{i}vely expected to be of $\mathcal{O}(\lambda)$, where $\lambda = \mathcal{O}(\lambda) = \mathcal{O}(0.2)$, we eventually arrive at

$$A_{\text{mix}}^\text{CP}(B_d \to J/\psi K_S) = -\sin \phi_d + \mathcal{O}(\lambda^3).$$

(5)

In 2001, the $B^0_d \to J/\psi K_S$ decay and similar modes led to the observation of CP violation in the $B$-meson system by the BaBar and Belle collaborations. The present status of $\sin \phi_d^{\text{SM}} \equiv \sin 2\beta$ is given as follows:

$$\sin 2\beta = \begin{cases} 0.741 \pm 0.067 \pm 0.033 & (\text{BaBar}) \\ 0.719 \pm 0.074 \pm 0.035 & (\text{Belle}) \end{cases},$$

(7)

yielding the world average

$$\sin 2\beta = 0.734 \pm 0.054,$$

(8)
which agrees well with the results of the “CKM fits” of the UT summarized in \( \langle 1 \rangle \), implying \( 0.6 \lesssim \sin 2\beta \lesssim 0.9 \).

In the LHC era, the experimental accuracy of the measurement of \( \sin 2\beta \) may be increased by one order of magnitude. Such a tremendous experimental accuracy would then require deeper insights into the \( \mathcal{O}(\lambda^3) \) corrections affecting \( \langle 6 \rangle \), which are caused by penguin effects. A possibility to control them is provided by the \( B_s \to J/\psi K_S \) channel. Moreover, as can be seen in \( \langle 5 \rangle \), also direct CP violation in \( B_d \to J/\psi K_S \) allows us to probe these effects. So far, there are no experimental indications for a non-vanishing value of \( A_{\text{dir}}^{CP}(B_d \to J/\psi K_S) \).

The agreement between the world average \( \langle 8 \rangle \) and the results of the “CKM fits” is striking. However, it should not be forgotten that NP may nevertheless hide in \( A_{\text{mix}}^{CP}(B_d \to J/\psi K_S) \). The point is that the key quantity is actually the \( B_0^d - B_0^d \) mixing phase \( \phi_d \) itself; we obtain the twofold solution

\[
\phi_d = (47^{+5}_{-4})^\circ \lor (133^{+4}_{-5})^\circ, \tag{9}
\]

where the former value is in perfect agreement with \( 40^\circ \lesssim 2\beta_{\text{SM}} \equiv \phi_d \lesssim 60^\circ \), which is implied by the “CKM fits”, whereas the latter would correspond to NP. The two solutions can obviously be distinguished through a measurement of the sign of \( \cos \phi_d \).

Unfortunately, their practical implementations are rather challenging. One of the most accessible approaches employs the time-dependent angular distribution of the \( B_d \to J/\psi[\to \ell^+\ell^-]K^*[\to \pi^0 K_S] \) decay products, allowing us to extract \( \text{sgn}(\cos \phi_d) \), if we fix the sign of a hadronic parameter \( \cos \delta_f \), which involves a strong phase \( \delta_f \), through factorization. This analysis is already in progress at the \( B \) factories. For hadron colliders, the \( B_d \to J/\psi \rho^0 \), \( B_s \to J/\psi \phi \) system is very interesting to probe the sign of \( \cos \phi_d \). As we will see in Subsection 4.3, there is an interesting connection between the two solutions for \( \phi_d \) in \( \langle 5 \rangle \) and constraints on \( \gamma \), which is offered through CP violation in \( B_d \to \pi^+\pi^- \) decays.

The preferred mechanism for NP to enter the CP-violating effects of the “gold-plated” \( B_d \to J/\psi K_S \) channel is through \( B_0^d - B_0^d \) mixing. However, NP may, in principle, also enter at the \( B \to J/\psi K \) amplitude level. Estimates borrowed from effective field theory suggest that these effects are at most \( \mathcal{O}(10\%) \) for a generic NP scale \( \Lambda_{\text{NP}} \) in the TeV regime; in order to obtain the whole picture, a set of appropriate observables can be introduced, employing \( B_d \to J/\psi K_S \) and its charged counterpart \( B^\pm \to J/\psi K^\pm \). These observables are already severely constrained through the \( B \)-factory data, and do not signal any deviation from the SM.

### 3.2. \( B \to \phi K \)

Decays of the kind \( B \to \phi K \), which originate from \( \bar{b} \to \bar{s}s\bar{s} \) quark-level transitions, provide another important testing ground for the SM description of CP violation. These modes are governed by QCD penguins but also their EW penguin contributions are sizeable. Since such penguin topologies are absent at the tree level...
in the SM, $B \rightarrow \phi K$ decays represent a sensitive probe to search for NP effects. Within the SM, we obtain the following relations: 51, 55–57

\[ A_{\text{dir}}(B_d \rightarrow \phi K_S) = 0 + \mathcal{O}(\lambda^2) \] (10)
\[ A_{\text{mix}}(B_d \rightarrow \phi K_S) = A_{\text{mix}}^{\text{SM}}(B_d \rightarrow J/\psi K_S) + \mathcal{O}(\lambda^2). \] (11)

As in the case of the $B \rightarrow J/\psi K$ system, 42 a combined analysis of $B_d \rightarrow \phi K_S$ and its charged counterpart $B^{\pm} \rightarrow \phi K^{\pm}$ should be performed in order to obtain the whole picture. 57 There is also the possibility of an unfortunate case, where NP cannot be distinguished from the SM. 5, 57

The present experimental status of CP violation in $B_d \rightarrow \phi K_S$ is given as follows:

\[ A_{\text{dir}}(B_d \rightarrow \phi K_S) = \begin{cases} -0.80 \pm 0.38 \pm 0.12 \text{ (BaBar)}^{58} \\ +0.56 \pm 0.41 \pm 0.16 \text{ (Belle)}^{59} \end{cases} \] (12)
\[ A_{\text{mix}}(B_d \rightarrow \phi K_S) = \begin{cases} +0.18 \pm 0.51 \pm 0.07 \text{ (BaBar)}^{58} \\ +0.73 \pm 0.64 \pm 0.22 \text{ (Belle)}^{59} \end{cases}. \] (13)

Since we have, on the other hand, $A_{\text{mix}}^{\text{SM}}(B_d \rightarrow J/\psi K_S) = -0.734 \pm 0.054$, we arrive at a puzzling situation, which has already stimulated many speculations about NP effects in $B_d \rightarrow \phi K_S$. However, because of the large experimental uncertainties and the recently reported results for the direct CP asymmetries in (12), it is probably too early to get too excited by the possibility of having large NP contributions to the $B_d \rightarrow \phi K_S$ decay amplitude. Moreover, a recent BaBar analysis of direct CP violation in $B^{\pm} \rightarrow \phi K^{\pm}$ and $B \rightarrow \phi K^*$ transitions does not signal any effect. 61

It will be very interesting to observe the evolution of the $B$-factory data, also on $B_d \rightarrow \eta^{'}/K_S$ and other related modes.

### 3.3. $B \rightarrow \pi\pi$

The $B^0_d \rightarrow \pi^+\pi^-$ channel is another prominent $B$-meson transition, originating from $\bar{b} \rightarrow \bar{u}d$ quark-level processes. In the SM, we may write

\[ A(B^0_d \rightarrow \pi^+\pi^-) \propto [e^{i\gamma} - de^{i\theta}], \] (14)

where the CP-conserving strong parameter $de^{i\theta}$ measures the ratio of the penguin to tree contributions. 62 In contrast to the $B^0_d \rightarrow J/\psi K_S$ amplitude 4, this parameter does not enter (14) in a doubly Cabibbo-suppressed way, thereby leading to the well-known “penguin problem” in $B_d \rightarrow \pi^+\pi^-$. If we had negligible penguin contributions, i.e. $d = 0$, the corresponding CP-violating observables were simply given as follows:

\[ A_{\text{dir}}(B_d \rightarrow \pi^+\pi^-) = 0, \quad A_{\text{mix}}(B_d \rightarrow \pi^+\pi^-) = \sin(\phi_d + 2\gamma)^{\text{SM}} = -\sin 2\alpha \] (15)

where we have used the SM expression $\phi_d = 2\beta$ and the unitarity relation $2\beta + 2\gamma = 2\pi - 2\alpha$ in the last identity. We observe that actually $\phi_d$ and $\gamma$ enter directly $A_{\text{mix}}(B_d \rightarrow \pi^+\pi^-)$, and not $\alpha$. Consequently, since $\phi_d$ can be fixed straightforwardly
through $B_d \to J/\psi K_S$, we may use $B_d \to \pi^+\pi^-$ to probe $\gamma$. \cite{39,62}\cite{63} Measurements of the CP-violating $B_d \to \pi^+\pi^-$ observables are already available:

\begin{equation}
A_{\text{dir}}^{\text{CP}}(B_d \to \pi^+\pi^-) = \begin{cases} -0.30 \pm 0.25 \pm 0.04 \text{ (BaBar)} \cite{64} \\ -0.77 \pm 0.27 \pm 0.08 \text{ (Belle)} \ cite{65} \end{cases} \ (16)
\end{equation}

\begin{equation}
A_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-) = \begin{cases} -0.02 \pm 0.34 \pm 0.05 \text{ (BaBar)} \ cit{64} \\ +1.23 \pm 0.41^{+0.07}_{-0.08} \text{ (Belle)} \ cite{65} \end{cases} \ (17)
\end{equation}

The BaBar and Belle results are unfortunately not fully consistent with each other. If we form, nevertheless, the weighted averages of (16) and (17), applying the rules of the Particle Data Group (PDG), we obtain

\begin{equation}
A_{\text{dir}}^{\text{CP}}(B_d \to \pi^+\pi^-) = -0.51 \pm 0.19 \ (0.23) \ (18)
\end{equation}

\begin{equation}
A_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-) = +0.49 \pm 0.27 \ (0.61), \ (19)
\end{equation}

where the errors in brackets are those increased by the PDG scaling-factor procedure.\cite{66} Direct CP violation at this level would require large penguin contributions with large CP-conserving strong phases, which are not suggested by the QCD factorization approach, pointing towards $A_{\text{dir}}^{\text{CP}}(B_d \to \pi^+\pi^-) \sim +0.1$.\cite{11} In addition to (18), a significant impact of penguins on $B_d \to \pi^+\pi^-$ is also indicated by data on the $B \to \pi K, \pi\pi$ branching ratios,\cite{39,63} as well as by theoretical considerations.\cite{17,67}

Consequently, it is already evident that we must take the penguin contributions to $B_d \to \pi^+\pi^-$ into account in order to extract information on the UT from the corresponding CP asymmetries. Many approaches to address this challenging problem were proposed.\cite{17,39,68} We shall return to this issue in Subsection 4.3, focusing on an approach to complement $B_d \to \pi^+\pi^-$ with the $B_s \to K^+K^-$ channel.\cite{62}

### 3.4. $B \to \pi K$

Decays of this kind originate from $\bar{b} \to \bar{d}s, \bar{u}s$ quark-level transitions, and may receive contributions both from penguin and from tree topologies, where the latter are associated with the UT angle $\gamma$. Because of the tiny value of the CKM factor $|V_{us}V_{ub}^*/(V_{ts}V_{tb}^*)| \approx 0.02$, $B \to \pi K$ modes are interestingly dominated by QCD penguins, despite the loop suppression of these topologies. As far as electroweak (EW) penguins are concerned, their effects are expected to be negligible in the case of the $B_d^0 \to \pi^-K^+, B^+ \to \pi^+K^0$ system, as they contribute here only in colour-suppressed form. On the other hand, EW penguins may also contribute in colour-allowed form to $B^+ \to \pi^0K^+$ and $B_d^0 \to \pi^0K^0$, and are hence expected to be sizeable in these modes, i.e. of the same order of magnitude as the tree topologies.

Interference effects between tree and penguin amplitudes allow us to probe $\gamma$, where we may eliminate hadronic matrix elements with the help of the flavour symmetries of strong interactions. As a starting point, we may use an isospin relation, suggesting the following $B \to \pi K$ combinations to determine $\gamma$: the “mixed”
observables, which are given as follows:

The “charged” $B^\pm \to \pi^\pm K^\mp$ system,\textsuperscript{27} 30 and the “neutral” $B_d \to \pi^0 K$, $B_d \to \pi^\mp K^\pm$ system.\textsuperscript{31} 33

All three $B \to \pi K$ systems can be described by the same set of formulae by just making straightforward replacements of variables.\textsuperscript{32} Let us first focus on the charged and neutral $B \to \pi K$ systems. In order to determine $\gamma$ and the corresponding strong phases, we have to introduce appropriate CP-conserving and CP-violating observables, which are given as follows:

\[
\begin{align*}
\{ R_c \} & = 2 \left[ \frac{\text{BR}(B^\mp \to \pi^0 K^\mp) \pm \text{BR}(B^- \to \pi^0 K^-)}{\text{BR}(B^\mp \to \pi^0 K^0) + \text{BR}(B^- \to \pi^- K^-)} \right] \\
\{ A_0^{n} \} & = 2 \left[ \frac{\text{BR}(B_d^{0} \to \pi^- K^+) \pm \text{BR}(B_d^{0} \to \pi^+ K^-)}{\text{BR}(B_d^{0} \to \pi^0 K^0) + \text{BR}(B_d^{0} \to \pi^0 K^0)} \right].
\end{align*}
\]

For the parametrization of these observables, we employ the isospin relation mentioned above, and assume that certain rescattering effects are small, which is in accordance with the QCD factorization picture.\textsuperscript{15} 17 Anomalously large rescattering processes would be indicated by data on $B \to KK$ modes, which are already strongly constrained by the $B$ factories, and could in principle be included through more elaborate strategies,\textsuperscript{30} 32 which are, however, beyond the scope of this brief review. Following these lines, we obtain

\[
\begin{align*}
R_{c,n} & = 1 - 2r_{c,n} (\cos \gamma - q) \cos \delta_{c,n} + (1 - 2q \cos \gamma + q^2) r_{c,n} \\
A_0^{n} & = 2r_{c,n} \sin \delta_{c,n} \sin \gamma,
\end{align*}
\]

where the parameters $r_{c,n}$, $q$ and $\delta_{c,n}$ have the following physical interpretation: $r_{c,n}$ is a measure for the ratio of tree to penguin topologies, and can be fixed through $SU(3)$ arguments and data on $B^\pm \to \pi^\pm \pi^0$ modes.\textsuperscript{32} yielding $r_{c,n} \sim 0.2$. On the other hand, $q$ describes the ratio of EW penguin to tree contributions, and can be determined through $SU(3)$ arguments, yielding $q \sim 0.7$.\textsuperscript{31} Finally, $\delta_{c,n}$ is the CP-conserving strong phase between the tree and penguin amplitudes.

We observe that $R_{c,n}$ and $A_0^{n}$ depend on only two “unknown” parameters, $\delta_{c,n}$ and $\gamma$. If we vary them within their allowed ranges, i.e. $-180^\circ \leq \delta_{c,n} \leq +180^\circ$ and $0^\circ \leq \gamma \leq 180^\circ$, we obtain an allowed region in the $R_{c,n} - A_0^{n}$ plane.\textsuperscript{3} 34 Should the measured values of $R_{c,n}$ and $A_0^{n}$ lie outside this region, we would immediately have a signal for NP. On the other hand, should the measurements fall into the allowed range, $\gamma$ and $\delta_{c,n}$ could be extracted. In this case, $\gamma$ could be compared with the results of alternative “direct” strategies and the range implied by the “CKM fits”, whereas $\delta_{c,n}$ would provide valuable insights into hadron dynamics.

Following Ref. 39 we show in Fig. 2 the allowed regions in the $R_{c,n} - A_0^{n}$ planes, where the crosses represent the averages of the most recent $B$-factory data.\textsuperscript{39} 70 As can be read off from the contours in these figures, both the charged and the neutral $B \to \pi K$ data favour $\gamma \gtrsim 90^\circ$, which would be in conflict with $\gamma$. On the other hand, the charged modes point towards $|\delta_c| \lesssim 90^\circ$ (factorization predicts $\delta_c$ to be close to $0^\circ$), whereas the neutral decays seem to favour $|\delta_n| \gtrsim 90^\circ$. Since
4. CP Violation in $B_s$ Decays

4.1. General Remarks

Since $\Upsilon(4S)$ states decay only to $B_{u,d}$ mesons, but not to $B_s$, these mesons are not accessible at the $e^+e^- B$ factories operating at the $\Upsilon(4S)$ resonance. On the other hand, plenty of $B_s$ mesons are produced at hadron colliders, so that the $B_s$ system can be considered as the “El Dorado” for $B$-decay experiments at such machines.

The most exciting aspect of $B_s$ studies is clearly the exploration of CP violation. However, also the measurement of the mass difference $\Delta M_s$ would be a very important achievement, since this quantity nicely complements its $B_d$-meson counterpart $\Delta M_d$, thereby allowing a particularly interesting determination of the side $R_t$ of the UT. So far, only experimental lower bounds on $\Delta M_s$ are available, which can be converted into upper bounds on $R_t$, implying $\gamma \lesssim 90^\circ$.14 In the near future, run II of the Tevatron should provide a measurement of $\Delta M_s$, thereby constraining the UT – and in particular $\gamma$ – in a much more stringent way. Interesting applications of $\Delta \Gamma_s$, which may be as large as $O(10\%)$,71 whereas $\Delta \Gamma_d$ is negligibly small, are
extractions of $\gamma$ from “untagged” $B_s$ data samples, where we do not distinguish between initially, i.e. at time $t = 0$, present $B^0_s$ or $\bar{B}^0_s$ mesons.

4.2. $B_s \to J/\psi \phi$

This decay is the $B_s$ counterpart of the “gold-plated” mode $B_d \to J/\psi K_S$, and originates from this channel if we replace the down spectator quark by a strange quark. Consequently, the phase structure of the $B_s \to J/\psi \phi$ decay amplitude is analogous to the one of (14). However, in contrast to $B_d \to J/\psi K_S$, the final state of $B_s \to J/\psi \phi$ is an admixture of different CP eigenstates. In order to disentangle them, we must measure the angular distribution of the $J/\psi \to \ell^+ \ell^-$, $\phi \to K^+ K^-$ decay products. The corresponding CP-violating observables are governed by quantities with the following structure:

$$\xi^{(s)}_\psi \propto e^{-i\phi_s} \left(1 - i \sin \gamma \times \mathcal{O}(\lambda^3) \right),$$

(24)

where the generic expansion parameter $\lambda$ was introduced in Subsection 3.1 and describes the impact of penguin contributions. Since we have $\phi_s = -2\lambda^2 \eta = \mathcal{O}(-0.03)$ in the SM, the extraction of $\phi_s^{\text{SM}}$ from mixing-induced CP-violating effects arising in the time-dependent $B_s \to J/\psi \to \ell^+ \ell^-$, $\phi \to K^+ K^-$ angular distribution is affected by generic hadronic uncertainties of $\mathcal{O}(10\%)$. These penguin effects, which may become an important issue for the LHC 10, can be controlled through $B_d \to J/\psi \rho^0$, exhibiting also other interesting features.

Because of the tiny mixing-induced CP asymmetries arising in $B_s \to J/\psi \phi$ within the SM, this mode is an interesting probe to search for NP contributions to $B^0_s - \bar{B}^0_s$ mixing. A detailed discussion of “smoking-gun” signals of sizeable values of $\phi_s$ was given in Ref. 47, where also methods to fix this phase unambiguously were proposed. The latter issue was also recently addressed in Ref. 76.

4.3. Complementing $B_d \to \pi^+ \pi^-$ through $B_s \to K^+ K^-$

As we have seen in Subsection 3.5, the extraction of UT angles from the CP-violating $B_d \to \pi^+ \pi^-$ asymmetries is strongly affected by hadronic penguin effects. In order to deal with this problem, the decay $B_s \to K^+ K^-$ offers an interesting avenue for $B$ experiments at hadron colliders 12. Within the SM, we may write the CP asymmetries provided by these modes in the following form:

$$A_{\text{CP}}^{\text{dir}}(B_d \to \pi^+ \pi^-) = \text{fct}(d, \theta, \gamma), \quad A_{\text{CP}}^{\text{mix}}(B_d \to \pi^+ \pi^-) = \text{fct}(d, \theta, \gamma, \phi_d)$$

(25)

$$A_{\text{CP}}^{\text{dir}}(B_s \to K^+ K^-) = \text{fct}(d', \theta', \gamma), \quad A_{\text{CP}}^{\text{mix}}(B_s \to K^+ K^-) = \text{fct}(d', \theta', \gamma, \phi_s),$$

(26)

where the hadronic quantities $d'$ and $\theta'$ are the $B_s \to K^+ K^-$ counterparts of the parameters introduced in (14). If we take into account that $\phi_d$ and $\phi_s$ can straightforwardly be fixed separately, we may use the CP-violating asymmetries of the $B_d \to \pi^+ \pi^-$ and $B_s \to K^+ K^-$ modes to determine $d$ and $d'$ as functions of $\gamma$, respectively. This can be done in a theoretically clean manner, i.e. without using...
flavour-symmetry or plausible dynamical assumptions. If we look at the corresponding Feynman diagrams, we observe that $B_d \to \pi^+\pi^-$ is related to $B_s \to K^+K^-$ through an interchange of all down and strange quarks. Because of this feature, the $U$-spin flavour symmetry of strong interactions implies

$$d' = d, \quad \theta' = \theta. \quad (27)$$

If we now apply the former relation, we may determine $\gamma$, as well as the strong phases $\theta'$ and $\theta$, which provide a nice consistency check of the latter $U$-spin relation [22]. This strategy is also very promising from an experimental point of view: at Tevatron-II and the LHC, experimental accuracies for $\gamma$ of $\mathcal{O}(10^3)$ and $\mathcal{O}(1^\circ)$, respectively, are expected [30, 11]. For a collection of other $U$-spin strategies, see Refs. [41, 49, 77].

Since $B_s \to K^+K^-$ is not accessible at the $e^+e^-$ factories operating at the $\Upsilon(4S)$ resonance, we may not yet implement this approach. However, $B_s \to K^+K^-$ is related to $B_d \to \pi^\pm K^{\mp}$ through an interchange of spectator quarks. Consequently, we may approximately replace $B_s \to K^+K^-$ through $B_d \to \pi^\mp K^{\pm}$ to deal with the penguin problem in $B_d \to \pi^+\pi^-$. [23] To this end, the quantity

$$H = \frac{1}{\epsilon} \left( \frac{f_K}{f_\pi} \right)^2 \frac{\text{BR}(B_d \to \pi^+\pi^-)}{\text{BR}(B_d \to \pi^\mp K^{\pm})} = \begin{cases} 7.4 \pm 2.5 & \text{(CLEO) [50]} \\ 7.8 \pm 1.2 & \text{(BaBar) [70]} \\ 7.1 \pm 1.2 & \text{(Belle) [70]}, \end{cases} \quad (28)$$

where $\epsilon \equiv \lambda^2/(1 - \lambda^2)$, is particularly useful. Applying (27), we may write

$$H = \text{fct}(d, \theta, \gamma). \quad (29)$$

Consequently, if we complement [26] with [29], we have sufficient information to determine $\gamma$, $d$ and $\theta$. In particular, we may eliminate $d$ in $A_{\text{CP}}^{\text{dir}}(B_d \to \pi^+\pi^-)$ and $A_{\text{CP}}^{\text{mix}}(B_d \to \pi^+\pi^-)$, so that these observables then depend – for a given value of $\phi_d$ – only on $\gamma$ and the strong phase $\theta$. If we vary these parameters within their allowed ranges, we obtain an allowed region in the $A_{\text{CP}}^{\text{dir}}(B_d \to \pi^+\pi^-)$ - $A_{\text{CP}}^{\text{mix}}(B_d \to \pi^+\pi^-)$ plane [59], which is shown in Fig. 3 for the most recent $B$-factory data. We observe

Fig. 3. The allowed regions in the $B_d \to \pi^+\pi^-$ observable space, where (a) was calculated for $\phi_d = 47^\circ$ and various values of $H$, and (b) corresponds to $\phi_d = 133^\circ$ and $H = 7.5$. The SM regions appear if we restrict $\gamma$ to $[0, \pi]$. Contours representing fixed values of $\gamma$ are also included.

\[ \text{Contours representing fixed values of } \phi \quad \text{and } \gamma \text{ are also included.} \]
that the experimental averages (18) and (19), represented by the crosses, overlap nicely with the SM region for $\phi_d = 47^\circ$, and point towards $\gamma \sim 60^\circ$. In this case, not only $\gamma$ would be in accordance with the results of the “CKM fits” (1), but also $\phi_d$. On the other hand, for $\phi_d = 133^\circ$, the experimental values favour $\gamma \sim 120^\circ$, and have essentially no overlap with the SM region. At first sight, this may look puzzling. However, since the $\phi_d = 133^\circ$ solution would definitely require NP contributions to $B^0_d \rightarrow \overline{B}^0_d$ mixing, we may no longer use the SM interpretation of $\Delta M_d$ in this case to fix the UT side $R_t$, which is a crucial ingredient for the $\gamma$ range in (1). Consequently, if we choose $\phi_d = 133^\circ$, $\gamma$ may well be larger than $90^\circ$. As we have already noted, the $B \rightarrow \pi K$ data seem to favour such values; a similar feature is also suggested by the small $B_d \rightarrow \pi^+\pi^-$ rate. Interestingly, the measured branching ratio for the rare kaon decay $K^+ \rightarrow \pi^+\nu\bar{\nu}$ seems to point towards $\gamma > 90^\circ$ as well (83) thereby also favouring the unconventional solution of $\phi_d = 133^\circ$ (79). Further valuable information on this exciting possibility can be obtained from the rare decays $B_{s,d} \rightarrow \mu^+\mu^-$. We could straightforwardly deal with this picture in a scenario for physics beyond the SM, where we have large NP contributions to $B^0_d \overline{B}^0_d$ mixing, but not to the $\Delta B = 1$ and $\Delta S = 1$ decay processes. Such NP was already considered several years ago (50), and can be motivated by generic arguments and within supersymmetry (79). Since the determination of $R_b$ through semileptonic tree decays is in general very robust under NP effects and would not be affected either in this particular scenario, we may complement $R_b$ with the range for $\gamma$ extracted from our $B_d \rightarrow \pi^+\pi^-$ analysis, allowing us to fix the apex of the UT in the $\rho$-$\eta$ plane. The results of this exercise are summarized in Fig. 4 following Ref. (79, where also numerical values for $\alpha$, $\beta$ and $\gamma$ are given and a detailed discussion of the theoretical uncertainties can be found. Note that the SM contours implied by $\Delta M_d$, which are included in Fig. 4 (a) to guide the eye, are absent in (b), since $B^0_d \overline{B}^0_d$ mixing would there receive NP contributions. In this case, also we may no longer simply represent $\phi_d$ by a straight line, as the one in Fig. 4 (a), which corresponds to $\phi^{SM}_d = 2\beta$, since we would now have $\phi_d = 2\beta + \phi^{NP}_d$, with $\phi^{NP}_d \neq 0^\circ$. However, we may easily read off the “correct” value of $\beta$ from the black region in Fig. 4 (b) (79). Interestingly, both black regions in
Fig. 4 (a) and (b) are consistent with the SM $\varepsilon_K$ hyperbola.

Because of the unsatisfactory status of the measured CP-violating $B_d \to \pi^+\pi^-$ observables, we may not yet draw definite conclusions from this analysis, although it illustrates nicely how the corresponding strategy works. However, the experimental picture will improve significantly in the future, thereby providing more stringent constraints on $\gamma$ and the apex of the UT. Another milestone in this programme is the measurement of the CP-averaged $B_s \to K^+K^-$ branching ratio at run II of the Tevatron, which will allow a much better determination of $H$ that no longer relies on dynamical assumptions. Finally, if also the direct and mixing-induced CP asymmetries of $B_s \to K^+K^-$ are measured, we may determine $\gamma$ through a minimal $U$-spin input, as discussed above. After important steps by the CDF collaboration, LHCb and BTeV should be able to fully exploit the rich physics potential of the $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$ system. There are several other promising $B_s$ decays, which we shall address in the discussion of the following section.

5. New, Theoretically Clean Strategies to Extract $\gamma$

As far as theoretically clean determinations of $\gamma$ are concerned, pure “tree” decays play the key rôle. In this context, we may distinguish between the following two cases: $B$ decays exhibiting interference effects, which are induced by subsequent $D^0, \bar{D}^0 \to f_D$ transitions, and channels, where neutral $B_q^0$ and $\bar{B}_q^0$ mesons may decay into the same final state $f_s$ so that we have interference effects between $B_q^0 - \bar{B}_q^0$ mixing and decay processes. Prominent examples are $B^\pm \to DK^\pm$, $B_s^\pm \to DD^\pm$, ... and $B_d \to D^{(*)}\pi^\mp$, $B_s \to D_s^{(*)}\pm K^\mp$, ... modes, respectively. Let us focus here on recently proposed new strategies.

5.1. $B_d \to DK_{S(L)}$ and $B_s \to D\eta^{(c)}, D\phi, ...$

Colour-suppressed $B_d^0 \to D^0 K_S$ decays and similar modes provide interesting tools to explore CP violation. In the following, we shall consider general transitions of the kind $B_s^0 \to D^0 f_r$, where $r \in \{s, d\}$ distinguishes between $b \to Ds$ and $b \to Dd$ processes. 

If we require $\langle CP \rangle |f_r\rangle = \eta_f |f_r\rangle$, $B_d^0$ and $\bar{B}_d^0$ mesons may both decay into $D^0 f_r$, thereby leading to interference between $B_d^0 - \bar{B}_d^0$ mixing and decay processes, which involve $\phi_q + \gamma$. In the case of $r = s$, corresponding to $B_d \to DK_{S(L)}$, $B_s \to D\eta^{(c)}, D\phi, ...$, these interference effects are governed by a hadronic parameter $x_{fs} e^{i\delta_f} \propto R_b \approx 0.4$, and are hence favourably large. On the other hand, for $r = d$, which describes $B_s \to DK_{S(L)}$, $B_d \to D_0^\pi, D_0^\rho, ...$, the interference effects are tiny because of $x_{fs} e^{i\delta_f} \propto -\lambda^2 R_b \approx -0.02$. Let us first focus on the $r = s$ case.

If we consider $B_q \to D_{\pm} f_s$ modes, where $\langle CP \rangle |D_{\pm}\rangle = \pm |D_{\pm}\rangle$, additional interference between $B_q^0 \to D^0 f_s$ and $\bar{B}_q^0 \to \bar{D}^0 f_s$ arises at the decay level, involving $\gamma$. The most straightforward observable we may measure is the “untagged” rate

$$\langle \Gamma(B_q(t) \to D_{\pm} f_s) \rangle \equiv \Gamma(B_q^0(t) \to D_{\pm} f_s) + \Gamma(\bar{B}_q^0(t) \to \bar{D}_{\pm} f_s)$$

(30)
which allows us to determine the following “untagged” rate asymmetry:

\[ \Delta \gamma = \left( \Gamma(B_q^+ \to D_{++}f_s) - \Gamma(B_q^- \to D_{--}f_s) \right) e^{-T r t} \equiv \left( \Gamma(B_q \to D_{\pm}f_s) \right) e^{-T r t} \]

which implies bounds on \( \gamma \). Second, taking into account that factorization suggests \( \cos \delta_{f_s} \approx 0 \), we obtain

\[ \text{sgn}(\cos \gamma) = \text{sgn}(\Gamma_{f_s}^+) \]

allowing us to decide whether \( \gamma \) is smaller or larger than \( 90^\circ \).

If we measure also the mixing-induced observables \( S_{f_s}^{\pm} \equiv A_{\text{CP}}^{\text{mix}}(B_q \to D_{\pm}f_s) \), we may determine \( \gamma \). To this end, it is convenient to introduce the quantities

\[ \langle S_{f_s} \rangle_{\pm} = \frac{S_{f_s}^{+} + S_{f_s}^{-}}{2} \]

Expressing the \( \langle S_{f_s} \rangle_{\pm} \) in terms of the \( B_q \to D_{\pm}f_s \) decay parameters gives rather complicated formulae. However, complementing the \( \langle S_{f_s} \rangle_{\pm} \) with \( \Gamma_{f_s}^+ \) yields

\[ \tan \gamma \cos \phi_q = \left[ \frac{\eta_{f_s} \langle S_{f_s} \rangle_{+}}{\Gamma_{f_s}^+} \right] + \left[ \eta_{f_s} \langle S_{f_s} \rangle_{-} - \sin \phi_q \right] \]

where \( \eta_{f_s} \equiv (-1)^L \eta_{\text{CP}}^L \), with \( L \) denoting the \( D_{f_s} \) angular momentum. Using this simple but exact relation, we obtain the twofold solution \( \gamma = \gamma_1 \lor \gamma_2 \), with \( \gamma_1 \in [0^\circ, 180^\circ] \) and \( \gamma_2 = \gamma_1 + 180^\circ \). Since \( \cos \gamma_1 \) and \( \cos \gamma_2 \) have opposite signs, allows us to fix \( \gamma \) unambiguously. Another advantage of \eqref{eq:gamma-finding} is that \( \langle S_{f_s} \rangle_{+} \) and \( \Gamma_{f_s}^+ \) are both proportional to \( x_{f_s} \approx 0.4 \), so that the first term in square brackets is of \( \mathcal{O}(1) \), whereas the second one is of \( \mathcal{O}(x_{f_s}^2) \), hence playing a minor rôle. In order to extract \( \gamma \), we may also employ \( D \) decays into CP non-eigenstates \( f_{\text{NE}} \), where we have to deal with complications originating from \( D^0, \bar{D}^0 \to f_{\text{NE}} \) interference effects. Also in this case, \( \Gamma_{f_s}^+ \) is a very powerful ingredient, offering an efficient, analytical strategy to include these interference effects in the extraction of \( \gamma \).

Let us briefly come back to the \( r = d \) case, corresponding to \( B_s \to DK_{s(L)} \), \( B_d \to D^{\pi^0}, D^{\rho^0} \ldots \) decays, which can be described through the same formulae as their \( r = s \) counterparts. Since the relevant interference effects are governed by \( x_{f_s} \approx -0.02 \), these channels are not as attractive for the extraction of \( \gamma \) as the \( r = s \) modes. On the other hand, the relation

\[ \eta_{f_s} \langle S_{f_s} \rangle_{-} = \sin \phi_q + \mathcal{O}(x_{f_s}^2) = \sin \phi_q + \mathcal{O}(4 \times 10^{-4}) \]

offers very interesting determinations of \( \sin \phi_q \).

Following this avenue, there are no penguin uncertainties, and the theoretical accuracy is one order of magnitude better.
than in the “conventional” $B_d \to J/\psi K_S$, $B_s \to J/\psi \phi$ strategies. In particular, $\phi_s^{SM} = -2\lambda^2\eta$ could, in principle, be determined with a theoretical uncertainty of only $O(1\%)$, which should be compared with the discussion given in Subsection 5.1.

The $B_d^0 \to D^0\pi^0$ mode has already been seen at the $B$ factories, with branching ratios at the $3 \times 10^{-4}$ level. Recently, the Belle collaboration has also announced the observation of $B_d^0 \to D^0 K^0$, with the branching ratio $(5.0^{+1.3}_{-0.6} \times 10^{-5})$.

### 5.2. $B_s \to D_s^{(*)}\pm K^\mp, \ldots$ and $B_d \to D^{(*)}\pm \pi^\mp, \ldots$

Let us now turn to the colour-allowed counterparts of the $B_q \to D_q$ modes discussed in Subsection 5.1, which we may write generically as $B_q \to D_q \pi_q$. The characteristic feature of these transitions is that both a $B^0$ and a $\bar{B}^0$ meson may decay into $D_q \pi_q$, thereby leading to interference between $B^0_q - \bar{B}^0_q$ mixing and decay processes, which involve the weak phase $\phi_q + \gamma$. In the case of $q = s$, which corresponds to $D_s \in \{D_s^+, D_s^{*-}, \ldots\}$ and $u_s \in \{K^+, K^{*-}, \ldots\}$, these interference effects are governed by a hadronic parameter $x_s e^{i\phi_s} \propto R_s \approx 0.4$, and hence are large. On the other hand, in the case of $q = d$, corresponding to $D_d \in \{D^+_d, D^{*-}_d, \ldots\}$ and $u_d \in \{\pi^+, \rho^+, \ldots\}$, they are described by $x_d e^{i\phi_d} \propto -\lambda^2 R_d \approx -0.02$, and hence are tiny. In the following, we shall only consider $B_q \to D_q \pi_q$ modes, where at least one of the $D_q$, $\pi_q$ states is a pseudoscalar meson; otherwise a complicated angular analysis has to be performed.

It is well known that such decays allow a determination of $\phi_q + \gamma$, where the “conventional” approach works as follows if we measure the observables $C(B_q \to D_q \pi_q) \equiv C_q$ and $C(B_q \to D_q u_q) \equiv C_q$ provided by the $\text{cos} (\Delta M_q t)$ pieces of the time-dependent rate asymmetries, we may determine $x_q$ from terms entering at the $x_q^2$ level. In the case of $q = s$, we have $x_s = O(R_s)$, implying $x_s^2 = O(0.16)$, so that this may actually be possible, though challenging. On the other hand, $x_d = O(-\lambda^2 R_d)$ is doubly Cabibbo-suppressed. Although it should be possible to resolve terms of $O(x_d)$, this will be impossible for the vanishingly small $x_d^2 = O(0.0004)$ terms, so that other approaches to fix $x_d$ are required.

In order to extract $\phi_q + \gamma$, we must measure the mixing-induced observables $S(B_q \to D_q \pi_q) \equiv S_q$ and $S(B_q \to D_q u_q) \equiv \bar{S}_q$ associated with the $\sin (\Delta M_q t)$ terms of the time-dependent rate asymmetries, where it is convenient to introduce

$$\langle S_q \rangle \equiv \frac{S_q + \bar{S}_q}{2}. \quad (37)$$

If we assume that the hadronic parameter $x_q$ is known, we may consider

$$s_+ \equiv (-1)^L \left[ \frac{1 + x_q^2}{2x_q} \right] \langle S_q \rangle_+ = + \cos \delta_q \sin (\phi_q + \gamma) \quad (38)$$

$$s_- \equiv (-1)^L \left[ \frac{1 + x_q^2}{2x_q} \right] \langle S_q \rangle_- = - \sin \delta_q \cos (\phi_q + \gamma), \quad (39)$$
yielding
\[
\sin^2(\phi_q + \gamma) = \frac{1}{2} \left[ (1 + s_+^2 - s_-^2) \pm \sqrt{(1 + s_+^2 - s_-^2)^2 - 4s_+^2} \right],
\]
which implies an eightfold solution for \( \phi_q + \gamma \); assuming \( \text{sgn}(\cos \delta_q) > 0 \), as suggested by factorization, a fourfold discrete ambiguity emerges. This assumption allows us also to extract the sign of \( \sin(\phi_q + \gamma) \) from \( \langle S_q \rangle_+ \). To this end, the factor \((-1)^L\) where \( L \) is the \( D_q \pi_q \) angular momentum, has to be properly taken into account.\(^{[57]}\)

This is crucial for the extraction of the sign of \( \sin(\phi_q + \gamma) \) from \( B_d \to D^{\pm}\pi^\mp \) modes, allowing us to distinguish between the two solutions shown in Fig. 4.

Let us now discuss new approaches to deal with \( B_q \to D_q \pi_q \) modes, following Ref.\(^{[57]}\) If \( \Delta \Gamma_s \) is sizeable, the “untagged” rates
\[
\langle \Gamma(B_q(t) \to D_q \pi_q) \rangle = \langle \Gamma(B_q \to D_q \pi_q) \rangle \times \cosh(\Delta \Gamma_q t/2) - A_{\Delta \Gamma}(B_q \to D_q \pi_q) \sinh(\Delta \Gamma_q t/2) \rangle e^{-\Gamma_q t}
\]
provide observables \( A_{\Delta \Gamma}(B_s \to D_s \pi_s) \equiv A_{\Delta \Gamma_s} \) and \( A_{\Delta \Gamma}(B_s \to \overline{D}_s u_s) \equiv \overline{A}_{\Delta \Gamma_s} \), which yield
\[
\tan(\phi_s + \gamma) = \left[ \frac{\langle S_s \rangle_+}{\langle A_{\Delta \Gamma_s} \rangle_+} \right] = \left[ \frac{\langle A_{\Delta \Gamma_s} \rangle_-}{\langle S_s \rangle_-} \right],
\]
where \( \langle A_{\Delta \Gamma_s} \rangle_{\pm} \) is defined in analogy to \( \langle S_q \rangle_{\pm} \). These relations allow an \textit{unambiguous} determination of \( \phi_s + \gamma \), if we employ again \( \text{sgn}(\cos \delta_q) > 0 \). Another important advantage of \( \langle S_q \rangle_{\pm} \) is that we have not to rely on \( \mathcal{O}(x_q^2) \) terms, as \( \langle S_q \rangle_{\pm} \) are proportional to \( x_q \). On the other hand, we need a sizeable value of \( \Delta \Gamma_q \). Measurements of untagged rates are also very useful in the case of vanishingly small \( \Delta \Gamma_q \), since the “unevolved” untagged rates in \( \langle 41 \rangle \) offer various interesting strategies to determine \( x_q \) from the ratio of \( \langle \Gamma(B_q \to D_q \pi_q) \rangle + \langle \Gamma(B_q \to \overline{D}_q u_q) \rangle \) and CP-averaged rates of appropriate \( B^\pm \) or flavour-specific \( B_q \) decays.

If we keep the hadronic parameter \( x_q \) and the associated strong phase \( \delta_q \) as “unknown”, free parameters in the expressions for the \( \langle S_q \rangle_{\pm} \), we obtain
\[
|\sin(\phi_q + \gamma)| \geq |\langle S_q \rangle_+|, \quad |\cos(\phi_q + \gamma)| \geq |\langle S_q \rangle_-|,
\]
which can straightforwardly be converted into bounds on \( \phi_q + \gamma \). If \( x_q \) is known, stronger constraints are implied by
\[
|\sin(\phi_q + \gamma)| \geq s_+, \quad |\cos(\phi_q + \gamma)| \geq s_-.
\]
Once \( s_+ \) and \( s_- \) are known, we may of course determine \( \phi_q + \gamma \) through the “conventional” approach, using \( \langle 40 \rangle \). However, the bounds following from \( \langle 41 \rangle \) provide essentially the same information and are much simpler to implement. Moreover, as discussed in detail in Ref.\(^{[57]}\) for several examples corresponding to the SM, the bounds following from \( B_s \) and \( B_d \) modes may be highly complementary, thereby providing particularly narrow, theoretically clean ranges for \( \gamma \).

If we look at the corresponding topologies, we observe that \( B^+_s \to D_s^{(*)+} K^- \) and \( B^+_d \to D^{(*)+} \pi^- \) are related to each other through an interchange of all down and
strange quarks. Consequently, the $U$-spin symmetry implies $a_s = a_d$ and $\delta_s = \delta_d$, where $a_s = x_s/R_s$ and $a_d = -x_d/(\lambda^2 R_s)$ are the ratios of hadronic matrix elements entering $x_s$ and $x_d$, respectively. There are various possibilities to implement these relations.\[^{57}\] For example, we may assume that $a_s = a_d$ and $\delta_s = \delta_d$, yielding

$$
\tan \gamma = - \left[ \frac{\sin \phi_d - S \sin \phi_s}{\cos \phi_d - S \cos \phi_s} \right] \phi_s = 0^\circ = - \left[ \frac{\sin \phi_d}{\cos \phi_d - S} \right],
$$

where

$$
S = -R \left[ \frac{(S_d)_+}{(S_s)_+} \right],
$$

with

$$
R = \left( \frac{1 - \lambda^2}{\lambda^2} \right) \left[ \frac{1}{1 + x_s^2} \right];
$$

$R$ can be fixed through untagged $B_s$ rates with the help of

$$
R = \left( \frac{f_K}{f_\pi} \right)^2 \left[ \frac{\Gamma(B^0 \rightarrow D_s^{(*)} \pi^-) + \Gamma(B^0_s \rightarrow D_s^{(*)} - \pi^+)}{\Gamma(B_s \rightarrow D_s^{(*)} + K^-) + \Gamma(B_s \rightarrow D_s^{(*)} - K^+)} \right].
$$

Alternatively, we may only assume that $\delta_s = \delta_d$ or that $a_s = a_d$, as discussed in detail in Ref.\[^{57}\] Apart from features related to multiple discrete ambiguities, the most important advantage in comparison with the “conventional” approach is that the experimental resolution of the $x_s^2$ terms is not required. In particular, $x_s$ does not have to be fixed, and $x_s$ may only enter through a $1 + x_s^2$ correction, which can straightforwardly be determined through untagged $B_s$ rate measurements. In the most refined implementation of this strategy, the measurement of $x_s/x_d$ would only be interesting for the inclusion of $U$-spin-breaking effects in $a_d/a_s$. Moreover, we may obtain interesting insights into hadron dynamics and $U$-spin-breaking effects.

### 6. Conclusions and Outlook

Thanks to the “gold-plated” mode $B_d \rightarrow J/\psi K_S$ and similar channels, CP violation is now a well established phenomenon in the $B$-meson system. Although the present world average $\sin \phi_d = 0.734 \pm 0.054$ agrees well with the SM, we obtain the twofold solution $\phi_d \sim 47^\circ \vee 133^\circ$, where the latter leaves us with the exciting possibility of having large NP contributions to $B_d \rightarrow B_d^*$ mixing. The $B$ factories allow us now to confront many strategies to explore CP violation with the first data, yielding the following present picture: the SM relation $A_{CP}^{\text{mix}}(B_d \rightarrow J/\psi K_S) = A_{CP}^{\text{mix}}(B_d \rightarrow J/\psi K_S)$ may not be satisfied, $B \rightarrow \pi K$ data point towards $\gamma \gtrsim 90^\circ$ and a “puzzling” picture for strong phases, CP violation in $B_d \rightarrow \pi^+ \pi^-$ could accommodate $\gamma > 90^\circ$ for $\phi_d = 133^\circ$. The experimental uncertainties do not yet allow us to draw definite conclusions, but the situation will significantly improve in the future.

The physics potential of the $e^+e^-$ $B$ factories operating at the $\Upsilon(4S)$ resonance is nicely complemented by $B$-decay studies at hadron colliders, run II of the Tevatron and the LHC, providing in particular access to the $B_s$-meson system. Already
a measurement of the mass difference $\Delta M_s$ would be a very important achievement, since $\Delta M_s/\Delta M_d$ provides a particularly interesting determination of the side $R_t$ of the UT. However, the most exciting aspects are related to the exploration of CP violation, which is offered by several promising $B_s$ modes. Here another “gold-plated” mode is given by $B_s \to J/\psi \phi$, allowing us to check whether $\phi_s$ is actually negligibly small, as expected in the SM, or whether this phase is enhanced through NP contributions to $B_s^0 - B_s^0$ mixing; also in the latter case, we could extract $\phi_s$ through mixing-induced CP violation in $B_s \to J/\psi \phi$. As we have seen, another very interesting channel is the penguin-dominated decay $B_s \to J/\psi K^\ast$, which complements $B_d \to \pi^+ \pi^-$, thereby offering a promising determination of $\gamma$. Moreover, there are theoretically clean strategies to extract $\gamma$, where pure “tree” decays of $B_s$ mesons play a key role. Here it is also of great advantage to complement $B_s$ with $B_d$ modes, considering, for example, the $B_s \to D_s^{(*)\pm} K^\mp$, $B_d \to D^{(*)\pm} \pi^\mp$ system. It will be very exciting to see whether discrepancies between the $B_s \to J/\psi K^\ast$, $B_d \to \pi^+ \pi^-$ and $B_s \to D_s^{(*)\pm} K^\mp$, $B_d \to D^{(*)\pm} \pi^\mp$ results for $\gamma$ will emerge. Detailed feasibility studies of the new strategies discussed above are strongly encouraged.

References

1. J.H. Christenson et al., Phys. Rev. Lett. 13, 138 (1964).
2. V. Fanti et al. (NA48 Collaboration), Phys. Lett. B465, 335 (1999);
   J.R. Batley et al. (NA48 Collaboration), Phys. Lett. B544, 97 (2002);
   A. Alavi-Harati et al. (KTeV Collaboration), Phys. Rev. Lett. 83, 22 (1999) and Phys.
   Rev. D67, 012005 (2003).
3. S. Bertolini, hep-ph/0206095.
4. A.J. Buras, TUM-HEP-435-01 [hep-ph/0109197].
5. For a detailed review, see R. Fleischer, Phys. Rep. 370, 537 (2002).
6. B. Aubert et al. (BaBar Collaboration), Phys. Rev. Lett. 87, 091801 (2001).
7. K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. 87, 091802 (2001).
8. The BaBar Physics Book, eds. P. Harrison and H.R. Quinn, SLAC-R-504 (1998).
9. K. Anikeev et al., FERMILAB-Pub-01/197 [hep-ph/0201071].
10. P. Ball et al., CERN-TH/2000-101 [hep-ph/0003238], in CERN Report on Standard
    Model physics (and more) at the LHC (CERN, Geneva, 2000) p. 305.
11. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
12. L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983).
13. A.J. Buras, M.E. Lautenbacher and G. Ostermaier, Phys. Rev. D50, 3433 (1994).
14. M. Battaglia et al., hep-ph/0304132, to appear as a CERN Report, eds. M. Battaglia,
    A.J. Buras, P. Gambino, A. Stocchi (2003).
15. M. Beneke et al., Phys. Rev. Lett. 83, 1914 (1999).
16. M. Beneke et al., Nucl. Phys. B591, 313 (2000).
17. M. Beneke et al., Nucl. Phys. B606, 245 (2001).
18. R. Aleksan et al., DAPNIA-02-380 [hep-ph/0301165].
19. H.-n. Li and H.L. Yu, Phys. Rev. D53, 2480 (1996);
   Y.Y. Keum, H.-n. Li and A.I. Sanda, Phys. Rev. D63, 054008 (2001);
   Y.Y. Keum and H.-n. Li, Phys. Rev. D63, 074006 (2001).
20. A. Khodjamirian, Nucl. Phys. B605, 558 (2001);
    A. Khodjamirian, T. Mannel and B. Melic, TTP03-12 [hep-ph/0304179].
21. C.W. Bauer, D. Pirjol and I.W. Stewart, *Phys. Rev. Lett.* **87**, 201806 (2001); C.W. Bauer, B. Grinstein, D. Pirjol and I.W. Stewart, *Phys. Rev.* **D67**, 014010 (2003).
22. H.-n. Li, IPAS-03-01 [hep-ph/0303116].
23. M. Gronau and D. Wyler, *Phys. Lett.* **B265**, 172 (1991).
24. D. Atwood, I. Dunietz and A. Soni, *Phys. Rev.* **D63**, 036005 (2001).
25. R. Fleischer and D. Wyler, *Phys. Rev.* **D62**, 057503 (2000).
26. M. Gronau, J.L. Rosner and D. London, *Phys. Rev. Lett.* **73**, 21 (1994).
27. R. Fleischer, *Phys. Rev. Lett.* **B365**, 399 (1996).
28. R. Fleischer and T. Mannel, *Phys. Rev.* **D57**, 2752 (1998).
29. M. Gronau and J.L. Rosner, *Phys. Rev.* **D57**, 6843 (1998).
30. R. Fleischer, *Eur. Phys. J.* **C6**, 451 (1999); *Phys. Lett.* **B435**, 221 (1998).
31. M. Neubert and J.L. Rosner, *Phys. Lett.* **B441**, 403; *Phys. Rev. Lett.* **81**, 5076 (1998).
32. M. Neubert, *JHEP* **9902**, 014 (1999).
33. A.J. Buras and R. Fleischer, *Eur. Phys. J.* **C11**, 93 (1999).
34. A.J. Buras and R. Fleischer, *Eur. Phys. J.* **C16**, 97 (2000).
35. R. Fleischer and J. Matias, *Phys. Rev.* **D61**, 074004 (2000).
36. J. Matias, *Phys. Lett.* **B520**, 131 (2001).
37. M. Bargiotti et al., *Eur. Phys. J.* **C24**, 361 (2002).
38. M. Gronau and J.L. Rosner, *Phys. Rev.* **D65**, 013004 [E: **D65**, 079901] (2002).
39. R. Fleischer and J. Matias, *Phys. Rev.* **D66**, 054009 (2002).
40. A.B. Carter and A.I. Sanda, *Phys. Rev. Lett.* **45**, 952 (1980); *Phys. Rev.* **D23**, 1567 (1981); I.I. Bigi and A.I. Sanda, *Nucl. Phys.* **B193**, 85 (1981).
41. R. Fleischer, *Eur. Phys. J.* **C10**, 299 (1999).
42. R. Fleischer and T. Mannel, *Phys. Lett.* **B506**, 311 (2001).
43. B. Aubert et al. (BaBar Collaboration), *Phys. Rev. Lett.* **89**, 201802 (2002).
44. K. Abe et al. (Belle Collaboration), *Phys. Rev.* **D66**, 071102 (2002).
45. Ya.I. Azimov, V.L. Rappoport and V.V. Sarantsev, *Z. Phys.* **A356**, 437 (1997); Y. Grossman and H.R. Quinn, *Phys. Rev.* **D56**, 7259 (1997); J. Charles et al., *Phys. Lett.* **B425**, 375 (1998); B. Kayser and D. London, *Phys. Rev.* **D61**, 116012 (2000); H.R. Quinn et al., *Phys. Rev. Lett.* **85**, 5284 (2000).
46. A.S. Dighe, I. Dunietz and R. Fleischer, *Phys. Lett.* **B433**, 147 (1998).
47. I. Dunietz, R. Fleischer and U. Nierste, *Phys. Rev.* **D63**, 114015 (2001).
48. R. Itoh, KEK-PREPRINT-2002-106 [hep-ex/0210025].
49. R. Fleischer, *Phys. Rev.* **D60**, 073008 (1999).
50. Y. Grossman, Y. Nir and M.P. Worah, *Phys. Lett.* **B407**, 307 (1997).
51. Y. Grossman and M.P. Worah, *Phys. Lett.* **B395**, 241 (1997).
52. D. London and R.D. Peccei, *Phys. Lett.* **B223**, 257 (1989); N.G. Deshpande and J. Trampetic, *Phys. Rev.* **D41**, 895 and 2926 (1990); J.-M. Gérard and W.-S. Hou, *Phys. Rev.* **D43**, 2909 (1991).
53. R. Fleischer, *Z. Phys.* **C62**, 81 (1994).
54. N.G. Deshpande and X.-G. He, *Phys. Lett.* **B336**, 471 (1994).
55. R. Fleischer, *Int. J. Mod. Phys.* **A12**, 2459 (1997).
56. D. London and A. Soni, *Phys. Lett.* **B407**, 61 (1997).
57. R. Fleischer and T. Mannel, *Phys. Lett.* **B511**, 240 (2001).
58. G. Hamel de Monchenault, talk at [http://moriond.in2p3.fr/EW/2003/](http://moriond.in2p3.fr/EW/2003/).
59. K. Abe et al. (Belle Collaboration), *Phys. Rev.* **D67**, 031102 (2003).
60. See, for instance, G. Hiller, *Phys. Rev.* **D66**, 071502 (2002); A. Datta, *Phys. Rev.* **D66**, 071702 (2002); M. Raidal, *Phys. Rev. Lett.* **89**, 231803 (2002);
B. Dutta, C.S. Kim and S. Oh, *Phys. Rev. Lett.* **90**, 011801 (2003);
M. Ciuchini, E. Franco, A. Masiero and L. Silvestrini, *Phys. Rev.* **D67**, 075016 (2003).

61. B. Aubert *et al.* (BaBar Collaboration), BABAR-CONF-03/11 [hep-ex/0303029], BABAR-CONF-03/002 [hep-ex/0303020].
62. R. Fleischer, *Phys. Lett.* **B459**, 306 (1999).
63. R. Fleischer, *Eur. Phys. J.* **C16**, 87 (2000).
64. B. Aubert *et al.* (BaBar Collaboration), *Phys. Rev. Lett.* **89**, 281802 (2002).
65. K. Abe *et al.* (Belle Collaboration), Belle preprint 2003-1 [hep-ex/0301032].
66. Particle Data Group (K. Hagiwara *et al.*), *Phys. Rev.* **D66**, 010001 (2002).
67. A.I. Sanda and K. Ukai, *Prog. Theor. Phys.* **107**, 421 (2002);
Y.-Y. Keum, DPNU-02-30 [hep-ph/0209208].
68. M. Gronau and D. London, *Phys. Rev. Lett.* **65**, 3381 (1990);
J.P. Silva and L. Wolfenstein, *Phys. Rev.* **D49**, 1151 (1994);
R. Fleischer and T. Mannel, *Phys. Lett.* **B397**, 269 (1997);
Y. Grossman and H.R. Quinn, *Phys. Rev.* **D58**, 017504 (1998);
J. Charles, *Phys. Rev.* **D59**, 054007 (1999);
M. Gronau, D. London, N. Sinha and R. Sinha, *Phys. Lett.* **B514**, 315 (2001);
M. Gronau and J.L. Rosner, *Phys. Rev.* **D66**, 053003 (2002).
69. A. Bornheim *et al.* (CLEO Collaboration), CLEO-03-03 [hep-ex/0302026].
70. J. Olsen, plenary talk at the 2nd Workshop on the CKM Unitarity Triangle, Durham, April 5–9, 2003, [http://ckm-workshop.web.cern.ch/ckm-workshop/](http://ckm-workshop.web.cern.ch/ckm-workshop/).
71. For a detailed overview, see M. Beneke and A. Lenz, *J. Phys.* **G27**, 1219 (2001).
72. I. Dunietz, *Phys. Rev.* **D52**, 3048 (1995);
R. Fleischer and I. Dunietz, *Phys. Rev.* **D55**, 259 (1997).
73. R. Fleischer and I. Dunietz, *Phys. Lett.* **B387**, 361 (1996).
74. A.S. Dighe, I. Dunietz, H.J. Lipkin and J.L. Rosner, *Phys. Lett.* **B369**, 144 (1996);
A.S. Dighe, I. Dunietz and R. Fleischer, *Eur. Phys. J.* **C6**, 647 (1999).
75. Y. Nir and D.J. Silverman, *Nucl. Phys.* **B435**, 301 (1990).
76. R. Fleischer, CERN-TH/2003-010 [hep-ph/0301255], to appear in *Phys. Lett.* **B**.
77. M. Gronau and J.L. Rosner, *Phys. Lett.* **B482**, 71 (2000);
P.Z. Skands, *JHEP* **0101**, 008 (2001).
78. G. D’Ambrosio and G. Isidori, *Phys. Lett.* **B530**, 108 (2002).
79. R. Fleischer, G. Isidori and J. Matias, CERN-TH/2003-039 [hep-ph/0302229], to appear in *JHEP*.
80. R. Aleksan, T.C. Petersen and A. Soffer, *Phys. Rev.* **D67**, 096002 (2003).
81. Y. Grossman, Z. Ligeti and A. Soffer, *Phys. Rev.** D67**, 071301 (2003).
82. M. Gronau, *Phys. Lett.* **B557**, 198 (2003).
83. A. Giri, Y. Grossman, A. Soffer and J. Zupan, [hep-ph/0303187].
84. I. Dunietz and R.G. Sachs, *Phys. Rev.** D37**, 3186 (1988) [E: *D39*, 3515 (1989)];
I. Dunietz, *Phys. Lett.* **B427**, 179 (1998);
M. Diehl and G. Hiller, *Phys. Lett.* **B517**, 125 (2001);
D.A. Suprun, C.W. Chiang and J.L. Rosner, *Phys. Rev.* **D65**, 054025 (2002).
85. R. Fleischer, CERN-TH/2003-011 [hep-ph/0301256], to appear in *Nucl. Phys. B*.
86. R. Fleischer, CERN-TH/2003-084 [hep-ph/0304027].
87. M. Gronau and D. London, *Phys. Lett.* **B253**, 483 (1991).
88. B. Kayser and D. London, *Phys. Rev.* **D61**, 116013 (2000).
89. K. Abe *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **88**, 052002 (2002);
T.E. Coan *et al.* (CLEO Collaboration), *Phys. Rev. Lett.* **88**, 062001 (2002);
B. Aubert *et al.* (BaBar Collaboration), BABAR-CONF-02/17 [hep-ex/0207092].
91. P. Krokovny et al. (Belle Collaboration), *Phys. Rev. Lett.* **90**, 141802 (2003).
92. D. London, N. Sinha and R. Sinha, *Phys. Rev. Lett.* **85**, 1807 (2000).
93. M. Gronau, D. Pirjol and D. Wyler, *Phys. Rev. Lett.* **90**, 051801 (2003).