AC Josephson Effect Induced by Spin Injection

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Pure spin currents can be injected and detected in conductors via ferromagnetic contacts. We consider the case when the conductors become superconducting. A DC pure spin current flowing in one superconducting wire towards another superconductor via a ferromagnet contact induces AC voltage oscillations caused by Josephson tunneling of condensate electrons. Quasiparticles simultaneously counterflow resulting in zero total electric current through the contact. The Josephson oscillations can be accompanied by Carlson-Goldman collective modes leading to a resonance in the voltage oscillation amplitude.

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I. INTRODUCTION

Electric and spin transport near ferromagnet-paramagnet interfaces received a large attention boost with the discovery of the giant magnetoresistance effect\textsuperscript{1} and the subsequent developments in magnetoelectronics and spintronics. Aronov\textsuperscript{2} and later Johnson and Silsbee\textsuperscript{3} theoretically predicted that an electric current through such an interface leads to an accumulation of nonequilibrium spin polarization with an accompanying spin current in the paramagnetic metal. A reverse effect also takes place, a pure spin current from the normal metal gives rise to an electric potential difference in the ferromagnet. The physics of these phenomena is quite simple. A sufficiently large difference of conductivities of spin-up and spin-down electrons in ferromagnets induces spin polarization of the electric current therein. Spin polarized currents passing through ferromagnet-paramagnet boundaries result in an accumulation of nonequilibrium magnetization near the interface. Both the spin injection and detection of this spin polarization has been experimentally demonstrated in Refs.\textsuperscript{4,5} in systems containing two or more junctions of thin normal metal wires with ferromagnets. One of them acts as a spin injector, while the other is a detector, where the voltage created by diffusing spins can be measured. Related spin-polarized transport phenomena have been investigated in many spintronic applications, such as giant magnetoresistance\textsuperscript{6}, spin Hall effects\textsuperscript{7}, current induced magnetization dynamics\textsuperscript{8}, spin-pumping\textsuperscript{9}, and spin caloritronics\textsuperscript{10}.

In the case of superconducting systems, spin injection and detection within a nonlocal setup similar to the one studied in Ref.\textsuperscript{4,5} was investigated both theoretically\textsuperscript{11,12} and experimentally\textsuperscript{13,14}. These studies have been focused on DC transport. They revealed a strong renormalization of spin-related transport parameters as compared to normal systems. These changes were mostly caused by the modified density of states in a superconductor. Beyond such quasi-particle transport properties, the macroscopic coherent state of the superconducting condensate can give rise to a quite different transport phenomenon associated with the spin-polarized transport.

Below we will consider an AC effect produced by a DC spin current towards a thin ferromagnetic contact. The DC potential induced by this polarization flux gives rise to an AC electric current of condensate electrons. Since in the considered experimental setup the total current of the superconducting and normal components must be zero, the AC condensate oscillations result in an AC potential difference between the opposite sides of the contact. A schematic of a possible experimental setup is shown in Fig. 1. A current is passed from a ferromagnet to a normal metal generating an associated spin accumulation and spin current therein. In the non-local geometry, this spin accumulation also diffuses transversely in a contacted normal metal towards another normal metal reservoir via a ferromagnet contact. The non-local potential $V$ increases with the injected DC current $I$ and the nonlocal resistance $R_{nl} = V/I$ describes the spin transport properties in the device. We will demonstrate that when the normal metals become superconducting, $R_{nl}$ acquires an AC component in addition to the DC comp-

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{A possible setup for observation of Josephson voltage oscillations. A spin current (dashed arrows) is injected by passing a DC current (solid arrow) from the ferromagnetic contact (brown). The DC spin current through the ferromagnetic contact between the right (R) and left (L) superconducting electrodes induces periodic oscillations of their electric potential difference $V$.}
\end{figure}
ponent in the normal state.

The outline of this paper is as follows. A model system used in our calculation is described in Sec. III. Also in this section we present a simple calculation of the AC voltage oscillations assuming a local thermodynamic equilibrium between quasiparticles and the condensate. A microscopic analysis based on coupled kinetic equations for the superconducting order parameter and the quasiparticle distribution function will be given in Sec. III. A discussion of results will be presented in Sec. IV.

II. AC VOLTAGE OSCILLATIONS IN A LOCAL THERMODYNAMIC EQUILIBRIUM

Our model system consists of two superconducting wires in contact via a spin-active barrier. We consider this contact to be weak in the form of a thin ferromagnetic layer with, if necessary, additional insulating layers. Such a barrier can be characterized by two resistances \( R_L \) and \( R_R \), corresponding to two spin eigenstates. We assume that a non-equilibrium spin polarization is created in the left wire (see Fig. 1), either by spin injection, as shown in Fig. 1, or by other means. Moreover, we assume that the electron’s energy relaxation is faster than their spin relaxation, so that up and down spin distributions can be characterized by the respective chemical potentials \( \mu_L^\uparrow \) and \( \mu_R^\uparrow \), resulting in the spin accumulation potential \( \delta \mu_s = \left( \mu_L^\uparrow - \mu_R^\uparrow \right) \). In the right (R) wire \( \delta \mu_s \) is much smaller, if the spin relaxation is faster than the influx of polarization from the left reservoir through the ferromagnetic contact. This is satisfied when the contact resistance is much larger than the resistance of the wire of the length \( l_s = \sqrt{D/\tau_s} \), where \( D \) is the diffusion constant and \( \tau_s \) is the spin relaxation rate. This is true in many practical cases, in particular, in the systems studied in Ref. 5,6.

We, therefore, simplify our model assuming \( \mu_R^\uparrow = \mu_R = -eV/2 \) and \( \mu_L^\uparrow + \mu_L^\downarrow = 2\mu + eV \), where \( \mu \) is the equilibrium chemical potential and \( V \) is the charge potential difference between two wires.

With these definitions the electric current through the contact in the normal state is

\[
I_n = \frac{V}{R_c} + \frac{\delta \mu_s}{2eR_s},
\]

where the inverse charge and spin resistances are given by \( R_c^{-1} = R_L^{-1} + R_R^{-1} \) and \( R_s^{-1} = R_L^{-1} - R_R^{-1} \), respectively. In an open circuit, electro-neutrality requires \( I = 0 \), and Eq. (1) gives \( V = -\delta \mu_s R_c/2eR_s = -\delta \mu_s P/2e \), where \( P = R_s^{-1}/R_c^{-1} \) is the spin current polarization of the contact. This is just the voltage induced by the spin current through the contact, as it has been experimentally demonstrated in Ref. 7,8.

Let us now consider this situation for superconducting wires. We assume that \( 2\delta \mu_s \ll k_BT_c \), so that the nonequilibrium spin polarization does not cause depairing.12 The difference between Cooper pair energies on opposite sides of the contact is 2eV. This potential difference gives rise to the AC Josephson current \( I_J \) of condensed electrons. In addition, electro-neutrality causes an oppositely directed current \( I_n \) of quasi-particles, so that the total electric current is zero. This results in DC and AC voltage differences between the left and right superconductors that we will now compute.

The simplest approach to this problem is based on the assumption that in the vicinity of the critical temperature \( T_c - T \ll T_c \), the quasiparticle current remains expressed by Eq. (1). We will discuss in the next section in which regime this approach is valid. Denoting the phase difference of the order parameters between the left and right wires as \( \phi \) and taking into account that \( d\phi/dt = -2eV/h \), electro-neutrality \( I_n + I_J = 0 \) and Eq. (1) dictates

\[
I_c \sin \phi - \frac{\hbar}{2eR_c} \frac{d\phi}{dt} + \frac{\delta \mu_s}{2eR_s} = 0,
\]

where \( I_c \) is the critical Josephson current. It is easy to see that when \( I_c \ll \delta \mu_s/2eR_c \), the Josephson current is dominated by harmonic oscillations with the frequency \( \omega = P \delta \mu_s/h \equiv 2eV_0/h \). Hence, the voltage induced by the spin current is

\[
V(t) = -V_0 - I_c R_c \sin \omega t.
\]

The DC component \(-V_0\) of this voltage is exactly the same as in the case of normal metals. Additionally, \( V(t) \) contains a term that oscillates with a frequency determined by the DC (normal state) contribution of the non-local signal. The magnitude of the oscillating voltage can be estimated by noting that at temperatures close to \( T_c \)

\[
I_c = \frac{\zeta}{R_c} \frac{\pi \Delta^2}{4ek_BT_c},
\]

where \( \Delta \) is the superconducting gap and \( \zeta \) is a dimensionless coefficient that takes into account the depairing effect inside the ferromagnetic contact layer, leading to exponential suppression of the Josephson current and, consequently to small values of \( \zeta \). It should be noted that according to Eqs. (3) and (4) the oscillation amplitude \( I_c R_c \) does not explicitly depend on the transmission coefficient, apart from the weak dependence through the depairing factor \( \zeta \).

III. NONEQUILIBRIUM EFFECTS AND COLLECTIVE MODES

The above analysis was based on the assumption that near the critical temperature, the current carried by quasiparticles can be represented by the expression in the normal state Eq. (1), ignoring small corrections associated with the gap in the quasiparticle spectrum. The small gap alone, however, does not justify this assumption. In particular, when quasiparticles are transmitted...
between the left and right wires they may not be in the local thermal equilibrium with the respective condensates, that have been assumed when deriving Eq. (2). To take into account nonequilibrium effects, one needs to consider time dependent transport and relaxation of the quasiparticles. There are two physical effects that determine kinetics of quasiparticles in the superconducting wires. The first one is the so-called charge (or branch) imbalance of electron and hole excitations. It is produced by quasiparticle tunneling between superconducting electrodes, leading to a quasiparticle distribution with a local chemical potential different from that of the condensate. This difference relaxes during a time much longer than the electron-phonon scattering time. Another effect is related to condensed space-time oscillations. It dominates over the charge imbalance relaxation when \( \omega \) is large enough. We will demonstrate that the spin injection then enables detection of collective condensate-quasiparticle modes, Carlson-Goldman modes which are characterized by oppositely directed oscillations of condensate and normal fluids. There is an important difference with respect to the usual Josephson effect, since our device requires no net current \( I = 0 \). The usual Josephson effect does not couple to Carlson-Goldman modes and is not reduced at low temperatures, \( T \rightarrow 0 \). In contrast, to provide a counterflow we need excitations that vanish at low temperatures. The coupling to the collective modes is enabled by a spin-driven battery effect induced by the spin injection.

Let us now detail the calculations. Assuming a small deviation from equilibrium we employ the linearized time dependent kinetic and Ginzburg-Landau equations in the diffusive regime, when the elastic mean free path is much less than the superconductor’s coherence length, as well as other relevant length scales. In this case, the isotropic quasiparticle distribution function \( f(E,t) \), where \( \sigma \) is the spin projection, depends only on the energy \( E \) and time \( t \). Within the linear theory the single condensate couples to the spin-independent part \( f(E,t) = \left( f_{\uparrow}(E,t) + f_{\downarrow}(E,t) \right) / 2 \) of the distribution function. Therefore, after ignoring small terms \( (\delta \mu / k_B T)^2 \), the unperturbed spin-independent distributions takes the form of Fermi equilibrium functions \( f^{eq}_{\uparrow/\downarrow}(E,t) \) of the left and right wires with respective electrochemical potentials \( eV/2 + \mu \) and \( -eV/2 + \mu \). In its turn, the corresponding gap functions of unperturbed condensates are \( \Delta \exp(i \phi/2 - 2 i \mu t) \) and \( \Delta \exp(- i \phi/2 - 2 i \mu t) \). It is easy to see that in this unperturbed state the spin independent contribution to the quasiparticle current through the contact is given by the first term in the right-hand side of Eq. (1). Taking into account above condensate functions one can easily obtain Eqs. (2) and (3). In the perturbed state we have \( f(E,t) = f^{eq}(E,t) + \delta f(E,t) \) (we will skip here and below the labels \( L \) and \( R \)). Since the perturbation violates the electron-hole symmetry, it gives rise to a spatially dependent potential \( \phi(r,t) \) near the contact. Also, a correction to the order parameter \( \delta \Delta(r,t) \) appears. In order to simplify the further analysis, we assume that \( \omega \ll \Delta \) and \( 1/\tau_E \ll \Delta \), where \( 1/\tau_E \) is the electron-phonon relaxation rate. Besides that, the critical supercurrent \( I_c \) is taken small enough, so that the time dependence of all functions is dominated by harmonic oscillations. Accordingly, we introduce the time Fourier components \( \delta f_{\omega}(r,E) \), \( \delta \Delta_{\omega}(r) \) and \( \phi_{\omega}(r) \). From Refs. [18,21] it follows that \( f_{\omega} \) obeys the kinetic equation

\[
(-i \omega N_1 - \tilde{D} \nabla^2 + 2 \Delta N_2) \delta f_{\omega} + \omega N_1 f_0 e^{i \phi} \delta \bar{\omega} - \omega N_2 f_0 \delta \Delta_{\omega} = I_{st},
\]

where \( f_0 = 1/(4 k_B T \cosh^2(E/2k_B T)) \) and \( I_{st} \) is the electron-phonon scattering integral, whose explicit form can be found in Ref. [18,21]. Furthermore, \( \tilde{D} = D(N_1^2 + N_2^2) \), where \( N_1 \) and \( N_2 \) are the spectral functions:

\[
2N_{1/2} = G_{1/2}(E + \bar{\omega}/2) + G_{1/2}(E - \bar{\omega}/2),
\]

where \( G_1(E) = E/\sqrt{E^2 - \Delta^2} \) and \( G_2(E) = i \Delta/\sqrt{E^2 - \Delta^2} \), with \( \bar{\omega} = \omega + i/\tau_E \). In its turn, the linearized Ginzburg-Landau equation takes the form

\[
-\omega \delta \Delta_{\omega} + \frac{8i k_B T \Delta}{\pi |\Delta|} \int dE N_2 \delta f_{\omega} = D \nabla^2 \delta \Delta_{\omega}.
\]

We will employ the above equations for the analysis of our model in two limiting cases of weak and strong energy relaxation versus the Josephson frequency, \( \tau_{E \omega} \ll 1 \) and \( \tau_{E \omega} \gg 1 \), corresponding to very different physical situations. In the former case slow time variations of \( \delta f \) may be ignored, so that the quasiparticle kinetics is dominated by the charge imbalance of electron and hole excitations. The deviation from the thermodynamic equilibrium decreases with increasing distance from the contact on the characteristic length scale \( \sqrt{D\tau_R} \), where \( \tau_R = 4k_B T_0 \tau_E / \pi \Delta \) is the charge imbalance relaxation time, that is much longer than \( \tau_E \). In the opposite high-frequency regime inelastic collision processes are not important, because the quasiparticle distribution oscillates fast. Therefore, one can neglect \( 1/\tau_E \) and \( I_{st} \) in Eqs. (17). In this case, since Josephson oscillations of the condensate take place at zero total current, they strongly couple to Carlson-Goldman modes. Therefore, one can expect such modes to be excited near the contact and propagate along the left and right wires. In both low-frequency and high-frequency regimes, using Eqs. (17) with a reduced form of \( I_{st} \) from Refs. [21] and [18] and taking into account the zero electric current condition, one arrives to the equation for the potential

\[
\kappa^2(\omega) \psi = \nabla^2 \psi, \tag{8}
\]

where \( \kappa^2(\omega) = 1/D \tau_R \) at \( \tau_{E \omega} \ll 1 \) and

\[
c_s^2 \kappa^2(\omega) = -\omega^2 - i \pi \omega \Delta^2 / 4 k_B T \tag{9}
\]

at \( \tau_{E \omega} \gg 1 \), where the sound velocity \( c_s = \sqrt{2D \Delta} \). Eq. (8) is well known. At \( \tau_{E \omega} \ll 1 \) it describes the charge imbalance relaxation [17,18], while in the opposite limit it gives the dispersion of Carlson-Goldman modes[22].
For our geometry, when $\kappa^{-1}$ is much larger than the width and thickness of the wire, $\varphi_\omega$ depends only on the coordinate $x$ along the wire. Then, at $\tau_E\omega \ll 1$, $\varphi_\omega$ exponentially decreases with increasing distance from the contact, while at $\tau_E\omega \gg 1$ it shows decaying oscillations. We assume that the left and right wires are of the same length $L$. Since the system is symmetric with respect to $x \to -x$, the oscillating part of the electrochemical potential is $-V_\omega/2 + \varphi_\omega(x)$ at $x > 0$ and $V_\omega/2 - \varphi_\omega(-x)$ at $x < 0$, where $V_\omega$ denotes the Fourier component of $V(t)$. The solution of Eq. (8) has the form

$$\varphi_\omega(x) = \alpha e^{\kappa x} + \beta e^{-\kappa x}$$

with the boundary conditions $\nabla_x \varphi_\omega(\pm L) = 0$ and

$$V_\omega - 2\varphi_\omega(0) = A\sigma \nabla_x \varphi_\omega(0),$$

where $A$ is the wire cross-section area and $\sigma$ is the normal state conductivity. These boundary conditions provide a zero electric current of quasiparticles at the wire ends and the current equal to the injected one at $x = 0$. From Eqs. (8), (10) - (11) one obtains a periodic part of the injected current

$$\frac{V_\omega - 2\varphi_\omega(0)}{R_c} = \frac{V_\omega}{R_c + 2R_w} = \frac{V_\omega}{R_{eff}(\omega)},$$

where

$$R_w = \frac{1}{A\sigma \kappa} \left(1 + e^{-2\kappa L}\right), \quad \text{Re}(\kappa) > 0.$$  

Hence, the result is a renormalization of $R_c$ in Eq. (11), such that $R_c \to R_c + 2R_w$. To find the voltage $V_\omega$, the quasiparticle current (12) must be equated with the Josephson current. By this way we obtain a new expression for a time dependent part of $V$, instead of the second term in the right-hand side of Eq. (3):

$$V + V_0 = -I_c[\cos \omega t \text{Im}R_{eff}(\omega) + \sin \omega t \text{Re}R_{eff}(\omega)]$$  

IV. DISCUSSION

Let us analyse above results in some limiting cases. Since $\kappa(\omega) \to \infty$ in both cases of high frequencies $\omega \to \infty$ and strong energy relaxation $\tau_E \to 0$, it follows from Eq. (13) that $R_w \to 0$. We thus obtain Eq. (8), that is an expected result, because in these limits a deviation from equilibrium is small. On the other hand, the nonequilibrium effect of quasiparticle’s kinetics becomes strong when $R_c \lesssim 2R_w$. The assumed linearization condition, however, restricts this inequality. This condition can be expressed in the form $\pi R_w/R_c \ll 1$. Therefore, the linear theory allows $R_c \lesssim R_w$ only at small $\xi$. It should be noted that, according to Eq. (12), $R_w$ can be enhanced due to resonances of Josephson oscillations with collective modes at $\text{Im}\kappa L = \pi n$, if they are not overdamped (if $\text{Re}\kappa L \ll 1$). $R_w$ also increases at small enough $L$, when $2|\kappa|L \ll 1$. In practice $R_w$ may be varied in quite wide range. In Al wires from Ref. (6) $V_0 = 10^{-6}$ V, resulting in $\omega^{-1} \approx 0.3 \cdot 10^{-9}$ s. Since $\omega^{-1} \approx \tau_E^{-1} \approx 10^{-9}$, a regime intermediate between charge imbalance relaxation and generation of Carlson-Goldman modes will be realized, with $\kappa^{-1}$ about several $\mu$m. Therefore, strong resonances in $R_w$ are not expected. One can evaluate $R_w \approx 50\Omega$, that is much less than $R_c = 600\Omega$. Hence, in the considered parameter range Eq. (3) remains valid.

In samples with higher polarizations $P$ and at larger spin current through the contact the Josephson oscillation frequency is expected to be large enough to produce noticeable collective resonances of $R_{eff}$ in Eq. (12).

The above calculations of Josephson voltage oscillations have been restricted to $\Delta \ll T_c$. At the larger gap the oscillation amplitude is expected to decrease, because less excitations are available to compensate the supercurrent through the contact. On the other hand, in this range one should take into account that besides quasiparticles the spin transport through the contact can be associated with triplet components of Cooper pair states that appear due to spin dependent tunneling and nonequilibrium spin polarization of superconducting wires. Further studies are needed to understand the effect of such transport.

In conclusion, we considered an AC Josephson effect induced by a DC spin current through the contact whose transmittance depends on the spin orientation of tunneling electrons. The oscillations of the voltage across the contact at zero electric current have, in certain parameter range, harmonic time dependence with the frequency proportional to the spin current. The amplitude and phase of these oscillations depend on coupled kinetics of quasiparticles and condensate in superconducting wires. The corresponding calculations have been performed within linearized kinetic equations at temperature close to $T_c$. We predict that at the high enough frequency the measured AC voltage will show up the resonance structure associated with excitation of Carlson-Goldman modes.

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