Prospect Theory and Interval-Valued Hesitant Set for Safety Evacuation Model

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Abstract. The study applies the research results of prospect theory and multi attribute decision making theory, combined with the complexity, uncertainty and multifactor influence of the underground mine fire system and takes the decision makers’ psychological behavior of emotion and intuition into full account to establish the intuitionistic fuzzy multiple attribute decision making method that is based on the prospect theory. The model established by this method can explain the decision maker’s safety evacuation decision behavior in the complex system of underground mine fire due to the uncertainty of the environment, imperfection of the information and human psychological behavior and other factors.

1. Introduction
Researchers have proposed various decision-making methods for multi attribute decision making problem. Most of them are based on Expected Utility Theory, namely that the decision maker is completely rational. In 1979, Kahneman and Tversky put forward the prospect theory [1] based on “Bounded rational people”. The prospect theory takes into account that the decision maker does not always pursue the maximum utility in behavior, but chooses the program that satisfies him most. The prospect theory well reflects the decision maker’s subjective risk preference and has been widely used in all kinds of research on multiple attribute decision making [2-4]. The multiple attribute decision making problem can be seen everywhere in the social economic system. However, in many decision-making method researches based on the prospect theory, the problem of group decision making is rare. The common group decision making method is to assemble the decision values of each expert by giving different weights. But when give different weights to each decision expert, subjective preferences will be involved. In order to avoid this defect, the decision experts will use anonymous methods to score decisions. Under the condition of anonymity, Torra and Norukawa put forward the hesitant fuzzy set [5] in view of how to deal with multiple values of each indicator attribute. The advantage of a hesitant fuzzy set is that it allows more than one element in the set. Therefore, under the condition of group decision, the attribute value of each decision index is a hesitant fuzzy element and each decision plan constitutes a hesitant fuzzy set. Many scholars study the fuzzy set and its application in decision making [6-8].

In case of sudden underground mine fire, the decision-making problem faced by the decision maker is multi-attribute decision making problem. The attribute information of complex multi-attribute decision making problem is usually not expressed in precise number due to its objective uncertainty
and complexity as well as subjective knowledge limitations of the decision maker. Fuzzy numbers and fuzzy sets provide tools for attribute value information representation. Intuitionistic fuzzy sets can explain the establishment of decision model based on the reasonable tree structure due to the decision maker’s subjectivity and the association between decision attributes. Because of the complexity and uncertainty of objective things, and people often have vague thinking in underground mine accidents, the attribute value of indicators is presented in the form of an indeterminate interval number instead of an exact value.

The study applies the research results of prospect theory and multi attribute decision making theory, combined with the complexity, uncertainty and multifactor influence of the underground mine fire system and takes the decision makers’ psychological behavior of emotion and intuition into full account to establish the intuitionistic fuzzy multiple attribute decision making method that is based on the prospect theory. The model established by this method can explain the decision maker’s safety evacuation decision behavior in the complex system of underground mine fire due to the uncertainty of the environment, imperfection of the information and human psychological behavior and other factors.

2. Establishment of Safety Evacuation Behavior Decision Model for Underground Mine

2.1. Model Description

Suppose there are m evacuation plans $A = (a_1, a_2, \cdots, a_m)$ in the underground mine fire environment; n evaluation indicators $C = (c_1, c_2, \cdots, c_n)$, written as $M = \{1, 2, \cdots, m\}$, $N = \{1, 2, \cdots, n\}$. The weighting of indicator attribute $c_j$ is $w_j$ and $0 \leq w_j \leq 1, \sum_{j=1}^{n} w_j = 1$, weighting vector $w = \{w_1, w_2, \cdots, w_n\}$. The state set of attributes $c_j$ is $\lambda = \{\lambda_1, \lambda_2, \cdots, \lambda_i\}$, so the attribute value of decision $a_i$ under the $\lambda_i$ state of attribute $c_j$ is $x^i_{ij}$ and its occurrence probability is $p^i_{ij}$.

2.2. Decision-Making Model

(1) Set intuitionistic fuzzy prospect. Suppose there are m prospects to be evaluated

$$\tilde{f} = (x_{i1}, p_{i1}; x_{i2}, p_{i2}; \cdots; x_{in}, p_{in});$$

$\tilde{x}_{ij}$ stands for the attribute value of prospect $a_i$ under the state $\lambda_i$ of attribute $j$, and $\tilde{p}_{ij}$ is the corresponding occurrence probability of $\tilde{x}_{ij}$, $1 \leq i \leq m, 1 \leq j \leq n$.

(2) Set the interval value of personal psychological factor

Interval intuitionistic fuzzy definition $\tilde{a} = [a^l, a^u] = \{x \mid a^l \leq x \leq a^u, a^l, a^u \in R\}$, and $\tilde{a}$ is an interval number. $\tilde{a}_{ij}$ locates in the middle of interval $[a^l_{ij}, a^u_{ij}]$, $(a^l_{ij} + a^u_{ij}) / 2$ is the most possible, so suppose $\tilde{a}_{ij}$ takes the median as the decision reference point, and the reference point shall be determined by the decision maker of the underground mine safety evacuation decision according to personal psychological characteristics and risk preference. Evacuation decisions influenced by individual psychological preferences are more real. In the process of safety evacuation, the evacuation behavior depends heavily on individual psychology influence.

$$\tilde{a}_{ij} = (a^l_{ij} + a^u_{ij}) / 2$$
(3) Calculate the prospect value function  
The benefit value and loss value function of reference point $\tilde{x}_y$ relative to reference point $\tilde{a}$:

$$
\nu(\tilde{x}_y) = \nu(\Delta x_i) = \begin{cases} 
    [d(x_i, \tilde{a})]^\gamma, & x_i - \tilde{a} \geq 0 \\
    [-d(x_i, \tilde{a})]^\gamma, & x_i - \tilde{a} < 0
\end{cases}
$$

(4) Calculate the decision weighting function based on the prospect theory  
The decision weighting function is obtained according to Kahneman and Tversky's parametric method:

$$
w_j^\delta = \frac{\delta(p_j^\delta)}{\delta(p_j^\delta) + (1 - p_j^\delta)} \\
\pi^+(w_j) = \frac{w_j^\delta}{(w_j^\delta + (1 - w_j^\delta))^{\nu_\delta}}, \quad \pi^-(w_j) = \frac{w_j^\delta}{(w_j^\delta + (1 - w_j^\delta))^{\nu_\delta}}
$$

(5) Calculate safety evacuation decision value $\nu(f_i)$  
The total prospect value $\nu(f_i)$ of safety evacuation decision is the sum of positive and negative prospect values, namely

$$
\nu^+(f_i) = \sum_{j=1}^{n} v_j^\delta \pi^+(w_j), \quad \nu^-(f_i) = \sum_{j=1}^{n} v_j^\delta \pi^-(w_j), \quad \nu(f_i) = \nu^+(f_i) + \nu^-(f_i)
$$

(6) Sort according to the value of $\nu_i$ to get the pros and cons of the plan.

3. Behavior Simulation Experiment of the Underground Mine Evacuation Personnel  
(1) Evacuation personnel agent behavior setting  
According to the setting behavior rules of Agent for evacuation personnel, set four types of Agent behavior individual, namely $A_{Se}$, $A_{Be}$, $A_{Ne}$, $A_{Fe}$.

(2) Determination of evacuation behavior decision  
Agent of the simulation system includes three types, L-A type, E-A type and C-A type. Take the decision-making process of L-A Type Agent as an example to elaborate how the evacuation personnel make evacuation behavior decision when underground mine fire suddenly happens.

In the simulation of this study, there are 3 evacuation paths in the underground mine fire situation, namely $a_1, a_2, a_3$. Now evaluate the three evacuation plans based on three decision indexes $c_1, c_2, c_3$, and the corresponding weight vector $\omega = (\omega_1, \omega_2, \omega_3) = (0.4, 0.1, 0.5)$. And then suppose the three decision indexes all have a state set $\lambda = \{\lambda_1, \lambda_2\}$, namely (safe, dangerous). The decision process of L-A type Agent is as follows.

Step 1: set intuitionistic fuzzy prospect, as shown in table 1.
Table 1. Decision Matrix and Reference Point

|      | $c_1$ |   | $c_2$ |   | $c_3$ |   |
|------|-------|---|-------|---|-------|---|
|      | $\lambda_1$ | $\lambda_2$ | $\lambda_1$ | $\lambda_2$ | $\lambda_1$ | $\lambda_2$ |
| $a_1$ | 0.4, 0.7, 0.9 | 0.1, 0.7, 0.8 | 0.5, 0.6, 0.7 | 0.3, 0.4, 0.7 | 0.2, 0.5, 0.6 | 0.3, 0.6, 0.9 |
| $a_2$ | 0.3, 0.5, 0.6 | 0.2, 0.4, 0.7 | 0.3, 0.4, 0.8 | 0.4, 0.5, 0.5 | 0.4, 0.6, 0.8 | 0.4, 0.5, 0.8 |
| $a_3$ | 0.5, 0.6, 0.7 | 0.4, 0.5, 0.7 | 0.2, 0.7, 0.8 | 0.3, 0.6, 0.7 | 0.3, 0.5, 0.8 | 0.4, 0.5, 0.7 |
| $p_{ij}$ | 0.37 | 0.62 | 0.37 | 0.62 | 0.37 | 0.62 |

Step 2: Set the risk preference interval value caused by personal psychological factor. According to the attribute characteristics of L-A type Agent, set the single reference point in the risk preference interval caused by the psychological state when evacuate in the fire: $\tilde{a} = (0.4, 0.5, 0.6)$.

Step 3: Calculate prospect value function $v_{ij}$, according to the value suggestion of Tversky and Kahneman’s parametric method, $\alpha = \beta = 0.88, \lambda = 2.25$ and we can get from formula (6).

$$v(\tilde{x}_j) = v(\Delta x_j) = \left\{ \begin{array}{ll} [d(x_j, \tilde{a})]^{0.88}, & x_j - \tilde{a} \geq 0 \\ [-2.25d(x_j, \tilde{a})]^{0.88}, & x_j - \tilde{a} < 0 \end{array} \right.$$

(7)

According to Hamming distance formula.

$$d(\tilde{a}_1, \tilde{a}_2) = \frac{1}{n} \left( \sum_{j=1}^{n} |\mu_{a_1} a_{ij} - \mu_{a_2} a_{ij}| + |(1 - v_{a_1}) b_{ij} - (1 - v_{a_2}) b_{ij}| \right)$$

(8)

Calculate according to formula (6) and we can get.

$$\Delta \tilde{v}_{ij}^1 = 0.167, \ \Delta \tilde{v}_{ij}^2 = 0.233, \ \Delta \tilde{v}_{ij}^3 = 0.067, \ \Delta \tilde{v}_{ij}^4 = -0.133, \ \Delta \tilde{v}_{ij}^5 = -0.067, \ \Delta \tilde{v}_{ij}^6 = 0.167,$$

$$\Delta \tilde{v}_{ij}^{11} = -0.033, \ \Delta \tilde{v}_{ij}^{12} = -0.400, \ \Delta \tilde{v}_{ij}^{13} = 0.133, \ \Delta \tilde{v}_{ij}^{22} = -0.033, \ \Delta \tilde{v}_{ij}^{23} = 0.200, \ \Delta \tilde{v}_{ij}^{23} = 0.133, \ \Delta \tilde{v}_{ij}^{33} = 0.067, \ \Delta \tilde{v}_{ij}^{33} = -0.067, \ \Delta \tilde{v}_{ij}^{33} = -0.033.$$

$\tilde{v}_{ij}^i$ stands for the prospect value function value of plan $a_i$ under the state $\lambda_i$ of attribute $c_j$, and $i = (1, 2, 3); j = (1, 2, 3), t = (1, 2)$.

$$\tilde{v}_{ij}^1 = 0.210, \ \tilde{v}_{ij}^2 = 0.096, \ \tilde{v}_{ij}^3 = 0.274, \ \tilde{v}_{ij}^4 = -0.378, \ \tilde{v}_{ij}^5 = -0.217, \ \tilde{v}_{ij}^6 = 0.210,$$

$$\tilde{v}_{ij}^{11} = -0.103, \ \tilde{v}_{ij}^{12} = -0.103, \ \tilde{v}_{ij}^{13} = 0.166, \ \tilde{v}_{ij}^{22} = -0.103, \ \tilde{v}_{ij}^{23} = 0.243, \ \tilde{v}_{ij}^{23} = 0.166,$$

$$\tilde{v}_{ij}^{31} = 0.096, \ \tilde{v}_{ij}^{32} = 0.046, \ \tilde{v}_{ij}^{33} = 0.243, \ \tilde{v}_{ij}^{33} = 0.466, \ \tilde{v}_{ij}^{33} = -0.209, \ \tilde{v}_{ij}^{33} = -0.103.$$

Step 4: Calculate the function value of decision weighting.

The weight function value is obtained according to the value suggestion of Tversky and Kahneman’s parametric method that parameter $\delta = 0.61$ and $\gamma = 0.69$. 

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$w_1^i = 0.297, w_2^i = 0.461,$
$w_1^j = 0.297, w_2^j = 0.461,$
$w_1^k = 0.297, w_2^k = 0.461$

Step 5: Calculate the total prospect value of alternative evacuation plans to get,

$$\nu(f_i) = 0.0296, \nu(f_j) = 0.0513, \nu(f_k) = 0.0175$$

Step 6: Sort according to the size of each plan $\nu(f_i)$ to get the pros and cons of the plan,

$$\nu(f_j) > \nu(f_i) > \nu(f_k)$$

According to the interval intuitionistic fuzzy multiple attribute decision making method based on the prospect theory, the evacuation paths have been analyzed when L-A type Agent meets underground mine fire according to his perception and judgment of the current environment and state combine with his knowledge base system. According to the decision model of fire evacuation behavior in underground mine fire, set the path parameter index and the physiological and psychological preference of L-A type Agent to carry out model calculation. Finally, the evacuation plans are sorted as $a_2 > a_1 > a_3$ when underground mine fire suddenly happens. E-A and C-A type Agent’s evacuation behavior decision plans can be respectively sorted according to the above method. The simulation process of underground mine fire is a dynamic process, and the environment of underground mine personnel is changing in real time. In the simulation process, calculate according to the decision model of fire evacuation behavior in underground mine fire so as to determine the decision process of L-A type, E-A type and C-A type Agent to $A_{SE}, A_{BE}, A_{NE}, A_F$ evacuation behavior.

4. Conclusion

The study takes into account the multi attribute safety evacuation problem whose attribute value is interval hesitant fuzzy set. Calculate the prospect value function according to the constructed model to construct the interval intuitionistic fuzzy multi-attribute evacuation behavior decision model based on the prospect theory so as to solve the problem of the decision maker’s safety evacuation decision behavior in the complex system of underground mine fire due to the uncertainty of the environment, imperfection of the information and human psychological behavior and other factors.

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