Construction Method of Geomagnetic Reference Map for Satellite Communication Navigation through Kriging Method

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Abstract. The construction of the geomagnetic reference map is the basis of geomagnetic navigation, but the geomagnetic reference map established directly from the measured data is generally difficult to meet the accuracy requirements of the geomagnetic navigation. A method of constructing geomagnetic reference map based on the combination of Kriging method and BP neural network is proposed. Through the establishment of simulation models and interpolation experiments, the relationship between the interpolation errors of the two algorithms and the total magnetic field gradient at the interpolation point is analyzed. The interpolation error of Kriging interpolation in the region with large gradient is greater than that of BP neural network, and the interpolation error in the region with small gradient is smaller than that of BP neural network. Kriging interpolation has greater errors in areas with higher gradients than that of BP neural network. As the gradient increases, the difference between the cumulative error of the BP neural network and the Kriging method has an obvious point of inflection, indicating that in the region where the gradient is greater than the point of inflection coordinates, the Kriging method interpolation result can be replaced by the BP neural network prediction value to improve the interpolation accuracy. Experimental result shows that the method is effective.

Keywords: Kriging, BP neural network, interpolation, reference map, geomagnetic navigation.

1. Introduction
Geomagnetic navigation \cite{1} has the advantages of good concealment, strong autonomy, and strong anti-interference ability. It can be used as an aided navigation method for inertial navigation. The geomagnetic matching method is applied to the detection and recognition of magnetic targets, and can make up for the high false alarm rate of the existing detection methods. Therefore, the geomagnetic matching method has broad development prospects \cite{2}.

The principle of geomagnetic matching \cite{3} is to compare the real-time data measured by the magnetic sensor with the known geomagnetic reference map of the area to obtain information such as the location of the target. Therefore, the construction of a high-precision local geomagnetic reference map is a prerequisite for geomagnetic matching.
The current methods for constructing geomagnetic reference map mainly include the geomagnetic model method and the spatial interpolation method. The two methods cannot meet the accuracy requirements of geomagnetic matching [4-5]. The construction of an accurate geomagnetic reference map is usually carried out by an aircraft or ship carrying sensors and navigating the target area. However, due to the large interval between survey lines, the small amount of data and some areas not convenient for aircraft or ship to be measured, the measured data cannot directly used to construct the geomagnetic reference map, and the measurement data needs to be spatially interpolated. Currently, the commonly used methods include Inverse Distance to a Power (IDP), Kriging interpolation algorithm [6]. Both are accurate interpolation based on the value of the sample points around the unknown point. The IDP is based on a specific formula. The calculation speed is fast, but the "bull's eye" effect is prone to occur. Kriging interpolation algorithm not only considers the distance of the surrounding sample points, but also considers the spatial correlation of the sample points. However, the Kriging interpolation algorithm has larger interpolation errors in areas with larger gradients. On these basis, the universal Kriging interpolation method avoids the second-order stationary assumption of the ordinary Kriging interpolation method [7]; Kriging interpolation method based on PSO-GA algorithm uses two algorithms alternately to optimize the Kriging algorithm, which has a smaller variance than using Kriging interpolation alone [8]; The result of interpolation algorithm based on BP neural network is relatively stable and the error value is relatively small. When factors such as elevation and time are further considered, the BP neural network modeling method will have more advantages [9]. This paper proposes a method of constructing geomagnetic reference map based on Kriging method and BP neural network, and verifies the effectiveness of the algorithm through simulation data and measured data.

2. Kriging Interpolation and BP Neural Network Interpolation Algorithm

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2.1. Introduction to Kriging Algorithm

Ordinary Kriging (OK) interpolation method is an interpolation algorithm based on the statistical characteristics of data. The algorithm considers the relationship between points in space and is the best unbiased estimate [8, 10].

The equation of OK algorithm to estimate the interpolation point is

\[ Z = \sum_{i=1}^{m} w_i Z_i \]  

\( Z \) is the estimated value of the geomagnetic element at the interpolation point, \( w_i \) is the weight coefficient of the \( i \)th measuring point, \( Z_i \) is the value of the geomagnetic element at the \( i \)th measuring point, and m is the number of spatial measuring points.

\( \hat{Z} \) is an unbiased estimate of \( Z_i \),

\[ \sum_{i=1}^{m} w_i E(Z_i) = E(\sum_{i=1}^{m} w_i Z_i) \]  

And \( \sum_{i=1}^{m} w_i = 1 \).

The semi-variance function fitting model used in this paper is a spherical model, and its formula is...
\[
\gamma(h) = \begin{cases} 
0, & h = 0 \\
C_0 + C \left( \frac{3h}{2a} - \frac{h^3}{2a^3} \right), & 0 < h < a \\
C_0 + C, & h > a
\end{cases}
\]

(3)

Where \(a\) is the correlation distance, \(h\) is the distance between two measurement points, \(C_0\) is the nugget value, and \(C_0 + C\) is the sill. Organize the above formula

\[
\sum_{i=1}^{m} w_i \gamma(h_{i,j}) - \mu = \gamma(h_{i,0})
\]

(4)

2.2. BP Neural Network

The neural network is composed of many neurons that operate in parallel. After training with a large amount of data, it can approximate complex nonlinear systems. In theory, it can approximate the nonlinear function defined in the density with arbitrary precision [11].

The essence of constructing a local geomagnetic map is to regard the magnetic characteristic value of the target area as a nonlinear system about longitude, latitude, height, time, etc. Through the measurement data of many known points, a nonlinear system is established, and then the magnetic characteristics of unknown points are estimated. Artificial neural network is suitable for modeling this kind of complex nonlinear system. This paper uses Back Propagation (BP) learning algorithm, namely BP neural network, as shown in Figure 1. It uses the output error of the latter layer to estimate the error of the previous layer and then obtain the error estimation of all layers, and realize the approximation of the nonlinear system through the training of a large amount of data.

![Figure 1. Sketch of BP neural network with two hidden layers.](image-url)
2.3. Parameter Selection of BP Neural Network

The detection area is generally in the range of several hundred to several kilometers. The changes in altitude and time can be ignored. The input layer is set to 2 neurons, which are longitude and latitude, and the output layer is 1 neuron, which is the magnetic field value. Considering that the magnetic environment is more complicated and there are many magnetic objects in the target area, a double hidden layer network structure is selected. Use the measured data to train in the matlab2012b environment, compare the root mean square error under different neuron numbers, activation functions and training functions, and select a set of parameters with the smallest error.

3. The Application of Two Algorithms in Dipole Simulation Data

A combination of several dipole magnetic sources is used to simulate the Magnetic anomaly background field. The parameters are set as: local magnetic declination angle \( D = 0^\circ \), magnetic inclination angle \( I = 45^\circ \), and the position of the three magnetic sources are \((500,800,-200)\), \((600,1200,-200)\), \((1000,400,-200)\), magnetic moments \( m = 2 \times 10^4 \text{Am}^2 \), \( d = 0^\circ \), \( i = 45^\circ \), the aircraft flies in the horizontal direction in plane \( z = 200 \), speed \( v = 360 \text{km/h} \), sampling rate 20Hz. The magnetic anomaly in the 2000m\( \times \)2000m measurement plane is shown in Fig. 2, and the magnetic field contour is shown in Fig. 3, with the unit nT. The straight line with arrows in Fig. 3 represents the plane survey line with a spacing of 100m, \( y = 0,100,200,\ldots,2000 \) .

![Figure 2. Amplitude of simulated magnetic anomaly data.](image)

![Figure 3. Contour of magnetic anomaly data.](image)
Based on the total magnetic field anomaly, add Gaussian white noise with a mean value of 0 and a variance of 1% of the average magnetic anomaly in the area. Both two algorithms are used to interpolate the known survey lines. The 50m×5m grid data will be generated from the 100m×5m magnetic field grid data. Kriging interpolation is based on surfer16 software, and BP neural network is based on matlab neural network toolbox. The double hidden layer network structure is selected. The number of neurons in both hidden layers is 15. The training function is Bayesian regularization algorithm. The activation function is a tangent sigmoid transfer function

\[
f(x) = \frac{2}{1 + e^{-x}} - 1 \quad (-1 < f(x) < 1)
\] (5)

The magnetic anomaly maps interpolated by two methods are shown in Figure 4 and 5.

![Figure 4. Magnetic anomaly map interpolated by Kriging.](image1)

![Figure 5. Magnetic anomaly map interpolated by BP neural network.](image2)

Because the amount of simulation data is sufficient and the measurement error is small, from a qualitatively point of view, the interpolation effect of the two methods is good and the image is smooth.
To analyze the suitability of the two interpolation methods better, the errors of the two interpolation methods are quantitatively compared from the three indicators of maximum interpolation error (MAX), mean absolute error (MAE) and root mean square error (RMSE).

\[
\text{MAE} = \frac{1}{N} \sum_{i=1}^{N} |\hat{Z}_i - Z_i| \quad (6)
\]

\[
\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\hat{Z}_i - Z_i)^2} \quad (7)
\]

The error of the two interpolation methods are shown in Table 1.

| Interpolation method | MAX/nT | MAE/nT | RMSE  |
|----------------------|--------|--------|-------|
| Kriging              | 0.00341| 0.00048| 0.00107|
| BP                   | 0.00563| 0.00084| 0.00114|

We can see from Table 1 that the interpolation errors of the two methods are not much different, and the Kriging method has a slightly smaller error than the BP neural network method.

Calculate the absolute errors of the two interpolation methods at each measuring point on the \(y = 50,150,250 \ldots 1950\)m survey line, define \(e = |\text{error(BP)}| - |\text{error(Kriging)}|\), which denotes the pros and cons of the two interpolation methods. When \(e > 0\), it can be considered that the interpolation error of the BP neural network at this point is better than Kriging. For this point, the Kriging method is better, and vice versa. Draw the value of \(e\) distribution of each interpolation point as shown in Figure 6 and 7. If the interpolation value is large, the Kriging method is better for this point, while the BP neural network method is better. Draw the \(e\) value of each interpolation point as shown in Fig. 6 and Fig.7.

The position and coordinates of the measurement points are shown in Figure 8. Because the magnetic field measurement points are discrete, it is impossible to calculate the accurate magnetic field gradient. The magnetic field intensity at point \((i, j)\) is \(H_{s,i,j}\). The magnetic field gradients in 8 directions can be approximately calculated according to the 8 points around point \((i, j)\). They are respectively

\[
\begin{align*}
\frac{H_{s,i+1,j} - H_{s,i-1,j}}{y_{i+1} - y_{i-1}}, & \quad \frac{H_{s,i+1,j} - H_{s,i+1,j-1}}{x_{i+1} - x_{i-1}}, & \quad \frac{H_{s,i+1,j} - H_{s,i+1,j+1}}{x_{i+1} - x_{i-1}}; \\
\frac{H_{s,i+1,j} - H_{s,i-1,j}}{y_{i+1} - y_{i-1}}, & \quad \frac{H_{s,i+1,j} - H_{s,i+1,j+1}}{x_{i+1} - x_{i-1}}; & \quad \frac{H_{s,i+1,j} - H_{s,i+1,j+1}}{x_{i+1} - x_{i-1}}.
\end{align*}
\]

Take the maximum value of the above 8 direction gradients as the gradient of point \((i, j)\). Draw the magnetic field gradient of each interpolation point as shown in Figure 9.

The Kriging interpolation error at the red circle in Figure 5 and Figure 6 is greater than that of the BP neural network. Comparing Figure 8 it can be found that the total magnetic field gradient at these points is large.

From Figure 6 and Figure 7 we can conclude that the gradient is larger in the area near the magnetic source, the error of the BP neural network is smaller than the Kriging, and the interpolation effect of the BP neural network is better. In the area with smaller gradient, the error of Kriging is smaller than that of BP, and the Kriging interpolation effect is better.

Arrange all interpolation points according to the gradient from small to large, and the cumulative value of the two error differences varies with the gradient as shown in Figure 10.
the Kriging method begins to decrease, indicating that the BP neural network interpolation error is less than the Kriging error, and the BP neural network interpolation result replaces Kriging the interpolation result can improve the accuracy of the geomagnetic map; when the gradient is less than $7.925 \times 10^{-4} \text{nT/m}$, the BP neural network interpolation error is greater than the Kriging error.

Figure 6. Error comparison for each measuring point.

Figure 7. Plan view of the error comparison for each measuring point.

Figure 8. Schematic diagram of measurement points.
4. Construction Method of Geomagnetic Reference Map Based on Kriging Method and BP Neural Network Interpolation

4.1. Method Ideas
Kriging interpolation is an accurate interpolation method based on sample points around unknown points. In addition to considering the distance relationship between the unknown point and the known point, it also considers the spatial characteristics of the sample, which can avoid the ‘bull's eye’ effect of the IDP method and can generate grid data with higher accuracy. But the speed of Kriging interpolation is slow and the interpolation result will be unstable in the area with large gradient, and the error will be big. BP neural network is suitable for situations with a large amount of measurement data. Use the known magnetic field data to train the BP neural network. If the measurement data is sufficient, the neural network will have high accuracy in predicting the unknown point and is not sensitive to gradients. But the BP neural network needs to adjust the appropriate network structure and parameters to achieve high-precision interpolation, the training speed is slow, and it takes a long time. It can be concluded from the simulation experiment in the previous section that the interpolation accuracy of the BP neural network is higher than the Kriging method in the large gradient area, and the interpolation accuracy is lower than the Kriging method in the small gradient area. Therefore, we can combine the advantages of the two methods. In large gradient areas, the BP neural network is
used for prediction, and in small gradient areas, the Kriging method is used for interpolation, which can improve the accuracy of the regional geomagnetic reference map.

4.2. Method Flow
The process of the construction method of geomagnetic reference map based on Kriging method and BP neural network proposed in this paper is shown in the figure 11.

Step 1 Obtain measured data, perform data preprocessing on measured data such as remove outliers, correct daily changes, and error compensation

Step 2 Interpolate with Kriging to obtain a preliminary geomagnetic reference map

Step 3 Draw contour map and gradient map, divide areas according to gradients, and select areas with larger gradients

Step 4 Use the measurement data to train the neural network to predict the magnetic field data of unknown points in the large gradient area

Step 5 Replace the magnetic field data of the larger gradient area in the Kriging interpolation with the predicted value of the neural network

Step 6 Combine the area with smaller gradient obtained by Kriging interpolation and the area with larger gradient predicted by BP neural network to complete the construction of the geomagnetic reference map.

The division of regions in this article is based on the difference between the sum of the root mean square errors of the Kriging method and the BP neural network method in different gradient regions: set a small gradient value, use this gradient to divide the area into two parts, and calculate the sum of the root mean square errors of the two algorithms in the large gradient area. If the error of Kriging method is smaller than that of BP neural network method, gradually increase the gradient critical value until the sum of the root mean square error of the Kriging method in the large gradient area is greater than or equal to that of BP neural network method, and this gradient value is used as the critical value of the boundary of the divided area.

In the large gradient region, the BP neural network of most interpolation points has smaller errors than the Kriging algorithm. According to Figure 7, the rectangular region in Figure 12 is selected from the large gradient region, and the interpolation result of the Kriging algorithm is replaced with the BP neural network predicted value to verify the effectiveness of the algorithm. The errors of the three interpolation methods are shown in Table 2.

Figure 11. Flow chart for construction of geomagnetic reference map.
It can be concluded from Table 2 that compared with using the two methods alone, combining the Kriging method in the small gradient area and the BP neural network in the large gradient area can reduce the MAE and RMSE.

5. Conclusion

After analyzing the applicable conditions of Kriging method and BP neural network in the problem of geomagnetic reference map interpolation, referring to the verification of the simulation data, the following conclusions can be drawn. The Kriging method has a good interpolation effect in the small gradient area, but the interpolation error is big in the large gradient area. There is no obvious difference in the interpolation error of BP neural network in different gradient regions. The interpolation effect of BP neural network is better than the Kriging method in the large gradient area, but worse in the small gradient.

A construction method of geomagnetic reference map based on the combination of Kriging method and BP neural network is proposed. This method avoids the shortcomings of the two methods and combines the advantages of the two methods in establishing geomagnetic reference maps in different gradient regions. The simulation results show that this method can reduce the interpolation error compared with when the two are used alone.

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