Identifying Majorana bound states at quantum spin Hall edges using a metallic probe

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We study the conductance afforded by a normal-metal probe which is directly contacting the helical edge modes of a quantum spin Hall insulator (QSHI). We show a $2e^2/h$ conductance peak at zero temperature in QSHI-based superconductor–ferromagnet hybrids due to the formation of a single Majorana bound state (MBS). In a corresponding Josephson junction hosting a pair of MBSs, a $4e^2/h$ conductance peak is found at zero temperature. The conductance quantization is robust to changes of the relevant system parameters and, remarkably, remains unaltered with increasing the distance between probe and MBSs. In the low temperature limit, the conductance peak is robust as the probe is placed within the localization length of MBSs. Our findings can therefore provide an effective way to detect the existence of MBSs in QSHI systems.

I. INTRODUCTION

A quantum spin Hall insulator (QSHI) is a two-dimensional (2D) topological insulator featuring topologically protected one-dimensional (1D) edge states [1–7]. These edge states are termed helical since they circulate in reversed directions with opposite spin orientations. QSHIs have been realized using HgTe [8–11] and InAs/GaSb [12] quantum wells, among others [13–15], and are attracting significant attention as a platform for the creation of quantum wells, among others [13–15], and are attracting significant attention as a platform for the creation of

A similar three-terminal setup with a metallic tip has been proposed to detect the helical nature of the edge states or to design novel applications [52–59]. Here, we focus on the detection of MBSs and consider both superconducting and magnetic regions, which open gaps on the helical edge channel forming the FS and SFS junctions featured in, respectively, Fig. 1(a) and Fig. 1(b). In the beam-splitter configuration, a bias voltage $V$ at the probe terminal injects a current $I$ into the helical edge state, propagating along it until is collected by distant grounded contacts. When the FS junction hosts a single MBS (or a pair for the SFS junction), the conductance $dI/dV$ at the probe terminal reaches a quantized plateau of $2e^2/h$ ($4e^2/h$) at zero bias. The conductance quantization is robust under the change of all relevant system parameters. Strikingly, the quantization is also independent of the position of the probe terminal along the edge, which is in contrast to the decaying local density of states of MBSs. The ZBCP at finite temperature is suppressed due to thermal broadening but remains observable in the low temperature limit as the probe is placed within the localization length of MBSs. Such a zero-bias coherent transport effect thus provides an experimental signature

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Schematics of the three-terminal setup. FS (a) and SFS (b) junctions based on a single QSHI edge, with a biased one-dimensional normal-metal probe terminal.}
\end{figure}
of MBSs in a QSHI.

II. MODEL AND FORMULAS

Model.—Our setup consists of a semi-infinite normal probe on the y axis and a QSHI edge lying along the x direction (Fig. 1). We set the origin at the FS boundary, or at the middle of the SFS, and place the contact point between the QSHI edge and the probe terminal at \((x_c, 0)\). The Hamiltonian is \(H = \frac{1}{2} \hat{\Psi}^\dagger \hat{H} \hat{\Psi} \), with \(\hat{H} = H_N + H_T + H_j\), \(\hat{\Psi} = (\psi^+_\uparrow, \psi^+_\downarrow, \psi^-_\uparrow, \psi^-_\downarrow)^T\), and \(\psi^\pm_\sigma\) the field operators for right (left) movers. \(H_N\) describes the normal probe terminal with a standard parabolic dispersion

\[
H_N = \left( -\frac{\hbar^2 \partial_y^2}{2m_N} - \mu_N \right) \hat{\tau}_z \Theta (y) + U \hat{\tau}_z \delta \left( y - 0^+ \right),
\]

where \(m_N, \mu_N\), and \(\hat{\tau}_\tau = x, y, z\) are the effective mass, chemical potential of normal probe, and Pauli matrices in the Nambu space. \(\Theta (y)\) and \(\delta \left( y - 0^+ \right)\) are the Heaviside and delta functions, respectively, and \(U\) is the barrier parameter between the probe and the QSHI. The bare helical edge states are described by the Hamiltonian

\[
H_T = -i \hbar v_L \hat{\sigma}_x \partial_x - \mu_T \hat{\tau}_z,
\]

where \(v_L\) is the Fermi velocity, \(\mu_T\) is the chemical potential measured from the Dirac point of the helical states, and \(\hat{\sigma}_\tau = x, y, z\) are Pauli matrices in the spin space. Finally, \(H_j\) describes the proximity-induced terms in the helical state with \(j = 1\) \((j = 2)\) corresponding to the FS (SFS) junction. Specifically,

\[
H_1 = M_x \hat{\sigma}_y \hat{\tau}_z \Theta (-x) - \Delta \hat{\tau}_y \Theta (x), \quad H_2 = M_x \hat{\sigma}_y \hat{\tau}_z \left( \frac{L}{2} - |x| \right) - \Delta \hat{\tau}_y e^{i \chi N} \Theta \left( -x - \frac{L}{2} \right) - \Delta \hat{\tau}_y e^{i \chi N} \Theta \left( x + \frac{L}{2} \right).
\]

Here, \(M_x\) is an exchange field from the coupling to a ferromagnetic insulator and \(\Delta\) is the proximity-induced pair potential from a conventional s-wave superconductor. For the SFS junction, \(L\) is the width of the F region and \(\chi, \lambda_N\) are the macroscopic superconducting phases, with \(\chi = \chi_L - \chi_R\) being their difference.

The scattering wave function for the normal probe is

\[
\Psi_{N\sigma} = \Phi_\sigma + b_\sigma \hat{B}_1 e^{i q y} + a_\sigma \hat{B}_1 e^{-i q y} + a_\sigma \hat{A}_1 e^{-i q y} + b_\sigma \hat{A}_1 e^{i q y},
\]

for an incident electron with spin \(\sigma = \uparrow, \downarrow\) and wave function \(\Phi_\sigma = (1, 0, 0, 0)^T e^{-i q y}\) \((\Phi_\sigma = (0, 1, 0, 0)^T e^{-i q y}\). Under the wide band approximation \((\mu_N \gg E)\), the wave vector is \(q = \sqrt{2m_N \mu_N / \hbar}\) and the spinors are \(\hat{B}_\sigma = (1, 0, 0, 0)^T, \hat{B}_\sigma = (0, 1, 0, 0)^T, \hat{A}_\sigma = (0, 0, 1, 0)^T\).

\(\hat{A}_\sigma\) = \((0, 0, 1, 0)^T\). The normal (Andreev) reflection amplitudes are \(b_{\sigma\sigma'}\) \((a_{\sigma\sigma'}\) for an incoming electron of spin \(\sigma\) scattered as an electron (hole) of spin \(\sigma'\).

At the edge of QSHI \([60–65]\), the solutions on the F region are \(\hat{F}_1 = N_1^{-1} (\hbar v_L \kappa_+ + \mu_T, M_x, 0, 0)^T e^{i \kappa_+ x}, \hat{F}_2 = N_1^{-1} (M_x, \hbar v_L \kappa_+ + \mu_T, 0, 0)^T e^{-i \kappa_+ x}, \hat{F}_3 = N_1^{-1} (0, 0, \hbar v_L \kappa_+ + \mu_T, M_x)^T e^{i \kappa_+ x}\), and \(\hat{F}_4 = N_1^{-1} (0, 0, M_x, \hbar v_L \kappa_+ + \mu_T)^T e^{-i \kappa_+ x}\), with wave vectors \(\kappa_{\sigma}(h) = \sqrt{\mu_T^2 - M_x^2 / \hbar v_L}\) for \(M_x < \mu_T\), and \(\kappa_{\sigma}(h) = \pm \sqrt{\mu_T^2 - M_x^2 / \hbar v_L}\) for \(M_x > \mu_T\). \(N_{1=4}\) are normalization factors. On the S region we find \(\hat{S}_1 = (u, 0, 0, v)^T e^{i k^+_y x}, \hat{S}_2 = (0, -u, v, 0)^T e^{-i k^+_y x}, \hat{S}_3 = (0, -v, 0, 0)^T e^{-i k^+_y x}, \) and \(\hat{S}_4 = (v, 0, 0, u)^T e^{i k^+_y x}\), with \(u(v) = \sqrt{(E \pm \sqrt{E^2 - \Delta^2}) / 2E}\). An incident spin-\(\sigma\) electron from the normal probe thus has a transmitted wave function \(\Psi_{T\sigma}(x)\) on the QSHI side, which is a suitable superposition of spinors \(\hat{F}_{i=1-4}\) and \(\hat{S}_{i=1-4}\).

To determine the scattering amplitudes, we match \(\Psi_{T\sigma}\) and \(\Psi_{T\sigma}\) at the contact using the boundary conditions \([59, 66, 67]\)

\[
\Psi_{N\sigma}(y = 0^+) = e \Psi_{T\sigma}(x^-), \quad e \Psi_{T\sigma}(x^-) = e \Psi_{T\sigma}(x^+),
\]

with \(K = h \hbar \sigma \tau_z \left( m_N - 1 \partial_y - 2U / \hbar^2 \right)\). The parameter \(Z = m_N U / (\hbar^2 q)\) describes the barrier strength between the probe and the helical edge, while the real and dimensionless number \(c\) represents the different microscopic details between them, such as the hopping integrals in the underlying lattice model \([59, 66, 67]\).

When a bias voltage \(V\) is applied to the normal probe, the conductance \(G\) at zero temperature is calculated using the formula \([68]\)

\[
G = \frac{e^2}{h} \sum_{\sigma\sigma'} (\delta_{\sigma\sigma'} + |a_{\sigma\sigma'}|^2 - |b_{\sigma\sigma'}|^2).
\]

Here, \(a_{\sigma\sigma'}\) and \(b_{\sigma\sigma'}\) are reflection amplitudes at \(E = eV\).

For the numerical calculations we choose \(v_N \approx 1.57 \times 10^6\) m/s and \(\mu_N \approx 7.0\) eV, corresponding to copper; \(v_1 \approx 5.5 \times 10^5\) m/s and \(\mu_T \approx 0.01\) eV for HgCdTe quantum wells; and \(\Delta \approx 0.1\) meV, which corresponds to a proximity-induced gap from a niobium superconductor. For simplicity, we only consider the short junction for the SFS geometry, with \(k_T L \ll \mu_T / 2\), and choose \(k_T L = 2\) so that the width of F is \(L \approx 70\) nm. We note that our main conclusions are not material dependent.

III. FS JUNCTION

We start with the FS junction [Fig. 1(a)] and analyze the conductance probed by a normal-metal terminal posi-
FIG. 2. Zero-temperature conductance on the normal-metal terminal placed at the FS boundary. (a) The conductance spectra at zero temperature as a function of the energy $eV$ and magnetization $M_x$ for $c = 1$ and $Z = 1$. (b), (c), and (d) show the zero-bias conductance as a function of $M_x$, $Z$, and $c$, respectively. We set $c = 1$ for (b) and (c), and $Z = 1$ for (d).

The robust $2e^2/h$ value indicates the presence of a single MBS. However, in the trivial phase ($M_x < \mu_T$) the conductance dependence on $c$ is very different. First, for $Z = 0$ the metallic probe is transparently coupled to the helical state and all the injected current at zero bias can either directly flow to the drain in F or undergo a perfect Andreev reflection on S. This results in a conductance value slightly bigger than $2e^2/h$, which can not be achieved in the absence of a tip when the bias is applied to the F region. For $Z \neq 0$, electrons incoming from the tip can backscatter, thus monotonically reducing the conductance as $Z$ increases. By contrast, the dependence on $c$ of the ZBCP displays a non-monotonic behavior in the trivial phase. The conductance is suppressed for $c \to 0, \infty$, since the probe is decoupled from the edge state in these limits, and thus exhibits a maximum in between, whose position depends on both $c$ and $Z$.

When the probe is at the FS boundary, we have shown a perfectly quantized ZBCP in the topological phase $M_x > \mu_T$. However, this exact position may be challenging to reach in experiments, so we now consider the spatial dependence of the conductance as the probe moves along the QSHI edge. First, it is convenient to define two characteristic length scales: $\xi_F = \hbar v_F/\mu_T$ inside F and $\xi_S = \hbar v_F/\Delta$ in S. Figure 3 shows the dependence on the coordinate $x$ of the conductance at the normal terminal probe. As expected, there is no quantized ZBCP in the trivial phase ($M_x < \mu_T$), see Fig. 3(a) and Fig. 3(b). In this regime, the tip conductance in the F region features small oscillations around a constant value, while it displays a gapped profile with large peaks at $|eV| = \Delta$ in the S region, induced by the density of states in a uniform superconductor.

By contrast, the topological phase ($M_x > \mu_T$) shown in Fig. 3(c) and Fig. 3(d) presents a remarkable result. The zero bias conductance is not only quantized at the
with $\Omega$ global phase factors $e^{i\chi}$ for $M_x = 0$ and $Z = c = 1$. (b) For fixed $\chi = \pi$, conductance versus $eV$ and $M_x$, with $Z = 0$ and $c = 1$. (c,d) Zero bias conductance as a function of $\chi$ for $M_x = 0.5\mu T$ and several values of $Z$ (c) or $c$ (d). We set $c = 1$ for (c) and $Z = 1$ for (d).

**FIG. 4.** Zero-temperature conductance of a normal-metal probe placed in the middle of an SFS junction. (a) Conductance versus applied voltage and phase difference $\chi$ for $M_x = 0$ and $Z = c = 1$. (b) For fixed $\chi = \pi$, conductance versus $eV$ and $M_x$, with $Z = 0$ and $c = 1$. (c,d) Zero bias conductance as a function of $\chi$ for $M_x = 0.5\mu T$ and several values of $Z$ (c) or $c$ (d). We set $c = 1$ for (c) and $Z = 1$ for (d).

FS boundary, but also maintains a constant value of $2e^2/h$ for every position along the QSHI edge. The emergence of such a robust ZBCP seems exotic, especially for $x_c \to +\infty$, i.e., several coherence lengths ($\xi_S$) away from the localized MBS. We note that the bulk proximitized superconducting region ($x_c \to +\infty$) is fully gapped at zero energy, without free quasiparticles and with the local density of the MBS greatly suppressed [60, 61, 63, 65, 78].

To further analyze this counter-intuitive result, we examine the zero-bias scattering amplitudes for $x_c > 0$ in the topological phase with $M_x > \mu T$, namely,

\[
\begin{align*}
 b_{\uparrow\uparrow} &= - \left[ e^4 (i + 2Z)^2 + \eta^2 \right] \Xi^{-1}, \\
 a_{\uparrow\downarrow} &= - e^{-2ik_T x_c} \Omega_+ \left[ e^4 + (2c^2Z - i\eta)^2 \right] \Xi^{-1}, \\
 b_{\uparrow\downarrow}(i\downarrow) &= \pm ie^{-2ik_T x_c} \Omega_+ \left[ e^2 (i + 2Z) \pm i\eta \right] \Xi^{-1}, \\
 a_{\uparrow\downarrow}(i\downarrow) &= - i \left[ e^4 (1 + 4Z^2) \mp 2c^2\eta + \eta^2 \right] \Xi^{-1},
\end{align*}
\]

with $\Xi = M_x / (\pm i\mu T + \sqrt{M_x^2 - \mu^2}), \quad (9a)$

\[
 b_{\uparrow\uparrow} = M_x / (\pm i\mu T + \sqrt{M_x^2 - \mu^2}), \quad (9b)
\]

\[
 a_{\uparrow\downarrow} = - e^{-2ik_T x_c} \Omega_+ \left[ e^4 + (2c^2Z - i\eta)^2 \right] \Xi^{-1},
\]

\[
 b_{\uparrow\downarrow}(i\downarrow) = \pm ie^{-2ik_T x_c} \Omega_+ \left[ e^2 (i + 2Z) \pm i\eta \right] \Xi^{-1},
\]

\[
 a_{\uparrow\downarrow}(i\downarrow) = - i \left[ e^4 (1 + 4Z^2) \mp 2c^2\eta + \eta^2 \right] \Xi^{-1},
\]

\[
\text{(9d)}
\]

The spacial dependence at zero energy appears as global phase factors $e^{\pm 2ik_T x_c}$, so the resulting conductance is thus quantized and independent of the position $x_c$. This effect provides an interesting signature of the existence of MBS in a QSHI.

**FIG. 5.** Spatial dependence of the probe conductance at zero temperature for an SFS junction. (a, b) For $\chi = 0$, (a) conductance spectra as a function of bias and position, and (b) three cuts corresponding to biases $eV = 0$, $\Delta$ and $2\Delta$. (c,d) Same as before for $\chi = \pi$. In all cases, we set $M_x = 0.5\mu T$, $Z = 1$, and $c = 1$.

### IV. SFS JUNCTION

We now connect the normal-metal terminal probe to the SFS junction. First, we place it in the middle of F and study the conductance as a function of $M_x$ and the superconducting phase difference $\chi$ in Fig. 4. At zero magnetization [Fig. 4(a)], a resonance peak inside the superconducting gap indicates the presence of a pair of MBSs exhibiting a protected crossing at $\chi = \pi$. The robustness of the crossing point is shown in Fig. 4(b), where the zero biased conductance is quantized with $G = 4e^2/h$ for $\chi = \pi$ and arbitrary $M_x$, similar to works in other topological Josephson junctions [80–82]. Indeed, the QSHI-mediated Josephson junction is in the topological phase independently of the magnetization [26, 72]. For nonzero $eV$, the transition between a dominant superconducting gap into a dominant magnetic one can be seen around $M_x = \mu T$; as the magnetic gap becomes dominant for $M_x > \mu T$, the conductance is suppressed. Figure 4(c) and Fig. 4(d) show the robustness of the ZBCP at $\chi = \pi$ against the coupling parameters $c$ and $Z$.

Next, we compare the spacial dependence of the probe conductance for $\chi = 0$ [Fig. 5(a) and Fig. 5(b)] and $\chi = \pi$ [Fig. 5(c) and Fig. 5(d)]. For $\chi = 0$, there is no ZBCP in the SFS junction since the MBSs have merged with the continuum. By contrast, at the crossing $\chi = \pi$, the zero-bias conductance becomes perfectly quantized to $4e^2/h$. As it was the case for the FS junction in the topological...
phase, the normal probe conductance remains quantized for any position across the whole SFS junction. However, the quantized ZBCP is now present for any value of $M_z$, and is broadened in the F region, cf. Fig. 4(c). Analytically, when the probe is on the left S side ($x_c < -|L/2|$, the coefficients are obtained as

$$b_{\uparrow\uparrow(\downarrow\downarrow)} = -\left(1 + e^{ix}\right) \Upsilon_1 / \left(\Upsilon_2 \pm \Upsilon_3\right),$$

(10a)

$$a_{\downarrow\uparrow(\uparrow\downarrow)} = \mp \Upsilon_4 / \left(\Upsilon_2 \pm \Upsilon_3\right),$$

(10b)

and $b_{\downarrow\downarrow} = b_{\uparrow\uparrow} = a_{\downarrow\uparrow} = a_{\uparrow\downarrow} = 0$, with $\Upsilon_1 = e^4 (i + 2Z)^2 + \eta$, $\Upsilon_2 = (1 + e^{ix}) \left(e^4 + 4e^4Z^2 + \eta^2\right)$,

$$\Upsilon_3 = 2e^2 \left(e^{ix} - 1\right) \eta e^{\Delta(L+2x_c)/2h\nu},$$

and $\Upsilon_4 = 2ie^2 \left(1 + e^{ix}\right) \eta - i\Upsilon_3$. As the phase bias is $\chi = \pi$, $b_{\sigma\sigma}$ becomes 0 while $a_{\sigma\uparrow} = a_{\uparrow\sigma} = 1$ exhibiting perfect Andreev reflection, regardless of the position of probe.

V. TEMPERATURE EFFECT

So far we have reported that the ZBCP always sticks to the quantized value at zero temperature. To obtain a result that can be directly compared to experiments, we next remark on the temperature effect. At zero temperature, we find that the topological ZBCP becomes sharper as the probe is placed far from the localized MBSs, see Fig. 3(c) and Fig. 5(c). This indicates that the height of the ZBCP would be lowered at finite temperatures due to the thermal broadening. We calculate the conductance $G$ at zero temperature given

$$G(x) = \int_{-\infty}^{+\infty} d\varepsilon G(\varepsilon) \left[4k_B T \cosh^2 \left(\frac{\varepsilon - eV}{2k_B T}\right)\right]^{-1},$$

(11)

where $G(\varepsilon)$ is the conductance at zero temperature given in Eq. (8). We find that the ZBCP height does not change significantly by increasing the temperature $T$ as the probe tip is placed at $x_c = 0$, i.e., the interface of the FS junction or the middle of the SFS junction, as shown in Fig. 6. However, a finite temperature has a drastic suppression effect on the ZBCP height as the tip moves far away from $x_c = 0$. This is consistent with recent experiments where a robust ZBCP was found around the FS interface at finite temperature [83]. Our results show that the weakened ZBCP is still observable if the probe is within the localization length of the MBSs at low temperature, e.g., $x_c = 0.1\xi_S$ (red line in Fig. 6(b)) in the SFS junction. Consequently, the unusual spatial independence of the ZBCP found in our calculation could be experimentally observed in low temperature measurements.

VI. CONCLUSION

We studied the conductance afforded by a normal-metal probe which directly contacts the helical edge modes of a quantum spin Hall insulator. We found a robust quantized zero-biased conductance peak in both FS and SFS junctions indicating the presence of Majorana bound states at each FS boundary. The conductance quantization is robust under variations of the parameters controlling the coupling between the tip and the helical edge state. Moreover, we found that the zero bias conductance quantization remains unchanged as we moved the probe along the edge, remarkably, even for distances much larger than the localization length of the Majorana states. This result can not be simply explained by the local density of states in the tunneling model [56–58]. Our analytical results suggested that the spatial independence of the zero-bias tip conductance results from a coherent coupling to the zero-energy MBSs. Finally, we discuss the temperature effect on the ZBCP to estimate the quality of the conductance quantization in actual experiments. Our proposal for the observation of Majorana states using a metallic probe is within reach of recent experimental advances implementing hybrid superconductor and magnetic structures on the quantum spin Hall insulator [15, 28, 29, 33, 83–87].

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FIG. 6. Zero-biased conductance as a function of temperature $T$ for different positions of the probe. (a) The probe is placed at $x_c = 0$, $-\xi_F$, and $\xi_S$ in the FS junction. The parameters of the FS junction are the same as Fig. 3(c). (b) The probe is placed at $x_c = 0, 0.1\xi_S$, and $\xi_S$ in the SFS junction with the same parameters as in Fig. 5(c). $\Delta_0$ is the superconducting gap at zero temperature.
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