WEAK LENSING, STRUCTURE FORMATION AND DARK ENERGY

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Abstract. We study how CMB bispectrum, produced by weak gravitational lensing and structure formation, can constrain the redshift evolution of the dark energy equation of state independently on its present value. Analyzing the line of sight contribution to the angular CMB bispectrum, we find that the relevant redshift at which the structure formation contributes to the signal is $0.1 \lesssim z \lesssim 2$ for multipoles $1000 \lesssim l \lesssim 100$: just the epoch when the dark energy starts to dominate the cosmological expansion rate. For scenarios having the same equation of state at the present, this turns out to be a new observable capable to discriminate between models of dark energy with different time evolution of equation of state. We assess the strength of this effect within the framework of tracking Quintessence trajectories obeying SUGRA and Ratra-Peebles potentials.

1 Introduction

In the last few years the cosmological parameters have been constrained in a narrow range of values by several experiments and the picture of the universe seems to be the following. The unexpected dimming of type Ia supernovae (SNIa), used as standard candles, is the evidence of an accelerating expansion of the universe, SNIa probes the cosmological expansion up to redshift $z \sim 2$ [1, 2]; in a Friedmann Robertson Walker universe this acceleration is provided by an amount of dark energy that is 70% of the whole energy content. Other independent indications of the presence of a missing energy come from Cosmic Microwave Background (CMB) anisotropies [3] and Large Scale Structures (LSS) [4], CMB data support a flat cosmological geometry while LSS indicate that 30% of critical density is clustered. The first candidate for the dark energy is the Cosmological Constant, which has two well known problems: the coincidence problem (why Cosmological Constant density and matter density are comparable today?) and the fine tuning problem (why Cosmological Constant scale is 120 orders of magnitude less than Planck scale?). To solve the latter problem a dynamical scalar field, known as Quintessence [5, 6], has been introduced as a minimally extension of Cosmological Constant. With this field we have a dynamical equation of state for the dark energy alleviating the fine tuning problem. Recently a strong constraint has been put for the value of equation of state at present, combining CMB, LSS, SNIa and HST observations we have $w_0 < -0.78$ [3]. The next challenge in cosmology is to constrain the time evolution of $w$; this can be done with future experiments like PLANCK for CMB [7] and future SNIa observations.

The weak lensing effect on CMB anisotropies has been studied [8] and the third order statistics, the bispectrum, has been used to constrain the effective dark energy equation of state [9]. If the primordial anisotropies are Gaussian the bispectrum is zero within cosmic variance; cosmic structures produce a non-Gaussian feature on CMB and a non vanishing bispectrum. In this work we study the weak lensing effect produced by structure formation on CMB pattern in Quintessence cosmology; the bispectrum is written in terms of a line of sight integral involving the redshift derivative of gravitational potential and cosmological distances. Since the dynamics of dark energy involves both perturbations growth and cosmological distances, also the bispectrum depends on the evolution of dark energy; here we show how different dark energies with same value of $w$ at present produce different features on the bispectrum, introducing a new observable capable to discriminate between different dark energies independently on the value of $w_0$ [10].
2 Quintessence, weak lensing and integrated Sachs-Wolfe effect

We have considered two kind of potential for Quintessence: the Ratra-Peebles inverse power law (hereafter RP [11]) and SUGRA [12]. In terms of scalar field $\phi$ the potentials are written as

$$V_{RP} = \frac{M^{(4+\alpha)}}{\phi^\alpha}, \quad V_{SUGRA} = \frac{M^{(4+\alpha)}}{\phi^\alpha} \cdot e^{4\pi G \phi^2};$$

(1)

with these potentials the problem of fine tuning is reduced: the only relevant condition for the initial value of scalar field is $\phi_i << M_{Pl}$ ($M_{Pl}$ is the Planck mass). In this way we can reach the present value $\phi_0$ for a wide set of initial condition [6]; in the case of a scalar field the equation of state $w_Q$, defined as $p_Q/\rho_Q$, evolves in time depending on the potentials.

We consider now the CMB anisotropies produced at decoupling and those produced by the integrated Sachs-Wolfe effect (hereafter ISW [13]) taking into account the weak lensing effect of growing structures. The CMB anisotropy in the direction $\hat{n}$ can be decomposed as

$$\Theta (\hat{n}) \simeq \Theta_{lss} (\hat{n}) + \Theta_{ISW} (\hat{n}) + \vec{\nabla} \Theta_{lss} (\hat{n}) \cdot \vec{\alpha},$$

(2)

the first term is the primordial anisotropy, the second one is the ISW anisotropy and the last one is the contribution due to weak lensing; $\alpha$ is the deflection angle. The potential perturbations that produces the ISW also produces the weak lensing; because of this the lensed CMB photons and those redded by ISW are correlated and a non-Gaussian feature on CMB pattern is produced, that give us a non null signal of bispectrum. To evaluate the non linear density power spectrum we have used a semi-analytical approach [14]. Throughout this work we used the best fit of cosmological parameters (excluding $w$) [15], but with a $n_s = 1$ value for the spectral index.

3 The CMB bispectrum

To calculate the bispectrum, we have used three equal multipoles $l$ probing the CMB pattern with equilateral triangles with size of about $\theta \sim 180^\circ/l$; this is the easiest choice but it is sufficient to show how the dynamics of dark energy affect the bispectrum. The CMB bispectrum is written as

$$B_l = 3l (l+1) \sqrt{\frac{(2l+1)^3}{4\pi}} \begin{pmatrix} l & l & l \\ 0 & 0 & 0 \end{pmatrix} C_l^P Q (l),$$

(3)

where $C_l^P$ is the primordial power spectrum at multipole $l$, the parenthesis are the Wigner’s 3J symbols and $Q (l)$ is

$$Q (l) \equiv \langle \left( d_{l,m}^{lens} \right)^* d_{l,m}^{ISW} \rangle \simeq 2 \int_0^{z_{lss}} \frac{dz}{r(z)} \left( \frac{r(z_{lss})}{r(z)} \right)^3 \left( \frac{\partial P_\Psi (k,z)}{\partial z} \right)_{k=\frac{r(z)}{H(z)}},$$

(4)

In the previous relation the quantity $P_\Psi (k,z)$ is the gravitational potential power spectrum, related to the density power spectrum by the relation $P_\Psi (k,z) = \left( \frac{3}{2} \Omega_M \right)^2 H^4 (k) P (k,z) (1+z)^2$, $z_{lss}$ is the redshift of last scattering surface and $d_{l,m}^{lens}$ and $d_{l,m}^{ISW}$ are respectively the coefficients of harmonics expansion of lensing contribution and ISW. A detailed outline on how write Eq. (3) and (4) can be found in [9] and references therein.

The integral $Q (l)$ is the most relevant quantity in this work, it describes how the weak lensing on CMB is produced by the forming structures along the line of sight. Analyzing the redshift behavior of the integrand of (4) we can study how the contribution of $Q (l)$ is distributed along the line of sight and how it depends on the time evolution of Quintessence equation of state. The integrand can be decomposed in two factors: a geometrical factor $\frac{r(z_{lss})}{r(z)} \left( \frac{r(z_{lss})}{r(z)} \right)^3$ and a derivative factor $\frac{\partial P_\Psi (k,z)}{\partial z}$. In the geometrical factor the redshift behavior of $w$ is averaged through the distance integral so we don’t have a significant sensitivity to this parameter while the derivative factor is more sensitive to the time evolution of Quintessence equation of state giving the most important contribution to the integrand of $Q (l)$. In Fig. 1 we have plotted the integrand of (4) for SUGRA model with $w_0 = -0.9$, once
we fix the multipole $l$; coming form high redshift, the integrand probes from large to small scales by
the relation $\lambda(z) = 2\pi r(z)/l$. As the redshift comes down from infinity, the scale $\lambda(z)$ decreases
until it meets the growing comoving scale of the horizon becoming a sub-horizon comoving scale; the
gravitational potential starts to decrease because of the free streaming and this feature is represented
by the positive peaks in Fig. 1 (the derivative factor is positive). As $z$ get smaller, the scale $\lambda(z)$
matches the scales which enter in the non-linear phase at that epoch and the power increase with
time (the derivative factor is negative); this feature is shown in Fig. 1 with a negative peak. When
$z$ approaches zero, the wavenumber tends to infinity and the power vanishes. $Q(l)$ has a non zero
contribution, yielded by the derivative factor, in the narrow range of redshift (between 0.1 and 2)
where the dark energy starts to dominate over the matter; therefore the integrand can be used as
a window function that selects just the relevant redshift to probe the cosmic acceleration. In this
pictures the dark energy equation of state is probed independently on its value today (the power is
zero at the present as remarked above) and only at the redshift where the cosmological expansion rate
is affected by it. In this way the correlation (4) is a powerful tool to study the redshift behavior of
dark energy equation of state; RP and SUGRA have a different evolution of equation of state, from
the tracking regime to $w_0$, and this difference is just in the range of redshift where the integrand is
not null.

The different behaviors of $w(z)$ for different models of dark energy are reflected in $Q(l)$ and also
in the bispectrum. In Fig. 2 we plotted the bispectrum for constant $w$, RP and SUGRA model, all
with $w_0 = -0.9$. The main feature of bispectrum is the cusp and represents the transition between
linear and non linear regime when density perturbations grows i.e. when the integral (4) is zero. The
position of cusp in $l$-space depends on the dark energy model considered; in SUGRA model the cusp
is shifted with respect to RP models of 50 multipoles in spite of the fact that both models have the
same value of $w_0$ (for SUGRA the cusp is located at $l = 412$, for RP at $l = 362$). This shift is due
to the fact that in SUGRA scenario the growth factor is sensibly larger than that of RP scenario at
almost all epochs [10]. Correspondingly, the change at low redshift (when the dark energy starts to
dominate the expansion) is stronger in SUGRA scenario producing a larger amplitude for the linear
regime in the line of sight integral (when the derivative factor is positive). The contribution to the
bispectrum coming from the non linear scales has lower amplitude because the rise due to the onset
of non-linearity has to overcome the gravitational potential decay which in SUGRA is stronger than
in RP. The net effect is that, for a given multipole $l$, the larger is the value of $w$ at the relevant
epoch, the larger is the contribution from the linear regime; the scale at which linear power balances

Figure 1: Integrand of (4) for SUGRA model with $w_0 = -0.9$ for different multipoles; solid line is
linear regime, dotted line is non linear regime. The graphics are qualitative the same for constant $w$
and RP.
the non-linear one is shifted at larger wavenumber (higher multipoles) as is shown in Fig. 2. The dependence on the dark energy of the position of the cusp in the bispectrum is a new feature that can be used to discriminate between different dark energies and must be take in consideration in future observations. For completeness, in Fig. 3 we plot the CMB power spectrum for the models considered, together with the WMAP data [15]; in fact, comparing Fig. 2 and Fig. 3, the degeneration between models is higher in the power spectrum than in the bispectrum [10].

4 Conclusions

We have studied the CMB bispectrum produced by weak lensing and ISW effect in Quintessence cosmology; we have found a new observable capable to discriminate between different models of dark energy with the same value of equation of state at the present but with different behavior in the past.

The correlation between weak lensing and ISW is made of two contributions yielding opposite signs after horizon crossing: in the linear regime the gravitational potential decreases in time, in the non-linear one we have a growth. Most of the contribution to the bispectrum signal come from the narrow range of redshift where the dark energy starts to dominate the expansion rate of the universe driving the acceleration and this epoch is that typical of structure formation. Because of these reasons, the bispectrum is sensitive to cosmic expansion rate at that redshift, i.e. to the equation of state at that time independently on its value today. To illustrate this effect we have used two kinds of dark energy: RP model and SUGRA model. Both RP and SUGRA have a time-variable equation of state but a different behavior in redshift. As result we have a geometry shift of the projected bispectrum, affecting in particular the cusp, that represent the balance between linear and non-linear regime; in the case of SUGRA, the cusp is located at higher multipoles with respect RP and this shift is about one order of magnitude greater than the shift in the CMB power spectrum.

We don’t explored the degeneration of the cusp position with respect other cosmological parameters, but we must keep in mind that this effect is due to a projection along the line of sight, so the shift can be mimicked by variations in the cosmological parameters inducing geometrical features in CMB anisotropies (curvature, $H_0$). On the other hand these variations will strongly affect both the bispectrum and the power spectrum.

The capability of the next CMB probes, PLANCK and CMBPol, to look for the effect we pointed out here is currently under study, as well as future weak lensing surveys [16].
Figure 3: Power spectra for constant $w$ (solid line), RP (dotted line) and SUGRA (dashed line) $w_0 = -0.9$, error bars are binned WMAP data.

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