Controlling Kerr nonlinearity with electric fields in asymmetric double quantum-dots

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Abstract

The control of Kerr nonlinearity with electric fields in an asymmetric double quantum-dot systems coupling with tunneling is investigated theoretically. It is found that, by proper tuning of two light beams and tunneling via a bias voltage, the Kerr nonlinearity can be enhanced and varied within a wide scale.

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Over the past decades, there have been extensive researches on nonlinear effects in multi-level systems, which mainly emphasized on the enhancement of nonlinear optical properties [1, 2, 3, 4, 5]. Recent evidences have shown that possible applications would include frequency conversion [6], four-wave mixing [7] and harmonic optical generation [8]. In nonlinear optics, third-order susceptibility plays an important role in information process. A typical scheme for giant Kerr effect is proposed by Schmidt and Imamoglu [9]. The principle is a four-level atomic system coupled by three fields: pump field, signal field and coupling field, which consist an N configuration. This proposal had been indirectly measured in the experiment [10]. Harris and Yamamoto [11] also proposed a four-level atomic system which existed a greatly enhanced Kerr nonlinearity, but vanishing linear absorption. Recently, Sun et al. [12] studied the enhanced Kerr nonlinearity in an asymmetric GaAs double quantum well via Fano interference. Noticeably, most of Kerr effects in above references are achieved by detuning of pump field or signal field. In a recent report by Sinclair and Korolkova [13], they investigated Kerr effect considering the detuning of coupling field.

On the other hand, Villas-Boas et al. [14] proposed a scheme of applying a tunneling voltage coupling two quantum-dots (QDs) (see Fig.1). Yuan and Zhu [15] demonstrated theoretically that there existed EIT in asymmetric double QD systems coupling with a bias voltage. Comparing with Ref. [2], we further investigate a four level system in such asymmetric double quantum-dots system. However, different from N-type in atomic systems, the upper two level are coupled by tunneling, more like ”n-type”. Because of the tunneling between QDs via a bias voltage, the advantage of using quantum-dots is that QDs allow direct control of its energy scales and physical properties [14].

The quantum dot molecule (QDM) consisting of vertically stacked pair of QDs coupled by tunneling via a bias voltage is shown in Fig.1(a). One layer of highly $n^+$-doped GaAs and one layer of undoped GaAs consist a QDM layer. Two layers are separated by 10 nm thick GaAs spacer [16]. The lower dot layer consists of 8ML $Ga_{0.5}In_{0.5}As$, and the upper layer 7ML. By applying a bias voltage between the $n$ contact and the Schottky gate, the axial electric field can be tuned from 0 to $\sim 250kV/cm$. We model asymmetric double QDs consisting of two dots with different band structure and the top two state $|3\rangle,|4\rangle$ are coupled by tunneling (see Fig.1(b)). In this model, one could see left dot (see Fig 1(b)) coupled with signal beam and the right dot coupled with pump beam. The tunnel barrier in the double QDs can be controlled by applying a bias voltage between two dots. With the
electromagnetic field, one exciton can be exited from the valence to the conduction band of the right dot, which would then tunnel to the left dot.

We consider such a Hamiltonian,

\[
H = \sum \varepsilon_{|j\rangle<br |j\rangle} + \hbar \Omega_p (|e^{-i\omega_{p}t}|^{3} |1\rangle + |e^{i\omega_{s}t}|^{1} |3\rangle) + \hbar \Omega_s (|e^{-i\omega_{s}t}|^{4} |2\rangle + |e^{i\omega_{s}t}|^{2} |4\rangle) + T_e (|3\rangle |e^{-i\omega_{s}t}|^{3} |4\rangle + |e^{i\omega_{p}t}|^{1} |3\rangle).
\]

(1)

where \(\varepsilon_{j}\) is the energy of the state \(|j\rangle = (j = 1, 2, 3, 4)\), \(T_e\) is the electron tunneling matrix element. \(\Omega_p\) and \(\Omega_s\) are Rabi frequency for pump light and signal light, respectively. Then we get,

\[
\rho_{24} = -(i\omega_{24} + \gamma_{24})\rho_{24} - i\Omega_s e^{-i\omega_{s}t}(\rho_{14} - \rho_{22}) + iT_e \rho_{23},
\]

(2)

\[
\rho_{13} = -(i\omega_{13} + \gamma_{13})\rho_{13} - i\Omega_p e^{-i\omega_{p}t}(\rho_{33} - \rho_{11}) + iT_e \rho_{14},
\]

(3)

\[
\rho_{34} = -(i\omega_{34} + \gamma_{34})\rho_{34} - i(\Omega_p e^{-i\omega_{p}t}\rho_{14} - \Omega_s e^{-i\omega_{s}t}\rho_{32}) - iT_e (\rho_{44} - \rho_{33}),
\]

(4)

\[
\rho_{12} = -(i\omega_{12} + \gamma_{12})\rho_{12} - i(\Omega_p e^{-i\omega_{p}t}\rho_{32} - \Omega_s e^{-i\omega_{s}t}\rho_{14}) + iT_e \rho_{13},
\]

(5)

\[
\rho_{14} = -(i\omega_{14} + \gamma_{14})\rho_{14} - i(\Omega_p e^{-i\omega_{p}t}\rho_{34} - \Omega_s e^{-i\omega_{s}t}\rho_{12}) + iT_e \rho_{13},
\]

(6)

\[
\rho_{23} = -(i\omega_{23} + \gamma_{23})\rho_{23} - i(\Omega_s e^{-i\omega_{s}t}\rho_{43} - \Omega_p e^{-i\omega_{p}t}\rho_{21}) + iT_e \rho_{24}.
\]

(7)

with \(\omega_{24} = \varepsilon_{2} - \varepsilon_{4}, \omega_{13} = \varepsilon_{1} - \varepsilon_{3}, \omega_{34} = \varepsilon_{3} - \varepsilon_{4}, \omega_{12} = \varepsilon_{1} - \varepsilon_{2}, \omega_{14} = \varepsilon_{1} - \varepsilon_{4}, \omega_{23} = \varepsilon_{2} - \varepsilon_{3}.

Making the substitution,

\[
\tilde{\rho}_{24} = \rho_{24} e^{i\omega_{s}t}, \tilde{\rho}_{13} = \rho_{13} e^{i\omega_{p}t}, \tilde{\rho}_{34} = \rho_{34},
\]

\[
\tilde{\rho}_{12} = \rho_{12} e^{i(\omega_{p} - \omega_{s})t}, \tilde{\rho}_{14} = \rho_{14} e^{i\omega_{p}t}, \tilde{\rho}_{23} = \rho_{23} e^{i\omega_{s}t}.
\]

The formulas can be rewritten as,

\[
\tilde{\rho}_{24} = -(i\Delta_{24} + \gamma_{24})\tilde{\rho}_{24} - i\Omega_s (\tilde{\rho}_{44} - \tilde{\rho}_{22}) + iT_e \tilde{\rho}_{23},
\]

(8)

\[
\tilde{\rho}_{13} = -(i\Delta_{13} + \gamma_{13})\tilde{\rho}_{13} - i\Omega_p (\tilde{\rho}_{33} - \tilde{\rho}_{11}) + iT_e \tilde{\rho}_{14},
\]

(9)

\[
\tilde{\rho}_{34} = -(i\omega_{34} + \gamma_{34})\tilde{\rho}_{34} - i(\Omega_p \tilde{\rho}_{14} - \Omega_s \tilde{\rho}_{32}) - iT_e (\tilde{\rho}_{44} - \tilde{\rho}_{33}),
\]

(10)

\[
\tilde{\rho}_{12} = -(i\Delta_{12} + \gamma_{12})\tilde{\rho}_{12} - i(\Omega_p \tilde{\rho}_{32} - \Omega_s \tilde{\rho}_{14}),
\]

(11)

\[
\tilde{\rho}_{14} = -(i\Delta_{14} + \gamma_{14})\tilde{\rho}_{14} - i(\Omega_p \tilde{\rho}_{34} - \Omega_s \tilde{\rho}_{12}) + iT_e \tilde{\rho}_{13},
\]

(12)

\[
\tilde{\rho}_{23} = -(i\Delta_{23} + \gamma_{23})\tilde{\rho}_{23} - i(\Omega_s \tilde{\rho}_{43} - \Omega_p \tilde{\rho}_{21}) + iT_e \tilde{\rho}_{24}.
\]

(13)

where \(\gamma_{13}, \gamma_{24}, \gamma_{34}\) represent decay rates for \(\rho_{13}, \rho_{24}, \rho_{34}\).
Under single electron approximation, the double quantum-dot system is initially in the ground state $|2\rangle$, so we assume that $\tilde{\rho}_{22}^{(0)} = 1, \tilde{\rho}_{11}^{(0)} = \tilde{\rho}_{33}^{(0)} = \tilde{\rho}_{44}^{(0)} = 0$. And at the initial, the pump light and the voltage are strong, so $\tilde{\rho}_{13}^{(0)} = 0, \tilde{\rho}_{34}^{(0)} = 0$.

This set of equations can be solved,

$$|M| = (i\Delta_{24} + \gamma_{24})(i(\Delta_{13} - \Delta_{24} - \omega_{34}) + \gamma_{12})(i(\Delta_{13} + \omega_{34}) + \gamma_{14})(i(\Delta_{24} - \omega_{34}) + \gamma_{23})$$

$$-(i\Delta_{24} + \gamma_{24})(i(\Delta_{13} + \omega_{34}) + \gamma_{14})\Omega_{p}^{2} - T_{e}^{2}\Omega_{s}^{2}, \quad (14)$$

$$\tilde{\rho}_{24} = \frac{i\Omega_{s}}{|M|}[(i(\Delta_{13} - \Delta_{24} - \omega_{34}) + \gamma_{12})(i(\Delta_{13} + \omega_{34}) + \gamma_{14})(i(\Delta_{24} - \omega_{34}) + \gamma_{23})$$

$$-(i(\Delta_{24} - \omega_{34}) + \gamma_{23})\Omega_{s}^{2} + (i(\Delta_{13} + \omega_{34}) + \gamma_{14})\Omega_{p}^{2}], \quad (15)$$

We then obtain

$$\chi^{(3)} = \frac{N\rho_{24}}{3E_{p}^{2}E_{s}} = \frac{N\mu^{4}}{3h^{3}}\chi_{\text{eff}}^{(3)}, \quad (16)$$

$$\chi_{\text{eff}}^{(3)} = \frac{i(i(\Delta_{13} + \omega_{34}) + \gamma_{14})}{\alpha + i\beta} = \frac{-(\Delta_{13} + \omega_{34})\alpha + i\beta(\Delta_{13} + \omega_{34}) + i\alpha\gamma_{14} + \beta\gamma_{14}}{\alpha^{2} + \beta^{2}} \quad (17)$$

with

$$\alpha = (\Delta_{13} - \Delta_{24} - \omega_{34})(\Delta_{13} + \omega_{34})(\Delta_{24} - \omega_{34})\gamma_{14}\gamma_{23}$$

$$-(\Delta_{13} + \omega_{34})\Delta_{24}\gamma_{12}\gamma_{23} - (\Delta_{24} - \omega_{34})\Delta_{24}\gamma_{12}\gamma_{14} - (\Delta_{13} - \Delta_{24} - \omega_{34})(\Delta_{13} + \omega_{34})\gamma_{23}\gamma_{24}$$

$$-(\Delta_{13} - \Delta_{24} - \omega_{34})(\Delta_{24} - \omega_{34})\gamma_{14}\gamma_{24} - (\Delta_{13} + \omega_{34})(\Delta_{24} - \omega_{34})\gamma_{12}\gamma_{24} + \gamma_{12}\gamma_{14}\gamma_{23}\gamma_{24}$$

$$+(\Delta_{24}(\Delta_{13} + \omega_{34}) - \gamma_{14}\gamma_{24})\Omega_{p}^{2} - T_{e}^{2}\Omega_{s}^{2}, \quad (18)$$

$$\beta = -(\Delta_{13} + \omega_{34})(\Delta_{13} - \Delta_{24} - \omega_{34})(\Delta_{24} - \omega_{34})\gamma_{23} - (\Delta_{24} - \omega_{34})\Delta_{24}(\Delta_{13} - \Delta_{24} - \omega_{34})\gamma_{14}$$

$$-(\Delta_{13} + \omega_{34})(\Delta_{24} - \omega_{34})\Delta_{24}\gamma_{12} + \Delta_{24}\gamma_{12}\gamma_{14}\gamma_{23} - (\Delta_{13} - \Delta_{24} - \omega_{34})(\Delta_{13} + \omega_{34})(\Delta_{24} - \omega_{34})\gamma_{24}$$

$$+(\Delta_{13} - \Delta_{24} - \omega_{34})\gamma_{23}\gamma_{14}\gamma_{24} + (\Delta_{13} + \omega_{34})\gamma_{23}\gamma_{12}\gamma_{24} + (\Delta_{24} - \omega_{34})\gamma_{12}\gamma_{14}\gamma_{24}$$

$$-(\Delta_{24}\gamma_{14} + (\Delta_{13} + \omega_{34})\gamma_{24})\Omega_{p}^{2}, \quad (19)$$

The real and imaginary parts of $\chi_{\text{eff}}^{(3)}$, representatively, yield the Kerr nonlinearity and nonlinear absorption. In the model we presented before, the nonlinear properties can be controlled through the intensity $\Omega_{p}$ and $\Omega_{s}$, detunings $\Delta_{13}$ and $\Delta_{24}$, and tunneling $T_{e}$ via a bias voltage. Since $\Omega_{s}$ is much smaller than $\Omega_{p}$, we focus on changing of $\Omega_{p}$. Fig.2 shows the normalized Kerr nonlinearity index and the nonlinear absorption as a function of the detuning $\Delta_{24}$ with tunneling $T_{e}$ equal to 10$\gamma_{24}$. We choose $\gamma_{12} = \gamma_{14} = \gamma_{23} = 0$, because they are much smaller than decay rate $\gamma_{24}$, which is usually the case. When the detuning $\Delta_{13} = 0,$
which means both signal and pump light are on resonance, Kerr nonlinearity would increase with increasing $\Omega_p$. Taking Fig.2(a) as an example, when detuning $\Delta_{24}/\gamma_{24} = 0.7$, the nonlinear absorption is approximately zero, while Kerr nonlinearity reaches its maximum. This result indicates that Kerr nonlinearity can be enhanced, while the nonlinear absorption is cancelled. Comparing Fig.2(a) with (c), if $\Omega_p$ is fixed, Kerr nonlinearity is larger under off-resonant condition. In order to obtain the maximal Kerr nonlinearity with a certain bias voltage, experimentally, one should choose both large $\Omega_p$ and $\Delta_{24}$.

The Kerr nonlinearity as a function of tunneling $T_e$ is plotted in Fig.3. Here are parameters we choose: the quantum-dot has a base length of $9nm$ and a height of $3.5nm$ and an area density of about $4 \times 10^{-10} cm^2$. The interband transition energy for each dot are $1.1eV$ and $1.3eV$ respectively. The bandwidth of signal laser is $100Gbits/s$, which corresponding to $\hbar\gamma_{24} = 66\mu eV$. $\hbar\Omega_p$ and $\hbar\Omega_s$ are $1eV$ and $0.1eV$, respectively, since pump beam is strong. When bias voltage is zero, the whole system turns to be EIT in QD systems, which we call Phonon Induced Transparency. Four lines give the off-resonance condition of both pump light and signal light. Comparing $\Delta = -3$ with $\Delta = 3$, and $\Delta = -6$ with $\Delta = 6$, if detuning is positive, Kerr nonlinearity begins from negative value. Conversely, Kerr nonlinearity would be positive at the beginning if detuning is negative. All four curves indicate that Kerr nonlinearities have a maximum point and a minimum point. When detuning is positive, Kerr nonlinearity first increases to maximum point, then decreases to minimum point as the tunneling $T_e$ increases. Eq.(17) exhibits that the contributions to the third-order susceptibility consists of two part: the first is the influence of the pump light; the second part is the transit between level $|1>$ and $|4>$ which is negligible. So in Fig.3, the off-resonant line shape is indeed determined by pump field. When a light beam travels in a homogeneous Kerr medium, positive nonlinearity ($>0$) always leads to self-focusing, while negative nonlinearity ($<0$) leads to self-defocusing. This figure shows that by properly controlling the bias voltage, the signal light can be either self-focusing, or self-defocusing.

In conclusion, we have theoretically investigated the enhancement of Kerr nonlinearity in asymmetric double QDs. The results show that by proper tuning of the bias voltage, giant Kerr nonlinearities can be realized while cancelling nonlinear absorption. This structure also indicates that a wide range of Kerr nonlinearity from negative to positive can be simply adjusted via a bias voltage, which would result in wide control of signal beam.

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Figure Captions

Fig.1. (a) Scheme of the setup. Signal light and pump light transit through QDs. $V_B$ is a bias voltage. (b) Energy level scheme for an asymmetric double QD system. $|3\rangle$ and $|4\rangle$ are coupled by the tunneling $T_e$.

Fig.2 The real part and imaginary part of the linear optical susceptibility as a function of the detuning $\Delta_{24}/\gamma_{24}$. The solid lines are Kerr nonlinearities, and the dashed lines are nonlinear absorption. Parameters are chosen as: $\omega_{34}/\gamma_{24} = 1, \Omega_s/\gamma_{24} = 0.1, T_e/\gamma_{24} = 10, \gamma_{12} = \gamma_{14} = \gamma_{23} = 0$.

Fig.3 The Kerr coefficient as a function of tunneling $T_e/\gamma_{24}$. On-resonance and near-resonance conditions are considered. Parameters are chosen as: $\omega_{34}/\gamma_{24} = 1, \gamma_{12} = \gamma_{14} = \gamma_{23} = 0$. $\hbar\Omega_p$ and $\hbar\Omega_s$ are 1$eV$ and 0.1$eV$, respectively.
FIG. 1:
FIG. 2:
FIG. 3: