Transversal and longitudinal gluon spectral functions from twisted mass lattice QCD with $N_f = 2 + 1 + 1$ flavors

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Abstract

I report on the first application of a novel, generalized Bayesian reconstruction (BR) method for spectral functions to the characterization of QCD constituents. These spectral functions find applications in off-shell kinetics of the quark-gluon plasma and in calculations of transport coefficients. The new BR method is applied to Euclidean propagator data, obtained in Landau gauge on lattices with $N_f = 2 + 1 + 1$ dynamical flavors by the “twisted mass at finite temperature” (tmfT) collaboration. The deployed reconstruction method is designed for spectral functions that can exhibit positivity violation (opposed to that of hadronic bound states). The transversal and longitudinal gluon spectral functions show a robust structure composed of quasiparticle peak and a negative trough. Characteristic differences between the hadronic and the plasma phase and between the two channels become visible. We obtain the temperature dependence of the transversal and longitudinal gluon masses.

Keywords: gluon propagator, gluon spectral function, lattice QCD, Bayesian reconstruction

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1 Introduction

Nowadays, the focus of lattice studies of extreme QCD is increasingly shifting from mere phase structure to an understanding of the real-time dynamics of strongly interacting matter. The aim is to learn how phase transitions proceed and how the essentially different degrees of freedom emerge. Already since long quasiparticle ideas play a role in phenomenological descriptions of the quark-gluon plasma in equilibrium [1], and later appeared in more ambitious off-shell kinetic approaches [2]. In these cases, the “in-medium” gluon and quark spectral functions are an important input. During the last years, functional approaches (FRG and DSE) to continuum QCD have been developed (for two recent FRG studies see [3, 4]). Among other objectives, this work has pointed out how quasiparticle spectral functions can be used in order to express transport coefficients of the quark-gluon plasma [5, 6].

In our work [7], that I am going to report on, the spectral function encoding gluon properties is calculated in a way that is applicable across the phase transition (crossover) of lattice QCD. In our concrete case this method is applied to simulation results of QCD with $N_f = 2 + 1 + 1$ dynamical quarks pursued by the tmfT collaboration [8, 9, 10].

The gluon spectral function witnesses the fact that the gluon is not a physical particle: the spectral function is (i) gauge dependent and it (ii) violates spectral positivity [11, 12]. The degree of violation is expected to be stronger in confinement than in the deconfinement phase. In the deconfined (quark-gluon plasma) phase, the quasiparticle picture of gluons and quarks is obviously useful. It comprises insight into the physics of the quark-gluon plasma complementary to what lattice gauge theory usually provides. The violation of spectral positivity precludes the application of standard algorithms like MEM [13] or standard Bayesian reconstruction [14] for the calculation of the gluon spectral function.
2 From Gluon Correlators to Gluon Spectral Functions

Our raw data come from a (Euclidean) simulation of dynamical QCD within the twisted mass approach, including strange and charm quarks with physical mass \[8\]. The simulations of the tmfT collaboration have been performed for various light quark masses heavier than physical. In order to test our reconstruction method we have selected ensembles generated for \(m_\pi \approx 370\) MeV at three different lattice spacings (\(\beta = 1.90,\ \beta = 1.95\) and \(\beta = 2.10\)). I present here only the results for the finest lattice.

After gauge-fixing the lattice configurations to the Landau gauge, we have calculated the gluon propagator, extended to non-zero Matsubara frequencies. Unfolding the real-time information from the ensemble average \[4\] through the ensemble average

\[
A^\mu_\rho(x + \frac{\hat{p}}{2}) = \frac{1}{2i a g_0} \left( U_{x,\mu} - U^\dagger_{x,\mu} \right) \Big|_{\text{traceless}},
\]

(1)

through the ensemble average

\[
D^{ab}_{\mu \nu}(q) = \langle \hat{A}_\mu^a(q) \hat{A}_\nu^b(-q) \rangle.
\]

(2)

In a thermal state (heat bath) characterized by some temperature, the gluon propagator can be decomposed into transversal (chromomagnetic) and longitudinal (chromoelectric) parts

\[
D^{ab}_{\mu \nu}(q_4, q) = \delta^{ab} \left( P^T_{\mu \nu} \ D_T(q_4, q) + P^L_{\mu \nu} \ D_L(q_4, q^2) \right)
\]

(3)

with the projectors \(P^T_{\mu \nu} = \left( \delta_{\mu \nu} - \frac{q_{\mu} q_{\nu}}{q^2} \right)\) - \(P^T_{\mu \nu}\) (longitudinal) and \(P^L_{\mu \nu} = (1 - \delta_{\mu \nu})(1 - \delta_{\mu \nu}) \left( \delta_{\mu \nu} - \frac{q_{\mu} q_{\nu}}{q^2} \right)\) (transversal). These gluon correlators are related to the respective spectral functions via the Källen-Lehmann representation linking Minkowski and Euclidean space

\[
D_{T,L}(q_4, q) = \int_{-\infty}^{\infty} \frac{1}{i q_4 + \omega} \rho_{T,L}(\omega, q) d\omega
\]

(4)

\[
D_{T,L}(q_4, q) = \int_0^{\infty} \frac{2 \omega}{q_4^2 + \omega^2} \rho_{T,L}(\omega, q) d\omega.
\]

(5)

The spectral function is understood to be antisymmetric, \(\rho(-\omega) = -\rho(\omega)\).

Inverting this relation for transversal and longitudinal propagators at any \(q\) is an inverse problem, which can be tackled with Bayesian methods. In this approach, the probability functional of a test spectral function \(\rho(\omega)\) can be written in factorized form

\[
P[\rho|D, I] \propto P[D|\rho, I] P[\rho|I].
\]

(6)

The first factor \(P[D|\rho, I] = \exp (-L)\) contains the \(\chi^2\) likelihood \(L\) that \(\rho\) reproduces the lattice data \(D_i\) (of \(D_T\) or \(D_L\)) for each Matsubara frequency,

\[
L = \frac{1}{2} \sum_{i,j=1}^{N_{q_4}} (D_i - D_i^\theta) C_{ij}^{-1} (D_j - D_j^\theta),
\]

(7)

where \(C_{ij}\) is the covariance matrix of the measured \(D_i\) and

\[
D_i^\theta = \sum_{l=1}^{N_\omega} K_{il} \ \rho_l \ \Delta \omega_l, \quad 0 \leq i \leq N_{q_4}
\]

(8)

is the binned integral form of the Källen-Lehmann representation for \(D_i\) (with a huge number of bins \(N_\omega >> N_{q_4}\), compared to the small number of data points (Matsubara frequencies). This makes a regularization indispensable.
The second factor contains the so-called prior probability in the form $P[\rho|I] = \exp(\alpha S)$. The prior data $I$ consists of the functional form of $S = S[\rho(\omega), m(\omega)]$, which depends on the applied test spectral function $\rho$ and a default model, the model function $m(\omega)$, as well as the binning made to enable the integration. In the case of the Maximal Entropy Method (MEM) the Shannon-Jaynes (relative) entropy plays this role

$$S_{MEM} = \int d\omega \left( \rho(\omega) - m(\omega) - \rho(\omega) \log \frac{\rho(\omega)}{m(\omega)} \right), \quad (9)$$

while in the case of standard Bayesian reconstruction

$$S_{BR} = \int d\omega \left( 1 - \frac{\rho(\omega)}{m(\omega)} + \log \left[ \frac{\rho(\omega)}{m(\omega)} \right] \right). \quad (10)$$

Both is not applicable if spectral positivity is violated. Instead we apply a generalized BR regulator $[7]

$$S_{BR}^g = \int d\omega \left( -\frac{\rho(\omega) - m(\omega)}{h(\omega)} + \log \left[ 1 + \frac{\rho(\omega) - m(\omega)}{h(\omega)} \right] \right). \quad (11)$$

The additional model function $h(\omega)$ presents the confidence in the default model function $m(\omega)$. The spectral reconstruction consists in finding the most probable spectral function according to

$$\frac{\delta P[\rho|D, I]}{\delta \rho(\omega)} \bigg|_{\rho=\rho_{Bayes}} = 0. \quad (12)$$

The generalized BR prior is the weakest among all similar priors, in contrast – for example – to a prior quadratic in $\rho(\omega) - m(\omega)$. Consequently, the spectral function is to a maximal amount determined by the lattice data $D$, and only minimally influenced by the regulator. Previously, the $T$-dependence of the electric and magnetic gluon propagators $[17, 18, 19]$ was discussed with attention restricted only to the gluon propagators at Matsubara frequency $q_4 = 0$. Here we are going to exploit the splitting of the propagator data between Matsubara frequencies. From this we shall infer the corresponding $\omega$-dependence of the reconstructed spectral functions. In previous attempts to the reconstruction problem it has been often assumed that the propagator satisfies some $O(4)$ invariance $[10]$ in momentum space, $D_{T,L}(q_4, |q|) \approx D_{T,L} \left( 0, \sqrt{q_4^2 + q^2} \right)$. As our correlator has shown, this assumption is justified within small deviations only close to $q_4 = 0$, but becomes problematic near to the end of the Brillouin zone.

## 3 Reconstructed spectra below and above $T_\chi$

I show in Fig. 1 the result of reconstructing the longitudinal and transversal gluon spectral functions for $T = 0.152$ GeV < $T_\chi$ from data obtained on a lattice with $N_\sigma = 48$ and a temporal extent $N_\tau = 20$
at $m_\pi = 370$ MeV and $\beta = 2.10$: We clearly observe at low temperature a structure of a peak and subsequent broader negative trough in both channels. The negative contribution appears slightly stronger in the transversal sector.

Next I show in Fig. 2 the result of reconstructing the longitudinal and transversal gluon spectral functions for $T = 0.305$ GeV > $T_\chi$ from a similar data set with $N_\sigma = 32$ and $N_\tau = 10$: The negative contribution is significantly reduced at this temperature $T > T_\chi$ in both channels.

One may use the peak position of the low-momentum structure of the longitudinal spectral function to define a longitudinal gluon dispersion relation $\omega^\text{max}_L(q)$ valid at the respective temperature. In Fig. 3 (left) I show the maximum position in energy vs. momentum for eight values of the temperature. Fig. 3 (right) shows a free–field fit for the lowest and the highest temperature. The intercept at $|q| = 0$ (the longitudinal gluon mass) is decreasing with rising temperature across the transition (which is, actually, a thermal crossover). Even for the highest temperature, the longitudinal gluon mass is larger than the Debye screening mass exhibited by the heavy quark potential at the same temperature in simulations with $N_f = 2 + 1$ flavors [20] (with no momentum dependence). A corresponding measurement of the Debye mass is not yet available from our twisted mass simulations with $N_f = 2 + 1 + 1$.

In Fig. 4 the transversal gluon dispersion relation $\omega^\text{max}_T(q)$ is shown. In the left panel I show the maximum position in energy vs. momentum for eight temperatures. Fig. 4 (right) shows a free–field fit for the lowest and the highest temperature.

Figure 2: The longitudinal and the transversal gluon spectral functions for different momenta at temperature $T = 305$ MeV > $T_\chi$.

Figure 3: Left: Momentum dependence of the longitudinal quasiparticle peak position at $\beta = 2.10$ showing a non-zero intercept. Right: Fit of the lowest and highest temperature curves with the free–field ansatz $\omega^\text{L}_L(|\vec{q}|) = A \sqrt{B^2 + |\vec{q}|^2}$. The gluon quasiparticle mass is defined as $m = AB$. The Debye mass from $N_f = 2 + 1$ lattice QCD [20] is given for comparison.
Figure 4: Left: Momentum dependence of the transversal quasiparticle peak position at $\beta = 2.10$ showing a non-zero intercept. Right: Fit of the lowest and highest temperature curves with the free-field ansatz $\omega_0^q(|q|) = A \sqrt{B^2 + |q|^2}$. The gluon quasiparticle mass is defined as $m = AB$. The Debye mass from $N_f = 2 + 1$ lattice QCD [20] is given for comparison.

Also in the transversal case, the intercept at $|q| = 0$ (the transversal gluon mass) is decreasing with rising temperature across the transition. Now, for the higher temperature, the transversal gluon mass drops below the Debye screening mass of the heavy quark potential at the same temperature obtained in simulations with $N_f = 2 + 1$ flavors [20].

4 Conclusion and Outlook

We have demonstrated the successful application of a new Bayesian method to reconstruct the gluon spectral function from gluon two-point correlator data in $N_f = 2 + 1 + 1$ flavor twisted mass QCD thermodynamics with $m_\pi = 370$ MeV across the deconfining and chiral restoration crossover.

Presently, we are preparing to repeat this analysis for twisted mass ensembles with a lower pion mass $m_\pi \approx 210$ MeV. A measurement of the static $Q\bar{Q}$ potential with $N_f = 2 + 1 + 1$ twisted mass flavors in the deconfinement phase and the determination of the Debye screening mass belongs to our nearest goals in a program of twisted mass lattice QCD thermodynamics.

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