Evidence for quantized magnetic flux in an axon

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Abstract

In December of 2018 I published my consolidated findings of a closed-form description of propagated signaling phenomena in the membrane of an axon [1]. Those results demonstrate how intracellular conductance, the thermodynamics of magnetization, and current modulation, function together in generating an action potential in a unified differential equation. At present, I report on a subsequent finding within this model. Namely, evidence of quantized magnetic flux $\Phi_0$ in an axon.

Keywords: Action potential; axon; cable theory; field-dependent current modulation; fluxoid; Hodgkin; Huxley; Langevin; intracellular magnetization; magnetic flux quantum; membrane depolarization; membrane electric field; membrane magnetic field; myelinated nerve; quantized magnetic flux.

I. Scope of this Article

To present evidence that the natural constant magnetic flux quantum $\Phi_0$ [4,5] is built-into the fabric of the action potential as per my recently published model [1].

II. Method

Melendy [1] demonstrated that the differential equation:

$$\frac{1}{\Gamma} V_m - u = 0 \quad (1a)$$

is a novel, closed-form quantification of the membrane action potential, $V_m$. (1a) is in contrast to the Hodgkin-Huxley quantification of $V_m$ [6] which requires numerically integrating four differential equations to solve for the membrane voltage. Here, $u = (67.9 \times 10^{-9}) \Gamma^{-1}$, where:

$$\Gamma = \frac{\varepsilon_0 \Delta R (\Delta \sin \pi r)}{(0.5 \pi)^2 \cosh (\pi r) / G_m \cosh (\pi \chi)} \quad (1b)$$

In review [1] (p. 107), $G_m$ is the leaky cable input conductance along the longitudinal length of neuronal fiber ($\Omega^{-1}$) and $X$ (Chi) is a normalized length (dimensionless) [7,8]. The hyperbolic tangent term (p. 108) is Langevin’s thermodynamic relation [9]. The sine term (p. 108-109) is the current-modulation function and is a magnetic field-dependent current. On p. 110 [1] the myelinated thickness of the axon is given as $\Delta r$ and $\varepsilon_0$ is the vacuum permittivity of free space ($8.854 \times 10^{-12}$ $F/m$). The relationship between $V_m$ and $\Gamma$ is given as $V_m = \Gamma E_m - 67.9 \times 10^{-9}$ V (p. 112), where $E_m$ is the membrane electric field (V/m).

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From classical electrodynamics [10], Melendy showed that (p. 110) $E_m = (2\pi a^2) \rho_m \sin(nt)$, where $\alpha$ is the axon radius ($\mu m$), $\rho_m$ is the longitudinal membrane resistivity ($\Omega$-m), $B_m$ is the membrane magnetic field (T or Wb/m$^2$), and $\mu_0$ is the vacuum permittivity of free space ($4\pi \times 10^{-7}$ H/m). From this relationship between $E_m$ and $B_m$ is Melendy’s origianlly-derived model (p. 109) but that includes identical numerical parameters as in (1b):

$$V_m = \frac{B_m^2 \frac{2\pi a}{\mu_0}}{(0.5\pi \sin(\frac{4\pi \mu_B t}{\pi}))} - 67.9 \times 10^{-3} V \quad (1c)$$

In other words, (1c) produces identical results to (4b) (p. 111) [1] for $B_m = 2.964(74) \times 10^{-5}$ Wb/m$^2$ and $\alpha = 4.710(94) \mu m$.

The laws of electrodynamics state that $B_m = \Phi_m / \pi a^2$, where $\Phi_m$ is the membrane magnetic flux (Wb). The axon cross-sectional area [1] is $\pi a^2 = 6.972(15) \times 10^{-11}$ m$^2$, resulting in a flux of $\Phi_m = 2.067(06) \times 10^{-15}$ Wb. This is very nearly the value of the magnetic flux quantum, $\Phi_0 = \hbar/2e$ [11-13], where $h = 6.626 069 934 \times 10^{-34}$ J/Hz is the currently reported measured value of Planck’s constant [14] and $e = 1.602 176 634 \times 10^{-19}$ C is elementary charge [15].

**III. Summary**

The development of an original, quantitative model of the membrane (action) potential $V_m$ was presented in [1]. This is a conductance-based model rooted in cable theory. It is independent of the chemistry and physics behind the contribution of sodium, potassium, and leakage ions to the action potential cycle. In this article, the electric field model (1a) was re-stated in terms of the membrane magnetic field $B_m$ and subsequently, the membrane magnetic flux, $\Phi_m$. It was discovered that $\Phi_m = 2.067(06) \times 10^{-15}$ Wb, which is very nearly the value of the magnetic flux quantum, $\Phi_0 = \hbar/2e$. This is evidence for quantized magnetic flux in the membrane of an axon.

**IV. Ethical Approval**

The conducted research reported in this article is not related to either human or animal use.

**V. Compliance with Ethical Standards**

Conflict of Interests: The author Robert F. Melendy, Ph.D. declares that I have no conflict of interest(s).

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