Logistic Growth for the Nuzi Cuneiform Tablets:
Analyzing Family Networks in Ancient Mesopotamia

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We reconstruct the year of publication of each cuneiform tablet of the Nuzi society in ancient Mesopotamia. The tablets, are on land transaction, marriage, loan, slavery contracts etc. The number of tablets seem to increase by logistic growth until saturation. It may show the dynamics of concentration of lands or other properties into few powerful families in a period of about twenty years. We reconstruct family trees and social networks of Nuzi and estimate the publication years of cuneiform tablets consistently with the trees and networks, formulating least squares problems with linear inequality constraints.

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The observed phase changes of phenomena in human sciences are sometimes well explained by using the models developed in physics. The neolithic transition (i.e., the shift from hunter-gatherer to agricultural economies), change of word ordering rules in natural languages, change of election systems in the present society, and many interesting problems.

The time evolution of Networks based on human interactions in economy, politics, transportation systems, Internet, family trees, etc. is drawing the attention to the present physicists. Here we make use of the interactions in the network of family trees to estimate the years of the contracts given in the Nuzi cuneiform tablets in ancient Mesopotamia to find the simplest nonlinear dynamics, logistic growth of the number of cuneiform tablets.

The Nuzi society, existed for five or six generations in about 15th century B.C. ‘Nuzi Personal Names’ (NPN) is the index for personal names with the kinships for the cuneiform tablets. We introduce the time coordinate of publication of each cuneiform tablet, as well as the birth year and the death year of each person listed in NPN. Fig 2. We formulating least squares problems with linear inequality constraints making use of conditions obtained by kinship relations and other information in NPN. For example we assume that a father is at least 15 years older than his son, contractors were living at the time of the contract, etc. It seems that publication of the cuneiform tablets (documents) are concentrated on a period of twenty years and that they continued to be published until saturation by logistic growth Fig 4.

The large portion of the documents are on land transaction, for example by false adoption given later JEN 208. The logistic growth seems to show the dynamics of the emergence of social hierarchy, which is well recognized in philological studies, namely the increase of concentration of lands or other properties into few powerful families, as the Tehiptilla family. The logistic growth seems to be natural to understand the accumulation of cuneiform tablets in Nuzi and support to convince the success of our numerical study. Our time obtained from the least square will reconstruct the life of Tehiptilla and other persons in Nuzi in ancient Mesopotamia.

Nuzi is known for its unique false adoption documents as shown below in Example 1, the cuneiform document JEN 208. The adopted members protected the adoptive father’s family in exchange for their properties, such as land estates. There exists a family called Tehiptilla’s family, and this family was involved in many contracts and transactions. JEN 208. is the contract between Iluija, a son of Hamattar and Tehiptilla a son of Puhisenni, with the names of witnesses and scribes. The documents JEN are given in, and JENu, given later, are unpublished Nuzi tablets excavated by the Iraq Museum and the American Schools of Oriental Research in 1926.

Example 1. (JEN 208) Adoption tablet. Iluija, a son of Hamattar. He made, Tehiptilla son of Puhisenni, for sonship. Iluija assigned Tehiptilla to as (his) share 2 imer and 3 awehari of land in the large standard, west of the dimtu of Imbi-ili-su, east of the dimtu of enilia. And Tehiptilla gave to Iluija as his gift 10 imer of barley. If the land gets a claimant, Iluija shall clear (it), to Tehiptilla he (shall) restore (it). The ilk service of the land only [I]luia [shall] bear/ [If Iluija infringes the agreement], [he shall furnish 1 mina of silver (and) 1 mina of gold]. (Rest of the obverse destroyed) L1.17-20:4 seals; some seals destroyed.

[ ]: broken
( ): added by original translator

dimtu: originally means a tower, but also means a region surrounded by city wall, and a district. Jancowska pointed out that there were 71 dimtu in Nuzi.

In NPN, the personal names originally written in cuneiform are included as the form of phonetic values with alphabet. For example a name Iluija in JEN 208 is expressed in three cuneiform characters and these phonetic values joined by hyphen as I-lu-ia. Cuneiform character sometimes has more than one phonetic value, and different values may be put by different readers or scholars such as I-lu-ya. We analyzed data only appears in NPN, we could avoid this reading difference.
Most names found in Nuzi documents are male and inscribed with his father, as Tehiptilla, son of Puhîshnûni. There are 124 persons named Iluia, 7 persons named Iluia, 1 person named Hamattar, etc. We take the sequence of phonetic values listed at first in NPN.

In NPN, individual names are listed in alphabetical order with the information in the original cuneiform documents. Example 2 is the part of NPN associated with two names "Iluiâ" and "Hamattar" who appear in JEN 208. We see that there is one Hamattar. There are 7 Iluia and some of them represent the same person.

**Example 2.** Description on Hamattar and Iluia in Nuzi personal names (A syllable is separated by −). The special character ¯ is described by h_(u). The ¯ is described by s* (v).)

H_(u)AMATTAR
H_(u)am-ma-at-ta-ar var. (2) H_(u)am-ma-at-ti-ir
1) f. of I-lu-ia JEN 208:2; (2) JENu 414; gf. of Ta-a-a JEN 369:4
ILUIA
Ilui-ia var. (2) I-lu-ia
1) s. of H_(u)am-ma-at-ta-ar (2) JEN 208: 8 11 13 14; 369:3; 10; H_(u)am-ma-at-ti-ir (2) JENu 414
2) scribes. of "(d)Sin-na-ap-s^(v)
3) s. of U^(v)-en-ur-se JEN 13:37
4) f. of S^(v)-ya-ar-ti-la JEN 640:13; 662:95; HSS II 7:31 (read so against Ili-iddina of copy); 35:38; RA XXXII 33:33; 50:43; 67:23
5) f. of Ta-a-a JEN 369:3 101
6) f. of Da-an-ni-mu-s^(v) JEN 345:5
7) scribe JENu 625; AASOR XVI 56:41

These information include a represented name, the scribal variations, kinship relations, and the line number in which the name appears. For example, JEN 208:2 means that the name appears in the line 2 of the cuneiform tablet JEN 208. In some case, only document is shown without a line number, like JENu 414. The contractor’s names are usually included in the first 10 lines, about 2 to 5 lines, in each documents, while the names of witnesses and scribes are given after the first 10 lines.

We list all personal names of JEN 208.

**JEN 208**

| line | name and kinship information |
|------|------------------------------|
| 1    | IluiaCs. of H _(u)am-ma-at-ta-ar |
| 2    | H _(u)am-ma-at-ta-arCf. of I-lu-ia |
| 3    | Pu-h _(u)am-ma-at-ta-arCf. of Te-h _(u)am-ma-at-ta-arCf. |
| 4    | Im-bi-li-s^(v) |
| 5    | E-ni-ia |
| 6    | IluiaCs. of H _(u)am-ma-at-ta-ar |
| 7    | Te-h _(u)am-ma-at-ta-arCf. of Te-h _(u)am-ma-at-ta-arCf. |
| 8    | Pu-h _(u)am-ma-at-ta-arCf. of Te-h _(u)am-ma-at-ta-arCf. |
| 9    | Te-h _(u)am-ma-at-ta-arCf. of Te-h _(u)am-ma-at-ta-arCf. |
| 10   | Pu-h _(u)am-ma-at-ta-arCf. of Te-h _(u)am-ma-at-ta-arCf. |
| 11   | IluiaCs. of H _(u)am-ma-at-ta-ar |
| 12   | IluiaCs. of H _(u)am-ma-at-ta-ar |
| 13   | Te-h _(u)am-ma-at-ta-arCf. of Te-h _(u)am-ma-at-ta-arCf. |
| 14   | IluiaCs. of H _(u)am-ma-at-ta-ar |
| 15   | A-ta-a-aCf. of Na^(v)U-tar(U) |
| 16   | It-h _(u)am-ma-at-ta-arCf. of Te-h _(u)am-ma-at-ta-arCf. |
| 17   | Ki-li-ip-s^(v) |
| 18   | Na-puCf. of Ki-li-ip-s^(v) |

We make a database just from NPN, which is an index of about ten thousands of individuals (about 95% of them are male) who appear in cuneiform tablets excavated from the site of Nuzi.

Several problem, however, should be solved when constructing a data base from NPN and identify the individuals,

1) Many persons share the same name,
2) Personal names might be written with different cuneiform characters,
3) Some names might be overlapping, for example, name A can be a single name, but also included in two, three or more generation family trees.

We reconstruct family trees from NPN. We apply the criterion that two persons with the same name are regarded as the same person,

(R1) if the both persons appear on the same line of the same document and regarding them as the same person does not make any contradiction.

or

(R2) if the both persons appear at least in three common documents and their relation is consistent.

For example from the above (R1), we find 1) and 5) of Iluia in Example 2 is the same person. After applying the two criterions (R1) and (R2), we obtained a total of 10343 family trees. We arrange sequentially the family trees obtained directly from the data, as f_1, f_2,..., f_10343. Let us say the family tree A and the family tree B are consistent with each other, if and only if at least two names are common in the A and the B and there is no contradiction in their kinship relations for the names both in the A and the B. Starting from the initial set of the 10343 family trees, we apply the sequential algorithm recursively to unify mutually consistent family trees. We continue the procedure until there is no family tree to be unified and get family trees as shown in Fig 1 which is a part of 16 members obtained by our procedure. The Tehiptilla family tree estimated by our sequential method, using just NPN, is about the half size of the one obtained directly from the data, as f_1, f_2,..., f_10343. Let us consider the family of Hamattar, in Example 1, 2, of three generations. Iluia, the son of Hamattar, is in the
The number of appearance of each name in documents is shown as 596 Tehiptilla. The birth year and death year of each name is as, Puhlisseni (39.06–99.06), Tehiptilla (59.06–114.06), Haistesu (63.27–114.06), Akiptasseni (74.35–134.27), Ennamati (74.06–114.06), Surkitilla (74.07–105.69), Arillumti (78.27–135.74), Umpia (78.27–114.06), Takku (89.06–139.78), Zike (89.06–116.70), Tarmitilla (89.07–137.56).

The dynamical system, for the logistic growth, has the solution
\[
\frac{df(t)}{dt} = \frac{1}{\beta} (1 - f(t)) f(t),
\]

which seems to explain the distribution of publication years of the cuneiform tablets Fig 2. We can imagine the land owner class appeared through the contracts, as in JEN 208 of Example 1, during the period and after the period the society changes to another stage of social hierarchy in which lands are owned by few families.

For example in population genetics the relative abundance of a advantageous mutant gene increases in a population until saturation \( f(t) = 1 \). Assuming the above dynamical system for the ratio \( f(t) \) of properties owned by powerful families as the Tehiptilla family, we have the probability density of logistic distribution function with mean \( \mu \) and scale parameter \( \beta \),

\[
\frac{e^{-\frac{t-\mu}{\beta}}}{\beta(1 + e^{-\frac{t-\mu}{\beta}})^2}.
\]
Although the obtained configuration is different depending on the initial lifespan given at random. We find a common social structure among the obtained configurations which would reflect what was going on in ancient Nuzi.

We formulate the optimization problem for unknown variables, the birth year $b_i$ and the death year $d_i$ of the person $i$, the id number of the father $f_i$ and the mother $m_i$ of the person $i$, and the published year $P_k$ of the document $k$. The quadratic optimization problem to minimize $\sum_i (d_i - b_i - \mu_i)^2$ finds a configuration, whose lifespans are closest to the initial lifespans $\mu_i$, in the least square sense, subject to that

(I) $b_i \geq b_f + g_f$, (a male becomes a father elder than $g_f$ years old for all person $i$ whose father appear in NPN), and $d_f \geq b_i$, $b_i \geq b_m + g_m$ (a female becomes a mother elder than $g_m$ years old, for all person $i$ whose mother appear in NPN), and $d_m \geq b_i$.

(II) $P_k \geq b_i + g_p$, $d_i \geq P_k$ (a person appeared in a document is supposed to be greater than $g_p$ years old for all documents $k$ and person $i$ in the document $k$, and $i$ lived at the time of contract).

(III) $P_{906} = 100$ to fix the origin of coordinate, where 906 is the id for JEN 525. JEN 525 contains 52 different individual names. We make a set of names, each of which is connected with directly or indirectly to one of the member through kinship relation or common contracts, and call it extended members of JEN 525. For each document we make extended members. JEN 525 has the largest extended members among all documents which includes Tehiptilla’s family.

The above least square problem was solved with a software package NUOPT (Mathematical Systems Inc.). The optimization result is depending on the initial set of lifespans. We conducted the computation for Fig 12 assuming $g_f = 15$, $g_m = 20$, and $g_p = 10$.

The result of our reconstructing family tree makes the number of variables and constraints smaller, as we find 1) and 5) of Iluia in ex.2 is the same person. A total of 10060 persons appears in our study of NPN. We denote by $\mu_i$ the lifespan of the person $i$, for $i = 1, ..., 10060$. The number of documents is 1662.

The convex quadratic program with $21782(= 10060 \times 2$ (representing $b_i$, $d_i$) + 1662 (representing $P_k$)) variables. We have 39357 constraints. The above (I) makes 4241 $\times$ 2 constraints of them, (II) makes 15437 $\times$ 2 constraints of them, and (III) makes 1 constraint of them.

We carried out the optimization 10 times independently, changing the initial span given at random. The structure of the optimal solutions are more or less the same regarding to the properties we reported here.

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