A Phenomenological Theory of Fermion Masses and Mixings

P. Q. Hung

Department of Physics, University of Virginia, Charlottesville, VA 22901
(September 6, 2018)

Abstract

A phenomenological theory of fermion masses and mixings is constructed within the framework of a four-family symmetry. It is found that the most favored set of relevant CKM elements are $|V_{us}| \approx 0.222$, $|V_{cb}| \approx 0.044$, $|V_{ub}/V_{cb}| \approx 0.082$, $|V_{ud}| \approx 0.974$, $|V_{cs}| \approx 0.9736$, $|V_{cd}| \approx 0.224$ with $\hat{B}_K \approx 0.8$. The top quark mass is predicted to be 258 GeV at 1 GeV with its physical mass approximately equal to 153 GeV. The Majorana scale associated with the fourth neutrino is bound from above to be 6.4 TeV.
Despite the impressive agreement of the standard model with experiment, it is clear that questions such as the origin of fermion masses and mixing angles cannot be answered solely within the framework of the standard model. At the present time, one does not know at what energy scale (called Family scale here) lies the solution to the mass problem. There is some belief that such a scale might be very near the Planck mass where it is hoped that all interactions (including perhaps gravity) are unified. This might be the case. On the other hand, it is also possible that the Family scale might not be too much higher than the electroweak scale. We shall explore this possibility below.

It is a fact that not only do we have inter-generation mass splitting but we also have intra-generation mass splitting. This last splitting breaks explicitly the so-called "custodial" $SU(2)$ symmetry (in the simplest version of the standard model) which guarantees the $\rho$ parameter to be equal to unity at tree level. What is the relationship between the "custodial" breaking and the behaviour of the fermion mass matrices? One first notices that, if the mass matrices for the Up and Down sectors were identical (in form and magnitude), the CKM matrix $[1]$ would become the unit matrix and all mixing angles would vanish. The fact that the CKM mixing angles are non-vanishing and small is an indication that the fermionic "custodial" breaking should be non-vanishing and small as well. By "small" we mean that the contribution to the $\rho$-parameter is small. We shall use this hint in the construction of our model.

To address the mass problem, it is obvious that one has to go beyond the minimal (three generations) standard model. We propose in this note a model with four generations and a family symmetry $[\mathcal{Z}]$. This (broken) family symmetry gives rise to mixing terms in the fermion mass matrices at one-loop level. One of the motivations for having a fourth family here is the fact that, in our model, the main contribution to the off-diagonal elements of the mass matrices come from one-loop diagrams with a scalar exchange, and large Yukawa couplings are preferable to guarantee that these elements are non-negligible. The nature of fermion masses and mixing in our framework is linked to the nature of the scalar vacuum expectation values and resulting scalar mixing. We shall show that the mixing terms in the
quark and lepton mass matrices are intrinsically related. As a consequence, we shall also show how a rare decay process such as $K_L \rightarrow \mu e$ can set an upper bound on the Majorana mass scale.

To set the stage for the construction of the mass matrices, we now list our assumptions. They are:

1) There is a vector-like $SU(4)$ family (gauge) symmetry.

2) The combined generation-electroweak symmetry is $SU(4) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. The existence of $SU(2)_R$ in our context is linked to the assumption that the family symmetry is vector-like.

3) There is a Higgs field, $\Phi = T^i \Phi^i$ with $i = 1, \ldots, 15$, which transforms as $(15, 2, 2, 0)$ and another one, $\phi$, which transforms as $(1, 2, 2, 0)$ under $SU(4) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. (We are ignoring here scalars which are electroweak singlets.)

4) The left and right-handed quarks transform respectively as $(4, 2, 1, 1/3)$ and $(4, 1, 2, 1/3)$ while the left and right-handed leptons transform as $(4, 2, 1, -1)$ and $(4, 1, 2, -1)$. The charged generator is $Q = T^3_L + T^3_R + (B - L)/2$.

5) There exists a Yukawa coupling of the form

$$L_Y = \overline{q}_L (h_1 \Phi + h_2 \tilde{\Phi} + h_1' \phi + h_2' \tilde{\phi}) q_R + h.c.$$  

$$+ \overline{l}_L (l_1 \Phi + l_2 \tilde{\Phi} + l_1' \phi + l_2' \tilde{\phi}) l_R + h.c. \quad (1)$$

where the notations used for quark and lepton fields are self-explanatory and where $(\Phi, \tilde{\phi}) \equiv (T^i_1 \tau_2 \Phi^i \tau_2, \tau_2 \phi^* \tau_2)$.

6) The Higgs fields have vacuum expectation values (VEV) \[2\]

$$\langle \Phi \rangle = (\langle \Phi_3 \rangle / \sqrt{2}) \text{diag}(1, -1, 0, 0) + (\langle \Phi_8 \rangle / \sqrt{6}) \text{diag}(1, 1, -2, 0) + (\langle \Phi_{15} \rangle / \sqrt{12}) \text{diag}(1, 1, 1, -3),$$  

\[2a\]

$$\langle \phi \rangle = \text{diag}(\kappa, \kappa'),$$  

\[2b\]

where
\[ \langle \Phi_{3,8,15} \rangle = \text{diag}(v_{3,8,15}, v'_{3,8,15}) \]  

(3)

The various VEV’s will in general be complex and can be parametrized generically as \( ve^{i\delta} \).

7) The breaking of the family symmetry leads to the following mass matrices for the scalars which connect the fourth family to the other three and to itself, and which are \( \Phi_{1,4} = (\Phi_9 + i\Phi_{10})/\sqrt{2} \), \( \Phi_{2,4} = (\Phi_{11} + i\Phi_{12})/\sqrt{2} \), \( \Phi_{3,4} = (\Phi_{13} + i\Phi_{14})/\sqrt{2} \), and \( \Phi_{44} = \Phi_{15} \),

\[
M^2 = \begin{pmatrix}
1 & a & 0 & 0 \\
0 & a & b & 0 \\
0 & b & 1 & c \\
0 & 0 & c & 1
\end{pmatrix}
\]

(4)

for the neutral sector, \( \Phi^0 \), and

\[
M^2 = \begin{pmatrix}
1 & a' & 0 & 0 \\
0 & a' & b' & 0 \\
0 & b' & 1 & c' \\
0 & 0 & c' & 1
\end{pmatrix}
\]

(5)

for the charged sector, \( \Phi^\pm \). The diagonal elements are (14), (24), (34) and (44) from top to bottom. The six coefficients \( a, \ldots, c' \) will determine the amount of mixing among different generations. It is found that the primed and unprimed coefficients are not too different from each other, which means that the amount of scalar ”isospin” breaking is small. We shall assume that the mixing among other generation-changing scalars, if it exists, is negligible compared with the above and hence the only dominant off-diagonal elements of the mass matrices are those generated by the above scalars.

8) Since \( \Phi \) transform as \( (2, 2) \) under \( SU(2)_L \otimes SU(2)_R \), each component \( \Phi^i \) takes the form

\[
\Phi^i = \begin{pmatrix}
\phi_1^{0i} & \phi_2^{1i} \\
-\phi_1^{*i} & \phi_2^{0i*}
\end{pmatrix}
\]

(6)

In the computation of the one-loop contribution to various off-diagonal elements, both \( \phi_1^i \) and \( \phi_2^i \) contribute. We will assume that they both have mass mixing of the form of Eqs.(4,5).

4
Furthermore, we will assume that the coefficients $a, \ldots, c'$ of the two sectors are proportional to each other, with a common proportionality coefficient denoted by $y$.

Flavor-changing neutral currents can be suppressed provided the masses of the family gauge bosons and mixing parameters in both the left-handed and right-handed quark ”rotation” matrices are appropriately chosen. Similar considerations apply to the scalar sector. It is beyond the scope of this paper to present such an analysis. We shall assume it can be done and shall concentrate here on the construction of the mass matrices.

Various mixing parameters and phases in the fermion mass matrices are explicitly computed in terms of parameters of the basic family symmetry. Once the mixing parameters are fixed, the phases are completely determined. These same parameters enter in the mass matrices of both quark and lepton sectors so that the mixing angles and phases of the two sectors are found to be related.

The mass mixing among the scalars are assumed to be such that only the following mixings occur: $\phi_{44} - \phi_{43}, \phi_{43} - \phi_{42}, \phi_{42} - \phi_{41}$ giving rise to a particular type of symmetric mass matrices. In a way, fermion mixing can be seen as a direct consequence of scalar mixing. If we recall Eq.(2), the breaking of the family symmetry is assumed to be such that all of the family changing scalars have vanishing vacuum expectation values (a familiar assumption). In this case, there is no direct fermion mixing at tree level coming from the symmetry breaking. On the other hand, a general gauge-invariant scalar potential will yield, after symmetry breaking, various mass mixings among the scalars. Couplings of the fermions to these same scalars will give rise to fermion mixing beyond the tree level in the mass matrices. The Yukawa couplings that enter these radiative corrections are much larger than the gauge couplings and, because of that feature, we shall ignore the contributions coming from the family gauge bosons.

The diagonalization of Eqs.(4,5) can be done analytically in a straightforward manner. Each of these four scalar mass eigenstates can now couple the fourth generation to all the other three, with the strength given by the elements of the eigenvectors. The trivial details are given elsewhere. The only remark one would like to make here is the fact that the mass
scale $M^2$ mentioned above do not enter the calculations given below because the results there (inside the logarithms) are given in terms of mass ratios.

At the one-loop level, the off-diagonal elements of the fermion mass matrices are generated by the exchange of the above four scalars and are finite. In our model, the twenty-four off-diagonal elements of the up- and down-symmetric mass matrices of the quark and lepton sectors can be computed in terms of the six parameters $(a, \ldots, c')$, the four fourth-generation masses, and a phase difference (to be specified below). Let us first concentrate on the following elements in a typical mass matrix: 4-3, 3-2, 2-1. For the up sector (for quarks), the one-loop contributions are given by

$$M_{43}^U = h_2^2 e^{i\delta_1} |M_U^0| (\lambda'' x e^{-i\Delta} - \frac{1}{r}) L_0'',$$

$$M_{32}^U = h_2^2 e^{i\delta_1} |M_U^0| (\lambda x e^{-i\Delta} - \frac{1}{r}) L_0,$$  

$$M_{21}^U = h_2^2 e^{i\delta_1} |M_U^0| (\lambda' x e^{-i\Delta} - \frac{1}{r}) L_0'',$$

where $r = h_2/h_1$ and $x = |M_D^0|/|M_U^0|$, the ratio of the fourth-generation quark masses. For the down sector, one has

$$M_{43}^D = h_1^2 e^{i\delta_2} |M_U^0| (\lambda'' e^{i\Delta} - r x) L_0'',$$

$$M_{32}^D = h_1^2 e^{i\delta_2} |M_U^0| (\lambda' e^{i\Delta} - r x) L_0,$$  

$$M_{21}^D = h_1^2 e^{i\delta_2} |M_U^0| (\lambda' e^{i\Delta} - r x) L_0'.$$

Here, $L_0^{(r,y)} = L_2^{(0,y)} - L_1^{(0,y)}$. Assumption (8) gives $L_2^{(0,y)} = r^2 L_1^{(0,y)}$ (here $y = r^2$) so that $L_0^{(r,y)} = (r^2 - 1)L_1^{(0,y)}$. The $L_0$’s are the one-loop contribution to the off-diagonal elements coming from neutral scalars. The parameters $\lambda^{(r,y)}$ represent the contribution of the charged scalars relative to the neutral ones. They are defined for the case when $r \neq 1$. Assumption (8) gives $L_2^{+(r,y)} = r^2 L_1^{+(r,y)}$. One then has $\lambda^{(r,y)} = (r/(r^2 - 1))(L_1^{+(r,y)}/L_1^{(0,y)})$. Notice that
the various $L$'s can be explicitly computed using the mass eigenstates and eigenvectors of Eqs. (4,5). They are typically of the form: $(A \ln (m_1/m_2) + B \ln (m_3/m_4))/16\pi^2$, where $A$ and $B$ are some combinations of the elements of the eigenvectors. The parameter $\Delta$ is defined as $\Delta = \delta_1 - \delta_2$ where $m_U^0 \equiv e^{i\delta_1}|m_U^0|$ and $m_D^0 \equiv e^{i\delta_2}|m_D^0|$. $\delta_{1,2}$ are functions of the phases of the complex VEV's.

The parameters $\lambda$'s and $L_0$'s depend entirely on the scalar sector and that is the reason why they appear in both Eq.(7) and Eq.(8). This will also be the reason why they also appear in the lepton mass matrices. There one just has to make the replacements: $h_{1,2} \rightarrow l_{1,2}$, $r \rightarrow r_l = l_2/l_1$ and $x \rightarrow x_l = |m_E^0|/|m_N^0|$ in Eqs. (7,8).

Notice that we seem to ignore elements such as $M_{42}$ and $M_{41}$ which can be computed at the one-loop level and other off-diagonal elements which arise at higher order. The reason is simply that they are numerically small compared with the above elements. Within the precision considered in this paper, they are found to be not so important. A more detailed and precise analysis to be carried out in a subsequent work will include these extra corrections. It is however important to state here the fact that these extra terms are perfectly calculable in terms of known parameters of the model.

Let us now examine Eqs. (7,8) in the case when the "isospin" breaking parameter $r = h_2/h_1$ becomes unity, i.e. no "isospin" breaking. A look at the diagonal masses obtained from Eqs. (1,2,3) reveals that when $r = 1$ or equivalently $h_2 = h_1$, one obtains $m_U^0 = m_D^0$, $m_t^0 = m_b^0$, $m_c^0 = m_s^0$, and $m_u^0 = m_d^0$. Now $m_U^0 = m_D^0$ means that the parameter $x = 1$ and that $\delta_1 = \delta_2$ giving $\Delta = 0$. Eqs. (8,9) then tell us that $M_{43}^u = M_{43}^d$, $M_{32}^u = M_{32}^d$, and $M_{21}^u = M_{21}^d$. (One can make similar statements for the other off-diagonal elements.) This means the the mass matrices for the up and down sectors are identical and so are the matrices $U_U$ and $U_D$ which diagonalize them. This now means that the generalized CKM matrix $V_{CKM} = U_U^{-1}U_D$ is equal to the unit matrix. All mixing angles vanish. This is precisely the point that we have mentioned above. This feature is true regardless of how much "isospin" breaking there is in the scalar sector, i.e. regardless of how much mass splitting there is between the charged and neutral scalars. In our model it turns out that the magnitude of the CKM elements is
a result of the interplay between the "isospin" breaking term of the Yukawa sector, \( r \), and that of the scalar sector as defined by the parameters \( \lambda^{(r,u)} \).

In order to carry out a numerical study of mass eigenvalues and CKM matrix elements, it is found to be more convenient to parametrize the off-diagonal elements of the mass matrices in a slightly different way although in principle one can just use directly Eqs. (7,8). One just has to vary the parameters described above in order to fit the generalized CKM matrix elements and the quark masses. This is a perfectly well-defined task albeit a very time-consuming one. To narrow down the range of values, we are guided by the hierarchical nature of the masses and the sizes of the CKM matrix elements. We shall parametrize the various M’s as: 

\[
M^U_{43} = c_U \lambda_U^2 e^{i\delta_U}, \quad M^U_{32} = r_t \lambda_U e^{i\delta_U}, \quad M^U_{21} = r_t \lambda_U^3 e^{i\delta_U}
\]

for the up sector, and 

\[
M^D_{43} = c_D \lambda_D^2 e^{i\delta_D}, \quad M^D_{32} = r_b \lambda_D e^{i\delta_D}, \quad M^D_{21} = r_b \lambda_D^3 e^{i\delta_D}
\]

for the down sector. The previous quantities are the elements of \( M_U \) and \( M_D \) where \( M_U = |M_U^0| e^{i\delta_1} M_U \) and \( M_D = |M_D^0| e^{i\delta_2} M_D \). The parameters \( r_t \) and \( r_b \) are defined as \( r_t = |m_U^0|/|m_U^1| \) and \( r_b = |m_D^0|/|m_D^1| \). The above phases are defined as \( tan\delta_U = \lambda x sin(\Delta/(1/r - \lambda x cos(\Delta))) \) and \( tan\delta_D = \lambda x sin(\Delta/(\lambda x cos(\Delta) - rx)) \) and similarly for the primed and double-primed quantities.

The phases \( \delta_{1,2} \) can be absorbed in a redefinition of the up and down quark fields. The parameters \( \lambda^{(r,u)}_{U,D} \) are defined in terms of those of the fundamental theory via Eqs. (7,8). In total the ten parameters \( \lambda_U, \lambda_D, c_U, c_D, \delta^{(r,u)}_{U,D} \) are computed interns of seven parameters \( r, \lambda^{(r,u)}_{U,D}, L_0/L_0, L''_0/L_0, \) and \( \Delta \). To complete the picture, one has to specify the diagonal elements coming from the other three generations for both up and down quarks. In general they have arbitrary phases and magnitudes. Although the phases can be similar to those of the fourth generation, we shall treat them as free parameters. It turns out that the attractive possibility of having all phases approximately equal to each other can actually be realized in our model.

After factoring out \( |M_U^0| e^{i\delta_1,2} \) in the up and down mass matrices, the first three diagonal elements can be written as \( r_u e^{i\alpha_1}, r_c e^{i\alpha_2}, r_t e^{i\alpha_3} \) for the up sector and \( r_d e^{i\beta_1}, r_s e^{i\beta_2}, r_b e^{i\beta_3} \) for the down sector, where \( \alpha_1 = \delta_u - \delta_1, \alpha_2 = \delta_c - \delta_1, \alpha_3 = \delta_t - \delta_1, \) and \( \beta_1 = \delta_d - \delta_2, \beta_2 = \delta_s - \delta_2, \beta_3 = \delta_b - \delta_2 \). Here \( r_i = |m^0_i|/|m^1_i| \). \( M_U \) and \( M_D \) which are symmetric matrices can be
made real by an appropriate redefinition of the quark phases provided $\alpha_1 = 2(\delta''_U + \delta''_D - \delta_U)$, $\alpha_2 = -2(\delta''_U - \delta_U)$, $\alpha_3 = 2\delta''_D$, $\beta_1 = 2(\delta''_D + \delta'_D - \delta_D)$, $\beta_2 = -2(\delta''_D - \delta_D)$, $\beta_3 = 2\delta''_D$. Assuming that $\delta_u, \delta_c, \text{etc...}$ satisfy the previous "reality" conditions, it will be seen that the only CP phases which enter the generalized CKM matrix are $\tilde{\Delta} = \delta_U - \delta_D$, $\tilde{\Delta}' = \delta_U - \delta'_D$, and $\tilde{\Delta}'' = \delta''_U - \delta''_D$. Phenomenologically, $\delta_U, \delta_D, \text{etc...}$ are found to be either close to (but not equal to) 0° or 180° so that $\delta_u, \delta_d, \text{etc...}$ are almost equal to $\delta_1$ and $\delta_2$. Such a possibility is rather attractive: all quark masses have roughly similar phases.

To bring $\mathcal{M}_U$ and $\mathcal{M}_D$ to a real form, we redefine the left-handed quark phases by a diagonal matrix of the form $Q_U = \text{diag}(1, e^{-i\phi'_U}, e^{-i\phi''_U})$ for the up sector and $Q_D = \text{diag}(1, e^{-i\phi'_D}, e^{-i\phi''_D})$ for the down sector, where $\phi_1 = 2\delta'' + \delta' - 2\delta$, $\phi_2 = \delta' - \delta$, $\phi_3 = \delta'' + \delta' - \delta$ with the appropriate subscripts for up and down. (There is also a redefinition of the right-handed quark phases but it is irrelevant for $V_{CKM}$ and we shall not discuss it here.) The eigenvalues of the real, symmetric 4x4 matrices denoted by $\mathcal{M}^R_U$ and $\mathcal{M}^R_D$ can be computed numerically. These eigenvalues are in turn used to construct the eigenvectors which are then used to obtain the matrices $R_U$ and $R_D$ which diagonalize $\mathcal{M}^R_U$ and $\mathcal{M}^R_D$ respectively. The generalized CKM matrix is then defined as $V_{CKM} = Q_U^{-1}R_U^{-1}R_DQ_D$. A full account of our analysis will be given in a separate publication. We shall illustrate our model with a few examples in this letter. A few typical elements are $V_{us} = \{\lambda^3_D/|\tilde{r}_s| - (\tilde{r}_u/\lambda^3_D) + i(2\tilde{\Delta}'' + \tilde{\Delta}' - 2\tilde{\Delta}) + O(\lambda^3)\}/N_{us}$, $V_{ub} = (\tilde{r}_u/\lambda^2_D)\{(K_u^{-1}e^{i(\tilde{\Delta}' - \tilde{\Delta})} + (K_b/\lambda_U\lambda_D)e^{i(2\tilde{\Delta}'' + \tilde{\Delta}' - 2\tilde{\Delta})} + O(\lambda^2)\}/N_{ub}$, $V_{cb} = \{-(K_b/\lambda_D)e^{i(2\tilde{\Delta}'' + \tilde{\Delta}' - 2\tilde{\Delta})} - (\lambda_U/\lambda_D)e^{i(\tilde{\Delta}' - \tilde{\Delta})} + O(\lambda^5)\}/N_{cb}$. All other elements of the generalized CKM matrix are easily computable and they will be given in an extensive version of this manuscript. The various $N$’s are normalization factors. The $\tilde{r}$’s are related to the ratios of the eigenvalues and $r_t$ or $r_b$. The quantities $K_i$’s are $K_i = 1 - \tilde{r}_i - (c_{i,D}\lambda^2_{U,D})^2/(r_{i,b}(1 - r_i))$ with $i = u, c, b$. One can immediately notice that the mixing of the "light" generations with the fourth generation creeps into $V_{ub}$ and $V_{cb}$ via various $K_i$’s which contain $c_U$ or $c_D$ which enter $\mathcal{M}_{43}$. This is because of the way the fourth generation mixes with the third one in our mass matrices. The CKM elements which do not involve the third generation—we are mainly concerned with the 3x3 sub-matrix here—are relatively
insensitive to the presence of the fourth generation. In this sense, the physics of the third generation (in particular B-physics) indirectly probes the existence or non-existence of a fourth generation. Recall that the fourth generation indirectly manifests itself through the parameter $x$ which enters in all elements of the mass matrices. It is however the resulting (dominant) mass mixing with the third generation that manifests itself most visibly in the CKM elements involving the third generation.

An extensive numerical analysis is underway. We shall give here some preliminary results. The inputs are given in the form of $M(r_1, r_2, r_3, 1)$ for the masses. We have $|M_U^0| = 1 TeV$, $x = |M_D^0|/|M_U^0| = 0.98, 1 TeV(-0.8 \times 10^{-5}, 0.0213, 0.54)$ for the up sector and $0.98 TeV(0.596 \times 10^{-5}, -0.952 \times 10^{-5}, 0.062)$ for the down sector, $\lambda_U = 0.064$, $\lambda_D = 0.15$, $c_U = 112.5$, $c_D = 0.325$, $r = 1.4637$. The previous values not only determine the mass eigenvalues but they also fix the phases in the following way. By equating the two ways of writing $M_{43}$, $M_{32}$, and $M_{21}$ for the up and down sectors and by taking the absolute value of these elements, we can fix the values of $\lambda'$ and $\lambda''$ once $\lambda$ and $\Delta$ are given. Let us recall that the phases $\delta_U$, $\delta_D$ and their primed and double-primed counterparts are given in terms of $x$, $r$ and $\lambda$, $\lambda'$, and $\lambda''$ respectively. Since the CKM elements depend on the phase differences $\Delta$, $\tilde{\Delta}$ and $\tilde{\Delta}''$ defined above, we have the following four sets of values: 1)$(1.47726, 1.262553, 1.4592)$ giving $(7.962^\circ, -181.124^\circ, -1.299^\circ)$, 2)$(1.477385, 1.26252, 1.459415)$ giving $(6.508^\circ, -180.92^\circ, -1.0631^\circ)$, 3)$(1.4775, 1.2625, 1.45957)$ giving $(5.0566^\circ, -180.715^\circ, -0.8269^\circ)$, 4)$(1.47753, 1.262495, 1.45965)$ giving $(4.334^\circ, -180.613^\circ, -0.7088^\circ)$, where in (1) to (4) the first and second sets correspond to $(\lambda, \lambda', \lambda'')$ and $(\Delta, \tilde{\Delta}, \tilde{\Delta}'')$ respectively. The results are: 1) $|V_{us}| = 0.22, |V_{cb}| = 0.0476, |V_{ub}/V_{cb}| = 0.0834, |V_{cs}| = 0.9739, |V_{cd}| = 0.222, |V_{ud}| = 0.9745, \hat{B}_K = 0.555$, 2) $|V_{us}| = 0.221, |V_{cb}| = 0.0459, |V_{ub}/V_{cb}| = 0.0827, |V_{cs}| = 0.9737, |V_{cd}| = 0.2228, |V_{ud}| = 0.9743, \hat{B}_K = 0.6385$, 3) $|V_{us}| = 0.222, |V_{cb}| = 0.0444, |V_{ub}/V_{cb}| = 0.082, |V_{cs}| = 0.9736, |V_{cd}| = 0.2237, |V_{ud}| = 0.9741, \hat{B}_K = 0.794$, 4) $|V_{us}| = 0.2225, |V_{cb}| = 0.04378, |V_{ub}/V_{cb}| = 0.0816, |V_{cs}| = 0.9736, |V_{cd}| = 0.224, |V_{ud}| = 0.974, \hat{B}_K = 0.913$, for the four cases. In all four cases, the mass eigenvalues are (at 1GeV) $m_U = 1.28 TeV$, $m_t = 258 GeV$,
\( m_c = 1.5 \text{GeV}, \ m_u = 5.16 \text{MeV}, \ m_D = 980 \text{GeV}, \ m_b = 6.15 \text{GeV}, \ m_s = 147 \text{MeV}, \ m_d = 8.6 \text{MeV}. \) The values of \( \hat{B}_K \) were obtained using the experimental value of \( \epsilon \) namely 0.00227. In this preliminary analysis the physical top quark mass is \( m_t^0 \approx 153 \text{GeV} \). A better way to present the above results would be to use a plot of the CKM elements versus \( \hat{B}_K \) but the trend can already be seen from the above numbers. The latest lattice computation of the Isgur-Wise function [4] give
\[ |V_{cb}| \sqrt{\tau_B/1.48 \text{ps}} = 0.038^{+0.008}_{-0.007} \] [5],
\[ |V_{cb}| \sqrt{\tau_B/1.53 \text{ps}} = 0.044 \pm 0.005 \pm 0.007 \] [5], and \( \hat{B}_K = 0.825 \pm 0.027 \pm 0.023 \) using staggered fermions [7]. Taken at face value, \( \hat{B}_K \) tends to favor
\[ |V_{us}| \approx 0.222, \ |V_{cb}| \approx 0.044, \ |V_{ub}/V_{cb}| \approx 0.082, \ |V_{ud}| \approx 0.974, \ |V_{cs}| \approx 0.9736, \ |V_{cd}| \approx 0.224. \] Further experimental and theoretical efforts are clearly needed to pinpoint these values.

One might also ask about the sensitivity of the top quark mass prediction to the value of the fourth-generation mass. Our preliminary results indicate that, for the fourth-generation quark mass not too far from 1 TeV, the result is not very sensitive to the precise value of that mass since having fixed \( \lambda_U \) and \( \lambda_D \), one has \( m_t \approx m_c/\lambda_U^2 \), and therefore \( m_t \) is more sensitive to \( \lambda_U \) and \( m_c \) than \( m_U \). A more detailed investigation of the dependence on \( m_U \) for a larger range of values will be carried out separately. The results presented here are for \( m_U \) around 1 TeV.

The lepton sector is identical in form to the quark sector and the results can be obtained with the replacement \( h_{1,2} \rightarrow l_{1,2} \). A rough estimate gives
\[ \lambda_E \approx 1.4 \lambda_D (m_b^0/m_t^0)(m_N^0/m_U^0)^3 \]
and \( \lambda_N \approx 1.4 \lambda_E (\lambda_U/\lambda_D) \). Here \( m_N^0 \) is the Dirac bare mass of the fourth-generation neutrino. Since \( |V_{us}| \approx \lambda_U + \lambda_D \), the leptonic version is \( |V_{\nu_{e}\mu}| \approx \lambda_N + \lambda_E \). A back of the envelope estimate gives \( BR(K_L \rightarrow \mu e) \approx (\lambda_N + \lambda_E)^2 BR(K_L \rightarrow \mu \mu) \). With \( BR(K_L \rightarrow \mu \mu) = 7.3 \times 10^{-8} \) and \( BR(K_L \rightarrow \mu e) < 9 \times 10^{-11} \), we find \( m_N^0 < 451 \text{ GeV} \). For the sake of estimate, the mass eigenvalue is assumed to be similar to that obtained for \( m_U \), namely a factor of 1.2 of its input, giving \( m_{N\text{phys}} < 541 \text{ GeV} \). Assuming a see-saw mechanism of the form \( m_{\nu_4} = m_{N\text{phys}}^2/\mathcal{M} \) and requiring that \( m_{\nu_4} > 46 \text{ GeV} \), one obtains \( \mathcal{M} < 6.4 \text{ TeV} \). A more detailed calculation would presumably gives a bound on the Majorana scale (at least for the fourth generation) not too far from the previous value. Also it is not hard to
arrange the masses of the other three neutrinos to be much lighter than the fourth one. A host of interesting phenomena might be studied such as an intriguing possibility of direct CP-violation in $\tau$-decay, presumably in some kind of $\tau$ factory.

This work was supported in part by the U. S. Department of Energy under Grant No. DE-A505-89ER40518.
REFERENCES

[1] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[2] P. Q. Hung, ”On The Origin of Family Replica: Unification of Horizontal and Vertical Symmetries”, Preprint INPP-UVA-7-92 (unpublished).

[3] For a discussion of fourth-generation quarks in that mass range, see P. Q. Hung, Phys. Rev. Lett. 69, 3143 (1992).

[4] N. Isgur and M. Wise, Phys. Lett. B232, 113 (1989); Phys. Lett. B237, 527 (1990).

[5] UKQCD Collaboration, S. P. Booth et al., preprint EDINBURGH-93-525, hep-lat/9308019.

[6] C. W. Bernard, Y. Shen and A. Soni, preprint BUHEP-93-15, hep-lat/9307005, to be published in Phys. Lett. B.

[7] S. Sharpe, preprint UW/PT-93-25 and references therein.