Stress-Dilatancy of Cambria Sand for Triaxial Tests at High Pressures

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Abstract. In this paper, the stress-dilatancy relationship of Cambria sand for drained triaxial compression and extension tests at high stress level is investigated. The stress dilatancy relationship is obtained by use of frictional state theory and experimental tests data published in literature. It is shown that stress-dilatancy relationship is bilinear, described by three parameters of frictional state theory: critical frictional angle and two other parameters. It is accepted that critical friction angle is independent of confining pressure. The two additional parameters are strongly dependent on confining pressure and different for initial and advanced stages. The point at which the values of these parameters change is termed as Transformation Shear Point. This point is not simply visible either in stress ratio-strain or the volume strain-shear strain relationship which are traditionally shown in soil mechanics papers. Transformation Shear Point is very characteristic in stress ratio-plastic dilatancy plane. Thus, stress ratio- plastic dilatancy is very important for describing stress-strain behaviour of soils. The relationship shown in the paper can be used in soil modelling in the future.

1. Introduction

Stress-dilatancy relations play a crucial role in understanding of the mechanical behaviour of soils and in development of realistic constitutive models for their response [1].

Particle crushing dominates the deformation behaviour of granular materials under significantly high compressive and shear stress [2]. Particle breakage depends on particle size distribution, particle shape, state of effective stress, stress path, void ratio, particle hardness and saturation of soil [3].

Coarse soils with angular particles show more particle crushing than soil with rounded fine particles [4]. Particle breakage can change the frictional properties of the material and modify the critical states as a consequence of the changes of grain size distribution [5-8].

The influence of particle breakage on dilatancy, shear strength and stress-dilatancy relationship were extensively investigated experimentally and theoretically in the past [e.g. 9-14]. It is noted that noticeable influence of particle crushing on stress -strain relationship for triaxial tests occurs once the confining pressure exceeds a certain stress level defined as reference crushing stress. For Toyoura sand the reference crushing stress equals 5.85MPa [2].

In this paper, basing on some experiments conducted by Yamamuro and Lade [15] the stress ratio-dilatancy relationships are shown for drained triaxial compression and extension tests at high stress level.
The frictional state theory developed by Szypcio [16] gives linear relationship between stress ratio ($\eta$) and plastic dilatancy ($D^p$). Straight line in $\eta$-$D^p$ plane is designed by critical state friction angle ($\phi^o$) and two parameters ($\alpha$) and ($\beta$).

It is shown that for analysed in the paper experimental data the stress ratio-dilatancy relationship is bilinear. In the initial and advanced stage values of parameters ($\alpha$) and ($\beta$) are significantly different. The point for which the parameters are changed is very characteristic for all tests. The position of this point in $\eta$-$D^p$ plane is a function of confining stress. This point is named Transformation Shear Point. It is not a visible characteristic point in ($\sigma_1$/$\sigma_3$ - $\varepsilon_1$) and ($\varepsilon_v$-$\varepsilon_1$) planes as traditionally used in soil mechanics papers. The stress-dilatancy relationship in $\eta$-$D^p$ plane is very important to fully describe soil behaviour and soil modelling.

2. Experimental data
Cambria sand consists of sub-rounded to well-rounded grains with diameter between 0.83 and 2mm. The maximum void ratio is 0.792, the minimum void ratio is 0.503, $D_{avg}$=1.66mm and $C_u$=1.30 [16].

![Figure 1](image.png)

**Figure 1.** Triaxial compression of Cambria sand: (a) relationships between major stress ratio and major principal strain, (b) relationship between volumetric strain and major principal strain
The series of drained triaxial tests were performed by Yamamuro and Lade [15] at relative density of 89.5%. The compression tests (TXC) were performed at various confining compression $\sigma_c=2.1, 4.0, 5.8, 8.0, 11.5, 15.0, 17.2, 26.0, 40.0$ and 52.0MPa. The extension tests (TXE) were performed at $\sigma_c=0.25, 1.0, 2.2, 4.0, 6.0, 8.0, 10.0, 12.0, 14.5, 17.5, 22.0, 24.0, 26.0, 35.0, 42.0$ and 52.0MPa. For very high confining pressure $\sigma_c>14.5$MPa the stress-strain behaviour of sand is very similar [15].

For clarity of consideration some of these tests were selected for later analysis. The dependence of effective principal stress ratio on major principal strain is shown in Figures 1a and 2a for TXC and TXE tests respectively. The dependence of volumetric strain on major principal strain for TXC and TXE is shown in Figures 1b and 2b respectively.

The physical meaning of marked characteristic points in Figure 1 and Figure 2 will be explained later.

**Figure 2.** Triaxial extension of Cambria sand: (a) relationship between major stress ratio and major principal strain, (b) relationship between volumetric strain and major principal strain
3. Stress-dilatancy for triaxial conditions

General stress-dilatancy relationship has the form [16]

\[ \eta = Q - A \cdot D^p \]  

where

\[ \eta = q/p' \]  

\[ Q = M^o - \alpha \cdot A^o \]  

\[ A = \beta \cdot A^o \]  

\[ D^p = \delta e_v^p / \delta e_q^p \]  

and

\[ M^o = M_e^o = 6 \sin \phi^o / \left(3 - \sin \phi^o\right) \] for triaxial compression

\[ M^o = M_e^o = 6 \sin \phi^o / \left(3 + \sin \phi^o\right) \] for triaxial extension

\[ A^o = A_v^o = 1 - \frac{1}{3} M_e^o \] for drained triaxial compression

\[ A^o = A_v^o = 1 - \frac{2}{3} M_e^o \] for drained triaxial extension

where \( \phi^o \) is the critical frictional state angle, \( \alpha \) and \( \beta \) are experimental parameters [18].

The frictional state is the state for which \( \alpha = 0.0 \) and \( \beta = 1.0 \) [17]. For many sands \( \phi^o = \phi_{cv} \) [18].

For triaxial conditions

\[ q = \sigma_1' - \sigma_3' \]  

\[ \delta e_q = \delta e_1 - \frac{1}{3} \delta e_v \]  

and for triaxial compression

\[ p' = \frac{1}{3} (\sigma_1' + 2 \sigma_3') \]  

\[ \delta e_v = \delta e_1 + 2 \delta e_3 \]  

\[ \frac{\sigma_1'}{\sigma_3'} = \frac{3 + 2 \eta}{3 - \eta} \]  

and for triaxial extension

\[ p' = \frac{1}{3} (2 \sigma_1' + \sigma_3') \]  

\[ \delta e_v = 2 \delta e_1 + \delta e_3 \]  

\[ \frac{\sigma_1'}{\sigma_3'} = \frac{3 + \eta}{3 - 2 \eta} \]
It is accepted that

\[ \delta e_r' = \delta e_r - \delta e_c' \]  \hspace{1cm} (14)

\[ \delta e_q' = \delta e_q - \delta e_c' \]  \hspace{1cm} (15)

where

\[ \delta e_c' = \frac{\kappa}{\nu} \frac{\delta p'}{p'} \]  \hspace{1cm} (16)

\[ \delta e_c = \frac{2}{9} \frac{1 + \nu}{1 - 2\nu} \frac{\kappa}{\nu} \frac{\delta q}{p'} \]  \hspace{1cm} (17)

\( \kappa \) - unload-reload slope of line in \( e \)-ln\( p' \) plane

\( \nu \) - Poisson’s ratio

\( \nu = 1 - e \) - specific volume.

In this paper it is accepted that for Cambria sand \( \phi^o=33.5^\circ \), \( \nu=0.25 \) and \( \kappa=0.0025 \). The parameters \( \alpha \) and \( \beta \) will be calculated from analysis of experimental data.

4. Calculation procedure

The selected tests graph of dependence between principal stress ratio \( (\sigma_1'/\sigma_3') \) and volumetric strain \( (\varepsilon_v) \) on major principal strain \( (\varepsilon_1) \) are shown in Figures 1 and 2 were approximated by a high degree of polynomial and theoretical dependence of stress ratio \( (\eta) \) on plastic dilatancy \( (D_p) \) was found using the frictional state theory. The appropriate values of parameters \( \alpha \) and \( \beta \) were found by use of the trial and error method. The fixed values of parameters give best conformity of calculated and experimental dependencies of \( \eta-D_p \) and \( \varepsilon_v-D_p \) for the analysed test data.

5. Stress ratio-dilatancy for Cambria sand

At the initial and advanced phase of shearing the fixed values of parameters \( \alpha \) and \( \beta \) termed as \( \alpha_o, \beta_o \) and \( \alpha_1, \beta_1 \) respectively are significantly different. The calculated and experimental dependences of \( \eta \) on \( D_p \) were shown in Figure 3 for TXC and Figure 4 for TXE tests respectively.

![Figure 3. Relationships between stress ratio and plastic dilatancy](image-url)
The characteristic points of shear deformations are shown in Figures 1-4. The point for which values of parameters $\alpha$ and $\beta$ were changed is named Transformation Shear Point (TSP). In reality, it is not a point but a state shown in figures not a TSP but TSP* is shown. The TSP* points are where the theoretical lines intersect. The Peak Point (PP) where stress ratio values are maximum and Ultimate Shear Points (USP) where tests were terminated are also shown in all figures.

![Figure 4. Relationships between stress ratio and plastic dilatancy: (a) confining pressure 0.25-6.0MPa, (b) confining pressure 12.0 and 42.0MPa](image)

The significant plastic part of global strain achieved at a very small value of major strain (0.1%) [19]. This point represents onset of yield [19] and is not exactly found by the calculation method adopted in the paper. It is unexpected that the line representing $\eta-\Delta p$ relationship at initial phase of shearing intersects horizontal axes ($\Delta p$) at values between 0.6 and 0.8 for almost all the analysed tests. It is consistent with experimental finding for other soils at smaller stress level [20]. Then it proves...
correctness of the frictional state theory and correctness of accepted in the paper parameters of Cambria sand.

Table 1. Characteristic values of tests and \( \eta - D^p \) relationship.

| Test | \( \sigma_c \) MPa | \( \phi^o \) Deg. | \( M^o \) | \( A^o \) | \( \alpha_o \) | \( \beta_o \) | \( \alpha_1 \) | \( \beta_1 \) |
|------|-----------------|-----------------|--------|--------|-------------|--------|--------|--------|
| TXC  |                 |                 |        |        |             |        |        |        |
| 5.8  | 33.5            | 1.353           | 0.549  | -60.0  | 100.0       | -0.60  | 2.50   |        |
| 8.0  | 33.5            | 1.353           | 0.549  | -13.3  | 18.0        | -0.45  | 1.90   |        |
| 11.5 | 33.5            | 1.353           | 0.549  | -8.0   | 9.0         | -0.25  | 1.50   |        |
| 40.0 | 33.5            | 1.353           | 0.549  | -3.4   | 4.25        | -0.08  | 1.35   |        |
|      | 0.25            | 33.5            | 0.932  | 0.378  | 1.0         | 2.6    | 0.30   | 1.0    |
|      | 2.2             | 33.5            | 0.932  | 0.378  | 0.07        | 3.0    | 0.30   | 0.95   |
|      | 4.0             | 33.5            | 0.932  | 0.378  | -0.90       | 3.6    | 0.10   | 0.90   |
|      | 6.0             | 33.5            | 0.932  | 0.378  | -8.0        | 15.0   | 0.0    | 0.75   |
|      | 12.0            | 33.5            | 0.932  | 0.378  | 4.05        | -2.7   | -0.10  | 0.65   |
|      | 42.0            | 33.5            | 0.932  | 0.378  | 3.2         | -1.2   | -0.08  | 0.45   |

Table 2. Characteristic points of \( \eta - D^p \) relationship.

| Test | \( \sigma_c \) MPa | \( \epsilon_1 \) \( \% \) | \( \sigma_1' / \sigma_3' \) | \( D^p \) | \( \epsilon_1 \) \( \% \) | \( \sigma_1' / \sigma_3' \) | \( D^p \) | \( \epsilon_1 \) \( \% \) | \( \sigma_1' / \sigma_3' \) | \( D^p \) |
|------|-----------------|----------------|----------------|--------|----------------|----------------|--------|----------------|----------------|--------|
| TXC  |                 |                 |                 |        |                 |                 |        |                 |                 |        |
| 5.8  | 2.25            | 2.19            | 0.61            | 35.0   | 3.44            | 0.192          | 40     | 3.40            | 0.167          |        |
| 8.0  | 3.25            | 2.04            | 0.80            | 35.5   | 3.39            | 0.169          | 40     | 3.35            | 0.120          |        |
| 11.5 | 4.50            | 1.81            | 1.03            | 38.0   | 3.46            | 0.108          | 40     | 3.44            | 0.076          |        |
| 40.0 | 4.30            | 1.68            | 1.15            | 30.0   | 3.39            | 0.016          | 40     | 3.06            | -0.072         |        |
|      | 0.25            | 3.60            | -0.40           | 2.20   | 5.48            | -0.83          | 2.20   | 5.48            | -0.830         |        |
|      | 2.2             | 2.70            | 0.10            | 2.85   | 3.90            | -0.515         | 2.85   | 3.90            | -0.515         |        |
|      | 4.0             | 2.62            | 0.36            | 3.75   | 3.90            | -0.305         | 4.00   | 3.80            | -0.327         |        |
|      | 6.0             | 2.58            | 0.56            | 4.00   | 3.69            | -0.109         | 4.25   | 3.66            | -0.170         |        |
|      | 12.0            | 2.24            | 1.23            | 5.40   | 3.27            | 0.230          | 6.00   | 3.22            | 0.373          |        |
|      | 42.0            | 2.05            | 1.96            | 7.00   | 3.08            | 0.710          | 8.50   | 2.48            | 0.412          |        |

6. Conclusions
For all the analysed tests \( \alpha \neq 0 \) and \( \beta \neq 1 \) so at high stress level Cambria sand does not exhibit frictional state. It is principally the effect of particle crushing.

On the basis of the test data analysis presented in this paper it is not possible to separate the influence of changes in volume from the influence of the energy consumed on particle breakage on stress-dilatancy relationship.

The Transformation Shear Point is not simply visible in major stress ratio and volumetric strain dependence on strain traditionally shown in soil mechanics.

The significant values of plastic dilatancy (\( D^p \approx 0.6-0.8 \)) (at onset of yielding) are exhibited at small major strain (\( \epsilon_1 < 0.1\% \)). The frictional state theory may be used to describe stress-dilatancy relationship for Cambria sand at high stress level.

More investigation is needed to prove correctness of the frictional state theory for different soils at different stress levels and paths in the future.

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