Analytic Relations between the Observed Gravitational Microlensing Parameters With and Without the Effect of Blending

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ABSTRACT
When a microlensing light curve is contaminated by blended light from unresolved stars near the line of sight to the lensed star, the light curve shape and corresponding parameterization for the event will differ from the values expected when the event is not affected by blending. As a result, blending makes it difficult to identify the major lens population and to estimate the amount of lensing matter. In order to estimate the effect of blending on the result of lensing experiments, it is, therefore, essential to know how the observed lensing parameters change depending on the fraction of blended light. Previously, the changed lensing parameters were obtained with a statistical method that not only required a large amount of computation time but also was prone to uncertainty. In this paper, we derive analytic relations between the lensing parameters with and without the effect of blending. By using these relations, we investigate the dependence of the observed lensing parameters on the amount of blended light, the impact parameter, and the threshold amplification for event detection.

Key words: gravitational lensing – dark matter – photometry

1 INTRODUCTION

Following the proposal of Paczyński (1986), searches for Massive Astronomical Compact Objects (MACHOs) by detecting light variations of stars caused by gravitational microlensing are being carried out by several groups (MACHO, Alcock et al. 1997; EROS, Ansari et al. 1996; OGLE, Udalski et al. 1997).

When an isolated source star is gravitationally microlensed, its flux is amplified by an amount

\[ A_0(u) = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}; \quad u = \left[ \beta^2 + \left( \frac{t - t_0}{t_E} \right)^2 \right]^{1/2}, \]  

(1.1)

where \( u \) is the lens-source separation in units of the angular Einstein ring radius, \( \theta_E \). The lensing parameters \( t_0, \beta, \) \( \) and \( t_E \) represent the time of maximum amplification, the lens-source impact parameter in units of \( \theta_E \), and the Einstein ring radius crossing time (Einstein time scale), respectively. These parameters are obtained by fitting model light curves in equation (1.1) to the observed one. Among these parameters, the Einstein time scale is related to the physical parameters of the lens by

\[ t_E = \frac{r_E}{v}; \quad r_E = \left( \frac{4GM}{c^2D_{\text{ol}}D_{\text{ls}}D_{\text{os}}} \right)^{1/2}, \]  

(1.2)

where \( M \) is the mass of the lens, \( v \) is the lens-source transverse velocity, \( D_{\text{ol}}, D_{\text{ls}}, \) \( D_{\text{os}} \) are the separations between the observer, lens, and source star, and \( r_E = D_{\text{ol}}\theta_E \) is the physical size of the Einstein ring radius. Since the Einstein time scale results from the combination of the physical parameters of the lens, it is difficult to obtain information about these parameters of individual lenses. However, one can still obtain information about the major population of lenses from the distribution of Einstein time scales by modeling the distribution and motion of lens matter constrained from other types of observation (Han & Gould 1996; Gould 1996; Zhao, Rich, & Spergel 1996; Han & Chang 1998). In addition, from the comparison of the experimentally determined optical depth, which is proportional to the summation of the Einstein time scales of individual events, i.e. \( \tau \propto \sum t_{E,i} \), to the theoretically expected value, experiments toward Magellanic Clouds allow one to constrain the MACHO dark matter fraction in the Galactic halo.

However, the observed light curves of a significant fraction of lensing events will differ from the light curve in equation (1.1). This is because current lensing experiments are being conducted toward very dense star fields such as the Magellanic Clouds and the Galactic bulge. While searches towards these crowded fields result in an increased event...
rate, it also implies that many of the observed light curves are blended by the light from unresolved stars that are not being lensed. When an event is affected by blending, the observed light curve becomes

\[ A_{\text{obs}}(u, B/F_0) = \frac{A_0(u)F_0 + B}{F_0 + B} = \frac{A_0(u) + B/F_0}{1 + B/F_0} \tag{1.3} \]

where \( F_0 \) is the unblended and unamplified flux of the lensed star and \( B \) is the amount of blended light. The problem of blending is that it is very difficult to detect the presence of a blend, and practically impossible to correct for the effect by purely photometrical means (Woźniak & Paczyński [1997], see also Figure 2 for example blended light curves). Since blending makes the observed parameters of an event differ from the values expected when the event is not affected by blending, blending leads to misidentification of the major lens population. In addition, the MACHO fraction determined from the lensing optical depth will be subject to great uncertainty. Throughout this paper, we consistently use term ‘observed lensing parameters’ to represent the lensing parameters determined from a blended light curve not knowing it is affected by blending.

Blending can be classified into several types depending on the origin of the blended light. The first type, regular blending, occurs when a bright source star registered on a template plate is lensed and its flux is affected by the light from faint unresolved stars below the detection limit imposed by crowding. Second, lens blending occurs when the origin of the blended light is the lens itself (Kamionkowski 1995; Buchalter & Kamionkowski 1997; Nemiroff 1997; Han 1999). Third, amplification-bias blending occurs when one of several unresolved faint stars in the seeing disk of a reference star is lensed, and the flux of the lensed star is associated with the flux from other stars in the integrated seeing disk (Nemiroff 1994; Han 1997a; Alard 1997). Finally, if a source is composed of binaries and one of the component is gravitationally amplified, the light from the other binary component simply contribute as blended light: binary-source blending (Griest & Hu 1992; Dominik 1998; Han & Jeong 1998).

In order to estimate the effects of the individual types of blending on the results of lensing experiments, one has to know how the observed lensing parameters change depending on the fractions of blended light. One of the methods to determine the changed lensing parameters is statistically fitting a series of unblended light curves to the blended light curve of interest (e.g., Woźniak & Paczyński 1997). In this case, determining the parameters requires a large amount of computation time for fitting process. Furthermore, for the full analysis of the blending effect, one should repeat the same procedure for events with all combinations of lensing parameters. In addition, there are inevitable statistical uncertainties that accompany the use of this method. Di Stefano & Esin (1995) attempted to develop analytic formalism for the treatment of blending. However, their so-called “blended Einstein ring radius” quantifies only how the maximum allowed impact parameter of lens-source encounter for event detection changes as a function of blended light fraction (see § 2). Since all information about lenses are obtained from lensing parameters, especially the Einstein time scale, simple analytic relations between the lensing parameters with and without blending will be very useful for the estimation of blending effect on the result of lensing experiments.

In this paper, we derive these relationships and use the relations to investigate the dependence of the observed lensing parameters on the amount of blended light, the impact parameter, and the threshold amplification for event detection.

## 2 CHANGES IN LENSING PARAMETERS DUE TO BLENDING

The shape of a microlensing event light curve is characterized by its height (peak amplification \( A_p \)) and width (event duration \( t_d \)), which are dependent on the lensing parameters: \( \beta \) for the height (see equation [2.2]) and \( t_b \) and \( \beta \) for the width (see equation [2.5]). Because of the change in the peak amplification due to blending, the observed impact parameter differs from the value determined from unblended light curve. Without blending, the impact parameter is related to the peak amplification of the unblended light curve by

\[ \beta = \left[ 2 \left( 1 - A_p^{-2} \right)^{-1/2} - 2 \right]^{1/2}. \tag{2.1} \]

When the event is affected by blending, on the other hand, the observed peak amplification decreases into

\[ A_{p,\text{obs}}(\beta, B/F_0) = \frac{A_p(\beta)B/F_0}{1 + B/F_0}, \quad A_p(\beta) = \frac{\beta^2 + 2}{\beta(\beta^2 + 4)^{1/2}} \tag{2.2} \]

As a result, the impact parameter obtained by ignoring blending effect becomes dependent on the amount of the blended light by

\[ \beta_{\text{obs}}(\beta, B/F_0) = \left[ 2 \left( 1 - A_{p,\text{obs}}^{-2} \right)^{-1/2} - 2 \right]^{1/2}. \tag{2.3} \]

Not only the peak amplification, but also the event duration is affected by blending. As an event is always amplified by more than 1 by definition, we define the event duration by half of the time a given event has an amplification equal to the threshold amplification for event detection. For a given threshold amplification for event detection, the maximum allowed impact parameter (threshold impact parameter) is determined by

\[ \beta_{th,0}(A_{th,0}) = \left[ 2 \left( 1 - A_{th,0}^{-2} \right)^{-1/2} - 2 \right]^{1/2}. \tag{2.4} \]

For example, if the adopted threshold amplification is \( A_{th,0} = 3/\sqrt{5} \), the threshold impact parameter becomes \( \beta_{th,0} = 1 \). According to this definition of a detectable microlensing event, the source star should enter the Einstein

* To avoid confusion, we note the difference between the event duration, \( t_d \), and the Einstein time scale, \( t_b \). For an unblended event detected with a threshold amplification of \( A_{th,0} = 3/\sqrt{5} \), \( t_d \) represent the half of time for the source to stay within the Einstein ring while \( t_b \) is the time scale for a source to cross the Einstein ring radius. The longest source star trajectory within the Einstein ring is the Einstein ring diameter. Therefore, the two time scales become equal only when the source crosses the center of the Einstein ring, but otherwise \( t_d \) is always less than \( t_b \).
Analytic Relations of Blending

The Einstein time scales ($t_E$) and the event duration ($t_d$) are marked by thick solid lines with arrows.

On the other hand, if the event is affected by blending, the same event must have a higher amplification because blending increases the threshold amplification to:

$$A_{th}(A_{th,0}, B/F_0) = A_{th,0}(1 + B/F_0) - B/F_0.$$  \hfill (2.6)

Blending also decreases the observed duration of the event to:

$$t_{d,obs}(A_{th,0}, t_E, \beta, B/F_0) = t_E \left(\beta_{th}^2 - \beta^2\right)^{1/2},$$  \hfill (2.7)

where

$$\beta_{th}(A_{th,0}, \beta, B/F_0) = \left[2 \left(1 - A_{th}^{-2}\right)^{-1/2} - 2\right]^{1/2}.$$  \hfill (2.8)

is the decreased threshold impact parameter caused by the increased threshold amplification $A_{th}$ in the presence of blending. We note that the formula of so-called “blended Einstein ring” of Di Stefano & Esin (1995) is identical to our threshold impact parameter in equation (2.8) for a special case for $A_{th,0} = 3/\sqrt{\pi}$.

As a result of the changes in the impact parameter and the duration of event, the Einstein time scale also changes. Not knowing an event is affected by blending, one will obtain the Einstein time scale from the relation in equation (2.5), but based on the observed impact parameter and the duration of event by

$$t_{E,obs} = \frac{t_{d,obs}}{(\beta_{th,0}^2 - \beta_{obs}^2)^{1/2}}.$$  \hfill (2.9)

In the equation, the threshold impact parameter $\beta_{th,0}$ is included instead of the decreased value of $\beta_{th}$ due to blending, because one still thinks he (or she) consistently applied the same threshold impact parameter of $\beta_{th,0}$. Since the observed duration of the event is related to the unblended lensing parameters by equation (2.7), one obtains the relation between the Einstein time scales with and without blending by

$$t_{E,obs} = t_{E} \left(\frac{\beta_{th}^2 - \beta^2}{\beta_{th,0}^2 - \beta_{obs}^2}\right)^{1/2},$$  \hfill (2.10)

where the definitions of the various types of impact parameter of $\beta$, $\beta_{obs}$, $\beta_{th,0}$, and $\beta_{th}$ are given in equations (2.1), (2.3), (2.4), and (2.8).

In Figure 2, we present several illustrative light curves with blending (solid curves) and the corresponding degenerate unblended light curves (dotted curves) whose lensing parameters (i.e., $\beta_{obs}$ and $t_{E,obs}$) are determined by our derived relations in equations (2.3) and (2.10). All of these events have the same unblended impact parameter $\beta = 0.2$ and the adopted threshold amplification is $A_{th,0} = 3/\sqrt{\pi}$ (represented by a solid line). The fraction of blended light and the fractional changes in the lensing parameters with and without blending are marked in each panel. As shown by Woźniak & Paczyński (1997), the light curves of blended events are very well fit by unblended light curves of different lensing parameters.

With the derived analytic relations, the effects of blending on the observed lensing parameters can be computed explicitly. We compute these changes in the lensing parameters due to blending with the equations derived above for the impact parameter (2.6) and for the Einstein time scale.
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Figure 2. Blended event light curves (solid curves) and the corresponding degenerate unblended light curves (dotted curves). The lensing parameters determined from the unblended fit to the blended light curves (i.e., $\beta_{\text{obs}}$ and $t_{E,\text{obs}}$) are obtained by using the derived equations (2.3) and (2.10). All of these events have the same unblended impact parameter ($\beta = 0.2$) and the Einstein time scale. The adopted threshold amplification is $A_{\text{th},0} = 3/\sqrt{5}$ (represented by a solid line). The fraction of blended light ($B/F_0$) and the fractional changes in the lensing parameters ($\beta_{\text{obs}}/\beta$ and $t_{E,\text{obs}}/t_E$ for the impact parameter and the Einstein time scale, respectively) with and without blending are marked in each panel. One finds that the blended light curves are very well fit by unblended light curves of different lensing parameters.

Figure 3. The fractional changes in the observed lensing parameters ($t_{E,\text{obs}}/t_E$ and $\beta_{\text{obs}}/\beta$ for the Einstein time scale and the impact parameter, respectively) as a function of the fraction of blended light ($B/F_0$) for events with different values of the impact parameter. We assume that the threshold amplification for event detection is $A_{\text{th},0} = 3/\sqrt{5}$. In the figure, the relation becomes discontinuous at a certain value of the blended light fraction $B/F_0$ because events more strongly affected by this amount of blended light cannot be amplified higher than $A_{\text{th},0}$ and would fall below the detection threshold.

3 CONCLUSION

We have derived analytic relations for the observed microlensing parameters with and without the consideration...
Figure 4. The dependence of the observed lensing parameters ($t_E_{obs}/t_E$ and $\beta_{obs}/\beta$ for the Einstein time scale and the impact parameter, respectively) on the fraction of blended light ($B/F_0$) for different values of the adopted threshold amplification $A_{th,0}$. This model event has an unblended impact parameter of $\beta = 0.2$. As the value of the threshold amplification increases, the decrease in the observed Einstein time scale becomes more important. Unlike the Einstein time scale, however, the observed impact parameter has the same dependence on $B/F_0$ regardless of the adopted values of $A_{th,0}$. Note that the four lines in the lower panel coincide.

of blending effect. We have used these relations to investigate the dependence of the observed lensing parameters on the fraction of blended light for events with various values of impact parameters under various selection criteria. The results of this investigation are as follows:

(i) The observed Einstein time scale decreases and the impact parameter increases with an increasing fraction of blended light. Although these trends in the observed lensing parameters are already known, this derivation is the first to analytically quantify this effect.

(ii) The observed lensing parameters depend in different ways on the fraction of blended light for events with different impact parameters. These changes in the observed lensing parameters are more important for events with smaller impact parameters, implying that the effects of blending becomes more important for events with higher amplifications.

(iii) The observed Einstein time scale depends on the threshold amplification for event detection. With increasing value of the adopted threshold amplification, the decrease in the observed Einstein time scale becomes more important. However, the observed impact parameter has the same dependence regardless of the adopted threshold amplification.

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