A NOVEL CAZAC Sequence Based Timing Synchronization Scheme for OFDM System

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Abstract—Several classical timing synchronization schemes have been proposed for the timing synchronization in OFDM systems based on the correlation between identical parts of OFDM symbol. These schemes show poor performance due to the presence of plateau and significant side lobe. In this paper we present a timing synchronization schemes with timing metric based on a Constant Amplitude Zero Auto Correlation (CAZAC) sequence. The performance of the proposed timing synchronization scheme is better than the classical techniques.

Index Terms—OFDM, CAZAC, Timing Synchronization, Differential normalization.

I. INTRODUCTION

In recent times there is an exponential rise in the demand of multimedia wireless services based on broadband standard. A common technology in most of the broadband standards designed to provide broadband services is Orthogonal Frequency Division Multiplexing [1,2]. OFDM is a multicarrier modulation technique, where high data rate serial bits are converted into low data rate parallel paths and the signals in each of the parallel paths modulate orthogonal sub-carriers. This process of transmission of OFDM signal converts frequency selective channel into frequency flat fading channel. In contrast to single carrier communication, OFDM combats the effects of frequency flat fading channel in frequency domain using bank of simple one tap equalizer. Due to the robustness of OFDM to multipath fading, it is adopted in WLAN, DVB-T, LTE-A, MB-OFDM UWB to provide wireless broadband services. However, OFDM is very sensitive of time and frequency synchronization error [2]. In an OFDM system, the timing and frequency of the received OFDM signal should be synchronized with the reference signal at the receiver. Frequency synchronization error in OFDM system results Inter Channel Interference (ICI). Timing synchronization error [2] in OFDM system results in Inter Symbol Interference (ISI), Inter Channel Interference (ICI) and change in the amplitude of the received signal.

Several timing synchronization schemes have been reported for OFDM systems which are mostly based on the auto-correlation between identical repeated parts of OFDM symbol. Schimdl [3] describes a training symbol having two identical sequences for time synchronization in OFDM system. However, the peak value of the timing metric which is used to determine the start of OFDM symbol has a plateau. Subsequently Minn [4] has proposed a training symbol having four repeated parts with 3rd and fourth part as negative of 1st and 2nd. The scheme due to Minn reduces the timing metric plateau. However, the mean square error of the estimator is large in ISI channel due to the timing metric values around the correct time to be almost same.

Park [5] has proposed training symbol having four repeated parts with first two parts symmetric and second two parts conjugate of first two parts. The first part of the OFDM symbol is obtained due to the DFT of the PN sequence. This said structure of the symbol results in the increase of difference between timing metric at correct time and other time. The timing metric is observed to be sharp, but there is a presence of side lobe with significant power. In order to improve the performance of timing synchronization, Ren [6] has proposed an OFDM symbol with two identical parts, where each part is generated from the DFT a CAZAC sequence and weighted by a PN sequence. It is observed the scheme due to Ren reduces the timing offset estimation error. However, the performance degrades significantly under multipath fading channel [7]. Fang [7] proposes an OFDM symbol with two equal parts, where second part is the DFT of a CAZAC sequence. The first part is equal to the second part sequence weighted by random exponential sequence. Scheme due to Fang observed to have timing metric with lower side lobe power and lower mean timing offset than the scheme due to Ren.

In this paper we propose a new OFDM symbol with repeated symmetric conjugate sequence based CAZAC sequence. We also propose a new timing metric for the timing synchronization utilizing the modified training symbol. We have determined timing metric, peak to side lobe ratio and mean of timing offset. The performance is observed to be better than the previous techniques.

The rest of the paper is organized as follows. Section II presents a brief description of OFDM system followed by proposed method in Section III. Performance of the proposed scheme is presented in Section IV and the paper is concluded in section V.
II. OFDM SYSTEM DESCRIPTION

An IFFT operation is carried out on a group of $N$ symbols to generate time domain OFDM symbol. The $n$th time-domain samples of OFDM signal transmitted through a fading channel is represented as

$$x[n] = \sum_{k=0}^{N-1} c_k e^{j(\frac{2\pi}{N})kn}$$  \hspace{1cm} (1)

where $N$ is the total number of orthogonal subcarriers, $c_k$’s are the $k$th complex information symbol which modulates $k$th subcarrier. The $n$th received sample from a multipath fading channel having channel impulse response $h(m)$ is given as

$$y[n] = \sum_{m=0}^{L-1} h[m] x[n-m], 0 < n < N$$  \hspace{1cm} (2)

where $L$ is the memory of the channel. In OFDM system, timing offset is considered as an unknown timing instant of received signal and frequency offset is considered as a phase rotation of the received data in the time domain. Considering these two uncertainties on received signal, the $n$th received signal sample in AWGN channel is given as

$$r[n] = y[n - n_c] e^{j\left(\frac{2\pi n_c}{N}\right)} + w[n]$$  \hspace{1cm} (3)

where, $n_c$ is the integer-valued unknown arrival time of a symbol $\theta_c$ is the frequency offset and $w(n)$ is the additive white Gaussian Noise (AWGN).

A. Timing Synchronization Schemes

Several classical timing synchronization schemes due to Schmidl, Minn, Park have been reported in literature. The scheme due to Schmidl employs two identical sequences, Minn and Park employ four identical sequences. Each of the sequences is obtained due to the DFT operation on the PN sequence. Timing synchronization and proposed timing metric for Schmidl schemes is based on the correlation between two identical parts of OFDM symbol normalized with the energy of the symbol. Timing metric described by the Eq. (4) is used to determine the start of the OFDM symbol. The sample for which the timing matrix is maximum is the start of the OFDM symbol. However, the timing metric observed to have a plateau with a duration related to cyclic prefix duration. This results in higher mean square error of the timing offset. Subsequently Minn has proposed a scheme consisting of several repeated parts in an OFDM symbol. The timing metric of this scheme indicate side lobes of higher magnitude and leads to poor timing synchronization performance.

Further enhancements in timing synchronization have been proposed by Park [5] and Fang [7] which are described below.

1) Park’s Scheme: To reduce the side lobe and increase the difference between the peak value of timing metric observed in the scheme due to Minn, Park have proposed a preamble consisting of conjugate and symmetric sequence in an OFDM symbol. The preamble design proposed by Park is given as where $C_{N/4}^*$ represents samples of length $N/4$ generated by IFFT of a PN sequence, and $C_{N/4}^*$ represents a conjugate of $C_{N/4}$ Symmetric $D_{N/4}$

The Timing Metric is given by:

$$M_{Park}(d) = \frac{|P_{Park}(d)|^2}{R_{Park}^2(d)}$$  \hspace{1cm} (4)

$$P_{Park}(d) = \sum_{k=0}^{N-1} r(d-k) \cdot r(d+k)$$  \hspace{1cm} (5)

$$R_{Park}(d) = \sum_{k=0}^{N-1} |r(d+k)|^2.$$  \hspace{1cm} (6)

In a multipath fading channel, its performances decrease due to the presence of side lobes. For better performance Fang proposed a method based on CAZAC sequence.

2) Fang Scheme: The correlation based synchronization method is based on auto-correlation property of PN (Pseudo-random Noise) -sequence. Compared to PN sequence, CAZAC sequence has better auto-correlation and cross-correlation property and hence, improves the timing synchronization performances. The scheme due to Fang assume sequence $s(k)$ as a CAZAC sequence with length of $N$ (even number). The Properties of CAZAC sequence are

$$s(k) = Costant, k = 0, 1, 2 \ldots N - 1$$  \hspace{1cm} (7)

$$\sum_{k=0}^{N-1} s(k)s^*(k+\tau) = \begin{cases} N, & \tau = 0 \\ 0, & \tau \neq 0. \end{cases}$$  \hspace{1cm} (8)

The CAZAC sequence $s(k)$ described in [8] is written as

$$s(k) = e^{j\left(\frac{2\pi k}{N}\right)}, k = 0, 1, 2 \ldots N - 1$$  \hspace{1cm} (9)

where, $\mu$ is a positive integer co-prime to $N$.

Synchrohronization Preamble Design: The property of CAZAC sequence doesn’t change after IFFT operation, so Fang proposed a preamble by repeating CAZAC sequence after IFFT which is shown by below:

$$TR_{Park} = C_{N/4} \quad D_{N/4} \quad C_{N/4} \quad D_{N/4}$$

Here $C_{N/2}$ is given by $C_{N/2} = \nu(i) \cdot D_{N/2}$, where $i = 0, 1, \ldots N/2 - 1, \nu(i) = exp(j\pi.\nu(i))$ is a random sequence and $\nu(n)$ is the uniformly distributed sequence ranging from $-0.2$ to $1$.

Timing Synchronization metric: According to Fang, the timing metric is expressed as

$$M_{Fang}(d) = \frac{|P_{Fang}(d)|^2}{R_{Fang}^2(d)}$$  \hspace{1cm} (10)
cyclic prefix), expressed as:

\[ f_{\text{Symmetry}} = \exp\left(\frac{j\pi m^2}{N}\right) \]

where \( N \) represents the length of CAZAC sequence, \( m \) is an integer between 0 and \( N/2 \).

The training sequence (excluding cyclic prefix), expressed as:

\[ f(n) = \begin{cases} \sum_{k=0}^{N/2-1} v_n^* (d + k) \cdot r(d + k) \cdot r^* (d + k + N/2) & \text{for } n \text{ odd} \\ 0 & \text{for } n \text{ even} \end{cases} \]

\( v_n \) is the normalized factor.

Timing synchronization based on differential absolute value as a normalized factor.

We also propose a new timing metric for mitigating side lobes by multiplication of weighting factor. However, the timing metric indicates the presence of small amount of side lobes. So, we propose a new timing synchronization scheme which reduces the side lobe to almost zero value and improves the probability of detection.

**III. PROPOSED METHOD**

In the proposed method weighted CAZAC sequence is utilized to generate an OFDM symbol with repeated conjugate symmetry sequence. We also propose a new timing metric for timing synchronization based on differential absolute value as a normalized factor.

**Synchronization Preamble Design:** Our method is the modified version of Park scheme. The training sequence (excluding cyclic prefix), expressed as:

\[
\begin{array}{c|c|c|c}
C_{N/4} & D_{N/4} & C_{N/4} & D_{N/4} \\
\hline
\end{array}
\]

where, \( C_{N/4} \) represents first quarter of CAZAC sequence of length \( N \), i.e., \( C_{N/4} = \exp\left(\frac{j\pi m^2}{N}\right) \), \( m = 0, 1, 2 \ldots \left(\frac{N}{4} - 1\right) \) and \( D_{N/4} \) is conjugate and symmetric to \( C_{N/4} \).

The \( n \)th received sample from a multipath fading channel having channel impulse response \( h(m) \) is given as

\[ r(n) = \sum_{m=0}^{L-1} h(m) x(n-m) + w(n), \]

\[ 0 < n < N. \] \hspace{1cm} (13)

**Timing Synchronization**: In conventional methods, a normalizing factor which is equal to the half energy of the window is used to determine the timing metric. It is observed that the timing metric has several side lobes with significant magnitude. In order to reduce magnitude of side lobes, we propose timing metric based on the difference of absolute value of samples as normalization factor presented in (6). The proposed normalization factor is expressed as

\[ R_{\text{proposed}}(d) = \sum_{k=0}^{N/2-1} \left(|r(d-k)| - |r(d+k)| \right)^2. \] \hspace{1cm} (14)

The timing metric based on the difference of magnitude as a normalized factor is given as

\[ M_{\text{proposed}}(d) = \frac{|P_{\text{proposed}}(d)|^2}{R_{\text{proposed}}(d)} \] \hspace{1cm} (15)

where,

\[ P_{\text{proposed}}(d) = \sum_{k=0}^{N/2-1} r(d-k) \cdot r(d+k). \] \hspace{1cm} (16)

The performance of the proposed technique is presented in next section.

**IV. PERFORMANCE EVALUATION**

It is essential to evaluate the performance of the proposed timing synchronization scheme for OFDM system and compare with the classical schemes. We considered an OFDM system with 1024 sub-carriers (\( N \)), cyclic prefix with \( N/8 \) samples, normalized carrier frequency offset = 0.1 to evaluate the performance of timing synchronization scheme in exponential decaying Rayleigh multipath fading channel. The performance metrics which are used to evaluate the performance are timing metric, Peak to side lobe ratio, MSE and Mean of the timing offset.

**Timing metric**: Fig. 1 shows the normalized timing metric for the schemes due to Park, Fang and the proposed scheme at 10 db SNR. It is observed from the timing metric for Park scheme that the side lobe power is of significant magnitude compared to the peak value. However, the timing metric for Fang scheme indicate side lobes with smaller magnitude compared to the peak value. The timing metric for the proposed scheme indicates side lobe magnitude is less than the scheme due to Fang and almost equal to zero. However the proposed scheme is more complex as compared to Park and Fang scheme as difference of absolute value is used as a normalized factor.

**Peak-to-Side Lobe Ratio vs. SNR**: Peak to Side lobe Ratio (PSR) vs. SNR in dB is presented in Fig. 2. It is observed that the PSR for Park, Fang and the proposed scheme increases with increase in SNR. However, the proposed scheme observed to have a higher PSR of 10 compared to PSR of 3 and 1.75 for Fang and park scheme respectively at 10 db SNR.
MSE of the Timing Offset: Fig. 3 shows the MSE of timing offset of different methods. The MSE performance of Park and Fang decreases with increase in SNR, but after 12 dB SNR, the MSE of proposed method becomes zeros. The very low value of MSE shows the superiority to other two methods.

Mean of the Timing Offset: Fig. 4 shows the mean of the timing offset. The mean value of Park and Fang decreases gradually towards zero at SNR of 16 dB and 11 dB respectively but in case of the proposed scheme it attains the mean value zero at SNR of 4 dB. This shows rapid acquisition of accurate estimation. This rapid acquisition of low mean value implies better performance of proposed scheme.

V. CONCLUSION

In this paper, we proposed a novel timing synchronization method for OFDM system based on CAZAC sequence and differential normalization method, which shows better performances than Park and Fang method. A good correlation property of CAZAC sequence, high impulse-shaped peak at the correcting timing and reduced side lobe makes the proposed method superior than others classical methods. The use of differential normalization allows the system to reduce side lobe and improves the performance.

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