Critical Higgs Mass and Temperature Dependence of Gauge Boson Masses in the SU(2) Gauge-Higgs Model

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We study the effective 3-D SU(2) Gauge-Higgs model at finite temperature for Higgs-masses in the range from 60 GeV up to 100 GeV. The first order electroweak phase transition weakens with increasing Higgs-mass and terminates at a critical end-point. For Higgs-mass values larger than about \( m_{H,c} = 75.4(6) \) GeV the thermodynamic signature of the transition is described by a crossover. Close to this Higgs-mass value we investigate the vector boson propagator in Landau gauge. The calculated W-boson screening masses are compared with predictions based on gap equations.

1. Introduction

The standard model of electroweak interactions predicts the existence of a phase transition between a low temperature symmetry broken and a high temperature symmetric phase \([1]\). Its thermodynamic properties lead to cosmological consequences. One might hope that the baryon asymmetry can be generated at the electroweak phase transition, if the transition is of strong first order. For values of the zero temperature Higgs-mass \( m_H \leq 70 \) GeV the phase transition is of first order \([2,3]\). As the Higgs-mass is increased further, the thermodynamic singularity at the electroweak phase transition weakens. It is even possible, that for large values of the Higgs-mass the phase transition loses its nonanalytical structure and is described by a crossover \([4]\). In the strict sense the electroweak phase transition would then cease to exist. The exact determination of the critical Higgs-mass value \( m_{H,c} \), at which the first order electroweak phase transition changes from first order to a crossover is important in view of its implications for the standard model.

Another aspect of the electroweak theory is the non-vanishing magnetic screening mass. It controls the infrared behavior of the theory at high temperatures and influences the nature of the phase transition itself.

2. The 3-D SU(2) Gauge-Higgs model on the lattice

This work is a continuation of earlier studies and for details on the formulation of the theory we refer to \([5]\). Here we recall the simulated 3 − \( D \) action functional:

\[
S_{lat}^{3D} = \sum_x \sum_y \sum_i \frac{1}{2} \text{Tr} U(x) U_{x+i} \Phi(x) \Phi(x+i) - \frac{1}{2 \kappa} \text{Tr} \Phi(x) \Phi(x) - \frac{1}{2 \lambda_3} \left( \frac{1}{2} \text{Tr} \Phi(x) \Phi(x) \right)^2. \tag{1}
\]

The relationship of the dimensionless lattice couplings \( \beta, \lambda_3, \kappa \) to the couplings of the \( T = 0 \) SU(2) Gauge-Higgs system can be found in \([5]\). Our simulation is performed at \( \beta = 9.0 \) and at 5 values \( \lambda_3 = 0.170190, 0.291275, 0.313860, 0.401087 \) and \( 0.498579 \). The hopping parameter is varied across the phase transition. These \( \lambda_3 \)-values correspond to Higgs-mass values of \( m_H = 59.2, 76.1, 78.9, 88.7 \) and 98.5 GeV, if the 1-loop parameter mapping of \([5]\) is used. For possible
correction terms inducing small corrections to the here cited Higgs-mass values we refer to a forthcoming publication.

3. Determination of the critical Higgs-mass

To illustrate the critical behavior as a function of $\lambda_3$, or the Higgs-mass, we display in figure 1 the maxima of the $\Phi^2$-susceptibility, which is given by

$$\chi_{\Phi^2} = V \left\langle (\Phi^\dagger \Phi - \langle \Phi^\dagger \Phi \rangle)^2 \right\rangle. \quad (2)$$

Figure 1. Maxima of the $\Phi^2$-susceptibility.

We investigate lattice sizes ranging from $L = 8$ up to $L = 48$. The labeling of the data in figure 1 corresponds to the one used in figure 2. The data at $\lambda_3 = 0.170190$ are consistent with a first order transition, the dotted line with slope 3 in the figure. At $\lambda_3 = 0.313860, 0.401087$ and $0.498579$ a crossover is observed, while at $\lambda_3 = 0.291275$ the behavior is almost critical, the solid scaling curve in figure 2.

The determination of the critical Higgs-mass value relies on the analysis of Fisher or Lee-Yang zeroes in the crossover-region of the theory. The partition function $Z$ is analytically continued into the complex plane as a function of the complex hopping parameter $\kappa$. Denoting with $z_0$ the lowest zero of $Z$, i.e. the zero in $\kappa$ with smallest length, we expect in the vicinity of the critical end-point the scaling law

$$\text{Im}(z_0) = CL^{-1/\nu} + R(\lambda_3). \quad (3)$$

Such a scaling behavior can also be observed for Lee-Yang zeroes in the high temperature phase of the Ising model. Our strategy to localize the end-point then is to determine the value of $\lambda_3$, at which the regular contribution $R$ to the scaling law vanishes: $R(\lambda_3, c) = 0$. In figure 2 we display $\ln\text{Im}(z_0)$ vs. $\ln L$. The solid curves in figure 2 correspond to fits with the scaling law.

4. Gauge Boson masses in Landau gauge

The W-boson mass is determined from the W-boson propagator in Landau-gauge, $|\partial^\mu A_\mu(x)|^2 = 0$. For a detailed discussion of the implementation of the Landau-gauge on a lattice see [5] and references therein. We investigate the W-boson propagator at $\lambda_3 = 0.291275$, which is very close to the critical Higgs-mass. Simulations have been performed...
Figure 3. The determination of $\lambda_{3,c}$. performed on a $16^2 \times 32$ lattice. The propagators have been analyzed in the same way as discussed in [5]. Our results are shown in figure 4.

Figure 4. W-boson screening masses, calculated on a $16^2 \times 32$ lattice at $\lambda_3 = 0.291275$. The full curve describes the fit to the data. The dashed curve represents the result obtained from gap equations.

In the high temperature phase the W-boson propagator stays constant. A fit to the data for $\kappa \leq \kappa_c$, the full triangles in the figure, yields $m_W(\kappa \leq \kappa_c) = 0.161(3)$. It is worthwhile to note, that in pure SU(2) gauge theory the magnetic screening mass has a value 0.165(12), the horizontal full and dotted lines in figure 4. In the symmetry broken phase the mass increases rapidly. In this region the data are very well described by the ansatz

$$m_W = 0.161 + a(\kappa - \kappa_c)^\beta \ \kappa \geq \kappa_c .$$

The fit to the full circles in the figure results into a value for the exponent $\beta$ of $\beta \approx 0.4$. We are now able to compare our results with predictions based on gap equations. Using similar parameters in the gap equations as in our study we observe a qualitative agreement, the dashed curve in the figure.

5. Summary

We have exploited the scaling behavior of partition function zeroes in the vicinity of the critical end-point in order to determine the critical Higgs-mass $m_{H,c}$. The electroweak phase transition looses its first order character at a Higgs-mass value of about $m_{H,c} = 75.4(6)$ GeV. Close to the critical Higgs-mass we have measured the W-boson propagator in Landau gauge. W-boson screening masses remain constant in the high temperature symmetric phase and increase in the low temperature Higgs phase. The agreement of these data with predictions based on gap equations is of qualitative nature.

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