Rule-based control of a semi-active suspension for minimal sprung mass acceleration: design and measurement

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ABSTRACT
In this paper, a rule-based controller is developed for the control of a semi-active suspension to achieve minimal vertical acceleration. The rules are derived from the results obtained with a model predictive controller. It is shown that a rule-based controller can be derived that mimics the results of the model predictive controller and minimises vertical acceleration. Besides this, measurements on a test vehicle show that the developed rule-based controller achieves a real-world reduction of the vertical acceleration, which is in agreement with the simulations.

1. Introduction
The design of a car suspension system is always a trade-off between comfort, dynamic vertical tire fore variations and available space. It is already shown, using a simple quarter-car model, that these properties are conflicting.[1,2] However, with active suspension components this trade-off can be influenced during driving. By controlling the (semi-) active suspension, the setting can be made depending on the driving condition. Various solutions have been developed over the years, such as adding an actuator to the suspension, thereby creating a fully active suspension.[3–5] With this type of active suspension, energy can be removed or added to the system, however, power consumption of such a system is often very large. A possible solution for this is a slow active suspension with continuously variable damping [6] which can still achieve a 43% reduction of RMS sprung mass acceleration compared to the selected passive suspension. Finally, semi-active suspension, which controls the damping coefficient of the system, requires the least power and still offers significant improvement over passive suspension systems.[7–9] In this paper, the latter system will be considered.

Numerous controllers for semi-active suspension systems have been developed over the years, with the most well-known strategy being sky-hook control.[8] This controller can be interpreted as a damper that connects the vehicle body to an inertial ground. In this way, the vertical acceleration at the sprung mass eigenfrequency is reduced. The sky-hook controller switches the damping coefficient based on the product of vertical sprung mass velocity and suspension velocity. An alternative is the acceleration-driven damper...
(ADD) [10] which switches based on the product of sprung mass vertical acceleration and suspension velocity. The ADD controller achieves a reduction of the sprung mass acceleration at higher frequencies than the sky-hook controller, it typically performs better than sky-hook beyond 2 Hz.

Since sky-hook and ADD achieve a reduction of the sprung mass acceleration in different frequency ranges, a mixed sky-hook ADD controller has been developed in [11]. Savaresi uses a frequency-dependent switch to decide which of the two control topologies (sky-hook or ADD) should be used. The performance of this controller is compared with the solution determined through minimisation of the sprung mass acceleration, which shows that it has better performance than either sky-hook or ADD, but some small performance gains are still possible. A second approach to merge sky-hook with ADD is performed in [12]. The authors present an energy based analysis which provides a switching rule that indicates when sky-hook damping should be applied and when ADD should be used. The problem with this approach, however, is that it requires the measurement of the absolute velocity of the sprung and unsprung mass, which are generally not available in practice.

A different approach that can be found in the literature is that of a clipped LQ controller. [13,14] With this control topology state feedback gains are calculated by solving a Riccati equation. Forces calculated by the controller are then saturated by the actuator limits. It is shown by measurements on a four-post-test-rig and measurements on a real car that an improvement in ride comfort can be achieved with this control topology. [13] Measurements on a high end sports car show that the LQ controller also lends itself for the control of multiple objectives, [14] in this case also taking into account suspension travel and tire compression. The authors find that good comfort as well as, a reduction in suspension travel can be achieved.

Model predictive control with different types of actuator constraints is discussed in [15]. A comparison of these controllers with soft and hard passive suspension is made, which shows that they perform better than either the hard or soft setting. The problem of implementing MPC controller in real time is discussed in [16]. The author argues that calculating one optimisation step takes too long to be suitable for real-time implementation. Therefore, controller actions are calculated offline for a grid of states. For real-time implementation, an interpolation on this grid is performed at each time to determine a suitable control action. By means of simulations, it is shown that this approach has good performance compared to the well-known sky-hook controller and clipped LQ controllers. The author concludes that the inclusion of prediction in controller design improves the performance of semi-active suspension systems.

The problem with the controllers proposed above is that they are either simple to implement and have only a small performance gain, or are difficult to implement in real-time, but approach the optimal performance for a given actuator within its limitations. In this paper, a controller that performs similar to the optimal performance, but is easy to implement in real-time, will be shown. For this a controller for a semi-active suspension system is developed using MPC with objective minimal sprung mass vertical acceleration. The vehicle states, output and control signals are then used to develop a controller that switches based on a set of rules, which can be implemented in real time. Real time implementation of this rule-based controller will be shown both in simulation and using measurements performed on the test vehicle equipped with semi-active suspension.
The outline of this paper is as follows. The vehicle for which the controller is developed is described in Section 2. Following this, the quarter car model that will be used throughout this paper is described. In Section 4, a semi-active suspension controller is derived using model predictive control with the objective of minimising sprung mass vertical acceleration. With these results a rule-based controller is derived in Section 5. The performance of the rule-based controller is shown in Section 6 and measurements with the test vehicle are presented in Section 6.4. Finally, conclusions and recommendations are given in Section 7.

2. Test vehicle

The vehicle for which the controller is developed is a 2001 BMW 318i. This vehicle is equipped with semi-active suspension designed by Tractive Suspension.[17] The car has sprung mass acceleration sensors at each corner and at the centre of gravity. Besides the acceleration sensors on the sprung mass, acceleration sensors are mounted on each unsprung mass and stroke sensors are installed at each corner between the wheel and chassis, measuring the suspension displacement. The suspension velocity is estimated using a Kalman filter that combines the acceleration difference and suspension displacement. All sensor signals are logged using a dSpace Autobox which samples at 2000 Hz. Processing of the signals happens in software, this means that no filtering of the signals is done before they enter the data acquisition device. Control of the dampers also runs via the dSpace system, with a PWM amplifier between the analog dSpace output and the dampers.

A change of the damping force is achieved by attenuating the position of a valve mounted on the piston of the damper. Depending on the position of the valve, the damper exhibits a different damper force. Note that all positions between soft (fully open) and hard (fully closed) are possible. The unique property of the damper is that it can switch from open to closed while moving in only 0.01 s. The range of adjustment for the front and rear damper is shown in Figure 1, here \( \dot{z}_s \) stands for the velocity of the chassis, \( \dot{z}_u \) for the velocity of the unsprung mass and \( F_d \) for the damper force.

The relevant parameters of the test vehicle are determined using a combination of static and dynamic measurements and are summarised in Table 1. Measurements show that the test vehicle has a near to perfect 50/50 weight distribution, therefore, only the parameters of one corner of the vehicle are shown.

3. Vehicle model

To model the vertical dynamics of the test vehicle, a quarter car model is chosen, as shown in Figure 2. This model consists of two masses, one representing a quarter of the total sprung mass and the unsprung mass the weight of the wheel, brake, upright and part of the suspension linkages. The two masses are connected by means of a spring and damper. Finally, the tire is modelled by a spring. It is assumed that road contact is maintained at all times. The adjustable damper is implemented by means of an actuator providing a force \( F_{act} \), the constraints on the actuator are defined below. The equations of motion are given as

\[
\dot{x}_q = A_q x_q + B_q1 z_r + B_q2 F_{act},
\]

\[
x_q = [z_s \quad \dot{z}_s \quad z_u \quad \dot{z}_u]^T,
\]

\[
y_q = a_z = C x_q + D_q1 z_r + D_q2 F_{act},
\]
Figure 1. Range of damper force as a function of suspension velocity.

Table 1. Parameters of the BMW 318i test vehicle, quarter car parameters are based on the rear of the vehicle.

| Name                        | Parameter | Value | Unit |
|-----------------------------|-----------|-------|------|
| Total mass, including two persons | $m_{\text{tot}}$ | 1636 kg |      |
| Unsprung mass               | $m_u$    | 29 kg |      |
| Sprung mass                 | $m_s$    | 380 kg |      |
| Spring stiffness            | $k_s$    | 21,500 N/m |    |
| Tire stiffness              | $k_t$    | 174,000 N/m |   |

Figure 2. Quarter car model.

with $y_q = a_z$ the output that has to be controlled. The state-space matrices are defined as

$$A_q = \begin{bmatrix}
0 & k_s & 1 & 0 & 0 \\
\frac{k_s}{m_s} & \frac{d_s}{m_s} & \frac{k_s}{m_s} & \frac{d_s}{m_s} \\
0 & 0 & 0 & \frac{k_s}{m_s} & \frac{d_s}{m_s} \\
\frac{k_s}{m_u} & \frac{d_s}{m_u} & \frac{k_s}{m_u} & \frac{d_s}{m_u} & \frac{1}{m_u}
\end{bmatrix},$$

$$B_{q1} = \begin{bmatrix}
0 & 0 & 0 & k_t & m_u
\end{bmatrix}^T.$$

(3)
\[
B_{q2} = \begin{bmatrix} 0 & \frac{1}{m_s} & 0 & \frac{1}{m_u} \end{bmatrix}^T,
\]
\[
C_q = \begin{bmatrix} -\frac{k_s}{m_s} & -\frac{d_{s0}}{m_s} & \frac{k_s}{m_s} & \frac{d_{s0}}{m_s} \end{bmatrix},
\]
\[
D_{q1} = [0], \quad D_{q2} = \begin{bmatrix} -\frac{1}{m_s} \end{bmatrix},
\]

with \(z_s\) being the sprung mass position and \(z_u\) the unsprung mass position. The vertical road disturbance is given by \(z_r\). The sprung mass is indicated by \(m_s\), the unsprung mass by \(m_u\). The stiffness that couples the sprung and unsprung mass equals \(k_s\), the vertical tire stiffness is \(k_t\). The nominal damping coefficient is represented by \(d_{s0}\) and the actuator force by \(F_{act}\). Since in this research semi-active suspension is considered, only the amount of dissipation of energy can be changed. This is captured in the following constraints:

\[
(d_{\text{min}} - d_{s0})(\dot{z}_s - \dot{z}_u) < F_{act} < (d_{\text{max}} - d_{s0})(\dot{z}_s - \dot{z}_u) \quad \text{if} \quad \dot{z}_s - \dot{z}_u \geq 0,
\]
\[
(d_{\text{max}} - d_{s0})(\dot{z}_s - \dot{z}_u) < F_{act} < (d_{\text{min}} - d_{s0})(\dot{z}_s - \dot{z}_u) \quad \text{if} \quad \dot{z}_s - \dot{z}_u < 0. \tag{4}
\]

By splitting up the semi-active suspension into a nominal part and an active part as shown in Figure 3, the non-linearities are captured in the constraints and the model of the system remains linear. Note that this implementation is a simplification of the true damper curves as shown in Figure 1. The implication of this simplification is a topic of future research.

The parameters of the quarter car model has been verified using measurements on the Eindhoven University of Technology campus road. For reasons of brevity, results are not displayed here. Detailed plots of the verification can be found in [18].

### 3.1. Control objective

When a vehicle drives over an uneven surface, road disturbances are transmitted to the vehicle body via the tires and suspension system. This results in motions of the vehicle body, mostly in the vertical direction, but vibrations in the roll and pitch direction also have a contribution on the ride comfort experienced by the driver. The general notion is that the lower the amplitude of these disturbances that are transmitted to the driver, the

**Figure 3.** Graphic representation of the semi-active suspension split into a nominal damping and a constrained active part.
better the comfort of the vehicle is. The ISO standard for expressing comfort is the acceleration of the vehicle body.[19] When expressing the comfort of the vehicle by only one number, the RMS sprung acceleration is used

$$a_{\text{RMS}} = \left( \frac{1}{T} \int_{0}^{T} a_z^2(t) \, dt \right)^{1/2},$$

(5)

with $a_z(t)$ being the acceleration as a function of time and $T$ the duration of the measurement, which is a good measure for comfort. In this paper, however, the improvement will be explored in more detail by looking at the power spectral density of the sprung mass acceleration. The minimisation of the vertical sprung mass acceleration, $a_z$, will be considered as value to be controlled. Besides the amplitude of the vertical acceleration, the frequency content of the disturbance has influence on the comfort and perception of the driver,[19] however, for simplicity reasons, this frequency weighting is omitted.

4. Suspension control

As already mentioned in the Introduction, numerous control techniques exist for the control of semi-active suspension. In this paper, it will first be explored what is possible with this semi-active suspension given the constraints of the actuator. To determine this, an MPC controller is developed to minimise the sprung vertical acceleration given the limitations of the semi-active suspension. Second, the performance of the MPC controller is analysed for a discrete range of frequencies to better understand its behaviour. With this knowledge, a numerical optimisation will be performed to find a rule-based controller that mimics the MPC controller and can be implemented in real time. Finally, the performance of the rule-based controller will be compared with known control strategies such as sky-hook, ADD and obviously the MPC controller itself. In the next sections, these steps will be discussed.

4.1. Model predictive suspension control

A suitable method for determining control actions for the semi-active suspension is model predictive control. With this control technique, the non-linear constraints on the actuator force can be implemented easily. With model predictive control, the control action is calculated based on future inputs and states of the model, for an efficient evaluation the model is transformed into discrete time

$$x_{qd}(k+1) = A_{qd}x_{qd}(k) + B_{qd1}z_t(k) + B_{qd2}F_{\text{act}}(k),$$

(6)

$$y_{qd}(k) = C_{qd}x_{qd}(k) + D_{qd1}z_t(k) + D_{qd2}F_{\text{act}}(k),$$

(7)

with

$$A_{qd} = e^{A_q t_s}, \quad B_{qdi} = A_q^{-1}(A_{qd} - I)B_{qi},$$

(8)

$$C_{qd} = C_q, \quad D_{qdi} = D_q.$$  

(9)

The sample time, $t_s$, is set to 0.004 s. Here, the subscript $i=1$ refers to the road input and the subscript $i=2$ to the control input as shown in Equation (1). The future states and
outputs can now be calculated as follows:

$$\begin{bmatrix} x_{qd}(k + 1) \\
... \\
x_{qd}(k + N + 1) \end{bmatrix} = A_{qh} x_{qd}(k) + B_{qh1} \begin{bmatrix} z_r(k) \\
... \\
z_r(k + N) \end{bmatrix} + B_{qh2} \begin{bmatrix} F_{act}(k) \\
... \\
F_{act}(k + N) \end{bmatrix}, \quad (10)$$

$$\begin{bmatrix} y_{qd}(k) \\
... \\
y_{qd}(k + N) \end{bmatrix} = C_{qh} x_{qd}(k) + D_{qh1} \begin{bmatrix} z_r(k) \\
... \\
z_r(k + N) \end{bmatrix} + D_{qh2} \begin{bmatrix} F_{act}(k) \\
... \\
F_{act}(k + N) \end{bmatrix}, \quad (11)$$

with

$$A_{qh} = \begin{bmatrix} A_{qd} \\
A_{qd}^2 \\
... \\
A_{qd}^N \end{bmatrix}, \quad B_{qhi} = \begin{bmatrix} B_{qdi} & 0 & \ldots & 0 \\
A_{qd} B_{qdi} & B_{qdi} & \ldots & 0 \\
... & ... & ... & \ldots \\
A_{qd}^{N-1} B_{qdi} & A_{qd}^{N-2} B_{qdi} & \ldots & B_{qdi} \end{bmatrix},$$

$$C_{qh} = \begin{bmatrix} C_{qd} \\
C_{qd} A_{qd} \\
... \\
C_{qd} A_{qd}^{N-1} \end{bmatrix}, \quad D_{qhi} = \begin{bmatrix} D_{qdi} & 0 & \ldots & 0 \\
C_{qd} B_{qdi} & D_{qdi} & \ldots & 0 \\
... & ... & ... & \ldots \\
C_{qd} A_{qd}^{N-2} B_{qdi} & C_{qd} A_{qd}^{N-3} B_{qdi} & \ldots & D_{qdi} \end{bmatrix}. \quad (12)$$

The performance objective has to be minimised at each time step, this objective is formulated as

$$J_c(k) = \frac{1}{N} \sum_{l=k}^{k+N} a_z^2(l), \quad (13)$$

with $N$ being the prediction horizon over which the objective has to be minimised. To capture all the relevant dynamics, the prediction horizon is chosen as $N = 60$, with a sample time of $t_s = 0.004 \text{ s}$ the preview is 0.24 s.

A typical road profile can be represented as white noise filtered with a low-pass filter.[20] This stochastic signal can also be considered as the sum of multiple sinusoids. However, for analysis of the control signal, this is not insightful. Therefore, a single sinusoidal road disturbance, $z_r$, is chosen as

$$z_r(k) = 0.02 \sin \left( 2\pi f \frac{k}{t_s} \right). \quad (14)$$

The amplitude of this sinusoidal disturbance is chosen such that on a real car non-linearities such as suspension stoke limitations are not met. The frequency $f$ is chosen from a range of discrete frequencies from 0.5 to 20 Hz, which captures all the relevant dynamics of the quarter car.
Figure 4. RMS sprung mass acceleration for MPC, nominal, minimum and maximum damping. Lower values mean better comfort.

For determining the MPC controller, it is assumed that full state of the quarter car model can be measured. Furthermore, knowledge of the input and its future are assumed. The following procedure is followed to calculate the MPC controller action at each time step $k$:

1. Use the optimisation tool MATLAB `fmincon` to calculate the control sequence $[F_{\text{act}}(k) F_{\text{act}}(k + 1) \ldots F_{\text{act}}(k + N)]^T$ that minimises the objective in Equation (13) with the future outputs defined by Equation (10). The optimisation is subjected to the constraints as formulated in Equation (4). Note that the optimisation might give a local minimum.
2. Apply the output $F_{\text{act}}(k)$ as determined in step (1) and calculate $x(k + 1)$.
3. As long as $k < t_{\text{end}}/t_s$, $k = k+1$. Go back to step (1) and use the calculated $x(k + 1)$ from the previous time step as current state, take $F_{\text{act}}(k)$ as starting point for the optimisation.

In Figure 4, the performance of the MPC controller can be seen for a range of sinusoidal road disturbances. From this plot, it becomes clear that the MPC controller has better performance (lower acceleration) at all frequencies when compared to the baseline system. It combines the best performance of the hard damping at low frequencies with the best performance of the soft damping at higher frequencies. Note that at the wheel-hop frequency, no actuation is possible due to the presence of an invariant point.[21]

An overview of the damping selected by the MPC controller for a 2 Hz disturbance is shown in Figure 5. It can be seen that the damping exclusively varies between minimum and maximum damping, which is expected, since the requirement is either to minimise the transmission of road vibrations due to compression to the sprung mass, or decrease the sprung mass velocity. It can also be seen that the MPC controller achieves a
reduction of vertical acceleration, $a_z$, in comparison to the system with constant nominal damping.

5. Rule-based control

The model predictive control solution that was presented in Section 4 shows the optimal solution for the objective of minimal vertical acceleration taking into account the constraints of the semi-active suspension and assuming that there is knowledge of future disturbances. In practice, however, this knowledge is normally not available. Besides this, model predictive control is computationally very expensive since an optimisation problem has to be solved each time step [16] and full state knowledge is required. This makes MPC infeasible for real-time implementation. However, the MPC controller can be determined and its control action calculated in a simulation environment. With this information, a rule-based controller can be developed that mimics the behaviour of the model predictive controller. For this, it is first assumed that the control actions of the model predictive controller can be approximated by a feedback controller, this approach will be discussed in the next section.
5.1. Feedback from measured outputs

With knowledge of the control signal, $u_{\text{MPC}}$, of the MPC controller, it is postulated that a feedback controller can be found such that

$$u_{\text{MPC}}(k) \approx K_1(k)y_s(k),$$

where the feedback gain $K_1(k)$ has to be determined. The output, $y_s(k)$, contains a combination of sensor signals such as the sprung mass acceleration, suspension travel and velocity and unsprung acceleration as measured in the test vehicle that is introduced in Section 2. For now $y_s(k)$ is taken as a product of the sprung mass acceleration, $a_z(k)$, and suspension velocity, $\dot{z}_s(k) - \dot{z}_u(k)$, similar to the approach taken by Savarese with the ADD controller.[22] All of these signals are directly measured on the test vehicle, allowing for simple implementation in a test set-up. Sky-hook control, for example, requires knowledge of the sprung mass velocity, which is not easily measurable. Since the damping coefficient selected by the MPC controller only varies between the minimum and maximum value, see Figure 5, only the sign of $y_s(k)$ is of relevance. Note that with the model predictive controller, the boundaries of the control range are selected almost exclusively. The measured output signal is thus chosen as

$$y_s(k) = \text{sign}(a_z(k)(\dot{z}_s(k) - \dot{z}_u(k))),$$

with $y_s(k)$ defined the feedback gain, $K_1(k)$ can be determined for each sample $k$ by minimising the error criterion

$$e(k) = \sum_{l=k-M}^{k+M} (K_1(l)y_s(l) - u_{\text{MPC}}(l))^2$$

$$= \sum_{l=k-M}^{k+M} (u(l) - u_{\text{MPC}}(l))^2,$$

here $e(k)$ is the sum of the squared difference between the MPC controller solution and the feedback controller in the sample interval $[-M, M]$ around $k$.

An overview of the result of the minimisation of Equation (17) with road excitation, $z_r$, frequencies of 1, 2 and 6 Hz is shown in Figure 6. Here, $M = 1$, which gives a width of three samples. From this figure, it becomes clear that for low frequencies the feedback gain varies between $K_1 = -500$ and $K_1 = 500$, similar to the difference between the minimum and nominal damping and maximum and nominal damping, respectively, as listed in Table 2. For high frequencies, i.e. 6 Hz, an almost constant feedback gain of 500 is found. This leads to the proposition that the feedback gain $K_1(k)$ is not time varying but constant and equal to the range of adjustment, $\Delta d_s$.

5.2. Constant gain feedback control

Based on the results obtained, a simplification of Equation (15) is implemented, where $K_1(k)$ is constant instead of time varying

$$u_{\text{MPC}}(k) \approx K_2y_s(k).$$
Figure 6. Calculated feedback gain $K$ as a function of time for 1, 2 and 6 Hz disturbances by minimising the error criterion (14), $M = 1$.

Table 2. Active damper parameters as measured by Tractive suspension.[17]

| Name                        | Parameter | Value | Unit |
|-----------------------------|-----------|-------|------|
| Nominal damping coefficient | $d_0$     | 1240  | Ns/m |
| Minimum damping coefficient | $d_{\text{min}}$ | 740   | Ns/m |
| Maximum damping coefficient | $d_{\text{max}}$ | 1740  | Ns/m |

Recall Figure 5 where it is clear that $K_1(k)$ is time varying between $-\Delta d_s$ and $\Delta d_s$. Assuming that this feedback gain is constant implies that the output $y_s(k)$ should be selected such that a constant feedback gain $K_2$ can be applied. This constant feedback gain can be found in sky-hook control and ADD control, which are formulated as

$$d_{\text{sky}} = \Delta d_s \text{sign}(\ddot{z}_s(\dot{z}_s - \dot{z}_u)),$$  \hspace{1cm} (19)

for sky-hook control and

$$d_{\text{ADD}} = \Delta d_s \text{sign}(\dddot{z}_s(\dot{z}_s - \dot{z}_u)),$$  \hspace{1cm} (20)
for the acceleration-driven damper controller. The sky-hook controller manages to suppress low-frequency disturbances, whereas the ADD controller suppresses vibrations at higher frequencies. The only difference between these controllers is that the sky-hook controller uses the sprung mass velocity, $\dot{z}_s$, and the ADD controller the sprung mass acceleration $\ddot{z}_s$. Since the sprung mass acceleration is the time derivative of the sprung mass velocity, a 90 degree phase shift exists between these two signals. Remember that up to now single sinusoidal disturbances with a known frequency are considered, this implies that a phase difference can be written as a time difference. Taking this into consideration, it is assumed that a phase-shifted version of the sprung mass acceleration signal, $\ddot{z}_s$, should be used to achieve a sky-hook-like behaviour at low frequencies. The new time-shifted acceleration is then expressed as

$$a_{zF}(t) = a_z(t - \tau), \quad (21)$$

where $a_{zF}$ is shifted in time with delay $\tau$ with respect to $a_z$ such that controller becomes

$$u(k) = K_2 \text{sign}(a_{zF}(k)(\dot{z}_s(k) - \dot{z}_u(k))), \quad (22)$$

here $K_2$ is chosen to be equal to $\Delta d_s$, the optimisation of Equation (17) can now be formulated as: find $\tau$ for each frequency such that Equation (17) is minimal under the assumption of a constant $K_2 = \Delta d_s$. The time delay that is found is converted into a phase difference using

$$\phi = \frac{\tau}{2\pi f}, \quad (23)$$

and can be seen from the black line in Figure 7. The output controller (22) is extremely simple, though obviously, the price is that it approximates the ‘optimal’ MPC controller less well than the controller (15).

![Figure 7. Phase difference and transfer function fit for control signal $d_{act}$ and the product of $a_z(\dot{z}_s - \dot{z}_u)$.](image-url)
Figure 8. Normalised error $e(k)$ (17) for sky-hook, ADD and the controller proposed in Equation (22) with the optimal time delay.

To show that the proposed controller performs better than the sky-hook and ADD controllers, the cumulative error in comparison with the optimal MPC controller solution is calculated for each frequency

$$e_c(f) = \sum_{l=1}^{k_{\text{end}}} (u(z_{rf}(f, l)) - u_{\text{MPC}}(z_{rf}(f, l)))^2, \quad \forall f.$$  

(24)

This error is calculated for sky-hook, acceleration-driven damper control and the rule-based controller proposed in this paper. The results are shown in Figure 8. For ease of reading, the errors are normalised to the largest value that occurs, in this case sky-hook at 3 Hz. From this figure, it is clear that the difference in the MPC controller can be reduced significantly compared to both sky-hook and ADD control. It is also visible that at low frequencies, the sky-hook controller approaches the performance of the MPC controller the best and that at high frequencies, the ADD controller matches the MPC controller the best. However, both controllers have larger errors in specific frequency ranges, compared to the controller proposed here. Note that the controller proposed by Savaresi [22] is a combination of the sky-hook and ADD controller. Figure 8 shows that the crossover frequency for this system should be at approximately 1.7 Hz.

### 5.3. Real-time implementation

A normal road obviously does not consist of only one sinusoid with one frequency and a fixed amplitude.[20] Therefore, the time delay that was calculated per frequency in the previous section is approximated by a transfer function. Based on the phase rotation as shown in Figure 7, a transfer function with four poles and four zeros is selected to describe the behaviour. With the general structure of the transfer function known, a fit can be made...
that approximates the phase rotation. A suitable candidate for the transfer function is

$$
\Phi(s) = \frac{a_z F}{a_z} = \frac{1.42 \times 10^{-6} s^4 + 6.15 s^3 + 0.009 s^2 + 0.11 s + 1}{1.45 \times 10^{-5} s^4 + 2.5 \times 10^{-4} s^3 + 0.08 s^2 + s}.
$$

(25)

Note that this fit is only valid for the car parameters as presented in Figure 1. A comparison of the transfer function $\Phi(s)$ with the phase delay as calculated before is shown in Figure 7. Note that only the phase of $\Phi(s)$ is of importance as only the sign of $\Phi(s)a_z(\dot{z}_s - \dot{z}_u)$ is used in the controller and not the amplitude. The difference between the time delay as calculated for the optimal controller and the transfer function $\Phi(s)$ is smaller than $15^\circ$ for all frequencies of interest. It is clear from Figure 7 that at low frequencies a $90^\circ$ phase lag is required, corresponding to the sprung velocity $\dot{z}_s$ for a sky-hook controller. At higher frequencies, it approaches a $0^\circ$ phase lag, thus being the same as the acceleration-driven damper.  

With the time delay described by a continuous transfer function $\Phi(s)$, the controller can now be formulated as

$$
F_{act} = d_{sact}(\dot{z}_s - \dot{z}_u) \quad \text{with} \quad \begin{cases} 
    d_{sact} = d_{\text{max}} - d_{s0} & \text{if} \quad \Phi(s)a_z(\dot{z}_s - \dot{z}_u) > 0, \\
    d_{sact} = d_{\text{min}} - d_{s0} & \text{if} \quad \Phi(s)a_z(\dot{z}_s - \dot{z}_u) \leq 0.
\end{cases}
$$

(26)

So, a rule-based controller has been found that switches between minimum and maximum damping value. Switching is applied based on a signal that consists of the product of vertical acceleration, $a_z$, and suspension velocity, $\dot{z}_s - \dot{z}_u$. The product of these signals is then passed through a filter which, depending on the frequency of the signal, gives a phase rotation to the product of vertical acceleration and suspension velocity. The filter has a phase lag of approximately $90^\circ$ at low frequencies and zero phase lag beyond $4$ Hz.

6. Results

Now that a controller has been synthesised, its performance will be evaluated for a number of different conditions. The comfort controller as described by (26) is implemented in MATLAB/Simulink with the quarter car as introduced in Equation (1). It is assumed that the suspension velocity $\dot{z}_s - \dot{z}_u$ can be measured in this model, for implementation on a real vehicle an observer that combines the measurements of the sprung and unsprung acceleration and the suspension travel can be used. To show that the developed controller is also valid for different road disturbances, three different inputs will be shown here. First, the sinusoidal disturbances with which the controller has been synthesised. Second, results of the controllers on a random road will be shown and finally measurements will be performed using the test vehicle.

6.1. Sinusoidal road disturbance

The road disturbance, $z_r$, is modelled as a sinusoidal disturbance with an amplitude of $0.02$ m. Similar to Equation (14), a frequency range of $0.5$–$20$ Hz is used. The results of
these simulation can be seen in Figure 9. As expected, from this plot it is clear that the
rule-based comfort controller matches the MPC controller quite closely up to the tire res-
onance frequency ($\approx 13$ Hz). It can also be seen that at low frequencies it behaves similar
to the sky-hook controller, as is already expected from the phase plot in Figure 7. Beyond
the sprung resonance frequency, it has a lower sprung mass acceleration than the sky-
hook controller. Finally, at 2.5 Hz and beyond it behaves similar to the acceleration-driven
damper controller.

### 6.2. Random road disturbance

The random road is represented by low-pass filtered white noise as was presented in [20].
The amplitude is scaled such that it matches the amplitude of an average road.[4] The
results of the simulations are shown in Figure 10 where the power spectral density of the
sprung mass acceleration is shown. The MPC controller results are obtained by running
the optimisation as described in Section 4 with the random road as input. From this plot, it
can be seen that in the frequency range between 1.2 and 4 Hz, the MPC controller performs
better than the controller proposed in this paper. The low-frequency difference is caused

![Figure 9. RMS sprung mass acceleration for MPC, $d_{s\min}$, $d_{s\max}$, sky-hook control, ADD, Savaresi controller
and the comfort controller proposed in Equation (26) for a sinusoidal disturbance.](image)
Figure 10. Power spectral density of the vertical acceleration $a_z$ for the MPC controller, nominal damping and the comfort controller proposed in this paper (26). Note that ADD and sky-hook have been omitted for reasons of clarity.

by a deviation in the transfer function, $\Phi(s)$, and the time delay as determined optimal for the best comfort. RMS sprung mass acceleration for the baseline system is 1.28 m/s$^2$, for the sky-hook and ADD controller these values are 1.16 m/s$^2$ and 1.22 m/s$^2$, respectively. For the comfort controller proposed in this paper, this value is 1.15 m/s$^2$, which shows that the proposed controller achieves a better performance than the existing rule-based controllers. Note that the RMS sprung mass acceleration of the sky-hook controller is almost similar to the controller proposed here. This is caused by the road input, which has a large contribution at low frequencies. This implies that changes at low frequencies influence the RMS value the most. Finally, the MPC controller achieves an RMS acceleration of 1.07 m/s$^2$, which is lower than all the other controllers. In relation to the RMS sprung mass acceleration numbers, note that a 5% difference in vertical acceleration is noticeable for a human.[23] This implies that the difference between the proposed controller and the baseline system is clearly noticeable.

### 6.3. Step disturbance

The response to a discrete disturbance is determined by simulating a step in the road input of 0.05 m. The approach of Section 4 is followed with the step disturbance to determine the MPC control action. Besides the MPC controller result, the nominal damping, sky-hook controller, ADD controller and the controller from this paper are shown in Figure 11. It can be clearly seen that all three controllers have better performance for the initial step upwards than the nominal system. In the negative acceleration part at 1.05 s, the controller proposed here first follows the sky-hook controller and then switches to a lower damping coefficient, similar to the behaviour of the ADD controller. Higher accelerations compared to the nominal system are seen at 1.1 s for all controllers.
6.4. Full car measurements

With the test vehicle as described in Section 2, measurements using the comfort controller proposed in this paper have been performed. As a test track, the Eindhoven University of Technology campus road is used, which consists of mostly well-worn bricks. Care was taken for the speed profile to be the same on each run. During each run, an average velocity of 9.6 m/s (= 35 km/h) was achieved. A separate controller was implemented at each corner of the vehicle, since at each corner the sprung acceleration, $a_z$, and suspension velocity, $\dot{z}_s - \dot{z}_u$, are measured. By minimising all four corner accelerations, it is assumed that the overall sprung mass acceleration is also reduced.

The results of these measurements can be seen in Figure 12 which shows the power spectral density of the left rear sprung mass acceleration for soft, hard and controlled damping. From this figure, it can be clearly seen that at the sprung mass resonance at 1.5 Hz, the sprung mass acceleration for the soft damping is higher than that for the controlled and hard damping. Beyond 2 Hz the sprung mass acceleration for the soft damper setting is lower than that of the hard damper setting. Most importantly though, from this figure it can be seen that the controlled vehicle combines the best properties of the soft and hard damping. This means that for low-frequency accelerations it behaves similar to the hard damping and for higher frequencies, it is similar to the soft damper setting.

The RMS sprung mass acceleration for each corner is shown in Table 3. As is visible, the controlled vehicle always has the lowest vertical acceleration compared to the soft and hard
Figure 12. Measured rear left sprung mass acceleration whilst driving on the TU/e campus roads for soft, hard and controlled damping.

Table 3. Sprung mass acceleration per corner.

|               | FL (m/s²) | FR (m/s²) | RL (m/s²) | RR (m/s²) |
|---------------|-----------|-----------|-----------|-----------|
| Soft          | 1.75      | 1.96      | 1.89      | 1.73      |
| Hard          | 1.67      | 1.93      | 2.12      | 2.01      |
| Comfort controller | 1.59    | 1.83      | 1.70      | 1.57      |

setting. Note that the front right suspension shows less performance compared to the other three corners. This is caused by a defective, leaking damper, resulting in less suspension stroke and thus causing worse performance.

Besides the objective evaluation of the suspension, which shows that the vertical acceleration is lower for the controlled vehicle, approximately 10 experts in the field, some of them responsible for shock absorber tuning, have driven with all three settings. The comment on the soft suspension was usually that they did like the general comfort, but did not like the sense of ‘instability’ that the vehicle exhibited. This sensation was attributed to the vehicle body moving too much at low frequencies. When switching from the soft to the hard setting the general opinion was that they felt more in control over the vehicle motion. Some argued that they felt the ride to be a bit harsh, but in general, the hard setting was preferred over the soft setting. However, when the controlled setting was turned on the drivers immediately preferred it, saying that they did feel in control of the vehicle motion as a result of steering input, whilst not feeling the harshness of the hard damper setting. Most drivers actually started driving faster after some time, because they felt more control of the vehicle.
7. Conclusion

In this paper, the objective was to develop a controller for a semi-active suspension that minimises the vertical acceleration of the sprung mass, thereby improving comfort. This is done by designing a rule-based controller that matches the performance of a model predictive controller, which is assumed to be optimal. This model predictive controller is developed for minimal vertical acceleration given the constraints of the semi-active suspension. It is shown that for a discrete set of frequencies, a direct correlation between the product of the sprung mass acceleration and suspension velocity with the control signal exists. For the other frequencies, a phase-shifted version of this signal has to be used. This phase shift is determined by minimising the error between the suggested control signal and the MPC control action. With this analysis, a simple to implement rule-based controller is found which includes a transfer function that filters the sprung acceleration.

Various tests show that the rule-based controller performs similar to the MPC controller and has better performance than existing controllers, such as sky-hook control and ADD. In fact, it mimics the behaviour of a sky-hook controller at low frequencies and an ADD controller at high frequencies. A small performance improvement over the mixed SH-ADD controller is achieved, however, the approach is significantly different. On sinusoidal disturbances, random road surface and a step disturbance the comfort is always better than existing control techniques. Furthermore, measurements were performed with a test vehicle equipped with semi-active suspension. On-road measurements show that a reduction of the sprung mass vertical acceleration is achieved at all four corners of the car. Furthermore, experts in the field acknowledged the improvement in comfort of the test vehicle with the newly developed controller.

Disclosure statement

No potential conflict of interest was reported by the authors.

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