Supergravity from the weakly coupled heterotic string∗†
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Abstract

The weakly coupled vacuum of $E_8 \otimes E_8$ heterotic string theory remains a promising candidate for the underlying theory of the Standard Model. The particle spectrum and the issue of dilaton stabilization are reviewed. Specific models for hidden sector condensation and supersymmetry breaking are described and their phenomenological and cosmological implications are discussed.

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1 Introduction

In this talk I review the motivation for the view that the weakly coupled heterotic string (WCHS), with Calabi-Yau (or a CY-like orbifold) compactification remains a prime candidate for the description of observed physical phenomena. Several years ago, dilaton stabilization – needed to fix the gauge coupling constant and thought to be an intractable problem in this context – was shown to be achievable by invoking nonperturbative corrections to the dilaton Kähler potential. I will review the properties of a class of models based on orbifold compactifications of the heterotic string, with supersymmetry broken by condensation in a hidden sector with a strongly coupled gauge group. New results, that incorporate an anomalous $U(1)$ in the effective supergravity theory, will be presented.

2 The case for the weakly coupled heterotic string

2.1 Bottom up approach

This approach starts from experimental data with the aim of deciphering what it implies for an underlying, more fundamental theory. One outstanding datum is the observed large hierarchy between the $Z$ mass, characteristic of the scale of electroweak symmetry breaking, and the reduced Planck scale $m_P$:

$$m_Z \approx 90 \text{GeV} \ll m_P = \sqrt{\frac{8\pi}{G_N}} \approx 2 \times 10^{18} \text{GeV},$$

which can be technically understood in the context of supersymmetry (SUSY). The conjunction of SUSY and general relativity (GR) implies supergravity (SUGRA). The absence of observed SUSY partners (sparticles) requires broken SUSY in the vacuum, and the observed particle spectrum constrains the mechanism of SUSY-breaking in the observable sector: spontaneous SUSY-breaking is not viable, leaving soft SUSY-breaking as the only option that preserves the technical SUSY solution to the hierarchy problem. This means introducing SUSY-breaking operators of dimension three or less–such as gauge invariant masses–into the Lagrangian for the SUSY extension of the Standard Model (SM). The unattractiveness of these ad hoc soft terms suggests they arise from spontaneous SUSY breaking in a “hidden sector” of the underlying theory. Based on the above facts, a number of standard scenarios have emerged. These include: 1) Gravity mediated SUSY-breaking, usually understood as “minimal SUGRA” (mSUGRA), which has been the focus of a number of talks at this meeting. This scenario is typically characterized by
2.2 Top down approach

This approach starts from a ToE with the hope of deriving the Standard Model from it; the present prime candidate ToE is string or M theory. The driving motivation is that superstring theory is at present the only known candidate for reconciling GR with quantum mechanics. These theories are consistent in ten dimensions; in recent years it was discovered that all the consistent superstring theories are related to one another by dualities, namely S-duality: $\alpha \rightarrow 1/\alpha$, and T-duality: $R \rightarrow 1/R$, where $\alpha$ is the fine structure constant of the gauge group(s) at the string scale, and $R$ is a radius of compactification from dimension D to dimension D − 1. Figure 1 shows how these dualities relate the various 10-D superstring theories to one another, and to the currently presumed ToE, M-theory. Another image of M-theory, the “puddle diagram” of Figure 2 indicates that all the known superstring theories, as well as D = 11 SUGRA, are particular limits of M-theory. Currently, there is a lot of activity in type I and II theories, or more generally in theories with branes. Similarly the Hořava-Witten (HW) scenario and its inspirations have received considerable attention. If one compactifies one dimension of the 11-D limit of M-theory, one gets the HW scenario with two 10-D branes, each having an $E_8$ gauge group. As the radius of this 11th dimension is shrunk to zero, the WCHS scenario is recovered. This is the scenario addressed here.

2.3 The $E_8 \otimes E_8$ Heterotic String

Here I outline the appealing aspects of the weakly coupled $E_8 \otimes E_8$ heterotic string theory. The zero-slope (infinite string tension) limit of superstring theory is ten dimensional supergravity coupled to a supersymmetric Yang-Mills theory with an $E_8 \otimes E_8$ gauge group. To make contact with the real world, six of these ten dimensions must be compact and here are assumed to be of order $m_P \sim$
10^{-32}\text{cm}. If the topology of the extra dimensions were a six-torus, which has a flat geometry, the 8-component spinorial parameter of $N = 1$ supergravity in ten dimensions would appear as the four two-component parameters of $N = 4$ supergravity in four dimensions. A Calabi-Yau (CY) manifold leaves only one of these spinors invariant under parallel transport; the group of transformations under parallel transport (holonomy group) is the $SU(3)$ subgroup of the maximal $SU(4) \cong SO(6)$ holonomy group of a six dimensional compact space. This breaks $N = 4$ supersymmetry to $N = 1$ in four dimensions. The only phenomenologically viable supersymmetric theory at low energies is $N = 1$, because it is the only one that admits complex representations of the gauge group that are needed to describe quarks and leptons. For this solution, the classical equations of motion impose the identification of the affine connection of general coordinate transformations on the compact space (described by three complex dimensions) with the gauge connection of an $SU(3)$ subgroup of one of the $E_8$’s: $E_8 \supseteq E_6 \otimes SU(3)$, resulting in $E_6 \otimes E_8$ as the gauge group in four dimensions. Since the early 1980’s, $E_6$ has been considered the largest group that is a phenomenologically viable candidate for a Grand Unified Theory (GUT) of the SM. Hence $E_6$ is identified as the gauge group of the “observable sector”, and the additional $E_8$ is attributed to a “hidden sector”, that interacts with the former only with gravitational strength couplings. Orbifolds, which are flat spaces except for points of infinite curvature, are more easily studied than CY manifolds, and orbifold compactifications that closely mimic CY compactification, and that yield realistic spectra with just three generations of quarks and leptons, have been found.\[13, 14\] In this case the surviving gauge group is $E_6 \otimes G_0 \otimes E_8$, $G_0 \in SU(3)$. The low energy effective field theory is determined by the massless spectrum, i.e., the spectrum of states with masses very small compared with the string tension and compactification scale. Massless bosons have zero triality under an $SU(3)$ which is the diagonal of the $SU(3)$ holonomy group and the (broken) $SU(3)$ subgroup of one $E_8$. The ten-vectors $A_M$, $M = 0, 1, \ldots, 9$, appear in four dimensions as four-vectors $A_\mu$, $\mu = M = 0, 1, \ldots, 3$, and as scalars $A_m$, $m = M - 3 = 1, \ldots, 6$. Under the decomposition $E_8 \supseteq E_6 \otimes SU(3)$, the $E_8$ adjoint contains the adjoints of $E_6$ and $SU(3)$, and the representation $(27, 3) + (\overline{27}, \overline{3})$. Thus the massless spectrum includes gauge fields in the adjoint representation of $E_6 \otimes G_0 \otimes E_8$ with zero triality under both $SU(3)$’s, and scalar fields in $27 + \overline{27}$ of $E_6$, with triality $\pm 1$ under both $SU(3)$’s, together with their fermionic superpartners. The number of $27$ and $\overline{27}$ chiral supermultiplets that are massless depends on the topology of the compact manifold. The important point for phenomenology is the decomposition under $E_6 \to SO(10) \to SU(5)$:

\[(27)_{E_6} = (16 + 10 + 1)_{SO(10)} = (\{\overline{5} + 10 + 1\} + \{5 + \overline{5} + 1\})_{SU(5)} .\] (2.1)
$3 + 10 + 1$ contains one generation of quarks and leptons of the SM, a right-handed neutrino and their scalar superpartners; a $5 + \overline{5}$ contains the two Higgs doublets needed in the supersymmetric extension of the SM and their fermion superpartners, as well as color-triplet supermultiplets. While all the states of the SM and its minimal supersymmetric extension are present, there are no scalar particles in the adjoint representation of the gauge group. In conventional models for grand unification, these (or other large representations) are needed to break the GUT group to the SM. In string theory, this symmetry breaking can be achieved by the Hosotani or “Wilson line”, mechanism in which gauge flux is trapped around “holes” or “tubes” in the compact manifold, in a manner reminiscent of the Arahonov-Bohm effect. The vacuum value of the trapped flux $\langle \int d\ell^m A_m \rangle$ has the same effect as an adjoint Higgs, without the difficulties of constructing a potential for large Higgs representations that actually reproduces the observed vacuum. When this effect is included, the gauge group in four dimensions is

$$G_{\text{obs}} \otimes G_{\text{hid}}, \quad G_{\text{obs}} = G_{\text{SM}} \otimes G' \otimes G_o, \quad G_{\text{SM}} \otimes G' \in E_6, \quad G_o \in SU(3),$$

$$G_{\text{hid}} \in E_8, \quad G_{\text{SM}} = SU(3)_c \otimes SU(2)_L \otimes U(1)_w.$$  \hfill (2.2)

There are many other four dimensional string vacua in addition to those described above. The attractiveness of the above picture is that the requirement of $N = 1$ SUSY naturally results in a phenomenologically viable gauge group and particle spectrum, and the gauge symmetry can be broken to a product group embedding the SM without introducing large Higgs representations. The $E_8 \otimes E_8$ string theory also provides a hidden sector needed for spontaneous SUSY-breaking. Specifically, if some subgroup $G_c$ of $G_{\text{hid}}$ is asymptotically free, with a $\beta$-function coefficient $b_c > b_{SU(3)}$, defined by the renormalization group equation (RGE)

$$\mu \frac{\partial g_c(\mu)}{\partial \mu} = -\frac{3}{2} b_c g_c^3(\mu) + O(g_c^5),$$  \hfill (2.3)

confinement and fermion condensation will occur at a scale $\Lambda_c \gg \Lambda_{\text{QCD}}$, and hidden sector gaugino condensation $\langle \lambda \lambda \rangle_{G_c} \neq 0$, may induce \cite{15} supersymmetry breaking. To discuss supersymmetry breaking in more detail, we need the low energy spectrum resulting from the ten-dimensional gravity supermultiplet that consists of the 10-D metric $g_{MN}$, an antisymmetric tensor $b_{MN}$, the dilaton $\phi$, the gravitino $\psi_M$ and the dilatino $\chi$. For the class of CY and orbifold compactifications described above, the massless bosons in four dimensions are the 4-D metric $g_{\mu \nu}$, the antisymmetric tensor $b_{\mu \nu}$, the dilaton $\phi$, and certain components of the tensors $g_{mn}$ and $b_{mn}$ that form the real and imaginary parts, respectively, of complex scalars known as moduli. The number of moduli is
related to the number of particle generations (# of $2^7$'s − # of $\overline{2^7}$'s). In three generation orbifold models there are at least three moduli $t_I$ whose vev’s $\langle \text{Re} t_I \rangle$ determine the radii of the three tori of the compact space. They form chiral multiplets with fermions $\chi^I$ obtained from components of $\psi_m$. The 4-D dilatino $\chi$ forms a chiral multiplet with with a complex scalar field $s$ whose vev $\langle s \rangle = g^{-2} - i\theta/8\pi^2$ determines the gauge coupling constant and the $\theta$ parameter of the 4-D Yang-Mills theory. The “universal” axion Im$s$ is obtained by a duality transformation from the antisymmetric tensor $b_{\mu\nu}$: $\partial_\mu \text{Im}s \leftrightarrow \varepsilon_{\mu\nu\rho\sigma} \partial_\nu b^{\rho\sigma}$. Because the dilaton couples to the (observable and hidden) Yang-Mills sector, gaugino condensation induces a superpotential for the dilaton superfield$^1$ $S$:

$$W(S) \propto e^{-S/(bc)}.$$  \hspace{1cm} (2.4)

The vacuum value $\langle W(S) \rangle = \infty$ $\langle e^{-S/(bc)} \rangle = e^{-g^{-2}/bc} = \Lambda_c$ is governed by the condensation scale $\Lambda_c$ as determined by the RGE (2.3). If it is nonzero, the gravitino acquires a mass $m_{3/2} \propto \langle W \rangle$, and local supersymmetry is broken.

3  A model for SUSY breaking

In this section I review the properties of a class of models,$^4$ based on affine level one orbifolds with three untwisted moduli $T^I$ and a gauge group of the form (2.2), with one factor $G_c \in \mathcal{G}_{hid}$ that becomes strongly coupled.

3.1  The Runaway Dilaton

The superpotential (2.4) results in a potential for the dilaton of the form $V(s) \propto e^{-2\text{Re}s/(bc)}$, which has its minimum at vanishing vacuum energy and vanishing gauge coupling: $\langle \text{Re}s \rangle \rightarrow \infty$, $g^2 \rightarrow 0$. This is the notorious runaway dilaton problem. The effective potential for $s$ is in fact determined from anomaly matching: $\delta L_{\text{eff}}(s, u) \leftrightarrow \delta L_{\text{hid}}(\text{gauge})$, where $u$, $\langle u \rangle = \langle \bar{\lambda}\lambda \rangle_{G_c}$, is the lightest scalar bound state of the strongly interacting, confined gauge sector. Just as in QCD, the effective low energy theory of bound states must reflect both the symmetries and the anomalies, i.e. the quantum induced breaking of classical symmetries, of the underlying Yang-Mills theory. It turns out that the effective quantum field theory (QFT) is anomalous under T-duality. Since this is an exact symmetry of heterotic string perturbation theory, it means that the effective QFT is

$^1$Throughout I use capital Greek or Roman letters to denote a chiral superfield, and the corresponding lower case letter to denote its scalar component.
incomplete. This is cured by including model dependent string-loop threshold corrections\cite{16} as well as a “Green-Schwarz” (GS) counter-term,\cite{17} analogous to the GS mechanism in 10-D SUGRA. This introduces dilaton-moduli mixing, and the gauge coupling constant is now identified as

\[ g^2 = 2 \langle \ell \rangle, \quad \ell^{-1} = 2 \text{Res} - b \sum_I \ln(2 \Re t_I), \quad (3.5) \]

where \( b \leq b_{E_8} = 30/8\pi^2 \) is the coefficient of the GS term, and and \( \ell \) is the scalar component of a linear superfield \( L \) that includes the two-form \( b_{\mu \nu} \) and is dual to the chiral superfield \( S \) in the supersymmetric version of the two-form/axion duality of Section 2.3. The GS term introduces a second runaway direction, this time at strong coupling: \( V \to -\infty \) for \( g^2 \to \infty \). The small coupling behavior is unaffected, but the potential becomes negative for \( \alpha = \ell / 2\pi > .57 \). This is the strong coupling regime, and nonperturbative string effects cannot be neglected; they are expected\cite{8} to modify the Kähler potential for the dilaton, and therefore the potential \( V(\ell, u) \). It has been shown\cite{8} that these contributions can indeed stabilize the dilaton. Retaining just one or two terms of the suggested parameterizations\cite{8} of the nonperturbative string corrections: \( a_n \ell^{-n/2} e^{-c_n/\sqrt{\ell}} \) or \( a_n \ell^{-n} e^{-c_n/\ell} \), the potential can be made positive-definite everywhere and the parameters \( a_n, c_n \) can be chosen to fit two data points: the coupling constant \( g^2 \approx 1/2 \) and the cosmological constant \( \Lambda \approx 0 \). This is fine tuning, but it can be done with reasonable (order 1) values for the parameters \( c_n, a_n \). If there are several condensates with different \( \beta \)-functions, the potential is dominated by the condensate with the largest \( \beta \)-function coefficient \( b_\pm \), and the result is essentially the same as in the single condensate case, except that a small mass is generated for the axion \( a = \text{Im} s \). In these models the presence of \( \beta \)-function coefficients generate mass hierarchies that have interesting implications for cosmology and the spectrum of sparticles—the supersymmetric partners of the SM particles.

### 3.2 Sparticle Spectrum

If the gauge group \( G_c \in E_8 \) is smaller than \( E_8 \), the moduli \( t_I \) are stabilized through their couplings to twisted sector matter and/or moduli-dependent string threshold corrections at a self-dual point, \( < t_I > = 1 \) or \( e^{i\pi/6} \), and their auxiliary fields vanish in the vacuum. SUSY-breaking is dilaton mediated, avoiding a potentially dangerous source of flavor changing neutral currents (FCNC). This result holds up to unknown couplings \( p_A \) of chiral matter \( \phi^A \) to the GS term: at the scale \( \Lambda_c \), \( m_0^A = m_3/2 \) if \( p_A = 0 \), while \( m_0^A = \frac{1}{2} m_{t_l} \approx 10 m_3/2 \) if the scalar \( \phi^A \) couples with the same strength as the T-moduli: \( p_A = b \). Gaugino masses are suppressed: \( m_1 \approx b_c m_3/7 \) at the scale \( \Lambda_c \)
in the tree approximation. As a consequence quantum corrections can be important, mimicking anomaly-mediated scenarios in some regions of parameter space. If \( p_A = b \) for some gauge-charged chiral fields, there are enhanced loop corrections to gaugino masses.\(^{[18]}\) Four sample scenarios were studied\(^{[19]}\): A) \( p_A = 0 \), B) \( p_A = b \), C) \( p_A = 0 \) for the superpartners of the first two generations of SM particles and \( p_A = b \) for the third, and D) \( p_A = 0 \) for the Higgs particles and \( p_A = b \) otherwise. Imposing constraints from experiments and the correct electroweak symmetry-breaking vacuum rules out scenarios B and C. Scenario A is viable for \( 1.65 < \tan \beta < 4.5 \), and scenario D is viable for all values of \( \tan \beta \), the ratio of Higgs vev’s in the supersymmetric extension of the SM. The viable range of parameter space is shown\(^{[20]}\) in Figure 3 for \( g^2 = 1/2 \). The dashed lines represent the possible dominant condensing hidden gauge groups \( G_c = G + \epsilon \in E_8 \) with chiral matter in the coset space \( E_8/G_{hid} \).

3.3 Modular Cosmology

The masses of the dilaton \( \sigma = \text{Res} \) and the complex t-moduli \( t^I = (\tau^I + ia^I)/\sqrt{2} \), are related to the gravitino mass by\(^{[4, 6]}\)

\[
m_{I} \approx \frac{1}{b_c} m_{3/2}, \quad m_{\tau, a} \approx \frac{(b - b_c) \mu_{\tau, a}(\langle \tau^I \rangle)}{b_c(1 + b < \ell >)} m_{3/2},
\]

(3.6)

where at the self-dual points \( \mu_{\tau} \approx 3, \mu_{a} \approx 0.5-1 \). Taking \( b = b_{E_8} \approx 0.38 \approx 10b_c \), gives a hierarchy of order \( m_{3/2} \sim 10^{-15} m_{P} \sim 10^3 \text{GeV} \) and \( m_{\tau} \approx 30 m_{3/2} \approx 30 \text{TeV} \), \( m_{a} \approx (5-10)m_{3/2} \approx 5-10 \text{TeV} \), \( m_{\sigma} \sim 10^{10} m_{3/2} \sim 10^6 \text{GeV} \), which is sufficient to evade the late moduli decay problem\(^{[21]}\) in nucleosynthesis.

If there is just one hidden sector condensate, the axion \( a = \text{Im}s \) is massless up to QCD-induced effects: \( m_a \sim (\Lambda_{QCD}/\Lambda) m_{3/2} \sim 10^{-9} \text{eV} \), and it is the natural candidate for the Peccei-Quinn axion. Because of string nonperturbative corrections to its gauge kinetic term, the decay constant \( f_a \) of the canonically normalized axion is reduced with respect to the standard result by a factor \( b_c \ell^2 \sqrt{6} \approx 1/50 \) if \( b_c \approx 0.1 b_{E_8} \), which may be sufficiently small to satisfy the (looser) constraints on \( f_a \) when moduli are present.\(^{[22]}\)

3.4 Flat Directions in the Early Universe

Many successful cosmological scenarios—such as an epoch of inflation—require flat directions in the potential. A promising scenario for baryogenesis suggested\(^{[23]}\) by Affleck and Dine (AD) requires flat directions during inflation in sparticle field space: \( \langle \tilde{q} \rangle, \langle \tilde{\ell} \rangle \neq 0 \), where \( \tilde{f} \) denotes the superpartner of the fermion \( f \). While flat directions are common in SUSY theories, they are
generally lifted in the early universe by SUGRA couplings to the potential that drives inflation. This problem is evaded in models with a “no-scale” structure, such as the classical potential for the untwisted sector of orbifold compactifications. Although the GS term breaks the no-scale property, quasi-flat directions can still be found. An explicit model for inflation based on the effective theory described above allows dilaton stabilization within its domain of attraction with one or more moduli stabilized at the vacuum value $t_I = e^{i\pi/6}$. One of the moduli may be the inflaton. The moduli masses are sufficiently large to evade the late moduli decay problem in nucleosynthesis, but unlike the dilaton, they are insufficient to avoid a large relic LSP density without violation of R-parity (which distinguishes SM particles from their superpartners). If R-parity is conserved, this problem can be evaded if the moduli are stabilized at or near their vacuum values—or for a modulus that is itself the inflaton. It is possible that the requirement that the remaining moduli be in the domain of attraction is sufficient to avoid the problem altogether. For example, if $\text{Im} t_I = 0$, the domain of attraction near $t_I = 1$ is rather limited: $0 < \text{Re} t_I < 1.6$, and the entropy produced by dilaton decay with an initial value in this range might be less than commonly assumed. The dilaton decay to its true ground state may provide partial baryon number dilution, which is generally needed for a viable AD scenario.

3.5 Relic Density of the Lightest SUSY Particle (LSP)

Two pertinent questions for SUSY cosmology are: 1) Does the LSP overclose the Universe? 2) Can the LSP be dark matter? As discussed by others at this meeting, the window for LSP dark matter in the much-studied mSUGRA scenario, has become smaller as the Higgs mass limit has increased; there is not much parameter space in which the LSP does not overclose the universe. The ratios of electroweak sparticle masses at the Plank scale determine the composition of the LSP (which must be neutral) in terms of the Bino (superpartner of the SM $U(1)$ gauge boson), the electrically neutral Wino (superpartner of the neutral SM $SU(2)$ gauge boson), and the higgsino (superpartner of the Higgs boson). The mSUGRA assumption of equal gaugino masses at the Planck scale leads to a Bino LSP with rather weak couplings, resulting in little annihilation and the tendency to overclose the universe, except in a narrow range of parameter space where the LSP is nearly degenerate with the next to lightest sparticle (in this case a stau $\tilde{\tau}$), allowing significant coannihilation. Relaxing this assumption allows a predominantly Bino LSP with a small admixture of Wino, that can provide the observed amount $\Omega_d$ of dark matter. In the condensation model, this occurs in the region indicated by fine points in Figure 8. Here the deviation from mSUGRA is due to loop corrections to gaugino masses giving a small Wino component in the LSP; its near degeneracy in mass with the
lightest charged gaugino also enhances coannihilation. For larger $b_c$ the LSP becomes pure Bino as in mSUGRA, and for smaller values it becomes Wino-dominated as in anomaly-mediated models which are cosmologically safe, but do not provide LSP dark matter, because Wino annihilation is too fast.

4 Incorporating an anomalous $U(1)$

Orbifold compactifications with the Wilson line/Hosotani mechanism needed to break $E_6$ to the SM gauge group generally have $b_c ≤ b ≤ b_{E_8}$. An example is a model,[13] hereafter called the FIQS model, with hidden gauge group $SO(10)$ and $b_c = b = b_{SO(10)}$. It is clear from (3.6) that this would lead to disastrous modular cosmology, since the $t$-moduli are massless. Moreover, in many orbifold compactifications, the gauge group $G_{obs} \otimes G_{hid}$ obtained at the string scale has no asymptotically free subgroup that could condense to trigger SUSY-breaking. However in many compactifications with realistic particle spectra,[13, 14] the effective field theory has[30] an anomalous $U(1)$ gauge subgroup, which is not anomalous at the string theory level. The anomaly is canceled[31] by a second GS counterterm. This results in a D-term that forces some otherwise flat direction in scalar field space to acquire a vacuum expectation value, further breaking the gauge symmetry, and giving masses of order $\Lambda_D$ to some chiral multiplets, so that the $\beta$-function of some of the surviving gauge subgroups may be negative below the scale $\Lambda_D$, typically an order of magnitude below the string scale. The presence of such a D-term was explicitly invoked in the above-mentioned inflationary model.[26]

4.1 The effective theory below the $U(1)$-breaking scale

The GS mechanism that restores invariance under the anomalous $U(1)_X$ gauge group induces a Fayet-Iliopoulos D-term that drives nonvanishing vev’s for the scalar components $\phi^A$ of $n \ U(1)_X$-charged chiral supermultiplets $\Phi^A$ that in turn break a total of $m$ gauge symmetries $U(1)_a$. The equations of motion for the auxiliary field components $D_a$ of the vector supermultiplets $V_a$ take the form:

$$D_a = \sum_A K_A q^a_A \phi^A - \delta_X a \delta_X / 2, \quad \delta_X = -\frac{\text{Tr} T_X}{48\pi^2}, \quad (4.7)$$
where \( q_X^A \) is the \( U(1)_X \) charge of the scalar field \( \phi^A \): \( T_X \phi^A = q_X^A \phi^A \) and \( \ell \) is the dilaton field introduced in (3.5) except that the duality relation in the classical limit now reads

\[
\ell^{-1} = 2\text{Re} s - b \sum_{I} \ln(2\text{Re} t^I) + c_X \delta_X/2,
\]

with \( c_X \) the scalar component of the vector superfield \( V_X \). The derivatives \( K_A = \partial K/\partial \phi^A \) of the Kähler potential \( K = k(\ell) + G(t + \bar{t}, |\phi^A|^2) \) are functions of the real moduli, and the vacuum conditions \( D_a = 0 \) determine the vev’s of \( \phi^A \) as functions of the dilaton and moduli: \( \langle \phi^A \rangle = \phi^A(\ell, t + \bar{t}) \). The vacuum values \( \langle \ell \rangle \) and \( \langle t \rangle \) remain undetermined, and SUSY and T-duality remain unbroken at the scale \( \Lambda_D \) where the \( U(1)_a \) gauge symmetries are broken. The effective theory obtained by integrating out the massive vector bosons should reflect these features. By promoting the conditions \( D_a = 0 \), with \( D_a \) given in (4.7), to superfield equations, it has been shown\([5]\) how to construct an effective theory below the \( U(1)_X \) breaking scale that has manifest local SUSY and T-duality, and preserves the correct linearity condition for the linear multiplet \( L \). This effective theory has several new features: 1) A modified Kähler potential for the dilaton, which can affect dilaton/axion cosmology, the gaugino/gravitino mass ratio, and the scales of SUSY breaking and of coupling constant unification. 2) Modified couplings of moduli to the GS term and to hidden sector matter that govern the moduli masses. 3) A modified effective Kähler metric for matter, which together with possible \( U(1)_a \) charges, can affect soft terms in the scalar potential below the scale of SUSY breaking. 4) Massless chiral multiplets (“D-moduli”) associated with the large vacuum degeneracy at the scale \( \Lambda_D \) of the D-term induced breaking of the \( U(1)_a \)’s that are potentially dangerous for a viable modular cosmology\([32]\).

4.2 The effective theory below the condensation scale

The effective theory below the scale of condensation in a strongly coupled hidden sector with gauge group \( G_c \) was studied\([6]\) for a class of models models in which either a minimal set \( n = m \) of scalar fields acquire vev’s \( \langle \phi^A \rangle \sim \sqrt{\delta_X} \) that break the \( m \) \( U(1)_a \)’s, or there are \( N \) replicas of minimal sets \( \phi^A \) with identical charges that acquire vev’s. If in addition we assume a minimal Kähler potential for matter:

\[
K(\Phi, \bar{\Phi}) = \sum_A x^A + \sum_M x^M, \quad x^{A,M} = e^{G^{A,M} + 2\sum_a q^a_M V_a |\Phi^{A,M}|^2},
\]

where by definition \( \langle \Phi^M \rangle = 0 \), and the functions \( G^{A,M}(t + \bar{t}) \) assure T-duality of the Kähler potential, the masses of the complex scalars \( \phi^M \), that include observable sector particles, are given.
by

\[ m_M^2 = \frac{m_3^2}{1 + 2z} \left[ \frac{1 - \zeta_M (1 + z)^2}{z} \right]^2 + 2z - \zeta_M (3 + z) \]

\[ \zeta_M = \sum_{a,A} q_M^a Q_A^a, \quad \sum_a Q_A^a q_M^a = \delta_A^A, \quad z = b_c \ell. \]  
(4.10)

Note that the D-term contribution to these masses is \( (m_M^2)_D = -\zeta_M z^{-2} m_3^2 [1 + O(z)] \). The leading order \( \sim z^{-2} \) terms linear in \( \zeta_M \), \( i.e. \) linear in the \( U(1)_a \) charges \( q_M^a \) are canceled by other terms in the scalar potential. As a result the squared masses are positive, \( m_M^2 > 0 \), over most of relevant parameter space. If \( z \ll 1 \) and \( \zeta_M \sim 1, \) \( m_M^2 \gg m_3^2 \), so the D-terms dominate over the contribution one gets in their absence \( (\zeta_M^M = 0) \). Since the gaugino masses are unchanged with respect to the model of Section 3, this results in an increased ratio \( m_0/m_1^2 \) that may be too large for a viable phenomenology. There are several possible cures for this. 1) If we take instead of (4.9)

\[ K(\Phi, \bar{\Phi}) = -\sum_{\alpha} C_{\alpha} \ln \left[ 1 - C_{\alpha}^{-1} \left( \sum_A x_A^A + \sum_M x_M^M \right) \right], \]  
(4.11)

There is little change in scalar masses \( m_M^M \), but the effective Kähler metric for \( \ell \) is modified in a way that can increase \( m_M^M \); for example, by up to a factor four in the FIQS model if \( C_{\alpha} = 1 \). If we relax the condition \( z \ll 1 \) we can significantly reduce the scalar masses. For example in the FIQS models, the smallest possible squark, slepton and Higgs masses are in the range \( 1.5 m_3^2 \leq m_0 \leq 18 m_3^2 \) if \( \ell = 1 \); this reduces to \( .3 m_3^2 \leq m_0 \leq 5.5 m_3^2 \) if \( \ell = 5 \). Since we need \( b_c \ll 1 \) to generate a gauge hierarchy, this would suggest strongish coupling, in other words a point in the Hořava-Witten scenario that is not quite the WCHS limit. However, in the presence of string nonperturbative effects (SNPE) to stabilize the potential at strong coupling, that is, to prevent \( V(\ell \to \infty) \to -\infty \).

For example, the parameterization

\[ f(\ell) = 2 \ell s(\ell) - 1 = \sum_n a_n x^n e^{-x}, \quad x = \beta / \sqrt{\ell}, \]  
(4.12)

was used in. A solution was found with vanishing cosmological constant, weak coupling \( g^2 \approx .5 \) and \( f \sim 1 \) at the vacuum. In the presence of D-terms we always have \( V(\ell \to \infty) \to +\infty \), but
SNPE are still needed to stabilize the dilaton at weak coupling and zero cosmological constant:

\[ \langle V \rangle \propto \ell k' (\ell) - 3 rz/(1 + z)^2 = 0, \quad (4.13) \]

where the parameter \( r \) depends on the choice of the Kähler potential for \( \phi^A \); \( r = 1 \) for the minimal choice \( (4.9) \) and can be larger for a nonminimal choice as in \( (4.11) \), e.g. \( r = 1 - 4 \) in the FIQS model with \( C_\alpha = 1 \). However there is now more freedom in choice of parameterization. For example, if we take the dilaton Kähler potential

\[ k = -\ln(2s) + \delta k - \ln[1 + h(s)], \quad s = s(\ell), \quad (4.14) \]

where the first term gives the classical relation, \( \delta k \) is the contribution from \( \langle x^A \rangle \sim \delta X \ell \), and \( h \) is the SNPE contribution. If \( 0 < 1 + h = \epsilon \ll 1 \), then \( \ell k' \sim \ell^{-1} \sim \epsilon^{-1} \). Choices of \( h \) similar to the parameterizations \( (4.12) \) of \( f \) used in \( [4] \) can give \( \ell \approx 5, \quad \ell k' \approx .25 \) with \( g^2 = s^{-1} = .5 \).

The D-moduli couplings to matter condensates lift some of the vacuum degeneracy at the \( U(1)_a \)-breaking scale to give masses to all of the real scalar D-moduli. While these are much larger than the gravitino mass if \( z \ll 1 \), pushing all of them up to cosmologically safe levels tends to conflict with the need to reduce the scalar/gaugino mass ratio in the observable sector in generic models. In addition one expects massless D-axions and/or massless D-fermions. For example in the FIQS model with three minimal, identically charged sets of six fields \( \phi^A \) acquiring vev’s to break six \( U(1)_a \)'s, one linear combination of these comprises the eaten Goldstone bosons, while the other two sets of chiral superfields acquire F-term masses such that the axions remain massless. In that model there are at least 12 additional states associated with flat directions for which the complex scalars acquire masses and the fermions do not. This particular model is not viable in any case, since it cannot reproduce the observed SM Yukawa textures, \([33]\) and in the present context it gives implausibly large values for \( m_3^2, \Lambda_c \).

Although the D-term modifies the dilaton metric \( k'/2\ell \), it is still suppressed by the vacuum condition \( (4.13) \) if \( z \ll 1 \), giving an enhanced dilaton mass \( m_\sigma \) and a suppressed axion coupling \( f_a \). Because the effective theory above the SUSY-breaking scale is modular invariant, one again obtains moduli stabilized at self-dual points giving FCNC-free dilaton dominated SUSY-breaking. An enhancement of the ratio \( m_\ell / m_{\frac{3}{2}} \) can result from couplings to condensates of \( U(1)_a \)-charged D-moduli, that also carry T-modular weights. For example in the FIQS model one gets \( m_\ell \approx 10m_{\frac{3}{2}}, \quad m_a \approx (2-4)m_{\frac{3}{2}} \).
5 Conclusions

The message of this talk is three-fold: 1) Quantitative studies with predictions for observable phenomena are possible within the context of the WCHS. 2) Experiments can place restrictions on the underlying theory, such as the parameter space of the strongly coupled hidden gauge sector, as shown in Figure 3, as well as the superpotential couplings, modular weights and $U(1)_a$ charges of D-moduli when an anomalous $U(1)$ is present. Experiments can also inform us about Plank scale physics, such as matter couplings to the GS term. The one-loop corrections to the soft scalar potential are also sensitive to the details of Plank scale physics. 3) Searches for sparticles should avoid restrictive assumptions, since explicit string-derived models have particle spectra that do not necessarily conform to conventional scenarios.

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$D = 11$

$D = 10$

$D = 9$

$T \leftrightarrow 1/T$

$S \leftrightarrow 1/S$

Figure 1: M-theory according to John Schwarz.

$11$-D SUGRA

IIA, IIB: D-Branes

$O(32)_{\text{II}}$

$O(32)_{\text{I}}$

$O(32)_{\text{H}}$

$E_8 \otimes E_8$ WCHS

HW theory: (very?) large extra dimension(s)

Figure 2: M-theory according to Mike Green.
Figure 3: Viable hidden sector gauge groups for scenario A of the condensation model. The swath bounded by lines (a) and (b) is defined by $.1 < m_\tilde{\chi}/\text{TeV}, \lambda_c < 10$, with $\lambda_c$ a condensate superpotential coupling constant. The fine points correspond to $.1 \leq \Omega_d h^2 \leq .3$, and the course points to $.3 < \Omega_d h^2 \leq 1$. $b_c^\alpha$ is the hidden matter contribution to $b_c$. 