Noise immunity of signal demodulation procedure in binary coded communication channels with a frequency-position multiplexing

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Abstract. The article presents the algorithms for demodulating binary channels of data transmission lines with frequency multiplexing used in remote control and warning systems. The noise immunity of the algorithms were investigated. It is shown that significant simplification and significant noise immunity increasing can be obtained by applying the majority decision rule.

1. Introduction
This paper discusses one of the ways to build a noise-immunity communication lines. These lines are designed to transmit information about the status of binary sensors of any devices. It is necessary, for example, in fire and burglar alarm systems, discrete control systems, and in selective warning systems. Such systems may also include warning systems using the transmission of infralow frequency signals through rock massives and preserving their operability in emergency situations. Basically, these systems use signals with discrete frequency modulation and amplitude modulation with a single sideband [1–7]. A comparative effectiveness estimate of using these signal generation methods in underground communication systems is given in [4], which shows the advantages of using discrete frequency modulation (FM) in underground warning systems. Discrete FM signals using at infralow frequencies and its coding for transmission through rock massives allows solving the problem of preserving warning system good condition after an accident. However, if a significant level of interference in the propagation medium is present, among which are both additive and multiplicative interference, it is necessary to apply signal processing techniques to improve the message transmission accuracy. In addition, narrow bandwidth of the channel force us to use two a bit differ one from another operating frequencies and the data transfer rate when generating an alert signal is rather small. This paper presents an investigation dedicated to research a noise-resistant warning system that uses multi-position discrete frequency shift keying method for transmitting information. Algorithms for demodulating signals have been synthesized with minimum of error probability in each binary channel and the algorithm was obtained that ensures the minimum error probability of the summary binary message of all binary channels.
2. Results and discussions

Total number of frequency positions required $M$ for a data transmission system with the number of channels is defined as $M = 2^N$.

The signal at the transmission of position number $r$ has the form

$$Z'(t) = U_c \cos \omega_r + U_s \sin \omega_r + n(t),$$

$$0 \leq t \leq T, \quad r = 1,2, ... M.$$  

Here $U_c = \cos \varphi; U_s = \sin \varphi$ - quadrature components of the amplitude position at the input of the demodulator. $U$ and $\varphi$ are, respectively, the amplitude of the initial phase of this position; $n(t)$ is an additive interference approximated by white noise with a zero mean value and a finite variance;

$$\omega_r = \omega_1 + \frac{2\pi d(r-1)}{T}, \quad d = 1,2, ...$$  

$\omega_r$ - frequency of position with number $r$; $\omega_1$ - is the minimum frequency signal adopted for the first position ($r = 1$); $d$ - integer; $T$ - the duration of the symbol. The choice of frequency positions according to the relation (1) ensures their orthogonality.

Unlike the multichannel lines that uses the method of phase-positional or amplitude-phase-positional interleaving, for frequency-positional interleaving the error probability is the same for all frequency channels [8]. There are two approaches for the optimal decision schemes synthesis. The first approach is to minimize the error probability in each binary channel. The second approach involves minimizing the error probability in the summary of all message transmitted over all channels [9-13].

First we will consider a decision-making algorithm that minimizes the error probability in each binary channel. Assuming a priori known intensity of white additive noise, it can be represented as

$$\eta_{m_1} = \sum_{r \in < r >_{m_1}} \exp((2D)^{-1}V_r^2) - \sum_{r \in < r >_{m_0}} \exp((2D)^{-1}V_r^2) \rightarrow 0$$  

(2)

Here $r$ and $m$ are respectively the signal frequency position number and the binary channel number; $r = 1,2, ..., M; m = 1,2, ..., N; D = V^2/\mu^2$ - subsets of positions that carry information about 1 and 0 in the binary channel with the number $m$, the arrow indicates the decision to accept one or zero with a corresponding inequality sign, $\mu$ - one-sided noise power spectral density; $V^2_r = \sqrt{X_r^2 + Y_r^2}$ - sample of voltage at the end of the reception of the next element in the $r$-th chain of optimal incoherent processing, where

$$X_r = \frac{2}{T} \int_0^T Z(T) \cos \omega_r \tau d\tau; \quad Y_r = \frac{2}{T} \int_0^T \sin \omega_r \tau d\tau.$$  

The implementation the algorithm (2) requires to know the magnitude of $D$. Overcoming a priori uncertainty is possible by using an asymptotic rule, which may be obtained from (2) by series expanding of the exponential functions and then limit the number of terms. In this case

$$\exp((1/2D)V_r^2) \equiv 1 + (1/2D)V_r^2.$$  

(3)

Using (3), we obtain the second demodulation algorithm, which is asymptotically optimal for low signal-to-noise ratio.

$$\eta_{m_1} = \sum_{r \in < r >_{m_1}} V_r^2 - \sum_{r \in < r >_{m_0}} V_r^2 \rightarrow 10.$$  

(4)
The third algorithm is obtained from (2) by replacing the unknown value $D$ with its maximum likelihood estimate $\hat{D}$, which can be obtained from the prehistory or at the next element reception interval. For the calculation $\hat{D}$, it is possible to use the fact that when processing orthogonal FM signals, the signal position is formed only at the output of one receiving path, while in the overs $(M - 1)$th paths at these moments the signal value are zero and the sample values are determined by only noise intensity. These $(M - 1)$th samples can be used to estimate the spectral density estimate of the noise power. The estimate can be found as

$$
\hat{D} = \frac{1}{2(M-1)} \sum_{r=1}^{M} V_r^2.
$$

(5)

When finding the estimate (5), it is assumed that information contained in signal position and transmitted in sample may be identified with the known maximum likelihood algorithm [8, 9]

$$
V_q > V_r, r \neq q, r, q = 1,2, \ldots, M,
$$

Then

$$
\eta_{m1} = \sum_{r < r_{m1}} \exp\left[(2\hat{D})^{-1}V_r^2\right] - \sum_{r \in < r_{m1}} \exp\left[(2\hat{D})^{-1}V_r^2\right]_{r \rightarrow 0} 0.
$$

(6)

The squaring operation in (6) can be omitted. It is saved here due to the fact that it is necessary in the formation of (5) and (6).

$$
V_q > V_r, r \neq q, r, q = 1,2, \ldots, M
$$

(7)

Then, using the code converter, values "1" or "0" in all $N$ binary channels are formed. The forming rule in $m$th binary channel is:

"1" if $r \in < r_{m1}$

"0" if $r \in < r_{m0}$

(8)

Then, the authors estimate the noise immunity of the reception according to the above algorithms in the absence and in the presence of Rayleigh fading. For a four-channel data transmission system with number of channel $N = 4$ and number of positions $M = 16$. Let assume that the duration of the signal element is $T = 0,1$ c. Then the minimum frequency division between adjacent frequency positions is 10 Hz, and the frequency band occupied by the signal will be slightly more than 1600 Hz. Let estimate the ratio of the average signal energy to the spectral density of the average noise power as

$$
H^2 = \frac{P_c T}{\nu^2},
$$

and calculate the error probabilities by the rules (2), (4), (6), (7), which are denoted respectively by $P_1, P_2, P_3, P_4, P_5$. In mathematical modeling of algorithms, the results of [14, 15] were used. The calculation results are summarized in table 1 and 2.

The error probability when using the algorithm with minimum error in the summary message is not significantly different from the one in the optimum reception. Thus because this demodulation algorithm is more simple compared to the one presented in (6), it can be recommended for implementation.

The considered receiving methods with FM multi-position signals need hard clock synchronization, which significantly complicates the demodulator's circuit. These difficulties may be reduced by using
the feature of the considered data transfer system. The things is that it does not require binary data continuous transmission for each binary channel, but only detection is necessary, and for quite a long time (of a second). Thus it is possible the asynchronous operation of the demodulator with a local clock generator which frequency is equal

$$F_{\text{clock}} = \frac{\omega_{r+1} - \omega_r}{2\pi} = \frac{d}{T} = \Delta F_{\text{min}}.$$ 

**Table 1.** Channel without Rayleigh fading

| $P$ | 1   | 2   | 3   | 3.5 |
|-----|-----|-----|-----|-----|
| $P_1$ | 0.40 | 0.22 | 0.024 | 0.007 |
| $P_2$ | 0.40 | 0.19 | 0.025 | 0.007 |
| $P_3$ | 0.38 | 0.27 | 0.051 | 0.023 |
| $P_4$ | 0.41 | 0.18 | 0.025 | 0.061 |
| $P_5$ | 0.37 | 0.086 | 0.002 | 0.0001 |

**Table 2.** Channel with Rayleigh fading

| $P$ | 1   | 2   | 3   | 3.5 |
|-----|-----|-----|-----|-----|
| $P_1$ | 0.30 | 0.060 | 0.017 | 0.06 |
| $P_2$ | 0.30 | 0.060 | 0.017 | 0.06 |
| $P_3$ | 0.033 | 0.093 | 0.020 | 0.09 |
| $P_4$ | 0.41 | 0.063 | 0.017 | 0.010 |
| $P_5$ | 0.37 | 0.010 | 0.0009 | 0.0003 |

Here $d$ is an integer. The clock frequency matches the minimum frequency spacing of adjacent positions $\Delta F_{\text{min}}$. In terms of bandwidth savings, it is recommended $d = 1$ when

$$F_{\text{clock}} = \frac{1}{T} = \Delta F_{\text{min}} = B, \text{ Hz}.$$ 

Here $B$ is manipulation speed. The clock generator phase may be arbitrary regarding changes in the information parameter in any binary channel. Due to the fact that the time interval after state change is much longer $T$, one can apply the majority principle of detecting the information parameter when the $L$ parameters of elements are taken in series and decision is made by the majority of “votes”. This means that when transmit, for example, "0" or "1" in any channel, a decision about presence is made if in $L$ elements the number of decisions equal "1" is greater than $\text{int}(L/2)$, where $\text{int}(X)$ denotes the integer part of the number $X$. This method of reception improves the noise immunity compared to usual elementwise reception. Indeed, if the error probability in element-by-element reception is equal to $P_4$, then the error probability in the majority method with independent distortions in the elements can be obtained in the form

$$P_5 = 1 - \sum_{l=0}^{\text{int}(L/2)} C_L^l P_4^l (1 - P_4)^{L-1}.$$ 

The calculation results for $P_5$ with this ratio for the minimum number $L-1$ are presented in Table 1. With $P_4$ of the order $(1 \ldots 5)10^{-2}$, the gain in error probability with the majority solution for $L = 3$ will be more than 10 times. In our case, when $T = 0.1\text{ s}$, the detection time of the state changing will be no more than $0.3\text{ s}$.
3. Conclusions
The noise immunity of binary demodulators of communication lines with frequency-positional division has been investigated by the method of mathematical modeling. It is shown that a significant simplification of demodulators with a significant increase in noise immunity can be obtained by applying the majority principle of decision making. In the presence of fluctuation and impulse noise, a scheme operating according to rule (7), (8) and using a limiter [7] may be recommended for use.

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