$\eta \to \pi^0\gamma\gamma$ decay in the three-flavor
Nambu–Jona-Lasinio model

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Abstract

We study the $\eta \to \pi^0\gamma\gamma$ decay via the quark-box diagram in the three-flavor Nambu–Jona-Lasinio model that includes the $U_A(1)$ breaking effect. We find that the $\eta$-meson mass, the $\eta \to \gamma\gamma$ decay width and the $\eta \to \pi^0\gamma\gamma$ decay width are in good agreement with the experimental values when the $U_A(1)$ breaking is strong and the flavor $SU(3)$ singlet-octet mixing angle $\theta$ is about zero. The photon energy and the photon invariant mass spectra in $\eta \to \pi^0\gamma\gamma$ are compared with those in the chiral perturbation theory.

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Chiral symmetry plays an essential role in the light-flavor QCD. Light pseudoscalar mesons are regarded as the Nambu-Goldstone (NG) bosons of spontaneous symmetry breaking of chiral $SU(N)_L \times SU(N)_R$ symmetry. Properties of the NG bosons have been studied in various chiral effective theories successfully.

Another interesting low-energy symmetry is the axial $U(1)$ symmetry, which is explicitly broken by the anomaly. The symmetry breaking is manifested in the heavy mass of $\eta'$ meson, which has been studied as the “$U_A(1)$ problem”. It is yet not clear how strong the anomaly effect is on the pseudoscalar spectrum mainly because of complicated interference of the explicit flavor symmetry breaking due to the strange quark mass. Two of the authors (M.T. and M.O.) argued in a previous paper\cite{1} that a strong $U_A(1)$ breaking and consequently a large flavor mixing are favorable for the $\eta$ meson. The argument is based on the analysis of the $\eta$ system as a $q\bar{q}$ bound state in the three-flavor Nambu–Jona-Lasinio (NJL) model. We claim that the $\eta$ mass as well as the $\eta \rightarrow \gamma\gamma$ decay rate supports a $U_A(1)$ breaking six-quark interaction much stronger than previously used\cite{2}. This interaction causes a large flavor mixing resulting almost pure octet $\eta \simeq \eta_8$.

The purpose of this paper is to extend the previous analysis to another $\eta$ decay process, $\eta \rightarrow \pi^0\gamma\gamma$ and to confirm our claim of the strong $U_A(1)$ breaking. We are interested in this process where the internal structure of the $\eta, \pi^0$ mesons plays essential roles because the photon does not couple to the neutral mesons directly. Furthermore, chiral perturbation theory (ChPT) gives too small prediction in the leading order and higher order terms are expected to be dominant.

The three-flavor NJL model is one of the phenomenologically successful chiral effective models of the low-energy QCD and the lagrangian density we have used is as follows,

$$
\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_4 + \mathcal{L}_6, 
$$

(1)
\[ L_0 = \overline{\psi}(i\partial_\mu \gamma^\mu - \hat{m})\psi, \]  
\[ L_4 = \frac{G_S}{2} \sum_{a=0}^8 \left[ (\overline{\psi}\lambda^a\psi)^2 + (\overline{\psi}\lambda^a i\gamma_5 \psi)^2 \right], \]  
\[ L_6 = G_D \left\{ \det_{(i,j)} \left[ \overline{\psi}_i (1 + \gamma_5) \psi_j \right] + \det_{(i,j)} \left[ \overline{\psi}_i (1 - \gamma_5) \psi_j \right] \right\}, \]

where \( \psi \) is the quark field, \( \hat{m} = \text{diag}(m_u, m_d, m_s) \) is the current quark mass matrix, and \( \lambda^a \) is the flavor \( U(3) \) generator \( (\lambda^0 = \sqrt{2/3}I) \). The determinant in \( L_6 \) is a \( 3 \times 3 \) determinant with respect to the flavor indices \( i, j = u, d, s \). The model involves the \( U_L(3) \times U_R(3) \) symmetric four-quark interaction \( L_4 \) and the six-quark flavor-determinant interaction \( L_6 \) incorporating effects of the \( U_A(1) \) anomaly.

Quark condensates and constituent quark masses are self-consistently determined by the gap equations in the mean field approximation. The covariant cutoff \( \Lambda \) is introduced to regularize the divergent integrals. The pseudoscalar channel quark-antiquark scattering amplitudes are then calculated in the ladder approximation. From the pole positions of the scattering amplitudes, the pseudoscalar meson masses are determined. We define the effective meson-quark coupling constants \( g_{\eta qq} \) and \( g_{\pi qq} \) by introducing additional vertex lagrangians,

\[ L_{\eta qq} = g_{\eta qq} \overline{\psi}_i \gamma_5 \lambda^\eta \psi \phi_\eta, \]  
\[ L_{\pi qq} = g_{\pi qq} \overline{\psi}_i \gamma_5 \lambda^3 \psi \phi_\pi^0, \]

with \( \lambda^\eta = \cos \theta \lambda^8 - \sin \theta \lambda^0 \). Here \( \phi \) is an auxiliary meson field introduced for convenience and the effective meson-quark coupling constants are calculated from the residues of the \( q\bar{q} \)-scattering amplitudes at the corresponding meson poles. Because of the \( SU(3) \) symmetry breaking, the flavor \( \lambda^8 - \lambda^0 \) components mix with each other. Thus we solve the coupled-channel \( q\bar{q} \) scattering problem for the \( \eta \) meson. The mixing angle \( \theta \) is obtained by diagonalization of the \( q\bar{q} \)-scattering
amplitude at the $\eta$-meson pole. The meson decay constant $f_M$ ($M = \pi, K, \eta$) is determined by calculating the quark-antiquark one-loop graph. The explicit expressions are found in [1].

The parameters of the model are the current quark masses $m_u = m_d, m_s$, the four-quark coupling constant $G_S$, the six-quark determinant coupling constant $G_D$ and the covariant cutoff $\Lambda$. We take $G_D$ as a free parameter and study $\eta$ meson properties as functions of $G_D$. We use the light current quark masses $m_u = m_d = 8.0$ MeV (same as in [1]). Other parameters, $m_s, G_D$, and $\Lambda$, are determined so as to reproduce the isospin averaged observed masses, $m_\pi, m_K$, and $f_\pi$.

We obtain $m_s = 193$ MeV, $\Lambda = 783$ MeV, the constituent $u, d$-quark mass $M_{u,d} = 325$ MeV and $g_{\pi qq} = 3.44$, which are almost independent of $G_D$. 

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Table 1: The parameters and $\eta \rightarrow \pi^0\gamma\gamma$ decay widths for each $G_D^{\text{eff}}$

| $G_D^{\text{eff}}$ | $G_S^{\text{eff}}$ | $M_\eta$ [MeV] | $\theta(M_\eta^2)$ [deg] | $g_{\pi qq}$ | $\Gamma$ [eV] |
|-------------------|-------------------|-----------------|--------------------------|-------------|---------|
| 0.00              | 0.73              | 138.1           | -54.74                   | 3.44        | 2.88    |
| 0.10              | 0.70              | 285.3           | -44.61                   | 3.23        | 2.46    |
| 0.20              | 0.66              | 366.1           | -33.52                   | 3.12        | 2.06    |
| 0.30              | 0.63              | 419.1           | -23.24                   | 3.11        | 1.71    |
| 0.40              | 0.60              | 455.0           | -14.98                   | 3.15        | 1.42    |
| 0.50              | 0.57              | 479.7           | -8.86                    | 3.20        | 1.20    |
| 0.60              | 0.54              | 497.3           | -4.44                    | 3.25        | 1.04    |
| 0.70              | 0.51              | 510.0           | -1.25                    | 3.28        | 0.92    |
| 0.80              | 0.47              | 519.6           | 1.09                     | 3.30        | 0.84    |
| 0.90              | 0.44              | 527.0           | 2.84                     | 3.31        | 0.77    |
| 1.00              | 0.41              | 532.8           | 4.17                     | 3.32        | 0.71    |
| 1.10              | 0.40              | 537.5           | 5.21                     | 3.32        | 0.67    |
| 1.20              | 0.35              | 541.3           | 6.02                     | 3.31        | 0.63    |
| 1.30              | 0.32              | 544.5           | 6.66                     | 3.30        | 0.61    |
| 1.40              | 0.29              | 547.2           | 7.17                     | 3.29        | 0.58    |
| 1.50              | 0.25              | 549.4           | 7.57                     | 3.28        | 0.56    |
| 1.60              | 0.22              | 551.4           | 7.90                     | 3.26        | 0.55    |
Table 1 summarizes the fitted results of the model parameters and the quantities necessary for calculating the $\eta \rightarrow \pi^0 \gamma \gamma$ decay width which depend on $G_D$. We define dimensionless parameters $G_D^{\text{eff}} \equiv -G_D(\Lambda/2\pi)^4 A N_c^2$ and $G_S^{\text{eff}} \equiv G_S(\Lambda/2\pi)^2 N_c$. When $G_D^{\text{eff}}$ is zero, our lagrangian does not cause the flavor mixing and therefore the ideal mixing is achieved. The "$\eta$" is purely $u\bar{u} + d\bar{d}$ and is degenerate to the pion in this limit.

It is found in [1] that the $\eta \rightarrow \gamma \gamma$ decay width is reproduced at about $G_D^{\text{eff}} = 0.7$. At this value the ratio $G_D(\bar{s}s)/G_S = 0.44$ indicates that the contribution from $\mathcal{L}_6$ to the dynamical mass of the up and down quarks is 44% of that from $\mathcal{L}_4$. The mixing angle at $G_D^{\text{eff}} = 0.7$ is $\theta = -1.3^\circ$ and that indicates a strong OZI violation and a large (u,d)-s mixing. This disagrees with the "standard" value $\theta \simeq -20^\circ$ obtained in ChPT [3]. This is due to the stronger $U_A(1)$ breaking in the present calculation. The difference mainly comes from the fact that the mixing angle in the NJL model depends on $q^2$ of the $\overline{q}q$ state and thus reflects the internal structure of the $\eta$ meson. On the contrary the analyses of ChPT [3] assume an energy-independent mixing angle, i.e., $\theta(M_{\eta}^2) = \theta(M_{\eta}^2)$.

We are now in a position to study whether the $\eta \rightarrow \pi^0 \gamma \gamma$ decay rate is consistent with our picture of $\eta$ with a large OZI mixing.

The experimental value of the $\eta \rightarrow \pi^0 \gamma \gamma$ decay width is [4]

$$\Gamma_{\text{exp}}(\eta \rightarrow \pi^0 \gamma \gamma) = 0.85 \pm 0.19 \text{ eV}. \quad (7)$$

We evaluate the quark-box diagram given in Fig 1. We follow the evaluation of the box-diagram performed in [3]. Other possible contributions will be discussed later.

The $\eta \rightarrow \pi^0 \gamma \gamma$ decay amplitude is given by

$$\langle \pi^0(p_{\pi})\gamma(k_1,\epsilon_1)\gamma(k_2,\epsilon_2)|\eta(p)\rangle = i(2\pi)^4\delta^4(p_{\pi} + k_1 + k_2 - p)\epsilon_{\mu}^{\pi} \epsilon_{\nu}^{\gamma} T_{\mu\nu}. \quad (8)$$
Figure 1: The quark-box diagram for $\eta \to \pi^0\gamma\gamma$

where $\epsilon_1$ and $\epsilon_2$ are the polarization vectors of the photons. $T_{\mu\nu}$ is given by a straightforward evaluation of the Feynman diagrams. After calculating traces in color and flavor spaces, we obtain

$$T_{\mu\nu} = -i \frac{1}{\sqrt{3}} (\cos \theta - \sqrt{2} \sin \theta) e^2 g_{\eta qq} g_{\pi qq} \int \frac{d^4q}{(2\pi)^4} \sum_{i=1}^{6} U^i_{\mu\nu},$$

(9)

with

$$U^1_{\mu\nu} = \text{Tr}^{(D)} \left\{ \gamma^5 \frac{1}{q - M + i\epsilon} \gamma^5 \frac{1}{q + \not{p} - \not{k}_1 - \not{k}_2 - M + i\epsilon} \right\},$$

(10)

$$U^2_{\mu\nu} = \text{Tr}^{(D)} \left\{ \gamma^5 \frac{1}{q - M + i\epsilon} \gamma^5 \frac{1}{q + \not{k}_2 - M + i\epsilon} \right\},$$

(11)

$$U^3_{\mu\nu} = \text{Tr}^{(D)} \left\{ \gamma^5 \frac{1}{q - M + i\epsilon} \gamma^5 \frac{1}{q + \not{k}_1 + \not{k}_2 - M + i\epsilon} \right\},$$

(12)
\[ U^4_{\mu\nu} = U^1_{\nu\mu}(k_1 \leftrightarrow k_2), \]  
\[ U^5_{\mu\nu} = U^2_{\nu\mu}(k_1 \leftrightarrow k_2), \]  
\[ U^6_{\mu\nu} = U^3_{\nu\mu}(k_1 \leftrightarrow k_2). \]  

Here $\text{Tr}^{(D)}$ means trace in the Dirac indices and $M$ is the constituent u,d-quark mass. Because the loop integration in (9) is not divergent, we do not introduce the UV cutoff. Then the gauge invariance is preserved. The inclusion of the cutoff that is consistent with the gap equation will break the gauge invariance and make the present calculation too complicated. Note that the strange quark does not contribute to the loop.

On the other hand the amplitude $T_{\mu\nu}$ has a general form required by the gauge invariance [6]

\[ T^{\mu\nu} = A(x_1, x_2)(k^\mu_1 k^\nu_2 - k_1 \cdot k_2 g^{\mu\nu}) + B(x_1, x_2) \left[ -M^2_\eta x_1 x_2 g^{\mu\nu} - \frac{k_1 \cdot k_2}{M^2_\eta} p^\mu p^\nu + x_1 k^\mu_2 p^\nu + x_2 p^\mu k^\nu_1 \right], \]  

with
\[ x_i = \frac{p \cdot k_i}{M^2_\eta}, \]  
and $M_\eta$ is the $\eta$ meson mass. With $A$ and $B$, the differential decay rate with respect to the energies of the two photons is given by

\[ \frac{d^2\Gamma}{dx_1 dx_2} = \frac{M^5_\eta}{256\pi^2} \left\{ \left[ A + \frac{1}{2} B \right]^2 \left[ 2(x_1 + x_2) + \frac{M^2_\pi}{M^2_\eta} - 1 \right] \right\}^2 + \frac{1}{4} |B|^2 \left[ 4x_1 x_2 - \left[ 2(x_1 + x_2) + \frac{M^2_\pi}{M^2_\eta} - 1 \right] \right]^2, \]  

where $M_\pi$ is the $\pi^0$ meson mass. Though the mass of $\eta$ as a $\bar{q}q$ bound state depends on $G^{\text{eff}}_{B\bar{B}}$, we use the experimental value $M_\eta = 547$ MeV in evaluating (18). The Dalitz boundary is given by two conditions:

\[ \frac{1}{2} \left( 1 - \frac{M^2_\pi}{M^2_\eta} \right) \leq x_1 + x_2 \leq 1 - \frac{M_\pi}{M_\eta}. \]
and
\[ x_1 + x_2 - 2x_1x_2 \leq \frac{1}{2} \left( 1 - \frac{M^2_x}{M^2_\eta} \right). \] (20)

In evaluating (10)-(15), one only has to identify the coefficients of \( p^\mu p^\nu \) and \( g^{\mu\nu} \).

Details of the calculation are given in [5]. Defining \( A \) and \( B \) by
\[
\int \frac{d^4q}{(2\pi)^4} \sum_{i=1}^{6} U_i^{\mu\nu} = -i \left( A g^{\mu\nu} + B \frac{p^\mu p^\nu}{M^2_\eta} + \cdots \right),
\] (21)
we find \( A \) and \( B \) as
\[
A = \frac{1}{\sqrt{3}} (\cos \theta - \sqrt{2} \sin \theta) e^2 g^{\eta\eta\eta\eta} \frac{2}{M^2_\eta} \left[ A - 2x_1x_2 \frac{B}{\sigma} \right],
\] (22)
\[
B = \frac{1}{\sqrt{3}} (\cos \theta - \sqrt{2} \sin \theta) e^2 g^{\eta\eta\eta\eta} \frac{2}{M^2_\eta} \frac{B}{\sigma},
\] (23)
with
\[
\sigma = \frac{(k_1 + k_2)^2}{M^2_\eta} = 2(x_1 + x_2) + \frac{M^2_\pi}{M^2_\eta} - 1. \] (24)

We evaluate \( A \) and \( B \) numerically and further integrate (18) to obtain the \( \eta \rightarrow \pi^0\gamma\gamma \) decay rate. The results are given in the last column of Table 1 and shown in Fig 2.

Our result is \( \Gamma(\eta \rightarrow \pi^0\gamma\gamma) = 0.92 \text{ eV} \) at \( G^\text{eff}_D = 0.70 \) where the \( \eta \rightarrow \gamma\gamma \) decay width is reproduced [4]. At \( G^\text{eff}_D = 1.40 \) which reproduces the experimental \( \eta \) meson mass, \( \Gamma(\eta \rightarrow \pi^0\gamma\gamma) = 0.58 \text{ eV} \). Both are in reasonable agreement with (7).

In ChPT [4], there is no lowest order \( O(p^2) \) contribution to the \( \eta \rightarrow \pi^0\gamma\gamma \) process because the involved mesons are neutral. Likewise the next order \( O(p^4) \) contributions do not exist. Thus the \( O(p^4) \) one-loop diagrams give the leading term in this process, but the contribution is two orders of magnitude smaller than the experimental value (7). This is because the pion loop violates the G-parity invariance and the kaon loop is also suppressed by the large kaon mass.

At \( O(p^6) \), there exists contribution coming from tree diagrams, one-loops and two-loops. The loop contributions are smaller than those from the order \( O(p^4) \).
Figure 2: Dependence of the $\eta \to \pi^0\gamma\gamma$ decay width on the dimension-less coupling constant $G_{D}^{\text{eff}}$. The horizontal solid line indicates the experimental value $\Gamma = 0.85$ eV and the dashed lines indicate its error widths.

At $O(p^8)$, more tree diagrams and a new type of loop corrections appear, but the loop corrections are also small.

In [7], coupling strengths of the tree diagrams are determined assuming saturation by meson resonance poles, such as $\rho, \omega, a_0$ and $a_2$. This gives

$$\Gamma(\eta \to \pi^0\gamma\gamma) = 0.42 \pm 0.20 \text{ eV},$$  

(25)

where the 0.20 eV is the contribution of $a_0$ and $a_2$, the sign of which is not known. The result is a factor two smaller than the experimental value.

The contributions of other mesons, such as $b_1(1235), h_1(1170), h_1(1380)$ [8] and other tree diagrams [9], are found to be small.

Although these results based on ChPT are not too far from the experimental value, it is noted that the higher order $O(p^6)$ terms in the perturbation expansion are larger than the leading $O(p^4)$ terms and the results contain ambiguous
parameters that cannot be determined well from other processes.

On the other hand in [10] the $O(p^6)$ tree diagrams are evaluated by using
the extended NJL (ENJL) model[11]. They calculated three contributions in
ENJL, namely, the vector and scalar resonance exchange and the quark-loop
contributions. Their result is $\Gamma(\eta \to \pi^0\gamma\gamma) \simeq 0.5$ eV. They further introduced
the $O(p^8)$ chiral corrections as well as the axialvector and tensor meson exchange
contributions, and finally obtained $\Gamma(\eta \to \pi^0\gamma\gamma) = 0.58 \pm 0.3$ eV.

The difference between our approach and that in [10] are as follows. The
ENJL model lagrangian has not only the scalar-pseudoscalar four quark interac-
tions but also the vector-axialvector four quark interactions. However, the $U_A(1)$
breaking is not explicitly included in their model and therefore the $\eta - \eta'$ mix-
ing is introduced by hand with the mixing angle $\theta = -20^\circ$. We stress that the
introduction of the $U_A(1)$ breaking interaction is important to understand the
structure of the $\eta$ meson.

There is another difference. The coupling constants of the chiral effective me-
son lagrangian predicted in the ENJL model are parameters of the Green function
evaluated at zero momenta. On the other hand we evaluate the quantities at the
pole position of the mesons.

Calculated spectrum of the photon invariant mass square $m_{\gamma\gamma}^2$ for the $\eta \to
\pi^0\gamma\gamma$ decay is shown in Fig 3. As this spectrum is compared with those calculated
by ChPT in [9], we find ours to be similar to the one for $d_3 = 4.5 \times 10^{-2}$ GeV$^{-2}$
in [9] which involves an additional $O(p^6)$ contribution to the original Lagrangian.

Spectrum of the photon energy $E_\gamma$ for the $\eta \to \pi^0\gamma\gamma$ decay is shown in Fig 4,
and given in [7] in ChPT. Both are also similar, though there is no experimental
result.

In our calculation of the $\eta \to \pi^0\gamma\gamma$ decay, we evaluate only the quark-box
diagram in Fig 1. Since the vector and axialvector four-quark interactions are not included in our model, the only other contribution to this process is the scalar resonance exchange. In the ENJL model the contribution of the scalar resonance exchange is small[10]. We expect that similar result will be obtained in our approach.

If one includes the vector and axialvector four-quark interaction in the NJL model, the pseudoscalar meson properties are affected through the pseudoscalar-axialvector channel mixing and the model parameters with and without the vector and axialvector four-quark interaction are different. We expect that the models with and without the vector-axialvector interaction predict similar results for the processes involving only the pseudoscalar mesons with energies much below the vector meson masses. It is further argued that the contribution of the quark-box

![Graph showing the spectrum of the photon invariant mass $m_{\gamma\gamma}^2$.](image)

Figure 3: Spectrum of the photon invariant mass $m_{\gamma\gamma}^2$. 

\[ \frac{d\Gamma}{dm_{\gamma\gamma}^2} \times 10^{-6} \text{ [MeV}^{-1}] \] 

\[ \times 10^3 \text{ [MeV}^2] \]
Figure 4: Spectrum of the photon energy $E_\gamma$.

diagram to the $\gamma\gamma \to \pi^0\pi^0$ process, that is similar to $\eta \to \pi^0\gamma\gamma$, is quite close to that of the vector meson exchange in the vector dominance model[12].

In summary, we have studied the $\eta \to \pi^0\gamma\gamma$ decay in the three-flavor NJL model that includes the $U_A(1)$ breaking six-quark determinant interaction. The $\eta$ meson mass, the $\eta \to \gamma\gamma$ decay width and the $\eta \to \pi^0\gamma\gamma$ decay width are reproduced well with a rather strong $U_A(1)$ breaking interaction, that makes $\eta_1 - \eta_8$ mixing angle $\theta \simeq 0^\circ$. Since the $\eta'$ meson is expected to be sensitive to the effects of the $U_A(1)$ anomaly, it is very important to study the $\eta'$ meson properties. It should be noted, however, that the NJL model does not confine quarks. While the NG bosons, $\pi, K$ and $\eta$, are strongly bound and therefore can be described in the NJL model fairly well, we do not apply our model to the heavy mesons such as $\rho, \omega$ and $\eta'$. Further study of the $U_A(1)$ breaking and the
$\eta_1 - \eta_8$ mixing will require a calculation including the confinement mechanism.

References

[1] M. Takizawa and M. Oka, Phys. Lett. B 359 (1995) 210; B 364 (1995) 249 (E).

[2] T. Hatsuda and T. Kunihiro, Phys. Rep. 247 (1994) 221.

[3] J.F. Donoghue, B.R. Holstein, and Y.-C.R. Lin, Phys. Rev. Lett 55 (1985) 2766; 61 (1988) 1527 (E).

[4] Particle Data Group, M. Aguilar-Benitez et al, Phys. Rev. D 50 (1994) 1173.

[5] J.N. Ng and D.J. Peters, Phys. Rev. D 47 (1993) 4939.

[6] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. B 303 (1988) 665.

[7] Ll. Ametller, J. Bijnens, A. Bramon and F. Cornet, Phys. Lett. B 276 (1992) 185.

[8] P. Ko, Phys. Rev. D 47 (1993) 3933.

[9] P. Ko, Phys. Lett. B 349 (1995) 555.

[10] S. Bellucci and C. Bruno, Nucl. Phys. B 452 (1995) 626.

[11] J. Bijnens, C. Bruno and E. de Rafael, Nucl. Phys. B 390 (1993) 501.

[12] J. Bijnens, S. Dawson and G. Valencia, Phys. Rev. D 44 (1991) 3555.