HOW TO USE SU(3)-FLAVOR SYMMETRY TO EXTRACT CP VIOLATING PHASES AND STRONG PHASE SHIFTS AT CLEO∗

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ABSTRACT

After a short introduction to the SM picture of CP violation, we discuss recent work on how to use SU(3) flavor symmetry, along with some dynamical approximations, to extract the CKM weak phases and the strong rescattering phases from experimental measurements alone. This surprising wealth of information depends on our two strongest assumptions: SU(3) invariance, and the relative unimportance of exchange and annihilation diagrams. We discuss soon to be measured decay rate measurements that will test the validity of these assumptions.

1. Introduction

1.1. Charge and Parity Conjugation

If asked the reason for the apparent left-right asymmetry in our bodies (location of our heart, liver, the asymmetry in our face, etc.) most of us would probably say that it has to do with random initial conditions, either evolutionary or developmental. Thus despite the fact that in our daily lives things are not completely left-right symmetric, before 1957 physicists took it for granted that the fundamental laws of physics were parity invariant. It came as quite a shock to everyone when in 1957 a left-right asymmetry was discovered in beta, pion, and muon decays. Since these same experiments established an asymmetry between the decays of positive and negative particles, an absolute (versus relative) difference between positive and negative charges was established.

Well then, if we can't have P-invariance or C-invariance how about CP-invariance. CP, along with P and C conjugation is a discrete symmetry of the Poincaré group,

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which was respected by the P and C violating beta, pion and muon decay experiments, and by all the macroscopic laws of physics.\footnote{Of course on the macroscopic level P and C are also good symmetries. So billiards in a mirror is indistinguishable from billiards in real life. And billiards with positively charged balls is in indistinguishable from billiards with negatively charged balls and they are both also equivalent to billiards with oppositely charged balls in the mirror.} But then in 1964 came the second surprise. The CP odd state $K_L$ decayed once every couple thousand times into the CP even state of two pions.

1.2. The Standard Model Unitary Triangle

So far the Kaon system is the only system where CP violation has been observed. In the future we will have a statistically large enough sample of $B$-mesons that their rare decays will also become a crucial testing ground for our ideas about CP violation. The Standard Model (SM) picture of CP violation, is based on phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix\footnote{This approximation is based on the observation that the elements of the CKM matrix obey a hierarchy in powers of the Cabibbo angle, $\lambda \approx 0.22$:}

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\sim
\begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & |V_{ub}| \exp(-i\gamma) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 & A \lambda^2 \\
|V_{td}| \exp(-i\beta) & -A \lambda^2 & 1
\end{pmatrix}.
\]

(1)

Here, $A$ is a parameter of $O(1)$, and $|V_{ub}|$ and $|V_{td}|$ are terms of order $\lambda^3$. In this approximation, the only non-negligible complex phases appear in the terms $V_{ub}$ and $V_{td}$. Unitarity of the CKM matrix implies, among other things, the orthogonality of the first and third columns:

\[
V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 .
\]

(2)

This relation can be represented as a triangle in the complex plane (the unitarity triangle), as shown in Fig. 1. In the Wolfenstein approximation, the angles in the unitarity triangle are given by $\beta = -\text{Arg}(V_{td})$, $\gamma = \text{Arg}(V_{ub}^*)$, and $\alpha = \pi - \beta - \gamma$.

The SM picture of CP violation can thus be tested by independently measuring the three angles $\alpha$, $\beta$ and $\gamma$ and seeing (i) that they are all different from 0 or $\pi$, and (ii) that they add up to $\pi$ radians.

2. Measuring CP Violation in $B$ Decays

2.1. Direct CP Violation

The most straightforward way to detect CP violation in $B$ decays would be to observe a difference between the decay of a $B$ meson to a final state $f$ and the CP-conjugate process:

\[
\Gamma(B \to f) \neq \Gamma(\bar{B} \to \bar{f})
\]

(3)
Let’s take for example the case $B^0 \to \pi^- K^+$. In the quark model representation of the mesons, this process is the sum of a tree and a penguin diagram in Fig. 2 dressed with gluons in all possible places. It is the contribution of these soft gluons which we can not calculate. Still the amplitude for this process is the sum of two complex numbers, one coming from the tree diagram $T$ and the other from the penguin diagram $P$. The phase associated with each diagram has a “weak contribution and a “strong contribution. The weak phases $\phi$ are due to the CKM matrix elements and they change sign in the CP conjugate process, whereas the strong phases $\delta$ are due to hadronization and final state rescattering effects and they do are the same for both the original decay and the CP conjugate process. This is because CP violation does not occur in the strong interactions (as upper bounds on the neutron electric dipole moment show), but only in the weak sector.

Thus we write

$$A(B^0 \to \pi^- K^+) = Te^{i\phi_T} e^{i\delta_T} + Pe^{i\phi_P} e^{i\delta_P},$$

$$A(B^0 \to \pi^+ K^-) = Te^{-i\phi_T} e^{i\delta_T} + Pe^{-i\phi_P} e^{i\delta_P}. \quad (4)$$

(In this case $\phi_T = \text{arg}(V_{ub}^* V_{us}) = \gamma$ and $\phi_P = \text{arg}(V_{tb}^* V_{ts}) = \pi$.) It is straightforward to show that the difference in the decay rates is

$$\Gamma(B^0 \to \pi^- K^+) - \Gamma(B^0 \to \pi^+ K^-) \sim \sin(\phi_T - \phi_P) \sin(\delta_T - \delta_P). \quad (5)$$

Note that, although this rate asymmetry is proportional to $\sin(\phi_T - \phi_P) \sim \sin \gamma$, it also depends on the strong phase difference $\sin(\delta_T - \delta_P)$. The problem is that these strong phases are incalculable. Thus, a measurement of the rate asymmetry in $B^0 \to \pi^- K^+$ does not provide clean information on the CKM phases. This is true of all processes which involve direct CP violation. However, we will soon see how to use flavor SU(3) to separate the weak and the strong phases, so that direct CP-violation measurements can in fact be used to extract the weak phases cleanly.

2.2 Indirect CP Violation

Suppose on chooses a final state $f$ to which both $B^0$ and $\bar{B}^0$ can decay. Then due to $B^0 - \bar{B}^0$ mixing, there will be interference between the two amplitudes $B^0 \to f$ and $B^0 \to \bar{B}^0 \to f$ which allows us to observe CP violation.

In order to be able to obtain clean CKM phase information, it is a necessary requirement that only one weak amplitude contribute to the decay otherwise direct
CP violation is introduced, ruining the cleanliness of the measurement. This is in fact the case for many $B$ decays such as $^{(-)}\bar{B}_d \to \pi^+\pi^-$, as shown in Fig. 3. Here, the tree diagram has the weak phase $V_{ub}^*V_{ud} (\sim \gamma)$, while that of the penguin diagram is $V_{tb}^*V_{td} (\sim \beta)$. In other words, in this decay, in addition to indirect CP violation, direct CP violation is present due to the interference of the tree and penguin diagrams. The presence of direct CP violation spoils the cleanliness of the measurement, hence the term “penguin pollution.” Thus a measurement of the CP asymmetry in this mode does not give access to a CKM phase ($\alpha$ in this case) but rather we measure a quantity that depends on the weak and strong phases of the tree and penguin diagrams, as well as on their relative sizes.

All is not lost, however. Even in the presence of penguin diagrams, it is still possible to cleanly extract the CKM phase $\alpha$ by using an isospin analysis. The idea is to use isospin to relate the three amplitudes $A(B_d^0 \to \pi^+\pi^-)$, $A(B_d^0 \to \pi^0\pi^0)$, and $A(B^+ \to \pi^+\pi^0)$, and similarly for the CP-conjugate processes. For all these decays, the final state has total isospin $I = 0$ or 2. In other words, there are two amplitudes for these decays: $\Delta I = 1/2$ and $\Delta I = 3/2$. Since there are two isospin amplitudes, but three $B$-decay amplitudes, there must be a triangle relation among the $B$ amplitudes. It is:

$$\frac{1}{\sqrt{2}}A^{+-} + A^{00} = A^{+0} . \quad (6)$$
Figure 4: Isospin triangles in $B \to \pi \pi$. 

There is a similar relation among the CP-conjugate processes:

$$\frac{1}{\sqrt{2}} \tilde{A}^{+} - \tilde{A}^{00} = \tilde{A}^{-0} .$$  \hspace{1cm} (7)$$

Note that the tree diagram has both $\Delta I = 1/2$ and $\Delta I = 3/2$ pieces, but the penguin diagram is pure $\Delta I = 1/2$. We can isolate the $\Delta I = 3/2$ contribution and remove the “penguin pollution” by using the triangle relations above. One measures the rates for the three decay processes and their CP conjugate processes (six in total) and constructs the triangles as in Fig. 4. (In this figure, the $\tilde{A}$’s are related to the $\bar{A}$’s by a rotation.) Thus, up to a discrete ambiguity (since one or both triangles may be flipped upside-down), this determines $\theta_{+-}$, the penguin pollution. With this knowledge the angle $\alpha$ can be extracted by measuring CP violation in $B_{0}^{0}(t) \to \pi^{+} \pi^{-}$. Therefore, even in the presence of penguins, $\alpha$ can be obtained cleanly by using the above isospin analysis.

3. SU(3) Relations Among Amplitudes

In general, our inability to calculate strong interaction effects hampers out ability to cleanly extract a CKM phase from a our measurement of decay processes. We have seen that SU(2) isospin allowed us to get around these problems in mixing-induced CP violation measurements, however difficult tagging and time dependent measurements are required. There is a way to use $B \to DK$ decays to obtain clean CP violation information without tagging. However the triangle that needs to be constructed is expected to be very thin; two of the sides will be an order of magnitude longer than the third. On top of this, only the angle $\gamma$ can be extracted this way.

The successful application of isospin symmetry in the $B \to \pi \pi$ analysis begs the question, “what information does flavor SU(3) allow us to extract”. The answer is “just about everything” 7. We will see that, together with a few simple approximations, SU(3) symmetry allows us to obtain all of the CKM weak phases and all of the strong phase shifts from time-independent measurements alone without tagging. Since tagging and time-dependent measurements are not necessary, our analysis allows CLEO to scoop the $B$ factory in the search for CP violation in the $B$ system.

In going from SU(2) to SU(3) the number of Goldstone bosons increases from 3
Figure 5: Diagrams describing decays of $B$ mesons to pairs of light pseudoscalar mesons. Here $\bar{q} = \bar{d}$ for unprimed amplitudes and $\bar{s}$ for primed amplitudes. (a) “Tree” (color-favored) amplitude $T$ or $T'$; (b) “Color-suppressed” amplitude $C$ or $C'$; (c) “Penguin” amplitude $P$ or $P'$ (we do not show intermediate quarks and gluons); (d) “Exchange” amplitude $E$ or $E'$; (e) “Annihilation” amplitude $A$ or $A'$; (f) “Penguin annihilation” amplitude $PA$ or $PA'$. 
Consider all the decays of $B$ mesons to pairs of light pseudoscalar mesons $\pi\pi$, $\pi K$ and $K\bar{K}$. The amplitudes for these decays can be expressed in terms of the following diagrams (see Fig. 3): a “tree” amplitude $T$ or $T'$, a “color-suppressed” amplitude $C$ or $C'$, a “penguin” amplitude $P$ or $P'$, an “exchange” amplitude $E$ or $E'$, an “annihilation” amplitude $A$ or $A'$, and a “penguin annihilation” amplitude $PA$ or $PA'$. Here an unprimed amplitude stands for a strangeness-preserving decay, while a primed contribution stands for a strangeness-changing decay. As noted in Refs. 7, this set of amplitudes is over-complete. The physical processes of interest involve only five distinct linear combinations of these six terms.

Now comes one of the main points. The diagrams denoted by $E$, $A$ and $PA$ can be ignored relative to the other diagrams. The reasons are as follows. First, the diagrams $E$ and $A$ are helicity suppressed by $(m_{u,d,s}/m_B)$ since the $B$ mesons are pseudoscalars. Second, annihilation and exchange processes, such as those represented by $E$, $A$, $PA$, are directly proportional to a factor of the $B$-meson wave function at the origin. Thus these diagrams are suppressed by a factor of $(f_B/m_B)<0.05$ relative to diagrams $T$, $C$ and $P$ (and similarly for their primed counterparts). This suppression should remain valid unless hadronization and rescattering effects are important. Such rescatterings could be responsible for certain decays of charmed particles, but should be less important for the higher-energy $B$ decays.

Neglecting the contributions of the above diagrams, we are left with the 6 diagrams $T$, $T'$, $C$, $C'$, $P$ and $P'$. These six complex parameters determine the 13 allowed $B$ decays to states with pions and kaons, as listed in Table 1. This table is derived by expressing the $B$ into pseudoscalar decay as graphs in terms of their quark level contributions, keeping track of minus signs and $\sqrt{2}$ factors in going from quarks to mesons. The primed and unprimed diagrams are not independent, but are related by CKM matrix elements. In particular, $T'/T = C'/C = r_u$, where $r_u \equiv V_{us}/V_{ud} \approx 0.23$. Assuming that the penguin amplitudes are dominated by the top quark loop, one has $P'/P = r_t$, with $r_t \equiv V_{ts}/V_{td}$. We therefore have 13 decays described by 3 independent graphs, implying that there are 10 relations among the amplitudes. These can be expressed in terms of 6 amplitude equalities, 3 triangle relations, and one quadrangle relation.

The three independent triangle relations and one quadrangle relation are

\[(T + C) = (C - P) + (T + P),\]

\[(T + C) = (C' - P')/r_u + (T' + P'/r_u),\]

\[(T + C) = (C - P) + (T + P) + (C' - P')/r_u + (T' + P'/r_u),\]
Table 1: The 13 decay amplitudes in terms of the 8 graphical combinations. The $\sqrt{2}(B^+ \rightarrow \pi^+ \pi^0)$ in the $-(T+C)$ column means that $A(B^+ \rightarrow \pi^+ \pi^0) = -(T+C)/\sqrt{2}$, and similarly for other entries. Processes in the same column can be related by an amplitude equality, e.g. the amplitudes for $B^+ \rightarrow K^+ \bar{K}^0$ and $B^0 \rightarrow K^0 \bar{K}^0$ are equal.

| $-(T+C)$ | $-(C-P)$ | $-(T+P)$ | $(P)$ |
|----------|----------|----------|-------|
| $\sqrt{2}(B^+ \rightarrow \pi^+ \pi^0)$ | $\sqrt{2}(B^0 \rightarrow \pi^0 \pi^0)$ | $B^0 \rightarrow \pi^+ \pi^-$ | $B^+ \rightarrow K^+ \bar{K}$ |
| $\sqrt{2}(B_s \rightarrow \pi^0 K^0)$ | $B_s \rightarrow \pi^+ K^-$ | $B^0 \rightarrow K^0 \bar{K}$ |

$-(T'+C'+P')$ | $-(C'-P')$ | $-(T'+P')$ | $(P')$ |
|----------|----------|----------|-------|
| $\sqrt{2}(B^+ \rightarrow \pi^0 K^+)$ | $\sqrt{2}(B^0 \rightarrow \pi^0 K^0)$ | $B^0 \rightarrow \pi^- K^+$ | $B^+ \rightarrow \pi^+ K^0$ |
| $B_s \rightarrow K^- K^+$ | $B_s \rightarrow K^0 \bar{K}$ |

$(T+C) = (T'+C'+P')/r_u - (P')/r_u$ ,

$(T'+P') - (P') = r_u(T+P) - r_u(P)$ .

For example, by using Table 1 we can rewrite the relation in Eq. (8) in terms of decay amplitudes as:

$$\sqrt{2}A(B^+ \rightarrow \pi^+ \pi^0) = \sqrt{2}A(B^0 \rightarrow \pi^0 \pi^0) + A(B^0 \rightarrow \pi^+ \pi^-) .$$

We have chosen to express this relation using $B^0$ and $B^+$ mesons only. However, by using the amplitude equalities from Table 1, we could equally have written the right side of the above relation in terms of $B_s$.

The surprising result is that the three triangle relations allow us to completely solve for the magnitudes and phases of the amplitudes $T, C, P$. In addition we will have enough independent determinations of the same quantities to be able to test our two assumptions, namely SU(3) symmetry and the neglect of the $E, A, PA$ diagrams.

Since the amplitude for $B \rightarrow \pi^+ \pi^0$ decay, given by $-(T+C)/\sqrt{2}$, is pure $\Delta I = 3/2$, the diagram $(T+C)$ has only one term, which we denote by $A_{i=2}e^{i\phi_2}e^{i\delta_2}$. Thus, for example, the triangle relation given in Eq. (8) becomes

$$A_{i=2}e^{i\phi_2}e^{i\delta_2} = (A_C e^{i\phi_C} e^{i\delta_C} - A_P e^{i\phi_P} e^{i\delta_P}) + (A_T e^{i\phi_T} e^{i\delta_T} + A_P e^{i\phi_P} e^{i\delta_P}) ,$$

and similarly for the other relations. As before, the $\phi_i$ are the weak phases and the $\delta_i$ are the strong phases. The $\delta_i$ are chosen such that the quantities $A_{i=2}, A_T, A_T', A_C, A_C', A_P$ and $A_P'$ are real and positive (only relative strong phase differences are physically meaningful). SU(3) symmetry implies that the strong phases for the primed and unprimed graphs are equivalent. Working within the Wolfenstein approximation of the CKM matrix, it is easy to see that the weak phases of the various amplitudes are: $\phi_2 = \phi_T = \phi_T' = \phi_C = \phi_C' = \gamma$, $\phi_P = -\beta$, and $\phi_P' = \pi$ (up to corrections of order $\lambda^2 \approx 0.05$). Also, $A_T'/r_u = A_T$ and $A_C'/r_u = A_C$. Finally, multiplying through on both sides by exp$(-i\gamma - i\delta_2)$, the 3 triangle relations become

$$A_{i=2} = (A_C e^{i\Delta_C} + A_P e^{i\gamma} e^{i\Delta_P}) + (A_T e^{i\Delta_T} - A_P e^{i\gamma} e^{i\Delta_P}) ,$$

$$A_{i=2} = (A_C e^{i\Delta_C} + A_P e^{-i\gamma} e^{i\Delta_P} / r_u) + (A_T e^{i\Delta_T} - A_P e^{-i\gamma} e^{i\Delta_P} / r_u) ,$$

$$A_{i=2} = (A_T e^{i\Delta_T} + A_C e^{i\Delta_C} - A_P e^{-i\gamma} e^{i\Delta_P} / r_u) + A_P e^{-i\gamma} e^{i\Delta_T} / r_u) .$$
where we have defined $\Delta_i \equiv \delta_i - \delta_2$.

Consider first the two triangle relations in Eqs. (15) and (16). These relations define two triangles which share a common base. Each triangle is determined up to a two-fold ambiguity, since it can be reflected about its base. Implicit in these two triangle relations is the relation

$$A_{I=2} = |T + C| = A_T e^{i\Delta_T} + A_C e^{i\Delta_C}. \quad (17)$$

Thus both of these triangles also share a common subtriangle with sides $T + C$, $C$ and $T$ as shown in Fig. 6. The key point is this: the subtriangle is completely determined, up to a four-fold ambiguity, by the two triangles in Eqs. (15) and (16). This is because both the magnitude and relative direction of $P'/r_u$ are completely determined by constructing the triangle in Eq. (16). Therefore the point where the vectors $C$ and $T$ meet is given by drawing the vector $P'/r_u$ from the vertex opposite the base [see Fig. 6]. (A similar construction would have given the same point if we had used the vector $T + P'/r_u$ instead of $P'/r_u$.) Thus by measuring the five rates
for

\[B^0 \rightarrow \pi^0 K^0 \text{ (giving } |C - P'/r_u|),\]
\[B^0 \rightarrow \pi^- K^+ \text{ (giving } |T + P'/r_u|),\]
\[B^+ \rightarrow \pi^0 K^+ \text{ (giving } |T + C + P'/r_u|),\]
\[B^+ \rightarrow \pi^+ K^0 \text{ (giving } |P'/r_u|),\] and
\[B^+ \rightarrow \pi^+ \pi^0 \text{ (giving } |T + C| = A_{1,2}, \text{ i.e. the triangle’s base),}\]

we can determine \(\Delta_P - \gamma, |T|\) and \(|C|\), up to a two-fold ambiguity and \(\Delta_C\) and \(\Delta_T\) up to a four-fold ambiguity. As we will discuss later, these discrete ambiguities can be at least partially removed through the knowledge of the relative magnitudes of \(|P|\), \(|C|\), \(|T|\) and \(|P'|\), and through independent measurements of the amplitudes and the strong and weak phases.

If we also measure the rates for the CP-conjugate processes of the above decays, we can get more information. These CP-conjugate decays obey similar triangle relations to those in Eqs. (13) and (16). However, recall that under CP conjugation, the weak phases change sign, but strong phases do not. Thus we can perform an identical analysis with the CP-conjugate processes, giving us another, independent determination of \(|T|\), \(|C|\), \(\Delta_C\) and \(\Delta_T\). But, instead of \(\Delta_P - \gamma\), this time we get \(\Delta_P + \gamma\). Thus we obtain \(\Delta_P\) and \(\gamma\) separately. Note that it is not, in fact, necessary to measure all 5 CP-conjugate processes. The rate for \(B^- \rightarrow \pi^- \pi^0\) is the same as that for \(B^+ \rightarrow \pi^+ \pi^0\), since they involve a single weak phase and a single strong phase. Similarly, the rates for \(B^+ \rightarrow \pi^+ K^0\) and \(B^- \rightarrow \pi^- K^0\) are equal. Therefore, in order to extract \(\gamma\), in addition to the above 5 rates, we need only measure \(\overline{B}^0 \rightarrow \pi^0 \overline{K}^0\) (giving \(|\bar{C} - \bar{P}'/r_u|\)), \(\overline{B}^0 \rightarrow \pi^+ K^-\) (giving \(|\bar{T} + \bar{P}'/r_u|\)), and \(B^- \rightarrow \pi^0 K^-\) (giving \(|\bar{T} + \bar{C} + \bar{P}'/r_u|\)). To sum up, by measuring the above 8 rates, the following quantities can be obtained: the weak phase \(\gamma\), the strong phase differences \(\Delta_T\), \(\Delta_C\) and \(\Delta_P\), and the magnitudes of the different amplitudes \(|T|\), \(|C|\) and \(|P'|\).

Note that the two triangles given by the relations in Eqs. (14) and (15) share a common base with each other and also with the sub-triangle in Eq. (17) (which still holds). The same is true for the two triangles constructed using the triangle relations in Eqs. (13) and (16). Unlike the first two-triangle construction, however, the shape of the sub-triangle is not yet fixed. Nevertheless, the point where the vectors \(C\) and \(T\) meet can still be determined by measuring the additional decays represented by \(P\), \(P'\), or \(|T + P'/r_u|\). A detailed explanation of these two constructions can be found in Ref. [4]. The point is that by measuring 7 rates we can extract \(\Delta_P + \alpha\), \(\Delta_P - \gamma\), \(\Delta_C\), and \(\Delta_T\), up to an eight-fold ambiguity, and \(|T|\) and \(|C|\) up to a four-fold ambiguity. Through the two quantities \(\Delta_P + \alpha\) and \(\Delta_P - \gamma\), we can then determine the weak phase \(\beta\) (using \(\beta = \pi - \alpha - \gamma\)), up to discrete ambiguities. As in the first two-triangle construction, all rates are time-independent. What is surprising, perhaps, about this particular construction is that it is not even necessary to measure the CP-conjugate rates in order to obtain \(\beta\). The reason is that SU(3) flavor symmetry implies the equality of the strong final-state phases of two different amplitudes, in this case \(P\).
and \( P' \). Subtracting the (strong plus weak) phase of one amplitude from the other then determines a weak phase. Usually, in a given process, without measuring the charge-conjugate rate one can only measure the sum of a weak and a strong phase.

If the CP-conjugate rates are also measured, we can obtain \( \Delta_P, \alpha, \) and \( \gamma \) separately. This provides another, independent determination of \( |T|, |C|, \Delta_C \) and \( \Delta_T \). As in the first construction, no observation of CP violation is necessary to make such measurements. Again, it is not necessary to measure all the CP-conjugate rates – only four can be different from their counterparts.

4. Testing Our Assumptions

The three constructions use \( B \) decays to \( \pi \pi, \pi K \) and \( K \bar{K} \) final states. At present, the decays \( B^0 \to \pi^+\pi^- \) and/or \( \pi^- K^+ \) have been observed, but the two final states cannot be distinguished\(^\text{13}\). The combined branching ratio is about \( 2 \times 10^{-5} \). Assuming equal rates for \( \pi^+\pi^- \) and \( \pi^- K^+ \), which seems likely, the amplitudes \( |T| \) and \( |P'| \) should be about the same size. On the other hand, the amplitude \( |C| \) is expected to be about a factor of 5 smaller: the amplitudes \( |T| \) and \( |C| \) are basically the same as \( |a_1| \) and \( |a_2| \), respectively, introduced in Ref.\(^\text{14}\), for which the values \( |a_1| = 1.11 \) and \( |a_2| = 0.21 \) have been found\(^\text{16}\). The ratio \( |P/T| \) has also been estimated to be small, \( \lesssim 0.20 \)\(^\text{15}\). Therefore all the decays used in these constructions should have branching ratios of the order of \( 10^{-5} \), with the exception of \( B \to K \bar{K} \) (\( P \)) and \( B^0 \to \pi^0 \pi^0 \) \( \sim (C - P) \), which are probably an order of magnitude smaller.

The knowledge that the amplitudes obey the hierarchy \( |P|, |C| < |T| < |P'/r_u| \) will also help in reducing discrete ambiguities. For example, in the first two-triangle construction [Fig. 6], we noted in the discussion following Eq. (17) that the subtriangle can be determined up to a four-fold ambiguity. However, two of these four solutions imply that \( |C| \) and \( |T| \) are both of order \( |P'/r_u| \), which violates the above hierarchy. Thus the four-fold ambiguity in the determination of the subtriangle is reduced to a two-fold ambiguity, and the discrete ambiguities in the determination of subsequent quantities such as \( \Delta_P, \gamma, \Delta_C \), etc., are likewise reduced. The ambiguities in the other two constructions can be partially removed in a similar way.

All three two-triangle constructions described above rely on two assumptions. The first is that the diagrams \( A, E \) and \( PA \) (and their primed counterparts) can be neglected. This can be tested experimentally. The decays \( B^0 \to K^+K^- \) and \( B_s \to \pi^+\pi^- \) can occur only through the diagrams \( E \) and \( PA \), and \( E' \) and \( PA' \), respectively. Therefore, if the above assumption is correct, the rates for these two decays should be much smaller than the rates for the decays in Table 1.

The second assumption is that of an unbroken SU(3) symmetry. We know, however, that SU(3) is in fact broken in nature. Assuming factorization, SU(3)-breaking effects can be taken into account by including the meson decay constants \( f_\pi \) and \( f_K \) in the relations between \( B \to \pi \pi \) decays and \( B \to \pi K \) decays\(^\text{15}\). In other words, the factor \( r_u \) which appears in two of the triangle relations should be multiplied by \( f_K/f_\pi \approx 1.2 \). One way to test whether this properly accounts for all SU(3)-breaking effects...
effects is through the rate equalities in Table 1. Even if it turns out that $f_K/f_π$ does not take into account all SU(3)-breaking effects, the large number of independent measurements is likely to help in reducing uncertainties due to SU(3) breaking. For example, note that, not counting the CP-conjugate processes, the last two constructions have six of their seven rates in common. This means that a measurement of only eight decay rates gives two independent measurements of $|T|$, $|C|$, $Δ_C$, $Δ_T$, $Δ_F - γ$ and $Δ_F + α$. In fact, these eight rates already contain the five rates of the first construction [Fig. I]. Thus we actually have three independent ways of arriving at $|T|$, $|C|$, $Δ_C$, $Δ_T$ and $Δ_F - γ$. Including also the CP-conjugate processes, we have a total of 13 $B$-decay rate measurements which give us six independent ways to measure $|T|$, $|C|$, $Δ_C$ and $Δ_T$, five ways to measure $Δ_F$, three independent ways to measure $γ$, and two ways to measure $α$. (If time-dependent measurements are possible, there are additional independent ways to measure $α$.) The point is that the three two-triangle constructions include many ways to measure the same quantity. This redundancy provides a powerful way to test the validity of our SU(3) analysis and reduces the discrete ambiguities in the determination of the various quantities.

A simpler system where a subset of these assumptions can be tested are the decays of $B$’s to one light pseudoscalar and one charmed meson. Here one can also test for the absence of exchange and annihilation graphs; there is no analogue of the penguin annihilation graph. Furthermore, the effects of decay constants and form factors in SU(3) breaking can be studied individually, whereas they occur together when both final-state mesons are light.

Assuming that exchange and annihilation contributions can be neglected, the following decay rates are expected to be equal:

(I) $V_{ud}^*V_{ud} \sim O(λ^2)$ processes:
(a) $B^0 \rightarrow π^+D^- = T + E$ and $B_s \rightarrow π^+D^- = T$;
(b) $\sqrt{2}(B^0 \rightarrow π^0D^0) = C - E$ and $B_s \rightarrow \bar{K}^0\bar{D}^0 = C$;

(II) $V_{us}^*V_{us} \sim O(λ^3)$ processes:
$B^0 \rightarrow K^+D^- = T' + E'$ and $B_s \rightarrow K^+D_s^- = T' + E'$;

(III) $V_{ub}^*V_{cs} \sim O(λ^4)$ processes:
(a) $B^+ \rightarrow K^+D^0 = -(\bar{C} + \bar{A})$ and $B^0 \rightarrow K^0D^0 = -\bar{C}$;
(b) $B^0 \rightarrow π^-D_s^+ = -\bar{T}, \sqrt{2}(B^+ \rightarrow π^0D_s^+) = -\bar{T}$, and $B_s \rightarrow K^-D_s^+ = -(\bar{T} + \bar{E})$;

(IV) $V_{ub}^*V_{cd} \sim O(λ^4)$ processes:
(a) $\sqrt{2}(B^+ \rightarrow π^0D^+) = -\bar{T}' + \bar{A}', B^0 \rightarrow π^-D^+ = -(\bar{T}' + \bar{E}')$, and $B_s \rightarrow K^-D^+ = -\bar{T}'$;
(b) $B^+ \rightarrow π^+D^0 = -(\bar{C}' + \bar{A}')$, $\sqrt{2}(B^0 \rightarrow π^0D^0) = -\bar{C}' + \bar{E}'$, and $B_s \rightarrow \bar{K}^0D^0 = -\bar{C}'$.

Here in the SU(3) limit $T'/T = C'/C = E'/E = \bar{T}'/\bar{T} = \bar{C}'/\bar{C} = \bar{E}'/\bar{E} = \bar{A}'/\bar{A} = |V_{us}/V_{ud}| = |V_{cd}/V_{cs}| = r_u = 0.23.
The following processes are expected to be suppressed by a term of order \((f_B/m_b)\) with respect to the color-favored processes of the same order in \(\lambda\):

(I) \(V_{cb}^*V_{ud} \sim O(\lambda^2)\) processes:
\[B^0 \rightarrow K^+D_s^- = E.\]

(II) \(V_{cb}^*V_{us} \sim O(\lambda^3)\) processes:
\[B_s \rightarrow \pi^+D^- = E' \text{ and } -\sqrt{2}(B_s \rightarrow \pi^0\bar{D}^0) = E'.\]

(III) \(V_{ub}^*V_{cs} \sim O(\lambda^3)\) processes:
\[B^+ \rightarrow K^0D_s^+ = \tilde{A}, \quad -(B_s \rightarrow \pi^-D^+) = \tilde{E}, \text{ and } \sqrt{2}(B_s \rightarrow \pi^0D^0) = \tilde{E}.\]

(IV) \(V_{ub}^*V_{cd} \sim O(\lambda^4)\) processes:
\[B^+ \rightarrow K^0\bar{D}_s^0 = \tilde{A}' \text{ and } B^0 \rightarrow K^-D_s^+ = -\tilde{E}'.\]

The effect of form factors in SU(3) symmetry breaking can be directly studied by comparing spectator quark processes in which strange and non-strange quarks combine with strange or non-strange quarks. For example consider the following ratios of rates.

\[\Gamma(B^0 \rightarrow K^+D^-)/\Gamma(B_s \rightarrow \pi^+D_s^-) = |T'/T|^2 \quad (18)\]
\[\Gamma(B^0 \rightarrow K^0D^0)/\Gamma(B_s \rightarrow K^0D^0) = |C'/C|^2 \quad (19)\]
\[\Gamma(B_s \rightarrow K^-D^+)/\Gamma(B^0 \rightarrow \pi^-D^+_s) = |\tilde{T}'/\tilde{T}|^2 \quad (20)\]
\[\Gamma(B_s \rightarrow \bar{K}^0D^0)/\Gamma(B^0 \rightarrow K^0D^0) = |\tilde{C}'/\tilde{C}|^2 \quad (21)\]

In the absence of SU(3) symmetry breaking they should all equal \(r_u^2 = |V_{us}/V_{ud}|^2\). (We are not interested in isospin symmetry breaking effects such as the deviation of the ratio \(\Gamma(B^+ \rightarrow \pi^0D^+_s)/\Gamma(B^0 \rightarrow \pi^-D^+_s)\) from 1/2) Deviations in \(r_u^2\) in Eq. (19) will tell us about form factor effects in strange versus non-strange combining with a heavy SU(3) singlet quark, whereas Eq. (21) is the same spectator process combing with a non-strange SU(3) anti-triplet. Deviations in Eq. (20) and Eq. (21) will measure the same thing; strange and non-strange combining with a non-strange and strange repectively. Thus if form factors are the main SU(3) breaking effects in Eq. (20) and Eq. (21), we would expect equal but opposite deviations from \(\lambda^2\) in these two processes.

Finally deviations in the following triangle relations test form factor SU(3) symmetry breaking effects:

\[O(\lambda^2) \text{process : } (B^+ \rightarrow \pi^+\bar{D}^0) = (B_s \rightarrow \pi^+D^-) + (B_s \rightarrow \bar{K}^0\bar{D}^0) \quad (22)\]
\[\quad (T + C) = (T) + (C)\]

\[O(\lambda^3) \text{process : } (B^+ \rightarrow K^+\bar{D}^0) = (B^0 \rightarrow \bar{K}^+\bar{D}^-) + (B_0 \rightarrow K^0\bar{D}^0) \quad (23)\]
\[\quad (T + C) = (T') + (C')\]
where, for example, on the left-hand side of Eq. (23) we have non-strange quarks combining with non-strange quarks, whereas on the right-hand side we have strange combining with non-strange.

5. Summary and Conclusions

We have also described in some detail the recent developments which provide a prescription for the measurement of all relevant quantities: weak and strong phases, and the sizes of the contributing diagrams. This analysis uses SU(3) flavour symmetry along with the important dynamical assumption that exchange and annihilation diagrams can be neglected. This method relies on several triangle relations which hold under these assumptions. Like $B \to DK$ decays, neither time-dependent measurements nor tagging are required. This analysis can therefore be carried out at a symmetric $B$-factory such as CLEO. Unlike $B \to DK$, however, the branching ratios for most of the processes involved are expected to be $O(10^{-5})$, so that the sides of the triangles are all roughly the same size. This method also provides enough redundancy to test the consistency of the assumptions.

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