The triplet vector boson (TVB) is a simplified new physics model involving massive vector bosons transforming as a weak triplet vector, which it has been proposed as a combined explanation to the anomalous $b \to s\mu^+\mu^-$ and $b \to c\tau\bar{\nu}_\tau$ data (the so-called $B$ meson anomalies). In this work, we carry out an updated view of the TVB model, including the Belle II perspectives. We perform a global fit to explore the allowed parameter space by the most current $b \to s\mu^+\mu^-$ and $b \to c\tau\bar{\nu}_\tau$ data, by considering all relevant low-energy flavor observables. Our results are confronted with the most recent LHC constraints. We also incorporate in our study the first measurement on the ratio $R(\Lambda_c) = BR(\Lambda_b \to \Lambda_c\tau\bar{\nu}_\tau)/BR(\Lambda_b \to \Lambda_c\mu\bar{\nu}_\mu)$ very recently obtained by LHCb. In particular, we show that the TVB model can provide an explanation to the $B$ meson anomalies; however, this framework is in strong tension with LHC bounds. In respect to future flavor measurements at Belle II, our results suggest that a small new physics window would be allow to solely explain the $b \to c\tau\bar{\nu}_\tau$ data in agreement with LHC constraints. Furthermore, the implications of our phenomenological analysis of the TVB model to some known flavor parametrizations are also discussed.

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I. INTRODUCTION

In the last ten years, approximately, the high-energy physics community has been a witness of discrepancies between experimental measurements and the Standard Model (SM) calculations in several observables involving $b \rightarrow s \mu^+ \mu^-$ (neutral-current) and $b \rightarrow c \tau \bar{\nu}_\tau$ (charged-current) transitions, which provide an important test of lepton flavor universality (LFU). Such inconsistencies indicate strong signals of LFU violation (for a very recent interesting reviews, see Refs. [1–3]). For the neutral-current $b \rightarrow s \mu^+ \mu^-$ transition, the ratio of semileptonic decay channels,

$$R_{K^{(*)}} = \frac{BR(B \rightarrow K^{(*)}\mu^+ \mu^-)}{BR(B \rightarrow K^{(*)}e^+ e^-)},$$

(1)

provides a test of $\mu/e$ LFU for different dilepton mass-squared range $q^2$ ($q^2$ bins) [4–10]. The ratio $R_K$ was first reported in 2014 by the LHCb collaboration [4],

$$R_{K}^{LHCb-14} = 0.745_{-0.074}^{+0.090} \pm 0.036, \quad q^2 \in [1.0, 6.0] \text{ GeV}^2,$$

(2)

which deviates from the SM prediction of $R_{K}^{SM} \approx 1$ at a 2.6$\sigma$ [4] level. Recently, the LHCb has released an updated measurement on $R_{K}^{[5]}

$$R_{K}^{LHCb-21} = 0.846_{-0.041}^{+0.044}, \quad q^2 \in [1.1, 6.0] \text{ GeV}^2,$$

(3)

and the new measurement $R_{K_S}$ [6]

$$R_{K_S}^{LHCb-21} = 0.66_{-0.14}^{+0.20} + 0.02, \quad q^2 \in [1.1, 6.0] \text{ GeV}^2,$$

(4)

which are 1.5$\sigma$ and 3.1$\sigma$ away from the SM prediction, respectively. The $R_K$ measurement improves the uncertainties of the previously reported value [7]. In 2019, the Belle experiment reported a less precise $R_K$ value of [8]

$$R_{K}^{Belle-19} = 1.03_{-0.24}^{+0.28} \pm 0.01, \quad q^2 \in [1.0, 6.0] \text{ GeV}^2,$$

(5)

in consistency with the SM expectations (because of the large uncertainties). Moreover, the flavor ratio $R_{K^*}$ was measured in 2017 by the LHCb Collaboration in the low and central $q^2$ bins [9],

$$R_{K^*}^{LHCb-17} = \begin{cases} 0.66_{-0.07}^{+0.11} \pm 0.03, & q^2 \in [0.045, 1.1] \text{ GeV}^2, \\ 0.69_{-0.07}^{+0.11} \pm 0.05, & q^2 \in [1.1, 6.0] \text{ GeV}^2, \end{cases}$$

(6)

respectively. These measurements differ from the SM in the two $q^2$ regions by $\sim 2.3\sigma$ and $\sim 2.5\sigma$, respectively [9]. In a wider $q^2$ bin, the LHCb has recently reported [6]

$$R_{K^{(*)}}^{LHCb-21} = 0.70_{-0.13}^{+0.18} + 0.03, \quad q^2 \in [0.045, 6.0] \text{ GeV}^2,$$

(7)

Likewise, the Belle experiment reported values on $R_{K^*}$ in different $q^2$ bins with still large uncertainties and hence in agreement with the SM (as well as LHCb) [10]. It is expected that Belle II experiment will be able to improve the current uncertainties [11].

Thus, the largest deviations of $R_K$ and $R_{K^*}$ have been observed by the LHCb, hinting toward LFU violation. These data can be explained if there is new physics (NP) effects in $b \rightarrow s \mu^+ \mu^-$, i.e., the hypothesis that NP couples selectively to the muons. Such discrepancies are strengthened by some additional anomalous observables (such as angular observables and differential branching fractions) related with $B \rightarrow K^* \mu^+ \mu^-$ and $B_\tau \rightarrow \phi \mu^+ \mu^-$ decays [12–17]. Several global fit analyses taking into account the most recent $b \rightarrow s \mu^+ \mu^-$ observables have been performed in the literature [1, 18–25]. Assuming a model-independent effective Hamiltonian approach, different scenarios with NP operators (dimension-six) have been surveyed concluding that the muon-specific Wilson coefficient (WC) solution $C_9^{bsq\mu} = -C_{10}^{bsq\mu}$, related with the operators $(\bar{s}P_L\gamma_\alpha b)(\bar{\mu}\gamma^{\alpha}\mu)$ and $(\bar{s}P_L\gamma_\alpha b)(\bar{\mu}\gamma^{\alpha}\gamma_5\mu)$, is firmly preferred by the data [1, 18–25].

On the other hand, simultaneously, the experimental measurements collected by the BABAR, Belle, and LHCb experiments on several charged-current $b \rightarrow c \tau \bar{\nu}_\tau$ observables, also indicate the existence of disagreement with respect to the SM predictions [26–40]. These measurements include the ratio of semileptonic $B$ meson decays

$$R(D^{(*)}) = \frac{BR(B \rightarrow D^{(*)}\tau \bar{\nu}_\tau)}{BR(B \rightarrow D^{(*)}\bar{\nu}_\tau)},$$

(8)
TABLE I. Experimental status and SM predictions on observables related to the charged-current transitions $b \to c\ell\bar{\nu}_\ell$ ($\ell = \mu, \tau$) and $b \to u\tau\bar{\nu}_\tau$.

| Transition | Observable | Expt. measurement | SM prediction |
|------------|------------|--------------------|---------------|
| $b \to c\tau\bar{\nu}_\tau$ | $R(D)$ | $0.340 \pm 0.027 \pm 0.013$ [39] | $0.299 \pm 0.003$ [39, 40] |
| $R(D^*)$ | $0.295 \pm 0.011 \pm 0.008$ [39] | $0.258 \pm 0.005$ [39, 40] |
| $R(J/\psi)$ | $0.71 \pm 0.17 \pm 0.18$ [37] | $0.2582 \pm 0.0038$ [41] |
| $P_1(D^*)$ | $-0.38 \pm 0.51^{+0.21}_{-0.16}$ [35, 36] | $-0.497 \pm 0.013$ [42] |
| $F_1(D^*)$ | $0.60 \pm 0.08 \pm 0.035$ [38] | $0.46 \pm 0.04$ [43] |
| $R(X_c)$ | $0.223 \pm 0.030$ [44] | $0.216 \pm 0.003$ [44] |
| $BR(B_{c}^{+} \to \tau^{-}\nu_{\tau})$ | $< 10\%$ [66], $< 30\%$ [65] | $(2.16 \pm 0.16)\%$ [116] |
| $b \to c\mu\bar{\nu}_\mu$ | $R_{D^{(*)}}^{\mu/e}$ | $0.995 \pm 0.022 \pm 0.039$ [122] | $0.9960 \pm 0.0002$ [121] |
| $b \to u\tau\bar{\nu}_\tau$ | $B_{-}^{-} \to \tau^{-}\bar{\nu}_\tau$ | $(1.09 \pm 0.24) \times 10^{-4}$ [123] | $(0.989 \pm 0.013) \times 10^{-4}$ |

with $\ell' = e$ or $\mu$ (the so-called $R(D^{(*)})$ anomalies) and

$$R(J/\psi) = \frac{BR(B_c \to J/\psi\tau\bar{\nu}_\tau)}{BR(B_c \to J/\psi\mu\bar{\nu}_\mu)},$$

(9)

as well as the polarization observables such as the $\tau$ lepton polarization $P_1(D^*)$ and the longitudinal polarization of the $D^*$ meson $F_1(D^*)$ related with the channel $B \to D^*\tau\bar{\nu}_\tau$, which have been observed by the Belle experiment [35, 36, 38]. In Table I we summarize the current experimental measurements [26–40] and the SM predictions [39–43]. Moreover, we collect in Table I the experimental and theoretical values of the ratio of inclusive decays $R(X_c) \equiv BR(B \to X_c\tau\bar{\nu}_\tau)/BR(B \to X_c\mu\bar{\nu}_\mu)$, which is generated via the same $b \to c\tau\bar{\nu}_\tau$ transition [44]. The $R(D^{(*)})$ anomalies still exhibit the largest deviation by 1.4$\sigma$ and 2.5$\sigma$, respectively. The other $b \to c\tau\bar{\nu}_\tau$ observables also show tension (moderate) with the data, although, some of them have large experimental uncertainties (such as $R(J/\psi)$ and $P_1(D^*)$). While the ratio $R(X_c)$ is in excellent agreement with the SM. In addition, the LHCb Collaboration has recently released the first measurement of the ratio of semileptonic $\Lambda_b$ baryon decays, namely [45]

$$R(\Lambda_c) \equiv \frac{BR(\Lambda_c^0 \to \Lambda_c^{\pm}\tau^{-}\bar{\nu}_\tau)}{BR(\Lambda_c^0 \to \Lambda_c^{\pm}\mu^{-}\bar{\nu}_\mu)} = 0.242 \pm 0.076,$$

(10)

in agreement at the ~1.2$\sigma$ level with the most recent SM calculation, $R(\Lambda_c)^{SM} = 0.324 \pm 0.004$ [46]. In Eq. (10) we have added in quadrature the statistical and systematic uncertainties, and the external branching ratio uncertainty from the channel $\Lambda_c^0 \to \Lambda_c^{\pm}\mu^{-}\bar{\nu}_\mu$) [45]. It is interesting to stress out that this new measurement is below the SM value, pointing to an opposite direction than the current $b \to c\tau\bar{\nu}_\tau$ data (see Table I).

Similarly to the case of $b \to s\mu^+\mu^-$ data, the $b \to c\tau\bar{\nu}_\tau$ one is also suggesting strong signals of LFU violation, and all possible NP four-fermion operators (involving either left-handed and right-handed neutrinos) of a given Lorentz structure (vector, scalar and tensor) have been extensively studied and constraints on their WCs have been derived by fitting them to the most recent $b \to c\tau\bar{\nu}_\tau$ data [1, 47–64]. Besides, bounds from the branching ratio of the tauonic $B_c$ decay, $BR(B_c^+ \to \tau^-\nu_\tau)$, must be taking into account [65, 66], and NP explanations (with left-handed neutrinos) have to confront additional constraints from the ratio of leptonic $\Upsilon(nS)$ ($n = 1, 2, 3$) decays, $R_{\Upsilon(nS)} \equiv BR(\Upsilon(nS) \to \tau^+\tau^-)/BR(\Upsilon(nS) \to \ell^+\ell^-)$ [67, 68] 1. All of these global fit studies show that the single WC $C_{\Upsilon}^{\ell\tau
u\tau}$ associated with the NP vector operator $(\bar{c}\gamma_\mu P_L b)(\bar{\tau}\gamma^\mu P_L \nu_\tau)$ provides the best good fit to the data [1, 47–64].

In addition to a model-independent effective approach of the $b \to s\mu^+\mu^-$ and $b \to c\tau\bar{\nu}_\tau$ anomalies, there has been a growing industry of attempts to include particular NP models as a way to adjust the current data, either separately or combined [1]. In the case of a simultaneous explanation, the key point is that scenarios with WCs $C_{\Upsilon}^{b\mu\mu} = -C_{\Upsilon}^{b\tau\nu}$ and $C_{\Upsilon}^{\ell\tau
u\tau}$, are strongly favored by the global fit analyses. Keeping this in mind, such a requirement can be generated by different tree-level heavy mediators with the adequate couplings, for example, extra gauge bosons or leptoquarks (for an extensive list of literature, see the theoretical status report presented in Ref. [1]). In this work

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1 Recently, in Ref. [69] has been proposed a new method to test LFU through inclusive dileptonic $\Upsilon(4S)$ decays.
we will pay particular attention to the common explanation provides by the so-called Triplet Vector Boson (TVB) model [70–77]², in which the SM is extended by including a color-neutral real $SU(2)_L$ triplet of massive vectors $W'$ and $Z'$ that coupled predominantly to left-handed (LH) fermions from the second- and third-generations [70–77]. The neutral boson $Z'$ is responsible for the $b \to s\mu^+\mu^-$ anomaly, while the charged boson $W'$ generates the $b \to c\tau\bar{\nu}_\tau$ one. Another possibility is that the $Z'$ transforms as a singlet of $SU(2)_L$, in this case it must be an extra neutral gauge boson associated with an additional symmetry $U(1)'$. In this direction, a great number of models have been proposed to address solely the $b \to c\tau\bar{\nu}_\tau$ and $b \to s\mu^+\mu^-$ data, we present an updated view by performing a global fit to explore the allowed parameter space of the couplings of the TVB model, including all relevant flavor observables that are also affected by them, such as $B_s - \bar{B}_s$, mixing, neutrino trident production, LFV decays $(B^+ \to K^\mp \tau^+\tau^-, B_s \to \mu^\mp\tau^\pm, \tau \to \mu\phi, Y(nS) \to \mu^\pm\tau^\mp)$, rare $B$ decays $(B \to K\nu\bar{\nu}, B \to K\tau^+\tau^-, B_s \to \tau^+\tau^-)$, and bottomonium ratios $R_{Y(nS)}$; as well as Large Hadron Collider (LHC) bounds from searches of high-mass dilepton resonances at the ATLAS experiment. We also incorporate the new LHCb measurement of the ratio $R(\Lambda_c)$ [45] and explore its effect into our analysis. Furthermore, we study the TVB model prospects in what concerns to the improvements of numerous flavor observables by future data at Belle II. Our study aims to complement and enlarge with new additional observables, the previous similar analysis performed in 2018 by Kumar, London, and Watanabe (KLW) [76], and extend the very recent analysis (done by some of us) where only the charged-current $b \to c\tau\bar{\nu}_\tau$ anomaly was addressed within this framework [67].

This work is structured as follows: in the Sec. II we discuss the main aspects of the TVB model to accommodate the $B$ meson anomalies. As a next step in Sec. III, we consider the most relevant flavor observables and present the TVB model contributions to them. The LHC bounds are also studied. We then perform our phenomenological analysis of the allowed parametric space in Sec. IV and our conclusions are presented in Sec. V.

II. THE TRIPLET VECTOR BOSON MODEL

In general, flavor anomalies have been boarded into the current literature as a motivation to build innovative models and to test well established New Physics (NP) models. In this section, we focus in the previously mentioned Triplet Vector Boson (TVB) model [70–77] as a possible explanations of these anomalies, that might accommodate the observed flavor experimental results. One significant feature of this model, is the inclusion of extra SM-like vector bosons with non-zero couplings to the SM fermions, that allow us to include additional interactions.

In the fermion mass basis, the most general lagrangian describing the dynamics of the fields can be written as

$$\Delta \mathcal{L}_V = g_{JQ}^2 \langle \overline{\psi}^L_Q \gamma^\mu \sigma^I \psi^Q_{J,L} \rangle V^I_{\mu} + g_{J\ell}^2 \langle \overline{\psi}^L_{\ell} \gamma^\mu \sigma^I \psi^\ell_{J,L} \rangle V^I_{\mu}$$

(11)

where, $V_{\mu}$ stands for the extra or new vector bosons that transform as $(1, 3, 0)$ under the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry and must be redefined as $W'^\pm, Z'$. On other side, SM fermions are arranged into the doublets $\Psi^Q_L$ and $\Psi^\ell_L$ given by

$$\Psi^Q_L = \left( V^{|u_L} \right), \quad \Psi^\ell_L = \left( \mu_L \right)$$

(12)

It is worth noticing here that in this particular model the CKM mixing matrix $V$ is applied on the up-type quarks.

In order to find the effective lagrangian for this model the heavy degrees of freedom corresponding to vector bosons introduced above must be integrated out. Introducing the definition for the currents $J_Q = \overline{\psi}^Q_{i,L} \gamma^\mu \sigma^I \psi^Q_{j,L}$ and $J_\ell = \overline{\psi}^\ell_{i,L} \gamma^\mu \sigma^I \psi^\ell_{j,L}$, the effective lagrangian is therefore

$$\mathcal{L}_{eff} = - \frac{(g_{JQ}^2 J_Q + g_{J\ell}^2 J_\ell)^2}{2M_V^2}$$

(13)

$$= - \frac{(g_{JQ}^2 J_Q)^2}{2M_V^2} - \frac{g_{JQ}^2 g_{J\ell}^2 J_Q J_\ell}{M_V^2} - \frac{(g_{J\ell}^2 J_\ell)^2}{2M_V^2}$$

(14)

The middle term of the right-hand side of the above equation corresponds to

$$\frac{g_{JQ}^2 g_{J\ell}^2 J_Q J_\ell}{M_V^2} = \frac{g_{JQ}^2 g_{J\ell}^2 \langle \overline{\psi}^Q_L \gamma^\mu \sigma^I \psi^Q_{J,L} \rangle \langle \overline{\psi}^\ell_L \gamma^\mu \sigma^I \psi^\ell_{J,L} \rangle}{M_V^2}$$

(15)

² Let us notice that in a recent work [78], the TVB model was implemented as an explanation to the Cabibbo angle anomaly and $b \to s\ell^+\ell^-$ data.
be mediated by extra bosonic charged fields, while the remaining terms are mediated by an extra neutral bosonic field. In this expression, we can identify that the first term expresses an effective interaction of the SM fields that should naively be considered to be (almost) degenerated which is required by electroweak precision data \[73\]. For simplicity, and without loosing generality, we are going to consider that the couplings \(g^{d}\) are real to avoid CP violation effects. Additionally, it is important to notice that we can write compactly the couplings of quarks to the vector boson fields with an explicit dependence on the couplings of the down sector and also, keeping in mind that the CKM matrix couples into the doublets to up-type quarks and that we should restrict the significant contributions for the second and third families. For this purpose, we restrict the relevant couplings of the down sector to \(g_{bb}, g_{ss}\) and \(g_{bb} = g_{ss}\) while other terms remain zero. This hypothesis that the couplings to the first generation of fermions (also in the leptonic sector) can be neglected has been widely accepted in the literature into the context of flavor anomaly explanations \[70–77\]. Lastly, the resultant compact form for the couplings of the quark sector to the \(W'\) that we obtained are

\[
g_{ab} = gbV_{ab} + gsV_{as},
\]

\[
g_{as} = gsV_{as} + gbV_{ab},
\]

where \(\alpha\) stands for \(u, c\) or \(t\) quark flavors. The same procedure described above must be implemented for a compact form of the couplings of up-type quarks to the \(Z'\) boson. In this case we find two possibilities: one on flavor conserving interaction given by

\[
g_{\alpha\alpha} = gbV_{ab}^2 + 2gbV_{as}V_{ab} + gsV_{as}^2;
\]

the another is related to flavor changing \(Z'\) couplings mediated by

\[
g_{\alpha\beta} = gbV_{\beta b}V_{ab} + gbV_{\beta s}V_{as} + gbV_{\beta b}V_{as} + gsV_{\beta s}V_{as},
\]

where \(\alpha \neq \beta\) labels \(u, c\) or \(t\) quark flavors.

To close this kind of parametrization, we mention that the terms of the r.h.s of equation (15) are responsible for and will be important to \(4q\) and \(4f\) interactions ruled by the lagrangian

\[
\mathcal{L}^{4q,4f}_{NP} = -\frac{g_{ij}g_{kl}}{2M_V^2}(\psi^Q_{iL}^\dagger \gamma^\mu \sigma^f \psi^Q_{jL})(\psi^Q_{kL}^\dagger \gamma^\mu \sigma^f \psi^Q_{lL}) - \frac{g_{ij}g_{kl}}{2M_V^2}(\psi^\ell_{iL}^\dagger \gamma^\mu \sigma^f \psi^\ell_{jL})(\psi^\ell_{kL}^\dagger \gamma^\mu \sigma^f \psi^\ell_{lL})
\]

A. Other parametrizations

In this subsection, we compare the previous parameterization explained above with others used in some representative references that had been studied widely in the TVB model.

On one hand, in the TVB model presented in refs \[70, 75\], the mixing pattern for quarks is enriched by the inclusion of mixing matrices that will rotate the fields from the gauge basis to the mass basis and a projector \((X, Y)\) that will ensure the dominance of the second and third families to explain anomalies. Particularly, the explicit form of these matrices for the down-type quarks and charged leptons and projectors are

\[
D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_D & \sin\theta_D \\ 0 & -\sin\theta_D & \cos\theta_D \end{pmatrix}, \quad L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_L & \sin\theta_L \\ 0 & -\sin\theta_L & \cos\theta_L \end{pmatrix}, \quad X = Y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

these matrices will leave an explicit dependence of these mixing angles \((\theta_D, L)\) into the couplings to the extra fields which by the experimental results coming from different observable can be constrained. The assumptions made into the introduction of these matrices was previously introduced in \[70\] and we can establish the full equivalence between
the notations of the angles by the relations \( \theta_D = \alpha_{sb} \) and \( \theta_L = \alpha_{\mu \tau} \). We also found that these couplings can be translated to the generic parameterization introduced at the beginning of this section. For this purpose, as it was explained before, the couplings of all the quark sector will be dependent on the couplings of the down-type quarks, particularly in this kind of parameterization, we can illustrate the way that the couplings are obtained through the effective charged lagrangian that will be given as

\[
\mathcal{L}_{\text{eff}}^{W'} = \frac{g_3^2 g_4^2}{M_W^2} [(V D^\dagger X D)_{ij}(\bar{u}_i L \gamma_\mu d_j L)(L^\dagger Y L)_{kl}(\bar{\ell}_k \gamma^\mu \nu_{L}) + \text{h.c.}];
\]

thus, we obtain the equivalence

\[
\begin{align*}
g_{bb} &\to g_3^2 \cos^2 \theta_D \\
g_{sb} &\to -g_3^2 \sin \theta_D \cos \theta_D \\
g_{ss} &\to g_3^2 \sin^2 \theta_D,
\end{align*}
\]

and for the leptonic sector

\[
\begin{align*}
g_{\tau \tau} &\to g_4^2 \cos^2 \theta_L \\
g_{\mu \tau} &\to -g_4^2 \sin \theta_L \cos \theta_L \\
g_{\mu \mu} &\to g_4^2 \sin^2 \theta_L.
\end{align*}
\]

The comparison and equivalence among parameterizations of different influential references can be found in Tables II, III, IV and V.

For our last comparison, we considered the parameterization given in Refs. \[71, 74\] where the couplings to the vector bosons have almost the same structure of the initial parameterization presented here, but its major difference consists in the dependence on flavor matrices denoted by the authors as \( \lambda^{q, \ell}_i \). This incidence of the flavor structure into the model can be shown using the charged effective lagrangian as we did before

\[
\mathcal{L}_{\text{eff}}^{W'} = \frac{g_q \theta_q}{2M_W^2} [(V \lambda)_{ij} (\bar{u}_i L \gamma_\mu d_j L)(\bar{\ell}_k \gamma^\mu \nu_{L}) + \text{h.c.}],
\]

to obtain the desired dominance of couplings to the second and third families using the flavor matrices mentioned before, the \( \lambda_{ij} \) belonging to the first family must be set to zero. Additionally, the values for \( \lambda_{bb} = \lambda_{\tau \tau} = 1 \) in order to maximize its contribution. However, as an illustration, we can make a complete relation of the implementation of the flavor matrices to the construction of couplings for the quark sector without any assumption in Tables II, III, IV and V.

**TABLE II.** Couplings to \( W' \) boson in different parameterizations of the TVB model

| Coupling | Parameterization in [76] | Parameterization in [70, 75] | Parameterization in [71, 74] |
|----------|----------------------------|-------------------------------|-------------------------------|
| \( g_{bb}^q \) | \( g_{bb} V_{ub} + g_{sb} V_{us} \) | \( g_3^2 (V_{ub} \cos^2 \theta_D - V_{us} \cos \theta_d \sin \theta_d) \) | \( g_q (V_{ub} + V_{ud} \lambda_{ub} + V_{us} \lambda_{us})/\sqrt{2} \) |
| \( g_{cc}^q \) | \( g_{cc} V_{cb} + g_{bc} V_{cs} \) | \( g_3^2 (V_{cb} \cos^2 \theta_B - V_{cs} \cos \theta_d \sin \theta_d) \) | \( g_q (V_{cb} + V_{cd} \lambda_{cb} + V_{cs} \lambda_{cs})/\sqrt{2} \) |
| \( g_{bb}^q \) | \( g_{bb} V_{tb} + g_{sb} V_{ts} \) | \( g_3^2 (V_{tb} \cos^2 \theta_D - V_{ts} \cos \theta_d \sin \theta_d) \) | \( g_q (V_{tb} + V_{td} \lambda_{tb} + V_{ts} \lambda_{ts})/\sqrt{2} \) |
| \( g_{bb}^q \) | \( g_{bb} V_{ub} + g_{sb} V_{us} \) | \( g_3^2 (V_{ub} \sin^2 \theta_D - V_{us} \cos \theta_d \sin \theta_d) \) | \( g_q (V_{ub} \lambda_{ub} + V_{ub} \lambda_{us} + V_{us} \lambda_{us})/\sqrt{2} \) |
| \( g_{bb}^q \) | \( g_{bb} V_{tb} + g_{sb} V_{ts} \) | \( g_3^2 (V_{tb} \sin^2 \theta_D - V_{ts} \cos \theta_d \sin \theta_d) \) | \( g_q (V_{tb} \lambda_{tb} + V_{tb} \lambda_{ts} + V_{ts} \lambda_{ts})/\sqrt{2} \) |

**TABLE III.** Flavor conserving couplings to \( Z' \) boson in different parameterizations of the TVB model

| Coupling | Parameterization in [76] | Parameterization in [70, 75] | Parameterization in [71, 74] |
|----------|----------------------------|-------------------------------|-------------------------------|
| \( g_{u u}^q \) | \( g_{u u} V_{ub} + 2 g_{ub} V_{us} V_{ub} + g_{ss} V_{ss} \) | \( g_3^2 (V_{ub}^2 \cos^2 \theta_d - 2 V_{us} V_{ub} \cos \theta_d \sin \theta_d + V_{ss}^2 \sin^2 \theta_d) \) | \( g_q \lambda_{uu}/\sqrt{2} \) |
| \( g_{c c}^q \) | \( g_{c c} V_{cb} + 2 g_{bc} V_{cs} V_{cb} + g_{ss} V_{ss} \) | \( g_3^2 (V_{cb}^2 \cos^2 \theta_B - 2 V_{cs} V_{cb} \cos \theta_d \sin \theta_d + V_{ss}^2 \sin^2 \theta_d) \) | \( g_q \lambda_{cc}/\sqrt{2} \) |
| \( g_{t t}^q \) | \( g_{t t} V_{tb} + 2 g_{tb} V_{ts} V_{tb} + g_{ss} V_{ss} \) | \( g_3^2 (V_{tb}^2 \cos^2 \theta_T - 2 V_{ts} V_{tb} \cos \theta_d \sin \theta_d + V_{ss}^2 \sin^2 \theta_d) \) | \( g_q \lambda_{tt}/\sqrt{2} \) |

We make emphasis that the results presented in tables II, III, IV and V, by one side, allows us to understand the differences and similarities for the parameterizations presented above in the context of the TVB model, and in another side, give us a complete interpretation of the variables present on each one and the possibilities to find adjustments to explain flavor anomalies.
TABLE IV. Flavor changing couplings to $Z'$ boson in different parameterizations of the TVB model.

| Coupling | Parameterization in [76] | Parameterization in [70, 75] | Parameterization in [71, 74] |
|----------|--------------------------|-------------------------------|-------------------------------|
| $g_{μμ}$ | $g_{μμ}$ | $g_2 \sin^2 \theta_L$ | $g_q (\lambda_{μμ}) / \sqrt{2}$ |
| $g_{μτ}$ | $g_{μτ}$ | $-g_2^2 \sin \theta_L \cos \theta_L$ | $2g_q (\lambda_{μτ}) / \sqrt{2}$ |
| $g_{ττ}$ | $g_{ττ}$ | $g_1^2 \cos^2 \theta_L$ | $g_q / \sqrt{2}$ |

TABLE V. Couplings of leptons to $Z'$ boson in different parameterizations of the TVB model.

### III. RELEVANT OBSERVABLES

In this section, we discuss the most important constraints on the TVB model couplings that accommodate simultaneously the $B$ meson anomalies, coming from the most relevant flavor observables.

#### A. $b \to c \ell^- \nu_\ell$ ($\ell = μ, \tau$) data

The $W'$ boson leads to additional tree-level contribution to $b \to c \ell^- \nu_\ell$ transitions involving leptons from second- and third-generation ($\ell = μ, \tau$). The total low-energy effective Lagrangian has the following form [116]

$$-\mathcal{L}_{\text{eff}}(b \to c \ell \bar{\nu}_\ell)_{\text{SM} + W'} = \frac{4G_F}{\sqrt{2}} V_{cb} \left[ 1 + C_V^{b c e F V} \right] \left( \bar{c} \gamma_\mu P_L b \left( \bar{\ell} \gamma_\mu P_L \nu_\ell \right) \right],$$

(26)

where $G_F$ is the Fermi coupling constant, $V_{cb}$ is the charm-bottom Cabibbo-Kobayashi-Maskawa (CKM) matrix element, and $C_V^{b c e F V}$ is the Wilson coefficient (WC) associated with the NP vector (left-left) operator. This WC is defined as

$$C_V^{b c e F V} = \sqrt{2} \frac{2(V_{cb} g_{q_1} + V_{cb} g_{q_2}) g_{\ell \ell}}{M_V} (\ell = μ, \tau),$$

(27)

with $M_V$ the heavy boson mass. The NP effects on the LFU ratios $R(X)$ ($X = D, D^*, J/ψ$), the $D^*$ and $τ$ longitudinal polarizations related with the channel $B \to D^* τ \bar{ν}_τ$, the ratio of inclusive decays $R(X_c) \equiv BR(B \to X_c τ \bar{ν}_τ)/BR(B \to X_c μ \bar{ν}_μ)$, and the tauonic decay $B_c^- \to τ^- \bar{ν}_τ$ can be easily parametrized as [67, 116]

$$R(X) = R(X)_{\text{SM}} \left| 1 + C_V^{b c e τ ν_τ} \right|^2,$$

(28)

$$F_L(D^*) = F_L(D^*)_{\text{SM}} \left| 1 + C_V^{b c e τ ν_τ} \right|^2,$$

(29)

$$P_\tau(D^*) = P_\tau(D^*)_{\text{SM}} \left| 1 + C_V^{b c e τ ν_τ} \right|^2,$$

(30)

$$R(X_c) = R(X_c)_{\text{SM}} \left( 1 + 2.294 \text{Re}(C_V^{b c e τ ν_τ}) + 1.147 |C_V^{b c e τ ν_τ}|^2 \right),$$

(31)

$$\text{BR}(B_c^- \to τ^- \bar{ν}_τ) = \text{BR}(B_c^- \to τ^- \bar{ν}_τ)_{\text{SM}} \left| 1 + C_V^{b c e τ ν_τ} \right|^2,$$

(32)

respectively, where $\tau_{D^*} = R(D^*)/R(D^*)_{\text{SM}}$. For $BR(B_c^- \to τ^- \bar{ν}_τ)$, we will use the bound < 10% [66]. Concerning to the ratio $R(Λ_c)$ very recently measured by LHCb [45], the SM contribution is also rescaled by the overall factor $\left| 1 + C_V^{b c e τ ν_τ} \right|^2$, namely [117]

$$R(Λ_c) = R(Λ_c)_{\text{SM}} \left| 1 + C_V^{b c e τ ν_τ} \right|^2.$$

(33)

A long term integrated luminosity of 50 ab$^{-1}$ is expected to be accumulated by the Belle II experiment [11], allowing improvements at the level of $\sim 3\%$ and $\sim 2\%$ for the statistical and systematic uncertainties of $R(D)$ and $R(D^*)$, respectively.
the leptonic decay

Using the result of the global fit, Eq. (41), this corresponds to

where

which is relevant for the couplings aiming to explain the \( C_P \) and \( C_R \) measurements of \( \tau \) respectively. Several global analyses of the most current \( b \)-quark oscillations have been performed, including the NP encoded in the WCs

agreement with the experimental value reported by Belle [122] (see Table I). The \( W' \) boson coupling to lepton pair \( \mu \nu \) modifies this ratio as

\[
R_D^{\mu/e} = |R_D^{\mu/e}|_{\text{SM}} |1 + C_V^{\mu\nu}\rangle^2.
\]

From this ratio we get the bound

\[
\left| \frac{(V_{cs}g_{ab}^q + V_{cb}g_{ab}^q)g_\mu^\ell}{M_V^2} \right| \leq 0.013 \, \text{TeV}^{-2},
\]

which is relevant for the couplings aiming to explain the \( b \to s\mu^+\mu^- \) anomaly (see Sec. III C).

**B. \( b \to u\tau\bar{\nu}_\tau \) data**

The TVB model can also induce NP contributions in the charged-current transition \( b \to u\tau\bar{\nu}_\tau \), such as the case of the leptonic decay \( B \to \tau\bar{\nu}_\tau \). Its branching fraction can be rescaled as

\[
\text{BR}(B^+ \to \tau^-\bar{\nu}_\tau) = \text{BR}(B^+ \to \tau^-\bar{\nu}_\tau)_{\text{SM}} |1 + C_V^{\mu\nu}\rangle^2,
\]

where

\[
C_V^{\mu\nu} = \frac{\sqrt{2}}{4G_FV_{ub}} \times \frac{2(V_{cs}g_{ab}^q + V_{cb}g_{ab}^q)g_\mu^\ell}{M_V^2},
\]

with \( V_{cs} \) and \( V_{ub} \) denoting the CKM matrix elements involved. In Table I we show the current experimental value reported by the Particle Data Group (PDG) [123], as well as its corresponding SM estimation. This theoretical value was obtained by using \( f_B = (190.0 \pm 1.3) \, \text{MeV} \) and \( V_{ub} = (3.94 \pm 0.36) \times 10^{-3} \) from PDG [123].

**C. \( b \to s\mu^+\mu^- \) data**

The NP effective Lagrangian responsible for the semileptonic transition \( b \to s\mu^+\mu^- \) can be expressed as

\[
\mathcal{L}(b \to s\mu^+\mu^-)_{\text{NP}} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^\ast (C_9^{bs\mu\mu} C_9^{bs\mu\mu} + C_9^{bs\mu\mu} C_9^{bs\mu\mu}) + \text{h.c.},
\]

where the NP is encoded in the WCs \( C_9^{bs\mu\mu} \) and \( C_{10}^{bs\mu\mu} \) of the four-fermion operators

\[
C_9^{bs\mu\mu} = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s}_\gamma \mu_P b)(\bar{\mu}_\gamma \mu),
\]

\[
C_{10}^{bs\mu\mu} = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s}_\gamma \mu_P b)(\bar{\mu}_\gamma \mu_\gamma \gamma_5),
\]

respectively. Several global analyses of the most current \( b \to s\mu^+\mu^- \) data have been recently performed suggesting various NP interpretations [1, 18–24]. Among the NP scenarios, the \( C_9^{bs\mu\mu} = -C_{10}^{bs\mu\mu} \) solution is strongly preferred by the data. In our analysis we will take into account the results obtained by Altmannshofer and Stangl [19], in which the best fit 1\( \sigma \) solution leads to

\[
C_9^{bs\mu\mu} = -C_{10}^{bs\mu\mu} \in [-0.46, -0.32].
\]

In the context of the TVB model, the \( Z' \) boson induces a tree-level contribution to \( b \to s\mu^+\mu^- \) transition via the WCs

\[
C_9^{bs\mu\mu} = -C_{10}^{bs\mu\mu} = -\frac{\pi}{\sqrt{2}G_F\alpha_{\text{em}}V_{tb}V_{ts}^\ast} \times \frac{g_\mu^q g_\mu^\ell}{M_V^2}.
\]

Using the result of the global fit, Eq. (41), this corresponds to

\[
- \frac{g_\mu^q g_\mu^\ell}{M_V^2} \in [4.8, 6.9] \times 10^{-4} \, \text{TeV}^{-2}.
\]
D. Bottomonium processes: $R_{T(nS)}$ and $\Upsilon \to \mu^+\tau^-$

Test of LFU has been also studied in the leptonic ratio $R_{T(nS)}$ (with $n = 1, 2, 3$) in connection with the reported hints of LFU violation in the charged-current transition $b \to c\tau\bar{\nu}_\tau$ [67, 68]. It is known that NP scenarios aiming to provide an explanation to the anomalous $b \to c\tau\bar{\nu}_\tau$ data, also induce effects in the neutral-current transition $bb \to \tau^+\tau^-$ [67, 68]. Experimentally, the BABAR and CLEO Collaborations have reported the values [118–120]

\[
R_{T(1S)} = \begin{cases} 
\text{BABAR-10: } 1.005 \pm 0.013 \pm 0.022 \text{ \[118\]}, \\
\text{SM: } 0.9924 \text{ \[68\]},
\end{cases}
\]

\[
R_{T(2S)} = \begin{cases} 
\text{CLEO-07: } 1.04 \pm 0.04 \pm 0.05 \text{ \[119\]}, \\
\text{SM: } 0.9940 \text{ \[68\]},
\end{cases}
\]

\[
R_{T(3S)} = \begin{cases} 
\text{CLEO-07: } 1.05 \pm 0.08 \pm 0.05 \text{ \[119\]}, \\
\text{BABAR-20: } 0.966 \pm 0.008 \pm 0.014 \text{ \[120\]}, \\
\text{SM: } 0.9948 \text{ \[68\]},
\end{cases}
\]

where the theoretical uncertainty is typically of the order $\pm O(10^{-5})$ [68]. These measurements are in good accordance with the SM estimations, except for the 2020 measurement on $R_{T(3S)}$ that shows an agreement at the 1.7σ level [120]. By averaging the CLEO-07 [119] and BABAR-20 [120] measurements we obtain $R_{T(3S)}^{\text{Ave}} = 0.968 \pm 0.016$, which deviates at the 1.7σ level with respect to the SM prediction [67].

The NP effects of the TVB model on the leptonic ratio can be expressed as [67, 68]

\[
R_{T(nS)} = \frac{1 - 4x_\tau^2}{|A_{V}^{SM}|^2} \left[ |A_{V}^{T(1S)}|^2 (1 + 2x_\tau^2) + |B_{V}^{T(1S)}|^2 (1 - 4x_\tau^2) \right],
\]

with $x_\tau = m_\tau/m_{T(nS)}$, $|A_{V}^{SM}| = -4\pi\alpha Q_b$, and

\[
A_{V}^{T(1S)} = -4\pi\alpha Q_b + \frac{m_{T(nS)}^2}{4} \frac{g_{\bar{b}b}(g_{\mu\tau})^*}{M_V^2},
\]

\[
B_{V}^{T(1S)} = -\frac{m_{T(nS)}^2}{2} \frac{g_{\bar{b}b}(g_{\mu\tau})^*}{M_V^2}.
\]

The neutral gauge boson also generates the LFV processes $\Upsilon \to \mu^+\tau^-$ ($\Upsilon \equiv \Upsilon(nS)$). The branching fraction is given by [75, 76]

\[
\text{BR}(\Upsilon \to \mu^+\tau^-) = \frac{f_\Upsilon^2 m_{\Upsilon}^2}{48\pi \Gamma_\Upsilon} \left( 2 + \frac{m_\mu^2}{m_{\Upsilon}^2} \right) \left( 1 - \frac{m_\mu^2}{m_{\Upsilon}^2} \right)^2 \left| \frac{g_{\bar{b}b}(g_{\mu\tau})^*}{M_V^2} \right|^2,
\]

where $f_\Upsilon$ and $m_\Upsilon$ are the Upsilon decay constant and mass, respectively. The decay constant values can be extracted from the experimental branching ratio measurements of the processes $\Upsilon \to e^- e^+$. Using current data from the Particle Data Group (PDG) [123], one obtains $f_{\Upsilon(1S)} = (659 \pm 17)$ MeV, $f_{\Upsilon(2S)} = (468 \pm 27)$ MeV, and $f_{\Upsilon(3S)} = (405 \pm 26)$ MeV. Experimentally, the Particle Data Group reported the UL BR($\Upsilon(1S) \to \mu^+\tau^-) < 6.0 \times 10^{-6}$, BR($\Upsilon(2S) \to \mu^+\tau^-) < 3.3 \times 10^{-6}$, and BR($\Upsilon(3S) \to \mu^+\tau^-) < 3.1 \times 10^{-6}$. In our analysis we will only take into account the UL from $\Upsilon(3S)$ that leads to the strongest bound, namely

\[
\frac{|g_{\bar{b}b}(g_{\mu\tau})^*|}{M_V^2} < 5.2 \text{ TeV}^{-2}.
\]

Belle II would be able to improve BR($\Upsilon(3S) \to \mu^+\tau^-)$ down to $\sim 10^{-7}$ [11], allowing us to set a bound of the order $|g_{\bar{b}b}(g_{\mu\tau})^*|/M_V^2 \sim 0.94 \text{ TeV}^{-2}$.

E. $\Delta F = 2$ processes: $B_s - \bar{B}_s$ and $D^0 - \bar{D}^0$ mixing

The interactions of a $Z'$ boson to quarks $\bar{s}b$ relevant for $b \to s\mu^+\mu^-$ processes also generate a contribution to $B_s - \bar{B}_s$ mixing [112, 113]. The NP effects to the $B_s - \bar{B}_s$ mixing can be described by the effective Lagrangian

\[
\mathcal{L}_{\Delta B=2}^{Z'} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \frac{1}{2} C_{sb}^{LL}(\bar{s}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L b) + \text{h.c.},
\]
where

\[ C_{sb}^{LL} = \frac{1}{4v^2G_F |V_{tb}V_{ts}^*|^2} \frac{|g_{sb}^q|}{M_{Z'}^2}. \]  

Thus, the NP contributions to the mass difference \( \Delta M_s \) of the neutral \( B_s \) meson can be expressed as [112]

\[ \frac{\Delta M_{s}^{SM+NP}}{\Delta M_{s}^{SM}} = \left( 1 + \frac{\eta_{\text{loop}}^{6/23}}{R_{\text{SM}}^{\text{loop}}} C_{sb}^{LL} \right), \]  

where \( \eta = \alpha_s(M_{Z'})/\alpha_s(m_b) \) accounts for running from the \( M_{Z'} \) scale down to the \( b \)-quark mass scale and the SM loop function is \( R_{\text{SM}}^{\text{loop}} = (1.310 \pm 0.010) \times 10^{-3} \) [112]. At present, \( \Delta M_s \) has been experimentally measured with great precision \( \Delta M_{s}^{\text{Exp}} = (17.757 \pm 0.021) \text{ ps}^{-1} \) [39, 112]. On the theoretical side, the most recent 2019 average is \( \Delta M_{s}^{\text{SM}} = (18.4^{+0.7}_{-1.2}) \text{ ps}^{-1} \) implying that \( \Delta M_{s}^{\text{SM}}/\Delta M_{s}^{\text{Exp}} = 1.04^{+0.04}_{-0.07} \) [112]. This value yields to

\[ 0.89 \leq 1 + \frac{\eta_{\text{loop}}^{6/23}}{R_{\text{SM}}^{\text{loop}}} C_{sb}^{LL} \leq 1.11, \]  

where in the TVB model translates into the important 2\( \sigma \) bound

\[ \left| \frac{g_{sb}^q}{M_V} \right| \geq 3.9 \times 10^{-3} \text{ TeV}^{-1}. \]  

In addition, the \( Z' \) boson can also admit \( c \to u \) transitions, consequently generating tree-level effects on \( D^0 - \bar{D}^0 \) mixing [76, 114]. The effective Lagrangian describing the \( Z' \) contribution to \( D^0 - \bar{D}^0 \) mixing can be expressed as [76, 114]

\[ \mathcal{L}_{Z'^{c=2}} = - \frac{|g_{uc}|^2}{2M_{Z'}} (\bar{c} \gamma_\mu P_L u)(\bar{c} \gamma^\mu P_L u) + \text{ h.c.}, \]  

where \( g_{uc} = g_{sb}^q V_{cb} V_{ub}^* + g_{sb}^q (V_{cs} V_{ub}^* + V_{cb} V_{us}^*) + g_{sb}^q V_{cs} V_{us}^* \) [76] (see also Table IV). Such a NP contributions are constrained by the results of the mass difference \( \Delta M_D \) of neutral \( D \) mesons. The theoretical determination of this mass difference is limited by our understanding of the short and long-distance contributions [76, 114]. Here we follow the recent analysis of Ref. [76] focused on short-distance SM contribution that sets the conservative (strong) bound

\[ \left| \frac{g_{sb}^q}{M_V} \right| \leq 3 \times 10^{-3} \text{ TeV}^{-1}. \]  

The couplings \( g_{sb}^q \) are less constrained by \( \Delta M_D \) [76], therefore, we will skip them in our study.

### F. Neutrino Trident Production

The \( Z' \) couplings to leptons from second-generation \((g_{\mu \mu} = g_{e\mu, e\mu})\) also generate contribution to the cross section of neutrino trident production (NTP), \( \nu_\mu, N \to \nu_\mu, N \mu^+ \mu^- \) [115]. The cross section is given by [88, 115]

\[ \frac{\sigma_{\text{SM+NP}}}{\sigma_{\text{SM}}} = \frac{1}{1 + (1 + 4s_W^2 \nu^2)} \left[ \left( 1 + \frac{v^2 g_{\mu \mu}^2}{M_V^2} \right)^2 + \left( 1 + 4s_W^2 + \frac{v^2 g_{\mu \mu}^2}{M_V^2} \right)^2 \right], \]  

where \( \nu = (\sqrt{2}G_F)^{-1/2} \) and \( s_W \equiv \sin \theta_W \) (with \( \theta_W \) the Weinberg angle). The existing CCFR trident measurement \( \sigma_{\text{CCFR}}/\sigma_{\text{SM}} = 0.82 \pm 0.28 \) provides the upper bound

\[ \left| \frac{g_{\mu \mu}}{M_{Z'}} \right| \leq 1.13 \text{ TeV}^{-1}. \]
G. LFV $B$ decays: $B^+ \to K^+ \mu^+ \tau^-$ and $B^0 \to \mu^+ \tau^-$

The $Z'$ boson mediates LFV transitions $b \to s\mu^+ \tau^-$ at tree level via the WCs [83]

$$C_9^{b\mu\tau} = -C_{10}^{b\mu\tau} = -\frac{\pi g_s^b(g_{\mu\tau}^f)^*}{\sqrt{2}G_F Q_{em} V_{tb} V_{ts}^* M_V^2}$$ \hspace{1cm} (61)

The branching ratio of $B^+ \to K^+ \mu^+ \tau^-$ can be written as [83]

$$\text{BR}(B^+ \to K^+ \mu^+ \tau^-) = (a_K |C_9^{b\mu\tau}|^2 + b_K |C_{10}^{b\mu\tau}|^2) \times 10^{-9},$$ \hspace{1cm} (62)

with $a_K = 9.6 \pm 1.0$ and $b_K = 10.0 \pm 1.3$. These numerical coefficients have been calculated using the $B \to K$ transitions form factors obtained from lattice QCD [70]. The decay channel with final state $\mu^- \tau^+$ can be easily obtained by replacing $\mu \leftrightarrow \tau$. The current experimental limits (90% C.L.) on the branching ratios are [123]

$$\text{BR}(B^+ \to K^+ \mu^+ \tau^-)_{\text{exp}} < 4.5 \times 10^{-5},$$ \hspace{1cm} (63)

$$\text{BR}(B^+ \to K^+ \mu^+ \tau^-)_{\text{exp}} < 2.8 \times 10^{-5}. $$ \hspace{1cm} (64)

Let us notice that LHCb Collaboration obtained a limit of $\text{BR}(B^+ \to K^+ \mu^+ \tau^-)_{\text{LHCb}} < 3.9 \times 10^{-5}$ [124] that is comparable with the one quoted above from PDG. The Belle II experiment is expected to reach a sensitivity of $\text{BR}(B^+ \to K^+ \mu^+ \tau^-) \sim 3.3 \times 10^{-6}$, for an integrated luminosity of 50 ab$^{-1}$ [11]. This corresponds to the bounds

$$B^+ \to K^+ \mu^+ \tau^- : \frac{|g_{sb}^q(g_{\mu\tau}^f)^*|}{M_V^2} < 7.2 \times 10^{-2} \text{ TeV}^{-2},$$ \hspace{1cm} (65a)

$$B^+ \to K^+ \mu^+ \tau^- : \frac{|g_{sb}^q(g_{\mu\tau}^f)^*|}{M_V^2} < 5.7 \times 10^{-2} \text{ TeV}^{-2},$$ \hspace{1cm} (65b)

while for the Belle II projection we have for both combination of quark-lepton couplings $|g_{sb}^q(g_{\mu\tau}^f)|/M_V^2 \lesssim 2 \times 10^{-2} \text{ TeV}^{-2}$.

As for the LFV leptonic decay $B_s \to \mu^+ \tau^-$, the branching ratio is [70]

$$\text{BR}(B_s^0 \to \mu^+ \tau^-) = \tau_{B_s} f_{B_s}^2 m_{B_s} m_B^2 \frac{2\pi^3}{32} a^2 G_F^2 |V_{tb} V_{ts}^*|^2 \left(1 - \frac{m_{\tau}^2}{m_{B_s}^2}\right)^2 (|C_9^{b\mu\tau}|^2 + |C_{10}^{b\mu\tau}|^2),$$ \hspace{1cm} (66)

where $f_{B_s} = (230.3 \pm 1.3)$ MeV is the $B_s$ decay constant [39] and we have used the limit $m_{\tau} \gg m_{\mu}$. Recently, the LHCb experiment has reported the first upper limit of $\text{BR}(B_s \to \mu^+ \tau^-) < 4.2 \times 10^{-5}$ at 95% CL. [125]. Thus, one gets the following limit

$$\frac{|g_{sb}^q(g_{\mu\tau}^f)^*|}{M_V^2} < 5.1 \times 10^{-2} \text{ TeV}^{-2},$$ \hspace{1cm} (67)

which provides a slightly stronger bound than $B^+ \to K^+ \mu^+ \tau^-$. 

H. Rare $B$ decays: $B \to K^{(*)}\nu\bar{\nu}$, $B \to K\tau^+\tau^-$ and $B_s \to \tau^+\tau^-$

Recently, the interplay between the di-neutrino channel $B \to K^{(*)}\nu\bar{\nu}$ and the $B$ meson anomalies has been studied by several works [114, 126–129]. In the NP scenario under study, the $Z'$ boson can give rise to $B \to K^{(*)}\nu\bar{\nu}$ at tree level. The effective Hamiltonian for the $b \to s\nu\bar{\nu}$ transition is given by [130]

$$\mathcal{H}_{\text{eff}}(b \to s\nu\bar{\nu}) = -\frac{\alpha_{\text{em}} G_F}{\sqrt{2} \pi} V_{tb} V_{ts}^* C_{L}^{ij} \langle \bar{s}\gamma^\mu P_L b \rangle (\bar{\nu}_i \gamma_\mu (1 - \gamma_5) \nu_j),$$ \hspace{1cm} (68)

where $C_{L}^{ij} = C_{L}^{SM} + \Delta C_{L}^{ij}$ is the aggregate of the SM contribution $C_{L}^{SM} \approx -6.4$ and the NP effects $\Delta C_{L}^{ij}$, that in the TVB framework read as

$$\Delta C_{L}^{ij} = \frac{\pi}{\sqrt{2} G_F Q_{em} V_{tb} V_{ts}^* M_V^2} g_{sb}^q g_{\mu\tau}^f,$$ \hspace{1cm} (69)
with $i, j = \mu, \tau$. By defining the ratio [130]

$$R_{K^*(\tau)}^{\nu}\equiv \frac{BR(B \to K^{(*)}\nu\bar{\nu})}{BR(B \to K^{(*)}\nu\bar{\nu})_{SM}},$$

(70)

the NP contributions can be constrained. In the TVB model this ratio is modified as

$$R_{K^*(\tau)}^{\nu}(\tau) = \sum_{ij} \frac{\delta_{ij} C_{ij}^{SM} + \Delta C_{ij}^{ij} |^2}{3|C_{ij}^{SM}|^2},$$

(71)

$$= 1 + \sum_{i} 2 C_{ij}^{SM} \Delta C_{ij}^{ij} + \sum_{ij} \frac{|\Delta C_{ij}^{ij}|^2}{3|C_{ij}^{SM}|^2},$$

(72)

From this expression we can observe that diagonal leptonic couplings $g_{i\mu}^i$ and $g_{i\tau}^i$ contribute to $b \to s\nu_\mu\bar{\nu}_\mu$ (relevant for $b \to s\mu^+\mu^-$ data) and $b \to s\nu_\tau\bar{\nu}_\tau$ (relevant for $b \to c\tau\bar{\nu}_\tau$ data), respectively. In addition, since the neutrino flavor is experimentally unobservable in heavy meson experiments, it is also possible to induce the LFV transitions $b \to s\nu_\mu\bar{\nu}_\tau$ (and $\nu_\tau\bar{\nu}_\mu$) through the off-diagonal coupling $g_{ij}^{\mu\tau}$.

On the experimental side, the Belle experiment in 2017 obtained the following ULs on the branching fractions $BR(B \to K\nu\bar{\nu}) < 1.6 \times 10^{-5}$ and $BR(B \to K^*\nu\bar{\nu}) < 2.7 \times 10^{-5}$ [131], resulting in limits on the ratios, $R_{K^*}^{\nu} < 3.9$ and $R_{K^*}^{\nu.2} < 2.7$ (90% C.L.), respectively [131]. In 2021, based on an inclusive tagging technique, the Belle II experiment reported the bound $BR(B^+ \to K^*\nu\bar{\nu}) < 4.1 \times 10^{-5}$ at 90% C.L. [132]. A combination of this new result with previous experimental results leads to the weighted average $BR(B^+ \to K^*\nu\bar{\nu}) = (1.1 \pm 0.4) \times 10^{-5}$ [133]. In turn, the ratio $R_{K^+}^{\nu}$ has been calculated to be, $R_{K^+}^{\nu} = 2.4 \pm 0.9$ [128].

The rare $B$ processes $B_s \to \tau^+\tau^-$ and $B \to K^\tau\tau^-$ (induced via $b \to s\tau^+\tau^-$ transition) are expected to receive significant NP impact. For the leptonic process $B_s \to \tau^+\tau^-$, the SM branching ratio is shifted by the factor

$$BR(B_s \to \tau^+\tau^-) = \left| 1 + \frac{\pi}{\sqrt{2} G_F \alpha_{em} V_{tb} V_{ts}^* C_{10}^{SM}} g_{10}^{\nu\tau^+\tau^-} \right|^2,$$

(73)

where $C_{10}^{SM} \simeq -4.3$. The strongest experimental bound on its branching ratio has been obtained by the LHCb, $BR(B_s \to \tau^+\tau^-) < 6.8 \times 10^{-3}$ at 95% confidence level [134], while its SM predictions is $BR(B_s^0 \to \tau^+\tau^-)_{SM} = (7.73 \pm 0.49) \times 10^{-7}$ [135]. At Belle II, the existing limits will be improved up to the order of $\sim 8.1 \times 10^{-4}$ for 5 ab$^{-1}$ data [11]. While, a projected sensitivity of $\sim 5 \times 10^{-4}$ is expected at LHCb with 50 fb$^{-1}$ [136]. The bound is

$$g_{10}^{\nu\tau^+\tau^-} |_{M_V^2} < 0.56 \text{ TeV}^{-2},$$

(74)

and $0.19 \text{ TeV}^{-2}$ (0.15 TeV$^{-2}$) for Belle II (LHCb) projected sensitivity.

As concerns the semileptonic decay $B \to K^\tau\tau^-$, a easy handle numerical formula for the branching ratio (over the whole kinematic range for the lepton pair invariant mass) has been obtained in Ref. [137], for the case of a singlet vector leptoquark explanation of the $B$ meson anomalies. Since the NP contribution is generated via the same operator, this expression can be easily (but properly) translated to the TVB model, namely

$$BR(B \to K^\tau\tau^-) \simeq 1.5 \times 10^{-7} + 1.4 \times 10^{-3} \left( \frac{1}{2\sqrt{2} G_F} \right) \text{Re} |g_{10}^{\nu\tau^+\tau^-}| \left( \frac{1}{M_V^2} \right) + 3.5 \left( \frac{1}{2\sqrt{2} G_F} \right)^2 |g_{10}^{\nu\tau^+\tau^-}|^2 \left( \frac{1}{M_V^2} \right).$$

(75)

This decay channel has not been observed so far and the present reported bound is $BR(B \to K^\tau\tau^-) < 2.25 \times 10^{-3}$ [123], and the planned Belle II sensitivity is $\sim 2.0 \times 10^{-3}$ for 50 ab$^{-1}$ data [11]. We obtained the following bound

$$g_{10}^{\nu\tau^+\tau^-} |_{M_V^2} < 0.83 \text{ TeV}^{-2},$$

(76)

that is weaker than the one get from $B_s \to \tau^+\tau^-$. The Belle II improvement in sensitivity would imply a bound of $\sim 1.8 \times 10^{-2}$ TeV$^{-2}$. 


I. $\tau$ decays: $\tau \rightarrow 3\mu$, $\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau$, and $\tau \rightarrow \mu \phi$

It is known that the TVB model can produce the four-lepton operators $(\bar{\mu} \gamma^\alpha P_L \tau)(\bar{\mu} \gamma^\alpha P_L \mu)$ and $(\bar{\mu} \gamma^\alpha P_L \tau)(\bar{\mu} \gamma^\alpha P_L \nu_\mu)$, thus yielding to tree-level contributions to the leptonic $\tau$ decays, $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu^- \mu^-$ ($\tau \rightarrow 3\mu$) and $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$, respectively [75, 76]. For the LFV decay $\tau \rightarrow 3\mu$, the expression for the branching ratio can be written as

$$\text{BR}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu^- \mu^-) = \frac{m_\tau^5}{1536 \pi^3 \Gamma_\tau} \left| \frac{g^\ell_\mu g^\ell_\mu}{M_V^6} \right|^2,$$  

(77)

where $\Gamma_\tau$ is the total decay width of the $\tau$ lepton. The current experimental UL obtained by Belle (at 90% CL) is $\text{BR}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu^- \mu^-) < 2.1 \times 10^{-8}$ [123]. This corresponds to

$$\left| \frac{g^\ell_\mu g^\ell_\mu}{M_V^6} \right| < 1.13 \times 10^{-2} \text{ TeV}^{-2}.$$  

(78)

The expected Belle II upper limit is $\sim 3.3 \times 10^{-10}$ with the 50 ab$^{-1}$ data sample [11] would lead to $|g^\ell_\mu g^\ell_\mu|/M_V^6 < 1.35 \times 10^{-3}$ TeV$^{-2}$.

The leptonic decay $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$ is a lepton flavor conserving and SM allowed process that receives tree-level contribution from both $Z'$ and $W'$ bosons [76]. Following the discussion presented in Ref [76], we will consider the conservative constraint

$$\left| \frac{g^\ell_\mu g^\ell_\mu}{M_V^6} \right| < 0.1 \text{ TeV}^{-2},$$  

(79)

in which small LFV coupling $g^\ell_\mu$ contribution has been neglected.

The leptonic decay $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$ is a lepton flavor conserving and SM allowed process that receives tree-level contribution from both $W'$ (via lepton flavor conserving couplings) and $Z'$ (via LFV couplings) bosons [76]. The branching ratio is given by [76]

$$\text{BR}(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau) = \text{BR}(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau)_{\text{SM}} \left( 1 + \frac{1}{2 \sqrt{2} G_F M_V^2} (2 |g^\ell_\mu g^\ell_\mu| - |g^\ell_\mu|^2) \right)^2 + |g^\ell_\mu|^2 \right)^2, \text{ (80)}$$

where $\text{BR}(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau)_{\text{SM}} = (17.29 \pm 0.03)\%$ [138]. The $Z'$ boson can also generates one-loop corrections, which can be safely ignored. This value has to be compared with the experimental value reported by PDG $\text{BR}(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau) = (17.39 \pm 0.04)\%$ [123].

Finally, the branching ratio of the LFV hadronic $\tau$ decay $\tau \rightarrow \mu \phi$ ($\tau \rightarrow \mu s \bar{s}$ transition), can be expressed as [75]

$$\text{BR}(\tau^- \rightarrow \mu^- \phi) = \frac{f_\phi^2 m_\phi^3}{128 \pi \Gamma_\tau} \left( 1 + 2 \frac{m_\phi^2}{m_\phi^2} \right) \left( 1 - \frac{m_\phi^2}{m_\phi^2} \right)^2 \frac{|g^\ell_\mu g^\ell_\mu|^2}{M_V^6}, \text{ (81)}$$

where $m_\phi$ and $f_\phi = (238 \pm 3)$ MeV [76] are the $\phi$ meson mass and decay constant, respectively. Currently, the UL on the branching ratio is $\text{BR}(\tau^- \rightarrow \mu^- \phi) < 8.4 \times 10^{-8}$ [123]. An envisaged limit by Belle II is $\sim 2 \times 10^{-9}$ [11]. The current UL is translated into the bound

$$\left| \frac{g^\ell_\mu g^\ell_\mu}{M_V^6} \right| < 1.8 \times 10^{-2} \text{ TeV}^{-2},$$  

(82)

whereas from Belle II we might expect improvements at the level of $\sim 3 \times 10^{-3}$ TeV$^{-2}$. Since the $D^0 - \bar{D}^0$ mixing imposes that $|g^\ell_\mu|^2/M_V \leq 3.3 \times 10^{-3}$ TeV$^{-1}$ (see Sec. III E) the constraint from $\tau \rightarrow \mu \phi$ can be easily fulfilled, even for the Belle II projections. We will not take into account this LFV process in further TVB model analysis.

J. LHC bounds

LHC constraints are always important for models with non-zero $Z'$ couplings to the SM particles [139]. In particular, in our study, it will set important constraints on the parametric space of the TVB couplings $(g^\ell_\mu, g^\ell_\mu)$ and $(g^\ell_\mu, g^\ell_\mu)$. 
By using the ATLAS search for high-mass dilepton resonances in the mass range of 250 GeV to 6 TeV, in proton-proton collisions at a center-of-mass energy of $\sqrt{s} = 13$ TeV during Run 2 of the Large Hadron Collider (LHC) with an integrated luminosity of 139 fb$^{-1}$ [140], we obtain for $M_V = 1$ TeV the lower limit on the parameter space from the intersection of the 95% CL upper limit on the cross-section from the the ATLAS experiment [140]. For the theoretical cross-section we use the one reported in Ref. [141]. The TVB model has zero couplings to the first family to avoid the strong LHC constraints from the coupling between a $Z'$ boson and valence quarks. The strongest restrictions come from $Z'$ production processes in the $b\bar{b}$ annihilation and the subsequent $Z'$ decay into muons ($\mu^+\mu^-$) and taus ($\tau^+\tau^-$). Further details are shown in Refs. [141–143]. Let us remark that within the TVB framework is also possible to consider the annihilation between quarks with different flavors (namely, $g_b^{\ell}$), however, we anticipate that according to our phenomenological analysis in Sec. IV this coupling is very small; therefore, we only consider production processes without flavor changing neutral currents. In the next Sec. IV we will show that the TVB parameter space is limited by LHC constraints to regions where the couplings of the lepton or the quarks are close to zero, excluding the regions preferred by the $B$ meson anomalies and low-energy flavor observables.

**IV. ANALYSIS ON THE TVB PARAMETRIC SPACE**

In this section we present the parametric space analysis of the TVB model addressing a simultaneous explanation of the $b \to s\mu^+\mu^-$ and $b \to c\tau\bar{\nu}_\tau$ anomalies. We define the pull for the $i$-th observable as

$$\text{pull}_i = \frac{O_{\text{exp}}^i - O_{\text{th}}^i}{\Delta C_i},$$

where $O_{\text{exp}}^i$ is the experimental measurement, $O_{\text{th}}^i \equiv O^i(\tilde{g}_b^{\ell}, \tilde{g}_b^{\ell}, g_b^{\ell}, g_\mu^i, g_{\tau}^i)$ is the theoretical prediction that include the NP contributions, and $\Delta C_i = ((\Delta O_{\text{exp}}^i)^2 + (\Delta O_{\text{th}}^i)^2)^{1/2}$ corresponds to the combined experimental and theoretical uncertainties. By means of the pull, we can compare the fitted values of each observable to their measured values. The $\chi^2$ function is written as the sum of squared pulls, i.e.,

$$\chi^2 = \sum_{i} (\text{pull}_i)^2,$$

where the sum extends over the number of observables ($N_{\text{obs}}$) to be fitted. To account for the experimental correlation between $R(D)$ and $R(D^*)$, we will use in our analysis the correlation value $-0.38$ from HFLAV [39, 40]. Likewise, we use the $p$-value to evaluate the fit-quality.

Our analysis is based on the flavor observables presented in the previous Sec. III. Let us recall that this dataset includes: $b \to c\tau\bar{\nu}_\tau$ and $b \to s\mu^+\mu^-$ data, bottomonium ratios $R_{\Upsilon(nS)}$, LFV decays ($B^+ \to K^+\mu^+\tau^-$, $B_s \to \mu^+\tau^-$, $\Upsilon(nS) \to \mu^+\tau^-$), rare $B$ decays ($B \to K^{(*)}\mu\bar{\nu}_\mu$), $B \to K^+\tau^+\tau^-$, $B_s \to \tau^+\tau^-$), and $\tau$ decays ($\tau \to 3\mu$, $\tau \to \mu\bar{\nu}_\mu\nu_\tau$). For the processes that only ULs have been experimentally reported so far, we will include them into the fit as a branching fraction of $0 \pm 1 \text{UL}/2$. In addition, the $B_s$ mixing and neutrino trident production are considered as constraints to be fulfilled. Our goal is to present an updated status of the TVB model as an explanation to the current $b \to c\tau\bar{\nu}_\tau$ and $b \to s\mu^+\mu^-$ data (referred to us as “current data”) hinting to LFU violation. In order to see the impact of the very recent LHCb measurement of $R(\Lambda_c)$, Eq. (10), into our analysis, we simply add $R(\Lambda_c)$ to the current data observables (referred to us as “current data + $R(\Lambda_c)$”). Furthermore, taking into account the Belle II envisaged improvements on different observables previously discussed in Sec. III, we investigate the impact of such sensitivities on the TVB model. For this purpose we will take the following considerations:

- Future Belle II sensitivities on the branching fraction of LFV decays ($B^+ \to K^+\mu^+\tau^-$, $B_s \to \mu^+\tau^-$, $\Upsilon(nS) \to \mu^+\tau^-$), and rare $B$ decays ($B \to K^{(*)}\mu\bar{\nu}_\mu$, $B \to K^+\tau^+\tau^-$, $B_s \to \tau^+\tau^-$).

- For the $b \to s\mu^+\mu^-$ data, we contemplate the future scenario with central values remaining the same, i.e., the $b \to s\mu^+\mu^-$ anomaly keeps in the future.

- With respect to $b\bar{b} \to \tau^+\tau^-$ processes, we assume that bottomonium leptonic decay ratios $R_{\Upsilon(nS)}$ keep the central values, particularly, the recently obtained by BABAR on $R_{\Upsilon(3S)}$.

- Finally, as concerns the $b \to c\tau\bar{\nu}_\tau$ data, we will pay particular attention to the Belle II prospects on $R(D^{(*)})$ [11] by considering two plausible scenarios: (1) Belle II measurements on $R(D^{(*)})$ keep the central values of Belle combination averages [34] with the projected Belle II sensitivities for 50 ab$^{-1}$ and (2) Belle II measurements...
versus the pulls at the best-fit point of the TVB model (Fig. 1(b)), we observe that there is an improvement in values by means of the pull defined in Eq. (83), as illustrated in Fig. 1. Comparing the pulls in the SM fit (Fig. 1(a)) still allow sizeable couplings, thus, leaving room for significant NP contributions.

II-P2 projections of the 1σ intervals of the five TVB couplings in conjunction with the projected Belle II sensitivities for 50 ab⁻¹. In general, it is found that in the quark sector this analysis requires small allowed intervals for the TVB couplings (Table VI). In the lepton sector, we have a total of 26 observables, implying to a number of degrees of freedom of $N_{dof} = 21$. For the remaining $b \to c\tau\nu_\tau$ observables ($R(J/\psi)$, polarizations $P_x(D^*)$ and $P_L(D^*)$, $R(X_c)$, and $R(\Lambda_c)$), we will assume that hold the same experimental values.

Regarding these two $R(D^{(*)})$ scenarios along with other observables perspectives we will refer to these data set of projections as “Belle II-P1” and “Belle II-P2”, respectively. Such implications were not explored in the KLW analysis [76], neither in other recent works.

By fixing the TVB mass to the benchmark value $M_V = 1$ TeV, there is a set of five TVB couplings ($g_{bs}^0, g_{bb}^0, g_{\mu\mu}^0, g_{\tau\tau}^0, g_{\mu\tau}^0$) to be fitted. For the current data we have a total of 26 observables, implying to a number of degrees of freedom of $N_{dof} = 21$. As for current data + $R(\Lambda_c)$, we have 27 observables and $N_{dof} = 22$. For these two sets of observables we find the best-fit point values by minimizing the $\chi^2$ function. For the fit of current data we get $\chi^2_{min} = 11.8$, with a good fit of the data $\chi^2_{min}/N_{dof} = 0.57$ corresponding to a $p$-value = 94%. While for current data + $R(\Lambda_c)$, we obtained $\chi^2_{min} = 14.6$ with a goodness $\chi^2_{min}/N_{dof} = 0.67$ ($p$-value = 87.5%). We notice that the inclusion of $R(\Lambda_c)$ into the analysis does not helps to the improvement of the fit, on the contrary, the goodness of the fit is reduced; however, still keeps as a good fit. In Table VI we report our results of the best-fit point values and 1σ intervals of TVB couplings. In general, it is found that in the quark sector this analysis requires small $g_{bs}^0$ coupling, $g_{bb}^0 = -4.5 \times 10^{-3}$, and opposite sign to $g_{\mu\mu}^0$ to be consistent with $b \to s\mu^+\mu^-$ data ($C^\text{mm}_9 = -C^\text{mm}_{10}$ solution) and $B_s - B_s$ mixing, and prefers large values for the $g_{bb}^0$ coupling. As a matter of fact, in order to avoid too large $g_{bb}^0$ coupling ($\sim \sqrt{4\pi}$) that would put the perturbativity of the model into question, the TVB mass can be as large as $M_V \sim 2$ TeV. As for the leptonic couplings, it is found that the lepton flavor conserving ones have a similar sign, $g_{\mu\mu}^0 \approx g_{\tau\tau}^0 \sim O(10^{-1})$, suggesting non-hierarchy pattern, while the LFV coupling $g_{\mu\tau}^0$ is negligible. Thus, the TVB model do not lead to appreciable LFV effects. On the other hand, we show in Table VII the Belle II-P1 and Belle II-P2 projections of the 1σ allowed intervals for the TVB couplings ($g_{bs}^0, g_{bb}^0, g_{\mu\mu}^0, g_{\tau\tau}^0, g_{\mu\tau}^0$). These projections would still allow sizeable couplings, thus, leaving room for significant NP contributions.

To further discussion, we contrast our fitted values with the $b \to c\tau\bar{\nu}_\tau$ observables evaluated at the best-fit point values by means of the pull defined in Eq. (83), as illustrated in Fig. 1. Comparing the pulls in the SM fit (Fig. 1(a)) versus the pulls at the best-fit point of the TVB model (Fig. 1(b)), we observe that there is an improvement in

| TVB couplings | Best-fit point | 1σ intervals |
|---------------|---------------|--------------|
| $g_{bs}^0$    | $-4.5 \times 10^{-3}$ | $[-5.8, -3.3] \times 10^{-3}$ |
| $g_{bb}^0$    | 2.54          | [1.55, 3.52]  |
| $g_{\mu\mu}^0$| 0.12          | [0.09, 0.15]  |
| $g_{\tau\tau}^0$| 0.45      | [0.29, 0.60]  |
| $g_{\mu\tau}^0$| $\sim 0$    | $[-0.14, 0.14]$ |

| TVB couplings | Belle II-P1 | Belle II-P2 |
|---------------|-------------|-------------|
| $g_{bs}^0$    | $[-3.6, -2.1] \times 10^{-3}$ | $[-2.8, -1.6] \times 10^{-3}$ |
| $g_{bb}^0$    | [1.69, 2.96]  | [0.47, 2.24]  |
| $g_{\mu\mu}^0$| [0.14, 0.23]  | [0.19, 0.31]  |
| $g_{\tau\tau}^0$| [0.24, 0.41] | [0.12, 0.33]  |
| $g_{\mu\tau}^0$| $[-11, 11] \times 10^{-3}$ | $[-8.2, 8.2] \times 10^{-3}$ |
the prediction of $R(D^{(*)})$ and there are only few observables for which the pull is larger than 1. Particularly, the observables that generate more tension are $R(\Lambda_c)$, $R(J/\psi)$, and $F_L(D^*)$, even though they have large experimental uncertainties. In the case of $R(\Lambda_c)$, it is interesting to remark that the experimental measurement is below the SM estimation, thus, pointing into an opposite direction than $R(D^{(*)})$ anomalies. In addition, we have also checked that pulls related with LFV and rare processes are less than 1.

In Fig. 2, we show the allowed regions of the most relevant two-dimension (2D) parametric space of current data for $M_V = 1$ TeV, where the 1σ and 2σ regions are shown in green and light-green colors, respectively. In each plot we are marginalizing over the rest of the parameters. The projections Belle II-P1 and Belle II-P2 for an integrated luminosity of 50 ab$^{-1}$ are illustrated by the red dotted and red solid contours, respectively. Furthermore, we include the LHC bounds (light-gray regions) obtained from searches of high-mass dilepton resonances at the ATLAS experiment [140]. As the main outcome, it is observed in Figs. 2(b) and 2(d) that TVB model is already excluded by the strong LHC bounds, even for the projections at Belle II. Except for the $(g_{1l}^3, g_{1r}^3)$ plane in which projection Belle II-P2 would still allow NP window in agreement with LHC. Therefore, in general, the TVB model can provide combined explanation of the $b \rightarrow c\tau\bar{\nu}_l$ and $b \rightarrow s\mu^+\mu^-$ anomalies (in consistency with other flavor observables); unfortunately, this seems to be strongly ruled out by the LHC bounds.

A similar recent analysis of the TVB model was reported by KLW by implementing the 2018 $b \rightarrow c\tau\bar{\nu}_\tau$ and $b \rightarrow s\mu^+\mu^-$ data [76]. KLW found that the TVB model is excluded as a possible explanation of the $B$ meson anomalies [76], which is in agreement with our result considering the current 2021 data. In addition, we have incorporated the bottomonium leptonic decay ratios $R_{T(J)}$ and the forthcoming sensitivity of Belle II on $R(D^{(*)})$ measurements. Thus, our present study extends and complements the previous analysis performed by KLW. We obtain that even with the current data, the TVB model is still excluded, as previously pointed out by KLW with the 2018 data [76].

### A. Implications to some flavor parametrizations

As the next step in our analysis, we will explore the implications to our previous phenomenological analysis on TVB model to some flavor parametrizations that have been already studied in the literature. For this we consider scenarios in which the transformations involve only the second and third generations [70, 75], as it was previously discussed in Sec. II. We found that the equivalence in the quark sector is Eq. (23) while for the leptonic sector we have Eq. (24). Taking into account the 1σ range solutions of TVB couplings obtained in Table VI, we get $1.01 \leq g_{1V}^{33} \leq 2.71$ and a very small mixing angle $1.9 \times 10^{-3} \leq |\theta_D| \leq 2.9 \times 10^{-3}$. Such a small mixing angle $(|\theta_D| \ll V_{cb})$ result is still in agreement with previous analysis [70, 75]. On the contrary, in the leptonic sector, since the LFV coupling is negligible ($g_{1\tau} \ll 1$), we obtained that is not possible to find a physical solution to the mixing angle $\theta_L$. As additional probe, we have performed a global fit to the current $b \rightarrow s\mu^+\mu^-$ and $b \rightarrow c\tau\bar{\nu}_\tau$ data, and the most relevant flavor observables, with $(g_{1V}^{33}, g_{1r}^{33}, \theta_D, \theta_L)$ as free parameters. For a fixed mass value $M_V = 1$ TeV, we obtained a very poor fit ($\chi^2_{\min}/N_{\text{dof}} \gg 1$), concluding that this kind of flavor setup is not viable within the TVB model.
FIG. 2. Allowed regions of the most relevant 2D parametric space of all data (current data + \(R(\Lambda_c)\)) for \(M_{\nu} = 1\) TeV, where the 1\(\sigma\) and 2\(\sigma\) regions are shown in green and light-green colors, respectively. In each plot we are marginalizing over the rest of the parameters. The inner red dotted and red solid contours illustrate the permitted regions by projections Belle II-P1 and Belle II-P2, respectively. The SM value is represented by the blue circle. In panels (b) and (d), the light-gray region corresponds to LHC bounds at the 95% CL.

V. CONCLUSIONS

We have presented an updated view and perspectives of the triplet vector boson (TVB) model as a simultaneous explanation of the \(B\) meson anomalies (\(b \to c\tau\bar{\nu}_\tau\) and \(b \to s\mu^+\mu^-\) data). We performed a global fit of the parameter space of this model with the available 2022 data, including the very recent LHCb measurement on the ratio \(R(\Lambda_c) = \frac{\text{BR}(\Lambda_c \to \Lambda_c \tau \bar{\nu}_\tau)}{\text{BR}(\Lambda_b \to \Lambda_c \mu \bar{\nu}_\mu)}\) and global fit analyses of the most current \(b \to s\mu^+\mu^-\) data (\(C_{bs\mu\mu}^9 = -C_{bs\mu\mu}^{10}\) solution). We have also included all relevant flavor observables such as \(B_s - \bar{B}_s\) mixing, neutrino trident production, LFV decays (\(B^+ \to K^+\mu^+\tau^-\), \(B_s \to \mu^+\tau^-\), \(\tau \to \mu\phi\), \(\Upsilon(nS) \to \mu^+\tau^-\)), rare \(B\) decays (\(B \to K\nu\bar{\nu}, B \to K\tau^+\tau^-, B_s \to \tau^+\tau^-\)), and bottomonium ratios \(R_{\Upsilon(nS)}\); as well as LHC bounds from searches of high-mass dilepton resonances at
the ATLAS experiment. Additionally, we have studied the perspectives on TVB model by taking into account the expected Belle II future improvements on an extensive array of flavor processes, with special attention to the Belle II prospects on \( R(D^{(*)}) \). Our analysis has shown that although the TVB model can accommodate the \( b \rightarrow c\tau\bar{\nu}_\tau \) and \( b \rightarrow s\mu^+\mu^- \) anomalies (in consistency with other flavor observables), this seems to be strongly disfavoured by the LHC bounds. As concerns the future flavor data at Belle II, our findings suggest that the projected scenario in which the experimental measurement on \( R(D^{(*)}) \) is reduced, it would allow a small NP window to solely explain the \( b \rightarrow c\tau\bar{\nu}_\tau \) data in agreement with LHC constraints.

We have also studied the consequences of our analysis of the TVB model to flavor parametrizations in which the transformations involve only the second and third generations. We obtained that such a flavor ansatz is not viable within the TVB model.

**ACKNOWLEDGMENTS**

J. H. M. is grateful to Oficina de Investigaciones de Universidad del Tolima for financial support of Project No. 290130517. The work of N. Q. has been financially supported by MINCIENCIAS and Universidad del Tolima through Convocatoria Estancias Postdoctorales No. 848-2019 (Contract No. 834-2020). E. R. acknowledges financial support from the “Vicerrectoría de Investigaciones e Interacción Social VIIS de la Universidad de Nariño,” Projects No. 1928 and No. 2172. We are grateful to Hector Gisbert for his comments on LFV effects in the dineutrino channels \( B \rightarrow K^{(*)}\nu\bar{\nu} \).
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