Data-Driven Enhanced Model Reduction for Bifurcating Models in Computational Fluid Dynamics

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We investigate various data-driven methods to enhance projection-based model reduction techniques with the aim of capturing bifurcating solutions. In particular, we use a three-step procedure of proper orthogonal decomposition (POD), dynamic mode decomposition (DMD) and manifold interpolation \cite{7} to compute parametrized reduced order models for a cavity undergoing Hopf bifurcations with varying Grashof number \cite{3}, \cite{6}. The approach can be extended to solution trajectories, which are not necessarily on the limit cycle but start from a common initial state for all trajectories corresponding to the considered parameters with the Hankel-DMD \cite{1}, \cite{5}. Additionally, it is possible to resolve multiple solutions for a channel flow undergoing a pitchfork bifurcation \cite{2} using localized model reduction and artificial neural networks \cite{4}, \cite{5}.

References

\cite{1} H. Arbabi and I. Mezic. Ergodic theory, dynamic mode decomposition, and computation of spectral properties of the koopman operator. \textit{SIAM Journal on Applied Dynamical Systems}, 16(4):2096–2126, 2017.
\cite{2} D. Drikakis. Bifurcation phenomena in incompressible sudden expansion flows. \textit{Physics of Fluids}, (9):76–87, 1997.
\cite{3} A. Gelfgat, P. Bar-Yoseph, and A. Yarin. Stability of multiple steady states of convection in laterally heated cavities. \textit{Journal of Fluid Mechanics}, 388:315–334, 1999.
\cite{4} M. W. Hess, A. Quaini, and G. Rozza. A comparison of reduced-order modeling approaches using artificial neural networks for PDEs with bifurcating solutions. \textit{ETNA - Electronic Transactions on Numerical Analysis}, 56:52–65, 2022.
\cite{5} M. W. Hess, A. Quaini, and G. Rozza. Data-driven enhanced model reduction for bifurcating models in computational fluid dynamics. \textit{https://arxiv.org/abs/2202.09250}, 2022.
\cite{6} M. W. Hess, A. Quaini, and G. Rozza. A data-driven surrogate modeling approach for time-dependent incompressible navier-stokes equations with dynamic mode decomposition and manifold interpolation. \textit{http://arxiv.org/abs/2201.10872}, 2022.
\cite{7} R. Zimmermann. Manifold interpolation. In P. Benner, S. Grivet-Talocia, A. Quarteroni, G. Rozza, W. Schilders, and L. M. Silveira, editors, \textit{Model Order Reduction, Volume 1: System- and Data-Driven Methods and Algorithms}, pages 229–274. De Gruyter, 2021.