Helicity Amplitudes for massive gravitinos in $\mathcal{N} = 1$ Supergravity

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Abstract. We develop the formal tools needed to construct helicity amplitudes for massive gravitino in $\mathcal{N} = 1$ SUGRA. We start by considering the helicity states for massive spin-3/2 particles, which involves the solutions of Rarita-Schwinger equation. These solutions are written using the modern spinor bra-ket notation and are used then to derive the interactions of gravitino with matter and gauge supermultiplets within $\mathcal{N} = 1$ Supergravity. The corresponding interactions of goldstinos are discussed too, relying on the goldstino-gravitino equivalence theorem.

1 Introduction

Great progress has been made in recent years to understand the amplitudes in gauge theories, including gravitation and Yang-Mills fields [1]-[10]. Impressive results for multi-leg amplitudes in the massless case have been derived [11, 12], which allow to evaluate multiparticle final states for maximal helicity violating amplitudes.

Some of the results have been derived for $\mathcal{N} = 4$ Super Yang Mills theory, which turns out to provide some regularities that make it to look as ”The simplest Quantum Field Theory” [13]. Exploring whether the local SUSY theory also knows as Supergravity, retains some of these properties would be quite interesting. One would like to have similar progress for the massive case, both from its formal relevance as well as the phenomenological implications namely, colliders like LHC are aimed to study massive states, such as the top quark, W, Z, Higgs, which have a mass that is not negligible as compared with the CM energy. Ideally, if possible we would like to understand the mass effects as perturbations from the massless case. It is possible that we can learn about this by studying specific cases.

With this aim we are interested in studying the application of helicity methods to treat the amplitudes involving the massive gravitino, which appears as the superpartner of the graviton in minimal $\mathcal{N} = 1$ Supergravity. Studying the gravitino has a relevance in its own in particle physics and cosmology, partly because when the Minimal SUSY extension of the standard model is embedded within SUGRA, the SUSY spectrum include the gravitino as the lightest SUSY particle, and therefore it could become a candidate for dark matter [14]. In fact supersymmetric extension of the standard model of particle physics have been thoroughly studied theoretically and its effects and predictions have been searched at low and high energies.

Studying the gravitino properties and its implications for both collider physics and the early universe, require the evaluation of many processes which could be quite involved due to the form of the spin-3/2 propagators and wave-functions for external legs. Some simplifications can arise for very light gravitinos, where one can rely on the equivalence theorem and replace the longitudinal components (spin-1/2) of the gravitino by the goldstino coming from the Chiral superfield appearing in the super-Higgs mechanism [15, 16, 17, 18, 19, 20].

We have already considered some aspects of gravitino phenomenology [21], in particular we studied the stop decay $\tilde{t} \to t W \tilde{\Psi} \mu$ [14], which already shows some complications. We would like to work with a formalism based on helicity methods to deal with such decays, as well as other process appearing in gravitino phenomenology. Some calculations dealing with gravitino were presented some time ago [22, 23, 24]; more modern methods have been incorporated into general programs such as Madgraph [25]. However, these methods have still some limitations, such as give only numerical outputs and not all vertices of the general SUGRA are included into the program.

In general, the incorporation of the massive case is not treated with full generality in the literature, which is one of the goals of this paper. Thus, we shall present the implementation of the Feynman rules for gravitino with an appropriate notation, which allows to reduce huge calculation which are very difficult to compute analytically using the traditional approach. In the language of Hunters, we want to show the guts and not only the skin of cross section and amplitudes.

After this introductory Section 1 let us present the organization of our paper. In Section 2 we shall discuss the solutions of the Rarita-Schwinger equation appropriate to be implemented with helicity methods. The helicity amplitudes with the full spin-3/2 gravitino and with the goldstino approximation are presented in Section 3; finally Section 4 includes some applications were we compared the helicity amplitudes for the 2-body neutralino decay with gravitino and goldstino in the final state as well as the reactions: $e^+ e^- \to GG$. Some details and conventions are left to the appendices.
2 Helicity Spinor Formalism for spin-3/2 gravitino field

In order to compute Scattering Amplitudes (SA) with spin-3/2 gravitino field in the final state, we shall use the marvelous advantages that the Spinor Helicity Formalism (SHF) \cite{26, 27, 28, 29} provides to handle perturbative calculation in quantum fields theories. In principle we want to compute \( S\alpha \) considering massive particles, hence it will be necessary to use to the Light Cone Decomposition (LCD) \cite{30, 31, 32, 33} which helps for expressing massive momenta in terms of massless ones. In Appendix A just by completeness we review some basics properties of the massless SHF that will also be useful for the massive extension.

The Rarita-Schwinger equation \cite{34, 35, 36} is equivalent to the following set of equations

\[
\gamma_\mu \tilde{\Psi}_\mu^\nu(p) = 0, \tag{1}
\]

\[
p_\mu \tilde{\Psi}_\mu^\nu(p) = 0, \tag{2}
\]

\[
(\rho - \tilde{m}) \tilde{\Psi}_\mu^\nu(p) = 0. \tag{3}
\]

The 4 polarization states of the gravitino in the momentum space (in terms of spin-1 and spin-1/2 components) that fulfill these equations are as follows

\[
\tilde{\Psi}^\mu_{+-}(p) = \epsilon^\mu_+(p)u_+(p), \tag{4}
\]

\[
\tilde{\Psi}^\mu_{-+}(p) = \epsilon^\mu_-(p)u_-(p), \tag{5}
\]

\[
\tilde{\Psi}^\mu_{-+}(p) = \sqrt{\frac{2}{3}} \epsilon^\mu_+(p)u_+(p) + \frac{1}{\sqrt{3}} \epsilon^\mu_-(p)u_-(p), \tag{6}
\]

\[
\tilde{\Psi}^\mu_{++}(p) = \sqrt{\frac{2}{3}} \epsilon^\mu_+(p)u_+(p) + \frac{1}{\sqrt{3}} \epsilon^\mu_-(p)u_-(p), \tag{7}
\]

It is known in literature how to express the polarization vectors \( \epsilon^\mu_\pm(p) \) as well as the massive Dirac spinors \( u_\pm(p) \) in terms of the modern and powerful bra-kets notation \cite{37} (see Ref. \cite{38} for a detailed review of massive SHF and its applications to QED, EWSM and Physics Beyond the Standard Model). It is straightforward to express the 4 gravitino states in this bra-kets notation, this are as follows

\[
\tilde{\Psi}^\mu_{++}(p) = \langle r|\gamma^\mu|q\rangle \left( |r\rangle + \tilde{m}|q\rangle |rq\rangle \right), \tag{8}
\]

\[
\tilde{\Psi}^\mu_{--}(p) = \langle q|\gamma^\mu|r\rangle \left( |r\rangle + \tilde{m}|q\rangle |rq\rangle \right), \tag{9}
\]

\[
\tilde{\Psi}^\mu_{+-}(p) = \sqrt{\frac{2}{3}} \left( \frac{r^\mu}{\tilde{m}} - \tilde{m} |q\rangle \langle s_{qr}| \right) \left( |r\rangle + \tilde{m}|q\rangle |rq\rangle \right) + \frac{1}{\sqrt{3}} \left( \frac{q^\mu}{\sqrt{2}|rq\rangle} \right) \left( |r\rangle + \tilde{m}|q\rangle |rq\rangle \right), \tag{10}
\]

\[
\tilde{\Psi}^\mu_{-+}(p) = \sqrt{\frac{2}{3}} \left( \frac{r^\mu}{\tilde{m}} - \tilde{m} |q\rangle \langle s_{qr}| \right) \left( |r\rangle + \tilde{m}|q\rangle |rq\rangle \right) + \frac{1}{\sqrt{3}} \left( \frac{r^\mu|q\rangle}{\sqrt{2}|rq\rangle} \right) \left( |r\rangle + \tilde{m}|q\rangle |rq\rangle \right), \tag{11}
\]

where the 4-momenta \( r^\mu \) and \( p^\mu \) are massless, and the Mandelstam variable is \( s_{qr} = -(q + r)^2 = -2q \cdot r \). Before go ahed one has to check if the 4 gravitino states in this new notation fulfill the equations \( \{11\}-\{3\} \) as well as the normalization condition

\[
\tilde{\Psi}_{\lambda\mu}(p)\tilde{\Psi}_{\lambda'\mu'}(p) = 2\tilde{m} \delta_{\lambda\lambda'} \delta_{\mu\mu'}, \tag{12}
\]

It shall be useful rearrange the 4 gravitino states as an expansion on the gravitino mass \( \tilde{m} \)

\[
\tilde{\Psi}_{++}(p) = \beta^\mu_1 |r\rangle + \beta^\mu_2 \langle q| \tilde{m}, \tag{13}
\]

\[
\tilde{\Psi}_{--}(p) = -\beta^\mu_1 |r\rangle + \beta^\mu_2 \langle q| \tilde{m}, \tag{14}
\]

\[
\tilde{\Psi}_{+-}(p) = \beta^\mu_1 |r\rangle + (\beta^\mu_2 \langle q| + \beta^\mu_2 |r\rangle) \tilde{m} + (\beta^\mu_2 |r\rangle + \beta^\mu_2 |q\rangle) \tilde{m}^2 + \beta^\mu_2 |q\rangle \tilde{m}^3, \tag{15}
\]

\[
\tilde{\Psi}_{-+}(p) = \beta^\mu_1 |r\rangle - (\beta^\mu_2 \langle q| + \beta^\mu_2 |r\rangle) \tilde{m} + (\beta^\mu_2 |r\rangle + \beta^\mu_2 |q\rangle) \tilde{m}^2 - \beta^\mu_2 |q\rangle \tilde{m}^3, \tag{16}
\]

the gravitino mass \( \tilde{m} \) is directly connected with the the SUSY breaking energy scale \( F \) as \( \tilde{m} = \frac{F}{\sqrt{M}} \), where \( M \) is the Plank mass. we have defined all the \( \beta^\mu_i \forall i = 1 \cdots 8 \) in the next table:
Table 1: Definitions of the $\beta^\mu_i \forall i = 1 \cdots 8$ with $\zeta = \frac{\sqrt{t}}{\sqrt{m}}$ and $s_{qr} = -(q + r)^2$.

Just by completeness we also express the 4 gravitino states $\tilde{\Psi}^\mu_{\lambda_\mu}(p)$ with $\lambda_\mu = +, -, +, -$, these take the following form:

\[
\tilde{\Psi}^\mu_+(p) = \beta^\mu_1 |r| + \beta^\mu_2 (q|\tilde{m}|, \quad (17)
\]
\[
\tilde{\Psi}^\mu_-(p) = -\beta^\mu_1 |r| + \beta^\mu_2 |q|\tilde{m}|, \quad (18)
\]
\[
\tilde{\Psi}^\mu_+(p) = \beta^\mu_3 |r| + (\beta^\mu_4 |q| + \beta^\mu_2 |r|)\tilde{m} + (\beta^\mu_6 |r| + \beta^\mu_2 |q|)\tilde{m}^2 + \beta^\mu_8 |q|\tilde{m}^3, \quad (19)
\]
\[
\tilde{\Psi}^\mu_-(p) = \beta^\mu_3 |r| - (\beta^\mu_4 |q| + \beta^\mu_2 |r|)\tilde{m} + (\beta^\mu_6 |r| + \beta^\mu_2 |q|)\tilde{m}^2 + \beta^\mu_8 |q|\tilde{m}^3. \quad (20)
\]

Having the massive gravitino states in this kind of basis it is even more simple to handle the helicity amplitudes. For example we can verify that the gravitino states fulfill the normalization condition Eq. (12) i.e. taking $\lambda_\mu = -$, we have:

\[
\tilde{\Psi}^\mu_+(p)\tilde{\Psi}^\mu_-(p) = (rq)(\beta^\mu_3 \beta^\mu_4 \beta^\mu_5 \beta^\mu_6 \beta^\mu_7 \beta^\mu_8 \beta^\mu_9 \beta^\mu_10 + \beta^\mu_6 \beta^\mu_7 \beta^\mu_8 \beta^\mu_9 \beta^\mu_7 \beta^\mu_8 \beta^\mu_9 \beta^\mu_10 - \beta^\mu_6 \beta^\mu_7 \beta^\mu_8 \beta^\mu_9 \beta^\mu_7 \beta^\mu_8 \beta^\mu_9 \beta^\mu_10) + c.c. \quad (21)
\]
\[
= (rq)\left(-\frac{2\zeta^2 |q||r| |q||r| |q||r| |q||r|^3}{s_{qr}} - \frac{\zeta^2 |q||r| |q||r| |q||r|^3}{2s_{qr}} \right) + c.c. \quad (22)
\]
\[
= 3\zeta^2 |q||r| |q||r| |q||r|^3 + c.c. \quad (23)
\]
\[
= 2\tilde{m}^2, \quad (24)
\]

as it can be noticed in the last calculation, the equations (13)-(16) and (17)-(20) are very convenient in order to handle huge and messy algebraic calculations.

3 Helicity Amplitudes

3.1 The goldstino equivalence theorem

For a light gravitino it is possible to discuss its properties using the equivalence theorem [15], and replace the longitudinal components of the gravitino by the derivatives of the Goldstino field. For the strict massless case one can simply apply the massless helicity methods, while for the massive case one needs to take into account the massive Dirac equation and the light-cone decomposition. Considering the gravitino 4-momentum in spherical coordinates

\[
p^\mu = (E, |\vec{p}| \sin \theta \cos \phi, |\vec{p}| \sin \theta \sin \phi, |\vec{p}| \cos \theta), \quad (25)
\]

with $p^2 = -\tilde{m}^2$. The polarization vectors take the following form

\[
\epsilon^\mu_+(p) = \frac{1}{\sqrt{2}}(0, \cos \theta \cos \phi - i \sin \phi, \cos \theta \sin \phi + i \cos \phi, - \sin \theta), \quad (26)
\]
\[
\epsilon^\mu_-(p) = -\frac{1}{\sqrt{2}}(0, \cos \theta \cos \phi + i \sin \phi, \cos \theta \sin \phi - i \cos \phi, - \sin \theta), \quad (27)
\]
\[
\epsilon^\mu_0(p) = \frac{1}{\tilde{m}}(|\vec{p}|, -E \sin \theta \cos \phi, -E \sin \theta \sin \phi, -E \cos \theta), \quad (28)
\]
when on takes the limit $|\vec{p}| \to \infty$, one has $E \approx |\vec{p}|$, which implies that
\begin{align}
\epsilon_{\mu+}(p)\rho^\mu &= -\epsilon_{0+}(p)\rho^0 + \tilde{\epsilon}_{+}(p) \cdot \vec{p} \\
&= -\epsilon_{0+}(p)|\vec{p}| + |\tilde{\epsilon}_{+}(p)||\vec{p}| \sin \theta,
\end{align}
for the condition $p_\mu \epsilon_{\mu+}(p) = 0$ implies $\epsilon_{0+}(p) = 0$ and $\epsilon_{\mu+}(p) = 0$ when $|\vec{p}| \to \infty$. However in this limit the polarization vector $\epsilon_{\omega}(p)$ has the following expression:
\begin{equation}
\epsilon_{\omega}(p) = \frac{p_\mu}{m}.
\end{equation}
Thus the helicity states of the gravitino Eqs. (4)-(7) are reduced when one takes into account high energy limit, and now the surviving gravitino states are only those of helicity $\pm 1/2$, namely
\begin{align}
\tilde{\Psi}^\mu_{++}(p) &= 0, \\
\tilde{\Psi}^\mu_{+-}(p) &= 0, \\
\tilde{\Psi}^\mu_{-+}(p) &= \sqrt{2/3} \epsilon_{0+}^\mu \rho_{-}(p) = \sqrt{2/3} \left(\frac{p_\mu}{m}\right) u_{-}(p), \\
\tilde{\Psi}^\mu_{++}(p) &= \sqrt{2/3} \epsilon_{0+}^\mu \rho_{+}(p) = \sqrt{2/3} \left(\frac{p_\mu}{m}\right) u_{+}(p).
\end{align}
To convert then in to coordinate space we need to replace $\rho^\mu \to i\partial^\mu$ in the gravitino field Eqs. (34)-(35) i.e. $\tilde{\Psi}_\mu(x) \to i\sqrt{2/3} \frac{\psi(x)}{m}$, where $\psi(x)$ is the so-called spin-1/2 goldstino state. After replacing the gravitino field as goldstino approximation in the lagrangian with gravitino $\Psi^\mu(x)$ one obtain an effective lagrangian describing the interaction of the goldstino with chiral superfields, this is given by $\frac{36}$:
\begin{equation}
\mathcal{L} = \frac{i(m_0^2 - m^2)}{\sqrt{3mM}}(\bar{\psi}_R \gamma^\mu \phi^\mu - \frac{im_\lambda}{8\sqrt{6mM}} \vec{\phi}[\gamma^\mu, \gamma^\nu] \lambda^{(a)} F_{\mu\nu}^{(a)} + h.c.)
\end{equation}

In this approximation when one assemble the HA’s from the Feynman rules, the goldstino field is just a Dirac spinor that is well known in literature.

### 3.2 Massless and massive gauge boson amplitudes

It is known that tree-level amplitudes that include $n$ massless gauge bosons of configurations $(+,+,\cdots,+)$ or $(-,\cdots,-)$ vanish exactly; one needs to have at least two helicities of each sign in order to have a non-vanishing amplitude, i.e. $(-,-,+,\cdots,+)$ or $(+,+,\cdots,-)$, as this case extends to amplitudes involving massless gravitons.

Now, it is the case that we are interested in evaluating processes involving massive gauge bosons, as one of the goals of LHC is to probe the mechanism of EWSB. Thus, it should be interesting to discuss to what extent the results of massless gauge boson scattering generalize to the massive case. In fact, this has addressed in [selection rules paper], with the finding that certain vanishing amplitudes for the massless case i.e. with $(-,-,+,\cdots,+)$ or $(+,+,\cdots,-)$ configuration become non-vanishing but with factors of the form $O(\frac{m^4}{M^4})$, with $m$ being the gauge boson mass and $E$ denoting the C.M. energy of the physical process under consideration. This result could also be understood by relying on the equivalence theorem, namely when the vector boson scattering is approximated by the pseudo-goldstone bosons.

### 3.3 Amplitudes for massive gravitinos

A similar result is expected to hold for the massive gravitino scattering. Namely, some amplitudes with some helicity configurations that vanish in the massless case would get corrections of the form $O(\frac{m^8}{M^8})$, where $\hat{m}$ denotes the gravitino mass, and $\hat{E}$ is the typical energy of the physical process.

Again this can be induced by relying on the SUGRA equivalence theorem, where the $\pm$ helicity states associated with the goldstino that arises from the Super-Higgs mechanism, that it is used to break supersymmetry and induce masses of the superpartners, including the gravitino.

As we are not aware of the corresponding discussions on the general case, we shall look at some specific process in order to identify the corresponding results, namely to look at the result for the MHV amplitude and try to identify the possible corrections, then to identify such non-MHV amplitudes that become non-vanishing and which amplitudes remain vanishing.
A comparison of amplitudes with gravitino and goldstino

4.1 The 2-body neutralino decay $\tilde{\chi}_0 \to \tilde{\psi} \gamma$ with LSP gravitino in the final state

One of the simplest processes that allows us to study the helicity configurations for scattering amplitudes is the 2-body decay of a neutralino into a gravitino and a photon. Using the interactions of the MSSM with gravity ($\tilde{\chi}_0 \to \tilde{\psi} \gamma$) we write the amplitude for the simple Feynman diagram Fig. (1) that contributes is as follows:

$$\mathcal{M}_{\lambda_q, \lambda_p, \lambda_k}^\chi = \frac{1}{4M} C_{\chi \gamma} \tilde{\psi}_{\mu \lambda_p} (\bar{p}_\nu [\gamma^\nu, \gamma^\mu] \sigma_{\lambda_k} (k) u_{\lambda_k} (q))$$

(37)

$$= \frac{1}{4M} C_{\chi \gamma} \tilde{\psi}_{\mu \lambda_p} (p) (\epsilon^\nu_{\lambda_k} (k) \gamma^\mu - k^\mu \sigma_{\lambda_k} (k)) u_{\lambda_k} (q)$$

(38)

with $C_{\chi \gamma} = U_{11} \cos \theta_W + U_{12} \sin \theta_W$. The momenta assignments for this decay is $p$ for the gravitino field ($\Psi_{\mu} (p)$), $q$ for the Neutralino ($\chi_0 (q)$) and $k$ for the photon ($\gamma (k)$) and $\lambda_p$, $\lambda_q$ and $\lambda_k$ are their helicities labels, we have also defined in Eq. (39) $X_{\lambda_k}^\chi (k) = \epsilon^\nu_{\lambda_k} (k) \gamma^\mu - k^\mu \sigma_{\lambda_k} (k)$. There are 16 helicity amplitudes to compute, but by complex conjugate symmetry we just need to calculate half of them.

Figure 1: Feynman diagram for gravitino interaction with neutralino and photon

The nonzero HA are shown in Table 2

| $\lambda_q, \lambda_p, \lambda_k$ | $\mathcal{M}_{\lambda_q, \lambda_p, \lambda_k}^\chi$ | $\mathcal{M}_{\lambda_q, \lambda_p, \lambda_k}^\gamma$ |
|-----------------|----------|----------|
| $--++,+$        | $\frac{C_{\lambda_q \lambda_p \lambda_k}}{m_{\chi_0}}$ | 0        |
| $--,--,-$       | $\frac{C_{\lambda_q \lambda_p \lambda_k}}{\sqrt{m_{\chi_0} m_{\gamma}}}$ | $\frac{C_{\lambda_q \lambda_p \lambda_k}}{\sqrt{m_{\gamma} m_{\chi_0}}}$ |

Table 2: Helicity Amplitudes for the 2-body Neutralino decay $\chi_0 \to \gamma G$. Here $\mathcal{M}_{\lambda_q, \lambda_p, \lambda_k}^\chi$ represent the helicity amplitudes complete, this means for the massive spin-3/2 gravitino and, $\mathcal{M}_{\lambda_q, \lambda_p, \lambda_k}^\gamma$ are for the approximation of the gravitino to goldstino.

It is quite remarkable that the “massless” approximation for the helicity amplitudes with gravitino is exactly the helicity amplitude for the goldstino with just one configuration of helicities, this is $\mathcal{M}_{--,-,-}^\chi \equiv \mathcal{M}_{--,-,-}^\gamma$.

The squared and averaged amplitude take the form

$$\langle |\mathcal{M}|^2 \rangle = \frac{C_{\chi \gamma}^2}{2M^2} \left( 2 |\mathcal{M}_{--,-,-}^\gamma|^2 + 2 |\mathcal{M}_{--,-,-}^\gamma|^2 \right)$$

(40)

$$= \frac{C_{\chi \gamma}^2}{M^2} \left( \frac{s^2 \lambda_2 \delta_{\lambda_0 \lambda_2}^2}{s_{\lambda_1 \lambda_2}} + \frac{s^2 \lambda_2 \delta_{\lambda_0 \lambda_2}^2}{3m^2 s_{\lambda_1 \lambda_2}} \right)$$

(41)

$$= \frac{C_{\chi \gamma}^2}{M^2} \left( \frac{(m_{\chi_0}^2 - \tilde{m}^2)^2}{3m^2} (3\tilde{m}^2 + m_{\chi_0}^2) \right)$$

(42)

$$= \frac{C_{\chi \gamma}^2 m^2_{\chi_0}}{M^2} \left( 1 - \frac{\tilde{m}^2}{m_{\chi_0}^2} \right) \left( \frac{1}{3} + \frac{\tilde{m}^2}{m_{\chi_0}^2} \right)$$

(43)
then the decay width $\Gamma$ for the 2-body neutralino decay ($\tilde{\chi}_0 \to \gamma \tilde{G}$) is as follows

$$\Gamma_{\tilde{\chi}_0 \to \gamma \tilde{G}} = \frac{C_s^2 m_{\tilde{\chi}_0}^5}{16\pi M^2 m^2} \left(1 - \frac{\tilde{m}_2}{m_{\tilde{\chi}_0}}\right)^3 \left(\frac{1}{3} + \frac{\tilde{m}_2}{m_{\tilde{\chi}_0}}\right),$$

where $s_{r_1r_2} = m_{\tilde{\chi}_0}^2$, $s_{r_2q_2} = m_{\tilde{\chi}_0}^2 - \tilde{m}_2^2$ and $s_{q_2r_1} = 0$.

### 4.2 Production of light gravitino at colliders: $e^+e^- \to \tilde{G}\tilde{G}$

We will compute the scattering amplitude for the reaction $e^- e^+ \to GG$ with the gravitino approximation to goldstino (massless) [40]. Each Feynman diagram of Fig. (2) contributes to the total amplitude:

$$\mathcal{M} = \mathcal{M}^c + \mathcal{M}^u + \mathcal{M}^t$$

where

$$\mathcal{M}^c = -\frac{m_{\tilde{\varepsilon}_{\lambda_1}}^2}{F^2} (T^t - T^u),$$

$$\mathcal{M}^t = -\frac{m_{\tilde{\varepsilon}_{\lambda_1}}^4}{F^2(t - m_{\tilde{\varepsilon}_{\lambda_1}})} T^t,$$

$$\mathcal{M}^u = \frac{m_{\tilde{\varepsilon}_{\lambda_1}}^4}{F^2(u - m_{\tilde{\varepsilon}_{\lambda_1}})} T^u,$$

where $m_{\tilde{\varepsilon}_{\lambda_1}}$ is the selectron masses, $\lambda_1 = \pm$.

![Feynman diagrams for gravitino production at $e^+e^-$ colliders](image)

The nonzero helicity amplitudes are shown in Table 3.

| $\lambda_1 \lambda_2 \lambda_3 \lambda_4$ | $\mathcal{M}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}$ |
|--------------------------------------|----------------------------------|
| $- + + -$                           | $-\frac{m_{\tilde{\varepsilon}_{-}}^4}{F^2(t - m_{\tilde{\varepsilon}_{-}}^2)} [31]$ [24] |
| $- + - +$                           | $-\frac{m_{\tilde{\varepsilon}_{-}}^4}{F^2(u - m_{\tilde{\varepsilon}_{-}}^2)} [41]$ [23] |

Table 3: Helicity Amplitudes for the reaction $e^- e^+ \to \tilde{G}\tilde{G}$

6
5 Conclusions

In this paper we have developed the formal tools needed to construct helicity amplitudes for massive gravitino in $\mathcal{N} = 1$ SUGRA. We started by considering the helicity states for massive spin-3/2 particles, which involves the solutions of Rarita-Schwinger equation. Adopting the helicity bra-ket notation for these solutions, they were expressed in a convenient way for assembling helicity amplitudes and were used to derive the interactions of the gravitino with matter and gauge fields within $\mathcal{N} = 1$ Supergravity. We also have studied the corresponding interactions of goldstinos, relying on the goldstino-gravitino equivalence theorem. Finally to appreciate the power of the method we have evaluated the cross sections for $e^+ e^- \rightarrow GG$ and the width decay $\chi_{0} \rightarrow Z \bar{\psi}_{\mu}$. It was shown in Tables 2 and 3 how the Spinor Helicity Formalism (massless and massive cases) reduce the expressions for scattering amplitudes, allowing to calculate effectively the physical observables.

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A Basics of the massless helicity formalism

In this appendix it is introduced the properties for the massless spinors that are use through this paper, most of them were taking from Ref. [27].

Using the powerful spinor bra-ket notation, the 4-component Dirac spinor are rewritten as follows

\begin{align}
  u_-(p) &= v_+(p) = [p], \\
  u_+(p) &= v_-(p) = [p], \\
  \bar{u}_+(p) &= \bar{v}_-(p) = [p], \\
  \bar{u}_-(p) &= \bar{v}_+(p) = [p],
\end{align}

which obey the relations

\begin{align}
  u_+(p)\bar{u}_+(p) &= \frac{1}{2}(1 + s\gamma_5)(-\bar{p}) \\
  v_+(p)\bar{v}_+(p) &= \frac{1}{2}(1 - s\gamma_5)(-\bar{p})
\end{align}

where $s = \pm$ indicates the helicity. Spinor products are antisymmetric

\begin{align}
  \bar{u}_+(p)u_-(k) &= [pk] = -[kp] = -\bar{u}_+(k)u_-(p), \\
  \bar{u}_-(p)u_+(k) &= [pk] = -[kp] = \bar{u}_-(k)u_+(p),
\end{align}

taking the last results into account one also have that the spinor product fulfill $[qq] = [qq] = 0$, the type of spinor products $[kp]$ and $[pk]$ are also null.

For real momenta these spinor products satisfy

\begin{align}
  \langle pk \rangle &= [kp]^*, \\
  [kp] &= \langle pk \rangle^*, \\
  [pq] &= s_{pq} = -(p + q)^2 = -2p \cdot q.
\end{align}

Other useful properties are the following

\begin{align}
  [k|\gamma^{\mu}|p] &= \langle p|\gamma^{\mu}|k], \\
  [k|\gamma^{\mu}|p]^* &= \langle p|\gamma^{\mu}|k] \\
  \langle p|k|q] &= -(pk)[kq], \\
  \langle p|\gamma^{\mu}|p] &= 2p^{\mu}. 
\end{align}

Fierz identity is also a useful property, this take the following form

\begin{align}
  \langle p|\gamma^{\mu}|q|\gamma^{\nu}|w] &= 2\langle pr|qw].
\end{align}

From the completeness relation, one is able to express $\rho$ as a product of spinors

\begin{align}
  \rho &= -(\rho|\rho + |p|p])
\end{align}
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