Effect of reaction products kinetic on stability of thermonuclear spark ignition

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Abstract. Thermonuclear spark instability driven by reaction products momentum transfer is considered in this paper. General theory of such instability was developed by Lykov [1] based on one-group particles transport equation solving. Main dimensionless parameter of this problem is $l/\bar{\lambda}$, where $\bar{\lambda}$ is the perturbation wavelength and $l$ is the reaction product mean free path. In two limiting cases of $l/\bar{\lambda}$ ratio we recalculated perturbations growth rates in a more precise manner, obtaining reaction products distribution function through Fokker-Planck equation. While our results are generally consistent with those of Lykov’s work, in stability growth rates predicted in this investigation are somewhat smaller than the previously reported values. By performing a sequence of hybrid kinetic-fluid simulations in code FRONT3D, we demonstrate their better agreement with our asymptotic predictions. However, it was shown that in a wide range of $l/\bar{\lambda}$ ratio the instability growth rates are well described by Lykov’s theory. We apply Chapman-Enskog method to describe plasma ions deviations away from Maxwellian distributions, which appears from interaction with highly non-maxwellian reaction products. We derive an equation, that connects concentration of non-equilibrium particles with such parameters as ion momentum relaxation time and reaction rate. As it was shown, for typical ICF parameters, non-equilibrium part does not exceed several percents even at the burn peak.

1. Introduction
Hydrodynamic instabilities of thermonuclear spark surface still remain one of the main problem in ICF [2]. According to general theory of spark ignition [3], hot spot threshold energy, after which ignition is achieved, defined through heat balance equation $W_{\text{fus}} - W_{r} - W_{\text{ec}} - W_{\text{ad}} \geq 0$. Here $W_{\text{fus}}$, $W_{r}$, $W_{\text{ec}}$ are hot spot volume weighted rates of thermonuclear heating, radiation and electron conductivity cooling respectively; $W_{\text{ad}}$ - mean rate of purely hydrodynamic cooling due to adiabatic expansion. Since $W_{\text{ec}}$ value is directly proportional to hot spot surface area, it could strongly increase due to hot spot spherical symmetry violation.

Main contribution to hot spot perturbations growth is caused by Rayleigh-Taylor instability, acting in deceleration phase of ICF targets implosion [4]. During this stage the hot spot ion temperature climbs to many kiloelectronvolts, so reaction products mean free path becomes close to hot spot own
length scale [5]. Nowadays the lack of agreement between ICF experiments and numerical simulations [6, 7, 8] is particularly associated with kinetic regime of reaction products energy transfer, therefore this question has received considerable attention [9]. Numerical simulations with alpha particles transport treated with multi-group diffusion model shows that re-deposition of alpha particles energy into the cold fuel shell leads to polishing effect and stabilization of high-mode perturbations[10].

In this work we are focusing on perturbation evolution caused by reaction products momentum transfer. Previous research [1] has attempted to describe reaction product transfer by employing one-group transfer equation. Although one-group approximation allows to obtain instability increment for arbitrary $l/\lambda$ ratios, it neglects the effect of stopping power varying during reaction product deceleration. Therefore we were strongly motivated to recalculate the part of previous results [1] in a more accurate and precise manner.

2. Growth rates

Let us consider two superposed incompressible, inviscid, irrotational, and immiscible liquids, in one of which reactions occurs (active medium). OXY plane is unperturbed medium interface. $Z$ axis is up, so active and passive mediums lie in half-space $z > 0$ and in half-space $z < 0$ respectively. The goal of this section is to describe evolution of a single mode interface perturbation.

2.1. Small $l/\lambda$ ratio

In this case reaction products produce non-zero momentum flux (and volume force respectively) only in very thin "Knudsen" layers adjacent to the interface. Since mean free paths in both medias are small and perturbation wavelength is large, every element of interface could be then considered like an element of radial surface with its own curvature.

While asymptotic smallness of Knudsen layers allows to neglect their width, potential flow of active and passive medias is described by a Laplace equation for the hydrodynamic potentials $\varphi_i$,

$$\Delta \varphi_i = 0,$$

where $\varphi = \nabla \varphi$ and the indices $i = 1, 2$ indicate active and passive media.

On the boundary between the mediums, described by its $z$ displacement, the normal component of the velocity must be continuous. Effect of volume force on flow can be written in terms of the pressure jump across the interface that gives the equation

$$\rho_{act} \left[ \frac{\partial \varphi_{act}}{\partial t} + \frac{1}{2} \left( \nabla \varphi_{act} \right)^2 \right]_{z = \eta} + \delta P_{z = \eta} = \rho_{pass} \left[ \frac{\partial \varphi_{pass}}{\partial t} + \frac{1}{2} \left( \nabla \varphi_{pass} \right)^2 \right]_{z = \eta}$$

To compute pressure jump $\delta P$ it is necessary to find reaction products distribution function $f(r, \nu, \mu, \theta)$ in cylindrical shells shown in figure 1, where $\mu$ is the cosine of angle between velocity and radius-vector and $\theta$ is the cosine of angle between velocity and $Z$ axis. $\delta P$ is then

$$\delta P = M_{rp} \int \mu f(r, \nu, \mu, \theta) d\nu dr$$

Figure 1. Interface perturbation curvature. Solid lines – perturbed interface, dotted lines – imaginary cylindrical continuations of interface, space between dashed lines – imaginary regions of nonzero acceleration. Figure shows only two cylinders for clarity, thought every point of the interface is associated with its own cylinder.

However, due to pressure additivity, $\delta P$ could be obtained from more simple consideration of spherical shell system with the same radii, shown in figure 2.
Figure 2. Considered spherical geometry. Shell radii and reference points are equal to radii of cylindrical shells and points, represented in figure 1.

In case of spherical symmetry the distribution function \( f(r, \nu, \mu, \theta) \) of reaction products is governed by the stationary Fokker-Planck equation with isotropic source term:

\[
\nu \left( \frac{\partial f}{\partial r} + \frac{1 - \mu^2}{r} \frac{\partial f}{\partial \mu} \right) - \frac{1}{\nu^2} \frac{\partial}{\partial \nu} \left( \nu^2 a(\nu) f \right) + q \frac{\delta(\nu - \nu_0)}{4\pi\nu^2} = 0,
\]

where only the dynamical friction term is retained for the present discussion. In our formulation coefficients of function \( a(\nu) \) could be varying from shell to shell and \( q \) is not equal to zero only for active medium shells. Inhomogeneous term in equation (4) could be extracted in boundary condition and reduce to Cauchy problem for homogenous equation. For uniform sphere radius \( R \) of active medium it results in

\[
f(r, \nu_0, \mu) = -\frac{q}{4\pi\nu_0^2 a(\nu_0)} \theta(R - r)
\]

Its solution could be obtained by the method of characteristics:

\[
f(r, \nu, \mu) = -\frac{q}{4\pi\nu^2 a(\nu)} \theta \left( R - r \left[ 1 - \mu^2 + \left( \mu - \frac{1}{r} \int_0^\nu \frac{\nu d\nu}{a(\nu)} \right) \right]^{\frac{1}{2}} \right)
\]

Since distribution functions are found in all other spherical shells, one can calculate pressure jump. It's known[11], that deceleration of reaction products due to interaction with plasma ions could be neglected comparing its deceleration by plasma electrons at initial stages of thermonuclear burning. Thus, reaction product deceleration could be written in following form:

\[
\frac{d\nu}{dt} = a(\nu) = -\nu \frac{8\sqrt{\pi}Z_{\text{RP}} e^{\frac{1}{2}} n_e \sqrt{m_e}}{3(\kappa T_e)^{3/2} M_{\text{RP}}} \ln \Lambda_{\nu,\text{RP}},
\]

where indices RP indicates reaction products quantities. By providing rather cumbersome integration, we obtain pressure jump for spherical systems, that were shown in figure 2:

\[
\begin{align*}
\delta P_1^s &= \frac{qM_{\text{RP}} v_0 l_{\text{act}}}{6} \left( 1 + \frac{l_{\text{act}} - l_{\text{pass}}}{8R} \right) \\
\delta P_2^s &= \frac{qM_{\text{RP}} v_0 l_{\text{act}}}{6} \left( 1 - \frac{l_{\text{act}} - l_{\text{pass}}}{8R} \right)
\end{align*}
\]

Minimum curvature of ellipse, resulting from section of cylinder with radius \( R \) by plane, makes an angle of \( \alpha \) with element of cylinder, is \( R / \sin^2 \alpha \). Providing angle integration, we obtain pressure jump in cylindrical systems.
\[
\begin{align*}
\delta P_1 &= \frac{qM_{RP} \nu_0 l_{act}}{6} \left( 1 + \frac{l_{act} - l_{pass}}{16R} \right) \\
\delta P_2 &= \frac{qM_{RP} \nu_0 l_{act}}{6} \left( 1 - \frac{l_{act} - l_{pass}}{16R} \right)
\end{align*}
\] (9)

Since curvature radius is approximately inversely proportional to second spatial derivative, two
equations of system (9) could be reduced to one, connecting interface perturbation and pressure jump
on it:
\[
\delta P(x,t) = \frac{qM_{RP} \nu_0 l_{act}}{6} \left( 1 - \eta''(x,t) \frac{l_{pass} - l_{act}}{16} \right)
\] (10)

This is similar to capillary wave problem[12], so corresponding perturbation growth rate is
\[
\gamma = \left[ \frac{1}{96} \frac{qM_{RP} \nu_0 l_{act} (l_{act} - l_{pass}) k^3}{(\rho_{pass} + \rho_{act})} \right]^{1/2}
\] (11)

Before comparing with previous results[1] we have to notice, that in one-group approximation
reaction product mean free path has the following definition:
\[
l_{IG}^{-1} = -3M_{RP} \left[ \frac{d(e_0)}{e_0} - \frac{1}{e_0} \int_0^{e_0} d(e) \frac{d(a(e))}{d(e)} d(e) \right]
\] (12)

Corresponding growth rate for the function \( a(\nu) \) taken in the form(7) is
\[
\gamma_{IG} = \left[ \frac{25}{432\pi} \frac{qM_{RP} \nu_0 l_{act} (l_{act} - l_{pass}) k^3}{(\rho_{pass} + \rho_{act})} \right]^{1/2}
\] (13)

2.2. Large \( l/\lambda \) ratio

When perturbation wavelength is much smaller than reaction product mean free paths, their
distribution functions stay close to their unperturbated values. Besides, due to strong surface
localization of gravitational waves (here waves on mediums surfaces in external gravitational field, not
waves of gravitational field itself), one can use \( f(0) \) instead of \( f(z) \) for the acceleration field
evaluation. Computing \( f(0) \) with neglecting deceleration on ions and using classical theory of
gravitational waves we obtain the following dispersion relation:
\[
\gamma = \left[ \frac{qM_{RP} \nu_0 k (l_{pass} - l_{act})}{4l_{pass} (\rho_{pass} + \rho_{act})} \right]^{1/2}
\] (14)

It should be compared with growth rates, obtained[1] in one-group approximation
\[
\gamma_{IG} = \left[ \frac{qM_{RP} \nu_0 k (l_{pass} - l_{act})}{\pi l_{pass} (\rho_{pass} + \rho_{act})} \right]^{1/2}
\] (15)

Parameter space, where both growth rates(11,14) are real is the parameter space of interface
instability. Nontrivial result here consists in instability development while vectors of density gradient
and acceleration are codirectional.

3. Numerical simulations

Validation of growth rates, obtained in previous section were provided by numerical simulation in
Euler code FRONT3D [13]. Account of reaction products propagation was provided by computing of
their fluxes from adjacent cells closer than free path maximum to given cell. In all our simulations ratio between reaction products mean free path in active and passive media is \( l_{\text{act}} = 2l_{\text{pass}} \). To simulate both considered limit cases, we varied \( l_{\text{pass}} / \lambda \) ratio from 0.04 to 10.24.

Analysis of data from figure 3 and figure 4 shows the justice of predicted asymptotic for instability growth rates. Since the results obtained in [1] are also in good agreement with presented numerical simulations, it may be concluded that the theory based on one-group kinetic equation solving is really good applicable to this problem. Some differences could be easy eliminated by correction multiplicators including.

\[
\gamma^2 = \frac{1}{96} \frac{qM_{\text{rp}}u_0}{(\rho_{\text{pass}} + \rho_{\text{act}})} \left( l_{\text{act}} - l_{\text{pass}} \right)^{k^3}.
\]

\[
\gamma^2 = \frac{1}{4} \frac{qM_{\text{rp}}u_0}{l_{\text{pass}}(\rho_{\text{pass}} + \rho_{\text{act}})} \left( l_{\text{act}} - l_{\text{pass}} \right)^{k^3}.
\]

4. Hydrodynamic approximation applicability

Due to interaction with energetic reaction products, plasma ions and electrons is falling out of equilibrium. Since ion relaxation time is much greater, than electron relaxation time [14], it is reasonable to consider non-equilibrium of plasma ions only.

Let us write the kinetic equation for plasma ion distribution function in case of spatial homogeneity and absence of external force:

\[
\frac{\partial f_i}{\partial t} = C(f_i, f_e) + C(f_i, f_i) + C(f_i, f_{\text{rp}})
\]

In the RHS of equation (16) appear collision terms, model the effect of collisions between plasma ions with themselves and all other species. Ion-ion relaxation time relates to ion-electron relaxation as approximately \( \sqrt{m_e T_e} / \sqrt{M T_i} \), so the first term could be neglected. Third term models the collisions between ions and reaction products and is given by the following Fokker-Planck form:

\[
C(f_i, f_{\text{rp}}) = \frac{K}{v^2} \frac{\partial}{\partial U} \left( F_U + D \frac{\partial f_i}{\partial U} \right),
\]

where \( F(U) = \frac{4\pi M_{\text{rp}}}{M_{\text{rp}}} \int_0^U dU' f_{\text{rp}}(U) \) and \( D(U) = \int_0^U dU' f_{\text{rp}}(U) + \frac{4\pi U^2}{3} \int_0^U dU' f_{\text{rp}}(U) \) [15].
Constant $K$ is $\frac{4 \pi Z_i^2 Z_{\beta}^2 e^4}{M_i}$, where $\ln \Lambda_{RP,j}$ – Coulomb logarithm for collisions between plasma ions and reaction products [16].

The solution of $f_i$ can be carried out via the Chapman-Enskog expansion of equation (16). We will restrict ourselves only to a first order expansion. Plasma ions distribution function then brakess into sum of Maxwell distribution $f_{i,0}$ and non-equilibrium addition $f_{i,1}$. Since Maxwell distribution vanishes collision terms, equation (16) becomes:

$$\frac{\partial f_{i,0}}{\partial t} + \frac{\partial f_{i,1}}{\partial t} = C(f_{i,0}, f_{i,1}) + C(f_{i,0}, f_{RP})$$  \hspace{1cm} (18)

Being focused on intensive burn stages, we consider the case of high plasma temperatures (hundreds keV). In this situation reaction products deceleration by plasma electrons could be neglected and therefore their distribution function is equal to $\frac{q}{4\pi\beta} \theta(\nu_0 - \nu)$, where $\beta = \frac{4\pi Z_i^2 Z_{\beta}^2 e^4 n_i}{M_i M_{RP}}$. Besides generation of non-equilibrium particles, collisions with reaction products leads to evolution of equilibrium distribution, that results in temperature growth

$$\frac{\partial f_{i,0}}{\partial t} = -3 \frac{M_i \nu_i^2}{2 kT} \left( \frac{1}{T} \right) \frac{1}{dt}$$

Evaluating the collision term for ions and reaction products (17) results in:

$$\frac{C(f_{i,0}, f_{RP})}{C(f_{i,0}, f_{RP})} = C(f_{i,0}, f_{RP})_{\nu_i \sim 0} = \frac{K f_{i,0}}{\beta} \left[ 4 \left( \frac{M_i \nu_i^2}{2kT} \right) \right]^2 +$$

$$+ \left[ \frac{2 M_i \nu_i^2}{3 kT} - 1 \right] \left[ \frac{M_i \nu_i^2}{kT} - \frac{M_i}{kT} - \frac{3}{2 \frac{M_i}{dt} Kq} \right]$$

Integration $C(f_{i,0}, f_{RP})$ with $\frac{1}{2} M_i \nu_i^2$ over ion velocity space gives the expected temperature growth rate:

$$\frac{3}{2} \frac{n_i}{dt} \frac{M_i \nu_i^2}{dt} = \frac{K q f_{i,0}}{2}$$

Therefore collision term, corresponding to non-equilibrium particles generation is:

$$\frac{C(f_{i,0}, f_{RP})}{C(f_{i,0}, f_{RP})} = \frac{K f_{i,0}}{\beta} \left[ 4 \left( \frac{M_i \nu_i^2}{2kT} \right) \right]^2 + \left[ \frac{2 M_i \nu_i^2}{3 kT} - 1 \right] \left[ \frac{M_i \nu_i^2}{kT} - \frac{M_i}{kT} \right]$$

Being function of ion velocity, collision term (21) can be positive or negative. Computing threshold ion velocity $\nu_{i,TH}$, which makes collision term (21) zero and using the fact of particle number conversation, we can write the expression for non-equilibrium particles concentration growth rate:

$$\int_{\nu_{i,TH}}^{\infty} \sqrt{\frac{2}{K q f_{i,0}}} d\nu_i = q \Omega(M_i, M_{RP})$$

Calculation $\Omega(M_i, M_{RP})$ for deuterium and tritium collisions with alpha particles gives ~0.99 and ~0.75 respectively. To evaluate collision term $C(f_{i,0}, f_{i,1})$, models collisions between equilibrium and non-equilibrium ions we use $\tau$-approximation. Therefore we can finally obtain an equation, connects concentration of non-equilibrium particles with such parameters as reaction rate and ion momentum relaxation time [14].
\[
\frac{dn_{\text{NE}}}{dt} = -\frac{n_{\text{NE}}}{\tau} + q\Omega
\]  

(23)

Comparison of non-equilibrium particles concentration, obtained by equation (23), solved with concentrations, obtained from numerical solution of Fokker-Planck equation in burning DT-plasma [17] is shown in figure 5.

One can conclude, that considered approach of non-equilibrium particles number estimation leads to result, that is similar to results from full numerical kinetic simulations.

5. Conclusion
The paper presents description of instability, driven by energetic reaction products momentum transfer. Analytical result was justified by FRONT3D code simulations. Besides it was shown, that previously reported growth rates, obtained from one-group kinetic treatment of reaction products, are also in rather good agreement. So one could confirm the previous work [1] conclusions about long-wavelength spark instability in ICF targets.

In the framework of purely kinetic analysis of this problem the equation that describes concentration of non-equilibrium particles dependence on such parameters as plasma ions momentum relaxation time and thermonuclear reaction rate was obtained. In case of infinite burning DT–plasma, its solution gives concentrations of non-equilibrium plasma ions that are close to results of direct numeric solution of Fokker-Planck kinetic equation.

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