D2D Enhanced Heterogeneous Cellular Networks with Dynamic TDD

Hongguang Sun, Matthias Wildemeersch, Member, IEEE,
Min Sheng, Member, IEEE, Tony Q. S. Quek, Senior Member, IEEE

Abstract

Over the last decade, the growing amount of UL and DL mobile data traffic has been characterized by substantial asymmetry and time variations. Dynamic time-division duplex (TDD) has the capability to accommodate to the traffic asymmetry by adapting the UL/DL configuration to the current traffic demands. In this work, we study a two-tier heterogeneous cellular network (HCN) where the macro tier and small cell tier operate according to a dynamic TDD scheme on orthogonal frequency bands. To offload the network infrastructure, mobile users in proximity can engage in D2D communications, whose activity is determined by a carrier sensing multiple access (CSMA) scheme to protect the ongoing infrastructure-based and D2D transmissions. We present an analytical framework to evaluate the network performance in terms of load-aware coverage probability and network throughput. The proposed framework allows to quantify the effect on the coverage probability of the most important TDD system parameters, such as the UL/DL configuration, the base station density, and the bias factor. In addition, we evaluate how the bandwidth partition and the D2D network access scheme affect the total network throughput. Through the study of the tradeoff between coverage probability and D2D user activity, we provide guidelines for the optimal design of D2D network access.

Index Terms

Small cell network, dynamic time-division duplex, carrier sensing multiple access, device-to-device, stochastic geometry

H. Sun and M. Sheng are with the State Key Laboratory of Integrated Service Networks, Institute of Information Science, Xidian University, Xi’an, Shaanxi, 710071, China. (email: hgsun@xidian.edu.cn, msheng@mail.xidian.edu.cn).
Matthias Wildemeersch is with the Singapore University of Technology and Design, Singapore. (e-mail: matthiaswildemeersch@sutd.edu.sg).
T. Q. S. Quek is with the Singapore University of Technology and Design and the Institute for Infocomm Research, Singapore. (e-mail: tonyquek@sutd.edu.sg).
I. Introduction

As social applications in current data-centric networks continue to increase, mobile operators need to address the exponential growth of data traffic. Deploying diverse low-power small cell access points (SAPs) to complement the conventional macrocell network has proven a cost-effective means to increase the network capacity and enhance coverage [1], [2]. Aside from the surge in data traffic, Internet services and video applications also lead to asymmetry and dynamic variations in the uplink (UL) and downlink (DL) traffic load. Time-division duplex (TDD) systems [3] have the capability to manage the UL/DL traffic asymmetry by adjusting the fraction of time dedicated to UL and DL transmissions, which we refer to as the UL/DL configuration, to the current traffic conditions. The UL/DL configuration in present-day LTE-TDD systems is identical for all base stations and the configuration is modified very infrequently, such that these deployments cannot respond to the instantaneous traffic conditions. To accommodate the instantaneous traffic load, dynamic TDD with variable UL/DL configuration is under consideration and allows to make better use of the resources [4], [5]. Another technique to address the surge of mobile data traffic is Device-to-Device (D2D) communications, which have recently been considered as an alternative communication method for emerging proximity-based services, such as local advertising and social networking applications. With this technology, mobile users in proximity can establish a direct link and bypass the base stations, thereby offloading the network infrastructure and providing increased spectral efficiency [6], [7]. Different from ad hoc networks, D2D communications are operator-controlled and can share the licensed band with cellular transmissions. Specifically, D2D users can utilize dedicated spectrum in overlay mode or share the same bandwidth with cellular users in underlay mode. Considering the scarce radio resources, the underlay mode has been shown to provide better network performance in terms of outage capacity [6].

In a two-tier HCN operating with universal frequency reuse, the major challenge is the cross-tier and co-tier interference. In a network operating with dynamic TDD, interference conditions can be severe and strong DL-to-UL (base station-to-base station) interference may lead to an unacceptable performance in the UL transmissions [8]. In addition, extra interference is imposed
to the cellular transmissions if underlaid D2D transmissions are admitted. Accurate network interference modeling is essential to assess the network performance and recently, stochastic geometry has received much attention as a powerful tool for interference modeling and network performance evaluation [9], [10]. With this tool, the distribution of DL (UL) SINR at an arbitrary mobile user (base station) was derived in a single tier dynamic TDD small cell network [11]. However, the effect of important parameters such as UL/DL configuration and base station density on the network performance was not analyzed. Considering a two-tier HCN, a cognitive hybrid division duplex (CHDD) scheme was proposed in [12], where the macrocells operate with frequency division duplex (FDD), and small cells operate dynamic TDD on both FDD bands. Without interference management, the authors in [12] demonstrate that universal frequency reuse leads to a significant deterioration in the UL signal quality, ascribed to the cross-tier and co-tier interference. To alleviate the cross-tier interference in a two-tier HCN, variable interference management schemes have been proposed, such as power control [13], sleep mode techniques [14], and spectrum allocation [15], [16]. For spectrum allocation, a distributed disjoint subchannel allocation policy is sensible especially in dense small cell networks [15]. To address the co-tier interference, medium access control (MAC) is an effective and widely used technique in distributed ad hoc/sensor networks [17]–[20]. Carrier sensing multiple access (CSMA) is a popular MAC protocol where the positions of simultaneously transmitting nodes can be modeled by a Matern Hard-core Process (MHP) [17]–[19]. In an MHP, each node respects a minimum exclusion distance with respect to each other so as to control the mutual interference. Carrier sensing is also employed in cognitive radio networks to limit the interference inflicted on primary users (PUs). In [20], secondary users (SUs) are modeled as a Poisson Hole Process (PHP), such that only SUs located outside the exclusion region of PUs can transmit.

Despite the fact that both the merit of dynamic TDD networks [4], [5] and the benefit of D2D communications in FDD networks [6], [7] have been widely discussed in literature, a unifying framework for D2D enhanced TDD networks is still missing. In this work, we consider a D2D enhanced two-tier HCN operating with dynamic TDD where macrocells and small cells operate on two orthogonal frequency bands to eliminate the cross-tier interference. D2D users share the
same bandwidth with the small cell tier and control their interference by means of a CSMA scheme. We model the activity of D2D users and quantify the effect of important parameters, such as UL/DL configuration, base station density, bias factor and bandwidth partition. Although the effect of base station density and bias factor on the coverage probability is well understood in FDD networks [21], the effect of these system parameters in dynamic TDD networks is still unclear. Furthermore, prior literature usually considers a fully-loaded model where every Voronoi cell has a single active user uniformly distributed over the cell area, which engenders strong correlation between the point processes of base stations and mobile users. In this work, we use an actual PPP to model the transmitting mobile users, which may lead to empty cells. As such, we consider a load-aware model, where the density of active cells is exactly derived. Our main contributions can be listed as follows:

• We propose a simple PPP model for the active D2D transmitters based on the combined effect of a PHP and an MHP process, and we illustrate the validity of the PPP approximation by means of extensive simulations.
• We define an association policy that leads to different tessellations in UL and DL, and present an analytical framework to derive the load-aware coverage probability and network throughput. The proposed framework allows to numerically quantify how the UL/DL configuration, the base station density, and the bias factor affect the coverage probability, and how the bandwidth partition between the tiers relates to the network throughput.
• We evaluate the effect of D2D network access scheme on network performance, and quantify its advantage over the random access scheme ALOHA.
• We study the tradeoff between coverage probability and D2D user activity. From the perspective of total network throughput, we provide guidelines for the optimal design of the network access scheme in D2D enhanced TDD networks.

The rest of the paper is organized as follows. In Section II, the system model is presented. In Section III, the load-aware coverage probability and network throughput are derived and the validity of the analytical framework is demonstrated by means of the numerical simulations. In Section IV, the impact of key parameters on the network coverage probability is evaluated and
operating guidelines for the practical network design are provided. In Section V, the effect of bandwidth partition between the tiers on the network throughput is discussed. In Section VI, the impact of D2D network access control on the network performance is evaluated. Conclusions are given in Section VII.

II. System Model

A. Network Model

We consider a two-tier HCN which consists of a first tier of macro base stations (MBSs) distributed according to a homogeneous PPP $\Phi_m$ with density $\lambda_m$, overlaid with a network of SAPs distributed according to a PPP $\Phi_s$ with density $\lambda_s$. Mobile users are scattered over $\mathbb{R}^2$ according to a PPP $\Phi_u$ with density $\lambda_u$. A fraction $\zeta$ of the mobile users have their target receiver within a close distance and are considered as potential D2D transmitters. As an independent thinning of $\Phi_u$ with probability $\zeta$, the set of potential D2D transmitters $\tilde{\Phi}_d = \{T_i\}$ forms a PPP with density $\zeta \lambda_u$. We assume that each potential D2D transmitter has an assigned receiver (not belonging to $\Phi_u$) at a fixed distance $r_d$ in a uniformly random direction.\footnote{We note that the potential D2D receivers are scattered according to a PPP with density $\zeta \lambda_u$, where the potential D2D receivers and $\Phi_u$ are dependent point processes.}

We consider orthogonal spectrum allocation where the total bandwidth $W$ is divided into two non-overlapping parts $\eta W$ and $(1 - \eta) W$ that are allocated to the macro tier and small cell tier as depicted in Fig. 1. The potential D2D users share the spectrum with the small cell tier, thus leading to coexistence issues with the small cell users and SAPs. Both the macro and small cell tier operate according to the dynamic TDD scheme where at each timeslot a cell configures flexibly in DL or UL mode. The transmission mode selection for macrocells and small cells is modeled by independent Bernoulli random variables (r.v.’s), such that macrocells and small cells are configured in DL mode with probability $q_{D,m}$ and $q_{D,s}$, respectively, while the corresponding UL mode probabilities are given by $1 - q_{D,m}$ and $1 - q_{D,s}$. The multiplexing probabilities $q_{D,m}$ and $q_{D,s}$ define the UL/DL configuration for the macro tier and small cell tier. The concurrent DL and UL transmissions in neighboring cells may lead to new types of inter-cell interference, i.e. DL-to-UL and UL-to-DL (user-to-user)
interference. Let $P_m, P_s, Q_m$ and $Q_s$ denote the transmit power of MBSs, SAPs, mobile users associated with the macro tier, and mobile users associated with the small cell tier. We use $Q_d$ to represent the transmit power of potential D2D users.

We consider a load-aware resource allocation model where each base station always has data to transmit if it has a mobile user within its coverage. We adopt orthogonal multiple access, such that within a cell only a single mobile user can be active at any given timeslot and subchannel. If several mobile users connect to the same base station, they will be served with an equal probability. To control the interference inflicted by D2D transmissions on small cell transmissions, we provide medium access control by means of a CSMA scheme. The channel model consists of path loss and fast fading. The fading power from a transmitter located at point $x$ to the typical receiver located at the origin is denoted by $h_{ox}$ and is assumed to be an independent and identically distributed (i.i.d.) exponential r.v., $h \sim \exp(1)$, which corresponds to Rayleigh fading. Note that the analysis can be generalized to an arbitrary fading distribution by means of the Fourier integral techniques proposed in [22]. The path loss function is given by $g(||x||) = ||x||^{-\alpha}$, with $\alpha > 2$ the path loss exponent. Motivated by the high density of transmissions in current heterogeneous networks, we consider the interference-limited regime, and ignore the thermal noise.

**B. Cell Association**

Each timeslot, a mobile user acts as a transmitter or receiver with probability $\mu$ and $1 - \mu$, respectively. Assuming open access, the association of a mobile user to a given tier is based on
the maximum biased received signal power averaged over fading. The bias in this association policy is used to increase the coverage areas of small cells, which is commonly known as range expansion. While range expansion can alleviate the cross-tier interference in UL, in DL it results in bad signal conditions in the range expanded areas if no additional interference mitigation is employed [23]. Therefore, we explicitly model different association policies in UL and DL.

1) Downlink Association Policy: A typical receiver is associated with the nearest base station in DL mode of tier $i$ if

$$i = \arg \max_{k \in \{m, s\}} P_k B_k D_{D,k}^{-\alpha},$$

where $B_k$ is the bias factor of tier $k$, and $D_{D,k}$ denotes the distance from the typical receiver to the nearest base station of $\Phi_k$ operating in DL mode with thinned density $q_{D,k} \lambda_k$.

2) Uplink Association Policy: A typical transmitter is associated with the nearest base station in UL mode of tier $i$ if

$$i = \arg \max_{k \in \{m, s\}} Q_k B_k D_{U,k}^{-\alpha},$$

where $D_{U,k}$ denotes the distance from the typical transmitter to the nearest base station of $\Phi_k$ operating in UL mode with thinned density $(1 - q_{D,k}) \lambda_k$.

For notational brevity, we define the normalized parameters of tier $k$ conditioned on the serving tier $i$.

$$\hat{\lambda}_k^{(i)} \triangleq \frac{\lambda_k}{\lambda_i}, \quad \hat{q}_{D,k}^{(i)} \triangleq \frac{q_{D,k}}{q_{D,i}}, \quad \hat{q}_{U,k}^{(i)} \triangleq \frac{1 - q_{D,k}}{1 - q_{D,i}}, \quad \hat{P}_k^{(i)} \triangleq \frac{P_k}{P_i}, \quad \hat{Q}_k^{(i)} \triangleq \frac{Q_k}{Q_i}, \quad \hat{B}_k^{(i)} \triangleq \frac{B_k}{B_i}.$$  

Using the association rules defined in (1) and (2), the set of base stations form different multiplicatively weighted Voronoi tessellations of the two dimensional plane in DL and UL.

**Definition 1.** The downlink and uplink association regions of an MBS or a SAP located at point $x$ are given by (4) and (5), respectively [24].

$$C_{D,x} = \left\{ y \in \mathbb{R}^2 | \|y - x\| \leq (\hat{P}_k^{(i)} \hat{B}_k^{(i)})^{-1/\alpha} \|y - X^*_{D,k}(y)\|, \forall k \in \{m, s\} \right\},$$

$$C_{U,x} = \left\{ y \in \mathbb{R}^2 | \|y - x\| \leq (\hat{Q}_k^{(i)} \hat{B}_k^{(i)})^{-1/\alpha} \|y - X^*_{U,k}(y)\|, \forall k \in \{m, s\} \right\}.$$
where $X_{D,k}^*(y)$ and $X_{U,k}^*(y)$ denote the corresponding distances from $y$ to the nearest DL base station and to the UL base station of tier $k$.

As a consequence, a mobile user may associate with different base stations for DL and UL traffic. Let $\mathcal{N}_{D,i}$ and $\mathcal{N}_{U,i}$ denote the DL load and UL load, defined as the number of mobile users served by a base station of tier $i$ operating in DL and UL.

**C. CSMA model of potential D2D users**

At the start of a timeslot, each potential D2D transmitter senses the active small cell transmissions which originate from SAPs in DL mode and transmitting mobile users associated with the small cell tier. Assuming channel reciprocity, the potential D2D transmitter calculates the interference inflicted on the small cell transmitters and refrains from transmitting if the interference exceeds the protection threshold $\rho_s$. As such, D2D transmissions respect an exclusion region around each small cell transmitter. The remaining potential D2D transmitters form a PHP\(^2\) which can be approximated by a PPP \(^{20}\). Carrier sensing is also performed with respect to the remaining potential D2D transmitters, where the energy from a nearby D2D transmitter is not allowed to surpass the contention threshold $\rho_d$. To resolve the collision among the D2D contenders, we use a back-off scheme. Specifically, each remaining D2D transmitter independently samples a random timer $t_i \sim \mathcal{U}[0, 1]$ and channel access is granted to the contender with the smallest timer within a contention region \(^{17}\).

Let $U_i$ be the retention indicator of the $i$-th potential D2D transmitter $T_i$, \(^3\) which is given by

$$U_i = \prod_{Y_j \in \Phi_{D}^s} 1\left(\frac{Q_d h_{ji}}{\|T_i - Y_j\|^{\alpha} < \rho_s}\right) \prod_{Z_l \in \Phi_{T_u,s}^s} 1\left(\frac{Q_d h_{li}}{\|T_i - Z_l\|^{\alpha} < \rho_s}\right) \prod_{T_k \in \Phi_{d} \setminus T_i} \left(1(t_i < t_k) + 1(t_i > t_k) 1\left(\frac{Q_d h_{ki}}{\|T_i - T_k\|^{\alpha} < \rho_d}\right)\right),$$

where $h_{ji}$ denotes the channel fading from $T_i$ to $Y_j$, $\{Y_j\} = \Phi_{D}^s$ denotes the set of active SAPs

\(^2\)Note that \(^{20}\) considers a fixed exclusion distance. In this work, we take into account the channel fading, and the exclusion region of each small cell transmitter is a function of the instantaneous channel gain.

\(^3\)We use $T_i$ to indicate the position of the transmitter, and the transmitter itself.
in DL, and \( \{ Z_i \} = \Phi_{u,a}^T \) represents the set of transmitting mobile users associated with the small cell tier. The first two products in (6) reflect that the interference inflicted by a potential D2D transmitter on an active small cell transmitter should be smaller than \( \rho_s \). The first term inside the last product corresponds to the event where the timer \( t_i \) is smaller than \( t_k \), while the second term corresponds to the event where the timer \( t_i \) is larger than \( t_k \), yet the interference from \( T_i \) to \( T_k \) is smaller than \( \rho_d \).

Define \( \beta \triangleq \Pr[U_i = 1] \) as the retaining probability of \( T_i \), which depends on the timer \( t_i \), the channel fading and the distance between \( T_i \) and small cell transmitters. The set of winning contenders forms a point process similar to the MHP\(^4\) where any two points respect a minimum exclusion distance determined by \( \rho_d \) and the instantaneous channel gain. It is known that the aggregate interference experienced by a user of an MHP can be approximated by the interference resulting from a PPP that has the same density as the MHP and exists outside the exclusion region \([18], [19]\). In this work, we assume that each potential D2D transmitter is retained independently with the probability \( \beta \). As a result, the retained D2D transmitters \( \Phi_d \) form a PPP with density \( \beta \zeta \lambda_u \), where the combined effect of PHP and MHP is captured by \( \beta \). Note that the retained D2D transmitters are the actually active D2D transmitters in the current timeslot.

III. PERFORMANCE ANALYSIS

In this section, we derive the load-aware coverage probability and network throughput, and we validate the theoretical model by means of simulations.

A. Association and Load Characterization

The probability that a typical receiving and transmitting mobile user is associated with tier \( i \) for DL and UL, is given by

\[
\mathcal{A}_{D,i} = \frac{q_{D,i} \lambda_i}{\sum_{k \in \{m, a\}} G_{D,k}^{(i)}},
\]

\(^4\)By definition, the MHP originates from a homogeneous PPP with some density \( \lambda \), where each node associates with a random mark, and a node is forbidden to transmit only if there is another node within a certain exclusion distance with a smaller mark \([13]\).
A_{u,i} = \frac{(1 - q_{D,i}) \lambda_i}{\sum_{k \in \{m,s\} G_{u,k}^{(i)}}}, \quad (8)

where \( G_{D,k} = q_{D,k} \lambda_k \left( \hat{P}_k \hat{B}_k \right)^{2/\alpha} \), \( G_{U,k} = (1 - q_{D,k}) \lambda_k \left( \hat{Q}_k \hat{B}_k \right)^{2/\alpha} \). The result is a corollary of Lemma 1 in [21] and extends the DL association policy to the dynamic TDD scheme. For the special case of \( \{q_{D,m}, q_{D,s}\} = \{0, 0\} \), we define \( \{A_{D,m}, A_{D,s}\} = \{0, 0\} \), the network changes into a two-tier UL network. For \( \{q_{D,m}, q_{D,s}\} = \{1, 1\} \), we define \( \{A_{U,m}, A_{U,s}\} = \{0, 0\} \), the network transforms to a two-tier DL network. The association probabilities defined in (7) and (8) indicate how the per tier association probability in a two-tier dynamic TDD network depends on the relative transmit power, bias factor, and base station density in the corresponding transmission mode. Note that the base station density affects the per tier association probability more than transmit power or bias factor.

Due to the small coverage of SAPs, the fully-loaded model may significantly overstate the network interference from small cells, leading to a pessimistic estimation on the coverage probability. By considering the traffic load, we derive a more accurate load-aware coverage probability. For each tier \( i \), we compute the void probability of a random base station in DL and UL, i.e. \( P_{D,i,e} \) and \( P_{U,i,e} \), and compare it with a threshold value to determine the network traffic load. When \( P_{D,i,e} < 10^{-4} \) and \( P_{U,i,e} < 10^{-4} \), we say tier \( i \) is fully-loaded, otherwise, partially-loaded. Denote \( \Phi_a^D \sim \text{PPP}(\lambda_a^D) \), \( \Phi_a^U \sim \text{PPP}(\lambda_a^U) \), \( \Phi_m^D \sim \text{PPP}(\lambda_m^D) \) and \( \Phi_m^U \sim \text{PPP}(\lambda_m^U) \) as the point processes of active DL MBSs, DL SAPs, UL MBSs, and UL SAPs, respectively, with corresponding densities \( \lambda_a^D, \lambda_a^U, \lambda_m^D \) and \( \lambda_m^U \). In the following lemma, we derive the void probability of a base station in tier \( i \), and we determine the exact density of active base stations in DL and UL.

**Lemma 1.** The probability that a cell of tier \( i \) is void for DL and UL transmissions is derived as

\[
P_{D,i} = (1 + \frac{(1 - \frac{1}{\mu})(1 - \zeta) \lambda_u A_{D,i}}{3.5 q_{D,i} \lambda_i})^{-3.5}, \quad P_{U,i} = (1 + \frac{\mu (1 - \zeta) \lambda_u A_{U,i}}{3.5 (1 - q_{D,i}) \lambda_i})^{-3.5}. \quad (9)
\]

Furthermore, the density of active base stations in DL and UL mode of tier \( i \) is given by

\[
\lambda_i^D = \lambda_i q_{D,i} (1 - P_{D,i}^e) \quad \text{and} \quad \lambda_i^U = \lambda_i (1 - q_{D,i})(1 - P_{U,i}^e). \quad (10)
\]
Proof: The results can be proved by a minor modification of Lemma 1 in [25]. Here we give the proof for completeness. The probability density function (PDF) of the area of a random Voronoi cell is given by

\[ f_X(x) = \frac{3.5^{3.5}}{\Gamma(3.5)} x^{2.5} e^{-3.5x} \]

where \( X \) denotes the area of a random Voronoi cell normalized by the value \( 1/q_{D,i} \lambda_i \) in DL and \( 1/(1-q_{D,i}) \lambda_i \) in UL mode. Taking the DL mode as an example, the PDF of the DL load \( N_{D,i} \) is given by

\[
\Pr[N_{D,i} = n] = \int_0^\infty \Pr[X = x] \cdot f_X(x) \, dx
\]

\[
= \int_0^\infty \frac{(\lambda_{u,i}/q_{D,i} \lambda_i)^n}{n!} e^{-\lambda_{u,i}/q_{D,i} \lambda_i} \cdot f_X(x) \, dx
\]

\[
= \frac{3.5^{3.5} \Gamma(n+3.5)}{n! \Gamma(3.5)} \left( \frac{\lambda_{u,i}}{q_{D,i} \lambda_i} \right)^n \times \left( 3.5 + \frac{\lambda_{u,i}}{q_{D,i} \lambda_i} \right)^{-n-3.5},
\]

where \( \lambda_{u,i} = (1-\mu)(1-\zeta) \lambda_{u,i} A_{D,i} \) denotes the density of the receiving mobile users associated with tier \( i \), \( q_{D,i} \lambda_i \) is the density of total DL base stations of tier \( i \), \( \Gamma(\cdot) \) is the gamma function, which is given by \( \Gamma(x) = \int_0^\infty t^{x-1} \exp(-t) \, dt \), (a) is due to the definition of Poisson distribution, and (b) taking the expectation with respect to the area distribution \( f_X(x) \). Substituting \( n = 0 \) into (11) derives the void probability \( P_{D,i}^e = \Pr[N_{D,i} = 0] = (1 + \frac{\lambda_{u,i}}{3.5q_{D,i} \lambda_i})^{-3.5} \), which concludes the proof. \( \square \)

B. Coverage Probability

With the void probability derived in (9), we accurately characterize the interference in the network and derive the load-aware coverage probability of tier \( i, i \in \{m, s\} \) in DL and UL as

\[
P_i^D = \Pr[SIR_i^D > \gamma_i^D], \quad P_i^U = \Pr[SIR_i^U > \gamma_i^U],
\]

where \( \gamma_i^D \) and \( \gamma_i^U \) denote the SIR thresholds of DL and UL transmissions in tier \( i \). Similarly, the coverage probability of a typical D2D receiver is \( P_d^b = \Pr[SIR_d > \gamma_d] \) with \( \gamma_d \) the SIR threshold of D2D user. The overall coverage probability of a mobile user associated with the infrastructure

\footnote{To compute the density of active base stations, we consider the void probability of a random cell, rather than of a typical cell as in [24].}
in DL and UL mode is
\[
\bar{P}_D = \sum_{i \in \{m,s\}} P_{D,i} A_{D,i}, \quad \bar{P}_U = \sum_{i \in \{m,s\}} P_{U,i} A_{U,i}.
\] (13)

Since we consider open access, the distance between a typical mobile user and its serving base station of tier \(i\) in DL or UL mode, \(Y_{D,i}\) or \(Y_{U,i}\), is not only influenced by \(\Phi_{D,i}\) or \(\Phi_{U,i}\), but also by \(\Phi_{D,k}\) or \(\Phi_{U,k}\), \(k \neq i\). The distance distribution is given by
\[
f_{Y_{D,i}}(y) = 2\pi q_{D,i} \lambda_i y \exp\left\{-\pi q_{D,i} \lambda_i y^2\right\}, \quad f_{Y_{U,i}}(y) = 2\pi (1 - q_{D,i}) \lambda_i y \exp\left\{\pi (1 - q_{D,i}) \lambda_i y^2\right\},
\] (14)
where the result is a modification of Lemma 4 in [21] for dynamic TDD networks.

The DL and UL SIR of a typical receiver associated with the macro tier is given by
\[
\text{SIR}_D = \frac{P_m h_{or} r^{-\alpha}}{I_{D \rightarrow D}^{(m)} + I_{U \rightarrow D}^{(m)}}, \quad \text{SIR}_U = \frac{Q_m h_{or} r^{-\alpha}}{I_{D \rightarrow U}^{(m)} + I_{U \rightarrow U}^{(m)}},
\] (15)
where \(h_{or}\) and \(r\) are the fading power and the typical link length, and
\[
I_{D \rightarrow D}^{(m)} = \sum_{y \in \Phi_{D,m} \setminus \{y_0\}} P_m h_{oy} y^{-\alpha}, \quad I_{U \rightarrow D}^{(m)} = \sum_{x \in \Phi_{U,m}} Q_m h_{ox} x^{-\alpha},
\]
\[
I_{D \rightarrow U}^{(m)} = \sum_{y \in \Phi_{D,m}^{0}} P_m h_{oy} y^{-\alpha}, \quad I_{U \rightarrow U}^{(m)} = \sum_{x \in \Phi_{U,m}^{0} \setminus \{x_0\}} Q_m h_{ox} x^{-\alpha},
\]
where \(y_0\) and \(x_0\) represent the position of typical transmitter in DL and UL mode, \(\Phi_{U,m}^{T}\) represents the set of transmitting mobile users associated with macro tier. Due to the orthogonal multiple access technology, there is a one-to-one mapping from the transmitting mobile users associated with macro tier to the active UL MBSs. Since the coupling between the location of MBSs and transmitting mobile users has little effect on the coverage probability [12], we neglect the coupling and model \(\Phi_{U,m}^{T}\) as a PPP with density \(\lambda_{U,m}^{T} = \lambda_{U}^{V}\). The simulation results in Section IV also validate the accuracy of the approximation.

\(^6\)To clarify the channel of a specific link, \(r\) in the subscript denotes the position of a transmitter.
Figure 2. In the small cell tier, the typical receiver locates at the origin, and the serving transmitter $y_0$ employs $\iota_s$ to form an exclusion region, within which no D2D transmitter can exist. (a) is for $\|y_0\| \leq \iota_s$ and (b) is for $\|y_0\| > \iota_s$.

The DL and UL SIR of a typical receiver associated with small cell tier is denoted by

$$
\text{SIR}_D^B = \frac{P_s h_{0y} r^{-\alpha}}{I_{0\rightarrow D}^{(a)} + I_{0\rightarrow D}^{(b)} + I_{d\rightarrow D}}, \quad \text{SIR}_U^B = \frac{Q_s h_{0y} r^{-\alpha}}{I_{0\rightarrow U}^{(a)} + I_{0\rightarrow U}^{(b)} + I_{d\rightarrow U}},
$$

(16)

where

$$
I_{0\rightarrow D}^{(a)} = \sum_{y \in \Phi D_s} P_s h_{0y} y^{-\alpha}, \quad I_{0\rightarrow D}^{(b)} = \sum_{x \in \Phi T_u} Q_s h_{0x} x^{-\alpha}, \quad I_{d\rightarrow D} = \sum_{z \in \Phi d} Q_d h_{oz} z^{-\alpha},
$$

$$
I_{0\rightarrow U}^{(a)} = \sum_{y \in \Phi D_s} P_s h_{0y} y^{-\alpha}, \quad I_{0\rightarrow U}^{(b)} = \sum_{x \in \Phi T_u} Q_s h_{0x} x^{-\alpha}, \quad I_{d\rightarrow U} = \sum_{z \in \Phi d} Q_d h_{oz} z^{-\alpha},
$$

where $\Phi_{T_u} \sim \text{PPP}(\lambda_{T_u})$ represents the set of transmitting mobile users associated with small cell tier with density $\lambda_{T_u} = \lambda_U$. As is shown in Fig. 2, the active D2D transmitters are distributed in the shaded region, i.e. the whole $\mathbb{R}^2$ plane except for the ball $B$ centered at $y_0$, which can be divided into two disjoint parts by the circle $\mathcal{H}$ with the center at the origin. Define $\mathcal{H} \triangleq b(0, \|y_0\| + \iota_s)$, $B \triangleq b(y_0, \iota_s)$, where $B$ is the exclusion region for D2D transmissions around each small cell transmitter. Note that the exclusion region is a function of $\rho_s$ and channel fading, and varies in different timeslot. For simplicity, we use an equivalent exclusion distance by imposing a small miss detection probability threshold $\epsilon$. The constraint is met when $\Pr[Q_d h_{x_0 z} / \iota_s > \rho_s] = \epsilon$, and solving for $\iota_s$ yields $\iota_s = \left(\frac{-\ln \epsilon}{\rho_s / Q_d}\right)^{1/\alpha}$, where $h_{x_0 z}$ denotes the fading power from a D2D transmitter $z$ to the typical small cell transmitter $x_0$. 
The SIR of a typical D2D receiver is given by

$$\text{SIR}_d = \frac{Q_d h_{or} r_d^{-\alpha}}{I_{D\to d}^{(s)} + I_{U\to d}^{(s)} + I_{d\to d}},$$  \hspace{1cm} (17)$$

where

$$I_{D\to d}^{(s)} = \sum_{y \in \Phi_{z_d}^s \setminus b(z_0, \tau_d)} P_s h_{oy} y^{-\alpha}, \quad I_{U\to d}^{(s)} = \sum_{x \in \Phi_{z_d}^t \setminus b(z_0, \tau_d)} Q_s h_{ox} x^{-\alpha}, \quad I_{d\to d} = \sum_{z \in \Phi_{z_d}^d \setminus b(z_0, \tau_d)} Q_d h_{oz} z^{-\alpha}.$$  

The exclusion region \(b(z_0, \tau_d)\) results from the protection threshold \(\rho_s\), and the exclusion region \(b(z_0, \tau_d)\) results from the contention threshold \(\rho_d\). The radius \(\tau_d\) is constrained by \(\epsilon\) as \(\Pr[Q_d h_{oz} / \tau_d^\alpha > \rho_d] = \epsilon\), and solving for \(\tau_d\) yields \(\tau_d = \left(\frac{-\ln \epsilon}{\rho_d/Q_d}\right)^{1/\alpha}\).

In the following lemma, we provide the retaining probability of each potential D2D transmitter and derive the density of active D2D transmitters.

**Lemma 2.** The retaining probability of a potential D2D transmitter is given by

$$\beta = \exp \left( - \left( \lambda_s^B + \lambda_u^T \right) K_{o,s} \right) \frac{1 - \exp \left( -\zeta \lambda_u K_{o,d} \right)}{\zeta \lambda_u K_{o,d}},$$  \hspace{1cm} (18)$$
and the corresponding density of active D2D transmitters is derived as

$$\lambda_d = \exp \left( - \left( \lambda_s^B + \lambda_u^T \right) K_{o,s} \right) \frac{1 - \exp \left( -\zeta \lambda_u K_{o,d} \right)}{K_{o,d}},$$  \hspace{1cm} (19)$$

where \(K_{o,s} = \frac{2\pi \Gamma(2/\alpha)}{\alpha \rho_s^{2/\alpha}}\) and \(K_{o,d} = \frac{2\pi \Gamma(2/\alpha)}{\alpha \rho_d^{2/\alpha}}\).

*Proof:* See Appendix A. \(\square\)

With the per tier association probability, we derive the overall coverage probability of a mobile user associated with the infrastructure and the coverage probability of a typical D2D receiver as follows.

**Theorem 1.** In a two-tier dynamic TDD heterogeneous network, the overall load-aware coverage probability of a mobile user associated with the infrastructure in DL and UL mode is given by

$$\bar{P}_D = P_{m,D,m} A_{D,m} + P_{g,D,s} A_{D,s}, \quad \bar{P}_U = P_{m,U,m} A_{U,m} + P_{g,U,s} A_{U,s},$$  \hspace{1cm} (20)$$
and the coverage probability of the typical D2D receiver is derived as

$$
\Pr_D = \exp \left( -\mathcal{I}_1 \left( \frac{\gamma_d r_d^{\alpha}}{Q_d}; P_s \right) - \mathcal{I}_2 \left( \frac{\gamma_d r_d^{\alpha}}{Q_d}; Q_s \right) - \mathcal{I}_3 \left( \frac{\gamma_d r_d^{\alpha}}{Q_d}; Q_d \right) \right),
$$

(21)

where

$$
\Pr^D_m = \frac{q_{D,m} \lambda_m}{\lambda_m A_{D,m} \delta(\gamma_m^D, \alpha) + \lambda_{u,m}^T A_{D,m} C(\alpha) \left( \frac{Q_s}{P_s} \right) \gamma_m^D \frac{2}{\alpha} + q_{D,m} \lambda_m},
$$

(22)

$$
\Pr^U_m = \frac{(1 - q_{D,m}) \lambda_m}{C(\alpha) \left( \frac{\gamma_m^U}{\lambda_m^D} \pi \frac{\lambda^D_m}{(Q_s P_s)} \frac{2}{\alpha} + \lambda_{u,m}^T \right) + (1 - q_{D,m}) \lambda_m},
$$

(23)

$$
\Pr^D_s = \frac{\pi q_{D,s} \lambda_s}{A_{D,s}} \left[ \int_0^{t_a^2} e^{-\pi u \mathcal{F} L_{t_a \rightarrow 0} \left( \frac{\gamma_s^D u v^2}{P_s} \right)} v \leq t^2_a dv + \int_{t_a^2}^{\infty} e^{-\pi u \mathcal{F} L_{t_a \rightarrow 0} \left( \frac{\gamma_s^D u v^2}{P_s} \right)} v > t^2_a dv \right],
$$

(24)

$$
\Pr^U_s = \frac{\pi (1 - q_{D,s}) \lambda_s}{A_{U,s}} \left[ \int_0^{t_a^2} e^{-\pi u \mathcal{G} L_{t_a \rightarrow 0} \left( \frac{\gamma_s^U u v^2}{Q_s} \right)} v \leq t^2_a dv + \int_{t_a^2}^{\infty} e^{-\pi u \mathcal{G} L_{t_a \rightarrow 0} \left( \frac{\gamma_s^U u v^2}{Q_s} \right)} v > t^2_a dv \right],
$$

(25)

$$
\mathcal{I}_1 (s; P_s) = \pi \lambda^D_s \left( t_a + r_d \right)^2 \delta \left( \frac{s P_s}{(t_a + r_d)^\alpha} \right) + \lambda^D_s Z_{0, l_{OE}}(s; P_s),
$$

(26)

$$
\mathcal{I}_2 (s; Q_s) = \pi \lambda^T_{u,s} \left( t_a + r_d \right)^2 \delta \left( \frac{s Q_s}{(t_a + r_d)^\alpha} \right) + \lambda^T_{u,s} Z_{0, l_{OE}}(s; Q_s),
$$

(27)

$$
\mathcal{I}_3 (s; Q_d) = \pi \lambda_d \left( t_d + r_d \right)^2 \delta \left( \frac{s Q_d}{(t_d + r_d)^\alpha} \right) + \lambda_d Z_{0, l_{OP}}(s; Q_d).
$$

(28)

The variables in (21)-(28) are defined as

$$
\mathcal{F} \triangleq \lambda^D_s \delta(\gamma_s^D, \alpha) + \lambda^T_{u,s} C(\alpha) \left( \frac{Q_s}{P_s} \right) \gamma_s^D \frac{2}{\alpha} + \frac{q_{D,s} \lambda_s}{A_{D,s}},
$$

$$
\mathcal{G} \triangleq C(\alpha) \left( \frac{\gamma_s^U}{\lambda^D_s} \pi \frac{\lambda^D_s}{(Q_s P_s)} \frac{2}{\alpha} + \lambda^T_{u,s} \right) + \frac{(1 - q_{D,s}) \lambda_s}{A_{U,s}},
$$

$$
\mathcal{L}_{t_a \rightarrow 0} (s \mid r \leq t_a) = \mathcal{L}_{t_a \rightarrow 0} (s \mid r \leq t_a) = \mathcal{L}_{t_{out}} (s \mid r) \exp \left( -\lambda_d Z_{0, l_{OP}}^{\pi r_d + r} (s; Q_d) \right),
$$

where
\[ \mathcal{L}_{I_d \rightarrow D}(s \mid r > t_a) = \mathcal{L}_{I_d \rightarrow U}(s \mid r > t_a) = \mathcal{L}_{I_{out}}(s \mid r) \times \exp \left( -\lambda_d (Z_{\Theta,0}^{I_{OD}}(s; Q_d) + Z_{\Theta,0}^{I_{UC}}(s; Q_d) + Z_{\Theta,0}^{I_{UC}+r}(s; Q_d)) \right), \]

\[ \mathcal{L}_{I_{out}}(s \mid r) = \exp \left( -\pi \lambda_d (t_a + r)^2 \delta \left( \frac{sQ_d}{t_a + r} \right) \right), \]

\[ Z_{\Theta,0}^{I_{UC}}(s; Q) = (sQ)^2 \int_{\phi_{\ell}}^{\phi_{u}} \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1 + u^2} dud\phi, \]

\[ C(\alpha) = \frac{2\pi/\alpha}{\sin(2\pi/\alpha)}, \quad \delta(\beta, \alpha) = \int_{\beta-2/\alpha}^{\infty} \frac{\beta^{2/\alpha}}{1 + u^2} du, \quad l_{OA} = \sqrt{t_a^2 - (r\sin\theta)^2 + r\cos\theta}, \]

\[ \Theta = \arcsin \left( \frac{t_a}{r} \right), \quad l_{OC} = \sqrt{t_a^2 - (r\sin\theta)^2} - r\cos\theta, \quad l_{OD} = r\cos\theta - \sqrt{t_a^2 - (r\sin\theta)^2}, \]

\[ l_{OE} = \sqrt{t_a^2 - (r_d\sin\theta)^2} + r_d\cos\theta, \quad l_{OF} = \sqrt{t_a^2 - (r_d\sin\theta)^2} + r_d\cos\theta. \]

**Proof:** The full proof is provided in Appendix B. \( F \) and \( G \), respectively, correspond to the interference inflicted by DL SAPs and transmitting mobile users on the typical small cell receiver in DL and UL. \( \mathcal{L}_{I_d \rightarrow 0}(s \mid r \leq t_a), \mathcal{L}_{I_d \rightarrow 0}(s \mid r > t_a) \) and \( \mathcal{L}_{I_{out}}(s \mid r \leq t_a), \mathcal{L}_{I_{out}}(s \mid r > t_a) \) are the Laplace transforms of interference incurred by the active D2D transmitters on the typical small cell receiver in DL and UL, distinguished by the exclusion distance \( t_a \). \( I_1(s; Q_a), I_2(s; Q_a) \) and \( I_3(s; Q_d) \) correspond to the interference incurred by the DL SAPs, transmitting mobile users and active D2D transmitters, respectively. \( \square \)

The adoption of the CSMA scheme in our analysis leads to elaborate expressions of the coverage probability for the small cell tier. For the most general case, it involves a single numerical integration along with a lookup table for \( Z \). In the following section, we present the asymptotic analysis related to the protection threshold \( \rho_a \), which simplifies the analysis substantially.
C. Asymptotic Analysis

1) No D2D transmissions: When $\rho_s \to 0$, we have $\lambda_s \to 0$. The interference to the typical small cell receiver only comes from active DL SAPs and transmitting mobile users associated with the small cell tier. We derive the coverage probability of small cell tier in DL and UL as

$$\lim_{\rho_s \to 0} P_{D_s} = \frac{q_{D_s} \lambda_s}{\lambda_s A_{D,s} \delta (\gamma_s, \alpha) + \lambda_{s,u} A_{D,s} C(\alpha) \left(\frac{Q_s}{P_s}\right)^{2/\alpha} + q_{D,s} \lambda_s},$$

$$\lim_{\rho_s \to 0} P_{U_s} = \frac{(1 - q_{D,s}) \lambda_s}{C(\alpha) (\gamma_s)^{2/\alpha} A_{U,s} \left(\lambda_s^2 \left(\frac{Q_s}{P_s}\right)^{2/\alpha} + \lambda_{s,u}^2\right) + (1 - q_{D,s}) \lambda_s}. \quad (29)$$

In the absence of D2D transmissions, the overall coverage probability can be found in closed-form by inserting (22), (23), (29) and (30) into (20).

2) No sensing for small cell transmissions: When $\rho_s \to \infty$, the active D2D transmitters form an MHP with the retaining probability $\beta = \frac{1 - \exp(-\zeta \lambda_u K_{o,d})}{\zeta \lambda_u K_{o,d}} \approx \frac{1}{\zeta \lambda_u K_{o,d}}$, where (a) comes from $\zeta \lambda_u K_{o,d} \gg 1$. Thereby the density of active D2D transmitters is given by $\lambda_d = \beta \zeta \lambda_u \approx \frac{1}{K_{o,d}} = \frac{\alpha \left(\frac{Q_s}{P_s}\right)^{2/\lambda}}{2\pi \Gamma(2/\alpha)}$. The coverage probability of small cell tier in DL and UL is respectively,

$$\lim_{\rho_s \to \infty} P_{D_s} \approx \frac{q_{D,s} \lambda_s}{\lambda_s A_{D,s} \delta (\gamma_s, \alpha) + C(\alpha) \left(\gamma_s^{2/\alpha} A_{D,s} \left(\lambda_{s,u}^2 \left(\frac{Q_s}{P_s}\right)^{2/\alpha} + \frac{\alpha \left(\rho_u/P_s\right)^{2/\alpha}}{2\pi \Gamma(2/\alpha)}\right) + q_{D,s} \lambda_s}, \quad (31)$$

$$\lim_{\rho_s \to \infty} P_{U_s} \approx \frac{(1 - q_{D,s}) \lambda_s}{C(\alpha) (\gamma_s^{2/\alpha} A_{U,s} \left(\lambda_s^2 \left(\frac{Q_s}{P_s}\right)^{2/\alpha} + \lambda_{s,u}^2\right) + (1 - q_{D,s}) \lambda_s}. \quad (32)$$

Accordingly, without sensing for small cell transmissions, we can derive the overall coverage probability in closed-form by inserting (22), (23), (31) and (32) into (20).

D. Network throughput

With the coverage probability obtained in Theorem 1, we derive the sum throughput of the two-tier network, where the bandwidth of each tier is normalized by $W$. We consider outage capacity with constant bit-rate coding, such that the total network throughput in DL and UL
mode can be written as

\[ T_D(\eta; \rho_s; \rho_d) = \eta T_m^D + (1 - \eta) \left( T_a^D + \frac{1}{2} T_d \right), \]

(33)

\[ T_U(\eta; \rho_s; \rho_d) = \eta T_m^U + (1 - \eta) \left( T_a^U + \frac{1}{2} T_d \right). \]

(34)

where \( T_m^D = \lambda_D^m P_m^D \log_2(1 + \gamma_i^D) \), \( T_m^U = \lambda_T^m P_m^U \log_2(1 + \gamma_i^U) \) and \( T_d = \lambda_D^p P_d \log_2(1 + \gamma_d) \). Note that half of the D2D outage capacity is included in the DL and UL network throughput, respectively.

E. Validation

In this section, we verify by means of simulations the validity of the theoretical model and the approximations therein made concerning the active D2D transmitters and the active transmitting mobile users. All simulations are performed over a square window of 5000 × 5000 m² with 10000 iterations. In each iteration, the locations of MBSs, SAPs and mobile users, and the channel gains are independently generated. The typical user is assumed to be located at the origin, and the corresponding serving base station for DL and UL transmissions is determined by (1) and (2). Unless otherwise specified, we use the default values of the system parameters as shown in Table I.
In Fig. 3, we study the validity of the PPP approximation of active D2D transmitters in terms of coverage probability by varying $\rho_s$. The approximation is caused by the following factors: i) modeling the combined effect of PHP and MHP with an independent thinning of a PPP, ii) neglecting the coupling between the locations of mobile users and base stations in the UL transmission, iii) replacing the instantaneous exclusion distance by $\iota_s$ and $\iota_d$ constrained by a small miss detection probability threshold $\epsilon$. Figure 3 indicates that the PPP approximation is accurate for low and high values of $\rho_s$. Low values of $\rho_s$ correspond to large exclusion distances $\iota_s$, leading to a low retaining probability $\beta$. The good agreement between simulation and analysis can be explained by the fact that the smaller density of D2D transmitters leads to little interference. For high values of $\rho_s$ and corresponding small exclusion distances $\iota_s$, the density of the active D2D transmitters approaches that of the initial PPP, which eliminates the inaccuracy caused by the approximation. The middle range of values of $\rho_s$ results in inaccuracy on the coverage probability. In this example, the maximum error is achieved at $\rho_s = -70$ dBm.
Figure 4. Comparison of coverage probability from simulation (markers) and theoretical analysis (solid lines) as a function of $q_{D,s}$. ($\lambda_s = 5\lambda_a$, $\lambda_a = 100\lambda_a$, $\{B_m,B_s\} = \{1,1\}$, $\zeta = 0.1$, $\rho_s = \rho_d = -60$ dBm).

However, the order of magnitude of the largest error is less than 10%. Similar effect can be seen for $\rho_d$, and we find that the PPP approximation by varying $\rho_d$ is more accurate than by varying $\rho_s$. This can be explained by the larger effect of $\rho_s$ on $\lambda_d$ than the effect of $\rho_d$ on $\lambda_d$.

Figure 4 represents the coverage probability as a function of $q_{D,a}$ and reflects the accuracy of the PPP approximation of transmitting mobile users in UL case. We observe that the accuracy of the approximation deteriorates as more UL transmissions take place. The order of magnitude of the largest error is approximately 5%. For D2D users, the inaccuracy is mainly caused by the use of fixed $\iota_s$, where the channel fading is averaged. Note that the D2D user coverage is dominated by the strong interference from DL active SAPs. As $q_{D,a}$ increases, the density of SAPs increases and the impact of channel fading on the approximation is more apparent. The comparison of the analytical framework with numerical simulations has been performed for a large set of all relevant parameters that affect the approximations. The reasonable order of magnitude of the worst-case errors validates our theoretical model and in the following, we will present results based on our analytical framework.
IV. Coverage Probability Evaluation

In this section, we evaluate how the important network parameters, such as UL/DL configuration, base station density and bias factor affect the load-aware coverage probability. In case of a fully-loaded network without D2D users, we derive the parameters that maximize the per tier coverage probability and overall coverage probability.

A. Effects of UL/DL configuration

In this section, we elucidate the non-trivial system behavior in dynamic TDD networks that results from the coexistence of UL and DL transmissions. How the UL/DL configuration $q_{D,i}$ affects $P_{D_i}$ and $P_{U_i}$ is not very explicit, because increasing $q_{D,i}$ gives rise to a reduction of the UL interference and a surge of the DL interference. In Fig. 4, the coverage probability is depicted as a function of the small cell UL/DL configuration $q_{D,s}$ for different values of the macro tier UL/DL configuration $q_{D,m}$. Figure 4 shows that the increase in $q_{D,s}$ leads to an improvement in $P_{D,m}$ but a deterioration in $P_{U,m}$. Similarly, notice the increase of $P_{D,s}$ and decrease of $P_{U,s}$ with $q_{D,s}$. This can be explained by the fact that as $q_{D,s}$ increases, more receiving mobile users are associated with DL SAPs, which is reflected by in the per tier association probabilities (7), (8) and distance distribution (14). Associating more receiving mobile users to the small cell tier reduces the interference in the macro tier, while the change of the distance distribution is in particular evident for macro users with low SIR. In the UL, the decrease of $q_{U,s} = 1 - q_{D,s}$ leads to the handover of more transmitting mobile users to the macro tier, which alters the distance distribution and results in a decline of $P_{U,m}$. Given $P_s > Q_s$, the D2D coverage probability $P_d$ decreases with $q_{D,s}$ as a result of the increasing DL small cell interference. From Fig. 4 we observe that $P_{U,m}$ and $P_{U,s}$ deteriorate with $q_{D,m}$ and $q_{D,s}$, respectively, which reflects that the DL interference dominates the UL coverage due to the higher transmit power of base stations. For each tier, both the relative transmit power $Q_i / P_{i,m}$ and the association probability $A_{D,i}$ determine whether $P_{D_i}$ is dominated by the DL interference or the UL interference. This can lead to non-trivial behavior where $P_{D_i}$ increases or decreases with $q_{D,i}$ depending on the value of $q_{D,k}$ with $k \neq i$. This is illustrated in Fig. 4 by the crossing curves marked with triangles and circles where
\( P_m \) decreases or increases with \( q_{D,m} \) depending on the value of \( q_{D,a} \). In a fully-loaded network with \( \rho_s \to 0 \), we derive the UL/DL configuration that optimizes the per tier coverage probability in DL and UL mode.

Optimization of per tier UL/DL configuration: In a fully-loaded network, i.e. \( P_{e,k}^{D} \to 0 \) and \( P_{e,k}^{U} \to 0 \), and for \( \rho_s \to 0 \), the UL/DL configuration that maximizes the per tier UL coverage probability \( P_m \) and \( P_s \) is given by \( q_{D,m}^{*} = 0 \) and \( q_{D,a}^{*} = 0 \), respectively. The optimal UL/DL configuration for the per tier DL coverage probability is derived as follows. Define \( \bar{q}_{D,k} = \hat{\lambda}_i (\hat{P}_i (k) \hat{B}_i (k))^{2/\alpha} (\frac{\delta (\gamma_i^{p})}{C(a) (\hat{P}_i (k) \hat{B}_i (k))^{2/\alpha}} - 1)^{-1}, k, i \in \{m, s\}, k \neq i. \)

1. If \( \frac{\delta (\gamma_i^{p})}{C(a) (\hat{P}_i (k) \hat{B}_i (k))^{2/\alpha}} \leq (\frac{Q_i}{P_i})^{2/\alpha} \), \( P_i \) is a monotone increasing function of \( q_{D,i} \), where \( P_i \) is dominated by the UL interference. The optimal UL/DL configuration is achieved at \( q_{D,i}^{*} = 1 \);

2. If \( \frac{\delta (\gamma_i^{p})}{C(a) (\hat{P}_i (k) \hat{B}_i (k))^{2/\alpha}} > (\frac{Q_i}{P_i})^{2/\alpha} \), the monotonicity of \( P_i \) with respect to \( q_{D,i} \) is determined by the range of \( q_{D,k} \). When \( q_{D,k} < \bar{q}_{D,k} \), \( P_i \) increases with \( q_{D,i} \), and we have \( q_{D,i}^{*} = 1 \); when \( q_{D,k} > \bar{q}_{D,k} \), \( P_i \) is a decreasing function of \( q_{D,i} \), where \( P_i \) is dominated by the DL interference. The optimal UL/DL configuration is achieved at the limiting case of \( q_{D,i}^{*} = 0 \).

The results can be obtained by taking the the first-order derivative of \( P_m \) and \( P_s \) with respect to \( q_{D,i} \). In a realistic scenario, the base station has larger transmit power than that of the transmitting mobile user, i.e., \( P_i > Q_i \). Therefore, we have \( \frac{\partial P_s}{\partial q_{D,i}} < 0 \), which means \( P_s \) always decreases with \( q_{D,i} \). Note that the UL and DL coverage probabilities are also related to the relative base station density and bias factor, such that these parameters also affect the optimal UL/DL configuration.

B. Effects of base station density

From (22) to (25) and the definition of \( A_{b,k} \) and \( A_{d,k} \), we observe that both the base station density and bias factor have similar effects on the coverage probability in DL and UL. Due to space limitations, we take DL coverage as an example and the conclusions can be directly applied to the UL case. We evaluate the variation of the DL load-aware coverage probability as a function of \( \lambda_a \), as depicted in Fig. 5. In terms of traffic load, the network evolves from a fully-loaded sparse network to a partially-loaded dense network. From Fig. 5a, we observe that \( P_m \) increases monotonously with \( \lambda_a \), which can be ascribed to the handover of macro mobile
users with low SIR to the small cell tier and the corresponding reduction of interference in the macro tier. With respect to the small cell tier, the small cell network interference increases with \( \lambda_s \), while the activity of D2D users diminishes exponentially with \( \lambda_s \) as can be verified in (18). These opposite effects are reflected in the load-aware coverage probability for the small cell tier. Figure 5(b) depicts the overall DL coverage probability \( \bar{P}_D \) as a function of \( \lambda_s \) and indicates that an optimal \( \lambda_s \) can be found in the feasible region of small cell densities. As \( \lambda_s \) increases, the network load moves into the lightly loaded regime, where the aggregate small cell interference is constrained by the density of mobile users. As \( \lambda_s \to \infty \), we have \( A_{D,s} \to 1 \) and \( \bar{P}_D \to 1 \). As opposed to the fully-loaded traffic model with constant coverage probability in the asymptotic regime [26], this result highlights the usefulness of the load-aware model to capture the coverage probability in realistic conditions. In addition, Fig. 5 shows that the optimal value of \( \lambda_s \) increases with \( \zeta \) in the fully-loaded regime, although the effect is modest. Given a good estimate of the user density, the proposed analytical framework allows us to find the small cell density within the realistic regime that optimizes the overall coverage probability. In the fully-loaded network, and for \( \rho_s \to 0 \), we derive the optimal \( \frac{\lambda_s}{\lambda_m} \) in DL and UL mode, denoted by \( \hat{\lambda}_s^{(m)D} \) and \( \hat{\lambda}_s^{(m)U} \) as follows.
Optimization of base station density: In the fully-loaded network, and for \( \rho_s \to 0 \), the optimal \( \lambda_{a}^{(m)}\big|_{D} \) and \( \lambda_{a}^{(m)}\big|_{U} \) are given by

\[
\hat{\lambda}_{a}^{(m)}\big|_{D} = \left( \frac{\delta (\gamma_{m}^{D}, \alpha) + \left( \frac{1}{q_{D,a}} - 1 \right) (\frac{Q_{a}}{P_{a}} \gamma_{m}^{D})^{2/\alpha}}{q_{D,a} (\hat{P}_{a}^{(m)} \hat{B}_{a}^{(m)})^{2/\alpha}} \right), \quad q_{D,a} \in (0, 1],
\]

and

\[
\hat{\lambda}_{a}^{(m)}\big|_{U} = \left( \frac{\delta (\gamma_{m}^{U}, \alpha) + \left( \frac{1}{q_{D,a}} - 1 \right) (\frac{Q_{a}}{P_{a}} \gamma_{m}^{U})^{2/\alpha}}{q_{D,a} (\hat{Q}_{a}^{(m)} \hat{B}_{a}^{(m)})^{2/\alpha}} \right), \quad q_{D,a} \in (0, 1].
\]

The optimal value can be found by taking the first-order derivative with respect to \( \hat{\lambda}_{a}^{(m)} \). We take \( \hat{\lambda}_{a}^{(m)}\big|_{D} \) as an example, and similar method can be used to derive \( \hat{\lambda}_{a}^{(m)}\big|_{U} \). Given \( \rho_s \to 0 \) and for fully-loaded network, \( \mathbb{P}_{D}^{b} = \left( b \lambda_{a}^{(m)} + a + 1 \right)^{-1} \left( \frac{Q_{a}}{P_{a}} \left( \frac{Q_{a}}{P_{a}} \right)^{2} + c + 1 \right)^{-1} \), where we define \( a \triangleq \delta (\gamma_{m}^{D}, \alpha) + \left( \frac{1}{q_{D,a}} - 1 \right) (\frac{Q_{a}}{P_{a}} \gamma_{m}^{D})^{2/\alpha} \), \( b \triangleq \hat{q}_{D,a} (\hat{P}_{a}^{(m)} \hat{B}_{a}^{(m)})^{2/\alpha} \), and \( c \triangleq \delta (\gamma_{m}^{U}, \alpha) + \left( \frac{1}{q_{D,a}} - 1 \right) (\frac{Q_{a}}{P_{a}} \gamma_{m}^{U})^{2/\alpha} \). The first-order derivative is given by

\[
\frac{\partial \mathbb{P}_{D}^{b}}{\partial \lambda_{a}^{(m)}} = \frac{b}{(1+b(1+c)\lambda_{a}^{(m)})^2} - \frac{b}{(1+a+b\lambda_{a}^{(m)})^2}.
\]

Let \( \frac{\partial \mathbb{P}_{D}^{b}}{\partial \lambda_{a}^{(m)}} = 0 \), we derive the single critical point \( \hat{\lambda}_{a}^{(m)}\big|_{D} = \frac{a}{bc} \). In addition, when \( \hat{\lambda}_{a}^{(m)}\big|_{D} < \frac{a}{bc} \), we have \( \frac{\partial \mathbb{P}_{D}^{b}}{\partial \lambda_{a}^{(m)}} < 0 \), and when \( \hat{\lambda}_{a}^{(m)}\big|_{D} > \frac{a}{bc} \), we have \( \frac{\partial \mathbb{P}_{D}^{b}}{\partial \lambda_{a}^{(m)}} > 0 \). Thus, \( \hat{\lambda}_{a}^{(m)}\big|_{D} = \frac{a}{bc} \) is the optimal point of \( \mathbb{P}_{D}^{b} \).

C. Effects of bias factor

Figure 6 depicts the DL coverage probability as a function of the bias factor \( B_{a} \). We observe that increasing the density of SAPs \( \lambda_{a} \) decreases the optimal \( B_{a} \). This is due to the fact that a larger \( \lambda_{a} \) inflicts more interference on the small cell mobile users, and decreasing \( B_{a} \) helps to increase the overall coverage probability by shifting small cell mobile users with low SIR to the macro tier. It shows that with the analytical framework, we can derive the optimal \( \frac{B_{a}}{B_{m}} \) or \( \hat{B}_{a}^{(m)} \) that maximize the overall coverage probability.

Optimization of bias factor: In the fully-loaded network, and for \( \rho_s \to 0 \), the optimal \( \hat{B}_{a}^{(m)}\big|_{D} \) and \( \hat{B}_{a}^{(m)}\big|_{U} \) for DL and UL transmissions are given by

\[
\hat{B}_{a}^{(m)}\big|_{D} = \left( \frac{\delta (\gamma_{m}^{D}, \alpha) + \left( \frac{1}{q_{D,a}} - 1 \right) (\frac{Q_{a}}{P_{a}} \gamma_{m}^{D})^{2/\alpha}}{q_{D,a} \hat{P}_{a}^{(m)} \hat{B}_{a}^{(m)}} \right)^{2/\alpha}, \quad q_{D,a} \in (0, 1].
\]
Bias Factor of SAPs, $B_s$ [dB]

DL Overall Coverage Probability

$x = 10^x \lambda_s$, $\{q_D, q_s\} = \{0.5, 0.5\}$,

$B_m = 1$, $\zeta = 0.01$, $\rho_s = \rho_d = -60$ dBm).

Figure 6. Overall downlink coverage probability as a function of bias factor of SAPs $B_s$, ($\lambda_s = 10^3 \lambda$, $\{q_D, q_s\} = \{0.5, 0.5\}$, $B_m = 1$, $\zeta = 0.01$, $\rho_s = \rho_d = -60$ dBm).

The optimal bias factor is derived by taking the first-order derivative with respect to $\hat{B}_s(m^* U) = \frac{\left(\gamma_U^{(m)}\right)^{2/\alpha}}{\left(q^{(m)}_D \lambda_s^{(m)} \hat{Q}_s^{(m)} \gamma_U^{(m)}\right)^{2/\alpha} \left(\left(1 - q_{D,m} \gamma_U^{(m)}\right) - 1\right) \left(\frac{P_s}{Q_s}\right)^{2/\alpha} + 1}^{\alpha/2}$, $q_{D,m}, q_D, a \in (0, 1)$. (38)

Remark 1: The results derived in (35)-(38) are also applicable to the partially-loaded case by means of a transformation on the network load state. Specifically, we transform the network into an equivalent fully-loaded network by replacing the per tier base station density $\lambda_i$ by the active base station density, i.e., $\lambda_i \triangleq \lambda_i^P + \lambda_i^C$, where $\lambda_i^P$ and $\lambda_i^C$ are derived in (10). As such, we treat the network as fully-loaded and we study the effect of these important parameters on the coverage probability theoretically.

V. NETWORK THROUGHPUT EVALUATION

In this section, we study the D2D enhanced network from a throughput perspective. We reveal the differences between the network performance in terms of coverage probability and network throughput, and we demonstrate that the D2D transmissions can substantially improve the network throughput.

Figure 7 presents the DL network throughput with and without D2D capabilities as a function of $\lambda_s$ for different values of the bandwidth partition factor $\eta$. Without D2D capabilities, the
potential D2D transmitters can only associate with the infrastructure in UL, and those potential D2D receivers are blocked in the current timeslot. Comparing the curves with and without D2D capabilities for the same $\zeta$, we observe that even a small D2D user fraction ($\zeta = 0.01$) results in a considerable throughput gain. In the D2D enhanced network, we observe that allocating more spectrum to the small cell tier leads to a larger network throughput, ascribed to the high outage capacity of D2D users and the spatial reuse gain from small cell transmissions. Figure 7 also illustrates that the network throughput benefits from a higher fraction of potential D2D users $\zeta$. Interestingly, the network throughput with $\zeta = 0.1$ features a convex behavior as a function of $\lambda_s$. The initial decrease of network throughput follows from the decline of the retaining probability with $\lambda_s$ as indicated in (18). Compared with Fig. 5, we also observe that the D2D user fraction $\zeta$ leads to a tradeoff between the overall coverage probability and the total network throughput for fully-loaded network. A larger $\zeta$ improves the network throughput, yet deteriorates the overall coverage probability due to the interference inflicted on the small cell tier.

The results presented in Fig. 7 indicate that the bandwidth partition strongly affects the network throughput. We optimize now the bandwidth partition, and consider as an example the DL network throughput presented in (33). If $T_{b} > T_{d} + \frac{1}{2}T_{d}$, $T_{d}(\eta; \rho_{b}; \rho_{d})$ is a monotone increasing
function of $\eta$, we have $\eta^* = 1$ and $T_D(\eta^*; \rho_a; \rho_d) = T_D^{\text{opt}}$; if $T_m < T_s^{\text{opt}} + \frac{1}{2} T_d$, $T_D(\eta; \rho_a; \rho_d)$ monotonously decreases with $\eta$, thereby $\eta^* = 0$ and $T_D(\eta^*; \rho_a; \rho_d) = T_s^{\text{opt}} + \frac{1}{2} T_d$; if $T_m = T_s^{\text{opt}} + \frac{1}{2} T_d$, $T_D(\eta; \rho_a; \rho_d)$ is a constant and does not change with $\eta$. The result is intuitive, which means giving more bandwidth to the dominant tier is beneficial to the total network throughput.

VI. NETWORK ACCESS DESIGN

In this work, we use the distributed network access scheme CSMA to control the channel access of D2D transmitters and protect the ongoing small cell transmissions. To emphasize the benefit of the network access scheme, we compare CSMA with the random access scheme ALOHA. We study the network access scheme both from a coverage and throughput perspective. We refer to Fig. 3 which depicts that both the coverage probabilities of small cell tier and D2D user deteriorate with $\rho_s$. Similar effect can be seen for $\rho_d$. This is due to the fact that the retaining probability and corresponding $\lambda_d$ increase with $\rho_a$ and $\rho_d$. Thus, in terms of the overall coverage probability for infrastructure based transmissions and typical D2D user, the optimal sensing threshold is given by $\rho_s^* = 0$ and $\rho_d^* = 0$. However, the absence of D2D transmissions results in reduced network throughput.

Figure 8 depicts the total network throughput as a function of $\rho_a$ and $\rho_d$. From Fig. 8(a),
we observe that the network throughput exhibits a concave behavior with respect to $\rho_s$. This is caused by the opposite effects of $\rho_s$ on $\lambda_d$ and the coverage probability of the small cell tier and typical D2D user, a tradeoff between coverage probability and D2D user activity that is made explicit in the expressions of the network throughput (33) and (34). From Fig. 8(b), we notice a similar effect of $\rho_d$ on the network throughput. As for the coverage analysis, the effect of $\rho_s$ on the network throughput is more evident than the effect of $\rho_d$. This can be understood by the effectiveness of the protection threshold $\rho_s$ in controlling the mutual interference and improving the coverage probability. In addition, we observe that the optimal $\rho_s^*$ is larger than $\rho_d^*$ due to the smaller effect of $\rho_d$ on the D2D retaining probability $\beta$. Figure 8 also shows that in the dense scenario ($\lambda_s = 100\lambda_m$), giving more bandwidth to the small cell tier can increase the total network throughput. The presented results show that the proposed analytical framework can be used to determine $\rho_s^*$ or $\rho_d^*$ that maximize the total network throughput.

In Fig. 9 we depict the coverage probability and DL network throughput as a function of retaining probability $\beta$ or access probability $p$ when D2D users utilize CSMA and ALOHA. We keep $\rho_d = -20$ dBm, and change $\rho_s$ to alter $\beta$. With ALOHA, each D2D transmitter activates its transmission with a certain probability $p$, $p \in (0, 1]$, such that the active D2D transmitters form a PPP with density $p\lambda_d$. To make a fair comparison, let $p = \beta$. From Fig. 9(a), we observe that the use of CSMA scheme leads to a higher coverage probability than the use of ALOHA, especially for the typical D2D receiver. The is due to the fact that with CSMA scheme, only D2D users outside the small cell exclusion regions can be active, decreasing the interference from small cell transmitters to a large extent. From Fig. 9(b), we observe that the total network throughput gain achieved by CSMA over ALOHA scheme reaches up to 310% and 285% with $\eta = 0.1$ and $\eta = 0.9$, respectively. Similar to CSMA, there exists a $p^*$ to maximize the total network throughput. From the results depicted in Fig. 9 we observe that the optimal retaining probability with CSMA is higher than the optimal value with ALOHA, which means that in terms of network throughput a careful network access design provides more access opportunities for

7Note that with the given parameters, the protection threshold $\rho_s = -20$ dBm leads to negligible contention among D2D transmitters such that the network performance is dominated by $\rho_d$. 


potential D2D transmitters than the random ALOHA scheme. The results shown in Fig. 9 indicate that better network performance can be achieved by a careful network access design.

![Comparison of coverage probability and DL total network throughput from CSMA and ALOHA as a function of retaining probability $\beta$ or access probability $p$.](image)

**Figure 9.** Comparison of coverage probability and DL total network throughput from CSMA and ALOHA as a function of retaining probability $\beta$ or access probability $p$, (a) for coverage probability, (b) for downlink total network throughput, ($\lambda_u = 10^4 \lambda_s$, $\lambda_m = 100 \lambda_s$, $\{q_D, q_m\} = \{0.5, 0.5\}$, $\zeta = 0.1$, $\rho_d = -20$ dBm).

**Optimization of $\rho_a$ and $\rho_d$:** We take the DL total throughput derived in (33) as an example. Take the first derivative of $T_D(\eta; \rho_a; \rho_d)$ with respect to $\rho_a$, we have

$$
\frac{\partial T_D(\eta; \rho_a; \rho_d)}{\partial \rho_a} = (1 - \eta) \left( \frac{\partial T_a}{\partial \rho_a} + \frac{1}{2} \frac{\partial T_d}{\partial \rho_a} \right).
$$

Note that $\rho_a$ is a monotone increasing function with $P_D$, and $\lambda_D$ does not change with $\rho_a$, we have $\frac{\partial T_a}{\partial \rho_a} < 0$. With regard to $T_d$, we have $\frac{\partial T_d}{\partial \rho_a} = \log_2(1 + \gamma_d) \left( \frac{\partial P_d}{\partial \rho_a} \frac{\partial \lambda_d}{\partial \rho_a} + \lambda_d \frac{\partial P_d}{\partial \rho_a} \right)$. As shown in (21), we observe that $\rho_a$ has opposite effects on $\lambda_d$ and $P_d$, i.e. $\frac{\partial \lambda_d}{\partial \rho_a} > 0$, $\frac{\partial P_d}{\partial \rho_a} < 0$. Since $T_D(\eta; \rho_a; \rho_d)$ is a continuous function of $\rho_a$ over $[0, \infty)$, there exists at least one optimal $\rho_a^*$ where $T_D(\eta; \rho_a^*; \rho_d)$ is maximized. By averaging the channel fading and consider the pathloss, we have $\rho_a^* \in [0, Q_d]$, which reduces the complexity of the search process. Similarly, we can get $\rho_d^* \in [0, Q_d]$. The joint optimization of $\rho_a$ and $\rho_d$ may be implemented with a two-stage optimization method where we first fix $\rho_d$ and optimize the network throughput with respect to $\rho_a$, followed by the optimization with respect to $\rho_d$.

**VII. Conclusion**

In this work, we studied a two-tier D2D enhanced HCN operating with dynamic TDD, where the D2D transmitters follow a CSMA scheme. We proposed a simple PPP model for the
active D2D users and verified the accuracy by extensive simulations. We presented an analytical framework to evaluate the load-aware coverage probability and network throughput. The proposed model allows us to analyze the non-trivial system behavior of dynamic TDD networks and to quantify the effect of most important network parameters such as the UL/DL configuration, base station density, and bias factor on the coverage probability, and the bandwidth partition on the total network throughput. We evaluated how much D2D network access based on CSMA outperforms the ALOHA random access scheme and we provided guidelines on the optimal design of the network access scheme. Possible future directions to extend this work are to include a dynamic traffic model in our framework and consider the spatio-temporal correlations in the dynamic TDD network.

APPENDIX

A. Proof of Lemma 2

Assuming $T_0$ locates at the origin and by Slivnyak’s Theorem, the point process of the D2D contenders forms a PPP. By using the Palm distribution $P_0$, we derive the retaining probability $\beta$ of a potential D2D transmitter as follows:

$$\beta = P_0[U_0 = 1]$$

$$= E_0 \left[ \prod_{y_j \in \Phi_s^D} I_{Q_2h_{o,j}/\|T_0 - y_j\| > \rho_u} \prod_{z_k \in \Phi_s^{\perp, a}} I_{Q_2h_{o,k}/\|T_0 - z_k\| \leq \rho_u} \right] \left[ \prod_{t_k \in \Phi_d \setminus T_0} \left( 1_{t_k \leq t_k} + 1_{t_k > t_k} \right) \right]$$

$$\approx E_0 \left[ \prod_{y_j \in \Phi_s^D} \Pr(h_{o,j} < \frac{\rho_u}{Q_2}\|Y_j\|^\alpha) \right] E_0 \left[ \prod_{z_k \in \Phi_s^{\perp, a}} \Pr(h_{o,k} < \frac{\rho_u}{Q_2}\|Z_k\|^\alpha) \right] \times \int_0^1 E_0 \left[ \prod_{t_k \in \Phi_d \setminus T_0} \left( 1_{t_k \leq t_k} + 1_{t_k > t_k} \right) \right] dt$$

$$\approx \exp \left( -\lambda_s^D \int_{\mathbb{R}^2} e^{-\frac{\rho_u}{Q_2}\|x\|^\alpha} dx \right) \exp \left( -\lambda_s^{\perp, a} \int_{\mathbb{R}^2} e^{-\frac{\rho_u}{Q_2}\|x\|^\alpha} dx \right) \int_0^1 \exp \left( -\lambda_o t \int_{\mathbb{R}^2} e^{-\frac{\rho_u}{Q_2}\|x\|^\alpha} dx \right) dt$$

$$\leq \frac{1 - \exp \left( -\lambda_s^D - \lambda_s^{\perp, a} \right) \mathcal{K}_{o,d}}{\lambda_o \mathcal{K}_{o,d}}$$

where $\mathcal{K}_{o,a} = \frac{2\pi \Gamma(2/\alpha)}{\alpha \left( \frac{\rho_u}{Q_2} \right)^{2/\alpha}}$ and $\mathcal{K}_{o,d} = \frac{2\pi \Gamma(2/\alpha)}{\alpha \left( \frac{\rho_u}{Q_2} \right)^{2/\alpha}}$. (a) is due to the independence of PPPs, (b) follows by expectation over the channel gains $h$, (c) is obtained by using the probability generating functional (PGFL) of the PPP \cite{9} of $\Phi_s^D$, $\Phi_s^{\perp, a}$ and $\Phi_d$, and (d) evaluating the given integrals by
changing to polar coordinates and the use of Gamma function.

**B. Proof of Theorem 1**

First, we derive the coverage probability of the macro tier as follows. For DL mode,

\[
\mathbb{P}^D_m = \int_0^\infty \mathbb{E}_{\Phi_u} \left[ \exp \left( -\frac{\gamma_{D,m}^{D}}{P_m} (I_{D\to D}^{(m)} + I_{U\to D}^{(m)}) \right) \right] f_{Y_{D,m}} (r) \, dr
\]

\[
= \int_0^\infty \mathbb{E}_{\Phi_u} \left[ \exp \left( -\frac{\gamma_{D,m}^{D}}{P_m} I_{D\to D}^{(m)} \right) \right] \mathbb{E}_{\Phi_u} \left[ \exp \left( -\frac{\gamma_{U,m}^{D}}{P_m} I_{U\to D}^{(m)} \right) \right] f_{Y_{D,m}} (r) \, dr
\]

\[
= \int_0^\infty \exp \left( -\pi r^2 \lambda_D^D \delta (\gamma_{D,m}^{D}, \alpha) \right) \exp \left( -\pi r^2 \lambda_{u,m}^R C (\alpha) \left( \frac{Q_m}{P_m} \gamma_{u,m}^r \right)^{2/\alpha} \right) f_{Y_{D,m}} (r) \, dr
\]

\[
= \frac{q_{D,m} \lambda_m^D \mathcal{A}_{D,m} \delta (\gamma_{D,m}^{D}, \alpha) + \lambda_{u,m}^R \mathcal{C} (\alpha) \left( \frac{Q_m}{P_m} \gamma_{u,m}^r \right)^{2/\alpha} + q_{D,m} \lambda_m^D}{C (\alpha) \left( \gamma_{u,m}^r \right)^{2/\alpha} \mathcal{A}_{u,m} \left( \lambda_m^D \left( \frac{P_m}{Q_m} \right)^{2/\alpha} + \lambda_{u,m}^R \right) + (1 - q_{D,m}) \lambda_m^D}.
\]

where \( C (\alpha) = \frac{2\pi/\alpha}{\sin(2\pi/\alpha)} \), and \( \delta (\beta, \alpha) = \int_0^\infty \frac{\beta^{2/\alpha}}{1+u^{2/\alpha}} \, du \), (a) follows by taking expectation over the channel gains \( h \), (b) is due to the independence of the PPPs. (c) results from the Laplace transform and the first exponential term is due to the DL association policy, where the active interfering MBSs can not stay within the disk \( b(0, r) \). Finally, (d) by integrating with respect to the PDF \( f_{Y_{D,m}} (y) \) as defined in [14]. For UL mode, the coverage probability is given by

\[
\mathbb{P}^U_m = \int_0^\infty \mathbb{E}_{\Phi_u} \left[ \exp \left( -\frac{\gamma_{U,m}^{U}}{Q_m} (I_{D\to U}^{(m)} + I_{U\to U}^{(m)}) \right) \right] f_{Y_{U,m}} (r) \, dr
\]

\[
= \int_0^\infty \exp \left( -\pi r^2 \lambda_m^D C (\alpha) \left( \frac{P_m}{Q_m} \gamma_{u,m}^r \right)^{2/\alpha} \right) \exp \left( -\pi r^2 \lambda_{u,m}^R C (\alpha) \left( \gamma_{u,m}^r \right)^{2/\alpha} \right) f_{Y_{U,m}} (r) \, dr
\]

\[
= \frac{(1 - q_{D,m}) \lambda_m}{C (\alpha) \left( \gamma_{u,m}^r \right)^{2/\alpha} \mathcal{A}_{u,m} \left( \lambda_m^D \left( \frac{P_m}{Q_m} \right)^{2/\alpha} + \lambda_{u,m}^R \right) + (1 - q_{D,m}) \lambda_m}.
\]

With regard to the small cell tier, the interferer consists of DL active SAPs, transmitting mobile users associated with small cell tier, and the active D2D transmitters. The Laplace transform of the aggregate interference from SAPs and transmitting mobile users in DL and UL mode can be derived similar to [39] and [40], respectively. In the following, we focus on the Laplace transform of the aggregate interference from active D2D transmitters. As is shown in Fig. 2, we have

\[
\mathcal{L}_I (s) = \mathbb{E}_{\Phi_d} \left[ \exp \left( -s \left( \sum_{z \in \Phi_d \cap \mathcal{H}} Q_d h_{oz} z^{-\alpha} \right) \right) \right] \mathbb{E}_{\Phi_d} \left[ \exp \left( -s \left( \sum_{z \in \Phi_d \cap \mathcal{H} \cap \mathcal{B}} Q_d h_{oz} z^{-\alpha} \right) \right) \right].
\]
where the first term is related to the interferer distributed outside the big circle of radius \( \| y_0 \| + \iota_a \), i.e. the shaded region \( \Phi_d \cap \overline{\mathcal{H}} \), and can be easily derived. The second term corresponds to the interferer scattered over the shaded region within the big circle, i.e. \( \Phi_d \cap \mathcal{H} \cap \overline{\mathcal{B}} \). Two cases can be distinguished for the calculation of the Laplace transform: \( \| y_0 \| \leq \iota_a \) (see Fig. 2(a)) and \( \| y_0 \| > \iota_a \) (see Fig. 2(b)). Denote the Laplace transform of the interference from active D2D transmitters in \( \Phi_d \cap \overline{\mathcal{H}} \) and \( \Phi_d \cap \mathcal{H} \cap \overline{\mathcal{B}} \) as \( L_{I_{\text{out}}} (s) \) and \( L_{I_{\text{in}}} (s) \), respectively. Conditioned on the typical small cell link length being \( \| y_0 \| \), we have

\[
L_{I_{\text{out}}} (s \mid \| y_0 \|) = \mathbb{E}_{\Phi_d} \left[ \exp \left( -s \left( \sum_{z \in \Phi_d \cap \overline{\mathcal{H}}} Q_d h_{oz} z^{-\alpha} \right) \right) \mid \| y_0 \| \right] \\
= (a) \exp \left( -2\pi \lambda_d \int_{\iota_a + \| y_0 \|}^{\infty} \frac{y}{1 + \frac{y^2}{sQ_d}} dy \right) \\
= (b) \exp \left( -\pi \lambda_d (\iota_a + \| y_0 \|)^2 \delta \left( \frac{sQ_d}{(\iota_a + \| y_0 \|)^\alpha} \right), \right) 
\]  

where (a) follows from the PGFL of the PPP, and (b) is resulted from \( \delta (\beta, \alpha) = \int_{\beta - 2/\alpha}^{\infty} \frac{\beta^{2/\alpha}}{1 + u^{\alpha/2}} du \), where \( \iota_a + \| y_0 \| \) is the distance from the nearest interferer staying within \( \Phi_d \cap \overline{\mathcal{H}} \).

In the following, we focus on the computation of \( L_{I_{\text{in}}} (s) \). For notational simplicity, we first define a \( Z \) function as

\[
Z_{\theta_l, \theta_u, \kappa_l, \kappa_u}^{\theta, \kappa} (s; Q) = (sQ)^{2/\alpha} \int_{\theta_l}^{\theta_u} \int_{\kappa_l}^{\kappa_u} \frac{1}{1 + u^{\alpha/2}} dud\theta, 
\]  

where \( Q \) represents the transmit power of interferer, \( \theta_l, \theta_u, \) and \( \kappa_l, \kappa_u \) denote the lower bound and upper bound of the angle and distance from the interferer distributed in the shaded region \( \Phi_d \cap \mathcal{H} \cap \overline{\mathcal{B}} \).

For \( \| y_0 \| \leq \iota_a \), denote \( l_{OA} = \| OA \| \), which is given by

\[
l_{OA} = \sqrt{l_a^2 - (\| y_0 \| \sin \theta)^2} + \| y_0 \| \cos \theta, \quad \theta \in [0, \pi].
\]  

(44)
The Laplace transform $\mathcal{L}_{I_{in}}(s \mid \|y_0\| \leq \iota_a)$ is derived as
\[
\mathcal{L}_{I_{in}}(s \mid \|y_0\| \leq \iota_a) = \mathbb{E}_{\Phi_d} \left[ \exp \left( -s \left( \sum_{z \in \Phi_d \cap H_{\|y_0\|}} Q_d h_{oz} z^{-\alpha} \right) \right) \mid \|y_0\| \leq \iota_a \right]
\]
\[
\equiv (a) \exp \left( -2\lambda_d \int_0^\pi \int_0^{\iota_s + \|y_0\|} \frac{y}{1 + \frac{y}{sQ_d}} dyd\theta \right)
\]
\[
\equiv (b) \exp \left( -\lambda_d \mathcal{Z}_{0,\iota_s + \|y_0\|}^{\iota_s,\iota_s + \|y_0\|} (s; Q_d) \right),
\]
where (a) follows by the PGFL of the PPP and the change of polar coordinates, and (b) by substituting the corresponding bounds of the angle and distance into (43). By combining (42) with (45), we have
\[
\mathcal{L}(s \mid \|y_0\| \leq \iota_a) = \mathcal{L}_{I_{out}}(s \mid \|y_0\|) \mathcal{L}_{I_{in}}(s \mid \|y_0\| \leq \iota_a),
\]
(46)

For $\|y_0\| > \iota_a$, denote $\Theta = \arcsin \left( \frac{\iota_s }{\|y_0\|} \right)$, $l_{OD} = \|OD\|$ and $l_{OC} = \|OC\|$, where $l_{OD}$ and $l_{OC}$ can be found by simple geometric formulas, similar to $l_{OA}$ given in (44). The Laplace transform $\mathcal{L}_{I_{in}}(s \mid \|y_0\| > \iota_a)$ is given by
\[
\mathcal{L}_{I_{in}}(s \mid \|y_0\| > \iota_a) \overset{(a)}{=} \exp \left( -2\lambda_d \left( \int_0^{\Theta} \int_0^{l_{OD}} \frac{y}{1 + \frac{y}{sQ_d}} dyd\theta 
\right. \right.
\]
\[
+ \int_0^{\Theta} \int_{l_{OC}}^{\iota_s + \|y_0\|} \frac{y}{1 + \frac{y}{sQ_d}} dyd\theta + \int_{\Theta}^{\pi} \int_0^{\iota_s + \|y_0\|} \frac{y}{1 + \frac{y}{sQ_d}} dyd\theta \left. \right)
\]
\[
= \exp \left( -\lambda_d \left( \mathcal{Z}_{0,0}^{\iota_s,\iota_s + \|y_0\|} (s; Q_d) + \mathcal{Z}_{0,l_{OD}}^{\Theta,\iota_s + \|y_0\|} (s; Q_d) + \mathcal{Z}_{\Theta,l_{OC}}^{\pi,\iota_s + \|y_0\|} (s; Q_d) \right) \right),
\]
(47)

where (a) is due to the fact that when $\theta < \arcsin \left( \frac{\iota_s }{\|y_0\|} \right)$, the line $OB$ passes through the whole exclusion region. By combining (42) with (47), we obtain
\[
\mathcal{L}(s \mid \|y_0\| > \iota_a) = \mathcal{L}_{I_{out}}(s \mid \|y_0\|) \mathcal{L}_{I_{in}}(s \mid \|y_0\| > \iota_a),
\]
(48)

For DL transmission, let $r = \|y_0\|$, substituting $I = I_{d \rightarrow D} = \sum_{z \in \Phi_d \setminus \{y_0, \iota_a\}} Q_d h_{oz} z^{-\alpha}$, we
derive

\[
P^D_a = \int_0^\infty \mathbb{E}_{\Phi_s^D, \Phi_{u_a}, \Phi_d} \left[ \exp \left( -\frac{\gamma^D_{u_a, \alpha}}{P_s} (I_{D \to u}^{(s)} + I_{u \to D}^{(s)} + I_{d \to D}) \right) \right] f_{Y_{D,a}}(r) \, dr
\]

\[
= \frac{\pi q_{D,a} \lambda_a}{\mathcal{A}_{D,a}} \left[ \int_0^{r_s^2} e^{-\pi u \mathcal{F}} \cdot \mathcal{L}_{I_{d \to D}}(\frac{\gamma^D_{u,a, \alpha} / 2}{P_s} | v \leq r_s^2) \, dv \\
+ \int_r^{\infty} e^{-\pi u \mathcal{F}} \cdot \mathcal{L}_{I_{d \to D}}(\frac{\gamma^D_{u,a, \alpha} / 2}{P_s} | v > r_s^2) \, dv \right],
\]

(49)

where (a) follows by inserting \( s = \frac{\gamma^D_{u,a, \alpha}}{P_s} \) and the change of variable \( r^2 = v \), and

\[
\mathcal{F} \triangleq \lambda^D_a \delta(\gamma^D_{u,a, \alpha}) + \lambda^D_{u,a} C(\alpha) \left( \frac{Q_s}{P_s} \right)^{2/\alpha} + \frac{q_{D,a} \lambda_a}{\mathcal{A}_{D,a}},
\]

(50)

where the first term and second term, respectively, relates to the interference from DL SAPs and transmitting mobile users, and the last term comes from \( f_{Y_{D,a}}(r) \) derived in (14).

For UL transmission, substituting \( I = I_{d \to u} = \sum_{z \in \Phi_{d} \setminus b(x_o, \tau_d)} Q_d h_o z^{-\alpha} \), we have

\[
P^U_a = \int_0^\infty \mathbb{E}_{\Phi_s^U, \Phi_{u_a}, \Phi_d} \left[ \exp \left( -\frac{\gamma^U_{u,a, \alpha}}{Q_s} (I_{D \to u}^{(s)} + I_{u \to D}^{(s)} + I_{d \to u}) \right) \right] f_{Y_{D,a}}(r) \, dr
\]

\[
= \frac{\pi (1 - q_{D,a}) \lambda_a}{\mathcal{A}_{U,a}} \left[ \int_0^{r_s^2} e^{-\pi u \mathcal{G}} \cdot \mathcal{L}_{I_{d \to u}}(\frac{\gamma^U_{u,a, \alpha} / 2}{Q_s} | v \leq r_s^2) \, dv \\
+ \int_r^{\infty} e^{-\pi u \mathcal{G}} \cdot \mathcal{L}_{I_{d \to u}}(\frac{\gamma^U_{u,a, \alpha} / 2}{Q_s} | v > r_s^2) \, dv \right],
\]

(51)

where (a) follows by inserting \( s = \frac{\gamma^U_{u,a, \alpha}}{Q_s} \) and the change of variable \( r^2 = v \), and

\[
\mathcal{G} \triangleq C(\alpha) \left( \frac{\gamma^U_{u,a, \alpha}}{Q_s} \right)^{2/\alpha} + \lambda^U_{u,a} + \frac{(1 - q_{D,a}) \lambda_a}{\mathcal{A}_{U,a}}.
\]

(52)

Furthermore, combining (1), (2) with (39), (40), (49) and (51), we obtain the overall load-aware coverage probabilities in DL and UL in (20).

At last, we obtain the D2D receiver coverage probability. Due to the small value of \( r_d \), we reasonably assume \( r_d \leq \tau_a \) and \( r_d \leq \tau_d \) to simplify the analysis. Due to the CSMA scheme, it is equivalent to draw two exclusion regions around the serving D2D transmitter with radius \( \tau_a \) and \( \tau_d \), respectively, within which, the small cell transmitters and D2D transmitters are not allowed to exist. The typical D2D receiver is assumed to be at the origin and the serving D2D
transmitter locates at \( z_0 \). The derivation of the coverage probability of typical D2D receiver is similar to the coverage probability of small cell tier with the case \( \| y_0 \| \leq \iota_{s} \), as shown in Fig. 2(a). Thus, we skip the details and give the results directly.

\[
P_d = \mathbb{E}_{\Phi_0, \Phi_u, \Phi_d} \left[ \exp \left( -\frac{\gamma_{d}^{\alpha} Q_d}{Q_d} (I_{0\rightarrow d} + I_{y_0 \rightarrow d} + I_{d \rightarrow d}) \right) \right]
\]

\[
= \exp \left( -\mathcal{I}_1 \left( \frac{\gamma_{d}^{\alpha} P_s}{Q_d} ; \gamma_{d}^{\alpha} Q_s \right) - \mathcal{I}_2 \left( \frac{\gamma_{d}^{\alpha} Q_d}{Q_d} ; \gamma_{d}^{\alpha} Q_d \right) - \mathcal{I}_3 \left( \frac{\gamma_{d}^{\alpha} Q_d}{Q_d} ; \gamma_{d}^{\alpha} Q_d \right) \right),
\]

where \( \mathcal{I}_1(s; P_s) \), \( \mathcal{I}_2(s; Q_s) \) and \( \mathcal{I}_3(s; Q_d) \) are given by (26), (27) and (28), respectively.

REFERENCES

[1] J. Hoydis, M. Kobayashi, and M. Debbah, “Green small-cell networks,” IEEE Veh. Technol. Mag., vol. 6, no. 1, pp. 37–43, Mar. 2011.

[2] T. Q. S. Quek, G. de la Roche, I. Gven, and M. Kountouris, Small Cell Networks: Deployment, PHY Techniques, and Resource Management. New York, NY, USA: Cambridge Univ. Press, 2013.

[3] S. Sesia, I. Toufik, and M. Baker, LTE: The UMTS Long Term Evolution: From Theory to Practice. New Jersey, NJ, USA: John Wiley & Sons, 2011.

[4] Z. Shen, A. Khoryaev, E. Eriksson, and X. Pan, “Dynamic uplink-downlink configuration and interference management in TD-LTE,” IEEE Commun. Mag., vol. 50, no. 11, pp. 51–59, Nov. 2012.

[5] M. S. ElBamby, M. Bennis, W. Saad, and M. Latva-aho, “Dynamic Uplink-Downlink Optimization in TDD-based Small Cell Networks,” in Proc. IEEE INFOCOM Workshops, Toronto, Canada, Apr. 27 - May 2, 2014, pp. 712–717.

[6] C.-H. Yu, K. Doppler, C. Ribeiro, and O. Tirkkonen, “Resource sharing optimization for device-to-device communication underlaying cellular networks,” IEEE Trans. Wireless Commun., vol. 10, no. 8, pp. 2752–2763, Aug. 2011.

[7] D. Feng, L. Lu, Y. Yuan-Wu, G. Ye Li, S. Li, and G. Feng, “Device-to-device communications in cellular networks,” IEEE Commun. Mag., vol. 52, no. 4, pp. 49–55, Apr. 2014.

[8] J. Li, S. Farahvash, M. Kavehrad, and R. Valenzuela, “Dynamic TDD and fixed cellular networks,” IEEE Commun. Lett., vol. 4, no. 7, pp. 218–220, Jul. 2000.

[9] W. K. D. Stoyan and J. Mecke, Stochastic geometry and its applications. Chichester: Wiley, 1995.

[10] M. Haenggi, J. G. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, “Stochastic geometry and random graphs for the analysis and design of wireless networks,” IEEE J. Sel. Areas Commun., vol. 27, no. 7, pp. 1029–1046, Sept. 2009.

[11] B. Yu, S. Mukherjee, H. Ishii, and L. Yang, “Dynamic TDD support in the LTE-B enhanced Local Area architecture,” in IEEE GLOBECOM Workshops, Anaheim, America, Dec. 3-7, 2012, pp. 585–591.

[12] Y. S. Soh, T. Q. S. Quek, M. Kountouris, and G. Caire, “Cognitive hybrid division duplex for two-tier femtocell networks,” IEEE Trans. Wireless Commun., vol. 12, no. 10, pp. 4852–4865, Oct. 2013.

[13] V. Chandrasekhar, J. G. Andrews, T. Muharemovic, Z. Shen, and A. Gatherer, “Power control in two-tier femtocell networks,” IEEE Trans. Wireless Commun., vol. 8, no. 8, pp. 4316–4328, Aug. 2009.
[14] M. Wildemeersch, T. Q. S. Quek, C. Slump, and A. Rabbachin, “Cognitive small cell networks: Energy efficiency and trade-offs,” IEEE Trans. Commun., vol. 61, no. 9, pp. 4016–4029, Sept. 2013.

[15] V. Chandrasekhar and J. G. Andrews, “Spectrum allocation in tiered cellular networks,” IEEE Trans. Wireless Commun., vol. 57, no. 10, pp. 3059–3068, Oct. 2009.

[16] W. C. Cheung, T. Q. S. Quek, and M. Kountouris, “Throughput Optimization, Spectrum Allocation, and Access Control in Two-Tier Femtocell Networks,” IEEE J. Sel. Areas Commun., vol. 30, no. 3, pp. 561–574, Apr. 2012.

[17] T. V. Nguyen and F. Baccelli, “A stochastic geometry model for cognitive radio networks,” The Computer Journal, vol. 55, no. 5, pp. 534 – 552, Jul. 2012.

[18] M. Haenggi, “Mean interference in hard-core wireless networks,” IEEE Commun. Lett., vol. 15, no. 8, pp. 792–794, Aug. 2011.

[19] H. ElSawy and E. Hossain, “A Modified Hard Core Point Process for Analysis of Random CSMA Wireless Networks in General Fading Environments,” IEEE Trans. Commun., vol. 61, no. 4, pp. 1520–1534, Apr. 2013.

[20] C. Han Lee and M. Haenggi, “Interference and outage in poisson cognitive networks,” IEEE Trans. Wireless Commun., vol. 11, no. 4, pp. 1392–1401, Apr. 2012.

[21] H.-S. Jo, Y. J. Sang, P. Xia, and J. G. Andrews, “Heterogeneous cellular networks with flexible cell association: A comprehensive downlink SINR analysis,” IEEE Trans. Wireless Commun., vol. 11, no. 10, pp. 3484–3495, Oct. 2012.

[22] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, “Stochastic analysis of spatial and opportunistic Aloha,” IEEE J. Sel. Areas Commun., vol. 27, no. 7, pp. 1105–1119, Sept. 2009.

[23] M. Wildemeersch, T. Q. S. Quek, M. Kountouris, A. Rabbachin, and C. H. Slump, “Successive interference cancellation in heterogeneous cellular networks,” in Proc. ICASSP, Florence, Italy, May 4-9, 2014.

[24] S. Singh, H. S. Dhillon, and J. G. Andrews, “Offloading in heterogeneous networks: Modeling, analysis, and design insights,” IEEE Trans. Wireless Commun., vol. 12, no. 5, pp. 2484–2497, May 2013.

[25] S. M. Yu and S.-L. Kim, “Downlink capacity and base station density in cellular networks,” in Proc. IEEE WiOpt, Tsukuba Science City, Japan, May 13-17, 2013, pp. 119–124.

[26] J. G. Andrews, F. Baccelli, and R. Ganti, “A Tractable Approach to Coverage and Rate in Cellular Networks,” IEEE Trans. Commun., vol. 59, no. 11, pp. 3122–3134, Nov. 2011.