Underground Structures Endangered by Explosion

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Abstract. Because of the increasing prevalence of the threat of explosions endangering structures, the effective protection of parts of the civil infrastructure, such as embassies, government buildings, airports, bridges, tunnels, as well as the effect of gas explosions and other accidents of that kind against blast loads, has become a critical issue. Hence, it is of great importance to assess the dynamic impacts of explosives, which are required by some contractors or developers to reduce the risk of structural damage due to shock waves. In order to fulfill these demands, accurate computation is necessary and on the other hand, also new protective structures have to be suggested and verified. Effects of the explosion on the damage to tunnels, which are a typical example of closed spaces, are studied using Navier-Stoke’s equations describing movement in the air and discontinuous boundary elements solving the damage to the solid structure, i.e. the lining of the tunnel emerged in the rock environment.

1. Introduction

Today's world has a very urbanized structure - more than a half of the population of 6.8 billion people lives in cities and underground spaces are one of the most prerogative zones for cities of modern civilization. Together with the urbanization, underground spaces have grown along with the risk of terrorism and its organization. Therefore, it is very important to predict the dynamic impacts of explosives in the underground spaces and the waves should be additionally transformed into a loading of the solid structures. In this paper, the impact of explosions and air shock waves is formulated and solved. The location of the center of the explosion is very relevant. It appears that if the location is at the solid surface, this part can be up to 30% of the total energy (in soft solids). The variables to be calculated are the mass density of a gas, the velocity of movements and the internal energy, or entropy. The latter covers the influence of the gas pressure, being given for the adiabatic state.

The expensive, time-consuming experiments should support the formulation and identification of necessary parameters in modeling the real behavior of interaction structure-shock waves. The models can then comprise improved effects of the interaction incorporating reflective and reverberating waves, attenuation of the air, changes in the air density, etc. Numerical simulation has become an important approach and plays vital roles in engineering applications and scientific investigations, but the methods, which follow from the accurate computations also have to be transformed to some fast algorithms being available for pre-designed decision of the consulting engineer. In general, one can say that the extreme design situation is causing the failure of the structure which was designed and built in accordance with the applicable regulations before its service life. The relevance of the above factors comes to the forefront especially, in the last decades, in connection with the extraordinary events that lead to the failure of the load-bearing structures.

In order to ensure the required bearing capacity and stability of structures exposed to an extreme load or possibly to provide enough time needed to evacuate people out of the touched area, any
structural complex must be designed with the required fire resistance and resistance against impact load, in accordance with applicable standards and regulations, or accepting the newest knowledge in the field envisaged.

The research is here focused on the effect of a type of support structures in the underground free spaces. Blast or explosion is a physical phenomenon, in which there is a sudden, very rapid release of energy, usually accompanied by a local increase in temperature and pressure. Depending on the source of the local release of energy, the explosion can be divided into many causes. In this study, mechanical explosion, as the main influence of the explosion, is dealt with.

Gas dynamics describing the dispersion of the impact shock wave is based on similar equations of hydrodynamics. One of the fundamental books on this issue is written by a famous couple of authors, Landau and Lipschitz, [1], and there is a comprehensive study on hydrodynamics put forward, and [2] it is a modern instruction on how to work out the super-, hyper-sonic behavior in the air as well. The books serve a starting point for the formulations of strongly non-linear equations governing the gas dynamics: the Euler equations, the Navier-Stoke equations. Starting with the latter equations, the shock waves cause discontinuities in mass density, velocity and entropy before and after the shock wave front. There is a couple of theoretical and practical papers, which formulate this phenomenon, [3-5]. The latter paper deals with Arbitrary-Lagrangian-Eulerian method, which seems to be one of the most attractive methods for researchers in the field of propagation of shock waves.

The model of damage states in the lining of underground structures is based on discontinuous boundary element method, [6-8], which is one of distinct element methods or the free hexagonal particles, and they enable us the definition of stress and strain states inside of each particle. In this paper, hexagonal particles, free in the space and connected with the neighboring particles by either springs (soft contact) or using the Uzawa algorithm (hard contact), are considered. Moreover, contrary to finite elements, boundary elements make it possible to create continuous relation between tractions and displacements along the interfacial boundary of the adjacent particles.

2. Discontinuous boundary element method – static case
Consider a mesh of arbitrarily selected hexagonal elements covering the domain of both air and the solid phase in the coordinate system $\Omega_{1,2}$. The elements are not-overlapping. Each element shares a common part of element boundaries – abscissa – with one of the adjacent elements. Let us denote $k_n$ the spring stiffness in the normal direction and the spring stiffness in the tangential direction as $k_t$ on the interface, see Figure 1.

Inside of each element, linear elasticity is described by boundary elements, which lead us to the well-known matrix relation (matrix $B$ is always regular and therefore admits an inversion):

$$Au(t) = Bp(t) + b(t) \Rightarrow Ku(t) = p(t) + Q(t), \quad K = B^{-1}A, \quad Q = B^{-1}b$$ (1)
Figure 1. Three adjacent elements (left) and denotation of contact conditions (right)

Both matrices, \( A, B \) are square, matrix \( B \) is always regular and therefore admits an inversion. \( u, p \) are boundary displacements and tractions, respectively, both uniformly distributed along the boundary abscissas. This is the advantage of boundary elements, as finite elements and higher order boundary elements have to create such a property in a complicated and not accurate way. Vector \( b \) involves volume weight on the one side and also lumped D’Alembert inertia forces. The advantage mentioned previously is also projected into relations defined in the next paragraph.

Namely, after deformation, \( [u]_1 \) is the difference between displacement \( u \) in the \( x_1 \) direction, while the difference between displacement \( u \) in the direction \( x_2 \) is \( [u]_2 \), see Figure 1. In the normal direction, adjacent elements are constrained by contact conditions, which state that no such elements can overlap each other and debonding when the tensile limit state is reached:

\[
[u]_n \geq 0, \quad p_n \leq p_n^+, \quad \text{if} \quad p_n > p_n^+ \Rightarrow p_n = 0
\]

where \( [u]_n \) is a jump in the displacements in normal direction, \( p_n \) are tractions in normal direction and \( p_n^+ \) is an admissible normal stress (traction). In the tangential (shear) direction the Mohr-Coulomb hypotheses is applied. A special form used in the free hexagon method can be recorded as:

\[
| p_t | \leq c \kappa (p_n^+ - p_n) - p_n \tan \phi,
\]

\[
\text{if} \quad | p_t | > c \kappa (p_n^+ - p_n) - p_n \tan \phi \Rightarrow p_t = p_n \tan \phi \text{sgn}[u],
\]

where \( p_t \) is shear traction, \( [u]_t \) is the jump in displacements in tangential direction, \( \phi \) and \( c \) are material constants, \( \kappa \) is the Heaviside function ensuring that when the condition of \( c = 0 \) is met, there is a debond between the adjacent particles.

The solution coming from the above-described relations consists of selecting a current particle and the other particles are fixed. Then the movement of this particle fulfills the equilibrium with the neighboring elements. The requirements are fulfilled:

- there is no penetration (mutual overlapping) of the elements,
- tensile strength \( p_n^+ \) cannot be exceeded on the interfaces. The tensile strength decides whether the split occurs in direction normal to the interfaces.
- Mohr–Coulomb hypothesis is fulfilled in tangential direction.
Discontinuous boundary element method (free hexagon method) provides a particle model, similar to classical PFC (Particle Flow Method), containing the following assumptions used specifically in this paper:

- All particles are hexagonal, and either regular partition of the domain is considered (honey combs), or the hexagons possess arbitrary convex shape.
- Linear material behavior is assumed inside of each particle and is described by the boundary element method, with a uniform distribution of both displacements and tractions along the boundaries of the particles.
- The contacts are always specified along the interfaces (abscissas) between two adjacent elements.
- Behavior along the contacts are considered as soft, based on the idea of a spring connection in both the normal and tangential directions with respect to the boundaries between adjacent elements.

It is worth noting that in comparison to the classical PFC, the free hexagon method allows for the definition of stresses along the interfaces.

3. Dynamic case

The equations of motion can be expressed as two types of vector equations, one of which relates the resultant forces to the normal (translational) and tangential motion and other of which relates the resultant moment to the shear (rotational) motion. If the global coordinate system is \(x_1, x_2\), in each particle the equation for translational motion and rotation have to be fulfilled and can be written in the vector form:

\[
F_i = m(\ddot{x}_i - g_i),
\]  

where \(F_i\) are the resultant forces, the sum of all externally applied forces acting on the current particle in \(i\)-th direction, i.e. in the direction \(x_1, x_2\), \(m\) is total mass of the particle, \(\ddot{x}_i\) (dot above the variables stands for derivative by time) is the acceleration and \(g_i\) is the body force acceleration vector (e.g. loading due to gravity). The translation is calculated as:

\[
\ddot{x}_i = \frac{1}{\delta t}(\ddot{x}_i^{t+\delta t/2} - \ddot{x}_i^{t-\delta t/2})
\]  

Inserting the velocities from the latter equation to the equations for motion and solving it for the velocities at time \((t + \delta t/2)\) results in:

\[
m[(\ddot{x}_i^{t+\delta t/2} - \ddot{x}_i^{t-\delta t/2}) / \delta t - g_i] = F_i^{t+\delta t/2}
\]  

Finally, the velocities are used to update the position of the particle center:

\[
\ddot{x}_i^{t+\delta t/2} = (x_i^{t+\delta t} - x_i^{t-\delta t}) / \delta t, \quad \ddot{x}_i^{t+\delta t/2} = (x_i^{t+\delta t} - x_i^{t}) / \delta t
\]  

where \(x_i\) are the coordinates of the nodes moved in time \(t\), actually the displacements

\[
u_i = x_i - x_i^0,
\]  

where \(x_i^0\) are the starting coordinates of the nodal point.

Our aim is to calculate new positions of nodes in each particle. For this aim, it is necessary to carry out transformations of coordinates from global and local coordinate systems. Note that in contradiction to PFC, the equilibrium of the forces distributed along the interfacial boundaries and the inertia forces have to be calculated from the movements or more exactly, as averages of the tractions. The stresses,
strains and displacements inside the particles are easily calculated from the tractions, as there are planes (abscissas), contrary to the PFC, where point touch of the particles is considered. Finally, the “soft contact” can be created in a simple way, as there is a one-to-one relation between the boundary displacement and tractions. A similar assertion is valid for the “hard contact” being formulated, using Uzawa’s algorithm.

Summing up, at the time \( t \) one knows the displacements \( x_i^t \) at the time \( t \) and the same quantities at the previous time-step \( x_i^{t-\delta t} \). From this, it is also known \( F_i^t \). Applying the former forces and the time-increment of loading, one gets \( x_i^{t+\delta t} \equiv x_i^{t+\delta t} \) from the algorithm defined above. Using the second formula (7), \( x_i^{t+\delta t/2} \) immediately follows. In order to get new inertia forces from the known position-dependent iteration, an explicit time-dependent iteration can be established which is in compliance with the description of propagation of shock wave in the air. Since the number of degrees of freedom may be very extensive, for calculation of accelerations at the time \( T \) the forces \( F_i^t \) are utilized. It means that (6) is applied in the sense of the mentioned assumption:

\[
 m[(\dot{x}_i^{t+\delta t/2} - \dot{x}_i^{t-\delta t/2})/\delta t - g_i] = F_i^t, 
\]

and new velocities of the particles \( \dot{x}_i^{T+\delta t/2} \) are available from the previous formulas. Since the new positions of the nodes \( x_i^{T} \) are known from the position iteration. The first relation (7) delivers

\[
 \dot{x}_i^{T-\delta t/2} = (x_i^{T} - x_i^{t})/\delta t. 
\]

Now \( F_i^{T+\delta t} \equiv F_i^T \) is obtainable from (4) setting \( t + \delta t := T + \delta t \) and \( t - \delta t := T - \delta t \). Updating all particles, the position iteration follows.

4. Example

An explosion in an underground tunnel with rectangular cross-section is considered. The charge is located in the center of the till. Symmetric case is solved. The radius of the charge \( R_c \) is 0.245 m, its mass \( q = 50 \) kg and the density of TNT \( \rho_{TNT} = 1620 \) kg/m\(^3\).

A simplification is introduced here that the material properties of the rock and concrete lining are uniform and given in a standard way by modules for linear elasticity: \( E = 208 \times 10^9 \) N/m\(^2\), \( \rho = 7833 \) kg/m\(^3\), \( \nu = 0.29 \), \( c = 0.9 \) MPa, \( \varphi = 30^\circ \).

In Figure 2, the geometry of the problem is seen together with the particle set up for the particular calculation. The number of particles in our problem is 1453; along x-axis symmetry is assumed, along y-axis symmetric boundary conditions. Since gravity has been set up a long time before the state in which the structure is assessed, movements do not have to be subtracted from that calculated in the virgin state. This is why the vertical displacements are almost negligible far from the face of the parking, while in the neighborhood of the face significant movements are observed, as seen from the next pictures.
In the next pictures on the left side, the distribution of moving particles is depicted and on the right side, the same with the vectors of displacements are displayed. The particular elements are shown themselves in undeformed state in order to distinguish the concentration of stresses or jump in the displacements (debond).

The following pictures show the displacements and their vectors from the undeformed state. The first state of the movements of particles is calculated at 0.01 sec and the results of the solution are seen in Figure 3. The pressure from the explosion is concentrated at the axis of symmetry, i.e. at the crossing of the $x$-axis and the face of the parking. Significant horizontal displacements and a small lift in the vertical direction are observed in the pictures.

After another 0.02 sec of development of the pressure from the explosion, a larger part of the face is touched by the explosion and concentration of the pressure is spread out along the face. In Figure 4, this situation is depicted. From this, one can see that larger displacements occur along the longer part of the face.

In Figure 5, the ceiling of the parking lifts more distinctly and the horizontal movements are very extensive. The pressure spreads out against the ceiling and the front wall, which is directly touched by
the explosion. Moreover, the movement back at the axis of symmetry starts to be in effect. This phenomenon appears fully in Figure 6, where the resistance of the rock causes backward movement. The process can continue till the local failure of the rock surrounding the parking or the stable state is reached.

Figure 4. Movement of the particles after 0.03 sec

Figure 5. Movement of the particles after 0.08 sec
5. Results and discussion
The use of discrete element methods has proven to be very successful in solving time-dependent problems, and especially in monitoring the structure's response to its behavior at different time phases. The method of discontinuous boundary elements belonging to the discrete method proves to be very useful mainly in monitoring the stress concentration and the development of the damage to the structure. The accompanying figures in this paper are designed so that the movements of the individual particles (hexagons) move in accordance with the movements achieved by the computation, but the deformation of the elements themselves is not depicted, allowing better observation and estimate of stress concentrations and particle movements.

When changing the direction of the shock wave due to reflection from a solid part of, for example, rock or concrete lining, or because of interaction with the secondary waves, there is a dilution in the transition area between the two media, air and solid phase, and thus tensile failure of the structure and spalling occur. This is a typical damage phenomenon and can even location can be identify where it begins. Note that if the discontinuous boundary element method is applied, it is not necessary to know the origin where the failure starts.

Spalling is not reflected in the figures in this publication, as a larger period of time needs to be observed for this phenomenon.

6. Conclusions
In this paper, very topical problem is proposed and solved, which is the effect of an explosion in a closed space – underground structure, tunnel lining. The effect of the explosion of a charge being situated at the side wall of the underground parking is formulated and solved. The free hexagon method is proposed as an appropriate numerical tool and developed for the time dependent D’Alembert principle involved to the formulation of the numerical procedure. It appears that in such an application, the method envisaged is very powerful and efficient as the computer time consumed for computation is relatively small.

Although short time steps seem to be required, the results from the example presented, in the frame of this paper, are reasonable for time step equal to 0.01 sec. This relatively rough discretization of the time interval does not harm seriously the results obtained by this procedure.

The time-dependent primary loading of the side wall of the underground structure is compared with the results from the previous studies of the behavior of strike waves in the air (obeying the gas
dynamics equations of conservation of mass density, velocity and energy, in the case of supersonic movement the shock waves have to obey ALE method). In these previous studies, the bearing system (side walls, ceiling) was considered as rigid and the resilience was simplified to refraction from a rigid body. If the solution in the air consumes time $n$ and in the solid body $m$ then obviously $n \times m$ will be the resulting computer time for solving the whole system. The problem still consists in a correct definition of the contact conditions between the air and the solid body, i.e. accurate definition of refractions, part absorption of the kinetic energy, possible whirls of the streaming air and such phenomena.

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