OUTFLOW STRUCTURE AND RECONNECTION RATE OF THE SELF-SIMILAR EVOLUTION MODEL OF FAST MAGNETIC RECONNECTION

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ABSTRACT

In order to understand the nature of magnetic reconnection in “free space,” which is free from any influence of external circumstances, I have studied the structure of spontaneous reconnection outflow using a shock tube approximation. The reconnection system of this case continues to expand self-similarly. This work aims to (1) solve the structure of reconnection outflow and (2) clarify the determination mechanism of the reconnection rate of the “self-similar evolution model” of fast reconnection. Many cases of reconnection in astrophysical phenomena are characterized by the huge dynamic range of expansion of the size ($\sim 10^7$ for typical solar flares). Although such reconnection is intrinsically time dependent, the specialized model underlying the situation has not been established yet. The theoretical contribution of this paper is in obtaining a solution for outflow structure that is absent in our previous papers proposing the above new model. The outflow has a shock tube–like structure, i.e., forward slow shock, reverse fast shock, and contact discontinuity between them. By solving the structure in a sufficiently wide range of plasma-$\beta$, $0.001 \leq \beta \leq 100$, we obtain an almost constant reconnection rate ($\sim 0.05$: this value is the maximum for spontaneous reconnection and is consistent with previous models) and boundary value along the edge of the outflow (good agreement with our simulation result), which is important to solve the inflow region. Note that everything, including the reconnection rate, is spontaneously determined by the reconnection system itself in our model.

Subject headings: Earth — ISM: magnetic fields — MHD — Sun: flares

1. INTRODUCTION

It is widely accepted that magnetic reconnection very commonly plays important roles as a powerful energy converter in astrophysical plasma systems. However, there still remain many open questions regarding not only the microscopic physics of the anomalous resistivity but also the macroscopic magnetohydrodynamical (MHD) structure. In this work, our attention is focused on the macroscopic evolutionary properties of magnetic reconnection, especially on the structure of the reconnection outflow.

We consider reconnection in an antiparallel magnetic configuration called a “current sheet system.” In this system, two similarly uniform magnetized regions are set in contact, divided by a boundary. We assume that the directions of the magnetic field on both sides are antiparallel to clarify the essence of the problem. In this case, the boundary carries a strong electric current; hence, we call it the current sheet. If resistivity is enhanced in this current sheet, it plays an important role in energy conversion from the magnetic form to others. In resistive plasmas, there are two fundamental processes of magnetic energy conversion. One is magnetic diffusion, and another is magnetic reconnection.

The best known process of energy conversion in resistive plasmas from magnetic to other forms is ohmic dissipation (magnetic diffusion). We must note, however, that plasmas are highly conductive in most astrophysical problems. Since the resistivity is very small, energy conversion by global magnetic diffusion takes a very long time and is not applicable to many astrophysical phenomena with very violent energy releases, e.g., solar flares. Even in such a case, the majority believes that magnetic reconnection can convert the magnetic energy very quickly. This is why we must study magnetic reconnection.

Such energy conversion in the current sheet system is very important in many astrophysical plasma systems. The most famous example is the relation between solar activity and geomagnetospheric activity, called solar flares and geomagnetospheric substorms. In this case, at least three current sheet systems are included. The first one is in the solar corona. The second and third are on the day side and the night side of the geomagnetosphere, respectively. Recent observations and numerical simulations suggest that similar phenomena seemingly activated by magnetic reconnection are universal, e.g., flares in accretion disks of young stellar objects (YSOs; see Koyama et al. 1994; Hayashi et al. 1999) and galactic ridge X-ray emissions (GRXE; see Koyama et al. 1986; Tanuma et al. 1999).

We must note that many cases of the actual magnetic reconnection in astrophysical systems usually grow over a huge dynamic range in its spatial dimension. For example, the initial scale of the reconnection system can be defined by the initial current sheet thickness, but this is too small to be observed in typical solar flares. We do not have any convincing estimate of the scale, but if we estimate it to be of the order of the ion Larmor radius, it is extremely small ($\sim 10^0$ m in the solar corona). Finally, the reconnection system develops to a scale of the order of the initial curvature radius of the magnetic field lines ($\sim 10^7$ m, $\sim 1.5\%$ of the solar radius for typical solar flares). The dynamic range of the spatial scale is obviously...
huge ($\sim 10^7$ for solar flares). For geomagnospheric substorms, their dynamic range of growth is also large ($\sim 10^4$ for substorms). Such a very wide dynamic range of growth suggests that the evolution of the magnetic reconnection should be treated as a development in “free space” and that the external circumstance does not affect the evolutionary process of magnetic reconnection, at least at the expanding stage just after the onset of reconnection.

Even a system is not completely free from the influence of the external circumstances; we can approximately treat it as a spontaneously evolving system if the evolution timescale (which is estimated as Alfvén transit time $\tau_A$ for the final system scale because the spontaneous expansion speed of the reconnection system is equivalent to the fast-mode propagation speed; see Nitta et al. 2001, hereafter Paper I) is much smaller than the timescale imposed by the external circumstances (e.g., convection timescale). For typical substorms, the evolution timescale of the reconnection is of the order of a second or a minute while the convection timescale to compress the current sheet is of the order of an hour. In such cases, the external circumstances simply play a role of triggering the onset of reconnection and the evolution itself is approximately free from the influence of the external circumstances. Although, of course, exceptional cases in which the external circumstances intrinsically influence the evolution can arise for particular situations (e.g., cases in which very fast convection drives the reconnection), the author believes that it is worthwhile to establish a new model applicable to the cases that are free from external influences.

However, no reconnection model evolving in a free space has been studied. We should note that most previous theoretical and numerical works on reconnection treated it as a boundary problem strongly influenced by external circumstances.

In actual numerical studies, there is a serious and inevitable difficulty: for magnetic reconnection to be properly studied, the thickness of the current sheet must be sufficiently resolved by simulation mesh size. Usually, the thickness of the current sheet is much smaller than the entire system. Hence, in previous works we have been forced to cut a finite volume out of actual large-scale reconnection systems for numerical studies.

Of course, we wish to realize that the boundary conditions of this type of finite simulation box reproduce the evolution in unbounded space. However, we should note that, in actual simulations, the boundary of such a simulation box necessarily affects the evolution inside the box, even if we use so-called free boundary conditions. This has an adverse influence on the evolutionary process of the numerically simulated magnetic reconnection: when the reconnection proceeds and physical signals propagating from the inner region cross the boundaries of the simulation box, the subsequent evolution is necessarily affected by the boundary conditions. This is because the information propagated through the boundary is completely lost and such an artificially cut-out simulation box will never receive the proper response of the outer regions. Additionally, numerical and unphysical signals emitted from the boundary may disturb the evolution. Such artificially affected evolution is obviously unnatural, and the resultant stationary state should differ from actual reconnection occurring in a free space.

Even in previous numerical studies aimed at clarification of the time evolution of reconnection (for example, series of studies originated by Ugai & Tsuda 1977 or Sato & Hayashi 1979), the evolution could not be followed for a long time. This is mainly due to restriction of the size of the simulation box; hence, application of the results was limited to spatial scales typically, say, 100 times the spatial scale of the diffusion region. We are interested here in an evolutionary process in free space without any influence of external circumstances. In such a system, the evolution and resultant structure would be quite different from these previous numerical models.

The same problem also appears in previous theoretical works. We must realize that, in many cases, astrophysical magnetic reconnection is essentially a nonstationary process because it grows in a huge spatial dynamic range. Most of these works, however, for mathematical simplicity, treat a stationary state of the reconnection in a finite volume (see, e.g., Priest & Forbes 1986). These are obviously a boundary problem and solutions must be influenced by boundary conditions. The problem is that we do not know how we should set the boundary condition in order to simulate external influences. In general, it is impossible to set. Hence, the situations argued in previous works are rather artificial and unnatural in many cases of astrophysical application.

As is discussed later (see § 5.6), if we cut the central region out from the entire expanding system, the central region tends to be a stationary state and is very similar to the well-known Petschek model; i.e., the system is characterized by the fast-mode rarefaction-dominated inflow and the X-shaped slow shock. The author thinks that stationary solutions are meaningful as the interior solution of the entire evolving system in free space although such stationary models are frequently treated as externally “driven” processes by boundary conditions.

We demonstrated the above process of the self-similar evolution model of fast reconnection by numerical simulation (see Paper I) and semianalytic study (see Nitta et al. 2002, hereafter Paper II). Here we provide an overview of such an evolutionary process in free space.

We suppose a two-dimensional equilibrium state with antiparallel magnetic field distribution, as in the Harris solution. When magnetic diffusion takes place in the current sheet by some localized resistivity, magnetic reconnection will occur, and a pair of reconnection jets is ejected along the current sheet. This causes a decrease in total pressure near the reconnection point. Such information propagates outward as a rarefaction wave. In a low-$\beta$ plasma ($\beta \ll 1$ in the region very distant from the current sheet [asymptotic region], as typically encountered in astrophysical problems), the propagation speed of the fast magnetosonic wave is isotropic and much larger than that of other wave modes. Thus, information about the decreasing total pressure propagates almost isotropically as a fast-mode rarefaction wave (FRW) with a speed almost equal to the Alfvén speed $V_A$ in the asymptotic region. Hence, the FRW front (FRWF) has a cylindrical shape except near the point where the FRWF intersects with the current sheet. When the FRWF sufficiently expands, the initial thickness of the current sheet becomes negligible compared with the system size $V_A t_i$, where $t_i$ is the time from the onset of reconnection. In such a case, there is only one characteristic scale, i.e., the radius of the FRWF ($V_A t_i$), which linearly increases as time proceeds. This is just the condition for self-similar growth.

In our semianalytic work (Paper II), the boundary condition along the edge of the outflow is artificially imposed approximating the result of our numerical simulation in Paper I. We set the boundary condition at $y = 0$ for simplicity because the outflow is very thin. This boundary condition denotes the junction condition to the reconnection outflow. Hence, the important question of how this boundary condition is spontaneously determined by the reconnection system still remains.
Our motivation for this work is to clarify the mechanism to determine this boundary condition.

The boundary condition at $y = 0$ determines the inflow region. Hence, the reconnection rate, which is usually denoted by the normalized inflow speed toward the diffusion region, is also determined. We must note that, in our self-similar evolution model, everything including the reconnection rate is spontaneously determined by the reconnection system itself. We try to clarify how the system regulates the reconnection.

This paper is organized as follows. We state a model of reconnection outflow as a kind of shock tube problem in §2. Basic equations for outflow structure and the numerical procedure to solve the equations are listed in §3. Properties of the result, e.g., the reconnection rate and boundary condition along the slow shock, are shown in §4. In §5 we summarize our study and discuss spontaneous structure formation in the reconnection outflow.

2. MODEL OF RECONNECTION OUTFLOW

We present a schematic picture of the reconnection outflow in our self-similar evolution model (see Fig. 1). A similar structure of reconnection outflow is shown in Figure 9 of Ugai (1999) as a simulation result. Because of symmetry with respect to $x$- and $y$-axes, we treat only the region $x, y > 0$. The outflow is composed of two different plasmas that have different origins. The rear-half region ($x < x_c$: reconnection jet) is filled with reconnected plasma coming from outside the current sheet (inflow region). The front-half region ($x > x_c$: plasmoid) is filled with the original current sheet plasma. These two regions are divided by a contact discontinuity located at $x = x_c$.

The entire outflow is surrounded by a slow shock that has a complicated "crab-hand" shape. The Petschek-like X-shaped slow shock (oblique shock) is elongated from the diffusion region with a slight opening angle $\theta$. There is a reverse fast shock (almost perpendicular shock) inside the reconnection jet ($x = x_f$). In front of the plasmoid, a forward V-shaped slow shock (oblique shock) forms. The opening angle and the locus (crossing point with $x$-axis) are $\phi$ and $x_s$, respectively.

The entire structure including several discontinuities is analogous to the one-dimensional shock tube problem. The forward shock (V-shaped slow shock) and the reverse shock (fast shock) are formed by the collision of the reconnection jet and the original current sheet plasma and propagate in both directions. Between these two shocks, the contact discontinuity forms. We approximate this reconnection outflow as a quasi-one-dimensional problem in order to solve it analytically. Such an approximation may be valid near the $x$- and $y$-axes because the system is symmetric with respect to these axes.

We focus our attention on the quasi-one-dimensional problem of the L-shaped region near the $x$- and $y$-axes. (We treat the region $x, y = \text{finite} \gg D$, where $D$ is the initial current sheet thickness. Note that this region apparently coincides with the $x$- and $y$-axes in the self-similar stage, which is a very late stage from the onset when we observe the evolution in a zoom-out coordinate; see Paper I.) Each region between two neighboring discontinuities is approximated to be uniform. We note in the upstream region $p$ just above the X-shaped slow shock that the region between the slow shock and the separatrix field line (X-shaped field line reaching the X-point) is approximated to be uniform and the $x$-component of the velocity vanishes. In this region, each reconnected field line has an almost straight shape and crosses the X-shaped slow shock while each field line has a hyperbolic shape in the region above the separatrix field line.
Region 3 between the contact discontinuity and the V-shaped slow shock looks to be nonuniform. Since we do not know how we can solve the two-dimensional structure of this region analytically, we approximate this region to be uniform. The effect of this rough treatment is estimated in § 5.5.

In the vicinity of the diffusion region of self-similar reconnection, the solution is quasi-stationary. Hence, the inflow speed toward the diffusion region must equal the magnetic diffusion speed. We here introduce the main assumption of this work: the magnetic diffusion speed is fully adjustable with inflow speed into the diffusion region. This is possible by tuning the thickness of the diffusion region if the resistivity is sufficiently large. Detailed discussion on the validity of this assumption is made in § 5.4. We note that the reconnection system in free space has only two parameters: the magnetic Reynolds number and the plasma-β at the asymptotic region (see Paper I). If the diffusion speed is adjustable, the inflow speed is not restricted by the diffusion speed. Hence, the reconnection may not depend on the magnetic Reynolds number (Yokoyama & Shibata 1994). Thus, our attention will be focused on the dependence on the plasma-β. We wish to emphasize that the resistivity does not regulate the system in such a situation (it may play a passive role) although electric resistivity is necessary in order to reconnect the magnetic field lines. We now wonder what mechanism does regulate the system. In order to answer this question, we consider the following shock tube–like model. We can treat the main part of such a problem by ideal (nonresistive) MHD.

The quantities denoting the initial uniform equilibrium at the asymptotic region are gas pressure $P_0$, mass density $\rho_0$, and magnetic field strength $B_0$. Plasma-β at the asymptotic region is defined as $\beta \equiv P_0/[B_0^2/(2\mu)]$, where $\mu$ is the magnetic permeability of the vacuum. In the rest of this paper we use the normalization of physical quantities as in Paper II. We define units for each dimension as follows: the unit of velocity is $V_{\infty}\equiv B_0/(\mu P_0)^{1/2}$ (Alfvén speed at the asymptotic region), the unit of the length is $V_{\infty}/r$, where $r$ is the time from the onset of reconnection, the unit of mass density is $\rho_0$, the unit of magnetic field is $B_0$, and the unit of pressure is $P_0 V_{\infty}^2/2$.

We set the one-dimensional shock tube–like problem as follows (see Fig. 1). The system has 22 unknown quantities: $P_p, \rho_p, \nu_p, B_{ap}, B_{ap}, \theta, \psi, \rho_1, \nu_1, B_1, x_1, x_f, \rho_2, \nu_2, B_2, \rho_3, \nu_3, B_3, \phi, \psi, \rho_4, \nu_4, B_4$. These unknowns should be related to each other via conditions coming from the integrated form of conservation laws (i.e., the Rankine-Hugoniot [R-H] conditions) or other relations. The set of relations is listed in the next section.

When $\beta > 0.8$, we must change the model because the reverse fast shock does not form in the reconnection outflow. The criterion of $\beta$ whether the reverse fast shock forms or not is in the range $0.7 \leq \beta \leq 0.8$. Although we did not precisely estimate the critical value of $\beta$, we can fairly match the solutions of these two schemes around $\beta \approx 0.75$. For the case including no fast shock, we remove $x_f$ from the unknowns and set $Q_1 = Q_2$ where $Q_3$ is representative of physical quantities at region 1 or 2. According to this alteration, the coupled equations of the unknowns are also altered, but we abbreviate their detailed forms because this alteration is trivial.

3. BASIC EQUATIONS AND NUMERICAL APPROACH

According to the above model of the reconnection outflow, we obtain the following 22 equations for 22 unknown quantities (Detailed forms of each equation are listed in the Appendix):

1. Relations between pre–X-shaped slow shock and asymptotic region:
   a) Frozen-in condition (eq. [A1]).
   b) Polytropic relation (eq. [A2]).

2. R-H jump conditions at X-shaped slow shock:
   a) Pressure jump (eq. [A3]).
   b) Density jump (eq. [A4]).
   c) Velocity jump (parallel component; eq. [A5]).
   d) Velocity jump (perpendicular component; eq. [A6]).
   e) Magnetic field jump (parallel component; eq. [A7]).
   f) Magnetic field jump (perpendicular component; eq. [A8]).

3. R-H jump conditions at reverse fast shock:
   a) Pressure jump (eq. [A9]).
   b) Density jump (eq. [A10]).
   c) Velocity jump (eq. [A11]).
   d) Magnetic field jump (eq. [A12]).

4. Magnetic flux conservation at X-point (eq. [A13]).

5. Force balance at contact discontinuity (eq. [A14]).

6. R-H jump conditions at forward V-shaped slow shock:
   a) Pressure jump (eq. [A15]).
   b) Density jump (eq. [A16]).
   c) Velocity jump (parallel component; eq. [A17]).
   d) Velocity jump (perpendicular component; eq. [A18]).
   e) Magnetic field jump (parallel component; eq. [A19]).
   f) Magnetic field jump (perpendicular component; eq. [A20]).

7. Boundary condition at the tip of the outflow (eq. [A21]).

8. Magnetic flux conservation all over the outflow (eq. [A22]).

We can solve the majority of these equations by hand, and by substituting the solutions into other equations, we can reduce equations. Finally, nine equations, (A3), (A4), (A5), (A6), (A7), (A11), (A14), (A21), and (A22), remain as complicated nonlinear coupled equations for nine unknowns, $\nu_1, B_{ap}, B_{ap}, \theta, P_1, \rho_1, x_f, \phi, \psi$, and $x_f$.

We solve these coupled nine equations by an iterative method (Newton-Raphson). In order to find the solution of nonlinear coupled equations, in general, a precise initial guess of the unknowns is required. Unfortunately, we cannot guess the solution so easily, so we adopt the following primitive method to find a good initial guess of the unknowns.

We estimate them from the result of our numerical simulation in Paper I for the case $\beta = 0.2$ in the asymptotic region. Of course, the numerical result is contaminated by numerical noise, and so we can treat our simulation result as simply a hint.

Each equation can be reduced to a form where the left-hand side of the equation equals 0. When we substitute the initial guess of physical quantities from our simulation result for $\beta = 0.2$ into the unknown quantities, the left-hand side does not vanish because these are not the true solutions. By defining the residue as the sum of absolute values of the left-hand side of each equation, we can plot a graph of the residue as a function of nine unknown quantities. Then we improve the guess of each unknown to diminish the residue. This process will be...
iteratively continued by hand until the residue settles to a value as small as possible (until the residue becomes 2 orders of magnitude smaller than residue for the simulation result). This is a very time-consuming procedure. Thus, we can reach a good initial guess for the unknowns in order to use the Newton-Raphson method for \( \beta = 0.2 \).

First, we start with the case \( \beta = 0.2 \). Once we find a converged solution of the Newton-Raphson procedure for the case \( \beta = 0.2 \), we treat it as the initial guess for the case, e.g., \( \beta = 0.19 \) or 0.21, and we have successively obtained the solutions for a range of 0.001 \( \leq \beta \leq 0.7 \).

For the case \( \beta > 0.8 \), we replace the numerical procedure with another one for the case of no reverse fast shock. When \( \beta = 0.8 \), we adopt the solution for \( \beta = 0.7 \) as the initial guess. Thus, we can continue the converging procedure to the case of very large \( \beta \) and obtain the result for \( \beta \leq 100 \).

4. RESULTS

4.1. Reconnection Rate

We investigated the structure of the reconnection outflow by using a shock tube approximation. The main issues of this work are (1) the reconnection rate of the self-similar evolution model and (2) the boundary condition along the edge of the outflow.

Most interest is focused on the reconnection rate. Let us show the \( \beta \) dependence of the reconnection rate \( R \). The reconnection rate is defined by the inflow magnetic flux per unit time: \( R = -V_{\|}B_{\|} \) in our dimensionless form. Figure 2 clearly shows that the reconnection rate is almost constant in a very wide range of \( \beta \). The reconnection rate varies in a dynamic range 0.038 \( \leq R \leq 0.057 \) for 0.001 \( \leq \beta \leq 100 \). The maximum reconnection rate \( R = 0.057 \) is attained at \( \beta = 0.6 \). In the low-\( \beta \) limit and the high-\( \beta \) limit, \( R \rightarrow 0.050 \) and \( R \rightarrow 0.038 \), respectively. The range of reconnection rate obtained here is consistent with previous classical models of fast reconnection, e.g., the Petschek model (\( 10^{-2} < R < 10^{-1} \); see Petschek 1964; Vasyliunas 1975).

Although we can calculate for the range \( \beta < 0.001 \) and \( \beta > 100 \) without any difficulty, it should be less meaningful because the curve of \( R \) shown in Figure 2 almost settles to each terminal value for both the low-\( \beta \) limit and high-\( \beta \) limit. Since the asymptotic behaviors of both limits \( \beta \rightarrow 0 \) and \( \beta \rightarrow \infty \) are easily expected, the author thinks that our result is enough.

4.2. Boundary Condition along the Slow Shock

In Paper II, the inflow region was discussed as a boundary problem by using the Grad-Shafranov (G-S) approach in which we solved a second-order partial differential equation (the G-S equation) of elliptic type for the magnetic vector potential \( A'_j \) of the perturbed field, where the perturbed field is a deviation from the original uniform magnetic field. In order to solve the G-S equation, we need a boundary condition along the slow shock (the locus of the boundary is approximated as \( y = 0 \) when we impose the boundary condition in Paper II because the opening angle of the X-shaped slow shock is very small). We artificially imposed this boundary condition to be analogous with the result of our numerical simulation in Paper I. Thus, an important question as to how we should impose the boundary condition for the slow shock boundary is still open. The answer is that this boundary condition is spontaneously determined by the shock tube problem discussed in this work. The perturbed magnetic vector potential \( A'_j \) can be calculated along the edge of the slow shock (see Fig. 3) with respect to the above solutions.

We can obtain the distribution of \( A'_j \) along the \( x \)-axis (\( y = 0 \)) from the solution. However, the boundary value we want is the distribution along the slow shock. Here we assume the shape of the slow shock as shown in Figure 3. In order to cancel the original uniform magnetic field, we add the correction term \(-B_0y\) to the value on \( y = 0 \). From Figure 3, the locus of the slow shock is denoted by \( y = x \tan \theta \) for \( 0 \leq x \leq x_s, y = [(1 - x_s^2) \tan \theta - x_s, tan \theta/(1 - x_s^2)]x \tan \theta \) for \( x_s \leq x \leq 1 \).

Thus, we obtain the distribution of \( A'_j \) as \( A'_j = B_j(x_f - x) + B_2(x_c - x_f) - B_0x \tan \theta \) for \( 0 \leq x \leq x_f \), \( A'_j = B_2(x_c - x) - B_0x \tan \theta \) for \( x \leq x \leq x_s \), \( -B_3(x - x_s) + (B_3 - B_0)(1 - x^2) \tan \theta - x_s, tan \theta/(1 - x_s^2)]x \tan \theta \) for \( x_s \leq x \leq 1 \).

We demonstrate the case \( \beta = 0.2 \), which we numerically simulated in Paper I, and compare the present analytic result with the previous numerical result in Figure 4. The dotted line and solid line show the simulation result of Paper I and the above result of this work, respectively. These two lines are qualitatively analogous but quantitatively somewhat different. We discuss the difference in \( \S \) 5.5.

5. SUMMARY AND DISCUSSION

5.1. Summary

We here briefly summarize the structure of the reconnection system obtained in our numerical simulation in Paper I. The feature of the inner region is very similar to fast reconnection regimes (Priest & Forbes 1986) of which the Petschek model is one member; i.e., we can find the fast-mode rarefaction-dominated inflow and X-shaped slow shock. However, the entire structure shows properties of time-dependent reconnection as follows. The reconnection outflow has a finite span that is increasing in proportion to the time from the onset. A V-shaped forward slow shock forms in the vicinity of the spearhead of the outflow. In region 3, plasmas are compressed by the reconnection jet (piston effect), and squeezed plasmas are gushing out to make a vertical outflow. These are properly included in the evolutionary process.

In this paper the structure of reconnection outflow in the self-similar evolution model has been studied in a shock tube...
approximation. We have obtained the reconnection rate and the required boundary condition to solve the inflow region from the structure of the reconnection outflow.

5.2. Self-Similarity of the Reconnection Outflow

We have solved the structure of the reconnection outflow for the self-similar evolution model of fast magnetic reconnection. The structure is spontaneously determined by the reconnection system itself as approximated by a kind of shock tube problem. The reconnection outflow is divided into several regions by discontinuities, e.g., forward shock, reverse shock, and contact discontinuity. We solved this shock tube problem by an iterative (Newton-Raphson) method. The moving speeds of these discontinuities are constant in time in our result. Thus, as expected from the well-known shock tube problem, the reconnection outflow extends self-similarly.

First, we argue for self-similarity of the evolution of the reconnection system in free space. In the discussion of Paper II, we need an a priori statement to authorize a self-similar solution for the inflow region so that the reconnection outflow (in other words, the boundary condition along the slow shock) also evolves self-similarly. The validity of this assumption was, however, unclear in previous papers. We must note that because we have found the self-similarly evolving solution for the reconnection outflow, we can ensure self-similar evolution of the entire reconnection system.

Comparison of the outflow structure with a shock tube model is also argued in Abe & Hoshino (2001). They obtained a similar structure to our result, but there is a critical difference. The V-shaped discontinuity as the forward shock is identified as an intermediate shock in their result instead of a slow shock in our case. The author cannot explain the reason for such a difference but notes that their attention is focused on a much earlier stage than our case. The scale of the plasmoid is the order of the current sheet thickness in their case, but we are interested in a much larger scale structure at the self-similar stage. Actually, the numerical result of Ugai (1999; see Fig. 9 of that paper) shows a similar structure to our result. The later evolution of the simulation of Abe & Hoshino (2001) may tend also to coincide with our result.

5.3. $\beta$ Dependence of the Reconnection Rate

Second, our attention is focused on the $\beta$ dependence of the reconnection rate $R$. In our result, $R$ is almost constant in the range $0.001 \leq \beta \leq 100$ (see Fig. 2). We could foresee
this behavior of the reconnection rate from the following intuition.

We consider the low-β limit at the asymptotic region. When the plasma-β at the asymptotic region far outside the current sheet becomes smaller, the Petschek-type X-shaped slow shock must be stronger. This is explained as follows. The gas pressure at the upstream region is very small relative to the magnetic pressure for a low-β plasma, while in the downstream region the gas pressure should be very large to keep total pressure equilibrium with the upstream region. Hence, the total pressure equilibrium imposes a large gas pressure jump at the X-shaped slow shock. This leads to a large jump of the mass density.

Next we consider the high-β limit at the asymptotic region. Both sides of the Petschek-type X-shaped slow shock are filled with gas pressure–dominated plasmas in this case and balanced with each other mainly by gas pressure. This results in a very small density jump at the slow shock (\( \chi \rightarrow 1 \)). Note that this does not imply the weak limit shock because the jump of the tangential component of velocity is significant (\( v_t \approx 1 \) while \( v_p = 0 \)). The \( \beta \) dependence of the compression ratio \( \chi \) of the X-shaped slow shock is shown in Figure 5 and is consistent with the above discussion.

The reconnection rate \( R \) is defined as \( R \equiv -v_p B_{xp} \). First, we discuss the low-β limit. The inflow speed \( -v_p \), which is determined by the magnetic Reynolds Number

\[ R \equiv -v_p B_{xp} \]

inflow speed, \( -v_p \), will increase as the slow shock strengthens (\( \beta \) decreases) for the following reason. The ejection speed of the reconnection jet is almost constant (\( -V_{A0} \)), while the mass density of the jet increases as the slow shock strengthens (the compression ratio \( \chi \) increases as \( \beta \) decreases). Thus, the inhalation of mass flux toward the slow shock is also strengthened. This results in an increase of \( -v_p \) (see Fig. 6) as \( \beta \) decreases because the mass density at the pre–slow shock region is almost constant (although weakly rarefied by the fast-mode rarefaction wave). As the inflow speed \( -v_p \) increases, bending of the magnetic field line becomes remarkable, and \( B_{xp} \), decreases (see Fig. 6). We can also understand the result for the high-β limit in the same way (vice versa of the low-β limit). Consequently, the product of \( -v_p \) and \( B_{xp} \), and hence the reconnection rate \( R \), is almost constant independent of \( \beta \).

Dependence of the reconnection rate on the plasma-β is also considered in other works, e.g., Ugai & Kondoh (2001). Their attention is focused on an implicit contribution of \( \beta \) via a current-driven anomalous resistivity model. The aim of our study is intrinsically different from theirs. Needless to say, anomalous resistivity is very important to confirm the validity of a localization of resistivity, which is necessary to establish fast reconnection (see Yokoyama & Shibata 1994). We have concentrated on a macroscopic process that can be treated in the ideal MHD regime and paid scant attention to how we should introduce the anomalous resistivity. We should note that we have no definite information on how to make a relevant macroscopic (MHD) model of anomalous resistivity, which should be treated in a microscopic regime.

5.4. Relation with Magnetic Reynolds Number

One may feel that our result is strange because, while the reconnection needs electric resistivity, we treat only ideal MHD and do not consider the dependence on the magnetic Reynolds number. We here discuss a regulation process of the energy conversion speed. As discussed in \( \S \) 2, we assume that the magnetic diffusion speed is fully adjustable with the spontaneous inhalation speed \( -v_p \), which is determined by the above shock tube problem. This assumption is valid for the case of sufficiently large electric resistivity (small magnetic Reynolds number) in which the spontaneous inhalation speed is smaller than the maximum diffusion speed. Such a large resistivity may be realized by anomalous resistivity. We need an assumption for the anomalous resistivity because we do not have a convincing model for the mechanism and estimated value of anomalous resistivity (Coppi & Friedland 1971; Ji et al. 1998; Shinozawa et al. 1998). Note that the diffusion speed is adjustable by a change of the diffusion region thickness even if the resistivity is fixed to be constant, but if there is a lower limit for the thickness (e.g., simulation mesh size or the ion Larmor radius), the diffusion speed is bound by an upper limit.

In Paper I we suppose the Reynolds number to be 24.5. One may think that this is an incredibly large resistivity. We should
be careful of what denotes the length in the definition of the magnetic Reynolds number. Usually speaking, the magnetic Reynolds number according to Spitzer resistivity for typical solar flares is extremely large:

$$Re_m \approx 10^{15} \frac{(L/10^7 \text{ m})(T/10^6 \text{ K})^{3/2}(B/10^{-3} \text{ T})}{n/10^{15} \text{ m}^{-3}},$$

(1)

where $L$, $T$, $B$, and $n$ are the dimension of the entire system, the temperature, the magnetic field strength, and the plasma number density, respectively. We must note that “the effective Reynolds number,” which is based not on the dimension of the entire system but on that of the diffusion region ($L \sim 10^6$ m: the ion Larmor radius for solar corona), is important. Hence, the effective Reynolds number $Re_e$ for Spitzer resistivity is $Re_e \approx 10^6$. If anomalous resistivity, which is much larger than the Spitzer resistivity (typically assumed as $\sim 10^{5-6}$ times Spitzer resistivity in many numerical MHD simulations), takes place, the effective Reynolds number is estimated as $Re_e \approx 10^4$. The magnetic Reynolds number adopted in Paper I (which is the effective Reynolds number: $Re_e = 24.5$) may be relevant for the case of anomalous resistivity.

A localized resistivity is assumed as a model of anomalous resistivity at the center of the reconnection system. We put the thickness of the initial current sheet $D$ as in Paper I. Here $D$ is supposed to be of the order of the ion Larmor radius; hence, it is close to the minimum value of the thickness. The dimension of the resistive region is set larger than $D$ (in Paper I, we set $2D$) in order to treat the change of the diffusion region size. Thus, the effective thickness of the diffusion region can vary between $D$ and $2D$. The maximum diffusion speed is achieved when the thickness becomes $D$.

The smallest effective magnetic Reynolds number $Re_{e,\text{min}}$ is defined as

$$Re_{e,\text{min}} = \frac{V_{A0}}{\eta/D},$$

(2)

where $\eta$ is the magnetic diffusivity. For the value $Re_{e,\text{min}} = 24.5$ in Paper I, the maximum diffusion speed into the diffusion region is determined by the Sweet-Parker model (Sweet 1958; Parker 1963): $v_{d,\text{max}} \approx \frac{V_{A0}}{Re_{e,\text{min}}} \approx 0.2$, while the spontaneous inhalation speed determined by the shock tube approximation is $-v_{yp} \approx 0.05$. The diffusion speed does not limit the inflow speed because $-v_{yp} < v_{d,\text{max}}$. This is why the reconnection rate of our model does not depend on the magnetic Reynolds number.

If the Reynolds number is not so small (small resistivity), say, $Re_{e,\text{min}} \geq 400$, the diffusion speed is smaller than the spontaneous inhalation speed and may limit the inflow speed. In such cases, the reconnection rate will be suppressed by the small diffusion speed and depends on the magnetic Reynolds number in ways discussed in the previous literature. This problem is very interesting; however, detailed discussion for such cases is beyond the scope of this work.

In this meaning, the value of $R$ plotted in Figure 2 will be the maximum value for spontaneous reconnection as a function of $\beta$ when the resistivity is sufficiently large ($Re_{e,\text{min}} \leq 400$). The system cannot exceed this value by spontaneous inhalation of the inflow plasma without an external injection.

A similar discussion about spontaneous reconnection was made by Ugai in his series of works (e.g., Ugai & Tsuda 1979; Ugai 1983). He obtained a result supporting Petschek’s prediction, i.e., $R \propto (\log Re_e)^{-1}$. We here compare with the result of Ugai (1983). His interest is focused on the steady state in a finite simulation box (several times the initial current sheet thickness). This is realized in a very late stage from the onset. At that stage, waves emitted from the central region propagate beyond the entire simulation box (Ugai paid attention to a stage later than roughly 7 times the Alfvén transit time with respect to the entire size of the simulation region).

We should note that this is completely different from our case. His resultant reconnection rate logarithmically varies in a range of $0.1-0.3$ for $2.5 \leq Re_e \leq 200$, and the inflow speed varies in a range of $0.1V_{A0}-0.3V_{A0}$ while the maximum diffusion speed (defined by the minimum mesh size) varies between $20 \geq v_{d,\text{max}} \geq 0.3$. The author cannot explain why such a very large inflow speed is attained (e.g., inflow speed $-v_{yp} \approx 0.05$ in our case), but we can see that their inflow speed is not so small compared with the maximum diffusion speed for $Re_e > 10^2$.

Thus, the inflow speed might be restricted by the diffusion speed and the reconnection rate might depend on the magnetic Reynolds number in Ugai’s result. The author thinks that such a situation is unnatural for astrophysical reconnection as discussed in our works (e.g., § 1 of this paper). Since there is no essential difference between Ugai’s numerical procedure and our procedure, if one sets a sufficiently wider simulation box than Ugai’s, one would obtain a similar result to ours.

### 5.5. Boundary Condition along the Slow Shock

Finally, we discuss the boundary condition for the inflow region, which remained an open question in Paper II. We obtain the distribution of the magnetic vector potential $A'_i$ for the perturbed magnetic field along the edge of the reconnection outflow (see Fig. 4).

The result is roughly consistent with the result of our numerical simulation, but it cannot explain the numerical result quantitatively. The major difference is in the region $x > x_f$. This is mainly due to the two-dimensional effects of the actual outflow. We here assume a quasi—one-dimensional model for the reconnection outflow for simplicity, but the actual outflow clearly has a two-dimensional structure. For example, the magnetic field lines in this region have a round-shaped upward convexity. If we make a precise model of the reconnection outflow and take account of this two-dimensional shape, such a discrepancy will be reduced. Note that, in our experience, the physical quantities in region $p$ or $1$, which are important to determine the reconnection rate, are insensitive to the model of the slow shock shape, and so we do not need to be nervous about this problem. Obviously, such further discussion cannot proceed without the help of a computer simulation and is beyond the scope of this work.

### 5.6. Relation to Stationary Models

As noted in § 5.1, the central region of the entire expanding system tends to be a stationary state that is similar to the Petschek model. The author thinks that the Petschek-like stationary state is unique as the inner solution of the spontaneously evoking reconnection system in free space with a locally enhanced resistivity (see Paper I). Even an external circumstance strongly influences the evolution, such that the Petschek-like central structure will hold a long duration of the order of 100 times the Alfvén transit time with respect to the proper scale of the external circumstance (e.g., the diameter of a magnetic flux tube in the case of solar flares). Because the induced inflow speed is of the order of $10^{-2} V_{A0}$ (see
Paper I), it needs a long time to change the inflow region by the influence from the external circumstance (see Paper I).

Other types of stationary state may occur in the following situations: (1) a very fast plasma flow (~$V_{A0}$; similar to or faster than the FRW propagation) is injected through the boundary, (2) the proper scale of the external circumstance is not much larger than the initial current sheet thickness (e.g., in the case of laboratory plasma), (3) the resistive region is not so localized, etc. However, our self-similar evolution model may be applicable to many cases in astrophysics.

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APPENDIX

EQUATIONS FOR STRUCTURE OF THE RECONNECTION OUTFLOW

The structure of the reconnection outflow is determined by the following equations. We assume that the region between the asymptotic region and the preshock region is filled with nonresistive plasma. Hence, the magnetic flux is frozen into the induced inflow. We also assume a polytropic variation in the induced inflow because there is no violent process in the fast-mode rarefaction. Thus, we impose the frozen-in condition

$$\frac{B_0}{\rho_0} = \frac{B_{sp}}{\rho_p}$$

and the polytropic relation

$$P_0 \rho_0^\gamma = P_p \rho_p^\gamma,$$

where $\gamma$ is the specific heat ratio. We assume $\gamma = 5/3$ (monoatomic ideal gas).

There are several discontinuities in the reconnection outflow, i.e., X-shaped slow shock, reverse fast shock, contact discontinuity, and forward V-shaped slow shock (see Fig. 1). We set jump conditions for both sides of each discontinuity. The following are the X-shaped slow shock Rankine-Hugoniot (R-H) jump conditions:  

1. Pressure jump

$$\frac{P_1}{P_p} = 1 + \frac{\gamma}{c_{sp}^2} (\cos \theta_{v1y})^2 (X - 1) \left\{ \frac{1}{X} - \frac{(\cos \theta_{v1y} + \sin \theta_{v1y})^2}{2} \right\}$$

where

$$c_{sp} = \sqrt{\frac{\gamma P_p}{\rho_p}}$$

$$V_{Apv} = \sqrt{\frac{(-\sin \theta_{v1y} + \cos \theta_{v1y})^2}{\mu \rho_p}}$$

$\mu$ is the magnetic permeability of vacuum, and $X$ is the compression ratio.

2. Density jump

$$\frac{\rho_1}{\rho_p} = X.$$  

3. Velocity (parallel) jump

$$\frac{\cos \theta_{v1} - v_0}{\sin \theta_{v1y} - v_0} = \frac{(\cos \theta_{v1y})^2 - V_{Apv}^2}{\cos \theta_{v1y}^2 - XV_{Apv}^2}.$$  

where $v_0 = v_{y1} B_{sp}/(B_{sp} \sin \theta - B_{tp} \cos \theta)$ is the shift speed of the de Hoffmann-Teller coordinate.

4. Velocity (perpendicular) jump

$$\frac{-\sin \theta_{v1}}{\cos \theta_{v1y}} = \frac{1}{X}.$$
5. Magnetic field (parallel) jump

\[
\sin \theta B_1 = \frac{(\cos \theta_{v_{yp}})^2 - V_{\lambda}^2}{(\cos \theta_{v_{yp}})^2 - X V_{\lambda}^2}. \tag{A7}
\]

6. Magnetic field (perpendicular) jump

\[
\frac{\cos \theta B_1}{- \sin \theta B_{yp} + \cos \theta B_{yp}} = 1. \tag{A8}
\]

The compression ratio \(X\) is defined by the following equation (third-order algebraic equation for \(X\)):

\[
(\sin \theta B_{yp} + \cos \theta B_{yp})^2 \left\{ (\cos \theta B_{yp} + \sin \theta B_{yp})^2 (\gamma - 1) \rho_p (\cos \theta v_{yp})^2 \\
+ (\sin \theta B_{yp} + \cos \theta B_{yp})^2 \left[ 2 \gamma P_p - \rho_p (\cos \theta v_{yp})^2 + \gamma \rho_p (\cos \theta v_{yp})^2 \right] \right\} \right)^3 \]

\[
+ (\sin \theta B_{yp} + \cos \theta B_{yp})^2 \left[ - \rho_p (\cos \theta v_{yp})^2 \left( - \sin \theta B_{yp} + \cos \theta B_{yp} \right)^2 (\gamma - 1) + (\cos \theta B_{yp} + \sin \theta B_{yp})^2 (\gamma - 2) \mu \rho_p (\cos \theta v_{yp})^2 \\
+ (\sin \theta B_{yp} + \cos \theta B_{yp})^2 \left( \cos \theta B_{yp} + \sin \theta B_{yp} \right)^2 (\gamma + 1) \\
+ \mu \left[ 2 \gamma P_p - \rho_p (\cos \theta v_{yp})^2 + \gamma \rho_p (\cos \theta v_{yp})^2 \right] \right\} \right)^2 \]

\[
X + \left[ (\gamma + 1) \mu^2 \rho_p^2 (\cos \theta v_{yp})^6 \right] = 0.
\]

This equation has three roots. We must choose a real positive root larger than unity.

The following are the reverse fast shock R-H conditions:

1. Pressure jump

\[
\frac{P_2}{P_1} = \zeta_f, \tag{A9}
\]

where

\[
\zeta_f = \gamma M_{1f}^2 \left( 1 - \frac{1}{\xi_f} \right) + \frac{1 - \xi_f^2}{\beta_1} + 1,
\]

with

\[
\xi_f = -l + \sqrt{l^2 + 2 / \beta_1 (2 - \gamma)(\gamma + 1) \gamma M_{1f}^2} / 2 / \beta_1 (2 - \gamma),
\]

\[
l = \gamma \left( \frac{1}{\beta_1} + 1 \right) + \frac{(\gamma - 1) \gamma M_{1f}^2}{2},
\]

\[
\beta_1 = \frac{P_1}{B_1^2 / (2 \mu)},
\]

\[
M_{1f} = \frac{v_1 - x_f V_{\lambda}}{c_f},
\]

\[
c_f = \sqrt{\gamma P_1 / \rho_1},
\]

and

\[
V_{\lambda} = \frac{B_0}{\sqrt{\mu \rho_0}}.
\]
2. Density jump

\[ \frac{\rho_2}{\rho_1} = \xi_f. \]  

(A10)

3. Velocity jump

\[ \frac{v_1 - x_f}{v_2 - x_f} = \xi_f. \]  

(A11)

4. Magnetic field jump

\[ \frac{B_2}{B_1} = \xi_f. \]  

(A12)

We must impose local magnetic flux conservation on both sides of region \( p \) and region 1.

5. Magnetic flux conservation at X-point

\[ v_{yp}B_{yp} + v_1B_1 = 0. \]  

(A13)

6. Force balance at contact discontinuity

\[ P_2 + \frac{B_2^2}{2\mu} = P_3 + \frac{B_{3z}^2 + B_{3z}^2}{2\mu}. \]  

(A14)

The following are the V-shaped forward slow shock R-H conditions:

1. Pressure jump

\[ P_3 = P_0 \left(1 + \frac{\gamma}{C_{s0}} (x_0 \sin \phi)^2 (X_h - 1) \left(1 - \frac{(B_0 \cos \phi)^2 - 2V_{A0}^2 (X_h + (x_0 \sin \phi)^2 (X_h + 1))}{\frac{1}{X_h} - \frac{1}{X_h^2}} \right) \right), \]  

(A15)

where \( C_{s0} = (\gamma P_0/\rho_0)^{1/2} \), \( V_{A0} = \left|\left[-\sin \theta B_0^2/(\mu \rho_0)\right]^{1/2} \right| \), and \( X_h \) is the compression ratio.

2. Density jump

\[ \frac{\rho_3}{\rho_0} = X_h. \]  

(A16)

3. Velocity (parallel) jump

\[ \frac{\cos \phi (v_{3z} - x_0 V_{A0}) + \sin \phi v_{3z}}{-V_{A0} x_0 \sin \phi} = \frac{v_{m0x} - V_{A0}^2}{v_{m0x} - X_h V_{A0x}^2}, \]  

(A17)

where \( v_{m0x} = x_0 \sin \phi \) and \( V_{A0} = B_0/(\mu \rho_0)^{1/2} \).

4. Velocity (perpendicular) jump

\[ \frac{-\sin \phi (v_{3z} - x_0 V_{A0}) + \cos \phi v_{3z}}{V_{A0} x_0 \sin \phi} = \frac{1}{X_h}. \]  

(A18)

5. Magnetic field (parallel) jump

\[ \frac{\cos \phi B_{3z} + \sin \phi B_{3z}}{B_0 \cos \phi} = \frac{(v_{m0x} - V_{A0x}^2) X_h}{v_{m0x} - X_h V_{A0x}^2}. \]  

(A19)
6. Magnetic field (perpendicular) jump

\[ \frac{-\sin \phi B_{x3} + \cos \phi B_{z3}}{-B_0 \sin \phi} = 1. \]  
(A20)

The compression ratio \( X_h \) is a solution of the following third-order algebraic equation:

\[
\left( B_0 \sin \phi \right)^2 \left\{ (B_0 \sin \phi)^2 (\gamma - 1) \rho_0 (x_s \sin \phi)^2 + (B_0 \sin \phi)^2 \left[ 2 \gamma P_0 - \rho_0 (x_s \sin \phi)^2 + \gamma \rho_0 (x_s \sin \phi)^2 \right] \right\} X_h^3
\]
\[ + \left\{ -\rho_0 (x_s \sin \phi)^2 (B_0 \sin \phi)^4 (\gamma + 1) + (B_0 \cos \phi)^2 (\gamma - 2) \mu \rho_0 (x_s \sin \phi)^2 \right\} X_h^2
\]
\[ + \left\{ \mu \rho_0^2 (x_s \sin \phi)^4 (B_0 \cos \phi)^2 \gamma + 2 (B_0 \sin \phi)^3 (\gamma + 1) + \mu \left[ 2 \gamma P_0 - \rho_0 (x_s \sin \phi)^2 + \gamma \rho_0 (x_s \sin \phi)^2 \right] \right\} X_h
\]
\[ + \left[ -(\gamma + 1) \mu^2 \rho_0^3 (x_s \sin \phi)^6 \right] = 0. \]

We must choose a real positive root larger than unity.

The tip of the reconnection outflow touches the FRWF (see Fig. 3); hence, \( A'_1 = 0 \) at this point. This condition reduces to the following equation:

\[ -B_{x3} (1 - x_c) + B_{z3} \left[ (1 - x_s) \tan \phi - x_c \tan \theta \right] - B_0 (1 - x_s) \tan \phi = 0. \]  
(A21)

From magnetic flux conservation, the injected magnetic flux must be redistributed in the reconnection jet. This leads to the following equation:

\[ v_{yp} B_{yp} + [x_f B_1 + (x_c - x_f) B_2] = 0. \]  
(A22)

We assume the following trivial relations:

\[ x_c = v_2 = v_{x3} \]

(definition from the contact discontinuity). We can solve equation \( A1 \) for \( \rho_{yp} \), equation \( A2 \) for \( P_{yp} \), equation \( A8 \) for \( B_1 \), equation \( A9 \) for \( P_2 \), equation \( A10 \) for \( P_2 \), equation \( A12 \) for \( B_2 \), equation \( A13 \) for \( v_{yp} \), equation \( A15 \) for \( P_3 \), equation \( A16 \) for \( \rho_3 \), equations \( A17 \) and \( A18 \) for \( v_{x3} \) and \( v_{z3} \), and equations \( A19 \) and \( A20 \) for \( B_{x3} \) and \( B_{z3} \) by hand and then substitute them into the other nine equations (eqs. \( A3 \), \( A4 \), \( A5 \), \( A6 \), \( A7 \), \( A11 \), \( A14 \), \( A21 \), and \( A22 \)) for the following unknowns: \( v_1, B_{x3}, B_{z3}, \theta, P_1, \rho_1, \phi, \) and \( x_c \). The only parameter included in this problem is the plasma-\( \beta \) value at the asymptotic region. By using a Newton-Raphson routine, with an initial guess for these unknowns, we obtain converged solutions. The procedure to obtain the series of converged solutions is discussed in § 3 in detail.

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