$U(1) \times SU(2)$ from the tangent bundle

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Abstract. Élie Cartan developed modern differential geometry as theory of moving frames. Particles do not enter the equations of structure and thus play a less fundamental role. A Kaluza-Klein (KK) space without compactification brings particles into the core of geometry by making propertime ($\tau$) the fifth dimension. $(x', \tau)$ emerges as the subspace for the quantum sector. This KK space does not make sense in SR, by virtue of non-orthogonality of $\tau$ to 3-space (actually), which brings a preferred frame to the fore. In contrast, propertime is perpendicular to 3-space in the “para-Lorentzian structure” with absolute time dilation (PL). Its $(x', \tau)$ subspace looks very much like $(t, x')$ in SR. And its $(t, x')$ sector is not made of orthogonal frames but does not cause contradictions with SR, which supporters of the thesis of conventionality of synchronizations have been claiming for many decades. In PL, the conjunction of the Clifford algebras of differential forms (Kaehler’s) and of their valuedness gives rise to a commutative algebra of primitive idempotents that embodies the $U(1) \times SU(2)$ symmetry. $SU(3)$ also emerges in the process, but we do not deal with this issue beyond proposing the geometric palette of quarks.

1. The big picture
This paper is one in a series devoted to show that we do not need auxiliary bundles for the type of physics that goes under the name of Yang-Mills theory. There is room for that in a structure directly related to the tangent bundle by looking at the metric as a natural lifting condition relating spacetime and time-space-propertime.

A previous paper titled “Real Units Imaginary in Kaehler’s Quantum Mechanics” is of the essence in the argument that we are about to present [1]. The main points that we made there were:

(a) Spin is an internal property only if one looks at particles independently of the field in which they are immersed. From the field’s perspective, spin is at par with orbital angular momentum.

(b) The approach to rotations through Euclidean algebra (read Clifford algebra) allows one to geometrize the units imaginary that appear in relativistic quantum mechanics.

This geometrization leads nowhere if we think that “spacetime is everything there is to spacetime”. Help comes to the rescue through the foundations of special relativity. Lest we are misunderstood, we state unequivocally that special relativity is here to stay, specially if we refer to its contents, abstraction made of the etymology of the term relativity. But its foundations may not be what have been believed to be so far. In the present paper, we fill a vacuum of related mathematical research. It has implications for Yang-Mills theory.

In a first part of the paper, we consider the foundations of differential geometry by interpreting the equations of structure of Euclidean space with the method of the moving frame. We find a vacuum here also. Dealing with it, we also fill the previously mentioned vacuum. Spacetime has
to be replaced by time-space-propertime as the arena of the physics, a most natural Kaluza-Klein type of space, but without compactification. In the process, groups become ever more loosely connected with what appears to be most relevant physically for particle phenomenology. On the one hand, their Lie algebras play more direct role in any dynamics based on connections, since it is in those algebras that connections take their values. But the generators can alternatively be seen as living in a Clifford-algebra-valued Clifford algebra of differential forms. In this last structure, idempotents rather than generators may play the decisive role in determining what fundamental particles exist.

2. Introduction

2.1. Use of term tangent bundle

In a previous paper [1], we started what should become a long process of “classical geometrization” of quantum mechanics (QM). We now continue that process with the replacing of $U(1) \times SU(2)$ Yang-Mills symmetry with $U(1) \times SU(2)$ tangent bundle symmetry, thus our use of the term classical geometrization. In the interest of brevity and simplicity, tangent bundle means here not only the bundle of tangent vectors, but also other bundles directly related to it. Thus, for example, a bundle of orthonormal frames made of tangent vectors also comes under the umbrella of this denomination. More to the point, we also consider as “tangent bundle” anything directly related to the extension of the spacetime manifold with propertime, $\tau$, to yield a 5-D Kaluza-Klein (KK) space, $(t, x, \tau)$. The dual vector of this fifth dimension “more than replaces” the 4-velocity of the paradigm.

Modern differential geometry is (or is equivalent to) a theory of moving frames [2], particles being absent from the equations of structure. Their motion is represented by the origins of a succession of frames, but this only becomes apparent in Finsler bundles [3] (See a summary of the argument after Eq. (3)). Giving particles a separate life independent of the frames may be viewed as a “mathematically overlooked issue”, and as an approach different from other approaches to 5-D KK spaces in the literature. Physically, this type of structure is specially revealing (we would even say that it is only possible) in context of the specific preferred frame scenario to which we shall refer as para-Lorentzian (PL). It appears to have advantage over special relativity (SR). We proceed to explain why.

2.2. Of relativity and orthogonality

The subspace $(x, \tau)$ is a natural one for quantum physics, but not without problems when the spacetime subsector is the relativistic one. The reason is that the role of the 4-velocity is now taken by the unit vector dual to the propertime coordinate lines. It is not orthogonal to spacetime, its components being the components of the “old” 4-velocity. The $(x, \tau)$ frames are thus velocity-dependent. But quantum physics, like the classical one, must at least look relativistic, given how much relativity permeates a good functioning paradigm.

Paradoxically, the relativistic format (i.e. orthogonality, velocity independence) emerges in the $(x, \tau)$ subspace in PL context. As for the non-orthogonality of PL in the spacetime subspace, it is without consequence, at least for virtually any physical situation or experiment conceived so far. For an example of analysis of a recent experiment where that “indistinguishability” is explicitly shown, see our Opera paper [5]. Notice that we claimed that, if PL had to do with the unexpected discrepancy, we could only provide an explanation based on ad hoc assumption about an alternative hypothetical dynamics. Superluminal velocities needed not be involved; new physics, perhaps. One now believes that there was not any new physics and thus for an ad hoc assumption producing the 60 nanoseconds.
2.3. Indistinguishability and conventionalism

That indistinguishability has been believed or known for a long time by philosophers of science subscribing to the so called thesis of conventionality of synchronizations, though the type of argument leaves room for disagreement as to the limits of its applicability. See for instance the interesting analysis of the resonance experiment in a centrifuge by Maciel and Tiomno [6]. An alternative analysis of this experiment confirming that result was later provided by this author in co-authored paper with Torr [7].

Occasionally, some renowned physicist has also reached similar conclusions about that indistinguishability on more physical argument [4], though without a discussion of the limits of its applicability. The argument has to do with Einstein’s synchronizations, which is unavoidably implemented on clocks in a PL world, even if they stay put or are in slow motion in the laboratory frame.

The conclusion of indistinguishability is counter intuitive to, say, those involved in the dynamics of high energy accelerators. Some of them will vehemently dispute it. In my doctoral dissertation, I had to suffer being caught between a supporter of the latter view and a conventionalist. It is fair to say that the limits of the thesis of conventionality of synchronizations remains a matter of controversy.

2.4. Substructure for special relativity

We posit that the relativistic-like behavior of the physics in both subspaces is a concomitant of the PL world, not of SR. The PL preferred frame structure makes possible velocity independence in the quantum sector (velocities may and will, however, be present in some role, but not in the form of the dynamical equations). It is thus more than just a competitor to SR. It may be the kinematical side of an underlying framework of a theory of which relativistic physics emerges as its facade.

In this new view, physics remains virtually what it appears to be —i.e. looking relativistic— except for being based on different foundations and except, perhaps, at the interface of the classical and quantum sectors, where a breakthrough may be very much needed. The Tajmar experiments may pertain to this interface, since they appear to suggest classical effects (specifically gravitational) of macroscopic quantum systems (superconductors) [8], [9], [10].

The proposed new scenario would concern only experts on very specialized areas of theoretical physics, the ones where a breakthrough is very much needed. It certainly would be theoretically interesting if $U(1)$ and $SU(2)$ could be viewed as spacetime related symmetries, provided that $su(3)$ could also be viewed in a similar light as $u(1)$ and $su(2)$. It will.

Since this paper touches sensitive points of modern physics, we shall let the mathematics speak, relegating physical issues for future papers. Readers reluctant to accept our results may always provide their own solution to the mathematical problem with which we launch our argument in the next section, and then draw their own consequences. Physicists who may feel the need to go deeper into these subjects and do not feel comfortable with the formal literature on differential geometry may wish to refer to our recent book [2]. Although it does not deal with Finsler geometry, readers can then go into a series of papers on the subject that I co-authored and which should then be easy reading [3], [11], [12], [13].

A different, less geometric approach to substructure for special relativity based on a preferred frame has been developed by Caban and Rembielinski [14]. The effort goes back to pioneering work by the last of these authors [15]. Their key word is “meta relativity” [16]. It emphasizes tachions, thus superluminal velocities (See for instance [17]). The driving idea is the same as in the series by this author, in frequent collaboration with D. G. Torr: there is more to structure in special relativity than meets the eye. But we do not see at this point a role for those velocities.

This paper is organized as follows. In section 3, we make the case for a Kaluza-Klein type of space directly related to the tangent bundle. Its development, which takes place in section
5, requires the justification and presentation of PL kinematics. In section 4, we show how this kinematics mimics SR. In section 6, we proceed to discuss the individual $U(1)$ and $SU(2)$ symmetries, as well as their coming together. In section 7, we start to consider electroweak stoichiometry. In section 8, we propose idempotents for the palet of quarks, but without yet going into any stoichiometry.

3. Kaluza-Klein space from tangent bundle

3.1. Motivation

In a paper that preceded his series on the theory of affine connections, Cartan [18] argued as follows. In Euclidean space, the change of the coordinates of a fixed point relative to a frame that experiences differential translation and rotation is given by

$$dx^i + x^j \omega_i^j + \omega^i = 0,$$

where $\omega^i$ represents the translation and $x^j \omega_i^j$ (with $\omega_{ij} + \omega_{ji} = 0$) the rotation. Exterior differentiating (1) and using (1) itself in the system resulting from the differentiation, one obtains

$$d\omega^i - \omega^j \wedge \omega^i_j + x^k (d\omega^i_k - \omega^j_k \wedge \omega^i_j) = 0.$$

Equation (2) is valid for all $x$, which results in the equations of structure of Euclidean space

$$d\omega^i - \omega^j \wedge \omega^i_j = 0, \quad d\omega^i_k - \omega^j_k \wedge \omega^i_j = 0, \quad \omega_{ij} + \omega_{ji} = 0.$$  

This argument makes it clear that the equations of structure are about a geometry where points remain fixed and frames “move”. Points are, so to speak, left out. We have explained through the Finslerian refibration of the frame bundle of spacetime that autoparallels are successions of origins of a succession of frames. The push-forward of the equation $d\mathbf{u} = 0$ to the Finsler bundle becomes $d\mathbf{e}_0 = 0$ or, equivalently, $\omega^i_0 = 0$ [3]. As for extremals, this is a concept not connected with the theory of moving frames, except when dealing with the Levi-Civita connection. The extremals then become autoparallels and then also respond to the equation $d\mathbf{e}_0 = 0$ in the Finsler bundle.

3.2. Natural lifting condition for KK space

We address the issue of incorporating particles deeper into the geometry through a KK type of space. In the theory of moving frames, the standard quadratic forms known as metrics can be viewed as involving a double product:

$$d\mathbf{P}(\otimes, \cdot) d\mathbf{P} = dx^\mu e_\mu(\otimes, \cdot) dx^\nu e_\nu = e_\mu \cdot dx^\mu \otimes dx^\nu = g_{\mu \nu} dx^\mu \otimes dx^\nu.$$  

where $d\mathbf{P}$ is the translation element in the spacetime 4-dimensional manifold. The translation element of our KK space is [19] ($c = 1$)

$$d\varphi = d\mathbf{P} + ud\tau = \omega^\mu e_\mu + ud\tau.$$  

The vector $u$ is not contained in spacetime, since $\tau$ now is an independent coordinate. As for $d$, it is not here what one modernly calls the exterior derivative but part of a symbol. The same comment applies to $d\varphi$.

In our KK space, (4) is replaced with

$$d\varphi(\cdot, \cdot) d\varphi = \omega^A e_A(\cdot, \cdot) \omega^B e_B = 0,$$  

where $\omega^A e_A$ and $\omega^B e_B$ are the elements of the bundle of KK space.
with \( \omega^5 = d\tau \). In (6), two Clifford products are involved, respectively for tangent vector fields and differential forms viewed as integrands. This view of differential forms is very explicit in [20], [21]. Modernly, differential forms are defined as antisymmetric multilinear functions of vectors, but such will not be the case here.

If the spacetime dynamics is governed from the KK \((t, x, \tau)\) manifold, the latter’s relevant curves are those that satisfy Eq. (6), which thus is a natural lifting condition. \( \tau \) thus relates to the proper time assigned to those trajectories. Recall that, on curves, all differential forms are multiples (we do not mean integer multiples) of one of them, and thus of \( d\tau \). But, in the \((x^i, \tau)\) submanifold, it is simply the evolution parameter of quantum systems.

### 3.3. Clifford algebras and Kaluza-Klein structure

Logically, we should view classical physics as being what it is because quantum physics is what it is. The sector \((x, \tau)\) should then determine what classical physics is like. But this is not actually so, due to historical accident.

He developed a exterior-interior calculus of differential forms for quantum mechanics. It does not resort to gamma matrices [21], [22]. We thoroughly rely on it given what we believe to be its enormous superiority over Dirac’s.

Following Kähler, we shall use the signature \( \eta = (-1, 1, 1, 1) \) for spacetime. In five dimensions, the metric will be postulated to be null, as per (6). The development of this equation yields

\[
0 = \eta_{\mu\nu} \omega^\mu \cdot \omega^\nu + d\tau \cdot d\tau + (\omega^\mu \cdot d\tau) u \cdot e_\mu. \tag{7}
\]

with

\[
u \cdot u = 1, \quad e_0 \cdot e_0 = -1. \tag{8}
\]

If we ad hoc set

\[
\omega^\mu \cdot d\tau = 0, \tag{9}
\]

and specialize to \( \omega^\mu = (dt, dx^i) \), the equation

\[
0 = dt \cdot dt - \sum_i dx^i \cdot dx^i + d\tau \cdot d\tau \tag{10}
\]

follows. We shall write it as

\[
0 = dt^2 - \sum_i (dx^i)^2 - d\tau d\tau, \tag{11}
\]

in parallel to writing in that way the metric tensor equation

\[
0 = dt \otimes dt - \sum_i dx^i \otimes dx^i + d\tau \otimes d\tau, \tag{12}
\]

which actually is an algebraic definition of the tensor \( d\tau \otimes d\tau \).

The dot product in (10) has to be understood in a Clifford algebra context of differential forms, i.e. Kähler’s algebra [21], [22]. We have, in the spacetime case,

\[
\omega^\mu \vee \omega^\nu + \omega^\nu \vee \omega^\mu = 2\eta^{\mu\nu}. \tag{13}
\]

In order to get to (13), we avoided the non-orthogonality of \( u \) and \( e_0 \) in the relativistic case by virtue of (12). But we do not see how anything of theoretical interest could emerge under that non-orthogonality. But, if we consider the PL structure, a new theoretical scenario reveals itself. It can actually be viewed as the infrastructure that hides and paradoxically justifies that the quantum physics sector looks relativistic. Readers should start getting to the idea that relativistic and orthonormal need not always go together.
4. Preferred frame kinematics

4.1. Para-Lorentzian structure

Consider regions of spacetime as small as needed to be viewed as regions of affine spacetime, thus enjoying translational space symmetry and uniformity of time. Assume a frame $\Sigma$ of isotropy, Einstein’s preferred frame [23]. For motion along the $x$ axis and under the assumption of homogeneity of spacetime and spatial isotropy in the preferred frame, the transformations must be of the form

$$
x' = a \cdot (X - VT), \quad t' = hX + jT, \quad y' = eY, \quad z' = eZ. \quad (14)
$$

Assume further that there is compliance with the interpretation by Robertson of the standard results attributed to the Michelson-Morley, Kennedy-Thorndike and Ives-Stilwell experiments. It implies [24]

$$
MM : a = \gamma e, \quad KT : j + hV = a\gamma^{-2}, \quad IS : j + hV = \gamma^{-1}.
$$

One readily checks that equality of the to and fro speeds of light requires

$$
h + Vj = 0, \quad (16)
$$

which, together with relations (15), makes (14) become the special Lorentz transformations.

Because of their relevance in applications of the KK theory of present interest, we shall consider the restriction of (14) by (16) only. One obtains:

$$
x' = \gamma \cdot (X - VT), \quad y' = Y, \quad z' = Z, \quad t' = hX + (\gamma^{-1} - Vh)T. \quad (17)
$$

Clearly, the option of absolute simultaneity with time dilation with respect to the preferred frame in the ratio $\gamma^{-1}$ is contained in this system of equations, specifically for $h = 0$:

$$
x' = \gamma \cdot (X - VT), \quad y' = Y, \quad z' = Z, \quad t' = \gamma^{-1}T. \quad (18)
$$

We shall refer to this option as the para-Lorentzian option, or simply PL. If the velocity is not a constant, we should not write these equations but rather

$$
\omega^1 = \gamma \cdot (dX - VdT), \quad \omega^2 = dY, \quad \omega^3 = dZ, \quad \omega^0 = \gamma^{-1}dT. \quad (19)
$$

Notice that the differentials of some of the right hand sides would not be zero if $V$ were viewed as a coordinate and thus not as a constant, which is why we did not write on the left hand sides $dx^\mu$ (since $ddx^\mu$ is zero). In SR, the role of the $\Sigma$ frame can be played by any Lorentzian frame, and we have

$$
\omega^1 = \gamma \cdot (dX - VdT), \quad \omega^2 = dY, \quad \omega^3 = dZ, \quad \omega^0 = \gamma \cdot (dT - VdX). \quad (20)
$$

By eliminating $(T, X^i)$ between (18) and the special Lorentz transformations, we get $t' = t + Vx, \quad x'^i = x^i$. Correspondingly, we have

$$
t' = t + V_i x^i, \quad x'^i = x^i \quad (21)
$$

for arbitrary direction of the velocity. The relation between SR and PL frames is then given by

$$
e'_0 = e_0, \quad e'_i = e_i - V_i e_0. \quad (22)
$$

The ten dimensional set of PL frames is obtained by applying the usual rotations $SO(3)$ to the primed frames.
The PL metric may be obtained from the SR metric

$$ds^2 = (\omega^0)^2 - \sum_i (\omega^i)^2 = (\omega^0 - V_i \omega^i)^2 - \sum_i (\omega^i)^2,$$

which, for mnemonic reasons, we shall write with the wrong notation

$$ds^2 = (dt' - V_i dx^i)^2 - \sum_i (dx^i)^2.$$  \hfill (23)

We may also use \(e'_\mu \cdot e'_\nu = g'_{\mu\nu}\) and \(e_\mu \cdot e_\nu = \eta_{\mu\nu}\).

4.2. Slow clock transport synchronization in SR and PL

We shall now look at slow clock transport synchronizations in a PL world. The argument about to be made was already made by Bohm decades ago [4]. Unfortunately, it is an argument that very few physicists appear to be aware of, including this author when he took his first steps as a researcher [25] (my blunder was that I overlooked that one factor in a key product went to infinity as the other factor went to zero). It helps keep a more open mind about the foundations of SR. We now provide an argument to the same effect, but where we take in PL the same steps as in SR. We start with the latter.

Let a clock move with a small velocity \(\epsilon\) with respect to a frame \(S(t, x)\) that in turn moves with respect to a frame \(\Sigma(T, X)\) with velocity \(V\). The proper time \(\tau\) of the moving clock will be given by

$$\tau \simeq T \sqrt{1 - (V + \epsilon)^2}; \quad T = \gamma_v \cdot (t + Vx), \quad x = \epsilon t,$$

with \(\gamma_v = (1 - V^2)^{-1/2}\). Therefore, in first order,

$$\tau \simeq \frac{\sqrt{1 - (V + \epsilon)^2}}{\sqrt{1 - V^2}} (t + Vx) \simeq (1 - V\epsilon)t(1 + V\epsilon) \simeq t.$$ \hfill (25)

This means that, to within terms of order \(\epsilon^2\), the proper time of the clock moving with speed \(v\) with respect to \(S\) coincides with the relativistic time at the point in front of him (i.e. with coordinate \(x = \epsilon t\)).

The same computation in a PL world yields

$$\tau \simeq T \sqrt{1 - (V + \epsilon)^2}; \quad T = \gamma_v t', \quad x = x' = \epsilon t,$$

with \(\gamma_v = (1 - V^2)^{-1/2}\). Therefore,

$$\tau \simeq \frac{\sqrt{1 - (V + \epsilon)^2}}{\sqrt{1 - V^2}} t' \simeq (1 - V\epsilon)(t + Vx) \simeq (1 - V\epsilon)t(1 + V\epsilon) \simeq t,$$ \hfill (27)

which shows that slow clock transport in a PL world performs a relativistic synchronization.

Actually, one need not perform this computation since \(d\tau\) is in PL the same as in SR, namely \(\gamma^{-1}T\). But the derivation just given makes our point more explicit. The bottom line is that, for most purposes, the kinematics of a PL world is unavoidably like in SR for most purposes. These considerations may not, however, apply if we had a Mossbauer experiment involving the comparison of frequencies of two identical clocks in a rotating disk, as this would not involve in any way a synchronization of clocks [6]. There is always room for doubt nevertheless, specially given the controversy surrounding rotating disks (Ehrenfest paradox).
5. Tangent bundle Kaluza-Klein spaces

5.1. SR, quantum mechanics and KK structure

The type of 5-dimensional space that is being introduced here has been overlooked, like spacetime was until the very early 20th century. Had it not been so, the subspace \((e_i, u)\) rather than \((e_0, e_i)\) would have been the arena for the dynamics of quantum systems, and the hydrogen atom in particular. Said differently, propertime, not time, matters. It has its own representation rather than just been an ad hoc chosen laboratory time so that the hydrogen atom will be at rest in it.

On curves, and by virtue of \(u\) being dual to propertime, the dot products \(u \cdot e^{\mu} (= g_{4\mu})\) have to be interpreted as components \(U_\mu\) of the 4-velocity in SR, thus not null. Hence \(u\) is not perpendicular to spacetime, and to 3-space in particular. We can certainly orthonormalize the vector basis but, in the normalized basis, the meaning of the unit vector would be lost. The relevant frame in this scenario for solving the hydrogen atom, then is \((e_i, u)\), which is not orthonormal. This introduces ab initio dependence of the dynamics with respect to velocity the laboratory of the physics itself. Of course, this issue is sidestepped by “bringing the laboratory to the hydrogen atom”, which is disguised as bringing the hydrogen atom to the laboratory.

It would be a waste of time to discuss the issue of the covariance of the Dirac equation as justification for the validity of, as we said, bringing the laboratory to the atom. Better to explore other avenues given that prominent physicists and mathematicians—even experts on the Dirac equation [26] (see his preface)—spontaneously admit that one does not quite understand this equation. The present author is totally convinced of the far greater naturalness and superiority of the Kähler equation [27], [22]. We shall use tangent vectors for the valuedness of the differential forms.

The negative definiteness of \(- \sum (dx^i)^2 - d\tau d\tau\) should not be considered a counter argument (at least at this point) against this KK space. One could, for example, tinker with the signs of relevant equations in order to get the right results. Thus, for instance, the placement of the unit imaginary in the Kähler and Dirac equations is different and non-equivalent, but they give the same fine structure for the hydrogen atom [27], [22]. They also yield the same classical type electromagnetic Hamiltonian [28]. Furthermore, the unit imaginary will be replaced with members of real Clifford algebras [1]. We must wait for further work on this subject before understanding whether there is any problem here.

To summarize, if dealing with a quantum problem we had to choose between the subspaces \((t, x)\) and \((x, \tau)\) because they yielded different results, it is clear that we should choose \((x, \tau)\) over \((t, x)\), if we are serious about this KK option.

5.2. Para-Lorentzian KK structure and preferred frame

We proceed to consider the very revealing KK space for the PL spacetime. Let \(E_\mu\) denote a basis in the preference frame (up to a 3D rotation) and let \(w\) be the unit vector dual to propertime. Physically it represents in the KK space what in spacetime is the 4-velocity of a particle. We choose the symbol \(w\) where we previously chose the symbol \(u\), which one might now erroneously replace with the particle’s velocity \(u’\) relative to \(S’\). Time dilations are absolute and determined by speed with respect to the preferred frame.

In parallel to (4), we now consider

\[
d\varphi = dP + d\tau w = dX^\mu E_\mu + d\tau w. \tag{29}
\]

We set

\[
w \cdot w = -1. \tag{30}
\]

which yields

\[
d\varphi(\cdot, \cdot)d\varphi = 0 = \eta_{\mu\nu} dX^\mu \cdot dX^\nu - d\tau \cdot d\tau + 2(dX^\mu \cdot d\tau)E_\mu \cdot w. \tag{31}
\]
Clearly
\[ \mathbf{E}_i \cdot \mathbf{w} = 0, \]  
(32)
since, otherwise, we would not have invariance under rotations in the preferred frame. It is a tenet of post-SR preferred frame theories that we must have
\[ 0 = -dT^2 + \sum_i (dX^i)^2 + d\tau^2. \]  
(33)

In order to get from (31) to (33), we need
\[ (dX^\mu \cdot d\tau)(\mathbf{w} \cdot \mathbf{E}_\mu) = d\tau \cdot d\tau. \]  
(34)
Using (32), we can write (34) as
\[ (dT \cdot d\tau)(\mathbf{E}_0 \cdot \mathbf{w}) = d\tau \cdot d\tau. \]  
(35)
The equation
\[ d\tau = dT (1 - w^2)^{1/2} = dT \gamma_w^{-1}, \]  
(36)
then allows to substitute \(dT\) in terms of \(dT\) in (35) and obtain
\[ \mathbf{E}_0 \cdot \mathbf{w} = \gamma_w^{-1}. \]  
(37)

5.3. Para-Lorentzian KK structure and moving frame
Assume that PL system \( \mathcal{S}' \) moves with velocity of components \((V, 0, 0)\) with respect to the preferred frame. From
\[ T\mathbf{E}_0 + X\mathbf{E}_1 = t'\mathbf{e}_0' + x'\mathbf{e}_1', \]  
(38)
we get, using (18),
\[ \mathbf{e}_0' = \gamma (\mathbf{E}_0 + V\mathbf{E}_1), \quad \mathbf{e}_1' = \gamma^{-1}\mathbf{E}_1, \]  
(39)
where \(\gamma\) is \((1 - V^2)^{-1/2}\). The covariant form of (39) under rotations is
\[ \mathbf{e}_0' = \gamma (\mathbf{E}_0 + V^i\mathbf{E}_i), \quad \mathbf{e}_i' = \gamma^{-1}\mathbf{E}_i, \]  
(40)
and, therefore,
\[ \mathbf{e}_i' \cdot \mathbf{w} = 0. \]  
(41)
Thus, regardless of the velocity \(\mathbf{V}\) of the frame \((\mathbf{e}_0', \mathbf{e}_i')\), the subspace spanned by \((\mathbf{e}_i')\) is perpendicular to the velocity \(\mathbf{w}\) of particles with respect to the preferred frame. In other words, the sets \((\mathbf{e}_i', \mathbf{w})\) constitute frames adapted to particles, as in Finsler bundles [3].

We further have, using the first of (40),
\[ \mathbf{e}_0' \cdot \mathbf{w} = \gamma (\mathbf{E}_0 \cdot \mathbf{w} + V^i \mathbf{E}_i \cdot \mathbf{w}) = \gamma \gamma_w^{-1}. \]  
(42)
When a quantum system is at rest in the laboratory, \(\gamma = \gamma_w\) and
\[ \mathbf{e}_i' \cdot \mathbf{w} = 1. \]  
(43)
The \(dx'^i\)'s of a moving particle are then set to zero in the classical representation, in the sense that \(\int dx'^i\) is zero. Hence, we have
\[ 0 = (dt' \mathbf{e}_0' + d\tau \mathbf{w})(\cdot \cdot)(dt' \mathbf{e}_0' + d\tau \mathbf{w}) = -dt' \cdot dt' - d\tau \cdot d\tau + 2(dt' \cdot d\tau) \]  
(44)
and, therefore,
\[ dt' = d\tau \] (45)
in this case. Furthermore, since
\[ d\phi = d\tau (e'_0 + w), \] (46)
the natural lifting condition implies \( d\phi = 0 \) and, therefore,
\[ w = -e'_0. \] (47)

These results speak of structural consistency. They will appear more natural when, in future papers, we are led to viewing this interplay of two algebras in the direction suggested in section 6.

Assume that a system \( S' \) moves with velocity \((V, 0, 0)\). The \( e'_i \) are not perpendicular among themselves since the process followed does not relate orthonormal 3D SR frames to orthonormal 3D PL frames. However, if the dot products are defined, one can always orthonormalize the 3-D basis. We emphasize that the crucial point here is the orthogonality of \( w \) and the subspace subtended by the \( e'_i \) regardless of what is the velocity \( V \) of the spacetime frame \((e'_0, e'_i)\) to which we attach a \((e'_i)\) spatial basis.

To summarize, we have uncovered a time-space-propertime KK structure with particles represented with the help of frames but not by frames.

5.4. Non-SR, yet Lorentzian Kaluza-Klein structures

The type of KK structure that we have developed is inconsistent with the foundations of SR since it is inconsistent even with a Lorentzian structure with preferred frame, i.e. without SR. We proceed to prove it.

The Lorentz transformations do not imply SR [29]. Assume an ab initio preferred frame \( \Sigma \). Assume that all frames are related to it by Lorentz transformations. If we hit without friction a body at rest in the preferred frame, it jumps by hypothesis to a Lorentz frame \( S_1 \) related to \( \Sigma \) by a Lorentz boost without rotation. Also from \( \Sigma \), we hit without rotation an identical body in a similar manner but with different force and in a different direction. It falls to rest in Lorentz frame \( S_2 \). Let \( v \) be the relative velocity of \( S_2 \) with respect to \( S_1 \). If we now hit, again without friction, the body at \( S_1 \) so that it falls into \( S_2 \) and assume SR, this body reaches there with a Thomas rotation with respect to the one already there by boost from \( \Sigma \). But a maker of worlds could in principle have created a world where the result of the blow from \( S_1 \) to \( S_2 \) is a Lorentz boost with rotation in such a way that the final state in \( S_2 \) is the same regardless of which of the two ways took as there. It is in such a context that we now proceed with the discussion.

\( w \) will again refer to the velocity of particles with respect to \( \Sigma \). As in PL, we would choose \( w \cdot w = -1 \). Let \( S \) be any other moving orthonormal frame that will play the role that \( S' \) played in the previous subsection. Equations (39) and (40) must now be replaced by Lorentz boots and Eq. (41) no longer applies. The same conclusion is reached if we had simply substituted the second of equations (15) in (41):
\[ (e_i - V_ie_0) \cdot w = 0. \] (48)

We thus have orthonormality and, therefore, presumed be velocity dependence of the dynamics. The fine structure of the hydrogen atom would depend on the velocity with respect to the laboratory. We are not speaking here of the shifted spectrum of an atom by virtue of its motion with respect to the laboratory frame. We would be computing in the propertime scenario that this KK theory blindly assigns to it.

The standard relativistic approach to the hydrogen atom based on the \((e_0, e_i)\) subspace at zero velocity with respect to the laboratory yields the right spectrum (up to second quantization corrections). The KK scenario validates PL. The relativistic approach for the hydrogen atom is, therefore, mimicking the PL physics in the \((x^i, \tau)\) subspace, equivalently \( e'_i, w \) subspace.
6. Partial geometric interpretation of $U(1) \times SU(2)$
We speak of partial interpretation because, going the full way with the interpretation, we fall into $SU(3)$. We want to follow the historical development—first electroweak, then $U(1) \times SU(2) \times SU(3)$—because that is the way in which it started to evolve in our own mind. In any case, going directly to $U(1) \times SU(2) \times SU(3)$ would be too much to digest at once. We leave it for a future paper, when this author will himself have a better grasp of the issues involved.

6.1. Rotations in a Clifford algebra and in its ideals
In Kähler’s approach to quantum mechanics, the equation that supersedes Dirac’s is called the Kähler equation. It is not about spinors but about a primordial field that is a member of an algebra. Let us refer to it as the Kähler algebra in order to distinguish it from other algebras to be considered. The Kähler algebra can be decomposed into ideals defined by symmetry properties. The field can correspondingly be decomposed into spinors.

The differential forms that are members of the Kähler algebra for the electromagnetic interaction are scalar valued. There are four basic mutually annulling primitive idempotents. They are associated with charge and chirality (or energy and spin if one considers the phase factors that multiply those idempotents). Not all symmetries of a system fit at the same time in the decomposition of the solutions. For example, only one component of spin fits in them even if there is spherical symmetry of the system. In Dirac’s theory, this has to do with whether generators commute or not.

Because of scalar valuedness, Kähler had no alternative but to use the unit imaginary to form idempotents. This is totally unnecessary when the differential forms are Clifford valued. They are associated with charge and chirality (or energy and spin if one considers the phase factors that multiply those idempotents). Not all symmetries of a system fit at the same time in the decomposition of the solutions. For example, only one component of spin fits in them even if there is spherical symmetry of the system. In Dirac’s theory, this has to do with whether generators commute or not.

For simplicity take $n = a_3$. Then, (48) becomes

$$e^{-\frac{1}{2}\phi a_1 a_2} A e^{\frac{1}{2}\phi a_1 a_2}.$$  \hspace{1cm} (50)

Left spinors $L$ of the same algebra, on the other hand, transform like

$$e^{-\frac{1}{2}\phi n^i a_j a_k} L.$$  \hspace{1cm} (51)

And, corresponding to (49),

$$e^{-\frac{1}{2}\phi a_1 a_2} L.$$  \hspace{1cm} (52)

Whether the rotation is represented by (48), i.e. $SO(3)$, or by (50), i.e. $SU(2)$, depends on the objects on which rotations act. It is then as legitimate to speak of rotations when dealing with $SU(2)$ as when dealing with $SO(3)$, even if the relation is 2 to 1.

6.2. $SU(2)$
Kähler’s QM is more revealing than Dirac’s regarding interactions beyond the electromagnetic one. The calculus on which he based his work follows long after his seminal work on exterior systems nowadays known as the Cartan-Kähler theory of differential equations. His here-crucial
treatment of symmetries is what it is because of his replacement of Dirac’s equation for spinors with an equation for a primordial field valued in some algebra. The spinors are present through the decomposition of the algebra into ideals.

He decomposed his Clifford algebra of differential forms through four commuting primitive idempotents, $\tau^\pm\tau^*$, where the asterisk means both signs independently of the sign in $\epsilon^\pm$. The $\tau^\pm$ and $\epsilon^\pm$ are

$$\tau^\pm \equiv \frac{1}{2}(1 \pm i dx dy), \quad \epsilon^\pm \equiv \frac{1}{2}(1 \mp i dt).$$

(53)

The symmetry is around the $z$ axis (actually by choice of axis). There are more pairs of mutually annulling primitive idempotents, say $\tau^\pm_{yz} \equiv \frac{1}{2}(1 \pm idy dz)$, $\tau^\pm_{zx} \equiv \frac{1}{2}(1 \pm idz dx)$,

(54)

but none of these commutes with $\tau^\pm$, to which we refer as $\tau^\pm_{xy}$ in the following.

With the geometrization of quantum mechanics that we have recently proposed [1], we make the replacement of

$$\tau^\pm_{ij} \equiv \frac{1}{2}(1 \pm i dx^i dx^j)$$

(55)

and $\epsilon^\pm$ with

$$\tau^\pm_{ij} \equiv \frac{1}{2}(1 \pm a^i a^j dx^i dx^j), \quad \epsilon^\pm \equiv \frac{1}{2}(1 \mp w d\tau),$$

(56)

where $\tau$ without indices is propertime. Clearly, no summation over repeated indices is understood.

All the six independent $\tau^\pm_{ij}$ commute, which does not mean necessarily that they can be used as factors in hypothetical solutions with spherical symmetry. More than that would be needed, as explained elsewhere [28]. One may nevertheless replace the family $\epsilon^\pm\tau^*$ with the family

$$\epsilon^\pm\tau^*_y, \quad \epsilon^\pm\tau^*_z, \quad \epsilon^\pm\tau^*_x.$$  

(57)

At this point, one may wonder what is the Lie product in the algebra that supposedly replaces the Lie algebra of the rotation group. The Clifford product would play that role when we multiplying two different factors. Define the natural order of the space bivectors and corresponding differential 2-forms as $a^i a^j$, $a^j a^k$, $a^k a^i$. Then:

$$(a^i a^j)(a^j a^k) = a^i a^k = -a^k a^i,$$

(58)

and, similarly,

$$(dx^i dx^j)(dx^j dx^k) = dx^i dx^k = -dx^k dx^i.$$  

(59)

The problem with this would appear to be that the product of $a^i a^j$ with itself (equivalently $dx^i dx^j$) does not yield zero. If need be, we recover an equivalent Lie algebra in both cases by defining a Lie product of $(a^i a^j)$ and $(a^j a^k)$ as

$$\frac{1}{2}[(a^i a^j)(a^j a^k) - (a^j a^k)(a^i a^j)],$$

(60)

and similarly for the $dx^i dx^j$.

Little by little it will become increasingly evident that the Lie products of the generators are irrelevant. What matters is the compatibility of idempotents, even if the algebra from which they are built is commutative.
6.3. $U(1)$ and $U(1) \times SU(2)$

Recall the opening mathematical argument of the paper. With the particle at rest, we obtained the equations of structure by moving the frame to which the particle is referred. The boosts of frames mimic the translation of the particles when, for instance, we consider this translation as being represented by autoparallels.

The translation of particles is now obviously represented by $\mathbf{w}$, but not in $(x, \tau)$ subspace, where it is multiplied by the differential $d\tau$ of the evolution parameter in the quantum sector $\tau$. Hence, in this sector, we have to see the $U(1)$ symmetry as pertaining to this evolution. And the relation to electrodynamics is obvious by running the argument of the previous subsection in reverse, from the primitive idempotents (56) to the $\epsilon^\pm \tau^*$. The $\epsilon^\pm$ are then the idempotents associated with propertime translation symmetry in $(x, \tau)$ subspace. In the full $(t, x, \tau)$ space and through the natural lifting condition, $\tau$ represents the (propertime of a) particle rather than the evolution parameter for particle-less quantum mechanical field theory. [32] is then the Lie algebra for the one-parameter group of propertime translation symmetry, $\mathbf{w}$ being the generator. But what matters is the associated idempotents (accompanied by the appropriate phase factor [28]).

At its deepest level, quantum physics is more directly concerned with its Lie algebra, which is where connections take their values. Thus the focussing on groups rather than on their Lie algebras may be specially misleading in the case of $U(1)$ because there is only one Lie algebra in dimension one. It does not matter whether the group is compact or not since the connection is a local concept.

Using rotations as an example, the point about different groups being locally equivalent was succinctly and authoritatively made by Donaldson and Kronheimer when they stated “For the purposes of local differential geometry, $SO(3)$ connections and $SU(2)$ connections are completely equivalent”, and further down in the same paragraph: “So there is not really much difference between working with the structure groups $SU(2)$ and $SO(3)$ [32] (page 42, no italics in the original). Of course, there are global differences, but these may be totally irrelevant for a physicist if, as pointed out with a specific case by Cartan in correspondence with Einstein “... every singularity-free solution of system (1), creates from the topological point of view, the continuum in which it exits” [33] (p. 103, italics as in the original; the system (1) to which Cartan refers is an example he proposed to make a point to Einstein).

A too narrow focus on groups reflects a perspective of symmetries inherited from classical physics, where restrictions imposed by lack of commutativity of operators do not enter the theory. Rotations are very illuminating also to illustrate this point. A Kähler equation may have as input a spherically symmetric potential, but the solutions with angular momentum different from zero do not have spherical symmetry because of reasons that go beyond commutativity or not of the operators. Of the essence is the form of the output of the equation (spinors, in the case of particle solutions as opposed to preferred frame solutions) rather than the symmetry properties of the input (the electromagnetic potential), greater than those of the input.

But the everyday manifestation of classical electrodynamics —except for the trained eye who sees its effects in determining the properties of matter through quantum laws— is in the classical domain. In this domain, the equation of motion of particles is given by $d\mathbf{u} = 0$. This is the case not only for the theory of gravitation but also for electrodynamics, once we accept that it has to do with geometry in the tangent bundle [30], [31]. In turn, $\mathbf{u}$ and $\mathbf{w}$ are intimately related through the natural lifting condition, as we already saw. See argument that starts after Eq. (42) and goes all the way down to equation (46). So, propertime translation symmetry (i.e. the $U(1)$ group in the quantum sector) also is related to classical electrodynamics.
7. Limitation of “electroweak” stoichiometry

By the term stoichiometry, we mean here the balancing of idempotents in particle reactions. By the qualification electroweak, we mean not only the absence of quark considerations, but also the absence of vector bosons. These have lifetimes like third generation quarks, thus highly non-nucleonic. We have put weak in electro“weak” between inverted commas for the same reason as to why we spoke of “partial” interpretation in the title of the previous section.

The general stoichiometry of elementary particles is based on equations such as

$$1 = \tau^+ + \tau^- = \frac{1}{2}(1 + idxdy) + \frac{1}{2}(1 + idxdy).$$

$$1 = \epsilon^+ + \epsilon^- = \frac{1}{2}(1 - idt) + \frac{1}{2}(1 + idt).$$

$$1 = \epsilon^+ \tau^+ + \epsilon^- \tau^- + \epsilon^+ \tau^- + \epsilon^- \tau^-. $$

All these equations are supposed to be multiplied by a solution of the primordial field on the left. One then brings $dt$ and $dxdy$ factors in the primordial field to the right to be absorbed by the idempotents through

$$dt \epsilon^+ = \pm \epsilon^+, \quad dxdy \tau^+ = \pm \tau^+. $$

Whether $\epsilon^- \tau^+$ or $\epsilon^- \tau^+ + \epsilon^- \tau^-(= \epsilon^-)$ represents an electron or not depends on what these idempotents are multiplied by in the end, i.e. when these operations have taken place. First, there has to be an appropriate phase factor. But that is not all. The factor that still remains must not contain $t$ and $dt$ in the case of the idempotents $\epsilon$. The case of $\epsilon^- \tau^+$ and $\epsilon^- \tau^-$ follows along similar lines, but involving more variables and differentials.

According to this scheme of things, neutron decay,

$$n \rightarrow p + e + \bar{\nu}_e,$$

is represented by

$$\tau^+ = \epsilon^+ + \epsilon^- - \tau^-.$$ 

This can be rewritten as

$$0 = \epsilon^+ + \epsilon^- - (\tau^+ + \tau^-).$$

At this level of theory, one cannot even distinguish between the idempotents for neutrinos from those for neutrons since $\tau^-$ represents a neutrino. But, in another reaction, it could represent an antineutrino. One needs more idempotents to specify $n$ and $p$, but this would be strong interaction stoichiometry.

The question then is, how does the electroweak case improve on the electromagnetic case in this regard. In the above scheme of things, one simply replaces $\tau^\pm$ and $\epsilon^\pm$ with

$$\tau_{ij}^\pm \equiv \frac{1}{2}(1 \pm a_i a_j dx^i dx^j), \quad \epsilon^\pm \equiv \frac{1}{2}(1 \mp d\tau).$$

Thus, equation (66) is now replaced with

$$\tau_{xy}^+ = \epsilon^+ + \epsilon^- - \tau_{xy}^-.$$ 

Very little has been gained, except that we also have

$$\tau_{yz}^+ = \epsilon^+ + \epsilon^- - \tau_{yz}^-; \quad \tau_{xz}^+ = \epsilon^+ + \epsilon^- - \tau_{xz}^-.$$
for neutron decay via the muon and tau channels.

One has to be careful with the handling of these equations before we deal with the strong interaction, at which point the different particles in the reaction will be represented by richer idempotent decompositions. One also must be careful not to mix equations of the types (65) and (66). One might be tempted to write the reactions

\[ W^- \rightarrow e + \bar{\nu}_e, \quad W^- \rightarrow \mu + \bar{\nu}_\mu, \quad W^- \rightarrow \tau + \bar{\nu}_\tau \]  

(71)

jointly as

\[ W^- = e + \bar{\nu}_e = \mu + \bar{\nu}_\mu = \tau + \bar{\nu}_\tau, \]  

(NO!)

This is clearly incorrect as a direct consequence of Eqs. (69)-(70).

But there is another very important reason. \( W^- \) is a boson. So, like the gammas, they must not appear in our equations. Bosons will be included in the primordial field, except that this field is more sophisticated in the electroweak case than in the electromagnetic case. In Eq. (63), the primordial electromagnetic field will be included in this equation only after that equation is multiplied by it. So, we may say that, in that equation, it is represented by the unity. In the same way, we shall now have

\[ 1 = \varepsilon^+ \tau^+_{yz} + \varepsilon^+ \tau^+_{yz} + \varepsilon^- \tau^+_{yz} + \varepsilon^- \tau^+_{yz} + \]  

\[ + \varepsilon^+ \tau^+_{xz} + \varepsilon^+ \tau^+_{xz} + \varepsilon^- \tau^+_{xz} + \varepsilon^- \tau^+_{xz} + \]  

\[ + \varepsilon^+ \tau^+_{xy} + \varepsilon^+ \tau^+_{xy} + \varepsilon^- \tau^+_{xy} + \varepsilon^- \tau^+_{xy}, \]  

(72)

the unit on the left representing the electroweak field. A more significant difference is that the photon is massless and the \( W^\pm \) and \( Z \) are not.

8. Beyond electroweak stoichiometry: quarks

The chirality operators are associated with spin, thus with rotational symmetry, but nothing has been said of the relation of the axis of rotational symmetry to the motion of particles with respect to the preferred frame. It may make a difference whether something spins in the direction of the motion or perpendicular to it. That is an element to be taken into account in building \( U(1) \times SU(2) \times SU(3) \) stoichiometry. It brings new factors into the idempotents that represent particles.

The three pairs \( \tau^\pm_{ij} \) are associated with spin/chirality in three orthogonal directions. One may think of quarks as organizations of the field that failed to be particles because they missed one of the elements needed to be so; they died while in gestation. “Failed” experiments of nature can be built by incorporating new idempotents as factors of the \( \varepsilon^\pm \tau^\pm_{ij} \). We have specifically in mind translational symmetry idempotents

\[ \chi_x \equiv \frac{1}{2}(1 + dx), \quad \chi_y \equiv \frac{1}{2}(1 + dy), \quad \chi_z \equiv \frac{1}{2}(1 + dz), \]  

(73)

since they are naturally correlated with rotational degrees of freedom and can be suggested by relevant velocities. The symbols \( dx, dy \) and \( dz \) are just notation for \( dx a_1, dy a_2 \) and \( dz a_3 \). They can participate similarly in the making of quarks.

Owing to reasons that we shall not enter at this point, one does not need a \( \pm \) here because it is the relation between \( \chi \)'s and \( \tau \)'s that matters, and these are taken care of already with just the plus signs in the \( \chi \)'s. We are also led to believe that idempotents (73) are associated with generations. Then, within each of these, color will be associated with rotational idempotents. But all that is for a future paper.
In order to see how (73) expands the algebra for any given generation, just notice that the subspace of our commutative algebra that \( \varepsilon^+ \tau_{xy}^+ \) spans is the same one as generated by

\[
1, \ d\tau w, \ dxdy, \ dx dy d\tau w.
\]  

In order to avoid distractions with a double use of the symbol tau, we do not write \( d\tau w \) as \( d\tau \). We can built copies of \( SU(2) \) with these generators in many different ways. In each of the columns of the table that follows, the product of any two of them in cyclic order amount to the negative of the third one:

\[
\begin{pmatrix}
1 & 1 & d\tau w & \varepsilon^+ & \tau_{xy}^+ \\
dxdy & -d\tau w & dxdy & dxdy\varepsilon^+ & -\tau_{xy}^+ d\tau w \\
-dxdy & d\tau w & -dxdy d\tau w & -dxdy\varepsilon^+ & \tau_{xy}^+ d\tau w
\end{pmatrix},
\]

(75)

as per (58). The fourth and fifth columns result from multiplication of the first and second ones by \( \varepsilon^+ \) and \( \tau_{xy}^+ \), respectively. We might as well have multiplied by \( \varepsilon^- \) and \( \tau_{xy}^- \).

Let us now put together the contents of the previous two paragraphs. We multiply the first two rows of the three last columns by \( dz \) to get the following copies of \( su(2) \):

\[
\begin{pmatrix}
dzd\tau w & dz\varepsilon^+ & d\varepsilon^+ dz \tau_{xy}^+ \\
dz dxdy & dzdxdy\varepsilon^+ & -dz\tau_{xy}^+ d\tau w \\
-dxdy d\tau w & -dxdy\varepsilon^+ & \tau_{xy}^+ d\tau w
\end{pmatrix}
\]

(76)

We now show that seven and only seven of these nine “generators” (of the algebra) are independent. An eighth one emerges from squaring the elements of the first column.

Consider the first two rows of (76). It is easy to find four independent elements among the six of them. There cannot be more than four since all of them are linear combinations of those in (74) (multiplied by \( dz \)). The elements of the third row, not containing the factor \( dz \), are independent of the elements of the first two rows. They also are independent among themselves since \( dx dy \varepsilon^+ \) (a) contains one term \( dx dy \) but the other two elements in the third row have it only multiplied by something else, and (b) these two are independent of each other (notice that \( d\tau w \) is present in \( \tau_{xy}^+ d\tau w \), but not in \( dx dy d\tau w \)). Together with the unity (which is the square of all terms in the first column), those eight independent elements span the eight dimensional algebra to which we have referred above. Thus the \( SU(3) \) idempotents for first generation quarks will be given by

\[
\chi_\varepsilon \varepsilon^* \tau_{ij}^{**}.
\]

There are twelve of them, i.e. \( 3(3 \times 2) \times (3) \times (2) \) between the three generations.

We are leaving for a future paper progress we have made on concepts such as, say, double strangeness, color and its relation to spin and generation jumping. Also, we have assigned actual physical directions to \( x, y \) and \( z \). We have not yet started on the stoichiometry.

9. Concluding remarks

In Kähler’s scheme of things, the phenomenology associated with the interactions should arise from the expression of a primordial field into a sum of members of ideals defined by idempotents. The range of phenomena classified as electroweak is very small, mainly because of the physical indefiniteness associated with the electroweak interaction (56). The idempotents should contain information speaking of how the different directions in the mathematical sense relate to actual physical direction(s). Incorporating quarks in this scheme of things is not enough. We leave for a future paper a more close implementation of the inner logic of a preferred frame scenario.
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