BFKL in forward jet production

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Abstract

In this talk I consider dijet production at large rapidity intervals in hadron collisions and forward-jet production in DIS as candidate signatures of a BFKL evolution. The state of the art on the measurements, and on the BFKL-motivated phenomenological analyses with emphasis on the different approximations involved, is reviewed.

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1 Dijet production at large rapidity intervals

1.1 The BFKL ladder

About ten years have past since Mueller and Navelet [1] have proposed to test in dijet production at hadron colliders the BFKL theory [2], modelling strong-interaction processes with two large and disparate scales, and my goal here is to describe how their original proposal has evolved. First, I shall briefly summarize what the BFKL theory is about: in the limit of center-of-mass energy much greater than the momentum transfer, $\hat{s} \gg |\hat{t}|$, any scattering process is dominated by gluon exchange in the cross channel, which occurs at $O(\alpha_s^2)$; thus, if we take parton-parton scattering as a paradigm process, the functional form of the amplitudes for gluon-gluon, gluon-quark or quark-quark scattering is the same; they differ only for the color strength in the parton-production vertices. To higher orders, we may resum the contribution of the radiative corrections to parton-parton scattering to leading logarithmic (LL) accuracy, in $\ln(\hat{s}/|\hat{t}|)$, through the BFKL equation, i.e. a two-dimensional integral equation which describes the evolution in transverse momentum space and moment space of the gluon propagator exchanged in the cross channel,

$$ \omega f_\omega(k_a, k_b) = \frac{1}{2} \delta^2(k_a - k_b) + \frac{\alpha_s N_c}{\pi^2} \int \frac{d^2 k_\perp}{k_\perp^2} K(k_a, k_b, k), $$

(1)

with $N_c = 3$ the number of colors, $k_a$ and $k_b$ the transverse momenta of the gluons at the ends of the propagator, and with kernel $K$,

$$ K(k_a, k_b, k) = f_\omega(k_a + k, k_b) - \frac{k_\perp^2}{k_\perp^2 + (k_a + k)_\perp^2} f_\omega(k_a, k_b), $$

(2)

where the first term accounts for the emission of a gluon of momentum $k$ and the second for the virtual radiative corrections. It must be noted, though, that eq. (1) has been derived in the multi-Regge kinematics, which presumes that the produced gluons, with momenta $p_a$, $p_b$ and $k$ are strongly ordered in rapidity and have comparable transverse momenta,

$$ \eta_a \gg \eta \gg \eta_b; \quad |p_a| \simeq |k_\perp| \simeq |p_b|, $$

(3)

This may be used as a diagnostic tool for discriminating between different dynamical models for parton-parton scattering. Namely, in the measurement of dijet angular distributions, models which feature gluon exchange in the cross channel, like QCD, predict a parton cross section, and therefore a dijet angular distribution, which doesn’t fall off as $\hat{s}/|\hat{t}|$ grows [3], [4], while models featuring contact-term interactions don’t have gluon exchange in the cross channel, and thus predict the dijet angular distribution to fall off as $\hat{s}/|\hat{t}|$ grows [5].
with \( p_{a \perp} = -k_{a \perp}, p_{b \perp} = k_{b \perp} \). The solution of eq. (1), transformed from moment space to \( \eta \) space, is

\[
\frac{d\omega}{2\pi i} e^{i\omega \eta} f_\omega(k_a, k_b) \approx \frac{1}{(2\pi)^2 k_{a \perp} k_{b \perp}} \sum_{n=-\infty}^{\infty} e^{in\tilde{\phi}} \int_{-\infty}^{\infty} d\nu e^{i\omega(\nu,n)\eta} \left( \frac{k_{a \perp}^2}{k_{b \perp}^2} \right)^{\nu},
\]

(4)

with \( \tilde{\phi} \) the azimuthal angle between \( k_a \) and \( k_b \), \( \eta \simeq \ln(\hat{s}/|\hat{t}|) \simeq \ln(\hat{s}/k_{\perp}^2) \) the evolution parameter of the propagator, with \( \eta \gg 1 \), and \( \omega(\nu,n) \) the eigenvalue of the BFKL equation whose maximum \( \omega(0,0) = 4\ln 2 N_c \alpha_s/\pi \) yields the known power-like growth of \( f \) in energy [2].

In inclusive dijet production in hadron-hadron collisions the resummed parton cross section for gluon-gluon scattering is [3],

\[
\frac{d\hat{\sigma}_{gg}}{dk_{a \perp}^2 dk_{b \perp}^2 d\phi} = \frac{\pi N_c^2 \alpha_s^2}{2k_{a \perp}^2 k_{b \perp}^2} f(k_a, k_b, \eta),
\]

(5)

with \( \phi \) the azimuthal angle between the tagging jets, \( \phi = \tilde{\phi} + \pi \). At the hadron level we convolute the parton cross section (5) with parton distribution functions (pdf), \( f(x, \mu_F^2) \). Thus, in the high-energy limit, and e.g. at fixed parton momentum fractions, we have

\[
\frac{d\sigma}{dx_A dx_B dk_{a \perp}^2 dk_{b \perp}^2 d\phi} = f_{eff}(x_A, \mu_F^2) f_{eff}(x_B, \mu_F^2) \frac{d\hat{\sigma}_{gg}}{dk_{a \perp}^2 dk_{b \perp}^2 d\phi}.
\]

(6)

with the effective pdf’s

\[
f_{eff}(x, \mu_F^2) = G(x, \mu_F^2) + \frac{4}{9} \sum_f \left[ Q_f(x, \mu_F^2) + \bar{Q}_f(x, \mu_F^2) \right],
\]

(7)

with the sum over the quark flavors. In order to detect evidence of a BFKL-type behavior, we’d like to see how \( f(k_a^2, k_b^2, \eta) \) grows with \( \eta \). In inclusive dijet production, \( \eta \) is the rapidity difference between the tagging jets, \( \Delta \eta = \ln(\hat{s}/k_{\perp}^2) = \eta_1 - \eta_2 \), and accordingly evidence of the BFKL dynamics is searched in dijet events at large rapidity intervals [4]. To obtain \( \Delta \eta \) as large as possible, we minimize \( k_{\perp} \), i.e. the jet transverse energy, and maximize \( \hat{s} \); since \( \hat{s} = x_A x_B s \) this may be achieved in two ways:

- by fixing the parton momentum fractions \( x \) and letting the hadron center-of-mass energy \( s \) grow, and then measuring e.g. the dijet production rate \( d\sigma/dx_A dx_B \) [4].

The theoretical advantage in this set-up is that pdf variations are minimised, while
variations in the parton dynamics, and thus in the eventual underlying BFKL behavior, are stressed. The experimental drawback is of course that one needs different colliding-beam energies, which have become just recently available with the colliding beams at Tevatron running at $\sqrt{s} = 630$ GeV, besides the usual data sample at $\sqrt{s} = 1800$ GeV.

• else, we may keep $s$ fixed and let the $x$’s grow. That is experimentally much easier to realize (and before the 630 GeV run at the Tevatron it was the only possibility), however, it is theoretically unfavourable because as the $x$’s grow the $pdf$’s fall off, so it is harder to disentangle the eventual BFKL-induced rise of the parton cross section from the $pdf$’s fall off. We may resort then to less inclusive observables: it was noted that the kinematic correlation between the tagging jets, which to leading order are supposed to be back-to-back, is diluted as the rapidity distance $\Delta \eta$ grows. This is due to the more abundant gluon radiation between the tagging jets, which blurs the information on the mutual position in transverse momentum space. Accordingly the transverse momentum inbalance \cite{7}, and the azimuthal angle decorrelation \cite{7}, have been proposed as BFKL observables.

The BFKL theory, being a LL resummation and not an exact calculation, makes a few approximations which, even though formally subleading, may be important for any phenomenological purposes. I shall list them in random order

i) The BFKL resummation is performed at fixed coupling constant, thus any variation in its scale, $\alpha_s(\nu^2) = \alpha_s(\mu^2) - b_0 \ln(\nu^2/\mu^2)\alpha_s^2(\mu^2) + \ldots$, with $b_0 = (11N_c - 2n_f)/12\pi$ and $n_f$ the number of quark flavors, would appear in the next-to-leading-logarithmic (NLL) terms, because it yields terms of $O(\alpha_s^2 \ln(\nu^2/\mu^2) \ln^{-1}(\hat{s}/|\hat{t}|))$.

ii) From the kinematics of two-parton production at $\hat{s} \gg |\hat{t}|$ we identify the rapidity interval between the tagging jets as $\Delta \eta \simeq \ln(\hat{s}/|\hat{t}|) \simeq \ln(\hat{s}/k^2_{\perp})$, however, we know from the exact kinematics that $\Delta \eta = \ln(\hat{s}/|\hat{t}| - 1)$ and $|\hat{t}| = k^2_{\perp}(1 + \exp(-\Delta \eta))$, therefore the identification of the rapidity interval $\Delta \eta$ with $\ln(\hat{s}/|\hat{t}|)$ is up to next-to-leading terms.

iii) Because of the strong rapidity ordering any two-parton invariant mass is large. Thus there are no collinear divergences in the LL resummation in the BFKL ladder; jets are determined only to leading order and accordingly have no non-trivial structure.\[\text{Modulo pathological behaviors of the BFKL ladder as } \nu^2 \to s, \text{ which seems to generate LL terms which are not in the BFKL ladder}\.]
iv) Finally, energy-momentum is not conserved, and since the momentum fraction $x$ of the incoming parton is reconstructed from the kinematic variables of the outgoing partons, the BFKL predictions may be affected by large numerical errors. In particular, if $n + 2$ partons are produced, we have

$$x_{A(B)} = \frac{p_{a\perp}}{\sqrt{s}}e^{(-)\eta_a} + \sum_{i=1}^{n} \frac{k_{i\perp}}{\sqrt{s}}e^{(-)\eta_i} + \frac{p_{b\perp}}{\sqrt{s}}e^{(-)\eta_b},$$

(8)

where the minus sign in the exponentials of the right-hand side applies to the subscript $B$ on the left-hand side. In the BFKL theory, the LL approximation and the kinematics (3) imply that in the determination of $x_A (x_B)$ only the first (last) term in eq. (8) is kept,

$$x_A^0 = \frac{p_{a\perp}}{\sqrt{s}}e^{\eta_a},$$

$$x_B^0 = \frac{p_{b\perp}}{\sqrt{s}}e^{-\eta_b}.$$

(9)

The ensuing violation of energy-momentum conservation and the neglected terms in eq.(9) are formally subleading. However, they may be important for any phenomenological purposes. Indeed, a comparison within dijet production of the three-parton production to $\mathcal{O}(\alpha_3^3)$ with the exact kinematics to the truncation of the BFKL ladder to $\mathcal{O}(\alpha_3^2)$ shows that the LL approximation may severely underestimate the exact evaluation of the $x$’s (8), and therefore entail sizable violations of energy-momentum conservation, even though the extent to which this is true depends on the specific production rate considered [10].

The phenomenological improvement of the BFKL ladder through feedback from the exact three-parton kinematics cures only partially the pathological violation of energy-momentum conservation. A systematic approach is to require energy-momentum conservation at each stage in the gluon emission in the BFKL ladder. This may be achieved through a Monte Carlo implementation of the BFKL equation (1) [11], [12]. In particular, one can define a scale $\mu$, such that $\mu \ll k_a, k_b$ and then split the real contribution to, i.e. the first term of, the BFKL kernel (2), into unresolved (for $k < \mu$) and resolved (for $k > \mu$) contributions; next, one combines the unresolved contribution, for which $f_\omega(k_a + k, k_b) \simeq f_\omega(k_a, k_b)$, with the virtual one, i.e. with the second term of eq. (2), in order to cancel the infrared singularities. One is then left over with the integral over $k > \mu$ for the resolved contribution, which is amenable to Monte Carlo resolution. Finally, one can check that the precise choice of the scale $\mu$ is immaterial [11], [12].
1.2 Dijet production at fixed parton momentum fractions

Now we go back to the first scenario outlined above, i.e. increasing the rapidity interval \( \Delta \eta \) by letting \( s \) grow at fixed \( x \)'s. We may compute the ensuing cross section \( d\sigma/dx_A dx_B \) through the BFKL resummation from eq. (6) by integrating over the jet transverse energies in various stages of approximation, and consider the ratio \( R \) of dijet production at \( \sqrt{s_1} = 1800 \) GeV and \( \sqrt{s_2} = 630 \) GeV,

\[
R(x_A, x_B; s_1, s_2) = \frac{d\sigma(s_1)/dx_A dx_B}{d\sigma(s_2)/dx_A dx_B},
\]

(10)

for values of \( x_{A,B} \), and therefore of \( \Delta \eta \), large enough that BFKL is a sensible approximation to consider. Thus, we choose \( x_A = x_B = 0.1 \) for which \( \Delta \eta(s_1) \approx 4.5 \) and \( \Delta \eta(s_2) \approx 2.3 \), and \( x_A = x_B = 0.2 \) for which \( \Delta \eta(s_1) \approx 5.9 \) and \( \Delta \eta(s_2) \approx 3.7 \).

\[ \text{a) The simplest, and least accurate, approximation consists in fixing the renormalization scale of } \alpha_s, \text{ the factorization scale of the pdf's as well as the momentum transfer at the jet minimum transverse energy, } \mu_R^2 = \mu_F^2 = |\hat{t}| = k_{\perp \text{min}}^2, \text{ and in determining the parton momentum fractions through eq. (9). Then the pdf's factor out of the integrals over transverse momentum which may be easily performed analytically. Thus the cross section (6) becomes} \]

\[
\frac{d\sigma}{dx_A^0 dx_B^0} = f_{\text{eff}}(x_A^0, k_{\perp \text{min}}^2) f_{\text{eff}}(x_B^0, k_{\perp \text{min}}^2) \frac{\pi N_c^2 \alpha_s^2}{2k_{\perp \text{min}}^2} f(\Delta \eta),
\]

(11)

with

\[
f(\Delta \eta) = \int_{-\infty}^{\infty} d\nu \frac{e^{\nu(\nu,n=0)\Delta \eta}}{\nu^2 + \frac{1}{4}}
\]

\[
\simeq \frac{e^{4\ln 2N_c\alpha_s\Delta \eta/\pi}}{\sqrt{7\zeta(3)N_c\alpha_s\Delta \eta/2}},
\]

(12)

with \( \zeta(3) = 1.202... \) and where in the second line we have performed a saddle-point evaluation of the integral over \( \nu \). Accordingly, we get \( R(x_A = x_B = 0.1; s_1, s_2) = 1.66 \) and \( R(x_A = x_B = 0.2; s_1, s_2) = 1.84 \).

\[ \text{b) Else, we may fix the renormalization/factorization scales and the momentum transfer in terms of the jet transverse energies, e.g. we may choose } \mu_R^2 = \mu_F^2 = |\hat{t}| = k_{a\perp} k_{b\perp} \text{ and then integrate eq. (6) numerically. For the sake of later comparison with the Monte Carlo results, it is convenient to consider the averaged} \]

\[ \text{Note then that } \alpha_s \text{ runs as a function of } k_{a\perp} \text{ and } k_{b\perp} \text{ in eq. (6), however, it is still fixed within the BFKL ladder (6), i.e. it is not a function of the transverse energies of the gluons emitted along the ladder.} \]
ratio $\bar{R}$, obtained by integrating numerator and denominator of the right-hand side of eq. (10) over a range of $x_{A,B}$, e.g. over $0.1 \leq x_{A,B} \leq 0.2$. One then finds $\bar{R}(0.1 \leq x_A, x_B \leq 0.2; s_1, s_2) \simeq 1.5$. As expected, the ratio is somewhat smaller than in approximation $a)$ since in that instance the choice $\mu_R^2 = \mu_F^2 = |\hat{t}| = k_{\text{min}}^2$ overestimates the pdf’s, the coupling constant and the size of the rapidity interval.

c) Finally, the averaged ratio $\bar{R}$ may be computed using a Monte Carlo evaluation of eq. (10), with energy-momentum conservation in the gluon emission along the BFKL ladder [11], [12]. It is possible to achieve that through a two-stage process, namely requiring energy-momentum conservation only on the kinematic part of the cross section (6), while still using the approximate parton momentum fractions (7) in determining $\hat{s}$ in the squared amplitudes, or else using the exact parton momentum fractions (8) everywhere in the cross section (6) [10]. The results of both these methods, though, agree with one another and with the one of approximation $b)$, within the statistical error, i.e. $\bar{R}(0.1 \leq x_A, x_B \leq 0.2; s_1, s_2) \simeq 1.5$ [13]. That entails apparently that energy-momentum conservation is not so relevant for the averaged ratio $\bar{R}$, however this conclusion is misleading since the absolute cross section $d\sigma/dx_A dx_B$ evaluated through the BFKL ladder increases with respect to the $\mathcal{O}(\alpha_s^2)$ i.e. the lowest order calculation within approximation $b$), while it decreases with respect to the $\mathcal{O}(\alpha_s^2)$ calculation in the Monte Carlo evaluation [13]. In addition, the averaged ratio is different in the approximation $b$) and in the Monte Carlo evaluation if other values of $x_A, x_B$ and/or of $s_1, s_2$ (like e.g. $\sqrt{s_1} = 14$ TeV and $\sqrt{s_2} = 1800$ GeV) are used [13]. Thus the insensitivity of $\bar{R}(0.1 \leq x_A, x_B \leq 0.2; s_1, s_2)$ to the implementation of energy-momentum conservation in the BFKL ladder seems to be an accident of the values of $x_A, x_B; s_1, s_2$ chosen above.

It remains to be seen what the result is for the ratio (10) if a standard Monte Carlo event generator, like HERWIG [14], that includes Altarelli-Parisi-evolved parton showers and coherence, but no BFKL evolution, is used.$^d$

$^d$The Monte Carlo event generator of ref. [12] includes also the option of running $\alpha_s$ within the BFKL ladder.

$^c$Before any comparison is made, it should be stressed that none of the BFKL Monte Carlo event generators and calculations described above includes hadronization effects.
1.3 Dijet production at fixed hadron energy

Let us consider now the second scenario outlined above, namely increasing the rapidity interval $\Delta \eta$ by letting $\hat{s}$ grow at fixed $s$. That entails an increase in the $x$'s, thus it is more convenient to measure the inclusive dijet rate at fixed rapidities $d\sigma/d\eta_1 d\eta_2$ (or equivalently, at fixed $\Delta \eta$ and rapidity boost $\bar{\eta} = (\eta_1 + \eta_2)/2$). The ensuing factorization formula in the high-energy limit is

$$
\frac{d\sigma}{d\Delta \eta d\bar{\eta} k_{a\perp}^2 d k_{b\perp}^2 d\phi} = x_A f_{a\perp}(x_A, \mu_F^2) x_B f_{b\perp}(x_B, \mu_F^2) \frac{d\hat{\sigma}_{gg}}{d k_{a\perp}^2 d k_{b\perp}^2 d\phi}.
$$

(13)

Since the $x$'s grow linearly with the transverse momenta (cf. eq. (8-9)), when integrating over the latter, the BFKL-induced growth of the dijet rate is upset by the falling parton luminosities. This is even more noticeable for the dijet rate calculated with the BFKL ladder than for the rate computed to lowest order, i.e. $\mathcal{O}(\alpha_s^2)$, because the additional gluon radiation generated by the BFKL ladder makes the dijet rate to run out of phase space more rapidly than the one computed to lowest order \[7\], \[8\], \[15\]. Accordingly, it was proposed to measure the transverse momentum imbalance \[7\], and the decorrelation in the azimuthal angle $\phi$ between the tagging jets \[7\], \[8\], as signatures of an eventual BFKL evolution. The azimuthal angle decorrelation turns out to be easier to measure \[7\], so we shall concentrate on that. The inclusive distribution $d\sigma/d\phi$ is centered around the peak $\phi = \pi$ since to lowest order kinematics require the jets to be back-to-back, $d\sigma/d\phi \sim \delta(\phi - \pi)$; the additional gluon radiation, induced by parton showers and hadronization, smears the $\delta$ function into a bell curve peaked at $\phi = \pi$. However, if we look at the distribution also as a function of the rapidity distance between the jets, $d\sigma/d\Delta \eta d\phi$, we expect that the larger $\Delta \eta$ the larger the smearing of the distribution \[7\], \[8\]. The reason is that the BFKL-induced gluon radiation, which is roughly constant per unit of rapidity, is so more abundant for a larger $\Delta \eta$, and accordingly the information on the mutual position of the jets in transverse momentum space is more diluted as $\Delta \eta$ grows. This has been experimentally confirmed \[7\]. From the calculational point of view, it is easier to evaluate the moments of $\phi$ \[8\],

$$
< \cos n(\phi - \pi) > = \frac{\int_0^{2\pi} d\phi \cos n(\phi - \pi) (d\sigma/d\Delta \eta d\phi)}{\int_0^{2\pi} d\phi (d\sigma/d\Delta \eta d\phi)}.
$$

(14)

For a $\delta$-function distribution at $\phi = \pi$, as occurs at the lowest order, all of the moments will equal one, while for a flat distribution all of the moments will equal zero for $n \geq 1$. Thus, the decay of the moments from unity is a good measure of the decorrelation in $\phi$.  

7
The decorrelation of the first moment has been measured for values of the rapidity interval up to $\Delta \eta = 5$ and for transverse momenta of the jets $k_{a\perp} = 50$ GeV and $k_{b\perp} = 20$ GeV \cite{6}, and preliminary results are available from a larger data sample for rapidity intervals up to $\Delta \eta = 6$ and for a symmetric configuration $k_{a\perp} = k_{b\perp} = 20$ GeV \cite{16}.

The evaluation of $<\cos(\phi - \pi)>$ using the BFKL ladder without energy-momentum conservation (approximation $b$ of sect. 1.2) yields too much decorrelation between the jets \cite{8}, \cite{17}, as compared to the data \cite{6}. This is mainly due to the parton momentum fractions \cite{9} sizeably underestimating the exact kinematics (8), and thus violating substantially energy-momentum conservation. As hinted in item iv) of sect. 1.1, a phenomenological improvement may be achieved by noting that since the identification $\Delta \eta \simeq \ln(\hat{s}/|\hat{t}|)$ holds up to next-to-leading terms, and the difference between the exact (8) and the approximate (9) kinematics resides also in the next-to-leading terms, we may use the three-parton production with the exact kinematics, i.e. eq. (8) with $n = 1$, to define an effective rapidity interval $\Delta \hat{\eta}$, such that if we replace $\Delta \eta \rightarrow \Delta \hat{\eta}$ in the BFKL ladder and truncate it to $O(\alpha_s^3)$ we reproduce the exact three-parton contribution to dijet production \cite{10}. Using the effective rapidity interval $\Delta \hat{\eta}$ in the BFKL ladder \cite{17} improves sizeably the agreement with the data \cite{6}, even though the BFKL ladder shows still too much decorrelation. Eventually the best solution is to require energy-momentum conservation for the emission of each gluon along the BFKL ladder, through a Monte Carlo evaluation of eq. (1) \cite{11}, \cite{12}. Accordingly the first moment $<\cos(\phi - \pi)>$ has been evaluated \cite{12} using the kinematic cuts of the more recent D0 analysis \cite{16}. The BFKL Monte Carlo generator still seems to yield too much decorrelation as compared to the data \cite{16}, even though it is possible to notice in the Monte-Carlo-generated curve the workings of energy-momentum conservation as the boundary of phase space is approached, i.e. for the largest values of $\Delta \eta$ kinematically attainable. However, it is premature to draw conclusions since the D0 data are preliminary and, as noted at the end of sect. 1.2, the BFKL Monte Carlo generators do not include hadronization effects.

It must also be stressed that the HERWIG Monte Carlo generator \cite{14} is in perfect agreement with the data for the first moment $<\cos(\phi - \pi)>$, which entails that the azimuthal angle decorrelation is not exclusive to the gluon radiation induced by the BFKL ladder, but can be found also, and with better agreement, in standard patterns of gluon radiation.
1.4 Conclusions for dijet production

The interplay between data and theory has allowed us to improve considerably the phenomenological predictions based on the BFKL ladder, however, we do not see yet, in the context of dijet production at the Tevatron, a clear signal of BFKL behavior. On one hand, the several approximations listed in sect. 1.1 and embodied in the LL resummation, on which the BFKL ladder is based, seem somewhat unreliable for the kinematic range available in dijet production at the Tevatron; to this effect a full NLL calculation, which is not available yet [18], should achieve a considerable improvement. On the other hand, the comparison with experiment has taught us that it may be not so easy to disentangle the effects of BFKL-induced gluon radiation from the ones of the more standard and well understood Altarelli-Parisi gluon radiation, based on collinear emission. In this search other eventual BFKL observables, like the transverse energy in the central rapidity region between the tagging jets [11], or the single particle $k_{\perp}$ spectra [19], as well as a deeper analysis of the existing ones, e.g. exploring moments higher than the first in the azimuthal angle decorrelation, should be sought after.

2 Forward jet production in DIS

2.1 Fixed-order calculations

A variant of dijet production at hadron colliders is the production in DIS of a jet close in rapidity to the proton remnants, and with transverse momentum comparable to the virtuality of the photon $k_{\perp}^2 \simeq Q^2$, in order to fulfil as closely as possible the constraints of the multi-Regge kinematics [8] [20], [21]. In this case the resummation parameter $\eta$ of the BFKL ladder [8] is $\eta = \ln(\hat{s}_{\gamma p}/k_{\perp}^2) \simeq \ln(\hat{s}_{\gamma p}/Q^2) \simeq \ln(x/x_{bj})$, with $\hat{s}_{\gamma p}$ the center-of-mass energy between the photon and the struck parton, $x_{bj} = Q^2/s$ the Bjorken variable ($s$ is the lepton-proton energy) and $x$ the momentum fraction of the struck parton. In order to have $\eta \gg 1$, we require that $x \gg x_{bj}$.

To clarify the picture of forward jet production in DIS, let us start from the parton model, where a quark is struck by the virtual photon. In this case the kinematics costrain $x$ to be $x = x_{bj}$, and a jet may form in the photon direction, usually termed the current fragmentation region. However, since we have required $x \gg x_{bj}$, there is no contribution to forward jet production in DIS from the parton model (just like for the structure function $F_L$).
The leading-order (LO) contribution to forward jet production comes to $\mathcal{O}(\alpha_s)$ from tree-level production of two partons. Then it is possible to compute either LO two-jet production, one of which required to be forward, or NLO forward-jet production; however requiring the jet to be forward makes the latter effectively LO since the one-loop one-parton production, which has $x = x_{bj}$, is kinematically forbidden. Therefore we expect that to $\mathcal{O}(\alpha_s)$ the two-jet (one forward) rate and the forward-jet rate are of the same size, since in the latter we are simply integrating over the kinematic variables of the extra jet, produced in the current fragmentation region.

To $\mathcal{O}(\alpha_s^2)$ we may produce three final-state partons at tree level, with the novel feature of gluon exchange in the cross channel, which in the high-energy limit $\hat{s} \gg \hat{t}$ dominates the production rate; thus the LO three-jet (one forward) rate turns out to be of the same size as the LO two-jet rate, even though it is one order higher in $\alpha_s$, and therefore in general expected to be an order of magnitude smaller \[22\]. Accordingly, the $\mathcal{O}(\alpha_s^2)$ NLO two-jet (one forward) rate, which is computed from tree-level three-parton and one-loop two-parton final states, turns out to be sizeably bigger than the $\mathcal{O}(\alpha_s)$ LO two-jet rate \[22\], because the latter does not have gluon exchange in the cross channel. Finally, since two-loop one-parton production, which has $x = x_{bj}$, is kinematically forbidden, it is also feasible to compute the NNLO forward-jet rate, because requiring the jet to be forward makes the rate effectively NLO; analogously to the $\mathcal{O}(\alpha_s)$ case we expect it to be of the same size as NLO two-jet production. In addition, since the forward-jet rates above are dominated by gluon exchange in the cross channel, which to $\mathcal{O}(\alpha_s^2)$ occurs in tree-level diagrams, the dependence on the factorization/renormalization scales is sizeable \[22\], even though the rates have been computed at NLO.

Since no new kinematic features appear to $\mathcal{O}(\alpha_s^3)$, we expect that the LO four-jet (one forward) rate should be markedly smaller than the LO three-jet one, while NLO and LO three-jet rates should be of the same order of magnitude, however, in the $\mathcal{O}(\alpha_s^3)$ calculation the dependence on the factorization/renormalization scales should be sensibly reduced.

### 2.2 The BFKL ladder

The BFKL ladder builds up the radiative corrections to gluon exchange in the cross channel, thus it appears to $\mathcal{O}(\alpha_s^2)$ in forward-jet production. The corresponding lepton-parton cross section is obtained by convoluting in $k_\perp$ space the BFKL ladder \[1\] with the
coefficient function $\gamma^* g^* \to q\bar{q}$, with off-shell gluon and photon,

$$
\frac{d\hat{\sigma}}{dx_{bj}dQ^2dk_{\perp}^2d\phi} = \sum_q e_q^2 \frac{N_c\alpha^2\alpha_s^2}{\pi^2(Q^2)^2k_{\perp}^2x_{bj}} \int \frac{dv_{\perp}^2}{v_{\perp}^2} f(v_{\perp}, k_{\perp}, \eta) F(v_{\perp}, Q^2, y),
$$

(15)

with $y = Q^2/x_{bj}s$ the electron energy loss, $F$ the off-shell coefficient function, $k_{\perp}$ and $v_{\perp}$ respectively the transverse momenta of the forward jet and of the gluon attaching to the $q\bar{q}$ pair, and with the sum over the quark flavors in the $q\bar{q}$ pair. After substituting the off-shell coefficient function $F$, the integral over $v_{\perp}$ on the right-hand side of eq. (15) may be performed, yielding \cite{23},

$$
\int \frac{dv_{\perp}^2}{v_{\perp}^2} f(v_{\perp}, k_{\perp}, \eta) F(v_{\perp}, Q^2, y)
= \frac{\pi}{8} \left( \frac{Q^2}{k_{\perp}^2} \right)^{1/2} \int_0^{\infty} d\nu \cos \left( \nu \ln \frac{Q^2}{k_{\perp}^2} \right) \frac{\sinh(\pi\nu)}{\cosh^2(\pi\nu)} \frac{1}{\nu(1 + \nu^2)}
\times \left( e^{\omega(\nu, 0)\eta} \left[ \frac{1}{2} \left( \nu^2 + \frac{9}{4} \right) \left[ 1 + (1 - y)^2 \right] + 2 \left( \nu^2 + \frac{1}{4} \right) (1 - y) \right]
- e^{\omega(\nu, 2)\eta} \cos(2\phi) \left( \nu^2 + \frac{1}{4} \right) (1 - y) \right).
$$

(16)

In the high-energy limit, and at fixed parton momentum fraction, the forward-jet rate is,

$$
\frac{d\sigma}{dx_{bj}dQ^2dxdk_{\perp}^2d\phi} = f_{efj}(x, \mu_F^2) \frac{d\hat{\sigma}}{dx_{bj}dQ^2dk_{\perp}^2d\phi},
$$

(17)

with the parton cross section \cite{13}. By producing the jet forward, we make $x$ large; $\eta$ is then made larger and larger by making $x_{bj}$ smaller and smaller. Thus the advantage of forward-jet production in DIS is that a fixed-energy $ep$ collider is nonetheless a variable-energy collider in the photon-proton frame \cite{20}, and we may realize the first scenario of sect. 1.1 (fixed $x$, variable $\hat{s}$) with a large range, in principle a continuum, of energies, instead of just two.

The H1 Collaboration at HERA \cite{24} has measured the forward-jet rate with $x > 0.025$, $2 \cdot 10^{-4} < x_{bj} < 2 \cdot 10^{-3}$ (thus with typical values of $2.5 < \eta < 4.8$), $5\text{GeV}^2 < Q^2 < 100\text{GeV}^2$, $k_{\perp} > 5 \text{GeV}$ and $0.5 < k_{\perp}^2/Q^2 < 4$. The $O(\alpha_s^3)$ forward-jet rate mentioned in sect. 2.1, based on the NLO Monte Carlo program MEPJET \cite{25}, is substantially larger than the $O(\alpha_s^2)$ one, as expected, however, it falls below the data by a factor $4 \div 5$ \cite{22}, which hints that the radiative corrections beyond $O(\alpha_s^2)$ are important. A calculation of the forward-jet rate through the BFKL ladder, performed like in approximation b) of sect. 1.2, i.e. by fixing $\mu_R^2 = \mu_F^2 = k_{\perp}^2$ and integrating eq. (17) numerically, shows a
good agreement with the data \cite{23}. On the other hand, the forward-jet rate computed by truncating the BFKL ladder to its lowest order, i.e. to $\mathcal{O}(\alpha_s^2)$ \cite{23}, is in good agreement with the exact $\mathcal{O}(\alpha_s^2)$ forward-jet rate \cite{22}, confirming that in the high-energy limit the exact $\mathcal{O}(\alpha_s^2)$ calculation is dominated by gluon exchange in the cross channel, which forms the lowest-order contribution to the BFKL ladder.

A more recent and larger data sample from the H1 Collaboration \cite{26}, with $x > 0.035, 5 \cdot 10^{-4} < x_{bj} < 3.5 \cdot 10^{-3}$ (thus with typical values of $2.3 < \eta < 4.2$), $k_\perp > 3.5$ GeV and $0.5 < k_\perp^2/Q^2 < 2$, basically confirms the earlier findings \cite{24}, namely the forward-jet rate computed through the NLO Monte Carlo program DISENT \cite{27} falls well below the data; while the rate based on the BFKL calculation \cite{23} fares better but tends to overshoot the data, at the lower values of $x_{bj}$. However, a caveat is in order: DISENT and the BFKL calculation do not include hadronization, while a simulation through the LEPTO Monte Carlo event generator \cite{28}, based on Altarelli-Parisi-evolved parton showers, shows that hadronization effects are important. It is also encouraging that the ARIADNE Monte Carlo generator \cite{29}, which does not have the typical Altarelli-Parisi-induced $k_\perp$ ordering but rather privileges comparable transverse momenta in accordance with the multi-Regge kinematics \cite{8}, is very close to the data.

Analogously, the ZEUS Collaboration has measured the forward-jet rate \cite{30}, with $x > 0.035, 5 \cdot 10^{-4} < x_{bj} < 5 \cdot 10^{-3}$ (thus with typical values of $1.9 < \eta < 4.2$), $k_\perp > 5$ GeV and $0.5 < k_\perp^2/Q^2 < 4$, and the same considerations done for ref. \cite{26} about the comparison with the different theoretical or phenomenological models apply here.

### 2.3 Conclusions for forward-jet production

The analyses of the H1 and ZEUS Collaborations at HERA seem to indicate that forward-jet production is a good candidate as a BFKL observable. In order to improve the agreement with the data, and enforce energy-momentum conservation, a BFKL Monte Carlo generator \cite{11}, \cite{12} should be used. However, some experimental issues like the hadronization effects mentioned above or the excess in the dijet rate as a function of $x_{bj}$ and $Q^2$ \cite{26}, \cite{30}, \cite{31}, which might imply a contamination from resolved-photon production, particularly at low $Q^2$, should be understood before firm conclusions on forward-jet production may be reached.

The study of other BFKL observables in DIS, like the single particle $k_\perp$ spectra \cite{13} mentioned above, or, in analogy to the analysis of sect. \cite{13}, the decorrelation of the azimuthal angle between the lepton and the jet, which shows a peculiar shift of the
distribution maximum from $\phi = \pi$ at large $x_{bj}$ to $\phi = \pi/2$ at small $x_{bj}$ [2], [32], [33], should also be pursued.

For the future, the exchange of a BFKL ladder might also be studied in the same theoretical framework as originally suggested in ref. [2], namely in virtual photon-photon scattering in $e^+e^-$ collisions [34]. This would provide a particularly clean environment, benefitting from the absence of QCD initial-state radiation.

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