Dzyaloshinskii-Moriya interaction induced extrinsic linewidth broadening of ferromagnetic resonance

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For a thin ferromagnetic film with the Dzyaloshinskii-Moriya interaction (DMI), we derive an expression of the extrinsic ferromagnetic resonance (FMR) linewidth in a quantum mechanical way, taking into account scatterings from structural inhomogeneity. In the presence of the DMI, the magnon dispersion exhibits rich resonant states, especially in small external magnetic fields and strong DMI strength. It is found that the FMR linewidth shows several characteristic features such as a finite linewidth at zero frequency and peaks in the low frequency range.

I. INTRODUCTION

Magnetic damping, which is parameterized by the Gilbert damping constant $\alpha$, describes the energy dissipation rate of magnetization dynamics and determines the performance of magnetic devices including magnetic random access memories and magnetic sensors. The ferromagnetic resonance (FMR) provides a way to measure the damping $\alpha$ from the FMR linewidth. A widely adopted method for the estimation of $\alpha$ is to measure the frequency-dependent change in the linewidth, assuming that the only intrinsic damping contribution is proportional to the frequency. However, this method is not always straightforward as the measured linewidth includes not only intrinsic damping contribution but also extrinsic contributions originating from inhomogeneity of the sample. Especially, the contribution of two-magnon scattering to the linewidth also depends on the frequency, demanding a detailed understanding of the frequency-dependent linewidth of two-magnon scattering.

Recently, there has been much interest in the spin-orbit coupling effect for a thin ferromagnetic layer in contact with a heavy metal layer because it permits the control and manipulation of electronic spin degree of freedom. In the presence of spin-orbit interaction combined with inversion symmetry breaking, the antisymmetric exchange interaction, known as Dzyaloshinskii-Moriya interaction (DMI), emerges in addition to the symmetric exchange interaction responsible for ferromagnetism. The DMI affects equilibrium spin textures by stabilizing chiral orders such as spin spirals, chiral ferromagnetism. The DMI affects equilibrium spin textures by stabilizing chiral orders such as spin spirals, chiral ferromagnetism.

We also make use of the $\hat{x}_3$ coordinate system. As shown in Fig. 1, the $\hat{x}_3$ axis is coincided with the equilibrium magnetization direction while the $\hat{x}_2$ axis is directed to the negative $x$-axis. Thus, $H_1(k) = \cos \theta_M H_{ext}^y(k)$ and $H_2(k) = -H_{ext}^x(k)$.

To be specific, the Hamiltonian $H_0$ consists of the demagnetization, exchange, anisotropy, and Zeeman energies: $H_0 = H_{demag} + H_{ex} + H_A + H_Z$. In a second-quantized form, $H_0$ is summarized as

$$H_0 = \sum_k \left[ \sum_{\kappa = 1, 2} m_\kappa(-k)\epsilon_{\kappa\kappa'}(k) m_{\kappa'}(k) - \sqrt{2\hbar\gamma M_S} \sum_{\kappa = 1, 2} m_\kappa(k)\mu_0 H_\kappa(-k) \right],$$

where $\gamma$ is the gyromagnetic ratio, $M_S$ is the saturation magnetization, and $\mu_0$ is the permeability of free space.
where
\[\epsilon_{11} = \epsilon_{11}^0 + \epsilon_M (\sin^2 \theta_M \cos^2 \phi_k - \cos 2\theta_M) + \epsilon_{ext} k^2 d^2,\]
\[\epsilon_{22} = \epsilon_{22}^0 + \epsilon_M (\sin^2 \phi_k + \sin^2 \theta_M) + \epsilon_{ext} k^2 d^2,\]
\[\epsilon_{12} = \epsilon_M \sin \phi_k \cos \phi_k \sin \theta_M + i \epsilon_{DM} k d \sin \phi_k \cos \theta_M,\]
\[\epsilon_{21} = \epsilon_{12}^*,\]
\[(2)\]

Here, we define \(\epsilon_M = \hbar \gamma \mu_0 M_S (1 - N_d)\) with the saturation magnetization \(M_S\) and the gyromagnetic ratio \(\gamma\), \(\epsilon_{ext} = 2 \hbar \gamma A / M_S d^2\) with the exchange stiffness constant \(A\), and \(\epsilon_{DM} = 4 \hbar \gamma D / M_S d^2\) with the DMI strength \(D\). \(\epsilon_{11}^0\) and \(\epsilon_{22}^0\) are \(k\)-independent terms and originate from both Zeeman and surface anisotropy energies with an anisotropy field \(H_S\);

\[\epsilon_{11}^0 = \hbar \gamma \mu_0 \left[ H_3 + (M_S + H_S) \cos 2\theta_M \right],\]
\[\epsilon_{22}^0 = \hbar \gamma \mu_0 \left[ H_3 - (M_S + H_S) \sin^2 \theta_M \right],\]
\[(3)\]

where \(H_3 = H_\text{ext} \cos(\theta_H - \theta_M)\) is the \(\hat{x}_3\)-component of magnetic field \(H_\text{ext}\). By requiring the \(M_x = 0\) in equilibrium, the magnetization angle \(\theta_M\) is determined by \(2H_\text{ext} \sin(\theta_H - \theta_M) = (H_S + M_S) \sin 2\theta_M\) for a given magnetic field.

We express transverse components of the magnetization, \(M_{1,2} = \sqrt{2\hbar \gamma M_S} m_{1,2}\) with bosonic annihilation and creation operators approximated from the Holstein-Primakoff setup\(\text{^2}\):

\[m_1(k) = \frac{1}{2} \left( c_k + c_k^* \right),\]
\[m_2(k) = \frac{1}{2i} \left( c_k - c_k^* \right),\]
\[(5)\]

satisfying usual bosonic commutation relations of \([c_k, c_k] = 0, [c_k^*, c_k^*] = 0,\) and \([c_k, c_k^*] = \delta_{k,k'}\).

Eigenenergy of magnon modes can be obtained by the Bogoliubov transformation of the Hamiltonian \(\mathcal{H}_0\). In terms of \(\epsilon_{nk}\), the eigenenergy at a \(k\) point is given by,

\[\epsilon(k) = \text{Im}[\epsilon_{12}(k)] + \sqrt{\epsilon_{11}(k) \epsilon_{22}(k)} - \text{Re}[\epsilon_{12}(k)]^2.\]
\[(7)\]

For two-magnon scattering of FMR experiment, we focus on the energy \(\epsilon(k = 0) = \sqrt{\epsilon_{11}^0 \epsilon_{22}^0}\) and its resonant states, namely a set of states at \(k \neq 0\) with the same energy. We plot the dispersion of the system at various DMI strengths in the upper panel of Fig. 2. In the figure, we display those resonant states with solid circles, satisfying the relation of \(\epsilon(k) = \sqrt{\epsilon_{11}^0 \epsilon_{22}^0}\). We find that the calculated dispersion is different from the DMI-free case mainly in two points; the resonant state with the largest \(|k|\) occurs around the angle \(\phi_k = -\pi/2\) (in the case of \(D < 0, \phi_k = \pi/2\)) and moreover, its value of \(kd\) is not small, but can be comparable to one. In the absence of DMI, resonant states are determined from the competition between exchange and demagnetization energies. In this case, the resonant state with the largest \(k\) occurs at \(\phi_k = 0\). In the presence of DMI, on the other hand, an additional decreasing energy is incorporated in the region of \(k \sin \phi_k < 0\) through \(\epsilon_{12}(k)\) of Eq. 2. Consequently, as \(k\) increases, this energy competes with an increasing part (i.e., the exchange energy) and eventually, the largest-\(k\) resonant state is formed around \(\phi_k = -\pi/2\).

In order to show the tendency clearly, we plot the evolution of resonant states in the lower panel of Fig. 2 for several DMI strengths. In a DMI-free case \((D = 0)\), one finds two lobes having the largest distances at \(\phi_k = 0\) and \(\pi\), respectively. As we increase DMI strength, the two lobes become one at a certain DMI energy (say \(\epsilon_{DM}^0\))
by creating a resonant state at \( \phi_k = -\pi/2 \). Eventually, for a sufficiently strong DMI, the resonant state with the largest \( k \) appears at \( \phi_k = -\pi/2 \).

According to the two-magnon theory, inhomogeneities allow a magnon mode at \( k = 0 \) to scatter into modes with finite wave vectors and couple these modes with different wave vectors. To describe this coupling, we add a perturbation term into the Hamiltonian as, \( \mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{pertub}}; \)

\[
\mathcal{H}_{\text{pertub}} = \sum_{k,k',\kappa,\kappa'} m_{\kappa}(k) U_{\kappa\kappa'}^{k,k'} m_{\kappa'}(k').
\]

In general, the potential \( U_{\kappa\kappa'}^{k,k'} \) is not diagonal in the \( k \)-space and thus causes a coupling between different \( k \)-states. The perturbation potential from inhomogeneities is zero in average, but it provides appropriate momentum and energy for magnon energy levels to be shifted and broaden.

In this work, we focus on DMI-induced perturbations. Inferring from the imaginary part of \( \epsilon_{12}(k) \) in Eq. (2) relevant to the DMI, we consider two feasible mechanisms as perturbation sources. One is a random spatial variation of the parameter \( D \) while the other is an abrupt change of the magnetization vectors, for example, at edges of terrace on sample surfaces, near magnetic and non-magnetic impurities, etc. If structural defects whose characteristic size is larger or equal to the film thickness \( d \), the magnetization will be slowly varying and the former is dominant for the fluctuation, whereas, in a scale much less than \( d \), the latter is dominant. In both cases, the fluctuation potential is modeled as,

\[
U_{11}^{k,k'} = U_{22}^{k,k'} = 0,
\]

\[
U_{12}^{k,k'} = -U_{21}^{k,k'} = \sum_j e^{-i(k-k')\cdot R_j} U^a(k,k'),
\]

where \( R_j \) is a random position vector of impurity (localized inhomogeneity). By defining \( \lambda_c \) as a characteristic length scale of the impurity, we model its localized potential with Fourier components of (see Appendix A for details),

\[
U^a(k,k') = \epsilon_{\text{DMI}} f_R(k-k') \cos \theta_M \mathcal{W}(k,k'),
\]

where \( \mathcal{W}(k,k') = i(k_x' + k_x) d/2 \) for \( \lambda_c \gg d \), whereas \( \mathcal{W} = d/\lambda_c \) for \( \lambda_c \ll d \). Here, \( f_R(k) \) is a form factor of the localized potential; for example, \( f_R(k) = e^{-k^2 \lambda_{\text{imp}}^2 \pi^2/2} \) if we assume Gaussian shapes with an impurity size \( \lambda_{\text{imp}} \) distributed in the \((L \times L \times d)\)-sized ferromagnetic film.

III. SUSCEPTIBILITY

We shall calculate FMR linewidth by examining the susceptibility of the system. According to equation of motion for \( m_{1,2}(k,t) \), the susceptibility for an external field with wave-number \( k' \) and time-varying \( e^{i\omega t} \) is given by,

\[
\left( \begin{array}{cc} \chi_{11} & \chi_{12} \\ \chi_{21} & \chi_{22} \end{array} \right) (k,k';\omega) = [G^{-1} \delta_{k,k'} - \mathcal{V}]^{-1},
\]

where

\[
\mathcal{G}(k;\omega) = h \gamma M_S \left( \frac{-i\hbar \omega + \epsilon_{21}(k)}{\epsilon_{11}(k)} \right)^{-1},
\]

\[
\mathcal{V}(k,k') = -\frac{1}{h \gamma M_S} \left( \begin{array}{cc} V_{21}(k,k') & V_{22}(k,k') \\ V_{11}(k,k') & V_{12}(k,k') \end{array} \right),
\]

with \( V_{\kappa\kappa'}(k,k') = (U_{\kappa\kappa'}^{k,k'} + U_{\kappa\kappa'}^{k,-k'})/2 \).

A. Average over impurities

In order to average the susceptibility over an ensemble of impurity, we expand the susceptibility with perturbation potential \( \mathcal{V}; \)

\[
\left( \begin{array}{cc} \chi_{11} & \chi_{12} \\ \chi_{21} & \chi_{22} \end{array} \right) = \mathcal{G} + \mathcal{G} \langle \mathcal{V} \rangle_{\text{imp}} \mathcal{G} + \mathcal{G} \langle \mathcal{V} \mathcal{G} \mathcal{V} \rangle_{\text{imp}} \mathcal{G} + \cdots
\]

\[
= [G^{-1} \delta_{k,k'} - C]^{-1}.
\]

where \( \langle \cdots \rangle_{\text{imp}} \) denotes an average over impurity and usually, \( \mathcal{V}_{\text{imp}} \) is 0 due to the randomness of \( \mathcal{V} \). We approximate a self-energy as \( \mathcal{C} = \langle \mathcal{V} \mathcal{G} \mathcal{V} \rangle_{\text{imp}} \) by neglecting crossing terms. Because the fluctuating potential is contributed from randomly located sites at \( \{ R_j \} \) and thus \( V_{\kappa\kappa'}(k,k') = \sum_j V^a_{\kappa\kappa'}(k,k') e^{-i (k-k') \cdot R_j} \), we arrive at the self-energy of

\[
C(k,k';\omega) = \delta_{k,k'} \frac{1}{h \gamma M_S} \left( \begin{array}{cc} \Sigma_{21} & \Sigma_{22} \\ \Sigma_{11} & \Sigma_{12} \end{array} \right)(k;\omega)
\]

where

\[
\left( \begin{array}{cc} \Sigma_{21} & \Sigma_{22} \\ \Sigma_{11} & \Sigma_{12} \end{array} \right)(k;\omega) = \frac{N_{\text{imp}}}{h \gamma M_S} \sum_k \left( \begin{array}{c} V_{21}^{a_k} \\ V_{11}^{a_k} \end{array} \right),
\]

\[
\mathcal{G}(k_1;\omega) \left( \begin{array}{c} V_{21}^{a_k} \\ V_{11}^{a_k} \end{array} \right)(k_1, k)
\]

with the number of impurities \( N_{\text{imp}} \). We note that the self-energy of \( C(k,k';\omega) \) is now diagonal over the \( k \)-space, meaning that a translation symmetry is recovered through the impurity average. Finally, the susceptibility is given as,

\[
\left( \begin{array}{cc} \chi_{11} & \chi_{12} \\ \chi_{21} & \chi_{22} \end{array} \right)(k,k';\omega) = h \gamma M_S \left( \begin{array}{cc} -i \hbar \omega + \tilde{\epsilon}_{21} & \tilde{\epsilon}_{22} \\ \tilde{\epsilon}_{11} & i \hbar \omega + \tilde{\epsilon}_{12} \end{array} \right)^{-1} \delta_{k,k'}
\]

with renormalized energy components of \( \tilde{\epsilon}_{\kappa\kappa'}(k,\omega) = \epsilon_{\kappa\kappa'}(k) - \Sigma_{\kappa\kappa'}(k;\omega) \).
B. Linewidths

Now we focus on the longitudinal susceptibility along the $x$-direction, $\chi_{xx} = \chi_{22}$, from Eq. (10):

$$\chi_{xx}(\mathbf{k}, \omega) = \frac{h\gamma M_S}{\omega} \frac{\epsilon_{11} e_{11} + \epsilon_{22} e_{22}}{|i\omega - \epsilon_{21}|^2 + \epsilon_{12}^2 + |i\omega + \epsilon_{12}|^2 + \epsilon_{11}^2 + \epsilon_{22}^2}. \quad (17)$$

In the absence of DMI, this result is consistent with that in Ref. [22]. At this stage, we insert the intrinsic damping contribution by adding $i\hbar\alpha$ to $\epsilon_{11}(\mathbf{k})$ and $\epsilon_{22}(\mathbf{k})$. In FMR experiments, since the frequency $\omega$ is fixed to $\omega_{\text{FMR}}$ and the dc field $H_{\text{ext}}$ is swept, a linewidth of the FMR resonance is equal to Lorentzian broadening of Eq. (17) at $\mathbf{k} = 0$. A straightforward calculation leads to the linewidth of,

$$h\gamma_0 \Delta H_{\text{ext}} = \frac{\alpha}{\cos(\theta_M - \theta_H)} h\omega_{\text{FMR}} + h\gamma_0 \Delta H^{(2)}(\omega), \quad (19)$$

where $\Delta H^{(2)}$ stands for the extrinsic contribution from the two-magnon scattering. Detailed forms of these quantities depend on the model of perturbation potential. For instance, in the case of Eq. (10), the extrinsic contribution is given by,

$$h\gamma_0 \Delta H^{(2)}(\omega) = \left( \frac{N_{\text{imp}} (\pi \lambda_{\text{imp}}^2)^2 \sigma_{\text{DM}}^2 \cos^2 \theta_M}{\lambda_c^2} \right) \Gamma(\omega). \quad (20)$$

Here, the amplitude (the part enclosed by the parenthesis) depends on the model of scattering potential and a dimensionless function $\Gamma(\omega)$ is determined purely by the magnon dispersion and a cut-off function through,

$$\Gamma(\omega) = \frac{1}{\cos(\theta_M - \theta_H)} \left( \frac{\epsilon_{12}(\mathbf{k})}{\epsilon_{22}(\mathbf{k})} \right) \left( \frac{d^2}{L^2} \right) \int d^2 k f_{\epsilon}(k)$$

$$= \frac{\epsilon_{11}(\mathbf{k})}{|i\omega - \epsilon_{21}(\mathbf{k})||i\omega + \epsilon_{12}(\mathbf{k})| + |i\omega + \epsilon_{12}(\mathbf{k})| + |i\omega + \epsilon_{12}(\mathbf{k})| + |i\omega + \epsilon_{12}(\mathbf{k})|}, \quad (21)$$

where $f_{\epsilon}(k) = e^{-k^2 \lambda_{\text{imp}}^2 (\lambda_c^2)^2}$ for a large-sized impurity model ($\lambda_c \gg d$) and $f_{\epsilon}(k) = e^{-k^2 \lambda_{\text{imp}}^2}$ for a small-sized model ($\lambda_c \ll d$). Other scattering models, such as anisotropy-induced scattering potential by surface roughness, can be reproduced by simply replacing $f_{\epsilon}(k) = 1$, but with the modified amplitude that includes detailed form-factors of impurity. In calculating the function $\Gamma(\omega)$, the summation over $\mathbf{k}$ is mainly contributed by those $\mathbf{k}$-points where the denominator, $|i\omega - \epsilon_{21}(\mathbf{k})||i\omega + \epsilon_{12}(\mathbf{k})| + |i\omega + \epsilon_{12}(\mathbf{k})| + |i\omega + \epsilon_{12}(\mathbf{k})|$, approaches zero. These $\mathbf{k}$-points correspond to the resonant states discussed in the previous section. Hence, the total number of resonant state is important in determining $\Gamma(\omega)$ and eventually the FMR linewidth.

IV. RESULTS AND DISCUSSION

Now we proceed to study behavior of the linewidth by varying the DMI strength $D$. In order to present calculated results in a simple way, the perturbation potential model by a small sized impurities of $\lambda_c \ll d$ is mainly discussed for a fixed film thickness of $d = 5$ nm. We note that $\Gamma(\omega)$ is much smaller for a large size impurity of $\lambda_c \gg d$. As an example, we choose parameters of impurity potential such as impurity density of $N_{\text{imp}}/L^2 = 0.03/(\pi \lambda_{\text{imp}}^2)$, $\lambda_c = 1$ nm, and $\lambda_{\text{imp}} = 0.5$ nm. For the exchange stiffness constant $A = 1.3 \times 10^{-11}$ J/m and DMI strength $D = 1.5$ mJ/m$^2$, the chosen parameters give $1.186 \cos^2 \theta_M$ meV for the amplitude of Eq. (20), which corresponds to $102.5 \cos^2 \theta_M$ G with $h\gamma = 0.1158$ meV/T.

In Fig. 3(a), we show the extrinsic linewidth broadening $\Delta H^{(2)}$ as a function of resonant frequency for various DMI strengths. Because of $\epsilon_{\text{DM}}^2$ dependence in Eq. (20), overall amplitudes of the linewidth become larger with an increased DMI strength, whereas, for a given DMI strength, the DMI-limited scattering potential exhibits characteristic behavior of the linewidth as a function of frequency. Namely, with increasing the DMI strength from zero, the linewidth changes slightly in a high frequency range while bumps or peaks are developed in a low frequency range. Furthermore, in a strong DMI case, for example, $D = 2.3$ mJ/m$^2$ (or $\epsilon_{\text{DM}}/\epsilon_{\text{DM}} \sim 1.0$) in the figure, one can see that the linewidth is finite even at zero frequency.

We first analyze the results in the range of high frequency in Fig. 3(a), where the linewidth shows $\epsilon_{\text{DM}}^2$ behavior as a function of frequency. In fact, as well as the $\epsilon_{\text{DM}}^2$ dependence, it is found that the linewidth has structured behavior resulted from the function $\Gamma(\omega)$ as shown in Fig. 3(b). For a given frequency, one can see that $\Gamma(\omega)$ increases with an increased DMI and then decreases above a certain DMI strength. Through detailed numerical examinations, we find that, at the turning point (call the DMI energy $\epsilon_{\text{DM}}^0$), the two lobes in equi-energy lines illustrated in the lower panel of Fig. 2 are about to be one by separating magnon modes at $\phi_k = -\pi/2$ into two resonant states. We derive $\epsilon_{\text{DM}}$ in terms of various energies in Eq. (22). In the case of $\theta_H = 0$, the DMI energy $\epsilon_{\text{DM}}^0$ is approximately given by,

$$\epsilon_{\text{DM}}^0 = \frac{h\gamma_0 (M_S + H_d) \epsilon_{\text{MS}}}{2 h\omega_{\text{FMR}}} \quad (22)$$

with $\epsilon_{\text{MS}} = h\gamma_0 M_S/2$.

In the case of $\epsilon_{\text{DM}} \lesssim \epsilon_{\text{DM}}^0$, we find that lobes of the equi-energy line consist of small points, namely $kd \lesssim 1$. Thus, in this weak DMI regime, we find that a calculated linewidth function $\Gamma(\omega)$ is consistent with the free-DMI results by Ref. [21]. A detailed expression of $\Gamma(\omega)$ for the weak DMI is written in Eq. (23). According to the equation, the increasing behavior in the weak DMI regime ($\epsilon_{\text{DM}} \lesssim \epsilon_{\text{DM}}^0$) of Fig. 3(b) originates from an enlarged
size of the equi-energy lobes in the k-space by increasing the DMI strength. In other words, the enlarged lobes are directly related to more available resonant states to give rise to the increased linewidth.

In the strong DMI case of $g_{\text{DM}} \gtrsim |\epsilon_{\text{DM}}|$, on the other hand, $\Gamma(\omega)$ shows a decreasing behavior as a function of DMI strength even though the size of equi-energy lobes becomes larger as shown in Fig. 2(b). In this regime, there are many resonant states with large k-points, namely $kd \geq 1$. At these k-points, the magnon dispersion is dominantly governed by the exchange energy, $\epsilon_{\text{ex}} k^2 d^2$. This is a rapidly increasing function of $kd$ and thus, results in small magnon density of states or less-available resonant states. This explanation is also consistent with the expression of $\Gamma(\omega)$ for $kd \geq 1$ in Eq. (2).

We next discuss the origin of bumps (appeared for the case $D < 1.2 \text{ mJ/m}^2$) and peaks (appeared for the case $D \geq 1.2 \text{ mJ/m}^2$) in Fig. 5(a). We find that the bumps appear when the DMI energy equal to $\epsilon_{\text{DM}}^0$. Thus, the appearance of resonant state at $\phi_k = -\pi/2$ is responsible for the bumps. On the other hand, the peaks in Fig. 5(a) have somewhat different origins, more complicated behavior of the energy dispersion. At the points where the peaks are developed, we find that there are dual solutions of the cubic equation derived from $\epsilon(\mathbf{k}) = \sqrt{\epsilon_{11}^2 + \epsilon_{22}^2}$. For example, in the case of $D = 1.5 \text{ mJ/m}^2$, we plot equi-energy lines (the right lobe in Fig. 2) in Fig. 4 around the external magnetic field exhibiting the peak. As the external magnetic field increases, the equi-energy lines are evolved to have a distorted shape, and eventually to touch the other (the left lobe). Then, the left and right lobes are separated into outer and inner ones. This modification of the lobes occurs at a lower magnetic field than that giving the appearance of a resonant state at $\phi_k = -\pi/2$. At the magnetic field where the two lobes interact each other, the peak is developed in the function $\Gamma(\omega)$. This is because such formation of the lobes gives rise to a very small group velocity of $\partial \epsilon(\mathbf{k})/\partial \mathbf{k}$. In other words, this means that magnon density of states at the point is very abundant and provides much possibility into which two magnons are scattered to increase the FMR linewidth.

As for a finite linewidth at $\omega_{\text{FMR}} = 0$, we find that a DMI strength is very large, roughly $\epsilon_{\text{DM}} > \sqrt{\gamma M_s(H_s + H_{\text{ext}})} \epsilon_{\text{ex}}$. Under this condition, there are lower energy states than a uniformly magnetized state even at $H_{\text{ext}} = 0$, making resonant states still possible. This can be understood by reminding the fact that spiral states are a ground state rather than a ferromagnetic state for systems accompanying a large DMI.

We now turn to the discussion about the linewidth in the out-of-plane geometry of applied magnetic fields. Figure 5(a) is the linewidth for different tilted angles of the external magnetic field. Because we consider a strong DMI case ($\epsilon_{\text{DM}} \simeq \epsilon_{\text{ex}}$), the peaks appear in the linewidth curves. We find that the peak positions are nearly unchanged with the tilting angle of magnetic field. In the range of high frequency apart from the peak structures, there is a critical frequency above which the linewidth becomes zero, a similar behavior found in DMI-free cases.

In Fig. 5(b), we plot the linewidth as a function of the tilting angle when the FMR frequency is fixed. For a small DMI case, the critical angle above which the linewidth is zero is determined by Eq. (2) and is simi-

![FIG. 3. (color online) In (a), the calculated linewidths for various DMI strengths $D$ are plotted as a function of FMR frequency. In (b), we show the variation of $\Gamma(\omega)$ with increasing DMI strength by fixing FMR frequency. The exchange stiffness constant, $A = 1.3 \times 10^{-11} \text{ J/m}$ is used and the external magnetic field is in-plane, $\theta_H = 0$. Other parameters are equal to those in Fig. 2.](image-url)
field. This may be not possible if a uniform ferromagnetic system such as DMI-free case is the ground state. However, in the case of strong DMI, a non-colinear spin state like spiral configuration has a lower energy than a ferromagnetic one. Consequently, in the presence of the strong DMI, there are still resonant states even at zero magnetic field and thus a uniform spin state excited by microwaves in FMR experiments is still scattered into resonant states by inhomogeneities scattering.

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Appendix A: DMI perturbation potential

Let’s consider fluctuation from a Rashba-type spin-orbit interaction:

$$\Delta E_D = \frac{4\hbar\gamma}{M_S} \int d\mathbf{r} D^d(\mathbf{r}) \left[ \hat{z} \cdot \frac{\partial \mathbf{m}}{\partial x} - \hat{x} \cdot \frac{\partial \mathbf{m}}{\partial z} \right]$$

where $D^d(\mathbf{r})$ is local DMI strength for each defect.

One of feasible models for the fluctuation of this energy is a random spatial variation of the parameter $D^d(\mathbf{r})$ with keeping the magnon propagating vector $\mathbf{m}$. This case may occur structural defects whose characteristic size is larger than or equal to the film thickness $d$ with weak DMI. Then, the fluctuation energy becomes, in Fourier space,

$$\Delta E_D = \sum_{\mathbf{k},\mathbf{k}'} V^d(\mathbf{k} - \mathbf{k}') ik'_z d [\mathbf{m}(-\mathbf{k}) \times \mathbf{m}(\mathbf{k}')] \cdot \hat{z} - \sum_{\mathbf{k},\mathbf{k}'} V^d(\mathbf{k} - \mathbf{k}') ik'_z d [\mathbf{m}(-\mathbf{k}) \times \mathbf{m}(\mathbf{k}')] \cdot \hat{x}.$$  \hfill (A1)

where the defect potential energy is defined by,

$$V^d(\mathbf{r}) = \frac{4\hbar\gamma}{M_S d} D^d(\mathbf{r}) = \sum_k V^d(k) e^{i\mathbf{k} \cdot \mathbf{r}}.$$  \hfill (A2)

As another model, we can take account of an abrupt change of the magnetization vectors, for example, at edges of terrace on sample surfaces, near magnetic and non-magnetic impurities, etc. Moreover, it is also known that the tilt magnetization state (spiral or skyrmion phases) rather than a ferromagnetic state is more stable for a strong DMI strength. For those cases, we expect that magnon modes are additionally modulated as $\tilde{m}(\mathbf{r}) e^{i(k + q) \cdot \mathbf{r}}$ near the defect ($\mathbf{q}$ is a pitch vector, for example, in a spiral state). Then, the fluctuation energy becomes similar to Eq. (A1), but replacing $k_z \rightarrow k_x + q_x$ and $k_z \rightarrow k_z + q_z$. By considering small defects with a
characteristic length $\lambda_0$ (order of an impurity size $\ll d$), we further set $k_x \to 1/\lambda_0$ and $k_z \to 1/\lambda_0$ in Eq. (A1).

We assume that the defect potential energy $V^d(r)$ is contributed from atom-like and Gaussian-shaped functions located at random sites;

$$V^d(r) = \sum_j v_0(r - R_j), \quad v_0(r) = \frac{4\hbar \gamma D}{Ms d} e^{-r^2/\lambda_{imp}^2} \tag{A2}$$

with a defect size, $\lambda_{imp}$. Then, by inserting Eq. (A2) into Eq. (A1), the fluctuation energy becomes, in the local coordinates,

$$\Delta E_D = \sum_{k, k'} V^d(k - k') \frac{i(k_x + k'_x + 2/\lambda_0)d}{2} \cos \theta_M \frac{\sum_{k, k'} m_k(-k)U^{a}(k)U^{o}(k') \cdot [\mathbf{m}(-k) \times \mathbf{m}(k')]}{\cos \theta_M}$$

$$= \sum_{k, k'} \sum_{k, k'} m_k(-k)U^{a}(k)U^{o}(k') \tag{A3}$$

where

$$U^{a}_{11} = U^{a}_{22} = 0, \quad U^{a}_{12} = -U^{a}_{21} = \frac{2\hbar \gamma D}{Ms d} f_{R}(k - k')W(k, k') \cos \theta_M \tag{A4}$$

Here, $U^{a}(k, k')$ is given by,

$$W(k, k') = \frac{1}{(k_x + k_z/2 d/2 \lambda_0 \ll d$, whereas $W = d/\lambda_0$ for $\lambda_0 \ll d$. Here, $f_{R}(k) = e^{-k^2\lambda_{imp}^2 \pi^2 / L^2}$ is a form factor of the atomic potential ($L$ is a side length of the ferromagnetic film).

**Appendix B: Critical angle of external magnetic fields**

Above a certain angle of external magnetic field, the linewidth of FMR becomes zero. The critical angle is determined by demanding zero resonant states, or tiny linewidth of FMR becomes zero. The critical angle is contributed from atom-like and Gaussian-shaped functions located at random sites;

Thus, for a small DMI of $\frac{C_1}{2C_2} \leq 1$, the critical angle is determined by solving the equation,

$$\left( \frac{C_1}{2C_2} \right)^2 + \frac{C_0}{C_2} = 0 \tag{B2}$$

while, $\frac{C_1}{2C_2} > 1$,

$$\left( \frac{C_1}{2C_2} \right)^2 + \frac{C_0}{C_2} - \left( \frac{C_1}{2C_2} - 1 \right)^2 = 0 \tag{B3}$$

**Appendix C: Evaluation of the linewidth function $\Gamma(\omega)$**

The linewidth function $\Gamma(\omega_{FMR})$ can be rewritten as, from Eq. (21),

$$\Gamma(\omega) = \frac{1}{\cos(\theta_M - \theta_H)} \frac{\epsilon_{\text{ex}}}{\pi d^2} \sum_{k} \frac{\epsilon_{0}^{22} + \epsilon_{0}^{11}}{L^2} \int \phi_{a} d\phi_{R}(\phi)$$

where

$$\epsilon(\mathbf{k}) = \frac{2}{\pi} \cos(\theta_M - \theta_H)(\epsilon_{0}^{22} + \epsilon_{0}^{11})[C_0] \int \phi_{a} d\phi_{R}(\phi)$$

and then, $\Gamma(\omega)$ is further simplified to

$$\Gamma(\omega) \simeq \frac{\epsilon_{\text{ex}}}{\pi d^2} \cos(\theta_M - \theta_H)(\epsilon_{0}^{22} + \epsilon_{0}^{11})[C_0] \int \phi_{a} d\phi_{R}(\phi)$$

Now we consider a strong DMI of $\frac{C_1}{2C_2} > 1$ where the dipole energy $\epsilon_{\text{MS}}$ is relatively unimportant. In this case, eigenenergy of magnon can be approximated by, from Eq. (17),

$$\epsilon(\mathbf{k}) = \text{Im}[\epsilon_{12}^{0}(\mathbf{k})] + \sqrt{\epsilon_{11}^{0}(\mathbf{k})^2 + \epsilon_{22}^{0}(\mathbf{k})^2}$$

By solving this, one can show that the resonant states appear at,

$$k_{R}(\phi_{k}) \simeq \frac{\epsilon_{DM}^{0} \epsilon_{ex} \sin \phi_{k} \cos \theta_{M}}{\epsilon_{ex}} \sin \phi_{k} \cos \theta_{M}$$

Then, magnon density of states at a $\mathbf{k}$ point is given by,

$$\delta(h\omega - \epsilon(\mathbf{k})) \approx \frac{1}{\epsilon_{11}^{0} + \epsilon_{22}^{0} - h\omega + \frac{\epsilon_{DM}^{0}}{\epsilon_{ex}} \delta(kd - k_{R}(\phi_{k}))d}$$
where we abbreviate $\tilde{\epsilon}_{DM} = \epsilon_{DM} |\sin \phi_k| \cos \theta_M$, and the associated linewidth function $\Gamma(\omega)$ becomes

$$
\Gamma(\omega) \simeq \frac{1}{4\pi \cos(\theta_M - \theta_H)} \int_{-\pi/2}^0 d\phi_k \frac{\epsilon_{ex}}{\epsilon_{11}^0 + \epsilon_{22}^0 - \hbar \omega + 2 \tilde{\epsilon}_{DM}^2}. \quad (C4)
$$
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