Electron density profile reconstruction on the TCABR sweeping reflectometer

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Abstract. A Frequency Sweeping Reflectometry System is under re-installation in the TCABR tokamak to provide the electron density profiles in this machine. This diagnostic operates in a frequency range between 18 GHz and 40 GHz, in Ordinary (O-mode) or in Extraordinary (X-mode) polarization modes. This diagnostic can cover electron densities between $0.4\times10^{19} m^{-3}$ and $2.0\times10^{19} m^{-3}$, when configured in O-mode. Electron density profiles can be inferred from the phase dependence of the probing frequency, $d\Phi(f)/df$, measured at the output of the reflectometry system. To evaluate the measuring capabilities and the requirements for using a frequency sweeping reflectometer, a numerical method to reconstruct the density profiles in the TCABR tokamak was implemented. The robustness of inversion method was tested through the following steps: i) Determination of $d\Phi(f)/df$ from a known density profile; ii) Reconstruction of the electron density profile by the method that will be presented here; iii) Comparison between the original and reconstructed electron density profiles to test the validity of the method. The method and analysis are applied for the X and O polarization modes. These computational analyses can be useful for the interpretation of reflectometer diagnostic data and the experimental data can be used to check the validity of the numerical model.

I – Introduction

Frequency sweeping reflectometry is a diagnostic technique frequently used to measure the density profiles in tokamak machines, successfully implemented for the O-mode [1] (when the electric field of the probing wave is parallel to the magnetic field in the plasma) and for the X-mode [2] (when the electric field of the probing wave is perpendicular to the magnetic field in the plasma). The density profiles can be inferred from the phase dependence of the probing frequency $d\Phi(f)/df$, measured at the output of the reflectometry system. In order to reduce the effect of turbulence in the density profile reconstruction, a fast sweeping local oscillator is used. In the TCABR tokamak, the sweeping reflectometer uses Hyperabrupt Varactor-Tuned (HTO) oscillator to provide density profiles in 8µs, faster than the typical period of the plasma fluctuations. During each shot, more than eleven thousand of sweeps are made and the output signals acquired in a dedicated Data
Acquisition System (DAS based in ATCA - Advanced Telecom Computing Architecture) with a sampling frequency up to 200 MHz.

To evaluate the measuring capabilities and the requirements for using a sweeping reflectometer, a numerical method to reconstruct the density profiles in the TCABR tokamak was implemented. The method and analysis are applied for the X and O modes of wave propagation in fusion plasmas. In this paper, a brief review of wave propagation conditions in cold plasma approximation and accessibility curves for the TCABR tokamak are presented in Section 2. The numerical method implemented for the reconstruction of density profiles is described in Section 3. The numerical result and comparison between the original and reconstructed electron density profile, to test the validity of the method, are also presented. In another example, the typical behaviors of phase and time of flight in the presence of a magnetic island are presented in Section 4. Finally, the main conclusions and future developments are discussed in Section 5.

II – Basic Principles

The phase delay of the reflected signal, measured at output of the reflectometer, is given by \( \Phi_m = \Phi_r + \Phi_p \), where \( \Phi_r \) is the reference phase (contribution from the waveguides and air/vacuum path lengths, which can be determined from calibration procedures or calculated numerically) and \( \Phi_p \) is the phase contribution from plasma, given by:

\[ \Phi_p(f) = \frac{4\pi f}{c} \int_{x_0}^{x_c} N(f, x) \, dx, \tag{1} \]

where \( N(f, x) \) is the wave refractive index and \( x_0 \) and \( x_c \) are the coordinates of the vacuum-plasma boundary (physical limiter in our case) and the cutoff layer, respectively. The plasma phase is obtained experimentally from the subtraction of the reference phase, \( \Phi_r \), of the measured phase of the reflected signal, \( \Phi_m \). The refractive indexes for the Ordinary and Extraordinary modes of propagation, in a cold plasma approximation and perpendicularly to the magnetic field [3], are given respectively by:

\[ N_0 = \left[ 1 - \left( \frac{f_{pe}}{f} \right)^2 \right]^{1/2}, \tag{2} \]
\[ N_X = \left[ 1 - \left( \frac{f_{pe}}{f} \right)^2 \frac{f^2 - f_{pe}^2}{f^2 - f_{ce}^2 - f_{pe}^2} \right]^{1/2}, \tag{3} \]

where \( f_{pe} = 8.98(n_e(x))^{1/2} \) is the plasma frequency, \( f_{ce} = 27.99 \times B \) is the electron cyclotron frequency (both in SI units), and B, in last expression, is the total magnetic field.

The total wave reflection inside the plasma column occurs when \( N_{0,X} = 0 \); for the O-mode the cutoff is given by \( f = f_{pe} \) and, for the X-mode, \( f_{R,L} = \frac{\pm f_{ce} + \sqrt{(f_{ce}^2 + 4f_{pe}^2)}}{2} \) \( f_{R,L} \) is the Right-Hand cutoff (plus sign in the last expression) and \( f_{L} \) is the Left-Hand cutoff (minus sign in the last expression). The accessibility conditions on the equatorial plane of the TCABR tokamak (main parameters [4]: Major radius \( R_0 = 0.615 \) m, Minor radius \( a = 0.18 \) m, Toroidal magnetic field \( B_0 = 1.0 \) T – 1.1 T, Plasma current \( I_p \leq 120 \) kA, Plasma current duration \( t_p \leq 150 \) ms, Peak electron temperature \( T_{e0} \leq 600 \) eV, Peak electron density \( n_{e0} \leq 6.0 \times 10^{19} \) m\(^{-3} \), as function of major radius \( R \), are presented in figure 1. The density [5,6] and temperature [7] profiles considered in the simulations are:

\[ n_e(R) = n_{e0} \left[ 1 - \left( \frac{R - R_{av}}{a} \right)^2 \right]^B \] and \[ T_e(R) = T_{e0} \left( \frac{R - R_{av}}{a} \right)^{8BT} \],

respectively. The total magnetic strength is given by \( B = \sqrt{B_{\theta}^2 + B_T^2} \), where \( B_{\theta} \) is the poloidal magnetic field and \( B_T = \frac{B_0 R_0}{r} \) is the toroidal magnetic field. The poloidal magnetic field is small, \( B_{\theta}/B_T < 4\% \), and so the change in the orientation of the magnetic field of \( \theta < 2.2^0 \) is considered negligible.
Figure 1: Accessibility conditions for typical density profiles in the TCABR tokamak: $f_{ce}$ and $2f_{ce}$ are the first and second Electron Cyclotron Emission, respectively, $f_R$ is the Right-Hand cutoff, $f_L$ is the Left-Hand cutoff and $f_{pe}$ is the plasma frequency.

The phase shift of the wave, for the rays travelling inside the plasma column along the horizontal axis, can be obtained integrating numerically equation (1). The total phase, $\Phi_m$, as function of the probing frequency, for two different central densities and for the O and X polarizations modes, are presented in figure 2(a) and figure 2(b), respectively. In this example, the reference phase, $\Phi_R$ (the distance between the antennas and the limiter is 18.5 cm), is represented by the blue color in figure 2.

Figure 2: Total phase of the wave versus probing frequency, for peak densities $n_{e0} = 1.0 \times 10^{19} \text{ m}^{-3}$ (curve 1) and $n_{e0} = 2.0 \times 10^{19} \text{ m}^{-3}$ (curve 2), and for the O-mode (part a) and X-mode polarization (part b). The reference phase, $\Phi_R$, is shown by the blue color straight line.

The total phase diverges from the reference phase at $f > 11 \text{ GHz}$ for the O-mode, and $f > 6 \text{ GHz}$ for the X-mode, indicated by A in the figures 2.a) and 2.b). The points B and C in figure 2.a) (O-mode polarization) represent the phase jumps when the frequency of the probing wave reaches the maximum value of plasma frequency, i.e. at values of 28.4 GHz (peak density of $n_{e0} = 1.0 \times 10^{19} \text{ m}^{-3}$) and 40.2 GHz (peak density of $n_{e0} = 2.0 \times 10^{19} \text{ m}^{-3}$), and corresponds to the transition between Reflectometry to Interferometry (or Refractometry) operation mode. In X-mode polarization, the Reflectometry system can operate in Interferometry mode between the maximum of the Left-Hand cutoff (point B in figure 2.b)) and the minimum of the Right-Hand cutoff (point C in figure 2.b)). In the same figure, the point C corresponds also to the first fringe ($f \sim 23.2 \text{ GHz}$) and this is very useful for the initialization of the density profile using the X-mode polarization. For frequencies higher than the second ECE harmonic at plasma edge, $f > 46.3 \text{ GHz}$, the wave launched towards the plasma column in the X-mode polarization will be absorbed.
III – Profile inversion numerical method

To evaluate the measuring capabilities and the requirements for using a sweeping reflectometer, a numerical method to reconstruct the density profiles in the TCABR tokamak was implemented and based on the method described elsewhere [8]. The cutoff positions are determined interactively, using the following equation

\[ X(i) = \left[ \frac{A(0) \times X(0) + (i-1) \times \text{mean}[X(1:(i-1)) \times (A(1:(i-1)) - A(0:(i-2)))] - \Phi_p(i)}{A(i-1)} \right] \]

where \( A(i) = \frac{\pi \times f(i) \times [N(i) + N(i-1)]}{c} \) and \( N \) is the refractive index defined by the equations (2) and (3). The term \( \Phi_p(i) \) is the contribution of the phase delay due to the plasma path. Once the cutoff position is determined, the electron density is easily obtained from well known relations (plasma frequency for the O-mode and equation (4) for the X-mode). To ensure numerical stability in the determination of the cutoff position, \( X(i) \) was replaced by \( X_i = \frac{X_i(i) + X_i(i-1)}{2} \).

To test the method, in the next step we will consider a typical density profile in the TCABR tokamak. In figure 3.a) is represented the density profile reconstructed from the phase of the plasma, shown in figure 2.a). In the same simulation, the cutoff position as a function of the probing frequency is depicted in figure 3.b). In these simulations, the range of frequency probing considered was from 0.1 GHz up to 50 GHz.

![Figure 3: Expected (black curve) and reconstructed (red curve) obtained for the O-mode reconstruction method: a) Density profile; b) Cutoff position as function of probing frequency.](image1)

The density profile reconstructed from the plasma phase shift in X-mode (represented in figure 2.b)), is shown in figure 4.a). In this polarization mode, the density profile can be reconstructed from the edge of the plasma column (the first fringe occurs at \( f = 23.2 \) GHz) up to value for the second ECE frequency at edge. In the X-mode polarization, the wave launched towards the plasma column will be absorbed at \( f > 46.4 \) GHz, that corresponds to the radial position \( R < 73.4 \) cm. The cutoff position as function of the probing frequency is shown in figure 4.b).
In the next figure are shown the errors in density and cutoff position for the O-mode (graphics at left) and X-mode (graphics at right).

Figure 5: Error due to the reconstruction method for density profile and cutoff position, in the O-mode (part a) and X-mode (part b).

A very good agreement between the original and the reconstructed profiles, in both wave polarization modes, and from the plasma edge and close to the centre of plasma column, was found as seen in figure 5. The uncertainty in the cutoff positions rises quite fast, near the centre of the plasma column, due to flatness of the plasma frequency profile (for the O-mode) and in the Right-Hand cutoff for the X-mode.

The O-mode polarization has easier accessibility to plasma phase measurements than using the X-mode. The main disadvantage of using the O-mode polarization is the difficulty to estimate correctly the edge contribution required to initialize the density profiles [9].

The effect of the presence of a magnetic island in the density profile reconstruction will be presented in the next section.

IV – Effect of magnetic island in the reconstruction of the density profile

The perturbation in the density profile due to the presence of a magnetic island (red curve in the figure 6.) is given by the equation $\tilde{n}_{int} = \tilde{n}_{e0} \exp \left[ \frac{[R-x]^2}{W^2/2} \right]$, where $\tilde{n}_{e0}$ is the maximum amplitude of perturbation, $x$ is the location of the rational surface and $\frac{W}{2}$ is the magnetic island half width. The
values used in the density profile perturbation, presented in figure 6, are: \( \bar{n}_e = 2 \times 10^{18} \text{m}^{-3}, \bar{R} = 0.76 \text{ m} \) and \( \bar{W} = 4.2 \text{ cm} \).

![Figure 6: Non-perturbed (black curve) and perturbed density profiles by the presence of magnetic island (red curve).](image)

The phase shift of the wave inside the plasma column as function of the probing frequency, in the presence of a magnetic island (blue curve) and non-perturbed profiles (red curve), for the X-mode polarization, is represented in right scale of figure 7. The cutoff position as function of the probing frequency, for a non-perturbed density profile (red curve) and in the presence of the magnetic island (blue curve), determined from the X-mode wave polarization reconstruction method, is presented in the left scale of figure 7. From this figure, we observe that the cutoff position recovers completely outside the magnetic island region but the same conclusion is not valid for the phase.

![Figure 7: The cutoff position (left side in graphic) and phase delay inside the plasma column (right side in graphic), in X-mode polarization mode, for a non-perturbed (red curves) and perturbed density profiles by the presence of a magnetic island. The waves are absorbed by the plasma column for \( f > 46.3 \text{ GHz} \).](image)
The density profile is perfectly reconstructed by numerical simulation, as shown in the figure 8.a). Also, the errors in the density and cutoff positions are negligible (see figures 8.b) and 8.c)).

The time of flight, determined by $\tau_g = \frac{1}{2\pi} \frac{\delta \varphi}{\delta f}$ and, for the X-mode, is presented in the next figure.

The effect of the magnetic island starts to appear at $f \sim 10$ GHz (in the Left-Hand cutoff) and at $f \sim 34$ GHz (in the Right-Hand cutoff), for the X-mode polarization, as shown in figure 9. The magnetic island effect in the time of flight reaches a maximum at $f \sim 12$ GHz and $f \sim 36.5$ GHz, for the Left-Hand cutoff and Right-Hand cutoff, respectively.

The time of flight as function of the cutoff position for the Left-Hand cutoff (curve A, occurs from the edge of the plasma column up to $R = 0.72m$) and for the Right-Hand cutoff (curve B, from the plasma edge up to $R \sim 0.73m$), is shown in figure 10. The red curves represent the effect of
magnetic island in the time of flight. The X-mode polarized wave will be absorbed by the second ECE harmonic, for plasma positions with $R \leq 0.73$ m.

![Diagram of time of flight as function of the cutoff position, in presence of a magnetic island (red curves) and in a non-perturbative density profile (black curve), obtained from the Left-Hand cutoff (curve A) and Right-Hand cutoff (curve B).](image)

From figure 10 on can be observed that the effect of magnetic island in the time of flight appears between $R \sim 72.0$ cm and $R \sim 78.0$ cm, but in fact, the region with the presence of the magnetic island is between $R \sim 74.0$ cm and $R \sim 78.0$ cm (see figure 6). This happens because the wave is influenced by the presence of magnetic island before the wave be reflected in the cutoff layer.

The main conclusions and the next perspectives are presented in the next Section.

V – Conclusions and next steps

In this paper was presented a numerical method used for the reconstruction of density profiles from some predetermined density profiles. The robustness of the inversion method was tested to validate the numerical method and these analyses were applied for the X and O modes of wave propagation in fusion plasmas. The phase dependence of the probing frequency, $d\Phi(f)/df$, measured at the output of the reflectometer system, will be tested in the numerical method in order to obtain the reconstruction of density profiles in the TCABR tokamak.

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