Velocity selective trapping of atoms in a frequency-modulated standing laser wave: wave function and stochastic trajectory approaches

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Abstract.

The wave function of a moderately cold atom in a stationary near-resonant standing light wave delocalizes very fast due to wave packet splitting. However, frequency modulation of the field may suppress packet splitting for some atoms having specific velocities in a narrow range. These atoms remain localized in a small space for a long time. We show that modulated field can not only trap, but also cool the atoms. We perform a numerical experiment with a large atomic ensemble having wide initial velocity and energy distribution. During the experiment, most of atoms leave the wave while trapped atoms have narrow energy distribution.

1. Introduction

Laser cooling and trapping of atoms and ions is a rapidly developing field of modern physics. Cold particles in a laser field are a common physical substrate used in numerous fundamental and applied issues such as Bose-Einstein condensates, quantum chaos, single-atom laser, quantum computer, etc. A significant number of methods of atomic cooling in a laser field were developed in the recent decades (the Doppler cooling [1, 2], the Sisyphus cooling [3, 4], the velocity selective coherent population trapping (VSCPT) [5], dynamical localization and trapping [6], etc. [7]).

In this paper we suggest a method of coherent laser cooling in the absence of spontaneous emission. When an atom moves in a near-resonant standing light wave, two periodic optical potentials form in the space [8]. When the atom crosses a standing wave node, it may undergo the Landau-Zener transition between these two potentials. Such transitions cause splitting of the wave packets [9] and rapid delocalization of the wave function [10]. In this paper we show that frequency modulation of the field may suppress the splitting of wave packets for atoms that have velocities in the specific narrow range determined by the field modulation parameters. We suppose that in a real experiment, this may significantly decrease the energy distribution of moderately cold atoms.

2. Equations of motion

Let us consider a two-level atom (with the transition frequency $\omega_a$ and mass $m_a$) moving in a strong standing laser wave with the modulated frequency $\omega_f(t)$. In absence of spontaneous emission (the atomic excited state must have long lifetime, or some experimental methods must
suppress the decoherence) the atomic motion may be described by the Hamiltonian [11]
\[ \hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} \hbar (\omega_a - \omega_f \tau) \hat{\sigma}_z - \hbar \Omega (\hat{\sigma}_- + \hat{\sigma}_+) \cos k_f \hat{X}, \]
where \( \hat{\sigma}_{\pm, z} \) are the operators of transitions between the atomic excited and ground states (the Pauli matrices), \( \hat{X} \) and \( \hat{P} \) are the operators of the atomic coordinate and momentum, and \( \Omega \) is the Rabi frequency.

Let us use the following dimensionless normalized quantities: momentum \( p \equiv P/\hbar k_f \), time \( \tau \equiv \Omega t \), position \( x \equiv k_f X \), mass \( m \equiv m_a \Omega/\hbar k_f^2 \) and detuning \( \Delta[\tau] \equiv (\omega_f[\tau] - \omega_a)/\Omega \). Let us suppose that the field modulation is harmonic,
\[ \Delta[\tau] = \Delta_0 + \Delta_1 \cos[\zeta \tau + \phi], \quad \zeta \ll 1, \quad \Delta_0 \ll \Delta_1 \ll 1. \]

Now let us obtain the equations for the probability amplitudes to find an atom with the normalized momentum \( p \) in the excited or ground state, \( a[p, \tau] \) and \( b[p, \tau] \), correspondently:
\[ i \dot{a}[p, \tau] = \left( \frac{p^2}{2m} - \frac{\Delta[\tau]}{2} \right) a[p] - \frac{1}{2} (b[p - 1] + b[p + 1]), \]
\[ i \dot{b}[p, \tau] = \left( \frac{p^2}{2m} + \frac{\Delta[\tau]}{2} \right) b[p] - \frac{1}{2} (a[p - 1] + a[p + 1]). \]

Here the dot designates the differentiation with respect to \( \tau \). For every value of \( p \), there is its own pair (3).

3. Wavefunction approach: numerical manifestations of velocity selective trapping
Let us choose the values of the parameters and initial conditions in order to perform the numerical simulation. The average initial atomic momentum \( \langle p[0] \rangle \) will be a variable condition for the purpose of this paper. All other conditions will be fixed: normalized mass \( m = 10^5 \) (by order of magnitude this corresponds to the experiments with Cs [12] and Rb [13] atoms, but for a stronger field \( \Omega \sim 10^{8-10} \text{Hz} \), field parameters \( \Delta_0 = -0.02, \Delta_1 = 0.047, \zeta = 0.00508, \phi = 0 \). The initial wave packet has a Gaussian form with \( \langle x[0] \rangle = 0 \) and the initial probability to find the atom in the excited state 0.5. The standard deviation of the atomic momentum is \( \sigma_p[0] = 5\sqrt{2} \). Therefore, in accordance with the Heisenberg relation, the standard deviation of the initial coordinate is \( \sigma_x[0] = 1/(2\sigma_p[0]) = 0.1/\sqrt{2} \) (it is much less than the normalized optical wavelength \( 2\pi \)). In numerical experiments, we use these initial conditions to simulate the system of 8000 equations (3) with \(-1000 \leq p \leq 1000 \). For larger values of \( |p| \), we put \( a[p, \tau] = b[p, \tau] = 0 \) due to the energy restrictions. Obtaining the solution in the momentum space we perform the Fourier transform and get the wave function in the coordinate space.

Let us study the effect of field modulation on atomic delocalization. In [9, 10], the atomic motion was studied in absence of modulation. At \( \Delta = 0 \) and \( |\Delta| \gtrsim 1 \) the atomic motion is simple. Atoms move in constant spatially periodic potentials. Slow atoms are trapped in potential wells and fast atoms move ballistically through the wave. At \( 0 < |\Delta| \ll 1 \) the atomic motion is more complex. The slowest atoms (\( |\langle p[0] \rangle| < \sqrt{2m} \)) are trapped in potential wells. Faster atoms (\( \sqrt{2m} \leq |\langle p[0] \rangle| < 2\sqrt{m} \)) perform a kind of random walk. Their wave packets split each time they cross standing-wave nodes, and this causes fast delocalization of the wave functions. The fastest atoms (\( |\langle p[0] \rangle| > 2\sqrt{m} \)) move ballistically through the wave. Their wave packets split, but all products move in the same direction, so the overall delocalization is slow.

In Fig. 1 we calculate the variance of the atomic position \( \sigma_x^2 \) after a relatively long time span of coherent evolution \( \tau = 5000 \) as a function of the initial atomic momentum \( \langle p[0] \rangle \). For the
Figure 1. The variance of atomic position $\sigma^2_x$ at $\tau = 5000$ as a function of initial atomic momentum $\langle p[0] \rangle$: curve — constant field $\Delta = -0.02$, triangles — modulated field $\Delta = -0.02 + 0.047 \cos[0.00508\tau]$

constant field (solid curve) this function shows fast delocalization of all atoms in the range of $\sqrt{2m} \approx 440 \lesssim \langle p[0] \rangle \lesssim 2\sqrt{m} \approx 640$ (cold atoms with velocities of the order of 1 m/s). Local peak at $\langle p[0] \rangle \approx 630$ is produced by moderately fast atoms having an uncertain scenario of either random walking or flying ballistically.

Now let us "switch on" the field modulation and see the changes. In Fig. 1 the analogous function of $\sigma^2_x$ is shown with triangles. This function has a more complex structure. In particular, it has a prominent additional minimum at $\langle p[0] \rangle = p_{tr} \approx 500$. These atoms are not trapped in potential wells in a strict sense (their energy is too high, see the theory in the next sections), but some mechanism significantly suppresses the delocalization of their wave functions (note that both functions are shown in a logarithmic scale).

4. Semiclassical approach: explanation of the effect

In order to explain the effect of velocity selective trapping, let us mention some analytical results from [9, 10, 11]. Near resonance, the atomic motion can be described in terms of two potentials

$$U^- = -\sqrt{\cos^2[x] + \frac{\Delta^2[\tau]}{4}}, \quad U^+ = \sqrt{\cos^2[x] + \frac{\Delta^2[\tau]}{4}}. \quad (4)$$

An atom moves in one of these potentials when it is far from standing-wave nodes ($x = \pi/2 + n$). The semiclassical energy and equations of motion are [11]

$$E = \frac{p^2}{2m} + U[x, \tau], \quad \dot{x} = \frac{p}{m}, \quad \dot{p} = -\text{grad}[U]. \quad (5)$$

If initial energy $E[0] \lesssim 0$ (for $x[0] = 0$, this corresponds to $|p[0]| \lesssim \sqrt{2m}$), then an atom cannot reach any standing-wave node. It is trapped in the bottom of the first potential well near $x = 0$. If initial energy is in the range of $0 \lesssim E \lesssim 1$ (for $x[0] = 0$, this corresponds to $\sqrt{2m} \lesssim |p[0]| \lesssim 2\sqrt{m}$), then an atom may either perform a random walk or being trapped (if $p[0] = p_{tr}$). Faster atoms with $E \gtrsim 1$ move ballistically through the wave in a constant direction.

When an atom crosses the node, the potential may change the sign (atom undergoes the Landau-Zener tunneling between potentials $U^\pm$) with the probability

$$W_{LZ} \approx \exp\frac{-\Delta^2 m \pi}{4 |p_{\text{node}}|}, \quad (6)$$

where $\langle p_{\text{node}} \rangle$ is an average momentum of an atom when it crosses the node. At $0 < |\Delta| \ll 1$ the tunneling causes splitting of wave packets (observed in numerical experiments [9, 10, 11]).
and fast delocalization of the wave function. However, at $\Delta = 0$ potentials coincide at nodes, so the probability of tunneling is equal to 1 and wave packets do not split. Delocalization of wave function is very slow and the potential takes the simplest form $U = \pm \cos[x]$.

Velocity-selective trapping of atom occurs if the field modulation is synchronized with the atomic mechanical motion. It is possible to choose such modulation parameters and atomic momentum (see analytical estimations in [11]) that $\Delta[\tau]$ takes zero values each time an atom crosses the node. With our parameters such synchronization occurs at $\langle p[0] \rangle = p_{tr} \simeq 500$. Therefore, packet splitting and delocalization are suppressed.

Note that for trapped atoms, equations (5) stay correct during entire evolution (even during node crossings), and the potential takes simplest form. Trapping occurs, if atom either does not crosses nodes at all, or node crossings take place when $\Delta[\tau] = 0$. Therefore, the term $\Delta^2/4$ in (4) is always neglible, and the atom moves in a constant potential with a constant energy

$$U \simeq -\cos[x], \quad \tilde{E} \equiv \frac{p^2}{2m} - \cos[x].$$

Initial conditions used in this paper correspond the negative sign of $U$ (conserved for trapped atoms).

5. Stochastic trajectory approach: modeling of atomic cooling process

In order to show that reported effect is not only trapping of atoms but also their cooling, we must simulate the dynamics of an atomic ensemble having wide initial velocity (and energy) distribution and show that the distribution goes narrow during the evolution. Such simulation with quantum equations requires a huge computational time. Therefore, we develop an alternative simplified model of atomic motion based on the following principles.

1. Atom is a dot-like particle having a particular trajectory.
2. Between standing-wave nodes, an atom moves in an effective potential $U^\pm[x, \tau]$ (4) having constant sign. Such motion is governed by semiclassical equations (5).
3. At initial time moment, the potential $U^\pm$ has negative sign. Any time when an atom crosses a node, the potential changes its sign with the probability (6).

In Fig. 2, we simulate the dynamics of an atomic ensemble (several thousands of atoms) with comparatively wide initial momentum distribution moving in positive direction with average velocity $\langle p[0] \rangle = 550$ (Fig. 2a). Corresponding energy distribution is shown in Fig. 2b (we calculate simplified energy $\tilde{E}$ (7), but it is equal to general energy $E$ (5) at initial time moment).

In order to show that velocity selective trapping really cools atoms, let us consider a small part of laser wave in a range

$$-\frac{3}{2}\pi < x < \frac{3}{2}\pi.$$  

(8)

At the beginning of the experiment, all the atoms have $x \simeq 0$. During the evolution, trapped atoms ($p[0] \lesssim 440, E[0] \lesssim 0$ and $p[0] \simeq p_{tr} = 500, E[0] \simeq 0.25$) stay in the range (8) while most of other atoms leave it (due to ballistic flight or random walk). Trapped atoms have wide momentum distribution because their momenums oscillate in a wide range. However, their energy distribution is very narrow. In Figs. 2c, d, there is a prominent peak near $\tilde{E} = 0.25$, and it is very narrow in comparison with initial energy distribution. This is because the majority of atoms with other initial values of energy leaved the wave. Note that simplified energy $\tilde{E}$ is conserved only for trapped atoms. Other atoms can change it during the evolution (see, for example, spontaneous peak at $\tilde{E} \simeq -0.6$, Fig. 2c). However, the number of such atoms in area (8) decays fastly, so they do not change the overall picture.
Figure 2. Cooling of an atomic cloud due to velocity selective trapping (statistics of atoms having positions in a range $-3\pi/2 < x < 3\pi/2$). Probability density $W$ to find an atom with a given momentum or energy is shown in arbitrary units.

6. Conclusion
Intensive coherent light produces significant mechanical action on cold atoms having velocities of the order of 1 m/s. There is a wide range of field parameters at which atom performs a kind of random walk accompanied by wave packets splitting and fast delocalization of wave function. However, a specific field modulation can suppress wave packet splitting for atoms with precisely selected velocities. These atoms oscillate in a small space, and their wave functions are almost completely localized. This effect cannot cool atoms in the sense of achieving zero velocity, but it can significantly decrease their mechanical energy distribution (see Fig. 2).

Note that the effect is purely coherent, it takes place only in absence of dissipation. However, we believe that this is just a quantitative technical limitation that may be overcome by an appropriate choice of atoms and hi-Q cavities.

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