Resonance Extraction from the SAID Analysis

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Abstract. Resonances are extracted from a number of energy-dependent and single-energy fits to scattering data. The influence of recent, precise EPECUR data is investigated. Results for the single-energy fits are derived using the L\textsuperscript{+}P method of analysis and are compared to those obtained using contour integration applied to the global energy-dependent fits.

INTRODUCTION

The SAID website has links to analyses and databases for a number of fundamental medium-energy reactions. These include $\pi N$, $NN$, $\pi d$ elastic scattering and the photo- and electro-production of pions. Fits to data have been either energy-dependent (ED) or single-energy (SE). In the ED fits, all data are fitted over the full energy range using a single parameterization. In the SE fits, data within narrow energy bins are fitted by varying the dominant partial-wave amplitudes, with the ED values taken as a starting point. The SE partial-wave analyses (PWA) of $\pi N$ elastic scattering have been analyzed for resonance content and have been used in a number of multi-channel analyses as PWA 'data' in lieu of actual $\pi N$ scattering data [1, 2, 3].

As the abovementioned multi-channel analyses have found resonances beyond those extracted from the global ED fits, efforts have been made to allow for additional resonance contributions in the SAID fits [4, 5, 6]. These we review, focusing on the center-of-mass energy region near 1.7 GeV, where the SAID extractions fail to reliably determine pole positions for the N(1710)$^{1/2+}$ and N(1700)$^{3/2+}$ (PDG 3-star) states [7].

The SAID SE analyses show more structure than the ED results and recent Laurent+Pietarinen fits to the SE amplitudes [8, 9] have found poles one could associate with the abovementioned missing resonances. These results are re-examined in light of recent, very precise, $\pi N$ elastic scattering data from the EPECUR collaboration [10] covering the energy range of interest. These measurements were motivated by the search for possible narrow $N^*$ states near 1.7 GeV, but also serve to distinguish between the existing PWA, and may also be sensitive to cusp structures associated with opening channels, such as $K\Sigma$.

ATTEMPTS TO ADD RESONANCES

As resonances, apart from the $\Delta(1232)$, have appeared in the SAID analysis as a result of the Chew-Mandelstam formalism, and are not inserted by hand, only those states with significant $\pi N$ couplings have been found, constituting a 'minimal' set of resonances. However, the classic Karlsruhe-Helsinki [12] (KH) and Carnegie-Mellon-Berkeley [13](CMB) analyses have found numerous additional (PDG 3-star and lower rated) resonances, with some of these now appearing more clearly in reactions with different final states (such as $KA$) [3].
In the 1990 SAID analysis [6], amplitudes from the KH and CMB fits were added as soft constraints, and fitted together with the scattering data, in order to force the SAID fit to more closely approximate the KH and CMB results. A number of new poles appeared in these constrained SAID analyses, but these were generally not in good agreement with the KH and CMB values. In particular, the $P_{11}$ and $D_{13}$ waves failed to produce poles near the expected $P_{11}(1710)$ and $D_{13}(1700)$ states, absent from the original SAID analysis.

A sensitivity test for resonance addition was made in the 1995 analysis [5]. Here the standard fit was augmented in a product S-matrix approach, $S_{Prior} S_{BW}$, with a Breit-Wigner state added and searched in each partial wave. This exercise found evidence for a second $P_{11}$ state somewhat higher in mass and broader than expected. In addition, a second $F_{15}$, $N(1860)5/2^-$, was found (a PDG 2-star state) which persists in current fits. Combining this search with the previous set of soft amplitude constraints was no longer feasible as amplitude constraints were being used in the iteration of fits constrained by forward and fixed-t dispersion relations.

Finally, a fit was made with explicit Chew-Mandelstam K-matrix poles inserted in each partial wave [4]. The initial expectation was for a significant increase in the number of T-matrix poles per partial wave. However, the actual result was a set of partial wave amplitudes nearly identical to those from a fit without explicit K-matrix poles. In effect, the fit generally moved the dominate structures to the poles added by hand, with secondary poles mainly appearing far from the physical axis. For example, the second $P_{11}$ pole appeared at $(1646 - i290)$ MeV with a real part near the expected value but double the expected imaginary part. In the $D_{13}$ wave, two poles appeared again with the expected real part but with imaginary parts that were either too small (accompanied by a very small residue) or too large, compared to values found in the KH and CMB fits.
LAURENT+PIETARINEN FITS

While it has proven challenging to incorporate additional pole structures into the SAID ED fits, the associated SE fits have, by design, added structures in energy beyond the global ED results. By analyzing narrow energy bins of data, without any smoothness constraints apart from the ED amplitude starting points, the SE amplitudes would be expected to fluctuate around the ED values. The original intent of these SE fits was to check for systematic structure missing from the ED parameterization. However, in order to determine whether the SE behavior is consistent with additional pole structure, it must first be fitted with a function that can be extrapolated into the complex energy plane. A convenient form is the Laurent+Pietarinen (L+P) parameterization of the $\pi N$ T-matrix,

$$ T(W) = \sum_{i=1}^{k} \frac{d_i^{(0)}}{W - W_i} + B(W), \quad (1) $$

where the non-pole term $B(W)$ is constructed from a conformal mapping of the cut energy plane onto the unit circle as described in Refs. [8, 9].

Fits of the L+P type have [8, 9] have found, for the $P_{11}$ partial wave, the first (Roper) state and the second $N(1710)$ with pole values in line with PDG estimates, starting from the WI08 SE amplitudes [4]. For the second $D_{13}$ state, the L+P fit finds a pole at $(1752 - i286)$ MeV, qualitatively in line with the deeper pole found in the explicit pole fit of Ref. [4] but outside the, rather broad, PDG imaginary-part range of approximately 50-200 MeV.

One might ask if the SE fluctuations are reliable or are due to possibly inconsistent data in particular energy bins, or are influenced by the choice of energy-bin width and center. We address both questions below by first considering the effect of including much more precise cross section data available from the EPECUR experiment [10], and then shifting the bin positions and widths.
MODIFIED ED AND SE FITS

As a first step, the new EPECUR data were included in a revised ED fit to update to the WI08 ED result. This ED solution was then used as the basis for new SE solutions. As the EPECUR data are very precise and have a small step size in energy, individual energy bins contained significantly more high-quality cross section data. This had the effect of reducing the SE errors by up to approximately 30 percent, depending on the partial wave.

The SE set for the $P_{11}$ partial wave was then fitted using a number of assumptions regarding the pole content, as displayed in Figs. 1 to 3. In Fig. 1, a 2-pole search was made with the second fixed at the value found in the corresponding ED fit, $(1659 - i262)$ MeV. The resulting chi-squared was 1.1 per datum for the L+P fit. Adding a third pole, with the second remaining fixed, as shown in Fig. 2, had little effect on the qualitative fit and chi-squared. However, allowing a search of the second pole resulted in a qualitatively different fit with a slightly better chi-squared per datum. The resulting second pole position was $(1725 - i100)$ MeV, which is in good agreement with the present PDG estimate of $(1720 - i115)$ MeV.

As a final exercise, the SE energy bin centers and widths were changed randomly to determine whether this would produce significantly different scatter in the SE solutions. The resulting SE set was again fitted with three searched poles. In this case, the second pole, which one would expect to correspond to the N(1710), was found at $(1653 - i84)$ MeV, significantly different from the previous determination. This then suggests re-binning effects should be taken into account when estimating extracted pole position uncertainties. Results for the N(1710) pole determinations are summarized in Table 1.
TABLE 1. Summary of N(1710) pole determinations from the KH [12], CMB [13], BnGa [3] and the L+P analyses: L+P(WI08), L+P(EPECUR), and L+P(Bin Shift).

| Fit               | Real   | -2 Imaginary |
|-------------------|--------|--------------|
| KH                | 1690   | 200          |
| CMB               | 1690±20| 80±20        |
| BnGa              | 1687±17| 200±25       |
| L+P(WI08)         | 1711±0.6| 84±20±2     |
| L+P(EPECUR)       | 1725±18| 200±37       |
| L+P(Bin Shift)    | 1653±22| 168±41       |

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