Heavy Scalar Tau Decays in the Complex MSSM:
A Full One-Loop Analysis

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Abstract

We evaluate all two-body decay modes of the heavy scalar tau in the Minimal Supersymmetric Standard Model with complex parameters (cMSSM) and no generation mixing. The evaluation is based on a full one-loop calculation of all decay channels, also including hard and soft QED radiation. The renormalization of the relevant sectors is briefly reviewed. The dependence of the heavy scalar tau decay on the relevant cMSSM parameters is analyzed numerically, including also the decay to Higgs bosons and another scalar lepton or to a tau and the lightest neutralino. We find sizable contributions to many partial decay widths and branching ratios. They are mostly of $O(5−10\%)$ of the tree-level results, but can go up to 20\%. These contributions are potentially important for the correct interpretation of scalar tau decays at the LHC and, if kinematically allowed, at the ILC or CLIC. The evaluation of the branching ratios of the heavy scalar tau will be implemented into the Fortran code FeynHiggs.
1 Introduction

Beside the Higgs boson search another important task at the LHC is to search for physics effects beyond the Standard Model (SM), where the Minimal Supersymmetric Standard Model (MSSM) [1] is one of the leading candidates. Two related important tasks are investigating the mechanism of electroweak symmetry breaking, as well as the production and measurement of the properties of Cold Dark Matter (CDM). The most frequently investigated models for electroweak symmetry breaking are the Higgs mechanism within the SM and within the MSSM. The latter also offers a natural candidate for CDM, the Lightest Supersymmetric Particle (LSP), i.e. the lightest neutralino, $\tilde{\chi}_1^0$ [2]. Supersymmetry (SUSY) predicts two scalar partners for all SM fermions as well as fermionic partners to all SM bosons. Contrary to the case of the SM, in the MSSM two Higgs doublets are required. This results in five physical Higgs bosons instead of the single Higgs boson in the SM. These are the light and heavy $CP$-even Higgs bosons, $h$ and $H$, the $CP$-odd Higgs boson, $A$, and the charged Higgs bosons, $H^\pm$. In the MSSM with complex parameters (cMSSM) the three neutral Higgs bosons mix [3–5], giving rise to the states $h_1, h_2, h_3$. The tree-level input parameters are the charged Higgs boson mass, $M_{H^\pm}$ and $\tan \beta$, the ratio of the two vacuum expectation values.

If SUSY is realized in nature and the scalar quarks and/or the gluino are in the kinematic reach of the LHC, it is expected that these strongly interacting particles are copiously produced. The primarily produced strongly interacting particles subsequently decay via cascades to SM particles and (if $R$-parity conservation is assumed, as we do) the LSP. One step in these decay chains is often the decay of a scalar tau, $\tilde{\tau}_{1,2}$, to a SM particle and the LSP, or as a ‘competing process’ the scalar taus decay to another SUSY particle accompanied by a SM particle. Also neutral and charged Higgs bosons can be produced this way. Via these decays some characteristics of the LSP and/or Higgs bosons can be assessed, see, e.g., Refs. [7,8] and references therein. At any future $e^+e^-$ collider (such as ILC or CLIC) a precision determination of the properties of the observed particles is expected [9]. (For combined LHC/ILC analyses and further prospects see Ref. [12].) Thus, if kinematically accessible, the pair production of scalar taus with a subsequent decay to the LSP and/or Higgs bosons can yield important information about the lightest neutralino and the Higgs sector of the model.

In order to yield a sufficient accuracy, one-loop corrections to the various scalar tau decay modes have to be considered. We take into account all two-body decay modes of the heavy scalar tau, $\tilde{\tau}_2^-$, in the MSSM with complex parameters (cMSSM), but we neglect flavor violation effects. More specifically, we calculate the full one-loop corrections to the partial decay widths:

$$\Gamma(\tilde{\tau}_2^- \to \tilde{\tau}_1^- h_n) \quad (n = 1, 2, 3),$$

$$\Gamma(\tilde{\tau}_2^- \to \tilde{\tau}_1^- Z),$$

$$\Gamma(\tilde{\tau}_2^- \to \tau^- \tilde{\chi}_k^0) \quad (k = 1, 2, 3, 4),$$

$$\Gamma(\tilde{\tau}_2^- \to \tilde{\nu}_\tau H^-),$$

1 Scalar taus can also be produced directly at the LHC, see for instance Ref. [6], where, however, only cross sections for a lighter stau where evaluated numerically.

2 It should be noted that the purely loop induced decay channels $\tilde{\tau}_2^- \to \tilde{\tau}_1^- \gamma$ have been neglected because they yield exactly zero, see Sect. [3] for further details.
\[ \Gamma(\tilde{\tau}_2^- \to \nu_\tau W^-) , \]
\[ \Gamma(\tilde{\tau}_2^- \to \nu_\tau \tilde{\chi}_j^-) \quad (j = 1, 2) , \]

where \( \tilde{\chi}_k^0 \) denotes the neutralinos, \( \tilde{\chi}_j^\pm \) the charginos, \( \tau \) and \( \nu_\tau \) the tau and tau-neutrino and \( Z \) and \( W^\pm \) the SM gauge bosons. The total decay width is defined as the sum of the partial decay widths (1) to (6), where for a given parameter point several channels may be kinematically forbidden.

As explained above, we are especially interested in the branching ratios (BR) of the decays involving a Higgs boson, Eqs. (1), (4) as part of an evaluation of a Higgs production cross section, or involving the LSP, Eq. (3) as part of the measurement of CDM properties at the LHC or a future \( e^+e^- \) collider. Consequently, it is not necessary to investigate three- or four-body decay modes. These only play a significant role once the two-body modes are kinematically forbidden, and thus the relevant BR’s are zero.

We also concentrate on the decays of \( \tilde{\tau}_2^- \) and do not investigate \( \tilde{\tau}_2^+ \) decays. In the presence of complex phases this would lead to somewhat different results. However, such an analysis of \( CP \)-violating effects is beyond the scope of this paper.

Scalar tau decays have been investigated in many analyses over the last decade. Most of them were restricted to tree-level evaluations. Existing loop corrections are restricted to the MSSM with real parameters (rMSSM). First tree-level results for stau decays in the rMSSM were published in Refs. [13–15]. Corresponding tree-level results are implemented in SDECAY [16]. Tree-level results in the cMSSM can be found in Ref. [17–19]. An analysis on three-body decays is given in Ref. [20]. Complete one-loop corrections to sfermion decays in the rMSSM involving SM fermions were presented in Ref. [21]. However, no explicit numerical results for the full one-loop corrections to stau decays are included in this paper. Full one-loop corrections to sfermion decays involving SM gauge bosons in the rMSSM are presented in Ref. [22]. However, again no numerical results for scalar tau decays are included. One-loop corrections to stau decays in the rMSSM, derived in a pure \( \overline{\text{DR}} \) scheme (see below) have been made available in the program package SFOLD [23].

Several methods have been discussed in the literature to extract the complex parameters of the model from experimental measurements. A determination of the trilinear Stau-Higgs coupling, \( A_\tau \), in the rMSSM from heavy MSSM Higgs decays was presented in Ref. [24]. \( CP \)-even observables to extract its phase, \( \varphi_{A_\tau} \), have been analyzed in Ref. [25]. \( CP \)-odd observables for this determination are investigated in Ref. [26] (with more details on the specific LHC analysis in Ref. [27]). Depending on the realized cMSSM parameter space and on some further assumptions on the LHC performance, it seems to be possible to obtain limits on, e.g., \( |A_\tau| \) and \( \varphi_{A_\tau} \).

In this paper we present for the first time a full one-loop calculation for all two-body decay channels of the heavier scalar tau in the cMSSM (with no generation mixing), taking into account soft and hard QED radiation. In Sect. 2 we briefly review the renormalization of all relevant sectors of the cMSSM. Details about the calculation can be found in Sect. 3 and the numerical results for all decay channels are presented in Sect. 4. The conclusions can be found in Sect. 5. The results will be implemented into the Fortran code FeynHiggs [28–31].
2 The relevant sectors of the complex MSSM

All the channels (1) – (6) are calculated at the one-loop level, including real QED radiation. This requires the simultaneous renormalization of several sectors of the cMSSM. In the following subsections we briefly review these sectors to make this article self-contained. Details about the renormalization of most of the sectors can be found in Refs. [32–35].

2.1 The tau lepton/slepton sector of the cMSSM

For the evaluation of the one-loop contributions to the decay channels in Eqs. (1) – (6) a renormalization of the scalar tau (τ̃) and τ-neutrino (ντ̃) sector is needed (we assume no generation mixing). The stau and tau sneutrino mass matrices Mτ̃ and Mντ̃ read

\[
M_{\tilde{\tau}} = \begin{pmatrix}
M_{L}^2 + m_\tau^2 + M_Z^2 c_{2\beta} (I_\tau^3 - Q_\tau s_\tau^2) & m_\tau X_\tau \\
& M_{R}^2 + m_\tau^2 + M_Z^2 c_{2\beta} Q_\tau s_\tau^2
\end{pmatrix},
\]

(7)

\[
M_{\tilde{\nu}_\tau} = M_{L}^2 + I_{\nu_\tau}^3 c_{2\beta} M_Z^2
\]

(8)

with

\[
X_\tau = A_\tau - \mu^* \tan \beta.
\]

(9)

M_{\tilde{\tau}_L} and M_{\tilde{\tau}_R} are the soft SUSY-breaking mass parameters, where M_{\tilde{\tau}_L} is equal for all members of an SU(2)_L doublet. m_\tau and Q_\tau are, respectively, the mass and the charge of the corresponding lepton, I_{\nu_\tau}^3 denotes the isospin of \(\nu/\nu_\tau\), and A_\tau is the trilinear soft-breaking parameter. M_Z and M_W are the masses of the Z and W boson, c_w = M_W/M_Z, and s_w = \sqrt{1 - c_w^2}. Finally we use the short-hand notations c_x = \cos(x), s_x = \sin(x). The mass matrix M_{\tilde{\tau}} can be diagonalized with the help of a unitary transformation U_\tilde{\tau},

\[
D_{\tilde{\tau}} = U_{\tilde{\tau}} M_{\tilde{\tau}} U_{\tilde{\tau}}^T = \begin{pmatrix}
m_{\tilde{\tau}_1}^2 & 0 \\
0 & m_{\tilde{\tau}_2}^2
\end{pmatrix}, \quad U_{\tilde{\tau}} = \begin{pmatrix}
U_{\tilde{\tau}_11} & U_{\tilde{\tau}_12} \\
U_{\tilde{\tau}_21} & U_{\tilde{\tau}_22}
\end{pmatrix}.
\]

(10)

The mass eigenvalues depend only on \(|X_\tau|\). The scalar tau masses will always be mass ordered, i.e. \(m_{\tilde{\tau}_1} \leq m_{\tilde{\tau}_2} \):

\[
m_{\tilde{\tau}_{1,2}}^2 = \frac{1}{2} (M_{L}^2 + M_{R}^2) + m_\tau^2 + \frac{1}{2} I_{\tau}^3 c_{2\beta} M_Z^2
\]

(11)

\[\pm \frac{1}{2} \sqrt{[M_{L}^2 - M_{R}^2 + M_Z^2 c_{2\beta} (I_\tau^3 - 2Q_\tau s_\tau^2)]^2 + 4m_\tau^2 |X_\tau|^2},
\]

\[
m_{\tilde{\nu}_\tau}^2 = M_{L}^2 + I_{\nu_\tau}^3 c_{2\beta} M_Z^2.
\]

(12)

2.1.1 Renormalization

The parameter renormalization can be performed as follows,

\[
M_{\tilde{\tau}} \rightarrow M_{\tilde{\tau}} + \delta M_{\tilde{\tau}}, \quad M_{\tilde{\nu}_\tau} \rightarrow M_{\tilde{\nu}_\tau} + \delta M_{\tilde{\nu}_\tau}
\]

(13)
which means that the parameters in the mass matrix $M_\tau$ are replaced by the renormalized parameters and a counterterm. After the expansion $\delta M_\tau$ contains the counterterm part,

$$
\delta M_{\tilde{\tau}_{11}} = \delta M^2_{\tilde{\tau}_{11}} + 2m_\tau \delta m_\tau - M^2_{\tilde{\tau}_{11}} c_{2\beta} \frac{Q_\tau}{s_{\tau}} \frac{\delta m^2}{s_w} + (I^3_{\tau} - Q_\tau s^2_{\tau})(c_{2\beta} \delta M^2_Z + M^2_Z \delta c_{2\beta}) ,
$$

$$
\delta M_{\tilde{\tau}_{12}} = (A^*_\tau - \mu \tan \beta) \delta m_\tau + m_\tau (\delta A^*_\tau - \mu \delta \tan \beta - \tan \beta \delta \mu) ,
$$

$$
\delta M_{\tilde{\tau}_{21}} = \delta M^*_{\tilde{\tau}_{21}} ,
$$

$$
\delta M_{\tilde{\tau}_{22}} = \delta M^2_{\tilde{\tau}_{22}} + 2m_\tau \delta m_\tau + M^2_{\tilde{\tau}_{22}} c_{2\beta} \frac{Q_\tau}{s_{\tau}} \frac{\delta m^2}{s_w} + Q_\tau s^2_{\tau}(c_{2\beta} \delta M^2_Z + M^2_Z \delta c_{2\beta}) ,
$$

$$
\delta M_{\tilde{\nu}_\tau} = \delta M^2_{\tilde{\nu}_\tau} + I^3_{\nu_\tau} (c_{2\beta} \delta M^2_Z + M^2_Z \delta c_{2\beta}) .
$$

Another possibility for the parameter renormalization of the staus is to start out with

$$
U_\tau \cdot M_\tau \cdot U^\dagger_\tau 
\rightarrow
U_\tilde{\tau} \cdot M_\tilde{\tau} \cdot U^\dagger_\tilde{\tau}
+ \delta M_\tau \cdot U^\dagger_\tau
= \begin{pmatrix}
m^2_{\tilde{\tau}_{11}} & Y_\tau \\
Y^*_\tau & m^2_{\tilde{\tau}_{22}}
\end{pmatrix}
+ \begin{pmatrix}
\delta m^2_{\tilde{\tau}_{11}} & \delta Y_\tau \\
\delta Y^*_\tau & \delta m^2_{\tilde{\tau}_{22}}
\end{pmatrix}
$$

where $\delta m^2_{\tilde{\tau}_{11}}$ and $\delta m^2_{\tilde{\tau}_{22}}$ are the counterterms of the stau mass squares. $\delta Y_\tau$ is the counterterm to the stau mixing parameter $Y_\tau$ (which vanishes at tree-level, $Y_\tau = 0$), and corresponds to the off-diagonal entries in $D_\tau = U_\tau \cdot M_\tau \cdot U^\dagger_\tau$, Eq. (10). Using Eq. (19) one can express $\delta M_\tau$ by the counterterms $\delta m^2_{\tilde{\tau}_{11}}$, $\delta m^2_{\tilde{\tau}_{22}}$, $\delta Y_\tau$ and $\delta Y^*_\tau$. Especially for $\delta M_{\tilde{\tau}_{12}}$ one finds

$$
\delta M_{\tilde{\tau}_{12}} = U^*_{\tilde{\tau}_{11}} \delta m^2_{\tilde{\tau}_{12}} U_{\tilde{\tau}_{12}} + U^*_{\tilde{\tau}_{11}} \delta Y_{\tau} U_{\tilde{\tau}_{12}} + U^*_{\tilde{\tau}_{11}} \delta Y^*_{\tau} U_{\tilde{\tau}_{12}}
$$

Eqs. (15) and (20) yield a relation between $\delta Y_{\tau}$, $\delta A_{\tau}$ and $\delta m_\tau$, see below.

For the field renormalization the following procedure is applied,

$$
\frac{\tilde{\tau}_1}{\tilde{\tau}_2} \rightarrow \left(1 + \frac{i}{2} \delta \bar{Z}_\tau\right) \frac{\tilde{\tau}_1}{\tilde{\tau}_2} \quad \text{with} \quad \delta \bar{Z}_\tau = \begin{pmatrix}
\delta \bar{Z}_{\tilde{\tau}_{11}} & \delta \bar{Z}_{\tilde{\tau}_{12}} \\
\delta \bar{Z}_{\tilde{\tau}_{21}} & \delta \bar{Z}_{\tilde{\tau}_{22}}
\end{pmatrix}
$$

This yields for the renormalized self-energies

$$
\hat{\Sigma}_{\tilde{\tau}_{11}}(p^2) = \Sigma_{\tilde{\tau}_{11}}(p^2) + \frac{1}{2} (p^2 - m^2_{\tilde{\tau}_{11}})(\delta \bar{Z}_{\tilde{\tau}_{11}} + \delta \bar{Z}^*_{\tilde{\tau}_{11}}) - \delta m^2_{\tilde{\tau}_{11}} ,
$$

$$
\hat{\Sigma}_{\tilde{\tau}_{12}}(p^2) = \Sigma_{\tilde{\tau}_{12}}(p^2) + \frac{1}{2} (p^2 - m^2_{\tilde{\tau}_{12}}) \delta \bar{Z}_{\tilde{\tau}_{12}} + \frac{1}{2} (p^2 - m^2_{\tilde{\tau}_{12}}) \delta \bar{Z}^*_{\tilde{\tau}_{12}} - \delta Y_{\tau} ,
$$

$$
\hat{\Sigma}_{\tilde{\tau}_{21}}(p^2) = \Sigma_{\tilde{\tau}_{21}}(p^2) + \frac{1}{2} (p^2 - m^2_{\tilde{\tau}_{21}}) \delta \bar{Z}_{\tilde{\tau}_{21}} + \frac{1}{2} (p^2 - m^2_{\tilde{\tau}_{21}}) \delta \bar{Z}^*_{\tilde{\tau}_{21}} - \delta Y^*_{\tau} ,
$$

$$
\hat{\Sigma}_{\tilde{\tau}_{22}}(p^2) = \Sigma_{\tilde{\tau}_{22}}(p^2) + \frac{1}{2} (p^2 - m^2_{\tilde{\tau}_{22}}) (\delta \bar{Z}_{\tilde{\tau}_{22}} + \delta \bar{Z}^*_{\tilde{\tau}_{22}}) - \delta m^2_{\tilde{\tau}_{22}} ,
$$

$$
\hat{\Sigma}_{\tilde{\nu}_\tau}(p^2) = \Sigma_{\tilde{\nu}_\tau}(p^2) + \frac{1}{2} (p^2 - m^2_{\tilde{\nu}_\tau}) (\delta \bar{Z}_{\tilde{\nu}_\tau} + \delta \bar{Z}^*_{\tilde{\nu}_\tau}) - \delta m^2_{\tilde{\nu}_\tau} .
$$

In order to complete the tau lepton/slepton sector renormalization also for the corresponding lepton (i.e. the $\tau$ mass, $m_\tau$, and the lepton fields $\tau_L$, $\tau_R$, $\nu_{\tau L}$) renormalization constants have to be introduced:

$$
m_\tau \rightarrow m_\tau + \delta m_\tau ,
$$

---

\footnote{The unitary matrix $U_\tau$ can be expressed by a mixing angle and a corresponding phase. Then the counterterm $\delta Y_\tau$ can be related to the counterterms of the mixing angle and the phase (see Ref. [36]).}
\[ \tau_{L/R} \rightarrow (1 + \frac{1}{2} \delta Z_{L/R}^{\tau}) \tau_{L/R} , \tag{29} \]
\[ \nu_{\tau_{L}} \rightarrow (1 + \frac{1}{2} \delta Z_{\nu_{\tau}}) \nu_{\tau_{L}}, \tag{30} \]

with \( \delta m_{\tau} \) being the tau mass counterterm and \( \delta Z_{L/R}^{\tau} \) being the \( Z \) factors of the left/right-handed charged lepton fields; \( \delta Z_{\nu_{\tau}} \) is the neutrino field renormalization. Then the renormalized self energy \( \hat{\Sigma}_{\tau} \) can be decomposed into left/right-handed and scalar left/right-handed parts, \( \Sigma_{L/R}^{\tau} \) and \( \Sigma_{SL/SR}^{\tau} \), respectively, while only the left-handed part exists for the self energy \( \hat{\Sigma}_{\nu_{\tau}} \) of the massless neutrino

\[
\hat{\Sigma}_{\tau}(p) = \not{p} \omega_{-} \hat{\Sigma}_{\tau}^{L}(p^2) + \not{p} \omega_{+} \hat{\Sigma}_{\tau}^{R}(p^2) + \omega_{-} \hat{\Sigma}_{\tau}^{SL}(p^2) + \omega_{+} \hat{\Sigma}_{\tau}^{SR}(p^2), \tag{31}
\]
\[
\hat{\Sigma}_{\nu_{\tau}}(p) = \not{p} \omega_{-} \hat{\Sigma}_{\nu_{\tau}}^{L}(p^2), \tag{32}
\]

where the components are given by

\[
\hat{\Sigma}_{\tau}^{L/R}(p^2) = \Sigma_{L/R}^{\tau}(p^2) + \frac{1}{2}(\delta Z_{L/R}^{\tau} + \delta Z_{L/R}^{\tau*}), \tag{33}
\]
\[
\hat{\Sigma}_{\tau}^{SL}(p^2) = \Sigma_{SL}^{\tau}(p^2) - \frac{m_{\tau}}{2}(\delta Z_{L}^{\tau} + \delta Z_{R}^{\tau*}) - \delta m_{\tau}, \tag{34}
\]
\[
\hat{\Sigma}_{\tau}^{SR}(p^2) = \Sigma_{SR}^{\tau}(p^2) - \frac{m_{\tau}}{2}(\delta Z_{R}^{\tau} + \delta Z_{L}^{\tau*}) - \delta m_{\tau}, \tag{35}
\]
\[
\hat{\Sigma}_{\nu_{\tau}}^{L}(p^2) = \Sigma_{\nu_{\tau}}^{L}(p^2) + \frac{1}{2}(\delta Z_{\nu_{\tau}}^{L} + \delta Z_{\nu_{\tau}}^{L*}), \tag{36}
\]

and \( \omega_{\pm} = \frac{1}{2}(\mathbb{1} \pm \gamma_{5}) \) are the right- and left-handed projectors, respectively. It should be noted that \( \text{Re} \hat{\Sigma}_{\tau}^{SR}(p^2) = (\text{Re} \hat{\Sigma}_{\tau}^{SL}(p^2))^{*} \) holds due to \( CPT \) invariance. \( \text{Re} \) denotes the real part with respect to contributions from the loop integral, but leaves the complex couplings unaffected.

### 2.1.2 The tau neutrino/sneutrino sector

We follow closely the renormalization presented in Ref. [32,33], slightly modified to be applicable to the tau/stau sector.

(i) The tau neutrino is defined on-shell (OS), yielding the one-loop field renormalization

\[
\text{Re} \delta Z_{\nu_{\tau}} = -\text{Re} \Sigma_{\nu_{\tau}}(0), \quad \text{Im} \delta Z_{\nu_{\tau}} = 0. \tag{37}
\]

(ii) The \( \tilde{\nu}_{\tau} \) mass is defined OS,

\[
\text{Re} \hat{\Sigma}_{\tilde{\nu}_{\tau}}(m_{\tilde{\nu}_{\tau}}^2) = 0. \tag{38}
\]

This yields for the tau sneutrino mass counter terms

\[
\delta m_{\tilde{\nu}_{\tau}}^2 = \text{Re} \Sigma_{\tilde{\nu}_{\tau}}(m_{\tilde{\nu}_{\tau}}^2). \tag{39}
\]

(iii) Due to \( m_{\nu_{\tau}} \equiv 0 \) no off-diagonal parameters in the sneutrino mass matrix have to be renormalized.
(iv) The diagonal tau sneutrino $Z$ factor is determined OS such that the real part of the residuum of the propagator is set to unity,

$$\widetilde{\text{Re}}\Sigma_{\bar{\nu}_\tau}(p^2)|_{p^2=m_{\bar{\nu}_\tau}^2} = 0 .$$

with $\Sigma'(p^2) \equiv \frac{\partial \Sigma(p^2)}{\partial p^2}$. This condition fixes the real part of the diagonal $Z$ factor to

$$\text{Re} \delta Z_{\bar{\nu}_\tau} = -\widetilde{\text{Re}}\Sigma_{\bar{\nu}_\tau}(p^2)|_{p^2=m_{\bar{\nu}_\tau}^2} ,$$

which is correct, since the imaginary part of the diagonal $Z$ factor does not contain any divergences and can be (implicitly) set to zero,

$$\text{Im} \delta Z_{\bar{\nu}_\tau} = 0 .$$

Including absorptive parts of self-energy type corrections into this $Z$ factor leads to new combined factors $\mathcal{Z}$ which are (in general) different for incoming particles/outgoing antiparticles (unbarred) and outgoing particles/incoming antiparticles (barred), see Refs. [33, 34] for more details. The combined diagonal tau sneutrino $Z$ factors read

$$\delta \mathcal{Z}_{\bar{\nu}_\tau} = -\Sigma'_{\bar{\nu}_\tau}(p^2)|_{p^2=m_{\bar{\nu}_\tau}^2} , \quad \delta \mathcal{Z}_{\bar{\nu}_\tau} = \delta \mathcal{Z}_{\bar{\nu}_\tau} .$$

(v) Due to $m_{\nu_\tau} \equiv 0$ no off-diagonal field renormalization for the tau sneutrino has to be performed.

### 2.1.3 The tau/stau sector

We choose the stau masses $m_{\tilde{\tau}_1}$, $m_{\tilde{\tau}_2}$ and the tau mass $m_\tau$ as independent parameters. Since we also require an independent renormalization of the scalar neutrino, this requires an explicit restoration of the $SU(2)_L$ relation, achieved via a shift in the $M_{\tilde{\tau}_L}$ parameter entering the $\tilde{\tau}$ mass matrix (see also Refs. [37,38]). Requiring the $SU(2)_L$ relation to be valid at the loop level induces the following shift in $M_{\tilde{\tau}_L}^2(\tilde{\tau})$

$$M_{\tilde{\tau}_L}^2(\tilde{\tau}) = M_{\tilde{\tau}_L}^2(\tilde{\nu}_\tau) + \delta M_{\tilde{\tau}_L}^2(\tilde{\nu}_\tau) - \delta M_{\tilde{\tau}_L}^2(\tilde{\tau})$$

with

$$\delta M_{\tilde{\tau}_L}^2(\tilde{\tau}) = |U_{\tilde{\tau}_{11}}|^2 m_{\tilde{\tau}_1}^2 + |U_{\tilde{\tau}_{12}}|^2 m_{\tilde{\tau}_2}^2 - U_{\tilde{\tau}_{12}} U_{\tilde{\tau}_{21}}^* Y_{\tau} - U_{\tilde{\tau}_{12}}^* U_{\tilde{\tau}_{21}} Y_{\tau}^* - 2m_\tau \delta m_\tau + M_{\tilde{Z}}^2 c_{2\beta} Q_\tau \delta s_{\omega}^2 - (I_\tau^3 - Q_\tau s_{\omega}^2)(c_{2\beta} \delta M_{\tilde{Z}}^2 + M_{\tilde{Z}}^2 \delta c_{2\beta}) , \quad \delta m_{\tilde{\tau}_L}^2(\tilde{\nu}_\tau) = \delta m_{\tilde{\nu}_\tau}^2 - I_{\nu_\tau}^3 (c_{2\beta} \delta M_{\tilde{Z}}^2 + M_{\tilde{Z}}^2 \delta c_{2\beta}) .$$

This choice avoids problems concerning UV- and IR-finiteness as discussed in detail in Ref. [32], but also leads to shifts in both stau masses, which are therefore slightly shifted away from their on-shell values. An additional shift in $M_{\tilde{\tau}_R}$ recovers at least one on-shell stau mass, which is now compatible with our choice of independent parameters

$$M_{\tilde{\tau}_R}^2(\tilde{\tau}_i) = \frac{m_\tau^2 |A_\tau + \mu \tan \beta|^2}{M_{\tilde{\tau}_L}^2(\tilde{l}) + m_\tau^2 + M_{\tilde{Z}}^2 c_{2\beta}(I_\tau^3 - Q_\tau s_{\omega}^2) - m_\tau^2 - M_{\tilde{Z}}^2 c_{2\beta} Q_\tau s_{\omega}^2 + m_{\tilde{\tau}_i}^2} .$$

6
The choice of stau for this additional shift, which relates its mass to the stau parameter $M_{\tilde{\tau}_R}$, also represents a choice of scenario, with the chosen stau having a dominantly right-handed character. A “natural” choice is to preserve the character of the staus in the renormalization process. With our choice of mass ordering, $m_{\tilde{\tau}_1} \leq m_{\tilde{\tau}_2}$ (see above), this suggests to recover $m_{\tilde{\tau}_1}$ for $M_{\tilde{\tau}_L}^2 > M_{\tilde{\tau}_R}^2$, and to recover $m_{\tilde{\tau}_2}$ for the other mass hierarchy. Consequently, for our numerical choice given below in Tab. 1 we insert $m_{\tilde{\tau}_2}$ into Eq. (47) and recover its original value from the re-diagonalization after applying this shift.

For the tau/stau sector we can now employ a “full” on-shell scheme, where the following renormalization conditions are imposed:

(i) The tau mass is defined on-shell, yielding the one-loop counterterm $\delta m_\tau$:

$$\delta m_\tau = \frac{1}{2} \hat{\text{Re}} \left\{ m_\tau \left[ \Sigma_{\tau}^L(m_\tau^2) + \Sigma_{\tau}^R(m_\tau^2) \right] + \left[ \Sigma_{\tau}^{SL}(m_\tau^2) + \Sigma_{\tau}^{SR}(m_\tau^2) \right] \right\}, \quad (48)$$

referring to the Lorentz decomposition of the self energy $\hat{\Sigma}_\tau(p)$, see Eq. (31).

The field renormalization constants are given by

$$\delta Z_{L/R}^\tau = \frac{1}{2} \hat{\text{Re}} \left\{ \Sigma_{\tau}^{L/R}(m_\tau^2) + m_\tau^2 \left( \Sigma_{\tau}^{L'}(m_\tau^2) + \Sigma_{\tau}^{R'}(m_\tau^2) \right) + m_\tau \left[ \Sigma_{\tau}^{SL'}(m_\tau^2) + \Sigma_{\tau}^{SR'}(m_\tau^2) \right] \right\}, \quad (49)$$

with $\Sigma'(m^2) \equiv \frac{\partial \Sigma(p^2)}{\partial p^2} \big|_{p^2=m^2}$.

(ii) The stau masses are also determined via on-shell conditions [29, 39], yielding

$$\delta m^2_{\tilde{\tau}_i} = \hat{\text{Re}} \Sigma_{\tilde{\tau}_i}(m^2_{\tilde{\tau}_i}) \quad (i = 1, 2). \quad (50)$$

(iii) The non-diagonal entry of Eq. (19) is fixed as [32, 39, 40]

$$\delta Y_\tau = \frac{1}{2} \hat{\text{Re}} \left\{ \Sigma_{\tilde{\tau}_{12}}(m^2_{\tilde{\tau}_1}) + \Sigma_{\tilde{\tau}_{12}}(m^2_{\tilde{\tau}_2}) \right\}, \quad (51)$$

which corresponds to two separate conditions in the case of a complex $\delta Y_\tau$. The counterterm of the trilinear coupling $\delta A_\tau$ can be obtained from the relation of Eqs. (15) and (20),

$$\delta A_\tau = \frac{1}{m_\tau} \left[ U_{\tilde{\tau}_{11}} U_{\tilde{\tau}_{21}}^* (\delta m^2_{\tilde{\tau}_1} - \delta m^2_{\tilde{\tau}_2}) + U_{\tilde{\tau}_{12}} U_{\tilde{\tau}_{22}}^* \delta Y_{\tau}^* + U_{\tilde{\tau}_{12}}^* U_{\tilde{\tau}_{21}} \delta Y_{\tau} - (A_\tau - \mu^* \tan \beta) \delta m_\tau \right] + (\delta \mu^* \tan \beta + \mu^* \delta \tan \beta). \quad (52)$$

So far undetermined are $\delta \tan \beta$ and $\delta \mu$, which are defined via the Higgs sector and the chargino/neutralino sector, see Ref. [33] for details.

(iv) The diagonal scalar tau $Z$ factors are determined OS such that the real parts of the residua of the propagators are set to unity,

$$\hat{\text{Re}} \Sigma_{\tilde{\tau}_i}'(p^2) \big|_{p^2=m^2_{\tilde{\tau}_i}} = 0 \quad (i = 1, 2). \quad (53)$$
This condition fixes the real parts of the diagonal $Z$ factors to

$$\text{Re} \delta Z_{\tilde{t}_i} = -\text{Re} \Sigma'_{\tilde{t}_i}(p^2)\big|_{p^2=m_{\tilde{t}_i}^2} \quad (i = 1, 2), \quad (54)$$

which is correct, since the imaginary parts of the diagonal $Z$ factors does not contain any divergences and can be (implicitly) set to zero,

$$\text{Im} \delta Z_{\tilde{t}_i} = 0 \quad (i = 1, 2). \quad (55)$$

Including absorptive parts of self-energy type corrections into these $Z$ factors leads to new combined factors $\bar{Z}$

$$\delta Z_{\tilde{t}_i} = -\Sigma'_{\tilde{t}_i}(p^2)\big|_{p^2=m_{\tilde{t}_i}^2}, \quad \delta \bar{Z}_{\tilde{t}_i} = \delta Z_{\tilde{t}_i}. \quad (56)$$

(v) For the non-diagonal $Z$ factors we impose the condition that for on-shell staus no transition from one stau to the other occurs,

$$\bar{\text{Re}} \tilde{\Sigma}_{\tilde{t}_12}(m_{\tilde{t}_1}^2) = 0, \quad \bar{\text{Re}} \tilde{\Sigma}_{\tilde{t}_21}(m_{\tilde{t}_2}^2) = 0 \quad (i = 1, 2). \quad (57)$$

This yields

$$\delta Z_{\tilde{t}_{12}} = +2 \frac{\bar{\text{Re}} \tilde{\Sigma}_{\tilde{t}_{12}}(m_{\tilde{t}_1}^2) - \delta Y^*_\tau}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)}, \quad \delta Z_{\tilde{t}_{21}} = -2 \frac{\bar{\text{Re}} \tilde{\Sigma}_{\tilde{t}_{21}}(m_{\tilde{t}_2}^2) - \delta Y^*_\tau}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)}. \quad (58)$$

Taking the absorptive parts of the self-energy type corrections into account, the conditions change to

$$\tilde{\Sigma}_{\tilde{t}_{12}}(m_{\tilde{t}_1}^2) = 0, \quad \tilde{\Sigma}_{\tilde{t}_{21}}(m_{\tilde{t}_2}^2) = 0 \quad (i = 1, 2). \quad (59)$$

This yields the following combined field renormalization constants for incoming particles/outgoing antiparticles (unbarred) and outgoing particles/incoming antiparticles (barred),

$$\delta Z_{\tilde{t}_{12}} = +2 \frac{\Sigma_{\tilde{t}_{12}}(m_{\tilde{t}_1}^2) - \delta Y^*_\tau}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)}, \quad \delta \bar{Z}_{\tilde{t}_{12}} = +2 \frac{\Sigma_{\tilde{t}_{21}}(m_{\tilde{t}_2}^2) - \delta Y^*_\tau}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)}, \quad (60)$$

$$\delta Z_{\tilde{t}_{21}} = -2 \frac{\Sigma_{\tilde{t}_{21}}(m_{\tilde{t}_2}^2) - \delta Y^*_\tau}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)}, \quad \delta \bar{Z}_{\tilde{t}_{21}} = -2 \frac{\Sigma_{\tilde{t}_{21}}(m_{\tilde{t}_2}^2) - \delta Y^*_\tau}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)}. \quad (61)$$

### 2.2 The Higgs and gauge boson sector of the cMSSM

The two Higgs doublets of the cMSSM are decomposed in the following way,

$$\mathcal{H}_1 = \begin{pmatrix} H_{11} \\ H_{12} \end{pmatrix} = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1 - i\chi_1) \\ -\phi_1^- \end{pmatrix}, \quad \mathcal{H}_2 = \begin{pmatrix} H_{21} \\ H_{22} \end{pmatrix} = e^{i\xi} \begin{pmatrix} v_2 + \frac{1}{\sqrt{2}}(\phi_2 + i\chi_2) \\ -\phi_2^- \end{pmatrix}. \quad (62)$$

Besides the vacuum expectation values $v_1$ and $v_2$, in Eqs. (62) a possible new phase $\xi$ between the two Higgs doublets is introduced. The Higgs potential $V_H$ can be written in powers of the Higgs fields,

$$V_H = \ldots + T_{\phi_1} \phi_1 + T_{\phi_2} \phi_2 + T_{\chi_1} \chi_1 + T_{\chi_2} \chi_2$$

8
\[-\frac{1}{2} \begin{pmatrix} \phi_1, \phi_2, \chi_1, \chi_2 \end{pmatrix} M_{\phi\phi\chi\chi} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \chi_1 \\ \chi_2 \end{pmatrix} - \begin{pmatrix} \phi_1^+, \phi_2^+ \end{pmatrix} M_{\phi^\pm\phi^\pm}^T \begin{pmatrix} \phi_1^- \\ \phi_2^- \end{pmatrix} + \ldots, \tag{63}\]

where the coefficients of the linear terms are called tadpoles and those of the bilinear terms are the mass matrices $M_{\phi\phi\chi\chi}$ and $M_{\phi^\pm\phi^\pm}$. After a rotation to the physical fields one obtains

\[V_H = \ldots + T_h h + T_H H + T_A A - \frac{1}{2} \begin{pmatrix} h, H, A, G \end{pmatrix} M_{hHAG}^{\text{diag}} \begin{pmatrix} h \\ H_A \\ A \\ G \end{pmatrix} - \begin{pmatrix} H^+, G^+ \end{pmatrix} M_{H^\pm G^\pm}^{\text{diag}} \begin{pmatrix} H^- \\ G^- \end{pmatrix} + \ldots, \tag{64}\]

where the tree-level masses are denoted as $m_h, m_H, m_A, m_G, M_{H^\pm}, M_{G^\pm}$. With the help of a Peccei-Quinn transformation $\mu$ and the complex soft SUSY-breaking parameters in the Higgs sector can be redefined such that the complex phases vanish at tree-level.

Concerning the renormalization we follow the usual approach where the gauge-fixing term does not receive a net contribution from the renormalization transformations. As input parameter we choose the mass of the charged Higgs boson, $M_{H^\pm}$. All details can be found in Refs. \[31, 33\] (see also Ref. \[43\] for the alternative effective potential approach and Ref. \[44\] for the renormalization group improved effective potential approach including Higgs pole mass effects).

Including higher-order corrections the three neutral Higgs bosons can mix \[3, 5, 31\],

\[(h, H, A) \rightarrow (h_1, h_2, h_3), \tag{65}\]

where we define the loop corrected masses according to

\[M_{h_1} \leq M_{h_2} \leq M_{h_3}. \tag{66}\]

A vertex with an external on-shell Higgs boson $h_n (n = 1, 2, 3)$ is obtained from the decay widths to the tree-level Higgs bosons via the complex matrix $Z$ \[31\],

\[\Gamma_{h_n} = [Z]_{n1}\Gamma_h + [Z]_{n2}\Gamma_H + [Z]_{n3}\Gamma_A + \ldots, \tag{67}\]

where the ellipsis represents contributions from the mixing with the Goldstone boson and the $Z$ boson, see Sect. \[3\] It should be noted that the ‘rotation’ with $Z$ is not a unitary transformation, see Ref. \[31\] for details.

Also the charged Higgs boson appearing as an external particle in a stau decay has to obey the proper on-shell conditions. This leads to an extra $Z$ factor,

\[\hat{Z}_{H^-H^+} = \left[1 + \text{Re} \hat{\Sigma}'_{H^-H^+}(p^2)\big|_{p^2=M_{H^\pm}^2}\right]^{-1}. \tag{68}\]

\[\]4 Corresponding to the convention used in FeynArts/FormCalc, we exchanged in the charged part the positive Higgs fields with the negative ones, which is in contrast to Ref. \[31\]. As we keep the definition of the matrix $M_{\phi^\pm\phi^\pm}$ used in \[31\] the transposed matrix will appear in the expression for $M_{H^\pm G^\pm}^{\text{diag}}$.\]
Expanding to one-loop order yields the $Z$ factor that has to be applied to the process with an external charged Higgs boson,

$$\sqrt{Z_{H-H^+}} = 1 + \frac{1}{2}\delta Z_{H-H^+}$$

with

$$\delta Z_{H-H^+} = -\text{Re} \left( \Sigma_{H-H^+}(M_{H^\pm}^2) \right) = -\text{Re} \left( \Sigma'_{H-H^+}(M_{H^\pm}^2) \right) - Z_{H-H^+}.$$  

As for the neutral Higgs bosons, there are contributions from the mixing with the Goldstone boson and the $W$ boson. This $Z$ factor is by definition UV-finite. However, it contains IR-divergences that cancel with the soft photon contributions from the loop diagrams, see Sect. 3.

For the renormalization of $\tan \beta$ and the Higgs field renormalization the $\overline{\text{DR}}$ scheme is chosen [31, 33]. This leads to the introduction of the scale $\mu_R$, which will be fixed later to the mass of the decaying particle.

### 2.3 The chargino/neutralino sector of the cMSSM

The mass eigenstates of the charginos can be determined from the matrix

$$X = \begin{pmatrix} M_2 & \sqrt{2} \sin \beta M_W \\ \sqrt{2} \cos \beta M_W & \mu \end{pmatrix}.$$  

In addition to the higgsino mass parameter $\mu$ it contains the soft breaking term $M_2$, which can also be complex in the cMSSM. The rotation to the chargino mass eigenstates is done by transforming the original wino and higgsino fields with the help of two unitary $2 \times 2$ matrices $U$ and $V$,

$$\tilde{\chi}_i^- = \left( \begin{array}{c} \psi_i^L \\ \psi_i^R \end{array} \right) \quad \text{with} \quad \psi_i^L = U_{ij} \left( \begin{array}{c} \tilde{W}^-_j \\ \tilde{H}^-_j \end{array} \right) \quad \text{and} \quad \psi_i^R = V_{ij} \left( \begin{array}{c} \tilde{W}^+_j \\ \tilde{H}^+_j \end{array} \right),$$

where the $i$th mass eigenstate can be expressed in terms of either the Weyl spinors $\psi_i^L$ and $\psi_i^R$ or the Dirac spinor $\tilde{\chi}_i^-$. These rotations lead to the diagonal mass matrix

$$M_{\tilde{\chi}^-} = V^* X^T U^\dagger = \text{diag}(m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}).$$

From this relation, it becomes clear that the mass ordered chargino masses $m_{\tilde{\chi}_1^\pm} < m_{\tilde{\chi}_2^\pm}$ can be determined as the (real and positive) singular values of $X$. The singular value decomposition of $X$ also yields results for $U$ and $V$.

A similar procedure is used for the determination of the neutralino masses and mixing matrix, which can both be calculated from the mass matrix

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z s_w \cos \beta & M_Z s_w \sin \beta \\ 0 & M_2 & M_Z c_w \cos \beta & -M_Z c_w \sin \beta \\ -M_Z s_w \cos \beta & M_Z c_w \cos \beta & 0 & -\mu \\ M_Z s_w \sin \beta & -M_Z c_w \sin \beta & -\mu & 0 \end{pmatrix}.$$  


This symmetric matrix contains the additional complex soft-breaking parameter $M_1$. The diagonalization of the matrix is achieved by a transformation starting from the original bino/wino/higgsino basis,

$$
\tilde{\chi}_k^0 = \left( \frac{\psi_k^0}{\psi_k^0} \right)
$$

with

$$
\psi_k^0 = N_{kl} (\tilde{B}_0, \tilde{W}_0, \tilde{H}_1^0, \tilde{H}_2^0)_l^T,
$$

$$
M_{\tilde{\chi}} = N^* Y N^T = \text{diag}(m_{\tilde{\chi}_1}^0, m_{\tilde{\chi}_2}^0, m_{\tilde{\chi}_3}^0, m_{\tilde{\chi}_4}^0),
$$

where $\psi_k^0$ denotes the two component Weyl spinor and $\tilde{\chi}_k^0$ the four component Majorana spinor of the $k$th neutralino field. The unitary $4 \times 4$ matrix $N$ and the physical neutralino masses result from a numerical Takagi factorization of $Y$. The symmetry of $Y$ permits the non-trivial condition of using only one matrix $N$ for its diagonalization, in contrast to the chargino case shown above.

Concerning the renormalization we use the results of Ref. [33, 45–48]. Since the chargino masses $m_{\tilde{\chi}^+}$, $m_{\tilde{\chi}^+_2}$ and the lightest neutralino mass $m_{\tilde{\chi}_1}^0$ have been chosen as independent parameters the one-loop masses of the heavier neutralinos are obtained from the tree-level ones with the shifts

$$
\Delta m_{\tilde{\chi}_k}^0 = - \text{Re} \left[ m_{\tilde{\chi}_k}^0 \tilde{\Sigma}_L^{k_1} (m_{\tilde{\chi}_k}^2) + \tilde{\Sigma}_L^{k_2} (m_{\tilde{\chi}_k}^2) \right]_{kk} \quad (k = 2, 3, 4),
$$

where the renormalized self energies of the neutralino have been decomposed into their left/right-handed and scalar left/right-handed parts as in Eq. (31). $\Delta m_{\tilde{\chi}_1}^0 = 0$ is just the real part of one of our renormalization conditions. Special care has to be taken in the regions of the cMSSM parameter space where the gaugino-higgsino mixing in the chargino sector is maximal, i.e. where $\mu \approx M_2$. Here $\delta M_2$ (see Eq. (180) in Ref. [33]) and $\delta \mu$ (see Eq. (181) in Ref. [33]) diverge as $(U_{11}^* U_{22} V_{11}^* V_{22}^* - U_{12}^* U_{21} V_{12}^* V_{21}^*)^{-1} \text{ and }$ and the loop calculation does not yield a reliable result. An analysis of various renormalization schemes was recently published in Ref. [49], where this kind of divergences were discussed. In Ref. [49] it was furthermore emphasized that in the case of the renormalization of two charginos and one neutralino mass always the most bino-like neutralino has to be renormalized in order to find a numerically stable result (see also Ref. [50]). In our numerical set-up, see Sect. 4, the lightest neutralino is nearly always rather bino-like. If required, however, it would be trivial to change our prescription from the lightest neutralino to any other neutralino.

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5 Similar divergences appearing in the on-shell renormalization in the sbottom sector, occurring for “maximal sbottom mixing”, have been observed and discussed in Refs. [32,33].

6 In Ref. [49] it was also suggested that the numerically most stable result is obtained via the renormalization of one chargino and two neutralinos. However, in our approach, this choice leads to IR divergences, since the chargino mass changes (from the tree-level mass to the one-loop pole mass) by a finite shift due to the renormalization procedure. Using the shifted mass for the external chargino, but the tree-level mass for internal charginos results in IR divergences. On the other hand, in general, inserting the shifted chargino mass everywhere yields UV divergences. Consequently, we stick to our choice of imposing on-shell conditions for the two charginos and one neutralino.
3 Calculation of loop diagrams

In this section we give some details about the calculation of the higher-order corrections to the partial decay widths of scalar taus. Generic diagrams are shown in Figs. 1 – 6. Not shown are the diagrams for real (hard or soft) photon radiation. They are obtained from the corresponding tree-level diagrams by attaching a photon to the electrically charged particles. The internal generically depicted particles in Figs. 1 – 6 are labeled as follows: \( F \) can be a tau \( \tau \), tau-neutrino \( \nu_\tau \), chargino \( \tilde{\chi}^\pm_j \) or neutralino \( \tilde{\chi}^0_k \), \( S \) can be a sfermion \( \tilde{f}_i \) or a Higgs boson \( h_n \), \( V \) can be a photon \( \gamma \) or a massive SM gauge boson, \( Z \) or \( W^\pm \). For internally appearing Higgs bosons no higher-order corrections to their masses or couplings are taken into account; these corrections would correspond to effects beyond one-loop order.

For external Higgs bosons, as described in Sect. 2.2, the appropriate \( Z \) factors are applied and on-shell masses (including higher order corrections) are used (as evaluated with \texttt{FeynHiggs} [28–31]).

Also not shown are the diagrams with a gauge–Higgs boson or a Goldstone–Higgs boson self-energy contribution on the external Higgs boson leg. They appear in the decay \( \tilde{\tau}^-_2 \rightarrow \tilde{\tau}^-_1 h_n \), Fig. 1 with a \( Z/G-h_n \) transition and in the decay \( \tilde{\tau}^-_2 \rightarrow \tilde{\nu}_\tau H^- \), Fig. 5 with a \( W^-/G^-–H^- \) transition. The corresponding self-energy diagram belonging to the process \( \tilde{\tau}^-_2 \rightarrow \tilde{\tau}^-_1 Z \) or \( \tilde{\tau}^-_2 \rightarrow \tilde{\nu}_\tau W^- \), respectively, yields a vanishing contribution for external on-shell gauge bosons due to \( \varepsilon \cdot p = 0 \) for \( p^2 = M_Z^2 \) (\( p^2 = M_W^2 \)), where \( p \) denotes the external momentum and \( \varepsilon \) the polarization vector of the gauge boson.

Furthermore, in general, in Figs. 1 – 6 we have omitted diagrams with self-energy type corrections of external (on-shell) particles. While the contributions from the real parts of the loop functions are taken into account via the renormalization constants defined by on-shell renormalization conditions, the contributions coming from the imaginary part of the loop functions can result in an additional (real) correction if multiplied by complex parameters (such as \( A_\tau \)). In the analytical and numerical evaluation, these diagrams have been taken into account via the prescription outlined in Sect. 2 and their numerical contributions are included in the results discussed in Sect. 4.

Finally it should be noted that the purely loop induced decay channels \( \tilde{\tau}^-_2 \rightarrow \tilde{\tau}^-_1 \gamma \) yield exactly zero due to the fact that the decay width is proportional to \( \varepsilon \cdot p \) and the photon is on-shell, i.e. \( \varepsilon \cdot p = 0 \).

The diagrams and corresponding amplitudes have been obtained with \texttt{FeynArts} version 3.7 [51]. The model file, including the MSSM counterterms, is largely based on Ref. [48], however adjusted to match exactly the renormalization prescription described in Sect. 2.2 see also Refs. [32–35]. The further evaluation has been performed with \texttt{FormCalc} version 7.3 (and \texttt{LoopTools} version 2.7) [52].

Ultraviolet divergences

As regularization scheme for the UV-divergences we have used constrained differential renormalization [53], which has been shown to be equivalent to dimensional reduction [54] at the one-loop level [52]. Thus the employed regularization scheme preserves SUSY [55, 56] and

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7 We found that using loop corrected Higgs boson masses in the loops leads to a UV divergent result.
8 From a technical point of view, the \( W^-/G^-–H^- \) transitions have been absorbed into the respective counterterms, while the \( Z/G-h_n \) transitions have been calculated explicitly.
guarantees that the SUSY relations are kept intact, e.g. that the gauge couplings of the SM vertices and the Yukawa couplings of the corresponding SUSY vertices also coincide to one-loop order in the SUSY limit. Therefore no additional shifts, which might occur when using a different regularization scheme, arise. All UV-divergences cancel to all orders in the final result.

**Infrared divergences**

The IR-divergences from diagrams with an internal photon have to cancel with the ones from the corresponding real soft radiation. They are included via analytical formulas following the description given in Ref. [57]. The IR-divergences arising from the diagrams involving a $\gamma$ are regularized by introducing a photon mass parameter, $\lambda$. All IR-divergences, i.e. all divergences in the limit $\lambda \to 0$, cancel to all orders once virtual and real diagrams for one decay channel are added.

However, in order to achieve the all order cancellation, special care has to be taken in the decay modes involving scalar neutrinos and a $W$ boson. Using tree-level stau masses yields a cancellation of IR divergences to all orders for all $\tilde{\tau}_2^-$ decay modes. However, inserting the one-loop corrected stau masses (see Sect. [2.1]), as required for consistency, we found...
cancellation to all orders of the related IR divergences, except for the decay mode $\tilde{\tau}_2^- \rightarrow \tilde{\nu}_\tau W^-$. Within this decay the tree-level relation required by the $SU(2)$ symmetry $M_{\tilde{\tau}_L}(\tilde{\tau}) = M_{\tilde{\nu}_L}(\tilde{\nu})$, corresponding to

$$|U_{\tilde{\tau}_11}|^2 m_{\tilde{\tau}_1}^2 + |U_{\tilde{\tau}_12}|^2 m_{\tilde{\tau}_2}^2 = m_{\tilde{\nu}_\tau}^2 + m_{\tau}^2 - M_Z^2 \cos 2\beta ,$$

has to be fulfilled to yield a cancellation of all IR divergences. On the other hand, the requirement of on-shell stau masses as well as an intact $SU(2)$ relation at the one-loop level leads to the necessity of a shift in the scalar tau masses, see Eq. (78). Therefore Eq. (78) is “violated” at the one-loop level, introducing a two-loop IR divergence in $\Gamma(\tilde{\tau}_2^- \rightarrow \tilde{\nu}_\tau W^-)$. In order to eliminate this two-loop IR divergence we introduced a counterterm in the $\tilde{\tau}_2 \tilde{\nu}_\tau W$ vertex,

$$\delta Z_{\text{int}} = \left( |U_{\tilde{\tau}_11}|^2 m_{\tilde{\tau}_1}^2 + |U_{\tilde{\tau}_12}|^2 m_{\tilde{\tau}_2}^2 - |U_{\tilde{\tau}_11}|^2 m_{\tilde{\nu}_\tau}^2 - |U_{\tilde{\tau}_12}|^2 m_{\tau}^2 \right) \times \text{IR div} ,$$

Eq. (78) has been deduced via

$$M_{\tilde{\tau}_L}^2(\tilde{\tau}) = |U_{\tilde{\tau}_11}|^2 m_{\tilde{\tau}_1}^2 + |U_{\tilde{\tau}_12}|^2 m_{\tilde{\tau}_2}^2 - M_Z^2 c_{2\beta} (\tan^2 \beta - Q_{\tau} s_{\tau}^2) - m_{\tau}^2 ,$$

$$M_{\tilde{\nu}_L}^2(\tilde{\nu}) = m_{\tilde{\nu}_\tau}^2 - M_W^2 c_{2\beta} M_Z^2$$

Figure 2: Generic Feynman diagrams for the decay $\tilde{\tau}_2^- \rightarrow \tilde{\tau}_1^- Z$. $F$ can be a tau, tau-neutrino, chargino, or neutralino, $S$ can be a sfermion or a Higgs boson, $V$ can be a $\gamma$, $Z$ or $W^\pm$. 
to restore the tree-level $SU(2)$ relation. The left term in Eq. (80) contains only ‘tree-level’ values, while the index ‘shift’ refers to inserting the one-loop masses and mixing matrices. The IR divergence has been taken from Eq. (B.5) of Ref. [58] (it can also be found in
Figure 5: Generic Feynman diagrams for the decay $\tilde{\tau}^-_2 \to \tilde{\nu}_\tau H^-$. $F$ can be a tau, tau-neutrino, chargino or neutralino, $S$ can be a sfermion or a Higgs boson, $V$ can be a $\gamma$, $Z$ or $W^\pm$. Not shown are the diagrams with a $W^+ - H^+$ or $G^+ - H^+$ transition contribution on the external Higgs boson leg.

Ref. [59]), and reads (in our case):

$$\text{IR div} = -\frac{\alpha}{2\pi} \frac{x_\tau \ln(x_\tau)}{m_{\tilde{\tau}_2} M_W (1 - x_\tau^2)} \ln \left( \frac{m_{\tilde{\tau}_2} M_W}{\lambda^2} \right)$$

with

$$x_\tau = \frac{\sqrt{1 - 4 m_{\tilde{\tau}_2} M_W / (m_{\tilde{\tau}_2}^2 + i0 - (M_W - m_{\tilde{\tau}_2})^2)} - 1}{\sqrt{1 - 4 m_{\tilde{\tau}_2} M_W / (m_{\tilde{\tau}_2}^2 + i0 - (M_W - m_{\tilde{\tau}_2})^2)} + 1},$$

where $i0$ denotes an infinitesimally small imaginary part. After including this tree-level relation restoring counterterm we find an IR finite results to all orders as required.

Tree-level formulas

For completeness we show here also the formulas for the tree-level decay widths:

$$\Gamma_{\text{tree}}(\tilde{\tau}^-_2 \to \tilde{\tau}^-_1 h_n) = \frac{|C(\tilde{\tau}^-_2, \tilde{\tau}^-_1, h_n)|^2 \lambda^{1/2}(m_{\tilde{\tau}_2}^2, m_{\tilde{\tau}_1}^2, m_{h_n}^2)}{16 \pi m_{\tilde{\tau}_2}^3} \quad (n = 1, 2, 3),$$

$$\Gamma_{\text{tree}}(\tilde{\tau}^-_2 \to \tilde{\tau}^-_1 Z) = \frac{|C(\tilde{\tau}^-_2, \tilde{\tau}^-_1, Z)|^2 \lambda^{3/2}(m_{\tilde{\tau}_2}^2, m_{\tilde{\tau}_1}^2, M_Z^2)}{16 \pi M_Z^2 m_{\tilde{\tau}_2}^3},$$

(81, 83, 84)
Figure 6: Generic Feynman diagrams for the decay $\tilde{\tau}^- \to \tilde{\nu}_\tau W^-$. $F$ can be a tau, tau-neutrino, chargino or neutralino, $S$ can be a sfermion or a Higgs boson, $V$ can be a $\gamma$, $Z$ or $W^\pm$.

\[
\Gamma^{\text{tree}}(\tilde{\tau}_2^- \to \tau^- \tilde{\chi}_k^0) = \left[ |C(\tilde{\tau}_2^-, \tau^-, \tilde{\chi}_k^0)_L|^2 + |C(\tilde{\tau}_2^-, \tau^-, \tilde{\chi}_k^0)_R|^2 \right] (m_{\tilde{\tau}_2}^2 - m_{\tau}^2 - m_{\tilde{\chi}_k}^2) \\
- 4 \text{Re}\{C(\tilde{\tau}_2^-, \tau^-, \tilde{\chi}_k^0)_L C(\tilde{\tau}_2^-, \tau^-, \tilde{\chi}_k^0)_R\} m_{\tau} m_{\tilde{\chi}_k} \times \\
\frac{\lambda^{1/2}(m_{\tilde{\tau}_2}^2, m_{\tau}^2, m_{\tilde{\chi}_k}^2)}{16 \pi m_{\tilde{\tau}_2}^3} \quad (k = 1, 2, 3, 4),
\]

(85)

\[
\Gamma^{\text{tree}}(\tilde{\tau}_2^- \to \nu_\tau \tilde{\chi}_j^0) = |C(\tilde{\tau}_2^-, \nu_\tau, \tilde{\chi}_j^0)_L|^2 + |C(\tilde{\tau}_2^-, \nu_\tau, \tilde{\chi}_j^0)_R|^2 \times \\
\frac{\lambda^{1/2}(m_{\tilde{\tau}_2}^2, 0, m_{\tilde{\chi}_j}^2)}{16 \pi m_{\tilde{\tau}_2}^3} \quad (j = 1, 2),
\]

(86)

\[
\Gamma^{\text{tree}}(\tilde{\tau}_2^- \to \tilde{\nu}_\tau H^-) = |C(\tilde{\tau}_2^-, \tilde{\nu}_\tau, H^-)|^2 \frac{\lambda^{1/2}(m_{\tilde{\tau}_2}^2, m_{\tilde{\nu}_\tau}^2, M_{H^\pm}^2)}{16 \pi m_{\tilde{\tau}_2}^3},
\]

(87)

\[
\Gamma^{\text{tree}}(\tilde{\tau}_2^- \to \tilde{\nu}_\tau W^-) = |C(\tilde{\tau}_2^-, \tilde{\nu}_\tau, W^-)|^2 \frac{\lambda^{3/2}(m_{\tilde{\tau}_2}^2, m_{\tilde{\nu}_\tau}^2, M_{W}^2)}{16 \pi M_{W}^2 m_{\tilde{\tau}_2}^3},
\]

(88)

where $\lambda(x, y, z) = (x - y - z)^2 - 4yz$, and the couplings $C(a, b, c)$ can be found in the FeynArts model files [60]. $C(a, b, c)_{L,R}$ denote the part of the coupling which is proportional to $(1 \mp \gamma_5)/2$. 

17
Comparison with other calculations

As discussed in the introduction, hardly any numerical results for stau decays at the loop level are available in the literature. We employed the program SFOLD [23] to obtain numerical results for scalar tau decays. SFOLD is based on a complete \(\overline{\text{DR}}\) renormalization at one-loop order (restricted to the rMSSM), but with the possibility also having OS masses (instead of the \(\overline{\text{DR}}\) masses) internal and/or external\(^{10}\). SFOLD uses a running electromagnetic coupling \(\alpha(Q)\) with \(Q\) denoting the \(\overline{\text{DR}}\) scale. This leads to a numerical value significantly higher than \(\alpha(0)\), see Eq. (95) below. Consequently, our tree-level results differ substantially. However, at the loop-level the two results are in better agreement as expected. This agreement improves for lower values of \(Q\), but differences at the level of 5% were found for \(Q \sim 2\) TeV.

4 Numerical analysis

In this section we present a numerical analysis of all 12 decay channels. In the various figures below we show the partial decay widths and their relative correction at the tree-level (“tree”) and at the one-loop level (“full”),

\[
\Gamma_{\text{tree}} \equiv \Gamma_{\text{tree}}(\tilde{\tau}_2^- \to xy), \quad \Gamma_{\text{full}} \equiv \Gamma_{\text{full}}(\tilde{\tau}_2^- \to xy), \quad \delta \Gamma/\Gamma_{\text{tree}} \equiv \frac{\Gamma_{\text{full}} - \Gamma_{\text{tree}}}{\Gamma_{\text{tree}}}, \tag{89}
\]

where \(xy\) denotes the specific final state. The total decay width is defined as the sum of all 12 partial decay widths,

\[
\Gamma_{\text{tot}}^\text{tree} \equiv \sum_{xy} \Gamma_{\text{tree}}(\tilde{\tau}_2^- \to xy), \quad \Gamma_{\text{tot}}^\text{full} \equiv \sum_{xy} \Gamma_{\text{full}}(\tilde{\tau}_2^- \to xy), \quad \delta \Gamma_{\text{tot}}/\Gamma_{\text{tot}}^\text{tree} \equiv \frac{\Gamma_{\text{full}}^\text{tot} - \Gamma_{\text{tree}}^\text{tot}}{\Gamma_{\text{tree}}^\text{tot}}. \tag{90}
\]

We also show the absolute and relative changes of the branching ratios,

\[
\Gamma_{\text{tot}}^\text{tree} \equiv \frac{\Gamma_{\text{tree}}(\tilde{\tau}_2^- \to xy)}{\Gamma_{\text{tot}}^\text{tree}}, \quad \Gamma_{\text{tot}}^\text{full} \equiv \frac{\Gamma_{\text{full}}(\tilde{\tau}_2^- \to xy)}{\Gamma_{\text{tot}}^\text{full}}, \quad \delta \Gamma_{\text{tot}}/\Gamma_{\text{tot}}^\text{tree} \equiv \frac{\Gamma_{\text{full}}^\text{tot} - \Gamma_{\text{tree}}^\text{tot}}{\Gamma_{\text{tree}}^\text{tot}}. \tag{91}
\]

The last quantity is relevant for an analysis of the impact of the one-loop corrections on the phenomenology at the LHC and the ILC.

4.1 Parameter settings

The renormalization scale \(\mu_R\) has been set to the mass of the decaying particle, i.e. \(\mu_R = m_{\tilde{\tau}_2}\). The SM parameters are chosen as follows, see also [61]:

- Fermion masses:
  
  \[
  m_e = 0.51099891 \ \text{MeV} , \quad m_{\nu_e} = 0 , \quad m_\mu = 105.658367 \ \text{MeV} , \quad m_{\nu_\mu} = 0 ,
  \]

\(^{10}\) It should be noted that we had to use \(\overline{\text{DR}}\) masses everywhere for our comparison.
\[ m_\tau = 1776.82 \text{ MeV} , \quad m_\nu_\tau = 0 , \]
\[ m_u = 62.8 \text{ MeV} , \quad m_d = 62.8 \text{ MeV} , \]
\[ m_c = 1.27 \text{ GeV} , \quad m_s = 101 \text{ MeV} , \]
\[ m_t = 172.0 \text{ GeV} , \quad m_b = 4.67 \text{ GeV} . \] (92)

\[ m_u \text{ and } m_d \text{ are effective parameters, calculated through the hadronic contributions to:} \]
\[ \Delta \alpha^{(5)}_{\text{had}}(M_Z) = \frac{\alpha}{\pi} \sum_{f=u,c,d,s,b} Q_f^2 \left( \ln \frac{M_Z^2}{m_f^2} - \frac{5}{3} \right) = 0.02793 . \] (93)

- The CKM matrix has been set to unity.
- Gauge boson masses:
  \[ M_Z = 91.1876 \text{ GeV} , \quad M_W = 80.399 \text{ GeV} . \] (94)
- Coupling constant:
  \[ \alpha \equiv \alpha(0) = 1/137.035999679 . \] (95)

The Higgs sector quantities (masses, mixings, etc.) have been evaluated using FeynHiggs (version 2.8.6) [28–31] [11].

We will show the results for some representative numerical examples. The parameters are chosen according to the scenario S, shown in Tab. [1] but with one of the parameters varied. For the scalar quark sector we have chosen \( M_\tilde{q}_L = M_\tilde{q}_R = \frac{1}{2} A_q = 1000 \text{ GeV} \) (\( q = u, c, t, d, s, b \)) to yield \( M_{h_1} \approx 120 \text{ GeV} \). The value of \( M_1 \) is fixed via the GUT relation \( M_1 = \frac{5}{3} \tan^2 \theta_W M_2 \approx \frac{1}{2} M_2 \). The scenarios are defined such that all decay modes are open simultaneously to permit an analysis of all channels, i.e. not picking specific parameters for each decay. We will start with a variation of \( m_\tilde{\tau}_2 \), and show later the results for varying \( \phi_{A_L} \). The scenarios are in agreement with the MSSM Higgs boson searches at LEP [62, 63], the Tevatron [64] and the LHC [65]. Furthermore the following exclusion limits [61] hold in our scenario:

\[ m_\tilde{\chi}_1^0 > 46 \text{ GeV} , \quad m_\tilde{\chi}_2^0 > 62 \text{ GeV} , \quad m_\tilde{\chi}_3^0 > 100 \text{ GeV} , \quad m_\tilde{\chi}_4^0 > 116 \text{ GeV} , \quad m_\tilde{\chi}_1^\pm > 94 \text{ GeV} . \] (96)

A few examples of the stau and sneutrino masses in S are shown in Tab. [2]. We assume SUSY mass scales that allow for the copious production of the colored particles at the LHC, with the subsequent cascade decay to uncolored particles we are interested in. Furthermore, in S the production of \( \tilde{\tau}_2^- \) at the ILC(1000), i.e. with \( \sqrt{s} = 1000 \text{ GeV} \), via \( e^+ e^- \rightarrow \tilde{\tau}_1^+ \tilde{\tau}_2^- \) will be possible, with all the subsequent decay modes (1) – (6) being open. The clean environment of the ILC would permit a detailed study of the scalar tau decays. We find, depending on the mixing in the stau sector, cross sections up to \( \sigma(e^+ e^- \rightarrow \tilde{\tau}_1^+ \tilde{\tau}_2^-) \sim 1 \text{ fb} \), where these larger cross sections are found for larger mixing. Even larger cross sections are found in the case of \( \sigma(e^+ e^- \rightarrow \tilde{\tau}_1^+ \tilde{\tau}_1^-) \). An integrated luminosity of \( \sim 1 \text{ ab}^{-1} \) would yield up to 1000 scalar taus for \( \sigma = 1 \text{ fb} \). The ILC environment together with such high numbers of

\[ ^{11} \text{As default value within FeynHiggs, } \mu_R = m_t \text{ is used.} \]
### Table 1: MSSM input parameters for the initial numerical investigation; all masses are in GeV.

| Scen. | tan \( \beta \) | \( M_{H^\pm} \) | \( m_{\tilde{\tau}_2} \) | \( m_{\tilde{\tau}_1} \) | \( M_{\tilde{q}_{L,R}} \) | \( \mu \) | \( A_t \) | \( A_q \) | \( M_1 \) | \( M_2 \) | \( M_3 \) |
|-------|------------------|----------------|------------------|------------------|------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( S \) | 5                | 200            | 550              | \( \frac{1}{2} m_{\tilde{\tau}_2} \) | 1000             | 150            | \( \frac{9}{5} m_{\tilde{\tau}_2} \) | 2000            | \( \sim \frac{1}{2} M_2 \) | 250            | 1500           |

In our analysis, \( M_{\tilde{q}_{L,R}} \) are chosen such that the values of \( m_{\tilde{\tau}_1} \) and \( m_{\tilde{\tau}_2} \) are realized. For the \( \tilde{\tau} \) sector the shifts in \( M_{\tilde{q}_{L,R}}(\tilde{\tau}) \) as defined in Eqs. (44) and (47) are taken into account. \( M_{\tilde{q}_{L,R}} \) denote the diagonal soft SUSY-breaking parameters in the scalar quark mass matrices, while \( A_q \) is the trilinear squark Higgs coupling, and \( M_3 \) denotes the gluino mass parameter. The values for \( A_f \) (\( f = \tau, t, b, \ldots \)) are chosen such that charge- and color-breaking minima are avoided [66].

**Table 2:** The stau and tau-sneutrino masses in \( S \) for the numerical investigation; at the right-hand side of the table the shifts as defined in Eqs. (44) and (47) have been taken into account. All masses are in GeV and rounded to one MeV.

| Without shifts | With shifts |
|----------------|-------------|
| \( m_{\tilde{\tau}_1} \) | \( m_{\tilde{\tau}_1} \) |
| \( m_{\tilde{\tau}_2} \) | \( m_{\tilde{\tau}_2} \) |
| \( m_{\tilde{\nu}_\tau} \) | \( m_{\tilde{\nu}_\tau} \) |
| 275.000         | 274.478     |
| 550.000         | 550.000     |
| 263.924         | 263.924     |

Produced staus would result in an accuracy of the relative branching ratio (Eq. (91)) close to the statistical uncertainty: a BR of 30% could be determined down to \( \sim 5\% \). Depending on the combination of allowed decay channels a determination of the branching ratios at the few per-cent level might be achievable in the high-luminosity running of the ILC(1000).

The numerical results we will show in the next subsections are of course dependent on choice of the SUSY parameters. Nevertheless, they give an idea of the relevance of the full one-loop corrections. As an example, the largest decay widths are \( \Gamma(\tilde{\tau}_2^- \rightarrow \tau^- \tilde{\chi}_1^0) \), dominating the total decay width, \( \Gamma_{\text{tot}} \), and thus the various branching ratios. This is due to the strong bino component in \( \tilde{\chi}_1^0 \) in combination with a relatively small mixing in the \( \tilde{\tau} \) sector. For other choices of \( \mu, M_1, M_2 \), e.g. \( \mu \ll M_{1,2} \) and/or larger mixing in the \( \tilde{\tau} \) sector, the light neutralinos would be higgsino dominated and the decay widths would turn out to be substantially smaller. Consequently, the corrections to the (other) decay widths would stay the same, but the branching ratios would look very different. Channels (and their respective one-loop corrections) that may look unobservable due to the smallness of their BR in the plots shown below, could become important if other channels are kinematically forbidden.

### 4.2 Full one-loop results for varying \( m_{\tilde{\tau}_2} \)

The results shown in this and the following subsections consist of “tree”, which denotes the tree-level value and of “full”, which is the partial decay width including all one-loop corrections as described in Sect. 3. We start the numerical analysis with partial decay widths of \( \tilde{\tau}_2^- \) evaluated as a function of \( m_{\tilde{\tau}_2} \), starting at \( m_{\tilde{\tau}_2} = 220 \) GeV up to \( m_{\tilde{\tau}_2} = 2 \) TeV,
which roughly coincides with the reach of CLIC. The upper panels contain the results for the absolute value of the various partial decay widths, $\Gamma(\tilde{\tau}_2^- \to xy)$ (left) and the relative correction from the full one-loop contributions (right). The lower panels show the same results for $\text{BR}(\tilde{\tau}_2^- \to xy)$.

Since in this section all parameters are chosen to be real no contributions from absorptive parts of self-energy type corrections on external legs can contribute. This will be different in Sect. 4.3.

In Fig. 7–9 we show the results for the process $\tilde{\tau}_2^- \to \tilde{\tau}_1^- h_n$ ($n = 1, 2, 3$) as a function of $m_{\tilde{\tau}_2}$. Here, as well as in the other channels some dips and peaks appear, which are due to various thresholds in self-energy or vertex diagram contributions. Three kinks that are present in principle in all decays (with the partial exception of $\tilde{\tau}_2^- \to \tilde{\nu}_\tau H^-$, see below), but that are only partially visible, appear at $m_{\tilde{\tau}_2} \approx 364$ GeV $\approx M_A + m_{\tilde{\tau}_1}$, 366 GeV $\approx M_{H^\pm} + m_{\tilde{\tau}_1}$, 373 GeV $\approx M_H + m_{\tilde{\tau}_1}$. The thresholds appear in the stau self-energies and thus enter via $\delta m_{\tilde{\tau}_2}^2$, $[\delta Z_{\tilde{\tau}}]_{12,22}$ and $\delta Y_{\tilde{\tau}}$. Visible in the plots is only the kink at $m_{\tilde{\tau}_2}$ $\approx$ 366 GeV.

In the decays $\tilde{\tau}_2^- \to \tilde{\tau}_1^- h_n$ at $m_{\tilde{\tau}_2} \approx 583$ GeV we find a kink due to $m_{\tilde{\tau}_1} \approx m_{\tau} + m_{\chi_4^0}$. One can see that the size of the corrections of the partial decay widths is especially large very close to the production threshold from which on the considered decay mode is kinematically possible. Away from this threshold relative corrections of $\sim +5\%, +6\%, +6\%$ are found for $h_1, h_2, h_3$, respectively. In (all) the plots the value of $m_{\tilde{\tau}_2}$ for which $m_{\tilde{\tau}_1} + m_{\tilde{\tau}_2} = 1000$ GeV is shown as a vertical line, i.e. the region where the heavier stau can be produced at the ILC(1000). In these regions the size of the corrections is only slightly smaller than the numbers above. The BRs are at the per-cent level for all three channels. The relative change in the BRs for the masses accessible at the ILC(1000) are about $-5\%, -4\%, -4\%$ for $h_1, h_2, h_3$, respectively. For larger masses, only accessible at CLIC, the one-loop corrections are even smaller. Depending on the MSSM parameters (and the channels kinematically allowed) the one-loop contributions presented here can be relevant for analyses at the ILC and potentially as well as at the LHC.

Next, in Fig. 10 we show results for the decay $\Gamma(\tilde{\tau}_2^- \to \tilde{\tau}_1^- Z)$. The dips due to the thresholds in $[\delta Z_{\tilde{\tau}}]_{12,22}$, $\delta Y_{\tilde{\tau}}$ and $\delta m_{\tilde{\tau}_2}^2$ are the same as before. Furthermore at $m_{\tilde{\tau}_2} \approx 416.7$ GeV a sign change in the stau mixing matrix takes place, and the tree-level result comes out zero for $U_{\tilde{\tau}} = I$. Consequently, around this value the loop corrections are substantially larger than the tree-level result, however, not invalidating the perturbative series. The relative corrections to the partial decay width in $S$ range between $+5\%$ at low $m_{\tilde{\tau}_2}$, i.e. in the “ILC(1000) regime”, to about zero at large $m_{\tilde{\tau}_1}$, with the exception of the region around $m_{\tilde{\tau}_2} \approx 416.7$ GeV.

Now we turn to the decays $\tilde{\tau}_2^- \to \tau^- \chi_k^0$ ($k = 1, 2, 3, 4$), with the results shown in Figs. 11–14. Since $\mu$, $M_1$ and $M_2$ are roughly of the same order, the four states are a mixture of gauginos and higgsinos, however, the two lighter states carry a substantial bino component, which is the only one in the case of small $\tilde{\tau}$ mixing, which is not suppressed by small lepton masses. Consequently, these two partial decay widths, $\Gamma(\tilde{\tau}_2^- \to \tau^- \chi_{1,2}^0)$ are found to be roughly the same and dominating above all other decay widths. Also $\Gamma(\tilde{\tau}_2^- \to \tau^- \chi_{3,4}^0)$

\[\text{Here and below we round most of the values to one GeV.}\]

\[\text{It should be noted that a calculation very close to threshold requires the inclusion of additional (non-relativistic) contributions, which is beyond the scope of this paper. Consequently, very close to threshold our calculation (at the tree- or loop-level) does not provide a very accurate description of the decay width.}\]
are roughly the same but significantly smaller, due to their small bino component, where the dominating higgsino component is proportional to the (suppressed) Yukawa coupling. Apart from the above mentioned general dips and thresholds, we find another threshold for $\Gamma(\tilde{\tau}_2^\pm \to \tau^0 \tilde{\chi}^{\mp}_{2,4})$ for $m_{\tilde{\tau}_2} = 300,576$ GeV, where $m_{\tilde{\chi}^{\pm}_{2,4}} = m_{\tilde{\tau}_1} + m_{\tau}$, respectively. This threshold appears in the stau self-energies and enters via the field renormalization of the neutralinos \[33\]. Small steps can be observed at $m_{\tilde{\tau}_2} = 524,528,544$ GeV in the decay $\tilde{\tau}_2^- \to \tau^- \tilde{\chi}^0_4$. At these values of the heavy stau mass kinks in the vertex loop function occurs. The larger partial decay widths in $S$ for the decay modes $\tilde{\tau}_2^- \to \tau^- \tilde{\chi}^0_k$ with $k = 1,2$ go up to $\sim 5$ GeV for $m_{\tilde{\tau}_2} = 2$ TeV. The radiative corrections are at the 8% (10%) level for $\tilde{\tau}_2^- \to \tau^- \tilde{\chi}^0_1(2)$, respectively. Since these decays are dominating the total width the effect in the corresponding branching ratios is small. For $\Gamma(\tilde{\tau}_2^- \to \tau^- \tilde{\chi}^0_3(4))$ the corrections are around 10% (8%), respectively. All four decay modes show a substantial one-loop correction in the region accessible at the ILC(1000).

Next in Figs. 15, 16 we present the results for $\tilde{\tau}_2^- \to \nu_{\tau} \tilde{\chi}^j_1 (j = 1,2)$. The size of the partial decay widths and branching ratios for $\tilde{\tau}_2^- \to \nu_{\tau} \tilde{\chi}^1_1 (\tilde{\tau}_2^- \to \nu_{\tau} \tilde{\chi}^2_2)$ are relatively small, below 0.1 (0.01) GeV, respectively. Apart from the general thresholds we find two additional ones for each decay, located at $m_{\tilde{\tau}_2} \approx 284.8,287.6 (593.8,597.0)$ GeV, where $m_{\tilde{\chi}^-_1} = m_{\tau} + m_{\nu_{\tau}}, m_{\tilde{\chi}^-_2} = m_{\nu_{\tau}}, m_{\tilde{\chi}^+_2} = m_{\nu_{\tau}}$. The thresholds occur in the respective chargino self-energies and thus enter via $\delta M_2, \delta m_{\tilde{\chi}^+_2}$ and the field renormalization constants. The corrections to the decay widths are $\sim +5\%, -5\%$ for $\tilde{\tau}_2^- \to \nu_{\tau} \tilde{\chi}^1_1$ and $\tilde{\tau}_2^- \to \nu_{\tau} \tilde{\chi}^2_2$ in the ILC(1000) relevant region and thus potentially relevant. For larger $m_{\tilde{\tau}_2}$ they become smaller in the case of the lighter chargino and larger for the heavier chargino.

We now turn to the decay mode $\tilde{\tau}_2^- \to \nu_{\tau} H^-$, which is shown in Fig. 17. In addition to the general dips and thresholds another one can be found at $m_{\tilde{\tau}_2} \approx 601$ GeV, where $m_{\tilde{\nu}_{\tau}} = m_{\tau} + m_{\tilde{\chi}^0_2}$ is realized, i.e. the threshold enters via the sneutrino field renormalization. The corrections to $\Gamma(\tilde{\tau}_2^- \to \nu_{\tau} H^-)$ is $\sim 4.5\%$ in the ILC(1000) regions, and thus potentially relevant, and rises to $\sim 8\%$ for large $m_{\tilde{\tau}_2}$. The relative correction to $\text{BR}(\tilde{\tau}_2^- \to \nu_{\tau} H^-)$ reaches $-5\%$ for low $m_{\tilde{\tau}_2}$ values.

Finally, results for the other decay mode involving scalar neutrinos, $\tilde{\tau}_2^- \to \nu_{\tau} W^-$, are shown in Fig. 18. We find two additional dips due to thresholds at $m_{\tilde{\tau}_2} = 291,601$ GeV, where $m_{\tilde{\nu}_{\tau}} = m_{\tau} + m_{\tilde{\chi}^0_{1,2}}$. Furthermore, as in the decay $\tilde{\tau}_2^- \to \tilde{\tau}_1^- Z$, at $m_{\tilde{\tau}_2} \approx 416.7$ GeV a sign change in the stau mixing matrix takes place, and the tree-level result comes out zero for $U_\tau = \mathbb{U}$. Consequently, around this value the loop corrections are substantially larger than the tree-level result, however, not invalidating the perturbative series (see above). The one-loop corrections to the decay width are found to be below the 3% level, except around the threshold at $m_{\tilde{\tau}_2} \approx 416.7$ GeV.
Figure 7: $\Gamma(\tilde{\tau}_2^- \to \tilde{\tau}_1^- h_1)$. Tree-level and full one-loop corrected partial decay widths are shown with the parameters chosen according to $S$ (see Tab. 1), with $m_{\tilde{\tau}_2}$ varied. The upper left plot shows the partial decay width, the upper right plot shows the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot shows the relative correction of the BR. The vertical lines indicate where $m_{\tilde{\tau}_2} + m_{\tilde{\tau}_1} = 1000$ GeV, i.e. the maximum reach of the ILC(1000).
Figure 8: $\Gamma(\tilde{\tau}_2^- \rightarrow \tilde{\tau}_1^- h_2)$. Tree-level and full one-loop corrected partial decay widths are shown with the parameters chosen according to $S$ (see Tab. 1), with $m_{\tilde{\tau}_2}$ varied. The upper left plot shows the partial decay width, the upper right plot shows the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot shows the relative correction of the BR. The vertical lines indicate where $m_{\tilde{\tau}_2} + m_{\tilde{\tau}_1} = 1000$ GeV, i.e. the maximum reach of the ILC(1000).
Figure 9: $\Gamma(\tilde{\tau}_2^- \rightarrow \tilde{\tau}_1^- h_3)$. Tree-level and full one-loop corrected partial decay widths are shown with the parameters chosen according to $S$ (see Tab. 1), with $m_{\tilde{\tau}_2}$ varied. The upper left plot shows the partial decay width, the upper right plot shows the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot shows the relative correction of the BR. The vertical lines indicate where $m_{\tilde{\tau}_2} + m_{\tilde{\tau}_1} = 1000$ GeV, i.e. the maximum reach of the ILC(1000).

Figure 10: $\Gamma(\tilde{\tau}_2^- \to \tilde{\tau}_1^- Z)$. Tree-level and full one-loop corrected partial decay widths are shown with the parameters chosen according to $S$ (see Tab. 1), with $m_{\tilde{\tau}_2}$ varied. The upper left plot shows the partial decay width, the upper right plot shows the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot shows the relative correction of the BR. The vertical lines indicate where $m_{\tilde{\tau}_2} + m_{\tilde{\tau}_1} = 1000$ GeV, i.e. the maximum reach of the ILC(1000).
Figure 11: $\Gamma(\tilde{\tau}_2^- \rightarrow \tau^- \tilde{\chi}_1^0)$. Tree-level and full one-loop corrected partial decay widths are shown with the parameters chosen according to $S$ (see Tab. 1), with $m_{\tilde{\tau}_2}$ varied. The upper left plot shows the partial decay width, the upper right plot shows the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot shows the relative correction of the BR. The vertical lines indicate where $m_{\tilde{\tau}_2} + m_{\tilde{\chi}_1} = 1000$ GeV, i.e. the maximum reach of the ILC(1000).
Figure 12: $\Gamma(\tilde{\tau}_2^- \to \tau^- \tilde{\chi}_2^0)$. Tree-level and full one-loop corrected partial decay widths are shown with the parameters chosen according to $S$ (see Tab. 1), with $m_{\tilde{\tau}_2}$ varied. The upper left plot shows the partial decay width, the upper right plot shows the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot shows the relative correction of the BR. The vertical lines indicate where $m_{\tilde{\tau}_2} + m_{\tilde{\tau}_1} = 1000$ GeV, i.e. the maximum reach of the ILC(1000).
Figure 13: $\Gamma(\tilde{\tau}_2^- \rightarrow \tau^- \tilde{\chi}_3^0)$. Tree-level and full one-loop corrected partial decay widths are shown with the parameters chosen according to $S$ (see Tab. [1], with $m_{\tilde{\tau}_2}$ varied. The upper left plot shows the partial decay width, the upper right plot shows the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot shows the relative correction of the BR. The vertical lines indicate where $m_{\tilde{\tau}_2} + m_{\tilde{\tau}_1} = 1000$ GeV, i.e. the maximum reach of the ILC(1000).
Figure 14: $\Gamma(\tilde{\tau}_2^- \to \tau^- \tilde{\chi}_4^0)$. Tree-level and full one-loop corrected partial decay widths are shown with the parameters chosen according to $S$ (see Tab. 1), with $m_{\tilde{\tau}_2}$ varied. The upper left plot shows the partial decay width, the upper right plot shows the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot shows the relative correction of the BR. The vertical lines indicate where $m_{\tilde{\tau}_2} + m_{\tilde{\chi}_1} = 1000$ GeV, i.e. the maximum reach of the ILC(1000).
Figure 15: $\Gamma(\tilde{\tau}_2^- \to \nu_\tau \tilde{\chi}_1^-)$. Tree-level and full one-loop corrected partial decay widths are shown with the parameters chosen according to $S$ (see Tab. 1), with $m_{\tilde{\tau}_2}$ varied. The upper left plot shows the partial decay width, the upper right plot shows the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot shows the relative correction of the BR. The vertical lines indicate where $m_{\tilde{\tau}_2} + m_{\tilde{\tau}_1} = 1000$ GeV, i.e. the maximum reach of the ILC(1000).
Figure 16: $\Gamma(\tilde{\tau}_2^- \to \nu_\tau \tilde{\chi}_2^-)$. Tree-level and full one-loop corrected partial decay widths are shown with the parameters chosen according to $\mathcal{S}$ (see Tab. 1), with $m_{\tilde{\tau}_2}$ varied. The upper left plot shows the partial decay width, the upper right plot shows the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot shows the relative correction of the BR. The vertical lines indicate where $m_{\tilde{\tau}_2} + m_{\tilde{\chi}_1} = 1000$ GeV, i.e. the maximum reach of the ILC(1000).
Figure 17: $\Gamma(\tilde{\tau}_2^- \to \tilde{\nu}_\tau H^-)$. Tree-level and full one-loop corrected partial decay widths are shown with the parameters chosen according to $S$ (see Tab. 1), with $m_{\tilde{\tau}_2}$ varied. The upper left plot shows the partial decay width, the upper right plot shows the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot shows the relative correction of the BR. The vertical lines indicate where $m_{\tilde{\tau}_2} + m_{\tilde{\tau}_1} = 1000$ GeV, i.e. the maximum reach of the ILC(1000).
Figure 18: $\Gamma(\tilde{\tau}_2^- \rightarrow \tilde{\nu}_\tau W^-)$. Tree-level and full one-loop corrected partial decay widths are shown with the parameters chosen according to $S$ (see Tab. [1]), with $m_{\tilde{\tau}_2}$ varied. The upper left plot shows the partial decay width, the upper right plot shows the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot shows the relative correction of the BR. The vertical lines indicate where $m_{\tilde{\tau}_2} + m_{\tilde{\tau}_1} = 1000$ GeV, i.e. the maximum reach of the ILC(1000).
4.3 Full one-loop results for varying $\varphi_{A_r}$

In this subsection we analyze the various partial decay widths\footnote{Again we note, that we do not investigate the decays of $\tilde{\tau}_2^+$ here, which would correspond to an analysis of $CP$-asymmetries, which is beyond the scope of this paper.} and branching ratios as a function of $\varphi_{A_r}$. The other parameters are chosen according to Tab. 1. Thus, within $S$ we have $m_{\tilde{\tau}_1} + m_{\tilde{\tau}_2} = 825$ GeV, i.e. the production channel $e^+e^- \to \tilde{\tau}_1^+\tilde{\tau}_2^-$ is open at the ILC(1000). Consequently, the accuracy of the prediction of the various partial decay widths and branching ratios should be at the same level (or better) as the anticipated ILC precision. It should be noted that already the tree-level prediction depends on $\varphi_{A_r}$ via the stau mixing matrix.

When performing an analysis involving complex parameters it should be noted that the results for physical observables are affected only by certain combinations of the complex phases of the parameters $\mu$, the trilinear couplings $A_f$ ($f = \tau, t, b, \ldots$) and the gaugino mass parameters $M_1, M_2, M_3$\footnote{In particular the phase $\varphi_{M_2}$ can appear in the couplings, contributions from absorptive parts of self-energy type corrections on external legs can arise, and they are included in the numerical results shown as “full”. The corresponding formulas for an inclusion of these absorptive contributions via finite wave function correction factors can be found in Sect. 2.} and the gaugino mass parameters $M_1, M_2, M_3$. It is possible, for instance, to rotate the phase $\varphi_{M_2}$ away. Experimental constraints on the (combinations of) complex phases arise in particular from their contributions to electric dipole moments of the electron and the neutron (see Refs. 68, 69 and references therein), of the deuteron 70 and of heavy quarks 71. While SM contributions enter only at the three-loop level, due to its complex phases the MSSM can contribute already at one-loop order. Large phases in the first two generations of sfermions can only be accommodated if these generations are assumed to be very heavy 72 or large cancellations occur 73, see however the discussion in Ref. 74. A recent review can be found in Ref. 75. Accordingly (using the convention that $\varphi_{M_2} = 0$, as done in this paper), in particular the phase $\varphi_{M_1}$ is tightly constrained 76, while the bounds on the phases of the third generation trilinear couplings are much weaker. The phases of $\mu$ and $A_{\tau, t, b}$ enter only in the combinations $(\varphi_{A_{\tau, t, b}} + \varphi_{\mu})$ (or in different combinations together with phases of $M_1$ or $M_3$). Setting $\varphi_{\mu} = 0$ (see above) as well as $\varphi_{M_1} = 0$ (we do not consider this phase in this paper) leaves us with the trilinear couplings as the only complex valued parameters.

The dependence on $\varphi_{A_b}$ and $\varphi_{A_t}$ on the partial decay widths involving scalar bottom and top quarks has been analyzed in detail in Refs. 32, 33, and these phases only enter via loop corrections into the prediction for the stau decays, whereas $A_r$ enters at the tree-level. Consequently, we focus on a complex $A_r$ and keep $A_t$ and $A_b$ real.

Since now a complex $A_r$ can appear in the couplings, contributions from absorptive parts of self-energy type corrections on external legs can arise, and they are included in the numerical results shown as “full”. The corresponding formulas for an inclusion of these absorptive contributions via finite wave function correction factors can be found in Sect. 2.

As before we start with the decays to Higgs bosons, $\tilde{\tau}_2^- \to \tilde{\tau}_1^- h_n$ ($n = 1, 2, 3$) shown in Fig. 19 – 21. The arrangement of the panels is the same as in the previous subsection. In Fig. 19 where the partial decay width $\Gamma(\tilde{\tau}_2^- \to \tilde{\tau}_1^- h_1)$ is given as a function of $\varphi_{A_r}$, one can see that the size of the correction to the partial decay width varies substantially with $\varphi_{A_r}$. The one-loop effects range from $+5\%$ to $+11\%$ in $S$. It should be kept in mind that the parameters are chosen such that $e^+e^- \to \tilde{\tau}_1^+\tilde{\tau}_2^-$ is kinematically possible at the ILC(1000) in $S$, where the knowledge of such a large variation can be very important. For $\tilde{\tau}_2^- \to \tilde{\tau}_1^- h_2$, shown in Fig. 20 the variation with $\varphi_{A_r}$ is even larger, ranging from $+5\%$ to $+16\%$ with similar conclusions for the ILC(1000) as above. The results for $\tilde{\tau}_2^- \to \tilde{\tau}_1^- h_3$...
can be found in Fig. 21. Also here the size of the corrections shows a large variation with \( \varphi_{A_\tau} \), similar to \( \Gamma(\tilde{\tau}_2^- \rightarrow \tilde{\tau}_1^- h_2) \). For the two heavier Higgs bosons, also the variation of the respective branching ratios with \( \varphi_{A_\tau} \) is substantial in \( S \), ranging from \(-4\%\) to \(+6\%\), potentially exceeding the ILC precision.

In Fig. 22 we present the phase dependence for the decay mode \( \tilde{\tau}_2^- \rightarrow \tilde{\tau}_1^- Z \). In our scenario \( S \) the effect of the one-loop corrections to \( \Gamma(\tilde{\tau}_2^- \rightarrow \tilde{\tau}_1^- Z) \) varies from \( \sim -2\%\) to \( \sim +10\%\), again relevant for the ILC precision. An effect of similar size can be observed for \( \text{BR}(\tilde{\tau}_2^- \rightarrow \tilde{\tau}_1^- Z) \).

In Figs. 23 – 26 we present the variation of \( \Gamma(\tilde{\tau}_2^- \rightarrow \tau^- \tilde{\chi}_k^{0}) \) \((k = 1, 2, 3, 4)\) as a function of \( \varphi_{A_\tau} \). As for the variation with \( m_{\tilde{\tau}_2} \) also here for \( k = 1, 2 \) larger values of the partial decay width are found in \( S \) with a similar size as before, again dominating the total width (see the discussions above). The one-loop effects on \( \Gamma(\tilde{\tau}_2^- \rightarrow \tau^- \tilde{\chi}_{1,2}^{0}) \) are about \( 8\%, 10\% \) with a small variation with \( \varphi_{A_\tau} \). For \( \Gamma(\tilde{\tau}_2^- \rightarrow \tau^- \tilde{\chi}_{3,4}^{0}) \), which are substantially smaller, the one-loop effects are of similar size, \( \sim 11\%, 7\% \), respectively, again with a small phase variation. Within \( S \), i.e. with the ILC(1000) accessible parameter space, the one-loop corrections to the various branching ratios is smaller than the effects on the partial widths. However, as discussed above, with a different combination of \( \mu, M_1 \) and \( M_2 \) the one-loop effects can still exceed the potential ILC precision, where, however, only a moderate phase dependence is observed.

The results for \( \Gamma(\tilde{\tau}_2^- \rightarrow \nu_\tau \tilde{\chi}_j^-) \) \((j = 1, 2)\) are shown in Figs. 27, 28. Both decay widths change substantially with \( \varphi_{A_\tau} \). The relative corrections are between \(+2\%\) and \(+9\%\) for \( \tilde{\tau}_2^- \rightarrow \nu_\tau \tilde{\chi}_1^- \), and between \(-7\%\) and \(+3\%\) for \( \tilde{\tau}_2^- \rightarrow \nu_\tau \tilde{\chi}_2^- \). Within \( S \) the variation of the branching ratios is slightly smaller for \( \tilde{\tau}_2^- \rightarrow \nu_\tau \tilde{\chi}_1^- \), and substantially larger, up to \(-16\%\) for \( \tilde{\tau}_2^- \rightarrow \nu_\tau \tilde{\chi}_2^- \), again with a strong variation with \( \varphi_{A_\tau} \), which can then be relevant for the ILC.

Finally we turn to the decay modes involving scalar neutrinos. In Fig. 29 the results for \( \Gamma(\tilde{\tau}_2^- \rightarrow \nu_\tau H^-) \) are presented. The relative correction to the decay width varies strongly between \(+4\%\) and \(+17\%\) with \( \varphi_{A_\tau} \), whereas \( \text{BR}(\tilde{\tau}_2^- \rightarrow \nu_\tau H^-) \) varies within \( S \) between \(-4\%\) and \(+7\%\), potentially exceeding the ILC precision.

The other decay mode involving scalar neutrinos, \( \tilde{\tau}_2^- \rightarrow \nu_\tau W^- \) is analyzed in Fig. 30. The size of the relative correction to the decay width is similar to the \( \tilde{\tau}_2^- \rightarrow \nu_\tau H^- \) channel, varying between \(+8\%\) and \(-2\%\) with \( \varphi_{A_\tau} \), with a corresponding variation in the branching ratio between 0 and \(-11\%\), which is potentially important for physics at the ILC.
Figure 19: $\Gamma(\tilde{\tau}_2^- \to \tilde{\tau}_1^- h_1)$. Tree-level (“tree”) and full one-loop (“full”) corrected partial decay widths (including absorptive self-energy contributions) are shown. The parameters are chosen according to $S$ (see Tab. 1), with $\varphi_{A_r}$ varied. The upper left plot shows the partial decay width, the upper right plot the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot the relative correction of the BR.
Figure 20: $\Gamma(\tilde{\tau}_2^{-} \rightarrow \tilde{\tau}_1^- h_2)$. Tree-level (“tree”) and full one-loop (“full”) corrected partial decay widths (including absorptive self-energy contributions) are shown. The parameters are chosen according to $\mathcal{S}$ (see Tab. 1), with $\varphi_{A_{\tau}}$ varied. The upper left plot shows the partial decay width, the upper right plot the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot the relative correction of the BR.
Figure 21: $\Gamma(\tilde{\tau}_2^\tau \rightarrow \tilde{\tau}_1^\tau h_3)$. Tree-level (“tree”) and full one-loop (“full”) corrected partial decay widths (including absorptive self-energy contributions) are shown. The parameters are chosen according to $S$ (see Tab. 1), with $\varphi_{A_r}$ varied. The upper left plot shows the partial decay width, the upper right plot the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot the relative correction of the BR.
Figure 22: $\Gamma(\tilde{\tau}_2^\pm \to \tilde{\tau}_1^\pm Z)$. Tree-level ("tree") and full one-loop ("full") corrected partial decay widths (including absorptive self-energy contributions) are shown. The parameters are chosen according to $\mathcal{S}$ (see Tab. I), with $\phi_{A_\tau}$ varied. The upper left plot shows the partial decay width, the upper right plot the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot the relative correction of the BR.
Figure 23: $\Gamma(\tilde{\tau}_2^- \rightarrow \tau^- \tilde{\chi}_1^0)$. Tree-level (“tree”) and full one-loop (“full”) corrected partial decay widths (including absorptive self-energy contributions) are shown. The parameters are chosen according to $\mathcal{S}$ (see Tab. 1), with $\varphi_{A_\tau}$ varied. The upper left plot shows the partial decay width, the upper right plot the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot the relative correction of the BR.
Figure 24: $\Gamma(\tilde{\tau}_2^- \rightarrow \tau^- \tilde{\chi}_2^0)$. Tree-level (“tree”) and full one-loop (“full”) corrected partial decay widths (including absorptive self-energy contributions) are shown. The parameters are chosen according to $S$ (see Tab. 1), with $\varphi_{A_\tau}$ varied. The upper left plot shows the partial decay width, the upper right plot the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot the relative correction of the BR.
Figure 25: $\Gamma(\tilde{\tau}_2^- \rightarrow \tau^- \tilde{\chi}_3^0)$. Tree-level ("tree") and full one-loop ("full") corrected partial decay widths (including absorptive self-energy contributions) are shown. The parameters are chosen according to $S$ (see Tab. [1]), with $\varphi_{A_\tau}$ varied. The upper left plot shows the partial decay width, the upper right plot the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot the relative correction of the BR.
Figure 26: $\Gamma(\tilde{\tau}_2^- \to \tau^- \tilde{\chi}_3^0)$. Tree-level (“tree”) and full one-loop (“full”) corrected partial decay widths (including absorptive self-energy contributions) are shown. The parameters are chosen according to $S$ (see Tab. [I]), with $\varphi_A$ varied. The upper left plot shows the partial decay width, the upper right plot the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot the relative correction of the BR.
Figure 27: $\Gamma(\tilde{\tau}_2^- \rightarrow \nu_\tau \tilde{\chi}_1^-)$. Tree-level (“tree”) and full one-loop (“full”) corrected partial decay widths (including absorptive self-energy contributions) are shown. The parameters are chosen according to $S$ (see Tab. 1), with $\varphi_A$ varied. The upper left plot shows the partial decay width, the upper right plot the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot the relative correction of the BR.
Figure 28: $\Gamma(\tilde{\tau}_2^− → \nu_τ\tilde{\chi}_2^-)$. Tree-level (“tree”) and full one-loop (“full”) corrected partial decay widths (including absorptive self-energy contributions) are shown. The parameters are chosen according to $S$ (see Tab.\[1\]), with $\varphi_{Aτ}$ varied. The upper left plot shows the partial decay width, the upper right plot the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot the relative correction of the BR.
Figure 29: $\Gamma(\tilde{\tau}_2^{-} \rightarrow \tilde{\nu}_\tau H^{-})$. Tree-level ("tree") and full one-loop ("full") corrected partial decay widths (including absorptive self-energy contributions) are shown. The parameters are chosen according to $\mathcal{S}$ (see Tab. I), with $\varphi_{A_t}$ varied. The upper left plot shows the partial decay width, the upper right plot the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot the relative correction of the BR.
Figure 30: $\Gamma(\tilde{\tau}_2^- \to \tilde{\nu}_\tau W^-)$. Tree-level (“tree”) and full one-loop (“full”) corrected partial decay widths (including absorptive self-energy contributions) are shown. The parameters are chosen according to $S$ (see Tab. I), with $\phi_A^\tau$ varied. The upper left plot shows the partial decay width, the upper right plot the corresponding relative size of the corrections. The lower left plot shows the BR, the lower right plot the relative correction of the BR.
4.4 The total decay width

Finally we show the results for the total decay width of $\tilde{\tau}^-_2$. In Fig. 31 the upper panels show the absolute and relative variation with $m_{\tilde{\tau}^2}$. The lower panels depict the result for varying $\varphi_{A_{\tau}}$. The dips and peaks visible (best) in the upper right panel have been described in Sect. 4.2. In $S$ the size of the relative corrections of $\Gamma_{\text{tot}}$ ranges between about +8% close to threshold and goes up to above +9% for large values of $m_{\tilde{\tau}^2}$. Such an effect should be detectable at the ILC(1000) or CLIC. The variation with $\varphi_{A_{\tau}}$ is found to be small in our numerical scenario, due to the dominance of $\Gamma(\tilde{\tau}^-_2 \to \tau^- \tilde{\chi}^0_{1,2})$, which shows a small variation with $\varphi_{A_{\tau}}$, see Sect. 4.3. The overall size of the effect of $\varphi_{A_{\tau}}$, shown in the lower row, are around +9%, again a value that should be detectable at a future LC.

5 Conclusions

We evaluate all partial decay widths corresponding to a two-body decay of the heavy scalar tau in the Minimal Supersymmetric Standard Model with complex parameters (cMSSM). The decay modes are given in Eqs. (1) – (6). The evaluation is based on a complete one-loop calculation of all decay channels, also including soft and hard QED radiation. Such a calculation is necessary to derive a reliable prediction of any two-body decay branching ratio. Three-body decay modes can become sizable only if all the two-body decay channels are kinematically (nearly) closed and have thus been neglected throughout the paper.

We first reviewed the one-loop renormalization procedure of the $\tau/\tilde{\tau}$ and $\nu_{\tau}/\tilde{\nu}_{\tau}$ sector (according to the analyses in Refs. [32, 34]) in the cMSSM, which is relevant for our calculation. The details for the Higgs boson and chargino/neutralino sector renormalization can be found in Ref. [33]. We have discussed the calculation of the one-loop diagrams, the treatment of UV- and IR-divergences that are canceled by the inclusion of soft QED radiation.

Our calculation set-up can easily be extended to other two-body decay modes in the cMSSM.

For the numerical analysis we have chosen a parameter set that allows simultaneously all two-body decay modes, i.e. not to maximize any loop effects. The masses of the scalar taus in these scenarios are 275 and 550 GeV for the lighter and the heavier stau, respectively. The production of colored particles at the LHC lead to the subsequent cascade decay production of scalar taus at the LHC. A decay of the heavy stau to a lighter stau (or sneutrino) and a neutral (or charged) Higgs boson can serve as a source of Higgs bosons at the LHC, thus a precise knowledge of stau branching ratios is desirable. The scenario also allows $\tilde{\tau}^+_1\tilde{\tau}^-_2$ production at the ILC(1000) or at CLIC, where statistically dominated experimental measurements of the heavy stau branching ratios will be possible (depending on the details of the MSSM parameters). Depending on the integrated luminosity a precision at the few per-cent level could be achievable.

In our numerical analysis we have shown results for varying $m_{\tilde{\tau}^2}$ and $\varphi_{A_{\tau}}$, the phase of the trilinear coupling $A_{\tau}$. In the results with varied $m_{\tilde{\tau}^2}$ only the lighter values allow $\tilde{\tau}^+_1\tilde{\tau}^-_2$ production at the ILC(1000), whereas the results with varied $\varphi_{A_{\tau}}$ have sufficiently light scalar taus to permit $e^+e^- \rightarrow \tilde{\tau}^+_1\tilde{\tau}^-_2$. In the numerical scenario we compared the tree-level partial widths with the one-loop corrected partial decay widths. In the analysis with $\varphi_{A_{\tau}}$ varied we explicitly included the effect of the absorptive parts of self-energy type...
Figure 31: $\Gamma_{\text{tot}}$. The tree level ("tree") and full one-loop ("full") corrected total decay widths shown with the parameters chosen according to $S$ (see Tab. 1). The upper left plot shows the total decay width, the upper right plot the corresponding relative size of the total corrections, with $m_{\tilde{\tau}_2}$ varied. The vertical lines indicate where $m_{\tilde{\tau}_2} + m_{\tilde{\tau}_1} = 1000$ GeV, i.e. the maximum reach of the ILC(1000). The lower plots show the same but with $\varphi_{A_\tau}$ varied (including absorptive self-energy contributions).
corrections on external legs. We also analyzed the relative change of the partial decay widths to demonstrate the size of the loop corrections on each individual channel. In order to see the effect on the experimentally accessible quantities we also show the various branching ratios at tree-level (all channels are evaluated at tree-level) and at the one-loop level (with all channels evaluated including the full one-loop contributions). Furthermore we presented the relative change of the BRs that can directly be compared with the anticipated experimental accuracy.

We found sizable, roughly $O(5 - 10\%)$, corrections in most of the channels. For some parts of the parameter space (not only close to thresholds) also larger corrections up to 15% or even up to 20% have been observed. The size of the full one-loop corrections to the partial decay widths and the branching ratios also depends strongly on $\varphi_A$. The one-loop contributions, again being roughly of $O(5 - 10\%)$, often vary by a factor of 2 as a function of $\varphi_A$. All results are given in detail in Sects. 4.2 and 4.3.

The numerical results we have shown are, of course, dependent on the choice of the MSSM parameters. Nevertheless, they give an idea of the relevance of the full one-loop corrections. The largest partial decay widths are $\Gamma(\tilde{\tau}_2^- \rightarrow \tau^- \tilde{\chi}^0_{1,2})$ in our scenario, dominating the total decay width, $\Gamma_{\text{tot}}$, and thus the various branching ratios. This is due to the strong bino component in $\tilde{\chi}^0_{1,2}$. For other choices of $\mu$, $M_1$, $M_2$, e.g. $\mu \ll M_{1,2}$, the light neutralinos would be higgsino dominated and the decay widths would turn out to be substantially smaller. Consequently, corrections to the partial decay widths would stay the same, but the branching ratios would look very different. Decay channels (and their respective one-loop corrections) that may look unobservable due to the smallness of their BR in our numerical examples could become important if other channels are kinematically forbidden.

Following our analysis it is evident that the full one-loop corrections are mandatory for a precise prediction of the various branching ratios. The results for the scalar tau decays will be implemented into the Fortran code FeynHiggs.

Acknowledgements

We thank F. Campanario, T. Hahn, W. Hollik, O. Kittel, K. Kovarik, F. von der Pahlen, H. Rzehak and G. Weiglein for helpful discussions. We furthermore thank H. Eberl for assistance with the code SFOLD and corresponding discussions. The work of S.H. was supported in part by CICYT (grant FPA 2010-22163-C02-01) and by the Spanish MICINN’s Consolider-Ingenio 2010 Program under grant MultiDark CSD2009-00064.
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