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Atom loss maximum in ultra-cold Fermi gases

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Abstract. Recent experiments on atom loss in ultra-cold Fermi gases all show a maximum at a magnetic field below Feshbach resonance, where the s-wave scattering length is large (comparable to inter-particle distance) and positive. These experiments have been performed over a wide range of conditions, with temperatures and trap depths spanning three decades. Different groups have come up with different explanations, including the emergence of Stoner ferromagnetism. Here, we show that this maximum is a consequence of two major steps. The first is the establishment of a population of shallow dimers, which is the combined effect of dimer formation through three-body recombination, and the dissociation of shallow dimers back to atoms through collisions. The dissociation process will be temperature dependent and is affected by Pauli blocking at low temperatures. The second is the relaxation of shallow dimers into tightly bound dimers through atom–dimer and dimer–dimer collisions. In these collisions, a significant amount of energy is released. The reaction products leave the trap, leading to trap loss. We have constructed a simple set of rate equations describing these processes. Remarkably, even with only a few parameters, these equations reproduce the loss rate observed in all recent experiments, despite their widely different experimental conditions. Our studies show that the location of the maximum loss rate depends crucially on experimental parameters such as trap depth and temperature. These extrinsic characters show that this maximum is not a reliable probe of the nature of the underlying quantum states. The physics of our equations also explains some general trends found in current experiments.

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1. Introduction

Three-body processes occupy a unique place in the study of ultra-cold atomic gases. On the one hand, while two-body elastic collisions are important for thermal equilibrium, three-body processes are crucial for producing molecules of interest, such as those in the Bose–Einstein condensates–Bardeen–Cooper–Schrieffer (BEC–BCS) crossover. On the other hand, three-body processes can also lead to atom loss from the trap, which limits the lifetime of the sample and constrains the time scale over which experiments can be performed. In addition, the three-body process can be used as a diagnostic tool. Recent advances in Efimov physics have resulted from the use of three-body loss to identify the presence of trimers in a bosonic gas of \(^7\)Li \([1, 2]\).

Theoretically, the study of the three-body problem dates back to the early days of quantum mechanics and has revealed many exciting new phenomena, such as the Thomas effect \([3]\) and the Efimov effect \([4]\). For the case of interest here, namely an equal population of two hyperfine states of fermionic quantum gas such as \(^6\)Li, the Efimov effect is irrelevant. What is important is the process of three-body recombination, in which three atoms collide to form a shallow dimer, with the third atom carrying away the energy released in the process. We shall refer to this as process (I) and denote it as \(A_\uparrow + A_\downarrow + A_\uparrow \rightarrow D + A_\uparrow\) and \(A_\downarrow + A_\uparrow + A_\downarrow \rightarrow D + A_\downarrow\), where \(D\) is the shallow dimer; \(A_\uparrow\) and \(A_\downarrow\) are the atoms in different hyperfine states, which we simply denote as \(\uparrow\) and \(\downarrow\). Note that process (I) occurs only when the two-body s-wave scattering length \(a_s\) is positive, since weakly bound states exist only at \(a_s > 0\). The reverse of process (I) is the dissociation of shallow dimers back to atoms, \(A_\uparrow + D \rightarrow A_\uparrow + A_\uparrow + A_\downarrow\), which we refer to as process (I'). The combined effect of (I) and (I') is to provide a net population of shallow dimers.

Once shallow dimers \(D\) are formed, they can further relax into deep bound states in the van der Waals potential through either atom–dimer collision, \(A + D \rightarrow D^* + A\) (referred to as process (II)), or dimer–dimer collision, \(D + D \rightarrow D^* + D^*\) (referred to as process (III)), where \(D^*\) is the deep bound state. These two processes are the key reasons for particle loss in a trapped gas, since both types of particles (dimer and atoms) in the final product will have very high
kinetic energies due to energy-momentum conservation, and will leave the trap. It should be noted that these two processes can only be activated after a population of shallow dimers is achieved as a result of processes (I) and (I'). Our study shows that the atom loss rate depends on a variety of factors such as trap depth and the temperature of the system. The location of maximum loss rate is not ‘intrinsic’ in the sense that it can change by varying the external conditions such as trap depth, even though the gas parameter, $k_F a_s$, is kept fixed, where $k_F$ is the Fermi wave vector in the center of the trap.

The paper is organized as follows. In section 2, we give an overview of the recent experiments and the range of parameters appearing in these studies. In section 3, we discuss the processes (I)–(III) mentioned above. We show that on general grounds, the loss rate will have a maximum at a magnetic field below Feshbach resonance. In section 4, we implement these processes into a set of rate equations, and give explicit expressions for the rate constants in these equations. In section 5, we show that the results obtained from the rate equations provide a good description for all current experiments that are performed under a variety of external conditions. Finally, in section 6, we further discuss the implications of the ‘two-step’ process (i.e. [(I) + (I')] and [(II) + (III)]) mentioned above.

2. Overview of recent experiments

The experiments we are going to discuss were performed in a mixture of two-component Fermi gases with density $n$ interacting with an s-wave scattering length $a_s$. The interaction strength of the system is characterized by the ‘gas parameter’ $k_F a_s$, where $k_F = (3\pi^2 n)^{1/3}$ is the Fermi wavevector. For positive scattering length, $a_s > 0$, a two-body system can be either in a scattering state or in a bound state. In the literature, these bound states are sometimes referred to as ‘Feshbach molecules’ or sometimes simply as ‘dimers’. The binding energy of the dimer is $E_b = \hbar^2/ma_s^2 > 0$. When the two-body system is in the scattering state (or in the bound state), it is referred to as in the ‘upper branch’ (or ‘lower branch’). The generalization of this terminology to the many-body case (for $a_s > 0$) is as follows. If the quantum gas is in thermal equilibrium, it is referred to as in the ‘lower’ branch. If the quantum gas is prepared in a state where dimers are absent, such as the case at sufficiently high temperature or sufficiently small $k_F a_s$, it is referred to as in the ‘upper branch’. By definition, the system in the upper branch is not in its ground state. However, if the rate of production of Feshbach molecules is sufficiently low, the atoms in the system can be considered to be in a quasi-equilibrium state during the time when experiments are performed.

To our knowledge, all recent experiments on atom loss in a trap were performed in the upper branch. They include many studies by Grimm’s group at Innsbruck [5]; the work of Jin’s group at JILA [6]; the work of Ketterle’s group at MIT [7, 8]; and the work of Salomon’s group at ENS [9]. Those experiments were performed under a wide range of conditions, with temperature $T$ spanning over two orders of magnitude, trapping depth $V_0$ spanning over three orders of magnitude and a wide range of Fermi temperatures $T_F$. The parameters of these experiments are listed in table 1. In all of these experiments, a maximum loss rate is found at a magnetic field $B^*$ below the magnetic field $B_\infty$ where Feshbach resonance occurs. The positive scattering length at which the maximum loss takes place will be denoted as $a_s^*$. The binding energy of the Feshbach molecule (or dimer) at $a_s^*$ will be denoted as $E_b^*$.

Different groups have different views about the physical origin of the maximal loss. The Innsbruck group interpreted their data as due to a two-step process, similar to the mechanism
Table 1. Experimental parameters.

| $V_0$ ($\mu$K) | $T$ ($\mu$K) | $T_F$ ($\mu$K) | $k_B a^*_s$ | $B^*$ (G) | $a^*_s$ (Å) | $E^*_b$ ($\mu$K) | $E^*_b/T$ | $E^*_b/V_0$ | $V_0/T$ | Reference  |
|----------------|-------------|----------------|-------------|----------|-------------|----------------|----------|-------------|-----------|------------|
| 7.1            | 0.28        | 1.4            | 2           | 790      | 4318        | 0.7            | 2.5      | 0.098       | 25.35     | MIT [7], Li |
| 10             | 0.67        | 0.79           | 0.83        | 201.4    | 990         | 1.64           | 2.44     | 0.164       | 14.9      | JILA [6], K |
| 175            | 22          | 21             | 1.6         | 680      | 700         | 17.88          | 0.81     | 0.102       | 7.95      | MIT [8], Li |
| 350            | 22          | 2.8            | 0.359       | 644      | 427         | 64             | 2.9      | 0.183       | 15.9      | Innsbruck [5], Li |
| 500            | 30          | 2.8            | 0.319       | 636      | 380         | 83             | 2.76     | 0.166       | 16.6      | Innsbruck [5], Li |
| 1000           | 60          | 2.8            | 0.286       | 629      | 342         | 104            | 1.73     | 0.104       | 16.6      | Innsbruck [5], Li |
| a              | 2.4         | 6              | 1.2         | 720      | 1207        | 7              | 2.9      | ENS [9], Li |

* Bourdel et al. [9] stated that the trap depth is of the same order as temperature. The specific value for it was not reported.

presented below, but without detailed formulations and calculations [5]. We shall discuss this picture in detail below. The JILA group noted that at the maximal loss, the heating of the system also reaches a maximum. However, no specific proposals were made on the origin of the observation. The ENS group considered the maximum loss to be a consequence of the rising three-body recombination rate at small $a_s$, and the decrease in binding energy of the Feshbach molecule as $a_s$ approaches infinity at resonance. It is argued that the latter reduces heating (and hence loss rate) because the energy release in the three-body recombination is small. However, no quantitative comparison had been made with experimental data using this picture. In the earlier experiment of the MIT group, no explanation was given for this maximum loss. In [7], which was performed at very low temperatures, the maximum loss rate is considered to be evidence of Stoner ferromagnetism. Their point is that if the system turns ferromagnetic, different spins will segregate, which will naturally lead to a vanishing three-body loss rate. The only problem, however, is that no segregation of up and down spins has ever been observed. This explanation is very different from all previous pictures because it makes use of the intrinsic property of the ground state rather than specific microscopic scattering processes. It raises the general question of how reliable it is to use three-body loss as a tool to probe the nature of the ground state, ferromagnetic or not. Very recently, the experiment of MIT [7] was also interpreted as a competition between the BCS pair instability and the Stoner instability, with the conclusion that the BCS pair instability is more important [11]. However, the paper [11] does not discuss the maximum atom loss in other experiments under very different conditions [5–9].

3. The two-step process

Let us first examine the loss channels available for the trapped gas. For simplicity, we start with a system that consists of only atoms, namely, a system in which no shallow dimers are present. In that case, the only loss channel at time $t = 0$ is three-body recombination (process (I)), which we have schematically denoted as

$$A_1^+ + A_1^+ + A_1^- \rightarrow A_1^+ + D$$

Process (I),

and a similar process where up and down spins are interchanged. Here, $A_1$ ($A_1^-$) stands for spin up (down) atoms and $D$ is a shallow dimer that consists of two opposite spins. We have
considered only those three-body recombinations that lead to shallow dimers. In principle, three-body recombinations can also lead to deep bound states that exist in the two-body van der Waals potential. However, the Franck–Condon factors for these transitions are so much smaller than those for shallow dimers that they render the transitions to the deep bound state negligible in comparison. Process (I) is an exothermic process. An important point to note is that the energy released in the process is of the order of the dimer binding energy \( E_b \), which is typically much smaller than the trap depth \( V_0 \), i.e. \( E_b \ll V_0 \) (see table 1). As a result, the shallow dimers \( D \) formed in process (I) do not leave the trap. We shall see that this has important consequences for all other processes to be discussed. In the literature, the rate of process (I) is often denoted as \( L_3 \). This rate has been calculated by Petrov [16]. The result is that \( L_3 \) is proportional to \( a_s^6 \) and depends linearly on the average kinetic energy of the particles.

Together with process (I), there is its reverse process in which a dimer dissociates back to atoms through collisions with other atoms and dimers. We denote the above process as

\[
A_{\uparrow \downarrow} + D \rightarrow A_{\uparrow \downarrow} + A_{\uparrow} + A_{\downarrow} \quad \text{Process (I').}
\]

This will lead to an increase in the number of atoms and a decrease in the number of shallow dimers \( D \) in the trap. Process (I') shares the same microscopic matrix element as (I). However, the density of states of the initial configuration of these two processes are entirely different. For the dissociation process (I'), it depends crucially on temperature and interaction parameters of the system. The population of shallow dimers in the trap is a result of the competing effects (I) and (I').

While both (I) and (I') change the numbers of atoms and dimers, none of these particles is lost from the trap during these processes, since \( E_b \ll V_0 \), as mentioned before. Of course, shallow dimers can also dissociate through dimer–dimer collision,

\[
D + D \rightarrow A_{\uparrow} + A_{\downarrow} + A_{\downarrow} + A_{\downarrow} \quad \text{Process (I'').}
\]

We shall ignore process (I''), since it has the same microscopic matrix element as its reverse process in which four atoms collide to form two shallow dimers, which is very small. While (I) and (I') do not cause particle loss from the trap, processes leading to the formation of a deep bound dimer state will. These processes arise from the collision between shallow dimers and atoms (referred to as process (II)) or the collision of shallow dimers with each other (referred to as process (III)). Schematically, they can be represented as

\[
A_{\uparrow \downarrow} + D \rightarrow A_{\uparrow \downarrow} + D^* \quad \text{Process (II)},
\]

\[
D + D \rightarrow D^* + D^* \quad \text{Process (III)},
\]

where \( D^* \) represents the deep bound state. Since the energy of the deep bound state is large and negative, the atoms and dimers in the final state in processes (II) and (III) will carry a very large kinetic energy and will leave the trap. The rates of processes (II) and (III), denoted as \( L_2 \) and \( L_m \), respectively, have also been calculated by Petrov et al [14, 15], who found that \( L_2 \propto a_s^{-3.3} \) and \( L_m \propto a_s^{-2.5} \).

The origin of the maximum atom loss rate. The processes above show that the loss of atoms from a trap proceeds in two steps. The first is to produce a density of shallow dimers, which is the combined effect of three-body recombination (I) and dimer disassociation (I'). Once the dimers are formed, processes (II) and (III) will be activated and produce atoms and dimers with kinetic energies high enough for them to leave the trap. As one approaches the resonance from
small $k_3a_s$, the rate of three-body recombination (process (I)) scales as $a_s^6$ [16]. The population
of the shallow dimer, and hence the atom loss rate, therefore increases with $a_s$. However, as one
gets closer to resonance, $k_3a_s \to +\infty$, the binding energy of the shallow dimer $E_b = \hbar^2/ma_s^2$
decreases rapidly, making the dissociation process (I') more and more effective, caused by
thermal effects at high temperatures or Pauli blocking effects in the quantum degenerate regime
(see, for example, [17] and discussions later in this paper). At some point, dimer dissociation
(I') will overwhelm the three-body recombination process (I), thereby quenching processes (II)
and (III) and reducing the loss rate.

4. Theoretical model

In this section, we express processes discussed in section 3 in terms of rate equations. Our basic
assumption is that there are well-defined degrees of freedom for the shallow dimers. We shall
denote the number of atoms of up and down spins as $n_\uparrow$ and $n_\downarrow$ and the number of atoms and
the number of dimers as $n_a$ and $n_m$, respectively; here $n_a = n_\uparrow + n_\downarrow$. The processes in section 3 imply

$$\frac{dn_m(t)}{dt} = [L_3(n_\uparrow^3(t)n_\downarrow(t) + n_\uparrow(t)n_\downarrow^2(t)) - qL_3n_a(t)n_m(t)] - L_2n_a(t)n_m(t) - 2L_mn_m(t)^2, \quad (6)$$

$$\frac{dn_\uparrow(t)}{dt} = [-L_3(n_\uparrow^3(t)n_\downarrow(t) + n_\uparrow(t)n_\downarrow^2(t)) + qL_3n_a(t)n_m(t)] - L_2n_\uparrow(t)n_m(t), \quad (7)$$

$$\frac{dn_\downarrow(t)}{dt} = [-L_3(n_\uparrow^3(t)n_\downarrow(t) + n_\uparrow(t)n_\downarrow^2(t)) + qL_3n_a(t)n_m(t)] - L_2n_\downarrow(t)n_m(t). \quad (8)$$

The first terms on the right-hand side of equation (6) describe the production of dimers through
three-body recombination (process (I)) with rate $L_3$. The second term describes the dissociation
process (I') through atom–dimer collision (hence proportional to $n_\uparrow n_m$). We have parameterized
the rate of the process as $L_3q$. As we shall see, $q$ depends on temperature $T$, scattering length
$a_s$ and the total number of particles $n = n_a + 2n_m$,

$$q = q(T, a_s, n), \quad (9)$$

since the ratio between the numbers of atom and shallow dimers in equilibrium depends on the
total particle number $n$ (see discussions later in this section). We have grouped the two terms
($L_3$ and $qL_3$) together in square brackets because they do not contribute to particle loss in the
trap (see discussions in the previous section). The third term in equation (6) describes the loss of
shallow dimers in process (II) through collisions between atoms and shallow dimers that leads to
the production of deeply bound states. The rate of this process is $L_2$. The last term in
equation (6) describes process (III), which describes the loss of shallow dimers due to collisions
between them to form deeply bound states. The rate of this process is $L_m$. Equations (7) and (8)
can be understood similarly, based on the picture that the dimers produced in process (I) do not
leave the system (since the binding energy of the dimer $E_b$ is much less than the typical trap
depth shown in table 1). As a result, the shallow dimers produced continue to participate in the
three-body recombination process and the dissociation process.

With $q$ being a function of $n = n_a + 2n_m = n_\uparrow + n_\downarrow + 2n_m$, equations (6)–(8) form a close
set of equations for $n_\uparrow, n_\downarrow$ and $n_m$, and allow us to study the time evolution of these quantities.
For the case of equal spin population, $n_\uparrow = n_\downarrow = n_a/2$, equations (6)–(8) can be simplified to

$$\frac{dn_m(t)}{dt} = \left[\frac{L_3}{4}n_\uparrow^3(t) - qL_3n_a(t)n_m(t)\right] - L_2n_a(t)n_m(t) - 2L_mn_m(t)^2, \quad (10)$$
\[
\frac{\partial n_a(t)}{\partial t} = -\left[ \frac{L_3}{2} a_3^2(t) - 2q L_3 n_a(t) n_m(t) \right] - L_2 n_a(t) n_m(t).
\]

Note that from equations (6) and (11), the total number of particles \( n = n_a + 2n_m \) decreases as
\[
\frac{\partial n}{\partial t} = -2L_2 n_a(t) n_m(t) - 2L_m n_m^2(t),
\]
which means that only processes (II) and (III) lead to particle loss.

To make use of these equations, we need to obtain the expressions of the parameters \( L_3, L_2, L_m \) and the function \( q(T, a_s, n) \). The typical values of \( L_3, L_2 \) and \( L_m \) can be obtained from existing experiments. Our strategy is to fix their values at one scattering length \( a_s \) and then use the scaling relation derived by Petrov et al [14, 15] to obtain the value at another scattering lengths. For \(^6\)Li, the values of \( L_3 \) and \( L_m \) are given by Jochim et al [5]. At the field \( B = 690 \) G, we have \( L_3(B = 690 \) G\) = 1 x 10⁻²⁵ cm³ s⁻¹. Thus, at a different magnetic field \( B \),

\[
L_3(B) = \left[ \frac{a_s(B)}{a_s(B = 690 \) G\)} \right]^6 L_3(B = 690 \) G\).
\]

The value of \( L_m \) was estimated to be 5 x 10⁻¹¹ cm³ s⁻¹ by Jochim et al [5] at the field \( B = 546 \) G. According to the calculation by Petrov et al [14, 15], at another magnetic field \( B \), the rate coefficient is given by

\[
L_m(B) = \left[ \frac{a_s(B)}{a_s(B = 546 \) G\)} \right]^{-2.5} L_2(B = 546 \) G\).
\]

We have not been able to find the precise value of \( L_2 \) in existing experiments. As commented in [10], the value of \( L_2 \) cannot be safely estimated, but a reasonable value can be estimated as \( L_2(B = 690 \) G\) = 1 x 10⁻¹³ cm³ s⁻¹, and thus at another magnetic field

\[
L_2(B) = \left[ \frac{a_s(B)}{a_s(B = 690 \) G\)} \right]^{-3.3} L_2(B = 690 \) G\).
\]

Note that, strictly speaking, the above formulae were derived for the regime \( k_F a_s < 1 \) [14–16]. We shall assume in our subsequent discussion that these expressions continue to give a reasonable approximation to actual rates in the region \( k_F a_s \sim 1 \).

To determine \( q \), we use the fact that in the absence of the terms \( L_2 \) and \( L_m \), the long time evolution of the equations above should establish chemical equilibrium between atoms and molecules. In that case, we find that

\[
n_{a,eq}^2 = 4q n_{m,eq},
\]
where \( n_{a,eq} = \lim_{t \to \infty} n_a(t) \), and similarly for \( n_{m,eq} \). Equation (16) shows that the quantity \( q \) is simply the ratio of atoms and molecules in an equilibrium mixture. To estimate \( n_{a,eq} \) and \( n_{m,eq} \) (and hence \( q \)), we use the simplest model of a non-interacting mixture, with the approximate Hamiltonian \( K = (h_a - \mu_a n_a) + (h_m - \mu_m n_m) \), where \( h_a \) and \( h_m \) are the Hamiltonians of the atoms and shallow dimers, respectively, and \( \mu_a \) and \( \mu_m \) are their chemical potentials (see also, for example, Kokkelmans et al [12] as well as Chin and Grimm [13]). The energy of the dimer will be denoted as \( E_b \). Equilibration between atoms and dimers is ensured through the relation between their chemical potentials,

\[
2\mu_a = -E_b + \mu_m,
\]
and the number of atoms $n_a$ and the number of dimers are constrained by the condition

$$n = n_{a,\text{eq}} + 2n_{m,\text{eq}}. \quad (18)$$

The explicit forms of $n_a$ and $n_m$ are

$$n_{a,\text{eq}} = 2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\beta(k^2/2m-\mu_a)} + 1}, \quad (19)$$

$$n_{m,\text{eq}} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\beta(k^2/4m-\mu_m)} - 1}, \quad (20)$$

where $\beta = 1/k_B T$ is the inverse temperature and $m$ is the mass of the atom.

**Pauli blocking.** In vacuum, we have $E_b = \hbar^2/(ma_s)$. In a many-body system, as the temperature drops to the quantum degenerate regime, the pair formation is affected by the presence of a Fermi sea, which has taken up some of the momentum states needed for the pair wave function. We note that the Pauli blocking effect becomes more prominent as temperature decreases, since the Fermi surface becomes more sharply defined, hence locking up more momentum states. At high temperature, each momentum state is barely occupied and the Pauli blocking effect is minimized. To capture the effect of Pauli blocking on the formation of a bound state, we consider the analogue of Cooper pairing in the presence of a Fermi sea at finite temperature. Denoting the quantum state of the Cooper pair as $|\Psi\rangle = \sum_k |\Psi_k\rangle = |k\uparrow, -k\downarrow\rangle$, the Schrödinger equation of the pair is

$$E_b |\Psi\rangle = 2\epsilon_k |\Psi\rangle + \frac{u_o}{\Omega} \sum_{k'} (1 - f_k)^2 |\Psi_{k'}\rangle, \quad (21)$$

where $|\Psi_k\rangle$ is the amplitude for the presence of a pair, $\epsilon_k = \hbar^2/(2m)$, $\Omega$ is the volume, $f_k$ is a Fermi function for the atom,

$$f_k = \frac{1}{e^{\beta(\epsilon_k - \mu_s)} + 1}, \quad (22)$$

and $u_o$ is a bare interaction parameter that is designed to reproduce the low-energy scattering amplitude and is related to the s-wave scattering length $a_s$ as

$$\frac{m}{4\pi \hbar^2 a_s} = \frac{1}{u_o} + \frac{1}{\Omega} \sum_k \frac{1}{2\epsilon_k}. \quad (23)$$

The reason for the power 2 in the Fermi exclusion factor $1 - f_k$ is because the scattered state consists of two particles $k'$ and $-k'$. The solution of equation (21) is

$$\frac{m}{4\pi \hbar^2 a_s} = \frac{1}{\Omega} \sum_k \left[ \frac{(1 - f_k)^2}{E_b - 2\epsilon_k} + \frac{1}{2\epsilon_k} \right]. \quad (24)$$

Equation (24) gives $E_b$ as a function of $T$, $a_s$ and $\mu_a$. It is easy to see that in the non-degenerate limit, $e^{\mu_a/T} \to 0$, we have $f_k \to 0$, and equation (24) reduces to the equation for bound state energy in vacuum, which gives the usual result $E_b = \hbar^2/(ma_s^2)$.

Equations (18) and (17) now imply

$$n = 2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\beta(k^2/2m-\mu_a)} + 1} + 2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\beta(k^2/4m-2\mu_a-E_b)} - 1}. \quad (25)$$
Since \( E_b \) is a function of \( T, a_s \) and \( \mu_a, E_b = E_b(a_s, T, \mu_a) \). Equation (25) gives \( \mu_a \) as a function of \( n, T \) and \( a_s \). Once \( \mu_a = \mu_a(n, T, a_s) \) is determined, we can then calculate \( n_a \) and \( n_m \) from equations (19) and (20) and obtain \( q \) of equation (16),

\[
q(T, a_s, n) = n_a^{2\text{eq}}/(4n_m^{\text{eq}}).
\]  

Before proceeding, let us comment on the assumptions of equations (10) and (11). (i) First of all, we have assumed well-defined degrees of freedom for the shallow dimers, which is only valid for small \( k_B a_s \) or high temperature. As a result, our equations will not be accurate very close to the resonance where the dimer degrees of freedom become less well defined due to many-body effects. (ii) The most important effect that is not captured by equations (10) and (11) is heating. In principle, there should be an equation of the form \( \partial T/\partial t = F(T, n_a, n_m; a_s) \), where \( F \) describes the effect of re-thermalization of the energy released from various processes. We have not attempted to construct this equation. The viewpoint we take is that since the physics of the two-step process has already provided an explanation for the loss maximum qualitatively (see section 5), it is useful to first find out how well these equations account for current experiments quantitatively so as to determine the validity and usefulness of these rate equations. The more elaborate effects of heating will be explored elsewhere. (iii) Throughout our discussion, we shall replace the quantum gas in a harmonic trap with a non-uniform density profile by the one in a square box with uniform density. The replacement is mainly for simplicity. While one can perform a full calculation using local density approximation, for the level of accuracy of our comparison, we believe that our simple replacement is sufficient.

5. Comparison with experiments

Essentially, all experiments on atom loss are performed in the following way. One starts with a sample with a fixed number of atoms \( n \) and no dimers at an initial magnetic field \( B_{\text{ini}} \) corresponding to a small and positive scattering length \( a_s^{\text{ini}} \). One then jumps to the magnetic field \( B_{\text{fin}} \), corresponding to the scattering length \( a_s \) of interest, and waits for a time interval \( \Delta t \). Within this time interval, atoms are converted to shallow dimers as well as deep bound states, which then escape from the trap. At the end of this interval, the system has \( n_a \) atoms and \( n_m \) shallow dimers. The number of atoms \( n_a \) is the quantity of interest. To image the number of remaining atoms after the interval \( \Delta t \), one pulls back the magnetic field \( B \) from \( B_{\text{fin}} \) to \( B_{\text{measure}} \), typically corresponding to a scattering length that is small and positive \( a_s^{\text{measure}} \). The rate of the pull back is such that all of the shallow dimers formed at the magnetic field \( B_{\text{fin}} \) are converted to a tightly bound state of size \( a_s^{\text{measure}} \) and therefore will not be counted by the imaging process for atoms. The number of atoms counted at \( B_{\text{measure}} \) (or at \( a_s^{\text{measure}} \)) therefore gives the number of atoms \( n_a \) at the end of the time interval \( \Delta t \).

We have applied the rate equations of section 4 to the different experiments listed in table 1. The results and the parameters used in our calculations are summarized below.

5.1. MIT/2002

In this experiment [8], the initial ensemble consists of \( 3 \times 10^5 \) \(^6\)Li atoms with a peak density of \( 3 \times 10^{13} \) cm\(^{-3}\). The temperature at which the final measurement is carried out is around \( T = 22 \mu K \), slightly higher than the Fermi temperature \( T_F = 21 \mu K \). The trapping parameters are \( \omega_r = 12 \) Hz, \( \omega_z = 200 \) Hz and the trap depth \( V_0 = 175 \mu K \). The magnetic field is turned on.
Figure 1. The calculated atom remaining in the trap for the MIT 2002 experiment. The initial density is given by \(n_0(t = 0) = 3 \times 10^{13} \text{ cm}^{-3}\) and \(n_m(t = 0) = 0\). The corresponding function \(q(T, a)\) for the appropriate density is shown as well.

within 4 ms to \(B_{\text{fin}}\) and one waits for 50 or 500 ms at \(B = B_{\text{fin}}\), and then the magnetic field is switched off within 100 \(\mu\)s and the cloud is probed by absorption imaging. The experimental findings are that, for an incoherent two-component Fermi gas, there is a strong loss around the magnetic field \(B = 680\) G. Also, for \(B > 680\) G and close to the unitarity \(B_0 = 834\) G, the loss decreases and saturates.

To describe the experiment, we choose as our initial conditions for the rate equations \(n = n_0(t = 0) = 3 \times 10^{13} \text{ cm}^{-3}\) and \(n_m(t = 0) = 0\), corresponding to the experiment. We run our rate equations for \(\Delta t = 0.045\) s. We have chosen the following parameters for the experiment: \(L_3(B = 690\) G\) = \(1 \times 10^{-24} \text{ cm}^6 \text{ s}^{-1}\), \(L_2(B = 690\) G\) = \(1 \times 10^{-13} \text{ cm}^3 \text{ s}^{-1}\) and \(L_m(B = 546\) G\) = \(5 \times 10^{-11} \text{ cm}^3 \text{ s}^{-1}\). The calculated fraction of atoms remaining in the trap is shown in figure 1. We note that the atom loss behavior in this experiment is quite insensitive to the value of \(L_m\).

5.2. Innsbruck/2003

This set of experiments [5] is described in detail in Jochim et al’s thesis [5]. One usually starts with about two million atoms in the lowest two hyperfine-Zeeman states of \(^{6}\text{Li}\) atoms. The samples are cooled to three different temperatures, 22, 30 and \(60\) \(\mu\)K, at a magnetic field of 300 G where the scattering length is large and negative. As a result, there are no shallow dimers in the initial state. One then ramps the system to close to Feshbach resonance within 50 ms. After waiting for 5–7 s, the magnetic field is ramped back to zero, at which point the number of atoms are measured. The atom loss is found to have a maximum at 636 G. To describe the experiment, we run the equation for \(\Delta t = 5\) s, with \(n = n_0(t = 0) = 3.8 \times 10^{13} \text{ cm}^{-3}\), \(n_m(t = 0) = 0\) estimated from [5], and \(L_3(B = 690\) G\) = \(8 \times 10^{-25} \text{ cm}^6 \text{ s}^{-1}\), \(L_2(B = 690\) G\) = \(1 \times 10^{-14} \text{ cm}^3 \text{ s}^{-1}\) and \(L_m(B = 546\) G\) = \(5 \times 10^{-11} \text{ cm}^3 \text{ s}^{-1}\). The calculated number of atoms remaining is shown in figure 2.

5.3. JILA/2004

The starting point of this experiment [6] is an ensemble of \(^{40}\text{K}\) atoms in the hyperfine-Zeeman states \(|9/2, -9/2\rangle\) and \(|9/2, -7/2\rangle\), with temperature \(T = 70\) nK and \(T/T_F = 0.22\). The peak density of the system is \(1.5 \times 10^{13} \text{ cm}^{-3}\). In this experiment, one uses radio-frequency spectroscopy to disassociate molecules in the states \(|9/2, -9/2\rangle\) and \(|9/2, -7/2\rangle\) to atoms in
Figure 2. The calculated fraction of atoms remaining in the trap for the Innsbruck 2003 experiment. The initial density is given by $n_a(t = 0) = 3.8 \times 10^{13}$ cm$^{-3}$ and $n_m(t = 0) = 0$. The corresponding function $q(T, a_s)$ for the appropriate density is shown as well.

Figure 3. The calculated decay rate for the JILA 2004 experiment. The initial density is given by $n_a(t = 0) = 1.5 \times 10^{13}$ cm$^{-3}$ and $n_m(t = 0) = 0$. The corresponding function $q(T, a_s)$ for the appropriate density is shown as well.

the hyperfine-Zeeman states $|9/2, -9/2\rangle$ and $|9/2, -5/2\rangle$. By measuring the atoms in the state $|9/2, -5/2\rangle$, one infers the number of molecules of the system.

The atom loss shows the expected non-monotonic behavior for both spin components. To study this experimental situation, we choose the temperature of the system to be 0.67 $\mu$K and run our rate equation for $\Delta t = 95$ ms, which is the holding time in the experiment. The maximum loss occurs at around 201.5 G. The following parameters are assumed for the experiment: $L_3(B = 200$ G$) = 1.5 \times 10^{-25}$ cm$^6$ s$^{-1}$, $L_2(B = 200$ G$) = 2 \times 10^{-12}$ cm$^3$ s$^{-1}$ and $L_m(B = 200$ G$) = 1 \times 10^{-11}$ cm$^3$ s$^{-1}$. We have taken $n = n_a(t = 0) = 1.5 \times 10^{13}$ cm$^{-3}$, $n_m(t = 0) = 0$. The calculated atom loss rate is shown in figure 3.

5.4. MIT/2009

In this experiment [7], the Fermi gas is prepared initially at a field $B$ around 600 G. The central density of the trapped gas is $n(r = 0) = 0.69 \times 10^{13}$ cm$^{-3}$. The trap frequencies are $\nu_x = \nu_y = 300$ Hz and $\nu_z = 70$ Hz. In this experiment, the field is ramped down within 4.6 ms to the desired final field. It is noted that for $k_F a_s > 1.8$, there is approximately 25% molecule population after the ramp, independent of temperature (within the range considered). A maximum atom loss similar to that from the observations of other groups is found, but it occurs around 780 G.
From the information in [7], we take $T = 0.3 \mu \text{K}$. To study this experiment using our rate equation, we choose $L_3(B = 690 \text{ G}) = 1 \times 10^{-29} \text{ cm}^6 \text{s}^{-1}$, $L_2(B = 690 \text{ G}) = 1 \times 10^{-13} \text{ cm}^3 \text{s}^{-1}$ and $L_m(B = 546 \text{ G}) = 1 \times 10^{-11} \text{ cm}^3 \text{s}^{-1}$. We run our equation for $\Delta t = 2 \text{ ms}$, since it was remarked in [7] that the rates are ‘measured within the first 2 ms’. We have run our equations with three different initial densities $n = n_0(t = 0)$ chosen as follows. Since we have replaced the trapped gas in a harmonic well by the one in a cubic box, we take $n$ as some average density of the density in the harmonic trap, i.e. setting $n = n_{TF}(r^*)$, where $n_{TF}(r)$ is the Thomas–Fermi density profile of the Fermi gas in the harmonic trap and $r^*$ is some radius less than the Thomas–Fermi radius $R_{TF}$ ($r^* < R_{TF}$). The calculated atom loss rate is shown in figure 4. The three curves in figure 4 are different choices of $n$ corresponding to different choices of $r^*$. The color scheme in figure 4 is that blue, purple and brown correspond to $r^* = \sqrt{2/5}R_{TF}$, $r^* = \sqrt{1/2}R_{TF}$ and $r^* = \sqrt{3/5}R_{TF}$, respectively.

5.5. ENS/2004

This experiment [9] starts with a gas of $7 \times 10^4$ $^6\text{Li}$ atoms in an anisotropic trap with $\omega_x = 2\pi \times 0.78 \text{ kHz}$, $\omega_y = 2\pi \times 2.1 \text{ kHz}$ and $\omega_z = 2\pi \times 2.25 \text{ kHz}$. The temperature of the system is estimated to be $T = 2.4 \mu \text{K}$, while the Fermi temperature is $T_F \approx 6 \mu \text{K}$. The system is prepared at low magnetic field and then evaporative cooling is performed at the field $B = 320 \text{ G}$, where the scattering length $a_s = -8 \text{ nm}$. In 10 ms, the magnetic field is ramped to anywhere between 600 and 850 G, and time-of-flight expansions are taken and the number of atoms is counted. It is found that the loss rate has a maximum around $B = 720 \text{ G}$. Interestingly, together with this loss maximum, the sign of the interaction energy changes at the same point.

The following numbers are used to fit the experiment. The central density is given by $n(r = 0) = 3.5 \times 10^{13} \text{ cm}^{-3}$. Note that $L_3(B = 690 \text{ G}) = 1.5 \times 10^{-26} \text{ cm}^6 \text{s}^{-1}$, $L_2(B = 690 \text{ G}) = 4 \times 10^{-13} \text{ cm}^3 \text{s}^{-1}$ and $L_m(B = 546 \text{ G}) = 1 \times 10^{-12} \text{ cm}^3 \text{s}^{-1}$. We run the equation for $\Delta t = 1.5 \text{ s}$. We have not been able to find the holding time in [9]. However, it is remarked in the cited reference that the lifetime of the gases ranges from 100 ms to a few seconds. Our choice of 1.5 s is a rough estimate of the holding time. We have checked that a different choice of reasonable holding time does not change the loss behavior, provided that we modify appropriately other parameters in the calculation. The calculated atom loss rate is shown in figure 5.

![Figure 4](http://www.njp.org/)
Finally, we would like to point out that in all of our studies, the emergence of a maximum atom loss as a function of $a_s$ is a robust phenomenon. Modest changes in input parameters as well as in the rates $L_2$, $L_3$ and $L_m$ do not change our results.

6. Conclusions

In this work, we have investigated the origin of the maximum atom loss as a function of scattering length observed in many experiments performed over a wide range of temperatures, trap depths and particle numbers. We find that the atom loss is the result of a two-step process. The first is the production of a population of shallow dimers that remains in the trap. The second is the conversion of these shallow dimers into deep bound states through collision processes. This second step, which causes particle loss from the trap, depends on the number of shallow dimers generated in the first step. The maximum atom loss is caused by a variation in the number of shallow dimers as a function of $a_s$. This number is the result of competition between three-body recombination and the dimer dissociation processes. While the former increases the dimer population at a rate that scales as $a_s^6$, the latter reduces it and becomes more and more effective as one approaches the resonance, since the binding energy of the dimer decreases rapidly. We have cast these processes in a set of simple rate equation, and show that they can account for the atom loss observed in all of the present experiments.

Apart from these agreements, we note from table 1 that $E_b^*/V_0 \sim 0.1$ for all the present experiments. We believe that this is not an accident. Typically, because of the evaporation process, the temperature of the system is lowered if the trapped depth is lowered. Let $a_s^*$ be the scattering length of the maximum atom loss at temperature $T$. By raising the temperature (caused by a higher trap depth $V_0$), one makes the dissociation process more effective. As a result, one will have to go to a small scattering length (hence larger binding energy $E_b^* = \hbar^2/ma_s^2$) to achieve the same ratio between the rate of three-body recombination and the rate of dissociation. This shows that the observed maximum atom loss is not purely a function of $k_F(a)_s$, but depends on external parameters such as trap depth and temperature. In other words, even though experiments are performed with the same atom density $k_F$, the location of the maximum loss will be different for different runs with different trap depths $V_0$, and the result would appear irreproducible if the variations of extrinsic factors are not fully taken into account. In any case,
our results show that the maximum atom loss cannot be used as a tool to determine the nature of the ground state of the system.

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