Is there anything wrong? The inconsistency of argument with the table of truth

Darmadi
Universitas PGRI Madiun

Corresponding author: darmadi.mathedu@unipma.ac.id

Abstract. The main character of the materials of mathematics is consistent. A problem emerged and became the focus of the research is obtained an inconsistency in mathematics. Research methods used are qualitative research methods. Data were taken with the use of interview-based tests and observation. The research results showed that: 1) there is an inconsistency in argumentation learning logic and mathematical set; 2) Inconsistency argument inspection results occur because of difference in taking for example; and 3) Lecture need to pay attention to the machining process of the students. As a result, in the study of cognitive conflicts can be happen in students and lecturers.

1. Introduction
Consistent is the main character in mathematics. Consistent means fixed (not changeable), basic obedience, or synch [1]. Inconsistency means not permanent or change, or don't obey the principle. The number of nines and imaginary numbers are not included in the system of real numbers because if placed in the real number system with general operation will be inconsistency. The consistency are the basis of important developments in science including mathematics.

One of the goals of learning courses logic and set or introduction to the basics of mathematics is students can analyze an argument [2]. In accordance with the name of his eyes, the analysis of the argumentation that stressed is to use logic and set. Arguing with the logic of more significance on reasoning by using the table of truth or proposition algebra as a basis of argument. However, in some other issues, arguing with the set is more reasoning by using the set as the basis for argumentation.

Argumentation is often also called the postulate. Argument is a set of propositions such that one of the propositions is defined on the basis of other proposition [3]. Propositions can be a single statement or compound statement. The statement is a sentence that has the truth value true or false but not both. Compound statement is a statement that is obtained from two or more basic proposition. A proposition or statement is often symbolized by lowercase letters p, q, or r. The set of propositions is commonly symbolized by the capital letter P, Q or R. Proposition affirmed on the basis of these other propositions called the conclusion. Meanwhile, proposition affirming the conclusion or conclusions are often referred to with the premise.

An argument can be consists of one or more of the premise and one or more conclusions. Premise can be grouped into premise major and premise minor. Premise major is obtained from two or more basic proposition. Premise minor generally consists of a statement. Based on an existing premise are drawn inferences or conclusions. To distinguish a premise with conclusions usually used the sign lines or $\Rightarrow$ for algebra proposition. The conclusion or the conclusion is the statement obtained from premise minor. Therefore, some experts interpret the argument more refers to the process of the withdrawal of the
conclusion. The ability to attract the right conclusions from the evidence that exists, and according to certain rules referred to the ability of reasoning [4].

An argument can be valid and may be invalid. An argument is said to be valid if the arguments stated in the implication such that its premises is the antecedent, conclusion is consistent, and the implication is a logical implications [3]. Legitimate or good argument is a valid argument.

An argument is said to be valid if it is a tautology. Tautology is always worth composite proposition is true for any value of the truth of the proposition [3]. Tautology in logic the set composite is always true for every possible happening. Some books do not mention directly that a valid argument or legitimate if in the form of tautology. However, it is understood that a valid or invalid argumentation form a tautology.

To check the validity of arguments, can use the table of truth or algebra proposition. Initial survey results show that 80% of students would rather prove rather than using algebraic propositions. A further interview results provide answers that the arguments by using the table of truth tends to be easier to use than using algebraic propositions.

The basics that used to use the table of truth covers "conjunction" or "and" denoted by "\(\land\)", "disjunction" or "or" denoted by "\(\lor\)", "negation" denoted by "\(!\)”, and "implications" or "if then" denoted by “\(\rightarrow\)”. Truth table of the basics [3] are as follows

| \(p\) | \(q\) | \(p \land q\) | \(p \lor q\) | \(p \rightarrow q\) |
|-------|-------|-------------|-------------|-----------------|
| \(T\) | \(T\) | \(T\)       | \(T\)       |                 |
| \(T\) | \(F\) | \(F\)       | \(T\)       | \(F\)           |
| \(F\) | \(T\) | \(F\)       | \(T\)       |                 |
| \(F\) | \(F\) | \(F\)       | \(T\)       |                 |

The symbol T is taken from the word “True”. The symbol F is taken from the word “False”. From the basic truth table, for further levels, we acquired the basics of algebra proposition.

The basics of algebra proposition include the commutative, associative, distributive, idempotent, and de Morgan. The idempotent nature on algebraic properties of conjunction is \(p \land p \equiv p\). Idempotent on algebraic properties of disjunction is \(p \lor p \equiv p\). Commutative on algebra proposition of conjunction is \(p \land q \equiv q \land p\). Commutative on algebra proposition of disjunction is \(p \lor q \equiv q \lor p\). Associative on algebra proposition of conjunction is \(p \land (q \land r) \equiv (p \land q) \land r\). Associative on algebra proposition of disjunction is \(p \lor (q \lor r) \equiv (p \lor q) \lor r\). Distributive of the proposition are algebra \(p \land (q \lor r) \equiv (p \land q) \lor (p \land r)\) and \(p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)\). De Morgan of algebra-proposition are \(~(p \land q) \equiv ~p \lor ~q\) and \(~(p \lor q) \equiv ~p \land ~q\). Additionally, there is the nature of implication algebra proposition i.e. \(p \rightarrow q \equiv ~p \lor q\). As well as the table of truth, this proposition algebra often used to check the validity of the argument.

A problem arises when learning logic and a set of program study of mathematics education of FKIP Universitas PGRI Madiun. The problem that arises is found inconsistencies result examination of argumentation. It is interesting to note an inconsistency because the main character in mathematics is consistency [5]. In addition, inconsistencies could bring cognitive conflicts for students. The initial problem is obtained when learning logic and set.

Mathematics is often referred to as an exact science. Therefore, in general mathematics is considered always true. The novelty of this paper is to discuss something that's not true in mathematics. The untruth may be due to less careful in workmanship. However, after the scrutiny until the paper is delivered has not been found by the author or the author's colleagues. Therefore, on this occasion was very good to discuss this issue so that the retrieved clarity. Input from various parties the author look forward to get the truth.
2. Research methods
Research methods used to know an inconsistent state math argument used qualitative research methods. Because of the problem finding on study of mathematics education courses of Universitas PGRI Madiun. The procedures used for this research are as follows: 1) the determination of the subject of research, 2) Development aid instruments, 3) data retrieval, 4) data validation, 4 and 5) Data analysis. The subject for this study are students of mathematics education courses semester 1 of FKIP of Universitas PGRI Madiun academic year 2018/2019 who follow logic courses and the set.

Methods of data retrieval using question form-based tasks and depth interview. The given tasks are as follows.

Check whether the following legitimate argument and give an explanation!

P1. If I was working, I didn't learn
P2. I learned or I pass the exam
P3. I pass the exam

..............................................................................

S1. Therefore, I learned

In addition, data retrieval is also done with the interview to get the depth and breadth of data. Data retrieval is performed in accordance with the schedule of lessons. This is done so that the research done does not interfere with learning. There are two classes, namely class A and class b. class A consists of 25 students and a class B consists of 38 students. Data retrieval is also done with in-depth interviews for depth data. The interview was conducted in the classroom or in the Office or in the laboratory in accordance with the time and conditions.

After the data collected do triangulation of the data. Triangulation of data is done with the coding to be able to browse the data. Reduction of data is also perpetrated by not paying attention to data that is not important. Data relevant to the research topic for the next is to exposure. Triangulation is used time and subject. Time triangulation is to guarantee saturation data. Triangulation of the subject is done by doing this the answer of the subject. From triangulation of data obtained a valid data.

After triangulation, the data subject are analyzed and categorized based on his character, the subject interviewed. Interview used is semi-structured to facilitate the exposure data. Triangulation data sources are performed based on the given data source or subject of the research. Data that less relevant is reduced or not used while the appropriate data analysis is used for research purposes. The coding is done for ease in search of data.

Data analysis was done with doing the categorization of answers of the students, conduct reduction and exposes data, triangulation and withdrawal of the conclusion so that the data obtained are saturated and the conclusion about the inconsistency of mathematical arguments.

3. Research results
Research results showed the existence of two kinds of ways or category completion given by students in solving the problem that are given.

The answer given by the model 1 is 85% of respondents (students). In this model, students answer taking example “p = I was working”, “q = I learned”, and “r = I pass the exam”. So, students get “¬q = I didn’t learn” and form simple premis and conclusions drawn as follows.

P1. p→¬q (If I was working, I didn’t learn)
P2. q ∨ r (I learned or I pass the exam)
P3. r (I pass the exam)

..............................................................................

S1. q (I learned)

Thus, the retrieved premise as follows: If I was working, I didn’t learn (p→¬q); I learned or I pass the exam (q∨r); and I pass the exam (r). With the conclusion: I learned (q). Next, the students plan to prove that ((p→¬q)∧(q∨r)∧r)→q is a tautology by making the following truth table.
However, a group of students based on the answers of the subject, 4.

of this discussion is the data that the student problem resolution by using the table of truth.

students wrong and some are not done yet. Thus, the data can be used and in accordance with the topic of this discussion is the data that the student problem resolution by using the table of truth.

Table 2. The truth table 1 model answers

| P | q | r | ~q | p→q | q ∨ r | (p→q)∧(q ∨ r) ∧ r | ((p→q)∧(q ∨ r) ∧ r) → q |
|---|---|---|----|------|------|---------------------|-----------------------|
| T | T | T | F  | F    | F    | T                   | T                     |
| T | T | F | F  | F    | F    | T                   | T                     |
| T | F | T | T  | T    | T    | T                   | T                     |
| T | F | F | T  | F    | F    | T                   | T                     |
| F | T | T | F  | F    | F    | T                   | T                     |
| F | T | F | T  | T    | T    | T                   | T                     |
| F | F | T | T  | F    | F    | T                   | T                     |

On the basis of the truth table, it appears that the final results of the table of truth showed true value to all. So, the students concluded that proposition \((p→q)∧(q ∨ r)∧r) → q\) is a tautology. Thus, the argumentation is valid.

Answer style 2 given by 15% of respondents (student). On this answer, the student model “p = I was working”, “q = I didn’t learn”, and “r = I pass the exam”. Because of “q = I didn’t learn”, so the students get “~q = I learned” a simple premise and conclusions drawn as follows.

1. \(p → q\) (If I was working, I didn’t learn)
2. \(q ∨ r\) (I learned or I pass the exam)
3. \(r\) (I pass the exam)

...………………………………………………………………………………

S1. \(~q\) (I learned)

Thus, the retrieved premise as follows: If I was working, I didn’t learn \((p → q)\); I learned or I pass the exam \((~q ∨ r)\); and I pass the exam \((r)\). Conclusion: Therefore, I am learned \((~q)\). Next, the students plan to prove that \((p→q)∧(~q ∨ r)∧r) → q\) is a tautology by making the following truth table.

Table 3. The truth table 2 model answers

| p | q | r | p→q | ~q | ~q ∨ r | (p→q)∧(~q ∨ r) ∧ r | ((p→q)∧(~q ∨ r) ∧ r) → ~q |
|---|---|---|-----|----|------|---------------------|----------------------|
| T | T | T | T   | F  | F    | F                   | T                    |
| T | T | F | T   | F  | F    | T                   | T                    |
| T | F | T | F   | T  | T    | T                   | T                    |
| T | F | F | T   | T  | T    | T                   | T                    |
| F | T | T | F   | T  | T    | T                   | T                    |
| F | T | F | T   | F  | F    | T                   | T                    |
| F | F | T | T   | T  | T    | T                   | T                    |

Based on the truth table, it appears that the final results table shows the value of truth is not true of all. So, the students concluded that proposition \((p→q)∧(~q ∨ r)∧r) → ~q\) is not a tautology. In other words, the argument is invalid.

To using truth table, several students are trying to prove by using algebra proposition. However, a student cannot resolve by using algebra proposition. Certain uses of the given proposition algebra students wrong and some are not done yet. Thus, the data can be used and in accordance with the topic of this discussion is the data that the student problem resolution by using the table of truth.

4. Discussions

Based on the answers of the subject, the problem solving of the task can be categorized into two groups: a group of students who answered the valid and the group of students that answer is not valid or invalid. However, after the scrutiny of the way students thinks, all seemed right.
Proposition \((p \rightarrow q) \land (\neg q \lor \neg r) \rightarrow \neg q\) gives proposition \(((p \rightarrow q) \land (\neg q \lor \neg r) \rightarrow \neg q\). Proposition \((p \rightarrow \neg q) \land (q \lor r) \lor r\rightarrow \rightarrow q\). However, in the composition of the truth values are the same, but the table of truth to the proposition \(((p \rightarrow q) \land (\neg q \lor \neg r) \rightarrow \neg q\) is not identical with the proposition \(((p \rightarrow \neg q) \land (q \lor r) \lor r\)\rightarrow q\). So, proposition \(((p \rightarrow q) \land (\neg q \lor \neg r) \rightarrow \neg q\) is not equivalent with proposition \(((p \rightarrow \neg q) \land (q \lor r) \lor r\)\rightarrow q\). Some problems of mathematics can be understood as a concept image and concept definition [6]. However, the problem is more on the completion of the procedure.

If \(X = q\), then \(\neg X = \neg q\) and proposition \(((p \rightarrow q) \land (\neg q \lor \neg r) \rightarrow \neg q\) being \(((p \rightarrow X) \land (\neg X \lor \neg r) \rightarrow \neg X\). If \(Y = \neg q\), then \(\neg Y = \neg (\neg q) = q\) and proposition \(((p \rightarrow q) \land (q \lor r) \lor r) \rightarrow q\) being \(((p \rightarrow Y) \land (\neg Y \lor r) \lor r) \rightarrow \neg Y\). If \(Z = X = Y\), then the proposition \(((p \rightarrow X) \land (\neg X \lor \neg r) \rightarrow \neg X\) equals the proposition \(((p \rightarrow Y) \land (\neg Y \lor r) \lor r) \rightarrow \neg Y\). Thus, supposedly, the proposition \(((p \rightarrow q) \land (\neg q \lor r) \rightarrow \neg q\) is identical to the proposition \(((p \rightarrow q) \land (q \lor r) \lor r) \rightarrow q\). However, the truth table shown in table 2 and table 3 show the other things. Things like this can be understood because some of the problems encountered in mathematics is still a mystery [7].

The difference in the way 1 way 2 is taking suppositions. Suppositions \(q = I\) do not learn on how to \(1\) and \(q = I\) learned on the way 2. The research results showed that the uptake of different in taking the example can provide different results. This shows the need to take appropriate in taking the example in argue. Mathematics requires logical reasoning power high (Higher Order Thinking) [8].

Both models have the students answer the steps the same settlement. Students solve a problem with the memisalkan to understand the problem, plan a workaround by using the table of truth, implement the plan by making a table of the truth, and rechecking so that conclusion. Troubleshooting steps performed student according the theory of Polya [9], namely: understanding, planning, implementations, and evaluate it.

Both models have the students answer given the similarities in stage plan. Students planning to use to resolve the problem of truth tables. The stage of completion of the given problem students can be presented as follows.

| Stage            | Model 1                       | Model 2                       |
|------------------|-------------------------------|-------------------------------|
| Understanding    | By taking the example         | By taking the example         |
|                  | \(p = I \text{ was working}\) | \(p = I \text{ was working}\) |
|                  | \(q = I \text{ learned}\)    | \(q = I \text{ didn’t learn}\) |
|                  | \(r = I \text{ pass the exam}\) | \(r = I \text{ pass the exam}\) |
| Planing          | With the table of truth       | With the table of truth       |
| Implementation   | Create the table of truth     | Create the table of truth     |
| Evaluating       | Give an answer                | Give an answer                |
|                  | \(\text{Valid}\)             | \(\text{Invalid}\)           |

More in-depth interviews showed that students feel that using tables the truth easier than algebra proposition to resolve the problem. This indicates that students tend to seek out an easy completion according to the student. The trend in the search for a settlement with the easiest retrieved because the research used the qualitative research is a demanding data must naturally. Completion of measures provided by the student shows the procedure of solving problems by using the table of truth. In addition to an understanding of the concept, one of the other capabilities that can be controlled by the students in learning mathematics is the understanding of the process [10].

The differences, in the final result, give the impact of learning in courses logic and set. One of the impacts that appear is the emergence of new problems for students. The problem can be make the cognitive conflict that some students feel confident and most of the other students are hesitant. Conflict is in which we are forced to decide between two undesirable alternatives [11]. The same problem can be
felt by the lecturer of courses. Therefore, the lecturer needs to pay attention to the machining process rather than just paying attention to the results.

Some of the problems found in the study to note because it may be the beginning of mathematical thinking. Mathematics depends on an assumption, which is widely adopted, implicitly if not explicitly [12]. If the assumptions used are wrong, then mathematics is studied it can be wrong.

An inconsistent state is discussed in learning in calculus and analysis [13]. As a lecturer in mathematics rather than a researcher in mathematics education, David Tall first became involved in the difficulties of students learning the calculus and analysis. At that time his belief, which he suspect remains common amongst professional mathematicians, was that the difficulties could be eased by preparing materials in a logical and coherent way for students to understand. Exploratory investigations into student’s conceptions revealed the inadequacy of this viewpoint as the inherent cognitive conflict in many of the concepts was exposed.

5. Conclusion
Three conclusions can be obtained from the discussion of the results of this research are as follows: 1) There is an inconsistency in argumentation learning logic and mathematical sets; 2) Inspection results an inconsistency of argument occurs because of the difference in taking the example; 3) The final result can be different but lecture need to pay attention to the machining process of the students.

6. Reference
[1] Team Dictionary Language Center. 2008. Indonesian Language Dictionary. Jakarta: Language Center
[2] The Team's Constituents. 2018. The Academic Handbook. Madiun: UNIPMA
[3] Sugiarto. 2011. The Basic Introductory Learning Materials For Mathematics (PDM), Department of Mathematical Science Faculty Guidance Means UNNES. Page: 72
[4] Toto 'Bara Setiawan. Mathematical Logic. Program: Mathematics Education Majors: educational Sciences faculties: pedagogy and educational sciences of the University: University of Jember.
[5] Darmadi. 2018. The Value And Character In The Learning Of Mathematics. The Proceedings Of The National Seminar On Syllogisms Of The Mathematics Education of Universitas PGRI Madiun. 18 July 2018.
[6] Tall, David; Vinner, Shlomo. 1981. "Concept image and concept definition in mathematics with particular reference to limits and continuity", Educational Studies in Mathematics, 12 (May, 1981), no. 2, 151–169.
[7] Mario Livio, Simon & Schuster Paperbacks, The Equation That Couldn't Be Solved: How Mathematical Genius Discovered the Language of Symmetry, New York • London • Toronto • Sydney
[8] Directorate General of teachers and Educational Personnel of the Ministry of education and culture. 2018. The Handbook of learning-oriented high level thinking skills. Author: Yoki Ariyana, Mt. Widyaiaiswara PPPPTK IPA Bandung Dr. Pudjiastuti Umbilical M. Pd. Widyaiaiswara PPPPTK PKn IPS Reisky Bestary Stone Town, M. Pd. Widyaiaiswara LPMP Riau Province, Prof. Dr. Zamroni, Ph.d. State University of Yogyakarta
[9] Polya. 1973. A New Aspect of Matematical Method. Second edition. Princeton, New Jersey: Princeton University Press
[10] Tall, David: (2013). How Humans Learn to Think Mathematically: Exploring the Three Worlds of Mathematics (Learning in Doing: Social, Cognitive and Computational Perspectives). Cambridge: Cambridge University Press. doi:10.1017/CBO9781139565202
[11] David Matsumoto 2009 The Cambridge Dictionary of Psychology. San Francisco State University. Published in the United States of America by Cambridge University Press, New York.
[12] Paul Ernest. The Philosophy of Mathematics Education. Studies in Mathematics Education
[13] David Tall. 1989. Inconsistencies in the Learning of Calculus and Analysis, *The Role of Inconsistent Ideas in Learning Mathematics*, AERA, New Orleans April 7 1989, published by Department of Math Ed, Georgia, pp 37–46.