Electrical resistivity in 2d Kondo lattice systems

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Abstract

I extend the calculations represented in [4] regarding the resistivity in Kondo lattice materials from 3d system to 2d systems. In the present work I consider a 2d system, and memory function is computed. However, results found in 2d case are different from 3d system. I find that in 2d in low temperature regime($k_B T \ll \mu_d$) resistivity shows power law($\frac{1}{T}$) behaviour and in the high temperature regime($k_B T \gg \mu_d$) resistivity varies linearly with temperature. In 3d these behaviours are as $\frac{1}{T}$ and as $T^{\frac{3}{2}}$ respectively.

1 Introduction

The electrical resistivity originates from spin dependent scattering processes [1, 2, 3, 4, 5]. These scattering events are considered between the localised $d$ or $f$-moments and conduction $s$-electron. The arrangements of spins of localized moments and mobile $s$-electrons construct Kondo-lattice system. In Kondo system at each lattice site a local moment interact via exchange coupling with the spin of conduction electron[3, 4, 5, 6, 7, 8, 9, 10]. These conduction electrons spins undergo the spin-flip scattering processes with the localized magnetic moment spins[11, 12, 13], thus this mechanism leads to resistivity.

The present work is dedicated to the computation of the electrical resistivity of 2d Kondo lattice system. I have found analytically that resistivity $\rho \propto \frac{1}{T}$ in low temperature regime ($k_B T \ll \mu_d$). However, in the high temperature regime ($k_B T \gg \mu_d$) resistivity scales to cube root of $T$($\rho \propto T^{\frac{3}{2}}$).

The recent published paper[4] has reported the resistivity in the 3d Kondo lattice materials using the memory function formalism. The electrical resistivity in such system shows power law behaviour ($\rho \propto \frac{1}{T}$) in the low temperature ($k_B T \ll \mu_d$) limit. In the high temperature regime ($k_B T \gg \mu_d$) resistivity scales to cube root of $T$($\rho \propto T^{\frac{3}{2}}$). The coupling Hamiltonian in the Kondo lattice system is the $s$-$d$ Hamiltonian[5]:

$$H_{sd} = \frac{J}{N} \sum_{k,k',\sigma} \left\{ a_{k'\sigma}^\dagger a_{k\sigma} S^- (k' - k) + a_{k'\sigma}^\dagger a_{k\sigma} S^+ (k' - k) + (a_{k'\sigma}^\dagger a_{k\sigma} + a_{k'\sigma}^\dagger a_{k\sigma}) S^z (k' - k) \right\}$$

(1)

Here $a_{k'\sigma}^\dagger (a_{k\sigma})$ are creation(annihilation) operators of $s$-electrons and $S^- (k' - k)$ is spin lowering operator of quasi $d$ or $f$-electrons. In the next section, I write directly imaginary part of the memory function(all the mathematical details are available in ref.[3]).
2 General expression of Memory function Formula

Writing the imaginary part of the memory function formula for s and d or f electrons

\[ M''(\omega) = \frac{J^2 m \pi}{N^2 \hbar^3 n V \omega} \sum_{k'k}(v_1(k') - v_1(k))^2 \{ f_{k'}^s (1 - f_k^s) \sum_{k_d, k_d'} (f_{k_d}^d - f_{k_d'}^d) - (f_k^s - f_{k'}^s) \sum_{k_d, k_d'} f_{k_d}^d (1 - f_{k_d'}^d) \} \]

\[ \cdot [\delta(\frac{\epsilon_{k'}}{\hbar} - \frac{\epsilon_k}{\hbar} - \omega_{k'k} + \omega) - \delta(\frac{\epsilon_{k'}}{\hbar} - \frac{\epsilon_k}{\hbar} - \omega_{k'k} - \omega)]. \]

(2)

Replacing velocity components for states k and k’ in terms of wavevectors \((v_1(k') = \frac{\hbar}{m} k')\). Next, write the momentum conservation \(\vec{k}' - \vec{k} = \vec{k}_d - \vec{k}_d = \vec{q}\). Insert an integral \(d\vec{q} \delta(\vec{q} - |\vec{k}' - \vec{k}|)\) over \(q\) into equation (2), which simplifies the magnitude of \(|\vec{k}' - \vec{k}|\) greatly. The spatial isotropy of the free electron writes the velocity as \(\vec{v} = (v_x^2 + v_y^2 + v_z^2)\). Finally, converting the summations into integrals for \(k\) and \(k'\) using \(\frac{1}{N} \sum \rightarrow \int \frac{d^3k}{(2\pi)^3}\), one gets

\[ M''(\omega) = \frac{J^2 \pi V}{3 N^2 m n} \int_{0}^{\infty} \frac{dq}{\omega} q^2 \int_{0}^{\infty} \frac{dk}{k} \int_{0}^{\infty} \frac{dk'}{(2\pi)^2} \delta(q - |\vec{k}' - \vec{k}|)F(f_k^s, f_{k'}^s, f_{k_d}^d, f_{k_d'}^d)[\delta(\epsilon_{kq} - \epsilon_k - \hbar \omega_q + \hbar \omega) - \delta(\epsilon_{kq} - \epsilon_k - \hbar \omega_q - \hbar \omega)]. \]

(3)

The function \(F(f_k^s, f_{k'}^s, f_{k_d}^d, f_{k_d'}^d)\) denotes the short hand notation for Fermi distribution functions. Writing the integral \(\int d^3 k = 2\pi \int kdk, \int d^2 k = \int k' dk' \int_0^{2\pi} d\phi\) (take \(k\) as pointing along the \(z\)-direction) and expression (3) changes to

\[ M''(\omega) = \frac{J^2 \pi V}{3 N^2 m n} (2\pi)^2 \int_{0}^{\infty} \frac{dq}{\omega} q^2 \int_{0}^{\infty} \frac{kdk}{k} \int_{0}^{\infty} \frac{k' dk'}{(2\pi)^2} \delta(q - \sqrt{k^2 + k'^2}) \sum_{k_d, k_d'} \int_{0}^{2\pi} d\phi \frac{F(f_k^s, f_{k'}^s, f_{k_d}^d, f_{k_d'}^d)}{\sqrt{k^2 - q^2}} \left[ \delta(\epsilon_{kq} - \epsilon_k - \hbar \omega_q + \hbar \omega) - \delta(\epsilon_{kq} - \epsilon_k - \hbar \omega_q - \hbar \omega) \right]. \]

(4)

The computation of \(\phi\) integral (appendix A) simplifies the eqn (4) as

\[ M''(\omega) = \frac{J^2 \pi V}{3 N^2 m n} (2\pi)^2 \int_{0}^{\infty} \frac{dq}{\omega} q^2 \int_{0}^{\infty} k \frac{dk}{\sqrt{k^2 - q^2}} \int_{0}^{2\pi} d\phi \frac{F(f_k^s, f_{k'}^s, f_{k_d}^d, f_{k_d'}^d)}{\sqrt{k^2 - q^2}} \left[ \delta(\epsilon_{kq} - \epsilon_k - \hbar \omega_q + \hbar \omega) - \delta(\epsilon_{kq} - \epsilon_k - \hbar \omega_q - \hbar \omega) \right]. \]

(5)

Converting \(k\) and \(k'\) integral into \(\epsilon\) and \(\epsilon'\) and replacing \(\epsilon'\) integral from appendix B

\[ M''(\omega) = \frac{J^2 A^2}{6 N^2 n V (2\pi)^3} \frac{(m \hbar^2)}{2m \hbar^4} \int_{0}^{2\pi} dq q^2 \int_{\epsilon_0}^{\infty} \frac{d\epsilon}{\sqrt{\epsilon - \epsilon_0}} \frac{1}{\omega} \left\{ f^s(\epsilon_k + \hbar \omega_q - \hbar \omega) - f^s(\epsilon_k + \hbar \omega_q + \hbar \omega) \right\} \left( 1 - f^s(\epsilon_k) \right) F^1_d(q) + \left\{ f^s(\epsilon_k + \hbar \omega_q - \hbar \omega) - f^s(\epsilon_k + \hbar \omega_q + \hbar \omega) \right\} F^2_d(q). \]

(6)
This is required general expression of imaginary part of the Memory Function, which is valid for all frequencies and all temperature regimes. Operating the limit $\omega \to 0$, we rewrite equation (7) as follows:

$$M''(\omega = 0, T) = \frac{J^2 \beta A^2 m^2}{6N^2 nV(2\pi)^3 \hbar^2 \sqrt{2m}} \int_{0}^{\infty} dq d\epsilon \int_{\epsilon_0}^{\infty} \frac{d\epsilon}{\sqrt{\epsilon - \epsilon_0}} \{ f^s(\epsilon + \hbar \omega_q)(1 - f^s(\epsilon + \hbar \omega_q)) \} 
\left[ (1 - f^s(\epsilon)) \sum_{k_d,k_d'} (f_{k_d} - f_{k_d'}) + \sum_{k_d,k_d'} f_{k_d}(1 - f_{k_d'}) \right].$$

(7)

The use of valid reasonable assumptions simplify the above expression: (1) The s-electrons fermi functions energy $k_BT \ll \mu_s$ (chemical potential for s-electrons, $\mu_s \simeq 10 eV$), which is much greater than room temperature($\sim 0.025eV$) (2) the energy scale of magnetic excitation $\hbar \omega_q \ll \mu_s$ ($meV$) is much less than the assumed s electrons chemical potential. Using the second assumption the Fermi function $f^s(\epsilon + \hbar \omega_q) = \frac{1}{e^{\beta(\epsilon + \hbar \omega_q - \mu_s)} + 1}$ approximates to $f^s(\epsilon)$ and the above expression becomes

$$M''(\omega = 0, T) = \frac{p_0}{k_BT} \int_{\epsilon_0}^{\infty} \frac{d\epsilon}{\sqrt{\epsilon - \epsilon_0}} \frac{1}{\beta} \delta(\epsilon - \mu_s) \left[ \int_{0}^{\infty} dq d\epsilon \sum_{k_d,k_d'} (1 - f^s(\epsilon))(f_{k_d} - f_{k_d'}) + f_{k_d}(1 - f_{k_d'}) \right].$$

(8)

For $\mu_s \gg \epsilon_0$ the above expression simplifies to

$$M''(\omega = 0, T) = \frac{p_0}{\sqrt{\mu_s}} \left[ \frac{1}{2} \int_{0}^{\infty} dq d\epsilon \sum_{k_d,k_d'} (f_{k_d} - f_{k_d'}) + \int_{0}^{\infty} dq d\epsilon \sum_{k_d,k_d'} f_{k_d}(1 - f_{k_d'}) \right].$$

(9)

where $p_0 = \frac{\beta^2 A^2 m}{3N^2 nV(2\pi)^3 \hbar^2 \sqrt{2m}}$. We take integral terms as $\mathbb{I}_1(q)$ and $\mathbb{I}_2(q)$ and substitute computed expressions from appendices C and D(eqn (32) (36)

$$M''(\omega = 0, T) = \frac{J^2 A^2 m}{12\pi^2 N^2 nV\hbar^3 \epsilon_0^3} \left[ \frac{A \lambda D}{4 \pi} \left\{ \beta \int_{0}^{\infty} \frac{d\epsilon e^{\beta(\epsilon - \mu_d)}}{(e^{\beta(\epsilon - \mu_d)} + 1)^3} + \beta^2 \int_{0}^{\infty} \frac{d\epsilon e^{\beta(\epsilon - \mu_d)}}{(e^{\beta(\epsilon - \mu_d)} + 1)^3} \right\} \right] + Am \lambda D \int_{0}^{\infty} \frac{d\epsilon e^{\beta(\epsilon - \mu_d)}}{(e^{\beta(\epsilon - \mu_d)} + 1)^3} + \beta^2 \int_{0}^{\infty} \frac{d\epsilon e^{\beta(\epsilon - \mu_d)}}{(e^{\beta(\epsilon - \mu_d)} + 1)^3} - 2s^2 \int_{0}^{\infty} \frac{d\epsilon e^{2\beta(\epsilon - \mu_d)}}{(e^{2\beta(\epsilon - \mu_d)} + 1)^3} \right\} \right].$$

(10)

In the above expression we have replaced $\sqrt{2m \mu_s} = \hbar q_s$ and $m_d = m \lambda$. The transformation of the variables in all integrands $x = \beta(\epsilon - \mu_d)$ changes the above expression as

$$M''(\omega = 0, T) = \frac{J^2 A^2 m}{12\pi^2 N^2 nV\hbar^3 \epsilon_0^3} \left[ \frac{A \lambda D}{4 \pi}\left\{ \int_{-\beta \mu_d}^{\infty} \frac{dx e^{x}}{(e^{x} + 1)^2} + \beta \int_{-\beta \mu_d}^{\infty} \frac{dx (\frac{x}{\beta} + \mu_d)e^{x}}{(e^{x} + 1)^2} \right\} \right] + Am \lambda D \int_{-\beta \mu_d}^{\infty} \frac{dx e^{x}}{6\pi\hbar^2 \epsilon_0^3 q_s^3 \beta} + \int_{-\beta \mu_d}^{\infty} \frac{dx (\frac{x}{\beta} + \mu_d)e^{2x}}{(e^{x} + 1)^3} \right\} \right].$$

In the above expression we have replaced $\sqrt{2m \mu_s} = \hbar q_s$ and $m_d = m \lambda$. The transformation of the variables in all integrands $x = \beta(\epsilon - \mu_d)$ changes the above expression as

$$M''(\omega = 0, T) = \frac{J^2 A^2 m}{12\pi^2 N^2 nV\hbar^3 \epsilon_0^3} \left[ \frac{A \lambda D}{4 \pi}\left\{ \int_{-\beta \mu_d}^{\infty} \frac{dx e^{x}}{(e^{x} + 1)^2} + \beta \int_{-\beta \mu_d}^{\infty} \frac{dx (\frac{x}{\beta} + \mu_d)e^{x}}{(e^{x} + 1)^2} \right\} \right] + Am \lambda D \int_{-\beta \mu_d}^{\infty} \frac{dx e^{x}}{6\pi\hbar^2 \epsilon_0^3 q_s^3 \beta} + \int_{-\beta \mu_d}^{\infty} \frac{dx (\frac{x}{\beta} + \mu_d)e^{2x}}{(e^{x} + 1)^3} \right\} \right].$$

(10)
\[
\frac{Aq^4}{20\pi q_s^5} \{ \int_{-\beta \mu_d}^{\infty} \frac{dx}{e^x + 1} + 2 \int_{-\beta \mu_d}^{\infty} \frac{dx(e^x + 1)^3}{(e^x + 1)^3} \}.
\]

Thus, we have obtained more simplified form of the expression computed under two important assumptions (mentioned as 1 and 2). We further analyse the general expression for low and high temperature limit.

3 Special cases:

3.1 Low temperature limit \((k_B T \ll \mu_d)\)

For limit \(\beta \mu_d \gg 1\) the expression (11) reduces to

\[
M''(T \to 0) \approx \frac{F^2 A^2 m}{12 \pi^2 N^2 n V \hbar^4} \left[ \frac{Aq^4}{40 \pi q_s^5} \right] \left\{ \int_{-\beta \mu_d}^{\infty} \frac{dx}{(e^x + 1)^2} + \beta \mu_d \int_{-\beta \mu_d}^{\infty} \frac{dx}{(e^x + 1)^3} \right\} - 2 \beta \mu_d \int_{-\beta \mu_d}^{\infty} \frac{dx}{(e^x + 1)^4}
\]

here we have replaced \(\sqrt{x + \beta \mu_d} \approx \sqrt{\beta \mu_d}\). Writing the dominating terms containing \(\frac{1}{T}\) factor with exponential integrals

\[
M''(T \to 0) \approx \frac{1}{T} f_0.
\]

The value of \(f_0 = \int_{-\beta \mu_d}^{\infty} \frac{dx}{(e^x + 1)^2} \left[ 1 - 2 \frac{e^x}{(e^x + 1)^2} + 2 \frac{e^x}{(e^x + 1)^3} - 4 \frac{e^x}{(e^x + 1)^4} \right]\) is constant \((f_0 = \frac{1}{6})\) in the low temperature limit. Thus we observe power law divergence behavior of quasi localized \(d\) or \(f\) electrons. Thus, DC resistivity is given by \(\rho(T) = \frac{m}{ne^2} M''(T) = \frac{m}{3ne^2 T} \) where \(T \ll \frac{\mu_d}{k_B}\).

3.2 High temperature limit \((k_B T \gg \mu_d)\)

In high temperature limit \(\beta \mu_d \ll 1\) the general expression (11) simplifies to

\[
M''(T = 0, T) = \frac{F^2 A^2 m}{12 \pi^2 N^2 n V \hbar^4} \left[ \frac{Aq^4}{40 \pi q_s^5} \right] \left\{ \int_0^{\infty} \frac{dx}{(e^x + 1)^2} + \int_0^{\infty} \frac{dx}{(e^x + 1)^3} - 2 \int_0^{\infty} \frac{dx}{(e^x + 1)^4} \right\}
\]

The above expression clearly contains the temperature dependence in the middle term. Thus,

\[
M''(k_B T \gg \mu_d) \approx C_0 T,
\]

where prefactor \(C_0 = \frac{A m \lambda}{12 \pi n} q_s^3 \sqrt{q_d} \). Therefore, in high temperature limit we observe that memory function varies linearly with temperature. Resistivity is given by \(\rho(T) = \frac{m}{ne^2} M''(T) = \frac{C_0 m T}{ne^2}\).
4 2d DC Resistivity in general case

The temperature dependent DC resistivity can be written from memory function formula using formula $\rho(T) = \frac{m}{n_{fe}} M''(T)$ in the general case as:

$$\rho_{2d}(T) = \frac{m^2 J^2 A m}{12 \pi^2 N^2 nV^2} \left[ \frac{q_D}{q_s} \right]^5 \left\{ \int_{-\beta_{\mu_d}}^{\infty} \frac{dx e^x}{(e^x+1)^2} + \beta \int_{-\beta_{\mu_d}}^{\infty} \frac{dx (\frac{\beta}{D} + \mu_d) e^{2x}}{(e^x+1)^2} \right. $$

$$ - 2 \beta \int_{-\beta_{\mu_d}}^{\infty} \frac{dx (\frac{\beta}{D} + \mu_d) e^{2x}}{(e^x+1)^2} + \frac{A q_D^2}{20 \pi} \left\{ \int_{-\beta_{\mu_d}}^{\infty} \frac{dx e^x}{(e^x+1)^3} + \beta \int_{-\beta_{\mu_d}}^{\infty} \frac{dx (\frac{\beta}{D} + \mu_d) e^{2x}}{(e^x+1)^3} \right. $$

$$ - 2 \beta \int_{-\beta_{\mu_d}}^{\infty} \frac{dx (\frac{\beta}{D} + \mu_d) e^{2x}}{(e^x+1)^4} \right\}. $$

(16)

This is the general formula for DC resistivity in a 2d Kondo lattice system.

5 Conclusion

The present work is the extension of the computation of the DC resistivity of 3d Kondo lattice materials. The results are as follows. The temperature of the system is compared with $\mu_d$ (chemical potentials of $d$ electrons). At low temperature, I find that DC resistivity proportional to $T^{-1}$. In the high temperature regime resistivity varies linearly with temperature ($\rho \propto T$). The difference is found only in the high temperature limit (in 3d $\rho \propto T^2$ and 2d $\rho \propto T$).

Appendix

A $\phi$ integral solution

In the presence of Fermi factors of the form $f_k^s(1-f_k^s)$ and at ordinary temperature $k_B T \ll \mu_s(\sim eV)$, one can replace $\epsilon$ and $\epsilon'$ inside the square root by $\mu_s$ for $s$ electrons ($\mu_s = \frac{k_B q_s^2}{2m}$) where $q_s$ is Fermi wave vector for $s$-electrons:

$$\mathbb{I}_\phi = \int_0^{2\pi} d\phi \delta(q - \sqrt{2m} \sqrt{(\epsilon' + \epsilon - 2\sqrt{2} \sqrt{\epsilon' \epsilon} \cos \phi)})$$

$$= \int_0^{2\pi} d\phi \delta \left( \frac{hq}{2\sqrt{2}me} \sin \left( \frac{\phi}{2} \right) \right). \quad (17)$$

Using property of delta function $\delta F(x) = \sum \frac{\delta(x-x_0)}{|F(x_0)|}$, $\mathbb{I}_\phi$ reduces to

$$\mathbb{I}_\phi = \frac{h}{2\sqrt{2}me} \int_0^{2\pi} d\phi \left( \frac{1}{2} \cos \frac{\phi}{2} \right) \delta(\phi - \phi_0) = \frac{h}{2\sqrt{2}me} \sqrt{1 - \frac{q_0^2}{2\sqrt{2}me}}. \quad (18)$$

put $\sqrt{2me} = \hbar q_s = \hbar k$ and $\frac{q_0^2}{2} = q_0$

$$\mathbb{I}(q,k) = \frac{1}{q_s \sqrt{1 - \frac{q^2}{4 q_s^2}}} = \frac{1}{\sqrt{k^2 - q_0^2}}. \quad (19)$$
B \( \mathbb{I}(k') \) integral solution

\[
\mathbb{I}(k') = \int_0^\infty k' dk' F(k_k', f_{k_d}', f_{k_d}'') \delta(\epsilon_{k+q} - \epsilon_k - \hbar \omega_q + \hbar \omega) - \delta(\epsilon_{k+q} - \epsilon_k - \hbar \omega_q - \hbar \omega),
\]

changing \( k' \) integral into \( \epsilon' \) integral by setting \( k' = \sqrt{2m \epsilon'/\hbar} \) and \( dk' = \frac{2m}{\hbar} d\epsilon' \)

\[
\mathbb{I}(\epsilon') = \frac{m}{\hbar^2} \int_0^\infty d\epsilon' \left\{ f^*(\epsilon_{k+q})(1 - f^*(\epsilon_k)) \sum_{kd} F^1_d(q) - (f^*(\epsilon_k) - f^*(\epsilon_{k+q})) \sum_{kd} F^2_d(q) \right\}
\]

\[
\left[ \delta(\epsilon_{k+q} - \epsilon_k - \hbar \omega_q + \hbar \omega) - \delta(\epsilon_{k+q} - \epsilon_k - \hbar \omega_q - \hbar \omega) \right],
\]

here \( F^1_d(q) = \sum_{kd} (f^d_{kd} - f^d_{kd}') \), \( F^2_d(q) = \sum_{kd} f^d_{kd}'(1 - f^d_{kd}') \) and \( \epsilon' = \epsilon_{k+q} \). The property of delta function \( \int dx f(x) \delta(x - a) = f(a) \) changes the above expression to

\[
\mathbb{I}(\epsilon) = \frac{m}{\hbar^2} \left\{ f^*(\epsilon_k + \hbar \omega_q - \hbar \omega) \left( 1 - f^*(\epsilon_k) \right) F^1_d(q) - \left( f^*(\epsilon_k) - f^*(\epsilon_{k+q}) \right) F^2_d(q) \right\}
\]

\[
- \left\{ f^*(\epsilon_k + \hbar \omega_q + \hbar \omega) \left( 1 - f^*(\epsilon_k) \right) F^1_d(q) - \left( f^*(\epsilon_k) - f^*(\epsilon_{k+q}) \right) F^2_d(q) \right\},
\]

simplifies to

\[
\mathbb{I}(\epsilon) = \frac{m}{\hbar^2} \left\{ f^*(\epsilon_k + \hbar \omega_q - \hbar \omega) - f^*(\epsilon_k + \hbar \omega_q + \hbar \omega) \right\} \left( 1 - f^*(\epsilon_k) \right) F^1_d(q) +
\]

\[
\left\{ f^*(\epsilon_k + \hbar \omega_q - \hbar \omega) - f^*(\epsilon_k + \hbar \omega_q + \hbar \omega) \right\} F^2_d(q).
\]

C Computation of \( \mathbb{I}_1(q) \)

\[
\mathbb{I}_1(q) = \int_0^{q_0} dq q^2 \sum_{kd,k_d} (f^d_{kd} - f^d_{kd}')
\]

write

\[
f^1_d(q) = \sum_{kd} [f^d(\epsilon_{kd}) - f^d(\epsilon_{kd}')].
\]

Small \( q \) expansion of \( f^1_d(q) \) gives

\[
f^1_d(q) = - \frac{A}{(2\pi)^2} \int_0^\infty k dk \int_0^{2\pi} d\phi \left[ q \frac{\partial f^d(\epsilon_{kd}')}{\partial q} \right]_{q=0} + \frac{q^2}{2!} \frac{\partial^2 f^d(\epsilon_{kd}')}{\partial q^2} \right]_{q=0} + \frac{q^3}{3!} \frac{\partial^3 f^d(\epsilon_{kd}')}{\partial q^3} \right]_{q=0} \ldots
\]
We have Fermi function \( f^d(\epsilon_{k_d}, \theta) = \frac{1}{e^{\beta_{k_d}^2} + e^{\beta_{k_d}^2} + e^{\beta_{k_d}^2}} \). For simplification, on performing Taylor’s expansion for small \((q \to 0)\) and converting summation into integral we getwe put \( \alpha = \beta(\frac{h}{2m_d} - \mu_d), \eta = \beta \frac{h^2}{2m_d} \) and \( \gamma = \beta \frac{h^2}{2m_d} \). The Fermi function set to

\[
f^d(q, \alpha, \eta, \gamma, \theta) = \frac{1}{e^{\alpha + \eta q + \gamma q \cos \theta}}, \quad \left. \frac{\partial f^d(\alpha, \gamma, \theta)}{\partial q} \right|_{q=0} = -\frac{e^\alpha \gamma \cos \theta}{(e^\alpha + 1)^2},
\]

(27)
on substituting derivatives of fermi functions for small q limit and performing \( \phi \) integral eqn [26] becomes

\[
f^1_d(q, k_d) = \frac{q^2 A}{2(2\pi)^2} \int_0^\infty k_d dk_d \left\{ \frac{e^\alpha}{(e^\alpha + 1)^2} \left[ 4\pi \eta + \gamma^2 \pi \right] - \frac{2\pi \gamma^2 e^{2\alpha}}{(e^\alpha + 1)^3} + \ldots \right\}.
\]

(28)
We convert \( k_d \) integral into energy integral \((\epsilon_d)\) and replace \( \alpha, \beta \) and \( \gamma \) with their respective terms

\[
f^1_d(q, \epsilon_d) = \frac{Amq^2}{8\pi \hbar^2} \left[ \frac{2\beta^2 h^2}{m} \int_0^\infty d\epsilon_d e^{\beta (\epsilon_d - \mu_d)} + \frac{2\beta^2 h^2}{m} \int_0^\infty d\epsilon_d e^{\beta (\epsilon_d - \mu_d)} \right]
\]

\[
\ldots - \frac{4 \beta^2 h^2}{m} \int_0^\infty d\epsilon_d e^{\beta (\epsilon_d - \mu_d)} \left( e^{\beta (\epsilon_d - \mu_d)} + 1 \right)^2
\]

simplifies to

\[
f^1_d(q, \epsilon_d) = \frac{Aq^2}{4\pi} \left[ \beta \int_0^\infty d\epsilon_d e^{\beta (\epsilon_d - \mu_d)} + \beta^2 \int_0^\infty d\epsilon_d e^{\beta (\epsilon_d - \mu_d)} \right] - 2\beta^2 \int_0^\infty d\epsilon_d e^{2\beta (\epsilon_d - \mu_d)}.
\]

(29)
Substituting the above expression in eqn [23] and performing integration over q we obtain

\[
I_1(q) = \frac{Aq^2}{20\pi} \left[ \beta \int_0^\infty d\epsilon_d e^{\beta (\epsilon_d - \mu_d)} + \beta^2 \int_0^\infty d\epsilon_d e^{\beta (\epsilon_d - \mu_d)} \right] - 2\beta^2 \int_0^\infty d\epsilon_d e^{2\beta (\epsilon_d - \mu_d)}.
\]

(30)

**D Computation of \( I_2(q) \)**

\[
I_2(q) = \int_0^{q_m} dq q^2 \sum_{k_d, k_d'} f^d_{k_d}(1 - f^d_{k_d'}), \quad (32)
\]
write

\[
f^2_d(q) = \sum_{k_d} f^d(\epsilon_{k_d})[1 - f^d(\epsilon_{k_d})].
\]

(33)
On performing Taylor’s expansion for small \((q \to 0)\) and converting summation into integral we get

\[
f^2_d(q) = \frac{A}{(2\pi)^2} \int_0^\infty k_d dk_d \int_0^{2\pi} d\phi f^d(\epsilon_{k_d})(1 - f^d(\epsilon_{k_d})) - \frac{A\pi q^2}{(2\pi)^2} \frac{2}{2} \int_0^\infty k_d dk_d \left\{ \frac{-e^\alpha}{(e^\alpha + 1)^3} \times \left[ 4\eta + \gamma^2 \right] + 2\gamma^2 \frac{e^{2\alpha}}{(e^\alpha + 1)^3} \right\}.
\]

(34)
We convert $k_d$ integral into energy integral ($\epsilon_d$) and replace $\alpha, \beta$ and $\gamma$ with their respective terms

$$f^2_d(q, \epsilon_d) = \frac{A m_d}{2 \pi \hbar^2} \int_0^\infty \frac{d \epsilon_d e^{\beta (\epsilon_d - \mu_d)}}{(e^{\beta (\epsilon_d - \mu_d)} + 1)^2} + \frac{A q^2}{4 \pi} \left[ \beta \int_0^\infty \frac{d \epsilon_d e^{\beta (\epsilon_d - \mu_d)}}{(e^{\beta (\epsilon_d - \mu_d)} + 1)^3} + \beta^2 \int_0^\infty \frac{d \epsilon_d d \epsilon_d e^{2 \beta (\epsilon_d - \mu_d)}}{(e^{\beta (\epsilon_d - \mu_d)} + 1)^3} \right]$$

On substituting the above expression into eqn(32) and performing $q$ integration we get

$$I^2(q) = \frac{A m_d q^2}{6 \pi \hbar^2} \int_0^\infty \frac{d \epsilon_d e^{\beta (\epsilon_d - \mu_d)}}{(e^{\beta (\epsilon_d - \mu_d)} + 1)^2} + \frac{A q^5}{20 \pi} \left[ \beta \int_0^\infty \frac{d \epsilon_d e^{\beta (\epsilon_d - \mu_d)}}{(e^{\beta (\epsilon_d - \mu_d)} + 1)^3} + \beta^2 \int_0^\infty \frac{d \epsilon_d d \epsilon_d e^{2 \beta (\epsilon_d - \mu_d)}}{(e^{\beta (\epsilon_d - \mu_d)} + 1)^3} \right]$$

(35)

(36)

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