Consideration of a Phase Change Model Based on Apparent Phase Equilibrium

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Abstract. It has been known that cavity volume is underestimated and there is a discrepancy between predicted and measured breakdown characteristics for the numerical simulation of unsteady cavitation around a hydrofoil at high angle of attack. Therefore, in this study, in order to predict the cavity volume with high accuracy, the phenomena that gas phase increases even at a pressure higher than saturated vapour pressure which is known as aeration is modelled, and applied to phase change term. It was assumed that the precipitation of dissolved air is promoted by mechanical stimulation such as Reynolds stress in unsteady flow. The effectivity of the proposed model is discussed through the comparison among some kinds of components of the pressure variation.

1. Introduction
In recent years, several cavitation models for numerical simulation have been proposed, and many commercial software which are implementing those cavitation models are now available. However, for the simulation of unsteady cavitating flow around a single hydrofoil at high angle of attack [1], it has been reported that the prediction accuracy of time averaged lift and drag is low, and the length of sheet cavity is prone to be underestimated. It can be thought that following two mechanisms contribute to the underestimation of lift coefficient. 1. Cavity volume is underestimated since pressure does not decrease, and 2. Pressure does not decrease since cavity volume is underestimated. In the former case, cavity volume increases as pressure decreases even mass fraction of gas phase does not increase which means evaporation rate does not increase. In the latter case, cavity volume increases as evaporation increases and it leads to reduction of pressure. In the present study, the latter case is assumed. Hence, increase of evaporation rate is studied in unsteady cavitation.

In conventional models for the numerical analysis of cavitation, evaporation and condensation is estimated by whether the local pressure takes lower or higher value than the local saturated vapour pressure. On the other hand, it is well known that the cavity incepts on the condition that the local pressure is higher than the saturated vapour pressure [2]. The phenomenon depends on Reynolds number, which is called scale effect. The author also shown by an experiment that the gas phase increase even at a pressure higher than the local saturated vapour pressure if the liquid contains dissolved air and exposed to disturbance. Therefore new phase change model which is different from phase change models based on phase equilibrium at the saturated vapor pressure is necessary in order to express the increase of gas phase in actual flow field by homogeneous model which has difficulty in treating bubble nucleus.

Thus, in this study, a phase change model which was suggested by the authors [3] is used for numerical simulation of unsteady cavitation in which the situation can be reproduced where the gas phase increase even above saturated vapor pressure. In this model, precipitation of dissolved air is not taken into account explicitly since the model is based on homogeneous model, but the effect of dissolved air
is taken into account implicitly for homogeneous model. The precipitation of dissolved air is assumed to be promoted by dynamic stimulation such as Reynolds stress or Baroclinic torque.

The flow field around single NACA0015 hydrofoil which is well known for benchmark simulation of cavitation flow, also known as the most difficult condition for prediction is selected. Through the comparison of time averaged pressure distribution on a hydrofoil between experimental data, the effectivity of apparent phase equilibrium model is considered.

2. Numerical Method
2.1 Two-phase locally homogeneous medium model
In this study, locally homogeneous model of gas-liquid two phase medium \(^\text{[4]}\) is used. This model treats two phase medium as a pseudo single-phase medium which has locally homogeneous void fraction. Considering compressibility for both gas phase and liquid phase, governing equations are given as follows:

\[
\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial (\mathbf{E}_j - \mathbf{E}_{ij})}{\partial x_j} = \mathbf{S} = \begin{bmatrix}
\rho \\
\rho u_i \\
\rho Y \\
\rho u_i u_j + \delta_{ij} \rho p \\
\rho u_i Y \\
\rho u_i \tau_{ij} \\
0 \\
0
\end{bmatrix}, \quad \mathbf{E}_{ij} = \begin{bmatrix}
0 \\
0
\end{bmatrix}, \quad \mathbf{S} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

(1)

where, \(\rho, p, u, \) are density, pressure, and velocity, respectively. \(Y\) is mass fraction of gas phase, and \(\Gamma\) in Eq. (1) is evaporation speed. In the present study, for stability of computation, phase equilibrium takes place instantaneously when pressure decreases under vapor pressure. In the model\(^\text{[5]}\), there is no empirical constant.

Additionally, the working fluid in the present numerical study is room temperature water for which the temperature distribution is known to be almost uniform. Thus, energy conservation equation can be omitted from the governing equations given in Eq. (1). The equation of state of two phase medium is expressed by using \(Y\) as follows:

\[
\rho = \frac{p(p + p_v)}{K_l(1-Y)p(T + T_c) + R_Y(p + p_v)YT}
\]

(2)

where, \(K_l, R_g, T_c, p_v\) are liquid constant, gas constant, liquid temperature constant, and liquid pressure constant, respectively. Void fraction \(\alpha\) is calculated as:

\[
\alpha = \frac{R_Y(p + p_v)YT}{K_l(1-Y)p(T + T_c) + R_Y(p + p_v)YT}
\]

(3)

Finite difference method is used for discretization of governing equation, and the explicit TVD-MacCormack method with 2nd-order accuracy in time and space is used for time integration and evaluation of the convection and viscous terms. In addition, turbulence model is not used since the effectiveness of RANS model was not observed for the simulation of cavitating flow in a previous research \(^\text{[8]}\). However, large scale velocity variation is reproduced which is originated by oscillation of unsteady cavitation.

2.2 Apparent phase equilibrium model
In this study, APE (apparent phase equilibrium) model \(^\text{[4]}\) is applied in which the situation that the gas phase increases even above saturated vapour pressure. This model, saturated vapour pressure in phase change term \(\Gamma\) is replaced by \(p_v^\text{sat}\) : apparent phase equilibrium pressure. The phase equilibrium pressure is considered to shift toward higher pressure side, \(p_v^\text{sat}\) is expressed as follows:

\[
p_v^\text{sat} = p_v^\text{sat} + f(\alpha) p_v^\text{turb}
\]

(4)

Function i : \(f(\alpha) = \tanh((0.5 - \alpha)2n)/2\tanh(n) + 0.5\)

Function ii : \(f(\alpha) = \alpha(1-\alpha)/0.25\)

(5)

where, \(p_v^\text{turb}\) is pressure variation term which changes according to local unsteady flow condition, and \(f(\alpha)\) is a weighting function which is a function of void fraction \(\alpha\). Two functions are compared in the present study. The Function i, which is previous function \(^\text{[5]}\), takes a value 1 at void fraction \(\alpha = 0\) and
takes 0 at α = 1. It represents the situation that apparent phase equilibrium takes place at start of evaporation and the phase change goes to real phase equilibrium \((p = p_{sat})\) at the end of evaporation. On the other hand, the Function ii takes 1 when void fraction \(α\) is 0.5 and takes 0 at \(α = 0\) and 1. It corresponds to the situation that there is no gas-liquid interface and phase change does not occur at \(α = 0\) and 1, then the effect of dissolved air does not appear at the condition. Both Function i and ii can represent the well-known fact that pressure inside fully developed attached sheet cavity becomes saturated vapour pressure \(6\).

\(p^turb\) is a component which contributes to the prediction of dissolved air, but we don’t know what kind of component acts on the phenomenon. Therefore, the numerical experiment is conducted in the present study. The following three kinds of variation components were compared,

\[
\begin{align*}
\text{Model1:} & \quad p^turb = C \rho u_i u_j \\
\text{Model2:} & \quad p^turb = C \rho \left( L \frac{1}{p^2} (\nabla p \times \nabla p) \right) \\
\text{Model3:} & \quad p^turb = \begin{cases} 
-0.5 C \rho u_i u_j, & (u_i u_j < 0) \\
0, & (u_i u_j > 0)
\end{cases}
\end{align*}
\]

These components are based on Reynolds stress, Baroclinic torque, and Q value which is the second invariant of velocity gradient. \(L\) is characteristic length which was used to adjust the dimension, and \(C\) is control parameter. In the present study, the above values are roughly estimated not from turbulence model but from calculation result which is grid scale disturbance caused by homogeneously represented unsteady cavitation.

2.3 Calculation condition

Simulations of cavitating flow around NACA0015 hydrofoil were performed. Main flow velocity, temperature, and void fraction are 8[m/s], 293.15[K], and 1%, respectively. Angle of attack is 8[deg], and cavitation number \(σ\) is set to be 1.4 since for this case the discrepancy between the predicted and measured breakdown characteristics is most prominent \(1\).

3. Results and Discussions

![Figure 1](image)

**Figure 1.** Time averaged pressure distribution on a NACA0015 hydrofoil (AoA = 8[deg], \(σ = 1.4\))
The numerical results of time averaged pressure distribution on a NACA0015 hydrofoil by the present APE model are shown with that of experimental data in Fig.1. Also the numerical result that without the present modification\cite{4} is shown. In the result without the modification, suction side pressure was fail to predict as much higher than that in experiment in the region after mid chord, which indicates that the cavity length was much shorter compared with the experiment. In this study, Function i was adopted for Model 1, and Function ii is adopted for Model 1, Model 2, and Model 3. From Model 1 with Function i in Fig. 1(a) and with Function ii in Fig. 1(b), by adopting Function ii, vapour pressure inside the cavity manages to remain at comparatively lower pressure when the pressure in the region after mid chord is decreased, although it is still insufficient. In Model 2 with Function ii in Fig. 1(c), the suction side pressure decreases sufficiently at mid-chord region, but the pressure inside the sheet cavity especially near the leading edge increased. In Model 3 with Function ii in Fig. 1(d), the pressure distribution showed similar trend to Model 2 using Function ii. In all Model with Function ii, the suction side pressure decreased in the region after the pressure recovery point to trailing edge which corresponds to the region where cloud cavity sheds, but the pressure inside the sheet cavity increased simultaneously in the region before the recovery point which corresponds to the region where sheet cavity develops.

4. Conclusions
In order to improve the prediction accuracy of transient cavitating flow around NACA0015 hydrofoil at higher angle of attack, APE (apparent phase equilibrium) model was applied, and numerical simulation was carried out. The results are summarized as follows:
- By applying the APE model, the pressure distribution on suction side of a NACA0015 hydrofoil was slightly improved in the region after pressure recovery point to trailing edge which corresponds the region where cloud cavity sheds.
- The cavity volume and the aspect of cavity shedding were successfully improved by the APE model.
- For the component which affects the apparent phase equilibrium assumption, Reynolds stress and Q value are comparatively better.
- By applying a weighting function which represents the situation that there is no gas-liquid interface and phase change does not occur and the effect of dissolved air does not appear at void fraction 0 and 1, the pressure distribution improved compared with previous function.

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