The Minimal Dimensionless Standard Model (MDSM) and its Cosmology

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ABSTRACT: Consider the minimal renormalizable extension of the Standard Model with purely dimensionless couplings, successful electroweak symmetry breaking (via the Coleman-Weinberg mechanism) and a see-saw mechanism for neutrino mass: we will call this the Minimal Dimensionless Standard Model (MDSM). In fact, 3 closely related models fit the bill: MDSM$_1$, MDSM$_2$ and MDSM$_3$. We analyze the theoretical and observational constraints on these models. We argue that, when they are minimally coupled to gravity, they can accomplish several important cosmological tasks (inflation, dark matter, leptogenesis) in a way that is economical, predictive and tightly woven into the fabric of known physics. One of the models (MDSM$_3$), which includes an extra $U(1)_{B-L}$ gauge symmetry, seems particularly promising.
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1. Introduction

Over the years, many physicists have been intrigued (for a variety of different reasons) by the idea that the laws of physics might, at fundamental level, be based on massless particles and dimensionless couplings; and that masses and other dimensionful quantities might, in some sense, be secondary or emergent. This has led some researchers to pay special attention to gauge field theories with purely dimensionless coupling constants \((e.g. \ [1, 2, 3])\). Although such theories possess conformal symmetry at the classical level, this symmetry is generally violated at the quantum level. Nevertheless, these theories are strictly renormalizable in the sense that \((e.g.\ if\ we\ use\ dimensional\ regularization)\) there are no ultraviolet divergences that require counterterms with dimensionful coupling constants; the dimensionless couplings just run logarithmically with scale. Some physicists have argued that the resolution of the standard model “hierarchy problem” may lie in the standard model’s proximity to a model with purely dimensionless couplings \((e.g.\ [2, 4])\).

In the standard model of particle physics (which we here take to include 3 right-handed neutrinos \(\nu_R\)), most of the coupling constants are dimensionless. Dimensionful couplings only appear in two places in the Lagrangian: (i) in the Majorana mass term \(\nu_R^T \gamma^0 \gamma^2 M_m \nu_R\) for the right-handed neutrinos; and (ii) in the quadratic self-coupling \(m^2 h^\dagger h\) of the Higgs doublet \(h\). (Here the 3 \(\times\) 3 matrix \(M_m\) and the constant \(m\) both have dimensions of mass.) Is it possible that these two dimensionful terms actually vanish, so that the Lagrangian only contains dimensionless couplings? The answer is no: this model (which we shall call the “Minimal Dimensionless Standard Model, Version 0” or “MDSM\(_0\)”) is ruled out. These two dimensionful terms each play an important phenomenological role in the standard model: the \(\nu_R^T \gamma^0 \gamma^2 M_m \nu_R\) term is responsible for the “see-saw mechanism,” the best available explanation for the smallness of the observed neutrino masses; and, crucially, the \(m^2 h^\dagger h\) term is needed to generate spontaneous breaking of electroweak symmetry. (One might wonder whether it is possible...
to set $m = 0$, and still have electroweak symmetry breaking via the Coleman-Weinberg mechanism \[1, 2\]; but, as we shall review in Sec. 4.2, this doesn’t work in MDSM$_0$ \[3\].

Instead, let us ask for the minimal renormalizable extension of the standard model that only contains dimensionless couplings in its Lagrangian, but nevertheless exhibits a see-saw mechanism for neutrino masses, and spontaneous electroweak symmetry breaking via the Coleman-Weinberg mechanism. In fact, there are three closely related variants that fit the bill: we will call them MDSM$_1$, MDSM$_2$ and MDSM$_3$. All three models have previously been discussed in the literature: MDSM$_1$ in \[3\], MDSM$_2$ in \[4, 5, 6\], and MDSM$_3$ in \[7\]. (For other previous work on dimensionless variants of the standard model, see \[8, 9, 10, 11, 12, 13, 14, 15, 16\].) In this paper, we re-assess these three models and their cosmological consequences, contributing a range of new results.

The layout of the paper is the following. Sections 2 and 3 are intended to establish our notations and conventions: in Section 2, we specify the three models MDSM$_1$, MDSM$_2$ and MDSM$_3$; and in Section 3 we present the tree-level mass spectra in these theories, which we will need for our subsequent analysis. In Section 4 we discuss spontaneous symmetry breaking in these theories via the Coleman-Weinberg mechanism. First, in Subsection 4.1, we present a convenient formalism, due to E. Gildener and S. Weinberg \[2\], for extracting some reliable results about Coleman-Weinberg symmetry breaking, without having to calculate the full renormalization-group-improved effective potential. Then in Subsections 4.2 and 4.3 we apply this formalism to our three models to obtain lower bounds on the new bosonic masses, and upper bounds on the heavy neutrino masses.

In Section 5 we discuss inflation. In our three models, the inflaton candidate is the scalar field $\psi$ which rolls along the “Gildener-Weinberg” direction connecting the origin in field space (the point of unbroken symmetry) to the symmetry-breaking VEV. In Subsection 5.1, we explain that inflation predicts a specific mass relation between the heaviest bosonic and fermionic particles in these theories; in Subsection 5.2, we present the observational predictions for the primordial perturbations generated by inflation in these models; in Subsection 5.3 we present the main decay channels at the end of inflation; and in Subsection 5.4 we obtain the reheating temperature after inflation. In Subsection 5.5 we consider whether these inflationary models are compatible with the mass bound presented in Subsection 4.2: in the case of MDSM$_3$ we find compatibility; in the cases of MDSM$_1$ and MDSM$_2$ we find incompatibility. In Subsection 5.6 we discuss the relation between this inflationary model, and some previous models that have been discussed in the literature.

In Section 6 we consider the possibility that the observed dark matter density and matter/anti-matter asymmetry were both produced directly by the decay of the inflaton into heavy neutrinos at the end of inflation. In this scenario, one of the heavy neutrinos
is the dark matter: we compute what its mass should be, find that it is automatically a non-thermal cold dark matter candidate, and show that if this scenario is right, one of the three light neutrinos must be essentially massless. Then we discuss the possibility that one of the other heavy neutrino species, directly produced during inflaton decay, can generate the cosmological matter/anti-matter asymmetry through its CP violating decay; this is (non-thermal) leptogenesis [18]. Again, we find this picture seems to be compatible with MDSM$_3$, but in tension or outright conflict with MDSM$_1$ and MDSM$_2$.

In Section 7 we draw the reader’s attention to other routes by which dark matter and the cosmological matter/anti-matter asymmetry may be produced in this model. Since these alternative scenarios do not involve an early epoch of inflation driven by $\psi\parallel$, the tension with MDSM$_1$ and MDSM$_2$ is relieved. In Section 8 we draw together and emphasize the particularly compelling features of MDSM$_3$, including several features that were not mentioned earlier in the paper. Finally, in Section 9 we discuss some directions for future work.

2. The base model and 3 minimal extensions

The three models we will be interested in (MDSM$_1$, MDSM$_2$ and MDSM$_3$) are all slight variants of a common (but non-viable) base model MDSM$_0$. It is convenient to first specify MDSM$_0$, and then describe the 3 variants in turn.

2.1 MDSM$_0$ (the base model)

In brief, the base model MDSM$_0$ is the standard model plus 3 right-handed neutrinos (i.e. the “νMSM” [19, 20]), augmented by the additional constraint of classical conformal invariance to eliminate all dimensionful couplings.

The base model has the same gauge group and field content as the minimal $SU(3)_C \times SU(2)_L \times U(1)_Y$ standard model, including 3 gauge-singlet right-handed neutrinos (one per generation). In other words, the fields and representations in MDSM$_0$ are summarized by the following table:

| Field | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ |
|-------|-----------|-----------|-----------|
| $q_L$ | 3         | 2         | +1/6      |
| $u_R$ | 3         | 1         | +2/3      |
| $d_R$ | 3         | 1         | −1/3      |
| $l_L$ | 1         | 2         | −1/2      |
| $\nu_R$ | 1       | 1         | 0         |
| $e_R$ | 1         | 1         | −1        |
| $h$   | 1         | 2         | +1/2      |

(2.1)
Here $q_L$ is the left-handed quark doublet, $u_R$ and $d_R$ are the right-handed quark singlets, $l_L$ is the left-handed lepton doublet, $\nu_R$ and $e_R$ are the right-handed lepton singlets, and $h$ is the scalar Higgs doublet. As usual, the fermion (quark and lepton) fields all come in 3 generations that are identical, apart from the values of their Yukawa couplings. Our base model is now obtained by writing down the most general renormalizable Lagrangian built from these ingredients, with classical conformal invariance (i.e. dimensionless couplings)

$$L_0 = -(1/4) B_{\mu \nu}^2 - (1/4) W^{(a)}_{\mu \nu}^2 - (1/4) G^{(a)}_{\mu \nu}^2 + (D_\mu h)^\dagger (D^\mu h) - \lambda_h (h^\dagger h)^2$$

$$+ i\bar{q}_L \not{D} q_L + i\bar{u}_R \not{D} u_R + i\bar{d}_R \not{D} d_R + i\bar{l}_L \not{D} l_L + i\bar{\nu}_R \not{D} \nu_R + i\bar{e}_R \not{D} e_R$$

$$- \bar{q}_L Y_u u_R \bar{h} - \bar{q}_L Y_d d_R \bar{h} - \bar{l}_L Y_\nu \nu_R \bar{h} - \bar{l}_L Y_e e_R \bar{h} + h.c.$$ (2.2)

where $Y_u$, $Y_d$, $Y_\nu$ and $Y_e$ are the four $3 \times 3$ Yukawa matrices, whose indices run over the 3 fermion generations; and here and throughout the rest of the paper, we will use the following notation for charge conjugate fields:

$$\bar{h} \equiv i\sigma^2 h^*,$$  
$$\nu_c \equiv i\gamma^2 \nu_R^*.$$ (2.3)

We will often work in “unitary gauge,” in which $h$ is written in the form

$$h = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_h + \rho_h \end{array} \right)$$ (2.4)

where $v_h$ is a constant (the VEV) and $\rho_h$ is a field (the displacement from the VEV).

The classical conformal invariance has the effect of removing two key terms that are otherwise present and play a phenomenologically important role in the $\nu$MSM Lagrangian: (i) first, the terms $\frac{1}{2} \bar{\nu}_c M m R + h.c.$ that give rise to see-saw masses for the neutrinos; and (ii) second, the term $m_h^2 h^\dagger h$ that generates spontaneous symmetry breaking at tree level.

Note that MDSM$_0$ is not viable by itself. As we shall review in Subsection 4.2, because the top quark is so heavy (relative to the $W$ and $Z$ bosons), radiative corrections destabilize the theory: the one-loop effective Higgs potential is unbounded below.

Let us now present three minimal extensions of this base model that retain its renormalizability and classical conformal invariance, but re-introduce see-saw masses for the neutrinos, and also achieve successful spontaneous electroweak symmetry breaking via the Coleman-Weinberg mechanism.

### 2.2 MDSM$_1$

MDSM$_1$ is obtained by adding a single real scalar field $\varphi$, which is a gauge singlet under $SU(3)_C \times SU(2)_L \times U(1)_Y$, and again writing down the most general renormalizable
Lagrangian, with classical conformal invariance (i.e. dimensionless couplings):

\[ \mathcal{L}_1 = \mathcal{L}_0 + \frac{1}{2} (\partial \phi)^2 - \frac{\lambda_2}{4} \phi^4 - \lambda_m (h^\dagger h) \phi^2 - \frac{1}{2} \frac{\phi}{\sqrt{2}} (\bar{\nu}_c Y^\dagger_m \nu_R + \text{h.c.}) \]  

(2.5)

where \( Y_m = Y_m^T \). As we did with \( h \), we split \( \phi \) into a constant \( v_\phi \) (the VEV) and a field \( \rho_\phi \) (the displacement from the VEV):

\[ \phi = v_\phi + \rho_\phi. \]  

(2.6)

### 2.3 MDSM\(_2\)

MDSM\(_2\) is the same as MDSM\(_1\), except now the gauge singlet scalar field \( \phi \) is complex rather than real, so the most general renormalizable Lagrangian with classical conformal invariance is

\[ \mathcal{L}_2 = \mathcal{L}_0 + |\partial \phi|^2 - \lambda_\phi |\phi|^4 - 2 \lambda_m (h^\dagger h) |\phi|^2 - \frac{1}{2} (\phi \bar{\nu}_c Y^\dagger_m \nu_R + \text{h.c.}) \]  

(2.7)

where again \( Y_m = Y_m^T \). We now write \( \phi \) as

\[ \phi = \frac{1}{\sqrt{2}} (v_\phi + \rho_\phi) e^{i a/v_\phi}. \]  

(2.8)

Note the presence in this model of the additional real field \( a \), which measures the complex phase of \( \phi \). This field, called the Majoron \([21]\), is a Goldstone boson at tree level. Refs. \([7, 8]\) argue that \( a \) obtains a mass via quantum effects, and from a phenomenological standpoint is very much like an axion\(^1\).

### 2.4 MDSM\(_3\)

MDSM\(_3\) is similar to MDSM\(_2\), except now we also add a new \( U(1)_X \) gauge symmetry, carried by a new gauge boson \( C_\mu \). Without loss of generality (see Subsection \([8, 1]\)) this new charge may be taken to be nothing but baryon number minus lepton number \((X = B - L)\); and in order to preserve see-saw neutrino masses, the scalar field \( \phi \) must couple to this new symmetry with charge +2. The fields and their representations are

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\(^1\)This offers the possibility that the strong CP problem may be solved in MDSM\(_2\). As far as we are aware, it is not solved in MDSM\(_1\) or MDSM\(_3\).
now summarized by the following table

| SU(3)_C | SU(2)_L | U(1)_Y | U(1)_X |
|---------|---------|---------|---------|
| q_L     | 3       | 2       | +1/6    | +1/3    |
| u_R     | 3       | 1       | +2/3    | +1/3    |
| d_R     | 3       | 1       | −1/3    | +1/3    |
| l_L     | 1       | 2       | −1/2    | −1      |
| ν_R     | 1       | 1       | 0       | −1      |
| ε_R     | 1       | 1       | −1      | −1      |
| h       | 1       | 2       | +1/2    | 0       |
| φ       | 1       | 1       | 0       | +2      |

(2.9)

and the most general renormalizable Lagrangian with classical conformal symmetry built from these ingredients is

\[ \mathcal{L}_3 = \mathcal{L}_0 + |D_\mu \varphi|^2 - \lambda_\varphi |\varphi|^4 - 2 \lambda_m (h^\dagger h) |\varphi|^2 - \frac{1}{2} (\varphi \bar{\nu}_c Y_m Y_{\nu_R} + h.c.) - \frac{1}{4} C_{\mu\nu}^2 - \frac{\kappa}{2} B_{\mu\nu} C^{\mu\nu} \]

(2.10)

where \( Y_m = Y_{m0} \). We fix the final \( U(1)_X \) gauge freedom by setting the imaginary part of \( \varphi \) to zero, and then write it as

\[ \varphi = \frac{1}{\sqrt{2}} (v_\varphi + \rho_\varphi). \]

(2.11)

3. Tree-level masses

In this section, we present the tree-level mass spectra in MDSM_1, MDSM_2 and MDSM_3. These will be needed in our subsequent analysis.

3.1 Scalar masses

MDSM_1 and MDSM_3 contain two physical scalar fields, \( \rho_h \) and \( \rho_\varphi \), that give rise to two physical mass eigenstates \( \rho_\parallel \) and \( \rho_\perp \); MDSM_2 has, in addition, a third scalar field \( a \) that is a massless Goldstone boson at the classical level, but obtains a mass via quantum effects and is similar to the usual QCD axion. Let us start by studying the \( \rho_\parallel \) and \( \rho_\perp \) bosons, since they are present in all 3 models. In all 3 models, the tree-level scalar potential becomes

\[ \frac{\lambda_\varphi}{4} (v_\varphi + \rho_\varphi)^4 + \frac{\lambda_m}{2} (v_\varphi + \rho_\varphi)^2 (v_h + \rho_h)^2 + \frac{\lambda_h}{4} (v_h + \rho_h)^4. \]

(3.1)

In describing the VEVs, it will be convenient to switch from cartesian to polar coordinates by introducing the notation (see Fig. [3])

\[ \begin{pmatrix} v_\varphi \\ v_h \end{pmatrix} = \begin{pmatrix} v \cos \chi \\ v \sin \chi \end{pmatrix}. \]

(3.2)
Figure 1: This figure depicts the relationship (in the space of scalar fields) between the scalar VEV (with coordinate \(\{v, v_h\}\)), the inflaton direction \(\psi\parallel\), and the scalar mass eigenstates (\(\rho\parallel\) and \(\rho\perp\)).

Since \(\rho\phi\) and \(\rho_h\) represent displacements from the VEV, the terms in (3.1) linear in \(\rho\phi\) and \(\rho_h\) must vanish, leading to the conditions

\[
\lambda\phi\cos^2\chi + \lambda_m\sin^2\chi = 0 \tag{3.3a}
\]
\[
\lambda_h\sin^2\chi + \lambda_m\cos^2\chi = 0. \tag{3.3b}
\]

When these conditions are satisfied, the quadratic terms in (3.1) reduce to

\[
\frac{1}{2}(m^2\rho^2\parallel + m^2\rho^2\perp) \tag{3.4}
\]

where the mass eigenstates and eigenvalues are (see Fig. 1)

\[
\begin{pmatrix}
\rho\parallel \\
\rho\perp
\end{pmatrix} =
\begin{pmatrix}
\cos\chi & \sin\chi \\
-\sin\chi & \cos\chi
\end{pmatrix}
\begin{pmatrix}
\rho\phi \\
\rho_h
\end{pmatrix}
\]

\[
m\parallel = 0 \quad m\perp = (-2\lambda_m)^{1/2}v. \tag{3.5}
\]

Finally, the axion-like field \(a\) in MDSM\(_2\) is a Goldstone boson at the classical level, with tree-level mass \(m_a = 0\).
3.2 Fermion masses

All 3 models have the same fermion content, and the same expressions for the fermion masses at tree level. To see this, note that in each model, when we expand around the scalar VEVs, the fermion mass terms are

\[-\frac{v_h}{\sqrt{2}} [(\bar{u}_L Y_u^+ u_R + \bar{u}_R Y_u u_L) + (\bar{d}_L Y_d^+ d_R + \bar{d}_R Y_d d_L) + (\bar{e}_L Y_e^+ e_R + \bar{e}_R Y_e e_L)]
\]

\[-\frac{v_h}{\sqrt{2}} (\bar{\nu}_L Y_\nu^+ \nu_R + \bar{\nu}_R Y_\nu \nu_L) - \frac{1}{2} \frac{v_\phi}{\sqrt{2}} (\bar{\nu}_c Y_m^+ \nu_R + \bar{\nu}_R Y_m \nu_c). \tag{3.6}\]

To obtain the masses corresponding to the terms on the first line of (3.6), we perform a singular value decomposition, \( Y_\alpha = U_\alpha y_\alpha V_\alpha^* \), where \( U_\alpha \) and \( V_\alpha \) are unitary matrices, and \( y_\alpha \) is diagonal with real, non-negative eigenvalues \( y_\alpha^{(1)} \leq y_\alpha^{(2)} \leq y_\alpha^{(3)} \). Then the masses of the up-type, down-type, and electron-type fermions are

\[ m_u^{(i)} = \frac{v_h}{\sqrt{2}} y_u^{(i)}, \quad m_d^{(i)} = \frac{v_h}{\sqrt{2}} y_d^{(i)}, \quad m_e^{(i)} = \frac{v_h}{\sqrt{2}} y_e^{(i)}. \tag{3.7}\]

To obtain the masses corresponding to the terms on the second line of (3.6) – i.e. the neutrino masses – let us start by collecting all of these terms together into a single term of the form

\[-\frac{1}{2} \nu^T \gamma^0 \gamma^2 M \nu + h.c. \tag{3.8}\]

Here \( \nu \) is a 6 component vector consisting of all 6 left-handed neutrinos (3 \( \nu_L \)'s and 3 \( \nu_c \)'s), and \( M \) is the corresponding 6 \( \times \) 6 neutrino mass matrix:

\[ \nu \equiv \begin{pmatrix} \nu_L \\ \nu_c \end{pmatrix}, \quad M \equiv \begin{pmatrix} 0 & M^T_
u \\ M_\nu & M_m \end{pmatrix} \tag{3.9} \]

where the 3 \( \times \) 3 matrices \( M_\nu \) and \( M_m \) are given by

\[ M_\nu \equiv \frac{v_h}{\sqrt{2}} Y_\nu, \quad M_m \equiv \frac{v_\phi}{\sqrt{2}} Y_m. \tag{3.10}\]

Now if we regard the 3 \( \times \) 3 matrix

\[ r \equiv M_m^{-1} M_\nu \tag{3.11}\]

as “small” (as it is in the see-saw mechanism), and block diagonalize the matrix \( M \) through 2nd order in \( r \), the result is

\[ M = S^T \begin{pmatrix} M_\alpha & 0 \\ 0 & M_N \end{pmatrix} S \tag{3.12} \]
where $M_n$ and $M_N$ are the $3 \times 3$ mass matrices for the light neutrinos ($n$) and the heavy neutrinos ($N$) respectively

$$
M_n = -r^T M_m r, \quad M_N = M_m + \frac{1}{2} r r^T M_m + \frac{1}{2} M_m r r^T
$$

and the orthogonal matrix that achieves the block diagonalization is

$$
S = \begin{pmatrix}
1 - (1/2)r^T r & -r^T \\
r & 1 - (1/2)rr^T
\end{pmatrix}.
$$

Thus, the relation between the original basis ($\nu_L, \nu_c$) and the block diagonal basis ($n, N$) for the neutrino states is

$$
\begin{pmatrix}
n \\
N
\end{pmatrix} = S \begin{pmatrix}
\nu_L \\
\nu_c
\end{pmatrix} \quad \begin{pmatrix}
\nu_L \\
\nu_c
\end{pmatrix} = S^T \begin{pmatrix}
n \\
N
\end{pmatrix}.
$$

### 3.3 Vector masses

In MDSM$_1$ and MDSM$_2$, the gauge bosons are exactly the same as in the ordinary standard model: associated with the unbroken factor $SU(3)_C$ there are 8 massless gluons; and associated with the spontaneously broken factor $SU(2)_L \times U(1)_Y$ there are 4 gauge bosons ($W^{(1)}_\mu, W^{(2)}_\mu, W^{(3)}_\mu, B_\mu$) that, after spontaneous symmetry breaking, become 3 massive vector bosons ($W^{+}_\mu, W^{-}_\mu, Z_\mu$) and one massless gauge boson (the photon, $\gamma_\mu$, which couples to electric charge: $Q = Y + T_3$):

$$
\begin{align*}
W^{\pm}_\mu &= (W^{(1)}_\mu \pm iW^{(2)}_\mu)/\sqrt{2} & m_W &= g_w v_h/2 \\
Z_\mu &= \cos \theta_W W^{(3)}_\mu - \sin \theta_W B_\mu & m_Z &= g_w v_h/2 \cos \theta_W \\
\gamma_\mu &= \sin \theta_W W^{(3)}_\mu + \cos \theta_W B_\mu & m_\gamma &= 0
\end{align*}
$$

where $\theta_W$ is the Weinberg angle:

$$
\cos \theta_W = \frac{g_w}{\sqrt{g_w^2 + g_y^2}}, \quad \sin \theta_W = \frac{g_y}{\sqrt{g_w^2 + g_y^2}}.
$$

In MDSM$_3$, the situation is slightly more complicated. Associated with the unbroken factor $SU(3)_C$ there are again 8 massless gluons. Associated with the spontaneously broken factor $SU(2)_L \times U(1)_Y \times U(1)_X$ there are 5 gauge bosons ($W^{(1)}_\mu, W^{(2)}_\mu, W^{(3)}_\mu, B_\mu, C_\mu$) that, after spontaneous symmetry breaking, become 4 massive vector bosons ($W^{(4)}_\mu, W^{(-)}_\mu, Z_\mu, Z'_\mu$) and one massless gauge boson (the photon, $\gamma_\mu$, which again couples to electric charge $Q = Y + T_3$). Let us write these 5 mass eigenstates and eigenvalues more explicitly. The two electrically charged states ($W^{+}_\mu$ and $W^{-}_\mu$) are exactly the same as before:

$$
W^{\pm}_\mu = \frac{1}{\sqrt{2}}(W^{(1)}_\mu \pm iW^{(2)}_\mu), \quad m_W = g_w v_h/2
$$
while the photon is given by
\[ \gamma_\mu = \sin \theta_W W^{(3)}_\mu + \cos \theta_W [B_\mu + \kappa C_\mu], \quad m_\gamma = 0. \quad (3.19) \]

The remaining two electrically neutral bosons \((Z_\mu = Z^-_\mu \text{ and } Z'_\mu = Z^+_\mu)\) are given by the relatively complicated formulae:
\[ Z^\pm_\mu = \frac{1}{d \pm} [a \pm W^{(3)}_\mu + b \pm B_\mu + c \pm C_\mu], \quad m^2 \pm = (1/4)v_h^2 \lambda \pm \quad (3.20) \]

where we have defined the constants \(p = 4g_x(v_\psi/v_h)\) and:
\[ \lambda \pm = \frac{\kappa^2 \lambda \pm - g^2_w}{\lambda \pm - p^2}, \quad a \pm = \frac{g_y g_w \lambda \pm}{\lambda \pm - g^2_w} - 1, \quad c \pm = \frac{\kappa^2 \lambda \pm}{\lambda \pm - p^2}, \quad (3.21b) \]
\[ d \pm = \sqrt{a^2 \pm + c^2 \pm + (1 - \kappa^2)} \quad (3.21c) \]

The linear transformation that relates \((W^{(3)}_\mu, B_\mu, C_\mu)\) to \((\gamma_\mu, Z_\mu, Z'_\mu)\) is defined by the requirement that, not only do the mass terms become diagonal, but kinetic terms also become diagonal and canonically normalized; it is only an orthogonal transformation when \(\kappa = 0\). When the ratio \(v_\psi/v_h\) is large (as it will be for us) the expressions simplify, at leading order, to:
\[ Z_\mu \approx \cos \theta_W W^{(3)}_\mu - \sin \theta_W [B_\mu + \kappa C_\mu], \quad m_Z \approx g_w v_h/2 \cos \theta_W \]
\[ Z'_\mu \approx \sqrt{1 - \kappa^2} C_\mu, \quad m_{Z'} \approx 2(1 - \kappa^2)^{-1/2} g_x v_\psi \quad (3.22) \]

4. Symmetry breaking via the Coleman-Weinberg Mechanism

4.1 The Gildener-Weinberg formalism

An elegant and useful formalism for analyzing Coleman-Weinberg symmetry breaking \[\square\] in theories with arbitrary scalar field content is developed by E. Gildener and S. Weinberg in Ref. \[\square\]. For a detailed justification of the following calculations, we refer the reader to Ref. \[\square\]. In this section, we just summarize the essential results. The first step is to run the renormalization group scale to a special scale \(\Lambda\) at which the tree-level scalar potential has a degenerate valley of minima along a ray extending out from the origin in field space: in our models, this is equivalent to working at the renormalization scale \(\Lambda\) at which the tree-level parameters in our effective potential
satisfy Eq. (3.3), with $\lambda_m < 0$. At this scale, define two constants, $A$ and $B$:

$$A \equiv \frac{1}{64\pi^2} \left[ \sum_s \frac{m_s^4}{v^4} \ln \frac{m_s^2}{v^2} - 4 \sum_f \zeta_f \frac{m_f^4}{v^4} \ln \frac{m_f^2}{v^2} + 3 \sum_v \frac{m_v^4}{v^4} \ln \frac{m_v^2}{v^2} \right]$$

$$B \equiv \frac{1}{64\pi^2} \left[ \sum_s \frac{m_s^4}{v^4} - 4 \sum_f \zeta_f \frac{m_f^4}{v^4} + 3 \sum_v \frac{m_v^4}{v^4} \right]$$

where, in each equation, the 3 sums are over all tree-level scalar, fermion, and vector masses, respectively; and $\zeta_f = 1$ for Dirac fermions and $1/2$ for Majorana or Weyl fermions. These sums are dominated by largest tree-level masses in the theory; so, in MDSM$_1$, MDSM$_2$ and MDSM$_3$, respectively, we have

$$B_0 \approx \frac{1}{64\pi^2 v^4} \left[ -12m_t^4 - 2 \sum_{i=1}^{3} m_{N,i}^4 + 6m_W^4 + 3m_Z^4 \right]$$

$$B_1 = B_2 \approx \frac{1}{64\pi^2 v^4} \left[ -12m_t^4 - 2 \sum_{i=1}^{3} m_{N,i}^4 + 6m_W^4 + 3m_Z^4 + m_\perp^4 \right]$$

$$B_3 \approx \frac{1}{64\pi^2 v^4} \left[ -12m_t^4 - 2 \sum_{i=1}^{3} m_{N,i}^4 + 6m_W^4 + 3m_Z^4 + m_\perp^4 + 3m_{Z'}^4 \right].$$

At tree-level, the effective potential had a degenerate valley of minima along the direction given by angle $\chi$ in field space; but the one-loop correction gives a radial shape to the potential along this direction:

$$V(\psi_\parallel) = \frac{B}{2} v^4 \left( 1 - \frac{\psi_\parallel^4}{v^4} + 4 \frac{\psi_\parallel^4}{v^4} \ln \frac{\psi_\parallel}{v} \right).$$

This lifts the degeneracy, and picks out a unique minimum a distance $v$ from the origin in scalar field space (see Fig. [1]). For our later cosmological applications, we have added an overall constant to the potential so that $V = 0$ at the minimum $\psi_\parallel = v$ ($\rho_\parallel = 0$). In other words, we do not solve the cosmological constant problem: we do not explain why the cosmological constant is small, but merely choose it to be tiny, in accordance with cosmological observations. The $\rho_\parallel$ boson was massless at tree level, but at one-loop obtains a mass

$$m_\parallel = (8B)^{1/2} v.$$ 

Finally note that the VEV $v$, the renormalization scale $\Lambda$, and the constants $A$ and $B$ are related to one another via the constraint

$$\ln \frac{v^2}{\Lambda^2} = -\frac{1}{2} - \frac{A}{B}. $$
4.2 Lower bounds on new bosonic masses

We must require $B > 0$ so that the one-loop effective potential (4.3) is bounded below. In MDSM$_0$, this condition is not satisfied ($B_0 < 0$). Thus, as previously mentioned in Section 1 and Subsection 2.1, the base model MDSM$_0$ is not viable. In MDSM$_1$ and MDSM$_2$, the requirement $B > 0$ implies a lower bound on the $\rho_\perp$ mass

$$m_\perp > \left[12m_t^4 - 6m_W^4 - 3m_Z^4\right]^{1/4} = 318.3 \text{ GeV}. \quad (4.6)$$

In MDSM$_3$ it implies, instead, a lower bound on a combination of the $\rho_\perp$ and $Z'$ masses:

$$m_\perp^4 + 3m_{Z'}^4 > (318.3 \text{ GeV})^4. \quad (4.7)$$

4.3 Upper bounds on heavy neutrino masses

One can also interpret the requirement $B > 0$ as an upper bound on the 3 heavy neutrino masses $m_{N,i}$; in MDSM$_1$ and MDSM$_2$ this bound is

$$2 \sum_i m_{N,i}^4 < m_\perp^4 - (318.3 \text{ GeV})^4 \quad (4.8)$$

and in MDSM$_3$ it becomes

$$2 \sum_i m_{N,i}^4 < 3m_{Z'}^4 + m_\perp^4 - (318.3 \text{ GeV})^4. \quad (4.9)$$

5. Inflation

5.1 Predicted mass relation

The inflaton candidate in the MDSM is the scalar field $\psi_\parallel$. If $\psi_\parallel$ was initially perched near the point of unbroken symmetry$^2$ ($\psi_\parallel = 0$), standard single-field slow-roll inflation occurs as it rolls along the $\psi_\parallel$ direction toward the minimum at $\psi_\parallel = v$. The shape of the effective potential along this direction is given by (4.3). This potential depends on two parameters ($B$ and $v$), but in order to match the observed amplitude$^{[24]}$ of the primordial scalar perturbations [$\Delta_R^2(k_*) = (2.43 \pm 0.1) \times 10^{-9}$ at $k_* = 0.002 \text{ Mpc}^{-1}$] these parameters must be related as shown in the first panel of Fig. 2. With this constraint, we can regard the potential (4.3) as depending on just a single free parameter $v$.

$^2$One may ask why the field was initially perched near $\psi_\parallel = 0$. One possibility comes from the “no-boundary wave function” (“NBWF”) proposal of Hartle and Hawking$^{[22]}$. Following the reasoning in$^{[23]}$, one can show that the “top-down” prediction of the NBWF for our Coleman-Weinberg-shaped potential is, in fact, precisely that the field should have been perched near $\psi_\parallel = 0$ initially.
Recall that \( B \) is related to the tree-level mass spectrum of the theory: see Eqs. (4.1b) and (4.2). In Subsections 4.2 and 4.3, we used the requirement \( B > 0 \) to obtain predicted bounds on some of the heaviest particles in the MDSM. Now, since inflation predicts a specific relationship between \( B \) and \( v \), and in particular always predicts \( B \sim 10^{-14} \) (see Panel 1 in Fig. 2), we can go further: the relationship between \( B \) and \( v \) shown in Panel 1 of Fig. 2 amounts to a specific prediction from inflation for a mass relation between the heaviest particles in the MDSM. Is there any way to confirm this prediction (or its relationship to the other predictions depicted in the subsequent panels of Fig. 2)?

### 5.2 Predictions for primordial perturbations

Using techniques that are by now standard (see e.g. [25]), we compute (numerically) the observable predictions for the primordial scalar (density) and tensor (gravitational wave) perturbations predicted by the inflaton potential (4.3), as a function of \( v \). In particular, in panels 2 through 4 of Fig. 2 we plot the predicted scalar spectral index \( n_s \), the running of the scalar spectral index \( \alpha_s = dn_s/d\ln k \), and the tensor to scalar ratio \( r \) as a function of \( v \). Other predictions agree with standard single-field slow-roll inflation: the scalar perturbations should be adiabatic (no isocurvature perturbations) and gaussian (negligible non-gaussianity).

Notice that \( n_s \) is predicted to be significantly “red” \((n_s < 1)\), and gets redder as \( v \) decreases. Current cosmological observations place a lower bound on \( n_s \) which therefore implies an observational lower bound on \( v \) if \( \psi_\parallel \) is the inflaton. In particular, for \( r \approx 0 \) (appropriate for the low \( v \) regime), observations currently favor \( n_s \approx 0.96 \), with a lower bound \( n_s \gtrsim 0.93 \) (at 2\( \sigma \)) or \( n_s \gtrsim 0.92 \) (at 3\( \sigma \)) [24]. From Panel 2 in Fig. 2 we see that this translates into a lower bound \( v \gtrsim 10^{13} \) GeV or \( v \gtrsim 10^{11} \) GeV, respectively. Forthcoming data from the Planck satellite should significantly tighten the constraints on \( n_s \), leading to significantly tighter constraints on \( v \). The observable \( \alpha_s \) is predicted to be negative, but with sufficiently small absolute value that its deviation will be difficult (though not impossible) to detect [29]. The observable \( r \) is only large enough to be detected when \( v > 10^{19} \) GeV.

### 5.3 Inflaton decay

At the end of inflation, the energy of the universe is initially stored in an oscillating condensate of \( \rho_\parallel \) bosons. In this subsection, we will present some of the main decay rates by which the \( \rho_\parallel \) decays to other scalar, spinor, and vector fields.

If the \( \rho_\parallel \) mass is more than twice the \( \rho_\perp \) mass, then it can decay to a pair of \( \rho_\perp \)'s via a 3-leg diagram coming from a term \(-m_\perp^2/v)\rho_\parallel\rho_\perp^2 \) in the Lagrangian. The
Figure 2: This figure depicts several predictions of the MDSM inflationary scenario, as a function of $v$, the scalar field VEV. The top 4 panels show, respectively, the predicted value for $B$, the primordial scalar spectral index $n_s$, the scalar spectral running $\alpha_s = dn_s/d\ln k$, and the primordial tensor-to-scalar ratio $r$. The four curves in the bottom panel show the following: (i) the red curve shows $V_{1/4}^{1/4}$, where $V_{\text{max}}$ is the height of the potential at its local maximum at the point of unbroken symmetry; (ii) the black curve shows $m_{\parallel}$, the mass of the $\rho_{\parallel}$ boson; (iii) the blue curve shows $T_{RH}$, the reheating temperature; and (iv) the purple curve shows the dark-matter mass $M_1$ (i.e. the mass of the stable right-handed neutrino which gives the correct dark matter abundance).

The corresponding decay rate is

$$\Gamma(\rho_\parallel \to \rho_\perp \rho_\perp) = \frac{m_\perp^4}{8\pi m_\parallel v^2} \left(1 - 4\frac{m_\perp^2}{m_\parallel^2}\right)^{1/2}.$$  \hspace{1cm} (5.1)

If the $\rho_\parallel$ mass is more than twice the mass of fermion $f$, then it can decay to an $f\bar{f}$ pair via a 3-leg diagram with vertex factor $-i(m_f/v)$. The decay rate $\Gamma(\rho_\parallel \to f\bar{f})$
depends on whether \( f \) is a quark \( q \) (a Dirac fermion with 3 colors), a charged lepton \( l \) (a colorless Dirac fermion), or a neutrino mass eigenstate \( \nu \) (a colorless Majorana fermion); the corresponding rates are

\[
\Gamma(\rho || \rightarrow q\bar{q}) = \frac{3m_q^2 m_{\rho ||}}{8\pi v^2} \left( 1 - 4 \frac{m_q^2}{m_{\rho ||}^2} \right)^{3/2}, \tag{5.2a}
\]

\[
\Gamma(\rho || \rightarrow \bar{l}l) = \frac{m_l^2 m_{\rho ||}}{8\pi v^2} \left( 1 - 4 \frac{m_l^2}{m_{\rho ||}^2} \right)^{3/2}, \tag{5.2b}
\]

\[
\Gamma(\rho || \rightarrow \nu\bar{\nu}) = \frac{m_\nu^2 m_{\rho ||}}{16\pi v^2} \left( 1 - 4 \frac{m_\nu^2}{m_{\rho ||}^2} \right)^{3/2}. \tag{5.2c}
\]

If the \( \rho || \) mass is more than twice the mass of vector \( V \), then it can decay into a pair of \( V \)'s via a 3-leg diagram. In particular, it can decay to: (i) a \( W^+W^- \) pair through the Lagrangian term \(-2(m_W^2/v)\rho || W^+W^-\); (ii) a \( ZZ \) pair through the term \(-(m_Z^2/v)\rho || Z^2\); (iii) or (in Model 3 only) a \( Z'Z' \) pair through the term \(-(m_{Z'}^2/v)\rho || Z^2\).

The corresponding rates are

\[
\Gamma(\rho || \rightarrow W^+W^-) = \frac{m_{\rho ||}^3}{16\pi v^2} \left( 1 - 4 \frac{m_W^2}{m_{\rho ||}^2} \right)^{1/2} \left[ 1 - 4 \frac{m_W^2}{m_{\rho ||}^2} + 12 \frac{m_W^4}{m_{\rho ||}^4} \right] \tag{5.3a}
\]

\[
\Gamma(\rho || \rightarrow ZZ) = \frac{m_{\rho ||}^3}{32\pi v^2} \left( 1 - 4 \frac{m_Z^2}{m_{\rho ||}^2} \right)^{1/2} \left[ 1 - 4 \frac{m_Z^2}{m_{\rho ||}^2} + 12 \frac{m_Z^4}{m_{\rho ||}^4} \right] \tag{5.3b}
\]

\[
\Gamma(\rho || \rightarrow Z'Z') = \frac{m_{\rho ||}^3}{32\pi v^2} \left( 1 - 4 \frac{m_{Z'}^2}{m_{\rho ||}^2} \right)^{1/2} \left[ 1 - 4 \frac{m_{Z'}^2}{m_{\rho ||}^2} + 12 \frac{m_{Z'}^4}{m_{\rho ||}^4} \right] \tag{5.3c}
\]

### 5.4 Reheating Temperature \( T_{RH} \)

To compute the reheating temperature \( T_{RH} \) (the temperature at the start of the radiation-dominated epoch), we need to calculate the total \( \rho || \) decay rate, \( \Gamma || \). First consider the mass of the \( \rho || \) boson, \( m_{\rho ||} = (8B)^{1/2}v \), the black curve in the top panel of Fig. 2. For values of \( v \) that are sufficiently large to be consistent with the observational lower bound on \( n_s \) (discussed in Subsection 5.2), we see that \( m_{\rho ||} \) is large relative to the \( W \) and \( Z \) boson masses; and then, from inspecting the decay rates presented in the preceding few subsections, we see that the leading \( \rho || \) decay channels are \( \rho || \rightarrow W^+W^- \) and \( \rho || \rightarrow ZZ \). Although the contributions of \( \Gamma(\rho || \rightarrow Z'Z') \) and \( \Gamma(\rho || \rightarrow N_iN_i) \) can be also be non-negligible in some circumstances, they are never large, and neglecting them does not significantly alter the calculations in this subsection. Thus, we can
approximate the total $\rho_\parallel$ decay rate by the simple formula

$$\Gamma_\parallel \approx \Gamma(\rho_\parallel \to W^+W^-) + \Gamma(\rho_\parallel \to ZZ) \approx \frac{3m^3_\parallel}{32\pi v^2}. \quad (5.4)$$

From here, we can compute $T_{RH}$ in the standard way, by requiring that the decay rate $\Gamma_\parallel$ should equal the Hubble expansion rate $H_{RH}$ at the start of the radiation era \cite{27,28,29}:

$$\Gamma_\parallel = \left[ \frac{8\pi G}{3} \frac{\pi^2}{30} g_*(T_{RH}) T_{RH}^4 \right]^{1/2}. \quad (5.5)$$

We can solve this for $T_{RH}$ to find

$$T_{RH} = \frac{3}{\pi} \left( \frac{5}{g_*(T_{RH})} \right)^{1/4} B^{3/4} M_{pl}^{1/2} v^{1/2}. \quad (5.6)$$

The bottom panel of Fig. 2 shows the reheat temperature $T_{RH}$ (blue curve), alongside the mass $m_\parallel = (8B)^{1/2} v$ of the $\rho_\parallel$ boson (black curve) and the height of the inflationary hilltop $V_{max} = (B/2)v^4$ [or its 4th root, $V^{1/4}_{max} = (B/2)^{1/4} v$, which has units of mass] (red curve).

### 5.5 Compatibility with MDSM$_3$; incompatibility with MDSM$_1$ and MDSM$_2$

Which of the 3 models (MDSM$_1$, MDSM$_2$, or MDSM$_3$) is compatible with inflation? On the one hand, accelerator constraints require us to have $v_h = 246$ GeV. On the other hand, we saw in Subsection 5.2 that, in order to be compatible with observational constraints on the primordial scalar spectral index $n_s$, the parameter $v$ must be $\gtrsim 10^{11}$ GeV. Thus, if we want to embed inflation in the MDSM, we are forced into the regime $\chi \ll 1$. In this regime, the $\rho_\perp$ boson is essentially the ordinary Higgs boson, and its mass is given by $\rho_\perp \approx (2\lambda h)^{1/2} v_h$.

In MDSM$_1$ and MDSM$_2$, the mass bound (4.6) then says that the mass of the ordinary Higgs boson must be greater than 318.3 GeV; and when combined with current LHC results, this lower bound increases to $\sim 450$ GeV. This lower bound implies that the dimensionless coupling $\lambda_h$ must be $\gtrsim 2$, which is uncomfortably large within perturbation theory. Furthermore, an ordinary Higgs mass above either 318 GeV or 450 GeV is in tension with indirect limits coming from precision electroweak data, which prefer a light Higgs, but are rather broad. Based on these considerations, it seems at this point that MDSM$_1$ and MDSM$_2$ are not good candidates to drive inflation; but it may be too strong to say that this possibility is strictly ruled out at present\textsuperscript{3}.

\textsuperscript{3}In fact, in the time since this paper first appeared on the arXiv, the ATLAS and CMS collaborations have announced the discovery of what is apparently the Higgs boson, with a mass near 126 GeV, so that one can now say with much greater certainty that MDSM$_1$ and MDSM$_2$ are ruled out as candidates to drive inflation.
By contrast, MDSM$_3$ is completely compatible with inflation: the mass bound (4.7) is readily satisfied, since it involves both the Higgs mass $m_\perp$ and the $Z'$ mass $m_{Z'}$. For example, if forthcoming LHC results determine that the Higgs mass is 135 GeV, there is no obstruction to setting $m_\perp$ to this value: this corresponds to $\lambda_h \approx 0.15$ (comfortably within the range of validity of perturbation theory) and is compatible with the mass bound (4.7) as long as we make the $Z'$ mass sufficiently large.

5.6 Relation to previous inflationary model building

The textbook spontaneous symmetry breaking potential has the form

$$V(\psi) = V_0 - m^2 \psi^2 + \lambda \psi^4 = V_0 \left[ 1 - \frac{\psi^2}{v^2} \right]^2.$$  (5.7)

If one imagines minimally coupling this potential to gravity, and using it to drive single-field inflation, one finds that it can generate observationally acceptable primordial perturbations, but only when the VEV $v$ is larger than the Planck scale ($v \gtrsim 10^{19}$ GeV); for $v < 10^{19}$ GeV, the primordial scalar spectral index becomes unacceptably “red” ($n_s < 0.9$). For this reason, the minimal standard model Higgs doublet $h$, minimally coupled to gravity, is not a viable inflaton candidate (since its VEV must be 246 GeV).

One way around this problem is to couple the ordinary Higgs boson to gravity non-minimally – i.e. by adding to the Lagrangian a coupling of the form $\xi (h^\dagger h) R$ [30]. But one finds that, to match observations, the dimensionless coupling $\xi$ must be very large ($\xi \sim 10^5$), which casts doubt on the desirability and reliability of this solution.

Another solution is to replace the textbook potential (5.7) by the Coleman-Weinberg potential (4.3), as we have done in this paper. Although the Coleman-Weinberg potential has played an important role in the history of inflation since its earliest days [31, 32], as far as we are aware it was Ref. [33] that first emphasized the property that, for us, is essential. Namely, [33] emphasized that, because of the particular shape of the Coleman-Weinberg potential (and, in particular, because its second derivative $V''(\psi)$ vanishes near the hilltop), it predicts observationally acceptable primordial perturbations ($0.9 < n_s < 1$) even when the VEV $v$ is orders of magnitude below the Planck scale. (This is illustrated in the second panel of our Fig. 2.)

In other words, a nice consequence of starting from a model with no dimensionful couplings is that the Coleman-Weinberg shape of the symmetry breaking potential automatically leads to a model that retains the simplicity, economy and predictivity of single-field inflation, but nevertheless allows inflation to take place over a range of field values that is much smaller than the Planck scale. This is achieved without the need for large dimensionless coupling constants: e.g. in MDSM$_3$, all of the dimensionless couplings are small.
6. Dark matter and leptogenesis via direct inflaton decay

In this section, we imagine that inflation has just taken place, and analyze the elegant possibility that the dark matter abundance and cosmic matter/anti-matter asymmetry can both be accounted for via direct, non-thermal production of heavy right-handed neutrinos during inflaton decay.

6.1 Prediction 1: Mass of the dark matter particle

Suppose that one of the heavy neutrinos ($N_1$) is less than half the $\rho_\parallel$ mass, so that it is directly produced in $\rho_\parallel$ decay after inflation. In order for the $N_1$ to be produced with the correct abundance to match the presently observed dark matter density, its mass $M_1$ must satisfy the condition [see e.g. Eq. (11) in [34]]

\[ 1 \approx 2 \times 10^9b \frac{M_1}{m_\parallel} \frac{T_{RH}}{\text{GeV}} \]  

(6.1)

where $b$ is average number of dark matter particles created per $\rho_\parallel$ decay: $b \approx \frac{4}{3}(M_1/m_\parallel)^2$. Solving for $M_1$ we find

\[ \frac{M_1}{\text{GeV}} \approx 3 \times 10^{-6} B^{1/4} \left( \frac{\nu}{\text{GeV}} \right)^{5/6}, \]  

(6.2)

which is the purple curve in the bottom panel of Fig. 2.

6.2 Prediction 2: Dark matter is cold

When $N_1$ is originally produced, it is relativistic, with initial $\gamma$ factor

\[ \gamma_i = \frac{m_\parallel}{2M_1} \approx \frac{2^{1/2} B^{1/4}}{3 \times 10^{-6}} \left( \frac{\nu}{\text{GeV}} \right)^{1/6}. \]  

(6.3)

As the universe cools, it redshifts and eventually becomes non-relativistic when the temperature of the radiation bath (from which it is decoupled) has reached:

\[ T_{NR} \approx \frac{T_{RH}}{\gamma_i} \approx \frac{9 \times 10^{-6}}{\pi \sqrt{2}} \left( \frac{5}{g^*(T_{RH})} \right)^{1/4} \left( \frac{\nu}{\text{GeV}} \right)^{1/3} B^{1/2} M_{pl}^{1/2} \text{GeV}^{1/2}. \]  

(6.4)

Thus, as soon as we choose the $N_1$ particle to have the mass $M_1$ that is required to obtain the right dark matter abundance today, we also automatically ensure that this particle becomes non-relativistic sufficiently early in cosmic history so that its free-streaming has a negligible effect on the growth of observed cosmic structure. In other words, once we choose the mass $M_1$ to get the right dark matter abundance, we automatically obtain cold dark matter, as favored by cosmological observations.
6.3 Prediction 3: One neutrino is (essentially) massless

In order to be the dark matter particle, the neutrino \( N_1 \) must have a lifetime much longer than the current age of the universe (and, indeed, longer than about \( 4 \times 10^{22} \) seconds \[35\]). To achieve this, its effective mixing angle with the 3 active neutrinos must be extremely small. From Subsection 3.1, we see that this implies that one of the 3 light neutrinos must either be massless, or essentially massless \( (i.e. \) with a mass orders of magnitude less than that of the other two light neutrinos, and much too small to be detected in practice). To see that this state of affairs is technically natural, note that if the Lagrangian were symmetric under \( N_1 \rightarrow -N_1 \), the \( N_1 \) particle would have zero mixing with light neutrinos, and one of the light neutrinos would be strictly massless.

6.4 Leptogenesis via inflaton decay

Now suppose that at least one of the other two heavy neutrinos \( (N_2) \) is also lighter than half the \( \rho_\parallel \) mass, so that it is also directly produced in \( \rho_\parallel \) decay after inflation. The decay of this neutrino produces a primordial lepton asymmetry; and if this primordial lepton asymmetry is produced early enough, it can be converted into the observed baryon asymmetry via sphaleron transitions. This mechanism for producing the observed matter/anti-matter asymmetry is known as (non-thermal) leptogenesis \[18\].

In order to have successful leptogenesis, we also must first choose the \( N_2 \) decay rate \( \Gamma_2 \) to satisfy the following conditions: when the \( N_2 \) decays, its mass must be higher than the temperature \( T_2 \) of the radiation bath (so that inverse decays are Boltzmann suppressed), and also \( T_2 \) must be \( \gtrsim 100 \) GeV (so that the sphalerons – whose energy scale is set by the \( W \) and \( Z \) masses – are still active). This may be achieved by choosing its effective mixing angle \( \theta_2 \) with active neutrinos to lie in the right range. Once these conditions are satisfied, the condition for obtaining the correct matter/anti-matter asymmetry becomes a condition on the mass \( M_2 \) of the \( N_2 \) particle:

\[
\frac{M_2}{\text{GeV}} \approx 2.5 \times 10^{-4} \left[ \frac{(n_B/s)}{C(L_1 - L_2)(r - \bar{r})} \right]^{1/2} B^{3/8} \left( \frac{v}{\text{GeV}} \right)^{5/4} \quad \text{(6.5a)}
\]

\[
\gtrsim 2.5 \times 10^{-9} B^{3/8} \left( \frac{v}{\text{GeV}} \right)^{5/4} \quad \text{(6.5b)}
\]

Here we are following the notation in \[29\]: the \( N_2 \) has two decay channels: \( N_2 \rightarrow hl \) (with lepton number \( L_1 \) and branching ratio \( r \)) and \( N_2 \rightarrow \bar{h}l \) (with lepton number \( L_2 \) and branching ratio \( 1 - r \)), and \( C \) is a factor of order unity quantifying the conversion of lepton number into baryon number by sphalerons. The exact prediction for \( M_2 \) depends on the amount of CP violation in the neutrino sector (and hence the difference between the branching ratio \( r \) and the branching ratio \( \bar{r} \) for the corresponding process involving
anti-particles). We leave the more detailed calculation of the parameter constraints from this leptogenesis scenario for future work. For the time being, we just note the following: independent of the details of CP violation in the neutrino sector, we can rewrite the constraint on $M_2$ as a lower-bound, as shown in (6.5a).

This bound once again leads to tension with MDSM$_1$ and MDSM$_2$ since, in those models, one can barely make $M_2$ bigger than 100 GeV before destabilizing the Coleman-Weinberg potential. For example, if we assume $m_{\perp} < 500$ GeV (corresponding to $\lambda_b < 2$), stability of the Coleman-Weinberg potential in Models 1 and 2 requires $M_2 < 400$ GeV. But, once again, MDSM$_3$ escapes a similar fate, since one has the freedom to make $M_2$ very large, as long as the $Z'$ mass is also very large.

Let us recapitulate what we have learned thus far. Although all three version of the MDSM may be made compatible with current collider physics, MDSM$_1$ and MDSM$_2$ are not good candidates to also drive a period of inflation in the early universe. By contrast, MDSM$_3$ can also drive a period of inflation in the early universe; and then, the decay of the inflaton can directly generate a non-thermal cold dark matter particle and leptogenesis, in a rather elegant and economical fashion.

7. Dark matter and leptogenesis: other possibilities

In the preceding section, we have described a scenario in which dark matter and leptogenesis are both generated directly, and non-thermally, via inflaton decay; and we have seen that, although this scenario meshes nicely with MDSM$_3$, it is in conflict/tension with MDSM$_1$ and MDSM$_2$. In this section, we wish to briefly draw the reader’s attention to possible alternative routes to dark matter and leptogenesis in these models. These alternative routes are important to mention, since they are not linked to a period of primordial inflation driven by $\psi_\parallel$, and hence do not lead to the same tension with MDSM$_1$ and MDSM$_2$.

Instead of producing heavy neutrinos via $\rho_\parallel$ decay after inflation, we can produce them via oscillations from left-handed neutrinos in the primordial thermal bath. The heavy neutrinos produced in this way can account for dark matter, leptogenesis, or both. In this picture, lightest of the three heavy neutrinos is a warm dark matter candidate with a mass of a few keV. For an analysis of these scenarios, see [19, 20].

In Model 2, there is yet another possibility: Refs. [7, 8] argue that the Majoran field $a$ obtains a mass via quantum effects, and behaves just like an axion. Just like the usual axion, this particle can offer a solution to the strong CP problem, and also act as a dark matter candidate (see [27, 23, 28]).
8. Compelling features of MDSM$_3$

In this section, we draw together a variety of nice features of MDSM$_3$, to emphasize that it seems to be a particularly compelling model. In particular, the arguments in Subsections 8.1 and Subsections 8.2 have not been mentioned yet in this paper; we know that they are not original to us, but we do not know what the correct original reference is, and would appreciate it if some reader could point us to it.

8.1 Extra gauge symmetry, non-trivial anomaly cancellation

An interesting theoretical route by which we can arrive at the model MDSM$_3$ is the following. Suppose we look for an extension of the standard model with the following properties. We want it to have the same fermion content as the standard model (including right-handed neutrinos); and we want those fermions to be grouped into 3 identical generations, as in the standard model. We also want the theory to include the usual Yukawa terms that couple the fermions to the Higgs doublet $h$. The fields will have the same $SU(3) \times SU(2)$ representations as in the standard model, but now we want to extend the gauge group to $SU(3) \times SU(2) \times U(1)_1 \times \ldots \times U(1)_n$, so that, instead of containing a single $U(1)$ factor, it now contains $n$ different $U(1)$ factors. Let us start by allowing the various fields to carry arbitrary charges under these various $U(1)$ gauge groups, and see what the above conditions imply. Thus, we have the following fields, with the following charges under $SU(3) \times SU(2) \times U(1)_1 \times \ldots \times U(1)_n$:

| Variable | SU(3)$_C$ | SU(2)$_L$ | U(1)$_1$ | $\ldots$ | U(1)$_n$ |
|----------|-----------|-----------|-----------|-----------|
| $q_L$    | 3         | 2         | $Q_{1,q}$ | $\ldots$ | $Q_{n,q}$ |
| $u_R$    | 3         | 1         | $Q_{1,u}$ | $\ldots$ | $Q_{n,u}$ |
| $d_R$    | 3         | 1         | $Q_{1,d}$ | $\ldots$ | $Q_{n,d}$ |
| $l_L$    | 1         | 2         | $Q_{1,l}$ | $\ldots$ | $Q_{n,l}$ |
| $\nu_R$  | 1         | 1         | $Q_{1,\nu}$ | $\ldots$ | $Q_{n,\nu}$ |
| $e_R$    | 1         | 1         | $Q_{1,e}$ | $\ldots$ | $Q_{n,e}$ |
| $h$      | 1         | 2         | $Q_{1,h}$ | $\ldots$ | $Q_{n,h}$ |

The standard model contains Yukawa terms like

\begin{align*}
\bar{q}_L h u_R + h.c. & \quad (8.2a) \\
\bar{q}_L h d_R + h.c. & \quad (8.2b) \\
\bar{l}_L h \nu_R + h.c. & \quad (8.2c) \\
\bar{l}_L h e_R + h.c. & \quad (8.2d)
\end{align*}
For the extended model to retain these terms, they must remain gauge invariant, which implies the following conditions:

\begin{align}
0 &= -Q_{i,q} + Q_{i,u} - Q_{i,h} \\
0 &= -Q_{i,q} + Q_{i,d} + Q_{i,h} \\
0 &= -Q_{i,l} + Q_{i,\nu} - Q_{i,h} \\
0 &= -Q_{i,l} + Q_{i,e} + Q_{i,h}
\end{align}

(8.3a - 8.3d)

for all \( i \in [1,n] \). A number of additional constraints come from requirement that the chiral and gravitational anomalies vanish; the constraints involving the \( U(1)'s \) are:

\begin{align}
A[U(1)_i] &= 0 = 3[6Q_{i,q} - 3Q_{i,u} - 3Q_{i,d} + 2Q_{i,l} - Q_{i,\nu} - Q_{i,e}] \\
A[U(1)_iU(1)_jU(1)_k] &= 0 = 3[6Q_{i,q}Q_{j,q}Q_{k,q} - 3Q_{i,u}Q_{j,u}Q_{k,u} - 3Q_{i,d}Q_{j,d}Q_{k,d} \\
&\quad + 2Q_{i,l}Q_{j,l}Q_{k,l} - Q_{i,\nu}Q_{j,\nu}Q_{k,\nu} - Q_{i,e}Q_{j,e}Q_{k,e}] \\
A[SU(3)^2U(1)_i] &= 0 = \frac{3}{2}[2Q_{i,q} - Q_{i,u} - Q_{i,d}] \\
A[SU(2)^2U(1)_i] &= 0 = \frac{3}{2}[3Q_{i,q} + Q_{i,l}]
\end{align}

(8.4a - 8.4d)

for all \( \{i,j,k\} \in [1,n] \). It may be shown by induction that the general solution is

\[ Q_{i,A} = \alpha_i Y_A + \beta_i (B - L)_A. \]

(8.5)

In other words, each \( U(1) \) charge can be a different linear combination of ordinary hypercharge \( Y \) and baryon-minus-lepton number \( (B - L) \). What is the interpretation of this result?

First of all, it tells us that only two of the \( U(1)'s \) are linearly independent; the rest are redundant. Thus we will henceforth assume just two \( U(1)'s \). Then we can, without loss of generality, redefine our fields so that the first \( U(1) \) factor is purely hypercharge \( Y \), and the second \( U(1) \) factor is purely \( (B - L) \); this is the convention we have followed in this paper.

Secondly, the system is overconstrained: it is suprising to find a solution at all, let alone one with so many free parameters. In the case of interest, \( n = 2 \), we have 18 equations for 14 unknowns, and find a solution with 2 free parameters. We may be inclined to interpret this apparent coincidence as theoretical evidence that the additional \( U(1)_{B-L} \) gauge symmetry of MDSM_3 is on the right track. [The vanishing of the \( U(1)_{B-L} \) gauge anomaly in the standard model (with right handed neutrinos) is, of course, well known, and may also be interpreted/understood in terms of the fact that the standard model may be embedded in a grand unified theory based on \( SO(10) \).]
8.2 A *raison d’etre* for right-handed neutrinos

The ordinary standard model makes sense, with or without the right-handed neutrino. In particular, since the right-handed neutrino is a singlet under the standard model gauge group, it does not contribute to anomaly cancellation: in the standard model, the anomalies cancel whether or not the right-handed neutrino exists. The right-handed neutrino has recently been tacked on to the standard model in order to account for the observed neutrino oscillations – but not for any independent theoretical reason.

It is important to emphasize that the situation is very different in MDSM\(_3\). In this case, the right-handed neutrino is charged under \(U(1)_{B-L}\), and thus plays an essential role in the anomaly cancellation argument in the previous subsection. Stated another way: if we tried to add an extra \(U(1)\) gauge symmetry to the *minimal* standard model (with no right handed neutrinos), in the manner described in the previous section, we would find that we couldn’t do it – there would be no solutions to the corresponding constraints. We would be led to add a fermion to each generation, with exactly the properties of the right-handed neutrino, in order to make the anomalies cancel.

In this sense, MDSM\(_3\) has more explanatory power than MDSM\(_1\), MDSM\(_2\), or the standard model: it gives the right-handed neutrino a genuine theoretical *raison d’etre*.

8.3 One scalar, many different roles

Relative to the standard model, MDSM\(_3\) contains two new fields: the complex scalar field \(\varphi\) and the \((B-L)\) gauge field \(C_\mu\). In this subsection, we would like to stress the elegantly economical way in which \(\varphi\) simultaneously accomplishes several important phenomenological tasks in this model.

On the one hand, its properties – *i.e.* its \(SU(3) \times SU(2) \times U(1)_{Y} \times U(1)_{B-L}\) charges shown in (2.9) – were determined by requirement that it should be able to provide a Majorana-like Yukawa coupling to right-handed neutrinos, in order to give a see-saw mechanism for neutrino mass. On the other hand, these same properties are precisely what is needed in order for \(\varphi\) to perform another crucial task: spontaneously break \((B-L)\) symmetry, and give mass to the \(C_\mu\) boson via the Higgs mechanism, without leaving any additional unwanted Goldstone bosons.

At the same time, we have seen that \(\varphi\) expands the scalar sector in a way that leads to successful electroweak symmetry breaking via the Coleman-Weinberg mechanism, an inflaton candidate, a dark matter candidate, and a possible explanation for the cosmological matter/anti-matter asymmetry.

Finally, this model contains 4 different quantities that all must be rather large, for very different phenomenological/cosmological reasons: (i) the mass of the \(Z'_{\mu}\) boson, (ii) the masses of the heavy neutrinos; (iii) the mass of the \(\rho_{\parallel}\) boson, and (iv) the inflaton
VEV. And yet, in MDSM$_3$, all 4 large dimensionful quantities have a common origin in the large VEV of the field $\varphi$.

### 8.4 Cosmology

Finally, as we argued, MDSM$_3$ seems to be a viable extension of the standard model of particle physics and, at the same time, seems able to perform a wider variety of important cosmological tasks than MDSM$_1$, MDSM$_2$, or the standard model.

Taken together, these reasons point to MDSM$_3$ as a particularly interesting and compelling extension of the standard model.

### 9. Discussion

Finally, let us mention some of the most important ways that we hope to improve and extend this analysis in future work. (i) First, we plan to perform a more complete analysis of leptogenesis, to flesh out the treatment given above. (ii) Second, in this paper we have used the Gildener-Weinberg formalism; but it would be better (especially in our analysis of inflation) to perform a more complete analysis, incorporating all renormalization-group effects, and using the full renormalization-group-improved effective potential. (iii) Third, we plan to examine the RG flow to find regions in parameter space that are “asymptotically safe” (or, at least, free of instabilities or Landau poles up to the Planck scale). (iv) Finally, in this paper we have taken the MDSM to be minimally coupled to gravity; this minimal coupling introduces a dimensionful constant (Newton’s gravitational constant $G$) which is arguably at odds with the original spirit (and classical conformal invariance) of the original non-gravitational MDSM. Thus, we are exploring the possibility that there are other couplings to gravity that are more aligned with the spirit of the MDSM, but do not spoil the most desirable features of the model that follow from the minimally-coupled analysis performed in this paper.

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