Studying the baryon properties through chiral soliton model at finite temperature and density

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We have studied the chiral soliton model in a thermal vacuum. The soliton equations are solved at finite temperature and density. The temperature or density dependent soliton solutions are presented. The physical properties of baryons are derived from the soliton solutions at finite temperature and density. The temperature or density dependent variation of the baryon properties are discussed.

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I. INTRODUCTION

The fundamental theory of the strong interaction is quantum chromodynamics (QCD). This theory has the essential properties of asymptotic freedom, hidden (spontaneously broken) chiral symmetry and confinement. For the high energy process, the perturbative QCD calculation works well due to the asymptotic freedom. However, in the low energy nuclear sector, it is a hard journey for the QCD to walk out the way to calculate the quantities of nuclear physics because of the nonperturbative features of broken chiral symmetry and confinement. There are different approaches in this direction, like lattice QCD, QCD sum rule, chiral perturbation theory and Dyson-Schwinger equation [1–5]. Another important approach is to use the effective models which have certain essential features of QCD. In studying the hadron properties, the bag models which has the essential feature of confinement have been often used. They are MIT, SLAC and Friedberg-Lee models which describe the static nucleon properties quite well [6, 7]. In these models the hadronic bound states is determined by the confinement mechanism. There is also other perspective about the hadronic bound states. It suggests that there is a separation of roles between the forces responsible for binding quarks in hadrons and those which give absolute confinement [8, 9]. The forces responsible for quark binding are strong Coulomb-type and related to vacuum condensation and dynamical chiral symmetry breaking. Thus these authors suggested using chiral models to study hadron properties [10]. The chiral model was solved in the semiclassical or mean-field approximation and the semiclassical solution was referred to a chiral soliton. In the environment of vacuum, they have studied the static properties of nucleon and delta [6, 7]. After that the subsequential work had been made to study hadron properties in vacuum [11,14].

In high energy physics, the nuclear matter is studied under extreme condition. The study of hot and dense medium created in high energy heavy ion collisions is of great interest [15, 16]. The thermal vacuum is very different to the vacuum at zero temperature and density. The vacuum condensation would be melted which will result in the chiral symmetry restoration and deconfinement of the system. So it is a very interest topic to study the hadron properties in the thermal vacuum. It could give us more insight into the complex vacuum structure of QCD. In the present work our purpose is to extend the study of the chiral soliton model to finite temperature and density. The chiral soliton and baryon properties will be studied at finite temperature and density.

At finite temperature and density the chiral soliton model which is also called linear sigma model has been used extensively to study chiral restoration in an uniform system [17, 21], while the studis of chiral soliton and hadron properties at finite temperature and density in the same model are not so many. In reference [22], the chiral solitons in linear sigma model and NJL model were studied in hot and dense medium by means of variational projection techniques. In reference [22], temperature effect to chiral soliton model was introduced by adding one loop potential to the Lagrangian. Here we will use the stand finite temperature field theory to introduce temperature and density effect. In most recent work [24], the chiral soliton model has been discussed through the formalism of finite temperature field theory. Their main purpose is to study the chiral soliton in chiral phase transition. Our goal is concentrate on the baryon properties at finite temperature and density, and our treatment about the baryon mass is quite different with theirs, so the result is quite different, which will be discussed later.

The organization of this paper is as follows: in section 2 the chiral soliton model is introduced. The field equations are solved at zero temperature and density. The baryon properties in vacuum are reproduced. In section 3, the chiral soliton equations are extended to finite temperature and density. The soliton solutions are discussed at finite temperatures and densities. In section 4, the baryon properties are derived from the soliton solutions. The temperature...
or density dependent variation of these properties are presented and discussed. The last section is the summary.

II. CHIRAL SOLITON MODEL AT ZERO TEMPERATURE AND DENSITY

We start from the Lagrangian of the chiral soliton model,

$$\mathcal{L} = \bar{\psi} [i\gamma_\mu \partial^\mu + g(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi})] \psi + \frac{1}{4}(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi}) - U(\sigma, \vec{\pi}),$$

where

$$U(\sigma, \vec{\pi}) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - \nu^2)^2 + H\sigma - \frac{m_\pi^4}{4\lambda} + f_\pi^2 m_\pi^2.$$  \hspace{1cm} (2)

\(\psi\) represents the two flavor light quark fields \(\psi = (u, d)\), \(\sigma\) is the isosinglet scalar field, and \(\vec{\pi}\) is the isovector pion field \(\vec{\pi} = (\pi_1, \pi_2, \pi_3)\). \(H\sigma\) is the explicit chiral symmetry breaking term and \(H = f_\pi m_\pi^2\), where \(f_\pi = 93\text{MeV}\) is the pion decay constant and \(m_\pi = 138\text{MeV}\) is the pion mass. The chiral symmetry is explicitly broken in the vacuum and expectation values of the meson fields are \(\langle \sigma \rangle = -f_\pi\) and \(\langle \vec{\pi} \rangle = 0\). The constitute quark mass in vacuum is \(M_q = g_\pi f_\pi\), and the sigma mass is \(m_\sigma^2 = m_\pi^2 + 2\lambda f_\pi^2\). The quantity \(\nu^2\) can be expressed as \(\nu^2 = f_\pi^2 - m_\pi^2/\lambda\). The last two constants in equation (2) ensure that the vacuum energy is zero. In our calculation we have followed the choice of the reference [9] and set the constituent quark mass and the sigma mass as \(M_q = 500\text{MeV}\) and \(m_\sigma = 1200\text{MeV}\) which determine the parameters \(g \approx 5.28\) and \(\lambda \approx 82.1\).

From the Lagrangian (1) the field equations could be derived in the following,

$$[i\gamma^\mu \partial_\mu + g(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi})] \psi = 0,$$

\hspace{1cm} (3)

$$\partial_\mu \partial^\mu \sigma - g\bar{\psi} \psi = -\frac{\partial U(\sigma, \vec{\pi})}{\partial \sigma},$$

\hspace{1cm} (4)

$$\partial_\mu \partial^\mu \vec{\pi} - ig\bar{\psi} \gamma_5 \vec{\tau} \psi = -\frac{\partial U(\sigma, \vec{\pi})}{\partial \vec{\pi}}.$$

\hspace{1cm} (5)

One could take the mean field approximation and the “hedgehog” ansatz which means,

$$\sigma(\vec{r}, t) = \sigma(r), \hspace{0.5cm} \vec{\pi}(\vec{r}, t) = \hat{r}\pi(r),$$

\hspace{1cm} (6)

$$\psi(\vec{r}, t) = e^{-i\varepsilon t} \sum_{i=1}^{N} q_i(\vec{r}), \hspace{0.5cm} q(\vec{r}) = \begin{pmatrix} u(r) \\ i\hat{\sigma} \cdot \hat{r} v(r) \end{pmatrix} \chi,$$

\hspace{1cm} (7)

where \(q_i\) are \(N\) identical quarks in the lowest s-wave level with energy \(\varepsilon\). \(N = 3\) is for baryons and \(N = 2\) for meson. \(\chi\) is the spinor which satisfies the condition

$$(\hat{\sigma} + \hat{r})\chi = 0.$$  \hspace{1cm} (8)

\(\hat{r}\) is the radial unit vector. The meson fields \(\sigma(r), \pi(r)\) and the quark functions \(u(r), v(r)\) are spherical symmetric and satisfy the following set of coupled nonlinear radial differential equations which could be obtained from equations (3)-(6),

$$\frac{du(r)}{dr} = -(\varepsilon - g\sigma(r)) v(r) - g\pi(r) u(r),$$

\hspace{1cm} (9)

$$\frac{dv(r)}{dr} = -\left(\frac{2}{r} - g\pi(r)\right) v(r) + (\varepsilon + g\sigma(r)) u(r),$$

\hspace{1cm} (10)

$$\frac{d^2 \sigma(r)}{dr^2} + \frac{2}{r} \frac{d\sigma(r)}{dr} + Ng(u^2(r) - v^2(r)) = \frac{\partial U}{\partial \sigma},$$

\hspace{1cm} (11)
\[
d\frac{d^2\pi(r)}{dr^2} + \frac{2d\pi(r)}{r} - \frac{2\pi(r)}{r^2} + 2Ng(r)v(r) = \frac{\partial U}{\partial \pi}, \tag{12}
\]

The quark functions should satisfy the normalization condition
\[
4\pi \int r^2 (u^2(r) + v^2(r)) dr = 1. \tag{13}
\]

The boundary conditions on the quark functions and meson fields are,
\[
v(0) = 0, \quad \frac{d\sigma(0)}{dr} = 0, \quad \pi(0) = 0, \tag{14}
\]
\[
u(\infty) = 0, \quad \sigma(\infty) = -f, \quad \pi(\infty) = 0. \tag{15}
\]

As mentioned above the parameters have been fixed to \( g \approx 5.28 \) and \( \lambda \approx 82.1 \). The lowest quark energy eigenvalue is \( \varepsilon = 30.5 \text{MeV} \) [9]. In our study we are mainly concerned about the properties of baryons which means \( N = 3 \).

The equations (9)-(12) together with normalization condition (13) and boundary conditions (14) and (15) could be numerically solved by a standard numerical package which is called COLSYS [25]. The forms of the fields in a chiral soliton are shown in Fig.1 From these soliton solutions one can further derived the general properties of a baryon, like the soliton energy or baryon mass \( M_B \), the root mean square charge radius \( r_B \), the magnetic moment \( \mu_B \) and the ratio of the axial to vector coupling constants \( g_A/g_V \), which could be calculated in the following,
\[
E = M_B = 3\varepsilon + 4\pi \int dr r^2 \left[ \frac{1}{2} \left( \frac{d\sigma}{dr} \right)^2 + \frac{1}{2} \left( \frac{d\pi}{dr} \right)^2 + \frac{\pi^2}{r^2} + U(\sigma, \pi) \right], \tag{16}
\]
\[
< r_B^2 >= 4\pi \int_0^\infty (u^2 + v^2)r^4 dr, \tag{17}
\]
\[
\mu_B = \frac{8\pi}{3} \int_0^\infty r^3 uv dr, \tag{18}
\]
\[
\frac{g_A}{g_V} = \frac{20\pi}{3} \int_0^\infty r^2 (u^2 - \frac{v^2}{3}) dr. \tag{19}
\]

In our calculation all these physical quantities could be well reproduced and the results are: \( M_B = 1140 \text{MeV} \), \( r_B = 0.717 \text{fm} \), \( \mu_B = 0.195 \text{efm} \) and \( g_A/g_V = 1.162 \) which are in agreement with the results in reference [9]. All the calculations and results in the above discussions are in the vacuum which means at zero temperature and density. In the following sections we will study the solitons and baryon properties at finite temperature and density.
III. CHIRAL SOLITON SOLUTIONS AT FINITE TEMPERATURES AND DENSITIES

We consider a baryon embedded in a thermal quark medium. The vacuum become a thermal vacuum. The equations (11) and (12) become,

\[ \partial_{\mu} \partial^{\mu} \sigma - g \bar{\psi} \psi = g \langle \bar{\psi} \psi \rangle - \frac{\partial U(\sigma, \vec{\pi})}{\partial \sigma}, \]  

\[ \partial_{\mu} \partial^{\mu} \vec{\pi} - i g \bar{\psi} \gamma_{5} \vec{\tau} \psi = g \langle i \bar{\psi} \gamma_{5} \vec{\tau} \psi \rangle - \frac{\partial U(\sigma, \vec{\pi})}{\partial \vec{\pi}}, \]  

in which, the quark source terms decompose into two parts: one is the usual source part of one baryon as \( g \bar{\psi} \psi \) or \( i g \bar{\psi} \gamma_{5} \vec{\tau} \psi \) on the left hand side of the equation; the other is the thermal quark medium source part as \( g \langle \bar{\psi} \psi \rangle \) or \( g \langle i \bar{\psi} \gamma_{5} \vec{\tau} \psi \rangle \) on the right hand side of the equation. The meson field functions \( \sigma(r) \) and \( \pi(r) \) are taken as the classic mean fields in the thermal vacuum. The thermal quark medium part is also the thermal vacuum average of the quark source which could be calculated by standard method in finite temperature theory and the results are,

\[ \langle \bar{\psi} \psi \rangle = -g \sigma \nu_{q} \int \frac{d^{3}p}{(2\pi)^{3}} \frac{1}{E_{q}} \left( \frac{1}{e^{\beta(E_{q}-\mu)} + 1} + \frac{1}{e^{\beta(E_{q}+\mu)} + 1} \right), \]  

\[ \langle i \bar{\psi} \gamma_{5} \vec{\tau} \psi \rangle = -g \vec{\pi} \nu_{q} \int \frac{d^{3}p}{(2\pi)^{3}} \frac{1}{E_{q}} \left( \frac{1}{e^{\beta(E_{q}-\mu)} + 1} + \frac{1}{e^{\beta(E_{q}+\mu)} + 1} \right), \]  

where \( \beta \) is the inverse temperature \( \beta = 1/T \), \( \mu \) is the quark chemical potential, \( \nu_{q} \) is a degenerate factor, \( \nu_{q} = 2(\text{spin}) \times 2(\text{flavor}) \times 3(\text{color}) \), and \( E_{q} = \sqrt{p^{2} + m_{q}^{2}} \) with an effective quark mass term \( m_{q} = g \sigma \). Furthermore we could make the following definitions,

\[ \langle \bar{\psi} \psi \rangle \equiv -\rho_{s}(T, \mu), \quad \langle i \bar{\psi} \gamma_{5} \vec{\tau} \psi \rangle \equiv -\vec{\rho}_{ps}(T, \mu), \]  

\[ \frac{\partial U(\sigma, \vec{\pi})}{\partial \sigma} + g \rho_{s}(T, \mu) \equiv \frac{\partial V_{\text{eff}}(T, \mu)}{\partial \sigma}, \quad \frac{\partial U(\sigma, \vec{\pi})}{\partial \vec{\pi}} + g \vec{\rho}_{ps}(T, \mu) \equiv \frac{\partial V_{\text{eff}}(T, \mu)}{\partial \vec{\pi}}, \]  

where \( \rho_{s} \) and \( \vec{\rho}_{ps} \) are the scalar and pseudoscalar densities of quarks and anti-quarks. \( V_{\text{eff}} \) is defined as a thermal effective potential of the quark medium and by its definition the form is

\[ V_{\text{eff}}(T, \mu) = U(\sigma, \vec{\pi}) - \frac{\nu_{q}}{\beta} \int \frac{d^{3}p}{(2\pi)^{3}} \ln(1 + e^{-\beta(E_{q}-\mu)} + \ln(1 + e^{-\beta(E_{q}+\mu)})). \]  

The effective potential here is identical to the one loop or mean field thermodynamical potential of the model which could be also derived through the partition function in imaginary time formalism of finite temperature field theory. As we have mentioned at the start of the section this uniform quark medium is regraded as the thermal background in which the baryon is embedded. At this time the meson radial equations at finite temperature and density could be derived,

\[ \frac{d^{2} \sigma(r)}{dr^{2}} + \frac{2}{r} \frac{d \sigma(r)}{dr} + N g (u^{2}(r) - v^{2}(r)) = \frac{\partial V_{\text{eff}}}{\partial \sigma}, \]  

\[ \frac{d^{2} \pi(r)}{dr^{2}} + \frac{2}{r} \frac{d \pi(r)}{dr} - \frac{2 \pi(r)}{r^{2}} + 2 N g u(r) v(r) = \frac{\partial V_{\text{eff}}}{\partial \pi}. \]  

Compared to the radial equations (11) and (12) at zero temperature and density, it is convenient to obtain the finite temperature and density equations by simply replacing the classical potential \( U(\sigma, \vec{\pi}) \) with the thermal effective potential \( V_{\text{eff}} \). The forms of the equations of quark functions do not change. However one should notice that those equations are coupled to the meson field equations at finite temperature and density, which implies that the baryon is embedded in the thermal quark medium. As a result the field functions \( u(r), v(r), \sigma(r) \) and \( \pi(r) \) are all functions of temperature and density. The boundary conditions will not change except for the sigma field. At zero temperature and density the sigma field asymptotically approaches its vacuum value \( \langle \sigma \rangle = -f_{\sigma} \) as \( r \to \infty \). While at finite temperature and density the sigma field asymptotically approaches the thermal vacuum value \( \langle \sigma \rangle = -\sigma_{v} \) which will
be determined by the absolute minimum of the thermal effective potential \[26\]. At certain temperature and density the field equations \[9\], \[10\], \[27\] and \[28\] together with the thermal effective potential \[26\] could be numerically solved by a modified COLSYS.

In Fig. 2 we have showed the soliton solutions for different temperatures at fixed zero density. It could be seen that the amplitudes of the soliton solutions decrease with the temperature increasing. At relatively low temperatures \(T \lesssim 100\text{MeV}\) the soliton solutions change slowly while at relatively high temperatures they change more and more quickly. It could be estimated that when temperature increasing from 0\text{MeV} to 100\text{MeV} the soliton amplitudes decrease by 1\% ~ 2\%; when temperature increasing from 100\text{MeV} to 160\text{MeV} the soliton amplitudes decrease by 6\% ~ 9\%.

In Fig. 3 we have showed the soliton solutions for different chemical potentials at fixed temperature \(T = 50\text{MeV}\). One can see that the amplitudes of the solitons decrease slightly with the chemical potential increasing. When chemical potential increasing from 0\text{MeV} to 400\text{MeV} the soliton amplitudes decrease only by 2\% ~ 5\%. When the temperature or density further increases, there will be chiral restoring phase transition in the system, which lies out of the scope of this work. In that circumstance the soliton solutions deserve a thorough investigation in our future work.

**IV. DISCUSSIONS OF BARYON PROPERTIES AT FINITE TEMPERATURES AND DENSITIES**

As the field functions \(u(r)\), \(v(r)\), \(\sigma(r)\) and \(\pi(r)\) at finite temperatures and densities have been obtained in the above discussion, in this section we will discuss the baryon properties at finite temperatures and densities. Substituting
the finite temperature and density field functions $u(r)$, $v(r)$, $\sigma(r)$ and $\pi(r)$ into the equations (16)-(19) one could calculate the baryon mass $M_B$, the root mean square charge radius $r_B$, the magnetic moment $\mu_B$ and the ratio of the axial to vector coupling constants $g_A/g_V$ at different temperatures and densities.

In Table I we show the baryon properties for the different temperatures at fixed zero chemical potential. One can see that all the physical quantities increase with temperature increasing. In particular the baryon mass increases relatively slowly at temperature $T \lesssim 100\text{MeV}$ while more and more rapidly after that.

In Table II the baryon properties for different chemical potentials at fixed temperature $T = 50\text{MeV}$ are presented. It could be seen that all the physical quantities are also increasing with chemical potential increasing. The baryon mass increases moderately and gradually with chemical potential increasing.

Here we make some discussions about the baryon mass. In reference [24], the baryon mass is decreasing with temperature or chemical potential increasing. This is because of the different schemes in calculating the baryon mass. In our calculation of the baryon mass as shown in equation (16), we have included the meson interaction energy $U(\sigma, \pi)$ while they have neglected this part and only consider the kinetic energies of $\sigma$ and $\pi$. If we neglect the meson interaction energy the baryon mass is also decreasing in our work. The results about the baryon mass from these two different schemes are indeed opposite. However we think that at finite temperature and density the meson interaction energy should be included as this energy is also a part of the energy of the baryon. Our results are in agreement with reference [23] in which they have also taken the meson interaction energy into account when calculating the baryon mass. This is an interesting problem of which we will make a study in detail in a separate work.

V. SUMMARY

In this paper we have extended the chiral soliton model to finite temperatures and densities. The chiral soliton equations are solved and the soliton solutions are discussed at finite temperatures and densities. The soliton amplitudes decrease with temperature or density increase. As a result the physical quantities of the baryon, like the baryon mass, the mean charge radius, the magnetic moment and the the ratio of the axial to vector coupling constants, are all increase with temperature or density increase. The physical quantities change more rapidly at relatively high temperatures than that in relatively low temperatures. Compared to the temperature dependent case, the chemical potential dependent baryon properties change relatively slowly.

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