Application of powerline noise cancellation method in correlation identification of controlled source electromagnetic method

Zhi Yang1,2, Jingtian Tang1,3,4,5, Xiao Xiao1,3,4,5,*, Qiyun Jiang1,3,4,5, Xiangyu Huang1 and Shuanggui Hu1

1 School of Geosciences and Infophysics, Central South University, Changsha, 410083, China
2 School of Computer and Electrical Engineering, Hunan University of Arts and Sciences, Changde, 415000, China
3 Key Laboratory of Metallogenic Prediction of Nonferrous Metals and Geological Environment Monitoring Ministry of Education, School of Geoscience and Infophysics, Central South University, Changsha, 410083, China
4 Hunan Key Laboratory of Nonferrous Resources and Geological Hazards Exploration, Central South University, Changsha, 410083, China
5 Technical Innovation Center of Coverage Area Deep Resources Exploration, Ministry of Natural Resources, Hefei, 230001, China

*Corresponding author: Xiao Xiao. E-mail: csuxiaoxj@csu.edu.cn

Received 30 January 2021, revised 17 March 2021
Accepted for publication 26 April 2021

Abstract
Powerline interference in the controlled source electromagnetic method has traditionally been one of the biggest conundrums plaguing geophysicists, and its conventional denoising methods primarily include filtering and noise estimation. The filter method leaches noise at specific frequency points, which might also filter useful signals; the noise estimation method significantly eliminates interference, whereas the premise is that the noise is stable after a short time and a recorder is necessary in the field. In the present study, using the periodicity and symmetry of powerline noise, we propose a subtraction and an addition method for cancellation of the powerline noise. First, the transmitted signal is optimized so that the equivalent transmitted signal is an m sequence; then the response signal is processed by using the cancellation method; subsequently, the correlation identification is applied and finally, we solve the earth impulse response by means of the Wiener filter deconvolution method. Simulation experiments and field data tests demonstrate that the powerline noise can be well suppressed by the cancellation method proposed in the present study, so that the system identification accuracy is greatly improved. The method is simple in principle and effective in removing powerline noise, which presents a novel perspective on noise elimination for system identification.

Keywords: controlled source electromagnetic method, source signature optimization, the earth impulse response, correlation identification, m sequence, powerline noise

1. Introduction
The controlled source electromagnetics method is very susceptible to various noises for the signal measured at the receiver. With the progress of industrialization, the powerline interference attributed to considerable high-voltage transmission lines and a range of electric facilities is almost everywhere, and nearly no electromagnetic area is generated without the influence of powerline noise (László Szarka...
In areas with intensive industrial power, powerline interference significantly reduces the accuracy of measurement data, thereby causing a significantly reduced signal-to-noise ratio. As a result, the identification accuracy of the earth system is seriously affected, and ultimately geodetic information cannot be correctly extracted. Accordingly, removing powerline interference is the basic prerequisite for high-precision geophysical exploration.

The conventional methods to remove powerline interference primarily consist of a notch method and powerline noise estimation method (Li et al. 2021). For the notch method, the interference of specific frequency points is directly filtered out in the frequency domain. Its principle is simple, whereas there are two obvious problems. First, the filter leaches the interference of specific frequency points while filtering useful signals at the identical frequency point (Butler & Russell 1993). Second, the practical notch filter may cause the frequency response distortion near a specific frequency point. The powerline noise estimation method is also recognized as a powerline noise subtraction method. The subtraction method covers the block subtraction method and the sinusoid subtraction method. The principle is to subtract the estimated powerline noise from the measurement signal. To be specific, the block subtraction method is to add a recorder alone for noise, while the sinusoid subtraction method exploits the recorded noise to assess the amplitude and phase of the powerline interference (Butler & Russell 2003; Saucier et al. 2006). The powerline noise subtraction method significantly eliminates powerline interference, whereas the premise is that the noise is stable after a short time and a noise recorder should be added when working in the field (Legchenko & Valla 2003).

Since the pseudo-random signal was introduced into the field of geophysical exploration by Duncan et al. (1980), such an excitation signal exhibits high autocorrelation and an advantage in suppressing noise that other excitation signals are incomparable. It has been studied in depth and used extensively by numerous geophysicists (Guo et al. 2020). Wright et al. (2001) and Ziolkowski et al. (2007, 2011) presented an m sequence in MTEM and achieved effective results in oil and gas resource exploration. Li et al. (2008) and Qi et al. (2015) adopted a pseudo-random sequence correlation identification method. They suppressed the noise by using the irrelevant nature of the pseudo-random sequence and noise. Ilyichev & Bobrovsky (2015) showed that the anti-noise ability of pseudo-random sequences was significantly improved compared with conventional methods. These researchers suppressed noise by optimizing the excitation signal and significantly affected noise (e.g. powerline interference), but they did not study the elimination of strong powerline interference in depth.

Besides the powerline interference removal method and noise suppression method, many other effective methods have also been proposed by geophysicists. Warden et al. (2012) used curvelet technology to separate powerline interference from the signal, whereas the prerequisite is to select the appropriate mother wavelet. Yuan et al. (2017) developed an optimal data length method to eliminate powerline noise, significantly suppressing powerline noise by regulating the m sequence transmission period. Based on frequency domain sparse decomposition, Tang et al. (2018) designed redundant dictionary, matching the interference signal but exhibiting insensitivity to useful signals. In addition, this dictionary combined the improved orthogonal matching pursuit algorithm to separate the powerline interference components in the frequency domain signal. These researchers used different methods to analyse and eliminate noise from different angles, and they presented many useful ideas to elevate the signal-to-noise ratio.

In the present study, a method is proposed to eliminate powerline noise by optimizing the excitation signal and then to perform correlation identification of the earth system. The idea of cancelling noise is first proposed according to the characteristics of powerline interference. Next, according to the principle of noise cancellation, the excitation signal is optimized from the aspects of transmission time and equivalent signal. Subsequently, the correlation identification and deconvolution algorithm in the identification of the earth system is derived. Last, simulation experiments and field noise verification experiments are performed with the method of cancelling powerline noise. As revealed from the results, the method in this study can effectively remove powerline and random noise, and significantly increase the accuracy of system identification.

2. Method theory

2.1. Problem description

The earth system identification model can be simplified as shown in figure 1, where \( x(t) \) is the current signal at the transmitter, \( h(t) \) is the unit impulse response of the earth system, \( n(t) \) is additive noise and \( y(t) \) is the voltage signal measured at the receiver.

![Figure 1. Simplified model of the earth system.](image)

The earth system can be expressed by the following mathematical relations:

\[
y(t) = x(t) * h(t) + n(t). \tag{1}
\]

The symbol ‘∗’ represents convolution.
The received signal $y(t)$ is composed of useful information and noise. This study aims to identify the earth impulse response from the noisy received voltage signal. The quality of the identified result is directly related to the signal-to-noise ratio of the receiver. The cleaner the noise that is removed, the higher the identification accuracy. In different application environments, the main types of noise are different. This study mainly considers strong powerline interference and random noise interference.

2.2. Method introduction

The source signature $x(t)$ consists of two groups of sequences, denoted by $x_1(t)$ and $x_2(t)$ respectively, assuming that the noise term $n_i(t)$ is only powerline noise. Thus,

$$y_1(t) = x_1(t) * h(t) + n_i(t), \quad 0 < t \leq T. \quad (2)$$

$$y_2(t + T) = x_2(t + T) * h(t) + n_i(t + T), \quad 0 < t \leq T. \quad (3)$$

The transmission time length of signal $x_1(t)$ and $x_2(t)$ both are $T$, $y_1(t)$ and $y_2(t)$, respectively, represent the noisy response of $x_1(t)$ and $x_2(t)$. As long as $T$ is large enough, $x_1(t)$ and $x_2(t)$ can be treated as two independent source signatures. Equation (2) can be obtained from equation (1), and similarly, equation (3). The $t + T$ in equation (3) simply means that there is no gap between the first source signature $x_1(t)$ and the second source signature $x_2(t)$, i.e. the $x_1(t)$ and $x_2(t)$ is continuous transmission. To solve the unit impulse response of the earth system with high accuracy, equations (2) and (3) indicate that whether the noise term can be effectively removed is the key to the problem. In the next section, the following two cases from the relationship between the duration $T$ of the respective group of source signatures and the fundamental period $T_1$ of the powerline noise are considered:

2.3. Case 1: $T$ is an integer multiple of $T_1$

As shown in figure 2, $T$ is an integer multiple of $T_1$, that is

$$T = KT_1, \quad K \in \mathbb{N}^+. \quad (4)$$

The relationship between the fundamental period of powerline noise and its harmonic period is:

$$T_1 = nT_n, \quad n \in \mathbb{N}^+. \quad (5)$$

where $T_n$ denotes the period of the $n$ harmonic. Substituting equation (5) into equation (4), we obtain equation (6)

$$T = nKT_n, \quad n \in \mathbb{N}^+. \quad (6)$$

It can be seen that as long as the transmission duration satisfies equation (4), it should satisfy equation (6); that is, if the transmission duration is a multiple of the fundamental period of powerline noise, it should be a multiple of each harmonic period.

As impacted by the periodicity of powerline noise, it is only necessary to ensure that there is no gap between the first source signature and the second source signature, i.e. the $x_1(t)$ and $x_2(t)$ is continuous transmission. Subsequently, the transmission start time for the fundamental wave and each harmonic at any moment is written as:

$$n_i(t + T) = n_i(t + nKT_n) = n_i(t), \quad n \in \mathbb{N}^+. \quad (7)$$

Then equation (2) minus equation (3) becomes:

$$y_1(t) - y_2(t + T) = x_1(t) * h(t) - x_2(t + T) * h(t). \quad (8)$$

Equation (8) indicates that the fundamental wave component and the harmonic components in the noise term are cancelled overall. If $T$ is longer than the system response adjustment time, $x_1(t)$ and $x_2(t)$ can be considered two independent source signatures at the identical time slot, and $y_1(t)$ and $y_2(t)$ are the corresponding responses. Subsequently, equation (8) can be simplified as:

$$y_1(t) - y_2(t) = [x_1(t) - x_2(t)] * h(t). \quad (9)$$

In this study, $x_1(t) - x_2(t)$ is called the equivalent source signature and $y_1(t) - y_2(t)$ is the equivalent received signal.

2.4. Case 2: $T$ denotes a non-integer multiple of $T_1$

As shown in figure 3, $T$ denotes an integer multiple of $T_1$ plus one-half $T_1$, and it yields

$$T = KT_1 + \frac{1}{2}T_1, \quad K \in \mathbb{N}^+. \quad (10)$$

![Figure 2](https://example.com/figure2.png)

**Figure 2.** $T$ is an integer multiple of the $T_1$ relationship diagram (the dotted line represents a sinusoidal signal with multiple integer cycles).

![Figure 3](https://example.com/figure3.png)

**Figure 3.** $T$ is a non-integer multiple of $T_1$ (the dotted line represents a sinusoidal signal with multiple integer cycles).
For odd harmonics:
\[ T_1 = (2n+1) T_{2n+1}, \quad n \in N. \] (11)

where \(2n+1\) denotes the harmonic order. By substituting equation (11) into equation (10), this yields:
\[ T = KT_1 + \frac{1}{2} T_1 = \left( K + \frac{1}{2} \right) ((2n+1) T_{2n+1}) \]
\[ = [(2n+1)K + n] T_{2n+1} + \frac{1}{2} T_{2n+1}, \quad K \in N^*, n \in N. \] (12)

From equation (12), it can be seen that as long as \(T\) is a multiple of \(T_1\) plus one-half period \(T_1\), it should be a multiple of each odd harmonic period plus one-half period. But this is not true for even harmonics, so this cancellation method is not suitable for situations with strong, even harmonics.

As impacted by the periodicity and symmetry of powerline noise, as long as \(x_1(t)\) and \(x_2(t)\) are continuous transmitted, the transmission start time for the fundamental wave and each harmonic at any moment satisfy equation (13):
\[ n_1(t + T) = n_1 \left( t + \left[ (2n + 1)K + n \right] T_{2n+1} + \frac{1}{2} T_{2n+1} \right) \]
\[ = -n_1(t), \quad K \in N^*, n \in N. \] (13)

Then equation (2) plus equation (3) becomes:
\[ y_1(t) + y_2(t + T) = x_1(t) \ast h(t) + x_2(t + T) \ast h(t + T). \] (14)

From equation (14), it can be seen that the fundamental components of powerline noise and the odd harmonic components in the noise term are all cancelled. On the premise that \(T\) is longer than the system response adjustment time, \(x_1(t)\) and \(x_2(t)\) can be considered two independent transmissions at the same time slot, and \(y_1(t)\) and \(y_2(t)\) are the corresponding responses. Equation (14) is simplified to:
\[ y_1(t) + y_2(t) = [x_1(t) + x_2(t)] \ast h(t). \] (15)

In this study, \(x_1(t) + x_2(t)\) is called the equivalent source signature and \(y_1(t) + y_2(t)\) is the equivalent received signal.

These two cases are considered from the aspect of the transmission time of the excitation signal, and the method to cancel the powerline noise is derived from the premise that the powerline noise is stable after a short time. In practice, the effect of residual noise on system identification should also be considered. The following considers the relationship between the source signature and the equivalent source signature to eliminate residual noise and improving system identification accuracy.

### 2.5. Design of source signature

The choice of the source signature should be considered for anti-noise capability and deconvolution accuracy. The deconvolution algorithm is introduced in the subsequent sections. For anti-interference ability, especially anti-random noise, the good autocorrelation characteristics of the m-sequence have ensure it is used extensively in correlation identification to suppress noise and it has achieved good identified results (Wang et al. 2016, 2020).

The preceding section assumes that noise is an ideal powerline interference. In practice, however, powerline noise cannot be ideal, and its amplitude and phase may be slightly distorted. In this scenario, the powerline noise can be considered ideal powerline noise plus random noise, as expressed next:
\[ n_1(t) = n_1(t) + n_r(t). \] (16)

where \(n_1(t)\) denotes the distorted powerline noise; \(n_r(t)\) represents the ideal powerline noise and \(n_r(t)\) is the random noise. Through the method of eliminating powerline noise proposed in this study, \(n_r(t)\) can be completely cancelled and the noise is only random noise.

In equation (9) and (15), the signals \([x_1(t) - x_2(t)]\) and \([x_1(t) + x_2(t)]\) (collectively referred to as equivalent source signature) can be designed as an m sequence, so random noise can be eliminated by solving the earth impulse response based on the correlation identification. Thus, the design principle of the source signature is to make the equivalent source signature an m sequence, and the source signature is easy to implement.

Due to the limitation of the instrument, two simple and practical source signatures are given by complying with the source signature design principles. The specific implementation steps are expressed as follows:

(i) For case 1 with the equivalent source signature of \([x_1(t) - x_2(t)]\), the steps to determine the source sequence are elucidated next:

Step 1: Determine the required m sequence \(x_m(t)\). The initial state, feedback coefficient, order, symbol width, amplitude and other parameters of the generated m sequence can be set according to needs;

Step 2: \(x_m(t) = x_m(t), \quad x_3(t) = -x_m(t)\). Thus, the equivalent source signature \([x_1(t) - x_2(t)]\) is an m sequence \(2x_m(t)\).

(ii) For case 2 with the equivalent source signature of \([x_1(t) + x_2(t)]\), the steps to determine the source sequence are:

342
Step 1: Determine the required m sequence \( x_m(t) \);

Step 2: Set \( x_1(t) = x_m(t), x_2(t) = x_m(t) \). Thus, the equivalent source signature \([x_1(t) + x_2(t)]\) is also an m sequence \(2x_m(t)\).

Figure 4 parts a and c represent situation (i), in which figure 4a indicates the designed source signature, consisting of two \(x_m(t)\) and \(-x_m(t)\) parts, figure 4c is the equivalent source signature \(2x_m(t)\) and the power of the equivalent signal refers to the sum of the designed source signature’s power.

Figure 4 parts b and d represent situation (ii). Figure 4b indicates the designed source signature, composed of two \(x_m(t)\) and \(x_m(t)\) parts, and figure 4d is the equivalent source signature \(2x_m(t)\), which is exactly the same as the equivalent source signature in the situation (i).

The design of the mentioned source signature complies with the two principles; that is, designing the equivalent source signature into an m sequence, as well as maximizing the use of the transmission power, i.e. after cancelling the powerline noise, the autocorrelation of the m sequence is adopted to eliminate random noise. When the powerline noise is cancelled, the specific relationship between the transmission period \(T\) and the powerline noise period \(T_1\) should be considered. When the two do not satisfy the specific relationship, zeros should be added after the original effective source signature to achieve the prerequisites for the powerline noise cancelling. In the next section, the specific principles and methods of zero padding are clarified.

### 2.6. Zero-padding method of source signature

The premise of eliminating powerline noise here is to continuously transmit two excitation signals, and the responses of the two excitation signals have no effect on each other, i.e. to ensure that each excitation signal is input to obtain their own sufficient response. Accordingly, a period of zero excitation is required after each effective signal excitation. This period should not just ensure that the source signature fully excites the earth system so the receiver obtains a complete excitation response. Moreover, the zero excitation time plus the effective excitation time should satisfy the specific relationship with the powerline noise period.

Assuming that the effective signal time is \(T_v\), and the zero-pad time is \(T_{pz}\), the relationship between the two and the total time \(T\) of one transmission is expressed as:

\[
T = T_v + T_{pz}. \tag{17}
\]

According to the relationship between the transmission time and the powerline noise period derived, equation (17)
and 

\[ n_{r} = h_{i} \times v_{t} + n_{r} \]  

Figure 5. Equivalent model of the earth system.

is rewritten as:

\[ T_{pz} = T - T_{v} = \begin{cases} KT_{1} - T_{v} & \text{case 1} \\ KT_{1} + \frac{1}{2} T_{1} - T_{v} & \text{case 2} \end{cases} \]  

(18)

where case 1 represents the case \( T \) as an integer multiple of \( T_{1} \), and case 2 indicates the case \( T \) is a non-integer multiple of \( T_{1} \). The solution is as follows:

\[ T_{v} = (N_{f} * L_{m} * N_{spb}) / f_{s} \]  

(19)

where \( N_{p} \) denotes the numbers of periods of the sequence; \( L_{m} \) represents the number of symbols in a sequence period; \( N_{spb} \) is the number of samples per symbol and \( f_{s} \) is the sampling frequency.

The zero-padding time can be determined by equation (18), and the number \( N_{pz} \) of zero padding can be determined by equation (20):

\[ N_{pz} = T_{pz} / T_{v} = (T - T_{v}) / f_{s} \]  

\[ = \begin{cases} (KT_{1} - T_{v}) / f_{s} & \text{case 1} \\ (KT_{1} + \frac{1}{2} T_{1} - T_{v}) / f_{s} & \text{case 2} \end{cases} \]  

(20)

The parameter \( K \) should ensure that \( N_{pz} \) reaches zero.

2.7. Correlation identification and deconvolution method

After the powerline noise is cancelled, residual random noise will remain. Since the equivalent source signature is an \( m \) sequence, its correlation characteristics can be exploited to identify the earth system based on the Wiener filter deconvolution method. Assume that the system after cancelling the powerline noise is equated with figure 5, as shown.

In figure 5, \( n_{r} \) denotes the residual random noise, \( s_{i} \) and \( V_{t} \), respectively, represent the equivalent source signature and equivalent received signal. Subsequently, there is the following relationship between the quantities in the figure 5:

\[ V_{t} = s_{i} \times h_{i} + n_{r} \]  

(21)

Both sides of equation (21) are correlated with the input signal \( s_{i} \) simultaneously, and it yields:

\[ \phi_{V_{t}}(t) = \phi_{s_{i}}(t) \times h_{i} + \phi_{n_{r}}(t), \]  

(22)

where \( \phi_{V_{t}}(t) \) denotes the cross-correlation function of the equivalent received signal and the equivalent source signature; \( \phi_{s_{i}}(t) \) represents the autocorrelation function of the equivalent source signature and \( \phi_{n_{r}}(t) \) is the cross-correlation function of random noise and the equivalent source signature. The equivalent source signature refers to an \( m \) sequence, and its correlation function with random noise is zero, i.e. the term \( \phi_{n_{r}}(t) \) is zero. Thus, equation (22) is simplified to:

\[ \phi_{V_{t}}(t) = \phi_{s_{i}}(t) \times h_{i}. \]  

(23)

By deconvolving equation (23) to solve the impulse function of the earth system, Ziolkowski (2013) deduced and proposed the Wiener filter algorithm as:

\[ \begin{bmatrix} A_{0} & A_{1} & \ldots & A_{n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n} & A_{n-1} & \ldots & A_{0} \end{bmatrix} \begin{bmatrix} h_{0} \\ \vdots \\ h_{n} \end{bmatrix} = \begin{bmatrix} B_{0} \\ \vdots \\ B_{n} \end{bmatrix}, \]  

(24)

where \( h_{i} \) denotes the estimated value of the earth impulse response; \( A_{s} \) represents the normalized autocorrelation value of the equivalent source signature \( s_{i} \) and \( B_{r} \) is the normalized cross-correlation value of the equivalent source signature \( s_{i} \) and the equivalent received signal \( V_{t} \). The equivalent source signature \( s_{i} \) and equivalent received signal \( V_{t} \) are expressed as follows:

\[ s_{i} = \begin{cases} x_{i}(t) - x_{i}(t) & \text{case 1} \\ x_{i}(t) + x_{i}(t) & \text{case 2} \end{cases} \]  

(25)

\[ V_{t} = \begin{cases} y_{i}(t) - y_{i}(t) & \text{case 1} \\ y_{i}(t) + y_{i}(t) & \text{case 2} \end{cases} \]  

(26)

The elements \( A_{s} \), in the square matrix on the left and the elements \( B_{r} \), in the matrix on the right in equation (24) are defined as follows:

\[ A_{s} = \sum_{i} s_{i} s_{i-t} \quad B_{r} = \sum_{i} V_{t} V_{t} \]  

(27)

(28)

By equations (24–28), the earth impulse response can be yielded, and with this method of solving using the characteristics of \( m \) sequence autocorrelation, its algorithm itself has strong anti-noise performance. Thus, the random noise mixed in the powerline noise can be well suppressed. The specific derivation process of the Wiener filtering algorithm (Robinson & Treitel 2006) is detailed in the Appendix.

3. Experimental verification

To test the feasibility of the denoising algorithm proposed in the present study, take the uniform half-space model as an example. The resistivity of the uniform half space is set to 100 \( \Omega \cdot m^{-1} \), the length of the transmitting electric dipole
source is set to 1 m, and the source-receiver offset is set to 1000 m.

The step response $E_s(t)$ of a uniform half space without noise is written as (Weir 1980):

$$E_s(t) = \frac{I dl \rho}{2 \pi r^3} \left[ \text{erf} \left( \frac{r}{2 \sqrt{\pi} c \sqrt{t}} \right) - \frac{2}{\sqrt{\pi}} \frac{r}{2 c \sqrt{t}} \exp \left( -\frac{r^2}{4c^2 t} \right) \right],$$

(29)

where $c^2 = \frac{\mu}{\rho}$, $I$ denotes the current intensity, $dl$ represents the electric dipole source length, $\rho$ is the resistivity, $r$ is the source-receiver offset, $\text{erf}$ is the error function and $\mu_0$ is the permeability.

The response field excited by the source signature is obtained by equation (30) (Yin et al. 2013):

$$E(t) = I(t) * h(t) = -\frac{\partial}{\partial t} [I(t)] * E_s(t).$$

(30)

Theoretically, the impulse response of a uniform half space can be determined by deriving the time $t$ by equation (29):

$$h_i(t) = \frac{I dl \rho}{8 \pi \sqrt{\pi} c^3} \exp \left( -\frac{t^2}{4c^2 t} \right) t^{-5/2}. $$

(31)

### 3.1. Simulation experiment

#### 3.1.1. Case 1: $T$ is an integer multiple of $T_1$.

The source signature $x_m(t)$ is set to a two-period eight-order m sequence. The feedback coefficient of m sequence generator is achieved as 747(Octal). The initial state of the m sequence shift register is $[1 0 \cdots 0]$, and the amplitude of the m sequence is $\pm 1A$, $x_1(t) = x_m(t)$, $x_2(t) = -x_m(t)$. Padding zeros after the effective signal to make $T$ are written as in equations (4) and (15), and the sampling frequency is 24 kHz.

The effective duration of $x_1(t)$ and $x_2(t)$ is expressed as $T$, the duration $T$ of one transmission and the number of zero padding $N_{pz}$ are solved as follows:

$$T = \frac{N_p \times L_m \times N_{spb}}{f_s},$$

where $T = \frac{(N_p \times L_m \times N_{spb})}{f_s}$, $f_s$ is the sampling frequency.

$$N_{pz} = T_{pz} \times f_s = (T - T_s) \times f_s = \left(0.1 - 0.085\right) \times 24000 = 360.$$

3.1.1. Case 1: $T$ is an integer multiple of $T_1$. The source signature $x_m(t)$ is set to a two-period eight-order m sequence. The feedback coefficient of m sequence generator is achieved as 747(Octal). The initial state of the m sequence shift register is $[1 0 \cdots 0]$, and the amplitude of the m sequence is $\pm 1A$, $x_1(t) = x_m(t)$, $x_2(t) = -x_m(t)$. Padding zeros after the effective signal to make $T$ are written as in equations (4) and (15), and the sampling frequency is 24 kHz.

The effective duration of $x_1(t)$ and $x_2(t)$ is expressed as $T$, the duration $T$ of one transmission and the number of zero padding $N_{pz}$ are solved as follows:

$$T = \frac{N_p \times L_m \times N_{spb}}{f_s},$$

where $T = \frac{(N_p \times L_m \times N_{spb})}{f_s}$, $f_s$ is the sampling frequency.

$$N_{pz} = T_{pz} \times f_s = (T - T_s) \times f_s = \left(0.1 - 0.085\right) \times 24000 = 360.$$

The actual source signature is presented in figure 6a, and figure 6b indicates the equivalent source signature, i.e. the signal $x_1(t) + x_2(t)$ in equation (7). According to the figure, the power of the equivalent source signature is the sum of the designed transmission power. The system response calculated by equation (26) is expressed in figure 6c. The picture indicates that the responses of the second half and the first half are symmetrical.

---

**Figure 6.** Source signature and corresponding system response in case 1. (a) Source signature. (b) Equivalent source signature of (a). (c) The response of the earth system corresponding to input signal (a).
Adding the ideal system response to the strong powerline noise interference $n_c(t) = 10^{-7} \cos(2\pi \times 50t)$, the noisy signal is presented in figure 7a, obviously demonstrating that the useful signal is completely submerged in the strong interference. The equal duration of the signal in figure 7a is split into two, which are represented by $y_1(t)$ and $y_2(t)$, respectively. After the two are subtracted, the equivalent excitation response shown in figure 7b is obtained. From the right-hand figure, it can be seen that the powerline noise has been cancelled out during the subtraction process.

As indicated by equation (9), after obtaining the equivalent source signature and the equivalent response signal, the earth impulse response to be identified can be generated. When deconvolution is performed to solve the earth impulse response, the equivalent signal acts as the m sequence. Moreover, through the correlation operation with random noise, the results of the two are approximately zero and the earth impulse response estimated value $ht$ is subsequently generated by inverse calculation by equation (22).

The figure 8b presents the error rate curve, and its solution is defined as equation (32), in which $R_{err}$ is the error rate. From the error rate graph, it is easy to see that in the early stage, the identification error is small, and the later identification error shows a fluctuating rapid increase. This is because the latter theoretical value is significantly small, and slight deviations in the identification will cause a large error rate. In addition, this reflects that a small amount of high-frequency interference remains in the denoising process.

$$R_{err} = \frac{ht(t) - h_t}{h_t} \times 100\%.$$  \hfill (32)

3.1.2. Case 2: $T$ is a non-integer multiple of $T_1$. The source signature $x_m(t)$ is set to a two-period eight-order m sequence. The feedback coefficient of m sequence generator is 747(Octal). The initial state of the m sequence shift register is [1 0 ⋯ 0], and the amplitude of the m sequence is $\pm 1A$, $x_1(t) = x_m(t)$, $x_2(t) = x_m(t)$. Padding zeros after the effective signal, so $T$ satisfies equations (10) and (17), and the sampling frequency $f_s$ is 24 kHz.

The effective duration of $x_1(t)$ and $x_2(t)$ is expressed as $T_e$, the duration $T$ of one transmission and the number of zero padding $N_{pz}$ are solved as follows:

$$T_e = (N_p \ast L_m \ast N_{spb})/f_s = (2 \ast 255 \ast 4)/24000 = 0.085s.$$  

$$T = KT_e + \frac{1}{2} T_e = 0.02K = 0.11, \quad K = 5.$$  

$$N_{pz} = T_e \ast f_s = (T - T_e) \ast f_s = (0.11 - 0.085) \times 24 000 = 600.$$
Figure 8. Identified result and identification error in case 1. (a) The results of identification (red dotted line) by the subtraction cancellation method proposed in this study are compared with the theoretical impulse response (black solid line). (b) The error rate between the identification result and the theoretical impulse response.

Figure 9. Source signature and corresponding system response in case 2. (a) Source signature. (b) Equivalent source signature of (a). (c) The response of the earth system corresponding to input signal (a).

The figure 9a is the source signature, the signal in the front half is \(x_1(t)\), and the signal in the back half is \(x_2(t)\). \(x_1(t)\) and \(x_2(t)\) are identical, and here they are both a two-period eight-order m sequence. The figure 9b is the equivalent source signature. It is easy to see from the figure that the equivalent source signature power is twice the designed source signature power. The figure 9c shows the system response corresponding to the designed source signature. As indicated in the figure, the second half of the response is identical to the first half.

Figure 10a illustrates the noisy response after adding strong powerline interference to the system response. Figure 10a falls into two parts, as represented by \(y_1(t)\) and \(y_2(t)\), respectively, and then the two are added together to
obtain the equivalent system response. Figure 10b presents the equivalent excitation response. Obviously, in the process of adding, the powerline noise is cancelled, whereas the equivalent response power is doubled.

Figure 11a shows the identified results. The black line is the theoretical impulse response of a uniform earth system, and the red line is the identified result under the case 2 here. As indicated by figure 11a, the identification effect is significantly high, showing nearly no difference from the theoretical value. Figure 11b is the identification error graph, indicating that there is almost no error in the identified result in the early stage, and the error will fluctuate in the later stage, whereas the error remains very small. According to the glitch phenomenon in the figure, high-frequency interference remains in the denoising process.
3.2. Field data

3.2.1. Verification of field noise. The measured noise data were collected at the latitude and the longitude (110.8110°, 31.2727°) in Xingshan County, Yichang City, Hubei Province on 21 June 2019. The sampling frequency is 24,000. There are civilian wires and high-voltage wires close to the collection point.

First, the actual data of case 1 are verified. A piece of measured noise data is taken, as illustrated in figure 12a, whereas figure 12b represents the frequency spectrum of figure 12a. The total noise duration is 0.8 s, and \( T = 0.4 \) s, and then the duration \( T \) denotes an integer multiple of the fundamental period of the powerline noise. In addition, figure 12c is generated by subtracting the two noise terms, where figure 12d represents the frequency spectrum of figure 12c. According to figure 12 parts a and b, noise is primarily composed of powerline noise and its harmonics, and also contains a small amount of spike noise. Figure 12 parts c and d show that the powerline noise and its various harmonics are significantly eliminated by using the subtraction cancellation proposed here, and the inherent errors of the measurement are cancelled as well. After removing the powerline noise, the noise item only refers to the spike noise and the random noise.

Table 1 quantitatively assesses the denoising effect of case 1. As suggested from the attenuation data of each frequency in the table, the attenuation of the fundamental wave is as high as 21.29 dB, the attenuation of the 3, 5, 7, 11, 13 and 17 multiples of the fundamental frequency are above 10 dB overall and the attenuation of the other multiple of the fundamental frequency are nearly 10 dB. It is therefore revealed that the subtraction cancellation has a very good denoising effect on the powerline fundamental wave and its multiple of the fundamental frequency interference.

To perform actual data verification for case 2, take a piece of measured noise data, as showed in figure 13a: figure 13b is the frequency spectrum of figure 13a. When the total noise duration is 0.82 s, and \( T = 0.41 \) s, i.e. the duration \( T \) is the integer and one-half times of the fundamental period of the powerline noise, and the two noise terms are added together to get as shown in figure 13c, figure 13d is the frequency spectrum of figure 13c. As shown in the time domain and frequency domain diagrams before and after denoising in figure 13, the use of the addition cancellation method proposed here also eliminates significantly the powerline fundamental frequency and its odd multiple of the fundamental frequency noise, whereas the inherent error of the system is not eliminated.
Table 2. Evaluation of denoising effect of case 2 in the present study

| Frequency (Hz) | 50  | 150 | 250 | 350 | 450 | 550 | 650 | 750 |
|---------------|-----|-----|-----|-----|-----|-----|-----|-----|
| Attenuation (dB) | 23.49 | 12.49 | 12.37 | 11.29 | 11.31 | 14.23 | 12.51 | 7.56 |
| Frequency (Hz) | 850 | 950 | 1050 | 1150 | 1250 | 1350 | 1450 | 1550 |
| Attenuation (dB) | 17.66 | 10.18 | 9.35 | 9.83 | 9.99 | 7.99 | 9.93 | 9.69 |

Table 2 quantitatively assesses the denoising effect of case 2. Thus, the addition cancellation method significantly eliminates the powerline fundamental wave and its odd multiples of the fundamental frequency noise; in particular, the attenuation of the fundamental wave reaches 23.49 dB. However, it can be viewed on the spectrogram that the energy of the even multiples of the fundamental frequency noise has increased. After the comparison of the energy before and after the even multiples of the fundamental frequency, it is reported that this method increases the energy by 3 dB. This is because the addition cancellation method has no denoising capability for even multiples of the fundamental frequency.

3.2.2. System identification with field noise. Obviously, a fixed measurement error is identified in the measured noise, so the system identification and denoising with the measured noise adopts the subtraction cancellation. The source signature adopts 12 cycles of an eight-order m sequence (zero padding performed after the m sequence), and the source signature current is 1 A. In addition, the uniform half-space model is adopted, and the resistivity is set to 100 \( \Omega \) m\(^{-1}\). Moreover, the length of the source electrode separation is 1 m and the source-receiver offset is 1000 m.

Figure 14 presents the time domain and corresponding frequency domain diagrams before and after noise cancellation at the receiver. Figure 14c shows the theoretical response plus the measured noise. It can be seen that the system response completely submerges in noise. Figure 14d indicates that the noise is primarily 50 Hz powerline noise and its multiple frequency noise. Figure 14e is the equivalent response signal after the use of the subtraction cancellation. The figure shows that after the removal of the powerline noise, the response contains random noise and spike interference as well. As suggested by figure 14f, the powerline fundamental frequency and its multiple frequency are removed significantly, and the equivalent response is determined by subtracting \( y_2(t) \) from the response \( y_1(t) \). It yields \( y_2(t) = -y_1(t) \) in the absence of noise (since the source signature is \( x_2(t) = -x_1(t) \)), and the equivalent response power after subtraction is nearly twice the original received power. As indicated by the comparison of figure 14 parts f and b, compared with the theoretical response spectrum, only high-frequency interference and random interference are added.

The application of the correlation identification and Wiener deconvolution algorithm to obtain the earth impulse response is presented in the red line in figure 15a, whereas the black line is the theoretical unit impulse response.
Figure 14. Time-frequency domain diagram before and after noise cancellation. (a) The theoretically equivalent response. (b) The spectrogram of (a). (c) The system response with the field noise. (d) The spectrogram of (c); (e) The equivalent response after denoising by using the subtraction cancellation. (f) The spectrogram of (e).

Figure 15. Identified result and identification error with field noise. (a) The results of identification (red dotted line) by the method proposed in this study are compared with the theoretical impulse response (black solid line). (b) The error rate between the identification result and the theoretical impulse response.

Figure 15a shows that the system identified result is excellent at the early stage, fluctuates over time at the late stage and the error increases rapidly. This finding can be seen in figure 15b.

3.3. Discussion

As indicated from the simulation and field noise test, for case 1, by setting the duration of each segment of the source signature to an integer multiple of the period of powerline noise, the two segments of source signature are subtracted to generate an equivalent source signature, and the response of the two segments of the system response are subtracted to generate an equivalent response. When the two-segment system response is being subtracted, the powerline noise is cancelled, and the signal-to-noise ratio at the receiver is significantly improved.
After the powerline noise is cancelled, the earth system is simplified to an equivalent system containing only random noise. By designing the equivalent source signature into an m sequence, the good autocorrelation of the m sequence is exploited to eliminate the effect of random noise, and finally the earth impulse response is obtained with the Wiener filter deconvolution method. According to the system identified results and identification errors, case 1 successfully eliminates the fundamental frequency of powerline noise and its multiple frequency noise. According to the final identified results, this method of cancelling powerline noise proves feasible.

For case 2, by setting the duration of each segment of the source signature to be an integer plus one-half of the period of powerline noise, exploiting the periodicity and symmetry of powerline noise, the response of the two segments is introduced to cancel the powerline noise. During addition, the equivalent response is strengthened, and its power refers to the sum of the original two-segment system response power. Moreover, the equivalent source signature has been strengthened, and its equivalent transmission power indicates the sum of the original transmission power. This significantly reduces the calculation errors of subsequent correlation operations and deconvolution operations, and significantly improves the accuracy of system identification.

As revealed from the simulation results, the identification effect of case 1 is generally worse than that of case 2, whereas the field noise data indicate that the collected data have system errors in many cases, and there are some errors for the transmitter or receiver (e.g. system inherent errors or external DC interference), and the subtraction cancellation can cancel the system error while cancelling the powerline interference. Furthermore, case 2 adopts the addition cancellation method, which will only superimpose and amplify the system error, whereas the even-multiple fundamental frequency of powerline noise cannot be cancelled. Obviously, for most field data denoising situations, the subtraction cancellation is more effective than the addition cancellation method.

4. Conclusion

In the present study, by adjusting the transmission time of the source signature, it shows a specific relationship with the fundamental wave period and harmonic period of the powerline noise as an attempt to exploit the powerline noise characteristics to cancel it. There are two types of cancellation method: subtraction cancellation and addition cancellation. As revealed from the results, the subtraction cancellation method has a very good denoising effect to eliminate the powerline fundamental frequency and its multiple frequency noises, while it can eliminate the influence of system errors; the addition cancellation method can more obviously denoise for the powerline fundamental frequency and its odd-multiple frequency noises, whereas the system error cannot be eliminated, and it exhibits no denoising ability for even-multiple frequency noise.

Second, this study optimizes the signal from the perspective of the equivalent signal. The transmitting source is designed as an m sequence for the equivalent source signature, thereby improving the anti-noise performance of the system, and the power of the equivalent source signature is doubled compared with the design source signature power; the effective power of the equivalent response is also doubled, the powerline noise is significantly eliminated, so the signal-to-noise ratio of the equivalent response signal is significantly improved.

In addition, with the correlation identification method, the good autocorrelation of the m sequence is adopted maximally to suppress noise, and the Wiener filter deconvolution method is used to increase the system identification accuracy. As revealed from the results, the correlation identification method can significantly help suppress random noise, and the Wiener filter deconvolution method exhibits higher identification accuracy than other deconvolution methods.

The method proposed in the present study is simple in principle, easy to implement and effective in removing specific noise, which presents a novel research insight into powerline noise elimination. The proposed method should be further optimized and verified, and it can be extended and applied to eliminate multiple periodic noises simultaneously.

Acknowledgements

This work appreciates the support of the National Key R&D Program of China (grant no. 2018YFC0603202), the National Natural Science Foundation of China (grant no. 42074087), the Research Foundation of Education Bureau of Hunan Province of China (grant no. 18C0741), the Postgraduate Research and Innovation of Central South University of Hunan Province of China (grant no. 2018zzts207).

Conflict of interest statement. None declared.

References

Butler, K.E. & Russell, R.D., 1993. Subtraction of powerline harmonics from geophysical records, Geophysics, 58, 898–903.
Butler, K.E. & Russell, R.D., 2003. Cancellation of multiple harmonic noise series in geophysical records, Geophysics, 68, 1083–1090.
Duncan, P.M., Hwang, A., Edwards, R. N., Bailey, R.C. & Garland, G.D., 1980. The development and applications of a wide band electromagnetic sounding system using a pseudo-noise source, Geophysics, 45, 1276–1296.
Guo, Z.W., Lai, J.Q., Zhang, K.N., Mao, X.C., Wang, Z.L., Guo, R.W., Deng, H., Sun, P.H., Zhang, S.H., Yu, M., Cui, Y.A. & Liu, J.X., 2020. Geosciences in Central South University: a state-of-the-art review, Journal of Central South University, 27, 975–996.
Ilyichev, P.V. & Bobrovsky, V.V., 2015. Application of pseudonoise signals in systems of active geoelectric exploration (results of mathematical simulation and field experiments), Seismic Instruments, 51, 53–64.
Legchenko, A. & Valla, P., 2003. Removal of power-line harmonics from proton magnetic resonance measurements, *Journal of Applied Geophysics*, 53, 103–120.

Li, G., He, Z.S., Tang, J.T., Deng, J.Z., Liu, X.Q. & Zhu, H.J., 2021. Dictionary learning and shift-invariant sparse coding denoising for controlled-source electromagnetic data combined with complementary ensemble empirical mode decomposition, *Geophysics*, 86, 1–14.

Li, G., Liu, X.Q., Tang, J.T., Deng, J.Z., Hu, S.G., Zhou, C., Chen, C.J. & Tang, W.W., 2020. Improved shift-invariant sparse coding for noise attenuation of magnetotelluric data, *Earth, Planets and Space*, 72, 45.

Li, M., Wei, W.B., Deng, M., Yuan, W.J. & Zhang, Q.S., 2008. Application model of pseudo-random correlation method in electrical prospecting, *Kybernetes*, 37, 1451–1456.

Qb, Y.F., Yin, C.C., Wang, R. & Cai, J., 2015. Multi-transient EM full-time forward modeling and inversion of m-sequences, *Chinese Journal of Geophysics*, 58, 2566–2577. (in Chinese).

Robinson, E.A. & Treitel, S., 2006. Principles of digital Wiener filtering, *Geophysical Prospecting*, 15, 311–332.

Saucier, A., Marchant, M. & Chouteau, M., 2006. A fast and accurate frequency estimation method for canceling harmonic noise in geophysical records, *Geophysics*, 71, V7–V18.

Szarka, L., 1988. Geophysical aspects of man-made electromagnetic noise in the earth—a review, *Surveys in Geophysics*, 9, 287–318.

Tang, J.T., Li, G., Zhou, C., Li, J., Liu, X.Q. & Zhu, H.J., 2018. Power-line interference suppression of MT data based on frequency domain sparse decomposition, *Journal of Central South University*, 25, 2150–2163.

Wang, X.X., Deng, J.Z. & Ren, J.L., 2020. Selection of code width and analysis of influencing factors in multitransient electromagnetic method, *Geophysical Journal International*, 220, 160–173.

Wang, X.X., Di, Q.Y., Wang, M.Y. & Deng, J.Z., 2016. A study on the noise immunity of electromagnetic methods based on m-pseudo-random sequence, *Chinese Journal of Geophysics*, 59, 1861–1874. (in Chinese).

Warden, S., Garambois, S., Sailhac, P., Jouniaux, L. & Bano, M., 2012. Curvelet-based seisemclectric data processing, *Geophysical Journal International*, 190, 1533–1550.

Weir, G.J., 1980. Transient electromagnetic fields about an infinitesimally long ground electric dipole on the surface of a uniform half-space, *Geophysical Journal International*, 61, 41–56.

Wright, D.A., Ziolkowski, A. & Hobbs, B.A., 2001. Hydrocarbon detection with a multi-channel transient electromagnetic survey, in *Proceedings of the 71st Annual International Meeting*, SEG, Expanded Abstracts, Society of Exploration Geophysicists, 1435–1438.

Yin, C.C., Huang, W. & Ben, F., 2013. The full-time electromagnetic modeling for time-domain airborne electromagnetic systems, *Chinese Journal of Geophysics*, 56, 3153–3162. (in Chinese).

Yuan, Z., Zhang, Y.M. & Wang, X.H., 2017. Improved data segmentation method for EM excited by m-sequence: a new approach in powerline noise reduction, *Journal of Applied Geophysics*, 143, 156–168.

Ziolkowski, A., 2013. Wiener estimation of the Green’s function, *Geophysics*, 78, W31–W44.

Ziolkowski, A., Hobbs, B.A. & Wright, D., 2007. Multitransient electromagnetic demonstration survey in France, *Geophysics*, 72, F197–F209.

Ziolkowski, A., Wright, D. & Mattsson, J., 2011. Comparison of pseudo-random binary sequence and square-wave transient controlled-source electromagnetic data over the Peon gas discovery, *Norway, Geophysical Prospecting*, 59, 1114–1131.

**Appendix**

The most basic principle of Wiener filtering is the least mean square criterion, which is to minimize the energy of the difference between the theoretical output $z_t$ and the actual output $y_t$. In other words, we look for the filter coefficient $f_t$ to minimize the value of the following equation.

$$I = E\{ (z_t - y_t)^2 \}. \quad (A-1)$$

To facilitate the calculation, we take enough but limited filter coefficients, namely

$$f_t = \{ f_0 f_1 f_2 \cdots f_{n-1} \}, \quad (A-2)$$

that is to say, the filter coefficient $f_t$ is approximated by $n + 1$ equally spaced coefficients.

The system output $y_t$ is the convolution of the filter coefficient $f_t$ and the input signal $x_t$:

$$y_t = x_t \ast f_t = \sum_{r=0}^{n} f_r x_{t-r}. \quad (A-3)$$

So, the energy error can be written as follows:

$$I = E \left\{ (z_t - \sum_{r=0}^{n} f_r x_{t-r})^2 \right\}. \quad (A-4)$$

To make equation (A-4) reach the minimum value, the partial derivative of each filter coefficient $f_r$ of equation (A-4) is required to be zero. The partial derivative of $f_1$ is solved as follows:

$$\frac{\partial I}{\partial f_1} = E \left\{ 2 \left( z_t - \sum_{r=0}^{n} f_r x_{t-r} \right) \frac{\partial}{\partial f_1} \left( z_t - \sum_{r=0}^{n} f_r x_{t-r} \right) \right\} \quad (A-5)$$

$$= 2E \left\{ -x_{t-1} x_t + \sum_{r=0}^{n} f_r x_{t-r} x_{t-1} \right\} \quad (A-5)$$

$$= 2 \left[ -E (x_{t-1} x_t) + \sum_{r=0}^{n} f_r E (x_{t-r} x_{t-1}) \right] \quad (A-5)$$

$$= 2 \left[ -\phi_{xx} (1) + \sum_{r=0}^{n} f_r \phi_{xx} (1 - r) \right]. \quad (A-5)$$

Let equation (A-5) be equal to zero, thus the following equation can be obtained:

$$\sum_{r=0}^{n} f_r \phi_{xx} (1 - r) = \phi_{xx} (1). \quad (A-6)$$
In the same way, we sequentially calculate the partial derivatives of $I$ to $f_j (j = 0, 1, 2, \ldots n)$, and finally we can obtain $(n + 1)$ linear simultaneous equations. We write it in the following form:

$$
\sum_{\tau=0}^{n} f_\tau \phi_{xx}(j - \tau) = \phi_{xx}(j), \quad j = 0, 1, 2 \ldots n, \quad (A-7)
$$

where $\phi_{xx}(\tau)$ is the autocorrelation of the input signal $x_t$, and $\phi_{zx}(\tau)$ is the cross-correlation between the output function $z_t$ and the input function $x_t$:

$$
\phi_{xx}(\tau) = E(x_t x_{t-\tau}) = \sum_i x_i x_{i-\tau}, \quad (A-8)
$$

$$
\phi_{zx}(\tau) = E(z_t x_{t-\tau}) = \sum_i z_i. \quad (A-9)
$$

In equation (A-7), we know $\phi_{xx}(\tau)$ and $\phi_{zx}(\tau)$, and the filter coefficient $f_\tau$ has $n + 1$ unknown coefficients, the equation (A-7) can be written as $n + 1$ simultaneous equations, and its matrix form is:

$$
\begin{bmatrix}
A_0 & A_1 & \cdots & A_n \\
A_1 & A_0 & \cdots & A_{n-1} \\
\vdots & \vdots & \ddots & \vdots \\
A_n & A_{n-1} & \cdots & A_0 \\
\end{bmatrix}
\begin{bmatrix}
f_0 \\
f_1 \\
\vdots \\
f_n \\
\end{bmatrix}
= 
\begin{bmatrix}
B_0 \\
B_1 \\
\vdots \\
B_n \\
\end{bmatrix}, \quad (A-10)
$$

where

$$
A_\tau = \frac{\phi_{xx}(\tau)}{\phi_{xx}(0)}, \quad (A-11)
$$

$$
B_\tau = \frac{\phi_{zx}(\tau)}{\phi_{zx}(0)}. \quad (A-12)
$$

So far, the optimal filter coefficient $f_j (j = 0, 1, 2 \ldots n)$ can be solved by equation (A-10).