Generalized Holographic Dark Energy Model

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Abstract

In this paper, the model of the holographic Chaplygin gas has been extended to two general cases: first the case of a modified variable Chaplygin gas and second the case of the viscous generalized Chaplygin gas. The dynamics of the model is expressed by the use of scalar fields and scalar potentials.

Keywords: Cosmological constant; cosmic coincidence problem; dark energy; holographic principle.

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1 Introduction

Astrophysical observations of type Ia supernovae [1, 2], galaxy redshift surveys [3], cosmic background radiation (CMB) data [4, 5, 6], large scale structure [7, 8] and gravitational lensing surveys [9] convincingly suggest that the observable universe is undergoing accelerated expansion. The observations also suggest that the transition from the earlier deceleration phase to the recent acceleration phase is marginally recent $z \lesssim 1$ [10]. The cause of this sudden transition and the source of this expansion has not yet been identified categorically. It is generally believed that some sort of ‘dark energy’ is pervading the universe. Consequently several questions arise: if dark energy dominates the universe, then why did it remain dormant until recently? Can it interact with other cosmic ingredients like matter and radiation? What is its equation of state (EoS)? and also what is the composition of this energy? The word ‘dark’ itself implies that our understanding about the nature of this energy is very modest, despite substantial progress both in the theoretical and observational fields. Several cosmological models have been proposed in recent years to explain dark energy including those based on the Chaplygin gas [11], scalar field models like quintessence [12, 13], k-essence [14, 15] and phantom energy [16], modified $f(R)$ gravity theories [17, 18] and variable constants approach [19], to name a few. It might be possible that the accelerated expansion of the universe is a manifestation of the inhomogeneity and anisotropy of the space itself and that dark energy may not be mandatory at all [20].

The problem of dark energy has also been addressed in the context of the holographic principle. The principle says that the number of degrees of freedom of a physical system should scale with its bounding area rather than with its volume. It is motivated from an observation that in quantum field theory, the ultraviolet cut-off $\Lambda$ could be related to the infrared cut-off $L$ due to the limit set by forming a black hole i.e. the total energy of a system of size $L$ should not exceed the mass of a system-size black hole [21, 22]:

$$L^3 \rho_\Lambda \leq M_{p}^2 L,$$

which yields

$$\rho_\Lambda = 3c^2 M_{p}^2 L^{-2}.$$

Here $c$ is a positive constant of the order unity and $3c^2$ is attached for convenience, $M_{p} \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass and $L_\Lambda$ is the largest cut-off chosen to convert into an equality. It has been shown that a Friedmann-Robertson-Walker (FRW) universe filled with matter and holographic dark energy (HDE) fits the supernova Ia data if the parameter $c$ is taken to be $c = 0.21$ or more generally $c < 1$ [23, 24]. We shall use the notation $\rho_\Lambda$ and $\rho_{de}$ synonymously. Eq. (2) represents the energy density corresponding to the HDE. Hence if the infrared cut-off is taken as the current size of
the universe i.e. $L_\Lambda = H^{-1}$ then the holographic energy density is close to the density of dark energy [25]. In [26], the author has developed the correspondence between HDE and tachyons while in [27], the connection between HDE and Gauss-Bonnet dark energy is established. In [28], the relationship between HDE and $f(R)$ theories is developed, and also in [29] the analogy between HDE and Brans–Dicke theory is proposed, while in [30], a correspondence of HDE with quintessence, tachyons and K-essence is obtained. All these correspondences lead to accelerated expansion solutions at late times. The HDE can also realize a quintom scenario i.e. it evolves from a quintessence phase to the phantom phase [31, 32]. The holographic dark energy has been tested and constrained by various observations, such as SNe Ia [33], CMB [34], X-ray gas mass fraction of galaxy clusters [35] and the differential ages of passively evolving galaxies [36]. The HDE also fairly alleviates some hard cosmological problems like the cosmic age problem [37], the cosmic coincidence problem [38, 39] and the fine tuning problem [40].

The holographic dark energy model was originally proposed by Nojiri and Odintsov [41]. Recently, the holographic dark energy model with Chaplygin gas [42] and with modified Chaplygin gas [43] have been investigated. We here extend their studies to two EoSs involving a modified variable Chaplygin gas and viscous dark energy in the context of the holographic principle, in the next two sections. These EoSs belong to a general class of inhomogeneous EoSs as discussed in [44].

## 2 Holographic modified variable Chaplygin gas model

We start by assuming the background to be spatially homogeneous and isotropic FRW spacetime, specified by the line element

$$\left( 3 \right)$$

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right].$$

Here $a(t)$ is the dimensionless scale factor and $k$ is the curvature parameter which takes the three possible values $+1, 0, -1$ which correspond to spatially closed, flat and open spacetimes, respectively. The corresponding Einstein field equation is given by

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2} \left[ \rho_{de} + \rho_m \right], \quad \left(4\right)$$

where $M_p^2 = (8\pi G)^{-1}$ is the modified Planck mass. The dimensionless density parameters corresponding to matter, dark energy and curvature are

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{\rho_m}{3H^2M_p^2}, \quad \Omega_{de} = \frac{\rho_{de}}{\rho_{cr}} = \frac{\rho_{de}}{3H^2M_p^2}, \quad \Omega_k = \frac{k}{(aH)^2}. \quad \left(5\right)$$
$\rho_{cr} = 3M^2_pH^2$ is the critical density. Note that from (4) and (5), we can write $\Omega_m + \Omega_{de} = 1 + \Omega_k$. The EoS representing the dark energy is the modified variable Chaplygin gas (MVG) and is given by [45, 46]

$$p_{de} = A\rho_{de} - \frac{B(a)}{\rho_{de}^2}, \quad B(a) = B_o a^{-n}.$$  \hspace{1cm} (6)

Here $0 \leq \alpha \leq 1$, $0 \leq A \leq 1$, $B_o$ and $n$ are constant parameters. The Chaplygin gas behaves like dust in the early evolution of the universe and subsequently grows to an asymptotic cosmological constant at late time when the universe is sufficiently large.

In the cosmological context, the Chaplygin gas was first suggested as an alternative to quintessence [47]. Later on, the Chaplygin gas state equation was extended to a modified form by adding a barotropic term [48, 49]. Recent supernovae data also favor the two-fluid cosmological model with Chaplygin gas and matter [50]. We assume the two species i.e. matter and dark energy to be non-interacting, thus the energy conservation equations are

$$\dot{\rho}_m + 3H\rho_m = 0, \quad \dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = 0.$$  \hspace{1cm} (7)

The evolution of the energy density of MVG is obtained by substituting (6) in (7) to get

$$\rho_{de} = \left[\frac{3(1+\alpha)B_o}{3(1+\alpha)(1+A) - n} \frac{1}{a^n} - \frac{C}{a^{3(1+\alpha)(1+A)}}\right]^\frac{1}{1+\alpha}.$$  \hspace{1cm} (8)

Here $C$ is a constant of integration.

Astrophysical observations suggest that the EoS parameter $\omega_{de}$ is a dynamical variable which favors a phantom-non-phantom transition in the recent past. This behavior of dark energy is best explained with the help of a dynamically evolving and minimally coupled scalar field [51]. Note that this kind of scalar field formalism for dark energy is motivated by the cosmological inflation models as well [52]. Consider a scalar field $\phi$ with potential $V(\phi)$, related with the energy density and the pressure of MVG as

$$\rho_{\phi} = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \left[\frac{3(1+\alpha)B_o}{3(1+\alpha)(1+A) - n} \frac{1}{a^n} - \frac{C}{a^{3(1+\alpha)(1+A)}}\right]^\frac{1}{1+\alpha}.$$  \hspace{1cm} (9)

$$p_{\phi} = \frac{1}{2}\dot{\phi}^2 - V(\phi) = A\left[\frac{3(1+\alpha)B_o}{3(1+\alpha)(1+A) - n} \frac{1}{a^n} - \frac{C}{a^{3(1+\alpha)(1+A)}}\right]^\frac{1}{1+\alpha} - \frac{B_o a^{-n}}{\left[\frac{3(1+\alpha)B_o}{3(1+\alpha)(1+A) - n} \frac{1}{a^n} - \frac{C}{a^{3(1+\alpha)(1+A)}}\right]^\frac{\alpha}{1+\alpha}}.$$  \hspace{1cm} (10)
From the last two equations, the kinetic and the potential terms are evaluated to be

\[
\dot{\phi}^2 = \rho_\phi + p_\phi = (1 + A) \left[ \frac{3(1 + \alpha)B_\phi}{[3(1 + \alpha)(1 + A) - n]} \frac{1}{a^n} - \frac{C}{a^{3(1 + \alpha)(1 + A)}} \right]^{\frac{3}{1 + \alpha}} \\
- \frac{3(1 + \alpha)B_\phi}{[3(1 + \alpha)(1 + A) - n]} \frac{1}{a^n} - \frac{C}{a^{3(1 + \alpha)(1 + A)}} \right]^{\frac{3}{1 + \alpha}} \\
\]  

(11)

\[
2V(\phi) = \rho_\phi - p_\phi = (1 - A) \left[ \frac{3(1 + \alpha)B_\phi}{[3(1 + \alpha)(1 + A) - n]} \frac{1}{a^n} - \frac{C}{a^{3(1 + \alpha)(1 + A)}} \right]^{\frac{3}{1 + \alpha}} \\
+ \frac{3(1 + \alpha)B_\phi}{[3(1 + \alpha)(1 + A) - n]} \frac{1}{a^n} - \frac{C}{a^{3(1 + \alpha)(1 + A)}} \right]^{\frac{3}{1 + \alpha}} \right]. \\
\]  

(12)

The size of the future event horizon \( R_h \) is defined as

\[
R_h(t) = a(t) \int_t^\infty \frac{dt'}{a(t')} = a(t) \int_0^{r_1} \frac{dr}{\sqrt{1 - kr^2}}. \\
\]  

(13)

We shall use the identity

\[
\int_0^{r_1} \frac{dr}{\sqrt{1 - kr^2}} = \frac{1}{\sqrt{|k|}} \sin^{-1}(\sqrt{|k|}r_1) = \begin{cases} 
\sin^{-1}r_1 & k = +1, \\
\sin^{-1}1 & k = 0, \\
\sin^{-1}(-r_1) & k = -1.
\end{cases} \\
\]  

(14)

Also \( L \) is defined as

\[
L = ar(t). \\
\]  

(15)

It is suggested in [32] that \( r(t) = R_h(t) \) as the infrared cut-off. From Eqs. (3), (13) and (15), we can write

\[
L = a(t) \frac{\sin[\sqrt{|k|}R_h(t)/a(t)]}{\sqrt{|k|}}. \\
\]  

(16)

Here \( \sin y = \sin y, \sin^{-1} y \) for \( k = +1, 0, -1 \) respectively, where \( y = \sqrt{|k|}R_h(t)/a(t) \). Combining the usual definitions of \( \Omega_{de} \) and \( \rho_{cr} \) yields

\[
HL = \frac{c}{\sqrt{\Omega_{de}}}. \\
\]  

(17)

Differentiating Eq. (17) with respect to \( t \) and using (16), we get

\[
\dot{L} = \frac{c}{\sqrt{\Omega_{de}}} - \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|}R_h/a). \\
\]  

(18)
Here $\cos y = \cos y, \cos^{-1} y$ for $k = +1, 0, -1$ respectively, where $y = \sqrt{|k|} R_h(t)/a(t)$. The EoS parameter for dark energy is given by

$$\omega_{de} = -\frac{1}{3} - \frac{2\sqrt{\Omega_{de}}}{3c} \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|} R_h/a). \quad (19)$$

Invoking the correspondence between Eqs. (2) and (8), we obtain

$$C = a^{3(1+\alpha)(1+A)} (3c^2 M_p^2 L^{-2})^{1+\alpha} - \frac{3(1+\alpha)B_o}{[3(1+\alpha)(1+A) - n] a^n} \frac{1}{a^n}. \quad (20)$$

The constant parameter $B_o$ is determined to be

$$B_o = a^n \rho_{de}^{1+\alpha} (A - \omega_{de}),
= a^n (3c^2 M_p^2 L^{-2})^{1+\alpha} \left[ A + \frac{1}{3} + \frac{2\sqrt{\Omega_{de}}}{3c} \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|} R_h/a) \right]. \quad (21)$$

Using Eq. (21) in (20), we get

$$C = a^{3(1+\alpha)(1+A)} (3c^2 M_p^2 L^{-2})^{1+\alpha} \left[ 1 - \frac{3(1+\alpha)}{[(1+\alpha)(1+A) - n]} \left\{ A + \frac{1}{3} + \frac{2\sqrt{\Omega_{de}}}{3c} \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|} R_h/a) \right\} \right]. \quad (22)$$

The EoS parameter $\omega_{de}$ is

$$\omega_{de} = \frac{p_{de}}{\rho_{de}} = \frac{1}{\rho_{de}} \left[ A \rho_{de} - \frac{B_o a^{-n}}{\rho_{de}^\alpha} \right] = A - \frac{B_o a^{-n}}{\rho_{de}^{1+\alpha}}. \quad (23)$$

From Eq. (11), the kinetic term is re-written to be

$$\dot{\phi}^2 = 2c^2 M_p^2 L^{-2} \left[ 1 - \frac{\sqrt{\Omega_{de}}}{3c} \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|} R_h/a) \right], \quad (24)$$

$$\dot{\phi} = \sqrt{(2c^2 M_p^2 L^{-2}) \left[ 1 - \frac{\sqrt{\Omega_{de}}}{3c} \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|} R_h/a) \right]}. \quad (25)$$

From Eq. (12), the potential term becomes

$$2V(\phi) = \frac{\rho_{de}^{1+\alpha} (1 - A) + B_o a^{-n}}{\rho_{de}^\alpha},
= 3c^2 M_p^2 L^{-2} \left[ \frac{4}{3} + \frac{2\sqrt{\Omega_{de}}}{3c} \frac{1}{\sqrt{|k|}} \cosn(\sqrt{|k|} R_h/a) \right]. \quad (26)$$
Hence the potential term turns out to be the same as that for the holographic Chaplygin gas \cite{42}. Using the relation with $x = \ln a$, we have
\[
\dot{\phi} = \phi' H, 
\] (27)
and we obtain
\[
\phi' = M_p \sqrt{2 \Omega_{de} \left[1 - \frac{\sqrt{\Omega_{de}}}{3c} \frac{1}{\sqrt{|k|}} \cos(n \sqrt{|k| R_h/a})\right]}. 
\] (28)
After integration, we get
\[
\phi(a) - \phi(a_o) = M_p \int_a^{\ln a} \sqrt{2 \Omega_{de} \left[1 - \frac{\sqrt{\Omega_{de}}}{3c} \frac{1}{\sqrt{|k|}} \cos(n \sqrt{|k| R_h/a})\right]} dx. 
\] (29)

3 Holographic viscous Chaplygin gas model

In this section, we shall consider dark energy with non-zero bulk viscosity. The role of viscosity has been widely discussed and is a promising candidate to explain several cosmological puzzles, especially dark energy. Notably, the viscous dark energy can explain the high photon to baryon ratio \cite{53}, and it can lead to an inflationary scenario in the early phase of the universe \cite{54}. The coefficient of viscosity should decrease as the universe expands; moreover, its presence can explain the current accelerated expansion \cite{55, 56, 57}. It has been pointed out that viscous dark energy can drive expansion so rapidly that it may result in a catastrophic big rip singularity \cite{58}. A cosmological model with bulk viscosity also rules out the possibility of a big bang singularity, which is as yet unexplained \cite{59}. This model is also consistent with astrophysical observations at the lower redshifts, and a viscous cosmic model favors a standard cold dark matter model with cosmological constant ($\Lambda$CDM) in the later cosmic evolution \cite{60}. The model also presents the scenario of phantom crossing (or phantom divide) i.e. the transition of parameter $\omega_{de} > -1$ to $\omega_{de} < -1$ \cite{61}. We consider the viscous dark energy with the EoS \cite{62, 63}
\[
p_{\text{eff}} = p_{de} + \Pi. 
\] (30)
Here $p_{de}$ is the barotropic pressure (which depends only on the energy density of the fluid) while $\Pi = -\xi(p_{de}) u^\mu_{\text{di}}$ is the viscous pressure, $u^\mu$ is the four-velocity vector and $\xi$ is the coefficient of bulk viscosity \cite{64}. In the FRW model, the viscous pressure takes the form $\Pi = -3H\xi \rho_{de}$ \cite{65}. In the cosmological context, the barotropic EoS can be chosen to be the Chaplygin gas \cite{66}. For the purpose of generality, we specify the $p_{de}$ by the generalized Chaplygin gas, so that (30) takes the form
\[
p_{\text{eff}} = \frac{\chi}{\rho_{de}^3} - 3H\xi(p_{de}). 
\] (31)
In general, the viscosity coefficient can be of power-law form, i.e. $\xi \sim \rho^n$ for $n \geq 0$ and hence it yields a power-law expansion of the scale factor \[67\]. We also assume the parametric form $\xi(\rho_{de}) = \nu \rho_{de}^{1/2}$, where $\nu$ is a constant parameter. Hence Eq. (31) becomes

$$p_{\text{eff}} = \frac{X}{\rho_{de}^\alpha} - 3\nu H \rho_{de}^{1/2}.$$  \(32\)

The EoS parameter gives

$$\omega_{de} = \frac{p_{\text{eff}}}{\rho_{de}} = \frac{X}{\rho_{de}^{1+\alpha}} - 3\nu H \rho_{de}^{-1/2},$$  \(33\)

or we can write

$$\chi = \rho_{de}^{1+\alpha}[\omega_{de} + 3\nu H \rho_{de}^{-1/2}].$$  \(34\)

The corresponding energy conservation equation is

$$\rho_{de} = \left[\frac{Na^{3(1-\nu\gamma)(1+\alpha)} - \chi}{1 - \nu\gamma}\right]^{\frac{1}{1+\alpha}}.$$  \(35\)

Here $D$ is a constant of integration. The parameters used are

$$\gamma = M_p^{-1}\sqrt{1 - r_m}, \quad r_m \equiv \rho_m/\rho_{de}.\quad (36)$$

Using Eqs. (2) and (19) in (35), we obtain

$$D = a^{3(1+\alpha)(1-\nu\gamma)}[\rho_{de}^{1+\alpha}(1 - \nu\gamma) + \chi].$$  \(37\)

The kinetic term becomes

$$\dot{\phi}^2 = \rho_{\phi} + p_{\phi},$$

$$= \frac{1}{\rho_{de}^{1+\alpha}}[\rho_{de}^{1+\alpha} + \chi - 3\nu H \rho_{de}^{\alpha+1/2}],$$

$$= (3c^2M_p^2L^{-2})\left\{\frac{2}{3} - \frac{2\sqrt{\Omega_{de}}}{3}\cos(\sqrt{|k|}R_h/a) + \frac{3\nu L}{c\sqrt{\rho_{cr}}}\right\} - \frac{3\nu c\sqrt{\rho_{cr}}}{L}.$$  \(38\)

Also, the potential term becomes

$$2V(\phi) = (\rho_{de} - p_{\text{eff}}),$$

$$= \frac{1}{\rho_{de}^{1+\alpha}}[\rho_{de}^{1+\alpha} - \chi + 3\nu H \rho_{de}^{\alpha+1/2}],$$

$$= (3c^2M_p^2L^{-2})\left\{\frac{4}{3} + \frac{2\sqrt{\Omega_{de}}}{3}\cos(\sqrt{|k|}R_h/a) - \frac{3\nu L}{c\sqrt{\rho_{cr}}}\right\} + \frac{3\nu c\sqrt{\rho_{cr}}}{L}.$$  \(39\)
From (38), we have
\[ \dot{\phi} = H M_p \left[ 3 \Omega_{de} \left\{ \frac{2}{3} - \frac{2}{3} \frac{\sqrt{\Omega_{de}}}{c} \frac{1}{\sqrt{|k|}} \cos(\sqrt{|k|R_h/a}) + \frac{3\nu L}{c\sqrt{\rho_{cr}}} \right\} - \frac{3\nu c \sqrt{\rho_{cr}}}{L} \right]^{1/2}. \] (40)

Making use of (40) in (27) gives
\[ \phi' = M_p \left[ 3 \Omega_{de} \left\{ \frac{2}{3} - \frac{2}{3} \frac{\sqrt{\Omega_{de}}}{c} \frac{1}{\sqrt{|k|}} \cos(\sqrt{|k|R_h/a}) + \frac{3\nu L}{c\sqrt{\rho_{cr}}} \right\} - \frac{3\nu c \sqrt{\rho_{cr}}}{L} \right]^{1/2}. \] (41)

On integration, we obtain
\[ \phi(a) - \phi(a_o) = \int_{a_o}^{a} M_p \left[ 3 \Omega_{de} \left\{ \frac{2}{3} - \frac{2}{3} \frac{\sqrt{\Omega_{de}}}{c} \frac{1}{\sqrt{|k|}} \cos(\sqrt{|k|R_h/a}) + \frac{3\nu L}{c\sqrt{\rho_{cr}}} \right\} - \frac{3\nu c \sqrt{\rho_{cr}}}{L} \right]^{1/2} \] (42)

\section{Conclusion and discussion}

The holographic dark energy presents the dynamical nature of the vacuum energy. This dynamical nature is manifested through the holographic parameter \( c \) which by varying gives an evolving dark energy. For instance, if \( c \geq 1 \) gives the quintessence where its state equation parameter lies in the range \(-1 \leq \omega \leq -1/3\), while \( c = 1 \) yields the cosmological constant phase and \( c < -1 \) gives the phantom energy dominated universe. Thus the whole range of \( c \) provides a quintom (quintessence to phantom) like model.

As discussed before, several authors have established the connection between the holographic dark energy and various theories of gravity. These correspondences are motivated to demystify the origin of dark energy and the evolution of the universe. In this context, we have presented the link between the holographic dark energy and the modified variable Chaplygin gas and the viscous generalized Chaplygin gas. The Chaplygin gas has been extensively used in recent literature on cosmology due to the fact that its predictions are consistent with the observational results. Moreover, it gives a unified picture of dark energy and dark matter, which helps in building and analyzing new cosmological models. For specific choices of parameters, our results in both models reduced to those discussed in \cite{42} and \cite{43}, thus our model is an extension of these previous studies. Finally, the dynamics of dark energy in our model is described by scalar fields with scalar potentials.

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