Evolution, pulsation and period change in the Cepheid SZ Tau

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Abstract — The study is devoted to radial pulsations in Population I Cepheids with masses from 5.4$M_\odot$ to 6$M_\odot$. Solution of the equations of radiation hydrodynamics and turbulent convection for nonlinear stellar pulsations was obtained with initial conditions as the core helium burning models of computed evolutionary sequences. For each value of the initial mass we considered stellar evolution from the zero age main sequence to central helium exhaustion with three values of the convective overshoot parameter: $\alpha_{ov} = 0.1$, 0.15 and 0.2. Models for the Cepheid SZ Tau with pulsation period $\Pi = 3.149$ day can be constructed only for $\alpha_{ov} = 0.1$ and $\alpha_{ov} = 0.15$. The star is at the evolutionary stage of the second crossing of the instability strip and pulsates in the first overtone just near the boundary between the fundamental mode and the first overtone. The mass, radius and age of the star are in the ranges $5.46 \leq M/M_\odot \leq 5.75$, $41.5 \leq R/R_\odot \leq 42.3$ and $6.9 \times 10^7$ yr $\leq t_{ev} \leq 8.0 \times 10^7$ yr, respectively. Predicted period change rates are of $\dot{\Pi} \approx -0.4$ s/yr.

Keywords: stars: variable and peculiar, pulsations of Galactic Cepheids.
INTRODUCTION

The pulsating variable star SZ Tau belongs to a small group of Cepheids that are members of open clusters. The first report that the Cepheid SZ Tau observed in the extended corona of NGC 1647 belongs to this cluster was made by Efremov (1964) on the basis of similar values of the distance and the radial velocity of the Cepheid and the stars of the cluster. Later this conclusion was confirmed by other observational data (Gieren 1985; Turner 1992; Anderson et al. 2013). The study of the Cepheid which belongs to the stellar cluster and determination of its fundamental parameters (the mass, radius and luminosity) by methods of stellar pulsation theory is of great importance since provides us with an independent method of evaluation of the distance and the age of the stellar cluster.

In the General Catalogue of Variable Stars (Samus et al. 2012) the star SZ Tau with a period $\Pi = 3.14873$ day is referred to Cepheids of type DCEPS with nearly symmetrical light curves of small amplitude. Turner (1992) estimated that the average absolute magnitude of SZ Tau corresponds to radial pulsations in the first overtone. Later this assumption was confirmed by Fourier analysis of the radial velocity curve (Antonello and Aikawa 1995). Another confirmation of the first overtone pulsation was provided in the work by Bonno et al. (2001) where the mean stellar radius $R = 45.6 R_\odot$ obtained from the radial velocity observations (Bersier et al. 1994) was compared with empirical period–radius relation of Galactic Cepheids. It should be noted that the mean radius of SZ Tau was evaluated by the Baade–Wesselink method in a number of studies (Gieren 1985; Moffett and Barnes 1987; Laney and Stobie 1995; Ripepi et al. 1997; Rastorguev and Dambis 2011) but the large spread in values of the radius ($34 R_\odot \leq R \leq 57 R_\odot$) is a serious handicap for determining the stellar mass from the period–mean density relation.

In contrast to most of the Cepheids the light curve of SZ Tau reveals the absence of strictly repetitive light variability and this feature substantially complicates the detection of period change due to stellar evolution. In particular, Szabados (1977) and Evans et al. (2015) found significant erratic variations in the $O - C$ diagram which do not allow for an unambiguous detection of the secular period change. However Berdnikov and Pastukhova (1995) showed that the $O - C$ diagram can be fitted by the quadratic function and therefore SZ Tau is on the evolutionary stage of the second crossing of the Cepheid instability strip and its period decreases with a rate of $\dot{\Pi} = -0.49$ s/yr.

The goal of the present study is to theoretically estimate the fundamental parameters of SZ Tau and to validate the conclusion of Berdnikov and Pastukhova (1995) on the evolutionary status of the star. To this end we carry out the consistent calculations of stellar evolution and nonlinear stellar pulsations and determine the conditions for pulsations with period of $\Pi = 3.149$ day, whereas the theoretical estimates of the period change rate are compared with
\[ \Pi = -0.49 \, \text{s/yr}. \] In this work we do not use the period–mean density relation and therefore evaluation of the stellar mass does not suffer from uncertainties in the stellar radius estimates obtained by the Baade–Wesselink method. Earlier the author employed such an approach for determination of the fundamental parameters and the age of the Cepheid \( \alpha \) UMi (Fadeyev 2015).

Basic equations and methods of their solution are described in previous papers of the author (Fadeyev 2013a, 2014, 2015). Results of computations presented below are obtained for the initial composition \( X = 0.7, Z = 0.02 \), where \( X \) and \( Z \) are the mass fractional abundances of hydrogen and of elements heavier than helium, respectively. Convection in evolving stars is treated according to the mixing length theory (Böhm–Vitense 1958) with a mixing length to pressure scale height ratio \( \alpha_\Lambda = 1.6 \). Enlargement of the convective core due to convective overshooting is taken into account by the parameter \( \alpha_{ov} = \Delta r/H_P \), where \( \Delta r \) is the radial extension of the outer boundary of the convective core and \( H_P \) is the pressure scale height.

The evolutionary computations were done for \( \alpha_{ov} = 0.1, 0.15, 0.2 \) and the parameter \( \alpha_{ov} \) was assumed to be the same from the main sequence to core helium exhaustion.

**RESULTS OF COMPUTATIONS**

To determine the ranges of the initial stellar mass \( M_{ZAMS} \) and the parameter \( \alpha_{ov} \) for which the radial pulsations with a period of \( \Pi = 3.149 \) day become possible during the second crossing of the instability strip we computed the grid of evolutionary tracks for stars with masses \( 5.4 M_\odot \leq M_{ZAMS} \leq 6 M_\odot \). Together with computation of stellar evolution we solved the linear adiabatic wave equation (see, e.g., Cox 1983) and evaluated eigenfrequencies of the fundamental mode and the first overtone. If the evolutionary model with effective temperature \( 5400 \, \text{K} \leq T_{\text{eff}} \leq 6700 \, \text{K} \) has the adiabatic period of one of these oscillation modes close to the value \( \Pi = 3.149 \) day then from 10 to 15 models of the evolutionary sequence were selected and were used as initial conditions for solution of the equations of hydrodynamics.

From results of hydrodynamic computations we primarily determined the edges of the pulsation instability strip, that is the age of the star \( t_{ev} \) corresponding to the zero growth rate of the kinetic energy. Subsequently for each hydrodynamic model we calculated the periods of the fundamental mode and the first overtone using the Fourier transform of the kinetic energy of pulsation motions. In the most of considered evolutionary sequences the mode switch from the fundamental mode to the first overtone occurs while the star crosses the instability strip. In such a case it was assumed that the mode switch is abrupt and the change of the pulsation period \( \Pi \) within the instability strip is described by two continuous functions. Determination of the mode switch boundary is discussed in our previous papers (Fadeyev, 2013b; 2015). Within
the evolutionary time interval with continuous change of the period the sequence of the values of \( \Pi \) obtained from hydrodynamic computations is fitted by the algebraic polynomial of the order \( 2 \leq n \leq 4 \). The choice of the polynomial order \( n \) obeys the condition that the approximation rms error cannot exceed 0.1%.

For Cepheid hydrodynamic models computed in the present study the most short period of the fundamental mode is \( \Pi \approx 4.3 \) day therefore the following discussion of SZ Tau model will be confined to consideration of the first overtone pulsators. The existence of the point of the evolutionary track with pulsation period \( \Pi = 3.149 \) day depends on both the initial stellar mass \( M_{\text{ZAMS}} \) and the overshoot parameter \( \alpha_{\text{ov}} \). Dependence on the initial mass is illustrated in Fig. 1 where the evolutionary tracks in the Hertzsprung–Russel (HR) diagram are shown for the models \( M_{\text{ZAMS}} = 5.5M_\odot \) and \( M_{\text{ZAMS}} = 5.8M_\odot \) computed with the overshoot parameter \( \alpha_{\text{ov}} = 0.15 \). For the sake of graphical clarity, we present in the figure only the blue loops corresponding to the core helium burning. Evolution along the shown tracks proceeds clockwise.

As soon as a star with an initial mass \( M_{\text{ZAMS}} = 5.5M_\odot \) crosses the red edge of the instability strip (\( T_{\text{eff}} = 5550 \) K) it becomes a fundamental mode pulsator. The pulsation switches to the first overtone at the effective temperature \( T_{\text{eff}} = 5840 \) K and the pulsation ceases at the blue edge of the instability strip (\( T_{\text{eff}} = 6300 \) K). As can be seen in Fig. 1 the model \( M_{\text{ZAMS}} = 5.5M_\odot \) with pulsation period \( \Pi = 3.149 \) day locates just near the boundary between the fundamental mode and the first overtone.

As the initial stellar mass increases the boundary of the mode switch from the fundamental mode to the first overtone moves to higher values of \( T_{\text{eff}} \) and for stars \( M_{\text{ZAMS}} \geq 5.8M_\odot \) becomes beyond the blue edge of the instability strip. Therefore, as seen in Fig. 1 the star with initial mass \( M_{\text{ZAMS}} = 5.8M_\odot \) remains the fundamental mode pulsator within the whole instability strip (\( 5360 \) K \( \leq T_{\text{eff}} \leq 6250 \) K). The pulsation periods are confined to the interval \( 4.45 \text{ day} \leq \Pi \leq 6.90 \text{ day} \).

Conditions for the occurrence of pulsations with period \( \Pi = 3.149 \) day on the evolutionary tracks computed with different values of the overshoot parameter \( \alpha_{\text{ov}} \) are illustrated in Fig. 2 for \( M_{\text{ZAMS}} = 5.6M_\odot \). As can be seen, the luminosity of the Cepheid increases with increasing \( \alpha_{\text{ov}} \) (mainly due to the larger mass of the convective core during the main sequence stage), whereas the maximum effective temperature of the blue loop decreases. As a result, among three evolutionary tracks shown in Fig. 2 only one (\( \alpha_{\text{ov}} = 0.15 \)) can be used for construction of the SZ Tau model, the star locating just near the boundary between the fundamental mode and the first overtone. The evolutionary track computed with \( \alpha_{\text{ov}} = 0.1 \) is not appropriate for the model of SZ Tau because of too short period (\( \Pi = 2.81 \) day) of the first overtone at the mode switch boundary. On the other hand, for the evolutionary track computed with \( \alpha_{\text{ov}} = 0.2 \)
the blue edge of the instability strip is beyond the turning point of the track so that during the stage of increasing effective temperature the Cepheid remains the fundamental mode pulsator.

It should be noted that for all evolutionary tracks \((5.4 M_\odot < M_{\text{ZAMS}} < 6 M_\odot)\) computed with \(\alpha_{ov} = 0.2\) the radial pulsations during the second crossing of the instability strip were found to be due to the instability in the fundamental mode. Therefore further discussion will be confined to results of hydrodynamical computations done with initial conditions obtained with \(\alpha_{ov} = 0.1\) and \(\alpha_{ov} = 0.15\).

Conditions of existence of the pulsation with period \(\Pi = 3.149\) day can be represented in more detail by the dependence of the period of the first overtone on the evolutionary time. The plots for several evolutionary sequences computed with \(\alpha_{ov} = 0.1\) are shown in Fig. 3. Each plot ends at the blue edge of the instability strip. For the sake of graphical convenience the evolutionary time \(t_{ev}\) is set to zero at the mode switch boundary.

For \(M_{\text{ZAMS}} \geq 5.9 M_\odot\) the star pulsates in the fundamental mode within the whole instability strip. On the other hand, for \(M_{\text{ZAMS}} \leq 5.6 M_\odot\) the period of the first overtone is smaller than \(\Pi = 3.149\) day. Therefore, the model of SZ Tau with the overshoot parameter \(\alpha_{ov} = 0.1\) can be constructed for the initial stellar mass \(M_{\text{ZAMS}}\) ranging from \(5.6 M_\odot\) to \(5.9 M_\odot\). The principal cause of the narrow mass interval is the close position of the Cepheid to the mode switch boundary.

Temporal dependences of the first overtone period in the evolutionary sequences computed with overshoot parameter \(\alpha_{ov} = 0.15\) show a similar behavior. The only difference in comparison with models computed with \(\alpha_{ov} = 0.1\) is that pulsations with period \(\Pi = 3.149\) day arise in stars with lower masses: \(M_{\text{ZAMS}} = 5.5 M_\odot\) and \(M_{\text{ZAMS}} = 5.6 M_\odot\). The narrow ranges of initial stellar masses allowed us to approximately evaluate the fundamental parameters of SZ Tau which are given in the table.

To compare our theoretical results with observations Fig. 4 shows the plots of \(\dot{\Pi}\) as a function of period \(\Pi\) for Cepheid models pulsating in the first overtone. Each curve in Fig. 4 represents the evolutionary track in the period–period change rate diagram and evolution proceeds as the pulsation period \(\Pi\) decreases. The position of SZ Tau on the diagram and its rms error are shown on the basis of data given by Berdnikov and Pastukhova (1995).

**CONCLUSIONS**

The Cepheid SZ Tau is located just near the mode switch from the fundamental mode to the first overtone. The proximity of the pulsation period \(\Pi = 3.149\) day to the boundary value of the period of first overtone is by an order of magnitude \(\delta \Pi / \Pi \sim 10^{-2}\). This is the principal cause that stellar evolution and nonlinear stellar pulsation calculations lead to the narrow
ranges of the fundamental parameters of the Cepheid. It should also be noted that models of
SZ Tau obtained for the overshoot parameters $\alpha_{ov} = 0.1$ and $\alpha_{ov} = 0.15$ give evidence in favor
of the small and intermediate convective overshooting during the main sequence and the helium
core burning stages.

Predicted period change rates obtained in the present study are in a good agreement with
the observational estimate ($\dot{\Omega} = -4.9\ s/yr$) by Berdnikov and Pastukhova (1995) and therefore
confirm a suggestion that SZ Tau is at the evolutionary stage of the second crossing of the
instability strip.

Theoretical estimates of the radius of SZ Tau ($41.5R_\odot \leq R \leq 42.3R_\odot$) given in the table
show better agreement with measurements by Ripepi et al. (1997). Authors of this work
employed the improved version of the Baade–Wesselink method (the CORS method) based on
the more correct relations between the color index and the surface brightness (Caccin et al.
1981). For two variants of the CORS method Ripepi et al. (1997) estimated the mean radius
as $R = 41.8R_\odot$ and $R = 44.8R_\odot$.

Hydrodynamic computations of nonlinear pulsations in SZ Tau located just at the mode
switch are accompanied with great numerical difficulties. This is mainly due to the small decay
rate of the fundamental mode which leads to slow relaxation of nonregular oscillations after
attainment of the first overtone limit cycle. Absence of strict regularity in the observed light
and radial velocity curves of SZ Tau might be due to slow decay of the fundamental mode.
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Models of Cepheids with period 3.149 day

| $\alpha_{\text{ov}}$ | $M_{\text{ZAMS}}/M_\odot$ | $M/M_\odot$ | $R/R_\odot$ | $L/L_\odot$ | $T_{\text{eff}}, \text{K}$ | $\dot{\Pi}, \text{s/yr}$ | $t_{\text{ev}}, 10^6 \text{ yr}$ |
|-----------------|-----------------|-------------|-------------|-------------|-----------------|-----------------|-----------------|
| 0.1             | 5.7             | 5.66        | 42.0        | 1878        | 5870            | -0.42           | 72.1            |
|                 | 5.8             | 5.75        | 42.3        | 2008        | 5950            | -0.43           | 69.2            |
| 0.15            | 5.5             | 5.46        | 41.5        | 1819        | 5860            | -0.38           | 80.2            |
|                 | 5.6             | 5.56        | 41.8        | 1968        | 5960            | -0.43           | 77.0            |
Figure captions

Fig. 1. Evolutionary tracks of core helium burning stars with initial masses $5.5M_\odot$ and $5.8M_\odot$ computed with the overshoot parameter $\alpha_{ov} = 0.15$. Parts of the track corresponding to the pulsational instability during the second crossing of the instability strip are shown by the dashed and dotted lines for the fundamental mode and the first overtone, respectively. The filled circle indicates the position of the Cepheid pulsating in the first overtone with period $\Pi = 3.149$ day.

Fig. 2. Same as Fig. 1 but for the initial stellar mass $M_{ZAMS} = 5.6M_\odot$ with $\alpha_{ov} = 0.1$, 0.15 and 0.2.

Fig. 3. The first overtone pulsation period $\Pi$ as a function of the evolutionary time $t_{ev}$ during the second crossing of the instability strip. Solid lines show evolutionary sequences computed for $\alpha_{ov} = 0.1$. The evolutionary time $t_{ev} = 0$ corresponds to the mode switch from the fundamental mode to the first overtone.

Fig. 4. The period change rate $\dot{\Pi}$ as a function of period $\Pi$ for Cepheids pulsating in the first overtone. The solid and dashed lines show the evolutionary sequences computed for the overshoot parameter $\alpha_{ov} = 0.1$ and $\alpha_{ov} = 0.15$, respectively. The values of $M_{ZAMS}$ are given at the curves. The position of the Cepheid SZ Tau corresponds to the observational estimates by Berdnikov and Pastukhova (1995).
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