On the using M-BCJR demodulation for partial response signalling followed by decoding of LDPC codes

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Abstract. This paper studies various sub-optimal versions of BCJR demodulator to examine the possibility of computational reduction. BCJR demodulator is used to deal with intentional intersymbol interference in bandwidth efficient systems. The complexity of BCJR demodulator can be reduced by using the approximation in the logarithm domain and by considering less number of trellis states instead of full states. To estimate the performance of different types of BCJR demodulator, we proposed the partial response signalling system with finite shaping pulse providing 10% gain of bandwidth efficiency. The simulation results of this study show that in the case without error correction coding the complexity of BCJR demodulator can be significantly decreased without energy losses. When low-density parity-check coding is used, it is possible to use these sub-optimal versions of BCJR demodulator, but the system may suffer additional energy losses.

1. Introduction

In the design of modern telecommunication systems, the key factor is to improve bandwidth efficiency, which is calculated as the data rate by bandwidth. Traditional systems use the shaping pulses following Nyquist intersymbol interference criterion [1]. Recent developments in electronics components allow us to exploit the idea of Faster-Than-Nyquist (FTN) signaling in order to gain bandwidth efficiency [2, 3]. Instead of using signals with orthogonal pulses, FTN system adds intentional intersymbol interference (ISI), which leads to either higher data rate or narrower bandwidth than traditional systems, but more complex receiver is required.

The baseband signal with linear modulation, which is generated from $N$ data symbols $\{C_n\}$, can be formulated as follows:

$$s(t) = \sum_{n=1}^{N} C_n a(t-nT),$$  \hspace{1cm} (1)$$

where $a(t)$ is impulse response and $T$ is the symbol interval. If the autocorrelation function of $a(t)$ equal to zero at the moments of $kT$ ($k = \pm 1, \pm 2, \pm 3, \ldots$), then it is the case of full response signaling (FRS). Otherwise, it is partial response signaling (PRS). For symmetrical signal constellations, the spectrum of signal $s(t)$ depends on the shape of the pulse $a(t)$. Therefore, we can optimize the pulse $a(t)$ by different criteria and under different constraints to improve bandwidth or/and energy
efficiencies [4, 5, 10-13]. As an example, the study in [5] proposes multicomponent signals with optimal finite pulses based on the truncated Fourier decomposition.

Because of ISI caused by the pulse \( a(t) \), BCJR or Viterbi demodulators are supposed to be used instead of symbol-by-symbol detection on the receiver side [6, 7]. These demodulators can limit the negative effect of ISI on the bit error rate (BER), but they require significant computational resources. The complexity of BCJR algorithm, which increases exponentially with the length of impulse response, can be reduced in several ways. The study in [8] considers the performance of a sub-optimal version of BCJR algorithm, which is simplified by calculating probability metrics only for \( M \) states per trellis stage and by applying the following approximation formula to avoid logarithm and exponent operations:

\[
\log \left( \sum_i \exp(x_i) \right) \approx \max_i (x_i)
\]  

This type of BCJR demodulator is denoted as MAX-LOG-M-BCJR. However, it is unclear how MAX-LOG-M-BCJR demodulator performs under poor signal-to-noise condition where state-of-the-art error correction coders work. For example, low-density parity-check (LDPC) coder in DVB-S2 standard works in the condition where \( E_b/N_0 \) is from 0 dB to 4 dB for QPSK modulation. By \( E_b/N_0 \) we mean the ratio of the energy per information bit to noise one-sided power spectral density.

Moreover, for low signal/noise ratios different criteria are applied in the optimization problem of pulse synthesis. Consequently, obtained optimal pulses provide good performance in poor condition using the optimal receiver. The question is whether sub-optimal versions of the receiver can be used for these pulses.

This paper aims to analyze the possibility of using the sub-optimal versions of the BCJR algorithm, which is able to demodulate the signal with additional ISI, in the presence of LDPC coding. For bandwidth efficiency gain, we use PRS signals with an optimal pulse in [5]. We consider the AWGN channel for the simulation model. The behaviors of BCJR demodulator in other types of channel are studied in [14, 15]. In the next sections, we present the simulation model, obtained results and conclusions.

2. Simulation model

In figure 1, we suppose a simplified simulation model to estimate noise immunity of the system with PRS pulses under AWGN channel. As seen in the model, the data bits are encoded by LDPC encoder and then modulated by QPSK constellation. Then, the real and imaginary parts of modulated symbols are filtered by the shaping pulse \( a(t) \) and summed to form the complex baseband signal \( y(t) = y_I(t) + jy_Q(t) \). After AWGN channel, real and image parts of the received signal are independently processed by two BCJR demodulator blocks. By this way, each BCJR demodulator works with BPSK and provides soft decisions about transmitted symbols. In general, for complex constellations, a posterior probabilities for complex symbols are computed by multiplying probabilities \( Pr(C_I^1) \) and \( Pr(C_Q^1) \) which are calculated from BCJR demodulators. In our case of BPSK, each BCJR demodulator provides probabilities (or logarithm of probabilities) for received bits in each branch. After that, these probabilities are converted into log-likelihood ratios (LLRs) of received bits and sent to LDPC decoder.
Parameters for LDPC coding are chosen from the DVB-S2 standard. We consider two code rates 3/5 and 4/5 with the same length of coded frame (64800 bits). Other smaller values of frame length (16200 bits for instance) may be used to decrease the delay of decoding, but it requires more energy to archive the same BER. The LDPC decoder uses the message-passing algorithm, and the maximum number of iterations is specified as 50. Output bits $d_{\text{rec}}$ of the decoder are compared with transmitted bits $d$ to obtain BER curves for different values of $E_b/N_0$.

For the shaping pulse $a(t)$ we use finite pulses obtained by solving the optimization problem of minimization of autocorrelation coefficients for multicomponent signals in [5]. In this experiment, we use the optimal pulse, which has the length of $8T$ and provides 10% of spectral efficiency gain in comparison with FRS pulses. Working with this pulse, the traditional full-state BCJR demodulator should consider $2^7 = 128$ states at each trellis stage.

In blocks “BCJR demodulator”, we use three types of BCJR demodulators. The first type is traditional full-state BCJR demodulator (True Full BCJR), which operates with all states of trellis (128 states). This type requires the most computational resources. Secondly, the MAX-LOG Full BCJR is considered by applying the formula (2). This demodulator also works with all states but in the logarithmic domain and is simpler to be implemented. Finally, the last one is MAX-LOG-M-BCJR demodulator, which calculates metrics through the trellis retaining the best $M$ paths at each stage and uses the formula (2). By this way, the smaller $M$ is used, the less computational resources are needed. In our case, MAX-LOG-M-BCJR with $M = 128$ is identical to the MAX-LOG Full BCJR. Notice that since the last two types of mentioned demodulators are sub-optimal versions of MAP algorithm, some level of BER performance losses may be expected.

3. Simulation results

The simulation experiment is conducted in MATLAB by Monte-Carlo method. We present obtained results by the figures of BER performance against $E_b/N_0$. For sufficient BER measurements, coded data frames are transmitted until 1000 erroneous frames are detected.

At first, the model without LDPC coding is used to examine behaviors of True Full BCJR, MAX-LOG Full BCJR and MAX-LOG-M-BCJR with different values of $M$. The outputs of demodulators are transformed into hard decisions. Figure 2 shows the BER curves of these demodulators. The results for the FRS system are presented for comparison with the proposed PRS system. On the left, it is clear that at $BER = 10^{-4}$ the curves of MAX-LOG-M-BCJR with consequently $M = 4$ and 8 overlap with the curve of FRS system. It means that the complexity of BCJR algorithm can be reduced significantly (by $128 \div 4 = 32$ times) without performance degradation. However, this situation changes in the area of lower $E_b/N_0$ where LDPC coding is supposed to be used. For each code rate, LDPC decoder requires a certain value of BER from demodulator to reach its maximal correction.
ability. This value of BER varies from \( \text{BER} = 0.2 \) down to \( \text{BER} = 0.01 \) for the code rates from 1/4 to 9/10 respectively. On the right of the figure, it is showed that these BCJR demodulators need more energy than FRS system to achieve the same BER in low \( E_b/N_0 \) area. To be more specific, we take \( \text{BER} = 0.05 \) as an example. Despite the approximation in (2), the True Full BCJR and MAX-LOG Full BCJR demodulators suffer the same energy loss of 0.07 dB, which is caused by using the PRS pulse. Finally, the MAX-LOG-M-BCJR demodulator shows significant energy losses when the number of considered states \( M \) is smaller than 8. Therefore, it seems that MAX-LOG-M-BCJR with \( M = 8 \) (or higher) can be a good candidate for the situation where LDPC coding is applied. We can see how the losses change in presence of LDPC coding in the next paragraphs.

Figure 2. BER performance without coding for True Full BCJR, MAX-LOG Full BCJR, MAX-LOG-M-BCJR and FRS matched filter demodulators.

Figure 3 illustrates BER performance in presence of LDPC coding for two code rates 3/5 (on the left) and 4/5 (on the right). At first, we consider the performance of True Full BCJR demodulator. It is interesting that the energy losses, which are caused by the transition from FRS to PRS, remains the same (about 0.07 dB) as in the case without LDPC coding at \( \text{BER} = 10^{-4} \) for both code rates. In other words, the LDPC coder does not cause any additional degradation. Notice that these losses of 0.07 dB are the trade-offs for 10% gain in bandwidth efficiency.

Now we consider the sub-optimal versions of BCJR algorithm. The difference of 0.02 dB in BER performance between MAX-LOG Full BCJR and True Full BCJR can be neglected. It means that the approximation formula (2) can be applied in the case with LDPC coding. However, the situation gets worse for MAX-LOG-M-BCJR. Simulation results show that it is required at least \( M = 64 \) for code rate 3/5 or \( M = 32 \) for code rate 4/5 to achieve the performance close to the True Full BCJR case at \( \text{BER} = 10^{-4} \). Moreover, the smaller number of states \( M \) is, the more seriously the losses become. Take \( M = 8 \) as an example, while there is no BER performance degradation in the case without coding, the losses become 1.73 and 0.98 dB (for code rates 3/5 and 4/5 respectively) in comparison with FRS in the case of LDPC coding. It can be explained by examining the input LLRs of LDPC decoder. For various versions of BCJR algorithm, these LLRs are different from each other, although they provide the same hard-decision results in the case without coding. This causes the additional losses since the LDPC decoder is sensitive to the input LLRs.
Figure 3. BER performance in presence of LDPC coding for two code rates 3/5 (on the left) and 4/5 (on the right) for True Full BCJR, MAX-LOG Full BCJR, MAX-LOG-M-BCJR and FRS matched filter demodulators.

By testing different shaping pulses, we confirm that the performance of all considered types of BCJR demodulators depends persistently on the energy distribution of the shaping pulse, which is called partial energy of pulse in [9]. In fact, the higher partial energy distributed in the first samples, the smaller number of $M$ states M-BCJR demodulator needs to be considered in order to achieve near ML performance.

4. Conclusions
In this paper, we consider different sub-optimal versions of BCJR algorithm to examine the possibility of computational minimization. It is showed that the BCJR demodulator with the approximation in the logarithm domain (2) can be used to simplify the reception algorithm in both cases with and without LDPC coding. Moreover, the complexity of BCJR algorithm can be significantly reduced by considering less number of states for each trellis stage in the case without error correction coding. To be more specific, the sub-optimal BCJR demodulator needs to consider only 8 states instead of 128 states, and this computational reduction does not cause any additional losses for the system with the proposed MCS optimal pulse providing 10% bandwidth efficiency gain. However, when LDPC coding is used, the sub-optimal BCJR demodulator suffers performance degradation even with a high number of considered states since the LDPC decoder is sensitive to soft-decision inputs. Therefore, this fact needs to be taken into account when these sub-optimal versions are implemented.

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