Collective field approach

to gauged principal chiral field at large $N$

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Abstract

The lattice model of principal chiral field interacting with the gauge fields, which have no kinetic term, is considered. This model can be regarded as a strong coupling limit of lattice gauge theory at finite temperature. The complete set of equations for collective field variables is derived in the large $N$ limit and the phase structure of the model is studied.
1 Introduction

The Kazakov–Migdal model \([1]\) was originally proposed as a model of induced QCD. Although this idea suffers from several problems \([2]\), there is another motivation for study of this model and it’s generalizations – the usual for matrix models interpretation in terms of random surfaces, 2d gravity and noncritical string theory. The Kazakov–Migdal model significantly simplifies in the large \(N\) limit and can be treated by saddle point methods \([3, 4]\), or by means of loop equations \([5]\). In this paper the modification of the Kazakov–Migdal model with the Hermitean matrix field replaced by the principal chiral one is considered:

\[
\mathcal{L} = N \text{tr} \left[ \frac{1}{2\gamma} (\nabla_\mu g)^\dagger (\nabla^\mu g) - V(g) - V(g^\dagger) \right], \quad \nabla_\mu g = \partial_\mu g - [A_\mu, g]. \tag{1.1}
\]

There \(g(x)\) is \(U(N)\) (or \(SU(N)\)) matrix, while \(A_\mu(x)\) is \(u(N)\) (or \(su(N)\)) gauge field.

This model, defined on a \(D\)-dimensional lattice, can be regarded as a strong coupling limit of the \((D + 1)\)-dimensional lattice gauge theory at finite temperature \([6, 7, 8]\). It can be shown in a most simple way within the Hamiltonian framework \([6]\). In the strong coupling limit only the electric part of the Kogut–Susskind Hamiltonian \([9]\)

\[
\mathcal{H} = -\frac{g^2}{2} \sum_{x,\mu} \Delta U_\mu(x) - \frac{2}{g^2} \sum \theta \left[ \text{tr} \left[ U(\Box) + U^\dagger(\Box) \right] \right] \tag{1.2}
\]

contributes to the partition function

\[
Z = \text{Tr} e^{-\frac{\mathcal{H}}{T}}. \tag{1.3}
\]

There \(\Delta U_\mu(x)\) is invariant Laplacian acting on the group variable \(U_\mu(x)\), attached to the link \((x, x + \mu)\) of a \(D\)-dimensional lattice. If it were not for the gauge invariance, the partition function \([1.3]\) would factorize in the strong coupling limit on the product over the links of the heat kernels on the group manifold, independently integrated over the gauge fields. However, one should first average over all gauges:

\[
Z_{\text{s.c.}} = \int \prod_x \prod_{x,\mu} D U_\mu(x) D g(x) \prod_{x,\mu} K \left( U_\mu(x), g^\dagger(x) U_\mu(x) g(x + \mu) \left| \frac{g^2 N}{T} \right. \right), \tag{1.4}
\]

where \(K(g, h|\tau)\) is a solution to the heat equation

\[
\frac{\partial}{\partial \tau} K(g, h|\tau) = \frac{1}{2N} \Delta g K(g, h|\tau) \tag{1.5}
\]

with initial condition

\[
K(g, h|0) = \delta(g, h). \tag{1.6}
\]

An explicit expression for \(K(g, h|\tau)\) in terms of a character expansion is

\[
K(g, h|\tau) = \sum_{R} \dim R \ e^{-\frac{2\pi}{\tau} \chi_R(gh^\dagger)}, \tag{1.7}
\]
where the sum is taken over all irreducible representations of the gauge group, with dim\(R\), \(c_R\) and \(\chi_R\) being the dimension, quadratic Casimir and the character of the representation \(R\).

The partition function (1.4) is nothing that the lattice regularization of (1.1) with zero potential and \(\gamma\) related to the gauge coupling \(g\) and the temperature \(T\) by
\[
\gamma = \frac{g^2 N}{T}.
\]

This model, with exponential Boltzmann weight instead of the heat kernel one and a kinetic term for the gauge fields added, was studied both numerically and in a mean field approximation for \(D = 3, 4\) and \(N = 2, 3\) [10]. The purpose of the present paper is to derive the large \(N\) equations of motion for collective variables in spirit of Refs. [11, 12] and to study the phase structure of the model in the large \(N\) limit.

It is well known that at some critical temperature lattice gauge theories undergo the deconfining phase transition [6, 13] associated with the center group symmetry breaking. In terms of the above model this is the symmetry with respect to the multiplication of all \(g(x)\) by an element of the center of the gauge group (this symmetry is spoiled by an addition of the potential term). At low temperatures (large \(\gamma\)) the model is in the symmetric (confining) phase and \(\langle \frac{1}{N} \text{tr } g(x) \rangle = 0\), while at some critical value of \(\gamma\) the symmetry is violated and the chiral field gains a nonzero expectation value. It will be shown below, that, at least for sufficiently large \(D\), confining and deconfining phases are separated by an intermediate one.

At high temperatures the fluctuations of the chiral field are suppressed, as \(\gamma\) is small, so it can be expanded around unity
\[
g(x) = e^{i\gamma \Phi(x)} \simeq 1 + i\gamma \Phi(x)\tag{1.9}
\]
and (1.4) reduces to the partition function for the Kazakov–Migdal model with critical potential [8].

2 Large \(N\) equations of motion

Consider the partition function (1.4) with a potential term added:
\[
Z = \int \prod_x Dg(x) \prod_{x,\mu} DU_\mu(x) e^{-\sum_x N^x [V(g(x)) + V(g^\dagger(x))]}
\times \prod_{x,\mu} K \left(g(x), U_\mu(x)g(x + \mu)U_\mu^\dagger(x) \Big| \gamma.\right)\tag{2.1}
\]
Integration over the gauge fields goes independently on each link, so, fixing the diagonal gauge: \(g(x) = \text{diag}(e^{i\alpha(x)})\) and integrating out the link variables, one is left with an effective action for the eigenvalues of \(g(x)\), which contains only local interactions:
\[
S_{eff} = \sum_x \left[ N \sum_i U(\alpha_i(x)) - \sum_{i<j} \log \sin^2 \frac{\alpha_i(x) - \alpha_j(x)}{2} \right]
\]
\[
- \frac{1}{2} \sum_{\mu = -D}^{D} \log I(\alpha(x), \alpha(x + \mu)|\gamma),
\]  

(2.2)

where

\[
U(\theta) = V(e^{i\theta}) + V(e^{-i\theta}),
\]  

(2.3)

the second term in (2.2) comes from the decomposition of invariant measure:

\[
Dg = D\Omega \prod_{i=1}^{N} d\alpha_i J^2(\alpha), \quad J(\alpha) = \prod_{i<j} \sin \frac{\alpha_i - \alpha_j}{2}
\]  

(2.4)

for \( g = \Omega e^{i\alpha} \Omega^\dagger \), and

\[
I(\alpha, \alpha'|\tau) = \int DU K(e^{i\alpha}, U e^{i\alpha'} U^\dagger |\tau).
\]  

(2.5)

It is convenient to introduce the density of the eigenvalues of the matrix \( g(x) \):

\[
\rho(\theta, x) = \frac{1}{N} \sum_{i=1}^{N} \delta(\theta - \alpha_i(x)).
\]  

(2.6)

In the large \( N \) limit \( \rho(\theta, x) \) becomes smooth \( 2\pi \)-periodic function, normalized to unity on the interval \((-\pi, \pi)\). For simplicity we restrict our consideration to the case of \( U(N) \) gauge group. For \( SU(N) \) the constraint \( \int_{-\pi}^{\pi} d\theta \theta \rho(\theta, x) = 0 \) should be imposed on the eigenvalue density and the Lagrange multiplier ensuring this condition should be introduced. This is achieved by adding the term \( \lambda(x)\theta \) to the potential (2.3).

The classical equations of motion for (2.2) are exact in the large \( N \) limit and can be written in terms of the eigenvalue density as follows:

\[
U'(\theta) + (D - 1)G(\theta, x) = \sum_{\mu = -D}^{D} F_\mu(\theta, x|\gamma),
\]  

(2.7)

where

\[
G(\theta, x) = \phi \int_{-\pi}^{\pi} d\theta' \rho(\theta', x) \cot \frac{\theta - \theta'}{2}
\]  

(2.8)

and we have defined

\[
F_\mu(\theta, x|\gamma) = \frac{1}{N^2} \frac{\partial}{\partial \theta} \frac{\partial}{\delta \rho(\theta, x)} \log I(\alpha(x), \alpha(x + \mu)|\gamma) + \frac{1}{2} G(\theta, x).
\]  

(2.9)

The one–link integral (2.3) is the unitary analog of the Itzykson–Zuber integral and can be treated by the same method, proposed in Ref. [14] for finite \( N \) and used to calculate the large \( N \) limit of the Itzykson–Zuber integral in Ref. [15]. This method exploits the relation of the matrix quantum mechanics to that of the free nonrelativistic fermions [16]. Really, \( K(g, h|\tau) \) is an imaginary time transition function for the free Hamiltonian on the \( U(N) \) group manifold. After the calculations, similar to that of Ref. [14], it can be shown, that \( J(\alpha)J(\alpha')I(\alpha, \alpha'|\tau) \) is the transition function for \( N \) noninteracting fermions.
on a circle. From which it immediately follows, that it can be written in a form of a Slater determinant:
\[ I(\alpha, \alpha' | \tau) = \text{const} \cdot \left| \frac{i \tau}{2\pi N} \right| \cdot \det_{kj} \frac{\partial}{\partial \alpha_k} \frac{\partial}{\partial \alpha'_j} \left( \frac{\alpha_k - \alpha'_j}{2\pi} \right) \cdot J(\alpha) J(\alpha'), \] (2.10)

where \( \vartheta(z, \tau) \) is the Riemann theta function.

In the large \( N \) limit the fermions behave semiclassically, so the transition function for them is dominated by a single classical trajectory given, in terms of the density \( \sigma(\theta, \tau) \), by Euler equations for an ideal fluid \([11, 12]\):
\[ \frac{\partial \sigma}{\partial \tau} + \frac{\partial}{\partial \theta} (\sigma s) = 0, \] (2.11)
\[ \frac{\partial s}{\partial \tau} + s \frac{\partial}{\partial \theta} (\sigma s) - \pi^2 \sigma \frac{\partial \sigma}{\partial \theta} = 0. \] (2.12)

The minus sign before the last term in (2.12) is because the time is imaginary. The function \( s(\theta, \tau) \) is a velocity of the Fermi fluid. Eqs. (2.11), (2.12), of course, can be derived directly by a reduction of the heat equation for (2.5) to the Hamilton–Jacoby equation for the collective field Hamiltonian by the calculations, analogous to that of Ref. [15].

To find (2.9) one should solve (2.11), (2.12) with boundary conditions
\[ \sigma(\theta, 0; x, \mu) = \rho(\theta, x), \] (2.13)
\[ \sigma(\theta, \gamma; x, \mu) = \rho(\theta, x + \mu). \] (2.14)

Then for (2.9) one has:
\[ F_{\mu}(\theta, x | \gamma) = s(\theta, 0; x, \mu). \] (2.15)

This formula follows from the definition (2.9), because the variation of the classical action with respect to the initial condition for the density (2.13) is equal to it’s conjugate variable, which is a potential of the velocity [17]. Reversing time in (2.11), (2.12) one also finds:
\[ F_{-\mu}(\theta, x | \gamma) = -s(\theta, \gamma; x - \mu, \mu). \] (2.16)

Thus in the large \( N \) limit the model reduces to the set of differential equations (2.11), (2.12) to be solved on each link with boundary conditions for the density given by (2.13), (2.14) and the sum of the jumps of the velocities at each lattice cite given by the l.h.s. of eq. (2.7). Eqs. (2.11), (2.12) are integrable, although in an implicit form, and the problem is, in principle, reducible to the purely algebraic one [15], differing from that for Hermitean matrix model in the periodic boundary conditions for \( \theta \) and the kernel in the integral (2.8) (the Cauchy one versus the Hilbert kernel in the Hermitean case).

In one–dimensional case the gauge field in (1.1) can be absorbed by a gauge transformation, so \( D = 1 \) model is equivalent to the unitary matrix quantum mechanics, solved in the large \( N \) limit in Refs. [12, 18]. It is instructive to reproduce these results from the
above lattice equations. One can suppose that \( \tau \) in eqs. (2.11), (2.12) changes from \(-\infty\) to \(+\infty\), while at the points \( \tau = n\gamma \) the velocity has a jump according to (2.7), (2.15), (2.16). However, in the continuum limit \( \gamma \) and \( U'(\theta) \) scales as \( a\gamma \) and \( aU'(\theta) \), respectively (\( a \) is the lattice spacing), so the discontinuities in \( s(\theta, \tau) \) can be absorbed by adding the term \( \frac{1}{\gamma}U'(\theta) \) to the r.h.s. of eq. (2.12). The magnitude of a deviation of the solution of resulting equations from the exact one on the time interval \((n\gamma, (n+1)\gamma)\) is of the order \((a\gamma)^2\) and can be neglected as \( a \to 0 \). Thus one obtains in the continuum limit:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} &+ \gamma \frac{\partial}{\partial \theta}(\rho v) = 0, \\
\frac{\partial v}{\partial t} &+ \gamma v \frac{\partial v}{\partial \theta} + \pi^2 \gamma \frac{\partial \rho}{\partial \theta} + U' = 0,
\end{align*}
\]

(2.17) (2.18)

where \( t \) is the real time, measured in an ordinary units (while \( \tau \) is measured in the units of \( \gamma \)). These are nothing that Jevicki–Sakita collective field theory equations [12].

In the multidimensional case the second term in the l.h.s. of eq. (2.7) do not vanish and is of order unity as \( a \to 0 \), so to obtain the continuum limit in the same way as in one dimension, the solutions should be considered with the velocity \( s(\theta, \tau; x, \mu) \) being by itself of order \( a^{-1} \) and \( \gamma \) scaling as \( a^2 \gamma \). Then the arguments, similar to that used to obtain (2.17), (2.18), give the following equations in the continuum limit (in the Minkowski space):

\[
\begin{align*}
\frac{\partial \rho}{\partial x^\mu} &+ \gamma \frac{\partial}{\partial \theta}(\rho v_\mu) = 0, \\
\frac{\partial v_\mu}{\partial x^\nu} &+ \gamma v_\mu \frac{\partial v_\nu}{\partial \theta} + (D-1)G + U' = 0.
\end{align*}
\]

(2.19) (2.20)

As in the case of Hermitean matrix model, it can be shown, that translationally invariant solution to these equations is unstable [13], the spectrum containing an infinite number of tachyons. Thus one has to look for other possibilities to take the continuum limit. In this respect the study of the phase structure of the model is of fundamental importance.

### 3 Strong coupling phase

Consider the model (2.1) without a potential term. Equations of the previous section always have a trivial solution \( \rho(\theta, x) = \frac{1}{2\pi} \) in this case (with \( s = 0 \) in (2.11), (2.12)). This solution is valid at large \( \gamma \). However, the instability in the continuum limit at \( \gamma = 0 \) shows, that at some \( \gamma_c \) the strong coupling solution becomes a maximum, rather than a minimum, of the effective action.

To find the spectrum of excitations about the strong coupling solution we write

\[
\begin{align*}
\rho(\theta, x) &= \frac{1}{2\pi} + \frac{1}{2\pi} \sum_{n \neq 0} a_n(x) e^{in\theta}, \quad a_n^* = a_{-n} \\
\sigma(\theta, \tau; x, \mu) &= \frac{1}{2\pi} + \frac{1}{2\pi} \sum_{n \neq 0} \alpha_n(\tau; x, \mu) e^{in\theta}, \quad \alpha_n^* = \alpha_{-n}
\end{align*}
\]

(3.1) (3.2)
\( s(\theta, \tau; x, \mu) = \sum_{n \neq 0} \beta_n(\tau; x, \mu) e^{in\theta}, \quad \beta_n^* = \beta_{-n} \) (3.3)

and linearize the equations of motion in \( a_n, \alpha_n \) and \( \beta_n \). The solution to eqs. (2.11), (2.12) then reads:

\[
\alpha_n(\tau; x, \mu) = \alpha_n^+(x, \mu) e^{\frac{\tau}{2}} + \alpha_n^-(x, \mu) e^{-\frac{\tau}{2}},
\]

\[
\beta_n(\tau; x, \mu) = i \left[ \alpha_n^+(x, \mu) e^{\frac{\tau}{2}} - \alpha_n^-(x, \mu) e^{-\frac{\tau}{2}} \right].
\] (3.5)

Substituting these expressions in the boundary conditions (2.13), (2.14), (2.15), (2.16) and (2.7) one obtains, after a simple algebra, the equation for the Fourier coefficients of the eigenvalue density:

\[
\sum_{\mu=1}^{D} \left[ a_n(x + \mu) - 2 \left( \cosh \frac{n\gamma}{2} - \frac{D-1}{D} \sinh \frac{n\gamma}{2} \right) a_n(x) + a_n(x - \mu) \right] = 0,
\] (3.6)

which leads to the following mass spectrum:

\[
M_n^2 = D \cosh \frac{n\gamma}{2} - (D-1) \sinh \frac{n\gamma}{2},
\] (3.7)

where \( M_n^2 = \sum_{\mu=1}^{D} \cosh \lambda_{n}^{\mu} \) for the solution of (3.6) of the form \( \exp(\sum_{\mu=1}^{D} \lambda_{n}^{\mu} x^{\mu}) \).

At the critical point

\[
\gamma_c = 2 \log(2D-1)
\] (3.8)

the lowest excitation becomes massless, and the strong coupling solution becomes unstable. In principle this critical point can be used to construct the continuum limit. However, it describes only one complex scalar particle with the mass

\[
m^2 = (D-1) \frac{\gamma - \gamma_c}{a^2},
\] (3.9)

while the masses of other excitations remain of the cut–off order. It is worth mentioning, that an obstacle to take the continuum limit at \( \gamma_c \) may occur due to the possibility for the model to undergo a first order phase transition into the weak coupling phase before the point (3.8) is reached (see the next section).

### 4 Weak coupling phase

As \( \gamma \to 0 \) the expansion (1.9) becomes legitimate, so one expects that the eigenvalue density \( \rho(\theta, x) \) is peaked about zero. Thus, in the zero approximation, the cotangent in (2.8) can be replaced by \( \left( \frac{\theta - \theta'}{2} \right)^{-1} \) and the equations of motion become the same as for the Kazakov–Migdal model with the action containing only the derivative squared term. It is known, that such model has a stable translationally invariant solution [4]. The fluctuations about it contains one massless mode [20] related to the shifts of the center of the eigenvalue density. It is straightforward to show (in the general case) that, if \( \rho(\theta, x) \) solves the equations of Sec. 2, then \( \rho(\theta + \theta(x), x) \) also does, provided that \( \theta(x) \) satisfies
lattice Laplace equation. So the Goldstone mode simply decouples and can be disregarded (one may exclude it considering SU(N) model).

Now let us take into account the next term in the expansion of the cotangent in (2.8) (the eigenvalue density is assumed to be an even function of $\theta$):

$$\int d\theta' \rho(\theta', x) \cot \frac{\theta - \theta'}{2} = 2 \int d\theta' \frac{\rho(\theta', x)}{\theta - \theta'} - \frac{1}{6} \theta - \ldots \tag{4.1}$$

After the substitution of (4.1) in the l.h.s. of eq. (2.7) the equations of motion become equivalent to that for the Kazakov–Migdal model with a mass term added. Note, that this mass is negative. Moreover, the residual term in (4.1) is given by the convergent, as $|\theta - \theta'| < 2\pi$, series in the odd powers of $\theta - \theta'$ with negative coefficients. Thus an account of the corrections coming from the nonlinearity of the chiral field leads to the $\rho$-dependent upside–down potential and (4.1) gives an upper bound for it. For sufficiently weak potential an interaction with the gauge fields stabilizes the fluctuations and provides a minimum for an effective action, but at some critical $\gamma_*$ the effective potential becomes strong enough to make a weak coupling solution unstable. An upper estimate on $\gamma_*$ can be obtained considering the Kazakov–Migdal model with the quadratic potential, an effective mass being given by (in the notations of Ref. [4]):

$$m_{eff}^2 = 2D - \frac{1}{6}(D - 1)\gamma, \tag{4.2}$$

the solution to which is known [4]:

$$\rho(\theta) \simeq \frac{1}{\pi} \sqrt{\mu - \frac{1}{4} \mu^2 \theta^2}, \tag{4.3}$$

$$\mu = \frac{m_{eff}^2(D - 1) + D\sqrt{m_{eff}^4 - 4(2D - 1)}}{(2D - 1)\gamma}. \tag{4.4}$$

This solution is stable until $m_{eff}^2 = 2\sqrt{2D - 1}$, when (4.4) becomes complex and the massless excitation appears in the spectrum [20]. This gives the following bound on $\gamma_*$:

$$\gamma_* < \frac{12(D - \sqrt{2D - 1})}{D - 1}. \tag{4.5}$$

It is worth mentioning that for $D = 2, 3$ the width of the support of eigenvalue density (4.3) becomes larger than $2\pi$ for smaller value of the coupling.

There are, in principle, three possibilities for the critical behavior:

i) $\gamma_* > \gamma_c$. In this case strong and weak coupling solutions can coexist and the first order phase transition takes place at some $\gamma$ between $\gamma_c$ and $\gamma_*$. ii) $\gamma_* = \gamma_c$. The phase transition is of the second order.

iii) $\gamma_* < \gamma_c$. This means that an intermediate phase exists, separated from strong and weak coupling ones by the second order phase transitions.
The third possibility is realized in the large $D$ limit. Really, $\gamma_c$ grows as $2 \log D$, while $\gamma_*$ tends to 12 as $D \to \infty$ (the estimate (4.3) gives an exact large $D$ asymptotics of $\gamma_*$, because the width of the eigenvalue distribution (4.3) behaves at the critical point as $4 \left( \frac{72}{D} \right)^{1/4}$ and corrections to (4.1) vanish). However, using an upper bound (4.5) it is possible to advocate the existence of an intermediate phase only for $D > 86$. It is worth mentioning that the value of $\theta$, at which the eigenvalue density (4.3) with critical $m^2_{\text{eff}}$ and $\gamma$, given by an upper bound (4.5), turns to zero, is, nevertheless, not small – $\theta_{\text{max}} = 0.66\pi$ for $D = 87$, so the approximation (4.1) remains rough at the point of the phase transition even for such large $D$. Thus more accurate treatment is necessary to distinguish between the possibilities i)–iii) for low dimensions.

For $D = 3$, the first possibility is favored by the Monte Carlo simulations of Ref. [10] and the large amount of numerical data on the finite temperature gauge theories. However, the second possibility do not contradict the numerical results, as we are dealing with the boundary of the phase diagram in $g^2$, $T$, and the line of the first order transitions may terminate at the point of the second order one. The third possibility is also not excluded, because an intermediate phase can disappear at some finite, but large $N$, and thus be invisible in the Monte Carlo simulations for $N = 2, 3$.

## 5 Conclusions

If we discard the possibility of the first order phase transition between the strong and the weak coupling phases, which is legitimate, at least for large $D$, as discussed in Sec. 4, the continuum limit of the model can be constructed. Unfortunately, corresponding continuum theory contains only finite number of degrees of freedom, if we approach the critical point from strong or from weak coupling phase, and thus cannot describe extended objects, which was one of the motivations for study of the model. However, this may not be the case for the model with a potential term added, or if one takes the continuum limit in the intermediate phase. It is also interesting to understand the origin of this phase from the point of view of thermal lattice gauge theory. To clarify these questions it is desirable to know an exact solutions to the model in the weak and the intermediate phases. To this end, it would be interesting to develop an alternative approach, based on Schwinger–Dyson equations, which proved to be useful in the context of the Kazakov–Migdal model [6].

The author is grateful to Yu. Makeenko for discussions. The work was supported in part by RFFR grant No. 94-01-00285.

## Note added

When this work was being prepared for publication, there appeared a paper by D.V. Boulataev [21], which also includes the treatment of the gauged chiral field model at large $N$. 


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