Shape Analysis of Particles by an Image Scanner and a Microcomputer: Application to Agglomerated Aerosol Particles

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Abstract

Algorithms are presented for feature extraction of the geometrical shape of particles. The system consists of a microcomputer and an image scanner which scans the micrograph of particles and transmits the compressed image data. Methods for shape analysis are: 1) calculation of fundamental particle shape parameters (Feret diameter, area, perimeter, first moment, second moment), 2) fractal analysis, 3) opening method and 4) separation of circular primary particles from an agglomerate. The connectivity of particle boundary is recognized for five cases (continuation, termination, creation, split and merge), and the fundamental shape features are calculated according to it. The validity and accuracy of these methods are examined by comparing the calculated and theoretical shape parameters for standard figures. A new index is proposed for the description of the structuring elements number of a two-dimensional particle shape in opening analysis.

These methods are applied to the shape analysis of agglomerated aerosol particles generated from an electric furnace and by the CVD method, and it is shown that the fractal dimension as a particle distribution is useful for the quantitative description of shape.

1. Introduction

Agglomerated aerosol particles are found in various fields, such as coal combustion, diesel exhaust gases and nuclear reactor safety. They have complicated shapes like chain or cluster aggregates. Investigation of the dynamic behavior of these agglomerated particles is required for evaluating the measurements on these aerosols or their effects on human health. The relationship between the geometrical shape and the dynamics of a non-spherical particle has not yet been studied sufficiently, except for particles having simple configurations. It is necessary to find a shape parameter (or parameters) that can clearly explain the relation between the geometrical shape and the dynamic behavior of irregular-shaped particles. Quantification of the geometrical shape of aerosol particles is performed mainly for two-dimensional microscopic images. Computer image processing is indispensable for the morphological analysis of a particle having a complicated shape.

Recently, image processing technology has significantly advanced so that even bad quality image can be processed at a high-speed using image signal processors. These commercially available image processing instruments are very expensive and cannot be used easily, while the image scanner, which is used for inputting images, such as drawings into a microcomputer, is relatively inexpensive. Moreover, it has the advantages of low image distortion and high resolution due to the use of Charge Coupled Devices (CCD). Therefore, the scanner is effective for processing micrograph images of particles with a microcomputer.

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† This report was originally printed in J. Aerosol Research, Japan, 2, 117-127 (1987) in Japanese, before being translated into English with the permission of the editorial committee of the Japan Association of Aerosol Science and Technology, Japan.
This paper discusses the methods for calculating various shape parameters from micrographs using an image scanner and a microcomputer. The methods are then applied to the shape analysis of agglomerated aerosol particles.

2. Image Processing Using Image Scanner

2.1 Calculation of fundamental particle shape parameters

The calculation of particle shape parameters is performed for a binary coded image that is converted from the gray picture of an original micrograph. The threshold value for binary coding is set at an appropriate level by changing the preset level of the image scanner (PC-INS01, NEC), and the binary coded images are displayed on a CRT by the microcomputer (PC-9801, NEC). The scanner reads the image data of a particle micrograph in the horizontal direction using CCD and transmits the compressed binary image data to the microcomputer by sliding the micrograph in the vertical direction with a roller. We have adopted sequential method for processing an image because the scanner transmits data, scanning the field of view in a raster scan mode, and the method also saves data storage area.

The following conditions should be satisfied for the 8-connected continuation of one pair of points (start point \( S_k(i) \) and end point \( E_k(i) \) of the \( k \)-th edge points) detected on the scan line \( i \), and the \( h \)-th edge points on the scan line \( (i-1) \):

\[
S_h(i-1) \leq E_k(i) \quad (1)
\]

\[
S_k(i) \leq E_h(i-1) \quad (2)
\]

The connectivity at each edge point is recognized as one of five cases shown in Fig. 1.

(1) Creation

No edge points on the scan line \((i-1)\) are continuous to the ones on line \(i\), and the object on line \(i\) is designated with a new label.

(2) Termination

No edge points on the scan line \((i+1)\) are continuous to the ones on line \(i\).

(3) Continuation

Only one pair of edge points on the scan line \((i-1)\) are continuous to the ones on line \(i\), and the object on line \(i\) is designated with the same label as the one on line \((i-1)\).

(4) Merge

Two or more pairs of edge points on the scan line \((i-1)\) are continuous to the single pair of edge points on line \(i\), and the same label is reassigned to the corresponding objects.

(5) Split

Two or more pairs of edge points on the scan line \(i\) are continuous to the single pair of edge points on line \((i-1)\), and the label of the object on line \((i-1)\) are assigned to the ones on line \(i\).

Based on the recognition described above, we can calculate the fundamental particle shape parameters, such as horizontal or vertical Feret diameter, area, perimeter, first moment and second moment.

(a) Horizontal Feret diameter:

\[
F_h = \max(jE - jS)
\]

where \(jS\) and \(jE\) are the edge points designated label \(j\).
(b) Vertical Feret diameter:
\[ F_v = \max(Y_r) - \max(Y_c) \]  
where \( Y_c \) and \( Y_r \) are the scan line numbers (Y coordinate) when conditions (1) and (2) are satisfied, respectively.

c) Area:
\[ A = \sum \{ E(i) - S(i) \} \]  
d) Perimeter:
\[ P = \sum [1 + \{ S(i) - S(i - 1) \}^2]^{1/2} + \sum [1 + \{ E(i) - E(i - 1) \}^2]^{1/2} \]  

The correction method proposed by Taniguchi\(^1\) is adopted to reduce errors caused by digitization of the image in the diagonal direction.

e) First moment:
\[ G_x = \sum \{ E(i) - S(i) \} y \]  
\[ G_y = \sum \{ S(i) + \cdots + E(i) \} y \]  

(f) Second moment:
\[ I_x = \sum \{ E(i) - S(i) \} y^2 \]  
\[ I_y = \sum \{ S(i)^2 + \cdots + E(i)^2 \} \]  
\[ I_{xy} = \sum \{ S(i) + \cdots + E(i) \} y \]  

Using these fundamental shape parameters, we can calculate diameter of the circle of equal projected area, diameter of the circle of equitl, center of gravity, radius of gyration, direction of major axis, circularity, anisometry and bulkiness. In cases of Merge and Split, the basic parameters are corrected. The number of particles equals the difference between the total event number of Creation and that of Merge for particle boundaries having different labels.

2.2 Fractal dimension

The shape parameters described in Section 2.1 cannot completely reflect the two structural characteristics of an agglomerated particle profile, that is, the boundary consists of fine primary particles and macro-scale irregular form. Kaye\(^2\) applied the concept of “fractal” proposed by Mandelbrodt\(^3\) to the shape analysis of powders and analyzed the structures using the structured walk method. To apply that concept to this image processing system, a new calculation method has been examined.

If the length of a particle boundary is measured with scale \( \lambda_n \) and \( n \) steps are required for measurement, the length \( L_n \) of the boundary is given by
\[ L_n = \lambda_n \cdot n \]  

When a straight line is measured for several values of \( \lambda_n \), a linear relationship holds between \( L_n \) and \( \lambda_n \). If the length of complicated boundary, such as a coastline, is measured in the same manner, the length increases with the shortening of \( \lambda_n \) because the fine structure of the boundary appears. This relationship can be expressed as,
\[ L_n = \alpha \lambda_n^{-\beta} \]  
for fractal boundaries, where \( \alpha \) is constant, and \( \beta \) is negative value. With Eq. (12) = Eq. (13), the following equation is obtained:
\[ n = \alpha \lambda_n^{-(1-\beta)} = \alpha \lambda_n^{-d} \]  

A log-log plot of \( n \) versus \( \lambda_n \) generates a linear line. The fractal dimension \( d \) represented below can be obtained from the slope of the line:
\[ d = 1 - \beta \]  
The fractal dimension is calculated by the following procedures which are different from the sequential method in Section 2.1.

a) First, a particle binary image is stored in the VRAM (resolution: 640x400) of the microcomputer, and the outline is extracted.

b) The profile can be extracted by detecting the edge points on a specific line and carrying out a logical operation between the image data on the specific scan line and on the upper one, as well as on the lower one.

c) Next, grid length \( \lambda_n \) is chosen as a basic value for scale conversion.

d) The number \( N \) of the intersection points of the particle boundary and grid lines (X and Y axes) is counted, as shown in Fig. 2, for a series of lengths \( \lambda_n \).

e) The fractal dimension \( d \) is obtained as the absolute value of the slope of the line in the log-log graph of \( N \) versus \( \lambda_n \).

However, \( N \) should be corrected to unity within specific grid interval if the boundary intersects two or more times.

A one-dimensional measure is used to determine the fractal dimension of a particle boundary, as shown in Fig. 2. If open space, such as a netlike agglomerate, exists in the particle, the
The following procedure can be applied in order to obtain the porosity, that is, the fractal dimension for a two-dimensional measure.

The gravity center of a particle image is picked, and then a series of nested squares of different sizes are placed around it. The number of particle pixels in each square is counted.

2.3 Opening method

Fractal dimension is effective for quantitatively describing the properties of the complicated boundary of an aerosol particle. However, the dimension cannot express the isotropic properties of a two-dimensional shape or the elemental particle numbers of agglomerated particle. To measure particle size distribution, Matheron has mathematically developed the concept “opening” (refer to reference 5 for details). Domon (6) has applied this for quantitatively describing the morphology of mouse parotid glands.

Since a square lattice is used in this system, \( n/2 \) operations of the erosion-dilation processing to a binary coded image on square grids can be considered as the opening process of size \( n \). If the total pixel number of binary images is \( P(n) \), the pixel number of particles for the opening size \( n \) is given by \( D(n) = P(n-1) - P(n) \). A two-dimensional image can be expressed using the number distribution of particles, that is, the distribution of \( D(n) \).

For instance, the distribution \( D(n) \) of a figure consisting of only one structuring element (a square in this system) has only one peak at \( n \) which corresponds to its side. The distribution has some peaks if the figure consists of some elements having different sizes, like chain agglomerates.

In practical processing, binary image data is stored in the VRAM of the microcomputer, and the number of pixels are counted after activating erosion-dilation operations \( n \) times.

2.4 Primary particle separation from an agglomerated particle

The method of erosion-dilation processing described in Section 2.3 can be applied to estimate original primary particles from the two-dimensional image of an agglomerated aerosol particle which consists of fine spherical particles if the degree of particle overlapping is slight. When the degree is significant, the separation is impossible using this method. The iterative method (7) or the method for feature extraction and hierarchical decomposition for a closed curve using averaging operations (8) has been applied to separate the above with satisfactory results (9). If these methods were to be applied to our system, all calculations would have to be processed by the software of the microcomputer, resulting in an overloading. Then, we have examined a new processing method.

When primary particles are assumed to be spherical, the coordinates of centers and radii must be determined. To obtain these, a part of their arcs should be estimated from the agglomerated particle. First, an outline of the two-dimensional image is extracted by the operation described in Section 2.2. The curvature of the boundary changes abruptly on the intersection points where different circles meet. This can be expressed by the tangent angle becoming negative at the points. Therefore, we calculate the tangent angles at each discrete outline point with the boundary following, and select the candidate points for calculating parameters of circles during positive tangent angles.

To reduce the effects of digitization errors, the original image is subjected to one dilation-erosion process, and the coordinates of the
boundary points are smoothed by the interpolation using the gravity center of three points. Next, three parameters (the coordinates of the center and the radius of a circle) are estimated by the non-linear least squares method. The initial value of each parameter is obtained as follows: the center is the intersection point of the perpendicular bisectors of two chords (one is formed by candidate points having an initial number and an intermediate number; the other is formed by candidate points having a final number and an intermediate number), the radius is the distance between the center and the candidate point having intermediate number.

In case that the parameters obtained by the above-mentioned steps should indicate a circle, the program may sometimes recognize them as indicating different circles. We define the distance function $P$, which depends on the distance between centers of each circle $\Delta r$, written as

$$P = 1 - \frac{\Delta r}{d_{th}} \quad (\Delta r \leq d_{th}) \\ P = 0 \quad (\Delta r > d_{th})$$

A threshold value of $P$ is determined by the discriminant analysis method in order to judge that some pairs of circles indicate an identical circle. If the pair of circles indicate an identical circle, the parameters are recalculated by reorganizing the candidate points on the boundary. In our calculation $d_{th}$ is assumed to be 10. If a large circle should include a smaller one, another judgement is done in order to reorganize the parameters.

The programs described in Section 2.1 are written in the assembly language when the programs (for controlling input/output to or from the image scanner and VRAM) require high-speed processing, and the others are in the C language. These programs are modularized.

3. Measurement Results for Standard Figures

3.1 Fundamental shape parameters

The measurement accuracy depends on two kinds of errors: one is caused by the algorithm itself, and the other is due to binary coding. The latter is attributed to the photographic contrast or the characteristics of the image scanner.

A study on the accuracy of the algorithm was carried out by Kuga and the authors using model images generated by a computer. They have reported that the largest error occurs in perimeter measurement. Since the threshold level of the image scanner can be only changed into 8 levels, we used photographs having higher contrast. In the following, we have discussed the errors caused by the fluctuation of light source and electrical circuits.

As shown in Fig. 3, the standard figures have been created and the shape parameters of these figures have been calculated. The sizes of figures were changed into 4 or 5 levels, and the average values of 20 measurements were obtained. The figures, except for a circle, are input by rotating them to some degree. Tables 1 and 2 list the mean ratio of the measured to the theoretical shape parameter and the coefficients of variation. The perimeter and the area represent the diameter equivalent to the circle perimeter and the area, respectively. The sizes shown in Tables 1 and 2 represent the pixel numbers equivalent to: the circle diameter; the side of a square and triangle; the diameter of the structuring element circle for a doublet and triplet (same size); and the smaller circle diameter for a doublet (different size). In this case, the diameter of the larger circle is twice that of the smaller one.

Table 1 shows that accuracy increases with the increase in the figure size. When the number of pixels exceeds 70, the difference be-
Table 1: Mean ratio of measured to theoretical shape parameters and coefficient of variation for three standard figures

| Figure | Size | Area   | Perimeter | Radius of gyration |
|--------|------|--------|-----------|--------------------|
| Circle | 11   | 1.05±0.030 | 1.04±0.027 | 1.05±0.028         |
|        | 20   | 1.01±0.012 | 1.01±0.015 | 1.01±0.012         |
|        | 38   | 1.01±0.0061 | 1.01±0.0080 | 1.01±0.0059       |
|        | 73   | 1.01±0.0046 | 1.01±0.0066 | 1.01±0.0045       |
|        | 127  | 1.00±0.0023 | 1.00±0.0032 | 1.00±0.0020       |
| Square | 10   | 1.02±0.019 | 1.03±0.026 | 1.03±0.018         |
|        | 19   | 1.01±0.011 | 0.98±0.027 | 1.01±0.012         |
|        | 37   | 1.00±0.0055 | 0.98±0.031 | 1.00±0.0054       |
|        | 72   | 1.01±0.0053 | 1.00±0.025 | 1.01±0.0052       |
|        | 128  | 1.01±0.0037 | 0.99±0.035 | 1.01±0.0033       |
| Triangle | 11  | 1.09±0.020 | 0.95±0.023 | 1.06±0.021         |
|         | 19   | 1.07±0.017 | 0.99±0.020 | 1.06±0.016         |
|         | 38   | 1.02±0.0068 | 0.98±0.0096 | 1.02±0.0068       |
|         | 72   | 1.01±0.0072 | 1.01±0.0069 | 1.03±0.0069       |
|         | 130  | 1.01±0.0045 | 1.00±0.0051 | 1.01±0.0041       |

Table 2: Mean ratio of measured to theoretical shape parameters and coefficient of variation for simple agglomerate models

| Figure | Size | Area   | Perimeter |
|--------|------|--------|-----------|
| Doublet (same size) | 10   | 1.02±0.020 | 0.98±0.026 |
|         | 19   | 1.01±0.0082 | 1.00±0.010 |
|         | 37   | 1.01±0.011 | 1.01±0.019 |
|         | 72   | 1.01±0.0019 | 1.01±0.0064 |
| Doublet (different size) | 8    | 1.02±0.015 | 0.99±0.026 |
|         | 15   | 1.02±0.0139 | 1.00±0.019 |
|         | 29   | 1.01±0.0046 | 1.01±0.0070 |
|         | 57   | 1.01±0.0042 | 1.01±0.0058 |
| Triplet | 10   | 1.02±0.017 | 0.99±0.018 |
|         | 19   | 1.01±0.012 | 0.99±0.015 |
|         | 37   | 1.01±0.0054 | 1.00±0.0080 |
|         | 73   | 1.01±0.0028 | 1.01±0.0043 |

3.2 Fractal dimension

In the same manner as in Section 3.1, standard figures are created, and image data is input by the image scanner to obtain the fractal dimension of the boundary. Figure 4 shows the results of some examples. In this graph, the abscissa indicates the grid length normalized by the horizontal Feret diameter. The theoretical dimensions of the circle and triadic Koch curve are 1 and 1.2618, respectively. The dimension of the figure shown in (c) is 1.11, obtained by Kaye’s method. Our calculation results are in good agreement with the above theoretical or measured values. For the other figures that Kaye used, our results agree with findings obtained by Kaye within a maximum error of 2%.

The Koch curve has an infinite self-similarity, however, a lower limit exists for drawing the figure or resolution of the computer display. Therefore, the fractal dimension is defined for the range that is larger than the lower limit mentioned above. The dimension is unity for the range below the lower limit. It should be noted that the fractal dimension is defined within a specific finite range because a figure has a finite scale.

3.3 Opening method

Figure 5 shows the results of a circle, square and rectangle with an aspect ratio of 2, obtained by the opening method. The abscissa indicates the size of opening normalized by the side length (square root of the area) of the square that is equivalent to the area of the tar-
get figure. The ordinate $D(\lambda)$ is normalized with the area of the target figure and $n(\lambda)$ indicates the number of squares of side $\lambda$. These have the following relationship: $n(\lambda) = D(\lambda)/\lambda^2$ where $D(\lambda)$ is not normalized with the area of the target figure.

By comparing the data of squares which are smoothed and not smoothed, shown in Fig. 5 (the square includes some ruggedness that cannot be shown in the figure), a peak in the number distribution of the smallest size is found in the graph for the unsmoothed square. This indicates that the smoothing operation is effective in reducing the effects of the digitization noise.

After the opening operation, the peak in the distribution for a square appears at the value of $\lambda$ corresponding to the length of its side ($\lambda = 1$). The peak for a rectangle appears at the size of its shorter side. The number distribution shows a good agreement with the theoretical value. The maximum opening size is the side length of the inscribed square for a circle, and the distribution of squares is determined so as to fill the remaining portion of the circle. Consequently, the squares distribute over a relatively wide range of size that is smaller than its peak position, as shown in Fig. 5.

Fig. 4 Plot of the normalized grid length by horizontal Feret diameter vs. $N$ for standard figures; (a) circle, (b) triadic Koch curve, (c) test powder2).

Fig. 5 Area and number distributions of openings for standard figures, (a) square (not smoothed), (b) square (after smoothing), (c) rectangle, (d) circle.
To evaluate the effectiveness of the opening method that divides a two-dimensional figure into structuring elements, it was applied to a triadic Koch island, to model figures of agglomerates, and to test powders that Kaye used to calculate fractal dimensions. As shown in Fig. 6, it was revealed for the Koch island that many smaller squares distribute around the central largest one, and the number of those smaller squares rapidly increases with the decrease in size. The results of the model figures of agglomerates shown in Fig. 6 indicate that the...
agglomerates are divided into their structuring elements. For the strongly fused agglomerate consisting of many structuring elements, a peak appears at a point which corresponds to the largest circle of the fused portion. The results shown in Fig. 7 also indicate that the test powders are satisfactorily divided into their structuring elements according to their shapes.

The area-weighted average of opening sizes is considered to represent a "squareness" using a standard figure as the quantitative evaluation of shape. Figure 8 indicates the average, except for the Koch island, shows a good relationship to the circularity that is generally used. By considering the ratio of the total number \( N \) of distributed figures to the peak value \( n_p \), an index as to the structuring elements of a figure, \( \rho = N/n_p \) (18), can offer the measure of structuring elements number with the total number \( N \) as shown in Table 3. For instance, when the agglomerates (c) and (d) in Fig. 6 are compared, the fractal dimension of agglomerate (d) is larger than that of (c), and \( \rho \) of (c) is larger than that of (d), indicating that figure (c) includes more structuring elements than (d). \( N \) of (d) is larger than that of (c). This fact indicates that agglomerate (c) consists of more elements of the same size than (d).

3. 4 Separation of primary particles from an agglomerate

As described in Section 3. 3, the opening method can estimate the size and the number of primary particles, but the method is not effective in estimating primary particles when they are strongly fused. Figure 9 illustrates the estimated results of structuring element circles from agglomerate models according to the method in Section 2. 4. In each model, the estimated results are satisfactory, and the estimation error of the radius is within 2%. However, some circles are not identical, though their arcs are on the same circumference. This was considered to be caused by the digitization errors resulting from the input condition. Therefore, we had to diallogically change the threshold value in the discriminant analysis in Section 2. 4.

4. Application to Agglomerated Aerosol Particles

4. 1 Particles generated by an electric furnace

A boat containing a granular lead with no silver was fed into an electric furnace. The generated aerosol particles were guided into a coagulation chamber by flowing \( N_2 \) gas, thereby obtaining agglomerated aerosol particles. Since the generated aerosol particles were of a chain structure consisting of spherical primary particles, we applied the separation method described in Section 2. 4.

As shown in Fig. 10, the size distribution of primary particles is log-normal, having a geometric mean diameter of 0.205 µm and a geometric standard deviation of 1.46. On the other hand, the measurements of about 500 primary
Fig. 9 Outlines of agglomerate models (left) and estimated primary circles (right).

Fig. 10 Cumulative size distributions of primary particles; separation method in this study (○), visual method (●). Material: Pb, temperature of electric furnance: 1050°C, N₂ flow rate: 16.7 cm³/s.

Particle diameters using calipers show a geometric mean diameter of 0.195 μm and a geometric standard deviation of 1.41. This difference suggests that the number of larger diameter particles decreased more than the result obtained by the separation method. This is caused by the following: the parameter estimation was difficult or the estimated diameter was larger than the real diameter because only a small numbers of pixels of an arc were used for estimation, or the change in curvature was too small as shown in Fig. 11. As a result, the discriminant function for reforming the arcs did not work effectively. It is required to en-

Fig. 11 Outlines of Pb fume generated from electric furnance (upper) and estimated primary particles (lower). Experimental condition is the same as in Fig. 10.
hance the resolution ability of the image scanner or to manually eliminate excessively large circles to analyze particles that were not separated sufficiently.

Fig. 12 TEM micrographs of aerosol particles generated by CVD method. AIAA concentration: 10 mol/m³, equivalence ratio: 2.5, CO flow rate: (a) 8.33, (b) 12.5, (c) 16.7 cm³/s.

4. 2 Particles generated by the CVD method

We also applied the above described method to the shape analysis of aerosol particles generated by the CVD (Chemical Vapor Deposition) method. The aerosol particles are generated as follows: the Al₂O₃ aerosol particles were generated using oxidation of aluminium acetylacetonate (AIAA) in CO-O₂ flame. AIAA is introduced into the flame in a finely sprayed ethanol solution by a ultrasonic nebulizer with CO. Clean air flowed from the peripheral portion of the burner to introduce the particles into the plenum chamber for stabilization. The aerosol particles were sampled using thermopositor and subjected to observation using an electron microscope. Figure 12 shows an example of TEM micrographs of the generated particles. The primary particles are approximately spherical in shape, and the size distribution is log-normal, having a geometrical mean diameter of 5 ~ 9 nm and a geometrical standard deviation of 1.2 ~ 1.3.

As shown in Fig. 13, the fractal dimension of the generated aerosol particle changes significantly at the point that corresponds to the size of the primary particles. This indicates that there are two shape structures of the agglomerated particle, i.e. the Euclidean boundary (d=1) of the spherical primary particle and the complicated boundary (d=1.34) of the entire agglomerated particle.

If the concentration of the solution and the equivalence ratio are constant, the particle
shape shifts from web to cluster aggregates with the increase in the CO flow rate. Circularity can be used as an index for quantifying such difference in shape of agglomerated particles. The circularity, however, is calculated for a single particle and has a specific distribution for each condition of particle generation. Thus, the obtained data was summarized using an arithmetic mean of circularities for 300 ~ 500 particles as shown in Fig. 14. When AlAA concentration is 10 mol/m$^3$, the circularity is proportional to the CO flow rate, coinciding with the visual recognition. However, for the concentration of 20 mol/m$^3$, the circularity is not proportional to the CO flow rate. When the number percentage of fine particles is large on a micrograph, significant error in the digital image processing occurs because of poor resolution. As a result, the percentage of particles having a circularity of nearly unity increases, hence the mean value of circularities cannot completely express shape irregularity. This leads to possible disparity with the visual recognition result.

The size distribution of primary particles is relatively monodisperse, so agglomerated particles are considered to be self-similar as a group. We select radius of gyration $r_g$ of an agglomerated particle as a basic scale, and perimeter $P$ or area $A$ as a measurement quantity. Then for the agglomerated particles, the relationship between $r_g$ and $P$ (or $A$) can be expressed as, 

$$P \text{ (or } A) \propto r_g^d$$

as shown in Fig. 15. Here, $d$ is the fractal dimension for a particle size distribution. This method can improve upon the lack of information, and it can eliminate erroneous information on the shapes of fine particles. The fractal dimensions in Fig. 14 are obtained from the relationship between the perimeter (in this case, diameter of the circle of equal perimeter) and the radius of gyration. The results show that the method can quantitatively represent the characteristics of shape that cannot be described with circularity. The dimension agrees satisfactorily with that obtained from a single agglomerate in Fig. 13, and this indicates the consistency of both methods.

5. Conclusions

By using an image scanner and a microcomputer, shape analysis of particles was performed relatively inexpensively for the microscopic images. The calculation of fundamental shape parameters from a micrograph containing many particles was sequentially carried out by com-

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**Fig. 14** Relationship between CO flow rate and shape factors of agglomerated aerosol particles generated by CVD method. $\Phi$: arithmetic mean circularity, $d$: fractal dimension. AlAA concentration: 10 mol/m$^3$ ($\circ$), 20 mol/m$^3$ ($\bullet$).

**Fig. 15** Relationship between radius of gyration and diameter of the circle of equal perimeter for agglomerated aerosol particles generated by CVD method. CO flow rate: 8.33 cm$^3$/s, equivalence ratio: 2.5, AlAA concentration: 10 mol/m$^3$. 

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**KONA** No. 6 (1988)
bining the classification of the connectivity of the particle boundary and the conventional calculation algorithm of shape parameters. The accuracy of this method is of approximately 2% error when the number of pixels is 40 or more. We have developed the fractal and opening analysis method of a single particle, and the separation method of spherical primary particles from an agglomerate particle. Moreover, the particle shape can be described by parameters that relate to the index of structuring elements number proposed in the opening analysis. In the applications of these methods to agglomerated aerosol particles generated by the two methods, we introduced the fractal dimension for particle groups, which is obtained for the entire range of the particle size distribution. The fractal dimension, in this case, is determined from the relationship between the radius of gyration of a single particle as a standard scale and the perimeter as a measurement value. The dimension can quantitatively express the characteristic change of shape caused by the different conditions of generation, while it cannot be described with the mean value of circularities.

Nomenclature

- \( A \) : area \((m^2)\)
- \( D(n) \) : number of pixels of particles for opening size \( n \) \((-)\)
- \( d \) : fractal dimension \((-)\)
- \( d_{th} \) : threshold level \((-)\)
- \( E_k(i) \) : \( k \)-th end (right) edge point at the scan line \( i \) \((-)\)
- \( F_h \) : horizontal Feret diameter \((m)\)
- \( F_v \) : vertical Feret diameter \((m)\)
- \( G \) : first moment \((m^3)\)
- \( I \) : second moment \((m^4)\)
- \( L \) : length of perimeter \((m)\)
- \( N \) : total number of structuring elements \((-)\)
- \( n \) : number, size \((-)\)
- \( P \) : perimeter, distance function defined in Eqs. (16) and (17) \((m, -)\)
- \( P(n) \) : total number of pixels after opening for size \( n \) \((-)\)
- \( r \) : normalized distance \((-)\)
- \( r_g \) : radius of gyration \((m)\)
- \( S_k(i) \) : \( k \)-th start (left) edge point at the scan line \( i \) \((-)\)
- \( x \) : abscissa \((-)\)
- \( y \) : ordinate \((-)\)
- \( \alpha \) : constant \((-)\)
- \( \beta \) : \( 1 - d \) \((-)\)
- \( \lambda \) : opening size \((-)\)
- \( \rho \) : index of structuring element number \((-)\)

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