Abstract
Coincidence summing effects arises when two or more γ-rays are emitted in a cascade from an excited nucleus and are detected within the resolving time of the detector. Without correction of such effects, the activity of radionuclides cannot be accurately determined. For the correction of summing effects, a new simulation method in GEANT4 was established to simulate the coincidence summing correction factors (CSF, \textit{simu}) for an HPGe detector. In the simulation, a cylindrical and Marinelli beaker source containing several radionuclides were used with different volumes, covering the energy range from 59.50 keV to 1836.01 keV. In the case of volumetric sources, the coincidence summing correction factors for two nuclides (\isotope[60]{Co} and \isotope[88]{Y}) were calculated from the efficiencies at different points throughout the source volume. The dependence of the coincidence correction factor on the sample density was also carried out for some particular nuclide and photon energy. The same methodology of coincidence summing correction factor was applied for the complex decay scheme of \isotope[133]{Ba} and \isotope[152]{Eu} obtained a good agreement with the experimental results.

Keywords
GEANT4, HPGe detector, Coincidence summing, Marinelli beaker sources

Introduction
\γ-ray spectrometry with HPGe detector is widely used to determine the activity of radionuclides in environmental samples. The accurate assessment of the activity of radionuclides would require a minimum source-detector distance to reduce the detection limit of the measuring system. The coincidence summing effect is more significant at a small source-detector distance because the probability of two \γ-rays reaching the detector at the same time cannot be negligible at such distance. The coincidence summing effect changes in the count from the peaks corresponding to the two \γ-rays and nuclides activity become inaccurate if no correction is performed. For the correction of such effects, the contribution of total efficiency is also required with the full energy peak efficiency. Various groups used different calibration techniques and obtained the coincidence summing correction factors (CSFs) from the total efficiency. Debertin & Schötzig \cite{1} used the experimental technique and calculated the CSF from the total efficiency (the ratio of the total number of pulses recorded to the
number of photons emitted by the source). Practically, the total efficiency curve is difficult to achieve due to the single γ-ray emitting nuclides and preparation of standard sources. Several authors used the analytical approaches for the calculation of the CSF from the total efficiency [2-9]. These approaches required information about the nuclear decay parameters such as the mode of parent nuclide decay, conversion factors, and the probability for the γ-ray transition from one energy level to another etc. Z Wang, et al. [10] used the Monte Carlo code MCNP and simulated the total efficiencies for the correction of coincidence summing effect. They used point source to test the coincidence summing correction method and observed a coincidence peak efficiency of at small source-detector distances. However such analysis is difficult to achieve for the close geometry measurements and large volume samples because in volumetric sources the contribution of the scattered γ-rays to the total efficiency cannot be neglected [11]. Many authors proposed an approach of point sources positioned in the matrix of the extended source for the calculation of peak, total efficiencies and CSF [4,12-15]. Tk Wang, et al. [4] include the effects of volume factor in the CSF values and observed a good agreement between calculated and experimental results. Recent techniques [16,17] in GEANT4 were good for the calculation of CSF, but such computational techniques required elaborate work in its implementation.

The aim of this paper is to develop a simplest and modest method in Geant4 for the coincidence summing correction factors (CSFsimu) of the extended sources. The CSFsimu values were compared with the calculated and experimental results reported by Wang, et al. [4] and obtained good agreements.

**Materials and Methods**

GEANT4 [18] toolkit includes simulation of the electromagnetic interaction of charged particle, gamma, and optical photons. The code follows the history of each individual primary photon until its energy dissipated in the detector and produces secondary particles as a result of photoelectric effect, Compton effect, pair production interaction, multiple scattering, bremsstrahlung, and ionization. The secondary electrons formed by photon interaction processes were also taken into consideration in the simulation. GEANT4 electromagnetic physics class was used in the simulation since the energy limit for the electromagnetic process is 10 keV to 100 TeV. Therefore, Ge X-rays of energy below 10 keV cannot be processed. GEANT4 also includes low-level electromagnetic processes that can simulate a particle down to 250 eV. The number of total histories (10⁷ primary photons) was considered for the simulation to obtain a statistical uncertainty of no more than 0.1%. All the photon energies emitted by the source were individually simulated for the source-detector geometries.

Only the γ-rays, which deposit their full energy

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**Figure 1:** A typical decay scheme to show the coincidence summing effect.
in the active volume of the detector, were considered for the evaluation of full energy peak efficiency. The simulated full energy peak efficiencies are obtained from

\[ \varepsilon = \frac{Q}{M} \]  \hspace{1cm} (1)

where \( \varepsilon \) is the full energy peak efficiency, \( Q \) is the number of counts that deposit their full energy in the active detector volume, and \( M \) is the number of total simulated \( \gamma \)-rays counts for a given energy, \( E \).

In order to simulate the total efficiencies and CSFs, a detailed decay scheme is considered as shown in Figure 1. The nuclide A decays to the two excited states of B. The two excited states deexcite by the emission of three \( \gamma \)-rays \( \gamma_1, \gamma_2, \gamma_3 \) with their respective probabilities as \( P_1, P_2, P_3 \).

In absence of coincidence summing, the count rate is given by;

\[ N_1 = A \cdot P_1 \cdot \varepsilon_1 \] \hspace{1cm} (2)

Where \( A \) is the source activity, \( P_1 \) is the emission probability with energy \( E_1 \) and \( \varepsilon_1 \) is peak efficiency for \( \gamma_1 \) with \( E_1 \).

The count rate \( N_1^* \) in the recorded full energy peak will be smaller than \( N_1 \). So the in presence of coincidence summing the count rate is given by

\[ N_1^* = A \cdot P_1 \cdot \varepsilon_1 - A \cdot P_1 \cdot \varepsilon_2 \] \hspace{1cm} (3)

Where \( \varepsilon_2 \) is the total detection efficiency for \( \gamma_2 \). The CSF for \( \gamma_2 \) is given by

\[ \text{CSF}_{\text{Simu}} = 1 - \varepsilon_{T1} \] \hspace{1cm} (4)

\[ \frac{N_1^*}{N_1} = 1 - \varepsilon_{T2} \] \hspace{1cm} (5)

or

\[ \text{CSF}_{\text{Simu}}^b = 1 - \varepsilon_{T2} \] \hspace{1cm} (6)

Similarly for \( \gamma_2 \),

\[ N_2 = A \cdot P_2 \cdot \varepsilon_2 \] \hspace{1cm} (7)

\[ N_2^* = A \cdot P_2 \cdot \varepsilon_2 - A \cdot P_1 \cdot \varepsilon_2 \cdot \varepsilon_{T1} \] \hspace{1cm} (8)

\[ \frac{N_2^*}{N_2} = 1 - \frac{P_1}{P_2} \cdot \varepsilon_{T1} \] \hspace{1cm} (9)

\[ \frac{N_2^*}{N_2} = 1 - \frac{P_1}{P_2} \cdot \varepsilon_{T1} \] \hspace{1cm} (10)

Or

\[ \text{CSF}_{\text{Simu}}^b = 1 - \frac{P_1}{P_2} \cdot \varepsilon_{T1} \] \hspace{1cm} (11)

Where \( \text{CSF}_{\text{Simu}}^a \) and \( \text{CSF}_{\text{Simu}}^b \) are the simulated coincidence summing correction factors, \( \varepsilon_{T1}^a \) and \( \varepsilon_{T2}^b \) are the simulated total efficiencies of 1173.24 keV (a) and 1332.50 keV (b) respectively, similarly for \( ^{68\text{Y}}, ^{133\text{Ba}} \) and \( ^{152\text{Eu}} \).

The coincidence summing effects become more complicated for the extended volume sources. In this case, the correction factor not only depends on the peak and total efficiencies but also on the source volume and the differential efficiency distributions within the source. For volume sources, the CSF is given by

\[ \text{CSF}_{\text{Simu}} = \int \rho \cdot \varepsilon_1 \left( 1 - \varepsilon_{T1} \right) d\rho / \int \rho \cdot \varepsilon_1 d\rho \] \hspace{1cm} (12)

\[ \text{CSF}_{\text{Simu}}^b = \int \rho \cdot \varepsilon_2 \left( 1 - \frac{P_1}{P_2} \cdot \varepsilon_{T1} \right) d\rho / \int \rho \cdot \varepsilon_2 d\rho \] \hspace{1cm} (13)

Or, as a summation,

\[ \text{CSF}_{\text{Simu}} = 1 - \left[ \sum \rho_i \cdot \varepsilon_1 \cdot \varepsilon_{T1} d\rho / \sum \rho_i \cdot \varepsilon_1 d\rho \right] \] \hspace{1cm} (14)

\[ \text{CSF}_{\text{Simu}}^b = 1 - \left[ \sum \rho_i \cdot \varepsilon_2 \frac{P_1}{P_2} \cdot \varepsilon_{T1} d\rho / \sum \rho_i \cdot \varepsilon_2 d\rho \right] \] \hspace{1cm} (15)

Where \( \rho_i \) are the radial positions of the point sources from the beaker axis. Eq 14 and Eq 15 can be written as

\[ \text{CSF}_{\text{Simu}}^a = 1 - \left\langle J_1 \right\rangle \] \hspace{1cm} (16)

For \( h_1 \),

\[ J_{1h_1} = \sum \rho_i \cdot \varepsilon_1 \cdot \varepsilon_{T1} d\rho / \sum \rho_i \cdot \varepsilon_1 d\rho \] \hspace{1cm} (17)

For the whole volume source height,

\[ \left\langle J_1 \right\rangle = \frac{\sum_{h_1} J_{1h_1}}{3} \] \hspace{1cm} (18)

Where \( h_1 \) are the different distances from the beaker bottom. Similarly,

\[ \text{CSF}_{\text{Simu}}^b = 1 - \left\langle J_2 \right\rangle \] \hspace{1cm} (19)

\[ J_{2h_1} = \sum \rho_i \cdot \varepsilon_2 \frac{P_1}{P_2} \cdot \varepsilon_{T1} d\rho / \sum \rho_i \cdot \varepsilon_2 d\rho \] \hspace{1cm} (20)
\[
\langle J_2 \rangle = \frac{\sum_{i=1}^{3} J_{2h_i}}{3}
\]  

(21)

Where \( \langle J_1 \rangle \) and \( \langle J_2 \rangle \) are the average of 15-point integration of efficiencies.

To calculate the coincidence summing correction for all volumes, first, the cylinder and Marinelli beaker volumes are divided into three volumes \( (h_1, h_2, \text{ and } h_3) \) and then further subdivided into 5 volume elements \( J_{1h_i} \) for each \( (h_1, h_2, \text{ and } h_3) \). Every single nuclide in \(^{60}\text{Co}, ^{88}\text{Y}, \text{ and } ^{152}\text{Eu} \) considered as a point source with their respective photon energies and placed at 15 positions within the source volume with three different distances \( (h_1, h_2, \text{ and } h_3) \) from the beaker bottom. To get \( J_{1h_1} \) for 898.02 keV or 1173.24 keV at volume source height \( h_1 \), first computed the \( \varepsilon \) and \( T_{\text{Simu}} \varepsilon \) (at 1836.01 keV or 1332.50 keV) values at 5 different positions in the source volume and then computed the 5-point integration (i.e., multiplied each value by \( \rho_i \), summed them, and divided by the sum of the \( \rho_i \), \( \varepsilon \)). Similarly, calculated \( J_{1h_2} \) (5-point integration of efficiencies) and \( J_{1h_3} \) (5-point integration of efficiencies) at height \( h_2 \) and \( h_3 \) respectively and averaged them to get \( \langle J_2 \rangle \) at 15 volume elements except for the axial position of the beaker. The \( \varepsilon \) and \( T_{\text{Simu}} \varepsilon \) value does not change with the further subdivision of the beaker volume. The same method was applied for 1836.01 keV and 1332.50 keV to obtain \( \langle J_2 \rangle \) but used \( T_{\text{Simu}} \varepsilon \) (898.02 keV and 1173.24 keV) respectively in this case. The CSF values were also obtained for \(^{133}\text{Ba} \) (276.39 keV, 302.85 keV) and \(^{152}\text{Eu} \) (778.9 KeV, 964.0 keV and 444.0 KeV) nuclides using the same procedures.

The detector considered for MC simulation was a p-type coaxial HPGe detector (Canberra). The main parameters of the detector provided by the manufacturer are shown in Figure 2. No information was available by the manufacturer about whether the Ge crystals had rounded edges. Sharp edges of the crystals were assumed in the simulation. First, a cylindrical beaker source of diameter \( (D = 43.4 \text{ mm}) \) filled with gamma radionuclides aqueous solution of volumes \( V_1 \) (50 mL), \( V_2 \) (100 mL), \( V_3 \) (200 mL), and \( V_4 \) (300 mL) was used to obtain the values. A Marinelli

![Figure 2: Schematic of the detector with Marinelli beaker source.](image)
Table 1: Single line and multi gamma ray nuclides with emission probability.

| Nuclide | Energy (keV) | P(%) |
|---------|-------------|------|
| 241Am   | 59.50       |      |
| 109Cd   | 88.02       |      |
| 65Zn    | 111.50      |      |
| 57Co    | 122.06      |      |
| 141Ce   | 145.44      |      |
| 139Ce   | 165.85      |      |
| 51Cr    | 320.08      |      |
| 113Sn   | 391.69      |      |
| 137Cs   | 661.66      |      |
| 54Mn    | 834.84      |      |
| 60Co    | 898.02      | 93.70|
|         | 1836.01     | 99.35|
| 133Ba   | 276.39      | 7.164|
|         | 160.61      | 0.645|
|         | 302.85      | 18.33|
|         | 80.99       | 34.06|
| 152Eu   | 778.9       | 13.06|
|         | 344.3       | 2.79 |
|         | 444.0       | 10.12|
|         | 1085.9      | 14.5 |
|         | 964.0       | 28.81|
|         | 121.8       |      |

In order to simulate the CSF, the total efficiency is always required with the full energy peak efficiency. The simulated full energy peak and total efficiency curves for cylindrical and Marinelli beaker sources with different volumes are shown in Figure 3 and Figure 4. The figures show that the full energy peak and total efficiency increases for the various volumes with the photon energy around 122.06 keV where the maximum values for the full energy peak and total efficiency were obtained. The full energy peak and total efficiency are close to each other at the low energy range because the absorption of the γ-rays in a single photoelectric interaction is predominated only for energies below about 145.44 keV as shown in figures. At high photon energy, the full energy peak efficiency drops off faster than the total efficiency because of the probability of Compton scattering followed by photoelectric absorption of the scattered photon is dominant than the absorption of the full photon energy in a single photoelectric event. As shown in figures the multiple scattering is the dominant contributor to the total efficiency over all but the lowest range of γ-ray energies. The total efficiency drops off slowly with the increased photon energy due to the less probability of scattered photon in the crystal active volume.

The 15-point integration of efficiency ⟨<J⟩⟩ values obtained with our simulation approach is simple and precise to be used to calculate the CSF. The ⟨J⟩ values for the nuclides 60Co and 88Y for the various source volumes are listed in Table 2. The ⟨J⟩ values for each source volumes are smaller at low energies and significantly increase at high energy range as shown in Table 2. The computed ⟨J⟩ value depends on the source volumes. In volumes (50-300 ml) and (450-1000 ml), the ⟨J⟩ values decrease with the increase of source volumes for each photon energy. For Marinelli beaker source the ⟨J⟩ value is greater because of the close contact and the small distance of the source inside in the Marinelli beaker to the detector is shown in Table 2.

The CSF values were simulated for cylindrical and Marinelli beaker sources filled with aqueous solution of density 1 g/cm³. The values of the simulated coincidence summing correction factor (CSF_simu) obtained from Eq. 16 and Eq. 19 for 60Co and 88Y) are shown in Table 3 and Table 4. The CSF_simu is independent of the detector count rate but it is strongly dependent on the full energy peak and total efficiency. The CSF_simu values were
Figure 3: Simulated peak and total efficiencies for cylindrical beaker source.

Figure 4: Simulated peak and total efficiencies for Marinelli beaker source.
nuclide, the \( \text{CSF}_{\text{simu}} \) value is somewhat greater at low photon energy because of the greater \( \langle J \rangle \) value at high photon energy, which means that there is an

table 2: Computed 15-point integration of efficiency values for cylindrical and Marinelli beaker sources.

| Energy (keV) | 15-point integration of efficiencies | Volume (ml) | Volume (ml) |
|-------------|-------------------------------------|-------------|-------------|
| 1173.24     | \( \langle J_1 \rangle \) \( \langle J_2 \rangle \) | 50          | 0.076       |
| 1332.50     | \( \langle J_1 \rangle \) \( \langle J_2 \rangle \) |             | 0.078       |
| 898.02      | \( \langle J_1 \rangle \) \( \langle J_2 \rangle \) |             | 0.043       |
| 1836.01     | \( \langle J_1 \rangle \) \( \langle J_2 \rangle \) |             | 0.099       |
| 1173.24     | \( \langle J_1 \rangle \) \( \langle J_2 \rangle \) | 100         | 0.074       |
| 1332.50     | \( \langle J_1 \rangle \) \( \langle J_2 \rangle \) |             | 0.075       |
| 898.02      | \( \langle J_1 \rangle \) \( \langle J_2 \rangle \) |             | 0.049       |
| 1836.01     | \( \langle J_1 \rangle \) \( \langle J_2 \rangle \) |             | 0.108       |
| 1173.24     | \( \langle J_1 \rangle \) \( \langle J_2 \rangle \) | 200         | 0.073       |
| 1332.50     | \( \langle J_1 \rangle \) \( \langle J_2 \rangle \) |             | 0.075       |
| 898.02      | \( \langle J_1 \rangle \) \( \langle J_2 \rangle \) |             | 0.058       |
| 1836.01     | \( \langle J_1 \rangle \) \( \langle J_2 \rangle \) |             | 0.120       |
| 1173.24     | \( \langle J_1 \rangle \) \( \langle J_2 \rangle \) | 300         | 0.072       |
| 1332.50     | \( \langle J_1 \rangle \) \( \langle J_2 \rangle \) |             | 0.074       |
| 898.02      | \( \langle J_1 \rangle \) \( \langle J_2 \rangle \) |             | 0.062       |
| 1836.01     | \( \langle J_1 \rangle \) \( \langle J_2 \rangle \) |             | 0.128       |

Table 3: Comparison between experimental and simulated coincidence summing correction factors for the cylindrical source.

| Volume (ml) | Energy (keV) | \( \text{CSF}_{\text{Exp}} \) | \( \text{CSF}_{\text{Cal}} \) | \( \text{CSF}_{\text{Simu}} \) | RD (%) |
|------------|-------------|------------------------------|------------------------------|------------------------------|--------|
| 50         | 1173.24     | 0.917                        | 0.923                        | 0.924                        | -0.7   |
|            | 1332.50     | 0.916                        | 0.921                        | 0.922                        | -0.6   |
|            | 898.02      |                             | 0.957                        |                              |        |
|            | 1836.01     |                             | 0.901                        |                              |        |
| 100        | 1173.24     | 0.921                        | 0.927                        | 0.926                        | -0.5   |
|            | 1332.50     | 0.905                        | 0.925                        | 0.925                        | -2.1   |
|            | 898.02      |                             | 0.951                        |                              |        |
|            | 1836.01     |                             | 0.892                        |                              |        |
| 200        | 1173.24     | 0.923                        | 0.935                        | 0.927                        | -0.4   |
|            | 1332.50     | 0.911                        | 0.934                        | 0.925                        | -1.5   |
|            | 898.02      |                             | 0.942                        |                              |        |
|            | 1836.01     |                             | 0.880                        |                              |        |
| 300        | 1173.24     | 0.922                        | 0.938                        | 0.928                        | -0.6   |
|            | 1332.50     | 0.905                        | 0.937                        | 0.926                        | -2.2   |
|            | 898.02      |                             | 0.938                        |                              |        |
|            | 1836.01     |                             | 0.872                        |                              |        |

compared with the experimental and calculated results and obtained good agreement with the relative deviation equal to 2%. For each multi \( \gamma \)-ray nuclide, the \( \text{CSF}_{\text{simu}} \) value is somewhat greater at low photon energy because of the greater \( \langle J \rangle \) value at high photon energy, which means that there is an
Table 4: Comparison between experimental and simulated coincidence summing correction factors for Marinelli beaker source.

| Volume (ml) | Energy (keV) | CSF_{Exp} | CSF_{Cal} | CSF_{Simu} | RD (%) |
|-------------|--------------|-----------|-----------|------------|--------|
| 450         | 1173.24      | 0.875     | 0.901     | 0.880      | -0.5   |
|             | 1332.50      | 0.858     | 0.900     | 0.867      | -0.1   |
|             | 898.02       |           |           | 0.909      |        |
|             | 1836.01      |           |           | 0.847      |        |
| 600         | 1173.24      | 0.874     | 0.906     | 0.882      | -0.9   |
|             | 1332.50      | 0.867     | 0.905     | 0.871      | -0.4   |
|             | 898.02       |           |           | 0.901      |        |
|             | 1836.01      |           |           | 0.831      |        |
| 800         | 1173.24      | 0.872     | 0.909     | 0.886      | -1.6   |
|             | 1332.50      | 0.868     | 0.908     | 0.875      | -0.8   |
|             | 898.02       |           |           | 0.896      |        |
|             | 1836.01      |           |           | 0.824      |        |
| 1000        | 1173.24      | 0.884     | 0.911     | 0.889      | -0.5   |
|             | 1332.50      | 0.877     | 0.910     | 0.891      | -1.5   |
|             | 898.02       |           |           | 0.893      |        |
|             | 1836.01      |           |           | 0.812      |        |

Table 5: Comparison of the simulated coincidence summing correction factors for different densities.

| Volume (ml) | Energy (keV) | CSF_{Simu} | Density | ρ (g/cm³) |
|-------------|--------------|------------|---------|-----------|
| 50          | 1173.24      | 0.920      | 0.924   | 0.945     |
|             | 1332.50      | 0.916      | 0.922   | 0.945     |
| 100         | 1173.24      | 0.922      | 0.926   | 0.950     |
|             | 1332.50      | 0.917      | 0.925   | 0.947     |
| 200         | 1173.24      | 0.923      | 0.927   | 0.962     |
|             | 1332.50      | 0.917      | 0.925   | 0.962     |
| 300         | 1173.24      | 0.924      | 0.928   | 0.971     |
|             | 1332.50      | 0.920      | 0.926   | 0.970     |
| 450         | 1173.24      | 0.859      | 0.880   | 0.931     |
|             | 1332.50      | 0.832      | 0.867   | 0.929     |
| 600         | 1173.24      | 0.828      | 0.882   | 0.937     |
|             | 1332.50      | 0.822      | 0.871   | 0.936     |
| 800         | 1173.24      | 0.810      | 0.886   | 0.948     |
|             | 1332.50      | 0.801      | 0.875   | 0.946     |
| 1000        | 1173.24      | 0.781      | 0.889   | 0.957     |
|             | 1332.50      | 0.772      | 0.891   | 0.956     |

inverse relationship between ⟨J⟩ and CSF_{simu} values.

To observe the sample density effect on the CSF_{simu} value, the simulation was performed for ethanol, water and sea sand sample (major component SiO₂) with densities (0.7, 1 and 2.5 g/cm³) respectively. The comparison of the CSF_{simu} values for cylindrical and Marinelli beaker sources with different sample density are shown in Table 5. When the density of sample increases the CSF_{simu} value increases because the minimum number of γ-rays
Table 6: Comparison of experimental and simulated coincidence summing correction factors of $^{133}$Ba for cylindrical source.

| Volumes (ml) | Energy (keV) | CSF$_{Exp}$ | CSF$_{Cal}$ | $\langle J_{1} \rangle$ $\langle J_{2} \rangle$ | CSF$_{Simu}$ | RD (%) |
|--------------|--------------|--------------|-------------|-----------------------------------|--------------|--------|
| 50           | 276.39       | 0.915        | 0.924       | 0.062                             | 0.938        | -2.5   |
|              | 302.85       | 0.937        | 0.959       | 0.033                             | 0.967        | -3.2   |
| 100          | 276.39       | 0.934        | 0.925       | 0.069                             | 0.931        | 0.3    |
|              | 302.85       | 0.944        | 0.959       | 0.042                             | 0.958        | -1.4   |
| 200          | 276.39       | 0.929        | 0.934       | 0.071                             | 0.929        | 0.0    |
|              | 302.85       | 0.952        | 0.965       | 0.050                             | 0.950        | 0.2    |
| 300          | 276.39       | 0.932        | 0.937       | 0.040                             | 0.960        | -3.0   |
|              | 302.85       | 0.958        | 0.967       | 0.030                             | 0.970        | -1.2   |

Table 7: Comparison of experimental and simulated coincidence summing correction factors of $^{133}$Ba for Marinelli beaker source.

| Volumes (ml) | Energy (keV) | CSF$_{Exp}$ | CSF$_{Calc}$ | $\langle J_{1} \rangle$ $\langle J_{2} \rangle$ | CSF$_{Simu}$ | RD (%) |
|--------------|--------------|--------------|--------------|-----------------------------------|--------------|--------|
| 450          | 276.39       | 0.927        | 0.918        | 0.086                             | 0.914        | 1.4    |
|              | 302.85       | 0.947        | 0.958        | 0.035                             | 0.965        | -1.9   |
| 600          | 276.39       | 0.920        | 0.923        | 0.062                             | 0.938        | -1.9   |
|              | 302.85       | 0.945        | 0.961        | 0.050                             | 0.950        | -0.5   |
| 800          | 276.39       | 0.932        | 0.924        | 0.064                             | 0.936        | -0.4   |
|              | 302.85       | 0.957        | 0.961        | 0.040                             | 0.960        | -0.3   |
| 1000         | 276.39       | 0.932        | 0.924        | 0.066                             | 0.934        | -0.2   |
|              | 302.85       | 0.958        | 0.961        | 0.031                             | 0.969        | -1.1   |

Scattered in the samples itself at greater density. This analysis shows that the CSF value increased with the self-absorption of the source matrix.

The proposed simulated method was also applied to obtain the CSF values of $^{133}$B and $^{152}$Eu. The CSF$_{simu}$ value for $^{133}$B (276.39 keV) was calculated using Eq.16 with total efficiency of 160.61 keV. Similarly, CSF$_{simu}$ value was calculated for 302.85 keV using Eq. 19 with emission probability ratio of (80.99 keV and 302.85 keV) and total efficiency of 80.99 keV. The simulated values were compared with the experimental results for cylindrical and Marinelli beaker sources as shown in Table 6 and Table 7. The simulated results agreed with the experimental results within 2% for all source volumes, except for the 50 ml and 300 ml where they are up to 3%. In the case of $^{152}$Eu, Eq.16 was used to calculate the CSF$_{simu}$ value for (778.9 keV, 964.0 keV and 444.0 KeV) with respect to the total efficiency of 344.3 keV, 1085.9 keV and 121.8 keV. The simulated results were compared with the experimental and calculated CSF values and obtained good agreements with experimental as shown in Table 8 and Table 9.

**Conclusions**

A new method was used in GEANT4 to calculate the coincidence summing correction factors from the peak and total efficiencies and obtained accurate results for $^{60}$Co, $^{88}$Y, $^{133}$Ba and $^{152}$Eu, the average discrepancies between the experimental and simulated results were less than 1%. The simulation was performed and obtained the coincidence summing correction factors for various source volumes and observed the dependence of correction factors value on different samples densities. An easy technique developed in this study for the calculation of coincidence summing correction factor of complex nuclides. The suggested simulation method avoids the preparation of a great variety of gaseous samples with several isotopes and has added the advantages to improve the detection efficiencies for...
Table 8: Comparison of experimental and simulated coincidence summing correction factors of $^{152}$Eu for cylindrical source.

| Volumes (ml) | Energy (keV) | $\text{CSF}_{\text{Exp}}$ | $\text{CSF}_{\text{Cal}}$ | $\langle J_{1}\rangle$ | $\text{CSF}_{\text{Simu}}$ | RD (%) |
|--------------|--------------|--------------------------|--------------------------|----------------|--------------------------|--------|
| 50           | 778.9        | 0.903                    | 0.906                    | 0.083          | 0.917                    | -1.5   |
|              | 964.0        | 0.920                    | 0.933                    | 0.065          | 0.935                    | -0.2   |
|              | 444.0        | 0.902                    | 0.884                    | 0.086          | 0.914                    | -3.0   |
| 100          | 778.9        | 0.894                    | 0.911                    | 0.091          | 0.909                    | -1.6   |
|              | 964.0        | 0.914                    | 0.935                    | 0.077          | 0.923                    | -0.9   |
|              | 444.0        | 0.914                    | 0.889                    | 0.083          | 0.917                    | -0.3   |
| 200          | 778.9        | 0.907                    | 0.921                    | 0.078          | 0.922                    | -1.6   |
|              | 964.0        | 0.925                    | 0.942                    | 0.067          | 0.933                    | -0.8   |
|              | 444.0        | 0.927                    | 0.901                    | 0.065          | 0.935                    | -0.8   |
| 300          | 778.9        | 0.916                    | 0.924                    | 0.071          | 0.929                    | -1.4   |
|              | 964.0        | 0.923                    | 0.946                    | 0.073          | 0.927                    | -0.4   |
|              | 444.0        | 0.932                    | 0.905                    | 0.058          | 0.942                    | -1.0   |

Table 9: Comparison of experimental and simulated coincidence summing correction factors of $^{152}$Eu for Marinelli beaker source.

| Volumes (ml) | Energy (keV) | $\text{CSF}_{\text{Exp}}$ | $\text{CSF}_{\text{Cal}}$ | $\langle J_{1}\rangle$ | $\text{CSF}_{\text{Simu}}$ | RD (%) |
|--------------|--------------|--------------------------|--------------------------|----------------|--------------------------|--------|
| 450          | 778.9        | 0.870                    | 0.887                    | 0.111          | 0.889                    | -2.0   |
|              | 964.0        | 0.894                    | 0.922                    | 0.922          | 0.912                    | -2.0   |
|              | 444.0        | 0.888                    | 0.859                    | 0.095          | 0.905                    | -1.9   |
| 600          | 778.9        | 0.967                    | 0.893                    | 0.030          | 0.970                    | -0.3   |
|              | 964.0        | 0.899                    | 0.927                    | 0.085          | 0.915                    | -1.7   |
|              | 444.0        | 0.894                    | 0.867                    | 0.088          | 0.912                    | -2.0   |
| 800          | 778.9        | 0.870                    | 0.896                    | 0.110          | 0.890                    | -2.2   |
|              | 964.0        | 0.895                    | 0.929                    | 0.090          | 0.910                    | -1.6   |
|              | 444.0        | 0.894                    | 0.870                    | 0.095          | 0.905                    | -1.2   |
| 1000         | 778.9        | 0.875                    | 0.897                    | 0.108          | 0.892                    | -1.9   |
|              | 964.0        | 0.904                    | 0.930                    | 0.087          | 0.913                    | -0.9   |
|              | 444.0        | 0.908                    | 0.972                    | 0.083          | 0.917                    | -0.9   |

the measurement of the activity of environmental samples.

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