Modeling and Analysis of Ferrofluid Transport in Porous Media

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Abstract. As a new kind of material, ferrofluid has received a huge amount of concern and researchers have tried to apply ferrofluid in the field of petroleum engineering. In this paper, the effect of polymer in ferrofluid has been taken into consideration in ferrofluid flooding, a comprehensive mathematical model has been developed. The mathematical model considers the magnetic force, the physical process of the mobilization of polymer through porous media. The results show that using ferrofluid as displacing fluid can greatly improve oil recovery.

1. Introduction

Water flooding is a common way to maintain reservoir pressure and has been used to enhance oil recovery. But the sweep efficiency in strongly heterogeneous reservoirs is extremely low. With the application of ferrofluid in many fields, like reservoir monitoring, researchers have come up with ferrofluid flooding. [1]

Ferrofluid is a kind of stable colloid composed of magnetic nanoparticles coated with a layer of dispersant and liquid carrier. Figure 1 is the schematic of ferrofluid composition. The dispersant is surfactant or polymer, which can prevent particles from sticking together. [2] Ferrofluid performs like true homogeneous fluid and the motion follows the rule of hydrodynamic law, yet is greatly affected by the external magnetic field. These unique characteristics enable ferrofluid to be manipulated by the external magnetic field.

![Figure 1. Schematic of the synthesis of ferrofluid.](image)

Ferrofluid, as one kind of material composed of magnetic nanoparticles, has been the subject of various experimental and numerical studies in the field of petroleum engineering. Oldenburg et al. introduced the magnetic term into the Darcy equation for the first time. [3] And then, Tao Huang et al.
studied the EOR performance of ferrofluid flooding in porous media.\[4\] But to the best of our knowledge, there is no published study on the numerical simulation considering the effect of dispersant in ferrofluid. In our work, we have considered the effects of free polymer existing in the ferrofluid. This paper presents a mathematical model of the ferrofluid and polymer flooding. The results show the huge potential of this new flooding method and the oil recovery has been greatly improved in comparison to water flooding.

2. Flow caused by magnetic forces
Considering ferrofluid as a uniform medium, the attractive force acting on particles is regarded as a body force acting on the ferrofluid on macro-level. Rosensweig gives an expression of the attractive force on the ferrofluid per unit volume: [2]

$$F_m = \mu_0 M \nabla H$$  

(1)

Where $\mu_0$ is the permeability of vacuum and $\mu_0 = 4\pi \times 10^{-7} N \cdot A^{-2}$. $H$ is the magnetic field strength. $M$ is the strength of magnetization and related to the property of magnetic material and the strength of $H$, which can be defined as $M = \alpha \tan^{-1}(BH)$.

Taking consideration of the magnetic force, an additional magnetic force term appears in the Darcy’s Equation. [3]

$$\nabla \cdot (\mu_f \nabla p - \mu_f \nabla D - \mu_0 M \nabla H)$$  

(2)

Where $K$ is permeability tensor, which becomes scalar $k$ in isotropic porous media, $k_r$ is the relative permeability, $\mu$ is fluid viscosity, $p$ is fluid pressure, $D$ denotes highness, $g$ is the gravitational acceleration.

3. Numerical simulation of ferrofluid and polymer flow in porous media

3.1. Governing equations
The mass conservation equation is represented as follows:

$$\frac{\partial}{\partial t}(\rho_\alpha \phi S_\alpha) + \nabla \cdot (\rho_\alpha v_\alpha) = q_\alpha \quad \alpha = o, ff$$  

(3)

Where $\rho_\alpha$ is the phase density, $v_\alpha$ is the phase velocity, $q_\alpha$ is the sink or source term, $\phi$ is the porosity of porous media, $S_\alpha$ is the phase saturation, and the subscript $ff$ represents the ferrofluid phase, $o$ represents the oleic phase.

The polymer behavior is described by an advection-dispersion equation:

$$\frac{\partial}{\partial t}(\phi_c S_c C_p + \rho_m (1 - \phi) C_{pm}) + \nabla \cdot \left[ \mu_f (v_{ff} - \phi S_{ff} D_v C_p) \right] = q_{ff}$$  

(4)

Where $\rho_m$ is the rock density, $C_p$ is the polymer concentration, $C_{pm}$ is the mass of polymer adsorbed by per unit mass rock, $D_v$ is the diffusion-dispersion tensor. Note that, taking the effects of polymer retention into consideration, the velocity of the ferrofluid phase can be modified as follows:

$$v_{ff} = -K \frac{k_{ff}}{R_t \mu_f} \left( \nabla p - \rho_f g VD - \mu_0 M \nabla H \right)$$  

(5)

3.2. Constitutive equations

3.2.1. Polymer rheology. The existence of polymer in ferrofluid increases the viscosity of the mixture, and the variation of viscosity is mainly related to the polymer concentration. Therefore, the following formula is applied to characterize the polymer component concentration to the ferrofluid. [5]

$$\mu_f = \mu_f^0 \left( 1 + a_1 C_p + a_2 C_p^2 + a_3 C_p^3 + \ldots \right)$$  

(6)
Where $\mu_0^{ff}, \mu_f$ are viscosities of the pure ferrofluid and the mixture of ferrofluid and polymer, respectively. And $a_1, a_2, a_3, \ldots$ are coefficients measured from experiments, this means they can be input directly.

### 3.2.2. Polymer Retention

The polymer retention lead to the loss of polymer in the reservoir. The absorbed polymers have effects on increasing the resistance to flow by reducing permeability. The adsorption is described as the Langmuir-type adsorption [6]:

$$C_p = C_{p,\text{max}} \frac{a C_p}{1 + b C_p}$$

Where $C_p$ is the polymer concentration, $C_{p,\text{max}}$ is the mass of polymer adsorbed by per unit mass rock, $C_{p,\text{max}}$ is the saturated adsorption concentration.

### 3.2.3. Inaccessible porous volume

Polymer molecules with long chains cannot enter into some regions of the porous media. Therefore, a factor called inaccessible porous volume is used to describe the volume of those pores volume that cannot be accessed by polymer molecules.[7]

$$\phi_p = (1 - IPV)\phi$$

Where $\phi_p$ is the porous space available for transport of polymer solution, $\phi$ is the original porous volume of the rock.

### 3.2.4. Permeability reduction

The reduction of permeability is a result of polymer retention. Once the polymer molecules come into contact with the surface of the rock, polymer molecules would be adsorbed or desorbed from the rock surface.[8]

$$R_i = 1 + \left( R_{i,\text{max}} - 1 \right) \frac{C_p}{C_{p,\text{max}}}$$

Where $R_i$ is the permeability reduction factor, $R_{i,\text{max}}$ is the maximum value of permeability reduction.

### 4. Examples and Discussion

**Table 1. Properties of the fluids in this simulation.**

| Name                        | Value     |
|-----------------------------|-----------|
| oil viscosity               | 15 mPa·s  |
| ferrofluid viscosity        | 5 mPa·s   |
| water viscosity             | 1 mPa·s   |
| oil density                 | 750 kg/m³ |
| ferrofluid density          | 1100 kg/m³|
| water density               | 1000 kg/m³|

Parameters for calculation of polymer viscosity $(a_1, a_2)$: -0.9113, 3.0881

Parameters for calculation of polymer viscosity $(C_{p,\text{max}}, a, b)$: 0.0002 kg/m³, 0.00012, 9

Considering the situation where there is a high permeable zone between the injector and the producer, the specific diagram is shown in Figure 2(a). The porosity and permeability of high permeable Zone 1 are 0.35 and 3000 mD, respectively. In Zone 2, the porosity and permeability are 0.3 and 500 mD. The fluid properties used in the computation are shown in Table 1. The ferrofluid phase relative permeability is $k_{ff} = S_{ff}^2$, the oil phase relative permeability is $k_{ro} = (1 - S_{ro})^2$. We presume that irreducible ferrofluid saturation and residual oil saturation are 0, initial oil saturation equals to 1. The injection and production rate of ferrofluid are 0.01Vp/day, where $V_p$ is the total pore volume. The polymer concentration in the injected ferrofluid is 1.5 kg/m³. In order to generate a magnetic field, we assume that there are two huge
magnets placed next to the seepage field, one at the top left and the other at the bottom right, and the distribution of the magnetic field strength is shown in Figure 2(b). Moreover, the magnetic parameters of ferrofluid in this simulation are $M_{\text{max}} = 1.596 \times 10^4$ A/m, $\alpha = 1 \times 10^4$ and $\beta = 3.5 \times 10^{-5}$.[9]

![Figure 2](image1.png)

**Figure 2.** The schematic of the physical model and the distribution of magnetic field strength log (H).

![Figure 3](image2.png)

**Figure 3.** Comparison of production index curves of water-flooding, ferrofluid-flooding, and ferrofluid-polymer flooding; (a) Recovery curves; (b) Water cut and ferrofluid cut curves.

Distributions of the water/ferrofluid saturation are presented in Figure 4. It can be seen that the sweep efficiency of injected fluid is low under the condition of water flooding because most of the injected water flows directly into the production well through the high permeability channel. The preferential water pathway will lead to low oil-displacing efficiency and difficulties in subsequent development. The results of ferrofluid-polymer flooding show that the swept area is enlarged, and the remaining oil in reservoir corners can be displaced, which can not be touched in water flooding.

As shown in Figure 3, because of the addition of polymer in the ferrofluid, the viscosity of the injected fluid increases and we can see the water breakthrough time is significantly delayed. Using ferrofluid with polymer for flooding can expand the swept area and enhance oil-displacing efficiency under an external magnetic field. And the oil recovery increased from 30% to 71% compared with water flooding.
5. Conclusion
This study focuses on numerical simulation on an EOR method that combines the advantages of the directional flow of ferrofluid and the effects of traditional polymer flooding. Firstly, the model is established and the fully implicit finite volume method is applied to solve this mathematical model. Then, the calculation results of a 2D example show that ferrofluid-polymer flooding has a great potential for enhancing oil recovery. It can be seen that the injection of ferrofluid-polymer can greatly improve oil recovery when the same volume of fluid is injected into the reservoir, especially when a high conductivity path exists in the reservoir. The water-free production time is prolonged compared and the swept area is expanded, the oil-displacing efficiency is enhanced under an external magnetic field.

Acknowledgments
This work was financially supported by National Natural Science Foundation of China (52004246) and Natural Science Foundation of Zhejiang Province of China (LQ20E040003).

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