Numerical simulation of filtration of a water–gas–oil mixture in a porous medium under the acoustic influence

Yu A Pityuk, S U Fazletdinov, R R Fakhreeva

Center for Micro and Nanoscale Dynamics of Dispersed Systems, Bashkir State University, Ufa, Russia

E-mail: pityukyulia@gmail.com

Abstract. The work is dedicated to the water-alternating-gas injection in oil reservoirs under the influence of a periodic acoustic field. The problem is solved numerically using the control volume method. The mathematical model includes the conservation equations describing the three-phase filtration of water with gas bubbles, oil, and free gas into an element of porous medium taking into account the growth of microbubbles, the formation of free gas, the fluid flow with slippage, the compressibility of a crack network, and the non-isothermal character of the process. Multiparametric calculations are conducted to obtain the dependence of the exposure duration, the residual effect, and amplitude of an acoustic field on the oil displacement. Numerical analysis showed that the acoustic field has a positive effect on the filtration properties of a fluid, which is important for developing the technology of oil recovery.

1. Introduction

Nowadays the development of oil and gas fields is due to an increase in the share of hard-to-recover oil reserves. The presence in the reservoir of different types and compositions of saturating fluids requires the simultaneous application of various exposure technologies.

A special place among the combination of methods for increasing oil recovery is occupied by the technology of water-alternating-gas (WAG) injection in the reservoir, which combines the positive aspects of the technology of oil displacement by high-pressure gas, and the technology of developing an oil reservoir by water flooding, and also eliminates their inherent disadvantages. When using the WAG, the injectivity profile of the near-well part of the formation is aligned near the injection well, the oil displacement coefficient increases and a significant increase in the factor of reservoir coverage is also observed. The WAG allows not only increasing the oil recovery coefficient but also to utilizing large volumes of hydrocarbon gas produced along with oil, which is often flared. In work [1], a calculation methodology of WAG injection into the reservoir was presented and its practical application was shown.

Another technology of enhanced oil recovery is acoustic influence [2], which increases well producibility due to two main effects. The first one is cleaning up the damaged near-wellbore rocks from substances. The second one is eliminating the blocking effects of the residual gas and oil. The authors of [3] presented the results of laboratory studies of the mechanism of oil displacement in the presence of an acoustic field. They showed that ultrasound releases gas from natural oil, which will increase the mobility of the residual oil. The released gas is concentrated in the form of a layer of microbubbles at the oil displacement front, which stimulates the displacement process.
In the present study, the influence of both technologies oil displacement is considered. We presented numerical simulation non-isothermal three-phase filtration of water with gas bubbles, oil, and free gas in an element of a porous medium in the presence of an acoustic field taking into account the growth of microbubbles in pore channels, the formation of free gas, fluid flow with slippage, and crack network compressibility.

2. Problem formulation

We consider the one-dimensional problem of joint non-isothermal displacement of oil by a bubbly liquid (water with gas bubbles) in an element of a porous medium with porosity \( \phi \). This volume of void space contains simultaneously oil saturation \( S_o \), water saturation \( S_w \) with bubble gas saturation \( S_b \) and free gas saturation \( S_g \). Hereinafter, the lower indices “o”, “w”, “b”, and “g” denote oil, water, gas bubbles, and free gas, respectively. According to the definition of saturation\(^{1}\):

\[
1 = S_o + S_w + S_b + S_g
\]

The displacing liquid is a water-gas mixture (the WGM, indicated with lower index «wb»), where water is the continuous carrier phase, and the bubbles are the disperse phase. Then the saturation condition of the WGM can be written as

\[
S_w + S_b = S_{wb}
\]

The WGM filtration rate \( \nu \) is defined as \(^{4}\)

\[
\nu_{wb} = -\frac{kk_{wb} \partial P}{\mu_{wb}} = \nu_w + \nu_b,
\]

where \( P \) is the pressure, \( \mu \) is the viscosity, \( k \) is the reservoir permeability, \( k_r \) is the relative permeability of a phase. The viscosity of the WGM is approximated by the generalized Einstein’s formula \(^{5}\):

\[
\mu_{wb} = \mu_w (1 + \chi \cdot R_b),
\]

where \( \chi = 2.5 \) is the empirical coefficient (this value corresponds to spherical bubbles). The filtration rates of bubbles and water are related to the filtration rate of the WGM in a porous medium as follows

\[
\nu_b = R_b \nu_{wb}, \quad \nu_w = (1 - R_b) \nu_{wb}.
\]

The intensity of the transition of bubbles into the free gas due to their combination is defined as

\[
q_g = \begin{cases} 
0, & R_b \leq R_b^*, \\
V_b \rho_b j_g (R_b - R_b^*), & R_b > R_b^*.
\end{cases}
\]

where \( j_g \) is the number of combined bubbles per unit volume, \( R_b^* \) is the critical value of the volumetric content of bubbles in the WGM, at which the bubbles begin to unite and pass into the free gas.

The mass conservation equations for the oil, water, bubbles in water, and free gas are as follows

\[
\frac{\partial}{\partial t} (\rho S_o \rho_o) - \frac{\partial}{\partial x} \left[ \frac{kk_{wo} \rho_o \partial P}{\mu_o} \right] = 0,
\]

\[
\frac{\partial}{\partial t} (\rho S_w \rho_w) - \frac{\partial}{\partial x} \left[ \frac{(1-R_b)kk_{wb} \rho_w \partial P}{\mu_{wb}} \right] = 0,
\]

\[
\frac{\partial}{\partial t} (\rho S_b \rho_b) - \frac{\partial}{\partial x} \left[ R_b \frac{kk_{wb} \rho_b \partial P}{\mu_{wb}} \right] = -q_g,
\]

\[
\frac{\partial}{\partial t} (\rho S_g \rho_g) - \frac{\partial}{\partial x} \left[ \frac{kk_{wg} \rho_g \partial P}{\mu_g} \right] = q_g,
\]
where \( \rho \) is the density. It is assumed that the liquid phases (water and oil) are incompressible, and the gas density changes according to the equation of state of an ideal gas

\[
\rho_g = \rho_o = \frac{PM}{RT},
\]

where \( T \) is the temperature, \( M \) is the molar mass of gas, \( R \) is the universal gas constant.

The change in temperature in a porous medium is described by the heat equation

\[
(c\rho)_f \frac{\partial T}{\partial t} + (c\rho)_f v_f \frac{\partial T}{\partial x} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right),
\]

where \((c\rho)_f = (c\rho)_o S_w + (c\rho)_o S_o + (c\rho)_g (S_b + S_g)\) is the volumetric heat capacity of the fluid, \((c\rho)_s = \varphi (c\rho)_f + (1 - \varphi) (c\rho)_s\) is the volumetric heat capacity of a saturated porous medium, \(\lambda = \varphi \lambda_w S_w + \lambda_o S_o + \lambda_g (S_b + S_g)\) is the coefficient of thermal conductivity of a saturated porous medium. Here, the index “\( f \)” refers to the saturated porous medium, “\( f \)” refers to the fluid, “\( S \)” refers to the rock.

The fluid filtration rate is calculated as

\[
V_f = V_o + V_{wb} + V_g
\]

The WGM can be considered as a liquid with effective permeability, which takes into account the slippage effect \( k = k_0 \frac{1 + 4b}{r_m} \), where \( b \) is the slip coefficient, \( r_m \) is the average radius of the pore channel, \( k_0 \) is the reservoir permeability, which depends on the pressure.

At the bottomhole, a variable pressure according to the harmonic law is considered

\[
P^b = P^a + P^s \sin(\omega t + \alpha),
\]

where \( P^b \) is the bottomhole pressure, \( P^a \) is the amplitude of bottomhole pressure change, \( \omega \) is the cycle frequency of the bottomhole pressure change, \( \alpha \) is the phase shift.

It is assumed that at the initial time, the porous medium is uniformly saturated with oil and water \( S_{o0}, S_{w0} \) at reservoir pressure \( P_0 \) and temperature \( T_0 \). On the left border, we set the saturations \( S_{o0}^{in}, S_{w0}^{in} \), the pressure \( P^{oi} \) in case of acoustic impact, or \( P^{in} \) in the case without acoustic field, the temperature \( T^{in} \) of the injected WGM. On the right border, we set constant pressure \( P^{out} \) and symmetry condition for saturation and temperature. The problem is solved numerically by the control volume method [5].

3. Results and discussion

We consider an element of a porous medium with the length \( L = 100 \text{ m} \) and the porosity \( \varphi = 0.18 \). At the initial time, the element at reservoir pressure \( P_0 = 200 \text{ atm} \) and temperature \( T_0 = 70^\circ \text{C} \) is evenly saturated with oil \( S_{o0} = 0.8 \) and water \( S_{w0} = 0.2 \). On the left boundary of the element, the WGM is injected with the volumetric content of bubbles \( S_{wb}^{in} = 0.1 \) at temperature \( T^{in} = 20^\circ \text{C} \) and initial bottomhole pressure \( P^{in} = 300 \text{ atm} \), which varies according to the harmonic law with amplitude ranged from 1 to 3 atm, and the frequency \( \omega / (2\pi) = 2 \text{ Hz} \). Such frequency is chosen according to efficiency of acoustic impact at low frequency [6] and optimal time step for the considered process. On the right boundary, we set constant pressure \( P^{out} = 200 \text{ atm} \) and the condition of symmetry in saturation and temperature.

To understand the physical process, we considered changes in such parameters as the viscosity and concentration of bubbles in the WGM (Figure 1), the distribution of pressure and temperature in the porous medium (Figure 2) at different times after the start of injection. It is seen from Figure 1 that
over time, the bubble content (a) in the element increases to a certain critical value, upon reaching which the gas bubbles begin to transform into free gas according to equation (3), the viscosity of the WGM also increases proportionally to the bubble content (b). Figure 2 shows that over time, the pressure curve deviates more and more from the linear law (a), and the cold front of the fluid moves in the porous medium (b).

![Figure 1](image1.png)

**Figure 1.** Distribution of the bubble content in the WGM (a) and the change in the viscosity of the WGM (b) in the porous medium element at 1, 2, and 4 hours after the injection.

![Figure 2](image2.png)

**Figure 2.** Distribution of the pressure and temperature in the porous medium element at 1, 2, and 4 hours after injection.

To determine the effect of the periodic acoustic field on the fluid filtration we conducted a series of calculations and analyzed the oil displacement. We considered 6 periods, 40 minutes each, during four hours of injection. In the first case, filtration was considered without acoustic impact (0/40). In the second case, the periodic acoustic impact was applied during 10 minutes with a break for 30 minutes per period (10/30). And in the third case, the periodic acoustic impact was applied during 20 minutes with a break for 20 minutes per period (20/20). Figure 3 shows that the longer the element is subjected to the acoustic field, the greater is the effect on the WGM filtration. Thus, in the last case (20/20), the residual oil saturation in the reservoir decreases.
Figure 3. Oil saturation (a) and WGM saturation (b) in the porous medium element at different acoustic impacts (20/20, 10/30, 0/40) at four hours after injection.

We examined the residual effect of periodic acoustic impact on the WGM filtration in the porous medium element. Based on numerical simulation, and analysis of the oil displacement is carried out without an acoustic field (Without AF), with a periodic acoustic field during 10 minutes with a break of 30 minutes in the first two hours of injection (AF(2h)), and for four hours (AF(4h)). Thus, three periods of the acoustic impact in the second case, and six periods in the third case are considered. It is found that during two hours of the periodic acoustic influence and then two hours without an acoustic field, the residual oil saturation (Figure 4) is less than without an acoustic field, which indicates the presence of a residual effect from the acoustic impact on the filtration in the reservoir.

Figure 4. Oil saturation (a) and WGM saturation (b) in the porous medium element at different acoustic impacts (4h, 2h, Without AF) at four hours after injection.

A multiparametric analysis was also carried out on the change in the amplitude of the acoustic field. The variable bottomhole pressure of 300 atm with amplitudes $P^a = 1.2, 3 \text{ atm}$ was considered. It is seen from Figure 5 that the impact of the acoustic field with the larger amplitude reduces the residual oil saturation in the reservoir. Thus, the larger acoustic field amplitude, the better are the filtration properties of the WGM.
Figure 5. Oil saturation (a) and WGM saturation (b) in the porous medium element at the different amplitudes of the acoustic influence (1 atm, 2 atm, 3 atm) at four hours after injection.

Conclusions
The multiparametric analysis of the WGM filtration in the porous medium element with periodic bottomhole pressure was conducted based on the developed program code. We presented the effect of the acoustic impact parameters on the oil displacement. It is found that the bubbly liquid displaces oil better for a longer application of the acoustic field since a residual effect from the acoustic impact on the reservoir. At that, the acoustic effect is most effective at a larger amplitude. The obtained numerical results confirmed the effectiveness and feasibility of applying an acoustic field on the oil reservoir during WGM injection.

Acknowledgments
The reported study was funded by RFBR according to the research project No.18-38-20102.

References
[1] Drozdov A N, Narozhnyy I M, Mereutsa A M 2019 IOP Conf. Series: Materials Science and Engineering 675 012023
[2] Kuznetsov O L, Simkin E M, Chilingar G V, Gorfunkel M V, Robertson J O 2002 Energy Sources 24 877
[3] Nikolayevskiy V N, Stepanova G S 2005 Acoustic Physics 51 131
[4] Mikhailov D N 2012 Journal of Applied Mechanics and Technical Physics 53 366
[5] Patankar S 1980 Hemisphere Publishing Corporation 152
[6] Nikolayevskiy V N 1992 Izv. Ross. Akad. Nauk, Ser. Mekh. Zhidk. Gaza 5 110