Error analysis for the rate azimuth platform INS

Fenglin Wang
School of Automation, Nanjing Institute of Technology, Nanjing 211167, China
Corresponding author: zdhxwfl@njit.edu.cn

Abstract. To lower the cost of passive gravity navigation and dynamic gravity measurement, a rate azimuth inertial platform with a gravity sensor on it was put forward to compose a navigator with gravity measuring. Some research about this rate azimuth platform inertial navigation system was carried out, the rate azimuth horizontal reference frame and actual platform reference frame were set up, the error equationaitions, which including position error, velocity error, platform angle error, course error were put up, which would contribute to the integrated system, and the simulation of error for the rate azimuth inertial platform was shown of system errors was shown.

1. Introduction
The inertial navigation system, which can provide overall navigation information, has strong autonomy and short-term high precision, is essential equipment for navigating. The rate azimuth platform inertial navigation system (RAPINS), compared with the typical 3-axis inertial platform navigation system, leaves out the azimuth ring and stabilized loop, uses rate gyro to measure the angular rate of the platform azimuth angle, and then integrates it to calculate the platform azimuth angle[1,2]. Compared with the 3-axis inertial platform navigation system, RAPINS has the advantage of simpler composition, smaller size and better reliability. If a gravity sensor is placed on the platform of RAPINS, a navigator with gravity measurement will be constituted. If combined with digital gravity map, this system would come to a lower cost and simple passive gravity navigation system [3,4]. If combined with GPS, this system would come to a dynamic gravity measuring system [5]. On the basis of the operating principle by the rate azimuth platform inertial navigation system, the rate azimuth horizontal reference frame is set up, and the error equationations, which including position error, velocity error, platform angle error, course error of working equationations were put up under this reference frame.

2. Rate azimuth platform reference frame
The Based on the working principle of rate azimuth platform navigation system, a new rate azimuth horizontal reference frame, which is $kE_kN_kU_k$, also called $kx_ky_kz_k$ is put up, and k reference frame is for short. The geodetic vertical reference frame is $ox_ty_tz_t$, also called $oENU$ reference frame, and $v$ reference frame is for short. The axis of $oE_k$ and $oN_k$ is in the same horizontal plane decided by east axis $oE$ and north axis $oN$, and $oU_k$ has the same direction as $oU$.
At the beginning of the navigation, the rate azimuth platform aims at the geodetic vertical reference frame, which means reference frame $k$ is a coincidence with reference frame $v$. In the process of navigation, due to earth's rotation and vehicle movement, the axis $oU_k$ of the rate azimuth platform will deviate from the axis $oU$, the deviating angle velocity is $\omega_{vkz}$, and the angle from $oN$ to $oN_k$ is called course angle $K$, which is shown as Fig.1. The angle from $oN_k$ to $oN$ is called azimuth angle $\alpha$.

Because the axis $koz$ is in the same direction as the axis $voz$, $k\alpha = -v\alpha$. The transformation matrix $k_vC$ from the rate azimuth horizontal platform reference frame to the geodetic vertical reference frame can be described as follows,

$$
\begin{bmatrix}
\cos \alpha & \sin \alpha & 0 \\
-\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

Due to all kinds of interference, such as error elements from gyro and accelerometer precision, initial navigation parameters error of position, velocity, course angle, installation error, the actual rate azimuth platform and ideal rate azimuth platform, which is the model for reference frame $oEN_kU_k$, do not overlap. The actual azimuth platform reference is $PPox y z$, and $p$ reference frame is for short. Platform angle error $\phi$ is used to describe the transformation from ideal rate azimuth platform reference frame to actual azimuth platform reference. Because the azimuth direction of the actual rate azimuth is open, no stable loop, no need to exert control moment gyroscopes, and therefore no error that exist in the closed loop circuit, then,

$$
\phi_z = 0
$$

In fig.1, the relation among the rate azimuth horizontal reference frame $oEN_kU_k$, the actual rate azimuth platform reference frame $oPPox y z$, and $oENU$ reference frame is given out. The direction cosine matrix from the rate azimuth horizontal reference frame $oEN_kU_k$ to the actual rate azimuth platform reference frame is $C_p^k$, then,

$$
C_p^k = \begin{bmatrix}
1 & 0 & -\phi_y \\
0 & 1 & \phi_x \\
\phi_y & -\phi_x & 1
\end{bmatrix}
$$

The rotating angular rate of the rate azimuth horizontal platform reference frame $ox_ky_kz_k$ to reference frame $oENU$ is described as $\omega_{vkx}$, then,
\[ \omega_{ik}^v = \begin{bmatrix} \omega_{ikx}^v \\ \omega_{iky}^v \\ \omega_{ikz}^v \end{bmatrix}^T = \begin{bmatrix} 0 \\ 0 \\ \dot{\alpha} \end{bmatrix}^T \]  

(4)

The biggest difference between rate azimuth platform inertial navigation system and general platform system is that azimuth gyro outputs angular rate \( \omega_{ikz} \), which is the sum of the vertical component of the earth rotation angle rate \( \omega_{ie} \) and the angular rate of azimuth axis turning around the vertical axis. Thus the azimuth angle can be show as, 

\[ \dot{\alpha} = \omega_{ikz} - \omega_{ie} \sin \varphi - V_k \tan \varphi R_N^{-1} \]  

(5)

Where \( \alpha \) is azimuth angle, \( \omega_{ikz} \) is the output angle rate of azimuth gyro, \( \omega_{ie} \) is the earth rotation angle rate, which marked as \( \dot{\Omega} \), \( \varphi \) is the geodetic latitude, \( V_k \) is the velocity of carrier along east under the geodetic vertical reference frame OENU, \( R_N \) is the prime vertical plane of the curvature radius, and \( \lambda \) is the geodetic longitude. The rotating angular rate of the reference frame \( \alpha x_k y_k z_k \) relative to inertial space is \( \omega_{ik}^k \), which can be described as \( \omega_{ik}^k = \omega_{ie}^k + \omega_{ik}^k \), then,

\[ \begin{bmatrix} \omega_{ikx}^k \\ \omega_{iky}^k \end{bmatrix} = \begin{bmatrix} \sin \alpha \cos \varphi \\ \cos \alpha \cos \varphi \end{bmatrix} + \begin{bmatrix} -V_k \tau^{-1} \\ -V_k R_N \tau^{-1} \end{bmatrix} \]  

(6)

Where \( R_x^{-1} = \cos^2 \alpha R_N^{-1} + \sin \alpha \sin^2 \alpha R_M^{-1} \), \( R_M \) is the curvature radius of the meridian plane, \( R_N^{-1} = \sin \alpha \sin^2 \alpha R_M^{-1} \), \( \tau^{-1} = (R_M^{-1} - R_N^{-1}) \sin \alpha \cos \alpha \), \( \omega_{ikx}^k \) and \( \omega_{iky}^k \) is the ideal platform instruction angular velocity for RAPINS, which is also the horizontal component of \( \omega_{ik}^k \). Because the rate azimuth platform inertial navigation system in the vertical direction is open, which is unnecessary to exert torque, simply need azimuth gyro to measure \( \omega_{ikz} \), which is azimuth angular rate output. On basis of Equation(5), azimuth gyro output \( \omega_{ikz} \) can be described as, 

\[ \omega_{ikz} = \dot{\alpha} + \omega_{ie} \sin \varphi + V_k \tan \varphi R_N^{-1} \]  

(7)

The velocity of carrier the reference frame \( \alpha x_k y_k z_k \) can be calculated by the following equationation,

\[ \begin{bmatrix} \dot{V}_E^k \\ \dot{V}_N^k \\ \dot{V}_E^k \end{bmatrix} = \begin{bmatrix} f_{E_k}^k \\ f_{N_k}^k \\ f_{E_k}^k \end{bmatrix} - \begin{bmatrix} 2 \omega_{ie} c_{32} + \omega_{eky} \\ \omega_{ie} c_{33} + \omega_{ekz} \end{bmatrix} V_k^k + \begin{bmatrix} \omega_{ie} c_{33} + \omega_{ekz} \end{bmatrix} V_N^k + \begin{bmatrix} 2 \omega_{ie} c_{31} + \omega_{ekx} \end{bmatrix} V_E^k \]  

(8)

\[ \begin{bmatrix} \dot{V}_E^k \\ \dot{V}_N^k \\ \dot{V}_E^k \end{bmatrix} = \begin{bmatrix} f_{E_k}^k \\ f_{N_k}^k \\ f_{E_k}^k \end{bmatrix} - \begin{bmatrix} 2 \omega_{ie} c_{31} + \omega_{ekx} \\ \omega_{ie} c_{32} + \omega_{eky} \end{bmatrix} V_N^k + \begin{bmatrix} 2 \omega_{ie} c_{32} + \omega_{eky} \end{bmatrix} V_E^k - g \]  

Where the force \( f_{E_k}^k, f_{N_k}^k \) and \( f_{E_k}^k \) can be measured by accelerometer, the azimuth axis rotating angular rate \( \omega_{ikx}^k \) can be measured by azimuth gyro. \( \omega_{ekx}^k \) and \( \omega_{eky}^k \) can be achieved by the following equationuation,

\[ \begin{bmatrix} \omega_{ekx}^k \\ \omega_{eky}^k \end{bmatrix} = \begin{bmatrix} -\tau^{-1} \\ -R_{y}^{-1} \end{bmatrix} \begin{bmatrix} V_E^k \\ V_N^k \end{bmatrix} \]  

(9)

3. The error equationuations of RAPINs

3.1 The equationuation of platform angle error

The instruction information given by computer, after the transformation, in the form of current is transmitted to the gyro torque to make gyroscope precession, by stabilizing circuit, so the platform
rotates. So, the actual platform motion is associated with instruction angular rate from computer and gyro drift. If the platform error angular rate is \( \omega_{ip} \), then
\[
\begin{bmatrix}
\dot{\phi}_x \\
\dot{\phi}_y \\
\dot{\phi}_z
\end{bmatrix} = \begin{bmatrix}
1 & 0 & -\phi_y \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\omega_{iks} \\
\omega_{iky} \\
\omega_{ikz}
\end{bmatrix} + \begin{bmatrix}
\dot{\phi}_x \\
\dot{\phi}_y \\
\dot{\phi}_z
\end{bmatrix}
\]
(10)
\( \omega_{ik} \) is the angular rate of ideal horizontal platform relative to inertial space in the ideal rate azimuth platform reference frame k, \( \omega_{ip} \) is the angular rate of actual platform relative to inertial space in the reference frame p, and \( \omega_{ip} = \begin{bmatrix}
\omega_{ips} \\
\omega_{ipy} \\
\omega_{ipz}
\end{bmatrix} \), on basis of Equation(3) and Equation(10), then,
\[
\begin{bmatrix}
\phi_x \\
\phi_y \\
\phi_z
\end{bmatrix} = \begin{bmatrix}
\phi_x + (\omega_{c,ix} - \omega_{ikx}) + \epsilon_x \\
\phi_y - (\omega_{c,iy} - \omega_{iky}) + \epsilon_y
\end{bmatrix}
\]
(11)
\( \omega_{px} \) and \( \omega_{py} \) is the actual platform angle rate in the inertial space, which is the horizontal gyro output. Using \( \omega_{c,ix} \) and \( \omega_{c,iy} \) as the actual platform gyro torque imposed by computer, \( \epsilon_x \) and \( \epsilon_y \) as gyro drift, so \( \omega_{px} \) and \( \omega_{py} \) are the sum of gyroscope control information of platform reference axis and gyroscope drift respectively. In view of Equation(11), then the actual platform angle error can be obtained as follows,
\[
\phi_x = \phi_x + (\omega_{c,ix} - \omega_{ikx}) + \epsilon_x
\]
(12)
The gyroscope control instruction information \( \omega_{c,ix} \) and \( \omega_{c,iy} \) can be calculated by Equation(6), which use navigation parameter calculated value \( V_{cx} \), \( V_{cy} \), \( V_{cz} \), \( \varphi_c \) and \( \alpha_c \) to instead of ideal value \( V_{ek} \), \( V_{nk} \), \( V_{uk} \), \( \varphi \) and \( \alpha \). Regarding the earth as a round body simply, that is \( R_M = R_N = R \), the calculation equationuation of actual platform angle error can be deduced as follows,
\[
\begin{bmatrix}
\phi_x \\
\phi_y
\end{bmatrix} = \begin{bmatrix}
\phi_x + \omega_{ikx} - \omega_x \sin\varphi \cos\alpha \Delta\varphi + \omega_x \cos\varphi \cos\alpha \Delta\alpha - \Delta V_{ek} R^{-1} + \epsilon_x \\
\phi_y - \phi_y + \omega_{iky} - \omega_y \sin\varphi \cos\alpha \Delta\varphi - \omega_y \cos\varphi \sin\alpha \Delta\alpha + \Delta V_{ek} R^{-1} + \epsilon_y
\end{bmatrix}
\]
(13)
Where \( \Delta\varphi \) is latitude error, \( \Delta\alpha \) is azimuth angle error, \( \Delta V_{ek} \) and \( \Delta V_{nk} \) are velocity error in the reference frame k.

3.2 The course angle error
Regarding \( \omega_{meas} \) as the azimuth gyro measuring value of the actual rate azimuth platform navigation, which the sum is of \( \omega_{ip} \) and gyro drift \( \epsilon_z \), in view of Equation(11), then,
\[
\omega_{meas} = \begin{bmatrix}
\phi_x & \phi_y & \phi_y
\end{bmatrix} = \begin{bmatrix}
\omega_{iks} & \omega_{iky} & \omega_{ikz}
\end{bmatrix} + \begin{bmatrix}
\epsilon_x \\
\epsilon_y \\
\epsilon_z
\end{bmatrix}
\]
(14)
In Equation(7), Applying calculated parameter value to instead of true value, using gyro measuring value \( \omega_{meas} \) to approximate gyro ideal output \( \omega_{ik} \), the azimuth angle calculation value \( \alpha_c \) can be obtained as \( \dot{\alpha}_c = \omega_{meas} - \omega_{ie} \sin\varphi_c - V_{Ec} \tan\varphi_c R_N^{-1} \). Because \( \Delta\dot{\alpha} = \dot{\alpha}_c - \dot{\alpha} \), in view of Equation(7) and Equation(14), then,
\[
\Delta\dot{\alpha} = \phi_y \omega_{iky} + \phi_y \omega_{iky} + \epsilon_z - \Delta V_{Ek} R_N^{-1} \tan\varphi_c - \omega_{ie} \cos\varphi + V_{E} (R_N \cos^2 \varphi)^{-1} \] \Delta\varphi
(15)
Using \( \phi_{uk} \) as the error of course angle \( k \), then 
\[ \dot{\phi}_{uk} = \Delta \dot{\phi} - \Delta \dot{\phi} . \]
Regarding the earth is round simply, on basis of Equation(15), then the course angle error in rate azimuth horizontal reference frame can be derived as,
\[ \dot{\phi}_{uk} = \omega_{ik} \phi_{ik} - \omega_{ik} \phi_{ik} + ( - \sin kV_{ik} + \cos kV_{ik} ) \Delta \phi + \cos kV_{ik} \Delta V_{ik} + \sin kV_{ik} \Delta V_{ik} + \omega_{ik} \cos \varphi V_{ik} \varphi_{ik} \Delta \varphi - \varepsilon . \]

(16)

3.3 The velocity error
The accelerometer output \( A_p \) on actual platform is the sum of specific force \( f_{pk} \) in the axis direction of platform frame reference \( p \), then 
\[ f^{pk} = c_{ik}^p f^k , \]
\( f^k \) is the specific force in \( k \) reference frame, that is, 
\[ f^k = \begin{bmatrix} f^{ik} \\ f^{ik} \\ f^{ik} \\ f^{ik} \\ f^{ik} \end{bmatrix} + \begin{bmatrix} \Delta A_i \\ \Delta A_i \\ \Delta A_i \\ \Delta A_i \\ \Delta A_i \end{bmatrix} \]
(17)

Where \( \Delta A_i \), \( \Delta A_i \) and \( \Delta A_i \) is accelerometer drift. Using local gravity value \( g \) to approximate using the actual platform accelerometer output \( px, py, pz \) to approximate \( Ekf, Nkf, Ukf \), using \( \omega_{med} \) to approximate \( \omega_{ik} \), in view of Equation(8), the velocity error equationuation in the rate azimuth horizontal reference frame can be derived as,
\[ \Delta V_{ik} = - \omega_{ik} V_{ik} \phi_{ik} + ( \omega_{ik} V_{ik} - f_{ik} ) \phi_{ik} + 2 \omega_{ik} \sin k \cos \varphi V_{ik} \phi_{ik} + V_{ik} \Delta V_{ik} + ( \omega_{ik} \sin \varphi + \omega_{ik} \varphi_{ik} ) \Delta V_{ik} - ( 2 \omega_{ik} \cos \varphi V_{ik} \varphi_{ik} ) \Delta V_{ik} + ( \omega_{ik} \cos \varphi V_{ik} + 2 \omega_{ik} \sin \varphi \cos kV_{ik} ) \Delta \varphi + V_{ik} \Delta \varphi + \Delta A_i \]
\[ \Delta V_{ik} = ( f_{ik} + \omega_{ik} V_{ik} ) \phi_{ik} - \omega_{ik} \Delta V_{ik} \phi_{ik} - 2 \omega_{ik} \cos k \cos \varphi V_{ik} \phi_{ik} - ( \omega_{ik} \sin \varphi + \omega_{ik} \varphi_{ik} ) \Delta V_{ik} - V_{ik} \cos k \cos \varphi V_{ik} \phi_{ik} - ( \omega_{ik} \sin \varphi + \omega_{ik} \varphi_{ik} ) \Delta V_{ik} - V_{ik} \Delta V_{ik} + ( \omega_{ik} \cos \varphi - 2 \omega_{ik} \sin \varphi \varphi_{ik} ) \Delta \varphi - V_{ik} \Delta \varphi + \Delta A_i \]
(18)

3.4 The position error
Because \( \varphi = V_{ik} ( R_{ik} )^{-1} \), \( \lambda = V_{ik} sec \varphi ( R_{ik} )^{-1} \), \( \dot{h} = V_{ik} \), the positon error equationuation can be derived as,
\[ \Delta \varphi = - V_{ik} R_{ik} \phi_{ik} - \sin kV_{ik} \Delta V_{ik} + \cos kV_{ik} \Delta V_{ik} \]
\[ \Delta \lambda = V_{ik} ( R \cos \varphi ) \phi_{ik} + \cos ( kV_{ik} ) \Delta V_{ik} + \sin ( kV_{ik} ) \Delta V_{ik} + V_{ik} \sin \varphi ( R \cos \varphi ) \Delta \varphi \]
(19)

\[ \Delta \dot{h} = \Delta V_{ik} \]
The gyro drift \( \varepsilon_x, \varepsilon_y, \varepsilon_z \) and the accelerometer drift \( \Delta A_i, \Delta A_i, \Delta A_i \) are all regarded as the combination of random constants and white noise, as well as the first-order Markova process.

4. Simulation
The static vehicle with RAPINS has initial conditions and model parameters as follows, initial position of longitude 125° and latitude 30°, RAPINS initial horizontal attitude (pitch and roll) error angles 45°, initial azimuth error angle 2°, initial position error 50m, velocity error 0.3m/s, integrated gyro bias drift and random walk are 0.01°/h and 0.005°/h, rate gyro bias drift and random walk are 0.05°/h and 0.03°/h respectively, three accelerometers bias drift error is 50 µg, the platform angle error curve of RAPINS in 48 hours is shown in Fig. 2, Fig. 3 and Fig. 4.
It can be seen that $\phi_x$ and $\phi_y$ are within $1.5^\circ$, the maximum error of course angle $\phi_U$ is $15^\circ$. At the same time, the maximum velocity error is within 4m/s, and the maximum error of Latitude is 35,000m, while the error of longitude increases to 160,000m with time.

5. Conclusion
Some research about rate azimuth platform inertial navigation system is carried out in this paper, the rate azimuth horizontal reference frame is set up, and the navigation parameter error equationations of this system are deduced, and simulation is given out. Based on these error equationations, kalman filter can be introduced to correct navigation parameter error, and so to keep the gravity sensor pointing vertically.

6. References
[1] Cai Tijing, G. I. Emeliantsev, 2005 Study on rate azimuth platform inertial navigation system Journal of Southeast University 121(1) p 29-32
[2] Wang Fenglin, Wen Xiulan, Cai Tijing, 2009 Federated Kalman filter for rate azimuth inertial platform/log/gravity matching integrated navigation system Journal of Southeast University Vol 39 p 49-54
[3] WEN Chaobin, WANG Yuegang, IANG Xinle 2015 Gravity Aided Navigation Precise Algorithm with Gauss SplineInterpolation Acta Geodaetica et Cartographica Sinica 44(1) p13-18
[4] Zhu Yanhua, Cai Tijing, Liu Ying, 2009 Information fusion method of IMU/log/gravity integrated navigation system Journal of Southeast University 39(6) p 1147-1150
[5] Dong ming, ZENG Xian chao, 2014 An Airborne Gravimeter Based on the Rate Azimuth Platform INS Navigation and control 3(4) p 7-10

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