Explaining $B$ decays anomalies in SUSY models

Dris Boubaa$^{1,2}$, Shaaban Khalil$^3$ and Stefano Moretti$^4$

$^1$Department of Physics, Faculty of Exact Sciences and Computing, Hassiba Benbouali University of Chlef, B.P 78C, Ouled Fares Chlef 02180, Algeria
$^2$Laboratoire de Physique des Particules et Physique Statistique, Ecole Normale Supérieure-Kouba, B.P. 92, 16050, Vieux-Kouba, Algiers, Algeria
$^3$Center for Fundamental Physics, Zewail City of Science and Technology, Sheikh Zayed,12588, Giza, Egypt
$^4$School of Physics and Astronomy, University of Southampton, Highfield, Southampton SO17 1BJ, UK

E-mail: d.boubaa@univ-chlef.dz, skhalil@zewailcity.edu.eg, s.moretti@soton.ac.uk.

Abstract. Recent measurements of certain $B$ decays indicate deviations from Standard Model (SM) predictions. We show that Supersymmetric effects can increase the Branching Ratios (BRs) of both $B \to D\tau\bar{\nu}_\tau$ and $B \to D^*\tau\bar{\nu}_\tau$ with respect to the SM rates, thereby approaching their newest experimentally measured values.

1. Introduction

Semileptonic decays $B \to D^{(*)}\tau\bar{\nu}_\tau$ have been widely studied in the last few years which provide a good opportunity for testing the SM and searching for possible New Physics (NP) Beyond the SM (BSM). In fact, there are continuous efforts being undertaken at $B$ factories, so that the BaBar, Belle and LHCb collaborations continue to update their measurements with ever better precision. The ratios of semileptonic $B$-decay rates,

$$R(D) = \frac{\text{BR}(\bar{B} \to D\tau\bar{\nu}_\tau)}{\text{BR}(\bar{B} \to Dl\bar{\nu}_l)}, \quad R(D^*) = \frac{\text{BR}(\bar{B} \to D^*\tau\bar{\nu}_\tau)}{\text{BR}(\bar{B} \to D^*l\bar{\nu}_l)}, \quad (l = e, \mu),$$

have been measured by the three groups between 2012 and 2019. All measurements are shown in Fig. 1. Combining the experimental data for $R(D)$ and $R(D^*)$ using data from BaBar [1, 2], Belle [3, 4, 5, 6, 7] and LHCb [8, 9, 10], the Heavy Flavor Averaging (HFLAV) Group determined the world averages for 2019 as [11]

$$R(D) = 0.340 \pm 0.027 \pm 0.013,$$

$$R(D^*) = 0.295 \pm 0.011 \pm 0.008,$$

which deviate by $1.4\sigma$ for $R(D)$ and $2.5\sigma$ for $R(D^*)$ from the SM expectations that are given by [11]

$$R_{\text{SM}}(D) = 0.299 \pm 0.003,$$

$$R_{\text{SM}}(D^*) = 0.258 \pm 0.005,$$
In this paper, based on [12], we argue that the recent experimental measurements of the so-called flavour anomalies $B \rightarrow D\tau\bar{\nu}_\tau$ and $B \rightarrow D^*\tau\bar{\nu}_\tau$ can be explained by BSM physics. Specifically, we discuss that SUSY contributions, as described in the Minimal Supersymmetric Standard Model (MSSM) with non-universal soft SUSY-breaking terms, might help to explain the discrepancy between the experimental results for $R(D)$ as well as $R(D^*)$ and the corresponding SM expectations.

2. $R(D)$ and $R(D^*)$ in the MSSM

The effective Hamiltonian for $b \rightarrow c\ell\bar{\nu}_\ell$ is

$$H_{\text{eff}} = \frac{4G_F V_{cb}}{\sqrt{2}}\left[(1 + g_{VL})[\bar{c}\gamma_\mu P_L b][\bar{\nu}_\mu P_L \nu \nu] + g_{VR}[\bar{c}\gamma_\mu P_R b][\bar{\nu}_\mu P_L \nu \nu] + g_{SL}[\bar{c}\mu P_L b][\bar{\nu}_\mu P_L \nu \nu] + g_{SR}[\bar{c}\mu P_R b][\bar{\nu}_\mu P_L \nu \nu] + g_{T}[\bar{c}\sigma^{\mu\nu}\gamma_5 P_L b][\bar{\nu}_{\mu\nu} P_L \nu \nu]\right],$$

(6)

where $G_F$ is the Fermi coupling constant, $V_{cb}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element between charm and bottom quarks while $P_{L/R} = (1 - / + \gamma_5)/2$ are the chirality projection operators. Furthermore, $g_i$ is defined in terms of the Wilson coefficients (see [13] for prospects of extracting these using optimal observables) $C_i$ as $g_i = C_i^{\text{SUSY}}/C_{\text{SM}}$, with $i \equiv VL, VR, SL, SR, T$ and $C_{\text{SM}} = \frac{4G_F V_{cb}}{\sqrt{2}}$. The amplitudes of possible NP contributions to $B \rightarrow D^{(*)}\ell\bar{\nu}_\ell$, $\mathcal{M} \equiv \langle D^{(*)}\ell\bar{\nu}_\ell|H_{\text{eff}}|B\rangle$, can be written in the form [14, 15]

$$\mathcal{M}^{\lambda_2^{(s)}, \lambda_1}_{S(L,R)} = -\frac{G_F}{\sqrt{2}}V_{cb} g_{S(L,R)} H_{S(L,R)}^{\lambda_2^{(s)}} L_{\lambda_1},$$

(7)

$$\mathcal{M}^{\lambda_2^{(s)}, \lambda_1}_{V(L,R)} = \frac{G_F}{\sqrt{2}}V_{cb} g_{V(L,R)} \sum_{\lambda} \eta_{\lambda} H_{V(L,R), \lambda}^{\lambda_2^{(s)}} L_{\lambda_1},$$

(8)

$$\mathcal{M}^{\lambda_2^{(s)}, \lambda_1}_{T} = -\frac{G_F}{\sqrt{2}}V_{cb} g_{T} \sum_{\lambda, \lambda'} \eta_{\lambda} \eta_{\lambda'} H_{\lambda\lambda'}^{\lambda_2^{(s)}} L_{\lambda_1},$$

(9)
The SM amplitude is given by
\begin{equation}
M_{SM}^{\lambda_2^{(+)}\lambda_1} = \frac{G_F}{\sqrt{2}} V_{tb} \sum_\lambda \eta_\lambda H_{VL,\lambda}^{\lambda_2^{(+)}\lambda_1},
\end{equation}

where \( \lambda_l \) is the helicity of the lepton \( l \) and \( \lambda, \lambda' = \pm, 0 \) are the helicity of virtual vector bosons. The \( D^{(s)} \)-meson is taken to be either a spin-0 \( D \)-meson, with \( \lambda_D = s \), or a spin-1 \( D^* \)-meson, with \( \lambda_D^{(s)} = \pm, 0 \). The summation is over the virtual vector boson helicities with the metric \( \eta_\pm = \eta_0 = -\eta_s = 1 \), \( H \)'s and \( L \)'s are the hadronic and leptonic amplitudes which are defined in Refs. \([16, 17, 18, 19, 20]\). Furthermore, one can also define the differential rate for the process \( \bar{B} \to D^{(s)} l \nu_l \) as
\begin{equation}
\frac{d\Gamma}{dq^2 d\cos \theta_l} = \frac{\sqrt{Q^2 Q^- v_l}}{256 \pi^3 m_B} |M(\bar{B} \to D^{(s)} l \nu_l)|^2,
\end{equation}
\begin{equation}
R(D) = \frac{\Gamma(\bar{B} \to D^\tau \nu_\tau)}{\Gamma(\bar{B} \to D l \nu_l)}, \quad R(D^*) = \frac{\Gamma(\bar{B} \to D^{*\tau} \nu_\tau)}{\Gamma(\bar{B} \to D^{*} l \nu_l)}.
\end{equation}

Using the explicit formulae of the hadronic and leptonic amplitudes in Refs. \([14, 15, 16, 17, 21, 22, 23]\) (where the \( l \) contribution is assumed to be described by the SM) and upon fixing the SM parameters and the form factors involved in the definition of the matrix elements to their central values as in Ref. \([2]\), we can cast the explicit dependence of \( R(D) \) and \( R(D^*) \) upon the Wilson coefficients in the MSSM as follows \([12]\):
\begin{align}
R(D) &= R(D)^{SM} \left[ 0.981 |g_{SR} + g_{SL}|^2 + |1 + g_{VL} + g_{VR}|^2 + 0.811 |g_T|^2 + 1.465 \text{ Re}[(1 + g_{VL} + g_{VR}) \times (g_{SR} + g_{SL})^*] + 1.074 \text{ Re}[(1 + g_{VL} + g_{VR}) g_T^*] \right], \\
R(D^*) &= R(D^*)^{SM} \left[ 0.025 |g_{SR} - g_{SL}|^2 + |1 + g_{VL}|^2 + |g_{VR}|^2 + 16.739 |g_T|^2 + 0.994 \text{ Re}[(1 + g_{VL} + g_{VR}) \times (g_{SR} - g_{SL})^*] + 6.513 \text{ Re}[g_{VR} g_T^*] \\
&\quad - 4.457 \text{ Re}[(1 + g_{VL}) g_T^*] - 1.748 \text{ Re}[(1 + g_{VL}) g_{VR}^*] \right].
\end{align}

Thus, in case of a dominant scalar contribution (and negligible vector and tensor ones), it is clear that \( R(D^*) \) cannot be significantly larger than the SM expectation, due to the smallness of the coefficient of this contribution, unless \( |g_{SR} - g_{SL}| \) is much larger than 1 (i.e., \( C^S_{SUSY} > C^S_{SM} \)), which is not possible. This conclusion is confirmed in Fig. 2, where we display the regions in the \((g_{SL}, g_{SR})\) plane that can accommodate the experimental results of \( R(D) \) and \( R(D^*) \) within 1\( \sigma \) and 2\( \sigma \) CL for, e.g., Belle, the experiment with predictions closer to the SM. From this figure, it is clear that the scalar contribution alone cannot account for both \( R(D) \) and \( R(D^*) \) simultaneously. In order to get \( R(D) \) and \( R(D^*) \) within 2\( \sigma \) of the aforementioned average results from the various experiments, \((g_{SL}, g_{SR})\) should lie between \((-0.75, -0.69)\) and \((-0.04, -1.65)\), respectively. In these conditions, either \( g_{SL} \) or \( g_{SR} \) is larger than 1, which is not possible.
Figure 2. The allowed regions in the \((g_{SL}, g_{SR})\) (left) and \((g_{VL}, g_{VR})\) (right) planes by the 1\(\sigma\) and 2\(\sigma\) experimental results on \(R(D)\) (green) and \(R(D^*)\) (orange) of the 2019 averages.

Figure 3. Triangle diagrams (penguins) contributing to, e.g., \(b \rightarrow c(\tau, e)\nu(\tau, e)\) affecting the leptonic vertex.

In case of a dominant vector contribution, as shown from the allowed regions of \((g_{VL}, g_{VR})\) in Fig. 2, one gets \(R(D)\) and \(R(D^*)\) inside the 2\(\sigma\) region of the averages if \((g_{VL}, g_{VR})\) varies between \((0.03, -0.03)\) and \((0.1, 0.02)\), respectively. Furthermore, it is remarkable that, unlike the scalar contribution, a small vector contribution, \(g_{VL} \sim \mathcal{O}(0.1)\) and \(g_{VR} \sim \mathcal{O}(0.01)\), can induce significant enhancement for both \(R(D)\) and \(R(D^*)\): e.g., \(R(D) \sim 0.336\) and \(R(D^*) \sim 0.277\) if \(g_{VL} \sim 0.05\) and \(g_{VR} \sim 0\), which, as we will see, are quite plausible values in the MSSM. Finally, the tensor contribution, which is typically quite small, may affect only \(R(D^*)\).

The SUSY contributions to \(g_{VL}\) are generated from the penguin corrections to the vertex \(W^\pm l\nu_l\) \((l = e, \mu, \tau)\) through the exchange of charginos and neutralinos alongside sleptons and sneutrinos, respectively, as displayed in Fig. 3. Let us now try to decode our results, by concentrating on the Wilson coefficient \(C_{VL}\), which sees contributions induced by the penguin topologies in Fig. 3. Firstly, we can confirm that the graph with neutral Higgs bosons is small while the other two are roughly comparable. Thus, the emerging \(C_{VL}^{\text{SUSY}}\) term is essentially

\[
C_{VL}^{\text{SUSY}} = C_{VL}^\tau + C_{VL}^{\tilde{\nu}} + C_{VL}^{(A^0, H^0, H^0)},
\]

where

\[
C_{VL}^\tau = \frac{\Gamma^L_{\tilde{\chi}_j^0 \tilde{\nu}_{\tau} \tilde{e}_L}}{16\pi^2 M_{W^\pm}^2} \left[ \Gamma^R_{\tilde{\chi}_j^0 \tilde{\tau} \tilde{e}_L} - \Gamma^L_{\tilde{\chi}_j^0 \tilde{\nu}_{e} \tilde{e}_L} \right] \begin{bmatrix} m_{\tilde{\tau}} C_0 (m_{\tilde{\tau}}^2, m_{\tilde{\nu}_{\tau}}^2, m_{\tilde{\nu}_{e}}^2) \\ m_{\tilde{\nu}_{\tau}} C_0 (m_{\tilde{\tau}}^2, m_{\tilde{\nu}_{\tau}}^2, m_{\tilde{\nu}_{e}}^2) \end{bmatrix},
\]

\[
C_{VL}^{\tilde{\nu}} = \frac{\Gamma^L_{\tilde{\chi}_j^0 \tilde{\nu}_{\tau} \tilde{e}_L}}{16\pi^2 M_{W^\pm}^2} \left[ \Gamma^R_{\tilde{\chi}_j^0 \tilde{\tau} \tilde{e}_L} - \Gamma^L_{\tilde{\chi}_j^0 \tilde{\nu}_{e} \tilde{e}_L} \right] \begin{bmatrix} m_{\tilde{\tau}} C_0 (m_{\tilde{\tau}}^2, m_{\tilde{\nu}_{\tau}}^2, m_{\tilde{\nu}_{e}}^2) \\ m_{\tilde{\nu}_{\tau}} C_0 (m_{\tilde{\tau}}^2, m_{\tilde{\nu}_{\tau}}^2, m_{\tilde{\nu}_{e}}^2) \end{bmatrix},
\]

\[
C_{VL}^{(A^0, H^0, H^0)} = \frac{\Gamma^L_{\tilde{\chi}_j^0 \tilde{\nu}_{\tau} \tilde{e}_L}}{16\pi^2 M_{W^\pm}^2} \left[ \Gamma^R_{\tilde{\chi}_j^0 \tilde{\tau} \tilde{e}_L} - \Gamma^L_{\tilde{\chi}_j^0 \tilde{\nu}_{e} \tilde{e}_L} \right] \begin{bmatrix} m_{\tilde{\tau}} C_0 (m_{\tilde{\tau}}^2, m_{\tilde{\nu}_{\tau}}^2, m_{\tilde{\nu}_{e}}^2) \\ m_{\tilde{\nu}_{\tau}} C_0 (m_{\tilde{\tau}}^2, m_{\tilde{\nu}_{\tau}}^2, m_{\tilde{\nu}_{e}}^2) \end{bmatrix},
\]
Figure 4. Behaviour of the last term in Eq. (17), \( m_\chi^2 C_0(m_\chi^2, m_\chi^2, m_\chi^2) \) (right) and \( C_0(m_\chi^2, m_\chi^2, m_\chi^2) \) (left), with \( m_\tau \) for degenerate chargino/neutralino masses.

The Wilson coefficients \( C_{iVL}^{(H^0, h^0)} \) can be obtained from \( C_{iVL}^{A^0} \) by exchanging \( A^0 \leftrightarrow (H^0, h^0) \). The corresponding couplings are given by

\[
\Gamma_{iVL}^{L} = g(Z_{L}^{1I} Z_{L}^{1I} + g \frac{m_{ij}}{\sqrt{2}} W_{\pm} M_{W} \cos \beta Z_{L}^{(I+3)I} Z_{L}^{(I-3)I},
\]

\[
\Gamma_{iVL}^{R} = \frac{g}{\sqrt{2}} Z_{L}^{iI} (\tan \theta_{W} Z_{N}^{kI} + Z_{N}^{kI}) - \frac{m_{ij}}{M_{W} \cos \beta} Z_{L}^{(I+3)I} Z_{L}^{(I-3)I},
\]

\[
\Gamma_{iVL}^{R} = \frac{g}{\sqrt{2}} Z_{\nu}^{iI} (\tan \theta_{W} Z_{N}^{kI} - Z_{N}^{kI}),
\]

\[
\Gamma_{iVL}^{R} = -g Z_{i}^{1I} Z_{\nu}^{I}, \quad \Gamma_{iVL}^{L} = -g Z_{i}^{1I} Z_{N}^{kI} - \frac{1}{\sqrt{2}} Z_{i}^{2I} Z_{N}^{kI},
\]

\[
\Gamma_{iVL}^{R} = -g Z_{i}^{1I} Z_{N}^{kI} - \frac{1}{\sqrt{2}} Z_{i}^{2I} Z_{N}^{kI}, \quad \Gamma_{iVL}^{L} = -g Z_{i}^{1I} Z_{N}^{kI}, \quad \Gamma_{iVL}^{A^0} = -g Z_{i}^{1I} Z_{N}^{kI}.
\]

where \( Z_{L}, Z_{\nu}, Z_{\pm}, Z_{N} \) and \( Z_{(H^0, h^0)} \) are the diagonalising matrices for slepton, sneutrino, chargino, neutralino and Higgs masses, respectively. In addition, the loop functions are given by [24].
\begin{align}
B_0(x,y) &= \eta_e - 1 + \log \frac{x}{\mu^2} - \frac{y \log \frac{y}{x}}{x-y}, \\
C_0(x,y,z) &= \frac{1}{y-z} \left( \frac{y \log \frac{y}{x}}{y-x} + \frac{z \log \frac{z}{x}}{x-z} \right), \\
C_{00}(x,y,z) &= \frac{1}{4} \left( \eta_e - \log \frac{x}{\mu^2} \right) + \frac{3}{8} + \frac{1}{y-z} \left( \frac{y^2 \log \frac{y}{x}}{4(x-y)} - \frac{z^2 \log \frac{z}{x}}{4(x-z)} \right),
\end{align}

with \( \eta_e = \frac{2}{\pi^2} + \log 4\pi\gamma_E \), which is subtracted in the modified Dimensional Regularisation/Reduction (DR) scheme, and \( \bar{\mu} \) the renormalisation scale with the dimensions of mass. Our calculation is based on FlavorKit [25], SARAH [26] and SPheno [27]. Here, a few comments are in order. (i) The loop function \( C_0(m^2_{\tilde{\chi}^0_k}, m^2_{\tilde{\chi}^0_j}, m^2_{\tilde{\tau}_j}) \) to 0 if \( m_{\tilde{\chi}^0_k}, m_{\tilde{\chi}^0_j} \) and \( m_{\tilde{\tau}_j} \) are very heavy, then \( m^2_{\tilde{\tau}_j} C_0(m^2_{\tilde{\chi}^0_k}, m^2_{\tilde{\chi}^0_j}, m^2_{\tilde{\tau}_j}) \) does not vanish, as this is not a decoupling limit since a light fermionic SUSY spectrum is assumed. Specifically, for \( m_{\tilde{\chi}^0_k} \sim m_{\tilde{\chi}^0_j} \), the loop function takes the form

\[ C_0(m^2_{\tilde{\chi}^0_k}, m^2_{\tilde{\chi}^0_j}, m^2_{\tilde{\tau}_j}) = \frac{1}{(m^2_{\tilde{\chi}^0_k} - m^2_{\tilde{\tau}_j})^2} \left[ m^2_{\tilde{\chi}^0_k} - m^2_{\tilde{\tau}_j} + m^2_{\tilde{\chi}^0_j} \log \left( \frac{m^2_{\tilde{\tau}_j}}{m^2_{\tilde{\chi}^0_k}} \right) \right]. \]

(iii) From Eq. (17), one can see that, if \( C_0(m^2_{\tilde{\chi}^0_k}, m^2_{\tilde{\chi}^0_j}, m^2_{\tilde{\tau}_j}) \neq 0 \), then the last term, proportional to \( m^2_{\tilde{\tau}_j} C_0(m^2_{\tilde{\chi}^0_k}, m^2_{\tilde{\chi}^0_j}, m^2_{\tilde{\tau}_j}) \), gives the dominant effect to \( C_{\tilde{V}L} \). These comments are explicitly displayed in Fig. 4. Thus, the typical values of the couplings \( \Gamma^{\tilde{V}L}_{\tilde{\chi}^0_k \tilde{\chi}^0_j \tilde{\tau}_j} \), \( \Gamma^{\tilde{V}L}_{\tilde{\chi}^0_k \tilde{\chi}^0_j \xi_1} \), \( \Gamma^{\tilde{V}L}_{\tilde{\chi}^0_k \tilde{\tau}_j \xi_1} \) and \( \Gamma^{\tilde{V}L}_{\tilde{\chi}^0_k \tilde{\tau}_j \tilde{\chi}^0_j} \) and the loop function \( C_0(m^2_{\tilde{\chi}^0_k}, m^2_{\tilde{\chi}^0_j}, m^2_{\tilde{\tau}_j}) \) at \( m_{\tilde{\chi}^0_k} \sim \mathcal{O}(100) \) GeV and \( m_{\tilde{\tau}_j} \sim \mathcal{O}(1) \) TeV imply that \( C_{\tilde{V}L} \sim \frac{2 \times 10^{-3}}{16 \pi^2 M_{W^\pm}} m^2_{\tilde{\tau}_j} C_0(m^2_{\tilde{\chi}^0_k}, m^2_{\tilde{\chi}^0_j}, m^2_{\tilde{\tau}_j}) \) is of order \( 10^{-8} \) GeV\(^{-2} \). Therefore, \( g_{\tilde{V}L} = C_{\tilde{V}L}/C_{\text{SM}} \), where \( C_{\text{SM}} \sim 1.38 \times 10^{-6} \) GeV\(^{-2} \), can be of order 0.01.

Finally, one should consider a possible constraint due to the direct measurement of the \( W^\pm \) boson decay widths that leads to [28]

\[ \Gamma(W \rightarrow \tau \nu)/\Gamma(W \rightarrow e\nu) = 1.043 \pm 0.024. \]

The SM prediction for this ratio is given by \( \sim 0.999267 \), which is consistent with the measured value. Similarly, constraints can also be obtained from [28]

\[ \Gamma(W \rightarrow \tau \nu)/\Gamma(W \rightarrow \mu \nu) = 1.07 \pm 0.026, \]

with which the SM is also consistent. Another important experimental measurement connected with lepton universality in \( \tau \) decay that should be considered here is of \( \tau \rightarrow \nu_l l\bar{l}_l \) with \( l = e, \mu \), which is given by the relation [29]

\[ \left( \frac{g_\mu}{g_e} \right)^2 = \frac{BR(\tau \rightarrow \mu \nu_\mu)}{BR(\tau \rightarrow e \nu_e)} \frac{f(m^2_{\mu}/m^2_{e})}{f(m^2_{e}/m^2_{\mu})}, \]

In the SM, the universal gauge interaction implies that

\[ \frac{\Gamma(\tau \rightarrow \mu \nu_\mu)}{\Gamma(\tau \rightarrow e \nu_e)} = \frac{f(m^2_{\mu}/m^2_{e})}{f(m^2_{e}/m^2_{\mu})} = 0.9726, \]
where \( f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log(x) \). The current experimental result for this ratio is 0.979 ± 0.004 [28], which gives \( \left( \frac{g_\mu}{g_e} \right)_\tau = 1.0032 ± 0.0002 \). With SUSY contributions, Eq. (34) can be written as

\[
\frac{\Gamma(\tau \to \mu \nu_\tau \nu_\mu)}{\Gamma(\tau \to e \nu_\tau \nu_e)} = 0.9726 \frac{|1 + g_{\nu L}^\mu|^2}{|1 + g_{\nu L}^e|^2},
\]

where \( g_{\nu L}^\mu = C_{\text{SUSY}}(\tau \to \nu_\tau \nu_\mu)/C_{\text{SM}}(\tau \to \nu_\tau \nu_\mu) \) with \( C_{\text{SM}}(\tau \to \nu_\tau \nu_\mu) = 2\sqrt{2} G_F \). (As we will show, this imposes stringent constraints on SUSY contributions to \( g_{\nu L} \).) Furthermore, SUSY loop effects induce a correction to the Fermi coupling via a potential breaking of \( m - e \) universality. In fact, using Eqs. (33) and (35), for \( g_{\nu L} \ll 1 \) one can find

\[
\left( \frac{g_\mu}{g_e} \right)_\tau = \frac{|1 + g_{\nu L}^\mu|}{|1 + g_{\nu L}^e|} = 1 + \Delta g_{\nu L}^{\mu,e},
\]

where \( \Delta g_{\nu L}^{\mu,e} = g_{\nu L}^\mu - g_{\nu L}^e \), so that the above experimental constraints impose that 0.0012 \( \leq \Delta g_{\nu L}^{\mu,e} \leq 0.0052 \). In our work, we will enforce \( g_\mu = g_e = g \), which satisfies Eq. (36).

Furthermore, the oblique Electro-Weak (EW) parameters \( S \), \( T \), and \( U \) [30] are useful to constraint NP that enters in self-energy corrections to a gauge boson propagator, denoted by \( \Pi_{ij} \), which represents the transition \( ij \) (\( i,j = W, Z, \gamma \)), as we have [28]

\[
\hat{\alpha}(M_Z)T = \frac{\Pi_{WW}^{NP}(0)}{M_W^2} - \frac{\Pi_{ZZ}^{NP}(0)}{m_Z^2},
\]

where \( \hat{\alpha}(M_Z) \) is the renormalised Electro-Magnetic (EM) coupling constant at the \( M_Z \) scale. Here, we are interested in the \( T \) parameter. In this respect, a related quantity known as the \( \rho \) parameter is defined as [28]

\[
\rho - 1 = \frac{1}{1 - \hat{\alpha}(M_Z)T} \simeq \hat{\alpha}(M_Z)T.
\]

In this work we take \( \Delta \rho^{\exp} = \rho - 1 = 0.0006 \pm 0.0009 \), which is extracted from the data on the \( T \) parameter (0.08 ± 0.12) [28]. While in the SM \( \rho \equiv \rho_0 = M_W^2/M_Z^2 \cos \theta_W = 1 \) at tree level, in our scan we obtain \( \Delta \rho^{\text{SUSY}} \in [0.0001, 0.0006] \). However, we will focus on the strongest constraint, which is in fact from the decay \( \tau \to \nu_\tau \nu_\tau \nu_\tau \), essentially because it carries the same one-loop corrections of the vertex \( W^\pm \nu_\tau \nu_\tau \) within the process \( b \to c \nu_\tau \nu_\tau \). In order to have sizable loop functions, we will enforce on our scans the condition \( m_{\tilde{\chi}_1^0} \approx m_{\tilde{\chi}_1^\pm} \lesssim 500 \text{ GeV} \). As mentioned, the enhancement of \( C_{\nu L}^T \) occurs mostly when the chargino and neutralino masses are light and similar, in addition to large \( \tan \beta \) and stau mass. Therefore, in our scan, we focus on benchmark points where the gaugino soft masses are given by \( M_1, M_2 \in [110, 500] \text{ GeV} \) and \( M_3 = 2 \text{ TeV} \). Also, we choose the \( \mu \) parameter in [100, 500] GeV, \( m_0^2 \in [0.25 \times 10^4] \text{ GeV}^2 \), the \( A \) terms in [−2000, −100] GeV, \( M_{Q_i} \), \( M_U \), and \( M_D \) are fixed in the TeV range while the slepton soft mass terms \( m_{\tilde{l}} \) and \( m_{\tilde{e}} \) in [100, 5000] GeV. Finally, we take \( \tan \beta \in [5, 70] \).

In Fig. 5 we present the correlation between \( R(D) \) and \( R(D^*) \) at one-loop due to the SUSY contributions to the lepton penguins alone. As can be seen from this plot, in presence of MSSM one-loop corrections, \( R(D) \) can reach 0.335 while \( R(D^*) \) extends to 0.277 (left panel), which are results rather consistent with the Belle measurements shown by the green ellipse (right frame) and not that far from the BaBar ones. Also, the MSSM one-loop corrections leads to rather consistent results for \( R(D) \) (somewhat less so for \( R(D^*) \)) with the averages represented by the red ellipse. This correlation can be understood from the fact that SUSY one-loop corrections give a significant contribution to \( g_{\nu L} \) only (of order 6%) and, hence, according to Eqs. (16)–(17), both \( R(D) \) and \( R(D^*) \) are affected by the same correction factor \( \propto (1 + g_{\nu L})^2 \) through a
common Wilson coefficient. It is also worth noting that the enhancements of $R(D)$ and $R(D^*)$ require a very peculiar region of parameter space of the MSSM, especially in terms of $m_{\tilde{\chi}^\pm_1}$ and $\tan\beta$, wherein, however, all experimental and theoretical constraints sensitive to the latter two quantities are taken into account and included in our scan and numerical analysis. To our knowledge, these enhancements in both $R(D)$ and $R(D^*)$ have never been accounted for before in any NP scenario.

It is also very relevant to extract the typical mass spectra which are responsible for the MSSM configurations yielding $R(D)$ and $R(D^*)$ values (potentially) consistent with experimental measurements, as these might be accessible during Run 3 at the LHC. As an indication, this is done in Fig. 6 for the case of the chargino and neutralino masses (left frame) as well as sneutrino and stau masses (right frame). The plot shows a predilection of the highest $R(D)$ and $R(D^*)$ points for MSSM parameter configurations with $m_{\tilde{\chi}^\pm_1} > m_{\tilde{\chi}^0_1}$ and $m_{\tilde{\tau}_1} > m_{\tilde{\nu}_1}$ while the absolute mass scale can cover the entire interval from 100 GeV to 400 GeV in the first case and from 200 GeV to 5 TeV in the second case. Further, the points with $R(D) > 0.33$ prefer both $m_{\tilde{\chi}^\pm_1}$ and $m_{\tilde{\chi}^0_1}$ below 300 GeV and require a rather large $\tilde{\tau}_1$ and $\tilde{\nu}_1$ masses (say, above 2.5 TeV as well as
large $\tan\beta$). This signals that there occurs an interplay between mass suppressions in the loops and enhancements in the couplings.

3. Conclusion

We have shown that the MSSM has the potential to explain data by BaBar and Belle revealing rather significant anomalies in $R(D)$ and $R(D^*)$. Within this BSM scenario, such excesses can be approached in presence of lightest neutralino/chargino mass degeneracy and large $\tilde{\tau}_1$ and $\tilde{\nu}_1$ masses. Altogether, we found a more than acceptable agreement with both Belle (especially) and BaBar (to a lesser extent) results.

Acknowledgments

DB was supported by the Algerian Ministry of Higher Education and Scientific Research under the PNE Fellowship. SK acknowledges partial support from the Durham IPPP Visiting Academics (DIVA) programme. SM is financed in part through the NExT Institute and the STFC consolidated Grant No. ST/L000296/1. The work of SK and SM was partially supported by the H2020-MSCA-RISE-2014 grant No. 645722 (NonMinimalHiggs).

References

[1] Lees J P et al. [BaBar Collaboration], 2012 Phys. Rev. Lett. 109, 101802.
[2] Lees J P et al. [BaBar Collaboration], 2013 Phys. Rev. D 88, no. 7, 072012.
[3] Huschle M et al. [Belle Collaboration], 2015 Phys. Rev. D 92, no. 7, 072014.
[4] Sato Y et al. [Belle Collaboration], 2016 Phys. Rev. D 94, no. 7, 072007.
[5] Hirose S et al. [Belle Collaboration], 2017 Phys. Rev. Lett. 118, no. 21, 211801.
[6] Hirose S et al. [Belle Collaboration], 2018 Phys. Rev. D 97, no. 1, 012004.
[7] Abdesselam A et al. [Belle Collaboration], 2019 Int. J. Mod. Phys. A 34, no. 32, 1950209.
[8] Bhattacharya S, Nandi S and Patra S K, 2016 Phys. Rev. D 93, 034011.
[9] Hagiwara K, Martin A D and Wade M F, 1990 Nucl. Phys. B 327, 569.
[10] Datta A, Duraisamy M and Ghosh D, 2012 Phys. Rev. D 86, 094025.
[11] Fajfer S, Kamenik J F and Nisandzic I, 2012 Phys. Rev. D 85, 094025.
[12] Crivellin A, Greub C and Kokulu A, 2012 Phys. Rev. D 86, 054014.
[13] Patrignani C, 2016 Chin. Phys. C 40, no. 10, 100001.
[14] Aubert B et al. [BaBar Collaboration], 2010 Phys. Rev. Lett. 105, 051602.
[15] Peskin M E and Takeuchi T, 1990 Phys. Rev. Lett. 65, 964.