Open Inflation With Scalar-tensor Gravity

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The open inflation model recently proposed by Hawking and Turok is investigated in scalar-tensor gravity context. If the dilaton-like field has no potential, the instanton of our model is singular but has a finite action. The Gibbons-Hawking surface term vanishes and hence, can not be used to make $\Omega_0$ nonzero. To obtain a successful open inflation one should introduce other matter fields or a potential for the dilaton-like fields.

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Recently, Hawking and Turok [1, 2] proposed that an open inflation can be obtained by the singular instanton describing quantum creation of a homogeneous open universe with ‘no-boundary’ proposal [3]. Their model does not require fine tuning of parameters to obtain the bubble formation and, at the same time, the slow-roll inflation required in earlier models [4]. However, the HT (Hawking-Turok) instanton solution is singular and the physical nature of this singularity is still controversial [5, 6, 7].

In this paper, we will extend their works to the case with scalar-tensor gravity. Since the quantum creation scenario of the universe is adequate at the Planck scale, it is natural to consider the extended gravity sector which is common to the unified theories such as supergravity, superstring and Kaluza-Klein theory [8]. For example, recently, effective low-energy four dimensional Lagrangians have been obtained from spherical compactifications of string/M-theory [9].

In the no-boundary quantum cosmology, the probability of creation of an universe is given by \( P \propto \exp(-2S_E) \), where \( S_E \) is the Euclidean four action of the instanton. In the simplest version of HT’s model the most probable universe is that with the present density parameter \( \Omega_0 = 0 \). Hence, it was required to introduce the anthropic principle or other matter fields to obtain the nonzero \( \Omega_0 \). The Gibbons-Hawking surface term also contribute to \( S_E \) and play a role in making \( \Omega_0 \) nonzero.

We here investigate how all these facts change when we consider the instanton
with scalar-tensor gravity.

Let us find the instanton solution of our model. The general Euclidean O(4) symmetric metric is

$$ds^2 = d\tau^2 + b^2(\tau)d\Omega_3^2,$$

where $d\Omega_3^2$ is the metric describing $S^3$ and $b$ is the cosmic scale factor. We consider the Euclidean four action which is given by\[10\]

$$S_E = -\int\sqrt{g}d^4x[\xi\phi^2R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - U(\phi) - \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - V(\sigma)],$$

where $\phi$ is a dilaton or Brans-Dicke like field and $\sigma$ is an inflaton field. In this frame $\phi$ couples to gravity non-minimally. For simplicity, we will set $U(\phi) = 0$. Then our gravity sector corresponds to the ordinary Brans-Dicke theory after a conformal rescaling. In this case Brans-Dicke parameter $\omega$, which is constrained to be greater than 500 by experiment, is equal to $1/8\xi$. Hence, $\xi \ll 1$.

From $S_E$ we obtain the equations of motion for $b$ and the scalar fields

$$\frac{\dot{b}^2}{b^2} = -\frac{2b\dot{\phi}}{b\phi} + \frac{1}{b^2} + \frac{1}{6\xi\phi^2}\frac{\dot{\phi}^2}{2} + \frac{\ddot{\phi}^2}{2} - V(\sigma),$$

$$\ddot{\phi} = -3\frac{\dot{b}\dot{\phi}}{b} - \frac{\ddot{\phi}}{\phi} - \frac{1}{1 + 12\xi}\frac{\dot{\sigma}^2}{\phi} + \frac{4}{\phi}V(\sigma),$$

$$\ddot{\sigma} = -3\frac{b\dot{\sigma}}{b} + V'(\sigma),$$

where the dots denote the derivative with respect to $\tau$ and $V'(\sigma) \equiv dV/d\sigma$. The instanton is a solution of the above equations with boundary conditions: $b = 0, \dot{b} = 1$, and $\dot{\phi} = 0 = \dot{\sigma} at \tau = 0$ and $b = 0$ again at some later $\tau = \tau_f$. 

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Fig. 1. shows numerical solutions of the equations with $V(\sigma) = m^2\sigma^2$, where the inflaton mass $m = 10^{13} \text{ GeV}$.

It is nontrivial to show the full behaviour of the system. Hence, we will restrict ourselves to the monotonically increasing and then decreasing solutions where $b$ increases maximally ($\dot{b} = 0$) at $\tau = \tau_m$ and $b = 0$ again at $\tau = \tau_f$ as shown in Fig. 1. Except for the first term the right hand side of Eq.(4) always makes negative contribution to $\ddot{\phi}$. However, for $\tau < \tau_m$ ($\dot{b} > 0$) this term can give only damping force to $\phi$, and $\dot{\phi} \leq 0$. For $\tau \geq \tau_m$, since $\dot{b} \leq 0$ and by continuity, $\dot{\phi}$ remains negative. Hence $\phi$ is a monotonically decreasing function of $\tau$ and becomes zero at the singularity ($\tau = \tau_f$).

The equation for $\sigma$ is the same as that with Einstein gravity. So $\phi$ can have an effect on $\sigma$ only through $b(\tau)$. This implies that it is nontrivial to avoid the singular behaviour of the HT instanton near $\tau = \tau_f$, even with scalar-tensor gravity. Near the singular region and for the sufficiently flat potential we can ignore the $V(\sigma)$ and $V'(\sigma)$ dependent terms, and $\dot{\sigma}$ goes like $b^{-3}$. Since $\dot{b}^2 \geq 0$ and $\phi$ approaches zero near the singularity, we expect

$$\frac{b(\dot{\phi}^2 + \dot{\sigma}^2)}{12\xi\phi^2} \gg \frac{2\dot{b}\dot{\phi}}{\phi},$$

if there is no miraculous cancellation between the terms in Eq.(3).

On the other hand, from the field equations one can obtain the following
equation for $b$:
\[
\ddot{b} = 2 \frac{\dot{b}\dot{\phi}}{\phi} - \frac{b}{6\xi \phi^2} \left[ \dot{\phi}^2 + \frac{1 + 6\xi}{1 + 12\xi} \dot{\sigma}^2 + \frac{1 - 12\xi}{1 + 12\xi} V(\sigma) \right].
\] (7)

In the case where $\xi \ll 1$, using Eq.(3) and Eq.(6) the above equation reduces to
\[
\ddot{b} \simeq -\frac{2\dot{b}^2}{b},
\] (8)

which has a solution $b(\tau) \propto (\tau_f - \tau)^{\frac{1}{3}}$ like in the HT instanton. With Eq.(3) this implies $\sigma(\tau) \propto \ln(\tau_f - \tau)$.

With the metric in Eq.(1) and the $O(4)$ symmetric fields, $S_E$ is given by
\[
S_E = \pi^2 \int d\tau \left\{ b^3 \left[ \frac{\dot{\phi}^2}{2} + \frac{\dot{\sigma}^2}{2} + V(\sigma) \right] + 6\xi \phi^2 (\ddot{b}b^2 + \dot{b}^2 b - b) \right\},
\] (9)

where we have inserted $R = -6b^{-2}(\ddot{b} + \dot{b}^2 - 1)$ into the equation and the integration was taken over the half of $S^3$. Using Eq.(3) in Eq.(9) and integrating by parts, we obtain
\[
S_E = \pi^2 \int_0^{\tau_f} d\tau \left[ b^3 V(\sigma) - 6\xi \phi^2 \dot{b} \right] + 2\pi^2 \xi (b^3)_{\tau = \tau_f},
\] (10)

where the last term should be canceled by the Gibbons-Hawking surface term.

In the approximation of constant fields ($\phi = \phi_0$, $\sigma = \sigma_0$) and the $O(5)$ symmetry ($b(\tau) \simeq H_0^{-1} Sin(H_0\tau)$), the integration yields the usual factor,

$-12\pi^2 M_P(\phi_0)^4/V(\sigma_0)$. Here $M_P(\phi_0) \equiv (2\xi \phi_0^2)^{\frac{1}{4}}$ is the reduced Planck mass corresponding to $\phi_0$ and $H_0$ is the corresponding Hubble parameter. The quantities with the subscript 0 are the values at $\tau = 0$. If we consider only the first term
in $S_E$, it has a minima at $\sigma_0 = 0$ and the most probable universe is that with $\Omega_0 = 0$.

Near the singularity, since $b(\tau) \propto (\tau_f - \tau)^{\frac{1}{3}}$, the integrand in Eq.(10) does not diverge and gives finite contribution to the action. Since $\phi$ goes to zero as $\tau$ approaches $\tau_f$, the surface term vanishes and can not be used to shift the minima of $S_E$ and to get nonzero $\Omega_0$. Therefore, the Euclidean action of the instanton is finite.

What is the role of the four-form field $F_{\mu\nu\rho\lambda}$ in our model? Since the role of the four-form fields is to give a constant contribution to the cosmological constant and the energy momentum tensor, the generic behavior of the other fields does not change.

Adding

$$S_F = \int d^4x \frac{\sqrt{g}}{48} F_{\mu\nu\rho\lambda} F^{\mu\nu\rho\lambda}$$

into $S_E$, one can get

$$S_E \approx -12\pi^2 M_p^4 (\phi_0) \left[ V(\sigma_0) - \frac{F^2}{48} \right],$$

where $F^2 = F_{\mu\nu\rho\lambda} F^{\mu\nu\rho\lambda}$.

However, one must add a total divergence term to obtain a stationary action under variations where the four-form is fixed on the boundary. This term can cancel the $F^2$ term\textsuperscript{[2, 6, 12, 13]}. So, including the four-form field one can not obtain the desired $\Omega_0$ value. But, the field can play a role in the cosmological constant problem.
The vanishing of $\phi$ at the singularity means that the effective gravitational constant diverges there and we could not ignore non-perturbative effects or higher order terms, if our theory is a low energy effective one of some fundamental theory. Hence, it might be necessary to consider the effects of nonzero $U(\phi)$, higher order correction terms, and other matter fields [2, 14, 15, 16].

In summary, in the context of the Brans-Dicke like gravity, we found the finite action instanton which is singular and has a vanishing Gibbons-Hawking surface term. Similar to HT’s model, our model requires additional matter fields or a dilaton potential to obtain a successful open inflation.

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**Figure Caption**

Fig.1 Numerical solutions of the field equations with $\xi = 5 \times 10^{-5}$. The scale of $b(\tau)$ is reduced by 50. The mass unit is the reduced Planck mass at present.
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