Approximated model of primary cosmic radiation

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Abstract. The galactic cosmic rays (CR) are the main ionization source at altitude of ~ 3 to 35 km in the atmosphere. An analytical model is proposed, which describes the intensity of galactic cosmic rays in the inner heliosphere (where energy losses have important role) and the outer heliosphere (where convection-diffusion processes are dominant) [Buchvarova et al, 2011]. On the base of computed model parameters for 1 AU integral CR spectra $D(\geq E)$ for the Earth and outer planet are obtained at solar minimum and maximum. The obtained results are compared with experimental data and theoretical results.

1. Introduction

The intensity of galactic cosmic rays (CR) with energies less than 40 GeV varies during 11-year solar cycle, having their maximum intensity at the solar minimum. The transport of galactic cosmic rays, with the following simplifying assumptions: a steady state flux and no sources of cosmic rays, is described by spherically symmetric transport equation in terms of the differential number density $U(r, E)$ [1]:

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 V U - r^2 k \frac{dU}{dr} \right) - \frac{1}{3r^2} \frac{d}{dE} \left( r^2 V \right) \frac{d}{dE} \left( \alpha \Phi \right) = 0$$

(1)

where $r$ is heliocentric distance and $E$ is particle kinetic energy. The solar wind speed is given by $V(r)$, the particle diffusion coefficient by $k(r, E)$, and $\alpha \Phi = (E + 2E_0) / (E + E_0)$; here $E_0$ the rest energy of a particle and $\alpha \Phi$ is troublesome factor [2]). The terms in equation (1) describe, from left to right, the convection, diffusion of the particles and energy loss in the expanding solar wind [3].

2. Parameterization of galactic cosmic ray spectrum

CRSA model [4] is given in the form:

$$D(E) = D_{\mathrm{LIS}}(E) \left( 1 + \left[ (e - 1) \delta \right] \frac{\mu E V}{E} \right)^{-3\Phi} \left( \frac{A}{\alpha} \right)^{\mu} \left( \frac{A/eZ}{k_2} \right)^{\nu}$$

(2)

where $D(E)$ is the differential intensity, $D_{\mathrm{LIS}}(E)$ is the Local Interstellar Spectrum (LIS), and $\Phi = (eZ/A)\varphi$ is the mean energy loss experienced by particles incoming from distance $R$ [5]; $\varphi$ is the force-field potential (in GV), and $A$ and $Z$ are the mass and charge numbers, respectively. Here $\mu = (1 - \nu)$ is energy loss factor and $\nu$ is convection-diffusion factor [4, 6]. The parameter $\alpha = [(e - 1)/(e - 1)]$, where $e$ is...
the Euler’s number [4]. The coefficient $\Lambda$ is approximately equal to 1.618 [4, 6] and $k_0=k_0(P)$ is a rigidity-dependent diffusion coefficient, $P$ is rigidity. The differential spectrum is given as the number of particles observed per (m$^2$.ster.MeV/nuc).

When $\nu$ is equal to 1 ($\nu=0$) the model equation (2) has the form of the convection-diffusion approximation to the cosmic ray transport equation (1) [4]. By substituting $\mu=1$ ($\nu=0$) equation (2) can be written as [4]

$$D(E) = D_{LIS}(E) \left(1 + \frac{\alpha}{E}\right)^{-\beta}$$

(3)

The quality $\alpha$ has the meaning of energy (at GeV), while $\beta=2\Phi(\Lambda/\alpha)^{\nu}$ is dimensionless one. The model equation (3) is a strictly convex function and it has a unique minimizer [4].

The differential spectrum of the protons and other groups of galactic CR nuclei with account of the anomalous CR is given in the form [7]:

$$D(E) = D_{LIS}(E) \left(1 + \frac{\alpha}{E}\right)^{-\beta} \left\{ \frac{1}{2} \left[1 + \tanh(\lambda (E_c - \mu))\right] \right\}$$

$$+ \left\{ \frac{x}{E^y} \right\} \left\{ \frac{1}{2} \left[1 - \tanh(\lambda (E_c - \mu))\right] \right\}$$

(4)

The first term in equation (4) is related to galactic CR modulation in energy range from ~ 30 MeV to 100 GeV. The second term takes into account anomalous component in energy range from 1.8 MeV to about 30 MeV [7]. The parameters $x$ and $y$ define anomalous cosmic ray modulation in energy range from ~ 1 MeV to 30 MeV. The members with tanh are smoothing functions between galactic and anomalous CR spectra. The physical meaning of $\mu$ [GeV/(nuc)] is the energy $E_{1/2}$ at which differential spectrum of galactic CR and differential spectrum of anomalous CR contribute to $D(E)$ with half of its values [7]. We denote by $E_c$ the energy at which the differential spectrum of galactic CR crosses the differential spectrum of anomalous CR. The dimensionless parameter $\lambda$ determines the contribution of the galactic and anomalous cosmic ray spectrum to energy $E_c$ [7, 8]. In our computations $\lambda = 100$ is chosen. In this case energy value $E_c$ is close to $E_{1/2}$ [8].

3. Cosmic Ray Integral Spectra

Using an analytical expressions for $D(E)$ we can obtain integral spectra of galactic CR [9]:

$$D(\geq E) = \int_{E}^{\infty} D(E) \, dE$$

(5)

$D(\geq E)$ is expressed by the number of particles per unit solid angle, square centimetre, and second, with kinetic energies at least $E$. The integration of the differential spectrum $D(E)$ begins at energy $E$ corresponding to the geomagnetic cut-off rigidity $P$ in the point of measurements [9]. The relationship between the kinetic energy of a particle $E$ and the corresponding geomagnetic cut-off rigidity $P$ is:

$$E = \left[ \left( \frac{eZ}{A} P \right)^2 + E_0^2 \right]^{1/2} - E_0$$

If we substitute the equation (4) into (5) we obtain the integral spectrum of the galactic cosmic rays with account of the anomalous component (1.8 - 30 MeV):

$$D_{GCR, ACR} (\geq E) = \int_{E}^{\infty} D_{LIS}(E) \left(1 + \frac{\alpha}{E}\right)^{-\beta} \left\{ \frac{1}{2} \left[1 + \tanh(\lambda (E_c - \mu))\right] \right\} \, dE$$

$$+ \int_{E}^{\infty} \left\{ \frac{x}{E^y} \right\} \left\{ \frac{1}{2} \left[1 - \tanh(\lambda (E_c - \mu))\right] \right\} \, dE$$

(6)
Experimental data \((E_i, D_i)\) in energy interval 1.8 MeV-100 GeV for protons for ascending part of solar cycle \(\# 20\) (1965-1969) are from \([10, 11]\). The Hillas' spectra \([10]\) are fitted to the model equation (4) with

\[
D_{\text{LIS}} = K(E + E_0)^{-\gamma}
\]

The parameters for protons \(K_p = 14.56 \text{ (GeV}^{2.63}/(\text{s.m}^2\text{ster.MeV})\) and \(\gamma_p = 2.63\) \([11]\) form the spectrum of the galactic cosmic rays above 20 GeV; \(E_0 = 0.938 \text{ GeV}\) is the rest energy of a proton. The calculation of the parameters \(a, \beta, x, y, \text{and } \mu\) is performed by Levenberg - Marquardt algorithm \([9, 12, 13]\) applied to the special case of a least squares. The computer code is realized in algorithmic language C++. On the bases of the received parameter values and equation (6), the calculated values \(D(>E)\) for protons at extreme solar activities for the Earth are obtained.

Mean distances \(r_a\) of the outer planets from the Sun and the parameters for each planet: \(P_{\text{GCR}}\) and \(P_{\text{ACR}}\) of increasing of galactic and anomalous CR intensity (because of solar modulation) are shown in table 1 \([14]\), respectively. The calculated values \(P_{\text{ACR}}\) for each planet are on the bases on average gradients for anomalous protons \(G_r = 7\%/\text{AU}\) \([15]\). The average radial gradient for galactic protons is \(-3\%/\text{AU}\) \([14, 16]\) for \(\geq 0.2 \text{ GeV} \geq 0.71 \text{ GV}\) in the range of intense modulation. From the data analyzed so far the gradient \(G_r\) shows a decrease in rigidity in the GV range \([16]\). We get an average radial gradient \(2\%/\text{AU}\) for \(\geq 0.2 \text{ GeV} \geq 0.71 \text{ GV}\) protons \([17]\). This value decreases with increasing energy (or rigidity) to \(1\%/\text{AU}\) for \(\geq 10 \text{ GeV}/(\text{nuc})\) protons.

Table 1. Values of planetary average distances \(r_a\) from the Sun, parameters \(P_{\text{GCR}}\) of increasing galactic CR intensity for a given average radial gradient \(G_r\) and parameters \(P_{\text{ACR}}\) of increasing anomalous CR intensity for protons \([14]\).

| Planet | Earth | Jupiter | Saturn | Uranus | Neptune |
|--------|-------|---------|--------|--------|---------|
| \(r_a, \text{ AU}\) | 1.00  | 5.2028  | 9.5388 | 19.1914| 30.0611 |
| \(P_{\text{GCR}}(G_r=1\%/\text{AU})\) | 1.00  | 1.05    | 1.10   | 1.19   | 1.30    |
| \(P_{\text{GCR}}(G_r=2\%/\text{AU})\) | 1.00  | 1.10    | 1.19   | 1.38   | 1.60    |
| \(P_{\text{GCR}}(G_r=3\%/\text{AU})\) | 1.00  | 1.16    | 1.29   | 1.58   | 1.90    |
| \(P_{\text{ACR}}(p)\) | 1.00  | 1.36    | 1.67   | 2.34   | 3.10    |

Figure 1. The modeled integral spectra \(D(>E)\) of galactic and anomalous CR protons for maximum and minimum levels of solar activity for Earth and Jupiter. The results are compared with the experimental data: + -Schopper \([18]\), \(\Delta\) - CREME96 model \([19]\) and the computations of Bobik \([20]\) for 2D transport model with drift.

Figure 2. The modeled integral spectra \(D(>E)\) of galactic and anomalous CR protons for maximum and minimum levels of solar activity for Saturn, Uranus and Neptune. The results are compared with the computations of Bobik \([20]\) for 2D transport model with drift.
Integral spectra of the planets: Jupiter, Saturn, Uranus and Neptune are calculated using the obtained integral spectra for the Earth and the values of the parameters $P_{GCR}$ and $P_{ACR}$ for the outer planets from table 1. The integral energy spectra $D(>E)$ of primary protons at solar minimum and maximum for the Earth and planets are given in figures 1 and 2. The computations are compared with experimental data: Schopper [18] and theoretical results: CREME96 [19] for the Earth and 2D stochastic model built from Bobik [20] for the Earth and the outer planets in figures 1 and 2. In our computations integral spectra are obtained only in first approximation. We assume average differential gradient of CR which is independent on the distance in the heliosphere, solar activity level and particle types [14].

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