Nonfactorization in Hadronic Two-body Cabibbo-favored
decays of $D^0$ and $D^+$

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Abstract

With the inclusion of nonfactorized amplitudes in a scheme with $N_c = 3$, we have studied Cabibbo-favored decays of $D^0$ and $D^+$ into two-body hadronic states involving two isospins in the final state. We have shown that it is possible to understand the measured branching ratios and determined the sizes and signs of nonfactorized amplitudes required.

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I. Introduction

In recent past there has been a growing interest [1-7] in exploring the role played by nonfactorized terms in the hadronic decays of charmed and beauty mesons. Ref. [1] and [2] have endeavored to calculate the nonfactorized contribution to two-body hadronic decays of the B meson. These calculations lend support to the \( N_c \to \infty \) rule in two-body hadronic B decays. Experimentally however, the evidence in support [3] of the \( N_c \to \infty \) rule which appeared to be there in the earlier B-decay data has since weakened [9] and the sign of the phenomenological parameter \( a_2 \) appears to be positive [9] contrary to the prediction of the \( N_c \to \infty \) rule.

More recently, the view that the phenomenological parameters \( a_1 \) and \( a_2 \) are effective and process-dependent has been pursued further [3-7]. The effective \( a_1 \) and \( a_2 \), evaluated with \( N_c = 3 \), depend on the nonfactorized contribution. In particular it was shown in Ref. [3] how the conundrum of the failure [10] of all popular models to explain the longitudinal polarization fraction in \( B_0 \to \psi K^*0 \) could be resolved in a scheme that uses \( N_c = 3 \) but allows a small nonfactorized amplitude. This idea was carried over to the charm sector in Ref. [6] where it was shown that with \( N_c = 3 \) allowing nonfactorized terms somewhat larger than in B decays (by nonfactorized terms ‘large’ or ‘small’ we mean: in relation to factorized terms), one could understand data in \( D_s^+ \to \phi \pi^+, \phi \rho^+ \) and \( \phi l^+ \nu_l \) decays. The introduction and description of nonfactorized terms is purely phenomenological in Refs. [3, 6] as is also the case in [3, 4, 7]. No attempt is made to calculate the nonfactorized terms but, rather, the emphasis is to glean some systematic behavior of these terms so that more can be learned about them in future.

With this objective we have studied those hadronic two-body Cabibbo-favored decays of \( D^0, D^+ \) mesons that involve two isospins in the final state in \( N_c = 3 \) scheme. These decays are: \( D \to \bar{K}\pi, \bar{K}^*\pi, \bar{K}\rho, \bar{K}a_1 \) and \( \bar{K}^*\rho \). By fitting data, we have calculated the size and the sign of the nonfactorized term in each decay. Annihilation terms wherever permitted have been neglected in \( D^0 \) decays due to the smallness of \( a_2 \) \((= C_2 + \frac{C_4}{N_c}) \) for \( N_c = 3 \). We have included final-state interaction phases wherever they have been determined experimentally, examples are the decays \( D^0 \to K^-\pi^+, \bar{K}^0\pi^0, D^0 \to K^-\rho^+, \bar{K}^0\rho^0 \) and \( D^0 \to K^*\pi^+, K^*0\pi^0 \). However, we have neglected inelastic final state interactions due to the ignorance of the rescattering parameters to be used in such an analysis.

For decays involving a single Lorentz scalar structure, such as \( D \to \bar{K}\pi, \bar{K}^*\pi, \bar{K}\rho \) and \( \bar{K}a_1 \), one can extract effective \( a_1 \) and \( a_2 \) which we show to be process-dependent. We also argue that color-suppressed decays are more likely to reveal presence or otherwise of nonfactorized effects. This paper is organized as follows: Section II contains the conventions and definitions used throughout. We discuss the decays \( D \to \bar{K}\pi \) in Section III, \( D \to \bar{K}^*\pi, \bar{K}\rho, \bar{K}a_1 \) in Section IV and \( D \to \bar{K}^*\rho \) in Section V. The results are discussed in Section VI.

II. Definitions

The effective Hamiltonian for Cabibbo-favored hadronic charm decays is given by

\[
H_w = \tilde{G}_F \left\{ C_1 (\bar{u}d) (\bar{s}c) + C_2 (\bar{u}c) (\bar{s}d) \right\},
\]

where \( \tilde{G}_F = \frac{G_F}{\sqrt{2}} V_{us} V_{cb}^* \) and \((\bar{u}d)\) etc. represent color-singlet (V-A) Dirac currents. \( C_1 \) and \( C_2 \) are the Wilson coefficients for which we adopt the following values,

\[
C_1 = 1.26 \pm 0.04, \quad C_2 = -0.51 \pm 0.05.
\]
The central values of $C_1$ and $C_2$ are taken from Ref. the and the errors are ours.

Fierz transforming the product of two Dirac currents of eqn.(1) in $N_c$-color-space, we get,

\[
(\bar{u}c) (\bar{s}d) = \frac{1}{N_c} (\bar{u}d) (\bar{s}c) + \frac{1}{2} \sum_{a=1}^{8} (\bar{u}\lambda^a d) (\bar{s}\lambda^a c), \quad (3)
\]

and an analogous relation for $(\bar{u}d) (\bar{s}c)$, where $\lambda^a$ are the Gell-Mann matrices. Using eqn.(3) and its analogue, we reduce the effective Hamiltonian of eqn.(1) to the following forms

\[
H_{w}^{\text{CF}} = \tilde{G}_F \left\{ a_1 (\bar{u}d) (\bar{s}c) + C_2 H_{\text{sc}}^8 \right\}, \quad (4)
\]
\[
\text{and} \quad H_{w}^{\text{CS}} = \tilde{G}_F \left\{ a_2 (\bar{u}c) (\bar{s}d) + C_1 H_{\text{sc}}^8 \right\}, \quad (5)
\]
to describe color-favored (CF) and color-suppressed (CS) decays respectively. The matrix elements of the first terms in (4) and (5) are expected to be dominated by factorized contributions; any nonfactorized part arising from them is parametrized as detailed in the text. The second terms, $H_{w}^{(8)}(\equiv \frac{1}{2} \sum (\bar{u}\lambda^a d)(\bar{s}\lambda^a c))$ and $\tilde{H}_{w}^{(8)}(\equiv \frac{1}{2} \sum (\bar{u}\lambda^a c)(\bar{s}\lambda^a d))$, involving color-octet currents generate nonfactorized contributions. We have defined here for $N_c = 3$,

\[
a_1 = C_1 + \frac{C_2}{3} = 1.09 \pm 0.04, \quad a_2 = C_2 + \frac{C_1}{3} = -0.09 \pm 0.05. \quad (6)
\]

It should be obvious from (4) and (5) that nonfactorized effects are more likely to manifest themselves in color-suppressed decays than in color-favored decays due to the fact that $C_1$ is much larger than $a_2$ in magnitude.

Further, in calculating the factorized amplitudes, we use the following matrix elements [8, 11] for the weak vector ($j_{\mu}^V$) and axial vector ($j_{\mu}^A$) currents between the vacuum and the pseudoscalar (P), vector (V) and axial vector (A) states

\[
\langle V(p, \varepsilon) | j_{\mu}^V | 0 \rangle = \varepsilon^*_\mu m_V f_V, \quad \langle A(p, \varepsilon) | j_{\mu}^A | 0 \rangle = \varepsilon^*_\mu m_A f_A, \quad \langle P(p) | j_{\mu}^A | 0 \rangle = -i f_P p_{\mu}, \quad (7)
\]

and the form factors for the transition of a pseudoscalar meson (M) to pseudoscalar (P) and vector (V) mesons,

\[
\langle P(p') | j_{\mu}^A | M(p) \rangle = \left\{ (p + p')_{\mu} - \frac{m_M^2 - m_P^2}{q^2} q_{\mu} \right\} F_{1}^{MP} \left( q^2 \right) + \frac{m_M^2 - m_P^2}{q^2} q_{\mu} F_{0}^{MP} \left( q^2 \right), \quad (8)
\]
\[
\langle V(p', \varepsilon) | j_{\mu}^V - j_{\mu}^A | M(p) \rangle = i \left\{ (m_M + m_V) \varepsilon^*_\mu A_{1}^{MV} (q^2) - \frac{\varepsilon^*_\mu q}{m_M + m_V} (p + p')_{\mu} A_{2}^{MV} (q^2) - 2m_V \frac{\varepsilon^*_\mu q_{\mu}}{q^2} \left( A_{3}^{MV} (q^2) - A_{0}^{MV} (q^2) \right) \right\} + \frac{2}{m_M + m_V} \varepsilon_{\mu \nu \rho \sigma} \varepsilon^{* \nu} p^\rho p^\sigma V^{MV} (q^2), \quad (9)
\]
where $q_\mu = (p - p')_\mu$. In addition, the following constraint applies at all $q^2$,

$$2 m_V A_3^{MV}(q^2) = (m_M + m_V) A_1^{MV}(q^2) - (m_M - m_V) A_2^{MV}(q^2).$$

(10)

The following relation is needed to cancel the poles at $q^2 = 0$,

$$A_0^{MV}(0) = A_3^{MV}(0).$$

(11)

For an axial vector meson, $A$, we define analogously to (9),

$$A(p', \varepsilon) | j_\mu^V - j_\mu^A | M(p) \rangle = -i \left\{ (m_M + m_A) \varepsilon_\mu V_1^{MA}(q^2) - \frac{\varepsilon \cdot q}{m_M + m_A} (p + p')_\mu V_2^{MA}(q^2) \right. 
- \left. 2 m_A \frac{\varepsilon \cdot q}{q^2} q_\mu \left( V_3^{MA}(q^2) - V_0^{MA}(q^2) \right) \right\} 
- \frac{2}{m_M + m_A} \varepsilon_{\mu \rho \sigma \tau} \varepsilon^{\rho \nu} p^\nu p^\sigma A^{MA}(q^2),$$

(12)

with the same conditions (10) and (11) imposed on $V_i^{MA}(q^2)$.

The branching ratio for $M \to M_1 M_2$, where $M_1$ and $M_2$ are pseudoscalar mesons, is given by

$$B(M \to M_1 M_2) = \tau_M \frac{|\bar{p}|}{8\pi m_M^2} |A(M \to M_1 M_2)|^2,$$

(13)

and that for $M \to V_1 V_2$, where $V_1$ and $V_2$ are vector mesons, is written as

$$B(M \to V_1 V_2) = \tau_M \frac{|\bar{p}|}{8\pi m_M^2} \sum_\lambda |A(M \to V_1 V_2)_{\lambda \lambda}|^2,$$

(14)

where $|\bar{p}|$ is the magnitude of final-state three-momentum in $M$-rest frame, $\tau_M$ is the life time of $M$ and $A(M \to M_1 M_2)$ etc. are the decay amplitudes. The branching ratio formula for $M \to PV$ decay is the same as (14) with a sum over polarizations of $V$.

In the following, we list some of the parameters we have used throughout this paper:

$$f_\pi = 130.7 \text{ MeV}, \quad f_K = 159.8 \text{ MeV},$$
$$f_\rho = 212.0 \text{ MeV}, \quad f_{K^*} = 221.0 \text{ MeV},$$
$$f_{a_1} = 212.0 \text{ MeV},$$
$$V_{cs} = 0.975, \quad V_{ud} = 0.975.$$  

(15)

**III. $D \to P_1 P_2$**

**A. $D^0 \to K^- \pi^+$, $K^0 \pi^0$ and $D^+ \to \bar{K}^0 \pi^+$**

To illustrate our method we write, using eqn. (4) for the effective Hamiltonian, the decay amplitude of $D^0 \to K^- \pi^+$ as,

$$A(D^0 \to K^- \pi^+) = G_F \left\{ a_1 \left< K^- \pi^+ |(\bar{s}c)(\bar{u}d)|D^0 \right> + C_2 \left< K^- \pi^+ |H^{(s)}_\omega|D^0 \right> \right\}.$$  

(16)
We write the first term as a sum of a factorized and a nonfactorized part,
\[
\langle K^{-}\pi^{+}|(\bar{s}c)(\bar{u}d)|D^{0}\rangle = \langle \pi^{+}|(\bar{u}d)|0\rangle \langle K^{-}|(\bar{s}c)|D^{0}\rangle + \langle \pi^{+}K^{-}|(\bar{s}c)(\bar{u}d)|D^{0}\rangle^{nf}
\]
\[
= -if_{\pi}(m_{D}^{2} - m_{K}^{2})(F_{0}^{DK}(m_{\pi}^{2}) + F_{0}^{(1)nf}(m_{\pi}^{2})) ,
\]
where we have defined the nonfactorized matrix element of the product of the color-singlet currents 
\((\bar{s}c)(\bar{u}d)\) as
\[
\langle K^{-}\pi^{+}|(\bar{s}c)(\bar{u}d)|D^{0}\rangle^{nf} \equiv -if_{\pi}(m_{D}^{2} - m_{K}^{2})F_{0}^{(1)nf}(m_{\pi}^{2}) .
\]
For the second term in (16) we write,
\[
\langle K^{-}\pi^{+}|H_{w}^{(8)}|D^{0}\rangle = -if_{\pi}(m_{D}^{2} - m_{K}^{2})F_{0}^{(8)nf}(m_{\pi}^{2}) .
\]
The decay amplitude of eqn. (16) is then written in the form,
\[
A(D^{0} \rightarrow K^{-}\pi^{+}) = -i\tilde{G}_{F}(a_{1}^{eff})_{K\pi}f_{\pi} \left( m_{D}^{2} - m_{K}^{2} \right) F_{0}^{DK}(m_{\pi}^{2}) ,
\]
where,
\[
(a_{1}^{eff})_{K\pi} = a_{1}\left( 1 + \frac{F_{0}^{(1)nf}(m_{\pi}^{2})}{F_{0}^{DK}(m_{\pi}^{2})} + \frac{C_{2}F_{0}^{(8)nf}(m_{\pi}^{2})}{a_{1}F_{0}^{DK}(m_{\pi}^{2})} \right) .
\]
This defines a process-dependent effective \(a_{1}\). We shall see that it is possible to do so for all decays involving a single Lorentz scalar structure. We notice also that as the coefficient \(C_{2}/a_{1}\) (\(\approx -0.47\)) is smaller than unity, the effect of the nonfactorized amplitude is suppressed relative to the factorized amplitude in color-favored decays. For the same reason, the nonfactorized term proportional to \(F_{0}^{(1)nf}\) could compete favorably with \(F_{0}^{(8)nf}\).

The decay amplitude for the color-suppressed decay \(D^{0} \rightarrow \bar{K}^{0}\pi^{0}\) by following an analogous procedure is given by,
\[
A(D^{0} \rightarrow \bar{K}^{0}\pi^{0}) = -\sqrt{2}G_{F}(a_{2}^{eff})_{K\pi}f_{K} \left( m_{D}^{2} - m_{\pi}^{2} \right) F_{0}^{D\pi}(m_{K}^{2}) ,
\]
where,
\[
(a_{2}^{eff})_{K\pi} = a_{2}\left( 1 + \frac{C_{1}F_{0}^{(8)nf}(m_{K}^{2})}{a_{2}F_{0}^{D\pi}(m_{K}^{2})} \right) .
\]
In writing (22) we have used,
\[
\langle \bar{K}^{0}\pi^{0}|(\bar{u}c)(\bar{s}d)|D^{0}\rangle = \langle \bar{K}^{0}|(\bar{s}d)|0\rangle \langle \pi^{0}|(\bar{u}c)|D^{0}\rangle + \langle \bar{K}^{0}\pi^{0}|(\bar{u}c)(\bar{s}d)|D^{0}\rangle^{nf} ,
\]
\[
\approx -i\frac{f_{K}}{\sqrt{2}}(m_{D}^{2} - m_{\pi}^{2})F_{0}^{D\pi}(m_{K}^{2}) ,
\]
\[
\left\langle K^0\pi^0|\bar{H}_w^{(8)}|D^0\right\rangle = -i\frac{f_K}{\sqrt{2}}(m_D^2 - m_{\pi}^2)F_0^{(8)nf}(m_K^2).
\]  

(25)

Now, as \(\frac{C_1}{a_2}\) in eqn. (23) is large \((\approx -14)\), the nonfactorized contribution arising from \(\bar{H}_w^{(8)}\) is greatly enhanced. In contrast, any possible nonfactorized effects in (24) are suppressed due to the smallness of \(a_2\). For this reason we have neglected the nonfactorized contribution in (24).

The amplitude for \(D^+ \rightarrow \bar{K}^0\pi^+\) decay is obtained from eqns. (20) and (22) via the isospin sum rule
\[
A(D^+ \rightarrow \bar{K}^0\pi^+) = A(D^0 \rightarrow K^+\pi^-) + \sqrt{2}A(D^0 \rightarrow K^0\pi^0).
\]  

(26)

In terms of isospin amplitudes \(A_{1/2}\) and \(A_{3/2}\) and the final-state interaction (fsi) phases,
\[
\begin{align*}
A(D^0 \rightarrow K^-\pi^+) &= \frac{1}{\sqrt{3}} \left( A_{3/2}exp(i\delta_{3/2}) + \sqrt{2}A_{1/2}exp(i\delta_{1/2}) \right) \\
A(D^0 \rightarrow \bar{K}^0\pi^0) &= \frac{1}{\sqrt{3}} \left( \sqrt{2}A_{3/2}exp(i\delta_{3/2}) - A_{1/2}exp(i\delta_{1/2}) \right) \\
A(D^+ \rightarrow \bar{K}^0\pi^+) &= \sqrt{3}A_{3/2}exp(i\delta_{3/2})
\end{align*}
\]  

(27)

The relative phase is known \([13]\) to be
\[
\delta_{1/2} - \delta_{3/2} = (86 \pm 8)^0
\]  

(28)

We determine \(A_{1/2}\) and \(A_{3/2}\) by equating eqns. (20) and (22) to eqn. (27) with the phases \(\delta_{1/2}\) and \(\delta_{3/2}\) set equal to zero; and then reinstate the phases to calculate the branching ratios from eqn.(13). This procedure is equivalent to assuming that the effect of fsi in this mode is simply to rotate the isospin amplitudes without effecting their magnitudes. For the form factors we have used the following normalizations at \(q^2 = 0\),
\[
\begin{align*}
F_{0}^{DK}(0) &= 0.77 \pm 0.04, \quad [12] \\
F_{0}^{D\pi}(0) &= 0.83 \pm 0.08. \quad [13, 14]
\end{align*}
\]  

(29)

In practice we have used only the central values of these form factors and extrapolated \(F_{0}^{DK}(q^2)\) and \(F_{0}^{D\pi}(q^2)\) as monopoles with \(0^+\) pole masses of 2.01 and 2.47 GeV respectively as in Ref. \([11]\). As these form factors are needed at a relatively small \(q^2 \approx m_{\pi}^2\) or \(m_{K}^2\), the results are not very sensitive to the manner of extrapolation.

The results are summarized below: Defining
\[
\chi_{K\pi} \equiv \frac{F_{0}^{(8)nf}}{F_{0}^{DK}(m_{\pi}^2)} + \frac{a_1}{C_2} \frac{F_{0}^{(1)nf}}{F_{0}^{DK}(m_{\pi}^2)} \quad \text{and} \quad \xi_{K\pi} \equiv \frac{\bar{F}_{0}^{(8)nf}}{F_{0}^{D\pi}(m_{K}^2)}
\]  

(30)

we get agreement with the data for nonzero \(\chi_{K\pi}\) and \(\xi_{K\pi}\) only with the fsi relative phase lying in the following range,
\[
77^0 \leq \delta_{1/2} - \delta_{3/2} \leq 103^0.
\]  

(31)
Taking $\delta_{1/2}^K - \delta_{3/2}^K = 90^0$, we determine the allowed ranges of $\chi_{K\pi}$ and $\xi_{K\pi}$ to be,

$$-0.16 \leq \chi_{K\pi} \leq -0.08, \quad -0.29 \leq \xi_{K\pi} \leq -0.26.$$  \hfill (32)

These ranges of $\chi_{K\pi}$ and $\xi_{K\pi}$ translate into the following ranges of $(a_{1}^{\text{eff}})_{K\pi}$ and $(a_{2}^{\text{eff}})_{K\pi}$,

$$1.13 \leq (a_{1}^{\text{eff}})_{K\pi} \leq 1.17, \quad \text{and} \quad -0.46 \leq (a_{2}^{\text{eff}})_{K\pi} \leq -0.42.$$  \hfill (33)

In particular, with $\delta_{1/2}^K - \delta_{3/2}^K = 90^0$, $\chi_{K\pi} = -0.12$ and $\xi_{K\pi} = -0.27$, we obtain,

$$B(D^0 \to K^{-}\pi^+) = 3.99\% \text{ (expt.}[15]: (4.01 \pm 0.14)\%)$$
$$B(D^0 \to \bar{K}^{0}\pi^0) = 2.17\% \text{ (expt.}[13]: (2.05 \pm 0.26)\%)$$
$$B(D^+ \to \bar{K}^{0}\pi^+) = 2.76\% \text{ (expt.}[13]: (2.74 \pm 0.29)\%)$$ \hfill (34)

Clearly, with nonfactorized contribution proportionately larger in $D^0 \to \bar{K}^{0}\pi^0$ than in $D^0 \to K^{-}\pi^+$, it is possible to understand data in a scheme with $N_c = 3$. The amount of nonfactorized amplitude needed is reasonably small. We wish to emphasize that an annihilation term, if present, would be much suppressed in our description since such a term would be proportional to $a_2$ which in $N_c = 3$ scheme is only $\approx -0.09$. Past estimates $[13, 14]$ of allowed annihilation terms were based on the $N_c \to \infty$ value of $a_2 = -0.51$. We shall return to a discussion of our numerical estimates of $\chi_{K\pi}$ and $\xi_{K\pi}$ (equivalently $(a_{1}^{\text{eff}})_{K\pi}$ and $(a_{2}^{\text{eff}})_{K\pi}$) in Section VI.

**IV. $D \to PV_1$**

**A. $D^0 \to K^{*-}\pi^+$, $\bar{K}^{*0}\pi^0$ and $D^+ \to \bar{K}^{*0}\pi^+$**

Using the definitions introduced in Section II and the method of calculation detailed for $D \to \bar{K}\pi$ decays, the amplitudes for the decays $D^0 \to K^{*-}$ are given by

$$A(D^0 \to K^{*-}\pi^+) = 2\tilde{G}_Ff_{\pi}m_K^*A_0^{DK^*}(m_\pi^2)(\epsilon^* . p_D)(a_{1}^{\text{eff}})_{K^*\pi},$$
$$A(D^0 \to \bar{K}^{*0}\pi^0) = \sqrt{2}\tilde{G}_Ff_{K^*}m_K^*F_1^{DK}(m_\pi^2)(\epsilon^* . p_D)(a_{2}^{\text{eff}})_{K^*\pi},$$
$$A(D^+ \to \bar{K}^{*0}\pi^+) = A(D^0 \to K^{*-}\pi^+) + \sqrt{2}A(D^0 \to \bar{K}^{*0}\pi^0).$$  \hfill (35)

where

$$\left(a_{1}^{\text{eff}}\right)_{K^*\pi} = a_1\left(1 + \frac{A_0^{(1)nf}}{A_0^{DK^*}(m_\pi^2)} + \frac{C_2}{a_1} \frac{A_0^{(8)nf}}{A_0^{DK^*}(m_\pi^2)}\right),$$
$$\left(a_{2}^{\text{eff}}\right)_{K^*\pi} = a_2\left(1 + \frac{C_1}{a_2} \frac{\tilde{F}_1^{(8)nf}}{\tilde{F}_1^{DK^*}(m_{K^*}^2)}\right).$$  \hfill (36)

In (35) and (36), in addition to (8) and (9), we have used the following definitions

$$\langle K^{*-}\pi^+|(\bar{s}c)(\bar{u}d)|D^0\rangle^{nf} = 2\tilde{G}_Ff_{\pi}m_K^*A_0^{(1)nf}(m_\pi^2)(\epsilon^* . p_D),$$
$$\langle K^{*-}\pi^+|H_8^{(8)}|D^0\rangle = 2\tilde{G}_Ff_{\pi}m_K^*A_0^{(8)nf}(m_\pi^2)(\epsilon^* . p_D),$$

and

$$\langle \bar{K}^{*0}\pi^+|H_8^{(8)}|D^0\rangle = \sqrt{2}\tilde{G}_Ff_{K^*}m_K^*\tilde{F}_1^{(8)nf}(m_\pi^2)(\epsilon^* . p_D).$$  \hfill (37)
It is known \([13]\) that fsi phases in this decay are large, \(\delta_{1/2}^{K^*\pi} - \delta_{3/2}^{K^*\pi} = (103 \pm 17)^0\). To take the fsi phases into account we follow a procedure similar to that for \(D \to K\pi\) decays; we calculate the isospin amplitudes by equating the amplitudes in (35) to those in (27) with phases set equal to zero. Having so determined \(A_{1/2}\) and \(A_{3/2}\), we reinstate the phases. For the form factors we have used the following normalizations at \(q^2 = 0\),

\[
A_{0}^{DK^*}(0) = 0.70 \pm 0.09, \quad [12, 13]
\]

\[
F_{1}^{D\pi}(0) = 0.83 \pm 0.08. \quad [13, 14]
\]

In actual calculation we have used only the central values and we have considered monopole (referred to as BSWI hereafter) as well as dipole (referred to as BSWII hereafter) forms for the \(q^2\)-extrapolation of the form factors \(A_{0}^{DK^*}(q^2)\) and \(F_{1}^{D\pi}(q^2)\) with pole masses 2.11 and 1.87 GeV respectively. The results are shown in Table 1 where data are fitted for \(\delta_{1/2}^{K^*\pi} - \delta_{3/2}^{K^*\pi} = 110^0\), with \(\chi_{K^*\pi}\) and \(\xi_{K^*\pi}\), defined in the Table, in the ranges indicated. We point out that non-empty domains of \(\chi_{K^*\pi}\) and \(\xi_{K^*\pi}\) were found for \(\delta_{1/2}^{K^*\pi} - \delta_{3/2}^{K^*\pi}\) lying in the interval \((54^0 - 125^0)\). The corresponding ranges of effective \(a_1\) and \(a_2\) in BSWI and BSWII scenarios are given as follows

|        | BSWI | BSWII |
|--------|------|-------|
| \(\chi_{K^*\pi}\) | \(1.77 \leq (a_1)^{\chi_{K^*\pi}} \leq 1.92\) | \(1.77 \leq (a_1)^{\chi_{K^*\pi}} \leq 1.91\) |
| \(\xi_{K^*\pi}\) | \(-0.52 \leq (a_2)^{\xi_{K^*\pi}} \leq -0.44\) | \(-0.42 \leq (a_2)^{\xi_{K^*\pi}} \leq -0.35\) |

A discussion of these results is given in Section VI.

**B. \(D^0 \to K^-\rho^+\), \(K^0\rho^0\) and \(D^+ \to K^0\rho^+\)**

We write, using the definitions given in II, the amplitudes for the decays \(D^0 \to K\rho\) as

\[
A(D^0 \to K^-\rho^+) = 2\tilde{G}_F f_{\rho} m_{\rho} (\varepsilon^*\cdot pD) F_{1}^{DK}(m_\rho^2)(a_{1}^{eff})_{\rho},
\]

\[
A(D^0 \to K^0\rho^0) = \sqrt{2}\tilde{G}_F f_{\rho} m_{\rho} (\varepsilon^*\cdot pD) A_0^{D\rho}(m_K^2)(a_{2}^{eff})_{\rho},
\]

and

\[
A(D^+ \to K^0\rho^+) = A(D^0 \to K^-\rho^+) + \sqrt{2}A(D^0 \to K^0\rho^0),
\]

where

\[
(a_{1}^{eff})_{\rho} = a_1 \left(1 + \frac{F_{1}^{(1)nf}}{F_{1}^{DK}(m_\rho^2)} + \frac{C_2}{a_1} \frac{F_{1}^{(8)nf}}{F_{1}^{DK}(m_\rho^2)} \right),
\]

\[
(a_{2}^{eff})_{\rho} = a_2 \left(1 + \frac{C_1}{a_2} \frac{A_0^{(8)nf}}{A_0^{D\rho}(m_K^2)} \right).
\]

We have also used, in addition to (8) and (9), the following definitions,

\[
\langle K^-\rho^+|\bar{s}c)(\bar{u}d)|D^0\rangle_{nf} = 2\tilde{G}_F f_{\rho} m_{\rho} F_{1}^{(1)nf}(m_\rho^2)(\varepsilon^*\cdot pD),
\]

\[
\langle K^-\rho^+|H_w^{(8)}|D^0\rangle = 2\tilde{G}_F f_{\rho} m_{\rho} F_{1}^{(8)nf}(m_\rho^2)(\varepsilon^*\cdot pD),
\]

\[
\langle K^0\rho^0|\bar{H}_w^{(8)}|D^0\rangle = \sqrt{2}\tilde{G}_F f_{K} m_{\rho} A_0^{(8)nf}(m_K^2)(\varepsilon^*\cdot pD).
\]
Fits to $D \to \bar{K}\rho$ data admit a solution with zero fsi phases \[13, 16\], thus we assume $\delta_{1/2}^{K\rho} - \delta_{3/2}^{K\rho} = 0$. We use $F_{1}^{DK}(0)$ from eqn. (29) and, for want of better information, the BSW \[11\] value of $A_{0}^{D\rho}(0) = 0.67$. In this decay also we have considered both monopole (BSWI) and dipole (BSWII) extrapolations of the form factors $F_{1}^{DK}(q^{2})$ and $A_{0}^{D\rho}(q^{2})$ with $1^{-}$ pole at 2.11 GeV and $0^{-}$ pole at 1.87 GeV respectively. In Table 2 we show the allowed ranges of the parameter $\chi_{K\rho}$ and $\xi_{K\rho}$, defined in the Table, for which the data could be fitted. The ranges of effective $a_{1}$ and $a_{2}$ in BSWI and BSWII scenarios are given as follows

\begin{align*}
\text{(BSWI)} & \quad 1.17 \leq \left(a_{1}^{\text{eff}}\right)_{K\rho} \leq 1.32, \quad -1.00 \leq \left(a_{2}^{\text{eff}}\right)_{K\rho} \leq -0.86 \\
\text{(BSWII)} & \quad 1.02 \leq \left(a_{1}^{\text{eff}}\right)_{K\rho} \leq 1.15, \quad -0.92 \leq \left(a_{2}^{\text{eff}}\right)_{K\rho} \leq -0.80
\end{align*}

(43)

A discussion of $\chi_{K^*\pi}$ and $\xi_{K^*\pi}$ is given in the last Section.

**C. $D^0 \to K^-a_1^+$, $\bar{K}^0a_1^0$ and $D^+ \to \bar{K}^0a_1^+$**

We write, using definitions given in II, decay amplitudes for $D \to K a_1$ as follows,

\begin{align*}
A(D^0 \to K^- a_1^+) &= 2\tilde{G}_{F} f_{a_1} m_{a_1} (\bar{\epsilon} . p_D) F_{1}^{DK}(m_{a_1}^2) (a_{1}^{\text{eff}})_{K a_1}, \\
A(D^0 \to \bar{K}^0 a_1^0) &= \sqrt{2}\tilde{G}_{F} f_{K} m_{a_1} (\bar{\epsilon} . p_D) C_1 \tilde{V}_{0}^{(8)nf}, \\
A(D^+ \to \bar{K}^0 a_1^+) &= A(D^0 \to K^- a_1^+) + \sqrt{2}A(D^0 \to \bar{K}^0 a_1^0),
\end{align*}

(44)

where

\begin{equation}
\left(a_{1}^{\text{eff}}\right)_{K a_1} = a_1 \left(1 + \frac{F_{1}^{(1)nf}}{F_{1}^{DK}(m_{a_1}^2)} + \frac{C_2}{a_1} \frac{F_{1}^{(8)nf}}{F_{1}^{DK}(m_{a_1}^2)}\right).
\end{equation}

(45)

In deriving (44), in addition to (8) and (12), we have used the following definitions,

\begin{align*}
\langle K^- a_1^+ | (\bar{s}c)(\bar{u}d) | D^0 \rangle^{nf} &= 2\tilde{G}_{F} f_{a_1} m_{a_1} (\bar{\epsilon} . p_D) F_{1}^{(1)nf}, \\
\langle K^- a_1^+ | H_{w}^{(8)} | D^0 \rangle^{nf} &= 2\tilde{G}_{F} f_{a_1} m_{a_1} (\bar{\epsilon} . p_D) F_{1}^{(8)nf}, \\
\langle \bar{K}^0 a_1^0 | \tilde{H}_{w}^{(8)} | D^0 \rangle^{nf} &= \sqrt{2}\tilde{G}_{F} f_{K} m_{a_1} (\bar{\epsilon} . p_D) \tilde{V}_{0}^{(8)nf}.
\end{align*}

(46)

In the decay amplitude for $D^0 \to \bar{K}^0 a_1^0$ we have retained only the nonfactorized contribution arising from $\tilde{H}_{w}^{(8)}$. The reason being that the factorized amplitude cannot be calculated in the BSW scheme, $a_1(1260)$ being a $3P_1$ state, unlike for $K^*$ which is a $3S_1$ state, BSW procedure does not define the null-plane wave function for $L = 1$ quark-antiquark pairs. However, the relevant form factor $V_{0}^{D_{a_1}}(q^{2})$ (see eq.(12)) can be calculated in the model proposed by Isgur, Scora, Grinstein and Wise \[17\] where it can be shown that it vanishes at the zero-recoil point. This does not imply that it vanishes everywhere but as it also comes multiplied by the rather small coefficient $a_2(\approx -0.09)$, we have neglected the factorized amplitude all together. In contrast, the nonfactorized term contributing to color-suppressed decay $D^0 \to \bar{K}^0 a_1^0$ which we retain in (44), is multiplied by a relatively large Wilson coefficient $C_1$. 

8
We use $F_1^{DK}(0)$ from eqn. (29) and both monopole (BSWI) and dipole (BSWII) forms for $q^2$ extrapolation of the form factor $F_1^{DK}(q^2)$ with $1^-$ pole at 2.11 GeV. The results are given in Table 3. The allowed range of effective $a_1$ is given as follows,

\[(\text{BSWI}) \quad 2.28 \leq (a_1)^{eff} \leq 2.66, \]
\[(\text{BSWII}) \quad 1.51 \leq (a_1)^{eff} \leq 1.76. \quad (47)\]

As $a_1 = 1.09$, it may be concluded from (47) that there are large nonfactorized contributions in $D \to \bar{K}a_1$ decays. This is not unanticipated as the final state particles are relatively slow in this process.

V. $D \to V_1V_2$

A. $D^0 \to K^*-\rho^+$, $\bar{K}^*\rho^0$ and $D^+ \to \bar{K}^*\rho^+$

Using the definitions given in Section II one can write the decay amplitudes for $D^0 \to K^*-\rho^+$, $\bar{K}^*\rho^0$ and $D^+ \to \bar{K}^*\rho^+$ as follows,

\[
A(D^0 \to K^*-\rho^+) = 2G_fm_\rho \varepsilon \left\{ (m_D + m_{K^*})\varepsilon_{K^*} \varepsilon_\rho (a_1A_1^{DK^*}(m_D^2) + a_1A_1^{(1)n_f} + C_2A_1^{(8)n_f}) \\
- \frac{\varepsilon_{K^*}(p_D - p_{K^*})\varepsilon_\rho(p_D + p_{K^*})}{m_D + m_{K^*}} a_1A_1^{DK^*}(m_D^2) \\
+ \frac{2i}{m_D + m_{K^*}} \varepsilon_{\mu\sigma\delta} \varepsilon_{K^*} (p_D^\sigma p_{K^*}^\delta a_1V^{DK^*}(m_D^2)) \right\},
\]

\[
A(D^0 \to \bar{K}^*\rho^0) = \sqrt{2}G_fm_{K^*} \varepsilon \left\{ (m_D + m_\rho)\varepsilon_{K^*} \varepsilon_\rho (a_2A_1^{D\rho}(m_D^2) + C_2A_1^{(8)n_f}) \\
- \frac{\varepsilon_\rho(p_D - p_\rho)\varepsilon_{K^*}(p_D + p_\rho)}{m_D + m_\rho} a_2A_1^{D\rho}(m_D^2) \\
+ \frac{2i}{m_D + m_\rho} \varepsilon_{\mu\sigma\delta} \varepsilon_{K^*} (p_D^\sigma p_{K^*}^\delta a_1V^{D\rho}(m_D^2)) \right\},
\]

and

\[
A(D^+ \to K^*\rho^+) = A(D^0 \to K^*-\rho^+) + \sqrt{2}A(D^0 \to \bar{K}^*\rho^0),
\]

(48)

where the quantities with super index 1 (e.g. $A_1^{(1)n_f}$) arise from the nonfactorized contribution to the matrix elements of the color-singlet currents $(sc)(\bar{u}d)$; those with super index 8 (e.g. $A_1^{(8)n_f}$) arise from $H_8^{(8)}$ made up of color-octet currents and tildaed quantities (e.g. $\tilde{A}_1^{(8)n_f}$) arise from $\tilde{H}_8^{(8)}$. In writing (48) we have retained nonfactorized contribution only to S-waves, i.e. $A_1^{(1)n_f}$, $A_1^{(8)n_f}$ and $\tilde{A}_1^{(8)n_f}$, and neglected all other nonfactorized contributions as we did in [5] and [3].

The decay rate can then be calculated using (14). For the form factors we use the following normalizations (only central values of the experimental numbers are used),

\[
A_1^{DK^*}(0) = 0.61 \pm 0.05, \quad A_2^{DK^*}(0) = 0.45 \pm 0.09, \quad V^{DK^*}(0) = 1.16 \pm 0.16 \quad [12],
\]
\[
A_1^{D\rho}(0) = 0.78, \quad A_2^{D\rho}(0) = 0.92, \quad V^{D\rho}(0) = 1.23 \quad [11],
\]

(49)

and extrapolate them to relevant $q^2$ with monopole forms with pole masses 2.53 GeV for $A_1^{DK^*}$ and $A_2^{DK^*}$, 2.11 GeV for $V^{DK^*}$, 2.42 GeV for $A_1^{D\rho}$ and $A_2^{D\rho}$ and 2.01 GeV for $V^{D\rho}$. We account
for nonfactorized contributions through two parameters $\kappa$ and $\tilde{\kappa}$,
\[ \kappa \equiv 1 + \frac{C_2}{a_1} \chi_{K^{*}\rho}, \quad \tilde{\kappa} \equiv 1 + \frac{C_1}{a_2} \xi_{K^{*}\rho}, \tag{50} \]
with
\[ \chi_{K^{*}\rho} = \frac{A_{i}^{(8)nf}}{A_{i}^{K^{*}}(m_{\rho}^{2})} + \frac{a_1}{C_2} \frac{A_{i}^{(1)nf}}{A_{i}^{pK^{*}}(m_{\rho}^{2})}, \quad \text{and} \quad \xi_{K^{*}\rho} = \frac{\tilde{A}_{i}^{(8)nf}}{A_{i}^{p}(m_{K^{*}}^{2})}. \tag{51} \]

We find that agreement of data with the calculated branching ratios is possible for $\xi_{K^{*}\rho}$ and $\chi_{K^{*}\rho}$ lying in the following range,
\[ 0.02 \leq \chi_{K^{*}\rho} \leq 0.80, \quad -0.31 \leq \xi_{K^{*}\rho} \leq -0.24. \tag{52} \]

In particular with $\chi_{K^{*}\rho} = 0.41$ and $\xi_{K^{*}\rho} = -0.28$ we get,
\[ B(D^{0} \to K^{*+}\rho^{+}) = 5.61\% \text{ (expt.}[15] : (5.9 \pm 2.4)\%) , \]
\[ B(D^{0} \to K^{*0}\rho^{0}) = 1.63\% \text{ (expt.}[15] : (1.6 \pm 0.4)\%) , \]
\[ B(D^{+} \to K^{*+}\rho^{+}) = 1.81\% \text{ (expt.}[15] : (2.1 \pm 1.4)\%) . \tag{53} \]

VI. Summary and Conclusions

We have carried out an analysis of those Cabibbo-favored two-body hadronic decays of $D^0$ and $D^+$ which involve two isospins in the final state in a formalism that uses $N_c = 3$ and includes nonfactorized amplitudes. These decays are: $D \to \bar{K}\pi$, $\bar{K}^{*}\pi$, $\bar{K}\rho$, $\bar{K}a_1$ and $\bar{K}^{*}\rho$. We have included the measured fsi phases in $\bar{K}\pi$ and $\bar{K}^{*}\pi$ decays but only in so far as they rotate the isospin amplitudes without affecting their magnitudes. We have ignored fsi phases in $\bar{K}^{*}\rho$ and $\bar{K}a_1$ decays while the relative phases is known to be consistent with zero in $\bar{K}\rho$ channel. We have also ignored annihilation terms and inelastic fsi. The rationale for the former is that these terms are proportional to $a_2$ in $D^0$ decays which in our scheme is only $\approx -0.09$, while the neglect of the latter is largely due to ignorance of the parameters to be used in implementing a believable calculation.

From the data, one only determines $(a_1)^{eff}$ and $(a_2)^{eff}$ which, as we and others [7] have shown, are process-dependent. The next question is: What effects contribute to $(a_1)^{eff}$ and $(a_2)^{eff}$ in a scheme that uses $N_c = 3$? We have tacitly assumed that these effects arise from three sources: the nonfactorized matrix elements of $H_{w}^{(8)} = \frac{1}{2} \sum_{a} (\bar{s}\lambda^{a}c)(\bar{u}\lambda^{a}d)$, $\tilde{H}_{w}^{(8)} = \frac{1}{2} \sum_{a} (\bar{s}\lambda^{a}d)(\bar{u}\lambda^{a}c)$ and the Hamiltonian made up of color-singlet currents $(\bar{s}c)(\bar{u}d)$. With these assumptions, we have extracted the relative size of the nonfactorized contribution in each specific channel. We now turn to a detailed discussion of specific decays.

From $D \to \bar{K}\pi$ decays we have determined the parameter $\xi_{K\pi}$ of (30) which is proportional to the matrix element of $\tilde{H}_{w}^{(8)}$ denoted by $\tilde{F}_{0}^{(8)nf}$, to lie in the range $-0.29 \leq \xi_{K\pi} \leq -0.26$. Cheng [7] determines the same parameter to be -0.36. The small difference could be due to the fact that we include the fsi phases. We also determine the parameter $\chi_{K\pi}$ of (30) which includes nonfactorized contributions from $H_{w}^{(8)}$ and $(\bar{s}c)(\bar{u}d)$, denoted by $\tilde{F}_{0}^{(8)nf}$ and $\tilde{F}_{0}^{(1)nf}$ respectively in (30). It would
be tempting to assume that \( F_{0}^{(8)nf} = \tilde{F}_{0}^{(8)nf} \) and hence extract the value of \( F_{0}^{(1)nf} \). However, such an assumption would be flawed since \( H_{w}^{(8)} \) and \( \tilde{H}_{w}^{(8)} \) are related by V-spin symmetry \((s \leftrightarrow u)\), but under the same transformation \(|D^{0}\rangle \rightarrow |D_{s}^{+}\rangle\) and \(|K^{-}\pi^{+}\rangle \rightarrow |K^{+}\bar{K}^{0}\rangle\). Thus V-spin symmetry leads to

\[
\langle K^{-}\pi^{+}|H_{w}^{(8)}|D^{0}\rangle = \langle K^{+}\bar{K}^{0}|\tilde{H}_{w}^{(8)}|D_{s}^{+}\rangle
\]  

and not to a relation between \( F_{0}^{(8)nf} \) and \( \tilde{F}_{0}^{(8)nf} \). Thus these parameters remain unrelated and very little can be concluded about the size of the nonfactorized contribution \( F_{0}^{(1)nf} \). All that can be said is that the nonfactorized amplitude in \( D^{0} \rightarrow K^{-}\pi^{+} \) decay is relatively smaller than in \( D^{0} \rightarrow K^{0}\pi^{0} \) decay. We also emphasize that the nonfactorized contribution in the color-suppressed decay \( D^{0} \rightarrow K^{0}\pi^{0} \) is enhanced relative to the factorized term by a factor of \( C_{1}/a_{2}(\approx -14) \) which is not the case in the color-favored decay \( D^{0} \rightarrow K^{-}\pi^{+} \). Thus the color-suppressed processes are more likely to reveal the presence of nonfactorized contributions than color-favored processes. Further, in the color-favored decay the nonfactorized amplitude arising from the color-singlet currents \((\bar{s}c)(\bar{u}\bar{d})\) (called \( F_{0}^{(1)nf} \) here) could be as important as the one from \( H_{w}^{(8)} \) (called \( F_{0}^{(8)nf} \) here).

In \( D \rightarrow \bar{K}^{*}\pi \) decays we find the parameter \( \xi_{K'\pi} \), defined in Table 1, to be in the range \(-0.34 \leq \xi_{K'\pi} \leq -0.28 \) for monopole form factors and in the range \(-0.26 \leq \xi_{K'\pi} \leq -0.21 \) for dipole form factors. These values are considerably smaller than \( \approx -0.61 \) given in [7]. The difference could again be due to our inclusion of the FSI phases which are large. Our estimate of \( \xi_{K'\pi} \) implies \(-0.52 \leq (a_{2}^{n}f)_{K'\pi} \leq -0.44 \) for monopole form factors and \(-0.42 \leq (a_{2}^{n}f)_{K'\pi} \leq -0.35 \) for dipole form factors. We also find significantly large nonfactorized effects in the color-favored decay \( D^{0} \rightarrow K^{-}\pi^{+} \) resulting in \( 1.77 \leq (a_{1}^{n}f)_{K'\pi} \leq 1.92 \). We recall that it had been shown in [13] that factorization assumption together with \( a_{1} = 1.26 \) and \( a_{2} = -0.51 \) had underestimated \( B(D^{0} \rightarrow K^{-}\pi^{+}) + B(D^{0} \rightarrow K^{0}\pi^{0}) \) and \( B(D^{+} \rightarrow \bar{K}^{0}\pi^{+}) \). It is the larger value of \( (a_{1}^{n}f)_{K'\pi} \) that allows a resolution of the problem.

In the color-suppressed decay \( D^{0} \rightarrow \bar{K}^{0}\rho^{0} \), we find large nonfactorized contributions: \(-0.72 \leq \xi_{K'\rho} \leq -0.61 \) for monopole form factor and \(-0.66 \leq \xi_{K'\rho} \leq -0.56 \) for dipole form factors. They result in \(-1.00 \leq (a_{2}^{n}f)_{K'\rho} \leq -0.86 \) and \(-0.92 \leq (a_{2}^{n}f)_{K'\rho} \leq -0.80 \) respectively. These parameters help resolve the problem with factorization assumption which predicted \[13\] too large a branching ratio for \( D^{+} \rightarrow K^{0}\rho^{+} \) if \( a_{1} = 1.26 \) and \( a_{2} = -0.51 \) were used. With a much larger effective \( a_{2} \) the rate for \( D^{+} \rightarrow K^{0}\rho^{+} \) is brought down to the experimental value due to a larger destructive interference between effective \( a_{1} \) and \( a_{2} \). The nonfactorized contribution to the color-favored decay \( D^{0} \rightarrow K^{-}\rho^{+} \) appears to be small leading to: \( 1.17 \leq (a_{1}^{n}f)_{K'\rho} \leq 1.32 \) for monopole form factors and \( 1.02 \leq (a_{1}^{n}f)_{K'\rho} \leq 1.15 \) for dipole form factors.

The decays \( D \rightarrow \bar{K}a_{1} \) have long posed a problem for the factorization model. Inclusion of nonfactorized amplitudes allows us to understand the branching ratios involved. Our picture suggests that the color-suppressed decay \( D^{0} \rightarrow \bar{K}^{0}a_{1}^{0} \) proceeds almost entirely through a nonfactorized amplitude whose size we limit by the experimental upper limit on \( B(D^{0} \rightarrow \bar{K}^{0}a_{1}^{0}) \). We are then able to understand the measured branching ratios \( B(D^{0} \rightarrow \bar{K}^{0}a_{1}^{0}) \) and \( B(D^{+} \rightarrow \bar{K}^{0}a_{1}^{0}) \) provided that: \( 2.28 \leq (a_{1}^{n}f)_{K'1} \leq 2.66 \) for monopole form factors and \( 1.51 \leq (a_{1}^{n}f)_{K'1} \leq 1.76 \) for dipole form factors. These large values of \( (a_{1}^{n}f) \) are not unexpected for this mode where the final-state particles are relatively slow.
For the decays $D \to K^* \rho$ (and, in general, for any $P \to VV$ decay) one cannot define $(a_{1}^{eff})$ and $(a_{2}^{eff})$ as the decay amplitude involves three independent Lorentz scalar structures and it is not possible to factor out an effective $a_1$ and $a_2$. However, retaining the nonfactorized effects only in S-wave final states, we find significant nonfactorized effects in the color-suppressed decay $D^0 \to K^{*0} \rho^0$ characterized by the parameter $\xi_{K^*\rho}$ of eqn. (50): $-0.31 \leq \xi_{K^*\rho} \leq -0.24$. The analogous parameter $\chi_{K^*\rho}$, eq.(50), which is a measure of nonfactorized contribution to the color-favored decay $D^0 \to K^{*-} \rho^+$ has the opposite sign, and could in principle, be very small: $0.02 \leq \chi_{K^*\rho} \leq 0.80$.

We conclude by saying that one can understand D decays in a picture with $N_c = 3$ but with the inclusion of nonfactorized amplitudes. This picture results in process-dependent effective $a_1$ and $a_2$, which ought to be complex as are all the nonfactorized amplitudes. We have not included the inelastic final-state interaction effects which would further complicate the analysis. The effort here was to parametrize the nonfactorized amplitudes and determine their size. The understanding of any systematics that emerge is yet to come.

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Table 1:  $D \rightarrow K^*\pi$

| Channel                  | Scenario 1 (BSWI) | Scenario 2 (BSWII) | Th.BR$^{(a)}$ | Expt.BR$^{(c)}$
|--------------------------|-------------------|-------------------|---------------|-------------------|
|                          | $\chi_{K^*\pi}$  | $\xi_{K^*\pi}$   | in %          | in %             |
| $D^0 \rightarrow K^{*-}\pi^+$ | -1.62 $\leq \chi_{K^*\pi} \leq$ -1.33 | -0.34 $\leq \xi_{K^*\pi} \leq$ -0.28 | 4.63          | 4.9 $\pm$ 0.6   |
| $D^0 \rightarrow \bar{K}^{*0}\pi^0$ | -1.60 $\leq \chi_{K^*\pi} \leq$ -1.34 | -0.26 $\leq \xi_{K^*\pi} \leq$ -0.21 | 4.62          | 4.9 $\pm$ 0.6   |

\[ \chi_{K^*\pi} \equiv \frac{A^{(6)nf}_{K^0K^0}(m_{\pi^0}^2)}{A^{(8)nf}_{K^0K^0}(m_{\pi^0}^2)} + \frac{A^{(1)nf}_{L_H}}{\xi_{L_H}} \]

\[ \xi_{K^*\pi} \equiv \frac{F^{(8)nf}_{K^0K^0}(m_{\pi^0}^2)}{F^{(8)nf}_{K^0K^0}(m_{\pi^0}^2)} \]

(a) for $\chi_{K^*\pi} = -1.47$ and $\xi_{K^*\pi} = -0.31$
(b) for $\chi_{K^*\pi} = -1.47$ and $\xi_{K^*\pi} = -0.23$
(c) Source Ref. [14]
### Table 2: $D \rightarrow K\rho$

| Channel                  | Scenario 1 (BSWI) | Scenario 2 (BSWII) |
|--------------------------|-------------------|--------------------|
|                          | $\chi_{K\rho}$    | $\xi_{K\rho}$      | $\chi_{K\rho}$    | $\xi_{K\rho}$      |
| $D^0 \rightarrow K^-\rho^+$ | -0.45 $\leq \chi_{K\rho} \leq$ -0.16 | -0.72 $\leq \xi_{K\rho} \leq$ -0.61 | 10.36 | 10.40 $\pm$ 1.30 |
| $D^0 \rightarrow \bar{K}^0\rho^0$ | -0.45 $\leq \chi_{K\rho} \leq$ -0.16 | -0.72 $\leq \xi_{K\rho} \leq$ -0.61 | 1.10  | 1.10 $\pm$ 0.18 |
| $D^+ \rightarrow \bar{K}^0\rho^+$  | -0.45 $\leq \chi_{K\rho} \leq$ -0.16 | -0.72 $\leq \xi_{K\rho} \leq$ -0.61 | 7.56  | 6.60 $\pm$ 2.50 |

$\chi_{K\rho} \equiv \frac{F^{(8)n_f}_{1D}(m_1^2)}{F^{(1)n_f}_{1D}(m_1^2)} + \frac{a_1}{C_2} \frac{F^{(1)n_f}_{1D}(m_1^2)}{A_0(m_K^2)}$

$\xi_{K\rho} \equiv \frac{\tilde{V}_{0}^{(8)n_f}_{1D}(m_1^2)}{A_0(m_K^2)}$

(a) for $\chi_{K\rho} = -0.30$ and $\xi_{K\rho} = -0.67$

(b) for $\chi_{K\rho} = 0.02$ and $\xi_{K\rho} = -0.61$

(c) Source Ref. [15]

### Table 3: $D \rightarrow K\alpha_1$

| Channel                  | Scenario 1 (BSWI) | Scenario 2 (BSWII) |
|--------------------------|-------------------|--------------------|
|                          | $\chi_{K\alpha_1}$ | $\tilde{V}_{0}^{(8)n_f}$ | $\chi_{K\alpha_1}$ | $\tilde{V}_{0}^{(8)n_f}$ |
| $D^0 \rightarrow K^-\alpha_1^+$ | -3.07 $\leq \chi_{K\alpha_1} \leq$ -2.34 | -1.54 $\leq \tilde{V}_{0}^{(8)n_f} \leq$ -0.89 | 7.87 | 7.9 $\pm$ 1.2 |
| $D^0 \rightarrow \bar{K}^0\alpha_1^0$ | -3.07 $\leq \chi_{K\alpha_1} \leq$ -2.34 | -1.54 $\leq \tilde{V}_{0}^{(8)n_f} \leq$ -0.89 | 0.62 | < 1.9 |
| $D^+ \rightarrow \bar{K}^0\alpha_1^+$  | -3.07 $\leq \chi_{K\alpha_1} \leq$ -2.34 | -1.54 $\leq \tilde{V}_{0}^{(8)n_f} \leq$ -0.89 | 7.19 | 8.1 $\pm$ 1.7 |

$\chi_{K\alpha_1} \equiv \frac{F^{(8)n_f}_{1D}(m_1^2)}{F^{(1)n_f}_{1D}(m_1^2)} + \frac{a_1}{C_2} \frac{F^{(1)n_f}_{1D}(m_1^2)}{A_0(m_K^2)}$

(a) for $\chi_{K\alpha_1} = -2.71$ and $\tilde{V}_{0}^{(8)n_f} = -1.22$

(b) for $\chi_{K\alpha_1} = -1.07$ and $\tilde{V}_{0}^{(8)n_f} = -1.12$

(c) Source Ref. [15]