Disturbance observer–based event-triggered tracking control of networked robot manipulator

Wei Bu¹, Ting Li²*, Jun Yang² and Yang Yi³

Abstract
We investigate the event-triggered tracking control of networked manipulator in the presence of external disturbance and a variety of system uncertainties. The event-triggered controller is designed by the nonlinear disturbance observer–based control approach, which can dynamically compensate for both errors caused by disturbances and the undesirable effects caused by event-triggering rules. A rigorous Lyapunov stability analysis method is proposed to show that the boundedness of all the signals in the closed-loop system can be guaranteed while in the absence of the input-to-state stability assumption related to measurement errors, tracking error can be constrained to an arbitrarily small set without escaping. Finally, by some simulation results, the feasibility and effectiveness of the proposed control approach is demonstrated.

Keywords
Networked manipulator, event-triggered tracking control, external disturbance and system uncertainties, disturbance observer–based control

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Introduction
Research on robot manipulators has received sustained attention in the field of control due to its important role in industrial applications such as advanced medical, and space and defense.¹⁻³ Meanwhile, control system is often realized through network in today’s research due to their advantages in terms of flexibility and cost.⁴⁻⁷ Therefore, some research results of the network robot have been published successfully. But the burden of signal receiving and transmission may be heavy, which may affect the performance of the robot.³,⁸

With the development of digitization, event-triggered control has been crucial due to resource constraints in networked control systems, such as limited computational power and communication bandwidth, as well as restricted energy resources.⁹,¹⁰ Its control mechanism is that control tasks are performed only when it is really necessary to ensure the stability and performance of the system.¹¹,¹² Motivated by the above-mentioned realities, varying inter-event times are required for a control scheme in order to guarantee the execution of the control tasks. This makes up for the defect of time-triggered control in resource utilization and so on.¹²⁻¹⁶ Several effective event-triggering mechanisms are proposed and applied in Liu and Jiang,¹⁷ Xing et al.,¹⁸,¹⁹ Donkers and Heemels,²⁰ Borgers and Heemels²¹ and the references therein. However, for most related research works, stability results are ensured only if the measurement errors meet the corresponding input-to-state stability (ISS) condition, which is usually impossible to satisfy for the systems with external disturbances.¹³,¹⁷ Although the recent results¹⁸,¹⁹ achieve the robust stabilization for uncertain nonlinear systems via event-triggered input, and eliminate the ISS assumption depending on measurement error, external disturbances and time-varying uncertainties make these design strategies invalid. Considering the complexity in practical application, robot manipulators are influenced by external disturbances, as well as a variety of system uncertainties,

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such as control coefficient uncertainty, parametric perturbations, as well as other static structured/unstructured modeling errors.\textsuperscript{22,23} As a result, how to handle time-varying external disturbances and a variety of system uncertainties for robot manipulators, as well as relaxing the ISS assumptions remains to be overcome within the framework of event-triggered control.

The control objective of the paper is to guarantee the tracking of given reference signal by robot manipulators subject to time-varying external disturbances and uncertainties using a event-triggered controller, while reducing the usage of the communication channel by limiting the amount of control input updates. Based on the fixed threshold strategy given in Xing et al.,\textsuperscript{18,19} an event-triggered controller is proposed to overcome the aforementioned objective. In the proposed event-triggering mechanism, the control signal is transmitted via communication network only when a predetermined lower threshold that depending on the control signal has been reached. Moreover, without the ISS assumption depending on the measurement errors, the nonlinear disturbance observer–based control (DOBC) approach\textsuperscript{24–27} is applied to effectively estimate the total disturbance. By the proposed disturbance observer–based event-triggered control method, it is shown that the global boundedness of the signals in closed-loop system is guaranteed, and the tracking error is ensured to converge to a set, which can be made as small as desired by adjusting control parameters. The major innovations or difficulties of the proposed control design in this paper are twofold as follows:

1. Only the observer gain needs to be adjusted in order to ensure a desired tracking error, meanwhile the transmission efficiency is increased;
2. By the virtue of the technique of DOBC, the negative impacts of the lumped disturbance and the measurement errors can be attenuated in the framework of event-triggered control without ISS assumption.

The remainder of this paper is organized as follows. Necessary preliminaries and the problem formulation are provided in section “Preliminaries and problem formulation.” Section “Main results” gives the event-triggered control design for networked manipulator and the resulting closed-loop system rigorously analyzed. The effectiveness of the proposed method is demonstrated by a numerical simulation in section “Simulation examples.” Section “Conclusion” concludes the paper.

**Preliminaries and problem formulation**

**Preliminaries**

Now, we give the following preliminary lemmas which are necessary in our design.

**Lemma 1.** For given $m > 0, n > 0$ and $a$, there exists $c > 0$ such that\textsuperscript{28}

$$|ax^my^n| \leq c|x|^{m+n} + \frac{n}{m+n} \left(\frac{m}{(m+n)c}\right)^{\frac{1}{2}}|a|^{\frac{m+n}{m+n-1}}|y|^{m+n}$$

for all $x \in R$ and $y \in R$.

**Lemma 2.** For given $r > 0$ and every $x \in R$, $y \in R$, there holds $|x + y|^r \leq c_x(|x|^r + |y|^r)$, where $c_x = 2^{r-1}$ if $r \geq 1$, and $c_x = 1$ if $r < 1$. If $r \geq 1$ is an odd integer, then $|x - y|^r \leq 2^{r-1} |x^r - y^r|$\textsuperscript{28}

**Lemma 3.** For any $\nu_1 \in R$ and $\nu_2 > 0$, there holds\textsuperscript{18}

$$0 \leq |\nu_1| - \nu_1 \tanh \left(\frac{\nu_1}{\nu_2}\right) \leq 0.2785 \nu_2$$

**Problem formulation**

Now, we consider the following dynamics of networked robot manipulator

$$D\dot{\theta} + C\theta + G = \tau + d$$

where the angle $\theta$ is the output signal; the torque $\tau$ is the control signal; the moment of inertia $D$ satisfies $D = (4ml^2/3)$, $m$ is the mass, $l$ denotes the distance from the centroid to the center of connecting rod rotation; $C$ represents the viscous friction coefficient; the gravity $G$ satisfies $G = mg\cos \theta$ with $g$ being the gravitational acceleration; and $d$ represents the external disturbance. Moreover, we denote $\theta_d$ as the reference signal of $\theta$.

As the system parameters $m$, $g$, $l$ and $C$ may not be accurately known especially in practical applications, all kinds of uncertainties usually need to be considered. We define $m_0$, $g_0$, $l_0$ and $C_0$ as the nominal values of $m$, $g$, $l$ and $C$, respectively.

With the help of the following coordinate transformations $x_1 = \theta - \theta_d$, the system (equation (1)) can be converted into

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= a_0 \tau + \omega \\
y &= x_1
\end{align*}$$

where $a_0 = (3/4m_0l_0^2)$ denotes the known nominal parameter, and $\omega = -(3C/4ml^2)\dot{\theta} - (3g/4l) \cos (\theta) + ((3/4ml^2) - (3C/4m_0l_0^2))\tau + (3/4ml^2)d + \dot{\theta}_d$ represents the total disturbance including model uncertainties and external disturbances.

The control objective is to construct an appropriate event-triggered controller without any ISS assumptions such that the global boundedness of the signals in closed-loop system is guaranteed, and the output signal $y(t)$ in equation (2) can be regulated to an arbitrarily small set. To this end, the following assumption is needed.
Assumption 1. The lumped disturbance $\omega(t)$ is continuously differentiable and satisfy $|\omega(t)| \leq \bar{\omega}_1$, $|\dot{\omega}(t)| \leq \bar{\omega}_2$, where $\bar{\omega}_1$ and $\bar{\omega}_2$ are positive constants.

Remark 1. The lumped disturbance $\omega$ in equation (2) may inevitably deteriorate the tracking performance, and the ISS assumptions related to measurement errors are difficult to verify. In order to reduce data transmission and guarantee the tracking performance, the event-triggering mechanism is employed in the digital communication channel between the controller and the actuator. In this setup, a new robust event-triggered tracking controller via DOBC for the dynamics (equation (2)) is proposed. Figure 1 shows the diagram of the event-triggered control schematic.

In the proposed control method, the DOBC is first designed to estimate the disturbances. In order to save the communication resource, the controller is transmitted only when the event-triggering mechanism is triggered.

Main results

Event-triggered control design via DOBC

In this part, a disturbance estimator is designed below to estimate the lumped uncertainties $\omega(t)$ in system (equation (2)), which is depicted by

$$
\xi = -L\xi - L(a_0\tau + Lx_2)
$$

where $\xi \in R$ is the state of the observer, $\dot{\omega} \in R$ is the estimate of the lumped uncertainty $\omega(t)$ and $L$ is a positive observer gain factor to be designed. The error vector of the estimator (equation (3)) is defined as follows: $\xi = \omega - \dot{\omega}$. Then, with the help of system (equation (2)) and observer (equation (3)), one can get the following estimation error system

$$
\dot{\xi} = -L\xi + \dot{\omega}
$$

Furthermore, let us assume that $t_1 = 0$ is the time when the first event is triggered, the sequence $\{t_k\}_{k \in Z^+}$ represents a set of event-triggering instants with $t_k$ being the kth event-triggering instant. As per the above design, a robust controller based on event-triggered input is constructed as follows

$$
\tau(t) = \tau^*(t_k), \forall t \in [t_k, t_{k+1})
$$

$$
t_{k+1} = \inf\{t > t_k | |\tau^*(t) - \tau(t_k)| \geq M\}
$$

where the continuous signal $\tau^*(t)$ denotes the signal before applying event-triggering strategy and satisfies

$$
\tau^*(t) = -\frac{1}{a_0}(k_1x_1 + (k_1 + 1)x_2) - \frac{1}{a_0}\dot{\omega} - M_1\tanh\left(\frac{M_1(x_1 + x_2)}{\varepsilon}\right)
$$

and $\varepsilon \leq 1$, $k_1 \geq 2$, $M, M_1 \geq M$ are positive design parameters.

Stability analysis

In the following, the stability performance of the closed-loop system (equations (2)-(3)-(5)-(6)) is summarized in Theorem 1.

Theorem 1. For system (equation (1)) satisfying Assumptions 1, the disturbance estimator (equation (3)), and the fixed control strategy (equations (5)-(7)), there exists a suitable observer gain $L$ such that the following performance of the closed-loop system can be guaranteed:

1. All the closed-loop signals are bounded, and the tracking error $x_1$ converges toward an arbitrarily small set;
2. There is a positive constant $\tau^*$ such that the inter-execution intervals $t_{k+1} - t_k \geq \tau^*, \forall k \in Z^+$.

Proof

1. It follows from equation (6), that there is a function $\delta(t)$ satisfying $|\delta(t)| \leq 1$ such that

$$
\tau(t) = \tau^*(t) + \delta(t)M, \quad \forall t \geq 0
$$

Define a Lyapunov function $V$ for the closed-loop system as

$$
V = (1/2)x_1^2 + (1/2)(x_1 + x_2)^2 + (1/2)\xi^2
$$

Then, from equations (5)-(8) and Lemma 3, we have

$$
\dot{V} = x_1x_2 + (x_1 + x_2)(x_2 + a_0\tau + \omega) + \xi \dot{\xi}
$$

$$
= x_1x_2 + (x_1 + x_2)(x_2 - (k_1x_1 + (k_1 + 1)x_2) - a_0M_1\tanh\left(\frac{M_1(x_1 + x_2)}{\varepsilon}\right) + \xi + a_0\delta M
$$

$$
+ \xi(-L\xi + \dot{\omega})
$$

Figure 1. Event-triggered control schematic.
\[ \leq -x_1^2 + x_1(x_1 + x_2) - k_1(x_1 + x_2)^2 - a_0 M_1 \]

\[ \times (x_1 + x_2) \tanh \left( \frac{M_1(x_1 + x_2)}{\varepsilon} \right) + a_0 M \]

\[ \times |x_1 + x_2| + (x_1 + x_2) \xi - L_\xi^2 + \frac{1}{2} L_\xi^2 + \frac{1}{2 L} \omega^2 \]

\[ \leq -\frac{1}{2} x_1^2 - (k_1 - 1)(x_1 + x_2)^2 - \frac{1}{2} (L - 1) \xi^2 \]

\[ + \frac{1}{2 L} \omega^2 + 0.2785 \varepsilon \]

Based on the above process, one can select \( L \) satisfying

\[ L \geq \max \left\{ \frac{\omega^2}{\varepsilon}, 3 \right\} \]

Then, it follows from equation (10) that

\[ V \leq -\frac{1}{2} x_1^2 - \frac{1}{2} (x_1 + x_2)^2 - \frac{1}{2} \xi^2 + 0.7785 \varepsilon \]

\[ \leq -V + 0.7785 \varepsilon \]

Therefore

\[ \dot{V} \leq -V + 0.7785 \varepsilon \]

which implies

\[ V \leq e^{-t} V(0) + 0.7785 \varepsilon (1 - e^{-t}) \]

By equation (13), one gets that \( V \) is uniformly bounded; that is, \( x_1, x_2 \) and \( \xi \) are bounded.

Moreover, from equation (13), we get that \( x_1, x_2 \) and \( \xi \) can converge exponentially to the compact sets \( \Omega_1 = \{ x_i | x_i | \leq 0.7785 \varepsilon \}, i = 1, 2, \) and \( \Omega_3 = \{ \xi | \xi | \leq 0.7785 \varepsilon \}, \) respectively.

Remark 3. It is worth mentioning that the nonlinearity estimator (equation (3)) is the key to compensate for the lumped disturbance \( \omega(t) \). Furthermore, although the event-triggered mechanism in this paper is designed based on the fixed threshold strategy, other strategies, such as the relative threshold strategy and the switching threshold strategy, can also be applied to deal with the event-triggered control problem for nonlinear system with external disturbance by the similar control design method.

Remark 4. The effects of the controller parameters on control precision and the triggering frequency of control action are summarized based on the following: (1) the greater the value of \( \varepsilon \), the bigger the ultimate bound of states, the slower the control rate, and the lower the triggering frequency; (2) the greater the values of \( M_1 \), the faster the control rate, and the higher the triggering frequency; and (3) the greater the values of \( M \), the slower the triggering frequency.

Simulation examples

In this section, we show the simulation results on the single-link robot manipulator. In order to demonstrate the effectiveness of the proposed event-triggered control approach, we also take another simulation case into account where the estimation and compensation of the lumped disturbance is not considered (we define this method as ETM without compensation).

The event-triggered continuous control signal without disturbance compensation becomes

\[ \tau^*(t) = - \frac{1}{a_0} (k_1 x_1 + (k_1 + 1) x_2) \]

\[ - M_1 \tan \left( \frac{M_1 (x_1 + x_2)}{\varepsilon} \right) \]

where \( k_1 \) is the feedback control gains to be designed, \( M_1 \) and \( \varepsilon \) are positive design parameters. The reminder of the design and the stability analysis of the ETM without observe compensation are very similar to the event-triggered control design in section “Main results”; hence, we omit them.

We choose the nominal values of the parameters in system (equation (1)) as follows: \( g_0 = 9.81 \text{ m/s}^2 \), \( l_0 = 0.5 \text{ m} \), \( C_0 = 2.00 \text{ N} \cdot \text{m/s} \cdot \text{rad} \), \( m_0 = 1.00 \text{ kg} \). In the simulations, we assume \( g = g_0, l = l_0, a(t) = 1 \) and the uncertainties \( C(t) \) and \( m(t) \) as follows

\[ C(t) = \begin{cases} 2 + 0.3(t - 5), & t \leq 7 \\ 2.5, & t > 7 \end{cases} \]
In the simulations, the selection of the design parameters $M$, $M_1$, $\varepsilon$ is the same for the following two different control methods, and we select $M_1 = M = 2$, $\varepsilon = 1$. Then, the initial states $[x_1(0), x_2(0), \xi(0)]$ are set as follows: $k_1 = 4$, $[x_1(0), x_2(0), \xi(0)] = [1, -1, 0]$ and the observer gain is selected as $L = 80$ for the ETM with compensation. The controller parameters and the initial states for the ETM without compensation are selected as $k_1 = 4$, $[x_1(0), x_2(0)] = [1, -1]$.

Under the two different control methods, the response curves of the tracking errors are depicted in Figure 2. From Figure 2, we can see that the control design in this paper can achieve the desired tracking control performance, but without disturbance observer compensating disturbances, the tracking error is quite unsatisfactory. Figure 3 shows the tracking performance of angle $\theta(t)$, with the angular velocity $\dot{\theta}(t)$ presented in Figure 4. In Figure 5, the control signals $\tau(t)$ given in equation (5) together with signals $\tau_c(t)$ in equation (7) is illustrated, which shows that compared with the continuous control design, the proposed event-triggered control method can significantly save the communication resource while guaranteeing a desirable tracking control performance. The triggered time intervals of each case are presented in Figure 6.

**Conclusion**

Robust tracking control with event-triggered input for networked manipulator in the presence of external disturbance and a variety of system uncertainties is investigated in this paper. Moreover, the ISS assumption depending on measurement errors is removed in the control design. Based on DOBC technique, a nonlinear disturbance observer-based event-triggered controller...
is proposed to dynamically compensate for measurement error and the lumped disturbances of the considered system. The desired control performances of the presented event-triggered control design have been successfully verified by the Lyapunov analysis method and further illustrated by simulation example. Based on the existence of measurement noise and time-delay for every physical system, the effect of these conditions should be further researched in the analysis of both the control properties and the computation properties of the system.

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