Comprehensive analysis of distributed energy resource penetration and placement using probabilistic framework

Priyanka Paliwal

Abstract
Integration of distributed energy resources (DERs) affect system performance in multitude of ways such as reliability, losses and voltage profile. Thus, increasing distributed energy resource penetration necessitates impact assessment particularly when DERs comprise of stochastic RES based distributed generators (DGs) and storage system. This paper presents a formulation for the analysis of DER penetration and placement on system losses and voltage profile in probabilistic framework. Penetration level has been defined, giving due consideration to stochastic behaviour of RES based distributed generators. The work presented incorporates analysis and consideration of two important factors in placement problem, that is, intermittent nature of RES based distributed generators and dual nature of storage units. In order to account for uncertainties inherently present in RES, probabilistic load flow has been used in this work. Placement problem is solved for three different penetration levels, that is, 20%, 40% and 60% using butterfly particle swarm optimisation. The impact of seasonal effect on system losses and voltage profile in correlation with penetration levels has also been investigated. Results provide useful insights for identification of optimum penetration level.

1 INTRODUCTION

Distributed energy resources (DER) are defined as small-scale generation units (DGs) and energy storage technologies connected at distribution network [1, 2]. Integration of DERs offer an array of benefits and provide a promising alternative to centralised generation [3]. One of the major benefits offered by DER integration is the reduction in line losses and improvement in voltage profile. Due to their proximity to load centres, DERs can positively contribute towards reduction in line losses; particularly if the feeder is heavily loaded. Integration of DERs can have significant impact on voltage profile of the distribution grid. The presence of DERs may raise system voltage slightly [4]. This will not create any problem, if the consumers connected to network had been experiencing low voltages wherein the DER integration will provide required voltage support and will contribute towards a flat voltage profile. However, if consumers had a normal voltage profile earlier, the DER integration may cause voltage to even cross-defined upper limit. Thus based on existing network and loading conditions, voltage profile has been used as an objective or a constraint in distributed generator (DG) planning problem. Liu et al. [5] have formulated a multi-objective problem considering different cost functions and voltage profile. Masoud et al. [6] and Yarahmadi et al. [7] have solved sizing and siting problem of wind generators considering voltage stability as the objective. A bi-level optimisation for PV and storage allocation has been framed by Sharma et al. [8]. Singh et al. [9] have used sensitivity for improvement in losses and voltage. Elnashar et al. [10] have used losses, voltage profile and short circuit level as objectives for DG sizing and siting problem. Danilo et al. [11] have proposed a mixed integer linear programming based approach for improving voltage and energy efficiency. Voltage has also been used as a constraint in various planning studies. Ugranlı and Karatepe [12] have reported placement of multiple-DG unit for power loss reduction constrained by voltage limits. Voltage profile has also been used as a constraint in placement problem by Kryonidis et al. [13] and Liu et al. [14]. However inappropriate size and
location can lead to increase in losses and deterioration of voltage profile [12]. Thus planning from the perspective of line loss reduction is a sizing, as well as, locational issue.

### 1.1 Literature survey

There has been significant contribution by researchers in determining optimum size and location of DERs for line loss reduction. The literature survey conducted on DER sizing and placement can be broadly classified under following groups:

(i) Studies involving only dispatchable DERs (such as, diesel generators, biomass):

Amongst the literature involving dispatchable DERs, Ugranli et al. [12] have investigated behaviour of multiple-DGs in terms of capacity and load uncertainty using ANN. Andrés et al. [15] have determined the optimal size and location using a modified teaching-learning based algorithm. Esmaili [16] have used fuzzification to address multi-objective problem of losses and voltage stability margin. Impact of reverse power flow in context of increasing DG penetration has been studied by Sgouras et al. [17]. An effective methodology for determining optimum DG location has been proposed by Singh and Parida [18] based on system planner’s decision and analytical hierarchy. Multi-criteria decision model [19] has also been used for determining the allocation of high impedance fault detectors on distribution feeder. Reddy et al. [20] have considered critical load pick up as the objective for solving optimal DG placement problem using fuzzy multi-criteria decision-making (MCDM). Voltage has been used as a constraint in this study. However, loss analysis has not been considered.

(ii) Studies involving RES based DERs which have stochastic behaviour (such as, solar and wind):

With renewable energy penetration on increasing trend, it is essential to analyse the impact of integration of RES based DERs on losses. Amongst the literature involving RES based DERs, Caampued et al. [21] have presented evaluation of wind power penetration on system losses. However, penetration level as defined in ref. [21] does not address the stochastic behaviour of wind energy. Garzon et al. [22] have studied voltage variation in presence of PV systems without storage. Ali et al. [23] have investigated the impact of penetration of wind and solar penetration on voltage profile. Impact on system losses has not been considered. Solar photovoltaic, biomass and wind system have been considered for integration in ref. [24] but effect of DG penetration has not been studied. Naghdí et al. [25] and Saric et al. [26] have used probabilistic models of renewable DGs for loss minimisation. A static voltage stability optimisation formulation has been developed by Liu et al. [27] with solar and wind based DERs. Analysis considering time series data of RES based sources has been carried out in ref. [5]. Optimal allocation of wind based DERs using probabilistic model has been done considering various types of time and voltage dependent-load models [7]. The influence of stochastic uncertainties has been handled by using affine arithmetic model by Zhao et al. [28]. Zio et al. [29] have used probability density functions to model load uncertainty but variability of DGs has not been considered. An effective analysis has been put forward by Quezada et al. [30] wherein effect of different DG technologies, penetration levels, DG dispersion and location on system losses has been examined. However, the penetration level as defined in the work does not ensure that for every considered time interval, penetration of DG units will be equal to the defined one. This is significant in context with RES based DGs due to their stochastic nature. Atwa et al. [31] proposed a methodology for optimally allocating different types of RES based DGs for loss reduction. The methodology is based on generating a probabilistic generation model that combines all possible operating conditions of RES based DGs along with their probabilities of occurrence. A multi-objective formulation using game theory has been proposed by Soleymani et al. [32] considering solar and wind based DERs. The impact of intermittency on penetration level has not been examined. Multi-criteria assessment of RES has been carried out by Janjic et al. [33] and Tanwar and Khatod [34] using analytical hierarchy process. An integrated decision making planning approach has been proposed by Kazmi et al. [35] which embeds MCDM with unanimous decision making for sizing and siting of PV and D-STATCOM.

(iii) Studies involving stochastic DERs and storage:

Amongst the literature involving stochastic DERs and storage, Qiu et al. [36], have addressed optimal allocation of energy storage to help integration of wind energy. El-Zonkoly [37] discussed optimal scheduling and placement of PV and storage based system. A bi-level optimisation framework has been developed considering RES based DGs and battery storage [8]. However, loss minimisation and voltage profile improvement has not been targeted in these papers. In ref. [38] a bi-level optimisation model has been proposed to determine the optimal location and sizing of battery energy storage system. However, the analysis is focused only on battery storage and does not consider the sizing/placement of RES based sources. Seyed et al. [39] have determined the optimal allocation of PV/wind/storage/fuel cell based micro-grid. Kalkhambkar et al. [40] and Chedid et al. [41] have determined allocation of solar/wind based DGs and storage. However, impact of penetration level has not been analysed. In order to have a swift perception of research gaps, a summary of literature survey is presented in Table 1.

### 1.2 Research gaps

Based on literature survey following conclusions can be drawn:

(i) Though impact of integration of conventional DG units on line losses and voltage profile has been widely studied, majority of studies fail to acknowledge the impact of
intermittency of RES based DERs (solar and wind) on line losses.

(ii) Due to intermittent nature of RES based DERs, integration of storage has been found essential in order to provide firm support to renewables. With the increasing inclination towards RES, storage integration is expected to gain increasing importance in near future. However, none of the analyses consider the impact of adding storage units along with RES based DERs on system losses and voltage profile.

(iii) With regards to studies analysing the impact of variation of penetration level on system losses and voltage profile, it has been observed that penetration level as defined in literature does not take into account the stochastic nature of RES.

Optimal placement becomes more challenging when DERs comprise of solar/wind based DERs and storage due to following reasons:

• The solar energy is not available for nearly half of the time. The wind energy has a highly intermittent nature. Thus correlation between intermittency and placement has to be adequately analysed.

• The storage is an energy limited source. The availability of energy in storage is dependent on excess energy available from RES based DERs. The major problem encountered in placement of storage is due to the fact that storage behaves as a source when it is in discharging mode and behaves as a load when in charging mode exhibiting dual nature. This can become particularly significant in systems with high DER penetration wherein power drawn and supplied by storage is quite high.

1.3 Contributions and organisation

In order to address research gaps discussed in Section 1.2, this paper proposes a generalised formulation embedded in probabilistic framework for impact assessment of DER penetration and placement on system losses and voltage profile. Though RES based DERs comprise of a variety of generating sources, wind and solar have been considered in this paper as they are emerging as most preferred technologies. In order to adequately address the stochastic behaviour of RES based DERs, the penetration level corresponding to a time segment is defined as a function of meteorological conditions of that particular time segment. The assessment of repercussion of penetration level on system losses and voltage profile has been investigated on hour by hour basis. The optimum placement problem is formulated with the objective of loss minimisation. In order to incorporate the uncertainties associated with RES, load flow has been
carried out using backward–forward sweep algorithm embedded in probabilistic framework. The proposed methodology has been applied to a 33-bus radial distribution feeder.

Remainder of the paper is organised as follows: Section 2 explains mathematical formulation used in the paper. A brief discussion on mathematical models of DERs has been provided. Formulation of dispatch strategy and active/reactive power injection matrix has also been discussed. Section 3 explains problem formulation. In Section 4 solution methodology in probabilistic framework has been presented. Section 5 presents a case study wherein the proposed formulation is implemented. A detailed discussion on analysis of result has been carried out. Section 6 presents the relevant conclusions and future scope of work.

2 | MATHEMATICAL FORMULATION

In order to address intermittent behaviour of RES based DERs and their corresponding correlation with storage units; a probabilistic model has been developed by author in previous work [42–44]. The detailed description of mathematical model can be obtained from [42]. The probabilistic model comprises of renewable energy source model (RESM) and battery storage model (BSM). RESM includes all possible combinations of solar irradiance and wind speed along with their associated probabilities. Solar irradiance is modelled using standard beta probability density function and wind speed using Weibull probability density functions. BSM comprises of multiple states of battery SOC and associated probabilities. The probabilistic BSM is integrated with RESM in order to correlate effect intermittency of RES with battery SOC. Thus, in accordance with above models, the system state corresponding to $b_{th}$ state of BSM and $k_{th}$ state of RESM for $t_{th}$ time segment is represented as $S_{k,b}^{t}$.

2.1 | Distributed energy resource penetration

With RES based DERs, penetration level is a function of meteorological parameters. Thus, it becomes essential to investigate penetration level on hour by hour basis. In this paper, penetration of DERs is meant to ensure a firm capacity addition which means that at all points of time and for all system states, DERs should be able to fetch a percentage of load defined by minimum penetration level [44].

In order to ascertain high standards of planning, analysis is carried out for minimum penetration level and maximum allowable penetration level which are explained as follows:

A. Minimum penetration level: This refers to minimum percentage of load that must be supplied by DERs for each time segment irrespective of system states. Thus, for any time segment $t'$ in study period,

$$\rho_{k,b}^{t'} \geq \rho_{\text{def}}^{\text{min}} \text{ for } \forall k \in \text{RESM}', \forall b \in \text{BSM}'. \quad (1)$$

The power purchased from grid is bounded by $\rho_{\text{def}}^{\text{min}}$ irrespective of states of RESM, and BSM. Hence maximum power which can be drawn from grid for $t_{th}$ time segment can be expressed as:

$$P_{G_{\text{max}}}^{t'} = (1 - \rho_{\text{def}}^{\text{min}}) L_{t}^{t'}.$$  \hspace{1cm} (2)

where, $P_{G_{\text{max}}}^{t'}$ = maximum permissible power from grid for $t_{th}$ time segment, kW.

A. Maximum allowable penetration level $\rho_{\text{def}}^{\text{max}}$: This refers to maximum percentage of load which DERs are allowed to supply for each time segment. Thus, for any time segment $t$ in study period,

$$\rho_{k,b}^{t} \leq \rho_{\text{def}}^{\text{max}} \text{ for } \forall k \in \text{RESM}', \forall b \in \text{BSM}'. \quad (3)$$

Due to highly stochastic nature of RES, a system designed to ensure $\rho_{\text{def}}^{\text{min}}$ has to take into account worst periods of wind and sun. However, system can definitely deliver much higher amount of generation during favourable meteorological conditions. Thus economically it makes all sense to allow higher amount of penetration from DERs than $\rho_{\text{def}}^{\text{min}}$ whenever it is feasible to do so. However, allowing higher amount of power from DERs can adversely affect system losses and voltage profile. The $\rho_{\text{def}}^{\text{max}}$ puts an upper limit to amount of power which DERs are allowed to supply so that system losses and voltage profile are constrained within specified limits. A thorough analysis of impact of $\rho_{\text{def}}^{\text{min}}$ and $\rho_{\text{def}}^{\text{max}}$ is essential in order to come up with a well-planned system.

2.2 | Dispatch strategy

For $t_{th}$ time segment load which must be supplied by DERs is constrained by minimum penetration level and is expressed as:

$$L_{t}^{t} = \rho_{\text{def}}^{\text{min}} \times L_{t}^{t}.$$ \hspace{1cm} (4)

For $t_{th}$ time segment, maximum load which DERs are allowed to supply is constrained by maximum allowable penetration level (explained in Section 2, Part B) and is expressed as:

$$L_{t}^{t} = \rho_{\text{def}}^{\text{max}} \times L_{t}^{t}.$$ \hspace{1cm} (5)

The assumptions made in this study are as follows:

(i) PV arrays are assumed to supply only active power.
(ii) WTGs are synchronous machines and are assumed to be operating at a lagging power factor of 0.8 or above [30].
(iii) The storage units are discharged at a lagging power factor of 0.8 or above and are charged at unity power factor.
(iv) All storage capacity is considered as a single unit and is not distributed along the feeder.
The DERs are dispatched in following order:

- PV arrays are first to be dispatched since they have lowest operating costs.
- WTG units come in next to PV arrays.
- Storage units are given least priority in dispatch order. The primary objective of storage integration in this work is to counteract intermittent nature of RES by providing a firm backup. Hence, they are dispatched only when power from RES is not available either due to unfavourable meteorological condition or a unit outage.

The dispatch strategy proposed in this work is enumerated as follows:

(i) Dispatch generator (as per dispatch order) to supply load constrained by \( L'_{\text{DER}_{\text{load}}} \).

(ii) If output power available from considered generator is more than \( L'_{\text{DER}_{\text{min}}} \), storage goes into charging mode.

(iii) If there is excess power available from considered generator after supplying \( L'_{\text{DER}_{\text{load}}} \) and duly charging storage units, then excess power is made to supply load constrained by \( L'_{\text{DER}_{\text{max}}} \).

Steps (i) to (iii) are repeated for all generating units.

(i) \( L'_{\text{DER}_{\text{max}}} \) remains unsupplied after dispatching all generating units then storage unit is dispatched. The power supplied from storage units is subjected to constraints on battery state of charge.

### 2.2.1 Active power injection matrix

The active power injected by PV arrays and WTG units is calculated based on dispatch strategy discussed in Section 2.2. For each system state \( S_{t,k,b} \), the PV arrays are dispatched first. The active power injected by \( m^b \) PV array is \( P_{PV_{t,k,b}} \). If the output power from PV arrays is more than that required by load, the excess power is used to charge battery. The active power injection from PV arrays can be expressed as \([P_{PV_{t,k,b} 1}, ..., P_{PV_{t,k,b} m^b}, ..., P_{PV_{t,k,b} N^P}]\). Once, the output power from PV arrays is exhausted, WTG units are dispatched. The active power injected by \( n^b \) WTG unit is \( P_{WTG_{t,k,b}} \). If the output power from WTG units is more than that required by load, the excess power is used to charge storage. The active power injection from WTG units can be expressed as \([P_{VTG_{t,k,b} 1}, ..., P_{VTG_{t,k,b} n^b}, ..., P_{VTG_{t,k,b} N^W}]\).

The battery can inject or draw active power based on power flows. If output power available from generators is more than \( L'_{\text{DER}_{\text{max}}} \), storage goes into charging mode. In case of power deficit, storage bank exhibit discharging mode and injects active power. The formulation of active power injection matrix based on proposed strategy is depicted in Figure 1. The active power injection matrix can be expressed as:

\[
P'_{k,b} = \begin{bmatrix}
P_{PV_{t,k,b} 1} & ... & P_{PV_{t,k,b} m^b} & ... & P_{PV_{t,k,b} N^P} \\
P_{VTG_{t,k,b} 1} & ... & P_{VTG_{t,k,b} n^b} & ... & P_{VTG_{t,k,b} N^W} \\
\pm P_{B_{t,k,b}}
\end{bmatrix}.
\]

### 2.2.2 Reactive power injection matrix

The PV arrays are assumed to be operating at unity power factor. Hence reactive power supplied by them is zero. The WTGs and storage units are assumed to supply reactive power at a power factor between 0.8 lag to unity based on reactive power requirement of load. Thus

\[ Q_{PV_{t,k,b}} = 0. \]

\[ Q_{WTG_{t,k,b}} = \tan(\varphi) \times P_{WTG_{t,k,b}}. \]

Thus reactive power injection matrix can be expressed as:

\[
Q_{k,b} = \begin{bmatrix}
Q_{PV_{t,k,b} 1} & ... & Q_{PV_{t,k,b} m^b} & ... & Q_{PV_{t,k,b} N^P} \\
Q_{WTG_{t,k,b} 1} & ... & Q_{WTG_{t,k,b} n^b} & ... & Q_{WTG_{t,k,b} N^W} \\
Q_{B_{t,k,b}}
\end{bmatrix}.
\]

### 3 PROBLEM FORMULATION

The system losses and voltage profile respond differently to different DER penetration levels. One of the major contributions of this work is analysis and consideration of two important factors in placement problem which have not been accounted in previous studies. The factors which drive placement problem in this work are as follows:

(i) Intermittent nature of RES based DGs: The generation from RES based DGs is highly unpredictable, thus, location which might be appropriate for injection of low power levels might be turned down at other instants when power availability increase during high periods of wind and sun.

(ii) Dual nature of storage units: The storage exhibits dual nature; behaving as load or source based on availability of power from RES and system loading conditions. Thus location which might be appropriate with storage as source might not be appropriate with storage as load.

Due to above factors, it is possible that DER penetration reduces losses at one instant of time and at some other instant losses may even rise above the losses without DER penetration. Similar perception holds true regarding voltage profile as well. Hence, an in depth analysis considering above factors...
FIGURE 1  Evaluation of active power injection based on proposed dispatch strategy
is required in order to achieve maximum benefits from DER penetration.

### 3.1 Objective function

The objective function for DER placement can be stated as:

\[
\text{Minimise } P_{\text{loss}}.
\]

where, \( P_{\text{loss}} = \text{Annual expected energy loss, kWh.} \)

### 3.2 Constraints

The problem of finding optimal placement of DERs is subjected to following constraints:

A. Constraint on load supplied by DERs: The load which is to be supplied by DERs for any time segment is constrained by defined minimum penetration level and maximum allowable penetration level.

\[
I_{\text{DER min}}^i \leq I_{\text{DER}}^i \leq I_{\text{DER max}}^i.
\]

B. Constraint on voltage limits on each bus: The magnitude of voltage at all buses in network must comply with defined voltage limit. Thus, voltage magnitude at \( i^{th} \) bus \( V_i \) is subjected to strict voltage constraints.

\[
V_{\text{min}} \leq V_i \leq V_{\text{max}} \quad \text{for } i = 1, 2, \ldots, N_{\text{bus}}.
\]

C. Constraint on battery parameters: In order to ensure optimum life, battery is subjected to constraints on battery state of charge as follows:

\[
SOC_{\text{min}} \leq SOC \leq SOC_{\text{max}}.
\]

### 4 SOLUTION METHODOLOGY

Figure 2 presents the block diagram of proposed methodology in probabilistic framework. A brief discussion of butterfly particle swarm optimisation (BP-PSO), probabilistic load flow (PLF) and evaluation of objective function in probabilistic framework is presented in following subsections.

#### 4.1 Optimisation technique

The problem of finding optimal placement of DERs is a constrained non-linear, discrete combinatorial optimisation problem. The optimal placement problem for each penetration level is solved by using BP-PSO.

BP-PSO is a modified version of PSO [45, 46]. The BF-PSO mimics the natural intelligence and information sharing mechanism which the butterflies exhibit during nectar search process. The butterflies use sense of smell to determine possible direction of food, communicate with each other and accordingly move in a cooperative manner towards food position. In addition to parameters of standard PSO (inertia weight and acceleration coefficients), BF-PSO incorporates following parameters:

(i) Sensitivity of butterfly towards flower (\( S \))
(ii) Probability of food (nectar) (\( P \))
(iii) Time varying probability coefficient (\( \alpha \))

These parameters considerably improve the ability of algorithm to find better quality solutions in minimum possible time. The ranges of sensitivity and probability are considered varying between 0.0 and 1.0 and are expressed as a function of iteration as follows:

\[
S_k = e^{-(ITER_{\text{max}} - ITER_k) / ITER_{\text{max}}},
\]

where, \( ITER_{\text{max}} = \text{maximum number of iterations} \) and \( ITER_k = k^{th} \) iteration count.

\[
P_k = \frac{FIT_{\text{glob},k}}{\sum_{i} FIT_{\text{pbest}_{i,k}}}
\]

where, \( FIT_{\text{pbest}_{i,k}} = \text{fitness of personal best solution of } i^{th} \text{ particle in } k^{th} \text{ iteration, } FIT_{\text{glob},k} = \text{fitness of global best solutions in } k^{th} \text{ iteration, } P_k \text{ is the probability of } k^{th} \text{ iteration.} \)
best (gbest) as follows [45]:

\[ r_{id}(k + 1) = w_k r_{id}(k) + S_k(1 - P_k) C_1 r_1(P_{best_{id}}(k) - x_{id}(k)) + P_k C_2 r_2 [gbest_{id}(k) - x_{id}(k)]. \] (17)

\[ x_{id}(k + 1) = x_{id}(k) + \alpha_k r_{id}(k + 1). \] (18)

where, \( r_{id}, x_{id}, P_{best_{id}} \) and gbest_{id} represent velocity, position, personal best and global best respectively of \( d \) th dimension of \( j \) th particle, \( w_k \) is inertia weight for \( k \) th iteration, \( C_1 \) and \( C_2 \) are acceleration coefficients and \( r_1 \) and \( r_2 \) are random number in the interval \([0, 1]\), \( P_k \) is the probability of global best (generally assume \( P_k = 1 \) for global solution) and \( \alpha_k \) is time varying probability coefficient. \( \alpha_k \) is calculated as follows:

\[ \alpha_k = \text{rand} \times P_k. \] (19)

where, \( \text{rand} \) is the random number \([0, 1]\).

4.2 Probabilistic load flow using backward–forward sweep

The deterministic load flow (DLF) has been traditionally used by system planners for determination of losses. DLF calculates power flows in network for specified generation. This is acceptable in system incorporating conventional generating units. However, for systems with RES based DGs, DLF fails to incorporate uncertainties associated with these sources.

In order to overcome difficulties associated with DLF, PLF has been used in this paper [47, 48]. In contrast with DLF which uses deterministic values, the input to PLF is specified by pdfs so that system uncertainties are given due consideration. In this work PLF is solved using backward/forward sweep algorithm [49].

4.3 Objective function evaluation in probabilistic framework

In order to incorporate uncertainties associated with DERs, calculation of objective function has been carried out in a probabilistic framework. The PLF employing backward–forward sweep algorithm estimates power flows and losses in network. The power flow and loss for system state corresponding to \( S'_{k,b} \) is a function of active power injection matrix \( P'_{k,b} \) and reactive power injection matrix \( Q'_{k,b} \). The probability of system state corresponding to \( S'_{k,b} \) can be expressed as:

\[ P(S'_{k,b}) = P(R'_{k,b}) \times P(SOC'_{k,b}) \] (20)

where, operator ‘\( P \)’ represent the probability.

The expected value of power loss for \( k \) th state of RESM’ and \( b \) th state of BSM’ can be expressed as:

\[ P_{\text{Loss}}(S'_{k,b}) = f(P'_{k,b}, Q'_{k,b}) \times P(S'_{k,b}) \] (21)

where, \( P_{\text{Loss}}(S'_{k,b}) \) is variable representing expected power loss for \( S'_{k,b} \) kW, \( f \) = function of \( P'_{k,b} \), \( Q'_{k,b} \), and \( L' \) to compute power losses in the system. For \( k \) th time segment, the amount of active and reactive power injections, given by Equations (6) and (10) respectively, are dependent on RESM RESM’, BSM BSM’ and load \( L' \). Hence, \( f \) has been considered as function of RESM’, BSM’ and \( L' \). Since RESM’, BSM’ and \( L' \) are variable during \( k \) th time segment, their function \( f \) is also a variable during same time segment kW.

The expected value of power loss for \( j \) th time segment can thus be calculated as:

\[ P_{\text{Loss}} = \sum_{k=1}^{N_{\text{RESM}}} \sum_{l=1}^{N_{\text{BSM}}} P_{\text{Loss}}(S'_{k,b}). \] (22)

The expected value of energy loss over entire study period can thus be calculated as:

\[ P_{\text{Loss}} = \sum_{t=1}^{T} P_{\text{Loss}}(s) \times l(t). \] (23)

where, \( l(t) = \) length of \( t \) th time segment, hours.

5 CASE STUDY: RESULTS AND DISCUSSIONS

The proposed formulation has been applied to a 12.66 kV, 33-bus distribution system derived from [50]. The schematic diagram of radial distribution feeder is shown in Figure 3. The distribution feeder is connected to grid at bus number 1 through substation and has a peak load of 3.755 MW. The distribution system has been assumed to be located at Jaisalmer, Rajasthan,
In order to analyse impact of penetration level of DERs, three different minimum penetration levels, for example, 20%, 40% and 60% have been considered in this study. The optimal sizing for different DER penetration levels has been carried out using the formulation presented by authors in refs. [43, 44]. The optimal placement problem has been solved using methodology explained in Section 4. The placement problem is solved for each minimum penetration level using BFPSO. Maximum voltage limit on each bus is taken as 1.05 p.u. Minimum voltage limit on each bus is constrained by base case minimum voltage on each bus. Table 2 presents losses and real and reactive power drawn from grid without DER penetration. The optimal sizing and placement results for different penetration levels are presented in Table 3.

As is evident from Table 3, for 20% minimum penetration configuration, though storage is integrated, its capacity is not substantial. Thus, placement of PV arrays and WTG units is not affected by dual nature of storage units for 20% minimum penetration configuration. However, for 40% and 60% minimum penetration configurations, the capacity of storage integrated into system is substantial. It is evident from optimal placement results for 40% and 60% minimum penetration configurations presented in Table 3, that half of the WTG capacity is placed along with storage units. The placement of WTG and storage units on the same bus for 40% and 60% minimum penetration configurations as suggested by optimal placement results are analysed as follows:

Since order of dispatching units is solar, wind and then storage, the availability of spare solar power after supplying load is quite low. Thus charging power to storage units is not usually supplied by PV arrays. However, WTG units have surplus availability during good periods of wind and also during sunshine hours. Thus WTG units are mostly responsible for charging storage units. During charging period, storage units pose as a load which is significant enough to cause an increase in power loss. This also results in voltage drop on the line thereby raising voltage issues. However, placement of storage unit on same bus as WTG unit solves this problem. Thus WTG units and storage are located on the same bus.

Hence, although optimal placement of storage with storage acting as source might have been different, considering it as a load requires placement of WTG unit on the same bus. Therefore, placement of DERs comprising of storage units is a challenging issue and requires detailed analysis in order to come up with an optimal solution.

Having determined optimal placement of DERs corresponding to each minimum penetration configuration, impact of varying maximum allowable penetration level is investigated. As discussed in Section 2.2, the system designed for a defined minimum penetration level is capable of delivering higher amounts of power during favourable meteorological conditions. Allowing higher $\rho_{\text{def}}$ amplifies economic viability of DER integration. In this section impact of allowing higher values of $\rho_{\text{def}}$ is studied on system losses and voltage profile with respect to all three minimum penetration configurations. The analysis is focused on determining following information:

(i) The effect of varying $\rho_{\text{def}}$ on annual active and reactive energy losses.

(ii) Analysis of losses and voltage profile on hour by hour basis.

This is essential in order to understand effect of intermittency.

### 5.1 Effect of variation of maximum allowable penetration level on active and reactive energy losses

Figures 4–6 shows effect of varying $\rho_{\text{def}}$ on annual active and reactive energy losses for $\rho_{\text{def}} = 20\%$, 40% and 60%, respectively.

It can be observed from Figure 4 that for $\rho_{\text{def}} = 20\%$, after an initial decline in losses, an increasing trend is observed beyond $\rho_{\text{def}} = 40\%$. However, as shown in Figures 5 and 6, for $\rho_{\text{def}} = 40\%$ and 60%, losses increase steadily.
Table 4 presents the optimum value of $\rho_{\text{def}}^{\text{max}}$ following objectives:

(i) Determination of $\rho_{\text{def}}^{\text{max}}$ which fetches minimum system losses.

(ii) Determination of $\rho_{\text{def}}^{\text{max}}$ which fetches maximum DER penetration constrained by base case losses.

It can be observed from Table 4, that increasing $\rho_{\text{def}}^{\text{max}}$ beyond 20% does not render any benefit from the perspective of loss minimisation. However, from the perspective of maximising DER penetration, $\rho_{\text{def}}^{\text{max}} = 20\%$ is the most suitable penetration level. The utilisation of resources is maximum in this case. At higher penetration levels, the system has enough redundancy in order to maintain reliability standards during worst periods of wind and sun. Hence, power which system can supply also increases. It is apparent from Table 4, that $\rho_{\text{def}}^{\text{max}}$ is minimum with $\rho_{\text{def}}^{\text{min}} = 60\%$.

### Table 4  Optimum value of $\rho_{\text{def}}^{\text{max}}$ for each $\rho_{\text{def}}^{\text{min}}$.

| $\rho_{\text{def}}^{\text{min}}$ | $\rho_{\text{def}}^{\text{max}}$ for minimum system losses | $\rho_{\text{def}}^{\text{max}}$ for maximum DER penetration (constrained by base case losses) |
|---|---|---|
| 20% | 40% | 95% |
| 40% | 40% | 84% |
| 60% | 60% | 80% |
FIGURE 7 Hourly variation of active energy losses for $\rho_{\text{def}} = 20\%$

![Figure 7](image_url)

FIGURE 8 Hourly variation of active energy losses for $\rho_{\text{def}} = 40\%$

![Figure 8](image_url)

5.2 Analysis of hourly variation of active energy losses

Figures 7–9 show the hourly variation of active energy losses for $\rho_{\text{def}} = 20\%, 40\%$ and $60\%$, respectively, w.r.t. $\rho_{\text{def max}}$ presented in Table 4. From Figures 7–9, following important observations can be made:

(i) In comparison with losses without DER penetration, losses with DER penetration are lower all year around for all the three considered $\rho_{\text{def min}}$ when $\rho_{\text{def max}} = \rho_{\text{def min}}$.

(ii) When the value of $\rho_{\text{def max}}$ is increased and chosen from perspective of maximising DER penetration, losses do not stay below the base level throughout the year. The losses are higher during summer and monsoon months as compared to base case losses. This is due to increase in availability of sunshine hours during summer months and better wind speed during monsoon months. Nevertheless, increase in losses beyond base case during summer and monsoon months is compensated by decrease in losses for rest of the time segments. Thus, total losses still stay below the base case level.

5.3 Analysis of voltage profile

Figures 10–12 show the shows voltage profile on each bus for $\rho_{\text{def}} = 20\%, 40\%$ and $60\%$ respectively w.r.t. $\rho_{\text{def max}}$ presented in Table 4.

From Figures 10–12, following important observations can be made:

(i) With DER penetration voltage profile shows improvement on all buses compared with when no DERs are placed.
FIGURE 10  Voltage profile at various buses for $\rho_{\text{def}} = 20\%$

FIGURE 11  Voltage profile at various buses for $\rho_{\text{def}} = 40\%$

FIGURE 12  Voltage profile at various buses for $\rho_{\text{def}} = 60\%$
(ii) On comparing voltage profile for $\rho_{\text{def}}=40\%$ and $\rho_{\text{def}}=60\%$ with voltage profile for $\rho_{\text{def}}=20\%$, it can be observed that minimum voltage achieved with $\rho_{\text{def}}=40\%$ and $60\%$ is higher in comparison with that achieved through $\rho_{\text{def}}=20\%$. This is attributed to increase in availability of power from DERs for higher $\rho_{\text{def}}$.

This can further be explained with reference to optimal placement results presented in Table 3. It can be seen that now storage and half of the WTG capacity are placed on the same bus. As discussed earlier, the charging power to storage units is mostly supplied by WTGs. Now, even when battery is in charging mode and storage is acting as a load, there is no drop in the line since storage and WTG units are placed on the same bus.

Thus, it can be concluded that placement of storage units in conjunction with RES based DGs require special considerations and has to be dealt carefully.

6 Conclusion and Future Scope

In view of increasing DER penetration, it is imperative to analyse the effect of different penetration levels on various system parameters. This paper presents a formulation for impact assessment of DER penetration level and placement on system losses and voltage profile. The main contributions of this paper can be summarised as follows:

(i) A generalised formulation embedded in probabilistic framework has been developed for impact assessment of DER penetration and placement on system losses and voltage profile.

(ii) The planning formulation presented in this paper is bestowed with following specific features:

- Acknowledgement of intermittent nature of RES based DGs in placement problem.
- Analysis of effect of dual nature of storage units in placement problem.

(iii) Penetration level has been defined and analysed so as to adequately address the stochastic behaviour of RES based DERs.

(iv) Assessment of effect of penetration level on system losses and voltage profile on hour by hour basis. A thorough analysis of $\rho_{\text{def}}$ for a particular $\rho_{\text{def}}$ has been carried out.

In order to establish the importance of investigating DER penetration, three different minimum penetration levels have been studied. Impact of variation of maximum allowable penetration level has been studied for each of the minimum penetration configurations. Following inferences have been drawn based on study carried out in this work:

- The storage exhibits dual nature; behaving as load or as a source based on availability of power from RES and system loading conditions. Thus location which might be appropriate with storage as source might not be appropriate with storage as load.
- Due to intermittent nature of RES based DGs it is possible that DER penetration reduces losses at one instant of time and at some other instant losses may even rise above losses without DER penetration.
- Increasing DER penetration beyond a certain level can result in degradation of line losses and voltage profile.

The analyses presented in this work will assist system planners to choose optimum DER penetration level. Nevertheless, grid integration of DERs calls for increased analysis. Thus, present study can be further extended in following research areas:

- This paper deals with the planning issues concerning DER penetration. However, the work can be further extended to incorporate operational issues as well.
- Formulation of control strategies considering the effect of DER penetration level can be done.
- The formulation presented in this paper is a generalised one and can be applied for analysis of large power systems. The analysis can be further extended to meshed distribution network as well. Feeder reconfiguration in conjunction with different DER penetration levels can also be studied for loss minimisation.

Nomenclature

| Symbol | Description |
|--------|-------------|
| $Q_{\text{B}_{b}}$ | Reactive power injection by storage in discharging mode for $S'_{b}$ |
| $L'_{\text{DER}_{\text{min}}}, L'_{\text{DER}_{\text{max}}}$ | Minimum and Maximum load respectively for $n^{th}$ time segment which must be supplied by DERs, kW |
| $\rho_{\text{def}_{\text{min}}}, \rho_{\text{def}_{\text{max}}}$ | Minimum and Maximum allowable penetration level respectively |
| $P_{\text{PV}_{b}}^\text{a}, Q_{\text{PV}_{b}}^\text{a}$ | Active and Reactive power injection respectively by PV array for $S'_{b}$ |
| $P_{\text{WTG}_{b}}^\text{a}, Q_{\text{WTG}_{b}}^\text{a}$ | Active and Reactive power injection respectively by WTG unit for $S'_{b}$ |
| $+P_{b_{k}} - P_{b_{k}}$ | Active power injection and drawn respectively by storage unit for $S'_{b}$ |
| $\varphi_{a}, \varphi_{b}$ | Power factor angle of $n^{th}$ WTG and storage unit respectively |

Acronyms

$b$ Index representing $b^{th}$ state of $BSM$

DER Distributed energy resources

DG Distributed generators

Indices

$k$ Index representing $k^{th}$ state of $RESM$

$L'$ Load during $n^{th}$ time segment, kW
\(N_{PV}, N_{WTG}\) Number of PV arrays and WTG units respectively.

**Parameters**

| Symbol | Description |
|--------|-------------|
| PV     | Photovoltaic |
| RES    | Renewable energy sources |
| SOC\(_{\text{min}}\), SOC\(_{\text{max}}\) | Lower and upper limit respectively for battery SOC |
| \(t\)  | Index representing \(t^{\text{th}}\) time segment |

**Variables**

| Symbol | Description |
|--------|-------------|
| \(V_{\text{min}}, V_{\text{max}}\) | Minimum and maximum permissible voltage magnitude respectively, p.u. |
| WTG    | Wind turbine generator |

**REFERENCES**

1. IEEE Std 1547-2018: IEEE Standard for Interconnection and Interoperability of Distributed Energy Resources with Associated Electric Power Systems Interfaces, (2018). https://doi.org/10.1109/IEEEPSIG.2018.8332112
2. Wankhede, S., et al.: Increasing penetration of DERs in smart grid framework: A state-of-the-art review on challenges, mitigation techniques and role of smart inverters, Journal of Circuits, Systems and Computers 29(16), 2030014, 2019 https://doi.org/10.1142/S0218126620300147.
3. Peri, P.G.V., et al.: ACMC-based hybrid AC/LVDC micro-grid. IET Renewable Power Gener. 11(4), 521–528 (2017). https://doi.org/10.1049/iet-rpg.2016.0389
4. Barker, P.: Determining the impact of distributed generation on power systems: Part 1- radial distribution systems. Int. 2000 IEEE Power Engineering Society Summer Meeting, Seattle, USA, June 16–20 July 2000 https://doi.org/10.1109/PES.2000.868775
5. Liu, K., et al.: Optimal siting and sizing of DGs in distribution system considering time sequence characteristics of loads and DGs. International J. Electrical Power Energy Syst. 69, 430–440 (2015). https://doi.org/10.1016/j.ijepes.2015.01.033
6. Masaud, T.M., et al.: Optimal placement and sizing of distributed generation-based wind energy considering optimal self VAR control. IET Renewable Power Gener. 11(3), 281–288 (2017). https://doi.org/10.1049/iet-rpg.2015.0391
7. Yarahmadi, M., Shakarami, M.R.: An analytical and probabilistic method to determine wind distributed generators penetration for distribution networks based on time-dependent loads. Int. J.Electrical Power Energy Syst. 103, 404–413 (2018). https://doi.org/10.1016/j.ijepes.2018.06.025
8. Sharma, S., et al.: Bilevel optimization framework for impact analysis of DR on optimal accommodation of PV and BESS in distribution system. International Transactions on Electrical Energy Systems 29(9), e12062 (2019). https://doi.org/10.1049/iet-tsee.2020.7038.12062
9. Singh, A.K., Parida, S.K.: Novel sensitivity factors for DG placement based on loss reduction and voltage improvement. International Journal of Electric Power & Energy Systems 74, 453–456 (2016). https://doi.org/10.1016/j.ijepes.2015.04.010
10. Elbashar, M.M., et al.: Optimum siting and sizing of a large distributed generator in a mesh connected system. Electric Power Syst. Research 80(6), 690–697 (2010). https://doi.org/10.1016/j.epsr.2009.10.034
11. Danilo, M.O., et al.: Relaxed convex model for optimal location and sizing of DGs in DC grids using sequential quadratic programming and random hyperplane approaches. International J. Electrical Power Energy Syst. 115, 105442 (2020). https://doi.org/10.1016/j.ijepes.2019.105442
12. Ugrani, F., Karatepe, E.: Multiple-distributed generation planning under load uncertainty and different penetration levels. International J. Electrical Power Energy Syst. 46, 132–144 (2015). https://doi.org/10.1016/j.ijepes.2012.10.043
13. Kryonidis, G.C., et al.: A nearly decentralized voltage regulation algorithm for loss minimization in radial MV networks with high DG penetration. IEEE Trans. Sustainable Energy 7(4), 1430–1439 (2016). https://doi.org/10.1109/TSTE.2016.2560090
14. Liu, J., et al.: Stochastic expansion planning of interconnected distribution networks with renewable sources considering uncertainties and power transfer capability. The J. Engl. 2017(3), 1600–1604 (2017). https://doi.org/10.1049/joe.2017.0602
15. Andrés Martín, G.J., José Gil, M.A.: Optimal distributed generation location and size using a modified teaching–learning based optimization algorithm. International J. Electrical Power Energy Syst. 50, 65–75 (2013). https://doi.org/10.1016/j.ijepes.2013.02.023
16. Esfami, M.: Placement of minimum distributed generation units observing power losses and voltage stability with network constraints. IET Generation Transmission Distribution 7(8), 813–821 (2013). https://doi.org/10.1049/iet-gtd.2013.0140
17. Sgouras, K.I., et al.: Impact of reverse power flow on the optimal distributed generation placement problem. IET Generation, Transmission Distribution 11(18), 4626–4632 (2017). https://doi.org/10.1049/iet-gtd.2016.1791
18. Singh, A.K., Parida, S.K.: Allocation of distributed generation using proposed DMSP approach based on utility and customers aspects under deregulated environment. International J. Electrical Power Energy Syst. 68, 159–169 (2015). https://doi.org/10.1016/j.ijepes.2014.12.076
19. Khani, M., et al.: Decision support system for optimal location of HIFDs in real distribution network using an integrated EPISO-fuzzy AHP model. IET Generation, Transmission Distribution 14(9), 1616–1626 (2020). https://doi.org/10.1049/iet-gtd.2018.6696
20. Reddy, G.H., et al.: Optimal distributed generation placement in distribution system to improve reliability and critical loads pick up after natural disasters. International Journal of Eng. Sci. Technol. 20(3), 825–832 (2017). https://doi.org/10.14419/ijest.v20i3.7590
21. Christian, C.P., et al.: Determination of penetration limit of wind distributed generation (DG) considering multiple bus integration. In: 2018 IEEE PES Asia-Pacific Power and Energy Engineering Conference (APPEEC), Kota Kinabalu, Malaysia, 7–10 October 2018 https://doi.org/10.1109/APPEnec.2018.8566589
22. Ruiz-Garzón, J., González-Tristanche, D.: Impact of changing location and power of a PV system in electrical distribution networks, integrating MAT-LAB and OpenDSS. DYNA 85, 125–131 (2018). 10.15446/dyna.v85n205.68846
23. Ali, B., et al.: Optimal oversizing of utility-owned renewable DG inverter for voltage rise prevention in MV distribution systems. International J. Electrical Power Energy Syst. 105, 500–513 (2019). https://doi.org/10.1016/j.ijepes.2018.08.040
24. Rao, G.S., Das, D.: Optimum placement and sizing of DGs considering average hourly variations of load. International J. Electrical Power Energy Syst. 66, 25–40 (2015). https://doi.org/10.1016/j.ijepes.2014.10.047
25. Nagdhi, M., et al.: A combined probabilistic modeling of renewable generation and system load types to determine allowable DG penetration level in distribution networks. International Transactions on Electrical Energy Systems 29(1), e2696 (2019). https://doi.org/10.1002/etep.2696
26. Sari, M., et al.: Distributed generation allocation considering uncertainties. International Transactions on Electrical Energy Systems 28(9), e2585 (2018). https://doi.org/10.1002/etep.2585
27. Liu, K., et al.: Simplified probabilistic voltage stability evaluation considering variable renewable distributed generation in distribution systems. IET Generation Transmission Distribution 9(12), 1464–1473 (2015). https://doi.org/10.1049/iet-gtd.2014.0840
28. Zhao, Q., et al.: Multi-objective optimal allocation of distributed generations under uncertainty based on D-S evidence theory and affine arithmetic. International Journal of Electrical Power Energy Syst. 112, 70–82 (2019). https://doi.org/10.1016/j.ijepes.2019.04.044
29. Zio, E., et al.: Monte Carlo simulation-based probabilistic assessment of DG penetration in medium voltage distribution networks. International J. Electrical Power Energy Syst. 64, 852–860 (2015). https://doi.org/10.1016/j.ijepes.2014.08.004
30. Méndez Quezada, V.H., et al.: Assessment of energy distribution losses for increasing penetration of distributed generation. IEEE Transaction on...
31. Arwa, Y.M., et al.: Optimal renewable resources mix for distribution system energy loss minimization. IEEE Transactions on Power System 25(1), 360–370 (2010). https://doi.org/10.1109/TPWRS.2009.2030276

32. Soleymani, S., et al.: Optimal placement of DG units for the enhancement of micro-grid networks performance using coalition game theory. IET Generation, Transmission Distribution 14(5), 853–862 (2020). https://doi.org/10.1049/iet-gtd.2019.0070

33. Janjic, A., et al.: Renewable energy integration in smart grids-multicriteria assessment using the fuzzy analytical hierarchy process. Turkish Journal of Electrical Engineering and Computer Sciences 23, 1896–1912 (2015). https://doi.org/10.3906/elk-1404-287

34. Tanwar, S.S., Khatod, D.K.: Techno-economic and environmental approach for optimal placement and sizing of renewable DGs in distribution system. Energy 127, 52–67 (2017). https://doi.org/10.1016/j.energy.2017.02.172

35. Kazmi, S.A., et al.: A techno-economic centric integrated decision-making planning approach for optimal assets placement in meshed distribution network across the load growth. Energies 13, 1444 (2020). https://doi.org/10.3390/en13061444

36. Qiu, J., et al.: Distributed generation and energy storage system planning for a distribution system operator. IET Renewable Power Gener. 12(12), 1345–1353 (2018). https://doi.org/10.1049/iet-rpg.2018.5115

37. El-Zonkoly, A.: Optimal placement and schedule of multiple grid connected hybrid energy systems. International J. Electrical Power Energy Syst. 61, 239–247 (2014). https://doi.org/10.1016/j.ijepes.2014.03.040

38. Xiao, J., et al.: Determination of the optimal installation site and capacity of battery energy storage system in distribution network integrated with distributed generation. IET Generation, Transmission Distribution 10(3), 601–607 (2016). https://doi.org/10.1049/iet-gtd.2015.0130

39. Seyed, H., et al.: Optimal sizing and siting of microgrid units under high renewables penetration considering demand response. IET Renewable Power Gener. 13(10), 1809–1822 (2019). https://doi.org/10.1049/iet-rpg.2018.6015

40. Kalkhambkar, V., et al.: Joint optimal allocation methodology for renewable distributed generation and energy storage for economic benefits. IET Renewable Power Gener. 10(9), 1422–1429 (2016). https://doi.org/10.1049/iet-rpg.2016.0014

41. Chedid, R., Sawwas, A.: Optimal placement and sizing of photovoltaics and battery storage in distribution networks. Energy Storage 1(4), e46 (2019). https://doi.org/10.1002/est2.46

42. Paliwal, P., et al.: A novel method for reliability assessment of autonomous PV-wind-storage system using probabilistic storage model. International Journal of Electrical Power and Energy Syst. 55, 692–703 (2014). https://doi.org/10.1016/j.ijepes.2013.10.010

43. Paliwal, P., et al.: Determination of reliability constrained optimal resource mix for an autonomous hybrid power system using particle swarm optimization. Renewable Energy 63, 194–204 (2014). https://doi.org/10.1016/j.renene.2013.09.003

44. Paliwal, P., et al.: Probabilistic indices for analyzing the impact of DER penetration on system reliability. IET Renewable Power Gener. 14(12), 2154 (2020). https://doi.org/10.1049/iet-rpg.2019.1214

45. Tong, L., et al.: A simple butterfly particle swarm optimization algorithm with the fitness-based adaptive inertia weight and the opposition-based learning average elite strategy. Fundam. Inform. 163(2), 205–223 (2018). https://doi.org/10.3233/FI-2018-1738

46. Kamdar, R., et al.: A hybrid multi-agent based BFPSO algorithm for optimization of benchmark functions. Journal of Circuits, Systems and Computers 29, 2050112 (2019). 29. https://doi.org/10.1142/S0218126620501121

47. Valverde, G., et al.: Probabilistic load flow with non-gaussian correlated random variables using gaussian mixture models. IET Generation, Transmission Distribution 6(7), 701–709 (2012). https://doi.org/10.1049/iet-gtd.2011.0545

48. Valverde, G., et al.: Stochastic monitoring of distribution networks including correlated input variables. IEEE Trans. Power Syst. 28(1), 246–255 (2013). https://doi.org/10.1109/TPWRS.2012.2201178

49. Bompard, E., et al.: Convergence of the backward/forward sweep method for the load-flow analysis of radial distribution systems. Int. Journal of Elect. Power Energy Syst. 22(7), 521–530 (2000). https://doi.org/10.1016/S0142-0615(00)00009-0

50. Acharya, N., et al.: An analytical approach for DG allocation in primary distribution network. International Journal of Electric Power and Energy Syst. 28, 669–678 (2006). https://doi.org/10.1016/S0266-0339(05)00013-9

51. India Meteorological Department. Solar Radiant Energy over India. Ministry of Earth Sciences (2009)

52. Mani, A., Wind Energy Resource Survey in India-II. Allied Publishers Limited (1992)

53. Grigg, C., et al.: The IEEE reliability test system-1996. A report prepared by the reliability test system task force of the application of probability methods subcommittee. IEEE Trans. Power Syst. 14(3), 1010–1020 (1999). https://doi.org/10.1109/TPWRS.1999.780914

How to cite this article: Paliwal Priyanka. Comprehensive analysis of distributed energy resource penetration and placement using probabilistic framework. IET Renewable Power Gener. 2021;15:794–808. https://doi.org/10.1049/rpg2.12069