Modelling of fracture phenomenon in case of composite materials reinforced with short carbon fibers

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Abstract. The research work presented in this paper describes the composite materials in terms of formation and propagation of cracks using an algorithm that imposes disproportional loads on composite samples. The required parameters that describe the composites fracture demand inputs as: load intensity, geometry features and relative loading direction. In order to obtain reliable results, it should be a good correlation between the model which describes the fracture propagation, the composition of the material and the structural homogeneity. The presented study is using a Functionally Graded Material with local homogeneity in fracture area, and a numerical model based on integration of interactions (Mori – Tanaka method). The parameters that describes the fracture behaviour, includes a factor of stress intensity which is important for establish the fracture direction. The model used in simulations is considering a composite sample with rectangular shape and 6 mm thickness. The sample is loaded with predefined stress \( \sigma_{ct} \) (MPa) above and under the fracture line. \( \sigma_{ct} \) represents the critical stress able to lead to fracture propagation. The main objective of this research work it was to generate a numerical model which describes the fracture behaviour of a composite material. The obtained model and its accuracy to describe the fracture behaviour of a composite material is presented in the final part of this paper.

1. Introduction

The composite materials have been used long time before there were known as “composite”. Due to permanently growing demands regarding performance and strength of structures in general, especially those for aeronautics and military applications, which require very severe condition during operation, oblige the composite developers to keep pace with [1].

The composite materials are becoming the adopted solution in achieving efficient structures, with applications in all industries. Their implementation in various production fields, as advantageous alternative to classical materials, are coming with some issues arising from their very complex structure and the production possibilities, also insufficiently known behaviour in various applications [2]. Special interest is given, among other things, determining the damages that may occur under loading conditions, their effect on the load bearing structures, as well as their behavior in harsh conditions (large variation of the working parameters). These are only some of the issues raised by researchers in the field, the multitude of published papers demonstrate that the problems are far from resolved [3].

The sustainability of matrix composites can be calculated by using uniaxial and biaxial loading of the composite, studying the wear phenomenon that occurs in these cases. However, this method of loading the composite material offers multiple ways of analyzing its resistance to cracking, because
the phenomenon of wear which is characteristic for this kind of material is composed from two important components, wearing of the composite matrix, but mostly from sliding wear and cracking/breaking of fibres [4].

In our days, the optimization of composites structure and properties, such as cracking resistance has become a real need in practice as result of the requirement of making optimizations of complex structures, such as those from aeronautics. To achieve an objective of this kind, there are available specialised software able to deliver very accurate results. The combination between solid modelling and structural analysis, leads to a specially designed system, which can solve engineering problems.

This paper presents a study regarding the modelling process of fracture initiation and propagation in case of polymeric composite materials reinforced with short carbon fibres. In recent times, there are multiple concerns regarding strength and fracture behaviour of composites due to their utilisation in complex structures such as those used in the field of aviation.

2. The calculation algorithm applied to the modelling of crack formation and propagation

Composite materials have been investigated, with regard to crack formation and propagation by using an algorithm which applies disproportional loads, in conjunction with the finite element method. The required parameters in order to describe the material fracture impose the introduction of some intensity factors for load, geometry and load direction [5]. The correspondence between the model which explains the behaviour of crack propagation, the composition and the structural homogeneity, it is done by using numerical algorithm for simulation.

Compared to aluminium based composite materials, already studied and considered "classic", the metal matrix composites reinforced with short carbon fibres, lead to an increase of cracking resistance by lowering the concentration factor in case of matrix loads [6].

Assuming that, at one moment, a symmetrical load will determine a crack generation in the matrix body, simultaneously several parallel lines will appear and cause crack deviation and enlargement, as schematized in figure 1. In this case, the stronger the material oppose, the larger is the cracks deflection.

![Figure 1](image)

**Figure 1.** Schematized model of composite loading a) - geometry and loading directions (units and force in mm and N); b) - transposition network model.

On the assumption that the composite behave as a material that has gradual properties, which can be considered to be of FGM (Functionally Graded Materials), but which has local homogeneity in the failure area, known models could be used, based on numerical methods, but are implying a integrating method of inter-reactions [7, 8]. The parameter that describes the behaviour of cracking includes a stress intensity factor which is important in describing the cracking direction on two ways of actions KI and KII.

For the modelling process has been considered a composite surface with rectangular geometry (figure1) and thickness of 6 mm. The model is loaded with a tension $\sigma_{cr}$ (MPa) above and below the
fracture line. The critical load \( \sigma_{cr} \) is defined as the value that leads to the crack propagation. Where reference is made to a planar loading state is considered that the in composite material has a zonal homogeneity, and it is treated from the tension point of view as a lamina. The obtained values are presented in table 1.

Table 1. Properties and characteristics of material.

| \( \xi \) | E (MPa) | \( \nu \) | \( K_{fr} \) (MPa \( \sqrt{m} \)) |
|------|------|------|----------------|
| 0.00 | 3000 | 0.35 | 1.2 |
| 0.17 | 3300 | 0.34 | 2.1 |
| 0.33 | 5300 | 0.33 | 2.7 |
| 0.58 | 7300 | 0.31 | 2.7 |
| 0.83 | 8300 | 0.30 | 2.6 |
| 1.00 | 8600 | 0.29 | 2.6 |

where: \( \xi \) - defines the point with coordinates \( x \) and \( y \) from the network; can range between 0.00 and 1.00, as shown in figure 1.

3. Establishing the cracks propagation model and the parameters used in simulation

Based on the crack propagation criterion of Cotterell and Rice, for fragile materials (\( K_{II} = 0 \)), it will be considered that the crack propagation is made with a gradient whose direction of development is perpendicular to the direction of crack. For modelling is considered a hyperbolic surface, with increasing values for the Young's modulus and for which is applied a fracture pattern, by bending, in three or four-point (figure 2).

The program that allow the description of cracking and fracture propagation, is using FRANC 2D element type (Fracture Analysis Code 2D) and is able to simulate a 2D cracks in similar conditions with the reality. The analysis is based on stepped propagation of the crack. This concept can be extended with good results, for materials that are not homogeneous (maximum 50% reinforced elements).

![Figure 2. General formulation of the isoparameters based on the use of finite element method.](image)

![Figure 3. Model development and representation of discontinuous crack propagation in composite materials.](image)

The material properties for Gauss points (PGP) are interpolated with the properties from nodal points (Pi) by equation PGP = \( \Sigma \) Ni Pi in which N represent a functions ordering.

In order to represent a single node using triangulation with four points and six nodes of type (T6qp), it is necessary to represent a singular area of tension with the origin O (r=1/2), having radial type and circular development (rings), for which the representation of propagation is done by defining circular sectors as it can be shown in figure 3.

The crack propagation geometry is translated into a mesh model, describing the crack propagation in case of a composite material. The numerical values for material properties are defined by a certain area that can be described by a restriction having the following form: \( 0 \leq \xi \leq 1 \).

The Young's modulus and Poisson's coefficient are presented in Cartesian coordinates \( X_1-X_2 \). The mesh consists from 818 elements of parallelepiped shape type (Q8), 106 nodes of triangulation (T6).
and 12 nodes of triangulation type (T6qp), having a total number of 936 elements of triangulation and 2827 mesh nodes.

The increment constant is $\Delta a = 1\text{mm}$. The material heterogeneous is has a significant influence on the trajectory of the fracture in case of a composite material. The precise determination of stress intensity factor SIFs is a prerequisite in order to achieve the prediction of initiation and propagation of the fracture. The integration methods of the interactions, taking into account the stress intensity factor are particularly laborious and involves the application of evaluation schemes, using finite element methods. For this activity it is compulsory to call some micromechanics models and simulation calculations of the mechanical properties of materials having gradual properties.

The sequential stages (figure 4) represent a classical procedure of evaluation:

- the surface geometry on which the initial crack will propagate (figure 4a);
- the development of cracking is done by deleting the mesh elements which are associated to the tension zone (figure 4b);
- the development of a triangulation with nodal elements of type (T6qp) around the crack initiation point (figure 4c);
- the triangulation elements are generated by specific algorithm, as is shown in figure 4d;
- the detailed representation of the modified mesh around the crack it is made by increasing the number of elements of triangulation, with representation by jumps in the mesh (figure 4e)

The stress intensity factor can be defined by the following formula:

$$
\sigma_y(r,\theta) = \frac{K_I}{\sqrt{2\pi r}} f_y^I(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f_y^{II}(\theta)
$$

where $\sigma$ defines the stress tensor $ij$, $K_I$ and $K_{II}$ represent the stress intensity factor for mode I and II, and $f_y^{II}(\theta)$ the angular positions. The stress intensity factor contains all the parameters required to characterize the behaviour of the composites in case of crack.
The coordinates and the contours of integration and calculation model.

The auxiliary displacement caused by the existence of tensions can be described by a formula having the following structure:

$$
\begin{align*}
\varepsilon_{aux} &= (\text{sim}\Delta) \cdot u_{aux} \\
&= \frac{K_{I}^{aux}}{\mu_{ip}} \sqrt{\frac{r}{2\pi g'(\theta)}} + \frac{K_{II}^{aux}}{\mu_{ip}} \sqrt{\frac{r}{2\pi g''(\theta)}}
\end{align*}
$$

where $K_{I}^{aux}$ and $K_{II}^{aux}$ are two methods of stress intensity factor to determine the stress, used in fracture of micromechanics.

The formulation by integration of the stress intensity factor can be defined by an integral known as integral $J$.

$$
J = \lim_{s \to 0} \int_{\Gamma} \left( W \delta_{ij} - \sigma_{j} u_{i} \right) n_{j} d\Gamma
$$

where $W$ is the density of deformation energy, expressed by the expression $W = \sigma_{j} \varepsilon_{j} / 2$, $n_{j}$ is the normal vector on $\Gamma$, outline (figure 5), in which the development of integration function is obtained by summing:

$$
J = \int_{\Lambda} \left( \sigma_{j} u_{i} - W \delta_{ij} \right) q_{j} dA + \int_{\Lambda} \left( \sigma_{j} u_{i} - W \delta_{ij} \right) j_{q} dA
$$

The integral $J$ obtained by extrapolating actual area and the auxiliary area where the crack is developing can be decomposed as $J' = J + J_{aux} + M$, in which $J_{aux}$ is described by the relation:

$$
J = \int_{\Lambda} \left( \sigma_{j} u_{i}^{aux} - W \delta_{ij} \right) q_{j} dA + \int_{\Lambda} \left( \sigma_{j} u_{i}^{aux} \frac{1}{2} \sigma_{ik} \varepsilon_{ik}^{aux} \delta_{ij} \right) j_{q} dA
$$

resulting in an integration formula (M) based on a state of non-equilibrium:

$$
M = \int_{\Lambda} \left( \sigma_{j} u_{i}^{aux} + \sigma_{j} u_{i}^{aux} - \sigma_{ik} \varepsilon_{ik}^{aux} \delta_{ij} \right) q_{j} dA + \int_{\Lambda} \left( \sigma_{j} u_{i}^{aux} + \sigma_{j} u_{i}^{aux} - \sigma_{ik} \varepsilon_{ik}^{aux} \delta_{ij} \right) j_{q} dA = M_{1} + M_{2}
$$

The last term of the second integral (M2) in equation (6) can be expressed by the following equality:

$$
\left( \sigma_{ik} \varepsilon_{ik}^{aux} \delta_{ij} \right) = \left( \sigma_{ik} \varepsilon_{ik}^{aux} \right) = \left( C_{ij} \varepsilon_{i}^{aux} \right) = C_{ij} \varepsilon_{i}^{aux} + \sigma_{ij}^{aux} \varepsilon_{ij} + \sigma_{ij} \varepsilon_{ij}^{aux}
$$
in which $C_{ijk}^{1}$ represent continuous and derivative functions of auxiliary stresses.

Inserting equation (7) in equation (6) and using the compatibility and equilibrium value for which $\sigma_{ij,j} = 0$ (there are no forces inside the solid) will get another value for $M$:

$$M = \int_A \left( \sigma_{ij}^{aux} u_{ij} + \sigma_{ij}^{aux} u_{ij} - \sigma_{ij} \varepsilon_{ij}^{aux} \right) \delta_j dA + \int_A \left( \sigma_{ij}^{aux} u_{ij} - C_{ijk} \varepsilon_{ij} \varepsilon_{jk}^{aux} \right) q dA$$

(8)

where the underlined term is a state of non-equilibrium, the introduction of a auxiliary stress, leading to the production of convergent solutions.

4. Conclusions
The fracture analysis is used to assess the durability of a model, having variable geometry defined by the finite elements, when simultaneously or sequentially loads are applied to it. There is possible to run analyses which are linear or nonlinear.

The model presented in this paper can achieve further development as modelling and verification of transient fatigue of composites. This approach can be used to calculate the dynamic stress and the specific effort, thus being able to estimate the wear lifetime of industrial products.

The proposed model offers efficient and flexible tools in order to generate, modify and display the mechanical and thermal properties, necessary for product design, simulation and manufacturing.

The analysis of a model with errors can consume a lot of time and money, and often, errors are not detected even after analysis. The proposed model provides a complete set of mathematical and graphical tools for checking and correcting of the model before its use in simulation.

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