Equilibrium Queueing Strategies of Three Types of Customers in A Single Queue

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Abstract. Different types of customers have different revenue and waiting costs per unit time. Although in queueing economics, research on customer equilibrium strategies has achieved fruitful results, most of these are based on a single customer type. Research on multiple types of customers is rare. We consider a single queue with three types of customers. We analyse system stationary distribution and obtain the equilibrium balking strategy of each type customers. In addition, the social benefit is discussed under the reward-cost structure in the full observable.

1. Introduction
In M/M/1 queue research, most considered the system with one type customer. However, in many real-life situations it may not be feasible. Customer differences are mainly reflected in the value of service benefits and waiting costs. The literature on multiples customers in queueing systems begins with Armony and Haviv[1], who studied an M/M/1 queueing model with different waiting costs in the fully observable case. Afanasyev and Mendelon[2] investigated two types customers with different revenue. Zhou[3] showed that how service organizations set optimal prices when they face different types of customers who have different revenue and waiting costs per unit of time. Guo[4] considered customer equilibrium strategy and optimal arrival rate based on the Markovian queuing model. Kotiah and Slater [4] calculated the expected waiting time in this two-server Poisson queue with two types of customers. Recently, Cheng Baoan[6] considered equilibrium strategies of queueing systems with two types of customers. Xu Xiuli[7] considered two types of parallel customers in the queuing system, and showed the equilibrium balking strategies of the two types of customers and the average social benefit s of the system are discussed under the reward-cost structure in the full observable. Alexandre Brandwajn and Thomas Begin[9] presented a simple approach to the solution of a multi-server FCFS queueing system with several classes of customers and phase-type service time distributions. Tang Yanli [10] considered a single queue with two identical servers and two types of customers and obtained the equilibrium queueing strategy for each type of customers. To the best of the authors’ knowledge, studies for multiple types customers in queueing system do not yet exist. It is the aim of this paper to study the equilibrium strategies of customers in the context of an observable M/M/1 queue. The paper is organized as follows. Descriptions of the model is given in Section 2. In Section 3,
the equilibrium strategies for fully observable is identified and the equilibrium social benefit are derived. Finally, in Section 4, some conclusions are given.

2. Model description
We consider the fully and partially observable single-server queue where customers arrive according to a Poisson process with rate \( \lambda \) and service times are assumed to be exponentially distributed with rate \( \mu \). There is only one waiter in the system and only one customer can be served at a time, and the first-come, first-served rule (FCFS). Suppose there are three types of customers in the system, arriving according to the Poisson process, with rate \( \lambda_1, \lambda_2, \lambda_3 \), and the arrival processes are independent of each other. In order to ensure the steady state of the system, suppose \( \lambda = \lambda_1 + \lambda_2 + \lambda_3 < \mu \). Upon arriving, the customers decide whether to join or balk the queue based on observation of the queue length, along with the consideration of waiting cost and the reward after finishing their service. After service, every customer receives a reward of \( R \) units. This may reflect his satisfaction or the added value of being served. On the other hand, there exists a waiting cost of \( C \) units per time unit when the customers remain in the system including the time of waiting in queue and being served. Customers are risk neutral and maximize their expected net benefit. Each customer can observe the number of customers ahead of him upon his arrival. Thus, we can represent the state of the system at time \( t \) by a pair \( (N(t), j) \), where \( N(t) \) and \( j \) denote the number of customers and \( j \) class customers \( j = 1, 2, 3 \). The process \( \{N(t), j\}: t \geq 0 \) is a three-dimensional continuous time Markov process. In order to better describe the customer's decision-making process, a quantitative method is adopted to establish a "revenue-cost" structure function. Assume that the revenue per completed service of the \( j \)th customer is \( R_j \); and the stay cost per unit time is \( C_j \). Firstly, customers make sure \( R_j \geq C_j \frac{1}{\mu} \). It is established to ensure that the arriving customers will choose to enter the system when the system is empty. Because the expected net income is positive, that is, the income after completing the service is greater than the cost brought by the service time. Otherwise, the expected net income is negative value, even if there is no customer in the system, no one will enter.

3. Equilibrium threshold strategies
In this section we shall find that there exist equilibrium threshold strategies in three type customers mentioned above. In the fully observable case where customers are informed both the type of the customer \( j \) and the number of the present customer \( N(t) \) at arrival time \( t \), a pure threshold strategy is specified by a \( n_{ej} \). Average stay time is \( \frac{n+1}{\mu} \), when a customer arrives the system. If the customer's expected net income is positive, the customer's best strategy is to enter. If the expected net income is zero, it doesn't matter if the customer enters or stops. If the expected net income is negative, the customer's best strategy is to balk. Assuming that customers enter the queuing system, if and only if their net income is non-negative. We have the following result.

3.1. Theorem 1
In the fully observable M/M/1 queue with balk there exist thresholds for three types customers

\[
n_{ej} = \left[ \frac{R_j \mu}{C_j} \right] - 1, j = 1, 2, 3
\]

(1)

It is obvious that for an arriving customer, his expected net reward if he enters is:

\[
R_j - C_j \left( n + 1 \right) \geq 0
\]

(2)

\[
(n + 1) \leq \frac{R_j \mu}{C_j}
\]

(3)

Solving Eq. (3) we obtain Eq. (1).

Each type customer knows \( N(t) \) when it arrives, in fully observable queueing system, the equilibrium threshold \( n_{ej} \) for each type of customer is independent of its arrival rate \( \lambda_j \). Because in
fully observable situation, each customer knows the system state and the type to which they belong. According to the "revenue-cost" structure function, the equilibrium threshold of each customer is derived from the equation without arrival rate $\lambda_j$. When $N(t) \leq n_{e_j}$, customer enters the queuing system, otherwise not. Therefore, the threshold strategy $n_{e_j}$ is the only equilibrium. Assuming $R_1/C_1 \leq R_2/C_2 \leq R_3/C_3$, then $n_{e_1} \leq n_{e_2} \leq n_{e_3}$, that is, the first type customer balk before the second type customer, and the second type of customer balk ahead of the third type customers.

For the stationary analysis of the system, note that if all customers follow the threshold strategy in (1) the system follows a Markov chain with state space $S_{f_o} = \{n | 0 \leq n \leq n_{e_3} + 1\}$. The stationary distribution $\{p_n \mid n \in S_{f_o}\}$ is obtained as the solution of the following system of balance equations:

\[ p_0 \lambda_1 = p_1 \mu \]  
\[ p_n \lambda_1 = p_{n+1} \mu, n = 1, 2, \ldots, n_{e_1} \]  
\[ p_n \lambda_2 = p_{n+1} \mu, n = n_{e_1} + 1, n_{e_1} + 2, \ldots, n_{e_2} \]  
\[ p_n \lambda_3 = p_{n+1} \mu, n = n_{e_2} + 1, n_{e_2} + 2, \ldots, n_{e_3} \]  
\[ p_{n-1} = \frac{\lambda_1}{\mu}, p_2 = \frac{\lambda_2}{\mu}, p_3 = \frac{\lambda_3}{\mu} \]  

Denote by $\rho_1 = \frac{\lambda_1}{\mu}, \rho_2 = \frac{\lambda_2}{\mu}, \rho_3 = \frac{\lambda_3}{\mu}$. From Eqs. (4), (5), (6), (7), we have:

\[ p_n = \rho_1^n p_0, n = 0, 1, \ldots, n_{e_1} + 1 \]  
\[ p_n = \rho_2^{n-n_{e_2}-n_{e_1}+1} \rho_1^{n_{e_1}+1} p_0, n = n_{e_1} + 2, n_{e_1} + 3, \ldots, n_{e_2} + 1 \]  
\[ p_n = \rho_3^{n-n_{e_3}-n_{e_2}-n_{e_1}+1} \rho_2^{n-n_{e_2}-n_{e_1}+1} \rho_1^{n_{e_1}+1} p_0, n = n_{e_2} + 2, n_{e_2} + 3, \ldots, n_{e_3} + 1 \]  

The balk probability of the first type customers is $\sum_{n=0}^{n_{e_3}+1} p_n$, and the balk probability of the second type customers is $\sum_{n=n_{e_2}+1}^{n_{e_3}+1} p_n$, and the balk probability of the third type customer is $p_{n_{e_3}+1}$. When all customers follow the threshold policy $n_{e_j}$ given in Theorem 1 equals, the social benefit is as follows, according to the balk probability of each type customer.

\[ SB_{f_o} = SB_{f_0} + SB_{2f_0} + SB_{3f_0} \]  

Where

\[ SB_{3f_0} = R_1 \lambda_1 \left( 1 - \sum_{n=n_{e_1}+1}^{n_{e_3}+1} p_n \right) - C_1 \sum_{n=0}^{n_{e_1}+1} n p_n \]
The social benefits increase with the arrival rate. More customers are willing to join the system with net profit increasing. But the social benefits decrease with the arrival rate when the system is crowded and stay payment becomes longer. So, the social benefits firstly increase and then decrease with arrival rate.

4. Conclusion

Customers can be categorized into three types based on the service rewards and the waiting time, arriving to the system according to a Poisson process. We consider a fully observable queueing system with one server where the customers choose whether to balk or to join. We study the equilibrium strategy for each type customers and our analysis show that the third customers are willing to bear is more than the second customers, all the second customers balk the queue as long as the first type customers balk. which can not only provide guidance for the multiple types of customer decision-making and reduce queuing losses, but also provide a theoretical reference for service organizations and governments to formulate reasonable strategies.

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