Computer-aided Linear Algebra Course on Jupyter-Python Notebook for Engineering Undergraduates

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Abstract. As an important basic course, linear algebra plays critical role for engineering undergraduates. However, traditional purely theoretical explanation makes students difficult to deeply understand the essence of linear algebra. In this paper, we introduce a new form of computer-aided linear algebra course with the application of a set of Python computer programs implemented on the Jupyter notebooks platform, and also aim to improve the development of innovative teaching methodologies for this course. In addition, in order to help students understand the intrinsic physical meanings of some concepts, we design some visual examples which demonstrate the practical applications of linear algebra. A serious of user-friendly Jupyter notebooks workflows developed in this work allow teachers–students to intuitively teach and learn theoretical concepts, description of practical problems, programming code, and simulation results with a visual output. The designed environment can help teachers/students to implement their simulations in Python language.

1. Introduction
Linear algebra course is generally taught in the first year of engineering undergraduates, it is very important due to its widely application in solving practical engineering problems. For example, the Singular Value Decomposition (SVD) and Eigen Decomposition are commonly used techniques in computer science, especially in artificial intelligence field. On one hand, the pure theories of linear algebra are hard to understand for the first year undergraduates, which hinders them to apply linear algebra concepts for solving real problems. On the other hand, in many machine learning algorithms of computer science, a serious of linear algebra concepts are often implemented by computer programming languages such as Python, C++ and Matlab, just list a few. Among those programming languages, python language is at the forefront of scientific computation for engineers, and there are a large number of artificial intelligence algorithms implemented by Python.

In this paper, we design and introduce a new linear algebra course by using Jupyter Notebooks for engineering undergraduates, which aims to help students understand the intrinsic physical meaning of some linear algebra concepts. This course is designed to give students a solid foundation in linear algebra in terms of the most useful aspects in an engaging and easy to follow way. In addition, some visual examples which demonstrate the practical applications of linear algebra knowledge are designed for undergraduates to strengthen the learned concepts.

After learning this course, the students should be able to understand the physic meaning of vector, matrix, multiplication of vectors, multiplication of matrices, linear transformation, SVD, eigen decomposition develop, inverse matrix, and finally, their implementation in several practical applications. As to teachers, the Jupyter-notebook environment provide a good platform to showcase theoretical concepts description of problems, programming code, and graphical simulation results. For
student, the Jupyter-notebook environment allows them to iteratively read the theory, modify inputs/outputs, and see the immediate effects in an intuitive manner. Moreover, the environment allows students to encode his/her simulations in Python language. The main workflow of the proposed online-supported linear algebra course based on Jupyter notebook is shown in Figure 1. The whole course can be used by teacher and different students in an interactive modeling computer environment.

![Figure 1](image_url)  
**Figure 1.** The main workflow of the online environment.

![Figure 2](image_url)  
**Figure 2.** Core software and tools for the implementation of the online course.

### 2. Related Works

As an open-source software, Jupyter-notebook has been increasingly used for coding and education, especially in higher education about computer science majors. In addition, due to its simplicity, and active support community with plotting and numerical libraries such as IPython, math, scipy, numpy, os, time, and matplotlib [1], the Jupyter-notebook environment has been used as one of the most popular programming platform which can conveniently process programming languages such as Python and Octave [2]. Especially, by using the IPython open-source module, interactively working can be easily realized with the web-cloud interface and Python language.

**Table 1.** Examples of courses based on Jupyter-notebook in different higher education majors.

| Major                     | Course contents                                                                 |
|---------------------------|---------------------------------------------------------------------------------|
| Artificial intelligence   | Artificial intelligence techniques about sustainable development [5]            |
| Computer science          | Python programming on Jupyter-notebook [6]                                      |
| Electronics               | Photonic integrated circuits, including Design, simulation, and fabrication on Jupyter notebooks [7] |
| Manufacturing             | Optimization problems of manufacturing teaching with code components of a Jupyter notebook [14] |
| Transport phenomena       | Analysis of transport phenomena and reaction in porous catalyst pellet based on a set of Jupyter notebooks [15] |
| Control                   | Laboratory oriented control theory courses [16]                                |
| Engineering               | Computational science education MOOC [8]                                        |
| Archives management       | generous archive interfaces designed with Jupyter notebook [12]                |
| Astronomy                 | Citation practices in the astronomy community [13]                             |
| Environment               | Jupyter notebook computing environment for participatory Green infrastructure design, visualization, and evaluation [9] |
| Chemical                  | Computational chemistry includes simulations, data processing, and visualization for undergraduate [10] |
| Agriculture               | Data mining and machine learning techniques for agriculture data analysis [11]  |
| Mechatronics              | Embedded systems in mechatronics for the engineering curricula [17]            |

As to higher education, Guerra et al. applied the Jupyter-notebook to a CS2-based course [3], which demonstrates how the Jupyter-notebook can be used for computer course. In [3], the well-known methodologies in industry and manufacturing are used to the teaching of laboratory classes in a computer science course. The "student-centered learning" paradigm is designed to present theoretical and laboratory computer courses. Interactive online course for learning the Python language is also
implemented by using Jupyter-notebook [4], which consists of three levels, including basic, intermediate, and advanced levels, and these levels are developed in Jupyter-notebook. Both the students and the teachers can benefit from this course. As to students, they can review any forgotten knowledge and gain new knowledge. As to teachers, they can follow the progress of students and check which part presented them with considerable difficulty. The students and teachers can act in an interactive manner, i.e., messages from students can be easily obtained by teachers while teachers can give the feedback to students timely. In Table 1, we list some examples of courses designed based on Jupyter-notebook in different higher education fields.

3. Detailed Course Implementation
The kernel of the course implementation is a visualized web browser with the Jupyter-notebook Integrated Development Environment (IDE). The environment for programming allows the integration of several tools in addition to Python language, which is helpful to realize an intuitive online course.

The function of different elements are as follows:
(1) The bookdown package [19] allows writing the course document, which includes the insertion of code for figures and quotations.
(2) The stands for Cascading Style Sheets CSS [20] code is included to modify the appearance of the website.
(3) The Latex code allows us to write the mathematical equations conveniently.
(4) The Github client allows us to upload our course documents and Python code to the website, then students can download the materials from the server.

The Python or Octave languages are used for coding, which implement some computation of linear algebra concepts.

For each concept of linear algebra, we first introduce its detailed theory with intuitive visual description, then it is implemented by Python language with some practical applications. Finally, the documents with Python code are uploaded to Github and students can learn the theory and run the code downloaded from Github. Figure 3 shows an example of the course structure.

4. Experimental Results
In this section, we give two course examples with simulations and qualitative satisfaction study of the designed course.

4.1. Course Examples
In each simulation, we use the Jupyter-notebook to running the Python code and the corresponding visual results can be timely obtained. The course examples are presented as follows:

1. Linear equation;

   \[ \begin{cases} 2x - y = 0 \\ -x + 2y = 3 \end{cases} \]

   \[ (1) \]

2. Principal component analysis;

   a) Linear equation. Supposing we have a linear equation with two unknown variables need to be solved, which is formulated as follows:

   \[ \begin{cases} 2x - y = 0 \\ -x + 2y = 3 \end{cases} \]

   \[ (1) \]
where $x$ and $y$ are two variables which we need to solve. By re-writing the variable coefficients of Eq. (1) into vector form, we have:

$$
\begin{bmatrix}
2 \\
-1
\end{bmatrix} x + \begin{bmatrix}
-1 \\
2
\end{bmatrix} y = \begin{bmatrix}
0 \\
3
\end{bmatrix}.
$$

(2)

Then we plot the three column vectors as follows: As can be seen from Figure 3, the red vector, i.e., $[0,3]^T$ can be obtained by adding the green vector, i.e., $[2,-1]^T$ to two times of the yellow vector, i.e., $[-1,2]^T$. Therefore, $x$ and $y$ can be intuitively understood as the linear combination coefficients of two vectors to construct a vector, which is the intrinsic meaning of linear equations.

**b) Principal component analysis.** As a popular machine learning algorithm, principal component analysis (PCA) is frequently used in many practical problems. In order to give a deep understanding for students, we first introduce the theoretical derivation of PCA and then give a real example on digits de-noising. Due to the page limitation, the document of theoretical derivation is omitted here and we directly show the visual example by using Jupyter-notebook. The detailed process of this course example are as follows:

1. Import Python Libraries. The implementation codes are shown in Table 2.
2. Read digits data. The implementation codes for data loading are shown in Table 3.

| Table 2. Python code for importing python libraries. | Table 3. Python code for loading data. |
|---|---|
| # Import Python Libraries | # Read digits data |
| import matplotlib.pyplot as plt | digits=datasets.load_digits() |
| import numpy as np | x=digits.data |
| # Import PCA Algorithm | y=digits.target |
| from sklearn.decomposition import PCA | noisy_digits=x+np.random.normal(0,2,size=x.shape) |
| # Import Dataset | ex=noisy_digits[y==0,][:10] |
| from sklearn import datasets | for num in range(1,10):
| | x_num=noisy_digits[y==num,][:10] |
| | ex=np.vstack([ex,x_num]) |
| | # Print the size of digits |
| | print(ex.shape) |

3. Show original digits with noise. Firstly, we give an intuitively showing of original noisy images, and the Python codes are shown in Table 4. The results are shown in Figure 4. As can be seen, original digits images are with noise.

| Table 4. Python code for showing original noisy data. | Table 5. Python code for de-noising with PCA algorithm. |
|---|---|
| # Intuitively showing of the digits | # Call the PCA algorithm for de-noising |
| def plot_digits(data): | pca = PCA(0.5) |
| fig,axes=plt.subplots(10,10,figsize=(10,10), | pca.fit(noisy_digits) |
| subplot_kw={"xticks":[]},"yticks":[]}, | a=pca.transform(ex) |
| gridspec_kw=dict(hspace=0.1,wspace=0.1)) | b=pca.inverse_transform(a) |
| for i,ax in enumerate(axes.flat): | plot_digits(b) |
| ax.imshow(data[i].reshape(8,8), | |
| cmap="binary",interpolation="nearest", | |
| clim=(0,16)) | |
| plt.show() | |
| plot_digits(ex) |

4. De-noising by using PCA algorithm. Finally, we call the PCA algorithm for de-noising of the digits images. The Python codes are shown in Table 5.

4.2. Qualitative Satisfaction Study

In order to qualitatively evaluate the designed linear algebra course, we carry out a satisfaction study test. In this experiments, 30 students of the first-year computer science major are chosen from the
school of computer science, China University of Geosciences, Wuhan, China. These students have no prior knowledge about linear algebra. Firstly, a linear algebra course instructor explain how the linear algebra course works to the students. Then we test if the designed course is a satisfactory complementary learning tool for undergraduate students. Therefore, students are asked to fill out a survey which consists of six questions after they finish the online course [18]. For each question, there are five choices, including (1) no, (2) rather no, (3) do not know, (4) rather yes, and (5) yes. The six questions are as follows:

- Is it useful of the proposed course to understand the linear algebra concepts?
- Easiness of learning with the linear algebra course.
- How the developed linear algebra course motivates you to learn other traditional courses?
- Does it increase your willingness to study at home?
- Does this linear algebra course allows you to simulate comfortably?
- Do you think this linear algebra course is a complementary tool for traditional course?

The survey results are presented in Table 6. As can be seen from the results, the students are satisfied with the new linear algebra course to a large extent. The designed course is regarded as a useful resource for learning linear algebra.

| Questions                                                                 | No % | Rather no | Do not know | Rather yes | Yes |
|---------------------------------------------------------------------------|------|-----------|-------------|------------|-----|
| Q1  Is it useful of the proposed course to understand the linear algebra  | 0    | 0%        | 0%          | 5%         | 95% |
| concepts?                                                                 |      |           |             |            |     |
| Q2  Easiness of learning with the linear algebra course.                  | 0    | 0%        | 0%          | 86%        | 12% |
| Q3  How the developed linear algebra course motivates you to learn other  | 0    | 1%        | 2%          | 83%        | 14% |
| traditional courses?                                                      |      |           |             |            |     |
| Q4  Does it increase your willingness to study at home?                   | 0    | 0%        | 1%          | 6%         | 93% |
| Q5  Does this linear algebra course allows you to simulate comfortably?    | 0    | 0%        | 0%          | 3%         | 97% |
| Q6  Do you think this linear algebra course is a complementary tool for   | 0    | 0%        | 1%          | 11%        | 88% |
| traditional course?                                                       |      |           |             |            |     |

5. Conclusions
In this paper, we introduce a linear algebra course based on Jupyter-notebook. The course is implemented on the Jupyter notebooks platform, and aims to improve the development of innovative teaching methodologies for linear algebra. In order to help students understand the intrinsic physical meaning of some concepts, we design some visual examples which demonstrate the practical applications of linear algebra. By learning the designed course, students can understand the hard concepts of linear algebra more easily and this course also improve their learning enthusiasm for other courses.

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