ANTI-GZK EFFECT IN ULTRA–HIGH-ENERGY COSMIC RAY DIFFUSIVE PROPAGATION

R. ALOISIO AND V. S. BEREZINSKY
Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali del Gran Sasso, Strada Statale 17/bis Km 18+910, I-67010 Assergi (AQ), Italy

Received 2005 January 13; accepted 2005 February 10

ABSTRACT

We discuss the anti-GZK effect in the diffusive propagation of ultra–high-energy protons in intergalactic magnetic fields, which consists of a jump-like increase of the maximum distance from which ultra–high-energy protons can reach an observer. The position of this jump, \( E_\gamma \approx 2 \times 10^{19} \text{ eV} \), is determined exclusively by energy losses (transition from adiabatic to pair-production energy losses), and it is independent of the diffusion parameters. The diffuse spectrum presents a low-energy steepening approximately at this energy, which is very close to the position of the second knee observed in the cosmic ray spectrum. The dip, seen in the universal spectrum as a signature of the interaction with the cosmic microwave background radiation, is also present in the case of diffusive propagation in magnetic fields.

Subject headings: cosmic rays — diffusion — ISM: magnetic fields

1. INTRODUCTION

The GZK cutoff (Greisen 1966; Zatsepin & Kuzmin 1966) is a steepening of the ultra–high-energy (UHE) protons spectrum as a result of interaction with the cosmic microwave background (CMB) radiation. The presence of an intergalactic magnetic field can modify the GZK cutoff to the point of negating it in the case of very strong magnetic fields (Sigl et al. 2004; Yoshiguchi et al. 2003; for a physical explanation of this effect, see Aloisio & Berezinsky 2004). The proton propagation in a magnetic field can affect the observed UHE proton spectrum also at energies (much) lower than the GZK cutoff. The crucial parameter that determines the modification of the spectrum is the distance \( d \) between sources. If this distance is much less than all propagation distances, such as energy-attenuation length \( l_\text{att} \) and diffusion length \( l_\text{diff} \) the spectrum is not distorted and has a universal (standard) shape (Aloisio & Berezinsky 2004). This statement has the status of a theorem.

All of these effects depend strongly on the strength of the large-scale intergalactic magnetic field (IMF), the knowledge of which still remains poor. The modes of the UHE-proton propagation vary between quasi-rectilinear propagation in a weak field and diffusive propagation in a strong magnetic field. The experimental data on IMF and the models of origin of these fields do not at present allow a choice, even between the two extreme propagation regimes mentioned above.

The most reliable observations of the IMF are based on the Faraday rotation of the polarized radio emission (for reviews see Kronberg 1994; Vallee 1997; Carilli & Taylor 2002). The upper limit on the Faraday rotation measure (RM) in the extragalactic magnetic field, obtained from the observations of distant quasars, gives \( RM < 5 \text{ rad m}^{-2} \). It implies an upper limit on the extragalactic magnetic field on each assumed scale of coherence length (Kronberg 1994; Vallee 1997; Ryu et al. 1998). For example, according to Blasi et al. (1999), for an inhomogeneous universe \( B_\perp < 4 \text{ nG} \) on a scale of coherence \( l_\perp = 50 \text{ Mpc} \).

According to observations of the Faraday rotations, the extragalactic magnetic field is strongest, on the order of 1 \( \mu \text{G} \), in clusters of galaxies and radio lobes of radio galaxies (Vallee 1997; Kronberg 1994; Carilli & Taylor 2002). The largest scale in both structures reaches \( l_\perp \approx 1 \text{ Mpc} \). Most probably, various structures of the universe have dramatically different magnetic fields, with very weak fields in voids and much stronger fields in the filaments (Ryu et al. 1998). Superclusters seem to be too young for the regular magnetic field to be formed in these structures on a large scale, \( l_\perp \approx 10 \text{ Mpc} \).

In the case of hierarchical magnetic field structures in the universe, UHE protons with \( E > 4 \times 10^{19} \text{ eV} \) can propagate in a quasi-rectilinear regime. Scattering of UHE protons occurs mostly in galaxy clusters, radio lobes, and filaments. Deflections of UHE protons can be large for some directions and small for the others.

The universe looks like a leaky, worm-holed box, and correlation with the sources can be observed [see Tinyakov & Tkachev 2001, in which correlations of ultra–high-energy cosmic rays (UHECR) with BL Lacertae objects are found]. Such a picture has been suggested by Berezinsky et al. (2002b, 2003).

A promising theoretical tool to predict the IMF in large-scale structures is given by magnetohydrodynamic (MHD) simulations. The main uncertainty in these simulations is related to the assumptions concerning the seed magnetic field.

The MHD simulations of Sigl et al. (2004, 2003) favor a hierarchical structure with strong magnetic fields. Assuming an inhomogeneous seed magnetic field generated by cosmic shocks through the Biermann battery mechanism, the authors obtain an \( \approx 100 \text{ nG} \) magnetic field in filaments and \( \approx 1 \text{ nG} \) in voids. In some cases they consider IMF up to a few micro-Gauss as allowed. In these simulations UHECR are characterized by large deflection angles, of the order of 20°, at energies up to \( E \approx 10^{20} \text{ eV} \) (Sigl et al. 2003, 2004). Thus, the scenario that emerges in these simulations seems to exclude the possibility of an UHECR astronomy. These simulations have some ambiguity related to the choice of magnetic field at the position of the observer (Sigl et al. 2003, 2004). The authors consider two cases: a strong local magnetic field, \( B \approx 100 \text{ nG} \), and a weak field, \( B \ll 100 \text{ nG} \). The different assumptions about the local magnetic field strongly affect the conclusions about UHECR spectrum and anisotropy.

The essential step forward in MHD simulations has been made by Dolag et al. (2004, 2005). In this work the local universe is simulated with the observed density and velocity field. This eliminates the ambiguity for the local magnetic field, which is found to be weak. The seed magnetic field, used in this simulation, is normalized by the observed magnetic field in rich clusters of...
galaxies. The results of these constrained simulations indicate weak magnetic fields in the universe, of the order of 0.1 nG in typical filaments and 0.01 nG in voids. The strong large-scale magnetic field, \( B \sim 10^6 \) nG, exists in clusters of galaxies, which however, occupy an insignificant volume of the universe. The picture that emerges from the simulations of Dolag et al. (2004, 2005) favors a hierarchical magnetic field structure characterized by weak magnetic fields. UHE protons with \( E > 4 \times 10^{19} \) eV can propagate in a quasi-rectilinear regime, with the expected deflection angles being very small, \(< 1^\circ\).

The case of strong magnetic fields up to 1 \( \mu \)G has been studied in Sigl et al. (1999), Lemoine et al. (1999), Stanek et al. (2000), Harari et al. (2002), Yoshiguchi et al. (2003), and Deligny et al. (2004). The interesting features found in these calculations are a small-scale clustering of UHE particles, as observed by Hayashida et al. (1996), Takeda et al. (1999), Uchiyama et al. (2000), and Glushkov & Pradine (2001), and an absence of the GZK cutoff in the diffusive propagation, when the magnetic field is very strong. Many aspects of the diffusion of UHECR have been studied in numerical simulation by Casse et al. (2002).

The small-scale clustering allows us to estimate the space density of the sources (Dubovsky et al. 2000; Fodor & Katz 2001). The recent Monte Carlo simulations (Yoshiguchi et al. 2003; Blasi & De Marco 2004; Kachelriess & Semikoz 2004) favor a hierarchical magnetic field structure characterized by weak magnetic fields. UHE protons with \( E > 4 \times 10^{19} \) eV can propagate in a quasi-rectilinear regime, with the expected deflection angles being very small, \(< 1^\circ\).

Diffusive propagation of extragalactic UHECR has been studied already in earlier work. The stationary diffusion from the Virgo Cluster was considered by Wdowczyk & Wolfendale (1979) and Giller et al. (1980), and nonstationary diffusion from a nearby source was studied by Berezinsky et al. (1990) and Blasi & Olinto (1999) using the Syrovatsky solution (Syrovatskii 1959) of the diffusion equation. In this case the GZK cutoff may be absent.

A very interesting phenomenon, caused by the propagation of UHE protons in the extragalactic magnetic fields, has been recently found by Lemoine (2004). It consists of a low-energy steepening of the spectrum of UHE protons at energies below \( 1 \times 10^{18} \) eV produced by a large diffusive propagation time (exceeding the age of the universe) to the nearby sources. In this paper, we discuss the anti-GZK effect in the diffusive propagation of UHE protons, which is responsible for this low-energy steepening, and discuss the transition from Galactic to extragalactic cosmic rays. In our calculations we follow, like Lemoine (2004), the theoretical approach of Aloisio & Berezinsky (2004).

### 2. DIFFUSIVE PROPAGATION IN THE ANALYTIC APPROACH

The analysis below is based on the analytical solution of the diffusion equation, found by Syrovatskii (1959). Using a distribution of sources on a lattice, the diffuse flux can be calculated as the sum over the fluxes from the discrete sources \( \mathcal{L} \):

\[
J_p^{\text{diff}}(E) = \frac{c}{4\pi} L_p K(\gamma_g) \frac{b(E)}{b(\epsilon)} \times \sum_i \int_{E_i}^{E_{\text{max}}} dE_s q_{\text{gen}}(E_s) \exp\left\{\frac{-r_i^2}{4\lambda(E_s, E)}\right\} \left\{\frac{4\pi\lambda(E_s, E)}{[4\pi\lambda(E_s, E)]} \right\}^{1/2},
\]

where \( b(E) = dE/dt \) is the proton energy loss, summation over all lattice vertexes, \( L_p \) is the proton luminosity of a source, \( q(E_s) = E^{-\gamma_g} \) is the generation function, and \( K(\gamma_g) \) is the normalization coefficient, equal to \( \gamma_g - 2 \) if \( \gamma_g > 2 \) and \( 1/\ln(E_{\text{max}}/E_{\text{min}}) \) if \( \gamma_g = 2 \) (all energies are measured in GeV), and

\[
\lambda(E, E_s) = \int_{E_s}^{E} d\epsilon \frac{D(\epsilon)}{b(\epsilon)}
\]

is the Syrovatsky variable, which has the physical meaning of the squared distance traversed by a proton in the observer direction, while its energy diminishes from \( E_s \) to \( E \). From equation (1), one can see that the sources at distances \( r > 2\lambda(E, E_s)^{1/2} \) give negligible contribution to the flux.

In our calculations we shall use also the second Syrovatsky variable, which can be understood as the time needed by a proton to diminish its energy from \( E_s \) to \( E \):

\[
\tau(E, E_s) = \int_{E_s}^{E} d\epsilon \frac{b(\epsilon)}{b(\epsilon)}.
\]

The Syrovatsky solution formally includes all propagation times up to \( t \to \infty \), and the generation energies are restricted from above only by the maximum acceleration energy, \( E_{\text{acc}} \), that a source can provide. In our case the propagation time from a source at a fixed distance \( r \) must be smaller than the age of the universe \( t_0 \), and because of this condition, one more upper limit on the maximum generation energy, \( E_{\text{acc}} \), emerges. This limit is given by the condition \( \tau(E, E_s) < t_0 \) and results in \( E_{\text{acc}}(E_s, t_0) \), which can also be calculated by evolving energy backward in time from \( E \) at \( t = 0 \) to \( E_s \) at \( t = t_0 \).

The upper limit \( E_{\text{acc}} \) in equation (1) is then the minimum between the two quantities \( E_{\text{acc}}(E_s, t_0) \) and the maximal acceleration energy \( E_{\text{acc}} \):

\[
E_{\text{acc}}(E_s) = \min\{E_{\text{acc}}(E_s, t_0), E_{\text{acc}}\}.
\]

At small energies, \( E < 2 \times 10^{18} \) eV, \( E_{\text{acc}}(E, t_0) < E_{\text{acc}} \), while at larger energies, \( E_{\text{acc}}(E, t_0) > E_{\text{acc}} \). In the calculations below we assume \( E_{\text{acc}} = 1 \times 10^{22} \) eV.

The crucial quantity in the following discussion, the proton energy loss \( \beta(E) = (1/E)dE/dt \), is shown in Figure 1. Note the
characteristic energy \( E_\beta \approx 2 \times 10^{18} \text{ eV} \), where the pair production energy losses \( \beta_{\text{pc}}(E) \) reach the adiabatic energy losses.

Using these energy losses, we can calculate \( E_{g,x}^{\text{max}}(E) \). The results are presented in Figure 2. At low energies \( E_{g}(E, t_0) \) increases because of adiabatic energy losses. At the end of this stage, the increase becomes more sharp, because at large time \( t \) the pair-production energy losses set in. Finally, at \( E \approx E_\beta \), \( E_{g}(E, t_0) \) abruptly increases up to \( E_{g,x}^{\text{max}} \), practically by a jump. The jump factor is roughly given by \( \exp(t_0/\tau) \), where \( \tau \) is the energy-loss time, which diminishes as the energy rises with the backward time. This behavior of \( E_{g,x}^{\text{max}}(E) \) is responsible for the anti-GZK effect, which is discussed in the next section.

We now specify the diffusion coefficient \( D(E) \), which determines \( \lambda(E, E_{g}) \) in equation (2). In the following discussion we also use the diffusion length definition \( l_d(E) = 3D(E)/c \).

We assume diffusion in a random magnetic field with a strength \( B_0 \) on the maximum coherent length \( l_c \), denoting this magnetic configuration by \( (B_0, l_c) \). This assumption determines the diffusion coefficient \( D(E) \) at the highest energies when the proton Larmor radius \( r_L(E) \gg l_c \),

\[
D(E) = \frac{1}{3} \frac{c r_L^2(E)}{l_c}. \tag{5}
\]

At low energies, when \( r_L(E) \leq l_c \), we shall consider three cases:

1. The Kolmogorov diffusion coefficient,

\[
D_K(E) = \frac{1}{3} cl_c \left[ \frac{r_L(E)}{l_c} \right]^{1/3}. \tag{6}
\]

2. The Bohm diffusion coefficient,

\[
D_B(E) = \frac{1}{3} c r_L(E). \tag{7}
\]

3. An arbitrary case, \( D(E) \propto E^\alpha \), with \( \alpha = 2 \) for the extreme energy regime.

In all cases we normalize the diffusion coefficient by \( \frac{1}{3} cl_c \) at \( r_L = l_c \). The characteristic energy \( E_c \) of the transition between the high-energy and low-energy regimes is determined by the condition \( r_L(E) = l_c \) and is

\[
E_c = 0.93 \times 10^{18} \left( \frac{B_0}{1 \text{ nG}} \right) \left( \frac{l_c}{\text{ Mpc}} \right) \text{ eV}. \tag{8}
\]

The smooth transition between the low-energy and high-energy diffusion regimes is provided with the help of an interpolation formula for the diffusion length,

\[
l_d(E) = \Lambda_d + \frac{r_{\text{diff}}^2(E)}{l_c}, \tag{9}
\]

with \( \Lambda_d = r_L(E) \) for the Bohm diffusion, and \( \Lambda_d = l_c(r_L/l_c)^{1/3} \) for the Kolmogorov regime.

For completeness we also give the numerical expression for the Larmor radius,

\[
r_L(E) = 1.08 \times 10^7 \frac{E}{1 \times 10^{20} \text{ eV}} \frac{1 \text{ nG}}{B} \text{ Mpc}. \tag{10}
\]

At distances \( r \leq l_d(E) \), the fluxes from individual sources \( i \) are calculated in the rectilinear approximation, and the diffuse flux is given by

\[
J_p^\text{re}(E) = \frac{L_p K(\gamma_0)}{(4\pi)^2} \sum_i q_{\text{gen}} \left[ \frac{g_{\text{gen}}(E, r_i)}{r_i^2} \right] \frac{dE_p(s, E, r)}{dE}, \tag{11}
\]

where \( dE_p/dE \) is given in Berezinsky et al. (2002a).

3. ANTI-GZK CUTOFF

In this section we demonstrate that, in contrast to the GZK cutoff, increasing of the proton energy losses at \( E \geq 1 \times 10^{18} \text{ eV} \) results, in the case of diffusive propagation, in an increase of the maximal distance from which protons can arrive.

Below we calculate \( \lambda(E, E_{g,x}^{\text{max}}) \), which according to equation (2) gives \( r_{\text{max}}^2(E) \), where \( r_{\text{max}}(E) \) is the maximal distance from which protons with the observed energy \( E \) can arrive, as it follows from equation (1):

\[
\lambda(E, E_{g,x}^{\text{max}}) = \int_{E_{g,x}^{\text{max}}}^E dE \frac{D(E)}{b(E)}. \tag{12}
\]

In two extreme limits, at low and high energies, \( \lambda(E, E_{g,x}^{\text{max}}) \) can be calculated analytically.

Let us start from the low-energy case, \( E \ll E_\beta \), when only adiabatic energy loss operates. Using \( D(E) \propto E^\alpha \), we obtain from equation (12)

\[
\lambda(E, E_{g,x}^{\text{max}}) = \frac{D(E)}{\alpha H_0} \left[ \frac{E_{g,x}^{\text{max}}}{E} \right]^\alpha - 1. \tag{13}
\]

The \( E_{g,x}^{\text{max}} \) found from the condition \( \tau(E, E_{g,x}^{\text{max}}) = t_0 \) is \( E_{g,x}^{\text{max}} = E \exp(H_0 t_0) \), which results in

\[
r_{\text{max}}(E) = 2 \left[ \frac{D(E)}{\alpha H_0} \right]^{1/2} \left( e^{\alpha H_0 t_0} - 1 \right)^{1/2}, \tag{14}
\]

where according to the WMAP data (Spergel et al. 2003), \( H_0 t_0 \approx 1 \).
In the extreme high-energy regime, \( E \geq 3 \times 10^{20} \text{ eV} \), \( \tau_\infty = E(dE/dt)^{-1} \approx 4.1 \times 10^7 \text{ yr} \) does not depend on energy, and from equation (12) we have

\[
r_{\text{max}}^{\text{diff}}(E) = \sqrt{2D_0 \tau_\infty} \left( \frac{E_{\text{acc}}^{\text{max}}}{E_c} \right) .
\]

Consider now the intermediate energies, when \( E \) approaches \( 1 \times 10^{18} \text{ eV} \), but \( E_s(E_0, t_0) \) remains less than \( E_s \approx 4 \times 10^{18} \text{ eV} \), where photopion production starts. One obtains in the case of \( D(E) \propto E^\alpha \),

\[
r_{\text{max}}^{\text{diff}}(E) \propto \sqrt{D_0 \tau_\infty \frac{E_s(E_0, t_0)}{E}}^{\alpha/2} ,
\]

where \( \tau_\infty \sim \beta^{-1} \). In this case \( r_{\text{max}}^{\text{diff}}(E) \) grows fast with \( E \) because of the fast growth of \( E_s(E_0, t_0) \) (see Fig. 2).

When \( E \) approaches \( E_\beta \approx 2 \times 10^{18} \text{ eV} \), the value of \( r_{\text{max}}^{\text{diff}} \) is determined by the energy interval between \( E_s \) and \( E_{\text{acc}}^{\text{max}} \), where \( D(E) \propto E^2 \). There \( E_{\text{acc}}^{\text{max}} \) grows by a jump to \( E_{\text{acc}}^{\text{max}}^{\text{max}} \), and \( r_{\text{max}}^{\text{diff}} \) also grows by a jump to the high-energy asymptotic value given by equation (15).

The accurate numerical calculations are displayed in Figure 3 for two different magnetic field configurations, (1 nG, 1 Mpc) and (100 nG, 1 Mpc).

In a diffusive regime of propagation, there is an additional upper limit for a distance to a source, which we refer to as the rectilinear maximal distance \( r_{\text{max}}^{\text{rect}}(E) \). It is defined as

\[
r_{\text{max}}^{\text{rect}}(E) = \begin{cases} c\tau E_s(E_{\text{acc}}^{\text{max}}) & \text{if } \tau < t_0, \\ c t_0 & \text{if } \tau > t_0. \end{cases}
\]

At small \( E \), \( r_{\text{max}}^{\text{rect}}(E) \equiv c\tau(E, E_{\text{acc}}^{\text{max}}) = (c/H_0) \ln (E_{\text{acc}}^{\text{max}}/E) \) is larger than \( c t_0 \) and \( r_{\text{max}}^{\text{diff}}(E) \), as one can see from Figure 3. At large \( E \), \( r_{\text{max}}^{\text{rect}}(E) \) is smaller than \( r_{\text{max}}^{\text{diff}}(E) \), and thus the rectilinear upper limit becomes restrictive.

The Syrovatsky solution, equation (1), does not automatically include the restriction due to \( r_{\text{max}}^{\text{rect}}(E) \), because propagation time there varies from 0 to \( \infty \). The restriction equation (17) must also be imposed in equation (1). This restriction is also valid in the case without a magnetic field, and numerically it is very close to the attenuation length \( l_{\text{att}}(E) = E(dE/dt)^{-1} \), which describes the ordinary GZK cutoff.

Figure 3 illustrates the anti-GZK effect, which we discuss here. While the energy-attenuation length \( l_{\text{att}}(E) = E(dE/dt)^{-1} \) (or maximal rectilinear distance \( r_{\text{max}}^{\text{rect}} \)) diminishes with energy \( E \) and has the sharp GZK steepening at \( E \approx 5 \times 10^{18} \text{ eV} \), the diffusive maximum distance \( r_{\text{max}}^{\text{diff}}(E) \) increases with energy and has a sharp jump at energy \( E_j \approx 2 \times 10^{18} \text{ eV} \). As we discussed above, this energy is determined entirely by energy losses, and it does not depend on the diffusion parameters.

The growth of \( r_{\text{max}}^{\text{diff}}(E) \) depends on the diffusive regime, as it directly follows from equation (14).

4. RESULTS AND DISCUSSION

The maximum distance, \( r_{\text{max}}(E) \), determines the number of sources that in principle can contribute to the observed diffuse flux \( J_d(E) \): the flux from the sources at distances \( r \) larger than \( r_{\text{max}} \) is suppressed as \( \exp \left( -r^2/r_{\text{max}}^2 \right) \). But inside the sphere with radius \( r_{\text{max}} \), the fluxes from the sources are suppressed by \( \lambda(E, E_j) \), which is less than \( \lambda(E, E_{\text{acc}}^{\text{max}}) \), and by \( E_j/\lambda(E, E_{\text{acc}}^{\text{max}}) \). For this reason, the jump in \( r_{\text{max}} \) does not produce a jump in the flux at energy \( E_j \). The situation is different at \( E < 2 \times 10^{18} \text{ eV} \), where \( r_{\text{max}} \) suppresses the diffuse flux, restricting the number of contributing sources.

In Figures 4, 5, and 6 we present the calculated diffuse spectra using equations (1) and (11), in the case of two configurations \((B_0, l)\) and for different distances \(d\) between sources.

In our calculations the sources are located in the vertices of a lattice, and summation is performed within the volume limited by \( r_{\text{max}}(E) \) as described in § 3. In fact, only the rectilinear limit is introduced by hand, while \( r_{\text{max}}^{\text{diff}}(E) \) at lower energies appears automatically.

As expected, the energy of the low-energy steepening \( E_\beta \) is nearly the same for all magnetic configurations and approximately coincides with the crossover of adiabatic and pair production energy losses \( E_\beta \) and with the position of jump \( E_j \). In accordance with \( r_{\text{max}}(E) \) given by equation (14), the flux below the low-energy cutoff is largest for the Kolmogorov diffusion (or \( D = \text{const} \) regime) and lowest for \( D(E) \propto E^2 \) diffusion, with the Bohm diffusion between them.
In the calculations for a reasonable magnetic field configuration with \( B_0 = 1 \) nG and \( l_c = 1 \) Mpc, we have used a separation between sources \( d = 30 \) Mpc and \( d = 50 \) Mpc, which corresponds to a source space density of \( 3.7 \times 10^{-5} \) Mpc\(^{-3}\) and \( 8.0 \times 10^{-6} \) Mpc\(^{-3}\), respectively. As discussed in § 1, the small-angle clustering favors a density \( n_s \sim (1 - 3) \times 10^{-3} \) Mpc\(^{-3}\), with some uncertainties. In the case of strong magnetic field \( B_0 = 100 \) nG, we have used a larger separation, \( d = 100 \) Mpc, to improve the agreement with observations.

In Figures 4 and 5 we show the spectra in the case of \( B_0 = 1 \) nG and \( l_c = 1 \) Mpc. The critical energy where the diffusion changes its regime is \( E_c \sim 1 \times 10^{15} \) eV, and the diffusion length at this energy is \( l_{\text{diff}} \approx 1 \) Mpc. The best fit to the observations is obtained for \( \gamma_d = 2.7 \). The energy of the steepening in both cases is \( E_s \sim 1 \times 10^{18} \) eV. The source luminosities \( L_s \) needed to provide the observed flux are very high, if one assumes a power-law generation spectrum from \( E_{\text{min}} \sim 1 \) GeV up to \( E_{\text{acc}} \sim 1 \times 10^{22} \) eV. For \( d = 50 \) Mpc, \( L_p = 1.5 \times 10^{46} \) erg s\(^{-1}\), and for \( d = 30 \) Mpc, \( L_p = 3.0 \times 10^{48} \) erg s\(^{-1}\). To reduce these luminosities, one can assume that the acceleration mechanism operates starting from some larger \( E_{\text{min}} \). Then the required luminosity is reduced by a factor \( E_{\text{acc}}^{(\gamma_d - 2)} \), which is \( 1.3 \times 10^{-3} \) for \( E_{\text{min}} = 1 \times 10^8 \) GeV, and \( 2.5 \times 10^{-6} \) for \( E_{\text{min}} = 1 \times 10^9 \) GeV. Another possible assumption is that the standard spectrum \( \propto 1/E^2 \) at \( E < E_{\text{min}} \), as Berezinsky et al. (2002b, 2003) have assumed.

Figures 4 and 5 show that the dip seen in the universal spectrum as a signature of the interaction with CMB (Berezinsky et al. 2002a) survives in the case of propagation in a magnetic field with configuration (1 nG, 1 Mpc). As shown below, the same is true for weaker and stronger magnetic fields.

The case of a strong magnetic field \((B_0, l_c) = (100 \) nG, 1 Mpc\) is shown in Figure 6. This is a very attractive case: the good agreement with the data is reached using the standard generation spectrum \( \propto 1/E^2 \) and \( d = 100 \) Mpc. The required luminosity is reasonable, \( L_p = 3 \times 10^{45} \) ergs s\(^{-1}\) for \( E_{\text{min}} \sim 1 \) GeV and \( E_{\text{max}} = 1 \times 10^{25} \) eV. The diffusion coefficient used in this case is \( D \approx \text{const} \) at \( E \leq E_c \) [the best fit in Fig. 6 is obtained for \( D(E) \propto E^{0.02} \)]. Unfortunately, the required magnetic field is much higher than that obtained in the MHD simulations by Dolag et al. (2004) and Sigl et al. (2004), although it does not contradict the existing observational upper limits.

Let us now consider the case of the very weak magnetic field \( B_0 \sim 0.1 \) nG favored in MHD simulations by Dolag et al. (2004). In this case \( E_c \approx 1 \times 10^{17} (l_c/1 \) Mpc\) eV and \( l_{\text{diff}}(E) \approx 100 E_c^{(\gamma_d - 1)} (1 \) Mpc/\( l_c) \) Mpc. Therefore, for \( l_c \leq 1 \) Mpc and \( E \gtrsim 3 \times 10^{18} \) eV, the protons propagate quasi-rectilinearly in the universe. In this case the distance between sources \( d \) is less than the propagation lengths \( l_{\text{diff}}(E) \) and \( l_{\text{diff}}(E) \), and the spectrum at least at energies \( (1-40) \times 10^{18} \) eV must be universal.

A note of warning should be made about the validity of the Syrovatsky solution at \( E < 1 \times 10^{19} \) eV. This solution is expected to work not perfectly well at these energies, because it is valid only in the case when the energy losses \( b(E) \) and diffusion coefficient \( D(E) \) are time independent.\(^1\) For the above-mentioned energies, this is not the case, because during the time of propagation

\(^1\) One should be careful with inserting ad hoc time-dependent quantities in the Syrovatsky solution, equation (1), because in this case it ceases to be a solution of the corresponding diffusion equation. For example, it is forbidden to introduce the cosmological scaling factor \( a(t) \), because it results in time-dependent energy losses \( b(E, t) \) or to consider \( \lambda \) in equation (1) as a function of \( E, E_p \) and \( t \).
the temperature of the CMB radiation changes appreciably, and hence the energy losses change too. The diffusion equation itself should be also modified as $t \rightarrow \tau_0$ by the cosmological relations between time and distance. However, the approximate agreement, which we obtained (to be discussed somewhere else) between the Syrovatsky solution in quasi-rectilinear regime and the exact rectilinear propagation, demonstrates the approximate validity of this solution at the discussed energies.

Another argument in favor of the Syrovatsky solution as a reasonable approximation at the discussed energies, $E \leq 1 \times 10^{19}$ eV, is the convergence to the universal spectrum (compare Fig. 4 and Fig. 5 for $d = 50$ Mpc and $d = 30$ Mpc, respectively). The universal spectrum is calculated in the case of time-dependent CMB temperature and for an expanding universe. The Syrovatsky solution converges to this spectrum with an accuracy better than 15% when $d \rightarrow 3-5$ Mpc (to be discussed in a forthcoming paper).

Following the papers by Berezinsky et al. (2004) and Lemoine (2004), we now discuss briefly the transition from Galactic to extragalactic cosmic rays. The remarkable feature of the diffusive spectra is the low-energy steepening at the fixed energy $E_s \sim 1 \times 10^{18}$ eV, which provide the transition from extragalactic to Galactic cosmic rays. This energy coincides approximately with the position of the second knee $E_{sk}$ and gives a nontrivial explanation of its value as $E_{sk} \sim E_\beta$, where $E_\beta$ is the characteristic energy of the cosmic rays.

As in the above-mentioned works, we assume that at $E \geq 1 \times 10^{15}$ eV the Galactic spectrum is dominated by iron nuclei and calculate their flux by subtracting the calculated flux of extragalactic protons from an all-particle Akeno spectrum. For these calculations we fix the spectrum with magnetic configuration (1 nG, 1 Mpc), the Bohm diffusion at $E < E_s$, and a separation between sources on the lattice $d = 30$ Mpc (see Fig. 5). The calculated spectrum of Galactic iron is shown in Figure 7 by the dashed curve. The fraction of iron nuclei in the total flux is shown in Table 1 as a function of energy. This prediction should be taken with caution because of the model-dependent calculations (assumption of Bohm diffusion) and uncertainties involved in the Syrovatsky solution. However, it is interesting to note that the iron-nuclei spectrum in Figure 7 practically coincides with the spectrum calculated by Berezinsky et al. (2004) for the model with the generation spectrum steepening. The iron nuclei spectra in both cases are well described by Hall diffusion (Ptuskin et al. 1993) in the Galactic magnetic field at energies above the knee.

We now compare our results with those obtained by Lemoine (2004), who also found low-energy steepening of the spectrum because of diffusion. Lemoine has limited his calculations to the case $B_0(l_c)^{1/2} \sim 2 \times 10^{-10}$ G Mpc$^{-1/2}$, while we demonstrated that this phenomenon is valid for a much wider range of parameters; for example, our configuration (100 nG, 1 Mpc) corresponds to the Lemoine parameter 2 orders of magnitude larger. We consider here a more realistic basic scale $l_c \sim 1$ Mpc and the various regimes of diffusion, while Lemoine limited himself to the $D(E) \propto E^2$ regime only. We have also obtained the important result that the energy of the steepening is the same, $E_s \sim 1 \times 10^{18}$ eV, for all diffusion regimes and distances between the sources, and that universalities is determined almost entirely by the proton energy losses. We discuss the diffusive anti-GZK effect, which we consider to be the most interesting observation of this work.

5. CONCLUSIONS

We have analyzed the anti-GZK effect in the diffusive propagation of ultra–high-energy protons. This effect consists of an increase of the maximum distance $r_{max}(E)$, from which ultra–high-energy protons can reach an observer, with an increasing of the energy $E$. This increase is terminated by a jump, which is located at energy $E_j \approx 2 \times 10^{19}$ eV. The position of the jump is determined exclusively by energy losses (transition from adiabatic to pair-production energy losses), and it is independent of the diffusion parameters. The position of the jump practically coincides with the position of the aforementioned transition and gives approximately the position of the second knee observed in the cosmic-ray spectrum (see below).

The observational consequence of the anti-GZK effect is the low-energy “cutoff” of the diffuse spectrum, which is in fact a steepening in the spectrum, as the GZK cutoff is. The steepening energy $E_s$ coincides approximately with the position of the jump, $E_s \approx E_j$, and it is also practically independent of the diffusion parameters, i.e., of the basic scale of magnetic field coherence $l_c$ and of the magnetic field $B_0$ on this scale. However, the shape of the steepening is determined by the diffusion regime: it is most steep in the case of $D(E) \propto E^2$ diffusion, most flat in the case of the Kolmogorov diffusion, with the Bohm diffusion between them.

In our calculations we have used the Syrovatsky solution to the diffusion equation, combined with rectilinear propagation at the appropriate distances. The sources are located in the vertexes

\[ E_s \approx E_j, \]
of a lattice with a spacing scale $d$ (the source separation). We have used mostly $d = 30$ Mpc and $d = 50$ Mpc, which correspond to a space density of the sources of $3.7 \times 10^{-5}$ Mpc$^{-3}$ and $8.0 \times 10^{-6}$ Mpc$^{-3}$, respectively. The observed small-angle clustering favors the density $n_s \sim (1-3) \times 10^{-5}$ Mpc$^{-3}$. The diffusion coefficient $D(E)$ is calculated for a random magnetic field with the basic scale $l_r$ and the coherent magnetic field on this scale $B_0$. Using this approach we have calculated the diffusive spectra for various magnetic configurations ($B_0$, $l_r$) and source separations $d$.

Physically, the most reasonable case corresponds to a magnetic field configuration (1 G, 1 Mpc) with a source separation of $d = 30$ Mpc and $d = 50$ Mpc. The calculated spectra are shown in Figures 4 and 5 in comparison with Akeno Giant Air Shower Array (AGASA) data. For a power-law generation spectrum with $y_g = 2.7$, the agreement is good but needs too high a luminosity of the sources $L_p$ if the power-law spectrum starts with low energy, $E_{\text{min}} \sim 1$ GeV. This problem can be ameliorated by assuming higher values of $E_{\text{min}}$.

The calculated diffusive spectra in the energy interval (1–80) $\times 10^{18}$ eV agree perfectly well with the universal spectrum and experimental data, showing the presence of the dip caused by $e^+e^-$ production.

An interesting case is given by the diffusion in a strong magnetic field with basic configuration (100 G, 1 Mpc) and source separation $d = 100$ Mpc. In this case (Fig. 6), the best fit of the spectrum is obtained for the standard acceleration spectrum $Q(E) \propto 1/E^2$ and $E_{\text{min}} \sim 1$ GeV. The required luminosity is $L_p = 3 \times 10^{45}$ erg s$^{-1}$. Up to energy $E \sim 1 \times 10^{20}$ eV, the predicted spectrum agrees with data of both detectors, AGASA and HiRes. The sharp cutoff at $E \sim 1 \times 10^{20}$ eV is produced by large distances, $r \sim d$, to the nearby sources. For the explanation of the AGASA excess at $E \gtrsim 1 \times 10^{20}$ eV, a new component of ultra–high-energy cosmic rays (e.g., from superheavy dark matter; see Aloisio et al. 2004) is needed.

At energies $E < E_r$, where $l_{\text{diff}}$ becomes much smaller than $d$, the diffusive spectrum exhibits a steepening, in contrast to the universal spectrum (see Figs. 4–6).

The steepening of the spectrum at $E_r \sim 1 \times 10^{18}$ eV provides a natural transition from Galactic to extragalactic cosmic rays. This energy coincides with the second knee observed in cosmic-ray spectra by most of the detectors. While the energy of the transition $E_r$ (and thus the position of the second knee) is predicted in a model-independent way, the shape of the proton spectrum below $1 \times 10^{18}$ eV and the fraction of Galactic iron nuclei are model dependent: they differ for various diffusion regimes.

We are grateful to Pasquale Blasi for useful discussion. We thank the transnational access to research infrastructures (TARI) program through the LNGS TARI grant contract HPRI-CT-2001-00149.