THE THERMAL EVOLUTION OF ULTRAMAGNETIZED NEUTRON STARS

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ABSTRACT

Using recently calculated analytic and numerical models for the thermal structure of ultramagnetized neutron stars, we estimate the effects that ultrastrong magnetic fields $B \geq 10^{14}$ G have on the thermal evolution of a neutron star. Understanding this evolution is necessary to interpret models that invoke “magnetars” to account for soft gamma-ray emission from some repeating sources.

Subject headings: stars: neutron — stars: magnetic fields — radiative transfer — X-rays: stars

1. INTRODUCTION

Neutron stars with extremely strong magnetic dipole fields ($B \geq 10^{14}$ G) may form if a helical dynamo mechanism operates efficiently during the first few seconds after gravitational collapse (Thompson & Duncan 1993) or through the conventional process of flux freezing if the progenitor star has a sufficiently intense core field. These “magnetars” initially rotate with periods $P \sim 1$ ms but would quickly slow down because of magnetic dipole radiation and cross the pulsar death line after about $10^5$–$10^6$ yr. With their strong magnetic fields, magnetars have been used to explain several phenomena, including gamma-ray bursts (Uslov 1992, Duncan & Thompson 1992) and soft gamma repeaters (Thompson & Duncan 1995).

Shibanov & Yakovlev (1996) have recently discussed how magnetic fields $B < 10^{13.5}$ G affect neutron star cooling. (Throughout, we will use the symbol $B$ to denote the field strength at the magnetic pole.) They find that below $B \sim 10^{12}$ G, the magnetic field suppresses the total heat flux radiated by a neutron star. However, for $B \approx 10^{12}$ G, the quantization of the electron energies enhances the conductivity along the field lines, resulting in a net increase in the heat flux. Here we extend these results into the ultramagnetized regime with $B = 10^{14}$–$10^{16}$ G.

In this Letter, we will discuss the cooling evolution of neutron stars that have not accreted significant material from their surroundings, i.e., neutron stars with iron envelopes.

2. MODEL ENVELOPES

Heyl & Hernquist (1997) have developed analytic models for ultramagnetized neutron star envelopes and find that the transmitted flux through the envelope is simply related to the direction and strength of the magnetic field and to the core temperature ($T_c$). Using these results as a guide, we numerically integrate several envelopes with $B = 10^{14}$–$10^{16}$ G for the case of parallel transport. We will present the detailed results of these calculations in a future article.

At the outer boundary, we apply the photospheric condition (see, e.g., Kippenhahn & Weigert 1990). In the nondegenerate regime, photon conduction dominates. For the range of effective temperatures considered, free-free absorption is the most important source of opacity, and we estimate the anisotropy factor due to the magnetic field using the results of Pavlov & Panov (1976) and Silant’ev & Yakovlev (1980). In the degenerate regime, electrons dominate the conduction; we use the conductivities of Hernquist (1984) and those of Potekhin & Yakovlev (1996) and present results using both these values. In the semidegenerate regime, both processes are important, so we sum the two conductivities.

Potekhin & Yakovlev (1996) give formulae to calculate electron conductivities in the liquid and solid regimes for arbitrary magnetic field strengths. The results of Hernquist (1984) are given for specific values of the field strength. We calculate the conductivities using the formalism and assumptions outlined in Hernquist (1984) and extend his calculations to stronger fields.

The conductivities of Hernquist (1984) and Potekhin & Yakovlev (1996) do not differ in the physical processes considered in their calculations but in the approximations employed. In the liquid state, the conductivities of Hernquist (1984) tend to be approximately 15% larger for the core Landau level and to up to 40% larger for the excited levels than those of Potekhin & Yakovlev (1996). In the solid state, the conductivities of Potekhin & Yakovlev (1996) exceed those of Hernquist (1984) by a factor of several; therefore, these two models span much of the uncertainty in these quantities.

In the liquid state, the differences arise from two sources. First, both the fits of Hernquist (1984) and of Potekhin & Yakovlev (1996) for the function $\phi(E)$ are inaccurate to $\sim 10%$. Second, Hernquist (1984) assumes that electron-ion scattering is screened by the ion sphere; this process dominates in the liquid regime. Potekhin & Yakovlev (1996) include Debye and electron screening, as well, which dominate in the gaseous regime. Their results are appropriate for both the gaseous and liquid regimes. In the solid regime, Hernquist (1984) does not take the Debye-Waller factor into account. This factor tends to increase the conductivity over a wide range of temperatures and densities (Naoki et al. 1984; Potekhin & Yakovlev 1996).

Our iron envelope models are calculated by adopting a plane-parallel Newtonian approximation. Hernquist (1985) found that using $Z = 26$ and $A = 56$ throughout is sufficient to accurately model the envelope. For simplicity, we fix $Z$ and $A$ to these values rather than using the equilibrium composition of Ba
c

\[ \text{For such strong fields, the models have a simple dependence on the angle } \psi; \text{ i.e., } F/g, \propto \cos^2 \psi \text{ (Greenstein & Hartke 1983; } \]

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Page 1995; Shibanov et al. 1995; Shibanov & Yakovlev 1996; Heyl & Hernquist 1997), and furthermore, the flux for a fixed core temperature is approximately proportional to $B^{1/8}$. With these two facts, we find that the average flux over the surface of a neutron star with a dipole field configuration is $0.4765$ times its peak value at the magnetic poles. We neglect the effects of general relativity, which tend to make the field more radial (Ginzburg & Ozerney 1964), on the field configuration, increasing the effects discussed here.

Using these models, we have calculated a grid of theoretical envelopes with average effective temperatures ranging from $10^{12}$ to $10^{6}$ K, corresponding to a factor of $\sim 10^3$ in transmitted flux. We take $g_\sigma = g_{\sigma,1} 10^{14}$ cm s$^{-2}$. Figure 1 (upper panel) depicts the ratio of the core temperature to the zero-field case (Hernquist & Applegate 1984) as a function of the magnetic field and the mean effective temperature $T_{\text{eff}}$ over the neutron star. In the zero-field case, to determine the core temperature for a given flux, we combine equations (4.7) and (4.8) of Hernquist & Applegate (1984) as a function of the magnetic field and the mean effective temperature $T_{\text{eff}}$ over the neutron star. In the zero-field case, to determine the core temperature for a given flux, we combine equations (4.7) and (4.8) of Hernquist & Applegate (1984), switching from the first relation to the second when the surface effective temperature drops below $4.25 \times 10^5$ K. The results do not depend qualitatively on whether equation (4.7) or (4.8) of Hernquist & Applegate (1984) is used. The lower panel of Figure 1 shows the ratio of the core temperature with a magnetic field to the zero-field case for the conductivities of Potekhin & Yakovlev (1996).

For a given core temperature, the magnetized envelopes transmit more heat than the unmagnetized envelopes. For example, an effective temperature of $3.5 \times 10^6$ K corresponds to a core temperature of $1.1 \times 10^4$ K for an unmagnetized envelope. With $B = 10^{16}$ G, the core temperature is $5.3 \times 10^6$ K for the Hernquist (1984) conductivities and $5.8 \times 10^8$ K for those of Potekhin & Yakovlev (1996). Because the Hernquist (1984) conductivities in the liquid phase are $\sim 20\%$ larger than those of Potekhin & Yakovlev (1996), we find that the effective temperature is slightly higher during the early cooling ($t \leq 10^3$ yr) of the neutron star if one uses the values of Hernquist (1984).

For lower core temperatures, the insulating envelope is thinner and the magnetic field has a stronger effect (Van Riper 1988); the difference in core temperatures may be even more extreme, by up to a factor of 4 or 10 (for Hernquist 1984 and Potekhin & Yakovlev 1996 conductivities, respectively) in the coolest envelopes considered here. For the cooler envelopes, the relationship between the effective temperature and the core temperature is strongly sensitive to the conductivities in the solid phase; consequently, the Debye-Waller factor is most important during the later cooling of the neutron star ($t \geq 10^5$ yr). Furthermore, during this late phase, the partial ionization of iron may affect the equation of state as well as the electron and photon conductivities. This area has not been thoroughly explored, especially at high $B$-values.

For $T_c > 10^8$ K, the dominant heat-loss mechanism is through neutrino emission (see, e.g., Shapiro & Teukolsky 1983) that has a cooling time proportional to $T_c^{-8}$ for the modified Urca process. Because of this steep power law, a factor of $1.9-2.1$ difference in the core temperature (the first example) would lead one to infer a cooling time of an unmagnetized neutron star 50-80 times greater than if one did not consider the effects of the magnetic field on heat transport. And since neutrino-cooling models generally have a cooling time proportional to $T_c^{-\alpha}$ with $\alpha = 4-6$ (see, e.g., Shapiro & Teukolsky 1983), one would generally underestimate the cooling ages of unmagnetized neutron stars by a large factor.

3. THERMAL EVOLUTION

For $t \geq 10^3$ yr, the neutron star interior has relaxed thermally (Nomoto & Tsuruta 1981), and we can use the flux-to-core-temperature relation for several values of $B$ including $B = 0$ to derive the relationship between $T_{\text{eff}}$ and the cooling time. The technique is straightforward during the epoch of neutrino cooling. However, since photon emission is enhanced, the epoch of photon cooling will begin slightly earlier, and the time dependence of the temperature will have a slightly different slope. For the neutrino-cooling model, we use the modified Urca process (see, e.g., Shapiro & Teukolsky 1983),

$$L_\nu = (3.5 \times 10^{39} \text{ ergs s}^{-1}) \left( \frac{M}{M_\odot} \left( \frac{\rho_{\text{en}}}{\rho} \right)^{1/3} \right) T_{\nu,0}^8, \quad (1)$$

where $T_\nu = T/10^3$ K.

To understand the evolution during the photon-cooling epoch, we take into account the surface thermal emission of photons,

$$L_\gamma = 4\pi R^2 \bar{T}_{\text{eff}}^4 = \pi (9.5 \times 10^{32} \text{ ergs s}^{-1}) \bar{T}_{\text{eff}}^4 \frac{M}{g_{\sigma,14} M_\odot}, \quad (2)$$

![Fig. 1.—Upper panel: ratio of the core temperature with $B \neq 0$ to the zero-field case (Hernquist & Applegate 1984) using the conductivities of Hernquist (1984). Lower panel: results for the Potekhin & Yakovlev (1996) conductivities.](image-url)
and we take the total thermal energy of the neutron star to be (Shapiro & Teukolsky 1983)

\[ U_n \approx (6 \times 10^{47} \text{ ergs}) \frac{M}{M_\odot} \left( \frac{\rho}{\rho_{\text{cuc}}} \right)^{2/3} T_{c,9}^4. \]  

(3)

Combining these equations yields

\[ \frac{dU_n}{dt} = -(L_s + L_g), \]

\[ \frac{dT_{c,9}}{dr} = -\frac{1}{4 \times 10^7 \text{ yr}} \frac{T_{c,9}^4}{T_{c,9}} \left( \frac{\rho}{\rho_{\text{cuc}}} \right)^{2/3} \]

\[ + \frac{1}{8 \text{ yr}} \left( \frac{\rho_{\text{cuc}}}{\rho} \right)^{1/3} T_{c,9}^8, \]

where \( \rho \) is the mean density of the neutron star and \( \rho_{\text{cuc}} = 2.8 \times 10^{14} \text{ g cm}^{-3} \).

Figure 2 shows the evolution of the core temperature and mean effective temperature at the surface. The evolution of the core temperature is unaffected by the magnetic field during the neutrino-cooling epoch. For fields approaching \( 10^{18} \text{ G} \), the magnetic field may begin to affect neutrino emission (see, e.g., Bander & Rubinstein 1993). However, after approximately \( 10^6 \text{ yr} \), photon emission from the surface begins to dominate the evolution.

The cooling is accelerated by the magnetic field. In the presence of a \( 10^{15} \text{ G} \) field, the core reaches a temperature of \( 10^7 \text{ K} \) in only \( 3-6 \times 10^3 \text{ yr} \) compared to \( 6 \times 10^4 \text{ yr} \) for an unmagnetized neutron star. The bold curves trace the cooling of the core using the Hernquist (1984) conductivities, and the light curves follow the results for the Potekhin & Yakovlev (1996) conductivities. The effect is more dramatic when one compares the effective surface temperatures of the models as a function of time. Again we present two sets of models. The lower panel of Figure 2 compares the cooling evolution using the conductivities of Hernquist (1984; bold curves) and using those of Potekhin & Yakovlev (1996; light curves) in the degenerate regime.

During the neutrino-cooling epoch, the ultramagnetized neutron stars (\( B = 10^{16} \text{ G} \)) have 45% higher effective temperatures and emit over 4 times more radiation. Because during neutrino cooling the effective temperature falls relatively slowly with time, one can make a large error in estimating the age of the neutron star from its luminosity. For example, an envelope with \( 10^{15} \text{ G} \) field remains above a given effective temperature 40 times longer than an unmagnetized envelope. For \( 10^{15} \text{ G} \), the timescale is increased by up to a factor of 10.

During the photon-dominated cooling era, the enhanced flux in a strong magnetic field reverses this effect. Photon cooling begins to dominate after about \( 10^5 \text{ yr} \) for \( 10^{15} \text{ G} \) compared to \( 10^6 \text{ yr} \) in the zero-field case. Once photon cooling begins to dominate, the stars with stronger magnetic fields cool more quickly. A star with a \( 10^{16} \text{ G} \) field reaches a given effective temperature 3–5 times faster than an unmagnetized star.

4. DISCUSSION

Magnetic fields, especially those associated with magnetars, have a strong effect on the observed thermal evolution of neutron stars. In agreement with Shibanov & Yakovlev (1996), we find that during the neutrino-cooling epoch, neutron stars with strong magnetic fields are brighter than their unmagnetized coevals. During the photon-cooling epoch, the situation is reversed. A strongly magnetized neutron star cools more quickly during this era and emits less radiation at a given age.

It is difficult to compare our results more quantitatively with
those of Shibanov & Yakovlev (1996), because, besides studying more weakly magnetized neutron stars, they make slightly different assumptions regarding the properties of the envelope, include general relativistic effects on the magnetic field geometry, and use the models of Van Riper (1988) that include Coulomb corrections to the equation of state. For the larger fields investigated here, we do find a stronger effect than do Shibanov & Yakovlev (1996); however, the effect is not as strong as a naive power-law extrapolation from $10^{15} \text{ G}$ (the largest field studied by Shibanov & Yakovlev 1996) to the ultramagnetized regime would indicate. Usov (1997) extrapolates results at lower field strengths and finds substantially larger photon luminosities during the neutrino-cooling epoch than we do. Again, a straightforward extrapolation from weaker fields overestimates the flux transmitted through a magnetized envelope.

We do not find the net insulating effect that Tsuruta & Qin (1995) find for weaker fields of $10^{12} \text{ G}$. Shibanov & Yakovlev (1996) find a similar, albeit much weaker effect, for fields of $\sim 10^{10} - 10^{12} \text{ G}$. At these field strengths, the classical decrement in the thermal conductivity transverse to the field direction decreases the transmitted flux for a given core temperature. At the much stronger field strengths examined here, the increase in conductivity along the field lines (caused by the quantization of the electron energies) dominates the decrease for perpendicular transport (in using the $\cos^2 \psi$ rule, we have neglected all heat transport perpendicular to the field lines).

Thompson & Duncan (1995) argue that soft gamma repeaters (SGRs) are powered by magnetic reconnection events near the surfaces of ultramagnetized neutron stars. Furthermore, Ulmer (1994) finds that a strong magnetic field can explain the super-Eddington radiation transfer in SGRs. Rothschild, Kulkarni, & Lingenfelter (1994) estimate the luminosity of SGR 0526−66 in the quiescent state to be approximately $7 \times 10^{35} \text{ ergs s}^{-1}$. Since SGR 0526−66 is located in a supernova remnant, they can also estimate the age of the source to be approximately 5000 yr. For an isolated neutron star cooling by the modified Urca process, after 5000 yr, one would expect $L_N = 6 \times 10^{36} \text{ ergs s}^{-1}$ for $B = 0$ and $L_N = 3 \times 10^{33} \text{ ergs s}^{-1}$ for $B = 10^{14} \text{ G}$ (we assume that the mass of the neutron star is $1.4 M_\odot$). Both of these estimates fall short of the observed value. Even if SGR 0526−66 is powered by an ultramagnetized neutron star, its quiescent X-ray luminosity does not originate entirely from the thermal emission from the surface of the neutron star unless either the age or luminosity estimates are in error by an order of magnitude, or possibly unless it has an accreted envelope.

5. CONCLUSIONS

We extend the previous studies of neutron star cooling into the ultramagnetized or magnetar regime ($B \sim 10^{15} - 10^{16} \text{ G}$) for iron envelopes. We find that such an intense magnetic field dramatically affects the thermal evolution of a neutron star. In the neutrino-cooling epoch, effective temperatures of ultramagnetized neutron stars are up to 40% larger than their unmagnetized coevals. If the nucleons in the neutron star core are superfluid, neutrino cooling is inhibited. This will also increase the surface temperature at a given epoch.

Furthermore, if one assumes an unmagnetized evolutionary track for an ultramagnetized neutron star, one would overestimate its age by up to a factor of 25. During the photon-cooling epoch, the effect is reversed: ultramagnetized neutron stars cool to a given effective temperature 3 times faster than their unmagnetized counterparts.

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