The rotation of polarization by gravitational waves

Valerio Faraoni

Physics Department, Bishop’s University
2600 College Street, Sherbrooke, Québec, Canada J1M 0C8
email: vfaraoni@ubishops.ca

Abstract

There are conflicting statements in the literature about the gravitational Faraday rotation of the plane of polarization of polarized electromagnetic radiation travelling through a gravitational wave. This issue is reconsidered using a simple formalism describing the rotation of the plane of polarization in a gravitational field, in the geometric optics approximation. It is shown that, to first order in the gravitational wave amplitude, the rotation angle is a boundary effect which vanishes for localized (astrophysically generated) gravitational waves and is non-zero, but nevertheless negligible, for cosmological gravitational waves.

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1 Introduction

There are many instances in astrophysics in which electromagnetic waves propagate through gravitational waves. The effects induced on a light beam traversing gravitational waves of astrophysical or cosmological origin include small deflections and frequency shifts, and they have been studied extensively in the literature (Wheeler 1960, Winterberg 1968, Zipoy 1966, Zipoy & Bertotti 1968, Dautcourt 1969, 1974, 1975a, b, 1977, Kaufmann 1970, Bergmann 1975, Bertotti 1971, Bertotti & Catenacci 1975, Burke 1975, 1981, Korotun 1970, Linder 1986, McBreen & Metcalfe 1988, Allen 1989, 1990, Braginsky et al. 1990, Kovner 1990, Faraoni 1992, 1993, 1996, 1998, Fakir 1993a, b, 1994a, b, 1996, 1997, Pogrebenko et al. 1994, 1996, Labeyrie 1993, Frieman et al. 1994, Durrer 1994, Pyne et al. 1996, Bar-Kana 1996, Marleau & Starkman 1996, Bracco 1997, Gwinn et al. 1997, Kaiser & Jaffe 1997, Faraoni & Gunzig 1998, Bracco & Teyssandier 1998, Damour & Esposito-Farèse 1998, Kopeikin et al. 1999, 2006, Larson & Schild 2000, Ragazzoni et al. 2003, Lesovik et al. 2005, Kopeikin & Korobkov 2005). The problem of whether, and how much, a gravitational wave rotates the plane of polarization of a polarized electromagnetic wave propagating through it (gravitational Faraday rotation or Skrotskii effect) has been considered now and again in astrophysics (Faraoni 1993, Surpi & Harari 1999, Cooperstock & Faraoni 1993, Kopeikin & Mashhoon 2002, Prasanna & Mohanty 2002). It is interesting to determine whether this effect is detectable with current or foreseeable technology in realistic astrophysical or cosmological situations, as a means of detecting gravitational waves through their interaction with light from distant sources. The effect considered here is different from the polarization of the cosmic microwave background, which is essentially Thomson scattering of photons of the cosmic microwave background by an anisotropic plasma, with the anisotropy caused by the shear associated with the gravitational wave. Instead, here we are interested in the geometric rotation of the plane of polarization caused directly by the presence of the gravitational wave. This effect was considered in previous literature in the context of lensing by “ordinary” gravitational lenses, i.e., localized mass distributions (Dyer & Shaver 1992), and in this context the extension to non-conventional lenses, such as gravitational waves, was considered. It was found that, to first order in the gravitational wave amplitudes, the gravitational Faraday rotation is absent (Faraoni 1993, Cooperstock & Faraoni 1993). This result has implications for the observation of lensed polarized radio sources (Kronberg et al. 1991). However, results that apparently contradict this conclusion have since appeared in the literature (Surpi & Harari 1999, Kopeikin & Mashhoon 2002, Prasanna & Mohanty 2002). The purpose of this note is to study these discrepancies and to re-examine the validity of the result of (Faraoni 1993). It is found that the different results of Surpi & Harari 1999), Kopeikin & Mashhoon 2002), and Prasanna &
Mohanty 2002) are due to the fact that different physical situations are studied, which is reflected in the different boundary conditions adopted. The first order systematic rotation of the polarization plane described by Surpi & Harari (1999) is a boundary effect which vanishes for localized gravitational waves. Unfortunately, for cosmological gravitational waves which are not localized between the source and the observer, the effect is too small to be observable. Moreover, it is not a differential effect, which makes it undetectable if no independent information is available on the polarization of light before propagation through gravitational waves. The effect reported in (Surpi & Harari 1999, Kopeikin & Mashhoon 2002 and Prasanna & Mohanty 2002) is too small to be observed and it disappears in the limit $\lambda \ll \lambda_{gw}$, where $\lambda$ and $\lambda_{gw}$ are the wavelengths of the electromagnetic and gravitational waves, respectively.

## 2 Polarized radiation crossing a gravitational wave

For simplicity, we consider a flat background described by the Minkowski metric $\eta_{\mu\nu}$, perturbed by localized gravitational waves. The resulting metric is $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ in an asymptotically Cartesian coordinate system $\{x^\alpha\}$, with $|h_{\mu\nu}| \ll 1$ and $h_{\mu\nu} \to 0$ as $r \equiv \sqrt{x^2 + y^2 + z^2} \to +\infty$. In the eikonal approximation, the wavelength $\lambda$ of light propagating through the gravitational wave (of wavelength $\lambda_{gw}$) satisfies $\lambda \ll \lambda_{gw}$. The electromagnetic four-potential is

$$A_\mu = \hat{A}_\mu (x^\alpha) \ e^{i\omega S(x^\alpha)} ,$$

where $\hat{A}_\mu$ is a slowly varying amplitude, $\omega$ is the angular frequency of the wave, and the rapidly varying eikonal $S$ and its gradient $S_\mu \equiv \nabla_\mu S$ satisfy

$$S_\mu S^\mu = 0 , \quad A_\mu S^\mu = 0 , \quad S^\mu \nabla_\nu S^\mu = 0 .$$

(2.2)

It is assumed that

$$S^\mu = S^{(0)\mu} + \delta S^\mu = (1, 0, 0, 1) + \delta S^\mu ,$$

(2.3)

where $S^{(0)\mu}$ is the unperturbed tangent to the null geodesic, and $\delta S^\mu$ is a small perturbation induced by the gravitational wave. The perturbed geodesic equation yields (e.g., Faraoni 1993)

$$\delta S^\mu = -2\left[h_0^{\mu\nu} + h_3^{\mu\nu}\right]_S^O + \frac{1}{2} \int_S^O d z \left(h_{00} + 2h_{03} + h_{33}\right)^{\mu\nu} + O(2) ,$$

(2.4)

where $\cdot^{\mu\nu}$ denotes partial differentiation with respect to $x^\mu$. The quantities on the right hand side of eq. (2.4) are evaluated between the light source $S$ and the observer $O$, and
O(2) denotes second order quantities in the gravitational wave amplitudes. The first term on the right hand side is a boundary term and vanishes for astrophysically generated gravitational waves localized between the source and the observer. The amplitude of the electromagnetic potential is further decomposed as (Stephani 2004) $\hat{A}^\mu \equiv a P^\mu$, where $a$ is a complex scalar and $P^\mu$ is a real vector satisfying (Stephani 2004)

$$\frac{1}{a} \frac{da}{d\sigma} = -\theta \equiv -\frac{\nabla_\alpha S^\alpha}{2}, \quad (2.5)$$

$$\frac{dP^\mu}{d\sigma} = \frac{1}{2} \left( \frac{P^\nu \partial_\nu a}{a} + \nabla_\nu P^\nu \right) S^\mu. \quad (2.6)$$

Here $\sigma$ is an affine parameter along the null geodesic with tangent $S^\mu$ and $\theta$ is the expansion of a congruence of null geodesics around a fiducial ray. The decomposition

$$a = a^{(0)} + \delta a = \frac{A}{\sigma} + \delta a, \quad (2.7)$$

$$P^\mu = P^{(0)\mu} + \delta P^\mu = (0, 1, 0, 0) + \delta P^\mu \quad (2.8)$$

(where $A$ is a complex constant and $P^{(0)\mu}$ corresponds to radiation polarized along the $x$-axis) yields

$$\frac{1}{a^{(0)}} \frac{d(\delta a)}{d\sigma} + \frac{1}{\sigma} \frac{\delta a}{a^{(0)}} + \delta \theta = 0, \quad (2.9)$$

$$\frac{(\delta P^\mu)}{d\sigma} = \frac{1}{2} \left[ P^{(0)\nu} \frac{\partial_\nu (\delta a)}{a^{(0)}} + \nabla_\nu P^\nu \right] S^{(0)\mu}, \quad (2.10)$$

where $\delta \theta = \theta - \theta^{(0)}$ is the perturbation of the expansion of a congruence of null geodesics. The four-divergence of $P^\nu$ is

$$\nabla_\nu P^\mu = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} P^\nu) = \partial_\mu (\delta P^\nu) + \frac{1}{2} \partial_\nu h, \quad (2.11)$$

where $h \equiv h^\mu_\mu$ and $\sqrt{-g} = 1 + h/2 + O(2)$. As a result,

$$\frac{d(\delta P^\mu)}{d\sigma} = \frac{1}{2} \left[ \frac{\partial_\nu (\delta a)}{a^{(0)}} + \partial_\alpha (\delta P^\alpha) + \frac{1}{2} \partial_\nu h \right] \left( \delta^\mu_{\nu} + \delta^\nu_{\mu} \right) + O(2). \quad (2.12)$$

Integration along the unperturbed photon path, instead of the perturbed one, only implies a second order error and yields

$$\delta P^\mu = \frac{1}{2} \left( \delta^\mu_{\nu} + \delta^\nu_{\mu} \right) \int^0_S dz \left[ \frac{\partial_\nu (\delta a)}{a^{(0)}} + \partial_\alpha (\delta P^\alpha) \right] + C^\mu + O(2) \quad (2.13)$$
in the transverse-traceless (TT) gauge in which $h = 0$, and where $C^\mu$ are integration constants. Equation (2.13) yields $\delta P^1 = \text{const.}$, $\delta P^2 = \text{const.}$ When the boundary conditions $h_{\mu\nu}(S) = h_{\mu\nu}(O) = 0$ describing localized gravitational waves are imposed, $\delta P^1 = \delta P^2 = 0$. Because $P^\mu$ is a purely spatial vector, $\delta P^0 = 0$, which implies that the integral on the right hand side of eq. (2.13) vanishes and then $\delta P^3 = \text{const.}$ as well.\footnote{Alternatively, one can rewrite eq. (2.13) as $dP^\mu/d\sigma = S^\mu \nabla^\nu A_\nu/(2\alpha)$ and note that the right hand side vanishes in the Lorentz gauge $\nabla^\mu A_\mu = 0$ and, therefore, in any gauge because this quantity is a scalar.}

3 Conclusions

The rotation angle is a boundary term effect which vanishes for intervening gravitational waves of astrophysical origin, which are localized between the source and the observer. Surpi & Harari (1999), instead, consider a cosmological gravitational wave, for which the boundary conditions consist of both $h_{\mu\nu}(S)$ and $h_{\mu\nu}(O)$ non-zero and $h_{\mu\nu}(S) \neq h_{\mu\nu}(O)$. They find the rotation of the polarization vector of electromagnetic radiation (Surpi & Harari 1999)

$$\delta \theta = \frac{1}{2} (1 + \mu) [h_\times (z_S, t_e) - h_\times (0, t_o)]$$

$$+ \frac{1}{4} (1 + \mu^2) \Delta h_+ \sin (2\varphi) + \frac{\mu}{2} \Delta h_\times \cos (2\varphi) , \quad (3.1)$$

where $\mu = \vec{k} \cdot \vec{k}_{gw} / k_{gw}$, $\vec{k}$ is the electromagnetic wave vector, and $\vec{k}_{gw}$ is the gravitational wave vector. $t_e$ and $z_S$ are the emission time and the source position, while $t_O$ is the time at which the light is observed at the location of the observer $z = 0$, and $h_+$ and $h_\times$ are the two independent polarizations of the gravitational wave in TT gauge. Unperturbed light propagates in the $z$-direction with wave vector $\omega S^{(0)} / \mu$ which has the projection $(\cos \varphi, \sin \varphi) \sqrt{1 - \mu}$ onto the $(x, y)$ plane. The effect described by eq. (3.1) is clearly an endpoint effect due entirely to the boundary conditions describing a cosmological gravitational wave and different from those corresponding to a localized wave employed in (Faraoni 1993).

While only the gauge-dependent four-potential $A_\mu$ is discussed here, one can conclude that because the gravitational wave induces no effect in $A^\mu$ to first order, to the same order there is no effect also in the (gauge-invariant) Maxwell field $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$ derived from it.

A careful analysis of gravitomagnetic effects in the propagation of electromagnetic waves through time-dependent gravitational fields, including gravitational waves, is per-
formed by Kopeikin & Mashhoon (2002) by explicitly expanding the gravitational field in multipoles. The rotation of the plane of polarization by quadrupolar gravitational waves is entirely given by endpoint terms (see eq. (138) of Kopeikin & Mashhoon 2002). Similarly, Prasanna & Mohanty (2002) consider the rotation of the plane of polarization of electromagnetic waves propagating parallel to a gravitational wave, in the specific case of pulses emitted by a binary pulsar. They find the dispersion relation

\[ k^2 = \omega^2 \left( 1 \pm \frac{G\mu_r d^2 \omega_{gw}^2}{3\omega_{gw}^2} e^{i\omega_{gw}(t-z)} \right), \]

(3.2)

where \( \mu_r \) is the reduced mass of the binary and \( d \) is a parameter describing the projected size of the orbit and related to the binary system semi-major axis (Prasanna & Mohanty 2002). This leads again to a rotation angle of the polarization vector consisting of boundary terms (eq. (19) of (Prasanna & Mohanty 2002). Moreover, the correction to the flat space dispersion relation \( k^2 = \omega^2 \) is of the order \( \left( \frac{\omega_{gw}}{\omega} \right)^2 h_{\mu\nu} \), and it becomes exceedingly small in the limit \( \lambda \ll \lambda_{gw} \). In the most optimistic case considered in (Prasanna & Mohanty 2002), the rotation angle of the polarization vector is of the order of \( 10^{-8} \) radians or less, leaving little hope for detection.

Another situation in which the gravitational wave is not localized, but the boundary conditions determine \( \delta P^\mu = 0 \) at the endpoints, occurs in laser interferometric detectors of gravitational waves. In (Cooperstock & Faraoni 1993), laser interferometers are studied by explicitly computing the perturbations of the Maxwell tensor by the gravitational wave. By contrast, in most of the literature on the subject, the phase shift between two different arms of the interferometer is computed by considering different travel times or different lengths travelled without explicitly considering the changes induced in the electromagnetic field, and in the approximation \( \lambda \ll \lambda_{gw} \). In an interferometer’s arm, an electromagnetic wave is not localized between a “source” and an “observer”, but it spans the entire length of the arm. In (Cooperstock & Faraoni 1993), no rotation of the polarization vector of electromagnetic radiation was found, to first order in the gravitational wave amplitude. This fact is again explained by the boundary conditions imposed at the endpoints: at these locations, reflection off perfect mirrors is assumed and the electromagnetic field describing a standing wave between the two mirrors, which are nodes, also vanishes. As a consequence, the endpoint effect vanishes too.

As a conclusion, the discrepancy between (Faraoni 1993) and (Surpi & Harari 1999, Kopeikin & Mashhoon 2002, and Prasanna & Mohanty 2002) is due to the different boundary conditions. The rotation angle always vanishes, to first order, for localized gravitational waves and is always reducible to an endpoint effect in (Surpi & Harari 1999, Kopeikin & Mashhoon 2002, Prasanna & Mohanty 2002). Unfortunately, the detection
of gravitational waves through the rotation of the plane of polarization of light from distant sources is not feasible with technology currently available or foreseeable in the near future. On the other hand, we should not worry about the plane of polarization of polarized radiation emitted from radio galaxies (Kronberg et al. 1991) being altered by gravitational waves along the line of sight, in the same way that gravitational Faraday rotation is negligible in weak lensing by ordinary gravitational lenses (Dyer & Shaver 1992, Faraoni 1993).

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