Partial Breaking of $\mathcal{N} = 2$ Supersymmetry and Decoupling Limit of Nambu-Goldstone Fermion in $U(N)$ Gauge Model

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Abstract

We study the $\mathcal{N} = 1$ $U(N)$ gauge model obtained by spontaneous breaking of $\mathcal{N} = 2$ supersymmetry. The Fayet-Iliopoulos term included in the $\mathcal{N} = 2$ action does not appear in the resulting $\mathcal{N} = 1$ action and the superpotential is modified to break discrete $R$ symmetry. We take a limit in which the Kähler metric becomes flat and the superpotential preserves non-trivial form. The Nambu-Goldstone fermion is decoupled from other fields but the resulting action is still $\mathcal{N} = 1$ supersymmetric. It shows the origin of the fermionic shift symmetry in $\mathcal{N} = 1$ $U(N)$ gauge theory.

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1 Introduction

It was conjectured in [1] that non-perturbative quantities in a low energy effective gauge theory can be computed by a matrix model. This conjecture was confirmed by [2] for the case of an $\mathcal{N} = 1$ $U(N)$ gauge theory with a chiral superfield $\Phi$ in the adjoint representation of $U(N)$. The $\mathcal{N} = 1$ action is obtained from “soft” breaking of $\mathcal{N} = 2$ supersymmetry by adding the tree-level superpotential

$$\int d^2\theta \text{Tr} W(\Phi).$$

(1.1)

The group $SU(N)$ is confined and there is a symmetry of shifting the $U(1)$ gaugino by an anticommuting c-number $W_\alpha \to W_\alpha - 4\pi \chi_\alpha$. It is called “fermionic shift symmetry”. Thanks to this symmetry, effective superpotential is written as

$$W_{\text{eff}} = \int d^2\chi F,$$

(1.2)

for some function $F$. The fermionic shift symmetry is due to a free fermion and should be related to a second, spontaneously broken supersymmetry.

Antoniadis-Partouche-Taylor (APT) constructed the $U(1)$ gauge model which breaks $\mathcal{N} = 2$ supersymmetry to $\mathcal{N} = 1$ spontaneously by electric and magnetic Fayet-Iliopoulos (FI) terms [3]. (See also [4].) The $U(N)$ generalization of the APT model was given in [5] which is described by $\mathcal{N} = 1$ chiral superfields and $\mathcal{N} = 1$ vector superfields. The Nambu-Goldstone fermion appears in the overall $U(1)$ part of $U(N)$ gauge group and couples with the $SU(N)$ sector because of the fact that the 3rd derivatives of the prepotential are non-vanishing [6]. A manifestly $\mathcal{N} = 2$ formulation of $U(N)$ gauge model [5] with/without $\mathcal{N} = 2$ hypermultiplets has been realized in [7]. It overcomes the difficulty in coupling hypermultiplets to the APT model. Partial breaking of local $\mathcal{N} = 2$ supersymmetry was discussed in a lot of papers [8, 9].

This paper is organized as follows. In section 2, We derive the resulting $\mathcal{N} = 1$ $U(N)$ action from the $\mathcal{N} = 2$ $U(N)$ gauge model [5]. In section 3, we take a limit in which the Kähler metric becomes flat, while the superpotential preserves its non-trivial form. After taking this limit the Nambu-Goldstone fermion is decoupled from other fields, but partial breaking of $\mathcal{N} = 2$ supersymmetry is realized as before. We get a general $\mathcal{N} = 1$ action discussed in [11, 2]. It shows that the fermionic shift symmetry is due to the decoupling limit of the Nambu-Goldstone fermion. In the appendix, we derive the resulting $\mathcal{N} = 1$ supercharge algebra.

\footnote{We follow the notation of [10]}
2 Spontaneous partial breaking of $\mathcal{N} = 2$ supersymmetry and resulting $\mathcal{N} = 1$ action

The on-shell action of the $\mathcal{N} = 2$ $U(N)$ gauge model \[5\] takes the following form:

$$\mathcal{L}_{\text{on-shell}}^{\mathcal{N} = 2} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{Pauli}} + \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{fermi}}^4,$$  
(2.1)

with

$$\mathcal{L}_{\text{kin}} = -g_{ab} D_m A^a D^m A^b - \frac{1}{4} g_{ab} v^a_{mn} v^{bmn} - \frac{1}{8} \text{Re}(\mathcal{F}_{ab}) \varepsilon^{mnpq} v^a_{mn} v^b_{pq}$$  
(2.2)

$$\mathcal{L}_{\text{pot}} = -\frac{1}{2} g_{ab} \left( \frac{1}{2} D_a + \sqrt{2} \xi \delta^a \right) \left( \frac{1}{2} D_b + \sqrt{2} \xi \delta^b \right) - g^{ab} \partial_a \bar{W} \partial_b W^*,$$  
(2.3)

$$\mathcal{L}_{\text{Pauli}} = i \frac{\sqrt{2}}{8} F_{abc} \psi^c \sigma^m \bar{a}^a \lambda^b v^b_{mn} + i \frac{\sqrt{2}}{8} F_{abc} \bar{a}^a \sigma^m \bar{a}^a \lambda^b v^b_{mn},$$  
(2.4)

$$\mathcal{L}_{\text{mass}} = \left( -\frac{i}{4} F_{abc} g^{cd} \partial_d W - \frac{1}{2} \partial_a \partial_b W \right) \psi^a \lambda^b + i \frac{1}{4} F_{abc} g^{cd} \partial_d W^* \lambda^b \lambda^c + c.c.,$$  
(2.5)

$$\mathcal{L}_{\text{fermi}}^4 = -\frac{i}{8} F_{abcd} \psi^c \psi^d \lambda^b \lambda^b + i \frac{1}{8} F_{*ab} \psi^c \psi^d \lambda^b \lambda^b + g a \bar{a} \bar{F}^{ab} + \frac{1}{2} g_{ab} \bar{F}^a \bar{F}^b$$

$$+ \frac{i}{4} F_{abc} \bar{F}^{dc} \psi^a \psi^b + \frac{i}{4} F_{abc} \bar{F}^{dc} \lambda^a \lambda^b + \frac{1}{2} F_{abc} \bar{F}^{dc} \lambda^a \lambda^b$$

$$- \frac{i}{4} F_{abc} \bar{F}^{dc} \psi^a \psi^b - \frac{i}{4} F_{abc} \bar{F}^{dc} \lambda^a \lambda^b + \frac{1}{2} F_{abc} \bar{F}^{dc} \psi^a \psi^b.$$

(2.6)

where $\bar{F}^a \equiv -\frac{\sqrt{2}}{4} g^{ab} \left( \mathcal{F}_{bcd} \psi^d \lambda^c + \mathcal{F}_{*bd} \bar{\psi}^d \bar{\lambda}^c \right)$, $\bar{F}^a \equiv \frac{i}{4} g^{ab} \left( \mathcal{F}_{bcd} \bar{\lambda}^c \bar{\lambda}^d - \mathcal{F}_{*bd} \bar{\psi}^c \bar{\psi}^d \right)$ and $W = eA^0 + m \mathcal{F}_0$. Let us examine the case with $\mathcal{F} = \sum_{k=0}^{n} \text{tr} \frac{g_k}{k!} \Phi^k$. The vacuum condition

$$\frac{\partial \mathcal{L}_{\text{pot}}}{\partial A^a} = 0$$

reduces to

$$\langle \mathcal{F}_{0a} \rangle = \frac{-e \pm i \xi}{m},$$

(2.7)

where $\langle \cdots \rangle$ denotes ... evaluated at $A^r = 0$ (indices $r$ represent non-Cartan generators).

For the sake of simplicity, we choose $+$ sign in (2.7) and this means $\xi > 0$. It is revealed in [6] that the Nambu-Goldstone fermion exists in the overall $U(1)$ part of $U(N)$ gauge group,

$$\langle \bar{\delta}_{\mathcal{N}=2} \left( \frac{\lambda^0 - \psi^0}{\sqrt{2}} \right) \rangle = -2im(\eta_1 + \eta_2),$$

$$\langle \bar{\delta}_{\mathcal{N}=2} \left( \frac{\lambda^0 + \psi^0}{\sqrt{2}} \right) \rangle = 0.$$  
(2.8)

3
We use $\langle \ldots \rangle$ for vacuum expectation values which satisfy (2.7). $\frac{\lambda_0 - \psi^a}{\sqrt{2}}$ is the Nambu-Goldstone fermion and it will be included in the overall $U(1)$ part of the resulting $\mathcal{N} = 1$ $U(N)$ vector superfield. The vacuum expectation value of the scalar potential $\mathcal{V} \equiv -\mathcal{L}_{\text{pot}}$ is $\langle \mathcal{V} \rangle = 2m\xi$. As is pointed out in [5], the second term in the RHS of the local version of $\mathcal{N} = 2$ supersymmetry algebra enables us to add a constant $2m\xi$ to the action (2.1) in order to set $\langle \mathcal{V} \rangle = 0$. In the formalism of harmonic superspace, this freedom to add a constant number comes from arbitrariness to choose the imaginary part of the magnetic FI term in [7].

To obtain the resulting $\mathcal{N} = 1$ action for the case that $U(N)$ gauge symmetry is not broken at vacua, we shift the scalar fields $A^a$ by its vacuum expectation value and mix the spinor fields $\psi^a$ and $\lambda^a$. We define

$$\tilde{A}^a \equiv A^a - \langle \langle A^0 \rangle \rangle \delta_0^a, \quad \lambda^{-a} \equiv \frac{1}{\sqrt{2}} (\lambda^a - \psi^a), \quad \lambda^{+a} \equiv \frac{1}{\sqrt{2}} (\lambda^a + \psi^a). \quad (2.9)$$

Substitute these into (2.1), we get the resulting $\mathcal{N} = 1 U(N)$ gauge action after spontaneous breaking of $\mathcal{N} = 2$ supersymmetry,

$$\mathcal{L}_{\mathcal{N} = 1, \text{on-shell}} = \tilde{\mathcal{L}}_{\text{kin}} + \tilde{\mathcal{L}}_{\text{pot}} + \tilde{\mathcal{L}}_{\text{Pauli}} + \tilde{\mathcal{L}}_{\text{mass}} + \tilde{\mathcal{L}}_{\text{fermi}}^4, \quad (2.10)$$

with

$$\tilde{\mathcal{L}}_{\text{kin}} = -\frac{1}{4} g_{ab} \delta_{mn} A^a D^m A^b - \frac{1}{2} g_{ab} b^a_{mn} v^{bn} - \frac{1}{8} \text{Re}(\tilde{F}_{ab}) \epsilon^{mpq} v_{mn} v_{pq} \quad (2.11)$$

$$- \frac{1}{2} \tilde{F}_{ab} \lambda^{-a} \sigma^m D_m \lambda^{-b} - \frac{1}{2} \tilde{F}_{ab} \lambda^{-a} \sigma^m \lambda^{-b} - \frac{1}{2} \tilde{F}_{ab} \lambda^{+a} \sigma^m \lambda^{+b} - \frac{1}{2} \tilde{F}_{ab} D_m \lambda^{+a} \sigma^m \lambda^{+b},$$

$$\tilde{\mathcal{L}}_{\text{pot}} = -\frac{1}{8} g^{ab} \delta_d \tilde{D}_d - g^{ab} \tilde{D}_b \tilde{W} \tilde{\partial}_d \tilde{W}^*, \quad (2.12)$$

$$\tilde{\mathcal{L}}_{\text{Pauli}} = \frac{i}{\sqrt{2}} \tilde{F}_{abc} \lambda^{+c} \sigma^m \lambda^{-a} \sigma^b_{mn} + i \frac{\sqrt{2}}{8} \tilde{F}_{abc} \lambda^{-a} \sigma^m \lambda^{+c} \sigma^b_{mn} \quad (2.13)$$

$$\tilde{\mathcal{L}}_{\text{mass}} = \left( -\frac{i}{4} \tilde{F}_{abc} g^{cd} \tilde{\partial}_d \tilde{W} - \frac{1}{2} \tilde{\partial}_a \tilde{\partial}_b \tilde{W} \right) \lambda^{+a} \lambda^{-b} - \frac{i}{4} \tilde{F}_{abc} g^{cd} \tilde{\partial}_d \tilde{W}^* \lambda^{+a} \lambda^{-b}$$

$$+ \left\{ \frac{1}{4} \tilde{F}_{abc} g^{cd} \tilde{\partial}_d + \frac{1}{\sqrt{2}} g_{abc} \tilde{F}_{d} \right\} \lambda^{+a} \lambda^{-b} + c.c., \quad (2.14)$$

$$\tilde{\mathcal{L}}_{\text{fermi}}^4 = -\frac{i}{8} \tilde{F}_{abcd} \lambda^{+c} \lambda^{+d} \lambda^{-a} \lambda^{-b} + \frac{i}{8} \tilde{F}_{abc} \lambda^{+c} \lambda^{+d} \lambda^{-a} \lambda^{-b} + \tilde{g}_{ab} \tilde{F}_{c} + \frac{1}{4} \tilde{g}_{ab} \tilde{D}^a \tilde{D}^b$$

$$+ \frac{i}{4} \tilde{F}_{abc} \tilde{F}_{d} \lambda^{+a} \lambda^{+b} + \frac{i}{4} \tilde{F}_{abc} \tilde{F}_{d} \lambda^{-a} \lambda^{-b} + \frac{1}{2} \tilde{F}_{abc} \tilde{D}^c \lambda^{+a} \lambda^{-b}$$

$$- \frac{i}{4} \tilde{F}_{abc} \tilde{F}_{d} \lambda^{-a} \lambda^{-b} + \frac{1}{2} \tilde{F}_{abc} \tilde{D}^c \lambda^{+a} \lambda^{+b} \quad (2.15)$$

2 In [3], such freedom comes from the electric FI term.
where

\[ \tilde{\mathcal{F}}(\tilde{A}) \equiv \langle \mathcal{F} \rangle + \langle \mathcal{F}_a \rangle \tilde{A}^a + \langle \mathcal{F}_{ab} \rangle \frac{\tilde{A}^a \tilde{A}^b}{2!} + \langle \mathcal{F}_{abc} \rangle \frac{\tilde{A}^a \tilde{A}^b \tilde{A}^c}{3!} + \cdots, \]  
\[ \tilde{\mathcal{F}}_a \equiv \frac{\partial^2 \tilde{\mathcal{F}}(\tilde{A})}{\partial \tilde{A}^a}, \]  
\[ \tilde{F}_{ab} \equiv \frac{\partial \tilde{\mathcal{F}}_a}{\partial \tilde{A}^b}, \]  
\[ \tilde{\mathcal{D}}_a \equiv -i \tilde{g}_{ab} \tilde{f}^b_{cd} \tilde{A}^c \tilde{A}^d, \]  
\[ \tilde{k}_a \equiv -i \tilde{g}_{bc} \frac{\partial}{\partial \tilde{A}^c} \tilde{\mathcal{D}}_a. \]

\[ \tilde{F}^a \equiv \frac{i}{4} \tilde{g}^{ab} \tilde{F}_{bc} \tilde{\lambda}^{e} \tilde{\lambda}^{-d} - \frac{i}{4} \tilde{g}^{ab} \tilde{F}_{bc} \tilde{\lambda}^{-e} \tilde{\lambda}^{d}, \]  
\[ \tilde{\mathcal{D}}^a \equiv -\frac{\sqrt{2}}{4} \tilde{g}^{ab} \tilde{F}_{bc} \tilde{\lambda}^{e} \tilde{\lambda}^{-d} - \frac{\sqrt{2}}{4} \tilde{g}^{ab} \tilde{F}_{bc} \tilde{\lambda}^{d} \tilde{\lambda}^{-e}. \]

Here we have used

\begin{align*}
&i \partial_a \mathcal{D}_b + i \partial_b \mathcal{D}_a - \frac{1}{2} g^{cd} \mathcal{F}_{abc} \mathcal{D}_d = 0, \quad g^{ab} \mathcal{D}_a \delta^0_b = 0, \quad (2.16) \\
&\mathcal{F}_{abc} \lambda^{a} \sigma^m \tilde{\sigma}^m \lambda^{b} \tilde{\sigma}^c v_{mn} = 0, \quad \mathcal{F}_{abcd} \lambda^{a} \lambda^{b} \lambda^{c} \lambda^{d} = 0. \quad (2.17)
\end{align*}

Take notice that we have added the constant \(2m\xi\) to \(\mathcal{L}_{\text{pot}}\) as mentioned above.

As a result, the action (2.10) agrees with the action (2.1) except for the superpotential term and FI term. There is no FI term in (2.10), and the superpotential \(W = eA^0 + m\tilde{F}_0\) get shifted to \(\tilde{W} = (e - i\xi)\tilde{A}^0 + m\tilde{F}_0\) (we neglected a constant term). Because the coefficient \((e - i\xi)\) in \(\tilde{W}\) is a complex number, (2.10) is not invariant under the discrete \(R\) transformation.

We can write the off-shell \(\mathcal{N} = 1\) action by introducing auxiliary fields \(\tilde{F}\) and \(\tilde{D}\),

\begin{align*}
\mathcal{L}_{\text{off-shell}}^{\mathcal{N} = 1} &= \\
&\tilde{g}_{ab} \mathcal{D}_a \tilde{A}^a \mathcal{D}_b \tilde{A}^b - \frac{1}{4} \tilde{g}_{ab} \tilde{t}^a_{mn} \tilde{v}^{bmn} - \frac{1}{8} \text{Re}(\tilde{\mathcal{F}}_{ab}) \tilde{e}^{mnpq} \tilde{t}^c_{mn} \tilde{v}^{pq} \\
&- \frac{1}{2} \tilde{\mathcal{F}}_{ab} \mathcal{D}_a \mathcal{D}_b \lambda^a \tilde{\lambda}^{-b} - \frac{1}{2} \tilde{\mathcal{F}}_{ab} \mathcal{D}_a \mathcal{D}_b \lambda^{-a} \tilde{\lambda}^b - \frac{1}{2} \tilde{\mathcal{F}}_{ab} \mathcal{D}_a \mathcal{D}_b \lambda^a \tilde{\lambda}^b - \frac{1}{2} \tilde{\mathcal{F}}_{ab} \mathcal{D}_a \mathcal{D}_b \lambda^{-a} \tilde{\lambda}^{-b} \\
&+ \tilde{g}_{ab} \tilde{F}^{a \tilde{F}^{b \dagger}} + \tilde{F}^{a \dagger} \tilde{\partial}_a \tilde{W} + \tilde{F}^{* \dagger} \tilde{\partial}_a \tilde{W}^{*} + \frac{1}{2} \tilde{g}_{ab} \tilde{D}^{a \dagger} \tilde{D}^{b \dagger} + \frac{1}{2} \tilde{D}^{a \dagger} \tilde{\mathcal{D}}^{a} \\
&+ \left( \frac{i}{4} \tilde{F}_{abc} \tilde{F}^{* \dagger} \right) \tilde{\partial}_a \tilde{\partial}_b \tilde{\partial}_c \tilde{\partial}_d \tilde{\lambda}^a \tilde{\lambda}^b + \left( \frac{i}{4} \tilde{F}_{abc} \tilde{F}^{* \dagger} \right) \tilde{\partial}_a \tilde{\partial}_b \tilde{\partial}_c \tilde{\partial}_d \tilde{\lambda}^{-a} \tilde{\lambda}^{-b} + \frac{1}{\sqrt{2}} \left( \tilde{g}_{ac} \tilde{h}^{\dagger} + \frac{1}{2} \tilde{F}_{abc} \tilde{D}^{c} \right) \tilde{\partial}_a \tilde{\partial}_b \tilde{\partial}_c \tilde{\partial}_d \tilde{\lambda}^a \tilde{\lambda}^{-b} \\
&+ \left( \frac{i}{4} \tilde{F}_{abc} \tilde{F}^{* \dagger} \right) \tilde{\partial}_a \tilde{\partial}_b \tilde{\partial}_c \tilde{\partial}_d \tilde{\lambda}^a \tilde{\lambda}^{-b} + \frac{1}{\sqrt{2}} \left( \tilde{g}_{ac} \tilde{h}^{\dagger} + \frac{1}{2} \tilde{F}_{abc} \tilde{D}^{c} \right) \tilde{\partial}_a \tilde{\partial}_b \tilde{\partial}_c \tilde{\partial}_d \tilde{\lambda}^{-a} \tilde{\lambda}^b \\
&- \frac{i}{8} \sqrt{2} \left( \tilde{F}_{abc} \lambda^{a} \sigma^m \tilde{\sigma}^m \lambda^{-a} \tilde{\sigma}^c \tilde{v}^{bmn} \right) \\
&- \frac{i}{8} \tilde{F}_{abcd} \lambda^a \lambda^b \lambda^c \lambda^{-d} \tilde{\lambda}^a \tilde{\lambda}^{-b}. \quad (2.18)
\end{align*}

\(^3R:\begin{pmatrix} \lambda^{-a} \\ \lambda^a \end{pmatrix} \rightarrow \begin{pmatrix} \lambda^a \\ -\lambda^{-a} \end{pmatrix}\)
Component fields \((\tilde{A}^a, \lambda^+ a, \tilde{F}^a)\) form massive \(\mathcal{N} = 1\) chiral multiplets \(\tilde{\Phi}^a\). Other component fields \((v^a_m, \lambda^- a, \tilde{D}^a)\) form massless \(\mathcal{N} = 1\) vector multiplets \(\tilde{V}^a\). The Nambu-Goldstone fermion \(\lambda^- 0\) is contained in the overall \(U(1)\) part of \(\tilde{V}^a\).

## 3 Reparametrization and scaling limit

We consider a limit in which the Nambu-Goldstone fermion \(\lambda^- 0\) is decoupled from other fields with \(\mathcal{N} = 2\) supersymmetry breaking to \(\mathcal{N} = 1\). If the prepotential \(\mathcal{F}\) is a second order polynomial, there are no Yukawa couplings in (2.18) and \(\lambda^- 0\) will be a free fermion. However, derivatives of the superpotential become zero, \(\tilde{\partial}_a \tilde{\partial}_b \tilde{W} = m \tilde{F}_{0ab} = 0\) and \(\tilde{\partial}_a \tilde{W} = (e - i \xi) \delta^0_a + m \tilde{F}_{0a} = (e - i \xi) \delta^0_a + m \langle \mathcal{F}_{0a} \rangle = 0\). This means that the superpotential does not contribute to (2.18) and it preserves \(\mathcal{N} = 2\) supersymmetry. This problem can be solved by a large limit of the parameters \((e, m, \xi)\), i.e. large limit of electric and magnetic FI terms.

We reparametrize \(g_k = \frac{g_k}{\Lambda}(k \geq 3)\) and \((e, m, \xi) = (\Lambda e', \Lambda m', \Lambda \xi')\). The prepotential \(\mathcal{F}\) is

\[
\mathcal{F} = \sum_{k=0}^{n} \text{tr} \frac{g_k}{k!} \Phi^k = \text{tr} \left( g_0 \mathbf{1} + g_1 \Phi + \frac{g_2}{2} \Phi^2 \right) + \frac{1}{\Lambda} \sum_{k=3}^{n} \text{tr} \frac{g'_k}{k!} \Phi^k ,
\]

(3.1)

and we see the \(\Lambda\) dependence of the following terms:

\[
\tilde{F}_{ab} = \langle \mathcal{F}'_{ab} \rangle + \frac{1}{\Lambda} \left\{ \langle \mathcal{F}'_{abc} \rangle \tilde{A}^c + \langle \mathcal{F}'_{abcd} \rangle \tilde{A}^c \tilde{A}^d + \cdots \right\} = -\frac{e + i \xi}{m} \delta_{ab} + \mathcal{O}(\Lambda^{-1}),
\]

\[
\tilde{F}_{abc} = \mathcal{O}(\Lambda^{-1}), \quad \tilde{F}_{abcd} = \mathcal{O}(\Lambda^{-1}), \quad \tilde{g}_{ab} = \frac{\xi}{m} \delta_{ab} + \mathcal{O}(\Lambda^{-1}),
\]

\[
\tilde{D}_a = -i \tilde{g}_{ab} f^b_{cd} \tilde{A}^c \tilde{A}^d = -i \frac{\xi}{m} \delta_{ab} f^b_{cd} \tilde{A}^c \tilde{A}^d + \mathcal{O}(\Lambda^{-1}) .
\]

(3.2)

where \(\mathcal{F}' = \text{tr} \left( g_0 \mathbf{1} + g_1 \Phi + \frac{g_2}{2} \Phi^2 \right) + \sum_{k=3}^{n} \text{tr} \frac{g'_k}{k!} \Phi^k\). Note that the scaling parameter \(\Lambda\) is cancelled out in the superpotential term:

\[
\tilde{\partial}_a \tilde{W} = (e - i \xi) \delta^0_a + m \tilde{F}_{0a} = m' \left\{ \langle \mathcal{F}'_{0ab} \rangle \tilde{A}^b + \frac{1}{2!} \langle \mathcal{F}'_{0abc} \rangle \tilde{A}^b \tilde{A}^c + \cdots \right\} ,
\]

(3.3)

\[
\tilde{\partial}_a \tilde{\partial}_b \tilde{W} = m' \left\{ \langle \mathcal{F}'_{0ab} \rangle + \langle \mathcal{F}'_{0abc} \tilde{A}^c \rangle + \frac{1}{2!} \langle \mathcal{F}'_{0abcd} \rangle \tilde{A}^c \tilde{A}^d + \cdots \right\} .
\]

(3.4)

Take a limit \(\Lambda \to \infty\), and the action (2.18) is converted into

\[
\mathcal{L} = \frac{\xi}{m} \delta_{ab} \left\{ -D_m \tilde{A}^a D^m \tilde{A}^{* b} - i \lambda^+ a \sigma^m D_m \tilde{A}^{* b} \right\} .
\]

6
The matrix form of the superpotential 
\[ \tilde{\mathcal{W}} \] which is known as a “soft” broken
where we define
\[ \tilde{\mathcal{W}} \]
We normalize the standard
\[ \tilde{\mathcal{W}} \]
We can rewrite the action (3.5) in superfield formalism as
\[ \tilde{\mathcal{W}} \equiv m' \left\{ \frac{1}{2!} \langle \mathcal{F}_{ab} \rangle \tilde{A}^a \tilde{A}^b + \frac{1}{3!} \langle \mathcal{F}_{abc} \rangle \tilde{A}^a \tilde{A}^b \tilde{A}^c + \cdots \right\} 
= m \left\{ \frac{1}{2!} \langle \mathcal{F}_{ab} \rangle \tilde{A}^a \tilde{A}^b + \frac{1}{3!} \langle \mathcal{F}_{abc} \rangle \tilde{A}^a \tilde{A}^b \tilde{A}^c + \cdots \right\} 
= m \sum_{k=1}^{n} \frac{g_k}{\sqrt{2N}} \left( \frac{\langle A^0 \rangle}{\sqrt{2N}} \right)^{k-1} \right\} 
= m \sum_{k=1}^{n} \frac{h_k}{k+1} \left( \frac{\langle A^0 \rangle}{\sqrt{2N}} \right)^{k-1} \right\} 
\text{where we define } h_k \equiv \frac{(k+1)}{\sqrt{2N}} \sum_{\ell=0}^{k-2} \frac{g_{k+\ell+2}}{(k+\ell+1)!} (k+\ell+1) C_{\ell} \left( \frac{\langle A^0 \rangle}{\sqrt{2N}} \right)^{\ell} . \right\} 
\text{Here the symbol } (k+\ell+1) C_{\ell} \text{ is a binomial coefficient.} \right\} 
\text{We can rewrite the action (3.5) in superfield formalism as}
\[ \mathcal{L} = \text{Im} \left[ \frac{-e + i \xi}{m} \left( 2 \int d^4 \theta \text{tr} \tilde{\Phi}^+ e \tilde{\Phi} + \int d^2 \theta \text{tr} \tilde{\mathcal{W}}^0 \tilde{\mathcal{W}}_0 \right) \right] + \left( \int d^2 \theta \tilde{\mathcal{W}} \left( \tilde{\Phi} \right) + \text{c.c.} \right), \] 
where \( \tilde{\mathcal{W}} \) is the field strength of \( \tilde{\mathcal{V}} \). The factor 2 in the first term comes from the normalization of the standard \( u(N) \) Cartan generators. Note that the Nambu-Goldstone fermion \( \lambda^0 \), which is contained in the overall \( U(1) \) part of \( \mathcal{N} = 1 \) \( U(N) \) vector superfields \( \tilde{V} \), is decoupled from other fields in (3.7). However \( \mathcal{N} = 2 \) supersymmetry is broken to \( \mathcal{N} = 1 \) because of existence of the superpotential. We get a general \( \mathcal{N} = 1 \) action (3.7), which is known as a “soft” broken \( \mathcal{N} = 1 \) action, from spontaneously broken \( \mathcal{N} = 2 \) supersymmetry. We conclude that the fermionic shift symmetry in [2] is related to the decoupling limit of the Nambu-Goldstone fermion.

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\footnote{4 We normalize the standard \( u(N) \) Cartan generators \( t_i \) as \( \text{tr}(t_i t_j) = \frac{1}{2} \delta_{ij} \), which implies that the overall \( u(1) \) generator is \( t_0 = \frac{1}{\sqrt{2N}} 1_{N \times N} \).}
A Supercharge algebra

The $\mathcal{N} = 2$ transformation rule are given by a combination of following transformation rules:

$$
\begin{align*}
\delta_m A^a &= \sqrt{2} \eta_1 \psi^a \\
\delta_m \psi^a &= i \sqrt{2} \sigma^m \eta_1 D_m A^a + \sqrt{2} \eta_1 (\hat{F}^a - g^{ab} \partial_b W^*) \\
\delta_m \lambda^a &= \frac{i}{2} g^{ab} \sigma^m \eta_1 \gamma^m \eta^a + i \eta_1 (\hat{D}^a - \frac{1}{2} g^{ab} (D_b + 2 \sqrt{2} \xi \delta^0)) \\
\delta_m v^a_m &= i \eta_1 \sigma^m \lambda^a - i \lambda^a \sigma^m \eta_1 \\
\delta_m \epsilon^a &= -\sqrt{2} \eta_2 \lambda^a \\
\delta_m \psi^a &= \frac{i}{2} g^{ab} \sigma^m \eta_2 v^a_m - i \eta_2 (\hat{D}^a + \frac{1}{2} g^{ab} (D_b - 2 \sqrt{2} \xi \delta^0)) \\
\delta_m \lambda^a &= -i \sqrt{2} \sigma^m \eta_2 D_m A^a - \sqrt{2} \eta_2 (\hat{F}^a - g^{ab} \partial_b W^*) \\
\delta_m v^a_m &= i \eta_2 \sigma^m \psi^a - i \psi^a \sigma^m \eta_2
\end{align*}
$$

where spinors $\eta_k (k = 1, 2)$ are transformation parameters. The $\mathcal{N} = 2$ supersymmetric transformation rules are $\delta_{\mathcal{N}=2} \lambda^a = \delta_m \lambda^a + \delta_{\eta} \lambda^a$. We can find the 1st supercurrent $S_{1a}^m$ from the action $\mathcal{L}$:

$$
S_{1}^m = -i g_{ab} \sigma^m \psi^a \bar{\lambda}^b v^a_m - \frac{1}{2} \sigma^m \lambda^a D_a + i \sqrt{2} (e \delta^0_e + m F^a_{\psi}) \sigma^m \bar{\psi}^c - \sqrt{2} \xi \sigma^m \lambda^0 - \sqrt{2} g_{ab} \sigma^m \bar{\psi}^a D_n A^{*b} + \cdots, \quad (A.1)
$$

where the dots denote terms involving three fermions. The 2nd supercurrent $S_{2a}^m$ is given by the discrete $R$ transformation of $S_{1a}^m$ with a flip of the sign of the FI parameter $\xi$,

$$
S_{2}^m = -i g_{ab} \sigma^m \psi^a \bar{\lambda}^b v^a_m - \frac{1}{2} \sigma^m \bar{\psi}^a D_a - i \sqrt{2} (e \delta^0_e + m F^a_{\psi}) \sigma^m \bar{\lambda}^c + \sqrt{2} \xi \sigma^m \bar{\psi}^0 + \sqrt{2} g_{ab} \sigma^m \bar{\lambda}^a D_n A^{*b} + \cdots. \quad (A.2)
$$

Supercharge algebra is derived by

$$
\begin{align*}
\delta_{\eta A} S_{Ba}^0 &= i \left[ \eta_A Q_A + \bar{\eta}_A \bar{Q}_A, \ S_{Ba}^0 \right] = i \eta_{A \beta} \left\{ Q_{A \beta}, \ S_{Ba}^0 \right\} + i \bar{\eta}_{A \beta} \left\{ \bar{Q}_{A \beta}, \ S_{Ba}^0 \right\}, \quad (A.3)
\end{align*}
$$

where $A, B = 1$ or 2. It may be irrelevant to denote supercharges as $Q_1, Q_2$ because $\mathcal{N} = 2$ supersymmetry is broken to $\mathcal{N} = 1$ spontaneously and the supercharge corresponding to the broken supersymmetry is ill-defined. We ignore this point here and write the divergent part explicitly. We obtain the central charge

$$
\begin{align*}
\{ Q_{1 \alpha}, \ Q_{2 \beta} \} &= \sqrt{2} i \epsilon_{\beta \alpha} \int dx^3 \partial_i \left\{ (A^{*b} Re F_{ab} - 2 i \partial_4 K) e^{0ijk} v^a_{jk} + 2 g_{ab} A^{*b} v^{a0i} \right\} \\
&+ 8 \xi \int dx^3 \partial_i \left\{ A^{*a} (\sigma^0 \epsilon)_{\beta \alpha} \right\}.
\end{align*}
$$

It is easy to give proof that $\delta_{\eta_2} \mathcal{L} = 0$ (up to total derivative) with the use of $\delta_{\eta} \mathcal{L} = 0$ and $R \mathcal{L} = \mathcal{L} |_{\xi \rightarrow - \xi}$. (See [5].) As in [11], the FI term does not break $\mathcal{N} = 2$ supersymmetry.
Here \( K = \frac{i}{2}(A^a F^*_a - A^a a^* F_a) \) is the Kähler potential. To get the resulting \( N = 1 \) supercharge algebra, we define \( Q^- \equiv \frac{1}{\sqrt{2}}(Q_1 - Q_2) \) and \( Q^+ \equiv \frac{1}{\sqrt{2}}(Q_1 + Q_2) \). Anti-commutators of \( Q^- (Q^+ \) and \( \bar{Q}^- \) (\( \bar{Q}^+ \)) are given as

\[
\{ Q^-_\alpha, \bar{Q}^-_\dot{\beta} \} = -i \int d^3 x \left[ \frac{i}{4} g^{ab} (g_{ac} v_{np} \sigma^n \sigma^p + i \mathcal{D}_a) \sigma^0 (g_{bd} v_{qr} \bar{\sigma}^q \sigma^r + i \mathcal{D}_b) - 2 i g^{ab} \partial_a W \partial_b \bar{W} \cdot \sigma^0 + \cdots \right]_{\alpha\dot{\beta}},
\]

\[
\{ Q^+_\alpha, \bar{Q}^+_\dot{\beta} \} = -i \int d^3 x \left[ \frac{i}{4} g^{ab} (g_{ac} v_{np} \sigma^n \sigma^p + i \mathcal{D}_a) \sigma^0 (g_{bd} v_{qr} \bar{\sigma}^q \sigma^r + i \mathcal{D}_b) - 2 i g^{ab} \partial_a \bar{W} \partial_b W \cdot \sigma^0 + \cdots \right]_{\alpha\dot{\beta}}
\]

\[
-8 m \xi \sigma^0_{\alpha\dot{\beta}} \int d^3 x,
\]

(A.5)

where the dots indicate terms involving fermion fields. This result agree with the supersymmetry algebra in [12]. Finally, we conclude that \( Q^- \) is the unbroken generator and \( Q^+ \) is the broken one.

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