An essentially "complete" description of the low-energy nuclear structure of the superfluid nucleus $^{120}$Sn and of its odd-$A$ neighbors is provided by the observations carried out with the help of Coulomb excitation and of one– and of two– particle
transfer reactions, specific probes of vibrations, quasiparticle and pairing degrees of freedom respectively, and of their mutual couplings. These experimental findings are used to stringently test the predictions of a similarly "complete" description of $^{119,120,121}$Sn carried out in terms of elementary modes of excitation which, through their interweaving, melt together into effective fields, each displaying properties reflecting that of all others, there individuality resulting from the actual relative importance of each one. Its implementation is done by solving the Nambu-Gor’kov equations including, for the first time, all medium polarization effects resulting from the interweaving of quasiparticles, spin and surface vibrations, taking into account, within the framework of nuclear field theory (NFT), the variety of processes leading to self-energy, vertex and Pauli principle corrections, and to the induced pairing interaction. Theory provides an overall quantitative account of the experimental findings. From these results one can, not only obtain strong circumstantial evidence for the inevitability for the dual origin of pairing in nuclei but also, extract information which can be used at profit to quantitatively disentangle the contributions to pairing correlations in general and to the pairing gap in particular, arising from the bare and from the induced pairing interactions.
Single-particle motion emerges from the same properties which characterize collective motion \cite{1}, their interweaving leading to observables like quasiparticle states and collective vibrations, and to renormalized interactions, in particular the dressed pairing interaction, sum of the bare $v_p^{\text{bare}}$ interaction and of the attraction resulting from the exchange of vibrational modes between nucleons moving in time-reversal states lying close to the Fermi energy. In this paper we calculate the pairing and the low–energy properties of $^{120}$Sn and of its neighboring odd-$A$ nuclei (see also \cite{2–5} and refs. therein). To this scope we diagonalize $v_p^{\text{bare}}$ in the BCS approximation making use of a basis composed of nucleons moving in the Hartree-Fock field resulting from the SLy4 interaction ($\varepsilon_j, m_k$). We have fixed the strength of $v_p^{\text{bare}}$ by adopting the pairing gap calculated by Lesinski et al. \cite{6} (see also \cite{7}) fitting their results with a standard monopole-monopole pairing force of constant matrix elements $G = 0.22$ MeV. We have calculated the vibrations of the system diagonalizing, in QRPA (two-quasiparticle basis) separable multipole-multipole interactions (natural parity density modes with $\lambda^\pi = 2^+, 3^-, 4^+, 5^-$) and the spin-dependent part of the SLy4 force (spin modes of both natural and unnatural parity, $\lambda^\pi = 2^\pm, 3^\mp, 4^\pm, 5^\mp$). Taking into account self-energy and renormalization processes according to the rules of NFT \cite{9–12} (Fig. 1), the dressed particle states ($\tilde{\varepsilon}_j, m_\omega, Z_\omega$) (cf. \cite{13}) and the induced pairing interaction $v_p^{\text{ind}}$ were calculated. Adding $v_p^{\text{ind}}$ to $v_p^{\text{bare}}$, the total effective pairing interaction $v_p^{\text{eff}}$ was determined. With these elements, the Nambu-Gor’kov (NG) equation \cite{8, 14–16} was solved selfconsistently using Green functions techniques, and the parameters characterizing the superfluid state determined. Within this framework, the quasiparticle energies $\tilde{E}_\nu$

\[1\] In this way one neglects the state dependence of the $NN+3N$ matrix elements which is of little consequence within the framework of the present paper (see e.g. \cite{8})
and the state-dependent pairing gap $\tilde{\Delta}_\nu$ are related by the generalized gap equation

$$\tilde{\Delta}_\nu = \Delta_\nu^{\text{bare}} + \Delta_\nu^{\text{ind}} = -Z_\nu \sum_{\nu'} \langle \nu' \bar{p} | v_p^{\text{bare}} + v_p^{\text{ind}} | \nu \bar{p} \rangle N_{\nu'} \tilde{\Delta}_{\nu'},$$

(1)

where $N_\nu = u_\nu^2 + v_\nu^2$ and $Z_\nu = \left(1 - \frac{\Sigma_\nu^{\text{odd}}}{E_\nu}\right)^{-1}$ ($\Sigma_\nu^{\text{odd}}$ being the odd part of the normal self-energy). Both the pairing gap and the lowest quasiparticle energies are displayed in Fig. 2 in comparison with the experimental findings. The contributions of $v_p^{\text{bare}}$ and $v_p^{\text{ind}}$ to $\tilde{\Delta}_\nu$ are about equal, density modes leading to attractive contributions which are partially canceled by that of spin modes\footnote{While we only display the results obtained with SLy4 to treat the spin modes, the calculations were also carried out with other two effective forces, namely SkM* and SAMi. The errors attributed to the theoretical estimates reflect the quantitative variation of the cancellation effects.}, as expected from general transformation properties of the associated operators entering the particle vibration coupling vertices [17]. It is also noticed that $^{120}\text{Sn}$ can be called a Migdal “superconductor”, in keeping with the fact that vertex corrections are small throughout [18].

The strength functions associated with the lowest quasiparticle states provide an overall account of the experimental findings, in particular concerning the $u_\nu^2$, $v_\nu^2$ occupation numbers. Making use of these quantities and of global optical parameters, the absolute one-particle transfer cross sections were calculated. As an example, the results associated with the more trying and less well reproduced strength function, namely that of the $E_{d_{5/2}}$ quasiparticle state are displayed in Fig. 3 together with the experimental data ($^{120}\text{Sn}(p,d)^{119}\text{Sn}$) [19]. Four $d_{5/2}$ fragments are predicted by theory at low energy ($< 2$ MeV) with a summed integrated cross section $\sum_{i=1}^{4} \sigma(2^\circ - 55^\circ) \approx 6.2$ mb, while five are experimentally observed with $\sum_{i=1}^{5} \sigma(2^\circ - 55^\circ) \approx 8$ mb $\pm$ 2 mb.

With the help of two-neutron spectroscopic amplitudes $B_{j_\nu} = (j_\nu + 1/2)^{1/2} u_{j_\nu} v_{j_\nu}$ and of global optical parameters the absolute differential cross section $^{120}\text{Sn}(p,t)^{118}\text{Sn}(\text{gs})$ was
calculated and is plotted in Fig. 4(a) in comparison with experimental data [20]. Theory
\( \sigma(7.6^\circ - 69.7^\circ) = 2360 \mu b \) reproduces the experimental integrated absolute cross section
\( \sigma(7.6^\circ - 69.7^\circ) = 2250 \pm 338 \mu b \) within experimental errors.

At the basis of the quantitative account of the experimental finding reported in Figs. 2, 3
and 4(a) one finds NFT renormalization processes in general, and induced pairing interaction
in particular. One could argue that the evidences on which this statement is based are all
related to virtual processes and thus amenable, in principle, to different interpretations. To
answer this objection we have used the strength of the coupling vertex \((h_{11/2} \otimes 2^+)_{15/2^- - 7/2^-}\)
employed above, and calculated the energy of the quintuplet of states arising from this
coupling [1, 9, 21]. Theory gives a good reproduction of the experimental data [22–25]
(Fig. 4(b)) in a situation in which the basic states (quasiparticle and vibration) involved
in the coupling are present asymptotically, the associated transfer processes making them
real. This is also the case for the electromagnetic decay of the quasiparticle states. As seen
from Fig. 5, theory also provides an overall quantitative picture of the Coulomb excitation
measurements [26].

Summing up, structure properties, Coulomb excitation and one- and two- particle trans-
fer reactions associated with the superfluid nucleus \( ^{120}\text{Sn} \) and with its neighboring odd-\( A \)
isotopes, have been calculated. Both the bare pairing interaction as well as medium polariza-
tion effects resulting from the coupling of quasiparticles states with density and spin modes
have been taken into account. The implementation of the program has been done using
the NFT rules of structure and reactions [27], within the framework of the Nambu-Gor’kov
equations (\textit{structure}), and of finite range, full recoil DWBA state of the art \((p, d)\) reaction
theory, and of successive and simultaneous \((p, t)\) transfer reaction channels, corrected for
non-orthogonality contributions, within the framework of second order DWBA (\textit{reactions})
This is a first in the study of pairing in nuclei, theory providing a quantitative description of the nuclear response to a wide variety of external fields. Arguably, it also provides the first \textit{bona fide} embodiment of the NFT renormalization program pioneered by Bohr, Mottelson, Bes and coworkers (cf. ref. \cite{30} and refs. therein). Furthermore, the above results contain, through their essential “completeness”, the basis to quantitatively assess the role played by \(v_p^{\text{bare}}\) and \(v_p^{\text{ind}}\) on the overall nuclear pairing correlations in general, and on the pairing gap in particular. In fact, changing the value of the different couplings contributing to \(v_p^{\text{ind}}\), while boosting at the same time \(v_p^{\text{bare}}\) so as to reproduce the empirical value of the (three-point formula) odd-even mass difference (solid arrow in Fig. \ref{fig:2}\(b\)) would not alter the overall average value of \(\tilde{\Delta}\), nor of the absolute two–particle transfer cross section (Fig. \ref{fig:4}\(a\)), but will change the splitting of the multiplet (Fig. \ref{fig:4}\(b\)) and the value of the \(B(E2)\) transition probabilities (Fig. \ref{fig:5}), as well as the fragmentation of the \(d_{5/2}\) quasiparticle state as revealed by the \(^{120}\text{Sn}(p,d)^{119}\text{Sn}(5/2^+)^\prime\) cross sections (Fig. \ref{fig:3}). Changing the different individual couplings in varied sequences and/or contemporaneities provides the basis for an operative protocol to disentangle the contribution of \(v_p^{\text{bare}}\) and \(v_p^{\text{ind}}\) to e.g. the pairing gap.

Within this context we conclude that pairing in typical superfluid nuclei lying along the stability valley like \(^{119,120,121}\text{Sn}\) has a dual origin, in which \(v_p^{\text{bare}}\) and \(v_p^{\text{ind}}\) play an essentially \textit{on par} role.

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FIG. 1. Resumé of the strategy followed to calculate dressed elementary modes of excitation and induced pairing interaction in terms of NFT diagrams propagated by Nambu-Gorkov equations.
FIG. 2. (a) Lowest quasiparticle energies for the five valence orbitals calculated for the nucleus $^{120}$Sn in comparison to the experimental energies (open circles), taken as the average of the quasiparticle states observed in $^{121}$Sn and $^{119}$Sn. The energies obtained including the bare interaction in the pairing channel with HF (SLy4) single–particle states, are shown by solid squares. The energies obtained by adding the coupling to surface modes are shown by solid diamonds. Adding vertex corrections leads to the results plotted in terms of open diamonds. The results obtained by adding the coupling to spin modes are displayed with solid dots. (b) Pairing gaps of the states lying around the Fermi energy. $\Delta_{BCS}^{bare} = 1.08$ MeV, horizontal straight line) was calculated with $v_p^{bare}$. The effect of the coupling to surface modes is displayed in terms of open diamonds as in (a). The coupling to spin modes has also been included making use of the Landau parameters associated with the SLy4 interaction (solid dots). The grey area shows the estimated errors of the calculations, due to variations associated with the choice of the (spin-spin) interaction and in the cutoff used to calculate the coupling to spin modes. The empirical value (three-point formula) of the pairing gap is equal to 1.46 MeV (arrow on the vertical axis).
FIG. 3. $^{120}\text{Sn}(p,d)^{119}\text{Sn}(5/2^+)$ absolute experimental cross sections [19] (dots), together with the DWBA fit carried out in the analysis of the data (right panel), in comparison with the finite range, full recoil DWBA calculations carried out with the help of state of the art optical potential and $v_{np}$ interaction (I. J. Thompson, private communication), making use of the NFT structure inputs as explained in the text.
FIG. 4. (a) Two particle transfer absolute cross section of $^{120}$Sn($p,t$)$^{118}$Sn reaction second order DWBA calculations compared to experiment [20]. (b) Energies of the quintuplet of states arising from the coupling of the $h_{11/2}$ quasiparticle state and the lowest quadrupole vibration of $^{120}$Sn (i.e. $(h_{11/2} \otimes 2^+)^{7/2^+,15/2^-}$) worked out using NFT diagrams. The experimental data is the average of $^{119}$Sn and $^{121}$Sn. The placement of the level $11/2^-$ member of the multiplet is from systematics [22–25].

FIG. 5. (left) Experimental level diagram of $^{119}$Sn and $B(E2 \downarrow)$ values in single-particle units, as measured by Coulomb excitation [26]. (right) Theoretical spectrum.
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