A toroidal trap for cold $^{87}\text{Rb}$ atoms using an rf-dressed quadrupole trap

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Abstract

We demonstrate the trapping of cold $^{87}\text{Rb}$ atoms in a toroidal geometry using a radio frequency (rf) dressed quadrupole magnetic trap formed by superposing a strong rf-field on a quadrupole trap. This rf-dressed quadrupole trap has the minimum potential away from the quadrupole trap centre on a circular path which facilitates trapping in toroidal geometry. In these experiments, the laser cooled atoms were first trapped in a quadrupole trap, then cooled evaporatively using a weak rf-field, and finally trapped in an rf-dressed quadrupole trap. The radius of the toroid could be varied by varying the frequency of the dressing rf-field. It has also been demonstrated that a single rf source and an antenna can be used for the rf-evaporative cooling as well as for the rf-dressing of atoms. The atoms trapped in the toroidal trap may have applications in the realization of an atom gyroscope as well as in studying the quantum gases in low dimensions.

Keywords: rf dressing, quantum optics, magnetic trap

(Some figures may appear in colour only in the online journal)

1. Introduction

The development of atom traps with sophisticated potential landscapes has catalysed research in the field of cold quantum gases. Optical lattices [1], double-well [2] and potentials confining atoms in low dimensions (two dimensions (2D) and one dimension (1D)) [3] are the examples where new physics has been elucidated in the recent past. Trapping atoms in the low dimensions is of interest for several reasons [4]. The phase transition to Bose-Einstein condensation (BEC), forbidden for a homogeneous Bose gas in 1D and 2D geometries, becomes feasible if the confining 1D or 2D potentials obey suitable power laws for the spatial variation. Atom trapping in the toroidal or ring shaped geometry is useful for the study of coherence, super-fluidity, Josephson oscillations of the quantum gases [5, 6] confined in low dimensions, besides having applications in the realization of an atom gyroscope [7].

The proposals and realizations of the ring shaped atom traps have been reported by several groups in recent past employing various techniques. The use of a static magnetic trap superimposed with a strong rf-field, called rf-dressed magnetic trap, appears promising due to the flexibility and control available on the generated trapping potentials [8]. The reports on the ring traps include the use of a specially designed magnetic coils for a ring shaped quadrupole trap [9], an rf-dressed magnetic trap in combination with a standing wave pattern of an optical beam [7], and an rf-dressed magnetic trap in combination with a sheet type dipole laser beam [10]. In an interesting work, using hollow beams and an optical lattice, Amico et al [11] have proposed and demonstrated the generation of a ring optical lattice for the trapping of a superfluid atomic system to be used as qubits.

In the rf-dressed magnetic trap, an atom experiences adiabatic potential, which is position dependent eigen-energy of the dressed-state of the atom, while it interacts with the static and oscillating magnetic fields. This type of adiabatic potentials, also referred to as rf-dressed potentials, can be handled with less complexity as compared to the potentials of optical beams. These adiabatic potentials also permit ample control over the generated potential landscape, which can be tuned by tuning the amplitude, frequency and phase of the time varying rf-fields [7, 12–20]. Depending upon the rf-field parameters, rf-dressed potentials can offer exquisite trapping geometries like multiple-wells, asymmetric arcs, ring traps and rotating (or oscillating) toroidal traps [21]. Such non-
trivial trapping geometries are important in conventional as well as miniaturised atom traps [22] for various applications.

In this work, we demonstrate the trapping of cold \( ^{87}\text{Rb} \) atoms in a toroidal geometry using an rf-dressed quadrupole magnetic trap. As compared to the earlier approaches, in which the toroidal trap was formed either using a rf-dressed magnetic trap and a dipole potential of a laser beam [10] or using a specially designed magnetic coils for ring shaped quadrupole traps [9], our method is simple to implement as it requires an rf-dressed quadrupole trap only. In our method, a laser cooled atom-cloud, trapped in the quadrupole trap in \( |F = 2, m_F = 2\rangle \) state, is first exposed to a weak rf-field (with frequency sweep) for evaporative cooling. Then a stronger rf-field, at a different frequency, is used for transferring the cloud to the rf-dressed potential. The rf-field used for dressing is also subjected to a small range frequency sweep (few MHz) to obtain a clean toroidal shape of the trapped cloud. It is also demonstrated that a single rf source and an antenna can be used for the evaporative cooling as well as for the rf-dressing to trap atoms in the toroidal geometry. The observed number of atoms in the rf-dressed trap is higher than that in a bare quadrupole trap, when rf-dressed state atoms are transferred to the bare trap. It is also experimentally demonstrated that the use of an additional rf-field along the quadrupole trap axis, in the presence of linearly polarized rf-field perpendicular to the trap axis, gives rise to asymmetry in the atom cloud trapped in the ring. These results are in agreement with the theoretical predictions. Since the position of the trapped cloud can be altered by altering the phase of the axial rf-field, the modulation of phase of this field can provide an opportunity to rotate the cloud along the ring [21]. This method of rotation can prove more simple than the use of a high power dipole laser beam for stirring the cloud. The rotation of the atom cloud on the ring path is useful to study super-fluidity in ultracold atomic samples. These experiments will be attempted by us in future.

The article is organized as follows. In section 2, the theoretical background required for the rf-dressed potentials is discussed. The section 3 gives the description of the experimental setup used for generation of the toroidal rf-dressed potentials in the present work. The main results of the work are discussed in section 4. Finally, the conclusions of the present work are given in section 5.

2. rf-dressed quadrupole trap

The trapping of neutral atoms in an inhomogeneous static magnetic field, superimposed with a time varying rf-field, can be well described using a semi-classical approach [23]. The rf-field couples various hyperfine Zeeman sub-levels of an atom and the rf-dressed potential is the effective potential experienced by the atom in static and time varying fields. The rf-dressed potential is also an energy eigen value of the Hamiltonian in the dressed state picture. Figure 1 shows a schematic diagram of the effect of an rf-field on the potential energy of an atom trapped in a quadrupole trap. After rf dressing, the potential minimum shifts away from the quadrupole trap centre and forms the avoided crossings shown in figure 1(b). In order to evaluate the potential energy \( V(\mathbf{r}) \) of an atom in the static and rf fields, we consider a quadrupole magnetic trap having the field distribution as,

\[
B^x(x, y, z) = B_y \left( \frac{x}{2z}, \frac{y}{2z} \right),
\]

where \( B_y \) is the radial field gradient and \( z \)-axis is the axis of quadrupole trap. We consider the rf-field of the form \( B^d(t) = \{ B_x \cos(\omega t), B_y \cos(\omega t - \alpha), B_z \cos(\omega t - \beta) \} \) for the dressing purpose, where \( B_x, B_y, \) and \( B_z \) are the amplitudes of the rf-field in three orthogonal directions, and, \( \alpha \) and \( \beta \) are the relative phases.

As is known, the trapping in the bare quadrupole trap results in Zeeman splitting of the hyperfine levels of \( ^{87}\text{Rb} \) atoms with transition frequency between adjacent levels is given by,

\[
\omega_0 = \frac{g_F \mu_B B_z}{\hbar} \sqrt{x^2 + y^2 + 4z^2},
\]

where \( g_F \) is the Landé’s g-factor, \( \mu_B \) is the Bohr magneton and \( \hbar \) is the reduced Planck’s constant. This transition frequency, known as Larmor frequency, is inherently position dependent and isotropic in the x-y plane. With the rf radiation of frequency \( \omega \), the transitions between adjacent Zeeman levels of a hyperfine level can be excited, if \( \omega \approx \omega_0 \). Because of the position dependent nature of the Larmor frequency, a single frequency rf-source does not excite transitions at every position in the trap. This spatially varying transition frequency in the quadrupole trap contributes to the position dependent nature of the rf-dressed potential energy \( V(\mathbf{r}) \).

For an atom in a Zeeman hyperfine sub-level \( m_F \), the potential energy \( V(\mathbf{r}) \) can be calculated using a rotating wave approximation formalism [10, 17, 21] and can be written, considering gravity opposite to the direction of y-axis, as

\[
V(\mathbf{r}) = m_F \hbar \sqrt{\delta^2 + |\Omega|^2} + mg_y.
\]

where

\[
\delta = \omega - \omega_0.
\]

Figure 1. Variation in potential energy with position for an atom: (a) in bare quadrupole trap and (b) in rf-dressed quadrupole trap. The potential in graph (b) (continuous curve) shows the avoided crossing at the regions of resonance due to rf-field induced coupling between the states. The potentials corresponding to the dressed states are denoted with the subscript A.
Here $\delta$ is detuning of the rf-field from the Larmor frequency $\frac{\Omega}{\hbar}$ and $\Omega$ is the Rabi frequency for coupling between the sub-levels $m_F$ and $m_F'$. $\Omega$ can be determined as \cite{21, 23}

$$\Omega^2 = \left( \frac{g_F \mu_B}{2 \hbar} \right)^2 \left[ \frac{4 \epsilon^2}{x^2 + y^2 + 4z^2} \left( \frac{B_z^2 x^2 + B_y^2 y^2}{x^2 + y^2} \right) + \frac{B_x^2 y^2 + B_z^2 y^2}{x^2 + y^2} + B_z^2 \right]$$

By choosing appropriate values of the field strengths $B_x$, $B_y$, $B_z$ and phases $\alpha$ and $\beta$, a variety of potentials can be generated \cite{21} and some of these are shown in figure 2. To predict the trapping geometries, the dressed state potentials (from equation (3)) are calculated numerically, using the values of detuning $\delta$ and Rabi frequency $\Omega$ associated with the rf-field. This helps in choosing various parameters for the desired trap geometry, by ensuring the potential depth suitable for an atom cloud at the given temperature.

From equations (3)-(5), it can be determined that, with a linear polarisation of rf-field (i.e. $B_z = 0$, $B_x = B_y = 0$), the rf-dressed potential is expected to be a double-well potential as shown in figure 2(a). These wells are formed on circumference of a ring. The positions of these wells on the ring circumference are symmetric, when effect of gravity is negligible. With a circular polarisation of rf-field (i.e. $B_x = 0$, $B_z = 0$, $B_y = 0$ and $\alpha = \pm \pi/2$), the resulting potential is expected to be a ring trap potential as shown in figure 2(b). An addition of the rf-field polarized along the quadrupole trap axis (i.e. $B_z = 0$), in presence of the linearly polarized rf-field (i.e. $B_z = 0$), results in an asymmetric ring potential with its local minimum on the ring as shown in figure 2(c). The spatial position of this local minimum is governed by the phase angle $\beta$ of the axial rf-field.

The shape of the rf-dressed potentials may be affected by gravity. The parameter describing the effect of gravity is $\kappa (= g y \mu_B \mu_B / m g)$, which is the ratio of the magnetic potential energy to the gravitational potential energy. In our experiments $\kappa \sim 13$, therefore the effect of gravity in the magnetic trap is negligible. Another factor which compares the strength of rf coupling with that of the gravity is the value of ratio $(\omega / \Omega)$ in comparison to $\kappa$ \cite{15}. If $(\omega / \Omega) < \kappa$, the potential is coupling dominant and its minimum occurs at the minimum of the coupling. In our experiments $(\omega / \Omega) \sim 9$, which ensures the trap is dominated by the coupling and not by gravity. Without the fulfilment of these conditions, the gravity may cause the distortion in the symmetry of the trapping potential (double-well or ring). Under the influence of gravity, the two wells in the double-well potential will not have same separation when measured along different arcs on the ring. The ring trapping potential will also be modified to an asymmetric ring potential, favouring the accumulation of the atom cloud in the lower half of the ring in the direction of gravity. But in our experiments, none of these distortions have been observed in the cloud images. Thus, effect of gravity seems negligible due to appropriate parameters chosen in the experiments.

3. Experimental realization

The experiments have been performed on a double magneto-optical trap (double-MOT) setup \cite{24–26}. The schematic of the setup is shown in figure 3. In this setup, a vapour chamber MOT (VC-MOT) of $^{87}$Rb atoms is formed in a chamber at a pressure of $\sim 1 \times 10^{-8}$ mbar and an ultra-high vacuum MOT (UHV-MOT) is formed in a glass cell at a pressure of $\sim 5 \times 10^{-11}$ mbar. The atom cloud in the VC-MOT works as a source of atoms to load the UHV-MOT. The UHV-MOT is loaded by using a push beam focused on the VC-MOT atom cloud. The three cooling laser beams each having 15 mW of power are used for the VC-MOT in a retro-reflection configuration. The re-pumping beam for the VC-MOT has $\sim 14$ mW power which is mixed with one of the three cooling beams. For the UHV-MOT, six independent cooling beams each having $\sim 8$ mW power are used. The re-pumping laser beam with total power of $\sim 18$ mW is divided and mixed with these cooling beams for the UHV-MOT. The cooling laser beams, for both the MOTs, are derived from a laser beam ($\sim 750$ mW power) which is the output of an amplifier (TAL-Boosta, Toptica, Germany). This amplifier is seeded by an external cavity diode laser (DL-100, Toptica, Germany). Several acousto-optic modulators (AOMs) are used for the control over the duration of laser beams as well as for the shifting of frequency of laser beams. In addition to AOMs, the mechanical shutters are also used in the path of laser beams, to completely block the leakage of laser emission from AOMs during the rf-dressing of the trapped atoms. This improves the life time of the trapped atoms.
The atoms in the UHV-MOT are evaporatively cooled down to 15-23 A, to increase the radial gradient of UHV-MOT magnetic field. After this, the magnetic field is switched off and the atoms are transferred to an optical molasses for 5 ms, in which the temperature of the cloud decreases to ~40 μK. After the optical molasses stage, the UHV-MOT laser beams are turned off, and optical pumping is done for 500 μs. After the optical pumping of the atom cloud, the current in the UHV-MOT coils (i.e. quadrupole trap coils) is switched-on (0 to 15 A in 2 ms) to trap atoms in the magnetic trap with a radial gradient of ~75 G cm⁻¹. Subsequently, the current in the quadrupole trap coils is ramped slowly in 1 s duration from 15 to 23 A, to increase the radial field gradient from ~75 to ~115 G cm⁻¹. This completes the formation of the quadrupole trap. The atom cloud in the final quadrupole trap has ~1 x 10⁷ atoms with an approximate temperature ~250 μK and trap lifetime ~18 s. The rf-field is switched-on after the atom cloud is settled for more than 50 ms in the quadrupole trap. This 50 ms time is long enough for untrappable hot atoms to escape the trap.

4. Results and discussion

In the experiments, the effect of the applied dressing rf-field on cold ⁸⁷Rb atoms trapped in the quadrupole magnetic trap has been studied. To start with, the atom cloud trapped in the quadrupole trap was first evaporatively cooled to reduce temperature from ~250 μK (number of atoms ~1 x 10⁷) to ~20 μK (number of atoms ~2.5 x 10⁵), by applying the rf-field using the antennae LA-2. The rf evaporation is implemented by ramping down the source frequency (ν = ω/2π) from 12 MHz to 3 MHz in a duration of 5 s. When this
evaporatively cooled atom cloud in the quadrupole trap was exposed to the dressing rf-field, emitted from the multi-turn antenna LA-1, the trapping of atoms in toroidal geometry was clearly observed after appropriate adjustment of power in the dressing rf-field. The estimated temperature of atoms in the toroidal trap was $\sim 40 \mu K$, which is higher than the initial temperature of $\sim 20 \mu K$ in the quadrupole trap. The increased temperature in the rf-dressed quadrupole trap is expected due to gain in energy by atoms from the modified potential, after the rf-dressing. The number of atoms trapped in the toroidal trap was typically $1.3 \times 10^5$, giving the transfer efficiency of $\sim 50\%$ from quadrupole trap to toroidal trap. Figure 4(a) shows the absorption image of the cloud in the quadrupole magnetic trap after the evaporative cooling and before the rf-dressing. Figure 4(b) shows the image of the atom cloud in the rf-dressed quadrupole trap. The low density in the centre of the absorption image of the cloud in the rf-dressed quadrupole trap (figure 4(b)) shows the trapping of atoms in the toroidal geometry. One may note that, since the dressing rf-field is linearly polarized, the rf-dressed potential in our case is expected to have a double-well structure on a circular ring (as shown in figure 2(a)), with two potential barriers separating the wells in the azimuthal direction. We have observed the toroidal (or ring shaped) atom cloud because the temperature ($\sim 40 \mu K$) of the atom cloud is higher than the height (few $\mu K$) of these azimuthal potential barriers. Due to higher temperature, atoms in the cloud can ride the barriers and move throughout the circumference of the ring. In our dressed quadrupole trap, under the harmonic potential approximation near the trap minimum on the ring, the ratio of axial and radial trap frequencies ($\omega_z/\omega_r$) is higher than 1 (estimated using the approach described in [3]). Hence the confinement along the $z$-direction is stronger than that in the radial direction. This indicates that the trapped cloud is quasi-2D type.

Due to the non-uniform potential depth around the toroid, distribution of atom cloud is expected to be non-uniform which is also observed experimentally. The observed asymmetric cloud distribution pattern was shot to shot repeatable, with negligibly small variations in the density distribution. In the measurements, a single image of the cloud was stored and analysed to retain the original density distribution intact, instead of taking multiple images and then averaging. The measurements were repeated for a number of times to check the reproducibility and derive the statistical errors.

To estimate the radius of the toroidal ring in the absorption image of the cloud, the optical density profile along a selected diameter across the image was plotted. The measured optical density profile was fit to two Gaussian profiles at the two ends of this diameter. The peak to peak separation between two Gaussian profiles has been used as the diameter (twice the radius) of the ring. An average over different diameters along different directions was performed to obtain the mean radius of the toroidal ring. The radius values obtained this way are shown in figure 5, as a function of the rf frequency $\omega$. The experimental and theoretical values shown in the figure are in reasonable agreement.

In further experiments, we have used coil LA-1 for evaporative cooling as well as for generation of the rf-dressed potential for cold atoms. The rf-frequency ($\nu = \omega/2\pi$) in this case is ramped down from 15 MHz to 1 MHz linearly in a duration of 5 s. The impedance matching circuit is kept resonant near the 1 MHz end of the ramp, which results in high rf-field amplitude only at lower frequencies ($\sim 1$ MHz). This makes it possible to use the same antenna for evaporative cooling as well dressing of the atoms in the trap. The current variation through the coil LA-1 with frequency is shown in figure 6. During the frequency ramp down from 15 MHz to 1 MHz, the rf-field of low amplitude (due to low current in the antenna) is radiated in the beginning which is suitable for rf evaporative cooling. In the trailing part of the ramp (near 1 MHz), a high amplitude rf-field is obtained due to multi-fold increase in the current in the antenna coil. This high amplitude rf-field converts the quadrupole trap into an rf-dressed quadrupole trap. Figure 7(a) shows a typical image of the cloud trapped in the toroidal rf-dressed

![Figure 4](image1.png) Observed absorption images of the cloud (in x, y-plane) after evaporative cooling in quadrupole trap: (a) before rf-dressing and (b) after rf-dressing. The curves show the profiles of optical density along the selected diameters across the image. The arrows indicate the approximate optical density at that position. Different colours in an absorption image from blue to red represent the optical density values in the increasing order.

![Figure 5](image2.png) Observed variation in the ring radius with frequency ($\omega/2\pi$) of the dressing rf-field. Points show the experimentally measured values and the error bars show the deviation in the value in repeated measurements. The line curve represents the theoretical prediction of radius considering experimental parameters. The inset shows the absorption images (optical density) of the atom cloud for different frequencies of dressing field.
potentially, using this single coil configuration for rf-dressing. In single coil configuration, the rf evaporation and rf-dressing are achieved starting from \( \sim 1 \times 10^7 \) atoms at temperature \( \sim 250 \) \( \mu \text{K} \) in the quadrupole trap. Finally \( \sim 2 \times 10^5 \) atoms at temperature of \( \sim 40 \) \( \mu \text{K} \) are obtained in the toroidal ring trap. Therefore single coil configuration is more efficient in terms of number of atoms than the double coil configuration. This is a expected result due to adiabatic nature of conversion of quadrupole trap into rf-dressed quadrupole trap in the case of single coil configuration.

Subsequently, the experiments have also been performed using both the coils LA-1 and LA-2 simultaneously with rf-field frequency ramped down from 15 MHz to 1 MHz. In this case, the rf-field from LA-1 is stronger than that from LA-2. The presence of the \( z \)-component of rf-field, due to coil LA-2, destroys the radial symmetry of the potential and shifts the atom cloud in one arc of the toroid, as shown in figure 7(b). This result is consistent with the results of calculations performed by incorporating \( z \)-component in the rf-field along with the linearly polarized rf-field (figure 2 (c)). The asymmetry arises due to inhomogeneity of the coupling strength around the toroid. As predicted by the earlier work [21], the position of the potential minimum can be changed by changing the value of phase \( \beta \). Thus, insertion of a phase modulation unit between the two amplifiers driving LA-1 and LA-2 can be used as a technique to modulate \( \beta \), and rotate the atom cloud along the toroidal ring [21, 27]. Such a rotation of the cloud will be attempted by us in future. This method of rotation appears simpler than the use of a laser dipole potential to stir the cloud, which is commonly used for super-fluidity experiments.

Next, we studied the loss of number of atoms trapped in the rf-dressed potential with time. For these measurements, atoms trapped in the quadrupole trap were exposed to an rf-field using the single coil configuration (coil LA-1) for rf evaporation and dressing. In this, the frequency of rf-field was ramped down from 15 MHz to 1 MHz as described before. At the end of the frequency ramp, the rf-field was left ON at 1 MHz frequency, and number of atoms in the trap was measured as a function of time (holding time). The measured variation in number of atoms in this rf-dressed trap with holding time is shown by circles in figure 8. For a comparison with the bare quadrupole trap, the rf-field was switched-off after the completion of the frequency ramp, and number of atoms was measured again as a function of holding time. These data are shown by crosses in figure 8.

The data in figure 8 shows the non-exponential decrease in number of atoms with time. In case of the rf-dressed trap, the decrease in number of atoms with time has been earlier attributed to an evaporation process via Landau-Zener losses at positions of weak rf-coupling [15]. We believe that similar mechanism could be responsible for the initial decrease in number of atoms in the rf-dressed trap in our case also (figure 8). However a different mechanism can be visualised for
the high loss of atoms (occurring initially) in the bare trap (shown by crosses in figure 8). Since the rf-dressed states are superposition of trappable as well as untrappable magnetic hyperfine spin states, the bare trap rejects atoms in untrappable states, leading to high loss of atoms when rf-dressed atoms are transferred to it. At longer holding time, the slow variation in number of atoms with time in both the traps can be attributed to the loss limited by background collisions (life time \(\sim 18\) s).

It is evident from the figure 8 that rf-dressed trap holds more number of atoms than the bare quadrupole trap over a long holding time. The results of comparison between the rf-dressed trap and the bare trap (figure 8) clearly establish the working of the rf-dressing in our experiments. The rf-dressed traps of this kind can be useful for rf evaporative cooling to achieve degeneracy in the cold gas samples in unconventional potential landscapes [10, 28–30].

5. Conclusion

The trapping of the cold \(^{87}\text{Rb}\) atoms in a toroidal geometry has been demonstrated in an rf-dressed quadrupole trap. It is also shown that toroidal trap can be formed using a single coil for both the purposes; evaporative cooling and rf-dressing. As predicted, by using two mutually perpendicular dressing rf-fields, the asymmetric ring trapping is also observed. The rf-dressed trap has shown lower loss rate for atoms than the bare magnetic trap. This kind of traps with further lower temperature can be used to study tunnelling and super-fluidity in low dimensions.

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