MODEL-INDEPENDENT ESTIMATIONS FOR THE CURVATURE FROM STANDARD CANDLES AND CLOCKS

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ABSTRACT

Model-independent estimations for the spatial curvature of the universe not only provide a test for the fundamental assumption of the Copernican principle, but also can effectively break the degeneracy between curvature and dark-energy properties. In this paper, we propose to achieve model-independent constraints on the spatial curvature from observations of standard candles and standard clocks, without assuming any fiducial cosmology or other priors. We find that, for the popular Union2.1 observations of Type Ia supernovae (SNe Ia), the spatial curvature is constrained to be $\Omega_k = -0.045^{+0.176}_{-0.172}$. For the latest joint light-curve analysis of SNe Ia observations, we obtain $\Omega_k = -0.140^{+0.161}_{-0.158}$. It is suggested that these results are in excellent agreement with a spatially flat universe. Moreover, compared to other approaches aiming for model-independent estimations of spatial curvature, this method also achieves constraints with competitive precision.

Key words: cosmological parameters – cosmology: observations

1. INTRODUCTION

The spatial curvature of the universe is one of the most fundamental issues in modern cosmology. Specifically, on one hand, estimating the curvature of the universe is a robust way to test the important assumption that the universe is exactly described by the homogeneous and isotropic Friedman–Lemaître–Robertson–Walker (FLRW) metric. On the other hand, the curvature of the universe is also closely related to some other important problems such as the evolution of the universe and the nature of dark energy. For instance, nonzero curvature may result in enormous effects on reconstructing the state equation of dark energy even though the true curvature might be very small (Ichikawa et al. 2006; Clarkson et al. 2007; Gong & Wang 2007; Virey et al. 2008), and any significant deviation from the flat case would lead to profound consequences for inflation models and fundamental physics. Moreover, possibilities for the failure of the FLRW approximation have been proposed to account for the observed late-time accelerated expansion (Ferrer & Räsänen 2006; Enqvist 2008; Ferrer et al. 2009; Räsänen 2009; Boehm & Räsänen 2013; Lavinto et al. 2013; Redlich et al. 2014). Therefore, observational constraints on the cosmic curvature from popular probes have been extensively studied in the literature (Eisenstein et al. 2005; Tegmark et al. 2006; Wright 2007; Zhao et al. 2007). We emphasize here that a spatially flat universe in the framework of the standard $\Lambda$CDM model is favored at a very high confidence level by the latest Planck 2015 results of observations of the cosmic microwave background (CMB) (Ade et al. 2015). However, none of these works measured the curvature in any direct geometric way. That is, curvature is primarily derived from a measurement of the angular diameter distance to recombination, which depends not only on curvature but also on the choice of cosmological model assumed in the analysis.

Clarkson et al. (2008) proposed to measure the spatial curvature of the universe or even test the FLRW metric in a model-independent way by combining observations of expansion rate and distance, which has been fully implemented with updated observational data (Shafieloo & Clarkson 2010; Mörtsell & Jönsson 2011; Li, et al. 2014; Sapone et al. 2014; Cai et al. 2016). In this method, the derivative of distance with respect to redshift $z$ is necessary to estimate the curvature, and this treatment introduces a large uncertainty. Therefore, Yu & Wang (2016) improved this method by confronting the distances derived from observations of expansion rate and baryon acoustic oscillations (BAO). However, the data on angular diameter distance and some measures of expansion rate used in their analysis are obtained from BAO observations, which are dependent on the assumed fiducial cosmological model and the prior for the distance to the last-scattering surface from CMB measurements (Audren et al. 2013; Audren 2014). Another method was also put forward to provide a similar test by using parallax distances and angular diameter distances (Räsänen 2014). In addition, the cross-correlation between foreground mass and gravitational shear of background galaxies has been proposed to be a practical measurement of the curvature of the universe, which relies purely on the properties of the FLRW metric (Bernstein 2006). More recently, the sum rule of distances along null geodesics of the FLRW metric has been put forth as a consistency test (Räsänen et al. 2015). It is interesting to note that, on one hand, the FLRW background will be ruled out if the sum rule is violated; on the other hand, if the observational data are consistent with the sum rule, then the test provides a model-independent estimation of the spatial curvature of the universe. In their analysis, by using the Union2.1 compilation of Type Ia supernovae (SNe Ia) (Suzuki et al. 2012) and data on strong gravitational lensing selected from the Sloan Lens ACS Survey (Bolton et al. 2008), the spatial curvature parameter was weakly constrained to be $\Omega_k = -0.55^{+1.18}_{-0.67}$ at 95% confidence level, which slightly favors a spatially closed universe. Actually, the distances used in their analysis from the Union2.1 SNe Ia are not completely cosmology-free, since the light-curve fitting parameters accounting for distance estimation are determined from a global fit in the assumed standard dark-energy model with the equation of state being constant.
Although such an effect is likely subdominant to the uncertainties, the measurement of spatial curvature was not as independent of the cosmological model as they claimed. Moreover, for the distance sum rule (Equation (4) in Rässänen et al. 2015) used to calculate the spatial curvature, it is argued that this formula is valid only for the case of $\Omega_K \geq 0$ (Hogg 1999).

In this paper, we first reconstruct a function of the Hubble parameter with respect to redshift $z$ from measurements of the expansion rate from cosmic chronometers (or standard clocks) by using a non-parametric smoothing (NPS) method (Li et al. 2016). This reconstructed function enables us to obtain the comoving distance directly by calculating its integral. Next, with the spatial curvature parameter taken into consideration, we transform these comoving distances into curvature-dependent luminosity distances. Then, by confronting them with luminosity distances depending on light-curve fitting parameters from observations of SNe Ia, we achieve constraints on the spatial curvature that are independent of the cosmological model. For the Union2.1 SNe Ia, we obtain $\Omega_K = -0.045^{+0.176}_{-0.122}$. When the latest joint light-curve analysis (JLA) of SNe Ia (Betoule et al. 2014) is used, the spatial curvature is constrained to be $\Omega_K = -0.140^{+0.161}_{-0.155}$. On one hand, these results consistently favor a spatially flat universe. On the other hand, in the context of model-independent estimations for spatial curvature, these constraints are comparable in terms of precision.

2. METHOD AND DATA

2.1. Distance from Expansion Rate Measurements

The expansion rate at any redshift $z = 0$, $H = \dot{a}/a$ where $a = 1/(1 + z)$, can be obtained from the derivative of redshift with respect to cosmic time, i.e., $H(z) \simeq -\frac{1}{1+z}\frac{\Delta z}{\Delta t}$. The difficulty of this approach is to estimate the change in the age of the universe as a function of redshift $\Delta t$. Jimenez & Loeb (2002) proposed to make this method practicable by calculating the age difference between two luminous red galaxies at different redshifts. In the literature, this method is usually referred to as differential age and the passively evolving galaxies from which $\Delta t$ is estimated are called cosmic chronometers. So far, 22 measurements of $H(z)$ based on this method (in the redshift range 0.070 $\leq z \leq 1.965$) have been obtained (Jimenez et al. 2003; Simon et al. 2005; Stern et al. 2010; Moresco et al. 2012, 2016; Moresco 2015). Although independent of the cosmological model, some of these estimates are sensitive to models of stellar population synthesis that influence $\Delta t$. It was found that this influence becomes important at $z \gtrsim 1.2$ (Verde et al. 2014). Therefore, we consider only 16 $H(z)$ measurements in the range $z < 1.2$, which, in practice, given the redshift distribution of the $H(z)$ data, means $z \leq 1.037$. In addition, we also slightly increase (20%) the error bar of the highest-$z$ point to account for the uncertainties in the models of stellar population synthesis (Verde et al. 2014). This ensures that the evolution of the reconstructed Hubble parameter as a function of redshift in the following analysis is dependent on neither the cosmology nor the model of stellar population synthesis.

In Li et al. (2016), we reconstructed a reasonable function of Hubble parameter versus redshift with the NPS method, which is an improved version of the smoothing process proposed by Shafieloo et al. (2006). The 16 measurements of the expansion rate from cosmic chronometers and the reconstructed function with 1σ confidence region are shown in the left panel of Figure 1. As proposed in Busti et al. (2014), by extrapolating this function to redshift $z = 0$, we can obtain a model-independent determination of the Hubble constant, $H_0$, from observations of intermediate-redshift cosmic chronometers. In their analysis, using the Gaussian processes (Seikel et al. 2012), they obtained a lower $H_0$ than the CMB value and hence increased the tension with the local measurement. Here, we find $H_0 = 69.407 \pm 6.031$ km s$^{-1}$ Mpc$^{-1}$, which is in agreement with both the latest CMB value (Ade et al. 2015) and local measurement (Ackermann et al. 2016). For consistency, this extrapolated $H_0$ is also used for distance estimation in the following analysis. Enlightened by the method that roughly transforms discrete $H(z)$ measurements into comoving distances by solving the integral with a simple trapezoidal rule (Holanda et al. 2013), we obtain the comoving distances at $z \leq 1.2$ by integrating the smoothed function of Hubble parameter with respect to redshift. It is known that the comoving distance $D_C$ is related to the luminosity distance $D_L$ via (Hogg 1999)

$$D_L = \begin{cases} \frac{D_H}{1 + z} & \Omega_K > 0 \\ \frac{D_H}{\sqrt{\Omega_K} \sinh [\sqrt{\Omega_K} D_C/D_H]} & \Omega_K = 0 \\ \frac{D_H}{\sqrt{\Omega_K} \sin [\sqrt{\Omega_K} D_C/D_H]} & \Omega_K < 0 \end{cases}$$

where $D_H = cH_0^{-1}$ and $c$ is the speed of light. With the extrapolated $H_0$, we obtain the curvature-free $D_C/D_H$ and the result is presented in the middle panel of Figure 1. Moreover, in the right panel of Figure 1, we illustrate the dependence of distance modulus, $\mu = 5 \log \left[ \frac{D_L}{M_{pc}} \right] + 25$, derived from observations of cosmic chronometers, on the spatial curvature.

2.2. Distance from SNe Ia Observations

The estimation of distance from SNe Ia data is on the basis of the empirical observation that these events form a homogeneous class whose remaining variability is reasonably well captured by two parameters. One of them describes the time stretching of the light curve ($x_1$) whereas the other describes the color of SNe Ia at maximum brightness ($c$). For the popular Union2.1 SNe Ia (Suzuki et al. 2012), where the SALT2 model is used to reconstruct light-curve parameters ($x_1$, $c$, and the observed peak magnitude in the rest-frame B band $m_B^*$), the distance estimator assumes that SNe Ia with identical color, shape, and galactic environment have on average the same intrinsic luminosity at all redshifts. This assumption can yield a linear expression to standardize the distance modulus:

$$\mu^{\text{SN}}(\alpha, \beta, \delta, M_b) = m_b^* - M_b + \alpha x_1 - \beta x_2 - c + \delta \times P(m_*^{\text{true}} < m_*^{\text{threshold}}),$$

where $\alpha$ and $\beta$ are nuisance parameters that characterize the stretch–luminosity and color–luminosity relationships, reflecting the well-known broader–brighter and bluer–brighter relationships, respectively. $M_b$ is another nuisance parameter and represents the absolute magnitude of a fiducial SN. In addition, the term $\delta \times P(m_*^{\text{true}} < m_*^{\text{threshold}})$ with $m_*^{\text{threshold}} = 10^{\delta m_*} m_*$ is introduced to account for the host-mass correction to SNe Ia luminosities (Sullivan et al. 2010).
In the latest JLA of SNe Ia (Betoule et al. 2014), light-curve parameters are also obtained with the SALT2 model and the distance modulus is estimated from an expression similar to Equation (2) but without the term for host-mass correction. Alternatively, they approximately correct for the effect of a dependence of the absolute magnitude $M_B$ on the properties of host galaxies, e.g., the host stellar mass ($M_{\text{stellar}}$), with a simple step function when the mechanism is not fully understood (Conley et al. 2011; Sullivan et al. 2011).

$$M_B = \begin{cases} M_B^1 & \text{if } M_{\text{stellar}} < 10^{10} M_\odot, \\ M_B^1 + \Delta M & \text{otherwise.} \end{cases}$$ (3)

In general, the light-curve fitting parameters, $\alpha$ and $\beta$, and $\delta$ are left as free parameters to be determined in the global fit to the Hubble diagram in the framework of the standard dark-energy model. This treatment results in the dependence of distance estimation on the cosmological model used in the analysis. Therefore, implications derived from observations of SNe Ia with the light-curve fitting parameters determined in the global fit to the Hubble diagram are somewhat dependent on the cosmological model.

3. RESULTS

In order to achieve a model-independent estimation for the cosmic curvature, rather than using the data on distance modulus versus redshift published in conventional samples of SNe Ia, we confront distances in Equation (2) that depend on light-curve fitting parameters with distances in Equation (1) that depend on curvature from observations of cosmic chronometers, by maximizing the following likelihood:

$$L(\Omega_k; P_{\text{SN}}) \propto \prod_{i=1}^{\text{SN}} \exp \left[ -\frac{(\mu^{{\text{CC}}}(z_i; \Omega_k) - \mu^{{\text{SN}}}(z_i; P_{\text{SN}}))^2}{2(\sigma^2_{\mu^{{\text{CC}}}} + \sigma^2_{\mu^{{\text{SN}}}k})} \right].$$ (4)

where $P_{\text{SN}}$ stands for parameters in distance estimation of SNe Ia, including light-curve fitting parameters ($\alpha$, $\beta$, and $\delta$, or $M_B$ and $\Delta M$), and $\sigma^2_{\mu^{{\text{SN}}}k}$ accounts for error in SNe Ia observations propagated from the covariance matrix (Amanullah et al. 2010; Conley et al. 2011). In our analysis, we use the covariance matrix with both the reported statistical and systematic errors. Then, we use emcee4 introduced by Foreman-Mackey et al. (2012), a Python module that includes the Markov chain Monte Carlo procedure, to get the best-fit values and their corresponding uncertainties for both $\Omega_k$ and parameters in distance estimation of SNe Ia by generating sample points of the probability distribution.

For the Union2.1 SNe Ia, 563 well-measured SNe Ia events remain for the likelihood estimation because of the redshift cutoff $z < 1.2$ for distance derived from expansion rate measurements. Results are shown in Figure 2 and summarized in Table 1. We find that, from the Union2.1 SNe Ia and observations of cosmic chronometers, the model-independent estimation for the spatial curvature is $\Omega_k = -0.045^{+0.176}_{-0.172}$. This is in full agreement with the constraints obtained from the latest Planck CMB measurements (Ade et al. 2015). Moreover, the precision of this estimation is more competitive than the model-independent test based on the distance sum rule (Räsänen et al. 2015). When the JLA SNe Ia are used, 737 well-measured events are distributed in the range $z < 1.2$. Results are shown in Figure 3 and summarized in Table 1. We obtain that the spatial curvature is model-independently constrained to be $\Omega_k = -0.140^{+0.101}_{-0.158}$. It is suggested that this is also consistent with the spatially flat universe. Moreover, compared with what is obtained from the Union2.1 SNe Ia, there is a subtle improvement in precision when the JLA SNe Ia are considered.

4. CONCLUSIONS

In this paper, we first reconstruct a function of Hubble parameter with respect to redshift, $H(z)$, with expansion rate measurements obtained from observations of cosmic chronometers. The non-parametric smoothing method and the cutoff of redshift $z < 1.2$ for observational data ensure that the reconstructed function depends on neither the cosmological model nor the model of stellar population synthesis. Next, we obtain the comoving distance by directly solving the integral of this reconstructed function. We present the function of $H(z)$ smoothed from observations of cosmic chronometers and the reconstructed comoving distance in units of $c/H_0^{-1}$ in the left and middle panels of Figure 1, respectively. Furthermore, with

4 https://pypi.python.org/pypi/emcee
and light-curve fitting parameters model-independently constrained from the Union2.1 SNe Ia and observations of cosmic chronometers.

Figure 2. Contours of 68% and 95% confidence levels for the spatial curvature and light-curve fitting parameters model-independently constrained from the Union2.1 SNe Ia and observations of cosmic chronometers.

Table 1

| Union2.1 SN Ia and Cosmic Chronometers | $\Omega_K$ | $\alpha$ | $\beta$ | $\delta$ | $M_B$ |
|--------------------------------------|-----------|---------|--------|---------|-------|
|                                      | -0.045$^{+0.176}_{-0.172}$ | 0.120$^{+0.009}_{-0.009}$ | 2.568$^{+0.071}_{-0.070}$ | 0.034$^{+0.033}_{-0.032}$ | -19.268$^{+0.017}_{-0.017}$ |

| JLA SN Ia and Cosmic Chronometers | $\Omega_K$ | $\alpha$ | $\beta$ | $M_B^*$ | $\Delta M$ |
|-----------------------------------|-----------|---------|--------|--------|--------|
|                                   | -0.140$^{+0.158}_{-0.158}$ | 0.140$^{+0.009}_{-0.009}$ | 2.983$^{+0.114}_{-0.112}$ | -19.040$^{+0.016}_{-0.016}$ | -0.038$^{+0.018}_{-0.018}$ |

the spatial curvature $\Omega_K$ taken into consideration, we can transform the comoving distance into the luminosity distance via Equation (1). The reconstructed distance modulus–redshift relations with different $\Omega_K$ considered are shown in the right panel of Figure 1. Clearly, these treatments are mathematical processes. Therefore, the curvature-dependent distance modulus reconstructed from observations of cosmic chronometers depends only on the assumption of a homogeneous and isotropic FLRW metric and has nothing to do with the matter–energy content of the universe. More conventionally, luminosity distance is measured from observations of SNe Ia. In this context, as shown in Equation (2), distance modulus is usually expressed as a linear combination of observed light-curve parameters ($m_B^*$, $x_t$, and $c$). Coefficients in this expression ($\alpha$, $\beta$, and $\delta$), which need to be calibrated, are termed light-curve fitting parameters. In order to dodge the reliance on any assumptions of a cosmological model, we directly confront the curvature-dependent distance (Equation (1)) from observations of cosmic chronometers with the distance (Equation (2)) from SNe Ia observations that depends on light-curve fitting parameters to obtain constraints on these undetermined coefficients. These results that are independent of cosmological model are shown in Figure 2, Figure 3, and Table 1. We find that the spatial curvature is constrained to be $\Omega_K = -0.045^{+0.176}_{-0.172}$ and $\Omega_K = -0.140^{+0.161}_{-0.158}$ when the Union2.1 and JLA SNe Ia are used, respectively. It is suggested that these are in excellent agreement with a spatially flat universe. Moreover, compared to the latest model-independent estimations of the spatial curvature based on geometric optics (Heavens et al. 2014; Räsänen et al. 2015), the results in our analysis are significantly improved in precision. This improvement might be very helpful in breaking degeneracies between the curvature and some other important problems such as the evolution of the universe and the nature of dark energy.

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