On the supersymmetry invariance of flat supergravity with boundary

Patrick Concha*, Lucrezia Ravera‡, Evelyn Rodríguez†

*Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4059, Valparaíso-Chile.
†INFN, Sezione di Milano, Via Celoria 16, I-20133 Milano-Italy.
‡Departamento de Ciencias, Facultad de Artes Liberales, Universidad Adolfo Ibáñez, Viña del Mar-Chile.

patrick.concha@pucv.cl, lucrezia.ravera@mi.infn.it, evelyn.rodriguez@edu.uai.cl

Abstract

The supersymmetry invariance of flat supergravity in four dimensions on a manifold with non-trivial boundary is explored. Using a geometric approach we find that the supersymmetry invariance of the Lagrangian requires to add appropriate boundary terms. This is achieved by considering additional gauge fields to the boundary without modifying the bulk Lagrangian. We also construct an enlarged supergravity model from which, in the vanishing cosmological constant limit, flat supergravity with a non-trivial boundary emerges properly.
1 Introduction

The presence of a boundary in (super)gravity theories has been of particular interest in the last 40 years [1–4]. The addition of boundary terms plays an important role in the so-called AdS/CFT duality [5–11]. Such duality between a quantum field theory living on the boundary and a string theory on asymptotically AdS spacetime implies in the supergravity limit a one to one correspondence between fields of the bulk supergravity theory and quantum operators in the boundary CFT. This requires to consider proper boundary conditions for the supergravity fields which act as sources for the operators of the CFT. Interestingly, the divergences of the bulk metric can be cancelled by adding appropriate counterterms at the boundary (holographic renormalization) [12–17].

The inclusion of boundary terms in supergravity has been studied by diverse authors in [18–33]. In particular, in [20, 21, 24, 25] it was pointed out that, unlike the Gibbons-Hawking prescription [2], the supersymmetry invariance of a supergravity action should be satisfied without imposing Dirichlet boundary conditions. Interestingly, for the $\mathcal{N}=1$ three-dimensional supergravity, it was proven that the boundary term reproduces not only the Gibbons-Hawking-York boundary term but also the counterterm allowing to regularize the action [25].

Recently, the authors of [27] have shown that the supersymmetry invariance of $\mathcal{N}=1$ and $\mathcal{N}=2$ four-dimensional supergravity with cosmological constant requires the presence of topological terms. Such result can be seen as the supersymmetric extension of those obtained in AdS gravity in which the addition of the topological Gauss-Bonnet term allows to regularize the action [34–38]. Subsequently, in [29, 33], using the same geometric approach used in [27], it was shown that the supersymmetric extension of a Gauss-Bonnet like gravity is required to restore the supersymmetry invariance of enlarged supergravities in the presence of a non-trivial boundary.\(^1\) Interestingly, the full supergravity actions obtained in [27, 29, 33] can be rewritten in terms of the super curvatures à la MacDowell-Mansouri [39].

Boundary conditions imposed at a finite value of the radial coordinate have been studied in $D=4$ supergravity with zero cosmological constant in [20, 22, 23]. However, the limit case of vanishing cosmological constant in the presence of a non-trivial boundary remains poorly explored. In particular, the flat supergravity Lagrangian is not supersymmetric when a non-trivial boundary is considered and, on the other hand, a flat limit cannot be naively applied to the MacDowell-Mansouri Lagrangian since it reduces only to boundary terms:

\[ \mathcal{L} = \epsilon_{abcd} R^{ab} R^{cd} + D\bar{\psi} \gamma_5 D\psi. \]  

(1.1)

This can be directly seen from the super curvatures $R^{ab}$ and $D\psi$ of the Poincaré superalgebra which do not allow the presence of the Einstein-Hilbert (EH) neither the Rarita Schwinger (RS) terms in the Lagrangian. This inconvenient appears only in presence of a boundary since the bulk flat Lagrangian can be recovered directly from the bulk $OSp(4|1)$ supergravity as a vanishing cosmological constant limit.

In this paper, we restore the supersymmetry invariance of the four-dimensional flat supergravity in presence of a non-trivial boundary. To this purpose, we introduce appropriate boundary terms to the Lagrangian such that the full supersymmetry in the bulk and boundary is recovered. This is

\(^1\)In presence of a non-trivial boundary of spacetime, that is when the boundary is not at infinity, the fields do not asymptotically vanish, and this has some consequences on the invariances of the theory, in particular on supersymmetry invariance.
achieved by adding new bosonic and fermionic gauge fields to the boundary in addition to the usual spin-connection, vielbein and gravitino. Interestingly, we find that the boundary values of the super curvatures are fixed by the field equations of the full Lagrangian. In particular, the full Lagrangian obtained can be rewritten in terms of super curvatures of a particular superalgebra known as Maxwell superalgebra \[\mathfrak{osp}(4|1)\]. We also present a proper vanishing cosmological constant limit from an enlarged supergravity theory with a non-trivial boundary. Such enlarged supergravity Lagrangian can be rewritten à la MacDowell-Mansouri for a deformation of the \[\mathfrak{osp}(4|1)\] superalgebra which corresponds to a new supersymmetric extension of the AdS-Lorentz algebra \[\mathfrak{oso}(4,1)\].

The paper is organized as follows: In Section 2, we present a brief review of the geometric approach we will adopt for the formulation of supergravity in superspace. The supersymmetry invariance of a supergravity Lagrangian in the presence of a non-trivial boundary is explicitly discussed within this framework. Section 3 and 4 contain our main results. In particular, in Section 3 we explore the supersymmetry invariance of flat supergravity in presence of a non-trivial boundary, while in Section 4 we construct an enlarged supergravity Lagrangian with a generalized cosmological constant term using the geometric approach. We show that the flat limit can be appropriately applied in presence of a boundary. We conclude our work with some comments and possible future developments.

2 Geometric approach and supersymmetry invariance in presence of a boundary

An interesting and powerful approach for constructing supergravity theories is the geometric or rheonomic approach \[\text{[43]}\]. The principal demand of any supergravity theory is the invariance of the action under supersymmetry transformations. In the rheonomic approach, a supergravity theory is given in terms of 1-form superfields defined on superspace (whose basis is given by the super-vielbein), and the supersymmetry transformations on spacetime correspond to diffeomorphisms in the fermionic directions of superspace. Thus, the principle of rheonomy makes the extension from spacetime to superspace uniquely defined and consequently allows for a geometric interpretation of the supersymmetry rules.

In this framework, the supersymmetry invariance of the Lagrangian is expressed by the vanishing of the Lie derivative of the Lagrangian for infinitesimal diffeomorphisms in the fermionic directions, that is to say

\[
\delta_\epsilon L = l_\epsilon L = l_\epsilon dL + d(l_\epsilon L) = 0. \tag{2.1}
\]

From condition 2.1 it is direct to see that when a supergravity Lagrangian is considered on spacetimes without boundary, it implies that \(l_\epsilon L|_{\partial\mathcal{M}} = 0\). Then, in the absence of a (non-trivial) boundary, \(l_\epsilon dL = 0\) results to be a sufficient condition for supersymmetry invariance. Nevertheless, when the background spacetime has a non-trivial boundary, the condition \(l_\epsilon L|_{\partial\mathcal{M}} = 0\) becomes non-trivial. Thus, in order to verify the invariance of the Lagrangian in presence of a non-trivial boundary, it is necessary to check it explicitly (see [27] for further details).

As it is well explained in [43], in the rheonomic approach the 1-forms fields \(\mu^A\) are extended from spacetime to superspace, such that the mapping \(\mu^A(x) \rightarrow \mu^A(x, \theta)\) is defined. Then, since the superspace equations of motion are given in terms of the superspace curvatures, they can be
analyzed by expanding the curvatures along a complete basis of 2-forms in superspace, such that

\[ R^A = R^A_{BC} \mu^B \mu^C. \]  

(2.2)

Expanding the curvatures \( R^A \) in superspace along the supervielbein basis \( \{ V^a, \psi \} \), we get

\[ R^A = R^A_{bc} V^b V^c + R^A_{a\alpha} V^a \psi^\alpha + R^A_{\alpha\beta} \psi^\alpha \psi^\beta. \]  

(2.3)

The rheonomy principle requires the outer components of the curvature 2-forms in superspace to be expressed in terms of the inner (or purely spacetime) components \( R^A_{ab} \). Furthermore, if we also assume Lorentz gauge invariance of the rheonomic parametrization for the curvature 2-forms and homogeneous scaling of all the terms involved, then the form of the superspace curvatures is completely determined, except for some constant coefficients which are fixed by the Bianchi identities. Along this work, we will see how the rheonomic approach is applied in order to restore the full supersymmetry of flat supergravity.

3 Flat supergravity in presence of a non-trivial boundary

It is well known that the four-dimensional flat supergravity Lagrangian

\[ L_{\text{bulk}} = \epsilon_{abcd} R^{ab} V^c V^d + 4 \bar{\psi} V^a \gamma_5 \gamma^a \rho, \]  

(3.1)

is simply given by the EH and RS terms, without involving a cosmological constant term. Here \( R^{ab} = d\omega^{ab} + \omega^{ac} \omega^b_c \) denotes the Lorentz super curvature 2-form and \( \rho = D\psi \) is the Lorentz covariant derivative of the gravitino. In particular, \( a, b, \ldots = 0, 1, 2, 3 \) are Lorentz indices and \( \epsilon_{abcd} \) is the four-dimensional Levi-Civita tensor. Note that the supergravity Lagrangian scales with \( \omega^2 \), being \( \omega^2 \) the scale-weight of the EH term. In fact, \( [\omega^{ab}] = \omega^0, [V^a] = \omega^1 \) and \( [\psi] = \omega^{1/2} \).

Such Lagrangian is on-shell invariant in the bulk under supersymmetry of the super-Poincaré group providing the vanishing of the supertorsion \( R^a = 0 \). Then, in the rheonomic approach, we have that \( \iota_e (dL_{\text{bulk}}) = 0 \) is satisfied. Nevertheless, the supersymmetry invariance of the Lagrangian is not guaranteed when a boundary is present. In particular, in presence of a non-trivial boundary, the condition

\[ \iota_e L|_{\partial M} = 0 \]  

(3.2)

is not necessarily satisfied and requires to be revised in order to restore supersymmetry invariance of the theory. To this purpose, we have to modify the bulk Lagrangian by adding terms that do not modify the dynamics but affect only the boundary. Thus, we have to consider boundary topological terms.

A good candidate for being considered as a boundary contribution should first scale homogeneously. In particular, each term must have the same scale-weight as the EH term. However, the only boundary terms that can be constructed using the spin-connection \( \omega^{ab} \), the vielbein \( V^a \) and the gravitino \( \psi \) are

\[ d \left( \omega^{ab} R^{cd} + \omega^a f^b \omega^{cd} \right) \epsilon_{abcd} = R^{ab} R^{cd} \epsilon_{abcd}, \]  

(3.3)

\[ d \left( \bar{\psi} \gamma^5 \rho \right) = \bar{\rho} \gamma_5 \rho, \]  

(3.4)

The “outer” components are those having at least one index along the \( \psi \) direction of superspace, while when the only non-vanishing components are along the bosonic vielbein they are called “inner”.

---

2 The “outer” components are those having at least one index along the \( \psi \) direction of superspace, while when the only non-vanishing components are along the bosonic vielbein they are called “inner”.
which scale with \( \omega^0 \) and \( \omega \), respectively. One could add arbitrary constants with appropriate scale-weight but this would imply the presence of a cosmological constant term in the bulk and would reproduce a sum of quadratic terms in the \( \text{osp}(4|1) \) covariant super field-strengths [27].

An alternative approach is to add new gauge fields with upper scale-weight than the present ones. The minimal content that can be added consists of a bosonic and a fermionic gauge field. In particular, we propose a new antisymmetric bosonic gauge field \( A_{ab} = -A_{ba} \) with scale-weight \( \omega^2 \) and an additional fermionic gauge field \( \chi \) with scale-weight \( \omega^{3/2} \). Naturally, one could consider additional bosonic gauge fields with scale-weight \( \omega \) and \( \omega^2 \) but, as we shall see, this will not be necessary to recover the supersymmetry invariance in the boundary.

The only boundary contributions constructed by using \( \{\omega^{ab}, V^a, A_{ab}, \psi, \chi\} \) that are compatible with parity, Lorentz invariance and that do not involve a scaling parameter are given by the following topological terms:

\[
d d \left( A_{ab} R^{cd} + \omega^a_f \omega^{fb} A^{cd} + 2\omega^a_f A^{fb} \omega^{cd} + \omega_{ab} F^{cd} \right) \epsilon_{abcd} = 2 R_{ab} F^{cd} \epsilon_{abcd},
\]
\[
d \left( D\bar{\psi} \gamma_5 \psi + D\bar{\chi} \gamma_5 \chi \right) = 2\sigma \gamma_5 \rho + \frac{1}{4} R^{ab} \chi \gamma^{cd} \psi \epsilon_{abcd}, \quad (3.5)
\]

where we have defined \( \sigma \equiv D\chi \) and \( F^{ab} \equiv DA_{ab} \) as the respective covariant derivatives of the new gauge fields. Thus, the boundary Lagrangian reads

\[
\mathcal{L}_{\text{bdy}} = \alpha \left( 2\sigma \gamma_5 \rho + \frac{1}{4} R^{ab} \chi \gamma^{cd} \psi \epsilon_{abcd} \right) + \beta \left( 2 R_{ab} F^{cd} \epsilon_{abcd} \right), \quad (3.6)
\]

where \( \alpha \) and \( \beta \) are constant parameters. Note that the boundary Lagrangian has scale-weight \( \omega^2 \) as the bulk Lagrangian.

Then, we have the following full Lagrangian:

\[
\mathcal{L}_{\text{full}} = \mathcal{L}_{\text{bulk}} + \mathcal{L}_{\text{bdy}} = \epsilon_{abcd} R^{ab} V^c V^d + 4 \bar{\psi} V^a \gamma_5 \rho \\
+ \alpha \left( 2\sigma \gamma_5 \rho + \frac{1}{4} R^{ab} \chi \gamma^{cd} \psi \epsilon_{abcd} \right) + \beta \left( 2 R_{ab} F^{cd} \epsilon_{abcd} \right) \quad (3.7)
\]

The supersymmetry invariance of the full Lagrangian requires

\[
\delta_\epsilon \mathcal{L}_{\text{full}} \equiv \iota_\epsilon \mathcal{L}_{\text{full}} = \iota_\epsilon d\mathcal{L}_{\text{full}} + d(\iota_\epsilon \mathcal{L}_{\text{full}}) = 0. \quad (3.8)
\]

Naturally, the boundary terms (3.5) that we have introduced do not affect the bulk and the supersymmetry invariance in the bulk is still satisfied such that \( \iota_\epsilon d\mathcal{L}_{\text{full}} = 0 \). Then, the supersymmetry invariance of the full Lagrangian \( \mathcal{L}_{\text{full}} \) requires to verify the condition \( \iota_\epsilon (\mathcal{L}_{\text{full}}) |_{\partial \mathcal{M}} = 0 \). In particular, we have

\[
\iota_\epsilon (\mathcal{L}_{\text{full}}) = \epsilon_{abcd} \iota_\epsilon \left( R^{ab} \right) V^c V^d + 4 \bar{\psi} V^a \gamma_5 \rho + 4 \bar{\psi} V^a \gamma_5 \iota_\epsilon (\rho) \\
+ \alpha \left\{ 2\iota_\epsilon (\sigma) \gamma_5 \rho + 2\iota_\epsilon \gamma_5 \iota_\epsilon (\rho) + \frac{1}{4} \left[ \iota_\epsilon \left( R^{ab} \right) \chi \gamma^{cd} \psi + R_{ab} \chi \gamma^{cd} \epsilon \right] \epsilon_{abcd} \right\} \\
+ 2\beta \left[ \iota_\epsilon \left( R^{ab} \right) F^{cd} \epsilon_{abcd} + R_{ab} \iota_\epsilon \left( F^{cd} \right) \epsilon_{abcd} \right]. \quad (3.9)
\]
In general, \( \iota_\epsilon \mathcal{L}_{\text{full}} \) is not zero; however, its projection on the boundary should be zero. One can see that the field equations acquire non-trivial boundary contributions coming not only from the boundary Lagrangian but also from the bulk Lagrangian (from the total differentials originating from partial integration), which implies the following constraints on the boundary:

\[
\begin{align*}
\mathcal{R}^{ab}_{\partial \mathcal{M}} &= 0, \\
\mathcal{F}^{ab}_{\partial \mathcal{M}} &= -\frac{1}{2\beta} \left( V^a V^b + \frac{\alpha}{4} \chi^{ab} \psi \right)_{\partial \mathcal{M}}, \\
\rho_{\partial \mathcal{M}} &= 0, \\
\sigma_{\partial \mathcal{M}} &= -\frac{2}{\alpha} (V^a \gamma^a \psi)_{\partial \mathcal{M}}.
\end{align*}
\]  

(3.10)

Upon use of (3.10) we find that

\[
\iota_\epsilon (\mathcal{L}_{\text{full}}) |_{\partial \mathcal{M}} = 0 \quad \forall \alpha, \beta.
\]  

(3.11)

Thus the supersymmetry invariance of the full Lagrangian is restored in the presence of a non-trivial boundary. The full Lagrangian can be then written in terms of (3.10) à la MacDowell-Mansouri [39] for \( \alpha = 4 \) and \( \beta = \frac{1}{2} \),

\[\mathcal{L}_{\text{full}} = \mathcal{R}^{ab} F^{cd} \epsilon_{abcd} + 8 \bar{\Xi} \gamma_5 \rho, \tag{3.12}\]

where we have defined

\[F^{ab} = \mathcal{F}^{ab} + V^a V^b + \bar{\chi} \gamma^{ab} \psi, \tag{3.13}\]

\[\Xi = \sigma + \frac{1}{2} V^a \gamma^a \psi. \tag{3.14}\]

It is important to point out that the supersymmetry invariance of the flat supergravity Lagrangian on the boundary can be restored by adding new gauge fields enlarging the Poincaré symmetry. In particular, this can be done for arbitrary values of \( \alpha \) and \( \beta \). Nevertheless, the full Lagrangian à la MacDowell-Mansouri appears for specific values of the constants \( \alpha \) and \( \beta \) (\( \alpha = 4 \) and \( \beta = \frac{1}{2} \)).

One could think that a dynamical Lagrangian can be recovered from the original MacDowell-Mansouri Lagrangian constructed from the \( OSp(4|1) \) covariant supercurvatures. Although the bulk Lagrangians can be related through a flat limit, the vanishing cosmological constant limit cannot be naively applied in presence of a non-trivial boundary. Indeed, in the flat limit, the only term that remains is a boundary topological term and corresponds to a Gauss-Bonnet term. This is mainly due to the presence of the \( \ell \) parameter (length scale) in every term of the bulk Lagrangian of the \( OSp(4|1) \) supergroup. Thus, in order to obtain the pure supergravity action without cosmological constant term using the MacDowell-Mansouri formalism, it is necessary to enlarge the symmetry. This enlargement, as we have shown, does not modify the bulk Lagrangian but affects only the boundary allowing to restore the supersymmetry invariance.

Let us observe that, interestingly, for \( \alpha = 4 \) and \( \beta = \frac{1}{2} \) we also find out that the curvatures (3.10) reproduce the minimal Maxwell covariant super curvatures [40,44], namely (3.13), (3.14), and

\[
\begin{align*}
R^{ab} &= \mathcal{R}^{ab}, \\
\Psi &= \rho, \\
R^a &= DV^a - \frac{1}{2} \bar{\psi} \gamma^a \psi, \tag{3.15}\end{align*}
\]
which satisfy the Bianchi identities

\begin{align*}
DR^{ab} &= 0, \\
DF^{ab} &= 2R^c_A A^{cb} + 2R^a V^b + \bar{\Xi} \gamma^{ab} \psi - \bar{\chi} \gamma^{ab} \Psi, \\
D\Psi &= \frac{1}{4} R^b_{\gamma ab} \psi, \\
D\Xi &= \frac{1}{4} R^b_{ab} \gamma^b \chi + \frac{1}{2} R^a \gamma_a \psi - \frac{1}{2} V^a \gamma_a \Psi, \\
D\bar{\phi} &= R^a_{b} V^b + \bar{\psi} \gamma^a \Psi.
\end{align*}

(3.16)

In particular, the super-Maxwell curvatures vanish at the boundary:

\begin{align*}
R^{ab}|_{\partial M} &= 0, \\
F^{ab}|_{\partial M} &= 0, \\
\Psi|_{\partial M} &= 0, \\
\Xi|_{\partial M} &= 0.
\end{align*}

(3.17)

Then, the full MacDowell-Mansouri like Lagrangian (3.12) can be rewritten as

\[ L_{\text{full}} = R^{ab} F^{cd} \epsilon_{abcd} + 8 \bar{\Xi} \gamma^a \Psi \]

(3.18)

in terms of the Maxwell super curvatures (3.13), (3.14), and (3.15).

Let us observe that the full Lagrangian obtained here cannot be directly obtained as a flat limit of the \( OSp(4|1) \) one [27] due to the presence of new gauge fields. Nevertheless, one could apply our approach to the supergravity Lagrangian in presence of the cosmological constant (and thus of a length scale \( \ell \)) by adding the extra gauge fields not only to the boundary Lagrangian but also to the bulk Lagrangian, such that the flat limit \( \ell \to \infty \) reproduces the full Lagrangian (3.12) obtained here.

Before studying the supersymmetry invariance of a supergravity theory with a generalized cosmological constant in the presence of a non-trivial boundary, let us provide the rheonomic parametrization of the Maxwell super curvatures and the supersymmetry transformation laws.

### 3.1 Maxwell supersymmetry

The minimal Maxwell superalgebra has been introduced in [40] in order to describe a generalized four-dimensional superspace in the presence of a constant abelian supersymmetric field-strength background. Such superalgebra has the particularity to extend the Maxwell symmetry \( \{ J_{ab}, P_a, Z_{ab} \} \) by adding two spinorial generators \( \{ Q_a, \Sigma_{\alpha} \} \). In particular, the generators satisfy the following
non-vanishing (anti)commutation relations:

\[
[J_{ab}, J_{cd}] = \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac} + \eta_{ad} J_{bc},
\]

\[
[J_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b,
\]

\[
[P_a, P_b] = Z_{ab},
\]

\[
[J_{ab}, Z_{cd}] = \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc},
\]

\[
[J_{ab}, Q_{\alpha}] = -\frac{1}{2} (\gamma_{ab}) Q_{\alpha} ,
\]

\[
[J_{ab}, \Sigma_{\alpha}] = -\frac{1}{2} (\gamma_{ab}) \Sigma_{\alpha} ,
\]

\[
[P_a, Q_{\alpha}] = -\frac{1}{2} (\gamma_a \Sigma) Q_{\alpha} ,
\]

\[
\{Q_{\alpha}, Q_{\beta}\} = (\gamma_{\alpha\beta}) Z_{ab} .
\]

Concerning the purely bosonic level, the Maxwell algebra has been introduced in [45, 46]. Such symmetry and its generalizations have been recently useful to recover General Relativity from Chern-Simons (CS) and Born-Infeld (BI) gravity formalisms [47–51]. More recently, there has been a new interest in exploring the three-dimensional Maxwell CS gravity [52–56]. At the supersymmetric level, the super-Maxwell family also appears in three spacetime dimensions allowing to reproduce CS supergravity models [57–59]. Further generalizations of the minimal Maxwell superalgebra can be found in [60–65] with diverse applications.

In the geometric approach, the most general ansatz for the super-Maxwell curvatures in the supervielbein basis \( \{V^a, \psi\} \) of superspace is given by

\[
R^{ab} = R_{\quad cd}^{ab} V^c \wedge V^d + \bar{\Theta}_{\quad c}^{ab} \psi \wedge V^c + \lambda_1 \bar{\psi} \wedge \gamma^{ab} \psi ,
\]

\[
R^a = R_{\quad bc}^a V^b \wedge V^c + \bar{\Theta}_{\quad c}^a \psi \wedge V^b + \lambda_2 \bar{\psi} \wedge \gamma^a \psi ,
\]

\[
F^{ab} = F_{\quad cd}^{ab} V^c \wedge V^d + \bar{\Lambda}_{\quad c}^{ab} \psi \wedge V^c + \lambda_3 \bar{\psi} \wedge \gamma^{ab} \psi ,
\]

\[
\Psi = \Psi_{\quad ab} V^a \wedge V^b + \lambda_4 \bar{\gamma}_a \psi \wedge V^a + \Omega_{\quad \alpha\beta} \psi^\alpha \wedge \psi^\beta ,
\]

\[
\Xi = \Xi_{\quad ab} V^a \wedge V^b + \lambda_5 \bar{\gamma}_a \psi \wedge V^a + \bar{\Omega}_{\quad \alpha\beta} \psi^\alpha \wedge \psi^\beta ,
\]

being the \( \lambda_i \)’s \( (i = 1, 2, \ldots, 5) \) constant parameters. Considering the on-shell condition \( R^a = 0 \) and studying the various sectors of the on-shell Bianchi identities (3.16) in superspace, one can show that the full set of super curvatures are parameterized as follows:

\[
R^{ab} = R_{\quad cd}^{ab} V^c V^d + \bar{\Theta}_{\quad c}^{ab} \psi V^c ,
\]

\[
R^a = 0 ,
\]

\[
F^{ab} = F_{\quad cd}^{ab} V^c V^d + \bar{\Lambda}_{\quad c}^{ab} \psi V^c ,
\]

\[
\Psi = \Psi_{\quad ab} V^a V^b ,
\]

\[
\Xi = \lambda_5 \bar{\gamma}_a \psi V^a ,
\]

with

\[
\bar{\Theta}_{\quad c}^{ab} = e^{abde} (\bar{\Psi}_{\quad cd\gamma_5 e\gamma_5} + \bar{\Psi}_{\quad ec\gamma_5 a\gamma_5} - \bar{\Psi}_{\quad de\gamma_5 c\gamma_5} ) .
\]
Such parametrization allows us to obtain explicitly the supersymmetry transformation laws. In particular, in the rheonomic approach, a 1-form superfield transformation on spacetime takes the form
\[ \delta \mu^A = (\nabla \epsilon)^A + \iota_\epsilon R^A, \]
where \( \epsilon^A \) are the gauge parameters and \( R^A \) are the parameterized super curvatures. Then, restricting us to supersymmetry transformation and considering the parametrization \((3.21)\) of the super-Maxwell curvatures, we find the following supersymmetry transformation laws:

\[
\begin{align*}
\delta \epsilon \omega^{ab} &= 2 \Theta^{ab} \epsilon V^c, \\
\delta \epsilon V^a &= \bar{\epsilon} \gamma^a \psi, \\
\delta \epsilon A^{ab} &= \bar{\epsilon} \gamma^{ab} \epsilon + 2 \bar{\lambda}^{ab} \epsilon V^c, \\
\delta \epsilon \psi &= D \epsilon, \\
\delta \epsilon \chi &= V^a \gamma_a \epsilon,
\end{align*}
\]

where we have set \( \lambda_5 = -\frac{1}{2} \).

4 Supergravity with generalized cosmological constant in presence of a non-trivial boundary

The presence of additional bosonic generators has been recently related to a generalization of the cosmological constant \([29, 44, 66, 67]\). Nevertheless, as we have previously seen in Section 3, the introduction of additional gauge fields does not necessarily imply the presence of a cosmological constant term in a (super)gravity action. In order to include a cosmological constant contribution to our supergravity model, it is necessary to switch on an explicit scale. In particular, we can write the following super curvatures:\(^3\)

\[
\begin{align*}
\mathfrak{R}^{ab} &\equiv d\omega^{ab} + \omega^a \omega^{cb} + \frac{1}{2\ell} \bar{\psi} \gamma^{ab} \psi, \\
\mathfrak{R}^a &\equiv DV^a + \frac{1}{\ell^2} A^a V^b - \frac{1}{2\ell} \bar{\psi} \gamma^a \psi - \frac{1}{\ell} \bar{\psi} \gamma^a \chi - \frac{1}{2\ell^2} \bar{\chi} \gamma^a \chi, \\
\rho &\equiv D \psi, \\
\mathfrak{F}^{ab} &\equiv DA^{ab} + V^a V^b + \bar{\chi} \gamma^{ab} \psi + \frac{1}{\ell^2} A^a A^{cb} + \frac{1}{2\ell} \bar{\chi} \gamma^{ab} \chi, \\
\Omega &\equiv D \chi + \frac{1}{2} V^a \gamma_a \psi + \frac{1}{2\ell} V^a \gamma_a \chi + \frac{1}{4\ell} A^a A_{cb} \gamma^{cb} + \frac{1}{4\ell^2} A^{ab} \gamma_{ab} \chi,
\end{align*}
\]

being, as usual, \( D = d + \omega \) the Lorentz covariant derivative. The next step is the explicit construction of the bulk Lagrangian.

\(^3\)Notice that these super curvatures can be obtained by considering the same scale-weight for the gauge fields as in the flat case. The main difference with the previous curvatures consists in the explicit presence of the length parameter \( \ell \).
4.1 Rheonomic construction of the bulk Lagrangian

The most general ansatz for the geometric bulk Lagrangian can be written as

\[ \mathcal{L} = \mu(4) + R^A\mu^{(2)}_A + R^A R^B \mu^{(0)}_{AB}, \]

where the upper index \((p)\) denotes the degree of the related \(p\)-forms. Here, the \(R^A\)'s are the super curvatures defined by eq. (4.1), which are invariant under the following rescaling:

\[ \omega^{ab} \rightarrow \omega^{ab}, \quad V^a \rightarrow \omega V^a, \quad A^{ab} \rightarrow \omega^2 A^{ab}, \quad \psi \rightarrow \omega^{1/2} \psi, \quad \chi \rightarrow \omega^{3/2} \chi. \]

In particular, the Lagrangian scales with \(\omega^2\), being \(\omega^2\) the scale-weight of the EH term. On the other hand, since we are interested in the construction of the bulk Lagrangian, we can set \(R^A R^B \mu^{(0)}_{AB} = 0\), since terms of this type correspond to boundary contributions. Then, considering the explicit form of the super curvatures (4.1) and applying the parity conservation law, we are left with the following Lagrangian:

\[ \mathcal{L} = \epsilon_{abcd} R^{ab} V^c V^d + \frac{\alpha_1}{\ell^2} \epsilon_{abcd} \tilde{\psi} V^c V^d + \frac{\alpha_2}{\ell} \tilde{\psi} \gamma_a \gamma_5 \rho V^a + \frac{\alpha_3}{\ell} \tilde{\psi} \gamma_a \gamma_5 \Omega V^a + \frac{\alpha_4}{\ell^2} \tilde{\psi} \gamma_a \gamma_5 \rho V^a + \frac{\alpha_5}{\ell^2} \epsilon_{abcd} V^a V^b V^c V^d + \frac{\alpha_6}{\ell} \epsilon_{abcd} \tilde{\psi} \gamma^{ab} \psi V^c V^d \]

\[ + \frac{\alpha_7}{\ell^2} \epsilon_{abcd} \psi \gamma^{ab} \chi V^c V^d + \frac{\alpha_8}{\ell^2} \epsilon_{abcd} \bar{\chi} \gamma^{ab} \chi V^c V^d, \]

where we have set the coefficient of the first term in (4.4) to 1. The \(\alpha_i\)'s constants are dimensionless parameters which can be determined from the equations of motion. Indeed, considering the variation of the Lagrangian with respect to the gauge fields we obtain the following field equations:

\[ \delta \omega \mathcal{L} = \delta A \mathcal{L} = 0 \rightarrow \epsilon_{abcd} \tilde{R}^c V^d = 0, \]

\[ \delta V \mathcal{L} = 0 \rightarrow 2 \epsilon_{abcd} \left( \tilde{R}^{ab} + \tilde{S}^{ab} \right) V^c + 4 \left( \tilde{\psi} + \frac{1}{\ell} \chi \right) \gamma_d \gamma_5 \rho + \frac{4}{\ell} \left( \tilde{\psi} + \frac{1}{\ell} \chi \right) \gamma_d \gamma_5 \Omega = 0, \]

\[ \delta \psi \mathcal{L} = \delta \chi \mathcal{L} = 0 \rightarrow 8 V^a \gamma_a \gamma_5 \left( \rho + \frac{1}{\ell} \Omega \right) + 4 \gamma_a \gamma_5 \left( \psi + \frac{1}{\ell} \chi \right) \tilde{R}^a = 0, \]

where we have set

\[ \alpha_1 = 1, \]

\[ \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 4, \]

\[ \alpha_6 = \alpha_7 = \alpha_9 = -\frac{1}{2}, \]

\[ \alpha_8 = -1. \]

Let us observe that the equations of motion (4.5) are a generalization of the \(OSp(4|1)\) ones. In particular, the first equation corresponds to the vanishing of the generalized supertorsion \(\tilde{R}^a\). With these particular values of the constants, the equations of motion of the Lagrangian admit an \(AdS\) vacuum solution with a generalized cosmological constant.
Thus, the bulk Lagrangian can be rewritten in terms of Lorentz-type curvatures as follows:

\[ \mathcal{L}_{\text{bulk}} = \epsilon_{abcd} R^{cd}_{\text{ab}} V^a + \frac{1}{\ell^2} \epsilon_{abcd} \tilde{F}^{ab}_{cd} V^d + 4 \tilde{\psi}_a \gamma_5 \rho V^a + \frac{4}{\ell} \tilde{\psi}_a \gamma_5 \Phi V^a + \frac{4}{\ell^2} \tilde{\chi}_a \gamma_5 \tilde{\chi}_a \Phi V^a + \frac{1}{\ell^2} \tilde{\chi}_a \gamma_5 \tilde{\chi}_a \Phi V^a, \]

(4.7)

with

\[ R^{ab} = d\omega^{ab} + \omega^a \omega^b, \]

\[ R^a = D V^a + \frac{1}{\ell^2} A^a_V - 2 \tilde{\gamma}^a \psi - \frac{1}{\ell} \tilde{\gamma}^a \chi - \frac{1}{2\ell^2} \tilde{\chi}^a \tilde{\chi}_a, \]

\[ F^{ab} = D A^{ab} + \frac{1}{\ell^2} A^a_{\omega^b} + \frac{1}{4\ell} A^a_{\omega^b} \psi + \frac{1}{4\ell} A^a_{\omega^b} \chi. \]

(4.8)

Let us note that the presence of the length parameter \( \ell \) in the super curvatures allows to introduce, in an alternative way, a generalized supersymmetric cosmological constant term in the Lagrangian. Such Lagrangian can be seen as a deformation of the usual supergravity Lagrangian for the \( \text{osp}(4|1) \) superalgebra. Interestingly, as for the case of the \( \text{osp}(4|1) \) supergravity, the vanishing cosmological constant limit \( \ell \to \infty \) leads to the flat supergravity Lagrangian. However, as we have discussed in Section 3, the supersymmetry invariance of flat supergravity on a manifold with boundary is recovered by adding non-trivial boundary (topological) terms. Then, it seems that the flat limit in presence of a boundary requires to consider a deformation of the full Lagrangian for the \( \text{osp}(4|1) \) superalgebra. Thus, the bulk Lagrangian (4.7) obtained here seems a good candidate to consider in presence of a boundary. Before studying the explicit boundary contributions required to assure supersymmetry invariance we shall provide the supersymmetry transformation laws under which the bulk Lagrangian (4.7) is invariant.

The supersymmetry transformations can be obtained from the rheonomic parametrization of the super curvatures which are determined from the study of the Bianchi identities of the Lorentz-type curvatures (4.8). In particular, the super curvatures (4.8) fulfill the following Bianchi identities:

\[ DR^{ab} = 0, \]

\[ D \tilde{\omega}^a = R^a_b V^b + \frac{1}{\ell^2} \tilde{F}^a_b V^b - \frac{1}{\ell^2} A^a_b \tilde{\omega}^b + \tilde{\gamma}^a \rho + \frac{1}{\ell^2} \tilde{\chi}^a \tilde{\chi} + \frac{1}{\ell^2} \tilde{\chi}^a \Phi + \frac{1}{\ell^2} \tilde{\chi}^a \Phi, \]

\[ D \rho = \frac{1}{4} R^{ab} \omega^a \psi, \]

\[ DF^{ab} = 2 R^a_c A^{cb} + \frac{2}{\ell^2} \tilde{F}^a_c A^{cb}, \]

\[ D \Phi = \frac{1}{4} R^{ab} \omega^a \Phi + \frac{1}{4} \tilde{F}^{ab} \omega^b \psi - \frac{1}{4} A^{ab} \omega^b \rho + \frac{1}{4} \tilde{F}^{ab} \omega^b \chi - \frac{1}{4\ell^2} A^{ab} \omega^b \Phi. \]

(4.9)

Notice that the supercurvatures above are defined in a superspace larger than the ordinary one. In the following, we will ask the curvatures parametrization to be well defined in ordinary superspace.
by exploiting the rheonomic approach. One can show that the Bianchi identities \((4.9)\) are satisfied by parameterizing (on-shell) the supercurvatures in the following way:

\[
\begin{align*}
R^{ab} &= \mathcal{R}_{cd}^{ab} V^c V^d + \Pi^{ab}_{\ c} \psi V^c + \varsigma_1 \bar{\psi} \gamma^{ab} \psi, \\
\mathcal{R}^a &= 0, \\
\mathcal{F}^{ab} &= \mathcal{F}_{cd}^{ab} V^c V^d + \Delta^{ab}_{\ c} \psi V^c, \\
\rho &= \rho_{ab} V^a V^b + \varsigma_2 \gamma^a \psi V^a, \\
\Phi &= \Phi_{ab} V^a V^b + \varsigma_3 \gamma^a \psi V^a,
\end{align*}
\]

where

\[
\begin{align*}
\Pi^{ab}_{\ c} &= \epsilon^{abde} \left( \bar{\rho}_{cd} \gamma^5 \gamma_e + \bar{\rho}_{ec} \gamma^5 \gamma_d - \bar{\rho}_{de} \gamma^5 \gamma_c \right), \\
\Delta^{ab}_{\ c} &= \epsilon^{abde} \left( \bar{\Phi}_{cd} \gamma^5 \gamma_e + \bar{\Phi}_{ec} \gamma^5 \gamma_d - \bar{\Phi}_{de} \gamma^5 \gamma_c \right), \\
\varsigma_1 &= -\varsigma_2 - \frac{1}{\ell}\varsigma_3,
\end{align*}
\]

and where we have set \(\mathcal{R}^a = 0\) (on-shell condition). Such parametrization of the supercurvatures in ordinary superspace provides us with the supersymmetry transformation laws for the 1-form gauge fields, which read as follows:

\[
\begin{align*}
\delta_\epsilon \omega^{ab} &= 2 \Pi^{ab}_{\ c} \epsilon V^c + 2 \varsigma_1 \bar{\epsilon} \gamma^{ab} \psi, \\
\delta_\epsilon V^a &= \bar{\epsilon} \gamma^a \psi - \frac{1}{\ell} \bar{\chi} \gamma^a \epsilon, \\
\delta_\epsilon A^{ab} &= 2 \Delta^{ab}_{\ c} \epsilon V^c, \\
\delta_\epsilon \psi &= D_\epsilon + \varsigma_2 \gamma^a \epsilon V^a, \\
\delta_\epsilon \chi &= \varsigma_3 \gamma^a \epsilon V^a.
\end{align*}
\]

It is important to point out that the spacetime Lagrangian \((4.7)\) is invariant up to boundary terms under the supersymmetry transformations \((4.12)\) of the gauge fields on spacetime. If the spacetime background has a non-trivial boundary, we have to check explicitly the supersymmetry invariance.

### 4.2 Supersymmetry invariance in presence of a boundary

Let us now consider the supergravity theory previously introduced in presence of a non-trivial spacetime boundary, and let us study the supersymmetry invariance of it. In particular, we shall see that appropriate boundary terms are required in order to restore the supersymmetry invariance of the full Lagrangian given by the bulk Lagrangian \((4.7)\) plus boundary contributions. Although the supersymmetry invariance in the bulk is satisfied on-shell, the invariance of the Lagrangian when a boundary is present, is not trivially satisfied:

\[\iota_\epsilon \mathcal{L}_{\text{bulk}}|_{\partial \mathcal{M}} \neq 0.\]

Then, for recovering the supersymmetry invariance of the theory, it is necessary to modify the bulk Lagrangian by adding suitable boundary terms. The only possible boundary contributions
compatible with parity and Lorentz invariance are given by
\[
d\left(\omega^{ab} R^{cd} + \omega^{a}_f \omega^{f} \omega^{cd}\right) \epsilon_{abcd} = \epsilon_{abcd} R^{ab} R^{cd},
\]
\[
d\left(A^{ab} R^{cd} + \omega^{a} \omega^{f} A^{cd} + 2 \omega^{a} A^{f} \omega^{cd} + \frac{1}{\ell^2} A^{a}_f A^{f} \omega^{cd} + \frac{2}{\ell^2} \omega^{a} A^{f} A^{cd}\right)
\]
\[
\frac{1}{\ell^2} A^{ab} R^{cd} + \frac{1}{\ell^2} A^{a}_f A^{f} A^{cd} \epsilon_{abcd} = \epsilon_{abcd} \left(2 R^{ab} R^{cd} + \frac{1}{\ell^2} R^{ab} R^{cd}\right),
\]
\[
d\left(\bar{\psi} \gamma^5 \rho + \frac{1}{8} R^{ab} \bar{\psi} \gamma^c d \psi \epsilon_{abcd}\right) = \rho \gamma_5 \rho + \frac{1}{8} R^{ab} \bar{\psi} \gamma^c d \psi \epsilon_{abcd},
\]
\[
\left(\bar{\psi} \gamma_5 \Phi + \bar{\chi} \gamma_5 \Phi + \frac{1}{4} R^{ab} \bar{\psi} \gamma^c d \chi \epsilon_{abcd} + \frac{1}{8} R^{ab} \bar{\psi} \gamma^c d \psi \epsilon_{abcd}\right)
\]
\[
+ \frac{1}{4} R^{ab} \bar{\psi} \gamma^c d \chi \epsilon_{abcd} + \frac{1}{8} \bar{\psi} \gamma^2 \phi \chi \epsilon_{abcd} + \frac{1}{8} \bar{\psi} \gamma^3 \phi \chi \epsilon_{abcd}.
\]

The boundary terms (4.14) allow us to write the following boundary Lagrangian:
\[
\mathcal{L}_{bdy} = \lambda \epsilon_{abcd} R^{ab} R^{cd} + \pi \left(\bar{\rho} \gamma_5 \rho + \frac{1}{8} R^{ab} \bar{\psi} \gamma^c d \psi \epsilon_{abcd}\right) + \mu \epsilon_{abcd} \left(2 R^{ab} R^{cd} + \frac{1}{\ell^2} R^{ab} R^{cd}\right)
\]
\[
\quad + \nu \left(2 \bar{\rho} \gamma_5 \Phi + \bar{\Phi} \gamma_5 \Phi + \frac{1}{4} R^{ab} \bar{\psi} \gamma^c d \chi \epsilon_{abcd} + \frac{1}{8} R^{ab} \bar{\psi} \gamma^c d \psi \epsilon_{abcd}\right)
\]
\[
\quad + \frac{1}{4} R^{ab} \bar{\psi} \gamma^c d \chi \epsilon_{abcd} + \frac{1}{8} \bar{\psi} \gamma^2 \phi \chi \epsilon_{abcd} + \frac{1}{8} \bar{\psi} \gamma^3 \phi \chi \epsilon_{abcd},
\]
where \(\lambda, \pi, \mu, \nu\) are constant parameters. In order to have a consistently defined limit \(\ell \to \infty\) (flat limit) at the level of the full Lagrangian, one should drop out the terms involving positive powers of \(\ell\). From the Lagrangian (4.15) we see that all terms have scale-weight \(\omega^2\) except those proportional to \(\lambda\) and \(\pi\). Thus, in order to have an appropriately scaled boundary Lagrangian one should define new (dimensionless) constants, \(\lambda'\) and \(\pi'\), such that \(\lambda = \ell^2 \lambda'\) and \(\pi = \ell \pi'\), which implies positive powers of \(\ell\). Then, the terms proportional to \(\lambda\) and \(\pi\) must be dropped out.

In this way, we are left with
\[
\mathcal{L}_{bdy} = \mu \epsilon_{abcd} \left(2 R^{ab} R^{cd} + \frac{1}{\ell^2} R^{ab} R^{cd}\right)
\]
\[
\quad + \nu \left(2 \bar{\rho} \gamma_5 \Phi + \bar{\Phi} \gamma_5 \Phi + \frac{1}{4} R^{ab} \bar{\psi} \gamma^c d \chi \epsilon_{abcd} + \frac{1}{8} R^{ab} \bar{\psi} \gamma^c d \psi \epsilon_{abcd}\right)
\]
\[
\quad + \frac{1}{4} R^{ab} \bar{\psi} \gamma^c d \chi \epsilon_{abcd} + \frac{1}{8} \bar{\psi} \gamma^2 \phi \chi \epsilon_{abcd} + \frac{1}{8} \bar{\psi} \gamma^3 \phi \chi \epsilon_{abcd}.
\]
Thus, let us consider the following full Lagrangian:

\[
\mathcal{L}_{\text{full}} = \mathcal{L}_{\text{bulk}} + \mathcal{L}_{\text{bdy}}
\]

\[
= \epsilon_{abcd} R^{ab} V^c V^d + \frac{1}{\ell^2} \epsilon_{abcd} \bar{\psi}^{ab} V^c V^d + 4 \bar{\psi} \gamma_a \gamma_5 \rho V^a + 4 \bar{\chi} \gamma_5 \rho V^a
\]

\[
+ \frac{4}{\ell} \bar{\psi} \gamma_a \gamma_5 \Phi V^a + \frac{4}{\ell^2} \bar{\chi} \gamma_a \gamma_5 \Phi V^a + \frac{4}{\ell} \bar{\chi} \gamma_5 \rho V^a
\]

\[
+ \frac{1}{2\ell^2} \epsilon_{abcd} V^a V^b V^c V^d + \frac{1}{\ell} \epsilon_{abcd} \bar{\psi} \gamma_{ab} \bar{\psi} V^c V^d + \frac{2}{\ell^2} \epsilon_{abcd} \bar{\chi} \gamma_{ab} \bar{\psi} V^c V^d
\]

\[
+ \frac{1}{\ell^2} \epsilon_{abcd} \bar{\chi} \gamma_{ab} \bar{\psi} V^c V^d + \mu \epsilon_{abcd} \left( 2 R^{ab} \bar{\cal F}^{cd} + \frac{1}{\ell^2} R^{ab} \bar{\psi} \gamma^c \gamma^d \bar{\psi} \epsilon_{abcd} \right)
\]

\[
+ \nu \left( 2 \bar{\rho} \gamma_5 \Phi + \frac{1}{\ell} \bar{\phi} \gamma_5 \Phi + \frac{1}{4} R^{ab} \bar{\psi} \gamma_{cd} \bar{\psi} \epsilon_{abcd} + \frac{1}{8 \ell} R^{ab} \bar{\cal F}^{cd} \bar{\psi} \gamma^c \gamma^d \bar{\psi} \epsilon_{abcd} \right)
\]

\[
\text{subject to } \ell \epsilon_{abcd} \bar{\cal F}^{cd} \big|_{\partial \mathcal{M}} = 0.
\]

Now we have to verify the supersymmetry invariance of the full Lagrangian. Clearly, \( \epsilon \big( \mathcal{L}_{\text{full}} \big) = 0 \), since the boundary terms are total derivatives. Then, considering the condition

\[
\delta \epsilon \mathcal{L}_{\text{full}} = \epsilon \big( \delta \mathcal{L}_{\text{full}} \big) + d (\epsilon \mathcal{L}_{\text{full}}) = 0,
\]

we have just to verify that \( \epsilon \big( \mathcal{L}_{\text{full}} \big) \big|_{\partial \mathcal{M}} = 0 \) is satisfied on the boundary. Considering (4.17), we have

\[
\epsilon (\mathcal{L}_{\text{full}}) = \epsilon_{abcd} \epsilon^{ab} \left( R^{ab} + \frac{1}{\ell^2} \bar{\cal F}^{ab} \right) V^c V^d + 4 \epsilon V^a \gamma_a \gamma_5 \rho + 4 \bar{\psi} V^a \gamma_a \gamma_5 \epsilon (\rho)
\]

\[
+ \frac{4}{\ell} \bar{\psi} V^a \gamma_a \gamma_5 \Phi + \frac{4}{\ell} \bar{\psi} V^a \gamma_5 \epsilon (\Phi) + \frac{4}{\ell} \bar{\chi} V^a \gamma_a \gamma_5 \epsilon (\Phi)
\]

\[
+ \frac{4}{\ell} \bar{\chi} V^a \gamma_5 \epsilon (\rho) + \left( \frac{2}{\ell} \bar{\psi} \gamma_{ab} \bar{\psi} V^c V^d + \frac{2}{\ell^2} \epsilon_{abcd} \bar{\chi} \gamma_{ab} \bar{\psi} V^c V^d \right) \epsilon_{abcd}
\]

\[
+ \mu \epsilon_{abcd} \epsilon^{ab} \left( 2 R^{ab} + \frac{1}{\ell^2} \bar{\cal F}^{ab} \right) \bar{\cal F}^{cd} + \mu \epsilon_{abcd} \left( 2 R^{ab} + \frac{1}{\ell^2} \bar{\cal F}^{ab} \right) \epsilon_{(abcd)}
\]

\[
+ 2 \nu \left[ \epsilon \left( \bar{\rho} \gamma_5 \Phi + \bar{\phi} \gamma_5 \epsilon (\Phi) + \frac{1}{\ell} \epsilon (\Phi) \gamma_5 \Phi \right) \right] + \nu \epsilon_{abcd} \left[ \frac{1}{4} \epsilon^{ab} \left( R^{ab} + \frac{1}{\ell^2} \bar{\cal F}^{ab} \right) \bar{\psi} \gamma_{cd} \gamma \bar{\psi} + \frac{1}{4} \epsilon^{ab} \left( R^{ab} + \frac{1}{\ell^2} \bar{\cal F}^{ab} \right) \bar{\cal F}^{cd} \bar{\psi} \right].
\]
Then, one can prove that \( \delta_{\text{full}} \bigg|_{\partial \mathcal{M}} = 0 \) leads to the following constraints on the boundary:

\[
\begin{align*}
R^{ab}|_{\partial \mathcal{M}} &= - \frac{\nu}{16 \mu \ell} \left( \bar{\psi} \gamma^{ab} \psi \right)_{\partial \mathcal{M}}, \\
F^{ab}|_{\partial \mathcal{M}} &= - \frac{1}{2 \mu} \left( V^a V^b + \frac{\nu}{4} \bar{\chi} \gamma^{ab} \psi + \frac{\nu}{8 \ell} \bar{\gamma} \gamma^{ab} \chi \right)_{\partial \mathcal{M}}, \\
\rho|_{\partial \mathcal{M}} &= 0, \\
\Phi|_{\partial \mathcal{M}} &= - \frac{2}{\nu} \left( V^a \gamma_a \psi + \frac{1}{\ell} V^a \gamma_a \chi \right)_{\partial \mathcal{M}}.
\end{align*}
\] (4.20)

Then, upon use of (4.20) we get

\[
i_\epsilon \left( L_{\text{full}} \right) |_{\partial \mathcal{M}} = 0 \iff \frac{\nu}{8 \mu} + \frac{4}{\nu} = 2.
\] (4.21)

This condition can be written as

\[
\nu = 8 \mu (1 + h), \quad h^2 = 1 - \frac{1}{2 \mu}; \quad (\nu \neq 0 \Rightarrow h \neq -1).
\] (4.22)

One can see that setting \( h = 0 \), we obtain

\[
\mu = \frac{1}{2} \quad \Rightarrow \quad \nu = 4.
\] (4.23)

Remarkably, with these values for \( \mu \) and \( \nu \), the full Lagrangian (4.17) can be written à la MacDowell-Mansouri as

\[
L_{\text{full}} = R^{ab} \bar{\sigma}^{cd} \epsilon_{abcd} + \frac{1}{2 \ell^2} \bar{\sigma}^{ab} \bar{\sigma}^{cd} \epsilon_{abcd} + 8 \Omega \gamma_5 \rho + \frac{4}{\ell} \Omega \gamma_5 \Omega,
\] (4.24)

written exactly in terms of the curvatures (4.1).

We have thus shown that the supersymmetric boundary terms given in (4.15) allow to recover the supersymmetry invariance of our supergravity model in the presence of a non-trivial boundary. One can notice, using (4.20), that the super curvatures (4.1) vanish at the boundary.

The flat limit \( \ell \to \infty \) is now well defined in the MacDowell-Mansouri formalism. Indeed, unlike the case of the full Lagrangian obtained for \( \text{osp}(4|1) \) supergravity in the presence of a non-trivial boundary \([27]\), the vanishing cosmological constant limit of the full Lagrangian (4.24) reproduces appropriately the flat supergravity model with boundary that we have considered initially. In particular, in this case not only the bulk Lagrangians are well related through the flat limit but also the boundary contributions. Furthermore, one can see that the super-Maxwell curvatures (3.13), (3.14), and (3.15) are recovered as a flat limit of the super curvatures (4.1). It is then natural to expect that the minimal Maxwell superalgebra (3.19) appears as a flat limit of a deformation of the \( \text{osp}(4|1) \) superalgebra.
4.2.1 AdS-Lorentz supersymmetry

The (anti)commutation relations of the superalgebra related to the super curvatures (4.1) are given by

\[
\begin{align*}
[J_{ab}, J_{cd}] &= \eta_{bc}J_{ad} - \eta_{ac}J_{bd} - \eta_{bd}J_{ac} + \eta_{ad}J_{bc}, \\
[J_{ab}, Z_{cd}] &= \eta_{bc}Z_{ad} - \eta_{ac}Z_{bd} - \eta_{bd}Z_{ac} + \eta_{ad}Z_{bc}, \\
[Z_{ab}, Z_{cd}] &= \frac{1}{\ell^2} (\eta_{bc}Z_{ad} - \eta_{ac}Z_{bd} - \eta_{bd}Z_{ac} + \eta_{ad}Z_{bc}), \\
[J_{ab}, P_c] &= \eta_{bc}P_a - \eta_{ac}P_b, \quad [Z_{ab}, P_c] = \frac{1}{\ell^2} (\eta_{bc}P_a - \eta_{ac}P_b), \\
[P_a, P_b] &= Z_{ab},
\end{align*}
\]

with \( a = 0, 1, 2, 3, \alpha = 1, 2, 3, 4 \), and where the generators \( J_{ab}, P_a, Z_{ab}, Q \) and \( \Sigma \) are respectively dual to the 1-form fields \( \omega^{ab}, V^a, A^{ab}, \psi, \) and \( \chi \). Interestingly, the present superalgebra corresponds to an alternative supersymmetric extension of the AdS-Lorentz symmetry (given by eq. (4.25)). Such bosonic algebra, also known as a semisimple extension of the Poincaré symmetry, was first presented in [41, 42]. Then, it was generalized to a family of \( \mathbb{C}_k \) algebras [68, 69] which have been useful to recover diverse (pure) Lovelock theories from CS and BI gravity theories [70–72].

Although it contains two spinorial charges as the minimal AdS-Lorentz superalgebras introduced in [44], the superalgebra given by eqs. (4.25) and (4.26) does not require additional bosonic generators with respect to the ones of the AdS-Lorentz algebra in order to satisfy the Jacobi identities. The closure of this superalgebra is guaranteed by the explicit form of the anticommutators in (4.26). Indeed, any subtle deformation of the aforementioned anticommutators requires to introduce additional bosonic generators as in [33, 44, 73]. On the other hand, it is important to signalize that the supersymmetrization of the AdS-Lorentz algebra is not unique. In particular, a super AdS-Lorentz symmetry with one spinor charge has also been considered in [41, 74, 75].

Note that in the limit \( \ell \to \infty \) the above superalgebra gives exactly the super-Maxwell algebra (3.19), without any auxiliary generator.

As a final remark, it is important to point out that in [29] a supergravity model in presence of a non-trivial boundary for the AdS-Lorentz superalgebra with one spinor charge has been presented. Nevertheless, such construction does not allow for a proper flat limit. In particular, although it is possible to recover a non-standard Maxwell superalgebra from the super AdS-Lorentz one.
considered in [29], the exotic anticommutation relation of the non-standard Maxwell does not allow a proper construction of a supergravity action. Such problematic comes from the fact that the four-momentum generators no longer appear as a result of the anticommutator of the fermionic generators. It seems that, as we have shown here, the only possibility to have a well defined vanishing cosmological constant limit for the AdS-Lorentz supergravity is to consider an additional fermionic charge. Remarkably, such flat limit allows to recover the usual flat supergravity with non-trivial boundary contributions.

5 Discussion

In this paper, we have studied the supersymmetry invariance of flat supergravity in presence of a non-trivial boundary. We have shown that supersymmetry invariance is achieved by adding proper topological terms in which additional gauge fields are considered. We have found that the full Lagrangian can be rewritten à la MacDowell-Mansouri in terms of Maxwell super curvatures. Interestingly, the bulk Lagrangian corresponds to the usual flat supergravity Lagrangian. Although the extra fields only appear in the boundary contributions, they are essential to recover supersymmetry invariance of the full Lagrangian. Furthermore, as the topological term in pure gravity allows to regularize the action, one could argue that the presence of the new gauge fields in the boundary would allow to regularize the supergravity action in the holographic renormalization language.

We have also explored the possibility of introducing a well-defined vanishing cosmological constant limit in a supergravity model in order to recover the Lagrangian obtained in the case of flat spacetime with a non-trivial boundary. As we have discussed, flat supergravity with boundary cannot be naively obtained from \textit{osp}(4|1) supergravity with boundary (MacDowell-Mansouri action). Indeed, this require to consider an enlarged supergravity with a generalized cosmological constant which has been obtained using the rheonomic approach. We have shown that, as in the flat case, supersymmetry invariance requires to add appropriate topological terms. The full supergravity obtained within this procedure can be rewritten in terms of the super curvatures of a particular AdS-Lorentz superalgebra. Remarkably, the flat limit of the full Lagrangian properly reproduces flat supergravity on a manifold with boundary.

It is important to observe that, as in [27,29,33], the full supergravity Lagrangian can be rewritten à la MacDowell-Mansouri. This would suggest, as it was pointed out in the bosonic case [76,77], a superconformal structure which is an interesting additional motivation to our study. On the other hand, our particular approach in which we consider additional gauge fields in the boundary could be useful in order to explore the supersymmetry invariance of supergravities with boundary coupled to matter or in higher dimensions. Indeed, it would be interesting to explore the boundary contributions required to recover supersymmetry invariance of a general matter coupled $\mathcal{N} = 2$ supergravity in four dimensions [78,79]. Naturally, one could consider first a supergravity theory coupled to scalar field. The extension of our results to $\mathcal{N} = 2$ is more subtle and will be studied in a future work. Finally, in view of the recent results presented in [65,80,81], it would be compelling to analyze the role of the additional gauge fields that we have introduced in the boundary in the context of the hidden gauge structure underlying supergravity theories in various dimensions.
6 Acknowledgment

The authors wish to thank L. Andrianopoli, R. D’Auria and M. Trigiante for enlightening discussions and their kind hospitality at DISAT of Politecnico di Torino (Italy), where the main discussion of this work was done. This work was supported by the Chilean FONDECYT Projects N°3170437 (P.C.) and N°3170438 (E.R.).

References

[1] J.W. York, Jr., Role of conformal three geometry in the dynamics of gravitation, Phys. Rev. Lett. 28 (1972) 1082.
[2] G.W. Gibbons, S.W. Hawking, Action Integrals and Partition Functions in Quantum Gravity, Phys. Rev. D 15 (1977) 2752.
[3] J.D. Brown, J.W. York, Jr., Quasilocal energy and conserved charges derived from the gravitational action, Phys. Rev. D 47 (1993) 1407. [gr-qc/9209012].
[4] P. Horava, E. Witten, Eleven-dimensional supergravity on a manifold with boundary, Nucl. Phys. B 475 (1996) 94. [hep-th/9603142].
[5] J.M. Maldacena, The Large N limit of superconformal field theories and supergravity, Int. J. Theor. Phys. 38 (1999) 1113. [hep-th/9711200].
[6] S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Gauge theory correlators from noncritical string theory, Phys. Lett. B 428 (1998) 105. [hep-th/9802109].
[7] E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. 2 (1998) 253. [hep-th/9802150].
[8] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri, Y. Oz, Large-N field theories, string theory and gravity, Phys. Rept. 323 (2000) 183. [hep-th/990511].
[9] E. D’Hoker, D.Z. Freedman, Supersymmetric gauge theories and the AdS/CFT correspondence. [hep-th/0201253].
[10] G. Comp`ere and D. Marolf, Setting the boundary free in AdS/CFT, Class. Quant. Grav. 25 (2008) 195014. arXiv:0805.1902 [hep-th].
[11] A. J. Amsel and G. Comp`ere, Supergravity at the boundary of AdS supergravity, Phys. Rev. D 79 (2009) 085006. arXiv:0901.3609 [hep-th].
[12] V. Balasubramanian, P. Kraus, A stress tensor for Anti-de Sitter gravity, Commun. Math. Phys. 208 (1999) 413. [hep-th/9902121].
[13] J. de Boer, E.P. Verlinde, H.L. Verlinde, On the holographic renormalization group, JHEP 0008 (2000) 003. [hep-th/9912012].
[14] E.P. Verlinde, H.L. Verlinde, RG flow, gravity and the cosmological constant, JHEP 0005 (2000) 034. [hep-th/9912018].
[15] J. de Boer, *The Holographic renormalization group*, Fortsch. Phys. 49 (2001) 339. [hep-th/0101026].

[16] S. de Haro, S.N. Solodkhin, K. Skenderis, *Holographic reconstruction of space-time and renormalization in the AdS/CFT correspondence*, Commun. Math. Phys. 217 (2001) 595. [hep-th/0002230].

[17] K. Skenderis, *Lecture notes on holographic renormalization*, Class. Quant. Grav. 19 (2002) 5849 [hep-th/0209067].

[18] G. Esposito, A.Y. Kamenshchik, K. Kirsten, *One loop effective action for Euclidean Maxwell theory on manifolds with boundary*, Phys. Rev. D 54 (1996) 7328 [hep-th/9606132].

[19] I.G. Moss, *Boundary terms for eleven-dimensional supergravity and M theory*, Phys. Lett. B 577 (2003) 71. [hep-th/0308159].

[20] P. van Nieuwenhuizen, D.V. Vassilevich, *Consistent boundary conditions for supergravity*, Class. Quant. Grav. 22 (2005) 5029. [hep-th/0507172].

[21] D.V. Belyaev, *Boundary conditions in supergravity on a manifold with boundary*, JHEP 0601 (2006) 047. [hep-th/0509172].

[22] P. van Nieuwenhuizen, A. Rebhan, D. V. Vassilevich and R. Wimmer, *Boundary terms in supergravity and supersymmetry*, Int. J. Mod. Phys. D 15 (2006) 1643. [hep-th/0606075].

[23] D. V. Belyaev and P. van Nieuwenhuizen, *Simple d=4 supergravity with a boundary*, JHEP 0809 (2008) 069. arXiv:0806.4723 [hep-th].

[24] D.V. Belyaev, P. van Nieuwenhuizen, *Tensor calculus for supergravity on a manifold with boundary*, JHEP 0802 (2008) 047. arXiv:0711.2272 [hep-th].

[25] D. Grumiller, P. van Nieuwenhuizen, *Holographic counterterms from local supersymmetry without boundary conditions*, Phys. Lett. B 682 (2010) 462. arXiv:0908.3486 [hep-th].

[26] P.S. Howe, T.G. Pugh, K.S. Stelle, C. Strickland-Constable, *Ectoplasm with an Edge*, JHEP 1108 (2011) 081. arXiv:1104.4387 [hep-th].

[27] L. Andrianopoli, R. D’Auria, *N=1 and N=2 pure supergravities on a manifold with boundary*, JHEP 1408 (2014) 012. arXiv:1405.2010 [hep-th].

[28] L. Di Pietro, N. Klinghoffer, I. Shamir, *On Supersymmetry, Boundary Actions and Brane Charges*, JHEP 1602 (2016) 163. arXiv:1502.05976 [hep-th].

[29] P.K. Concha, M.C. Ipinza, L. Ravera, E.K. Rodríguez, *On the supersymmetric extension of Gauss-Bonnet like gravity*, JHEP 09 (2016) 007. arXiv:1607.00373 [hep-th].

[30] D.Z. Freedman, K. Pilch, S.S. Pufu, N.P. Warner, *Boundary Terms and Three-Point Functions: An AdS/CFT Puzzle Resolved*, JHEP 1706 (2017) 053. arXiv:1611.01888 [hep-th].

[31] P. Benetti Genolini, D. Cassani, D. Martelli, J. Sparks, *Holographic renormalization and supersymmetry*, JHEP 1702 (2017) 132. arXiv:1612.06761 [hep-th].
[32] L. Andrianopoli, B.L. Cerchiai, R. D’Auria, M. Trigiante, *Unconventional supersymmetry at the boundary of AdS$_4$ supergravity*, JHEP **1804** (2018) 007. arXiv:1801.08081 [hep-th].

[33] A. Baunadi, L. Ravera, *Generalized AdS-Lorentz deformed supergravity on a manifold with boundary*, arXiv:1803.08738 [hep-th].

[34] R. Aros, M. Contreras, R. Olea, R. Troncoso, J. Zanelli, *Conserved charges for gravity with locally AdS asymptotics*, Phys. Rev. Lett. **84** (2000) 1647. [gr-qc/9909015].

[35] R. Aros, M. Contreras, R. Olea, R. Troncoso, J. Zanelli, *Conserved charges for even dimensional asymptotically AdS gravity theories*, Phys. Rev. D **92** (2000) 044002. [hep-th/9912045].

[36] P. Mora, R. Olea, R. Troncoso, J. Zanelli, *Finite action principle for Chern-Simons AdS gravity*, JHEP **0406** (2004) 036. [hep-th/0405267].

[37] R. Olea, *Mass, angular momentum and thermodynamics in four-dimensional Kerr-AdS black holes*, JHEP **0506** (2005) 023. [hep-th/0504233].

[38] D.P. Jatkar, G. Kofinas, O. Miskovic, R. Olea, *Conformal Mass in AdS gravity*, Phys. Rev. D **89** (2014) 124010. arXiv:1404.1411 [hep-th].

[39] S.W. MacDowell, F. Mansouri, *Unified Geometric Theory of Gravity and Supergravity*, Phys. Rev. Lett. **38** (1977) 739.

[40] S. Bonanos, J. Gomis, K. Kaminura, J. Lukierski, *Maxwell superalgebra and superparticle in constant Gauge background*, Phys. Rev. Lett. **104** (2010) 090401. arXiv:0911.5072 [hep-th].

[41] D.V. Soroka, V.A. Soroka, *Semi-simple extension of the (super) Poincaré algebra*, Adv. High Energy Phys. **2009** (2009) 234147. [hep-th/0605251].

[42] J. Gomis, K. Kaminura, J. Lukierski, *Deformations of Maxwell algebra and their Dynamical Realizations*, JHEP **0908** (2009) 039. arXiv:0906.4464 [hep-th].

[43] L. Castellani, R. D’Auria, P. Fré, *Supergravity and superstrings: A geometric prespective. Vol. 1 and 2*, World Scientific, Singapore (1991).

[44] P.K. Concha, E.K. Rodríguez, P. Salgado, *Generalized supersymmetric cosmological term in N=1 Supergravity*, JHEP **08** (2015) 009. arXiv:1504.01898 [hep-th].

[45] H. Bacry, P. Combe, J.L. Richard, *Group-theoretical analysis of elementary particles in an external electromagnetic fields. 1. The relativistic particle in a constant and uniform field*, Nuovo Cim. A **67** (1970) 267.

[46] R. Schrader, *The Maxwell group and the quantum theory of particles in classical homogeneous electromagnetic fields*, Fortsch. Phys. **20** (1972) 701.

[47] J.D. Edelstein, M. Hassaine, R. Troncoso, J. Zanelli, *Lie-algebra expansions, Chern-Simons theories and the Einstein-Hilbert Lagrangian*, Phys. Lett. B **640** (2006) 278. [hep-th/0605174].

[48] F. Izaurieta, E. Rodríguez, P. Minning, P. Salgado, A. Perez, *Standard General Relativity from Chern-Simons Gravity*, Phys. Lett. B **678** (2009) 213. arXiv:0905.2187 [hep-th].
[49] P.K. Concha, D.M. Peñafiel, E.K. Rodríguez, P. Salgado, *Even-dimensional General Relativity from Born-Infeld gravity*, Phys. Lett. B 725 (2013) 419. arXiv:1309.0062 [hep-th].

[50] P.K. Concha, D.M. Peñafiel, E.K. Rodríguez, P. Salgado, *Chern-Simons and Born-Infeld gravity theories and Maxwell algebras type*, Eur. Phys. J. C 74 (2014) 2741. arXiv:1402.0023 [hep-th].

[51] P.K. Concha, D.M. Peñafiel, E.K. Rodríguez, P. Salgado, *Generalized Poincaré algebras and Lovelock-Cartan gravity theory*, Phys. Lett. B 742 (2015) 310. arXiv:1405.7078 [hep-th].

[52] P. Salgado, R.J. Szabo, O. Valdivia, *Topological gravity and transgression holography*, Phys. Rev. D 89 (2014) 084077. arXiv:1401.3653 [hep-th].

[53] S. Hoseinzadeh, A. Rezaei-Aghdam, *(2+1)-dimensional gravity from Maxwell and semisimple extension of the Poincaré gauge symmetric models*, Phys. Rev. D 90 (2014) 084008. arXiv:1402.0320 [hep-th].

[54] R. Caroca, P. Concha, O. Fierro, E. Rodríguez, P. Salgado-Rebolledo, *Generalized Chern-Simons higher-spin gravity theories in three dimensions*, Nucl. Phys. B 934 (2018) 240. arXiv:1712.09975 [hep-th].

[55] L. Avilés, E. Frodden, J. Gomis, D. Hidalgo, J. Zanelli, *Non-Relativistic Maxwell Chern-Simons Gravity*, JHEP 05 (2018) 047. arXiv:1802.08453 [hep-th].

[56] P. Concha, N. Merino, O. Miskovic, E. Rodríguez, P. Salgado-Rebolledo, O. Valdivia, *Extended asymptotic symmetries of three-dimensional gravity in flat space*, arXiv:1805.08834 [hep-th].

[57] P.K. Concha, O. Fierro, E.K. Rodríguez, P. Salgado, *Chern-Simons supergravity in D=3 and Maxwell superalgebra*, Phys. Lett. B 750 (2015) 117. arXiv:1507.02335 [hep-th].

[58] P.K. Concha, O. Fierro, E.K. Rodríguez, *Inönü-Wigner contraction and D=2+1 supergravity*, Eur. Phys. J. C 77 (2017) 48. arXiv:1611.05018 [hep-th].

[59] P. Concha, D.M. Peñafiel, E. Rodríguez, *On the Maxwell supergravity and flat limit in 2+1 dimensions*, arXiv:1807.00194 [hep-th].

[60] J.A. de Azcarraga, J.M. Izquierdo, J. Lukierski, M. Woronowicz, *Generalizations of Maxwell (super)algebras by the expansion method*, Nucl. Phys. B 869 (2013) 303. arXiv:1210.1117 [hep-th].

[61] J.A. de Azcarraga, J.M. Izquierdo, *Minimal D=4 supergravity from superMaxwell algebra*, Nucl. Phys. B 885 (2014) 34. arXiv:1403.4128 [hep-th].

[62] P.K. Concha, E.K. Rodríguez, *Maxwell superalgebras and Abelian semigroup expansion*, Nucl. Phys. B 886 (2014) 1128. arXiv:1405.1334 [hep-th].

[63] P.K. Concha, E.K. Rodríguez, *N=1 Supergravity and Maxwell superalgebras*, JHEP 1409 (2014) 090. arXiv:1407.4635 [hep-th].

[64] D.M. Peñafiel, L. Ravera, *On the Hidden Maxwell Superalgebra underlying D=4 Supergravity*, Fortsch. Phys. 65 (2017) 1700005. arXiv:1701.04234 [hep-th].
[65] L. Ravera, *Hidden role of Maxwell superalgebras in the free differential algebras of $D = 4$ and $D = 11$ supergravity*, Eur. Phys. J. C **78** (2018) 211. arXiv:1801.08860 [hep-th].

[66] J.A. de Azcarraga, K. Kamimura, J. Lukierski, *Generalized cosmological term from Maxwell symmetries*, Phys. Rev. D **83** (2011) 124036. arXiv:1012.4402 [hep-th].

[67] D.M. Peñaafiel, L. Ravera, *Generalized cosmological term in D=4 supergravity from a new AdS-Lorentz superalgebra*, arXiv:1807.07673 [hep-th].

[68] P.K. Concha, R. Durka, N. Merino, E.K. Rodríguez, *New family of Maxwell like algebras*, Phys. Lett. B **759** (2016) 507. arXiv:1601.06443 [hep-th].

[69] R. Durka, *Resonant algebras and gravity*, J. Phys. A **50** (2017) 145202. arXiv:1605.00059 [hep-th].

[70] P.K. Concha, R. Durka, C. Inostroza, N. Merino, E.K. Rodríguez, *Pure Lovelock gravity and Chern-Simons theory*, Phys. Rev. D **94** (2016) 024055. arXiv:1603.09424 [hep-th].

[71] P.K. Concha, N. Merino, E.K. Rodríguez, *Lovelock gravity from Born-Infeld gravity theory*, Phys. Lett. B **765** (2017) 395. arXiv:1606.07083 [hep-th].

[72] P. Concha, E. Rodríguez, *Generalized Pure Lovelock Gravity*, Phys. Lett. B **774** (2017) 616. arXiv:1708.08827 [hep-th].

[73] S. Bonanos, J. Gomis, K. Kamimura, J. Lukierski, *Deformations of Maxwell Superalgebras and Their Applications*, J. Math. Phys. **51** (2010) 102301. arXiv:1005.3714 [hep-th].

[74] D.V. Soroka, V.A. Soroka, *Semi-simple o(N)-extended super-Poincaré algebra*, arXiv:1004.3194 [hep-th].

[75] O. Fierro, F. Izaurieta, P. Salgado, O. Valdivia, *Minimal AdS-Lorentz supergravity in three-dimensions*, arXiv:1401.3697 [hep-th].

[76] O. Miskovic, R. Olea, *Topological regularization and self-duality in four-dimensional anti-de Sitter gravity*, Phys. Rev. D **79** (2009) 124020. arXiv:0902.2082 [hep-th].

[77] O. Miskovic, R. Olea, M. Tsoukalas, *Renormalized AdS action and Critical Gravity*, JHEP **08** (2014) 108. arXiv:1404.5993 [hep-th].

[78] L. Andrianopoli, M. Bertolini, A. Ceresole, R. D’Auria, S. Ferrara, P. Fré, T. Magri, *N=2 Supergravity and N=2 super Yang-Mills theory on general scalar manifolds: Symplectic covariance, gaugings and the momentum map*, J. Geom. Phys. **23** (1997) 111. [hep-th/9605032].

[79] L. Andrianopoli, P. Concha, R. D’Auria, E. Rodríguez, M. Trigiante, *Observations on BI from $\mathcal{N} = 2$ Supergravity and the General Ward Identity*, JHEP **11** (2015) 061. arXiv:1508.01474 [hep-th].

[80] L. Andrianopoli, R. D’Auria and L. Ravera, *Hidden Gauge Structure of Supersymmetric Free Differential Algebras*, JHEP **1608** (2016) 095. arXiv:1606.07328 [hep-th].

[81] L. Andrianopoli, R. D’Auria and L. Ravera, *More on the Hidden Symmetries of 11D Supergravity*, Phys. Lett. B **772** (2017) 578. arXiv:1705.06251 [hep-th].