An investigation of a self-similarity for local vorticity and velocity components in tip vortex cores of a rotor wake

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Abstract. This experimental work is aimed at investigating the self-similarity of the local axial vorticity and azimuthal velocity in the tip vortices core in the wind turbine rotor wake. In the current work, the averaged local axial vorticity and azimuthal velocity profiles of the tip rotor vortex were measured. These profiles were tested to a self-similarity and compared with the Lamb-Oseen vortex.

1. Introduction
In recent years, the wind turbines application has grown to industrial use in wind farms. The interest to wind power tasks has increased significantly. The aerodynamics of the vortex wake behind a rotor of the wind turbines is essentially determined by dynamics of concentrated helical vortices which generated by tip of the rotor blades [1]. These helical vortices were the subject of many theoretical, experimental and numerical studies [2-9]. For example, in [10-11] the problem of the stability of a helical vortex system was considered analytically. In [12] it is shown that any two-dimensional vorticity distribution in the core tends to the Lamb-Oseen vortex. In numerical simulations of the rotor wake [13], an evolution of the tip helical vortex generated by a single-bladed rotor was tested to discover a self-similarity of the velocity and vorticity in the core of the helical vortex. The self-similar behavior of the vortex core was revealed in variables associated to the local azimuth angle changing along the center of the vortex core with an elliptical cross-section shape.

The current experimental work is aimed at investigating the self-similarity of the vorticity and axial velocity component in the core of the helical tip vortices in the rotor wake. For the first time, the azimuthally averaged vorticity profiles and the velocity component will be processed from our experimental velocity field measured behind rotor. The results will be compared with the Lamb-Oseen vortex.

2. Modelling
The profiles of velocity and vorticity components are self-similar if their profiles can be scaled and represented in the form:

\[ U/U_s = f(r/\eta), \]
\[ \omega/\omega_s = g(r/\eta). \]

Here \( U \) is axial velocity, \( \omega \) is vorticity, \( r \) is radius, \( \eta \) is radial variable of similarity, \( f \) and \( g \) are some functions.
These profiles, although they are different in their amplitude and expansion, have a similar shape after the scaling. The radius, velocity, and vorticity, scaled by the suitable functions, reduce to the family of profiles describing by a single curve. The scaling profiles become independent on time and space [14].

In fluid dynamics, the Lamb–Oseen vortex models a vortex that decays due to viscosity. This vortex is named after Horace Lamb and Carl Wilhelm Oseen [15]. The mathematical model for the flow velocity in the circumferential θ–direction in the Lamb–Oseen vortex is:

\[
V_\theta(r,t) = \frac{\Gamma}{2\pi R} \left[ 1 - \exp\left( -\frac{r^2}{r_c^2(t)} \right) \right],
\]

where \( r \) is radius, \( r_c(t) = \sqrt{4\nu t} \) is core radius of vortex, \( \nu \) is viscosity, and \( \Gamma \) is circulation contained in the vortex. The associated vorticity distribution [16] of the vortex-filament is:

\[
\omega_z(r,t) = \frac{\Gamma}{\nu r_c(t)^2} \exp\left( -\frac{r^2}{r_c^2(t)} \right)
\]

An alternative definition is to use the peak azimuthal velocity in the vortex core rather than their total circulation:

\[
V_\theta(r,t) = V_{\theta\max} \left[ 1 + \frac{1}{2\alpha} \frac{r_{\max}}{r} \left( 1 - \exp\left( -\alpha \frac{r^2}{r_{\max}^2} \right) \right) \right],
\]

where \( r_{\max} = \sqrt{\alpha} r_c(t) \) is the radius at which \( V_{\theta_{\max}} \) is attained, and the number \( \alpha = 1.25643 \), see [16].

In the paper the terminology and designations like it was specified in the article [13]. The authors [13] defined a local coordinate system (fig. 1.) placed at the center of the single tip vortex (in the axial cross-section). Authors followed its evolution in a frame rotating with the blade (i.e., using the transformation \( \theta = \Omega t; \ z = h \Omega t/2\pi \), \( h \) being the helix pitch). In [13] \( u_\rho, u_\theta \) and \( u_z \) denote, respectively, the radial, azimuthal and axial velocities in the local coordinate system and \( \omega_\zeta \) denotes the local vorticity normal to the axial plane (it corresponds to \( \omega_\theta \) in the global cylindrical coordinate system. With the transformation mentioned above the (locally) azimuthally (along \( \Theta \)) averaged profiles of these different quantities are functions of \( \rho \) (the radial distance from the center of the considered vortex in the axial plane) and the time \( t \).

![Figure 1.](image-url) The coordinate system with the center attached to a center of the tip vortex [13].

Let the averaged azimuthal velocity in the core and the azimuthal speed scale are correspondingly defined as:

\[
U_\theta = \frac{1}{2\pi} \int_0^{2\pi} u_\theta d\Theta,
\]

\[
U_\theta^s(t) = \max_{\rho = h/2} U_\theta(\rho, t).
\]

The radial variable of similarity is defined as:
\[ \eta = \frac{\rho}{\rho_c}, \quad (8) \]

where \( \rho_c \) is local radius at which \( U_\theta \) takes maximum.

\[ U_\theta(\eta, t) / U'_\theta(t) = \tilde{U}_\theta(\eta). \quad (9) \]

Therefore, the scaled value of \( \tilde{U} \) as a function of \( \eta \) does not depend on \( t \). Function \( \tilde{U}_\theta(\eta) \) is defined as:

\[ \tilde{U}_\theta(\eta) = \alpha(1 - e^{-\eta^2})/\eta. \quad (10) \]

Azimuthally averaged vorticity (normal to the axial plane) in the vortex core is noted:

\[ \omega = \frac{1}{2\pi} \int_0^{2\pi} \omega_z d\Theta. \quad (11) \]

The vorticity scale was defined as:

\[ \omega_\alpha(t) = \omega(\rho = 0, t). \quad (12) \]

The variable \( \eta \) given by equation (8) remains a similarity variable for the vorticity, so:

\[ \omega(\eta, t) / \omega_\alpha = \tilde{\omega}(\eta). \quad (13) \]

Function \( \tilde{\omega}(\eta) \) was defined as:

\[ \tilde{\omega}(\eta) = \exp(-\eta^2). \quad (14) \]

3. Experimental Methods and Results

A model of three-bladed rotor was specially manufactured for qualitative and quantitative flow visualizations downstream of the rotor [17]. The rotor diameter was \( 2R = 0.376 \) m, and the blade length was 0.159 m with the CD7003 blade profile [18]. The blade chord and angle of attack along span were calculated by the Glauert’s theory for an optimal wind turbine [19] with the tip speed ratio (TSR) \( \lambda = 5 \), where \( \lambda = \Omega R/V \), and \( \Omega \) is angular speed of the rotor. The Reynolds number was calculated as \( Re = \rho \Omega b / \mu \). Here \( \rho \) and \( \mu \) are the density and dynamic viscosity of the working fluid (tap water), \( b \) is the length of the blade chord (1 cm). The Reynolds number for all experiments was about 20 000 at the working temperature of 20ºС in the water flume. The water flume had a length of 35 m, a width of 3 m and the operative height was 0.9 m. A 3-m long test section with transparent walls of optical resolution was installed at a distance of 20 m from the channel inlet; the test section walls and bottom were made of glass.

The flow was studied by the Dantec Stereoscopic 2D-3C PIV system which gives all three velocity components throughout widow of the light sheet [20-21]. An Nd:YAG laser was used as a light source with the following characteristics: 120 mJ of energy in a single pulse, the wavelength is 532 nm, operation frequency is 15 Hz. The 2 mm thick light sheet was sent in vertically into the channel from the bottom and directly aiming at the rotor axis (see Fig. 2a). The images were recorded by two Dantec HiSense II cameras with 1344 ÷ 1024 pixels resolution. The 3D velocity field was calculated using Dantec Dynamic Studio 2.21. The cameras were placed perpendicularly to each other on the different sides of the flume and at angle of 45º to the walls (Fig. 2b). Water-filled optical prisms were installed between the cameras and the test section to reduce the distortions having from the camera inclination to the wall. Since the cameras were placed at an angle to the light sheet, the focus plane was adjusted using Scheimpflug adapters. The experimental errors has been minimized during calibration and test experiments by finding a trade-off between width of the light sheet, and the time between pulses, and the interrogation area. The stereoscopic PIV system was calibrated using a target with a well-defined dot-pattern which was translated and registered by the cameras in a number of well-defined positions.
at the light sheet. The measuring error of stereoscopic PIV was at the level of 3±5 %. The final size of
the total 3D velocity field was 1.03·0.29 m.

For each measuring window the ultimate velocity field was obtained by phase averaging of 100
realizations, which were recorded in the moment of a triggered signal by a light pulse per the one
revolution of the rotor. The angular encoder ROTACAM ASR58 with angular resolution 0.1°
installed on the rotor hub has formed the triggered pulse when one of the blades passed through the
light sheet. The stochastic errors vanish by this phase averaging. Moreover, this approach eliminates
the drifting error due to non-stationary flow regimes too.

Figure 2. Sketch of stereoscopic PIV measurements in the
transverse cross section and vortices visualization.

The processing of the images has been resulted in three uniform zones to yield the local velocity
field. The angles from which the synchronization was done were α = 0,15,30,45,60,75,90,105 degrees
of the rotor blade rotation. The more description of the experimental setup and the experimental
method can be found in[22-25]. Figure 3 shows the phase averaged distributions of three velocity
components for two regimes with a velocity of λ = 5.

Figure 3. The iso-contours of axial (U),
radial (V), tangential (W), velocity
components for TSR λ = 5.

Figure 4. The evolution of tip vortices in
different wake images.

The distributions of the components of the axial and radial velocity were measured in the different
planes of the laser knife (Fig. 4) and also the vorticity field was calculated from the measured velocity
fields. The cross sections of the tip vortex cores are well fixed(Figs. 3-4) which make it possible to
determine the centers of the tip vortices in the cross-sections. The regular vortex structure destroyed
for x between 1.8 R to 3.6 R and pairing with nearest turns takes place behind 3 R. If each cross-
section of tip vortex cores is numbered from 1 to 9, then the vortex cores No. 1, 4, 7 correspond to the
first blade, cores No. 2, 5, 8 correspond to the second blade, and cores No. 3, 6, 7 correspond to the
third blade. It can be seen that diffusion of tip vortices is take place. First 6 core images have strong concentration of vorticity and located approximately at equal distances, and the vorticity concentration in the cores No. 7, 8, 9 became smaller and placed at much more distances than the first 6 ones.

![Figure 5](image)

**Figure 5.** *In left:* Original profiles of the azimuthal velocity in the vortex core for different angles along the helical vortex. *In right:* The same profiles are scaled by equation (9). Equation (10) is plotted by solid line.

At first step the self-similarity of the local azimuthal velocity profiles were investigated. The angles 0° and 105° were taken as an interval of the changing the azimuth angle along the helical vortex. The numbers of the cross-sections of the vortex core were taken 2, 5, and 8. The velocity profiles are presented on fig. 5. The original profiles of the local azimuthal velocity in the vortex core cross-sections of different rotational angles are presented on the left plot. The profiles scaled using equation (9) is presented in the right plot. The comparison of the profiles shows a good agreement of the self-similarity for the azimuthal velocity in the core.

![Figure 6](image)

**Figure 6.** *In left:* Profiles of the original vorticity in the vortex core for different for different angles along the helical vortex. *In right:* The same profiles are scaled by equation (13). Vorticity of a Lamb-Oseen vortex (equation (14)) is plotted by solid line.

The original vorticity profiles in the tip vortex of the rotor operating with tip speed ratio $\lambda = 5$ is presented on figure 6 (left). The same profiles scaled using equation (13) are presented on the right side. The comparison of the profiles indicates much more inclination from the self-similarity as it was found for the velocity which can be explained by the additional error of the processing the vorticity from the measured velocity. It must be also mentioned that for the local velocity profiles the comparison with the Lamb-Oseen vortex is rather good, though, there is a little discrepancy in region $\eta = 1.8-2.4$. For the local vorticity we could not see the same perfect agreement with the Lamb-Oseen vortex, and the discrepancy of experimental points and the solution is not so small. This fact can be also explained by the additional errors of the vortex field reconstructions. So in further experiments it is necessary to measure velocity fields with less error and better spatial resolution.
4. Conclusions
The near wake flow with the strong tip helical vortices behind the three-bladed rotor has been studied by PIV measurements. The local azimuthal vorticity profiles and the local axial velocity component were investigated to discover the self-similarity in the tip vortex cores and also the good correlation with the Lamb-Oseen vortex was found.

Acknowledgments
Research was supported by the Russian Science Foundation (Project № 14-19-00487).

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