Polarizing primordial gravitational waves by parity violation

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We study primordial gravitational waves (PGWs) in the Horava-Lifshitz (HL) theory of quantum gravity, in which high-order spatial derivative terms, including the ones violating parity, generically appear in order to be UV complete. Because of the parity violation and non-adiabatic evolution, a large polarization of PGWs becomes possible, and it could be well within the range of detection for the forthcoming cosmic microwave background (CMB) observations.

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I. INTRODUCTION

With the arrival of the precision era of cosmological measurements, temperature and polarization maps of CMB with unprecedented accuracy will soon become available \cite{1}, and shall provide a wealth of data concerning the physics of the early universe, including inflation \cite{2}, a dominant paradigm, according to which primordial density and PGWs were created from quantum fluctuations in the very early universe. The former grows to produce the observed large-scale structure, and meanwhile creates CMB temperature anisotropy, which was already detected by the Cosmic Background Explorer (COBE) almost two decades ago \cite{3}. PGWs, on the other hand, produce not only temperature anisotropy, but also a distinguishable signature in CMB polarization. In particular, decomposing the polarization into two modes: one is curl-free, the E-mode, and the other is divergence-free, the B-mode, one finds that the B mode pattern cannot be produced by density fluctuations. Thus, its detection would provide a unique signature for the existence of PGWs \cite{4}. With such motivations, the studies of PGWs have attracted a great deal of attention, and various aspects have been explored \cite{11}.

In this Letter, we investigate the possibility to detect the chirality of PGWs by four potential CMB observations: Planck satellite \cite{5}, CMBPol mission \cite{6}, the ideal CMB experiment, where only the reduced lensed B-mode contaminations can be well removed.

To polarize the PGWs, an effective way is parity violation \cite{12}. In this Letter, we choose to work with the recently proposed HL theory of quantum gravity, in which parity violation happens generically \cite{10}, although our final conclusions are applicable to any theory in which the dispersion relation of the PGWs is described by Eq.(4). Current 7-year Wilkinson Microwave Anisotropy Probe (WMAP) observations give the tight constraint on the tensor-to-scalar ratio $r < 0.36$, which corresponds to the amplitude of the PGWs $\Delta_h < 0.87 \times 10^{-9}$ \cite{11,12}. However, they impose no constraint on their chirality \cite{13}.

In \cite{14}, the general covariance theory of the HL gravity without the projectability condition was proposed, in which the gravitational sector has the same degree of freedom as that in general relativity, i.e., only spin-2 massless gravitons exist. Such a theory is realized by assuming that: (i) The marginal part of the gravitational potential satisfies the detailed balance condition. (ii) The theory possesses an enlarged symmetry, $U(1) \times \text{Diff}(M, F)$, where $U(1)$ denotes a local $U(1)$ symmetry, and $\text{Diff}(M, F)$ the foliation-preserving diffeomorphisms. (iii) It respects the parity and time-reflection symmetries. To polarize gravitational waves, in this Letter we discard the spatial derivative terms, $\Delta S = \alpha_1 \omega_3 (r) / M_* + (\alpha_0 R_{ij} R_{ij} + \alpha_2 e^{i kn} R_{ij} R_{ij} / \sqrt{g}) / M_*^3 + \ldots$, into the action, where $\alpha_i$’s are dimensionless coupling constants, $M_*$ denotes the energy scale above which the high-order derivative terms become important, and $\ldots$ is the part proportional to $a_i$, which vanishes for tensor perturbations. Then, the total action for the gravitational sector takes the form, $S_{\text{total},g} = S_g - \alpha_1^2 / M_*^2 \int d^4 x \sqrt{-g} \Delta S$, where $S_g$ is given explicitly in \cite{11,14}. In this Letter, we shall adopt the definitions and conversations directly from \cite{14}, and refer readers to it for more detail.

In the Friedmann-Robertson-Walker flat universe, $\dot{N} = a(\eta), \dot{h}_{ij} = a^2(\eta) \delta_{ij}$, and $\dot{N}^i = \dot{\phi} = \dot{A} = 0$, we consider the tensor perturbations, $\delta g_{ij} = a^2 h_{ij}(\eta, x)$, where $h_{ij}$ is divergence-free and transverse, $\partial^i h_{ij} = 0, h^i_i = 0$. Then, the quadratic part of the total action is given by

$$S^{(2)}_{\text{total},g} = \xi^2 \int d^4 x \left\{ \frac{a^2}{4} (h_{ij})^2 - \frac{1}{4} a^2 (\partial_k h_{ij})^2 \right\}$$

$$- \frac{\gamma_3}{4 M_*^2} (\partial^2 h_{ij})^2 - \frac{\gamma_5}{4 M_*^2 a^2} (\partial^2 \partial_k h_{ij})^2$$

$$- \frac{\alpha_1 \alpha e^{ijk}}{2 M_*} (\partial h_{ij} \partial_m \partial h_{jk} - \partial h_{im} \delta_{ij} \partial h_{jk})$$

$$- \frac{\alpha_2 \alpha e^{ijk}}{4 M_*^2 a^2} \partial^2 h_{ij} \left( \partial^2 h_{jk} \right)_{i,j} + \frac{\alpha_0 H}{8 M_*^2 a} (\partial_k h_{ij})^2 \right\}, \quad (1)$$

II. THE MODEL
where $\partial^2 = \delta^{ij} \partial_i \partial_j$, $H = a'/a$, and $\gamma_i = \frac{\gamma_i}{M_i^2}$, $\gamma_5 = \frac{\gamma_5}{M_5^2} + \frac{\gamma_5}{M_5^2}$, with $\xi^2 = M_5^2$. To avoid fine-tuning, $\alpha_a$ and $\gamma_0$ should be all in the same order. Then, the field equations read,

$$h''_{ij} + 2Hh'_{ij} - \alpha^2 \partial^2 h_{ij} + \frac{\gamma_3}{aM_2^2} \partial^4 h_{ij} - \frac{\gamma_5}{aM_2^3} \partial^6 h_{ij} + e_{ik} \left( \frac{2a_1}{Ma} + \frac{a_2}{M^2a^3} \partial^2 \right) \left( \partial^2 h_{jk} \right), = 0,$$

where $\alpha^2 = 1 - 7a_0 H/(2M_5^2a)$, $h_{ij}(\eta, x)$ can be expanded over spatial Fourier harmonics as,

$$h_{ij}(\eta, x) = \sum_{A=R,L} \int_{A=-K_{L,F}} \frac{d\Omega^A}{(2\pi)^3} \psi^A(\eta) e^{ik \cdot A}_i f_{ij}(k),$$

where $P^A_i(k)$ are the circular polarization tensors and satisfy the relations: $i k r e^{i \xi \eta} P^A_i = k p^A F^A$ with $\rho^0 = 1$, $\rho^L = -1$, and $P^A_i \rho^L = \delta^A \delta^i$. Substituting it into Eq. (2), we find it can be written in the form,

$$v''_{A,ph} + a^2 \left( \omega_{A,ph}^2 + a^2 \alpha^2 / a^3 \right) v_{A,ph} = 0, \quad v_{A,ph} = a \psi_{A},$$

with $\omega_{A,ph} = \omega_{A} / a$, given by

$$\omega_{A,ph}^2 = \omega_{A}^2 - \rho^L \omega_{A,ph} \left( 2M_2^2 a_1 - \alpha_2 \frac{k_{ph}^2}{M_2^2} \right) / M_5^3,$$

where $\omega_{A,ph} = \omega_{A,ph} / (a^2 k_{ph}^2)$ and reads,

$$\omega_{A}^2 = 1 + \rho^A \left( \delta_1 y + \delta_3 y^3 \right) + \delta_2 y^2 + \delta_4 y^4,$$

with $\delta_1 = \frac{-2(\alpha_1 / \alpha^2) \xi_{HL}}{\xi_{HL}}$, $\delta_2 = \frac{(-\gamma_3 / \alpha^2) \xi_{HL}}{\xi_{HL}}$, $\delta_3 = \frac{\alpha_2 / \alpha^2 \xi_{HL}}{\xi_{HL}}$, $\delta_4 = \frac{\gamma_5 / \alpha^2 \xi_{HL}}{\xi_{HL}}$, where $\xi_{HL} = H/M_5 \ll 1$. Since $\alpha^2 \simeq 1$, without loss of generality, in the following we set $\alpha = 1$, or equivalently $\alpha_0 = 0$. Following (15), we choose the initial state at $y = y_i < 0$ as:

$$\phi_{A,0} = \frac{1}{\sqrt{2\omega_A}} d\phi_{A,0} / dy = i \sqrt{\omega_A/2},$$

which minimizes the energy density of the field.

Before proceeding further, let us note that the case studied in (9) corresponds to the choice, $\beta = \delta_2$, $\gamma = \delta_3 / (2\delta_2)$, $\delta_1 = 0$ and $\delta_4 = \delta_3^2 / (4\delta_2)$. To obtain sizable polarization of PGWs for future observations, (9) showed that $\beta$ and $\gamma$ should be of order one, which implies $\gamma_3 = O \left( \xi_{HL}^{-2} \right)$, $\alpha_2 = O \left( \xi_{HL}^{-3} \right)$, and $\gamma_5 = O \left( \xi_{HL}^{-4} \right)$. Clearly, this represents fine-tuning.

To see the effects of the parity-violated terms, let us first consider two representative cases: (i) $\delta_2 = \delta_3 = 0$; (ii) $\delta_1 = \delta_2 = 0$. In each of them, we can obtain $\omega_{R,ph}$ from $\omega_{L,ph}$ by flipping the sign of $\delta_1$ or $\delta_3$. Thus, without loss of generality, we assume that $\delta_3 \geq 0$. Then, from Eq. (2) we can see that $\omega_{R,ph}$ is a monotonically increasing function of $a$, and the equation, $\omega_{R,ph} = H$, has only one real positive root, as shown by Curve (a) in Fig. 1. Then, the WKB approximations are applicable in the region $\omega_{R,ph} > H$, and the mode function $v_R$ is given by

$$v_R = \frac{1}{\sqrt{2\omega_{R}}} e^{-i \int_{y_i}^{y_f} \omega_R(k, \eta) d\eta'}, \quad \omega_{R,ph} > H, \quad D_+ a(\eta) + D_- a(\eta) \int_{y_i}^{y_f} \frac{d\eta'}{a(\eta')}, \quad \omega_{R,ph} < H,$$

where $D_\pm(k)$ are uniquely determined by the boundary conditions at the horizon crossing $\omega_R = a(\eta_f) H$.

On the other hand, $\omega_{L,ph} = H$ can have one, two or three positive roots, depending on the ratio $\delta_1 / \delta_3$ or $\delta_3 / \delta_2$. If it has only one root, as shown by Curve (b), the WKB approximations are applicable in the region $\omega_{L,ph} > H$, and $v_L$ is given by Eq. (7). In the case with two real roots, we have $\eta_1 = \eta_2$, and Region II in Fig. 1 does not exist. As a result, $v_L$ is also given by Eq. (7). But, when it has three real roots, as shown by Curve (c), the WKB approximations are not applicable in Region II, and the evolution becomes non-adiabatic. Then, $v_L$ is given by

$$v_L = \frac{1}{\sqrt{2\omega_{L}}} e^{-i \int_{y_i}^{y_f} \omega_L(k, \eta') d\eta'}, \quad \omega_{L,ph} > H, \quad D_+ a(\eta) + D_- a(\eta) \int_{y_i}^{y_f} \frac{d\eta'}{a(\eta')}, \quad \omega_{L,ph} < H,$$

where $\Theta_{A}(k, \eta) \equiv \int_{y_i}^{y_f} \omega_A(k, \eta') d\eta'$. The coefficients $C_{\pm}(k), D_{\pm}(k), \alpha_0, \beta_0$ are uniquely determined by the boundary conditions (10). Note that due to the non-adiabaticity of the evolution in Region II, particles are created, where their occupation number $n_k$ is given by...
\[ n_k = |\beta_k|^2. \] To have the energy density of such particles be smaller than that of the background, one must require \( |\beta_k|^2 < (M_{pl}/M_\ast)^2 \varepsilon_{HL}. \) In the case without the \( U(1) \) symmetry, the study of the PPN corrections requires \( M_\ast \lesssim 10^{15} \text{ GeV} \) [18]. Although such a consideration with the \( U(1) \) symmetry has not been carried out so far, it is expected that a similar upper bound exists. Then, for \( \varepsilon_{HL} \approx M_\ast/M_{pl} \sim 10^{-5}, \) we have \( |\beta_k|^2 \approx O(1) \) [18].

Assuming that all the above conditions hold, we can see that in both of Cases (i) and (ii) there are only two distinguishable combinations, described, respectively, by Curves (a)+(b) and Curves (a)+(c) in Fig. 1. In the former, the power spectrum of PGWs and the circular polarization are given by,

\[ \Delta^2_h = \frac{k^3(\psi_R^2 + \psi_L^2)}{(2\pi)^2} = \frac{H^2}{4\pi^2} \left[ 1 + 21\alpha_1^2\varepsilon_{HL} + O(\varepsilon_{HL}^3) \right], \]

\[ \Pi = \left| \psi_R^2 - \psi_L^2 \right| \left| \psi_R^2 + \psi_L^2 \right| \left( 1 + \Delta_k^L \right)^2 \varepsilon_{HL} + O(\varepsilon_{HL}^3), \]

where \( \Delta_k^{\pm} = |\beta_k^{\mp}|^2 = |\beta_k^R|^2 + |\beta_k^L|^2. \) Again, since \( \Delta_k^R (A = R, L) \) can be as large as of order one, a large \( \Pi \) now also becomes possible, as shown by Fig. 2(c).

### IV. DETECTABILITY

In the case of the two-point statistics, the CMB temperature and polarization anisotropies are completely specified by six (TT, EE, BB, TE, TB, EB) power spectra. Usually, the PGWs produce the TT, EE, BB and TE spectra, but the spectra of TB and EB should vanish due to the parity conservation of the PGWs. However, if the linearized gravity is chiral as in the current case, the power spectra of right-hand and left-hand PGWs can have different amplitudes, and thus induce non-vanishing TB and EB correlation in large scales [8]. This provides the opportunity to directly detect the chiral asymmetry of gravity by observations, which has been discussed in some detail in [8, 13, 22]. Different from them, here we consider three information channels, BB, TB, and TT, to contain the chiral PGWs by determining both parameters \( r \) and \( \Pi. \) In Fig. 3(top panel), we show the CMB power spectra produced by PGWs with \( r = 0.1 \) and \( \Pi = 1. \) Note that the background cosmological parameters are chosen as 7-year WMAP best-fit values [11], and \( n_t = 0 \) is fixed throughout this Letter.

In order to determine the uncertainties of the parameters by the potential observations, we use the Fisher matrix technique to avoid the Monte Carlo simulations. The Fisher matrix is

\[ F_{ij} = \sum_{l} \sum_{XX',YY'} \frac{\partial C_{XX'}^{YY'}}{\partial p_l} C^{-1} \left( D_l^{XX'}, D_l^{YY'} \right) \frac{\partial C_{YY'}^{YY'}}{\partial p_j}, \]
where $C_{l}^{XX'}$ are the CMB power spectra and $D_{l}^{XX'}$ the corresponding estimators. $p_i$ are the parameters to be determined, which are $r$ and $\Pi$ in the present case. The covariance matrix of the estimators is given by

$$C(D_{l}^{XX'}, D_{l'}^{XY'}) = \frac{C_{l}^{XY} C_{l'}^{XY'} + C_{l}^{XY'} C_{l'}^{XY}}{(2l + 1)f_{\text{sky}}},$$

where $C_{l}^{XY} = C_{l}^{XY} + N_{l}^{XY}$, and the noise power spectra $N_{l}^{XY}$ include the instrumental noises, lensed B-mode polarization, and the CMB power spectra generated by density perturbations. $f_{\text{sky}}$ is the sky-cut factor, which will be taken $f_{\text{sky}} = 0.65$ for Planck, 0.8 for CMBPol and ideal experiments, and 1.0 for the cosmic variance limit. Throughout our calculations, we choose $t_{\text{max}} = 2000$. Once the Fisher matrix is calculated, the uncertainties of the parameters can be evaluated by $\Delta p_i = \sqrt{F_{ii}^{-1}}$.

As mentioned in [22], the determination of $r$ is mainly from the observation of BB information channel, but not from TB or EB channels, even if the PGWs are completely chiral. In order to quantify the detection abilities of the experiments, we define the signal-to-noise ratio $S/N \equiv r/\Delta r$. Similar to the discussions in the previous works [23, 24], we find that if the condition $S/N > 3$ is required, i.e. a definite detection, $r > 0.03$ is needed for Planck satellite. While CMBPol mission can detect the chirality if $r > 0.4 \times 10^{-3}$, and the ideal experiment can detect it if $r > 0.8 \times 10^{-5}$.

It should be noted that the actual detectability of the experiments are always worse than these evaluations, due to other contaminations, including various foregrounds, E-B leakage, systematics and so on (See for instance [24]). In particular, the constraint on $\Pi$ is mainly from the TB and EB channels, where the cosmic variances caused by TT, EE and BB are dominant. So, the values of $\Delta \Pi$ are nearly independent of $\Pi$ in the fiducial model. In Fig. 3 (Low panel), we present the uncertainties $\Delta \Pi$ as a function of $r$ for the four measurements, where we have set $\Pi = 1$ in the fiducial model. The results are quite close to those presented in [13, 22], which show that the free parameter $r$ can not significantly affect the determination of $\Pi$. It also shows that if $\Delta \Pi < 0.3$, the value of $r$ should be larger than 0.3 for Planck, which has nearly been excluded by the current observations [11]. While for CMBPol mission, the situation is a little better, in which $r > 0.12$ is needed, while for the ideal measurement, it requires $r > 0.09$. Even if we consider the extreme cosmic variance limit, $r > 0.04$ is still required. So, we conclude that, if $\Pi < 0.3$, the determination of the chirality of PGWs is quite difficult, unless the tensor-to-scalar ratio $r$ is large enough. However, if the PGWs are fully chiral, i.e. $\Pi \sim \pm 1$, the detection becomes much easier. We find that to get $\Delta \Pi < 1$, we only need $r > 0.05$ for Planck, $r > 0.014$ for CMBPol, $r > 0.01$ for the ideal experiment, and $r > 0.004$ for the cosmic variance limit.

V. CONCLUSIONS

In this Letter, we have studied the evolution of PGWs, described by the dispersion relation [14], obtained from the HL theory of quantum gravity [14]. From the analytical results given by Eqs.(9), (10) and (11), one can see that the polarization of PGWs is precisely due to the parity violation and non-adiabatic evolution of the mode function in Region II of Fig. 1 in which particles are created, where their occupation number is given by $n_k = |\beta_k|^2$. Fig. 2 on the other hand, shows clearly that the polarization is considerably enhanced by the third- and four-order spatial derivative terms of Eq.(4). The effects of the fifth-order terms were studied in [9], and are showed explicitly here that their contributions to the polarization are sub-dominant, and are quite difficult to be detected in the near future. The detectability of the polarization caused by other terms, on the other hand, seems very optimistic, as shown in Fig. 3 (Low panel).

It should be noted that, although the dispersion relation [14] is obtained from the HL theory [14], our results are actually applicable to any theory where the mode function of PGWs are described by Eqs.(5), (6) and (7), including the trans-Planck physics [12].

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