Theoretical constraints on brane inflation and cosmic superstring radiation

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We analyze theoretical constraints on the radiation modes of cosmic superstrings. Given that cosmic superstrings are formed at the end of brane inflation, we first investigate the implications of recently elucidated supergravity constraints on brane inflation models. We show that both D3/D7 and D3/D3 brane inflation are subject to non-trivial constraints. Both inflationary models can be shown to satisfy those constraints, but for the case of D3/D7 there seem to be important consequences for the dynamics of the inflationary mechanism. Bearing this in mind, we analyze the theoretical constraints on the nature of the allowed radiation by cosmic superstrings in the context of a warped background where brane-antibrane inflation takes place. Clearly such constraints do not apply to field theoretic cosmic strings, or to cosmic strings that arise in a background without warping. We argue that in a warped background where one might expect axionic radiation to be enhanced relative to gravitational radiation, neither F-strings nor D-strings can emit axionic radiation, and FD-strings cannot give rise to Neveu-Schwarz–Neveu-Schwarz particle emission, while their Ramond-Ramond particle emission is not well-defined.

Contents

I. Introduction 2

II. Brane inflation within string theory
   A. Brane inflation models
   B. Constraints from supergravity
      1. D3/D7 inflation
      2. D3/D3 inflation

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I. INTRODUCTION

Cosmological inflation [1] explains the large-scale homogeneity, isotropy and flatness of the universe as observed today. It also provides a model for structure formation (via fluctuations of the inflaton field) whose predictions [2] for the nature of the inhomogeneities in the Cosmic Microwave Background (CMB) are in impressive agreement with experiment [3]. However, it is not known why inflation occurred or which of the many models of inflation is correct. Inflation often seems to require very special initial conditions [4]. Theories of inflation, based on quantum field theory combined with general relativity, can depend very sensitively on Ultra-Violet (UV) physics in the sense that some details of the inflationary dynamics are controlled by Planck-suppressed contributions to the effective action [5]. One should therefore study inflation in a UV-complete theory, such as string theory, in which one has the hope of computing all Planck-suppressed contributions to the inflaton action.

Although it arguably does not arise naturally, inflation can be realized in various ways in string theory [5, 6], and can give rise to observational signatures, such as deviations from scale invariance, gaussianity and adiabaticity of the CMB. In particular, brane inflation models (in which the inflaton is given by the separation between branes with an attractive potential between them) generically results in the production of cosmic superstrings. Observational consequences of cosmic superstrings can serve as constraints on or signals of the underlying string theory model [7, 8]. Many studies have been done on cosmic superstring networks, their evolution [9] and radiation [10, 11]. However, one must keep in mind that in order for
any predictions to be useful, they must be consistent with the details of the string theory model under consideration. In some cases, careful analysis reveals strong constraints on the possible observational consequences of cosmic superstrings from brane inflation. In what follows, we revisit these models and their consequences, clarifying the possible signatures of brane inflation via radiation from cosmic superstrings, within a consistent theoretical framework.

This paper consists of two parts. In Section II we examine the implications of recent supergravity constraints on brane models of inflation, in which cosmic superstrings are produced. Specifically we compare the well-known models of D3/D7 inflation and D3/D3 inflation. Since, as we will show, D3/D3 inflation can be made compatible with the supergravity constraints without spoiling the inflationary dynamics, while for D3/D7 the D-term inflationary mechanism appears problematic, in Section III we concentrate only on cosmic string radiation in warped backgrounds in which brane-anti-brane inflation takes place. Consistent compactification of such backgrounds leads to further constraints on the allowed modes of radiation by these strings. We end with our conclusions.

II. BRANE INFLATION WITHIN STRING THEORY

In what follows we summarize the two classes of string theory motivated brane inflationary models. We then examine the constraints imposed by supergravity consistency conditions on these two classes of models. The conclusions we will draw may lead to important consequences for the feasibility of the models, and therefore the phenomenology of cosmic superstrings formed at the end of brane inflation.

A. Brane inflation models

Brane inflation models in string theory fall into two classes:

- D3/D7 inflation [12, 13] is a string realization of D-term inflation where the attraction between the branes is due to the breaking of supersymmetry by the presence of a non-self-dual flux on the D7-brane, which plays the rôle of a Fayet-Iliopoulos (FI) term. D3/D7 inflation takes place in an unwarped background: it is realized in type IIB string theory compactified on $K3 \times \mathbb{T}^2 / \mathbb{Z}_2$, where $\mathbb{Z}_2$ involves orbifold as well as orientifold operations. [We refer the reader to Refs. [14, 15], for the details of the compactification including moduli stabilization.]

- By contrast, brane-antibrane inflation [16], of which D3/D3 [17] is the best studied example, is a string realization of F-term inflation, where the attractive potential between the branes is due to warping. D3/D3 inflation takes place in a warped throat, such as
the Klebanov-Strassler (KS) geometry \[18\], which is a deformed conifold geometry with fluxes. This brane inflation model necessitates a warped flux compactification \[19, 20\].

In both D3/D7 and D3/D3 models, flat directions for the inflaton field can appear as a consequence of shift symmetry with respect to the inflaton field.

### B. Constraints from supergravity

Several recent papers have addressed the consistency of having constant (field-independent) FI terms in a supergravity theory \[21, 22\]. Although there is no problem with having such terms in a supersymmetric theory, inconsistencies arise when such a theory is gauged. \(^1\) These inconsistencies can be understood in different ways \[21, 22\], but in each case the conclusion is the same: field-independent FI terms are inconsistent in supergravity.

The same arguments rule out non-exact Kähler forms. In an equivalent way, theories with a Kähler form that is not exact, or which have a field-independent FI term, do not have a globally well-defined Ferrara-Zumino (FZ) multiplet \[21, 22\].

To ensure that the FZ-multiplet exists and it remains well-defined, the quantum moduli space must be such that the Kähler two-form \(J\) is given by \(dA\), for some one-form \(A\) which is globally well-defined. If this is the case, \(\int J \wedge J \wedge J \cdots\) vanishes for any compact cycle, giving a vanishing volume for the moduli space of the internal manifold if it is compact. Thus, the statement that the Kähler form must be exact is equivalent to the statement that the moduli space cannot be compact \[22\]. \(^2\) In conclusion, the constraints from supergravity are two-fold:

- field-independent FI terms are not allowed
- the moduli space cannot be compact.

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1. This holds in flat space. We refer the reader to Section IB2 for a discussion in the case of an AdS space, as studied in Ref. \[40\].

2. If the theory has a continuous R-symmetry, one can couple to supergravity using the so-called \(R\)-multiplet. However, in both cases — gauging the theory by using either the FZ-multiplet or the \(R\)-multiplet — the resulting on-shell theory has a continuous global symmetry \[21\], and gravity theories with continuous global symmetries are expected to be inconsistent (see e.g., Ref. \[23\], where it is argued that in quantum gravity models there are no global symmetries and all continuous gauge groups are compact). When one has an FI term but no R-symmetry, one can couple to the S-multiplet, which interpolates between the FZ- and R-multiplets. The resulting supergravity theory contains an additional massless chiral superfield \(\Phi\). It can equivalently be obtained by first adding such a field to the rigid theory so that the system including \(\Phi\) has an FZ multiplet and can be coupled to supergravity as usual. This renders the FI term field-dependent and the moduli space non-compact \[21, 22\].

3. This restriction on the moduli space is also indicated in earlier work \[24\].
In the following we investigate the implications of these (not unrelated) conditions for the models of brane inflation discussed above.

1. **D3/D7 inflation**

D3/D7 inflation [12, 13] is a stringy realization of D-term inflation, a type of hybrid inflation. In this model, a D3-brane is parallel to a D7-brane in the four non-compact directions, with the other legs of the D7 wrapping K3. The full compact manifold is K3×T^2/Z_2. If there is a non-self-dual flux on the D7-brane, the action of the system can be mapped to the action of a D-term inflationary model in \( \mathcal{N} = 1 \) supergravity, with the world-volume gauge field \( \mathcal{F} \) playing the role of the FI term (from the point of view of the field theory living on the brane). An attractive potential is then produced between the branes. The inflaton field is given by \( \phi = x^4 + ix^5 \) where \( x^4 \) and \( x^5 \) are the directions perpendicular to both branes, along which they feel the attractive potential. Inflation ends in the waterfall stage, when strings stretching between the branes become tachyonic. Supersymmetry is restored and \( U(1)_{\text{FI}} \) is spontaneously broken in the final state, in which the D3-brane is dissolved as an instanton into the D7-brane [12]. Spontaneous symmetry breaking occurs when the waterfall fields roll to a minimum at the end of this process, which results in the production of cosmic superstrings [12]. For D3/D7 inflation to take place, it is necessary that the volume modulus be stabilized at some large value for the K3 manifold. Failing to stabilize the compactification modulus may result in rapid decompactification instead of inflation. Further, the effective gauge coupling on the D7 world-volume is given by \( \tilde{g}_3 \), where

\[
\frac{1}{\tilde{g}_3^2} = \frac{\text{vol}(K3)R^4}{g_7^2},
\]

and \( R \) is an overall length scale of the K3. Thus it is inversely proportional to the volume of the K3. This means that we can neglect the interactions due to the D7 gauge fields as long as the K3 manifold is taken to be fixed at a large volume. In addition, the warp factor is of order 1 in this limit, allowing one to safely ignore warping effects [12]. In conclusion, volume modulus stabilization allows one to ignore warping effects and neglect interactions due to gauge fields from any additional D7-branes, and ensures that the FI-term is non-zero, which is necessary for D-term inflation to take place. The volume modulus was left unfixed in Ref. [12, 13] but was assumed to be large. In Ref. [14] the volume modulus was stabilized with a non-perturbative superpotential due to gaugino condensation on a stack of D7-branes wrapping the K3.

4 The recently proposed model of flux-brane or D7/D7 brane inflation [25] is also a string embedding of D-term inflation and should be subject to the same general constraints, but we do not consider it here. An attractive property of this model is that cosmic strings produced at the end of flux-brane inflation are consistent with the observational bound on their tension, in contrast to D3/D7.
Let us investigate the impact of the supergravity constraints discussed above on D3/D7 inflation. In Ref. [26] we argued that the FI term in D3/D7, which arises because of a non-self-dual flux on the D7 brane, is field-dependent and given by

\[ \xi = \frac{\delta_{\text{GS}}}{\text{vol}(\text{K3})}, \] (2)

where \( \delta_{\text{GS}} \) is the Green-Schwarz (GS) parameter [27]. This is fairly generic in string theory, where field-dependent FI terms arise from GS anomaly cancellation [28–30], and the connection between non-self-dual flux and a field-dependent FI term was alluded to in Refs. [31, 32], where it was pointed out that the rôle of the axion in the GS mechanism in Type IIB will be played by the field dual to the 4-form \( C_{(4)} \), in the same multiplet as the volume modulus. Thus the rôle usually played by the axion-dilaton is played by the Kähler modulus \( s = \text{vol}(\text{K3}) + \text{i}C_{(4)} \), giving the dependence of \( \xi \) on \( \text{vol}(\text{K3}) \).

From this dependence, the first supergravity constraint (i.e., that field-independent FI terms are not allowed) seems to be satisfied, but for this to be true the real part of the Kähler modulus, \( \text{vol}(\text{K3}) \), must be left unfixed. Specifically, the volume of the K3 cannot be stabilized above the SUSY-breaking scale in a consistent way [22]. However, the SUSY breaking scale in D3/D7 is given by \( \xi \):

\[ V \sim g^2 \xi^2 = g^2 \frac{\delta_{\text{GS}}^2}{[\text{vol}(\text{K3})]^2} = \frac{\delta_{\text{GS}}^2}{[\text{vol}(\text{K3})]^3}, \]

so \([\text{vol}(\text{K3})]^{-1/6}\) (the scale of the compactification) must necessarily be stabilised above the SUSY breaking scale which is approximately \([\text{vol}(\text{K3})]^{-3}\).

Phrasing this another way, stabilizing the Kähler modulus at a large finite value (as required for a successful model of inflation) above the supersymmetry breaking scale would make the FI term constant, thus problematic. Moreover this would amount to making the moduli space compact, which is inconsistent with the second supergravity constraint: In D3/D7 inflation, as in all proposed brane inflation models, the moduli space of the effective world-volume theory is the moduli space of the compactification manifold itself, fibered by the Kähler moduli. The open string moduli fields are the positions of the mobile brane on the internal manifold that encode the geometry of the compact base space while the closed string moduli are the Kähler moduli of the manifold which encode the geometry of the fibration, e.g. breathing modes of the internal manifold. These Kähler modes play an important rôle in the model. Although the K3 (four-dimensional Kähler manifold) surface is compact, the universal Kähler modulus, or volume modulus, fibers over it yielding a non-compact moduli space, given by \( \mathbb{R}^+ \times M_{19,3} \) [33], where the \( \mathbb{R}^+ \) factor corresponds to the overall volume modulus and the \( M_{19,3} \) factor describes a space of dimension \( 19 \times 3 = 57 \). If the volume modulus is fixed, the moduli space is rendered compact. As opposed to the brane-anti-brane inflationary model to be discussed in the next paragraph, the unavoidable constraint here is the constant FI-term.
Given this analysis, one might conclude that as it stands D3/D7 is not consistent with the supergravity constraints presented in Ref. [21, 22]. If the volume of K3 is allowed to vary, we no longer have perturbative control of the theory, do not necessarily get D-term inflation, and cannot neglect warping with impunity. However, the distance $r$ between the branes is an unfixed modulus in the theory, which the FI term $\xi$ appears to depend on. There are two ways to see this, described in the following.

Firstly, the real part of the Kähler modulus, $s_R$, is not given only by $\text{vol}(K3)$ when quantum corrections are taken into account. As explained in Refs. [14, 15], the physical warped volume of K3 also depends on the D3 brane position, i.e., on the brane separation $r$. This is in fact necessary in order to avoid the so-called rho problem [35–37] and ensure that the definitions of the volume modulus $\rho$ and of the D7 gauge coupling are consistent. Using Eq. (2), this means that $\xi$ also depends on $r$, which cannot be fixed as the branes move towards each other to give inflation.

Secondly, one can find the $r$-dependence of $\xi$ directly from the supergravity solution for the D3/D7 brane system [13]. The FI term is given by the non-self-dual part of the flux on the D7, namely

$$\frac{g^2\xi^2}{2} = \frac{1}{8R^{12}g_7} \int_{K3} F^- \wedge \star F^- ,$$

(3)

where the Hodge star is on the K3 wrapped by the D7 brane, $R$ represents the overall length scale of the compact K3, while $g$ and $g_7$ are the $U(1)_{\text{FI}}$ and D7 gauge coupling constants respectively, and $F^- = F - \star F$ where $F = F - B$. One finds that

$$\int_{K3} F^- \wedge \star F^- = \int_{K3} d^4x \left( \frac{H_2}{H_1} (B_{67} - \sqrt{g} B_{89})^2 + \frac{H_1}{H_2} (B_{89} - \sqrt{g} B_{67})^2 \right) ,$$

(4)

using

$$F^- = -(B_{67} - \sqrt{g} B_{89}) dx^6 \wedge dx^7 - (B_{89} - \sqrt{g} B_{67}) dx^8 \wedge dx^9 ,$$

(5)

$$\sqrt{g} = Z_7^{-1} H_1 H_2 ,$$

(6)

with the B-field given by

$$B_{67} = -\tan \theta_1 Z_7^{-1} H_1 ,$$

(7)

$$B_{89} = -\tan \theta_2 Z_7^{-1} H_2 ,$$

(8)

where $H_i(r)$ are the harmonic functions corresponding to the branes, and $Z_7$ is an $r$-dependent factor defined below. Neglecting terms of order $(\sin \theta)^3$ and higher, as well as taking $r$ to be large, we find

$$\int_{K3} F^- \wedge \star F^- \approx \int d^4x Z_7^{-3}(\sin \theta_1 - \sin \theta_2)^2 ,$$

(9)

Note that, as in Ref. [26], we use the notation of Ref. [14] multiplied by a factor of $i$ in order to be consistent with the notation of e.g., Ref. [34].
where \( Z_7 = 1 - 2c_7 \ln (r/\Lambda) \) gives the \( r \)-dependence of this term. It is worth noting that if only the leading order constant piece is identified with the FI term, namely \( \xi^2 \sim (\sin \theta_1 - \sin \theta_2)^2 \), then the supergravity constraint \([21, 22]\) applies, as the FI term is now field-independent, or, alternatively, the moduli space is compact. This implies theoretical inconsistency of the brane inflation model. If \( \xi \) depends on \( r \), the FI term is no longer field-independent and, furthermore, the SUSY breaking scale does not depend only on \( \text{vol}(K3) \) so it may be possible to fix the volume below the SUSY-breaking scale. However, the D-term inflationary mechanism seems to be affected, in the sense that it is not immediately clear how to interpret an \( r \)-dependent \( \xi \) in D-term inflation. More precisely, since the bifurcation point and the Hubble constant during inflation both depend on \( \xi \), an \( r \)-dependence of the FI term would affect the inflationary mechanism.

2. **D3/\( \overline{D3} \) inflation**

D3/\( \overline{D3} \) inflation is a stringy realization of F-inflation, which is in general plagued by the \( \eta \)-problem. A D3/\( \overline{D3} \) system lives at a specific point of a Calabi-Yau (CY) manifold. Supersymmetry breaking results in a net attractive force between the two branes, with the brane separation playing the rôle of the inflaton. To accommodate sufficient e-foldings, the first and second slow-roll parameters (\( \epsilon \) and \( \eta \), respectively) must be small. However, since the separation between the two branes cannot be greater than the size of the CY, one cannot achieve \( \eta \ll 1 \) in flat space. To evade this problem, one should consider the D3/\( \overline{D3} \) system in a warped geometry \([17]\). In this model, usually referred to as KLMT, D3/\( \overline{D3} \) inflation takes place in a warped throat, in contrast to the D3/D7 inflation case. The \( \overline{D3} \) brane is fixed; it sits at the tip of the throat where its energy is minimized, while the D3-brane feels a small attractive force towards the \( \overline{D3} \) brane and moves towards it. In the KLMT model the inflaton field is given by the separation between the D3 brane and the \( \overline{D3} \) brane. Inflation ends when the strings stretching between the branes become tachyonic and the branes annihilate, with fundamental IIB strings and D1-branes, localized in the throat, being naturally produced \([39]\). Due to the large gravitational red-shift at the throat, the inflationary scale and the string tension measured by a four-dimensional observer are suppressed by the warping factor, as opposed to those measured by a ten-dimensional inertial observer which are close to the four-dimensional Planck scale. \(^7\)

\(^6\) Alternative proposals that have been suggested are to consider branes at angles \([38]\) or collisions of multiple branes. In the first case the number of e-foldings depends on the collision angle, while in the second one the slow-roll condition is abandoned.

\(^7\) Note that the original KLMT model may be plagued by insufficient reheating. The U(1) gauge field on the stabilizing \( \overline{D3} \) is the only massless degree of freedom in the inflationary throat, and as such it is the only one that couples to the inflaton. As a result, at reheating almost all of the energy goes to the U(1)
Let us investigate whether D3/D3 is consistent with the supergravity constraints put forward recently in Refs. [21, 22]. The theory has no FI term, so it is only the second SUGRA constraint that we have to check, namely that the moduli space of the theory is non-compact.

In the compactifications of Refs. [19, 20], the volume modulus is unfixed. However, in the D3/D3 inflation model, the size of the internal manifold, which is a dynamical field, can become large too fast and spoil the slow-roll conditions [17]. Many efforts have been made to stabilize the volume in order to solve this problem. More precisely, one looks for a mechanism to stabilize the volume while leaving the D3 brane free to move in the CY. Since the moduli space of the theory is given by the manifold (in which the D3 brane is free to move), fibered by the volume modulus which must be stabilized, one might conclude that the moduli space is compact, making the model inconsistent with the KS constraints. Specifically, it is not consistent to stabilize the closed string modulus (the volume) at a scale larger than the SUSY breaking scale — in other words, without SUSY breaking. It can be seen, however, that KLMT D3/D3 inflation satisfies the supergravity consistency constraints [40], as we briefly discuss below.

The supergravity constraints discussed in Ref. [22] hold for sigma models in flat space, whereas D3/D3 inflation takes place in a dS vacuum, as constructed in KKLT [41]. Recall that in flat space rigid $\mathcal{N} = 1$ supersymmetry does not impose any such conditions — it is only when SUSY is gauged that the constraints detailed by Komargodski and Seiberg arise [22]. However, as was recently shown [40], the KS conditions not only extend to the case of $\mathcal{N} = 1$ AdS$_4$ compactifications, but they in fact arise as consistency conditions for unbroken $\mathcal{N} = 1$ SUSY on AdS$_4$ (i.e., before gauging to supergravity). These conditions follow from basic properties of SUSY in AdS$_4$.

The Kähler potential $K(\phi, \bar{\phi})$ and the superpotential $W(\phi)$ are mixed by Kähler transformations

$$K(\phi, \bar{\phi}) \rightarrow K(\phi, \bar{\phi}) + f(\phi) + \bar{f}(\bar{\phi}), \quad W(\phi) \rightarrow W(\phi) - \lambda f(\phi),$$

in the AdS$_4$ case. Hence, requiring that the action be invariant under Kähler transformations leads to a mixed potential of the form [40]:

$$V(\phi, \bar{\phi}) = g^{ij}(W_i + \lambda K_i)(\bar{W}_j + \lambda K_j) - 3\lambda \bar{W} - 3\lambda W - 3\lambda^2 K, \quad (10)$$

where $g^{ij}W_i\bar{W}_j$ is just the scalar potential in flat space and $-3\lambda^2$ stands for the cosmological constant. The mixing of the Kähler potential and superpotential in rigid AdS$_4$ implies that,

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8 This was achieved by first constructing a SUSY-preserving AdS vacuum with all moduli fixed, and then adding a D3 brane which is fixed at the bottom of the throat, in order to lift the vacuum to dS (breaking SUSY).

9 We thank McAllister for pointing us to Ref. [40].
even when the superpotential is zero, the SUSY vacua of this theory are generically a discrete set of points, as opposed to the situation for flat-space sigma models where the moduli space for vanishing superpotential is the full manifold. For large volume flux compactification — so that a perturbation expansion in $\lambda/M_{Pl}$ is valid — to AdS$_4$, with a small cosmological constant $\lambda$, the moduli have masses proportional to $\lambda$, and are therefore light. However, if one wishes to consider this compactification in an inflationary context one has to stabilize the Kähler moduli at a large value while keeping the open string moduli (inflaton) nearly flat.

The authors of Ref. [40] explicitly consider the example of a theory with a mobile D3 brane and Kähler modulus, still in the AdS$_4$ vacuum, i.e., before the addition of an anti-D3 brane as in KKLT [41]. The Kähler potential is of the form $K(\rho, \bar{\rho}, \phi, \bar{\phi}) = -3 \log(\rho + \bar{\rho} - k(\phi, \bar{\phi}))$. Both $\rho$, the volume modulus, and $\phi$, the D3-brane position, receive Vacuum Expectation Values (VEVs) with that of the volume still proportional to $\lambda$, though parametrically larger than the AdS$_4$ scale. This still corresponds to a moduli space composed of a set of discrete points, in which inflation cannot happen. The moduli space spanned by $\rho$ and $\phi$, denoted $\hat{X}$, is consistent with the constraints either when the volume is unfixed, giving a non-compact moduli space $\hat{X}$, or when both the brane position and the volume take VEVs, since a moduli space which is a discrete set of points has no compact cycles. If $\rho$ becomes massive while the $\phi$ remain massless, $\rho$ can be integrated out, leaving the compact moduli space $X$, which is just the manifold. Since, as we have seen, this is inconsistent, it is thus necessary to lift both the Kähler modulus and the brane moduli, by breaking SUSY.

This is in fact the method proposed in KLMT [17] for stabilizing the volume modulus while leaving the brane position unfixed. We have seen that adding a D3 brane to break SUSY and give the D3 brane a nearly flat potential is insufficient if one wants to fix the volume. One can introduce a non-perturbative superpotential to stabilize the volume; although this generically gives rise to a large mass for the inflaton, spoiling inflation, it is possible to achieve inflation with a fine-tuned superpotential which also depends on the inflaton. 10 (This dependence is expected because the Kähler transformations away from flat space mix the Kähler potential and the superpotential). Alternatively, the authors of Ref. [17] suggest stabilising $r \sim \rho + \bar{\rho} - k(\phi, \bar{\phi})$ directly via corrections to the Kähler potential.

Both these methods involve SUSY breaking, so that beneath the SUSY scale one can adjust the volume modulus as necessary, while contriving to safeguard the inflationary behaviour of the system: this means that in the absence of interactions the D3 brane should be free to move around the compact CY. The crucial difference between this case and that of the D3/D7 system in this regard is that the SUSY breaking scale for D3/D3 is not tied to the scale of the volume. This makes it possible to circumvent the consistency constraints by fixing the volume below the SUSY breaking scale, which is not possible for D3/D7 with

10 Many further attempts to stabilize the volume without ruining the flatness of the inflaton potential can be found in the literature, for instance in Refs. [42, 43].
FI term given by Eq. (2), as we have seen. Thus, although it may require fine tuning to pull it off, a D3/D3 inflationary scenario with stabilized volume can be consistent with the constraints discussed in Refs. [22, 40].

C. Summary

We have seen that the supergravity consistency conditions detailed in Refs. [21, 22] impose stringent constraints on models of brane inflation in string theory. Assuming that these constraints hold in the full string theory, we are forced to do a careful analysis of the details of compactification and moduli stabilization in brane inflation models to ensure they are consistent with these constraints.

Most generally, as explained in Ref. [22], it is difficult to allow open string degrees of freedom to remain massless while stabilizing closed string degrees of freedom, such as the volume of the compactification manifold. This can generally lead to a compact moduli space for the theory, which is inconsistent [22]. However, as we argued above, this problem can be evaded in the case of D3/D3.

In the presence of an FI term, as in D-term models such as D3/D7, the story is more complicated. Once again it is necessary and difficult to stabilize the volume of the compact manifold, and furthermore stabilizing the volume of K3 would make the FI term, at first glance proportional to 1/\text{vol}(K3), field-independent and therefore the brane inflation model would be inconsistent. The alternative, leaving the volume unfixed, means that inflation may be difficult to achieve, and one can neither neglect warping nor gauge interactions from additional D7 branes. These difficulties are avoided if, as argued above, \( \xi \) depends on the unfixed modulus \( r \), the brane separation. However, it is not immediately clear whether inflation can proceed as usual if this dependence is taken into account.

III. COSMIC SUPERSTRING RADIATION IN WARPED BACKGROUNDS

A. Cosmic superstrings

Cosmic strings are one-dimensional topological defects produced generically during phase transitions in the early universe, whenever the resulting vacuum manifold \( \mathcal{M} \) has non-trivial first homotopy group \( \pi_1(\mathcal{M}) \). Because they carry energy, cosmic strings can seed gravitational instabilities and were therefore investigated throughout the 80s and

\footnote{This assumption may turn out to be false. Supergravity is a low-energy description of string theory, which does not include \( \alpha' \) or higher derivative corrections. Thus, it is conceivable that the constraints from supergravity are weakened when these corrections are taken into account, and/or the brane inflation models need modifications from their current description.}
90s as sources for structure formation. In particular, GUT-scale cosmic strings lead to an adiabatic spectrum of primordial density fluctuations, in agreement with COBE-DMR measurements. However, data from BOOMERanG, DASI, MAXIMA and SASKATOON in the late 90s, and more recently data from WMAP, showed that inflation, which gives rise to an adiabatic scale-invariant power spectrum with Gaussian statistics, must be the dominant source of primordial density fluctuations, with cosmic strings giving a sub-dominant (but not negligible) contribution \[46\]. Despite the fact that GUT-scale cosmic strings are not the dominant source of primordial density perturbations, data is consistent with their sourcing up to 10% of these perturbations and with a string tension satisfying the bound \[47\]

\[
G\mu \leq 2 \times 10^{-7},
\]

where \(G\) is Newton’s constant. These strings could be observed via the gravitational radiation they emit or via gravitational lensing.

Interest in cosmic strings then waned until it was shown that cosmic superstrings are generically produced at the end of brane inflation \[39\] and could provide an observational window on string theory. In contrast to their gauge theory analogues, cosmic superstrings in general have Planck-scale energies, leading to \(G\mu \sim 1\), an inconsistently high tension. These objects are therefore undesirable in string theories of the early universe, and models in which they are produced and remain stable must be ruled out. In a warped geometry \[12\], however, such as in the case of the throat in which brane-antibrane inflation takes place, the tension of cosmic superstrings can easily be lowered to within the bound given in Eq. (11) above \[48\]. In the case of D3/D7 inflation, the problem of an unacceptably high string tension can be solved by making the strings semi-local \[12\], since the upper bound on the tension of semi-local strings is higher than that for local abelian strings, or by suppressing the string production by taking higher order corrections to the Kähler potential into account, as in Ref. \[14\].

Cosmic superstrings, produced at the end of brane inflation \[39\], can be F-strings (the fundamental strings of string theory), D-strings (one-dimensional Dirichlet branes) or \((p, q)\) strings, which are bound states of \(p\) F-strings and \(q\) D-strings. Stable or metastable strings arising from wrapped D-branes, Neveu-Schwarz (NS) branes and M-branes have also been constructed \[49, 50\]; these are higher-dimensional branes that are wrapped on compact cycles resulting in only one remaining non-compact dimension. Thus, the cosmic superstrings of string theory come in many more varieties than those of a GUT and with different interactions and a richer spectrum \[49, 51\]. This realization opened the possibility of a new observational window on string theory models and led to a renewal of interest in the subject. It is our aim to explore this window via radiation from cosmic superstrings. Taking a consistent compactification into account imposes certain constraints on the allowed radiation.

\[12\] The tension can also be lowered in models with large extra dimensions, which we do not consider here.
from these strings, as we will see below. Having examined possible constraints on the brane inflation models in which cosmic superstrings can be produced, we proceed with the allowed channels of cosmic superstring radiation in warped backgrounds.

B. Warped compactifications

In a warped geometry, the metric takes the form
\[ ds^2 = e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g_{mn} dy^m dy^n , \]  
(12)
where \( e^{2A(y)} \) is the warp factor. Here and in the rest of the paper greek indices run over the 0, 1, 2, 3 directions; \( y^m, y^n, \ldots \) denote the internal directions. The internal metric includes a throat-like region such as the warped deformed conifold of the Klebanov-Strassler geometry [18], for which the metric and fluxes are well known. In the warped deformed case the conifold singularity is smoothed away by fluxes.

Such warping can be produced by a stack of D3-branes (as in the AdS/CFT correspondence [52]; AdS_5 can be represented as a four-dimensional space plus radial direction with warp factor) and provides a way to obtain hierarchies of scale in four dimensions [53]. However, the warped KS-type throat in which brane-antibrane inflation takes place is non-compact. In order to include it in a string theory setting, the throat must be “glued” to a compact geometry which gives rise to reasonable four-dimensional physics (e.g. a finite Planck scale in four dimensions). This is achieved using negative tension objects such as orientifold planes, which are needed to satisfy the integrated field equations and result in stringent constraints on the fields and fluxes in the theory [19, 20]. Compactifying on an orientifold results in a truncated spectrum: the orientifold action projects out certain fields [54, 55].

In addition, the dimensional reduction of different light fields is non-trivial in flux compactifications [56]. For instance, consider the universal volume or Kähler modulus, which in the unwarped case corresponds to a rescaling
\[ g_{mn} \rightarrow e^{2u} g_{mn} , \]  
(13)
and fluctuation
\[ ds^2 = e^{-6u(x)} g_{\mu\nu} dx^\mu dx^\nu , \]  
(14)
and which pairs with the universal axion given by
\[ C_4 = \frac{1}{2} a(x) J \wedge J , \]  
(15)
where \( J \) is the Kähler form of the CY, into the complex field \( \rho = a + i e^{4u} \). In a warped background of the form Eq. (12), it is not immediately clear how to define the fluctuations \( u \) or \( a \). Naive attempts for \( u \), such as writing
\[ ds^2 = e^{2A(y)} e^{-6u(x)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} e^{2u(x)} g_{mn}(y) dy^m dy^n , \]  
(16)
do not solve the ten-dimensional Einstein equations. It turns out that additional components (called compensators) in the metric will be required \[56\], complicating the dimensional reduction. Similarly, (two) compensators are required for definition of the universal axion, and these enter in the equations of motion while obeying defining constraints. Thus, the universal Kähler modulus and universal axion wave-functions can be given, at least formally. The same is not true in the case of the non-universal axion which, as we shall see, is the only possible mode for axion emission from \((p,q)\) strings in a throat.

C. Allowed radiation from cosmic superstrings in a throat

We are interested in the types and signatures of radiation from superstring networks. For cosmic string networks to reach a scaling solution, in which the characteristic length scale of the network scales with time, the networks must lose energy. For conventional gauge strings, the dominant mode of this energy loss occurs when loops radiate away into gravitational radiation. Gravitational radiation is also possible (but sub-dominant) for other processes in the evolution of these networks, such as reconnection.

For cosmic superstrings, which are charged under the two-forms \(B_2\) and \(C_2\), axionic radiation is also possible, because these two-forms are Hodge dual to axionic scalars in four dimensions. In the case of D-strings, it has been argued \[11\] that axionic radiation can be the dominant mode of energy loss in a warped geometry, because the warp factor does not couple to the Chern-Simons part of the action, in which \(C_2\) appears.

It might seem that this result would translate to the case of \((p,q)\) strings in a warped background, which is where they are expected to be produced at the end of brane-antibrane inflation. This would give a clear observational difference between cosmic superstrings and gauge theory cosmic strings. However, taking the details of the string compactification into account leads to difficulties with the argument in Ref. \[11\]. More precisely, \(C_2\) and \(B_2\) are projected out of the spectrum, and the axion wave-function is non-trivially modified by the warp factor. We find that there is no well-defined axionic mode for the \((p,q)\) strings in a throat.

D. Gravitational radiation

A \((p,q)\) string in a KS throat can be constructed by wrapping a D3-brane with suitable charges on a 2-cycle (which is stabilized by fluxes) \[57\]. The action is given by

\[
S_{D3} = -\frac{\mu_3}{g_s} \int d^4x \sqrt{-g_{ab} + \mathcal{F}_{ab} + \mu_3 \left( C_2 \wedge \mathcal{F} + \frac{1}{2} C_0 \mathcal{F} \wedge \mathcal{F} \right)},
\]

where \(\mu_3\) is the D3-brane tension, the integral and \(a, b\) indices run over the four-dimensional world-volume 0, 1, 2, 3, where 2 and 3 are the coordinates on the 2-cycle the D3-brane wraps, and \(\mathcal{F}_{ab} = B_{ab} + \lambda F_{ab}\), where \(\lambda = 2\pi\alpha'\) and \(\mu_3 = 1/(2\pi)^3\alpha'\).
Here the metric reads

$$ds^2 = h^2 \eta_{\mu\nu} dx^\mu dx^\nu + ds_6^2,$$

where $h$ is the warp factor (which is a function of the internal coordinates and has thus been absorbed into $ds_6^2$), and the necessary fluxes are

$$F_{23} = \frac{q}{2} \sin \theta d\theta d\phi,$$

$$\tilde{F}^{01} = -\frac{p}{4\pi},$$

$$B_{23} \neq 0,$$

where $\tilde{F}^{\mu\nu}$ denotes the conjugate of the electric field. Hence, we can find the stress-energy tensor $T_{\text{DBI}}^{\mu\nu}$ derived from the Dirac-Born-Infeld (DBI) action:

$$T_{\text{DBI}}^{\mu\nu} = -\int d^2 \zeta \frac{\mu_3}{g_s} \left( g_{22} g_{33} + F_{23}^2 \right)^{1/2} h^4 \frac{1}{(h^4 - \lambda^2 F_{10}^2)^{1/2}} h^4 [\dot{X}^\mu \dot{X}^\nu - X^\mu X^\nu] .$$

Upon minimization, the $(00)$-component

$$T^{00} = \int d^2 \zeta \frac{\mu_3}{g_s} \left( g_{22} g_{33} + F_{23}^2 \right)^{1/2} h^4$$

leads to:

$$T_{(p,q)} = \frac{h_I^2}{2\pi \alpha'} \sqrt{\frac{q^2}{g_s^2} + \left( \frac{bM}{\pi} \right)^2 \sin^2 \left( \frac{\pi(p - qC_0)}{M} \right)}$$

$$= \sqrt{T_D^2 + T_F^2} ,$$

with $T_D, T_F$ denoting the tensions of the D-string and F-string, respectively. This reduces to the flat space expression $T_{(p,q)} = T_{F1} \sqrt{\left( \frac{q^2}{g_s^2} \right) + p^2}$ when $b = h_I = 1$ (and $C_0 = 0$). Thus, the DBI part of the action has the same effect as the usual Nambu-Goto action, except that the tension of the string is modified. Similarly, the string equation of motion arises from varying the action with respect to $\delta X^\mu$, so neither the Chern-Simons terms nor the $F_{ab}$ factor will contribute to these equations. Given $T^{\mu\nu}$, the gravitational radiation from $(p, q)$ strings is obtained from

$$\frac{dP_n}{d\Omega} = \frac{G\omega^2}{\Pi} \left[ T^{\mu\nu}_\ast(\omega_n, \vec{k}) T^{\mu\nu}(\omega_n, \vec{k}) - \frac{1}{2} |T^{\mu\nu}(\omega_n, \vec{k})|^2 \right],$$

where

$$T^{\mu\nu}(\omega_n, \vec{k}) = \frac{2}{L} \int_0^{L/2} dt e^{i\omega_n t} \int d^3 x e^{-i\vec{k}\vec{x}} T^{\mu\nu}(\vec{x}, t).$$

Thus, the gravitational power radiated per unit solid angle is proportional to $T_{(p,q)}^2$, with $T_{(p,q)}$ given by Eq. (22). Hence, it is the same as that radiated by a network of F-strings and a network of D-strings considered separately, in analogy with the result found for cosmic strings in a junction.
E. Axionic radiation

Because \((p, q)\) strings are charged under \(B_{NS}^2\) and \(C_{RR}^2\), the NS-NS and Ramond-Ramond (RR) 2-forms, it should be possible for them to lose energy via emission of massless RR or NS-NS particles. The RR particles are often referred to as axions, because the RR two-form \(C_2\) is Hodge dual in four dimensions to a pseudo-scalar known as the axion.

1. RR particle emission

In flat space, the power radiation via RR particle emission by D-strings is comparable to the radiation by emission of gravitational waves. However, in a warped background, only the DBI part of the action couples to the warp factor, so that it is possible for energy loss via RR radiation to dominate over energy loss by gravitational radiation \([11]\). We would like to check this result for a general network of \((p, q)\) strings.

Considering axion emission from D-strings, Ref. \([11]\) gives a comparison between the gravitational and axionic radiation arising from the IIB D-string action in four space-time dimensions

\[
S_{D,4dim} = \frac{M_p^2}{2} \int d^4x \sqrt{-g} \left( R - \frac{\beta g_s}{12} F_3^2 \right) - \mu_{eff} \int dt dx \sqrt{-\gamma} + \mu_1 \int dt dx C_2 ,
\]

where \(g, \gamma, R, F_3^2, M_P\) are all four-dimensional quantities, \(\gamma_{ab}\) is the pull-back of the four-dimensional metric on the string world-sheet (with the warp factor pulled out), and

\[
\mu_{eff} = h^2 \mu_1 g_s^{-1} ,
\]

\[
M_p^2 = \frac{1}{\kappa_{10}^2} \int d^6y \sqrt{g_6} h^2(y) ,
\]

with \(\mu_1\) the string charge, and \(\mu_1 = (2\pi\alpha')^{-1}\). Using well-known results \([60, 61]\) for field theory cosmic strings, one finds \([11]\)

\[
P_{RR} = \frac{\Gamma_{RR} \mu_1^2}{\pi^2 g_s \beta M_P^2} ,
\]

\[
P_g = \Gamma_g G \left( \frac{h^2 \mu_1}{g_s} \right)^2 ,
\]

leading to the ratio

\[
\frac{P_{RR}}{P_g} = \left( \frac{8 \Gamma_{RR}}{\pi \Gamma_g} \right) g_s \beta h^4 ,
\]

where we have used the fact that \(8\pi G = M_P^{-2}\). Note that \(\Gamma_{RR}\) and \(\Gamma_g\) are numerical factors of the same order \(\mathcal{O}(50)\), and \(\beta\) parametrizes the difference in normalizations between the Chern-Simons and the Einstein-Hilbert term in the presence of warping:

\[
\beta = \frac{\int d^6y \sqrt{g_6} h^2(y)}{\int d^6y \sqrt{g_6} h^2(y)} .
\]
Thus, we see that in the limit where $g_s \ll 1$ and warping is negligible (i.e., $\beta \approx 1$), power loss by gravitational radiation is dominant. However in a warped geometry, for instance a Klebanov-Strassler background, $h = e^{2\pi K/(3g_s M)}$ can be much less than 1 and $P_{RR}$ can be dominant. (Note that $K$ and $M$ are integers specifying the flux background, namely

$$\left(4\pi^2\alpha'\right)^{-1} \int_B H_3 = -K \quad \text{and} \quad \left(4\pi^2\alpha'\right)^{-1} \int_{S^3} F_3 = M ;$$

the RR flux $F_3$ wraps the $S^3$ while the NS-NS flux $H_3$ wraps the Poincaré dual 3-cycle $B$ that generates the warped throat.)

2. NS particle emission

In the case of F-strings, a kinetic term for $B_2$ must be included. It appears in the ten-dimensional (Einstein-frame) action as follows \(^{13}\):

$$S_{IIB} = \frac{1}{2\kappa_{10}} \int d^{10}x \sqrt{-G} \left[ R - \frac{1}{12g_s} H_{(3)}^2 \right] + S_{local} ,$$

where $H = dB$ and the $S_{local}$ part of the action is given by

$$S_{local} = -\mu_1 \int d^2\sigma \sqrt{-|\gamma_{ab}|} + \mu_1 \int d^2\sigma B_2^{NS} .$$

Following Ref. [11] we compare the four-dimensional action to that in Ref. [61], finding (we have to take $B_2 \rightarrow \tilde{B}_2 = (M_P/2)\sqrt{\beta/g_s B_2}$):

$$P_{NS-NS} = \Gamma_{NS} \frac{\mu_1^2 g_s}{\pi^2 \beta M_P^2} .$$

Then from Eqs. (28) and (35), the ratio $P_{NS}/P_g$ goes like $g_s^3$:

$$\frac{P_{NSNS}}{P_g} = \left( \frac{8\Gamma_{NSNS}}{\pi \Gamma_g} \right) \frac{g_s^3}{\beta h^4} ,$$

so this radiation is suppressed compared to the RR particle radiation.

One might wonder what these ratios, Eqs. (30) and (36), would be for $(p, q)$ strings, since if it is possible for particle radiation to be dominant over graviton emission, this would give an important observable difference between cosmic superstrings and gauge strings, at least in the case where they are produced in a warped throat. However, as we will show below, the warped throat construction results in severe constraints on the allowed radiation. In what follows, we will examine the validity of the result claimed in Ref. [11], and its possible extension in the case of $(p, q)$ strings, taking into account the constraints from the orientifold projection imposed by a consistent flux compactification.

\(^{13}\) The IIB action can also be given in $SL(2, \mathbb{Z})$ invariant form in terms of $G_3 = F_3 - \tau H_3$ where $\tau = C_0 + ie^{-\phi}$, but we consider the D- and F-string cases separately here.
3. Constraints from the orientifold projection

The enhancement of RR particle emission claimed for D-strings [11] is due to the effect of warping. We reviewed above the construction and tension of \((p, q)\) strings in a Klebanov-Strassler throat given in Ref. [57]. This is the relevant construction to study if we want to answer the question of whether the enhancement of RR radiation observed in Ref. [11] for D-strings carries over to the case of \((p, q)\) strings. However, a careful consideration of the throat geometry reveals some subtleties with the construction. The only known consistent compactification of the Klebanov-Strassler geometry is the flux compactification given by Giddings, Kachru and Polchinski (GKP) [19] (see also Ref. [20]), which involves an orientifold projection

\[ O = (-1)^{F_L} \Omega_p \sigma^* , \]  

where \(\sigma^*\) is the pull-back of an isometric and holomorphic involution \(\sigma\) which leaves the Kähler form \(J\) invariant \((\sigma^* J = J)\) but acts non-trivially on the holomorphic three-form \(\Omega\) \((\text{for } O3/O7\text{-orientifold planes } \sigma^* \Omega = -\Omega)\). \(\Omega_p\) is the world-sheet parity and \(F_L\) is the space-time fermion number in the left-moving sector. [We refer the reader to, for example, Refs. [55, 63] for a detailed explanation of the orientifold action.] The action of \(O\) on the NS-NS and RR two-forms is

\[ OB_2 = -\sigma^* B_2 \quad \text{and} \quad OC_2 = -\sigma^* C_2 , \]  

respectively. Since \(\sigma\) is an internal symmetry which acts on the internal manifold and leaves the four-dimensional non-compact space invariant, the NS-NS and RR two-forms with legs in the non-compact directions are projected out [54]. This means that there can be no massless RR or NS-NS axion, since the zero modes of \(B_{\mu\nu}\) and \(C_{\mu\nu}\) do not appear in the spectrum [49]. At this point it is worth noting that although this implies that D, F or \((p, q)\) strings will not saturate the Bogomol’nyi-Prasad-Sommerfeld (BPS) bound, because there is no gauge group for them to be charged under, they will be stable against annihilation with their image under the orientifold because this annihilation is suppressed by the warping: the strings stretching between them are massive [49].

---

14 Here we consider the orientifold projection corresponding to inclusion of \(O3\) and \(O7\)-planes, which is the case for KS. Another possible orientifold, corresponding to the case of \(O5\) and \(O9\) planes, has orientifold action \(\sigma^* \Omega = +\Omega\). This would result in a different spectrum, since \(C_2\) in the non-compact directions is not projected out in this case. However, at present no brane inflation model in a consistent compactification corresponding to this orientifold case is known to us. Furthermore, we remain at the orientifold limit, corresponding to the constant dilaton case. More complicated F-theory compactifications, in which the configuration of the \(O7\) planes and \(D7\)-planes can vary, are possible for the non-constant \(\tau\) case [19, 20], but we do not consider this case here. We expect that any states which are able to survive the orientifold projection in this case will be massive [13, 62], so they will not affect our argument.
One might thus wonder if axionic radiation is possible at all. To check this, we decompose a general two-form:

\[ D_2(x) = d_2(x^\mu) + d(x)\Omega_2 + V_1(x) \wedge \alpha_1 , \]

where \( d(x) \) is a scalar, \( d_2(x^\mu) \) is a two-form in the non-compact directions, \( \Omega_2 \) is a two-form in the internal directions, and \( \alpha_1 \) a one-form in the internal directions. While \( d_2(x^\mu) \) is projected out by the orientifold action, \( \Omega_2 \) is not as long as it is in the \(-1\) eigenspace of the involution \( \sigma^* \) in the orientifold projection. Because \( \mathcal{O}D_2 = -\sigma^*D_2 \), this component of the two-form will survive the orientifold projection, and indeed \( d(x) \) is known in the literature as a model-dependent axion [64]. For a D-string charged under \( C_{01} \) this makes no difference, as \( C_{01} \) is projected out. Since there is no massless RR mode, there can be no axionic RR radiation from a D-string in a warped background, so it does not make sense to compare this radiation to the gravitational radiation from a D-string in a throat. As a consequence, the result found in Ref. [11] is inapplicable. Similarly, an F-string charged under \( B_{01} \) will have no axionic zero modes in a consistent warped geometry. In conclusion, neither D- nor F-strings can lead to significant axionic emission, since by construction of the consistent brane inflationary model that will lead to their formation, such objects will not have any massless axionic radiation.

However, for a \((p, q)\) string which is actually a wrapped D3-brane with fluxes, as in Ref. [57], the situation may be different. The model-dependent axion \( d(x) \) described above is possible when \( D_2 \) has internal legs. Here \( C_{\theta \phi} \) (or \( C_{23} \)) is allowed, and couples to the string, as long as the internal 2-cycle is odd under \( \sigma^* \). There is no such axion for the NS-NS two-form, which must be along the 0,1 directions, so NS-NS particle radiation is entirely ruled out.

For the \((p, q)\) string arising from a wrapped D3-brane, we should also consider an analogous decomposition for the RR four-form \( C_4 \):

\[ C_4(x^M) = c_4(x^\mu) + c_2(x^\mu) \wedge \Omega_2 + c_1(x^\mu) \wedge \Omega_3 + c(x^\mu) \wedge \Omega_4 \quad \text{with} \quad \mathcal{O}C_4 = \sigma^*C_4 . \]  

In our case, in which the D3-brane is wrapped on a 2-cycle, it is the term \( c_2(x^\mu) \wedge \Omega_2 \) which is of interest, and which is allowed as long as \( \Omega_2 \) is odd under the involution.

Harmonic \((p, q)\) forms are in one-to-one correspondence with the elements of the cohomology groups \( H^{(p,q)} \) which split into two eigenstates under the action of \( \sigma^* \):

\[ H^{(p,q)} = H_+^{(p,q)} \oplus H_-^{(p,q)} . \]

The \( \pm \) subscripts refer to even/odd behavior under \( \sigma^* \). Thus, in order for any RR mode to survive, it must be of the form \( C_2 \wedge F_2 \) on the D3-brane, where \( F_2 \) has legs in the \( \theta \)- and \( \phi \)-directions on a two cycle \( \Omega_2 \in H_-^{(1,1)} \).

Such a cycle is certainly allowed for a general \( CY_3 \). There is only one two-form on the \( S^2 \) wrapped by the D3-brane, the volume form \( \omega \) which can be written as

\[ \omega = x_1 dx_2 \wedge dx_3 + x_2 dx_3 \wedge dx_1 + x_3 dx_1 \wedge dx_2 \]
and is odd under, e.g., $x_1 \leftrightarrow x_2$. As long as such an involution is holomorphic and isometric on the full CY, these two modes are allowed.

4. **The axionic wave-function**

To determine allowed modes for radiation by cosmic superstrings in a warped geometry, it is necessary not only to take into account which modes survive the orientifold projection, but also to consider the correct dimensional reduction of these modes, which is a non-trivial task. In Ref. [56], the equations of motion including warping (which includes compensating terms) were given for the universal Kähler (volume) modulus and its axionic partner $a$, the universal axion, which arises from the four-form as

$$C_4 = a J \wedge J,$$

(41)

where $J$ is the Kähler form. The action of the orientifold on $C_4$ is just $OC_4 = \sigma^* C_4$ and the involution $\sigma^*$ leaves the Kähler form invariant, so this universal axion is allowed in a compactified throat geometry. However, the legs of the four-form are all in the internal manifold, so this axion cannot be sourced by the wrapped D3-brane. Note that the problem is not solved by wrapping a D5-brane (say) on a 4 cycle, because this 4 cycle would have to be odd under the action of $\sigma^*$ and cannot therefore be given by $J \wedge J$.

Thus the only allowed axions are the non-universal axions. We have seen that only these could couple to the $(p, q)$ string constructed from a wrapped D3-brane.

For the axion arising from $C_4 = c_2(x) \wedge \Omega_2$, the analysis used in Ref. [56] should be applicable, but it is not known how to solve the equations of motion for this case. An added complication is the fact that the compensators needed, mentioned in Section III B, result in mixing between $C_2$ and $C_4$. This is quite generic in dimensional reduction on warped geometries, because the backgrounds break diffeomorphism invariance, so that the correct gauge-invariant object to consider is a combination of different fields [65]. The resulting wave-function would affect the magnitude of any radiation in this mode, so without it it is not possible to quantify the amplitude of the radiation.

F. **Dilaton radiation**

Let us briefly mention the possibility of dilatonic radiation by cosmic superstrings. As noted in Ref. [11], the dilaton is expected to be massive, making dilaton radiation by cosmic superstrings negligible compared to the massless axionic radiation modes. The dilaton mass is constrained by cosmological and astrophysical observations (see e.g. Refs. [66, 67]), and is compatible with observations only if it acquires a VEV early in the history of the universe (before the end of nucleosynthesis). Indeed, the dilaton gets a non-trivial potential in the GKP compactification, because it enters the action in combination with the RR and NS-NS
fluxes as $G_3 = F_3 - \tau H_3$, where $\tau = C_0 + i e^{-\phi}$ and $G_3$ must be imaginary selfdual. However, as was shown in, e.g., Refs. \cite{68,69}, the mass of a dilaton in the throat will be suppressed by the warp factor, namely

$$m \sim \frac{e^A}{\sqrt{\alpha'}}$$

deep in the throat, with $A$ denoting the warp factor. It is therefore conceivable that dilatons could be produced, particularly by high energy processes like the decay of metastable strings via monopole production \cite{70}. 15

Constraints on cosmic superstring tension as a function of the dilaton mass were obtained in Ref. \cite{67}. In Ref. \cite{71} it was shown that in a warped geometry, these constraints are weakened because, not only is the mass suppressed by the warp factor, but the dilaton wavefunction will be localised in the throat, with an exponential fall off in the bulk, which will increase the strength of the coupling $\alpha$ of the dilaton to matter \cite{68,69,72,74,75}. 16

Dilaton radiation from cosmic strings was studied in Refs. \cite{76,77,67} via the coupling of the dilaton field to the matter fields forming the strings. The interaction term in the Lagrangian taking the form

$$L_{\text{int}} \sim \frac{\alpha}{M_{Pl}} \phi T^\mu_\mu,$$  \hspace{1cm} (42)

with $\alpha$ the coupling constant, the power of dilaton radiation was found to be proportional to the square of the coupling:

$$P_d \sim \Gamma \alpha^2 G\mu^2,$$  \hspace{1cm} (43)

where $\Gamma$ is a numerical factor of order 30. In the case of F- and D-strings the constraints on the string tension from dilaton emission can be further weakened depending on whether they couple to matter or not \cite{71}.

IV. CONCLUSIONS

In this article we have set out to examine theoretical constraints on brane inflation models and the radiation channels of cosmic superstrings formed at the end of brane inflation. Cosmic superstrings can be produced at the end of brane inflation, whether it be D3/D7 inflation which takes place in an unwarped geometry, or brane-antibrane inflation (for instance

\footnote{For it to make sense to consider only the lightest dilaton mode, one needs to check that the lowest mass is much smaller than the KK mass gap. Otherwise this lowest mass mode should be integrated out \cite{69}. The KKLT throat is a “short throat” in the terminology of Ref. \cite{69}, which means that the mass gap allows one to keep the lowest mass dilaton mode, while the suppression of the mass compared to the unwarped case still holds.}

\footnote{There is a possible complication here: axion-dilaton fluctuations in a warped background will mix with metric fluctuations \cite{73}. This will affect the wave-function. According to Ref. \cite{69} though, the contribution of compensators to the axion-dilaton mass will be negligible as long as the throat is long enough.}
D3/D3) which takes place in a throat. Cosmic superstrings can arise in many different forms in string theory, as F-, D-, or FD-strings (or wrapped higher-dimensional branes) and correspondingly have more possible radiation modes than gauge theory cosmic strings. However, the constraints on both the brane inflationary models leading to cosmic superstring formation, as well as the radiation modes of cosmic superstrings coming from consistency of the string theory embedding, are quite dramatic.

In the first part of this article we considered the implications of recent SUGRA constraints on models of brane inflation. It is necessary to check carefully that such models are consistent with these constraints, since in general in brane inflation models it is desirable to leave the brane positions unfixed while the volume of the compactification manifold is stabilized. In the case of D-term inflation models, a constant (field-independent) FI term is inconsistent with the SUGRA constraints, so these must be analyzed carefully.

We found that D3/D3 inflation is consistent with the SUGRA constraints since the volume modulus can be fixed below the SUSY breaking scale. In D3/D7 inflation, which relies on the existence of an FI term, it is necessary for consistency of the model that the FI term depend on a modulus which is unfixed. We argue that this is the case – that the FI term depends on the separation between the branes – but point out that such a dependence may have implications for the inflationary dynamics in this model. In the second part of the article we focussed on radiation channels of cosmic superstrings in a warped background, which are also subject to strong constraints. It is in warped throats that cosmic superstring tension can be lowered to within observationally acceptable bounds (without the need for the strings to be semilocal, for instance). Furthermore, it was argued in Ref. [11] that warping could result in enhancement of axionic radiation by D-strings as compared to gravitational radiation. We have seen that the results of Ref. [11] do not translate easily to the case of \((p, q)\) strings in a throat. To begin with, axionic radiation from either F- or D-strings is ruled out in a consistent warped compactification because the modes \(B_2\) and \(C_2\) are projected out by the orientifold action. Thus, it is not possible for axionic radiation from D-strings in a throat to be enhanced relative to gravitational radiation. For \((p, q)\) strings which are actually branes wrapped on suitable cycles (e.g. a D3-brane wrapped on \(\Omega_2\) where \(\Omega_2\) is odd under the involution \(\sigma^*\)) axionic radiation is still possible in principle. However, the wave-function of the axionic zero mode is non-trivially modified by the warping, and only limited progress has been made in taking these modifications into account. The equations of motion for the universal axion can be written down, but the same is not yet true for the non-universal axion. We do not see any way for a \((p, q)\) string in a throat to couple to the universal axion, which means that it is not currently possible to calculate correctly the power of the Ramond-Ramond radiation by these strings.
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