Effect of Healthy Life Campaigns on Controlling Obesity Transmission: A Mathematical Study

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Abstract. In this study, we present a mathematical model that describes how obesity spread among the human population, considering human awareness levels to describe the difference in lifestyle of humans, in which the transition between this group depends on the media campaign from the authority about the importance of healthy lifestyles and persuasive capability of individuals who quit obesity. The model constructs as four-dimensional nonlinear ordinary differential equations. Possible equilibrium points are investigated regarding their existence and local stability criteria. Basic reproduction number ($R_0$) of the model obtained from the next-generation matrix approach. It has been shown that the obesity-free equilibrium is locally asymptotically stable if $R_0$ is less than one and unstable otherwise. A transcritical bifurcation when $R_0 = 1$ was investigated using the Castillo-Song bifurcation theorem. From the elasticity analysis, we find that the social contact rate is the most influential parameter in determining the magnitude of $R_0$, followed by a healthy life campaign from the government. A short discussion to understand the possible scenario in the field obtained numerically based on our analytical results conducted at last.

1. Introduction

The World Health Organization defines obesity as any abnormal or excessive fat distribution that presents a health risk. There are several ways to categorize someone as obese; one of the most common methods is using Body Mass Index (BMI). According to WHO, someone classified as overweight and obese when the BMI is equal to or more than 25 [1]. Some factors that can cause obesity are lack of exercise or any other physical activity that aims to burn calories and increase the consumption of foods with high fat and sugar. As a result, there is an imbalance between calories consumed and calories expended [2].

Obesity is associated with chronic diseases, such as diabetes mellitus, stroke, hypertension (high blood pressure), cancer, and heart disease [2,3]. Also, it can damage their self-esteem and cause depression [4]. Obesity can be reduced by choosing to eat healthier foods and limiting consumption fats or sugars, make regular physical activity, and diet behaviors consulted with experts [3].

In 2016, more than 1.9 billion people 18 years and older were overweight, and over 650 million of them were obese; out of the world’s population, there were 13% obese adults aged 18 years and over [2]. In Indonesia, in 2018, the average number of adults ages 18 years and over who are obese is 21.8%, with the highest obesity rate being recorded in North Sulawesi, at 30.2%, and the lowest obesity rate being recorded in East Nusa Tenggara, at 10.3% [5]. This result tends to increase from the previous years.
Therefore, to overcome the problem of obesity, many programs have been initialized, such as WHO developing new guidelines on physical activity and sedentary behavior [6]. Promoting physical activity programs to bring more active people for a healthier world, regular health check-ups, motivates individuals to go on a healthy diet by more eating vegetables, fruit, and reducing sugar consumption have been done around the world. In Indonesia, the Ministry of Health has a campaign for a healthy lifestyle (GERMAS, CERDIK, GENTAS) to increase awareness of obesity [7–9].

The spread of obesity has been studied in [10] by Christakis and Fowler; the result obtained is the chance of individuals becoming obese increased if they have spouse, siblings, or friends who are also obese. Because of this, it is said that obesity spreads through social ties. This approach has also resulted in obesity studied as a social epidemic [10–14]. Authors in [11] used numerical analysis to study infant obesity caused by high-frequency consumption of bakery, fried meals, and soft drinks that depend on a sociocultural characteristic. In [12], the authors conducted a simulation to see interactions in a randomly mixed population between healthy people, overweight people, and obese people. The intervention program is implemented in this paper, such as treatment for obese people and dietary programs with a health life campaign for overweight people. The authors claimed that a healthy life campaign and rehabilitation program could significantly reduce the number of obese people. In [13] for society with the number of obesity is low, obesity can occur due to spread through social contact with the overweight and obese population rather than lack of a campaign. However, the most effective strategy to reduce obesity in societies with a very high amount of obesity is campaign/education. The authors in [14] said that it is more prominent to make the population sensitive about the benefits of having a balanced and healthy diet and making informative policies related to public prevention.

Different from the mentioned references before, here in this article, we construct a mathematical model to describe how obesity spread among the closed population through direct social contact. We consider the low and high risk of an individual to describe the difference in lifestyle of humans, in which the transition between this group depends on the media campaign from the authority about the importance of health-life style. In the next section, we will construct the model carefully considering some assumptions. A Mathematical analysis conducted in the third section to see the effect of the basic reproduction number ($R_0$) in determining the existence and local stability of all equilibrium points. Our results show the existence of a transcritical bifurcation phenomenon on $R_0 = 1$. In the fourth section, sensitivity and elasticity analysis are given to see the most important parameter that can be controlled to reduce $R_0$. Some numerical simulations and conclusions are given in the fifth and sixth sections, respectively.

2. Mathematical model construction and assumptions

Here, we created a long term scenario of the spread of obesity with a healthy life campaign using a system of differential equations. We assumed that the human population is divided into four subpopulations, that is, normal-weight population with low awareness of obesity ($N_l$), normal-weight population with high awareness of obesity ($N_h$), obese population ($O$), and recovered population or ex-obese ($Q$). The total human population, denoted by $H$.

In this model, the number of people born per unit of time and the natural death rate are assumed to be equal. However, we also consider death from disease caused by obesity, so the population is not constant. The total birth rate, natural death rate, and obesity-related death rate, respectively, denoted by $A$, $\mu$, $\sigma$. The social contact that occurs in this model is persuasive, which means other people tend to follow the lifestyle of someone who influences them. People who can influence are obese people and recovered people. When normal-weight people with low awareness of obesity make intensive social contact with obese people, the transmission of obesity can occur through social contact.

Furthermore, social contact with recovered people intensively can increase awareness of
obesity through social contact. In addition, we applied a healthy life campaign in this model to promote activity and healthy food consumption. This healthy life campaign can increase awareness of obesity at a rate of $\alpha$. The decreased awareness of obesity can occur at a rate of $\delta$, and the recovery rate from the obese population to the recovered population is $\gamma$.

The transmission of obesity occurs through social contacts. If we assumed the average number of meetings each person with other people regardless of their health status is as much as $b$ per unit of time, then the number of meetings that occur between normal-weight people with low awareness of obesity and obese people per unit of time is $b\frac{N_l}{H}O$, where $\frac{N_l}{H}$ is the proportion of normal-weight people (low awareness) towards the total population. Assuming the successful transmission probability rate from obese people to normal-weight people with low awareness of obesity is $\bar{\beta}_1$, the transition rate from the normal-weight population with low awareness of obesity to the obese population is given by $\bar{\beta}_1 b\frac{N_l}{H}O$.

We set $\beta_1 = \bar{\beta}_1 b$. The process of increased awareness of obesity through social contact can be described as $\beta_2$. Afterward, $b\frac{N_l}{H}Q$ is the number of meetings between normal-weight people with low awareness of obesity and recovered people per unit of time. Assuming the successful rate of increasing awareness of obesity through social contact from recovered people to normal-weight people with low awareness of obesity is $\bar{\beta}_2$, then $\bar{\beta}_2 b\frac{N_l}{H}Q$, is the number of individual who become normal-weight people with high awareness of obesity per unit of time. Here, we have set $\beta_2 = \bar{\beta}_2 b$.

From the assumption above, the mathematic model of the spread of obesity with healthy life campaign is governed by following set of differential equations:

$$\begin{align*}
\frac{dN_l}{dt} &= A - \beta_1 \frac{N_l}{H}O + \delta N_h - \beta_2 \frac{N_l}{H}Q - \alpha N_l - \mu N_l, \\
\frac{dN_h}{dt} &= \beta_2 \frac{N_l}{H}Q + \alpha N_l - \delta N_l - \mu N_h, \\
\frac{dO}{dt} &= \beta_1 \frac{N_l}{H}O - \gamma O - (\mu + \sigma)O, \\
\frac{dQ}{dt} &= \gamma O - \mu Q,
\end{align*}$$

(1)

wherein $H = N_l + N_h + O + Q$. Note that this system equipped with non-negative initial conditions

$N_l(t = 0) = N_l(0) \geq 0$, $N_h(t = 0) = N_h(0) \geq 0$, $O(t = 0) = O(0) \geq 0$, $Q(t = 0) = Q(0) \geq 0$, and all parameters are positive. Note that above systems fulfill the following properties regarding their positiveness and boundedness of the solution.

**Theorem 1** Let the initial condition of each variables in system 1 are non-negative. Then the solution of $N_l(t), N_h(t), O(t)$ and $Q(t)$ are non-negative for all time $t > 0$.

**Theorem 2** Let $N_l(t), N_h(t), O(t)$ and $Q(t)$ be the solution of system 1 with non-negative initial conditions, and closed set

$$\Pi = \left\{(N_l(t), N_h(t), O(t), Q(t)) \in \mathbb{R}_+^4, N_h < \frac{A}{\mu}\right\}$$

then $\Pi$ is positively invariant and attracting under the flow described by system 1.
This both theorems, guarantee that our model will always have a biological interpretation, that each variables describe the number of human which is always positive.

3. Equilibrium points and the basic reproduction number

To calculate the equilibrium points of system 1, we have to set the right-hand side of system 1 equal to zero and solve it with respect to each variables. To be mentioned, let

\[
\begin{align*}
A - \beta_1 \frac{N_l}{N_l + N_h + O + Q} O + \delta N_h - \beta_2 \frac{N_l}{N_l + N_h + O + Q} Q - \alpha N_l - \mu N_l &= 0, \\
\beta_2 \frac{N_l}{N_l + N_h + O + Q} Q - \alpha N_l - \delta N_l - \mu N_h &= 0, \\
\beta_1 \frac{N_l}{N_l + N_h + O + Q} O - \gamma O - (\mu + \sigma) O &= 0, \\
\gamma O - \mu Q &= 0.
\end{align*}
\]

The system (2) has a trivial equilibrium, namely \( E_0 = (N_{l0}, N_{h0}, O_0, Q_0) = \left( \frac{A(\delta + \mu)}{\mu(\delta + \mu + \alpha)}, \frac{A\alpha}{\mu(\delta + \mu + \alpha)}, 0, 0 \right) \) as an obesity-free equilibrium (OFE). It can be seen that in this equilibrium, all infected population disappears from the population. We can also see that the ratio between \( N_l \) and \( N_h \) is given by

\[
\frac{N_l}{N_h} = \frac{\delta + \mu}{\alpha}.
\]

It is clear that the purpose of controlling obesity with media campaigns is to increase the number of humans who are aware of the dangers of obesity, which, in this case, is described by reducing this ratio. Reducing this ratio can be achieved by increasing \( \alpha \) (media campaign about health-life style), or reducing \( \delta \) (drop-out rate from high to low awareness susceptible individual).

Having the obesity-free equilibrium, we are set to calculate the respective basic reproduction number. The basic reproduction number is defined as the average number of new cases of an infection caused by one typical infected individual in a population consisting of susceptible only [15]. The basic reproduction number (\( R_0 \)) of our model is computed using the method described in [15]. First, we linearize the obesity subsystem of nonlinear ordinary differential equations around the obesity-free equilibrium. Then the Jacobian matrix is obtained as follows:

\[
J = \begin{bmatrix}
\beta_1 \frac{(\delta + \mu)}{\delta + \mu + \alpha} - \gamma - \mu - \sigma
\end{bmatrix}.
\]

Second step is decomposing the matrix (3) as \( T + \sum \), where \( T \) is the transmission part and \( \sum \) is the transition part, which yield

\[
T = \begin{bmatrix}
\beta_1 \frac{(\delta + \mu)}{\delta + \mu + \alpha}
\end{bmatrix},
\]

\[
\sum = \begin{bmatrix}
-\gamma - \mu - \sigma
\end{bmatrix}.
\]

Next, we compute the next generation matrix. This matrix is denoted by \( K \) which given by

\[
K = -T \sum^{-1},
\]

\[
= \begin{bmatrix}
\beta_1 \frac{(\delta + \mu)}{(\gamma + \mu + \sigma)(\delta + \mu + \alpha)}
\end{bmatrix}.
\]

\[
(4)
\]
The basic reproduction number \( R_0 \) obtained from the spectral radius \( \rho \) of (4) is given by

\[
R_0 = \frac{\beta_1 (\delta + \mu)}{(\gamma + \mu + \sigma) (\delta + \mu + \alpha)}.
\]  

(5)

For reader intent to see more example about the calculation of the basic reproduction number using the next-generation matrix can see [16–24].

This basic reproduction number can be described as a multiplication between two component, that is (i) \( \frac{\beta_1}{\gamma + \mu + \sigma} \), which describe the total number of infection produced by an infected individual during the infection period \((\gamma + \mu + \sigma)^{-1}\), and (ii) the proportion of susceptible individual with high risk of obesity with the total of the human population (without death induced by obesity). Therefore, reducing the infection rate \( \beta_1 \) and increasing media campaign \( \alpha \) is a very reasonable way to reduce the basic reproduction number. Further discussion on this analysis regarding \( R_0 \) will be discussed in the next section.

The next equilibrium point is the non-trivial equilibrium point, which is sometimes called the endemic equilibrium point is given by \( E_1 = (N_1^*, N_h^*, O^*, Q^*) \), where

\[
\begin{align*}
N_1^* &= \frac{A (R_0 (\gamma + \mu) (\delta + \mu + \alpha) + \gamma \beta_2 (\gamma + \mu + \sigma))}{\mu \beta_1 (R_0 (\gamma + \mu) + (R_0 - 1) \sigma) (\delta + \mu + \alpha) + \gamma \beta_2}, \\
N_h^* &= \frac{A \gamma (R_0 - 1) (\gamma + \mu + \sigma) (\delta + \mu + \alpha)}{(R_0 (\gamma + \mu) + (R_0 - 1) \sigma) (\delta + \mu + \alpha) + \gamma \beta_2}, \\
O^* &= \frac{A \gamma (R_0 - 1) (\gamma + \mu + \sigma) (\delta + \mu + \alpha)}{(R_0 (\gamma + \mu) + (R_0 - 1) \sigma) (\delta + \mu + \alpha) + \gamma \beta_2}, \\
Q^* &= \frac{A \gamma (R_0 - 1) (\gamma + \mu + \sigma) (\delta + \mu + \alpha)}{(R_0 (\gamma + \mu) + (R_0 - 1) \sigma) (\delta + \mu + \alpha) + \gamma \beta_2}.
\end{align*}
\]

(6)

We can see that the condition (6) corresponds to the basic reproduction number \( R_0 > 1 \), which is discussed in detail in the next section. Thus, for \( R_0 > 1 \), there exists the endemic equilibrium point. We conclude our results in this section in the following theorem.

**Theorem 3** System 1 always has an obesity-free equilibrium point which exist without condition and the endemic equilibrium point which exist when the basic reproduction number is larger than unity.

4. Stability and the transcritical bifurcation analysis

We continue our previous results by analyzing the long-term behavior of our model, especially regarding the stability of each equilibrium point. The first result is about the stability of the obesity-free equilibrium point, which is given in the following theorem.

**Theorem 4** The obesity-free equilibrium point of system 1 \( (E_0) \) is locally asymptotic stable if \( R_0 < 1 \), and unstable if \( R_0 > 1 \).

**Proof.** Evaluated Jacobian matrix in the obesity-free equilibrium to get the eigenvalues as follows:

\[
J_{OFE} = \begin{bmatrix}
-\alpha - \mu & \delta & -\frac{\beta_1 (\delta + \mu)}{\delta + \mu + \alpha} & -\frac{\beta_2 (\delta + \mu)}{\delta + \mu + \alpha} \\
\alpha & -\delta - \mu & 0 & \frac{\beta_2 (\delta + \mu)}{\delta + \mu + \alpha} \\
0 & 0 & \frac{\beta_1 (\delta + \mu)}{\delta + \mu + \alpha} - \gamma - \mu - \sigma & 0 \\
0 & 0 & \gamma & -\mu
\end{bmatrix},
\]

(7)
Note that, if $R_0 < 1$, then $\lambda_4 < 0$ and therefore $E_0$ is locally asymptotic stable, while $R_0 < 1$, $\lambda_4 > 0$ and $E_0$ is unstable.

Next, we analyze the stability of the endemic equilibrium point. Because of the complexity form of $E_1$, we prefer to use the center-manifold approach to analyze it stability. We will use Theorem 4.1 of [25], to obtain the local asymptotic stability of the endemic equilibrium. Let, $N_1 = x_1, N_2 = x_2, O = x_3,$ and $Q = x_4,$ and let $\beta_1$ be the bifurcation parameter. The system (2) becomes

\[
\begin{align*}
\frac{dx_1}{dt} &= A - \beta_1 \frac{x_1}{x_1 + x_2 + x_3 + x_4} x_3 + \delta x_2 - \beta_2 \frac{x_1}{x_1 + x_2 + x_3 + x_4} x_4 - \alpha x_1 - \mu x_1, \\
\frac{dx_2}{dt} &= \beta_2 \frac{x_1}{x_1 + x_2 + x_3 + x_4} x_4 + \alpha x_1 - \delta x_2 - \mu x_2, \\
\frac{dx_3}{dt} &= \beta_1 \frac{x_1}{x_1 + x_2 + x_3 + x_4} x_3 - \delta x_2 - \mu x_2, \\
\frac{dx_4}{dt} &= \gamma x_3 - \mu x_4,
\end{align*}
\]

with $R_0$ corresponding to $\beta_1 = \beta_1^* = \frac{(\delta + \mu + \alpha)(\gamma + \mu + \sigma)}{\delta + \mu + \alpha}$.

The linearization matrix of system (8) around the obesity-free equilibrium when $\beta_1 = \beta_1^*$ is

\[
D_x f = \begin{bmatrix} -\alpha - \mu & \delta & -\gamma - \mu - \sigma & -\frac{(\delta + \mu)\beta_2}{\delta + \mu + \alpha} \\ \alpha & -\mu - \delta & 0 & \frac{(\delta + \mu)\beta_2}{\delta + \mu + \alpha} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma & -\mu \end{bmatrix},
\]

it is clear that from the Jacobian matrix (9), 0 is a simple eigenvalue. Using a technique in Castillo-Chavez and Song, matrix $D_x f$ has a right eigenvector associated with 0 eigenvalue, where

\[
\begin{align*}
w_1 &= w_1 < 0, \\
w_2 &= \frac{((\gamma + \mu + \sigma)(\delta + \mu + \alpha)\alpha - \gamma \beta_2(\delta + \mu)) w_1}{(\delta + \mu)((\delta + \mu + \alpha)(\gamma + \mu + \sigma) + \gamma \beta_2)}, \\
w_3 &= -\frac{\mu (\delta + \mu + \alpha)^2 w_1}{(\mu + \delta)((\delta + \mu + \alpha)(\gamma + \mu + \sigma) + \gamma \beta_2)}, \\
w_4 &= -\frac{\gamma (\delta + \mu + \alpha)^2 w_1}{(\mu + \delta)((\delta + \mu + \alpha)(\gamma + \mu + \sigma) + \gamma \beta_2)}.
\end{align*}
\]

Further, the left eigenvector associated with 0 eigenvalues is

\[
\begin{align*}
v_1 &= 0, \\
v_2 &= 0, \\
v_3 &= v_3 > 0, \\
v_4 &= 0.
\end{align*}
\]
Since \( v_1 = v_2 = v_3 = 0 \), we only need to compute the partial derivative of \( f_3 \) at OFE. The nonzero partial derivative of \( f_3 \) at OFE from system (8) are given by

\[
\frac{\partial^2 f_3}{\partial x_1 \partial x_3} = \frac{\partial^2 f_3}{\partial x_3 \partial x_1} = \frac{\alpha \mu \beta_1}{A (\delta + \mu + \alpha)},
\]
\[
\frac{\partial^2 f_3}{\partial x_2 \partial x_3} = \frac{\partial^2 f_3}{\partial x_3 \partial x_2} = \frac{\partial^2 f_3}{\partial x_3 \partial x_4} = \frac{\partial^2 f_3}{\partial x_4 \partial x_3} = -\frac{\mu (\delta + \mu) \beta_1}{A (\delta + \mu + \alpha)},
\]
\[
\frac{\partial^2 f_3}{\partial x_3 \partial x_3} = -2 \frac{\mu (\delta + \mu) \beta_1}{A (\delta + \mu + \alpha)},
\]
\[
\frac{\partial^2 f_3}{\partial x_3 \partial \beta_1} = \frac{\delta + \mu}{\delta + \mu + \alpha}.
\]

Hence,

\[
a = -\frac{2 \mu^2 \beta_1 ((\delta + \mu + \alpha) (\gamma + \mu) + \gamma \beta_2 (\delta + \mu + \alpha)^2 w_1^2 v_3}{A (\delta + \mu) ((\delta + \mu + \alpha) (\gamma + \mu + \sigma) + \gamma \beta_2)},
\]
\[
b = -\frac{\mu (\delta + \mu + \alpha) w_1 v_3}{(\delta + \mu + \alpha) (\gamma + \mu + \sigma) + \gamma \beta_2}.
\]

Because \( w_1 < 0 \) and \( v_3 > 0 \), thus \( a < 0, b > 0 \). So by the Theorem 4.1 (iv) in [25], we can give the following result

**Theorem 5** The endemic equilibrium of the system 1 is locally asymptotic stable for \( R_0 \) larger but close to one.

To illustrate our above results, we give a numerical example regarding the existence of equilibrium points and the transcritical bifurcation simulation in Figure 1. To reach bifurcation diagram in Figure 1, we use the following parameters values, and leave \( \beta_1 \) as the bifurcation parameter.

\[
A = \frac{1000}{65}, \beta_2 = 0.005, \alpha = 0.035, \delta = 0.019, \gamma = 0.001, \mu = \frac{1}{65}, \sigma = 0.009.
\]

Figure 1, shows that when \( \beta_1 < 0.05122354155 \), we have that \( R_0 \) is less than unity. Therefore, based on Theorem 4, we have that \( E_0 \) is stable, and the endemic equilibrium not yet exist. When \( \beta_1 \) reach 0.05122354155, we have that \( R_0 \) equal to 1, and a simple zero eigenvalue appear. Change of stability (and the existence of endemic equilibrium) occur. When \( \beta_1 > 0.05122354155 \), we have that the \( R_0 > 1 \). Based on Theorem 3 and Theorem 5, endemic equilibrium appears and stable, while the obesity-free equilibrium becomes unstable when \( R_0 > 1 \).

As we notice, \( \beta_2 \) which describes the increase of awareness on obesity caused by social contact with individual whom had obesity experience, and already recovered, do not effect the size of \( R_0 \). Therefore, we can conclude that \( \beta_2 \) do not effect the final condition of the population, whether it will tends to obesity-free, or endemic equilibrium condition. However, \( \beta_2 \) has an impact on determining the size of each variable in the endemic equilibrium point. To analyze this, let us differentiate each variable in the endemic equilibrium respect to \( \beta_2 \), which yield

\[
\frac{\partial N_i^*}{\beta_2} = \frac{A \gamma (\gamma + \mu + \sigma)}{\mu \beta_1 ((R_0 (\gamma + \mu) + (R_0 - 1) \sigma) (\delta + \mu + \alpha) + \gamma \beta_2) - A \gamma (R_0 (\gamma + \mu) (\delta + \mu + \alpha) + \gamma \beta_2 (\gamma + \mu + \sigma))}{\mu \beta_1 ((R_0 (\gamma + \mu) + (R_0 - 1) \sigma) (\delta + \mu + \alpha) + \gamma \beta_2)^2},
\]

\[
\frac{\partial N_i^*}{\beta_2} = \frac{A \gamma (\gamma + \mu + \sigma)}{\mu \beta_1 ((R_0 (\gamma + \mu) + (R_0 - 1) \sigma) (\delta + \mu + \alpha) + \gamma \beta_2) - A \gamma (R_0 (\gamma + \mu) (\delta + \mu + \alpha) + \gamma \beta_2 (\gamma + \mu + \sigma))}{\mu \beta_1 ((R_0 (\gamma + \mu) + (R_0 - 1) \sigma) (\delta + \mu + \alpha) + \gamma \beta_2)^2}.
\]
Figure 1. Transcritical bifurcation of system 1 in $\mathcal{R}_0 = 1$.

\[
\frac{\partial N^*_\beta_1}{\beta_2} = \frac{A \gamma (\gamma + \mu + \sigma) \left( \mathcal{R}_0 - 1 + \frac{\mathcal{R}_0 \alpha}{\delta + \mu} \right)}{\mu \beta_1 \left( \left( \mathcal{R}_0 (\gamma + \mu) + (\mathcal{R}_0 - 1) \sigma \right) (\delta + \mu + \alpha) + \gamma \beta_2 \right)} - A \gamma^2 \left( \beta_1 (\mu + \alpha) + \beta_2 \left( \mathcal{R}_0 - 1 + \frac{\mathcal{R}_0 \alpha}{\delta + \mu} \right) (\gamma + \mu + \sigma) \right),
\]

\[
\frac{\partial O^*}{\beta_2} = -\frac{\left( \left( \mathcal{R}_0 (\gamma + \mu) + (\mathcal{R}_0 - 1) \sigma \right) (\delta + \mu + \alpha) + \gamma \beta_2 \right)^2}{\mathcal{R}_0 (\gamma + \mu) + (\mathcal{R}_0 - 1) \sigma} < 0,
\]

\[
\frac{\partial Q^*}{\beta_2} = -\frac{A \gamma^2 \left( \mathcal{R}_0 - 1 \right) (\gamma + \mu + \sigma) (\delta + \mu + \alpha)}{\left( \mathcal{R}_0 (\gamma + \mu) + (\mathcal{R}_0 - 1) \sigma \right) (\delta + \mu + \alpha) + \gamma \beta_2} < 0.
\]

From the above analysis, it can be seen that the final sizes of $O$ and $Q$ are getting smaller whenever $\beta_2$ increases. This result means that encouraging quitter individual who had an experience about obesity, to influence high-risk individual to aware about the danger of obesity. This can be done in various ways, such as from media social, forming small communities in an effort to stop obesity, active in social community, etc.

5. Numerical experiments

In this section, we will analyze the obesity model (system 1). The first subsection will analyze the sensitivity and elasticity of the basic reproduction number ($\mathcal{R}_0$). Then numerical simulation of the system 1 will follow to provide a prediction of long-term dynamics.

5.1. Sensitivity and elasticity analysis of the basic reproduction number

Elasticity analysis of the basic reproduction number is used to calculate the derivatives of the projection result in the parameters. The result is to identify parameters that have a significant impact on the model. Elasticities are proportional sensitivities. The system is simulated for the following set of parameters in Table 5.1.

In [26] the elasticity of $\mathcal{R}_0$ with respect to $P$ is define as:

\[
\varepsilon_{\mathcal{R}_0}^P = \frac{\partial \mathcal{R}_0}{\partial P} \times \frac{P}{\mathcal{R}_0}. \tag{10}
\]
| Par   | Description                                      | Value         |
|-------|--------------------------------------------------|---------------|
| A     | Number of people born per unit of time            | $\frac{1000}{65}$ |
| $\beta_1$ | Infection rate through social contact           | 0.05122354155 |
| $\beta_2$ | Increased rate of awareness through social contact | 0.005         |
| $\alpha$ | Healthy life campaign rate                        | 0.035         |
| $\delta$ | Decreased rate of awareness                      | 0.019         |
| $\gamma$ | Recovery rate                                    | 0.001         |
| $\mu$ | Natural death rate                               | $\frac{1}{65}$ |
| $\sigma$ | Obesity-related death rate                       | 0.009         |

Table 1. Parameters description and value.

In our case, $P$ represents all parameters in basic reproduction number. Therefore, in our case, we have:

$$
\varepsilon_{R_0}^{\beta_1} = \frac{\partial R_0}{\partial \beta_1 R_0} = 1, \quad \varepsilon_{R_0}^\mu = \frac{\partial R_0}{\partial \mu R_0} = \frac{(-\delta(\delta + \mu + \alpha) - \mu(\delta + \mu) + \alpha(\gamma + \sigma))\mu}{(\delta + \mu(\delta + \mu + \alpha)(\gamma + \mu + \sigma)} ,
$$

$$
\varepsilon_{R_0}^{\alpha} = \frac{\partial R_0}{\partial \alpha R_0} = -\frac{\alpha}{\delta + \mu + \alpha}, \quad \varepsilon_{R_0}^\delta = \frac{\partial R_0}{\partial \delta R_0} = \frac{\delta}{(\delta + \mu(\delta + \mu + \alpha)},
$$

$$
\varepsilon_{R_0}^\gamma = \frac{\partial R_0}{\partial \gamma R_0} = -\frac{\gamma}{\gamma + \mu + \sigma}, \quad \varepsilon_{R_0}^\sigma = \frac{\partial R_0}{\partial \sigma R_0} = -\frac{\sigma}{\gamma + \mu + \sigma}.
$$

Input parameters value in Table 5.1 except $\alpha$, which in this simulation, the value of $\alpha$ is conditioned, so that $R_0$ is smaller or greater than 1. We set $\alpha = 0.04$ for $R_0 < 1$, while $\alpha = 0.03$ for $R_0 > 1$. The estimated value from elasticity of $R_0$ towards all parameters in $R_0$ is given by Table 2.

Table 2. The elasticity of $R_0$ with respect to parameters.

| $R_0$ | $\varepsilon_{R_0}^{\beta_1}$ | $\varepsilon_{R_0}^{\alpha}$ | $\varepsilon_{R_0}^\delta$ | $\varepsilon_{R_0}^\gamma$ | $\varepsilon_{R_0}^\sigma$ | $\varepsilon_{R_0}^\mu$ |
|-------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| $R_0 < 1$ | 1.02971435 | -0.5377456 | -0.3654585 | -0.0393939 | -0.3545455 |
| $R_0 > 1$ | 1.2574712 | -0.4659498 | -0.3975819 | -0.0393939 | -0.3545455 |

According to Table 2, the positive values ($\beta_1, \delta$) indicate positive correlation with $R_0$. However, the negative values ($\alpha, \mu, \gamma, \sigma$) indicate negative correlation with $R_0$. The order of parameter that have a large impact to $R_0$, when $R_0 < 1$ or $R_0 > 1$ are $\beta_1, \alpha, \mu, \sigma, \delta, \gamma$. The fact that $\varepsilon_{R_0}^{\beta_1} = 1$ means that, if the value of $\beta_1$ increases by 1%, it will produce 1% increase in $R_0$. Otherwise, with the fact that $\varepsilon_{R_0}^{\alpha} = -0.5377456$ means that, if the value of $\alpha$ increases by 1%, it will produced 0.5377456% decrease in $R_0$. This explanation applies to all parameters in Table 2. As a result, the elasticity of $R_0$ is positive if $R_0$ increasing with respect to parameters, and negative if $R_0$ decreasing with respect to parameters.
Figure 2. The elasticity of $R_0$ between $\beta_1$ versus $\alpha$.

The level set of $R_0$ with respect to the value of infection through social contact ($\beta_1$) and healthy life campaign rate ($\alpha$) shown in Figure 2. The value of $R_0$ is increasing when a transition to the obese compartment is also increasing. Moreover, the value of $R_0$ will decrease when a health life campaign rate is increasing. To reduce the $R_0$, most effectively could be done by reducing $\beta_1$. That means minimizing the possibility of obese people make persuasive social contact and influence normal-weight with low awareness of obesity with unhealthy lifestyles. Another alternative way to reduce $R_0$ could be with increasing $\alpha$, which is related to maximizing the healthy life campaign.

5.2. Autonomous-system simulation

The first simulation was conducted to answer the question of how the effects of counseling to make people more wiser in socialize. This question will be answered by reducing $\beta_1$. We use the parameter values as presented in Table 5.1, except $\beta_1$, which adjusted according to $R_0$. For $\beta_1 = 0.06$ then $R_0 = 1.171336424 > 1$ and for $\beta_1 = 0.04$ then $R_0 = 0.7808909492 < 1$. Using the Runge-Kutta approach, the solution of system 1 can be illustrated in Figure 3.

The purpose of counseling is that individuals expected to follow good habits and stay away from bad habits related to lifestyle, by reducing the amount of obesity transmission ($\beta_1$) value from 0.06 to 0.04. As a result, the number of low awareness and high awareness of obesity are increasing. It can be seen in Figure 3 (a) and Figure 3 (b). Figure 3 (c) shows that counseling has succeeded in minimizing the number of obesity itself. Because the number of obese people is reduced, so the number of people recovering from obesity also decreases (see Figure 3 (d)).

Therefore, if counseling is successful at increasing awareness about not following an unhealthy lifestyle, then the number of obese people can be suppressed. However, intervention on social contact is carried out to prevent the transmission of obesity from one individual to another. But the feasibility of this intervention is very subjective because it depends on each individual. It is necessary to consider other interventions as alternatives.

The next simulation was conducted to answer how the effect of a healthy life campaign to increase community awareness of obesity, can be successful efforts to suppress the number of obese individuals. In our model, the increasing $\alpha$ will answer the question. Based on our elasticity analysis in Table 2, increasing 10% of $\alpha$ can reduce 5.37% of $R_0$. Using similar parameters value as the previous simulation in elasticity analysis, except $\alpha$, which used for
Figure 3. Autonomous simulation toward variations of $\beta_1$.

several values (0.03 and 0.04), we conduct the solution of system 1 using the Runge-Kutta approach in Figure 4.

Figure 4. Autonomous simulation toward variations of $\alpha$.

Figure 4 (a) and Figure 4 (c) show that increasing healthy life campaigns can make the number of normal-weight individuals with low awareness of obesity and obese individuals reduced. Because the number of obese people has decreased, then automatically, the number of people recovering from obesity also decreases, as we can see from Figure 4 (d). On the other hand, there is an increase in the number of normal-weight individuals with high awareness of obesity (see Figure 4 (b)).

Providing healthy life campaigns, such as encouraging exercise and eating healthy foods, indicates the success of increasing obesity awareness. This phenomenon appears because people become more aware of the importance of living a healthy lifestyle. Therefore, this makes the transmission of obesity more difficult, and the number of obese individuals can be suppressed.

The final simulation was conducted to answer the question of what if ex-obese individuals influence normal-weight people about their experiences of being obese individuals due to unhealthy lifestyles. The expected result is that normal-weight people become more aware
of obesity and not adopt an unhealthy lifestyle. The question will be answered by increasing the \( \beta_2 \) value. In this simulation, we use the various values of \( \beta_2 (\beta_2 = 0.05 \times k, \ k = 1, 2, 3, 4, 5) \) while the other parameters are in Table 5.1. The results of numerical simulation of system 1 is given in Figure 5.

![Figure 5](image.png)

**Figure 5.** Autonomous simulation toward variations of \( \beta_2 \).

When the value of \( \beta_2 \) is getting larger, it is clear from Figure 5 (a) and (c) that due to the increasing rate of awareness through social contact, both the normal-weight with low awareness of obesity and obese compartment decrease. Because the number of obese is reducing, then the number of recovered individuals from obesity is also decrease (see Figure 5 (d)). In Figure 5 (b), the Figure shows that the number of normal-weight individuals with high awareness is increasing when the value of \( \beta_2 \) increases. This situation means that the better ex-individuals give influences, the more people aware of obesity and take precautions to avoid obesity. In this stage, obesity transmission is more difficult.

6. Conclusions

In this research, a mathematical model has been proposed to understand how obesity spread among the population, by defining human classes based on their weight status, i.e., normal-weight population with low awareness \( N_l(t) \), normal-weight population with high awareness \( N_h(t) \), obese population \( O(t) \) and the ex-obese population \( Q(t) \). The model incorporates the effect of media campaigns on a healthy lifestyle in order to increase human awareness of the danger of obesity. Mathematical analysis has been conducted to show that our model is biologically well-posed. The threshold number, known as the basic reproduction number (\( R_0 \)) conducted using the next-generation matrix. Elasticity analysis of \( R_0 \) is conducted numerically to show the robustness of \( R_0 \) to the value of the change of parameters in the model, which have been shown in Table 2. In addition, the local stability of the obesity-free equilibrium verified when \( R_0 < 1 \). The model undergoes transcritical bifurcation in \( R_0 = 1 \), which indicates the local stability of the obesity-endemic equilibrium point when \( R_0 > 1 \), but close to one. Our results indicate the most effective intervention to reduce the spread of obesity is by counseling to make people more wiser in socialize. Another effective intervention is to encourage human awareness of the danger of obesity by giving an intensive media campaign about a healthy lifestyle.
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References

[1] World Health Organization 2015 Obesity https://www.who.int/
[2] World Health Organization 2020 Obesity and overweight https://www.who.int/
[3] The Ministry of Health of the Republic of Indonesia 2019 Obesitas - Direktorat P2PTM http://p2ptm.kemkes.go.id/
[4] Wardle J and Cooke L 2005 Best Pract. & Res. Clinical Endocrinology & Metabolism 19 421
[5] The Ministry of Health of the Republic of Indonesia 2018 Hasil Utama Riskesdas 2018
[6] World Health Organization 2019 Developing new guidelines on physical activity and sedentary behaviour https://www.who.int/
[7] The Ministry of Health of the Republic of Indonesia 2020 GERMAS - Gerakan Masyarakat Hidup Sehat - Direktorat Promosi Kesehatan Kementerian Kesehatan RI http://promkes.kemkes.go.id/
[8] The Ministry of Health of the Republic of Indonesia 2017 Buku pedoman gerakan nusantara tekan angka obesitas (GENTAS)
[9] Rokom 2020 Perilaku CERDIK: Masa Muda Sehat, Hari Tua Nikmat, Tanpa Penyakit Tidak Menular - Sehat Negeriku http://sehatnegeriku.kemkes.go.id/
[10] Christakis N and Fowler J 2007 New England J. of Med. 357 370
[11] Jódar, L, Santonja and F J and González-Parra G 2008 Comp. and Math. with Appl. 56 679
[12] Aldila D, Rasasati N, Nuraini N and Soewono E 2014 Int. J. of Math. and Math. Sci. 2014 1
[13] Oh C and MA M 2015 J. of Appl. Math. 2015 1
[14] Lozano-Ochoa E, Camacho JF and Vargas-De-León C 2017 Qualitative stability analysis of an obesity epidemic model with social contagion Discrete Dynamics in Nature and Society 2017
[15] Diekmann O, Heesterbeek J and Roberts M 2009 J. of The Royal Society Interface 7 873
[16] Bustamam A, Aldila D and Yuwanda A 2018 J. of Appl. Math. 2018 1
[17] Putri YE, Rozi S, Tasman H and Aldila D 2017 Assessing the effect of extrinsic incubation period (EIP) prolongation in controlling dengue transmission with wolbachia-infected mosquito intervention AIP Conference Proceedings 2017 Mar 27 1825
[18] Aldila D 2018 Mathematical model for HIV spread control program with ART treatment Journal of physics: Conf. series 974
[19] Aldila D and Asrianti D 2019 A deterministic model of measles with imperfect vaccination and quarantine intervention Journal of Physics: Conf. Series 1218 012044
[20] Aldila D, Handari B D, Widyah A and Hartanti G 2020 Commun. Math. Biol. Neurosci. 2020
[21] Handari B, Vatra F, Ahya R, Nadya S T and Aldila D 2019 Advances in Difference Equations 2019
[22] Aldila D, Khoshnaw S H, Safitri E, Anwar Y R, Bakry A R, Samiadj B M, Anugerah D A, GH M F A, Ayulani I D and Salim S N A mathematical study on the spread of covid-19 considering social distancing and rapid assessment: The case of Jakarta, Indonesia 2020 Chaos, Solitons and Fractals, 139 110042
[23] Aldila D, Ndii M Z and Samiadj B M 2020 Optimal control on COVID-19 eradication program in Indonesia under the effect of community awareness In Press on Mathematical Biosciences and Engineering 2020
[24] Aldila D 2020 Commun. in Math. Biology and Neurosci. 2020 1
[25] Castillo-Chavez C and Song B 2004 Math. Biosci. and Engine. 1 361
[26] Martcheva M 2015 An introduction to mathematical epidemiology (New York; Heidelberg; Springer)