Acceleration methods for modelling the static modes of electronic devices

E U Chye¹, A B Shein² and A V Levenets¹

¹Department of Automation and System Engineering, Pacific National University, 136, Tikhookeanskaya Street, Khabarovsk, 680035, Russia
²Faculty of Radio Electronics and Automation, Chuvash State University, 15, Moscow Avenue, Cheboksary, 428015, Russia

E-mail: levalvi@bk.ru

Abstract. The modelling of the static modes of complicated electronic devices on the basis of the solution of the differential equations is their long integration by known numerical methods until establishment of model periodically repeating decisions. For slowly fading transition processes it leads to the increase of modelling time. The article considers the algorithms of the accelerated calculation of the set modes of the electro technical devices described by the differential equations with periodically changing coefficients. The first algorithm based on the solution of the state equation with previously defined values of the initial state vector is transferring the decision to the established values area. The second algorithm is based on a finding of an entry conditions vector for formulas of the exact solution of the state equations transferring the decision to the established values area by use for this purpose of exact decision formulas.

1. Introduction

When calculating the energy characteristics of electrical devices, most of which are devices of periodic action, it is often necessary to determine the device static characteristics.

The use of conventional numerical methods requires the integration of differential equations describing the operation of the device for transient conditions over a long period of time sufficient to damp the transients arising from the operation of this device. This method is used if transients fade out quickly [1-3]. However, for slightly damped objects, for example, the frequency converters feeding induction heaters and characterized by a low Q-factor the situation is changed. For such object transients are slowing down and for integrating of differential equations systems describing electromagnetic processes in devices requires a lot of computer time. In this case, it is necessary to have a mathematical apparatus that allows studying the steady state operation of the device more efficiently and economically [4-8].

2. Problem statement

Consider the methods of accelerated calculation of the steady state operation of electrical devices of periodic action, assuming that the solution of the vector-matrix equation of state of the electronic device

\[ x'(t) = A(t)x(t) + B(t)y(t), \quad x(0) = x_0 \]  

(1)
where \( A(t) \) is a continuous, periodic with a period \( T \), structural-parametric matrix of a device of size \( n \times n \) so that \( A(t + T) = A(t) \); \( B(t) \) is the coupling matrix of input and output parameters of an \( n \times m \) device: \( B(t + T) = B(t) \); \( v(t) \) is the vector of input actions on a device of size \( m \times 1 \), which, in the general case, is a periodic function with the time period \( T \), can be expressed through the transition matrix \( \Phi \) \((t, t_0)\) in the following form:

\[
x(t) = \Phi(t, t_0)x(t_0) + \int_{t_0}^{t} \Phi(t, \tau)B(\tau)v(\tau)d\tau.
\]

It is obvious that the transient process in the device is determined by the transition function \( \Phi(t, t_0) \), which performs the transformation and translates the initial state \( x(t_0) \) of the device to a certain state \( x(t) \) for the instant \( t \). Moreover, if the solution of the device state equations with periodic coefficients is known at any time interval \([t, t + T]\) equal to the period, then it can be found for all other values of \( t \). If the solution is known for some time point \( t \), then it can be determined for other time points separated with time point \( t \) by an interval multiple of the period \( T \). Various methods for solving equations of state (1) are described in works [6, 9].

3. The decision of the problem

3.1. Algorithm 1

As it is shown above, the solution of the matrix equation (1) can be expressed in terms of the transition matrix \( \Phi(t) \). Obviously, the solution \( x(t) \) at any moment of time \( t \) uniquely depends on the initial state vector \( x_0 \). It is required to find such a vector \( x_0 = x(0) \), which, when the condition of periodicity of the processes occurring in the device is fulfilled, \( x(0) = x(T) \) immediately allows finding the steady state of its working mode.

The condition of periodicity with period \( T \) gives equality

\[
x(T) = \Phi(T)x_0 + \int_{0}^{T} \Phi(T + \tau)B(\tau)v(\tau)d\tau = x_0,
\]

from which it is easy to get an equation to determine the values of the vector \( x_0 \)

\[
x_0 = [E - \Phi(T)]^{-1} \int_{0}^{T} \Phi(T + \tau)B(\tau)v(\tau)d\tau.
\]  

(2)

To calculate the definite integral entering into equation (2), it is possible to use the parabolas formula (Simpson's formula) [10]. In this case, the time interval \([0, T]\) is divided into \(2l\) intervals by points \( t_j \) \((j = 0, 1, 2, ..., 2l)\):

\[
0 = t_0 < t_1 < t_2 < ... < t_{2l-1} < t_{2l} = T.
\]

The integrand is replaced by a Newton's second-degree interpolation polynomial on each time interval \([t_{2j-2}, t_{2j-1}, t_{2j}]\) and the approximate value of the integral is calculated in each segment with length \(2h\), where \( h = T/(2l) \). Then the value of the integral on the time interval \([0, T]\) is defined as the sum of the values integrals of these sections.

\[
\int_{0}^{T} \Phi(T + \tau)B(\tau)v(\tau)d\tau = \frac{T}{6l} [\Phi(T)B(0)v(0) + \Phi(0)B(T)v(T) + 4(\Phi(T - h)B(h)v(h)]
\]

\[
+ \Phi(T - 3h)B(3h)v(3h) + ... + \Phi(T - (2l - 1)h)B((2l - 1)h)v((2l - 1)h)
\]

\[
+ 2[\Phi(T - 2h)B(2h)v(2h) + \Phi(T - 4h)B(4h)v(4h)]
\]

\[
+ ... + \Phi(T - (2l - 2)h)B((2l - 2)h)v((2l - 2)h)]
\]  

(3)

The computational error of the value of a definite integral by formula (3) is small and is estimated by the value of [10]
\[ R(f) = -\frac{T^5}{180(2)^4} f^{(4)}(\xi), \]
where \( \xi \in [0, T] \). \( f(t) \) is the integrand function of time. Applied to this case, the computational error of the value of a certain integral can be found by the formula
\[ R(f) = -\frac{h^5}{90} f^{(4)}(\xi). \]

The values of \( \Phi(t) \) can be determined by the fourth-order Runge-Kutt numerical method up to \( O(h^4) \), since if the orders of the residual terms of the fourth Simpson formula and the fourth-order Runge-Kutt formula are compared, it is possible to conclude that the Runge-Kutt rule has the same accuracy of calculations as the Simpson formula and does not reduce the accuracy of determining the values of the vector \( x_0 \). For the case under consideration, the Runge-Kutt formula has the form
\[ \Phi(t_{i+1}) - \Phi(t_i) = \left[ K'_1 + 2K'_2 + 2K'_3 + K'_4 \right]/6, \]
where
\[ K'_1 = hA(t_i)\Phi(t_i); \quad K'_2 = hA(t_i + h/2)\Phi(t_i) + h^2A(t_i + h/2)A(t_i)\Phi(t_i)/2; \]
\[ K'_3 = hA(t_i + h/2)\Phi(t_i) + h^2A^2(t_i + h/2)\Phi(t_i)/2 + h^3A^2(t_i + h/2)A^2(t_i)\Phi(t_i)/4; \]
\[ K'_4 = hA(t_i + h)\Phi(t_i) + h^2A^2(t_i + h)A(t_i + h/2)\Phi(t_i) + h^3A(t_i + h)A^2(t_i + h/2)\Phi(t_i)/2 \]
\[ + h^4A(t_i + h)A^2(t_i + h/2)A(t_i)\Phi(t_i)/4. \]

The Runge-Kutt formula (4) can be represented in a more useful form for performing the calculations
\[ \Phi(t_{i+1}) = P(t_i) \Phi(t_i), \]
where
\[ P(t_i) = E + \left[ h[A(t_i) + 4A(t_i + h/2) + A(t_i + h)] + h^2[A(t_i + h/2)A(t_i) + A^2(t_i + h/2) \right. \]
\[ \left. + A(t_i + h)A(t_i + h/2) + 0.5h^3[A^2(t_i + h/2)A(t_i) + A(t_i + h)A^2(t_i + h/2)] \right] \]
\[ + h^4A(t_i + h)A^2(t_i + h/2)A(t_i)/4)/6. \]

Taking into account that \( \Phi(t_0) = \Phi(0) = E \), all the matrices \( \Phi(t_i) \) and, in particular, \( \Phi(T) \) can be calculated using formulas (5) and (6). After that, formula (3) determines the value of a certain integral, and then by formula (2) it is possible to determine the values of the initial state vector \( x_0 \), which translates the solution of the equation of state of the device (1) into the region of steady-state values. In order to calculate the steady state operation of the device with the initial conditions found, formulas for solving the system of differential equations (1) obtained above can be used or the integration of this system by some numerical method.

The proposed method allows, without loss of accuracy, calculating the steady state operation of an electronic device by solving the equation of state (1) with predetermined values of the initial state vector, which translates the solution into the region of steady values. In this case, the computation cost for the calculation of the steady state operation of the device is minimal.

3.2. Algorithm 2

The essence of the algorithm is to determine the vector of initial conditions for exact solution formulas of the equations of state. This vector is translating the solution into the domain of steady-state values by using themselves exact solution formulas [5].

Let the formula for the numerical solution of the equations of state for the case of replacing the function \( v(t) \) by a continuous piecewise constant function is given as the initial one:
\[ x((k + 1)h) = Fx(kh) + F_0Bhv(kh), \]
where \( F = e^{Ah}; \quad F_0 = (e^{Ah} - E)(Ah)^{-1}. \)
It is possible to solve the equations of state using this formula, assigning \( k \) values 0, 1, 2, ..., \( n \) and expressing each time the solution through the vector of the initial conditions \( x(0) \)

\[
k = 0, x(h) = Fx(0) + F_0 Bhv(0);
k = 1, x(2h) = F^2x(0) + FF_0 Bhv(0) + F_0 Bhv(h);
k = 2, x(3h) = F^3x(0) + F^2F_0 Bhv(0) + FF_0 Bhv(0) + F_0 Bhv(2h);
k = 3, x(4h) = F^4x(0) + F^3F_0 Bhv(0) + F^2F_0 Bhv(0) + F_0 Bhv(3h);
\]

\[
k = n, x((n + 1)h) = F^{n+1}x(0) + F^nF_0 Bhv(0) + F^{n-1}F_0 Bhv(h) + F^{n-2}F_0 Bhv(2h) + \ldots + F_0 Bhv(nh).
\]

The decision process is stopped at \( (n + 1)h = T \), where \( T \) is the period of steady state operation of the device, i.e. if it makes the condition \( n = T/h - 1 \):

\[
x(T) = (E - F^{n+1})x(0) + (E - F^n)\ldots + E)F_0 Bhv(0)
\]

If the authors take the initial formula for the numerical solution of the equations of state for the case of replacing \( v(t) \) by a piecewise linear function, i.e. by formula

\[
x((k + 1)h) = Bx(kh) + G_0 Bhv(kh) + G_1 Bhv((k + 1)h),
\]

where

\[
F = e^{Ah}; \ G_0 = [e^{Ah}(Ah - E) + E](Ah)^2; \ G_1 = [e^{Ah} - (E + Ah)](Ah)^2,
\]

then, by assigning \( k \) the value 0, 1, 2, ..., \( n \), we therefore find

\[
k = 0, x(h) = Fx(0) + G_0 Bhv(0) + G_1 Bhv(h);
k = 1, x(2h) = F^2x(0) + FG_0 Bhv(0) + (FG_1 + G_0) Bhv(h) + G_1 Bhv(2h);
k = 2, x(3h) = F^3x(0) + F^2G_0 Bhv(0) + F(2G_1 + G_0) Bhv(h) + (FG_1 + G_0) Bhv(2h) + G_1 Bhv(3h);
k = 3, x(4h) = F^4x(0) + F^3G_0 Bhv(0) + F^2(2G_1 + G_0) Bhv(h) + F(2G_1 + G_0) Bhv(2h) + \ldots + (FG_1 + G_0) Bhv(3h) + G_1 Bhv(4h);
\]

\[
k = n, x((n + 1)h) = F^{n+1}x(0) + F^nG_0 Bhv(0) + F^{n-1}(FG_1 + G_0) Bhv(h) + F^{n-2}(2G_1 + G_0) Bhv(2h) + \ldots + (FG_1 + G_0) Bhv((n + 1)h).
\]

Since \( (n + 1)h = T \), the equation of the form is obtained

\[
x(T) = (E - F^{n+1})x(0) + F^nG_0 Bhv(0) + F^{n-1}(FG_1 + G_0) Bhv(h) + F^{n-2}(2G_1 + G_0) Bhv(2h) + \ldots + (FG_1 + G_0) Bhv(T - h) + G_1 Bhv(T).
\]

From which, using the periodicity condition \( x(T) = x(0) \), it is found

\[
x(0) = (E - F^{n+1})[F^0G_0 Bhv(0) + F^{n-1}(FG_1 + G_0) Bhv(h) + F^{n-2}(2G_1 + G_0) Bhv(2h) + \ldots + (FG_1 + G_0) Bhv(T - h) + G_1 Bhv(T)].(9)
\]

The expression \( FG_1 + G_0 \) can be simplified by using the following transformation:
FG_1 + G_0 = F_0^2.

Then the formula (9) can be written in the following form:

\[
x(0) = \left( E - F^{n+1} \right)^{n} \left[ F^n G_0 Bhv(0) + F^{n-1} F_0^2 Bhv(h) + F^{n-2} F_0^2 Bhv(2h) + \ldots + F_0^2 Bhv(T - h) + G_1 Bhv(T) \right].
\]

(10)

If the input action \( v(t) = \text{const} = v(0) \), then formula (10) takes the form

\[
x(0) = \left( E - F^{n+1} \right)^{n} \left[ F^n G_0 + \left( F^{n-1} + F^{n-2} + \ldots + E \right) F_0^2 + G_1 \right] Bhv(0).
\]

Thus, the algorithm for the accelerated calculation of the steady state mode of operation of electronic devices of periodic action based on the formulas for the exact solution of the equations of state favorably differs from the others in its visibility and ease of implementation.

4. Conclusion

The calculations of the steady state operation of a static frequency converter working at induction heater showed complete adequacy of the results. For this case, the error is determined by the method of solving equation (1), used to obtain information from \((n - 1)\) or one period of the transition process in the devices under study. The computational cost is minimal, since to reach the steady state using the developed programs, information about the transition state of the device is required from no more than \((n - 1)\) periods is needed.

Thus, the proposed algorithms for accelerated calculation of the steady-state modes of complex electronic devices, described by differential equations with periodically varying coefficients are presented. The proposed algorithms are convenient for computer numerical implementation.

References

[1] Bosch van den P P J and Klauw van den A C 1994 Modeling Identification and Simulation of Dynamical System (London: CRC-Press)
[2] Fritzon P 2011 Introduction to Modeling and Simulation of Technical and Physical Systems (New York: IEEE Press Wiley)
[3] Hespanha J P 2009 Linear System Theory (Princeton: Princeton University Press)
[4] Leon O Ch and Pen-Min L 1975 Computer-aided Analysis of Electronic Circuits: Algorithms and Computational Techniques (New Jersey: Prentice Hall)
[5] Shein A B and Chye E U 2015 Electrotechnical Complexes and Control Systems 1(37) 5–8
[6] Demirchan K S and Butyrin P A 1998 Modelling and computing electrical circuit (Moscow: Vysshaya shkola)
[7] Ajuev B I, Davydov V V and Neymin V G 2008 Electricity 8 2–14
[8] Bernstein J B et al 2006 Microelectronics Reliability 46 1957–79
[9] Chye E U and Shein A B 2014 Electronic design and technology 1 17–20
[10] Krylov V I, Bobkov V V and Monastyrnyi P I 1976 Computational Methods vol 1 (Moscow: Nauka)