Gate control of spin dynamics in III–V semiconductor quantum dots

Rogerio de Sousa and S. Das Sarma
Condensed Matter Theory Center, Department of Physics,
University of Maryland, College Park, MD 20742-4111
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We show that the g-factor and the spin-flip time \( T_1 \) of a heterojunction quantum dot is very sensitive to the band-bending interface electric field even in the absence of wave function penetration into the barrier. When this electric field is of the order of 10^5 V/cm, \( g \) and \( T_1 \) show high sensitivity to dot radius and magnetic field arising from the interplay between Rashba and Dresselhaus spin-orbit interactions. This result opens new possibilities for the design of a quantum dot spin quantum computer where \( g \)-factor and \( T_1 \) can be engineered by manipulating the spin-orbit coupling through external gates.

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Understanding and controlling the behavior of spins in semiconductor heterostructures may lead to a whole new class of devices ranging from spin-polarized p-n junctions to a quantum dot spin quantum computer. Electrical control over spin-orbit coupling parameters in a quantum well has long been suggested as an effective means to manipulate spins and was recently experimentally demonstrated in \( \text{InAs} \) heterostructures. This was possible because the Rashba and the Dresselhaus spin-orbit interactions are sensitive to the electric field providing vertical confinement to a two dimensional electron gas (2DEG), which is approximately proportional to the 2DEG density and can be controlled through a gate voltage. When additional gates provide lateral confinement, a single electron can be trapped in a quantum dot (QD) whose orbital states in an external magnetic field perpendicular to the 2DEG are the well known Fock-Darwin states. Here we consider the effect of Rashba and Dresselhaus spin-orbit interactions in the spin-doublet ground state of a Fock-Darwin QD. Using an effective mass approximation and exact diagonalization of a Fock-Darwin subspace we show that the ground state \( g \)-factor and spontaneous phonon emission rate \( 1/T_1 \) can be substantially manipulated by varying the heterojunction electric field in the range of \( 10^5 - 10^6 \text{ V/cm} \) (corresponding to 2DEG density of the order of \( 10^{12} - 10^{13} \text{ cm}^{-2} \)).

Recently, electrical control over GaAs quantum well \( g \)-factor has been achieved by forcing the electron wave function to overlap with the AlGaAs barrier. Here we intentionally neglect barrier penetration, to show that overlap with a different material is not a necessary condition to achieve substantial electrical control over quantum dot \( g \)-factor. Moreover, by avoiding barrier penetration one can suppress an additional spin-lattice relaxation mechanism due to interface motion. These results open new possibilities in the design of a quantum dot quantum computer: for example, spin qubits can be brought in and out of resonance to a global spin resonance field by gate control of their \( g \)-factor a speed up in quantum computer initialization (setting all spins up for example) can be achieved by decreasing the spin-flip time \( T_1 \) with a gate voltage.

The Hamiltonian for a single electron bound to an heterojunction quantum dot can be divided into five parts,

\[
\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_z + \mathcal{H}_{\text{fr}} + \mathcal{H}_{\text{D1}} + \mathcal{H}_{\text{D2}}. \tag{1}
\]

The first contribution corresponds to a single 2D electron confined in the \( xy \) plane by a parabolic potential and subject to a magnetic field \( \mathbf{B} \),

\[
\mathcal{H}_0 = \frac{\mathbf{P}^2}{2m^*} + \frac{1}{2}m^*\omega_0^2 r^2 + \frac{1}{2}g_0\mu_B\sigma_z B, \tag{2}
\]

where the kinetic momentum \( \mathbf{P} = \mathbf{p} + e/cA \) is written with the canonical momentum \( \mathbf{p} = -i\hbar(\partial_x, \partial_y, 0) \) and vector potential \( \mathbf{A} = B/2(-y, x, 0) \) confined to the 2D plane. Here \( e \) is the electron charge, \( c \) is the velocity of light, \( m^* \) is the conduction band edge effective mass, \( \omega_0 \) is the parabolic confining potential frequency, \( g_0 \) is the Bulk \( g \)-factor, \( \mu_B \) is the Bohr magneton, and \( \sigma_z \) is the diagonal Pauli matrix. \( \mathcal{H}_0 \) is diagonal when written as a function of the Fock-Darwin number operators \( n_\pm = a_\pm^\dagger a_\pm \)

\[
\mathcal{H}_0 = \hbar\omega_+ \left( n_+ + \frac{1}{2} \right) + \hbar\omega_- \left( n_- + \frac{1}{2} \right) + \frac{1}{2}g_0\mu_B\sigma_z B, \tag{3}
\]

\[
a_\pm = \frac{1}{2\ell} (x \pm iy) - \frac{\ell}{2} (\partial_x \pm i\partial_y), \tag{4}
\]

\[
a_\pm = \frac{1}{2\ell} (x \mp iy) + \frac{\ell}{2} (\partial_x \mp i\partial_y). \tag{5}
\]

Here \( \omega_\pm = \Omega \pm \omega_c/2 \), with \( \Omega = \sqrt{\omega_0^2 + \omega_r^2/4} \) and \( \omega_c = eB/m^*c \) being the renormalized dot frequency and cyclotron frequency respectively, with \( \ell = \sqrt{\hbar/m^*\Omega} \) being the Fock-Darwin radius which sets the length scale for the eigenstates \( |n_\pm n_- \rangle \) (\( \sigma = \pm 1 \) represents the spin up/down states in the \( z \) direction). The second term in the Hamiltonian \( \mathcal{H}_z \) represents the quantum well confinement in the growth \( z \) direction, \( \mathcal{H}_z = \hbar^2/2m^* + V(z) \), where \( V(z) \) is a triangular well, \( V(z) = eEz \) for \( z \geq 0 \) and \( V(z) = \infty \) for \( z < 0 \). A simple numerical calculation
leads to the $H_z$ ground state

$$\Psi_{0z}(z) = 1.4261\kappa^{1/2} \mathrm{Ai}(\kappa z + \zeta_1),$$

where $\zeta_1 = -2.3381$ is the first zero of the Airy function $\mathrm{Ai}$, while the inverse length scale $\kappa$ is set by

$$\kappa = (2m^*eE/\hbar^2)^{1/3},$$

and the ground state energy is $E_{0z} = -\zeta_1 eE/\kappa$. In the discussion below we will make use of the average momentum squared in the state $\langle p_z^2 \rangle = 0.7794(\hbar\kappa)^2$, and the average position $\langle z \rangle = 1.5587/\kappa$ (which is the thickness of the 2DEG). We now turn to the spin-orbit interactions, third to fifth terms in Eq. $\Psi$. A $k \cdot p$ band structure calculation for zincblende material tends to the bulk conduction band spin-orbit interaction

$$\mathcal{H}_{\text{Bulk}} = \gamma_c/(2\hbar^3) \sigma \cdot \mathbf{P},$$

where $\mathbf{P}_x = P_x(P_y^2 - P_z^2) + \text{h.c.}$, $\mathbf{P}_y$ and $\mathbf{P}_z$ can be obtained by cyclic permutations. Notice that Eq. $\Psi$ is hermitian and gauge invariant. The value of $\gamma_c$ is determined by the band structure parameters of the III–V semiconductors (Table I). By averaging Eq. $\Psi$ over the quantum well ground state [Eq. $\Psi$] we get two spin-orbit terms, linear and cubic in momenta (here the quantum well growth direction is assumed to be [001]).

$$\mathcal{H}_{D1} = 0.7794\gamma_c \kappa^2/\hbar (-\sigma_x P_x + \sigma_y P_y),$$

$$\mathcal{H}_{D2} = \gamma_c/\hbar^3 (\sigma_x P_x P_y - \sigma_y P_y P_z^2) + \text{h.c.}.$$}

The structural inversion asymmetry in $V(z)$ leads to the Rashba interaction\textsuperscript{a}

$$\mathcal{H}_R = \alpha_R eE/\hbar (\sigma_x P_y - \sigma_y P_x),$$

which for the triangular well considered here is directly proportional to $E$ ($\alpha_R$ depends on band structure parameters)\textsuperscript{b} It is useful to condense $\mathcal{H}_R$ and $\mathcal{H}_{D1}$ in a single Hamiltonian written as a function of the Fock-Darwin operators,

$$\mathcal{H}_{D1} + \mathcal{H}_R = \hat{\mathcal{V}} \sigma_+ + \hat{\mathcal{V}} \sigma_-,$$

with $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$ and spin-orbit energy scales defined as

$$\alpha_\pm = \alpha_R eE \xi_\pm,$$

$$\beta_\pm = 0.78 \gamma_c \kappa^2 \xi_\pm,$$

$$\xi_\pm = 1/\ell \pm eE/2\hbar c.$$}

Therefore $\mathcal{H}_R$ and $\mathcal{H}_{D1}$ couples Fock-Darwin levels differing by one quantum number. The amount of spin up/down admixture in the ground state is given by the ratios $\alpha_\pm/\hbar \omega_0$, $\beta_\pm/\hbar \omega_0$, which at $B \approx 0$ are directly proportional to the dot radius $l_0 = \sqrt{\hbar/m^* \omega_0}$. Therefore larger dots will be more sensitive to spin-orbit coupling, at least within perturbation theory (see below). Moreover, the question of whether Rashba or Dresselhaus dominates depends only on material parameters and electric field, since $\alpha_\pm/\beta_\pm = \alpha_R/\gamma_c (\hbar^2/m^*)^2/3(eE)^{1/3}$. If the electric field ranges from $10^4$ to $10^6$ V/cm this ratio equals 0.1 – 0.7 for GaAs, 0.2 – 1.0 for GaSb, 1.5 – 6.8 for InAs, and 5.6 – 26 for InSb. Therefore III–V semiconductor quantum dots it is important to consider the interplay between Rashba and Dresselhaus spin-orbit interactions. High electric field GaSb heterojunctions might realize the condition $\alpha_\pm = \beta_\pm$, leading to an interesting simplification of Eq. $\Psi$, which becomes proportional to

FIG. 1: Quantum dot $g$-factor divided by the bulk $g$-factor as a function of the dot radius $l_0 = \sqrt{\hbar/m^* \omega_0}$ for a GaAs heterojunction. Each curve corresponds to a different electric field: From top to bottom, $E = 10^4, 1.2, \ldots, 10 \times 10^4$. The magnetic field applied in the growth [100] direction is assumed to be 1 Tesla. At large $l_0$ the two lowest energy states have same spin, hence the level crossing. The results for GaSb are similar, except that the level crossing occurs for smaller $l_0$.

FIG. 2: Quantum dot $g$-factor as a function of the magnetic field $B$ for a GaAs heterojunction. The electric fields are the same as in Fig. 1. Dot radius is $l_0 = 20$ nm. At large $B$ a similar level crossing as in Fig. 1 takes place.
with \( g_{\lambda} \) interaction Eq. (10) is neglected. No spin-lattice relaxation, as long as the cubic spin-orbit couples the state \( |n_+ - \sigma, n_- \rangle \) with \( n_+ + \sigma, n_- - 2\sigma, -\sigma \) which are degenerate at \( \omega_c = \omega_0/\sqrt{2} \) (the exact location of the anticrossing depends on Zeeman splitting). The magnitude of the anticrossing is given by

\[
\Delta E \approx \frac{2\gamma_2}{\sqrt{3}} \frac{\gamma_c}{\omega_0} \sqrt{n_+(n_+ + 1)(n_- + 2)}.
\]

For a GaAs dot with \( l_0 = 10 \text{ nm} \), \( \Delta E \approx 0.1 \text{ meV} \), while for GaSb, InAs, and InSb it can reach 1 meV. Note, however, that this anticrossing appears only for \( n_+ + n_- > 1 \). It may have interesting consequences for spin dependent transport through a few electron QD. The third to sixth terms in Eq. (17) are linear in \( a_\pm \) and \( a_\pm^\dagger \) leading to an enhancement of the spin-orbit effect [Eq. (13)].

Our QD ground state \( g \)-factor is defined by \( g = (E_2 - E_1)/(\mu_B E) \), with \( E_1 \) and \( E_2 \) being the ground and first excited states including spin. Considering Eq. (12) as a second order perturbation to Eq. (2) with \( B \) field in the [001] direction we get

\[
g \approx g_0 + 2\frac{m_e m^*}{\hbar^2} \left[ 0.6 \gamma_2^2/\kappa^4 (1 - \delta) - \alpha^2 \kappa^2 e^2 E^2 (1 + \delta) \right] \ell_0^2
- \frac{1}{2} \frac{m_e m^*}{\hbar^2} \left\{ 0.6 \gamma_2^2/\kappa^4 (1 - \delta + \delta^2 + \delta^3)
- \alpha^2 \kappa^2 e^2 E^2 (1 + \delta + \delta^2 - \delta^3) \right\} \omega_c \ell_0
+ O \left( \omega_c/\omega_0 \right)^4.
\]

Here \( \delta = g_0 m^*/m_e \), this expression being valid up to second order in \( \omega_c/\omega_0 \) and the spin-orbit admixtures.

Clearly one sees that \( g \)-factor displays a rich behavior as a function of QD radius and electric field. In particular, \( g - g_0 \) will be positive and proportional to \( E^{4/3} \) if the Dresselhaus spin-orbit interaction is dominating (GaAs and GaSb), but negative and proportional to \( E^2 \) when Rashba dominates (InAs, InSb). In addition these effects increase with increasing dot radius \( l_0 \) and there is a \( B^2 \) dependence at higher magnetic fields.

We study the \( g \)-factor behavior at high electric and magnetic fields, and large dot radius by resorting to exact diagonalization of the full Hamiltonian [Eq. (10)] (similar calculations for Landau levels in a quantum well are available)\(^{22}\). Our basis consists of nine Fock-Darwin shells \( (n_+ + n_- \leq 8) \), which together with spin leads to...
to a $90 \times 90$ matrix for the Hamiltonian. Figs. 1 and 2 show results for GaAs, Figs. 3 and 4 for InAs. GaSb shows very similar behavior to GaAs, while InSb is similar to InAs. The differences between the two sets of materials is attributed to Dresselhaus spin-orbit interaction dominating in Figs. 1 and 2, but Rashba dominating in Figs. 3 and 4 (the Rashba interaction appears to be dominant in SiGe heterostructures, therefore $g$-factor behavior should be similar to the InAs case considered here, except that $\alpha_R$ is three orders of magnitude smaller and $g_0 \approx 2$; hence the effects considered here should be quite small for Si heterostructures: we estimate $g - g_0 \sim -10^{-3}$ for $E \sim 10^5$ V/cm).\textsuperscript{21} Our result suggests $g$-factor can be controlled by a gate voltage (that either changes the longitudinal electric field $E$ or the lateral confinement $l_0$) as long as $E \sim 10^5$ V/cm. An important feature of Figs. 1–4 is that a level crossing takes place for large $l_0$ and $B$. In this regime the two lowest energy states of the QD are approximate Landau levels with the same spin, leading to extremely fast phonon emission rates (see below). Hence a QD quantum computer should operate away from this level crossing, which is actually a smooth anti-crossing for InAs (see Figs. 3 and 4). Note that Figs. 1–4 plot the ratio of QD to bulk transition rates (see below). Hence a QD quantum computer the same spin, leading to extremely fast phonon emission rates (see below). Hence a QD quantum computer should operate away from this level crossing, which is actually a smooth anti-crossing for InAs (see Figs. 3 and 4). Note that Figs. 1-4 plot the ratio of QD to bulk $g$-factor ($g/g_0$), therefore the corresponding deviation in $g$ Zeeman energy from the bulk value will be rather appreciable for GaSb, InAs, and InSb since these materials have $g_0 \sim -10$ (Table I).

We now turn to calculations of the transition rate between the two lowest energy states due to spontaneous phonon emission. The electron-piezophonon interaction\textsuperscript{12}

$$U_{e-ph}^\alpha = \sqrt{\frac{\hbar}{2pV\omega_\alpha}} e^{i(q \cdot r - \omega_\alpha t)} e^{iA_{\alpha\beta}b_{\beta}^\dagger} + \text{h.c.},$$

(20)

couples these states in the presence of spin-orbit admixture. Here $b_{\beta}^\dagger$ creates an acoustic phonon with wave vector $q$ and polarization $\mathbf{e}_\beta$ ($\alpha = L, T_1, T_2$), $\rho$ is the material density, and $V$ the volume of the sample. $A_{\alpha\beta}$ is the amplitude of the electric field created by the phonon strain, which is given by $\tilde{q}\tilde{q}e_{\alpha\beta\gamma}e_{\alpha}^{\gamma}$, with $\tilde{q} = q/q$, $e_{\alpha\beta\gamma} = e_{I44}$ (see Table I) for $i \neq k$, $i \neq j$, and $j \neq k$. The polarization directions are

$$\mathbf{e}_L = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta),$$

$$\mathbf{e}_{T1} = (\cos \theta \cos \phi, \cos \theta \sin \phi, -\sin \theta),$$

$$\mathbf{e}_{T2} = (-\sin \phi, \cos \phi, 0).$$

The transition rate is given by Fermi’s golden rule,

$$\frac{1}{T_1} = \frac{V}{(2\pi)^2\hbar} \sum_{\lambda} \int d^3 q \left| \langle 1 | U_{e-ph}^\alpha | 2 \rangle \right|^2 \delta(\hbar\omega_\alpha - E_2 + E_1),$$

(24)

which under the same perturbative approximation as

FIG. 5: Spin-lattice relaxation rate for the spin-doublet ground state of a 20 nm one-electron quantum dot as a function of the external magnetic field. Notice for all materials $1/T_1 \propto B^3$ at low $B$ fields. At high $B$ the Zeeman phonon wavelength becomes smaller than the dot radius and the rate is strongly suppressed. This effect is evident at $B \sim 1$ Tesla for the narrow gap materials, which have quite large bulk $g$-factors. Here we assumed $E = 10^4$ V/cm. The curves terminate at the level crossing, when the rate is extremely enhanced because the transition states have the same spin.

| Parameter | GaAs | GaSb | InAs | InSb |
|-----------|------|------|------|------|
| $g_0$     | -0.44 | -7.8 | -15  | -50.6|
| $m^*/m_e$ | 0.067 | 0.0412 | 0.0239 | 0.0136 |
| $\alpha_R$ | 4.4   | 33   | 110  | 500  |
| $\gamma_c$ | 26    | 187  | 130  | 228  |
| $c\hbar_{14}$ | 2.34  | 1.5  | 0.54  | 0.75  |
| $s_L$ | 5.14  | 4.30 | 4.20 | 3.69 |
| $s_T$ | 3.03  | 2.49 | 2.35 | 2.29 |
| $\rho$ | 5.3176 | 5.6137 | 5.6670 | 5.7747 |

TABLE I: Parameters used in our calculations.\textsuperscript{8,22}

Eq. (19) leads to

$$\frac{1}{T_1} \approx \frac{4}{105\pi} \left( \frac{1}{s_T^3} + \frac{3}{4} \frac{1}{s_L^3} \right) \frac{4m^* e^2 (e_{I44})^2}{\rho \hbar^3} \times \left( 0.61\kappa^2 + \alpha_R^2 \right)^2 \left( \frac{g_0 \mu_B B}{\hbar} \right)^5 l_0^4 \times (n_{ph} + 1) \left[ 1 + O(\omega_c/\omega_0)^2 \right].$$

(25)

Here $s_T$ and $s_L$ are the transverse and longitudinal acoustic phonon velocities respectively. The spin-flip rate is extremely sensitive to QD radius and external magnetic field. At temperatures lower than Zeeman splitting, the emitted phonon occupation number $n_{ph}$ is much smaller than one, and Eq. (25) is independent of temperature. At higher temperatures Raman processes will dominate.\textsuperscript{12}

It is interesting to study deviations from the perturbative approximation Eq. (25). In particular, at high magnetic fields the resonant phonon wavelength $\lambda_Z = \hbar s/E_Z$
Eq. (20) can be approximated by $1 + \eta(\theta, \phi) = q\ell \sin \theta e^{i\phi}/2$ depends on the polar angles of the phonon wave vector $q$ [because of this dependency, we have to perform the angular integrals in Eq. (24) numerically]. Each of the exponents in Eq. (26) is expanded in powers of $\eta$, but we note that within our subspace $n_\pm \leq 8$, therefore only up to the ninth power needs to be retained. We checked the convergence of our calculations by reducing the Fock-Darwin subspace and noting that no appreciable change takes place for $n_+ + n_- \geq 4$. Our results agree with perturbation theory [Eq. (20)] at low $B$ and $E$. Fig. 5 shows the spin-flip rate as a function of the magnetic field. It is evident that materials such as InAs and InSb deviate from perturbation theory by more than three orders of magnitude when $B$ is as low as 1 Tesla. This happens because taking into account the full electron-phonon Hamiltonian leads to an exponential decrease in the rate when $q\ell \gg 1$, since Eq. (20) oscillates appreciably in this regime. Note, however, that $1/T_1 \propto B^5$ at low enough $B$ for all materials. Fig. 6 shows the dependency of the spin-flip rate with lateral confinement radius $l_0$ in a GaAs QD. As the electric field increases, the dependency with $l_0$ displays an striking behavior, which happens due to the sign change in $g$-factor shown in Fig. 1. A small Zeeman energy implies negligible phonon density of states, and hence the rate is zero for $l_0 \approx 50$ nm and $E = 7 \times 10^5$ V/cm in Fig. 6. We expect $1/T_1$ will behave similarly to Fig. 6 when $g$ changes sign due to barrier penetration in AlGaAs; this property being extremely useful in the initialization and decoherence suppression of a QD quantum computer.

We now discuss possible corrections to the simple model discussed here. At strong confinement in the 100 direction one expects $\Gamma$-$X$ valley mixing to become important. For GaAs, $E_{\Gamma-X} = 0.48$ eV, which is comparable to $E_{0z}$ [Eq. (1)] only when the electric field is the highest considered here, $E > 10^8$ V/cm. This also holds true for InAs and InSb, but in GaSb $\Gamma$-$X$ coupling will be important for $E > 5 \times 10^5$ V/cm. Therefore even though a full $\mathbf{k} \cdot \mathbf{p}$ calculation would yield some corrections, we do not expect it to change our results qualitatively in the range considered here. The same holds true for the inadequacy of the Rashba Hamiltonian, which starts deviating from Eq. (11) when $E \sim 10^8$ V/cm.

Before concluding, we discuss the approximations and the limitations of this work. The most essential approximation of our model, the use of the $\mathbf{k} \cdot \mathbf{p}$ perturbation theory within an effective mass approximation scheme to describe the conduction band, has been extensively used in the literature and should be well-valid for the problem we study. We have made two additional non-essential approximations in our theory in order to simplify our numerical computations: The triangular well approximation for the $z$-confinement of the wave function and the parabolic well Fock-Darwin confinement approximation in the 2D $x$–$y$ plane. These approximations are reasonable enabling us to produce numerical results for a range of system parameters in several different semiconductor structures, which would have been difficult, if not impossible, to carry out had we used more realistic (and thereby numerically more demanding) quantum dot confinement potentials (The triangular well approximation was employed recently to derive new results regarding D’yakonov-Perel’ relaxation anisotropy for conduction electron spins confined by heterojunctions with $E \gtrsim 10^5$ V/cm).

Our most important qualitative result, establishing the viability of controlling the spin dynamics of III–V semiconductor quantum dots (both $g$-factor and $T_1$ engineering) by using external gates to suitably manipulate the spin-orbit coupling through Dresselhaus and Rashba effects, should be completely independent of these approximations. In fact, we expect that the use of more sophisticated confinement models may actually make the gate control effects we predict somewhat stronger by pushing the required electric fields (and consequently 2D carrier densities) to somewhat lower values than our predicted $10^5$ V/cm range. The main limitation of our predicted spin-orbit coupling induced gate control effect is, in fact, the rather large electric fields ($\sim 10^5$ V/cm) and the associated 2D carrier densities ($\sim 10^{12}$ cm$^{-2}$) that are required to produce significant gate control effects.
In conclusion, we show that quantum dot longitudinal $g$-factor and spin-flip time $T_1$ can be controlled electrically even in the absence of wave function overlap with a different material. These parameters show a striking dependence with dot radius and magnetic field when the 2DEG confinement is strong (electric field $E \sim 10^5$ V/cm). For example, the $g$-factor changes sign and $T_1$ is extremely sensitive to the dot radius. $g$-factors for one-electron dots can be measured using transport spectroscopy. We show that $T_1$ is drastically increased in narrow gap materials (InAs, InSb) due to deviations from the dipolar approximation in the electron-phonon interaction, suggesting these materials are promising for the fabrication of a quantum dot spin quantum computer. The relative quantitative importance of the Dresselhaus effect in III–V nanostructures should have considerable significance not only in the $g$-factor engineering and the spin relaxation time control of relevance to the spin quantum computer architecture (that we consider in this work), but also in the fabrication of the Datta-Das spintronic transistors where spin-orbit coupling is used to modulate a spin-polarized current in a field effect transistor configuration. The authors acknowledge discussions with A. Kaminski and I. Żutić. This work is supported by ARDA, LPS, US-ONR, and NSF.

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