Precise strength of the $\pi NN$ coupling constant

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Abstract. We report here a preliminary value for the $\pi NN$ coupling constant deduced from the GMO sumrule for forward $\pi N$ scattering. As in our previous determination from np backward differential scattering cross sections we give a critical discussion of the analysis with careful attention not only to the statistical, but also to the systematic uncertainties. Our preliminary evaluation gives $g^2_{\pi}(GMO) = 13.99(24)$.

1. INTRODUCTION

The crucial coupling of low energy hadron physics is the $\pi NN$ coupling constant, which for the pseudoscalar interaction of a charged pion has the approximate value $g^2_{\pi} \approx 14$. One would like this quantity to be determined experimentally to a precision of about 1% for accurate tests of chiral symmetry predictions, such as the Goldberger-Treiman relation. Determinations of the coupling constant in later years are given in Table 1.

The Nijmegen group pointed out some years ago that the earlier determinations from the 1980’s had important systematic uncertainties and they have since advocated values about 5% lower than the previous ones, mainly based on their analysis of $NN$ interactions $[1]$. However, these later determinations are, in general, not transparently linked to the underlying data and the systematic errors in the analysis are unknown. An exception is the GMO analysis by Arndt et al. $[2]$. Important physical constants are generally determined directly from experimental data with transparent, refutable procedures. The $\pi NN$ coupling constant should be no exception. We have therefore started a program of such determinations $[3, 4]$. A first approach is based on single energy backward np differential cross sections, dominated by pion pole contributions. The extrapolation to the pion pole at $t = -q^2 = m^2_\pi$ gives directly $g^4$. This is based on an old idea of Chew, which has not been workable in practice for the following reasons: 1) previous data were not precise enough and in particular lacked absolute normalization, 2) the original

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Table 1
Some important determinations of the pion-nucleon coupling constant

| Source                  | Year | System | $g_{\pi NN}^2$     | Reference                        |
|-------------------------|------|--------|-------------------|----------------------------------|
| Karlsruhe-Helsinki      | 1980 | $\pi p$ | 14.28(18)         | Nucl. Phys. A336, 331 (1980).    |
| Kroll et al.            | 1981 | pp     | 14.52(40)         | Physics Data 22-1(1981).         |
| Nijmegen [1]            | 1993 | pp, np | 13.58(5)          | Phys. Rev. C 47, 512 (1993).     |
| VPI                     | 1994 | pp, np | 13.7              | Phys. Rev. C 50, 2731 (1994).    |
| Nijmegen                | 1997 | pp, np | 13.54(5)          | IIN Newsletter 13, 96 (1997).    |
| Timmermans              | 1997 | $\pi^+p$ | 13.45(14)        | IIN Newsletter 13, 80 (1997).    |
| VPI [2]                 | 1994 | GMO, $\pi p$ | 13.75(15)      | Phys. Rev. C 49, 2729 (1994).    |
| Uppsala [3]             | 1995 | np→pn | 14.62(30)         | Phys. Rev. Lett. 75, 1046 (1995).|
| Uppsala [4]             | 1998 | np→pn | 14.52(26)         | Phys. Rev. C 57, 1077 (1998).    |

extrapolation method requires a polynomial expansion with a large number of terms, which makes systematics in the extrapolation obscure.

These deficiencies have been largely eliminated [3,4]. High precision absolutely normalized differential np cross sections have recently been measured at 96 and 162 MeV by the Uppsala neutron group. Furthermore, we have replaced the original Chew method by a Difference Method for which the extrapolation is required only for the difference between the actual cross section and that of a model with a known value for the coupling constant. The extrapolation now only concerns a correction and can be done with far greater simplicity and confidence. Figure 1 demonstrates concretely how we make such an extrapolation. Note the strong improvement in the quality of the experimental data from the older Bonner [3] data to the new Uppsala data at the same energy. How good is this method? We have tested it using over 10000 pseudoexperiments generated from models with known coupling constant with 'experimental' points equivalent to actual observed ones. The original coupling constants are regenerated with an accuracy of about ±1%. The method is therefore well under control.

The experimental differential cross sections have closely similar shape over a wide band of energies and any energy is as good as another for extrapolation purposes. The experimental data from Uppsala have been obtained in dedicated measurements, in contrast with previous data. They agree accurately with the shape of similar experiments at other energies by the PSI group [6], but differ in shape with data, mainly from Los Alamos [4]. This discrepancy is presently not fully resolved. (For a different opinion on the Uppsala data and the extrapolation procedure, see the Comment by de Swart et al. and our rebuttal, in Phys. Rev. Letters 81 issue 22, November 30, 1998). A critical discussion of the experimental situation has been made by Blomgren et al. [7]. Using the most recent Uppsala data gives $g_{\pi}^2 = 14.52(26)$ [4].

2. THE GMO RELATION

In order to obtain additional model-independent information we (T. E. O. Ericson, B. Loiseau, A. W. Thomas) evaluate at present the Goldberger-Miyazawa-Oehme (GMO) sumrule for $\pi N$ forward scattering [8] in terms of the $\pi N$ scattering lengths and total
Figure 1. Extrapolations of the Chew function $y(q^2)$ to the pion pole at 162 MeV with the Difference Method using Nijmegen 93 as comparison model and different polynomial order $n$. Left figure Uppsala data, right figure Bonner data. For $n=2$ $g_c^2$(Uppsala)=14.52(26); for $n=3$ $g_c^2$(Bonner)=12.95(37); the Bonner data are normalized to SM95.

cross sections. Assuming only charge symmetry:

\[ g_c^2 = -4.50 J^- + 103.3 \frac{a_{\pi^-p} - a_{\pi^+p}}{2}. \] (1)

Here $J^-$ is given in mb by the integral $J^- = -(1/4\pi^2) \int_0^\infty (\sigma^T_{\pi^-p} - \sigma^T_{\pi^+p})/\sqrt{k^2 + m^2_\pi} dk$ and $a_{\pi^\pm p}$ are expressed in units of $m^{-1}_{\pi^\pm}$.

Everything is in principle measurable to good precision. Still this expression has not been too useful in the past because the scattering lengths were theoretically constructed from the analysis of scattering at higher energies. Recent splendid experiments at PSI determine the $\pi^-p$ and $\pi^-d$ energy shifts and widths in pionic atoms and from that the corresponding scattering lengths follow accurately [9]. We have critically examined the situation with careful attention to errors. In particular, we have examined the accuracy of the constraints due to pion-deuteron data.

In order to get a robust evaluation we write the relation as

\[ g_c^2 = -4.50 J^- + 103.3 a_{\pi^-p} - 103.3 \frac{a_{\pi^-p} + a_{\pi^+p}}{2}. \] (2)

Using $J^- = -1.077(47)$ mb [10,11] and the experimental $\pi^-p$ scattering length [9]

\[ g_c^2 = 4.85(22) + 9.12(8) - 103.3 \frac{a_{\pi^-p} + a_{\pi^+p}}{2} = 13.97(23) - 103.3 \frac{a_{\pi^-p} + a_{\pi^+p}}{2}. \] (3)

Here the last term is a small quantity which we can evaluate with small statistical and systematic uncertainties from the experimental $\pi^-d$ scattering length. The cross section
integral $J^-$ is presently the largest source of error. Uncertainties from the small deuteron term will not have a major impact on the result which is stable. Evaluating this last term from the impulse approximation only would increase $g_c^2$ by 1.25(5). However, double s wave scattering decreases $g_c^2$ by $-1.08$, while smaller correction terms come from the p wave Fermi motion ($+0.24$), the dispersive correction from absorption ($-0.18(4)$) and the s-p wave double scattering interference term ($-0.21$) [12]. To exploit the present experimental precision the dominant double scattering term must be controlled to better than 10%, while other corrections require little more than estimates. Of these terms the s-p interference term is presently not fully elucidated. It depends on short range behavior and may be partly spurious. Using the correction terms from refs. [12] and [13] we find a preliminary value $g_c^2 = 13.99(24)$ including the s-p interference term and $14.20(24)$ excluding it.

In conclusion, we have now two independent methods with controllable errors for the coupling constant. The Difference Method gives 14.52(26) or a 2% error. Its future expected improvements are a) a full angular range, which will give normalization to ±1% (now ±2%) and b) several incident neutron energies (which in principle should contain very similar information) from which the future precision is expected to reach ±1.5%. The GMO relation gives the preliminary value $13.99(24)$ or ±2%. The expected improvements are in the dispersion integral evaluation, now ±4.6% to ±2 to 3%, which leads to a precision in the coupling constant of ±(1 to 1.5)%.

In summary, the two model independent methods which have been critically examined here provide no support for the low value for the coupling constant, close to 13.5, which has been advocated elsewhere. The lower value cannot be completely excluded at present, but better data and careful analysis should settle the issue.

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