Renormalization group flow for Yang-Mills fields interacting with matter

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Abstract

We show an application of the Wilson Renormalization Group (RG) method to a SU(2) gauge field theory in interaction with a massive fermion doublet. By choosing suitable boundary conditions to the RG equation, i.e. by requiring the relevant monomials not present in the classical action to satisfy the Slavnov-Taylor identities once the cutoffs are removed, we succeed in implementing the local gauge symmetry. In this way the so called fine-tuning problem, due to the assignation of boundary conditions in terms of the bare parameters, is avoided. In this framework, loop expansion is equivalent to the iterative solution of the RG equation; we perform one loop calculations in order to determine whether and, if so, how much the fermionic matter modifies the asymptotic form of the couplings. Then we compute the β-function and we check gluon transversality. Finally, a proof of perturbative renormalizability is shown.

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1 Introduction

For a long time U.V. divergences appearing in gauge field theories have been treated by those regularizations which were able to preserve the local gauge symmetry; in this way, indeed, it has been possible to take into account only the counter terms the symmetry itself allowed.

This is why dimensional regularization [1] is the most used one in dealing with such a theory. However, dimensional regularization is bounded to perturbative regimes and, moreover, it cannot be easily extended to chiral gauge theories, since one should define Dirac’s matrix $\gamma_5$ in complex space-time dimensions.

Therefore, it would be useful to find a new regularization scheme avoiding these problems: the Wilson renormalization group (RG) formulation [2] has just these features since it works in four dimensions and it is, in principle, non-perturbative. Nevertheless, gauge symmetry breaking, due to the cutoff introduction, is the price to be payed. Polchinski, in 1984, [3] proposed an original way of computing Green functions by a suitable RG equation. A few years later, Becchi [4] analysed the breaking induced by the cutoff in a $SU(2)$ gauge field theory and found a way of compensating it by a suitable choice of non gauge-invariant counter terms (fine-tuning equation).

Recently Bonini et al. [5] succeeded in avoiding the fine-tuning problem by choosing in a proper way boundary conditions to the RG equation.

In this paper we extend RG formulation to a $SU(2)$ gauge field theory interacting with a fermionic doublet by a minimal coupling.

The first step is the analysis of the classical action ($S_{cl}$) invariance under BRS transformations [6]. This invariance forces Green functions to satisfy the Slavnov-Taylor (ST) identities, the analogous of Ward’s in the non-abelian case. These identities can be expressed in a compact form by a functional identity generating them by successive functional derivations.

The second step is the introduction of a regularization scheme consisting in assuming the free propagators in momentum representation to vanish for $p^2 > \Lambda_0^2$ and $p^2 < \Lambda^2$ so that $\Lambda$ and $\Lambda_0$ represent respectively the infrared and ultraviolet cutoffs. This procedure is equivalent to Wilson’s since the Wilsonian effective action generates propagators having the same features. Obviously, the cutoff introduction breaks the gauge symmetry; as a consequence, a suitable set of non gauge-invariant counter terms has to be added and, moreover, the cutoff effective action, $\Gamma_{\{\varphi, \Lambda\}}$, does not satisfy the ST identities at the generic scale $\Lambda$.

The successive step is the derivation of the RG equation together with boundary conditions, which can be fixed in different ways. One can assign the bare parameters, i.e.
the couplings at the U.V. scale, giving rise to the fine-tuning problem, or, in order to
avoid it, one can fix different boundary conditions according to the different nature of
the couplings.

The irrelevant couplings, with negative mass dimensions, are assumed to vanish at
the U.V. scale, while the relevant ones, with non-negative mass dimensions, are fixed
at the physical point \( \Lambda = 0, \Lambda_0 \to \infty \) to be the physical masses and the coupling and
wave function constants. Since the interaction generates also terms with no classical
analogous, the relative conditions are given by requiring the relevant part of the ef-
tective action to satisfy the ST identities at the physical point; this is the first step
towards gauge symmetry recovery at the quantum level.

The RG equation obtained in this way is exact; once boundary conditions are given, it
allows to compute all the vertices of the theory (and, so, all the physical observables)
either by using numerical techniques or by solving it perturbatively.

In the last framework we compute, at one loop, all the couplings involved in the ver-
tex functions; starting from them, we derive the one loop \( \beta \)-function, related to the
asymptotic freedom, and we check the gluon transversality as a consequence of one of
the ST identities in the physical limit.

This paper is organized in the following way: in section 2 we analyse BRS symmetry
and its consequences over Green functions; then we derive the RG equation and we fix
suitable boundary conditions. In section 3 we discuss in detail the fermionic sector,
only quoting the other results. In section 4 we give a schematic proof of perturbative
renormalizability.

2 Renormalization group flow

2.1 Classical action, BRS symmetry and ST identities

The classical euclidean action in the Feynman gauge is

\[
S_{cl}(A_{\mu}, \bar{c}, c, \bar{\Psi}, \Psi) = \int dx \left[ i \bar{\Psi} \left( i \partial \Psi - m \right) + \frac{1}{4} F_{\mu\nu} \cdot F_{\mu\nu} + \frac{1}{2} \left( \partial_{\mu} A_{\mu} \right)^2 + \bar{c} \cdot \partial_{\mu} D_{\mu} c \right]
\]

where

\[
D_{\mu} = \partial_{\mu} - ig A_{\mu}
\]

\[
F_{\mu\nu} \cdot F_{\mu\nu} = F_{\mu\nu} F_{\mu\nu}^a
\]

\[
F_{\mu\nu}^a = \partial_{\mu} A_{\nu}^a - \partial_{\nu} A_{\mu}^a + g (A_{\mu} \wedge A_{\nu})^a
\]

\[
D_{\mu} c = \frac{1}{g} \partial_{\mu} c + A_{\mu} \wedge c
\]

the last two terms in \( \Box \) representing respectively the gauge-fixing and the Faddev-
Popov determinant parametrization.
This action is invariant under the non-linear, nihilpotent BRS transformations
\[ A_\mu = A'_\mu + \bar{\eta}D_\mu c' \]
\[ \bar{c} = \bar{c'} - \bar{\eta} \partial_\mu A'_\mu \]
\[ c = c' - \frac{\bar{\eta}}{2} c' \wedge c' \]
\[ \Psi = \Psi' + i\bar{\eta} c' \Psi' \]
\[ \bar{\Psi} = \bar{\Psi'} + i\bar{\eta} \bar{\Psi} c' \]
\[ \bar{\eta} \text{ being a Grassmann parameter.} \]

Owing to this invariance, vertex functions have to satisfy the ST identities, whose functional formulation can be simplified by adding to \( S_{cl} \) other four terms, obtained as scalar products between field variations and their coupled sources\(^1\).

\[ S_{BRS} = S_{cl} - \int dx \left( k_\mu \cdot D_\mu c + \frac{1}{2} L \cdot c \wedge c + i \bar{\lambda} c \Psi + i \bar{\Psi} c \lambda \right) \]

The generating functional of Green functions is
\[ Z\{j\} = \exp W\{j\} = \int D\varphi \exp \left[ -S_{BRS}[\varphi] + \int dx j(x) A(x) \varphi A(x) \right] \]

where
\[ \varphi \doteq (A_\mu, c, \bar{c}, \Psi, \bar{\Psi}) \]
\[ j \doteq (j_\mu, X, -X, \bar{\eta}, -\eta)^2 \]

In order to obtain the ST identities we perform the substitution (3) in (5), noting the invariance of the integration measure and of the action as well.

By a first order \( \bar{\eta} \)-expansion\(^3\) we obtain
\[ \int dx \left[ j_\mu \cdot \delta + \bar{\eta} \cdot \delta + \partial_\mu \delta j_\mu \cdot \eta - \bar{\lambda} \delta + \lambda \delta \right] Z\{j\} = 0 \]

(8) can be expressed in terms of the effective action \( \Gamma \), obtained by taking the functional Legendre transform of \( W\{\varphi\} \), defined in (3).

\[ \int dx \left[ -\frac{\delta \Gamma'}{\delta A_\mu} \delta A_\mu - \frac{\delta \Gamma'}{\delta c} \delta c + \frac{\delta \Gamma'}{\delta L} \delta L - \frac{\delta \Gamma'}{\delta \Psi} \delta \Psi - \frac{\delta \Gamma'}{\delta \bar{\Psi}} \delta \bar{\Psi} \right] = 0 \]

where \( \Gamma' = \Gamma + \frac{1}{2} \int dx (\partial_\mu A_\mu)^2 \).

The ST identity validity holds a great importance in order to assign the boundary conditions to the RG flow equation.

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\(^1\) (4) is still invariant under (3) because of the BRS operator nihilpotency.

\(^2\) This form of \( j \) takes into account the exchange properties of fields and sources.

\(^3\) The expansion can be made up to the first order because of the Grassmann nature of \( \bar{\eta} \).
2.2 RG equation

In order to regularize the theory we will adopt a scheme formally equivalent to Wilson’s; it consists in multiplying the quadratic part of the BRS action by a cutoff function \( K_{\Lambda A_0}(p) \) defined in momentum space as:

\[
K_{\Lambda A_0}(p) = \begin{cases} 
1 & \Lambda^2 \leq p^2 \leq \Lambda_0^2 \\
\supp K_{\Lambda A_0} \subset A & A \ni \{ p | \Lambda^2 \leq p^2 \leq \Lambda_0^2 \} 
\end{cases}
\]  
(10)

The resulting cutoff action \( S^{\Lambda A_0}_2 \) consists of two terms: \( S^{\Lambda A_0}_2 \), which represents the quadratic part of the action, and \( S^{\Lambda A_0}_{\text{int}} \), which contains all the Lorentz scalar \( SU(2) \) singlet monomials whose dimensions are not higher than four, according to the power counting.

The symmetry breaking, due to the cutoff introduction, makes the interaction generate even non-gauge-invariant monomials, which, of course, have no classical analogous.

\[
S^{\Lambda A_0}_2 = \int dx dy K^{-1}(y-x) \left[ -\frac{1}{2} A_\mu(y) \cdot \partial^2_a A_\mu(x) + i\bar{\Psi}(y)(i\partial - m) x \Psi(x) + \frac{1}{y} \bar{c}(y) \cdot \partial^2_c c(x) \right] = \frac{1}{2} \int dx dy \varphi_A(y) D_{AB}^{-1}(y,x;\Lambda) \varphi_B(x)
\]  
(11)

\[
S^{\Lambda A_0}_{\text{int}} = \int dx \left[ \frac{1}{2} A_\mu \cdot \left( \delta_{\mu\nu}(\sigma_{\Lambda A_0}(0) - \sigma_{\Lambda A_0}(0)) \partial^2 - \sigma_{\Lambda A_0}(0)(\delta_{\mu\nu} \partial^2 - \partial_\mu \partial_\nu) \right) A_\nu + i\bar{\Psi} \left( -\sigma_{m\psi}(\Lambda_0) + \sigma_{\psi}(\Lambda_0)(i\partial - m) \right) \Psi - \sigma_{\omega\epsilon}(\Lambda_0) \omega_\mu \cdot \partial_\mu c + \sigma_{3A}(\Lambda_0) \partial_\mu A_\nu \cdot (A_\mu \wedge A_\nu) + \frac{1}{4} \sigma_{4A}(\Lambda_0) \left( (A_\mu \wedge A_\nu) \cdot (A_\mu \wedge A_\nu) + \sigma_{\omega\epsilon A}(\Lambda_0) \omega_\mu \cdot (c \wedge A_\mu) + \frac{1}{2} \sigma_{4A}(\Lambda_0) \right) \bar{\Psi} A\Psi - \sigma_{Lc}(\Lambda_0) L \cdot c \wedge c - i\sigma_{\lambda c}(\Lambda_0) \left[ \lambda c \Psi + \bar{\Psi} c \lambda \right] + \frac{\sigma_{4A}(\Lambda_0)}{8} \left[ 2(A_\mu \cdot A_\nu)(A_\mu \cdot A_\nu) + (A_\mu \cdot A_\mu)(A_\nu \cdot A_\nu) \right] \right]
\]  
(12)

By \( S^{\Lambda A_0} \) the regularized generating functional \( Z(\varphi; \Lambda) \) is constructed and by noting that all its \( \Lambda \)-dependence comes from \( K_{\Lambda A_0}(p) \) the RG equation is easily derived.

\[
\Lambda \partial_\Lambda \Pi\{ \varphi; \Lambda \} = -\frac{1}{2} \int dk dl \det M_{DE}(l,k) \Delta_{DE}(k,l)
\]  
(13)

where

\[
\Pi\{ \varphi; \Lambda \} = -\Gamma\{ \varphi; \Lambda \} - S^{\Lambda A_0}_2
\]

\[
M_{DE}(l,k) = \int dx dy \Delta_{EA}(l-y) \left[ \Lambda \partial_\Lambda D_{AB}^{-1}(y,x;\Lambda) \Delta_{BD}(x-k) \right]
\]  
(14)

\( ^4 K_{\Lambda A_0}(p) \) regularity is treated in [1].

\( ^5 \)Only the \( \Lambda \)-dependence will be explicitly written in \( Z \) as well as in the other functionals, since we are always interested in the limit \( \Lambda_0 \to \infty \).
\( \Delta_{AB}(x - y) \) being the full propagator. The auxiliary functional \( \bar{\Gamma}_{BF} \) satisfies the integral equation

\[
\bar{\Gamma}_{BF}(y, \xi; \varphi) = (-1)^{\delta_{\varphi}+1} \Gamma_{BF}^{int}(y, \xi; \varphi) + \int dk dz \Delta_{CD}(z - k) \Gamma_{DF}(k, \xi; \varphi) \Gamma_{BC}^{int}(y, z; \varphi)
\]

where \( \Gamma_{BC}^{int}(y, z; \varphi) \) is defined by the relation

\[
\frac{\delta^2 \Gamma}{\delta \varphi_C(z) \delta \varphi_B(y)} = -\Delta_{BC}^{-1}(y - z) + \Gamma_{BC}^{int}(y, z; \varphi)
\]

The \( \bar{\Gamma} \) vertices can be computed in terms of the proper ones by expanding (15) in field powers.

### 2.3 Boundary conditions

Following [5] we assign different types of boundary conditions according to the nature of couplings; since \( \Pi{\varphi; \Lambda} \) contains both relevant and irrelevant couplings, the former ones will be indicated by \( \sigma(\Lambda) \) and \( \sigma'(\Lambda) \) whereas the latter ones by \( \Sigma(\Lambda) \). The relevant couplings having classical analogous, \( \sigma(\Lambda) \), will be given their physical values at \( \Lambda = 0 \) and \( \Lambda_0 \to \infty \); the relevant couplings not present in \( S_{cl} \), \( \sigma'(\Lambda) \), will be determined by requiring vertex functions to satisfy the ST identities at the same physical point. Finally, the irrelevant couplings, \( \Sigma(\Lambda) \), will be set to zero at the U.V. scale \( \Lambda = \Lambda_0 \). In this way the fine-tuning problem is avoided.

As an example, we will examine the boundary conditions for the couplings generated by introducing the fermion doublet in the action:

\[
\begin{align*}
\sigma_m^\Psi(\Lambda = 0) &= 0 & \sigma_\Psi(\Lambda = 0) &= 0 & \sigma_{\bar{\Psi}A\Psi}(\Lambda = 0) &= g \\
\sigma'_{\bar{\Psi}\lambda}(\Lambda = 0) &= \sigma'_{\bar{\Psi}c\lambda}(\Lambda = 0) &= 1
\end{align*}
\]

(17)

\( \sigma'_{\bar{\Psi}\lambda}(\Lambda = 0) \) and \( \sigma'_{\bar{\Psi}c\lambda}(\Lambda = 0) \) are computed by the following ST identity:

\[
\gamma_\mu \Gamma^\Psi(\Psi(k, p, q)) \Gamma_{\bar{\Psi}c}(p) - \Gamma^\Psi(\Psi(k)) \Gamma_{\bar{\Psi}c}(k, p, q) + \Gamma^\Psi(-q) \Gamma_{\bar{\Psi}c}(k, p, q) = 0
\]

obtained by differentiating (9) with respect to the fields \( \bar{\Psi}, \Psi \) and \( c \).

### 3 Loop expansion: results

It is possible to compute all the vertices of the theory, at least order by order, by solving the RG equation iteratively.

\( ^6 \)The vertices must be computed at the point where the irrelevant parts vanish.
One loop contributions can be derived by expanding the r.h.s. of (13) at the tree level:

\[ \Lambda \partial_\Lambda \Pi^{(1)}(\varphi; \Lambda) = \frac{1}{2} \int \frac{dq}{(2\pi)^4} \left[ \Lambda \partial_\Lambda D_{ED}(q; \Lambda) \right] \tilde{\Gamma}^{(0)}_{DE}(-q, q; \varphi) \]  

(19)

Obviously, one loop corrections to the single vertex can be obtained by differentiating (19) with respect to the fields marking the vertex itself and, then, by solving the resulting equation.

We are going to examine in detail the fermionic sector, while the other results will be only quoted. Then the \( \beta \)-function and the gauge symmetry recovery will be taken into account.

3.1 One loop couplings

The corrections to the inverse fermion propagator take the form:

\[ \Pi_f(p, \Lambda) = \sigma_{m\psi}(\Lambda) + \sigma_{\psi}(\Lambda)(\not{p} + m) + \Sigma_f(p, \Lambda) \]  

(20)

with \( \Sigma_f(\not{p} = -m, \Lambda) = 0 \) and \( (\partial_{\mu} \Sigma_f)(p = 0, \Lambda) = 0 \). By substituting (20) in (19) after differentiating it with respect to \( \bar{\Psi} \Psi \) one easily obtains

\[ \sigma_{m\psi}(\Lambda) = \int_0^\Lambda d\lambda \partial_\lambda \Pi_f(p, \lambda) \bigg|_{\not{p} = -m} \]

\[ \gamma_\mu \sigma_{\psi}(\Lambda) = \partial_{\mu p} \int_0^\Lambda d\lambda \partial_\lambda \Pi_f(p, \lambda) \bigg|_{p = 0} \]

\[ \Sigma_f(p, \Lambda = 0) = - \left[ \int_0^\Lambda d\lambda \partial_\lambda \Pi_f(p, \lambda) - \sigma_{m\psi}(\Lambda_0) - \sigma_{\psi}(\Lambda_0)(\not{p} + m) \right] \]

(21)

In order to compute these integrals the Feynman parameter technique has been used; owing to some approximations concerning products of cutoff functions, the following results hold only in the limit of large \( \Lambda \).

\[ \sigma_{m\psi}^{(1)}(\Lambda) = \frac{3}{4} \left[ \frac{g^2}{16\pi^2} m \log \frac{\Lambda^2}{m^2} \right] + O(1) \]

\[ \sigma_{\psi}^{(1)}(\Lambda) = \frac{3}{4} \left[ \frac{g^2}{16\pi^2} \log \frac{\Lambda^2}{m^2} \right] + O(1) \]

\[ \Sigma_f^{(1)}(p, \Lambda = 0) = O(1) \]  

(22)
The same technique can be used to compute all the other couplings:

\[
\sigma^{(1)}_{\omega c}(\Lambda) = \frac{g^2}{16\pi^2} \log \frac{\Lambda^2}{\mu^2} \\
\Sigma^{(1)}_{\omega c}(\Lambda = 0) = \frac{g^2}{16\pi^2} \log \frac{p^2}{\mu^2} \quad \sigma^{(1)}_A(\Lambda) = \frac{g^2}{24\pi^2} \left[ 5 \log \left( \frac{\Lambda^2}{\mu^2} \right) - \log \left( \frac{\Lambda^2}{m^2} \right) \right]
\]

\[
\lim_{m \to 0} \Sigma^{(1)}_{L}(p^2 = \mu^2, \Lambda = 0) = \frac{g^2}{16\pi^2} \log \frac{\mu^2}{\mu^2} \\
\Sigma^{(1)}_{L}(p, \Lambda = 0) = \mathcal{O}(1) \\
\sigma^{(1)}_{\omega c A}(\Lambda) = -\frac{g^2}{16\pi^2} \log \frac{\Lambda^2}{\mu^2} \\
\sigma^{(1)}_{\omega c A}(l, p, k, \Lambda = 0) \equiv \mathcal{O}(1)
\]

\[
\sigma^{(1)}_{A c}(\Lambda) = \frac{g^2}{64\pi^2} \frac{11}{2} \log \frac{\Lambda^2}{m^2} \\
\Sigma^{(1)}_{A c}(l, p, k, \Lambda = 0) = \mathcal{O}(1) \\
\sigma^{(1)}_{\lambda c}(\Lambda) = \sigma^{(1)}_{\Psi c}(\Lambda) = -\frac{g^2}{16\pi^2} \log \frac{\Lambda^2}{\mu^2} \\
\Sigma^{(1)}_{\lambda c}(\Lambda = 0) \equiv \Sigma^{(1)}_{\Psi c}(\Lambda = 0) = \mathcal{O}(1)
\]

(23)

\(\sigma^{(1)}_{3 A}\) and \(\sigma^{(1)}_{3 A}\) have not been computed since they receive no contribution from radiative corrections involving matter fields. As regards the couplings in the four vectorial field vertex, we only checked that matter fields did not modify the \(\Lambda\)-power behavior.

### 3.2 One loop \(\beta\)-function

Two renormalizable field theories, involving different subtraction points, can be connected in such a way as to describe the same physics. Each one, indeed, can be reconstructed from the other by introducing suitable counter terms. This procedure is equivalent to redefining coupling constants and scale constants \(Z\) so that the following scale equations are satisfied:

\[
\Gamma_{nA,m\omega c,l\Psi}(g', \mu') = Z_{A}^{-\frac{\Phi}{2}} Z_{C}^{-m} Z_{\Psi}^{-l} \Gamma_{nA,m\omega c,l\Psi}(g, \mu)
\]

(24)

where \(\Gamma_{nA,m\omega c,l\Psi}\) denotes the vertex with \(n\) vector fields, \(m\) pairs \(\omega\mu\), and \(l\) pairs \(\Psi\) at the physical point.

By using (24), it is possible to write down three one loop equations, involving \(\Gamma_{AA}\), \(\Gamma_{A\Psi}\), and \(\Gamma_{\Psi\Psi}\) terms.
\[ Z_A^{(1)} = 1 + \frac{g^2}{16\pi^2} \frac{8}{3} \log \frac{\mu'^2}{\mu^2} \quad Z_\Psi^{(1)} = \frac{g'}{g} \left[ 1 + \frac{g^2}{16\pi^2} \log \frac{\mu'^2}{\mu^2} \right] \]

By differentiating (25), the \( \beta \)-function in the zero mass limit is obtained

\[ \beta^{(1)}(g) \equiv \mu' \frac{\partial g'}{\partial \mu'} \bigg|_{\mu' = \mu} = -\frac{5}{12\pi^2} g^3 \]  

The same result can be computed by using other scale equations, involving different vertices.

### 3.3 Gluon transversality

We will show a check of the ST identity involving the inverse vector and ghost propagators at one loop order. Differentiating (9) with respect to the fields \( A_{\mu c} \) gives rise to

\[ \left[ \Gamma_{\mu\nu}(p, \Lambda = 0) + p_\mu p_\nu \right] \Gamma^{c}_{\mu c}(p, \Lambda = 0) = 0 \]  

which forces the longitudinal irrelevant part of the inverse vector propagator, \( \Sigma_L(p, \Lambda) \), to be zero at the physical point.

One finds

\[ \lim_{\Lambda_0 \to \infty} \Sigma_L^{(1)}(p, \Lambda = 0) = -\int_0^\infty d\lambda \partial_\lambda \left[ \Pi_L(p, \lambda) - \Pi_L(0, \lambda) - p^2 \frac{\partial}{\partial p^2} \Pi_L(\bar{p}, \lambda) \bigg|_{\bar{p}^2 = \mu^2} \right] = 0 \]

Owing to the irrelevant nature of \( \Sigma_L \), the leading term as \( \Lambda_0 \to \infty \) can be nothing but a constant; so it is sufficient to show this constant to be zero.

### 3.4 Perturbative renormalizability

In order to prove the theory to be perturbatively renormalizable one has to show that RG equation has a nontrivial limit as \( \Lambda_0 \to \infty \).

Starting from the integral form of the RG equation (13), so as to embody boundary
conditions, we analyse the behavior at large scales of proper vertices, whose form can be obtained, order by order, by successive iterations. If their \( \Lambda \)-dependence is such that power counting is satisfied at any order, perturbative renormalizability is proved. In order to simplify dimensional analysis by neglecting every momentum dependence, we introduce the norm \[3\]:

\[
\left| \Gamma(n_{\bar{\psi} \Psi}, n_{\nu \mu}, n_L, n_{\bar{\lambda} \lambda}) \right|_\Lambda = \max_{p_i^2 \leq c \Lambda^2} \left| \Gamma(p_1, \ldots, p_n; \Lambda) \right|
\]

where the generic \( n_\varphi \) is the number of \( \varphi \) fields or sources marking the vertex. Since

\[
dim \Gamma_{n_\varphi_1 \cdots n_\varphi_n} (p_1, \ldots, p_n) = 4 - n_A - 2n_{\bar{\psi} \Psi} - 2n_{\nu \mu} - 2n_L - 3n_{\bar{\lambda} \lambda} \pm 4 - \sum_i \alpha_i n_{\varphi_i}
\]

we assume, at \( l \)-loop order and for large \( \Lambda \), that

\[
\left| \Gamma^{(l)} (n_{\varphi_1}, \cdots, n_{\varphi_n}) \right|_\Lambda = \mathcal{O}(\Lambda^4 - \sum_i \alpha_i n_{\varphi_i})
\]

We want to show this \( \Lambda \)-dependence to be preserved at \((l + 1)\)-loop order. Since, by definition, auxiliary vertices have the same dimension as the proper ones, the following relations hold:

\[
\left| \bar{\Gamma}^{(l)}_{abn_{\varphi_1} \cdots n_{\varphi_n}} \right|_\Lambda = \left\{ \begin{array}{ll} \mathcal{O}(\Lambda^4 - \sum_i \alpha_i n_{\varphi_i} - 2) \\ \mathcal{O}(\Lambda^4 - \sum_i \alpha_i n_{\varphi_i} - 3) \end{array} \right.
\]

where the upper relation holds if \( ab \) refer to ghost or vectorial fields, while the lower holds if \( ab \) refer to matter fields. From \(32\) and \(33\) one gets

\[
\left| \Pi^{(l+1)}_{n_{\varphi_1} \cdots n_{\varphi_n}} \right|_\Lambda = \left\{ \begin{array}{ll} \mathcal{O}(\Lambda^{2 - \sum_i \alpha_i n_{\varphi_i} - 4 - 2}) \\ \mathcal{O}(\Lambda^{1 - \sum_i \alpha_i n_{\varphi_i} + 4 - 1}) \end{array} \right.
\]

by taking into account the different contributions \( M_{ED} \) gives according to the nature of the \( ED \) fields as well as the different integration domains, related to the relevant and irrelevant parts \( \Pi \) is made by (see also \[3\] for QED). Since \(31\) is satisfied at \( l = 0 \), by induction on the loop number we prove perturbative renormalizability.
4 Conclusions

A RG application to a non-abelian gauge theory interacting with a massive doublet by a minimal coupling has been shown.

We analysed in detail a mathematical technique which enables the Green function determination when the regularization breaks the gauge symmetry. This approach is, therefore, very useful to deal with chiral gauge theories, since in this case symmetry breaking, due to the regularization, is unavoidable. Moreover this method let us obtain an exact RG equation, describing the effective action evolution at any energy scale. Already in 1983 Becchi [4] proposed an exact RG equation in differential form together with boundary conditions fixed on the bare couplings: this fixing gave rise to the so called fine-tuning problem concerning the determination of the boundary conditions related to couplings not present in the classical action. In order to avoid this problem, boundary conditions are assigned on the renormalized theory.

By using this technique we computed, at one loop order, all the couplings at large scales: they perfectly agree with the results obtained by dimensional regularization. By evaluating couplings at the U.V. scale one achieves the relations between bare and renormalized couplings.

The gauge symmetry recovery at the physical point has been analysed: we checked, at one loop order, gluon transversality as a consequence of one of the ST identities; it would be interesting to verify the ST identities to be satisfied at any loop.

Finally we computed the one loop $\beta$-function, describing the asymptotic behavior of the theory. This result coincides with the one obtained by dimensional regularization in the independent mass prescription.

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