Co-evolution of Rumor Diffusion and Structural Stability in Signed Social Networks

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Abstract—Prediction and control of rumor diffusion in social networks (SNs) are closely tied to the underlying connectivity patterns. Contrary to most existing efforts that exclusively focus on positive social user interactions, the impact of contagion processes on the temporal evolution of signed SNs with distinctive friendly (positive) and hostile (negative) relationships yet, remains largely unexplored. In this paper, we study the interplay between social link polarity and propagation of intentionally fabricated information coupled with user alertness. In particular, we propose a novel energy model built on Heider’s balance theory that relates the stochastic susceptible-alert-infected-susceptible rumor epidemic model with the structural balance of signed SNs to substantiate the trade-off between social tension and rumor spread. Moreover, the role of hostile social links in the formation of disjoint friendly clusters of alerted and infected users is analyzed. Using three real-world datasets, we further present a time-efficient algorithm to expedite the energy computation in our Monte-Carlo simulation method and show compelling insights on the effectiveness and rationality of user awareness and initial network settings in reaching structurally balanced local and global network energy states.

Index Terms—Signed networks, rumor diffusion, balance theory, user awareness, epidemic modeling, energy function.

1 INTRODUCTION

Quantitative analysis of stochastic processes such as rumor and (mis-)information spreading over physical and online social networks (SNs) has stimulated intense research activities [1], [2]. Owing to the pervasive use of social media and the abundance of data extracted from several such networks, which for long were merely unavailable, the theoretical perception of epidemic dynamics driven by nodal interactions has refined substantially in recent years [3], [4]. While the vast majority of research has scrutinized only positive social relationships, user pairs may also signify enmity or distrust as perceived in reality. Subsequently, a user may decline to receive and propagate information from a hostile contact [5]. Accounting for heterogeneity in social relationships is therefore, crucial in characterizing social link valence evolution under the influence of individual user’s attitudes towards spreading the rumor. Unlike conventional networks, signed SNs evolve based on the structural balance theory, pioneered by Heider [6], where the relationship between any two users in a triad (3-clique) can be impacted by the third user [7]. That is to say, the theory posits that if “the friend of my friend is my friend” and “the friend of my enemy is my enemy”, then the resulting triad will be balanced and will constitute an odd number of friendly links. Evidently, signed SNs converge to structurally balanced states with minimum social tension by flipping the link polarity to maximize the number of balanced triads [8].

In the jargon of networked epidemics, there exist a handful of works dedicated to edge sign reconfiguration under the influence of evolving user states. The conditions to attain opinion convergence in generic SNs are obtained in [9] using monotone dynamical systems. Further extended in [10], Shi et al. analyze the asymptotic user state evolution affected by deterministic weights on pairwise interactions by formulating a relative-state-flipping model for consensus dynamics over signed random networks and prove the conditions leading to almost sure convergence and divergence of the network states. They use sample-path analysis to reveal the pivotal role of relative weights on negative social links in driving the network divergence. Their analysis builds on the assumption that the initial network structure is always balanced which in truth, may not always be the case. Saeedian et al. [11] study the non-trivial behavior of the two coupled dynamics over a complete signed graph using an energy function. The authors adopt the susceptible-infected (SI) epidemic model to study the local and global energy minima of the system irrespective of the possibility of recovery to susceptibility or rumor alertness. Lee et al. [12] introduce an adaptive susceptible-infected-susceptible (SIS) epidemic model to re-inforce transitivity by rewiring the links between susceptible and infected nodes rather than their signs. Though insightful, the emergent behaviors of the parallel processes in [12] are limited to the population level and do not address the microscopic dynamics inherent in user interactions. Li et al. [13] propose a non-stochastic computational model for maximizing polarity-related linear influence diffusion in SNs.

Thus far, there exists no work that investigates the intriguing co-evolution of SN structure and epidemic dynamics of a reversible rumor spreading process in conjunction with user alertness. Besides the analytical merit, such a projection model may serve beneficial to network administrators and social influencers in devising optimal control measures for...
better information flow in practice. In view of this gap, the main contributions of this work are summarized as follows:

- Inspired by Heider’s balance theory, a novel energy-based framework is proposed to jointly minimize the number of unbalanced triads that contribute to the social tension while mitigating the rumor virality in SNs. To capture users’ response to rumors, we formally define the susceptible-alert-infected-susceptible (SAIS) epidemic model [14] as a continuous-time Markov chain (CTMC) and derive the steady-state probabilities to investigate the virtues of promoting awareness on the network structural evolution.

- By incorporating a tuning parameter, we then analyze cases for which the initial fraction of positive links and the initially infected users induce natural immunization by altering unbalanced triads and infected users into two clusters interconnected via unfriendly links.

- The proposed model is evaluated on three datasets extracted from real signed SNs by employing a time-efficient Monte Carlo simulation method under different parametric settings.

The rest of this paper is organized as follows: Section 2 outlines the proposed network energy framework, followed by the definition of the corresponding CTMC and stationary probabilities in Section 3. In Section 4, an efficient algorithm for faster energy computation in the employed Monte Carlo method is presented. Experimental validation of the model is discussed in Section 5. Finally, Section 6 concludes the paper.

2 THE COUPLED NETWORK MODEL

In this section, we first formulate the stochastic SAIS model re-purposed for rumor diffusion in our analysis. The proposed energy model is then detailed subsequently.

2.1 Rumor Diffusion Model Description

We consider an undirected signed SN, represented by the graph $G_t = (V, E_t)$, with a set $V = \{1, 2, \ldots, n\}$ of $n$ users that form friendly (1), hostile (−1), or no (0) social links. The link polarity of user pair $(i, j) \in E_t$ at any given time $t$ is denoted by $A_{i,j}(t) \in \{-1, 0, 1\}$. For rumor propagation over $G_t$, the SAIS model in Fig. 1 is adopted, where each user is in the susceptible (S), alert (A), or infected (I) state at time $t$. User $i$ is said to be susceptible if he/she is completely unaware of the rumor. Since rumors do not propagate over negative links in signed networks [15], a susceptible user gets infected with rate $\beta_+ \in \mathbb{R}^+$ times the number of its infected friendly contacts [14]. A user aware of the rumor however, is less likely to get infected, with a lower infection rate $0 \leq \beta_- < \beta_+$, as compared to a susceptible user. Unlike the irreversible SI model in [11], a susceptible user becomes aware of the rumor with the rate $\kappa \in \mathbb{R}^+$ times the number of direct infected friends and all infected users may eventually forget about the rumor and recover back to susceptibility with rate $\delta \in \mathbb{R}^+$. For all $i \in V$, the network state can thus, be expressed mathematically as the CTMC $\{X_i(t); t \geq 0\}$, where:

$$X_i(t) = \begin{cases} 1; & \text{if user } i \text{ is susceptible at time } t, \\ 0; & \text{if user } i \text{ is alert at time } t, \\ -1; & \text{if user } i \text{ is infected at time } t. \end{cases}$$

Using (1), we now can define the probability of user $i$ being in one of the three epidemic states as $S_i(t) = \Pr[X_i(t) = 1]$, $A_i(t) = \Pr[X_i(t) = 0]$, and $I_i(t) = \Pr[X_i(t) = -1]$ such that for $1 \leq i \leq n$, $S_i(t) + A_i(t) + I_i(t) = 1$ always holds.

2.2 Pairwise Spreading Energy Function

We now delineate the sign evolution of user interactions in the context of energy. Given the three epidemic states (S, A, I) and binary link signs (−, +), there exist 12 distinct user pair configurations as shown in Fig. 2. We characterize the rumor spreading potency by mapping each pair configuration $(i, j)$, where $i, j \in V$, to the energy landscape as follows:

$$E_{i,j}^p(t) \triangleq \begin{cases} A_{i,j}(t) \frac{(X_i(t) - X_j(t))^2}{4} & \text{if } |X_i + X_j| \text{ mod } 2 = 0, \\ A_{i,j}(t) \frac{1 - X_i(t) - X_j(t)}{2} & \text{otherwise}. \end{cases}$$

Based on the functional value of (2), the configurations depicted in Fig. 2 can be classified as follows:

- **Balanced edges**: As long as the configurations S−I and A−I do not flip their edge signs while evolving, the users $i$ and $j$ are in a balanced social relationship and do not engage in rumor propagation. Therefore, they exhibit a pairwise energy of $E_{i,j}^p(t) = -1$.

- **Unbalanced edges**: Cases in which a susceptible or alert user is in a friend-like relationship with an infected user are socially unstable and are bound to change with time. In our model, S+I and A+1 serve as feasible links for rumor diffusion and thus, the users are in an unbalanced state with pairwise energy of $E_{i,j}^p(t) = 1$.

- **Neutral edges**: Irrespective of the edge sign, configurations S±S, I±I, A±A, and S±A do not contribute to the rumor propagation, and therefore, exhibit zero pairwise energy, i.e., $E_{i,j}^p(t) = 0$.

Accordingly, the total pairwise spreading energy of network $G_t$, denoted by $E_p(G_t)$, can be computed as:

$$E_p(G_t) = \frac{1}{n^2} \sum_{i \neq j} E_{i,j}^p(t).$$

2.3 Triad Structural Energy Function

Along with the users’ epidemic states, edge sign evolution is also driven by Heider’s structural balance criterion. A triad of users in $G_t$, denoted by $(i, j, k)$, is said to be balanced if the product of $A_{i,j}(t) \cdot A_{j,k}(t) \cdot A_{k,i}(t)$ is positive. In other words,
an unbalanced triad will always have an odd number of negative edges. Hence, network $G_t$ is fully balanced only if all the constituent triads are balanced. Conforming to Heider’s balance theory, the structural status of any triad $(i, j, k)$ can be mapped to the energy landscape as [6]:

$$E_{i,j,k}(t) \equiv -A_{i,j}(t) \cdot A_{j,k}(t) \cdot A_{k,i}(t).$$

(4)

With $\nabla$ in place, the energy contribution of any balanced (unbalanced) triad in $G_t$ is $E_{i,j,k}(t) = -1$ ($E_{i,j,k}(t) = 1$). Fig. 3 showcases all the balanced triads for the SAIS and the SIS (baseline) rumor models. In order to converge towards lower energy states (which corresponds to more social stability), users in unbalanced triads tend to flip their link signs which in turn, affects the configuration of other triads that share common edges with them. Consequently, the total normalized energy of $G_t$, denoted by $E_\Delta(G_t)$, is:

$$E_\Delta(G_t) = \frac{1}{(|G_t|)} \sum_{i,j,k \in G_t} E_{i,j,k}(t),$$

(5)

where $-1 \leq E_\Delta(G_t) \leq 1$. Users in triads decide on whether or not to alter their relationships only if the total triad energy of the resulting network is further reduced. Apparently, signed networks that manifest triad energy values closer to $-1$ tend to be socially more stable.

### 2.4 Weighted Network Energy Function

We now define the total energy of network $G_t$, given by $E(G_t)$, as the weighted sum of the overall pairwise and triad energy functions derived in [3] and [4], respectively:

$$E(G_t) = \alpha \cdot E_\Delta(G_t) + (1 - \alpha) \cdot E_p(G_t),$$

(6)

where $0 \leq \alpha \leq 1$ is the tuning parameter used to adjust the energy trade-off between the rumor epidemic ($\alpha = 0$) and the structural balance ($\alpha = 1$) in the network. Hence, if $G_t$ is fully balanced (fully unbalanced), then $E(G_t) = 1$ ($E(G_t) = -1$). Note that for some fixed $\alpha$ value, attaining the global (local) energy minimum state in which $G_t$ is fully (or nearly) balanced depends on the initial fractions of infected users ($0 \leq \rho_0 \leq 1$) and friendly links ($0 \leq \tau_0 \leq 1$) [11].

### 3 Steady-State Probability Distribution

Assuming that only one event is triggered in each time step, $\Delta t \ll 1$, i.e., either the epidemic state of exactly one of the users in $(i, j)$ changes or the edge sign is flipped, the rules defining the user pair transitions are shown in Fig. 4. The balanced edges $S \rightarrow I$ and $I \rightarrow S$ transition to $S \rightarrow S$ and $A \rightarrow S$, respectively, with probability $\delta \cdot \Delta t$ or change to $S + I$ and $A + I$, respectively, with probability $1 - \delta \cdot \Delta t$. For unbalanced edges, $S + I$ changes to $S - I$ with probability $1 - \delta \cdot \Delta t - (\beta + \kappa) \cdot \Delta t \cdot (1 - 2\delta \cdot \Delta t)$ or switches to states $A + I, I + I, I - I, S + S, S - S$, $A + A$, and $S - A$ flip their edge signs with probability 1 in each time step, $S + A$ changes to either $S - A$ or $A + A$ with probabilities $1 - \kappa \cdot \Delta t \cdot (1 - \beta \cdot \Delta t)$ and $\kappa \cdot \Delta t \cdot (1 - \beta \cdot \Delta t)$, respectively. Finally, $I \rightarrow S$ changes to $I + I$ with probability $1 - 2\delta \cdot \Delta t \cdot (1 - \beta \cdot \Delta t)$ and $1 - 2\delta \cdot \Delta t$, respectively, and to $S + I$ with probabilities $\delta \cdot \Delta t \cdot (1 - \delta \cdot \Delta t)$ and $\delta \cdot \Delta t$, respectively.

In general, given the user pair $(i, j)$, we can define a trivariate CTMC of the form $\{Z_{i,j}(t); t \geq 0\}$, where $Z_{i,j}(t) = (X_i(t), X_j(t), A_{i,j}(t))$ to formally express these conditional transition probabilities as:

$$P_{c,c'}(\Delta t) \equiv \Pr[Z_{i,j}(t + \Delta t) = c' | Z_{i,j}(t) = c],$$

(7)

where $c = (x, y, z)$, $c' = (x', y', z')$, and $x, y, z, x', y', z' \in \{-1, 0, 1\}$. Based on (7), the steady-state probability distribution is derived in Theorem 1.

**Theorem 1.** Let $\{\pi_{x,y,z}| x, y, z \in \{-1, 0, 1\}\}$ be the stationary probabilities for the CTMC $Z_{i,j}(t)$ defined above, then we have the following in steady-state:

(i) The fraction of susceptible users $(s_{\infty})$ is $\sum_{y,z} \pi_{1,y,z}$.
(ii) The fraction of infected users $(I_{\infty})$ is $\sum_{y,z} \pi_{1,y,z}$.
(iii) The fraction of alerted users $(a_{\infty})$ is $\sum_{y,z} \pi_{0,y,z}$.
(iv) The fraction of friendly links $(r_{\infty})$ is $\sum_{x,y} \pi_{x,y,1}$.

Proof. We use [7] to obtain the elements of the infinitesimal generator matrix $Q = [q_{c,c'}]$ of order $27$ as follows:

$$q_{c,c'} = \begin{cases} \lim_{\Delta t \rightarrow 0} \frac{P_{c,c'}(\Delta t) - 1}{\Delta t}; & \text{if } c' = c, \\ \lim_{\Delta t \rightarrow 0} P_{c,c'}(\Delta t); & \text{if } c' \neq c. \end{cases}$$

(8)

From [8], we can now obtain the stationary probabilities by solving $\Pi \cdot Q = 0$ and $\Pi \cdot 1 = 1$, where $\Pi = \{\pi_{x,y,z}| x, y, z \in \{-1, 0, 1\}\}$. The stationary distribution of the epidemic process is

$$\pi_{1,y,z} = \frac{1}{2} \left[ \left( \frac{1 - \delta}{1 - 2\delta - \kappa} \right) + \frac{\delta \cdot \Delta t \cdot (1 - \delta \cdot \Delta t)}{1 - 2\delta \cdot \Delta t} \right],$$

(9)

$$\pi_{0,y,z} = \frac{1}{2} \left[ \left( \frac{\delta \cdot \Delta t}{1 - 2\delta \cdot \Delta t} \right) + \frac{\kappa \cdot \Delta t \cdot (1 - \beta \cdot \Delta t)}{1 - 2\delta \cdot \Delta t} \right],$$

(10)

$$\pi_{-1,y,z} = \frac{1}{2} \left[ 1 - \left( \frac{\delta \cdot \Delta t}{1 - 2\delta \cdot \Delta t} \right) - \frac{\kappa \cdot \Delta t \cdot (1 - \beta \cdot \Delta t)}{1 - 2\delta \cdot \Delta t} \right].$$

(11)

Note that for $\delta = 0$, this reduces to the results in [7].

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**Fig. 3.** All possible balanced triads in the SAIS and the SIS models.

**Fig. 4.** Transition probabilities for temporal evolution of states in Fig. 2.
Algorithm 1 Time-efficient Network Energy Calculation

Input: \( G_0 = (V, \mathcal{E}_0), \alpha, \rho_0, r_0, \beta, \beta_a, \kappa, \) and \( \delta. \)
Output: \( E_{\text{min}}(G). \)

Initialization: \( \forall (i, j), A_{i,j}(0) \in \mathcal{E}_0 = -1, \) and \( c = 1. \)
1: Calculate \( E(G_0) \) using (6).
2: \( c \leftarrow E(G_0). \)
3: for \( t \leftarrow \Delta t \) to \( T \cdot \Delta t \) do
4: Randomly select an edge \( (i, j) \in \mathcal{E}_{t-\Delta t}. \)
5: Change the state of edge \( (i, j) \) according to Fig. 4
6: Update \( E(G_t) \) using (9), (11), and (12).
7: if \( E(G_t) = e \) then
8: \( c \leftarrow E(G_t) \) with probability 0.5.
9: else if \( E(G_t) < e \) then
10: \( c \leftarrow E(G_t) \)
11: return \( E_{\text{min}}(G) \)

\[ \{-1, 0, 1\}. \] Denoted by \( s_\infty = \sum_{i=1}^n S_i(\infty)/|V|, \) the fraction of susceptible users is computed as:
\[ s_\infty = \pi_1,1,1 + \pi_1,1,0 + \pi_1,1,-1 + \pi_1,0,1 + \pi_1,0,0 + \pi_1,0,-1 + \pi_1,-1,1 + \pi_1,-1,0 + \pi_1,-1,-1 \]
\[ = \sum_{y,z} \pi_{1,y,z}. \]

Similarly, the steady-state probabilities for \( \rho_\infty, a_\infty, \) and \( r_\infty \) can be obtained straightforwardly.

4 Monte Carlo Method

Starting from an initial network state at \( t = 0, \) where \( \rho_0 \) fraction of users are randomly infected, we select an edge \( (i, j) \) at random in each evolution step of the simulation and change its state according to Fig. 4. Doing so affects the energy states of all triads that share the edge \( (i, j) \) in the long-term which successively, alters the total network energy state. Convergence towards the new network structure transpires as long as the new energy state decreases in each time step \( t \). To expedite the computation, we instead propose Algorithm 1 that evaluates the energy difference of the selected edge \( (i, j) \) between consecutive time steps, \( t' \) and \( t'' \) \((t'' = t' + \Delta t) \), in \( O(1) \) time as below:
\[ \Delta E(i, j) = \alpha \cdot \Delta E_\Delta(i, j) + (1 - \alpha) \cdot \Delta E_p(i, j), \quad (9) \]
where \( \Delta E_\Delta(i, j) \) is the difference in the triad energy, i.e.,
\[ \Delta E_\Delta(i,j) = \frac{1}{(\gamma_1) \gamma_2} \sum_{j'k'} \left( E_{\Delta,j',k'}^i(t'') - E_{\Delta,j',k'}^i(t') \right) \]
\[ = \frac{1}{(\gamma_1)^2} \left( A_{i,j}(t'') - A_{i,j}(t') \right) \sum_{k' \neq i,j} A_{i,k}(t') \cdot A_{j,k}(t'). \quad (10) \]

If the state transition does not flip the edge sign, then \( E_\Delta(G_t) \) remains unaltered. Otherwise, flipping the edge sign implies that \( A_{i,j}(t'') = -A_{i,j}(t') \), which further reduces (10) to:
\[ \Delta E_\Delta(i, j) = \begin{cases} 
-2A_{i,j}(t'') \sum_{k' \neq i,j} A_{i,k}(t') \cdot A_{j,k}(t'); & \text{if } A_{i,j}(t'') = -A_{i,j}(t'), \\
0; & \text{if } A_{i,j}(t'') = A_{i,j}(t').
\end{cases} \quad (11) \]

5 Simulation Results and Discussions

In this section, we evaluate the proposed energy framework with respect to \( r_0, \rho_0, \) and \( \alpha. \) For Case Study I, we generate a complete network of \(|V| = 180|V| \) users to entail the maximum number of triads in our analysis. We then employ the publicly available Slashdot0811106 (SL, \(|V| = 200, |E| = 870\) [16], Bitcoin-OTC (BTC, \(|V| = 161, |E| = 547\) [16], and US Congress co-sponsorship (CS, \(|V| = 100, |E| = 3960\) [17] datasets in Case Study II. To ensure better prediction, all Monte Carlo simulation results are averaged over 200 runs on a PC with 3.2 GHz Intel Core i9-9900 CPU and 16 GB memory.

5.1 Case Study I: Results

For the synthetic network, Fig. 5 plots the trajectories for a rumor outbreak under varying \((\beta, \kappa)\) values with \( r_0 = 0.25, \rho_0 = 0.15, \alpha = 0.5, \beta_a = 0.3 \beta, \) and \( \delta = 9. \) As depicted in Fig. 5, unlike the SIS baseline \((\kappa = 0)\), the susceptible fraction under the SAIS rumor model drops to zero with rate proportional to \( \kappa \). As a result, the users are either influenced by the rumor or alerted thus making them less likely to fall prey in the long run since \( \beta_a \ll \beta \). The impact of \( \kappa \) on the alerted and infected user densities are, respectively, shown in Fig. 5 and Fig. 5C. For higher values of \( \kappa \), a larger susceptible fraction is made aware of the rumor which, in turn, diminishes the size of the infected population. For instance, in contrast to the baseline, Fig. 5C vividly shows that the infected cluster size decreases by nearly 14% when \( \kappa = 2. \) In spite of setting \( \beta = 0, \) note that there exists a non-zero infected population in the network. This clearly implies that for smaller values of \( r_0, \) the proposed model partitions the network into two clusters: one comprising of alerted users and the other containing infected
users. While the users within each cluster maintain a friendly relationship with each other, they are hostile towards users in the other cluster. However, the rumor virality gradually dies out with rise in $r_0$ and the two clusters eventually merge into a single cluster of alert users. The impact of $\kappa$ on the number of friendly links is shown in Fig. 8a. Driven by the transitions given in Fig. 4, such behavior is not far from expectation as users are inclined to detach from friends influenced by the rumor and instead, befriend those who are informed or share common views on the rumor to attain social stability.

Fig. 6 shows how the control parameter $\alpha$ arbitrates the rumor spread and social tension trade-off for varying $r_0$ values in steady-state. In Fig. 6a, we observe that the system gravitates towards the jammed states ($E(G) < -1$), where mitigating the rumor spread is favored over attaining structural balance, for lower ($\alpha, r_0$) values. As $\alpha$ and $r_0$ increase beyond 0.5 however, it is evident in Fig. 6b that the network tends towards the global minimum energy state to become structurally robust at the expense of further rumor diffusion. Interestingly, due to the reversible nature of the SAIS rumor model, the fully balanced complete network progresses to be rumor-free at $\alpha = r_0 = 1$ as all users eventually become aware of the rumor. The network therefore, tends to exploit the negative links to naturally immunize the susceptible users by separating them from the cluster of rumor-infected users. Therefore, for any given setting, there exists an optimal ($\alpha, r_0$) pair for which the network would contain minimum number of rumor-infected users in steady-state and yet, not necessarily exhibit a socially balanced structure.

The significance of $\rho_0$ in the coupled evolution is shown in Fig. 7. For $0 \leq \rho_0, r_0 \leq 0.5$, Fig. 7a showcases the impact of negative links in controlling the rumor. Further increase in $\rho_0$ however, yields a fixed fraction of infected users as most of the triads have evolved into a balanced state. Full recovery is attained when $\rho_0 \leq 0.5$ and $r_0 > 0.5$ due to the small infection prevalence and the low infection rate relative to $\kappa$ and $\delta$. We also observe that for low $\rho_0$, the steady-state infection density ($\rho_\infty$) decreases to zero, irrespective of $r_0$. The rumor diffuses at its maximum when $\rho_0$ and $r_0$ are both high. Fig. 7a shows the trivial effect of $\rho_0$ on the net network energy for high $r_0$, where the triad structural energy is dominant. Around $r_0 = 0.5$, the network struggles to become balanced as flipping the edge sign conceivably creates more unbalanced triads as compared to other values on the $r_0$ spectrum.

Fig. 8 shows the $E_p(G)$ and $E_\Delta(G)$ functions against $r_0$ for different ($\beta, \kappa$) values in steady-state. As seen in Fig. 8a, the minimum achievable pairwise and triad energy values increase with $\kappa$ for $r_0 < 0.6$, whereas the $\rho_\infty$ inversely goes down. But for larger $r_0$, the impact of $\beta$ and $\kappa$ on the energy and $\rho_\infty$ is minimal. That is to say, $E_p(G)$ in Fig. 8a increases to zero as the number of infected users becoming aware rapidly grows with increase in friendly links. Contrarily, $E_\Delta(G)$ in Fig. 8b falls to the global energy minimum due to the fact that the triads gradually evolve to become socially stable as most users have already formed friendly relationships.

### 5.2 Case Study II: Results

We now assess our model using the sparsely-connected SL ($\approx 4.37\%$), BC ($\approx 4.25\%$), and the dense CS ($\approx 74.67\%$) SN datasets. The total number of triads existing in SL, BC, and CS are 1927, 569, and 74140, respectively.

Fig. 9 compares the steady-state experimental results of the datasets with respect to $\beta$ and $\kappa$. Consistent with Fig. 6a and Fig. 7a, we see in Fig. 9a that $\rho_\infty$ reduces with increase in awareness for $\rho_0 = 0.15$, while it remains high and almost the same for $\rho_0 = 0.75$. As a resultant, the density of alerted users increases with $\kappa$ as shown in Fig. 9b. It is noteworthy that, irrespective of $\rho_0$, the gap in $\omega_{cs}$ between CS and the other two datasets gradually decreases. Despite the fewer nodes in CS, the plots reveal the profound impact of $\kappa$ on the nodal states of densely connected CS that contains more number of triads. Summing up the infected and alerted fractions for each ($\beta, \kappa$) pair also justifies the natural immunization induced by the formation of two-cluster networks. Changes in the rates of $\beta$ and $\kappa$ however, do not seem to influence the fraction of friendly links ($r_\infty$) in steady-state. Though the epidemic state of nodes are decisive in link sign evolution,

1. In general, the sharp upper bound on the number of triads in a graph with $n$ nodes and $m$ edges is $\frac{m}{3}(2n - n + 1)^{3/2}$.
we see that for all scenarios the ultimate number of friendly ties is the same. Upon reaching steady-state, the steady-state fraction of balanced triads, denoted by \(|\Delta B|\), is close to 100% in Fig. 9d, which indicates a balanced network structure in accordance with Heider’s balance theory. Furthermore, the results empirically show that although the structural energy of triads approach a global minimum, \(E(\mathcal{G})\) does not reach the global energy minimum state. The reason is because the pairwise energy tends towards a local energy minimum state when \(\alpha = 0.5\). Also, since \(\kappa\) affects the pairwise spreading energy, \(E(\mathcal{G})\) is slightly lower for higher \(\kappa\) rates as plotted in Fig. 9c. Comparing the trends in both rows of Fig. 9, the results for \(r_\infty\) and \(|\Delta B|\) are almost alike. With increase in \(\kappa\), \(E(\mathcal{G})\) of CS is higher by roughly 28% for \(\rho_0 = 0.15\) because of the larger population of alerted users being intrinsically immunized as compared to the setting with \(\rho_0 = 0.75\), where the impact of increasing \(\kappa\) is relatively inconsequential.

6 CONCLUSION

In this paper, coupled dynamics of the SAIS rumor diffusion model and the structural evolution of signed SNs was studied. Inspired by Heider’s balance theory, a network energy framework was formulated to capture rumor spreading via pairwise user interactions in conjunction with social stability in triad configurations. The superiority of incorporating user awareness in the classical SIS model was fully validated by the Monte Carlo simulation results. Moreover, it was shown that a complete network splits into two clusters of alerted and infected users upon reaching a local energy minimum. The alerted cluster density was found to grow with increase in the initial number of friendly links and a fully balanced network becomes rumor-free only when the triad energy is considered and all initial user relationships are friendly.

One interesting future direction of this work is to leverage network centrality measures other than user degree distribution in probing the trade-off between opposing rumors and social stability in single and composite signed networks.

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