Single-Image Geometric-Flow X-Ray Speckle Tracking

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We develop a speckle-tracking method for x-ray phase contrast imaging, based on the concept of geometric flow. This flow is a conserved current associated with deformation of illuminating x-ray speckles induced by passage through a sample. The method provides a rapid, efficient and accurate algorithm for quantitative phase imaging. It is highly photon efficient and able to image dynamic objects, since a single radiograph of the sample is sufficient for the phase recovery. We experimentally quantify the resolution and contrast of the approach with both two-dimensional and three-dimensional phase imaging applications using x-ray synchrotron radiation. Finally, we discuss adaptations of the method to imaging with compact x-ray sources that have a large source size and significant spectral bandwidth.

I. INTRODUCTION

Increasing contrast in x-ray imaging is of fundamental importance in many scientific fields such as material sciences, cultural heritage and medical imaging. X-ray Phase Contrast Imaging (XPCI) [1] significantly increases the contrast between materials and tissues of very close composition, as for instance in distinguishing tumorous from healthy biological soft tissue [2, 3]. While conventional x-ray imaging is based on the local attenuation of the photon beam, XPCI is sensitive to the real part of the complex optical refractive index of a material, which is responsible for light refraction. Despite significant advances in the field, XPCI remains costly at large scale facilities and very challenging on conventional x-ray sources since these latter experimental systems require high stability and precision optics [4].

Near-field speckle based x-ray phase imaging techniques appeared recently, and were quickly demonstrated attractive with respect to other XPCI techniques, on account of their simplicity of experimental implementation [5, 6]. See Zdora [7] for a comprehensive recent review. Unlike other modulation-based methods, a simple object with small random features is necessary to generate the modulating speckle observable in the bright field of an x-ray beam. In the Fresnel regime of hard x-rays, speckles resemble in size the object generating them over large distances, and such “near field” speckles can be tracked between different images taken at different points in time or locations in space. Furthermore, spatially random intensity modulation can be generated either from interference effects of the light scattered from a speckle mask containing small randomly distributed grains—the resulting intensity structures are in that case true fully-developed speckle—or by absorption contrast from a mask with randomly distributed attenuating apertures. Such features have made speckle-based methods readily compatible with low coherence sources, enabling them to rapidly spread beyond synchrotrons, to be demonstrated applicable with laboratory sources [8]. In parallel, scientists have developed various speckle-tracking processing methods, usually requiring several acquisitions, to optimize the technique’s sensitivity and resolution whilst also accessing the so-called dark-field signal [9].

In spite of their various advantages and successful applications, the requirement for several sample exposures to achieve high resolution raises strong challenges on the sample and speckle mask positioning reproducibility, which are for instance beyond what is acceptable when imaging living patients or dynamic samples.

Herein, we propose to solve this drawback using a speckle-tracking approach that is based on optical energy conservation and geometric flow. With respect to other speckle-tracking phase contrast techniques, the method described here intrinsically senses both lens and derivative terms of the phase. These simultaneously accounted-for terms are associated with propagation-induced phase contrast and differential phase contrast, respectively. The implementation of the method is fast, robust and extremely efficient. These attractive aspects are experimentally illustrated using reconstructions from data collected at an x-ray synchrotron. Finally, we discuss future
by the presence of strong intensity gradients such as are provided by an illuminating speckle field. To this prism term is added a “lensing term” \( I_R(x,y) \nabla_\perp \cdot D_\perp(x,y) \) corresponding to local concentration or rarefaction of intensity. Modelling the intensity flow via Eq. 1 takes both effects into account, implying the method developed below to simultaneously utilise both differential phase contrast and propagation-based phase contrast.

Following Teague [10] and Paganin and Nugent [11, 12], assume that the flow has a transverse current density \( I_R(x,y) \nabla_\perp \cdot D_\perp(x,y) \) that may be written as the gradient of a scalar auxiliary function \( \Lambda(x,y) \):

\[
I_R(x,y) \nabla_\perp \cdot D_\perp(x,y) \approx \nabla_\perp \Lambda(x,y). \tag{2}
\]

This approximation amounts to neglecting the curl of a vector potential which would otherwise need to be added to the right side of the above equation, in the Helmholtz decomposition of the vector field on the left. Stated differently, we have assumed the previously-mentioned geometric flow to be a gradient flow. Physically, this amounts to the assumption that the angular-momentum density of the flow is much smaller in magnitude than the linear-momentum density (cf. Schmaltz et al. [12]).

Our auxiliary function transforms Eq. 1 into:

\[
I_S(x,y) - I_R(x,y) = \nabla_\perp^2 \Lambda(x,y). \tag{3}
\]

Since the left side is known from measurement data, one may solve this Poisson equation using a variety of numerical methods (e.g. multigrid methods, relaxation methods etc.). Appropriate boundary conditions may be either measured or known a priori. For example, if the object is entirely immersed within the field of view of the illuminating speckle field, zero Dirichlet boundary conditions (and/or zero Neumann boundary conditions) may be assumed. While we implicitly assume this case below, such an assumption is easily relaxed.

We now give a method for solving Eq. 1 which has some parallels with that of Paganin and Nugent [11, 13] in a different context. Fourier transform Eq. 3 with respect to \( x \) and \( y \), then use the Fourier derivative theorem:

\[
\mathcal{F}[I_S(x,y) - I_R(x,y)] = -(k_x^2 + k_y^2)\mathcal{F}[\Lambda(x,y)]. \tag{4}
\]

Here, \( \mathcal{F} \) denotes Fourier transformation with respect to \( x \) and \( y \), \((k_x, k_y)\) are the corresponding Fourier coordinates, and we have used the Fourier-transform convention from Paganin [13]. Solving for \( \Lambda(x,y) \) then gives:

\[
\Lambda(x,y) = \mathcal{F}^{-1} \left\{ \frac{\mathcal{F}[I_R(x,y) - I_S(x,y)]}{k_x^2 + k_y^2} \right\}. \tag{5}
\]

The division-by-zero Fourier-space singularity, which corresponds to an irrelevant constant offset in the auxiliary function, implies that the point \((k_x, k_y) = (0,0)\) should be excluded from the above expression. This amounts to taking the Cauchy principal value of the integrals which must be performed in evaluating the above expression.

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**FIG. 1.** Generic setup for x-ray speckle tracking development of the method, whose potential adaptation to laboratory x-ray sources is highly promising.

### II. THEORY

Consider Fig. 1 where paraxial forward-propagating x-rays illuminate a speckle mask, before passing through a thin object and then traversing a distance \( Z \) to the planar surface of a position-sensitive detector. Let \( I_R(x,y) \) be the reference speckle, i.e. the image taken in the absence of the object, where \((x,y)\) are Cartesian coordinates in the plane perpendicular to the optic axis \( z \). The image in the presence of the sample is denoted by \( I_S(x,y) \). Assume the sample to be a thin perfectly x-ray-transparent object, whose presence geometrically distorts the reference x-ray speckles. Assume this geometric distortion to conserve the integrated intensity of the reference speckle image, both locally and globally. Hence one can describe the deformation of \( I_R(x,y) \) into \( I_S(x,y) \) as a geometric flow with a conserved current, namely a transverse flow of intensity that obeys the continuity equation. This flow corresponds to the continuous warping of the intensity distribution in the \( z = Z > 0 \) plane perpendicular to the optic axis, effected by continuously evolving the speckled intensity distribution over the plane \( z = Z \) in absence of the object, into the intensity distribution over the plane \( z = Z \) in the presence of the object.

This permits us to write

\[
I_S(x,y) - I_R(x,y) \approx \nabla_\perp \cdot [I_R(x,y)D_\perp(x,y)] \tag{1},
\]

where \( \nabla_\perp \) denotes the gradient operator in the \((x,y)\) plane, and \( D_\perp(x,y) = (D_x(x,y),D_y(x,y)) \) is the displacement field that distorts each feature in the reference image \( I_R(x,y) \) into the corresponding feature in the image \( I_S(x,y) \) taken in the presence of the sample. Note that expansion of the right side of Eq. 1 gives a “prism term” \( \nabla_\perp I_R(x,y) \cdot D_\perp(x,y) \) corresponding to the transverse motion of speckles in each of two orthogonal directions, in a contribution to the signal which is amplified...
Numerically, one simply omits the zero-frequency pixel in Fourier space, from the domain of integration.

The Fourier derivative theorem implies that the transverse gradient operator may be written as [13]:

\[ \nabla_\perp = i\mathcal{F}^{-1}(k_x, k_y)\mathcal{F}, \]

where all operators act from right to left. Apply \( \nabla_\perp \) to both sides of Eq. [5] then use Eq. [6] on the right-hand side and Eq. [2] on the left-hand side; finally, divide both sides of the resulting expression by \( I_R(x, y) > 0 \), to give:

\[ \mathbf{D}_\perp(x, y) = \frac{i}{I_R(x, y)} \mathcal{F}^{-1}\left((k_x, k_y) \frac{\mathcal{F}[I_R(x, y) - I_S(x, y)]}{k_x^2 + k_y^2}\right). \]

In analogy with the concept of a velocity potential introduced by Lagrange into potential flow theory for classical irrotational fluids, assume the velocity field of the flow—in the mapping deforming \( I_R(x, y) \) into \( I_S(x, y) \)—to be irrotational. This allows us to write the displacement field as the gradient of a scalar potential \( d(x, y) \):\]

\[ \mathbf{D}_\perp(x, y) = (D_x(x, y), D_y(x, y)) \approx \nabla_\perp d(x, y). \]

We can obtain \( d(x, y) \) from \( \mathbf{D}_\perp(x, y) \) using the Fourier transform based algorithm [14, 15]:

\[ d(x, y) = \mathcal{F}^{-1}\left\{ \frac{\mathcal{F}[D_x(x, y) + iD_y(x, y)]}{ik_x - k_y}\right\}. \]

Thus far we have not utilised any particular assumptions related to optical imaging, implying significant generality in the preceding development. The physical reason underpinning this generality is the fact that the local flows we consider are conserved currents embodying a local conservation principle (via the continuity equation), a setting that is far more generally applicable than a particular differential equation governing a particular non-dissipative conserved field. Notwithstanding the desirability of the level of generality with which we have hitherto worked, we now utilise some assumptions pertinent to speckle-tracking using hard x-rays. We restrict attention to this case for the remainder of the paper.

For paraxial quasi-monomochromatic complex scalar x-ray radiation with wavelength \( \lambda = 2\pi/k \), the geometry of Fig. 1 implies that \( d(x, y) \) is related to the transverse phase shift \( \phi(x, y) \) which the thin non-absorbing sample imparts upon the transmitted x-ray beam:

\[ d(x, y) = \frac{Z}{k} \phi(x, y). \]

The transverse gradient of the above expression gives:

\[ \nabla_\perp d(x, y) = \frac{Z}{k} \nabla_\perp \phi(x, y) = Z(\alpha_x(x, y), \alpha_y(x, y)), \]

where \( (\alpha_x(x, y), \alpha_y(x, y)) \) are the deflection angles that the object imparts on the traversing x-ray beam, in the \( x \) and \( y \) directions, respectively.

Equations [10] and [11] allow the preceding very general derivation, for reconstructing the scalar potential \( d(x, y) \) and the displacement field \( \mathbf{D}_\perp(x, y) \), to be converted into an algorithm for reconstructing the phase shift \( \phi(x, y) \) of the object, together with the associated deflection angles \( (\alpha_x(x, y), \alpha_y(x, y)) \). The formula for reconstructing the phase shift \( \phi(x, y) \) is:

\[ \phi(x, y) = \frac{k}{Z} \mathcal{F}^{-1}\left\{ \frac{\mathcal{F}[\hat{x} + i\hat{y}] \cdot \mathbf{D}_\perp(x, y)}{ik_x - k_y}\right\}, \]

where \( \hat{x} \) and \( \hat{y} \) are unit vectors in the \( x \) and \( y \) directions, respectively, and \( \mathbf{D}_\perp(x, y) \) is given by Eq. [7]. While the reconstruction of phase shifts is strictly only meaningful for monochromatic radiation, this requirement may be considerably relaxed when one instead reconstructs deflection angles. These angles \( (\alpha_x(x, y), \alpha_y(x, y)) \) may be obtained directly from Eq. [7] via:

\[ (\alpha_x(x, y), \alpha_y(x, y)) = \frac{\mathbf{D}_\perp(x, y)}{Z}. \]

Note that the magnitude of the Fourier-space filter, in the denominator of equations [9] and [12] is the inverse of the Ramachandran–Lakshminarayanan (“Ram–Lak”) filter. Hence if our speckle-tracking method is combined with tomography, the filter in braces in equations [9] or [12] will be multiplied by \( (k_x^2 + k_y^2)^{1/2} \). The fact that \( (k_x^2 + k_y^2)^{1/2}/(ik_x - k_y) = \exp[i\Phi(k_x, k_y)] \) where \( \Phi(k_x, k_y) \) is a polar angle in Fourier space, implies that in speckle-tracking tomography using our method, one can simply omit the Ram–Lak filter and replace division by \( (ik_x - k_y) \) with multiplication by the vortical unit modulus function \( \exp[i\Phi(k_x, k_y)] \). This makes the x-ray speckle-tracking tomography much more local, and more stable, than filtered-backprojection. Note also that the assumption that \( \mathbf{D}_\perp(x, y) \) be irrotational (see Eq. [8]) is exact if \( \phi(x, y) \) is continuous and single-valued, a condition that is guaranteed if the projection approximation [14] is valid and the sample’s refractive index is continuous.

III. MATERIALS AND METHODS

A. Synchrotron x-ray experiments

Two experiments were conducted at the ID17 Biomedical beamline of the European Synchrotron (ESRF), demonstrating applications of the method.

In the first experiment, a double Si(111) crystal system in Laue–Laue configuration was used to select a quasi-monomochromatic beam with an energy spread \( \epsilon \approx 10^{-4} \) at the energy \( E = 52 \) keV. Collimation was defined by the natural divergence of the 21-pole wiggler synchrotron source which is about < 1 mrad horizontally and < 0.1 mrad vertically. The geometrical configuration of the experiment resembles Fig. 1 in concept. The x-ray photons
FIG. 2. Geometric-flow x-ray speckle tracking applied for 2D radiography of 150 µm diameter nylon wires. Refraction angles measured in the (a) $x$-plane ($\alpha_x(x, y)$) and (b) $y$-plane ($\alpha_y(x, y)$). (c) Integrated phase $\phi(x, y)$. (d), (e) and (f) are profile plots along the segments marked in yellow in (a), (b) and (c) respectively.

first passed through the speckle-generator membrane before traversing the sample located 900 mm away and finally impinging onto the detector placed $Z = 12$ m further downstream. An indirect detection system was used, consisting of a scientific CMOS coupled to magnifying optics to image a scintillator screen made of a gadolinium oxysulfide sheet of 60 µm thickness. The resulting pixel size was of $\simeq 6$ µm. The images obtained during this experiment, of a home-made phantom, are shown in Fig. 2. The phantom was composed of several orthogonally crossed nylon wires with a 150 µm diameter.

To study the potential of this speckle-based methodology with a polychromatic beam, a second experiment was carried out using a white synchrotron beam filtered with 0.5 mm of Al and 0.35 mm of Cu. The resulting beam spectrum (see Fig. 4(a) of the supplementary material) corresponds to a pink beam with a peak centred at $E=37.3$ keV and a spectral bandwidth of approximately 20 keV ($\Delta E/E \simeq 0.5$). For this experiment, the detector system was the same scientific CMOS camera as before but coupled to a different magnifying optics which provided a resulting pixel size of $\simeq 3$ µm. The sample was a domestic cicada dried under natural conditions for 10 months after its natural death. The setup configuration was equivalent to the one employed to obtain the images presented in Fig. 2 but the sample-to-detector and the membrane-to-sample distances were set to 4 m and 1 m respectively. The tomography data consisted of 3000 projections collected periodically during a 360 degree scan of the sample. The centre of rotation was transversely off-centred by 200 pixels to operate a so-called half acquisition tomography scan. Such common procedure allows one to extend the 3D field of view. The phase images were calculated for each projections using equations 7 and 12. The 3D CT reconstruction was performed using the back projection implementation described in Mirone et al. 17 and the results are presented in Fig. 3.

B. Numerical implementation

Our algorithm was implemented in Python 3 and is now available under the GNU General Public Licence at https://github.com/labrieth/spytlab/. The code is not optimized in terms of computation time but provides a readable and understandable implementation of the method. Raw experimental data are provided as supplementary material and can be downloaded at https://github.com/labrieth/spytlab/.

IV. RESULTS

A. Radiography

The geometric-flow method for x-ray speckle tracking was first applied in a 2D radiography mode to the nylon-wire phantom. The total data consist of one reference speckle image $I_R(x, y)$ in the absence of the sample, and one speckle image $I_S(x, y)$ in the presence of the phantom. Figure 2 shows 2D maps of the recovered refraction angles (a) $\alpha_x(x, y)$ and (b) $\alpha_y(x, y)$, together with (c) the recovered phase shifts $\phi(x, y)$.

The figure highlights the method quantitativeness for the recovery of a phase shift. Such aspect allows one to extract the various indexes of refraction composing a sample when using a monochromatic beam, and enable a better distinction of the different materials. The profile plots of the figure present a high signal to noise ratio and the standard deviation of the reconstruction error in a region with no sample is below 100 nrad. Such value underlines the good stability of the method with respect to noise. Besides, no blurring effect that occurs with most speckle tracking techniques is observed here [18]. In short, the combined high sensitivity and high resolution of the method permits to image with high-fidelity the sample-induced phase shift from a single sample ex-
to manufacture), no resolution limitation and finally the requirements on the beam coherence are low. It is also radiation dose efficient because no absorbing element is used between the sample and the detector meaning that all photons passing through the sample eventually contribute to the image formation. To summarize, the experimental complexity of PCI is translated in x-ray speckle tracking to only the numerical processing side. Up to now the primary limitations of speckle tracking were that the technique necessitates a large number of sample exposures and time-consuming computer algorithms to achieve a high resolution and sensitivity. The method implemented in this work overcomes these limitations as we could show quantitative results using only a single sample exposure.

The method only requires two fast Fourier transforms (FFTs) per projection in order to reconstruct the displacement field \( D(x,y) \) per projection if both \( D(x,y) \) and the phase \( \phi(x,y) \) are required (using equations 7 and 12). Again, only one image per projection is needed, once the reference image \( I_R(x,y) \) in the absence of the sample has been taken. The method is therefore fast with respect to previously available speckle tracking methods and many other phase sensitive approaches. It is comparable in terms of calculating resources and speed to the widely used propagation-based phase contrast method of Paganin et al., which is now optimized for GPU calculations. Although the long object-to-detector propagation distances of a few metres are difficult to access at other facilities, our results show that in a single image the method is already competitive with other scanning techniques that require multiple images. The photon energy used for this experiment also demonstrates the potential of the approach for a range of imaging applications where penetrating x-rays or reduced deposited dose are strongly required. The computationally simple reconstruction method of the present paper may be used as a first iterate which is subsequently refined, e.g. if the distortions due to the object are strong, and/or if there is some Fresnel fringing. A variety of methods for such iterative refinement exist, such as those based on conjugate gradients, genetic algorithms, simulated annealing, machine learning and neural networks.

Given its successful application to data obtained using a broad-band polychromatic beam with energy spread \( \Delta E \) due to the absorption implied by the use of optical elements.

X-ray near-field speckle tracking is a novel, recently developed XPCI technique sensitive to the first derivative of the phase \( \partial \phi / \partial z \). The technique set-up besides its simplicity of implementation has the main advantages of having no field-of-view limitation (large diffusers are easy

B. Computed tomography

In a second experiment, we used geometric-flow x-ray speckle tracking to perform phase contrast tomography of a cicada using a polychromatic x-ray beam. A calculation of the broad x-ray energy spectrum as well as a speckle reference field and a profile cut are provided in the supplementary Fig. 4. Figure 3 shows (a) a differential phase gradient projection, (b) a computed tomography (CT) slice with (c) an inset in the compound-eye region and (d) a 3D volume rendering. In an attempt to probe the limitations of the method, the dataset consisted of only one pair of speckle images \( I_R(x,y) \) and \( I_S(x,y) \) for each projection. In practice, \( I_R(x,y) \) needs to be measured only once at the beginning or end of the scan and a single image \( I_S(x,y) \) is taken for each projection. Despite the reduced total exposure of the sample to the x-rays, the method successfully renders the phase gradient images of the cicada head with a limited amount of noise. This noise eventually cancels out in the tomographic reconstruction as one can observe in the reconstructed phase-shift slice (cf. our earlier comment regarding the reduction of the Ram–Lak filter and the associated stability for tomography). A zoom in to the cicada eye region in (c) illustrates the high resolution of the image and the subtle details obtained. Thus, the 3D volume reconstruction offers a full visualization of the cicada anatomy at high resolution (antennas, compound eyes, clipesus and the labrum) which was made possible by the method despite the low absorption of the sample whose content in water is low.

V. DISCUSSION AND CONCLUSION

XPCI emerged two and a half decades ago and later demonstrated its ability to provide an unprecedented sensitivity for all types of material including biological tissues. Yet, numerous challenges remain on the path for phase contrast imaging to become a unique non-invasive tool with laboratory set-ups. Indeed, although the methods currently available at laboratories usually feature a good sensitivity, these performances are balanced by the need of complex experimental set-ups and/or high stability and high precision optics. Recent developments in the field permitted to lift technological barriers but many key issues still need to be addressed. For instance, the current methods remain incompatible with CT in term of dose due to the absorption implied by the use of optical elements.

X-ray near-field speckle tracking is a novel, recently developed XPCI technique sensitive to the first derivative of the phase \( \partial \phi / \partial z \). The technique set-up besides its simplicity of implementation has the main advantages of having no field-of-view limitation (large diffusers are easy

posture.

The method only requires two fast Fourier transforms (FFTs) per projection in order to reconstruct the displacement field \( D(x,y) \) via Eq. 7 or four FFTs per projection if both \( D(x,y) \) and the phase \( \phi(x,y) \) are required (using equations 7 and 12). Again, only one image per projection is needed, once the reference image \( I_R(x,y) \) in the absence of the sample has been taken. The method is therefore fast with respect to previously available speckle tracking methods and many other phase sensitive approaches. It is comparable in terms of calculating resources and speed to the widely used propagation-based phase contrast method of Paganin et al., which is now optimized for GPU calculations. Although the long object-to-detector propagation distances of a few metres are difficult to access at other facilities, our results show that in a single image the method is already competitive with other scanning techniques that require multiple images. The photon energy used for this experiment also demonstrates the potential of the approach for a range of imaging applications where penetrating x-rays or reduced deposited dose are strongly required. The computationally simple reconstruction method of the present paper may be used as a first iterate which is subsequently refined, e.g. if the distortions due to the object are strong, and/or if there is some Fresnel fringing. A variety of methods for such iterative refinement exist, such as those based on conjugate gradients, genetic algorithms, simulated annealing, machine learning and neural networks.

Given its successful application to data obtained using a broad-band polychromatic beam with energy spread \( \Delta E \), it is evident that the coherence requirements for the method are rather weak. The method nowhere relies on interferometric phase contrast, but rather only on the transverse redistribution of optical energy on account of the geometric distortion of the speckle field that is induced by its passage through an object. While wavefield phase is referred to in several of the above equations, this always ultimately appears in the form \( \nabla \phi / k \), which we know to be a ray deflection angle, from Eq. 11. This emphasizes that it is geometric optics and geometric distortion of optical rays that underpin the present formalism, which nowhere relies on interference and which
therefore—as already mentioned—has limited coherence requirements. It would be interesting in future work to investigate the minimum coherence requirements for the method, with a view to seeing its application to a wide variety of low-brilliance sources.

Speed, simplicity and breadth of applicability are attractive features of our speckle-tracking method based on the concept of geometric flow. Since the reference image $I_R(x, y)$ need only be measured once, the method can be easily applied to dynamic data, where $I_S(x, y, t)$ is a function of time $t$. Both the Lagrangian and Eulerian viewpoints for the flow field can be reconstructed. Another direction for future work would be to compare the relative efficacy of a random absorption mask, instead of true interference speckle, being used to generate the single reference image $I_R(x, y)$ required by the method. We conjecture the method to be relatively insensitive to the nature of the speckle that constitutes $I_R(x, y)$, if this intensity distribution is highly structured spatially. Indeed, often one will have a mixture of both classes of speckle, especially at compact x-ray sources. Speckle methods based on an absorbing mask were demonstrated efficient \[19\]. Absorption masks are in greater need at high energies where the transverse coherence lengths are smaller and/or with sources of larger size. The idea of self-referencing objects, in which the flowing speckle is provided by the sample itself, would also be an interesting avenue for future research e.g. by tracking the speckles formed when sufficiently coherent x-rays pass through lung tissue. Finally, another avenue for future work could be to apply this approach to imaging at different length scales, e.g. for clinical imaging and industrial inspection.

In conclusion, our study shows that the use of a geometric-flow approach can address many limitations of existing speckle-based differential x-ray phase contrast techniques. This technique may be applied to lower-coherence sources such as laboratory set-ups, thereby spreading its impact beyond synchrotron sources.

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SUPPLEMENTARY MATERIAL

The supplementary figure shows characteristics of the beam used for the tomographic experiment. (a) presents a simulation of the beam spectrum and (c) displays the beam profile along the central line of the speckle image $I_R(x, y, t)$ shown in (b). The beam was polychromatic with a spectrum FWHM of approximately 50% demonstrating that monochromaticity is not a stringent condition of the method. Moreover the beam also shows a spatial energy spread, with higher energies at the centre of the beam. The reference image $I_R(x, y, t)$ features speckle of approximately 5 pixels in size, and a global visibility of $\sim 17\%$.

FIG. 4. Beam configuration used for the tomography experiment. (a) Energy spectrum of the x-rays. (b) One of the reference speckle fields used and (c) intensity profile in the middle of the frame as marked by the dashed line in (b).