Emergent reconfigurable mechanical metamaterial tessellations with an exponentially large number of discrete configurations

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HIGHLIGHTS
• Modular metamaterials are proposed with an exponentially large number of configurations.
• Each unit cell can switch into 4 patterns with multiple acoustic properties.
• Cellular metamaterial can be in situ set as wave filter, guide, lens and cloak.
• Extrinsic effect is reconceptualized as a design opportunity to form reconfigurable metamaterials.

GRAPHICAL ABSTRACT

ABSTRACT

Metamaterials are a class of engineered materials that often violate the routine assumptions that apply to ordinary materials. While metamaterials are typically assembled from carefully designed mesoscale units, the intended bulk-scale functionality can be obscured by unintended emergent effects driven by non-additive unit-unit interactions. These interactions are often sensitive to the number of units and their overall arrangement, making them extrinsic to the unit-scale design. As such, the emergence of extrinsic effects adds a significant hurdle for the development of general-purpose metamaterial technologies. Here, we reconceptualize bulk-scale extrinsic properties as a design opportunity and develop an approach that repurposes them in a new class of exponentially reconfigurable origami-inspired mechanical metamaterials. We illustrate the use of extrinsic properties to design a single general-purpose structure that can be transformed into a variety of passive mechanical devices including a waveguide, a wave lens, and a wave cloak. Bench-top experiments validate the core concepts of our approach and show how unintentional extrinsic effects become useful for applications of reconfigurable metamaterials.

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1. Introduction

Mechanical metamaterials are a class of materials designed with a wide range of unusual responses to external forces [1–19]. Typically, they are built by modularly assembling mesoscale units with the use of geometry to control mechanical function. Whether through origami-like folding sheets [1,3,4,14,15,18–22] or selective removal of material [4,6,8,23–25], the geometry of each module is the principle driver of the unusual “negative” phenomena, such as negative Poisson’s ratio, negative thermal expansion, negative compressibility, and negative stiffness. These materials have been investigated for their potential application to sound control, biomedical devices, protective systems,
impact energy absorption, and wave attenuation [26,27]. For example, Zhu et al. [28] proposed 2D metamaterial tessellation based on kirigami units with subwavelength flexural wave manipulation for non-destructive evaluations and structural health monitoring. While the unit cell design was made by cutting and folding a thin metallic plate, this application highlights a key feature of metamaterial engineering that makes it highly attractive for industrial applications. Specifically, bulk-scale response functions of metamaterials are encoded into properties intrinsic to individual units such as geometric angles and lengths without making chemical or molecular modification.

In making this observation about the nature of metamaterial response functions being encoded into the unit cell’s properties, we used the term “intrinsic” in the way it is used in thermodynamics, where intrinsic properties depend on the material in question rather than the amount of material involved. The classic example often used to explain the concept of an intrinsic property is density, whereas its extrinsic equivalent is mass, which depends critically on the amount of material involved. The basic idea is to show that individual origami units, two half-units, and one full unit are identical. The two half-units may be arranged in two ways: (a) parallel with the [1,0] crease configuration, and (b) perpendicular to the [1,0] crease configuration. The half-units joined along edges HI and HF produce a full unit. Two DOFs \( \Theta_1, \Theta_2 \) are defined by \( \Theta_1 = 0^\circ - \Theta_2 \). Practically, the convenience offered by \( \Theta_1, \Theta_2 \) is to express a formulation that matches hands-on intuition for the PERMuTE unit’s physical behavior, especially regarding the motion of the flanges. Mathematically, this convenience means we use variables that are linearly coupled to one another according to Eq. 2. If we insert the contours defined by \( \Theta_1 = \Theta_2 = 360^\circ - \Theta_1 \) into the expressions for \( \Theta_1 \) and \( \Theta_2 \) we indeed confirm \( \Theta_1, \Theta_2 \) form an orthogonal basis. Of course, these geometric relations for the DOF become more complicated when material bending is introduced. This complication leads to a situation where the practical benefits and conceptual conveniences of \( \Theta_1, \Theta_2 \) ultimately outweigh the mathematical formulations of \( \Theta_1, \Theta_2 \).

Detailed analytic derivations supplementing Finite Element Method (FEM) simulations of unit-scale PERMuTE mechanical properties are

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**Fig. 1.** Design of a mechanical metamaterial unit exhibiting extrinsic properties when tessellated. (a) Geometry and folding pattern of a half-unit. Red and blue lines denote mountain and valley folding creases, respectively. Edge CD joins with CD’. (b) Half-unit assembled. (c) Two half-units join along edges HI and HF to create a full unit. Two DOFs \( \Theta_1, \Theta_2 \) quantify whether flanges are up (pink) or down (brown). Four distinct configurations are denoted by \([0,0]\), \([1,0]\), \([0,1]\), \([1,1]\). Dihedral angle \( \alpha \) along VAV is used for measuring transverse compression. (d) Energetics of deformation in the \( \Theta_1, \Theta_2 \) configuration space. Diagonal lines are accessible strictly by crease deformations. Off-diagonal region is accessible by bending along \( \mathbf{G} \) and \( \mathbf{G}’ \). (e) Energetics of crease deformation (diagonal lines from (d)) show the basic unit (inset) is tristable with the \([1,0]\) and \([0,1]\) configurations being identical curves. \( |\Theta_2| = |\Theta_1 - 180^\circ| \) measures deformations away from the degenerate point \( \Theta_1, \Theta_2 \). (f) Energetics of the same unit embedded in a \( 3 \times 3 \) tessellation are rather different. (g) On inspection, the embedded unit (inset) is now quadstable due to extrinsic unit-unit interactions. (h) Frequency-dependent band structure is sensitive to the unit’s configuration. A band gap in the \([1,1]\) configuration (highlighted gray) vanishes in the \([0,0]\) configuration. (i) DOS shows configuration-specific vibration transmission properties. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
Throughout the simulation and experimental analysis of single- and multi-unit structures, we select a variety of frequency ranges to highlight notable acoustic properties. While these frequency ranges are generally non-overlapping, the overall design strategy proposed here is insensitive to these differences because it prioritizes: (i) generating metamaterial structures that can be reconfigured, and (ii) performing a computational search on this “menu of configurations” to identify which configuration has desired metamaterial properties. Even though computational efficiency is a separate question beyond the scope of this work that relates to details of software implementation, we note the benefits of this design strategy are cumulative over time as the number of analyzed configurations (and therefore the size of the “menu”) increases. This strategy is generalizable and can be applied to any reconfigurable metamaterial structure, thereby bypassing the challenges of extrinsic properties that arise in any metamaterial pattern.

2.2. Simulated PERMuTE frequency filter mechanics

Vibrational dynamics for the $1 \times 3$ PERMuTE material (Figs. 2 and S3) were calculated in COMSOL 5.1 using a Normal-size free tetrahedral mesh with the geometry of each PERMuTE unit in variable configurations. We chose free boundary conditions to mimic the effect an applied force to an unconstrained structure, and fixed properties to mimic fibrous pulp materials. Specifically, we set the Poisson’s ratio $\nu = 0.3$, Young’s modulus $Y = 3.64$ GPa, mass density $\rho = 871$ kg/m$^3$, sheet thickness to 2 mm, and geometric parameters (SI text) $\ell_m = 15$ mm, $\ell_n = 17.5$ mm, and $\ell_q = 50$ mm. Mass dampering was introduced and set to 100 to avoid unphysical exponentially-growing strains at resonant frequencies. The ratio of input oscillation amplitude to output response amplitude is a function of this dampering and can be tuned accordingly across a wide range of physically-plausible values. The initial displacement field and velocity field were all zero. The boundary loading type for the input was set to “face excitation” (Figs. 2 and S3, input force applied at “*”). We use a time-dependent solver in COMSOL from 0 to 0.1 s, with a step size of $5 \times 10^{-4}$ s. The resulting amplitude is obtained from the “total displacement” value (Figs. 2 and S3, output measured at “**”). While time-dependent transient oscillations appear early on, they are damped out by $t = 0.05$ s, and FEM computations produce steady oscillations for the remainder of the simulation. Thus, in plots we show results from $0.05 \leq t \leq 0.10$.

2.3. Simulated PERMuTE $9 \times 9$ bulk mechanics

The PERMuTE $9 \times 9$ material (Figs. 3 and S4) was analyzed in COMSOL 5.1 using the same methods as the $1 \times 3$ PERMuTE frequency filter. However, these results were computed in the frequency domain with vibration deformations incorporated.

2.4. Experimental prototype fabrication

Prototype PERMuTE structures were fabricated using Strathmore 500 Series 3-ply Bristol card stock that was laser cut using a PERMuTE design pattern generated in Mathematica 10.2 (Fig. S5). To join edges for each unit’s assembly, additional card stock was mounted with super glue (LOCTITE 431) to the facets so that the crease mechanics were identical to folds elsewhere in the structure. To join units into a $3 \times 3$ tessellation, a long card stock strip was used along the perimeter.
The intersection of creases at vertices are often found to be mechanically complex due to the presence of material stretching. We therefore removed a small circular domain at each vertex to avoid such effects and allow for creasing- and bending-driven material properties to dominate the measured response.

2.5. Experimental compression measurements

In compression measurements, force was applied to various PERMuTE structures at a constant loading speed of 0.5 mm/s. Longitudinal force-displacement measurements were performed with the force applied only to the central PERMuTE unit in a $3 \times 3$ tessellation (Fig. 4a). Transverse force-displacement measurements were performed on an isolated PERMuTE unit (Fig. 4b) as well as a $3 \times 3$ structure (Fig. 4c). In transverse-compression of an isolated PERMuTE unit, the stress-free size of the unit was 49 mm across, and compression was increased until the unit was 34 mm. In all cases, experiments were repeated three consecutive times for each configuration and averaged. Error estimates reported in the main text (Figs. 4a–c) are the minimum and maximum values across all repeated measurements, demonstrating a high degree of reproducibility in bench-top PERMuTE prototypes. The experimentally accessible range for the folding angle $\phi$ was different between an isolated unit ($45^\circ \leq \phi \leq 75^\circ$) and the $3 \times 3$ tessellation ($30^\circ \leq \phi \leq 60^\circ$). When units are combined and inserted into the testing apparatus, $\phi$ decreases under compression from the structure’s own weight. Nevertheless, in both cases, we were still able to probe $\approx 30^\circ$ in compression. Simultaneous theoretical fits to all compression data were produced using an elastic model (Fig. 4b, lines; SI text; Fig. S9). Parameter values were extracted and found to be mutually self-consistent with each other.

2.6. Experimental frequency-sweep measurements

Frequency-dependent mechanical experiments were performed with an Arbitrary Waveform Generator (20 MHz function / Agilent 33220A) and vibrator used to apply transverse oscillations to a $3 \times 3$ PERMuTE structure in various configurations (Figs. 4d and e). Displacement responses were measured with an IL-065 laser displacement detector. Frequency sweeps from 1 to 1000 Hz were performed within 5 s for an input wave with an amplitude of 0.5 mm so that the full range of displacement was 1 mm. Displacement measurements were Fourier transformed and the results plotted in frequency-space.

3. Results and discussion

3.1. Emergence of extrinsic properties in the PERMuTE unit

We demonstrate our strategy for addressing unintended extrinsic metamaterial effects by first introducing a mesoscale unit comprised of a thin foldable material bonded at the edges (Figs. 1a–c) (SI text; Fig. S1). The unit’s design was created using standard origami and kirigami techniques previously developed for modular metamaterial construction. While seemingly similar to other origami and kirigami structures [3,22], we note this present design is different in a number of important respects including: (i) different symmetries, (ii) distinct topologies, (iii) different number and direction of creases, (iv) different number of degrees of freedom, (v) different 3D tessellation patterns, and (vi) different geometric compatibilities (see extended discussion in SI). This unit has two symmetric flanges that actuate using weakly-coupled degrees of freedom (Fig. 1c, $\theta_1$ and $\theta_2$; $\theta_1$ is the dihedral angle of facets BCFG and BCF’G’; $\theta_2$ defined similarly) (SI). Each flange has two extreme states resulting in four configurations: both flanges up (Fig. 1c, configuration [1], black square), both flanges down (Fig. 1c, configuration [0], black triangle), and a symmetric pair of configurations with one flange up and one flange down (Fig. 1c, [0,1] and [1,0], black diamond and circle). Because the current work is primarily interested in emergent extrinsic properties in tessellations of this unit, we will address unit-level mechanics insofar as it advances us toward a better understanding of the more general problem central to this work (see SI text for additional unit-level details).

In accordance with convention and empirical observations on bench-top prototypes (Methods; SI text), we model folding creases as

Fig. 3. Combining units into a $9 \times 9$ PERMuTE material produces a general-purpose platform for designing mechanical metamaterial devices. (a) Waveguide device transmits input oscillations along the ‘+’ shaped channel (pink [1,1] units), but nowhere else (purple [0,0] units). (b) Wave lens device transmits input oscillations along a line of input units (left-most triangular edge of pink [1,1] units), and focuses oscillations to a single point on the opposite side of the device (right-most tip of triangle). (c) Wave cloak device insulates central $3 \times 3$ region (pink [1,1] units) from input oscillations on the outer edge (pink [1,1] units) with a rectangular annulus of non-transmitting units (purple [0,0] units). Upper: 3D renderings of each device. Lower: frequency-dependent response efficiency $\eta(f)$ for each device. Arrows point to optimal frequency, $f^*$. Left inset: schematic showing where longitudinal signal was input (orange), target output was measured (green), and reference output for calculating $\eta$ was measured (black); $\eta$ is average maximum-amplitude at green divided by average maximum-amplitude at black. Right inset: heatmap of normalized oscillation amplitudes at $f = f^*$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
linear torsional hinges and compute the energetics of geometrically-allowed configurations assuming ideal rigid facets (Fig. 1d, black diagonal lines $\theta_2 = \theta_1$ and $\theta_2 = 360^\circ - \theta_1$). These crease-only deformations are a subspace in a larger energetic landscape where the facet material is allowed to bend (Fig. 1d, colour corresponds to elastic potential energy from folding plus bending; see SI text for derivations) [3,14,15,18,21]. Regardless of whether facet bending is permitted, an isolated unit is intrinsically tristable with: (i) strong energetic minimum for the [1] configuration; (ii) weak energetic minima for the [0,1] and [1,0] configurations; and (iii) no minimum corresponding to the [0,0] configuration (Fig. 1e; Movie S1). However, when a unit is embedded in an $N = 3 \times 3 = 9$ unit tessellation where it interacts with its 8
the same PERMuTE structure to generate the \([1] [1] [0,0]\) configuration. Analytic calculations show the base of the structure \(H_{II’P}H’\) plays a role in this new-found stability of \([0,1]\) by coupling the mechanics of units within the tessellation (see SI text for derivation). Thus, extrinsic unit-unit interactions generate a new stable state with popping transitions between the various four configurations available to each unit (SI text; Movie S2).

Beyond statics, unit-unit interactions also have implications in a dynamic regime. To demonstrate the consequences of extrinsic interactions, we performed vibrational analysis of the mesoscale unit using FEM simulations of a thin fibrous pulp material (see SI text for FEM details including consideration of other thin sheet materials). These results show the band structure resulting from the extrinsic multistability has a gap around 2.9 kHz in the FX direction that can be reversibly opened or closed depending on whether the unit is in \([1] [1]\) or \([0,0]\) (Fig. 1h, red and blue lines; band gap highlighted by gray rectangle) \([10-12,16,17,43]\). This bandgap exhibits directionality, consistent with the unit’s orthotropic construction. Examining a broader frequency range shows this configuration-specific opening and closing of band gaps repeatedly occurs throughout the 1–10 kHz range (Fig. S2). Going one step further and integrating the band structure shows the Density of States (DOS) also exhibits a high degree of sensitivity to the module’s configuration (Fig. 1i, red and blue lines). Since both directionality and density of vibrational modes are strongly configuration-specific, the extrinsic interactions of PERMuTE provide an opportunity to develop dynamic metamaterial-based devices from its exponentially large \((-4^2 = 2^{2n})\) menu of configurations.

### 3.2. Linear PERMuTE structures have reconfigurable vibration transmission

Examining the properties of a \(1 \times 3\) PERMuTE material illustrates its potential as a platform for developing general-purpose reconfigurable metamaterials. Again using FEM, we input a time-dependent longitudinal force generated by 11 equally-spaced frequencies, \(F(t) = F_0 \sum_{n} \sin \left[2\pi n \left( f - f_i \right) t \right]\) (Fig. 2a; SI text; Fig. S3). Whereas continuous frequency ranges are well-suited for characterizing band structure, discrete input functions such as the frequency comb used here are better able to highlight the potential functionality of this device.

When the PERMuTE material is in the \([0,0][0,0][0,0]\) configuration (Fig. 2b, top), we find it functionally behaves as a vibrational filter (Fig. 2c, top) that suppresses 10 of the 11 input frequencies (Fig. 2d, top). We then reconfigure the PERMuTE material to \([0,0][1][0,0]\) (Fig. 2b, middle) by popping its middle unit, and find it now transmits a more complex waveform (Fig. 2c, middle) with two well-pronounced frequencies mixed with low-amplitude side-band contributions (Fig. 2d, middle). Popping another unit (Fig. 2b, bottom) of the same PERMuTE structure to generate the \([1][1][0,0]\) configuration then leads to another new output waveform (Fig. 2c, bottom) that again consists of two well-pronounced frequencies (Fig. 2d, bottom). However, in this third configuration, the output frequency composition has substantially changed relative to the previous two settings. With only 3 of the \(4^2 = 64\) configurations examined (SI text; Fig. S3), these results already illustrate how extrinsic factors affecting the PERMuTE material’s multistability lead to configuration-specific frequency filtering.

Interestingly, some portion of this \(1 \times 3\) PERMuTE structure’s frequency response may itself be an extrinsic effect separate from the configuration-dependent aspects already highlighted. We see this in the reciprocity of \([1][0,0][0,0]\) and \([0,0][0,0][1]\) compared to the non-reciprocity of \([1][1][0,0]\) and \([0,0][1][1]\) \((\text{Figs. 2 and S3})\) \([8]\). Such non-reciprocal phenomena in the transfer function are known to arise from nonlinear interactions, though whether in this case it is the same non-additive unit-unit interactions enabling multistability is an open question.

Noting extrinsic properties are sensitive to the number of modules and their macroscopic assembly, we recognize the specific band structure for this \(1 \times 3\) tessellation is distinct from that of an isolated unit or another size structure. We therefore expect a 2D PERMuTE material to have similar functional properties, but with a new degree of design freedom.

### 3.3. Planar PERMuTE structures as reconfigurable devices

Using similar methods to those applied to the linear PERMuTE structure, we analyzed vibrational properties of a \(3 \times 9\) PERMuTE material to determine what types of devices can be found among the configurations of this structure. Of the \(4^{3\times3} \approx 5.8 \times 10^{48}\) possibilities available (SI text; Figs. S4 and S5), we focus here on three. For the first device, we popped a “+” shape of units to \([1]\) and set the remainder to \([0,0]\) (Fig. 3a, top). This pattern was chosen to produce a 1-input/3-output waveguide that we tested by oscillating the input edge unit at various frequencies \(f\) (Fig. 3a, bottom; orange unit) while measuring the response amplitude throughout the structure (Fig. 3a, bottom; red heatmap). Defining the response signal efficiency \(\eta(f)\) as the average output amplitude \((\text{Fig. 3a, bottom; green units})\) divided by the average perimeter amplitude \((\text{Fig. 3a, bottom; black units})\), we find \(\eta(f)\) can be quite large \((\text{Fig. 3a, bottom; } \eta(1.007 \text{ Hz}) \approx 14)\). These large efficiencies arise when input oscillations excite the specific configuration’s band structure resonances.

For the second device, we reset all units to \([0,0]\) and then popped a triangular shaped region into \([1]\) (Fig. 3b, top). This reconfiguration programed the PERMuTE material to function as a vibrational wave lens that focuses a distributed line of input oscillation \((\text{Fig. 3b, bottom; orange units})\) onto a single output unit \((\text{Fig. 3b, bottom; green unit})\).

Again, sweeping frequency while measuring the response signal efficiency, we found a range of functional values where \(\eta(f) > 1\) \((\text{Fig. 3b, bottom; } \eta(658 \text{ Hz}) \approx 4)\).

For the third device, we reset the configuration to a rectangular annulus of \([0,0]\) units with the goal of creating an interior region isolated from vibrations \((\text{Fig. 3c, top; central region is targeted for vibration isolation})\). Measuring \(\eta(f)\), we found values generally \(<1\) with the best performance leading to \(\approx 80%\) vibration suppression \((\text{Fig. 3c, bottom; } \eta(89.9 \text{ Hz}) \approx 0.2)\). In this type of cloaking device, a resonant mode confines input oscillations to the edge of the structure and therefore prevents resonant frequency waves from propagating through the
annulus. Because the wave cloaking capabilities are tied to the band structure's resonances, reconfigurations of the device affecting the band gaps can be used to tune the cloaking properties. Inspired by electromagnetic metamaterials that cloak sensors [44], we can foresee this type of structure being useful for protecting sensitive equipment within the annulus from external driving at potentially harmful frequencies.

While the functional range of frequencies for these three devices vary, the basic unit's geometry remains the same. However, this does not mean the functional properties cannot be further adjusted from their current baseline. For example, modifications to the three configurations by changing various units will affect the band structure in difficult-to-predict-but-possibly-useful ways. Deeper computational exploration would be required to know exactly what these changes are, and whether the device's effects fall within the application's desired frequency range. Similarly, as with other classes of metamaterials, the mesoscale unit geometry is independent from the base material's composition, which makes its density and modulus free parameters for additional control over the resonances affecting \( n(f) \) (SI text). While the three devices shown here provide specific examples over specific frequency ranges, the ultimate limits are unknown and would make for an interesting computationally-driven material discovery exploration.

3.4. Bench-top experiments with PERMuTE structures and configuration-specific extrinsic properties

In light of the insights gained by these FEM studies (Figs. 1-3), we fabricated PERMuTE devices from laser cut cardstock (Fig. S5) and tested the mechanical properties in bench-top experiments (Fig. 4). We first verified an isolated unit is intrinsically bistable (Movie S1), whereas units in a \( 3 \times 3 \) tessellation are quasistable (Movie S2). We then verified the mechanical properties were dependent on extrinsic interactions with a longitudinal force-displacement measurement of the central unit (Fig. 4a, photos) that demonstrated a sensitivity to the configuration of adjacent units (Fig. 4a, dark and light green data). We also found the extrinsic multistability separating \([0,0]\) from \([1]\) vanished from this force-displacement measurement when testing an isolated unit (Fig. 4a, gray data). Additional transverse compression measurements of a single unit in the \([0,0]\), \([0,1]\), \([1,0]\), and \([1]\) configurations verified the predicted symmetry between \([0,1]\) and \([1,0]\) (Figs. 1e and 4b). The same transverse compression on a \( 3 \times 3 \) tessellation with varying number of units in the \([0,0]\) configuration showed the zero-frequency mechanical properties could be easily and reversibly configured due to the extrinsic multistability of the individual units (Fig. 4c; SI text; Fig. S6). Collectively, these experiments demonstrate two distinct examples for how emergent extrinsic effects can be repurposed. First, they stabilize new configurations of the PERMuTE tessellation's units, and second, they contribute to the bulk metamaterial's mechanical properties.

Transitioning to frequency-dependent measurements, we tested a \( 3 \times 3 \) PERMuTE structure to verify the anticipated functionality of extrinsic properties. Performing a frequency sweep on the tessellation in five distinct configurations showed resonant peaks that shifted up and down while maintaining a nearly constant central frequency (Fig. 4d, peaks marked \( P_1 \), \( P_2 \), and \( P_3 \); SI text; Figs. S7 and S8). These measurements demonstrate a range of extrinsic behavior depending on how many units were set to \([0,0]\), with \( P_2 \) vanishing in one configuration, while \( P_3 \) jumped 5-fold between its extremes. At higher frequencies, we also found that varying configurations caused resonance peaks to discretely shift their center frequency by \( \Delta f_1 = (9 \pm 1) \) Hz and \( \Delta f_2 = -(9 \pm 1) \) Hz (Fig. 4e). Interestingly, this shift groups configurations so that 0 and 1 units in \([0,0]\) have similar resonance peaks (Fig. 4e, gray and orange lines), while configurations with 7 and 9 units in \([0,0]\) are almost identically shifted (Fig. 4e, purple, black lines shifted by \( \Delta f_1 \) and \( \Delta f_2 \)). On one hand, these configuration-specific shifts experimentally realize an extrinsic frequency-filtering metamaterial (Fig. 2d). On the other hand, the reversible generation, enhancement, and alteration of resonance peaks (combined results of Fig. 4d and e) are the critical ingredients necessary for constructing the waveguide, wave lens, and wave cloak devices (Fig. 3). Even though these physical experiments only explored 5 of the \( 4^3 = 262,144 \) possible configurations over a limited frequency range, the experimental evidence validates the existence of extrinsic properties stemming from extrinsic multistability and demonstrates how their realization is practically implemented with PERMuTE.

Metamaterial design focuses on developing geometric patterns that can be embedded in conventional materials. An advantage to engineering materials this way is that the pattern's parameter space provides wide flexibility in the downstream properties. The cost of this flexibility is a challenging multi-objective parameter optimization problem when developing real-world applications of metamaterials due to emergent and extrinsic effects. In response to this challenge, we proposed, and subsequently demonstrated with PERMuTE designs as a specific example, how extrinsic effects can be embraced and repurposed into a menu of properties that can be selectively chosen from for device functionality. Actuating between these configurations is an application-specific extension of the concept, but can include motors, pneumatics, shape memory alloys, or even manual manipulation. By emphasizing post-fabrication reconfiguration of a structure, this metamaterial design approach contrasts with current and conventional methodologies, which instead rely on fixing gradients in unit geometry to statically program bulk-scale mechanics [5,6,8,14,15,18]. While this commonly-used approach allows one to achieve metamaterial functionality, it has little ability to respond to changing user needs or to circumvent the emergence of non-additive effects.

Real-world use-cases of our approach therefore involves: (i) choosing a base material to enhance with metamaterial properties; (ii) determining tolerances of the intended fabrication method in order to set the maximum allowable metamaterial pattern density; (iii) pre-computing response functions over a range of frequencies and configurations; and (iv) selecting desirable configurations for a given application. Because the band structure generally varies with material, tessellation size, and configuration, these steps must be repeated should any of these details change. Additional simulations and experimental results studying these variations with the PERMuTE geometry serve to reinforce how the extrinsic design approach is applied, though such effort likely offers little new generalizable insight beyond what's already presented here. For example, a 2.9 kHz gap in an infinite tessellation (e.g., Fig. 1h) may not necessarily appear in a \( 1 \times 3 \) or \( 9 \times 9 \) tessellation (e.g., Figs. 2 and 3), making material- and structure-specific response both open challenges and opportunities for extrinsic metamaterial design. The cumulative benefit of this computationally intensive design strategy is apparent: a single well-characterized structure can be used in numerous applications through simple reconfigurations, which circumvents the challenges to conventional metamaterials that have to be re-designed, re-computed, and re-fabricated if new properties are desired outside its original application. As such, the extrinsic design strategy enhances the predictability, flexibility, and programmability of metamaterials, increasing their potential impact in future applications.

4. Conclusion

In this work, we proposed the PERMuTE mechanical metamaterial unit cell, and studied its properties in 1D bar-like structures and 2D tessellations. We investigated the multistability and vibrational band gap in a unit cell, as well as the emergence of a new stable state when the basic unit was tessellated into a larger structure. These findings inspired us to design the 1D wave filter, 2D waveguide, 2D wave lens, and 2D cloak where non-additive extrinsic effects were essential for stabilizing the configuration. These results show the multistability of a simple unit is enhanced when it is embedded into a 2D tessellation through extrinsic unit-unit interactions, and this multistability can be utilized to design
mechanical metamaterial devices. Importantly, the proposed PERMuTE metatmater possesses exponentially programmable patterns with different “0/1” combinations of units, which opens up new avenues for designing new smart structures and novel devices in the fields of mechanics, energy, optics, aerospace, and electronics. While our findings are specific to the PERMuTE geometries, the concepts we used to direct the research are more general. The notion of “intrinsıc” and “extrinsic” properties can be found in the earliest work of thermodynamics and finds new relevance with mechanical metamaterials where “intrinsıc” properties can be attributed directly the unit cell’s geometric design, whereas “extrinsic” properties arise at the bulk-scale due to nonlinear and non-additive interactions. Whether in the context of orthotropic periodic tessellations as ours or periodic patterns such as a Penrose tiling, the distinction of intrinsic and extrinsic effects will be critical for developing metamaterial applications.

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Author contributions

N.Y. and J.L.S. designed research; N.Y. and C.C. performed research; N.Y., J.K., and J.L.S. analyzed data; N.Y. and J.L.S. wrote the paper; J.K. and J.L.S. supervised the research. The authors declare no conflict of interest. N.Y. and J.L.S. contributed equally to this work.

Data and materials availability

All data, code, and materials used in this work are freely available upon request.

Credit author statement

The corresponding author (N.Y.) ensures that the descriptions are accurate and agreed by all authors.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work presented in this paper.

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