Transient MHD Natural Convective Flow Past a Heated Vertical Non-porous Surface with Thermal Radiation and Double-Diffusion Effects

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Authors’ contributions

This work was carried out in collaboration between both authors. Author WIAO designed the study, wrote the protocol and the first draft of the manuscript. Author WIAO managed the literature searches. Author TO solve the problem and did the programming for the results. Author WIAO managed the analyses of results. Both authors read and approved the final manuscript.

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ABSTRACT

The problem of transient MHD natural convective flow past a vertical plate with thermal radiation, cross-diffusion and zero suction effects is investigated. The governing one-dimensional spatial and non-linear partial differential equations of the Boussinesq form are non-dimensionalized to, among others, bring out the necessary parameters. The evolving dimensionless equations are transformed into ordinary differential equations using the similarity transformation, and linearized using the regular perturbation expansion series solutions. The linearization leads to the zeroth and first order equations, and are solved semi-analytically using the Mathematica 11.0 computational software. Expressions for the temperature, concentration, velocity, Nusselt number, Sherwood number and Skin friction are obtained, computed and presented graphically. The analysis of results, amidst others, shows that the increase in the Dufour number increases the temperature, but decreases the Nusselt number; the increase in the Soret number decreases the concentration, but increases the Sherwood number; the increase in the Prandtl number decreases the temperature and...
concentration, but increases the Nusselt number and Sherwood number; the increase in the Hartmann number decreases the velocity and skin friction. These results are benchmarked with the existing reports in literatures, and are in good agreement.

Keywords: Double-diffusion; MHD; natural convection; thermal radiation.

NOMENCLATURE

\( C \): The fluid concentration (quantity of material being transported)
\( D \): The diffusion coefficient
\( f \): The similarity velocity parameter
\( g \): The gravitational field vector acting in the reverse direction of the flow
\( s \): The suction parameter
\( T \): The time
\( T^* \): Fluid temperature
\( U \): The fluid characteristic velocity
\( u \): The velocity components in the \( x \) direction
\( v \): The velocity components in the \( y \) directions, and is the velocity with which the fluid is sucked/injected
\((x, y)\): The Cartesian coordinates
\( B_o \): The uniform magnetic field strength
\( C_m \): The fluid ambient concentration at the wall of the plate
\( C_w \): The plate concentration at the wall
\( C_p \): The specific heat capacity at constant pressure
\( C_s \): The concentration susceptibility
\( D_f \): The Dufour number;
\( Gr/Gr_c \): The Grashof number due to temperature and concentration differences, respectively
\( k \): The thermal conductivity
\( k_r^2 \): The rate of chemical reaction of the fluid
\( k_T^2 \): The thermal diffusivity ratio of the fluid, and
\( M \): The magnetic field force
\( N \): The temperature difference parameter;
\( Pr \): The Prandtl number;
\( q_r \): The radiative heat flux
\( Sc \): The Schmidt number;
\( S_r \): The Soret number,
\( T_m \): The fluid ambient temperature
\( T_m \): The mean temperature of the fluid.
\( T_w \): The plate temperature at the wall
\( \alpha \): The optical property of the thin medium in which the heat is propagating
\( \rho \): The fluid density
\( \eta \): The independent variable
\( \nu \): The kinematic viscosity
\( \Theta \): The dimensionless temperature
\( \Theta \): The dimensionless concentration
\( \mu \): The viscosity of the fluid
\( \kappa \): The permeability parameter of the porous medium
the flow produces induced fields to modify the magnetic field. The flow-modifying effect of magnetic field on the flow is over vertical plates has been greatly researched on. The flow-modifying impact of magnetic field on the highly interacting flow has been investigated. Gnaneshwar et al. [3] examined the Soret and Dufour effects on the steady magneto-hydrodynamic natural convective flow over a semi infinite accelerating vertical plate in a porous medium with viscous dissipation; Shateyi et al. [4] considered the thermal radiation, Hall currents, Dufour and Soret effects on the magneto-hydrodynamic mixed convective flow over a vertical plate in porous media; Seethamalahakshmi et al. [5] investigated the effects of Soret/diffusion thermal on the unsteady MHD natural convective flow over an infinite porous plate with variable suction. Pattnaik et al. [6] examined the effects of Dufour and Hall currents on an unsteady magneto-hydrodynamic flow over an infinite vertical plate using a special function $H_h(z)$, and observed that injection enhances the flow, Hartmann number reduces the skin friction, heavier species decreases the velocity in both suction and injection cases; Hall currents slightly increase the flow velocity in the case of constant temperature and concentration. Okuyade et al. [7] studied the unsteady MHD free convective flow over a vertical plate with focus on the effects of chemical reaction, thermal radiation, Dufour, Soret, Hartmann number, Grashof number and constant suction using similarity transformation and perturbation series solutions methods, and observed that the Hartmann number and Grashof number increase the velocity; Dufour number increases the temperature, Nusselt number and velocity; chemical reaction rate decreases the temperature, but increases the concentration, velocity and Nusselt number; Soret number increases the concentration and velocity; the suction decreases the temperature, concentration and velocity, but increases the Nusselt number.

1. INTRODUCTION

MHD convective flow over vertical plates has applications in science and engineering. It is relevant in the new type of energy conversion devices; stellar and ionospheric phenomena; geophysics; chemical engineering.

Based on the level of interactions on the particles in a flow system, flow situations are classified as highly moderately less and non-interactive. In highly interacting flow systems where magnetic flux, convection and chemical reaction are significant, heat and mass transfer must occur simultaneously to produce cross/double-diffusion effects. Double-diffusion has attendant influence on the fluid buoyancy. More so, energy and mass fluxes are further generated by concentration and temperature differentials, respectively. These differentials produce the Dufour (thermo-diffusion) and Soret (diffusion-thermo/thermo-phoresis) effects. When the differences in the concentration and temperature are large or when the difference in molecular mass of two elements in a binary mixture is large the coupled interaction becomes significant.

A lot of reports exist in literature in this domain of study. Some considered the highly interacting flow with diverse parameters effects, and others the less. For the highly interacting flow, Awad et al. [1] studied the free convective flow with double diffusive convection over a vertical infinite plate using the successive linearization method; Srinivasacharya et al. [2] considered a mixed convective heat and mass transfer flow along a wavy surface in a Darcy porous medium in the presence of cross-diffusion effects using similarity transformation and numerical method.

Magneto-hydrodynamics has a wide range of applications in industries, geothermal, power generating system, liquid metal and highly temperature plasma, Magneto-hydrodynamic flows provide the interface between electrically electrolytes and magnetic field, a force is developed by the flow produces induced fields to modify the magnetic field.

\[
\begin{align*}
\sigma & \quad : \text{The Stefan-Boltzman constant;} \\
\phi & \quad : \text{A non-constant small temperature correction factor} \\
\beta_1 & \quad : \text{The volumetric expansion coefficient for temperature} \\
\beta_2 & \quad : \text{The volumetric expansion coefficient for concentration} \\
\mu_m & \quad : \text{The magnetic permeability of the fluid} \\
\sigma_e & \quad : \text{The electrical conductivity of the fluid} \\
\chi^2 & \quad : \text{The porosity parameter}
\end{align*}
\]

\[\text{\sigma}, \text{\phi}, \text{\beta}_1, \text{\beta}_2, \text{\mu}_m, \text{\sigma}_e, \text{\chi}^2\]
For the less interacting flow, in which case the thermal diffusion and thermo-phoresis effects are negligible, Soundalgekar et al. [8] worked on the two-dimensional unsteady free convective flow over an infinite vertical porous plate with oscillating wall temperature and constant suction. Sattar [9] studied the unsteady free convective flow through a porous plate with time-dependent temperature and concentration, and variable suction using the method of similarity transformation, and observed that the velocity increases with the increase in the convection current. Singh et al. [10] investigated unsteady mixed convective flow over an infinite vertical plate with periodic suction; Bakier [11] examined the radiation effects on mixed convection flow over an isothermal vertical surface in a saturated porous medium, and obtained self-similar solution; Das et al. [12] numerically examined the unsteady free convective flow over an accelerating vertical plate with suction and heat transfer flux. Halem [13] examined the unsteady mixed convective flow over a vertical porous plate with heat and mass transfer, periodic suction and oscillatory free stream using separation of variables and successive perturbation techniques, and found that the Stanton number increases as the porosity parameter increases but decreases as the Schmidt number increases.

Furthermore, the effects of MHD on the non-highly interacting flows were also investigated. For example, Gersten and Gross [14] investigated the flow past a vertical porous plate in the presence of transverse MHD and sinusoidal suction; Bansal et al. [15] investigated the unsteady MHD free convective flow over a moving infinite porous vertical plate; Alagoa et al. [16] considered the MHD free convection flow of an incompressible and optically transparent porous medium with time-dependent suction and radiation heat transfer using asymptotic approximation, and observed that the radiation, porosity and magnetic field parameters affect the flow. Abdel-Naby [17] considered the MHD free convective flow over a vertical plate in the presence of radiation and surface temperature using numerical approach. Sarangi and Jose [18] examined the unsteady MHD free convective flow through a porous medium with variable suction and constant heat flux. Chen [19] examined the MHD free convective flow past an inclined permeable surface with variable wall temperature and concentration, and saw that increase in: magnetic field decreases the flow velocity; angle of inclination decreases the effect of buoyancy force; Prandtl number increases the heat transfer rate. Kim [20] studied the unsteady MHD natural convective flow with heat transfer past a semi-infinite vertical and moving plate with variable suction. Das and Mitra [21] studied the unsteady MHD mixed convective flow and mass transfer over an accelerating infinite vertical plate with suction. Das et al. [22] investigated the effects of mass transfer on MHD flow and heat transfer over a vertical porous plate with oscillatory suction and heat source. Pal and Talukdar [23] worked on unsteady MHD heat and mass transfer with heat source, constant suction and radiation over a vertical porous plate using a perturbation technique, and noticed that unsteadiness is caused by time-dependent surface temperature and concentration. Narayana et al. [24] studied the effects of viscous dissipation and thermal radiation on an unsteady MHD free convective flow over an infinite vertical plate constant suction and heat sink. Gundagani et al. [25] examined the thermal radiation effects on unsteady MHD flow past a vertical porous plate with variable suction using finite element method, and noticed that when velocity function is increased by the increase in the radiation parameter and porosity parameter but is decreased by the increase in Hartmann number, Prandtl number and Schmidt number; the temperature function increases with the increase in the radiation parameter, Hartmann number but is decreased with the increase in the Prandtl number; concentration is increased by the increase in the Schmidt number. Acharya et al. [26] studied the transient free convective MHD flow of a viscous incompressible fluid over hot a porous plate under the attendant effects of heat source, viscous dissipation and variable temperature using the method of regular perturbation, and noticed that porosity has influential effect on the flow and viscous dissipation enhances the heating and cooling of the plate due to convective current. Reddy [27] considered the MHD natural convective flow over an inclined permeable surface with variable temperature, momentum and concentration. Acharya et al. [28] studied free convective fluctuating hydro-magnetic flow through porous media past a vertical porous plate with variable temperature and heat source. Dei et al. [29] considered the unsteady MHD mixed convective flow over a moving infinite vertical porous plate with suction in the presence of radiation and heat absorption/generation using similarity transformation and fourth order Runge-Kutta and shooting methods, and observed that the velocity increases with the increase in Grashof number.
but decreases through the increase in the suction, magnetic field parameters; the cooling of the plate is faster when the suction, radiation parameters and Prandtl number become large; heat source and sink enhances the temperature field; the skin friction, Nusselt number and Sherwood number increase as the suction parameter increases; the concentration decreases as the Schmidt number or suction parameter increases. Baiyeri et al. [30] studied the MHD natural convective flow over a vertically inclined plate in the presence of viscous dissipation and heat source/sink using implicit numerical scheme, and found that increase in: Grashof numbers increases the flow velocity; heat source parameter increases the velocity and temperature; Schmidt number increases the velocity and concentration.

Examining the influence of chemical reaction, Seddeek et al. [31] investigated the effects of chemical reaction and variable viscosity on hydro-magnetic mixed convective heat and mass transfer for Hiemenz flow through a Darcy porous medium in the presence of radiation and magnetic field; Gireesh et al. [32] considered the effects of chemical reaction on a flow over an infinite vertical plate with constant suction and heat sink; Sudheer and Satya [33] considered the effects of chemical reaction and radiation on an unsteady MHD free convection flow past a semi-infinite vertical plate with heat source and variable suction. Kesavaiah et al. [34] investigated the unsteady MHD heat and mass transfer in a flow over vertical porous plate with chemical reaction and thermal radiation. Anand and Shivaiah [35] examined numerically the effects of chemical reaction on an unsteady MHD free convection flow over an infinite vertical plate with constant suction. Anand and Shivaiah [36] investigated the chemical reaction effects on an unsteady MHD free convective flow past a semi-infinite porous plate with viscous dissipation; Gangadhar and Reddy [37] considered the chemically reacting MHD boundary layer flow of convective heat and mass transfer flow over an infinite vertical porous plate with suction. Sarada and Shanker [38] studied numerically the effects of chemical reaction in an unsteady MHD natural convective heat and mass transfer flow over an infinite vertical porous plate whose temperature oscillates with the same frequency as the variable suction, and noticed that the increase in: magnetic field decreases the velocity, skin friction; Prandtl number decreases the velocity, temperature, skin friction and Nusselt number; Schmidt number decreases the velocity, concentration, skin friction and Sherwood number; chemical reaction rate decreases the velocity, concentration, skin friction and Sherwood number; both thermal and solutal Grashof numbers increase the skin friction.

In Okuyade et al. [7], we considered the unsteady flow of a chemically reacting fluid over a vertically accelerating plate with constant suction, which is greater than zero using the similarity solution. With the aim of getting the picture of the problem under different suction situations, this paper studies the case when suction is zero.

This paper is organized in the following manner: section 2 is the materials and method; section 3 gives the results and discussion, while section 4 gives the conclusion.

2. METHODOLOGY

This problem is formulated on the assumptions that the fluid is incompressible, viscous,

![Fig. 1. The schematic model of a moving vertical surface in a fluid](image)
Newtonian, magnetically susceptible and optically thin; the surface/plate is vertical and is accelerating upwards with the lower edge \( y=0 \) as the origin; the \( x \)-axis is normal to the plate; the pressure does not vary in the axial direction such that it is equal to the hydrostatic pressure (i.e. \( p_o(x) = 0 \)); the flow is naturally convective; the suction is zero such that the plate is impermeable; the magnetic field is constant and is maintained in the \( x \)-direction; the problem is highly interactive such that the Dufour and Soret effects are significant. Upon the unsteady one-dimensional flow theory, the physical variables in this model in the Cartesian coordinates system are functions of \( y \) and \( t \) only. Furthermore, we assumed the plate wall concentration and temperature are sufficiently high to affect radiative heat transfer. So, if the axial velocity of the fluid is \( u \); the velocity of the fluid in the \( y \)-direction is \( v \), and it is the velocity at which the fluid is sucked at the wall, by the Boussinesq approximations, the governing continuity, momentum, energy and diffusion equations are as follows:

\[
\frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
\rho \left( \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_e B^2_u}{\mu_m} u + \rho g \beta_f (T - T_w) + \rho g \beta_c (C - C_w) \tag{2}
\]

\[
\rho C_p \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} \right) = \frac{\partial^2 T}{\partial y^2} - \frac{Dk_e}{C_e} \frac{\partial^2 C}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{3}
\]

\[
\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} \tag{4}
\]

with the boundary conditions:

\[
u = 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0 \tag{5}
\]

\[
u \to 0, \quad T \to T_w, \quad C \to C_w \quad \text{at} \quad y \to \infty \tag{6}
\]

Radiative heat transfer is relevant in a high temperature regime and is equivalent to convective heat transfer. From Vincentti and Krugger [39], the gradient of a radiative heat flux for an optically thin fluid is defined as

\[
\frac{\partial q_r}{\partial y} = -4\sigma_a \frac{\partial^2}{\partial y^2} (T^4 - T_w^4) \tag{7}
\]

in which case \( \alpha \ll 1 \). This is a physical assumption for an optically thin/transparent fluid, and it depicts that the depth of penetration of the radiant rays \( \alpha \) is far less than one, and the density is relatively low to allow long photon travel.

More so, we take the difference in temperature within the flow to be sufficiently small that we can state \( \phi = T - T_w \), where \( \phi \) is a non-constant small temperature correction factor; writing \( T^4 \) as a linear function of the temperature, we can express \( T^4 \) in a Taylor series about \( T_w \) as

\[
T^4 = 4T_w^3T - 3T_w^4 \tag{8}
\]

In situations like this, we say the fluid is slowly radiating and as such \( Ra < 1 \), (see [16]). Moreover, equation (1) shows that the velocity is not a function of \( y \). But, from Singh and Soundalgekar [40], the velocity \( v \) is prescribed as:

\[
v = s \left( \frac{v}{t} \right)^{1/2} \tag{9}
\]

where \( s \), the suction parameter is a function of time. Motivated to investigate the flow characteristic behaviour when the suction is neither a function of \( y \), \( t \) nor constant, in this paper, we assume the suction is zero such that the plate is impermeable and \( v = s = 0 \).

Introducing the similarity solution, dimensionless quantities, equations (7) and (8) in association with the suction condition into equations (1)-(6), we get

\[
f'' + 2n f f' - M \frac{\partial^2 f}{\partial \Theta^2} = -Gr \Theta - Gr \Phi \tag{10}
\]

\[
\Theta'' + 2 Pr \eta \Theta' = Ra \left[ 3(\Theta + \eta)^2(\Theta^2 + (\Theta + \eta) \Theta) \right] + \frac{D \Phi''}{2} \tag{11}
\]

\[
\Phi'' + 2 Sc \eta \Phi' = S \eta \Theta'' \tag{12}
\]

with the boundary conditions

\[
f(0) = 1, \quad \Theta(0) = 1, \quad \Phi(0) = 1 \tag{13}
\]
\[ f(\infty) = 0, \quad \Theta(\infty) = 0, \quad \Phi(\infty) = 0 \]  
(14)

Where

\[ u = Uf(\eta), \quad \eta = \frac{y}{2\sqrt{u}} \]  
(15)

are the similarity solutions,

\[ \Theta = \frac{T - T_w}{T_m - T_w}, \quad \Phi = \frac{C - C_m}{C_w - C_m}, \]  
\[ Gr = \frac{4g\beta_\nu(T_m - T_w)}{U}, \]  
\[ Gc = \frac{4g\beta_\nu(C_w - C_m)}{U}, \quad M^2 = \frac{\sigma B^2 \nu}{\mu \mu_m} \]  
(16)

\[ Pr = \frac{\nu}{k_o}, \quad Sc = \frac{\nu}{D}, \quad N = \frac{T_m}{(T_m - T_w)} \]  
\[ D_r = \frac{Dk_o(C_w - C_m)}{k_o C_s(T_m - T_w)} \quad Ra = \frac{16\sigma \alpha(T_m - T_w)}{\nu k} \]  

are the dimensionless quantities. It is noteworthy that there are many versions of similarity transformation equations. Equation (15) is used when the normal velocity \( v \) is assumed to be the suction, which can be a constant (zero or non-zero) or a variable function.

Equations (10)-(12) are nonlinear and highly coupled. So, to obtain analytic solutions we assume the fluid is slowly radiating such that \( Ra < 1 \) and perturb the equations. We seek for perturbation series solutions of the form

\[ n = n_o + n_1 \times Ra^2 + ... \]  
(17)

where \( n \) stands for the flow variables and \( n_i \) the order of perturbation.

Substituting equation (17) into equations (10)-(14), then collecting the coefficients of each power of \( Ra \), and equating to zero, we get

\[ f_1'' + 2\eta f_1' - M_1^2 f_1 = -Gr\Theta_1 - Gr\Phi_1 \]  
(18)

\[ \Theta_1'' + 2Pr\eta \Theta_1' = -D_1 \Phi_1' \]  
(19)

\[ f_o'' + 2\eta f_o' - M_o^2 f_o = -Gr\Theta_o - Gr\Phi_o \]  
(20)

with the boundary conditions

\[ f_o(0) = 1, \quad \Theta_o(0) = 1, \quad \Phi_o(0) = 1 \]  
(21)

\[ f_o(\infty) = 0, \quad \Theta_o(\infty) = 0, \quad \Phi_o(\infty) = 0 \]  
(22)

for the zeroth order

\[ f_1'' + 2\eta f_1' - M_1^2 f_1 = -Gr\Theta_1 - Gr\Phi_1 \]  
(23)

\[ \Theta_1'' + 2Pr\eta \Theta_1' = (\Theta_o^3 + 3N\Theta_o^2 + 3N^2\Theta_o + N^2)\Theta_1' + (3N\Theta_o^2 + 6N\Theta_o + 3N^2)\Theta_o^2 + D_1 \Phi_1'' \]  
(24)

\[ \Phi_1'' + 2Sc\eta \Phi_1' = S_r\Theta_1' \]  
(25)

with the boundary conditions

\[ f_1(0) = 0, \quad \Theta_1(0) = 0, \quad \Phi_1(0) = 0 \]  
(26)

\[ f_1(\infty) = 0, \quad \Theta_1(\infty) = 0, \quad \Phi_1(\infty) = 0 \]  
(27)

for the first order.

Additionally, we express the rate heat transfer \( (Nu) \) and rate mass transfer \( (Sh) \) as

\[ Nu = k \frac{2q_s \sqrt{u}}{(T_m - T_w)} = -\Theta|_{\eta=0} \]  
(28)

\[ Sh = D \frac{2s_s \sqrt{u}}{(C_m - C_w)} = -\Phi|_{\eta=0} \]  
(29)

\[ \tau_o = \mu \frac{\partial u}{\partial \eta}|_{\eta=0} = f'|_{\eta=0} \]  
(30)

Where

\[ q_s = -k \frac{\partial T}{\partial y}|_{y=0}, \quad s_s = -D \frac{\partial C}{\partial y}|_{y=0} \]  

Equations (18)-(30) are solved and the solutions plotted using the Mathematica 11.0 computational software. The analysis of the solutions shows that Mathematica 11.0 has
difficulty in evaluating the definite integrals of the evolving Error and Hyper-geometric functions, which encase the Grashof numbers, Prandtl number, Schmidt number, Dufour and Soret number in the zeroth order velocity solution. Upon this, the roles of these parameters on the velocity factor are not quantifiable. Similarly, the analysis of the solutions shows that the order one equations, that is, equations (23)-(27) has empty or zero solutions within the specified region $\eta = 0$ and $\eta = \infty$. Therefore, the roles of the heat exchange parameter (N) and Raleigh number are not feasible in this problem.

3. RESULTS AND DISCUSSION

In this study, the influence of Dufour number, Soret number, Prandtl number, Schmidt number and magnetic field strength on the flow characteristics when the suction is zero (or the plate impermeable) are investigated and the results are shown graphically in Fig. 2 – Fig. 15. For positively varied values of

$$D_r = 0.1, 0.3, 0.5, 0.7, 0.9, 1.2; \quad S_r = 0.1, 0.3, 0.5, 0.7, 0.9, 1.2; \quad Pr = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0; \quad Sc = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0 \quad \text{and} \quad M^2 = 0.1, 0.3, 0.5, 0.7, 0.9, 1.2$$

The distributions are presented below.

The effects of Dufour number on the flow are shown in Figs. 2, 3. Fig. 2 depicts that the increase in the Dufour number increases the fluid temperature. The Dufour number has the potential of driving small fluid particles towards the hot surface. The small fluid particles driven towards the hot surface are energized; their kinetic energy, and of course, their temperature is increased. This may account for the increase in the fluid temperature. This result agrees with Okuyade et al. [7]. Fig. 3 shows that the increase in the Dufour number reduces the rate at which heat is transferred to the fluid mainstream.
The effects of the Soret number on the flow are shown in Fig. 4 and Fig. 5. Fig. 4 show that the increase in the Soret number decreases the fluid concentration. The Soret number has the tendency of drifting small fluid particles away from the hot surface to less hot region; as seen in Acharya et al. [26]. This result aligns with Okuyade et al. [7]. Fig. 5 depicts that the increase in the Soret number increases the rate of mass transfer to the main stream of the fluid system.

The roles of the Prandtl number on the flow variables are seen in Fig. 6 – Fig. 9. As in radiation phenomenon, the effects of the Prandtl number are noticeable when the temperature regime of the flow system is high. Fig. 6 and Fig. 7 show respectively, that the increase in the Prandtl number decreases the temperature and concentration. Additionally, Fig. 8 and Fig. 9, respectively, show that the increase in the Prandtl number leads to the increase in the rates of heat and mass transfer. These results are in consonance with Gundagani et al. [25], Acharya et al. [26], Gangadhar and Reddy [37].

The influence of the Schmidt number on the flow variables are seen in Fig. 10, Fig. 13. Figure 10 shows that a twist occurs in the temperature distribution. In the region $\eta<1.3$, the temperature increases as the Schmidt number increases. At $\eta=1.3$, the temperature distribution curves converge and a twist occurs. The twist may be due to some adverse temperature gradient effect. And, at $\eta>1.3$, the temperature decreases as the Schmidt number increases. Fig. 11 shows that the concentration decreases as the Schmidt number increases; Fig. 12 illustrates that the rate of heat transfer in the fluid decreases as the Schmidt number increases; Fig. 13 depicts that the rate of mass transfer increases as the Schmidt number increases. These results are similar to those of Gundagani et al. [25], Acharya et al. [26], Gangadhar and Reddy [37].
The flow-modifying effects of magnetic field, which may be due to the interaction between the Earth magnetic field or other-wise, and the chemical content of the fluid are seen in Figs. 14, 15 and Fig. 14 shows that the increase in the Hartmann number decreases the fluid velocity;
Fig. 15 depicts that the increase in the Hartmann number decreases the skin friction or the force exacted on the plate by the fluid.

4. CONCLUSION

We considered the transient MHD natural convective flow of an incompressible and optically thin fluid past a hot vertical plate in the presence of thermal radiation, thermo-diffusion, diffusion-thermo and zero suction effects using similarity transformation, perturbation series solutions, and Mathematica11.0 software. The analyses of results show that increase in:

- \(D_t\) increases the fluid temperature, but decreases the Nusselt number;
- \(S_r\) Soret number decreases the concentration, but increases the Sherwood number;
- \(P_r\) Prandtl number decreases the temperature and concentration, but increases the Nusselt number and Sherwood number;
- \(S_C\) Schmidt number leads to a twist in the temperature distribution, decreases the concentration and Nusselt number, but increases the Sherwood number;
- \(M^2\) Hartmann number decreases the the velocity and skin friction.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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