Breaking of $L_\mu - L_\tau$ Flavor Symmetry, Lepton Flavor Violation and Leptogenesis

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Abstract

A supersymmetric see-saw model obeying the flavor symmetry $L_\mu - L_\tau$, which naturally predicts quasi-degenerate neutrinos, is investigated. Breaking of the symmetry is introduced in the Dirac mass matrix because it is the most economic choice in the sense that all interesting low and high energy phenomenology is made possible: we analyze the predictions for the low energy neutrino observables, for leptogenesis and for lepton flavor violating decays such as $\mu \to e\gamma$, where the SPS benchmark points for the SUSY parameters are used. It is outlined how these decays in connection with the requirement of successful leptogenesis and with correlations between the neutrino observables depend on the way the symmetry is broken.

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1 Introduction

Explanation of the peculiar neutrino mass and mixing schemes is one of the most interesting tasks of particle physics. Motivated by spectacular experimental results, a very large number of models has been proposed in recent years [1]. Typically, the see-saw mechanism [2] is the starting point of most analyzes:

\[ m_\nu = -m_D^T M_R^{-1} m_D , \]

where \( M_R \) is the mass matrix of three heavy Majorana neutrinos \( N_{1,2,3} \) and \( m_D \) is a Dirac mass matrix resulting from the coupling of the Higgs doublet to the lepton doublets and the \( N_i \). The light neutrino mass matrix \( m_\nu \) is diagonalized by the PMNS matrix \( U \) defined via

\[ U^T m_\nu U = \text{diag}(m_1, m_2, m_3) . \]

It can be parametrized as

\[ U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -c_{23} s_{12} - s_{23} s_{13} c_{12} e^{i\delta} & c_{23} c_{12} - s_{23} s_{13} s_{12} e^{i\delta} & s_{23} c_{13} \\ s_{23} s_{12} - c_{23} s_{13} c_{12} e^{i\delta} & -s_{23} c_{12} - c_{23} s_{13} s_{12} e^{i\delta} & c_{23} c_{13} \end{pmatrix} P , \]

where \( P = \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)}) \) and \( c_{ij}, s_{ij} \) are defined as \( \cos \theta_{ij} \) and \( \sin \theta_{ij} \), respectively. By making assumptions for the unknown neutrino parameters (in particular the mass scale, ordering and phases), one can reconstruct \( m_\nu \) with the help of our current knowledge of \( U \) and the mass differences [3]. Atmospheric neutrino mixing is close to maximal, \( \theta_{23} \approx \pi/4 \), and corresponds to a large \( \Delta m_{\text{atm}}^2 \approx 2.5 \times 10^{-3} \text{eV}^2 \) whereas solar neutrino mixing is large but non-maximal, \( \theta_{12} \approx \pi/5 \), and corresponds to a small \( \Delta m_{\text{sol}}^2 = m_2^2 - m_1^2 \approx 8 \times 10^{-5} \text{eV}^2 \). The third mixing angle is known to be smaller than roughly \( \pi/15 \). Nothing is known about the mass scale, the mass ordering (sign of \( \Delta m_{\text{atm}}^2 \)) and the phases. Several interesting hints towards the structure of \( m_\nu \) can thereby be obtained, for instance the possibility of a \( \mu-\tau \) exchange symmetry [1, 4]. One other possible point of view is that a simple Abelian \( U(1) \) symmetry is directly or effectively working on \( m_\nu \). Conservation of a flavor charge is implied by the conservation of this \( U(1) \) and well-known cases are \( L_e \) [4] and \( L_\mu - L_\tau \) [7], which lead to a normal \( (m_3^2 \gg m_{1,2}^2) \) and inverted \( (m_2^2 \approx m_1^2 \gg m_3^2) \) mass hierarchy, respectively. Recently the case \( L_\mu - L_\tau \) has been found to be also possible [5]. A low energy mass matrix conserving \( L_\mu - L_\tau \) has the form

\[ m_\nu = m_0 \begin{pmatrix} a & 0 & 0 \\ \cdot & 0 & b \\ \cdot & \cdot & 0 \end{pmatrix} \]

and for \( a \approx b \) one is lead to quasi-degenerate light neutrinos, i.e., masses \( m_3 \approx m_2 \approx m_1 \equiv m_0 \lesssim \text{eV} \) much larger than the mass splittings. The neutrino mixing as predicted by the above matrix corresponds to \( \theta_{13} = \theta_{12} = 0 \) and \( \theta_{23} = \pi/4 \), which reflects the \( \mu-\tau \) symmetry
|\(\nu_e, e_L\) | \(\nu_\mu, \mu_L\) | \(\nu_\tau, \tau_L\) | \(\bar{N}_1, e_R\) | \(\bar{N}_2, \mu_R\) | \(\bar{N}_3, \tau_R\) | \(\Phi\) |
|---|---|---|---|---|---|---|
| 0 | 1 | -1 | 0 | 1 | -1 | 0 |

Table 1: Particle content and charge under the \(U(1)\) symmetry corresponding to \(L_\mu - L_\tau\). Here \(\Phi\) denotes the Higgs-doublet, which is responsible for the Dirac mass term.

inherent in a matrix conserving \(L_\mu - L_\tau\). We remark here that \(L_\mu\) and \(L_e - L_\mu - L_\tau\) do not possess \(\mu-\tau\) symmetry. Note further that besides \(\theta_{12} = 0\) also \(\Delta m^2_3 = 0\) holds. However, due to the quasi-degeneracy of the neutrinos, breaking of the symmetry with small parameters allows to easily overcome these shortcomings \[5, 8, 9\].

The flavor symmetry \(L_\mu - L_\tau\) can be incorporated in a see-saw model \[5\]. The relevant Lagrangian reads

\[
-\mathcal{L} = \bar{N}_i (m_D)_{ia}(\nu_\alpha)_L + \frac{1}{2} \bar{N}_i(M_R)_{ij} \bar{N}^c_j + h.c. \tag{5}
\]

Here the superscript \(^c\) denotes charge conjugation. The charge assignment of the particles under \(L_\mu - L_\tau\) is given in Table 1. As a consequence, the charged lepton mass matrix is diagonal and in terms of mass matrices, we have (with \(v_u = v \sin \beta\), \(v = 174\) GeV, \(\tan \beta\) the ratio of the up- and down-type Higgs doublets and \(M\) the high mass scale of the heavy singlets)

\[
m_D = v_u \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & d \end{pmatrix} \quad \text{and} \quad M_R = M \begin{pmatrix} X & 0 & 0 \\ \cdot & 0 & Y \\ \cdot & \cdot & 0 \end{pmatrix}. \tag{6}
\]

One eigenvalue of \(M_R\) has a mass \(MX\) and there is a Pseudo-Dirac pair with masses \(\pm MY\). The low energy neutrino mass matrix is given by

\[
m_\nu = -m_D^T M^{-1} m_D = \frac{v_u^2}{M} \begin{pmatrix} \frac{a^2}{X} & 0 & 0 \\ \cdot & 0 & b d \\ \cdot & \cdot & \frac{Y}{0} \end{pmatrix}. \tag{7}
\]

Note that the form of \(M_R\) corresponds to the form of \(m_\nu\) from Eq. (4). The parameters \(a, b, d, X, Y\) are allowed by the symmetry and are therefore naturally of order one. As mentioned above, we need to break the symmetry in order to reproduce a non-zero atmospheric mass squared difference and a non-zero solar neutrino mixing angle. In addition, as we will see, successful leptogenesis and the existence of Lepton Flavor Violating (LFV) charged lepton decays such as \(\mu \to e\gamma\) also require breaking terms. The possibilities to break the symmetry are numerous: we can
• break $L_\mu - L_\tau$ in the charged lepton sector. This will allow only the generation of $	heta_{12} \neq 0$ (note that large mixing has to be generated) and for LFV decays;

• break $L_\mu - L_\tau$ in $M_R$. This will allow only for $\theta_{12} \neq 0$, $\Delta m_\alpha^2 \neq 0$ and for leptogenesis. Breaking in $M_R$ has previously been analyzed in [5, 8, 9];

• break $L_\mu - L_\tau$ in $m_D$. This will allow for $\theta_{12} \neq 0$, $\Delta m_\alpha^2 \neq 0$, for leptogenesis and for LFV decays.

We conclude that breaking $L_\mu - L_\tau$ in $m_D$ is the most economic choice when one wants to generate all interesting observables. Of course, one would expect breaking in all possible sectors, but this will lead to little predictivity. For the sake of definiteness, we therefore consider only breaking in $m_D$.

In the following we will consider the case that the entries in $M_R$ are complex and in $m_D$ are real. The heavy neutrino mass matrix is

$$\frac{M_R}{M} = \left( \begin{array}{ccc} X e^{i\omega} & 0 & 0 \\ \cdot & 0 & Y e^{i\phi} \\ \cdot & \cdot & 0 \end{array} \right) = V_R^* M_R^{\text{diag}} V_R^\dagger \equiv P_R \tilde{V}_R Q_R \left( \begin{array}{ccc} X & 0 & 0 \\ \cdot & Y & 0 \\ \cdot & \cdot & Y \end{array} \right) Q_R \tilde{V}_R^T P_R, \quad (8)$$

where we have defined

$$\tilde{V}_R = \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ 0 & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{array} \right), \quad P_R = \left( \begin{array}{ccc} e^{i\omega/2} & 0 & 0 \\ 0 & e^{i\phi} & 0 \\ 0 & 0 & 1 \end{array} \right) \quad \text{and} \quad Q_R = \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & 1 \end{array} \right). \quad (9)$$

For real entries in $M_R$ the matrix $P_R$ is the unit matrix. When the breaking of $L_\mu - L_\tau$ takes place only in $m_D$ we can quantify this as

$$m_D = v_u \left( \begin{array}{ccc} a & \epsilon_1 & \epsilon_2 \\ \eta_1 & b & \epsilon_3 \\ \eta_2 & \eta_3 & d \end{array} \right), \quad (10)$$

with $\epsilon_i, \eta_i \ll 1$. For symmetric $m_D$ it would hold that $\epsilon_i = \eta_i$. With real entries in the Dirac mass matrix, there is only one physical phase, namely $\omega - \phi$. Consequently, low and high energy $CP$ violation will be intimately related. We are therefore allowed to set $\phi$ to zero and keep only the phase $\omega$. In the remaining part of this Section we will give the relevant expressions for the general form of $m_D$ from Eq. (10), before considering more minimal braking scenarios in the next Section.

The parameters and the breaking are introduced at high scale. Consequently, radiative corrections, both below and in between the see-saw scales, can have impact on the results.
It has been shown in Ref. [8], however, that in the see-saw model basing on $L_\mu - L_\tau$ typically only $\theta_{12}$ gets corrected and that $\theta_{23}$ and $|U_{e3}|$, on which we later focus on, witness only little effects. Moreover, the textures of the mass matrices do only slightly change, i.e., small perturbations (over which we will scan numerically) remain small. We therefore neglect radiative effects, which should be a good approximation for our purposes.

In supersymmetric frameworks with universal boundary conditions there is an important possibility to probe the see-saw parameters, namely lepton flavor violating decays of charged leptons [10]. In the leading-log approximation one can obtain for the branching ratios of the decays $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu \gamma$ the following formula [10]:

$$B(\ell_i \to \ell_j \gamma) \simeq \frac{\alpha_{em}^2}{G_F m_S^2 v^4} \left| \frac{(3m_0^2 + A_0^2)}{8\pi^2} \right|^2 \left| \left( \tilde{m}_D^\dagger L \tilde{m}_D \right)_{ij} \right|^2 \tan^2 \beta , \quad (11)$$

where $\ell_i = e, \mu, \tau$ for $i = 1, 2, 3$. Here $m_0$ is the universal scalar mass, $A_0$ the universal trilinear coupling parameter, $m_S$ represents a SUSY particle mass and $L = \ln \delta_{ij} M_i/M_X$, with $M_i$ the heavy Majorana masses and $M_X = 2 \cdot 10^{16}$ GeV. The branching ratios have to be evaluated in the basis in which the heavy Majorana neutrinos are real and diagonal. To get into this basis we have to rotate $m_D$ to obtain $\tilde{m}_D$. Having defined the diagonalization of $M_R$ in Eq. (8) as $M_R = V^R_M M^\text{diag}_R V_R^T$, then

$$m_D \to \tilde{m}_D = V_R^T m_D . \quad (12)$$

At 90% C.L., the current limit on the branching ratio of $B(\mu \to e\gamma)$ is $1.2 \cdot 10^{-11}$ [11] and future improvement by two orders of magnitude is expected [12]. In most of the relevant soft SUSY breaking parameter space, the expression $m_S^2 \simeq 0.5 m_0^2 m_{1/2}^2 (m_0^2 + 0.6 m_{1/2}^2)^2$, with $m_{1/2}$ being the universal gaugino mass, is an excellent approximation to the results obtained in a full renormalization group analysis [13]. Apparently, the branching ratios depend crucially on the SUSY masses. We choose here to use as examples the SPS benchmark points from Ref. [14] as given in Table 2. Denoting $(3m_0^2 + A_0^2)^2/m_S^2$ with $1/m_4^2$, we can write

$$B(\mu \to e\gamma) \simeq 1.2 \cdot 10^{-9} \left( \frac{200 \text{ GeV}}{m_S} \right)^4 \left| \left( \tilde{m}_D^\dagger L \tilde{m}_D \right)_{21} \right|^2 \frac{1}{v_u^4} \tan^2 \beta , \quad (13)$$

which has to be smaller than $10^{-11}$. As we will see below, this can constrain the way $L_\mu - L_\tau$ should be broken.

It proves useful to consider also the double-ratios

$$\frac{B(\mu \to e + \gamma)}{B(\tau \to e + \gamma)} \simeq \left| \frac{\left( \tilde{m}_D^\dagger L \tilde{m}_D \right)_{21}}{\left( \tilde{m}_D^\dagger L \tilde{m}_D \right)_{31}} \right|^2 \quad \text{and} \quad \frac{B(\mu \to e + \gamma)}{B(\tau \to \mu + \gamma)} \simeq \left| \frac{\left( \tilde{m}_D^\dagger L \tilde{m}_D \right)_{21}}{\left( \tilde{m}_D^\dagger L \tilde{m}_D \right)_{32}} \right|^2 , \quad (14)$$

which are essentially independent on the SUSY parameters.
Table 2: SPS benchmark values for the mSUGRA parameters according to Ref. [14]. The values of $m_0$, $m_{1/2}$ and $A_0$ are in GeV.

| Point | $m_0$  | $m_{1/2}$ | $A_0$  | $\tan \beta$ |
|-------|--------|-----------|--------|-------------|
| 1a    | 100    | 250       | −100   | 10          |
| 1b    | 200    | 400       | 0      | 30          |
| 2     | 1450   | 300       | 0      | 10          |
| 3     | 90     | 400       | 0      | 10          |
| 4     | 400    | 300       | 0      | 50          |
| 5     | 150    | 300       | −1000  | 5           |

With the most general breaking structure in $m_D$ given in Eq. (10) and with using $L_3 = L_2$, the off-diagonal entries of $\tilde{m}_D^\dagger L \tilde{m}_D$ read

\[
\begin{align*}
(\tilde{m}_D^\dagger L \tilde{m}_D)_{12} &= a \epsilon_1 L_1 + (b \eta_1 + \eta_2 \eta_3) L_2 , \\
(\tilde{m}_D^\dagger L \tilde{m}_D)_{13} &= a \epsilon_2 L_1 + (d \eta_2 + \epsilon_3 \eta_1) L_2 , \\
(\tilde{m}_D^\dagger L \tilde{m}_D)_{23} &= \epsilon_1 \epsilon_2 L_1 + (b \epsilon_3 + d \eta_3) L_2 .
\end{align*}
\] (15)

If $L_\mu - L_\tau$ would be broken only in the heavy neutrino sector (as in Refs. [5, 8, 9]), then $\tilde{m}_D^\dagger L \tilde{m}_D$ would be diagonal and the decays would be extremely suppressed. If we break $L_\mu - L_\tau$ only in the charged lepton sector, then $\tilde{m}_D = m_D U_\ell$, where $U_\ell$ diagonalizes the (now non-diagonal) charged lepton mass matrix. In this case $\tilde{m}_D^\dagger L \tilde{m}_D$ will have off-diagonal entries, but leptogenesis, to be discussed in the next paragraph, will not be possible.

Another very helpful and interesting aspect of see-saw models is the possibility to generate the baryon asymmetry of the Universe with the help of the leptogenesis mechanism [15]. In the case of thermal leptogenesis the baryon asymmetry is given by (for a review see, e.g., [16])

\[
\eta_B = \frac{n_B}{n_\gamma} \simeq -1.04 \cdot 10^{-2} \kappa \epsilon_1 ,
\] (16)

where $\epsilon_1$ is the $CP$-violating asymmetry in the decay of the lightest right-handed Majorana neutrino $N_1$ having the mass $M_1$, and $\kappa$ is an efficiency factor calculated by solving the Boltzmann equations. A simple approximate expression for the efficiency factor $\kappa$ in the case of thermal leptogenesis was given in [14]:

\[
\frac{1}{\kappa} \simeq \frac{3.3 \cdot 10^{-3} \text{ eV}}{\tilde{m}_1} + \left( \frac{\tilde{m}_1}{0.55 \cdot 10^{-3} \text{ eV}} \right)^{1.16} ,
\] (17)

where the important parameter $\tilde{m}_1$ is given by

\[
\tilde{m}_1 \equiv \frac{(\tilde{m}_D \tilde{m}_D^\dagger)_{11}}{M_1} .
\] (18)
The CP-violating decay asymmetry $\varepsilon_i$ has the form (with $x_j = M_j^2/M_i^2$):

$$\varepsilon_i = \frac{1}{8\pi v_i^2} \frac{1}{(\tilde{m}_D \tilde{m}_D^\dagger)_{ii}} \sum_{j \neq i} \text{Im} \left\{ \left( \tilde{m}_D \tilde{m}_D^\dagger \right)_{ji} \left( \frac{2}{1 - x_j} - \ln \left( \frac{1 + x_j}{x_j} \right) \right) \right\}. \quad (19)$$

For neutrinos close in mass the (self-energy) term proportional to $(1 - x_j)^{-1}$ dominates [18].

It is important to note here that the Pseudo-Dirac pair of mass $Y M$ generates no decay asymmetry. The decay asymmetry is therefore generated by the decay of the neutrino with mass $M X$. It holds that $\tilde{m}_D \tilde{m}_D^\dagger = V_T R m_D m_D^\dagger V^R$. In case of $X \sim Y$ we have

$$\varepsilon_X \simeq -\frac{1}{4\pi} \frac{1}{a^2 + \epsilon_1^2 + \epsilon_2^2} \frac{1}{Y/X - X/Y} \frac{2}{X} \left( b \epsilon_1 + a \eta_1 + \epsilon_2 \epsilon_3 \right) \sin \omega \quad (20)$$

and

$$\tilde{m}_1 = \frac{v_2^2}{M} \frac{a^2 + \epsilon_1^2 + \epsilon_2^2}{X}. \quad (21)$$

Note that for no breaking of $L_\mu - L_\tau$ (i.e., $\eta_i = \epsilon_i = 0$) the decay asymmetry vanishes. In addition, if we break the symmetry only in the charged lepton sector we would have no decay asymmetry either, because $\tilde{m}_D \tilde{m}_D^\dagger$ would remain diagonal.

Numerically, $\eta_B$ should be given by $6 \cdot 10^{-10}$ [19], where the small error is on the 5% level. The formalism described above has however several sources of uncertainty. First, recall that expression (17) holds only for hierarchical heavy neutrinos. The wash-out effect of the neutrinos with mass $\pm Y M$ is therefore not properly taken into account. Second, it has recently been realized [20] that flavor effects in leptogenesis can significantly affect the results. Taking these issues into account would require a thorough study and solution of the Boltzmann equations, which is surely beyond the scope of this letter. Instead, when we in the next Section calculate the baryon asymmetry for a specific breaking scenario, we consider the calculation as successful, when the result is $4 \cdot 10^{-10} \leq \eta_B \leq 8 \cdot 10^{-10}$, which is presumably still a very conservative range.

2 Breaking of $L_\mu - L_\tau$: Concrete Examples

Up to now we gave the relevant expressions for the Dirac mass matrix from Eq. (10), i.e., we used the most general breaking scenario. With six arbitrary breaking parameters in $m_D$, however, there is little predictive power in what regards the observables and in order to make interesting statements more simplification is needed. We therefore turn to minimal breaking scenarios in the sense of having as few parameters as possible. To constrain the possibilities even more, we require the presence of both low and high energy CP violation. If there is low energy CP violation in oscillation experiments can be checked most easily by

\footnote{For extremely degenerate and therefore fine-tuned heavy neutrinos one should use a different formula [18].}
calculating the following invariant \[21\], to which any \(CP\) violation in neutrino oscillations has to be proportional:

\[
\text{Im} \{ h_{12} h_{23} h_{31} \} = \Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2 J_{CP} ,
\]

where \( h = m^\dagger \nu m \) and \( J_{CP} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta \).  

With only one perturbative parameter in \( m_D \), this expression always vanishes. Hence, we should analyze scenarios with two non-zero perturbations in \( m_D \), for which there are 15 possibilities. Except for one case, low energy mass matrices with one or two zeros are generated. Only one of them is ruled out by neutrino data, namely when the 23 and 32 elements of \( m_D \) are filled with non-zero entries. A low energy mass matrix with zeros in the 12 and 13 element would result, which can not reproduce the data \[22\]. More cases can be ruled out when we require the presence high energy \(CP\) violation, i.e., leptogenesis. Recall that the decay asymmetry is proportional to \((b \epsilon_1 + a \eta_1 + \epsilon_2 \epsilon_3)(d \epsilon_2 + a \eta_2 + \epsilon_1 \eta_3)\). Asking this expression to be non-zero rules out 8 more cases, leaving us with 6 remaining ones. In what regards the results of the breaking scenarios, we are interested in particular in the branching ratios of the LFV decays, some of which will be forbidden by certain scenarios. We have summarized in Table 3 all 15 possibilities together with their predictions for the branching ratios of the LFV processes, for low energy \(CP\) violation, for \( \eta_B \), and with a correlation for the oscillation parameters as obtained in \[22, 23\]. All obtained cases with two zeros are only possible for quasi-degenerate neutrinos \[22\]. Cases with one zero entry are in general possible also for other allowed mass hierarchies \[23\], but here we focus only on quasi-degenerate neutrinos. The one-zero matrices always come together with zero \( \eta_B \) and are disregarded anyway. From the six cases allowing for a non-zero baryon asymmetry one case has no correlation for the low energy observables and all branching ratios are non-zero. Four cases generating two zeros in the low energy mass matrix have indistinguishable neutrino phenomenology, but differ in the predictions for the branching ratios, except for 2 cases which predict identical results. From the six matrices allowing for leptogenesis, five also predict the decay \( \mu \rightarrow e\gamma \). We would like to remark here that some of the 15 possibilities have the amusing feature that there is low energy \(CP\) violation but no leptogenesis. There are no cases in which it is the other way around. Note finally that the rate for neutrinoless double beta decay (which is proportional to the \( ee \) element of \( m_\nu \)) is always non-zero.

Let us discuss one example in detail, namely the following form of \( m_D \) and the resulting low energy mass matrix \( m_\nu \):

\[
m_D = v_u \begin{pmatrix} a & \epsilon_1 & 0 \\ 0 & b & 0 \\ 0 & \eta_3 & d \end{pmatrix} \Rightarrow m_\nu = -e^{-i\omega} \frac{\bar{\nu}_u}{M} \begin{pmatrix} \frac{a^2}{X} & \frac{a \epsilon_1}{X} & 0 \\ \frac{\epsilon_1^2}{X} + 2 \frac{b \eta_3}{Y} e^{i\omega} & \frac{b d}{Y} e^{i\omega} & 0 \\ 0 & 0 & 0 \end{pmatrix} .
\]

The \( ee \) and the \( \mu\tau \) elements are allowed by \( L_\mu - L_\tau \) and, as it should, the additional non-zero entries are suppressed by the small breaking parameters. The expressions relevant for
high and low energy $CP$ violation are

$$\varepsilon_X \simeq -\frac{1}{2\pi} \frac{1}{Y/X - X/Y} \frac{b}{a^2} \epsilon_1^2 \eta_3 \sin \omega$$

and

$$\text{Im} \{h_{12} h_{23} h_{31}\} = \left(\frac{v_u^2}{M}\right)^6 \left(\frac{a^4 b^3 d^2}{X^3 Y^3}\right) \epsilon_1^2 \eta_3 \sin \omega.$$  \hfill (24)

The decay $\tau \rightarrow e\gamma$ is forbidden, whereas the branching ratio for $\mu \rightarrow e\gamma$ ($\tau \rightarrow \mu\gamma$) is proportional to $|a \epsilon_1 L_1|^2$ ($|d \eta_3 L_2|^2$). In what regards these LFV decays, let us return to Eqs. (13) and (15). Given the fact that $a$ is of order one, it is apparent that $|\langle \tilde{m}^U_D L \tilde{m}^D \rangle_{12}|^2/v_u^4$ is of the order of $\epsilon_1^2 L_1^2 \sim 10 \epsilon_1^2$. From Eq. (13) we see that for typical values of $\tilde{m}_S \simeq 200$ GeV and $\tan^2 \beta \simeq 10^2$, the branching ratio for $\mu \rightarrow e\gamma$ is roughly given by $10^{-6} \epsilon_1^2$. This indicates small values of $\epsilon_1$, which however also decreases the decay asymmetry parameter $\varepsilon_1$, which is relevant for leptogenesis. With this crude estimate we can see that the requirement of successful leptogenesis makes the branching ratio of $\mu \rightarrow e\gamma$ in general rather large, thereby snookering such scenarios. In principle one could let the heavy neutrino masses be extremely degenerate, so that the decay asymmetry is large even for small perturbative parameters, but this is regarded as fine-tuning. The underlying reason for the potentially too large branching ratios (for more model-independent analyzes, see for instance [24]) is that the entries allowed by the symmetry in $m_D$ are all of order one. It is therefore a generic issue of the framework.

We next perform a numerical search for successful parameters $a, b, d, X, Y$ (which are required to be of order one) and for the two perturbative parameters (which are required to be at least one order of magnitude smaller). The neutrino oscillation observables are required to lie within their $3\sigma$ ranges from Ref. [3]. We also demand $1 - X/Y \geq 0.1$ so that the heavy neutrinos are not too close in mass, i.e., Eq. (19) can still be used. We checked that the corrections to Eq. (19) are indeed subleading in this case. The upper left plot in Fig. 1 shows $B(\mu \rightarrow e\gamma)$ against $\eta_B$ for the SPS benchmark points 1a, 2 and 5. It turns out that points 1a and 1b generate practically identical results, and also points 2 and 3 are indistinguishable. The results for point 4 lie between points 2 and 5. The correlation between $\eta_B$ and $B(\mu \rightarrow e\gamma)$ is rather strong because both $\varepsilon_X$ and the branching ratio are proportional to $\epsilon_1^2$. The upper right plot shows the ratio of the two non-zero branching ratios, which is below one for successful leptogenesis. We included the current and a future bound on the branching ratio and also indicated how many points lie in the range $4 \cdot 10^{-10} \leq \eta_B \leq 8 \cdot 10^{-10}$. Except for the SPS point 2, which includes TeV scale parameters, $B(\mu \rightarrow e\gamma)$ is typically too large$^2$. As mentioned before, reducing the order of magnitude of the small perturbative parameters will strongly reduce $\eta_B$. A way to evade this problem is either to assume the SUSY parameters to be very large or to assume a breaking scheme of $L_\mu - L_\tau$ with zero $B(\mu \rightarrow e\gamma)$.

$^2$We remark that point 5 leads to a too small Higgs mass anyway [25].
Such an example is\(^3\)
\[
m_D = v_u \begin{pmatrix} a & 0 & \epsilon_2 \\ 0 & b & \epsilon_3 \\ 0 & 0 & d \end{pmatrix} \Rightarrow m_\nu = -e^{-i\omega} \frac{v_u^2}{M} \begin{pmatrix} a^2 & 0 & a\epsilon_2 \\ 0 & b d & e^{i\omega} \\ \cdot & \cdot & \frac{\epsilon^2}{X} + 2d\epsilon_3 e^{i\omega} \end{pmatrix}. \tag{25}
\]
This example has no decay \(\mu \rightarrow e\gamma\), and the branching ratio for \(\tau \rightarrow e\gamma\) \((\tau \rightarrow \mu\gamma)\) is proportional to \(|a\epsilon_2 L_1|^2 \left( |b\epsilon_3 L_2|^2 \right)\). \(CP\) violation is governed by
\[
\varepsilon_X \simeq -\frac{1}{2\pi} \frac{1}{Y/X - X/Y} \frac{d}{a^2} \epsilon_2 \epsilon_3 \sin \omega \quad \text{and} \\
\Im \{h_{12} h_{23} h_{31}\} = - \left( \frac{v_u^2}{M} \right)^6 \frac{2a^4 d^3 b^2}{X^3 Y^3} \epsilon_2^2 \epsilon_3 \sin \omega. \tag{26}
\]
The lower left plot of Fig.1 shows \(B(\tau \rightarrow \mu\gamma)\) against \(\eta_B\). We included the current \((6.8 \cdot 10^{-8} \text{ [26]}\) and a future bound \((5 \cdot 10^{-9}, \text{ see [25]}\) on the branching ratio and also indicated how many points lie in the range \(4 \cdot 10^{-10} \leq \eta_B \leq 8 \cdot 10^{-10}\). We see that \(\tau \rightarrow \mu\gamma\) lies in an observable range unless the SUSY masses are in the TeV range. The correlation between \(\eta_B\) and the branching ratio is weaker than in the previous example, because \(\varepsilon_X \propto \epsilon_3\) but \(B(\tau \rightarrow \mu\gamma) \propto \epsilon_3^2\). The lower right plot shows the ratio of the two non-zero branching ratios, which is above one.

It is of course possible to diagonalize the mass matrices Eqs. (25, 26) and express the observables in terms of the parameters appearing in \(m_D\) and \(M_R\), but the resulting expressions are rather cumbersome and little insight is gained. We rather note that from the condition that the \(e\tau\) and \(\tau\tau\) entries (or the \(e\mu\) and \(\mu\mu\)) vanish, one can obtain \([22]\)
\[
||U_{e3}|| \cos \delta \tan 2\theta_{23} \simeq \frac{\Delta m^2_{\odot}}{2\Delta m^2_{\text{AA}}} \sin 2\theta_{12}. \tag{27}
\]
Since \(\Delta m^2_{\odot}/\Delta m^2_{\text{AA}} \ll 1\), this expression means that \(\theta_{23}\) can not be exactly maximal: \(\sin^2 \theta_{23} \neq \frac{1}{2}\). Moreover, if \(|U_{e3}|\) is sizable then \(\cos \delta\) must be small, and therefore large \(CP\) violation is expected in this case: \(J_{CP} \simeq |U_{e3}|/4\). These features are nicely illustrated in Fig.2 where we have plotted \(|U_{e3}|\) against \(J_{CP}\) and against \(\sin^2 \theta_{23}\). Atmospheric neutrino mixing can not be exactly maximal and if \(|U_{e3}|\) is large, \(CP\) violation is also large. Identical results occur for Eq. (23). Another interplay of variables occurs when \(\theta_{23}\) is close to maximal. This implies again from Eq. (27) that \(\cos \delta\) is small and \(J_{CP}\) is large. Large \(J_{CP}\), in turn, implies from Eq. (26) that the decay asymmetry is large, because both \(\varepsilon_X\) and \(J_{CP}\) are proportional to \(\sin \omega\). Hence, the closer \(\theta_{23}\) is to \(\pi/4\), the smaller becomes \(\eta_B\). This is illustrated in Fig.3. We indicated the values \(\sin^2 \theta_{23} = 0.45\) and 0.55, which are the approximate lower and upper limits in order to still have successful leptogenesis.

We stress here that both examples, Eqs. (23, 25), predict basically identical neutrino phenomenology, but differ dramatically in their predictions for the LFV decays.

\(^3\)The remaining three cases with interesting correlations of the neutrino observables are found to be very fine-tuned, i.e., the numerical search for successful parameter values hardly finds any points.
3 Summary and Conclusions

A supersymmetric see-saw model obeying the flavor symmetry $L_\mu - L_\tau$ was analyzed. In the low energy sector this generates quasi-degenerate neutrinos, vanishing $\theta_{13}$ and maximal atmospheric neutrino mixing. With strict conservation of the symmetry both leptogenesis and LFV are not possible and in addition $\theta_{12}$ and the atmospheric $\Delta m^2$ is zero. Possibilities to break $L_\mu - L_\tau$ were considered and it was found that the most economic possibility is to include breaking only in $m_D$. Two small breaking parameters are required in order to allow for low energy $CP$ violation. Generation of the baryon asymmetry via leptogenesis is possible with heavy neutrino masses of similar size. We discussed how the breaking of the symmetry reflects in low energy observables, and in particular in the predictions for the LFV decays $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$. Scenarios with indistinguishable neutrino phenomenology can lead to drastically different relations between the branching ratios.

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| $m_D$ | $m_\nu$ | $\mu \rightarrow e\gamma$ | $\tau \rightarrow e\gamma$ | $\tau \rightarrow \mu\gamma$ | $J_{CP}$ | $\eta_B$ | Correlation |
|-------|-------|----------------|----------------|----------------|---------|--------|-------------|
| $a \ e_1 \ e_2$ | 0 0 d | x x | x | x | x | x | if QD; sin $\alpha = 0$ ⇒ $\langle m \rangle \simeq m_0$ |
| 0 0 c3 | 0 0 d | | | | | | |
| $a \ e_1$ | 0 0 d | x | 0 | x | x | 0 | QD; both orderings $|U_{e3}| \simeq R \left| \frac{\cot 2\theta_{23}}{\cot 2\theta_{13}} \right| \sin 2\theta_{12}$ |
| 0 0 c3 | 0 0 d | | | | | | |
| $a \ e_1$ | 0 0 d | x | x | x | x | x | QD; both orderings $|U_{e3}| \simeq R \left| \frac{\cot 2\theta_{23}}{\cot 2\theta_{13}} \right| \sin 2\theta_{12}$ |
| 0 0 c3 | 0 0 d | | | | | | |
| $a \ e_1$ | 0 0 d | x | x | 0 | x | x | QD; both orderings $|U_{e3}| \simeq R \left| \frac{\cot 2\theta_{23}}{\cot 2\theta_{13}} \right| \sin 2\theta_{12}$ |
| 0 0 c3 | 0 0 d | | | | | | |
| $a \ e_1$ | 0 0 d | x | x | x | x | x | QD; both orderings $|U_{e3}| \simeq R \left| \frac{\cot 2\theta_{23}}{\cot 2\theta_{13}} \right| \sin 2\theta_{12}$ |
| 0 0 c3 | 0 0 d | | | | | | |
| $a \ e_2$ | 0 0 d | x | x | 0 | x | x | QD; both orderings $|U_{e3}| \simeq R \left| \frac{\cot 2\theta_{23}}{\cot 2\theta_{13}} \right| \sin 2\theta_{12}$ |
| 0 0 c3 | 0 0 d | | | | | | |
| $a \ e_2$ | 0 0 d | x | x | 0 | x | x | QD; both orderings $|U_{e3}| \simeq R \left| \frac{\cot 2\theta_{23}}{\cot 2\theta_{13}} \right| \sin 2\theta_{12}$ |
| 0 0 c3 | 0 0 d | | | | | | |
| $a \ e_2$ | 0 0 d | x | x | 0 | x | x | QD; both orderings $|U_{e3}| \simeq R \left| \frac{\cot 2\theta_{23}}{\cot 2\theta_{13}} \right| \sin 2\theta_{12}$ |
| 0 0 c3 | 0 0 d | | | | | | |
| $a \ e_2$ | 0 0 d | x | x | 0 | x | x | QD; both orderings $|U_{e3}| \simeq R \left| \frac{\cot 2\theta_{23}}{\cot 2\theta_{13}} \right| \sin 2\theta_{12}$ |
| 0 0 c3 | 0 0 d | | | | | | |
| $a \ e_2$ | 0 0 d | x | x | 0 | x | x | QD; both orderings $|U_{e3}| \simeq R \left| \frac{\cot 2\theta_{23}}{\cot 2\theta_{13}} \right| \sin 2\theta_{12}$ |
| 0 0 c3 | 0 0 d | | | | | | |
| $a \ e_2$ | 0 0 d | x | x | 0 | x | x | QD; both orderings $|U_{e3}| \simeq R \left| \frac{\cot 2\theta_{23}}{\cot 2\theta_{13}} \right| \sin 2\theta_{12}$ |
| 0 0 c3 | 0 0 d | | | | | | |
| $a \ e_2$ | 0 0 d | x | x | 0 | x | x | QD; both orderings $|U_{e3}| \simeq R \left| \frac{\cot 2\theta_{23}}{\cot 2\theta_{13}} \right| \sin 2\theta_{12}$ |
| 0 0 c3 | 0 0 d | | | | | | |
| $a \ e_2$ | 0 0 d | x | x | 0 | x | x | QD; both orderings $|U_{e3}| \simeq R \left| \frac{\cot 2\theta_{23}}{\cot 2\theta_{13}} \right| \sin 2\theta_{12}$ |
| 0 0 c3 | 0 0 d | | | | | | |

Table 3: Dirac mass matrices with two non-zero breaking parameters, the resulting low energy mass matrix $m_\nu$, the implications for $\ell_j \rightarrow \ell_i \gamma$, for low energy $CP$ violation, for $\eta_B$, and a correlation of the neutrino observables resulting from the form of $m_\nu$. QD means quasi-degenerate neutrinos with a common mass scale $m_0$ and $R$ is defined as $\Delta m^2_{\odot}/\Delta m^2_{\text{atm}}$. 

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Figure 1: Magnitude of a LFV decay and the ratio of the non-zero branching ratios against the baryon asymmetry. The two upper plots are for Eq. (23) and the two lower plots are for Eq. (25).
Figure 2: Neutrino oscillation observables for Eq. (25). Plotted is $|U_{e3}|$ against $J_{CP}$ and against $\sin^2 \theta_{23}$. Atmospheric neutrino mixing can not be exactly maximal and if $|U_{e3}|$ is large, CP violation is also large. The results for Eq. (23) are identical.

Figure 3: Atmospheric neutrino oscillation observable $\sin^2 \theta_{23}$ against the baryon asymmetry $\eta_B$ for Eq. (25). The closer $\theta_{23}$ is to $\pi/4$, the smaller becomes $\eta_B$. 