Growth Dynamics of Value and Cost Trade-off in Competitive Temporal Networks

Sheida Hasani\textsuperscript{a}, Razieh Masoomi\textsuperscript{b}, Jamshid Ardalankia\textsuperscript{c}, Mohammadbashir Sedighi\textsuperscript{a}, Hamid Jafari\textsuperscript{b}

\textsuperscript{a}Department of Management, Science and Technology, Amirkabir University of Technology, Tehran, Iran
\textsuperscript{b}Department of Physics, Shahid Beheshti University, G.C., Evin, Tehran, 19839, Iran
\textsuperscript{c}Department of Financial Management, Shahid Beheshti University, G.C., Evin, Tehran, 19839, Iran

Abstract

The question is: What does happen to the real-world networks which cause them not to grow permanently? The idea here is that real-world networks have to pay the cost of growth. We investigate the growth and trade-off between value and cost in the networks with cost and preferential attachment together. Since, the preferential attachment in BA model does not consider any stop against the infinite growth of networks, we introduce a modified version of preferential attachment of BA model. This idea makes sense because the growth of real networks may be finite. In the present study, by combining preferential attachment in the science of temporal networks (interval graphs), and, the first order differential equations of value and cost of making links, future equilibrium of an evolving network is illustrated. During the process of achieving a winning position, the variables against growth such as the competition cost, besides with the internally structural cost may emerge. At the end, by applying this modified model, we found the circumstances which a trade-off between value and cost emerge.

Keywords: Temporal Network, Cost and Value Trade-off
1. Introduction

In the real world, networks inevitably pay the cost of growth. That is why any event corresponding to growth is not out of charge. Hence, real networks may not have infinite growth. A phenomenon like friction against growth -say cost of making new attachments- emerges in the networks. In this study, we investigate the growth dynamics of networks which emerge trade-off between value and cost. Firstly, we consider a network which its growth is affected by preferential attachment \([1-5]\) based on node activity \([6]\). The value and the cost of the aforementioned network are separately described by first order coupled differential equations, so we obtain a modified preferential attachment of Barabasi-Albert model. Applying this modified model to such a network helps to clarify and predict the circumstances which the network stops growing or continues to grow, or, there may exist some trade-off. As a background corresponding to present research, Jafari et al\([6]\) considered two types of drivers which play role in the network’s dynamics such as node activity and also memory effects. Indeed, memory and cost play the role of friction against the growth of network size. They found a critical time-scale, so-called ‘characteristic time’, which the attachment regime alters. Besides this, they found that the high event-wise temporal density -which implies higher cumulative degree, or similarly, the cumulative sum of past participation of a node in making links- cause more distinct and distinguishable critical time.

Other methods for creating links are popular. Some of these are selecting adequate connections to intensify collaboration among the weaker agents and also connection based on networks’ preferences not just nodes’ preferences, directed or undirected \([7]\), peripheral to peripheral nodes, peripheral to central nodes, central to peripheral nodes, central to central nodes in competing networks and increasing centrality in an individual sub-network \([8]\). Also Zhang et al \([9]\) considered the influential nodes based on their participation in arising the global efficiency of the complex network.

Also, some other researchers studied on the networks with accelerating growth
Among them, Safdari et al. [13] and Liu et al. [10] investigated the accelerating growing networks (AGN) together with the aging effects by applying a factor to nodes with older time stamps. In addition, Ikeda [12] focused on circumstances which the rate of accelerating growth in the network contributes to scale-free networks and investigated some accelerating scenarios and showed nonlinearity between the rate of adding new nodes and the growth of the network. Zhao et al. [14] investigated the evolutionary of instability of financial markets.

From a microscopic point of view, on the contrary to investigation on the evolution of eigen constituents for extracting information of dynamic behaviors and trends reversal [15, 16]. We introduce differential equations for the drivers (value and cost). Due to microscopic and internal dynamics (value and cost) in a competitive atmosphere, each node interacts with the newcomer links. In comparison with Iranzo et al. [7], we evaluate the behaviors of possible wandered links of the new comer temporal nodes without collaboration with the hosted network. Accordingly, the networks here are considered permanently growing, dead, or living in a trade-off among negative growth and permanent positive growth. This idea makes sense because in a non-collaborative economic world [17] constituents seek to maximize their utility. Furthermore, in the investment world, cost -as a barrier against networks’ growth- can be interpreted as the attraction of parallel markets and opportunities. Hence, the investment opportunities without considering other parallel markets does not make sense, and also, evaluating corporate strategies without considering customers’ earned value and cost aspects in a market is not precise.

2. Value and Cost Trade-off and Competing Communities

Some researchers [7, 17–19] investigated temporal network’s topology in financial and marketing atmosphere. To do this, Grauwin et al. [17] and Iranzo et al. [7] investigated networks containing sub-networks which include strong (high centrality) and weak (low centrality) networks in a game of achieving the
strength (more centrality) and resulted that under circumstances based on Nash equilibrium and eigenvector centrality, the strongest is exposed to get weaker in a temporal process. The trade-off for being the strongest besides bearing the threats (such as a situation which weaker competitors cooperate with each other to overcome a certain stronger network) leads to a characteristic time. So, in the fate of the competition, dominant community is in danger of some disturbing dynamic costs such as:

- Originated internally (like costly structures or conflict of inner interests due to *Agency Theory* in some growth level of financial [20–23], and social and behavioral systems [24–26]);
- Or, originated externally (like competitors changing strategy and underestimating the collective profitability of the united weaker ones [7] in *statue quo*).

A latent factor of these dynamics is agents’ utility function which in some scales will finally form the future state of the networks [17, 27].

In our study, we discuss on 3 main concepts such as:

- growth failure at the beginning;
- permanent growth;
- trade-off between stop or continue to grow.

### 3. Phase Transitions and Percolation Theory

Potential dynamics of interests in such systems emerge due to collective and collaborative effects -or simultaneously- emerge due to individual nature of links and sub-networks. This spectrum [17] on one side, is supposed to maximize its own utility in a selfish manner, and on the other side of the spectrum, is supposed to minimize the global energy of the system in a cooperative and efficient manner. Based on the arrangement of constituents’ interests among the spectrum of selfish-cooperative trade-off, transition phase between growth and barriers against growth (like *cost*) appears. Accordingly, while constituents try to reach the highest satisfaction, they may no longer face with their maximum satisfaction, as Schelling’s segregation model proves [17].
Actually, in such networks, pairwise interactions among agents should be replaced by interaction networks to reveal integrated behaviors. To illustrate phase transitions and the interactions’ strength of linked clusters in a random graph, percolation theory is fruitful. Also, by percolation theory in transition time of networks, the giant components may dominate the general behavior of the network against attacks and diffusion [28]. Hence, the transition and breakdown in the networks depend on the giant components’ critical behavior, the integrated behaviors of clusters, and the global efficiency in networks.

Dorogovtsev and Mendes, sec. 11.3 [11], by applying percolation theory thresholds, investigated the effects of sudden intentional random attack and damage (preferential elimination of nodes with highest degree which means more exposure to be targeted) in an undirected network. In this way, a certain network grows and are suddenly exposed to permanent damage. Hence, this approach is the giant component’s preferential damage against preferential attachment. The calculation of the random damage threshold (which leads to network’s breakdown) in scale-free networks was initially evaluated by Cohen et al [29]. They analytically showed that by applying a preferential removal of a fraction of nodes in scale-free networks, the percolation transition of breakdown will be dependent on the power-law and network size. Interestingly, in the percolation transition, the effects of different power-laws are more significant than network-size effects. Another approach for investigating breakdown’s predecessors is working on the phenomenon of removing links rather than removing influential nodes [9]. Our approaches toward the mentioned question is somehow different. On the contrary to a discrete evaluation of just failure or just growing, we are supposed to evaluate the trade-off between values and costs of attachments.

Indeed, a question arises here. Why do we seek value and cost of creating links in a network? A description for this approach relates to investment behavior and marketing strategies, and as a whole, a trade-off between possible scenarios in multi-agent situations. When a firm applies a platform, by passing time, managers may find it not cost effective and may leave it. What does the
term “cost effective” mean? indeed, managers may find that the cost is more significant rather than the earned value. This consciousness has a great deal of importance. Although, as soon as the links get to know that the cost of presence in the network is more than the value which is earned, they will no longer be motivated for presence in the network and they are not likely to attach it. In other words, once the cost gets larger than value, the whole network stops growing. So, with the help of knowledge about value and cost behaviors, one can describe possible scenarios of network growth.

4. Methodology

The main concept behind Barabasi-Albert model is preferential attachment which states that nodes with more links are more exposed to be joined by new links [3, 4]. Initially, there exist $N$ nodes with the initial Node Activity of $k_0$. For creation of an evolving process, at each time step $m$ nodes among $N$ nodes are selected based on a uniform random distribution. Then, each of the selected nodes attaches to its destination node with the probability proportional to its node activity [6]. A certain node with higher node activity, is more probable to attract a new attachment. We will have:

$$k_i(t) = \sum_{t=0}^{t} L_i(t);$$

where $L_i(t)$ means the number of links added to the node $i$ due to adding new nodes during $t$ to $t + dt$ time interval. Considering the initial number of links in the network as $m_0$, after $t$ time-step, we will have $m_0 + mt$ links. Hence, during an ‘intermediary process’ (adding links between constant number of previously existed nodes)[12], the evolving rate of node activity is as follows [6]:

$$\frac{dk_i(t)}{dt} = m + \frac{mk_i(t)}{\sum_{j \neq i} k_j(t)} .$$

Since each link possesses two ends, the attachment of one end is selected by a random uniform distribution. However, the attachment of the other end is created by preferential attachment based on higher activity [6], Eq. 1. In this
regard, at each time step, higher node activity of a certain node causes higher rate of attachment to it by new comers. Consequently, there would be some competitions among nodes for attracting more links. From relations above, it is obvious that during these competitions, nodes with higher activity in the network, have probably more chance to create links rather than nodes with less activity.

By applying an analytical solution to above discussions, following relation is provided for value behavior:

\[ k_{v,i}(t) = 2mt + c\sqrt{t} \]  

(3)

Where \( c \) is constant and depends on the initial conditions as below:

\[ c = \frac{k_0 - 2mt_0}{\sqrt{t_0}} \]  

(4)

If \( t \to \infty \) in Eq.3, the behavior of the network is more dominant by \( 2mt \). However, for \( t \to 0 \), the effects of \( c\sqrt{t} \) will be more significant.

\[
\begin{aligned}
  t \rightarrow \infty & : k_v \sim t \\
  t \rightarrow 0 & : k_v \sim \sqrt{t}
\end{aligned}
\]

(5)

There is a cross-over time, \( t^* \), which is simultaneous with the transformation of value behavior of the network [6]. This cross-over time step is a separating point between two behavioral regimes in Eq.3 and it is calculated by Eq.6:

\[ t^* = \frac{c^2}{4m^2}. \]

(6)

The behavioral transformation of value indicates that by increasing \( m \), while the network crosses its cross over, \( t^* \), the growth of the network increases with a greater acceleration [6, 11]. Since, total number of links grows nonlinearly [12] faster rather than linearly passing time steps, this phenomenon is “cumulative growth”.
Noteworthy, each agent has its own value and cost equations (Eq.7 and Eq.8).

Also, the fate of network will yield to the situations below:

\[
\begin{align*}
    k_v(t) - k_c(t) > 0 & : \text{Positive Growth (before characteristic time)} \\
    k_v(t) - k_c(t) \leq 0 & : \text{No Growth (after characteristic time)},
\end{align*}
\]

(7)

where \(k_v(t)\) and \(k_c(t)\) refer to value and cost of making links, respectively. For the sake of negative growth, some researchers applied preferential damage[11]. In this study we applied the cost of further growth.

From now on, we consider that links not only grow due to value, but also they are exposed to costs of attachments which also change by time. For now, we consider a model which indicates changes of degrees in the existence of cost, and, \(\alpha\) is phase space. We will have [6]:

\[
\begin{align*}
    k_v(t) &= 2mt + c\sqrt{t} \\
    k_c(t) &= (\alpha + m)t
\end{align*}
\]

(8)

Characteristic time is a temporal moment which cost curve intersects value curve. Consequently, at this moment value and cost are equal, \(k_v = k_c\). Hence, in the intersection we have:

\[
2mt + c\sqrt{t} = (\alpha + m)t
\]

\[
t_{\text{characteristic}} = \left(\frac{c}{\alpha - m}\right)^2.
\]

(9)

5. Results

Fig.1 presents cross-over time, \(t^*\), corresponding to different \(k_0\) and \(m\) parameters. As illustrated, when initial degree, \(k_0\), is smaller, the rise of links \(m\) in each time step will shorten the cross-over time, \(t^*[6]\). On the other hand, larger initial degree in the network needs more rate of link creation, \(m\), to shorten cross-over time, \(t^*\). This finding is crucial. This implies that the initial degree, \(k_0\), acts like inertia against the cumulative growth and accordingly it takes more time-steps for the network to pass the cross-over time, \(t^*\).
Figure 1: Cross-over time, $t^*$, for different initial degrees of the network, ‘$k_0$’, versus the number of links, ‘$m$’, which is created at each time step is demonstrated. Accordingly, the variations of $k_0$ and $m$ for $t^* = 600$, 1200 and 2400 is depicted.

Figure 2: Left) Contour plot for ln(Characteristic Time) considering the changes of $\frac{m}{k_0}$ versus the changes $\frac{\alpha}{k_0}$. Right) Contour plot for ln(Characteristic Time) considering the changes of $\frac{k_0}{m}$ versus the changes $\frac{\alpha}{m}$.

Again, as seen in the Fig.1, the rise of $m$ causes the cumulative growth of the network occurs sooner. This implies that lowering the cross-over time will cause the network to escape from initial failure (this fact will be further shown by Fig.3, too).

In Fig.2, characteristic time and its contours are illustrated for different ratios of $m/k_0$ vs. $\alpha/k_0$ (left panel), and $k_0/m$ vs. $\alpha/m$ (right panel). Since no existence of characteristic time is the best scenario for the network’s growth, it is vital for the network’s topology to encounter the links creation factor $m$ equal to $\alpha$. When the initial degree $k_0 \to 0$ (or $m/k_0 \to \infty$ and $\alpha/k_0 \to \infty$), the left and right panel together clarify the situation. As shown in the right panel of Fig.2, when $k_0 \to 0$ (or $k_0/m \to 0$) the characteristic time will interestingly be more sensitive to the changes of ratio of $\alpha/m$ (around $\frac{\alpha}{m} = 1$) rather than
larger $k_0/m$. Hence, when the initial degree $k_0$ is extremely low, the trade-off between $\alpha$ and $m$ is more crucial. On the other hand, when the initial degree $k_0$ is extremely high, the aforementioned trade-off among $\alpha$ and $m$ is less sharper. It leads us to the consciousness that the higher initial degree in the network’s birth, lower the sensitivity of the network to future $\alpha$ parameter. As a typical rule for the trade-off among $\alpha$ and $m$, the higher difference among them lower the characteristic time. Hence, the life of network will be shortened, as Fig.2 shows. Conversely, in the case of $\alpha \to 0$, again, the $t^{\text{Characteristic}} \to \infty$.

As illustrated in Fig.2, due to activity functions of nodes for different $m$, the higher $m$, the shorter cross-over time, $t^*$ [6]. This implies that by increasing the ability of whole nodes to find each other, the evolution of whole network proceeds faster. Hence, network will reach the cross-over time sooner.

However, more efficient communication and advertising tools will cause faster processes in market research, awareness phase and popularity cycles of goods and services in a market. Accordingly, businesses may experience faster growth phase, maturity phase, and possibly faster occurrence of death phase. When it comes to the real-world networks, faster information translation and links creation will contribute to higher frequency in business cycles. In such entangled markets, economic firms should lower their inertia internally and externally and permanently plan to develop new features on their goods and services. This will lower their network’s cost toward growth and postpone occurrence of characteristic time in the state of trade-off between growth of value and cost. According to issues raised, one may consider 3 scenarios for the network’s growth. Plus, the phase space relating to each aforementioned scenarios is presented as demonstrated in Fig.3.

**Scenario of Failure ‘a’**. In the present scenario, the cost of making new links among temporal nodes is always more than value of that. Hence, characteristic time tends to zero. This phenomenon occurs for $\alpha >> m$, as panel a.2 in Fig.3 proves.
Upon this scenario, in the investment world the cost and expected return of selecting a trading position, is higher than future value.

**Scenario of Ever-growing Conquer ‘b’**. As a matter of fact in this situation, panel b.1 and b.2 in Fig.3, the network has successfully fulfilled all the circumstances. In other words, the value behavior has accelerated enough and during ‘all’ circumstances, cost is less than the value which is earned by making links. Accordingly, constituents totally are satisfied! Hence, the network ‘continuously’ grows. In two areas along $\frac{\alpha}{m}$, the network has the chance of “Ever-growing Conquer”. These areas conclude $\frac{\alpha}{m} \to 1$, as panel b of Fig.3 and Eq.9 proves.

In the stock markets this situation occurs when the future value of the stock in the investors’ point of view will be continuously better. In this state, in-spite of probable rise in cost (like fear, liquidity problems, bad news), the collective effects of opinions toward value of the market as a whole overcome the cost. In the marketing, this scenario occurs when the firm has been launched properly, and has then passed the value behavior’s cross-over and the firm’s competitive advantage is stable during the investigated scale.

**Scenario of Trade-off**. The trade-off scenario can be explained by two events which both occur between failure and ever-growing. Accordingly, by equalizing the characteristic time and the cross-over time, $t^*$, we have two answers:

$$
\begin{cases}
\alpha = 3m \\
\alpha = -m
\end{cases}
$$

where second answer is ineligible. Hence, two states emerge:

- Based on Fig.3, panel c, the configuration of network is formed. Yet, before acceleration of value behavior in the network, cost outpaces the value of making links. Hence, before that the network has the chance of changing the value behavior, its growth stops.

The phase space of this state is obtained as following:

$$
t_{\text{characteristic}} < t^* \rightarrow \alpha > 3m.
$$
In the world of financial markets, this scenario occurs during speculation, or in the case of sudden (relating to investment horizon) bad news after trading, or trading on minor trends. Noteworthy, in marketing networks, it can imply that a platform has been properly launched but the environment of the industry changes before the firm grows enough.

- Again, based on Fig.3, panel d, another state of the trade-off can be described as follows. The network may pass value behavior’s cross-over and consequently, it is able to rise acceleration of the value behavior. Yet, after passing the value behavior’s cross-over, the cost of making newcomer temporal links boosts insofar as the cost outpaces the value. Eventually, the network’s growth stops. Noteworthy, this phenomenon may not occur soon enough to avoid the state of ever-growing. It actually depends on the targeted period.

To obtain phase space in this state we have:

\[ t_{\text{characteristic}} > t^* \rightarrow \alpha < 3m. \]  

(12)

In the stock market, this is the situation that the investors do not prefer to stay in the trading position. In the marketing, the business cycle is started to downturn and will be no longer a boosting industry as it was in the past and newer competitors may refuse to enter the industry.

It is notable that the scaling features are significant in the trade-off scenario. Needless to say, to some scales, the network’s growth can be a cumulative positive amplifier for its growth, and on the other scales it may be as a barrier to growth anymore.

To avoid the network to be vulnerable to “failure”, it can be kept safely in the range of \(0 \leq \frac{\alpha}{m} \leq 3\).

The existence of a friction-like factor (such as memory [6] and cost) against growth of the network earned value leads the whole system to smaller sub-networks and more wanderer agents. Interestingly, the emergence of wanderer agents, their memory, and the ability of sub-networks to absorb them, will determine the fate of the whole. For the sake of forecasting possible scenarios of such systems, it is crucial to investigate the cost of growth of the sub-networks.
6. Conclusion

We found that some networks may fail at the beginning phase since their cost overcomes their value from the early beginning. Some are successful at the beginning phase, however, they stop growing before reaching the positive acceleration of value behavior (cross-over phenomenon). The trade-off scenario happens when networks successfully pass their beginning phase. However, they are exposed to failure after the emergence of characteristic time, because their network’s cost of growth overcomes its value simultaneous to the characteristic time.

Individual nodes’ or sub-networks’ interests may be latent and be against collective motivations [17] of sub-networks and communities. This phenomenon is understood -for example- as internal cost of a certain sub-network, like agency theory in corporate finance. Another criterion of cost can be structural cost of aged links corresponding to previous customers and after sales services. Also to some extent, when a network grows, covering highly distant customers amplifies customer-company costs, which one or both of them need to cover it.

When it comes to financial markets, growth can be interpreted as investors mo-
tivation to gain future returns. This may contribute to a collective interest in the market to lead other stocks. This collective behavior illustrates leader of the market [30–32] which determines general market trends. Then, the fear of possible turning point in the market trend, causes investors to be careful about the stability of trend. These are examples of what we call the cost of growth of the strongest eigenvector [7]. Hence, a characteristic time (like turnover time) for stopping current trend comes to mind.

Our proposed model implies that coexistence of cumulative value in preferential attachment versus cost of attachments may cause the network to be not ever-growing, and under some circumstances growth of the network stops. Hence, depending on the quality of changes, a trade-off scenario is possible. In some scales the network’s growth accelerates itself and in others, it may cause some barrier against itself to grow more.

**Growth of the network (as an Amplifier).** The cumulative value which is earned from increase in number of links, can help in developing the network. Development of the network rises earned value of constituents and contributes to attracting newcomers as Barabasi-Albert preferential attachment proves.

The increase in the number of links around certain nodes may cause the emergence of monopoly. On the other hand, the network may be more vulnerable to the cost of any further growth. In this atmosphere, the competitors may be more focused on the network. Hence, the attraction of competitors can be considered as the cost of attachments to the network. In long term periods, the attraction of proposed network decreases. As a result, probable links of newer time-steps may diverge from preferential attachment and cost effects become more significant.

Aforementioned issues due to certain businesses can be considered in equations.
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