Toward pole inflation and attractors in supergravity: Chiral matter field inflation

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Received August 30, 2017; Revised October 25, 2017; Accepted October 31, 2017; Published December 23, 2017

In string-inspired supergravity theory, the Kähler metric of chiral matter fields often has a pole. Such a Kähler metric is interesting from the viewpoint of the framework of the pole inflation, where the scalar potential can be stretched out to be flat around the pole for a canonically normalized field and inflation can be realized. However, when the Kähler metric has a pole, the scalar potential can also have a pole at the same point in supergravity theory. We study such supergravity models with a pole, and provide numerical analysis of inflationary dynamics and resultant density perturbation. In contrast with the usual pole inflation models, inflation in this supergravity-based model occurs not on the pole but in a region apart from the pole. We show that the existence of the pole in the scalar potential is crucial nevertheless. We also examine attractor behavior of our model.

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1. Introduction

The exponentially accelerated expansion of spacetime in the very early Universe, so-called inflation, solves various problems in standard big bang cosmology (Ref. [1]). Nowadays the inflationary cosmological model is the standard paradigm in cosmology. In most of the early studies (Refs. [1–4]), it had been thought that the Higgs field in a grand unified theory provides a false vacuum energy for inflationary expansion. Since it had been pointed out that any gauge singlet scalar field with a chaotic initial condition can derive inflationary cosmic expansion if its scalar potential is flat enough (Ref. [5]), a scalar field to drive inflation, called the inflaton, \(\phi\), has been thought to be not necessarily a Higgs field related to any gauge theory.

When we refer recent observational results of temperature fluctuation in the cosmic microwave background (CMB; Refs. [6,7]), so-called Higgs inflation, where the Higgs field in the standard model of particle physics with nonminimal coupling to gravity plays the role of inflaton, fits the observational data well (Ref. [8]). The secret of the success of Higgs inflation is the Weyl rescaling from the Jordan frame to the Einstein frame; the exponential function factor for this frame transformation makes the scalar potential in the Einstein frame \(V_E\) for the canonical normalized inflaton \(\chi\) exponentially flatter:

\[
V_E(\chi) \sim \left(1 - \exp\left(-\frac{2\chi}{\sqrt{6}M_p}\right)\right)^2,
\]

(1)
with $M_P$ being the reduced Planck mass. An exponential-like potential can fit the observational data particularly well, because for such a model there is an interesting relation,

$$n_s \simeq 1 - \frac{2}{N_e}, \quad (2)$$

where $n_s$ and $N_e$ denote the spectral index of its density perturbation and the number of e-folds, respectively. Then, for a sensible number of e-folds $N_e \simeq 60$, the best fit value of $n_s \simeq 0.97$ can be automatically reproduced. This property can also be seen in the so-called Starobinsky inflation model proposed back in 1980, where inflaton as a scalar degree of freedom appears through the Weyl rescaling and, as the common result, has an exponential-dependent scalar potential (Ref. [9]). Hence, as is well known, both Higgs inflation and Starobinsky inflation explain observational results well.

The generality of this feature with an exponential function in the scalar potential can be classified and well understood in the framework of an $\alpha$-attractor model, where the scalar potential is expressed as

$$V_E(\chi) \sim \left(1 - \exp\left(-\frac{2}{3\alpha M_P} \chi\right)\right)^2, \quad (3)$$

with a new model parameter $\alpha$ (Refs. [10–13]). The original Starobinsky inflation model corresponds to $\alpha = 1$. In Ref. [13], the attractor behavior of the parameter $\alpha$ has been shown. Since it was recognized that the essence of the above key feature comes from the fact that the kinetic term in the original field or the original frame has a singular pole, this classification of models was extended and those models have been regarded as pole inflation models (Refs. [14,15]). If a field has a singularity in the kinetic term, the field is infinitely stretched around the singular point of the kinetic term by canonical normalization. In the case that the scalar potential has no singularity at the kinetic singular point, the scalar potential is also stretched infinitely. Then, we can realize a sufficiently flat potential, where inflation by the canonical field takes place. Thus, the pole inflation is interesting, and some extensions have also been studied (see, e.g., Ref. [16,17]).

Indeed, the Kähler metric of chiral matter fields can often have a pole within the framework of string-inspired supergravity theory, which typically leads to the following form of the kinetic term,

$$\mathcal{L}_{KE} = K_{\phi \bar{\phi}} |\partial_\mu \phi|^2 = \frac{p}{(1 - |\phi|^2)^2} |\partial_\mu \phi|^2, \quad (4)$$

where $K_{\phi \bar{\phi}}$ is the Kähler metric and $p$ is a real number. This kinetic term has a singular point at $\phi = 1$. Here we use the unit with the Planck scale $M_P = 1$. The above Kähler metric can be obtained from the Kähler potential $K = -p \ln(1 - |\phi|^2)$. Thus, the supergravity model with this Kähler potential would be a good candidate for realizing the pole inflation. Starobinsky inflation models in no-scale supergravity have been studied, e.g., in Refs. [18–27]. However, the corresponding scalar potential can have a similar pole at the same point, because the $F$-term scalar potential is given by

$$V = e^K \left[K^{IJ}(D_I W)(D_J \bar{W}) - 3 |W|^2\right], \quad (5)$$

by Kähler potential $K$ and superpotential $W$, with $D_I W \equiv K_I W + W_I$, and the overall factor $e^K$ is singular at the same point as the kinetic term. In fact, inflation cannot take place on the pole of the
kinetic term. That is, the realization of the inflation potential is not trivial within the framework of supergravity models with a singular kinetic term.

In this paper, motivated by the fact that the kinetic term of Eq. (4) often appears through a compactification of higher-dimensional stringy supergravity models, we consider inflationary potentials consisting of the $F$-term scalar potential (5) with a logarithmic Kähler potential, constant superpotential, and additional nonvanishing $F$-term component by another field, and a constant (or sufficiently flat) term in the scalar potential. We find that the potential has a flatter region apart from the pole and, by use of it, the inflationary expansion can be realized. Unlike the original pole inflation model (Refs. [14,15]), our inflation occurs at regions apart from the pole of the kinetic term because the scalar potential also has a pole at the same point. Nevertheless the existence of the pole is important to realize the flat region in the potential in our model. We show examples of an inflationary potential construction under the situation that both the kinetic term and the scalar potential have a pole at the same place, which is a case in a class of supergravity.

This paper is organized as follows. In Sect. 2, we explain the structure of scalar potential in our model. In Sect. 3, first, we show the numerical analysis of inflationary observables and the attractor behavior of parameter dependence in a model with a $\phi$-independent superpotential and an uplifting term. In Sect. 4, we include inflaton dependence in our superpotential and show that it does not spoil successful inflationary expansion. In Sect. 5, we consider a case of a $\phi$-dependent uplifting term. Section 6 is devoted to conclusions and discussions.

2. Inflation Potential

In superstring theory, the following type of Kähler potential,

$$K = -p \ln(T + \bar{T} - A|\phi|^2),$$

appears in the consequence of compactification of extra dimensions, where $T$ is a Kähler modulus, $\phi$ is a chiral superfield, and $A$ is a constant coefficient. Here we use the notation that the superfield and its lowest component are denoted by the same letter. We assume the Kähler modulus $T$ to be stabilized with a sufficiently heavy mass by a certain mechanism, and replace $T$ with its vacuum expectation value $\langle T \rangle$ below such a mass scale. Then, by rescaling $\phi$, we obtain the Kähler potential

$$K = -p \ln(1 - |\phi|^2).$$

The kinetic term of the $\phi$ is given by Eq. (4), which has a pole at $|\phi| = 1$. The $F$-term scalar potential in the framework of supergravity is given by Eq. (5).

In this section, we assume a nonvanishing $F$-term $|F|^2 \neq 0$ of other fields and a constant potential $V_C$ due to a $D$-term scalar potential and/or explicit supersymmetry breaking effects, and hence we obtain the scalar potential

$$V = e^K \left[ K^{\phi\bar{\phi}} |D_\phi W|^2 - 3 |W|^2 + |F|^2 \right] + V_C. \quad (8)$$

At first, for simplicity, we consider the $\phi$-independent superpotential $W = W_0$. In the following analysis, we often use the dimensionless scalar potential

$$\tilde{V} \equiv \frac{V}{|W_0|^2} = \frac{1}{(1 - |\phi|^2)^{\mu}} \left( p|\phi|^2 - 3 + \tilde{F}^2 \right) + \tilde{V}_C, \quad (9)$$

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Fig. 1. The shapes of the potentials (9) and (12) with $p = 3$, $\tilde{F}^2 = 0.08$, and $\tilde{V}_C = 625$. The pole at $\phi_r = 1$ in Fig. 1a corresponds to the pole at $\chi = \infty$ in Fig. 1b.

with $\tilde{F}^2 = |F|^2/|W_0|^2$ and $\tilde{V}_C = V_C/|W_0|^2$, normalizing the scalar potential (8) by $|W_0|^2$. We also define

$$\tilde{V}_1 = e^K \left( K\phi \partial_D W^2 \right) / |W_0|^2, \quad \tilde{V}_2 = e^K \left( -3|W|^2 + |F|^2 \right) / |W_0|^2;$$

(10)

i.e., $\tilde{V} = \tilde{V}_1 + \tilde{V}_2 + \tilde{V}_C$. Since we have assumed a $\phi$-independent superpotential, the scalar potential depends only on the radial component$^1$ of $\phi, \phi_r$; hence, without loss of generality, we can identify the inflaton direction with $\phi_r$, which is sometimes simply denoted by $\phi$. The form of potential is shown in Fig. 1a with special values of $\tilde{F}^2$ and $\tilde{V}_C$. The values of $\tilde{F}^2$ and $\tilde{V}_C$ are chosen in such a way that the minimum lies between $0 < \phi < 1$ and its vacuum energy vanishes there. Since $\tilde{V}_1$ decreases and $\tilde{V}_2$ increases divergently toward the pole at $\phi_r = 1$, the potential around the origin tends to be flat compared with the depth of the potential at the stationary point. (Thus, there is an approximate shift symmetry for the small $\phi_r$ region.)

A canonically normalized field $\chi$ that has canonical kinetic energy $L_{KE} = \frac{1}{2}(\partial_\mu \chi)^2$ can be defined by

$$\phi = \tanh \left( \frac{\chi}{\sqrt{2} p} \right).$$

(11)

The pole $\phi_r = 1$ corresponds to $\chi = \infty$. In terms of the $\chi$, the potential (9) is expressed by

$$\tilde{V} = \cosh^2 p \left( \frac{\chi}{\sqrt{2} p} \right) \left[ p \tanh^2 \left( \frac{\chi}{\sqrt{2} p} \right) - 3 + \tilde{F}^2 \right] + \tilde{V}_C,$$

and is shown in Fig. 1b. The potential of $\chi$ is not flat around the pole, but singular. This is because of the factor $e^K$ in the supergravity scalar potential. This is different from the simplest pole inflation, where $V(\phi)$ is not singular at the pole of the kinetic term (Ref. [14]). However, different singular behaviors between $\tilde{V}_1$ and $\tilde{V}_2$ create a minimum and make its depth deeper. Then, the potential for the smaller $\chi$ region is still flat compared with the depth of the minimum. Around such a flat region, we can realize the inflation.

The stationary condition of the potential $V = |W_0|^2 \tilde{V}$,

$$V_\chi = \frac{dV}{d\chi} = \frac{d\phi}{d\chi} V_\phi = 0,$$

(13)

$^1$ When the $\phi_r$-direction is stabilized and the $\theta$-direction is slightly sloped, then helical phase inflation due to $\theta$ can occur (Refs. [28–30]).
Fig. 2. $\tilde{F}$-dependence of the potential $\tilde{V}$ with $p = 3$ and $\tilde{V}_C = 0$ with respect to the canonical field $\chi$.

can be expressed by

$$\dot{\phi} = \frac{2p\phi[(p - 1)\phi^2 + \tilde{F}^2 - 2]}{(1 - \phi^2)^{p+1}} = 0,$$

by using the potential (9). A solution of Eq. (14) is

$$\phi^2 = 2 - \frac{\tilde{F}^2}{p - 1},$$

for $p \neq 1$, and the condition for the stationary point to be present at $0 < \phi < 1$ is

$$3 - p < \tilde{F}^2 < 2.$$

Figure 2 shows the role of $\tilde{F}^2$ in the potential (12). A smaller $\tilde{F}^2$ corresponds to a deeper vacuum, and the potential becomes sharp around the minimum. Hence, as $\tilde{F}^2$ becomes smaller, the potential becomes flatter except around the minimum.

3. Numerical analysis

In this section, we study the inflation behavior of our potential numerically.

3.1. Models with $p = 2, 3$

The shape of the potential of inflaton is restricted by observations of the fluctuation of temperature anisotropy in the CMB. Slow-roll parameters are defined by

$$\epsilon = \frac{1}{2} \left( \frac{V_{\chi}}{V} \right)^2, \quad \eta = \frac{V_{\chi\chi}}{V},$$

and the power spectrum, the spectral index of the power spectrum of density perturbation, and the tensor-to-scalar ratio are expressed by

$$P_\zeta = \frac{V}{24\pi^2 \epsilon}, \quad n_s = 1 + 2\eta - 6\epsilon, \quad r = 16\epsilon,$$

respectively. According to the BICEP2/Keck Array and Planck data (Refs. [6,7]), these quantities are measured or bounded as

$$P_\zeta = (2.20 \pm 1.10) \times 10^{-9}, \quad n_s = 0.9655 \pm 0.0062, \quad r < 0.11.$$
Fig. 3. \((n_s, r)\)-plots for \(p = 2\) (blue) and \(p = 3\) (red). The two vertical dashed lines represent the lower and upper bounds for \(n_s\) in the observational data (19). The values of variables at sample points (a)–(e) are shown in Table 1.

Table 1. The values of variables at each point (a)–(e) in Fig. 3. The first four variables are input parameters in this model, and the last two are output quantities.

| Point | \(p\) | \(\tilde{F}^2\) | \(V_C\) | \(|W_0|\) | \(n_s\) | \(r\) |
|-------|------|-----------------|-------|---------|-------|------|
| a     | 2    | 1.001           | 1.00 \times 10^3 | 2.59 \times 10^{-7} | 0.9676 | 2.03 \times 10^{-3} |
| b     | 2    | 1.013           | 7.69 \times 10^8 | 8.44 \times 10^{-7} | 0.9607 | 1.63 \times 10^{-3} |
| c     | 3    | 0.005           | 1.60 \times 10^5 | 1.33 \times 10^{-8} | 0.9670 | 0.87 \times 10^{-3} |
| d     | 3    | 0.055           | 1.32 \times 10^4 | 1.58 \times 10^{-7} | 0.9799 | 1.01 \times 10^{-3} |
| e     | 3    | 0.170           | 1.38 \times 10^2 | 4.82 \times 10^{-7} | 0.9608 | 0.96 \times 10^{-3} |

In addition, the number of e-folds,

\[
N_e \equiv -\ln \left( \frac{a_{end}}{a} \right) \simeq \int_{\chi_f}^{\chi_{N_e}} \frac{V}{V_C} d\chi,
\]

is required to be larger than about 50 to solve the flatness and horizon problems. Here \(\chi_{N_e}\) and \(\chi_f\) are the field values at the corresponding \(N_e\) and the final point of inflation, respectively. In all plots in this paper, we take \(P_\xi = (2.20 \pm 1.10) \times 10^{-9}\) and \(N_e = 60\) for normalization of those quantities.

There are three variables, \(p\), \(\tilde{F}\), and \(\tilde{V}_C\), in the potential (12). The value of \(p\) is quantized such as \(p = 1, 2, 3\) in superstring theory. Here, we use \(p = 2, 3\) because of the stationary condition (14) in this model. The value of \(\tilde{F}\) controls the depth of the potential at the stationary point, which corresponds to the magnitude of flatness of the potential in the region near the origin too. The value of \(\tilde{V}_C\) is adjusted uniquely to make the vacuum energy vanish at the minimum. The overall coefficient \(|W_0|\) of \(V = |W_0|^2\tilde{V}\) is determined by the amplitude of \(P_\xi\).

Figure 3 shows \((n_s, r)\)-plots for \(p = 2, 3\) with a parameter \(\tilde{F}^2\). The points (a) and (c) correspond to the lower limit of \(\tilde{F}^2\) in inequality (16), and both of the curves flow to the smaller \(n_s\) direction as \(\tilde{F}^2\) increases. Table 1 shows variables at each sample point (a)–(e) in Fig. 3. In both cases \(p = 2, 3\), there are parameter regions of \(\tilde{F}^2\) consistent with the observations (19). At each point (a)–(e), the superpotential \(|W_0|\) takes almost the same scale \(10^{-7}M_p^3 \simeq (10^{16} \text{GeV})^3\), and the constant potential \(V_C \simeq (10^{15} \text{GeV})^4\). The mass of inflaton becomes \(m_\chi^2 \simeq (10^{15} \text{GeV})^2\). The values of \(\tilde{V}_C\), \(|W_0|\), and \(m_\chi^2\) are analytically calculable by using approximations, and will be derived in Sect. 3.3.

3.2. Attractor

We showed the numerical analysis for \(p = 2, 3\) in Fig. 3. In this section, we treat \(p\) as not a realistic integer but a continuous parameter in order to examine attractor properties.
Fig. 4. \((n_s, r)\)-plots for different \(\tilde{F}^2\). The blue, red, and black curves represent \(\tilde{F}^2 = 1, 0, -2\), respectively. The number written next to each point on the curves represents the value of \(p\) at that point.

Fig. 5. \((n_s, r)\)-plots for different \(\delta\). The number written next to each point on the curves represents the value of \(p\) at that point. In the region near \(n_s \simeq 0.967\), three curves almost coincide with each other, and they take almost the same \(r\) at \(p \gtrsim 100\).

Figure 4 shows \((n_s, r)\)-plots by varying \(p\) continuously for several values of \(\tilde{F}^2\). The value of \(p\) is bounded by the stationary condition (16). Each curve flows to smaller \(n_s\) direction as \(p\) increases. It is found that the spectral index \(n_s\) becomes realistic values around \(3 - p \approx \tilde{F}^2\).

It is convenient to define \(\delta\) by

\[
\delta \equiv \tilde{F}^2 + p - 3. \tag{21}
\]

The stationary point (15) and the stationary condition (16) can be rewritten as

\[
\phi^2 = 1 - \frac{\delta}{p - 1}, \quad 0 < \delta < p - 1, \tag{22}
\]

respectively. A smaller \(\delta\), which makes the stationary point closer to the pole, realizes a deeper vacuum. Figure 5 shows \((n_s, r)\)-plots by varying \(p\) continuously for several \(\delta\). For a sufficiently large \(p\), the spectral index \(n_s\) becomes realistic. These values do not change and approach \(n_s \simeq 0.967\) as \(p\) increases. Also, as \(p\) increases, \(r\) becomes smaller. That is, we find attractor behavior. The point at \(p = 130\) is not a fixed point. For instance, \(r = O(10^{-6})\) can be realized by \(p = 1000\) on each curve. These results show that \(\tilde{F}^2 \approx 3 - p\) is favorable. In the limit \(\tilde{F}^2 \to 3 - p\) and \(\delta \to 0\), the potential becomes sharp around the minimum. Note that although the limit \(\delta \to 0\) is favorable, point (e) with \(\delta = 0.17\) in Table 1 also leads to \(n_s\) consistent with the Planck result.

3.3. Analytical calculations

We can examine analytically the attractor behavior shown in Sect. 3.2. First, we show the attractor behavior. Next, we derive values of the quantities mentioned in the last paragraph in Sect. 3.1.
The potential (12) is rewritten as
\[ \tilde{V} = -p \cosh^{2(p-1)} \left( \frac{\chi}{\sqrt{2p}} \right) + \delta \cosh^{2p} \left( \frac{\chi}{\sqrt{2p}} \right) + \tilde{V}_C, \tag{23} \]
by using \( \delta \) defined in Eq. (21). We can neglect the factor \( \exp(-\chi/\sqrt{2p}) \) in \( \cosh(\chi/\sqrt{2p}) \) in the region related to inflation, and we obtain
\[ \tilde{V} = -\frac{1}{2^{2p}} \left( 4p \exp \left( -\sqrt{\frac{2}{p}} \chi \right) - \delta \right) \exp \left( \sqrt{2p} \chi \right) + \tilde{V}_C, \]
\[ \tilde{V}_\chi = -\frac{\sqrt{2p}}{2^{2p}} \left[ 4(p-1) \exp \left( -\sqrt{\frac{2}{p}} \chi \right) - \delta \right] \exp \left( \sqrt{2p} \chi \right), \]
\[ \tilde{V}_{\chi\chi} = -\frac{1}{2^{2p-1}} \left[ 4(p-1)^2 \exp \left( -\sqrt{\frac{2}{p}} \chi \right) - p \delta \right] \exp \left( \sqrt{2p} \chi \right). \tag{24} \]
Being aware of \( \tilde{V} \simeq \tilde{V}_C \) during inflation and with Eq. (24), the number of e-folds is rewritten as
\[ N_e = \int_{\chi_f}^{\chi_{N_e}} \frac{V}{V_\chi} d\chi \]
\[ \simeq -\frac{2^{2p-2} \tilde{V}_C}{\sqrt{2p}(p-1)} \int_{\chi_f}^{\chi_{N_e}} d\chi \exp \left( -\sqrt{2p}(1 - \frac{1}{p}) \chi \right) \left[ 1 - \frac{\delta}{4(p-1)} \exp \left( \sqrt{\frac{2}{p}} \chi \right) \right]^{-1}. \tag{25} \]
In the limit of \( \delta \to 0 \) (or \( p \to \infty \)), the terms independent of \( \delta \) in Eq. (25) are dominant and we obtain
\[ N_e = \frac{2^{2p-3} \tilde{V}_C}{(p-1)^2} \exp \left( -\sqrt{2p}(1 - \frac{1}{p}) \chi_{N_e} \right). \tag{26} \]
Note that the factor \( \exp \left( -\sqrt{2p}(1 - \frac{1}{p}) \chi_f \right) \) is negligible because \( \chi_f \) is sufficiently large. By using Eq. (26) with \( \delta = 0 \), slow-roll parameters (17) are recast as
\[ \epsilon |_{\chi = \chi_{N_e}} = \frac{p}{4(p-1)^2 N_e^2}, \quad \eta |_{\chi = \chi_{N_e}} = -\frac{1}{N_e}. \tag{27} \]
Observable quantities (18) are expressed with \( N_e \) as
\[ n_s = 1 - \frac{2}{N_e} - \frac{3p}{2(p-1)^2 N_e^2}, \quad r = \frac{4p}{(p-1)^2 N_e^2}. \tag{28} \]
With a realistic value of \( p \), e.g., \( p = 3 \), and \( N_e = 60 \), we obtain
\[ n_s = 0.9664, \quad r = 0.83 \times 10^{-3}. \tag{29} \]
These are consistent with point (c) in Table 1. For a larger \( p \), instead we obtain
\[ n_s \simeq 0.9667 - \frac{1}{p} \times 10^{-3}, \quad r \simeq \frac{1}{p} \times 10^{-3}. \tag{30} \]
The second term in \( n_s \) can be neglected for sufficiently large \( p \), and we obtain \( n_s \simeq 0.9667 \). Equation (30) is consistent with \((n_s, r) = (0.9669, 1.01 \times 10^{-5})\) at \( p = 130 \) in Fig. 5, and shows attractor behavior.
Next, we estimate the values of $\tilde{V}_C$, $|W_0|$, and $m^2_\chi$. The stationary point $\chi_0$ is found from $\tilde{V}_\chi = 0$ in Eqs. (24) as

$$\exp\left(-\sqrt{\frac{\pi}{p}} \chi_0\right) = \frac{\delta}{4(p - 1)}. \quad (31)$$

Since the constant potential $\tilde{V}_C$ is determined to realize vanishing vacuum energy $\tilde{V}|_{\chi = \chi_0} = 0$, we obtain

$$\tilde{V}_C = \left(\frac{p - 1}{\delta}\right)^{p-1}. \quad (32)$$

The size of superpotential $|W_0|$ is determined by the amplitude of the power spectrum $\mathcal{P}_\zeta$. By using Eqs. (19), (27), and (32), we obtain

$$|W_0|^2 = 24\pi^2\mathcal{P}_\zeta \cdot \frac{p}{4(p - 1)^2N_\zeta^2} \cdot \left(\frac{\delta}{p - 1}\right)^{p-1} \simeq \frac{3p\delta^{p-1}}{(p - 1)^{p+1}} \times 10^{-11}. \quad (33)$$

The mass of inflation $m^2_\chi$ is obtained by

$$m^2_\chi = V_{XX}|_{\chi = \chi_0} = |W_0|^2 \tilde{V}_{XX}|_{\chi = \chi_0} = \frac{6\pi^2\mathcal{P}_\zeta p\delta^{p-1}}{N_\zeta^2(p - 1)^{p+1}} \cdot \frac{2(p - 1)^p}{\delta^{p-1}}$$

$$\simeq \frac{6p}{p - 1} \times 10^{-11}. \quad (34)$$

4. $\phi$-dependent superpotential

While we have considered a $\phi$-independent superpotential in previous sections for simplicity, in general superpotential is a function of $\phi$ as well. One expects that sufficiently small $\phi$-dependence in $W$ does not affect inflation in the previous sections. In this section, we include $\phi$-dependence in $W$ and analyze the inflation behavior of the scalar potential.

As an example of $\phi$-dependence, we take the following superpotential:

$$W = W_0(1 + a\phi^2 + b\phi^3), \quad (35)$$

where $a$ and $b$ are assumed to be real constants. Then, in the scalar potential, the CP of the $\phi$ field can be violated. Compared to a $\phi$-independent $W$ case, $V$ depends on the phase direction of $\phi$, $\theta$, and at the potential minimum its direction becomes massive.

We define the canonically normalized field of the imaginary direction as $\lambda$. We have confirmed that, at the field region of the very early stage of inflation ($\chi_{ini} < \chi_N$, $\lambda = 0$), $V_\lambda = 0$ and $|V_{XX}| > |V_{XX}| \times \mathcal{O}(10)$ with $V_{XX}$, $V_{XX} < 0$. Thus, the evolution of the $\lambda$ direction is negligible and we can regard the $\chi$ direction as the inflaton.

Figure 6 shows $(n_s, r)$-plots for different, fixed $a$ and $b$ with $p = 3$ and the parameter $\tilde{F}^2$. The endpoint of the right-hand side of each curve represents the lower bound for $\tilde{F}^2$ by inequality (16), i.e., $\delta \to 0$, and each curve flows from the point to the outside direction of the figure as $\tilde{F}^2$ increases. We can see that $\phi$-dependence in the superpotential does not spoil inflation even if $a, b = \mathcal{O}(1)$.
Fig. 6. \((n_s, r)\)-plots and different \(a\) and \(b\) with \(p = 3\). The black curve is the same as Fig. 3.

(a) \(p = 2, \alpha > 0\)

(b) \(p = 2, \alpha < 0\)

(c) \(p = 3, \alpha > 0\)

(d) \(p = 3, \alpha < 0\)

Fig. 7. \((n_s, r)\)-plots for \(p = 2, 3\) with positive and negative \(\alpha\). The black curve in each figure is the same as Fig. 3.

5. **D-term potential**

In Sect. 3, we assume the constant \(V_C\) in the potential (8). Since the role of \(V_C\) is to uplift the potential to give vanishing vacuum energy, we can consider another uplifting term depending on the field \(\chi\). In this section, we show one example, that is the \(D\)-term scalar potential written as

\[
V_D = \frac{1}{2} g^2 D^2
\]

with \(g\) being a gauge coupling. For earlier works on such attempts in a different context, see, e.g., Refs. [31,32] for \(D\)-term uplifting and Ref. [33] for inflation. When this is independent of \(\phi\), it corresponds to a constant \(V_C\). Here, we consider the case that the gauge coupling \(g\) depends on \(\phi\) as

\[
g^{-2} = g_0^{-2} + \beta \ln \left( \frac{\phi}{M_P} \right). \tag{36}
\]

The second term depends on \(\phi\) and such a dependence would appear when charged matter fields become massive by its expectation value of \(\phi\). The terms \(g_0\) and \(\beta\) denote the \(\phi\)-independent part of the gauge coupling and the corresponding beta function coefficient. Also, we assume that \(D\) itself is
independent of $\phi$ and just a constant. Then, the full scalar potential is written

$$V = e^K \left[ K \phi D_\phi W^2 - 3 |W|^2 + F^2 \right] + V_D. \tag{37}$$

After redefinition of parameters, we simplify the $D$-term potential as

$$V_D = \frac{D^2}{1 + \alpha \ln \phi}. \tag{38}$$

Since a sufficiently small $\alpha$ makes this $D$-term sufficiently flat in order not to disturb inflation, $V_D$ plays the role of $V_C$ in the potential (9).

Figure 7 shows $(n_s, r)$-plots for different fixed $\alpha$ with the parameter $\tilde{F}^2$. The point at the center of the figure corresponds to $\delta \to 0$, and each curve flows from the point to the outside direction of the figure as $\tilde{F}^2$ increases. Thus, even if we add $\phi$-dependent $V_D$, there is no change in the limit $\delta \to 0$ from the results in Sect. 3, and the region with small enough $\delta$ is favorable for any value of $p$ and $\alpha$.

We can discuss a more generic form $V(\phi)$. Judging from the above results, we can expect that successful inflationary potential is realized not only by adding the constant term $V_C$, but also by adding a sufficiently flat potential around small enough $\delta$.

6. Conclusion

We have studied inflationary dynamics and the resultant density perturbations in supergravity models, whose Kähler metric has a pole and whose scalar potential also has a pole at the same field value. We found that successful inflation is possible by appropriate choice of parameters. In particular, the parameter region around $\delta \approx 0$ is favorable. For a $\phi$-dependent $V_D(\phi)$ term in the scalar potential instead of constant uplifting, this property is unaffected and the parameter region around $\delta \approx 0$ is still favorable.

We have also studied attractor behavior by changing $p$ as a continuous parameter. The parameter region around $\delta \approx 0$ is favorable, again. For large enough $p$, as $p$ increases, $n_s$ does not change approaching to $n_s \simeq 0.967$ and only $r$ decreases.

Acknowledgements

The authors would like to thank Naoya Omoto for useful discussions. This work is supported in part by the Grant-in-Aid for Scientific Research No. 26247042 (T.K.), No. 26400243 (O.S.) from the Ministry of Education, Culture, Sports, Science and Technology in Japan.

Funding

Open Access funding: SCOAP$^3$.

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