A New Analysis of $^{14}$O Beta Decay: Branching Ratios and CVC Consistency

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The nucleus $^{14}$O decays predominantly by a Fermi transition to the 2.313 MeV $0^+$ state in $^{14}$N. Weak Gamow-Teller (GT) branches are evident to the $1^+$ state at 3.95 MeV in $^{14}$N (branching ratio, $R_{GT} = 0.055 \%$) and to the $1^+$ ground state, with $R_{GT} \sim 0.6 \%$. This ground-state GT transition is strongly inhibited: its $f$-value is roughly $10^4$ times larger than is typical for favored $0^+ \rightarrow 1^+$ transitions. (Even more inhibition is evident in the analog $^{14}$C $\rightarrow^{14}$N transition.) The inhibition is attributed to accidental cancellation in the allowed GT matrix element for this transition. Because the allowed matrix elements are so small, the induced terms – particularly “weak magnetism” – as well as the relativistic and the second-forbidden terms, are expected to contribute appreciably to the decay probability. As a consequence, many of the usual assumptions in the allowed approximation may not be valid. For example, the spectrum shape may deviate markedly from the allowed (or statistical) spectrum shape.

To date there has only been one measurement, by Sidhu and Gerhart [1, 2], of the detailed shape of the beta spectrum from $^{14}$O decay. It was performed with an iron-free, beta-ray spectrometer, and was published 40 years ago! More recently, calculations by Garcia and Brown [3] could not satisfactorily fit the observed beta spectrum, which led the authors to suggest that there might be some systematic problem with Sidhu and Gerhart's measurement. This conclusion would have a serious impact on the branching ratio for the Fermi transition. For this reason, we have reanalyzed [4] the data of Sidhu and Gerhart, which we obtained from a copy of Sidhu's Ph.D. thesis [2]. Our conclusion is that the $^{14}$O spectrum shape can be understood, but only if renormalized operators are used in the shell-model calculations of the nuclear matrix elements. Our re-analysis [4] yields a ground-state branching ratio of 0.54(2) %, compared with the originally published result [1] of 0.61(1) % – a large shift in terms of the uncertainties quoted.

Our re-analysis used shell-model calculations for the relevant nuclear matrix elements in model spaces that included both $|2h\rangle$ and $|4h-2p\rangle$ configurations relative to a closed-shell $^{16}$O core. As was noted by Garcia and Brown [3] the GT transition strength to the lowest $1^+$ state is very small compared to the strength of the transition to the second $1^+$ state at 3.95 MeV excitation energy. Thus, any small mixing between these two $1^+$ states in the model will have a large effect on the weak transition rate. To make use of this fact, we write the wave function for the lowest $1^+$ state in $^{14}$N as

$$|1^+_{\text{low}}> = \alpha |1^+ (1)> + \beta |1^+ (2)>,$$

with $\alpha^2 + \beta^2 = 1$. Here (1) and (2) refer to the first and second model states obtained in the shell-model calculation. Our strategy is then to adjust $\alpha$ to minimize the $\chi^2$ between the calculated and experimental shape-correction function. The magnitude of this function yields the GT matrix element while the slope depends on the interference between the weak magnetism and the GT matrix elements. The difficulty encountered by Garcia and Brown [3] was that the weak magnetism matrix element determined from the slope did not agree with the M1 matrix element determined from the electromagnetic transition between isobaric analog states and the ground state of $^{14}$N – a requirement of the conserved vector current (CVC)
hypothesis. We resolved this dilemma by recognizing that renormalized rather than free-nucleon operators should be used in the shell-model calculations of these matrix elements.

In the mirror decay of $^{14}$C a similar situation occurs. Consistency between the $^{14}$C lifetime, the slope of the shape-correction function and the M1 matrix element from gamma decay can only be achieved with renormalized operators in the shell-model calculation.

[1] G.S. Sidhu and J.B. Gerhart, Phys. Rev. 148, 1024 (1966).
[2] G.S. Sidhu, *An experimental study of the spectrum shape for the Gamow-Teller transition $^{14}$O $\rightarrow$ $^{14}$N*, Ph.D. thesis, University of Washington (1966).
[3] A. Garcia and B.A. Brown, Phys. Rev. C 52, 3416 (1995).
[4] I.S. Towner and J.C. Hardy, Phys. Rev. C 72, 055501 (2005).