Optimization based internal model control design

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Abstract. Control of nonlinear and/or constrained processes is a challenging and important task, especially chemical processes that need special attention to maximize the product thereby the profit. In this work, an optimization-based internal model control design is presented for both linear and nonlinear systems. For controlling nonlinear systems, the process dynamics are linearized around the operating point. The use of an optimizer as an internal model controller will enable the control engineer to deal with the constraints posed by the various process parameters and the effect of various process parameter variations in the closed-loop response is reported. Simulation case studies are presented viz., a first-order LTI system, and Continuous Stirred Tank Reactor (CSTR) to prove the efficacy of the optimization based internal model controller in controlling linear and nonlinear constrained processes.

Index terms—Internal model control, Process constraints, Optimization, Parameter uncertainty, Non-linearity

1. Introduction

The traditional PID controller is a proven and widely used control strategy in the chemical process industry [1]. But, the design procedure of a PID controller is a bit tedious and time-consuming when one has to deal with complex, nonlinear, and constrained processes. Model-based control algorithms are a better alternative for control of the nonlinear process. Model predictive control and internal model control are the popular model-based control strategies that were reported by many control researchers in the past. Both the above mentioned model-based control strategies are highly dependent on the process model used to compute the dynamic behavior of the process. In the control effort calculation, model predictive control algorithms use an optimizer to minimize the error between the reference and process variable because of which model predictive control algorithms attracted the control research community much. Whereas, internal model control design depends on model
inversion by excluding the portion of process dynamics that affect closed-loop stability and one may not be able to include process constraints in the IMC design procedure.

Over the last few couple of years, internal model control has a strong impact on control engineering researchers and rapidly changed the control engineering field. In cases, IMC is realized in terms of the PID controller and hence, called IMC-PID control strategy. Simple PID, like structure and one degree of freedom in terms of filter parameter $\lambda$, made IMC very popular in process control applications. The IMC tuning procedure has proven to be very good for both reference tracking and disturbance rejection[2][3][4][5].

Considering the advantages offered, the simplicity of the design procedure and PID, like structure has made the IMC a very key control strategy. The IMC-PID design procedure was reported by many numbers of researchers in the past for systems with different dynamic characteristics such as positive open loop zeros, dead time, open-loop unstable and inherently nonlinear. The goal of IMC is to address the problems associated with large dead time, integral windup ,and interaction between the process variables [6]. The idea of representing IMC parameters in the line of PID controller was first introduced by [7], in which the authors discussed the reformulation IMC design by closely following the design procedure of widely used conventional PID controller for processes whose dynamics are represented by the integrator, first order , and second-order dynamic equations. In[8], the IMC-PID design procedure reported by including series filter transfer function to have physically realizable control efforts ,and this way of tuning the IMC-PID controller is grabbed the attention of researchers to find the suitable PID tuning parameters based IMC design. As a result, a huge number IMC-PID control strategies such as robust[9], [10], [11], [12],[13],[14], [15]and nonlinear [11], [13] versions of IMC have already been successfully developed and implemented. However, none of the above-mentioned control structures can handle the constraints as a result of the physical meaning of various process parameters and variables. Hence, the central and key idea of this article is to use an optimizer as an internal model control which will allow the process to operate around the operating point even in the presence of process constraints.

2. Internal model control

The very basic idea of the IMC design procedure is to design a controller that will ensure the cancellation of the respective terms in the model of the process. The basic structure of IMC is depicted in Fig. 1, where $G_m(s)$ is the process model and $G_i(s)$ is the internal model controller. According to the fundamental goal of any control system, in IMC also the goal is to make the process follow the reference trajectory. In IMC, this reference tracking is achieved by inverting the process to form the controller. i.e.,

$$G_i(s) = \frac{1}{G_m(s)}$$  \hspace{1cm} (1)

![Figure 1. Basic IMC structure.](image-url)
During the IMC design procedure, one has to take care of some special issues related to the process model such as delay time, positive zeros which may result in physically unrealistic and unstable response[16]. The structure of IMC is as shown in Fig. 2. \( G_p(s), G_m(s), \) and \( G_c(s) \) are process, process model, and controller transfer function respectively. The process-model mismatch \( (d) \) is fed back to the controller to act on the resulting error. IMC is composed of two terms, one is the inverted model of process and the other is a suitable filter. The filter in IMC is mainly placed exactly after the controller transfer function to make the controller to physically realizable. i.e.,

\[
G_c(s) = \frac{1}{G_m(s)} F(s)
\]  

(2)

Filter \( F(s) \) takes the following form depending on the order of the process,

\[
F(s) = \frac{1}{(\lambda s + 1)^N}
\]  

(3)

Where \( N \) is the order of the filter, which is to make the controller realizable and \( \lambda \) is the filter parameter, which is the only parameter left to the control engineer to tune for better performance of the resulting closed-loop systems. Generally, the filter parameter \( \lambda \) will take care of the quality of the close-loop response, uncertainty in model parameters and plant-model mismatch[16]. The IMC design procedure is easily realized as a conventional PID controller particularly for linear processes with minimum changes in the control structure[16].

![Figure 2. Schematic of Internal Model Controller.](image)

3. Optimization based IMC
The Optimization based IMC design procedure deals with replacing the \( G_c(s) \) part of the IMC with a suitable optimizer. The idea of replacing \( G_c(s) \) by the optimizer is inspired by the design procedure of Model Predictive Controller (MPC), because of which the MPC has gained attention of control researchers. Use of optimizer as a controller is key to the constraint handling capacity of the MPC and the control design engineer will have the freedom to select the optimal objective function of his/her interest depending on the key issues such as product maximization, minimization of energy resources and minimum utilization of manpower.

The optimization based IMC design procedure can be regarded as an inverse process of identification of a dynamic model of the process[17]. Assuming that the process model around the operating point is available to the control designer, one can think of calculating the required control effort which will follow the specified closed-loop requirements by following the constraints posed by a different process, profit, and experimental conditions.

The idea of using an optimizer as an integral part of the IMC design procedure is as shown in Fig. 3, the required optimal control input calculation is exactly the reverse calculation of process identification(refer Fig. 2). In the present study, quadratic optimization objective function is considered to ensure the global optimal solution of the following form,
4. Results

To prove the efficiency of the optimization based IMC, both linear and nonlinear systems are considered for simulation experimentation. All the simulation experiments have been implemented in MATLAB.

4.1. First order LTI system

To demonstrate the design procedure of optimization based IMC, a simple first-order LTI system with the following dynamics is considered,

\[ G(s) = \frac{0.5}{0.5s + 1} \]  

(5)

Process static gain is 0.5 and the time constant is 0.5. It is also observed that the process shows significant variations in open-loop dynamic response for uncertainties in both process gain and time constant.

Figure 5. Servo control of LTI systems using optimization based IMC: Process variable (black line: reference signal, blue line: constrained, red line: unconstrained).
Figure 6. Servo control of LTI systems using optimization based IMC: Control Input (blue line: constrained, red line: unconstrained).

Figure 7. Servo control of LTI system with uncertainty in the process gain using optimization based IMC: process variable. (black line: reference signal, blue line: k=1 red line: k=2 green line: k=3)
Fig. 5 and Fig. 6 depicts the performance of both constrained and unconstrained IMC. In both cases, the IMC design is able to track the reference signal without any error and much faster than the open-loop response of the system. In Fig. 5, the respective control efforts are plotted along with the upper and lower bounds on the control signal. The constrained IMC is able to perform as well as the unconstrained one by following the constraints on the control signal.

**Figure 8.** Servo control of LTI system with uncertainty in the process gain using optimization based IMC: control input. (blue line: $k=1$ red line: $k=2$ green line: $k=3$)

**Figure 9.** Servo control of LTI system with uncertainty in the process time constant using optimization based IMC: control input. (black line: reference, blue line: $\tau = 1$ red line: $\tau = 1.25$)
Robustness of the optimization based IMC is tested and reported for various uncertainties in both process gain and time constant. Form Fig. 7 and Fig. 8 one can easily observe the capabilities of optimization based IMC performing acceptably for \( k = 1 \), \( k = 2 \), and \( k = 3 \) the reference tracking capabilities are good with a large time constant. It is also observed a corresponding increment in the closed-loop time constant as the process model mismatch increased in terms of model static gain.

The effect of variation in process time constant is depicted in Fig. 9 and Fig. 10. A significant destruction is observed in the closed-loop response with \( \tau = 1 \) and \( \tau = 1.25 \). The resulting control input also suffers from sharp changes at the time of set-point change.

4.2. Nonlinear Continuous Stirred Tank Reactor (CSTR)

CSTR is a highly nonlinear process, where an irreversible exothermic reaction takes place to form a component B from component A (A \( \rightarrow \) B). The inherent nonlinear dynamic behavior of CSTR is mainly due to the exothermic reaction taking place in a reactor with a fixed volume, which has thermal insulation with a coolant who’s temperature and flow rate can be manipulated [18, 19].

The mechanistic model of the CSTR is expressed by a set of two nonlinear differential equations with constant coefficients by the equations given below [18][19].

\[
\frac{dC_A(t)}{dt} = \frac{q(t)}{V}(C_{A0}(t) - C_A(t)) - k_C A(t)e^{\frac{-E}{RT(t)}} \tag{6}
\]

\[
\frac{dT(t)}{dt} = \frac{q(t)}{V}(T_{in}(t) - T(t)) - \frac{(-\Delta H)k_C A(t)}{\rho C_p} e^{\frac{-E}{RT(t)}} + \frac{\rho C_p}{\rho C_p V} \left[1 - e^{\frac{-\Delta H}{\rho C_p V}}\right] (T_{in}(t) - T(t)) \tag{7}
\]

Where \( C_A(t) \), \( T(t) \) dynamic states of the system and the operating parameters and various constants of the CSTR process are given in Table 1. In the present study, \( q_C(t) \) is considered as input and \( T(t) \) as output. The two differential equations(Eq. 7 and Eq. 8) are solved using MATLAB ode
solver to obtain the true output of the process. The open-loop dynamic response of the CSTR process is depicted in Fig. 10 by administrating step changes in the input coolant flow rate on either side of the operating point, from Fig. 10 one can observer that the CSTR process shows a significant change in process gain, inverse response and oscillatory dynamic response around the operating point.

### Table 1: Operating parameters of the CSTR process

| CSTR parameter                   | Value at operating point |
|----------------------------------|--------------------------|
| Process flow rate ($q$)          | 100 l/min                |
| Feed concentration ($C_{in}$)    | 1 mol/l                  |
| Feed temperature ($T_0$)         | 350 K                    |
| Inlet coolant temperature ($T_{co}$) | 350 K               |
| CSTR volume ($V$)                | 100 l                    |
| Heat transfer coefficient ($hA$)  | $7 \times 10^5$ cal/(minK) |
| Reaction rate constant ($k_0$)   | $7.2 \times 10^{10}$ min$^{-1}$ |
| Activation energy ($E/R$)        | $1 \times 10^9$ K        |
| Heat of reaction ($-\Delta H$)    | $-2 \times 10^5$ cal/mol  |
| Liquid density ($\rho, \rho_c$)  | $1 \times 10^3$ g/l      |
| Specific heats ($C_p, C_{pc}$)    | 1 cal/(gK)               |

**Figure 11.** Open-loop response of CSTR process by regulating step input change in coolant flow rate.

In the optimization based IMC design procedure, the nonlinear dynamics of the process are linearized around the operating point and the linearized model of the CSTR process is given by the following set of equations (Operating point considered: $q_c = 103 l/min$, $C_{ass} = 0.0989$ mol/l, $T_{ss} = 438.7763$ K):
\[
\frac{d \tilde{C}_A(t)}{dt} = -10.153 \tilde{C}_A(t) - 0.046833 \tilde{T}(t)
\] (8)

\[
\frac{d \tilde{T}(t)}{dt} = 180.6 \tilde{C}_A(t) + 7.3323 \tilde{T}(t) - 0.88094 \tilde{q}_c(t) + 1.028825 \tilde{T}_{co}(t)
\] (9)

\[
y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{C}_A(t) \\ \tilde{T}(t) \end{bmatrix}
\] (10)

Where \( \tilde{C}_A(t), \tilde{T}(t), \tilde{q}_c(t), \tilde{T}_{co}(t) \) are deviation variables.

### 4.2.1. Servo response

The servo response of the CSTR process with optimization based IMC is depicted in Fig. 11, different changes in the set-point are introduced on either side of the nominal operating temperature. A significant reduction in the oscillations in the process variable is observed at shifted operating points. Moreover, when the set-point is changed to 434 \( ^\circ K \) (high resonating zone) the resulting closed-loop control output much better than that of the open-loop response of the CSTR process and then settles to a stable reference value much faster than that of open-loop dynamics. Both the constrained and unconstrained IMC are shown in Fig. 11, the corresponding control efforts are plotted in Fig. 12. In the case of constrained IMC, the bound constraints are imposed on control effort through \( 93 \leq u^* \leq 113 \). The Integral Square Error (ISE) values between the set-point and the process output for both unconstrained and constrained IMC are reported as low as 1.1676 and 1.6703 respectively.

![Figure 12. Servo response of CSTR process with optimization based IMC: Process output (black line: reference signal, blue line: unconstrained, red line: constrained).](image-url)
4.2.2. Servo regulatory response

In this part simulation study, both set-point and a load disturbance are introduced to test the capabilities of the proposed IMC in the absence and presence of process constraints. Load disturbance is introduced in terms of feed temperature at different sampling instants as shown in Fig. 15. A disturbance in terms of feed temperature is introduced at sampling instant 10, sampling instant 80 by
keeping the set-point constant at its nominal value. At sampling instant 160, a set-point change is introduced by bringing back the feed temperature to its nominal value. Fig. 13 and Fig. 14 shows the performance of the closed-loop response of the optimization based IMC for changes in both reference and load. It is very clearly evident from the Fig. 13 and Fig. 14 that the disturbance rejection capabilities of IMC is very good in nominal operating conditions and shifted operating conditions. The Integral Square Error (ISE) values between the set-point and the process output for both unconstrained and constrained IMC are reported as low as 0.2347 and 0.3428 respectively.

Figure 15. Servo regulatory response of CSTR process with optimization based IMC: Controller output (black line: lower and upper bounds on control signal, blue line: unconstrained, red line: constrained).

Figure 16. Load change in feed temperature of CSTR process.
5. Conclusion

In this article, the authors have presented a simple way of handling constraints offered by various process parameters and experimental conditions through optimization based IMC. The proposed controllers efficacy in controlling both linear and nonlinear systems is presented. In the case of LTI systems, the effect of variations of the process parameters is reported. The capabilities of optimization based IMC on controlling the nonlinear CSTR process which is highly nonlinear, resonating around the operating point and shows the inverse response. The efficacy of the proposed IMC strategy is reported with the support of extensive simulation case studies, both at the nominal operating point and shifted operating points.

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