We discuss the properties of light pseudoscalar particles, the so-called pseudoaxions, within Little Higgs models, focusing on their phenomenology at the ILC. We especially discuss a method of how to distinguish between the two basic classes of Little Higgs models, the product and simple group models, by a specific production channel and decay mode. These are strictly forbidden in the product group models.

1 Little Higgs Models

Little Higgs Models [2] provide a solution to the hierarchy problem, as they stabilize the Higgs boson against quadratic divergences at the one-loop level by the mechanism of collective symmetry breaking: the Higgs is charged under two global symmetry groups, which both need to be broken in order to lift the flat direction in the potential of the Higgs boson and make it a pseudo-Nambu-Goldstone boson (PNGB). During the last years a bewilderment of different models has been developed.

These models can be classified in three different categories, the so-called moose models with a moose diagram structure of links of global and local symmetry groups, the product-group models and the simple-group models. In the product-group models (the most-studied case is the Littlest Higgs) the electroweak gauge group is doubled, broken down to the group $SU(2)_L$, while the Higgs shares together with the other PNGBs an irreducible representation of the coset space of the global symmetry breaking. On the other hand, in simple-group models the electroweak gauge group is enlarged to a simple $SU(N)$ group, while the Higgs is distributed over several multiplets of the global symmetry group, which usually has a product group structure similar to chiral symmetries in QCD [3]. For an overview, see [4].

The two crucial scales in the Little Higgs set-up are the cut-off scale $\Lambda$ where the models are embedded in a UV-complete theory (usually a strongly-interacting theory with a partonic substructure of the PNGBs) and the intermediate scale $F$ which determines the masses and decay constants of the PNGBs (except for the Higgs which is down at $v$ by the collective symmetry breaking mechanism). Electroweak precision observables and direct search limits [5] tell us that the scale $F$ must be at least of the order of $1 - 2$ TeV. Paradoxically, the Higgs boson in Little Higgs models tends to be quite heavy compared to the Standard LCWS/ILC 2007
Model or the MSSM, of the order of $200 - 600 \text{ GeV}$ [3]. For Little Higgs model scales that high most new particles will be produced close to the kinematical limit at the LHC, such that a precision determination of their parameters might be difficult. Furthermore, also the sensitivity of the ILC in indirect measurements might be limited, if the new physics does couple to SM fermions only very weakly [7]. A method to distinguish between different models, especially at the LHC, is highly welcome. Such a method will be presented here.

2 Pseudoaxions in Little Higgs models

Little Higgs models generally have a huge global symmetry group, which contains not only products of simple groups but also a certain number of $U(1)$ factors. These Abelian groups can either be gauged, in which case they lead to a $Z'$ boson, or they are only (approximate) global symmetries. In the latter case there is a pseudo-Goldstone boson attached to that spontaneously broken global $U(1)$ factor [8]. The number of pseudoaxions in a given model is determined by the mismatch between the rank reduction in the global and the local symmetry group, since it gives the number of uneaten bosons. In the Littlest Higgs, e.g., there is one such pseudoaxion, in the Simplest Little Higgs [9] there is one, in the original simple group model there are two, in the minimal moose model there are four, and so on.

These particles are electroweak singlets, hence all couplings to SM particles are suppressed by the ratio of the electroweak over the Little Higgs scale, $v/F$. There mass lies in the range from several GeV to a few hundred GeV, being limited by a naturalness argument and the stability of the Coleman-Weinberg potential. For the Simplest Little Higgs, on whose phenomenology we will concentrate here, there is a seesaw between the Higgs and the pseudoscalar mass [5], determined by the explicit symmetry breaking parameter $\mu$, where $m_\eta \approx \sqrt{2}\mu$. Since the pseudoaxions inherit the Yukawa coupling structure from the Higgs bosons, they decay predominantly to the heaviest available fermions in the SM, and because of the absence of the $WW$ and $ZZ$ modes, the anomaly-induced decays $gg$ and $\gamma\gamma$ are sizable over a wide mass range, cf. Fig. [1] From this, one can see that as soon as the decay to $HZ$ is kinematically allowed, it dominates completely. Such a $\eta HZ$ coupling, which is possible only after electroweak symmetry breaking and hence proportional to $v/F$, is only allowed in simple group models and is forbidden to all orders in product group models. One can factor out the $U(1)_\eta$ group from the matrix of pseudo-Goldstone bosons. We use $\xi = \exp[i\eta/F]$ for the pseudo-axion field and $\Sigma = \exp[i\Pi/F]$ for the non-linear representation of the remaining Goldstone multiplet $\Pi$ of Higgs and other heavy scalars. Then, for product group models.
with the Baker-Campbell-Hausdorff identity, one gets for the term in parentheses in Eq. (2):

$$Z \eta$$

global simple group forces them to be traceless as well. We conclude that the coefficient of entries of $\Sigma$, while the electroweak gauge bosons reside in the upper left corner of $U$.

This vanishes due to the zero trace of $SU$.

Next, we consider the simple group models, where we use the following notation for the $A$ which is the key to the Little Higgs mechanism: $D_\mu \Sigma = \partial_\mu \Sigma + A^a_{1,\mu} (T^a_1 \Sigma + \Sigma(T^a_1)^T) + A^a_{2,\mu} (T^a_2 \Sigma + \Sigma(T^a_2)^T)$, where $T^a_i, i = 1, 2$ are the generators of the two independent $SU(2)$ groups, and $A^a_{\mu} = W^a_\mu +$ heavy fields. Neglecting the heavy gauge fields and extracting the electroweak gauge bosons, we have $Tr [(D^\mu \Sigma)^i \Sigma] \sim W^a_\mu Tr [(T^a_1 + T^a_2) + (T^a_1 + T^a_2)^*] = 0$. This vanishes due to the zero trace of $SU(2)$ generators. The same is true when we include additional $U(1)$ gauge group generators such as hypercharge, since their embedding in the global simple group forces them to be traceless as well. We conclude that the coefficient of the $ZH\eta$ coupling vanishes to all orders in the $1/F$ expansion.

Next, we consider the simple group models, where we use the following notation for the nonlinear sigma fields: $\Phi \zeta$, where $\Phi = \exp[\Sigma/F]$ and $\zeta = (0, \ldots, 0, F)^T$ is the vev directing in the $N$ direction for an $SU(N)$ simple gauge group extension of the weak group. Thus, in simple group models the result is the $N, N$ component of a matrix:

$$L_{\text{kin}} \sim F^2 D^\mu (\zeta \Phi^\dagger D_\mu (\Phi \zeta) = \ldots + iF(\partial_\mu \eta) (\Phi^\dagger (D_\mu \Phi) - (D_\mu \Phi^\dagger) \Phi)_{N,N} .$$

We separate the last row and column in the matrix representations of the Goldstone fields $\Sigma$ and gauge boson fields $V_\mu$: the Higgs boson in simple group models sits in the off-diagonal entries of $\Sigma$, while the electroweak gauge bosons reside in the upper left corner of $V_\mu$. With the Baker-Campbell-Hausdorff identity, one gets for the term in parentheses in Eq. (2):

$$V_\mu + iF [\Sigma, V_\mu] - \frac{1}{2F^2} [\Sigma, [\Sigma, [\Sigma, V_\mu]]] + \ldots$$

$$= (W_\mu \quad 0) + i \left( \begin{array}{cc} 0 & -W_\mu h \\ h^\dagger W_\mu & 0 \end{array} \right) - \frac{1}{2F^2} \left( \begin{array}{cc} hh^\dagger W + W hh^\dagger & 0 \\ 0 & -2h^\dagger W h \end{array} \right) + \ldots$$

Figure 3: ILC cross section, left: small-mass $\eta$ with final state $Hbb$, right intermediate mass with final state $Hgg$. The dotted line is without the $Z/Z'$ interference, the dashed one the SM background.

$$L_{\text{kin}} \sim F^2 Tr [(D^\mu (\zeta \Phi^\dagger D_\mu (\Phi \zeta))] = \ldots - 2F(\partial_\mu \eta) \ ImTr [(D^\mu \Sigma)^i \Sigma] + O(\eta^2),$$

where we write only the term with one derivative acting on $\xi$ and one derivative acting on $\Sigma$. This term, if nonzero, is the only one that can yield a $ZH\eta$ coupling.
The \( N, N \) entry can only be nonzero from the third term on. The first term, would be a mixing between the \( \eta \) and the Goldstone boson(s) for the \( Z' \) state(s) and cancels with the help of the many-multiplet structure. If the \( N, N \) component of the second term were nonzero, it would induce a \( ZH \eta \) coupling without insertion of a factor \( v \). This is forbidden by electroweak symmetry. To see this, it is important to note that in simple group models the embedding of the Standard Model gauge group always works in such a way that hypercharge is a linear combination of the \( T_{N2-1} \) and \( U(1) \) generators. This has the effect of canceling the \( \gamma \) and \( Z \) from the diagonal elements beyond the first two positions, and preventing the diagonal part of \( W_\mu \) from being proportional to \( \tau^3 \). The third term in the expansion yields a contribution to the \( ZH \eta \) coupling,

\[
(\partial^\mu \eta) h^1 W_\mu h \sim v H Z_\mu \partial^\mu \eta.
\]

The crucial observation is that the matrix-representation embedding of the two non-Abelian \( SU(2) \) gauge groups, and especially of the two \( U(1) \) factors within the irreducible multiplet of the pseudo-Goldstone bosons of one simple group (e.g. \( SU(5) \) in the Littlest Higgs), is responsible for the non-existence of this coupling in product group models. It is exactly the mechanism which cancels the quadratic one-loop divergences between the electroweak and heavy \( SU(2) \) gauge bosons which cancels this coupling. In simple group models the Higgs mass term cancellation is taken over by enlarging \( SU(2) \) to \( SU(N) \), and the enlarged non-Abelian rank structure cancels the quadratic divergences in the gauge sector – but no longer forbids the \( ZH \eta \) coupling. Hence, its serves as a discriminator between the classes of models.

### 3 ILC phenomenology

The pseudoaxion can be discovered at the LHC in gluon fusion and observed in the rare decay mode \( \gamma \gamma \) [5]. But the \( \eta H Z \) coupling can be observed at the LHC only if either one of the decays \( H \rightarrow Z \eta \) or \( \eta \rightarrow ZH \) is kinematically allowed. This leaves large holes in parameter space, which can be covered by a 500 – 1000 GeV ILC, depending on the masses. Here, we focus on the discovery potential of the ILC for the pseudoaxions, assuming the presence of the \( ZH \eta \) coupling. We focus on the Simplest Little Higgs with parameters chosen to fulfill the low-energy constraints. The production happens via an \( s \)-channel \( Z \) exchange, in association with a Higgs boson like in a two-Higgs-model. Fig. 2 shows the cross section as a function of \( \sqrt{s} \) for three different values of the \( \eta \) mass. The simulations for the processes discussed here have been performed with the WHIZARD/OMEGA package [11, 12, 13], which is ideally suited for physics beyond the SM [14]. In the following, we assume that the Higgs properties are already known from LHC, and that the Higgs can be reconstructed. In Fig. 5 and 6, we show the three different possible final states, depending on the dominant branching ratio of the pseudoaxion: for low masses (up to approx. 150 GeV) this is \( b\bar{b} \), in an interme-

**Figure 4:** ILC cross section for a high-mass \( \eta \) with final state \( ZHH \). The dotted line is without the \( Z/Z' \) interference, the dashed one the SM background.
mediate range (between 150 GeV and 270 GeV) the $gg$ (hence dijet) mode is largest, while $ZH$ sets in above masses of 270 GeV. The figures show the effect of a destructive $Z/Z'$ interference, which brings cross sections down by a factor of two at the peak, but never endangers visibility. SM backgrounds are nowhere an issue, from marginal for $Hjj$ to negligible in the $Hbb$ case. Interesting is the $ZH H$ final state which is important for measuring the triple Higgs coupling [15]. In the SM the cross section is at the borderline of detectability, but Fig. 4 shows that rates are larger by factors two to six in the Simplest Little Higgs with the intermediate pseudoaxion. In conclusion, the ILC provides an ideal environment for discovering pseudoaxions and measuring their properties. The $ZH \eta$ coupling provides tool for the discrimination between simple and product group models.

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