The Theoretical Power Law Exponent for Electron and Positron Cosmic Rays: 
A Comment on the Recent Letter of the AMS Collaboration

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In a recent letter, the AMS collaboration reported the detailed and extensive data concerning the 
distribution in energy of electron and positron cosmic rays. A central result of the experimental 
work resides in the energy regime 30 GeV < E < 1 TeV wherein the power law exponent of 
the energy distribution is measured to be α(experiment) = 3.17. In virtue of the Fermi statistics 
obeyed by electrons and positrons, a theoretical value was predicted as α(theory) = 3.151374 in 
very good agreement with experimental data. The consequences of this agreement between theory 
and experiment concerning the sources of cosmic ray electrons and positrons are briefly explored.

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INTRODUCTION

In a recent letter\[1\], the AMS collaboration reported the energy distribution of the lepton sector, i.e. the 
energy probability density ρ(E) of the electron and positron contribution to cosmic rays. In particular, the AMS col- 
laboration measured the slope of the log-log plot of the 
energy distribution ρ(E) versus the single cosmic ray particle energy E,

\[ -\alpha(E) = \frac{d\ln \rho(E)}{d\ln E} = \left[ \frac{E}{\rho(E)} \right] \left[ \frac{d\rho(E)}{dE} \right]. \tag{1} \]

We have previously argued\[2\] that for cosmic rays emitted as a particle wind evaporating from a compact stellar 
source, such as a neutron star, the function α_{Fermi}(E) may be computed from quantum statistical thermodynamics. The argument is reviewed in what follows below. For fermions -in the asymptotic high energy region- we 
predicted that the power law exponent would be given by

\[ \alpha_{Fermi}(\text{theory}) = 3.151374, \tag{2} \]

while in the experimental region, 30 GeV < E < 1 TeV, we have

\[ \alpha(\text{experiment}) = 3.170 \pm 0.008(\text{stat + syst}) \]

\[ \pm 0.008(\text{energy scale}), \tag{3} \]

wherein the agreement between Eqs.\[2\] and \[3\] is more 
than satisfactory. Incidentally, were leptons a classical relativistic Maxwell-Boltzmann gas, the index would 
have been

\[ \alpha_{MB} = 3, \tag{4} \]

instead of the corresponding Fermi-Dirac index given in Eq.\[2\]. The difference is small and positive

\[ \alpha_{FD} - \alpha_{MB} \approx +0.151374, \tag{5} \]

that properly accounts for the repulsion between fermions inherent in their quantum statistics.

In the concluding section, we briefly discuss the implications of this agreement between theory and experiment.

STATISTICAL THERMODYNAMICS

The energy distribution of particles evaporated from compact stellar objects is determined by the entropy per 
evaporated particle s(E) via ρ(E) ∝ exp[−s(E)/k_B]. Together with Eq.\[1\], we thereby have

\[ \alpha(E) = \left[ \frac{E}{k_B} \right] \frac{ds(E)}{dE} = \frac{E}{k_B T(E)}, \tag{6} \]

wherein the thermodynamic relationship \( T = dE/ds \) has been employed. It is computationally more simple to find the energy as a function of temperature \( E(T) \) and later find the temperature as a function of energy \( T(E) \) as an inverse function. In virtue of a relativistic ideal Fermi gas 
with a density of states per unit energy per unit volume \( g(\epsilon) \), one obtains

\[ g(\epsilon) = \frac{\epsilon \sqrt{c^2 - m^2 c^4}}{\pi^2 h^3 c^3} \quad \text{for} \quad \epsilon \geq mc^2, \]

\[ f(\epsilon) = \frac{1}{\exp(\epsilon/k_BT) + 1}, \]

\[ E = \overline{\epsilon} = \frac{\int \epsilon g(\epsilon) f(\epsilon) d\epsilon}{\int g(\epsilon) f(\epsilon) d\epsilon}. \tag{7} \]

For the ultra relativistic regime wherein electron mass effects are small, i.e. \( mc^2 \ll k_BT \), \( mc^2 \ll E \) and \((E/k_BT) = \text{constant} \), our theoretical prediction in Eq.\[2\] is recovered from Eqs.\[6\] and \[7\].
RADIATION DAMPING

If the energy distribution of evaporating electrons and positrons from compact stellar sources is as described above, then the question arises as to whether this energy distribution changes appreciably due to radiation damping as these cosmic leptons propagate from the source to the laboratory detectors built within our solar system. We here note that the accelerations of these charged particles due to random cosmic electromagnetic fields of the order of micro-Gauss do not radiate appreciable energy for propagation distances of galactic proportions.

To see what is involved, consider an electron or positron with energy $E = mc^2\gamma$ moving along a circular arc in a magnetic field $B$. Due to radiation damping, it is well known that the radiation energy loss obeys

$$\frac{d\gamma}{dt} = -\frac{1}{\tau}(\gamma^2 - 1),$$

with a characteristic time scale $\tau$ determined by

$$\tau \approx 5.15868 \times 10^{20} \text{ sec} \left(\frac{10^{-6} \text{ Gauss}}{B}\right)^2.$$

With an initial energy $E_i = mc^2\gamma_i = mc^2 \coth \chi$, the exact solution of the radiation energy loss is thereby

$$\gamma(t) = \coth \left(\frac{t}{\tau} + \chi\right) = \frac{\tanh(t/\tau) + \gamma_i}{1 + \gamma_i \tanh(t/\tau)}.$$

It is thereby evident that

$$1 \gg \gamma_i \tanh(t/\tau) \quad \text{and} \quad \gamma_i \gg 1 \quad \text{implies} \quad \gamma(t) \approx \gamma_i.$$

For the energy range of importance in the AMS experiment and for cosmic magnetic fields of the order of micro-Gauss, the time of flight for an electron or positron emitted from a compact source to a detector within our solar system without appreciable radiation damping of energy is the upper time limit $t < t^* \sim 10^{14}$ sec. The time $t^*$ is by a large margin more than the time taken for a speed of light signal to transverse our galaxy.

CONCLUSION

In the baryon sector, the heavy nuclear cosmic ray particles can be bosons or fermions, the least massive cosmic ray particles being protons that are fermions. Bosons (such as bosonic nuclei) and fermions each have their own power law exponent $\alpha$ in the high energy regime depending purely upon statistics. A likely source of these evaporating cosmic rays are compact stellar objects such as neutron stars. Such objects would also radiate a copious amount of electrons (directly or from neutron decay) and electron-positron pairs (if for no other reason than that fast charged particles, be they electrons or baryons, when scattering through any background matter or radiation will produce such pairs). It is of central importance that the AMS collaboration has measured the appropriate fermion power law exponent here characteristic of the lepton sector of cosmic rays.

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[2] A. Widom, J. Swain and Y. N. Srivastava, “Concerning the Nature of the Cosmic Ray Power Law Exponents”, arXiv:1410.6498v1 [hep-ph] 15 Oct 2014.
[3] L. D. Landau and E. M. Lifschitz, “The Classical Theory of Fields”, 4th edition, Pergamon Press, Oxford, 1975.