Study of rotor dynamics considering a fluid film bearing

I I Ivanov¹,², V V Belousov¹, V Y Myasnikov¹,², N N Serebriakov¹ and D V Shadrin¹,²

¹ Dynamics and strength of aviation engines department, Central Institute of Aviation Motors, 111116 Aviamotornaya Street 2, Moscow, Russian Federation
² Applied Mechanics Department, Bauman Moscow State Technical University, 105005 2nd Baumanskaya Street 5, Moscow, Russian Federation

iiivanov@ciam.ru

Abstract. Fluid film bearings are widely used in supports of gas turbines, pumps and compressors. They also present in test benches designed for overspeed tests of aircraft engine’s discs. Stiffness and damping of these supporting elements are nonlinear. Moreover fluid film bearing reactions include circulation forces that can bring to well-known types of self-excited vibrations – whirl and whip. Magnitudes of those oscillations are almost not influenced by unbalances and are determined by the system’s properties, in particular – properties of damping and circulation forces. If this type of vibrations does not arise and only forced oscillations present, the corresponding resonance frequencies will depend on unbalances because of fluid film bearing nonlinear stiffness and damping. The listed considerations bring to the necessity of account of nonlinear non-conservative support’s properties in modeling of rotor dynamics. The paper describes the appropriate approach that includes joint use of finite element model of rotor and nonlinear models of supports. The vibrations of rotor with rolling and fluid film bearings in supports are considered as example. Rolling bearings were modeled by nonlinear exponential dependencies of forces on displacements. Those dependencies were computed from the Hertz contact theory. Fluid film bearing was modeled with analytical solution of the Reynolds equation. The problem was solved in nonlinear nonstationary formulation considering mutual influence of rotor displacements and bearing reactions. Results of multiple simulations performed for different values of rotor speed are presented as a frequency response function and a waterfall plot. The results were verified with experimental data.

1. Introduction
At present, simplified linearized bearing models are mostly used in dynamic models of system «rotor – bearings – casings». This assumption is acceptable for the case of aircraft engine as its casings are more flexible than bearings and its dynamical behavior is not strongly sensitive to bearing properties. However one should consider bearing nonlinear properties when modeling land-based units such as gas turbines, pumps and compressors. Reaction of this bearing includes circulation force that can bring to growth of well-known types of self-excited vibrations – whirl and whip [1]. Self-oscillation magnitudes cannot be defined if using linear models. Therefore it’s preferable to apply nonlinear rotor support models for study of those oscillations. Besides of self-excited vibrations there always present forced vibrations caused by rotor unbalance in the system. Frequency of those vibrations is synchronous to rotor speed. Because stiffness and damping of bearings are nonlinear the resonance
frequencies of the system «rotor – bearings – casings» are dependent on unbalances. In other words, rotor design and material properties do not completely determine the operating mode ranges with increased level of vibrations and rotor unbalances also influence the resonance frequencies. The listed argumentation brings to necessity of use the nonlinear models of rolling bearings and journal bearings when solving the rotor dynamics problems. Analogous considerations are true for the rotors with squeeze-film dampers in supports [2].

The common approach to study forces in fluid film of journal bearing is to solve the Reynolds equation [3] with boundary conditions corresponding to design of specific bearing. Classical solutions [3] of the equation for the so-called «long» and «short» fluid film bearings were derived correspondingly by Sommerfeld and Ocvirk. The limitations of those solutions are the assumption of ideal seals in long-bearing approximation and the assumption of full oil leakage in short-bearing approximation. There are also hypotheses of «2π-film» and «π-film» boundary conditions on cavitation region [4], [5]. Those assumptions do not allow use of the listed solutions for arbitrary sizes and designs of bearings because the cavitation region size is a priori unknown and the real end-seal properties usually do not match to long-bearing and short-bearing approximations. Therefore numerical solutions of the model equations are widely used: either finite difference method [6] or finite element method [7]. The detailed description of end-seal models is adduced in [6].

The cavitation regions strongly influence stiffness and damping of fluid film bearings. The most frequently used boundary conditions on cavitation regions are the conditions suggested by Sommerfeld (2π-film), Günbel (π-film), Swift and Stieber [8] and JFO boundary condition (Jakobsson, Floberg, Olsson) [9]. The last one is the only listed condition that does not break the mass conservation law [8] at the boundary between oil region and cavitation region. The direct use of the JFO boundary condition is inconvenient because determination of the cavitation region’s moving curvilinear boundary is poorly formalized within standard finite difference method and finite element method. The original approach to avoid that limitation was developed by Elrod [10].

We may conditionally classify the modern researches in the field of fluid film bearing modeling on two categories: developing of numerical methods for solution of the Reynolds equation, in particular – modifications of the Elrod’s algorithm [11], [12]; improvement of analytical approaches [13], [14]. Although the numerical methods are surely able to give high accuracy of oil thin film pressure computation, they can bring to unreasonably high computation costs when solving the problems of rotor dynamics. Therefore models of the system «rotor – bearings – casing» often use simplified analytical solutions of the Reynolds equation [15].

The common approach to study rotor dynamics considering nonlinear and non-conservative support properties is the mathematical simulation of the system «rotor – bearings – casing» dynamics [15], [16], where external forces and bearing reactions are applied to rotor and casing models. The iterative procedure of refinement of model displacement values is performed at every time step till the convergence criterion will be fulfilled. At each iteration the nonlinear bearing models determine reactions, and then the displacements are computed from the dynamic models of rotor and casing. Those displacements are further used at the next iteration for computation of bearing reactions. Thus the described approach accounts the mutual influence of support reactions and rotor and casing displacements in bearing positions. If the rotor has only rolling bearings the same approach is applied [17]-[21], but more simple nonlinear elastic models of the bearings are used. Those models are built from the Hertz contact theory.

Although there are a number of publications on the subject, the problem of vibrations of the closed-loop dynamic system «rotor – bearings - casing» has been not studied enough yet, because usually simplified model rotors are studied. Those rotors’ designs are significantly unlike the real designs. In the current research the approach described in the previous paragraph was applied to study the dynamical behavior of rotor of a test bench designed for aircraft engine disc overspeed tests. The 3D finite element (FE) model of rotor was developed and its dynamical behavior was simulated in non-stationary formulation with account of nonlinear supports. An analytical solution [22] was used to
model fluid film bearing. The results of computations are the time dependencies of displacements and also summary dependencies of oscillation magnitudes on rotor speed.

2. Mathematical model

The following analytical model of fluid film bearing was used [22]:

\[
p(\varphi, z) = \lambda p_0(\varphi) \left( \frac{z}{L} + \frac{1}{2} \right) + p_s \left( \frac{1}{2} \frac{z}{L} \right) + 6\mu \left( \frac{L^2}{c} \right) \left( \frac{2 \cos \varphi + \varepsilon \left( \omega - \Omega / 2 \right)}{1 + \varepsilon \cos \varphi} \right) \left( \frac{z}{L} \right) - \frac{1}{4} 
\]

where \( \lambda \) – dimensionless end-leakage factor, \( \lambda = 0 \ldots 1 \); \( p_0(\varphi) \) – pressure using the long-bearing approximation, MPa; \( \varphi \) – circumferential coordinate measured from the position of maximum film thickness, rad; \( z \) – axial coordinate, mm; \( L \) – bearing length, mm; \( p_s \) – oil supply pressure, MPa; \( \mu \) – oil viscosity, MPa·s; \( c \) – radial clearance, mm; \( \varepsilon \) – eccentricity ratio (relative radial displacement / radial clearance); \( \omega \) – angular velocity of shaft precession in bearing position, rad/s; \( \Omega \) – rotor speed, rad/s; \( R \) – bearing radius, mm.

Adjustment of parameter \( \lambda \) value allows the model (1), (2) to account non-ideal end-seal properties of bearings. Specifying \( \lambda \) and \( p_s \), one can [5] model fluid film bearings with different designs of end-seals and oil feed grooves. In the particular case, when there are no rotor vibrations and there is no oil leakage through the end-seals, model (1), (2) gives an uniform pressure distribution equal to oil supply pressure \( p_s \).

We account cavitation as the following: if pressure \( p \) will be below the value of cavitation pressure \( p_{cav} \) (we assume \( p_{cav} = 0 \)), we take \( p = p_{cav} \). That assumption is acceptable because we consider the integral values - fluid film resultant forces.

The journal bearing reactions were computed as integrals of expression (1) through the bearing’s surface with account of direction:

\[
F_r = \int_{0}^{2\pi} d\varphi \int_{-L/2}^{L/2} p(\varphi, z) R \cos \varphi d\varphi
\]

\[
F_t = \int_{0}^{2\pi} d\varphi \int_{-L/2}^{L/2} p(\varphi, z) R \sin \varphi d\varphi
\]

where \( F_r, F_t \) – correspondingly radial (along the direction of relative displacement of shaft and casing in the bearing’s position) and circumferential bearing forces, N. Those forces further were projected on axes of the global stationary rectangular coordinate system.

Mathematical model of ball bearing is given by the following expression [23]:

\[
\delta_r = 4.36 \cdot 10^{-4} \frac{Q_{max}^{2/3}}{D^{1/3} \cos \alpha}
\]

where \( \delta_r \) – relative displacement of shaft and casing in the bearing’s position, mm; \( Q_{max} \) – maximum force on one rolling element, N; \( D \) – ball diameter, mm; \( \alpha \) – contact angle in the ball bearing.

The force \( Q_{max} \) was determined from the Stribeck’s theory [23] that gives the following simplified expression for the maximum force with account of clearances in the bearing:
\[
Q_{\text{max}} = \frac{5F_r}{z \cos \alpha}
\]  

(6)

where \(F_r\) – resultant bearing radial force, N; \(z\) – number of rolling elements.

Substitute of (6) to (5) gives the following expression for \(F_r\):

\[
F_r = C\delta_r^n
\]

(7)

where \(C\) – the constant depending on the number and the sizes of rolling elements and the contact angle in the bearing; \(n\) – the exponent, for roll – \(n = 10/9\), for ball – \(n = 3/2\). The \(C\) value was determined from Hertz’s and Stribeck’s theories [23].

Simulation of rotor dynamics was performed in finite element application MSC Nastran by means of solution of the following system:

\[
[M]\{\ddot{q}\} + [D]\{\dot{q}\} + [G]\{\dot{q}\} + [K]\{q\} = \{f\}
\]

(8)

where \([M]\) – mass matrix, \([K]\) – stiffness matrix, \([D]\) – damping matrix, \([G]\) – gyroscopic matrix, \([f]\) – force vector. \([f]\) includes bearing reactions and centrifugal forces caused by rotor unbalance. The Guyan’s condensation method was applied to decrease computation time. The FE model was reduced to condensation nodes that were situated at the rotor axis and were connected to 3D FE model through multi-point constraints. The algorithm of simulation of fluid film bearing was based on the expressions (1)-(4) and implemented in C++ code that was linked to the model in FE application.

The nonlinear problem was solved iteratively at every time step. The algorithm of solution is following. First, the equation (8) gives rotor displacements. Then bearing reaction are computed from equations (1)-(4), (7). Third, the new iteration begins with new values of forces. Iterative procedure continues till convergence.

3. Description of design and model

The design of the rotor is depicted in the figure 1. The rotor is situated along the vertical axis and its disc 1 is in the bottom. All rotor’s parts with unspecified material (see figure 1) were made from steel. The details of design are shown in the figure 2, where positions 3 are square fits without initial interference. Those fits transfer only torsional moment and shear forces and do not transfer bending moments and axial force. Position 4 in the figure 2 is cylindrical fit transferring only shear forces.

The section of rotor FE model is shown in the figure 3. The model was built of elements with quadratic approximation of displacements. The figure 4 shows FE model of rotor with specified positions of supports. The model includes 7 nonlinear elements: 6 ball bearings and 1 fluid film bearing. Spherical hinges were used to model fits without interference (see figures 1, 2).

It was assumed that unbalance was situated in the disc. Parameters of fluid film bearing and rolling bearings are adduced in the tables 1 and 2 correspondingly. Summary parameters of rotor bench are shown in the table 3.

Figure 1. Design of the rotor bench

Br – bronze parts, Ti – titan parts, 1 – disc, 2 – fluid film bearing, A – connection of disc shaft and drive shaft, set of bearings
3 – square fits without interference, 4 – cylindrical fit without interference

**Figure 2.** Zoom of fragment of rotor bench design

Figure 3. Rotor finite element model

1-6 – rolling bearings, 7 – fluid film bearing.

**Figure 4.** Section of rotor finite element model with specification of support positions

| Table 1. Parameters of fluid film bearing |
|------------------------------------------|
| Parameter                  | Value          |
| Clearanace $c$ (mm)         | 0.0425         |
| Length $L$ (mm)             | 16             |
| Radius $R$ (mm)             | 19             |
| Oil viscosity $\mu$ (MPa·s) | $2.72 \times 10^{-8}$ |
| End-leakage factor $\lambda$| 1.0            |
| Oil supply pressure $p_s$ (bar) | 1.0        |
| Cavitation pressure $p_{cav}$ (bar) | 0.0     |

| Table 2. Parameters of rolling bearings and parameters of corresponding mathematical models |
|---------------------------------------------------------------------------------------------|
| Parameter       | Bearings in supports 1, 2 | Bearings in supports 3,4,5,6 |
|-----------------|----------------------------|-----------------------------|
| $D$ (mm)        | 4.762                      | 5.556                       |
| $z$             | 10                         | 10                          |
| $\alpha$ (deg)  | 26                         | 26                          |
| $C$ (N/mm$^{3/2}$) | $3.75 \times 10^5$        | $3.97 \times 10^6$         |

| Table 3. Summary parameters of rotor bench |
|---------------------------------------------|
| Length (mm)       | Disc diameter (mm) | Rotor mass (kg) | Unbalance (g·mm) |
|-------------------|-------------------|----------------|-----------------|
| 545.0             | 152.4             | 5.2            | 3.8             |

4. Simulation results

The rotor’s translational displacements at the position of the fluid film bearing were computed for different values of rotor speed. The time dependencies and specters corresponding to rotor speed values 1000 RPM and 3000 RPM are presented in the figures 5 and 6. We can see from the figure 5 that oscillations are limited and there is only synchronous rotor frequency at the specter. Figure 6 shows an oscillation magnitude increase and also that subsynchronous frequency dominates in the specter. That frequency is below $\frac{1}{2}$ of rotor speed, this is known marker of self-oscillations [24] excited due to fluid film forces.
Figure 5. Time dependence of displacements (left) and displacement specter (right) computed for rotor speed 1000 RPM

Figure 6. Time dependence of displacements (left) and displacement specter (right) computed for rotor speed 3000 RPM

The waterfall plot in the figure 7 summarizes the results of simulations for multiple rotor speed values. There are two sets of extreme in the figure: synchronous vibration and self-excited subsynchronous vibration. Those features are in good agreement with known classical solutions [24].

Further the computed frequency response is adduced in the figure 8 and the experimental frequency response is adduced in the figure 9. The table 4 shows the comparison of parameters of computed and measured frequency response functions.

| Parameter                                      | Computation | Experiment |
|------------------------------------------------|-------------|------------|
| Resonance frequency (RPM)                      | 900         | 850        |
| Resonance magnitude (μm)                       | 25          | 92         |
| The bottom frequency of self-oscillation region (RPM) | 3000        | 3400       |
| The maximum vibration magnitude in region of self-oscillations (μm) | 35          | 38         |
Figure 7. Waterfall plot

Figure 8. Computed nonlinear frequency response

Figure 9. Experimental time dependencies of rotor speed and vibration magnitude

We can see two regions of increased vibrations on the both computed and measured frequency response functions: resonance about 900 RPM and self-oscillations at rotor speed values higher than 3000 RPM. The significant difference of the resonance vibration magnitude is because of the
difference between residual unbalance after balancing procedure and actual unbalance during the experiment. Other parameters of frequency response functions, such as resonance frequency, the bottom frequency of self-oscillation region, the maximum vibration magnitude in region of self-oscillations are in good agreement.

5. Conclusion

The model of the system «rotor – bearings - casing» was developed in the study. It includes FE model of the rotor and nonlinear models of rolling bearings and fluid film bearing. The written C++ code implements the model of the fluid film bearing to general purpose FE application MSC Nastran. The study of unbalanced rotor vibrations was performed using the developed model. It was shown that model can predict the forced vibrations caused by the unbalance and also the self-oscillations caused by non-conservative forces in bearing fluid film. Multiple simulations gave the dependence of vibration magnitudes on rotor speed. The dependence was verified with experimental data. The approach may be used for study rotors with squeeze-film dampers in supports.

References

[1] Genta G 2007 Dynamics of rotating systems (Springer Science & Business Media)
[2] Inozemtsev A A and Sandratskiy V L 2006 Gazoturbinnyye dvigateli (Perm: Aviadvigatel)
[3] Hamrock B J 1994 Fundamentals of Fluid Film Lubrication (New York: McGraw-Hill)
[4] Belousov A I, Balyakin V B and Novikov D K 2002 Teoriya i proyektirovaniye gidrodinamicheskikh dempferov opor rotorov (Samara: SNC RAN)
[5] Leontiev M 1996 Damper supports. Rotor-Bearing Dynamics Technology Design Guide
[6] Marmol R A and Vance J M 1978 J. Mech. Des. 100 (1) 139-146
[7] Temis Y M and Temis M Y 2007 J. Fric. Wear 28 (2) 128-137
[8] Dowson D and Taylor C M 1979 Cavitation in bearings Ann. Rev. Fl. Mech. 11 (1) 35-65
[9] Floberg L 1961 ASLE transactions 4 (2) 282-286
[10] Elrod H G 1981 A cavitation algorithm J. Lubr. Tech. 103 (3) 350-354
[11] Nowald G, Schmoll R and Schweizer B 2016 Conference Vibr. Rot. Mach. 11 ( UK).
[12] Miraskari M, Henmati F, Jalali A, Alwaradawi M Y and Gadala M S 2017 J. Tribol. 139 (3) 031703
[13] Vignolo G, Barilá D and Quinzani L 2011 Tribol. Intern. 44 (10) 1089-99
[14] Chasalevris A and Sfyris D 2013 Tribol. Intern. 57 216-234
[15] De Castro H, Cavalcá K and Nordmann R 2008 JSV 317 (1-2) 273-293
[16] Bonello P and Hai P M 2010 J. Eng. Gas Turbines Power 132 (3) 032504
[17] Leontiev M K, Snetkova E I and Degtyarev S A 2013 Aerospace MAI J. 20 (1) 95-105
[18] Villa C V S, Sinou J J and Thouverez F 2007 J. Brazil. Soc. Mech. Sc. Eng. 29(1) 14-20
[19] Gunduz A, Dreyer J T and Singh R 2012 MSSP 31 176-195
[20] Cao L, Sadeghi F and Stacke L E 2017 J. Tribol. 139 (6) 061102
[21] Maraini D and Nataraj C 2018 JSV 420 227-241
[22] Dede M M, Dogan M and Holmes R 1985 J. Tribol. 107 (3) 411-418
[23] Harris T A 2001 Rolling Bearing Analysis (NY: John Wiley)
[24] Muszynska A 2005 Rotordynamics (CRC press)