Further Extended Theories of Gravitation: Part II*

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Abstract: We shall present and analyze two examples of extended theories of gravitation in Palatini formalism with matter that couples to the connection. This will show that the class of Further Extended Theories of Gravitation introduced in [1] does not trivially reduce to $f(R)$ models. It will also produce an example of theory that on-shell endows spacetime with a non-trivial Weyl geometry where the connection is not induced by the metric structure (though it is compatible with it in the sense of Ehlers-Pirani-Schild; see [2]).

1. Introduction

In a recent paper [1] we introduced the class of Further Extended Theories of Gravitation (FETG) and showed that it encompasses $f(R)$ theories. We shall here present two examples of FETG which are not $f(R)$ theories in Palatini framework nor equivalent to them.

These examples could of course be ruled out by some physical principle independent of the EPS axioms, or should be analyzed to check whether they could fit observational data.

As in [1] $M$ is considered as a connected and paracompact differential manifold of dimension 4, which allows global Lorentzian metrics. Axioms in EPS (see [2]) are assumed to hold and the corresponding Weyl geometry is induced on $M$.

In particular example 2 will provide an authentic non-trivial example of Weyl geometry endowed naturally by a relativistic field theory; in fact we shall show that on-shell the connection in that model will be so much as non-metric. In this model from a kinematical point of view the affine structure of spacetime is determined by the metric structure together with four additional degrees of freedom, hence with more freedom than the conformal freedom obtained in $f(R)$ theories. Non-metricity in Palatini framework has been considered (see for example [3], [4]); here, however, the model is considered within FETG framework which relies on EPS which enhances physical interpretation of the model by establishing a direct connection with observations.

This also shows that there is no logical obstruction to obtain from a variational principle a non-metric affine structure which is allowed by EPS axioms on a kinematical stance.

2. Example 1

Let us first consider on $M$ a metric field $g$, a torsionless connection $\Gamma$ and a tensor density $A$ of rank 1 and weight $-1$. The covariant derivative of $A_\mu$ is then defined as

$$\nabla_\mu A_\nu = \partial_\mu A_\nu - \Gamma^\lambda_{\nu\mu}A_\lambda + \Gamma^\lambda_{\lambda\mu}A_\nu$$

(2.1)

* This paper is published despite the effects of the Italian law 133/08 (http://groups.google.it/group/scienceaction). This law drastically reduces public funds to public Italian universities, which is particularly dangerous for free scientific research, and it will prevent young researchers from getting a position, either temporary or tenured, in Italy. The authors are protesting against this law to obtain its cancellation.
Accordingly, we have

\[ \nabla_{(\mu} A_{\nu)} = d_{(\mu} A_{\nu)} - \left( \Gamma_{\nu \mu}^{\alpha} - \delta_{(\nu}^{\alpha} \Gamma_{\mu)\lambda}^{\lambda} \right) A_{\alpha} = d_{(\mu} A_{\nu)} - u_{\mu \nu} A_{\nu} \]  

(2.2)

where we set \( u_{\mu \nu} := \Gamma_{\mu \nu}^{\alpha} - \delta_{(\mu}^{\alpha} \Gamma_{\nu)\lambda}^{\lambda} \).

Let us consider the following Lagrangian (density)

\[ L = \frac{1}{\kappa} \sqrt{g} f(R) + gg^{\mu \nu} \nabla_{\mu} A_{\nu} \]  

(2.3)

where \( g = |\det(g_{\mu \nu})| \), \( R = g^{\mu \nu} R_{\mu \nu} (\Gamma) \) is the scalar curvature of \((g, \Gamma)\), \( \kappa = 16 \pi G \) is a constant and \( f \) a generic (analytic) function; see [5].

By variation of this Lagrangian and usual covariant integration by parts one obtains

\[ \delta L = \frac{\sqrt{g}}{\kappa} (f'(R)R_{(\alpha \beta)} - \frac{1}{2} f(R)g_{\alpha \beta} - \kappa T_{\alpha \beta}) \delta g^{\alpha \beta} + \]

\[ - gg^{\alpha \beta} A_{\lambda} \delta u_{\alpha \beta}^{\lambda} + \frac{\sqrt{g}}{\kappa} g^{\alpha \beta} f'(R) \nabla_{\lambda} \delta u_{\alpha \beta}^{\lambda} + gg^{\mu \nu} \nabla_{\mu} \delta A_{\nu} = \]

\[ = \frac{\sqrt{g}}{\kappa} (f'(R)R_{(\alpha \beta)} - \frac{1}{2} f(R)g_{\alpha \beta} - \kappa T_{\alpha \beta}) \delta g^{\alpha \beta} - \frac{1}{\kappa} \Gamma_{\lambda} \left( \sqrt{g} g^{\alpha \beta} f'(R) + \kappa g g^{\alpha \beta} A_{\lambda} \right) \delta u_{\alpha \beta}^{\lambda} + \]

\[ - \nabla_{\mu} (gg^{\mu \nu}) \delta A_{\nu} + \nabla_{\lambda} \left( \frac{\sqrt{g}}{\kappa} g^{\alpha \beta} f'(R) \delta u_{\alpha \beta}^{\lambda} + gg^{\lambda \nu} \delta A_{\nu} \right) \]

(2.4)

where we used the well-known identity \( \delta R_{\alpha \beta} = \nabla_{\lambda} \delta u_{\alpha \beta}^{\lambda} \) and we set for the energy-momentum tensor \( T_{\alpha \beta} := \sqrt{g} \left( g_{\alpha \beta} g^{\mu \nu} \nabla_{\mu} A_{\nu} - \nabla_{(\alpha} A_{\beta)} \right) \).

Field equations are

\[ \left\{ \begin{array}{l} f'R_{(\alpha \beta)} - \frac{1}{2} f g_{\alpha \beta} = \kappa T_{\alpha \beta} \\ \nabla_{\lambda} \left( \sqrt{g} g^{\alpha \beta} f' \right) = \alpha_{\lambda} \sqrt{g} g^{\alpha \beta} f' \\ \nabla_{\mu} (gg^{\mu \nu}) = 0 \end{array} \right. \]  

(2.5)

where we set \( \alpha_{\lambda} := -\kappa \sqrt{g} A_{\lambda} \). Notice that the third equation (that is the matter field equation) is not enough to fix the connection due to the contraction. Notice also that these are more general than field equations of standard \( f(R) \) theories due to the rhs of the second equation (that is originated by the coupling between the matter field \( A \) and the connection \( \Gamma \)). Nevertheless one can analyze these field equations along the same lines used in \( f(R) \) theories. Let us hence define a metric \( h_{\mu \nu} = f' g_{\mu \nu} \) and rewrite the second equation as

\[ \nabla_{\lambda} \left( \sqrt{h} h^{\alpha \beta} \right) = \alpha_{\lambda} \sqrt{h} h^{\alpha \beta} \]  

(2.6)

According to the analysis of EPS-compatibility done in [1] this fixes the connection as

\[ \Gamma_{\beta \mu}^{\alpha} := \left\{ h \right\}_{\beta \mu}^{\alpha} - \kappa \frac{1}{2f'} \left( h^{\alpha \tau} h_{\tau \mu} - 2 \delta_{(\beta}^{\alpha} \delta_{\mu)}^{\tau} \right) a_{\tau} \]  

(2.7)

where for notational convenience we introduced the 1-form \( a_{\tau} := \sqrt{g} A_{\tau} \). For later convenience let us notice that we have

\[ K_{\beta \mu}^{\alpha} = \Gamma_{\beta \mu}^{\alpha} - \left\{ h \right\}_{\beta \mu}^{\alpha} = - \kappa \frac{1}{2f'} \left( h^{\alpha \tau} h_{\tau \mu} - 2 \delta_{(\beta}^{\alpha} \delta_{\mu)}^{\tau} \right) a_{\tau} \]  

(2.8)
Now we can define the tensor $H^\alpha_{\beta\mu} := \Gamma^\alpha_{\beta\mu} - \{g\}^\alpha_{\beta\mu}$ and obtain

\[ H^\alpha_{\beta\mu} = K^\alpha_{\beta\mu} - \frac{1}{2} \left( g^{\alpha\lambda} g_{\beta\mu} - 2 \delta^\alpha_{(\beta} \delta^\lambda_{\mu)} \right) \partial_\lambda \ln f' = -\frac{1}{2T} \left( g^{\alpha\mu} g_{\beta\mu} - 2 \delta^\alpha_{(\beta} \delta^\lambda_{\mu)} \right) \left( \kappa a_\epsilon + \partial_\epsilon f' \right) \]  

(2.9)

By substituting into the third field equation we obtain

\[ \nabla^\mu (gg^{\mu
u}) + g \left( H^\nu_{\lambda\mu} g^{\lambda\nu} + H^\nu_{\lambda\mu} g^{\mu\lambda} - 2H^\nu_{\lambda\mu} g^{\mu\nu} \right) = 0 \]

\[ \Rightarrow H^\nu_{\lambda\mu} h^{\lambda\mu} - H^\nu_{\lambda\mu} h^{\mu\nu} = 0 \]

\[ \Rightarrow -\frac{1}{2T} \left( \left( h^{\alpha\nu} h_{\lambda\mu} - 2 \delta^\alpha_{(\lambda} \delta^\nu_{\mu)} \right) h^{\mu\nu} - \left( h^{\alpha\nu} h_{\lambda\mu} - 2 \delta^\alpha_{(\lambda} \delta^\nu_{\mu)} \right) h^{\mu\nu} \right) \left( \kappa a_\epsilon + \partial_\epsilon f' \right) = 0 \]

\[ \Rightarrow -\frac{3}{T} h^{\mu\nu} \left( \kappa a_\epsilon + \partial_\epsilon f' \right) = 0 \]

\[ \Rightarrow a_\epsilon = -\frac{1}{\kappa} \partial_\epsilon f' \]

(2.10)

where $\nabla^\mu$ is now the covariant derivative with respect to the metric $g$. Hence the matter field $A_\epsilon = \sqrt{g} a_\epsilon = -\frac{\sqrt{3}}{\kappa} \partial_\epsilon f'$ has no dynamics and it is completely determined in terms of the other fields.

We can also express the connection as a function of $g$ alone (or, equivalently, of $h$ alone).

\[ \Gamma^\alpha_{\beta\mu} := \{h\}^\alpha_{\beta\mu} + \frac{1}{2} \left( h^{\alpha\nu} h_{\beta\mu} - 2 \delta^\alpha_{(\beta} \delta^\nu_{\mu)} \right) \partial_\nu \ln f' \equiv \{g\}^\alpha_{\beta\mu} \]

(2.11)

This behaviour, which has been introduced by the matter coupling, is quite peculiar; the model resembles in the action an $f(R)$ theory but in solution space the connection is directly determined by the original metric rather than by the conformal metric $h$ as in $f(R)$ theories. Still the metric $g$ obeys modified Einstein equations. In fact, we have the first field equation which is now depending on $g$ alone, since the matter and the connection have been determined as functions of $g$.

The master equation is obtained as usual by tracing (using $g^{\alpha\beta}$)

\[ f' R - 2f = \kappa T \quad \Rightarrow f = \frac{1}{2} \left( f' R - \kappa T \right) \]

(2.12)

where we set $T := T_{\alpha\beta} g^{\alpha\beta}$. Notice that in this case we obtain explicitly

\[ T_{\alpha\beta} = \frac{1}{\kappa} \left( \nabla_\alpha \nabla_\beta f' - g_{\alpha\beta} \Box f' \right) \]

\[ T = 4 \nabla_\alpha a_\beta g^{\alpha\beta} - g^{\alpha\nu} \nabla_\nu a_\alpha = -\frac{2}{\kappa} \Box f' \]

(2.13)

The master equation is then $f' R - 2f = -3 \Box f'$. Then substituting back into the first field equation we obtain

\[ f' \left( R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} \right) - \frac{1}{2} \Box f' g_{\alpha\beta} = \nabla_\alpha \nabla_\beta f' - \Box f' g_{\alpha\beta} \]

\[ \Rightarrow R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = \frac{1}{f'} \left( \nabla_\alpha \nabla_\beta f' - \frac{1}{2} \left( \Box f' + f' R \right) g_{\alpha\beta} \right) \]

(2.14)

where now the curvature and covariant derivatives refer to $g$. These are exactly the field equations obtained in the corresponding purely-metric $f(R)$ theory.

Hence we have that, regardless of the function $f$, when there is no matter field other than the field $A$ all these models behave exactly as metric $f(R)$ theories. Unlike in $f(R)$ theories, however, there is no conformal metric around; everything refers to the original metric $g$.

Obviously in this theory one can use the purely metric model for polytropic star as a solution to find a possible way around the no-go theorems formulated for Palatini extended theories; see [6]. Another possible way around will be presented in [7].
3. Example 2

The analysis of Example 1 is based on the assumption that the matter field $A$, is fundamental (or equivalently that $\delta A$ are independent of other field variations); on the other hand the EPS-compatibility is based on the geometric character of the matter field $A$. Let us now consider what happens when the tensor density $A$ is obtained as an object derived from $g$ and other, more fundamental, matter fields. Let us for example consider a (real) scalar field $\phi$ and set $A_\alpha = \frac{1}{\sqrt{g}} \nabla_\alpha \phi$. (Notice that the covariant derivative of $\phi$ is in fact independent of any connection since for scalars $\nabla_\mu = \partial_\mu$.)

Accordingly, let us consider a second model with the Lagrangian

$$L = \frac{1}{2} \sqrt{g} f(R) + g g^{\mu \nu} \nabla_\mu \left( \frac{1}{\sqrt{g}} \nabla_\nu \phi \right)$$

By variation of this Lagrangian and usual covariant integration by part one obtains

$$\delta L = \frac{\sqrt{g}}{\kappa} \left( f'(R) R_{(\alpha \beta)} - \frac{1}{2} f(R) g_{\alpha \beta} - \kappa T_{\alpha \beta} \right) \delta g^{\alpha \beta} +$$

$$- \frac{1}{2} \left( \Gamma^\alpha_\lambda \left( \sqrt{g} g^\alpha_{\beta \delta} f'(R) \right) + \kappa g g^{\alpha \beta} A_\lambda \right) \delta u^{\alpha \beta} + \nabla_\nu \left( \frac{1}{\sqrt{g}} \Gamma^\alpha_\mu \left( g g^{\mu \nu} \right) \right) \delta \phi +$$

$$+ \Gamma^\alpha_\lambda \left( \frac{\sqrt{g}}{\kappa} g^{\alpha \beta} f'(R) \delta u^{\alpha \beta} + \frac{\sqrt{g}}{\kappa} g^{\alpha \beta} \nabla_\nu \phi \delta g^{\alpha \beta} + \sqrt{g} g^{\alpha \beta} \nabla_\nu \delta \phi - \frac{1}{\sqrt{g}} \Gamma^\alpha_\mu \left( g g^{\mu \nu} \right) \right) \delta \phi \right)$$

where we set $T_{\alpha \beta} := \sqrt{g} \left( g_{\alpha \beta} g^{\mu \nu} A_\mu - \nabla_\alpha A_\beta \right) - \frac{1}{\sqrt{g}} \Gamma^\alpha_\mu \left( g g^{\mu \nu} \right) \nabla_\nu g_{\alpha \beta}$. Here we denote by $\nabla_u$ the covariant derivative wrt the connection $\Gamma$, while $\nabla_\mu$ is used for the special cases in which the covariant derivative turns out to be independent of any connection and reduces to a partial derivative (as it happens for scalars, vector densities of weight 1, and so on).

Field equations are

$$\begin{align*}
\Gamma^\alpha_\beta := & \nabla_\alpha \left( h^\beta_\gamma - \nabla^\gamma \left( h^{\alpha \nu} h_{\beta \nu} - 2 \delta^\alpha_\beta \delta^\nu_\gamma \right) \right) \\
\nabla_\nu \left( \frac{1}{\sqrt{g}} \Gamma^\alpha_\mu \left( g g^{\mu \nu} \right) \right) &= 0
\end{align*}$$

where we set again $\alpha_\lambda := - \kappa \frac{\sqrt{g}}{f} A_\lambda$. The second equation fixes again the connection

$$\Gamma^\alpha_\beta := \left\{ h \right\}^\alpha_\beta$$

where as usual we set $h_{\mu \nu} = f' g_{\mu \nu}$.

However, the third equation does not force the covector $\alpha_\epsilon = - \frac{\sqrt{g}}{3} \nabla_\epsilon \phi$ to be a closed form; thus the connection is not metric.

To see this, notice that the third equation is in the form $d \ast \beta = 0$ for a covector $\beta = \beta_\mu dx^\mu$. Here $\ast$ denotes the Hodge duality on forms. In fact, the third equation can be recasted as

$$\nabla_\mu \left( \frac{3 \sqrt{g}}{f'} g^{\nu \epsilon} \nabla_\epsilon \left( \kappa \phi + f' \right) \right) = 0 \quad \Rightarrow \ast \beta = - \frac{3 \sqrt{g}}{f'} g^{\nu \epsilon} \nabla_\epsilon \left( \kappa \phi + f' \right) d\nu$$

The general solution of this equation is

$$\ast \beta = d\theta + \omega$$
for a closed \((m-1)\)-form \(\omega = \omega^\mu ds_\mu\) and for some \((m-2)\)-form \(\theta = \frac{1}{2} \sqrt{g} \theta^{\mu\nu} ds_\mu ds_\nu\). The closed form \(\omega\) is defined modulo exact forms and they are classified in terms of spacetime cohomology. Accordingly, the third equation implies
\[
\kappa \nabla_\epsilon \phi = \frac{1}{3} g_{\nu\lambda} \frac{f'}{\sqrt{g}} \left[ \nabla_\lambda \left( \sqrt{g} \theta^{\lambda\nu} \right) + \omega^\nu \right] - \nabla_\epsilon f'
\] (3.7)
Consequently,
\[
\Gamma^\alpha_{\beta\mu} := (h)_{\beta\mu} - \frac{1}{6} \frac{1}{\sqrt{g}} \left( g^{\alpha\mu} g_{\beta\lambda} - 2 \delta^\alpha_{(\beta} \delta^\nu_{\mu)} \right) g_{\nu\lambda} \left[ \nabla_\lambda \left( \sqrt{g} \theta^{\lambda\nu} \right) + \omega^\nu \right] + \frac{1}{2} \left( g^{\alpha\mu} g_{\beta\lambda} - 2 \delta^\alpha_{(\beta} \delta^\nu_{\mu)} \right) \nabla_\nu \ln f'
\] (3.8)
which corresponds to
\[
\alpha_\epsilon = \nabla_\epsilon f' - \frac{1}{3} g_{\nu\lambda} \frac{1}{\sqrt{g}} \left[ \nabla_\lambda \left( \sqrt{g} \theta^{\lambda\nu} \right) + \omega^\nu \right]
\] (3.9)
The connection \(\Gamma\) is metric iff the covector \(\alpha = \alpha_\epsilon dx^\epsilon\) is closed. However, there is nothing here forcing this form to be closed (while of course it can be closed for specific choices of the arbitrary \(\theta^{\mu\nu}\), e.g. \(\theta = 0\)). For example, if \(g\) is Minkowski metric, \(\omega = 0\) and \(\theta = \sqrt{g(x^1)^2} dx_{12}\) one can prove that \(d\alpha \neq 0\) holds.

The field \(A\) can then be written as
\[
A_\epsilon = \frac{1}{\sqrt{g}} \nabla_\epsilon \phi = \frac{f'}{3 \kappa g} g_{\nu\lambda} \nabla_\lambda \left( \sqrt{g} \theta^{\lambda\nu} \right) - \frac{1}{\kappa \sqrt{g}} \nabla_\epsilon f'
\] (3.10)
Let us stress that now the matter field \(\phi\) is not completely determined by the other fields (there is in fact a freedom in the choice of the form \(\theta^{\mu\nu}\)).

The master equation induced by the first field equation is in this case
\[
f' R - 2 f = \kappa T_{\alpha\beta} g^{\alpha\beta} =: \kappa T \quad \Rightarrow \quad f = \frac{1}{2} \left( f' R - \kappa T \right)
\] (3.11)
which can be used back into the first field equation to obtain (when \(f' \neq 0\))
\[
R_{(\alpha\beta)} - \frac{1}{2} R g_{\alpha\beta} = \frac{\kappa}{f} \left( T_{\alpha\beta} + \frac{1}{2} \kappa g_{\alpha\beta} \right)
\] (3.12)
Similar examples are obtained any time that one can define a tensor density \(A_\epsilon\) of weight \(-1\) from any choice of fundamental fields.

4. Conclusions and Perspectives

We do not pretend here to propose any realistic physical model. In order to do that one should study specific models; for example in their cosmological mini-superspace or other astrophysical situations and try fitting observational data; see [8], [9], [10], [11].

We are here just considering the possibility to use EPS compatibility in order to constrain extended theories of gravitation. Since EPS criteria allow for non-metric connections it is interesting to notice that in fact a specific model (Example 2) can be presented in which non-metric connections appear naturally.

Of course, these examples are defined \textit{ad hoc} and may have no physical meaning whatsoever; however, this is hard to be seen as a critic. In fact once one accepts to introduce exotic dynamics (if not even considering Hilbert-Einstein gravitation as a \textit{special} model) then it is difficult to set
a point not to be crossed and any exotic model should be discussed in view of its own prediction. From this point of view it is quite interesting to notice that EPS criteria are a natural crosspoint to unphysical models. EPS axioms are quite concrete and physically well-based. This does not imply of course that they are a complete set of hypotheses. There could be further reasons to exclude the models we presented here, possibly by adding new criteria to what should be meant by “physical connections”. However, in this case such principles should be explicitly formulated and discussed.

Moreover, EPS setting provides a natural framework for relativistic theories of gravitation in Palatini formalism. Let us stress that further investigations are needed in order to provide a truly relativistic operational definition of measurements in this generalized setting; see [12], [13]

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