Unsupervised Learning of Multi-level Structures for Anomaly Detection

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Abstract

The main difficulty in high-dimensional anomaly detection tasks is the lack of anomalous data for training. And simply collecting anomalous data from the real world, common distributions, or the boundary of normal data manifold may face the problem of missing anomaly modes. This paper first introduces a novel method to generate anomalous data by breaking up global structures while preserving local structures of normal data at multiple levels. It can efficiently expose local abnormal structures of various levels. To fully exploit the exposure of multi-level abnormal structures, we propose to train multiple level-specific patch-based detectors with contrastive losses. Each detector learns to detect local abnormal structures of corresponding level at all locations and outputs patchwise anomaly scores. By aggregating the outputs of all level-specific detectors, we obtain a model that can detect all potential anomalies. The effectiveness is evaluated on MNIST, CIFAR10, and ImageNet10 dataset, where the results surpass the accuracy of state-of-the-art methods. Qualitative experiments demonstrate our model is robust that it unbiasedly detects all anomaly modes.

1. Introduction

Detecting anomalies or out-of-distribution (OOD) data is very important in different application domains, such as fraud detection for credit cards, defect detection of industrial products, lesion detection based on medical images, security check based on X-ray images, and video monitoring [59, 7, 46, 26, 2]. Although it has been studied for long, the anomaly detection for image data is still difficult to handle [51]. Machine learning models like deep neural networks often have a vast hypothesis space and rely on the i.i.d. (independent and identically distributed) assumption. By optimizing the training objectives, many functions can be found with desirable outputs on the in-distribution data but undefined outputs on the OOD data. Similarly, we cannot explicitly constraint an anomaly detector on those anomaly modes not used for training, and their predictions may not be desirable.

In low-dimensional tasks, it is easy to generate data that cover all anomaly modes from a noise distribution supported on the whole data space. We can train a binary classifier that learns the statistics of the normal data by contrasting the normal data with the generated noise data, like Noise Contrastive Estimation (NCE) [18, 17]. However, in high-dimensional tasks, there are two problems: (i) Almost all noise samples contain only low-level anomalous features, as shown in Fig. 1(a). They are inefficient to expose higher-level abnormal structures. (ii) Classifiers may degenerate to learn the statistics of some certain regions. For example, they can successfully discriminate the normal data from the generated data based on only a few pixels in the center of an image, without learning anything about other parts [14].

To address the first problem, we first propose to use images from Markov Random Field (MRF) approximations of multiple levels with suitable gaps to expose local abnormal structures of various levels (as shown in Fig. 1). We then develop a novel generation method called entropy-
regularized PatchGAN to obtain such images. To address the second problem (which also exists in images from MRF approximations), we propose to use multiple level-specific patch-based detectors. We train each level-specific detector to learn patch-wise anomaly scores and calculate the final anomaly score by aggregating the results over all patches and levels. We call the proposed model Multi-Level Structure Anomaly Detection (MLSAD).

MLSAD is a robust model that unbiasedly detects all anomaly modes, since it sees diverse local abnormal structures of various levels and efficiently learns to detect them at any location. Such robustness is very important for practical usage, especially for safety-critical environments and tasks where we do not know which type or level of anomalies will appear. MLSAD takes no assumption on the image type, so it is a general method for both objects and textures. It achieves state-of-the-art performance compared with methods requiring neither external OOD dataset nor prior knowledge on the image type.

2. Related works

Many deep unsupervised anomaly detection approaches have been developed in recent years. Some of them combine deep networks with traditional methods like DeepSVDD [44], which learns a neural network that maps the representations of normal data into a hypersphere with minimal volume. Besides, there are many other types of methods, which will be reviewed in the following.

Reconstruction based. Training autoencoders (AEs) [4, 2, 3] on normal data is a popular method for deep unsupervised anomaly detection. A well-trained AE is supposed to produce lower reconstruction errors on the normal data than the anomalous data. However, in practice, it may also reconstruct anomalies very well or even better [39]. In order to reconstruct only the normal data, AnoGAN [47] learns a generator mapping the latent space to the in-distribution manifold and finds the closest in-distribution sample for a given input by searching in the latent space. OCGAN [38] learns a bounded latent space that exclusively generates only in-distribution samples. In [36], it is argued that AEs tend to regress the conditional mean rather than the actual multi-modal distribution and propose to learn a multi-hypotheses autoencoder that finds the closest reconstruction for the input. GPND [40] trains an adversarial autoencoder and improves the detection performance by combining the reconstruction error with latent likelihood.

Confidence based. These methods [21, 23, 13, 29, 30] detect OOD data using the softmax activation statistics of multi-class models trained on normal data, with the assumption that anomalies will tend to have lower classification confidence than the correctly classified normal samples. To obtain classifiers with unlabeled normal data, [13] train networks to classify the geometric transformations applied to inputs. This idea is combined with adversarial training to improve both anomaly detection performance and robustness of classifiers in [23]. It is also extended to non-image data using random affine transformations by [6].

Density based. Another appealing idea for unsupervised anomaly detection is estimating the density of the in-distribution via deep generative models. However, recent works [11, 34] show that these models assign higher likelihoods to OOD samples even than in-distribution samples. [50] proposes to address it by subtracting the input complexity from the original OOD score. [22] finds training against an auxiliary dataset of outliers significantly improves their performance. [41] proposes a background contrastive score that captures the significance of the semantics compared with the background model for OOD detection. [27] shows that flow models tend to learn local pixel correlations rather than semantic structures and propose to modify the architecture of flow coupling layers.

Anomaly generation based. A common issue for most previous methods is that there is no direct constraint on those OOD samples, and thus some anomalies may have lower anomaly scores than the normal samples’. [20] shows that ReLU type neural networks always have high confidence predictions on samples far away from the training data and propose to use uniform noise or adversarial noise samples to enforce low confidence on the OOD data. But noise distributions will be less efficient when applied to high dimensional data. Some works [45, 57, 54] proposed to generate anomalous samples near the boundary of the in-distribution and thus obtain hard samples that are more efficient for learning a tight decision boundary. However, there is still no guarantee for handling all anomalies (e.g., the anomalies far away from the in-distribution data).

Unlike Geom [13], Rot [23] and the recent contrastive-learning-based model CSI [53], MLSAD does not rely on discriminating geometrically transformed images or contrasting with (manually selected) dataset-dependent shifted instances. Therefore it is less limited by the image type of the in-distribution. Compared with previous anomaly generation based methods, we explicitly take the multi-level anomalous structures into consideration and thus design a multiple level-specific patch-based detection framework to overcome the problem of missing anomaly modes.

3. Approaches

3.1. Exposing Multi-level Abnormal Structures

The main insight into exposing multi-level abnormal structures is breaking up the global structures while preserving local structures at various levels. Take text data as an example. We can expose spelling errors and grammar errors by recombining the elements of correct sentences at letter level and word level, respectively:
Let $X$ be the image variable, $c \in C^w$ be an image patch of $w \times w$ pixels, where $C^w$ denotes the set of all such patches. We use $X_c$ to denote the patch variable on $c$. We define the MRF approximations as following:

**Definition 1 (MRF approximations)** A distribution $q_X$ is called the $w$th-level MRF approximation of a given image distribution $p_X$, if for each patch $c$, the marginal distribution of $X_c$ from $q_X$ is identical to the one from $p_X$: $\forall c \in C^w, q_{X_c}(x_c) = p_{X_c}(x_c)$, and the density function of $q_X$ can be factored as the product of functions of patch variables: $q_X(x) = \prod_{c \in C^w} \phi_c(x_c)$. Particularly, the uniform distribution is defined to be the 0th-level MRF approximation of any image distribution.

We will use $p_X$ to denote the distribution of normal images and $q_X^w$ to denote the $w$th-level MRF approximation of it. By Definition 1, we have $q_{X_c}^w(x_c) = p_{X_c}(x_c)$ for any patch $c \in C^w$. It means images from $q_X^w$ are indistinguishable from normal images via any window of $w \times w$ pixels, and the local structures of normal images are well kept. Besides, there are no long-range interactions among patches since $q_X^w$ can be factored as $\prod_{c \in C^w} \phi_c(x_c)$. It breaks the global structures as much as possible and makes the local structures recombined more randomly. Moreover, we conclude that all recombinations can be sampled from $q_X^w$ with nonzero probability by the following theorem.

**Theorem 1** Let $S_w = \{x | \forall c \in C^w, p_{X_c}(x_c) > 0\}$, we have $\text{supp}(q_X^w) = S_w$. In other words, the support set of $q_X^w$ is identical to the set of all images where each patch of $w \times w$ pixels is normal.

All proofs are provided in Appendix A. However, just as letter-level recombinations of sentences can efficiently expose the word-level errors but few sentence-level errors, $q_X^w$ will be only efficient to expose the abnormal structures of levels slightly higher than $w$. This paper assumes that we can efficiently expose all local abnormal structures of all levels by MRF approximations of multiple levels with suitable gaps.

### 3.1.2 Entropy-regularized PatchGAN.

By definition 1, the $0$th-level MRF approximation $q_X^0$ can be obtained by the uniform distribution, and the $1$st-level MRF approximation $q_X^1$ can be easily obtained by shuffling pixels of the normal images along the batch axis. To generate the higher-level ($w \geq 2$) MRF approximations $q_X^w$, we propose a Generative Adversarial Networks (GAN [15]) based approach. Like previous works [24, 52, 8] that generate images with all locals realistic or tile image patches seamlessly, we train the generator $G$ that maps a noise $z$ to an image together with a patch-based discriminator $D$. $D$ is implemented with a Fully Convolutional Network (FCN [32]) with stride 1. For the $w$th-level MRF approximation, $D$ has a receptive filed size of $w \times w$ pixels, and outputs a score $D_c$ for each patch $c \in C^w$. The adversarial losses are averaged over all patches for both $D$ and $G$:

$$
\mathcal{L}_{adv}(D) = - \frac{1}{|C^w|} \sum_c |\mathbb{E}_x \log D_c(x) + \mathbb{E}_z \log (1 - D_c(G(z)))|,
$$

$$
\mathcal{L}_{adv}(G) = - \frac{1}{|C^w|} \sum_c |\mathbb{E}_z \log D_c(G(z))|.
$$

We feed coordinate information into $D$ like [31], since $p_X$ may be varied with patch $c$. Using the adversarial losses alone can ensure the local visual structures are well kept, while the global structures may not be thoroughly broken, since there are often some inherent global structures caused by the inductive bias of $G$.

**Theorem 2** The maximum entropy distribution satisfying $\forall c \in C^w, q_{X_c}(x_c) = p_{X_c}(x_c)$ can be factored as $\prod_{c \in C^w} \phi_c(x_c)$.

By Theorem 2 and Definition 1, we can know that the maximum entropy distribution preserving local structures in windows of $w \times w$ pixels is exactly the $w$th-level MRF approximation of $p_X$. So we turn to maximize the entropy of the outputs of the generator while training it with a GAN loss. Similar to [28], we adopt the recently proposed mutual information maximization technique. It relies on two facts: (i) The entropy of the generated distribution $H(G(Z))$ is identical to the mutual information between $Z$ and $G(Z)$, because $I(G(Z), Z) = H(G(Z)) - H(G(Z)|Z)$ and $H(G(Z)|Z) = 0$ since $G$ is deterministic. (ii) The mutual information between $X$ and $Z$ can be estimated by the neural mutual information measure [5]:

$$
\mathcal{T}_\theta(X, Z) = \max_{T \in T_{\text{is}}} \left[ \mathbb{E}_{p(x,z)}[T(X, Z)] - \log(\mathbb{E}_{p(x)p(z)}[e^{T(X, Z)}]) \right].
$$
The supremum is approximated using gradient descent on the parameters of a statistics network \( T \) (also updated in synchronization with \( G \)). Thus the total loss for \( G \) becomes:

\[
\mathcal{L}_{\text{total}}(G) = \mathcal{L}_{\text{adv}}(G) - \beta \mathcal{L}_{\Phi}(G(Z), Z). \tag{2}
\]

We can interpret the first term as an objective for preserving local structures and the second term for breaking global structures. These two terms work together to create diverse and seamless images. They will be used to expose local abnormal structures of levels slightly higher than \( w \). We will refer to this generation approach as entropy-regularized PatchGAN in the following.

### 3.2. Multi-Level Structure Anomaly Detection

Without loss of generality, we assume the image size is \( 2^W \times 2^W \) pixels, and that local abnormal structures of all levels can be efficiently exposed by MRF approximations with suitable level gaps \( \{q_{X}^{s}\mid s = 1, 2, 4, \ldots, 2^W\} \). However, we remark that any MRF approximation cannot efficiently expose any image with anomalous structures of multiple levels, and that images from MRF approximations (especially for low levels) often show abnormal cues at almost all locations. An image-level detector trained to discriminate between such images and the normal images may degenerate to learn the statistics of a few regions and cannot generalize to all anomalous images. To fully exploit the exposed local abnormal structures, learn the visual structures of all levels and at all locations, and handle various anomaly modes, we propose to train multiple level-specific patch-based detectors and aggregate their outputs for a final prediction, as shown in Fig. 2. We name the proposed method as Multi-Level Structure Anomaly Detection (MLSAD).

#### 3.2.1 The \( s \)th-level detector

The MLSAD model consists of \( M + 1 \) level-specific detectors, \( \{A^s\mid s = 1, 2, 4, \ldots, 2^M\} \), where \( A^s \) is the \( s \)th-level detector and \( M \leq W \). \( A^s \) outputs an anomaly score \( A^s_e(x) \in [0, 1] \) for each patch \( x \) in \( C^s \) by an FCN (with stride of \( 1 \times 1 \) and receptive field size of \( s \times s \) pixels). A coordinate map is used as the auxiliary input to give \( A^s_e \) the potential to learn position-dependent statistics, see Fig. 2(a).

#### 3.2.2 Training Phase

For each level-specific detector \( A^s \), we train it to detect the position-dependent patch abnormalities between level \( \lfloor \frac{s}{2} \rfloor + 1 \) and \( s \). The basic idea is to use the fake image \( \hat{\mathbf{x}}^{\lfloor \frac{s}{2} \rfloor} \) from \( \lfloor \frac{s}{2} \rfloor \)th-level MRF approximation for contrast,
and force $A^s$ to discriminate the normal image $x$ and $\hat{x}^{\lfloor \frac{s}{2} \rfloor}$ in each window of $s \times s$ pixels. However, we observed $A^s$ tend to focus on learning the common statistics shared among patches and utilize very little position information (See Appendix B). In the perspective of the output neurons of $A^s$, they compute the anomaly score for each patch using the same convolution kernels, and struggle to learn a position-aware score by modeling the dependence between the patch content and the coordinates. Such dependence can also be viewed as some kind of high-level structure and may be hard to learn. To alleviate this concern, we propose to use the normal image $x$ paired with a fake coordinate map $\hat{r}^s$ as additional contrastive data. These contrastive data provide samples breaking the dependence between position and local content.

The overall training method is shown in Fig. 2(b). The normal image paired with true coordinate map $(x, r^s)$ is designed as the normal pair, while both the structure anomaly pair $(\hat{x}^{\lfloor \frac{s}{2} \rfloor}, \hat{r}^s)$ and the position anomaly pair $(x, \hat{r}^s)$ are designed as the anomalous pairs. We train $A^s$ to (patchwise) classify the anomalous pairs as positive class and the normal pair as negative class via the following loss:

$$
\mathcal{L}_{contra} = -\frac{1}{|C^s|} \sum_c \left[ \mathbb{E}_{(x, r^s)} \log (1 - A^c_c(x, r^s)) + \alpha_1 \mathbb{E}_{(x, \hat{r}^s)} \log A^c_c(x, \hat{r}^s) + \alpha_2 \mathbb{E}_{(\hat{x}^{\lfloor \frac{s}{2} \rfloor}, \hat{r}^s)} \log A^c_c(\hat{x}^{\lfloor \frac{s}{2} \rfloor}, \hat{r}^s) \right],
$$

where $\alpha_1, \alpha_2 \in [0, 1]$. The elements of $r^s$ are assigned with coordinate values of its corresponding position. Each $\hat{r}^s$ is filled with randomly sampled values in coordinate range. We feed the coordinate maps into $A^s$ at an intermediate layer, whose following layers have strides of $1 \times 1$. This prevents $A^s$ from simply discriminating $(x, r)$ from $(x, \hat{r}^s)$ via the local discontinuities in $\hat{r}^s$.

### 3.2.3 Evaluation Phase

By setting $M=W$, $\bigcup_{s=1,2,4,...,2^M} \{ \lfloor \frac{s}{2} \rfloor + 1, \lfloor \frac{s}{2} \rfloor + 2, \lfloor \frac{s}{2} \rfloor + 3, \ldots, s \}$ can cover all possible levels. Then every anomalous image will trigger at least one level-specific detector with a high response at some location, since every anomalous image contains at least one abnormal structure of some level. Intuitively, we can use max pooling to aggregate the outputs of all levels and locations and obtain a final score that probably detects all possible anomalies. However, in practice, max pooling relies too much on the score of the most anomalous region, which may result in a false positive detection when the test normal image is slightly corrupted. We notice that average pooling and softmax pooling tend to work better [55]. Here, we simply perform average pooling over locations and levels successively:

$$
A(x) = \frac{1}{M+1} \sum_s \left[ \frac{1}{|C^s|} \sum_c A^c_c(x, r^s) \right].
$$

In practice, the detector of very high levels may not be necessary, and setting $M < W$ may also work well, since the structures below a certain level are often enough to capture the most information of normal images [9, 10], especially for texture-like images.

### 4. Experiments

In this section, we will first demonstrate the generation results of entropy-regularized PatchGAN, then report the anomaly detection results on various datasets, and finally investigate the robustness of MLSAD and existing methods. Meanwhile, ablation studies on each component are performed. The details in the following experiments can be found in Appendix C.
Table 1. Anomaly detection results on CIFAR10 dataset using AUROC metric.

| Method          | plane | car   | bird  | cat   | deer  | dog   | frog  | horse | ship | truck | mean  |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|------|-------|-------|
| Limited by image type |       |       |       |       |       |       |       |       |      |       |       |
| Geom[13]        | 0.747 | 0.957 | 0.781 | 0.724 | 0.878 | 0.878 | 0.834 | 0.955 | 0.933| 0.913 | 0.860 |
| Rot[23]         | 0.783 | 0.943 | 0.862 | 0.808 | 0.894 | 0.890 | 0.889 | 0.951 | 0.923| 0.897 | 0.884 |
| CSI[53]         | 0.899 | 0.991 | 0.931 | 0.864 | 0.939 | 0.932 | 0.951 | 0.987 | 0.979| 0.955 | 0.942 |
| Not limited by image type |       |       |       |       |       |       |       |       |      |       |       |
| OCSVM [48]      | 0.630 | 0.440 | 0.649 | 0.487 | 0.735 | 0.500 | 0.725 | 0.533 | 0.649| 0.508 | 0.585 |
| KDE [37]        | 0.658 | 0.520 | 0.657 | 0.497 | 0.727 | 0.496 | 0.758 | 0.564 | 0.680| 0.640 | 0.609 |
| AnoGAN [47]     | 0.671 | 0.547 | 0.529 | 0.545 | 0.651 | 0.603 | 0.585 | 0.625 | 0.758| 0.665 | 0.617 |
| DeepSVDD [44]   | 0.617 | 0.659 | 0.508 | 0.591 | 0.609 | 0.657 | 0.677 | 0.673 | 0.759| 0.731 | 0.648 |
| OCGAN [38]      | 0.757 | 0.531 | 0.640 | 0.620 | 0.723 | 0.620 | 0.723 | 0.575 | 0.820| 0.554 | 0.656 |
| DROCC [16]      | 0.817 | 0.767 | 0.667 | 0.671 | 0.736 | 0.744 | 0.744 | 0.714 | 0.800| 0.762 | 0.742 |
| MSLAD (Ours)    | 0.819 | 0.937 | 0.746 | 0.725 | 0.804 | 0.812 | 0.905 | 0.925 | 0.884| 0.907 | 0.845 |

Table 2. Anomaly detection results on ImageNet10 dataset using AUROC metric. Where 0 ∼ 9 denote image classes Tench, English Springer, Cassette Player, Chainsaw, Church, French Horn, Garbage Truck, Gas Pump, Golf Ball and Parachute.

| Method                    | 0   | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | mean  |
|---------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-------|
| Nearest Neighbor          | 0.656 | 0.564 | 0.477 | 0.452 | 0.614 | 0.505 | 0.542 | 0.474 | 0.704 | 0.759 | 0.5746 |
| DeepSVDD [44]             | 0.651 | 0.665 | 0.605 | 0.594 | 0.563 | 0.531 | 0.622 | 0.567 | 0.722 | 0.814 | 0.6333 |
| DROCC [16]                | 0.702 | 0.705 | 0.712 | 0.686 | 0.675 | 0.770 | 0.691 | 0.699 | 0.707 | 0.935 | 0.7283 |
| MSLAD (Ours)              | 0.879 | 0.843 | 0.829 | 0.642 | 0.841 | 0.725 | 0.867 | 0.664 | 0.729 | 0.801 | 0.7820 |

Table 3. Anomaly detection results on MNIST dataset using AUROC metric. The results are averaged over all digit classes.

| Method      | KDE [37] | AnoGAN [47] | DeepSVDD [44] | OCSVM [48] | AND [1] | OCGAN [38] | MSLAD_{α_1=0} | MSLAD (Ours) |
|-------------|----------|--------------|---------------|------------|----------|-------------|----------------|--------------|
| Mean        | 0.8143   | 0.9127       | 0.9480        | 0.9513     | 0.9671   | 0.9750      | 0.9139         | 0.9834       |

4.1. Anomaly Generation

We test entropy-regularized PatchGAN on three normal image sets: MNIST, CIFAR10, and CIFAR10 dog images. All images are preprocessed into a shape of $32 \times 32 \times 3$ by resizing and color converting if necessary. We generate MRF approximations of levels 2, 4, and 8 for each normal set. For comparison, we also train generators without the maximum entropy regularization for level 2.

Results. As shown in Fig. 3, the generations of entropy-regularized PatchGAN at high levels tend to show recognizable patterns and semantic-level anomalous structures. The generations at low levels tend to be more disordered and show low-level anomalous cues. Diverse and multi-level local abnormal structures at various locations can be observed. By contrast, the generations without maximum entropy regularization tend to contain simple global patterns and expose much fewer recombination modes.

4.2. Anomaly Detection

We evaluate the proposed MSLAD on the handwritten symbol, natural object, and industrial texture images. More specifically, the datasets used are MNIST, CIFAR10, ImageNet10 [12] and textures in MVTec AD [7]. For MNIST, CIFAR10 and ImageNet10, we preprocess the images into $32 \times 32 \times 3$. As the widely used evaluation methodology in prior works [16, 44, 38], we simulate the unsupervised anomaly detection setting by choosing one out of ten classes as the normal class and the rest as the anomalous class. The original training split of the known class is used for training/validation, and the testing split of all classes is used for testing. We compare MSLAD with deep unsupervised anomaly detection methods using the metric of AUROC (Area Under the Curve of Receiver Operating Characteristics curve). The MSLAD here consists of five level-specific detectors, $\{A_1, A_2, A_4, A_8, A_{16}\}$, where the image-level detector $A_{16}$ is not used since we find it contributes little to the final AUROC. For textures in MVTec AD, we preprocess the images into $256 \times 256$ pixels, and likely, we use only the first five level-specific detectors, since we find $\hat{x}_{16}$ can already capture almost all the structures of normal images. We train generators with small batch sizes and produce anomalous images with the size of $256 \times 256$, while level-specific detectors are trained with randomly cropped patches (size of $32 \times 32$) to avoid running out of GPU memory. We obtain the pixel-level anomaly detec-
Table 4. An ablation study of different levels. We choose frog, dog, and horse class in CIFAR10 as the normal class, various types of other image sets as anomalous class: rCIFAR10 (the rest classes in CIFAR10), CIFAR100, SVHN, MNIST, and Noise (uniform).

|      | A1  | A2  | A4  | A8  | A16 | rCIFAR10 | CIFAR100 | SVHN | MNIST | Noise | AUROC (frog/dog/horse) |
|------|-----|-----|-----|-----|-----|----------|----------|------|-------|-------|------------------------|
| ✓    |     |     |     | ✓   | ✓   | 0.730/0.645/0.765 | 0.712/0.681/0.792 | 0.696/0.592/0.807 | 0.487/0.345/0.836 | 1.000/1.000/1.000 |
| ✓    |     |     |     | ✓   |     | 0.785/0.741/0.804 | 0.737/0.749/0.816 | 0.878/0.787/0.859 | 0.992/0.787/0.900 | 1.000/1.000/1.000 |
| ✓    | ✓   |     |     |     | ✓   | 0.766/0.804/0.851 | 0.768/0.811/0.879 | 0.896/0.877/0.786 | 0.977/1.000/0.977 | 0.674/0.974/0.985 |
| ✓    | ✓   | ✓   |     |     |     | 0.894/0.821/0.917 | 0.890/0.838/0.928 | 0.850/0.836/0.871 | 0.950/0.914/0.944 | 1.000/1.000/1.000 |
| ✓    |     |     | ✓   | ✓   |     | 0.878/0.780/0.904 | 0.880/0.805/0.909 | 0.946/0.806/0.932 | 0.992/1.000/0.928 | 0.864/0.928/0.939 |
| ✓    | ✓   | ✓   | ✓   | ✓   | ✓   | 0.905/0.812/0.925 | 0.901/0.835/0.936 | 0.961/0.863/0.943 | 0.997/1.000/0.992 | 1.000/1.000/1.000 |

Figure 4. Qualitative results on the ‘grid’ textures in MVTec AD. (a) Training normal images (top) and multi-level anomalous images $\tilde{x}^2, \tilde{x}^4$ and $\tilde{x}^8$ (bottom). Where $\tilde{x}^0$ and $\tilde{x}^1$ are not shown. (b) Test anomalous images and the aggregated pixel-level anomaly scores.

4.3. Investigating the robustness.

Although previous results have demonstrated the effectiveness of MLSAD to some degree, whether it is robust enough to detect all potential anomalies is still unclear. Quantitative evaluation using available anomaly datasets is always biased [51] or even misleading [43], since we cannot collect all representative anomalies for a test. To probe the behavior of the MLSAD model on samples outside the normal image distribution, we search in the image space and try to find images with lower anomaly scores than 95% of the test normal images’. We then visualize the found images and check if they do look like the normal images. Similar to prior works [35, 49], we find such images via gradient descent on the calculated anomaly score. To search as broadly
Figure 5. Images found with lower anomaly scores than 95% test normal images via gradient descend. The detectors used here are trained on CIFAR10 horse images. The top line shows the initial images of the optimizations. The following lines are the images found in various anomaly detection models. The images found in MLSAD have the smallest perceptual differences from the real horse images. We remark that better results for MLSAD may be obtained by using $A^{32}$ and softmax pooling aggregation. See Appendix E for additional results.

as possible, we start the search from various types of images outside the in-distribution and use a variant of the pull-away term (PT) as a repelling regularizer:

$$L_{PT}(X_B) = -\frac{1}{N(N-1)} \sum_i \sum_{j \neq i} ||x_i - x_j||_1,$$

where $X_B = \{x_1, x_2, ..., x_N\}$ denotes a batch of images being optimized. For comparison, we also perform the search on ablated MLSADs, AE, DeepSVDD, and CSI.

**Results.** Fig. 5 shows the images found in various horse anomaly detectors. The ablated MLSAD using $A^1$ alone, AE, and DeepSVDD all fail to produce any horse-like images. CSI and the ablated MLSAD using $A^8$ alone indeed produce some images with horse-like patterns. It seems they learned some class discriminative features of the normal data like supervised multi-class classifiers. By contrast, the images found in MLSAD are more recognizable and normal-looking, no matter what images we start from. It indicates that MLSAD tends to unbiasedly detect anomalous features of various levels, and learn better and detailed generative features. As far as we know, deep neural networks widely used today cannot produce such normal-looking images by simply optimizing their outputs. Considering that similar results can not be observed in ablated MLSAD detectors, we conclude that the multi-level setting is necessary to build a robust model that probably detects various types of anomalies.

5. Conclusion

We first introduced the MRF approximations and argued that images from multi-level MRF approximations can efficiently expose the local abnormal structures of various levels. Then we developed entropy-regularized PatchGAN to generate such images. To fully exploit the local abnormal structures for anomaly detection, we proposed to train multiple level-specific patch-based detectors and calculate the overall anomaly score by aggregating the results over all patches and levels. Diverse multi-level abnormal structures were observed in the generations of entropy-regularized PatchGAN. The results on MNIST, CIFAR10, ImageNet10, and texture datasets show that MLSAD is a general, robust and effective anomaly detection model. They also indicate that unsupervised learning of multi-level visual structures using MRF approximations as contrastive distributions is feasible, which may be extended to broader areas like representation learning. However, MLSAD is computationally expensive for large images due to the multi-level setting and strides of $1 \times 1$. In the future, We will investigate the performance of using larger stride sizes, lower resolutions for high-level detectors, and larger level gaps.
References

[1] Davide Abati, Angelo Porrello, Simone Calderara, and Rita Cucchiara. Latent space autoregression for novelty detection. In CVPR, pages 481–490, 2019. 6
[2] Samet Akcay, Amir Atapour-Abarghouei, and Toby P Breckon. Ganomaly: Semi-supervised anomaly detection via adversarial training. In ACCV, 2018. 1, 2
[3] Jinwon An and Sungzoon Cho. Variational autoencoder based anomaly detection using reconstruction probability. In SNU Data Mining Center, Tech. Rep., 2015. 2
[4] Jerone TA Andrews, Edward J Morton, and Lewis D Griffin. Detecting anomalous data using auto-encoders. International Journal of Machine Learning and Computing, 6(1):21, 2016. 2
[5] Mohamed Ishmael Belghazi, Aristide Baratin, Sai Rajeswar, Sherjil Ozair, Yoshua Bengio, Devon Hjelm, and Aaron C. Courville. Mutual information neural estimation. In ICML, 2018. 3, 13
[6] Liron Bergman and Yedid Hoshen. Classification-based anomaly detection for general data. In ICLR, 2020. 2
[7] Paul Bergmann, Michael Fauser, David Sattlegger, and Carsten Steger. Mvtec ad—a comprehensive real-world dataset for unsupervised anomaly detection. In CVPR, 2019. 1, 6
[8] Urs Bergmann, Nikolay Jetchev, and Roland Vollgraf. Learning texture manifolds with the periodic spatial gan. In ICML, 2017. 3
[9] Ashish Bora, Eric Price, and Alexandros G. Dimakis. Ambientgan: Generative models from lossy measurements. In ICLR, 2018. 5
[10] Wieland Brendel and Matthias Bethge. Approximating cnns with bag-of-local-features models works surprisingly well on imagenet. In ICLR, 2019. 5
[11] Hyunsun Choi, Eric Jang, and Alexander A Alemi. Waic, but why? generative ensembles for robust anomaly detection. In arXiv preprint arXiv:1810.01392, 2018. 2
[12] Jia Deng, W. Dong, R. Socher, L. Li, K. Li, and Li Fei-Fei. Imagenet: A large-scale hierarchical image database. In CVPR, 2009. 6
[13] Izhak Golan and Ran El-Yaniv. Deep anomaly detection using geometric transformations. In NeurIPS, 2018. 2, 6
[14] Ian Goodfellow, Yoshua Bengio, and Aaron Courville. Deep Learning. MIT Press. http://www.deeplearningbook.org, page 625. 1
[15] Ian J. Goodfellow, Jean Pouget-Abadie, M. Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair, Aaron C. Courville, and Yoshua Bengio. Generative adversarial nets. In NIPS, 2014. 3
[16] Sachin Goyal, Aditi Raghunathan, Moksh Jain, H. Simhadri, and Prateek Jain. Drocc: Deep robust one-class classification. In ICML, 2020. 6
[17] Michael Gutmann and Aapo Hyvärinen. Learning features by contrasting natural images with noise. In International Conference on Artificial Neural Networks, 2009. 1
[18] Michael Gutmann and Aapo Hyvärinen. Noise-contrastive estimation: A new estimation principle for unnormalized statistical models. In Proceedings of the Thirteenth International Conference on Artificial Intelligence and Statistics, 2010. 1
[19] Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image recognition. In CVPR, 2016. 13
[20] Matthias Hein, Maksym Andriushchenko, and Julian Bitterwolf. Why relu networks yield high-confidence predictions far away from the training data and how to mitigate the problem. In CVPR, 2018. 2
[21] Dan Hendrycks and Kevin Gimpel. A baseline for detecting misclassified and out-of-distribution examples in neural networks. In ICLR, 2017. 2
[22] Dan Hendrycks, Mantas Mazeika, and Thomas G. Dietterich. Deep anomaly detection with outlier exposure. In ICLR, 2018. 2
[23] Dan Hendrycks, Mantas Mazeika, Saurav Kadavath, and Dawn Song. Using self-supervised learning can improve model robustness and uncertainty. In NeurIPS, 2019. 2, 6
[24] Phillip Isola, Jun-Yan Zhu, Tinghui Zhou, and Alexei A Efros. Image-to-image translation with conditional adversarial networks. In CVPR, 2017. 3
[25] Tero Karras, Samuli Laine, Miika Aittala, Janne Hellsten, Jaakko Lehtinen, and Timo Aila. Analyzing and improving the image quality of stylegan. In CVPR, 2020. 12
[26] Rithesh Kumar, Anirudh Goyal, Aaron C. Courville, and Yoshua Bengio. Maximum entropy generators for energy-based models. ArXiv, abs/1901.08508, 2019. 3
[27] Kimin Lee, Honglak Lee, Kibok Lee, and Jinwoo Shin. Training confidence-calibrated classifiers for detecting out-of-distribution samples. In ICLR, 2018. 2
[28] Shiyu Liang, Yixuan Li, and R. Srikant. Enhancing the reliability of out-of-distribution image detection in neural networks. In ICLR, 2018. 2
[29] Rosanne Liu, Joel Lehman, Piero Molino, Felipe Petroski Such, Eric Frank, Alex Sergeev, and Jason Yosinski. An intriguing failing of convolutional neural networks and the coordconv solution. In NeurIPS, 2018. 3
[30] Jonathan Long, Evan Shelhamer, and Trevor Darrell. Fully convolutional networks for semantic segmentation. In CVPR, pages 3431–3440, 2015. 3
[31] Takeru Miyato, Toshiki Kataoka, Masanori Koyama, and Yuichi Yoshida. Spectral normalization for generative adversarial networks. In ICLR, 2018. 13
[32] Eric T. Nalisnick, Akihiro Matsukawa, Yee Whye Teh, Dilan Görür, and Balaji Lakshminarayanan. Do deep generative models know what they don’t know? In ICLR, 2018. 2
[33] Anh M Nguyen, Jason Yosinski, and Jeff Clune. Deep neural networks are easily fooled: High confidence predictions for unrecognizable images. In CVPR, 2015. 7, 8
[36] Duc Tam Nguyen, Zhongyu Lou, Michael Klar, and Thomas Brox. Anomaly detection with multiple-hypotheses predictions. In ICML, 2019. 2

[37] E. Parzen. On estimation of a probability density function and mode. Annals of Mathematical Statistics, 33:1065–1076, 1962. 6

[38] Pramuditha Perera, Ramesh Nallapati, Bing Xiang, and NONE. Ocgan: One-class novelty detection using gans with constrained latent representations. In CVPR, 2019. 2, 6

[39] Stanislav Pidhorskyi, Ranya Almohsen, and Gianfranco Doretto. Generative probabilistic novelty detection with adversarial autoencoders. In NeurIPS, 2018. 2

[40] Stanislav Pidhorskyi, Ranya Almohsen, and Gianfranco Doretto. Generative probabilistic novelty detection with adversarial autoencoders. In NeurIPS, 2018. 2

[41] Jie Ren, Peter J. Liu, Emily Fertig, Jasper Snoek, Ryan Poplin, Mark A. DePristo, Joshua V. Dillon, and Balaji Lakshminarayanan. Likelihood ratios for out-of-distribution detection. In NeurIPS, 2019. 2

[42] Olaf Ronneberger, Philipp Fischer, and Thomas Brox. U-net: Convolutional networks for biomedical image segmentation. In International Conference on Medical image computing and computer-assisted intervention, 2015. 12

[43] Lukas Ruff, J. Kaufmann, Robert A. Vandermeulen, Grégoire Montavon, W. Samek, Marius Kloft, Thomas G. Dietterich, and K. Müller. A unifying review of deep and shallow anomaly detection. ArXiv, abs/2009.11732, 2020. 2, 7

[44] Lukas Ruff, Robert Vandermeulen, Nico Goernitz, Lucas Deecke, Shoaib Ahmed Siddiqui, Alexander Binder, Emanuel Müller, and Marius Kloft. Deep one-class classification. In ICML, 2018. 2, 6

[45] Mohammad Sabokrou, Mohammad Khalooei, Mahmood Fathy, and Ehsan Adeli. Adversarially learned one-class classifier for novelty detection. In CVPR, 2018. 2

[46] Daisuke Sato, Shouhei Hanaoka, Yuukihiro Nomura, Tomomi Takenaga, Soichiro Miki, Takeharu Yoshikawa, Naoto Hayashi, and Osamu Abe. A primitive study on unsupervised anomaly detection with an autoencoder in emergency head ct volumes. In Medical Imaging 2018: Computer-Aided Diagnosis, 2018. 1

[47] Thomas Schlegl, Philipp Seeböck, Sebastian M. Waldstein, Ursula M Schmidt-Erfurth, and Georg Langs. Unsupervised anomaly detection with generative adversarial networks to guide marker discovery. In IPMI, 2017. 2, 6

[48] Bernhard Schölkopf, John C Platt, John Shawe-Taylor, Alex J Smola, and Robert C Williamson. Estimating the support of a high-dimensional distribution. Neural computation, 13(7):1443–1471, 2001. 6

[49] Lukas Schott, Jonas Rauber, M. Bethge, and W. Brendel. Towards the first adversarially robust neural network model on mnist. In ICLR, 2019. 7

[50] Joan Serrà, David Álvarez, Vicenç Gómez, Olga Slizovskaia, José F. Núñez, and Jordi Luque. Input complexity and out-of-distribution detection with likelihood-based generative models. In ICLR, 2020. 2

[51] Alireza Shafaei, Mark Schmidt, and James J. Little. A less biased evaluation of out-of-distribution sample detectors. In BMVC, 2019. 1, 7

[52] Tamar Rott Shaham, Tali Dekel, and Tomer Michaeli. Simgan: Learning a generative model from a single natural image. In ICCV, 2019. 3

[53] Jihoon Tack, Sangwoo Mo, Jongheon Jeong, and Jinwoo Shin. Csi: Novelty detection via contrastive learning on distributionally shifted instances. In NeurIPS, 2020. 2, 6

[54] Sachin Vernekar, Ashish Gaurav, Váhdat Abdelzad, Taylor Denouden, Rick Salay, and Krzysztof Czarnecki. Out-of-distribution detection in classifiers via generation. NeurIPS workshop, 2019. 2

[55] Yun Wang, Juncheng Li, and Florian Metze. A comparison of five multiple instance learning pooling functions for sound event detection with weak labeling. In ICASSP 2019-2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2019. 5

[56] Jihun Yi and S. Yoon. Patch svdd: Patch-level svdd for anomaly detection and segmentation. In ACCV, 2020. 2

[57] Yang Yu, Wei-Yang Qu, Nan Li, and Zimin Guo. Open-category classification by adversarial sample generation. In IJCAI, 2017. 2

[58] Junbo Zhao, Michael Mathieu, and Yann LeCun. Energy-based generative adversarial networks. In ICLR, 2017. 8

[59] Panpan Zheng, Shuhan Yuan, Xintao Wu, Jun Yu Li, and Aidong Lu. One-class adversarial nets for fraud detection. In AAAI, 2018. 1
6. Appendix

A. Proofs

Theorem 1 Let \( S_w = \{x | \forall c \in C^w, p_{X_c}(x_c) > 0\} \), then \( \text{supp}(q_w^p) = S_w \). In other words, the support of \( q_w^p \) is identical to the set of all images where each patch of \( w \times w \) pixels is normal.

Proof. Firstly, we prove \( \text{supp}(q_w^p) \subseteq S_w \).
For all \( x \in \text{supp}(q_w^p) \), we have \( q_w^p(x) > 0 \). Let \( \bar{c} \) be the pixels outside the patch \( c \). Since

\[
q_w^p(x) = q_w^p(x_c)q_{X_c|X}(x_c|x_e),
\]

we have \( q_w^p(x_c) > 0 \). By the definition of MRF approximations (Definition 1), we know that \( \forall c \in C^w, q_w^p(x_c) = p_{X_c}(x_c) \). Therefore, \( \forall x \in \text{supp}(q_w^p), p_{X_c}(x_c) > 0 \) holds for all \( c \in C^w \). Because \( S_w = \{x | \forall c \in C^w, p_{X_c}(x_c) > 0\} \), for all \( x \in \text{supp}(q_w^p) \) we have \( x \in S_w \). It follows that \( \text{supp}(q_w^p) \subseteq S_w \).

Then, we prove \( S_w \subseteq \text{supp}(q_w^p) \).
By the definition of MRF approximations, we know that the \( w \)-th level MRF approximation can be written as

\[
q_w^p(x) = \prod_{c \in C^w} \phi_c(x_c). \tag{6}
\]

Since

\[
q_w^p(x_c) = \int_{x_e} q_w^p(x_c, x_{\bar{c}})dx_{\bar{c}},
\]

we have

\[
q_w^p(x_c) = \int_{x_e} \prod_{c' \in C^w} \phi_{c'}(x_{c'})dx_{\bar{c}} = \phi_c(x_c)\int_{x_e} \prod_{c' \in C^w \setminus \{c\}} \phi_{c'}(x_{c'})dx_{\bar{c}} \tag{7}
\]

where \( \psi_c(x_{\bar{c}}) := \int_{x_e} \prod_{c' \in C^w \setminus \{c\}} \phi_{c'}(x_{c'})dx_{\bar{c}} \). Then for all \( x \) such that \( \forall c \in C^w, p_{X_c}(x_c) > 0 \), since \( q_w^p(x_c) = p_{X_c}(x_c) \), we have \( q_w^p(x_c) > 0 \). Then using Equation 6 and Equation 7, we can get \( \forall c \in C^w, \phi_c(x_c) > 0 \) and \( q_w^p(x) > 0 \). Hence \( \forall x \in S_w, x \in \text{supp}(q_w^p) \) holds and thus \( S_w \subseteq \text{supp}(q_w^p) \).

Finally, we conclude that \( \text{supp}(q_w^p) = S_w \).

Theorem 2 The maximum entropy distribution satisfying \( \forall c \in C^w, q_{X_c}(x_c) = p_{X_c}(x_c) \) can be factored as \( \prod_{c \in C^w} \phi_c(x_c) \).

Proof. We prove it with the calculus of variations and Lagrange multipliers. Firstly, we can have the functional constraints:

\[
G_c[q_X] := q_{X_c}(x_c) - p_{X_c}(x_c) = 0, \forall c \in C^w.
\]

Additionally, we define a functional constraint that our density must sum to 1:

\[
G_0[q_X] := \int_X q_X(x)dx - 1 = 0.
\]

Let us use \( H \) to denote entropy, then the differential entropy functional can be written as

\[
H[q_X] := -\int_X q_X(x)\log q_X(x)dx.
\]

To maximize entropy \( H \) subject to \( G_0 \) and a serial of patch constraints \( \{G_c|c \in C^w\} \), we put them together and consider the functional:

\[
J[q_X] = \int_X -q_X(x)\log q_X(x)dx - \lambda_0[\int_X q_X(x)dx - 1] - \sum_c \lambda_c(x_c)[\int_{x_e} q_X(x)dx_e - p_{X_e}(x_e)]dx_c.
\]
where $\lambda_0$ and $\lambda_c$ are Lagrange multipliers. The final term can be arranged into

$$\sum_c \int_x \lambda_c(x_c) q_X(x) dx - \int_x \lambda_c(x_c) p_{X_c}(x_c) dx_c.$$ 

The entropy attains an extremum when the functional derivative is equal to zero:

$$\frac{\delta J}{\delta q_X} = -1 - \log q_X(x) - \lambda_0 - \sum_c \lambda_c(x_c) = 0.$$ 

We can get

$$\lambda_0 = -1, q_X(x) = e^{-\sum_c \lambda_c(x_c)}.$$ 

Therefore the maximum entropy distribution $q_X(x)$ can be factored as $\prod_c \phi_c(x_c)$.

B. Discussion on the position anomaly pair

We train a pixel-level detector $A^1$ and an ablated $A^1$ by removing the position anomaly pair (denoted by w/o PA) on images of MNIST digit 7, and compare them by visualizing their score maps. We first test both detectors on a noise image. The results are shown in Figure 6. We find that both the detectors can output high anomaly scores for almost all the colorful pixels. We then test both detectors on an image whose all pixels are white, as shown in Figure 7. Since the border regions of digit 7 images are always black, a perfect pixel-level anomaly detector should output high anomaly scores for such regions. It is observed that both the detectors (trained with and without the position anomaly pair) work nicely to a certain extent. However, we find that the detector trained with the position anomaly pair (w/ PA) outputs a score map with a more concrete shape that looks like the average image. Considering that the pixel values in MNIST images are almost binary, the pixel brightness in the average image can be treated as the probability that the corresponding pixel is white, and the observations in Figure 6, we conclude that the ablated pixel-level detector (w/o PA) focus more on learning the common color statistics shared in various locations, and the position anomaly pair indeed helps the pixel-level detector to learn the position-dependent abnormal cues.

Figure 6. Both the pixel-level detectors trained with and without the position anomaly pair can detect out almost all the colorful pixels.

Figure 7. The position anomaly pair helps the pixel-level detector learn a better position-aware anomaly score.

C. Experimental details

C.1 Maximum entropy-regularized PatchGAN.

We use Pytorch library to build and train our models. The intensity range of the images is normalized into $[-1, 1]$. As StyleGAN2 [25], we use minibatch standard deviation layers in the discriminators, and use bilinear upsampling layers in the generators. The generators are based on the UNet [42] architecture. Both SN and batch normalization are used at all non-output layers. For MNIST, CIFAR10 and ImageNet10 dataset, we generate images with a size of $32 \times 32$, the network
C.2 Details of level-specific detectors.

Level-specific detectors use no normalization operation for all. We add a Gaussian noise with a variance of $\sigma = 1$, which will not overwhelm the local structure term: 

$$
\beta = \min\left(\frac{\|g_{adv}\|}{\|g_{I}\| + 10^{-8}}, 1\right)
$$

(8)

where $g_{adv}$ is the gradient of $L_{adv}$ with respect to $G$, and $g_{I}$ is the gradient from $L_{\theta}$. The statistics network $T$ takes the concatenation of $z$ and $G(z)$ as input and uses a multi-scale pooling operation to calculate the final output, which is expected to be better for producing diverse recombinations. We train them with the Adam optimizer, a batch size of 32, and a max epoch of 300. The optimizer settings are shown in Table 9:

Table 5. Discriminator network for 2th-level MRF approximation.

| Layer         | Input shape | Kernel size | Stride size | Activation function | Normalization function |
|---------------|-------------|-------------|-------------|---------------------|-----------------------|
| Input $x,r$   | 32 x 32 x 5| -           | -           | LReLU               | SN                    |
| Convolution   | 31 x 31 x 64| 2 x 2       | 1 x 1       | LReLU               | SN                    |
| Minibatch stddev| 31 x 31 x 68| -           | -           | LReLU               | SN                    |
| Convolution   | 31 x 31 x 128| 1 x 1      | 1 x 1       | LReLU               | SN                    |
| ResBlock(256) | 31 x 31 x 128| -           | -           | LReLU               | SN                    |
| Convolution   | 31 x 31 x 1 | 1 x 1       | 1 x 1       | Sigmoid             | None                  |

Table 6. Discriminator network for 4th-level MRF approximation.

| Layer         | Input shape | Kernel size | Stride size | Activation function | Normalization function |
|---------------|-------------|-------------|-------------|---------------------|-----------------------|
| Input $x,r$   | 32 x 32 x 5| -           | -           | LReLU               | SN                    |
| Convolution   | 29 x 29 x 128| 4 x 4      | 1 x 1       | LReLU               | SN                    |
| Minibatch stddev| 29 x 29 x 132| -           | -           | LReLU               | SN                    |
| Convolution   | 29 x 29 x 256| 1 x 1      | 1 x 1       | LReLU               | SN                    |
| ResBlock(256) | 29 x 29 x 256| -           | -           | LReLU               | SN                    |
| Convolution   | 29 x 29 x 1 | 1 x 1       | 1 x 1       | Sigmoid             | None                  |

Table 7. Discriminator network for 8th-level MRF approximation.

| Layer         | Input shape | Kernel size | Stride size | Activation function | Normalization function |
|---------------|-------------|-------------|-------------|---------------------|-----------------------|
| Input $x,r$   | 32 x 32 x 3| 16 x 16 x 64| 4 x 4       | LReLU               | BN                    |
| Convolution   | 8 x 8 x 128| 8 x 8 x 128| 1 x 1       | LReLU               | BN                    |
| Minibatch stddev| 4 x 4 x 256| 4 x 4 x 256| 1 x 1       | LReLU               | BN                    |
| Convolution   | 1 x 1 x 512| 4 x 4 x 4   | 1 x 1       | LReLU               | BN                    |
| Convolution   | 4 x 4 x 256| 4 x 4 x 256| 1 x 1       | LReLU               | BN                    |

Table 8. Generator network for every level

| Layer         | Input shape | Kernel size | Stride size | Activation function | Normalization function |
|---------------|-------------|-------------|-------------|---------------------|-----------------------|
| Input $x,r$   | 32 x 32 x 3| 16 x 16 x 64| 4 x 4       | LReLU               | BN                    |
| Convolution   | 8 x 8 x 128| 8 x 8 x 128| 1 x 1       | LReLU               | BN                    |
| Minibatch stddev| 4 x 4 x 256| 4 x 4 x 256| 1 x 1       | LReLU               | BN                    |
| Convolution   | 1 x 1 x 512| 4 x 4 x 4   | 1 x 1       | LReLU               | BN                    |
| Convolution   | 4 x 4 x 256| 4 x 4 x 256| 1 x 1       | LReLU               | BN                    |

Table 9. Optimizer settings for level-specific detectors.

| Layer         | Input shape | Kernel size | Stride size | Activation function | Normalization function |
|---------------|-------------|-------------|-------------|---------------------|-----------------------|
| Input $x,r$   | 32 x 32 x 5| -           | -           | LReLU               | SN                    |
| Convolution   | 29 x 29 x 64| 4 x 4       | 1 x 1       | LReLU               | SN                    |
| Minibatch stddev| 29 x 29 x 68| -           | -           | LReLU               | SN                    |
| Convolution   | 29 x 29 x 128| 1 x 1      | 1 x 1       | LReLU               | SN                    |
| ResBlock(256) | 29 x 29 x 128| -           | -           | LReLU               | SN                    |
| Convolution   | 29 x 29 x 1 | 1 x 1       | 1 x 1       | Sigmoid             | None                  |

Table 10. Generator network for every level

| Layer         | Input shape | Kernel size | Stride size | Activation function | Normalization function |
|---------------|-------------|-------------|-------------|---------------------|-----------------------|
| Input $x,r$   | 32 x 32 x 3| 16 x 16 x 64| 4 x 4       | LReLU               | BN                    |
| Convolution   | 8 x 8 x 128| 8 x 8 x 128| 1 x 1       | LReLU               | BN                    |
| Minibatch stddev| 4 x 4 x 256| 4 x 4 x 256| 1 x 1       | LReLU               | BN                    |
| Convolution   | 1 x 1 x 512| 4 x 4 x 4   | 1 x 1       | LReLU               | BN                    |
| Convolution   | 4 x 4 x 256| 4 x 4 x 256| 1 x 1       | LReLU               | BN                    |

Table 10. Generator network for every level

For both MNIST and textures in MVTec AD, we add a Gaussian noise with a variance of $\sigma = 1$, and the learning rates decay by a factor of 0.3 at 50th and 75th epoch. For both CIFAR10 and ImageNet10, the learning rates decay by a factor of 0.1 at 100th and 200th epoch. We train level-specific detectors for a total of 100 epochs on MNIST and textures, and 250 epochs on CIFAR10 and ImageNet10, all with a batch size of 8. The hyper-parameters $\alpha_1$ and $\alpha_2$ are set to 0.6 and 0.4 for all levels and all datasets. We perform a non-exhaustive search for the hyper-parameters in the ranges shown in Table 10.
Table 9. Optimizer settings for training GANs.

| Network            | Learning rate | $\beta_1$ | $\beta_2$ |
|--------------------|---------------|-----------|-----------|
| Discriminators     | $4 \times 10^{-4}$ | 0         | 0.9       |
| Generators         | $1 \times 10^{-4}$ | 0         | 0.9       |
| Statistics networks| $4 \times 10^{-4}$ | 0         | 0.9       |

Table 10. Hyper-parameter search ranges for training detectors.

| hyperparameter | range |
|----------------|-------|
| batch size     | $\{8, 16, 32, 64\}$ |
| weight decay   | $\{1E-3, 5E-4, 1E-4, 5E-5\}$ |
| noise          | $\{0, 0.05, 0.1\}$ |
| $\alpha_1$     | $\{0.4, 0.6, 1.0\}$ |
| $\alpha_2$     | $\{0.4, 0.6, 1.0\}$ |
| num_block      | $\{1, 2\}$ |

Figure 8. The networks of level-specific detectors.

C.3 Details of investigating the robustness.

The total objective for the fooling test is to find $N$ images $X_B = \{x_1, x_2, ..., x_N\}$ that minimize $\frac{1}{N} \sum_i A(x_i) + \lambda L_{PT}$. Where $\lambda = 10^{-5}$ is used in our experiments. We initialize $x_i$ with images sampled from various datasets. Then we update them using the Adam optimizer with the learning rate of $0.02$, $\beta_1 = 0.5$ and $\beta_2 = 0.9$. The optimization will always be constrained in the image space by the clipping operation after each update: $x_i = \text{clip}(x_i, -1.0, 1.0)$ or $\text{clip}(x_i, 0.0, 1.0)$ (depending on the input range of the tested model).

D. More Results on Textures

Figure 9. Qualitative results on the textures in MVTec AD. The line 1 and line 3 are the test anomalous images of 'carpet' category and 'tile' category, respectively. The line 2 and line 4 are the corresponding (aggregated) pixel-level anomaly scores. We remark that the results can be further improved by using softmax pooling for the multi-level aggregation.
We present the additional visualizations of the pixel-level detection results (using average pooling for the multi-level aggregation) on textures of MVTec AD in Fig. 9.

E. More results on robustness

We present the additional results on the robustness in Fig. 10 and Fig. 11, the detectors tested here are trained on CIFAR10 classes ’bird’, ’plane’, ’frog’, ’car’ and ’ship’. We compare the MLSAD with AE, DeepSVDD and CSI. Results demonstrate the MLSAD can learn the more detailed and complete statistics of the normal data.

![Figure 10. Images found with lower anomaly scores than 95% test normal images’ via gradient decent. The images found in MLSAD have the smallest perceptual differences from the normal classes.](image)

![Figure 11. Images found with much more gradient decent steps (until the images have almost no changes). We again find that MLSAD produces the most normal-looking and recognizable images.](image)