Viscosity, heat conductivity and Prandtl number effects in Rayleigh-Taylor Instability

Feng Chen\textsuperscript{1*}, Aiguo Xu\textsuperscript{2,3†}, Guangcai Zhang\textsuperscript{2}

\textsuperscript{1}, School of Aeronautics, 
Shan Dong Jiaotong University, 
Jinan 250357, China

\textsuperscript{2}, National Key Laboratory of Computational Physics, 
Institute of Applied Physics and Computational Mathematics, 
P. O. Box 8009-26, 
Beijing 100088, China

\textsuperscript{3}, Center for Applied Physics and Technology, 
MOE Key Center for High Energy Density Physics Simulations, 
College of Engineering, Peking University, 
Beijing 100871, China

(Dated: November 21, 2021)

\textsuperscript{*} Corresponding author. E-mail: shanshiwycf@163.com

\textsuperscript{†} Corresponding author. E-mail: XuAiguo@iapcm.ac.cn
Abstract

Two-dimensional Rayleigh-Taylor (RT) instability problem is simulated with a multiple-relaxation-time discrete Boltzmann model with gravity term. The viscosity, heat conductivity and Prandtl number effects are probed from the macroscopic and the non-equilibrium views. In macro sense, both viscosity and heat conduction show significant inhibitory effect in the reacceleration stage, and the inhibition effect is mainly achieved by inhibiting the development of Kelvin-Helmholtz instability. Before this, the Prandtl number effect is not sensitive. Based on the view of non-equilibrium, the viscosity, heat conductivity, and Prandtl number effects on non-equilibrium manifestations, and the correlation degrees between the non-uniformity and the non-equilibrium strength in the complex flow are systematic investigated.

PACS numbers: 47.11.-j, 51.10.+y, 05.20.Dd

**Keywords:** discrete Boltzmann model/method; multiple-relaxation-time; Rayleigh-Taylor instability; non-equilibrium
I. INTRODUCTION

The Rayleigh-Taylor (RT) instability\cite{1, 2} occurs when a heavy fluid lies above a lighter one in a gravitational field with gravity pointing downward. The RT instability can be observed in a wide range of astrophysical and atmospheric flows, and has great significance in both fundamental research and practical applications. Since the existence of sharp interfaces and their evolutions, the flow system is out of equilibrium.

Over the decades, many numerical methods have been developed to simulate RT instability, such as flux-corrected transport method\cite{3}, level set method\cite{4}, front tracking method\cite{5}, marker-and-cell method\cite{6}, smoothed particle hydrodynamics method\cite{7}, boundary integral method\cite{8}, direct numerical simulations\cite{9, 10}, large-eddy simulations\cite{11}, and phase-field method\cite{12}. The influences of different factors on the evolution of RT instability have been studied more and more deeply. R. Betti et al.\cite{13} investigated the effect of vorticity accumulation on Ablative Rayleigh-Taylor Instability. M.R. Gupta et al.\cite{14} investigated the effect of magnetic field, compressibility and density variation on the nonlinear growth rate of RT instability. P.K. Sharma et al.\cite{15} analyzed the RT instability of two superposed fluids taking the effect of small rotation, suspended dust particles and surface tension. Rahul Banerjee et al.\cite{16} investigated the combined effect of viscosity and vorticity on the growth rate of the bubble associated with single mode RT instability. To cite but a few. To our knowledge, these numerical methods are based on the Euler or Navier-Stokes equations, but Euler and Navier-Stokes models fall short of describing the nonequilibrium effects. Consequently, the rich and complex nonequilibrium effects in the RT flow system are rarely investigated. At the same time, the molecular dynamic simulations can present helpful information on the nonequilibrium state\cite{17}, but due to the limitation of compute capacity, the spatial and temporal scales it can access are far from large enough.

Besides the numerical methods mentioned above, the Lattice Boltzmann (LB) method\cite{18–29} provides an alternative efficient tool for simulating complex fluid flows, and has been implemented in the RT instability study\cite{30, 39}. For instance, Nie et al. simulated the RT instability using a lattice Boltzmann model for multicomponent fluid flows, and Guo et al. investigated the effects of the Prandtl number on the mixing process in RT instability of incompressible and miscible fluids based on a double-distribution-function lattice Boltzmann method. But up to now, in most of previous studies this LB method works as a kind of new
scheme to solve partial differential equations such as the Euler equations and Navier-Stokes equations.

Recently, some scholars have re-positioned the method, and regard it as a kind of new mesoscopic and coarse-grained kinetic model of complex physical systems, which is juxtaposed with the traditional hydrodynamic method and called as Discrete Boltzmann Method (DBM). Compared with the first category, DBM possess more kinetic information which is beyond the description of the Navier-Stokes, and bring new physical insights into the physical system. The first DBM description appeared in a review article published in 2012. In the work, the authors pointed out how to investigate both the Hydrodynamic Non-Equilibrium (HNE) and Thermodynamic Non-Equilibrium (TNE) simultaneously in complex flows via the DBM. Subsequently, DBM has been gradually extended and applied to the combustion and detonation system, multiphase flow system, and fluid instability system. The finer physical structures of shock waves revealed by DBM have been confirmed and supplemented by the results of non-equilibrium molecular dynamics simulations.

In this paper, we present a multiple-relaxation-time (MRT) DBM with gravity. Two dimensional RT instability problem is simulated, and the results are compared with those in previous studies. The relaxation rates of the various kinetic moments due to particle collisions may be adjusted more physically in the MRT version. This overcomes some obvious deficiencies of the Single-Relaxation-Time(SRT) version, such as a fixed Prandtl number. Compared with previous studies on RT instability, the viscosity, heat conductivity, and Prandtl number effects on macro-dynamics and non-equilibrium manifestations are investigated simultaneously in the DBM model. With the increase of viscosity or heat conduction, various non-equilibrium components increase. When the RT instability develops into the turbulent mixing stage, the global average Thermodynamic NonEquilibrium (TNE) strength and Non-Organized Energy Flux(NOEF) strength have a decrease. The correlation degrees between density non-uniformity and the global average TNE strength, temperature non-uniformity and the global average NOEF strength, are numerically probed. And the simulation results show that heat conduction plays a major role on the correlation degree. The modeling of non-equilibrium feature is a helpful and effective complement to the macroscopic description. They two, together, provide new insights into complex flow systems.
The following part of the paper is planned as follows. Section II presents the MRT Discrete Boltzmann model with gravity. Systematic numerical simulations of RT instability and non-equilibrium characteristics are shown and analyzed in Section III. A brief conclusion is given in Section IV.

II. DESCRIPTION OF THE MRT DBM WITH GRAVITY

The MRT discrete Boltzmann equation with gravity term read as follows

$$\frac{\partial f_i}{\partial t} + v_{ia} \frac{\partial f_i}{\partial x_a} = -M_{ik}^{-1}\hat{\mathbf{S}}_k(f_k - f_k^{eq}) - g_\alpha \frac{(v_{ia} - u_\alpha)}{RT} f_i^{eq},$$

(1)

where $v_i$ is the discrete particle velocity, $i = 1, \ldots, N$, $N$ is the number of discrete velocities. The matrix $\hat{\mathbf{S}} = diag(s_1, s_2, \ldots, s_N)$ is the diagonal relaxation matrix. $f_i$ and $\hat{f}_i$ ($f_i^{eq}$ and $\hat{f}_i^{eq}$) are the particle (equilibrium) distribution function in the velocity space and the kinetic moment space respectively, the mapping between moment space and velocity space is defined by the linear transformation $M_{ij}$, i.e., $\hat{f}_i = M_{ij} f_j$, $f_i = M_{ij}^{-1} \hat{f}_j$. $g_\alpha$ is the acceleration, $u_\alpha$ is the macroscopic velocity, $T$ is the temperature.

Chapman-Enskog analysis indicates that it is independent of the Discrete Velocity Model (DVM). Therefore, the choosing of DVM has a high flexibility. Here, the following two-dimensional discrete velocity model is used

$$\begin{align*}
(v_{i1}, v_{i2}) = \begin{cases} 
\textbf{cyc} : c(\pm1,0), & \text{for } 1 \leq i \leq 4, \\
 & c(\pm1,\pm1), & \text{for } 5 \leq i \leq 8, \\
\textbf{cyc} : 2c(\pm1,0), & \text{for } 9 \leq i \leq 12, \\
 & 2c(\pm1,\pm1), & \text{for } 13 \leq i \leq 16,
\end{cases}
\end{align*}$$

(2)

where cyc indicates the cyclic permutation, $\eta_i = \eta_0$ for $i = 1, \ldots, 4$, and $\eta_i = 0$, for $i = 5, \ldots, 16$.

Transformation matrix and the corresponding equilibrium distribution functions in the Kinetic Moment Space are constructed according to the seven moment relations. Specifically, transformation matrix $\mathbf{M} = (m_1, m_2, \ldots, m_{16})^T$, $m_i = (1, v_{ix}, v_{iy}, (v_{ix}^2 + \eta_i^2)/2, v_{iy}^2, v_{ix}v_{iy}, v_{iy}^2, (v_{ix}^2 + \eta_i^2)v_{ix}/2, (v_{iy}^2 + \eta_i^2)v_{iy}/2, v_{ix}^3, v_{ix}^2v_{iy}, v_{ix}v_{iy}^2, v_{iy}^3, (v_{ix}^2 + \eta_i^2)v_{ix}^2/2, (v_{iy}^2 + \eta_i^2)v_{ix}v_{iy}/2, (v_{ix}^2 + \eta_i^2)v_{iy}^2/2)$. The corresponding equilibrium distribution functions in KMS: $\hat{f}_1^{eq} = \rho$, $\hat{f}_2^{eq} = j_x$, $\hat{f}_3^{eq} = j_y$, $\hat{f}_4^{eq} = e$, $\hat{f}_5^{eq} = P + \rho u_x^2$, $\hat{f}_6^{eq} = \rho u_x u_y$, $\hat{f}_7^{eq} = P + \rho u_y^2$, $\hat{f}_8^{eq} = (e + P)u_x$, $\hat{f}_9^{eq} = (e + P)u_y$, $\hat{f}_{10}^{eq} = \rho u_x (3T +$
FIG. 1: Schematic of the discrete-velocity model.

\[ \hat{f}_{11}^{\text{eq}} = \rho u_y (T + u_x^2), \quad \hat{f}_{12}^{\text{eq}} = \rho u_x (T + u_y^2), \quad \hat{f}_{13}^{\text{eq}} = \rho u_y (3T + u_x^2), \quad \hat{f}_{14}^{\text{eq}} = (e + P)T + (e + 2P)u_x^2, \quad \hat{f}_{15}^{\text{eq}} = (e + 2P)u_x u_y, \quad \hat{f}_{16}^{\text{eq}} = (e + P)T + (e + 2P)u_y^2, \]

where pressure \( P = \rho RT \), energy \( e = b\rho RT/2 + \rho u_x^2/2 \).

By using the Chapman-Enskog expansion on the two sides of the discrete Boltzmann equation (see Appendix for details), the final NS equations with gravity term for both compressible fluids and incompressible fluids can be obtained:

\[ \frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_\alpha)}{\partial x_\alpha} = 0, \tag{3a} \]

\[ \frac{\partial (\rho u_\alpha)}{\partial t} + \frac{\partial (\rho u_\alpha u_\beta)}{\partial x_\beta} + \frac{\partial P}{\partial x_\alpha} = \frac{\partial}{\partial x_\beta} [\mu (\partial u_\alpha/\partial x_\beta + \partial u_\beta/\partial x_\alpha - 2b \partial u_\chi/\partial x_\chi \delta_{\alpha\beta})] - \rho g_\alpha, \tag{3b} \]

\[ \frac{\partial e}{\partial t} + \frac{\partial}{\partial x_\alpha} [(e + P)u_\alpha] = \frac{\partial}{\partial x_\beta} [\lambda (\partial T/\partial x_\beta) + \mu (\partial u_\alpha/\partial x_\beta + \partial u_\beta/\partial x_\alpha - 2b \partial u_\chi/\partial x_\chi \delta_{\alpha\beta}) u_\alpha] - \rho g_\alpha u_\alpha, \tag{3c} \]

where \( \alpha, \beta, \chi = x, y \), the viscosity \( \mu = \rho RT/s_v \), \( s_v = s_5 = s_6 = s_7 \), the heat conductivity \( \lambda = (b/2 + 1)\rho R^2 T/s_T \), \( s_T = s_8 = s_9 \).

III. NUMERICAL SIMULATIONS

A. Performance on discontinuity

In order to check the performance of difference scheme on discontinuity, we construct this
FIG. 2: Density profiles with various difference schemes at \( t = 12 \).

The problem

\[
\begin{align*}
(p, u_1, u_2, T) &= (1.71429, 0.0, 0.697217, 1.26389), & y &\leq L/2, \\
(p, u_1, u_2, T) &= (1.0, 0.0, 0.0, 1.0), & L/2 < y &\leq L.
\end{align*}
\]

\((4)\)

\(L\) is the length of computational domain. The physical quantities on the two sides satisfy the Hugoniot relations, and specific heat ratio \( \gamma = 1.4 \). In the \( y \) direction \( f_i = M_{ij}^{-1} \hat{f}_j^{eq} \), and the macroscopic quantities adopt the initial values. In the \( x \) direction, the periodic boundary condition is adopted. Fig.2 shows the simulation results of density at time \( t = 12 \) using different space discretization schemes. The parameters are \( c = 2, \eta_0 = 4, dx = dy = 0.2, dt = 10^{-4}, s_i = 10^4, i = 1, \ldots, 16 \). The simulations with Lax-Wendroff scheme have strong unphysical oscillations in the shocked region. The second order upwind scheme results in unphysical ‘overshoot’ phenomena at the shock front. The simulation result with WENO scheme is much more accurate, and decreases the unphysical oscillations at the discontinuity.

B. Macro-characteristics of Rayleigh-Taylor instability

Numerical simulations of Rayleigh-Taylor instability are performed in the section. The computational domain is a two-dimensional box with height \( H = 80 \) and width \( W = 20 \), and the initial hydrostatic unstable configuration is given by:

\[
\begin{align*}
T_0(y) &= T_u; \rho_0(y) = \rho_u \exp(-g(y - y_s)/T_u); & y &\geq y_s \\
T_0(y) &= T_b; \rho_0(y) = \rho_b \exp(-g(y - y_s)/T_b); & y &< y_s
\end{align*}
\]

\((5)\)
where $y_s = 40 + 2 \cos(0.1 \pi x)$ is the initial small perturbation at the interface. To be at equilibrium, the same pressure at the interface should be required

$$p_0 = \rho_u T_u = \rho_b T_b,$$

(6)

where $T_u < T_b$, $\rho_u > \rho_b$. In order to have a finite width of the initial interface, all numerical experiments will be performed by preparing the initial configuration plus a smooth interpolation between the two half volumes. The initial temperature profile is therefore chosen to be:

$$T_0(y) = \frac{T_u + T_b}{2} + \frac{T_u - T_b}{2} \times \tanh(\frac{y - y_s}{w})$$

(7)

where $w$ denotes the initial width of the interface. Initial density $\rho_0(y)$ are then fixed by the initial settings (Eqs (5)-(6)) combined with the smoothed temperature profile. In the simulation, the bottom condition is solid condition, the top condition is free condition (that is to say, outflow condition), and the left and right boundaries are periodic boundary conditions. The fifth-order WENO scheme is used for space discretization, while the time evolution is performed through the third-order Runge-Kutta scheme.

In order to verify the validity of calculation, grid convergence study is conducted in different grids, $N_x \times N_y = 100 \times 400$ (grid I) and $N_x \times N_y = 200 \times 800$ (grid II). The initial condition is $\rho_b = 1$, $T_b = 1.4$, $\rho_u = 2.33333$, $T_u = 0.6$, $g_x = 0$, $g_y = 0.005$, $w = 0.8$, $\gamma = 1.4$, the Atwood number is $A = 0.4$. Fig.3 shows the density and temperature distributions along the line $x = 5$ at time $t = 200$, where $c = 1$, $\eta_0 = 3$, $dt = 10^{-3}$, all of the collision parameters are $10^3$. As one can see, the agreement is good, and grid I is enough to simulate the RT problem.

Figure 4 shows the evolution of the fluid interface at time $t = 0, 100, 200, 300, 400$. The bubble amplitude, spike amplitude, bubble growth rate, and spike growth rate can be seen in Fig.5, and represented by the black lines. When the amplitude of the perturbation is much smaller than the wave length, the perturbation of the fluid interface has an exponential growth. In the spike formation stage, the heavy and light fluids gradually penetrate into each other as time goes on, the light fluid rises to form a bubble and the heavy fluid falls to generate a spike. The interface becomes more acute and the growth rate is approximately linearly increased. Subsequently, the Kelvin-Helmholtz instability begins to develop and leads to the accumulation of heavy fluid at the top of the spike. The interface gradually becomes blunt, even eddy under certain conditions. The spike growth rate is reduced, and the
bubble growth rate reaches a constant velocity after a small attenuation. This is the nonlinear stage. Taylor derived an empirical formula for the constant velocity: \( v_b = C \sqrt{AgW/2} \), where \( C = 0.32 \). In the simulation, the fitting constant speed of bubble is 0.05329, thus \( C = 0.3768 \). The difference is due to the free condition at the top. In a test of solid wall condition at the top, the fitting constant velocity is 0.04622, and \( C = 0.3268 \). This agrees well with Taylor and Layzer’s results\(^\text{[52]}\). At a later time, the extrusion from two sides leads to the formation of the secondary spikes, and the growth rate increases again (reacceleration stage). The shapes of the fluid interface in the current study compare well with those in previous studies\(^\text{[53, 54]}\). The amplitude of spike is greater than that of the bubble, and the ratio is changing with time. After full development of the interface, the ratio is between \( 1.5 - 1.7 \), which is consistent with the numerical results of Youngs\(^\text{[55]}\).

Figure 6 shows the vertical distribution curve of heavy fluid \( m(y) \) at different times, which is defined as

\[
m(y) = \frac{\sum_{i=1}^{N_x} \rho(i, y)}{NX}.
\]

The occurrence and growth of the peak value of the heavy fluid vertical distribution at time \( t=150, 200 \), represent the accumulation of heavy fluid at the tip of the spike. Under the extrusion action from two sides, the interface along the two vortices is stretched, the peak value of heavy fluid vertical distribution decreases gradually, and the distribution tends to be approximate equilibrium.

FIG. 3: Grid convergence study: the density and temperature profiles at the line \( x = 5 \) at time \( t = 200 \).
FIG. 4: Evolution of the fluid interface from a single mode perturbation.

FIG. 5: Amplitude and growth rate with different viscosity or thermal conductivity.

The effects of viscosity and thermal conductivity on RT instability are also shown in Figure 5, (a) $\gamma = 1.4$, (b) $\gamma = 1.667$. The black curves correspond to simulation results of $s = 10^3$ ($Pr = 1$), the red curves correspond to simulation results of $s_v = 50$ (other collision parameters are $10^3$, $Pr = 20$), and the green curves correspond to $s_T = 10^2$ (other collision parameters are $10^3$, $Pr = 0.1$). Solid and dotted lines denote bubble and spike, respectively.
Before entering the reacceleration stage, the effects of viscosity and thermal conductivity on RT instability are negligible. At the reacceleration stage, both viscosity and thermal conductivity show significant inhibitory effect. In Figure 7, we can find the explanations. (a), (b) and (c) correspond to $Pr = 20$, $Pr = 1$ and $Pr = 0.1$ respectively. With the decrease of $s_v$ or $s_T$, the viscosity or thermal conductivity increases, the complicated secondary vortices generated by the Kelvin-Helmholtz instability are suppressed, and then the evolution of RT instability is suppressed. That is to say, the inhibition effect of viscosity and thermal conductivity on the RT instability is mainly achieved by inhibiting the development of KH instability in the RT instability.

C. Non-equilibrium characteristic of Rayleigh–Taylor instability

In the MRT model, the deviation from equilibrium can be defined as $\Delta_i = \hat{f}_i - \hat{f}_{ieq} = M_{ij}(f_j - f_{jeq})$. $\Delta_i$ contains the information of macroscopic flow velocity $u_\alpha$. Furthermore, we replace $v_i\alpha$ by $v_i\alpha - u_\alpha$ in the transformation matrix $M$, named $M^*$. $\Delta_i^* = M_{ij}^*(f_j - f_{j(eq)}^*)$ is only the manifestation of molecular thermal motion and does not contain the information of macroscopic flow. In order to make the meaning of $\Delta_i^*$ more clear, we introduce some symbols as $\Delta_{2xx}^* = \Delta_5^*$, $\Delta_{2xy}^* = \Delta_6^*$, $\Delta_{2yy}^* = \Delta_7^*$, $\Delta_{(3,1)x}^* = \Delta_8^*$, $\Delta_{(3,1)y}^* = \Delta_9^*$, $\Delta_{3xx}^* = \Delta_{10}^*$, $\Delta_{3xy}^* = \Delta_{11}^*$, $\Delta_{3yy}^* = \Delta_{12}^*$, $\Delta_{3xyy}^* = \Delta_{13}^*$, $\Delta_{(4,2)xx}^* = \Delta_{14}^*$, $\Delta_{(4,2)xy}^* = \Delta_{15}^*$, $\Delta_{(4,2)yy}^* = \Delta_{16}^*$. Here $\Delta_{2xx}^*$ and $\Delta_{2yy}^*$ describe the departures of the internal energies in the x and y degrees of freedom from their average, $\Delta_{2xy}^*$ is concerned with the shear effects, $\Delta_{3xxx}^*$, $\Delta_{3xyy}^*$ and $\Delta_{(3,1)x}^*$ are related to the internal energy flow caused by microscopic fluctuation in x direction, $\Delta_{3xx}^*$, $\Delta_{3yy}^*$ and $\Delta_{(3,1)y}^*$ are associated with the internal energy flow caused by microscopic
FIG. 7: Velocity vector plots of RT instability ($\gamma = 1.4$) in the part of $[0, 50] \times [41, 320]$ at time $t = 350$, (a) $s_v = 50$, (b) $s = 10^3$, (c) $s_T = 10^2$.

fluctuation in y direction. Compared with the macroscopic equations, $\Delta^{*}_{2\alpha\beta}$ and $\Delta^{*}_{(3,1)\alpha}$ correspond to the viscous stress tensor in the momentum equation and the heat flux term in energy equation, which are named as Non-Organized Momentum Flux (NOMF), Non-Organized Energy Flux (NOEF), respectively [56].

To provide a rough estimation of TNE, we follow the idea used in refs. [43], and define a non-dimensional “TNE strength” function

$$d(x, y) = \sqrt{\Delta^{*2}_{2\alpha\beta}/T^2 + \Delta^{*2}_{(3,1)\alpha}/T^3 + \Delta^{*2}_{3\alpha\beta\gamma}/T^3 + \Delta^{*2}_{(4,2)\alpha}/T^4}$$

where $d = 0$ in the thermodynamic equilibrium, and $d > 0$ in the thermodynamic nonequilibrium state. $D_{TNE} = \bar{d}$ is the global average TNE strength. Then we define $D_2 = \sqrt{\Delta^{*2}_{2\alpha\beta}}$ and $D_{(3,1)} = \sqrt{\Delta^{*2}_{(3,1)\alpha}}$, $D_2$ and $D_{(3,1)}$ are the global average NOMF strength and NOEF strength. Correspondently, a macroscopic non-uniformity function is defined

$$\delta W(x, y) = \sqrt{(W - \bar{W})^2}$$

where $W = (\rho, U, T)$ denotes the macroscopic distribution, $\bar{W}$ is the average value of a small cell around the point $(x, y)$. 

12
FIG. 8: Physical quantities and their gradients in the line $x = 10$ at time $t = 225$.

FIG. 9: Non-equilibrium characteristics in the line $x = 10$ at time $t = 225$ in four cases.
Here we first give some results of $\Delta^*_i$ in the evolution of RT instability. The initial physical quantities ($\rho, T, u_x, u_y$) are given the same values as those in Fig 4. Figure 8 shows the simulation results of physical quantities and their gradients in the line $x = 10$ at time $t = 225$. Fig. 9 shows the non-equilibrium characteristics of RT instability with different viscosity or heat conduction. The first line corresponds to $s = 10^3$ (case I), the second line corresponds to $s = 10^2$ (case II), the third line corresponds to $s_v = 10^2$ (case III), and the fourth line corresponds to $s_T = 10^2$ (other collision parameters are $10^3$, case IV). A vertical dashed line is plotted in each panel to guide the eye for the peak of spike. From Figs. 8 and 9, we can get the following information.

1) $\Delta^*_{2xx,2yy,(3,1)y,3xxy,3yy}$ in case II, $\Delta^*_{2xx,2yy}$ in case III, and $\Delta^*_{(3,1)y}$ in case IV are much larger than the values in case I. This is because that the relaxation time recovering to balance is inversely proportional to $s_i$. As $s_i$ decreases, the corresponding mode will take more time to restore equilibrium, and the deviation degree from the equilibrium increases. Physically, the viscosity and heat conductivity of the physical system in case II, the viscosity in case III, and the heat conductivity in case IV are larger than the values in case I, which increase the nonequilibrium behaviors of system.

2) $\Delta^*_{(3,1)x,(3,1)y,3xxx,3xxy,3xyy,3yy}$ in case III are similar to the values of case I, $\Delta^*_{2xx,2yy,3xxy,3yy}$ in case IV are smaller than the values of case I. It can be explained as follow. The relaxation parameters $s_i$ ($i = 8, 9, 10, 11, 12, 13$), density gradient and temperature gradient in case III are consistent with case I. The relaxation parameters $s_i$ ($i = 5, 7, 11, 13$) in case IV are the same as case I, but the larger heat conductivity leads to a decrease in density gradient and temperature gradient, which reduce the nonequilibrium effect. There is a competition between the viscosity, heat conduction and the gradient of physical quantities.

3) $\Delta^*_{2xy,(3,1)x,3xxx,3xyy}$ in case I and III are equal to zero, but the values in case II and IV are not equal to zero. The reason is that, there is neither shear effect nor energy flux in $x$ direction in case I and III ($u_x = 0$), so $\Delta^*_{2xy,(3,1)x,3xxx,3xyy} = 0$. In case II and IV, it’s the opposite.

Figure 10 shows the viscosity, heat conductivity and Prandtl number effects on the global average non-equilibrium characteristics, (a) $Pr = 0.5$, (b) $Pr = 1.0$, (c) $Pr = 2.0$. With the increase of viscosity and heat conduction, $D_{TNE}$, $D_2$, and $D_{(3,1)}$ will increase. The change of TNE strength is more significant when heat conduction changes. The growth of $D_2$ and $D_{(3,1)}$ depend on the viscosity and thermal conductivity, respectively. This further proves
the correspondence between $\Delta^*_{\alpha,\beta}$ and the viscosity term, and the correspondence between $\Delta^*_{(3,1)\alpha}$ and the heat conduction term in NS equation. When the spike arrives at the bottom of the calculation domain, or the RT instability develops into the turbulent mixing stage, the global average TNE strength and NOEF strength begin to decrease, and the global average NOMF strength growth is slowing. The inclined dashed lines roughly show the time that spikes reach the bottom boundary of the calculation domain. When the viscosity and heat conduction are relatively small, the spike develops relatively quickly and reaches the bottom earlier. This is consistent with the previous conclusion.

Figure 11 shows the snapshots of density non-uniformity $\delta\rho$ and TNE strength $d$ at time $t = 200$ and $t = 400$. $\delta\rho$ and $d$ demonstrate the HNE and TNE behaviours of the system, respectively. In the position far from the perturbation interface, $\delta\rho$ and $d$ are basically 0. Around the interface, particles with different density mix with each other, and the exchanges of kinetic energy and momentum are produced, $\delta\rho$ and $d$ are greater than zero. The characteristics of density non-uniformity $\delta\rho$ and TNE strength $d$ are quite consistent. HNE and TNE are ‘the two-sides of a coin’. In addition, both $\delta\rho$ and $d$ can be used to
FIG. 11: Snapshots of density non-uniformity $\delta \rho$ (a) and TNE strength $d$ (b) at time $t = 200$ and $t = 400$.

capture the interface.

Fig. 12 shows the correlation degrees between macroscopic non-uniformities and various global average nonequilibrium strength in the case of $s_v = 300$, $s_T = 150$. In the figure, considerably higher correlation degrees are founded between density non-uniformity and the global average TNE strength $D_{TNE}$, temperature non-uniformity and the global average NOEF strength $D_{(3,1)}$, which are approximate to 1. The correlation degree between the velocity non-uniformity and the global average NOMF strength $D_2$ is higher than that with other nonequilibrium strength.

In Fig. 13(a), we can find, the correlation degree between $\delta \rho$ and $D_{TNE}$ varies with the viscosity and heat conduction. Before the turbulent mixing stage, heat conduction plays a major role. The greater the heat conduction, the higher the degree of correlation. With the increase of heat conduction, the correlation degree gradually tends to 1. (Fig. 13b). The trend can be expressed by a exponential decay function (Fig. 13c),

$$C_{\delta \rho - D_{TNE}} = 1 - 0.102 \exp(-H_{2b} \times 10^5 / 5.22), H_{2b} = \sqrt{gk/s_T},$$  

where $H_{2b}$ is a relative thermal conductivity, $k$ is wave number. In the turbulent mixing stage, the effect of viscosity is reflected. When the heat conduction is constant, the higher the viscosity is, the higher the degree of correlation. When the correlation degree between the function A and B is equal to 1, there is a linear relationship between A and B, that is
FIG. 12: Correlation degrees between the macroscopic non-uniformities and various global average nonequilibrium strength. $\delta \rho$, $\delta T$ and $\delta U$ are density non-uniformity, temperature non-uniformity and velocity non-uniformity, respectively.

$B = \alpha A + \beta$. Fig. 13d shows the linear relationship between $\delta \rho$ and $D_{TNE}$. The solid lines are the fitted curves. As can be seen in the figure, the slope $\alpha$ of the linear relationship is determined by the heat conduction, $\alpha_1 = 0.025 + 240 \times H_{2b}$.

In Fig. 14, we can find, the correlation degree between $\delta T$ and global average NOEF strength $D_{(3,1)}$ also varies with the viscosity and heat conduction. Before the time $t = 200$, heat conduction plays a major role. The greater the heat conduction, the higher the degree of correlation. The effect of viscosity in the nonlinear stage is more obvious than that in the linear stage. A linear relationship between $\delta T$ and $D_{(3,1)}$ is also found, and the slope is also determined by the heat conduction, $\alpha_2 = -0.001 + 452 \times H_{2b}$.

IV. CONCLUSIONS

With a MRT discrete Boltzmann model, two-dimensional Rayleigh-Taylor instability with different viscosity, thermal conductivity and Prandtl number are simulated. Both viscosity and heat conduction show significant inhibitory effect on RT instability, and the inhibition effect is mainly achieved by inhibiting the development of Kelvin-Helmholtz instability in the reacceleration stage. Before this, the Prandtl number effect is not sensitive. The nonequilibrium characteristics of system are mainly probed. With the increase of viscosity or heat conduction, different non-equilibrium components increase. There is a competition
FIG. 13: The correlation degree between $\delta \rho$ and $D_{TNE}$, (a) the effects of viscosity and heat conduction, (b) (c) the variation of correlation degree with heat conduction, (d) the linear relationship between $\delta \rho$ and $D_{TNE}$, (e) the slope $\alpha$ of the linear relationship.

FIG. 14: The correlation degree between $\delta T$ and $D_{(3,1)}$, (a) the effect of viscosity and heat conduction, (b) the linear relationship between $\delta T$ and $D_{(3,1)}$, (c) the slope $\alpha$ of the linear relationship.
between the viscosity, the heat conduction and the gradient of physical quantities. When the RT instability develops into the turbulent mixing stage, the global average TNE strength and NOEF strength have a decrease. Correlation degrees between macroscopic non-uniformities and various global average nonequilibrium strength are analyzed. The correlation degrees between density non-uniformity and the global average TNE strength, temperature non-uniformity and the global average NOEF strength, are approximate to 1. Heat conduction shows a major role on the correlation degree.

Acknowledgements

FC acknowledges support of National Natural Science Foundation of China [under Grant Nos. 11402138]. AX and GZ acknowledge support of Foundation of LCP and National Natural Science Foundation of China (under Grant No. 11475028).

Appendix A: CE expansion for the MRT DBM with gravity

Using the Chapman-Enscog expansion on the two sides of discrete Boltzmann equation, the Navier–Stokes equations with gravity term can be derived.

We define

\[ \frac{\partial f_i}{\partial t} + v_{i\alpha} \frac{\partial f_i}{\partial x_\alpha} = -S_{il} (f_i - f_{eq}^l) - f_i \], \quad (A1a)\]

\[ f_i = f_i^{(0)} + f_i^{(1)} + f_i^{(2)}, \quad (A1b) \]

\[ \frac{\partial}{\partial t} = \frac{\partial}{\partial t_1} + \frac{\partial}{\partial t_2}, \quad (A1c) \]

\[ \frac{\partial}{\partial x} = \frac{\partial}{\partial x_1}, \quad (A1d) \]

where \( f_i^F = g_a \left( \frac{v_{i\alpha} - u_{i\alpha}}{RT} \right) f_{i\alpha} \), non-equilibrium parts \( f_i^{(l)} = O(\epsilon^l) \), and the partial derivatives \( \partial / \partial t_l = O(\epsilon^l) \), \( \partial / \partial x_l = O(\epsilon^l) \), \( (l = 1, 2, \ldots) \). Equating the coefficients of the zeroth, the first, and the second order terms in \( \epsilon \) gives

\[ f_i^{(0)} = f_{eq}^l, \quad (A2a) \]

\[ \left( \frac{\partial}{\partial t_1} + v_{i\alpha} \frac{\partial}{\partial x_{1\alpha}} \right) f_i^{(0)} = -S_{il} f_i^{(1)} - f_i^F, \quad (A2b) \]

\[ \frac{\partial}{\partial t_2} f_i^{(0)} + \left( \frac{\partial}{\partial t_1} + v_{i\alpha} \frac{\partial}{\partial x_{1\alpha}} \right) f_i^{(1)} = -S_{il} f_i^{(2)}. \quad (A2c) \]
They can be converted into moment space to obtain:

\[ \hat{f}^{(0)} = \hat{f}^{eq}, \]  

\[ (\frac{\partial}{\partial t_1} + \hat{E}_a \frac{\partial}{\partial x_{1a}})\hat{f}^{(0)} = -\hat{S}\hat{f}^{(1)} - \hat{F}, \]  

\[ \frac{\partial}{\partial t_2} \hat{f}^{(0)} + (\frac{\partial}{\partial t_1} + \hat{E}_a \frac{\partial}{\partial x_{1a}})\hat{f}^{(1)} = -\hat{S}\hat{f}^{(2)}, \]

where \( \hat{E}_a = M(v_{ia}I)M^{-1}. \)

From Eq.\( (A3b) \) we obtain

\[ \frac{\partial}{\partial t_1} \hat{f}^{eq}_1 + \frac{\partial}{\partial x_1} \hat{f}^{eq}_2 + \frac{\partial}{\partial y_1} \hat{f}^{eq}_3 = -\hat{F}_1, \]  

\[ \frac{\partial}{\partial t_1} \hat{f}^{eq}_2 + \frac{\partial}{\partial x_1} \hat{f}^{eq}_5 + \frac{\partial}{\partial y_1} \hat{f}^{eq}_6 = -\hat{F}_2, \]  

\[ \frac{\partial}{\partial t_1} \hat{f}^{eq}_3 + \frac{\partial}{\partial x_1} \hat{f}^{eq}_6 + \frac{\partial}{\partial y_1} \hat{f}^{eq}_7 = -\hat{F}_3, \]  

\[ \frac{\partial}{\partial t_1} \hat{f}^{eq}_4 + \frac{\partial}{\partial x_1} \hat{f}^{eq}_8 + \frac{\partial}{\partial y_1} \hat{f}^{eq}_9 = -\hat{F}_4, \]  

\[ \frac{\partial}{\partial t_1} \hat{f}^{eq}_5 + \frac{\partial}{\partial x_1} \hat{f}^{eq}_{10} + \frac{\partial}{\partial y_1} \hat{f}^{eq}_{11} = -s_5\hat{f}_5^{(1)} - \hat{F}_5, \]  

\[ \frac{\partial}{\partial t_1} \hat{f}^{eq}_6 + \frac{\partial}{\partial x_1} \hat{f}^{eq}_{11} + \frac{\partial}{\partial y_1} \hat{f}^{eq}_{12} = -s_6\hat{f}_6^{(1)} - \hat{F}_6, \]  

\[ \frac{\partial}{\partial t_1} \hat{f}^{eq}_7 + \frac{\partial}{\partial x_1} \hat{f}^{eq}_{12} + \frac{\partial}{\partial y_1} \hat{f}^{eq}_{13} = -s_7\hat{f}_7^{(1)} - \hat{F}_7, \]  

\[ \frac{\partial}{\partial t_1} \hat{f}^{eq}_8 + \frac{\partial}{\partial x_1} \hat{f}^{eq}_{14} + \frac{\partial}{\partial y_1} \hat{f}^{eq}_{15} = -s_8\hat{f}_8^{(1)} - \hat{F}_8, \]  

\[ \frac{\partial}{\partial t_1} \hat{f}^{eq}_9 + \frac{\partial}{\partial x_1} \hat{f}^{eq}_{15} + \frac{\partial}{\partial y_1} \hat{f}^{eq}_{16} = -s_9\hat{f}_9^{(1)} - \hat{F}_9. \]

From Eq.\( (A3c) \) we obtain

\[ \frac{\partial}{\partial t_2} \hat{f}^{eq}_1 = 0, \]  

\[ \frac{\partial}{\partial t_2} \hat{f}^{eq}_2 + \frac{\partial}{\partial x_1} \hat{f}^{(1)}_5 + \frac{\partial}{\partial y_1} \hat{f}^{(1)}_6 = 0, \]  

\[ \frac{\partial}{\partial t_2} \hat{f}^{eq}_3 + \frac{\partial}{\partial x_1} \hat{f}^{(1)}_6 + \frac{\partial}{\partial y_1} \hat{f}^{(1)}_7 = 0, \]  

\[ \frac{\partial}{\partial t_2} \hat{f}^{eq}_4 + \frac{\partial}{\partial x_1} \hat{f}^{(1)}_8 + \frac{\partial}{\partial y_1} \hat{f}^{(1)}_9 = 0. \]
Adding Eqs. (A4a)-(A4d) and (A5a)-(A5d) leads to the following equations,

$$\frac{\partial}{\partial t} \hat{f}^{eq}_1 + \frac{\partial}{\partial x} \hat{f}^{eq}_2 + \frac{\partial}{\partial y} \hat{f}^{eq}_3 = -\hat{f}^F,$$

(A6a)

$$\frac{\partial}{\partial t} \hat{f}^{eq}_2 + \frac{\partial}{\partial x} \hat{f}^{eq}_5 + \frac{\partial}{\partial y} \hat{f}^{eq}_6 = -\hat{f}^F - \frac{\partial}{\partial x} \hat{f}^{(1)}_5 - \frac{\partial}{\partial y} \hat{f}^{(1)}_6,$$

(A6b)

$$\frac{\partial}{\partial t} \hat{f}^{eq}_3 + \frac{\partial}{\partial x} \hat{f}^{eq}_6 + \frac{\partial}{\partial y} \hat{f}^{eq}_7 = -\hat{f}^F - \frac{\partial}{\partial x} \hat{f}^{(1)}_6 - \frac{\partial}{\partial y} \hat{f}^{(1)}_7,$$

(A6c)

$$\frac{\partial}{\partial t} \hat{f}^{eq}_4 + \frac{\partial}{\partial x} \hat{f}^{eq}_8 + \frac{\partial}{\partial y} \hat{f}^{eq}_9 = -\hat{f}^F - \frac{\partial}{\partial x} \hat{f}^{(1)}_8 - \frac{\partial}{\partial y} \hat{f}^{(1)}_9.$$

(A6d)

It is easily shown that function $f^F_i$ satisfies the similar moments.

$$\int \int f^F dv d\eta = 0 = \sum f^F_i,$$

(A7a)

$$\int \int f^F v_\alpha dv d\eta = \rho g_\alpha = \sum f^F_i v_{i\alpha},$$

(A7b)

$$\int \int f^F (v^2 + \eta^2) dv d\eta = \rho g_{\alpha u_\alpha} = \sum f^F_i (v^2 + \eta^2)/2,$$

(A7c)

$$\int \int f^F v_{i\alpha} v_{j\beta} dv d\eta = \rho g_{\alpha \beta} + \rho g_{\beta \alpha} = \sum f^F_i v_{i\alpha} v_{j\beta},$$

(A7d)

$$\int \int f^F (v^2 + \eta^2) v_\alpha dv d\eta = \rho [g_{\alpha u_\alpha} u_{\beta} + (b + 2RT + u^2/2)g_\alpha]$$

$$= \sum f^F_i (v^2 + \eta^2)/2 v_{i\alpha}.$$

(A7e)

Eqs. (A7a)-(A7e) can be written in a matrix form, i.e., $\hat{f}^F = Mt^F$, where $\hat{f}^F_1 = 0, \hat{f}^F_2 = \rho g_x, \hat{f}^F_3 = \rho g_y, \hat{f}^F_4 = \rho (g_x u_x + g_y u_y), \hat{f}^F_5 = 2\rho g_x u_x, \hat{f}^F_6 = \rho (g_x u_y + g_y u_x), \hat{f}^F_7 = 2\rho g_y u_y, \hat{f}^F_8 = \rho [g_x u_x^2 + g_y u_x u_y + g_x (b + 2RT + u^2/2)], \hat{f}^F_9 = \rho [g_y u_y^2 + g_x u_x u_y + g_y (b + 2RT + u^2/2)],$ and the others $(i = 10, \ldots, 16)$ are 0.

Using the definitions of $\hat{f}^{eq}_i$ and $\hat{f}^F_i$, we can obtain:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_\alpha)}{\partial x_\alpha} = 0,$$

(A8a)

$$\frac{\partial (\rho u_\alpha)}{\partial t} + \frac{\partial (\rho u_\alpha u_\beta)}{\partial x_\beta} + \frac{\partial P}{\partial x_\alpha} = \frac{\partial}{\partial x_\alpha} [\mu (\frac{\partial u_\alpha}{\partial x_\alpha} + \frac{\partial u_\beta}{\partial x_\beta} - \frac{2}{b} \frac{\partial u_\chi}{\partial x_\chi} \delta_{\alpha \beta})] - \rho g_\alpha,$$

(A8b)

$$\frac{\partial e}{\partial t} + \frac{\partial}{\partial x_\alpha} [(e + P)u_\alpha] = \frac{\partial}{\partial x_\beta} [(\frac{b}{2} + 1) \chi' R \frac{\partial T}{\partial x_\beta} + \chi (\frac{\partial u_\alpha}{\partial x_\alpha} + \frac{\partial u_\beta}{\partial x_\beta} - \frac{2}{b} \frac{\partial u_\chi}{\partial x_\chi} \delta_{\alpha \beta})u_\alpha] - \rho g_\alpha u_\alpha,$$

(A8c)

(\alpha, \beta, \chi = x, y).
where $\mu = \rho RT/s_v$, $(s_v = s_5 = s_6 = s_7)$, $\lambda' = \rho RT/s_T$, $(s_T = s_8 = s_9)$.

By modifying the collision operators of the moments related to energy flux:

\[ \hat{S}_{88}(\hat{f}_8 - \hat{f}_8^{eq}) \Rightarrow \hat{S}_{88}(\hat{f}_8 - \hat{f}_8^{eq}) + (s_T/s_v - 1)\rho T u_x \]
\[ \times \left( \frac{2}{b} \frac{\partial u_x}{\partial x} - 2 \frac{\partial u_y}{\partial x} - \frac{2}{b} \frac{\partial u_y}{\partial y} \right) + \frac{1}{s_T/s_v - 1} \rho T \left( \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \right), \]  

(A9a)

\[ \hat{S}_{99}(\hat{f}_9 - \hat{f}_9^{eq}) \Rightarrow \hat{S}_{99}(\hat{f}_9 - \hat{f}_9^{eq}) + (s_T/s_v - 1)\rho T u_x \]
\[ \times \left( \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} \right) + \frac{1}{s_T/s_v - 1} \rho T \left( 2 \frac{\partial u_y}{\partial y} - \frac{2}{b} \frac{\partial u_x}{\partial x} - \frac{2}{b} \frac{\partial u_y}{\partial y} \right), \]  

(A9b)

we get the following energy equation:

\[ \frac{\partial e}{\partial t} + \frac{\partial}{\partial x_\alpha} [(e + P)u_\alpha] = \frac{\partial}{\partial x_\beta} \left[ \lambda \frac{\partial T}{\partial x_\beta} + \mu \left( \frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha} - \frac{2}{b} \frac{\partial u_x}{\partial x_\alpha} \delta_{\alpha\beta} \right) u_\alpha \right] - \rho g_\alpha u_\alpha, \]  

(A10)

where $\lambda = (b^2 + 1)RX'$.

[1] L. Rayleigh, Investigation of the character of the equilibrium of an incompressible heavy fluid of variable density, Proc. London Math. Soc., 1882, s1-14(1): 170

[2] G. Taylor, The Instability of Liquid Surfaces when Accelerated in a Direction Perpendicular to their Planes. I, P. Roy. Soc. A, 1950, 201(1065): 192

[3] W.H. Ye, W.Y. Zhang, G.N. Chen, et al., Numerical simulations of the FCT method on Rayleigh-Taylor and Richtmyer-Meshkov instabilities, Chin. J. Comput. Phys., 1998, 15(3):277

[4] X.L. Li, B.X. Jin, J. Glimm, Numerical study for the three dimensional Rayleigh-Taylor instability through the TVD/AC scheme and parallel computation, J. Comp. Phys., 1996, 126: 343

[5] G. Tryggvason, B. Bunner, A. Esmaeeli, et al., A front-tracking method for the computations of multiphase flow, J. Comput. Phys., 2001, 169(2): 708

[6] Y.K. Li, A. Umemura, Mechanism of the large surface deformation caused by Rayleigh-Taylor instability at large Atwood number, Journal of Applied Mathematics and Physics, 2014, 2(10): 971

[7] W.H. Tang, Y.M. Mao, SPH Simulation of Rayleigh-Taylor Instability, J. Univ. Sci. Technol of China, 2004, 26(1): 21

22
[8] L. Duchemin, C. Josserand, and P. Clavin, Asymptotic behavior of the Rayleigh-Taylor instability, Phys. Rev. Lett, 2005, 94(22): 224501
[9] A.W. Cook, and P.E. Dimotakis, Transition stages of Rayleigh-Taylor instability between miscible fluids, J. Fluid Mech., 2001, 443: 69
[10] A. Celani, A. Mazzino, and L. Vozella, Rayleigh-Taylor turbulence in two dimensions, Phys. Rev. L, 2006, 96(13): 134504
[11] W. Cabot, Comparison of two- and three-dimensional simulations of miscible Rayleigh-Taylor instability, Phys. Fluids, 2006, 18(4): 045101
[12] A. Celani, A. Mazzino, P. Muratore-Ginanneschi, and L. Vozella, Phase-field model for the Rayleigh-Taylor instability of immiscible fluids, J. Fluid Mech., 2009, 622: 115
[13] R. Betti, J. Sanz, Bubble acceleration in the ablative Rayleigh-Taylor instability, Phys. Rev. Lett, 2006, 97(20): 205002
[14] M.R. Gupta, L. Mandal, S. Roy and M. Khan, Effect of magnetic field on temporal development of Rayleigh-Taylor instability induced interfacial nonlinear structure, Phys.Plasmas, 2010, 17(1): 012306
[15] P.K. Sharma, R.P. Prajapati and R.K. Chhajlani, Effect of Surface Tension and Rotation on Rayleigh-Taylor Instability of Two Superposed Fluids with Suspended Particles, Acta. Phys. Pol. A, 2010, 118(4): 576
[16] R. Banerjee,L.K. Mandal,S Roy,M Khan and M R Gupta, Combined effect of viscosity and vorticity on single mode Rayleigh-Taylor instability bubble growth, Phys.Plasmas, 2011, 18(2): 022109
[17] H. Liu, W. Kang, Q. Zhang, Y. Zhang, H. Duan, and X.T. He, Molecular Dynamics Simulations of Microscopic Structure of Ultra Strong Shock Waves in Dense Helium, Front. Phys. 2016, in press
[18] S. Succi, The Lattice Boltzmann Equation for Fluid Dynamics and Beyond, Oxford: Oxford University Press, 2001
[19] R. Benzi, S. Succi, and M. Vergassola, The lattice Boltzmann equation: Theory and applications, Phys. Rep., 1992, 222(3): 145
[20] A. Xu, G. Gonnella, and A. Lamura, Phase-separating binary fluids under oscillatory shear, Phys. Rev. E, 2003, 67(5): 056105
[21] A. G. Xu, G. Gonnella, and A. Lamura, Morphologies and flow patterns in quenching of
lamellar systems with shear, Phys. Rev. E, 2006, 74(1): 011505

[22] A. G. Xu, G. Gonnella, and A. Lamura, Simulations of complex fluids by mixed lattice Boltzmann-finite difference methods, Physica A, 2006, 362(1): 42

[23] X. Shan, H. Chen, Lattice Boltzmann model for simulating flows with multiple phases and components, Phys. Rev. E, 1993, 47(3): 1815

[24] X. Shan, H. Chen, Simulation of nonideal gases and liquid-gas phase transitions by the lattice Boltzmann equation, Phys. Rev. E, 1994, 49(4): 2941

[25] G. Gonnella, E. Orlandini, and J. M. Yeomans, Spinodal decomposition to a lamellar phase: Effects of hydrodynamic flow, Phys. Rev. Lett., 1997, 78(9): 1695

[26] H. Fang, Z. Wang, Z. Lin, and M. Liu, Lattice Boltzmann method for simulating the viscous flow in large distensible blood vessels, Phys. Rev. E, 2002, 65(5): 051925

[27] Z. Guo and C. Shu, Lattice Boltzmann Method and Its Applications in Engineering (advances in computational fluid dynamics), World Scientific Publishing Company, 2013

[28] A. Xu, G. Zhang, Y. Li, and H. Li, Modeling and Simulation of Nonequilibrium and Multiphase Complex Systems-Lattice Boltzmann kinetic Theory and Application, Prog. Phys., 2014, 34(3): 136

[29] R. Zhang, Y. Xu, B. Wen, N. Sheng, and H. Fang, Enhanced Permeation of a Hydrophobic Fluid through Particles with Hydrophobic and Hydrophilic Patterned Surfaces, Sci. Rep., 2014, 4: 5738

[30] X.B. Nie, Y.H. Qian, G.D. Doolen, and S.Y. Chen, Lattice Boltzmann simulation of the two-dimensional Rayleigh-Taylor instability, Phys. Rev. E, 1998, 58(5): 6861

[31] X.Y. He, S.Y. Chen, and R.Y. Zhang, A Lattice Boltzmann Scheme for Incompressible Multiphase Flow and Its Application in Simulation of Rayleigh-Taylor Instability, J. Comput. Phys., 1999, 152(2): 642

[32] X.Y. He, R.Y. Zhang, S.Y. Chen, and G.D. Doolen, On the three-dimensional Rayleigh-Taylor instability, Phys. Fluids, 1999, 11(5): 1143

[33] R.Y. Zhang, X.Y. He, and S.Y. Chen, Interface and surface tension in incompressible lattice Boltzmann multiphase model, Comput. Phys. Commun., 2000, 129(1-3): 121

[34] Q. Li, K.H. Luo, Y.J. Gao, and Y.L. He, Additional interfacial force in lattice Boltzmann models for incompressible multiphase flows, Phys. Rev. E, 2012, 85(2): 026704

[35] G.J. Liu, and Z.L. Guo, Effects of Prandtl number on mixing process in miscible Rayleigh-
Taylor instability: A lattice Boltzmann study, Int. J. Numer. Method. H., 2013, 23(1): 176

[36] H. Liang, B.C. Shi, Z.L. Guo, and Z.H. Chai, Phase-field-based multiple-relaxation-time lattice Boltzmann model for incompressible multiphase flows, Phys. Rev. E, 2014, 89(5): 053320

[37] M. Sbragaglia, R. Benzi, L. Biferale, H. Chen, X. Shan, and S. Succi, Lattice Boltzmann method with self-consistent thermo-hydrodynamic equilibria, J. Fluid Mech., 2009, 628: 299

[38] A. Scagliarini, L. Biferale, M. Sbragaglia, K. Sugiyama, and F. Toschi, Lattice Boltzmann methods for thermal flows: Continuum limit and applications to compressible Rayleigh-Taylor systems, Phys. Fluids, 2010, 22(5): 055101

[39] L. Biferale, F. Mantovani, M. Sbragaglia, A. Scagliarini, F. Toschi, and R. Tripiccione, Reactive Rayleigh-Taylor systems: Front propagation and non-stationarity, Europhys. Lett. 94(5): 54004

[40] A. Xu, G. Zhang, Y. Gan, F. Chen, X. Yu, Lattice Boltzmann modeling and simulation of compressible flows, Front. Phys., 2012, 7(5): 582

[41] B. Yan, A. Xu, G. Zhang, Y. Ying, H. Li, Lattice Boltzmann model for combustion and detonation, Front. Phys., 2013, 8(1): 94

[42] C. Lin, A. Xu, G. Zhang, Y. Li, Polar Coordinate Lattice Boltzmann Kinetic Modeling of Detonation Phenomena, Commun. Theor. Phys., 2014, 62(5): 737

[43] A. Xu, C. Lin, G. Zhang, Y. Li, Multiple-relaxation-time lattice Boltzmann kinetic model for combustion, Phys. Rev. E, 2015, 91(4): 043306

[44] A. Xu, G. Zhang, Y. Ying, Progress of discrete Boltzmann modeling and simulation of combustion system, Acta Phys. Sin., 2015, 64(18): 184701

[45] C. Lin, A. Xu, G. Zhang, Y. Li, Double-distribution-function discrete Boltzmann model for combustion, Combustion and Flame, 2016, 164: 137

[46] Y. Zhang, A. Xu, G. Zhang, C. Zhu, C. Lin, Kinetic modeling of detonation and effects of negative temperature coefficient, Combustion and Flame (in press, 2016), DOI:10.1016/j.combustflame.2016.04.003.

[47] Y. Gan, A. Xu, G. Zhang, S. Succi, Discrete Boltzmann modeling of multiphase flows: hydrodynamic and thermodynamic non-equilibrium effects, Soft Matter, 2015, 11(26): 5336

[48] C. Lin, A. Xu, G. Zhang, Y. Li, S. Succi, Polar-coordinate lattice Boltzmann modeling of compressible flows, Phys. Rev. E, 2014, 89(1): 013307

[49] F. Chen, A. Xu, G. Zhang, Y. Wang, Two-dimensional MRT LB model for compressible and
incompressible flows, Front Phys., 2014, 9(2): 246

[50] H. Lai, A. Xu, G. Zhang, Y. Gan, Y. Ying, S. Succi, Thermo-hydrodynamic non-equilibrium effects on compressible Rayleigh-Taylor instability, 2015, arXiv:1507.01107

[51] H. Liu, W. Kang, Q. Zhang, Y. Zhang, H. Duan, X. T. He, Molecular dynamics simulations of microscopic structure of ultra strong shock waves in dense helium, Front. Phys., 2016, 11(6): 115206

[52] D. Layzer, On the Instability of Superposed Fluids in a Gravitational Field, Astrophysical Journal, 1955, 122: 1

[53] X. Y. He, S. Y. Chen, R. Y. Zhang, A Lattice Boltzmann Scheme for Incompressible Multiphase Flow and Its Application in Simulation of Rayleigh-Taylor Instability, J. Comput. Phys., 1999, 152(2): 642

[54] S. F. Li, W. H. Ye, Y. Zhang, S. Shu, A. G. Xiao, High order FD-WENO schemes for Rayleigh-Taylor instability problems, Chinese J. Comput. Phys., 2008, 25(4): 379

[55] D. Youngs, Numerical simulation of turbulent mixing by Rayleigh-Taylor instability, Phys. D, 1984, 12(1-3): 32

[56] Y. D. Zhang, Modeling and research of detonation based on discrete Boltzmann method, A Dissertation Submitted for the Degree of Master, Beihang University, 2015