Propagation of SH-waves in piezoelectric structures with functionally graded coating from different materials

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Abstract. The paper proposes an approach to model piezoelectric structures with a coating of various piezoelectric materials. The coating is a layer of functionally graded piezoelectric material, the properties of which continuously change from the parameters of one material to another. In contrast to the layered model with gaps in the change of properties at the interfaces of the materials, the proposed model implements the adjustment of the thickness of the coating of the transition zone of one material to another. The choice of functional dependences of the change in properties was determined by the ratio of the moduli of the materials that make up the coating, the localization of inclusions and the size of the area of mutual penetration of materials. The studies were carried out for piezoelectric structures made of ferroelectric materials based on PZT. In this paper, using the example of a model of coating by two and three piezoelectric materials, we studied the influence of the nature and localization of the inhomogeneity of the coating on the features of the propagation of surface shear horizontally polarized waves.

1. Introduction

The fundamentals of creating acoustoelectronic devices using inhomogeneous ferroelectrics were laid in the 1960s – 1970s [1–4]. The area of their use is largely determined by the characteristics of the propagation and localization of surface acoustic waves (SAW) in inhomogeneous structures. This makes its necessary the modeling of such structures using functionally graded components [5–8], layered materials with various kinds of inclusions and coatings [9–11], structures made of artificial bi- and multi-materials. The complexity of modeling of functionally graded materials (FGM) is based on the impossibility of obtaining a rigorous analytical solution, which leads to the need to simplify the model. In [5, 6, 8–10], the assumption is used that the properties of a material vary according to one law and one spatial variable. This allows to get an analytical solution. When modeling FGM with an arbitrary functional dependence, division into layered elements with a linear or quadratic change of properties, the technique of expansion in power series, approximation by differentiable functions and polynomials is used. When modeling bi-and multi-materials, as a rule, the numerical methods are used. A generalized model of a prestressed ferroelectric structure consisting of a homogeneous half-space with an inhomogeneous coating was proposed in [12–14]. The coating is modeled either by a layer of functionally graded piezoelectric materials (FGPM), or by a package of both homogeneous and inhomogeneous layers [13]. In [14], the effect of the nature, intensity, and localization region of an inhomogeneity on the propagation of SAW was studied. In [15, 16], structures from FGPM are considered, which properties change in a piecewise continuous manner. The influence of the properties of the inner layer and the orientation of the polarization vectors of the coating and the substrate on the features of the distribution of SAW and the coefficient of electromechanical coupling...
are shown. In contrast to [14], in the present work, it is assumed that the physical parameters of the coating of FGPM continuously vary in thickness from the values of one material to the values of the inclusion material. The choice of functional dependences and the intensity of changes in properties is determined by the ratio of the moduli of the materials that make up the coating, the localization of inclusions and the size of the area of mutual penetration of materials. The models of coatings consisting of two and three different piezoelectric materials are considered. The effects of the nature and localization of coating inhomogeneity on the characteristics of the propagation of SAW in piezoelectric structures are investigated.

2. Formulation of the problem

The boundary problem on the oscillations of the composite electroelastic medium is described by equations [12–15]:

\[
\nabla \cdot \boldsymbol{\Theta}^{(n)} = \rho^{(n)} \dot{\mathbf{u}}^{(n)}, \quad \nabla \cdot \mathbf{\Lambda}^{(n)} = 0
\]

for \( x_1 \leq H \) : 

\[
\Delta \phi^{(0)} = 0
\]

for vacuum \( x_2 > H \)

with the boundary conditions on the surface:

– the absence of mechanical actions: \( n \cdot \boldsymbol{\Theta}^{(1)} \bigg|_{x_2=H} = 0 \)

– electrically free surface

\[
\nabla \cdot \mathbf{\Lambda}^{(1)} \bigg|_{x_2=H} = n \cdot \mathbf{\Lambda}^{(0)} \bigg|_{x_2=H}, \quad \phi^{(1)} \bigg|_{x_2=H} = \phi^{(0)} \bigg|_{x_2=H}
\]

– metalized surface:

\[
\phi^{(1)} \bigg|_{x_2=H} = 0
\]

at the interface boundary:

\[
\mathbf{u}^{(1)} \bigg|_{x_2=0} = \mathbf{u}^{(2)} \bigg|_{x_2=0}, \quad n \cdot \boldsymbol{\Theta}^{(1)} \bigg|_{x_2=0} = n \cdot \boldsymbol{\Theta}^{(2)} \bigg|_{x_2=0}, \quad n \cdot \mathbf{\Lambda}^{(1)} \bigg|_{x_2=0} = n \cdot \mathbf{\Lambda}^{(2)} \bigg|_{x_2=0}
\]

at infinity:

\[
\mathbf{u}^{(2)} \bigg|_{x_2 \to \infty} \to 0, \quad \mathbf{u}^{(6)} \bigg|_{x_2 \to \infty} \to 0
\]

here \( \nabla \) is Hamilton operator, \( \mathbf{u}^{(n)} = \{ \mathbf{q}^{(n)}, \mathbf{u}^{(n)} = \phi^{(n)} \} \) is the augmented vector of displacements, \( \phi^{(n)} \) is the function of the electrical potential, \( n \) is the vector of the external normal to the medium surface. \( \rho^{(n)} \) is the material density of the \( n \) component of structure, \( \Delta = \partial^2 / \partial x_1^2 + \partial^2 / \partial x_2^2 \) is Laplace operator. Stress tensor \( \boldsymbol{\Theta}^{(n)} \) and induction vector \( \mathbf{\Lambda}^{(n)} \) are represented by expressions:

\[
\phi_{\mathbf{g}}^{(n)} = c_{\mathbf{g},\mathbf{p}}^{(n)} + e_{\mathbf{g},\mathbf{p}}^{(n)} \phi^{(n)} + D_{\mathbf{g}}^{(n)} = e_{\mathbf{g},\mathbf{p}}^{(n)} \mathbf{u}_{\mathbf{p},\mathbf{p}}^{(n)} - e_{\mathbf{g},\mathbf{p}}^{(n)} \phi^{(n)}
\]

\( c_{\mathbf{g},\mathbf{p}}^{(n)}, e_{\mathbf{g},\mathbf{p}}^{(n)}, e_{\mathbf{g},\mathbf{p}}^{(n)} \) – the components of the tensors of elastic constants, piezoelectric modules and the dielectric constants.

In the study of the problem of the propagation of surface shear horizontally polarized waves, we assume that the piezoelectric structure occupies the region: \( x_2 \leq H, \quad |x_1|, |x_1| \leq \infty \), piezoelectric half space - \( x_2 \leq 0 \), inhomogeneous piezoelectric coating - \( 0 < x_2 \leq H \), the direction of wave propagation - along the axis \( x_1 \), the change in the properties of the coating is determined by the expressions [12–16]:

\[
\rho^{(1)} = \rho_0 f_p^{(1)}(x_2), \quad \epsilon_0^{(1)} = \epsilon_0^{(0)} f_e^{(1)}(x_2), \quad \epsilon_0^{(1)} = \epsilon_0^{(0)} f_e^{(1)}(x_2), \quad \epsilon_0^{(1)} = \epsilon_0^{(0)} f_e^{(1)}(x_2)
\]

\( \rho_0 = \rho^{(2)}, \epsilon_0^{(2)} = \epsilon_0^{(2)}, \epsilon_0^{(2)} = \epsilon_0^{(2)}, \epsilon_0^{(2)} = \epsilon_0^{(2)} \) are density and elastic, piezoelectric and dielectric moduli of the «base» material, respectively. It is assumed that the structure is made on the basis of the 6mm class piezoelectric, the symmetry axis of which is directed along axis \( x_1 \), the polarization vectors of the
half-space and the cover have the same direction. Harmonic oscillations of the medium are induced by the action of a distant source, the mode of oscillations is steady-state, the dynamic process satisfies conditions:

$$u_i^{(n)} = u_2^{(n)} = 0, \ u_3^{(n)}(x_1, x_2), u_3^{(0)} = 0, k = 3,4, n = 0,1,2$$

(10)

Further, the formulation of boundary value problems, the solution and the results of the research are given in dimensionless parameters [12-16]:

$$l = l/h, \ \rho = \rho^{(n)} / \rho^{(2)}, \ c_{ij}^{(n)} = c_{ij}^{(n)} / c_{44}^{(2)},$$

$$e_{ij}^{(n)} = e_{ij}^{(n)} \xi / c_{44}^{(2)}, \ e_{ij}^{(0)} \xi^2 / c_{44}^{(2)}, \ \xi = 10^{10} \left( \text{V} \cdot \text{m}^{-1} \right), \ e^{(0)} = \text{vacuum permittivity.}$$

In this paper we use dimensionless frequencies

$$\nu_0 = \frac{2}{V_s^{(2)}, V_s^{(2)} - \text{velocity of bulk shear waves without and with regard to piezoelectric properties.}$$

Next, we omit the strokes.

In the framework of this paper, the following problems are considered:

**problem I** is the problem with a free surface; it is described by motion equations (1), (2) with boundary conditions (3), (4) and (6), (7) - open case;

**problem II** is the problem with a metallized surface; it is described by motion equations (1) with boundary conditions (3), (5)–(7) - short case.

3. Solution method

The solution of problems I and II taking into account (9), (10) are built in the space of Fourier images ($\alpha$ - the transformation parameter to the coordinate $x_1$):

for homogeneous half-space and vacuum:

$$U_p^{(2)}(\alpha, x_2) = \sum_{k=1}^{4} f_{pk}^{(2)} c_k^{(2)} e^{\alpha x_1}, \ U_4^{(0)}(\alpha, x_2) = c_1^{(0)} e^{-\alpha x_1}$$

(11)

for inhomogeneous coating ( $p = 3,4$ ) [13 – 16]:

$$U_p^{(1)}(\alpha, x_2) = \sum_{k=1}^{4} f_{kp}^{(1)} c_k^{(1)}(\alpha, x_2)$$

(12)

The parameters $c_k^{(2)}$ and coefficients $f_{pk}^{(2)}$ involved in representation (11) are given in [14], functions $y_{kp}^{(1)}(\alpha, x_2)$ (12) are linearly independent solutions of the Cauchy problem with initial conditions

$$y_{kp}^{(1)}(\alpha, 0) = \delta_{kp}$$

for the equation

$$Y^{(1)} = M^{(1)}(\alpha, x_2) Y^{(1)}, \ Y^{(1)} = \begin{pmatrix} Y_2^{(1)} \\ Y_4^{(1)} \end{pmatrix}, \ Y_2^{(1)} = \begin{pmatrix} \Theta_2^{(1)} \\ D_2^{(1)} \end{pmatrix}, \ Y_4^{(1)} = \begin{pmatrix} U_3^{(1)} \\ U_4^{(1)} \end{pmatrix}$$

(13)

$\Theta_2^{(1)}, D_2^{(1)}, U_4^{(1)}$ – Fourier transforms of the stress tensor component, induction vector (8), extended displacement vector respectively; $\delta_{kp}$ - Kronecker symbol. The type of matrix $M^{(1)}(\alpha, x_2)$ is given in [12-14]. Unknown coefficients $c_k^{(n)}$ in representations (11), (12) are determined when the boundary conditions are satisfied. To solve system (13), we used the Runge – Kutta – Merson method.

4. Numerical results

The studies were carried out for structures made of ferroelectric materials based on PZT: $m_1$ (PZT-5 [1]), $m_2$ (PZT DL-61HD [14]), $m_3$ (PZT DL-40 [14]) and a hypothetical material $m_4$.  

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### Table 1. Material parameters

| $\rho \cdot 10^3$ (kg·m$^{-3}$) | $c_{ij}\cdot 10^4$ (N·m$^{-2}$) | $e_{ij} \cdot C\cdot m^{-2}$ | $e_{ij} / e_{ij}^{(0)}$ |
|---|---|---|---|
| $m_1$ | 7.75 | 1.21 | 0.754 | 0.752 | 1.11 | 0.211 | 12.3 | -5.40 | 15.8 | 916 | 830 |
| $m_2$ | 8.20 | 1.46 | 0.960 | 1.00 | 1.30 | 0.390 | 33.1 | -15.8 | 25.3 | 2810 | 2520 |
| $m_3$ | 7.70 | 1.78 | 1.01 | 0.920 | 1.24 | 0.230 | 6.20 | -0.10 | 9.00 | 290 | 210 |
| $m_4$ | 7.60 | 1.13 | 0.684 | 0.668 | 1.05 | 0.151 | 5.28 | -1.89 | 12.6 | 276 | 259 |

Velocity characteristics of materials:
- $m_1$ - "base material" $V_{Sm_1}^n = 1650\text{ (m·s$^{-1}$)}, V_{Se}^{m_1} = 2265\text{ (m·s$^{-1}$)}$;
- $m_2$ - "high speed material" $V_{Sm_2}^n = 2181\text{ (m·s$^{-1}$)}, V_{Se}^{m_2} = 3182\text{ (m·s$^{-1}$)}$;
- $m_3$ - "low speed material" $V_{Sm_3}^n = 1728\text{ (m·s$^{-1}$)}, V_{Se}^{m_3} = 2221\text{ (m·s$^{-1}$)}$;
- $m_4$ - hypothetical "low speed material 2" $V_{Sm_4}^n = 1407\text{ (m·s$^{-1}$)}, V_{Se}^{m_4} = 1865\text{ (m·s$^{-1}$)}$;

We introduce the notation: $V_{I}^{(2)} = V_{Sm_1}^n / V_{Se}^{m_1}, V_{II}^{(2)} = V_{Sm_2}^n / V_{Se}^{m_2}, V_{II}^{(2)} / V_{I}^{(2)} = 0.999999987, V_{II}^{(2)} / V_{II}^{(2)} = 0.88301211$. Here $V_{GB}^{(2)}$, $V_{GB}^{(2)}$ are the Gulyaev-Bluestein wave velocities of the "reference" material of the problems I and II. In contrast to [12 - 14], it is assumed that all coating parameters vary in thickness from the values of the material parameters $m_1$ to the values of the parameters of the inclusion material $m_2$, $m_3$ or $m_4$ (Table 1). The choice of functional dependences of changes in the properties of a coating is determined by the values of the parameters of the materials of its components, the localization region and the gradient of the transition of one material to another. In figure 1 (a) and (b) shows the laws of variation of the values of the dimensionless parameter for models of two (figure 1(a)) and three materials (figure 1(b)) with a different area of localization of the inclusion material. In figure 1 (a), solid lines indicate the localization of inclusion materials in the middle of the coating (curves 1, 2), dashed lines – in the top of the coating (curves 1', 2'), dotted lines – in the bottom of the coating (curves 1'', 2''). In the case of a model of three materials (figure 1 (b)), a solid line indicates the combination of materials with localization of supply (low-speed) material in the upper part of the coating, and hard (high-speed) material in its lower part (curve 3). The reverse combination of materials is marked with a dashed line (curve 4).

Figure 2 illustrates the effect of localization of inclusions on the relative phase velocities of SAW ($V_{II}^{(2)} / V_{II}^{(2)}$). Curves are calculated for presented on figures 1 structures in the case of the short case (figures (a) and (c)) and for the open case (figures (b) and (d)). The figures (a) and (b) shows the graphs for the structure of two materials, the figures (c) and (d) shows the graphs for the structure of
three materials. The notation for the curves in figure 2 correspond to the notation of the curves in figure 1. As follows from the graphs, the nature of the inclusions and their localization significantly affects the structure of the surface wave field and the properties of SAW.

In the short case for the two material structure of the wave field substantially depends on the localization of the inclusion. The main influence of the localization of the inclusion is realized in the low-frequency range. Here, the type of inclusion and its localization can lead either to a decrease in the phase velocity (curves 2 and 2'), or to an increase in it (curves 1, 1', 2') up to the transition of the SAW into the bulk wave (curve 1'). It should be noted that at high frequencies the phase velocity of the SAW does not depend on the nature of the inclusion and its location. An exception is the localization of the inclusion in the near-surface zone. Compliant inclusion in the middle and lower part of the coating (figures 2 (a), (c)) leads to the appearance of higher modes. The location of hard and supply

![Figure 2](image)

**Figure 2.** SAW velocities in piezoelectric structures with inhomogeneous coatings of two ((a), (b)) and three ((c), (d)) materials.

Inclusions in the structure of three materials significantly affects both the structure of the surface wave field and the properties of SAW. As follows from the graphs, the localization of the hard inclusion in the surface region leads to an increase the SAW velocity in the low-frequency region, as well as to a decrease in the frequency of appearance of the second mode. When hard inclusion is localized in the lower part of the coating, a decrease the SAW velocity in the low-frequency region and an increase in the frequency of the second mode are observed.

![Figure 3](image)

**Figure 3.** Low-frequency fragment of the coating structure influence on the SAW velocity. Problem I.
In the open case, the condition for the existence of SAW is the presence of a supply inclusion. In the structure of two materials, a supply inclusion determines the existence of SAW in an unlimited frequency range (figure 2(b)). Its localization in the coating leads either to a slight increase (curve 20) or to a certain decrease (curve 21) of the SAW velocity. In the structure of three materials, the condition for the existence of a wave is the localization of a supply inclusion in the near-surface region (figure 2(d)). Other variants for the location of inclusions determine the existence of SAW in a limited range of ultra-low frequencies (figure 3).

5. Conclusion

The paper considers a model of a structure with a coating of FGPM, which properties continuously vary from the parameters of one material to another. For short case, laws are established for the influence of the nature and localization of the coating heterogeneity on the features of propagation of surface SH-waves. For open case, it is shown that the presence of a high-speed inclusion \((V_{Se}^m > V_{Se}^{(2)})\) in the coating leads to a limitation of the range of frequencies of the existence of the SAW. The cut-off frequency appears, the value of which is determined by the ratio of the velocities \(V_{Se}^m\) and \(V_{Se}^{(2)}\), as well as the size of the transition region of materials. A certain combination of low- and high-speed inclusions in the coating leads to the appearance of a range of frequencies where there is no SAW.

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