Investigation of coherent Čerenkov radiation generated by 6.1 MeV electron beam passing near the dielectric target

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Abstract. Coherent Čerenkov radiation generated by the 6.1 MeV bunched electron beam travelling near dielectric target (teflon) have been investigated theoretically and experimentally. It has been shown that the longitudinal bunch form-factor plays important role in the characteristics of coherent Čerenkov radiation. We have also compared the characteristics of coherent Čerenkov radiation and the ones of coherent diffraction radiation from a metal target in the similar conditions and have shown that coherent Čerenkov radiation is more intensive. These facts make coherent Čerenkov radiation a promising mechanism for a new noninvasive beam diagnostic technique.

1. Introduction
Čerenkov radiation (ČR) that appears while a charged particle travels in a medium with a velocity that exceeds the speed of light in this medium is developed both theoretically and experimentally and is widely used in particle detectors for nuclear physics. The radiation cone in transparent medium is defined by the following condition:

$$\cos \theta = \frac{1}{\beta n}$$

where $\theta$ is the radiation angle, $\beta$ is the electron velocity in the speed of light units, $n = \sqrt{\varepsilon}$ is the refractive index.

ČR may also appear while a charged particle moves in the vacuum in a vicinity of the medium due to the fact that this interaction is caused by the charged particle’s electromagnetic field that have the transverse dimensions of about $\gamma \lambda$ ($\gamma$ is the particle Lorentz-factor, $\lambda$ is the radiation wavelength). The latter is used in the schemes of Free Electron Laser (FEL) based on relativistic beam ($\gamma \approx 2$) moving in a cylindrical waveguide [1].

In such well-known radiation mechanism there are still a couple of open questions while dealing with ČR caused by the moderately relativistic electrons traveling near the dielectric
medium. The problem is a coherency of ČR. In reference [2] it was claimed that the longitudinal beam size does not influence on the radiation intensity that makes it impossible to define the longitudinal beam size. This point of view contradicts to the data presented in reference [3], where authors measured properties of coherent ČR (CCR) and [4] where authors used ČCR for the measurement of electron bunch longitudinal size.

The main goal of this paper is to show theoretically and experimentally that the longitudinal beam size influences drastically on ČCR spectrum and intensity. We have also carried out a comparison at the similar conditions of the characteristics of ČCR and coherent diffraction radiation (CDR) that is nowadays used in beam diagnostics.

2. Theoretical background
In our theoretical estimations we will follow a recent paper [5] in which rather simple and physically clear method for the solution of a problem of ČR and DR that may appear simultaneously is developed. We will apply this method to ČCR problem to show that the longitudinal bunch size plays an important role. One should specify that in this theoretical estimation we are interested in the radiation inside a target bulk to simplify the problem.

According to the mentioned method [5] the magnetic field of polarization radiation \( H_{\text{pol}}(r, \omega) \) that appears while a charged particle travels in the vacuum near a target with arbitrary shape and characteristics may be written in a general as:

\[
H_{\text{pol}}(r, \omega) = \text{curl} \frac{1}{c} \int_{V_T} j_{\text{pol}}^{(0)}(r', \omega) e^{i \sqrt{\varepsilon(\omega)} |r-r'|/c} |r-r'| \, d^3r'.
\]  

(2)

We should mention that this formula is the exact solution of Maxwell equations without any assumptions or simplifications. Here \( c \) is the speed of light; \( j_{\text{pol}}^{(0)}(r', \omega) = \sigma(\omega) E_e(r', \omega) \) is the polarization current density, \( \sigma(\omega) = \frac{(\varepsilon(\omega)-1)\omega}{4\pi} \) is the conductivity of the target material (\( \varepsilon(\omega) \) is the permittivity); \( E_e(r', \omega) \) is the external field, in our case electron one; \( e^{i \sqrt{\varepsilon(\omega)} |r-r'|/c} |r-r'| \) is the Green function with \( r' \) – the radiation point and \( r \) – the detection point. The integration is performed over the whole target volume \( V_T \).

In the case when the electron (or a bunch of electrons) travels near the target with impact-parameter \( h \) (see Fig. 1) and in one direction the target may be assumed to be an infinite one (see
Fig. 1) and the detector is situated in the far-field zone the previous formula may be simplified:

\[
H^{\text{pol}}(r, \omega) = \frac{2\pi i e \sqrt{\epsilon(\omega)^3}}{c r} k \times \int_0^b dz' \int_{-a}^a dy' j^{(0)}_{\text{pol}}(k_x, y', z', \omega) e^{-i(k y' + k z')}.
\] (3)

Here \( k \) is the radiation wave-vector, \( j^{(0)}_{\text{pol}}(k_x, y', z', \omega) \) is the Fourier transform of the polarization current density, \( a \) and \( b \) are the target sizes.

From now let us assume that we have a longitudinal distribution of \( N_e \) electrons \( (N_e \gg 1) \) that move in one direction with the uniform velocity. In this case the bunch current density that is needed to find the electrical bunch field may be written as:

\[
j(r, t) = e v \sum_n \delta(x) \delta(y - h) \delta(z - z_n - vt).
\] (4)

Here \( e \) is the electron charge, \( r_n = \{0, h, z_n\} \) is the position of \( n \)-th electron in the bunch, \( v = \{0, 0, v\} \) is the electrons velocity vector, \( h \) is the impact-parameter.

Full Fourier-transform of the bunch current density may be written as:

\[
j(k, \omega) = \frac{e v}{(2\pi)^3} \sum_n \delta(x) \delta(y - h) \delta(z - z_n) e^{-i k z n}.
\] (5)

Full Fourier-transform of the bunch electrical field \( E_{\text{e}}(k, \omega) \) that is convenient to use in order to find partial Fourier-transform \( E_{\text{e}}(k_x, y, z, \omega) \) that is needed for radiation field determination may be written as [6]:

\[
E_{\text{e}}(k, \omega) = \frac{2e i}{(2\pi)^2} \frac{v \omega^2}{k^2 - \omega^2/c^2} e^{-i k y h} \sum_n e^{-i k z n}.
\] (6)

Partial Fourier-transform \( E_{\text{e}}(k_x, y, z, \omega) \) may be found as:

\[
E_{\text{e}}(k_x, y, z, \omega) = -\frac{ie}{2\pi v} \exp \left[ -(h - y) \sqrt{k_x^2 + \frac{\omega^2}{v^2} \gamma^2} \right] \left\{ k_x, -i \sqrt{k_x^2 + \frac{\omega^2}{v^2} \gamma^2}, -\omega \frac{\gamma - 2}{v \gamma - 1} \right\} \left( \sum_n e^{i \frac{\omega}{v} (z - z_n)} \right).
\] (7)

Here we assumed that all electrons moves outside of the target, that made us to choose \( h > 0 \). The radiation field in the medium according to Eq. (3) taking into account Eq. (7) and \( k = w/c\sqrt{\epsilon e} \) may be written as:

\[
H_{\text{pol}}^{\text{pol}} = \frac{\epsilon \beta \gamma}{4\pi c} \sqrt{\epsilon(\omega - 1)} e^{\sqrt{\epsilon(\omega - 1)} v/c} h e^{ib(1 - \sqrt{\epsilon(e_0)} \gamma e_x)} - 1 \exp \left[ -h \sqrt{\frac{1 - \beta^2}{\epsilon(\omega - 1)}} \frac{e^{\frac{i}{\epsilon}\sqrt{\epsilon(\omega - 1)} \gamma e_y + \sqrt{1 + \epsilon(\beta^2 e_x)}} - 1}{\sqrt{1 + \epsilon(\beta^2 e_x)^2}} \right] \sum_n e^{-i \frac{\omega}{v} z_n}.
\] (8)
where:
\[
\mathbf{h} = \{ \gamma^{-1} e_y + i e_z \sqrt{1 + \varepsilon (\beta \gamma e_x)^2}, e_x (\beta \gamma \sqrt{\varepsilon} e_z - \gamma^{-1}),
\]
\[-e_x (i \sqrt{1 + \varepsilon (\beta \gamma e_x)^2} + \beta \gamma \sqrt{\varepsilon} e_y) \},
\]
and
\[
\mathbf{e} = \{ \sin \theta \sin \phi, \sin \theta \cos \phi, \cos \theta \}
\]

Thus, the spectral-angular density of radiation may be written as:
\[
\frac{d^2 W}{\hbar d\omega d\Omega} = \frac{c r^2}{\hbar} \frac{1}{\sqrt{\varepsilon}} |\mathbf{H}^{\text{pol}}|^2
\]

From Eqs. (8) and (11) one may see that only one term in spectral-angular density of radiation that is ČR term depends on the target longitudinal size \(b\). If the ČR condition is fulfilled and \(b\) is finite the pole may be eliminated as follows [5]:
\[
\left\| \frac{e^{ib\frac{\gamma}{\beta \gamma e_x}} - 1}{1 - \beta \sqrt{\varepsilon} e_x} \right\|^2 \rightarrow \left| \frac{\beta \gamma e_x}{\beta c} \right|^2.
\]

From this equation one may clearly see that the dependence of spectral-angular density of ČR from target longitudinal size is quadratic for the Čerenkov angle and fixed radiation wavelength.

The sum in Eq. (8) may be treated in the following way:
\[
\left| \sum_{n=1}^{N_e} e^{-i \frac{\omega}{\beta \gamma} z_n} \right|^2 = \left\{ \begin{array}{ll}
N_e \quad n = m; \\
N_e-1 \sum_{n=1}^{N_e-1} \sum_{m=1}^{N_e} e^{i \frac{\omega}{\beta \gamma} z_n z_m}, & n \neq m.
\end{array} \right.
\]

In the case when \(n = m\) we have simple incoherent radiation. In the case when \(n \neq m\) we have coherent radiation and may treat the sums in the following way:
\[
\sum_{n=1}^{N_e-1} e^{-i \frac{\omega}{\beta \gamma} z_n} = \sum_{n=1}^{N_e-1} \int_{-\infty}^{\infty} \delta(z - z_n) e^{i \frac{\omega}{\beta \gamma} z} dz =
\]
\[
(N_e - 1) \int_{-\infty}^{\infty} \rho(z) e^{i \frac{\omega}{\beta \gamma} z} dz
\]

Where, in the case of a Gaussian bunch with rms \(\sigma_z\), one may obtain:
\[
\rho(z) = \frac{1}{N_e - 1} \sum_{n=1}^{N_e-1} \delta(z - z_n) \simeq \frac{1}{\sqrt{2\pi} \sigma_z} \exp \left[ -\frac{z^2}{2\sigma_z^2} \right]
\]

The total spectral-angular density of polarization radiation from the bunch of \(N_e\) electrons may be written as:
\[
\frac{d^2 W}{\hbar d\omega d\Omega} = \frac{c r^2}{\hbar} \frac{1}{\sqrt{\varepsilon}} |\mathbf{H}^{\text{pol}}|^2 N_e \left( 1 + (N_e - 1)|f_z(\sigma_z)|^2 \right),
\]

where \(|f_z(\sigma_z)|^2 = \int_{-\infty}^{\infty} \rho(z) e^{-i \frac{\omega}{\beta \gamma} z} dz|^2\) is the form-factor of the electron bunch in \(z\) direction.

For the Gaussian bunch with rms \(\sigma_z\) the form-factor looks like following:
\[
|f_z(\sigma_z)|^2 = \exp \left[ -\frac{\omega^2 \sigma_z^2}{\beta^2 c^2} \right]
\]
From Eq. (16) one may clearly see that for a case when the bunch travels near the target the longitudinal form-factor drastically changes the intensity of polarization radiation. One may also see that longitudinal form-factor does not depend on ČR condition that contradicts the predictions made in reference [2]. This simple estimation proves the fact that the coherency of any kind of polarization radiation strongly depends on the longitudinal bunch profile.

3. Experiment

We have carried out experimental investigations in Tomsk Polytechnic University at Nuclear Physics Institute microtron. The experimental scheme is shown in Fig. 2. The electron beam extracted into air through 50 µm Be foil was used. The train of bunches with electron energy 6.1 Mev (γ ≈ 12), consisting of n₀ = 10526 bunches (the maximal bunch population is about Nₑ = 10⁸ electrons) with τ = 4 µs duration travels near the teflon target. The average beam current was about 30 mA. The transverse sizes of electron beam in the extraction point are about σₓ × σᵧ = 4 × 4 mm². The longitudinal distribution of electrons in the bunch is a Gaussian with rms σ_z = 1.1 mm [9] (there is a misprint in paper [9] and “σ_z = 6.8 mm” should be read as “6σ_z = 6.8 mm”).

The detecting system consisted of so-called “telescope”, which represented a paraboloidal mirror (diameter 170 mm, focal distance 151 mm) in focus of which the detector was set up. Such telescope allows to measure the angular radiation characteristics equal to wave-zone ones [10]. The radiation from each train has been detected using DP-21M detector. The last is based on a wide-band antenna, high-frequency low barrier diode and preamplifier. The average sensitivity of the detector in the radiation wavelength region 11÷17 mm is approximately equal to 0.3 Volt/mWatt [11]. The measured region was limited by coherent threshold in the smaller wavelengths and by the beyond-cutoff waveguide (diameter 15 mm) that passes wavelengths lower 25 mm used to decrease accelerator RF background. Incoherent radiation can not be measured by the detector. The measured radiation yield was averaged over 20 trains. The statistic error was less than 10% during the experiments. The wire scanner was used to find the electron beam axis which was used as a starting point to determine the impact-parameter that was equal to h = 25 mm. ČR in diffraction geometry should be polarized only in horizontal plane, i.e. the plane of electron velocity vector and radiation wave-vector. During the experiment a grid polarizer was used. The Faraday cup was used to measure beam current.

Teflon (Polytetrafluoroethylene – PTFE) target with the length equal to 247 mm and the height equal to 74 mm was used during the experiment. The target was a prism based on right-angled isosceles triangle in order to decrease ČR refraction losses. Teflon refractive index in abovementioned wavelength region is equal to n = 1.45 [7] and n = 1.439 in region less than 5 mm according to [8].

At the first we have found the ČČR peak position while scanning over θ angle. Fig. 3
shows the result of $\theta$ scan. The measured horizontal polarization component of radiation is shown in Fig. 3 by the red circles. The theoretical simulation result obtained using Eq. (16) taken into account Snell’s law for $n = 1.45$ is shown by the solid line. Fig. 3 also shows the vertical polarization component of CCR by the blue squares. From Fig. 3 one may clearly see that the vertical polarization component is suppressed as expected. The measured peak is situated at the angle of $\theta = 46.5^\circ \pm 0.05^\circ$ while theoretical value is $\theta = 46.7^\circ$ that confirms the nature of measured radiation. The measured peak is wider than theoretical: measured FWHM = $4.3^\circ \pm 0.14^\circ$ and simulated FWHM = $2.72^\circ$. That may be explained by the angular divergence of the electron beam that is due to beam scattering in the beryllium foil and in the air and by the finite angular acceptance of the telescope. The beam divergence was not taken into account during theoretical simulations. The comparison of the peak power of measured and simulated CCR was not performed because of simplification made during theoretical estimation such as infinite target size in $x$ direction (see Fig. 1).

There are main point of interest to check for CCR as it was mentioned before. That is the coherency of radiation and its nature. In order to find whether the radiation is coherent or not the beam current dependence was measured. The latter is shown in Fig. 4. Measured radiation is shown by the red cirles and the solid line shows the fit by the function $y = a + bx + cx^2$. One may see from Fig. 4 that the fit function agrees very well with measured data and, therefore, radiation is coherent.

One may argue that such coherency is caused by the transverse beam size, not longitudinal one, just as it was considered in reference [2]. In the case when electron beam travels in a vicinity of the target the transverse form-factor is not so critical as longitudinal one if $\sigma_y \leq h \ll \gamma \lambda$. These conditions were fulfilled for our measurements ($\sigma_y \approx 4\text{ mm}$, $h = 25\text{ mm}$, $\gamma \lambda \approx 150\text{ mm}$). If longitudinal form-factor plays some role and our estimations are true the radiation coherent threshold should be at the wavelength of about 9 mm. Fig. 5 shows by the red histogram bars the spectrum of CCR measured at the angle of radiation maximum. For spectrum measurements the low-pass filters, described in [9] were used. The filters were set up in order instead of beyond-cutoff waveguide at the detector horn. The solid line shows the radiation form-factor from

![Figure 3](image_url)

**Figure 3.** The angular dependence of CCR: the red circles – horizontal polarization component, the blue squares – vertical polarization component, the solid line – theoretical simulation. Target No.1
Figure 4. The current dependence of CCR: the red circles – experimental data, the solid line – quadratic fit \( y = a + bx + cx^2 \).

Figure 5. The spectrum of CCR measured at radiation peak angle \( \theta = 46.5^\circ \). The red histogram bars – experimental data, the solid line – form-factor from Eq. (17) \( (\sigma_z = 1.13 \text{ mm}, \sigma_{lt} = 2 \text{ mm}) \). Target No.1

Eq. (17) for \( \sigma_z = 1.1 \text{ mm} \). One may see from Fig. 5 that there is no radiation with wavelengths less than 5 mm that corresponds to the form-factor dependence. The lack of radiation in the region from \( \lambda = 5 \) to \( \lambda = 11 \) mm comparing with form-factor dependence is no actually clear. It may be explained by the detector sensitivity that is unknown in the waveband 6 to 11 mm. In spite of some disagreement, one may clearly see that the coherency of \( \tilde{\text{C}} \text{R} \) is caused by the longitudinal bunch size, not transverse one.

In order to compare CCR and CDR from a metal target which is used widely for beam diagnostics we have measured both in the corresponding conditions. The CDR target was a 20\( \mu \)m thick copper on a woven-glase reinforced substrate of a square form with 170 mm side. The CDR target was inclined at an angle of 45\(^\circ\) with respect to the beam direction. The impact-parameter was equal to 25 mm, too. The measured distributions are shown in the same
Fig. 6 for convinience. One may clearly see in Fig. 6 that the measured power of ČCR is larger than CDR one in a factor of 2.5. The ČCR angular distribution is narrower than CDR one: $\text{FWHM}_{\text{ČCR}} = 4.3 \pm 0.14^\circ$ and $\text{FWHM}_{\text{CDR}} = 12 \pm 0.3^\circ$. It may be shown that the angular width of ČCR peak almost doesn’t depend of the particle Lorentz-factor in contrast of CDR one. Due to this fact a power of a part of ČCR cone (as in our case) may be higher than one from a whole cone of CDR for low values of Lorentz-factor. The theoretical FWHM of CDR is equal to $8.3^\circ$. One should mention that for both ČCR and CDR measured FWHM is larger than simulated one in a factor of 1.5.

4. Conclusion
In conclusion we would like to point that the coherency of polarization radiation and its dependence on the longitudinal bunch profile seems to be a basic feature that does not depend on the radiation mechanism (transition radiation, diffraction radiation, Čerenkov radiation, etc.). From this point of view the predictions made in reference [2] concerning that coherent Čerenkov radiation does not depend on longitudinal bunch size seems to be incorrect as it was shown theoretically. We demonstrated experimentally for 6.1 MeV bunched electron beam the square dependence of coherent Čerenkov radiation intensity on a bunch population.

The “longitudinal coherency” and high linear polarization may be used for new diagnostic methods based on the Čerenkov radiation beam. Development of the similar noninvasive methods seems to be promising due to the fact that the coherent Čerenkov radiation distribution is narrower and more intensive than the distribution of the coherent diffraction radiation from the metal target which is used nowadays for the same purpose.

The diagnostic scheme with coherent Čerenkov radiation that goes at an angle to beam direction seems to be more promising that the scheme proposed in [4], where coherent Čerenkov radiation goes in forward direction and one needs additional mirrors to extract it. This additional optics adds some radiation (e.g., transition one) that may interfere with coherent Čerenkov radiation and increase a background.

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Reference

[1] J.E. Walsh, T.C. Marshall, S.P. Shlesinger, Phys. Fluid. 20(4), 709 (1977).
[2] J. Zheng, C. X. Yu, Z. J. Zheng and K. A. Tanaka, Phys. Plasmas 12, 093105 (2005).
[3] T. Takahashi, Y. Shibata, K. Ishii, M. Ikezawa, M. Oyamada, Y. Kondo, Phys. Rev. E 62(6), 8606 (2000).
[4] S.V. Shchelkunov, T. C. Marshall, J. L. Hirshfield, and M. A. LaPointe, Phys. Rev. ST Accel. Beams 8, 062801 (2005)
[5] D.V. Karlovets, A.P. Potylitsyn, Pis'ma v ZhETP 90(5), 368 (2009) (D.V. Karlovets, A.P. Potylitsyn, JETP Lett. in press)
[6] D.V. Karlovets, A.P. Potylitsyn, ZhETP 134(5), 887 (2008) (D.V. Karlovets, A.P. Potylitsyn, JETP 107(5), 755 (2008))
[7] James W. Lamb, International Journal of Infrared and Millimeter Waves 17(12), 1997 (1996)
[8] Mohammed N. Afsar, Hua Chi, and Igor I. Tkachov, in Proceedings of Millimeter and Submillimeter Waves II, 1995, edited by Mohammed N. Afsar (SPIE) 2558, 73 (1995)
[9] B. N. Kalinin, D.V. Karlovets, A.S. Kostousov, G.A. Naumenko, A.P. Potylitsyn, G.A. Saruev and L.G. Sukhikh, Nucl. Instrum. Methods Phys. Res., Sect. B 252, 62 (2006).
[10] B. N. Kalinin, G. A. Naumenko, A. P. Potylitsyn, G. A. Saruev, L. G. Sukhikh and V. A. Cha, Pis'ma v Zh. Eksp. Teor. Fiz. 84(3), 110 (2006) [JETP Lett. 84(3), 110 (2006)].
[11] V.A. Cha, B.N. Kalinin, E.A. Monastyrev, G.A. Naumenko, A.P. Potylitsyn, G.A. Saruev and L.G. Sukhikh, in Proceedings of International Conference on Charged and Neutral Particles Channeling Phenomena II, 2006, edited by Sultan B. Dabagov (SPIE). 6634, 663416 (2007).