Coupled Numerical Analysis of Three-Dimensional Unsteady Flow with Pitching Motion of Reentry Capsule
—Investigation of the Third Harmonics of the Aerodynamic Force—*

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Computational fluid dynamics (CFD) analysis coupled with pitching motion of a reentry capsule is performed, and a model equation for the aerodynamic force coincident with the CFD result is proposed. The self-excitation of pitching oscillation and the subsequent limit-cycle oscillation are reproduced in a fine-grid CFD simulation. The axis of the vortex ring in the wake extracted by the phase average is displaced to the lower side of the capsule base when the pitch angle $\alpha = 0$ and $\alpha > 0$. Such a displacement induced the dynamic component of pitching moment around $\alpha = 0$. Subsequently, the pitching moment coefficient is decomposed into a Fourier series, where the amplitude of the third harmonics is larger than the dynamic component of the fundamental frequency. The proposed model equation for the pitching moment, which fully includes the third harmonics, reproduces the same amplitude and the same frequency of the CFD result in the case of limit-cycle oscillation. Compared to conventional models, the present model was found to give a better approximation of the dynamic component $\alpha C_{\text{Mdy}}$ of the unsteady aerodynamic work per unit time.

Key Words: Compressible Flows, Computational Fluid Dynamics, Reentry Capsule, Model Equation for Aerodynamic Force, Cartesian Cut-Cell Method

Nomenclature

- **H**: Heaviside function
- **Q**: conservative variables
- **E, F, G**: flux vectors
- **n**: unit normal vector
- **$\sigma$**: wall flux vector
- **t**: time
- **x, y, z**: Cartesian coordinates
- **$\phi$**: level set function
- **p**: pressure
- **u**: velocity
- **$\rho$**: density
- **e**: energy
- **$\alpha$**: pitch angle
- **$\dot{\alpha}$**: pitch angular velocity
- **Ma**: Mach number
- **D**: diameter of capsule
- **$\Delta x$**: grid width
- **$C_D$**: drag coefficient
- **$C_L$**: lift coefficient
- **$C_M$**: pitching moment coefficient
- **f**: frequency
- **M**: pitching moment
- **$\phi$**: phase advance

Subscripts

- **st**: static component
- **dy**: dynamic component
- **lim**: value in the limit-cycle oscillation
- **0**: initial value of the growing oscillation

1. Introduction

Dynamic instability is one of the unsettled problems in the reentry of a blunt-body capsule.1) In the worst-case scenario, the transonic instability may cause the failure of the parachute to open. A dynamically stable reentry during transonic conditions before parachute opening is essential to safe landing; furthermore, a stable parachute opening at low altitude in subsonic conditions improves the landing point accuracy owing to the reduced surface wind effect.

Dynamic instabilities of blunt-body capsules are often investigated in a typical wind-tunnel experiment called the free-rotation test. Hiraki et al.2,3) and Abe et al.4) conducted one degree-of-freedom (1-DoF) free-rotation tests in the same capsule shape with different supporting systems, revealing that the phase delay between the pitch angle and the aftbody surface-pressure fluctuation causes a dynamically unstable moment around a zero pitch angle.

Hiraki’s experiment5) reported the presence of two knots in the Lissajous figure of the pressure difference between two measurement points on the aftbody surface and pitch angle. The pressure difference is related to the pitching moment. Considering that the base pressure fluctuation contributes to the dynamically unstable moment, it can be regarded
that the third harmonics have a non-negligible effect on the pitching moment. A precise time history, which can be obtained through a coupled numerical analysis, is essential for an in-depth discussion of the third harmonics of the pitching moment.

Hiraki et al.\textsuperscript{2,3} developed a model equation for the aerodynamic force based on the van der Pol equation. Abe et al.\textsuperscript{4} and Kazemba et al.\textsuperscript{1,5} established model equations introducing phase delay effects called “temporal delay” and “lag time factor,” respectively, with that of Abe et al. being an extended Hiraki model. Although these model equations reproduced well most of the pitch-angle oscillations in the experiments, the third harmonics of the pitching moment was not sufficiently discussed. It is imperative to discuss this phenomenon in detail, as it plays a crucial role in the limit-cycle oscillation.

In the present study, a large-scale numerical analysis of the three-dimensional unsteady flow coupled with the 1-DoF pitching motion of a reentry capsule is performed. From the analysis, the pitching moment coefficient along with the divergence terms, are substituted respectively by the volume fraction of the cell and area fractions of the cell interface in each direction, which are subsequently evaluated by means of the Cartesian cut-cell method. The cell-merging technique\textsuperscript{8} is applied at each small cell having a small gas volume fraction.

The variable $\sigma$ on the right-hand side of Eq. (1) represents the wall flux vector (the interaction between the capsule and the air), which is directed into the air from the capsule, and is given by the following equation:

$$\sigma = \begin{pmatrix} 0 \\ pm_x \\ pm_y \\ pm_z \\ p(u_w n_x + u_w n_y + u_w n_z) \end{pmatrix}.$$  \hspace{1cm} (5)

Mass and heat transfer does not exist in the problems of the present paper.

The conserved quantities $Q$, and the fluxes $E, F,$ and $G$ in Eq. (1) are defined as follows:

$$Q = \begin{pmatrix} \rho \\ \rho u_x \\ \rho u_y \\ \rho u_z \\ e \end{pmatrix}, \quad E = \begin{pmatrix} \rho u_x \\ \rho u_x^2 + p \\ \rho u_x u_y \\ \rho u_x u_z \\ \rho u_x \end{pmatrix},$$

$$F = \begin{pmatrix} \rho u_y \\ \rho u_y^2 + p \\ \rho u_y u_z \\ \rho u_y \end{pmatrix}, \quad G = \begin{pmatrix} \rho u_z \\ \rho u_z^2 + p \\ \rho u_z \end{pmatrix}.$$ \hspace{1cm} (6)

The conservation Eq. (1) is discretized by the cell-centered finite volume method. The advection flux of the compressible Euler equation at the cell interface is calculated using the SLAU method.\textsuperscript{9} Velocity components on the cell interface are interpolated by the sixth-order accuracy essentially non-oscillatory (WENO) scheme.\textsuperscript{10} The interpolated velocity components are modified using the modification proposed by Thornber et al.\textsuperscript{11} The other primitive variables ($\rho, p$) on the cell interface are interpolated by the fifth-order monotone upwind scheme for conservation lows (MUSCL).\textsuperscript{12} Also, the time integration to the second-order accuracy is calculated by means of the TVD Runge–Kutta method.\textsuperscript{13} In the coupled analysis, the equation of angular motion is calculated using the fourth-order Adams–Bashforth method.

### 3. Numerical Analysis of the Flow around Capsule Model with Fixed Pitch Angle

CFD simulations of the flow around a capsule model D45S having a spherical base is performed to validate the simulation code. The pitch angle of D45S is fixed. Experimental data of the wind-tunnel testing with fixed pitch-angle measurements of the D45S spherical-based model and pitch-
ing oscillation experiments of the D45 frustoconical-based model could be found in Hiraki’s study.3) By contrast, data of fixed-angle measurements of the D45 model are not open to the public; thus, the fixed-angle problem of D45S are discussed in this section. In Hiraki’s study, the dynamic characteristics of D45 and D45S model are similar at $Ma > 1$. An unsteady numerical simulation of the flow around the D45S model is sufficient to validate the simulation code to reproduce the dynamical characteristics of the D45 model. A schematic representation of the D45S scale model is shown in Fig. 1, and the simulation conditions decided according to Hiraki’s experiments3) are summarized in Table 1. The uniform grid domain is applied with grid width $\Delta x/D$ and a domain width interval $(-0.12 \, \text{m} \leq x \leq 0.36 \, \text{m}, -0.12 \, \text{m} \leq y \leq 0.12 \, \text{m}, -0.12 \, \text{m} \leq z \leq 0.12 \, \text{m})$. Outside of the uniform grid domain, the grid width ratio between the neighboring computational cells is set at 10%.

Figure 2 shows the flow separation at the shoulder and the finite vortex structures in the wake. Isosurfaces of the second invariant of the velocity gradient tensor are generated at a certain distance from the shoulder, not at the vicinity of the shoulder. The pressure rises around the stagnation point at the nose and then gradually drops to the shoulder at the pitch angle $\alpha = 0^\circ$.

Figures 3(a) and 3(b) display the typical flow structure in supersonic flows, which includes bow shock, an expansion wave on the shoulder, and a recompression shock in the wake. The standing shock wave is formed on the forebody of the D45S in the case of $\alpha = 30^\circ$.

The aerodynamic coefficients are obtained as time-averaged values of the unsteady simulation. Sampling of the time-averaged value is conducted after the initial disturbance converges. The sampling time is 0.16 s which is 100 times larger than $D/u$. As shown in Figs. 4–6, coefficients of drag, lift, and pitching moment fairly agree with Hiraki’s experimental data.3) For the fine-grid simulations, the drag and lift coefficients are close to the experimental values in the decreasing grid width, while the moment coefficient agreed well with the experimental values, which has undulation. The aerodynamic coefficients of the coarse-grid simulations reproduces the pitch-angle dependency of the experimental results. Practically, the coarse-grid simulation is likewise acceptable for the fixed CFD.

Table 1. Simulation conditions for the flow around the D45S model.

| $\alpha$ | $Ma$ | $\frac{1}{2}p_0\alpha^2$ [kPa] | $\Delta x/D$ |
|---|---|---|---|
| $0^\circ$–$15^\circ$ | 1.1 | 58.9 | $0.0185, 0.00923$ |
| $20^\circ$–$30^\circ$ | 70.6 | | |

![Fig. 1. Schematic of the D45S scale model.](image1)

![Fig. 2. Isosurfaces of the second invariant of the velocity gradient tensor around the D45S model ($\alpha = 0^\circ$).](image2)

![Fig. 3. Pseudo-Schlieren images by the density gradient of the fixed pitch-angle simulation. (a) $\alpha = 0^\circ$](image3a)

![Fig. 3. Pseudo-Schlieren images by the density gradient of the fixed pitch-angle simulation. (a) $\alpha = 30^\circ$](image3b)
Coupled CFD Analysis with Pitching Rotation of the Capsule

Coupled CFD analysis with 1-DoF pitching rotation of the capsule is conducted for the frustoconical-based D45 model in Fig. 7, which is a scale model of Hayabusa reentry capsule in 2010. The axis of pitching rotation is set parallel to the y-axis at 50% position of the total length of the center axis, according to which the pitching moment of inertia is 

\[ J_{p} = \frac{0.07}{C_{2}^{2}} \frac{1}{C_{0}} \frac{4}{C_{1}x} \]

The upstream Mach number is 1.3, and the grid-width-to-shoulder-diameter ratio \( \Delta x / D \) are 0.02 (Coarse) and 0.01 (Fine). Moreover, grid width \( \Delta x \) is applied on the uniform grid domain \(-0.10 \leq x \leq 0.41 \text{ m}, -0.125 \leq y \leq 0.125 \text{ m}, -0.125 \leq z \leq 0.125 \text{ m}\). Outside the uniform grid domain, the grid width ratio between the neighboring computational cells is set at 10%. The initial condition for the coupled CFD analysis is obtained from the fixed CFD analysis with the D45 model at \( \alpha = 0 \), whereas the initial disturbance of the pitching oscillation is not added.

A comparison of the results of the 1-DoF coupled CFD analysis and Hiraki’s wind-tunnel experiment is summarized in Fig. 8. Note that the amplitude of the experimental result continuously grows by up to 20°, although, to ensure visibility, it is omitted in Fig. 8. Self-excitation of pitching oscillation occurs in the fine-grid simulation as well as the experiment. Subsequently, limit-cycle oscillation occurs in the fine-grid simulation. The amplitude fluctuations are found as like as that in the experiment during the limit-cycle oscillation. Oscillation frequency is obtained with the elapsed times of the four and eight periods, respectively, of the pitching oscillation. The axis of pitching rotation is set parallel to the y-axis at 50% position of the total length of the center axis, according to which the pitching moment of inertia is 

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the limit-cycle oscillations of the coupled CFD analysis and the experiment. The corresponding limit-cycle frequencies of the coupled CFD analysis and the experiment are 17.75 Hz and 17.98 Hz, showing a well-represented agreement. However, the amplitude in the fine-grid simulation is almost half that in the experiment and more remarkable in the coarse-grid simulation, in which the amplitude is much lower than in the experiment.

For the fine-grid simulation, the result qualitatively agrees with the experimental data. Consequently, the time history of the pitch angle and the pitching moment are obtained and are judged sufficient to elucidate in detail the growing and the limit-cycle oscillations. However, the reason for the quantitative disagreement of the limit-cycle amplitude is remains unclear.

The phase average of the flow field at \( \alpha = 0^\circ \) and \( \dot{\alpha} > 0 \) in the pitch-up condition is shown in Fig. 9. The stagnation point in the wake is the vicinity of the axis of the capsule in the coarse-grid simulation and is different to that in fine-grid simulation.

As shown in Fig. 10 the base pressure distribution of the capsule in the coarse-grid simulation is almost constant, whereas the base pressure at the upper side is lower than that at the lower side in the fine-grid simulation. Essentially, this distribution is the source of the unsteady component of pitching moment around \( \alpha = 0^\circ \).

Fine vortex structures in the wake in the fine-grid simulation realize the limit-cycle oscillation in the same order amplitude with the experiment, whereas the self-excitation of oscillation in the coarse-grid simulation is very weak, suggesting that the coupled CFD analysis with the capsule oscillation requires a finer grid than for the CFD analysis around a fixed-angle capsule.

5. Fourier Analysis of Limit-cycle Oscillation

The pitching moment during the limit-cycle oscillation can be decomposed into the Fourier series as follows:

\[
M = M_0 C_M, \quad M_0 = \frac{1}{2} \rho \alpha^2 SD, \quad C_M = a_1 \cos 2\pi f_{lim}(t - t_1) + b_1 \sin 2\pi f_{lim}(t - t_1)
+ a_3 \cos 6\pi f_{lim}(t - t_1) + b_3 \sin 6\pi f_{lim}(t - t_1) + \cdots \tag{7}
\]

Here, the constant \( t_1 = 0.3185 \) is a reference time when \( \alpha = 0 \) and \( \dot{\alpha} > 0 \). The fundamental components, third harmonics, and sum of the higher harmonics are given in Fig. 11. The sine component of the fundamental frequency is a major part of the pitching moment. Moreover the cosine and sine components of the third harmonics are stronger than the cosine component of the fundamental frequency. To elucidate the limit-cycle oscillation, investigating the third harmonics is imperative.

The angular velocity can be derived from the integral of the angular momentum equation \( \vec{I} \ddot{\alpha} = M \) as follows:

\[
\dot{\alpha} = \frac{M_0}{2\pi f_{lim} I} \left\{ a_1 \sin 2\pi f_{lim}(t - t_1) - b_1 \cos 2\pi f_{lim}(t - t_1) + a_3 \sin 6\pi f_{lim}(t - t_1) - b_3 \cos 6\pi f_{lim}(t - t_1) + \cdots \right\} \tag{9}
\]

where \( I \) denotes moment of inertia of the capsule. Further, integral of \( \ddot{\alpha} \) gives
The limit-cycle oscillation. This relation shows that the amplitude $A_{\lim}$ of $\alpha$ during limit-cycle oscillation is equal to $-b_1/C_1$. Using this relation, a model equation for the aerodynamic force is constructed as follows:

$$C_M = -C_1 \alpha + \frac{\alpha e}{A_{\lim}} \left\{ A_{\lim}^2 - \alpha^2 - \left( \frac{\alpha}{2\pi f_{\lim}} \right)^2 \right\}$$

$$+ \frac{8a_3}{9A_{\lim}^3} \frac{\alpha}{2\pi f_{\lim}} \left\{ \left( \frac{\alpha}{2\pi f_{\lim}} \right)^2 - 3\alpha^2 \right\}$$

$$+ \frac{8b_3}{9A_{\lim}^3} \alpha \left\{ 3 \left( \frac{\alpha}{2\pi f_{\lim}} \right)^2 - \alpha^2 \right\},$$

where the second and third terms of the right-hand side are the source of the growing oscillation, that do not affect the limit-cycle oscillation for any values assigned for the constants $a_e$ and $b_e$.

The limit-cycle frequency $f_{\lim}$ is $17.75$ Hz is decided as a result of the coupled CFD analysis with the capsule oscillation, with a corresponding $C_1 = 0.152$. Meanwhile, the Fourier coefficients $b_1 = -0.0277$, $a_1 = 0.00110$, and $b_3 = -0.00134$ are decided by the Fourier series expansion. Besides the initial frequency $f_0$ is reproduced by the fitting parameter $b_1$ in the growing oscillation, which can be estimated as follows:

$$b_1 = \left( 1 - \frac{f_0^2}{f_{\lim}^2} \right) b_1 = 0.00575$$

where $f_0 = 19.5$ Hz is decided by the results of the coupled CFD analysis. Another fitting parameter $a_e$ reproduces the envelope curve of the growing oscillation and is estimated to be 0.00481.

Hiraki’s model of the pitching moment coefficient, which is based on the van der Pol equation, can be expressed as follows:

$$C_M = a \alpha + b \alpha^3 + c \alpha \left( A_{\lim}^2 - 4\alpha^2 \right).$$

Here, the variables $a$, $b$, and $c$ are the fitting parameters. Hiraki’s model coincides with Eq. (14) if and only if $a_e = 8a_3/9$ and $b_e = -8b_3/3$; however, these values do not give a good approximation for the growing oscillation. Indeed, the lack of the terms $\alpha^2 \alpha$ and $\alpha^3$ in Hiraki’s model is remarkable.

The numerical solutions of the initial value problem $\ddot{\alpha} = M_o C_M$ are obtained using the present model and Hiraki’s model. Figure 12 shows the time history of the pitch angle $\alpha$ obtained from the coupled CFD analysis, the present model, and Hiraki’s model. Here, the parameters $a = -0.176$, $b = 0.95$, and $c = 0.007$ are used in Hiraki’s model. As observed, the time history of $\alpha$ in the CFD analysis was reproduced well by the numerical solutions of the two models.
Figure 13 shows the time history of the pitching moment coefficient $C_M$ obtained from the coupled CFD analysis, the present model, and Hiraki’s model. Similar to Fig. 12, the time history of CFD analysis is reproduced well by the numerical solution of $C_M$ of the present model. On the other hand, Hiraki’s model reproduces the time history of the growing oscillation, but the waveform distortion at $|C_M| > 0.02$ and an overestimation of the peak value of the limit-cycle oscillation is observed.

The moment coefficient is decomposed into static and dynamic components represented by the following equations:

$$C_M = C_{Mst} + C_{Mdy}, \quad C_{Mst} = -C_1\alpha. \quad (17)$$

Figure 14 shows the work per unit time $\alpha C_{Mdy}$ of the dynamic component of the moment during the limit-cycle oscillation. Asymmetry in the CFD results is caused by a shortage of the phase-averaged cycles, whereas it is expected to reduce with the increment of the averaged cycles. The present model reproduces the major part of $\alpha C_{Mdy}$ acting on the capsule during the limit-cycle oscillation. Additionally, the $\alpha C_{Mdy}$ around the maximum (or the minimum) $\alpha$ is positive, implying a fair qualitative consistency with the result of the coupled CFD analysis. Alternatively, the quantitative disagreement between the coupled CFD analysis and the present model is caused by ignoring the higher harmonics, which is done for simplicity. On the other hand, Hiraki’s model overestimates the $\alpha C_{Mdy}$, while the local maximum phase is appreciably different from the CFD result. Also, the $\alpha C_{Mdy}$ of the Hiraki’s model is not positive around the maximum (or the minimum) $\alpha$, implying a qualitative disagreement in the $\alpha C_{Mdy}$ between Hiraki’s model and the CFD analysis.

Figure 15 shows the dynamic component $C_{Mdy}$ of the moment coefficient of the numerical solution of the present model plotted on the $\alpha-\alpha/(2\pi f)$ phase plane. The sign of $C_{Mdy}$ changes in the fundamental frequency during the growing oscillation and in the third harmonics frequency in the limit-cycle oscillation.

Considering the clockwise time development of the state point in the phase plane, the local maximum phase of $C_{Mdy}$ could be seen to advance from the vertical axis. The dynamic component could be derived as $C_{Mdy} = C_M + C_1\alpha$ from Eq. (14). In the case of the limit-cycle oscillation, $C_{Mdy}$ would assume a trigonometric function by substituting Eqs. (9) and (10) into the right-hand side of this relation. Neglecting the small terms, $C_{Mdy}$ would be reduced further to $\cos[3(2\pi f_{lim}t + \psi_{lim})]$ with a certain coefficient. On the other hand, in the case of growing oscillation, $C_{Mdy}$ would be reduced to $a_{D}(\alpha/(2\pi f_{lim})/A_{lim} - b_\alpha/\lambda_{lim})$ during the initial stage. Applying this approximation for the initial stage of growing oscillation to the angular momentum equation $\dot{M} = I\ddot{\alpha}$, and gives the solution as follows:

\[ M = \frac{I}{\lambda_{lim}} \]
\[
\alpha = q_1 \exp \left( \frac{a_e \pi f_{\text{lim}}}{A_{\text{lim}} C_1} \right) \times \sin \left[ 2\pi f_{\text{lim}}(t - t_1) \left\{ 1 - \left( \frac{a_e}{2A_{\text{lim}} C_1} \right)^2 + \frac{b_e}{A_{\text{lim}} C_1} \right\} \right].
\]

(18)

Substituting this solution into the simplified equation of \( C_{\text{Mdy}} \) and neglecting small order terms of \( \Theta(a_e/A_{\text{lim}} C_1) \) and \( \Theta(b_e/A_{\text{lim}} C_1) \), we obtain

\[
C_{\text{Mdy}} = q_1 \sqrt{a_e^2 + b_e^2} \exp \left( \frac{a_e \pi f_{\text{lim}}}{A_{\text{lim}} C_1} \right) \times \cos(2\pi f_{\text{lim}}(t - t_1) + \varphi_0).
\]

(19)

The initial phase advance \( \varphi_0 \) in the growing oscillation and phase advance \( \varphi_{\text{lim}} \) in the limit-cycle oscillation are evaluated as follows:

\[
\varphi_0 = \tan^{-1} \left( \frac{b_e}{a_e} \right), \quad \varphi_{\text{lim}} = \frac{1}{3} \tan^{-1} \left( \frac{b_3}{a_3} \right).
\]

(20)

The value of the phase advance of \( C_{\text{Mdy}} \) shifts from the initial value \( \varphi_0 = 50.1^\circ \) to the limit-cycle value \( \varphi_{\text{lim}} = 16.9^\circ \); this suggests that the progress of oscillation growth up to the limit-cycle oscillation could be specified in the order of \( \varphi_{\text{lim}} \leq \varphi \leq \varphi_0 \). Note that the value of the phase advances is not universal but specified with the numerical (experimental) conditions. Being one of the potential criteria that quantify the dynamic stability of the reentry capsule, phase advance is ought to be validated by several experiments and numerical simulations data.

7. Conclusions

The coupled CFD analysis with the 1-DoF pitching motion of a scale-model of a reentry capsule is performed. The following findings are obtained.

Numerical simulations of the flow around the D45S capsule with fixed pitch angle is performed before the 1-DoF coupled analysis. The aerodynamic coefficients coincides with the experiment.3

The coupled CFD analysis with the pitching oscillation of the D45S capsule using a fine grid reproduces the same frequency and a half amplitude of Hiraki’s experimental data.2,3 The axis of the vortex ring in the wake of the phase average is displaced to the lower side of the capsule base when \( \alpha = 0 \) and \( \dot{\alpha} > 0 \), while the pressure on the upper side of the capsule base decreases, inducing the dynamic component of the unsteady pitching moment.

The pitching moment coefficient is decomposed into a Fourier series, where the amplitude of the third harmonics is larger than the cosine component of the fundamental frequency.

A model equation for the pitching moment which properly considers the third harmonics is developed. The numerical solution of \( \alpha \) by the present model reproduces almost the same oscillation as the coupled CFD analysis and Hiraki’s model.2,3 Moreover, the dynamic component \( C_{\text{Mdy}} \) of the unsteady aerodynamic work per unit time of the present model gives a better estimate than Hiraki’s model, although the present model equation does not require an increase of the number of fitting parameters. Consequently, the present model equation improves Hiraki’s model equation.

Acknowledgments

Part of the experimental results in this research were obtained using supercomputing resources at the Cyberscience Center, Tohoku University.

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