A proposal on complementary determination of the electro-weak mixing angles $\sin^2\theta_W$ at a super $Z$-factory

Xu-Chang Zheng

Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China.
School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China.

Chao-Hsi Chang

Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China.
School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China.
CCAST (World Laboratory), Beijing 100190, China.

Tai-Fu Feng

Department of Physics, Hebei University, Baoding 071002, China.
Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China.

A proposal on determination of the effective electro-weak mixing angle $\sin^2\theta^f_W$ ($f = c, b$) at a super $Z$-factory (an $e^+e^-$ collider designed with the possible highest luminosity and running around the center mass energy $\sqrt{s} = m_Z$) is proposed. It is to determine how the effective mixing angles depend on the flavors, especially the flavors $c, b$ (with the determined effective ones, the electro-weak mixing angles $\sin^2\theta^f_W$ ($f = c, b$) of Standard Model can be derived out precisely). The key point of the proposal is that at such a super $Z$-factory, via measuring the forward-backward ($A_{FB}$), left-right ($A_{LR}$) and combined left-right forward-backward ($A_{LRFB}$) asymmetries of the produced doubly heavy-flavored hadrons (such as $B_c, B_c^*$ and the baryons $H_{QQ'q}; \Xi_{c\bar{c}}, \Xi_{bc}$ and $\Xi_{bb}$ etc) in the relevant production processes $e^+e^- \rightarrow B_c + \cdots$ and $e^+e^- \rightarrow H_{QQ'q} + \cdots$ one may complement the final determination of the mixing angle $\sin^2\theta^f_W$ ($f = c, b$) of Standard Model. The advantage of the proposed way to determine the effective electro-weak mixing angles is that the doubly heavy flavor(s) and the out-going direction of the produced doubly-heavy hadron can be experimental determined precisely, i.e. there are no such errors caused by missing identification of the heavy flavor(s) and by determining the so-called thrust axis of the produced jets, which can not be avoided in the early determination at LEP-I and SLC.

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I. INTRODUCTION

The electro-weak mixing angle ($\sin \theta_W$) is an essential parameter in the Standard Model (SM). It describes the mixing from the gauged boson states $W^0$ and $B^0$ into the physical states, the photon $\gamma$ and the boson $Z$, which couple to the electromagnetic currents and neutral weak currents respectively, so according to the model, the mixing angle also appears in the couplings of $Z$-boson to up- and down-type leptons and quarks in a definite manner. Thus to determine the values of the couplings and the ‘flavored electro-weak mixing angle’ (the mixing angle appears in the coupling of the $Z$-boson and the flavored fermion) experimentally, i.e. to see how it depends on the coupled flavors, must be a test of the Standard Model (SM). Namely if experimentally it is found that the mixing angle deviates from that required by SM, it will mean that there may be something new beyond SM. In the paper as a complement we seriously consider a possible way to determine the mixing angles at an $e^+e^-$ collider running at the $Z$ resonance with very high luminosity (a super $Z$ factory) experimentally, which may offer the informations about the dependence of the mixing angle, especially, on the heavy flavors $c$ and $b$.

For convenience, in the paper the ‘effective flavored electro-weak mixing angle’ (appearing in the couplings of $Z$-boson to the relevant $f$-flavored fermion and relating to the experimental measurements directly) is denoted as $\sin^2\theta^f_W = \kappa_f \sin^2\theta_W$, where $\kappa_f$, being the possible electro-weak (EW) correction, can be computed within the SM, so the value of $\sin^2\theta^f_W$ can be extracted from the measured $\sin^2\theta^f_W$.

In SM, the mixing angle $\sin^2\theta^f_W$ does not depend on the flavor $f$ i.e. is the same for various flavors. Hence to determine the value of $\sin^2\theta^f_W$ experimentally so as to extract the value of $\sin^2\theta^f_W$ can be a serious test of SM.

The weak-boson $Z$ couples to the up- and down-type leptons and quarks in definite manners, and depend on the mixing angle and the fermion flavor, therefore in the production $e^+e^- \rightarrow ff$ at $Z$-
boson resonance (a Z-factory) there are ‘asymmetries’ in distributions. Especially, a lot of experimental errors and theoretical uncertainties for the asymmetries: forward-backward ($A_{FB}^{f}$), left-right ($A_{LR}^{f}$) and combined left-right forward-backward ($A_{LR}^{F,FB}$), whose definition will be given precisely, are canceled, thus at LEP-I and SLC, the experimental collaborations via measuring the forward-backward asymmetry $A_{FB}^{f}$ in the production $e^+e^-\rightarrow f\bar{f}$ at Z-boson resonance, determine the effective mixing angle $\sin^2\theta_{\text{eff}}^{q}$, and at SLC with polarized colliding beams via measuring the left-right asymmetries $A_{LR}^{\text{left}}(\text{left}=e)$ and the combined left-right forward-backward $A_{LR}^{\text{left,FB}}(\text{left}=e)$ determines the effective mixing angle $\sin^2\theta_{\text{eff}}^{q}$. The results obtained by the collaborations for the effective mixing angle $\sin^2\theta_{\text{eff}}^{q}$ are consistent with SM.[4] To combine the measurements of the asymmetries at LEP-I and SLC, $\sin^2\theta_{\text{eff}}^{q}=0.23153\pm 0.00016$, $\sin^2\theta_{\text{eff}}^{b}=0.281\pm 0.016$ and $\sin^2\theta_{\text{eff}}^{t}=0.2355\pm 0.0059$ were obtained[9]. The values for $\sin^2\theta_{\text{eff}}^{q}$ extracted from measuring $A_{FB}^{f}$ at LEP-I and those extracted from measuring the left-right asymmetries $A_{LR}^{\text{left}}$ at SLC are different by 3.2 standard deviations, and the determined value $\sin^2\theta_{\text{eff}}^{q}=0.281\pm 0.016$ is larger than the value $\sin^2\theta_{\text{eff}}^{q}=0.23293\pm 0.00025$ determined from the other experiments[9].

The determination at LEP-I and SLC needs to identify the flavors and to determine the so-called thrust axis of the produced jets in the production $e^+e^-\rightarrow f\bar{f}$ at Z-boson resonance, and the systematic errors owing to the miss-identifying flavor, the efficiency on identifying the flavors and determination of the the thrust axis for the jet production $e^+e^-\rightarrow f\bar{f}$ at Z-boson resonance are hard and cannot be improved further, especially, to determine the thrust axis of each jet is crucial for the determination of the forward-backward and combined left-right forward-backward asymmetries, thus to avoid the experimental errors we try to consider some way else for determining the electron-weak mixing angle $\sin^2\theta_{\text{eff}}^{q}$. Namely we suggest to determine the mixing angle by measuring the produced doubly heavy hadrons $B_c$ or $QQ'$ in the processes $e^+e^-\rightarrow B_c+\cdots$ and $e^+e^-\rightarrow HQQ'+\cdots$ ($Q,Q'=b,c; q=u,d,s$) at Z-pole. It is because that the flavor(s) and the out-going direction of the produced doubly heavy hadrons can be well-determined.

The calculations on the production of $e^+e^-\rightarrow B_c+\cdots$ and $e^+e^-\rightarrow HQQ'+\cdots$ at Z-pole in Ref.[11] show the production cross-sections are not very small and the differential cross-sections behave asymmetric. In addition, the experimental results in Ref.[12] indicates that the production mechanism adopted in Ref.[11] works well, thus in the paper we quantitatively investigate the capability possibility via measuring the production of $e^+e^-\rightarrow B_c+\cdots$ and $e^+e^-\rightarrow HQQ'+\cdots$ at Z-boson resonance to determine the mixing angle $\sin^2\theta_{\text{eff}}^{q}$. Since that as shown by LEP-I[1,4] and SLC[3,8], the theoretical uncertainties for the asymmetries $A_{FB}^{f}, A_{LR}^{f}$ and $A_{LR}^{F,FB}$ of the production $e^+e^-\rightarrow f\bar{f}$ at Z-boson resonance are cancelled greatly, so having the experiences of LEP-I and SLC, in the paper we are seriously considering to determine the effective mixing angle $\sin^2\theta_{\text{eff}}^{q}$ via measuring the asymmetries $A_{FB}^{f}, A_{LR}^{f}$ and $A_{LR}^{F,FB}$ of the produced doubly heavy flavor hadrons in the processes $e^+e^-\rightarrow B_c+\cdots$ and $e^+e^-\rightarrow HQQ'+\cdots$. Finally one will see that it can be a complement to the determination by LEP-I[1,4] and SLC[3,8] which is via measuring the asymmetries $A_{FB}^{f}, A_{LR}^{f}$ and $A_{LR}^{F,FB}$ of the jet production $e^+e^-\rightarrow f+\cdots$ at a Z-factory.

To be a complement in determination of the effective mixing angle $\sin^2\theta_{\text{eff}}^{q}$, especially for $f=c,b$, in the paper we suggest determining the electro-weak mixing angle via measuring the asymmetries $A_{FB}^{f}, A_{LR}^{f}$ and $A_{LR}^{F,FB}$ of the produced explicit doubly heavy flavor hadron, $B_c$ meson or a double heavy baryon $HQQ'$ ($Q=c,b;Q'=b,c; q=u,d,s$), in the process $e^+e^-\rightarrow B_c+\cdots$ or $e^+e^-\rightarrow HQQ'+\cdots$ at a Z-factory, instead of that via measuring the asymmetries $A_{FB}^{f}, A_{LR}^{f}$ and $A_{LR}^{F,FB}$ of the produced heavy flavor ($Q$) jet in the production $e^+e^-\rightarrow Q+\cdots$ as done by LEP-I and SLC. We also pointed out to be the complement to those of LEP-I and SLC, to suppress the statistic errors, the luminosity of the Z-factory should be so high $L\sim 10^{35}-36\text{cm}^{-2}\text{s}^{-1}$ (a super Z-factory) as FCC-ee, CEPC and ILC etc which are under consideration.

Note that recently one of the $\Xi_{c}^{*+}$ as the first doubly heavy baryon $HQQ'$ has been observed by LHCb[13,14]. The news also motivates us to propose the proposal.

The paper is organised as follows: Following the Introduction, in Section II, based on effective theory NRQCD[17], the adopted formulas as those in Ref.[11] for calculating the production of the various doubly heavy hadron and the definition of the asymmetry quantities which we are calculating are presented. In Section III, we show that the uncertainties, such as that caused by various values of the heavy quark masses taken, are suppressed greatly, so having the experiences of LEP-I and SLC, to suppress the statistic errors, the luminosity of the Z-factory should be so high $L\sim 10^{35}-36\text{cm}^{-2}\text{s}^{-1}$ (a super Z-factory) as FCC-ee, CEPC and ILC etc which are under consideration.

Section IV is reserved for discussions on the proposal and conclusions.

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1 The doubly heavy baryon $\Xi_{c}^{*+}$ was reported to be observed by SELEX collaboration several years ago[13,14], but the SELEX observation was not confirmed later on.
II. THE APPROACH TO THE PRODUCTION

In the SM, the fermion couplings to physical gauge fields \( A_\mu \) and \( Z_\mu \) are given by

\[
V_{\psi\bar{\psi}\gamma} = e \sum_f Q_f \bar{\psi}_f \gamma^\mu \psi_f A_\mu, \tag{1}
\]

\[
V_{\psi\bar{\psi}Z} = \frac{g}{2\cos\theta_W} \sum_f Q_f \bar{\psi}_f \gamma^\mu (g^f_V - g^f_A \gamma^5) \psi_f Z_\mu, \tag{2}
\]

where \( e = g \sin\theta_W \) is the electric charge carried by positron, and \( Q_f \) is the charge of the fermion \( \psi_f \) in units of \( e \); \( g^f_V \) and \( g^f_A \) are the couplings for the vector and axial-vector currents of fermions to the weak boson \( Z \):

\[
g^f_V = t_{3L}(f) - 2Q_f \sin^2\theta_W, \tag{3}
\]

\[
g^f_A = t_{3L}(f), \tag{4}
\]

where \( t_{3L}(f) \) is the weak isospin of the fermion \( f \).

In order to take into account higher order EW corrections, one may adopt the running QED coupling \( \alpha(s) \) and \( G_F \) to the LO cross section, and replace the tree-level couplings of \( Z f \bar{f} \) couplings with the effective couplings\( ^{22} \)

\[
\bar{g}^f_V = \sqrt{\rho_f} (t_{3L}(f) - 2Q_f \kappa_f \sin^2\theta_W), \tag{5}
\]

\[
\bar{g}^f_A = \sqrt{\rho_f} t_{3L}(f). \tag{6}
\]

The effective weak mixing angle is defined as\( ^{22} \)

\[
\sin^2\theta^f_{\text{eff}} = \kappa_f \sin^2\theta_W. \tag{7}
\]

One of the typical Feynman diagrams for the production of the \( B_c(B_c^*) \) meson is shown in Fig. 1 and one of the typical Feynman diagrams for the production of the \( (QQ'q) \) baryon is shown in Fig. 2. The way to calculate the cross sections of the \( B_c \) meson production, \( e^+e^- \rightarrow (\gamma/Z) \rightarrow B_c(B_c^*) + b + \bar{c} \), can be found in Ref.\( ^{11} \), so we will not repeat it here; whereas the way to calculate the doubly heavy baryon production should be illustrated here. In the most references, such as those in Refs.\( ^{18–20} \), the production is about the relevant double heavy diquark as a core inside the doubly heavy baryon to be produced. In the reference\( ^{21} \), the further process from the produced doubly heavy diquark to the doubly heavy baryon is considered. Thus here the doubly heavy baryon production will be calculated as that in reference\( ^{21} \).

The strategy in Ref.\( ^{21} \) to consider the production of the doubly heavy baryon, \( e^+e^- \rightarrow (\gamma/Z) \rightarrow (QQ'q) + Q + Q' + q \), is to divide it into two steps: one is the production of the core of the doubly heavy baryon \( (QQ'q) \), the relevant diquark \( \langle QQ' \rangle_3 \) in color anti-triplet \( 3 \) and with the other suitable quantum numbers, and the second step is the fragmentation from the diquark \( \langle QQ' \rangle_3 \) to the doubly heavy baryon \( (QQ'q) \), here \( Q, Q' \) denote heavy quarks, \( q \) denotes a light quark. As for the first step, based on the factorization of NRQCD\( ^{17} \), the production of the heavy diquark \( (QQ') \) with suitable quantum numbers, similar to the \( B_c \) production, is computed. As for the second step, similar to the fragmentation of a heavy quark \( Q \) into a heavy meson \( (Qq) \), the fragmentation of the produced diquark \( \langle QQ' \rangle_3 \) into a doubly heavy baryon \( (QQ'q) \) through catching a light quark \( q \) from the environment is computed.

Now let us illustrate the way to compute the production of a doubly heavy baryon at a \( Z \)-factory precisely. Based on NRQCD, the diquark production can be written as the follows:

\[
d\sigma \left( e^+e^- \rightarrow (\gamma/Z) \rightarrow \langle QQ' \rangle_3 + \bar{Q} + Q' \right) = \sum_n d\sigma \left( e^+e^- \rightarrow (\gamma/Z) \rightarrow (QQ'[n] + \bar{Q} + Q') \right) \cdot \langle OQQ'[n] \rangle, \tag{8}
\]

where the long-distance matrix element \( \langle OQQ'[n] \rangle \) represents the transition probability from the two quark state \( (QQ')_3[n] \) into the diquark state \( (QQ')_3 \). Note here that since the wave function of the diquark state \( (QQ')_3 \) should be totally antisymmetric under the exchange of the two quarks, so particularly when \( Q' = Q \), a diquark \( (QQ')_3 \) at the ground state (in \( S \)-wave and anti-color-triplet \( 3 \)) must be in spin-triplet \( S = 1 \).

\(^2\)According to QCD, in short distance two quarks in color anti-triplet will be attractive strongly, but in color sextet will be repulsive strongly, so here only the case in color anti-triplet is considered.
As for the second step, for the fragmentation of the heavy diquark \((QQ')_3\) into a doubly heavy baryon \((QQ'q)\), the ‘inspired Peterson model’ is employed. According to the model, the fragmentation function is assumed as

\[
D^H_{(QQ')_3}(z) = \frac{N^H_{(QQ')_3}}{z[1 - 1/z - e_H/(1 - z)]^2},
\]

where the energy fraction \(z = E_{(QQ')_3}/E_{(QQ'q)}\), \(H\) denotes the doubly heavy baryon \((QQ'q)\) and \(e_H \approx m_q^2/m^2_{(QQ')_3}\). The normalization factor \(N^H_{(QQ')_3}\) is fixed through

\[
\int_0^1 D^H_{(QQ')_3}(z)dz = R^H_{(QQ')_3},
\]

where \(R^H_{(QQ')_3}\) is the fragmentation probability for the diquark \((QQ')_3\) fragments to doubly heavy baryon \(H\). The fragmentation probabilities for different doubly heavy baryons with different light constituent quark may be estimated by the Lund model, which gives \(R^{(QQ')_3}\), \(R^{(QQ')_3}: R^{(QQ')_3} = 1 : 1 : 0.3\). The heavy quark production in the soft fragmentation is comparatively small so it is ignorable, thus \(R^{(QQ')_3} = R^{(QQ')_3} = 43.478\%\) and \(R^{(QQ')_3} = 13.044\%\) are obtained. With these probabilities, we can determine the values of \(N^H_{(QQ')_3}\). In the numerical calculations, the light quark masses \(m_u = m_d = 0.3\) GeV and \(m_s = 0.5\) GeV are adopted. Now the production of the doubly heavy baryons may compute out via the convolution of the diquark differential cross section with the fragmentation function:

\[
\frac{d\sigma(H(z))}{dz} = \int_z^1 dy \frac{d\sigma(e^+e^- \to (QQ')_3(y) + \bar{Q} + \bar{Q}')}{dy} \cdot D^H_{(QQ')_3}(y/z) .
\]

Now being observables for present proposal, let us define the asymmetries in production of doubly heavy hadrons at a Z-factory. The forward-backward asymmetry \((A_{FB})\) is defined as

\[
A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B},
\]

where \(\sigma_F (\sigma_B)\) is the cross section for the produced doubly heavy-flavored hadron travels in the forward (backward) with respect to the direction of the initial electron.

If the collision beams are polarized, there is left-right asymmetry \((A_{LR})\) which is defined as

\[
A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R},
\]

\[3\] The original model is for a heavy quark to fragment to a heavy meson, whereas here we extend it for a doubly heavy diquark fragmenting into a baryon inspired.
masses of the heavy quarks, and we try to take the two possible choices of the mass values as in Eq. [13] to see it.

Input – 1 : \( m_b \approx 4.9 \text{ GeV}, \ m_c \approx 1.5 \text{ GeV}, \)
Input – 2 : \( m_b \approx 5.1 \text{ GeV}, \ m_c \approx 1.8 \text{ GeV}. \) \( (18) \)

Firstly we calculate the differential cross-sections for the production of the doubly heavy baryons \( \Xi_{bc} \) with the reasonable masses as Input-1 or Input-2 (Eq. [18]), and the obtained results are presented in Fig[3]. From Fig[3] one may see that the differences of the differential cross-sections caused by the taken heavy quark masses are quite large, so we think that one may determine the heavy quark masses appearing in the mechanism of the production by fitting the differential cross-section when the experimental data are available in the future. If the differences of the differential cross-sections are considered as the theoretical ‘uncertainty’ for the production caused by the taken values of the heavy quark masses, so the so-called ‘uncertainty’ is quite large for the differential cross-sections of the production. Thus bearing the value of the mixing angle \( \sin^2 \theta_{\text{eff}} \) which is considered as that of SM and is about 0.232 determined by the exist experiments in mind, we further calculate the asymmetries \( A_{FB}, A_{LR} \)

and \( A^\prime_{FB,R} \) of the production with the masses Input-1 or Input-2 (Eq. [18]) as input. In order to see the asymmetries \( A_{FB}, A_{LR} \) and \( A^\prime_{FB,R} \) how sensitive to the relevant flavored effective electro-weak mixing angles, we calculate the asymmetries with ‘the flavored effective electro-weak mixing angles’ taken the SM ‘common value’, except one of the relevant flavored effective electro-weak mixing angles which takes the SM ‘common value’ 0.232 but with an assumed error (±0.010), namely as the three cases: i) \( \sin^2 \theta_{\text{eff}}^{b,c} = 0.232 \pm 0.010, \) ii) \( \sin^2 \theta_{\text{eff}}^b = 0.232 \pm 0.010, \) and iii). \( \sin^2 \theta_{\text{eff}}^c = 0.232 \pm 0.010, \) \( \sin^2 \theta_{\text{eff}}^{b,c} = 0.232. \) The obtained results are collected in tables (Tabs. [III][IV][III], here the errors of the asymmetries in the tables are due to the input ‘error’ for the electro-weak mixing angles as that in the three cases i), ii), iii). From the tables one may see clearly that with Input-1 or with Input-2 as input, the results of the asymmetries are very closed to each other for the two inputs of the heavy quark masses, that means the so-called theoretical ‘uncertainty’ of the asymmetries caused by inputting various heavy quark masses (Input-1 or Input-2) are cancelled greatly i.e. the asymmetries predicted theoretically are not sensitive to the heavy quark masses.

| Hadrons | \( A_{FB} \) | \( A_{LR} \) | \( A^\prime_{FB,R} \) |
|---------|-------------|-------------|----------------|
| \( B_c (\text{Input-1)} \) | \(-9.0^{+5.03}_{-4.94} \times 10^{-2} \) | 0.143±0.078 | -0.634±0.000 |
| \( B_c (\text{Input-2)} \) | \(-8.7^{+8.41}_{-4.76} \times 10^{-2} \) | 0.143±0.078 | -0.611±0.000 |
| \( B_c^1 (\text{Input-1)} \) | \(-9.44^{+5.23}_{-5.14} \times 10^{-2} \) | 0.143±0.079 | -0.659±0.000 |
| \( B_c^2 (\text{Input-2)} \) | \(-9.20^{+5.10}_{-5.01} \times 10^{-2} \) | 0.143±0.079 | -0.643±0.000 |
| \( (cc)^3 S_1^1 (\text{Input-1)} \) | \(7.03^{+3.83}_{-3.89} \times 10^{-2} \) | 0.143±0.078 | 0.496±0.000 |
| \( (cc)^3 S_1^2 (\text{Input-2)} \) | \(7.00^{+3.81}_{-3.87} \times 10^{-2} \) | 0.143±0.078 | 0.488±0.000 |
| \( (bc)^3 S_0^0 (\text{Input-1)} \) | \(9.40^{+5.11}_{-5.20} \times 10^{-2} \) | 0.143±0.078 | 0.656±0.000 |
| \( (bc)^3 S_1^1 (\text{Input-2)} \) | \(9.21^{+5.03}_{-5.12} \times 10^{-2} \) | 0.143±0.078 | 0.642±0.000 |
| \( (bc)^3 S_1^2 (\text{Input-1)} \) | \(9.64^{+5.25}_{-5.34} \times 10^{-2} \) | 0.143±0.079 | 0.673±0.000 |
| \( (bc)^3 S_2^2 (\text{Input-2)} \) | \(9.52^{+5.18}_{-5.27} \times 10^{-2} \) | 0.143±0.079 | 0.664±0.000 |
| \( (bb)^3 S_1^1 (\text{Input-1)} \) | \((8.97^{+4.88}_{-4.96} \times 10^{-2} \) | 0.143±0.078 | 0.626±0.000 |
| \( (bb)^3 S_1^2 (\text{Input-2)} \) | \((8.91^{+4.86}_{-4.93} \times 10^{-2} \) | 0.143±0.079 | 0.622±0.000 |

| TABLE I. The values of the asymmetries with \( \sin^2 \theta_{\text{eff}} = 0.232 \) \( (f = b,c) \) and the range of \( \sin^2 \theta_{\text{eff}} = 0.232 \pm 0.010 \), where “Input-1” and “Input-2” denote the input values of the heavy quark masses as those in Eq. [18]. |

Note here that strong coupling \( \alpha_s \) and the wave functions at origin for the doubly-heavy hadrons/\( d\) quarks, as factors, appear in the formulas for the production, so the values achieved by computation of the production asymmetries are independent on the values of \( \alpha_s \) and the wave functions at origin at all, thus the input values of \( \alpha_s \) and the wave functions at origin do not affect the computed asymmetries.

To see how great the cross-sections of the production, we also to calculate the production cross-sections for the production at an \( e^+e^- \) collider running at Z-boson resonance (a Z-factory) with the heavy quark masses as Input-1 and in addition for doubly-heavy baryons the masses of ‘light quarks’ are taken as \( m_u \approx m_d \approx 0.3 \text{GeV}, m_s \approx 0.5 \text{GeV} \). The obtained results are put in the table TAB IV. One may see from the table Tab IV that the production cross-sections for the doubly heavy hadrons can be in the magnitude order \( pb \), i.e. they are
Owing to the recent observation of Ξ_{cc}^{++} at LHCb, one may like to know more about the production of the doubly heavy baryons, such as the differential energy distributions for the production of the doubly heavy baryons not very small, so at a super Z-factory the doubly-heavy hadrons can be produced numerous and the measurements of the asymmetries can reach to quite accurate level in statistics error.

To see the sensitivity of the asymmetries $A_{FB}$, $A_{LR}$ and $A_{FB}^P_{LR}$ of the various doubly-heavy hadron production to the values of the effective specific-flavored mixing angles $\sin^2 \theta_{\text{eff}}^f (f = \text{lept}, c, b)$ quantitatively, we also calculate the asymmetries of the doubly heavy hadron production as one of the effective flavored mixing angles (with the rest relevant ones being fixed by the common value 0.232), and present the results in Figs. 4, 5, 6 accordingly.
In order to put all results into one figure, the results for $B_\text{c}$ and $B_\text{c}^*$ are multiplied by a factor -1; (c) The forward-backward asymmetries of the production of the doubly heavy-flavored hadrons as functions of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$, whereas $\sin^2 \theta_{\text{eff}}^{\text{b}}$ and $\sin^2 \theta_{\text{eff}}^{\text{c}}$ are fixed as 0.232. In order to put all results into one figure, the results for $B_\text{c}$ and $B_\text{c}^*$ are multiplied by a factor -1.

FIG. 5. (a) The forward-backward asymmetries of the production of the doubly heavy-flavored hadrons as functions of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$, whereas $\sin^2 \theta_{\text{eff}}^{\text{b}}$ and $\sin^2 \theta_{\text{eff}}^{\text{c}}$ are fixed as 0.232; (b) The forward-backward asymmetries of the production of the doubly heavy-flavored hadrons as functions of $\sin^2 \theta_{\text{eff}}^{\text{b}}$, whereas $\sin^2 \theta_{\text{eff}}^{\text{c}}$ are fixed as 0.232. In order to put all results into one figure, the results for $B_\text{c}$ and $B_\text{c}^*$ are multiplied by a factor -1; (c) The forward-backward asymmetries of the production of the doubly heavy-flavored hadrons as functions of $\sin^2 \theta_{\text{eff}}^{\text{c}}$, whereas $\sin^2 \theta_{\text{eff}}^{\text{b}}$ and $\sin^2 \theta_{\text{eff}}^{\text{c}}$ are fixed as 0.232. In order to put all results into one figure, the results for $B_\text{c}$ and $B_\text{c}^*$ are multiplied by a factor -1.

FIG. 6. (a) The left-right forward-backward asymmetries of the production of the doubly heavy-flavored hadrons as functions of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$, whereas $\sin^2 \theta_{\text{eff}}^{\text{b}}$ and $\sin^2 \theta_{\text{eff}}^{\text{c}}$ are fixed as 0.232; (b) The left-right forward-backward asymmetries of the production of the doubly heavy-flavored hadrons as functions of $\sin^2 \theta_{\text{eff}}^{\text{b}}$, whereas $\sin^2 \theta_{\text{eff}}^{\text{c}}$ are fixed as 0.232. In order to put all results into one figure, the results for $B_\text{c}$ and $B_\text{c}^*$ are multiplied by a factor -1; (c) The left-right forward-backward asymmetries of the production of the doubly heavy-flavored hadrons as functions of $\sin^2 \theta_{\text{eff}}^{\text{c}}$, whereas $\sin^2 \theta_{\text{eff}}^{\text{b}}$ and $\sin^2 \theta_{\text{eff}}^{\text{c}}$ are fixed as 0.232. In order to put all results into one figure, the results for $B_\text{c}$ and $B_\text{c}^*$ are multiplied by a factor -1.

with different light constituent quark at a Z-factory, so we also calculate the energy distribution of the production of doubly heavy baryons with different light constituent quark. The results on the differential energy distribution $d\sigma/dz$ for the doubly heavy baryons are presented in Fig 7. As a reference, the distributions for $(QQ')$-diquark are also presented in this figure, and one can see that the distributions of the doubly heavy baryons are
changed obviously in comparison with the distributions of the corresponding diquark. The maximum point of the distributions of the doubly heavy baryons (QQ’q) is shifted to a smaller value of energy fraction in comparison with that of the distributions of the corresponding diquark ⟨QQ’⟩.

![Diagram](image_url)

FIG. 7. (a) The differential energy distribution for doubly charmed baryons (ccq) with different light constituent quark; (b) The differential energy distribution for doubly charmed baryons (bcq) with different light constituent quark. (c) The differential energy distribution for doubly charmed baryons (bbq) with different light constituent quark.

In addition, from the dependence on sin²θ eff (sin²θ b eff) shown in the figures and tables, at such a super Z-factory via the forward-backward asymmetry and left-right forward-backward asymmetry of Ξ cc , Ω cc (Ξ bb , Ω bb ) production is more ideal for measuring sin²θ eff (sin²θ b eff) than via asymmetries of the other hadron production. It is because that the forward-backward symmetry and left-right forward-backward asymmetry of Ξ cc , Ω cc (Ξ bb , Ω bb ) only depends on the two effective flavored mixing angles sin²θ eff and sin²θ eff (sin²θ b eff) whereas the effective lepton-flavored mixing angles sin²θ lept can be determined very accurate via the asymmetries of the leptonic production e⁺e⁻ → l¯l at the Z-factory.

IV. DISCUSSIONS AND CONCLUSION

Bearing the problem on test of SM relating to the electro-weak mixing angles sin²θ W lept , sin²θ W and sin²θ b of the doubly heavy-flavored hadrons (Bc meson, Ξ cc , Ω cc , Ξ bb , Ω bb . . . baryons) at an e⁺e⁻ collider running at Z-resonance, especially, focus the capability and possibility to measure the effective mixing angles at an e⁺e⁻ collider with high luminosity and running at Z-resonance (super Z-factory). The relevant numerical results of the investigation are presented in tables and figures.

From the tables Tabls. III, IV, V, the advantages via measuring the asymmetries AFB, ALR, ALEP of the production of the various doubly-heavy hadrons at a Z-factory so as to determine the effective heavy-flavored mixing angle(s) can be realized well. Such as the advantages include that the important theoretical uncertainty caused by the masses of the relevant heavy flavor (c and/or b) is cancelled greatly and it is very sensitive to determining sin²θ eff and sin²θ b eff etc.

The results in Tabl. IV mean that the total cross-sections of the doubly-heavy hadron production are not very small, and as long as the luminosity of collider is so high as L ≥ 10^{35} cm^{-2} s^{-1} (a super Z-factory such as FCC-ee etc) the asymmetries may be measured quite precisely.

Of the doubly heavy baryons, although only the one Ξ₈⁺ has been observed experimentally, we also calculate the differential energy distribution for the production of the doubly heavy baryons (QQ’q) with the heavy quark Q = c or b, Q’ = c or b, q = u or d or s and put the results in Fig. 7(s) so one may see the characters of the production.

To see the sensitivity on the flavors by measuring the asymmetries to determine the mixing angles, we have computed the dependence of the asymmetries of the doubly-heavy hadron production on the effective flavored electro-weak angles sin²θ eff and sin²θ b eff (Q=c or b) by assuming one relevant flavored effective electro-weak angle to have ‘common value’ (0.232) with a artificial error (0.010), but the rest relevant one to have the ‘common value’ exactly in turn, and the obtained results as curves are presented in figures Figs. 7(a), 7(b), 7(c).

In summary the curves in the figures (Figs. 7(a), 7(b), 7(c)) and the values in the tables (Tabls. III, IV, V, VI) show clearly that how sensitive the asymmetries AFB, ALR, ALEP of the doubly heavy hadron production at a super Z-factory to the effective flavored mixing angles sin²θ eff (Q=c or b) and sin²θ lept. One may see that not only the experimental errors but also the theoretical uncertainties may be canceled a lot if the asymmetries of the production are adopted as observables. Moreover as the angles

4 The value is obtained in Ref. [10].
\[ \sin^2 \theta_{\text{eff}} \] can be determined via measuring the asymmetries of the production \( e^+ e^- \rightarrow \tau^+ \tau^- \) at the Z-factory very well, that have been done at SLC[5–8], so we propose to determine the effective electro-weak mixing angles \( \sin^2 \theta_Q \) \((Q=c, b)\), that was not determined so well via measuring the asymmetries of the jet production \( e^+ e^- \rightarrow Q Q \) \((Q = b, c)\) at the Z-factory, i.e. it would be a complement to the way that via measuring the asymmetries of the jet production at a Z-factory as done at LEP-I and SLC[1–8]. Finally we would like to note here that from Figs. 4, 5, 6 and Tables I, II, III, via the asymmetry \( A_{FB} \) one can determine \( \sin^2 \theta_{\text{eff}} \) \((Q=c \text{ or } b)\) much accurately, thus for the present purpose it would be better that the Z-factory will offer polarized colliding beams.

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