Design, Modeling and Analysis of a XY Nanopositioning Stage for High Speed Scanning

Shenglong Lin, Xianmin Zhang and Benliang Zhu

Guangdong Province Key Laboratory of Precision Equipment and Manufacturing Technology, South China University of Technology, Guangdong Guangzhou 510640, China

Corresponding author: zhangxm@scut.edu.cn

Abstract. In order to increase the imaging speed of scanning probe microscopy (SPM), especially atomic force microscopy (AFM) where needs a high-bandwidth moving stage possessing high resonant frequency and low cross-coupling, the paper proposes a kind of XY nanopositioning stage achieving about 10kHz resonant frequency, 15um × 15um workspace and well decoupled performance. Considering the design objective, a compliant nanopositioning stage is built with doubly clamped beam and parallelogram hybrid beam for overcoming the problem of low natural frequency and cross-coupling performance. By establishing mathematical model of the proposed stage including stiffness model and resonant frequencies model, the paper solves the highest natural frequency with its optimal dimensions of beams by applying optimization. Finally the designed stage is imported to Workbench for the validation of mathematical model by simulation, where presents the FEA results can nicely match the analytical results.

1. Introduction

Nowadays, micro and nanopositioning stage plays an indispensable role in both commercial and scientific field including SPM [1], biological science [2], nano-metrology [3], nano-machining and nano-fabrication [4], optical calibration [5] and micro-nano manipulation [6] due to its high accuracy and fine repeatability. However, high speed nanopositioning stage is extremely needed in various research fields especially in AFM, one kind of SPM recent years. For instance, in the area of biological research, it’s so urgent to acquire biospecimen image in AFM at a video-rate speed because the dynamic process of the sample in nanoscale would be a great benefit for the further study of bioscience [2, 7]. But the dynamic process of sample is always hard to be achieved because of the low resonant frequency of traditional nanopositioning stage in AFM, such as sectored piezoelectric tube scanner [8]. However, compliant mechanism with logical design seems to be a solution for the task, which owns advantages of monolithic structure, ease of processing, no friction and no backlash compared with traditional rigid mechanism, utilizing the elastic deformation of flexible hinges providing with accurate movements [9]. In general, the maximum bandwidth of stage is only 1/100th to 1/10th of the resonant frequency in open-loop [10]. Hence, it’s necessary to design a nanopositioning stage with high resonant frequency which can undistortedly track to a high frequency signals with proper control strategy. Aiming at high-bandwidth moving stage, many researchers have studied much researches in this field. For example, Brian et al. [11] designed a kind of high-bandwidth
nanopositioner with natural frequencies of 24.2 kHz and 6.0 kHz along X and Y axes respectively and a stroke of 9 × 9 μm², which can achieve fine accuracy track under 100 Hz triangular wave. Kam et al. [12] proposed a compliant positioning stage with a 9 × 9 μm² travel range and resonant frequencies of 4.5 kHz and 20 kHz in slow and fast axis respectively. Cai et al. [13] designed a compliant scanning platform with parallel structure, whose natural frequencies of X and Y axes are both 5.6 kHz and the stroke is close to 9 × 9 μm². Readers are also referred to more concerning stage by paper [14]. Based on the previous studies involved with high speed stage, a high bandwidth, relative large travel range and low cross-coupling nanopositioning stage is designed and optimized in the article, which adopts doubly clamped beam and parallelogram hybrid beam.

The remaining contents of this article are as follow. In Sec. II, the details of the structural design and analysis of stage are shown including the establishment of the initial structure and mathematical models of the developed stage. In Sec. III, this part introduce how to figure out the highest natural frequency of developed stage and obtain the optimal dimension of relevant compliant units by optimization in MATLAB. Then, finite element analysis (FEA) of the optimized mechanism is presented for static and dynamic analysis. In final section, the results of the paper and future works are illustrated.

2. The Design and Analysis Of Stage

In accordance with the kinematic structures, a XY flexure-based stages are categorized into two main configurations: serial-kinematic [2, 15, 16] stage and parallel-kinematic [13, 17-21] stage shown in figure 1. The serial-kinematic stage is created by nesting one flexure-based nanopositioner into another, which is more effective and economic with only one fast axis but generally have disadvantages of cumulative error and parasitic motion that hard to be measured [11]. The parallel-kinematic stage is constructed by symmetrical configurations, usually performing high motion accuracy, high mechanical stiffness and low inertia of the moving sample platform helpfully contributing to high resonance frequency. The latter configuration would be employed in our paper because of its potential in high resonance frequencies along both X and Y axis as well as advantage in temperature compensation. Besides, novel non-raster scan methods such as spiral- [22], cycloid- [23], and Lissajous-scan patterns [24] can be applied on parallel-kinematic stage.

![Figure 1. The Serial- and parallel-kinematic stage: (a) Serial-kinematic model; (b) Parallel-kinematic model.](image)

For achieving high resonance frequency while possessing a relative large travel and low coupling errors, the initial structure is presented at figure 2. As shown in figure 2(a), the piezoelectric actuators (PZTs) coloured green are applied to drive the compliant mechanism for nanoscale positioning. The main flexible elements of the stage cover with doubly clamped beam and parallelogram hybrid beam. The former beam is constant-width beam with both ends fixed, providing with a high stiffness
compared to parallelogram hybrid beam; the latter beam is made up of a thicken beam in one end in order to strengthen the bending stiffness of the middle part of doubly clamped beam as well as a lumped compliance hinge in another end for increasing the compliance of end-effector motion. The major component of the two kinds of beam is width-constant beam having advantage of large deformation, compact structure and easy processing. In the developed stage, the symmetrical configuration is adopted to reduce the cross-coupling error shown in figure 2(b).

![Figure 2](image)

**Figure 2.** The structure of developed stage: (a) The structure of XY stage; (b) The schematic diagram of XY stage in x direction.

### 2.1. Stiffness Analysis of Stage

The stiffness of stage is a crucial factor for acquiring high resonance frequency so it’s significant to precisely establish the stiffness mathematical model of stage. Due to the symmetrical configuration of stage, only one kinematic chain need to be analyzed in the paper. And the stiffness analysis of each kinematic chain can be divided into two sections, including doubly clamped beam and parallelogram hybrid beam.

As to build the stiffness of cantilever beam, the paper employed Castigliano’s second theorem, one of effective method to derive the relationship of external load and deformation when the beams deform elastically and comply with Hooke’s law [25]. According to the theorem, the force-displacement relation of the variable cross-section beam in i direction can be expressed as

$$u_i = \frac{\partial U}{\partial F_i}$$  \hspace{1cm} (1)

where $u_i$ is the displacement in i direction, $F_i$ is the external force in i direction and $U$ denote the total strain energy of the beam, including bending strain energy, shearing strain energy, axial strain energy and torsional strain energy, which can be shown as

$$U = U_b + U_s + U_a + U_t$$  \hspace{1cm} (2)

The beams of proposed stage are mainly influenced by bending moment $M_z$ and force $F_y$ which are presented in figure 3. Therefore, $U_a$ and $U_t$ is equal to zero, $U_b$ and $U_s$ can be presented as

$$U_b = \int L \frac{M_z^2}{2EI} ds,$$  \hspace{1cm} (3)

$$U_s = \int \frac{\alpha F_y^2}{2GA} ds$$  \hspace{1cm} (4)
The $E$, $I_z$ and $G$ denote the Young’s modulus of beam material, inertia moment respect to $Z$ axis and the shear modulus of beam material respectively. There would be gain a relationship between force and displacement when we take the partial derivative of $U$ with respect to $M_z$ and $F_y$.

$$u_y = C_{y-F_y} F_y + C_{y-M_z} M_z$$

(5)

$$C_{y-F_y} = \frac{12}{Eh} \int^L_0 \frac{x^2}{t(x)^3} dx + \frac{\alpha}{Gh} \int^L_0 \frac{1}{t(x)} dx$$

(6)

$$C_{y-M_z} = \frac{12}{Eh} \int^L_0 \frac{x}{t(x)^3} dx$$

(7)

$t(x)$ represent the sectional dimension and $h$ is the height of proposed stage.

The doubly clamped beam is directly connected to the PZTs and parallelogram hybrid beam, which provide the stage with accurate motion and high stiffness. When the equation (5) comes to the doubly clamped beam, it can be equivalent to the fixed-guided beam shown in figure 3(a), whose form can be directly used in Castigliano’s second theorem with invariant thickness. Therefore, the stiffness of single doubly clamped beam can be obtained by

$$C_f = \frac{3L_f^3}{16EhT_f^3} + \frac{\alpha L_f}{4GhT_f}$$

(8)

For the parallelogram hybrid beam presented in figure 3(b), the segmented function with respect to its variable thickness would be listed below:

$$t(x) = \begin{cases} 
T_p - 2x(2R - x) & x \in [0,2R] \\
T_p & x \in [2R, 2R + L_p] \\
T & x \in [2R + L_p, 2R + L_p + L]
\end{cases}$$

(9)

Take $R$, $T$ and $L$ as 0.3(mm), $T_p+1.6$(mm) and 0.5(mm). Similarly, we can get the compliance of parallelogram hybrid beam by substituting equation (9) into (5).

2.2. The Resonance Frequency Estimation of the Stage

As discussed previously, due to the great significance of improving resonance frequency of stage it’s important to correctly build natural frequency model of proposed stage ready for the optimization. The natural frequency of stage contain two aspects including stage itself and stage attached PZTs shown in
For the stage itself, it’s assumed that the stage is a two-degree-of-freedom undamped system, wherefore the Lagrange equation can be obtained as follow [26]:

\[ M \ddot{q} + Kq = 0 \]  \hspace{1cm} (10)

Here, \( M = \text{diag} (M_x, M_y) \), \( K = \text{diag} (K_x, K_y) \), \( q = [x, y]^T \). \( M'_x \) and \( M'_y \) represent the equivalent mass of stage in \( x \) and \( y \) direction respectively while \( K_x \) and \( K_y \) denote the stiffness along \( x \) and \( y \) direction respectively. Due to the symmetrical structure, \( M'_x = M'_y = M \), \( K_x = K_y \). If \( q = A \sin(\omega t + \theta) \), \( A = [A_x, A_y]^T \), we can get the generalized eigenvalue problem \( (K - \omega^2 M)A = 0 \), then the necessary and sufficient condition for \( A \) having non-zero solution is \( |K - \omega^2 M| = 0 \). Finally the angular natural frequency of stage \( \omega_1, \omega_2 \) can be obtained based above equation and the natural frequency of the stage is

\[ f_{\text{res}} = \frac{\min[\omega_1, \omega_2]}{2\pi} \]  \hspace{1cm} (11)

![Figure 4. One-degree-of-freedom second order undamped system.](image)

Similarly, when PZTs are considered to the model, the stage can be equivalent to one-degree-of-freedom second order undamped system as shown in figure 4. \( M'_p \), \( M'_s \) represent the equivalent mass of PZTs and stage respectively while \( K_p, K_s \) denote the stiffness of PZTs and stage. Hence, the resonant frequency of stage attached PZTs can be expressed by

\[ f_{\text{res}} = \frac{\sqrt{(K_p + K_s)(M_p + M_s)}}{2\pi} \]  \hspace{1cm} (12)

### 3. Dimensional Optimization of Stage Model

Generally, a rigid and compact structure would greatly contribute to a high resonant frequency due to its high stiffness but it would sacrifice a part of stage’s travel range. In other words, the high stiffness and large stroke of the stage are contradictory. What’s more, the accuracy and stiffness also exist conflict [27] but the relationship won’t be taken into account in this article. Based on the mentioned above, the paper attempts to optimally design a kind of XY nanopositioning stage whose resonance frequency can achieve 10kHz and travel range can be around 15\( \mu \)m. Beside, a good decoupled performance between \( x \) and \( y \) axis are expected.

**Table 1. The piezoelectric stack actuators technical parameters.**

| Model       | Length*Width*Height | Driving Voltage | Nominal Stroke | Stiffness |
|-------------|----------------------|-----------------|----------------|-----------|
| NAC2014-H14 | 7*7*14 mm³           | 150V            | 19.8\( \mu \)m | 104N/\( \mu \)m |

In order to achieve the objective referred above, optimization theory [28] is employed for obtaining max first resonant frequency by adjusting the dimensions of beam. For the workspace of stage, two
suitable piezoelectric stack actuators would be applied on the article, whose specific parameters are shown in table 1. In the meantime, aluminium alloy 7075 are employed for its Young’s modules, density, and yield stress up to $71.7 \text{ GPa}, 2810 \text{ kg/m}^3$, and 500 MPa, respectively. The target function and optimization variable can be expressed by $min f(x) = f_{res}$ and $x = [L_p, T, L_p, T_p]$, respectively. The constraints of the optimization model are listed as follow:

1) The constraint of stage’s travel range: As a crucial performance index of nanopositioning stage, the stroke of stage is largely influenced by the nominal stroke of piezoelectric stack actuator $R_{nom}$, stiffness of piezoelectric stack actuator $K_p$, stiffness of the stage $K_s$ and preload force $F_{pre}$. So the max displacement of stage in one direction is

$$D = \frac{K_s R_{nom} - F_{pre}}{K_s + K_p} \geq 15 \mu \text{m}$$

(13)

2) Stiffness constraint between doubly clamped beam and parallelogram hybrid beam: Doubly clamped beam provides the stage with displacement and a majority of the stiffness of whole stage while the parallelogram hybrid beam plays an important role in guiding as well as lower cross-coupling errors. Hence, confining the compliance ratio between parallelogram hybrid beam and doubly clamped beam to a proper range would be greatly beneficial for decoupling performance. The stiffness ratio in paper is

$$15 \leq \frac{C_p}{C_f} \leq 30$$

(14)

3) The max stress constraint: In the deformation process of beams, the maximum stress should be less than the allowable stress of the material. The maximum stress of doubly clamped beam $\sigma'_{max}$ and parallelogram hybrid beam $\sigma''_{max}$ can be derived as follow.

$$\sigma'_{max} = \frac{3L_p R}{2T^2 h C_f} \leq [\sigma]$$

(15)

$$\sigma''_{max} = \frac{6K_sK_p\arcsin\left(\frac{R}{R + \sqrt{R^2 - 4R}}\right)}{(T_p - 2R)^2} \leq [\sigma]$$

(16)

$R$ represent the maximum displacement of stage, coefficients $K_\theta$ and $K_\varphi$ concerning size and material of flexure hinge[29] while $[\sigma]$ is always set to be a half yield stress of the material.

4) The constraint of valuable range: In order to ensure the sufficient stiffness in the z-axis direction, we set the thickness of the stage to 10 mm. The lower limit of optimal variables is given to assure that the optimized size is not too small, which will not only affect the assembly of the platform but also increase the difficulty of processing. Moreover, The upper limit also need to be set for a compact stage.

$$20 \text{ mm} < L_p < 45 \text{ mm}, 1.5 \text{ mm} < T < 5 \text{ mm}$$

$$5 \text{ mm} < L_p < 15 \text{ mm}, 0.8 \text{ mm} < T_p < 1.5 \text{ mm}$$

(17)

Aiming at solving the optimal value of nonlinear multivariate functions with constraints, the "fmincon" optimization function provided by MATLAB R2016a optimization toolbox is adopted. Taking the initial value as $x_0 = [30.3, 10.1, 2] \text{ mm}$, the max natural frequency is 10.04 kHz without PZTs and 17.00 kHz with PZTs when the optimized variable is $x^* = [26.1, 2.5, 6.95, 0.8] \text{ mm}$.

### 4. Simulation for Validation

After the dimensional optimization of developed stage above, both static and dynamic simulation is indispensable for validating optimization results. In order to verify the accuracy of optimization results, finite element analysis in ANSYS WORKBENCH 16.0 software is adopted in this paper, by which the stiffness, travel range, dynamic performance, cross-coupling error, and stress analysis can be explicitly presented as follow.
Table 2. The comparison of theoretical results and FEA results.

|                        | Stiffness(N/um) | Workspace(um²) | Resonance frequency(Hz) |
|------------------------|-----------------|----------------|-------------------------|
| Theoretical results    | 22.75           | 15.07×15.07    | 10036.78/10036.78       |
| FEA results            | 22.28           | 15.13×15.13    | 9618.00/9626.8          |
| Error(%)               | 2.07%           | 0.40%          | 4.17%/4.08%            |

Table 2 shows the contrast between theoretical results and FEA results, showing a goodness of fit. The reason accounting for the errors among theoretical results and FEA results mainly results from the assumption of parallelogram hybrid beam whose axis is parallel to the actuation direction being a rigid body. In fact, there still exist tiny deformation in these beam leading to the loss of stiffness and resonance frequency in FEA. The relationship of force and displacement of stage are presented by figure 5(a). As shown in the figure, the X-axis displacement primarily actuated by the X-axis force while the Y-axis force has almost no effect on X-axis displacement exhibiting a good decoupled performance of stage. In figure 5(b), the maximum equivalent stress are far less than he allowable stress of the material, 250MPa. What’s more, by comparing to modal analysis of traditional stage in identical dimensions in figure 6(b), the proposed stage in figure 6(a) performs better decoupling performance in high frequency without sacrificing much natural frequency, the ratio of coupling errors between them is about 1:15.

![Figure 5](image-url)
1st modal 9618Hz  2nd modal 9627Hz  3rd modal 11526Hz

(a)

1st modal 9725 Hz  2nd modal 9732 Hz  3rd modal 11779Hz

(b)

Figure 6. The first three order modal of proposed stage and traditional stage in identical size. (a) The proposed stage using parallelogram hybrid; (b) The traditional stage using parallelogram beam.

5. Result and Future Work

The article proposes a XY nanopositioner by employing optimization method with advantages of high dominant frequency, relative large workspace and low cross-coupling, corresponding to 10kHz, 15×15μm². In the following work, experiment system on the stage would be built and experiments concerning bandwidth would be conducted with suitable control strategy.

References
[1] R. Wang, X. Zhang, IEEE Trans. Ind. Electron 65, 3 (2017).
[2] T. Ando, N. Kodera, E. Takai, D. Maruyama, K. Saito, A. Toda. PNAS 98, 22 (2001).
[3] A. D. Mazzeoa, A. J. Steina, D. L. Trumpera, R. J. Hockenb, Precision Engineering 33, 2 (2009).
[4] S. Verma, W. Kim, H. Shakir, IEEE Trans Ind Appl 41, 5 (2005).
[5] H. Marth, B. Lula, Proc. SPIE 62731K (2006).
[6] M. L. Culpepper, G. Anderson, Precis. Eng 28, 4 (2004).
[7] H. Y. Kim, D. H. Ahn, D. G. Gweon, Rev. Sci. Instrum 83, 5 (2012).
[8] G. Schitter, A. Stemmer, IEEE Trans. Control Syst. Technol 12, 3 (2004).
[9] M. Liu, X. Zhang, S. Fatikow, Rev. Sci. Instrum 87, 5 (2016).
[10] G. M. Clayton, S. Tien, K. K. Leang, Q. Zou, S. Devasia, J. Dyn. Sys., Meas., Control 131, 6 (2009).
[11] B. J. Kenton, K. K. Leang, IEEE/ASME Trans. Mech 17, 2 (2012).
[12] S. P. Wadikhaye, Y. K. Yong, S. O. R. Moheimani, Rev. Sci. Instrum 85,10 (2014).
[13] K. Cai, X. Xu, Y. Tian, X. Liu, D. Zhang, B. Shirinzadeh, Microsystem Technologies 24, 7 (2018).
[14] Y. K. Yong, S. O. R. Moheimani, B. J. Kenton, K. K. Leang, Rev. Sci. Instrum 83,12 (2012).
[15] K. K. Leang, A. J. Fleming, Asian Journal of Control 11, 144 (2009).
[16] T. Ando, N. Kodera, T. Uchihashi, A. Miyagi, R. Nakakita, H. Yamashita, e-Journal of Surface Science and Nanotechnology 3, 3 (2006)
[17] T. Lu, D. C. Handley, Y. K. Yong, C. Eales, Industrial Robot: An International Journal 31, 4 (2004).
[18] S. Salapaka, A. Sebastian, J. P. Cleveland, M. V. Salapaka, Rev. Sci. Instrum 73, 9 (2002).
[19] G. Schitter, K. J. Åström, B. E. Demartini, P. J. Thurner, K. L. Turner, P. K. Hansma, IEEE Trans. Control Syst. Technol 15, 5 (2007).
[20] Y. K. Yong, S. S. Aphale, S. O. R. Moheimani, IEEE Trans. Nano 8, 1 (2009).
[21] C. X. Li, G. Y. Gu, M. J. Yang, L. M. Zhu, Rev. Sci. Instrum 84, 12 (2013).
[22] I. A. Mahmood, S. O. R. Moheimani, B. Bhikkaji, IEEE Trans. Nano 10, 2 (2011).
[23] Y. K. Yong, S. O. R. Moheimani, I. R. Petersen, Nanotechnology 21, 36 (2010).
[24] A. Bazaei, Y. K. Yong, S. O. R. Moheimani, Rev. Sci. Instrum 83, 6 (2012).
[25] R. Lin, X. Zhang, S. Fatikow, Rev. Sci. Instrum 84, 8 (2013).
[26] L. S. Jacobsen, R. S. Ayre, Engineering vibration. McGraw-Hill (1994).
[27] S. Awtar, A. H. Slocum, E. Sevincer, J. Mech. Des 129, 6 (2007).
[28] J. Nocedal, S. J. Wright, Numerical Optimization. Springer (2006).
[29] J. W. Ryu, D. Gweon, K. S. Moon, Precis. Eng 21, 1 (1997)