Focusing of Radially polarized Light Using Gaussian laser beam Near its focal length

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Abstract. The Richards-Wolf diffraction model is used to examine focusing of RPGB radially polarized Gaussian beam of low & large numerical aperture. The longitudinally polarized component becomes predominant at high numerical aperture and high radius. Numerical calculations for varied beam characteristics, numerical aperture (NA), and beam radius ($\theta_0$) show the total optical intensity distribution in the focal region. The whole optical intensity distribution in the focal region is illustrated using numerical computations for various beam parameters, numerical aperture (NA), and beam radius ($\theta_0$). The suggested technique appears to produce subwavelength focal spot of $0.51\lambda$ & significant focal depth of $21\lambda$. Optical trapping, data storage, biomedical imaging, laser cutting, microscopy, and the manipulation of optical traps with high refractive index particles can all benefit from such beams with a small spot size and large focal depth.

Keywords: Radial polarization, Focal length, Focal spot, Wavelength, Numerical aperture, Beam radius, Beam amplitude.

1. Introduction

One of the most important subjects in optical research and applications is focusing light on a very tight spot. Microscopy, data storage, imaging, and lithography are all applications that benefit from a small focal spot \cite{1}. With the rising aperture angle and decreasing wavelength, the size of a focal spot decreases. As a result, the characteristics of laser beam focused by large NA lens have gained a lot of attention\cite{2-4}. Wang et al. \cite{5} use binary-phase and large NA lens to focus a radially polarized Bessel–Gaussian beam into a uniform and non-diffracting axial light beam with a sub-diffraction beam size of $0.43\lambda$. On focusing of RPB, a sub-wavelength light needle with a depth of focus of around $4\lambda$ was recently created \cite{6}. Another study used dual-beam focusing to create a sub-wavelength light needle with a greater depth of focus (over $9.5\lambda$)\cite{7,8} showed an optical needle with a tight focal spot formed by the longitudinal electric field. The effects of pupil function \cite{9} and apodizations \cite{10} on the focal spot size are presented in a way that is near to reality and can be used as a guide for experimentation. Zha et al. \cite{11} demonstrated that a radially polarized BG beam may be tightly focused by high NA lens & ternary optical element to produce longitudinally polarized beam. “A radially and azimuthally polarized Gaussian beam is proposed for illuminating the pupil plane of the objective lens to achieve a sub-wavelength longitudinal beam with a large focal depth [12]-[14]”

Laser beams with radial polarization are becoming more popular and its large NA focusing capabilities and applicability as large resolution probes have attracted researchers’ attention\cite{15} -[18]. An electric field with a radially polarized electric field is characterized by a strong axial component due to beam polarization symmetry. This characteristic can be utilized in various scientific fields like High-resolution microscopy, microlithography, metrology & nonlinear optics\cite{2}.

We look at how a radially polarized Gaussian laser beam behaves near its focal length in this work. We consider both low and high numerical aperture to analyze the field distributions of Gaussian laser beam.
near its focus and briefly discuss some of its recent applications. To evaluate the focusing of a Gaussian laser beam, we utilized the same Gaussian beam size, different beam diameters, and adjusted numerical aperture and radius throughout this study.

Theoretical modelling of electric field distribution for a RPGB presented in section 2. Section 3 covers numerical analysis for focusing radially polarized Gaussian beams using different parameters of beams. Section 4 discusses some of these beams’ potential applications.

2. Theoretical Modelling

The Richards-Wolf diffraction method is used to investigate the field distribution around the focus of highly focused polarized beams [19] - [20]. Figure 1 shows the schematic diagram that corresponds to this situation. The unit vector \( g_0 \) can be represented by the following way:

\[
g_0 = \cos \phi \hat{i} + \sin \phi \hat{j}
\]

\( g_0 \) represents radial component in the object space. The radial and azimuthal components of the incident field of the pupil plane can then be resolved:

\[
\vec{E}_i(r, \varphi, \phi) = l_0 \left[ e_\varphi^0 g_0 + e_\phi^0 (g_0 \times \vec{k}) \right]
\]

where \( l_0 \) is the radially varying relative amplitude of the field.

According to the Richards-Wolf theory, \( \vec{E}_i(r, \varphi, \phi) \) is [20].

\[
\vec{E}_i(r, \varphi, \phi) = \frac{-i k}{2\pi} \int \int d\theta d\varphi \tilde{a}(\theta, \varphi) \exp [ik(\vec{s} \cdot \vec{r})] \sin \theta \, d\varphi
\]

Where \( \theta_{\text{max}} \) is maximum angle specified by objective lens of NA, \( k \) wavenumber, and \( \tilde{a}(\theta, \varphi) \) is the field strength factor which is given by:

\[
\tilde{a}(\theta, \varphi) = l_0 f P(\theta) \left[ e_\varphi^0 g_1 + e_\phi^0 (g_1 \times \vec{s}) \right]
\]

The polarization unit vector is also affected by the refraction of the focusing lens. The unit vector \( g_1 \) is perpendicular to \( \vec{s} \), the ray’s propagation direction. The radial polarization unit vector is written by [17]:

\[
g_1 = \cos \theta (\cos \phi \hat{i} + \sin \phi \hat{j}) + \sin \theta \hat{k}
\]

where \( \theta \) indicates the polar angle. Then \( \vec{s} \cdot \vec{r} \) is:

\[
\vec{s} \cdot \vec{r} = Z \cos \theta + r \sin \theta \cos(\varphi - \phi)
\]
Figure 1. Schematic diagram of focused RPB. Objective focal length. Inplane, $Q(r, \varphi)$ is an observation point.

The electric field vector in the focus region is obtained by selecting a suitable field amplitude $\mathbf{a}$. We use $e_{\phi}^{(0)} = 0$ in equation (2) for radially polarized illumination. The electric field vector's Cartesian components near the focus can therefore be represented as follows:

$$
\vec{E}(r, \varphi, \phi) = \begin{bmatrix}
E_x(r, \varphi, \phi) \\
E_y(r, \varphi, \phi) \\
E_z(r, \varphi, \phi)
\end{bmatrix}
$$

$$
= -\frac{iA}{\pi} \int_{0}^{\theta_{\text{max}}} d\theta \int_{0}^{2\pi} \sin \theta \sqrt{\cos \theta} l_0 \exp \left[ik(z \cos \theta + r \sin \theta \cos(\varphi - \phi))\right] d\varphi d\theta
$$

The following transformations are used to create the axis of $\vec{E}(r, \varphi, \phi)$:

$$
\vec{E}_r = \cos \phi \mathbf{e}_x + \sin \phi \mathbf{e}_y
$$

and

$$
\vec{E}_\phi = \cos \phi \mathbf{e}_y - \sin \phi \mathbf{e}_x
$$

The azimuthal axis is 0 in the image space. Then the two axis are given as:

$$
\vec{E}_r(r, \varphi, \phi) = -\frac{iA}{\pi} \int_{0}^{\theta_{\text{max}}} d\theta \int_{0}^{2\pi} \sin \theta \sqrt{\cos \theta} l_0 \cos \varphi \cos(\varphi - \phi) \exp \left[ik(z \cos \theta + r \sin \theta \cos(\varphi - \phi))\right] d\phi d\theta
$$

And

$$
\vec{E}_z(r, \varphi, \phi) = -\frac{iA}{\pi} \int_{0}^{\theta_{\text{max}}} d\theta \int_{0}^{2\pi} \sqrt{\cos \theta} (\sin \theta)^2 l_0 \exp \left[ik(z \cos \theta + r \sin \theta \cos(\varphi - \phi))\right] d\phi d\theta
$$
The identity below can be used to complete the integration over $\phi$:

$$\int_0^{2\pi} \cos(n \varphi) \exp[ikr \sin \theta \cos \varphi] d\varphi = 2\pi i^n J_n(kr \sin \theta)$$

Where $J_n(kr \sin \theta)$ is a first order Bessel function.

Electric fields at the focus of a pupil illuminated by radial polarization take the following form:

$$\vec{E}_r(r, z) = A \sqrt{\cos \theta} l_0 J_0(kr \sin \theta) \exp[i k z \cos \theta] d\theta$$

And

$$\vec{E}_x(r, z) = 2i A \sqrt{\cos \theta} (\sin \theta) l_0 J_0(kr \sin \theta) \exp[i k z \cos \theta] d\theta$$

The constant $A$ is given by:

$$A = \frac{\pi f l_0}{\lambda}$$

At all distances $z$, precisely as it does with the paraxial solutions[17]. Equations 13 and 14 can be written in terms of $P(\theta)$ as follows:

$$\vec{E}_r(r, z) = A \int_0^{\theta_{\text{max}}} P(\theta) \sin(2\theta) J_1(kr \sin \theta) \exp[i k z \cos \theta] d\theta$$

And

$$\vec{E}_x(r, z) = 2i A \int_0^{\theta_{\text{max}}} P(\theta) (\sin \theta)^2 l_0 J_0(kr \sin \theta) \exp[i k z \cos \theta] d\theta$$

The radial & longitudinal components near the focus of $z=0$ are $\vec{E}_r(r, z)$ and $\vec{E}_x(r, z)$, respectively. The amplitude modulated annular multi-Gaussian beam is described by $P(\theta)$, which is given by [12]:

$$P(\theta) = \left(\frac{\theta}{\theta_0}\right)^m \sum_{n=-N}^N \exp\left[-\left(\frac{\theta - \theta_c - n\omega_0}{\omega_0}\right)^2\right]$$

$\theta$ is the converging semiangle. The maximum converging semiangle is denoted by symbol $\theta_{\text{max}}$, & related to NA by the formula $\alpha = \arcsin (NA)$. The value of $\theta_0$ is commonly selected to be slightly less than $\alpha$. The radial translation of $P(\theta)$ is determined by $\theta_c. \omega_0$ is given by:

$$\omega_0 = \frac{1}{2} \times \frac{\alpha}{N + [1 - \ln(\sum_{n=-N}^{N} \exp(-n^2))]^{1/2}}$$

The total field is expressed by using following equation[5], [7]:

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\[ \vec{E}_t(r, z) = E_r \vec{e}_r + E_z \vec{e}_z \]

The amplitudes of the two orthogonal components given in equations 13 and 14 are \( E_r \) and \( E_z \), respectively, and \( \vec{e}_r \) and \( \vec{e}_z \) are their corresponding unit vectors.

3. Numerical Analysis

Figure 2 depicts the amplitude profile of the Gaussian beam waist at \( \omega_0 = 0.14 \) beam size, as well as the different Gaussian beam radius, \( \theta_0 \). Figures 2 (a) and 2(b) use a numerical aperture of 0.6, while figures 2 (c) and 2(d) use a numerical aperture of 0.95. The AD of Gaussian beam of numerical aperture 0.95 and 0.6 is shown to examine the Beam Amplitude profile of a Gaussian beam with beam size \( \omega_0 \) and beam radius \( \theta_0 \). The Gaussian beam width grows with increasing radius for both numerical apertures.

![Figure 2](image-url)

Figure 2. Amplitude profile of the Gaussian beam at the output pupil for \( \omega_0 = 0.14 \). (a) and (b) are for the numerical aperture, NA of 0.6 and (c) and (d) is for the NA0.95. \( \theta_0 \) for (a), (b), (c)& (d) is 0.2, 0.3, 0.4 and 0.5 respectively.

Figure 3 deals focusing of electric density distributions for RPGB focused by NA=0.6 and n=1, where NA and n stand for Numerical Aperture and Refractive Index, respectively. \( \alpha = \sin \left( \frac{NA}{n} \right) = 1.25 \) rad. For \( \theta_0 = \sin \left( \frac{NA}{n} \right) \), \( \theta_0 \) is no larger than \( \alpha \) and assuming that the Gaussian profile is retained. \( E_r(0, z) \) is zero due to the Bessel function’s nature, although \( J_1(0) = 0 \) is also zero. As a result, for different \( z \), the
greatest field intensity occurs at location of \( r \neq 0 \). Indeed, the RP axis \( E_r(r, z) \) widen the focusing spot’s transversal width. It means that by lowering the high intensity of RP field & modifying point of highest intensities emerge, a small focusing spot obtained. \( J_0(0) = 1 \) for \( r = 0 \), and \( E_z(0, z) \) is high for a given \( z \). In these scenario, RP axis has no effect on the focal spot minimization[21].

The radial axis dominates for the polarized Gaussian beam with \( \theta_0 = 0.2 \), as seen in Figure 3(a)-(d). The radially polarized component has a maximum intensity nearly 0.98 at \( z = 1.12 \lambda \), which is higher than the longitudinally polarized component, which has a high intensity of about 0.21 at \( z = 0 \). Large total field intensity will appear at \( z = 1.12 \lambda \) because it cannot occur along the optical axis. The radial polarization axis dominates around \( z = 0 \). As a result, the focusing spot at the specified focal point cannot be reached. It also shows that the 0.6 Numerical Aperture at \( \theta_0 = 0.2 \) fails to converge the PGB with small \( \theta_0 \). Bright spots denote high electric intensity regions, while the dark areas denote low electric intensity regions.

![Contour plots of the focusing of electric density distributions in the r–z plane for GB that is radially polarized & used NA 0.6. \( \omega_0 = 0.14 \) and \( \theta_0 = 0.2 \) for (a) Radial component, \( |E_r|^2 \), (b) Longitudinally polarized component, \( |E_z|^2 \), and (c) Total electric density distribution, \( |E_r|^2 + |E_z|^2 \). (d) Focusing of electric density distributions in the plane focus \( z = 0 \) for GB that is radially polarized and focused by a lens for Radial component, \( |E_r|^2 \), Longitudinally polarized component, \( |E_z|^2 \), & Total electric density distribution, \( |E_r|^2 + |E_z|^2 \). Radial axis, longitudinal axis, & total fields are represented by red, blue, and black colors respectively.](image-url)
Electric density distributions inr–z plane for a Gaussian beam that is radially polarized and large NA 0.95, beam size ($\omega_0 = 0.14$), and Gaussian beam radius ($\theta_0 = 0.2$) are shown in Figure 4. The radially polarized component dominates the focusing field, hence the focusing spot is impossible to achieve at this focal point. The low radius of the Gaussian beam prevents a longitudinally polarized focusing field from being obtained, even if the numerical aperture is high.

Figures 5(a)-(d) illustrate the overall intensity distribution in r–z plane. We employed a high NA of 0.95 & large GB radius of $\theta_0 = 0.8$. With the growth of $\theta_0$, the maximum intensities longitudinal axis increases. As a result, longitudinally polarized component progressively becomes dominant. With rise of $\theta_0$, the beam quality improves. It indicates using a radially polarized Gaussian beam of high $\theta_0$, longitudinally polarized focusing field is generated. Figure 5(a)-(d) shows that for high NA & Gaussian beam radius, the produced focal field is focused.

**Figure 4.** Contour plot of focusing of electric density distributions in r–z plane for GB that is radially polarized & focused by lens with NA of 0.95, $\omega_0 = 0.14$, & $\theta_0 = 0.2$ for (a) Radial component, $|E_r|^2$, (b) Longitudinally polarized component, $|E_\theta|^2$, and (c) Total electric density distribution, $|E_r|^2 + |E_\theta|^2$. (d) Focusing of electric density distributions in the plane of focus $z = 0$, for GB that is radially polarized and focused by a lens for Radial component, $|E_r|^2$, Longitudinally polarized component, $|E_\theta|^2$, & total
electric density distribution, $|E_r|^2 + |E_z|^2$. Radial axis, longitudinal axis, & total fields are represented by red, blue, and black colors respectively.

**Figure 5:** Contour plots of the focusing of electric density distributions $r-z$ plane of GB that is radially polarized & focused by lens & NA of 0.95, $\omega_0 = 0.14$, and $\theta_0 = 0.8$ for (a) Radial component, $|E_r|^2$, (b) Longitudinally polarized component, $|E_z|^2$, and (c) Total electric density distribution, $|E_r|^2 + |E_z|^2$. (d) Focusing of electric density distributions in the plane of focus $z = 0$ for GB that is radially polarized and focused by a lens for Radial component, $|E_r|^2$, Longitudinally polarized component, $|E_z|^2$, & total electric density distribution, $|E_r|^2 + |E_z|^2$. Radial axis, longitudinal axis, and total field are represented by red, blue, and black colors respectively.

**4. Conclusion**

Finally, focusing of RPG beams is investigated using Richards’ formulas at various numerical apertures and Gaussian beam radiiuses. Numerical computations for varied beam parameters of $NA$, $\omega_0$, and $\theta_0$ show the overall optical intensity distribution. The suggested technique appears to produce a subwavelength focal spot (0.51$\lambda$) & significant focal depth (21$\lambda$). It believed that a beam with such
asmall spot size and extended focal depth will find widespread application in optical trapping, data storage, biomedical imaging, laser cutting, microscopy, & the control of high refractive index particle optical traps.

The Gaussian beam's Beam Amplitude profile is evaluated at low and high numerical apertures. For both low and large numerical apertures, it was demonstrated that as the Gaussian beam's radius increases, the width of the Gaussian beam also increases. The radially polarized component dominates the focusing fields in the focusing of electric field intensity, the polarized Gaussian beam at low NA and $\theta_0$. The radially polarized component dominates the focusing field around $z=0$. Due to the dominance of RP axis at this focal place, the focusing spot cannot be attained at low NA and $\theta_0$. The longitudinally polarized component becomes prominent in the focusing field as the radius of the Gaussian beam increases, and the focusing spot occurs at a large numerical aperture and radius of GB.

Impact of Gaussian beam's radius on focusing was addressed and examined. Both the Gaussian beam radius and the Gaussian beam waist at beam size define the magnitude distribution of GB. Maximum intensity of field is determined by the radius of the Gaussian beam. Sub-wavelength focusing spot is obtained at high radius of the Gaussian beam, overcoming the diffraction limit.

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