Comparison of standard ruler and standard candle constraints on dark energy models

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Abstract. We compare the dark energy model constraints obtained by using recent standard ruler data (baryon acoustic oscillations (BAO) at \(z = 0.2\) and 0.35 and cosmic microwave background (CMB) shift parameters \(R\) and \(l_a\)) with the corresponding constraints obtained by using recent type Ia supernovae (SnIa) standard candle data (ESSENCE + SNLS + HST from astro-ph/0701510). We find that, even though both classes of data are consistent with \(\Lambda\)CDM (CDM: cold dark matter) at the 2σ level, there is a systematic difference between the two classes of data. In particular, we find that for practically all values of the parameters \((\Omega_0^m, \Omega_b)\) in the 2σ range of the three-year WMAP data (WMAP3) best fit, \(\Lambda\)CDM is significantly more consistent with the SnIa data than with the CMB + BAO data. For example for \((\Omega_0^m, \Omega_b) = (0.24, 0.042)\) corresponding to the best fit values of WMAP3, the dark energy equation of state parameterization \(w(z) = w_0 + w_1(z/1+z)\) best fit is at a 0.5σ distance from \(\Lambda\)CDM\((w_0 = -1, w_1 = 0)\) using the SnIa data and 1.7σ away from \(\Lambda\)CDM using the CMB + BAO data. There is a similar trend in the earlier data (SNLS versus CMB + BAO at \(z = 0.35\)). This trend is such that the standard ruler CMB + BAO data show a mild preference for crossing of the phantom divide line \(w = -1\), while the recent SnIa data favor \(\Lambda\)CDM. Despite this mild difference in trends, we find no statistically significant evidence for violation of the cosmic distance duality relation \(\eta \equiv d_L(z)/d_A(z)(1+z)^2 = 1\). For example, using a

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prior of $\Omega_{0m} = 0.24$, we find $\eta = 0.95 \pm 0.025$ in the redshift range $0 < z < 2$, which is consistent with distance duality at the $2\sigma$ level.

**Keywords:** CMBR theory, dark energy theory, supernova type Ia

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## 1. Introduction

The accelerated expansion of the universe has been confirmed during the last decade by several observational probes [1]–[7]. The origin of this acceleration may be attributed as either dark energy with negative pressure, or a modification of general relativity that makes gravity repulsive at recent times on cosmological scales. In order to distinguish between these two possibilities and identify in detail the gravitational properties of dark energy or modified gravity two developments are required [8]:

1. Detailed observation of linear cosmic density perturbations $\delta(z) = \delta\rho(z)/\rho$ at recent redshifts.
2. Detailed mapping of the expansion rate $H(z)$ as a function of the redshift $z$.

The latter is equivalent to identifying the function $w(z)$ defined as

$$w(z) = -1 + \frac{1}{3}(1+z)\frac{d\ln(\delta H(z)^2)}{d\ln z},$$

where $\delta H(z)^2 = H(z)^2/H_0^2 - \Omega_{0m}(1+z)^3 - \Omega_{0r}(1+z)^4$ accounts for all terms in the Friedmann equation not related to matter and radiation. If the origin of the accelerating expansion is dark energy then $w(z)$ may be identified with the dark energy equation of state parameter $w(z) = p_X/\rho_X$. The cosmological constant ($w(z) = -1$) corresponds to a constant dark energy density.

It has been shown [9] that a $w(z)$ observed to cross the line $w(z) = -1$ (phantom divide line) is very hard to accommodate in a consistent theory in the context of general relativity. On the other hand, such a crossing can be easily accommodated in the context
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of extensions of general relativity [8]. Therefore, the crossing of the phantom divide line $w = -1$ could be interpreted as a hint in the direction of modified gravity. Such a hint would clearly need to be verified by observations of linear density perturbation evolution through e.g. weak lensing [10] or the redshift distortion factor [11].

There are two classes of probes that may be used to observe the expansion rate $H(z)$ or equivalently $w(z)$

- **Standard candles** are luminous sources of known intrinsic luminosity which may be used to measure the luminosity distance which, assuming flatness, is connected to $H(z)$ as

$$d_L(z) = c(1 + z) \int_0^z \frac{dz'}{H(z')}.$$  \hspace{1cm} (1.2)

Useful standard candles in cosmology are type Ia supernovae [1], [12]–[14] (SnIa) and the less accurate but more luminous gamma ray bursts [15]

- **Standard rulers** are objects of known comoving size which may be used to measure the angular diameter distance which, in a flat universe, is related to $H(z)$ as

$$d_A(z) = \frac{c}{1 + z} \int_0^z \frac{dz'}{H(z')}.$$  \hspace{1cm} (1.3)

The most useful standard ruler in cosmology is the last scattering horizon, the scale of which can be measured either directly at $z \simeq 1089$ through the CMB temperature power spectrum or indirectly through baryon acoustic oscillations (BAO) on the matter power spectrum at low redshifts. Clusters of galaxies [16,17] and radio galaxies [18] may also be used as standard rulers under certain assumptions but they are less accurate than CMB + BAO.

Early SnIa data put together with more recent such data through the Gold data set [1,12] have been used to reconstruct $w(z)$, and have demonstrated a mild preference for a $w(z)$ that crossed the phantom divide line [19,20]. A cosmological constant remained consistent but only at the 2σ level. However, the Gold data set has been shown to suffer from systematics due to the inhomogeneous origin of the data [21]. More recent SnIa data (SNLS [14], ESSENCE [22], HST [12]) re-compiled in [13] have demonstrated a higher level of consistency with ΛCDM and showed no trend for a redshift dependent equation of state.

On the other hand, the use of standard rulers (CMB + BAO) has rarely been studied independently of SnIa due to the small number of data points involved (see however [23]–[25]). It has been pointed out [26] that the latest BAO data ‘require slightly stronger cosmological acceleration at low redshifts than ΛCDM’. This statement is equivalent to a trend towards a $w(z) < -1$ at low $z$, and therefore a possibility of crossing the PDL $w = -1$. The goal of this paper is to quantify this statement in some detail by comparing the best fit form of $w(z)$ obtained from the SnIa data to the corresponding form obtained from the CMB + BAO data. This comparison is done quantitatively by identifying the quality of fit of ΛCDM in the context of each data set. In particular, we consider the Chevalier–Polarski–Linder (CPL) [27,28] parameterization

$$w(z) = w_0 + w_1 \frac{z}{1 + z}.$$  \hspace{1cm} (1.4)
and, assuming flatness, we identify the ‘distance’ in units of $\sigma$ ($\sigma$-distance) of the parameter space point $(w_0, w_1) = (-1, 0)$ corresponding to $\Lambda$CDM from the best fit point $(w_0, w_1)$ for each data set (SnIa standard candles or CMB + BAO standard rulers) and for several priors of $(\Omega_0m, \Omega_b)$. We thus identify an interesting systematic difference in trends between the two data sets.

We also discuss the implications of this difference in trends for the distance duality relation

$$\eta(z) \equiv \frac{d_L(z)}{d_A(z)(1+z)^2} = 1,$$

which measures quantitatively the agreement between luminosity and angular diameter distances. This relation has been shown to be respected when clusters of galaxies are used as standard rulers [17].

2. Likelihood calculations

We assume a CPL parameterization for $w(z)$ and apply the maximum likelihood method separately for standard rulers (CMB + BAO) and standard candles (SnIa) assuming flatness. The corresponding late time form of $H(z)$ for the CPL parameterization is

$$H^2(z) = H^2_0[\Omega_0m(1 + z)^3 + (1 - \Omega_0m)(1 + z)^3(1 + w_0 + w_1)e^{-3w_1z/(1+z)}]. \quad (2.1)$$

At earlier times this needs to be generalized taking into account radiation, i.e.,

$$E^2(a) \equiv \frac{H(a)^2}{H^2_0} = \Omega_m(a + a_{eq})a^{-4} + \Omega_{de}X(a), \quad (2.2)$$

where $a = 1/(1 + z)$, $\Omega_{de} = 1 - \Omega_m - \Omega_{rad}$ and

$$X(a) = \exp \left[-3 \int_1^a \frac{(1 + w(a'))}{a'} \, da'\right] = a^{-3(1 + w_0 + w_1)}e^{-3w_1(1-a)}, \quad (2.3)$$

with the CPL parameterization $w(a) = w_0 + w_1(1 - a)$.

2.1. Standard rulers

2.1.1. CMB. We use the data points $(R, l_a, \Omega_b h^2)$ of [25] where $R, l_a$ are two shift parameters:

- The scaled distance to recombination

$$R = \sqrt{\Omega_0m \frac{H^2_0}{c^2}} r(z_{\text{CMB}}), \quad (2.4)$$

where $r(z_{\text{CMB}})$ is the comoving distance from the observer to redshift $z$ and is given by

$$r(z) = \frac{c}{H_0} \int_{0}^{z} \frac{dz}{E(z)}, \quad (2.5)$$

with $E(z) = H(z)/H_0$. 

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- The angular scale of the sound horizon at recombination

\[ l_a = \pi \frac{r(a_{\text{CMB}})}{r_s(a_{\text{CMB}})}, \tag{2.6} \]

where \( r_s(a_{\text{CMB}}) \) is the comoving sound horizon at recombination given by

\[ r_s(a_{\text{CMB}}) = \frac{c}{H_0} \int_{0}^{a_{\text{CMB}}} \frac{c_s(a)}{a^2 E(a)} \, da, \tag{2.7} \]

with the sound speed being \( c_s(a) = 1/\sqrt{3(1 + R_b a)} \) and \( a_{\text{CMB}} = 1/1 + z_{\text{CMB}} \), where \( z_{\text{CMB}} = 1089 \). Actually, \( z_{\text{CMB}} \) has a weak dependence on \( \Omega_m \) and \( \Omega_b \) [29] but we have checked that the sound horizon changes only to less than 0.1%. The quantity \( R_b \), is actually the photon–baryon energy density ratio, and its value can be calculated using

\[ R_b = \left( \frac{3}{4} \Omega_b h^2 / \Omega_c h^2 \right) = 31,500 \Omega_b h^2 (T_{\text{CMB}}/2.7 \text{ K})^{-4}. \]

For a flat prior, the three-year WMAP data (WMAP3) [2] measured best fit values are [25]

\[ \bar{V}_{\text{CMB}} = \left( \begin{array}{c} R - 1.70 \\ l_a - 302.2 \\ \Omega_b h^2 \end{array} \right) = \left( \begin{array}{c} 0.022 \pm 0.00082 \\ 1 \\ 0.6283 \end{array} \right). \tag{2.8} \]

The corresponding normalized covariance matrix is [25]

\[ C_{\text{CMB}}^{\text{norm}} = \left( \begin{array}{ccc} 1 & -0.09047 & -0.01970 \\ -0.09047 & 1 & -0.6283 \\ -0.01970 & -0.6283 & 1 \end{array} \right), \tag{2.9} \]

from which the covariance matrix can be found to be

\[ (C_{\text{CMB}})^{ij} = (C_{\text{CMB}}^{\text{norm}})^{ij} \sigma_{\bar{V}_{\text{CMB}}^i} \sigma_{\bar{V}_{\text{CMB}}^j}, \tag{2.10} \]

where \( \sigma_{\bar{V}_{\text{CMB}}} \) are the 1σ errors of the measured best fit values of equation (2.8).

We thus use equations (2.8), (2.4) and (2.6) to define

\[ X_{\text{CMB}} = \left( \begin{array}{c} R - 1.70 \\ l_a - 302.2 \\ \Omega_b h^2 - 0.022 \end{array} \right), \tag{2.11} \]

and construct the contribution of CMB to the \( \chi^2 \) as

\[ \chi^2_{\text{CMB}} = X_{\text{CMB}}^T C_{\text{CMB}}^{-1} X_{\text{CMB}}, \tag{2.12} \]

with

\[ C_{\text{CMB}}^{-1} = \left( \begin{array}{ccc} 1131.32 & 4.8061 & 5234.42 \\ 4.8061 & 1.1678 & 1077.22 \\ 5234.42 & 1077.22 & 2.4814 10^6 \end{array} \right). \tag{2.13} \]

Notice that \( \chi^2_{\text{CMB}} \) depends on four parameters (\( \Omega_{0m}, \Omega_b, w_0 \) and \( w_1 \)). Due to the large number of parameters involved, in what follows we will consider various different priors on the parameters \( \Omega_{0m}, \Omega_b \).
2.1.2. BAO. As in the case of the CMB, we apply the maximum likelihood method using the data points [26]

\[ \mathbf{V}_{\text{BAO}} = \begin{pmatrix} \frac{r_s(z_{\text{CMB}})}{D_V(0.2)} = 0.1980 \pm 0.0058 \\ \frac{r_s(z_{\text{CMB}})}{D_V(0.35)} = 0.1094 \pm 0.0033 \end{pmatrix} \text{,} \]  

(2.14)

where the dilation scale

\[ D_V(z_{\text{BAO}}) = \left( \int_0^{z_{\text{BAO}}} \frac{dz}{H(z)} \right)^2 \frac{z_{\text{BAO}}}{H(z_{\text{BAO}})} \]  

(2.15)

encodes the visual distortion of a spherical object due to the non-Euclideanity of an FRW spacetime, and is equivalent to the geometric mean of the distortion along the line of sight and two orthogonal directions. We thus construct

\[ \mathbf{X}_{\text{BAO}} = \begin{pmatrix} \frac{r_s(z_{\text{dec}})}{D_V(0.2)} - 0.1980 \\ \frac{r_s(z_{\text{dec}})}{D_V(0.35)} - 0.1094 \end{pmatrix} \text{,} \]  

(2.16)

and using the inverse covariance matrix [26]

\[ \mathbf{C}_{\text{BAO}}^{-1} = \begin{pmatrix} 35059 & -24031 \\ -24031 & 108300 \end{pmatrix} \]  

(2.17)

we find the contribution of BAO to \( \chi^2 \) as

\[ \chi^2_{\text{BAO}} = \mathbf{X}_{\text{BAO}}^T \mathbf{C}_{\text{BAO}}^{-1} \mathbf{X}_{\text{BAO}} \text{.} \]  

(2.18)

2.2. Standard candles

2.2.1. SnIa. We use the SnIa data set of Davis et al [13] consisting of four subsets: ESSENCE [22] (60 points), SNLS [14] (57 points), nearby [1] (45 points) and HST [12] (30 points).

These observations provide the apparent magnitude \( m(z) \) of the supernovae at peak brightness after implementing the correction for galactic extinction, the K-correction and the light curve width–luminosity correction. The resulting apparent magnitude \( m(z) \) is related to the luminosity distance \( D_L(z) \) through

\[ m_{\text{th}}(z) = \bar{M}(M,H_0) + 5 \log_{10}(D_L(z)), \]  

(2.19)

where in a flat cosmological model

\[ D_L(z) = (1 + z) \int_0^z \frac{dz'}{H(z');\Omega_{0m},w_0,w_1} \]  

(2.20)

is the Hubble free luminosity distance \((H_0d_L)\), and \( \bar{M} \) is the magnitude zero-point offset and depends on the absolute magnitude \( M \) and on the present Hubble parameter \( H_0 \) as

\[ \bar{M} = M + 5 \log_{10} \left( \frac{H_0^{-1}}{M_{\text{pc}}} \right) + 25 = M - 5 \log_{10} h + 42.38. \]  

(2.21)

The parameter \( M \) is the absolute magnitude which is assumed to be constant after the above mentioned corrections have been implemented in \( m(z) \).
The SnIa data points are given, after the corrections have been implemented, in terms of the distance modulus
\[
\mu_{\text{obs}}(z_i) \equiv m_{\text{obs}}(z_i) - M.
\]
(2.22)

The theoretical model parameters are determined by minimizing the quantity
\[
\chi^2_{\text{SnIa}}(\Omega_0, w_0, w_1) = \sum_{i=1}^{N} \frac{(\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i))^2}{\sigma^2_{\mu_i}},
\]
(2.23)

where \( N = 192 \) and \( \sigma^2_{\mu_i} \) are the errors due to flux uncertainties, intrinsic dispersion of SnIa absolute magnitude and peculiar velocity dispersion. These errors are assumed to be Gaussian and uncorrelated. The theoretical distance modulus is defined as
\[
\mu_{\text{th}}(z_i) \equiv m_{\text{th}}(z_i) - M = 5 \log_{10}(D_L(z)) + \mu_0,
\]
(2.24)

where
\[
\mu_0 = 42.38 - 5 \log_{10} h
\]
(2.25)

and \( \mu_{\text{obs}} \) is given by (2.22). The steps that we followed for the minimization of (2.23) are described in detail in [19, 20, 30].

2.3. Results

We consider separately the standard ruler data (\( \chi^2_{\text{SR}} \equiv \chi^2_{\text{CMB}} + \chi^2_{\text{BAO}} \)) and the standard candle data (\( \chi^2_{\text{SnIa}} \)), and perform minimization of the corresponding \( \chi^2 \) with respect to the parameters \( w_0 \) and \( w_1 \) for various priors of \( \Omega_0 \) and \( \Omega_b \) in the 2\( \sigma \) range of the WMAP3 best fit, i.e. 0.17 \( \leq \) \( \Omega_0 \) \( \leq \) 0.31, 0.034 \( \leq \) \( \Omega_b \) \( \leq \) 0.049.

It should be stressed, however, that the reconstructed values of \( w_0 \) and \( w_1 \) are very sensitive to the chosen values of \( \Omega_b \) and \( \Omega_0 \) for the case of CMB + BAO and to the chosen value of \( \Omega_0 \) for the case of SnIa data. So choosing the wrong priors for \( \Omega_0 \) and \( \Omega_b \) can lead to very misleading values of \( w_0 \) and \( w_1 \). Results from simulated data for known fiducial models show this fact clearly in [31]–[33] where it was shown that the wrong choice of matter density could be completely misleading in the reconstruction of dark energy properties. In view of this important fact we are faced with two options:

(a) Use the WMAP constraints as priors for \( \Omega_0 \) and \( \Omega_b \) at the risk of getting misleading results if the WMAP estimates are incorrect due to statistical fluctuations.

(b) Perform minimization with respect to a larger parameter space thus significantly increasing the 1\( \sigma \) error regions of the estimated parameters.

In this paper we have made the choice (a). However, in what follows we also apply the maximum likelihood method in the larger parameter space in order to demonstrate the dramatic increase of the 1\( \sigma \) error regions when priors are avoided at the expense of increasing the parameter space.

In figure 1 we show the 68.3% and 95.4% \( \chi^2 \) confidence contours in the \((w_0, w_1)\) parameter space for the two data set categories (standard ruler and standard candle data) for \( \Omega_0 = 0.24 \) and \( \Omega_b = 0.042 \) (the best fit of the WMAP3 CMB data [2]). Figure 1(a) shows the \((w_0, w_1)\) contours obtained using SnIa data [13] (standard candles)
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Figure 1. The 68.3% and 95.4% $\chi^2$ confidence contours in the $(w_0, w_1)$ parameter space for each data set category: (a) for SNIa (based on data from [13]) and (b) for CMB + BAO data (based on data from [25, 26]) for $\Omega_{0m} = 0.24$ and $\Omega_b = 0.042$ (best fits of WMAP3). The blue dots correspond to the $(w_0, w_1)$ best fit, while the yellow dots correspond to $\Lambda\text{CDM} (-1, 0)$.

while figure 1(b) shows the corresponding contours assuming CMB + BAO data [25, 26] (standard rulers). The blue dots correspond to the $(w_0, w_1)$ best fit, while the yellow dots correspond to $\Lambda\text{CDM} (w_0, w_1) = (-1, 0)$. The distance in units of $\sigma$ ($\sigma$-distance $d_\sigma$) of the best fit to $\Lambda\text{CDM}$ was found by converting $\Delta \chi^2 = \chi^2_{\Lambda\text{CDM}} - \chi^2_{\text{min}}$ to $d_\sigma$, i.e. solving

$$1 - \Gamma(1, \Delta \chi^2/2)/\Gamma(1) = \text{erf}(d_\sigma/\sqrt{2})$$

(2.26)

for $d_\sigma$ ($\sigma$-distance), where $\Delta \chi^2$ is the $\chi^2$ difference between the best fit and $\Lambda\text{CDM}$ and erf( ) is the error function. Notice that $\Lambda\text{CDM}$ is consistent at less than 1$\sigma$ level according to the SNIa data ($d_{\sigma}^{\text{SNIa}} \approx 0.5$ in figure 1(a)), while the corresponding consistency level reduces to $d_{\sigma}^{\text{SR}} \approx 1.7\sigma$ for the standard ruler CMB + BAO data. This mild difference in trends between standard candles and standard rulers persists also for all values of $\Omega_{0m}$ in the 2$\sigma$ range of WMAP3 best fit. This is demonstrated in figure 2 where we show the $\sigma$-distance $d_{\sigma}^{\text{SR}}$ superposed with $d_{\sigma}^{\text{SNIa}}$ as a function of $\Omega_{0m}$ for $\Omega_b = 0.034$ (figure 2(a)), $\Omega_b = 0.042$ (figure 2(b)) and $\Omega_b = 0.049$ (figure 2(c)). These values of $\Omega_b$ span the 2$\sigma$ range of the corresponding WMAP3 best fit. Notice that the $\sigma$-distance between best fit values and $\Lambda\text{CDM}$ values is consistently larger when using standard ruler data (colored lines are consistently above black lines). However, according to figure 2(c), when both $\Omega_{0m}$ and $\Omega_b$ are at the 2$\sigma$ edge of the WMAP3 range (this could happen with probability less than 1%) then $\Lambda\text{CDM}$ and CMB + BAO are in excellent agreement with each other and equally favor $\Lambda\text{CDM}$.

As an additional test, we have also performed the $\chi^2$ minimization in the full parameter space ($\Omega_{0m}, \Omega_b, w_0, w_1$) and we find that the best fit values for $\Omega_{0m}$ are different in the two cases (standard candle and standard ruler) but they are consistent with each
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Figure 2. The $\sigma$-distance of the best fit parameters $(w_0, w_1)$ to $\Lambda$CDM $(-1,0)$ for SnIa data (black lines) and for CMB–BAO data (colored lines), as a function of $\Omega_{0m}$ for $\Omega_b = 0.034$ (a), $\Omega_b = 0.042$ (b) and $\Omega_b = 0.049$ (c).

Figure 3. The ratios of the $\sigma$-distances of the best fit parameters $(w_0, w_1)$ to $\Lambda$CDM $(-1,0)$ for SnIa standard candles over the one for CMB–BAO standard ruler data as defined in equation (2.27), as a function of $\Omega_{0m}$ for various values of $\Omega_b$.

Other at the 1$\sigma$ level. In particular for the SnIa data we find $(\Omega_m = 0.35 \pm 0.14, w_0 = -1.10 \pm 0.44, w_1 = -1.23 \pm 5.50)$, while for the BAO + CMB data we have $(\Omega_m = 0.242^{+0.030}_{-0.027}, \Omega_b = 0.042^{+0.003}_{-0.003}, w_0 = -1.41^{+0.62}_{-0.22}, w_1 = 1.31^{+0.10}_{-3.06})$. In order to make a meaningful comparison of the $\sigma$ distances ($d_\sigma$) to $\Lambda$CDM we have to use a common value of the parameter $\Omega_m$ for standard rulers and standard candles. If we choose the value favored by the CMB + BAO data $\Omega_m = 0.242$ then we find $d_\sigma(\text{SnIa}) = 0.45$ and $d_\sigma(\text{CMB+BAO}) = 0.90$. If on the other hand we choose the value favored by the SnIa data $\Omega_m = 0.35$ then we find $d_\sigma(\text{SnIa}) = 1.53$ and $d_\sigma(\text{CMB+BAO}) = 11.37$, which is in agreement with figure 3 as their ratio is $r = d_\sigma(\text{CMB+BAO})/d_\sigma(\text{SnIa}) = 11.37/1.53 = 7.4$ (see the blue line which is for $\Omega_b = 0.0458$). In both cases $\Lambda$CDM is significantly more consistent with the SnIa data than with the CMB + BAO data. Therefore, our conclusions persist also in the context of a full $\chi^2$ minimization.

An interesting possibility that could have arisen at this point is that the derived values of $d_\sigma$ for the two cases be comparable but achieved for significantly different values
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Figure 4. The 68.3% and 95.4% $\chi^2$ confidence contours in the $(w_0, w_1)$ parameter space for each of the old data set categories [14, 37] for $\Omega_0 = 0.24$. The green dots correspond to the $(w_0, w_1)$ best fit, while the yellow dots correspond to $\Lambda$CDM.

of $\Omega_0$. This would mean that either there exists an inconsistency between the data sets or there is something wrong with the assumed theoretical model [35]. However, as pointed out above, our results for the $d_\sigma$ difference persist also in the case of full minimization without priors while the results for the $\Omega_m$ parameter are consistent with each other at $1\sigma$. Therefore, there is no statistically significant inconsistency between standard rulers and standard candles even though there is a clear difference in trends.

An alternative way to see this mild difference in trends between standard candles and standard rulers is to plot the ratios $r$ of the $\sigma$-distances defined as

$$r(\Omega_0) \equiv \frac{d_{\sigma}^{SR}}{d_{\sigma}^{SNIA}}.$$  (2.27)

These plots are shown in figure 3 for five values of $\Omega_b$ spanning the 2$\sigma$ range of WMAP3. Notice that the colored lines are consistently above the line $r = 1$ indicating that the $\sigma$-distance is found to be consistently larger when using standard ruler data. An exception to this rule is the case corresponding to high values of both $\Omega_b$ and $\Omega_0$ set to values of $2\sigma$, or more, away from their best fit (see magenta line corresponding to $\Omega_b = 0.049$, for $\Omega_0 > 0.29$).

The above plots reveal a consistent trend of the standard ruler CMB + BAO data for a mild preference for crossing of the phantom divide line $w = -1$, while the recent SnIa data seem to favor $\Lambda$CDM. Figures 4 and 5 show corresponding plots obtained using earlier data (SNLS [14] versus CMB + BAO [36, 37] at $z = 0.35$), where a similar consistent trend is observed.

An interesting feature of the contours of figure 1(b) is the deformation appearing for relatively large values of $w_1$. There is a simple way to understand this deformation. For
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Figure 5. (a) The ratio of the σ-distance of the best fit parameters \((w_0, w_1)\) to ΛCDM \((-1, 0)\) for SnIa over the one for CMB–BAO data, as a function of Ω₀ᵐ for the early CMB + BAO data [37]. (b) The σ-distance of the best fit parameters \((w_0, w_1)\) to ΛCDM for SnIa data (black line) and for CMB + BAO data (red line), as a function of Ω₀ᵐ.

\((w_0, w_1)\) parameter values satisfying \(w_0 + w_1 \gtrsim 0\), the dark energy equation of state is approximately constant and positive at early times and therefore the corresponding dark energy density dominates over the matter density. As a result the Hubble expansion rate is significantly modified over the whole range from \(z = 0\) to \(z_{\text{CMB}}\) and the corresponding integral of the shift parameters becomes very sensitive to parameter changes. The effect is even more significant for the shift parameter \(l_a\) which involves the sound horizon \(r_s\) in the denominator (see equation (2.6)). The sound horizon drops more dramatically than the shift parameter \(R\) when the dark energy dominates at early times because the corresponding integral (2.7) depends only on the early time behavior of the expansion rate \(H(a)\). This effect is demonstrated by plotting the \(w_0 + w_1 = 0\) line in figure 1(b) which coincides approximately with the region where the contour deformation starts. The same line is also plotted in figure 4(b) corresponding to early CMB + BAO data [37] and involving only one shift parameter \((R)\). In this case the deformation effect is milder because the \(z\) integral corresponding to \(R\) spreads over a wide range of redshifts from \(z = 0\) to 1089 and the effect of dark energy domination is somewhat smeared out.

We also construct the likelihood contours using the combined SnIa + CMB + BAO data for \(\Omega_{0m} = 0.24\) and \(\Omega_b = 0.042\) corresponding to the best fit WMAP3 parameter values. As expected, the σ-distance between best fit and ΛCDM is at about 1σ, i.e. intermediate between the standard candle and standard ruler cases (see figure 6). Since the value \(\Omega_{0m} = 0.24\) practically coincides with the standard ruler (CMB + BAO) best fit from the full minimization while it is well within 1σ from the standard candle (SnIa) best fit, the results of figure 6 are not biased with respect to the value of \(\Omega_{0m}\).

3. Conclusions–discussion

We have demonstrated that there is a systematic difference in trends between standard candle (SnIa) and standard ruler (CMB + BAO) data. The former data are significantly more consistent with ΛCDM than the latter for practically all \((\Omega_{0m}, \Omega_b)\) parameter priors.
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Figure 6. The 68.3% and 95.4% $\chi^2$ confidence contours in the $(w_0, w_1)$ parameter space for the combined data sets SNIa + CMB + BAO for $\Omega_{0m} = 0.24$. The blue dot corresponds to the $(w_0, w_1)$ best fit while the red dot corresponds to $\Lambda$CDM ($-1, 0$).

within the $2\sigma$ range of WMAP3. In fact, the standard ruler data demonstrate a mild preference for a best fit $w(z)$ that crosses the phantom divide line $w = -1$.

This systematic difference in trends can be attributed to one of the following:

- **Statistical effects.** There is an $(\Omega_{0m}, \Omega_b)$ parameter range where the two data sets are consistent with each other and with $\Lambda$CDM at the $2\sigma$ level (see e.g. figure 3(b) with $\Omega_{0m} \simeq 0.25$). Therefore, for these parameter values the two data sets are consistent with each other and with $\Lambda$CDM at the $1\sigma$–$2\sigma$ level and the trend that we observe could well be a statistical fluctuation.

- **Systematic physical effects.** As discussed in [38] distances based on standard candles and standard rulers should agree as long as three conditions are met: (1) photon number is conserved, (2) gravity is described by a metric theory and (3) photons are traveling on unique null geodesics. If at least one of these conditions is not met then equations (1.2) and (1.3) will lead to generically different forms for the Hubble expansion rate $H(z)$ due to the violation of the distance duality relation (1.5). For example, lensing of SNIa by compact objects, if not properly accounted for, would tend to violate condition (3) and induce artificial brightening of distant SNIa. Alternatively, photon number violation (due e.g. to photon mixing [39]) would lead to artificial dimming of the SNIa.

In order to investigate the possible existence of systematic physical effects, we have used our results to test the cosmic distance duality relation (1.5). In particular, we use our results for the best fit parameter values ($(w_0, w_1)$) and their error bars obtained from each data set to derive constraints on the parameter $\eta(z)$. These constraints are shown in figure 7(a) for $\Omega_{0m} = 0.24$ and in figure 7(b) $\Omega_{0m} = 0.27$. Clearly, the anticipated value $\eta = 1$ is within $2\sigma$ for both priors used. Assuming a prior of $\Omega_{0m} = 0.24$ and taking an average value for $\eta(z)$ in the range $0 < z < 40$ (as for large enough $z\eta(z)$ converges (see
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Figure 7. The constrains on $\eta(z) = d_L(z)/d_A(z)(1 + z)^2$ for $\Omega_{0m} = 0.24$ in (a) and for $\Omega_{0m} = 0.27$ in (b). Clearly the anticipated value $\eta = 1$ is within 1–2$\sigma$ for both priors used.

Figure 7), yielding the value $\bar{\eta} = 0.96 \pm 0.07$ which is within 1$\sigma$ from the anticipated value $\eta = 1$. The consistency is somewhat reduced if we average over a more recent redshift range. In the range $1 < z < 2$ we find $\bar{\eta} = 0.95 \pm 0.025$ which is consistent with the anticipated value $\eta = 1$ at the 2$\sigma$ level. Similar results are obtained for other priors of $\Omega_{0m}$ within 2$\sigma$ from the WMAP3 best fit. Therefore, despite the mild difference in trends between SniA standard candles and CMB + BAO standard rulers, we find no statistically significant evidence for violation of the distance duality relation.

An interesting extension of this work would be the inclusion of more data from both categories. For example gamma ray bursts [15] could also be included as standard candles and x-ray profiles of clusters [17] or radio galaxies [18] could be included as standard rulers in order to investigate whether the mild difference in trends that we have identified persists in more general categories of data.

The Mathematica files with the numerical analysis of the paper can be found at http://leandros.physics.uoi.gr/rulcand/rulcand.htm.

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