Creation of a brane world with a bulk scalar field

Koh-suke Aoyanagi$^1$ and Kei-ichi Maeda$^{1,2,3}$

1 Department of Physics, Waseda University, Shinjuku-ku, Tokyo 169-8555, Japan
2 Advanced Research Institute for Science and Engineering, Waseda University, Shinjuku-ku, Tokyo 169-8555, Japan
3 Waseda Institute for Astrophysics, Waseda University, Shinjuku-ku, Tokyo 169-8555, Japan
E-mail: aoyanagi@gravity.phys.waseda.ac.jp, aoyanagi@asagi.waseda.jp and maeda@gravity.phys.waseda.ac.jp

Received 24 December 2005
Accepted 20 February 2006
Published 13 March 2006

Online at stacks.iop.org/JCAP/2006/i=03/a=012
doi:10.1088/1475-7516/2006/03/012

Abstract. We investigate the creation of a brane world with a bulk scalar field. We consider an exponential potential of a bulk scalar field: $V(\phi) \propto \exp(-2\beta\phi)$, where $\beta$ is the parameter of the theory. This model is based on a supersymmetric theory, and includes the Randall–Sundrum model ($\beta = 0$) and the five-dimensional effective model of the Hořava–Witten theory ($\beta = 1$). We show that for this potential a brane instanton is constructed only when the curvature of a brane vanishes, that is, the brane is flat. We construct an instanton with two branes and a singular instanton with a single brane. The Euclidean action of the singular instanton solution is finite if $\beta^2 > 2/3$. We also calculate perturbations of the action around a singular instanton solution in order to show that the singular instanton is well defined.

Keywords: cosmology with extra dimensions, cosmological phase transitions, string theory and cosmology, gravity
Contents

1. Introduction .............................................. 2
2. Action and equations of motion .......................... 3
3. Flat brane instanton .................................... 6
   3.1. Two-brane instanton ................................ 6
   3.2. Singular single-brane instanton .................... 7
4. Quadratic term of the perturbed action ............... 8
5. Toward an inflating brane instanton .................... 10
6. Conclusion ............................................... 12

Acknowledgments ............................................ 13

Appendix: quadratic term of the perturbed action ....... 13

References ................................................. 16

1. Introduction

A new paradigm of cosmology based on superstring/M-theory, the so-called ‘brane world’, has been discussed for the last few years. The prototype of a brane world was first discussed in [1,2]. More recently, this prototype has been combined with the idea of the D-brane found by Polchinski in string theory [3], and a new paradigm of a brane world has been developed [4]–[6]. Here, one of the most interesting approaches is that of Randall and Sundrum [5, 6]. They considered a pure five-dimensional (5D) Einstein gravity only with a cosmological constant in a bulk. In this scenario the effective 4D Einstein equations are obtained by projecting the 5D metric onto the brane [7, 8].

Whereas in a string/M-theory context one would also expect additional scalar fields, associated with the many moduli fields, as fundamental fields. Those fields will, in principle, also propagate in the bulk. For example, Lukas et al [9] derived an effective 5D action by a dimensional reduction from 11-dimensional M-theory. It contains a scalar field in the 5D bulk, which corresponds to moduli associated with compactification of six extra dimensions onto a Calabi–Yau space. In the model of a brane world with a bulk scalar fields, the effective four-dimensional Einstein equations are obtained covariantly by Maeda and Wands [10]. A brane world inflation with bulk scalar fields is discussed by Himemoto and his collaborators [11]. They proposed a ‘bulk inflaton model’ in which the inflation on the brane is caused by a scalar field in the bulk, but the bulk itself is not inflating. Cosmological perturbations are also discussed on the basis of dilatonic brane worlds [12]. The bulk scalar field yields a power-law inflation on the brane. In this model, perturbation equations are solved analytically. Another analysis of cosmological perturbations in the bulk inflaton models with an exponential potential is given in [13].
If a brane world describes our universe, we need to consider the creation of a brane world. Already, many authors have studied the creation of the Universe in four-dimensional spacetime. First the quantum creation of the universe was suggested by Vilenkin [14]. This approach is based on the picture wherein the Universe spontaneously nucleates in a de Sitter space. The mathematical description of this nucleation is analogous to quantum tunnelling through a potential barrier [15]. Another approach to quantum cosmology was developed by Hartle and Hawking [16]. They proposed that the wavefunction of the Universe is given by a path integral over compact Euclidean geometries with an appropriate boundary condition, which is called a ‘no-boundary’ boundary condition. Here the wavefunction of the Universe is expected to be proportional to $e^{-S_E}$, where $S_E$ is the Euclidean action.

In this paper we consider the creation of a brane world with a bulk scalar field using an instanton solution which is given by solving the 5D Euclidean Einstein equations. In order to construct a compact Euclidean manifold, we have to glue two copies of a finite patch of a bulk spacetime with a brane boundary by use of Israel’s junction condition [17]. Garriga and Sasaki [18] first constructed an instanton including an inflating brane in the Randall–Sundrum model. Hawking et al consider the creation of a brane world using an instanton, and discuss inflation and fluctuations during the de Sitter phase in the model containing the quantum correction term called a trace anomaly on the brane [19,20]. A model with similar quantum corrections was also analysed by Nojiri and Odintsov [21], and instanton solutions with a Gauss–Bonnet term in the bulk were discussed in [22,23].

The plan of this paper is as follows. In section 2, we present the Euclidean action and Euclidean equations of motion in the brane model with a bulk scalar field. In section 3, we obtain an instanton solution with two branes and that with a single brane. We then evaluate the Euclidean action of these instanton solutions. Although the single-brane instanton has a singularity, we find that the action is finite. In section 4, in order to show that this singularity is harmless, we consider perturbations of the action around the instanton solution. Our conclusions and remarks follow in section 6.

2. Action and equations of motion

We consider a scalar field in a bulk as well as gravity. The Euclidean action is given by

$$\begin{align*}
S_E &= -\frac{1}{2\kappa_5^2} \left\{ \int_{\mathcal{M}} d^5x \sqrt{g} \left[ R - \frac{1}{2} \partial_a \phi \partial^a \phi - V(\phi) \right] - \sum_i \int_{r=r_i} d^4x \sqrt{\gamma(i)} \left[ K^\pm_{(i)} - \lambda_{(i)}(\phi) \right] \right\},
\end{align*}$$

(2.1)

where $\kappa_5^2$ is the five-dimensional gravitational constant, $r_i$ is the position of the $i$th 3-brane, $\lambda_{(i)}$ denotes its tension, and $K^\pm_{(i)}$ denotes its extrinsic curvature. The suffix $i$ corresponds to the numbering of branes. For a two-brane model, $i = 1$ and $i = 2$ stand for a negative and a positive tension brane, respectively. Also for a single-brane model, we use $i = 0$ to stand for a brane (see figure 1).

Since we are looking for an instanton solution, we assume a highly symmetric Euclidean spacetime, whose metric is given by

$$\begin{align*}
ds_E^2 &= dr^2 + b(r)^2 \gamma_{\mu\nu} \, dx^\mu \, dx^\nu,
\end{align*}$$

(2.2)
Creation of a brane world with a bulk scalar field

Figure 1. Schematic diagrams of brane instantons. We construct instanton solutions by gluing two copies of a finite patch of a bulk spacetime with a brane boundary by use of the junction condition. The thick vertical circle at \( r = r_i \) (\( i = 0, 1, 2 \)) represents the four-dimensional maximally symmetric brane at which the two identical five-dimensional bulk spaces are glued.

where \( \gamma_{\mu\nu} \) is the metric of four-dimensional maximally symmetric space, which is classified into three types by the signature of curvature, i.e., \( k = 0 \) (zero), \( 1 \) (positive), or \( -1 \) (negative). These correspond to the curvature sign of the Friedmann universe after creation. Since the Euclidean space must be compact when we discuss its creation, in the case of \( k = 0 \) or \( -1 \), we have to make a space compact by identification. Then the flat spacetime is a four-dimensional torus, and that with \( k = -1 \) has a more complicated topology.

The equations of motion with these ansätze are given by

\[
(b')^2 = k + \frac{1}{12}b^2 \left( \frac{1}{2}(\phi')^2 - V(\phi) \right), \tag{2.3}
\]

\[
b'' = -\frac{1}{12}b \left[ \frac{3}{2}(\phi')^2 + V(\phi) + 2\lambda_i(\phi)\delta(r - r_i) \right], \tag{2.4}
\]

\[
\phi'' + \frac{4}{b}b'\phi' = \frac{dV}{d\phi} + \frac{d\lambda_i(\phi)}{d\phi}\delta(r - r_i), \tag{2.5}
\]

where the prime denotes the derivative with respect to \( r \), and \( k \) is a curvature of the brane.

In what follows, we specify the form of a scalar field potential. We assume that the bulk potential for a scalar field and the tension of the \( i \)th brane are given by

\[
V(\phi) = \left( \beta^2 - \frac{2}{7} \right)v^2e^{-2\beta\phi}, \tag{2.6}
\]

\[
\lambda_i(\phi) = \pm2\sqrt{2}ve^{-\beta\phi}, \tag{2.7}
\]

where \( v (>0) \) describes a typical energy scale, and \( \beta \) is a parameter of the model \([24, 25]\). The signature of equation (2.7) corresponds to the sign of tension of a brane.

This form includes the Randall–Sundrum model \((\beta = 0)\) and the five-dimensional effective model derived from M-theory via the Hořava–Witten theory by Lukas et al \((\beta = 1)\) \([9]\). If we assume supersymmetry and the superpotential given by an exponential potential, we find the above forms of a scalar field potential in a bulk and a tension on
Creation of a brane world with a bulk scalar field

Hence, our ansatz is rather generic in the context of a supersymmetric theory. This form of potential has also been used in [24]–[28].

By using the above potential and tension, we rewrite our basic equations. The equations of motion in the bulk are

\[(b')^2 = k + \frac{1}{12} b^2 \left[ \frac{1}{2} (\phi')^2 - (\beta^2 - \frac{2}{3}) v^2 e^{-2\beta\phi} \right], \tag{2.8}\]

\[b'' = -\frac{1}{12} b \left[ \frac{3}{2} (\phi')^2 + (\beta^2 - \frac{2}{3}) v^2 e^{-2\beta\phi} \right], \tag{2.9}\]

\[\phi'' + 4 b' \frac{\phi'}{b} = -2\beta \left( \beta^2 - \frac{2}{3} \right) v^2 e^{-2\beta\phi}. \tag{2.10}\]

Equations (2.9) and (2.10) are dynamical equations, while equation (2.8) is a constraint equation.

On each brane, we have to impose a boundary condition. Here we assume $Z_2$ symmetry on each brane as a conventional brane world model. If we adopt a different condition, our result may be changed.

From $Z_2$ symmetry, we have jump conditions for $b'$ and $\phi'$ on each brane as

\[b'(r_i) = \mp \epsilon_i \sqrt{\frac{2}{6}} b(r_i) v e^{-\phi} \bigg|_{r=r_i}, \tag{2.11}\]

\[\phi'(r_i) = \mp \epsilon_i \sqrt{2} v e^{-\phi} \bigg|_{r=r_i}, \tag{2.12}\]

where the upper (lower) sign applies to the first boundary at $r = r_1$ (the second boundary at $r = r_2$) and $\epsilon_i = \pm 1$ corresponds to the sign of tension of the $i$th brane. For a single-brane model, we take the lower sign.

For a single-brane model, we also have to impose another boundary condition at $r = 0$. Here we adopt the ‘no-boundary boundary condition’ [16]. This condition comes from regularity of the five-dimensional geometry. For $k = 1$, it simply gives the boundary condition of $b(0) = 0$, $b'(0) = 1$, $\phi'(0) = 0$. However, for $k = 0$ and $-1$, the same boundary condition does not give a regular spacetime. Hence we may have to impose a different boundary condition such as $b(0) \neq 0$, $b'(0) = 0$ with identification at $r = 0$. That is, we have to consider five-dimensional compactified manifold with topology different from that in figure 1(a) such as a five-dimensional inhomogeneous torus with a brane at $r = r_0$. In this paper, we just focus on instanton solutions with topology shown in figure 1(a). For instanton solutions with a different topology, we will give an analysis in a separate paper.

Although the condition $b(0) = 0$ for $k = 0$ and $-1$ causes a singularity as mentioned above, we may be able to construct a singular instanton such as the Hawking–Turok instanton [29]. Then we also discuss a singular instanton in this paper.

Before constructing an instanton solution, we show that a brane must be flat. The constraint equation (2.8) should be satisfied for any point $r$, and the boundary conditions (2.11) and (2.12) should also be satisfied at the position of a brane ($r = r_i$). Substituting equations (2.11) and (2.12) into equation (2.8), we find that $k$ must vanish. Therefore, we construct only an instanton solution with a flat brane. This condition, $k = 0$, may be understood from the following fact. In the present model, the effective
cosmological constant on the brane is given by

\[ (4)\Lambda = \frac{1}{4} \left[ V(\phi) + \frac{1}{12} \lambda(\phi)^2 - \frac{1}{8} \left( \frac{d\lambda(\phi)}{d\phi} \right)^2 \right] , \]  

(2.13)

(see [10]). Substituting our potential form (2.6) and tension (2.7) into equation (2.13), we find that the effective four-dimensional cosmological constant vanishes. Therefore the brane should be flat. This is because of supersymmetry. In what follows, we restrict our analysis to a flat brane model.

3. Flat brane instanton

In this section, we present flat brane instantons. We discuss a two-brane model and a single-brane one separately.

3.1. Two-brane instanton

Here we construct a two-brane instanton solution. We solve the equations of motion in the bulk, i.e., equations (2.9) and (2.10). The solutions are given by

\[ b(r) = b_0 r^{1/6} \beta^2 , \]  

(3.1)

\[ \phi(r) = \frac{1}{\beta} \ln \left( \sqrt{2} v \beta^2 r \right) . \]  

(3.2)

When the tension of the first brane is negative and that of the second brane is positive, this bulk solution satisfies the jump conditions (2.11) and (2.12) at any point \( r \). In other words, the positions of branes are freely chosen. We could put branes anywhere we like. This is because branes are flat (no curvature).

In order to find the most preferable positions of branes, we evaluate the Euclidean action (2.1). The Euclidean action (2.1) is rewritten using equations of motion (2.8) and (2.9) as

\[ S_E = -\frac{1}{2\kappa_5^2} \left\{ \int_M d^5x \sqrt{g} \left[ \frac{2}{3} V(\phi) \right] + \sum_i \int_{r_i}^{r_{i+1}} d^4x \sqrt{g_i} \left[ \frac{1}{3} \lambda_i(\phi) \right] \right\} \]

\[ = -\frac{V_4^\gamma}{2\kappa_5^2} \left\{ 2 \int_{r_1}^{r_2} dr b^4(r) \frac{2}{3} V(\phi) + \sum_i \frac{1}{3} b^4(r) \lambda_i(\phi) \right\} , \]  

(3.3)

where \( V_4^\gamma = \int dx^4 \sqrt{\gamma} \) is the volume of the manifold with a 4-metric \( \gamma_{\mu\nu} \). The factor 2 in front of the integral in (3.3) is required since our instanton solution is constructed from two copies of Euclidean manifolds (see figure 1). Performing the integration with solutions (3.1) and (3.2), we obtain

\[ S_E = -\frac{V_4^\gamma}{2\kappa_5^2} b_0^4 \left\{ \frac{2}{3} \left( r_1^{(2/3)^2} - 1 - r_2^{(2/3)^2} - 1 \right) \right\} = 0 . \]  

(3.4)

Unfortunately, this action vanishes and then does not have any minimum (or maximum) with respect to the positions of branes \( r_1 \) and \( r_2 \). Hence the positions of branes are not determined by the least action principle. This result may be related to the problem of moduli stabilization. We may need some additional mechanism to stabilize the distance between two branes [30].
3.2. Singular single-brane instanton

In this section, we consider a singular instanton such as the Hawking–Turok instanton [29]. For a single-brane instanton, from the bulk solution (3.1), we find that $b$ vanishes at $r = 0$. At this point, $r = 0$, the spacetime curvature of the five-dimensional manifold diverges and a scalar field also does so. Therefore this instanton solution has a singularity at $r = 0$. However, this singularity is ‘mild’ since the Euclidean action of this instanton solution is finite for some range of $\beta$.

To show this explicitly, we evaluate the Euclidean action. First we divide the action into three terms: $S_E^\text{total} = S_E^\text{bulk} + S_E^\text{brane} + S_E^\text{singularity}$, where $S_E^\text{brane}$ also includes a contribution from the Gibbons–Hawking term [31] at the positions of branes. $S_E^\text{singularity}$ is a boundary term at the singularities. Although we do not know an appropriate boundary condition at a singularity, our situation is the same as the case for a 4D singular instanton. For the 4D singular instanton, Vilenkin [32] adopted the Gibbons–Hawking term at the singularity. Here we also adopt the Gibbons–Hawking term. We will discuss later on the ambiguity of a boundary condition at a singularity.

In order that we perform the integration for $[0, r_0]$, first we integrate for an interval $[\varepsilon, r_0]$ ($\varepsilon \ll r_0$) and then take a limit of $\varepsilon \to 0$. The Euclidean action of the bulk is given by

$$S_E^\text{bulk} = -\frac{1}{2\kappa_5^2} \int_M dx^5 \sqrt{g} \left[ R - \frac{1}{2} \partial_a \phi \partial^a \phi - V(\phi) \right]$$

$$= -2 \frac{V_4^2}{2\kappa_5^2} \lim_{\varepsilon \to 0} \int_\varepsilon^{r_0} \frac{2}{3} b(r)^4 V(\phi) \, dr$$

$$= \lim_{\varepsilon \to 0} \frac{V_4^2}{2\kappa_5^2 \beta^2} \left( \frac{r_0^{(2/3\beta^2)-1}}{6(2/3\beta^2)} - \frac{\varepsilon^{(2/3\beta^2)-1}}{6(2/3\beta^2)} \right),$$

where the factor 2 in (3.5) is required since our instanton contains two copies of the Euclidean manifold (see figure 2).

The brane action is evaluated as

$$S_E^\text{brane} = -\frac{1}{2\kappa_5^2} \int_{r=r_0} \, dx^4 \sqrt{g(4)} \left[ K_{r=r_0}^\pm - \lambda_0(\phi) \right]$$

$$= -\frac{V_4^2}{2\kappa_5^2} \frac{1}{3} b(r_0)^4 \lambda_0(\phi)$$

$$= -\frac{V_4^2}{2\kappa_5^2 \lambda_0^2} \frac{2}{3} b(r_0)^{4(2/3\beta^2)-1}.$$ (3.7)

Finally the Gibbons–Hawking term at the singularity is given by

$$S_E^\text{singularity} = -2 \lim_{\varepsilon \to 0} \int_{r=\varepsilon} \, dx^4 \sqrt{g(4)} \left( \frac{1}{\kappa_5^2} K \right)$$

$$= \lim_{\varepsilon \to 0} \frac{V_4^2}{\kappa_5^2 \beta^2} \frac{4b_0^4}{6(2/3\beta^2)} \varepsilon^{(2/3\beta^2)-1},$$ (3.8)

where the factor 2 in (3.8) is also required since this instanton solution has two singularities.
Creation of a brane world with a bulk scalar field

Figure 2. Singular brane instanton. The thick vertical line at $r = r_0$ represents the $T^4$ brane at which the two identical five-dimensional bulk spaces are glued. The cross marks at $r = 0$ represent singular points.

We find that the total action is evaluated as

$$S_{\text{total}} = \frac{V_4}{\kappa_5^2} \frac{b_0^4}{\beta^2} \varepsilon^{(2/3)\beta^2 - 1} \left|_{\varepsilon \to 0} \right. \quad (3.10)$$

If $\beta^2 \leq 2/3$, this action converges. In that case, our instanton with a singularity could be allowed, which is called a singular instanton.

This action also has no minimum with respect to $r$. Therefore the position of a brane is not fixed, just as in the case of a two-brane instanton.

Although we have adopted the Gibbons–Hawking term at the singularity, the boundary condition at the singularity is somewhat ambiguous. For instance, when we consider the action of the asymptotically flat or asymptotically AdS spacetime, we need to add the counter-term in order for the action to be well defined [33]–[36].

However, in our present model, even if the Gibbons–Hawking term at the singularity is dropped from the action, our result does not change significantly. The action converges for $\beta^2 \leq 2/3$, which is the same condition as for the case with the Gibbons–Hawking term. Moreover, when we calculate the quadratic term of the perturbed action (fluctuations around the solution), which will be explicitly shown in next section, it will converge. This guarantees that such an instanton solution is well defined and harmless, just as the Hawking–Turok instanton is. Therefore we may conclude that our result does not depend on the choice of the boundary condition.

4. Quadratic term of the perturbed action

Here, we investigate perturbations around a singular instanton solution, whose Euclidean action is finite. Although a singular instanton contains a singularity, it is well defined if fluctuations around the instanton solution do not diverge near a singularity. Such an instanton solution could be realized under some circumstances [37].

We then calculate the quadratic term of the perturbed action near the singular point. As a useful form of a singular instanton for calculating the variations, we adopt the conformal frame, i.e.,

$$ds_E^2 = b^2(R) \left[ dR^2 + \gamma_{\mu \nu} dx^\mu dx^\nu \right]. \quad (4.1)$$
Under this coordinate system, the singular instanton solution (equations (3.1) and (3.2)) is given by

for $\beta^2 \neq 1/6$,

$$b(R) = \hat{b}_0 \left[ (1 - \frac{1}{6\beta^2}) R \right]^{1/(6\beta^2-1)},$$

$$\phi(R) = \phi_{1}^{c} + \frac{6\beta^2}{6\beta^2 - 1} \ln \left[ (1 - \frac{1}{6\beta^2}) R \right],$$

for $\beta^2 = 1/6$,

$$b(R) = b_0 e^{b_0 R},$$

$$\phi(R) = \phi_{2}^{c} + \frac{1}{\beta} R,$$

where $\hat{b}_0$, $\phi_{1}^{c}$, and $\phi_{2}^{c}$ are constants. For simplicity, we have introduced a new variable $\phi(R) \equiv \sqrt{2\kappa_5} \varphi(R)$. Note that the singular point of the instanton corresponds to $R = 0$ for $\beta^2 > 1/6$, while $R = -\infty$ for $\beta^2 \leq 1/6$.

The detailed calculation is given in the appendix. Here we show just the result. Up to total divergence terms, the quadratic term of the perturbed action (A.19) is

$$\delta_2 S_E = \frac{1}{2} \int d^5 x \left[ \dot{f}^2 + f_{\mu\nu} f^{\mu\nu} + \frac{\ddot{z}}{z} f^2 \right],$$

where a dot (\dot{}) denotes the derivative with respect to $R$, a vertical line (|) denotes the covariant derivative with respect to $\gamma_{\mu\nu}$, $f \equiv b^{3/2} [\delta \varphi + (\dot{\varphi}_0 / \mathcal{H}) \psi]$ is a gauge-invariant combination of perturbations of a scalar field and of the metric, and $z \equiv b^{3/2} \dot{\varphi}/\mathcal{H}$. $\mathcal{H} = b/b$ and $\psi$ are the Hubble parameter and one of the scalar modes of the metric perturbations (see equation (A.13)), respectively.

For our case,

$$\frac{\ddot{z}}{z} = \frac{3}{4} \frac{5 - 12\beta^2}{(6\beta^2 - 1)^2 R^2} \quad \text{for } \beta^2 \neq \frac{1}{6}$$

$$= \frac{3b_0^2}{4} \quad \text{for } \beta^2 = \frac{1}{6}.$$

Equation (4.6) is exactly the same as the form of a scalar field in flat spacetime with an $r$-dependent mass $\ddot{z}/z$. Note that $\gamma_{\mu\nu}$ is the metric of four-dimensional flat space. For $\beta^2 > 5/12$, the ‘mass’ term is negative. Hence we expect that such an instanton is unstable with respect to ‘time’ $r$. However, there might be an instanton solution with a different metric ansatz (e.g., with different topology), whose action is lower than that of the present solution. Although we may be able to conclude that there is no instanton for $\beta^2 > 5/12$, it is unlikely. Hence, in what follows, we consider only the case of $\beta^2 \leq 5/12$.

Varying equation (4.6) with respect to $f$, we obtain the equation of motion for $f$:

$$\ddot{f} + \Delta f - (\ddot{z}/z) f = 0.$$
Creation of a brane world with a bulk scalar field

The separation of variables, i.e., \( f = f_\ell(R)Y_\ell(x^\mu) \), gives two ordinary differential equations:

\[
(\Delta + \ell^2)Y_\ell = 0, \quad (4.10)
\]

\[
\ddot{f}_\ell - \left( \ell^2 + \frac{\ddot{z}}{z} \right) f_\ell = 0, \quad (4.11)
\]

where \( \ell \) is an eigenvalue and \( Y_\ell \) is its eigenfunction.

For each eigenmode \( \ell \), the quadratic term of the perturbed action is given by

\[
\delta_2 S_E = \frac{1}{2} \int dR \left[ \dot{f}_\ell^2 + \ell^2 + \frac{\ddot{z}}{z} \right] \int d^4x Y_\ell^2. \quad (4.12)
\]

Analysing the behaviour of \( f_\ell \) near the singularity, we evaluate this action. For \( \beta^2 > 1/6 \), toward the singularity \((R \to 0)\), the \( \ell^2 \) term dominates in equation (4.11). Then the regular solution of \( f_\ell \) behaves as

\[
f_\ell \propto R^{3/2} \left( 6\beta^2 - 1 \right). \quad (4.13)
\]

Inserting this solution into the action (4.12), we obtain

\[
\delta_2 S_E \propto R^{3/2(6\beta^2 - 1)} + \frac{3C_1}{2(6\beta^2 - 1)} R^{2(2 - 3\beta^2)/(6\beta^2 - 1)}, \quad (4.14)
\]

where \( C_1 \) is an integration constant. Near the singularity \((R \to 0)\), (4.14) is finite if \( 1/6 < \beta^2 \leq 2/3 \).

For \( \beta^2 < 1/6 \), near the singularity \((R \to -\infty)\), the \( \ddot{z}/z \) term dominates in equation (4.11). The regular solution of \( f_\ell \) behaves as

\[
f_\ell \propto e^{\omega R}, \quad (4.15)
\]

where \( \omega^2 = \ell^2 \). We also find the same form of the solution for \( \beta^2 = 1/6 \) if we set \( \omega^2 = \ell^2 + 9k_0^2/4 \).

Near the singularity \((R \to -\infty)\), the integration gives the asymptotic behaviour of the action as

\[
\delta_2 S_E \propto 2\omega^2 e^{2\omega R}. \quad (4.16)
\]

This is finite near a singularity.

We conclude that the singular instanton is well defined for \( \beta^2 \leq 5/12 \).

5. Toward an inflating brane instanton

As we have shown in section 2, a de Sitter brane instanton is not possible for the present potential form and tension (equations (2.6) and (2.7)). This is because our potential is based on supersymmetry. However, a de Sitter brane may be preferred from the point of view of cosmology. Supersymmetry is also broken in the present universe. Hence, in this section, we examine other potentials which contain some correction terms. Such corrections may be expected from quantum effects via a SUSY breaking process. They may provide us with an inflating brane instanton\(^4\).

\(^4\) If we include higher curvature terms such as the Gauss–Bonnet term, which are also expected to exist as quantum correction terms, we can construct a de Sitter brane instanton. See [22].
To be concrete, we consider the model with the following tension:
\[
\lambda = \pm (2\sqrt{2}ve^{-\beta \phi} + \lambda_i),
\]
where \(\lambda_i\) is regarded as a vacuum energy on the \(i\)th brane, e.g., the vacuum energy of quantum matter fields on the brane. We assume that \(\lambda_i\) is independent of a scalar field \(\phi\). Another model that we discuss is the one with tension such that
\[
\lambda = \pm 2\sqrt{2}\alpha ve^{-\beta \phi},
\]
where \(\alpha\) describes a deviation from the tension of the original model (2.7). This model is equivalent to the model with a modified bulk potential. In fact, when we perform the transformation such that \(v \rightarrow v/\alpha\) and \(\alpha \rightarrow [(\beta^2 - 2/3)/(\beta^2 - 2/3 + \delta)]^{1/2}\), we find that the model (5.2) is the same as the model with the bulk potential \(V = (\beta^2 - 2/3 + \delta) \exp[-2\beta \phi]\), where \(\delta\) is a deviation from the original bulk potential [12].

These two modifications give a non-vanishing cosmological constant on the brane. For some range of parameters \(\lambda_i\) and \(\alpha\) (e.g., \(\lambda_i > 0\) or \(\alpha < 1\)), we find a positive cosmological constant, which guarantees the existence of a de Sitter solution on the brane.

First we discuss the case of the tension (5.1). The junction condition in this case is
\[
b'(r_i) = \frac{b}{12} \left(2\sqrt{2}ve^{-\beta \phi} + \lambda_i\right)
\]
\[
\phi'(r_i) = \sqrt{2}v\beta e^{-\beta \phi}.
\]
When we define the new variable
\[
J \equiv \phi' - \sqrt{2}v\beta e^{-\beta \phi},
\]
the junction condition for a scalar field is given by \(J = 0\) at \(r = r_i\). Using the equations of motion (2.8)–(2.10) and the junction condition (5.4), the derivative of \(J\) with respect to \(r\) at \(r = r_i\) is given by\(^5\)
\[
J'(r_i) = -\frac{4}{3} \beta v^2 e^{-2\beta \phi} \left(1 + \frac{18}{b^2 v^2} e^{2\beta \phi}\right)^{1/2} + \frac{4}{3} \beta v^2 e^{-2\beta \phi}\bigg|_{r=r_i} < 0.
\]
\(J'\) is always negative at the position of the brane. This means that \(J = 0\) is satisfied on either the brane at \(r = r_1\) or at \(r = r_2\). Hence, if \(J = 0\) on one brane, e.g., \(r_1\), then we cannot impose \(J = 0\) on the other brane (\(r_2\)). This means that we cannot construct a two-brane instanton in this model. For a single-brane model, we find \(J < 0\) at \(r = 0\) from the no-boundary boundary condition. \(J = 0\) is not satisfied at any position of the brane.

Next we consider the model with tension (5.2). Then the junction condition is given by
\[
b'(r_i) = \frac{\sqrt{2}}{6} \alpha ve^{-\beta \phi}\bigg|_{r=r_i}
\]
\[
\phi'(r_i) = \sqrt{2}\alpha ve^{-\beta \phi}\bigg|_{r=r_i}.
\]
\(^5\) Here we have assumed \(b' > 0\). If the brane at \(r = r_1\) (\(r = r_2\) or \(r_0\)) has negative (positive) tension, which is a natural ansatz, we can prove analytically that this condition is obtained for \(\beta^2 > 2/3\). For \(\beta^2 < 2/3\), we have confirmed it numerically.
We rewrite this condition as follows:

\[ b'(r_i) = \frac{1}{6\beta} \phi'(r_i) b(r_i) \]  
\[ \phi'(r_i) = \sqrt{2} \alpha v e^{-\beta \phi} \bigg|_{r=r_i}. \]  

(5.9)  
(5.10)

Defining a new variable \( W \) by

\[ W \equiv b'(r_i) - \frac{1}{6\beta} \phi'(r_i) b(r_i), \]  
(5.11)

and using the equations of motion (2.9) and (2.10), we obtain the derivative of \( W \) with respect to \( r \) as

\[ W' = -3 \frac{b'}{b} W + \frac{3}{b} \bigg|_{r=r_i}. \]  
(5.12)

Since \( W = 0 \) at the position of the brane \( (r = r_i) \), \( W' \) is always positive. We cannot construct a two-brane instanton solution, because \( W = 0 \) and \( W' > 0 \) are not satisfied at both boundaries \( (r = r_1, r_2) \) simultaneously. For a single-brane model, \( W = 1 \) at \( r = 0 \) from the no-boundary boundary condition. In this case \( W = 0 \) is also not satisfied because \( W' > 0 \) at any position of the brane \( (W = 0) \).

As we have shown above, we cannot construct a de Sitter (inflating) brane instanton for the present types of modification. We may need different a type of correction term.

### 6. Conclusion

We have presented instanton solutions in the model with a bulk scalar field. For an exponential potential of a bulk scalar field and tension, which includes the Randall–Sundrum model \( (\beta = 0) \) and the five-dimensional effective Hořava–Witten theory \( (\beta = 1) \), we construct flat brane instanton solutions; one is a brane instanton solution with two flat branes, and the other is that with a single brane. We find that a single-brane instanton always has a singularity. However, the Euclidean action of such an instanton is finite. As a result, this singular instanton could be realized just like the Hawking–Turok singular instanton. In order to guarantee that the instanton is well defined, we also analyse the behaviour of the action perturbed around the singular instanton. We find that the quadratic term of the perturbed action is finite if \( \beta^2 < 2/3 \). This guarantees that the singular instanton is well defined. The second variation equation of the perturbed action is of the same form as that of a scalar field in flat spacetime with a ‘time \( (R) \)’ dependent mass. For \( \beta^2 > 5/12 \), this mass term becomes negative, which probably means that the instanton is unstable. We then conclude that a singular single-brane instanton is possible if \( \beta^2 \leq 5/12 \).

The action of the instanton solutions does not have any minimum with respect to the positions of branes. Thus we cannot adopt the least action principle to predict the initial state of a brane universe.
Taking into account some quantum corrections for a bulk potential or tension via a SUSY breaking process, we have also investigated the possibility of a de Sitter brane instanton solution. However, we could not find appropriate instanton solutions. In order to obtain a de Sitter brane instanton, we may need to include other important effects, such as the Casimir energy, which are not taken into account here, or higher curvature correction terms discussed in [22].

For a flat brane instanton that we constructed, we have to consider the evolution of the brane universe after its creation. We may not need inflation just after creation [38], or may have the KKLMMT type of inflation [39]. These issues are left to future study.

Acknowledgments

We would like to thank S Mizuno for useful discussion. KA acknowledges T Torii and N Okuyama for valuable comments. This work was partially supported by a Grant-in-Aid for Scientific Research Fund from the JSPS (No. 17540268) and another from the Japan–UK Research Cooperative Programme, and by Waseda University Grants for Special Research Projects and for The 21st Century COE Programme (Holistic Research and Education Centre for Physics Self-Organization Systems) at Waseda University.

Appendix: quadratic term of the perturbed action

We begin with the original Euclidean action:

\[ S_E = -\int d^5x \sqrt{g} \left[ \frac{1}{2\kappa_5^2} R - \frac{1}{2} \nabla_a \varphi \nabla^a \varphi - V(\varphi) \right]. \] (A.1)

We perturb the metric and a scalar field as

\[ g_{ab} = g^{(0)}_{ab} + h_{ab}, \quad \varphi = \varphi_0 + \delta \varphi. \] (A.2)

We then expand the total action up to second-order perturbations as

\[ \delta_2 (\sqrt{g}R) = \sqrt{g}^{(0)} \left\{ h^c_{\ a} h^c_{\ b} - \frac{1}{2} h h^{ab} + \left( \frac{1}{8} h^c_{\ a} h^c_{\ b} + \frac{1}{16} h^c_{\ d} h^d_{\ c} \right) g^{(0)ab} \right\} R^{(0)}_{ab} \]
\[ - \sqrt{g}^{(0)} \left( h_{ab} - \frac{1}{2} h g^{(0)ab} \right) \delta R_{ab} + \sqrt{g}^{(0)} g^{(0)ab} \delta^2 R_{ab}, \] (A.3)

where

\[ \delta R_{ab} = \delta \Gamma^c_{\ abc} - \delta \Gamma^c_{\ acb} + 2 \left( \delta \Gamma^a_{\ c} \delta \Gamma^d_{\ cb} - \delta \Gamma^d_{\ cb} \delta \Gamma^a_{\ cd} \right), \] (A.4)

\[ \delta^2 R_{ab} = \delta^2 \Gamma^c_{\ abc} - \delta^2 \Gamma^c_{\ aceb} + \delta \Gamma^d_{\ c} \delta \Gamma^e_{\ ab} - \delta \Gamma^e_{\ ab} \delta \Gamma^d_{\ ce} \delta \Gamma^c_{\ de}. \] (A.5)
Creation of a brane world with a bulk scalar field

\[ \delta \Gamma_{ab}^c = \frac{1}{2} \{ h^a_{c;b} + h^c_{b;a} - h_{ab}^c \}, \]  

(A.6)

\[ \delta^2 (\sqrt{g} R) = \sqrt{g^{(0)}} \left[ \left(h^{ac} h^b_{;c} - \frac{1}{2} h h^{ab} \right) R^{(0)}_{ab} + \left( \frac{1}{8} h^2 - \frac{3}{4} h_{cd} h^{cd} \right) R^{(0)} \right. \]

\[ + \frac{1}{4} \left( 2 h^{abc} h_{ac;b} - h^{abc} h_{ab;c} - 2 h^{ab} : h_{;a} + h_{;a} h^{a} \right) \]

\[ , \]  

(A.7)

\[ \delta^2 (\sqrt{g} \left[ - \frac{1}{2} g^{ab} \partial_a \varphi_0 \partial_b \varphi_0 - V(\varphi_0) \right]) \]

\[ = \sqrt{g^{(0)}} \left[ - \frac{1}{2} \frac{g^{a c}}{h^{a c}} + \frac{1}{4} h h^{ab} \right] \partial_a \varphi_0 \partial_b \varphi_0 \]

\[ + \left( \frac{1}{8} h^2 - \frac{3}{4} h_{cd} h^{cd} \right) \left( - \frac{1}{2} g^{(0) ab} \partial_a \varphi_0 \partial_b \varphi_0 - V(\varphi_0) \right) \]

\[ - \frac{1}{2} g^{(0) ab} \partial_a \varphi \partial_b \varphi + \left( h^{ab} - \frac{1}{2} h g^{(0) ab} \right) \partial_a \varphi_0 \partial_b \varphi - \frac{1}{2} h \frac{dV}{d\varphi} \bigg|_0 \delta \varphi \]

\[ - \frac{1}{2} \frac{d^2 V}{d\varphi^2} \bigg|_0 \delta \varphi^2 \right] . \]  

(A.8)

With the metric ansatz (4.1), a background spacetime is given by the equations of motion:

\[ \mathcal{H}^2 - \mathcal{H} - k = \frac{\kappa^2}{3} \varphi^2 , \]  

(A.9)

\[ \mathcal{H} + \mathcal{H}^2 - k = -\frac{\kappa^2}{6} \left[ \varphi^2_0 + 2 b^2 V(\varphi_0) \right] , \]  

(A.10)

\[ \varphi_0 + 3 \mathcal{H} \varphi_0 - b^2 V(\varphi_0) = 0 , \]  

(A.11)

where a dot denotes the derivative with respect to \( R \), \( k \) is the curvature of the brane, and \( \mathcal{H} = b / b \) is the ‘Hubble’ parameter. Using these equations, the second-order variation is given by

\[ - \frac{1}{2 \kappa^2} \delta^2 (\sqrt{g} R) - \delta^2 \left( \sqrt{g} \left[ - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - V(\varphi) \right] \right) \]

\[ = - \sqrt{g^{(0)}} \left[ - \frac{1}{8 \kappa^2} \left( h^2 - 2 h_{cd} h^{cd} \right) \frac{1}{b^2} \left( \mathcal{H} + 3 \mathcal{H}^2 - 3 k \right) \right. \]

\[ + \frac{1}{8 \kappa^2} \left( 2 h^{abc} h_{ac;b} - h^{abc} h_{ab;c} - 2 h^{ab} : h_{;a} + h_{;a} h^{a} \right) \]

\[ - \frac{1}{2} g^{(0) ab} \partial_a \varphi \partial_b \varphi + \left( h^{ab} - \frac{1}{2} h g^{(0) ab} \right) \partial_a \varphi_0 \partial_b \varphi \]

\[ - \frac{1}{2} h \frac{dV}{d\varphi} \bigg|_0 \delta \varphi - \frac{1}{2} \frac{d^2 V}{d\varphi^2} \bigg|_0 \delta \varphi^2 \right] . \]  

(A.12)

Since scalar, vector, and tensor modes of perturbations are decoupled in our model, we focus just on scalar perturbations. The scalar mode of metric perturbations is described as

\[ \Delta s^2_E = b^2 (R) \left[ \left( 1 + 2 A \right) d\mathbf{R}^2 + 2 B_{\mu \nu} dx^\mu dx^\nu + \left\{ (1 - 2 \psi) \gamma_{\mu \nu} + 2 E_{\mu \nu} \right\} dx^\mu dx^\nu \right] , \]  

(A.13)

where \( A, B, E, \) and \( \psi \) are metric components of perturbations.
The quadratic term of the action for scalar perturbations reads (see [40, 41])

\[ \delta_2 S_E = -\frac{1}{2\kappa_5^2} \int d^5x \sqrt{-\gamma} \left\{ b^3 \left[ 12\dot{\psi}^2 + 24\mathcal{H}A\dot{\psi} + 3(3\mathcal{H}^2 + \dot{\mathcal{H}})A^2 - 6\psi_{\mu}(A - \dot{\psi})^{\mu} \right] + 2\kappa_5^2 \left( -\dot{\phi}_0\dot{\delta}\varphi - 4\dot{\phi}_0\delta\varphi\dot{\psi} - 2AB^2 \frac{dV}{d\varphi}\delta\varphi \right) + \kappa_5^2 \left( -\dot{\varphi}^2 - \delta\varphi_{\mu}\delta\varphi^{\mu} - \frac{d^2V}{d\varphi^2}\delta\varphi^2 b^2 \right) + 6\Delta(B - \dot{E}) \left( \mathcal{H}A - \frac{\kappa_5^2}{3} \dot{\phi}_0\delta\varphi + \dot{\psi} \right) + k \left( 3\alpha^2 + 24A\psi - 24\psi^2 - 3(B - \dot{E})\Delta(B - \dot{E}) \right) \right\} + D_1 + D_2 \right\}, \quad (A.14) \]

where \( D_1 \) and \( D_2 \) are total divergence terms:

\[ D_1 = \frac{\partial}{\partial R} \left\{ b^3 \left[ -4\mathcal{H}A^2 - 8\mathcal{H}A\psi + 2\mathcal{H}A\Delta E + 16\mathcal{H}\psi^2 - 8\mathcal{H}\psi\Delta E + \mathcal{H}(\Delta E)^2 \right] - \mathcal{H}B_{\mu}B^{\mu} - 8\mathcal{H}\psi^2 + 4\mathcal{H}\psi\Delta E - 2\mathcal{H}E_{\mu\nu}E^{\mu\nu} + A_{\mu}B^{\mu} - 4\dot{\psi}\Delta B + \Delta E\Delta B + 8\kappa_5^2\psi\dot{\phi}_0\delta\varphi - 2\kappa_5^2\Delta E\dot{\phi}_0\delta\varphi + 2\kappa_5^2A\dot{\phi}_0\delta\varphi \right\}, \quad (A.15) \]

and

\[ D_2 = \left\{ b^3[\Delta B B^{\mu} - 2\Delta B\dot{B}^{\mu} + \Delta B\dot{E}^{\mu} + E^{[\mu\nu\rho]}E_{\nu\rho} - E^{[\mu\nu]}E^{[\mu\nu]} - A\dot{B}^{\mu} + B^{[\mu}\dot{B}^{\nu]} - \Delta B^{\mu} - 4\dot{\psi}\dot{B}^{[\mu} + \Delta E\dot{B}^{\mu} + 12\dot{\psi}B^{[\mu} - 3\Delta E\dot{B}^{\mu} \right] - 6\mathcal{H}AB^{\mu} + 2\kappa_5^2\dot{\varphi}_0B^{\mu} + k(12\dot{\psi}E^{[\mu} - 6AE^{[\mu} - E^{[\mu]}E_{[\mu]} \right] - 2\Delta EE^{[\mu} + 3(B - \dot{E})(B - \dot{E})^{[\mu]} \} \right\}. \quad (A.16) \]

Here we consider the model with \( k = 0 \) because we obtain only a flat brane solution. By varying equation (A.14) with respect to \((B - \dot{E})\), we get the following constraint equation:

\[ \dot{\psi} = \frac{\kappa_5^2}{3} \dot{\phi}_0\delta\varphi - \mathcal{H}A. \quad (A.17) \]

We introduce a gauge-invariant combination of perturbations of a scalar field and of the metric [40]

\[ f = b^{3/2} [\delta\varphi + (\dot{\phi}_0/\mathcal{H})\psi] = b^{3/2} \left[ \delta\varphi^{(G)} + (\dot{\phi}_0/\mathcal{H})\Psi \right], \quad (A.18) \]

where \( \delta\varphi^{(G)} \) is the gauge-invariant scalar field perturbation. Using (A.17) and (A.18) to replace \( \dot{\psi} \) and \( \delta\varphi \) with \( A, \psi, \) and \( v \), we obtain the following quadratic action:

\[ \delta_2 S_E = \frac{1}{2} \int d^5x \left[ \dot{f}^2 + f_{\mu\nu}f^{\mu\nu} + \frac{z}{z} f^2 + D_1 + D_2 + D_3 \right], \quad (A.19) \]

where \( z = b^{3/2}\dot{\phi}_0/\mathcal{H} \) and \( D_3 \) is a total divergence term:

\[ D_3 = \frac{\partial}{\partial R} \left\{ 2b^{3/2}\dot{\phi}_0Af - 2b^{3/2}\frac{\dot{\phi}_0}{\mathcal{H}}A\psi - \frac{3}{2}\mathcal{H}f^2 - \frac{\kappa_5^2}{3} \frac{\dot{\varphi}_0^2}{\mathcal{H}}f^2 - 2b^{3/2}\dot{\phi}_0f\psi + 2b^{3/2}\dot{\phi}_0f\psi - b^{3/2}\frac{\dot{\phi}_0}{\mathcal{H}}\psi^2 + b^{3/2}\dot{\phi}_0\frac{\dot{\phi}_0}{\mathcal{H}}\psi^2 - \frac{3b^2}{\kappa_5^2} \frac{\psi_{\mu\nu}^{[\mu}\psi^{\nu]}}{\mathcal{H}} \right\}. \quad (A.20) \]
This Euclidean action is the same as that of a massive scalar field \( f \) with a ‘time \( (R) \)’ dependent mass term, which is \((\dot{z}/z)f^2\), in a five-dimensional flat Euclidean space.

References

[1] Akama K, 1982 Lect. Notes Phys. 176 267
[2] Rubakov V A and Shaposhnikov M E, 1983 Phys. Lett. B 152 136 [SPIRES]
[3] Polchinski J, 1995 Phys. Rev. Lett. 75 4724 [SPIRES]
[4] Arkani-Hamed N, Dimopoulos S and Dvali G, 1998 Phys. Rev. Lett. 82 262 [SPIRES]
[5] Polchinski J, 1995 Phys. Rev. Lett. 75 4724 [SPIRES]
[6] Himemoto Y, Tanaka T and Sasaki M, 2002 Phys. Rev. D 65 024014 [SPIRES]
[7] Himemoto Y, Tanaka T and Sasaki M, 2002 Phys. Rev. D 65 104020 [SPIRES]
[8] Himemoto Y, Tanaka T and Sasaki M, 2002 Phys. Rev. D 65 104020 [SPIRES]
[9] Himemoto Y, Tanaka T and Sasaki M, 2002 Phys. Rev. D 65 104020 [SPIRES]
[10] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[11] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[12] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[13] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[14] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[15] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[16] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[17] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[18] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[19] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[20] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[21] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[22] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[23] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[24] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[25] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[26] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[27] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[28] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[29] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[30] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[31] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[32] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[33] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[34] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[35] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[36] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
[37] Vilenkin A, 1998 Phys. Rev. D 59 086004 [SPIRES]
Creation of a brane world with a bulk scalar field

[38] Linde A, 2004 J. Cosmol. Astropart. Phys. JCAP10(2004)004 [SPIRES]
Linde A, 2005 J. Phys. Conf. Ser. 24 151

[39] Kachru S, Kallosh R, Linde A, Maldacena J, McAllister L and Trivedi S P, 2003 J. Cosmol. Astropart.
Phys. JCAP10(2003)013 [SPIRES]

[40] Mukhanov V F, Feldmann H A and Brandenberger R H, 1992 Phys. Rep. 255 203 [SPIRES]

[41] Garriga J, Montes X, Sasaki M and Tanaka T, 1998 Nucl. Phys. B 513 343 [SPIRES]