Observation of a phononic higher-order Weyl semimetal

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Weyl semimetals (WSMs)1 exhibit phenomena such as Fermi arc surface states, pseudo-gauge fields and quantum anomalies that arise from topological band degeneracy in crystalline solids for electrons1 and metamaterials for photons2 and phonons3. Here we report a higher-order Weyl semimetal (HOWSM) in a phononic system that exhibits topologically protected boundary states in multiple dimensions. We created the physical realization of the HOWSM in a chiral phononic crystal with uniaxial screw symmetry. Using acoustic pump–probe spectroscopies, we observed coexisting chiral Fermi arc states on two-dimensional surfaces and dispersive hinge arc states on one-dimensional hinge boundaries. These topological boundary states link the projections of the Weyl points (WPs) in different dimensions and directions, and hence demonstrate the higher-order topological physics4–8 in WSMs. Our study further establishes the fundamental connection between higher-order topology and Weyl physics in crystalline materials and should stimulate further work on other potential materials, such as higher-order-topological nodal-line semimetals.

Symmetry plays a crucial part in the physics of topological materials. Symmetry-based indicators and topological quantum chemistry have been demonstrated as powerful theoretical tools for the diagnosis and prediction of topological insulators and semimetals. Recent studies uncovered symmetry-enforced phononic WSMs and nodal-line semimetals in three-dimensional (3D) natural and artificial crystals9,10. In particular, artificial phononic crystals with remarkable controllability provide an excellent platform towards various topological phases protected by crystalline symmetries. Excellent examples include the lately discovered higher-order (which includes 2D11–19 and 3D20–24 systems) topological insulators in phononic crystals. However, the recently proposed connection between the two fundamental classes of topological materials, the WSMs and higher-order topological insulators25,26, has only been tested in systems made of coupled 2D higher-order topological insulators27. Possibilities beyond such a limit are yet to be explored and may yield rich material properties.

To fill this gap, we realized experimentally the HOWSM phase, which exhibits simultaneously Weyl physics and higher-order topology using tetragonal lattices with uniaxial screw symmetry. We adopted a symmetry-based approach that provides material candidates beyond tight-binding models and thus gives access to material properties (such as dispersive hinge states) that cannot be described by tight-binding models. In principle, symmetry-based approaches enable richer phases of HOWSMs as well as efficient searches of their material candidates in both electronic and classical systems.

In conventional WSMs, WPs carry non-trivial topological chiral charge \( N_C \) and the band topology is manifested as the 2D chiral Fermi arcs that link the projections of the WPs with different chiral charges (Fig. 1a,b). The chiral charge \( N_C \), being equal to the change of the \( k \)-dependent Chern number (Fig. 1c), indicates that WPs are the monopole sources or sinks of the Berry flux. In HOWSMs, WPs can simultaneously carry the chiral charge \( N_C \) and the higher-order charge \( N_{Ch} \) (Fig. 1d,e). The higher-order charge \( N_{Ch} \), being equal to the change of the \( k \)-dependent higher-order topological numbers, reveals that WPs can be the pumping sources or annihilation sinks of the higher-order topology (Fig. 1f and Supplementary Note 1). For instance, in Fig. 1d such higher-order WPs (HOWPs) act as the transitions between the partial bandgaps with finite Chern numbers and the partial bandgap with a non-trivial quadrupole topological number. As a consequence, the HOWPs lead to 1D hinge arc states that link the projections of the HOWPs with opposite \( N_{Ch} \) in addition to the 2D chiral Fermi arcs (Fig. 1e). The coexisting chiral and higher-order charges of HOWPs in HOWSMs lead to rich physics, as elaborated below. Meanwhile, in the bulk, the interesting properties of WPS, for example, pseudo-gauge fields and chiral anomaly, remain intact for HOWPS, as these properties originate from the chiral charges.

We designed an airborne phononic crystal to realize the HOWSM in its phononic analogue. The phononic crystal, fabricated via a 3D-printing technology using photosensitive resin (Fig. 2a,b), formed a tetragonal lattice with lattice constants \( a = 20 \text{ mm} \) and \( a_c = 28 \text{ mm} \) for the \( x-y \) plane and the \( z \) direction, respectively. The phononic crystal is a layer stacking of planar periodic structures of square cavities with a chiral screw symmetry \( S_{4c} := (x, y, z) \rightarrow (y, -x, z + \frac{a}{4}) \), as illustrated in the right panel of Fig. 2c. Each square cavity has an air region with a geometry of \( 18.5 \text{ mm} \times 18.5 \text{ mm} \times h_1 \) \( (h_1 = 5 \text{ mm}) \) as surrounded by the resin walls of width \( d = 1.5 \text{ mm} \) in the \( x-y \) plane and those of thickness \( h_2 = 2 \text{ mm} \) in the \( z \) direction. In our design, each cavity does not couple to adjacent cavities in the same layer. However, each cavity couples to nearby cavities in the adjacent layers through the air cylinder at the middle of the \( x-y \) plane and that at the hinges of the unit cell. These cylinders extend periodically along the \( z \) direction. The radius of the former (latter) cylinder is \( r_1 = 1.5 \text{ mm} \) \( (r_2 = 4 \text{ mm}) \). These cylinders realize simultaneously the nearest-neighbour and next-nearest-neighbour couplings in a screw tetragonal manner. These couplings are crucial for the realization of the HOWSM phase, as elaborated in Supplementary Note 1.

The crucial symmetries in our phononic crystal are the following: the screw symmetry along the axis, \( S_{4c} \), the rotation symmetries along the \( x \) and \( y \) directions, respectively, \( C_{2x} := (x, y, z) \rightarrow (x, -y, z) \) and \( C_{2y} := (x, y, z) \rightarrow (x, y, -z) \). This symmetry-based approach provides material candidates in both electronic and classical systems.

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Letters

C2y := (x, y, z) → (−x, y, −z − ½), and the time-reversal symmetry T. As proved in Wang et al.25, the quadrupole topological index $q_{xy}$ is quantized to 0 or $\frac{1}{2}$ by the $C_{2x}T$ and $C_{2y}T$ combined symmetries. In our design, both the Weyl physics and the higher-order topology are induced by the chiral crystalline symmetry (space group 91: $P4_122$). In Supplementary Note 1, we show in detail that the $k_z$-dependent topological indices can be calculated through the symmetry indicators of the phononic Bloch wavefunctions. As a basis for the $k_z$-dependent band topology, we also verify in Supplementary Note 1 the $k_z$-dependent frequency gap by band-structure and density-of-states calculations. The band-structure calculation shows that there are three WPs between the second and third bands, a quadratic Weyl point (QWP) at $k = (0,0,0)$ and two HOWPs at $k = (\pi,\pi,\pm k_{WP})$. We found that for $|k_z| < k_{WP}$ ($k_{WP} = 0.55\pi/a_z$), a band-gap between the second and third bands emerged, which carried a topological quadrupole index $q_{xy} = \frac{1}{2}$ and a vanishing Chern number $C = 0$. In contrast, for $|k_z| > k_{WP}$, the second and third bands were also gapped where the Chern number was finite, $C = \pm \text{sign}(k_z)$, but the quadrupole index is trivial, $q_{xy} = 0$ (see Supplementary Note 1 for the details). The configuration of the WPs and $k_z$-dependent topological indices are exactly the same as that depicted in Fig. 1.

In this letter, we denote the $|k_z| < k_{WP}$ region with finite Chern numbers as the Chern partial gaps (CPGs), and the $|k_z| > k_{WP}$ region with the topological quadrupole index $q_{xy} = \frac{1}{2}$ as the higher-order partial gap (HOPG). The HOWPs with simultaneous chiral and higher-order charges lie at the boundaries between the CPGs and the HOPG (Fig. 2d–f). The QWP with a chiral charge $N_C = 2$ and higher-order charge $N_{q} = 0$ separates the CPGs with opposite Chern numbers (Fig. 2d–f). Figure 2d shows that the phononic crystal has a clean spectrum, which is ideal for the study of the properties of HOWSMs.

To verify the existence of the HOWPs, we first performed the analysis of the phononic bulk band structures using pump–probe measurements. For this purpose, an acoustic source (that is, a headphone) was placed at the centre of the phononic crystal sample, which can excite acoustic waves from 0.5 to 6.0 kHz. The phononic...
A phononic crystal sample had 20 unit-cell periods in the x, y, and z directions. A detector (that is, a small microphone) was inserted into the phononic crystal to scan the acoustic wavefunctions in the whole sample. The detected acoustic wavefunctions were then Fourier transformed to obtain the phononic bulk band structure (see Supplementary Note 2 for details). The measured phononic bulk spectrum along the high-symmetry lines is presented in Fig. 2g. From the figure, phononic dispersions that are consistent with the band-structure calculations are observed, especially when the dissipation and finite-size effects are considered, as analysed in detail in Supplementary Note 3. In particular, QWP and HOWPs are obviously witnessed between the second and third bands. To visualize the HOWP more clearly, we present the measured phononic bulk spectrum at \( k_z = k_{WP} \) in Fig. 2h. In the figure, the band crossing between the second and third bands as the HOWP is visualized.

We then used surface pump–probe spectroscopy to measure the phononic spectrum at the YZ surface of the same phononic crystal sample. The experimental set-up for the surface pump–probe spectroscopy is depicted in Fig. 3a. A subwavelength acoustic source \( S_1 \) was placed at the centre of the YZ surface where a resin plate was fabricated to form a hard-wall surface boundary. A small hole was opened at the centre of the surface boundary to insert the headphone as the acoustic source. A tiny microphone was inserted into the phononic crystal to probe the surface acoustic fields right below the hard-wall boundary. By scanning the acoustic field distributions underneath the surface boundaries for various frequencies and then Fourier transforming the detected acoustic fields, we can extracted the phononic dispersions for the surface states. Figure 3b shows the detected acoustic field distribution right below the YZ surface when the source \( S_1 \) has a frequency of 4.7 kHz.
The highly directional acoustic field pattern, which implies the openness of the isofrequency contour of the surface acoustic waves, is an indication of the Fermi arc surface states\(^3\). The measured dispersions of the surface acoustic waves for various \(k_z\) are presented in Fig. 3c–f, which show a reasonable agreement with the simulation, particularly when the dissipation and finite-size effects are taken into account, as elaborated in the detailed analysis in Supplementary Note 4. In Fig. 3c, with \(k_z = 0\), the measured phononic dispersions confirm the existence of the QWP at the \(\Gamma\) point (red sphere). In Fig. 3d, with \(k_z = 0.3\pi/a_x\), the CPG with Chern number \(C = 1\) is visualized experimentally through the gapless chiral edge states. In Fig. 3e, with \(k_z = k_{WP} = 0.55\pi/a_x\), the HOWP is found as the gap closing point at \(k_y = \pi/a_y\) (blue spheres). Although the bulk bandgap closes, the edge states still appear and merge into the HOWP, which can be understood as the residue effect of the helicoid Fermi arc surface states\(^3\). In Fig. 3f, at \(k_z = \pi/a_z\), the edge states become gapped, a feature consistent with the higher-order topology at \(|k_z| > k_{WP}\). From these measured dispersions, the transitions from the CPG to HOPG are visualized where the HOWP at \(k_z = k_{WP}\) serves as the transition point between them.

To visualize directly the Fermi arc surface states, we present the isofrequency contours of the surface acoustic waves in Fig. 3g at various frequencies. From 5.175 to 4.515 kHz, the emergence of the long chiral Fermi arcs that link the projections of the QWP and the HOWPs is clearly visible. Such very long chiral Fermi arcs (\(\pi/a\)) develop due to the highly chiral structure of the phononic crystal. The winding of the chiral Fermi arcs around the QWP is consistent with the picture of helicoid surface states in the WSMs\(^28\). In addition to the chiral Fermi arcs, gapped surface states due to higher-order topology emerge at \(|k_z| > k_{WP}\), as shown in Fig. 3f. The isofrequency contours of the YZ surface waves for various frequencies to visualize the chiral Fermi arc surface states at \(|k_z| < k_{WP}\) and the gapped surface states at \(|k_z| > k_{WP}\). Grey-shaded regions represent the calculated bulk bands. Black curves denote the calculated isofrequency contours of the YZ surface states.

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Fig. 3 | Topological surface states of the HOWSM. a, Illustration of the experimental set-up for the surface pump–probe spectroscopy. b, Measured amplitude of the acoustic pressure \(|p|\) right below the YZ surface as excited by the source \(S_1\) at the centre of the YZ surface. c–f, Measured dispersions of the surface waves (coloured) with \(k_z = 0\) (c), 0.3\(\pi/a_x\) (d), 0.55\(\pi/a_x\) (e) and \(\pi/a_x\) (f), separately. Grey curves represent the calculated bulk dispersions. Black curves denote the calculated dispersions of the YZ surface states. Red and blue spheres denote the projections of the QWP and HOWPs, respectively. g, Measured isofrequency contours (coloured) of the YZ surface waves for various frequencies to visualize the chiral Fermi arc surface states at \(|k_z| < k_{WP}\) and the gapped surface states at \(|k_z| > k_{WP}\). Grey-shaded regions represent the calculated bulk bands. Black curves denote the calculated isofrequency contours of the YZ surface states.
contour at 4.075 kHz indicates gapped surface states that do not connect the projections of the QWP and the HOWPs. An intermediate case with a frequency of 4.075 kHz shows the crossover between the chiral Fermi arc states and the gapped surface states.

Before the hinge states were measured, it was necessary to show that the hinge states were solely due to the higher-order topology. In our phononic crystal, if the intercell and intracell couplings are switched (by interchanging the geometry parameters $r_1$ and $r_2$), the higher-order topology and the hinge states can be eliminated. Although the bulk phonon dispersions remain the same, such an alteration leads to a conventional WSM (see Supplementary Note 5 for details). We then verified numerically the bulk-hinge correspondence by simulating the acoustic wave propagation when a source is placed at the top end of the hinge boundary (white stars, $S_2$). Figure 4a shows the propagation of the acoustic wave excited by the same source is well-confined by and propagates along the hinge boundary.

The emergence and propagation of the hinge states in the HOWSM was also confirmed in experiments at an excitation frequency of 4.0 kHz. As shown in the left panel of Fig. 4b, the detected acoustic wavefunctions were localized around the hinge boundary in the XOY plane, which was 235 mm below the source at the top end of the hinge boundary. The right panel of Fig. 4b indicates the propagation of the hinge arc states along the hinge boundary. The observation of the surface arc and hinge arc states in the same sample confirms one of the most important properties of HOWSMs: the coexistence of these topological boundary states at different dimensions (additional experimental evidence is given in Supplementary Note 6).

We then measured the dispersions of the phononic hinge arc states using the hinge pump–probe set-ups in the same phononic crystal sample. In such set-ups, the acoustic source is placed at either the bottom or the top end of the hinge boundary, and the detector scans the acoustic wavefunctions along the hinge. By setting the source at the bottom end of the hinge, one can only excite the hinge arc states with positive group velocities, that is, the hinge arc states that propagate upwards. Figure 4c shows the measured phononic spectrum for the hinge pump–probe set-up. Here, it is important to bear in mind that the phononic spectrum measured in the hinge pump–probe set-up may still contain contributions from the bulk and surface states beside that from the hinge states. As analysed in details in Supplementary Note 7, only the part of the measured...
spectrum in the frequency range from 3.86 to 4.66 kHz is dominated by the hinge states. From Fig. 4c, indeed, in this frequency range the measured phononic dispersion agrees with the calculated hinge dispersion for the hinge arc states with positive group velocities (the left branch of the black curves). However, by placing the acoustic source at the top end of the hinge boundary, we could excite the downward-propagating hinge states (the right branch of the black curves). The measured spectrum of such hinge arc states with negative group velocities also agrees with the band-structure calculation (as shown in Fig. 4d) when realistic conditions of the hinge pump–probe measurements are considered (as analysed in detail in Supplementary Note 7). The robustness of the hinge arc states against disorders that preserve the crystalline symmetry was investigated through the simulations shown in Supplementary Note 8. Supplementary Video 1 shows the dynamic propagation of hinge arc waves from the experimental measurements, as explained in Supplementary Note 9. Our symmetry-based design enables dispersive hinge arc states with a considerable bandwidth (about 800 Hz). In contrast, tight-binding models often create systems with flat hinge states\(^{25–27}\), which are more sensitive to losses and difficult to use in real life.

We proposed theoretically and observed experimentally a phononic HOWSM that exhibits intriguing multidimensional topological phenomena. The ability to integrate the hinge arc states on the 1D hinges with the Weyl points in the 3D bulk and the chiral Fermi arc states on the 2D surfaces in a single material may yield interesting material properties and potential applications. Realizing HOWSMS in electronic systems may lead to materials with fractional electronic charges at the hinge boundaries\(^{25}\) and other interesting properties\(^{25}\). Along these lines, future material studies on higher-order topological semimetals, such as higher-order Dirac semimetals\(^{26}\) and higher-order nodal-line semimetals\(^{26}\), are on the horizon.

### Online content

Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at [https://doi.org/10.1038/s41563-021-00985-6](https://doi.org/10.1038/s41563-021-00985-6).

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Methods

Simulations. All the simulations in this work were implemented with the acoustics module of COMSOL Multiphysics. The speed of sound and the air density used are 343 m s⁻¹ and 1.29 kg m⁻³, respectively. To obtain the bulk bands (Fig. 2d), boundaries of the unit cell in three directions were set to be periodic. To calculate the dispersions of the surface wave (Fig. 3), hard boundary conditions in the x direction and periodic boundary conditions in the remaining directions were applied to the supercell composed of 15 unit cells along the x direction. The surface wave dispersions were obtained by scanning the wavevectors in the entire surface Brillouin zone. To simulate the acoustic field distributions at the hinge, we calculated the frequency response of the conventional WSMs and HOWSMs with 8, 8 and 15 periods in the x, y and z directions, respectively (Fig. 4). The acoustic sources (white stars in Fig. 4a) were set at the top end of the hinge. To obtain the dispersions of the hinge states, a supercell with 11, 11 and 1 periods along the x, y and z directions, respectively, was constructed. Moreover, periodic boundary and the hard-wall boundary conditions were imposed in the z direction and two adjacent side surfaces, respectively. The remaining two side surfaces were set as radiation boundary conditions.

Experiments. The sample with 20, 20 and 20 periods (402 mm x 404 mm x 560 mm) in the x, y and z directions, respectively, was manufactured by a 3D printing technology using photosensitive resin. The 2-mm-thickness boundaries covered three sides of the sample, whereas the other sides were kept open. To show the bulk Weyl points, a headphone (diameter 6 mm) used for sound excitation was embedded in the centre of the sample. The microphone was inserted into the sample by a stainless-steel rod to measure the acoustic wave profiles using the network analyser (Keysight E5061B). The measurement set-up is shown in Supplementary Note 2. To measure the surface state, the headphone was placed in the cavity near the YZ surface of the sample (Fig. 3a, denoted as S). The microphone scanned the sample with scanning steps of 20 and 7 mm in y and z directions, respectively. By implementing the 2D Fourier transformations to the measured acoustic fields, the surface dispersion and surface Fermi arcs (Fig. 3) were obtained. For the hinge states, the acoustic source was placed at the top or bottom end of the hinge, and then the microphone scanned the acoustic fields along the hinge. The dispersions of the hinge states (Fig. 4) were unfolded by performing the 1D Fourier transformation on the measured acoustic pressure fields along the hinge.

Data availability

All data are available in the main text and the Supplementary Information. Additional information is available from the corresponding authors upon reasonable request.

Code availability

We used the commercial software COMSOL MULTIPHYSICS to perform the acoustic wave simulations and eigenstates calculations. Requests for the computation details can be addressed to the corresponding authors.

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Author contributions

J.-H.J. initiated the project. J.-H.J. and F.L. guided the research. J.-H.J., H.-X.W., B.J. and Z.-K.L. established the theory. H.-X.W. and L.L. performed the numerical calculations and simulations. L.L., Y.W., J.-H.J. and F.L. designed and achieved the experimental set-up and the measurements. All the authors contributed to the discussions of the results and the manuscript preparation. J.-H.J., H.-X.W., Z.K.L. and F.L. wrote the manuscript and the Supplementary Information.

Competing interests

The authors declare no competing interests.

Additional information

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