The effect of geometric characteristics on mechanical properties of asperities

Lin Huang¹,³, Cheng Zhao¹,²,³*, Bo Li³, Jinquan Xing¹ and Haoyu Pan¹

¹Department of Geotechnical Engineering, Tongji University, Shanghai, 200092, China
²Key Laboratory of Geotechnical and Underground Engineering of Ministry of Education, Tongji University, Shanghai, 200092, China
³Key Laboratory of Rock Mechanics and Geohazards of Zhejiang Province, Shaoxing University, Shaoxing, China

* Corresponding author: zhaocheng@tongji.edu.cn

Abstract. The deformation and failure behavior of fractured rock mass is closely related to the unevenness of the fracture surface. The deformation behavior of asperities on rough fracture surfaces has not been studied comprehensively. This paper aims to study the effect of geometric parameters on the mechanical properties of asperities under normal load. In this work, the uniaxial compression experiment using the discrete element software PFC3D was used to investigate the strength and stiffness law of asperities. The results show that compressive strength and stiffness are less affected by the height-width ratio of spherical asperities. However, the curvature of the asperities tip has a significant impact on the peak compressive strength and stiffness. As the curvature of asperities decreases, the peak compressive strength and stiffness rise. In addition, the strength and stiffness of the unevenness of the surface are also affected by the size effect. The peak compressive strength and stiffness increase as the size of the unevenness of the surface grows. The results show that the mechanical properties of asperities are significantly affected by the curvature and the size effect.

1. Introduction

Rock fissures are ubiquitous in underground rock masses and significantly affect the mechanical and hydraulic behavior of rock. The compression deformation, damage, and failure law fractured rock mass under external force are the basis for various studies related to the mechanical behavior of rock fractures. The deformation and destruction of fractured rock mass are often caused by the deformation of the unevenness of the fracture surface. The deformation behavior of asperities is closely related to the geometric morphology of asperities. Therefore, to analyze the deformation characteristics of fractured rock mass under normal force, it is necessary to study the mechanical properties of asperities with geometric parameters.

Extensive studies on contact deformation of rough fracture surfaces have been conducted in the past decades. A calculation model for elastic contact of multiple asperities was first introduced by Greenwood...
and Williamson [1], which is referred to as the GW model. Since then, many scholars have improved and expanded the GW model, and many new contact models suitable for rough fracture surfaces are proposed [2, 3]. The change in the height of the unevenness of the fracture surface and coalescence between adjacent growing contact zones is not considered. The asperities on rough surfaces have the same curvature under the action of external force, so these models are only suitable for lower pressure conditions [3]. The second type of model initiated by Persson [4] does not consider the notion of the unevenness of rough surfaces. It is built based on the relationship between contact pressure distribution and surface height. The model is more accurate in predicting contact pressure distribution [5], but it is less successful in predicting contact area [6]. With the improvement of computer technology, numerical calculation methods have gradually been applied to the field of contact mechanics and have become a commonly used research method of contact mechanics. The direct finite element method [7], the boundary element method based on the half-space method [8-11], and the discrete element method (DEM) [12, 13] are commonly used numerical calculation methods for the mechanical behavior of fractured rock mass. Hopkins [8] proposed a numerical method based on force-balance, using cylinders of different heights to simulate the rough fracture surface and considering the upper and lower surfaces of the crack and the elastic deformation of the cylindrical asperities. The geometric characteristics of the fracture surface and the local plastic deformation of the fracture are considered in the half-space method based on Boussinesq’s solution that treats the fracture surface as a continuous surface [9]. Kling [14] and Zou [11] used the theory of elastoplastic deformation to analyze the stress-displacement curve of the rough fracture surface and the damage zone. They verified that the numerical solution and experimental results are in good agreement. The effect of joint geometrical parameters on the mechanical properties of a non-persistent joint was studied by the DEM [12], and the mechanical properties of rock mass are significantly affected by the joint orientation with respect to principal stress direction. Valdez [13] proposed a methodology to produce rough self-affine rock joints using DEM and considered the impact of self-affine uneven joints, height variance, and self-affine correlation length on the closure and shear behaviors of 3D self-affine rock joints.

The damage of the fracture surface mainly concentrates in the area where the fracture surfaces are in contact with each other. The asperities in the contact area will be damaged and destroyed under load. As a result, the stiffness of the contact area of the fractured surface will change, which will cause damage and deformation of the fracture surface. At present, most studies consider the overall deformation behavior of the rough fracture surface. Although these studies have extensively promoted fractured rock mass damage, the changes in the strength and stiffness of the asperities in the contact area between the fracture surfaces are less considered. The strength and stiffness of asperities are closely related to their geometric parameters. Therefore, this paper takes the spherical unevenness of the surface with different geometric parameters as the research object. The uniaxial compression experiment based on a bonded particle model was conducted to study the change law of the strength and stiffness of the asperities under normal load.

2. Numerical method

2.1. Mechanism of the bonded particle model

The bonded particle model (BPM) [12, 15] has been used extensively to analyze rock damage and the mechanical behavior of rock [16]. The BPM is a collection of non-uniformly sized spherical particles that cemented when they contact each other. There are two contact forms to provide normal and tangential stiffness in BPM, as shown in figure 1. Particles considered to be rigid are allowed to overlap under compression in the first form. The normal and shear stiffness between particles are $k_n$ and $k_s$, respectively. The second contact form assumes that the contact plane provides the parallel bond normal and shear stiffness $\bar{k}_n$ and $\bar{k}_s$. In addition, parallel bonds have normal and shear strength, which resist tensile and shear forces. The parallel bond will be destroyed, and the bond stiffness will be removed when the normal or shear stress is greater than the bond strength. When the bonds break under loading, micro-
cracks will occur, propagate and coalesce.

![Diagram of the mechanism of the BPM](image)

**Figure 1.** Schematic diagram of the mechanism of the BPM [12, 15].

### 2.2. Micro-parameters of the BPM

When using the BPM to simulate rock materials, it is necessary to determine the micro-geometric and micromechanical parameters of the particles and bonds. The geometric parameters of the particles include the smallest radius $R_{\text{min}}$, the ratio of the largest radius to the smallest radius $R_{\text{max}}/R_{\text{min}}$. The mechanical parameters of the particles include Young's modulus $E$, the proportion of normal and shear stiffness of the particles $k_{n}/k_{s}$, and friction coefficient $\mu$. The microscopic parameters of bonds are radius multiplier $\bar{x}$, Young's modulus $\bar{E}$, normal and shear stiffness ratio $\bar{k}_{n}/\bar{k}_{s}$, tensile strength $\bar{\sigma}$, shear strength $\bar{\tau}$, and friction angle $\bar{\phi}$.

Since it is difficult to obtain the micro-parameters, it is necessary to adjust the microscopic parameters of the BPM to make the macroscopic mechanical properties of the numerical simulation consistent with the experimental results. A $\varnothing50\times100$ mm cylindrical specimen was generated by the DEM and was compressed under different confining pressures. The microscopic parameters of the BPM are shown in table 1, and the calibration results, which are Young's modulus $E$, uniaxial compressive strength $UCS$, Poisson's ratio $\nu$, cohesion $c$, and internal friction angle $\phi$, are shown in table 2.

**Table 1.** Micro-parameters of bonded particle model.

| Ball parameters | Parallel bond parameters |
|-----------------|--------------------------|
| $\rho$ (kg/m$^3$) | $2380$ | $8.0$ |
| $R_{\text{min}}$ (mm) | $1.0$ | $40.2$ |
| $R_{\text{max}}/R_{\text{min}}$ | $1.66$ | $1.0$ |
| $E$ (GPa) | $8.0$ | $23.1$ |
| $\mu$ | $0.577$ | $30$ |
| $k_{n}/k_{s}$ | $2.5$ | $2.5$ |
Table 2. Result for calibration of BPM.

|        | E (GPa) | UCS (MPa) | ν   | c (MPa) | φ (°) |
|--------|---------|-----------|-----|---------|-------|
| Experimental | 7.20    | 40.42     | 0.23 | 10.44   | 30    |
| Numerical   | 7.06    | 38.8      | 0.23 | 10.66   | 30.96 |

2.3. Verification of the numerical method

In order to evaluate the simulation effect of DEM on the mechanical properties of asperities, a uniaxial compression test was carried out on a spherical specimen with a diameter of 50 mm using the micro-parameters in Table 1. The load-displacement curve and failure of the sphere were obtained in the laboratory [17]. The numerical calculation results and laboratory results of the spherical sample are shown in figure 2. The microcracks appear at the contact point and gradually grow. Finally, a cutting-through crack appears in the vertical direction of the sphere. Comparing the numerical simulation and the experimental result, the failure mode of the sphere is almost the same. As the load increases, the load-displacement curves obtained by the experimental and DEM have similar growth trends. When the displacement exceeds 0.5 mm, the slopes of the two curves are stable and the same, so the rigidity of the sphere is almost the same. The peak compressive strength obtained by numerical simulation is greater than the laboratory results. The reason is that the numerical simulation of the spherical specimen uses the micro-parameters of a cylindrical specimen. The size of the spherical sample is smaller than that of the cylindrical sample, and the same size particle is used, so the size effect is different in the two samples. The compressive strength obtained by numerical simulation is slightly higher than the laboratory results, but the total displacement and stiffness of the sphere are almost the same. Thus, the numerical simulation and experimental results are relatively consistent. Overall, the mechanics and damage behavior of gypsum materials can be described by the DEM.

![Figure 2. Numerical and experimental results of point-surface contact.](image)

3. Numerical experiment of asperities with different geometric parameters

3.1. Geometric Morphology of asperities

In order to study the influence of different geometric characteristics on the mechanical mechanism of asperities, the asperity models with different curvatures, height-width ratios (H/W), and sizes are shown in figure 3. The radius of the asperity tip is $R$. The curvature represents the reciprocal of radius. The height-width ratio is the ratio of the height to the width of the asperity. Samples (series A to D) with
varying curvatures are used to analyze the effect of curvature on the mechanical properties of asperities. To study the impact of the height-width ratio, Samples D to G have the same curvature and different height-width ratios. Finally, the size effect is investigated by samples D, H to J with the same height-width ratio. It is worth noting that the tip radius of the asperities in samples A to C is greater than the radius of the cylindrical sample, and the bottom of the samples is completely composed of a sphere with a curvature. The value of H/W will change with the different curvatures, so it is meaningless to give the height-width ratios of the asperities. Therefore, the height-width ratios of asperities in samples A to C are represented by “N/A” in Figure 3.

3.2. Results

The load-displacement curve of asperities with different curvatures is shown in figure 4(a). As the load increases, the displacement gradually increases. As the curvature of asperities drops, the peak compressive strength and the slope of the curve rise. The reason is that the overall rigidity of the unevenness of the surface increases when the curvature decreases. As a result, it can be seen that the curvature of asperities has a significant influence on the peak UCS and stiffness of asperities.

The load-displacement curve of asperities with different height-width ratios is shown in figure 4(b). In the case of the same asperity radius, the compressive strength of the unevenness of the surface with different height-width ratios is basically the same. However, with the change of the height-width ratio, the slope of the load-displacement curve has a certain change. The possible reason for this phenomenon is that the distribution of the collection of non-uniformly sized spherical particles at the tip of asperities is different. The different particle distribution leads to the different local stiffness of the contact tip, making the slopes of the load-displacement curve different. It shows that the height-width ratio has little effect on the mechanical properties of asperities.

| Series | A | B | C | D | E | F | G | H | I | J |
|--------|---|---|---|---|---|---|---|---|---|---|
| R (mm) | 100 | 50 | 33.3 | 25 | 25 | 25 | 25 | 18 | 12.5 | 8 |
| Curvature (mm⁻¹) | 0.01 | 0.02 | 0.03 | 0.04 | 0.04 | 0.04 | 0.04 | 0.056 | 0.08 | 0.125 |
| Height-width ratio | N/A | N/A | N/A | 0.5 | 0.375 | 0.25 | 0.167 | 0.5 | 0.5 | 0.5 |

Figure 3. Morphology of asperities.
Figure 4. The load-displacement curve of asperities. (a) different curvatures, (b) different height-width ratios.

The load-displacement curve of asperities with different sizes is shown in figure 5. As the normal displacement increases, the load growth rate gradually slows down, and the load-displacement curve before failure fluctuates up and down. The reason is that micro-cracks inside the asperities will be generated, aggregated, expanded, and lead to the load decreasing rapidly. As the displacement continues to increase, the asperities locally form a relatively stable structure, causing the curve to rise again. Due to the growth and propagation of cracks inside the asperities, the overall growth trend of the load-displacement curve will gradually slow down. When the height-width ratio of asperities is the same, the peak compressive strength and the slope of the curve increase with the growth of R. The reason is that the overall rigidity of the unevenness of the surface will increase as the size increases.

Figure 5. The load-displacement curve of asperities with different sizes.

4. Conclusions
Based on the discrete element numerical analysis method, compression experiments of asperities with different height-width ratios, curvatures, and sizes are carried out. The main conclusions are as follows:

1. From the sphere compression test, it can be seen that the peak strength and stiffness of asperities calculated by the DEM are consistent with the experimental results in the laboratory. The compression
deformation behavior of the unevenness of the surface can be simulated well by the discrete element numerical analysis method based on the BPM.

2. The curvature and size effect have a significant influence on the mechanical properties of asperities. As the curvature of the unevenness of the surface increases, the peak compressive strength and stiffness decrease. The peak compressive strength and rigidity increase with the rise of the size of asperities. In a sense, the size and curvature of asperities have the same physical meaning. The size effect and the curvature have the same impact on the mechanical properties of the unevenness of the surface.

3. The peak compressive strength and stiffness of asperities are less affected by the height-width ratio. The damage of asperities is mainly concentrated on the tip of the unevenness, so the mechanical properties of asperities are mainly affected by the curvature. When the curvature of the asperities remains unchanged, and the height-width ratio is changed, the damage position of asperities does not vary. Therefore, the peak strength and stiffness almost unaffected.

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