Schrödinger Equation for Joint Bidirectional Evolution in Time: Astrophysical Applications

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Abstract. A straightforward extension of quantum mechanics and quantum field theory is proposed that can describe physical systems comprising two interacting subsystems: one subsystem evolves forward in time, the other, backward. The space of quantum states is the direct sum of the states representing the respective subsystems, whereupon there are two linearly independent vacuum states, one each for the forward and backward subspace. An indefinite metric is imposed on the space of quantum states such that purely forward (respectively, purely backward) states have positive (respectively, negative) norms. Hamiltonians are self-adjoint operators with respect to the metric, such that interactions/transitions between the subspaces can be accounted for. Given suitable definitions of input and of output states at the two ends of a time interval, input and output states separately have positive norms such that probability is conserved, and hence $S$-matrices are unitary. A discussion of the physics entailed in the proposed formalism is undertaken. Then as an application, a simple model of a relativistic quantum field theory is proposed. In this model, the expected vacuum energy (thought to be associated with the cosmological constant) almost vanishes uniformly for times in an interval due to cancellation of the energies of the forward and backward vacuum states; this cancellation holds whatever be the input vacuum state at the ends of the time interval. A model is advanced wherein magnetic monopoles live exclusively in backward-evolving states, and interact with forward-evolving electric charges in a certain way. Proposals for further research, particularly concerning the possible detection of advanced gravitational waves, and a conjecture on the physics of dark matter and dark energy, conclude the report.

1. INTRODUCTION

This report comprises a sketch of, and an elaboration of, material published previously [1]. In the the present Sec. 1, we shall review the construction proposed in [1] for an extension of quantum mechanics that can describe physical systems in which both forward and reverse causality in time can occur jointly and interactively.

Let $\mathcal{H}^F$ be the Hilbert space incorporating quantum states of the contents of the known world, comprising the quantum fields describing all possible states and occupations of electrons and other leptons, protons and other hadrons, photons and other vector bosons, etc. (Note: we shall exclude gravitation from the quantum world, and treat the gravitational field as governed by classical field equations, which determine the structure of the background space-time within which quantum phenomena evolve.) This world is known to evolve forward (hence the superscript $F$) in time; that is, given the physical state at an earlier time one can predict the probabilities of various outcomes at a later time. Now suppose that there is another Hilbert space $\mathcal{H}^B$, where the “$B$” stands for “backward”, wherein physical phenomena evolve contrariwise, that is, given the quantum state of a system at a later time, one can predict the probability of various outcomes
at an earlier time. We can call the latter a world “antiparallel” to our own. An intuitive, geometrical picture of the system comprises two parallel Minkowski spacetimes identified point-by-point in a metric-preserving way, such that certain interactions, or quantum jumps, can occur between particles living on different subspaces, but such that the dynamical time evolution in the two spacetimes is oppositely directed. The notion of these parallel spaces being embedded in some higher-dimensional spacetime is superfluous, and indeed is inconsistent with their oppositely directed causality in time.

Suppose further that the universe of quantum states comprises the direct sum of the above two spaces,

$$\mathcal{H} = \mathcal{H}^F \oplus \mathcal{H}^B.$$  \hspace{1cm} (1)

This means that if $\Psi^F$ is a state in $\mathcal{H}^F$ and $\Psi^B$ is a state in $\mathcal{H}^B$, then a general quantum state $\Psi \in \mathcal{H}$ looks like

$$\Psi = \begin{bmatrix} \Psi^F \\ \Psi^B \end{bmatrix}.$$  \hspace{1cm} (2)

Now let $M$ be a metric operator on $\mathcal{H}$ made up of the identity operators $I^{FF}$ and $I^{BB}$ on $\mathcal{H}^F$ and $\mathcal{H}^B$, respectively, as follows:

$$M = \begin{bmatrix} I^{FF} & 0 \\ 0 & -I^{BB} \end{bmatrix} = M^{-1} = M^\dagger.$$  \hspace{1cm} (3)

where the “$\dagger$” is the usual Hilbert space adjoint. Then if $\Phi$ and $\Psi$ are two quantum states in $\mathcal{H}$ their inner product $(\Phi, \Psi)$ with respect to $M$ is taken to be

$$(\Phi, \Psi) = \Phi^\dagger M \Psi,$$  \hspace{1cm} (4a)

$$= \left[ (\Phi^F)^\dagger \ (\Phi^B)^\dagger \right] \begin{bmatrix} I^{FF} & 0 \\ 0 & -I^{BB} \end{bmatrix} \begin{bmatrix} \Psi^F \\ \Psi^B \end{bmatrix}$$  \hspace{1cm} (4b)

$$= (\Phi^F)^\dagger \Psi^F - (\Phi^B)^\dagger \Psi^B,$$  \hspace{1cm} (4c)

$$= \langle \Phi^F | \Psi^F \rangle_F - \langle \Phi^B | \Psi^B \rangle_B,$$  \hspace{1cm} (4d)

where $\langle \Phi^F | \Psi^F \rangle_F$ and $\langle \Phi^B | \Psi^B \rangle_B$ are the Hilbert space inner products in Dirac notation.

We presume a Schrödinger equation of the form

$$i\hbar \frac{\partial \Psi}{\partial t} (t) = H(t) \Psi(t),$$  \hspace{1cm} (5)

where $t$ is the time, $\Psi(t)$ is as in (2), and $H(t)$ has the form

$$H(t) = \begin{bmatrix} H^{FF} (t) & H^{FB} (t) \\ H^{BF} (t) & H^{BB} (t) \end{bmatrix};$$  \hspace{1cm} (6)

note that $H^{FB} (t)$ is an interaction Hamiltonian, and performs a Hilbert space map $\mathcal{H}^B$ into $\mathcal{H}^F$, while $H^{BF}$ performs such a mapping in reverse. As usual herein, the superscripts are to be read from right to left: the right superscript refers to the operand,
alias domain space, the left to the image space. The Hamiltonian is to be self-adjoint with respect to the metric operator \( M \), that is

\[
H(t) = MH(t)\dagger M,
\]

so that

\[
(H_{FF}(t))\dagger = H_{FF}(t), \quad (H_{BB}(t))\dagger = H_{BB}(t), \quad H_{BF}(t) = -(H_{FB}(t))\dagger.
\]

In (8c), the adjoint, denoted with a “\( \dagger \)”, has the usual mathematical definition of the adjoint of a mapping of one Hilbert space into another, that is

\[
\langle (H_{FB}(t))\dagger |\Psi_B(t) \rangle_B = \langle \Psi_F(t)|H_{FB}(t)\Psi_B(t) \rangle_F
\]

defines a unique adjoint operator such that the above is true for all admissible \( \Psi_B(t) \in \mathcal{H}^B \) and \( \Psi_F(t) \in \mathcal{H}^F \).

Now suppose that the state vectors in (4a) are both time dependent, and are both solutions of (5). We take the time derivative of their inner product:

\[
i\hbar \frac{\partial}{\partial t} \langle \Phi(t), \Psi(t) \rangle = i\hbar \frac{\partial \Phi}{\partial t}(t) M \Psi(t) + i\hbar \Phi(t)\dagger M \frac{\partial \Psi}{\partial t}(t)
\]

\[
= \Phi(t)\dagger (-H\dagger M + MH) \Psi(t)
\]

\[
= 0,
\]

where the last line results from (7) and (3). Therefore the inner product of any two solutions of the Schrödinger equation is time independent; in particular, the \( M \)-norm of a time dependent quantum state, deconstructed as in (2), is conserved in time if \( \Psi(t) \) satisfies (5). Let \([t_a, t_b]\), with \( t_a < t_b \), be an interval in time. Then from (4d) we must have for \( \Psi(t) \) that

\[
\langle \Psi_F(t_b)|\Psi_F(t_b) \rangle_F - \langle \Psi^B(t_b)|\Psi^B(t_b) \rangle_B = \langle \Psi_F(t_a)|\Psi^F(t_a) \rangle_F - \langle \Psi^B(t_a)|\Psi^B(t_a) \rangle_B.
\]

Rearranging, we have

\[
\langle \Psi^F(t_b)|\Psi^F(t_b) \rangle_F + \langle \Psi^B(t_a)|\Psi^B(t_a) \rangle_B = \langle \Psi^F(t_a)|\Psi^F(t_a) \rangle_F + \langle \Psi^B(t_b)|\Psi^B(t_b) \rangle_B.
\]

Both sides of (12) are positive definite. This result suggests the following construction and interpretation, as pictured in Fig. 1: let the input and output to a physical process that takes place for \( t_a \leq t \leq t_b \) be the composite state vectors

\[
\Psi_{in}(t_a, t_b) = \begin{bmatrix} \Psi^F(t_a) \\ \Psi^B(t_b) \end{bmatrix},
\]

\[
\Psi_{out}(t_a, t_b) = \begin{bmatrix} \Psi^F(t_b) \\ \Psi^B(t_a) \end{bmatrix}.
\]


This makes physical sense: the input to a physical process in the time interval \([t_a, t_b]\) comprises the \(F\)-type signal at the beginning \(t_a\) of the interval and the \(B\)-type signal at the end \(t_b\), while the output comprises the \(F\)-type signal at the end and the \(B\)-type signal at the beginning of the time interval.

An analogous imposition of boundary conditions applies in generalized classical dynamics: for \(F\)-type particles we prescribe their positions and velocities at the beginning of a time interval, while for \(B\)-type particles we should specify their positions and velocities at the end of a time interval. Correspondingly, it is an extension of observed properties of objects in the real world that an external perturbation on an otherwise isolated particle or system of particles affects only the future trajectories of \(F\)-type particles, and would affect only the past trajectories of \(B\)-type particles (neglecting internal feedback interactions of a system in time).

Note that (12) implies that the positive-definite Hilbert-space norms of the input and output states are equal:

\[
\Psi_{\text{out}}(t_a, t_b) \Psi_{\text{out}}^\dagger(t_a, t_b) = \Psi_{\text{in}}(t_a, t_b) \Psi_{\text{in}}^\dagger(t_a, t_b).
\]  

Therefore, nonnegative probabilities are defined and conserved overall, notwithstanding the indefinite metric \(M\) introduced for instantaneous state vectors. The instantaneous \(M\)-norm (4a) of a state vector represents the flow of probability across the given \(t=\text{constant}\) surface: \(F\)-type states’ probability flows forward in time, \(B\)-states’ probability flows backward in time. The \(S\)-operator effects a transition from the input state to the output state:

\[
\Psi_{\text{out}}(t_a, t_b) = S(t_a, t_b) \Psi_{\text{in}}(t_a, t_b)
\]  

and must therefore be unitary:

\[
S(t_a, t_b)^\dagger S(t_a, t_b) = I_{\mathcal{H}},
\]

where \(I_{\mathcal{H}}\) is the identity operator in \(\mathcal{H}\). We now have a framework that is globally, but not instantaneously, consistent with ordinary quantum mechanics, and makes a physically plausible way to describe temporally “lumped” processes in which both forward and backward evolution in time, including transitions between the subspaces, can occur jointly.
I have not been able to discover an instantaneous density matrix formalism and associated dynamics that admits of a rearrangement to a mapping of input into output density matrices, thereby generalizing the state vector transformation between (11) and (12). I infer that computation of the evolution of mixtures from input to output must be subordinate to the treatment of the evolution of each of an assembly of pure states, which can then be combined into mixtures at input and, therefore, at output. Entanglements between F- and B-type states at input, or separately at output, can be described in this formalism.

We observe also that for instantaneous expectation values of a physical observable to be real, it must correspond to an operator self-adjoint with respect to the metric \( M \). That is, let \( Z(t) \) be a linear operator in \( \mathcal{H} \) for each time \( t \) such that for all \( \Psi(t) \in \mathcal{H} \) we have

\[
Z(t) = MZ(t)M.
\]

If we define the expectation value of \( Z(t) \) at time \( t \) in the state \( \Psi(t) \) as

\[
\langle Z(t) \rangle_{\Psi(t)} = \Psi(t)MZ(t)\Psi(t) = (\Psi(t), Z(t)\Psi(t)),
\]

then \( \langle Z(t) \rangle_{\Psi(t)} \) can easily be shown to be real; in fact,

\[
(\Psi(t)^\dagger MZ(t)\Psi(t))^* = \Psi(t)^\dagger Z(t)M^\dagger\Psi(t) = \Psi(t)^\dagger MZ(t)\Psi(t).
\]

We shall construe the expectation value of an operator at an instant of time as the net flow of the corresponding quantity (probability, energy, momentum, electric charge, baryon number, etc.) across the complete spacelike surface. This viewpoint therefore incorporates and generalizes the conventional interpretation of expectation values, as now quantities can flow backward as well as forward in time. In particular, as noted by Pauli ([2], Sec. 2), operators with only positive eigenvalues can have negative expectation values, corresponding to the backward-in-time flow of positive “stuff”. We remark that, as discussed in [1], Sec. 3, for time-independent, energy-conserving Hamiltonians we shall assume the unperturbed \( FF \) and \( BB \) sub-Hamiltonians both have positive eigenvalues, so that transitions between \( F \) and \( B \) states can be caused by time-independent \( FB \) interactions. (In this respect the present model differs from the Klein-Gordon and Dirac wave equations, where there are negative energy states in the unperturbed eigenvalue spectrum. Those respective equations yield indefinite and positive-definite conserved norms, respectively.)

This concludes our review of the mathematical physics underlying the proposed scheme. More details can be found in [1]. The remainder of this report is organized as follows: in Sec. 2, we shall discuss the physics of joint bidirectional motion in time, including \( F \leftrightarrow B \) transitions, a possible structure for a theory of measurement including the “grandfather paradox”, and a suggested treatment for gravitational radiation. In Sec. 3 we shall consider a simple model quantum field theory in which \( F \leftrightarrow B \) transitions can occur, remark on the unavoidable cancellation of vacuum energies in the model, and make a suggestion on the possible generation of advanced gravitational waves. Sec. 4 addresses the question of the existence of magnetic monopoles in \( B \)-type states (and not in \( F \)-type states). Sec. 5 concludes the report with suggestions for further research along these lines.
2. SOME RELEVANT PHYSICS

In this section we shall discuss some physics of the direct sum construction, outline an (at present, undeveloped) theory of measurement with reference to the “grandfather paradox”, and discuss how the present scheme might be made consistent with gravitation in the form of Einstein’s theory.

The direct sum construction of (1) and (2) has a different physical meaning from the direct product construction (as, for example, in the wave function for two or more particles) that is usual in combining subsystems in quantum mechanics, or combining different quantum fields to represent different families of physical particles. We are saying that there is really only one physical system, which has available quantum states within one of two antiparallel worlds. Furthermore both of these worlds coexist on the same underlying four-dimensional, pseudo-Riemannian, space-time manifold. The latter assumption imposes constraints on gravitational waves, since in Einstein’s geometric theory of gravitation, to which we shall try to accommodate our argument, the space-time manifold has a metric structure that alone accounts for gravitational phenomena. Matter, and indeed vacuum energy, momentum, and pressure of matter fields, in both $F$ and $B$ states will be taken to be sources of gravity. We shall propose a decomposition of the energy-momentum-pressure tensor into two tensor parts, one of which is coupled to gravitation by radiating retarded (diverging as time advances) gravitational waves, the other by radiating advanced (converging) gravitational waves. We shall discuss these matters further later in this Section and in Appendices A and B.

The hypothesis of coexisting antiparallel worlds of quantum states will also have consequences when we try to make a dynamical theory for a family of particles that locally obeys Lorentz/Poincaré invariance and can make transitions between the subspaces: the two subspaces of quantum states will be constrained to resemble one another to a certain degree beyond the assumption that they have in common the gravity-generated four-dimensional space-time—see the discussion in [1], Sec. 5.

Let us now turn to the subject of measurements in the extended quantum theory. A possible setup is displayed in Fig. 2, below. We shall not attempt a thorough quantitative analysis here. This is the scenario: Let $t_a < t_b < t_c < t_d$. The specified overall input signal comprises the direct sum of the vectors $\Psi^F(t_a)$ and $\Psi^B(t_d)$, the quantum output (to be determined) is the direct sum of $\Psi^F(t_d)$ and $\Psi^B(t_a)$. In an intermediate time interval $t_b \leq t \leq t_c$ measurements are performed on the system. We presume, as shown in Fig. 2, that the measurements are done separately on the $F$ and $B$ components of the state vector; no reflections in the time dimension, that is, no $F \leftrightarrow B$ transitions, are to be caused by the measurement process. Reflections can occur within interaction zones 1 and 2, so that the “trajectory” of any particle in the system has a possible feedback loop $1 \rightarrow F \rightarrow 2 \rightarrow B \rightarrow 1$ or $2 \rightarrow B \rightarrow 1 \rightarrow F \rightarrow 2$.

Suppose that the physical system consists of just one particle: The particle can traverse this loop zero, one, two, three, … times before exiting. If the measurement zones $F$ and $B$ do nothing more than record the particle’s passing through a respective zone, we will be able to count the number of times the system has gone around the feedback loop. We will need to distinguish types of predictions, according to whether or not we specify if, and how many, feedback loops are undergone by the particle between entry and exit of the overall time interval $[t_a, t_d]$.
Note also that this feedback property compels a revision of the traditional picture of the “collapse” of a wave function at the time of a measurement. To be sure, it is plausible that if a measurement is performed in the $F$ zone the wave function will collapse in part between $\Psi^F(t_b)$ and $\Psi^F(t_c)$, although there is the possibility that the particle crosses a given $t =$constant plane at more than one spatial position in an $F$-type state due to feedback. (An analogous partial collapse can occur in the measurement that takes $\Psi^B(t_c)$ into $\Psi^B(t_b)$.) Such a measurement also cannot simultaneously wipe out the $B$ component of the wave function, as a reflection can occur later in zone 2 that ensures a nonzero chance of the system crossing measurement zone $B$ at the same time that the particle is detected in zone $F$. A mathematical description of the input-to-output of this system must be done in a self-consistent manner, such that the information gleaned, and used or not used, in the intermediate measurements is reckoned as part of the output. I have made a tentative analysis of a classical analog to such a feedback system [3], but have not yet undertaken a careful analysis of a quantum system of the type described.

Assuming no intermediate measurements, a simple version of the grandfather paradox ([4], [5]) might be encapsulated as follows with respect to Fig. 2: given a partial input $\Psi^F(t_a) \neq 0$ and zero partial input at the later time ($\Psi^B(t_d) = 0$), a certain partial output $\Psi^F(t_d)$ will ensue. Can one now superimpose another partial input $\Psi^B(t_d)$ so that the resultant partial output $\Psi^F(t_d) = 0$? It is plausible, given suitable reflectivity properties of the interaction zones, that the partial output generated at $t = t_d$ by the two partial inputs might be made to cancel exactly by quantum interference. This cancellation could
not plausibly occur in a classical probabilistic linear system where the probabilities are all nonnegative. (Some complex, nonlinear classical probability scheme might do the job, however.) It seems, therefore, that realization of the grandfather paradox would be easier to effect (at least in a thought experiment) when quantum, as opposed to classical, mechanics holds.

We shall end this section with a preliminary discussion of the role of gravitational waves. It has been inferred from the gradual winding down of the orbital motion of a binary star (pulsar plus ordinary neutron star companion—see [6]) that this system loses energy in the form of gravitational radiation, in accord (to about \( \pm 0.2\% \)) with the quantitative predictions of Einstein’s theory ([7], [8]). This outcome causes a problem in the present context: as discussed in the next section, we will argue that the expectation value of the vacuum energy due to the \( F \)-type and \( B \)-type vacuum inevitably tend to cancel one another to high accuracy, and that this cancellation has the observable consequence that the cosmological constant is very small. This means that we are assuming that, while that part of the world in \( B \)-type states is invisible in the sense that it doesn’t emit/absorb electromagnetic waves of \( F \) type, at least, the part of the world in \( B \) states does interact with the gravitational field, the same field that we, living in \( F \)-type states, interact with. If gravitational waves are always of retarded type, however, there would be an asymmetry of gravitational phenomena with respect to time. Such a lopsidedness would tend to make the whole construct doubtful. There is a way around the problem, though, as we shall argue \textit{ad hoc}, by assuming that the matter in certain quantum transitions emits (or rather from our viewpoint of view, absorbs) gravitational radiation that converges on the spacetime-localized transition. Let me say that this is the most promising physical test that I have been able to formulate as yet to access the truth or falsity of the hypothetical structure described herein. This means that there would be gravitational waves present in our universe that have no visible source, as they are converging on a future event that only emits light forward in time, at least within the \( F \)-type world that we inhabit.

Quantitatively, the hypothesis concerning gravitational radiation looks as follows. Suppose that \( \kappa, \lambda, \mu, \nu = 0,1,2,3 \) are space-time coordinate indices. Let \( T^{\mu\nu}(r) \) be the operators representing the energy-momentum-pressure tensor components, where the Hamiltonian \( H \) is ([1], Eq. (63))

\[
H = \iiint_{\text{space}} \, d^3 r \, T^{00}(r).
\] (20)

Einstein’s field equations for gravity take the classical energy-momentum-pressure tensor field to be the source of gravitation. We shall construe this classical tensor field as expectation values of the quantum operators, that is,

\[
\langle T^{\mu\nu}(t,r) \rangle = \Psi(t)^\dagger M T^{\mu\nu}(r) \Psi(t),
\] (21)

where \( \Psi(t) \) is the quantum state of the universe at time \( t \) except gravity—we are not attempting anything more than a classical treatment of gravitational phenomena here. Note also that the operators \( T^{\mu\nu}(r) \) are presumed to incorporate the factor \( 16\pi G/c^4 \). The classical metric tensor field \( g_{\mu\nu}(t,r) \) obeys a set of nonlinear partial differential equations; the source terms are those of (21). Given a suitable combination of boundary
conditions, and given the source terms, we can solve the field equations for the metric tensor field. (Gravity also affects the evolution of the sources, so that a self-consistent procedure is required to get physical solutions of matter plus gravity.)

At modest (say, galactic) scales space-time looks pretty flat, so let us consider gravitational waves generated by some source involving matter in \( F \) or \( B \) states on a background flat spacetime, so that \( g_{\mu \nu} = \text{diag}(-c^2, 1, 1, 1) \). We represent these waves as a perturbation \( \delta g_{\mu \nu}(t, \mathbf{r}) \) on the flat-space metric. To first order in perturbation theory the \( \delta g_{\mu \nu} \) must be linear functionals of the source terms of (21). The functional operators that take the sources into the generated gravitational waves are known as Green’s functions. We have distinct choices for these Green’s functions, called retarded (left superscript \( R \)) and advanced (left superscript \( A \)):

\[
\begin{align*}
R G_{\mu \nu \kappa \lambda}(t, \mathbf{r}; t', \mathbf{r}'), \\
A G_{\mu \nu \kappa \lambda}(t, \mathbf{r}; t', \mathbf{r}').
\end{align*}
\] (22)

The retarded Green’s function \( R G \) is zero for \( t < t' \) and says that a source generates outgoing gravitational waves, while the advanced Green’s function \( A G \) is zero for \( t > t' \) and generates waves converging on the source as conventional time advances. Furthermore, let us assume that we can break up the source term (21) into two parts, as follows:

\[
\langle T^{\mu \nu}(t, \mathbf{r}) \rangle = \langle R T^{\mu \nu}(t, \mathbf{r}) \rangle + \langle A T^{\mu \nu}(t, \mathbf{r}) \rangle.
\] (23)

We shall discuss in the Section 3 and in Appendix A how to effect such a splitting of the matter field in a particular quantum field theory model. The upshot is that the perturbation of the metric tensor is presumed to be

\[
\delta g_{\mu \nu}(t, \mathbf{r}) = \sum_{\kappa, \lambda = 0}^{3} \iiint_{\text{spacetime}} dt' d^3 r' \left[ R G_{\mu \nu \kappa \lambda}(t, \mathbf{r}; t', \mathbf{r}') \langle R T^{\kappa \lambda}(t', \mathbf{r}') \rangle + A G_{\mu \nu \kappa \lambda}(t, \mathbf{r}; t', \mathbf{r}') \langle A T^{\kappa \lambda}(t', \mathbf{r}') \rangle \right].
\] (24)

Plausibly this formalism can be generalized to the case that the “zeroth order” background gravitational metric is some cosmological or other nontrivial case, such that the gravitational waves are generated as first-order perturbations on this background. I don’t know how to formulate this mixed retarded/advanced gravitational radiation criterion as, say, boundary conditions in the context of Einstein’s full, nonlinear field equations.

We shall discuss the possible detection of advanced gravitational waves in Sec. 5.

3. MODEL QUANTUM FIELD THEORY

A model quantum field theory for Lorentz scalar, electrically neutral particles was established in [1], Sec. 4. The \( FF, BB, FB \) and \( BF \) interaction Hamiltonian densities were all presumed to be proportional to \( \phi(\mathbf{r})^4 \)—see [1], Eqs. (62b) and (63c). We shall not review these basics here, but elaborate on the results of [1], Secs. 4 and 5.

The first question we want to address is whether any separation of the expectation value of the energy-momentum-pressure tensor into advanced and retarded sources—see (23) above—is Lorentz invariant on the presumed flat background spacetime. The
Lorentz transformation properties of the full energy-momentum-pressure tensor were asserted, based on unpublished work, in [1], following Eq. (95). The computation is nontrivial mainly for Lorentz "boosts", the generators of which are given by [1], Eq. (68d). Second, we recapitulate the argument of [1] that the expectation value of the vacuum energy tends to cancel to a high precision. And third, we shall make a plausible speculation concerning the nonrelativistic gravitational interactions between, and dynamics of, two point particles.

With respect to the Lorentz-invariant separation of the energy-momentum-pressure tensor into two nontrivial parts: There proves to be essentially one way to do this, modulo fixed linear combinations of the constituent tensors in the split. Let us define projection operators in the space of quantum states:

\[
P_F = \begin{bmatrix} I_{FF} & 0 \\ 0 & 0 \end{bmatrix}, \tag{25a}
\]

\[
P_B = \begin{bmatrix} 0 & 0 \\ 0 & I_{BB} \end{bmatrix}. \tag{25b}
\]

Then if we define

\[
D T^\mu\nu(r) = p_F T^\mu\nu(r) p_F + p_B T^\mu\nu(r) p_B, \tag{26a}
\]

\[
N T^\mu\nu(r) = p_F T^\mu\nu(r) p_B + p_B T^\mu\nu(r) p_F, \tag{26b}
\]

the separate expectation value arrays both prove to transform as second rank symmetric tensors with respect to the Lie algebra generators ([1], Eq. (70)) of the Poincaré group—we shall omit the details of the proof of this statement here. Unlike the summed tensor on the lhs of (23), however, the separate contributions on the rhs do not have zero four-divergence, that is, do not represent conserved currents of energy and momentum for the matter fields; therefore, we cannot use these entities directly in (24). What is needed is to subtract from the rhs of (26a) and add to the rhs of (26b) a Lorentz-invariant quantity so that both resulting tensors have zero four-divergence—the latter entities can then be applied to (24) using the advanced and retarded Green’s functions derived from inverting d’Alembert’s operator. We exhibit some mathematical steps along these lines in Appendix A.

It was argued in [1] that the net vacuum energy density is forced by the dynamics to be very small uniformly within a time interval (see [1], Eq. (94b)) when the discriminant of [1] Eq. (75) is negative. In physical terms, this means that the magnitude of the $F \leftrightarrow B$ energy term is greater than the magnitude of the splitting between the energies of the unperturbed $F$ and $B$ vacua. When this inequality occurs, the energy eigenvalues associated with the two vacuum states are nonreal (and are complex conjugates of one another), so that the quantum states are closed-channel states, such as often occur in nonrelativistic scattering theory. The corresponding largeness of the off-diagonal $FB$ and $BF$ matrix elements of the Hamiltonian compared to the splitting of $FF$ and $BB$ energies seems to mean that most of the low-lying energy levels of matter in the universe are also closed channels. Only quantum states with relatively large energy differences between the uncoupled $F$ and $B$ levels therefore should correspond to open-channel, nontransient states having real energy levels—it is intuitively plausible that our observed
universe comprises a mixture of nontransient, open-channel energy states. Therefore, there must be substantial differences in the complexes of energy states occupied by the F-type subworld and the B-type subworld, in order that the vacuum coupling does not overwhelm the cosmology. I have not yet undertaken to do cosmological modeling by seeing which, if any, of the input parameter values of [1], Eq. (94b) are such that the present hypotheses can be chosen to be consistent with what is known.

Let us neglect all nongravitational interactions for the moment, and consider what happens in the nonrelativistic, weak field limit of the mutual interaction of two gravitating particles. Consistent with Einstein’s geometrization of gravity, we presume that the principle of equivalence of inertial and gravitational mass holds in any circumstance. It is also plausible, on the basis of Einstein’s theory, that Newton’s equations hold to a first approximation for the equations of motion of two particles of masses \(m_1\) and \(m_2\), and positions \(\mathbf{r}_1\) and \(\mathbf{r}_2\), whether they might be both of F type, or both of B type, or of mixed type:

\[
m_1 \frac{d^2 \mathbf{r}_1}{dt^2} = -G m_2 \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3},
\]  
\[
m_2 \frac{d^2 \mathbf{r}_2}{dt^2} = -G m_1 \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|^3}.
\]

(See Appendix C for further development of the above dynamics.) Particles of type F will have positive energy expectation values, and hence positive inertial and gravitational masses; particles of type B will have negative energy expectation values, and hence negative inertial and gravitational masses. If (i) both \(m_1 > 0\) and \(m_2 > 0\), the particles accelerate toward one another; if (ii) both \(m_1 < 0\) and \(m_2 < 0\), the particles accelerate away from one another. If (iii) \(m_1 > 0\) and \(m_2 < 0\) the F-type particle 1 accelerates away from the B-type particle 2, while particle 2 accelerates toward particle 1. If (iv) \(m_1 < 0\) and \(m_2 > 0\), the latter tendencies are reversed. Combining (28a) and (28b), we infer that

\[
\frac{d^2 (\mathbf{r}_2 - \mathbf{r}_1)}{dt^2} = -G (m_1 + m_2) \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|^3}.
\]

If in case (iii) \(m_1 > -m_2 > 0\), the relative motion of the particles exhibits acceleration toward one another, while if \(-m_2 > m_1 > 0\), the relative motion shows acceleration away from one another. I have not attempted to model the gravitohydrodynamics of fluid mixtures of F-type and B-type matter.
4. MAGNETIC MONOPOLES

In this section we shall address the question of the existence of (at present, hypothetical) magnetic monopoles [11], [12], [13].

Magnetic monopoles have not been convincingly detected in experiments, albeit not for want of trying—see [11], Sec. II. This asymmetry in electromagnetic physics and in Maxwell’s equations for the electromagnetic fields has long been noted and questioned, despite the resulting mathematical convenience of being able to define an electromagnetic vector potential field without Dirac string singularities or other contrivances. Dirac ([14], reproduced in [11]) noted that the existence of just one magnetic monopole would ensure the quantization of electric charge. Thomson ([15], reproduced in [11]) noted that there is angular momentum in the combined field of a fixed electric monopole and magnetic monopole: if the electric charge is $e$, the magnetic charge is $g$, and the vector from the electric charge to magnetic charge is $\mathbf{r}$, then using the right-hand rule for converting pseudovectors to vectors we find, in Gaussian units, that

$$L = egr/(c|\mathbf{r}|).$$ (30)

Note that the magnitude $|L|$ of the angular momentum is independent of the magnitude $|\mathbf{r}|$ of the separation distance of the monopoles. We believe that angular momentum in all forms is quantized, so that in quantum theory, as observed by Saha [16],

$$|L| = \lambda \hbar,$$ (31)

where it is uncertain whether to choose $\lambda = 1/2$, $\lambda = 1$, or possibly some other such value. Dirac [14] recommended $\lambda = 1/2$, while Schwinger [17] argued for an even integral value, say $\lambda = 2$; more recent works (see papers reproduced in [11]) come down in favor of $\lambda = 1/2$ or $\lambda = 1$.

Provided that electric charges emit retarded electromagnetic signals and magnetic charges emit advanced signals, (30) has an (at least approximate) relativistic generalization. Let a classical electric charge have the spacetime trajectory given by $(t_e, \mathbf{r}_e(t_e))$; then the forward light cone from each point on this trajectory intercepts the spacetime trajectory of the magnetic monopole at a unique point $(t_m(t_e), \mathbf{r}_m(t_m(t_e)))$, such that the four-vector from electric pole to magnetic pole is a null vector. The two poles communicate, as it were, back and forth along this one-parameter family of null vectors. Let us now introduce some group representation theory. The so-called “little group”, in the sense of Wigner ([18], see also [19]) as the subgroup of the Poincaré group that leaves this null vector invariant, is known to be isomorphic to the three-parameter group of translations and rotations in the Euclidean plane. In fact, let us first translate, rotate, and boost a corresponding pair of trajectory points so that the electric pole is at $(0, 0, 0, 0)$ and the magnetic pole at the remove $(a/c, 0, 0, a)$, the latter being a null vector. Then clearly rotations in the $xy$ plane leave the two charges unmoved. There is also another, two-parameter, family of joint rotations and boosts that leaves the two positions unchanged, but we shall not display those transformations here. Rotations in the $xy$ plane correspond to rotations around the $z$ axis, along which the null vector has its spatial projection. If we quantize the angular momentum conjugate to rotations around the $z$-axis, we achieve a relativistic generalization of the quantization entailed in (30) and (31).
As noted, Thomson showed that the angular momentum associated with the circulating electromagnetic field momentum for a spatially fixed electric-magnetic monopole pair is given by (30). However, we are assuming herein that there are no direct electromagnetic interactions between particles in *F* states and those in *B* states; hence the magnetic field generated by a magnetic monopole in a *B* state would live entirely in the *B* subworld, and similarly for the electric field of an electric monopole in an *F* state. *I propose now to drop the assumption that the relative interaction between electric and magnetic charges and currents results from direct electromagnetic interaction, and to take as *ad hoc*, tentative physical axioms that magnetic monopoles exist in, and only in, *B*-type states and that each electric-magnetic monopole pair has a relative, and relativistic, angular momentum of the type described in the previous paragraph. Although the physical electromagnetic fields associated with the two charges, in particular, the radiation fields associated with an accelerating charge, do not interact directly with the other charge in such a pair, the two charges must nevertheless interact: their movements must be correlated so that this axial angular momentum plus any relativistic orbital angular momentum associated with the particles’ motion is conserved componentwise—there are six components of angular momentum in a four-dimensional spacetime (the three rotations plus three Lorentz boosts make up the six-parameter “rotation” group in Minkowski spacetime). The product of elementary units of electric and magnetic charge would therefore be quantized, and close encounters of electric and magnetic monopoles would result in mutual scatterings. I do not presently know how to formulate this hypothetical interaction in terms of quantum field theory, such that the presumed angular momentum conservation and charge quantization are natural consequences entailed by the mathematics. [Note added: PHYSICS TODAY of July, 2006 [20] contains a news article on a recent search, with a negative outcome, for magnetic monopoles at the Fermilab Tevatron.]

5. FURTHER RESEARCH

The picture proposed herein of two antiparallel but somehow disjoint spacetimes may be convenient for visualization, but is dispensible. As remarked above, a picture more in accord with quantum mechanics and with Einstein’s geometric theory of gravitation is that there is just one four-dimensional spacetime, within which there inhere two families of quantum states with their dynamical time evolutions being oppositely directed, and no direct electromagnetic interactions between the two subfamilies. The picture of just two families of quantum states, apart from dynamical considerations, can be construed as an extreme specialization of the much-discussed scenario of extra continuous dimensions to spacetime. In the latter case there would be infinitely many families of quantum states (countably infinite if spacetime is compact in the extra dimensions). I believe this proliferation of Lorentz-congruent families of particles to be excessive and unphysical; in the present scenario any “extraness” to spacetime or to particle families is discrete, finite, and small.

It is a consequence of the arguments leading to (24) that certain transitions between *F* and *B* states would give rise to advanced gravitational waves (and in our subworld, retarded electromagnetic waves). The waves, to be observable by us, would necessarily
arise from intense astrophysical events. This gravitational radiation would appear to us to be converging on the future explosion/implosion, and if it exists, should by our hypotheses be detectable with gravitational wave detectors. Two such detectors are under development or in the early stages of testing—they are known by the acronyms LISA [21] and LIGO [22], respectively Laser Interferometer Space Antenna and Laser Interferometer Gravitational Wave Observatory. Both of these detectors have directional capability. The spherical waves associated with both remote astrophysical events would locally resemble plane waves, so it would not be possible to distinguish exploding from imploding waves directly. What might be discovered, however, is gravitational waves that have no visible source in the heavens, that is such that there are no light signals plausibly associated with a gravitational wave by having the same direction and approximately the same arrival time. Such waves could, despite our normal guess that a concealed explosion is the source, actually be converging on an implosion in the diametrically opposite direction in space and time. (Admittedly, this search for invisible-future-sourced gravitational waves could be confused by the presence of invisible-past-sourced gravitational waves, which can arise from violent events within the $B$-subworld, for according to (26a) and (24), such events give rise to retarded gravitational waves; perhaps the detailed structure of the signals plus mathematical models could make it possible to distinguish these two sorts of waves.) I propose, therefore, that gravitational wave detectors spend a modest effort in searching for gravitational waves propagating with directions chosen more or less at random, in particular in directions that might be unpromising in a search for explosions in the $F$ world.

In the absence of more detailed modeling, I cannot guess how concentrations of $F$-type matter (stars, galaxies, cosmos) might interact with concentrations of $B$-type matter, so as to provide scenarios in which the latter might show their existence to us. The hypothesized interaction between magnetic and electric monopoles might lead to observable effects. A moving magnetic monopole in a $B$ state passing through ordinary matter would to an extent simulate the effects of a moving monopole in an $F$ state, not through its magnetic field but through the presumed axial angular momentum between the two kinds of monopole. Such a search for moving $F$-type magnetic monopoles could just as well be applied to search for $B$-type monopoles; a scattering would occur, but no magnetic monopole field would be present, and the monopole would enter and exit the experiment leaving no other trace of its presence. Although I have not analyzed this scenario in detail, it is plausible that a $B$-type magnetic monopole would gain energy (in our positive time direction) when passing through $F$-type matter, as its second law of thermodynamics is operating in reverse. In this connection, note that the future behavior of a $B$-type particle is to a great extent predetermined by the presumed boundary conditions on the problem: perturbations or measurements on the particle, will change its past, but not (apart from complex feedback-loop mechanisms) its future, evolution. Therefore “trapping” a magnetic monopole for long-term observation would be impractical, as random forces on it would likely be associated with anti-damping of its motion and its eventual escape as time goes forward.

The model described above, of exactly one $F$-type world and exactly one $B$-type world coexisting on the same four-dimensional space-time manifold, admits of straightforward generalizations to larger (or lesser) numbers of parallel/antiparallel worlds. A more complicated case is specified and discussed briefly in Appendix B. We have in the above kept
to the minimum nontrivial numbers in both cases in order to exhibit the beginnings of a plausible physical theory to describe phenomena involving both forward and reverse causation in time. In this connection—see Appendix B—a second parallel $F$-type (as opposed to an antiparallel $B$-type) subworld might contain matter that interacts mainly through (straightforwardly attractive) gravitation with matter in our subworld. The proposals for dark matter in galaxies [23] and in galaxy clusters in the form of weakly interacting massive particles (WIMPs)—see e.g. [24]—resemble this parallel-world scenario, except that the latter entails a subworld unto itself, with possibly complex structure and mutual interactions among its constituents. I do not presently know how to reconcile either of these hypotheses with the apparent absence of WIMPs or invisible $F$-type particles in stars.

Much theoretical work remains to be done along these lines, as is made explicit in the text above, to develop the extent and show the consistency of the model. We have described some possible ways to subject these hypotheses to experimental confirmation or disconfirmation. Perhaps more, and more feasible, tests will emerge from further study.

**APPENDIX A: GREEN’S FUNCTIONS**

The linearized theory of gravity is presented in [25], Ch. 18—see particularly Eqs. (18.7)—(18.8c) and Box 18.2. A consistent formalism of the desired type (24), above, is obtained if we (i) break up the energy-momentum-pressure tensor into two Lorentz-invariant summands that each have zero four-divergence, (ii) break up the perturbation on the background Minkowski metric into two summands, each of which satisfies the Lorentz condition ([25], Eq. (18.8a)), (iii) use Eq. (18.8b) for each corresponding summand in the source and the metric perturbation, and (iv) apply (24). In the following, $x = (x^0, \mathbf{x})$, $x^0 = ct$. Greek indices range from 0 to 3, and the summation convention applies. We use the Minkowski metric $\eta_{\mu\nu} = \text{diag}(-1,1,1,1)$ to be consistent with [25].

The Green’s function for d’Alembert’s operator satisfies

$$-\eta^{\mu\nu} \frac{\partial^2}{\partial x^\mu \partial x^\nu} G^\pm(x; y) = \delta^4(x - y).$$

Then we have

$$G^\pm(x; y) = -\frac{1}{(2\pi)^4} \iiint d^4k \exp[-ik^0(x^0 - y^0) + i\mathbf{k} \cdot (x - y)][(k^0 \pm i\epsilon)^2 - \mathbf{k} \cdot \mathbf{k}]^{-1}$$

where $\epsilon$ is the usual small positive quantity. The upper sign gives rise to a retarded Green’s function, the lower sign an advanced Green’s function. We then take for the Green’s functions of (22) the following:

$$R_G(x; y)_{\mu\nu\kappa\lambda} = \eta_{\mu\kappa} \eta_{\nu\lambda} G^+(x; y),$$

$$A_G(x; y)_{\mu\nu\kappa\lambda} = \eta_{\mu\kappa} \eta_{\nu\lambda} G^-(x; y).$$

(34a) (34b)
Let us now define
\[
\langle \Delta T^{\mu\nu}(x) \rangle = \eta^{\mu\kappa} \frac{\partial}{\partial x^\kappa} \int \int \int \int d^4y G^+(x,y) \frac{\partial}{\partial y^\lambda} \langle N T^{\lambda\nu}(y) \rangle \\
+ \eta^{\nu\kappa} \frac{\partial}{\partial x^\kappa} \int \int \int \int d^4y G^+(x,y) \frac{\partial}{\partial y^\lambda} \langle N T^{\mu\lambda}(y) \rangle \\
+ \eta^{\mu\kappa} \eta^{\nu\lambda} \frac{\partial^2}{\partial x^\kappa \partial x^\lambda} \int \int \int \int \int \int \int d^4y d^4z G^+(x,y) G^+(y,z) \frac{\partial^2}{\partial z^\mu \partial z^\nu} \langle N T^{\rho\sigma}(z) \rangle. \tag{35}
\]

Then if we take
\[
\langle R T^{\mu\nu}(x) \rangle = \langle D T^{\mu\nu}(x) \rangle - \langle \Delta T^{\mu\nu}(x) \rangle, \tag{36a}
\]
\[
\langle A T^{\mu\nu}(x) \rangle = \langle N T^{\mu\nu}(x) \rangle + \langle \Delta T^{\mu\nu}(x) \rangle, \tag{36b}
\]
both rhs’s have zero net four-divergence and can be applied to (24) with (34). Note that the choice of, among other possibilities, \( G^+ \) on the rhs of (35) has the physical property of concentrating sources of advanced gravitational waves in the future of an astrophysical event that generates a nonzero \( \langle N T^{\mu\nu}(x) \rangle \). The mathematics of (24) with (35) entails the convolution of two or three Green’s functions; I do not know whether or not this procedure is mathematically justifiable.

A possible way, which I have not investigated, to avoid the mathematically questionable procedure entailed by (35) might be to try to construct less singular, but not Lorentz invariant, forms of \( \langle \Delta T^{\mu\nu}(x) \rangle \), and redefine the generators of Lorentz boosts ([1], Eq. (68c)) to include infinitesimal gauge transformations ([25], Box 18.2B) so that the commutators and transformation properties are restored.

**APPENDIX B: FURTHER DISCUSSION**

We consider now that spacetime supports the direct sum of three families of quantized fields: (1) the forward-evolving fields (denoted \( F_1 \)) corresponding to conventional non-dark matter as it is displayed to us by nongravitational as well as by attractive gravitational interactions, (2) a second family of forward-evolving fields (denoted \( F_2 \)), the presence of which is detectable primarily through its attractive gravitational interactions with matter in our subworld, and (3) a third family of, in this case, backward-evolving fields (denoted \( B \)) that also makes its presence in space-time known primarily through attractive/repulsive gravitational interactions with matter in the \( F_1 \) and \( F_2 \) subworlds—see (27), et seq. We advance the conjecture that the second and third families are candidates for dark matter and dark energy, respectively.

A natural question is, how can one distinguish physically between taking the direct sum of, as contrasted with the direct product of, the fields \( F_1 \) and \( F_2 \) (and possibly \( B \)) as the space of states of the system? I think an irreconcilable difference is that in the direct sum, there is more than one vacuum state, exactly one for each subworld. A general vacuum state is thus a superposition or mixture of this presumably small number of separate vacua, where the time evolution of the superposition or mixture is controlled by the dynamics and the boundary conditions, as in [1], Eq. (80), et seq.
The above scenario therefore differs from the hypothesis of the existence of WIMPs in that there is presumed to be a second vacuum state associated with the $F_2$ subworld, and also associates the existence of dark energy with a third vacuum state, that belonging to the B subworld.
APPENDIX C: NEWTONIAN GRAVITATIONAL DYNAMICS WITH A POSITIVE AND A NEGATIVE MASS PARTICLE

(Note: The material in this Section augments the paper published in [26].)

Suppose that, in (28), \( m_1 > 0 \) and \( m_2 < 0 \), so that, respectively, these particles belong to the \( F \) and \( B \) subspaces. Then in establishing boundary value problems for the Newtonian equations of motion in a time interval, we take it as a physical axiom that we must specify, respectively, the initial and the final positions and velocities of the \( F \)- and \( B \)-type particle. Suppose that \( t_a \leq t \leq t_b \), and let it be given that

\[
\begin{align*}
\mathbf{r}_1(t_a) &= \mathbf{r}_{1a}, & \mathbf{r}'_1(t_a) &= \mathbf{u}_{1a}, \\
\mathbf{r}_2(t_b) &= \mathbf{r}_{2b}, & \mathbf{r}'_2(t_b) &= \mathbf{u}_{2b}.
\end{align*}
\]

The desired functions then are the solutions of the coupled integral equations

\[
\begin{align*}
\mathbf{r}_1(t) &= \mathbf{r}_{1a} + \mathbf{u}_{1a}(t-t_a) + Gm_2 \int_{t_a}^{t} dt' (t-t') \frac{[\mathbf{r}_2(t') - \mathbf{r}_1(t')]}{r_2(t')^3}, \\
\mathbf{r}_2(t) &= \mathbf{r}_{2b} - \mathbf{u}_{2b}(t-t_b) - Gm_1 \int_{t}^{t_b} dt' (t'-t) \frac{[\mathbf{r}_2(t') - \mathbf{r}_1(t')]}{r_2(t')^3}.
\end{align*}
\]

If we define

\[
\tilde{\Delta}(t_a,t_b) = \int_{t_a}^{t_b} dt' \frac{[\mathbf{r}_2(t') - \mathbf{r}_1(t')]}{r_2(t')^3},
\]

then

\[
\begin{align*}
\mathbf{r}'_1(t_b) &= \mathbf{u}_{1a} + Gm_2 \tilde{\Delta}(t_a,t_b), \\
\mathbf{r}'_2(t_a) &= \mathbf{u}_{2b} + Gm_1 \tilde{\Delta}(t_a,t_b).
\end{align*}
\]

Since energy is conserved, if we take \( t = t_a \) well before, and \( t = t_b \) well after, the time of closest approach, we have equality of the kinetic energies with small potential energy:

\[
(m_1/2)|\mathbf{r}_1(t_b)|^2 + (m_2/2)|\mathbf{r}_2(t_b)|^2 \approx (m_1/2)|\mathbf{r}_1(t_a)|^2 + (m_2/2)|\mathbf{r}_2(t_a)|^2.
\]

Since \( m_2 < 0 \), the above implies

\[
(m_1/2)|\mathbf{r}_1(t_b)|^2 + (m_2/2)|\mathbf{r}_2(t_a)|^2 \approx (m_1/2)|\mathbf{r}_1(t_a)|^2 + (m_2/2)|\mathbf{r}_2(t_b)|^2
\]

that is, the sum of the absolute values of the output kinetic energies is equal to the sum of the absolute values of the input kinetic energies—runaway solutions are impossible so long as the interparticle distance is bounded away from zero.

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