The Role of Time for Reparametrization-Invariant Systems

Vesselin G. Gueorguiev*

Institute for Advanced Physical Studies,
21 Montevideo Street, Sofia 1618, Bulgaria

We discuss reparametrization-invariant systems, mainly the relativistic particle and its $D$-dimensional extended-object generalization to $d$-branes. For a $d$-brane that doesn’t alter the background fields, we define non-relativistic equations assuming integral sub-manifold embedding of the $d$-brane. We argue for a one-time-physics as an essential ingredient for a non-relativistic limit. The mass-shell constraint and the Klein-Gordon equation are shown to be universal when gravity-like interaction is present. Our approach to the Dirac equation follows Rund’s technique for the algebra of the $\gamma$-matrices that doesn’t rely on the Klein-Gordon equation [1].

We discuss some aspects of the non-relativistic, relativistic, and a la Dirac-equation quantization of reparametrization-invariant systems. In its canonical form, the matter Lagrangian for reparametrization-invariant systems contains well known interaction terms, such as electromagnetism and gravity. For a reparametrization-invariant systems there are constraints among the equations of motion, which is a problem when attempting to quantize such system. Nevertheless, there are procedures for quantizing such theories [1, 2, 3, 4, 5]. Here, we will demonstrate another approach ($v \rightarrow \gamma$) that takes advantage of the fact that the corresponding Hamiltonian is identically zero ($H \equiv 0$) for reparametrization-invariant systems.

Furthermore, we argue that a one-time-physics is needed to assure causality via finite propagational speed in case of point particles. For $d$-branes the one-time-physics reflects separation of the internal from the external coordinates when the $d$-brane is considered as a sub-manifold of the target space manifold $M$. The non-relativistic limit is considered to be the case when the $d$-brane is embedded as a sub-manifold of $M$.

Some arguments for 4D space-time are based on geometric and differential structure of various brane and target spaces [6, 7]. All these are reasons why the spacetime seems to be four dimensional. Here we present an argument that only one-time-physics is consistent with a finite propagational speed. Thus, resulting in 1+3 Minkowski space-time.

In summary, we discuss the structure of the matter Lagrangian ($L$) for ex-

*e-mail: vesselin.gueorguiev@iaps.institute
tended objects. Imposing reparametrization invariance of the action $S$ naturally leads to a first order homogeneous Lagrangian. In its canonical form, $L$ contains electromagnetic and gravitational interactions, as well as interactions that are not clearly identified yet.

The non-relativistic limit for a $d$-brane has been defined as those coordinates where the brane is an integral sub-manifold of the target space. This gauge can be used to remove reparametrization invariance of the action $S$ and make the Hamiltonian function suitable for canonical quantization. For the 0-brane (the relativistic particle), this also has a clear physical interpretation associated with localization and finite propagational speed.

The existence of a mass-shell constraint is universal. It is essentially due to the gravitational (quadratic in velocities) type interaction in the Lagrangian and leads to a Klein-Gordon equation. Although the Klein-Gordon equation can be defined, it is not the only way to introduce the algebra of the $\gamma$-matrices needed for the Dirac equation. The algebraic properties of the $\gamma$-matrices may be derived using the Lie group structure of the coordinate bundle; these properties are closely related to the corresponding metric tensor $g^{\alpha\beta} = \{\gamma^\alpha, \gamma^\beta\}$ and may restrict the number of terms in the Lagrangian. Once the algebraic properties of the $\gamma$-matrices are defined, one can use $v \to \gamma$ quantization in the Hamiltonian function $H = pv - L(x,v)$ to obtain the Dirac equation.

References

[1] Rund H., *The Hamilton-Jacobi theory in the calculus of variations: its role in mathematics and physics*, Van Nostrand, Huntington, N.Y., 1966.

[2] Dirac P. A. M., *Generalized Hamiltonian Dynamics*, Proc. Roy. Soc. A 246 (1958) 326-332.

[3] Teitelboim C., *Quantum mechanics of the gravitational field*, Phys. Rev. D 25 (1982) 3159-3179.

[4] Henneaux M., Teitelboim C., *Quantization of gauge systems*, Princeton University, Princeton, N.J., 1992.

[5] Sundermeyer K., *Constrained dynamics*, Springer-Verlag, New York, 1982.

[6] Gueorguiev V. G., *The Relativistic Particle and its d-brane Cousins*, Proc. Geom. Int. Quant. (2003) 168-177 (math-ph/0210021).

[7] Sachoglu C., *Fake $R^4$s, Einstein spaces and Seiberg-Witten monopole equations*, Class. Quantum Grav. 18 (2001) 3287-3292.