Pattern similarities of vector matrices

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Abstract. A concept of vector matrix or matrix of vector is introduced. It is a well-known matrix where the elements are vectors. An example of vector matrix is a digital image represented in a matrix \( A_{m \times n} \). The elements of \( A \) are vectors in \( \mathbb{R}^3 \) represent color in red, green, blue, and \( m \times n \) is the number of pixels. Similarity of two matrices is defined by the similarity of each corresponding elements. Such definition is very strict, two similar matrices must have the same vector for each corresponding elements. Applying for digital images, two similar matrices are actually the same image. Less similarity concept is introduced, namely pattern similarity. Pattern similarity is a generalization of strict similarity by applying a pattern function. An application of pattern similarity is for object recognition.

1. Motivation
The concept of vector matrix is relatively new. It is originally motivated by a representation of digital images. The concept of vector matrix or matrix of vectors initially was introduced by Cahyono [1]. An application on digital image has been reported in [2]. An introduction of digital image can be found in [3] or in some standard textbook on digital image processing, e.g. [4, 5, 6].

Modern human almost always see images on computer monitors of gadgets. A digital image is formed by tiles of pixel, say \( m \) number of rows and \( n \) number of columns. Each pixel shows one color. The color is represented by a red, green, blue (r,g,b) data, where \( r, g, b \in \{0,1,2, \cdots, 255\} \). Here are some basic colors in (r,g,b) data. Black is \((0,0,0)\), white is \((255,255,255)\), red is \((255,0,0)\), green is \((0,255,0)\), blue is \((0,0,255)\), yellow is \((255,255,0)\), and purple is \((255,0,255)\).

Hence, a digital image can be assumed as a matrix. The elements of the matrix are vectors, in this case three dimensional vectors.

2. Vector matrices
This section is started by defining vector matrix

Definition 1
Let \( k, m, n \in \mathbb{N} \), for \( k, m, n > 1 \). Vector \( \mathbf{u}_{i,j} \in \mathbb{R}^k \) for \( 1 \leq i \leq m \) and \( 1 \leq j \leq n \). Vector matrix \( m \) by \( n \) is defined
 Such matrix is also written as $A_{m \times n} = \begin{bmatrix} u_{1,1} & u_{1,2} & \cdots & u_{1,n} \\ u_{2,1} & u_{2,2} & \cdots & u_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ u_{m,1} & u_{m,2} & \cdots & u_{m,n} \end{bmatrix}$. 

For the application on digital image, $m \times n$ represents the number of pixels, and $k = 3$. The elements of vector matrix are three dimensional vectors $u_{i,j} = (r_{i,j}, g_{i,j}, b_{i,j})$, where $r_{i,j}, g_{i,j}, b_{i,j} \in \{0,1,2,\cdots,255\}$. The element $u_{i,j} = (r_{i,j}, g_{i,j}, b_{i,j})$ represents the color of pixel in the position $(i,j)$. One may refer to [3] for a short introduction of digital image.

Figure 1 shows a digital image of Faculty of Mathematics and Natural Sciences of Universitas Halu Oleo. The picture was taken by Dani Rofianto [2], in 620 $\times$ 1108 pixels. Hence, this picture is represented in a vector matrix $B_{620 \times 1108}$. This matrix is too big to show in this paper. However, the first three rows and two columns of $B_{620 \times 1108}$ is expressed in vector matrix $A$, where

$$A = \begin{bmatrix} (132,172,207) & (138,177,210) \\ (135,170,210) & (139,169,207) \\ (136,175,208) & (135,174,205) \end{bmatrix}. \quad (1)$$

Figure 1: Digital image of FMIPA UHO building in 620 $\times$ 1108 pixels.

Some algebraic operation may be introduced for vector matrices. However, in this paper only few will be discussed, that is the one related to some applications. Below is scalar multiplication of a vector and a vector matrix.

**Definition 3**

Let $A$ be an $m \times n$ matrix, and $\mathbf{v}$ be a vector in $\mathbb{R}^k$. Scalar multiplication of $\mathbf{v}$ and $A$ (denoted by $\mathbf{v} \times A$ ) is a scalar matrix $C$, where the elements are scalar multiplication of $\mathbf{v}$ and the corresponding vector elements of $A$.

Example
Let \( \mathbf{v} = (0.277, 0.566, 0.144) \) and \( \mathbf{A} \) be given by (1). Scalar multiplication of \( \mathbf{v} \) and \( \mathbf{A} \) is

\[
\mathbf{v} \ast \mathbf{A} = \begin{bmatrix}
163.724 & 168.648 \\
163.855 & 163.965 \\
166.674 & 165.399
\end{bmatrix}
\]

Rounding each elements of \( \mathbf{v} \ast \mathbf{A} \), it gives matrix \( \mathbf{C} \) in the form

\[
\mathbf{C} = \begin{bmatrix}
164 & 169 \\
164 & 164 \\
167 & 165
\end{bmatrix}
\]

Figure 1 in grey scale is represented by rounding each element of a scalar matrix \( \mathbf{B} \times 620 \times 1108 \), where \( \mathbf{B} \times 620 \times 1108 = \mathbf{v} \ast \mathbf{B} \times 620 \times 1108 \). Hence, matrix \( \mathbf{C} \) is the first three rows and two columns of scalar matrix that represents the grey scale of Figure 1. Note that \( \mathbf{B} \times 620 \times 1108 \) (written in bold) is vector matrix, but \( \mathbf{B} \times 620 \times 1108 \) (not bold) is a scalar matrix.

Similarity of two vector matrices is defined by the similarity of the size and every corresponding element of the two.

**Definition 4**

Let \( \mathbf{A}_{m \times n} = [\mathbf{u}_{i,j}]_{m \times n} \) and \( \mathbf{B}_{m \times n} = [\mathbf{v}_{i,j}]_{m \times n} \) be vector matrices. \( \mathbf{A} \) is similar to \( \mathbf{B} \) and written by \( \mathbf{A} = \mathbf{B} \) if only if \( m_1 = m_2, n_1 = n_2 \) and \( \mathbf{u}_{i,j} = \mathbf{v}_{i,j} \) for every \( i, j \).

This definition of similarity is very strict, which does not work for application. Less strict similarity is necessary for application. Hence, less strict similarity will be introduced, namely pattern similarity. It will be start by defining an area of a matrix.

**Definition 5**

Let \( \mathbf{A} \) be an \( m \times n \) matrix. An area of matrix \( \mathbf{A} \), \( D(\mathbf{A}) \) is defined by a set consisting of the indices of the elements of \( \mathbf{A} \), \( D(\mathbf{A}) = \{(i,j): 1 \leq i \leq m, 1 \leq j \leq n\} \).

Elements of \( D(\mathbf{A}) \) is ordered pairs where the absis is the row of \( \mathbf{A} \), and the ordinate is the column of \( \mathbf{A} \). By this definition, vector matrices \( \mathbf{A} \) and \( \mathbf{B} \) of the same dimension have the same area. It means that \( D(\mathbf{A}) = D(\mathbf{B}) \). This is an area covered by \( m \times n \) tiles as illustrated by Fig. 2.

![Figure 2: An area representing a matrix.](image-url)
The concept of matrix area will be important to define pattern similarity of two vector matrices, where a non empty subset of matrix area becomes the focus of similarity.

**Definition 6**
Let $A$ and $B$ be $m \times n$ matrices, and $\Omega$ be non empty set in $D(A)$. Let $\varepsilon$ be a function from $D(A)$ to $R$, such that $\varepsilon(x) > 0$ for $x \in \Omega$ but $\varepsilon(x) = 0$ for $x \in D(A)/\Omega$. $A$ and $B$ have pattern similarity with respect to $\varepsilon$ denoted by $A \cong B$ if $|u_x - v_x| \leq \varepsilon(x)$ for $x \in \Omega$.

**Example**
Let $A$ be vector matrix given by (1), $\Omega = \{(1,1), (2,2), (3,1)\}$, $\varepsilon((1,1)) = 10$, $\varepsilon((2,2)) = 5$, and $\varepsilon((3,1)) = 4$. A vector matrix $B$ is given by

$$B = \begin{bmatrix} 130,172,205 & 0,0,210 \\ 15,10,10 & 139,170,207 \\ 135,176,207 & 250,200,100 \end{bmatrix}.$$  

(2)

Hence $A$ and $B$ have pattern similarity with respect to $\varepsilon$.

The subset $\Omega$ is the focus of similarity. In the application on the digital image, this is the image area of the concerning object. Hence, the image area outside $\Omega$ is not important. The function $\varepsilon$ represents strictness of similarity of the object such as brightness of the image. Obviously, an image has pattern similarity to itself. Mathematically speaking, if $A = B$, then $A \cong B$ as summarized in proposition below.

**Proposition**
Let $A$ and $B$ be $m \times n$ matrices, and $A = B$. Then $A \cong B$ with respect any function $\varepsilon$ defined on $D(A)$.

**Proof**
Since $A = B$, then $|u_x - v_x| = 0$ for any $x$ in $D(A)$. Hence, $|u_x - v_x| \leq \varepsilon(x)$ for any $x \in \Omega \subseteq D(A)$.

This theorem ends this section.

3. **Discussion and further research**
Matrices where the elements are vectors have been discussed. The concept of such matrices is relatively new. Operations and concepts of similarity have been proposed, and the applications of digital image also presented. This may open a new way to look in a digital image and to work on digital image processing.

Future research will be focus on the theory and applications. On the theory, the concept of pattern similarity of matrices will be designed not necessarily for matrices in the same dimension. On the application, the focus is on the implementation of computer program for object recognition.

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