Sequential Procurement Auctions and Their Effect on Investment Decisions

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Procurement Auctions

- Markets designed for the purchase of goods (typically of high cost)
- Used both in public and private sector
- Finding ways to **reduce total expenditures** is a question of first-order relevance:
  - OECD countries’ public procurement expenditures in 2011 accounted for 19% of their GDP
  - Chile: Transactions performed through *Chilecompra* 10.000 million USD in 2013 (∼ 4% GDP)
  - Also a relevant question in the private sector
Main Features

- These mechanisms are used repeatedly over time
- Tasks sometimes involve a high degree of expertise (*know-how*) ⇒ Group of sellers does not change too much
- Sellers can invest in improving their technologies. Specialized tasks ⇒ Relationship-specific investments
Two ways through which total expenditures can be reduced are:

1. Inter-temporal incentives: design of **dynamic mechanisms** that smooth out costs across time.
2. Incentivizing sellers to **invest in cost-reducing technologies**.

We derive the **optimal contract** (i.e., optimal auction + optimal level of investment) chosen by a buyer in an environment where:

- She must purchase two goods sequentially over time and can fully commit to a two-period mechanism.
- The winner of the first auction can invest in a cost-reducing technology for the second auction.
Main Results

- The optimal mechanism gives an advantage to the first-period winner in the second auction
  - Advantage decreases with the number of sellers, but it never disappears
- In this dynamic setting, commitment induces over-investment
- Investment observability is irrelevant for cost minimization and surplus maximization
- More generally, in dynamic environments awarding advantages
  - Can induce more competition among sellers ⇒ reduce current costs
  - Can incentivize sellers to invest more in cost-reducing technologies ⇒ reduce future costs
Literature

- Myerson (1981): Optimal (one-shot) auction design. Better competitors suffer a **disadvantage** in the optimal mechanism.

- Arozamena y Cantillón (2004): Investment stage before a one-shot auction takes place. **Underinvestment** in sealed-bid procurement auctions.

- Pesendorfer and Jofre-Bonnet (2014): Sequential auctions with exogenous distributions.
Contents

● Model

● Efficiency

● Cost Minimization

● Conclusions

● Lack of Commitment
Basics

- A buyer (she) must purchase two goods sequentially over time
- There are $n$ risk-neutral sellers that are ex-ante identical
- A Seller’s cost to produce each good is his private information
- Costs are independent across sellers, and also independent across time
- We are interested in mechanism design, i.e., the buyer can commit to a two-period mechanism at time zero
  - Since costs are i.i.d. across time, the revelation principle also holds when the buyer lacks commitment
Distributions of Costs

- In the first period a seller’s cost is drawn from a c.d.f. $F(\cdot)$, with density $f(\cdot)$ and support $C = [c, \bar{c}]$.
- First-period losers maintain $F(\cdot)$ for the second period.
- The first-period winner instead can invest in a cost-reducing technology between auctions:
  - Investing $I \geq 0 \Rightarrow$ Cost distribution becomes $G(\cdot, I)$, with density $g(\cdot, I)$ and support $C'$.
  - Investing is costly: $\Psi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ differentiable, strictly increasing and strictly convex, with $\Psi(0) = \Psi'(0) = 0$. 
Regularity Assumptions Over $F(\cdot)$ and $G(\cdot, \cdot)$

Assumption:

(i) $c + F(c)/f(c)$ is strictly increasing in $c$.

(ii) $F(c) \leq G(c, 0)$ for all $c \in C$.

(iii) For each $c \in C$, $I \mapsto G(c, I)$ is twice continuously differentiable, strictly increasing (FOSD) and concave. Furthermore, $\frac{\partial G}{\partial I}(c, 0) > 0$ for all $c \in C$.

**Obs:** The following are sufficient for (ii) and (iii):

(a) MLRP: For all $c' < c \in C$ and $0 \leq I' < I \in \mathbb{R}$,

$$\frac{f(c')}{f(c)} \leq \frac{g(c', I')}{g(c, I')} < \frac{g(c', I)}{g(c, I)}.$$

(b) Hazard-rate ordering: For all $c \in C$ and $0 \leq I' < I$

$$\frac{g(c, I)}{G(c, I)} \leq \frac{g(c, I')}{G(c, I')} \leq \frac{f(c)}{F(c)}$$. 
Timeline

- $t=0$: The rules of both procurement auctions are set
- $t=1$: First procurement auction takes place
- $t=2$: (1) Investment takes place. (2) Second procurement auction takes place
Direct Mechanisms

Definition

A direct mechanism that implements $I \geq 0$, $\Gamma(I)$, corresponds to a tuple $\Gamma(I) = (t^1(\cdot), q^1(\cdot), t^2_w(\cdot;I), q^2_w(\cdot;I), t^2_\ell(\cdot;I), q^2_\ell(\cdot;I))$ where

\[
\begin{align*}
  t^1 & : C^n \rightarrow \mathbb{R}^n \text{ (transfer at } t=1) \\
  q^1 & : C^n \rightarrow \Delta_n \text{ (allocation rule at } t=1) \\
  t^2_w(\cdot;I) & : C^n \rightarrow \mathbb{R} \\
  q^2_w(\cdot;I) & : C^n \rightarrow [0, 1] \\
  t^2_\ell(\cdot;I) & : C^n \rightarrow \mathbb{R}^{n-1} \\
  q^2_\ell(\cdot;I) & : C^n \rightarrow [0, 1]^{n-1}
\end{align*}
\]

such that $q^2_w(c;I) + \sum_{i \neq w} q^2_{\ell,i}(c;I) = 1$ for all $c \in C^n$, and such that the first-period winner finds it optimal to invest $I \geq 0$ between auctions.
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Ex-Post Allocative Efficiency

- Planner observes \( I \) and realized costs, and maximizes total surplus
- Efficient mechanism \( \Gamma^e \)

\[
q^{t,e}_i(c) = \begin{cases} 
1 & c_i < c_j \ \forall j \neq i \\
0 & \end{cases}
\]  

(1)

- Social cost:

\[
C(\Gamma^e, I) = n \int c[1 - F(c)]^{n-1} f(c) dc \\
+ \int c[1 - F(c)]^{n-1} g(c, I) dc \\
+(n - 1) \int c[1 - F(c)]^{n-2}[1 - G(c, I)] f(c) dc \\
+ \Psi(I)
\]  

(2)
Socially Efficient Investment

The planner solves

\[
\min_{I \geq 0} C(\Gamma^e, I)
\]

Proposition

The socially efficient level of investment, \( I^e \), is the solution to

\[
\max_{I \geq 0} \int \left[ (1 - F(c))^{n-1} G(c, I) dc - \Psi(I) \right]
\]

Furthermore, it can be induced using two SPA regardless of the observability of the investment decision.

- Observe that (3) \( \Leftrightarrow \max_{I \geq 0} \int \left[ (1 - F(c))^{n-1} \frac{G(c, I)}{g(c, I)} g(c, I) dc - \Psi(I) \right]

- Hidden investment: \( I^e \in \arg \max_{I \geq 0} \int \Pi^2, e_w(c, c) g(c, I) dc - \Psi(I) \) and

\[
\Pi^2, e_w(c, c) = \Pi^2, e_w(\bar{c}, \bar{c}) + \int_{c}^{\bar{c}} Q^2, e_w(s) ds
\]
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Cost Minimization Under Full Commitment

- Buyer must purchase two goods sequentially at the lowest possible cost
- She can commit to the rules of both auctions before these take place
- Suppose that investment is observable
Notation

\[
T^1_i(c'_i) = \int_{c_{-i}} t^1_i(c'_i, c_{-i}) f^{n-1}(c_{-i}) dc_{-i}
\]

\[
Q^1_i(c'_i) = \int_{c_{-i}} q^1_i(c'_i, c_{-i}) f^{n-1}(c_{-i}) dc_{-i}
\]

\[
\Pi^1_i(c_i, c'_i, I; I) = T^1_i(c'_i) - c_i Q^1_i(c'_i) + Q^1_i(c'_i) \int_C \Pi^2_w(c, c; I) g(c, I) dc
\]

\[
+ [1 - Q^1_i(c'_i)] \int_C \Pi^2_{\ell,i}(c, c; I) f(c) dc
\]

\[
\Pi^2_w(c, c'; I) = T^2_w(c'; I) - c Q^2_w(c'; I)
\]  

(4)
The Buyer’s Problem

The buyer minimizes

\[ C = \sum_{i=1}^{n} \int_{c}^{C} T_{1i}^1(c) f(c) dc + \int_{C}^{C} T_{2w}(c; I) g(c, I) dc + \sum_{j \neq w} \int_{C}^{C} T_{\ell,j}^2(c; I) f(c) dc \]  

subject to

- Incentive-compatibility constraints
- Individual rationality (i.e. voluntary participation)
Incentive Compatibility ($I$ is observable)

$IC_o : \begin{cases} 
\Pi^2_w(c_w, c_w; I) \geq \Pi^2_w(c_w, c'_w; I), \forall c_w, c'_w \in C. \\
\Pi^2_{\ell,i}(c_i, c_i; I) \geq \Pi^2_{\ell,i}(c_i, c'_i; I), \forall c_i, c'_i \in C, \forall i \neq w. \\
\Pi^1_i(c_i, c_i, I; I) \geq \Pi^1_i(c_i, c'_i, I; I), \forall c_i, c'_i \in C, \forall i \in N.
\end{cases}

Lemma

A mechanism $\Gamma(I)$ is IC if and only if

(i) $Q^1_i(\cdot)$ is non increasing and, for all $c_i \in C$,

$$
\Pi^1_{i,I}(c_i, c_i) = \Pi^1_{i,I}(\bar{c}, \bar{c}) + \int_{c_i}^{\bar{c}} Q^1_i(s) ds
$$

(ii) $Q^2_k(\cdot; I)$ is non increasing, $k = w, (\ell, i), i \neq w, i \in N$,

$$
\Pi^2_k(c_k, c_k; I) = \Pi^2_k(\bar{c}, \bar{c}; I) + \int_{c_k}^{\bar{c}} Q^2_k(s; I) ds.
$$
Participation Constraints

- Participation in the second period is ensured by assuming that

\[
PC^2(I) : \begin{cases} 
\Pi_w^2(c_w, c_w; I) - \Psi(I) \geq 0, \forall c_w \in C \\
\Pi_{\ell,i}^2(c_i, c_i; I) \geq 0, \forall c_i \in C, i \neq w.
\end{cases}
\]

- We follow Pesendorfer and Jofre-Bonet (2014):

\[
PC^1(I) : \Pi_i^1(c_i, c_i, I; I) \geq \int_C \Pi_{\ell,i}^2(c, c; I) f(c) dc, \forall c_i \in C, \forall i \in N,
\]

Intuition:

- Buyer wants to induce the participation of all sellers in both auctions
- But she cannot prevent the participation at \( t = 2 \) of a seller that skipped the first auction
Optimal Mechanism

Proposition

Suppose that the buyer wants to implement a level \( I \geq 0 \). The cost-minimizing mechanism, \( \Gamma^*(I) \), is given by

\[
q_1^{1*}(c_1, \ldots, c_n) = \mathbb{1}\{c_i < c_j, \forall j \neq i\},
\]

\[
q_w^{2*}(c_w, c_{-w}) = \mathbb{1}\{c_w < c_i + \left(1 + \frac{1}{n-1}\right) \frac{F(c_i)}{f(c_i)}, \forall i \neq w\},
\]

\[
t_w^{2*}(c_1, \ldots, c_n) = \mathbb{1}\{c_w < k(c_i), \forall i \neq w\} \min\{k(c_i); i \neq w\},
\]

\[
t_i^{1*}(c_i, c_{-i}; I) = \mathbb{1}\{c_i < c_j, \forall j \neq i\} \left[\min\{c_j; j \neq i\} - (\Pi_w^{2*}(I) - \Psi(I) - \Pi_{\ell}^{2*}(I))\right] - \Pi_{\ell}^{2*}(I)
\]

where \( k(c) := c + \left(1 + \frac{1}{n-1}\right) \frac{F(c)}{f(c)} \) and

\[
\Pi_w^{2*}(I) := \int_C \Pi_w^{2*}(c, c; I) g(c, I) dc \quad \text{and} \quad \Pi_{\ell}^{2*}(I) := \int_C \Pi_{\ell}^{2*}(c, c; I) f(c) dc.
\]
Intuition and Remarks

- First auction is efficient; the second is inefficient (advantage gap)
- \( \Gamma^*(I) \) is optimal even when \( \Psi \equiv 0 \). Intuition for the bias then?
  - Incentive to reduce \( \Pi_2^{\ell}(I) \) so as to relax \( \Pi_i^1(c_i, c_i, I; I) \geq \Pi_2^2(I) \)
  - \( t_i^1 (c_i, c_{-i}; I) = \frac{1}{c_i < c_j, \forall j \neq i} \left[ \min \{c_j; j \neq i\} - (\Pi_w^2(I) - \Pi_\ell^2(I) - \Psi(I)) \right] - \Pi_\ell^2(I) \)
  - Transfer to the winner at \( t = 1 \) is reduced by \( \Pi_w^2(I) - \Pi_\ell^2(I) \to \text{Buyer} \) extracts this extra rent, i.e., increased competition at \( t = 1 \)

- Advantage gap \( k(c) = c + \left(1 + \frac{1}{n-1}\right) \frac{F(c)}{f(c)} : \)
  - Is independent of \( G(\cdot, I) \)
  - Never disappears: \( k(c) \to c + \frac{F(c)}{f(c)} \) as \( n \to \infty \): Isolates the cost-smoothing property of dynamic auctions (In fact, \( I^*(n) \to 0 \) as \( n \to \infty \))
Proposition

When investment is observable, the buyer chooses an investment level $I^* > 0$ that solves

$$
\max_{I \geq 0} \int_{C} [1 - F(k^{-1}(c))]^{n-1} \frac{G(c, I)}{g(c, I)} g(c, I) dc - \Psi(I),
$$

(6)

where $k(c) = c + \left(1 + \frac{1}{n-1}\right) \frac{F(c)}{f(c)}$, $c \in C$. Moreover, $I^* > I^e$, so over-investment occurs.

**Intuition:** The winner gets the second project more often that under the efficient mechanism, i.e. $1 - F(k^{-1}(c)) > 1 - F(c)$, which is costly. Hence, it is optimal to make him win with an even lower average cost.
Hidden Investment: Constraints

- Incentive compatibility:

\[ I \in \arg \max_{K \geq 0} \int_C \Pi^2_w(c, c; I) g(c, K) dc - \Psi(K) \]

\[
\begin{cases} 
\Pi^2_w(c_w, c_w; I) \geq \Pi^2_w(c_w, c'_w; I), \quad \forall c_w, c'_w \in C \\
\Pi^2_{\ell, i}(c_i, c_i; I) \geq \Pi^2_{\ell, i}(c_i, c'_i; I), \quad \forall c_i, c'_i \in C, \quad \forall i \neq w \\
\Pi^1_i(c_i, c_i, I; I) \geq \Pi^1_i(c_i, c'_i, I; I), \quad \forall c_i, c'_i \in C, \quad \forall i \in N.
\end{cases}
\]

- Participation constraints: As before
Optimal Contract

Proposition

$\Gamma^*(I^*)$ induces the winner to invest $I^*$. Hence, it is optimal when investment is hidden, and $I^*$ can be implemented at no additional cost. Over-investment occurs.

Proof:

$$\max_{I \geq 0} \int_C \Pi^2_w(c, c; I^*)g(c, I)dc - \Psi(I) = \max_{I \geq 0} \int_C Q^2_w(c)g(c, I)dc - \Psi(I)$$

$$= \int_C [1 - F(k^{-1}(c))]^{n-1} G(c, I)dc - \Psi(I).$$

Intuition: Incentives on the margin are stepper
Remarks: Full-Commitment Case

- Cost minimization: Investment incentives are aligned under the optimal mechanism.
- Surplus maximization: Investment incentives are aligned under the efficient mechanism.
- Is it the same under any arbitrary mechanism (i.e., a consequence of risk neutrality)? \textbf{No}:

Proposition

Let $n = 2$ and consider the IC mechanism $q^{2}_{w,I}(c_w, c_l) = 1_{c_w < g(c_l)}$, with $g'(\cdot) \geq 0$, $g(c) = c$ and $g(c) \leq c + 2 \frac{F(c)}{f(c)}$, $\forall c \in C$, with strict inequality on a subset of $C$ with non-zero measure. Then, the buyer chooses an investment level that is larger than the one chosen by the first-period winner.
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Conclusions

- In dynamic contexts, mechanisms serve a dual role:
  - Inter-temporal cost smoothing
  - Induce incentives to invest
- Commitment generates **over-investment** via awarding **advantages** to previous winners
- When the buyer has full commitment not observing investment is irrelevant under optimal contracts (e.g.: cost minimization or surplus maximization). This is not the case when the buyer lacks commitment (**hold-up** effect)
- World is more complicated: although providing an advantage increases investment, it can creates barriers to entry
- **Challenging question:** fully dynamic environment with experience accumulation and history-dependent advantages
Thank you!
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Lack of Commitment

- In this case the buyer can change the rules of the second auction after the first one has taken place.

- We solve the problem using sequential rationality:
  - Observable investment: Stackelberg game in which the buyer treats investment as sunk.
  - Hidden investment: Simultaneous-move game in which the buyer takes into account the winner’s incentives to invest.

- Assume $c \mapsto c + \frac{G(c,I)}{g(c,I)}$ is increasing.
Observable Investment

- After investment becomes sunk → standard one-shot auction problem (Myerson, 1981) at $t = 2$. Call this mechanism $\hat{\Gamma}^2(I)$.

Proposition

Suppose that winner invests $I \geq 0$. Then, $\hat{\Gamma}^2(I)$ satisfies

$$\hat{q}^2_{w}(c_w, c_{-w}; I) = \begin{cases} 
1 & c_w + \frac{G(c_w, I)}{g(c_w, I)} < \min_{i \neq w} \left\{ c_i + \frac{F(c_i)}{f(c_i)} \right\} \\
0 & \sim 
\end{cases}$$

The investment induced in this setting, $\hat{I}$, satisfies

$$\max_{I \geq 0} V(I) = \int \left[ 1 - F(v^{-1}(h(c, I))) \right]^{n-1} G(c, I) dc - \Psi(I)$$

with $h(c, I) = c + \frac{G(c, I)}{g(c, I)}$ and $J(c) = c + \frac{F(c)}{f(c)}$. Hence, $\hat{\Gamma}^2(\hat{I})$ arises in equilibrium, and the winner suffers a disadvantage.
Hidden Investment: Simultaneous-Move Game

- Winner’s action space: \( I \in [0, +\infty) \).
- Buyer’s action space: \( BR_b = \{\hat{\Gamma}^2(I) | I \geq 0\} \) (rationalizability argument)
- Focus on pure-strategy equilibria

Proposition

*In this context, a pure-strategy equilibrium corresponds to a tuple* \((\hat{\Gamma}^2(\hat{I}), \hat{I}) \in BR_b \times [0, +\infty)\) *that solves*

\[
\begin{align*}
\min_{\hat{\Gamma}(I) \in BR_b} C^2(\hat{\Gamma}(I), J) \\
\text{s.t.} J \in \arg\max_{K \geq 0} \int_C Q^2_{w,I}(c)G(c,K)dc - \Psi(K)
\end{align*}
\]
Equilibrium Characterization and the Impact of Commitment on Investment Incentives

Proposition

The exists a unique equilibrium is pure-strategies $(\hat{I}^2(\hat{I}), \hat{I})$ where $\hat{I}$ is characterized by

$$\frac{\partial}{\partial I} \left( \int_C \left[ 1 - F(v^{-1}(h(\hat{I}, c))) \right]^{n-1} G(c, I) dc - \Psi(I) \right) \bigg|_{I=\hat{I}} = 0$$

Proposition

The following ranking holds: $\hat{I} < \hat{I} < I^e < I^*$