The study of distributed computing algorithms by multithread applications

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Abstract

The material in this note is used as an introduction to distributed algorithms in a four year course on software and automatic control system in the computer technology department of the Komsomolsk-on-Amur state technical university. All our the program examples are written in Borland C/C++ 5.02 for Windows 95/98/2000/NT/XP, and hence suit to compile and execute by Visual C/C++. We consider the following approaches of the distributed computing:

The conversion of recursive algorithms to multithread applications
A realization of the pairing algorithm
The building of wave systems by Petri nets and object oriented programming.

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1 Threads and the conversion of recursive programs

A thread is declared by its descriptor. We create the thread by the API function

HANDLE h=CreateThread (NULL, 0, Thr, (void *)p, 0, 0);

which loads the subroutine

DWORD WINAPI Thr (void *p)
{
    ...
}

and returns the descriptor h of the thread if the creation is successful and h=NULL otherwise. The argument p points to a list of parameters of the function Thr(). The subroutine call

WaitForSingleObject(h, INFINITE);

1
leads to the waiting of the thread termination.

If a program has a recursive function, then we can convert all the function calls to the thread creations. This conversion leads to a multithread application. We consider the Hoare quick sort for example:

```c
#include <stdlib.h>
#include <time.h>
#include <iostream.h>
#define N 100
int x[N];
void q_sort(int l, int u)
{
    int i, j, m, t; int temp;
    if(l < u)
    {
        t = x[l]; m = l;
        for (i = l + 1; i <= u; i++)
            if(x[i] < t)
                {m++; temp = x[m]; x[m] = x[i]; x[i] = temp;}
        temp = x[l]; x[l] = x[m]; x[m] = temp;
        q_sort(l, m - 1); q_sort(m + 1, u);
    }
}
main()
{
    int i; randomize();
    for (i = 0; i < N; i++) x[i] = random(100);
    q_sort(0, N - 1);
    for (i = 0; i < N; i++) cout << " " << x[i];
}
```

The subroutine q_sort() has two parameters. Hence we must define a structure

```c
struct arg
{
    int left, right;
};
```

for the parameters passing. We obtain the following text after the conversion:

```c
#include <windows.h>
#include <stdlib.h>
#include <time.h>
#include <iostream.h>
#include <conio.h>
```
#define N 100  
struct arg  
{   
    int left, right;  
};  
int x[N];  
DWORD WINAPI q_sort(void* p)  
{   
    int l=((arg *)p)->left, u=((arg *)p)->right;   
    int i, j, m, t, temp;   
    if (l<u)   
    {   
        t = x[l]; m = l;   
        for (i=l+1; i<=u; i++)   
            if (x[i]<t)   
                {   
                    m++; temp=x[m]; x[m]=x[i]; x[i]=temp;   
                }   
        temp = x[l]; x[l]= x[m]; x[m]=temp;   
        arg *r1 = new arg, *r2 = new arg;   
        HANDLE H[2];   
        r1->left=l; r1->right= m-1;   
        r2->left= m+1; r2->right = u;   
        H[0]= CreateThread(0,0,q_sort, (void *)r1,0,0);   
        H[1]= CreateThread(0,0,q_sort, (void *)r2,0,0);   
        for (i=0; i<2; i++)   
            WaitForSingleObject(H[i],INFINITE);   
        delete r1; delete r2;   
    }  
    return 1;  
}  
void main()  
{   
    int i;   
    randomize();   
    for (i=0; i<N; i++) x[i]=random(100);   
    for (i=0; i<N; i++) cout << " " << x[i];   
    cout << "\n";   
    arg a;   
    a.left = 0; a.right = N-1;   
    q_sort(&a);   
    for (i=0; i<N; i++) cout << " " << x[i];   
    getch();  
}
The first call of the recursive function remains in the main program whereas the calls in \texttt{qsort()} convert to \texttt{CreateThread()}. A project for this program must include the options "console application" and the "multithread application". Hence the creation of the project is given by the menu command \textit{File $\rightarrow$ New $\rightarrow$ Project} with Target Model "Console" and the option "Multithread". The files ".def" and ".rc" must be deleted from the project.

The program puts a sequence of 100 random numbers and then these numbers are displayed in the undecreasing order.

2 The pairing algorithm implementation

It is possible the implementation of a multiply applied associative operation by the recursive subroutines. The subroutine has two parameters with numbers of first and last elements of an array:

\begin{verbatim}
int sum(int l, int r) // x[l] + ... + x[r]
{
  if(l == r) return x[l];
  else return sum(l, (l+r+1)/2 - 1) + sum((l+r+1)/2, r);
}
\end{verbatim}

This subroutine is called in the main program by \texttt{s=sum(0,n-1)}. Hence, we obtain by the conversion to the multithread application the following text:

\begin{verbatim}
#include <windows.h>
#include <stdlib.h>
#include <time.h>
#include <iostream.h>
#include <conio.h>
#define N 32

struct arg
{
  int l, r, rez;
};

int x[N];

DWORD WINAPI sum(void* s)
{
  int i, l=((arg *)s)->l, r=((arg *)s)->r;
  if (l==r) ((arg *)s)->rez = x[l];
  else
    {
      arg *r1 = new arg, *r2 = new arg;
      HANDLE H[2];
      r1->l=1; r1->r= (1+r+1)/2-1;
      r2->l=(1+r+1)/2; r2->r = r;
      H[0]= CreateThread(0,0,sum, (void *)r1,0,0);
      H[1]= CreateThread(0,0,sum, (void *)r2,0,0);
      H[0]->WaitForSingleObject(0);
      //... all other threads...
      H[1]->CloseHandle();
    }
}
\end{verbatim}
H[1]= CreateThread(0,0,sum, (void *)r2,0,0);
for (i=0; i<2; i++)
    WaitForSingleObject(H[i],INFINITE);
((arg *)s)->rez = (r1->rez)+(r2->rez);
delete r1; delete r2;
}
return 1;
}
int sum0()
{
int s=0, i;
for (i=0; i<N; i++) s+=x[i];
return s;
}
void main()
{
int i;
randomize();
for (i=0; i<N; i++) x[i]=random(100);
arg t;
t.l = 0; t.r = N-1;
sum(&t);
cout << "\n sum obtained = "<< t.rez;
cout << "\n sum is = "<< sum0();
getch();
}

Consider the pairing algorithm for the computing of the polynomial 
\( p(x) = a_0 + a_1 x + \cdots + a_n x^n \) values by Horner’s scheme:

\[ p_0 = a_0; \quad p_1 = xp_0 + a_{n-1}; \quad p_2 = xp_1 + a_{n-2}; \quad \cdots \quad p_n = xp_{n-1} + a_0. \]

with \( p(x) = p_n \). If it possible to write \( p_k = xp_{k-1} + a_{n-k} \) as the equation

\[
\begin{pmatrix}
p_k \\
1
\end{pmatrix} =
\begin{pmatrix}
x & a_{n-k} \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
p_{k-1} \\
1
\end{pmatrix}.
\]

Therefore

\[
\begin{pmatrix}
p(x) \\
1
\end{pmatrix} =
\begin{pmatrix}
x & a_0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
x & a_1 \\
0 & 1
\end{pmatrix}
\cdots
\begin{pmatrix}
x & a_{n-1} \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
a_n \\
1
\end{pmatrix},
\]
and we can to compute the values of \( p(x) \) by the pairing algorithm for the matrix multiplications.

Now we write the recursive subroutine of the matrix multiplication. The subroutine get parameters contained in a structure

struct arg
The data are read from the external array \(a[i]\) and the variable \(x\). We have a text of the subroutine:

```c
void prod(void *s)
{
    int i, l = ((arg*)s) -> l, r = ((arg*)s) -> r;
    if(l == r)
    {
        ((arg*)s) -> p = x;
        ((arg*)s) -> q = x[l] ;
    }
    else
    {
        arg *r1 = new arg, *r2 = new arg;
        r1 -> l = l; r1 -> r1 = (l + r + 1)/2 - 1;
        r2 -> l = (l + r + 1)/2; r2 -> r = r;
        ((arg *)s) -> p = (r1 -> p)(r2 -> p);
        ((arg *)s) -> q = (r1 -> p)(r2 -> q) + (r1 -> q);
        delete r1; delete r2;
    }
}
```

Converting this subroutine we obtain the following multithread application where the values \(a_0 + a_1 x + a_2 x^2 + \cdots + a_N x^N\) are calculated by the pairing of the Horner scheme

```c
#include <windows.h> // multithread opportunity
#include <stdlib.h> // random numbers generation
#include <time.h> // the first random number
#include <iostream.h> // input/output
#include <conio.h> // for the getch()
#include <dos.h> // debugging by sleep()
define N 20 // degree of the polynomial
double x; // the value of the variable
struct arg // for the parameter settings
{
    int l, r; // left and right numbers of 22-marix
    double p, q; // the first matrix string entries
};
int a[N+1]; // coefficients of the polynomial
DWORD WINAPI prod(void* s)// the thread subroutine
{

int i, l=((arg *)s) -> l, r=((arg *)s) -> r;
sleep(random(2));    // the delay for the debugging
if (l == r)
{
    ((arg *)s) -> p = x;  // the coefficient
    ((arg *)s) ->q = a[r]; // settings
}
else
{
    arg *r1 = new arg, *r2 = new arg;
    HANDLE H[2];    // descriptors
    r1 -> l = l; r1 -> r = (l + r + 1)/2 - 1; // boundaries
    r2 -> l = (l + r + 1)/2; r2 -> r = r;
    H[0] = CreateThread(0,0,prod, (void *)r1, 0,0); // create
    H[1] = CreateThread(0,0,prod, (void *)r2,0,0); // threads
    for (i = 0; i < 2; i++) // wait of
        WaitForSingleObject(H[i],INFINITE);    // terminates
    ((arg *)s) -> p = (r1 -> p)*(r2 -> p);
    ((arg *)s) -> q = (r1 -> p)*(r2 -> q)+(r1 -> q);
    delete r1; delete r2;
}
    return 1;
} 
void main()
{
    int i,j;
    double w=0, v, z;
    randomize();
    for (i=0; i <=N; i++)
        a[i] = random(10);
    arg t;    // parameter structure
    for (x = 0; x < 10; x += 1) // values of x
    {
        t.l = 0; t.r = N;    // boundaries
        prod(&t);    // call of the subroutine
        cout << "\n Polynomial value = " << t.q; // output
        z = 0;
        for (i = 0; i <= N; i++)// checking cycle
        {
            v = 1;
            for (j = 0; j < i; j++) v = v*x; // computation of xi
            z += v*a[i];    // the polynomial values
        }
        cout << " == " << z; // displays for the checking
        getch(); // wait of the keyword press
    }
}
In fact the pairing algorithm computes in this application the product of $N+1$ matrices. The first string entries of the product will be equal to $x^{N+1}$ and $p \cdot a[N] + q$, where $p$ and $q$ are the first string entries of the product of $N$ matrices. Thus the program displays the values $p(x) = p \cdot a[N] + q$, and values obtained by the single thread program for the checking:

Polynomial value = 9 == 9
Polynomial value = 53 == 53
Polynomial value = 1.99675e+06 == 1.99675e+06
Polynomial value = 4.74122e+09 == 4.74122e+09
Polynomial value = 1.31457e+12 == 1.31457e+12
Polynomial value = 1.07123e+14 == 1.07123e+14
Polynomial value = 3.96656e+15 == 3.96656e+15
Polynomial value = 8.47447e+16 == 8.47447e+16
Polynomial value = 1.20753e+18 == 1.20753e+18
Polynomial value = 1.26117e+19 == 1.26117e+19

3 The synchronizing mechanism

Suppose that a few threads work with one variable in the global memory. It is possible the access of one at most a thread to this variable in any time. The access synchronization of these threads is called a serialization problem. A solution of the serialization problem is based on a semaphore mechanism. A semaphore is the data structure which consists of one integer number $s \geq 0$ and two functions $P(s)$ and $V(s)$ which are possible to unite in the following class:

```cpp
class Semaphore
{
    volatile int s; // a semaphore counter

    public:
        Semaphore(int init): s(init) {} // initial counter value
        void P() {while (s==0); s--;} // wait and locks
        void V() {s++;} // signal operation
}
```

Suppose that the functions of the semaphore class are realized by the hardware. Semaphores allow us solve the serialization problem. For example, consider the concurrent computing of the vector $(x_1, x_2, \cdots, x_n)$ length square. We load $n$ threads for the independent calculations of $x_i \cdot x_i$. Every thread locks the global variable $s$ after the calculation of $d = x_i \cdot x_i$ and yields the action $s = s + d$.

We obtain the following program scheme:

```cpp
double x[n];  // vector coordinates
Semaphore s(1); // initialize s = 1
double d = 0;   // the global variable
```
main ()
{
    /* here the threads are loaded ... */
}

The thread subroutine can be contains the following operators:

double y = x[i]*x[i];
s.P(); // the lockout of d resource

d = d + y;
s.V(); // the signal operation

Our aim is to write this program in the Borland C/C++. We will be use the following subroutines for the semaphore operations. The part of the constructor is belong to the function:

HANDLE s = CreateSemaphore (NULL, i, n, NULL);

where i is the initial value of the semaphore counter whereas n is the maximal value. The role of the call s.P() is played by

WaitForSingleObject(s, INFINITE) ;

and there call s.V() is executed by

ReleaseSemaphore(s, 1, NULL).

These subroutines allow us to write the following solution of the serialization problem:

#include <windows.h>
#include <stdlib.h>
#include <time.h>
#include <iostream.h>
#include <conio.h>
define n 100
volatile int j=0;
HANDLE mut;
double d=0;
double x[n];

DWORD WINAPI Thr(LPVOID v)
{
    double  *w = (double *)v; double u=(*w)*(*w);
    WaitForSingleObject(mut, INFINITE);
    d = d+u; j++;
    ReleaseSemaphore(mut,1, NULL);
    return 1;
}

double sum0()
double s0=0; int k;
for (k=0; k<n; k++)
    s0+= x[k]*x[k];
return s0;

void main()
{
    int i;
    randomize();
    for (i=0; i<n; i++) x[i]=(0.+random(100))/100+5;
    mut = CreateSemaphore(NULL,1,1, NULL);
    for (i=0; i<n; i++)
    {
        CreateThread(NULL,0,Thr, (void *)&x[i],0,0);
    }
    while (j<n);
    cout << "\nValue obtained by threads = " << d;
    cout << "\nValue computed by function = " << sum0();
    getch();
}

4 A channel class for the producer and consumer problem

Using the idea the Occam we define a class which objects are channels. Each channel is a data queue organized as an array. Suppose that this array contains the double precision numbers. The access to the queue is provided by the overload operations << for the writing and >> for the reading. Using a classical solution we define the class of the channel. We write this definition into the file channel.h:

// channel.h
template<class Type>
class channel
{
    Type *buf; // the buffer for a queue
    int size; // the buffer size
    HANDLE s, // the access semaphore
        empty, // number of free places in the queue
        full; // number of full places in the queue
    int countr, countw; // read/write counters
public:
    channel (int n): size(n) // constructor
    {
buf = new Type [n]; // the memory of the buffer
countr=0; countw=0;
s=CreateSemaphore(NULL,1,1,NULL); // semaphore initialization
empty=CreateSemaphore(NULL,n,n,NULL); // n free places
full=CreateSemaphore(NULL,0,n,NULL); // no full places
}

void operator << (Type d) // write to channel
{
  WaitForSingleObject(empty, INFINITE); // wait free places
  WaitForSingleObject(s, INFINITE); // take buffer
  buf[countw++]=d; // write to queue
  if (countw==size) countw=0;
  ReleaseSemaphore(s,1,NULL);
  ReleaseSemaphore(full,1,NULL); // full++
}

void operator >> (Type& d) // read from channel
{
  WaitForSingleObject(full, INFINITE); // wait of data
  WaitForSingleObject(s, INFINITE); // take buffer
  d = buf[countr]; countr++; // read
  if (countw==size) countw=0;
  ReleaseSemaphore(s,1,NULL);
  ReleaseSemaphore(empty,1,NULL); // empty++
}

It is a generalization with the template type instead of double. Now we have
the following solution of the producer and consumer problem:

#include <windows.h>
#include <stdlib.h>
#include <time.h>
#include <iostream.h>
#include <conio.h>
#include "channel.h"

double out[20];
DWORD WINAPI producer(LPVOID v)
{
  int j;
double d1;
  for (j=0; j<20; j++)
  {
    d1 = (double)(1+j);
    *(channel<double>*)v<<d1;
  }
  return 1;
}
DWORD WINAPI consumer(LPVOID v)
{
    int k;
    for (k=0; k<20; k++)
    {
        *(channel<double>*)v>>out[k];
    }
    return 1;
}

void main()
{
    channel<double> c0(10);
    int i=0;
    CreateThread(NULL,0,producer, (LPVOID) &c0,0,0);
    CreateThread(NULL,0,consumer, (LPVOID) &c0,0,0);
    cout<<"\n"; for(i=0;i<20;i++) cout << " " << out[i];
    getch();
    cout<<"\n"; for(i=0;i<20;i++) cout << " " << out[i];
    getch();
}

Here the producer writes to the channel the sequence of numbers. The consumer reads this sequence and writes it to the array out[]. The application displays 20 values of 0, and the sequence 1, 2, · · ·, 20 after the key input.

5 Wave systems and their Petri nets

A wave system is the generalization of the pipeline. Consider the computing of the values $y_n = f(g(x_n))$ with $n = 0, 1, 2, \cdots$. It is possible to load the threads which states are described by the following Petri net:

\[
\begin{array}{cccccc}
M & p1 & T1 & p2 & T2 & p3 & T3 \\
\end{array}
\]

where the main program $M$ and threads $T1$, $T2$, $T3$ are executed concurrently. The main program generated random numbers $x_n$ and writes $x_n$ to the channel corresponding to the channel $p1$. The thread $T1$ computes $g(x_n)$ and writes it to the channel $p2$. The thread $T2$ computes $f(g(x_n))$ and the thread $T3$ gets the data from the channel $p3$ and puts to the display. Such a system is called the pipeline and has the following generalization. For example, consider the computation of values

\[
x_n = u_n + sin(v_n^2), \quad y_n = exp(sin(u_n - v_n)).
\]
Applicate the flow control with the channel communications. The factorization of these operations leads us to the 10 threads executed concurrently. The threads correspond to the transitions of the Petri net which is shown in Figure 1.

The Petri net consists of 11 places corresponding to the channels and 10 transitions corresponding to the threads. Input of \((u_n, v_n)\) and output of \((x_n, y_n)\) will be yield in the main program. Thus the computations will be yield in the six threads. The array of channels

\[
\text{Channel } c[11] = \{10, 10, 10, 10, \ldots , 10\};
\]

is declared in the main program. Each channel is a queue of 10 elements of the array of double precision numbers. These channels connect by global pointer declared as \textit{channel \,*ps[11]}. The threads execute concurrently. Each of threads waits an input data and then computes an operation. For example, the subroutine \textit{mult()} gets the data from the channels \(c[0]\) and \(c[1]\) and sends their product to the channel \(c[4]\).

```c
#include <windows.h>
#include <stdlib.h>
#include <time.h>
#include <iostream.h>
#include <conio.h>
#include <math.h>
#define n 20
channel \,*pc[11];
DWORD WINAPI mult(LPVOID) // multiplication thread
{
  int j;
  double d1, d2;
  for (j=0; j<n; j++)
  {
    *pc[0]>>d1; *pc[1]>>d2; // get data from channels 0 and 1
    *pc[4]<<(d1*d2); // multiply and send to 4
  }
  return 1;
}
DWORD WINAPI minus(LPVOID) // subtract thread
{
  int j;
  double d1, d2;
  for (j=0; j<n; j++)
  {
    *pc[2]>>d1; *pc[3]>>d2; // get data from channels 2 and 3
    *pc[5]<<(d1-d2); // subtract and send to 5
  }
  return 1;
}
```

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Figure 1: The Petri net of the wave system
DWORD WINAPI sinus(LPVOID) // sinus thread
{
    int j;
    double d1;
    for (j=0; j<n; j++)
    {
        *pc[4]>>d1;                // get from 4
        *pc[7]<<sin(d1);          // send to 7
    }
    return 1;
}
DWORD WINAPI sinus2(LPVOID) // the second thread for sinus
{
    int j;
    double d1;
    for (j=0; j<n; j++)
    {
        *pc[5]>>d1;                // get from 5
        *pc[8]<<sin(d1);          // send to 8
    }
    return 1;
}
DWORD WINAPI plus(LPVOID) // thread for add
{
    int j;
    double d1, d2;
    for (j=0; j<n; j++)
    {
        *pc[6]>>d1; *pc[7]>>d2;    // get from 6 and 7
        *pc[9]<<((d1+d2);          // send to 9
    }
    return 1;
}
DWORD WINAPI expo(LPVOID) // thread for the exponent
{
    int j;
    double d1;
    for (j=0; j<n; j++)
    {
        *pc[8]>>d1;                // get from 8
        *pc[10]<<exp(d1);          // send to 10
    }
    return 1;
}
void main() // main program

{ 
channel c[11]={10,10,10,10,10,10,10,10,10,10,10};
// initialize threads
int i; // by 10 elements
for (i=0; i<11; i++) pc[i] = &c[i]; // pointers to channels
CreateThread(NULL, 0, mult, 0, 0, 0); // load the threads
CreateThread(NULL, 0, minus, 0, 0, 0);
CreateThread(NULL, 0, sinus, 0, 0, 0);
CreateThread(NULL, 0, sinus2, 0, 0, 0);
CreateThread(NULL, 0, plus, 0, 0, 0);
CreateThread(NULL, 0, expo, 0, 0, 0);
double u,v,x,y;
for (i=0; i<n; i++)
{
    u=i*i; v=i+1;
    c[0]<<v; c[1]<<v; // send to channels 0,1,2,3,6
    c[2]<<u; c[3]<<v; c[8]<<u;
}
for (i=0; i<n; i++)
{
    c[9]>>x; c[10]>>y; cout<<"\n"<<x;
    cout<<"=":; // read the results
    cout<<(double)(i*i+sin(0+(i+1)*(i+1))) ; // comparison
    cout<< "<<y"="<<(double)(exp(sin(i*i-i-1)));
}
getch();
}

The application displays:

0.841471 == 0.841741 0.431076 == 0.431076
0.243198 == 0.243198 0.431076 == 0.431076
4.41212 == 4.41212 2.31978 == 2.31278
8.7121 == 8.7121 0.383305 == 0.383305
15.8676 == 15.8676 0.367883 == 0.367883
24.0062 == 24.0822 1.16169 == 1.16169
35.0462 == 35.0462 0.514977 == 0.514977
49.92 == 49.92 0.853318 == 0.853318
63.3701 == 63.3701 0.36797 == 0.36797
80.4936 == 80.4936 2.58844 == 2.58844
100.999 == 100.999 2.36332 == 2.36332
120.509 == 120.509 2.26312 == 2.26312
143.398 == 143.398 0.444145 == 0.444145
169.94 == 169.94 0.417556 == 0.417556
195.07 == 195.07 0.392016 == 0.392016
224.001 == 224.001 2.70869 == 2.70869
255.973 == 255.973 1.26705 == 1.26705
The comparison of the results shows that the wave system is well defined.

We conclude that there is the well opportunity for the study of the concurrent algorithms on the every personal computer.

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