Supersymmetric $SU(2)_L \times SU(2)_R \times SU(4)_c$ and observable neutron-antineutron oscillation

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(February, 1998)

We show that in a large class of supersymmetric $SU(2)_L \times SU(2)_R \times SU(4)_c$ models with the see-saw mechanism for neutrino masses and an R-parity conserving vacuum, there are diquark Higgs bosons with masses $(M_{qq})$ near the weak scale even though the scale of $SU(2)_R \times SU(4)_c$ symmetry breaking is around $10^{10}$ GeV. This happens because these masses $(M_{qq})$ arise out of higher dimensional operators needed to stabilize the charge conserving vacuum in the model. This feature has the interesting implication that the $\Delta B = 2$ processes such as neutron-anti-neutron oscillation can have observable rates while at the same time yielding neutrino masses in the range of current interest.

UMD-PP-98-94

I. INTRODUCTION

A hallmark of the successful standard model of electroweak interactions is the automatic conservation of baryon and lepton number, a property obeyed by all known processes involving elementary particles. Even before the well-known experimental triumphs of the model, this property was recognized as very desirable and appealing. On the other hand, its supersymmetric extension, the minimal supersymmetric standard model (MSSM) which promises to explain two of the major unsolved problems of the standard model i.e. the origin and stability of the weak scale is plagued by uncontrollable amounts of both baryon and lepton number violation, known as R-parity violation. Thus a heavy price is paid to understand the symmetry breaking of the standard model if one insists on staying within the MSSM.

A model that preserves both the nice properties of the MSSM while at the same time solving the R-parity violation problem is the supersymmetric left-right model (SUSYLR) with the see-saw mechanism for neutrino masses [1]. Needless to say that the recent hints for neutrino masses provide an extra motivation for studying this model in any case.

A detailed analysis of this model has been the subject of several recent papers which explore its vacuum structure and resulting particle spectrum [2, 3]. Such investigations are essential to establish the viability of the model since constraints of supersymmetry are known to seriously alter the nature of general field theories compared to their nonsupersymmetric versions. A very important result of these investigations is that the requirement of electric charge conservation by vacuum imposes stringent constraints on the scale of left-right symmetry breaking, $v_R$ (or the $W_R$ scale) In a large class of models, essentially two possibilities emerge: (i) the $W_R$ mass is in the TeV range and R-parity is broken spontaneously [3] by the vev of $v^c$ or (ii) if R-parity is conserved by the vacuum, the $W_R$ scale is above $10^{10}$ GeV [2, 4]. In case (ii), when the $W_R$ scale is close to its minimum allowed value, there are light doubly charged bosons and fermions with masses in the 100 GeV range. There is a simple group theoretical way to understand this. The essential point is that the requirement of holomorphy of the superpotential enhances the global symmetry of the theory (making it bigger than the gauge symmetry). After the supersymmetry breaking terms are switched on, the minimum of the theory violates electric charge forcing one to include the nonrenormalizable terms in the superpotential. They then lead to lower limits on the $W_R$ mass following from the lightness of the pseudo-Goldstone (PG) states (since $M_{PG} \approx v^c_R/M$). Thus the low energy model in these theories is the familiar MSSM with automatic R-conservation plus massive neutrinos and doubly charged particles. This provides an experimental way to distinguish the SUSYLR models from the MSSM.

When the SUSYLR model is embedded into the $SU(2)_L \times SU(2)_R \times SU(4)_c$ gauge group with symmetry breaking implemented by the Higgs multiplets suggested in Ref. [2], the arguments leading to the above constraint on the $W_R$ scale carry over and one has $v_R \equiv M_c \geq 10^{10}$ GeV ($M_c$ being the $SU(4)_c$ breaking scale). The enlargement of the gauge group however has a new and important physical implication that we study in this paper. Due to the larger dimensionality of the Higgs multiplets, the global symmetry of the superpotential becomes larger leading to light doubly colored fields (or the di-quarks) with masses in the 100 GeV range even though the $SU(4)_c$ scale is in the range of $10^{10}$ GeV or so. This result is sharply different from the corresponding nonsupersymmetric case where the diquark bosons “tag” the $SU(4)_c$ scale and has the following experimental manifestations.

The existence of diquark Higgs bosons in the $SU(2)_L \times SU(2)_R \times SU(4)_c$ was shown in 1980 [5] to imply $\Delta B = 2$
II. THE MODEL

The gauge group $SU(2)_L \times SU(2)_R \times SU(4)_c$ (to be denoted shorthand when needed as $G_{224}$). In this simplest model with only singlets, the strong coupling becomes nonperturbative around $10^{15}$ GeV or so. We therefore extend the model in such a way that the strong coupling $M$ between $10^{15}$ to $10^{18}$ GeV. The relevant part of the superpotential is

$$W = i f (\Psi^c, \tau_2 \Delta^c \Psi^c + \Psi^T \tau_2 \Delta \Psi) + (M_0 + \lambda S) Tr(\Delta^c \Delta^c) + (M_0 - \lambda S) Tr(\Delta \Delta) + \mu_S S^2 + A [Tr(\Delta^c \Delta^c)]^2 + B Tr(\Delta^c \Delta^c) Tr(\Delta \Delta)$$

In the above equation, the $A$, $B$, $f$, $\lambda$ and $M_0$ are parameters of the theory with $A$ and $B$ are of order $1/M$. To this one must add the soft supersymmetry breaking terms, which have mass scale in the range of few hundred GeV's. Since supersymmetry must remain a good symmetry down to the weak scale, the $F$ terms for all the fields must be proportional to $m_{3/2}$, the SUSY breaking parameter.

Before turning to discuss the diquark mass spectrum, we point out a very crucial property of these models found in Ref. 3 and already alluded to in the introduction. The requirement of electric charge conservation by the vacuum state implies that one must include the $A$ and $B$ terms given in Eq.1. Due to the enhanced global symmetry of the renormalizable part of $W$, the model has light charged and/or colored fields, whose masses arise from the $B$ term and are therefore proportional to $v_R/M$. Since present collider data imply that there are no such particles below 50 to 100 GeV, this enables one to derive a lower limit on scale of $v_R$ to be $10^{10}$ GeV for $M = 2 \times 10^{18}$ GeV and slightly weaker otherwise. In what follows, we will use $10^{10}$ GeV as a generic lower limit on $v_R$.

Using Eq. 1, one can give a group theoretical argument for the existence of light doubly charged and doubly colored particles in the supersymmetric limit as follows. For this purpose let us first ignore the higher dimensional terms $A$ and $B$ as well as the leptonic couplings $f$. It is then clear that the superpotential has a complexified $U(30)$ symmetry (i.e. a $U(30)$ symmetry whose parameters are taken to be complex) that operates on the $\Delta^c$ and $\Delta^c$ fields. This is due to the holomorphy of the superpotential. After one component of each of the above fields acquires vev’s and supersymmetry guarantees that both vev’s are parallel, the resulting symmetry is the complexified $U(29)$. This leaves 118 massless fields. Once we bring in the $D$-terms and switch on the gauge fields, 18 of these fields become massive as a consequence of the Higgs mechanism of supersymmetric theories. That leaves 100 massless fields in the absence of higher dimensional terms. In the presence of the higher dimensional operators in the superpotential, they lead to 50 complex light fields which consist of 18 $\Delta^c_{qq}$ plus 18 $\Delta^c_{qd}$ fields; the two doubly charged fields of Ref. 4 and 12 leptoquark fields of type $(u^c e^c + d^c \nu_e^c)$, $d^c e^c$ and their complex conjugate states. The detailed analysis of the potential leading to these light fields in the presence of soft SUSY breaking is identical to that given Ref. 3. So we do not repeat it here. The important point is that their masses arise from the higher dimensional term $B$ and are given by $v_R^2/M$, as already mentioned.

In this simplest model with only singlets, the strong coupling becomes nonperturbative around $10^6$ GeV or so which is much below the $W_R$ scale of $10^{10}$ GeV or so. We therefore extend the model in such a way that the strong coupling
remains perturbative above the $v_R$ scale. The simplest way to do this is to add $SU(4)_c$ singlet but $SU(2)$ triplet fields (denoted by $\delta$ and $\delta^c$) to the model. The parity odd singlet will lift the left-handed part to the $W_R$ scale and make it phenomenologically innocuous at low energies. The resulting theory is described by a superpotential given by $W + W'$ with $W$ given above and

$$W' = \lambda'' S(\delta^2 - \delta^c 2) + M'(\delta^2 + \delta^c 2) + \lambda' (\Delta \delta \Delta + \Delta' \delta^c \Delta^c)$$  \hspace{1cm} (2)$$

The point of the extra field is that in the absence of the higher dimensional terms, this reduces the global symmetry to $U(10) \times SU(2)$ in the righthanded sector. The vevs of $\Delta^c$ and $\delta^c$ break this group down to $U(9) \times U(1)$. This leaves after gauge symmetry breaking 24 real massless states or 12 complex states. They are easily identified to be the twelve color symmetric diquark states $\Delta u_w u^c$ and $\Delta d_w d^c$. As before, the inclusion of the same higher dimensional terms in the superpotential gives mass of order 100 GeV to the $u^c u^c$ fields for $v_R \simeq 10^{10}$ GeV. The remaining diquark fields have masses of order of $< \delta^c >$. We will choose the tree level parameters of the potential such that $< \delta^c > \simeq (10^{-3} - 10^{-2}) v_R$ in the following discussion.

An alternative possibility is to include $SU(4)_c$ singlet but $SU(2)$ quintet fields (denoted here as $\Sigma \oplus \Sigma^c$).

$$W'' = M'' (\Sigma^c \Sigma^c + \Sigma \Sigma) + \lambda'' (\Delta^c \Sigma^c \Delta^c + \Delta \Sigma \Delta) + \lambda'' S(\Sigma^2 - \Sigma^c 2)$$  \hspace{1cm} (3)$$

The light particle counting in this case is more subtle since all terms in the superpotential do not take part in determining the vacuum state. By explicit calculation we have checked that the particles that are light in this case are: $\Delta u_w u^c$, $\Delta d_w d^c$, $\Delta d_d d^c$ and $\Delta_d d^c$, $\Delta_d d^c$. It is easily checked that their masses come entirely from the higher dimensional terms in the superpotential. In this case also, the strong coupling becomes nonperturbative below $v_R$.

Neutron-Anti-neutron oscillation

To see how $N - \bar{N}$ oscillation arises in the various models described above, let us include in the superpotential the following higher dimensional terms involving the $\Delta^c$ fields:

$$W' = \lambda' \frac{1}{M^{pprr}} e^{q' q} e^{r' r} \Delta^c_{ppr} \Delta^c_{qq} \Delta^c_{r' r} \Delta^c_{s' s} + \Delta^c \rightarrow \Delta + \text{ terms involving } \Delta^c$$  \hspace{1cm} (4)$$

The $SU(2)$ indices have been suppressed for brevity. We have scaled the nonrenormalizable terms by the same scale, $\lambda'$ used earlier. So in making estimates for the $\Delta B = 2$ amplitudes, we will vary this scale between the two values of $10^{15}$ to $10^{18}$ GeV. Now note that in conjunction with the $\Delta^c$ mass and coupling terms in the superpotential $W$, this gives rise to a four scalar $\Delta^c$ coupling with strength $\lambda_{eff}$ to be estimated below. As noted in Ref. [3], the diagram in Fig. 1 leads to the six quark $\Delta B = 2$ coupling $u^c d^c d^c d^c d^c$ with a strength

$$G_{\Delta B=2} \simeq \frac{\lambda_{eff} v_R f^3}{M^2_{d^c d^c} M^2_u u^c}$$  \hspace{1cm} (5)$$

There are also diagrams involving the exchange of two $u^c d^c$ type Higgs bosons in combination with one $d^c d^c$ boson. These are suppressed compared to the diagram in Figure 1 since the $M_{u^c u^c} \sim v_R$. In order to estimate $G_{\Delta B=2}$, we need to know the value of $\lambda_{eff}$. This will depend on whether we are considering the triplet or the quintet case.

(i) Triplet case:

This case is the most interesting since all the gauge couplings remain perturbative until $v_R$ and we therefore discuss it first. From the superpotential of the model it is easy to see that,

$$\lambda_{eff} = \lambda_2 < M_0 + \lambda S - \lambda' \delta^c > / M$$  \hspace{1cm} (6)$$

whereas the F-term condition gives the equation for exact supersymmetry below $v_R$ to be

$$M_0 + \lambda S + \lambda' \delta^c = 0$$  \hspace{1cm} (7)$$

The change in the sign of the coefficient of the $\delta^c$ term is due to the fact that $\Delta u_w u^c$ and $\Delta d_d d^c$ have opposite $I_{3R}$. Thus we find that

$$\lambda_{eff} M \equiv \lambda_2 (M_0 + \lambda S - \lambda' < \delta^c >) \simeq < \delta^c >$$  \hspace{1cm} (8)$$

From this we estimate $\lambda_{eff} \simeq 10^{-11} - 10^{-7}$ depending on whether we choose the nonrenormalizable term to be scaled by $M_{Pl}$ or $M_U$. 

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Taking $M_{a^u} \approx 100\text{ GeV}$, $M_{d^d} \approx 10^{-3} v_R$, we get $G_{\Delta B=2} \approx (10^{-30} - 10^{-33}) f^3 \text{ GeV}^{-5}$. To convert this into a $N - \bar{N}$ transition amplitude $\delta m_{N-\bar{N}}$, one must multiply it with the hadronization factor [10] usually estimated by various methods to be around $10^{-4} \text{ GeV}^6$. This leads to an neutron-anti-neutron oscillation time equal to $\tau_{N-\bar{N}} \approx 6 \times (10^9 - 10^{12}) \text{ sec}$. where we have chosen $f \approx 1$. On the other hand, if we chose $< \delta^c > \approx 10^{-2} v_R$, then we would have $\tau_{N-\bar{N}} \approx 6 \times (10^{12} - 10^{15}) \text{ sec}$. These estimates for $\tau_{N-\bar{N}}$ will go down by a factor of $f^3$ if we assume $M_{qq} \sim < \delta^c >$. We thus see that for plausible values of parameters of the theory, one can obtain observable $N - \bar{N}$ transition times. We find it very encouraging that we get numbers within the observable range of a recently proposed experiment at Oak Ridge [9].

(ii) Quintet case

This case has the drawback that the strong coupling becomes nonperturbative below the $v_R$ scale. If we however ignore this point, observable $\tau_{N-\bar{N}}$ comes out more easily in this case, since both $\Delta a^u$ and $\Delta d^d$ are in the 100-1000 GeV range. In this case, $\lambda_{eJ} M \approx M_0 + \lambda S + \lambda' \Sigma_{00}$. It therefore vanishes in the supersymmetric limit and is of order $m_{3/2}$ after soft SUSY breaking terms are included. We then get $\lambda_{eJ} \approx \frac{m_{3/2}}{\Delta^2}$. Now taking $M_{a^u} \approx M_{d^d} \approx 1000 \text{ GeV and } m_{3/2} \approx 1000 \text{ GeV, we get}$

$$G_{\Delta B=2} \approx (10^{-20} - 10^{-23}) f^3 \text{ GeV}^{-5}$$

Choosing $f \approx 0.1$, we get $\tau_{N-\bar{N}} \approx 3 \times 10^5 - 3 \times 10^8 \text{ sec}$. (using the hadronic factor to be $10^{-4} \text{ GeV}^6$) again in the observable range.

Let us end with a few comments:

(i) In general, the quark couplings to the diquark fields can lead to flavor changing neutral currents. The point is that the f coupling connects to all generations; as a result if we denote $a, b$ as the generation indices, then the $\Delta S = 2$ amplitude are induced by $\Delta d^d$ exchange at the tree level. However, in the triplet model, the diquark fields of $d^d$ type are naturally superheavy. As a result, there are no dangerous tree level flavor changing neutral currents. On the other hand, in the quintet model, the $d^d d^d$ diquark fields are light. We therefore have to resort to fine tuning such as $f_{12} = 0$ and $f_{11} = f_{22}$ to prevent large flavor changing neutral currents.

(ii) Furthermore, our conclusion is independent of the way supersymmetry is broken in the hidden sector i.e. whether it is gravity or gauge mediated. Again the arguments for the gauge mediated case are similar to the ones given in [3].

In conclusion, we have found that in a class of simple supersymmetric $SU(2)_L \times SU(2)_R \times SU(4)_c$ models, even though the $v_R$ scale is dictated by supersymmetry to be near or above $10^{10} \text{ GeV, some of the sextet diquark fields are forced to be light (in the 100 GeV range). The presence of these diquark fields can lead to observable neutrino-anti-neutrino oscillation while at the same time allowing neutrino masses to be in the currently favored eV range.}

Acknowledgements

This work is supported by the National Science Foundation grant no. PHY-9421385. We thank B. Brahmachari for help in drawing the figure.

\[\text{References}\]

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FIG. 1. The Feynman diagram responsible for $N - \bar{N}$ oscillation. The unlabelled dashed lines are the scalar diquark bosons with appropriate quantum numbers.