A model for predicting stress distribution and strain-force characteristics in regular braided ropes

Zhang Yujing, Meng Zhuo, Du Chengjie, Yao Linlin and Sun Yize

Abstract
Owing to the good mechanical properties of braided structures, regular braided ropes are increasingly being used in various fields, including marine exploration, aloft work, recreation activities, and oil prospecting. However, under certain severe conditions, they could break, a situation that is absolutely undesired. Thus, predicting their stress distribution and strain-force characteristics when they are subjected to different tensile loads is a pre-requisite for their application. Therefore, in this study, a mathematical model for ropes with regular braided structures is developed, and based on the model, this study reveals that uneven stress distributions in the different strands of regular braided ropes generate different stress distributions and strain-force characteristics in each of the strands. Additionally, the uneven stress distributions in the different strands also induce mechanical failure more readily. Finally, to ensure the reliability of braided ropes in different applications, different strand parameters are compared.

Keywords
Braided rope, stress distribution, regular braid, strand, yarn

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Introduction
Rope braiding possibly dates as far back as 28,000 years ago. During the braiding processes, two sets of braiding strands are each moved in the clockwise and counter-clockwise directions such that they interlace with each other. Three types of braided structures exist, these are: diamond, regular, and Hercules braided structures. Most common are regular braided structures, which can be produced with circular braiding machines. Even though they are simple, the tensile properties and internal stress characteristics of their constituent strands are not. The degree of complexity exhibited depends on the specific architecture of the braiding strands as well as many other braiding parameters.

Most previous studies published as far back as the 1950s focused on the geometrical modeling of braids, on the basis of which subsequent finite element analysis (FEA) as well as the analysis of the tensile properties of braids could be performed. Brunnschweiler and Goff propose an idealized geometry for diamond braided structures, which assume the concept of constituent strands. Later, Goff and Douglass
investigate braided structures with respect to different braid parameters, and recently, Rawal et al.\textsuperscript{8–10} focused on the
yarn path of braided structures, and they propose a series of
CAD models. Kyosev\textsuperscript{11} proposes a generalized geometrical
approach for the modeling of braided structures, and based
on the assumption that braiding strands follow a helical
path, Hristov et al.\textsuperscript{12} studies the braided structure and
mechanical behavior of circular hybrid braids. Additionally,
based on the results of studies conducted by Goff\textsuperscript{2} and Wu
et al.,\textsuperscript{13} Alpyildiz\textsuperscript{14} proposes a modified geometrical model
for the strand path of biaxial braided structures. For braided
ropes consisting of low twisted multi-filaments, Phoenix\textsuperscript{15}
proposes a diamond tubular braid model, which takes into
consideration constituent sinusoidal strands, and based on
this model, the actual strand length, effective cross-sectional
area of the strand that is normally directed along the braid
axis, crimp angle, and other braid assembly mechanical
parameters could be analyzed.

These proposed models that are primarily designed for
diamond braided structures are unsuitable for regular braided
structures, which are different. However, a few studies on
regular braided structures are conducted for example, for
regular braided structures, Lomov et al.\textsuperscript{16} propose a geomet-
rical model that takes into account structure skewness.

By establishing a geometric model, and thereafter solv-
ing it using a FEA software, more accurate rope mechanical
parameters could be obtained. Therefore, a professional
software, such as Texmind Braider, which can be applied
in braided structure modeling, is required\textsuperscript{17,18}; otherwise,
the task would be very cumbersome. In this light, the pur-
pose of this study is to establish a simplified model, and
from it, directly obtain approximate mechanical properties
that can be useful in preliminary design processes.

Thus, a simplified regular braided rope model that can
be used to compute the uneven stress distribution in differ-
ent braided rope strands is proposed. Thereafter, to better
illustrate the impact of the uneven stress distribution in dif-
f erent regular braided rope applications, different braided
rope parameters are compared.

### Mathematical modeling

Regular braided ropes have a loop structure along their
length. Figure 1 shows an example of such a regular
braided rope consisting of 12 strands.

In Figure 2, the strands of a one loop structure are shown,
and it gives a perspective view along the strand direction. It
shows that the yarns in the rope are under a tensile force
$T_{\text{total}}$, and the strands in each loop structure are divided into
four parts, referred to as units 1, 2, 3, and 4. The possible
internal forces are marked $F$, $f$, $P$, $Q$, and $T$.

The basic structure of regular braided ropes can be
studied using Figure 2, and for convenience and numerical
analysis, and based on practical engineering, the following
assumptions regarding the model are made.

1. Assume all strand cross-sections are circular
(radius $= r_0$ ) without stress; Under the influence of
the different forces, the circular shape is deformed
approximately into an elliptical shape with axes $r_a$
and $r_b$. The values of $r_a$ and $r_b$ could change with
the magnitude of the rope pulling force, but the
deformed cross-sections always keep oval shape.

2. Although there are shear forces during the process
of braiding and forming, they quickly change into
shear thermal form and are released owing to that
yarn and fluid have similar mechanical properties
to a degree. After forming the braided structure
completely, yarns are in a soft state. Therefore, the
internal forces $f$, $F$, $P$, and $Q$ are ignored.

![Figure 1. A regular braided rope with 12 strands.](image1)

![Figure 2. Perspective view of regular braided structures along the strand direction.](image2)
(3) As a consequence of the cylindrical symmetry, the stresses of each strand are periodically and symmetrically distributed.

(4) Given that strands are braided together closely, the multifilament effect inside each strand is ignored.

**Force equilibrium equations for regular braided structures**

Outer radius, $R_o$; braiding pitch, $L_o$; and number of strands, $N$ are considered as rope parameters, while the elastic modulus along and perpendicular to the strand centerline, $E_s$ and $E_p$, respectively, as well as Poisson’s ratio, $\nu$ are considered as strand parameters. After assuming that the centerline of the strands followed a sinusoidal path, and establishing the coordinate system shown in Figure 2, the equation of the centerline $\Gamma$, is given by:

$$z = \frac{R_o - 2r_b}{2} \cos \frac{N\pi}{2L_o} x, \quad x \geq 0 \quad (1)$$

As expected, the strands adjacent to the centerline are staggered by a one unit position. The cross-section consists of $N$ strands at each point, representing $N$ internal forces, among which there are $N/4$ sets of four different internal force values. For instance, while $N=12$, one loop of each strand consists of 12 units, and each unit has an equivalent internal force. However, most of the equivalent internal force values are the same. Due to symmetry, the internal force of the 12 strands could be divided into three same groups by a step of one-unit-position length, and each group has four different internal force values.

The total internal force at any cross-section is equal to $T_{total}$, and the projective value of the internal forces in the $xz$ plane is given by equation (2).

$$\frac{N}{4} \sum_{i=1}^{4} T_{0i}(x) \cos \theta_i(x) = T_{total};$$

$$\frac{\partial}{\partial x} \left( \sum_{i=1}^{4} T_{0i}(x) \cos \theta_i(x) \right) = 0 \quad (2)$$

where $i = 1, 2, 3, 4; \quad k = \left\{ i + 1, \quad i = 1, 2, 3 \right\} 1, \quad i = 4$; and $T_{0i}(x)$ represents the internal forces of the $i$-th unit at a distance $x$ along the rope direction. Particularly, when $x = 0$, $\frac{N}{4} \left( T_{s0} + T_{t0} + T_{c0} + T_{a0} \right) = T_{total}$; and $\theta_i(x) = \arctan \left( -\frac{N\pi (R_o - 2r_b)}{4L_o} \sin \frac{N\pi}{2L_o} \left( x + (i - 1)L_0 \right) \right)$.

Additionally, the internal force equilibrium equations for each unit are given by:

$$\sin \theta_i(x_i) \left( (p_{hi} + p_{ci}) ds_i \right) = T_{hi}(x_i) \cos \theta_i(x_i)$$

$$= T_{hi}(x) \cos \theta_i(x)$$

*Physical properties of regular braided structures*

Based on elastic mechanics and the Hertz contact theory,\(^2\) the equations for physical properties of the regular braided structure could be derived such that

$$r^2 = (1 + v_{rs})(1 - v_{rs})r_0; \quad r_b = (1 - v_s)(1 - v_{rs})r_0 \quad (4.1)$$

$$\sigma_s = \frac{\sigma_s r_0}{E_s}; \quad \varepsilon_s = \frac{\sqrt{2N_0} v_s}{\pi E_s (1 - v_{rs})^{1/2} (1 - v_s)^{1/2}} \quad (4.2)$$

$$\varepsilon_c = \frac{(1 - v_{rs})^{1/2} - 1}{(1 - v_{rs})^{1/2}} \quad (4.3)$$

where $\alpha = \arctan \left( \frac{N\pi (R_o - 2r_b)}{2(R_o - 2r_b)} \omega \right)$ represents the braid angle, $v_{rs}$, the traction speed; and $\omega$, the angular velocity of horn gears. $E_b$ represents a variable that is associated with strand twist variation; $\sigma_s$, $\varepsilon_s$ represents the stress and strain along the centerline strand, respectively, $\varepsilon_c$ represents the strain perpendicular to the centerline strand, $\varepsilon_c$ represents the strain along the rope direction, $\varepsilon_c$ represents the strain along the radius of the rope; and $A$ represents the total rope cross-sectional area.

**Deformation compatibility relationship in regular braided structures**

Figure 3 shows a sectional view of the cross section along the centerline strand. Based on the symmetry of the braided...
structures, the deformation compatibility relationship requires that at point C in the cross-section, $S_1$ should be on the same horizontal line as the point O.

The angle between the cross section $S_1$ and $S_2$ is $\gamma = \pi - 2\alpha$, the space coordinate of the point A is $\left(\frac{3L_0}{2N}, 0, -\frac{\sqrt{2}(R_0 - 2r_b)}{4}\right)$; and

$$\theta_A = \arctan\left(-\frac{\sqrt{2}N\pi(R_0 - 2r_b)}{8L_0}\right)$$

is the horizontal angle at point A.

The space volume equation of the strand is expressed as:

$$V_1 : \frac{y^2}{r_a^2} + \frac{\cos N\pi}{2L_0} \left(\frac{x \cos \theta(x) - z \sin \theta(x)}{r_b^2}\right) \leq 1$$

where $\theta(x) = \arctan\left(-\frac{N\pi(R_0 - 2r_b)}{4L_0} \sin \frac{N\pi}{2L_0} x\right)$.

The plane $\Pi_1$, which intersect vertically with the strand at point A had a normal vector $n_1 = (\sin(\pi - 2\alpha), \cos(\pi - 2\alpha), 0)$, and its equation is given by

$$\Pi_1 : \sin(\pi - 2\alpha)(x - \frac{3L_0}{2N}) + \cos(\pi - 2\alpha)y = 0$$

The projective area $\Omega_1$ of the strand onto the $\Pi_1$ plane, is given by

$$\begin{cases}
\frac{y^2}{r_a^2} + \frac{\cos N\pi}{2L_0} \left(\frac{x \cos \theta(x) - z \sin \theta(x)}{r_b^2}\right) & \leq 1 \\
\sin(\pi - 2\alpha)(x - \frac{3L_0}{2N}) + \cos(\pi - 2\alpha)y & = 0
\end{cases}$$

Based on the deformation compatibility relationship, the cross-sectional shape of $\Omega_1$ is found to be the same as that of $S_1$. To calculate the length of the straight line segment $AC$, the cross section $\Omega_1$ is fully considered. Beginning from point A, then to the origin, and rotating about the x axis at an angle $\theta_A$, a new cross section equation $\Omega_1'$ is obtained.

$$\begin{align*}
\frac{y^2}{r_a^2} + \frac{1}{r_b^2} & \left(\frac{y \sin \theta_A + z \cos \theta_A}{\cos \theta}(x - \frac{R_0 - 2r_b}{2}) + \frac{\cos N\pi}{2L_0} \left(\frac{x \cos \theta(x) - z \sin \theta(x)}{r_b^2}\right)\right) & \leq 1 \\
\sin(\pi - 2\alpha)(x - \frac{3L_0}{2N}) + \cos(\pi - 2\alpha)y & = 0
\end{align*}$$

By intersecting the new cross section $\Omega_1'$ and the $\Pi_2$ plane ($x = 0$), the straight line segment, $\Gamma_{DE}$ with end points $D(0, y_D, z_D)$, $E(0, y_E, z_E)$ is obtained, and its simplified equation is given by:

$$\begin{align*}
\Gamma_{DE} : \frac{y^2}{r_a^2} + \frac{(y \sin \theta_A + z \cos \theta_A)^2}{r_b^2} & \leq 1 \\
y \cos \theta_A & = z \sin \theta_A
\end{align*}$$

where $\theta_A = \arctan\left(-\frac{\sqrt{2}N\pi(R_0 - 2r_b)}{8L_0}\right)$.

The length of $\Gamma_{DE}$ and $AC$ are found to be the same that is,

$$\begin{align*}
(y_D - y_E)^2 + (z_D - z_E)^2 & = \frac{2 - \sqrt{2}}{4}(R_0 - 2r_b)
\end{align*}$$

**Results and discussion**

Combining equations (2) to (4) and (10), the stress distribution in regular braided rope strands could be determined using Matlab.

However, even though each basic structure consists of four basic units, only two different types of stress distributions are identified in counter-clockwise and clockwise strands. Based on the different parameters of regular braided ropes, the stress distributions in the strands are analyzed. Using aramid fibers, the braiding parameters are determined as: $R_0 = 200mm$; $N = 12$; $r_0 = 57.7mm$; $L_0 = 2000mm$; $\nu = 0.3$; $p = 0$; $v_b = 10mm/s$; $\omega = 300r/min$; $E_f = 124GPa$; and $E_b = 50MPa$. In Figures 4 and 5 the stress distributions, as well as other geometrical parameters of the strands are shown, and the tensile force,
When $T_{\text{total}}$ varies within the range $[1.1,5] \times 10^8$ N. However, given that the equations are piecewise functions, some singular points are observed at the boundaries of the different units during the numerical analysis (Figure 5).

As expected, as the tensile force increases, the geometrical parameters of the strands also increase; likewise, their stress distribution fluctuating amplitude also increases (Figure 5). However, the fluctuation ratio decreases from 18.8% to 16.4%, indicating that the rope could become tighter under a larger tension.

Additionally, given that the strand strain and the stress at point A, are important references in the failure criteria, they are studied under different $E_s$ conditions ranging between $[150,350]$ GPa as shown in Figures 6 and 7.

Stresses at point A did not significantly change under different elastic modulus ($E_s$) conditions (Figure 6(a)). However, average $N_s$ stresses change significantly as the

![Figure 4. Some strand geometrical parameters with varying tensile force: (a) clockwise strands and (b) counter-clockwise strands.](image-url)
$E_s$ varies (Figure 6(b)). Additionally, strain decreases as $E_s$ increases (Figure 7).

For different braiding applications, strand numbers could be varied (e.g. 8, 12, 16, 20). In Figures 8 and 9 the strand strains as well as the stress at point A for different strand numbers are shown. Here, $E_s = 124$ GPa.

Both, the stress in each strand as well as that in all the strands decreases as the number of strands increases (Figure 8). Additionally, the strain in both directions of the cross section decreases, while the strain of the rope increases as the number of strands increases (Figure 9).

Moreover, in Figures 7 and 9, it can be observed that curves do not start at zero. The cause of this phenomenon probably is that a certain amount of yarn tension is required when braiding. If there is not enough yarn tension, they cannot form a tight braid. Therefore, after braiding the rope, an initial pulling force exists to maintain the fixed braided structure, otherwise the rope state is uncertain and parameters could change, which could induce the simulation calculation could not analyze.

**Conclusions**

In this study, by dividing ropes into basic loops consisting of four different units, a regular-braided rope mathematical model that can be applied to mono-filaments is proposed. Using this model, the different tensile properties of regular braided ropes are investigated, and for numerical analysis, the rope strands are classified as clockwise or counter-clockwise, and their stress distribution, strain, as well as their other geometrical parameters are investigated.

The results show that an increase in the tensile force on the strands results in an increase in the fluctuating...
amplitude of the stress distributions in the strands. However, the fluctuation ratio decreases from 18.8 to 16.4% resulting in a tighter rope. Additionally, a comparison of Figures 4 and 5 shows that the points where the stress is maximum are the outermost points. This observation suggests that for regular braided ropes, an extra safety margin of approximately 20% should be allowed, owing to the uneven distribution of stresses in strands during their application.

Additionally, this proposed mathematical model could also be used to calculate the strain under different braiding parameters. This property is vital when dealing with some fragile strand materials for example, depending on the application, if a higher durability is required, the number of strands could be increased, and if a limited strain coefficient is required, the number of strands could be reduced.
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ORCID iD
Zhang Yujing https://orcid.org/0000-0003-3578-2455

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