Isospin equilibration processes and dynamical correlations: study of the system $^{40}{\text{Cl}} + ^{28}{\text{Si}}$ at 40 MeV/nucleon

M.Papa$^a$* and G.Giuliani $^b$

$^a$ Istituto Nazionale Fisica Nucleare-Sezione di Catania, V. S.Sofia 64 95123 Catania Italy and
$^b$ Dipartimento di Fisica e Astronomia, Università di Catania V. S.Sofia 64 95123 Catania Italy

Abstract

The asymptotic time derivative of the total dipole signal is proposed as an useful observable to investigate on Isospin equilibration phenomenon in multi-fragmentation processes. The study has been developed to describe charge/mass equilibration processes involving the gas and liquid “phases” of the total system formed during the early stage of a collision. General properties of this observable and the links with others isospin dependent phenomena are discussed. In particular, the $^{40}{\text{Cl}} + ^{28}{\text{Si}}$ system at 40 MeV/nucleon is investigated by means of semiclassical microscopic many-body calculations based on the CoMD-II model. The study of the dynamical many-body correlations produced by the model also shows how the proposed observable is rather sensitive to different parameterizations of the isospin dependent interaction.

PACS numbers: 25.70.Pq, 02.70.Ns, 21.30.Fe, 24.10.Cn

* e-mail: papa@ct.infn.it
I. INTRODUCTION

An interesting subject related to Heavy Ions Isospin physics is the process leading to the equilibration of the charge/mass ratio between the main partners of the reaction as well described in Ref. [2]. The so-called "isospin diffusion" phenomenon is the relevant mechanism acting between the reaction partners in binary processes [3, 4, 5, 6, 7]. In particular, in the collision of the 124 and 112 Tin isotopes at 50 MeV/nucleon, evidence of partial equilibrium in the charge/mass ratios of the quasi-projectile and quasi-target has been deduced through the study of the iso-scaling parameters related to the isotopic distributions. In this case dynamical calculations, based on the Boltzmann-Uehling-Uhlenbeck model [8, 9], show that the degree of equilibration depends on the behavior of the symmetry potential $U^\tau$ as a function of the density.

In this work we want to extend the study of the isospin equilibration processes, looking at the system in a global way, by using the following quantity:

$$\vec{V}(t) = \sum_{i=1}^{Z_{tot}} \vec{v}_i.$$  

At a microscopic level, the sum on the index $i$ is performed on all the $Z_{tot}$ protons (bound and free) of the system. $\vec{V}(t)$ corresponds, apart from the elementary charge $e$, to the time derivative of the total dipole of the system. The velocities $\vec{v}_i$ are computed in the center of mass (c.m.) reference frame. We note that, as due to the total momentum conservation, the global effect related to the motion of the neutral particles (bound and free) is also implicitly contained in $\vec{V}(t)$. Therefore we can expect, in a quite general way, a peculiar dependence of the behavior of $\vec{V}(t)$ from the iso-vectorial interaction.

Several studies were based on this dynamical variable to describe pre-equilibrium Giant Dipole Resonance (GDR) $\gamma$-ray emission (see Ref. [10, 11, 12] and references therein). Various reasons suggest us to use the same variable to also describe isospin equilibration in complex processes. The basic guideline starts just from these studies on pre-equilibrium radiative emission in essentially binary processes induced by charge/mass asymmetric partners. The ensemble average of the time derivative of the total dipole, in the initial and final stages of the collision, can be approximated through the so-called Molecular component [13]:

$$\langle \vec{V} \rangle \approx \langle \vec{V}_M \rangle \equiv \frac{1}{2} \langle \mu_{PT} \rangle \langle (\beta_T) - (\beta_P) \rangle \langle \vec{v}_{PT} \rangle$$  \hspace{1cm} (1)

where $\mu_{PT}$ is the reduced mass of the projectile or quasi-projectile (P) and target or quasi-
target \((T)\) of the binary system, \(\beta_{P,T} = \frac{N_{P,T} - Z_{P,T}}{A_{P,T}}\) represents the associated relative neutron excess \((N_{P,T}, Z_{P,T}, A_{P,T}\) are the neutron, proton and mass numbers respectively), \(\vec{v}_{P,T}\) indicates the relative velocity. During the interaction stage of the collision, if the partners have exchanged charge and mass for enough time in such a way to equilibrate, on average, the charge/mass ratio, then the final value of \(\langle \vec{V} \rangle\) will be zero. In this case the yield of the pre-equilibrium dipolar \(\gamma\)-ray emission, which satisfies the well known selection rule on the isospin quantum number \(T\) \((\Delta T = 1, \text{no isospin mixing})\) of the involved intermediate states, reaches the maximum value. For more complex mechanisms, as the ones leading to the substantial stopping of the two partners and to the formation of one hot source, the dipolar signal related to the other particles and fragments can not be neglected. However, also in these cases we can use the same definition of isospin equilibration through the condition \(\langle \vec{V} \rangle = 0\). As we will show in the following, this definition leads to other conditions concerning the production of differential flow between neutral and charged nucleons \([14, 15]\) and/or the relative neutron excesses of the main produced fragments and free particles.

In the following we briefly discuss some simple examples and properties of \(\langle \vec{V} \rangle = 0\) aiming to illustrate the information which is potentially contained in this quantity and the relation with charge/mass equilibration processes:

- (i) By taking into account only the effects associated to the strong interaction, after the pre-equilibrium stage, starting from the time \(t_{pre}\), when a second stage characterized by an average isotropic or symmetric emission of the secondary sources (statistical equilibrium) takes place, the ensemble average of \(\langle \vec{V}(t) \rangle\) satisfies the following relation:

\[
\langle \vec{V}(t_{pre}) \rangle = \langle \vec{V}(t > t_{pre}) \rangle \equiv \langle \vec{V} \rangle \quad [11].
\]

The average value of this dynamical variable at \(t_{pre}\) is invariant with respect to statistical processes and therefore the value of \(\langle \vec{V} \rangle\) is determined only by the complex dynamics which characterizes the early stage of the collision, when fast changes of the average nuclear density are expected. In particular, \(\langle \vec{V} \rangle\) can be expressed as a function of the charge \(Z\), mass \(A\), average multiplicity \(\langle m_{Z,A} \rangle\) and the average value of the mean momentum \(\langle \vec{P}_{Z,A} \rangle\) per unit of mass (expressed in \(fm^{-1}\)) of the detected particles in the generic event:

\[
\langle \vec{V} \rangle = \sum_{Z,A} \frac{Z}{A} \langle m_{Z,A} \rangle \langle \vec{P}_{Z,A} \rangle C_{Z,A}^{Z,A} \quad (2)
\]

\[
C_{Z,A}^{Z,A} = \frac{\langle m_{Z,A} \vec{P}_{Z,A} \rangle}{\langle \vec{P}_{Z,A} \rangle \langle m_{Z,A} \rangle} \quad (3)
\]
$C_{P Z A}^{Z A}$ is the correlation function between the multiplicity and the mean momentum. This
correlation function plays a key role for the invariance property and therefore requires for
an event by event analysis in which many-body correlations can not be neglected. For
symmetry reasons, $\langle \vec{V} \rangle$ lies on the reaction plane. It is directly linked with a weighted
mean of the charge/mass ratio, as Eq.(3) suggests. It also takes into account the average
isospin flow direction through the momenta $\vec{P}_{Z A}$. The long range Coulomb interaction
can produce differences between the value of $\langle \vec{V}(t_{pre}) \rangle$ and the observed asymptotic value.
These changes, however, are rather small at the involved energies (see Sec.III) and can be
evaluated with the necessary precision by taking into account the corrections due to the
Coulomb repulsion up to the asymptotic stage.

(ii) In the general case, we find attractive the following decomposition: $\langle \vec{V} \rangle = \langle \vec{V}_G \rangle +
\langle \vec{V}_L \rangle + \langle \vec{V}_{GL} \rangle$ where $\langle \vec{V}_G \rangle$ and $\langle \vec{V}_L \rangle$ are the average dipolar signals associated to the gas
"phase" (light charged particles) and to the "liquid" part [16] corresponding to the motion of
the produced heavy fragments. The signal $\langle \vec{V}_{GL} \rangle$ is instead associated to the relative motion
of the two "phases". By supposing, for simplicity, the gas "phase" formed by neutrons and
protons, $\langle \vec{V} \rangle$ can be further decomposed as:

$$\langle \vec{V} \rangle = \langle \frac{A_G (1 - \beta_G^2)}{4} \vec{v}_{PN} \rangle + \langle \frac{\mu_{GL} (\beta_L - \beta_G)}{2} \vec{v}_{cm, LG} \rangle + \langle \vec{V}_{r,L} \rangle$$

(4)

In the above expression the first term represents the contribution related to the proton-neutron relative motion of the gas "phase" composed of $A_G$ nucleons, expressed through the
relative velocity $\vec{v}_{PN}$; the second term concerns the relative velocity $\vec{v}_{cm, LG}$ between the
centers of mass of the "liquid" complex and the "gas", $\mu_{GL}$ is the reduced mass associate
to the two sub-systems; the last term represents the contribution produced by the relative
motion of the fragments. A similar expression can be obtained including other light particles
in the gas "phase" as, for example, light Intermediate Mass Fragments (IMF). From this
decomposition we can see how the isospin equilibration condition ($\langle \vec{V} \rangle = 0$), for the total
system, requires a delicate balance which depends on the average neutron excess of the
produced "liquid drops" $\langle \beta_L \rangle$, on the one associated to the gas "phase" $\langle \beta_G \rangle$, and on the
relative velocities between the different parts. To enlighten the role played by some of the
terms reported in Eq.(4), we can discuss the idealized decay of a charge/mass asymmetric
source through neutrons and protons emission (or the case in which the liquid drops are
produced through a statistical mechanism $\langle \vec{V}_{r,L} \rangle = 0$). This example can also schematically
describe the case of the complete stopping in heavy ion collisions. Moreover, for simplicity, we can consider uncorrelated fluctuations between the velocities, masses and neutron excesses. In absence of pre-equilibrium emission or for identical colliding nuclei the second term of Eq.(4) is zero (\(\langle \vec{v}_{cm, LG} \rangle = 0\)), and the isospin equilibration requires a neutron or proton gas ”phase” or absence of relative neutron-proton motion. For non-identical colliding nuclei, if pre-equilibrium emission exists, then \(\langle \vec{v}_{cm, LG} \rangle \neq 0\). In this case, if \(\langle \beta_G \rangle \neq \langle \beta_L \rangle\), due, for example, to the isospin ”distillation” phenomenon, the first term has to be necessarily different from zero and it will contribute to the neutron-proton differential flow (see also Sect.III). We remark that both the isospin ”distillation” phenomenon \([17]\) and the production of neutral-charged particles differential flow \([15]\) depend in a peculiar way from the iso-vectorial forces.

Therefore, according to our description, the understanding of the isospin equilibration process for the total system requires the gas ”phase” contribution to be taken into account. This term can be regarded as a kind of ”dissipation” with respect to the system formed by the liquid part. In this work, as an example, we will discuss the results obtained through the Constrained Molecular Dynamics-II approach (CoMD-II) \([18, 19]\) applied to the charge/mass asymmetric \(^{40}Cl + ^{28}Si\) system at 40 MeV/nucleon. The study is performed by using different options for the iso-vectorial potential term.

Before to show the results of our calculations, in the following section we briefly recall the way in which the isospin dependence of the nuclear interaction is introduced in the CoMD-II model.

II. SYMMETRY INTERACTION AND CORRELATIONS

According to the results shown in Ref.\([18, 20]\), starting from a Skyrme type two-body microscopic interaction, the two-body effective potential in CoMD-II model can be expressed through the nucleon-nucleon overlap integral \(\rho^{ij} = \int \int d^3r_j d^3r_i \delta(\vec{r}_i - \vec{r}_j) \hat{\rho} \hat{\rho}\). \(\vec{r}_i, \vec{r}_j\) represent the nucleon spatial coordinates. \(\hat{\rho}\) is the Gaussian distribution in the coordinate space related to the generic nucleonic wave-packet. The microscopic iso-vectorial interaction for the Stiff2 option is given by the following expression: \(V^\tau = \frac{a_0}{2\rho_0} \sum_{i \neq j=1}^{A} (2\delta_{\tau_i, \tau_j} - 1) \delta(\vec{r}_i - \vec{r}_j)\), \(A\) is the total mass number, \(\tau_i\) represent the generic third nucleonic isospin component and \(\rho_0\) is the one-body density at the saturation point. The coefficient \(a_0 = 72 MeV\) deter-
mines the strength of the iso-vectorial interaction at the saturation density. As shown in Ref. [20] the structure of $V^\tau$ can be obtained by taking into account that the two-body nuclear forces, in $S$ wave, around the ground state (g.s.) density, are less attractive in isospin triplet states ($T = 1$) with respect the singlet states ($T = 0$). From the above expressions it results that the associated effective interaction $U^\tau$ (after the convolution with the nucleonic wave-packets) can be expressed as a function of the average overlap integrals per couple of neutrons ($nn$), $\bar{\rho}^{nn}$, protons ($pp$) $\bar{\rho}^{pp}$, and neutron-proton ($np$) $\bar{\rho}^{np}$. As discussed in Ref. [20] for small asymmetries we can assume $\bar{\rho}^{nn} \approx \bar{\rho}^{pp} \approx \bar{\rho}^{np}$. To characterize the differences associated to the nucleon-nucleon dynamics, at a two-body level, we can introduce the correlation coefficient $\alpha$ in such a way $\bar{\rho}^{np} = (1 + \alpha)\bar{\rho}$. $\alpha$ depends on both $\bar{\rho}$ and the asymmetry parameter $\beta$. Results on nuclear matter simulations [20] show that the behavior of $\alpha$ as a function of $\beta$ can be approximated for moderate asymmetries by a parabolic law.

In this case, eqs.(6,7) of Ref.[20] give the following expression for the effective isovectorial potential in the Non Local (N.L.) approximation $U^\tau_{N.L.}$

\begin{equation}
U^\tau_{N.L.} \approx \frac{\alpha_0}{2\rho_0} \hat{A}^2 F'(s)[(1 + \frac{1}{2}\alpha_0 - \alpha')\beta^2 - \frac{1}{2}\alpha_0]
\end{equation}

\begin{equation}
\alpha' = \frac{1}{4} \left. \frac{\partial^2 \alpha}{\partial \beta^2} \right|_{\beta=0}
\end{equation}

$\beta^4$ terms are neglected in the previous expression. $\alpha_0 \equiv \alpha(\hat{\rho}, \beta = 0)$ represents the correlation coefficient related to the difference in the dynamics of the $np$ couples with respect to the $nn$ and $pp$ ones for symmetric nuclear matter.

It depends on the average overlap integral per couple of nucleons $\hat{\rho}$ which reflects the degree of compression. $s = \frac{2}{3A} \sum_{i\neq j} \hat{\rho}^{i,j}$ is associated to the total overlap integral per nucleon. $F'$ is a form factor which modulates the changes of the iso-vectorial interaction as a function of the average overlap integral $s$. For the Stiff1 option we use $F' = \frac{2s}{s_g+s}$, for the Stiff2 case $F' = 1$ and for the Soft option $F' = (\frac{s_g}{s})^{1/2}$. These form factors correspond to the following values of the $K_{sym}$ parameter as defined in Ref. [14, 15]: 94 MeV for the Stiff1 option, -27 MeV for the Stiff2 option and -88 MeV for the Soft one. This choice corresponds also to an $S_0$ value [14] of about 40 MeV. From Eq.(5) we note that in our approach the iso-vectorial forces generates, beyond the $\beta$ dependent potential, also another iso-vectorial density dependent term, independent on $\beta$ and proportional to the degree of correlation $\alpha$ evaluated for symmetric systems. As discussed in [20, 21], at small asymmetries, this term
determines the high sensitivity of the experimental observables to the different functional forms of $F'$. The finite value of $\alpha$, which is of the order of 15\%, apart from the Pauli principle and the Coulomb interaction, is strongly affected by the iso-vectorial interaction itself. In fact the neutron-proton couples, at variance with the other ones, suffer the more attractive singlet interaction.

For clarity, we have to note that the $np$ correlations generated by similar kind of forces represent a rather interesting subject for the description of the extra-bind energies in $N \simeq Z$ nuclei. These studies are performed with static sophisticated quantal approaches. The $np$ pairing correlations, for example, play a main role to microscopically describe the Wigner energy in symmetric nuclei \cite{22, 23, 24}. However, apart from effects induced by the Pauli principle in phase-space (as treated in our approach) and from the usage of an iso-vectorial potential which reflects a two-body interaction dependent on isospin quantum numbers, the description and the propagation of the many-body correlations in our model calculations are essentially classical. The $np$ correlations discussed in the present work can represent a dynamical classical analogous which includes Pauli principle and isospin effects of more complex quantal effects. Therefore the effects here discussed can not be compared with the ones able to go beyond this classical analogous and having therefore a pure quantal nature.

In fact our approach can not describe effects that, for example, could specifically arise from details of the single particles wave functions or also from details concerning the propagation and symmetries of the Slater determinant structure of the many-body wave functions, which instead characterizes the Fermionic molecular dynamics approaches \cite{25, 26}.

The case corresponding to vanishing values of the correlation $\alpha$ represents, in our framework, the so-called Iso-vectorial Mean Field Approximation (I.M.F.A.). In this case the average overlap integrals per couple of nucleons related to neutron-neutron, proton-proton and neutron-proton interactions have the same values ($\bar{\rho}^{nn} = \bar{\rho}^{pp} = \bar{\rho}^{np}$) and the iso-vectorial interactions generate only the usual symmetry potential term which depends on $\beta^2$. 
III. CALCULATION RESULTS

A. An example: the $^{40}Cl + ^{28}Si$ system at 40 MeV/nucleon

Now we discuss, as an example, the results concerning the isospin equilibration process for the $^{40}Cl + ^{28}Si$ system at 40 MeV/nucleon. In Fig. 1 we show the average total dipolar signals evaluated through CoMD-II calculations along the $\hat{z}$ beam direction $\langle V^z \rangle$ and along the impact parameter direction $\hat{x}$, $\langle V^x \rangle$, respectively. The reference frame is the c.m. one. The impact parameter $b$ is equal to 3 fm, in panels (a) and (c) and 1.5 fm in panels (b) and (d). In Fig. 1(a) and Fig. 1(c) the average dipolar signals are shown for the first 140 fm/c. Different lines refer to different iso-vectorial potentials, according to Ref. [18]. In the first 150 fm/c in all the cases wide oscillations exist. They are responsible for the pre-equilibrium $\gamma$-ray emission [10, 11]. The damped oscillations converge towards smaller and almost constant values. This can be seen in Fig. 1(b) and Fig. 1(d) in which the dynamical evolution is followed from 110 fm/c up to 300 fm/c. The inclusion of corrections due to Coulomb interaction at longer time is of the order of some percent. The time interval in which the almost stationary behavior is reached is related to the lifetime of the coherent dipolar collective mode and it is strictly linked with the average time for the formation of the main fragments and pre-equilibrium emission. Fig. 2 shows the asymptotic values of the $\hat{z}$ and $\hat{x}$ components of the average dipolar signal, evaluated for different reduced impact parameters $b_r = \frac{b}{b_{\text{max}}}$ ($b_{\text{max}} \simeq 7.5$ fm) and different interaction options. The figure enlightens the sensitivity of the $\langle -\rightarrow V \rangle$ observable to the iso-vectorial interaction.

In the following we would like to discuss the behaviors of the different components of the total dipolar signal along the $\hat{z}$ direction as a function of the impact parameter according to the scheme depicted in the previous section and related to Eqs. (1,4). In Fig. 3(a) the ratio $R_{GL} = \frac{\langle V^z_G \rangle}{\langle V^z_L \rangle}$ between the $\hat{z}$ asymptotic components of the dipolar signal associated to the light particles $\langle V^z_G \rangle$ and the one corresponding to the two biggest fragments $\langle V^z_L \rangle$ is plotted as a function of the reduced impact parameter $b_r$ for the Stiff2 option. The other options produce similar results. For peripheral collisions we see the predominance of the signal carried by the two biggest fragments ($|R_{GL}| < 1$). In these cases the liquid part is dominated by the so-called ”molecular” component $V_M$ (see Eq.(1)). It has a negative sign because the quasi-projectile has the largest neutron excess with respect to the quasi-target. The ”gas”
component produces a small negative value which indicates, on average, a dominance of the pre-equilibrium emission of nucleons from the target light partner. For smaller impact parameters the molecular component reduces its value because of the increasing stopping (reduction of $v_{PT}$ in Eq.(1)) and as a consequence of the charge/mass equilibration between the two partners. The asymptotic behavior of $R_{GL}$ between $b_r = 0.6 - 0.75$ is due to the change of sign of the ”liquid” component which evolves in a rather continuous way passing through zero. The ”liquid” part has a positive sign for more central collisions, because now the process can be roughly described as one charged source emitting pre-equilibrium charged particles mostly on the target side. The average recoil of the source in the projectile direction produce the positive value of the dipolar signal. The absolute value of $R_{GL}$ increases for more central collisions due to the increasing contribution of the pre-equilibrium particles.

As discussed in the introductory section, when the collision partners are not identical nuclei the isospin equilibration process produces a neutron-proton differential flow contribution if the neutron and proton ”gases” have different c.m. velocities. These pre-conditions are verified for the studied collision.

We briefly recall that the neutron-proton differential flow is an observable that has been proposed to investigate symmetry potential effects \[14, 15\]. It is expressed as the average difference between the transverse momenta of the emitted free neutrons and protons having rapidity $y'$:

$$F_{np} = \frac{1}{A(y')} \sum_{i=1}^{A(y')} w_i p_{ix}(y')$$

with $w_i = 1$ for neutrons and $w_i = -1$ for protons. For $b = 3$ fm and for the Stiff1 and Soft options, in Fig. 3(b) we show the neutron-proton differential flow $F_{np}$, expressed in $c$ units, as a function of the particles rapidity, $y$, normalized to the projectile one $y'_{beam}$. The rapidity values are evaluated in the c.m. of the total system reference frame. The dashed vertical lines indicate the projectile and target reduced rapidity.

From the figure we can understand that, by averaging on the rapidity, the neutron-proton transversal velocity has a negative value. This reflects the average ”bending” of the relative motion between the c.m. of the emitted neutrons and protons, through the half-plane opposite to the impact parameter direction. In Fig. 3(c) we display the average number of nucleons as a function of the rapidity for the two options. In both the cases the collision produces rather similar ”stopping”. The results shown in Fig. 3(b) can be compared with
the calculations displayed in Fig. 3(d) obtained by subtracting, event by event, the c.m. relative neutron-proton motion related to the "gas" phase. As can be seen, similarly to the case of identical nuclei, this correction restores (within the errors associated to the statistics of simulations) the almost specular behavior of $F_{np}$ with respect to the rapidity axes. The correction acts also along the beam direction. The largest absolute values of $F_{np}$ around $|y| \simeq 1$ for the Soft case is mostly due to the largest neutron excess of the "gas" phase with respect to the one obtained for the Stiff1 option. Finally, in Fig. 3(e) we show the vectors associated to the average proton-neutron relative motion $\vec{v}_{pn}$ for the two options. These quantities determine the correction on the differential flow due to the isospin equilibration process and are associated to the "gas" component of the total dipolar signal (see first term of Eq.(4)).

B. Sensitivity to the different options of the iso-vectorial interaction

In the following we want to discuss in some detail the sensitivity of the dipolar signal to different options concerning the iso-vectorial interaction including also the role played by the finite value of the correlation coefficient $\alpha$.

According to the nature of the discussed quantity, we can expect a particular sensitivity to the iso-vectorial forces because it contains implicitly information on the global relative motion of the charge particles with respect the neutral ones. In Fig. 4(a) we show as a function of the reduced impact parameter $b_r$ the asymptotic values of $\langle V^x \rangle$ and $\langle V^z \rangle$. The arrow indicates the direction of increasing impact parameters corresponding to the marked points with a step $\Delta b_r = 0.1$. Different symbols represent different options. In the region of the bending of the lines, around $b = 3$ fm, we observe the greater sensitivity to the iso-vectorial interactions. This result is particularly evident by studying the ratios $R = \langle V^x \rangle/\langle V^z \rangle$. In Fig. 4(b) we in fact show the relative change $r = \frac{\Delta R}{R}$ between couples of different options. We can see that for $b_r$ less than about 0.6 large changes are predicted according to the different shapes of the form factor $F'$. This impact parameter region produces a substantial stopping of the incident nuclei and is clearly dominated by large overlap between projectile and target which gives rise to processes changing from incomplete fusion reactions to IMF production. The region of intermediate impact parameters show the higher sensitivity when the mechanism evolves with respect to the essentially binary
processes which take place at the higher impact parameters. For $b_r$ greater than 0.6, in fact, the sensitivity is strongly reduced.

In particular, according to what previously observed (see for example point (ii) of Sec. I), we have evaluated the partial contributions $\langle V_x^L \rangle$ and $\langle V_z^L \rangle$ related to the two main fragments. As an example, for $b = 3$ fm and for the Stiff2 option, the "liquid" asymptotic values are $\langle V_x^L \rangle = -0.120c$ and $\langle V_z^L \rangle = 0.162c$ while the total contributions are $\langle V^x \rangle = 0.044c$ and $\langle V^z \rangle = -0.027c$. Therefore, it results that the contributions carried by the two main fragments only partially contribute to the isospin equilibration process. The remaining part ("gas"), which in this case we have associated to particles and to the IMF, generates a term with opposite sign and similar strength for both directions. It contributes in a decisive way to the global equilibration process. For the same impact parameter, in Fig. 4(b) we show with the star symbol the sensitivity parameter $r$ evaluated by changing the option from Stiff1 to Stiff2 and by only taking into account the contributions of the two main fragments. As we can see, the partial contribution shows a rather reduced sensitivity to the different options as compared to the case obtained by using the global information on the system.

Finally, in the following we show the role of the correlation coefficient $\alpha$, introduced in Sec. II, into determine the sensitivity of the investigated observable to the density dependence of the iso-vectorial interaction. For this aim, in Fig. 4(c), we compare, for different impact parameters, the values of $r$ obtained for the Stiff2-Stiff1 options with the ones obtained in the I.M.F.A. case. Fig. 4(c) clearly shows that in the I.M.F.A. case, at the investigated energies, the sensitivity of the isospin equilibration process to the behavior of the symmetry interaction is rather reduced. The I.M.F.A. also strongly affects the values of $\langle V \rangle$. In particular, independently from the used options, it produces for $b_r=0.4$, values of $|\langle V^z \rangle|$ about four times larger than the ones obtained with full CoMD-II calculations indicating a reduced capacity to obtain isospin equilibration along the $\hat{z}$ direction.

IV. SUMMARY AND CONCLUSIVE REMARKS

In summary, in this work the isospin equilibration process has been investigated by studying the ensemble average of the time derivative of the total dipole $\langle \bar{V} \rangle$ evaluated through CoMD-II calculations. Some general properties of this quantity have been discussed. In particular, it allows to generalize the definition of isospin equilibration also in complex reactions
evolving through multi-fragmentation processes. As an example, calculations performed for
the asymmetric charge/mass system $^{40}\text{Cl} + ^{28}\text{Si}$ at 40 MeV/nucleon show that the asymptotic
values of $\langle V \rangle$ for these processes are quite sensitive to different options for the iso-vectorial
potential; moreover, in central and mid-peripheral collisions, the dipolar contribution asso-
ciated to the pre-equilibrium emission of charged particles is relevant to determine the value
of $\langle V \rangle$ and the related sensitivity to different density dependent form factors. Semiclassical
CoMD-II calculations performed in the so-called I.M.F.A. scheme also highlights the promi-
nent role which could be played by the two-body neutron-proton correlations in the study
of the dynamics leading to the isospin equilibration processes.

[1] W.U.Schröder and J.Töke, in Nonequilibrium Physics at Short Times, edited by K.Morawetz,
Springer-Verlag, Berlin, Heidelberg, New York, 2004, p.417.
[2] L. Shi and P.Danielewicz, Phys. Rev. C. 68 064604 (2003).
[3] M.B.Tsang et al, Phys. Rev. Lett. Lett. 92, 062701 (2004).
[4] Betty Tsang and Lijun Shi, Nucl. Phys. A738 115 (2004).
[5] Andrew W.Steiner and Bao-An Li, Phys. Rev. C. 72 041601(R) (2005).
[6] V.Baran, M.Colonna, M.Di Toro, M.Zielinska-Pfabé and H.H.Wolter, Phys. Rev. C. 72 064620
(2005).
[7] E.Galichet et al, arXiv:0812.2786.
[8] G.Bertsch and S.Das Gupta, Phys.Rep. 160, 189 (1988).
[9] M.B.Tsang et al, Eur. Phys. J. A30, 129-139 (2006).
[10] G.Giuliani and M.Papa, Phys. Rev. C 73, 031601(R) (2006).
[11] M.Papa et al, Phys. Rev. C 72, 064608 (2005) and references there in.
[12] F.Amorini et al, Phys.Rev. C69 014608 (2004).
[13] M.Papa et al, Eur.Phys.J. A4 69 (1999).
[14] Bao-an Li, Phys. Rev. Lett. Lett. 85, 20 (2000).
[15] Bao-an Li, Phys. Rep. 464, 113-281 (2008).
[16] M.F. Rivet et al, Nucl. Phys. A 749, 73c (2005).
[17] M. Colonna V. Baran M. Di Toro and H. Wolter, Phys. Rev. C 78 064618 (2008).
[18] M.Papa, T.Maruyama and A.Bonasera, Phys. Rev. C 64, 024612 (2001), and references
therein.

[19] M. Papa, G. Giuliani and A. Bonasera, J Comput. Phys. 208, 403 (2005).

[20] M. Papa and G. Giuliani, Eur. Phys. J. 39, 117 (2009).

[21] Amorini, F. et al., Phys. Rev. Lett. 102, 112701 (2009).

[22] D. S. Brenner, C. Wesselborg, R. F. Casten, D. D. Warner and J. Y. Zhang, Phys. Lett. B 243, 1 (1990).

[23] W. Satula, D. J. Dean, G. Gary, S. Mizutori, W. Nazarewicz, Phys. Lett. B 407, 103 (1997).

[24] W. A. Friedman and G. F. Bertsch, Phys. Rev. C 76, 057301 (2007).

[25] H. Feldmeier, Nucl. Phys. A 515, 147 (1990).

[26] A. Ono, H. Horiuchi, Toshiki Maruyama, A. Ohnishi, Phys. Rev. Lett. 68, 2898 (1992).
Fig. 1 - Average dipolar signals $\langle \tilde{V} \rangle$ for $b=3$ fm along the $\hat{z}$ direction are plotted as a function of time in the intervals 0-140 fm/c (panel (a)) and 110-300 fm/c (panel (b)). Different lines indicate different options for the iso-vectorial interaction (see the text). Panels (c) and (d) display the average dipolar signals along the $\hat{x}$ direction for $b=1.5$ fm and in the same time intervals like panels (a) and (b) respectively.

Fig. 2 - Asymptotic values $\langle V^x \rangle$ and $\langle V^z \rangle$ evaluated for different values of the $b_r$ parameter (see the text). Different symbols indicate different options for the iso-vectorial interaction.

Fig. 3 - In panel (a) the $R_{GL} = \frac{\langle V_z^G \rangle}{\langle V_z^L \rangle}$ ratio (see the text) is plotted as a function of the reduced impact parameter for the Stiff2 option. Panels (b) and (c) display the neutron-proton differential flow $F_{np}$ and the number of free nucleons $A_G$ as a function of the c.m. rapidity respectively. In the the panel (d), $F_{np}$ is plotted after the correction for the relative c.m. velocity of the neutron and proton “gases” $v_{PN}$. The vertical dotted lines represent the target and projectile rapidity. Different symbols refers to different options. In the panel (e) the average $\langle v_{PN} \rangle$ contributing to the isospin equilibration process is represented for the Stiff1 and Soft options.

Fig. 4 - (a) $\langle V^x \rangle$ is plotted as a function of the corresponding $\langle V^z \rangle$ value for different reduced impact parameter $b_r$ values ($b_{max} \simeq 7.5$ fm and $\Delta b_r = 0.1$) and different options (different symbols). The arrow indicates the direction of increasing impact parameters. -(b) relative changes $r$ for the ratio $R$ (see the text) evaluated for different couples of options as a function of $b_r$. -(c) Values of $r$ evaluated for the Stiff1-Stiff2 options as a function of $b_r$ are plotted in the case of full CoMD-II calculations and in the case of I.M.F.A. approximation (see the text). The lines which join the points are meant only to guide the eye through the shown trend.
