Oscillatory Convection of a Colloidal Suspension in a Horizontal Cell

I. N. Cherepanov\(^a\)\(^*,\) and B. L. Smorodin\(^a\)\(^**,\)

\(^a\)Perm State National Research University, Perm, 614990 Russia

\(^*\)e-mail: che-email@yandex.ru

\(^**\)e-mail: bsmorodin@yandex.ru

Received June 9, 2020; revised July 13, 2020; accepted July 15, 2020

Abstract—Numerical simulation of convection by the finite difference method has been performed for the inhomogeneously heated colloidal suspension (Hyflon MFA) filling a horizontal cell with a finite length. The cell has rigid and impermeable boundaries and is heated from below. A linear temperature distribution is maintained at the side walls. Owing to negative Soret coupling and gravitational sedimentation, a heavy impurity comprised of nanoparticles is accumulated at the hot lower boundary, while convection transfers it to the inside of the cell. The experimentally observed transient flows and stable oscillatory nonlinear convection regimes are analyzed. If the initial distribution of nanoparticles is uniform, then steady convection occurs in the cell. With accumulation of the concentration inhomogeneity, oscillatory perturbations begin to grow. A modulated traveling wave can appear in the layer. Stable oscillatory modes exist when the Rayleigh number exceeds critical value \(R_S\), which depends, as calculations have shown, on the cell length. The spatial structure of the concentration field and the time evolution of the convective characteristics of the colloid suspension are determined. The numerical study results agree well with published experimental data. The behavior of the colloidal suspension in the modes of modulated traveling waves, including localized traveling waves and waves that change their direction, and transient flows near convection threshold \(R_S\) is simulated and clarified. Based on the results of analyzing the behavior of the vortex world lines and the impurity concentration field, the formation of defects in the form of vortex coalescence is established. It is found that the defect is formed as a result of merging two nearest vortices with the same direction of rotation.

Keywords: convection, colloidal suspension, traveling waves, numerical simulation

DOI: 10.1134/S0021894421070063

1. INTRODUCTION

Impurity transport mechanisms, such as diffusion and thermal diffusion, operate in molecular mixtures [1–3]. In colloidal suspensions, the size of particles is a few orders of magnitude larger than the size of molecules; therefore, sedimentation in a gravitational field [4, 5] and magnetophoresis [6] can be added to the modes of transport. All of them lead to the redistribution of the impurity and the formation (under certain conditions) of an oscillatory convective instability. As a result of the evolution of oscillatory perturbation in binary mixtures, a large number of nonlinear convection regimes are formed, such as: extended and localized states; steady-state and wave structures, including standing and traveling waves [1, 7–15], which are actively studied experimentally [1, 8–10], theoretically [7, 11, 12], and on the basis of numerical simulation [13–15]. Diffusion, thermal diffusion, convective, and gravitational flows affect the structure of the concentration field, change the buoyancy forces, and have an effect on the evolution of nonlinear flows [7, 12, 13]. In colloidal suspensions, the diffusion coefficients are substantially—by two to three orders of magnitude—lower than those in molecular binary mixtures. As a result, prolonged transient processes between convective states can take place.

This study analyzes the transitions between the mechanical equilibrium regime and the regime of nonlinear oscillations of a colloidal suspension, which have been previously studied experimentally for the case of a closed cell [1]. By direct numerical simulation, the spatial structure and time evolution of traveling waves are studied. A convective state is found, in which two waves traveling in opposite directions are present in the cell.
2. PROBLEM STATEMENT

We consider a horizontal cell filled with a colloidal solution (colloidal suspension) with height $h$ and length $l_x$, which is heated from below and located in gravity field $g = -gn_z$ ($n_z$ is a unit vector directed upwards). Further, a colloidal suspension of nanoparticles of the Hyflon MFA fluoropolymer in water will be used as an example [1]. The radius of the nanoparticles is 22 nm, their density is $\rho_s = 2.12 \, \text{g/cm}^3$, and the density of the carrier liquid is $\rho_f = 1 \, \text{g/cm}^3$. The condition for adhesion of the mixture is satisfied on the cell walls, which possess perfect thermal conductivity and are impermeable to impurity, and a linear temperature profile is maintained at the lateral walls. The $x$ axis of the Cartesian coordinate system is directed along the horizontal boundaries of the layer, and the $z$ axis is perpendicular to them. Given that temperature gradient is applied to the horizontal boundaries of the layer in the mixture, there is a transport of nanoparticles associated with thermal diffusion (Soret’s effect) [2]. In the case of an aqueous suspension of Hyflon MFA, the nanoparticles are coated with a double electric layer, due to which the impurity exhibits anomalous thermal diffusion, i.e., the heavy component tends to the hot region.

The equation of state of a colloidal suspension with a slight deviation of temperature $T$ and mass concentration $C$ of a heavy impurity from certain average values of $T_0$ and $C_0$ can be approximated by linear dependence

$$\rho = \rho_0 (1 - \alpha (T - T_0) + \beta (C - C_0)), \quad (1)$$

where $\alpha$ and $\beta$ are positive coefficients of thermal and concentration expansion, respectively:

$$\alpha = -\frac{1}{\rho_0} \left( \frac{\partial \rho}{\partial T} \right)_0, \quad \beta = \frac{1}{\rho_0} \left( \frac{\partial \rho}{\partial C} \right)_0. \quad (2)$$

We can write down the convection equations in dimensionless form using the following scales: $h$ for the length, $\sqrt{\chi}$ for the time ($\chi$ is the thermal diffusivity of a colloidal suspension), $\chi/h$ for the velocity, $\Theta$ for the temperature, $\alpha \Theta / \beta$ for the concentration, and $\rho \chi^2 / h^2$ for the pressure. Let us introduce the stream function related to fluid velocity $v$ by the following relations:

$$v_x = \frac{\partial \Psi}{\partial z}, \quad v_z = -\frac{\partial \Psi}{\partial x}, \quad \Phi = (\text{curl} \, v), \quad (3)$$

$$\frac{\partial \Phi}{\partial t} + \left[ \frac{\partial \Psi}{\partial z} \frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial x} \frac{\partial \Phi}{\partial z} \right] = \text{Pr} \left[ \Delta \Phi + R \left( \frac{\partial T}{\partial x} - \frac{\partial C}{\partial x} \right) \right], \quad \Phi = \Delta \Psi, \quad (4)$$

$$\frac{\partial T}{\partial t} + \left[ \frac{\partial \Psi}{\partial z} \frac{\partial T}{\partial x} - \frac{\partial \Psi}{\partial x} \frac{\partial T}{\partial z} \right] = \Delta T, \quad (5)$$

$$\frac{\partial C}{\partial t} + \left[ \frac{\partial \Psi}{\partial z} \frac{\partial C}{\partial x} - \frac{\partial \Psi}{\partial x} \frac{\partial C}{\partial z} \right] = \text{Le} \left[ \Delta (C + \psi T) + \frac{1}{l} \frac{\partial C}{\partial z} \right]. \quad (6)$$

Here, $R = \frac{\alpha gh^3 \Theta}{\psi S \beta}$, $\psi = \frac{S \beta}{\alpha \chi}$, $\text{Le} = \frac{D}{\chi}$, $\text{Pr} = \frac{\nu}{\chi}$ are the thermal Rayleigh number, the separation ratio of the mixture, the Lewis number, and the Prandtl number, respectively; furthermore, $\nu$ is the kinematic viscosity, $D$ is the diffusion coefficient of nanoparticles, and $S_r$ is Soret’s thermal diffusion coefficient. The following designations are also introduced: $l = l_{\text{sed}} / h$ is the dimensionless sedimentation length, where $l_{\text{sed}} = \frac{kT_0}{(\rho_s - \rho_f)Vg}$ is the characteristic scale on which the concentration decreases by a factor of $e$ with the barometric distribution [4] ($k$ is the Boltzmann constant, and $V$ is the volume of the nanoparticle). Aspect ratio $L = l_{x} / h$, i.e., the ratio of the cell length to its height, is another parameter of the problem.

We can write down the boundary conditions for problem (4)–(6) as follows:

$$0 \leq x \leq L: \quad \Psi (x,0) = \Psi (x,1) = 0, \quad \Psi_z (x,0) = \Psi_z (x,1) = 0, \quad T(x,0) = 0.5, \quad T(x,1) = -0.5 \quad (7a)$$
on the solid and ideally heat-conducting horizontal cell boundaries;

$$0 \leq z \leq 1: \quad \Psi (0,z) = \Psi (L,z) = 0; \quad \Psi_z (0,z) = \Psi_z (L,z) = 0 \quad (7b)$$
on the vertical cell boundaries.
On the horizontal cell boundaries, their impermeability requires setting the following zero normal component of the substance flow:

$$z = 0, 1: \frac{\partial C}{\partial z} + \psi \frac{\partial T}{\partial z} + \frac{1}{l} C = 0. \quad (8a)$$

The impermeability of the vertical cell boundaries requires the following condition:

$$x = 0, L: \frac{\partial C}{\partial x} + \psi \frac{\partial T}{\partial x} = 0. \quad (8b)$$

We consider the case of large anomalous thermal diffusion (for a Hyflon MFA suspension, the separation ratio of the mixture is 30 or more times higher than the values of the same parameter for molecular mixtures of the alcohol–water type [7]) when the thermal diffusion and sedimentation flows are codirected. With the gravitational sedimentation small in comparison with the thermal diffusion sedimentation, its influence on the flow behavior is insignificant, so it can be neglected. In the major part of the calculations, we further exclude the terms proportional to $1/l$ and responsible for the gravitational sedimentation of nanoparticles in Eq. (6) and boundary condition (8a). We will discuss below how this affects the result.

The experiments described in [1] showed that both stable flows and long-lived transient flows can arise and exist with an initial uniform distribution of nanoparticles in colloidal suspensions. They are caused by emerging substantial concentration inhomogeneities that are formed during diffusion time $\tau_D = h^2/(2/\chi) [5]$. For colloidal liquids, the ratio of the characteristic thermal and diffusion times is four orders of magnitude, i.e., $\tau_D/\tau_D = \chi/D = 10^4$. This can explain that the flows of a colloidal suspension are flows of a homogeneous liquid (deviations of the concentration of the mixture from the mean value have no effect on the characteristics of the convective flow) with times much shorter than the diffusion time. However, the concentration inhomogeneity formed in an inhomogeneous thermal field due to large values of the separation ratio changes the distribution of buoyancy forces with further evolution of the flow and, as a result, leads to the appearance of wave regimes.

To obtain information about finite-amplitude flows and to find the regions of existence of convective solutions, it is necessary to solve nonlinear problem (4)–(6) with boundary conditions (7) and (8). In the numerical simulation, the values of the dimensionless parameters of the problem were chosen so as to correspond to the real colloidal Hyflon MFA suspension [1]: $Le = 8.84 \times 10^{-5}$, $\psi = -7.5$, and $Pr = 6.0$. The Prandtl number equal to $Pr = 10.0$, which characterizes the colloidal suspension at a lower mean temperature, was also used in the calculations.

Upon the transition to discrete analogs of the equations of motion and heat conduction, the spatial derivatives were approximated by central differences. Concentration equation (6) must satisfy the law of conservation of mass. To achieve this, the equation was written in a conservative form and approximated using the control volume method [16]. For discrete representation of the convective term, central differences were used. The Poisson equation was solved by the method of expansion into Fourier series [16] with the fast Fourier transform [17]. A detailed description of the numerical algorithm is given in [14]. In the main calculations, a cell with an aspect ratio of $L = 4.14$ and a grid with $342 \times 82$ nodes was taken (Fig. 1). Verification calculations with less detailed ($256 \times 64$) and more detailed ($512 \times 128$) grids were also performed, which confirmed the results obtained with the $342 \times 82$ grid. The maximum values of the stream function fixed at a local point with coordinates $(x = 1/4, z = 1/2)$ were $\Psi = 2.4531$ with the main grid, $\Psi_{loc} = 2.3361$ (difference 4.7%) with a less detailed grid, and $\Psi_{loc} = 2.4532$ (difference 0.004%) with a more detailed grid. In addition, calculations were performed for other aspect ratios, namely: $L = 4, 5, 6, 7, 8$, and 8.14. In the calculations, the spatial step between the grid nodes was retained, i.e., the number of horizontal nodes in the computational domain increased with an increase in the cell length.

3. STABLE AND TRANSIENT CONVECTIVE FLOWS

The results of numerical simulation are discussed below.

The state of a given convective system and the distribution of the concentration of impurities depend on the initial conditions. If a steady-state concentration distribution is specified against the background of a resting colloidal suspension, then all perturbations attenuate in an oscillatory manner, as predicted by the linear theory for the case with $\psi < -1$ [7]. With a uniform initial distribution of the impurity concentration, stable convective flows are possible if the Rayleigh number exceeds a critical value of $R_0 = 1708$. 

JOURNAL OF APPLIED MECHANICS AND TECHNICAL PHYSICS Vol. 62 No. 7 2021
At the initial stage, the convection is a classical flow of a homogeneous liquid, the intensity of which depends only on the temperature gradient. Moreover, the rate at which the flow reaches a steady-state level is determined by the exceedance of the critical value. After some time, concentration inhomogeneities accumulate as a result of the thermal diffusion transport of the impurity and the steady-state flow loses its stability. As a result of the evolution of oscillatory perturbations, a traveling wave is formed. If the heating intensity is less than the next critical value $R_{\text{s}}$, then the traveling wave eventually loses its stability and the flow completely fades out after the transient process, for example, at $L = 4$ and $R_S = 2450$ (see Fig. 2). Changing the Prandtl number from 6 to 10 has no effect on the $R_S$ value.

Figure 2 shows the final fragment of the flow evolution in the form of a traveling wave: the maximum ($C_{\text{max}}$) and minimum ($C_{\text{min}}$) concentrations, and $\Psi_{\text{loc}}$ when the Rayleigh number is below the critical value ($R = 2400 < R_S$). As can be seen from Fig. 2, the traveling wave exists only for a certain interval of time. The amplitude of convective flows decreases, slowly at the initial moment and then rapidly. Moreover, concentration difference $\delta C = C_{\text{max}} - C_{\text{min}}$ increases with a rate that increases with a decrease in the convective transfer (the $\Psi$ value). The oscillation frequency also increases. The time of flow decay with the Rayleigh number slightly less than $R_S$ is rather long and sharply decreases with a decrease in the $R$ value. For example, the decay time of a traveling wave is $\tau_1 = 140$ for number $R = 2400$, $\tau_1 = 71$ for $R = 2300$, and $\tau_1 < 10$ for $R = 2000$. For a cell of a colloidal suspension of Hyflon MFA ($\chi = 1.47 \times 10^{-3} \text{ cm}^2/\text{s}$) with a height of $h = 0.29$ cm, which was used in the experiment described in [1], the unit of time is $[t] \approx 57$ s. Hence, the corresponding dimensional transition times are as follows: $\tau_1 \approx 8009 \text{ s} \approx 2.2 \text{ h}$, $\tau_1 \approx 4061 \text{ s} \approx 1.1 \text{ h}$, and $\tau_1 \approx 570 \text{ s} \approx 9.5 \text{ min}$, respectively.

Stable traveling waves are established in the cells with $R > R_S$. The characteristics of such a mode at $R = 2800$ are shown in Fig. 3. The phase portrait (Fig. 3a) corresponds to a modulated traveling wave. Oscillations of the stream function at a point with coordinates $x = 1/4$, $z = 1/2$ (Fig. 3b) demonstrate...
weak anharmonicity. The trajectories of the stream function extrema (nodes of the vertical convective velocity) on the world line plots $t - x$ (Fig. 3c) indicate the presence of a wave traveling from the left side of the cell to its right side. Moreover, convective vortices of opposite rotation are sequentially generated near the left boundary. As the suspension moves into the depth of the cell under study, the modulus of the stream function increases, reaches its maximum value in the middle of the cell, and then decreases to zero (Fig. 3d).

With an increase in the cell length, the threshold for the occurrence of a traveling wave shifts towards larger $R_S$ values (Fig. 3). In particular, $R_S = 2800$ for an aspect ratio of $L = 8.14$ corresponding to the aspect ratio used in the published experimental study [1]. Experiments [1] in a flat cell with a length-to-height ratio of $L = 8.14$ give the limit for the existence of a stable oscillatory flow regime at $R_S = 3400$, which differs from the results of the numerical simulation carried out by the authors of this work by 17%.

If we take into account the terms responsible for the gravitational settling of nanoparticles in the system of equations (4)–(6) and boundary conditions (7) and (8), as was done in [11, 12] for an infinite layer, then an even better match with $R_S = 3350$ (difference 1.5%) is obtained for the boundary of stable and unstable regimes of traveling waves in a finite-length cell. In the calculations, it is necessary to set additional parameters that bring the calculation closer to the experiment [1]: dimensionless sedimentation length $l = 3.5$ and mean concentration of nanoparticles in the layer $\tilde{C} = 4$ wt %.

As can be seen from Fig. 4, the frequency of the traveling wave increases with an increase in the aspect ratio starting from $L = 6.0$. In long cavities, qualitatively new effects that are not observed in short cells appear. The time evolution (Figs. 5–8) and the structure of transient currents become more complex. For

---

**Fig. 3.** Characteristics of a traveling wave in a cell with parameters of $L = 4.14$, $R = 2800$, $Pr = 10$, and $\psi = -7.5$: (a) the phase portrait of oscillations; (b) oscillations of the stream function at a point with coordinates $x = 1/4$, $z = 1/2$; (c) the positions of the coordinates of the centers of convective rolls on the characteristic plane; (d) extreme values of the stream function for convective rolls running through the cell.
example, the transient processes take place in a cavity with $L = 8.14$ with a Rayleigh number below critical value, which are characterized by various convective modes, such as traveling waves, localized traveling waves, and waves traveling in opposite directions. At a fixed point of the convective cell ($x = 1/4$, $z = 1/2$), the oscillations of the stream function (Fig. 5) are completely different from those shown in Fig. 3. In particular, the intensity of motion drops to almost zero at some moments of time ($t = 10, 35, 65$; Fig. 5b) and then increases again. Convection disappears only at $t > 80$.

One can see regions of strong (bright lines, $\Psi > 1$) and weak (faded lines, $\Psi < 1$) convective motions on the ($t - x$) world line plots (Fig. 6). Regions in which the traveling wave moves in the cell in different directions appear on the characteristic plane (Fig. 6a). In addition, there are time intervals ($6 < t < 15$, see Fig. 6b), during which there is strong convective motion in one part of the cell, but no motion in the other part. This corresponds to a localized traveling wave. At $t = 10$, it exists in the range of $3 < x < 10$.

There are defects associated with the coalescence of vortices on the world line plots (Fig. 7a). Evolution of the concentration field characterizing the behavior of the convective structure near a similar defect formed as a result of merging two vortices rotating counterclockwise is shown in Figs. 7b–7d. Hereinafter, regions enriched with a heavy impurity (for example, regions I and J) are shown in blue, and regions depleted in it are shown in green (see region 2). As can be seen from Figs. 7b–7d, the convective motion entrains impurity nanoparticles from the near-boundary hot layer, in which they were accumulated due to thermal diffusion, and draws it into the centers of vortices rotating counterclockwise. Vortices enriched and depleted in impurity alternate and move generally from the left side to the right side. Moreover, the vertical coordinate of the center of the vortices far from the defect corresponds to the half-height of the layer ($z = 1/2$). In general, this kind of behavior resembles a strongly nonlinear traveling wave in an alcohol–water mixture [7].

At time $t = 7.5$ (Fig. 7b), vortices I and J with the coordinates of the centers at $x = 1.5$ and $x = 2.5$, which take part in the formation of the defect, are separated by impurity-depleted region 2 rotating clockwise (vortex center coordinate $x = 2.0$). Further ($t = 7.75$, see Fig. 7c), all the traveling wave vortices are shifted to the right, and the centers of the merging vortices I and J are shifted vertically downward, and
the center of oppositely rotating vortex 2 is shifted vertically upward. Moreover, vortex 1 is accelerated relative to the group of vortices on the right side of the cell, while vortex 3 is decelerated, and the impurity gradually flows from vortex 1 to vortex 3. Eventually ($t = 8.0$, see Fig. 7d), the final merging of vortices 1 and 3 occurs, the center of the formed vortex shifts to the left (against the direction of the wave), which is reflected in Fig. 7a as the end of the blue line. Region 2 depleted in impurity decreases in size and disappears.

In the course of flow evolution, a regime of localized waves arises. Figure 8b shows the concentration field corresponding to the localized traveling wave regime, which exists in region $1.5 < x < 6$ capable of providing space for three wavelengths. In this case, the formation and destruction of convective vortices occurs far from the boundaries of the cavity. Outside the region of existence of a localized traveling wave, the liquid is nearly immobile. The localized traveling wave regime is unstable. Over time, the localized state decays with the formation of a regime in which two traveling waves diverge in different directions from the middle of the cell to its edges. Figure 8c illustrates the displacement of convective vortices that draw in a heavy impurity (shown by arrows) in different directions from the center in a time interval of

---

**Fig. 6.** Characteristics of a traveling wave in a cell with parameters of $L = 8.14$, $R = 2500$, $Pr = 10$, and $\psi = -7.5$: (a) the positions of the coordinates of the centers of convective rolls on the world line plots, and (b) the bottom part enlarged.

**Fig. 7.** (a) Defects on the world line plots and the corresponding change in the structure of the concentration field at times of (b) 7.5, (c) 7.75, and (d) 8.0 in a cell with parameters of $L = 8.14$, $R = 2500$, $Pr = 10$, and $\psi = -7.5$. 

---
A vortex moving to the right has an initial coordinate of $x = 4.5$ and a final coordinate of $x = 7$; a vortex moving to the left has an initial coordinate of $x = 2.4$ and a final coordinate of $x = 0.2$. Subsequently, the localized traveling wave moves from the right side to the left side.

4. CONCLUSIONS

Nonlinear oscillatory convection regimes of a colloidal suspension in a rectangular cavity elongated in the horizontal direction with heating from below are numerically investigated. The properties of the medium were selected so as to be close to those of a real suspension of Hyflon MFA. The suspension has a negative separation ratio; therefore, heavy nanoparticles are accumulated in the heated lower part of the cavity under the action of thermal diffusion, from where they are washed out by the convective flow. The results show that a stable oscillatory convective regime exists in the cavity when the Rayleigh number exceeds the critical value, i.e., $R > R_c$. The dependence of critical Rayleigh number $R_c$ on the cell length is determined. If the characteristic size of the cavity coincides with the size of the experimental cell [1],

![Fig. 8.](image)
then a good agreement was obtained between the results of numerical simulation and laboratory experiment. In the subcritical region \( R < R_* \), transient wave regimes have been discovered and investigated. Structural defects (due to the coalescence of vortices of the same direction of rotation), localized waves, and a change in the direction of traveling wave propagation are analyzed.

**FUNDING**

This work was supported by the Russian Foundation for Basic Research (project no. 20-01-00491).

**REFERENCES**

1. Donzelli, G., Cerbino, R., and Vailati, A., Bistable heat transfer in a nanofluid, *Phys. Rev. Lett.*, 2009, vol. 102, p. 104503.
   https://doi.org/10.1103/PhysRevLett.102.104503

2. Landau, L.D. and Lifshitz, E.M., *A Course of Theoretical Physics*, Vol. 6: Fluid Mechanics, Oxford: Pergamon, 1987.

3. Gershuni, G.Z. and Zhukhovitskii, E.M., *Convective Stability of Incompressible Fluids*, Jerusalem: Keter, 1976.

4. Mason, M. and Weaver, W., The settling of small particles in a fluid, *Phys. Rev.*, 1924, vol. 23, pp. 412–426.
   https://doi.org/10.1103/PhysRev.23.412

5. Raikher, Yu.L. and Shliomis, M.I., On the kinetics of establishment of the equilibrium concentration in a magnetic suspension, *J. Magn. Magn. Mater.*, 1993, vol. 122, pp. 93–97.
   https://doi.org/10.1016/0304-8853(93)91047-B

6. Shliomis, M.I. and Smorodin, B.L., Convective instability of magnetized ferrofluids, *J. Magn. Magn. Mater.*, 2002, vol. 252, pp. 197–202.
   https://doi.org/10.1016/S0304-8853(02)00712-6

7. Lücke, M., Barten, W., Büchel, P., Fütterer, C., Hollinger, St., and Jung, Ch., Pattern formation in binary fluid convection and in systems with through flow, in *Evolution of Spontaneous Structures in Dissipative Continuous Systems*, Berlin: Springer, 1998, pp. 127–196.
   https://doi.org/10.1007/3-540-49537-1_3

8. Putin, G.F., in *Proceedings of the 11th Riga Meeting on Magnetohydrodynamics*, Riga: Zinatne, 1984, vol. 3, pp. 15–18.

9. Glukhov, A.F. and Demin, V.A., Thermal convection of binary mixtures in vertical layers and channels when heated from below, *Vestn. Perm. Univ., Fiz.*, 2009, no. 27 (1), pp. 16–26.

10. Winkel, F., Messlinger, S., Schöpf, W., Rehberg, I., Siebenbürger, M., and Ballauff, M., Thermal convection in a thermosensitive colloidal suspension, *New J. Phys.*, 2010, vol. 12, p. 053003.
    https://doi.org/10.1088/1367-2630/12/5/053003

11. Smorodin, B.L., Cherepanov, I.N., Myznikova, B.I., and Shliomis, M.I., Traveling-wave convection in colloids stratified by gravity, *Phys. Rev. E*, 2011, vol. 84, p. 026305.
    https://doi.org/10.1103/PhysRevE.84.026305

12. Smorodin, B.L. and Cherepanov, I.N., Convection of colloidal suspensions stratified by thermodiffusion and gravity, *Eur. Phys. J. E*, 2014, vol. 37, p. 118.
    https://doi.org/10.1140/epje/i2014-14118-x

13. Smorodin, B.L. and Cherepanov, I.N., Convection in a colloidal suspension in a closed horizontal cell, *J. Exp. Theor. Phys.*, 2015, vol. 120, pp. 319–326.
    https://doi.org/10.1134/S1063776115010161

14. Cherepanov, I.N., Colloid flow in a horizontal cell subjected to heating from sidewall, *Vychisl. Mekh. Splosh. Sred*, 2016, vol. 9, no. 2, pp. 135–144.
    https://doi.org/10.7242/1999-6691/2016.9.2.12

15. Lyubimova, T.P. and Zubova, N.A., Onset and nonlinear regimes of convection of ternary mixture in a rectangular porous cavity taking into account Soret effect, *Vychisl. Mekh. Splosh. Sred*, 2019, vol. 12, no. 3, pp. 249–262.
    https://doi.org/10.7242/1999-6691/2019.12.3.21

16. Roache, P.J., *Computational Fluid Dynamics*, Albuquerque: Hermosa, 1976.

17. Marchuk, G.I., *Methods of Numerical Mathematics*, Berlin: Springer, 1982.

Translated by O. Kadkin