Application of compact grid-characteristic schemes for acoustic problems

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Abstract. This work is devoted to the numerical solution of the acoustic problem. It has a lot of important applications, for example, direct and inverse problems of the seismic survey process. The grid-characteristic approach was used to transform the problem into a set of independent linear transport equations. In one-dimensional case explicit numerical schemes for continuous and discontinuous materials were derived. Using differential sequences of initial equations, the new compact scheme on the two-point spatial stencil was proposed. A set of numerical experiments with different initial conditions and materials were carried out. Direct numerical simulations proved the third order of the approximation.

1. Introduction
The system of linear acoustic equations appears in many fundamental and applied problems: the earthquake forecast, the seismic survey process for oil and gas deposits, the sound wave propagation. With the development of modern high-performance computing systems, it is possible to simulate complex dynamic processes in domains with arbitrary geometries [1]. That is why the development of robust and precision numerical schemes is in a high priority.

Numerical solution of the differential equations system on the computational mesh is the basis of a lot of different simulation software. It includes not only research programs for scientific experiments but applied commercial software too. One of the important cases is the hyperbolic systems of equations. It describes a wide range of dynamic processes in continuum mechanics: deformable body mechanics, acoustics, aerodynamics, etc. To solve govern equations correct numerical methods have to be stable and accurate. That is why the developing of new ones with high orders of the spatial and time approximation is the important problem.

Expansion of the scheme stencil [2] is a common way of increasing the approximation order. Differential consequences of initial equations also can be used [3, 4]. These schemes are called a compact (or bicom pact). In work [5] supersonic flows past simple aerodynamics forms were simulated. Lubricant transfer from media to head during heat-assisted magnetic recording writing was predicted with compact schemes [6]. For elliptic equations this approach was applied in work [7]. The compact scheme was successfully applied for incompressible Navier-Stokes equations in two-dimensional case [8]. The same approach was applied for third order boundary value problem [9].
The monotonicity is an important property of any numerical methods, because it guarantees the physical nature of all solution oscillations. The third-order monotonic implicit compact scheme was constructed [10]. The grid-characteristic monotonicity criterium was used to obtain a stable bicom pact scheme in papers [11, 12].

In this work the bicom pact third-order scheme constructed with the polynomial interpolation for one-dimensional acoustic equations is described. The cases of continuous and discontinuous media are investigated. It was shown that the explicit identification of the parameter gap can be done while keeping the high-order approximation. The convergence tests were carried out.

This work is organized as follows. In Section 2 the three-dimensional acoustic system of equations is formulated. With splitting along the coordinate axes, it is reduced to a set of one-dimensional systems. The grid-characteristic approach is applied to obtain independent linear transport equations. In Subsection 2.1 the procedure of taking into account the medium discontinuity is presented. The method is based on the pressure continuity across the spatial domain. In Subsection 2.2 the bicom pact scheme on the two-point spatial stencil is derived. The differential consequences of initial equations are used. In Section 3 the results of convergence tests are presented for different initial conditions. The final pressure profiles obtained with Courant-Isaacson-Rees and bicom pact schemes are compared.

2. Materials and methods

Let’s consider the system of acoustic equations

\[ \nu_t + \frac{1}{\rho} \nabla p = 0, \quad (1) \]
\[ p_t + \rho c^2 (\nabla \cdot \nu) = 0, \quad (2) \]

where \( \nu \) is the velocity vector, \( p \) is the pressure, \( \rho \) is the material density, \( c \) is the wave propagation velocity and the subscript means a partial derivative.

Taking into account the vector of unknown variables

\[ \mathbf{U} = \begin{pmatrix} \nu \\ p \end{pmatrix}, \quad (3) \]

we can rewrite the initial system (1) – (2) into the canonical matrix form

\[ \mathbf{U}_t + \mathbf{A}_x \mathbf{U}_x + \mathbf{A}_y \mathbf{U}_y + \mathbf{A}_z \mathbf{U}_z = 0. \quad (4) \]

This system can be solved with the method of splitting along coordinate directions. We have to solve consequently three one-dimensional systems (with different matrixes \( \mathbf{A} \)) in the form of

\[ \mathbf{U}_t + \mathbf{A} \mathbf{U}_x. \quad (5) \]

In this paper we concentrated only on the one-dimensional case, but the generalization for the origin three-dimensional system of equations (1) – (2) can be done without significant effort. So, the govern system of equations is

\[ \nu_t + \frac{1}{\rho} p_x = 0, \quad (6) \]
\[ p_t + K \nu_x = 0, \quad (7) \]

where \( p(x,t) \) is the pressure, \( \nu(x,t) \) is the velocity, \( \rho(x) \) is the density and \( K(x) \) is the bulk modulus. We denote \( Z = \sqrt{K \rho} \) – the acoustic impedance and \( c = \sqrt{\frac{K}{\rho}} \) – the wave propagation velocity. It is valid, that \( K = c Z, Z = c \rho \).

The explicit form of the system of equations is

\[ \mathbf{U}_t + \mathbf{A} \mathbf{U}_x = 0, \quad (8) \]

where
For simplicity, we deal with the three consequently eigenvalues and eigenvectors. These vectors are
\[
(Z, c) = \begin{cases} (Z_l, c_l) & \text{if } x < x_\alpha \\ (Z_r, c_r) & \text{else } x > x_\alpha \end{cases}
\]  

(11)

The physically correct conditions across the discontinuity of the material are:
\[
[p] = 0, \quad [v] = 0.
\]

(12)  
(13)

The system of equations (6) – (7) is hyperbolic and the matrix \( \mathbf{A} \) has the complete system of real eigenvalues and eigenvectors. These vectors are
\[
\mathbf{r}_1 = \left( \begin{array}{c} -Z \\ 1 \end{array} \right), \quad \mathbf{r}_2 = \left( \begin{array}{c} Z \\ 1 \end{array} \right).
\]

(14)

The matrix \( \mathbf{A} \) can be written as \( \mathbf{A} = \mathbf{R}\Lambda \mathbf{R}^{-1} \), where \( \mathbf{R} \) is the matrix of eigenvectors
\[
\mathbf{R} = \mathbf{r}_1 \mathbf{r}_2 = \left( \begin{array}{cc} -Z & Z \\ 1 & 1 \end{array} \right),
\]

(15)

and \( \mathbf{R}^{-1} \) is the inverse matrix
\[
\mathbf{R}^{-1} = \frac{1}{2Z} \left( \begin{array}{cc} -1 & Z \\ 1 & Z \end{array} \right).
\]

(16)

The matrix \( \Lambda \) of eigenvectors is
\[
\Lambda = \left( \begin{array}{cc} -c & 0 \\ 0 & c \end{array} \right).
\]

(17)

We introduce new unknown variable (Riemann invariants)
\[
\mathbf{\omega} = \left( \begin{array}{c} \omega^- \\ \omega^+ \end{array} \right) = \mathbf{R}^{-1} \mathbf{U} = \frac{1}{2Z} \left( \begin{array}{c} -p + Zv \\ p + 2Zv \end{array} \right).
\]

(18)

With this replacement the system (6) – (7) is a set of independent linear transport equations
\[
\omega_1^+ + c \omega_2^+ = 0, \\
\omega_1^- - c \omega_2^- = 0.
\]

(19)  
(20)

If its solution \( \omega^{n+1} \) is found the inverse transformation can be easily done
\[
\mathbf{U} = \left( \begin{array}{c} p^n \\ 0 \end{array} \right) = \mathbf{R} \mathbf{\omega} = \left( \begin{array}{cc} -Z & Z \\ 1 & 1 \end{array} \right) \left( \begin{array}{c} \omega^- \\ \omega^+ \end{array} \right) = \left( \begin{array}{c} Z(\omega^+ - \omega^-) \\ \omega^+ - \omega^- \end{array} \right).
\]

(21)

For the process of the numerical solution, we use the time and space discretization with \( \tau \) and \( h \), consequently. We denote with the index \( n \) the current time layer and with \( m \) – the spatial node index. For simplicity, we deal with the three-points scheme stencil
\[
(t^n, x_{m-1}), (t^n, x_m), (t^n, x_{m+1}), (t^{n+1}, x_m).
\]

(22)

On the figure 1 the example of the stencil is depicted, when the point of the parameter discontinuity \( x = x_\alpha \) is placed in the mesh node. Without the loss of the generality, we choose \( x_\alpha = 0 \). In this case, \( \omega^+ \) goes to the left part and \( \omega^- \) – to the right part.
Figure 1. The stencil used in time and space and the contact border of materials.

If left and right parameters are the same, i.e. \((Z_l, c_l) = (Z_r, c_r) = (Z, c)\), the algorithms of calculation \(U_{n+1}\) is

1) Calculate the Riemann invariants \(\omega^n = R^{-1}U^n\);
2) Use any method (the interpolation, grid-characteristic or finite-volume methods) solve one-dimensional linear transport equations and calculate \(\omega^{n+1}\);
3) Do the inverse transformation \(U_{n+1} = R \omega^{n+1}\).

It should be noticed, that this algorithm can be rather effectively implemented. It is connected with the sparseness of \(R\) and \(R^{-1}\), and the fact that it is enough to store the values of \(\omega\) only at stencil nodes.

2.1. Discontinuous medium

In the case of parameter discontinuity, the described algorithm is not valid, because we can’t put matrixes \(R^{-1}\) and \(R\) under the derivative signs. Let’s split the material into two independent domains and deal with them separately (see figure 2).

We begin with the left part of the domain. Assume, that the point \(x_m\) is on the contact boundary with the right half-space with parameters \(Z_r, c_r\), and denote the external characteristic line with \(\omega_i\).

We don’t know its value directly, but we can propose the relationship \(\omega_i^{-n+1} = \alpha_l \omega_i^{n+1}\), where \(\alpha_l\) is the scalar value. Then, for the value of the vector \(U_{n+1}\) at the next time step:

\[
p_i^{n+1} = Z_i (\omega_i^{n+1} - \alpha_l \omega_i^{n+1}) = Z_i \omega_i^{n+1} (1 - \alpha_l),
\]

\[
v_i^{n+1} = \omega_i^{n+1} + \alpha_l \omega_i^{n+1} = \omega_i^{n+1} (1 + \alpha_l).
\]

The same procedure can be done for the other part of the domain. In this case we denote the external characteristic line as \(\omega_i^{-n+1} = \alpha_r \omega_i^{-n+1}\), where \(\alpha_r\) is the other scalar value. And the second equation for \(U_{n+1}\) can be written as
Initially, we have linear transport equations

\[ p^{n+1}_i = Z_r(\alpha_r \omega^{-n+1} + \omega^{-n+1}) = -Z_r \omega^{-n+1}(1 - \alpha_r), \]
\[ v^{n+1}_i = \alpha_r \omega^{-n+1} + \omega^{-n+1} = \omega^{-n+1}(1 + \alpha_r). \]

(25)
(26)

To calculate \( \alpha_l \) and \( \alpha_r \), we have to use the discontinuous condition \([\textbf{U}] = 0\), i.e. \( \textbf{U}_l^{n+1} = \textbf{U}_r^{n+1} \)

\[ p^{n+1}_l = p^{n+1}_r, \]
\[ v^{n+1}_l = v^{n+1}_r. \]

(27)
(28)

\[ Z_l(\omega^{n+1}_l(1 - \alpha_l) = -Z_r \omega^{-n+1}(1 - \alpha_r), \]
\[ \omega^{n+1}_l(1 + \alpha_l) = \omega^{-n+1}(1 + \alpha_r). \]

(29)
(30)

And unknown scalar coefficients are

\[ \alpha_r = 2 \frac{\omega^+}{\omega} \frac{Z_l}{Z_l + Z_r} - \frac{Z_l - Z_r}{Z_l + Z_r}, \]
\[ \alpha_l = 2 \frac{\omega^-}{\omega} \frac{Z_r}{Z_l + Z_r} + \frac{Z_l - Z_r}{Z_l + Z_r}. \]

(31)
(32)

In the initial variables \( p \) and \( v \):

\[ p^{n+1} = 2 \frac{Z_l}{Z_l + Z_r} (\omega^+ - \omega^-), \]
\[ v^{n+1} = 2 \frac{1}{Z_l + Z_r} (Z_l \omega^+ + Z_r \omega^-). \]

(33)
(34)

2.2. Bicompact scheme

There is the way to increase the spatial approximation order using appropriate differential sequences. Initially, we have linear transport equations

\[ \omega^+_t + c \omega^+_x = 0, \]
\[ \omega^-_t - c \omega^-_x = 0. \]

(35)
(36)

We can apply the operator \( \frac{\partial}{\partial x} \) and introduce new definitions

\[ v^+(x, 0) = \frac{\partial \omega^+}{\partial x}(x, 0). \]
\[ v^+(x, t) = \frac{\partial \omega^+}{\partial x}(x, t). \]
\[ v^-(x, 0) = \frac{\partial \omega^-}{\partial x}(x, 0). \]
\[ v^-(x, t) = \frac{\partial \omega^-}{\partial x}(x, t). \]

(37)
(38)
(39)
(40)

If all functions are enough smooth, so \( \frac{\partial^2 \omega}{\partial t \partial x} = \frac{\partial^2 \omega}{\partial x \partial t} \), then we obtain

\[ v^+_t + cv^+_x = 0, \]
\[ v^-_t - cv^-_x = 0. \]

(41)
(42)

We can use the same approach as for equations (23) – (26). Then for the left subdomain

\[ p^{n+1}_{x, l} = Z_l(v^{n+1} - \alpha_l v^{n+1}) = Z_l v^{n+1}(1 - \alpha_l), \]
\[ v^{n+1}_{x, l} = v^{n+1} + \alpha_l v^{n+1} = v^{n+1}(1 + \alpha_l). \]

(43)
(44)

And for the right subdomain
\[ p_{r,l}^{n+1} = Z_l (\alpha_l v^{-n+1} + v^{-n+1}) = -Z_r v^{-n+1} (1 - \alpha_r), \]  
(45)  
\[ v_{r,l}^{n+1} = \alpha_r v^{-n+1} + v^{-n+1} = v^{-n+1} (1 + \alpha_r), \]  
(46)  

where \( \alpha_l \) and \( \alpha_r \) are unknown scalar coefficients.  

At the point of the material discontinuity \([A U_a] = 0\), so  
\[ \frac{1}{\rho} p_{x,l} = 0, \]  
(47)  
\[ [K v_{x,l}] = 0. \]  
(48)  

And the contact condition is  
\[ Z_l v^+ v^{-1} (1 - \alpha_l) = -Z_r v^{-n+1} (1 - \alpha_r), \]  
(49)  
\[ K_l v^+ v^{-1} (1 + \alpha_l) = K_r v^{-n+1} (1 + \alpha_r). \]  
(50)  

The coefficients \( \alpha_l \) and \( \alpha_r \) are  
\[ \alpha_l = \frac{v^+ (K_l Z_l \rho_l - K_l Z_r \rho_r) + 2 v^- K_l Z_r \rho_l}{v^+ (K_l Z_l \rho_l + K_l Z_r \rho_r)}, \]  
(51)  
\[ \alpha_r = \frac{2 v^+ K_l Z_r \rho_r + v^- (K_l Z_r \rho_l - K_l Z_r \rho_r)}{v^+ (K_l Z_r \rho_l + K_l Z_r \rho_r)}. \]  
(52)  

And finally,  
\[ \frac{p_{x,l}}{\rho_l} = \frac{p_{x,r}}{\rho_r} = \frac{2 c Z_l v^+ - c_r Z_r v^-}{Z_l + Z_r}, \]  
(53)  
\[ K_l v_{x,l} = K_r v_{x,r} = \frac{2 c Z_l v^+ + c_r Z_r v^-}{Z_l + Z_r}. \]  
(54)  

If we know values of the function in two points and its derivative too, then we can restore the cubic polynomial solution. For the left subdomain we obtain  
\[ \omega^+ (x) = ax^3 + bx^2 + kx + l, \]  
(55)  
\[ v^+ (x) = 3ax^2 + 2bx + k, \]  
(56)  
\[ a = \frac{v_{m,n}^+ + v_{m-1,n}^+}{h^2} - 2 \frac{\omega_{m+1,n}^+ - \omega_{m,n}^+}{h^3}, \]  
(57)  
\[ b = \frac{2v_{m,n}^+ + v_{m-1,n}^+}{h} - 3 \frac{\omega_{m+1,n}^+ - \omega_{m,n}^+}{h^2}, \]  
(58)  
\[ k = v_{m,n}^+, \]  
(59)  
\[ l = \omega_{m,n}^+, \]  
(60)  
\[ \omega_{m+1,n}^+ = \omega^+ (x_m - c \tau), \]  
(61)  
\[ v_{m+1,n}^+ = v^+ (x_m - c \tau). \]  
(62)  

And for the right subdomain:  
\[ \omega^-(x) = ax^3 + bx^2 + kx + l, \]  
(63)  
\[ v^- (x) = 3ax^2 + 2bx + k, \]  
(64)  
\[ a = \frac{v_{m,n}^- + v_{m,n}^-}{h^2} - 2 \frac{\omega_{m+1,n}^- - \omega_{m,n}^-}{h^3}, \]  
(65)  
\[ b = \frac{v_{m,n}^- + 2v_{m,n}^-}{h} + 3 \frac{\omega_{m+1,n}^- - \omega_{m,n}^-}{h^2}, \]  
(66)
7

$k = \nu_m^n$, \hfill (67)

$l = \omega_m^n$, \hfill (68)

\[ \omega_m^{n+1} = \omega^{-n}(x_m + ct), \] \hfill (69)

\[ \nu_m^{n+1} = \nu^{-n}(x_m + ct). \] \hfill (70)

3. Results and discussion

The described above approach was implemented as a research software with C++ language. The staggered grid (see figure 3) was used. A set of numerical experiments were conducted to verify the proposed method.

Initially, we simulate a medium contained two layers with different impedances. The spatial region \( x \in [-1; 1] \), with the point of the discontinuity at \( x = 0 \). Parameters were \( c_l = 1.0, \rho_l = 1.0, c_r = 0.5, \rho_r = 3.0 \) and appropriate impedances \( -Z_l = 1.0, Z_r = 1.5 \). The initial data was used in the form of

\[ (x, 0) = e^{100(x+0.5)^2}. \] \hfill (71)

The spatial step was \( h = 0.00125 \), the time step was \( \tau = 0.00025 \). Accordingly, Courant numbers were 0.2 (left) and 0.1 (right). The resulting pressure distributions are depicted at figure 4 for two different schemes: the Courant-Isaacson-Rees (1st order) and the compact scheme. The last one reproduces the analytical solution with the better precision.

Secondly, the convergence of the compact scheme was investigated with a two set of initial data. The first one is the data from the equation (71) and the second one is

\[ f(x, 0) = \begin{cases} 
\sin^4\left(2\pi(x - 0.3)\right), & -0.7 \leq x \leq -0.2 \\
0, & \text{otherwise}
\end{cases}. \] \hfill (72)

Totally \( T = 1.0 \) was simulated. Due to the impedance difference, the reflected wave occurred with the amplitude equals to \( \frac{Z_r-Z_l}{Z_r+Z_l} = \frac{1}{5} \) of the initial perturbation. The amplitude of transmitted wave is
\[ \frac{2z_r}{z_r+z_l} = \frac{6}{5} \] of the initial perturbation. To estimate the convergence rate, we carried out a set of experiments on meshes with \( N = 101, 201, 401, 1601, 3201, 6401 \) nodes. Two different norm were used: \( L_1 = \sum |x_i| \times h, \ L_\infty = \max |x_i| \), where \( x_i = u_i - u_i^\text{theor} \) – the difference of analytical and numerical solutions. The results are summarized in tables 1, 2 and figure 5 for pressure and velocity values.

| \( N \) | \( L_1 \) | \( \text{order } L_1 \) | \( L_\infty \) | \( \text{order } L_\infty \) |
|-------|-------|-----------------|-------|-----------------|
| CIP   | 101   | 4.20E-03        | —     | 3.96E-02        | —     |
|       | 201   | 6.08E-04        | 2.81  | 6.75E-03        | 2.57  |
|       | 401   | 7.84E-05        | 2.97  | 8.91E-04        | 2.93  |
|       | 801   | 9.88E-06        | 2.99  | 1.13E-04        | 2.99  |
|       | 1601  | 1.24E-06        | 3.00  | 1.41E-05        | 3.00  |
|       | 3201  | 1.55E-07        | 3.00  | 1.76E-06        | 3.00  |
|       | 6401  | 1.94E-08        | 3.00  | 2.21E-07        | 3.00  |
| CIR   | 101   | 1.10E-01        | —     | 6.73E-01        | —     |
|       | 201   | 7.97E-02        | 0.47  | 5.66E-01        | 0.25  |
|       | 401   | 5.22E-02        | 0.61  | 4.08E-01        | 0.47  |
|       | 801   | 3.13E-02        | 0.74  | 2.66E-01        | 0.62  |
|       | 1601  | 1.76E-02        | 0.84  | 1.57E-01        | 0.76  |
|       | 3201  | 9.38E-03        | 0.91  | 8.70E-02        | 0.86  |
|       | 6401  | 4.86E-03        | 0.95  | 4.60E-02        | 0.92  |

\( p \)

| \( N \) | \( L_1 \) | \( \text{order } L_1 \) | \( L_\infty \) | \( \text{order } L_\infty \) |
|-------|-------|-----------------|-------|-----------------|

\( v \)

| \( N \) | \( L_1 \) | \( \text{order } L_1 \) | \( L_\infty \) | \( \text{order } L_\infty \) |
|-------|-------|-----------------|-------|-----------------|

Table 1. Grid convergence for the (71) initial data.
Table 2. Grid convergence for the (72) initial data.

| p     | N     | $L_1$  | order $L_1$ | $L_0$  | order $L_0$ |
|-------|-------|--------|--------------|--------|--------------|
| CIP   | 101   | 3.93E-03 | —            | 2.65E-02 | —            |
|       | 201   | 5.39E-04 | 2.89         | 3.88E-03 | 2.79         |
|       | 401   | 6.92E-05 | 2.97         | 5.04E-04 | 2.95         |
|       | 801   | 8.67E-06 | 3.00         | 6.33E-05 | 3.00         |
|       | 1601  | 1.09E-06 | 3.00         | 7.93E-06 | 3.00         |
|       | 3201  | 1.36E-07 | 3.00         | 9.92E-07 | 3.00         |
|       | 6401  | 1.69E-08 | 3.00         | 1.24E-07 | 3.00         |
| CIR   | 101   | 1.17E-01 | —            | 6.88E-01 | —            |
|       | 201   | 8.35E-02 | 0.49         | 5.34E-01 | 0.37         |
|       | 401   | 5.38E-02 | 0.64         | 3.80E-01 | 0.49         |
|       | 801   | 3.19E-02 | 0.76         | 2.39E-01 | 0.67         |
|       | 1601  | 1.76E-02 | 0.85         | 1.37E-01 | 0.80         |
|       | 3201  | 9.32E-03 | 0.92         | 7.38E-02 | 0.89         |
|       | 6401  | 4.80E-03 | 0.96         | 3.84E-02 | 0.94         |

| v     | N     | $L_1$  | order $L_1$ | $L_0$  | order $L_0$ |
|-------|-------|--------|--------------|--------|--------------|
| CIP   | 101   | 2.70E-03 | —            | 1.76E-02 | —            |
|       | 201   | 3.69E-04 | 2.89         | 2.58E-03 | 2.79         |
|       | 401   | 4.74E-05 | 2.97         | 3.36E-04 | 2.95         |
|       | 801   | 5.94E-06 | 3.00         | 4.22E-05 | 3.00         |
|       | 1601  | 7.43E-07 | 3.00         | 5.29E-06 | 3.00         |
|       | 3201  | 9.28E-08 | 3.00         | 6.62E-07 | 3.00         |
|       | 6401  | 1.16E-08 | 3.00         | 8.27E-08 | 3.00         |
| CIR   | 101   | 8.62E-02 | —            | 4.59E-01 | —            |
|       | 201   | 6.11E-02 | 0.50         | 3.56E-01 | 0.37         |
|       | 401   | 3.92E-02 | 0.64         | 2.53E-01 | 0.49         |
|       | 801   | 2.31E-02 | 0.76         | 1.59E-01 | 0.67         |
|       | 1601  | 1.28E-02 | 0.86         | 9.12E-02 | 0.80         |
|       | 3201  | 6.74E-03 | 0.92         | 4.92E-02 | 0.89         |
|       | 6401  | 3.47E-03 | 0.96         | 2.56E-02 | 0.94         |
4. Conclusion

The problem of the acoustic wave propagation in discontinuous media was investigated. The method of splitting along coordinate axes was applied to reduce its dimensions. The grid-characteristic approach was used to transform the one-dimensional problem into a set of independent linear transport equations. Based on it and the continuity of the pressure field, explicit numerical schemes for continuous and discontinuous materials were derived. Using differential sequences of initial equations, the new compact scheme on the two-point spatial stencil was proposed. The gap repair procedure was developed. The locality of the computational algorithm provides the freedom for effective parallel implementations on modern high-performance computing systems. A set of numerical experiments with different initial conditions and materials were carried out. Direct numerical simulations proved the third order of the approximation.

The further research shall be oriented on the extension of this approach to two-dimensional and three-dimensional problems for acoustic and elastic media. The appropriate boundary conditions have to be developed too for keeping the approximation order.

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