New Physics Near 1 TeV and Above

Roland E. Allen
Department of Physics, Texas A&M University
College Station, Texas 77843

Abstract. A new theory makes testable predictions: (1) Higgs fields have an unconventional equation of motion. (2) Fermions have a second-order coupling to gauge fields. (3) Fermion propagators are modified at high energy. (4) There are new scalar bosons which are supersymmetric partners of spin 1/2 fermions. (5) Since W-bosons gravitate differently from fermions and massless gauge bosons, there is a very small violation of the equivalence principle. Other features of the theory have implications for cosmology, including the values of cosmological parameters, a mechanism for scale-invariant density fluctuations, and a candidate for dark matter.

1. Introduction

Superstring theory represents the best effort to extend conventional physics, including Lorentz invariance, to the Planck scale [1]. Unfortunately, however, superstring theory has not yet yielded a single prediction that can be tested against experiment.

Recently a different approach was introduced [2], in which Lorentz invariance is not presumed to hold at arbitrarily high energies. Instead, a very simple form is postulated for the action, and Lorentz invariance is then found to emerge as a good approximation at energies that are well below the Planck scale.

This new theory turns out to lead to a large number of testable predictions, several of which are presented here for the first time:

- The Higgs boson has an unconventional equation of motion.
• Fermions have a second-order coupling to gauge fields.
• Fermion propagators are modified at high energy.
• There are new scalar bosons which are supersymmetric partners of spin 1/2 fermions.
• Since W-bosons gravitate differently from fermions and massless gauge bosons, there is a very small violation of the equivalence principle.
• In cosmology, the density parameter $\Omega_m$ is less than one, the universe is flat in the limit $t \to \infty$, and the deceleration parameter approaches zero as $t \to \infty$.
• There is a mechanism for producing scale-invariant density fluctuations which is different from the mechanism in standard inflationary scenarios.
• The cold dark matter consists of superheavy, sterile, right-handed neutrinos, produced by a nonthermal mechanism in the very early universe.

In the following sections, Ref. 2 will be cited as I, and equations from this paper will be denoted with the prefix I – e.g., I(3.1).

2. Equation of Motion of Higgs Fields

First consider an analogy. Suppose that an ordinary superfluid has an order parameter

$$\psi_s = \exp (i\theta) n_s^{1/2}$$

so that its velocity is $v_s^\mu = m^{-1} \nabla_s \theta$. Suppose also that a nonrelativistic particle has a wavefunction $\Psi_p(\vec{r}) = \exp (i \vec{p} \cdot \vec{r}) \Psi(0)$, so that its velocity is $V_p = m^{-1} \vec{p}$ (with the masses taken to be equal for simplicity). Finally, suppose that an observer is moving with the superfluid. In the frame of reference of this observer, the particle has a velocity $\vec{v}_p = \vec{V}_p - \vec{v}_s$ and a wavefunction $\psi_p$ given by

$$\Psi_p = \exp (i\theta) \psi_p$$

where the prefactor $\exp (i\theta)$ represents a local Galilean boost from $\vec{v}_p$ to $\vec{V}_p$.

In the present theory, $\psi_s$ is replaced by a GUT Higgs condensate $\Psi_s$, and the particle wavefunction $\Psi_p$ is replaced by a fundamental field $\Psi_a$. One of the principal new features of the present theory is that $\Psi_s$ and $\Psi_a$ have two
components rather than one (in addition to the SO(10) gauge symmetry discussed below), so they exhibit U(1)×SU(2) rather than U(1) rotations in 4-dimensional spacetime. These rotations of the order parameter as a function of \(x^\mu\) are described by a 2×2 matrix \(U\):\[
\Psi_s = Un_s^{1/2}\eta \tag{3}
\]
where \(\eta\) is a constant 2-dimensional vector. Since human observers live in the GUT condensate \(\Psi_s\), and are moving with it, one must transform the initial field \(\Psi_a\) to the field \(\psi_a\) seen by human observers. Just as in the analogy above, this is done by writing
\[
\Psi_a = U\psi_a \tag{4}
\]
with the prefactor \(U\) representing a boost in the U(1)×SU(2) rotations of \(\psi_a\).

Let us begin with the bosonic excitations \(\Phi_b\) defined by I(3.1), written as \(\Phi_b = U\phi_b\), \(\phi_b(x_A, x_B) = \sum_r \phi_r(x_A) \psi^B_r(x_B)\) (5) in the notation of I(5.18). Repeating the arguments in Section 8 of I (with appropriate modifications) leads to the Lagrangian density
\[
L_b = -\frac{1}{2} h \left\{ \hat{h}^{\mu\nu} \left[ \frac{1}{2m} \left( \hat{D}_\mu \phi \right)^\dagger \left( \hat{D}_\nu \phi \right) - \frac{1}{2} m v_\mu^\alpha v_\nu^\sigma \phi^\dagger \phi \right] + \frac{1}{2} b' \left( \phi^\dagger \phi \right)^2 \right\} . \tag{6}
\]
(This is the Lorentzian version of I(8.32), but with \(\Phi_b\) now written in the form (5), and \(b' = (2m)^{-2}\tilde{b}\). A proper derivation of (6) will be given elsewhere.) Here \(\phi\) is the vector with components \(\phi_r\) and
\[
\hat{D}_\mu = \partial_\mu + i A^a_\mu t_1 + i m v_\mu^\alpha \sigma_\alpha \tag{7}
\]
Also, \(h^{\mu\nu}\) is the initial metric tensor of I(2.16), except that it has been transformed from Euclidean to Lorentzian spacetime: \(h^{\mu\nu} = \eta^{\mu\nu}\), where \(\eta^{\mu\nu}\) is the Minkowski metric tensor \(\text{diag}(-1, 1, 1, 1)\).

In the present cosmological model \([2-4]\), the “superfluid velocity” of the GUT Higgs condensate has the form
\[
\nu^\mu_\alpha = h^{\mu\nu} v_\nu^\alpha = \lambda \delta^\mu_\alpha \tag{8}
\]
in regions of spacetime where the gravitational field is weak. (Since I(3.24) is not valid for a pure state, (8) provides the real justification for the simple form of the Bernoulli equation I(3.26).) After an integration by parts and the use of (8), the Lagrangian can then be simplified to
\[
L_b = -\frac{1}{2} h \left[ \phi^\dagger \left( \frac{1}{\sqrt{m}} g^{\mu\nu} D_\mu D_\nu + ie^a_\mu \sigma^a D_\mu \right) \phi + \frac{1}{2} b' \left( \phi^\dagger \phi \right)^2 \right] + \text{h.c.} \tag{9}
\]
where \( D_\mu = \partial_\mu + i A_\mu \), \( A_\mu = A_{\mu}^i t_i \), \( \overline{m} = 2\lambda^2 m \), \( e_\alpha^\mu = \nu_\alpha^\mu \), and “h.c.” means “Hermitian conjugate”. (Since I(8.31) is nonzero, this equation replaces I(8.40).) As in I, \( e_\alpha^\mu \) is interpreted as the gravitational vierbein, so that \( g^{\mu\nu} = \eta^\alpha_\beta e_\alpha^\mu e_\beta^\nu = \lambda^2 \delta^{\mu\nu} \).

Also as in I, suppose that radiative corrections lead to small mass terms. Let us focus on a particular field \( \phi_h \) with a negative mass term:

\[
L_h = -\frac{1}{2} g \left[ -\Phi^\dagger_h (g^{\mu\nu} D_\mu D_\nu + \overline{m} e_\alpha^\mu \sigma^\alpha) \Phi_h - \mu_h^2 \Phi^\dagger_h \Phi_h + \frac{1}{2} \overline{b} \left( \Phi^\dagger_h \Phi_h \right)^2 \right] + h.c. \tag{10}
\]

where \( \Phi_h = \overline{m}^{-1/2} \lambda^2 \phi_h \), \( \overline{b} = (2m)^2 b' \), and \( A_\mu \) now represents just those gauge fields that act on \( \Phi_h \). We have used the fact that \( g = (- \det g_{\mu\nu})^{1/2} = \lambda^{-4} \lambda(h_{\mu\nu})^{1/2} = \lambda^{-4} h \). For \( \partial_\mu \Phi_h = 0 \), (10) becomes

\[
L_h = -g \left[ \Phi^\dagger_h (g^{\mu\nu} A_\mu A_\nu + \overline{m} A_\mu e_\alpha^\mu \sigma^\alpha) \Phi_h - \mu_h^2 \Phi^\dagger_h \Phi_h + \frac{1}{2} \overline{b} \left( \Phi^\dagger_h \Phi_h \right)^2 \right]. \tag{11}
\]

Suppose that the total Lagrangian is postulated to contain both (11) and a term having the form of its charge conjugate. Then the contributions that are odd in \( A_\mu \) will cancel in the condensed phase, leaving

\[
L'_h = -g \left[ \Phi^\dagger_h (g^{\mu\nu} A_\mu A_\nu) \Phi_h - \mu_h^2 \Phi^\dagger_h \Phi_h + \frac{1}{2} \overline{b} \left( \Phi^\dagger_h \Phi_h \right)^2 \right] \tag{12}
\]

In this way we arrive at a Lagrangian for the Higgs field \( \Phi_h \) which has the same form as in the standard electroweak theory.

Notice, however, that \( g^{\mu\nu} \) is not to be regarded as a dynamical variable, since it was merely used to rescale the field and volume element. This means that there is no cosmological constant due to condensed Higgs fields [3].

Also, even though (12) has a conventional form, the equation of motion for a Higgs field does not. Extremalizing \( L_h \) with respect to \( \Phi^\dagger_h \) gives

\[
- (g^{\mu\nu} D_\mu D_\nu + \overline{m} e_\alpha^\mu \sigma^\alpha D_\mu) \Phi_h - \mu_h^2 \Phi^\dagger_h \Phi_h + \overline{b} \left( \Phi^\dagger_h \Phi_h \right) \Phi_h = 0. \tag{13}
\]

At very high energies and field strengths, this goes over to the usual form

\[
- g^{\mu\nu} D_\mu D_\nu - \mu_h^2 + \overline{b} \left( \Phi^\dagger_h \Phi_h \right) \Phi_h = 0. \tag{14}
\]

At low energies and field strengths, however, it becomes

\[
\left[ i e_\alpha^\mu \sigma^\alpha D_\mu + \overline{\mu} - \overline{b} \left( \Phi^\dagger_h \Phi_h \right) \right] \Phi_h = 0 \tag{15}
\]

where \( \overline{\mu} = \mu_h^2 / \overline{m} \) and \( \overline{b} = \overline{b}/\overline{m} \). According to the statement at the end of Ref. 3, \( \overline{m} \) must be greater than roughly 0.1 TeV. If \( \mu_h \) is \( \sim 0.1 \) TeV,
therefore, the prediction represented by (13) or (15) should be testable in the near future.

Although (15) resembles the equation of motion of a fermion obtained in the next section, $\Phi_h$ is a bosonic field which transforms as a scalar under rotations.

The essential reason for the linear term in (13) is easily understood through an analogy: Suppose that hypothetical observers in a two-dimensional spacetime are aware that they are living in a superfluid (which corresponds to the GUT Higgs condensate) and wish to determine whether it is rotating. The conjectured rotation results from a vortex of strength $a$ centered on $r = 0$, so the superfluid velocity is given by $v_s = a/mr$.

$E = \frac{1}{2}mv_s^2 + mv_s v$ (17)

rather than simply $E = \frac{1}{2}mv^2$. The term that is linear in $v$ is a signature that the observers are living in a rotating rather than a stationary condensate.

In 4-dimensional spacetime, with the topology $R^1 \times S^3$, the vortex is replaced by its simplest generalization, an SU(2) instanton, and the U(1) rotations of the order parameter in two dimensions are replaced by SU(2) rotations. The two terms in the energy of (17) are completely analogous to the first two terms in the Lagrangian (10) or the equation of motion (13).

In short, the linear term in (13) is a signature of the SU(2) rotations of the Higgs condensate $\Psi_s$ which are predicted by the present theory.

3. Coupling of Fermions to Gauge Fields

Because of the symmetry between bosons and fermions in the present theory, (9) also holds for the fermion field $\psi$ of I(6.16). However, since $\psi$ is an anticommuting Grassmann field with $(\psi^\dagger \psi)^2 = -(\psi^\dagger)^2 \psi^2 = 0$, the self-interaction term vanishes, leaving

$$\mathcal{L}_f = -\frac{1}{2}h \left[ -\psi^\dagger \left( m^{-1} g^{\mu\nu} D_\mu D_\nu + ie^{\mu}_{\sigma} D_\mu \right) \psi \right] + h.c.$$ (18)
or
\[ \mathcal{L}_f = \frac{1}{2} g \psi'^\dagger \left( i e_\alpha^n \sigma^\alpha D_\mu + \overline{m}^{-1} g^{\mu\nu} D_\mu D_\nu \right) \psi' + h.c \] (19)
where \( \psi' = \lambda^2 \psi \) and we have again used \( g = \lambda^{-4} h \). The equation of motion for massless fermions is then
\[ [i e_\alpha^n \sigma^\alpha D_\mu + \overline{m}^{-1} g^{\mu\nu} D_\mu D_\nu] \psi' = 0. \] (20)

At low fermion energies and low field strengths, the second term can be neglected, and we obtain the usual Dirac equation for massless fermions:
\[ i e_\alpha^n \sigma^\alpha D_\mu \psi' = 0. \] (21)

On the other hand, at high field strengths there is a second-order coupling to the gauge fields: If \( \partial_\mu \psi' \) is still small compared to \( \overline{m} \), but \( A_\mu \) is comparable to or larger than \( \overline{m} \), \( \mathcal{L}_f \) contains a term
\[ - g \psi'^\dagger \left( \overline{m}^{-1} g^{\mu\nu} A_\mu A_\nu \right) \psi'. \] (22)

This second-order coupling should be observable in both real and virtual processes. For example, there will be seagull diagrams leading to the production of two gauge bosons, just as for nonrelativistic electrons interacting with photons. Notice, however, that (19) is still gauge invariant, since both terms in \( \mathcal{L}_f \) involve the covariant derivative \( D_\mu \).

4. Fermion Propagators at High Energy

The Lagrangian density for a free massless fermion is
\[ \mathcal{L}_f = \frac{1}{2} g \psi'^\dagger \left( i e_\alpha^n \sigma^\alpha \partial_\mu + \overline{m}^{-1} g^{\mu\nu} \partial_\mu \partial_\nu \right) \psi' + h.c. \] (23)
For right- and left-handed fields coupled by a Dirac mass \( m_f \), this becomes
\[ \mathcal{L}_D = \psi_R'^\dagger \left( i \sigma^\mu \partial_\mu + \overline{m}^{-1} \eta^{\mu\nu} \partial_\mu \partial_\nu \right) \psi_R \] (24)
\[ + \psi_L'^\dagger \left( i \sigma^\mu \partial_\mu + \overline{m}^{-1} \eta^{\mu\nu} \partial_\mu \partial_\nu \right) \psi_L \] (25)
\[ - m_f \psi_R'^\dagger \psi_L - m_f \psi_L'^\dagger \psi_R \] (26)
in a locally inertial coordinate system, with \( \sigma^0 = \sigma^0 \) and \( \sigma^k = -\sigma^k \). The terms which are second-order in \( \partial_\mu \) lead to a modified form for the fermion propagator \( S_0 \),
\[ S_0 = \left( -\overline{m}^{-1} \eta^{\mu\nu} p_\mu p_\nu + \not{p} - m_f + i\epsilon \right)^{-1}. \] (27)
Again, this modification should be observable in both real and virtual processes. For example, it will affect the values of running coupling constants between the electroweak and Planck scales. With the supersymmetry of the present theory included, it will be interesting to see whether \( \alpha_1^{-1} \), \( \alpha_2^{-1} \), and \( \alpha_3^{-1} \) meet at a common energy near \( 10^{13} \) TeV.
5. New Scalar Bosons

The fundamental action $I(2.7)$ of the present theory has an unconventional kind of supersymmetry, with an equal number of scalar bosons and spin 1/2 fermions. (These are the only fundamental fields, with gauge bosons and gravitons arising as excitations of the GUT Higgs condensate.) Some of the initial bosonic fields undergo condensation, as exemplified by the minimal scheme [5-7]

$$SO(10) \rightarrow SU(5) \rightarrow SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1)$$

with the required multiplets of bosonic fields indicated for each stage of symmetry-breaking. The corresponding energy scales are presumably in the vicinity of $m_P \sim 10^{15}$ TeV to $m_{GUT} \sim 10^{13}$ TeV for the first two stages, and $m_{ew} \sim 0.25$ TeV for the last. The formation of the condensate $\Psi_s$ begins with the first stage, but at low energies all three sets of condensed bosonic fields contribute.

Suppose that there are initially 3 families of fermions, each with 16 fields (corresponding to the spinorial representation of $SO(10)$). There are then the same number of fundamental bosonic fields (each with two complex components), for a total of $3 \times 16 \times 2 \times 2 = 192$ real degrees of freedom. Many of these will be lost near the GUT scale, due to condensation and the growth of gauge-boson masses. Four of them will participate in the symmetry-breaking of the standard model at the electroweak scale. Others, however, will behave as normal scalar bosons.

If the supersymmetry of the present theory is to prevent the usual quadratic divergence for the electroweak Higgs boson, then there must be new scalar bosons with masses near 1 TeV. The present theory thus predicts supersymmetric partners of spin 1/2 fermions which may be observable at Tevatron or LHC energies.

6. W-bosons and the Equivalence Principle

The first term in (12) leads, as usual, to a mass term in the equation of motion of $W^\pm$ and $Z^0$ bosons. Recall, however, that the original form is

$$\mathcal{L}_w = -\lambda^{-2} h \Phi^\dagger h A_\mu A_\nu \Phi h.$$  \hfill (29)

It is valid to assume that $\lambda^{-2} h \approx g g^{\mu\nu}$ only in those regions of space-time where the gravitational field is weak. (The assumption $g^{\mu\nu} = \lambda^2 h^{\mu\nu}$ is supposed to hold only in such regions — e.g., in interstellar space during the current epoch of cosmological time.) In a gravitational field, $g^{\mu\nu}$ is not proportional to $h^{\mu\nu}$, and $\mathcal{L}_w$ will consequently not assume its conventional form in a locally inertial coordinate system.
Since ordinary matter contains only a very small concentration of virtual W-bosons, the resulting violation of the equivalence principle should also be very small, but it is in principle observable.

7. Cosmological Parameters

In discussing the implications for cosmology, let us begin with a hypothetical universe which contains no matter or radiation. Since it costs action (in a Euclidean picture) or energy (in a Lorentzian picture) to form new instantons, the topological charge is fixed as a function of the time $x^0$. According to Ref. 4, this implies that

$$ R(t) = (m/a) (x^0)^2 \quad \text{with} \quad dt = e_0 dx^0 $$

(30)

where

$$ e_0 = e_{\mu=0}^{\alpha=0} \quad \text{and} \quad e^0 = e_{\alpha=0}^{\mu=0} \quad \text{so that} \quad e_0 e^0 = 1. $$

(31)

The arguments of Refs. 3 and 4 also imply that $e_0 = (m/a) x^0$ for a universe in which the GUT Higgs condensate $\Psi_s$ is fully formed. Since the density of matter $\rho$ declines as $R^{-3}$, both of the above assumptions are valid in the remote future, and we have

$$ \frac{dR}{dt} = 2 \quad \text{in the limit} \quad t \to \infty. $$

(32)

The present theory thus predicts a coasting universe with

$$ q \equiv -\frac{2d^2R/dt^2}{H^2R} \to 0 $$

(33)

as $t \to \infty$. In the early universe, however, gravity dominates and Einstein’s field equations are satisfied to a very good approximation. Suppose that $\Omega_m \approx 0.3$ [8, 9], where

$$ \Omega_m = \frac{\rho}{\rho_c}, \quad \rho_c \equiv \left( \frac{3}{8\pi G} \right) H^2, \quad \text{and} \quad H \equiv \frac{dR/dt}{R}. $$

(34)

Suppose also that $h \approx 0.65$ [8, 9] where $H_0 = 100 h$ km s$^{-1}$Mpc$^{-1}$. An interpolative model then gives

$$ q_0 \approx 0.15 \quad \text{and} \quad t_0 \approx 12.5 \text{ Gyr}. $$

(35)

One could, of course, add a cosmological constant which would make $q_0$ negative and would yield a larger age for the universe, but such a term would be just as ad hoc in the present theory as it is in standard cosmology. Recent observations of Ia supernovae suggest an accelerating universe [11, 12], or a negative value of $q_0$, but there are still uncertainties in the interpretation of this data [13-15].
8. Scale-Invariant Density Fluctuations

In the very early universe, there is again negligible matter and radiation, so the form (30) still holds. As described in Refs. 2 and 4, this form follows from the assumption of a cosmological instanton centered (in four-dimensional Euclidean spacetime) on the point \( x^0 = 0 \). The “superfluid velocity” \( v^k_a \) (with \( k, a = 1, 2, 3 \)) diverges at this point, with

\[
v^k_a = \delta^k_a \pi/m x^0.
\]

(36)

However, this singularity is precisely analogous to the singularity at the center of a vortex in an ordinary superfluid, with

\[
n_s = \Psi_s^\dagger \Psi_s \to 0 \quad \text{as} \quad t \to 0.
\]

(37)

The Big Bang singularity at \( t = 0 \) is thus physically admissible in the present theory.

Since the initial cosmological instanton has a large strength \( \pi \), it at first appears that the condensate density \( n_s \) will remain very small until a late time \( x^0 \sim \pi/m \): With \( \Psi_s = n_s^{1/2} U \eta \), \( r = x^0 \), and the scalings \( \rho = r/\xi \), \( f = (n_s/\pi)^{1/2} \), \( \xi = (2m\mu)^{-1/2} \), \( \pi_s = \mu/b \), the generalized Bernoulli equation I(3.26) becomes

\[
- \frac{1}{f} \frac{1}{\rho^2} \frac{d}{d\rho} \left( \rho^2 \frac{df}{d\rho} \right) + \frac{3\pi^2}{\rho^2} + f^2 = 1
\]

(38)

in the original Euclidean picture of I. The asymptotic solution is

\[
f = C\rho^n \quad \text{as} \quad \rho \to 0
\]

(39)

where \( C \) is a constant and \( n = \left(1 + 3\pi^2\right)^{1/2} - 1 \), so

\[
n_s \propto (x^0)^{2n} \quad \text{as} \quad x^0 \to 0.
\]

(40)

For \( x^0 \ll \pi/m \), therefore, \( n_s \) is very flat and very nearly equal to zero.

The above behavior, however, assumes no topological defects other than the instanton and no fields other than \( \Psi_s \). As mentioned in I, there may also be monopole-like defects which act as sources of the \( U(1) \) velocity \( v^\alpha_0 \). Although it costs action or energy to form such defects, there is a net saving because the term involving \( v^\alpha_0 \) tends to cancel the term involving \( v^k_a \) in the Bernoulli equation

\[
P + \frac{1}{2}m \left( -v^0_\alpha v^\alpha_0 + v^k_a v^k_\alpha \right) + V = \mu
\]

(41)

so the GUT Higgs field can condense far more rapidly. (The above equation is the Lorentzian form of I(3.26), with (38) as the Euclidean version when
Suppose that, very near $x^0 = 0$, such defects are produced by quantum fluctuations at a rapid rate which is proportional to the 3-volume and constant with respect to $x^0$. Then $e^0$ grows linearly with $x^0$:

$$e^0 = \frac{1}{2} \bar{H} x^0 \quad \text{with} \quad \bar{H} = \text{constant}. \quad (42)$$

This means that $e_0 = 2/(\bar{H} x^0)$, so that $dt = e_0 dx^0 = 2 dx^0/ (\bar{H} x^0)$ and

$$t = \left(\frac{2}{\bar{H}}\right) \log x^0 + \text{constant}. \quad (43)$$

It follows that $x^0 (t) = x^0 (0) \exp (\bar{H} t/2)$ and, according to (30),

$$R(t) = R(0) \exp (\bar{H} t). \quad (44)$$

There is then an exponential increase in the cosmic scale factor $R$ as a function of the proper time $t$, just as in inflationary scenarios. Since the fermion fields $\psi$, the scalar boson fields $\phi$, the gauge fields $A^\mu_i$, and the gravitational field $g_{\mu\nu}$ experience $t$ as the physical time (to a good approximation), (44) provides a potential mechanism for producing the scale-invariant Harrison-Zel’dovich density fluctuations which are observed in the cosmic background radiation [16-19].

9. Superheavy Sterile Neutrinos as Dark Matter

Since the present theory contains an SO(10) grand unified gauge theory, it predicts an additional right-handed field $\nu^R$ for each family of fermions, in addition to the 15 fields of the standard model. Suppose that this additional field has a large Majorana mass $M_\nu$ which results from a Yukawa coupling to a Higgs field $H$ near the GUT scale, involving a term with the form

$$\nu^R C \nu^R H. \quad (45)$$

Such a term may arise from radiative corrections, and it will lead, through the seesaw mechanism, to naturally small masses for ordinary left-handed neutrinos [5-7].

In addition, however, there will be superheavy right-handed neutrinos. Sterile neutrinos of this kind will interact only gravitationally, so they will be stable for cosmologically long times. Such neutrinos are a candidate for dark matter, provided they can be produced in the right abundance in the early universe [20, 21].

In the “inflationary” scenario of the preceding section, $\langle H \rangle$ is very small near $t = 0$. It follows that the right-handed neutrinos $\nu^R$ will have very small masses during the period when $R(t) \propto \exp (\bar{H} t)$, and that they can
be easily produced by quantum fluctuations. After this period, formation of the condensate \( \langle H \rangle \) causes each \( \nu_R \) to grow a mass near the GUT scale, allowing these particles to play the role of cold dark matter in subsequent structure formation.

Although superheavy sterile neutrinos do not appear to be directly observable in terrestrial dark-matter or neutrino-oscillation experiments, they might have a noticeable effect on proton decay.

10. Other Features of the Theory

The present theory begins with the simplest imaginable action for a world which contains both bosons and fermions. The continuum version is given by I(2.7):

\[
S = \int d^{D}x \left[ \frac{1}{2m} \partial^{M} \Psi^{\dagger} \partial_{M} \Psi - \mu \Psi^{\dagger} \Psi + \frac{1}{2} b (\Psi^{\dagger} \Psi)^{2} \right].
\]  

(46)

\( \Psi \) is called a statistical superfield because its Euclidean action has the same form as \( \beta H \) for standard models in statistical mechanics. The Euclidean path integral, of course, has the same form as a partition function in statistical mechanics:

\[
Z = \int \mathcal{D} \Psi \mathcal{D} \Psi^{\dagger} e^{-S}.
\]  

(47)

Although the initial action is extremely simple, it can lead to the full richness of nature because of spontaneous symmetry-breaking and the formation of topological defects. Three types of topological defects are of central importance: the cosmological U(1) \( \times \) SU(2) instanton discussed in Refs. 2 and 4; an internal instanton which gives rise to the SO(10) gauge symmetry; and Planck-scale instantons which account for the curvature of both gauge fields and the gravitational field.

In addition to its simplicity, the present theory has several other favorable features, including the following:

- In conventional higher-dimensional theories, it is difficult to understand why the internal space with \((D - 4)\) dimensions is a compact manifold with a size comparable to the Planck length \( \ell_{P} \). In the present theory, however, the effective internal space is automatically a d-sphere with volume

\[
V_{d} \sim \ell_{P}^{d},
\]  

(48)

as indicated in I(7.23).
• With $d = 9$, one automatically obtains an SO(10) gauge group. The present theory thus implies neutrino masses, with about the right magnitudes to explain recent and established experiments involving atmospheric and solar neutrinos [22-27].

• Family replication results naturally if the initial group $G$ of I is slightly larger than Spin(10). I.e., the gauge group is determined by the nature of the internal space $V_d$, but the number of fermion species is determined separately, by the number of components in the original fermion field $Ψ_f$ of I(2.13).

• The theory is economical, in that it does not lead to vast numbers of new fields or particles. At the same time, it implies a wide variety of new phenomena in high energy physics and astrophysics.

11. Conclusion

The present theory leads to a large number of testable predictions. Perhaps the most promising for the near future is represented by (13): a scalar Higgs boson with an unconventional equation of motion. In addition, the supersymmetry of the theory implies a set of new scalar bosons near and above 1 TeV. The other predictions listed in Section 1 are also directly relevant to experiments and observations planned for the next 5-10 years, which promise to be an exceptionally productive period for both astrophysics and high energy physics.

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