Pushdown Exception-Flow Analysis of Object-Oriented Programs

Shuying Liang\textsuperscript{1}, Matthew Might\textsuperscript{1}, Thomas Gilray\textsuperscript{1}, and David Van Horn\textsuperscript{2}

\textsuperscript{1} University of Utah, Salt Lake City, Utah, USA, \{liangsy, might, tgilray\}@cs.utah.edu, \\
\textsuperscript{2} University of Northeastern University, dvanhorn@ccs.neu.edu

Abstract. Statically reasoning in the presence of and about exceptions is challenging: exceptions worsen the well-known mutual recursion between data-flow and control-flow analysis. The recent development of pushdown control-flow analysis for the $\lambda$-calculus hints at a way to improve analysis of exceptions: a pushdown stack can precisely match catches to throws in the same way it matches returns to calls. This work generalizes pushdown control-flow analysis to object-oriented programs and to exceptions. Pushdown analysis of exceptions improves precision over the next best analysis, Bravenboer and Smaragdakis’s Doop, by orders of magnitude. By then generalizing abstract garbage collection to object-oriented programs, we reduce analysis time by half over pure pushdown analysis. We evaluate our implementation for Dalvik bytecode on standard benchmarks as well as several Android applications.

1 Introduction

Exceptions are not exceptional enough. Thrown exceptions—or the possibility thereof—pervade the control-flow structure of modern object-oriented programs. A static analyzer grappling with Java must concede that even innocent-looking expressions like

$$x / \text{in.read()}$$

could throw four exceptions: \texttt{ArithmeticException} (for divide by 0); \texttt{IOException} (for reading); \texttt{NullPointerException} (for dereferencing \texttt{in}); and technically even \texttt{MethodNotFoundException} (if the \texttt{read} method was removed after this file was compiled).

To make sense of a program, a static analyzer must exploit data-flow information to rule out exceptions (such as \texttt{NullPointerException} and \texttt{MethodNotFoundException} in the prior expression). Yet, precise data-flow information requires a precise analysis of exceptions. Co-analyzing data- and exception-flow is essential for precision. Yet, even then, many exceptions (like \texttt{IOException} or \texttt{ArithmeticException} in the prior expression) cannot be ruled out statically. It is critical to precisely match catchers to throwers.
Exception-flow fundamentally follows the structure of the program stack at run-time. Because the stack can grow without bound, traditional analysis regimes like \( k \)-CFA \cite{27} and its many variants implicitly or explicitly finitize the stack during abstraction. In effect, analyzers carve up dynamic return points and exception-handling points among a finite number of abstract return contexts. When two dynamic return points map to the same abstract context, the analyzer loses the ability to distinguish them. This confusion is a control-flow analog of the classic data-flow value merging problem.

To ground this discussion, consider the following Java fragment:

```java
try {
    maybeThrow(); // Call 1
} catch (Exception e) {
    System.err.println("Got an exception"); // Handler 1
}
maybeThrow(); // Call 2
```

Under a monovariant abstraction like 0CFA \cite{27}, where the distinction between different invocations of the same procedure are lost, it will seem as though exceptions thrown from Call 2 can be caught by Handler 1.

The fundamental problem with the analysis of exceptions is that the abstract program stack is finite. Our message is that pushdown analysis, which does not finitize the program stack, is critical for precise analysis of exception-handling, yet it remains computable. Simply switching to pushdown analysis yields orders of magnitude improvements in precision over Doop \cite{6}, the current state of the art exception-flow analysis.

Spotting an easy opportunity to improve running time, we reduce the state-space via abstract garbage collection \cite{17}. We then further improve running time and precision by combining abstract garbage collection with live-range analysis. In the end, we cut the time cost of pushdown exception-flow analysis by half.

Our implementation for Java (which targets the Dalvik virtual machine for Android) is publicly available:

[https://github.com/shuyingliang/pushdownoo](https://github.com/shuyingliang/pushdownoo)

1.1 Contributions

We make several contributions:

1. The first application of the abstracting abstract machines (AAM) methodology \cite{29} to create a static analyzer for Java.
2. A pushdown flow analysis for precisely co-analyzing data-, control- and exception-flow.
3. An empirical evaluation demonstrating two orders of magnitude of precision improvement over the current best analysis for exception-flow within reasonable analysis time.
2 The setting: An object-oriented bytecode

In this section, we define an object-oriented bytecode language closely modeled on the Dalvik virtual machine to which Java applications for Android are compiled. Subsequent sections develop our analysis for this language.

2.1 Syntax

The syntax of the bytecode language is given in Figure 1. Statements encode individual actions for the machine; atomic expressions encode atomically computable values; and complex expressions encode expressions with possible non-termination or side effects. There are four kinds of names: Reg for registers, ClassName for class names FieldName for field names and MethodName for method names. There are two special register names: ret, which holds the return value of the last function called, and exn, which holds the most recently thrown exception.

The syntax is largely usual for an Java-like bytecode, but let us explain the statements related to exceptions in method-def in more detail:

- (throws class-name . . . ) indicates that a method makes a throws declaration.
- (push-handler class-name label) pushes a handler frame on the stack. The frame will catch exceptions of type class and divert execution to label.
- (pop-handler) pops the top-most handler frame off the stack.

With respect to a given program, we assume a syntactic metafunction $S : Label \rightarrow Stmt^*$, which maps a label to the sequence of statements that start with that label.

2.2 Concrete semantics

Interpretation of bytecode programs is defined in terms of a CESK-style machine model. States of this machine consist of a series of statements, a frame pointer, a heap, and a stack. The evaluation of a program is defined as the set of machine configurations reachable by machine transitions from the initial program. Formally, the evaluation function, $E : Stmt^* \rightarrow P(Conf)$, is defined as:

$$E(s) = \{ c : I(s) \Rightarrow^* c \}.$$ 

This function injects, using $I : Stmt^* \rightarrow Conf$, an initial program sequence into an initial machine configuration. From this initial configuration, evaluation is defined by the set of configurations reached by the reflexive, transitive closure of the machine transition relation, $\Rightarrow \subseteq Conf \times Conf$. The next section describes the details of machine configurations; the subsequent section defines the machine transition relation, $(\Rightarrow)$. 
program ::= class-def . . .
class-def ::= (attribute . . . class class-name extends class-name
(fld-def . . .) (meth-def . . .))
field-def ::= (field attribute . . . field-name type)
method-def ∈ MethodDef ::= (method attribute . . . method-name (type . . .) type
(throws class-name . . .) (limit n) s . . .)
s ∈ Stmt ::= (label label) | (nop) | (line int) | (goto label)
| (if f (goto label)) | (assign name [f | ce]) | (return f)
| (field-put fn, field-name fn) | (field-get $name fn, field-name)
| (push-handler class-name label) | (pop-handler) | (throw f)
f ∈ AExp ::= this | true | false | null | void | name | int
| (atomic-op f . . . f) | (instance-of (f, class-name))
ce ::= (new class-name) | (invoke-kind (f . . . f) (type0 . . . typen))
invoke-kind ::= invoke-static | invoke-direct | invoke-virtual | invoke-interface | invoke-super
type ::= class-name | int | byte | char | boolean
attribute ::= public | private | protected | final | abstract.

Fig. 1: An object-oriented bytecode adapted from the Android specification [21].

2.3 Concrete configuration-space

Figure 2 presents the machine’s concrete configuration-space. The machine has an explicit stack, which under structural abstraction will become the stack component of a pushdown system. The stack contains not only call frames, but also mini-frames for exception handlers. The FramePointer is the environmental component of the machine: by pairing the frame pointer with a register name, it forms the address of its value in the store.

The initial configuration consists of the program, the initial frame pointer, an empty heap, and an empty stack:

\[ c_0 = I(s) = (s, fp_0, [], []). \]

2.4 Concrete transition relation

In this section, we describe the essential cases of the (⇒) relation, which deal with objects and exceptions. The remaining cases are in Appendix A.2.

The machine relies on helper functions for evaluating atomic expressions, looking up field values, and allocating memory:

- \( A : \text{AExp} \times \text{FramePointer} \times \text{Store} \rightarrow \text{Val} \) evaluates atomic expressions:

\[ A(name, fp, \sigma) = \sigma(fp, name) \quad \text{[variable look-up]} \]
\[c \in \text{Conf} = \text{Stmt}^* \times \text{FramePointer} \times \text{Store} \times \text{Kont}\] configurations
\[\sigma \in \text{Store} = \text{Addr} \rightarrow \text{Val}\] stores
\[a \in \text{Addr} = \text{RegAddr} + \text{FieldAddr}\] addresses
\[ra \in \text{RegAddr} = \text{FramePointer} \times \text{Reg}\] allocations
\[fa \in \text{FieldAddr} = \text{ObjectPointer} \times \text{FieldName}\] allocation
\[\kappa \in \text{Kont} = \text{Frame}^*\] continuations
\[\phi \in \text{Frame} = \text{CallFrame} + \text{HandlerFrame}\] continuations
\[\chi \in \text{CallFrame} ::= \text{fun}(fp, s)\] continuations
\[\eta \in \text{HandlerFrame} ::= \text{handle}(\text{class-name}, label)\] continuations
\[d \in \text{Val} = \text{ObjectValue} + \text{String} + \mathbb{Z}\] values
\[ov \in \text{ObjectValue} = \text{ObjectPointer} \times \text{ClassName}\] values
\[fp \in \text{FramePointer} \text{ is an infinite set of frame pointers}\] harms
\[op \in \text{ObjectPointer} \text{ is an infinite set of object pointers}\] harms

Fig. 2: The concrete configuration-space.

- \(A_f : \text{AExp} \times \text{FramePointer} \times \text{Store} \times \text{FieldName} \rightarrow \text{Val}\) looks up fields:

\[
A_f(\alpha, fp, \sigma, \text{name}) = \sigma(\text{op}, \text{name}) \quad \text{[field look-up]}
\]

where \((\text{op}, \text{class-name}) = A(\alpha, fp, \sigma)\).

Allocation FramePointer and ObjectPointer determine addresses for RegAddr and FieldAddr respectively. We need to specify how to allocate these pointers:

- allocFP : Conf \rightarrow FramePointer chooses a fresh frame pointer for newly invoked method.
- allocOP : Conf \rightarrow ObjectPointer, allocates a fresh object pointer in the instantiation site.

For the sake of defining a concrete semantics, these could allocate increasingly larger natural numbers. Under abstraction, these parameters provide the knob to tune the polyvariance, context-sensitivity and object-sensitivity of the resulting analysis.

New object creation Creating an object allocates a new object pointer, creates a fresh address for the register and initializes the fields:

\[c \in \text{Conf} = \text{Stmt}^* \times \text{FramePointer} \times \text{Store} \times \text{Kont}\] configurations
\[\sigma \in \text{Store} = \text{Addr} \rightarrow \text{Val}\] stores
\[a \in \text{Addr} = \text{RegAddr} + \text{FieldAddr}\] addresses
\[ra \in \text{RegAddr} = \text{FramePointer} \times \text{Reg}\] allocations
\[fa \in \text{FieldAddr} = \text{ObjectPointer} \times \text{FieldName}\] allocation
\[\kappa \in \text{Kont} = \text{Frame}^*\] continuations
\[\phi \in \text{Frame} = \text{CallFrame} + \text{HandlerFrame}\] continuations
\[\chi \in \text{CallFrame} ::= \text{fun}(fp, s)\] continuations
\[\eta \in \text{HandlerFrame} ::= \text{handle}(\text{class-name}, label)\] continuations
\[d \in \text{Val} = \text{ObjectValue} + \text{String} + \mathbb{Z}\] values
\[ov \in \text{ObjectValue} = \text{ObjectPointer} \times \text{ClassName}\] values
\[fp \in \text{FramePointer} \text{ is an infinite set of frame pointers}\] harms
\[op \in \text{ObjectPointer} \text{ is an infinite set of object pointers}\] harms

New object creation Creating an object allocates a new object pointer, creates a fresh address for the register and initializes the fields:

\[
\text{assign name (new class-name)) : s, fp, \sigma, \kappa} \Rightarrow (s, fp, \sigma'', \kappa), \text{ where} \\
\text{op} = \text{allocOP}(c) \\
\sigma' = \sigma[(fp, \text{name}) \mapsto (op, \text{class-name})] \\
\sigma'' = \text{initObject}(\sigma', \text{class-name}).
\]
The helper function, \( \textit{initObject} : \text{Store} \times \text{Name} \rightarrow \text{Store} \), initializes the field addresses in the provided store.

**Instance field reference/update** Referencing a field gets the object pointer and then grabs the field value as an offset:

\[
\begin{align*}
\mathrm{[[\text{field-get name } x_o \ \text{field-name} : s], fp, \sigma, \kappa]} & \Rightarrow (s, fp, \sigma', \kappa), \\
\text{where } \sigma' &= \sigma[(fp, \text{name}) \mapsto \mathcal{A}_x(x_o, fp, \sigma, \text{field-name})].
\end{align*}
\]

Updating a field grabs the object, extracts the object pointer and updates the associated field in the store:

\[
\begin{align*}
\mathrm{[[\text{field-put } x_o \ \text{field-name } x_v : s], fp, \sigma, \kappa]} & \Rightarrow (s, fp, \sigma', \kappa), \\
\text{where } \sigma' &= \sigma[(op, \text{name}) \mapsto \mathcal{A}(x_v, fp, \sigma)] \\
(op, \text{class-name}) &= \mathcal{A}(x_o, fp, \sigma).
\end{align*}
\]

**Method invocation** Method invocation involves all four components of the machine. Since the language supports inheritance, method resolution requires a traversal of the class hierarchy. This traversal is not of interest, so we focus on the helper function that performs method application: \( \textit{applyMethod} \). The function \( \textit{applyMethod} \) takes a method definition, arguments, a frame pointer, a store and a continuation and produces the next configuration:

\[
\begin{align*}
\textit{applyMethod} : \text{MethodDef} \times \text{AExp}^* \times \text{FramePointer} \times \text{Store} \times \text{Kont} \rightarrow \text{Conf}.
\end{align*}
\]

It looks up the values of the arguments, binds them to the formal parameters of the method, creates a new frame pointer and a new continuation:

\[
\text{applyMethod}(m, \alpha, fp, \sigma, \kappa) = (s, fp', \sigma', (fp, s) : \kappa), \text{where } fp' \text{ is fresh, }
\sigma' = \sigma[(fp', \text{name}_i) \mapsto \mathcal{A}(x_i, fp, \sigma)].
\]

Finally, the transition looks up the method \( m \) and then passes it to \( \textit{applyMethod} \):

\[
\begin{align*}
\mathrm{[[\text{invoke-kind } (\alpha_0 \ldots \alpha_n)(\text{type}_0 \ldots \text{type}_n)]] : s, fp, \sigma, \kappa]} & \Rightarrow \textit{applyMethod}(m, \alpha, fp, \sigma, \kappa).
\end{align*}
\]

**Procedure return** Returning a value restores the caller’s context and puts the return value in the dedicated return register, ret.

\[
\begin{align*}
\mathrm{[[\text{return } \alpha]] : s, fp, \sigma, \text{fun}(fp', s') : \kappa]} & \Rightarrow (s', fp', \sigma', \kappa), \text{where } \\
\sigma' &= \sigma[(fp', \text{ret}) \mapsto \mathcal{A}(\alpha, fp, \sigma)].
\end{align*}
\]

If a \( \text{HandlerFrame} \) is on top of the stack, the transition will pop it without changing any other part of the state:

\[
\begin{align*}
\mathrm{[[\text{return } \alpha]] : s, fp, \sigma, \text{handle}(\text{class-name label}) : \kappa]} & \Rightarrow (\mathrm{[[\text{return } \alpha]] : s, fp, \sigma, \kappa}).
\end{align*}
\]
Pushing and popping exception handlers Pushing and popping exception handlers is straightforward:

\[
([], \text{push-handler } \text{class-name } \text{label}) : s, fp, \sigma, \kappa \Rightarrow (s, fp, \sigma, \text{handle}(\text{class-name } \text{label}) : \kappa),
\]

\[
([], \text{pop-handler}) : s, fp, \sigma, \text{handle}(\text{class-name } \text{label}) : \kappa \Rightarrow (s, fp, \sigma, \kappa).
\]

Throwing and catching exceptions The \textit{throw} statement peels away layers of the stack until it finds a matching exception handler:

\[
([], \text{throw } \text{exn}) : s, fp, \sigma, \kappa \Rightarrow \text{handle}(\text{exn}, s, fp, \sigma, \kappa),
\]

where the function \textit{handle} : \textsf{AExp} \times \textsf{Stmt}^* \times \textsf{FramePointer} \times \textsf{Store} \times \textsf{Kont} \to \textsf{Conf} does the peeling. If a matching handler is found, that is, \textit{class-name} is a subclass of \textit{class-name}', where \((\text{op, class-name}) = \mathcal{A}(\text{exn}, \sigma)\) and \textit{class-name}' is from the top \textit{HandlerFrame}, the execution flow jumps to code block of the handler:

\[
\text{handle}(\text{exn}, s, fp, \sigma, \text{handle}(\text{class-name}' \text{label}) : \kappa') = \left(\text{S}(\text{label}), fp, \sigma[(fp, \text{exn}) \mapsto (\text{op, class-name}'], \kappa')\right).
\]

The last thrown exception object value will be put in the dedicated exception register \textit{exn}.

If the exception type does not match or it’s a call frame, then \textit{handle} transits to a configuration with the control state unchanged but with the top frame popped:

\[
\text{handle}(\text{exn}, s, fp, \sigma, \text{handle}(\text{class-name}' \text{label}) : \kappa') = ([], \text{throw } \text{exn}) : s, fp, \sigma, \kappa')
\]

\[
\text{handle}(\text{exn}, s, fp, \sigma, \text{fun}(fp', s') : \kappa') = ([], \text{throw } \text{exn}) : s, fp, \sigma, \kappa').
\]

The abstraction of these “multi-pop” transition relations will require modification of the algorithm used for control-state reachability (Section 6.1).

3 Pushdown abstract semantics

With the concrete semantics in place, it is time to abstract them into an analysis. To achieve a pushdown analysis, we abstract less than we normally would. Specifically, we conduct a structural abstraction of the concrete state-space and leave the stack height unbounded rather than thread frames through the heap.

3.1 Abstract semantics

Abstract interpretation is defined in terms of a structural abstraction of the machine model of Section 2. The evaluation of a program is defined as the set of \textit{abstract} machine configurations reachable by an abstraction of the machine
transitions relation. Largely, abstract evaluation, \( \hat{E} : \text{Stmt}^{*} \rightarrow \mathcal{P}(\text{Conf}) \), mimics its concrete counterpart:

\[
\hat{E}(s) = \{ \hat{c} : \hat{I}(s) \leadsto^{*} \hat{c} \}.
\]

Abstract evaluation is defined by the set of configurations reached by the reflexive, transitive closure of the \((\leadsto)\) relation, which abstracts the \((\Rightarrow)\) relation.

### 3.2 Abstract configuration-space

Figure 3 details the abstract configuration-space. We assume the natural element-wise, point-wise and member-wise lifting of a partial order across this state-space.

\[
\hat{c} \in \text{Conf} = \text{Stmt}^{*} \times \text{FramePointer} \times \hat{\text{Store}} \times \hat{\text{Kont}} \quad \text{[configurations]}
\]

\[
\hat{\sigma} \in \hat{\text{Store}} = \text{Addr} \rightarrow \hat{\text{Val}} \quad \text{[stores]}
\]

\[
\hat{a} \in \hat{\text{Addr}} = \text{RegAddr} + \text{FieldAddr} \quad \text{[addresses]}
\]

\[
\hat{\text{ra}} \in \hat{\text{RegAddr}} = \text{FramePointer} \times \text{Reg}
\]

\[
\hat{fa} \in \hat{\text{FieldAddr}} = \text{ObjectPointer} \times \text{FieldName}
\]

\[
\hat{k} \in \hat{\text{Kont}} = \text{Frame}^{*} \quad \text{[continuations]}
\]

\[
\hat{\phi} \in \hat{\text{Frame}} = \text{CallFrame} + \text{HandlerFrame} \quad \text{[stack frames]}
\]

\[
\hat{\chi} \in \text{CallFrame ::= fun}(\hat{fp}, s)
\]

\[
\hat{\eta} \in \text{HandlerFrame ::= handle}(\text{class-name}, \text{label})
\]

\[
\hat{d} \in \hat{\text{Val}} = \mathcal{P} \left( \text{ObjectValue} + \text{String} + \hat{\mathcal{Z}} \right) \quad \text{[abstract values]}
\]

\[
\hat{\text{ov}} \in \text{ObjectValue} = \text{ObjectPointer} \times \text{ClassName}
\]

\[
\hat{fp} \in \text{FramePointer} \text{ is a finite set of frame pointers} \quad \text{[frame pointers]}
\]

\[
\hat{op} \in \text{ObjectPointer} \text{ is a finite set of object pointers} \quad \text{[object pointers]}
\]

Fig. 3: The abstract configuration-space.

To synthesize the abstract state-space, we force frame pointers and object pointers (and thus addresses) to be a finite set, but crucially, we leave the stack untouched. When we compact the set of addresses into a finite set, the machine may run out of addresses to allocate, and when it does, the pigeon-hole principle will force multiple abstract values to reside at the same address. As a result, we have no choice but to force the range of the \( \hat{\text{Store}} \) to become a power set in the abstract configuration-space.
3.3 Abstract transition relation

The abstract transition relation has components analogous to those from the concrete semantics:

- $I: \text{Stmt}^* \rightarrow \hat{\text{Conf}}$ injects an sequence of instructions into a configuration:
  \[ \hat{c}_0 = \hat{I}(s) = (s, \hat{fp}_0, [], []). \]

- $\hat{A}: \text{AExp} \times \text{FramePointer} \times \text{Store} \rightarrow \hat{\text{Val}}$ evaluates atomic expressions:
  \[ \hat{A}(\text{name}, \hat{fp}, \sigma) = \sigma(\hat{fp}, \text{name}) \quad \text{[variable look-up]} \]

- $\hat{A}_F: \text{AExp} \times \text{FramePointer} \times \text{Store} \times \text{FieldName} \rightarrow \hat{\text{Val}}$ looks up fields:
  \[ \hat{A}_F(\text{x}, \hat{fp}, \sigma, \text{field-name}) = \bigcup \tilde{\sigma}(\tilde{op}, \text{field-name}) \quad \text{[field look-up]} \]
  where $(\tilde{op}, \text{class-name}) \in \hat{A}(\text{x}, \hat{fp}, \sigma)$.

Because there are an infinite number of abstract configurations, a naive implementation of the $\hat{E}$ function may not terminate.

Appendix A.14 discusses abstractions of $\text{allocFP}$ and $\text{allocOP}$ that allow the selection of different analyses such as $k$-CFA or polymorphic splitting.

The rules for the abstract transition relation $(\rightarrow) \subseteq \hat{\text{Conf}} \times \hat{\text{Conf}}$ largely mimic the structure of the concrete relation $(\Rightarrow)$. The biggest difference is that the structural abstraction forces the abstract transition to become non-deterministic. We detail these rules below and illustrate the differences from its concrete counterpart. Again, we only cover rules involving objects and exceptions. Appendix A.3 contains the remaining rules.

New object creation

Creating an object allocates a potentially non-fresh object pointer and joins the newly initialized object into that store location:

\[
\begin{aligned}
\hat{c} &= \hat{I}(s) = (s, \hat{fp}_0, [], []). \\
\tilde{op}' &= \text{allocOP}(\hat{c}) \\
\tilde{\sigma}' &= \tilde{\sigma} \sqcup [(\hat{fp}, \text{name}) \mapsto (\tilde{op}', \text{class-name})] \\
\tilde{\sigma}'' &= \text{initObject}(\tilde{\sigma}', \text{class-name})
\end{aligned}
\]

where the helper $\text{initObject}: \text{Store} \times \text{ClassName} \rightarrow \text{Store}$ initializes fields.

Instance field reference/update

Referencing a field uses $\hat{A}_F$ to evaluate the field values and join the store for destination register:

\[
\begin{aligned}
\hat{c} &= \hat{I}(s) = (s, \hat{fp}_0, [], []). \\
\tilde{op}' &= \text{allocOP}(\hat{c}) \\
\tilde{\sigma}' &= \tilde{\sigma} \sqcup [(\hat{fp}, \text{name}) \mapsto (\tilde{op}', \text{class-name})] \\
\tilde{\sigma}'' &= \text{initObject}(\tilde{\sigma}', \text{class-name})
\end{aligned}
\]

where

\[
\begin{aligned}
\tilde{\sigma}' &= \tilde{\sigma} \sqcup [(\hat{fp}, \text{name}) \mapsto (\tilde{op}', \text{class-name})] \\
\tilde{\sigma}'' &= \text{initObject}(\tilde{\sigma}', \text{class-name})
\end{aligned}
\]
Updating a field first finds the abstract object values from the store, extracts its object pointer from each of all the possible values, then pairs this object pointer with the field name to get the field addresses, and finally joins the extensions to the store:

\[
\text{([[field-put } w_o \text{ field-name } w_r] : s], \hat{fp}, \hat{\sigma}, \hat{\kappa}) \leadsto (s, \hat{fp}, \hat{\sigma}', \hat{\kappa}), \text{ where}
\]

\[
\hat{\sigma}' = \hat{\sigma} \sqcup [([\hat{op}, \text{field-name}] \mapsto \hat{A}(w_r, \hat{fp}, \hat{\sigma})]
\]

\[
(\hat{op}, \text{class-name}) \in \hat{A}(w_o, \hat{fp}, \hat{\sigma}).
\]

**Method invocation** Like the concrete semantics, method invocation also involves all four components of the machine. The main difference is that, for non-static methods invocation, there can be a set of possible objects that are invoked, rather than only one as in its concrete counterpart. This also means that there could be multiple method definitions resolved for each object. For each such method \( m \):

\[
\text{([[invoke-kind } (w_0 \ldots w_n) \text{ (type}_0 \ldots \text{ type}_n)] : s, \hat{fp}, \hat{\sigma}, \hat{\kappa}) \leadsto \text{applyMethod}(m, w, \hat{fp}, \hat{\sigma}, \hat{\kappa}), \text{ where,}
\]

\[
\text{applyMethod}(m, w, \hat{fp}, \hat{\sigma}, \hat{\kappa}) = (s, \hat{fp}', \hat{\sigma}', (\hat{fp}, s) : \hat{\kappa}), \text{ where}
\]

\[
\hat{fp}' = \text{allocFP}(\hat{c})
\]

\[
\hat{\sigma}' = \hat{\sigma} \sqcup [(\hat{fp}', \text{name}_i) \mapsto \hat{A}(w_i, \hat{fp}, \hat{\sigma})].
\]

**Procedure return** Procedure return pops off the top-most fun frame:

\[
\text{([[return } w] : s], \hat{fp}, \hat{\sigma}, \text{fun}(\hat{fp}', s') : \hat{\kappa}) \leadsto (s', \hat{fp}', \hat{\sigma}', \hat{\kappa}), \text{ where}
\]

\[
\hat{\sigma}' = \hat{\sigma} \sqcup [(\hat{fp}', \text{ret}) \mapsto \hat{A}(w, \hat{fp}, \hat{\sigma})].
\]

If the top frame is a handle frame, the abstract interpreter pops until the top-most frame is a fun frame:

\[
\text{([[return } w] : s, \hat{fp}, \hat{\sigma}, \text{handle}(\text{class-name label}) : \hat{\kappa}) \leadsto ([[\text{return } w]) : s, \hat{fp}, \hat{\sigma}, \hat{\kappa}).
\]

**Pushing and popping handlers** Handlers push and pop as expected:

\[
\text{([[push-handler } \text{class-name label}] : s, \hat{fp}, \hat{\sigma}, \hat{\kappa}) \leadsto (s, \hat{fp}, \hat{\sigma}, \text{handle}(\text{class-name label}) : \hat{\kappa})
\]

\[
\text{([[pop-handler]) : s, \hat{fp}, \hat{\sigma}, \text{handle}(\text{class-name label}) : \hat{\kappa}) \leadsto (s, \hat{fp}, \hat{\sigma}, \hat{\kappa}).
\]
Throwing and catching exceptions  The throw statement peels away layers of the stack until it finds a matching exception handler:

\[
\text{handle}(\text{throw } \alpha) : \text{s}, \hat{fp}, \sigma, \kappa \rightarrow \text{handle}(\alpha, \text{s}, \hat{fp}, \sigma, \kappa),
\]

where the function \( \text{handle} : \text{AEExp} \times \text{Stmt}^* \times \text{FramePointer} \times \text{Store} \times \text{Kont} \rightarrow \text{Conf} \) behaves like its concrete counterpart when the top-most frame is a compatible handler:

\[
\text{handle}(\alpha, \text{s}, \hat{fp}, \sigma, \text{handle}(\text{class-name' label}) : \kappa') = (S(\text{label}), \hat{fp}, \sigma \sqcup [(\hat{fp}, \text{exn}) \mapsto (\hat{op}, \text{class-name})], \kappa').
\]

Otherwise, it pops a frame:

\[
\text{handle}(\alpha, \text{s}, \hat{fp}, \sigma, \text{fun}(\_ \_ \_ ) : \kappa') = (\text{[(throw } \alpha)] : \text{s}, \hat{fp}, \sigma, \kappa').
\]

4 The shift: From abstract CESK to pushdown systems

In the previous section, we constructed an infinite-state abstract interpretation of the CESK-like machine to analyze exception flows for object-oriented languages. The infinite-state nature of the abstraction makes it difficult to answer static analysis questions: How do you compute the reachable states if there are an infinite number of them? Fortunately, a shift in perspective reveals that the machine is in fact a pushdown system for which control-state reachability is decidable.

If we take \( \text{Stmt}^* \times \text{FramePointer} \times \text{Store} \) as the finite set of control states and \( \text{Kont} \) is the set of stacks, then it is immediately apparent that the abstract semantics that we have created is a pushdown system. This is the object-oriented analog of Earl et al.'s observation for the \( \lambda \)-calculus \cite{earl}. This shift permits the use of a control-state reachability algorithm in place of exhaustive search of the configuration-space. [Appendix Figure 9 defines the program-to-RPDS conversion function \( \text{PDS} : \text{Stmt}^* \rightarrow \text{RPDS} \) in detail.]

4.1 Mini-evaluation

In Table 2, when we compare the resulting analysis to Bravenboer and Smaragdakis’s finite-state analysis of exceptions \cite{bravenboer}, we find a solid improvement in precision, but a substantial slowdown in time. This is not surprising: computing the reachable states in a pushdown system is cubic in the number of states. In the next section, we improve the running time by porting another powerful technique from abstract interpretation of the \( \lambda \)-calculus: abstract garbage collection.
Abstract garbage collection

Abstract garbage collection for objects

Abstract garbage collection is known to yield order-of-magnitude improvements in precision, even as it drops run-times by cutting away false positives. Adapting abstract garbage collection seemed like the right tool to fix the performance problem of the previous section. We directly benefit from that line of work on the \( \lambda \)-calculus, which developed a class of introspective pushdown machines as a means of combining pushdown analysis with abstract garbage collection [10]. Introspective pushdown systems are pushdown systems that have read access to the entire stack during a transition. Since the root set for garbage collection depends on the entire stack, we need an introspective pushdown systems to use abstract garbage collection. [Appendix A.12 formalizes the injection into an introspective pushdown system.]

It’s natural to think that the combined technique will benefit exception-flow analysis for object-oriented languages. However, as we shall demonstrate, we must conduct a careful and subtle redesign of the abstract garbage collection machinery for object-oriented languages to gain the promised analysis precision and performance.

In the following, we present how to adapt abstract garbage collection to work under abstract semantics defined in Section 3. Abstract garbage collection discards unreachable elements from the store. It modifies the transition relation to conduct a “stop-and-copy” garbage collection before each transition. To do so, we define a garbage collection function \( \hat{G} : \hat{Conf} \rightarrow \hat{Conf} \) on configurations:

\[
\hat{G}(s, \hat{fp}, \hat{\sigma}, \hat{\kappa}) = (s, \hat{fp}, \hat{\sigma} | \text{Reachable}(\hat{e}), \hat{\kappa}),
\]

where the pipe operation \( f|S \) yields the function \( f \), but with inputs not in the set \( S \) mapped to bottom—the empty set. The reachability function \( \text{Reachable} : \hat{Conf} \rightarrow \mathcal{P}(\hat{Addr}) \) first computes the root set, and then the transitive closure of an address-to-address adjacency relation:

\[
\hat{\text{Reachable}}(s, \hat{fp}, \hat{\sigma}, \hat{\kappa}) = \left\{ \hat{a} : \hat{a}_0 \in \text{Root}(\hat{e}) \text{ and } \hat{a}_0 \xrightarrow{\hat{\sigma}} \hat{a} \right\},
\]

where the function \( \text{Root} : \hat{Conf} \rightarrow \mathcal{P}(\hat{Addr}) \) finds the root addresses:

\[
\text{Root}(s, \hat{fp}, \hat{\sigma}, \hat{\kappa}) = \{(\hat{fp}, r) : (\hat{fp}, r) \in \text{dom}(\hat{\sigma})\} \cup \text{StackRoot}(\hat{\kappa}),
\]

The \( \text{StackRoot} : \hat{\text{Kont}} \rightarrow \mathcal{P}(\hat{Addr}) \) function finds roots down the stack. However, only \( \text{CallFrame} \) has the component to construct addresses, so we define a helper function \( \hat{\mathcal{F}} : \hat{\text{Kont}} \rightarrow \text{CallFrame}^* \) to extract only \( \text{CallFrame} \) out from the stack and skip over all the handle frames. Now \( \text{StackRoot} \) is defined as

\[
\text{StackRoot}(\hat{\kappa}) = \{(\hat{fp}_i, r) : (\hat{fp}_i, r) \in \text{dom}(\hat{\sigma}) \text{ and } \hat{fp}_i \in \hat{\mathcal{F}}(\hat{\kappa})\},
\]
and the relation:

\[(\rightarrow) \subseteq \hat{Addr} \times \hat{Store} \times \hat{Addr}\]

connects adjacent addresses:

\[\hat{a} \rightarrow \hat{a}' \text{ iff there exists } (\hat{op}, \text{class-name}) \in \hat{\sigma}(\hat{a}) \]

such that \[\hat{a}' \in \{(\hat{op}, \text{field-name}) : (\hat{op}, \text{field-name}) \in \text{dom}(\hat{\sigma})\} .\]

**Example runs with abstract garbage collection** Table 1 presents the example results of running pushdown analysis with and without abstract garbage collection, as described. It shows that abstract garbage collection further improves the precision, but the effect is not as large as we had predicted, especially with respect of analysis time, where on functional programs, abstract garbage collection can bring order-of-magnitude reductions in both imprecision and time.

The next section teases out the problem and develops a solution: combining abstract garbage collection with live range analysis.

| Benchmark | Opts | Nodes | Edges | VarPointsTo | E-C links | Time(sec) |
|-----------|------|-------|-------|-------------|-----------|-----------|
| lusearch  | pdcfa| 91574 | 105154| (1423, 3)   | 76        | 5520      |
| +gc       | 26365| 30426 | (1086, 2)| 63          | 4800      |

Table 1: Example analysis result by (introspective) pushdown system. **VarPointsTo** measures how many objects can a variable possibly points to; it is presented as a tuple \((a, b)\), where \(a\) is the total entries, \(b\) is the average objects being invoked on; **E-C links** is the number of pairs of an instruction that can throw exceptions and a handler that can possibly handle the exception. These metrics are used by Fu, et al. [11], and Bravenboer and Smaragdakis [7].

### 5.1 Live register analysis (LRA) for AGC

Even though pushdown analysis with/without garbage collection promises to increase analysis precision, the analysis time is not satisfying, as shown in Table 1. The benchmark lusearch with abstract garbage collection still takes more than an hour. By manual inspection on some other benchmarks we have run, we find that in the register-based byte code, there are cases that the same register is reassigned multiple times at different sites within a method. Therefore, abstract object values are unnecessarily “merged” together. The result is that unnecessary state space is explored and analysis time is prolonged.

The direct adaptation of AGC to an object-oriented setting in Section 5 cannot collect these registers between uses. For object-oriented programs, we want to collect registers that are reachable, but not without an intervening assignment.

As it turns out, the fix for this problem is a classic data-flow analysis: live-register analysis (LRA). LRA can compute the set of registers that are alive at
each statement within a method. The garbage collector can then more precisely collect each frame.

Since LRA is well-defined in the literature \cite{1}, we skip the formalization here, but the Root is now modified to collect only living registers of the current statement Lives\{s\}_0:\n
\text{Root}(s, \hat{fp}, \hat{\sigma}, \hat{\kappa}) = \{(\hat{fp}, r') : (\hat{fp}, r') \in \text{dom}(\hat{\sigma}) \text{ and } r' \in \text{Lives}\{s\}_0\} \cup \text{StackRoot}(\hat{\kappa}).

Section 7 presents the complete results running on the suite of the benchmarks based on the joint analysis (denoted as \texttt{+gc+lra} in Table 2).

6 Extending pushdown reachability to exceptions

With the formalism in previous sections, it is not hard to translate the abstract semantics into working code. We use the Dyck State Graph synthesis algorithm—a purely functional version of the Summarization algorithm—for computing reachable pushdown control states \cite{10}.

6.1 Synthesizing a Dyck State Graph with exceptional flow

The Dyck State Graph (DSG) of a pushdown system is the subset of a pushdown system reachable over legal paths. (A path is legal if it never tries to pop a when a frame other than a is on top of the stack.) To synthesize a Dyck State Graph (DSG) from an (introspective) a pushdown system, Earl et al. present an efficient, functional modification of the pushdown summarization algorithm \cite{10}.

The algorithm iteratively constructs the reachable portion of the pushdown transition relation by inserting \(\epsilon\)-summary edges whenever it finds empty-stack (e.g., push a, push b, pop b, pop a) paths between control states.

For pushdown analysis \textit{without exception handling}, only two kinds of transitions can cause a change to the set of \(\epsilon\)-predecessors: an intraprocedural empty-stack transition and a frame-popping procedure return. With the addition of handle frames to the stack, there are several new cases to consider for popping frames (and hence adding \(\epsilon\)-edges).

The following subsections highlight how to handle exceptional flow during DSG synthesis, particularly as it relates to maintaining \(\epsilon\)-summary edges. The figures in these section use a graphical scheme for describing the cases for \(\epsilon\)-edge insertion. Existing edges are solid lines, while the \(\epsilon\)-summary edges to be added are dotted lines.

\textbf{Intraprocedural push/pop of handle frames} The simplest case is entering a \texttt{try} block (a \texttt{push-handler}) and leaving a \texttt{try} block (a \texttt{pop-handler}) entirely intraprocedurally—without throwing an exception. Figure 4 shows such a case: if there is a handler push followed by a handler pop, the synthesized (dotted) edge must be added.
Locally caught exceptions Figure 5 presents a case where a local handler catches an exception, popping it off the stack and continuing.

Exception propagation along the stack Figure 6 illustrates a case where an exception is not handled locally, and must pop off a call frame to reach the next handler on the stack. In this case, a popping self-edge from control state $q'$ to $q''$ lets the control state $q'$ see frames beneath the top. Using popping self-edges, a single state can pop off as many frames as necessary to reach the handle—one at a time.

Control transfers mixed in try/catch Figure 7 illustrates the situation where a procedure tries to return while a handle frame is on the top of the stack. It uses popping self-edges as well to find the top-most call frame.

Uncaught exceptions The case in Figure 8 shows popping all frames back to the bottom of the stack—indicating an uncaught exception.
7 Evaluation

We evaluated our pushdown exception flow analysis on standard Java benchmarks from the DaCapo suite \[4\] that we were able to port to Android; we have also used some native Android applications. We ran these benchmarks on OS X 10.8.2 with a 64GB DDR3 memory, 2 Six-Core Intel Xeon X5675 CPUs, 3.07GHz machine. Table 2 lists the results for all applications. To compare, we adopt metrics (and implementations) used by previous work \[11,7\] for object-oriented programs:

- \textbf{VarPointsTo}: Given a variable, to how many types may it point? Smaller sets indicate higher data-flow precision.
- \textbf{ThrowPointsTo}: At a throw, how many types of exceptions could be thrown? Smaller sets indicate higher data-flow precision.
- \textbf{Exception-Catch-Link (E-C Link)}: A pair of instructions in which second catches the first. Fewer E-C links indicate higher exception-flow precision.

The analysis result on running on Android applications of different size have already demonstrated the promise of our analytic techniques, with the average one to three on VarPointsTo and ThrowPointsTo, and small number of E-C links.

The evaluation conducted on standard Java benchmarks helps us compare results between our techniques and prior work. We use the same version of benchmark suite, the DaCapo benchmark programs, v.2006-10.MR2, which is used in \[7\]. However, only antlr, lucene, and pmd run on Dalvik bytecode, due to the Android SDK having class/interface naming clashes with the ones that are originally defined in Java SDK.

We contacted the authors for access to the original tool \texttt{Doop} \[7\] to run the above benchmarks and recompute the relative metrics. Specifically, we ran \texttt{Doop} Revision 958, on JRE 1.5 and Xubuntu 12.10 inside VirtualBox 4.2.2. The metrics we compute are VarPointsTo, E-C Links and analysis run time, with the option of context-sensitivity 1-Call+H and object-sensitivity 1-Obj+H respectively. These options are the closest to the allocation strategy in our analysis: 1-call-site sensitivity for calls, and 1-object-sensitivity for object allocation. In order to eliminate differences between the Dalvik and Java byte code, the VarPointsTo metric computes how many types can be invoked on at each call site.

The comparison result is shown in the first three rows in Table 2—the DaCapo benchmarks. We could not get \texttt{Doop} to operate properly on Android programs.

As we can see that the pushdown exception-flow analysis produces almost two orders of magnitude improvement to the precision of points-to information and E-C Links for all three benchmarks over \texttt{Doop}. We have reported running times for completeness, but these numbers can’t be compared as directly as precision. \texttt{Doop} used a high-performance Datalog engine to solve flow constraints; our implementation in Scala is asymptotically efficient, but it is not optimized; it incurs a significant constant-factor overhead.

The effect of analysis time varies from different benchmarks. But take into consideration of the difference of running environment, \texttt{Doop} demonstrated less
analysis than our analysis does. However, the co-analysis of pushdown system and augmented abstract garbage collection has demonstrated the best precision/performance trade-offs.

In Table 2 adding garbage collection and live-range analysis restriction \((+gc+lra)\) improves analysis time more significantly for Android application than Java applications. The reason is that Android applications are more sensitive to the LRA due to Android’s multi-entry points structure. However, the results on the DaCapo benchmarks clearly indicate improvements over Doop in precision.

| Benchmark | LOC  | Opt | Nodes | Edges | VarPointsTo | Throws | E-Cs | Time |
|-----------|------|-----|-------|-------|-------------|--------|------|------|
| antlr     | 35000| pdcfa +gc+lra | 1212 | 1251 | (681, 2)    | (78,2) | 65   | 4135 |
|           |      | 1-Call+H | -    | -    | (40503, 614) | -      | 2277 | >4h  |
|           |      | 1-Obj+H  | -    | -    | (41339,626)  | -      | 2203 | >3h  |
| lusearch  | 87000| pdcfa +gc+lra | 91574| 105154| (916, 3)    | (309, 3) | 76   | 5520 |
|           |      | 1-Call+H  | 14646| 16045| (709, 2)    | (213,2) | 59   | 2785 |
|           |      | 1-Obj+H   | -    | -    | (22970, 348) | -      | 2378 | 2796 |
|           |      |           | -    | -    | (24225,367)  | -      | 2304 | 1548 |
| pmd       | 55000| pdcfa +gc+lra | 59173| 61162| (1432, 3)   | (103, 3) | 51   | 4351 |
|           |      | 1-Call+H  | 3537 | 4035 | (1017, 2)   | (61,2)  | 38   | 1323 |
|           |      | 1-Obj+H   | -    | -    | (25286, 383) | -      | 2284 | 3375 |
|           |      |           | -    | -    | (26049,395)  | -      | 2212 | 3413 |
| Butane    | 2506 | pdcfa +gc+lra | 20140| 20334| (463,3)     | (2,1)  | 2    | 920  |
|           |      | 1-Call+H  | 724  | 740  | (322, 2)    | (2,1)  | 2    | 676  |
|           |      | 1-Obj+H   | 948  | 951  | (465, 2)    | (2,1)  | 2    | 28   |
| UltraCoolMap | 2605 | pdcfa +gc+lra | 17044| 17047| (552, 2)    | (2,1)  | 2    | 1056 |
|           |      | 1-Call+H  | 948  | 951  | (465, 2)    | (2,1)  | 2    | 28   |
|           |      | 1-Obj+H   | -    | -    | (26049,395)  | -      | 2212 | 3413 |
| Sysmon    | 4429 | pdcfa +gc+lra | -    | -    | (684, 2)    | (10,1) | 10   | 1534 |
| TodoList  | 6092 | pdcfa +gc+lra | -    | -    | (864, 2)    | (10,1) | 10   | 1534 |
|           |      | 1-Call+H  | -    | -    | (25286, 383) | -      | 2284 | 3375 |
|           |      | 1-Obj+H   | -    | -    | (26049,395)  | -      | 2212 | 3413 |
| SwiFTP    | 6521 | pdcfa +gc+lra | -    | -    | (634,2)     | (3,1)  | 3    | 4277 |
|           |      | 1-Call+H  | 147785| 155656| (634,2)   | (3,1)  | 3    | 4277 |
|           |      | 1-Obj+H   | 4711 | 5053 | (564, 2)    | (1,1)  | 1    | 1871 |
| MediaFun  | 7815 | pdcfa +gc+lra | -    | -    | (674, 2)    | (13,1) | 10   | 3032 |
| AndroidGame | 63755| pdcfa +gc+lra | -    | -    | (674, 2)    | (13,1) | 10   | 3032 |
|           |      | 1-Call+H  | 3851 | 4001 | (405, 3)    | (2,1)  | 2    | 989  |
|           |      | 1-Obj+H   | 1111 | 1180 | (363, 2)    | (2,1)  | 2    | 246  |
| ConnectBot | 68382| pdcfa +gc+lra | 14484| 15147| (543, 3)    | (5,1)  | 5    | 3012 |
|           |      | 1-Call+H  | 3739 | 4063 | (368, 1)    | (5,1)  | 5    | 1896 |

Table 2: Benchmark results: \(\text{VarPointsTo}\) and \(\text{Throws}\) is presented as tuples \((a, b)\), where \(a\) is the total entries, \(b\) is the average types being invoked on in \(\text{VarPointsTo}\) case, and average exception objects thrown in \(\text{Throws}\) case. All times are in seconds. \(\infty\) denotes the analysis did not finish within 6000 seconds.
8 Related Work

Precise and scalable context-sensitive points-to analysis has been an open problem for decades. Progress in general has been gradual, with results like object-sensitivity [18,19] intermittently providing a leap for most programs. Most results target improvements for individual classes of programs. The techniques we present here broadly target at all programs, and it is orthogonal to and compatible with results like object-sensitivity.

Much work in pointer analysis exploits methods to improve performance by strategically reducing precision. Lattener et al. show that an analysis with a context-sensitive heap abstraction can be efficient by sacrificing precision under unification constraints [13].

In full-context-sensitive pointer analysis, the literature has sought context abstractions that provide precise pointer information while not sacrificing performance. Milanova found that an object-sensitive analysis [19] is an effective context abstraction for object-oriented programs. This is confirmed by the extensive evaluation by Lhotak [15]. He and other researchers have also argued for using context-sensitive heap abstraction to improve precision [20].

BDDs have been used to compactly represent the large amount of redundant data in context-sensitive pointer analysis efficiently [3,31,33]. Specifically, Xu and Routev’s work [33] reduces the redundancy by choosing the right context abstractions. Such advancements could be applied to our pushdown framework, as they are orthogonal to its central thesis.

**Finite-state analysis of exceptions** The main contribution of the paper is significantly improved analysis precision via pushdown systems that analyze the exceptional control-flow of object-oriented programs.

The bulk of the previous literature has focused on finite-state abstractions for Java programs, i.e., k-CFA and its variants. Specifically, for the work that handles exception flows, the analysis is based on context-insensitivity or a limited form of context-sensitivity, which makes them unable to differentiate the contexts of where an exception is thrown and what handlers precisely can handle the exception. Robillard et al. [25] presents a truly interprocedural exception-flow analysis, but exceptions propagate via imprecise control flows by using class hierarchy analysis. The same is true for Jo et al. [32], and its extension for concurrent Java programs [26]. Leroy and Pessaux [14] use type systems to model exceptions, specifically to analyze uncaught exceptions. Limited context-sensitivity is employed for the purpose of more precise results on polymorphic functions. Fu et al. [11] proposed the E-C link metric to evaluate exception-flow precision. They also documented the exception handler matching problem caused by an imprecise control flow graph. They approach the problem by employing points-to information to refine control-flow reachability. Bravendoer and Smaragdakis [7] propose to join points-to analysis and exception flow analysis to improve precision and analysis run time in their Doop framework, based on the optimized analysis engine using Datalog [8]. They have conducted extensive comparison of
different options for polyvariance. It is the most precise and efficient exception-flow analysis compared to other work, with respect of points-to and E-C links. We conduct our comparison with respect to their work, and found the pushdown approach can yield significant improvement in precision, but the run-time is not comparable to their work, partly due to their mature optimization methodology for Datalog.

**Pushdown analysis for the λ-calculus** Vardoulakis and Shivers’s CFA2 [30] is the precursor to the pushdown control-flow analysis [9]. CFA2 is a table-driven summarization algorithm that exploits the balanced nature of calls and returns to improve return-flow precision in a control-flow analysis. While CFA2 uses a concept called “summarization,” it is a summarization of execution paths of the analysis, roughly equivalent to Dyck state graphs.

In terms of recovering precision, pushdown control-flow analysis [9] is the dual to abstract garbage collection: it focuses on the global interactions of configurations via transitions to precisely match push-pop/call-return, thereby eliminating all return-flow merging. However, pushdown control-flow analysis does nothing to improve argument merging.

This work directly draws on our previous work on pushdown analysis for higher-order programs [9] and introspective pushdown system (IPDS) for higher-order programs [10]. IPDS has tackled the challenge of incorporating abstract garbage collection [17] into pushdown system and improving the summarization algorithm for efficiency. That work shows significant improvements in precision and analysis time for the λ-calculus. We extend the introspective work in two dimensions: (1) we generalize the framework (including abstract garbage collection) to an object-oriented language, and (2) we adapt the Dyck state graph synthesis algorithm to handle the new stack change behavior introduced by exceptions.

**CFL- and pushdown-reachability techniques** In previous work, Earl et al. [10] develop a pushdown reachability algorithm suitable for the pushdown systems that we generate. It essentially draws on CFL- and pushdown-reachability analysis [5,12,23,24]. For instance, c-closure graphs, or equivalent variants thereof, appear in many context-free-language and pushdown reachability algorithms. Dyck state graph synthesis is an attractive perspective on pushdown reachability because it is purely functional, and it allows targeted modifications to the algorithm.

CFL-reachability techniques have also been used to compute classical finite-state abstraction CFAs [16] and type-based polymorphic control-flow analysis [22]. These analyses should not be confused with pushdown control-flow analysis, which is computing a fundamentally different kind of CFA.

**Pushdown exception-flow analysis** There is little work on pushdown analysis for object-oriented languages as a whole. Sridharan and Bodik proposed demand-driven analysis for Java that matches reads with writes to object fields.
selectively, by using refinement [28]. They employ a refinement-based CFL-reachability technique that refines calls and returns to valid matching pairs, but approximates for recursive calls. They do not consider specific applications of CFL-reachability to exception-flow.

9 Conclusion

Poor analysis of exceptions pollutes the interprocedural control-flow analysis of a program. In order to model exceptional control-flow precisely, we abandoned traditional finite-state approaches (e.g. k-CFA and its variants). In its place, we generalized pushdown control-flow analysis from the $\lambda$-calculus [10] to object-oriented programs, and made it capable of handling exceptions in the process. Pushdown control-flow analysis models the program stack (precisely) with the pushdown stack. Computing the reachable control states of the pushdown system (its Dyck state graph) yields combined data- and control-flow analysis of a program. Comparing this approach to the state-of-the-art [6], shows substantially improved precision. To improve time, we adapted abstract garbage collection to object-oriented program analysis. The end result is an improvement in data- and control-flow precision of roughly two orders of magnitude when soundly reasoning in the presence of exceptions.

References

1. Aho, A. V., Sethi, R., and Ullman, J. D. *Compilers: Principles, Techniques and Tools*. Addison-Wesley, 1988.
2. APK Tool, android-apktool, a tool for reverse engineering android apk files. [http://code.google.com/p/android-apktool/](http://code.google.com/p/android-apktool/)
3. Berndl, M., Lhotáč, O., Qian, F., Hendren, L., and Umanee, N. Points-to analysis using bdds. In *Proceedings of the ACM SIGPLAN 2003 conference on Programming language design and implementation* (New York, NY, USA, 2003), PLDI ’03, ACM, pp. 103–114.
4. Blackburn, S. M., Garner, R., Hoffman, C., Khan, A. M., McKinley, K. S., Bentzur, R., Diwan, A., Feinberg, D., Frampton, D., Guyer, S. Z., Hirzel, M., Hosking, A., Jump, M., Lee, H., Moss, J. E. B., Phansalkar, A., Stefanović, D., VanDrunen, T., von Dincklage, D., and Wiedermann, B. The DaCapo benchmarks: Java benchmarking development and analysis. In *OOPSLA ’06: Proceedings of the 21st annual ACM SIGPLAN conference on Object-Oriented Programming, Systems, Languages, and Applications* (New York, NY, USA, Oct. 2006), ACM Press, pp. 169–190.
5. Bouajjani, A., Esparza, J., and Maler, O. Reachability analysis of pushdown automata: Application to Model-Checking. In *Proceedings of the 8th International Conference on Concurrency Theory* (London, UK, UK, 1997), CONCUR ‘97, Springer-Verlag, pp. 135–150.
6. Bravenboer, M., and Smaragdakis, Y. Exception analysis and points-to analysis: Better together. In *ISSTA ’09: Proceedings of the Eighteenth International Symposium on Software Testing and Analysis* (New York, NY, USA, 2009), ACM, pp. 1–12.
7. Bravenboer, M., and Smaragdakis, Y. Exception analysis and points-to analysis: Better together. In ISSTA ’09: Proceedings of the Eighteenth International Symposium on Software Testing and Analysis (New York, NY, USA, 2009), ACM, pp. 1–12.
8. Bravenboer, M., and Smaragdakis, Y. Strictly declarative specification of sophisticated points-to analyses. In Proceedings of the 24th ACM SIGPLAN conference on Object oriented programming systems languages and applications (New York, NY, USA, 2009), OOPSLA ’09, ACM, pp. 243–262.
9. Earl, C., Might, M., and Horn, D. V. Pushdown control-flow analysis of higher-order programs. In Proceedings of the 2010 Workshop on Scheme and Functional Programming (Scheme 2010) (Montreal, Quebec, Canada, August 2010).
10. Earl, C., Sergey, I., Might, M., and Van Horn, D. Introspective pushdown analysis of higher-order programs. In Proceedings of the 17th ACM SIGPLAN International Conference on Functional Programming (New York, NY, USA, 2012), ICFP ’12, ACM, pp. 177–188.
11. Fu, C., Milanova, A., Ryder, B. G., and Wonnacott, D. G. Robustness Testing of Java Server Applications. IEEE Trans. Softw. Eng. 31, 4 (Apr. 2005), 292–311.
12. Kodumal, J., and Aiken, A. The set constraint/CFL reachability connection in practice. SIGPLAN Not. 39 (June 2004), 207–218.
13. Lattner, C., Lenhart, A., and Adve, V. Making context-sensitive points-to analysis with heap cloning practical for the real world. In Proceedings of the 2007 ACM SIGPLAN conference on Programming language design and implementation (New York, NY, USA, 2007), PLDI ’07, ACM, pp. 278–289.
14. Leroy, X., and Pessaux, F. Type-based analysis of uncaught exceptions. ACM Trans. Program. Lang. Syst. 22, 2 (Mar. 2000), 340–377.
15. Lhoták, O., and Hendren, L. Evaluating the benefits of context-sensitive points-to analysis using a bdd-based implementation. ACM Trans. Softw. Eng. Methodol. 18, 1 (Oct. 2008), 3:1–3:53.
16. Melski, D., and Reps, T. W. Interconvertibility of a class of set constraints and context-free-language reachability. Theoretical Computer Science 248, 1-2 (Oct. 2000), 29–98.
17. Might, M., and Shivers, O. Improving flow analyses via Gamma-CFA: Abstract garbage collection and counting. In ICFP ’06: Proceedings of the 11th ACM SIGPLAN International Conference on Functional Programming (New York, NY, USA, 2006), ACM, pp. 13–25.
18. Milanova, A. Light context-sensitive points-to analysis for java. In Proceedings of the 7th ACM SIGPLAN-SIGSOFT workshop on Program analysis for software tools and engineering (New York, NY, USA, 2007), PASTE ’07, ACM, pp. 25–30.
19. Milanova, A., and Ryder, B. G. Parameterized object sensitivity for points-to analysis for java. ACM Trans. Softw. Eng. Methodol 14 (2005), 2005.
20. Nyström, E. M., Kim, H.-S., and Hwu, W.-M. W. Importance of heap specialization in pointer analysis. In Proceedings of the 5th ACM SIGPLAN-SIGSOFT workshop on Program analysis for software tools and engineering (New York, NY, USA, 2004), PASTE ’04, ACM, pp. 43–48.
21. Project, T. A. O. S. Bytecode for the dalvik vm. http://source.android.com/tech/dalvik/dalvik-bytecode.html.
22. Rehof, J., and Fähndrich, M. Type-based flow analysis: From polymorphic subtyping to CFL-reachability. In POPL ’01: Proceedings of the 28th ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages (New York, NY, USA, 2001), ACM, pp. 54–66.
23. Reps, T. Program analysis via graph reachability. *Information and Software Technology* 40, 11-12 (Dec. 1998), 701–726.

24. Reps, T., Schwoon, S., Jha, S., and Melski, D. Weighted pushdown systems and their application to interprocedural dataflow analysis. *Science of Computer Programming* 58, 1-2 (Oct. 2005), 206–263.

25. Robillard, M. P., and Murphy, G. C. Static analysis to support the evolution of exception structure in object-oriented systems. *ACM Trans. Softw. Eng. Methodol.* 12, 2 (Apr. 2003), 191–221.

26. Ryu, S. Exception analysis for multithreaded Java programs.

27. Shivers, O. G. *Control-Flow Analysis of Higher-Order Languages*. PhD thesis, Carnegie Mellon University, Pittsburgh, PA, USA, 1991.

28. Sridharan, M., and Bod’ik, R. Refinement-based context-sensitive points-to analysis for Java. *SIGPLAN Not.* 41, 6 (June 2006), 387–400.

29. Van Horn, D., and Might, M. Abstraction of abstract machines. In *ICFP ’10: Proceedings of the 15th ACM SIGPLAN International Conference on Functional Programming* (2010), ICFP ’10, ACM Press, pp. 51–62.

30. Vardoulakis, D., and Shivers, O. CFA2: A Context-Free approach to Control-Flow analysis. In *Programming Languages and Systems*, A. D. Gordon, Ed., vol. 6012. Springer Berlin Heidelberg, Berlin, Heidelberg, 2010, ch. 30, pp. 570–589.

31. Whaley, J., and Lam, M. S. Cloning-based context-sensitive pointer alias analysis using binary decision diagrams. In *Proceedings of the ACM SIGPLAN 2004 conference on Programming language design and implementation* (New York, NY, USA, 2004), PLDI ’04, ACM, pp. 131–144.

32. Wu Jo, J., Mo Chang, B., Yi, K., and Moo Choe, K. An uncaught exception analysis for java, 2002.

33. Xu, G., and Rountev, A. Merging equivalent contexts for scalable heap-cloning-based context-sensitive points-to analysis. In *Proceedings of the 2008 international symposium on Software testing and analysis* (New York, NY, USA, 2008), ISSTA ’08, ACM, pp. 225–236.
A Optional Appendix

This appendix is not required reading. We provide it for background, refreshment and deeper discussion.

A.1 Notational conventions

We strive to stick to conventional notation and names wherever possible. For less common concepts, we define them here.

The functional update operation $f[x \mapsto y]$ extends a function with a new binding:

$$f[x \mapsto y](x) = y$$
$$f[x \mapsto y](x') = f(x') \text{ if } x \neq x'.$$

For objects like stores, we lift the least upper bound $\sqcup$ point-wise:

$$(\hat{\sigma} \sqcup \hat{\sigma'})(\hat{a}) = \sigma(\hat{a}) \cup \sigma'(\hat{a}).$$

A.2 Additional concrete transition relations

Stepping over nops and labels : The simplest instruction nop does not change any component in the configuration state:

$$\xrightarrow{\text{nop}}: (s, fp, \sigma, \kappa) \Rightarrow (s, fp, \sigma, \kappa).$$

label and line statement shares the same transition form.\(^3\)

Unconditional jumps : This kind of statement forces program to jump to the target statement sequence:

$$\xrightarrow{\text{goto label}}: (s, fp, \sigma, \kappa) \Rightarrow (S(label), fp, \sigma, \kappa).$$

where the function $S : \text{Label} \rightarrow \text{Stmt}^*$ maps a label to the statement sequence starting with that label.

\(^3\) The line statement is mainly for instrumenting context information to the statements that are actually interpreted.
**Conditionals**  The if-goto is not much more complicated than a nop or goto, but it needs to evaluate the conditional expression:

\[
\begin{align*}
&c \quad \Rightarrow \quad \begin{cases} 
    (s, fp, \sigma, \kappa) & A(\alpha e, fp, \sigma) \neq \text{false} \\
    (S(label), fp, \sigma, \kappa) & \text{otherwise}
  \end{cases}
\end{align*}
\]

**Atomic assignments** Atomic assignment statements assign the value of an atomic expression to a variable (register). This involves evaluating the expression, calculating the frame address to modify and then updating the store.

\[
\begin{align*}
&c \quad \Rightarrow \quad \begin{cases} 
    (s, fp, \sigma) & \sigma' = \sigma[[fp, $name] \mapsto A(e, fp, \sigma)] 
  \end{cases}
\end{align*}
\]

Note that a large set of instruction statements are transformed into *assign* form. For example, *(move-result $name)* is transformed to *(assign $name ret)* form.

**A.3 Other abstract transition relations**

- **Stepping over nops and labels**: Abstract transition relations for this kind of statement is almost like correspondent concrete semantics (Section 2.4)

\[
\begin{align*}
&\hat{c} \quad \Rightarrow \quad \begin{cases} 
    (s, fp, \sigma, \kappa) & \sigma' = \sigma[[fp, $name] \mapsto A(e, fp, \sigma)] 
  \end{cases}
\end{align*}
\]

The same right hand side of transition relation for label and line statement.

- **Unconditional jumps**: Like nop statement and label statements, no much difference but with abstract components replaced.

\[
\begin{align*}
&\hat{c} \quad \Rightarrow \quad \begin{cases} 
    (s, fp, \sigma, \kappa) & \sigma' = \sigma[[fp, $name] \mapsto A(e, fp, \sigma)] 
  \end{cases}
\end{align*}
\]

- **Conditionals** The if-goto is not much more complicated than a nop or goto, but it needs to evaluate the conditional expression:

\[
\begin{align*}
&\hat{c} \quad \Rightarrow \quad \begin{cases} 
    (s, fp, \sigma, \kappa) & \text{false} \notin \hat{A}(\alpha e, \hat{fp}, \hat{\sigma}) \\
    (S(label), \hat{fp}, \hat{\sigma}, \hat{\kappa}) & \text{otherwise}
  \end{cases}
\end{align*}
\]
Atomic assignments: The main change to the abstract transition relation for atomic assignments resides in the operation to the store component:

\[
\hat{c} (\text{assign } \$\text{name } e) \Rightarrow (\hat{s}, \hat{fp}, \hat{\sigma}', \hat{\kappa}) \text{ where }
\]
\[
\hat{\sigma}' = \hat{\sigma} \cup [(\hat{fp}, \$\text{name}) \rightarrow \hat{A}(e, \hat{fp}, \hat{\sigma})]
\]

A.4 Syntactic sugar for pushdown systems

When a triple \((x, \ell, x')\) is an edge in a labeled graph:

\[
x \xrightarrow{\ell} x' \equiv (x, \ell, x').
\]

Similarly, when a pair \((x, x')\) is a graph edge:

\[
x \rightarrow x' \equiv (x, x').
\]

We use both string and vector notation for sequences:

\[
a_1a_2\ldots a_n \equiv (a_1, a_2, \ldots, a_n) \equiv a.
\]

A.5 Stack actions, stack change and stack manipulation

Stacks are sequences over a stack alphabet \(\Gamma\). To reason about stack manipulation concisely, we first turn stack alphabets into “stack-action” sets; each character represents a change to the stack: push, pop or no change.

For each character \(\gamma\) in a stack alphabet \(\Gamma\), the **stack-action** set \(\Gamma_\pm\) contains a push character \(\gamma^+\); a pop character \(\gamma^-\); and a no-stack-change indicator, \(\epsilon\):

\[
g \in \Gamma_\pm ::= \epsilon \quad \text{[stack unchanged]}
\]

\[
\quad \mid \gamma^+ \quad \text{for each } \gamma \in \Gamma \quad \text{[pushed } \gamma\text{]}
\]

\[
\quad \mid \gamma^- \quad \text{for each } \gamma \in \Gamma \quad \text{[popped } \gamma\text{].}
\]

In this paper, the symbol \(g\) represents some stack action.

When we develop introspective pushdown systems, we are going to need formalisms for easily manipulating stack-action strings and stacks. Given a string of stack actions, we can compact it into a minimal string describing net stack change. We do so through the operator \([\cdot] : \Gamma_\pm^* \rightarrow \Gamma_\pm^*,\) which cancels out opposing adjacent push-pop stack actions:

\[
[g \gamma^+ \gamma^- g'] = [g g'] \quad [g \epsilon g'] = [g g'],
\]

so that \([g] = g\), if there are no cancellations to be made in the string \(g\).
We can convert a net string back into a stack by stripping off the push symbols with the stackify operator, \( [\cdot] : \Gamma^*_\pm \rightarrow \Gamma^* \):

\[
[\gamma_+ \gamma'_+ \ldots \gamma_+^{(n)}] = \langle \gamma^{(n)}, \ldots, \gamma', \gamma \rangle,
\]

and for convenience, \( [g] = [\lfloor g \rfloor] \). Notice the stackify operator is defined for strings containing only push actions.

### A.6 Pushdown systems

A pushdown system is a triple \( M = (Q, \Gamma, \delta) \) where:

1. \( Q \) is a finite set of control states;
2. \( \Gamma \) is a stack alphabet; and
3. \( \delta \subseteq Q \times \Gamma^*_\pm \times Q \) is a transition relation.

The set \( Q \times \Gamma^* \) is called the configuration-space of this pushdown system. We use \( \text{PDS} \) to denote the class of all pushdown systems.

For the following definitions, let \( M = (Q, \Gamma, \delta) \).

- The labeled transition relation \( (\longrightarrow_M) \subseteq (Q \times \Gamma^*) \times \Gamma^*_\pm \times (Q \times \Gamma^*) \) determines whether one configuration may transition to another while performing the given stack action:

\[
(q, \gamma) \xrightarrow{\gamma^+} (q', \gamma) \text{ iff } q \xrightarrow{\gamma} q' \in \delta \quad \text{[push]}
\]

\[
(q, \gamma : \gamma) \xrightarrow{\gamma^-} (q', \gamma) \text{ iff } q \xrightarrow{\gamma} q' \in \delta \quad \text{[pop]}
\]

\[
(q, \gamma) \xleftarrow{\gamma^+} (q', \gamma : \gamma) \text{ iff } q \xleftarrow{\gamma} q' \in \delta
\]

- If unlabelled, the transition relation \( (\rightarrow_M) \) checks whether any stack action can enable the transition:

\[
\mathrm{c} \xrightarrow{\frac{g}{M}} \mathrm{c}' \text{ iff } \mathrm{c} \xrightarrow{\frac{g}{M}} \mathrm{c}' \text{ for some stack action } g.
\]

- For a string of stack actions \( g_1 \ldots g_n \):

\[
c_0 \xrightarrow{g_1} c_1 \xrightarrow{g_2} \cdots \xrightarrow{g_{n-1}} c_{n-1} \xrightarrow{g_n} c_n,
\]

for some configurations \( c_0, \ldots, c_n \).

- For the transitive closure:

\[
c \xrightarrow{\ast_M} c' \text{ iff } \mathrm{c} \xrightarrow{\frac{g}{M}} \mathrm{c}' \text{ for some action string } g.
\]
A.7 Rooted pushdown systems

A rooted pushdown system is a quadruple \((Q, \Gamma, \delta, q_0)\) in which \((Q, \Gamma, \delta)\) is a pushdown system and \(q_0 \in Q\) is an initial (root) state. \(\text{RPDS}\) is the class of all rooted pushdown systems.

For a rooted pushdown system \(M = (Q, \Gamma, \delta, q_0)\), we define the reachable-from-root transition relation:

\[
c \xrightarrow{g} c' \iff (q_0, \langle \rangle) \xrightarrow{\ast} c \text{ and } c \xrightarrow{g} c'.
\]

In other words, the root-reachable transition relation also makes sure that the root control state can actually reach the transition.

We overload the root-reachable transition relation to operate on control states:

\[
q \xrightarrow{g} q' \iff (q, \gamma) \xrightarrow{\ast} (q', \gamma') \text{ for some stacks } \gamma, \gamma'.
\]

For both root-reachable relations, if we elide the stack-action label, then, as in the un-rooted case, the transition holds if there exists some stack action that enables the transition:

\[
q \xrightarrow{g} q' \iff q \xrightarrow{g} q' \text{ for some action } g.
\]

A.8 Computing reachability in pushdown systems

A pushdown flow analysis can be construed as computing the root-reachable subset of control states in a rooted pushdown system, \(M = (Q, \Gamma, \delta, q_0)\):

\[
\left\{ q : q_0 \xrightarrow{g} q \right\}
\]

Reps et al and many others provide a straightforward “summarization” algorithm to compute this set [5,12,23,24]. Our preliminary report also offers a reachability algorithm tailored to higher-order programs [9].

A.9 Nondeterministic finite automata

In this work, we will need a finite description of all possible stacks at a given control state within a rooted pushdown system. We will exploit the fact that the set of stacks at a given control point is a regular language. Specifically, we will extract a nondeterministic finite automaton accepting that language from the structure of a rooted pushdown system. A nondeterministic finite automaton (NFA) is a quintuple \(M = (Q, \Sigma, \delta, q_0, F)\):
– \( Q \) is a finite set of control states;
– \( \Sigma \) is an input alphabet;
– \( \delta \subseteq Q \times (\Sigma \cup \{\epsilon\}) \times Q \) is a transition relation.
– \( q_0 \) is a distinguished start state.
– \( F \subseteq Q \) is a set of accepting states.

We denote the class of all NFAs as \( \mathbb{NFA} \).

A.10 Introspective pushdown systems

An introspective pushdown system is a quadruple \( M = (Q, \Gamma, \delta, q_0) \):

1. \( Q \) is a finite set of control states;
2. \( \Gamma \) is a stack alphabet;
3. \( \delta \subseteq Q \times \Gamma^* \times \Gamma_\pm \times Q \) is a transition relation; and
4. \( q_0 \) is a distinguished root control state.

The second component in the transition relation is a realizable stack at the given control-state. This realizable stack distinguishes an introspective pushdown system from a general pushdown system. \( \text{IPDS} \) denotes the class of all introspective pushdown systems.

Determining how (or if) a control state \( q \) transitions to a control state \( q' \), requires knowing a path taken to the state \( q \). Thus, we need to define reachability inductively. When \( M = (Q, \Gamma, \delta, q_0) \), transition from the initial control state considers only empty stacks:

\[
q_0 \xrightarrow{g} q \text{ iff } (q_0, \langle \rangle, g, q) \in \delta.
\]

For non-root states, the paths to that state matter, since they determine the stacks realizable with that state:

\[
q \xrightarrow{g} q' \text{ iff there exists } g \text{ such that } q_0 \xrightarrow{g} q \text{ and } (q, [g], g, q') \in \delta,
\]

where \( q \xrightarrow{g_1, \ldots, g_n} q' \) iff \( q \xrightarrow{g_1} q_1 \xrightarrow{g_2} \cdots \xrightarrow{g_n} q' \).

A.11 Computing reachability in introspective pushdown system

We cast our reachability algorithm for introspective pushdown systems as finding a fixed-point, in which we incrementally accrete the reachable control states into a “Dyck state graph.”

A Dyck state graph is a quadruple \( G = (S, \Gamma, E, s_0) \), in which:
1. $S$ is a finite set of nodes;
2. $\Gamma$ is a set of frames;
3. $E \subseteq S \times \Gamma \times S$ is a set of stack-action edges; and
4. $s_0$ is an initial state;

such that for any node $s \in S$, it must be the case that:

$$(s_0, \langle \rangle) \xrightarrow{s} (s, \gamma) \text{ for some stack } \gamma.$$ 

In other words, a Dyck state graph is equivalent to a rooted pushdown system in which there is a legal path to every control state from the initial control state. We use $\text{DSG}$ to denote the class of Dyck state graphs. (Clearly, $\text{DSG} \subset \text{RPDS}$.)

Our goal is to compile an implicitly-defined introspective pushdown system into an explicit-constructed Dyck state graph. During this transformation, the per-state path considerations of an introspective pushdown are “baked into” the Dyck state graph. We can formalize this compilation process as a map, $\text{DSG} : \text{IPDS} \to \text{DSG}$.

Given an introspective pushdown system $M = (Q, \Gamma, \delta, q_0)$, its equivalent Dyck state graph is $\text{DSG}(M) = (S, \Gamma, E, q_0)$, where $s_0 = q_0$, the set $S$ contains reachable nodes:

$$S = \left\{ q : q_0 \xrightarrow{g} q \text{ for some stack-action sequence } g \right\},$$

and the set $E$ contains reachable edges:

$$E = \left\{ q \xrightarrow{g} q' : q \xrightarrow{g} q' \right\}.$$

Our goal is to find a method for computing a Dyck state graph from an introspective pushdown system.

### A.12 Garbage collection in introspective pushdown systems

Having augmented the abstract garbage collection with respect of objects, we are now ready to embed it into introspective pushdown systems. Using the function $\text{IPDS} : \text{Stmt}^* \to \text{IPDS}$ as presented in Fig 10.

---

4 We chose the term *Dyck state graph* because the sequences of stack actions along valid paths through the graph correspond to substrings in Dyck languages. A Dyck language is a language of balanced, “colored” parentheses. In this case, each character in the stack alphabet is a color.
\( \overline{PDS}(e) = (Q, \Gamma, \delta, q_0) \), where

\[ Q = \text{Stmt}^* \times \text{FramePointer} \times \text{Store} \]

\( \Gamma = \text{Frame} \)

\( (q, e, q') \in \delta \) iff \( (q', \hat{k}) \) for all \( \hat{k} \)

\( (q, \hat{\phi}, q') \in \delta \) iff \( (q', \hat{k}) \) for all \( \hat{k} \)

\( (q, \hat{\phi}, q') \in \delta \) iff \( (q', \hat{k}) \) for all \( \hat{k} \).

\( \overline{IPDS}(e) = (Q, \Gamma, \delta, q_0) \)

\[ Q = \text{Stmt}^* \times \text{FramePointer} \times \text{Store} \]

\( \Gamma = \text{Frame} \)

\( (q, \hat{k}, e, q') \in \delta \) iff \( \hat{G}(q, \hat{k}) \) for all \( \hat{k} \)

\( (q, \hat{\phi}, q') \in \delta \) iff \( \hat{G}(q, \hat{k}) \) for all \( \hat{k} \)

\( (q, \hat{\phi}, q') \in \delta \) iff \( \hat{G}(q, \hat{k}) \) for all \( \hat{k} \).

Fig. 9: \( \overline{PDS} : \text{Exp} \rightarrow \text{RPDS} \).

Fig. 10: \( \overline{IPDS} : \text{Exp} \rightarrow \text{IPDS} \).

**A.13 Introspective reachability via Dyck state graphs**

Compiling an introspective pushdown system into a Dyck state graph for exception-flow analysis does not require special modification with respect of the iterative method: The function \( F : \overline{IPDS} \rightarrow (\overline{DSG} \rightarrow \overline{DSG}) \) generates the monotonic iteration function we need:

\[ F(M) = f, \text{ where} \]

\[ M = (Q, \Gamma, \delta, q_0) \]

\[ f(S, \Gamma, E, s_0) = (S', \Gamma, E', s_0), \text{ where} \]

\[ S' = S \cup \left\{ s' : s \in S \text{ and } s \overset{g}{\rightarrow}_M s' \right\} \cup \{ s_0 \} \]

\[ E' = E \cup \left\{ s \overset{g}{\rightarrow}_M s' : s \in S \text{ and } s \overset{g}{\rightarrow}_M s' \right\} \].

Our implementation of the function \( \overline{DSG} \) corresponds exactly what’s defined as above. In section 6.1, we will show details of computing Dyck state graph in the presence of exception flows.

**A.14 Allocation: Polyvariance, context-sensitivity and object-sensitivity**

In the abstract semantics, the abstract allocation functions take the form: \( \overline{allocFP} : \text{Stmt} \times \overline{Conf} \rightarrow \text{FramePointer} \) and \( \overline{allocOP} : \text{Stmt} \times \overline{Conf} \rightarrow \text{ObjectPointer} \). The two allocation functions determine the polyvariance and object-sensitivity of the analysis. (In control-flow analysis, polyvariance literally refers to the number of abstract addresses (variants) there are for each variable.) All of the following allocation approaches can be used with abstract semantics:

- **Monovariance: Pushdown 0CFA** Pushdown 0CFA passes the statement itself for abstract addresses, meaning that \( \text{FramePointer} \) will be passed the
call site statement, and $\text{ObjectPointer}$ the instantiation site statement:

\[
\begin{align*}
\text{FramePointer} &= \text{Stmt} \\
\text{allocFP}(s, \hat{c}) &= s \\
\text{ObjectPointer} &= \text{Stmt} \\
\text{allocOP}(s, \hat{c}) &= s
\end{align*}
\]

– **Pushdown 1CFA** Pushdown 1CFA pairs the statement with current statement to get an abstract address:

\[
\begin{align*}
\text{FramePointer} &= \text{Stmt} \times \text{Stmt} \\
\text{allocFP}(s, (s', \hat{fp}, \hat{\sigma}, \hat{\kappa})) &= (s, s'_0) \\
\text{ObjectPointer} &= \text{Stmt} \times \text{Stmt} \\
\text{allocOP}(s, (s', \hat{fp}, \hat{\sigma}, \hat{\kappa})) &= (s, s'_0)
\end{align*}
\]

– **Pushdown k-CFA** Pushdown k-CFA looks beyond the current state and at the last \(k\) states. By concatenating the statements in the last \(k\) states together, and pairing this sequence with a variable we get pushdown k-CFA:

\[
\begin{align*}
\text{FramePointer} &= \text{Stmt} \times \text{Stmt}^k \\
\text{allocFP}(s, \langle (s_1, \hat{fp}, \hat{\sigma}, \hat{\kappa}), \ldots \rangle) &= (s, (s_{10}, \ldots, s_{k0})) \\
\text{ObjectPointer} &= \text{Stmt} \times \text{Stmt}^k \\
\text{allocOP}(s, \langle (s_1, \hat{fp}, \hat{\sigma}, \hat{\kappa}), \ldots \rangle) &= (s, (s_{10}, \ldots, s_{k0}))
\end{align*}
\]

In addition, there is much static context information after getting Abstract Syntax Tree (AST), such as for each statement, we can know its line number, what class and method it belongs to. By default, we also take advantage and instrument these information as complementary to the above context formalized.

### A.15 System architecture

We have implemented the analytic framework in Scala. Figure 11 presents the system architecture: `apktool` extracts `.dex` file from Android applications. `JDex2Sex` extracts class files from the `.dex` file to generate an S-expression encoding the `dex` file. The S-expression IR is then fed into Dalvik Parser and parsed into a Dalvik AST. The `Transformer` takes another pass on the Dalvik AST to instrument `push-handler` statements and `pop-handler` pseudo-statements, and attach some other context information to statements. `Preanalysis`, specifically, live register analysis, is performed right after `Transformer`. It is an intra-procedural backward data flow analysis on instructions for each method.

The core pushdown analytic components starts from the second row in Fig 11. The implementation of each component follows its correspondent formulation: `Stack-based CESK machine` embodies the abstract state space as shown in Fig 8.
and abstraction transition relations in Section 3.3. (I)PDCFA Machinery injects the program into a rooted pushdown system (Figure 10). A --gc flag determines whether we use PDCFA Machinery or (I)PDCFA Machinery. Dyck State Graph Machinery implements the fixed-point synthesis algorithm (summarized in Appendix A.13). In the following section, we will focus on the details of summarization algorithm in this machinery in handling exception flows.

![System Architecture of (i)pushdown exception flow analysis. (Lines without arrows indicates the components are implicitly connected)]