The new computer program for three dimensional relativistic hydrodynamical model

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Abstract. An effective computer program for three dimensional relativistic hydrodynamical model has been developed. It implements a new approach to the early hot phase of relativistic heavy-ion collisions. The computer program simulates time-space evolution of nuclear matter in terms of ideal-fluid dynamics. Equations of motions of hydrodynamics are solved making use of finite difference methods. Commonly-used algorithms of numerical relativistic hydrodynamics RHLLE and MUSTA-FORCE have been applied in simulations. To speed-up calculations, parallel processing has been made available for solving hydrodynamical equations. The test results of simulations for 3D, 2D and Bjorken expansion are reported in this paper. As a next step we plan to implement the hadronization algorithm by implementing the continuous particle emission for freeze-out and comparing it with Cooper-Frye formula.

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1. Introduction

The hydrodynamic approach to multiparticle production assumes the initial state a very hot and dense strongly interacting matter soon after the collision, which then expands hydrodynamically and lasts until the stage when the picture of continuous medium has cased to be valid. That stage, so-called freeze-out, is typically described by the so-called Cooper-Frye prescription (CFp) that treats the system at the decay stage of evolution as a locally equilibrated ideal gas at some hypersurface. However, this prescription produces some serious problems if the hypersurface is, at least
partially, not space-like, and - what is even more important - the results of many studies based on cascade (transport) models and experimental data contradict the idea of sudden freeze-out. The particles escape from the system during the whole period of its evolution and do not exhibit signs of a local equilibration at its late stages. Attempts to overcome the problem is made by hybrid hybrid "hydro" + "cascade" models. [1]

2. The relativistic hydrodynamic equations

After the first stage of a heavy ion collision, state of matter can be described in terms of fluid dynamic. Evolution of fluid dynamics is simulated using finite difference methods. Resolving differential equations on 3D lattice of physical cells represents time-space evolution of matter. The relativistic hydrodynamics equations represent energy-momentum and charge conservation. For heavy ion collisions, the conserved charge is e.g. the (net) baryon number, the (net) strangeness etc. Provided that matter is in local thermodynamical equilibrium, the energy-momentum tensor and the charge current assume ideal fluid. [2]

\[ \partial_{\mu} T^{\mu \nu} = 0 \] (1)
\[ \partial_{\mu} N^{\mu} = 0 \] (2)

The equations of ideal fluid-dynamics are completed by specifying an equation of state (EoS) for the matter under consideration in the form \( p = p(e,n) \)

The laboratory quantities \( R \) (mass density), \( M \) (momentum density), and \( E \) (energy density) are related to the quantities in the local rest frame: \( e \) (energy density) and \( n \) (mass density) and to the fluid velocity \( v \) through a set of non-linear transformations.

\[ \partial N \equiv \partial \dot{R} + \nabla \cdot (Rv) = 0 \] (3)
\[ \partial T^{\mu \nu} \equiv \partial \dot{E} + \nabla \cdot [(E+p)v] = 0 \] (4)
\[ \partial T^{\mu i} \equiv \partial \dot{M} + \nabla \cdot (M^i v) + \delta_i p = 0 \] (5)

The ideal gas equation of state (EoS); \( p = (\gamma - 1) (\epsilon - n) \) and its ultrarelativistic limit \( (\epsilon \gg n) \); \( p = (\gamma - 1) \) with the adiabatic index \( 1 \leq \Gamma \leq 2 \) [3]

3. Ellipsoidal flow

In [4] a new class of analytic solutions of the relativistic hydrodynamics was proposed for 3D asymmetric flows at soft EoS \( p=\text{const.} \). In general this type of solution describes expansion with the ellipsoidal flow. If one defines the initial conditions on the hypersurface of constant time, say \( t = 0 \), then \( t \) is a natural parameter of the evolution. Such a representation of the solutions similar to the Bjorken and Hubble ones with velocity field \( v_i = a_i x_i / t \) has property of an infinite velocity increase at \( x \rightarrow \infty \). A real fluid, therefore, can occupy only the space-time region...
where $|\mathbf{v}| < 1$. In [4] a solution of the relativistic hydrodynamics was proposed for anisotropic expansion of finite system. Proper choice of parameters in such solution induces formally Hubble-like velocity profile. This new class of analytic solutions for 3D relativistic expansion with anisotropic flows can describe the relativistic expansion of finite systems towards vacuum. They can be utilized for a description of the matter evolution in central and non-central ultra-relativistic heavy ion collisions, especially during deconfinement phase transition and the final stage of evolution of hadron systems. Also, the solutions can serve as a test for numerical codes describing 3D asymmetric flows in the relativistic hydrodynamics.

4. Algorithms for numerical hydrodynamics

Solving three dimensional hydrodynamics equations is done by using operator splitting method - by solving a sequence of one-dimensional equations of type:

$$\partial_t U + \partial_x F(U) = 0$$

(6)

using finite differences scheme:

$$U^{n+1}_i = U^n_i - \Delta t \frac{\Delta x}{\Delta x} \left[ F\left(U^*_{i+1/2}^n\right) - F\left(U^*_{i-1/2}^n\right) \right]$$

(7)

where: $F\left(U^*_{i+1/2}^n\right)$, $F\left(U^*_{i-1/2}^n\right)$ - intercell numerical fluxes.

4.1. Numerical fluxs

4.1.1. HLLE

HLLE (Harten-Lax-van Leer-Einfeldt) [4] is a Godunov-type algorithm, i.e. it does not apply the full solution of the Riemann problem but approximates it by a region of constant flow. In general, the good accuracy of Godunov-type fluxes results from the opening of the Riemann fan and picking up a single value at cell interface. Complete (exact or approximate) Riemann solvers recognize all waves in the Riemann fan and therefore provide good resolution of delicate features of the flow, such as contact discontinuities. Incomplete Riemann solvers (e.g. HLLE flux) do not recognize the intermediate waves in the Riemann fan and lump them all in one (averaged) state. The HLLE method is very efficient and it is exact for single shocks, but can be very dissipative - especially at contact discontinuities, what can be important during simulations of expansion of matter towards vacuum.

4.1.2. Musta-Force

A very simple and general approach to the construction of numerical fluxes, which combines the simplicity of centered Fluxes and the good accuracy of the Godunov method, is the Multi-Stage (MUSTA) approach [5]. The MUSTA approach develops
upwind numerical fluxes by utilizing centered fluxes (FORCE flux in our case) in a multi-stage predictor-corrector fashion. Effectively, MUSTA can be regarded as an approximate Riemann solver in which the predictor step opens the Riemann fan (self-similar solution) and the corrector step makes use of the information extracted from the opened Riemann fan. This is precisely the information needed for the upwind numerical flux. The key idea of the original MUSTA is to open the Riemann fan by solving the local Riemann problem numerically rather than analytically, i.e. solving it by evolving the initial state in time via the governing equations. It denotes that we do not explicitly make use of wave propagation information in the construction of the numerical flux.

5. Parallel processing solutions for solving hydro equations

Even with optimised code, complete three-dimensional calculation of hydrodynamic is an extremely time-consuming task. In order to mitigate this problem, attempts
are being made to take advantage of properties of numeric algorithms we use - namely, the fact they only operate on a fragment of computation space at a time - and increase simulation speed through parallel computation. Two different approaches are being evaluated and tested independently to obtain optimal results.

5.1. PVMFlower project

Takes advantage of the Parallel Virtual Machine framework, an established solution for distributed computing. Source with more information: [http://www.csm.ornl.gov/pvm/](http://www.csm.ornl.gov/pvm/)

5.2. Hydrogrid project

This is a scalable network computers solution for hydro program simulation, based on TCP/IP network protocol. This project consists only of standard Unix/Linux communication components.

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Fig. 2. The comparison of exact and numerical solutions (using Musta-Force flux) for ellipsoidal flow with constant pressure $p=0$ for time $t=4$ fm/c, arbitrary units. Non-zero value of velocity for $|r| > 4.5$ fm (where the maximal range of matter defined by velocity of light is 4.5 fm) is caused by numerical dissipation of the matter.