ANALYTICAL AND MONTE CARLO RESULTS FOR THE SURFACE-BRIGHTNESS–DIAMETER RELATIONSHIP IN SUPERNOVA REMNANTS

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ABSTRACT

The surface-brightness–diameter relationship for supernova remnants is explained by adopting a model of direct conversion of the flux of kinetic energy into synchrotron luminosity. Two laws of motion are adopted: a power-law model for the radius–time relationship and a model that uses the thin layer approximation. The fluctuations in the log–log surface diameter relationship are modeled by a Monte Carlo simulation. In this model, a new probability density function for the density as a function of the galactic height is introduced.

Key words: ISM: supernova remnants – radio continuum: galaxies

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1. INTRODUCTION

The correlation between the radio surface brightness, \( \Sigma_{1\text{GHz}} \), and diameter \( D \) in parsecs in supernova remnants (SNRs) is parameterized as

\[
\Sigma_{1\text{GHz}} = C_{\Sigma}(D)^{-\beta_{\Sigma}} \text{W m}^{-2} \text{Hz}^{-1} \text{sr}^{-1},
\]

where \( \beta_{\Sigma} \) and \( C_{\Sigma} \) are found through radio observations (Shklovskii 1960; Urošević et al. 2005). This is an observational relationship which has been explained by early theoretical models (Lequeux 1962; Poveda & Woltjer 1968; Kesteven 1968; Clark & Caswell 1976; Milne 1979). A theoretical interpretation of the \( \Sigma–D \) relationship was introduced by Duric & Seaquist (1986) by incorporating the Sedov blast wave solution for SNR expansion, the generation and evolution of the magnetic field as outlined by Güll (1973), and the acceleration of relativistic electrons by shocks as formulated by Bell (1978a, 1978b).

The time-dependent, nonlinear kinetic theory for cosmic ray production was adopted by Berezhko & Völk (2004) in order to derive a theoretically predicted brightness–diameter relation in the radio range for the Sedov phase. Recently, Bandiera & Petruk (2010) found that SNRs cease to emit effectively in radio at a stage near the end of their Sedov evolution, and that models of synchrotron emission with constant efficiencies in particle acceleration and magnetic field amplification do not provide a close match to the data. These previous models leave a series of questions unanswered or merely partially answered:

1. Is it possible to deduce a theoretical \( \Sigma–D \) relationship on the basis of power-law expansion for SNRs?
2. Is it possible to deduce a classical equation of expansion for an SNR with two adjustable parameters that can be found from a numerical analysis of the radius–time relationship?
3. Is the theoretical \( \Sigma–D \) relationship sensitive to the density of the interstellar medium (ISM), which decreases as a function of height above the galactic plane?

In order to answer these questions, we develop a theoretical derivation of the \( \Sigma–D \) relationship by modeling the expansion of a synchrotron-emitting shell. In Section 2, we show that it is reasonable to fit a power-law expansion relationship to a known young SNR, SN 1987J in M81, for which the growth of the shell-like structure is available over a period of 10 years. In Section 3, we discuss a modification of the power-law expansion model for the effects of the standard gradient in density in the ISM. In Section 4, we complete our derivation of the \( \Sigma–D \) relationship by adopting a direct conversion of the flux of kinetic energy into radiation calibrated to the observational data (Section 5). In Section 6, we address the observed fluctuations in the \( \Sigma–D \) relationship by utilizing a new probability density function (PDF) for the contrast of in situ densities of SNRs. This new PDF is derived from the standard number distribution of SNRs with galactic height.

2. THE POWER-LAW MODEL OF EXPANSION

A standard point of view assumes that the SNR expands at a constant velocity until the surrounding mass is of the order of solar mass. This timescale can be modeled by

\[
t_M = 186.45 \frac{M_\odot}{M_{10}} \frac{\sqrt{M_\odot}}{v_{10000}} \text{ yr},
\]

where \( M_\odot \) is the number of solar masses in the volume occupied by the SNR, \( n_0 \) is the number density expressed in particles cm\(^{-3}\), and \( v_{10000} \) is the initial velocity expressed in units of 10,000 km s\(^{-1}\); see McCray (1987). To model the time-dependent expansion of SNRs, we use

\[
R(t) = C t^\alpha,
\]

where the two parameters \( C \) and \( \alpha \) can be found from the observations. Ten years of observations of SN 1993J (Marcaide et al. 2009) allow us to fit these two parameters, which are reported in Table 1 and Figure 1. The quality of the fit is measured by the merit function

\[
\chi^2 = \sum_j \frac{(R_{\text{th}} - R_{\text{obs}})^2}{\sigma_{\text{obs}}^2},
\]

where \( R_{\text{th}}, R_{\text{obs}}, \) and \( \sigma_{\text{obs}} \) are the theoretical radius, the observed radius, and the observed uncertainty, respectively. This observed relationship allows us to express the radius as a function of time by

\[
R(t) = 0.0155 t^{0.828} \text{ pc},
\]
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Figure 1. Theoretical radius as given by the power-law model with data as in Table 1 (solid line), and astronomical data of SN 1993J with vertical error bars.

Table 1

| Parameters | Power law | Nonlinear radius |
|------------|-----------|------------------|
| $N$        | $\alpha = 0.82 \pm 0.0048$ | $d = 2.93; r_0 = 0.019$ pc; $t_0 = 0.249$ yr; $v_0 = 100,000$ km s$^{-1}$ |
| $\chi^2$   | $6364$    | $276$            |

Note. $N$ represents the number of free parameters.

where the time $t$ is expressed in yr. The velocity in this model is

$$V(t) = 12587.67 t^{-0.171} \text{ km s}^{-1}. \quad (6)$$

Figure 2 displays the observed instantaneous velocity as deduced from the finite difference method and the best fit according to Equation (6). Another interesting SNR is SN 1006, which started to be visible in 1006 AD and has an actual radius of $15'$ according to Green (2009). The radius in parsecs is a function of the adopted distance; as an example, Katsuda et al. (2010) quote a distance of $2.2$ kpc, and Winkler et al. (2003) $2.18$ kpc. Here we adopt a distance of $2.2$ kpc; consequently, the observed radius, $R_{\text{obs}}$, is

$$R = 9.59 D_{22} \text{ pc}, \quad (7)$$

where $D_{22}$ is the distance expressed in units of $2.2$ kpc. The proper motion of the remnant’s edge is a weak function of the azimuth angle, and the average value is

$$\overline{P_{\text{obs}}} = 0.5 \text{ arcsec yr}^{-1}. \quad (8)$$

The average velocity of the expansion is

$$V_{\text{obs}} = 5441 D_{22} \text{ km s}^{-1}. \quad (9)$$

Equation (3) and the connected velocity can be solved for the two unknown variables $\alpha$ and $C$

$$\alpha = 0.000001022 \frac{V_{\text{obs}}}{R_{\text{obs}}},$$

$$C = \frac{R_{\text{obs}}}{0.000001022 \frac{V_{\text{obs}}}{R_{\text{obs}}}}, \quad (10)$$

where $V_{\text{obs}}$ is expressed in km s$^{-1}$, $R_{\text{obs}}$ in pc, and $t$ in yr. Table 2 reports the numerical results.

3. MOMENTUM CONSERVATION AND GALACTIC HEIGHT

The presence of a slow wind from the SN progenitor makes it reasonable for us to assume that the SNR evolves in a previously ejected medium phase in which density is considerably higher than the ISM. The assumption used here is that the density of the ISM around the SNR has the following two piecewise dependencies:

$$\rho(R) = \begin{cases} 
\rho_0 & \text{if } R \leq R_0 \\
 f \rho_0 \left( \frac{R}{R_0} \right)^d & \text{if } R > R_0,
\end{cases} \quad (11)$$

where $f$ is a parameter that models the jump in density, $0 < f \leq 1$, and is connected with the vertical profile in density as a function of the galactic height. In this framework, the density decreases as an inverse power law with an exponent $d$ that can be fixed from the observed temporal evolution of the radius, with $d = 0$ meaning constant density; further on, the parameter $f$ regulates the density at $R = R_0$. The swept mass in the interval $0 \leq r \leq R_0$ is

$$M_0 = \frac{4}{3} \rho_0 \pi R_0^3. \quad (12)$$

The swept mass in the interval $0 \leq r \leq R$ with $r \geq R_0$ is

$$M = -4 f r^3 \rho_0 \pi \left( \frac{R_0}{r} \right)^d (d - 3)^{-1} \left( \frac{R - R_0}{r} \right) + 4 f \rho_0 \pi R_0^3 \frac{d}{d - 3} + \frac{4}{3} \rho_0 \pi R_0^3. \quad (13)$$

Momentum conservation requires that

$$Mv = M_0 v_0, \quad (14)$$
where $v$ is the velocity at $t$ and $v_0$ is the velocity at $t = t_0$. The velocity as a function of the radius is

$$v = \frac{v_0 r_0^3 (3 - d)}{3 r_0^d R^{3-d} f + r_0^3 (-d + 3 - 3 f)}.$$  

In this first-order differential equation in $R$, the variables can be separated and a term-by-term integration yields the following nonlinear equation:

$$\mathcal{F}_{NL} = (12 R_0^3 f - 3 R_0^3 f d + 7 R_0^3 d - R_0^3 d^2 - 12 R_0^3 R - 3 R_0^3 f R^{4-d} + 7 R_0^3 v_0 d t_0 + R_0^3 v_0 d^2 t - R_0^3 v_0 d t_0 + R_0^3 d^2 + 12 R_0^4 - 9 R_0^4 f - 7 R_0^4 d - 7 R_0^4 v_0 dt - 12 R_0^4 v_0 t + 3 R_0^4 f d + 12 R_0^3 v_0 t).$$  

An approximate solution of $\mathcal{F}_{NL}(r)$ can be obtained assuming that $3 R_0^3 R^{4-d} f \gg R_0^3 (4 - d) (d - 3 + 3 f) R$, as follows:

$$R(t) = \left( R_0^{4-d} - \frac{1}{3 f} (d - 3 + 3 f) R_0^{4-d} (d - 3)\right)^{1/3} + \frac{1}{3 f} (d - 3 + 3 f) v_0 R_0^{4-d} (3 - d) (t - t_0).$$  

The physical units have not been specified; pc for length and yr for time are perhaps acceptable astrophysical choices. With these units, the initial velocity $v_0$ is expressed in pc yr$^{-1}$ and should be converted into km s$^{-1}$; this means that $v_0 = 1.02 \times 10^6 v_1$, where $v_1$ is the initial velocity expressed in km s$^{-1}$.

A one-dimensional solution of Equation (16) can be found with the FORTRAN subroutine ZRIDDR (Press et al. 1992). A plot of this radial solution, using a parameter value of $f = 1$, is shown in Figure 3. Plots of other solutions with different values of $f$ can be found in Figure 4.

### 4. $\Sigma$–$D$ IN THE POWER-LAW MODEL

In this section, we review the timescale of synchrotron losses, the timescale of acceleration, and the inequality that allows us to use the in situ acceleration of electrons. The relation between the radio surface brightness and diameter of SNRs is deduced assuming direct proportionality between the radio luminosity and the flux of kinetic energy.

#### 4.1. The Synchrotron Emission

An electron that loses its energy due to synchrotron radiation has a lifetime of

$$\tau_e \approx \frac{E}{P_r} \approx 500 E^{-1} H^{-2} s,$$

where $E$ is the energy in erg, $H$ is the magnetic field in Gauss, and $P_r$ is the total radiated power (Lang 1999, Equation (1.157)). The energy is connected to the critical frequency (Lang 1999, Equation (1.154)) as

$$v_c = 6.266 \times 10^{18} H^2 E^2 Hz.$$

The lifetime for synchrotron losses is

$$\tau_{syn} = 39.660 \frac{1}{H \sqrt{Hv}} yr.$$

Following Fermi (1949, 1954), the gain in energy in a continuous form for a particle that spirals around a line of force is proportional to its energy, $E$,

$$\frac{dE}{dt} = \frac{E}{\tau_{\Omega}},$$

where $\tau_{\Omega}$ is the typical timescale, and

$$\frac{1}{\tau_{\Omega}} = \frac{4}{3} \left( \frac{u^2}{c^2} \right) \left( \frac{c}{L} \right),$$

where $u$ is the velocity of the accelerating cloud, $c$ is the velocity of the light, and $L$ is the mean free path between clouds (Lang 1999, Equation (4.439)). The mean free path between the accelerating clouds in the Fermi II mechanism can be found from the following inequality in time:

$$\tau_{\Omega} < \tau_{syn},$$

which corresponds to the following inequality for the mean free path between scatterers:

$$L < \frac{1.72 \times 10^5 u^2}{H \sqrt{Hv} c^2} pc.$$
The mean free path length for SN 1993J at \( v = 1 \) GHz gives

\[
L < 1.83 \times 10^{-5} t^{-0.34} \text{pc},
\]

where the velocity is given by Equation (6) and \( H = 65.1 \) G (Martí-Vidal et al. 2011). When this inequality is verified, the direct conversion of the flux of kinetic energy into radiation can be adopted. Recall that the Fermi II mechanism produces an inverse power-law spectrum in the energy of the type \( N(E) \propto E^{-\gamma} \) or an inverse power law in the observed frequencies \( N(\nu) \propto \nu^{-\alpha} \) with \( \alpha = (\gamma - 1)/2 \) (Lang 1999; Zaninetti 2011).

4.2. The Dimensional Approach

The source of synchrotron luminosity is here assumed to be the flux of kinetic energy

\[
L_m = \frac{1}{2} \rho A V^3,
\]

where \( A \) is the emitting surface area (de Young 2002, Equation (A28)). Assuming that the surface area is like that of a sphere, the luminosity is

\[
L_m = \frac{1}{2} \rho 4\pi R^2 V^3,
\]

where \( R \) is the instantaneous radius of the SNR and \( \rho \) is the density in the advancing layer in which the synchrotron emission takes place. We assume that the density of the advancing layer scales as \( R^{-d} \), which means that

\[
L_m \propto R^{2-d} V^3.
\]

The time dependence is eliminated by utilizing Equation (3)

\[
L_m = L_0 \left( \frac{R}{R_0} \right)^{-\frac{d-3+\alpha}{\alpha}},
\]

where \( L \) is the luminosity at \( R = R_0 \) or

\[
L_m = L_0 \left( \frac{D}{D_0} \right)^{-\frac{d-3+\alpha}{\alpha}},
\]

where \( D \) is the actual diameter and \( D_0 = 2R_0 \). Assuming that the observed luminosity, \( L_0 \), in a given band denoted by the frequency \( \nu \) is proportional to the mechanical luminosity we obtain

\[
L_0 = L_{0,\nu} \left( \frac{D}{D_0} \right)^{-\frac{d-3+\alpha}{\alpha}} \text{Jy kpc}^2,
\]

where \( L_{0,\nu} \) is the observed radio luminosity in a given band. The observations are generally represented in the form

\[
L = C_L D^{-\beta_L},
\]

where \( L \) and \( C_L \) are parameters deduced from the radio observations (Guseinov et al. 2003; Urošević et al. 2005). Given the two observational parameters \( \alpha \) and \( \beta_L \), we can derive

\[
d = \frac{\beta_L \alpha + 5 \alpha - 3}{\alpha}.
\]

The radio surface brightness of a remnant is defined as

\[
\Sigma = \frac{S_{1\text{GHz}}}{\theta^2},
\]

where \( S_{1\text{GHz}} \) is the detected flux of a remnant at 1 GHz and \( \theta \) is the observed angle. Because \( \theta \propto D^2 \), we have

\[
\Sigma_{1\text{GHz}} = \Sigma_{0,1\text{GHz}} \left( \frac{D}{D_0} \right)^{-\frac{d-1+\alpha}{\alpha}} \text{W m}^{-2} \text{Hz}^{-1} \text{sr}^{-1},
\]

where \( \Sigma_{0,1\text{GHz}} \) is the surface brightness at \( D = D_0 \). According to the basic observational relationship (1),

\[
d = \frac{\beta_S \alpha + 3 \alpha - 3}{\alpha}.
\]

5. OBSERVATIONS IN THE POWER-LAW MODEL

SNRs in our Galaxy have two luminosity–diameter relationships (Guseinov et al. 2003); the first is for SNRs which have \( L > 5300 \) Jy kpc\(^2\), \( D < 36.5 \) pc, and the second is for SNRs having \( L \leq 5300 \) Jy kpc\(^2\), \( D \geq 36.5 \) pc:

\[
L = 2.45 \times 10^{13} D^{-0.43} \text{ Jy kpc}^2, \quad D < 36.5 \text{ pc}
\]

and

\[
L = 5.38 \times 10^9 D^{-3.84} \text{ Jy kpc}^2, \quad D \geq 36.5 \text{ pc}
\]

The \( \Sigma–D \) relationship, similarly, is

\[
\Sigma = 2.7^{+2.1}_{-1.4} \times 10^{-17} D^{-2.47^{+0.76}_{-0.79}} \text{ W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1} \quad D < 36.5 \text{ pc},
\]

and

\[
\Sigma = 8.4^{+19.5}_{-6.3} \times 10^{-12} D^{-5.99^{+0.37}_{-0.23}} \text{ W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1} \quad D \geq 36.5 \text{ pc}.
\]

The observations of the surface brightness of SNRs in other galaxies were analyzed by Urošević et al. (2005) with data available at CDS. Figure 5 reports the observational data as well the fitting curve of all the radio SNRs in external galaxies. The results of our numerical analysis for extragalactic SNRs give

\[
\Sigma = (8.8 \pm 3.2) \times 10^{-16} D^{-3.1 \pm 0.11} \text{ W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}, \quad \times D < 450 \text{ pc},
\]

which agrees with the results of Urošević et al. (2005). The values of \( d \) from the \( \Sigma–D \) relationship are reported in Table 3.
6. A MONTE CARLO MODEL FOR $\Sigma$–$D$

The significant fluctuations observed in the $\Sigma$–$D$ relationship can be explained by correlating the well-known probability of having an SNR at galactic height $z$ with density. This conversion can be done using the nonlinear relationship between $z$ and density as given by the self-gravitating disk.

The radius function we derived in Section 3 (Equation (17)) from momentum conservation allows us to deduce an analytical expression for the $\Sigma$–$D$ relationship:

$$\Sigma_{\text{GHz}} = \Sigma_{0,1\text{GHz}} \left(\frac{1}{f^3}\right) \left(D/D_0\right)^{-\frac{d-3}{a}} \text{W m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}. \tag{42}$$

The $\Sigma$–$D$ relationship now has an inverse cubic dependence for the contrast parameter $f$.

6.1. The Profile of the ISM

The vertical number density distribution of galactic H I has the following three-component behavior as a function of the galactic height $z$, the distance from the galactic plane in parsecs:

$$n(z) = n_1 e^{-z^2/H_1^2} + n_2 e^{-z^2/H_2^2} + n_3 e^{-|z|/H_3}. \tag{43}$$

We set the densities in Equation (43) to $n_1 = 0.395$ particles cm$^{-3}$, $n_2 = 0.107$ particles cm$^{-3}$, $n_3 = 0.064$ particles cm$^{-3}$, and the scale heights to $H_1 = 127$ pc, $H_2 = 318$ pc, and $H_3 = 403$ (Lockman 1984; Dickey & Lockman 1990; Bisnovatyi-Kogan & Silich 1995). This distribution of galactic H I is valid in the range $0.4 < R < R_0$, where $R_0 = 8.5$ kpc and $R$ is the distance from the galaxy center. The previous empirical relationship can be modeled by a theoretical one. The density profile of a thin self-gravitating disk of gas which is characterized by a Maxwellian distribution in velocity and distribution that varies only in the $z$-direction has the following number density distribution:

$$n(z) = n_0 \text{sech}^2 \left(\frac{z}{2z_0}\right), \tag{44}$$

where $n_0$ is the density at $z = 0$ and $z_0$ is a scaling parameter (Bertin 2000; Padmanabhan 2002). Figure 6 displays a comparison between the empirical function sum of three exponential disks and the theoretical function as given by Equation (44).

The previous equation can be expressed with our contrast parameter $f$:

$$f = \text{sech}^2 \left(\frac{z}{2z_0}\right), \tag{45}$$

and the inversion gives $z$ as a function of $f$

$$z = 2 \text{arccosh} \left(\sqrt{f}\right)z_0. \tag{46}$$

6.2. The Distribution of SNRs

The PDF, $p(z)$, having an SNR as a function of galactic height $z$, is characterized by an exponential PDF

$$p(z) = \frac{1}{b} \exp \left(-\frac{z}{b}\right). \tag{47}$$

We briefly review how a PDF $p(z)$ changes to $g(f)$ when a new variable $f(z)$ is introduced. The rule for transforming a PDF is

$$g(f) = \frac{p(f(z))}{|dz/dz'|}. \tag{49}$$

Once the previous rule is implemented we obtain

$$g(f) = \frac{e^{-z \text{sech}^2(z/2z_0)/B}}{bf^{3/2} \sqrt{1/f - 1} \sqrt{1/f + 1}}. \tag{50}$$

The average value of $f$ when $b = 83$ pc and $z_0 = 90$ is $ar{f} = 0.7927$.

6.3. Monte Carlo Simulation

We are ready to build a Monte Carlo model for the $\Sigma$–$D$ distribution of SNRs for those SNRs characterized by the following constraints:

1. The lifetime of the SNR is generated between 0 and $t_{\max}$.
2. The time is converted in diameter, $D = R \times 2$, adopting Equation (5).
3. A value of $f$ is randomly generated according to the PDF, Equation (50).
4. In the previous points we have generated one $D$ and one $f$, and as a consequence a value of $\Sigma_{1\text{GHz}}$ as given by Equation (42) can be generated.

The results of this simulation are displayed in Figure 7 for the Galactic distribution, and in Figure 8 for the extragalactic distribution. In this section we derive the profile of the density in our Galaxy as a function of galactic height $z$ in the framework of the self-gravitating disk. This new relationship allows us to find a new PDF that characterizes the effect of SNRs expanding into a medium of varying density. The great fluctuations present in the galactic $\Sigma$–$D$ relationship are therefore explained by the SNR evolution in a medium with a lower density.
Figure 7. Monte Carlo log–log $\Sigma$–$D$ diagram at a frequency of 1 GHz, which simulates the Galactic SNRs for $D < 36.5$ pc. The parameters are $t_{\text{max}} = 5055$ yr, $\alpha = 0.82$, $b = 83$ pc, and $\Sigma_0 = 90$ pc. The Monte Carlo simulation gives $C_L = 2.71 \times 10^{-17}$ and $\beta_L = 2.45$.

Figure 8. Monte Carlo log–log $\Sigma$–$D$ diagram at a frequency of 1 GHz (green circles) and extragalactic SNRs as given by Urošević et al. (2005) for $D < 36.5$ pc (empty red stars). The parameters are $t_{\text{max}} = 2960$ yr, $C = 0.17$, $a = 0.57$, $b = 83$ pc, and $\Sigma_0 = 60$ pc. The Monte Carlo simulation gives $C_L = 8.84 \times 10^{-16}$ and $\beta_L = 3.65$.

(A color version of this figure is available in the online journal.)

7. CONCLUSIONS

The relation between the radio surface brightness and diameter of the galactic and extragalactic SNRs presents a linear relationship in the log($\Sigma$)–$D$ plane. Superposed on this observed linear behavior is a fluctuation of the variable density with the galactic height of the ISM. The major results of this paper are summarized as follows:

1. The power-law model for the expansion of an SNR coupled with a decreasing density medium allows us to build a simple expression for the luminosity as well as the radio surface brightness; see Equations (35) and (29).

2. A more sophisticated approach to the law of motion as given by the momentum conservation in a medium with variable density allows us to obtain the same result, as in Equation (42), once an asymptotic behavior for the law of motion is given, as in Equation (17). In this model, it is possible to introduce a contrast parameter $f$ that allows us to state that both the radius and the velocity increase as $1/f$ and the surface brightness increases as $1/f^3$.

3. The value of $f$ for each SNR is given by a new PDF (Equation (50)) derived from the observed PDF for SNR as a function of galactic height (see Equation (47)) and the physical profile of density of the ISM as given by Equation (44).

4. A Monte Carlo simulation of the radio surface brightness explains the fluctuations visible in the $\Sigma$–$D$ relationship as a probabilistic effect of having an SNR with a given galactic height or $f$.

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