Probing proton fluctuations with asymmetric rapidity correlations

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Abstract

Intrinsic fluctuations of the proton saturation momentum generate asymmetric rapidity distributions on an event-by-event basis. We argue that the asymmetric component, \( \langle a_1^2 \rangle \), of the orthogonal polynomial decomposition of the two-particle rapidity correlation function is a sensitive probe to this distribution of fluctuations. We present a simple model connecting the experimentally measured \( \langle a_1^2 \rangle \) to the variance, \( \sigma \), of the distribution of the logarithm of the proton saturation scale. We find that \( \sigma \approx 0.5 - 1 \) describes the asymmetric component of the rapidity correlations recently measured by the ATLAS collaboration.
I. INTRODUCTION

There has been considerable interest in the study of collective phenomena in small colliding systems since the initial discovery of long-range azimuthally collimated di-hadron correlations in high multiplicity proton-proton collisions at the LHC [1]. While the systematics of the measured ridge-like correlations have been confronted by both hydrodynamic and Color Glass Condensate inspired models (see [2] for a recent review) the origin and nature of the high multiplicity events are far from being understood. The strength of the near-side azimuthal correlation is observed to grow monotonically with event activity up to the highest measured multiplicities having about ten times the mean number of charged tracks which can only be accounted for by rare fluctuations of the proton wave function.

A recent work has shown that a combination of impact parameter and color charge fluctuations are insufficient in describing the broad width of multiplicity distribution in p+p collisions [3]. In order to accommodate the data additional event-by-event fluctuations of the saturation scale in the proton must be included. While this goes beyond the conventional Color Glass Condensate framework, the existence of such fluctuations has been known for almost a decade [4–7] as recently emphasized in [8] where it was shown that saturation scale fluctuations of the proton are required to understand the centrality dependence of the charged particle rapidity distribution in p+Pb collisions. By including event-by-event fluctuations of the initial proton saturation momenta within classical Yang-Mills simulations the authors of [9] were able to describe the broad tail of the multiplicity distribution lending credence to the importance of rare fluctuations in generating dense nuclear configurations.

The goal of this work is to demonstrate that the two-particle correlation function in rapidity, \( C_2(y_1, y_2) \), can serve as a sensitive probe of the distribution of saturation scale fluctuations of the proton. It provides constraints on the variance, \( \sigma \), of the logarithm of the saturation scale about its mean (i.e. minimum bias) position due to fluctuations occurring in the tail of the dipole scattering amplitude.

![FIG. 1. Event-by-event charged particle rapidity distribution in p+p collisions for \( \lambda = 0.32 \) and \( \sigma = 0.5 \). The event averaged \( \langle dN/d\eta \rangle \) is shown as the thicker blue curve. The conversion between rapidity \( y \) and pseudo-rapidity \( \eta \) was done in the same manner as [8].](image-url)
The basic idea is that partonic fluctuations in the proton result in a spread of saturation scales about the mean value. Since the saturation scale of the two protons involved in the collision fluctuate independently one expects an asymmetric single particle rapidity distribution, $dN/dy$, on a per-event basis. Figure 1 shows a model calculation of the rapidity distribution from a subset of events drawn from the distribution of equation 6. The event averaged $\langle dN/d\eta \rangle$ is shown as the thicker blue curve.

On a per-event basis the single particle rapidity distribution can be characterized by fluctuations about the mean rapidity distribution $dN/dy = \langle dN/dy \rangle (1 + a_0 + a_1 y + \ldots)$ (1) In the above expression $a_0$ characterizes event-by-event fluctuations in the multiplicity while $a_1$ captures the asymmetry in the rapidity distribution on a per-event basis, due in our model to the unequal saturation scales of the colliding protons. We should stress that strictly speaking equation 1 characterizes the per-event rapidity distribution in the limit of a large number of particles when statistical fluctuations are negligible. In order for experiments to obtain information on the dynamical fluctuations encoded in $a_i$ and disentangle these from statistical fluctuations it is necessary to measure the two-particle correlation function. By construction the event averaged $\langle a_i \rangle$, for $i \geq 0$ must vanish and we therefore focus on the root-mean-square values of the event-by-event $a_i$ via the quantities $\langle a_i a_j \rangle$, which can be extracted from the two-particle correlation function $C_2(y_1, y_2) = \langle d^2N/dy_1 dy_2 \rangle - \langle dN/dy_1 \rangle \langle dN/dy_2 \rangle$. (3)

As is evident from figure 1 the per-event $a_0$ and therefore $\langle a_0^2 \rangle$ is highly sensitive to saturation scale fluctuations. These event-by-event multiplicity fluctuations are related to the multiplicity distribution itself, and while saturation scale fluctuations have recently been shown necessary to explain the tail of the multiplicity distribution additional sources must also be included such as impact parameter fluctuations, fluctuations due to conservation laws, statistical fluctuations, etc. Unambiguously disentangling saturation scale fluctuations from other possible sources will require additional observables.

The asymmetric correlator $\langle a_0a_1 \rangle$ vanishes for symmetric colliding systems, such as p+p as considered in this work. Due to the independently fluctuating saturation scales of the two protons an event-by-event asymmetry with respect to $y$ is seen in figure 1. The nature of the event-by-event asymmetric rapidity distribution can be captured by the coefficient $\langle a_1^2 \rangle$ in the above expansion of the two-particle correlation.

One advantage of looking at rapidity asymmetric fluctuations, is that many of the more mundane sources of fluctuations that contribute to the multiplicity will not contribute to the coefficient $\langle a_1^2 \rangle$ due to their rapidity symmetric particle production. This will be discussed further in section III.

In what follows a simple model is presented, that will allow us to derive a relation between the experimentally measured $\langle a_1^2 \rangle$ and the variance, $\sigma$, of the proton saturation scale.
II. RAPIDITY DISTRIBUTION IN PROTON-PROTON COLLISIONS

Consider the single inclusive rapidity distribution for a generic asymmetric nucleus-nucleus collision \([11]\) valid outside of the fragmentation region of the two nuclei,

\[
\frac{dN}{dy} \propto S_\perp \text{Min}[Q^2_1, Q^2_2] \left( 2 + \ln \frac{\text{Max}[Q^2_1, Q^2_2]}{\text{Min}[Q^2_1, Q^2_2]} \right). \tag{4}
\]

The proportionality constant will cancel in the ratio in equation (2) and therefore is not needed in this work. The rapidity dependence enters through the evolution of the saturation scale with \(y\) as

\[
Q^2_1 = Q^2_{o,1}e^{+\lambda y}, \quad Q^2_2 = Q^2_{o,2}e^{-\lambda y} \tag{5}
\]

where \(Q_{o,1}\) and \(Q_{o,2}\) are the initial saturation scales in the two projectiles before evolution to the final saturation scales \(Q_1\) and \(Q_2\) respectively. As shown in [4–7] the dispersion in the final saturation scales are caused by fluctuations in the low density tail of the initial condition and can be realized by averaging over the initial saturation momenta drawn from a Gaussian distribution in the logarithm of the saturation scale,

\[
P[\rho] = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{\rho^2}{2\sigma^2} \right] \quad \text{where} \quad \rho \equiv \ln \left( \frac{Q^2}{Q^2_o} \right). \tag{6}
\]

Our treatment extends the work of [8] to include fluctuations in both nucleons in which case the event averaging can be performed according to

\[
\langle O \rangle = \int_{-\infty}^{+\infty} d\rho_1 d\rho_2 P[\rho_1]P[\rho_2] \langle O[\rho_1, \rho_2] \rangle, \tag{7}
\]

where the two saturations scale \(Q^2_{o,1}\) and \(Q^2_{o,2}\), fluctuate event-by-event around the mean saturation scales \(Q^2_{o,1}\) and \(Q^2_{o,2}\) respectively. In this work we are considering symmetric proton-proton collisions and therefore \(\bar{Q}^2_{o,1} = \bar{Q}^2_{o,2}\) which we set to be equivalent to \(\bar{Q}^2_o\). By defining,

\[
\rho_1 \equiv \ln \frac{Q^2_{o,1}}{Q^2_o}, \quad \rho_2 \equiv \ln \frac{Q^2_{o,2}}{Q^2_o} \tag{8}
\]

we can re-express equation (4) as

\[
\frac{1}{S_\perp Q^2_o} \frac{dN}{dy} \propto \begin{cases} \exp[\rho_1 + \rho_2 + \lambda y (2 + \rho_2 - \rho_1 - 2\lambda y)], & \text{if } 2\lambda y < \rho_2 - \rho_1 \\ \exp[\rho_1 - \rho_2 - \lambda y (2 + \rho_1 - \rho_2 + 2\lambda y)], & \text{if } 2\lambda y \geq \rho_2 - \rho_1 \end{cases} \tag{9}
\]

With the above expressions in hand one can calculate the quantities \(\langle dN/dy \rangle\) and \(\langle d^2N/dy_1dy_2 \rangle\). As the final expressions are rather formidable we have included them in the Appendix. Near mid-rapidity one can derive a rather simple expression for the coefficient \(\langle a^2_1 \rangle\) for minimum bias proton-proton collisions,

\[
\langle a^2_1 \rangle \simeq \frac{\lambda^2 \sigma^2}{2} \frac{4\pi (1 + 2\sigma^2) \exp[\sigma^2] \text{Erfc}[\sigma] - 8\sqrt{\pi} \sigma}{\left( \sqrt{\pi} (\sigma^2 - 2) \text{Erfc}\left[ \frac{\sigma}{2} \right] - 2\sigma \exp\left[ -\frac{\sigma^2}{4} \right] \right)^2}, \tag{10}
\]

where Erfc is the complementary error function. Equation (10) is the main result of this work. It provides a direct relation between asymmetric rapidity fluctuations \(\langle a^2_1 \rangle\) and the
FIG. 2. Plot of $\sqrt{\langle a_1^2 \rangle}$ from equation (10) as a function of $\sigma$ for $\lambda = 0.35$ and 0.25. The horizontal bar is representative of the data on $\sqrt{\langle a_1^2 \rangle}$ in minimum bias p+p collisions recently available by the ATLAS collaboration [15].

variance, $\sigma$, of saturation scale fluctuations in the proton. We plot equation (10) as a function of $\sigma$ for two representative values of $\lambda = 0.25, 0.35$. Phenomenological fits of Deep Inelastic Scattering data at small-$x$ [12–14] constrain $\lambda$ within this range. The horizontal band in figure 2 is representative of the data from minimum bias proton-proton collisions at $\sqrt{s} = 13$ TeV. The band is centered at $\langle a_1^2 \rangle = 0.098$ corresponding to the ATLAS collaboration measurement [15] at $N_{ch} = 17.6$ and the thickness of the band is two standard deviations $\pm 0.012$. We should caution the reader that the ATLAS collaboration has used very narrow centrality classes and that a more direct comparison with data would use the same centrality cuts as the experiment. This is left to future work.

Inspection of figure 2 shows that a value of $\sigma$ in the range $0.5 - 1$ is consistent with the experimental data. This is in qualitative agreement with the values obtained by fitting p+p multiplicity distributions [9] ($\sigma = 0.5$) and p+Pb rapidity distributions [8] ($\sigma = 1.55$).

III. DISCUSSION AND CONCLUSIONS

There are a number of caveats that should be discussed before a quantitative extraction of $\sigma$ is undertaken. First, the model under consideration in this work only includes intrinsic fluctuations of the proton saturation momentum. It is well known that in order to have quantitative agreement with the multiplicity distribution both impact parameter and color charge fluctuations must be included. However, it is plausible that many sources of fluctuations that contribute to the multiplicity have a smaller effect on the asymmetric coefficient $\langle a_1^2 \rangle$. For example, impact parameter fluctuations from a radially symmetric proton will produce a symmetric rapidity distribution in each event and will therefore not contribute to $\sqrt{\langle a_1^2 \rangle}$. The color charge fluctuations as included in [9] will also not contribute – the results of [9] are boost invariant in each event. More generally color charge fluctuations, with quan-
tum evolution, may introduce a forward-backward asymmetry on an event-by-event basis but we expect them to be suppressed by $1/(Q_s^2 S_\perp)$ where $Q_s^2$ represents the length scale of the color charge fluctuations and $S_\perp$ the collision overlap area.

In this paper we calculated the two-particle correlation function originating from the intrinsic fluctuations of the saturation scales in two colliding protons. Clearly this mechanism correlates not only two particles but also leads to multi-particle rapidity correlations. In reference [16] it was argued that higher order correlation functions naturally remove unwanted short-range rapidity correlations, e.g., resonance decays. In this work we focused on correlations after the experimental subtraction of the short range component.

In conclusion, we demonstrated that intrinsic fluctuations of the proton momentum saturation scale leads to an event-by-event asymmetric single particle distribution, $dN/dy$, and consequently to nontrivial rapidity correlations. We extracted the asymmetry coefficient $\sqrt{\langle a_1^2 \rangle}$ that directly measures the width of saturation scale fluctuations, that is, $\sqrt{\langle a_1^2 \rangle}$ is roughly proportional to $\lambda \sigma$, the width of Gaussian fluctuations in logarithm of the saturation scale, see figure 2. When compared to the preliminary ATLAS data we conclude that $\sigma$ is of the order of $0.5 - 1$.

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[10] A. Bzdak and D. Teaney, Phys. Rev. C87, 024906 (2013) arXiv:1210.1965 [nucl-th].
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Appendix A: Equations for the one- and two-particle rapidity distributions

In this section we collect analytic expressions for the one- and two-particle rapidity distributions and discuss the derivation of equation 10. Using equations 6, 7, 9 the event averaged multiplicity distribution is

\[
\frac{1}{S_{Q_0}^2} \langle dN/dy \rangle = \frac{1}{\sqrt{\pi}} \exp \left[ \frac{\sigma^2}{4} - \frac{\lambda y^2}{\sigma^2} \right] + \left( 1 + \lambda y - \frac{\sigma^2}{2} \right) \exp \left[ \frac{\sigma^2}{2} - \lambda y \right] \text{Erfc} \left[ \frac{\sigma}{2} - \frac{\lambda y}{\sigma} \right] + \{y \rightarrow -y\} \quad \text{(A1)}
\]

where Erfc = 1 – Erf is the complementary error function. The two particle rapidity correlation is more cumbersome and the final expression reads

\[
\frac{1}{(S_{Q_0}^2)^2} \langle d^2N/dy_1dy_2 \rangle = \begin{cases} 
N_2(y_1, y_2), & \text{if } y_2 > y_1 \\
N_2(y_2, y_1), & \text{if } y_2 \leq y_1 
\end{cases} \quad \text{(A2)}
\]

where we defined

\[
N_2(y_1, y_2) = 2 \left( \frac{\sigma^2}{2} + (\lambda y_2 + 1)(\lambda y_1 - 1) \right) \text{Erf} \left[ \frac{\lambda y_1}{\sigma} \right] \exp \left[ \sigma^2 + \lambda(y_1 - y_2) \right] \\
+ 2 \left( \frac{\sigma^2 - \lambda^2}{2} + (\lambda y_2 + 1)(\lambda y_1 + 1) \right) \text{Erfc} \left[ \frac{\sigma - \lambda y_1}{\sigma} \right] \exp \left[ 2\sigma^2 - \lambda(y_1 + y_2) \right] \\
- \frac{4\sigma}{\sqrt{\pi}} \left( \frac{\sigma^2}{2} + \lambda y_1 - 1 \right) \exp \left[ \sigma^2 + \lambda(y_1 - y_2) - \frac{\lambda^2 y_2^2}{\sigma^2} \right] \\
+ \{(y_1, y_2) \rightarrow (-y_2, -y_1)\} \quad \text{(A3)}
\]

Equation 10 is an approximation strictly valid in the limit \( \lambda Y \rightarrow 0 \). Most generally \( C_2(y_1, y_2) \) can be expressed in terms of orthogonal polynomials [10]

\[
\frac{C_2(y_1, y_2)}{\langle dN/dy_1 \rangle \langle dN/dy_2 \rangle} = \sum_{i,k=0} \langle a_i a_k \rangle T_i(y_1)T_k(y_2), \quad \text{(A4)}
\]

where following the notation employed in [15, 17]

\[
T_k(y) = Y \sqrt{\frac{2k + 1}{3}} P_k(y/Y), \quad P_k(x) = \frac{1}{2^k k!} \frac{d^k}{dx^k} (x^2 - 1)^k. \quad \text{(A5)}
\]
In the above expression $P_k(x)$ are Legendre polynomials (e.g. $P_0(x) = 1, P_1(x) = x, \ldots$) and the rapidity distribution is constrained to be measured in the interval $-Y \leq y \leq +Y$. Instead of using the full expression

$$
\langle a_1^2 \rangle = \left( \frac{3}{2Y^3} \right)^2 \int_{-Y}^{Y} dy_1 dy_2 \frac{C_2(y_1, y_2)}{\langle dN/dy_1 \rangle \langle dN/dy_2 \rangle} T_1(y_1) T_1(y_2),
$$

we have used a Taylor series expansion around mid-rapidity (see Eq. 2)

$$
\langle a_1^2 \rangle \simeq \frac{d}{dy_1} \frac{d}{dy_2} \frac{C_2(y_1, y_2)}{\langle dN/dy_1 \rangle \langle dN/dy_2 \rangle} \bigg|_{y_1 = y_2 = 0}.
$$

(A6)

This results in $\langle a_1^2 \rangle - \sqrt{21} \langle a_1 a_3 \rangle + (21/4) \langle a_3^2 \rangle + \ldots$, however, the higher components are small for small values of $\lambda Y$. We have checked and even for the acceptance of $-2.4 \leq y \leq 2.4$ these corrections amount to less than 10% on equation 10.