Theoretical Expectations in Radiative Top Decays

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We summarize the Standard Model predictions for the three-body decays of the top quark $t \to WbX$, where $X = Z, H, g$ or $\gamma$. Because of strong phase space suppression, we find that the branching ratios for the $Z$ and $H$ final states are of order a few times $10^{-7}$, rendering them invisible at Tevatron Run II. On the other hand, the decays to $g$ and $\gamma$ are suppressed only by the expected factor of $\alpha_s$ or $\alpha_{em}$.

According to the Standard Model, the decay $t \to Wb$ is by far the dominant decay mode of the top quark. Nevertheless, it is worthwhile to search for the other predicted decay modes of the top quark in order to more completely test the Standard Model. In this talk, we will examine the 3-body radiative decays $t \to WbX$, where $X$ can be a $Z$ boson, Higgs boson, gluon or photon.

An amusing coincidence involves the masses of the top quark, the bottom quark, and the two heavy gauge bosons. We observe that

$$M_t \sim M_W + M_b + M_Z. \quad (1)$$

Using the masses tabulated by the Particle Data Group, this relation reads

$$173.8 \pm 3.5 \pm 3.9 \text{ GeV} \sim 176.1 \pm 0.5 \text{ GeV}, \quad (2)$$

where the uncertainty on the right hand side is entirely due to the ambiguity in the $b$-quark mass. Thus, the on-shell decay of a top quark to a $WbZ$ final state is on the verge of being allowed. As a consequence, effects of the finite width of the top quark become crucial in calculating this decay rate.

In the stable-particle limit, there are three Feynman diagrams which contribute to the decay $t \to WbZ$, since the $Z$ may be radiated from any of the $t$, $W$, or $b$. To account for the finite widths of the $W$ and $Z$ bosons, we compute diagrams which include the decay products of the $W$ and $Z$, employing the Breit-Wigner form of the propagators in the unitary gauge. For example, we take the $W$ propagator to be

$$\frac{-i}{W^2 - m_W^2} \left( g^{\mu\nu} - \frac{W^\mu W^\nu}{m_W^2} \right). \quad (3)$$

where $m_W \equiv m_W + \frac{1}{2}i\Gamma_W$. This form of the propagator respects the necessary gauge invariance of the $t \to Wb\gamma$ amplitude.

To simplify the computation, we consider only the decays $W \to \mu^+\nu_\mu$ and $Z \to e^+e^-$, and extract the rate by dividing by the appropriate branching fractions:

$$\Gamma(t \to WbZ) = \frac{\Gamma(t \to \mu^+\nu_\mu be^+e^-)}{B(W \to \mu^+\nu_\mu)B(Z \to e^+e^-)}. \quad (4)$$

In all, a total of nine Feynman diagrams must be considered.

The first three diagrams (Fig. 1) are simply the stable particle diagrams for $t \to WbZ$ with the $W$ and $Z$ decays tacked on. We refer to them as resonant diagrams since for a heavy enough top quark, the $W$ and $Z$ are both on-shell. These are the dominant contributions to the total width. However, consistency demands that we include additional diagrams. For example, everywhere a $Z$ appears in Fig. 1, we must also substitute a photon (see Fig. 2). These diagrams contribute to the irreducible background to the process of interest. Their effect is minimal once we insist that the $e^+e^-$ pair we observe reconstructs to a $Z$ boson. In our calculation, we have required that the $e^+e^-$ pair mass be at least $0.8M_Z$. Finally, we also have the diagrams in Fig. 3. The kinematics of these diagrams are such that they are suppressed: compared to the primary set of Fig. 1, the diagrams of Fig. 3 contain an additional highly off-shell propagator. Thus, they contribute very little to the rate.

Fig. 4 shows our results for both the full calculation as well as the so-called “narrow-width approximation,” which is defined by forcing the $W$ and $Z$ to be on mass shell. Hence, the narrow-width rate goes to zero precisely at threshold. On the other hand, for very large top mass,
Figure 2: Irreducible background contributions to the process $t \rightarrow WbZ \rightarrow \mu^+\nu_\mu e^+e^-$ where a photon appears instead of a Z boson. These diagrams may be suppressed by cutting on the invariant mass of the $e^+e^-$ pair.

Figure 3: Non-resonant background contributions to the process $t \rightarrow WbZ \rightarrow \mu^+\nu_\mu e^+e^-$. These diagrams contain an extra highly off-shell propagator.

Figure 4: The ratio $\Gamma(t \rightarrow WbZ)/\Gamma(t \rightarrow Wb)$ as a function of the top quark mass, with an $e^+e^-$ invariant mass cut of $0.8M_Z$. The solid curve is the full calculation including the $W$ and $Z$ width effects, while the dotted curve is the narrow width approximation. For reference, the top quark mass range from the 1998 Review of Particle Properties is indicated.

Figure 5: Feynman diagrams for the process $t \rightarrow WbH \rightarrow \mu^+\nu_\mu bH$. Since the fourth diagram is suppressed by a tiny $\mu\mu H$ coupling and an off-shell $W$ propagator, we ignore it.

Both the full and narrow-width calculations reproduce the stable particle results presented in Ref. 3.

Compared to the uncertainty in the top quark mass, the uncertainty in the $b$ quark mass is negligible. Corresponding to the range of masses from the 1998 Review of Particle Properties we obtain

$$\frac{\Gamma(t \rightarrow WbZ)}{\Gamma(t \rightarrow Wb)} = (5.4^{+4.7}_{-2.0}) \times 10^{-7}. \quad (5)$$

Thus, the Standard Model prediction for this decay is well beyond the sensitivity of Tevatron Run II or even Run III. Its observation would imply new physics.

We treat the decay $t \rightarrow WbH$ in analogous fashion to $t \rightarrow WbZ$, except that now we take the Higgs boson to be stable. Thus, we consider only the four diagrams shown in Fig. 5. We ignore the diagram where the Higgs is emitted from the muon since it is suppressed by a very small $\mu\mu H$ coupling and an additional off-shell propagator. Our results appear as a function of the Higgs mass in Fig. 6, where the plotted error bars only account for the (dominant) uncertainty in the top quark mass. We have explicitly verified that our calculation agrees with the literature in the limit of large top mass.

Two of the four LEP collaborations have published 95% C.L. lower limits on the Higgs mass based on the 1997 run at $\sqrt{s} = 183$ GeV: L3 finds that $M_H > 87.6$ GeV, while ALEPH reports $M_H > 87.9$ GeV. Taking into account these limits, we see that $\Gamma(t \rightarrow WbH)/\Gamma(t \rightarrow Wb)$ is at most a few times $10^{-7}$. Once again, this rate is so tiny that observation of this decay would imply new physics.

Ref. 4 presents results for $t \rightarrow WbZ$ which are in disagreement with our results as well as those in Ref. 3.

For Higgs bosons light enough for this decay to be nearly on-shell, the corresponding Higgs width is negligible compared to the width of the $W$ boson.
mode in Runs II or III would imply non-Standard Model physics.

For completeness, we will say a few words about the decays \( t \to W bg \) and \( t \to W b \gamma \). These decays have been well-documented in Refs. 3 and 8. Both of these amplitudes are infrared divergent. Hence, the observed rate will depend in detail upon issues like the detector resolution and (in the case of \( W bg \)) the jet isolation algorithm. In addition, the shift in gluon or photon energy caused by the boost from the top quark rest frame to the lab frame will introduce a dependence on how the tops were originally produced. Thus, a careful calculation of these rates would include the full production process as well as a complete detector simulation. Nevertheless, we may get a feel for the behavior of these branching ratios by considering the (idealized) situation where we simply cut on the gluon or photon energy in the top quark rest frame. From Fig. 7 we see that these decays hold no theoretical surprises: the rates are approximately

\[
\begin{align*}
\Gamma(t \to W bg) &\sim O(\alpha_s) \times \Gamma(t \to Wb), \\
\Gamma(t \to Wb\gamma) &\sim O(\alpha_{em}) \times \Gamma(t \to Wb).
\end{align*}
\]

(6)

It is well-known that the presence of the gluonic radiative decay (as well as initial state gluon radiation) complicates the issue of determining the top quark mass accurately. In fact, extra soft jets are so common an occurrence that one could argue that there is a sense in which the decay to \( W bg \) has been already observed, although not unambiguously isolated. On the other hand, the decay to \( W b\gamma \) is a bit easier to get a handle on. The values indicated in Fig. 7 suggest that evidence for this decay mode may be accessible in Run II.

References

[*] Talk presented at the Thinkshop on Top-Quark Physics for Run II, held at the Fermi National Accelerator Laboratory, October 16–18, 1998.
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