Simulating the quark-gluon plasma

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Abstract. We present results for the equation of state of quark-gluon plasma in 2+1 flavor QCD with physical strange quark mass and close to the physical light quark masses. We also study the chiral condensate and susceptibility which are quantities sensitive to the chiral symmetry restoration aspects of the transition between ordinary matter and the quark-gluon plasma.

1. Introduction

The quark-gluon plasma is the state in which the matter of the early universe existed microseconds after the Big Bang. As the universe cooled down to temperatures about five orders of magnitude higher than that of the core of the Sun, the quark-gluon plasma underwent a transition into the ordinary matter we observe today. Currently, the primordial conditions which allow for the quark-gluon plasma to form are recreated in the heavy-ion collision experiments conducted at RHIC (The Relativistic Heavy-Ion Collider at the Brookhaven National Laboratory) and elsewhere. To interpret the experimental data, knowledge of the quark-gluon equation of state (i.e. the dependence of energy density and pressure on temperature) is essential. We present here the results for the equation of state obtained by the HotQCD collaboration using the only nonperturbative, first-principles method currently available – lattice QCD, the discretized version of the fundamental theory of the strong interactions. We simulate the QCD thermal statistical ensemble on a discrete four dimensional lattice representing the space-time. To approximate the physical world as closely as possible, we need to perform our computations on fine enough lattices that the discretization errors are small and well controlled. To gain a deeper understanding of the discretization effects we used two different lattice actions, the p4 and the asqtad actions [1], to perform two independent calculations of all the physical quantities of interest. The discretization effects for the two actions are not the same, but the results obtained with them should show consistency if the these effects are small. With both actions we simulate a quark-gluon system with two light degenerate sea quarks and one heavy sea quark whose mass, $m_s$, is tuned to the physical strange quark mass. The light quark masses, $m_l$, are one-tenth of $m_s$. To check for discretization errors, we also compare the equation of state obtained with two different temporal extents of

1 A. Bazavov, T. Bhattacharya, M. Cheng, N.H. Christ, C. DeTar, S. Ejiri, Steven Gottlieb, R. Gupta, U.M. Heller, K. Huebner, C. Jung, F. Karsch, E. Laermann, L. Levkova, C. Miao, R.D. Mawhinney, P. Petreczky, C. Schmidt, R.A. Soltz, W. Soeldner, R. Sugar, D. Toussaint and P. Vranas.

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the lattice, \( N_\tau = 8 \) and 6. For both actions the discretization effects are smaller as \( N_\tau \) becomes larger. Larger \( N_\tau \) also increases the expense of the calculation in terms of computer power.

2. The QCD equation of state

We employ the so called integral method of the determination of the equation of state of the quark-gluon plasma. At the basis of this method is the calculation of the energy-momentum tensor trace anomaly \( \Theta_{\mu\nu} \):

\[
\frac{\Theta_{\mu\nu}(T)}{T^4} = \frac{\epsilon - 3p}{T^4} = T \frac{\partial}{\partial T} \left( \frac{p}{T^4} \right) ,
\]

(1)

where \( \epsilon \) is the energy density, \( p \) is the pressure and \( T \) is the temperature. From the above it follows that the pressure can be obtained by integrating the trace anomaly:

\[
\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \frac{1}{T_0^4} \Theta_{\mu\nu}(T') .
\]

(2)

The integration is along lines of constant physics [1]. The lower integration limit \( T_0 \) is chosen such that \( p(T_0) \) is small. The energy density can be obtained as a simple linear combination:

\[
\frac{\epsilon(T)}{T^4} = \frac{3p(T)}{T^4} + \frac{\Theta_{\mu\nu}(T)}{T^4} .
\]

(3)

Figure 1 shows our results for the trace anomaly. From the data we can conclude that away from the peak region, we have a good agreement between the two actions and the two \( N_\tau \)'s. In

![Figure 1. The trace anomaly vs the temperature in MeV (lower x-axis) and in units of the scale parameter \( r_0 = 0.318 \) fm (upper x-axis), determined form the static quark potential [1].](image-url)
the peak region the two action show some differences. The p4 action gives a higher peak due to somewhat larger discretization effects. Figure 2 shows in more detail the trace anomaly in the high- and low-temperature regions. The behavior of the trace anomaly at low temperatures is commonly approximated with a hadron resonance gas model. In the left panel of Fig. 2 we compare it with our data at two resonance cutoffs of 1.5 and 2.5 GeV. In both cases the lattice results stay below the theoretical prediction. We attribute this discrepancy to the fact that both actions distort the hadronic spectrum at low temperature where the discretization error is larger than at higher $T$. The high-temperature behavior can be studied from the right panel of Fig. 2, where we can see that the two actions and the $N_t = 6$ and 8 cases show a good consistency. Also shown there are results with the p4 action at $N_t = 4$, which disagree with the rest of the data due to the large discretization errors at such a short temporal extent.

The energy density and pressure results are presented in Fig. 3. Both of these bulk thermodynamics quantities have a similar behavior: within the region of the transition they show a rapid growth which slows down as the temperature increases further. The rapid growth in the transition region is due to deconfinement which leads to the proliferation of new colored degrees of freedom. Still, even at temperatures of about 550 MeV the pressure and energy density do not reach the Stefan-Boltzmann ideal gas values, which means that strong interactions persist in the plasma above the transition temperature. The two actions give results which differ by no more than 10% at about 200 MeV and less than 5% at higher temperatures.

Of special interest to the hydrodynamics models of heavy-ion collisions is the speed of sound in the quark-gluon plasma. We extract it from the ratio of the pressure and energy density $p/\epsilon$, which is obtained from the ratio of interpolating curves for the trace anomaly and pressure. The speed of sound, $c_s$, is given by:

$$c_s^2 = \frac{dp}{d\epsilon} = \epsilon \frac{d(p/\epsilon)}{d\epsilon} + \frac{p}{\epsilon}. \quad (4)$$
Figure 3. Energy density and three times the pressure with the p4 and asqtad actions at $N_T = 8$. The error bars on the pressure shown at two temperatures indicate the systematic errors arising from the particular integration schemes used. The black filled bar shows the systematic shift to the data needed to match with hadron resonance gas at low temperature. For more details refer to [1].

Figure 4 presents our results for $p/\epsilon$ and $c_s^2$, where the former is also compared with the hadron resonance gas prediction with a cutoff of 2.5 GeV. The determination of the speed of sound is sensitive to the details of the interpolating techniques and the spread of the curves can be interpreted as an estimate of the systematic errors involved in the calculation.

3. Chiral symmetry restoration

QCD has a "chiral" symmetry that is spontaneously broken at low $T$ and restored at high $T$, when the quarks are massless. The transition between ordinary matter and a quark-gluon plasma combines features of both deconfinement and chiral symmetry restoration. In the previous section we presented our results for thermodynamic quantities, such as energy density and pressure, which are sensitive to the deconfining properties of the transition. Here we discuss observables which are related to the chiral symmetry restoration, namely the chiral condensate $\langle (\bar{\psi}\psi)_{q}\rangle$, $q = l, s$ denotes quark flavor) and the chiral susceptibility $\langle \chi_{q} \rangle$. The chiral condensate is an order parameter for the spontaneous symmetry breaking in the case of massless quarks. In our calculations (as in the real world) the quarks are not massless, which means that the chiral condensate is at most an approximate order parameter. To avoid ultraviolet singularities and a multiplicative renormalization in the chiral condensate, on the lattice we study the observable $\Delta_{l,s}$ constructed as:

$$\Delta_{l,s}(T) = \frac{\langle \bar{\psi}\psi \rangle_{l,T} - m_l}{m_s}\frac{\langle \bar{\psi}\psi \rangle_{s,T}}{\langle \bar{\psi}\psi \rangle_{l,0} - m_l}\frac{\langle \bar{\psi}\psi \rangle_{s,0}}{\langle \bar{\psi}\psi \rangle_{s,0}}.$$  (5)
Figure 4. The ratio of pressure and energy density, $p/\epsilon$ and the square of the speed of sound, $c_s^2$, obtained at $N_\tau = 6$ and 8. The curves with no data points represent the square of the speed of sound. The hadron resonance gas prediction for $p/\epsilon$ with a cutoff of 2.5 GeV is given by the dash-dotted curve.

In the left panel of Fig. 5 we show the results for $\Delta_{l,s}$. The data shows some small differences between the two actions and the two $N_\tau$’s. The vertical band denotes the temperature region where we would place the chiral transition. This band has been determined from the peak in the chiral susceptibility shown on the right panel of Fig. 5. The chiral susceptibility measures the fluctuations in the chiral condensate, which become infinite at the transition temperature in the chiral theory. In our case of massive quarks, the peak has a finite height and it shows a shoulder-like shape which complicates the definition of a single value for the transition temperature. Instead we choose to refer to the transition as happening in a range of temperatures: $180 \text{ MeV} \lesssim T \lesssim 200 \text{ MeV}$. We expect this region to shift downward in temperature by about 10 MeV when the full continuum and the physical quark mass limits are taken.

References
[1] Bazavov A. et al. [HotQCD Collaboration] 2009 Equation of state and QCD transition at finite temperature arXiv:0903.4379
Figure 5. The subtracted and normalized chiral condensate $\Delta_{l,s}$ (left panel) and the disconnected chiral susceptibility $\chi_{\text{disc}}/T^2$ (right panel).