Non-demolishing measurement of a spin qubit state via Fano resonance

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(Dated: July 4, 2008)

Fano resonances are proposed to perform a measurement of a spin state (whether it is up or down) of a single electron in a quantum dot via a spin-polarized current in an adjacent quantum wire. Rashba-like spin-orbit interaction in a quantum dot prohibits spin-flip events (Kondo-like phenomenon). That ensures the measurement to be non-demolishing.

Keywords: quantum computer, spin qubit, quantum wire, quantum dot

I. INTRODUCTION

The solid state structures seem quite promising to implement quantum computers. The first proposal of the solid state quantum computer based on quantum dots was put forward in 1998 by D. Loss and P. DiVincenzo \textsuperscript{[1]}. The quantum calculations were offered to be performed on single electron spins placed in quantum dots. To read out the result one should measure the state of the quantum dot register consisting of single spins. Several possibilities were proposed there. In particular, two of them exploited a spin blockade regime. One could use spin dependent tunneling of a target electron into a quantum dot with a definite spin orientation of a reference electron. The final charge state of the dot (whether tunneling occurred or not) could be tested by a single electron transistor (SET) quite sensitive for an environment charge.

In the same year Kane offered a solid state quantum computer based on $^{31}$P atoms embedded in Si substrate \textsuperscript{[2]}. The computation was to be performed on $^{31}$P nuclei spins mediated by outer shell electrons. The inventive procedure to transfer a resulting nucleus spin state into an electron spin state was proposed. The latter could be measured by single electron tunneling into the reference atom, i.e. almost with the same means as that in Ref. \textsuperscript{[1]}. This idea was much developed in Ref. \textsuperscript{[3]} where it was proposed to test a single electron spin in a quantum dot by a spin-polarized current passing through this dot sandwiched between leads. The inspiring experiment demonstrating such a readout of a single electron spin placed in an open quantum dot by detecting a current passing through the dot was presented in \textsuperscript{[4]}. Far earlier than the first solid state quantum computer implementations were put forward a set of scanning tunneling microscopy experiments to observe an evolution of a single spin with the help of spin-polarized current emanating from a magnetic tip had begun \textsuperscript{[3, 6, 7, 8]}. Most of proposals of spin-based quantum computers relate just to electron spins although their relaxation is much faster than that of nucleus spins. It is caused by relatively easier and faster operation upon them and a possibility of measurement of individual electron spin state. In general, most of proposals of spin-based quantum computers relate to electron spins although their relaxation is much faster than that of nucleus spins. It is caused by relatively easier and faster operation upon them and a possibility of measurement of individual electron spin state.

II. QUANTUM WIRE TO MEASURE A SINGLE SPIN STATE

We suggest a measurement which allows via detecting the current to conclude whether the electron in a quantum dot has the same spin orientation as electrons in the quantum wire or opposite one.
Dirac distribution function

The structure under consideration is schematically depicted in Fig. 1. It consists of a quantum wire transmitting a spin polarized current coupled to a nearby quantum dot via tunneling. The current through the wire is determined by well-known Landauer formula

$$I(V_{sd}) = \frac{e}{h} \sum_{i=0}^{\infty} \int dE T_i(E) [f_s(E) - f_d(E)],$$

where summation is fulfilled over all modes of transversal quantization in the wire, $T_i(E)$ is a transmission coefficient for $i$-th mode dependent on the total energy $E$, the factor $e/h$ arises from a conductance quantum $e^2/h$ for spin polarized current in a ballistic wire, $f(E)$ is Fermi-Dirac distribution function

$$f(E) = \frac{1}{1 + \exp \frac{E - \mu}{kT}},$$

providing the chemical potentials in the source contact $\mu_s$ and in the drain contact $\mu_d$ are shifted by bias: $\mu_s - \mu_d = eV_{sd}$. The transmission coefficients $T_i(E)$ may be different for different spin state of the quantum dot electron with respect to spin polarization of electrons in quantum wire. This results in different current. The measurement is operated by potentials on gate electrodes.

Recently the proposals appeared to exploit Fano resonances for a measurement of an individual electron spin state with the help of quantum wire or quantum constriction which in fact could be regarded as a merely short quantum wire [13, 14]. Indeed, Fano resonances make such a measurement more sensitive.

Fano effect in the considered structure means the following. An electron moving along the quantum wire can partially penetrate into the quantum dot due to tunneling. The interference between two routes, one of which passes through the dot and another doesn’t, determine the transmission coefficient of an electron through the wire. In other words, the discrete energy spectrum in the dot interferes with a continuum in the wire. This interference becomes destructive when the energy of an electron in wire coincides with that in a dot. This results in backscattering which can be detected by a dip on a current-voltage curve, so called Fano antiresonance.

Transmission coefficient $T$ for an electron with the energy detuning from the resonance $\varepsilon$ is supplied by the expression

$$T = \frac{\left| \frac{\varepsilon + q\Gamma}{\varepsilon^2 + \Gamma^2} \right|^2}{\varepsilon^2 + \Gamma^2},$$

where $\Gamma$ stands for the level broadening and $q$ is the Fano asymmetry factor. In general $q$ is a complex number depending on scattering and relaxation in the system. For a ballistic quantum wire and a quantum dot without relaxation $q \approx 0$. Here we assume that this is the case.

However, the model put forward in [13] does not take into account the possibility of spin flip, i.e. Kondo-like phenomenon. Indeed, the spin exchange between an incident electron and an electron in quantum dot takes the same time required for an incident electron to be backscattered. Therefore, the initial spin state of the measured electron becomes demolished after only one incident electron is backscattered. Here we endeavor to circumvent such a shortcoming introducing a spin-orbit interaction into the system.

However, for the sake of clarity we firstly discard a spin-orbit interaction to highlight the disadvantages of such an approach to measurement. The general way of measurement looks like follows. A conducting electron (an electron at the Fermi level) in the quantum wire tunnels to an excited state in a quantum dot which is split with respect to the total spin $S$ because just a total spin determines the exchange energy. We also suppose that the state $S = 1$ has a smaller exchange energy as the state $S = 0$ (Fig. 2). This is valid, at least, for the excited state with orbital moment $L = 1$. It should be noted that the Lieb-Mattis theorem is not valid for excited states. This theorem only claims that the ground state definitely has $S = 0$ for even number of electrons. Anyhow, this suggestion is not crucial for the measurement.

We suppose that the initial state of the system is the following. For the sake of definiteness, the spin of electron in quantum wire is always directed up ($\uparrow$) and the spin of target electron in the ground state of quantum dot is directed up ($\uparrow$) or down ($\downarrow$). Evidently, only mutual spin
FIG. 2: An electron tunnels from a quantum wire (QW) to the excited state in a quantum dot (QD) with total spin $S = 1$.

The orientation of both electrons matters. Due to tunneling this state evolves into some excited state of two electrons in the quantum dot.

The Hamiltonian describing tunneling of an electron from the quantum wire into an exited level in the quantum dot in absence of spin-orbit interaction reads

$$H_0 = \varepsilon_0 + (\varepsilon_1 + U_C - J\vec{S}_0\vec{S}_1) a_1^+ a_1 + \sum_j (T_j (a_1^+ b_j + b_j^+ a_1) + \varepsilon_j b_j^+ b_j) \quad (4)$$

where $\varepsilon_0$ is a ground level energy of a single electron in the quantum dot, $\varepsilon_1$ is an excited level energy, $U_C$ is a direct Coulomb interaction between an electron in the ground state and an electron in the excited state, $a_1^+$ and $a_1$ are operators of creation and annihilation of an electron in the excited level in the quantum dot, $b_j^+$ and $b_j$ are operators of creation and annihilation of an electron in the quantum wire in $j$-th state, in general, the longitudinal momentum and the transversal quantization subband (mode) number are ascribed to the index $j$, $J$ is the strength of exchange interaction, $\vec{S}_0$ and $\vec{S}_1$ are spin operators for an electron in the ground level and that in excited level, respectively. In general, the sign of exchange energy (therefore, the sign of $J$) could be positive or negative. It is only known for sure that the ground state of the system composed of two electrons definitely corresponds to the total spin $S = 0$ owing to Lieb-Mattis theorem.

It is convenient to present the spin Hamiltonian in the form

$$\vec{S}_0\vec{S}_1 = S_0^z S_1^z + S_0^+ S_1^- \quad (5)$$

where $S_0^z$ and $S_1^z$ are operators of $z$-projection, $S_0^+$ and $S_1^-$ are raising and lowering operators of $z$-projection. The second term describes spin-flip processes, i.e. Kondo-like phenomena.

Hereafter, we incorporate into consideration only three lowest states of two electrons in the quantum dot: the ground state with total spin $S = 0$ and two excited states corresponding to the same space wave function but the different total spin $S = 0$ and $S = 1$. The energies of these excited states differ due to exchange interaction.

There are two triplet states with $S = 1$ which can originate after an electron with spin up tunnels from a quantum wire into a quantum dot

$$\uparrow_D \uparrow_D \quad (6)$$

for parallel spins and

$$[\uparrow_D \downarrow_D + \downarrow_D \uparrow_D]/\sqrt{2} \quad (7)$$

for antiparallel spins. There could also occur a singlet state with $S = 0$

$$[\uparrow_D \downarrow_D - \downarrow_D \uparrow_D]/\sqrt{2} \quad (8)$$

for antiparallel spins. The above formulae are sensitive to position: the first place corresponds to the ground state.

Tunneling to the ground state from a quantum wire seems inappropriate for the measurements because the target electron may escape from the quantum dot the same way as the reference electron.

Therefore, tunneling to the excited state is preferable as it leaves the target electron in the dot.
First of all we must discuss the initial state of the system when only a target electron is situated in the ground state in a quantum dot and there is a reference electron in a quantum wire.

For parallel spin orientation

$$\uparrow_{D}\downarrow_{W}$$

This state relates to a definite value of total spin $S = 1$ and there is no entanglement between spin and space coordinates.

For antiparallel spins the state is

$$\downarrow_{D}\uparrow_{W}$$

Indeed, this state is a superposition of $S = 0$ and $S = 1$, moreover, there exists an entanglement of space and spin coordinates. Worth noting both space components of this state do not interfere because they are orthogonal with respect to total spin.

When the conditions shown in Fig. 2 are effectuated and electrons from the Fermi level in quantum wire can tunnel to the state with $S = 1$ in a quantum dot the reflection coefficient for the antiparallel spins (10) is exactly twice smaller than that for parallel spins (9). This looks like the basis of a spin state measurement.

Actually, it is an illusion that foregoing procedure is already satisfactory for the measurement. Really, Kondo-like phenomena, i.e. spin-flip events should be involved into consideration. The spin-flip occurs within the period of time $\tau \sim (J/\hbar)^{-1}$. The broadening of levels with $S = 0$ and $S = 1$ depends on tunneling rate, so that $\Gamma \approx T$. The claim for these levels to be clearly distinguished requires $J \gg T$. It follows that the spin flip process must be much faster than tunneling.

We propose to employ spin-orbit interaction inside a quantum dot to prevent spin-flip. Hereafter, we analyze the simplest case when a coin-like quantum dot is formed of a symmetric quantum well. It is sketched in Fig. 3. The arrows indicate the interface electric field at different interfaces caused by conduction (or valence) band discontinuity. For instance, this kind of dot could be fabricated by etching with a subsequent overgrowing of a host semiconductor with a wider gap. Rashba interaction is widely used for a 2DEG originating at a unique interface in a heterostructure, for example, at common GaAs/AlGaAs interface. We adopt this description to introduce a spin-orbit interaction caused by interface field at side wall of the dot. To that end, we propose a model Rashba-like Hamiltonian

$$\hat{H}_{RS} = \beta [\hat{S} \times \hat{r}] \hat{r} = \beta [\hat{k} \times \hat{r}] \hat{S} = \beta \hat{L} \hat{S},$$

where $\alpha_{R}$ is Rashba constant, $\hat{k} = -i\frac{\partial}{\partial \vec{r}}$ is an operator of in-plane moment, $\hat{S}$ is a spin operator, and a unit vector $\vec{r}$ is directed perpendicular to 2DEG plane. The Rashba constant $\alpha_{R}$ is non-zero for a 2DEG in non-symmetric quantum well. Unfortunately, Rashba Hamiltonian results in an entanglement of spin and space variables for an electron state in a quantum dot cut of a quantum well. It occurs even in a ground state. This kind of a quantum dot is not suitable for a spin qubit application. The appropriate quantum dot should be made of a symmetric quantum well with zero original Rashba term.

Rashba Hamiltonian is widely used for a 2DEG originating at a unique interface in a heterostructure, for example, at common GaAs/AlGaAs interface. We adopt this description to introduce a spin-orbit interaction caused by interface field at side wall of the dot. To that end, we propose a model Rashba-like Hamiltonian

$$H_{RS} = \beta L_{1Z} S_{1Z},$$

where $L_{1Z} = \pm 1$ is a z-projection of orbital moment, $S_{1Z} = \pm 1/2$ is a z-projection of electron spin in the excited state. The spin-orbit term should be added to the Hamiltonian to acquire the resultant Hamiltonian

$$H = H_{0} + H_{RS}.$$
FIG. 4: Two-electron energy levels in a quantum dot split by exchange and spin-orbit interaction. The arrow indicates the level to which an electron tunnels from a quantum wire.

The eigen-states of the Hamiltonian relevant to two electrons in the dot are depicted in Fig. 4. Tunneling of an electron from a quantum wire to a quantum dot occurs if only the latter has the same spin orientation as the measured electron in dot (the arrow in Fig. 4 marks the proper level).

Spin-orbit splitting makes impossible the spin-flip due to energy conservation law. As it was reported in Ref. [18] Rashba spin-orbit splitting in $A_{III}B_{V}$ heterostructures may attain to several meV. At the same time, the Zeeman energy splitting proposed in Ref. [9] for the same reason to suppress spin-flip processes in a dot approximately equals $0.3\text{meV}$ even in rather big magnetic field $5T$.

In accordance with expression (3) the mean reflection coefficient $R = 1 - T$ in the range $-\Gamma < \varepsilon < +\Gamma$ is approximately $1/3$. It means that the relative decrease in current for parallel spins is around $1/3$ if the bias $V \leq \Gamma/e$. For antiparallel spins it is twice smaller, i.e. around $1/6$. For the sake of better sensitivity the optimal bias $V$ for a ballistic quantum wire should be chosen around $V = \Gamma/e$. Then the absolute value of current could be roughly estimated for a single mode quantum wire as $I = G_0 V$, where $G_0 = e^2/h = (26k\text{Ohm})^{-1}$ is a conductance quantum for spin-polarized current in the wire. The possible level broadening $\Gamma$ is restricted only by spin-orbit splitting. Supposing the latter as several meV we are able to choose the broadening as $1\text{meV}$. Substituting $V = 1\text{mV}$ one arrives at the current equal to $4 \cdot 10^{-8}A$ which could be easily measured by up to date equipment. Moreover, this current exceeds that in a single electron transistor (SET). The greater is the current the faster is its measurement. One more significant advantage of a quantum wire is that it can be emptied during computing and, therefore, unlike to a SET the quantum wire does not introduce an additional decoherence in the system that time.

Worth noting a quantum wire allows to perform a partial measurement of the state of two adjacent spin qubits like in [9] or even distant ones: whether they are parallel or antiparallel. This also provides a possibility of quantum computation without organizing a perfectly controllable interaction between qubits.

If for some reason the reflection coefficient is too low, hopefully, the sensitivity may be augmented when $N$ identical qubits are placed in series along the wire. When qubits are situated randomly and, therefore, interference does not matter the sensitivity may rise as $\sim N$. When there is an order in qubit positions and the interference is significant the sensitivity increases as $\sim N^2$.

The opportunity to perform the proposed measurement is confirmed by findings in Ref. [19]. Schematically the structure was almost the same as that in Fig. 1. The combined Fano-Kondo anti-resonances were observed in the $I-V$ curve and exploited to test relaxation in multi-electron quantum dot. In principle, this set up
could serve as a prototype of our proposal.

III. CONCLUSION

We have examined the possibility to use Fano-Rashba resonances for non-demolishing measurement of a spin state (whether it is up or down) of a single electron in a quantum dot (spin qubit) via a spin-polarized current in an adjacent quantum wire. The spin-orbit interaction in a quantum dot prohibits spin-flip events (Kondo-like phenomenon). That makes the measurement non-demolishing.

Acknowledgments

The research was supported by NIX Computer Company (science@nix.ru), grant F793/8-05, via the grant of The Royal Swedish Academy of Sciences, and also by Russian Basic Research Foundation, grants # 08-07-00486-a-a and # 06-01-00097-a.

[1] D. Loss and D. DiVincenzo, Phys. Rev. A A57, 120 (1998).
[2] B. Kane, Nature 393, 133 (1998).
[3] P. Recher, E. Sukhorukov, and D. Loss, Phys.Rev.Lett. 85, 1962 (2000).
[4] M. Ciorga et al., Physica E 11, 35 (2001).
[5] Y. Manassen, R. J. Hamers, J. E. Demuth, A. J. Cestelano, Jr., Phys.Rev.Lett. 62, 2531 (1989).
[6] D. Shachal, Y. Manassen, Phys. Rev. B 46, 4795 (1992).
[7] Y. Manassen, I. Mukhopadhyay , Phys. Rev. B 61, 16223 (2000).
[8] Z. Nussinov, M. F. Cromrie, A. V. Balatsky, Phys. Rev. B 68, 085402 (2003).
[9] H. Engel and D. Loss, Science 309, 586 (2005).
[10] M. Sarovar, K. C. Young, T. Schenkel, and K. B. Whaley, Quantum non-demolishing measurement of single spins in semiconductors (15 Nov 2007), arXiv: 0711.2343.
[11] V. Vyurkov, V. Fedirko, and L. Fedichkin, Phys. Low-Dim. Structures 3/4, 209 (1999).
[12] V. Vyurkov, A. Vetrov, and A. Orlikovsky, Proc. SPIE. 1st Int. Symp. Quantum Informatics. Moscow. Russia. 5128, 164 (2003).
[13] L. Mourokh, V. Puller, A. Smirnov, and J. Bird, Appl.Phys.Lett. 87, 192501 (2005).
[14] V. Vyurkov, L. Gorelik, and A. Orlikovsky, Int. Symposium “Quantum Informatics’2005”, Moscow, 3rd-7th Oct., 2005, Book of Abstracts p. O27 (2005).
[15] E. Rashba, Sov.Phys.Semicond. 2, 1109 (1960).
[16] A. Zakharova, F. T. Vasko, and V. Ryzhii, J. Phys.: Condens. Matter 6, 7537 (1994).
[17] B. L. Wissinger, U. Rossler, R. Winkler, B. Jusserand, and D. Richards, Phys. Rev. B 58, 15375 (1998).
[18] J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys.Rev.Lett. 78, 1335 (1997).
[19] M. Sato, H. Aikawa, K. Kobayashi, S. Katsumoto, and Y. Iye, Phys.Rev.Lett. 95, 068801 (2005).