Non Newtonian Dynamics in Galaxies and Satellite Galaxies

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Abstract

Recent studies have thrown up a big enigma. On the one hand they point to satellite galaxies rotating faster than they should with the usual theory. On the other hand they also go against explanation of Newtonian gravity with dark matter. In this brief note it is argued that the varying $G$ cosmology which explains all the observed General Relativistic effects and the Pioneer anomaly reconciles the controversial conclusions of the latest observations.

Keywords: Newton, Dynamics, Galaxies.

1 Introduction

It is well known that F. Zwicky introduced the concept of dark matter to account for the anomalous rotation curves of the galaxies [1, 2]. The problem was that according to the usual Newtonian Dynamics the velocities of the stars at the edges of galaxies should fall with distance as in Keplarian orbits, roughly according to

$$v \approx \sqrt{\frac{GM}{r}}$$

(1)
where $M$ is the mass of the galaxy, $r$ the distance from the centre of the galaxy of the outlying star and $v$ the tangential velocity of the star. Observations however indicated that the velocity curves flatten out, rather than follow the law (1). This necessitated the introduction of the concept of dark matter which would take care of the discrepancy without modifying Newtonian dynamics. However even after nearly eight decades, dark matter has not been detected, even though there have been any number of candidates proposed for this, for example SUSY particles, massive neutrinos, undetectable brown dwarf stars, even black holes and so on.

Very recent developments are even more startling. These concern the rotating dwarf galaxies, which are satellites of the Milky Way [3, 4]. These studies throw up a big puzzle. On the one hand these dwarf satellites cannot contain any dark matter and on the other hand the stars in the satellite galaxies are observed to be moving much faster than predicted by Newtonian dynamics, exactly as in the case of the galaxies themselves. Metz, Kroupa, Theis, Hensler and Jerjen conclude that the only explanation lies in rejecting dark matter and Newtonian gravitation. Indeed a well known Astrophysicist, R. Sanders from the University of Groningen commenting on these studies notes [5], “The authors of this paper make a strong argument. Their result is entirely consistent with the expectations of modified Newtonian dynamics (MOND), but completely opposite to the predictions of the dark matter hypothesis. Rarely is an observational test so definite.” Moreover Vahe Petrosian of the Kavli Institute in the February 10 issue of the Astrophysical Journal, rules out dark matter on the basis of studying the interaction of electrons at the galactic edge with starlight.

In this note we point out that this could indeed be so, though not via Milgrom’s ad hoc modified dynamics [6, 7, 2], according to which a test particle at a distance $r$ from a large mass $M$ is subject to the acceleration $a$ given by

$$a^2/a_0 = MGr^{-2},$$

where $a_0$ is an acceleration such that standard Newtonian dynamics is a good approximation only for accelerations much larger than $a_0$. The above equation however would be true when $a$ is much less than $a_0$. Both the statements in (2) can be combined in the heuristic relation

$$\mu(a/a_0)a = MGr^{-2}$$

In (3) $\mu(x) \approx 1$ when $x >> 1$, and $\mu(x) \approx x$ when $x << 1$. It must be stressed that (2) or (3) are not deduced from any theory, but rather are an ad
hoc prescription to explain observations. Interestingly it must be mentioned that most of the implications of Modified Newtonian Dynamics or MOND do not depend strongly on the exact form of $\mu$.

It can then be shown that the problem of galactic velocities is now solved \[6, 7, 8, 9, 10\]. Nevertheless, most physicists are not comfortable with MOND because of the ad hoc nature of (2) and (3).

2 Varying $G$ Dynamics

We now come to the cosmological model described by the author in 1997 (Cf.ref.\[11, 2\] and several references therein), in which the universe, under the influence of dark energy would be accelerating with a small acceleration. Several other astrophysical relations, some of them hitherto inexplicable such as the Weinberg formula giving the pion mass in terms of the Hubble constant were also deduced in this model (Cf.also ref.\[12\] and references therein). While all this was exactly opposite to the then established theory, it is well known that the picture was observationally confirmed soon thereafter through the work of Perlmutter and others (Cf.ref.\[12\]). Interestingly, in this model Newton’s gravitational constant varied inversely with time.

Cosmologies with time varying $G$ have been considered in the past, for example in the Brans-Dicke theory or in the Dirac large number theory or by Hoyle \[13, 14, 15, 16, 17\]. In the case of the Dirac cosmology, the motivation was Dirac’s observation that the supposedly large number coincidences involving $N \sim 10^{80}$, the number of elementary particles in the universe had an underlying message if it is recognized that

$$\sqrt{N} \propto T$$

(4)

where $T$ is the age of the universe. Equation (4) too leads to a $G$ decreasing inversely with time as we will now show. We follow a route slightly different from that of Dirac.

From (4) it can easily be seen that

$$T = \sqrt{N}\tau$$

(5)

where $\tau$ is a typical Compton time of an elementary particle $\sim 10^{-23}secs$, because $T$, the present age of the universe is $\sim 10^{17}secs$. We also use the
following relation for a uniformly expanding Friedman Universe

\[ \dot{R}^2 = \frac{8\pi}{3} G R^2 \rho \]  

(6)

where \( R \) is the radius of the universe and \( \rho \) its density. We remember that

\[ \rho = \frac{3M}{4\pi R^3} \]  

and \( M = Nm \)  

(7)

where \( M \) is the mass of the universe, and \( m \) is the mass of an elementary particle \( \sim 10^{-25} \text{gm} \) (Cf. ref. [18]).

Use of (7) in (6) leads to another well known relation [19]

\[ R = \frac{GM}{c^2} \]  

(8)

because \( \dot{R} = c \). Further dividing both sides of (5) by \( c \) we get the famous Weyl-Eddington relation

\[ R = \sqrt{Nl} \]  

(9)

where \( l = \tau/c \) is a typical Compton length \( \sim 10^{-13} \text{cms} \).

Use of (7) and (9) in (8) now leads to

\[ G = \frac{c^2 l}{\sqrt{Nm}} = \left( \frac{c^2 \tau m}{m} \right) \cdot \frac{1}{T} \equiv \frac{G_0}{T} \]  

(10)

Equation (10) gives the above stated inverse dependence of the gravitational constant \( G \) on time, which Dirac obtained. On the other hand this same relation was obtained by a different route in the author’s dark energy – fluctuations cosmology in 1997. This work, particularly in the context of the Planck scale has been there for many years in the literature (Cf. [12, 2, 20] and references therein). Suffice to say that all the supposedly so called accidental Large Number Relations like (9) as also the inexplicable Weinberg formula which relates the Hubble constant to the mass of a pion, follow as deductions in this cosmology. The above references give a comprehensive picture.

The Brans-Dicke cosmology arose from the work of Jordan who was motivated by Dirac’s ideas to try and modify General Relativity suitably. In this scheme the variation of \( G \) could be obtained from a scalar field \( \phi \) which would satisfy a conservation law. This scalar tensor gravity theory was further developed by Brans and Dicke, in which \( G \) was inversely proportional to the
variable field $\phi$. (It may be mentioned that more recently the ideas of Brans and Dicke have been further generalized.)

In the Hoyle-Narlikar steady state model, it was assumed that in the Machian sense the inertia of a particle originates from the rest of the matter present in the universe. This again leads to a variable $G$. The above references give further details of these various schemes and their shortcomings which have lead to their falling out of favour.

In any case, our starting point is, equation (10) where $T$ is time (the age of the universe) and $G_0$ is a constant. Furthermore, other routine effects like the precession of the perihelion of Mercury and the bending of light and so on are also explained with (10) as will be briefly discussed below. We will also see that there is observational evidence for (10) (Cf. also [21] which described various observational evidences for the variation of $G$, for example from solar system observations, from cosmological observations and even from the palaeontological studies point of view).

With this background, we now mention some further tests for equation (10). This could explain the other General Relativistic effects like the shortening of the period of binary pulsars and so on (Cf. ref.[12, 2, 22, 23] and other references therein). Moreover, we could now also explain, the otherwise inexplicable anomalous acceleration of the Pioneer space crafts (Cf. ref.[2] for details). We will briefly revisit some of these effects later.

We now come to the problem of galactic rotational curves mentioned earlier (cf.ref.[1]). We would expect, on the basis of straightforward dynamics that the rotational velocities at the edges of galaxies would fall off according to

$$v^2 \approx \frac{GM}{r}$$

which is (11). However it is found that the velocities tend to a constant value,

$$v \sim 300 \text{km/sec}$$

This, as noted, has lead to the postulation of the as yet undetected additional matter alluded to, the so called dark matter. (However for an alternative view point Cf.[24]). We observe that (10) can be written for an increase $t$, in time, small compared to the age of the universe, now written as $t_0$

$$G = \frac{G_0}{t_0 + t} = \frac{G_0}{t_0} \left(1 - \frac{t}{t_0}\right)$$

(13)
Using (13), let us consider the gravitational potential energy $V$ between two masses, $m_1$ and $m_2$ by:

$$V = \frac{G m_1 m_2}{r_0} = \frac{G}{t_0} \frac{m_1 m_2}{r_0}$$  \hspace{1cm} (14)$$

After a time $t$ this would be, by (13),

$$V = \frac{G}{t_0} \left(1 - \frac{t}{t_0}\right) \frac{m_1 m_2}{r}$$ \hspace{1cm} (15)$$

Equating (14) and (15) we get,

$$r = r_0 \left(\frac{t_0}{t_0 + t}\right)$$ \hspace{1cm} (16)$$

The relation (16) has been deduced by a different route by Narlikar [1]. From (16) it easily follows that,

$$a \equiv \dot{r} \approx \frac{1}{t_o} (t o + 2 \dot{r}) \approx -2 \frac{r_0}{t^2}$$ \hspace{1cm} (17)$$

as we are considering intervals $t$ small compared to the age of the universe and nearly circular orbits. In (17), $a$ or the left side of (17) gives the new extra effect due to (13) and (16), this being a departure from the usual Newtonian gravitation. Equation (17) shows (Cf.ref[22] also) that there is an anomalous inward acceleration, as if there is an extra attractive force, or an additional central mass.

So, introducing the extra acceleration (17), we get,

$$\frac{G M m}{r^2} + \frac{2 m r}{t^2} \approx \frac{m v^2}{r}$$ \hspace{1cm} (18)$$

From (18) it follows that

$$v \approx \left(\frac{2 r^2}{t^2} + \frac{G M}{r}\right)^{1/2}$$ \hspace{1cm} (19)$$

So (19) replaces (11) in this model. This shows that as long as

$$\frac{2 r^2}{t^2} \ll \frac{G M}{r}, \hspace{1cm} (20)$$
Newtonian dynamics holds. But when the first term on the left side of (20) becomes of the order of the second (or greater), the new dynamical effects come in.

For example from (19) it is easily seen that at distances well within the edge of a typical galaxy, that is \( r < 10^{23}\text{cms} \) the usual equation (11) holds but as we reach the edge and beyond, that is for \( r \geq 10^{24}\text{cms} \) we have \( v \sim 10^7\text{cms per second} \), in agreement with (12). In fact as can be seen from (19), the first term in the square root has an extra contribution (due to the varying \( G \)) which exceeds the second term as we approach the galactic edge, as if there is an extra mass, that much more.

We would like to stress that the same conclusions will apply to the latest observations of the satellite galaxies (without requiring any dark matter). Let us for example consider the Megallanic clouds [25]. In this case, as we approach their edges, the first term within the square root on the right side of (19) or the left term of (20) already becomes of the order of the second term, leading to the new nonNewtonian effects.

3 Remarks

We have already noted that the varying \( G \) dynamics given in (10) explains all the General Relativistic effects. This work has been available in the literature for many years (Cf.[22, 12, 20, 2] for full details and further references). However we repeat some examples briefly to give an idea.

We could explain the correct gravitational bending of light. Infact in Newtonian theory too we obtain the bending of light, though the amount is half that predicted by General Relativity[1, 26, 27, 28]. In the Newtonian theory we can obtain the bending from the well known orbital equations (Cf.also[29]),

\[
\frac{1}{r} = \frac{GM}{L^2}(1 + e\cos\Theta)
\]  

(21)

where \( M \) is the mass of the central object, \( L \) is the angular momentum per unit mass, which in our case is \( bc \), \( b \) being the impact parameter or minimum approach distance of light to the object, and \( e \) the eccentricity of the trajectory is given by

\[
e^2 = 1 + \frac{c^2L^2}{G^2M^2}
\]  

(22)
For the deflection of light $\alpha$, if we substitute $r = \pm \infty$, and then use (22) we get

$$\alpha = \frac{2GM}{bc^2}$$

This is half the General Relativistic value.

We now note that the effect of time variation of $r$ is given by equation (16)(cf.ref. 22). Using this the well known equation for the trajectory is given by,

$$u'' + u = \frac{GM}{L^2} + u \frac{t}{t_0} + 0 \left( \frac{t}{t_0} \right)^2$$

where $u = \frac{1}{r}$ and primes denote differentiation with respect to $\Theta$.

The first term on the right hand side represents the Newtonian contribution while the remaining terms are the contributions due to (16). The solution of (24) is given by

$$u = \frac{GM}{L^2} \left[ 1 + e \cos \left\{ \left(1 - \frac{t}{2t_0} \right) \Theta + \omega \right\} \right]$$

where $\omega$ is a constant of integration. Corresponding to $-\infty < r < \infty$ in the Newtonian case we have in the present case, $-t_0 < t < t_0$, where $t_0$ is large and infinite for practical purposes. Accordingly the analogue of the reception of light for the observer, viz., $r = +\infty$ in the Newtonian case is obtained by taking $t = t_0$ in (25) which gives

$$u = \frac{GM}{L^2} + e \cos \left( \frac{\Theta}{2} + \omega \right)$$

Let us compare (26) with the Newtonian solution. This shows that the Newtonian $\Theta$ is replaced by $\frac{\Theta}{2}$, whence the deflection obtained by equating the left side of (26) to zero, is

$$\cos \Theta \left(1 - \frac{t}{2t_0}\right) = -\frac{1}{e}$$

where $e$ is given by (22). The value of the deflection from (27) is twice the Newtonian deflection given by (23). That is the deflection $\alpha$ is now given not by (23) but by the formula,

$$\alpha = \frac{4GM}{bc^2}$$
The relation (28) is the correct observed value and is the same as the General Relativistic formula which however is obtained by a different route [28, 30, 31].

Next, we come to the inexplicable anomalous accelerations of the Pioneer spacecrafts already alluded to, which have been observed by J.D. Anderson and coworkers at the Jet Propulsion Laboratory for well over a decade [32, 33] and have posed a puzzle. This can be explained in a simple way as follows: In fact from the usual orbital equations we have [34]

\[ v \dot{v} \approx -\frac{GM}{2tr^2}(1 + e\cos\Theta) - \frac{GM}{r^2} \dot{r}(1 + e\cos\Theta) \]

\( v \) being the velocity of the spacecraft and \( t \) is the time in general. It must be observed that the first term on the right side is the new effect due to (10).

There is now an anomalous acceleration given by

\[ a_r = \langle \dot{v} \rangle_{\text{anom}} = -\frac{GM}{2trv}(1 + e\cos\Theta) \]

\[ \approx -\frac{GM}{2t\lambda}(1 + e)^3 \]

where

\[ \lambda = r^4\dot{\Omega}^2 \]

If we insert the values for the Pioneer spacecrafts we get

\[ a_r \sim -10^{-7}\text{cm/sec}^2 \]

This is the anomalous acceleration reported by Anderson and co-workers.

We will next deduce that equations like (16) explains correctly the observed decrease in the orbital period of the binary pulsar PSR 1913 + 16, which has also been attributed to as yet undetected gravitational waves [35].

It may be observed that the energy \( E \) of two masses \( M \) and \( m \) in gravitational interaction at a distance \( L \) is given by

\[ E = \frac{GMm}{L} = \text{constant} \]  

(29)

We note that if this energy decreases by any mechanism, for example by the emission of gravitational waves, or by the decrease of \( G \), then because of (29), there is a compensation by the decrease in the orbital length and
orbital period. This is the standard General Relativistic explanation for the binary pulsar PSR 1913 + 16. We will show that the same holds good, if we are given instead, (10). That is, we will not invoke gravitational waves. In this case we have, from (29)

\[
\frac{\mu}{L} \equiv \frac{GMm}{L} = \text{const.} \tag{30}
\]

We can now write, for a time increase \( t \),

\[
\mu = \mu_0 - Kt \tag{31}
\]

where we have

\[
K \equiv \dot{\mu} \tag{32}
\]

In (32) \( \dot{\mu} \) can be taken to be a constant in view of the fact that \( G \) varies very slowly with time. Specifically we have

\[
G(T + t) = G(T) - \frac{tG}{T} + \frac{t^2 G}{2 T^2} + \cdots \approx G(T) - \frac{tG}{T} \tag{33}
\]

where \( T \) is the age of the universe and \( t \) is an incremental time. Whence using (33), \( K \) in (32) is given by

\[
K \propto \frac{G}{T}
\]

and so

\[
\dot{K} \sim \frac{G}{T^2} \approx 0
\]

So (30) requires

\[
L = L_0(1 - \alpha K)
\]

Whence on using (31) we get

\[
\alpha = \frac{t}{\mu_0} \tag{34}
\]

Let us now consider \( t \) to be the period of revolution in the case of the binary pulsar. Using (31) it follows that

\[
\delta L = -\frac{L_0 t K}{\mu_0} \tag{35}
\]
We also know (Cf.ref. [34])

\[ t = \frac{2\pi}{h} L^2 = \frac{2\pi}{\sqrt{\mu}} \]  

(36)

\[ t^2 = \frac{4\pi^2 L^3}{\mu}, \]  

(37)

\( h \) being the usual unit angular momentum and \( \mu \) has the units \( gm \ cm^4 \ sec^{-1} \).

Using (35), (36) and (37), a little manipulation gives

\[ \delta t = -\frac{2t^2 K}{\mu_0} \]  

(38)

(35) and (38) show that there is a decrease in the size of the orbit, as also in the orbital period. Such a decrease in the orbital period has been observed in the case of binary pulsars in general [35, 36].

Let us now apply the above considerations to the specific case of the binary pulsar PSR 1913 + 16 observed by Taylor and coworkers (Cf.ref. [36]). In this case it is known that, \( t \) is 8 hours while \( v \), the orbital speed is \( 3 \times 10^7 \) cms per second. It is easy to calculate from the above

\[ \mu_0 = 10^4 \times v^3 \sim 10^{26} \]

which gives \( M \sim 10^{33} gms \), which of course agrees with observation. Further we get using (4) and (31)

\[ \Delta t = \eta \times 10^{-5} \ sec/yr, \eta < \approx 8 \]  

(39)

Indeed (39) is in good agreement with the carefully observed value of \( \eta \approx 7.5 \) (Cf.refs. [35, 36]).

It should also be remarked that in the case of gravitational radiation, there are some objections relevant to the calculation (Cf.ref. [35]).

Finally, we may point out that a similar shrinking in size with time can be expected of galaxies themselves, and in general, gravitationally bound systems.

To consider the above result in a more general context, we come back to the well known orbital equation [34]

\[ d^2u/d\Theta^2 + u = \frac{\mu_0}{h^2} \]  

(40)
where $\mu_0 = GM$ and $u$ is the usual inverse of radial distance. $M$ is the mass of the central object and $h = r^2 d\Theta/dt$ - a constant. The solution of (40) is well known,

$$lu = 1 + e\cos\Theta$$

where $l = h^2/\mu_0$.

It must be mentioned that in the above purely classical analysis, there is no precession of the perihelion.

We now replace $\mu_0$ by $\mu$ and also assume $\mu$ to be varying slowly because $G$ itself varies slowly and uniformly, as noted earlier:

$$\dot{\mu} = d\mu/dt = K, \text{ a constant} \quad (41)$$

remembering that $\dot{K} \sim 0(1/T^2)$ and so can be neglected.

Using (41) in (40) and solving the orbital equation (40), the solution can now be obtained as

$$u = 1/l + (e/l)\cos\Theta + Kl^2\Theta/h^3 + Kl^2e\Theta\cos\Theta/h^3 \quad (42)$$

Keeping terms up to the power of \( e \) and \((K/\mu_0)^2\), the time period \( \tau \) for one revolution is given to this order of approximation by

$$\tau = 2\pi L^2/h \quad (43)$$

From (42)

$$L = l - K\frac{l^4\Theta}{h^3} \quad (44)$$

Substituting (41) in (43) we have

$$\tau = \frac{2\pi}{h} \left(l^2 - \frac{2Kl^5\Theta}{h^3}\right) \quad (45)$$

The second term in (45) represents the change in time period for one revolution. The decrease of time period is given by

$$\delta\tau = 8\pi^2l^3K/\mu_0^2 \quad (46)$$

The second term in (44) indicates the decrease in latus-rectum.

For one revolution the change of latus-rectum is given by

$$\delta l = 2\pi Kl^{2.5}/\mu_0^{1.5} \quad (47)$$
In the solar system, we have,

\[ K = 898800 \text{ cm gm} \]

Using \( K \) and \( \mu_0 \) to find the change in time period and the latus rectum in the varying \( G \) case by substituting in (46) and (47) respectively for Mercury we get

\[ \delta T = 1.37 \times 10^{-5} \text{ sec/rev} \]
\[ \delta l = 4.54 \text{ cm/rev} \]

We observe that the equations (46), (47) or (48) show a decrease in distance and in the time of revolution. If we use for the planetary motion, the General Relativistic analogue of (40), viz.,

\[ \frac{d^2 u}{d\Theta^2} + u = \frac{\mu_0}{h^2} (1 + 3h^2 u^2), \]

then while we recover the precession of the perihelion of Mercury, for example, there is no effect similar to (46), (47) or (48). On the other hand this effect is very minute– just a few centimeters per year in the case of the earth– and only protracted careful observations can detect it. Moreover these changes could also be masked at least partly, by gravitational and other perturbations. However as noted, the decrease of the period in (46) has been observed in the case of Binary Pulsars.

4 Conclusion

The point is that the above varying \( G \) scheme described in (10) or (19) reproduces all the effects of General Relativity as noted above, as also the anomalous acceleration of the Pioneer space crafts in addition to the conclusions of MOND regarding an alternative for dark matter, and is applicable to the latest observations of satellite galaxies. The satellite galaxy rotation puzzle is thus resolved.

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