Correction factors for primary beta dosimetry

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Abstract
Several correction factors for primary beta dosimetry in radiation protection have been determined by means of measurements and simulations for both the well-known Böhm primary standard extrapolation chamber developed at PTB (which has been commercially available for a long time) and a new primary standard extrapolation chamber also developed at PTB. The correction factors $k_{el}$ for electrostatic attraction of the entrance window due to the collecting voltage and $k_{sat}$ for ionisation collection losses due to ionic recombination have been measured. The following correction factors have been determined by simulations for normal (0°) as well as oblique (15°, 30°, 45° and 60°) radiation incidence: $k_{ba}$ for backscatter from the collecting electrode, $k_{pe}$ for perturbation by the chamber’s side walls, and $k_{ia}$, $k_{ac}$ and $k_{ih}$ for inhomogeneity inside the collecting volume. The latter three have been combined to $k_{ih}$. Six more correction factors adopted from the literature are described to complete the necessary corrections. A comparison with values given in ISO 6980-2:2004 (for 0°), all determined at that time by means of measurements, is shown as well as the application of the new factors to measured data. The newly determined correction factors are ready to be implemented in an updated version of ISO 6980-2.

Keywords: electron dosimetry, ionization chambers, measurement units and standards, computer modeling and simulation

(Some figures may appear in colour only in the online journal)

1. Introduction
Primary beta dosimetry in radiation protection has, for a long time, been based on the ISO series ISO 6980 [1–3]. Therein, extrapolation chamber measurements, the application of many correction factors, and subsequent determination of the absorbed dose to tissue, $D_h$, at a certain depth in a tissue equivalent phantom finally yield the radiation protection quantities such as $H_p(0.07)$ and $H'(0.07)$. Decades ago, many correction factors described in ISO 6980-2 [2] were determined by means of actual measurements. In this work, some of these correction factors have been re-determined either by measurements or by Monte Carlo particle transport simulations for both normal (0°) as well as oblique (15°, 30°, 45° and 60°) radiation incidence.

Section 2 describes the measurement and simulation methods and basic geometries used in the simulations; section 3 outlines the methods to determine the different corrections factors as well as the corresponding results and their comparison with the values contained in ISO 6980-2 [2]. Section 4 summarizes some more correction factors adopted from the literature while, finally, section 5 presents the application of the new data. Appendix A describes the newly developed primary extrapolation chamber from PTB and appendix B presents tables with the calculated correction factors.

All correction factors covered in this work have been determined for reference beta-particle radiation fields according to ISO 6980, for all beta radiation fields of the PTB developed Beta Secondary Standard BSS 2 [4–6], and for some additional beta radiation fields.
2. Measurements, simulations and verification

2.1. Measurements

2.1.1. Extrapolation chambers. Two different extrapolation chambers are currently used at PTB: the Böhm chamber [7] (Beta Primary Standard, BPS1) and a new extrapolation chamber (BPS2), both developed at PTB. For illustration purposes, figure 1 shows the two extrapolation chambers. The Böhm chamber was described in detail earlier [7], the new chamber is described in appendix A. For both BPS, Keithly 642 high precision electrometers were used to determine the ionization current, while for BPS2 a newly PTB developed electrometer was also used [8].

2.1.2. Correction factors measured. The correction factors for the electrostatic attraction of the entrance window, \( k_{el} \), and for the ionization losses due to recombination, \( k_{sat} \), cannot be determined by normal particle transport simulations as the electric field inside the chamber is relevant. Therefore, the factors were investigated by measurements, see sections 3.1 and 3.2, using both extrapolation chambers.

2.2. Simulations

2.2.1. Method and simulation parameters. The simulations were carried out using the Monte Carlo particle transport code package EGSnrc [9] with BEAMnrc [10, 11] utilising EGSpp [12]. The transport parameters were chosen as follows: The cross-sectional data for the electron transport (where the condensed history technique is applied) are: Standard EGSnrc (based on the Bethe-Bloch theory) for collision stopping power and Bethe-Heitler cross sections for radiative stopping power. For photons, the XCOM cross sections are used. The maximum energy loss per electron step is 20\% (ESTEPE = 0.20) and photons and electrons are followed down to an (kinetic) energy of 1 keV for the \(^{147}\)Pm source and 10 keV for the others.

To make the simulations efficient, the sources were not simulated directly but phase space files produced in earlier simulations were used as input [13]. These files contain the full information of a radiation field in a certain cross-sectional plane, e.g. 15 cm from the source, by storing the position, direction, energy, and type of particle (electron or photon) of millions of particles. The same method (use of the phase space files) had been applied earlier [14].

2.2.2. Geometries and substances. Figure 2 shows the geometry of this work’s simulations. A surrounding air cylinder with a diameter of \( \Theta = 1.2 \) m and height of \( z = 0.8 \) m forms the outer boundaries. The geometry is shown exemplarily for an angle of radiation incidence of \( \alpha = 0^\circ \) and for the BPS2. In the simulation producing the phase space files the front of the radiation source was located at the origin (0/0/0) [13]. The front of the extrapolation chamber’s entrance foil was located at different distances per specification of the respective radiation field, which are given in table 1. Therefore, the factors were investigated by measurements, see sections 3.1 and 3.2, using both extrapolation chambers.

The collecting volume (i.e. the active volume of the chamber) of both chambers is located directly behind the entrance window, has a diameter of 30 mm and can be varied in depth.
Figure 2. Overview of the simulation geometry, to scale.

Table 1. Radiation field geometries of the BSS 2 for which the simulations were performed.

| Radionuclide (source) | 11 cm, without filter | 20 cm, without filter | 20 cm, with filter | 30 cm, without filter | 30 cm, with filter | 50 cm, without filter | 50 cm, with filter |
|-----------------------|-----------------------|-----------------------|-------------------|----------------------|-------------------|----------------------|-------------------|
| $^{147}$Pm            | x                     | x                     |                   | x                    |                   | x                    |                   |
| $^{85}$Kr             |                       |                       |                   |                      |                   |                      |                   |
| $^{90}$Sr/$^{90}$Y    | x                     | x                     |                   |                      |                   |                      |                   |
| $^{106}$Ru/$^{106}$Rh | x                     | x                     |                   |                      |                   |                      |                   |
| Distance of the phase space file plane from the front of the source | 2 cm | 2 cm | 15 cm | 15 cm | 15 cm | 15 cm | 15 cm |
| Distance of the phase space file plane from the phantom surface | 9 cm | 18 cm | 5 cm | 15 cm | 35 cm |

from approximately 50 µm up to 10 mm. All simulations were performed
(a) for the two primary standards BPS1 and BPS2,
(b) for the field geometries shown in table 1 (14 source combinations),
(c) at chamber depths $l$ from 250 µm up to 2500 µm in steps of 250 µm (10 depths) and
(d) at angles of incidence $\alpha$ from 0° up to 60° in steps of 15° (5 angles of incidence).

This is in total 1400 simulations for each chamber geometry.

2.2.3. Uncertainty of the simulations. The non-statistical standard uncertainty of the calculated dose values is assumed to be in the order of 2% with the main contribution attributed to the uncertainty of the radiative stopping power of electrons [15]. However, the correction factors calculated, see below, are ratios of two doses which are highly correlated as the simulation program and materials are the same for all calculated doses (only some details of the extrapolation chamber are different). Therefore, the non-statistical uncertainty of the correction factors is assumed to be negligible and only the uncertainty due to statistics supplied by EGSnrc is considered. The simulation times were chosen so that the statistical standard uncertainty of the correction factors is less or equal to 0.2%
up to 0.5% resulted (the larger the angle of incidence the larger the uncertainty).

2.2.4. Verification of the simulations. As outlined in section 2.2.1, the method of utilizing phase space files as a starting point and then only varying the object the radiation impinges on, i.e. the extrapolation chamber, had been used earlier [14]. At that time, detailed comparisons with measurements showed good agreement with the simulations [14]. Therefore, the results of these simulations in this work are also considered to be reliable.

2.2.5. Correction factors calculated. The correction factors for the backscatter from the collecting electrode, $k_{ba}$, the perturbation from the chamber’s side walls, $k_{pe}$, and for the inhomogeneity inside the collecting volume, $k_{inh}$, cannot or not easily be determined by measurements as the side walls of the chamber cannot be removed during the measurements (necessary to determine $k_{pe}$) and as the collecting volume cannot be reduced (necessary to determine $k_{inh}$). Therefore, these factors were investigated by simulations, see sections 3.3, 3.4 and 3.5.

3. Determination of correction factors and their results

3.1. Correction for the electrostatic attraction of the entrance window, $k_{ei}$

The collecting volume of the chamber could be reduced by the electrostatic attraction of the entrance window due to the applied electric field of $10 \text{ V mm}^{-1}$. Therefore, the capacity of the chamber was measured at field strengths from 3 up to $100 \text{ V mm}^{-1}$ at chamber depths from 250 $\mu$m up to 2500 $\mu$m for both BPS1 and BPS2 (at 3, 7, 10, 30, 70, and 100 $\text{ V mm}^{-1}$). There was no significant change of the capacity up to $30 \text{ V mm}^{-1}$, i.e. the distance between the entrance foil and the back electrode, was measured (below 0.1%). At $70 \text{ V mm}^{-1}$ and $100 \text{ V mm}^{-1}$ a change in the order of 0.2% and 0.5% was measured at 250 $\mu$m and 1000 $\mu$m, respectively; but no significant change at 1750 $\mu$m and 2500 $\mu$m.

As mentioned above, the electric field during normal operation is $10 \text{ V mm}^{-1}$. Therefore, $k_{ei} = 1.00 \pm 0.01$ was concluded for both BPS1 and BPS2.

3.2. Correction for ionization losses due to recombination, $k_{sat}$

The ionization current is usually measured at an electric field strength of $10 \text{ V mm}^{-1}$. Due to ionization losses the current may be smaller than that, as if the electric field were infinity. Therefore, the current was measured

(a) using the two primary standards BPS1 and BPS2,
(b) from both a $^{88}\text{Kr}$- and a $^{90}\text{Sr/}^{90}\text{Y}$-source,
(c) at electric fields of 10, 15, 20, 30 and 60 $\text{ V mm}^{-1}$ and
(d) at chamber depths of 1000 and 2500 $\mu$m.

To obtain the current equivalent to an infinite electric field, the measured values were plotted in Jaffé diagrams, i.e. a reciprocal plot of the ionization current ($1/I$) against the reciprocal of the electric field ($1/E$). The linear regions were extrapolated to $1/(E = 0)$ being equivalent to $E = \infty$ resulting in the current $i.e. (a \cdot l)$. $a$, the effective collecting electrode area, $U_s$ the absolute value of the collecting voltage, $E_1$ = 4.4 $\text{V m}$, $e$, the elementary charge, $T$, the air temperature, expressed in kelvins, $k_b^*$, the Boltzmann’s constant.

According to to ISO 6980-2 [2] the total estimated uncertainty of $k_{sat}$ is lower than 0.2%.

3.3. Correction for backscatter from the collecting electrode, $k_{ba}$

The aim of measurement is to determine the absorbed dose to tissue, $D_a$. As extrapolation chambers are made of material different to tissue the correction for the difference in backscatter between tissue and the material of the collecting electrode needs to be assessed. This was achieved by comparing two simulations: (i) the simulation of the chamber with the collecting electrode made of tissue and (ii) the simulation of the real chamber. The correction factor is given by their ratio

$$k_{ba} = \frac{D_{\text{tissue}}}{D_{\text{chamber}}}$$

with $D_{\text{tissue}}$, the calculated dose in the active volume with a tissue back in the chamber and $D_{\text{chamber}}$, the calculated dose in the active volume with the real chamber material.

It turned out that the values of $k_{ba}$ are independent of the chamber depth $l$ (250 $\mu$m up to 2500 $\mu$m). Therefore, figure 3 shows the calculated values (mean for all ten chamber depths) of $k_{ba}$ depending on the source, i.e. on the radiation’s field mean energy, and the angle of incidence, $\alpha$, for both BPS1 and BPS2; the corresponding data are listed in tables B1 and B2 in appendix B. The back electrode of BPS1 is made of Polymethyl methacrylate (PMMA) while the one of
BPS2 is made of polyether ether ketone (PEEK). ISO 6980-2 [2] contains experimentally determined values for PMMA for $\alpha = 0^\circ$ which are shown in figure 3 in comparison with the calculated values for BPS1. The ISO-values are smaller than the calculated ones by 1.6%, 0.8%, and 0.8% for $^{147}$Pm, $^{85}$Kr, and $^{90}$Sr/$^{90}$Y, respectively, compared to the calculated ones. The measured data are uncertain by about 2% [17], however, a systematic effect can be supposed but not explained as all measurements are smaller than the calculations. Further more, it is obvious that the values for BPS2 are significantly smaller than those for BPS1, especially for $^{147}$Pm. The probable reason is that the backscatter contribution from PMMA is smaller than that of PEEK, especially in the sub 0.1 MeV energy region, which was apparent from simple simulations using EGSnrc [9, 12].

3.4. Correction for perturbation by the chamber’s side wall, $k_{pe}$

The side wall of an extrapolation chamber perturbs the electron flux by scattering and absorption. In the past rings with the
same diameter of the chamber, with varying thicknesses, were placed in front of the chamber to assess this influence indirectly as it is not possible to use the chamber without side wall \[2, 18\]. Nowadays, the direct determination of this influence is possible by simulating the chamber without a side wall. The correction factor is then given by their ratio

\[
k_{pe} = \frac{D_{\text{wallless chamber}}}{D_{\text{chamber}}} \tag{4}
\]

with \(D_{\text{wallless chamber}}\), the calculated dose in the active volume with the chamber wall replaced by air and \(D_{\text{chamber}}\), the calculated dose in the active volume with the real chamber material.

Figure 4 shows the resulting correction factor for \(^{90}\text{Sr}/^{90}\text{Y}\), with a beam flattening filter, at a distance of 30 cm from the source for the BPS1 and BPS2 depending on the chamber depth \(l\). At zero chamber depth no side wall is present resulting in no influence. Therefore, the calculated data points were fitted by a regression resulting for \(l = 0 \mu m\) in \(k_{pe} = 1.0\), i.e. through the point (0/1). For this, 2nd order polynomial regressions were suited much better than linear regressions, see figure 4. The side walls’ influence is larger for BPS1 than

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**Figure 4.** Calculated correction factor \(k_{pe}\) for BPS1 (top) and BPS2 (bottom) for the source \(^{90}\text{Sr}/^{90}\text{Y}\), 30 cm, with filter’ depending on the chamber depth and the angle of incidence, \(\alpha\), see legend. Symbols: calculation; solid lines: 2nd order polynomial regression through (0/1); dashed lines: linear regression through (0/1). The uncertainty bars represent the uncertainty of the single simulations (\(\leq 0.25\%\)).
for BPS2 which is plausible as the chamber diameters are 6 cm and 8 cm respectively, while for both chambers the active volume is 3 cm in diameter. For small angles of incidence, the correction is smaller than unity as radiation scattered at the side wall enlarges the signal in the active volume. In contrast, for angles above 45°, the side wall is partly located between the source and the active volume (shadowing), resulting in absorption in turn resulting in corrections larger than unity. This effect becomes smaller as the distance between the source and the chamber decreases, because the opening angle under which the chamber appears from the source position increases with decreasing distance. The 2nd order polynomial regression through (0/1) is represented by

\[
k_{pe} = 1 + f_7 \cdot l + f_8 \cdot l^2
\]

with \( f_7 \) and \( f_8 \), parameters obtained from a 2nd order polynomial regression through (0/1) and \( l \), the chamber depth.

The same equation is used in ISO 6980-2 \[2\]. The parameters \( f_7 \) and \( f_8 \) obtained from the regression are listed for all sources in tables B3 and B4 for BPS1 and BPS2, respectively, in appendix B.

In general, the correction becomes larger as the chamber depth becomes larger as the side wall increases with growing chamber depths. Therefore, to get an overview for all sources, figure 5 shows \( k_{pe} \) for the largest chamber depth, i.e. for \( l = 2500 \mu m \), depending on the radiation’s field mean energy and the angle of incidence \( \alpha \). For both BPS1 and BPS2, the behaviour of \( k_{pe} \) is similar with larger effects for BPS1 which is plausible as outlined above. The dependence on the mean energy results from the mixture of the contributions of scattering and absorption in both the chamber wall as well as in the air between the source and the chamber. For example, electrons with small energies are significantly scattered in the air between the source and the chamber resulting in a much broader angular distribution of the electrons in the chamber than for larger electron energies. This effect was investigated in detail earlier \[13\].

ISO 6980-2 gives only values for normal incidence (\( \alpha = 0° \)) which were measured at that time using BPS1 \[18\] (see open red diamonds in the top part of figure 5). For all radionuclides apart from \( ^{85} \)Kr the values from ISO and those calculated in this work agree quite well. The disagreement for \( ^{85} \)Kr could be attributed to the fact that, instead of \( ^{85} \)Kr beta radiation from the nuclide, \( ^{208} \)Tl was measured at that time and its values were assumed to be valid for \( ^{85} \)Kr, too \[2, 18\].

3.5. Correction for inhomogeneity inside the collecting volume, \( k_{ih} \)

As the absorbed dose is a point quantity, the dose at a single point is of interest while the measurement is performed using a finite active volume. In ISO 6980-2 \[2\], this effect is assessed by three different correction factors: the correction for the radial non-uniformity of the absorbed-dose rate in the radiation field, \( k_{ra} \), the correction for the attenuation of beta particles in the collecting volume, \( k_{ac} \), and the correction for axial non-uniformity, \( k_{ai} \). At the time of ISO 6980-2, \( k_{ra} \) was determined by measurements \[19\], \( k_{ac} \) by a combination of measurements and an analytic method while \( k_{ai} \) was determined purely analytically. In this work, their combination, \( k_{ih} = k_{ra} \cdot k_{ac} \cdot k_{ai} \), was determined by simulating the real chamber and from this simulation two doses were obtained: (i) the dose in the total active volume of 3 cm diameter and the whole chamber depth \( l \), \( D_{\text{total act. vol.}} \), and (ii) the dose in the centre front part of the active volume of 0.5 cm diameter and 100 \( \mu m \) depth, \( D_{\text{part of act. vol.}} \). The latter was used as a representative for the dose in the single centre point at the front of the active volume. The correction factor is then given by their ratio

\[
k_{ih} = \frac{D_{\text{part of act. vol.}}}{D_{\text{total act. vol.}}}
\]

Figure 6 shows the resulting correction factor for \( ^{147} \)Pm, with a beam flattening filter, at a distance of 20 cm from the source for the BPS1 and BPS2 depending on the chamber depth \( l \). This dependence can be fitted by a linear regression, see figure 6. General trends are as follows.

(a) \( k_{ih} \) increases linearly with increasing chamber depth, \( l \), which is plausible as the difference in the two volumes, part of the active volume vs. total active volume, increases linearly with increasing \( l \).

(b) \( k_{ih} \) is largest for normal incidence, i.e. for \( \alpha = 0° \), taking values above unity. That means, the dose in a point is larger than the one measured using the real, finite active volume. This is caused by the fact that the dose rate decreases outside of the radiation fields’ centre which was already seen when simulating the radiation fields of the BSS 2 \[13\].

(c) \( k_{ih} \) decreases with increasing \( \alpha \), even below unity. The reason is that for increasing values of \( \alpha \), the chambers’ side wall is located closer to the axis between the radiation source and the active volume. Thus, radiation is scattered from the side wall into the active volume. This in-scattered contribution is larger at the side of the active volume than in its centre, i.e. \( D_{\text{total act. vol.}} \) increases more than \( D_{\text{part of act. vol.}} \) resulting in \( k_{ih} < 1.0 \).

The linear regression mentioned above is represented by

\[
k_{ih} = b + m \cdot l
\]

with \( b \) and \( m \), parameters obtained from a linear regression and \( l \), the chamber depth.

The parameters \( b \) and \( m \) obtained from the regression are listed for all sources in tables B5 and B6 for BPS1 and BPS2, respectively, in appendix B.

To get an overview for all sources, figure 7 shows the mean of \( k_{ih} \) for all chamber depths together with its largest value (which it always takes for the largest \( l = 2500 \mu m \)) and its smallest value (which it always takes for the smallest \( l = 250 \mu m \)) depending on the radiation’s field mean energy and the angle of incidence \( \alpha \). For both BPS1 and BPS2, the behaviour of \( k_{ih} \) is the same with larger values for BPS1 as already seen in figure 6. At small distances, especially at 11 cm from the source, \( k_{ih} \) takes values farthest from unity, i.e. at 0° largest and at 60° smallest, as at small distances the radiation fields
are most inhomogeneous \cite{13}. Similarly, the radiation fields of $^{147}$Pm are most inhomogeneous and, therefore, $k_{ih}$ shows the largest spread for this radionuclide while no further distinct energy dependence is present.

For all correction factors, ISO 6980-2 gives only values for normal incidence ($\alpha = 0^\circ$). $k_{ac}$ and $k_{di}$ can be determined analytically according to ISO 6980-2 for nearly all source combinations while $k_{ra}$ was determined only for source combinations with a beam flattening filter \cite{19}. These three correction factors taken from ISO were multiplied to obtain $k_{ih} = k_{ra} \cdot k_{ac} \cdot k_{di}$ for comparison. The resulting values are given as open red diamonds in figure 7 for comparison with the values calculated in this work. The ISO values, determined for BPS1 at that time, and those calculated in this work agree within the uncertainties.

4. Further correction factors

4.1. Overview

For the application of the correction factors described in section 3 to measured extrapolation curves, i.e. measured ionization current depending on the chamber depth, further correction factors are necessary. The following subsections give details for the remaining correction factors.
Figure 6. Calculated correction factor $k_{ih}$ for BPS1 (top) and BPS2 (bottom) for the source $^{147}$Pm, 20 cm, with filter depending on the chamber depth and the angle of incidence, $\alpha$, see legend. Symbols: calculation; lines: linear regression. The uncertainty bars represent the uncertainty of the single simulations ($\leq 0.4\%$).

4.2. Correction factors taken from ISO 6980-2:2004

The following correction factors were directly taken from ISO 6980-2:2004 [2]:

(a) correction for the effect of humidity of the air in the collecting volume on the average energy required to produce an ion pair, $k_{hu}$

(b) correction for interface effects between the air in the collecting volume and the adjacent entrance window and collecting electrode, $k_{in}$

(c) correction for radioactive decay of the beta-particle source, $k_{de}$

4.3. Correction factor for the effect of bremsstrahlung from the beta-particle source, $k_{br}$

In ISO 6980-2:2004 [2] values of $k_{br}$ for a limited number of source combinations are given but not for all sources listed in table 1. Therefore, table C1 in appendix C lists values of $k_{br}$ for all source combinations. The values are indirectly taken from the literature [5] where values for the bremsstrahlung, x-ray, and gamma contribution, $D_{br}$, to the total dose, $D_{tot}$, are given: $\tau_{br} = D_{br}/D_{tot}$ with the total dose, $D_{tot}$, i.e. the dose due to betas and photons and the dose due to bremsstrahlung, $D_{br}$. The values of $\tau_{br}$ were used to calculate $k_{br}$ via $k_{br} = 1 - \tau_{br}$. 
Figure 7. Calculated correction factor $k_{ih}$ for BPS1 (top) and BPS2 (bottom) depending on the source, i.e. on the radiation’s field mean energy, and the angle of incidence, $\alpha$, see legend. The data points represent the mean for the 10 chamber depths from $l = 250 \mu m \ldots 2500 \mu m$ while the vertical bars represent the maximum and minimum value for the 10 chamber depths.

4.4. Correction factor for the air density in the collecting volume, $k_{ad}$

The density of the air in the collecting volume of the extrapolation chamber influences the collected current because, for a constant absorbed-dose rate, it is proportional to the number of air molecules available to be ionized, which is itself proportional to the air density. Thus, the measured ionization must be corrected to the air density at reference conditions. To a good approximation, the density of air at ambient conditions, $\rho_{a}$, can be expressed as [5]

$$
\rho_{a} = \frac{1}{\left[T + 273.15^\circ C\right] \cdot 287.05 \frac{\text{K}}{\text{kg} \cdot \text{K}}} \cdot \left(1 - \frac{287.05}{461.495}\right) \cdot r \cdot 611.213 \text{Pa} \cdot \exp\left\{17.5043 \cdot \frac{T}{241.2^\circ C + T}\right\}
$$

(8)

with $T$ the absolute temperature, expressed in $^\circ$C, of the air in the collecting volume,
Figure 8. Correction factor $k_{SA}$ depending on the chamber depth $l$ for some sources, see legend. Data points: calculated values, see. Equation (14); lines: 2nd order polynomial regressions, see eq. (15)

$p$ the air pressure, expressed in Pa,
$r$ the relative humidity of the air, expressed as a fraction and $\rho_0$ the air density for reference conditions which are defined for the following parameters:
$T_0 = 20 \, ^\circ\text{C}$; $p_0 = 101 \, 325 \, \text{Pa}$; $r_0 = 0.65$. Under these conditions this results in $\rho_0 = 1.197 \, 40 \, \text{kg} \, \text{m}^{-3}$.

Equation (8) is valid for air pressures from zero to several bar, for humidities from 0% to 100%, and for temperature up to 30 °C [20]. The correction factor for the variations of the air density within the collection volume, $k_{ad}$, is then given by

$$k_{ad} = \frac{\rho_0}{\rho_a} \quad (9)$$

4.5. Correction factor for variations in the attenuation and scattering of beta particles between the source and the collecting volume due to variations from reference conditions and for differences of the entrance window to a tissue-equivalent thickness of 0.07 mm, $k_{abs}$

In ISO 6980-2 the correction factor $k_{abs}$ accounts for variations in the attenuation and scattering of beta particles between the source and the collecting volume due to variations from reference conditions and for the difference of the entrance window to a tissue-equivalent thickness of 0.07 mm [2]. The reference thickness of the entrance window of the extrapolation chamber is $d_0 = 0.07 \, \text{mm}$ or 3 mm of ICRU tissue, i.e. an areal density of 7 mg · cm$^{-2}$ or 300 mg · cm$^{-2}$. The reference thickness of the air layer between the source and the surface of the extrapolation chamber, i.e. the distance, is $y_0$ at reference conditions, i.e. an air density of $\rho_0 = 1.1974 \cdot 10^{-3} \, \text{g} \, \text{cm}^{-3}$ due to an air pressure, temperature and relative humidity of $p_0 = 1013.25 \, \text{hPa}$, $T_0 = 20 \, ^\circ\text{C}$ and $h_0 = 65\%$, respectively. This air thickness corresponds to a tissue depth of $\eta_{at} = \rho_0 \cdot y_0 / \rho_1$ with the scaling factor $\eta_{at}$ relating the absorbed dose due to betas determined in one material, e.g. air (a), to that in another, e.g. tissue (t) [21] and $\rho_1 = 1.0 \, \text{g} \, \text{cm}^{-3}$ the density of ICRU tissue. Any deviation of the entrance window to a tissue-equivalent thickness of 0.07 mm, or of the ambient air density, $\rho_a$, from the reference air density, $\rho_0$, results in a different absorption of the beta radiation compared with reference conditions. The correction factor, $k_{abs}$, which accounts for this is given by

$$k_{abs} = \frac{T(d_0)}{T \left( \frac{\eta_{at} \cdot [\rho_a - \rho_0] \cdot y_0 + \eta_{m,t} \cdot \rho_m \cdot d_m}{\rho_1} \right)} \quad (10)$$

with $\eta_{m,t} \cdot \rho_m \cdot d_m / \rho_1$ the tissue-equivalent thickness of a window of medium $m$, thickness $d_m$ and density $\rho_m$.

$\eta_{at} : [\rho_a - \rho_0] / y_0 / \rho_1$ the tissue-equivalent difference from the reference air path $y_0$ and $\alpha$ the angle of radiation incidence.

The measured depth dose curves due to beta radiation, i.e. the transmission functions $T(d)$, are adequately represented by functions of the form [5, 20, 22]

$$T(d) = \frac{\sum_{i=0}^{8} T_i \cdot \cos \left[ i \cdot \arccos \left( X(d) \right) \right] - \tau_{br}}{1 - \tau_{br}}$$

(11)

with $X(d) = 2 \cdot \frac{\log_{10}(d + \delta) - \log_{10}(d_{min} + \delta)}{\log_{10}(d_{max} + \delta) - \log_{10}(d_{min} + \delta)} - 1$ a variable transformation from $d$ to $X(d) \in [-1; 1]$ and $\tau_{br}$ the bremsstrahlung contribution, $D_{tot}$, to the total dose, $D_{tot}$, i.e. $\tau_{br} = D_{br} / D_{tot} = 1 - k_{br}$. See table C1 in appendix C and section 4.3 for $k_{br}$ and further details.

Values for the parameters $T_i$, $i = 0...8$, as well as for $d_{min}$, $d_{max}$ and $\delta$ for several reference fields defined in ISO 6980-1...
Figure 9. Residuals of the ionization current to their linear regressions measured with the BPS1 for the source $^{85}$Kr, 30 cm, with filter’ at $\alpha = 0^\circ$ (top) and $\alpha = 45^\circ$ (bottom) depending on the chamber depth and the correction factors used, see legend. Symbols: measurements; lines to guide the eye. The uncertainties are in the order of the symbol size and, therefore, not shown.

are shown in table C2 in appendix C for $\alpha = 0^\circ$ [5]. These values were obtained as fits of measurements of transmission through PET foils and PMMA absorbers [20, 22]. The values for $\alpha = 0^\circ$ are to be applied to extrapolation curve measurements at all angles of incidence, i.e. at $\alpha = 0^\circ$ and at $\alpha \neq 0^\circ$.

4.6. Correction factor for the source to chamber distance at different phantom depth, $k_{ph}$

During the measurements, the distance $y_0$ between the source and the front entrance of the chamber is kept constant. Once an absorber of thickness $d_{abs}$ is placed in front of the chamber, the distance between the source and the front of the absorber is smaller than $y_0$ by the absorber thickness. This is taken into account by the correction factor [20]

$$k_{ph} = \left(1 - \frac{d_{abs}}{y_0}\right)^2.$$

Note 1 Absorbers in front of the chamber represent the front part of a tissue-equivalent phantom while the chamber represents the remaining part of such a phantom. Therefore, the correction is called $k_{ph}$ with the index ‘ph’ for ‘phantom’.
Figure 10. Residuals of the ionization current to their linear regressions measured with the BPS1 for the source $^{90}$Sr/$^{90}$Y, 30 cm, with filter at $\alpha = 0^\circ$ (top) and $\alpha = 60^\circ$ (bottom) depending on the chamber depth and the correction factors used, see legend. Symbols: measurements; lines to guide the eye. The uncertainties are in the order of the symbol size and, therefore, not shown.

4.7 Correction factor for the stopping power ratio at different phantom depths, $k_{Sta}$

The stopping power ratio, $s_{t,a}$, accounts for the fact that the measurement volume is filled with air, not tissue. The value of $s_{t,a}$ is equal to the ratio of the energy transfer of electrons in tissue and air. It is not a constant as it depends on the energy $E$ of the electrons, which decreases continuously as they pass deeper into the phantom $d$ due to the energy loss of the electrons along their path. From the spectral fluences at different depths in tissue, $d_t$, and the stopping powers of tissue and air.
for mono-energetic electrons, the correction factor can be formulated as [20]

\[
k_{\text{Sta}} = \frac{s_{\text{Sta}}(0)_{\text{BG}} + a \cdot (d_l/\mu m)^b}{s_{\text{Sta}}(0)_{\text{BG}}} \tag{13}
\]

with \(s_{\text{Sta}}(0)_{\text{BG}}\) the stopping power ratio due to Bragg-Gray taken from table C3 in appendix C and \(a\) and \(b\) taken from table C4 in appendix C. For a tissue depth of 70 \(\mu m\) this results in values for \(k_{\text{Sta}}\) of 1.0011, 1.0010, 1.0012 and 1.0009 for \(^{147}\text{Pm}\), \(^{85}\text{Kr}\), \(^{90}\text{Sr}/^{90}\text{Y}\) and \(^{106}\text{Ru}/^{106}\text{Rh}\), respectively.

4.8. Use of the Spencer-Attix theory via correction factor, \(k_{\text{SA}}\)

The Spencer-Attix cavity theory is considered to be more accurate than the Bragg-Gray one as it accounts for the variation in the response measured as a function of cavity dimension \(l\) whereas the Bragg-Gray theory does not [23]. The beta reference radiation fields from the beta secondary standard 2, BSS 2 [4, 5] have been determined and are freely available [13]. From these data corresponding Spencer-Attix stopping power ratios, \(s_{\text{Sta}}(l)_{\text{SA}}\), depending on the chamber depth, \(l\), have been calculated [23]. Their ratio to the

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**Figure 11.** Residuals of the ionization current to their linear regressions measured with the BPS1 for the source \(^{147}\text{Pm}\), 20 cm, with filter at \(\alpha = 45^\circ\) (top) and \(^{85}\text{Kr}\), 30 cm, with filter at \(\alpha = 60^\circ\) (bottom) depending on the chamber depth and the correction factors used, see legend. Symbols: measurements; lines to guide the eye. The uncertainties are in the order of the symbol size and, therefore, not shown.
Bragg-Gray stopping power ratio, \( s_{1,2}(0) \text{BG} \), see table C3 in appendix C and the literature [20], can be interpreted as correction factor \( k_{SA} \) applied to the Spencer-Attix theory:

\[
k_{SA} = \frac{s_{1,2}(l)_{SA}}{s_{1,2}(0)_{\text{BG}}}
\]  

(14)

This ratio can be fitted to a 2nd order polynomial function depending on the chamber depth \( l \) of the form

\[
k_{SA} = c_0 + c_1 \cdot l + c_2 \cdot l^2
\]  

(15)

with \( c_0 \), \( c_1 \) and \( c_2 \), parameters obtained from a 2nd order polynomial regression and

\( l \), the chamber depth.

The parameters \( c_0 \), \( c_1 \) and \( c_2 \) obtained from the regression are listed for several sources in table C5 in appendix C for BPS1. Figure 8 shows the data from the literature [23, 24] and the corresponding polynomial regressions according equation (15). For a chamber depth of 1000 \( \mu m \) this results in values for \( k_{SA} \) of 1.0036, 1.0045, 1.0057 and 1.0074 for \( ^{147}\text{Pm} \) at a distance of 20 cm, \( ^{85}\text{Kr} \) at a distance of 30 cm, \( ^{90}\text{Sr}^{90}\text{Y} \) at a distance of 30 cm and \( ^{106}\text{Rh}^{106}\text{Rh} \) also at a distance of 30 cm, all with a beam flattening filter, respectively.

**5. Application of correction factors**

It is well known that the ionization current in extrapolation chambers does not always linearly depend on the chamber depth although it should. This is indeed the case for extrapolation curves of the nuclide \( ^{85}\text{Kr} \). As a reason, imperfect corrections can be assumed: e.g. for \( ^{85}\text{Kr} \) it is assumed that the perturbation correction, \( k_{pe} \), is not appropriate as the original measurements had been performed using \( ^{204}\text{Tl} \) [18]. This nuclide emits a beta spectrum which is quite similar to the one of \( ^{85}\text{Kr} \) but not the same. Therefore, in ISO 6980-2, besides a linear regression, a polynomial regression of 2nd order is recommended to determine the slope of the ionization current at zero chamber depth [2]. However, the use of appropriate corrections to obtain a linear extrapolation curve has always been envisaged.

Figures 9 and 10 show the residuals of the ionization currents to their linear regressions for \( ^{85}\text{Kr} \) and \( ^{90}\text{Sr}^{90}\text{Y} \), respectively, for different angles of incidence, \( \alpha \), measured with the BPS1. Each graph has the same scale (for an easy comparison by eye) and they show the measured data with different corrections applied, see legends:

(a) ISO 6980-2:2004,
(b) ISO 6980-2:2004 except \( k_{pe} \), \( k_{ad} \), \( k_{sh} \), \( k_{ph} \) and \( k_{SA} \) are used as described in sections 4.3 to 4.7,
(c) as described at ii and in addition \( k_{sh} \) and \( k_{pe} \) are based on simulations, see sections 3.3 and 3.4,
(d) as described at iii and in addition \( k_{ph} \) is based on simulations, see section 3.5, and
(e) as described at iv and in addition \( k_{SA} \) is based on simulations, see section 4.8.

In the right part of the legends the dose rate divided by the dose rate obtained applying ISO-6980-2:2004 is shown. From figure 9 it can be seen that, applying the correction factors given in ISO 6980-2:2004, the non-perfect linearity of the chamber current vs. chamber depth is even worse for oblique radiation incidence, i.e. for \( \alpha \neq 0^\circ \) compared to normal incidence. That was expected as the correction factors stated in ISO 6980-2:2004 are only stated for normal incidence, i.e. for \( \alpha = 0^\circ \). From figure 9 it is obvious that the correction factors determined in the course of this work are more appropriate compared to those from ISO for both normal and oblique radiation incidence. Further, it can be seen that as of option iii and the following options, i.e. once the correction for perturbation by the chamber’s side wall, \( k_{pe} \), is applied, linearity is practically reached. In addition, a comparison of options iv) and v) shows that applying the Spencer-Attix theory, i.e. using \( k_{SA} \), leads to practically the same curve form and to slightly larger dose rates when compared to not applying it. It shall also be noted that in a few cases the linearity of the extrapolation curves become slightly worse when applying all corrections, i.e. also options iv) and v), see top part of figure 10. However, in most cases, especially at oblique radiation incidence, linearity becomes significantly better applying the correction factors determined within this work.

Finally, figure 11 shows two examples where the linearity is not satisfied by the application of the corrections from this work. However, in no cases does the deviation become worse when applying the corrections from ISO 6980-2:2004. Note that the scale is slightly different compared to figures 9 and 10.

**6. Conclusions**

A complete set of correction factors for primary beta dosimetry in radiation protection, i.e. for extrapolation curves measurements, are presented in this paper. Several of them have been re-determined, some by means of measurements and most of them by means of simulations. For the first time, also correction factors for oblique radiation incidence are presented.

In conclusion, it can be stated that the corrections determined in this work significantly improve the evaluation of extrapolation curve measurements at both normal and oblique radiation incidence. The data are ready to be implemented in the current revision of ISO 6980-2 [2].

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Figure A1. Geometry of the extrapolation chamber. The absorber in front of the chamber is only 50 µm in thickness and, therefore, very difficult to see. The entrance foil (polypropylene, PP) and the electric contacts (aluminium, Al) are not shown as they are too thin. The figure is shown to scale.

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Appendix A. Description of PTB’s new extrapolation chamber

A.1. Geometry, substances and electronic components of the chamber

Figure A1 shows the geometry of the extrapolation chamber and table A1 lists the materials used and their relevant properties. Figure A2 shows some further details of the chamber.

Both electrodes, the collecting electrode at the back of the chamber as well as the front electrode are covered with layers of Aluminium on the inner side of the chamber. The collecting electrode is contacted by electrically conductive polyether ether ketone (PEEK ELS), see figure A1. Below the active volume is a 2 mm thick isolating layer of PEEK and located below that is an electrically conductive 19 mm layer of PEEK ELS to prevent betas penetrating, i.e. electrons, to form room charges behind the active volume. Behind that is another 32 mm of PEEK, see bottom of figure A1.

In figure A1, the active collecting volume, 30 mm in diameter, is located at the figure top while in figure A2 it can be seen on the right. It is surrounded by a 15 mm wide guard ring to ensure a homogeneous electrical field inside the active volume. Located outside of this guard ring is another 10 mm wide ring divided into three segments. The electric capacity of these three segments can be measured independently to make sure the back and front electrode are parallel.

The chamber depth can be varied from 0.05 mm up to 10.5 mm, see figure A2 to investigate the chamber’s behaviour at such extreme depths. However, for routine use, the depth is varied from 0.5 mm up to 2.5 mm.

The ionization current is measured using a PTB developed electrometer with a leakage current of less than $10^{-16}$ A [8].

A.2. Software to control the measuring stand and software for the data evaluation

Both the software to control the hardware as well as the software for the data evaluation were developed by PTB.

During a measurement, all components of the measuring stand are adjusted and directly afterwards independently controlled by another component. This high level of quality assurance is implemented for the radiation source, its beam flattening filter, its shutter, the chamber distance from the source, the chamber depth, the absorber in front of the chamber, the ambient conditions (air temperature, pressure and humidity), the charge measurement as well as the measurement time. Using this software, both the extrapolation curves as well as the depth dose curves, can be measured automatically and all information is stored in a corresponding data file.

The software for the data evaluation to arrive at the dose rate and depth dose curve reads all information stored from a measurement and applies the corrections described in sections 3 and 4.
Table A1. Density and composition of substances used in the simulations. In the table entries fractions per mass are given in %.

| Name          | Density in g cm\(^{-3}\) | Al (Z = 13) | Polyethylene terephthalate: PET (Mylar, Hostaphan) | Polyether ether ketone, electrically conductive: PEEK ELS\(^a\) | Polycarbonate: PC (Makrolon, Lexan) | Air, 50% relative humidity |
|---------------|---------------------------|-------------|---------------------------------------------------|---------------------------------------------------------------|-------------------------------------|---------------------------|
| H (Z = 1)     | 14.4                      | 2.7         | 4.2                                               | 4.2                                                           | 3.6                                  | 5.5                       | 0.08                     |
| C (Z = 6)     | 85.6                      | 0.90        | 62.5                                              | 79.2                                                          | 82.3                                 | 75.6                      | 0.01                     |
| N (Z = 7)     | 74.99                     |             | 1.27                                              | 74.99                                                         | 74.99                                |                           |                          |
| O (Z = 8)     | 33.3                      |             | 16.6                                              | 33.3                                                          | 14.2                                 | 18.9                      | 23.65                    |
| Al (Z = 13)   | 100                       |             |                                                   |                                                               |                                      |                           |                          |
| Ar (Z = 18)   | 1.27                      |             |                                                   |                                                               |                                      |                           |                          |

\(^a\)Electrically conductive as carbon nanotubes are contained.

Figure A2. Details of the extrapolation chamber. The left part shows a side view while a three-dimensional view is given in the right part. The colour code is different to the one used in figure A1 and the figures are not to scale.
### Table B1. Calculated values of the backscatter factor $k_{ba}$ for BPS1 (PMMA back electrode): mean and standard deviation of the values of the 10 chamber depths (from $l = 250 \mu m$ up to $2500 \mu m$), see section 3.3.

| source       | $E_{mean}$/MeV | 0°  | u(0°) | 15°  | u(15°) | 30°  | u(30°) | 45°  | u(45°) | 60°  | u(60°) |
|--------------|----------------|-----|-------|------|--------|------|--------|------|--------|------|--------|
| Pm 20 cm wf  | 0.065          | 1.015 | 0.002 | 1.014 | 0.002  | 1.015 | 0.003  | 1.015 | 0.004  | 1.015 | 0.003  |
| Pm 11 cm w/of| 0.078          | 1.016 | 0.002 | 1.016 | 0.002  | 1.016 | 0.001  | 1.016 | 0.002  | 1.015 | 0.002  |
| Kr 50 cm wf  | 0.25           | 1.018 | 0.001 | 1.018 | 0.002  | 1.017 | 0.002  | 1.016 | 0.002  | 1.015 | 0.002  |
| Kr 30 cm wf  | 0.25           | 1.019 | 0.001 | 1.017 | 0.001  | 1.016 | 0.002  | 1.016 | 0.002  | 1.015 | 0.002  |
| Sr 50 cm wf  | 0.80           | 1.017 | 0.003 | 1.015 | 0.001  | 1.015 | 0.002  | 1.014 | 0.002  | 1.011 | 0.002  |
| Sr 30 cm wf  | 0.80           | 1.016 | 0.001 | 1.016 | 0.002  | 1.016 | 0.002  | 1.014 | 0.002  | 1.010 | 0.002  |
| Sr 20 cm wf  | 0.84           | 1.015 | 0.002 | 1.016 | 0.001  | 1.016 | 0.002  | 1.014 | 0.002  | 1.011 | 0.002  |
| Sr 11 cm w/of| 0.85           | 1.015 | 0.001 | 1.015 | 0.001  | 1.016 | 0.002  | 1.014 | 0.001  | 1.011 | 0.002  |
| Ru 50 cmwf   | 1.14           | 1.014 | 0.001 | 1.014 | 0.001  | 1.014 | 0.001  | 1.013 | 0.003  | 1.010 | 0.003  |
| Ru 30 cmwf   | 1.16           | 1.014 | 0.002 | 1.014 | 0.002  | 1.014 | 0.002  | 1.014 | 0.002  | 1.010 | 0.002  |
| Ru 20 cmw/of | 1.15           | 1.014 | 0.002 | 1.013 | 0.002  | 1.013 | 0.002  | 1.013 | 0.002  | 1.010 | 0.002  |
| Ru 11 cm w/of| 1.15           | 1.014 | 0.003 | 1.013 | 0.002  | 1.013 | 0.002  | 1.013 | 0.002  | 1.010 | 0.002  |

### Table B2. Calculated values of the backscatter factor $k_{ba}$ for BPS2 (PEEK back electrode): mean and standard deviation of the values of the 10 chamber depths (from $l = 250 \mu m$ up to $2500 \mu m$), see section 3.3.

| source       | $E_{mean}$/MeV | 0°  | u(0°) | 15°  | u(15°) | 30°  | u(30°) | 45°  | u(45°) | 60°  | u(60°) |
|--------------|----------------|-----|-------|------|--------|------|--------|------|--------|------|--------|
| Pm 20 cm wf  | 0.065          | 0.993 | 0.006 | 0.994 | 0.004  | 0.994 | 0.006  | 0.993 | 0.006  | 0.991 | 0.006  |
| Pm 11 cm w/of| 0.078          | 1.002 | 0.003 | 1.000 | 0.004  | 1.000 | 0.003  | 1.001 | 0.004  | 0.999 | 0.005  |
| Kr 50 cm wf  | 0.25           | 1.012 | 0.001 | 1.011 | 0.002  | 1.011 | 0.002  | 1.011 | 0.005  | 1.009 | 0.001  |
| Kr 30 cm wf  | 0.25           | 1.012 | 0.001 | 1.012 | 0.001  | 1.013 | 0.002  | 1.010 | 0.001  | 1.009 | 0.004  |
| Sr 50 cm wf  | 0.80           | 1.011 | 0.002 | 1.011 | 0.002  | 1.011 | 0.003  | 1.010 | 0.003  | 1.008 | 0.003  |
| Sr 30 cm wf  | 0.80           | 1.012 | 0.001 | 1.012 | 0.003  | 1.011 | 0.002  | 1.010 | 0.003  | 1.010 | 0.007  |
| Sr 20 cm w/of| 0.82           | 1.012 | 0.001 | 1.011 | 0.003  | 1.012 | 0.002  | 1.011 | 0.003  | 1.009 | 0.005  |
| Sr 11 cm w/of| 0.83           | 1.012 | 0.002 | 1.011 | 0.002  | 1.011 | 0.001  | 1.011 | 0.002  | 1.011 | 0.006  |
| Ru 50 cmwf   | 1.14           | 1.010 | 0.002 | 1.010 | 0.001  | 1.011 | 0.004  | 1.011 | 0.003  | 1.006 | 0.006  |
| Ru 30 cmwf   | 1.16           | 1.010 | 0.002 | 1.011 | 0.003  | 1.012 | 0.001  | 1.010 | 0.002  | 1.008 | 0.006  |
| Ru 20 cmw/of | 1.15           | 1.010 | 0.001 | 1.010 | 0.001  | 1.011 | 0.002  | 1.010 | 0.002  | 1.007 | 0.006  |
| Ru 11 cm w/of| 1.15           | 1.010 | 0.001 | 1.010 | 0.003  | 1.010 | 0.002  | 1.010 | 0.002  | 1.007 | 0.004  |
Table B3. Values of the fit parameters $f_7$ and $f_8$ for the calculation of the perturbation factor, $k_{pe}$, for the BPS1, see equation (5) in section 3.4. The parameters were determined via 2nd order polynomial regression through (0/1) of the calculated perturbation factor, $k_{pe}$.

| source          | $E_{\text{mean}}$ /MeV | $f_7(0^\circ)$ / (mm$^{-1}$ · $10^{-3}$) | $f_7(15^\circ)$ / (mm$^{-1}$ · $10^{-3}$) | $f_8(0^\circ)$ / (mm$^{-2}$ · $10^{-3}$) | $f_8(15^\circ)$ / (mm$^{-2}$ · $10^{-3}$) | $f_7(30^\circ)$ / (mm$^{-1}$ · $10^{-3}$) | $f_7(45^\circ)$ / (mm$^{-1}$ · $10^{-3}$) | $f_8(45^\circ)$ / (mm$^{-2}$ · $10^{-3}$) | $f_8(60^\circ)$ / (mm$^{-2}$ · $10^{-3}$) | $f_8(60^\circ)$ / (mm$^{-2}$ · $10^{-3}$) |
|-----------------|------------------------|-------------------------------------------|-------------------------------------------|-------------------------------------------|-------------------------------------------|-------------------------------------------|-------------------------------------------|-------------------------------------------|-------------------------------------------|-------------------------------------------|
| Pm 20 cm wf     | 0.065                  | −1.22                                     | 1.82                                      | −1.95                                     | 2.04                                      | −1.30                                     | 1.66                                      | −2.93                                     | 2.89                                      | −3.80                                     | 3.72                                      |
| Pm 11 cm w/of   | 0.078                  | 2.13                                      | 0.0163                                    | 0.431                                     | 0.750                                     | 0.497                                     | 1.37                                      | 0.693                                     | 1.69                                      | −2.14                                     | 4.31                                      |
| Kr 50 cm wf     | 0.25                   | 3.12                                      | 2.31                                      | 2.22                                      | 2.80                                      | 5.74                                      | 2.00                                      | 2.15                                      | 5.94                                      | 5.10                                      | 7.05                                      |
| Kr 30 cm wf     | 0.25                   | −0.191                                    | 2.93                                      | 0.515                                     | 2.69                                      | −0.919                                    | 4.55                                      | 3.59                                      | 4.66                                      | 6.49                                      | 6.92                                      |
| Sr 50 cm wf     | 0.80                   | −3.93                                     | −1.04                                     | −6.69                                     | 0.203                                     | −7.89                                     | 1.52                                      | −5.28                                     | 2.56                                      | 2.32                                      | 4.91                                      |
| Sr 30 cm wf     | 0.80                   | −3.70                                     | −1.40                                     | −6.77                                     | −0.299                                    | −9.19                                     | 1.30                                      | −10.5                                    | 3.73                                      | 0.0160                                    | 4.74                                      |
| Sr 50 cm w/of   | 0.82                   | −3.76                                     | −1.32                                     | −8.19                                     | 0.689                                     | −6.36                                     | 0.387                                     | −7.88                                     | 3.10                                      | 2.45                                      | 4.49                                      |
| Sr 30 cm w/of   | 0.83                   | −5.89                                     | −0.762                                    | −5.37                                     | −1.07                                     | −8.31                                     | 0.418                                     | −8.35                                     | 2.08                                      | −0.692                                    | 4.32                                      |
| Sr 20 cm w/of   | 0.84                   | −3.35                                     | −1.97                                     | −4.43                                     | −1.59                                     | −9.72                                     | 0.813                                     | −10.3                                    | 2.24                                      | −4.65                                     | 4.66                                      |
| Sr 11 cm w/of   | 0.85                   | −3.39                                     | −1.54                                     | −2.71                                     | −1.88                                     | −5.89                                     | −0.797                                    | −10.3                                    | 0.969                                     | −12.1                                     | 4.51                                      |
| Ru 50 cm wf     | 1.14                   | −6.25                                     | −0.409                                    | −6.10                                     | −0.457                                    | −5.48                                     | −0.892                                    | −9.91                                    | 1.95                                      | −6.37                                     | 5.02                                      |
| Ru 30 cm wf     | 1.16                   | −3.98                                     | −1.75                                     | −5.23                                     | −1.36                                     | −7.62                                     | −0.558                                    | −9.45                                    | 0.871                                     | −7.87                                     | 4.60                                      |
| Ru 20 cm w/of   | 1.15                   | −1.92                                     | −2.69                                     | −3.43                                     | −2.20                                     | −5.85                                     | −1.44                                     | −9.32                                    | 0.111                                     | −9.96                                     | 3.78                                      |
| Ru 11 cm w/of   | 1.15                   | −2.88                                     | −1.68                                     | −4.03                                     | −1.19                                     | −4.08                                     | −1.91                                     | −8.37                                    | −0.732                                    | −12.5                                     | 2.59                                      |

Note 1: The unit 1/(mm$^{-1}$ · $10^{-3}$) $\equiv$ l/(m$^{-1}$) while the unit 1/(mm$^{-2}$ · $10^{-3}$) $\equiv$ l/(m$^{-2}$ · $10^{3}$).

Note 2: An entry of ‘−1.22’ means $f_7 = −1.22$ · $10^{-3}$ mm$^{-1} = −0.00122$ mm$^{-1}$

Note 3: An entry of ‘1.82’ means $f_8 = 1.82$ · $10^{-3}$ mm$^{-2} = 0.000182$ mm$^{-2}$
Table B4. Values of the fit parameters \( f_7 \) and \( f_8 \) for the calculation of the perturbation factor, \( k_{pe} \), for the BPS2, see equation (5) in section 3.4. The parameters were determined via 2nd order polynomial regression through (0/1) of the calculated perturbation factor, \( k_{pe} \).

| source       | \( E_{\text{mean}} \) /MeV | \( f_7(0^\circ) \) | \( f_8(0^\circ) \) | \( f_7(15^\circ) \) | \( f_8(15^\circ) \) | \( f_7(30^\circ) \) | \( f_8(30^\circ) \) | \( f_7(45^\circ) \) | \( f_8(45^\circ) \) | \( f_7(60^\circ) \) | \( f_8(60^\circ) \) |
|--------------|-----------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Pm 20 cm wf  | 0.065                       | −1.44             | 0.701             | −1.41             | 0.782             | −2.15             | 1.33              | 1.95              | −0.330            | −1.80             | 1.17              |
| Pm 11 cm w/of| 0.078                       | −0.448            | 0.516             | 1.48              | −0.639            | −2.03             | 1.20              | −0.260            | 0.730             | 0.763             | 0.273             |
| Kr 50 cm wf  | 0.25                        | −1.11             | 1.73              | 1.03              | 0.962             | −1.14             | 2.71              | 1.71              | 2.37              | 1.16              | 4.59              |
| Kr 30 cm wf  | 0.25                        | −1.69             | 1.74              | −0.507            | 1.39              | 2.19              | 0.79              | 2.62              | 1.90              | −2.83             | 6.75              |
| Sr 50 cm wf  | 0.80                        | −2.06             | −0.876            | −1.92             | −0.834            | −2.73             | −0.361            | −5.11             | 1.90              | 0.761             | 3.55              |
| Sr 30 cm wf  | 0.80                        | −1.05             | −1.64             | 0.0276            | −2.06             | −5.07             | 0.299             | −4.99             | 1.12              | −3.05             | 3.88              |
| Sr 50 cm w/of| 0.82                        | −2.01             | −1.25             | −4.27             | −0.0585           | −2.72             | −0.408            | −4.35             | 1.63              | −4.34             | 5.73              |
| Sr 30 cm w/of| 0.83                        | −1.55             | −1.45             | −2.63             | −0.825            | −6.10             | 0.657             | −4.40             | 0.666             | 0.405             | 1.73              |
| Sr 20 cm w/of| 0.84                        | −3.46             | −0.599            | 0.582             | −2.50             | −0.438            | −2.06             | −3.63             | −0.403            | −6.75             | 4.00              |
| Sr 11 cm w/of| 0.85                        | −1.01             | −1.11             | −1.09             | −1.02             | −2.21             | −1.06             | −2.49             | −1.12             | −8.05             | 1.72              |
| Ru 50 cm wf  | 1.14                        | −1.49             | −1.28             | −3.22             | −0.630            | −4.63             | −0.301            | −4.95             | 0.306             | −8.81             | 5.27              |
| Ru 30 cm wf  | 1.16                        | −1.74             | −1.25             | 0.912             | −2.45             | −2.25             | −1.53             | −7.50             | 0.430             | −8.11             | 3.59              |
| Ru 20 cm w/of| 1.15                        | −0.190            | −1.82             | −1.28             | −1.48             | −1.10             | −2.11             | −7.13             | 0.472             | −9.47             | 3.14              |
| Ru 11 cm w/of| 1.15                        | 0.847             | −1.72             | −0.0457           | −1.32             | −3.28             | −0.413            | −3.87             | −1.04             | −10.2             | 1.33              |

Note 1: The unit 1/(mm\(^{-1} \cdot 10^{-3}) \equiv 1/(m^{-1}) \) while the unit 1/(mm\(^{-2} \cdot 10^{-3}) \equiv 1/(m^{-2} \cdot 10^{3}) \).

Note 2: An entry of ‘−1.44’ means \( f_7 = −1.44 \cdot 10^{-3} \) mm\(^{-1}\) = −0.00144 mm\(^{-1}\).

Note 3: An entry of ‘0.701’ means \( f_8 = 0.701 \cdot 10^{-3} \) mm\(^{-2}\) = 0.0000701 mm\(^{-2}\).
Table B5. Values of the fit parameters $m$ and $b$ for the calculation of the inhomogeneity factor, $k_{ih}$, for the BPS1, see equation (7) in section 3.5. The parameters were determined via linear regression of the calculated inhomogeneity factor, $k_{ih}$.

| source         | $E_{\text{mean}}$ /MeV | $b(0^\circ)$ | $m(0^\circ)/l$ (mm$^{-1} \cdot 10^{-3}$) | $b(15^\circ)$ | $m(15^\circ)/l$ (mm$^{-1} \cdot 10^{-3}$) | $b(30^\circ)$ | $m(30^\circ)/l$ (mm$^{-1} \cdot 10^{-3}$) | $b(45^\circ)$ | $m(45^\circ)/l$ (mm$^{-1} \cdot 10^{-3}$) | $b(60^\circ)$ | $m(60^\circ)/l$ (mm$^{-1} \cdot 10^{-3}$) |
|----------------|------------------------|--------------|------------------------------------------|--------------|------------------------------------------|--------------|------------------------------------------|--------------|------------------------------------------|--------------|------------------------------------------|
| Pm 20 cm wf    | 0.065                  | 1.014        | 17.4                                     | 1.009        | 16.6                                     | 0.999        | 16.3                                     | 0.987        | 15.0                                     | 0.971        | 12.8                                     |
| Pm 11 cm w/of  | 0.078                  | 1.030        | 15.3                                     | 1.022        | 16.0                                     | 1.006        | 16.9                                     | 0.986        | 15.0                                     | 0.971        | 12.8                                     |
| Kr 50 cm wf    | 0.25                   | 1.002        | 3.53                                     | 0.998        | 4.77                                     | 1.000        | 3.02                                     | 0.996        | 6.64                                     | 0.993        | 7.04                                     |
| Kr 30 cm wf    | 0.25                   | 0.997        | 5.48                                     | 0.995        | 5.26                                     | 0.993        | 6.95                                     | 0.999        | 1.68                                     | 0.993        | 6.78                                     |
| Sr 50 cm wf    | 0.80                   | 1.003        | -1.82                                    | 0.999        | 0.0133                                   | 0.999        | -0.327                                   | 1.001        | -0.268                                   | 0.996        | 3.91                                     |
| Sr 30 cm w/of  | 0.82                   | 1.000        | 1.50                                     | 0.998        | 0.895                                    | 0.998        | 1.30                                     | 0.995        | 3.14                                     | 0.999        | 2.05                                     |
| Sr 30 cm w/of  | 0.83                   | 1.002        | 1.05                                     | 1.000        | 2.24                                     | 0.999        | 1.27                                     | 0.997        | 1.75                                     | 0.999        | 1.38                                     |
| Sr 20 cm w/of  | 0.84                   | 1.000        | 4.99                                     | 1.001        | 3.81                                     | 1.003        | 2.21                                     | 0.995        | 3.58                                     | 0.997        | 4.13                                     |
| Sr 11 cm w/of  | 0.85                   | 1.015        | 5.82                                     | 1.013        | 5.44                                     | 1.013        | 3.04                                     | 1.006        | 3.57                                     | 1.003        | 3.38                                     |
| Ru 50 cm wf    | 1.14                   | 0.997        | 2.49                                     | 1.000        | -0.372                                   | 0.997        | 1.10                                     | 0.999        | -0.383                                   | 0.996        | 2.49                                     |
| Ru 30 cm wf    | 1.16                   | 0.999        | 1.68                                     | 0.998        | 1.56                                     | 0.997        | 1.78                                     | 0.998        | 0.616                                    | 0.996        | 3.12                                     |
| Ru 20 cm w/of  | 1.15                   | 1.006        | 1.71                                     | 1.003        | 2.08                                     | 1.000        | 2.17                                     | 1.000        | 0.502                                    | 1.001        | 0.824                                    |
| Ru 11 cm w/of  | 1.15                   | 1.017        | 4.32                                     | 1.012        | 5.29                                     | 1.008        | 5.26                                     | 1.004        | 2.94                                     | 0.999        | 3.95                                     |

Note 1: The unit $1/(\text{mm}^{-1} \cdot 10^{-3}) \equiv 10^3 \text{m}^{-1}$.

Note 2: An entry of '17.4' means $m = 17.4 \cdot 10^{-3} \text{mm}^{-1} = 0.0174 \text{m}^{-1}$.
Table B6. Values of the fit parameters $m$ and $b$ for the calculation of the inhomogeneity factor, $k_{ih}$, for the BPS2, see equation (7) in section 3.5. The parameters were determined via linear regression of the calculated inhomogeneity factor, $k_{ih}$.

| source        | $E_{\text{mean}}$ (MeV) | $b(0^\circ)$ | $m(0^\circ)/b(0^\circ)$ | $m(15^\circ)/b(15^\circ)$ | $m(30^\circ)/b(30^\circ)$ | $m(45^\circ)/b(45^\circ)$ | $m(60^\circ)/b(60^\circ)$ |
|---------------|--------------------------|--------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| Pm 20 cm wf   | 0.065                    | 1.000        | 17.4                      | 0.991                     | 20.5                      | 0.985                     | 17.4                      | 0.967                     | 19.6                      | 0.954                     | 18.6                      |
| Pm 11 cm w/of | 0.078                    | 1.020        | 16.3                      | 1.017                     | 12.8                      | 0.999                     | 13.2                      | 0.981                     | 12.3                      | 0.964                     | 11.7                      |
| Kr 50 cm wf   | 0.25                     | 0.994        | 2.80                      | 0.995                     | 1.36                      | 0.994                     | 2.30                      | 0.985                     | 1.89                      | 0.990                     | 3.87                      |
| Kr 30 cm wf   | 0.25                     | 0.994        | 2.69                      | 0.992                     | 3.18                      | 0.993                     | 2.97                      | 0.986                     | 4.95                      | 0.990                     | 2.62                      |
| Sr 50 cm wf   | 0.80                     | 0.998        | −0.238                    | 0.999                     | −1.33                     | 0.998                     | −0.662                    | 0.994                     | 0.590                     | 0.999                     | −1.38                     |
| Sr 30 cm wf   | 0.80                     | 0.997        | 1.08                      | 0.997                     | 1.45                      | 0.996                     | 0.463                     | 0.996                     | 0.0761                    | 0.993                     | 1.41                      |
| Sr 50 cm w/of | 0.82                     | 0.997        | 2.25                      | 0.997                     | 0.454                     | 0.998                     | 0.124                     | 0.995                     | 1.81                      | 0.991                     | 3.31                      |
| Sr 30 cm w/of | 0.83                     | 0.996        | 3.02                      | 0.998                     | 0.606                     | 0.999                     | −0.826                    | 0.999                     | −1.20                     | 0.998                     | −0.793                    |
| Sr 20 cm w/of | 0.84                     | 1.001        | 3.15                      | 1.001                     | 1.87                      | 0.999                     | 1.30                      | 0.998                     | 0.841                     | 0.993                     | 3.60                      |
| Sr 11 cm w/of | 0.85                     | 1.013        | 5.81                      | 1.012                     | 4.46                      | 1.010                     | 3.05                      | 1.006                     | 1.84                      | 0.995                     | 3.86                      |
| Ru 50 cm wf   | 1.14                     | 0.996        | 2.10                      | 0.996                     | 0.0820                    | 0.995                     | 0.403                     | 0.997                     | −0.590                    | 0.991                     | 2.49                      |
| Ru 30 cm wf   | 1.16                     | 0.994        | 3.45                      | 0.994                     | 3.04                      | 0.996                     | 0.786                     | 0.992                     | 1.28                      | 0.992                     | 1.72                      |
| Ru 20 cm w/of | 1.15                     | 1.001        | 3.70                      | 0.998                     | 3.83                      | 1.001                     | 0.491                     | 1.002                     | 0.312                     | 1.001                     | −2.32                     |
| Ru 11 cm w/of | 1.15                     | 1.012        | 5.50                      | 1.008                     | 5.83                      | 1.007                     | 3.63                      | 1.004                     | 1.71                      | 0.996                     | 2.18                      |

**Note 1:** The unit $1/(\text{mm}^{-1} \cdot 10^{-3}) \equiv 1/\text{m}^2$.

**Note 2:** An entry of '17.4' means $m = 17.4 \cdot 10^{-3} \text{ mm}^{-1} = 0.0174 \text{ m}^{-1}$. 
|                  | $^{106}$Ru + $^{106}$Rh | $^{106}$Ru/$^{106}$Rh | $^{106}$Ru/$^{106}$Rh | $^{90}$Sr/$^{90}$Y | $^{90}$Sr/$^{90}$Y | $^{90}$Sr/$^{90}$Y | $^{85}$Kr | $^{85}$Kr | $^{147}$Pm | $^{147}$Pm |
|------------------|--------------------------|------------------------|------------------------|-------------------|-------------------|-------------------|--------|--------|----------|----------|
|                  | without filter at 11 cm  | without filter at 20 cm| without filter at 30 cm| without filter at 50 cm| without filter at 11 cm| without filter at 20 cm| without filter at 30 cm| without filter at 50 cm| without filter at 11 cm| without filter at 20 cm|
| $k_{br}$        | 0.9980                  | 0.9977                 | 0.9983                 | 0.985             | 0.9996            | 0.9996            | 0.9995            | 0.99938   | 0.9990   | 0.99975   | 0.99972   | 0.9999   |
| $\sigma(k_{br})$| 0.10%                   | 0.12%                  | 0.09%                  | 0.8%              | 0.02%             | 0.02%             | 0.02%             | 0.03%        | 0.02%    | 0.05%     | 0.01%     | 0.01%    | 0.23%    |
Table C2. Fit parameters for the transmission functions $T(d)$ for $\alpha = 0^\circ$, see equation (11), to calculate, $k_{abs}$, see equation (10) in section 4.5.

|                          | $T_0$     | $T_1$     | $T_2$     | $T_3$     | $T_4$     | $T_5$     | $T_6$     | $T_7$     | $T_8$     | $d_{\text{min}}$ | $d_{\text{max}}$ | $\delta$ |
|--------------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|------------------|-------------------|-----------|
| $^{147}$Pm with filter at $y_0 = 20$ cm | 1.6722    | -2.50959  | 0.92046   | 0.05396   | -0.16166  | 0.01417   | 0.01564   | 0.00889   | -0.00705  | -1                | 558               | 25        |
| $^{147}$Pm with no filter at $y_0 = 11$ cm | 0.4835    | -0.89062  | 0.69086   | -0.43061  | 0.1937    | -0.02682  | -0.04924  | 0.0565    | -0.03276  | 0                | 11 408            | 110       |
| $^{85}$Kr with filter at $y_0 = 30$ cm | 0.40505   | -0.57459  | 0.15718   | 0.04294   | -0.03292  | 0.00369   | -0.00423  | 0.0027    | 0.00049   | -2               | 2315              | 585       |
| $^{85}$Kr with filter at $y_0 = 50$ cm | 0.30701   | -0.49681  | 0.27256   | -0.06098  | -0.0271   | 0.02547   | -0.0121   | -0.00158  | -0.00663  | 0                | 3880              | 1135      |
| $^{90}$Sr/$^{90}$Y with filter at $y_0 = 30$ cm | 0.52485   | -0.62644  | -0.02725  | 0.17243   | -0.01523  | -0.03018  | -0.00441  | 0.00589   | 0.00096   | -4               | 11 460            | 1090      |
| $^{90}$Sr/$^{90}$Y with filter at $y_0 = 50$ cm | 0.50226   | -0.61408  | 0.00142   | 0.15208   | -0.01592  | -0.02762  | -0.00524  | 0.00671   | 0.00122   | -3               | 11 398            | 1085      |
| $^{90}$Sr/$^{90}$Y with no filter at $y_0 = 11$ cm | 0.5769    | -0.62535  | -0.11587  | 0.17945   | 0.01967   | -0.02365  | -0.01608  | 0.00368   | 0.00152   | 0                | 11 411            | 710       |
| $^{90}$Sr/$^{90}$Y with no filter at $y_0 = 20$ cm | 0.56035   | -0.6312   | -0.08208  | 0.18187   | 0.00225   | -0.02563  | -0.01118  | 0.00432   | 0.00157   | 0                | 11 409            | 910       |
| $^{90}$Sr/$^{90}$Y with no filter at $y_0 = 30$ cm | 0.5477    | -0.62973  | -0.06156  | 0.17749   | -0.00474  | -0.02559  | -0.00879  | 0.00446   | 0.00108   | 0                | 11 409            | 985       |
| $^{90}$Sr/$^{90}$Y with no filter at $y_0 = 50$ cm | 0.53207   | -0.62387  | -0.04086  | 0.16804   | -0.00728  | -0.02604  | -0.00796  | 0.00463   | 0.00171   | -2               | 11 409            | 985       |
| $^{106}$Ru/$^{106}$Rh with filter at $y_0 = 30$ cm | 0.68318   | -0.53488  | -0.29439  | 0.01712   | 0.11269   | 0.05504   | -0.00718  | -0.02101  | -0.01169  | -1               | 18 676            | 60        |
| $^{106}$Ru/$^{106}$Rh with filter at $y_0 = 50$ cm | 0.57637   | -0.6138   | -0.10818  | 0.1611    | 0.04297   | -0.03413  | -0.0197   | 0         | 0.00688   | -10              | 21 448            | 511       |
| $^{106}$Ru/$^{106}$Rh with no filter at $y_0 = 11$ cm | 0.55902   | -0.63364  | -0.07661  | 0.18264   | 0.00571   | -0.03113  | -0.0133   | 0.00681   | 0.00177   | 0                | 21 457            | 860       |
| $^{106}$Ru/$^{106}$Rh with no filter at $y_0 = 20$ cm | 0.57495   | -0.6277   | -0.10174  | 0.17107   | 0.01784   | -0.02446  | -0.01323  | 0.00363   | 0.00104   | -1               | 18 676            | 910       |
Table C3. Bragg-Gray stopping power ratio, $s_{A}(0)_{BG}$, at 0 μm phantom depth for different radionuclides [20], see equation (13) in section 4.7 for details. The standards uncertainty is about 0.6%.

|     | $^{147}$Pm | $^{85}$Kr | $^{90}$Sr/$^{90}$Y | $^{106}$Ru/$^{106}$Rh |
|-----|-------------|------------|--------------------|-----------------------|
| 1.124 | 1.121 | 1.110 | 1.099 |

Table C4. Fit parameters $a$ and $b$ to calculate $k_{Sa}$ at different phantom depths, see equation (13) in section 4.7 for details.

|     | $^{106}$Ru/$^{106}$Rh | $^{90}$Sr/$^{90}$Y | $^{85}$Kr | $^{147}$Pm |
|-----|-----------------------|--------------------|------------|------------|
| $a$ | 7.38·10$^{-3}$         | 1.57·10$^{-4}$     | 3.81·10$^{-4}$ | 6.17·10$^{-4}$ |
| $b$ | 0.613                 | 0.506              | 0.249       | 0.159      |

Table C5. Fit parameters $c_0$, $c_1$, and $c_2$ to calculate $k_{Sa}$ to use the Spencer-Attix theory for the BPS1, see equation (15) in section 4.8. The parameters were determined via a 2nd order polynomial regression of the ratio of the calculated Spencer-Attix theory factor to the Bragg-Gray ratios, $s_{A}$, see equation (14) in section 4.8.

| source | $E_{\text{mean}}$ /MeV | $c_0$ | $c_1$ | $c_2$ |
|--------|----------------------|------|------|------|
| Pm 20 cm wf | 0.065 | 1.0053 | -3.10 | 0.712 |
| Pm 11 cm w/of | 0.078 | 1.0018 | -2.50 | 0.487 |
| Kr 50 cm wf | 0.25 | 1.0058 | -2.51 | 0.487 |
| Kr 30 cm wf | 0.25 | 1.0060 | -2.88 | 0.649 |
| Sr 50 cm wf | 0.80 | 1.0066 | -2.01 | 0.328 |
| Sr 30 cm wf | 0.80 | 1.0071 | -3.05 | 0.732 |
| Sr 50 cm w/of | 0.82 | 1.0062 | -4.16 | 1.147 |
| Sr 30 cm w/of | 0.83 | 1.0053 | -2.79 | 0.644 |
| Sr 20 cm w/of | 0.84 | 1.0047 | -1.90 | 0.382 |
| Sr 11 cm w/of | 0.85 | 1.0048 | -2.01 | 0.328 |
| Ru 50 cm wf | 1.14 | 1.0087 | -2.66 | 0.551 |
| Ru 30 cm wf | 1.16 | 1.0088 | -2.71 | 0.596 |
| Ru 20 cm w/of | 1.15 | 1.0088 | -3.23 | 0.772 |
| Ru 11 cm w/of | 1.15 | 1.0089 | -3.81 | 0.993 |

Note 1: The unit 1/(mm$^{-3}$ · 10$^{-3}$) ≡ 1/(m$^{-1}$) while the unit 1/(mm$^{-2}$ · 10$^{-3}$) ≡ 1/(m$^{-2}$ · 10$^{3}$).
Note 2: An entry of ‘−3.10’ means $c_1 = -3.10 · 10^{-3}$ mm$^{-1} = -0.00310$ mm$^{-1}$.
Note 3: An entry of ‘0.712’ means $c_2 = 0.712 · 10^{-3}$ mm$^{-2} = 0.0000712$ mm$^{-2}$.

Appendix B. Tables with the calculated correction factors

Tables B1 and B2 list the backscatter factors, $k_{br}$, for BPS1 and BPS2, respectively, together with their uncertainties. Tables B3 and B4 list the fit parameters $f_{s}$ and $f_3$ for the calculation of the perturbation factors, $k_{pc}$. Finally, tables B5 and B6 list the inhomogeneity factors, $k_{inh}$, for BPS1 and BPS2, respectively, together with their uncertainties. Regarding the nomenclature ‘w/of’ means ‘with a beam flattening filter 10 cm from the source’ between the source and the plain of the radiation field while ‘w/of’ mean ‘without beam flattening filter’.

Appendix C. Tables with further data and correction factors beyond ISO 6980-2:2004

In the following tables, the correction factors for bremsstrahlung, $k_{br}$, in table C1, the fit parameters for the transmission functions $T(d)$, in table C2, the Bragg-Gray stopping power ratio at 0 μm phantom depth, in table C3, the fit parameters to calculate $k_{Sa}$ at different phantom depths, in table C4, and the fit parameters to calculate $k_{SA}$ to use the Spencer-Attix theory, in table C5, are given.

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