Notes on an interacting holographic dark energy model in a closed universe

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Abstract. We consider an interacting holographic dark energy model in Friedmann–Robertson–Walker spacetime with positive spatial curvature and investigate the behavior of the geometric parameter and dark energy density in an accelerated expanding epoch. We also derive some conditions needed to cross the phantom dividing line in this model.

Keywords: dark energy theory, inflation, gravity, physics of the early universe
1. Introduction

To describe the present acceleration of the universe [1] different models have been proposed. If we adopt the Einstein theory of gravity, this acceleration is only possible when approximately 70% of the universe is filled with a component with negative pressure dubbed as dark energy. A straightforward candidate for dark energy is the vacuum energy which suffers from conceptual problems such as fine-tuning and coincidence problems [2]. The amount of the dark energy density assessed in this model differs by 120 orders of magnitude from the observational value. Some present data seem to favor a dark energy component with an equation of state (EoS) parameter, \( w_d \), evolving from a value greater than \(-1\) in the past to \( w_d < -1 \) in the present epoch [3]. This dynamical behavior cannot be explained by the cosmological constant which possesses a constant EoS parameter: \( w_d = -1 \). Observations also show that the dark energy and dark matter densities are of the same order at the present epoch (known as the coincidence problem). This would not be true if there were no interactions between these components. Indeed, in dark energy models, as the universe expands, the ratio of matter to dark energy density is expected to decrease rapidly (proportional to the scale factor). To solve these problems, one can adopt an evolving dark energy with suitable interaction with (dark) matter [4].

One of the models proposed to describe the present accelerated expansion of the universe, and the dynamical behavior of the EoS parameter, is the holographic dark energy model [5,6]. This model is based upon the fact that the formation of a black hole requires a relation between the ultraviolet and infrared cutoffs of the system which leads us to assume that the total dark energy contained in a system must not exceed the mass of a black hole of the same size [7]. In this way the dark energy density may be related to the dynamical infrared cutoff of the system [5]. Note that, besides the late-time acceleration, the holographic dark energy model also may be used to study the inflationary and post-inflationary epochs of the universe [8].

In this paper we consider an interacting holographic dark energy model [9] in a Friedmann–Robertson–Walker spacetime with positive spatial curvature. We do not restrict ourselves to only the small-curvature limit and discuss time evolution of dark energy and dark matter densities. We investigate the behavior of the geometric parameter in an accelerated expanding epoch. We allow the dark energy to exchange energy with...
(dark) matter and discuss conditions needed to cross the phantom dividing line, by considering the second law of thermodynamics. We show that at the transition time there may be an upper limit for dark energy density, which depends upon the interaction parameters as well as the geometric parameter which may be regarded as a geometrical correction (due to their departure from flatness) to our previous flat case results in [10]. Our results may also alleviate the coincidence problem.

Throughout this paper we use $\hbar = c = G = k_B = 1$ units.

2. Holographic dark energy

2.1. Properties and evolution

We consider a Friedmann–Robertson–Walker (FRW) spacetime described by the metric

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right),$$

in the comoving coordinates. $a(t)$ is the scale factor and $k$ determines the spatial curvature of the spacetime. The universe is assumed to be filled with perfect fluid(s) at large scale. The Hubble parameter, $H$, is related to energy density, $\rho$, via the Friedmann equation

$$H^2 = \frac{8\pi}{3} \rho - \frac{k}{a^2},$$

(2)

We have also the evolution equation

$$\dot{H} = -4\pi (P + \rho) + \frac{k}{a^2},$$

(3)

where $P$ is the pressure. When the total density is equal to the critical density defined by $\rho_c = 3H^2/8\pi$, the universe is spatially flat, i.e. $k = 0$. We assume that the universe is dominated by pressureless dark matter (denoted by the subscript m) and dark energy component (denoted by the subscript d). In this paper we restrict ourselves to positively curved space with three-dimensional spatial spherical geometry and take $k = 1$. The relative densities defined by $\Omega_m = \rho_m/\rho_c$ and $\Omega_d = \rho_d/\rho_c$ satisfy

$$\Omega_m + \Omega_d - \Omega_k = 1,$$

(4)

where the geometric parameter, $\Omega_k$, is defined as $\Omega_k = 1/(aH)^2$. The equation of state parameter of the dark energy, $w_d$, given by $P_d = w_d \rho_d$, satisfies

$$w_d = \frac{w(1 + \Omega_k)}{\Omega_d},$$

(5)

where $w$ is the EoS parameter of the universe. The time evolution of $\Omega_k$ is obtained as

$$\dot{\Omega}_k = -2H \Omega_k \left( 1 + \frac{\dot{H}}{H^2} \right).$$

(6)

We consider a model of dark energy and dark matter interacting via the source term $(\lambda_m \rho_m + \lambda_d \rho_d)H$. So there is energy exchange between the dark matter and dark energy.
components. While these components are not conserved
\[ \dot{\rho}_d + 3H\rho_d(1 + w_d) = -(\lambda_m\rho_m + \lambda_d\rho_d)H, \]
\[ \dot{\rho}_m + 3H\rho_m = (\lambda_m\rho_m + \lambda_d\rho_d)H, \]
the total density satisfies the continuity equation
\[ \dot{\rho} + 3H\rho(1 + w) = 0. \]
To study how the ratio of $\Omega_m$ to $\Omega_d$ changes with time, one can use
\[ \dot{r} = H(1 + r)((\lambda_m + 3w)r + \lambda_d), \]
where
\[ r = \frac{\rho_m}{\rho_d} = \frac{\Omega_m}{\Omega_d}. \]
In the absence of interaction ($\lambda_m = \lambda_d = 0$), $r$ is a decreasing (increasing) function of time, when $w < 0 (> 0)$. But in the presence of interaction $r$ may be increasing even in an accelerating phase.

The time derivative of the ratio $\mathcal{P} := \Omega_k/\Omega_d = (3/8\pi)(1/a^2\rho_d)$ has the same sign as $(1 + 3w_d + \lambda_d + \lambda_m)r$. To verify this claim one can use (7). When the components are non-interacting and in a (non-)accelerating phase, $w_d > (<) -\frac{1}{3}$, we have $\mathcal{P} > (<) 0$. In the presence of an interaction this claim is not generally true and the behavior of $\mathcal{P}$ depends upon the interactions and conditions considered in the model.

We take the dark energy component as a holographic dark energy determined through
\[ \rho_d = \frac{3c^2}{8\pi L^2}, \]
where $c$ is a numerical constant and $L$ is an infrared cutoff which may be chosen as follows. Assume a light signal which is emitted from $r$ at $t$ will arrive at the origin at $t = \infty$: as the light signal propagates along the geodesic $ds^2 = 0$, we have
\[ \int_t^\infty \frac{dt}{a(t)} = \int_0^r \frac{dr}{\sqrt{1 - r^2}} = \sin^{-1} r. \]
We choose $L$ as the radius of the event horizon measured on the sphere of the horizon (see the second reference in [5]), hence $L = a(t)r$. Defining $R_h = a(t)\int_t^\infty dt/a(t)$, we obtain $L = R_h\sin y$, where $y = R_h/a(t)$. In the flat case, $k = 0$ and $L$ reduces to $L = R_h = a(t)\int_t^\infty dt/a(t)$. One can assign an entropy to the universe characterized by the cutoff $L$ as
\[ S = \pi L^2. \]
The time derivative of $L$ can be shown to be
\[ \dot{L} = HL - \cos y. \]
Hence the second law of thermodynamics, $\dot{S} \geq 0$, is valid whenever
\[ 0 < \frac{\Omega_d^{1/2}}{c} \cos y \leq 1, \]
or in terms of $\Omega_k$

$$\Omega_d \leq c^2(1 + \Omega_k).$$  \hfill (15)

Note that $\ddot{L} = -L/a^2 + (HL)^\prime$. For $\dot{\Omega}_d > 0$, we have $(HL)^\prime < 0$, which leads to $\ddot{L} < 0$. But if one requires that the entropy attributed to the cutoff $L$ is increasing, he finds $\dot{L} > 0$, then either $\lim_{t \to \infty} \dot{L} = 0$ or $\dot{L}$ becomes positive after a finite time, i.e. $\Omega_d > 0$ will no longer be valid.

The equation of state parameter of the compact universe is

$$w = -1 - \frac{2}{3} \frac{(\dot{H}/H^2) - \Omega_k}{1 + \Omega_k},$$  \hfill (16)

which leads to $1 + \dot{H}/H^2 = -\frac{1}{2}(1 + \Omega_k)(1 + 3w)$. Therefore like the flat case, we have $\dot{a} > 0$ when $w < -\frac{1}{3}$. If $\Omega_k \neq 0$, we obtain $w = -\frac{1}{3} + \dot{\Omega}_k/(3H\Omega_k(1 + \Omega_k))$. Thus the sign of $\dot{\Omega}_k$ determines whether the universe is in an accelerated phase ($w < -\frac{1}{3}$) or not. The super-accelerated universe $\dot{H} > 0$ corresponds to $w < -\frac{1}{3}(1 + 2/(1 + \Omega_k))$.

Taking the time derivative of both sides of $HL = c\Omega_d^{1/2}$ (which may be derived from (10)) leads to $\dot{H}L + H^2L + (c/2)\Omega_d^{-3/2}\dot{\Omega}_d = H\cos y$. Therefore from (16) we obtain

$$w = \frac{1}{3} - \frac{2\cos y}{3c(1 + \Omega_k)}\Omega_d^{1/2} + \frac{1}{3H(1 + \Omega_k)}\frac{\dot{\Omega}_d}{\Omega_d}.$$  \hfill (17)

For $w < -\frac{1}{3}$, from the above equation we deduce $\dot{\Omega}_d \leq (2/c)H\Omega_d^{3/2}\cos y$, which by considering the second law of thermodynamics results in $\dot{\Omega}_d \leq 2H\Omega_d$, implying $\Omega_d \leq 2H(1 + \Omega_k)$. It can be shown that

$$\dot{r} = \frac{\Omega_k}{\Omega_d} - \frac{1 + \Omega_k}{\Omega_d^2}. \hfill (18)$$

Comparing this result with (9) yields

$$w = \frac{\dot{\Omega}_k\Omega_d}{3H\Omega_m(1 + \Omega_k)} - \frac{\dot{\Omega}_d}{3H\Omega_m} - \frac{1}{3} \left( \lambda_m + \frac{\lambda_d\Omega_d}{\Omega_m} \right).$$  \hfill (19)

Using (17) and (19) and

$$\Omega_k = H\Omega_k(1 + 3w)(1 + \Omega_k),$$  \hfill (20)

$w$ and $\dot{\Omega}_d$ may be obtained as

$$w = -\frac{2\cos y}{3c(1 + \Omega_k)}\Omega_d^{3/2} + \frac{\lambda_m - \lambda_d - 1}{3(1 + \Omega_k)}\Omega_d - \frac{\lambda_m}{3},$$

$$\frac{\dot{\Omega}_d}{\dot{H}} = -\frac{1 + \lambda_d - \lambda_m\Omega_d^2}{1 + \Omega_k} + \frac{2}{c} \left( 1 - \frac{\Omega_d}{1 + \Omega_k} \right)\Omega_d^{3/2} + (1 - \lambda_m)\Omega_d + \Omega_d\Omega_k(1 + 3w).$$  \hfill (21)
Note that study of this model is more complicated with respect to the flat case where the right-hand side of the above equation (besides $\lambda_m$, $\lambda_d$ and $c$) depends only on $\Omega_d$ [10]:

$$w = -\frac{2\Omega_d^{3/2}}{3c} + \frac{\lambda_m - \lambda_d - 1}{3} \Omega_d - \frac{\lambda_m}{3},$$

$$\frac{\dot{\Omega}_d}{H} = \Omega_d \left( \frac{2}{c} \Omega_d^{1/2} + 3w + 1 \right).$$

By (21) and (14), we obtain $(3w + \lambda_m)(r + 1) + 3 + \lambda_d - \lambda_m \geq 0$ or

$$\frac{\dot{r}}{H(1 + r)} \geq -3(1 + w).$$

Hence, if $w < -1$ then $\dot{r} > 0$, indicating that the ratio of dark matter to dark energy increases. For $w < -\frac{1}{3}$, we obtain $\dot{r} \geq -2H(1 + r)$. The evolution of the ratio of $\Omega_k$ to $\Omega_d$, represented by $P$, can be given by

$$\dot{P} = \frac{3H}{8\pi} \frac{(1 + 3w_d + \lambda_d)\rho_d + \lambda_m\rho_m}{a^2\rho_d^2}$$

$$= (1 + 3w_d + \lambda_d + \lambda_m)\dot{P}$$

$$= -\frac{2}{c} H\dot{P} \Omega_d^{1/2} \cos y.$$ (24)

Hence, if the second law of thermodynamics in the form (14) is valid then $\dot{P}$ must be a decreasing function of time.

### 2.2. $w = -1$ crossing

In order that the effective EoS parameter crosses $w = -1$, we must have $w_d < -1 - r$, which requires $\Omega_m < -(1 + w_d)\Omega_d$ or $\Omega_k < -1 - w_d\Omega_d$. If the transition is assumed to be from quintessence to the phantom phase, then $\dot{w}$ must be negative at $w = -1$. From (21) we have

$$\dot{w} = \frac{\dot{\Omega}_d}{1 + \Omega_k} \left( -\frac{1}{c} \Omega_d^{1/2} \cos y - \frac{c}{3} \Omega_d^{-1/2} - \frac{1}{3} (1 + \lambda_d - \lambda_m) \right)$$

$$+ \frac{\dot{\Omega}_k}{(1 + \Omega_k)^2} \left( \frac{2}{3c} \Omega_d^{1/2} \cos y + \frac{1}{3} (1 + \lambda_d - \lambda_m)\Omega_d + \frac{c}{3 \cos y} (1 + \Omega_k)\Omega_k\Omega_d^{1/2} \right).$$

Using (20), (25) becomes

$$\dot{w} = -\frac{H\Omega_d}{1 + \Omega_k} \left[ X^2 + \left( \frac{\lambda_d - \lambda_m + 1}{3} + \frac{\alpha}{6}(\Omega_k + 3) \right) X + \frac{\alpha}{6}(\lambda_d - \lambda_m + 1) + \frac{\Omega_k}{3} \right],$$

where $X = (1/c)\Omega_d^{1/2} \cos y$ and $\alpha = 1 + 3w$. At $w = -1$, (26) reduces to

$$\dot{w} = -\frac{H\Omega_d}{1 + \Omega_k} \left[ (X - 1) \left( X + \frac{1}{3}(\lambda_d - \lambda_m + 1 - \Omega_k) \right) \right].$$

(27)
The second law of thermodynamics implies that \( X \leq 1 \). For \( X = 1 \), we obtain \( \dot{w} = 0 \) at \( w = -1 \). But
\[
\dot{X} = H[X^2 + \frac{1}{2}(1 + \Omega_k)(1 + 3w)X + \Omega_k],
\]
and therefore, if \( X = 1 \) at \( w = -1 \), then we also must have \( \dot{X} = 0 \). In the same way, using (26) one can show that \( d^nX/dt^n = 0 \), which results in \( d^nw/dt^n = 0 \) at \( w = -1 \). Hence \( X = 1 \) at \( w = -1 \) implies that \( \dot{X} \), and higher derivatives of \( X \) must also be zero at that point (denoted as the point of infinite flatness). By considering that \( X \) is an analytic function, we conclude that infinite flatness may only occur at \( t \to \infty \). Hence if the transition from quintessence to the phantom phase is allowed we must have
\[
\Omega_1 \geq \frac{1}{2} d \cos y \leq c \left( -\lambda_d + \lambda_m - 1 + \Omega_k \right),
\]
\( 0 < \Omega_1 \leq c \).

Note that the validity of the above inequalities necessitates \( \lambda_d - \lambda_m + 1 < \Omega_k \). In the flat case (29) becomes
\[
\Omega_d^{1/2} \leq \frac{c}{3}(\lambda_m - \lambda_d - 1),
\]
\( 0 < \Omega_d^{1/2} < c \).

Hence in this situation, \( \lambda_d - \lambda_m + 1 \) may be only negative.

In terms of \( \Omega_d \), (29) may be written as
\[
\Omega_d < c^2 \left( \Omega_k + \min \left\{ 1, \left( \frac{\lambda_m - \lambda_d - 1 + \Omega_k}{3} \right)^2 \right\} \right),
\]
which imposes an upper bound on \( \Omega_d \) at transition time. At \( w = -1 \), we also have
\[
\Omega_d^{3/2} \cos y + \frac{c}{2}(\lambda_d - \lambda_m + 1)\Omega_d + \frac{c}{2}(\lambda_m - 3)(1 + \Omega_k) = 0.
\]

In order that the transition occurs it is necessary that (32) has at least one real root. By considering (31) and (32), we arrive at
\[
\frac{3 - \lambda_m}{\Omega_d} < \frac{1}{1 + \Omega_k} \left( (\lambda_d - \lambda_m + 1) + 2 \min \left\{ 1, \frac{\lambda_m - \lambda_d - 1 + \Omega_k}{3} \right\} \right).
\]

This inequality can be written as
\[
(1 + r)(3 - \lambda_m) < \gamma,
\]
or
\[
r < \frac{\gamma}{(3 - \lambda_m)} - 1, \quad \text{if } \lambda_m < 3,
\]
\[
r > \frac{\gamma}{(3 - \lambda_m)} - 1, \quad \text{if } \lambda_m > 3,
\]
where we have defined \( \gamma = (\lambda_d - \lambda_m + 1) + 2 \min\{1, (\lambda_m - \lambda_d - 1 + \Omega_k)/3\} \). For example, if the parameters of the interaction (i.e. \( \lambda_m \) and \( \lambda_d \)) satisfy \( \lambda_m > 3 \), \( 0 < \lambda_m - \lambda_d - 1 + \Omega_k < 3 \) and \( (3r_0 + 2)\lambda_m + \lambda_d - 9r_0 + 2\Omega_k > 8 \), where \( r_0 \) and \( \Omega_k \) are the values of \( r \) and \( \Omega_k \) at transition time, then (35) is satisfied. As another example, in a model...
characterized by $\lambda_m > 3$, $3 < \lambda_m - \lambda_d + \Omega_{k0} - 1$ and $(\lambda_m - 3)r_0 + \lambda_d > 0$, the required condition (35), for crossing the $w = -1$ line, is satisfied. For example, for a closed universe with $\{\Omega_{k0} = 0.02, r_0 = \frac{3}{7}\}$, all models whose interaction parameters satisfy $\{\lambda_m > 3.7\lambda_d + 3\lambda_m > 9, \lambda_m - \lambda_d > 3.98\}$ fulfill the condition (35).

Note that, for $\lambda_m < 3$, (35) implies $\gamma > 3 - \lambda_m$. For negative $\gamma$’s, the second inequality in (35) may be utilized to alleviate the coincidence problem. Indeed it may pose a positive lower bound on $r$ in the transition epoch.

3. Conclusion

In the present paper we have studied the holographic dark energy model in a closed FRW universe. We have considered an interaction between (dark) matter and dark energy (see (7)). By considering the second law of thermodynamics, corresponding to the entropy assigned to the horizon of the universe (see (12)), some relations for relative densities of dark energy ($\Omega_d$) and dark matter ($\Omega_m$), and the geometric parameter ($\Omega_k$), have been obtained. We have found that in a super-accelerated universe (phantom phase), $r = \Omega_m/\Omega_d$ is an increasing function (see (23)), but for an accelerated universe (quintessence phase), depending on the interaction involved in the theory, $r$ may be a decreasing or an increasing function of comoving time (see (9)). We have also shown that $\Omega_k/\Omega_d$ is decreasing, provided that the second law of thermodynamics is satisfied (see (24)). Using the expression obtained for the equation of state parameter in (21), we have obtained some necessary conditions required for the transition from quintessence to the phantom phase (see (29)). These conditions pose some bounds on the dark energy density at the transition time which can alleviate the coincidence problem (see (35)). Note that these bounds depend on the geometric parameter, $\Omega_k$, as well as on the interaction parameters.

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