The effect of fibers-matrix interaction on the composite materials elongation

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Abstract. Failure process in composite materials is the basis of various actual researches. It study is the almost treated with fiber bundle model (FBM), in the framework of local load sharing rule (LLS), were the breakdown mechanisms is controlled by different parameters. Almost tentative investigates only fiber failure process and neglects the effect of the matrix, as a second crucial component of composite. Effectively, quantification of fibers-matrix interactions is not generally clear. Furthermore, the diversity of composite intrinsic proprieties complicates this quantification. The originality of our investigations is to quantify all interactions between fibers and matrix in a single parameter. Likewise, this latter gives us a different results than the ones obtained with FBM where the interaction amplitude is neglected. Moreover, interaction decelerates the failure process; it subdivides the avalanche phenomena on two consecutive regimes separated by delaying duration. These results are more similar to the ones obtained regular fibers substations. Therefore any fiber have its own elongation, the mean fiber elongation produces an elastic energy. The temporal variation of these latter presents two extremum separated by time duration; at this separated period, the material self-rearranging. These results are more similar to the ones obtained by the regular fibers substitution process.

Keywords: Fiber bundle model, elastic energy, matrix-fibers interaction, composite materials.

1. Introduction

Recently, composite materials present various benefits in different application areas. These composites constitute of two non-miscible homogeneous components and present crucial physical and chemical characteristics [1–4].Moreover, failure phenomena of composites and structures, as a perplexed process in industry and science community, produces an enormous destructions and shedding (natural catastrophes such as mine fall-in, earth tremor and mudslide) in various daily human activities [5–7]. Diverse efforts have been depleted to present an explicit physical description of breakdown process. Nevertheless the huge damages produce engineers and scientific comity to investigate for the basic mechanisms of the fracture process [8–10]. So that the loss may be minimized by furnishing a forerunner: the elongation of composite is a substantial propriety in order to make enormous structures
like constructions, buildings, bridges, etc.). It has been indicates that the disorder performs a crucial character in determination of the material elongation and also in the avalanche phenomena [11–13]. The avalanche phenomena in composites generally can’t be described by a basic linear equation for the crucial non-regularity and the disorder in composites. As an outcome, the theoretical method of statistical physics is mostly involved to investigate characteristics of the failure process. However, the purpose for this point is most probably the emergence of the computer as an investigation tool, rendering issues, which were over the domain of systematic theoretical researches, now accessible. Furthermore, we observed that the fluctuation rather than the average feature performs an important role in the explanation of the breakdown mechanism [14–18]. Correspondingly to the way how the applied load is subdivided between the intact fibers; different models are proved in the literature.

We can cite, numerical and analytical approaches which have been studied [19–21], for example Monte Carlo simulation [22–24], fuse networks [25–27] and molecular dynamics [24–29]. Various statistical researches are relying to fiber bundle model (FBM), they investigating breakdown of disorder materials. In the most cases, this approach shows correctly proprieties of failure states in composites. That is to say that this model (FBM) has also proved to be a valuable method to explore the dynamical fracture in composite materials [30–32]. The approach of the FBM is perspicuous and approximately basic to reach numerically persuasive statistical properties of breakdown material or to recover analytically adequate results [30].

Under a constant applied load, most composites have a time-dependent response and fails in finite time. Over its great industrial utilization, the conception of such rupture process needs several essential issues for statistical physics too. The complexity of studying breakdown mechanism appears from the conditional circumstance on the materials type, it may have a various microscopic origins, from the applied load, through thermally activate, existence of frictional interfaces and components viscoelasticity, to the interactions between matrix and fibers [31]. Preceding investigations [1-15] study only the impact of applied load, thermal noise and system size. Hence, the inventiveness of our research is studying other parameters by quantifying the fibers-matrix interactions in a single term: interaction amplitude $\eta$.

To investigate the failure processes in composite materials, we may treat fiber bundle models, with random thresholds value for fibers formed our system; the widely exploited models [6–13], are commanded by accurate ruling for how load generated by a damaged ingredient is transferred to unbroken fibers. Almost investigations, by using this approach, may be applied to an inadequate domain for perplex materials.

Generally, the FBM is assumed to be composed by a set of fibers whose break strengths are assumed to comply with a certain statistical law, such as uniform distribution [32]. The first ascertainment confirms that the fiber will break when it overcomes the threshold value. Under stress-commanded loading condition, the system is loaded parallel to the fiber orientation. The stress delivered by each damaged fiber is transferred among the surviving fibers. As a result, a series of load redistribution may produce two consecutive avalanches, which may occasionally finish after an infallible number of successive fractures, preserving the plurality of the system, or may be catastrophic, leading a macroscopic damage of the totality of the material [30]. Depend to elongation of transverse connection in the failure process [28-30]. It delivered belong the unbroken fibers may be arranged into various categories, as though we cited the equal load sharing (ELS) or global load sharing (GLS), the local load sharing (LLS), and so on.

In this paper, we investigate numerically, the effect of the matrix-fibers interaction on the failure process of the composite materials in the framework fiber bundle model by using the LLS approach. So, this paper is organized as follow: the adopted model in this research is developed in section 2. The obtained results are reported and discussed in section 3. Finally, we summarized our results in the conclusion section.
2. Fiber bundle model with random oriented fibers:

Fiber bundle model explicates the description of composite material’s fracture. It also approved as a steady approach that elucidates the damage of a diverse category of disordered materials of numerous interacting constituents to different natures of external effect i.e. charge, temperature, system size, diverse interactions... etc. [32]. The fiber bundle model considers composite materials as a bundle, which matrix reinforces fibers. Then the studied system is obtained by introducing the two principal constituents in various geometrical configurations. The performant proprieties of composite materials impose new industry; specifically technology, aerospace and modern automotive, to operate them. They offer two particular benefits: they support high elongation while it relative low mass and they preserve these qualities also below extreme conditions (high value of temperature, pressure). They have the flexibility to produce modifications in conception and customization conforming to the requested implementations. The capacity to command the characteristics of our materials permits us to investigate the damage of our composite by inspecting fracture of fibers and discussing the impact of interaction between fibers and matrix. All fibers are delimited by matrix of the composite. When a fiber fails, the fiber-matrix interface disintegrates in proximity of the failure. As result of fragmental contact at the interface, the imposed force rise up against as a load restoration length, hence the fractured fiber involves to the global stress capacity of the material.

Amid the theoretical approaches to the problematic, the fiber bundle model (FBM) shows an essential role, and then it captures the focal elements of the rupture of disordered composites while it is still simple enough to simplify analytical calculations. Conferring to the charge transfer mode, fiber bundle model is separated into two groups: Global load-sharing (GLS) or democratic FBM and local load-sharing (LLS). In democratic mode intact fibers bear the charge equally and in local load-sharing the charge of the broken fiber is given to the intact neighbors. Actual developments exhibit old comportment of GLS bundle envisioning variation in applied charge. As well, giving noise-generated failure probability of fibers, old comportment is stretched in a homogeneous material under GLS with constant load [22]. In opposition to the fuse model [9], the fiber bundle model offers a satisfactory balance between being numerically well-regulated and analytically dissolvable. In its basic configuration, the global load sharing fiber bundle model, essentially each factor can be found. In other advanced models, the localized load sharing, some characteristics can be found analytically in this model, but others cannot [7-22]

We envisage a randomly material which is elaborated by discrete set of N0 fibers. Each fiber has identical initial elongation l0 and is randomly oriented in all directions confiding to the vertical one, and is introduced on a regular lattice of size L. The structure is connected at the first end and in the second end we imposed an exterior force [18]. The FBM is a relevant model that contributes to more describing breakdown in composites. Fibers are subject to an exterior force parallel to the ends of our structure. The totality of charge sustains by the fibers. While the fiber-matrix interface and matrix determine mainly the load transfer between fibers [28-29]. Each fiber i is inclined correspond to a constant angle

\[ \alpha_i \in \left[ \frac{0}{2}, \frac{\pi}{2} \right] \]

\[ P(\alpha) = \int_0^\alpha p(y)dy = \alpha \]  \hspace{1cm} (1)

The initial fibers elongation relies on fibers orientation. The fibers which are deposited vertically \((\alpha_i = 0)\) or strongly inclined \((\pi/4 < \alpha_i \leq \pi/2)\) have identical initial elongation l0 and are clamped with well-fixed clamps belonging to the both ends of the material. The fibers which are weakly inclined \((0 < \alpha_i \leq \pi/4)\) have another initial elongation. Which is chosen with similar method that these fibers also stay bounded to the two lateral areas by clamps.

The fibers are climbed between two hard clamps. The matrix assures the material cohesion and permits each fiber preserving the same direction over the application of force [16]. Each fiber i has its proper breaking threshold fi. Fibers crack individually when stretching threshold value is reached. If
the elastic load proceeds readily, it overtakes \( f_i \) then it cracks. Composites in the bundle form, composed of an important number of elastic fibers with low mass, are well known to be lucid templates of critical systems that show important fracture proprieties [17-22]. The fibers may merely sustain longitudinal displacement below an exterior charge. In our investigation we consider that the applied load \( f_i \) on each fiber produces an axial longitudinal deformation. This component presents an axial displacement \( \xi \) which is expressed by the Hooke’s law:

\[
\xi_i = f_i / k
\]  

(2)

where \( k \) denotes the stiffness, we consider that its value is the same for all fibers. In order to take into account of thermal noise in the failure process, it is supposed that the localized charge on fibers has time dependent on fluctuations, and then we introduce the corresponding local elongation \( \xi_r \) of each fiber due to the presence of thermal noise [6-11]

\[
\xi_r = \chi l_0 \sqrt{K_g T}
\]  

(3)

where \( K_g \) is the Boltzmann constant and \( \chi \) is the coefficient of proportionality between elongation and thermal energy.

The presence of the thermal noise has a quasi-static effect on the failure process in the composite material, yet that will affect the lifetime of the system [9-15]. Then, the actual total length arising \( x \) of each fiber is written as:

\[
l = \xi + \xi_r
\]  

(4)

If the elongation of fiber \( i \) overtakes the failure threshold elongation \( \xi_c \), the fiber fails and its charge is transferred to its neighboring intact fibers and so on [25]. In this paper, the broken threshold elongation \( \xi_c \) is randomly chosen and given with the density of probability \( p(x) \) and the uniform distribution \( P(x) \) which is the most studied threshold distributions [28-32]:

\[
P(x)=\frac{1}{b-a} \int_{a}^{b} p(y)dy=1 \quad \text{with} \quad p(x)=1
\]  

(5)

The fibers behave linearly elastic until they crack at a failure threshold elongation [29]. The randomness of failure thresholds elongation is presumed to show heterogeneous materials disorder. The number of intact fibers \( N_i \) is:

\[
P(x)=1-N_i / N_0
\]  

(6)

Effectively the intact fiber density is:

\[
\rho = \frac{N_0 - N}{N_0}
\]  

(7)

The imposed force is considered as a constant quantity. Among the failure, the load of an individual broken fiber discharges and distributes to other intact fibers. This load may produce avalanche phenomenon where the consecutive feedback process may generate the entire material damage [28].
When forces are applied to mechanical materials, constituents of this system store an elastic potential energy if they are deformed. In our study the variation of elongation $l$ produces an elastic energy. As cited previously $k$ is the stiffness of our material. So we explicit elastic energy by:

$$ E_e = \frac{1}{2} kl^2 $$

(8)

3. Results and discussions

The avalanche phenomenon observed in the failure process in composite materials can characterize clearly by different parameters. In the following part of this investigation, and for a first moment, we will give and discuss the results corresponding to the variation intact fiber density with load, matrix-fibers interaction and system size. In a second moment we will present the corresponding results characterizing the avalanche phenomenon and. In the last we present the elastic energy variation and system lifetime variation versus applied load and interaction amplitude.

Almost materials present a time-dependent response under a constant external load and its fail in a finite time. Creep breakdowns process is almost used in various technological applications, investigating creep phenomenon need different crucial topics for statistical physics [7-18]. Creep fracture perplexity comes from the moment that the occurrence that it can have various microscopic origins: thermally noise, fibers-matrix interactions, friction in interfaces, components viscoelasticity and system size.

In first moment, we imposed a constant exterior force $f$ on our material composed on a fiber bundle in a lattice of size $L \times L$. Firstly weak fibers broke and their loads are transferred to the intact neighboring fiber in the framework the LLS as redistribution load mode. We have calculated the intact fiber density $\rho$ versus time for system with $500 \times 500$ fibers and for different values of the applied load. The matrix-fibers interaction amplitude is fixed $\eta = 0.5$. We represent the number of broken fibers $N_b$ and the initial number of intact fibers $N_0$. The obtained results for different applied load and for temperature value $T = 0.03 T_c$ are plotted in figure 1.

We note that the considered material cracks as the applied load increases. However, failure process is triggered by breaking only the weak fibers and the total load is delivered on the nearly surviving fibers affecting a secondary breakdown. In the additional, the breaking process of the composite materials is different to the ones obtained by the classical model [28–30] where the effect of the matrix on the breaking fibers is neglected.

So, the introduction of the noise interaction matrix-fiber subdivided the avalanche breaking on two successively stages separated by delaying duration $\Delta t$. The temporal evolution of the intact fiber density, as presented in figure 1, is characterized by two regimes: a decreasing one with two different velocities, and a constant one. The two observed regimes are separated by a cross over time $\Delta t$. So, in the first time when the applied load is less than the threshold value ($f < f_r$), no fiber failed. While the load attains the failure threshold value of weak fibers, they crack and their loads are redistributed to the intact neighboring fibers according to the LLS rule. That is why the density of intact fiber decreases. After a certain time, a less intact fibers number amortizes the failure process; finally intact fiber density becomes constant by achieving its constant value. For the higher regime load, the avalanche breaking is similar to the one observed in the classical model when the interaction matrix-fiber is neglected. So the interaction matrix-fiber is not an important effect in the higher load regime.

We should affirm that with low force value the density reaches zero: all fibers broke, isn’t the case for high forces which the density that’s not reaching zero, material damaged before all fibers crack. We conclude clearly that the presence of fibers-matrix interaction make our composite more resistant.
Figure 1: the time evolution of the intact fiber density for different values of applied load

In result, the extremity of the fiber has not any contact with the clamps but they undergo the stress by local sharing load of its nearly surviving fibers. Thus, the intact fibers density decreases to reach zero value for less force domain, but with the same system without including the interaction amplitude, the density value attains 0.55 as indicated in [35]. Furthermore, the avalanche time for our studied composite is higher than one calculated for the same composite without introducing fibers-matrix interaction as established in [1]. Although, in the higher force domain, our results is analogues to the one without matrix-fiber interaction as obtained in [28-30].

The first important ingredient (fibers) constituting our system is merged as a bundle. These fibers are regularly linked by matrix which constitutes the second crucial ingredient strengthening the materials. The fibers-matrix interface is capable to sustain different charge types. It summarizes categories and figures of imposed charge: compression, elongation or intricate deformation [28].

Temperature and charge are not solely the important parameters which command the breakdown process of composites, yet various other elements may contribute the fatigue fracture of these systems: chemical effect, weather effects … etc. Actually, there has been an interim to integrate all noise modes via a unique parameter which may affect the breakdown probability of the each constituent, such a breakdown probability at any imposed force [29]. So, the breakdown process in composites should be considering the interaction between these two essential ingredients. Due to the large composite characteristics and the stochastic phenomena of these systems, we have included a randomly interaction where $\eta$ is the matrix-fibers interaction amplitude.

However, interaction amplitude may contain all details referring the load sharing between fibers and matrix. So, to expose the effect of interaction in homogeneity on thermally activated breakdown, we have calculated the density of intact fiber observed during the avalanche phenomenon with three different interaction amplitudes. We applied a constant load $f=0.2f_c$ on a system constituted of $500 \times 500$ fibers below a value of temperature equal to $T=0.03T_c$. So, we plot the intact fiber density versus time. The numerical corresponding results are plotted in figure 2. In the first observation, the results present identical schemes as found for different forces value. Hence, the intact fibers density decreases in time and decreases with the increasing interaction amplitude. Though, if interaction amplitude increases, the intact fibers density attains the zero in long time duration and the system breaks down slowly.

The intact fiber density evolution in time, as showed in figure 2, is characterized by two stages: a decreasing one with two different velocities, and a constant one. A cross over time $t_c$ distinguishes the two obtained stages. So, when the interaction attains the failure threshold value of weak fibers, they...
fail and their charge is transferred to the intact neighboring fibers by using the LLS mode. Effectively, that the reason why intact fibers density decreases. After a certain time, a less intact fibers number mitigates the breakdown phenomena; the composite doesn’t break. Finally, the intact fibers density becomes zero when all fibers crack. So the fibers-matrix interaction decelerates the failure process in the studied materials. Matrix-fibers interaction dissipates the imposed charge and the fluctuations appropriate to thermal activated. It delays the breakdown mechanism and finally, it reinforces our composite.

![Figure 2: the time evolution of the intact fiber density for different values of noise interaction](image1)

The avalanche distribution size is related on the adequate imposed force on the material, we determine the maximal charge $F/N$ as a maximal applied load which system can support until it fails totally. Another mode but homologue process is to apply a constant load $F$ to the composite, thus forthwith all fibers with a threshold elongation shorter than $F/N$ fail. The correspond fiber elongation is given by the maximal $F/N$ value whether doesn’t produce an entire failure of our composite [28-35]

![Figure 3: the time evolution of the intact fiber density for different values system sizes](image2)

To study the scaling law in the failure process on composites, we have calculated the intact fibers density evolution in time for different system sizes, under $f=0.2f_c$ with interaction amplitude $\eta=0.5$ in temperature equal to $0.03T_c$. The corresponding results are presented in figure 3. Moreover, the obtained profiles collapse on a unique curve which attests that the introduction of the matrix-fibers interaction doesn’t affect the scaling law detected in [38–41]
In the elementary literature of the model a homogeneous fibers assemblage is treated, with all the fibers have the same failure elongation, which is then under a constant sub-critical exterior charge. The material growth is operated by thermal noise: the local charge on fibers has thermally provoked fluctuations which can generate breakdown. The principal benefit of this elementary model is that various important macroscopic systems characteristic may be studied analytically. Then, it has been proved in [42] which the lifetime of the material depends on temperature by an Arrhenius law, even when the fibers have disordered elongation. More than that, Guarino in [31] proposed an elementary fiber bundle model, which has demonstrated very effective to get a theoretical results assuring comprehension of the applied load effect produced fractures accumulation on the creep failure process.

Our investigation is based on the consideration of an unbroken composite with N0 intact fibers distributed in random directions relative to the vertical axis. Each fiber is under an exterior charge. For the early time regime, the material resists to the force and after a threshold time value tf where the mean elongation fibers becomes greater than the threshold elongation, the breakdown process appear as micro-cracks which enhance rapidly in time producing an avalanche phenomena, we observe similar results as proved in new investigations [1-4].

As discussed previously, each fiber has an initial elongation l0. Hence when we applied a constant charge value, fibers elongate. This effect produce an elastic energy depends on fiber elongation. To more describing the breakdown process we calculate the elastic energy, obtained from elongation, as a crucial parameter. We plotted on figure 4 the temporal elastic energy variation for different applied load values. This energy pursues two stages and presents two extremums: a minimum at the time tmin and a maximum at the time tmax, the two times are separated by duration of time Δt. After attaining the threshold breakdown value, in the first stage fibers fail, the number of broken fibers increases, and as effect elastic energy increases to attain the maxima. We should note that LLS mode accelerate the breakdown mechanism, it correspond to the transfer of supplementary broken fibers charge. Although, in the second stage the mean fibers elongation decreases to a minimal value. As a consequence, the elastic energy decreases to reach a minimum value. We should note that the less broken fibers number, in this stage, neglect the LLS effect. Moreover, the intact fibers self-rearranged according to the self-criticality theory.

![Figure 4](image)

Figure 4: the time evolution of the elastic energy $E_e$ for different applied load values.

The maximal value of elastic energy decreases with increasing applied load, however the corresponding time to maximum tmax increases with decreasing applied load. Moreover, the minimal of elastic energy increases with decreasing applied load, hence the corresponding time to minimum tmin decreases with increasing applied load. These results are more similar to the ones obtained by a new investigation in regular fibers substitution process [1]. To confirm this remark, we calculate on
figure 5 the separated duration of time variation with applied load values. We observe that this separated duration decreases rapidly with increasing applied load values and changes the decreasing monotony at $t_e$ (is the inflexion time at which micro-crack process changes the velocity).

Therefore, the rearrangement of fibers arises at an earlier stage of failure when the surface was obviously unbroken [16]. Matrix contributes, in both boards, to the high elongation and preserves the articular fiber from rupture. Although, random fibers can reply more appreciatively to higher charge before failure, a random disposition of each fiber within composite is more adequate in standard conditions.

![Figure 5: The variation of separated duration of time $\Delta t$ versus applied load](image)

Figure 5: The variation of separated duration of time $\Delta t$ versus applied load

![Figure 6: the time evolution of the elastic energy $E_E$ for different interaction amplitude.](image)

Figure 6: the time evolution of the elastic energy $E_E$ for different interaction amplitude.

These facts will regularly be the origins for the structure of the ensemble fibers-matrix interaction on composites. So, we presented on figure 6 the temporal evolution of elastic energy for different noise values. Inversed behavior is observed than the obtained one with different applied load value: two extremums appear; the extremums has the same value for different noise values. The mean fibers elongation increases with increasing matrix-fibers interaction. However, the time corresponding to
extremum increases with increasing noise values. All observation confirm that interaction ensure the material resistance.

To finalize our study, we plot on figure 7 the variation of separated time duration $\Delta t$ versus interaction amplitude. We observed that this duration increase exponentially with increasing interaction. It delayed the elongation process, and the separated time corresponding to fibers rearrangement stage. We can say that the interaction amplitude decelerates the avalanche phenomenon and enhances the resistance of our composite. As result, after the first fracture, the charge of broken fibers has been transferred on the neighboring unbroken ones, and then the interaction sustains the load for a long time by slowing down the breakdown mechanism and preserving fibers.

![Figure 7: The variation of separated duration of time $\Delta t$ versus interaction amplitude](image)

**Conclusion**

In this paper, we adapt fiber bundle model, treated in LLS mode, to investigate failure process on composite materials. Evenly, previous researches study only the fibers breakdown without taking in account the impact of the intimate assemblage between fibers and matrix. As an originality of our investigation, we produce a single parameter quantifying all interactions between fibers and matrix.

To attain our aims, we pursue the temporal evolution of intact fiber density versus applied forces, fibers-matrix interaction amplitudes and system sizes. Therefore, we have proved that failure process dividing the avalanche failure on two consecutively periods separated by delaying duration. The impact of the interaction amplitude becomes fewer significant when enhancing applied load, thus the avalanche process is condensed to a one single regime. These results are different to the ones obtained without introducing interaction amplitude, but they are more similar to the ones acquired by the regular fibers substitution process. Moreover, fibers have it owing elongation; the variation of the mean elongation generates elastic energy. Effectively, we present the temporal evolution of this energy versus different applied loads and interaction amplitudes. Elastic energy increases to attain a maximum, then it decreases to reach a minimum and finally increases again.

As a general conclusion, our investigation presents the benefit of introducing matrix-fibers interaction amplitudes. Then these interaction sustains the charge for a long time, decelerates the breakdown mechanism, it blocks the breakdown of fiber and therefore it preserves the system. Hence, it also enhances the resistance of material and its lifetime. This impact expands the mechanical performance of composites. Moreover, during the time separated the two avalanches failure of fibers due to interaction amplitude, the material is self-rearranged and resists for longtime.
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