Estimation of Gas Holdup Using the Gassed to Ungassed Power Ratio of an Oxygen–Water System in a Stirred and Sparged Tank Contactor

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ABSTRACT: Gas holdup ($e_g$) and power correlations in gas–liquid (G–L) systems, apart from the physicochemical properties of the liquid phase, are dependent on impeller–sparger–vessel geometry. To date, reported correlations do not specifically address this issue, and it must be investigated with a unified approach. Here, we propose a correlation via the use of a normalized $e_g$ that involves the impeller–sparger system geometry for a vessel of standard geometry expressed as a function of an easily measurable and independent operational parameter, that is, $\left(1 - \frac{P_{gg}}{P_l}\right)$, where $P_{gg}/P_l$ is the gassed to ungassed power ratio. Furthermore, our work demonstrates that $P_{gg}/P_l$ can be used as a tool for the identification of hydrodynamic regimes. Radial and axial impellers with ring spargers were used in a stirred and sparged contactor (SSTC) of 0.25 m diameter containing 1 × 10⁻² m³ water. The oxygen flowrate ($Q_{O2}$) was varied from 2.5 to 40 LPM or (4.17 to 66.7) × 10⁻⁵ m³ s⁻¹, and the agitation intensity ($N_g$) was varied from 1.67 to 50 rps at the temperature ($\theta$) = 313 K under atmospheric pressure. This novel correlation is easy to use, offers reasonable precision, and can serve as a valuable alternative to more complex correlation models.

1. INTRODUCTION

1.1. Background. Gas transfer rates in gas–liquid (G–L) systems are often rate-limiting in which one of the reactants is a dissolved gas. G–L mass transport rates are dependent on hydrodynamic parameters, as shown in eq 1.

$$K_gS \propto \frac{P_g}{V_g} \left(\frac{u_s}{d_b}\right)^{2/3} P_{O2}$$

Considering that the gas side resistance to transport in G–L systems is negligible, the oxygen concentration $O^-$ at the G–L interface may be expressed by Henry’s law. On rearrangement of eq 1, it may be shown that volumetric mass transfer rates ($K_gS$) are influenced by gassed power ($P_g$) per unit liquid volume ($V_g$) and superficial gas velocity ($u_s$).

$$K_gS \propto \frac{P_g}{V_g} \left(\frac{u_s}{d_b}\right)^{2/3}$$

Therefore, G–L mixing enhancement by agitation and/or gas sparging is generally practiced to enhance mass transfer rates. Accordingly, stirred and sparged tank contactor (SSTC) systems consisting of baffles, stirrers, and spargers are the preferred mode of contact for such applications. In an SSTC, gas is introduced from a sparger underneath the impeller and subsequently the sparged gas bubbles are broken up and uniformly dispersed throughout the system due to the hydrodynamic regimes. To date, reported correlations do not specifically address this issue, and it must be investigated with a unified approach. Here, we propose a correlation via the use of a normalized $e_g$ that involves the impeller–sparger system geometry for a vessel of standard geometry expressed as a function of an easily measurable and independent operational parameter, that is, $\left(1 - \frac{P_{gg}}{P_l}\right)$, where $P_{gg}/P_l$ is the gassed to ungassed power ratio. Furthermore, our work demonstrates that $P_{gg}/P_l$ can be used as a tool for the identification of hydrodynamic regimes. Radial and axial impellers with ring spargers were used in a stirred and sparged contactor (SSTC) of 0.25 m diameter containing 1 × 10⁻² m³ water. The oxygen flowrate ($Q_{O2}$) was varied from 2.5 to 40 LPM or (4.17 to 66.7) × 10⁻⁵ m³ s⁻¹, and the agitation intensity ($N_g$) was varied from 1.67 to 50 rps at the temperature ($\theta$) = 313 K under atmospheric pressure. This novel correlation is easy to use, offers reasonable precision, and can serve as a valuable alternative to more complex correlation models.
argued that the overall $K_s \rho$ is directly influenced by mixing power: $P_l$ in the case of pure liquids and $P_g$ and sparging rates ($Q_g$) in the case of G–L systems.\(^6\) $K_s \rho$ may be enhanced by the directed manipulation of $N_a$ and the oxygen flow rate ($Q_g$) to overcome mass transfer limitations for achieving optimum kinetics.

Alternatively, based on film theory, the overall $K_s \rho$ dependence may also be expressed, as shown in eq 3, as

$$K_s \rho \propto \left( \frac{D_{in}}{t_f} \right)$$

where $t_f$ represents the film thickness, $D_{in}$ represents the diffusivity of oxygen in water, and $d_b$ represents the oxygen bubble diameter. We may assume that $K_s$ is linearly dependent (eq 3) on $\varepsilon$ which allows us an easier method indicative of mass transfer rates. Experimental determination of $K_s \rho$ is a bit tedious; therefore, a more straightforward and easily measurable parameter like $\varepsilon$ is preferred.\(^1\) Accordingly, in this paper, we develop a correlation for $\varepsilon_s$ and assess the applicability of $\varepsilon_s$ across the range of hydrodynamic regimes. Successful scale-up of SSTCs, therefore, requires identification of G–L hydrodynamic regimes to determine $\varepsilon_s$ on a benchtop setup and extrapolate to larger-scale systems within the applicable hydrodynamic regimes.\(^8\)

1.3. This Work. The factors influencing $\varepsilon_s$ include system geometry, superficial gas velocity, shape and size of the agitator, the intensity of agitation, and the physicochemical properties of the fluid. It is difficult to theoretically derive a clear-cut relationship between these factors and the observed hydrodynamic behavior in G–L dispersions. However, it is possible to relate system variables in the form of empirical correlations and dimensional analysis, and in a span of more than 70 years (1944–2016), several $\varepsilon_s$ correlations have been proposed by researchers for air–aqueous, nonelectrolytic systems.\(^3,5,7,9,10\) These $\varepsilon_s$ correlations are multiparametric and involve impeller diameter ($D$), tank diameter ($T$), liquid volume ($V_L$), liquid viscosity ($\mu$), liquid density ($\rho_L$), dispersion density ($\rho_D$), surface tension ($\sigma$), $Q_g$, $u_g$, and $N_a$. The first empirical correlation proposed by Foust, Mack, and Rushston (1944)\(^7\) is a function of $P_g$ and $u_g$ using a curved blade (CB). This was followed by similar correlations by Calderbank (1958)\(^10\) and Loiseau, Midoux, and Charpentier (1977)\(^11\) using disk turbines (DTs), and Stenberg and Andersson (1988)\(^3\) using paddle turbines (PD) with varied exponential dependence and forms of equations based on $P_g$, $P_l$, and $u_g$. Dimensionless analysis-based $\varepsilon$ correlations were reported by Hassan and Robinson (1977)\(^3\) and Hughmark (1980)\(^7\) using flat blade turbines (FBTs) and PD impellers. Focusing on liquid physicochemical properties, the effect of $\sigma$ has been studied extensively by Calderbank (1958)\(^10\), Loiseau, Midoux, and Charpentier (1977)\(^11\), Hassan and Robinson (1977)\(^3\), Hughmark (1980)\(^7\), and Paglianti et al. (2000);\(^12\) the values of the exponents are $-0.50, -0.36, -0.65, -0.25$, and $-0.17$, respectively. Generally, it has been observed that an increase in $\sigma$ results in a decreased $\varepsilon$. No definitive conclusions are reported about the effects of $\mu$, although Loiseau, Midoux, and Charpentier (1977)\(^11\) gave the exponential dependence as $-0.056$ and Paglianti et al. (2000)\(^12\) showed that an increase in $\mu$ led to an increase in $\varepsilon_s$ followed by a sharp decrease beyond a critical $\mu$. Calderbank (1958)\(^10\), on the contrary, found no significant effects of $\mu$ on $\varepsilon_s$. $\varepsilon$ correlations reported in the literature are specific to operational parameters and fluid properties and usually do not consider the impeller and sparger system geometry. Moreover, they are multiparametric, which introduces error and leads to reduced levels of prediction accuracy. Furthermore, it is known that $\varepsilon$ and $P_g/P_l$ correlations, apart from fluid physicochemical properties, are dependent upon the impeller geometry.$^{3,5,10,14}$
It may be, therefore, argued that, from an operational point of view, apart from geometrical dependence,
\[
\varepsilon_g \propto (1 - \frac{P_g}{P_0})
\]  
(6)

Accordingly, we proposed a correlation involving both impeller–sparger geometrical parameters and a simple operational parameter for a standard vessel geometry\(^6\)\(^{15}\) to have a fairly representative \(\varepsilon_g\) as shown in eq 7.
\[
\varepsilon_g = C_1 f\left(N_p, \Delta_s\right) f\left(1 - \frac{P_g}{P_0}\right)
\]  
(7)

where \(C_1\) is a constant, \(N_p\) represents impeller power number, and \(\Delta_s\) is a parameter representing the ring sparger–impeller geometry.
\[
\Delta_s = \sqrt[2]{\left(\frac{D - D_s}{D}\right)}
\]  
(8)

where \(D\) represents the impeller diameter, \(D_s\) the sparger ring diameter, and \(\Delta_s\) a ratio representing the relative diametrical position/radial distance of the ring spargers with respect to the impeller diameter.

On further simplification, the expression was normalized so that the normalized gas holdup (\(\varepsilon_g^{\text{norm}}\)) is independent of the impeller–sparger geometry effects
\[
\varepsilon_g^{\text{norm}} = \frac{\varepsilon_g}{f\left(N_p, \Delta_s\right)} = C_1 f\left(1 - \frac{P_g}{P_0}\right)
\]  
(9)

Therefore, our proposed correlation, eq 9, uses \(\varepsilon_g^{\text{norm}}\) which is expressed as a function of an easily measurable parameter, that is, \((1 - P_g/P_0)\), which in turn reflects the combined effects of \(N_p\) and \(Q_g\) on \(\varepsilon_g^{\text{norm}}\).\(^3\)

The operational parameter, \(P_g/P_0\) is also used as a tool for the identification of hydrodynamic regimes. When correlations are developed, it is important to consider the various possible hydrodynamic regimes (within the by-pass, loading, transition, and total recirculation regimes). Correlations extending beyond the recirculation regimes into the flooding regimes may not be well-defined due to the unpredictable hydrodynamic behavior. The description of these hydrodynamic regimes is summarized here, and the corresponding hydrodynamic patterns are shown in Figure 1a–e: (1) bypass, Figure 1a, defined as gas escaping to the liquid surface with negligible dispersion in the liquid, results in negligible \(\varepsilon_g^{\text{norm}}\) (2) loading, Figure 1b, as when gas is dispersed by the impeller but is poorly distributed;\(^10\) (3) transition, Figure 1c, wherein recirculation loops begin to form increasing the gas distribution;\(^11\)\(^,\)\(^12\)\(^,\)\(^16\)\(^,\)\(^18\)\(^,\)\(^19\) (4) total re-circulation, Figure 1d, representing circulation by loop formation, which indicates intimate G–L contacting;\(^8\)\(^,\)\(^12\)\(^,\)\(^16\)\(^,\)\(^18\)\(^,\)\(^19\) and (5) onset of flooding, Figure 1e, marked by constant \(\varepsilon_g\) accompanied by a reduction in \(K_{iS}\) and jumps/surges in \(P_g^{11}\)\(^,\)\(^12\)\(^,\)\(^16\)\(^,\)\(^18\)\(^,\)\(^19\). A detailed review of hydrodynamic regimes and their patterns based on the literature reported data is referenced.\(^1\)

2. RESULTS AND DISCUSSION

To propose meaningful correlations, we must first identify hydrodynamic regimes, namely bypass, loading, transition, and total recirculation, characterized by different hydrodynamic patterns. Beyond this, at the onset of the flooding regime (where the hydrodynamic behavior is unpredictable), data correlation would not be relevant.

Referring to the hydrodynamic patterns in Figure 1a–e and eq 6, the bypass regime (Figure 2) has negligible gas engagement (0 < \(N_g < N_b\)). However, loading and transition regimes (Figure 3) represent an initiation of engagement of gas in the liquid phase, as indirectly evidenced by a decrease in \(P_g/P_0\) ratios due to increased \(\varepsilon_g\) (\(N_g < N_b < N_b\)). Similarly, the onset of flooding (Figure 5a) is characterized by the initiation of disengagement of gas from the liquid phase, which is indirectly evidenced by an increase in power ratios that tend to increase and ultimately reach closer to 1 (\(N_b > N_b\)). The most preferred regime of operations being the “total recirculation regime” is characterized by intimate G–L contacting (Figure 4a) that leads to enhanced mass transfer rates (indirectly indicated by minimum \(P_g/P_0\) \(N_g < N_b < N_b\)). As a supporting strategy, the ratios of the dissolved oxygen concentration to its maximum observed value (DO/DO\(_{\text{max}}\)) profiles for DT are also plotted in Figure 4b confirming the hypothesis.

2.1. Identification of Hydrodynamic Regimes. Considering \(\varepsilon_g\) as a function of \(P_g\) and \(u_1\) based on eqs 2 and 3 and \(P_g/P_0\) in turn, being a function of \((1 - \varepsilon_g)\) as shown in eq 6, it may be argued that for a given impeller–sparger–vessel geometry, \(P_g/P_0\) is dependent on \(N_g\) and \(Q_g\). It is important to note that
the dependence of $\varepsilon_g$ on $N_0$ and $u_i$ has been studied and reported in the literature.\textsuperscript{1} A generic equation depicting the exponent of $\sim 2\sim 3$ for $N_0$ and $\sim$ less than 0.29 for $u_i$ shows the dependence being clearly dominated by $N_0$.\textsuperscript{1} Hence, it is logical to conclude that hydrodynamic regimes are predominantly decided by $N_0$ ranges.

Accordingly, data were collected to study the effects of $N_0$ in the range of 1.67–50 rps and $Q_k$ in the range of 2.5–40 LPM or 4.17–66.7 × 10$^{-5}$ m$^3$ s$^{-1}$ at $\theta = 303$ K under atmospheric pressure on $P_y/P_l$. The $P_y/P_l$ profiles for all the impeller–sparger geometry combinations were analyzed over the $N_0$ and $Q_k$ ranges to identify bypass, loading, transition, total recirculation, and flooding hydrodynamic regimes based on the combined approach involving our understanding about hydrodynamic patterns from the literature reported work as elaborated above and\textsuperscript{1,11,12,16,18,19} the graphical concepts\textsuperscript{11,20} along with the theoretical estimations as suggested by Nienow, cited in the work of Kapic and Hiendel (2006)\textsuperscript{8} and Smith and Warmoeskerken\textsuperscript{16,17} for turbine impellers (esp. DT). Supporting DO/DO$\max$ profiles were also obtained to verify the observations further; as a result, the individual $N_0$ regions for each hydrodynamic regime were identified; here, we present the detailed data analysis for the DT impeller type; however, the understanding developed may also be extended to other types of impellers as well.

The regime-wise data analyses were done based on segmental plots and are presented here.

2.2. Bypass Regime ($0 < N_0 < N_B$). The representative hydrodynamic pattern is shown in Figure 1a.\textsuperscript{1} The bypass regime is defined as having negligible $\varepsilon_g$.\textsuperscript{1,11} Hence, correlating this with the $P_y/P_l$ ratio, we have the following relationship.

\[ P_y/P_l \propto (1 - \varepsilon_g) \] as in eq 6; therefore, $P_y/P_l \approx 1$ in the bypass regime.

$P_y/P_l$ being dimensionless and dependent on $N_0$ and $Q_k$, its profile was plotted against the dimensionless aeration number ($N_A = Q_k/N_0D^2$) to determine $N_B$ such that the segmental region 0 to $N_B$ would represent the bypass regime. For a given $Q_k$, $N_0$ was varied till $N_B$ to obtain $N_B$ for the segmental $P_y/P_l$ plot representing the bypass regime. It was repeated for the entire range of $Q_k$ from 2.5 to 40 LPM or 4.17 to 66.7 × 10$^{-5}$ m$^3$ s$^{-1}$. The segmental plot thus obtained for the bypass regime is clearly represented by $P_y/P_l \approx 1$ till 6.67 rps, as shown in Figure 2. In fact, at 6.67 rps, we see the start of perturbation of the $P_y/P_l$ profile values. To test this hypothesis further, the Figure 2 plot was extended till 10 rps; we see a clear drop, that is, $P_y/P_l$ profile < 1 indicating that $N_B$’s limiting value is lower than 10 rps (appears to be 6.67 rps). As a cross-check, an attempt was made to theoretically estimate the limiting $N_B^{\max}$ value for the $Q_k$ range studied using Nienow’s\textsuperscript{8} formula in eq 10 which was found also to be approximately 6.67 rps (lower than 10 rps, supporting our hypothesis). Thus, we conclude that, for our set of experiments and what appears from theoretical calculations and also for all practical purposes, 0–6.67 rps represents the bypass regime (i.e., $N_0 = N_B = N_B^{\max}$).

\[
N_B^{\max} = \frac{Q_k}{D^2} \left( \frac{g}{30} \right)^{1/3} \left( \frac{T}{D} \right)^{5/3}
\]

(10)

2.3. Loading Regime ($N_0 < N_0 < N_L$). As $N_0$ increases further beyond $N_B$, the gas starts dispersing in the liquid phase due to the impeller action. Although the gas is successfully dispersed by the impeller, it is still poorly distributed throughout the vessel in the loading regime however, which indicates that the $P_y/P_l$ profile should start to decrease from one.\textsuperscript{16} The representative hydrodynamic pattern is shown in Figure 1b.\textsuperscript{1} The segmental region $N_B$ to $N_L$ represents the loading regime. The corresponding segmental plot is shown in Figure 3, which shows the decreasing tendency of $P_y/P_l$ as expected. Using the $Q_k$ range of 2.5–40 LPM or 4.17–66.7 × 10$^{-5}$ m$^3$ s$^{-1}$ in our studies, as a cross-check, an attempt was made to theoretically estimate the limiting value of $N_L^{\max}$ which was determined to be approximately 10 rps using the relationship proposed by Nienow\textsuperscript{8} in eq 11.

\[
N_L^{\max} = 2.94 \left( \frac{Q_k}{D^2} \right)^{2/3} \left( \frac{g}{30} \right)^{1/3} \left( \frac{T}{D} \right)^{1/3}
\]

(11)

Considering that $N_B$ is 6.67 rps (previous subsection), we conclude that for our set of experiments and as supported also by theoretical calculations, the range 6.67–10 rps represents the loading regime (i.e., $N_0 = N_L = N_L^{\max}$).

2.4. Transition Regime ($N_L < N_0 < N_T$). In the transition regime, the system becomes significantly oxygenated. The gas flow widens its path and spreads throughout the vessel, which results in an increased contact between the gas and liquid phases and leads to an increased distribution of bubbles. Circulation loops also begin to form partially.\textsuperscript{17} The representative hydrodynamic pattern is shown in Figure 1c.\textsuperscript{1} Logically, it is expected that the $P_y/P_l$ profile should start to decrease further as compared to the loading regime. Estimations of the operating range for this regime are indirect, assumed to begin at the end of the loading regime (i.e., $N_0 = N_L = N_L^{\max}$) and end with the start of the total recirculation regime (i.e., $N_0 = N_R = N_R^{\max}$). Hence, the transition regime

Figure 4. (a) Effect of $N_A$ on segmental $P_y/P_l$ profile for the identification of the re-circulation regime. The points represent experimental data. (b) Effect $N_0$ on the DO/DO$\max$ profile at $Q_k = 7.5$ LPM for the DT impeller using gas with oxygen composition ranging from 21 to 100 vol % for the identification of the recirculation regime. The points represent the experimental data.

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ranges between \( N_0 \) values from \( N_e \) up to \( N_F \). As a cross-check, using the theoretical calculations, the limiting value of the total recirculation regime \( N_{F_{\text{max}}} \) (mentioned in the next subsection) is estimated for the \( Q_e \) range studied. It is therefore understood that the transition regime ends at \( \sim 12.53 \) rps, which also marks the beginning of the subsequent total recirculation regime. As the loading regime exists between 6.67 and 10.00 rps, we suggest that the transition regime starts above 10.00 rps. Therefore, we conclude that the transition regime exists between 10.00 and 12.53 rps. The segmental plot representing the combined loading and transition regimes \( (N_R \text{ to } N_F) \) obtained using the graphical plot is shown in Figure 3. Here, the \( P_g/P_l \) ratio is <1 showing the expected gradual buildup of \( \varepsilon_g \) \((\propto 1 - P_g/P_l)\) in the range of 6.67–11.67 rps, which is just below the theoretically calculated value of 12.53 rps \( (i.e., \ N_0 = N_R = N_{F_{\text{max}}} \) based on eq 12) supporting the hypothesis of an expected increase in both the dispersion during the loading regime and distribution during the transition regime resulting in an increase in \( \varepsilon_g \).

2.5. Total Recirculation Regime \( (N_F < N_0 < N_e) \). The system is oxygenated and uniformly distributed throughout the vessel. Circulation loops are well-developed, and the gas recirculates very well throughout the entire vessel. As \( N_F \) is high, the impeller dominates the effect of gas sparging.\(^{16,18}\) In this regime, circulation loops are developed fully and are responsible for intimate G–L contacting, as also indicated in Figure 1d.\(^1\) We hypothesize that the maximum mass transfer between the gas and liquid phases occurs in this regime with minimum power consumption, also confirmed by minimum \( P_g/P_l \) values in the segmental plot shown in Figure 4a. We have cross-checked the theoretical limiting value of \( N_R \) for the \( Q_e \) range studied as a value that represents the start of the total recirculation regime using eq 12. From our experiments, in \( Q_e \) ranging from 2.5 to 40 LPM or 4.17 to 66.7 \( \times 10^{-3} \) m\(^3\) s\(^{-1}\), we found, using eq 12,\(^6\) that above 12.53 rps, that is, 13.36 rps, in our case, marks the beginning of the total recirculation regime \( (N_F) \). Also, we see from the calculations in the next subsection that the onset of flooding starts from approximately 21.72 rps \( (\text{limiting } N_{F_{\text{max}}} \text{ value}) \), which indicates that the total recirculation regime ranges from \( N_F \) to \( N_e \) \((i.e., 13.36–21.72 \text{ rps}) \) for our experimental conditions as shown in Figure 4a.

\[
N_{F_{\text{max}}} = 0.6 \left( \frac{Q_e}{D^3} \right)^{1/5} \left( \frac{g}{D} \right)^{2/5} \left( \frac{T}{D} \right)
\]  

(12)

To verify these findings, we also tracked the profile of the \((\text{DO/DO}_{\text{max}})\) ratios for DT over the \( N_e \) range at a fixed \( Q_e \) of 7.5 LPM, as shown in Figure 4b. The profiles were collected for different gas compositions ranging 21–100 vol % oxygen. It can be seen that \( \text{DO/DO}_{\text{max}} = 1 \) is achieved in the region starting from 11.67 rps and remains till 26.67 rps, which is quite similar for the \( P_g/P_l \) profile range \( (13.36 < N_0 < 21.72 \text{ rps}) \) in Figure 4a. We purposely selected \( N_F \) as 21.72 rps \((\text{minimums of 21.72 and 26.67 rps}) \) so that we are sure of not interfering with the onset of the flooding regime while selecting the total recirculation regime. The upper limit of \( \text{DO/DO}_{\text{max}} = 1 \) at the recirculation regime, irrespective of gas composition, is indicative of intimate G–L contact, leading to a maximum concentration of oxygen in the liquid phase. Taking all these together, these data indicate that the total recirculation regime occurs between 13.36 and 21.72 rps. Therefore, kinetic studies should ideally be carried out in the total recirculation regime with its minimum mass transfer resistances and similarly with minimum power consumption.\(^{16,19}\)

2.6. Onset of Flooding \( (N_0 > N_F) \). An additional increase in \( N_0 \) usually is accompanied by a surge/jump in power consumption due to the onset of flooding.\(^{16}\) Trends in the segmental plot starting from 23.38 rps \((\text{above 21.72 rps})\), shown in Figure 5a indicate the surge/rise in the \( P_g/P_l \) ratio to 0.95 or higher \((\text{as compared to 0.8 as seen previously})\), indicating the expected initiation of gas disengagement. Furthermore, the \( \text{DO/DO}_{\text{max}} \) ratios in Figure 4b also show a decline after the total recirculation regime confirming the initiation of disengagement of gas from the liquid phase responsible for the expected reduction in oxygen transfer rates, supporting our hypothesis.\(^{19}\)

As a cross-check, supporting theoretical estimates were also made. Using eq 13 by Smith and Warmoeskerken,\(^6,17\) we estimated the limiting value of \( \text{DO/DO}_{\text{max}} \) \((\text{for our } Q_e \text{ range}) \). Calculations show that the onset of flooding is approximately at 21.72 rps \((\text{limiting } N_{F_{\text{max}}} \text{ value}) \) supporting our hypothesis.

\[
N_{F_{\text{max}}} = 263.16 \left( \frac{Q_e}{D^3} \right) \left( \frac{\mu^2}{g} \right) \left( \frac{D}{T} \right)^{0.5}
\]  

(13)

Taking together, these data indicate that the onset of the flooding regime occurs at 21.72 rps. Hence, the region above \( N_F \) may be considered as flooding. Furthermore, a conﬁrmatory \( P_g/P_l \) profile in Figure 5b beyond 37.5 rps also conﬁrms that it resembles the proﬁle of the bypass regime, indicating signiﬁcant disengagement of the gas from the liquid phase due to the developed flooding regime.

2.7. Correlation Development. Detailed experimentation was carried out using FBT, DT, pitched blade turbine (PBT), and marine impeller (MI) with different ring sparger sizes over a range of \( N_F \) and \( Q_e \) as shown in Table 1a–d and 3b, to study
Table 1. Gas Holdup ($\varepsilon_g$) and Gassed to Ungassed Power Ratio ($P_g/P_i$) Data for a Oxygen (g)–Water (l) System Using (a) FBT, (b) DT, (c) PBT, and (d) MI Impeller at Temperature $\theta = 313$ K and Pressure $p = 0.101$ MPa$\,a,b,c,d$

| s no. | impeller | $N_o$ (rps) | $D_i$ (m) | $Q_g$ (LPM) | $\varepsilon_g$ (%) | $1 - P_g/P_i$ |
|-------|----------|-------------|-----------|-------------|---------------------|--------------|
| 1     | FBT      | 1.67        | 0.037     | 3.75        | 0.0                  | 0.00         |
| 2     | FBT      | 3.33        | 0.037     | 3.75        | 0.0                  | 0.00         |
| 3     | FBT      | 5.01        | 0.037     | 3.75        | 0.0                  | 0.00         |
| 4     | FBT      | 6.67        | 0.037     | 3.75        | 0.0                  | 0.00         |
| 5     | FBT      | 8.33        | 0.037     | 3.75        | 0.0                  | 0.00         |
| 6     | FBT      | 10.00       | 0.037     | 3.75        | 0.4                  | 0.03         |
| 7     | FBT      | 11.67       | 0.037     | 3.75        | 0.8                  | 0.05         |
| 8     | FBT      | 13.33       | 0.037     | 3.75        | 1.2                  | 0.07         |
| 9     | FBT      | 15.00       | 0.037     | 3.75        | 2.0                  | 0.08         |
| 10    | FBT      | 16.67       | 0.037     | 3.75        | 2.2                  | 0.09         |
| 11    | FBT      | 1.67        | 0.037     | 7.50        | 0.0                  | 0.00         |
| 12    | FBT      | 3.33        | 0.037     | 7.50        | 0.0                  | 0.00         |
| 13    | FBT      | 5.01        | 0.037     | 7.50        | 0.4                  | 0.04         |
| 14    | FBT      | 6.67        | 0.037     | 7.50        | 0.8                  | 0.05         |
| 15    | FBT      | 8.33        | 0.037     | 7.50        | 1.6                  | 0.08         |
| 16    | FBT      | 10.00       | 0.037     | 7.50        | 2.4                  | 0.10         |
| 17    | FBT      | 11.67       | 0.037     | 7.50        | 3.6                  | 0.13         |
| 18    | FBT      | 13.33       | 0.037     | 7.50        | 4.8                  | 0.15         |
| 19    | FBT      | 15.00       | 0.037     | 7.50        | 5.5                  | 0.16         |
| 20    | FBT      | 16.67       | 0.037     | 7.50        | 6.6                  | 0.16         |
| 21    | FBT      | 1.67        | 0.066     | 15.0        | 0.0                  | 0.00         |
| 22    | FBT      | 3.33        | 0.066     | 15.0        | 0.0                  | 0.00         |
| 23    | FBT      | 5.01        | 0.066     | 15.0        | 0.4                  | 0.04         |
| 24    | FBT      | 6.67        | 0.066     | 15.0        | 0.8                  | 0.05         |
| 25    | FBT      | 8.33        | 0.066     | 15.0        | 1.6                  | 0.07         |
| 26    | FBT      | 10.00       | 0.066     | 15.0        | 2.4                  | 0.09         |
| 27    | FBT      | 11.67       | 0.066     | 15.0        | 3.6                  | 0.11         |
| 28    | FBT      | 13.33       | 0.066     | 15.0        | 4.8                  | 0.13         |
| 29    | FBT      | 15.00       | 0.066     | 15.0        | 5.9                  | 0.17         |
| 30    | FBT      | 16.67       | 0.066     | 15.0        | 7.0                  | 0.17         |
| 31    | FBT      | 1.67        | 0.116     | 15.0        | 0.0                  | 0.00         |
| 32    | FBT      | 3.33        | 0.116     | 15.0        | 0.0                  | 0.00         |
| 33    | FBT      | 5.01        | 0.116     | 15.0        | 0.4                  | 0.04         |
| 34    | FBT      | 6.67        | 0.116     | 15.0        | 0.8                  | 0.05         |
| 35    | FBT      | 8.33        | 0.116     | 15.0        | 1.6                  | 0.07         |
| 36    | FBT      | 10.00       | 0.116     | 15.0        | 2.4                  | 0.09         |
| 37    | FBT      | 11.67       | 0.116     | 15.0        | 3.6                  | 0.11         |
| 38    | FBT      | 13.33       | 0.116     | 15.0        | 4.8                  | 0.13         |
| 39    | FBT      | 15.00       | 0.116     | 15.0        | 5.9                  | 0.17         |
| 40    | FBT      | 16.67       | 0.116     | 15.0        | 7.0                  | 0.17         |
| 41    | FBT      | 1.67        | 0.166     | 15.0        | 0.0                  | 0.00         |
| 42    | FBT      | 3.33        | 0.166     | 15.0        | 0.0                  | 0.00         |
| 43    | FBT      | 5.01        | 0.166     | 15.0        | 0.4                  | 0.04         |
| 44    | FBT      | 6.67        | 0.166     | 15.0        | 0.8                  | 0.05         |
| 45    | FBT      | 8.33        | 0.166     | 15.0        | 1.2                  | 0.05         |
| 46    | FBT      | 10.00       | 0.166     | 15.0        | 2.4                  | 0.08         |
| 47    | FBT      | 11.67       | 0.166     | 15.0        | 3.6                  | 0.12         |
| 48    | FBT      | 13.33       | 0.166     | 15.0        | 4.8                  | 0.14         |
| 49    | FBT      | 15.00       | 0.166     | 15.0        | 5.9                  | 0.15         |
| 50    | FBT      | 16.67       | 0.166     | 15.0        | 6.6                  | 0.17         |
| 51    | FBT      | 1.67        | 0.200     | 15.0        | 0.0                  | 0.00         |
| 52    | FBT      | 3.33        | 0.200     | 15.0        | 0.0                  | 0.00         |
| 53    | FBT      | 5.01        | 0.200     | 15.0        | 0.4                  | 0.04         |
| 54    | FBT      | 6.67        | 0.200     | 15.0        | 0.8                  | 0.05         |
| 55    | FBT      | 8.33        | 0.200     | 15.0        | 1.2                  | 0.06         |
| 56    | FBT      | 10.00       | 0.200     | 15.0        | 2.4                  | 0.08         |
| 57    | FBT      | 11.67       | 0.200     | 15.0        | 3.6                  | 0.10         |
| 58    | FBT      | 13.33       | 0.200     | 15.0        | 4.8                  | 0.12         |
| 59    | FBT      | 15.00       | 0.200     | 15.0        | 5.9                  | 0.14         |

Note: $\varepsilon_g$ is the gas holdup, $P_g$ is the gassed power, and $P_i$ is the ungassed power.
### Table 1. continued

| s no. | impeller | \( N_0 \) (rps) | \( D_i \) (m) | \( Q_g \) (LPM) | \( \varepsilon_g \) (%) | \( 1 - P_g/P_l \) |
|-------|---------|----------------|--------------|----------------|-------------------|-----------------|
| 58    | DT      | 13.33          | 0.166        | 3.75           | 3.75              | 0.0             |
| 59    | DT      | 15.00          | 0.166        | 3.75           | 3.75              | 0.4             |
| 60    | DT      | 16.67          | 0.166        | 3.75           | 3.75              | 1.0             |
| 61    | DT      | 1.67           | 0.037        | 3.75           | 3.75              | 0.0             |
| 62    | DT      | 3.33           | 0.037        | 3.75           | 3.75              | 0.0             |
| 63    | DT      | 5.01           | 0.037        | 3.75           | 3.75              | 0.0             |
| 64    | DT      | 6.67           | 0.037        | 3.75           | 3.75              | 0.0             |
| 65    | DT      | 8.33           | 0.037        | 3.75           | 3.75              | 0.0             |
| 66    | DT      | 10.00          | 0.037        | 3.75           | 3.75              | 0.0             |
| 67    | DT      | 11.67          | 0.037        | 3.75           | 3.75              | 0.0             |
| 68    | DT      | 13.33          | 0.037        | 3.75           | 3.75              | 0.0             |
| 69    | DT      | 15.00          | 0.037        | 3.75           | 3.75              | 0.0             |
| 70    | DT      | 16.67          | 0.037        | 3.75           | 3.75              | 0.0             |
| 71    | DT      | 1.67           | 0.066        | 7.50           | 7.50              | 0.0             |
| 72    | DT      | 3.33           | 0.066        | 7.50           | 7.50              | 0.0             |
| 73    | DT      | 5.01           | 0.066        | 7.50           | 7.50              | 0.0             |
| 74    | DT      | 6.67           | 0.066        | 7.50           | 7.50              | 0.0             |
| 75    | DT      | 8.33           | 0.066        | 7.50           | 7.50              | 0.0             |
| 76    | DT      | 10.00          | 0.066        | 7.50           | 7.50              | 0.0             |
| 77    | DT      | 11.67          | 0.066        | 7.50           | 7.50              | 0.0             |
| 78    | DT      | 13.33          | 0.066        | 7.50           | 7.50              | 0.0             |
| 79    | DT      | 15.00          | 0.066        | 7.50           | 7.50              | 0.0             |
| 80    | DT      | 16.67          | 0.066        | 7.50           | 7.50              | 0.0             |
| 81    | DT      | 1.67           | 0.066        | 15.0           | 15.0              | 0.0             |
| 82    | DT      | 3.33           | 0.066        | 15.0           | 15.0              | 0.0             |
| 83    | DT      | 5.01           | 0.066        | 15.0           | 15.0              | 0.0             |
| 84    | DT      | 6.67           | 0.066        | 15.0           | 15.0              | 0.0             |
| 85    | DT      | 8.33           | 0.066        | 15.0           | 15.0              | 0.0             |
| 86    | DT      | 10.00          | 0.066        | 15.0           | 15.0              | 0.0             |
| 87    | DT      | 11.67          | 0.066        | 15.0           | 15.0              | 0.0             |
| 88    | DT      | 13.33          | 0.066        | 15.0           | 15.0              | 0.0             |
| 89    | DT      | 15.00          | 0.066        | 15.0           | 15.0              | 0.0             |
| 90    | DT      | 16.67          | 0.066        | 15.0           | 15.0              | 0.0             |
| 91    | DT      | 1.67           | 0.066        | 3.75           | 3.75              | 0.0             |
| 92    | DT      | 3.33           | 0.066        | 3.75           | 3.75              | 0.0             |
| 93    | DT      | 5.01           | 0.066        | 3.75           | 3.75              | 0.0             |
| 94    | DT      | 6.67           | 0.066        | 3.75           | 3.75              | 0.0             |
| 95    | DT      | 8.33           | 0.066        | 3.75           | 3.75              | 0.0             |
| 96    | DT      | 10.00          | 0.066        | 3.75           | 3.75              | 0.0             |
| 97    | DT      | 11.67          | 0.066        | 3.75           | 3.75              | 0.0             |
| 98    | DT      | 13.33          | 0.066        | 3.75           | 3.75              | 0.0             |
| 99    | DT      | 15.00          | 0.066        | 3.75           | 3.75              | 0.0             |
| 100   | DT      | 16.67          | 0.066        | 3.75           | 3.75              | 0.0             |

**Notes:**
- \( N_0 \): speed of the impeller in revolutions per second (rps).
- \( D_i \): diameter of the impeller in meters (m).
- \( Q_g \): gas flow rate in liters per minute (LPM).
- \( \varepsilon_g \): gas holdup in percent (\%)
- \( 1 - P_g/P_l \): gas-to-liquid ratio.
| s no. | impeller | $N_0$ (rps) | $D_1$ (m) | $Q_0$ (LPM) | $c_ε$ (%) | $1 - P_ε/P_l$ |
|-------|----------|-------------|-----------|------------|------------|----------------|
| 37    | PBT      | 11.67       | 0.066     | 15.0       | 2.8        | 0.11           |
| 38    | PBT      | 13.33       | 0.066     | 15.0       | 3.2        | 0.13           |
| 39    | PBT      | 15.00       | 0.066     | 15.0       | 3.6        | 0.14           |
| 40    | PBT      | 16.67       | 0.066     | 15.0       | 4.0        | 0.14           |
| 41    | PBT      | 16.67       | 0.116     | 15.0       | 0.0        | 0.00           |
| 42    | PBT      | 3.33        | 0.116     | 15.0       | 0.0        | 0.00           |
| 43    | PBT      | 5.01        | 0.116     | 15.0       | 0.0        | 0.00           |
| 44    | PBT      | 6.67        | 0.116     | 15.0       | 0.0        | 0.00           |
| 45    | PBT      | 8.33        | 0.116     | 15.0       | 0.0        | 0.00           |
| 46    | PBT      | 10.00       | 0.116     | 15.0       | 0.0        | 0.00           |
| 47    | PBT      | 11.7        | 0.116     | 15.0       | 0.0        | 0.00           |
| 48    | PBT      | 13.33       | 0.116     | 15.0       | 0.0        | 0.00           |
| 49    | PBT      | 15.00       | 0.116     | 15.0       | 0.0        | 0.00           |
| 50    | PBT      | 16.67       | 0.116     | 15.0       | 1.2        | 0.06           |
| 51    | PBT      | 15.00       | 0.116     | 15.0       | 0.0        | 0.00           |
| 52    | PBT      | 3.33        | 0.116     | 15.0       | 0.0        | 0.00           |
| 53    | PBT      | 5.01        | 0.116     | 15.0       | 0.0        | 0.00           |
| 54    | PBT      | 6.67        | 0.116     | 15.0       | 0.0        | 0.00           |
| 55    | PBT      | 8.33        | 0.116     | 15.0       | 0.0        | 0.00           |
| 56    | PBT      | 10.00       | 0.116     | 15.0       | 0.0        | 0.00           |
| 57    | PBT      | 11.67       | 0.116     | 15.0       | 0.0        | 0.00           |
| 58    | PBT      | 13.33       | 0.116     | 15.0       | 0.0        | 0.00           |
| 59    | PBT      | 15.00       | 0.116     | 15.0       | 0.0        | 0.00           |
| 60    | PBT      | 16.67       | 0.116     | 15.0       | 0.0        | 0.00           |
| 61    | PBT      | 15.00       | 0.116     | 15.0       | 0.0        | 0.00           |
| 62    | PBT      | 3.33        | 0.116     | 15.0       | 0.0        | 0.00           |
| 63    | PBT      | 5.01        | 0.116     | 15.0       | 0.0        | 0.00           |
| 64    | PBT      | 6.67        | 0.116     | 15.0       | 0.0        | 0.00           |
| 65    | PBT      | 8.33        | 0.116     | 15.0       | 0.0        | 0.00           |
| 66    | PBT      | 10.00       | 0.116     | 15.0       | 0.0        | 0.00           |
| 67    | PBT      | 11.67       | 0.116     | 15.0       | 0.0        | 0.00           |
| 68    | PBT      | 13.33       | 0.116     | 15.0       | 0.0        | 0.00           |
| 69    | PBT      | 15.00       | 0.116     | 15.0       | 0.0        | 0.00           |
| 70    | PBT      | 16.67       | 0.116     | 15.0       | 0.0        | 0.00           |
| 71    | PBT      | 15.00       | 0.116     | 15.0       | 0.0        | 0.00           |
| 72    | PBT      | 3.33        | 0.116     | 15.0       | 0.0        | 0.00           |
| 73    | PBT      | 5.01        | 0.116     | 15.0       | 0.0        | 0.00           |
| 74    | PBT      | 6.67        | 0.116     | 15.0       | 0.0        | 0.00           |
| 75    | PBT      | 8.33        | 0.116     | 15.0       | 0.0        | 0.00           |
| 76    | PBT      | 10.00       | 0.116     | 15.0       | 0.0        | 0.00           |
| 77    | PBT      | 11.67       | 0.116     | 15.0       | 0.0        | 0.00           |
| 78    | PBT      | 13.33       | 0.116     | 15.0       | 0.0        | 0.00           |
| 79    | PBT      | 15.00       | 0.116     | 15.0       | 0.0        | 0.00           |
| 80    | PBT      | 16.67       | 0.116     | 15.0       | 0.0        | 0.00           |

| s no. | impeller | $N_0$ (rps) | $D_1$ (m) | $Q_0$ (LPM) | $c_ε$ (%) | $1 - P_ε/P_l$ |
|-------|----------|-------------|-----------|------------|------------|----------------|
| 1     | MI       | 1.67        | 0.037     | 3.75       | 0.0        | 0.00           |
| 2     | MI       | 3.33        | 0.037     | 3.75       | 0.0        | 0.00           |
| 3     | MI       | 5.01        | 0.037     | 3.75       | 0.0        | 0.00           |
| 4     | MI       | 6.67        | 0.037     | 3.75       | 0.0        | 0.00           |
| 5     | MI       | 8.33        | 0.037     | 3.75       | 0.0        | 0.00           |
| 6     | MI       | 10.00       | 0.037     | 3.75       | 0.0        | 0.00           |
| 7     | MI       | 11.67       | 0.037     | 3.75       | 0.0        | 0.00           |
| 8     | MI       | 13.33       | 0.037     | 3.75       | 0.0        | 0.00           |
| 9     | MI       | 15.00       | 0.037     | 3.75       | 0.0        | 0.00           |
| 10    | MI       | 16.67       | 0.037     | 3.75       | 0.0        | 0.00           |
| 11    | MI       | 15.00       | 0.037     | 3.75       | 0.0        | 0.00           |
| 12    | MI       | 3.33        | 0.037     | 3.75       | 0.0        | 0.00           |
| 13    | MI       | 5.01        | 0.037     | 3.75       | 0.0        | 0.00           |
| 14    | MI       | 6.67        | 0.037     | 3.75       | 0.0        | 0.00           |
| 15    | MI       | 8.33        | 0.037     | 3.75       | 0.0        | 0.00           |

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Table 1. continued

| s no. | impeller | \( N_0 \) (rps) | \( D_i \) (m) | \( Q_s \) (LPM) | \( \varepsilon_s \) (%) | \( 1 - P_s / P_i \) |
|-------|----------|-----------------|--------------|----------------|------------------------|------------------|
| 76    | MI       | 10.00           | 0.166        | 15.0           | 0.8                    | 0.07             |
| 77    | MI       | 11.67           | 0.166        | 15.0           | 0.8                    | 0.08             |
| 78    | MI       | 13.33           | 0.166        | 15.0           | 1.2                    | 0.09             |
| 79    | MI       | 15.00           | 0.166        | 15.0           | 1.6                    | 0.11             |
| 80    | MI       | 16.67           | 0.166        | 15.0           | 2.4                    | 0.12             |

Table 2. Summary of RMSD/%

| author(s) | expression | FBT | DT | PBT | MI | overall |
|-----------|------------|-----|----|-----|----|---------|
| Foust et al. (1944) | \( \varepsilon_g = \frac{\phi}{1 + \phi} \) \[ \phi = 1.54 \left( \frac{P_i}{V_i} \right) ^{0.45} \] u_s | 6.23 | 8.97 | 7.36 | 7.20 | 7.31 |
| Calderbank (1958) | \( \varepsilon_g = \left( \frac{u_s}{u_i} \right) ^{0.5} + 0.0216 \left[ \left( \frac{u_s}{u_i} \right) ^{0.4} \rho_c^{0.2} \sigma^{0.6} \right] \) | 4.21 | 4.89 | 5.10 | 4.72 | 4.67 |
| Hassan & Robinson (1977) | \( \varepsilon_g = C_2 \left( \frac{Q_s N_i^2}{\sigma} \right) \) ; \( C_2 = 0.111 ; z = 0.57 \) | 1.36 | 3.41 | 3.49 | 3.73 | 2.81 |
| Loiseau et al. (1977) | \( \varepsilon_g = 0.011 u_s \left[ \frac{0.35}{u_i} - 0.56 \mu - 0.06 \left( \frac{P_i}{V_i} + \frac{P_i}{V_c} \right) ^{0.27} \right] \) | 6.01 | 6.60 | 6.85 | 6.88 | 6.52 |
| Hughmark (1980) | \( \varepsilon_g = 0.74 \left[ \frac{Q_s}{N_i V_i} \right] ^{1/2} \left( \frac{N_i D^4}{g D V_i 2.75} \right) ^{1/2} \left( \frac{d_s N_i D^4}{\sigma V_i 2.75} \right) \) | 1.13 | 2.20 | 1.54 | 1.40 | 1.52 |
| Stenberg & Andersson (1988) | \( \varepsilon_g = \left[ \beta_i \frac{P_i}{V_i} \right] ^{0.5} ; \beta_i = 1.98 ; \beta_1 = 40.9 ; \beta_1 = 0.21 \) | 2.73 | 2.79 | 2.70 | 3.05 | 2.87 |
| Paglianti et al. (2000) | \( \varepsilon_g = 0.31 \left( \frac{\mu N_i}{g \sigma} \right) ^{0.5} \left( \frac{\mu N_i}{g \sigma} \right ) ^{0.5} + 0.45 \left( \frac{N_i - N_i \text{Nd}}{\sqrt{T}} \right) \left( \frac{D}{T} \right) \) | 3.26 | 6.15 | 6.20 | 6.15 | 5.20 |
| Average | \( \varepsilon_g^{\text{norm}} = 100 \left( 0.90 \left( 1 - \frac{P_s}{P_i} \right) ^2 + 0.036 \left( 1 - \frac{P_s}{P_i} \right) \right) \) | 3.56 | 5.00 | 4.79 | 4.74 | 4.42 |

Table 1. continued

The effect of the impeller and sparger geometry on \( \varepsilon_s \) was varied in the range of 3.75–15 LPM or 4.17–66.7 \times 10^{-5} \text{ m}^3 \text{ s}^{-1}, and \( N_0 \) was varied in the range of 1.67–33.4 rps at \( \theta = 313 \) K under atmospheric pressure. The calculations and trends in Figure 4a,b indicate that the total recirculation regime existed between 13.36 and 21.72 rps. To ensure that the \( \varepsilon_s \) correlation was limited to the total recirculation regime, we strictly considered data up to 16.70 rps (within the range of 13.36–21.72 rps) for the correlation development, that is, bypass, loading, transition, and total recirculation regimes.

Three hundred and sixty experimental data points were analyzed (FBT (120); DT (80); PBT (80); and MI (80)) to develop the proposed correlation. The experimental \( \varepsilon_s \) values obtained using FBT, DT, PBT, and MI, in combination with various ring sparger sizes, are reported in Table 1a-d. \( \varepsilon_s \) was also predicted by using the literature-reported correlations,\(^{3,5,7,8-12}\) and their extents of fit are shown in Table 2. The extent of fit is expressed in terms of root mean square deviation (rmsd), as defined in eq 14, which ranged from (1.25 to 8.97)%. The comparatively lower RMSD values shown in Table 2 for the FBT cases maybe because the original correlation development studies used FBT/PD type impellers.

This demonstrates that the geometry of an impeller plays an important role in \( \varepsilon_s \) calculations. Likewise, the sparger geometry likely plays a significant role. It was necessary to develop a unified approach for making the correlations less dependent on the impeller–sparger geometry. Accordingly, we propose a normalized \( \varepsilon_s \) correlation, which is the impeller–sparger geometry nonspecific for the oxygen–water system, using a single operating parameter (i.e., \( (1 - P_s / P_i) \)). To incorporate the effects of the impeller and sparger geometry, two geometrical features were incorporated: impeller power number \( (N_{Re}) \) and \( \Delta_s \). \( N_{Re} \) may be considered as a constant for the cases in which Reynold’s number \( (N_{Re}) > 10,000\).\(^{6}\)

\[
N_{Re} = \frac{\rho c N_i D^2}{\mu}
\]

(14)

Calculations of \( N_{Re} \) (eq 14) indicate that for \( N_0 > 1.29 \text{ rps} \), \( N_{Re} > 10,000 \). The data range for \( N_0 \) in our study starts from 1.67 rps, as shown in Table 1a-d. Hence, \( N_{Re} \) may be treated as a constant (6.1 for FBT; 4.1 for DT; 1.8 for PBT; and 0.9 for MI) for a practical range of operations for correlation development. Equation 16 depicts the formulation for the generic dependence of (1) the impeller and sparger geometry...
and (2) the operational parameter on \( \epsilon_g \). An effort was made to rationalize the correlation on \( \epsilon_g \) using the effects of both types of the impeller (radial and axial) and \( \Delta_s \).

Different forms of functions were considered for correlation development (e.g., linear, polynomial, or quadratic types). The objective error function [relative rmsd (RRMSD)] expressed in eq 15 was minimized using combinations of goal seek and solver functions of Microsoft Excel, and the corresponding parametric values for the exponents \( \alpha \) and \( \beta \) were estimated.

\[
RRMSD = 100 \left( \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\epsilon_{g,\text{exp}} - \epsilon_{g,\text{pred}}}{\epsilon_{g,\text{exp}}} \right)^2 \right)^{1/2}
\]  

(15)

Accordingly, the data in Table 1a–d were used to develop an easy-to-use, normalized \( \epsilon_g^{\text{norm}} \) predictive correlation based on the \((1 - P_g/P_l)\) experimental data shown in Figure 6. Based on overall RRMSD = 25.75% and a coefficient of regression value \( (R^2) = 0.91 \), the quadratic form was found to be the best fit, as shown in eq 16. Parametric values for the exponents \( \alpha \) and \( \beta \) were estimated as 0.281 and –0.106, respectively, and constants \( C_1 \), \( C_2 \), and \( C_3 \) were 100, 0.90, and 0.036, respectively, with an overall RMSD of 0.43%.

\[
\epsilon_g^{\text{norm}} = C_1 \left( C_2 \left( \frac{1 - P_g}{P_l} \right)^2 + C_3 \left( 1 - \frac{P_g}{P_l} \right) \right)
\]  

(16)

\[
\epsilon_g^{\text{norm}} = \frac{\epsilon_g}{(N_0 \alpha \Delta_s \beta)}
\]  

(17)

It is evident from our results that the incorporation of geometrical aspects improves the predictions by significantly reducing the overall RMSD (Table 2) from 4.42 to 0.43% with \( R^2 = 0.91 \) (Figure 6). These improvements in predictions, compared to the published correlations, indicate that our correlation merits consideration as an alternative and more straightforward approach involving fewer operational parameters. Furthermore, the data analysis and regime calculations make the correlation valid for bypass, loading, transition, and total recirculation regimes and hence are expected to be scalable for similar operating hydrodynamic regimes. The usefulness of the generic form of the correlation is depicted by Figure 6, which demonstrates that for a given \((1 - P_g/P_l)\) experimental value, the corresponding \( \epsilon_g^{\text{norm}} \) can be determined from the plot. Depending on the type of impeller and sparger geometry, the \( \epsilon_g \) can then be quickly estimated.

3. CONCLUSIONS

This paper presents an alternative method to predict \( \epsilon_g \) for an oxygen–water system using dimensionless impeller–sparger geometrical parameters for a vessel of a standard geometry and fewer operational parameters. The correlation is valid across few different hydrodynamic regimes. Identification of hydrodynamic regimes using \( P_g/P_l \) profiles, therefore, is also essential.

The overall residual RRMSD of 25.75% (RMSD of 0.43%) for our work may be attributed to the ideality of our assumption for simplifying eq 3, which led to the deviation from nonideality while assuming constant \( t_0 \), \( D_s \) and \( d_s \) over the entire range of studies.

4. EXPERIMENTAL METHODS

4.1. Materials. Oxygen, CAS no 7732-44-7 (O₂, 99.999 v/v %), from M/s Gupta Industrial Gases, India, and nitrogen, CAS no 7727-37-9 (N₂, 99.999 v/v %), from M/s Sigma Gases, India, and high pressure liquid chromatography (HPLC) grade water, CAS no 7732-18-5 (H₂O, 99.980 w/w % with conductivity 0.00 S m⁻¹), from Merck, India, were procured for the experimental studies. These chemicals were used in experimental studies without any further purification.

4.2. Experimental Setup. The experiments were conducted in a jacketed SSTS of 0.25 m (T) with a hemispherical bottom, which was fully baffled (four baffles each 1/10th of T). The \( V_0 \) was 1 × 10⁻³ m³. \( \theta \) was maintained by using a thermostatically controlled water bath to circulate water in the jacket. Four different impellers (FBT, DT, PBT, and MI) were used. The impeller blade dimensions and all other geometrical factors correspond with the standard configurations.6,15 The impellers were located at half the liquid depth (\( H_l \)).15 The \( H_l \) was maintained equal to \( T \).15 A variable speed electric motor drove the impeller shaft, and its power consumption (for both \( P_g \) and \( P_l \)) was recorded using an energy meter. Since the developed correlation is of single parametric dependence, that is, \((1 - P_g/P_l)\), an average of \( P_g/P_l \) from at least three readings (and preferably five) is recommended for better precision. \( Q_g \) of the gases was metered through mass flow controllers (MFCs), in pure or blended forms (as required), and released through a ring sparger system (four different ring sizes) of 30 holes with a \( 2 \times 10^{-3} \) m hole size (four different sizes) located just below the impeller. \( Q_g \) ranged from 2.5 to 40 LPM or 4.17 to 66.7 × 10⁻⁵.
m³ s⁻¹. \( N_o \) ranged from 1.67 to 50 rps. All experiments were conducted at \( \theta = 303−313 \) K under atmospheric pressure.

To aid the studies on hydrodynamic regimes and \( \varepsilon_g \) correlation development, systematic data collection of process parameters like \( \varepsilon_g \), \( P_l \), \( P_g \), \( \theta \), \( p \), and dissolved oxygen concentration (DO) was a part of the experimentation plan.

To study the effect of the impeller−sparger system geometry on \( \varepsilon_g \), a vessel with standard dimensional ratios was selected as described in Figure 7 and Table 3a. Two radial (FBT and DT) and two axial impellers with center downward flow (PBT and MI) were used to make the experiments generalizable.

Ring-type spargers were used because they offer the most uniform and homogenous dispersion of gas in a liquid medium. The dimensional details of the spargers are shown in Table 3b.

4.3. Estimation Methods. The uncertainty of measurement is a parameter associated with the dispersion of the values that could reasonably be attributed to the measurand. The standard uncertainty of a data of a parameter obtained during experiments is expressed as the standard deviation (\( \sigma_D \)). The \( \sigma_D \) was calculated using eq 18.

\[
\sigma_D = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i^{\exp} - x_{\text{mean}})^2}
\]  

(18)

The RMSD from a fitted curve is defined in eq 19. Unusual and abnormal data was omitted during calculations.

\[
\text{RMSD} = \frac{1}{N} \sum_{i=1}^{N} (\varepsilon_i^{\exp} - \varepsilon_i^{\text{pred}})^2
\]  

(19)

To increase in average height above the quiet air-free liquid height is called “gas holdup.” The \( \varepsilon_g \) was calculated using eq 20.

\[
\varepsilon_g = \frac{H_D - H_L}{H_D} \times 100
\]  

(20)

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■ SYMBOLS

\( B_T \), baffle thickness, m  
\( C_i \), impeller clearance, m  
\( C_s \), sparger clearance, m  
\( C_1 \), constant  
\( C_2 \), constant  
\( C_3 \), constant  
\( d_b \), bubble diameter, m  
\( d_s \), sparger hole size, m  
\( D_g \), diffusivity of gas in the liquid, m s⁻²  
\( D_s \), diameter of ring sparger, m
\( D_i \), impeller diameter, m

\( H_{D_{\infty}} \), height of dispersion, m

\( H_L \), height of liquid, m

\( K_{m_{\text{G-L}}} \), overall mass transfer coefficient based on the liquid side, m s\(^{-1}\)

\( O_i \), oxygen concentration at the G–L interface, mol m\(^{-3}\)

\( N_i \), number of data points

\( N_{\text{mb}} \), intensity of mixing, rps

\( N_{\text{mb}} \), upper boundary of the by-pass regime, rps

\( N_{\text{mb}} \), theoretical upper boundary of the by-pass regime for turbine impellers, rps

\( N_{\text{mb}} \), upper boundary of loading regime, rps

\( N_{\text{mb}} \), theoretical upper boundary of loading regime for turbine impellers, rps

\( N_{\text{b}} \), start of the re-circulation regime, rps

\( N_{\text{b}} \), theoretical start of the re-circulation regime for turbine impellers, rps

\( N_{\text{b}} \), start of onset of flooding regime, rps

\( N_{\text{b}} \), theoretical start of onset of flooding regime for turbine impellers, rps

\( \rho \), pressure, Pa

\( P_{\text{O}_2} \), oxygen partial pressure, Pa

\( P_{\text{p}} \), power consumption in the gassed system, W

\( P_{\text{g}} \), power consumption in the un-gassed system, W

\( Q_{\text{p}} \), volumetric gas flow rate, m\(^3\) s\(^{-1}\)

\( R^2 \), coefficient of regression

\( S_{\text{i}} \), interfacial area, m\(^2\) m\(^{-3}\)

\( T_f \), film diameter, m

\( t_f \), film thickness, m

\( u_i \), standard uncertainties

\( u_o \), superficial gas velocity, m s\(^{-1}\)

\( u_b \), bubble rise velocity, m s\(^{-1}\)

\( V_L \), liquid volume, m\(^3\)

\( x_i \), experimental parameter

\( \exp \) experimental

\( i \), \( G–L \) interface

\( \text{pred} \), predicted

\( \text{max} \), maximum

\( \text{un} \), un-gassed

\( \text{g} \), gassed

\( \alpha \), constant exponent of \( N_p \)

\( \beta \), constant exponent of \( \Delta_i \)

\( \varepsilon_{\text{g}} \), gas holdup

\( \varepsilon_{\text{g}} \), normalized gas holdup

\( \mu \), liquid viscosity, kg m s\(^{-1}\)

\( \rho_L \), liquid density, kg m\(^{-3}\)

\( \rho_D \), dispersion density, kg m\(^{-3}\)

\( \sigma \), surface tension, N m\(^{-1}\)

\( \theta \), temperature, K

\( \sigma_{\text{rms}} \), root mean square deviation

\( \Delta_S \), a ratio representing the relative diametrical position/radial distance of the ring spargers with respect to the impeller diameter

\( \text{ABBREVIATIONS} \)

CB, curved blades

DO, dissolved oxygen concentration

DT, disk turbine

\( f \), a function of

FBT, flat blade turbine

G–L, gas–liquid

H\(_2\)O, water

HPLC, high pressure liquid chromatography

LPM, liter per minute

MFC, mass flow controller

MI, marine impeller

\( N_i \), nitrogen

\( N_t \), number of data points

\( O_2 \), oxygen

PBT, pitched blade turbine

PD, paddle blade turbine

\( \text{rmsd} \), root mean square deviation

\( \% \), relative \( \text{rmsd}, \% \)

\( \text{rpm} \), revolutions per sec

SSTC, stirred and sparged tank contactor

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