The Solar X-Ray Limb

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Abstract

We describe a new technique to measure the height of the X-ray limb with observations from occulted X-ray flare sources as observed by the RHESSI (the Reuven Ramaty High-Energy Spectroscopic Imager) satellite. This method has model dependencies different from those present in traditional observations at optical wavelengths, which depend upon detailed modeling involving radiative transfer in a medium with complicated geometry and flows. It thus provides an independent and more rigorous measurement of the “true” solar radius, which means that of the mass distribution. RHESSI’s measurement makes use of the flare X-ray source’s spatial Fourier components (the visibilities), which are sensitive to the presence of the sharp edge at the lower boundary of the occulted source. We have found a suitable flare event for analysis, SOL2011-10-20T03:25 (M1.7), and report a first result from this novel technique here. Using a four-minute integration over the 3–25 keV photon energy range, we find $R_{\odot}\text{lim}=960.11 \pm 0.15 \pm 0.29$ arcsec, at 1 au, where the uncertainties include statistical uncertainties from the method and a systematic error. The standard VAL-C model predicts a value of 959.94 arcsec, which is about 1σ below our value.

Key words: Sun: flares – Sun: fundamental parameters – Sun: X-rays, gamma rays – techniques: miscellaneous

1. Introduction

We normally determine the outer boundary of the Sun with observations at optical wavelengths, essentially making use of the radial distance of the tangent ray for which the optical depth $\tau$ is unity at a standard wavelength, such as 5000 Å. Such a definition depends upon details of the atmospheric structure and of the physics of the radiative transfer (absorption and scattering) within that structure. The standard reference for the value, as cited by Allen (1973), appears to be the fundamental observational work of Auwers (1891), who made extensive visual observations with “heliometers.” These were used to measure the angular separation between the two ends of a given solar diameter. The measurements gave the value $1919.26 \pm 0.10$ arcsec. A more recent standard was set by Brown & Christensen-Dalsgaard (1998), who used timing measurements of meridian transits in a special-purpose telescope operated between 1981 and 1987. This yielded a value of $1919.359 \pm 0.018$ arcsec for the observed near-equatorial diameter. These angular measures for the location of the solar limb need an adjustment to provide estimates of the solar radius itself, expressed in SI units, because the observed limb lies several scale heights above the physical radius. The current best value for the radius itself, incorporating many sources, gives $R_{\odot} = 695.658 \pm 140$ km (Haberreiter et al. 2008). This value has been rounded off and adopted within errors as the IAU nominal value $R_{\odot}^{\text{nom}} = 695.7$ km (Prša et al. 2016) as one of a set of self-consistent values for solar and planetary properties.

What do such measurements mean physically? From the point of view of stellar structure, we may wish to use such a measurement to characterize the spatial distribution of the solar material, and so we would like to interpret the opacity measurement as a radial distance from the Sun center, for example. For reference, the point in a semi-empirical atmospheric model such as VAL-C (Vernazza et al. 1981) corresponding to the nominal radial distance $R_{\odot}$ has an optical depth $\tau_{5000\text{Å}} = 1$ as viewed radially from above. This optical depth conventionally defines the location of the photosphere or the solar radius. According to the VAL-C model, the total mass of the atmosphere above that point is about $10^{-10} M_{\odot}$, and so a sphere at this radial distance from Sun center (i.e., within $R_{\odot}$) contains most of the mass of the Sun. The increase of optical depth for oblique rays means that the observed limb always lies above the conventional photosphere. The difference amounts to several photospheric scale heights (about 350 km; see Brown & Christensen-Dalsgaard 1998; Haberreiter et al. 2008, for a discussion of this point). At longer and shorter wavelengths (relative to the “opacity minimum” at about 1.5 μm), the opacity generally increases, but its spectral behavior is greatly complicated by atomic and molecular transitions. The radial distance of the apparent limb thus generally increases as the opacity increases slowly toward the infrared and more rapidly in the UV.

Because of the presence of flows and oscillations on small scales, the surface of the Sun is not smooth. The height of the limb thus depends on the dynamics of the upper solar atmosphere at a given wavelength, an interesting problem normally described by numerical models and relevant to such matters as the chemical composition (e.g., Asplund et al. 2009). Much of the literature (see, e.g., Rozelot et al. 2015) has been devoted to characterizing the limb and its dynamics at all wavelengths. In this paper, we report a first attempt to make precise measurements of the X-ray limb, by a technique that is more directly related to the mass distribution, rather than to the opacity structure at longer wavelengths. This method is based on locating the sharp lower edge of partially occulted flare sources as observed by the Reuven Ramaty High-Energy Solar Spectroscopic Imager (RHESSI) spacecraft (Lin et al. 2002). The limb thus observed is in absorption, which is due to the photoeffect and Compton scattering. The photoelectric...
absorption, which is important below about 12 keV, depends upon the elemental abundances and the ionization state of the medium. At higher energies, Compton scattering becomes dominant. In the scattering regime, we have a particularly simple interpretation: the absorption ideally just depends upon the column density of electrons, free or bound, along the line of sight. \textit{RHESSI} is well-suited for observing the sharp occulted edge of the source because of its natural interpretation in terms of discrete spatial Fourier components (Hurford et al. 2002), as discussed for this application in Section 2. Note that Wanner et al. (1983) used an analogous technique to study the limb at mm wavelengths. In Section 3, we describe observations of a single partially occulted flare chosen as the best case after extensive searches. The calculation of the limb height and comparison with predictions from models are given in Section 4.

2. Visibility Theory

\textit{RHESSI} is a collimator-based Fourier imager. The incident photon flux from a solar flare is time-modulated by two grids as the spacecraft spins around its axis. \textit{RHESSI} has nine detectors and, accordingly, nine pairs of grids (G1 through G9) with different pitch-widths corresponding to angular resolutions from as small as 2.26 arcsec in the case of G1 up to 183.2 for G9 (Hurford et al. 2002; Lin et al. 2002). Our technique converts the \textit{RHESSI} time series of a given X-ray source into discrete two-dimensional Fourier components \(V(u, v)\) that we call visibilities. The directness of the sensitivity of visibility-based techniques to an occulted source is suggested by the relation between visibilities and the spatial profile of a source. Let \(I(x)\) be the intensity of a two-dimensional source projected onto the \(x\)-axis. Then the relationship between the projected intensity and the set of visibilities \(V(u)\), whose spatial frequencies \(u\) are in the \(x\)-direction, is given by

\[ V(u) = \int I(x) \exp(-2\pi iux) \, dx, \]  

and therefore

\[ I(x) = \int V(u) \exp(2\pi iux) \, du. \]  

This illustrates how the source profile can be reconstructed from a set of visibility measurements. Spatially differentiating Equation (2) yields

\[ \frac{dI(x)}{dx} = 2\pi \int iu V(u) \exp(2\pi iux) \, du \]  

\[ = 2\pi \int V_u(u) \exp(2\pi iux) \, du, \]  

which implies that the spatial derivative of the source profile, \(I'(x) = dI(x)/dx\), can be reconstructed from a set of modified visibilities, \(V_u(u) = iu V(u)\). The modified visibilities \(V_u(u)\) are thus derived from the measured visibilities, \(V(u)\), by applying a 90° phase shift (the factor \(i\)) and weighting by the spatial frequency \(u\). \(V_u(u)\) is given by

\[ V_u(u) = \int I'(x) \exp(-2\pi iux) \, dx. \]  

Numerically evaluating Equation (1) for an unocculted Gaussian source profile results in the behavior shown by the black solid curve in Figure 1. As a smaller source becomes over-resolved at high spatial frequencies, the visibility amplitudes decrease rapidly. For a smooth source that is occulted abruptly at \(x = x_0\), \(I'(x)\) has a sharp peak—approaching a delta function for an arbitrarily sharp limb—and evaluating Equation (5) yields \(V_u(u) = \exp(-2\pi iux_0)\), from which \(V(u) = (1/\pi u) \times \exp(2\pi iux_0)\). In other words, at high spatial frequencies, we find a contribution to the visibility amplitude that decreases as \(1/\text{spatial frequency}\). This is numerically confirmed in Figure 1 for various fractional occultations of a Gaussian source (colored solid lines). Note that the phase of this visibility depends on the location of the occulted edge of the source. As Figure 1 suggests, the presence of an occulted edge to an otherwise smooth source results in visibilities with significantly enhanced amplitudes at the higher spatial frequencies. However, this enhancement only applies to visibilities whose spatial frequencies are orthogonal to the occulted edge. Thus the direct signature of occultation is a spike in the amplitudes of visibilities at high spatial frequencies, when the direction of these visibilities approaches orthogonality to the occulted edge of the source. This is illustrated in the right panel of Figure 1, where the visibility amplitudes as a function of position angle for two sources, with 12 arcsec FWHM and 6 arcsec FWHM that are occulted at an angle of 20°, are shown. Note how the width of the peak in amplitude depends on the source size. To summarize, visibility analysis allows us to distinguish the sharp, precisely oriented emission must be large enough to provide sufficient counting statistics for \textit{RHESSI}. Note the trade-off with the depth of occultation, which results in a larger visibility enhancement (see Figure 1), but also in lower observable emission. We have found an ideal candidate, SOL2011-10-20T03:25 (M1.7). Images from \textit{SDO}/AIA at 131 Å suggest that the flare source was indeed occulted. In addition, \textit{RHESSI}'s detector 2 was performing well enough for its data to be used for this kind of analysis, hence we can use the modulation of G2 as an independent measure. As a feasibility demonstration, we analyzed the visibilities over an energy band of 3–25 keV, and over the four-minute time interval from 03:15:00 to
03:19:00 UT, which was chosen to maximize the signal-to-noise ratio. \textit{RHESSI}'s G1 with a spatial resolution of 2\arcsec 26 (FWHM), and G2 with a resolution of 3\arcsec 92, have the highest spatial frequencies and therefore are best suited to observe the spike in visibility amplitude described in Section 2. Figure 2 shows an AIA 131 \AA image overlaid with the \textit{RHESSI} source. Because the signature of the occulted edge of the source is limited to a small azimuthal range for the highest resolution subcollimators, images made with “traditional” imaging algorithms cannot be readily interpreted in terms of the edge location and properties. In particular, none of the traditional imaging algorithms use an image basis that is appropriately optimized for an occulted source, but use some form of circular or Gaussian symmetry for the reconstruction. This is why we must use visibilities directly. The visibility amplitudes as a function of position angle for collimators 1 and 2 (G1 and G2) are also shown in Figure 2.

We have applied the tests described in Section 2 to these observations. The first and most important test is for there to be distinct peaks in both G1 and G2 at the correct location and well in excess of the uncertainty estimates derived from the point-by-point measurements of the complex amplitudes. The data clearly pass this test. Unfortunately, the other tests have some ambiguities and uncertainties. Note that the expectations for these tests must refer to a model that may be inappropriate in some details. In particular, we do not know the actual width of the source nor the fraction by which it is occulted. A full image made via the visibility forward-fitting technique gives a FWHM of 11\arcsec. Hence, for comparison, an occultation of 50%...
4. Observed and Predicted Limb Height

4.1. Limb Height from Visibility Analysis

For a given subcollimator, the information on the radial position $R_L$ of the limb is contained in the phase $\phi$ of the complex visibility, where

$$R_L = C_M + \frac{(\phi_L - \pi/2)}{2\pi} \times p,$$  (6)

where $C_M = \sqrt{x^2 + y^2}$ is the position of the map center used for the visibility calculation, and $p$ is the pitch of the collimator that is used for the analysis (4.52 arcsec for G1 and 7.84 arcsec for G2, which are used here). The location of the map center is determined by the metrology of the RHESSI grid optics (essentially, the RHESSI plate scale). Figure 4 shows the visibilities of G1 and G2 in the complex plane, which is the basis of the analysis. The visibility phase is the angle measured counter-clockwise from the positive $x$-axis; the amplitude is the radial distance from the origin of the coordinate system. The visibilities dominated by the limb signal were identified as the points around the peak with an amplitude larger than its 3σ uncertainty. They are summed up vectorially to give the phase that represents the limb signal. From this, the vectorial sum of all other visibilities (“background”) was subtracted. The resulting phase was then used in Equation (6) to calculate the radial limb position. The mean limb location, derived in this way from the observed phase information, is given in Table 2.

We show two sets of errors: the formal errors of the visibility fit, based explicitly upon the counting statistics; and a second set that estimates the systematic error. The former is derived from the uncertainty of the phase measurement. Note that, even though the resolution of the grids is limited to a few arcseconds, the visibility phases can be measured with a much higher accuracy (compare Figure 4). The systematic uncertainty consists of three components. The first one uses the empirical scatter of the significant phase measurements at our binning (i.e., the standard deviation of the visibility phases associated with the limb red points in Figure 4). For the weighted mean, we also incorporate two possible instrumental uncertainties: the length of the metering tube that holds the grids, and the accuracy of the Sun-center determination from the solar aspect system. For the former, we adopt a 3σ error of 0.026 (Zehnder et al. 2003), and for the latter, as an upper limit, we adopt 0.15 (Fivian et al. 2002).
4.2. Predictions

Figure 5 displays our result in terms of height of the observed limb above the photosphere in comparison with the height of the visible limb. Given the fundamental difference between an X-ray absorption signature of the limb location, its visible location, and the known wavelength-dependence of the solar radius (e.g., Rozelot et al. 2015), our results are within our expectation regarding the difference between X-ray limb height and optical limb. We also compare the observed limb height with the predicted X-ray limb height from a model atmosphere. For this first result we compare with the standard VAL-C semi-empirical model atmosphere (Vernazza et al. 1981). We have calculated the X-ray opacity of the photospheric plasma from weighted means of the mass attenuation coefficients of the contributing elements (H, He, C, N, O, and Fe), and incorporated both the photoelectric effect and electron (Compton) scattering. Note that this approach omits any fine structure associated with flows (leading to corrugations) and sphericity. The predicted limb height varies with photon energy, as shown in Figure 5, for the photospheric solar elemental abundances of Caffau et al. (2011) and lies below the observed height with a 2σ significance (see Section 5). The electron scattering just depends upon the total line-of-sight electron density, including atomic electrons, at X-ray photon energies. The photoeffect, on the other hand, depends upon the bound-state populations of different ionic states, and requires a detailed model. We have assumed negligible ionization in the photosphere, and spherical symmetry, to determine the impact factor for 1/e attenuation of the occulted source. Note that the predicted height only varies slowly with photon energy above the Fe K-edge (visible as discontinuity at 7 keV in Figure 5) because of the dominance of electron scattering for the X-ray opacity in the photosphere. Nevertheless, the Fe feature does reveal itself significantly, so that with better observations one could obtain a clean determination of a mean Fe abundance in this way.

5. Discussion

The technique we have explored by analyzing the RHESSI observations of SOL2011-10-20T03:25 (M1.7) has a particularly simple physical interpretation in terms of the actual mass distribution with height in the atmosphere. Despite this, and
ties from the visibility uncertainty. The purple point gives the vectorially added background visibilities. The blue point and arrow give the phase of the limb.

\[
L \leq \frac{\text{FWHM}}{2} \pm \text{statistical and systematic uncertainties.}
\]

Several factors contribute, with the most important in this case being the relative locations of the pointing axis (a few arcminutes west of the Sun center), the Sun center, and the flare location. The measurement of visibilities whose orientations match the direction of a line from the pointing axis to the limb source can be compromised by an incomplete phase sampling (Figure 4 of Hurford et al. 2002). This factor could be alleviated by non-uniform pointing variations (at the relevant roll angle) on a scale of a few arcseconds. Unfortunately in this case, RHESSI’s pointing was too stable. For visibilities tangent to the limb, there is not a problem because of the rapid variation of the modulation. Other lesser problems with the data include the simple problem of counting statistics; only a small fraction of the data apply to just the visibility components needed. Finally, the theoretical visibility pattern depends, to a certain extent, on an unknown fine-scale image structure that may compete with the sharp edge expected for the occultation.

Why is the measured X-ray limb location slightly above its predicted height? Of course, the prediction requires model assumptions. In particular, our prediction ignores the presence of physical corrugations of the limb (Simon & Zirin 1969; Gu et al. 1997). The structures would need to be in excess of the hydrostatic scale height of about 100 km in the photosphere, and on small angular scales. The likeliest sources for such corrugations would be p-mode or convective flows in the photosphere (Kuhn et al. 2000), or some lower-atmosphere counterpart of the spicule structure seen in the chromosphere (e.g., Ewell et al. 1993; Ayres 2002). We think it unlikely that they have an influence on the model or observations, based on present knowledge, but look forward to higher-resolution optical observations and to more definitive modeling.

Other issues have to do with possible time variability of the limb location, as well as the physical nature of the particular patch of the Sun involved in our single determination; a quiet-Sun model such as VAL-C may simply be inappropriate. Such a local interpretation might also help to resolve the unexpectedly low altitude found for the hard X-ray sources in a single well-observed flare on the visible hemisphere (Martínez Oliveros et al. 2012). The Sun itself may be aspherical, but we note that the observed oblateness term is not large enough to be relevant here (Fivian et al. 2008).

6. Conclusion

The results presented here, to our knowledge are the first X-ray measurements of the solar radius, offer great promise. In spite of the caveats discussed above, we are optimistic that this
technique can generate a rigorous and independent measure-
ment of the true solar radius; the X-ray measurements provide a
means to determine the actual mass distribution rather than the
more model-dependent distribution of opacity that underlies
classical optical techniques. The result (Table 2) does not quite
match the expectations from the standard VAL-C model. The
discrepancy between observation and prediction, at about 1σ
significance, could imply errors in our use of the VAL-C model
(for example by application to an area of the Sun near a major
active region). Corrugations of the atmosphere at these
temperature-minimum heights could also play a role. The
technique is sound enough for us to encourage further
exploration of the RHESSI database for this purpose, with the
ultimate goal of establishing better solar radius determinations
this way. If this becomes possible, improved observations
using the X-ray technique could, in principle, help determine
solar iron abundance, taking advantage of the effect of the
K-edge visible in Figure 5.

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