Research on mechanics model verification and skyhook semi-active control of magneto-rheological damper

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Abstract. Aim at the need of vibration attenuation of some light military wheel cross-country vehicle, design valve model magneto-rheological damper (MRD) and deduce its damping force flat model. Conduct performance test of MRD and build relative mechanics model, apply the least square method to verify the model based on three indexes of relative error, relative coefficient and standard deviation. In order to test the performance of vibration attenuation of MRD, conduct platform experiment with the skyhook ON-OFF semi-active control algorithm. The result indicates that the error between the MRD model built and the result tested is small, the MRD can decrease vehicle body vertical acceleration beside vehicle body resonance area effectively, and improve ride comfort under the semi-active control.

1. Introduction

Vehicle suspension system is used to support vehicle body, ease the shock of road roughness, and attenuation of vibration energy [1]. Semi-active suspension realizes vibration reduction through changing damping, the energy is not involved in the vibration control directly, so it has the advantages of low energy consumption, damping adjustable, easy to control and the feature of "fail - safe", its control performance is close to the active suspension, so it is widely followed [2-3].

Magneto-rheological Damper (MRD) is a new kind of shock absorber damping, which has continuous adjustable damping, as the Magneto-rheological Fluid (MRF) occurs the action of rheological effect under the applied magnetic field, the working principle of the device is using this feature to change the damping force, realize variable damping [4]. MRD has quick response and usually a millisecond, small volume, and low energy consumption, it has good development prospects in the construction, vehicle and aerospace and other fields [5-6]. In order to control MRD, it is needed to set up a corresponding mechanics model, while conduct verification is an important step to verify the accuracy of mechanics model. At present, the verification of mechanics model does not cause enough attention, judging the fitting degree of damping force curve between build model and test data through simple compare can’t judge the accuracy of model scientifically.

This paper take a light-duty military wheel cross-country vehicle as model, design valve type MRD and build its damping force expression derived from the tablet model, conduct verification for polynomial mechanics model of MRD, fitting on the test data based on the least squares method, and use skyhook ON-OFF control algorithm to analyze the performance of vibration attenuation of MRD.
2. Structure design and modelling of MRD
The MRD of valve type is mainly constituted by the piston, piston rod, coil and steel cylinder etc, its structure figure is shown in figure 1. The piston is composed of inside damping piston and outside guide piston, this structure can shorten the axial size, the magnetic circuit around back from the guide piston can reduce the magnetic flux leakage. The up and down two sides of guide piston open MRF flow hole, while the damping gap applies the structure of annular aperture, and the working mode of MRF is valve type [7]. During the process of tensile and compression, there pressure difference produced on both ends of the piston, MRF flow through the damping gap under the action of pressure difference. At this time, if give electricity to the coil, produce magnetic field, so as to change the viscosity of MRF in the damping gap, then change the damping force, realize variable damping, the damping continuously adjustable can be realized by changing the size of the current. For that in the process of tensile and compression, the piston rod occupies part of volume, use float piston type volume compensate device to compensate this part of volume.

Figure 1. Diagram of MRD structure.

2.1. Model of MRD damping force
The MRD working mode is valve type, the literature [8] pointed out that if the circular aperture is approximation as parallel plate model, the maximum error is less than 5%. Therefore, use the tablet model derive damping force model, the valve type plate flow work mode diagram is shown as figure 2.

Figure 2. Sketch figure of valve flat flow work mode. Figure 3. Schematic diagram between flow velocity and stress distribution of MRF during the tablet. Where \( w \) is slab width, \( L \) is plate length; \( h \) is the gap height, \( Q \) is the volume flow of MRF, \( \Delta p \) is the pressure difference, \( H \) is magnetic field strength.

Establish a coordinate system \( xOz \), the zero \( O \) is on the surface of the down tablet, \( Ox \) is along with the flowing direction of MRF, \( Oz \) is along with the flow of fluid in the vertical direction. The schematic diagram between flow velocity and the stress distribution of MRF during the tablet is shown in figure 3.

Then deduce pressure gradient formula as

\[
\frac{\partial \tau(z)}{\partial z} = \frac{\partial p}{\partial x}
\]  

(1)

Where \( \tau(z) \) is shear stress, \( z \) is longitudinal axis coordinates, \( p \) is the pressure along with the flowing direction of MRF, \( x \) is coordinate of the flowing direction of MRF. Conduct integral on both sides of formula (1)

\[
\tau(z) = \int_{\tau_1}^{\tau_2} \frac{dp}{dx} \, dz + D_1
\]  

(2)

Where\( D_1 \) is constant. By \( \tau_{h_1} = \tau_y \), \( \tau_{h_2} = -\tau_y \) and formula (4),we can conclude
\[ D_i = -\frac{1}{2}(h_2 + h_i) \frac{dp}{dx}, \quad (3) \]
\[ \tau_y = -\frac{1}{2}(h_2 - h_j) \frac{dp}{dx}. \quad (4) \]

We can conclude the velocity of each area of I, C and II from deducing as

\[ u_c(z) = \begin{cases} 
\frac{1}{2\eta} \frac{dp}{dx} [h_1^2 - (h_1 - z)^2] & 0 \leq z \leq h_1 \\
\frac{1}{2\eta} \frac{dp}{dx} h_2^2 & h_1 \leq z \leq h_2 \\
\frac{1}{2\eta} \frac{dp}{dx} [(h_2 - h_1)^2 - (z - h_1)^2] - v_0 & h_2 \leq z \leq h 
\end{cases} \quad (5) \]

Where \( u_c \) is the flow rate of MRF. The volume flow \( Q \) of MRF is

\[ Q = -\frac{w}{2\eta} \frac{dp}{dx} h_1 \left( h - \frac{1}{3}(h + h_1 + h_2) \right) - \frac{w}{3} v_0 (h - h_2) \quad (6) \]

Where \( A_p \) is cross-sectional area of the piston, \( v_p = v_0 \) is the velocity of the piston. During the valve type flow model, the pressure gradient can be expressed as

\[ \Delta p = P_L - P_o = -L \frac{dp(x)}{dx} \quad (7) \]

Where \( L \) is the effective length of the magnetic field. The total damping force is \( F_v = \Delta p A_p, F_d = \Delta p A_p, \) unite formula (5)~(7), we can conclude that

\[ F_d = F_v + F_i = \frac{12\eta QA_L L}{wh^3} + \frac{6\eta QL}{h^3} + \frac{\tau_c A_L L}{h} \quad (8) \]

Where \( F_v \) is the viscous damping force, \( F_i \) is the adjustable damping force, \( F_i = \tau_c A_L L/h, \) \( F_v = 12\eta QA_L L/wh^3 + 6\eta QL/h^3, c = 2.07 + 1/\left(1 + 0.4wh^2\tau_c /12\eta Q\right). \)

Through formula (8), we can know that the MRD damping force is mainly composed of viscous damping force and adjustable damping force, when the MRD geometry size and MRF performance is determined, the size of the viscous damping force is only affected by the velocity of suspension; The adjustable damping force is a function which is related with \( \tau_y, \) and it is influenced by the magnetic field intensity, through changing the load current in coil size, we can adjust the value of the adjustable damping force to achieve variable damping.

2.2. Characteristic test of MRD

In order to control MRD, it is needed to test its damping characteristics and mechanics properties. According to the automobile shock absorber test standard QC/T545-1999, conduct performance test for MRD. Using sine excitation, amplitude ±25 mm, then test characteristic curve of damping force with displacement, damping force with velocity respectively, the velocity is 0.1, 0.2, 0.3, 0.4 and 0.5 m/s, while the load current is 0, 0.25, 0.75, 1.0, 1.5 and 2 A. The characteristic diagram of damping force with displacement, velocity and current under the velocity of 0.1 m/s is shown in figure 4.

![Characteristic diagram of damping force](image)

**Figure 4.** Characteristic diagram of damping force.
Figure 4 shows that the damping force with displacement indicator curve is full, floating piston volume compensation device can effectively compensate the volume difference generated during stretching and compression. The area surrounded by indicator diagram increases along with the increasing of current, which shows that the ability of power consumption of MRD is increasing. During the velocity range of 0.1 m/s~0.5 m/s, along with the increase of velocity, the damping force increases as linear type, this part of the damping force is mainly produced by the viscous damping force; While when the velocity is certain, and the current unceasingly increases, the damping force increases, this part of damping force is provided by the adjustable damping force. When the magnetic loop is not saturate, growth rate is fast, after 1A, as magnetic circuit saturate gradually, the growth slows down.

2.3. MRD Mechanics model and its verification

As the precision of the polynomial model can be improved through improving the fitting order, and it can reflect the hysteresis characteristics of damping force with velocity, take the polynomial model as the MRD mechanics model [9]. Take the polynomial fitting for order 3, then the relation between the damping force, current and velocity is

$$F_d = \begin{cases} 
(53.64 I_1^3 - 273 I_1^2 + 479 I_1 + 58) + (58.92 I_1^3 - 285.6 I_1^2 + 380.3 I_1 + 663.1) & \nu > 0 \\
(-61.53 I_1^3 + 286.6 I_1^2 - 485.8 I_1 - 47.71) + (91.35 I_1^3 - 414.6 I_1^2 + 509.1 I_1 + 673.9) & \nu < 0 
\end{cases}$$

(9)

Where $I$ is the control current, its scope is 0~2A, and $\nu$ is the relative velocity of suspension.

During the research on the accuracy of mechanics model, scholars usually only judge the accuracy through comparing the model built and the experimental data, this method is difficult to analysis the fitting error scientifically. In order to check up the fitting accuracy, the idea of calibrating the mechanics model of MRD fitted using the least squares method, set up the objective function, minimize the sum of squared residuals of the objective function is put forward, the expression is

$$\min_j J = \sum_{i=1}^{N} \left[ F_d(t_i) - F_d(t_i) \right]^2$$

(10)

Where $F_d(t_i)$ is the damping force measured at the moment of $t_i$, and $F_d(t_i)$ is the damping force calculated according to the MRD mechanics model, the total of data is $N$.

In order to compare the fitting accuracy of the mechanics model built and the experimental data, three indexes including relative error, correlation coefficient and standard deviation is set up. The relative error can reflect the error through the value of each data point; and the correlation coefficient can reflect the correlation, the larger the correlation coefficient is, the greater the combination of correlation is, and the better the model can accurately describe the MRD mechanics properties too; while the standard deviation reflects the degree of discrete of data sets. The expression of three indexes including relative error, correlation coefficient, standard deviation are shown as:

$$e(t) = \frac{|F_d(t_i) - F_d(t_i)|}{F_d_{\text{max}} - F_d_{\text{min}}}$$

(11)

$$r = \frac{N \sum_{i=1}^{N} F_d(t_i)F_d(t_i) - \left( \sum_{i=1}^{N} F_d(t_i) \right)^2}{\sqrt{N \sum_{i=1}^{N} F_d(t_i)^2 - \left( \sum_{i=1}^{N} F_d(t_i) \right)^2} \sqrt{N \sum_{i=1}^{N} F_d(t_i)^2 - \left( \sum_{i=1}^{N} F_d(t_i) \right)^2}}$$

(12)

$$S = \sqrt{\frac{\sum_{i=1}^{N} (F_d(t_i) - F_d(t_i))^2}{N-1}}$$

(13)

Where $F_d_{\text{max}}$ and $F_d_{\text{min}}$ are the maximum and minimum value of damping force under the given test condition respectively.

For the limitation of article, result of three indexes under positive velocity is shown in figure 5.

Figure 5(a) shows that except the condition of 0.25A current and 0.25 m/s velocity, the maximum relative error of the damping force with velocity is 10.62%, the rest are mostly under 5%, the relative error is relatively small. From figure 5(b), we can get that the correlation coefficient of velocity increase first, and then decreases, but all above 0.995, the linear correlation between the fitting function and experimental data is very well. Figure 5(c) shows that as the velocity increases, the standard deviation value increases, while the data discrete more, which indicates that the working stability of MRD under large velocity is worse than under small velocity.
Fitting characteristic figure of damping force with velocity and current is shown in figure 6, we can get that the fitness with figure 4 is very well, which indicates that the model of MRD is relatively accurate. The damping force increases along with the increase of velocity and current.

![Fitting characteristic figure of damping force with velocity and current](image)

Figure 6. Characteristic diagram of damping force.

3. Semi-active control of MRD

3.1. A quarter car dynamic model

The suspension of this type of military vehicle is independent suspension, it is assumed that the body mass distribution coefficient is 1, then we can use quarter car two degree of freedom model to describe the dynamic model of vehicle suspension. The motion equation of suspension system is

\[
\begin{align*}
\dot{m}_s \ddot{x} + k_s (x - x_s) &= F_d \\
\dot{m}_t \ddot{x}_t - k_t (x - x_s) + k_r (x - x_r) &= -F_d
\end{align*}
\]

(14)

Where \(m_s\) is the vehicle body mass, \(m_t\) is the wheel mass, \(k_s\) is suspension equivalent stiffness, \(k_t\) is tire equivalent stiffness, \(x_s\) is the vertical displacement of the vehicle body, \(x_t\) is the wheel vertical displacement, \(x_r\) is road motivate vertical displacement, take vertical up as positive.

Suspension parameters of quarter car is shown in table 1.
Table 1. Suspension parameters of quarter car.

| Parameter | $m_s$ (kg) | $m_t$ (kg) | $k_s$ (kN/m) | $k_t$ (kN/m) | $c_p$ (N.s/m) |
|-----------|------------|------------|--------------|--------------|---------------|
| Value     | 322.5      | 46.2       | 22.5         | 193.6        | 1550          |

Where $c_p$ is the passive damping coefficient, $[f_d]$ is the allowable suspension travel, $[f_d]=0.12$m.

3.2. Vibration control experiment

The vibration control experiment system of MRD is shown in figure 7. Experiment system mainly includes acceleration sensor and displacement sensor, which is used to acquire vehicle state information, the data acquisition, control and MRD current drive integration module set together with the data collect block, control block and the MRD current driver interface card, concurrently possess the function of data acquisition, control and power supply of MRD, the power source supply power for modules and components. Use the skyhook ON-OFF semi-active control algorithm [10].

![Figure 7](image)

**Figure 7.** Object diagram of MRD vibration control experiment system.

(1) Time domain analysis

Under the condition of velocity of 10 m/s and random road degree of D, the time domain results are shown in table 2 and figure 8.

Table 2. Statistics result of indexes under random stimulation response.

|                          | VBVA (m/s) | Improve (%) | SDT (m)   | Improve (%) |
|--------------------------|------------|-------------|-----------|-------------|
| Passive                  | 1.0425     | -           | 0.0102    | -           |
| ON-OFF                   | 0.9317     | 10.63       | 0.0134    | -31.37      |

![Figure 8](image)

**Figure 8** Time domain figure.

Compare with the passive suspension, the skyhook ON-OFF semi-active control can reduce the vehicle body vertical acceleration (VBVA) root mean square value by 10.63%, while the suspension dynamic travel (SDT) root mean square value deteriorated by 31.37%, but it does not exceed the allowable travel $f_d = 1/3[f_d] = 0.04$m, which means the MRD can improve the ride comfort effectively.

(2) The analysis of frequency domain
Frequency domain analysis diagram of random road is shown in Figure 9. The attenuation to VBVA is mainly concentrated in low-frequency region of the resonance zone beside vehicle body under the control of semi-active control, while failed in the rest of frequency. The SDT is mainly concentrated in low and middle frequencies, while deterioration is more obvious in the frequency near resonance zone of vehicle body. The analysis conclusion above is corresponding to the analysis results of time domain.

(a) VBVA  
(b) SDT

Figure 9 Frequency domain figure.

4. Conclusions
1) The damping force of MRD is mainly influenced by the relationship between the shear yield stress of MRF and the intensity of magnetic field. The damping force of MRD increases along with the increasing of current, and increase faster before 1A, while the magnetic circuit achieves the magnetic saturation after 1A, therefore, it is proper to ensure the load current below 1A.

2) Using the least square method to calibrate the polynomial mechanics model of MRD fitted, the correlation coefficient, relative error and standard deviations are small, the MRD model can describe the performance of MRD accurately.

3) The experiment shows that the MRD can attenuate the vibration effectively, the control algorithm enjoys good control effect in low frequency beside resonance region of vehicle body, which is fit for military off-road vehicles.

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