Phase diagram of a short-range interacting SO(3) quantum spin-glass

C.M.S. da Conceição¹ and E.C. Marino²

¹Departamento de Física Teórica, Universidade do Estado do Rio de Janeiro, Rio de Janeiro, RJ 20550-013, Brazil
²Instituto de Física, Universidade Federal do Rio de Janeiro, Cx.P. 68528, Rio de Janeiro, RJ 21941-972, Brazil
E-mail: marino@if.ufrj.br

Abstract. We present a stable approximate solution for a quantum, short-range interacting, SO(3) Heisenberg disordered spin system on a square lattice, with a Gaussian distribution of couplings centered on an anti-ferromagnetic coupling $\bar{J}$. The phase diagram $T \times \bar{J}$ contains a Néel (AF) phase at $T = 0, \bar{J} > \bar{J}_0$ and a spin-glass (SG) phase below a critical line, namely, $T < T_c(\bar{J}), \bar{J} < \bar{J}_0$. The rest of the phase diagram is occupied by the paramagnetic (PM) phase.

1. Introduction

The physics of disordered magnetic systems has attracted a lot of interest since a long time. The occurrence of a spin-glass (SG) phase in magnets with quenched disorder, in particular has been the object of many studies [1]. A lot is known about SG models with a long-range magnetic interaction, however the same is not true in the more physical case of short-range interacting SG.

In this paper, we consider a two-dimensional SO(3) quantum disordered magnetic system, described by a nearest neighbour interacting Heisenberg hamiltonian in a square lattice, with random exchange couplings associated to a Gaussian distribution centered in an anti-ferromagnetic coupling, in such a way that in the limit of vanishing disorder we recover a pure anti-ferromagnetic system on a square lattice.

The physical motivation for the model comes from the high-Tc superconducting cuprates, whose phenomenology in the low doping regime is compatible with this model.

Our main results are in Section 4. Sections 2 and 3 are included in order to make the paper self-contained [2, 3].

2. The model

The hamiltonian is

$$\hat{H} = \sum_{(ij)} J_{ij} \hat{S}_i \cdot \hat{S}_j,$$

(1)
on a $2D$ square lattice of spacing $a$. The nearest neighbours exchange couplings are random and associated with the distribution

$$P[J_{ij}] = \frac{1}{\sqrt{2\pi(\Delta J)^2}} \exp \left[ -\frac{(J_{ij} - \bar{J})^2}{2(\Delta J)^2} \right],$$

(2)

where $\bar{J} > 0$.

We consider the case in which the disorder is quenched and use the replica method in order to describe it. The average free-energy is given by

$$F = -k_B T \lim_{n \to 0} \frac{1}{n} ([Z^n]_{av} - 1),$$

(3)

where $[Z^n]_{av}$ is the disorder-averaged replicated partition function.

3. The free-energy

In order to obtain the above free-energy for this model, we use coherent spin states, $|\Omega^\alpha_i(\tau)\rangle$, having the property

$$\langle \Omega^\alpha_i(\tau)|\hat{S}^\alpha_i|\Omega^\alpha_i(\tau)\rangle = S \Omega^\alpha_i(\tau),$$

(4)

where $S$ is the spin quantum number.

Taking the continuum limit we obtain [2, 3]

$$[Z^n]_{av} = \int \mathcal{D}n\mathcal{D}Q\mathcal{D}\lambda \exp \left\{ -\int d\tau L_{J,\Delta}[n^\alpha, Q^{\alpha\beta}, \lambda^\alpha] \right\},$$

(5)

where the lagrangian density is

$$L_{J,\Delta} = \frac{1}{2} |\nabla n^\alpha|^2 + \frac{1}{2c^2} |\partial_\tau n^\alpha|^2 + i\lambda n^\alpha (|n^\alpha|^2 - \rho_s)$$

$$+ \frac{D}{2} \int d\tau' \left[ Q^{\alpha\beta}_{ab}(\tau, \tau')Q^{\alpha\beta}_{ab}(\tau, \tau') \right. \left. - \frac{2}{\rho_s} n^\alpha_a(\tau)Q^{\alpha\beta}_{ab}(\tau, \tau')n^\beta_b(\tau') \right],$$

(6)

with $D = S^4(\Delta J)^2/a^2$ ($a$: lattice parameter) and $\rho_s = S^2 \bar{J}$. In the above expression, summation on the replica indices $\alpha, \beta = 1, \ldots, n$ is understood.

The field $n^\alpha = (\sigma^\alpha, \vec{\pi}^\alpha)$ is the continuum limit of the (staggered) spin $\Omega^\alpha$ and satisfies the constraint $n^\alpha \cdot n^\alpha = \rho_s$, which is implemented by integration on $\lambda^\alpha$.

We now decompose the field $Q^{\alpha\beta}$ into replica diagonal and off-diagonal parts,

$$Q^{\alpha\beta}_{ab}(\vec{r}, \tau, \tau') \equiv \delta_{ab}[\delta^{\alpha\beta}\chi(\vec{r}; \tau, \tau') + q^{\alpha\beta}(\vec{r}, \tau, \tau')]$$

(7)

where $q^{\alpha\beta} = 0$ for $\alpha = \beta$. Subsequently, we integrate on the transverse components $\vec{\pi}^\alpha$ and use the stationary phase approximation on the resulting effective $\sigma$-field theory. We thereby obtain $[Z^n]_{av} = e^{-nS_{\text{eff}}}$, whereupon $\beta F = S_{\text{eff}}$.

The resulting average free-energy density is most conveniently expressed in the space of Matsubara frequencies $\omega_r = 2\pi r T, r \in \mathbb{Z}$, namely,

$$\tilde{f} \left[ \sigma^\alpha, m^2, q^{\alpha\beta}(\omega_r), \chi(\omega_r) \right] = \frac{m^2}{2} \left[ \frac{\sigma^2}{n} - \rho_s \right]$$
\[
- \frac{D}{n\rho_s} \left[ \chi(\omega_0) \delta^{\alpha\beta} + q^{\alpha\beta}(\omega_0) \right] \sigma^\alpha \sigma^\beta \\
+ 3DT \sum_{\omega_r} \left[ \chi(-\omega_r)\chi(\omega_r) + \frac{1}{n} q^{\alpha\beta}(-\omega_r)q^{\alpha\beta}(\omega_r) \right] \\
+ T \sum_{\omega_r} \int \frac{d^2 k}{(2\pi)^2} \left[ \ln \left( \frac{k^2 + M_r}{k^2 + M_r} \right) - \frac{A\bar{q}(\omega_r)}{k^2 + M_r} \right]
\]

where

\[ M_r \equiv M(\omega_r) = \omega_r^2 + m^2 - A[\chi(\omega_r) - \bar{q}(\omega_r)], \]

and \( \chi(\omega_r) \) and \( q^{\alpha\beta}(\omega_r) \) are, respectively, the Fourier components of \( \chi(\tau - \tau') \) and \( \bar{q}(\tau - \tau') \), with

\[ \bar{q} = \lim_{n \to 0} \frac{1}{n(n-1)} \sum_{\alpha\beta} q^{\alpha\beta} \]

4. The phase diagram

From the average free-energy density (8), we obtain the stationary phase equations by varying with respect to the different fields. By solving these equations, we are able to determine the phase diagram of the system [2, 3]. This presents an ordered Néel phase at \( T = 0 \), for \( \rho_s > \rho_0 \), a SG phase for \( \rho_s < \rho_0 \) and \( T < T_c \) and a paramagnetic phase in the remaining regions of the phase diagram. This is presented in Fig. 1, for different values of the distribution width.

We evaluate the static magnetic susceptibility \( \chi(\omega_0) \), as a function of the temperature, in each phase. This presents a Curie behavior in the paramagnetic phase.
at high temperatures, diverges in the ordered AF phase at \( T = 0 \) and shows a sharp cusp at the PM-SG transition at a finite temperature.

The quantum critical point depends on the parameters of the disorder distribution and is given by

\[
\rho_0 = \frac{\Lambda}{2\pi} \left[ 1 + \frac{1}{\gamma} \left[ 1 + \frac{1}{2} \ln(1 + \gamma) \right] \right], \tag{11}
\]

where, \( \Lambda = 1/\alpha \) is the momentum cutoff and \( \gamma = 3\pi \left( \frac{J}{\Delta J} \right)^2 \).

5. Stability

A necessary and sufficient condition for the mean field solution to be a local minimum is to have all the principal minors of the Hessian positive. This would be equivalent to having all the eigenvalues of the Hessian positive. The principal minors are the determinants of the matrices obtained from the original matrix by successively striping the last line and the last column, starting from the matrix itself and ultimately reaching the (11) element.

We have carefully evaluated each of these determinants in the limit \( n \to 0 \), for \( \sigma = 0 \) (PM and SG phases) and showed they are all positive [2, 3].

6. Final remarks

We have presented an approximate solution for a quantum, short-range interacting SG system with SO(3) symmetry, which is stable in spite of the fact that it has replica symmetry. This result brings us to the issue of the origin of the instability of the Sherrington-Kirkpatrick solution [4, 5]. It is conceivable that the constant, long-range interaction, rather than the replica symmetry might be the cause of it.

We would like to stress the fact that our results are far beyond the mean field approximation since the quantum fluctuations of the transverse part of the staggered field have been fully considered.

References

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