Memory switching in a neural network with chaotic neurons and synaptic depression

Zheng Li, Zhijie Wang, Shihuang Shao
College of Information Science and Technology, Donghua University, 200051, Shanghai, China

Abstract An associative neural network with chaotic neurons and synaptic depression (CNSD) is constructed. Quick memory switching phenomenon in the network is demonstrated. It is also shown that the memory retrieval frequency of CNSD distributes nearly uniformly among the stored patterns, and the rate of memory retrieval of CNSD is much higher than that of a chaotic neural network. A simplified analysis framework for the memory switching phenomenon of CNSD is presented, and the phenomenon is discussed within this framework.

1. Introduction
It is well known that an autoassociative neural network can store memories through its synaptic connections. Unlike a conventional autoassociative neural network which evolves to settle at a stable steady state [7][8][14], a chaotic associative neural network generates peculiar behavior that the network retrieves some stored patterns and escapes from them in the transient phase [2][3][13]. Such a chaotic associative memory is constructed with chaotic neuron models interconnected through a conventional auto-associative matrix of synaptic weights. On the other hand, once the weights of the synapses in a conventional autoassociative neural network are determined by learning rules, they are fixed during the memory retrieval process. This implies that the synapses of the neurons are assumed to be 'static', i.e., that they change their weights only on the slow time scale. However, it has been discovered that synaptic plasticity occurs across many time scales—from the order of days to the order of milliseconds [9]. Most of the models of short-time synaptic plasticity are constructed in the context of spiking neurons and under the principle of synaptic transmission resources [1][12]. Rate-code models of short-time synaptic plasticity are also deduced recently[11]. The dynamic association in a neural network with a rate-code model of synaptic depression is investigated [11]. Similar to a chaotic associative memory, the output of such a network transits among the stored patterns during the retrieval process.

In this paper, we explore the dynamics of an associative neural network with chaotic neuron models and synaptic depression (CNSD). We find a quick memory switching phenomenon in this kind of network. Meanwhile, we find that the rate of memory retrieval of CNSD is much higher than that in a chaotic neural network [2], and the retrieval frequency among the stored patterns in CNSD is nearly uniformly distributed. A simplified analysis framework for CNSD is presented. The distribution of the connection weights of the framework is calculated. After that, depending on the description of the dynamics of CNSD, the contribution of both the refractoriness of the chaotic neuron and the synaptic depression to the quick memory switching phenomenon is explained within this framework.
2. Models of the neural network with chaotic neurons and dynamic synapses

The chaotic neuron model in this paper is described by the following equation [2] [3]:

\[ x(t+1) = f[A(t) - \alpha \sum_{d=0}^{t} k^d g\{x(t-d)\} - \Theta], \]

where \( t \) is a discrete time step \((t = 0, 1, 2, \ldots)\), \( x(t) \) is the neuronal output with an analog value between 0 and 1 at the discrete time \( t \), \( f \) is the activation function, \( A(t) \) is the external stimulations at the time \( t \), \( g \) is the refractory function, \( \alpha \) is the refractory scaling parameter, \( k \) is the refractory decay parameter, and \( \Theta \) is the threshold.

The term \( k^d g\{x(t-d)\} \) represents the influence of the refractoriness due to the neuronal output of \( d \) time steps ago.

An internal state of the neuron can be defined as follows:

\[ y(t+1) = A(t) - \alpha \sum_{d=0}^{t} k^d g\{x(t-d)\} - \Theta. \]

A reduced difference equation describing dynamics of the internal state can be represented as follows:

\[ y(t+1) = k y(t) - \alpha g[f\{y(t)\}] + \theta(t), \]

where \( \theta(t) = A(t) - k A(t-1) - \Theta(1-k) \).

When we set \( g(x) = x, \theta(t) = a \) which is temporally constant, and \( f(y) = 1/(1+\exp(-y/\varepsilon)) \), which is the logistic function with the steepness parameter \( \varepsilon \), the model shows not only periodic response but also chaotic response according to the parameter values [2] [3].

The chaotic neural network is composed of the chaotic neurons shown above, which is described by Eqs.4-6 [2]:

\[ x_i(t+1) = f[\eta_i(t+1) + \zeta_i(t+1)], \]

\[ \eta_i(t+1) = \sum_{j=1}^{N} w_{ij}(t) x_j(t) + k_j \eta_j(t), \]

\[ \zeta_i(t+1) = k_i \zeta_i(t) - \alpha \xi_i(t) + \alpha_i, \]

where \( w_{ij}(t) \) is the synaptic weight to the \( i \) th neuron from the \( j \) th neuron, \( k_f \) is the decay parameter for the feedback inputs, and \( k_r \) is the decay parameter for the refractoriness, \( \alpha_i \) denotes the sum of the threshold and the temporally constant external input to the \( i \) th neuron.

The weight of the dynamic synapse is separated into two parts as follows [11]:

\[ w_{ij}(t) = w_{ij}^s \cdot r_j(t), \]

where \( w_{ij}^s \) is a static term that is determined by the stored patterns; \( r_j(t) \) is the term that is caused by the synaptic depression.

\( w_{ij}^c \) is defined as follows:

\[ w_{ij}^c = 1 - N_j \sum_{j=1}^{N} (\mu_i^p - m)(\mu_j^p - m), \]
where $\mu_i^p$ is the $i$th component of the $p$th stored pattern, $N_i$ is the number of the stored patterns and $m$ is the average firing rate of the network for the $N_i$ patterns. The four stored patterns are shown in Fig. 1 (see [2] for details). As the component of the stored patterns $\mu_i^p$ takes a binary value, the $i$th component of the output pattern of the network is defined as follows:

$$o_i = \begin{cases} 
1 & x_i \geq 0.5, \\
0 & x_i < 0.5.
\end{cases}$$

Fig. 1 The four stored patterns. The output pattern of the 100 neurons is displayed in the form of a 10 by 10 matrix where the size of each square is proportional to the magnitude of the neuron output.

The model of the synaptic depression can be described as follows (see [11] for details):

$$r_i(t+1) = r_i(t) + \Delta t \cdot (1 - r_i(t)) / \tau - U \cdot r_i(t) \cdot x_i(t),$$

where $r_i(t)$, which affects the weight of the dynamic synapse (see Eq. 7), is the recovered synaptic resources of the neuron $i$; $\tau$ is the recovery constant; $\Delta t$ is the time step of discretization. At every time step, a fraction of unavailable resources ($(1 - r_i(t))/\tau$) is recovered, while a fraction of available resources $(U r_i(t) x_i(t))$ becomes unavailable due to the activity of the presynaptic neuron.

3. Simulation results

Since CNSD is composed of chaotic neurons and dynamic synapses, the parameters of CNSD are classified into two groups. The group of $\varepsilon$, $\alpha$, $k_f$, $k_r$ and $a_i$ is for the chaotic neuron model, and the other group of $\tau$ and $U$ is for the synaptic depression. As the dynamics of the chaotic neural network [2] can be studied by changing the parameter of the refractoriness $k_r$, and the effect of the synaptic depression can be adjusted by the parameter of $U$ in Eq. 9, we explore the characteristic of the memory retrieval behavior of the CNSD by changing these two parameters. If the refractoriness and the synaptic depression are both strong, the behavior of CNSD seems somewhat like random. Therefore, we fix one of the two parameters at a small value, increasing the other parameter from a small value to a large value. Other parameters are fixed following related literatures [2][11]. We fix $\varepsilon = 0.015, \alpha = 6, k_f = 0.1, a_i = 2$ and $\tau = 10$ in this paper.

Fig. 2 Memory retrieval of CNSD under the parameters of $\varepsilon = 0.015, \alpha = 6, k_f = 0.1, k_r = 0.6, a_i = 2$, $\tau = 10$ and $U = 0.08$. The time step $t$ is from 0 to 100.
Fig. 2 shows an example of the memory retrieval process of CNSD. CNSD visits a stored pattern (reversed stored pattern), escapes from the stored pattern, searches for next stored pattern, and once again visits another stored pattern. Furthermore, if the parameter values of CNSD are appropriately adjusted, ANCNSD switches from one stored pattern to another stored pattern more quickly than a chaotic neural network without synaptic depression. For example, it take only about 15 time steps for the network to complete the memory switching in Fig. 2, implying that it switches more rapidly than a chaotic neural network (see [2] for details). The other subject in this paper is to explore the memory (stored pattern and reverse stored pattern) retrieval frequency, and Tab. 1, Tab. 2 and Tab. 3 show the simulation results.

Tab. 1 is the retrieval frequency of the stored patterns and reverse stored patterns with \( U \) fixed at 0.01 and \( k_r \) increased from 0.7 to 0.8. Tab. 2 is the memory retrieval frequency with \( k_r \) fixed at 0.65 and \( U \) increased from 0.07 to 0.1. Tab. 3 is the memory retrieval frequency with four randomly generated initial conditions. To characterize the memory retrieval frequency of CNSD, we define the rate of memory retrieval as follows:

\[
R = \frac{\text{the time steps of the network visiting the stored patterns}}{\text{the total time steps of the simulation}}
\]  

We decide that the neural network is visiting a stored pattern (reverse stored pattern) if the Hamming distance of the output pattern of the neural network equals that of the stored pattern exactly.

Tab. 1, Tab. 2 and Tab. 3 show that with various parameter value settings and with various initial conditions, the retrieval frequency of CNSD distributes nearly uniformly among the four stored patterns, and the rate of memory retrieval in CNSD is around 0.29, which is much higher than that in a chaotic neural network [2] (around 0.03).

**Tab. 1** Memory (stored patterns and reverse stored patterns) retrieval frequency with \( U \) fixed at 0.01 and \( k_r \) increased from 0.7 to 0.8. The total time step of the simulation is 5000

| \( k_r \) | 0.7 | 0.75 | 0.8 |
|---|---|---|---|
| Pattern | Stored | Reverse | Stored | Reverse | Stored | Reverse |
| 172 | 200 | 141 | 185 | 188 | 195 |
| 224 | 422 | 270 | 270 | 156 | 169 |
| 58 | 229 | 183 | 365 | 118 | 167 |
| 282 | 310 | 184 | 185 | 241 | 205 |
| \( R \) | 0.38 | 0.36 | 0.29 |

**Tab. 2** Memory (stored patterns and reverse stored patterns) retrieval frequency with \( k_r \) fixed at 0.65 and \( U \) increased from 0.07 to 0.1. The total time step of the simulation is 5000

| \( U \) | 0.07 | 0.09 | 0.08 | 0.1 |
|---|---|---|---|---|
| Pattern | Stored | Reverse | Stored | Reverse | Stored | Reverse | Stored | Reverse |
| 183 | 221 | 91 | 92 | 91 | 90 | 167 | 166 |
| 75 | 223 | 90 | 180 | 197 | 144 | 125 | 166 |
| 185 | 219 | 46 | 91 | 54 | 92 | 171 | 170 |
| 452 | 255 | 182 | 45 | 288 | 183 | 208 | 86 |
| \( R \) | 0.36 | 0.16 | 0.23 | 0.26 |
Tab. 3 Memory (stored patterns and reverse stored patterns) retrieval frequency in four runs. Each run starts with randomly generated initial conditions. $k_r$ is fixed at 0.8 and $\mu$ is fixed at 0.01. The total time step of the simulation is 5000.

|       | Run 1 | Run 2 | Run 3 | Run 4 |
|-------|-------|-------|-------|-------|
| Pattern Stored | Reverse | Stored | Reverse | Stored | Reverse |
| 184 | 199 | 174 | 187 | 174 | 191 | 183 | 193 |
| 164 | 175 | 162 | 173 | 154 | 169 | 165 | 176 |
| 114 | 157 | 120 | 163 | 128 | 172 | 113 | 161 |
| 245 | 208 | 238 | 206 | 243 | 215 | 245 | 211 |
| $R$ | 0.29 | 0.28 | 0.28 | 0.28 | 0.29 |

4. Qualitative analysis of the memory switching phenomenon

A memory switching phenomenon with following three properties is demonstrated in section 3:
1) CNSD switches from one stored pattern to another stored pattern rapidly.
2) Memory retrieval frequency of the stored patterns in CNSD is high.
3) Memory retrieval frequency is distributed nearly uniformly among the stored patterns.

We next make a qualitative analysis of the properties of (1) and (2). The analysis of the property of (3) is similar to that of (1) and (2), and is omitted in this paper.

4.1. The framework of the analysis
Suppose the neurons in the network are labeled as 1, 2, \cdots N. For simplicity, suppose that $a_i = a \cdot n_i(0) = 0, \xi_i(0) = 0$ and $r_i(0) = 1$ for $i = 1, 2, \cdots N$. Let $\Omega$ be the set that is composed of all the labels of the neurons: $\Omega = \{1, 2, \cdots N\}$. As shown in Fig. 3, let $A, B, C$ be three stored patterns.

Suppose pattern $A$ overlaps pattern $B$ and $E$ be the area of the overlap between $A$ and $B$. $A \subset \Omega, B \subset \Omega, C \subset \Omega, E = A \cap B$. Suppose that pattern $C$ does not overlap pattern $A$ or pattern $B$. Let $D = A \cup B \cup C, A' = \overline{B} \cap A, B' = \overline{A} \cap B$. Patterns $A, B$ and $C$ are stored in CNSD through the connection weights that are determined by Eq. 13. As shown in Fig. 3, let $w_{A'B'}$ denote the connection weight with one neuron from $A'$ and the other neuron from $B'$. Let $q_1 = w_{A'B'}^c$. By Eq. 8, we have $q_1 = w_{A'B'}^c = -\frac{2}{3} + \frac{m}{3(1-m)}$. Similarly, $w_{AC}^c = w_{BC}^c = w_{ED}^c = -\frac{2}{3} + \frac{m}{3(1-m)} = q_1$.

$q_2 = w_{A'E}^c = w_{B'E}^c = -\frac{1-m}{3m} - \frac{1}{3} + \frac{m}{3(1-m)}$, $q_3 = w_{CC}^c = w_{A'A'}^c = w_{BB'}^c = -\frac{1-m}{3m} + \frac{2m}{3(1-m)}$, $q_4 = w_{EE}^c = -\frac{2(1-m)}{m} + \frac{m}{3(1-m)}$, $q_5 = w_{CD}^c = w_{A'D}^c = w_{B'D}^c = -\frac{1}{3} + \frac{2m}{3(1-m)}$, and $w_{CE}^c = -1$. As $m$ is a small value in most associative neural networks, we have $q_1 < 0, q_2 > 0, q_3 > 0, q_4 > 0$, and $q_5 < 0$. 


4.2. Qualitative analysis of the properties of the memory switching

Since the neurons in the network can be divided into five areas: \( A^-, B^-, C, D, \) and \( E \) (see Fig. 3), we consider the output of the neurons in these areas respectively. Suppose the network sticks at pattern \( A^- \), namely, \( x_i(0) \approx 1 \) for \( i \in A \) and \( x_i(0) \approx 0 \) for \( i \notin A \).

(1) For neurons in \( A^- \)

According to Eqs. 4-6, for \( i \in A^- \),

\[
\eta(t+1) = \sum_{j=1}^{N} w_{ij} r_j(t)x_j(t) + k_i \eta_i(t)
\]

\[
= \sum_{j \in A} w_{ij} r_j(t)x_j(t) + \sum_{j \in B} w_{ij} r_j(t)x_j(t) + \sum_{j \in C} w_{ij} r_j(t)x_j(t) + \sum_{j \in D} w_{ij} r_j(t)x_j(t) + k_i \eta_i(t)
\]

\[
= \sum_{j \in A} \sum_{i \in A} |q_{ij}| r_j(t) x_j(t) + \sum_{j \in B} -|q_{ij}| r_j(t) x_j(t) + \sum_{j \in C} \sum_{i \in C} q_{ij} r_j(t) x_j(t) + \sum_{j \in D} \sum_{i \in D} \eta_i(t)
\]

\[
\zeta_i(t+1) = k_i \zeta_i(t) - \alpha \zeta_i(t) + a_i
\]

As the network sticks at pattern \( A^- \), therefore:

\[
\eta(t+1) \approx \sum_{j \in A} q_{ij} r_j(t) + \sum_{j \in B} q_{ij} r_j(t).
\]

\[
\zeta_i(t+1) \approx k_i \zeta_i(t) - \alpha + a_i,
\]

\[
r_i(t+1) \approx r_i(t) + \Delta \cdot (1 - r_i(t)) / \tau - U \cdot r_i(t),
\]

\[
x_i(t+1) = f(\eta_i(t+1) + \zeta_i(t+1)).
\]
From Eq. 14, since \( \alpha > a_i \), \( \zeta_i(t+1) \), which characterizes the effect of refractoriness, decreases with time. As \( r_i(0) = 1, r_i(t) \) of Eq. 15 decreases with time as well due to the synaptic depression. This causes \( \eta_i(t+1) \), which is the feedback input from other neurons to neurons in \( A^- \), to decrease with time. Therefore, the effect of refractoriness, together with the effect of the synaptic depression, makes the output of neurons in \( A^- (x_i(t+1) \) in Eq. 16) decrease. With the decreasing of the output of neurons in \( A^- \), the network no longer sticks at pattern \( A \).

It is worth noting from Eq. 14 that the effect of refractoriness itself can drive the network away from the pattern \( A \) if the parameter of \( k_r \) is large enough. If the network sticks at pattern \( A \), the effect of the refractoriness and the external stimulation of the \( i \)th neuron accumulates with time as follows (see Eq. 14):

\[
\eta_i(t+1) = k_r \eta_i(t) + \sum_{j \in A^-} q_i |r_j(t) x_j(t)| + \sum_{j \in B^-} q_i |r_j(t) x_j(t)| + \sum_{j \in C^+} q_i |r_j(t) x_j(t)| + \sum_{j \in C^+} q_i |r_j(t) x_j(t)|.
\]

If \( k_r \) is large enough, \( |\zeta_i(t+1)| \) becomes so large that neuron \( i \) stops firing due to the negative term of \( \zeta_i(t+1) \) in Eq. 16. Therefore, the chaotic neural network without synaptic depression can also escape from pattern \( A \). However, it is obvious that CNSD escapes from pattern \( A \) more quickly with contributions from both the effect of refractoriness and the effect of synaptic depression. On the other hand, though a large value of \( k_r \) can make a chaotic neural network without synaptic depression escape from the fixed point quickly (see Eq. 17), we argue in next section that a large value of \( k_r \) causes the memory retrieval frequency to decrease.

(2) For neurons in \( B^- , C , D , \) and \( E \)

Similar to the calculation of the neurons in \( A^- \), for \( i \in B^- \):

\[
\eta_i(t+1) = k_r \eta_i(t) + \sum_{j \in A^-} q_i |r_j(t) x_j(t)| + \sum_{j \in B^-} q_i |r_j(t) x_j(t)| + \sum_{j \in C^+} q_i |r_j(t) x_j(t)| + \sum_{j \in C^+} q_i |r_j(t) x_j(t)|.
\]

For \( i \in C \):

\[
\eta_i(t+1) = k_r \eta_i(t) + \sum_{j \in A^-} q_i |r_j(t) x_j(t)| + \sum_{j \in B^-} q_i |r_j(t) x_j(t)| + \sum_{j \in C^+} q_i |r_j(t) x_j(t)| + \sum_{j \in C^+} q_i |r_j(t) x_j(t)|.
\]

For \( i \in B^- \) and \( i \in C \):

\[
\zeta_i(t+1) = k_r \zeta_i(t) - \alpha \eta_i(t) + a_i,
\]

\[
x_i(t+1) = f\{\eta_i(t+1) + \zeta_i(t+1)\}.
\]
kept between neurons in $B^−$ and neurons in $C$. When the network is visiting pattern $A$, namely, $x_i(0) \approx 1$ for $i ∈ A$ and $x_i(0) \approx 0$ for $i \not∈ A$. According to Eq. 18 and Eq. 19:

$$\eta_i(t+1) = \sum_{j ≠ i} -|q_i| r_j(t)x_j(t) + \sum_{j ≠ i} q_j |r_j(t)x_j(t)|$$ for $i ∈ B^−$, (22)

$$\eta_i(t+1) = \sum_{j ≠ i} -|q_i| r_j(t)x_j(t) + \sum_{j ∈ E} -r_j(t)x_j(t)$$ for $i ∈ C$, (23)

$$\zeta_i(t+1) ≈ k_r \zeta_i(t) + a_i$$ for $i ∈ B^−$ and $i ∈ C$. (24)

As discussed above, the neurons in $A^−$ begin to stop firing due to both the effect of refractoriness and synaptic depression after the network stays at pattern $A$ for a certain period of time. Consequently, $\eta_i(t+1)$ (see Eq. 22 and Eq. 23) increases for the neurons in $B^−$ and $C$ since the negative quantity of the feedback input from neurons in $A^−$ decreases. $\zeta_i(t+1)$ also increases with time as $\zeta_i(t+1)$ accumulates the external input with time according to Eq. 24. Since the feedback input from neurons in $E$ ($E ⊂ A$) to neurons in $B^−$ is positive (the term of $\sum_{j ∈ E} q_j |r_j(t)x_j(t)|$ in Eq. 22), while the feedback input from neurons in $E$ ($E ⊂ A$) to neurons in $C$ is negative (the term of $\sum_{j ∈ E} -r_j(t)x_j(t)$ in Eq. 23), it is the neurons in $B^−$ other than the neurons in $C$ begin to fire with the increasing of $\eta_i(t+1)$ and $\zeta_i(t+1)$. Thus the neurons in $B^−$ win the competition of firing against the neurons in $C$. The firing of neurons in $B^−$ makes further the neurons in $A^−$ stop firing through the negative connection weight (the term $\sum_{j ∈ E} -|q_i| r_j(t)x_j(t)$ in Eq. 11), and makes the neurons in $B^−$ fire more easily (see the term of $\sum_{j ∈ E} q_j |r_j(t)x_j(t)|$ in Eq. 18). The neural network thus becomes a positive feedback system that prevents firing of the neurons in $A^−$ and enhances firing of the neurons in $B^−$. Due to this positive feedback mechanism, the output of the neurons in $A^−$ decreases to nearly 0 and the output of the neurons in $B^−$ rises to nearly one rapidly.

As neurons in $B^−$ win the competition of firing against the neurons in $C$, if the parameter value of $k_r$ is set appropriately, the neurons in $C$ keep quiet due to the negative feedback input from neurons in $B^−$ (see Eq. 19). However, if $k_r$ is too large, neurons in $C$ can not keep quiet. Suppose neurons in $C$ keep quiet, $\zeta_i(t+1)$ accumulates with time until it approach $\frac{a_i}{1−k_r}$ (see Eq. 24). If $k_r$ is large enough, $\zeta_i(t+1)$ will get large enough to cause the neurons in $C$ to fire. Similar to the discussion of the neurons in $C$, the neurons in $D$ keep quiet as well if $k_r$ is set appropriately. If $k_r$ is too large, both the neurons in $C$ and the neurons in $D$ no longer keep quiet. The firing of neurons in $C$ and $D$ destroys the completeness of pattern $A$ or pattern $B$. Thus too large a $k_r$ makes the memory retrieve frequency decrease.

Similar to the aforementioned analysis, $\eta_i(t+1)$ for $i ∈ E$ is positive and is large enough to keep the neurons in $E$ firing, when the network sticks at either the pattern $A$ or pattern $B$.

Combined the discussion in (1) with that in (2) in this section, we conclude that CNSD switches from pattern $A$ to pattern $B$ quickly due to both the effect of refractoriness and the synaptic
depression. Since the neurons in $C$ and $D$ are kept quiet after CNSD switches from pattern $A$ to pattern $B$, the memory retrieval frequency is high for CNSD. While for a chaotic neural network without synaptic depression, though a large $k_r$ can make the network escape from pattern $A$ quickly, the memory retrieval frequency of the network is low, as the neurons in $C$ and $D$ can not be kept quiet for a large value of $k_r$.

5. Conclusion and Discussion

A neural network with chaotic neurons and dynamic synapses (CNSD) is constructed. The associative dynamics in such a network is similar to that in a chaotic neural network [2]. However, CNSD transits more quickly among the stored patterns than a chaotic neural network. Furthermore, the rate of memory retrieval of CNSD is much higher than that in a chaotic neural network [2], and the retrieval frequency among the stored patterns in CNSD is nearly uniformly distributed. The memory switching phenomenon are believed to have relationship with the dynamical behavior of the olfactory system. For instance, the phenomenon is linked to the spatiotemporal encoding scheme of the olfactory cortex [5], the recognition of olfactory information [5], rapid and unbiased access to the learned patterns in the olfactory system [4], or classifying and learning novel patterns in the olfactory system [4]. The memory switching phenomenon may also be used to explain the mechanism of the goal-directed behaviors in working memory [10]. Therefore, the peculiar properties of CNSD, namely the quick memory switching and nearly uniformly distributed memory retrieval frequency of CNSD, might help CNSD to play an important functional role in real neural systems.

As a formal analysis of CNSD is extremely difficult, we have discussed the properties of CNSD in a simplified framework. It is worth noting that the discussion could be extended to a more general situation, for instance, to a situation with the average firing rate $m$ being any a small value, and to a situation with the stored patterns $A$, $B$, and $C$ overlapping with each other. According to the analysis of the properties of CNSD in section 4, due to both the effect of refractoriness and that of synaptic depression, CNSD can switch from pattern $A$ to another stored pattern more rapidly than a chaotic neural network with only the effect of refractoriness [2], than a network with dynamical thresholds [6] which only includes a kind of refractoriness, and than a network with only the effect of synaptic depression [11].

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