D-brane Bound States and the Generalised ADHM Construction

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ABSTRACT

We discuss the sigma model description of a D-string bound to \( k \) D-fivebranes in type I string theory. The effective theory is an \((0,4)\) supersymmetric hyper-Kähler with torsion sigma model on the moduli space of \( Sp(k) \) instantons on \( \mathbb{R}^4 \). Upon toroidal compactification to five dimensions the model is related to the type II picture where the target space is a symmetric product of \( K3 \)'s.

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1. Introduction

In the past few years a tremendous amount of progress has been made towards understanding the non-perturbative features of string theory. The main insights have been obtained by studying the BPS states and in particular, BPS bound states. A central theme in the present work has been the emergence of classical geometry as a derived concept from the underlying gauge theories and conformal field theories.

D-branes are responsible for a large number of the recent advances. In an elegant paper [1] Douglas showed that a D-string probe placed parallel to $k$ D-fivebranes in type I string theory has Witten’s ADHM massive sigma model [2] as an effective action. The 4 massless modes correspond to the positions of the D-string in the space transverse to the D-fivebrane and the $4k$ massive modes correspond to stretched strings between the fivebranes and the string. To obtain the low energy action for the massless modes of the D-string (which are the worldsheet fields of a heterotic string) one must integrate over the massive modes. This leads to an $(0,4)$ supersymmetric heterotic sigma model with a four dimensional target space containing an ADHM instanton with instanton number $k$. Thus, viewed as a probe of the D-fivebrane geometry, the D-string explicitly shows the equivalence of a spacetime instanton of instanton number $k$ with a configuration of $k$ D-fivebranes within type I string theory.

However this picture contains another phase [2,3]. When the spacetime instanton is shrunk to zero size another branch of the vacuum moduli space appears with a correspondingly different low energy description. If the D-string sits on the D-fivebranes then there is a branch of vacua where the $4k$ stretched strings are massless and the 4 transverse fields massive. In this phase the D-string is bound to the D-fivebrane. If we now calculate the conformal fixed point theory describing the massless modes we find a heterotic sigma model with a $4k$ dimensional target space containing an “ADHM instanton of instanton number one”†.

† Precisely what is meant by an ADHM instanton in $4k$ dimensions will be given below.
In this paper we will focus on the bound phase of the ADHM sigma model. We will provide an explicit description of the D-string/D-fivebrane bound state of type I string theory in terms of the sigma model for the D-string massless modes. Since most of the work on D-brane bound states has concentrated on type II strings it will be helpful to see some of the details in the type I case and compare the two. In particular for the type IIA string on $K3$ the corresponding bound state is described by a $(4,4)$ supersymmetric sigma model with a symmetric product of $K3$’s as the target space. We will be able to relate this picture to the ADHM sigma model thus providing a check on type I/type II string duality and the description of a gas of D-zero-branes [4].

In a different but not unrelated context, recent studies of the matrix description of M theory have involved some $(4,4)$ supersymmetric analogs of the models considered here [5, 6]. In gauge theory terminology of the non-chiral $(4,4)$ models the probe and bound phases described here correspond to the Coulomb and Higgs branches of the D-string’s vacuum moduli space respectively. However there are some crucial distinctions. In the models considered below there are no vector multiplets and hence no Coulomb branch (the two phases are both Higgs branches). A second distinction arises from the chiral nature of the theories considered here. It was argued in [6] that there are no quantum (i.e. $\alpha'$) corrections on the Higgs branch of $(4,4)$ sigma models so that the classical solution obtained by the hyper-Kähler quotient construction is exact. However this argument fails for the chiral models considered here because there are anomalies which lead to an infinite series of $\alpha'$ corrections to the metric and anti-symmetric tensor. Indeed we shall discuss the first order corrections in the next section.
2. The Bound State Sigma Model

We consider \(k\) D-fivebranes located at \(x^6 = x^7 = x^8 = x^9 = 0\) with a D-string worldsheet placed in the \((x^0, x^1)\) plane and use the notation of [1]. D-brane quantisation rules tell us that the massless modes (in the probe phase) of the D-string are given by \((1|1)\) strings which yield 8 bosons \(b^{AY}, b^{AA'}\) and their superpartners \(\psi_+^{A'Y}, \psi_+^{AA'}\) \((A, A', Y = 1, 2)\). These simply describe the motion of the D-string in the space transverse and tangent to the D-fivebranes respectively. Since we are in type I string theory there are also \((1|9)\) strings which yield the fermions \(\lambda_+^M\) \((M = 1, 2, ..., 32)\) taking values in the spacetime \(SO(32)\) gauge bundle. These make up the worldsheet degrees of freedom of a heterotic string. However the presence of the D-fivebranes induces \(4k\) massive modes \(\phi_{+m}\) (here \(m = 1, 2, ..., 2k\) labels the fivebranes), their superpartners \(\chi_{-m}\) and the additional fermions \(\lambda_{+m}^Y\) on the D-string. These given by the stretched \((1|5)\) strings (with a mass proportional to the distance to the D-fivebranes) and introduces a potential which breaks the \((0, 8)\) worldsheet supersymmetry to \((0, 4)\).

In this paper we are only interested in the case of vanishing instanton size (which corresponds to setting the \((5|9)\) strings of the D-fivebranes to zero [10]). The effective action for D-string is given by Witten’s ADHM sigma model [1,2]

\[
S = S_{\text{free}} - \int d^2 x \left\{ \frac{im}{2} \lambda_+ Y_{m} \phi_{B'}^m \psi_{-}^{B'Y} + \frac{im}{2} \lambda_+ Y_{m} b_{B'}^m Y \chi_{-m} + \frac{m^2}{8} \phi^2 b^2 \right\}. \tag{2.1}
\]

Note the \(b^{AA'}\) and \(\psi_+^{AA'}\) fields are free because of translational symmetry in the \(x^2, x^3, x^4, x^5\) plane and the \((1|9)\) fields \(\lambda_+^M\) have decoupled in the limit of vanishing instanton size.

The probe phase corresponds to choosing the vacuum \(\phi_{+m} = 0\). Here the \(b^{AY}\) fields are massless and describe the location of the D-string relative to the D-fivebranes. By integrating over the massive modes one obtains an action for the

* By a \((p|q)\) string we mean a string with one end on a D-p-brane and the other on a D-q-brane.
massless fields. This turns out to be a \((0,4)\) supersymmetric sigma model with a four dimensional target space containing zero sized instanton gauge field with instanton number \(k\) along with gravitational corrections \([2,7,8,9]\). Furthermore the construction of the effective action parallels the ADHM construction of instantons (by turning on the \((5|9)\) strings one finds the full ADHM construction of finite sized instantons \([1,10]\)).

The bound phase corresponds to choosing the vacuum \(b^{AY} = 0\). In this case the \(\phi^{Am}\) fields are massless and describe the internal state of the D-string within the D-fivebranes. Now the \(b^{AY}\) fields are massive indicating that the D-string has become bound to the D-fivebranes. If we then integrate over the massive fields to obtain the effective action for the massless modes we find an \((0,4)\) supersymmetric sigma model with a 4\(k\) dimensional target space. However, as in the probe phase a non-trivial gauge field appears in the low energy effective action. Furthermore the construction of this gauge field again parallels the ADHM construction of instantons but this time generalised to 4\(k\) dimensions \([11,12]\) where we will find a zero-sized instanton of instanton number one.

Before proceeding it is necessary to clarify we mean by instantons and instanton number in 4\(k\) dimensions. It was shown in \([12]\) that the ADHM construction may be generalised to 4\(k\) dimensions. This procedure then produces a gauge field whose curvature \(F\) is self-dual in the sense that

\[
F_{A'mB'n} = \epsilon_{A'B'}F_{mn},
\]

(2.2)

with \(F_{[mn]} = 0\). Such a gauge field then automatically solves the Yang-Mills field equations. In contrast to the four dimensional case where the ADHM construction is known to produce all self-dual gauge fields, in higher dimensions there is no such uniqueness. This can be most easily understood by noting that above four dimensions any notion of self-duality must break rotational symmetry down to some subgroup. There are a variety of possible subgroups \([13]\) and hence a variety of notions of self-duality. However, for the purposes of this paper we shall only need
those configurations which are produced by this generalised ADHM construction for which $SO(4k)$ is broken to $SU(2) \times Sp(k)$. Note also that the definition (2.2) makes no reference to the dimension and so can be most readily generalised.

The next point to consider is the definition of instanton number. In four dimensions gauge fields are divided into topological sectors labelled by their first Pontryagin index. In general there is no such topological classification for higher dimensional gauge fields. However it turns out that the instantons we shall consider may be divided into topological sectors labelled by the first Pontryagin index \cite{12}

$$p_1 = \frac{1}{16\pi^2} \int_M \text{Tr}(F \wedge F),$$

where $M$ is any of the four dimensional coordinate hyperplanes in $R^{4k}$. We therefore define the instanton number to be $p_1$ for any $k$.

To obtain the conformal fixed point action of (2.1) it is helpful to first blow up the instanton to a finite size. This is done by adding additional interactions to (2.1), parameterised by a size $\rho$, which are still consistent with $(0, 4)$ supersymmetry and preserve the bound state (Higgs) branch of the vacuum (there is an essentially unique way to do this \cite{2}). This is analogous to the four dimensional models found in the probe phase \cite{1,9}. One then looks for the massless fields which induce a self-dual connection (as constructed by the ADHM method) to appear in the classical low energy effective action \cite{1,11}. Only when the size is non-zero, and the solution is everywhere smooth can the instanton number be easily seen to be one. The field strength for the fixed point is then found by taking the limit of vanishing instanton size; $\rho \to 0$. The generalised ADHM construction can produce finite size instantons although it is not clear what if anything this corresponds to physically. The only fields we have set to zero are the $(5, 9)$ strings and if we were to turn these on then the vacuum states $b^{AY} = 0$ would disappear and the bound state phase would no longer exist. Indeed we will see below that in the bound phase these $(5, 9)$ strings are in fact massive.
A comparison of the field content we are using with the definitions in [12] shows that we are producing ADHM instantons with instanton number one in $4k$ dimensions with gauge group $Sp(k)$ (specifically we have $r = k$ and $l = 1$ in the notation of [12]). To be more explicit we obtain the following gauge field strength on $\mathbb{R}^{4k}$ [11]

$$F_{pq}^{mn} = -\frac{4}{\phi^6} \left( \phi^2 \delta_{(m}^p - 2 \epsilon_{C'D'} \phi^C \phi_{(m}^{D'} \right) \left( \phi^2 \delta_{n)}^q - 2 \epsilon_{E'F'} \phi^{E'} \phi_{n)}^{F'} \right). \quad (2.4)$$

The $(0, 4)$ supersymmetric sigma model conformal fixed point for the D-string in it’s bound state is therefore (to zeroth order in $\alpha'$)

$$S = \int d^2 x \left\{ \epsilon_{AB} \epsilon_{A'B'} \partial_+ b_{+}^{A'} \partial_+ b_{+}^{B'} + i \epsilon_{AB} \epsilon_{A'B'} \psi_{-}^{A'} \partial_\pm \psi_{-}^{B'} + i \delta_{MN} \lambda_{+}^{M} \partial_\pm \lambda_{-}^{N} + g_{A'mB'n} \partial_+ \phi_{-}^{A'} \partial_+ \phi_{-}^{B'} + i g_{A'mB'n} \chi_{-}^{A'm} \nabla_{+}^{(-)} \chi_{-}^{B'n} + i \epsilon_{YZ} \epsilon_{mn} \lambda_{+}^{Ym} \nabla_{+}^{Zn} - \frac{1}{2} \chi_{-}^{A'm} \chi_{-}^{B'n} \epsilon_{A'B'} \epsilon_{YZ} F_{pq}^{mn} \lambda_{+}^{p} \lambda_{+}^{q} \right\}, \quad (2.5)$$

where $\nabla$ is the covariant derivative of the gauge connection. Due to it’s chiral nature this theory is anomalous and generates a torsion at order $\alpha'$ which can be calculated as the Chern-Simons term for the gauge connection. This appears in (2.5) via the covariant derivative with torsion $\nabla^{(-)}$. The model (2.5) will in addition receive higher order corrections to the metric and dilaton in the form of finite local counter terms need to ensure finiteness (which is equivalent to requiring that the renormalisation prescription preserves the extended supersymmetry [14]). To first order in $\alpha'$ the metric is [11]

$$g_{A'mB'n} = \left( 1 + \frac{16 \alpha'}{\phi^2} \right) \epsilon_{A'B'} \epsilon_{mn} - \frac{16 \alpha'}{\phi^2} \epsilon_{A'B'} \epsilon_{C'D'} \phi_{m}^{C'} \phi_{n}^{D'}, \quad (2.6)$$

In contrast the gauge field is exact to all orders in $\alpha'$.

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* The factor of 16 corrects a factor of 12 in [9,11].
Now that we have found the bound state sigma model we should compare it with the description obtained from the D-fivebrane’s point of view. It’s worldvolume dynamics is that of a $D = 6$, $N = 1$ gauge theory and when $k$ D-fivebranes collapse to a point we obtain the gauge group $Sp(k)$ [10]. There is a vector multiplet and a hypermultiplet whose bosonic components $X^{AY}$ describe the location of the D-fivebrane which come from the $(5,5)$ strings. There are also hypermultiplets $h^M_{Am}$ coming from the $(5,9)$ strings [10] although we have set these to zero in the bound state phase. In fact it was argued in [3] that the $D = 6$, $N = 1$ gauge theory on the D-fivebrane worldsheet has the same potential as the D-string which is

$$V = \phi^2 (h^2 + X^2) . \quad (2.7)$$

Thus in the vacuum $h^M_{Am} = X^{AY} = 0$, where the D-string is bound to the D-fivebrane, both the $(5,5)$ and $(5,9)$ strings are massive and so cannot be turned on.

Now the D-string appears as a 1-brane soliton corresponding to an instanton of instanton number one in the space transverse to the string. The moduli space of $Sp(k)$ instantons with instanton number one is $4k + 4$ dimensional, which is precisely the number of bosonic degrees of freedom in the model (2.5) [3]. We therefore interpret the target space of the sigma model (2.5) to be the moduli space of instanton number one self-dual gauge fields on $\mathbb{R}^4$ with gauge group $Sp(k)$. In particular the $b^{AA'}$ fields describe the centre of the instanton and the $\phi^{A'm}$ fields describe its internal structure. The self-dual gauge field $F_{A'mB'n}^{pq}$ can hence be thought of as an “instanton on instanton moduli space”.

Let us examine this interpretation in the simplest case, $k = 1$, which is well known as the zero-sized gauge fivebrane [7,8]. Here $F_{mn}^{pq}$ vanishes everywhere (except at $\phi^{A'm} = 0$ where it is singular). As mentioned above the $b^{AA'}$ fields simply describe the position of the centre of the instanton in the D-fivebrane. The four $\phi^{A'm}$ then describe the $Sp(1) \cong S^3$ orientation of the instanton and its size. Thus, even though we started by shrinking the spacetime $SO(32)$ instanton to zero size, we recover the full moduli space of instantons on the D-fivebrane’s worldsheet.
Note that in the D-fivebrane metric (2.6) the zero size instantons appear at an infinite distance in the moduli space. This region, which can be described by a $SU(2)$ WZW model [8,15], is a stringy version of the collar neighbourhood in Donaldson’s moduli space. Furthermore, since this WZW model is free of higher order sigma model corrections it should provide a good description until the massive D-string fields become too light to be ignored.

Let us factor out the action of translation in the full moduli space of instantons, with instanton number one, on $\mathbb{R}^4$ (i.e. include rotations by the gauge group):

$$\mathcal{M}(\mathbb{R}^4) = \mathbb{R}^4 \times \mathcal{M}_1(\mathbb{R}^4),$$  \hspace{1cm} (2.8)

where $\mathcal{M}_1(\mathbb{R}^4)$ is the internal structure of the instanton and $\mathbb{R}^4$ is the location of its centre. The manifold $\mathcal{M}_1(\mathbb{R}^4)$ then possesses a singular point at the origin where the instantons are shrinking to zero size, which we have already discussed. One may also think of infinity as a singular point, where the instanton is spreading out over all of $\mathbb{R}^4$. At this point the action of translating the instanton centre has a fixed point and one cannot simply factor out the $\mathbb{R}^4$ from (2.8). However in the metric (2.6) both these points are at an infinite distance and the space $\mathcal{M}_1(\mathbb{R}^4)$ is smooth. Thus the heterotic sigma model smooths out the moduli space by moving the singular points to spatial infinity.

Another point of note is that the metric which appears in the sigma model (2.5) is hyper-Kähler with torsion, whereas with BPS monopole moduli spaces the natural metric (that is inherited from the Energy density functional) is simply hyper-Kähler. However since we are considering string theory on these manifolds it is natural to interpret the target space as a stringy generalisation of the moduli space metric. (Recently hyper-Kähler with torsion geometries have also appeared on black hole moduli spaces [16].) In a sense the gauge fivebrane metric may be viewed as a hyper-Kähler with torsion blow up of the moduli space. In particular the $SU(2)$ WZW model describes the sigma model in the region of the blow up at the origin (see also [17] where WZW models make a similar appearance). Most
often only hyper-Kähler resolutions of singular points are considered, which may always be embedded in both the heterotic and type II string theories. However the fact that both types of resolution appear in string theory can be easily seen by T-duality. For example consider a closed string theory on $\mathbb{R}^4/\mathbb{Z}_2$. As is well known the twisted sector states act to blow up the singularity into the hyper-Kähler Eguchi-Hanson space. However this space contains a compact killing direction and so can be T-dualised into a form of the symmetric fivebrane. This latter structure, which is equivalent and moreover indistinguishable within string theory to the Eguchi-Hanson space, is hyper-Kähler with torsion.

For $k \geq 2$ the model (2.5) becomes more complicated. In contrast to the $k = 1$ case the gauge field is non-trivial and the metric is no longer conformally flat and rotationally invariant. In higher dimensions there are $k$ natural four dimensional coordinate subplanes where the metric simply reduces to the $k = 1$ case and the four fermion term vanishes (if the fermions are taken to lie in the tangent space to the hyperplane). Clearly this corresponds to taking the instanton to lie in some $SU(2)$ subgroup of $Sp(k)$. In these models the structure is again hyper-Kähler with torsion but now the torsion is no longer a closed form (because the field strength and hence the chiral anomaly is nonzero) and these solutions cannot be embedded in type II theories. Although in the next section we will see that they may in some sense be thought of as dual to type IIA hyper-Kähler structures.

We could generalise the previous construction by considering $l$ D-strings bound together. In this case we would need to examine the infra-red fixed point of an $SO(l) \times Sp(k)$ gauge theory in two dimensions [3]. The vacua of this theory correspond to the moduli space of $Sp(k)$ instantons with instanton number $l$ and the low energy effective action that we would obtain for the D-strings would provide a hyper-Kähler with torsion metric on this moduli space. It is natural to suppose that these fixed point actions would involve $Sp(k)$ ADHM instantons of instanton number $l$ in $4(k + 1)l$ dimensions.
3. Type I/Type II Duality

In the previous section we examined the sigma model description of the bound state of a type I D-string with \( k \) D-fivebranes, let us now consider an application. To this end suppose we compactify to five dimensions by wrapping the D-string/D-fivebrane bound state around \( T^5 \). By string/string duality type I string theory on \( T^5 \) is equivalent to either the type IIA or type IIB string theories on \( K3 \times S^1 \). From the type IIB point of view this bound state arises as \( k \) D-strings wrapped around \( S^1 \), bound to a single D-fivebrane wrapped around \( K3 \times S^1 \). Whereas in the IIA string theory the bound state arises from \( k \) D-zero-branes bound to a single D-fourbrane wrapped around \( K3 \). In the type II string theory Vafa [4] has argued that the low energy effective action for the massless bound state degrees of freedom (i.e. the \((0|4)\) or \((1|5)\) strings [18]) is a hyper-Kähler \((4,4)\) supersymmetric sigma model with the target space

\[
\mathcal{M}_k(K3) = (K3)^k/S^k.
\] (3.1)

Here \( \mathcal{M}_k(K3) \) is (up to topological equivalence) the moduli space of \( SU(2) \) instantons on \( K3 \) with instanton number \( k \) [17]. The states which preserve 1/2 of the supersymmetry of the sigma model on \( \mathcal{M}_k(K3) \) can be identified with black hole microstates which preserve 1/4 of the spacetime supersymmetry in the five dimensional theory [19]. Furthermore by counting these states, that is states composed of a right moving vacuum tensored with an arbitrary number of left movers, one can reproduce the Beckenstein-Hawking entropy formula [19]. Under string/string duality, the BPS states of this type II sigma model must be in a one-to-one correspondence with states of the \((1|5)\) strings in the ADHM sigma model compactified on \( T^5 \) which preserve all of the worldsheet \((0,4)\) supersymmetry.

* Note that string/string duality relates electrically charged states to magnetically charged ones and hence interchanges the roles of fivebranes and strings.
Note that the type IIB description arises from the effective action of the D-fivebranes. The ground states, which preserve one half of the spacetime supersymmetry, correspond to D-string states with no D-fivebranes present and thus do not have an equivalent in the ADHM sigma model constructed above, since there would then be no massive terms in the action (2.1). However, under duality these states are mapped to states of the free heterotic string preserving all of the (0,8) supersymmetry with charge $k$. These are in turn given by a tensor product of the right moving ground state with an arbitrary left moving state at level $k$. It was observed in [17] that the degeneracy of these states, which is the degeneracy of the left moving modes of a bosonic string at level $k$, is precisely the Euler number of the space $\mathcal{M}_k(K3)$. But this is also the number of vacua of the effective sigma model on $\mathcal{M}_k(K3)$ in the type II theory and thus provides a highly non-trivial test of string/string duality [4].

Let us examine the correspondence between the (1|5) strings of the ADHM model and the (1|5) strings in the type IIB model more closely. After compactifying the coordinates $x^1, ..., x^5$ on $T^5$ the ADHM sigma model fields $\phi^{A'm}$ also become periodic and the field strength and metric found above for no longer hold. However, given our interpretation of the target space we can still hope to make some comments. First we note that since $Sp(k)$ is simply connected there are no instantons of instanton number one on $T^4$ (see for example [20]). Thus the D-string cannot appear as a single (non-singular) instanton in the transverse space. Furthermore it cannot appear as an instanton with a higher instanton number since the dimension of the target space would then be larger than $4k$. Therefore we will assume that the D-string can only appear as a (singular) zero sized instanton. The moduli space for the (1|5) strings is then just a choice of the vacuum transverse to the D-string. This is analogous to the construction of the collar region in Donaldson’s moduli space by attaching zero sized instantons to a flat connection, only now there are no moduli corresponding to size and gauge rotations which would serve to blow up the instanton. Thus the target space for the $\phi^{A'm}$ can be identified with the space of Wilson lines.
An arbitrary Wilson line for the group $Sp(k)$ on $T^4$ can be diagonalised to

$$W_i = \text{diag}(e^{ia_1^i}, \ldots, e^{ia_k^i}, e^{-ia_1^i}, \ldots, e^{-ia_k^i}) ,$$

(3.2)

where the $a_1^i, \ldots, a_k^i$ parameterise $k$ copies of the torus $T^4$. However we must also divide out by the action of the Weyl group $[10]$. This is generated by $k \mathbb{Z}_2$ actions $a_p^i \rightarrow -a_p^i$ and also an $S_k$ action which permutes the $p = 1, \ldots, k$ indices. Thus we find the moduli space of Wilson lines to be

$$\mathcal{W}_k = \frac{(T^4/\mathbb{Z}_2)^k}{S^k}. \quad (3.3)$$

Therefore we need to consider the $(1|5)$ fields $\phi^A m$, $\chi_A^m$ and $\lambda_Y^m$ of the ADHM sigma model on the target space $\mathcal{W}_k$. We need to count states which are in the right moving vacuum with the left movers arbitrary. Although there is no left moving supersymmetry there are equal numbers of fermions and bosons and therefore the left movers are in a one-to-one correspondence with the left movers of the $(4,4)$ sigma model on $\mathcal{M}_k(K3)$. Next we need to count the number of right moving vacua. The right moving modes have the target space which is an orbifold limit of $(K3)^k/S^k$. Following the discussion in the previous section we expect the the sigma model metric is a hyper-Kähler with torsion resolution of $\mathcal{W}_k$. However the number of vacuum states is determined by the cohomology of this space and it is reasonable to assume that this is the same as a hyper-Kähler resolution of $\mathcal{W}_k$, i.e. $\mathcal{M}_k(K3)$. Thus we find that indeed there are the same number of BPS states as a $(4,4)$ sigma model on $\mathcal{M}_k(K3)$, in agreement with the type II picture $[4]$.

Finally we note that the ADHM sigma model (2.5) also contains a free action for the massless $(1|1)$ and $(1|9)$ fields $b^{AA'}$, $\psi_A^{AA'}$ and $\lambda^{M}$. These are the same worldsheet fields as a heterotic $SO(32)$ string on a torus $T^4$. In fact these are remnants of the full ten dimensional string theory and describe the internal string states in the compact dimensions. Under string/string duality these are mapped to type IIB $(1|1)$ strings which remain massless when the D-strings are bound to the
D-fivebrane where they form a $(4, 4)$ sigma model on $K3$. It is a central observation of heterotic/type II duality that the BPS states of both of these theories are in a one-to-one correspondence with the even self-dual charge lattice $\Gamma^{21,5}$. On the heterotic side these arise as the Narian lattice of internal momentum modes while on the type IIA side these arise as the wrapping/momentum modes of the type II D-branes around $K3 \times S^1$.

4. Comments

In this paper we have discussed some details of the bound state of a D-string with $k$ D-fivebranes in type I string theory. This provided an interpretation of the ADHM construction in higher dimensions within string theory and was related to the instanton moduli space on $K3$. Finally we wish to conclude with some comments.

The $k = 1$ and $k = 2$ sigma models discussed in section two may also be embedded directly into heterotic string theory in ten dimensions as $p$-brane solitons by simply adding on $(10-4k)$-dimensional Minkowski space. The $k = 1$ case is the well known gauge fivebrane [7,8] preserving $1/2$ of the $D = 10$, $N = 1$ supersymmetry. Here we have provided this sigma model with another interpretation as a string in the moduli space of $SU(2)$ instantons on $\mathbb{R}^4$. The $k = 2$ case may be interpreted as a string soliton preserving $1/4$ of the $D = 10$, $N = 1$ supersymmetries [11] and possessing a non-vanishing spacetime gauge field. In fact the notion of self-dual gauge fields in higher dimensions contains a variety of examples, each of which may be embedded as a heterotic string soliton preserving some supersymmetry. In this way one can also obtain 1-branes [21] and 2-branes [22,23] preserving $1/16$ and $1/8$ of the spacetime supersymmetry respectively (there may be other possibilities which have not yet been constructed [13]). To date the meaning of these solitons has remained obscure. One might think of them as intersecting branes but the singularity in the metric is at a single point in the transverse space and makes this identification difficult. Furthermore their mass per unit $p$-volume diverges. Since
we have provided the ADHM 1-brane with a rather different role describing bound
states this raises the possibility that these additional ‘exotic’ branes may also have
a similar interpretation as effective actions for D-string bound states, possibly after
compactification on a manifold with reduced holonomy.

One final point is that one can apparently pass smoothly from the spacetime
interpretation of the D-string’s effective action (probe phase) into a region where
it describes the internal state of the D-string inside instanton moduli space (bound
phase), although in our case these two phases are infinitely far from each other.
From the D-fivebrane point of view the D-string has “dissolved” [3] in the bound
state phase. On the D-string worldsheet this phenomenon is reflected by the re-
interpretation of the target space as the moduli space of instantons rather than
as spacetime. A similar phenomenon appears in the D-brane picture of black
holes. In this case the point where a D-string becomes bound to a D-fivebrane
\[(x^6 = x^7 = x^8 = x^9 = 0)\] corresponds to the event horizon in the low energy
supergravity effective action. Classically the D-string may then further fall beyond
the event horizon into the singularity. However it was suggested in [18] that in
this case the zero-modes of the D-string are internal states and do not represent
the it’s motion in physical spacetime, where it remains fixed at the event horizon.
This blurring of space and moduli space is very intriguing and is perhaps pointing
to a more fundamental understanding of string theory.

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