Color-octet mechanism and

$J/\psi$ polarization at LEP

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Abstract

Polarized heavy quarkonium productions in $Z^0$ decays are considered. We find that polarizations of the produced quarkonia are independent of that of the parent $Z^0$ provided that one considers the energy distribution or the total production rate. Produced $J/\psi$'s via the color-octet and the color-singlet mechanisms are expected to be 19% and 29% longitudinally polarized, respectively. The energy dependence of $\eta_{L,8}(x) \equiv \frac{d\Gamma_{L,8}}{dx} / \frac{d\Gamma_{1,8}}{dx}$ is very sensitive to the production mechanism, and therefore the measurement of $\eta(x)_{exp}$ will be an independent probe of the color-octet mechanism.

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Since Braaten and Fleming put forward the idea of the color-octet mechanism \[1\] as a possible solution to the so-called $\psi'$ puzzle at the Tevatron \[2\], there have been many activities applying this idea to other processes: heavy quarkonium (both $S$- and $P$-wave charmonium and bottomium) productions at the Tevatron \[3\], in $B$ decays \[1\], fixed target experiments \[3\], $\gamma p$ collisions \[3\], $e^+e^-$ annihilations at CLEO \[7\], and $Z^0$ decays at LEP \[8–10\]. Polarized heavy quarkonium productions were also considered as an independent check of the color-octet mechanism \[11,12\]. It is adopted in the calculations of the $\psi'$ polarizations at the Tevatron \[13\]. A new way to regularize the ultraviolet/infrared divergences in heavy quarkonium calculations was proposed in Ref. \[14\]. Also, some NRQCD matrix elements relevant to $S$- and $P$-wave heavy quarkonium decays were calculated on the lattice \[15\]. Some reviews of earlier literatures can be found in Ref. \[16\].

In the color-singlet model, the prompt $J/\psi$ production rate in $Z^0$ decays is dominated by charm quark fragmentation \[17\]. However, recent reports by OPAL collaboration \[18\] claim that they have observed an excess of events for $Z^0 \to \Upsilon(nS) + X$ (for $n = 1, 2, 3$), larger than the theoretical expectation by a factor of $\sim 10$, compared to the $b$–quark fragmentation contribution \[17\]. Similar excess was also observed in the prompt $J/\psi$ and $\psi'$ production in $Z^0$ decays, although the experimental errors are quite large \[19\]. It turns out that the color-octet gluon fragmentation suggested by Braaten and Fleming could fix this discrepancy through $Z^0 \to q\bar{q} + g$ followed by color-octet gluon fragmentation into $J/\psi$ with emission of soft gluons \[8,11\]. In Refs. \[8,11\], the energy distribution of the produced $J/\psi$ via the color-octet $c\bar{c}(3S_1^{(8)})$ intermediate state was shown to be dramatically different from that of the $J/\psi$ produced via the color singlet mechanism. Therefore, the $J/\psi$ energy distribution in the $Z^0$ decays could be another good test of the idea of the color-octet mechanism. In Ref. \[10\], the present authors have considered the angular distribution of $J/\psi$'s in $Z^0$ decays, and one can find whether the color-octet mechanism is working or not.

In this work, we suggest another observable, the polarization of $J/\psi$ at LEP produced via the color-singlet and the color-octet mechanisms. In short, $J/\psi$'s produced via the two channels, $c\bar{c}(3S_1^{(1)}) \to J/\psi$ and $c\bar{c}(3S_1^{(8)}) \to J/\psi$, have distinctively different polarizations.
The $J/\psi$ produced by the color-octet mechanism is about 19% longitudinally polarized, whereas $J/\psi$ by the color-singlet mechanism is about 29% longitudinally polarized.

When treating polarized quarkonium productions, one should take care of the soft process $Q\overline{Q}(^{2S+1}L,J) \rightarrow H$. Recently, Braaten and Chen developed a method for treating the polarized heavy quarkonium production [11] which we use here. Beneke and Rothstein also pointed out that the interference among different $^3P_J$ states occurs in a polarized heavy quarkonium production [12]. In general, one expresses the free particle amplitude, where $Q\overline{Q}$ makes a transition into the physical quarkonium state, in a power series of the relative momentum $\mathbf{q}$ of $Q$ and $\overline{Q}$ in the $Q\overline{Q}$ rest frame. Then, one can find out what specific spectroscopic state of the $Q\overline{Q}$ pair is initially produced in the hard process amplitude. Finally, one may consider the soft transition in which the initially produced $Q\overline{Q}$ system transforms into the physical heavy quarkonium state in which one is interested. In the case of heavy quarkonium production by the color-singlet mechanism, the soft process does not change any spectroscopic quantum numbers such as color and angular momentum up to $v^2$ order correction (which contains relativistic correction and double $E1$ transitions):

\[ Q\overline{Q}(^3S_1^{(1)}) \rightarrow Q\overline{Q}(^3P_J^{(8)}) \rightarrow H(^3S_1^{(1)}). \]  

(1)

In $Z^0$ decay, the color-singlet production process mainly comes from the Feynman diagram shown in Fig. 1, whereas the color-octet production process mainly comes from the soft process $Q\overline{Q}(^3S_1^{(8)}) \rightarrow J/\psi$ as shown in Fig. 2. If we consider $J/\psi$ polarization, we should take into account the relation of the $J/\psi$ polarization and the angular momentum of the initial $Q\overline{Q}$ pair. If $J/\psi$ is produced via the color-singlet mechanism

\[ Q\overline{Q}(^3S_1^{(1)}) \rightarrow J/\psi(^3S_1^{(1)}), \]  

(2)

the polarization vector of $J/\psi$ is identical to spin wavefunction of the initially produced $Q\overline{Q}$ pair. The polarization vector of the $J/\psi$ produced via the color-octet mechanism through double $E1$ transitions,

\[ Q\overline{Q}(^3S_1^{(8)}) \rightarrow Q\overline{Q}(^3P_J^{(8)}) \rightarrow J/\psi(^3S_1^{(1)}), \]  

(3)
is the same as the spin polarization vector of the initially produced \( \bar{Q}Q^{(3S_1^{8})} \), since the E1 transition conserves spin and the total angular momenta of the \( \bar{Q}Q^{(3S_1^{8})} \) and \( J/\psi \) are the same. Therefore, in these two channels, there is no problem, even though we treat the polarization vector of the produced \( J/\psi \) and the spin wavefunction of the \( \bar{Q}Q \) pair to be the same. The only factor involving the polarization of the produced \( J/\psi \) is the hard process shown in Figs. 1 and 2 which produce a \( \bar{Q}Q \) pair at short distance. Since the \( \bar{Q}Q^{(3S_1^{8})} \) produced via the color-octet mechanism comes from the gluon propagator, it seems to be strongly transversely polarized. The quantitative number for the color-octet produced \( J/\psi \) polarization can be obtained only after the full calculations, which will be presented below along with the numerical results.

Before presenting the results for the \( J/\psi \) polarization at LEP, we first argue that the \( Z^0 \) polarization at LEP does not affect the \( J/\psi \) polarization in \( Z^0 \) decays. The \( Z^0 \) produced at LEP is polarized as a result of unequal vector and axial vector couplings between electron and \( Z^0 \) boson. Therefore, the density matrix \( \rho_{\mu\nu}^{Z^0} \) of \( Z^0 \) is given by

\[
\rho_{\mu\nu}^{Z^0} = \frac{1}{3} I^{\mu\nu} - \frac{i}{2 M_Z} \varepsilon_{\mu\nu\lambda\tau} Z_\lambda P_\tau - \frac{1}{2} Q^{\mu\nu},
\]

(4)

where \( I^{\mu\nu} \equiv -g^{\mu\nu} + \frac{Z^{\mu} Z^{\nu}}{M_Z^2} \), \( Z^{\mu} \) is 4-momentum of \( Z^0 \). \( P^{\mu} \) and \( Q^{\mu\nu} \) are vector and tensor polarization of a \( Z^0 \) boson:

\[
P^{\mu} = \frac{\Delta^{\mu}}{M_Z} \left( \frac{g_V^2}{g_V^2 + g_A^2} - \frac{g_A^2}{g_V^2 + g_A^2} \right),
\]

(5)

\[
Q^{\mu\nu} = -\frac{1}{3} I^{\mu\nu} + \frac{\Delta^{\mu} \Delta^{\nu}}{M_Z^2},
\]

(6)

where \( \Delta^{\mu} \equiv (k_1 - k_2)^{\mu} \) with \( k_1 \) and \( k_2 \) being four-momenta of \( e^- \) and \( e^+ \) at LEP, and \( g_{V,A} \) are the vector and the axial vector couplings between \( e \) and \( Z^0 \) boson. We can write the decay rate of \( Z^0 \) as

\[
d\Gamma = \frac{1}{2 M_Z} \rho_{\mu\nu}^{Z^0} \int [dp] \int d_2(PS) H_{\mu\nu},
\]

(7)

where \([dp] \equiv \frac{d^4p}{(2\pi)^3 2p} \) is the invariant phase space of the produced heavy quarkonium with four-momentum \( p_\mu \). By the Lorentz covariance, the integration \( \int d_2(PS) H_{\mu\nu} \) gives terms
proportional to $g_{\mu\nu}, Z_{\mu}Z_{\nu}, Z_{\mu}p_{\nu}, Z_{\nu}p_{\mu}, p_{\mu}p_{\nu}$, and $\epsilon_{\mu\nu\alpha\beta}Z^\alpha p^\beta$. Here, $\epsilon_{\mu\nu\alpha\beta}Z^\alpha p^\beta$ is the only non-vanishing term after being contracted with the vector polarization term in $\rho^\mu_{Z\nu}$, and the result is proportional to $\cos \theta^*$, where $\theta^*$ is the angle between the initial electron-beam and the produced quarkonium directions. When they are contracted with the tensor polarization contribution in $\rho^\mu_{Z\nu}$, only $p_{\mu}p_{\nu}$ gives a nonzero quantity, proportional to $3 \cos^2 \theta^* - 1$. In calculating the energy distribution or the total decay rate, we integrate over the angle $\theta^*$ by which all of the contributions from the polarization dependence vanish. Therefore $\rho^\mu_{Z\nu}$ can be safely replaced by $\frac{1}{3}I^\mu_{\nu}$ in our calculations, effectively.

When one considers the $J/\psi$ polarization, it is convenient to define $\eta$ to be the ratio of the production rate ($\Gamma_L$) of the longitudinal $J/\psi$ to the total production rate ($\Gamma_{TOT} \equiv \Gamma_L + \Gamma_T$) as follows:

$$\eta \equiv \frac{\Gamma_L}{\Gamma_{TOT}} = \frac{\Gamma_L}{\Gamma_L + \Gamma_T}. \quad (8)$$

This ratio $\eta$ can be determined experimentally from the measurement of the angular distribution of the leptons in the subsequent decay $J/\psi \rightarrow l^+l^-$ [21]. Defining $\theta^*_l$ to be the angle between the three momentum of $J/\psi$ in the $Z^0$ rest frame and the three momentum of the daughter lepton (say $l^-$) in the rest frame of $J/\psi$, the angular distribution of a lepton in the decaying $J/\psi$ rest frame has the form

$$\frac{d\Gamma(J/\psi \rightarrow l^+l^-)}{d \cos \theta^*_l} \propto 1 + \alpha \cos^2 \theta^*_l, \quad (9)$$

where

$$\alpha = \frac{1 - 3\eta}{1 + \eta}. \quad (10)$$

The unpolarized $J/\psi$ corresponds to $\eta = 1/3$, and $\alpha = 0$.

The polarized $J/\psi$ production in $Z^0$ decays in the color-singlet model was calculated in Ref. [21] using the fragmentation approximation. In that paper, the authors showed that the asymmetry $\alpha$ is rather small, i.e. $\sim 5\%$. Also, $\alpha$ is independent of the produced quarkonium mass so that $\alpha$’s are the same both for $J/\psi$ and $\Upsilon$ production in their fragmentation approach [21].
In our work, we calculated all the Feynman diagrams without any fragmentation approximation in the color-singlet and color-octet contribution. We recover their results in the limit of \( \lambda \equiv m_{J/\psi(\Upsilon)}^2/M_Z^2 \to 0 \). Our results shown in the Appendix depend explicitly on the parameter \( \lambda \), and numerical values are shown in Table I. Note that \( \alpha \)'s are considerably different for \( Z^0 \to J/\psi + X \) and \( Z^0 \to \Upsilon + X \). Also, the fragmentation approximation is not that accurate at calculating the \( \Upsilon \) polarization in the \( Z^0 \) decays because of a rather large mass of \( \Upsilon \). \( \alpha \)'s are enhanced compared to those calculated in the fragmentation approximation.

When we compare the polarization of \( J/\psi \) produced via the color-singlet and the color-octet mechanisms, we observe that there is a considerable difference between \( \alpha_1 = 0.10 \) for the singlet and \( \alpha_8 = 0.36 \) for the octet \( c\bar{c} \) contribution to \( J/\psi \) production. Adding the singlet and the octet contributions, we get \( \alpha_{\text{tot}}^{J/\psi} = 0.31 \), which is appreciably different from \( \alpha_1^{J/\psi} = 0.10 \) or \( \alpha_{\text{frag}}^{J/\psi} = 0.053 \). These numerical values are consistent with those mentioned in Ref. [22]. We have used the following numerical values for the matrix elements of NRQCD appearing in the \( J/\psi \) production rates from the \( Z^0 \) decays:

### Equations

\[
\langle 0 \vert O_1^{J/\psi}(3S_1) \vert 0 \rangle = 0.73 \text{ GeV}^3 \tag{11}
\]

\[
\langle 0 \vert O_8^{J/\psi}(3S_1) \vert 0 \rangle = 0.015 \text{ GeV}^3 \tag{12}
\]

We remark that both \( \alpha_1^{J/\psi} \) and \( \alpha_8^{J/\psi} \) are independent of these NRQCD matrix elements, since they cancel in the numerator and the denominator when we take the ratio in \( \eta \). On the other hand, \( \alpha_{\text{tot}}^{J/\psi} \) does depend on the numerical values of NRQCD matrix elements in Eqs. (11)-(12), each of which is known only within a factor of \( \sim 2 \). Therefore, the definite test of the color-octet mechanism in \( Z^0 \to J/\psi + X \) will be a deviation of the measured \( \alpha_{\text{exp}}^{J/\psi} \) from the singlet prediction, \( \alpha_1^{J/\psi} \) or \( \alpha_8^{J/\psi} \), in the direction of a larger value of \( \alpha_{\text{exp}}^{J/\psi} \). Deviation of the \( J/\psi \) polarizations (or \( \alpha \)) from the color-singlet prediction (\( \alpha_1^{J/\psi} = 0.10 \)) may be used as a probe to check the color octet mechanism in heavy quarkonium productions, once a few thousand decays of \( J/\psi \to l^+l^- \) are observed in \( Z^0 \) decays. In the case of \( Z^0 \to \Upsilon + X \), \( \alpha^{\Upsilon} \) is not so sensitive to the singlet/octet mechanisms : \( \alpha_8^{\Upsilon} = 0.28 \) and \( \alpha_1^{\Upsilon} = 0.23 \), which are
considerably larger than the prediction $\alpha_{\text{frag}} = 0.053$ based on the fragmentation approach.

The energy dependence of $\eta(x)$ (with $x \equiv 2E_{J/\psi}/M_Z$) differs greatly depending on the $J/\psi$ production mechanisms, as we can see in Fig. 3 for the case of $J/\psi$ and Fig. 4 for the case of $\Upsilon$. The $J/\psi$’s produced via the color-singlet mechanism are almost unpolarized in almost the entire energy range, while the $J/\psi$’s produced via the color-octet mechanism are highly transversal (especially at high energy). Therefore if we observe quarkonium of a particular energy range, we can greatly increase the polarization sensitivity if there are enough data. For example, if we observe the $J/\psi$ only in the range of $0.7 \leq x \leq 0.9$, where most of color-singlet $J/\psi$ is produced, $\eta_{\text{J}/\psi}^8 = 0.076$ and $\eta_{\text{J}/\psi}^1 = 0.30$. These values correspond to $\alpha_{\text{J}/\psi}^8 = 0.72$ and $\alpha_{\text{J}/\psi}^1 = 0.077$. In the same energy range, $\eta_{\Upsilon}^8 = 0.12$ and $\eta_{\Upsilon}^1 = 0.28$ for the case of $\Upsilon$, which correspond to $\alpha_{\Upsilon}^8 = 0.57$ and $\alpha_{\Upsilon}^1 = 0.13$, respectively.

We can also observe $\Upsilon$’s in the energy range $0.3 \leq x \leq 0.4$, where most of color-octet $\Upsilon$’s are produced. In this energy range, color-singlet $\Upsilon$ is more transversal than color-octet $\Upsilon$, where $\eta_{\Upsilon}^8 = 0.28$, and $\eta_{\Upsilon}^1 = 0.12$, corresponding to $\alpha_{\Upsilon}^8 = 0.13$ and $\alpha_{\Upsilon}^1 = 0.57$ respectively.

In conclusion, we have calculated polarization in heavy quarkonium ($J/\psi$ and $\Upsilon$) productions in $Z^0$ decay. The polarization of $J/\psi$’s produced via the color octet mechanism is more transversal compared to those produced via the color singlet mechanism (Table I). Therefore, the measurement of polarizations provides another independent test of the idea of the color-octet mechanism.

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In this appendix, we show the analytic forms of $\Gamma_{1(8),\text{TOT}}$ and $\Gamma_{1(8),\text{TOT}}$, defined in Eq. (8).

$$
\Gamma_{8,\text{TOT}} = \frac{\alpha_s^2(2m_c)}{18} \Gamma(Z \rightarrow q\bar{q}) \frac{\langle O^{J/\psi(3S_1)} \rangle}{m_c^3} 
\times \int_{2\sqrt{x}}^{1+\lambda} dx \left\{ \log \left( \frac{x + \sqrt{x^2 - 4\lambda}}{x - \sqrt{x^2 - 4\lambda}} \right) \frac{[x^2 - 2x + 2 + 2\lambda(2 - x) + 2\lambda^2]}{x} - 2\sqrt{x^2 - 4\lambda} \right\},
$$

(13)

$$
\Gamma_{8,L} = \frac{2\alpha_s^2(2m_c)}{9} \Gamma(Z \rightarrow q\bar{q}) \frac{\langle O^{J/\psi(3S_1)} \rangle}{m_c^3} 
\times \int_{2\sqrt{x}}^{1+\lambda} dx \left\{ \log \left( \frac{x + \sqrt{x^2 - 4\lambda}}{x - \sqrt{x^2 - 4\lambda}} \right) \frac{[x - 1 + \lambda(x - 2) - \lambda^2]}{x} 
+ \frac{1}{2\lambda}(1 + \lambda)(1 + \lambda - x)\sqrt{x^2 - 4\lambda} \right\}.
$$

(14)

$$
\Gamma_{1,\text{TOT}} = \frac{\alpha_s^2(2m_c)}{243} \Gamma(Z \rightarrow c\bar{c}) \frac{\langle O^{J/\psi(3S_1)} \rangle}{m_c^3} 
\times \int_{2\sqrt{x}}^{1} dx \left\{ 4\lambda \log \left( \frac{x\sqrt{1 - x + \lambda} + \sqrt{(x^2 - 4\lambda)(1 - x)}}{x\sqrt{1 - x + \lambda} - \sqrt{(x^2 - 4\lambda)(1 - x)}} \right) \right\} 
\times 10\lambda^3(x^2 + 4) + \lambda^2(-5x^4 + 20x^3 + 8x^2 - 80x + 80)
+ \lambda(9x^5 - 59x^4 - 8x^3 + 68x^2 - 128x + 64) + 4x^2(5x^2 - 4)
+ \frac{g_V^2 - g_A^2}{g_V^2 + g_A^2}(2\lambda^3(x^2 + 4) + \lambda^2(5x^4 - 60x^3 + 24x^2 - 48x + 48)
+ \lambda x^2(-9x^3 + 73x^2 - 76) + 32x^4(-x + 1)) \right\} / \left( x^3(2 - x)^2 \right)
- 8\sqrt{\frac{(x^2 - 4\lambda)(1 - x)}{1 - x + \lambda}} \left\{ 2\lambda^4(x + 2)(5x^3 - 38x^2 + 60x - 40)
+ \lambda^3(-5x^6 + 66x^5 - 286x^4 + 888x^3 - 992x^2 + 960x - 480)
+ 6\lambda^2(2x^7 - 25x^6 + 118x^5 - 324x^4 + 384x^3 - 360x^2 + 288x - 96)
+ \lambda(-5x^8 + 76x^7 - 411x^6 + 1168x^5 - 1384x^4 + 1248x^3 - 1456x^2 + 1024x - 256)
- 4x^2(x - 1)^2(5x^4 - 32x^3 + 72x^2 - 32x + 16)
+ \frac{g_V^2 - g_A^2}{g_V^2 + g_A^2}(2\lambda^4(x + 2)(x^3 + 18x^2 + 12x - 8)
+ \lambda^3(5x^6 - 78x^5 + 274x^4 - 840x^3 + 384x^2 + 448x - 224)
+ 4\lambda^2(-3x^7 + 45x^6 - 211x^5 + 528x^4 - 504x^3 + 44x^2 + 144x - 48)
+ 8)
\[
+\lambda x^2(x - 1)(5x^5 - 73x^4 + 328x^3 - 712x^2 + 560x - 80)\bigg/\left(x^2(2 - x)^6\right)
\]

(15)

\[
\Gamma_{1,\ell} = \frac{\alpha_s^2(2m_c)}{243} \Gamma(Z \to c\bar{c}) \frac{\langle O_{1/2}^{1/2}(3S_1) \rangle}{m_c^3}
\times \int_{2\sqrt{\lambda}}^1 dx \left[-4\log \left(\frac{x\sqrt{1 - x + \lambda}}{x\sqrt{1 - x + \lambda} - \sqrt{(x^2 - 4\lambda)(1 - x)}}\right) \right] \\
24\lambda^4(x^2 + 4) + 8\lambda^3(x^4 - 2x^3 - x^2 - 24x + 28) \\
+\lambda^2(-3x^6 + 2x^5 - 128x^4 + 64x^3 - 112x^2 - 128x + 128) \\
+\lambda x^2(-5x^5 + 3x^4 + 56x^3 + 60x^2 - 64) + 4x^4(-5x^2 + 4) \\
+\frac{g_V^2 - g_A^2}{g_V^2 + g_A^2} \lambda(8\lambda^3(-x^2 - 4) + 8\lambda^2(4x^4 - 12x^3 + 7x^2 + 12) \\
+\lambda x^2(3x^4 - 52x^3 + 176x^2 - 208x + 16) + x^4(x + 2)(x^2 - 3x + 6)\bigg/\left(x^3(x^2 - 4\lambda)(2 - x)^2\right) \\
+8\left(\frac{(x^2 - 4\lambda)(1 - x)}{1 - x + \lambda}\right) \right] \quad (16)
\]

with

\[
\lambda \equiv \frac{4m_c^2}{M_Z^2}.
\]

The same formulas apply to the \( Y \) case, with the substitution of \( m_b \) for \( m_c \), the corresponding change of couplings \( g_V \) and \( g_A \), and the corresponding long-range matrix elements.
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TABLE I. Longitudinal production fraction $\eta$ of quarkonium produced in the $Z^0$ decay and the asymmetry $\alpha$ of the angular distribution of the quarkonium decay in its rest frame.

| Fragmentation (Color Singlet) [21] | $\eta^{J/\psi}/\eta^{\Upsilon}$ | $\alpha^{J/\psi}/\alpha^{\Upsilon}$ |
|-----------------------------------|---------------------------------|-----------------------------------|
| Color Singlet (this work)         | 0.31 / 0.31                    | 0.053 / 0.053                     |
| Color Octet (this work)           | 0.29 / 0.24                    | 0.10 / 0.23                       |
| Octet + Singlet (this work)       | 0.19 / 0.22                    | 0.36 / 0.28                       |
|                                   | 0.21 / 0.22                    | 0.31 / 0.28                       |
Fig. 1  Feynman diagrams for the color-singlet mechanism for $Z^0 \rightarrow (c\bar{c})(^3S_1^{(1)}) + c\bar{c}$.

Fig. 2  Feynman diagrams for the color-octet mechanism for $Z^0 \rightarrow q\bar{q} + J/\psi$ with $q = u, d, c, s, b$.

Fig. 3  Energy dependence of $\eta(x)$ in case of $Z^0 \rightarrow J/\psi + X : \eta_{J/\psi}^s(x)$ in the solid curve, $\eta_{J/\psi}^1(x)$ in the dashed curve, and $\eta_{J/\psi}^{\text{frag}}(x)$ in the dotted curve.

Fig. 4  Energy dependence of $\eta(x)$ in case of $Z^0 \rightarrow \Upsilon + X : \eta_{\Upsilon}^s(x)$ in the solid curve, $\eta_{\Upsilon}^1(x)$ in the dashed curve, and $\eta_{\Upsilon}^{\text{frag}}(x)$ in the dotted curve.
FIG. 3.

Octet
Singlet
Frag.

FIG. 4.

η(x)

ψ

Frag.
Singlet
Octet

Y

Frag.
Singlet
Octet

15