COSTI: a New Classifier for Sequences of Temporal Intervals

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Abstract—Classification of sequences of temporal intervals is a part of time series analysis which concerns series of events. We propose a new method of transforming the problem to a task of multivariate series classification. We use one of the state-of-the-art algorithms from the latter domain on the new representation to obtain significantly better accuracy than the state-of-the-art methods from the former field. We discuss limitations of this workflow and address them by developing a novel method for classification termed COSTI (short for Classification of Sequences of Temporal Intervals) operating directly on sequences of temporal intervals. The proposed method remains at a high level of accuracy and obtains better performance while avoiding shortcomings connected to operating on transformed data. We propose a generalized version of the problem of classification of temporal intervals, where each event is supplemented with information about its intensity. We also provide two new data sets where this information is of substantial value.

Index Terms—classification, sequences of temporal events, multivariate temporal datasets

I. INTRODUCTION

This paper is concerned with classifying sequences of temporal intervals, a type of data that consists of sequences of different intervals limited by start and end timestamps. Real-life applications of this representation include music classification [1], sign language transcription [2], mining business transaction patterns [3] and human activity recognition and monitoring [4]. The interval representation is a natural choice when data consists of different types of events in time, possibly overlapping with one another.

Analysis of sequences of temporal intervals can be conducted with different goals in mind. This paper is devoted to supervised classification of sequences of temporal intervals. We noticed that despite the fact that we found numerous classification methods published in recent years, they did not point out the similarities between classification of sequences of temporal intervals and multivariate time series classification. Our work aims to bridge this gap between the two problems by creating a generalized version of sequences of temporal intervals, transforming data between the two problems and developing a new method of classification. Our aim was to show that while it was possible to transfer ideas for classification methods between the two problem types, doing it efficiently required creating a new algorithm.

The contribution of this paper is threefold. Firstly, we introduce a methodology for the generalization of classification of sequences of temporal intervals by adding information about the event’s intensity. This generalization widens the scope of practical applications of algorithms from this domain. We share two new data sets concerning music analysis and meteorology where intensity values are of substantial, informative importance. The proposed scheme can be applied to a plethora of real-world domains. Example novel areas of application include:

- healthcare (one sequence corresponds to one patient, events correspond to symptoms or reactions to drugs, each symptom/reaction can be described with its intensity that can vary in time),
- business process mining (one sequence corresponds to one process, intensity depicts the number of people participating in a given stage/event in the process),
- smart homes (one sequence corresponds to one house, intensity values can be used to describe number of persons in a room, energy consumption, etc.).

The list above is not exclusive. The new approach opens the room for novel studies on temporal events analysis where it is essential to measure the time of a change with high accuracy.

Secondly, we show how to convert data available in a generalized representation format to multivariate time series, a widely known time-varying phenomena representation form. This transformation allows using algorithms developed for the latter data representation format to be applied to the former. We show that using MiniRocketMultivariate [5], one of the multivariate time series classification methods, on the converted data gives considerably better results than dedicated state-of-the-art methods in the classification of sequences of temporal intervals (SMILE [6] and STIFE [7]). However, we can only apply the described transformation between data representation formats after establishing a sampling step, which is a challenging problem in itself. Moreover, utilization of sampling introduces time complexity depending on the duration of the sequences divided by the sampling step. When high accuracy is needed, and the duration of sequences is

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very high, this leads to a very high execution time. Based on the observed facts, we postulate that there is a need for an algorithm that would work directly on interval data (without sampling), not depend on the magnitude of timestamps and perform at a high level of accuracy.

The culminating aspect of the contribution is a new algorithm termed COSTI (short for Classification of Sequences of Temporal Intervals) for generalized classification of sequences of temporal intervals, adapting the core ideas of the MiniRocketMultivariate algorithm in a new method that works very well on a different type of input data, with some necessary conceptual changes and a completely new implementation. While retaining a high level of accuracy, the presented algorithm does not depend on the duration of sequences in time complexity but rather on the number of intervals present in the data. As a result, the new algorithm is faster than MiniRocketMultivariate for each considered real-world dataset; it is over 50 times faster in two cases. We show that the new algorithm is well-suited for sequences of temporal intervals. Not only is the new method applicable to the generalized version of the problem, but it also performs on a level of accuracy higher than state-of-the-art methods when the constrained version of the problem is concerned.

An indispensable part of the study presented here is the implementation of the algorithm. It was written in Python and compiled via Numba [8]. Our code was made available at https://bit.ly/3sTiHup. In the paper, we address an empirical study concerning eight datasets, which are at the moment deemed as benchmark datasets in this domain [6].

The remainder of the paper is structured as follows. Section III introduces preliminary notions and relevant literature positions. Section IV demonstrates the proposed data format transformation procedure and discusses its limitations. Section V describes the proposed new method of classification of sequences of temporal intervals. Section VI introduces two new data sets and identifies assumptions made in the experiments part. Section VII examines results of the conducted experiments. Section VIII is devoted to conclusions and critical discussion.

II. PRELIMINARY NOTE

Classification of data collected over time is a thriving field of research. The default design assumption of most existing methods is that they were constructed for univariate time series. This limits the scope of practical applications of algorithms from this domain. As argued by Gorecki and Luczak [9], in many cases extending the applicability of univariate time series classifiers to a multivariate case is methodologically sound. This paper presents original reasoning that led from a univariate time series classifier to a classifier dedicated to sequences of temporal intervals. Two transitions had to be made:

- Generalization from univariate time series classifier to multivariate time series classifier.
- Generalization from multivariate time series classifier to a classifier dedicated to sequences of temporal intervals.

III. EXISTING SOLUTIONS

A. Problem statement: multivariate time series classification

A common assumption in data analysis of time-varying phenomena is that the data are present as a sequence of real numbers, and successive values were collected with some fixed, evenly spaced interval. We were not given an analytic form of the relation describing the phenomenon in time. Still, we consider it as some continuous real-valued function that we can approximate by analyzing data points collected in a time series. Technically, we can represent such data as a vector containing real numbers indexed with a timestamp. We can say that this vector includes data from a single channel of information. In a situation where a given time-varying phenomenon is observed using more than one channel, we can store such data in a matrix, where successive columns correspond to consecutive information channels, and a timestamp is used to index the rows. An important assumption is to maintain the same sampling rate in each channel.

B. Multivariate time series classification

As Pasos Ruiz et al. [10] suggest, Dynamic Time Warping (DTW)-based approaches are deemed to be the ‘gold standard’ solution to time series classification. It is often used as a bottom-line method with which more sophisticated approaches are compared. DTW, in its simple form, is suitable for univariate time series. However, the literature proposes a couple of generalizations that allow to apply it in a scheme aiming at multivariate time series classification. As Shokohi-Yekta et al. [11] outline, the most straightforward adaptation is to run the procedure independently for all variables in the time series and then combine the results. The second strategy assumes that warping is the same across all dimensions [12]. Some approaches adaptively incorporate independent and dependent warping strategies [13].

When it comes to algorithms dedicated to multivariate data, we shall mention Multiple Representation Sequence Learner, MrSEQL [14] which is an ensemble operating on symbolic time series representation and deep neural architectures addressed by Fawaz et al. [15].

Finally, we shall mention that univariate time series classifiers can be adapted to deal with multivariate time series. We can list several studies addressing this line of research, where for example, HIVE-COTE [16] or RISE [17] algorithms were applied in a new role. The problems arise less in terms of classification quality but in classifiers’ performance. Importantly, algorithms focusing on time series feature extraction such as catch22 [18] or NetF [19] can be conveniently adapted to deal with multivariate data. In this paper, we present a novel adaptation of a classifier that was at first designed to deal with univariate time series to a different role. Please note that there is an abundance of state-of-the-art algorithms in both univariate and multivariate time series classification. However, we aim to show that adapting just one of these methods for the task of classification of sequences of temporal intervals allows to greatly outperform the state-of-the-art in that field. Thus, we
focus on the chosen method and abstain from describing others in more detail.

C. Univariate time series classification: Rocket

Rocket [20], introduced in 2019, achieved a state-of-the-art accuracy while requiring only a fraction of the training time required by the existing methods of univariate time series classification. Rocket uses random convolutional kernels to create a set of features. Convolutional kernel is a concept that was first utilized in convolutional neural networks. A convolutional kernel \( f \) as used in Rocket can be expressed as (version with no padding):

\[
f(X) = \left\{ \sum_{k=0}^{L-1} w_k \cdot X[t + dk] \right\}_{t=1,...,T-(L-1)d}
\]

where \( X \in \mathbb{R}^T \) is the input vector, \( L \in \mathbb{N} \) is the length of the kernel, \( w_k \in \mathbb{R} \) is its \( k \)-th weight, \( b \in \mathbb{R} \) is bias and \( d \in \mathbb{N} \) is dilation. When padding is enabled, a number of zeros is appended to \( X \) at its beginning and end, so that the iteration of \( t \) can start with the middle weight getting multiplied by \( X[1] \) and end by the middle weight getting multiplied by \( X[T] \). Please note that \( f : \mathbb{R}^T \rightarrow \mathbb{R}^{T-dL} \) for \( f : \mathbb{R}^T \rightarrow \mathbb{R}^T \), depending on whether padding is used, so one convolutional kernel applied to one sequence produces many real numbers.

Rocket generates a high number of random kernels, i.e., kernels with random length, weights, bias, dilation, and padding. After a kernel is applied on a time series, two statistics are calculated based on the output – maximum value and proportion of positive values. These two statistics become new features describing the sequence. Normally, a very high number of kernels is used (10,000), generating a set of 20,000 features for each sequence. Then, a classifier is used to perform the final classification. Most often, Ridge Classifier is used, sometimes replaced by a single softmax layer when the number of examples is very high.

D. Univariate time series classification: MiniRocket

Even though Rocket was extremely fast comparing to other algorithms in the field, its authors were able to come up with a version up to 75 times faster on larger data sets. MiniRocket [5], introduced in 2020, was able to retain the same level of accuracy as Rocket, while greatly reducing the time of execution and making the whole process almost deterministic (or fully deterministic, at an additional computational expense). This was done mainly by reducing to a bare minimum the random space from which the convolutional kernels were generated. Length of the kernel in MiniRocket is fixed to 9 and all random kernels must contain three weights equal to \(-2\) and six weights equal to \(1\), creating a set of 84 possible kernels.

E. Multivariate time series classification: MiniRocketMulti

Although MiniRocket was developed as an algorithm for univariate time series, a basic extension for multivariate case was also created by the same authors. The basic premise of this algorithm is treating data in the same way as in MiniRocket, only now instead of multiplying the weight by one element, it gets multiplied by the sum of elements from a subset of channels. This subset is chosen at random individually for each feature.

F. Problem statement: classification of sequences of temporal intervals

The subsequent generalization considered in this paper is to move to the case where the classification task concerns a sequence of temporal events with assigned duration. Each event is assigned a type, a start time, and an end time. The end time is always greater than or equal to the start time. When it is equal, according to Kotsifakos et al. [21], we say that an event is instantaneous. We assume that no events of the same type can overlap, while events of different types can.

It is possible to extend this description by adding a weight to an event, representing its intensity. Additionally, we can describe the absence of an event as an event with a weight value equal to zero. With this encoding, it is possible to represent this type of phenomena in a manner analogous to the case of multivariate time series–the proposed generalization results in an event being interpreted as an information channel. Fig. 1 provides a visual comparison between the data representations used in both problems.

G. Classification of sequences of temporal intervals

Junlin Lin [3] was one of the firsts to point out that it may be beneficial to represent business transaction data taking into account duration of the transactions. He considered data where each record consisted of a starting time and an ending time of a transaction and presented a method to mine maximal frequent intervals, time intervals that contain a number of transactions higher than a given threshold. In [22], authors pointed out that interval sequences can be used to approximate the behaviour of multivariate time series, and presented a method that uses Allen’s interval logic to mine temporal patterns. Another publication concerning the problem of pattern mining in interval sequences was [23], where authors searched for frequent sequences of events. Just like other methods mentioned in this paragraph, the method extracted features from sequences of temporal intervals, but did not perform classification.

A frequency-based method utilizing Allen’s interval logic was also presented by Patel et al. [24] and Moskovitch and Shahar [25]. In the latter paper, the authors propose a framework named Karma–Lego that considered transformation of multivariate time series data to sequences of temporal intervals, which were then subjected to classification process. The authors performed data transformation from multivariate time series to sequences of temporal intervals not only to use a different method of classification, but also to unify the data with some possible external temporal intervals data. In Section IV, we transform data between the two problems in the exact opposite direction. Karma–Lego, as well as other, similar methods, [26], [27], [28] despite using interval classification
methods, targeted multivariate time series, so they are not competitors of the method presented in this paper.

One way of constructing a simple classification method for sequences of temporal intervals is to use a distance measure and a k-NN classifier. Various distance measures have been developed with this purpose in mind. Artemis [29] compared two interval sequences using a bipartite graph mapping. While it focused on shared temporal relations as a means of comparison, it ignored the time duration of individual events.

This was pointed out as a potential weakness by authors of IBSM [21] – a distance measure that relied on transforming the interval sequences to a binary matrix representation and then comparing the matrices. To allow for a meaningful comparison of sequences of different maximal timestamp, a bilinear interpolation was used to make the size of compared matrices the same. IBSM was claimed to dominate the state-of-the-art of nearest neighbor classification in terms of accuracy and runtime. The method was later extended for subsequence matching in a framework called ABIDE [30]. Although k-NN classifiers were later outperformed by STIFE [7] and SMILE [6], both algorithms incorporated a distance measure as one of their core parts.

Nearest neighbour classifiers were foundations on which two classification frameworks, STIFE [7] and SMILE [6], were built. In both of them, static features and temporal relations were incorporated as means of feature extraction, generating data for a general-purpose classifier in the last step. The two algorithms were claimed to constitute the current state-of-the-art in classification of sequences of temporal intervals. Their mode of operation is explained in more detail in Section III-H.

Overall, the most important differences between data in multivariate time series classification and classification of sequences of temporal intervals, as considered in the presented publications, are: a) multivariate time series contain fixed, evenly spaced intervals between measurements, b) sequences of temporal intervals operate on binary values (an event is either taking place or not), c) because of the reduction to binary values in classification of sequences of temporal intervals, additional emphasis can be put on detecting complex sequential patterns between different event types. However, as we show in Section VII, this potential advantage did not allow to achieve a higher modelling power than a state-of-the-art method originally designed for multivariate time series classification.

H. Classification of sequences of temporal intervals: STIFE and SMILE

SMILE works by creating a vector of features for each sample. Extracted features are then used to perform classification using Random Forest [31]. Features are generated in four ways:

1) **Static features**: a set of 14 predefined statistics. These range from very simple, like the total number of events in a sequence, to more complex, like summed duration of time intervals when there was a maximal number of events active at the same time.

2) **Class-based medoid distance features**: distances to medoids representing each class. Medoids are chosen from training data based on IBSM, a distance metric for sequences of temporal intervals, introduced by Kotisfakos et al. [21].

3) **Interval relation-pair features**: numbers of occurrences of seven Allen’s temporal relations, introduced by Allen [32]. These relations can occur between pairs of events and the number of their occurrences is calculated for each configuration of different types of events (e.g. relation “X precedes Y” between event of type 2 and event of type 4 occurs 19 times in the sequence, 19 is the feature). Only 75 features from this group are chosen to be included in the final set of features. This choice is made based on information gain calculated on the training set.

4) **Interval segment features**: a set of distances between the sequence and 75 subsequences randomly extracted from the training set. The function used to measure distance is called ABIDE [30].

IV. Classification through transformation between data representation formats

The introduced data transformation method opens the possibility to apply methods of classification operating on the latter type of data, which can be very beneficial given a high development of the field of multivariate series classification. In addition, we show that using MiniRocketMultivariate on the transformed data gives considerably higher accuracy than the current state-of-the-art described in Section III-H.

A. Transforming sequences of temporal intervals into multivariate time series

Figure 1 illustrates an example of multivariate time series data and an event sequence to indicate how conceptually transparent the transition between one format and the other is. The general idea is to sample channels (events) at a chosen frequency. We can only do this after choosing a step in time. Using that step, consecutive rows in the matrix can be populated with values of channels, according to the formula: \( M[r, c] = X_c[r \cdot s] \), where \( r \) is row, \( c \) is column and \( s \) is the chosen step. In a particular case of MiniRocketMultivariate, we want all matrices to be of the same height, as the
| Model                     | Task | Description                          |
|--------------------------|------|--------------------------------------|
| MiniRocketMultivariate   | MTS  | A very fast state-of-the-art method   |
| IBSM                     | STI  | Distance measure used for k-NN classification |
| STIFE                    | STI  | State-of-the-art classifier          |
| SMILE                    | STI  | Extended version of STIFE             |
| Data transformation+     | STI  | New approach                         |
| MiniRocketMultivariate   | STI  | New method                           |
| COSTI                    | STI  | New method                           |

Fig. 2. Description of methods relevant for this publication. MTS – Multivariate time series classification. STI – Classification of sequences of temporal intervals. Methods introduced in this article are underlined.

algorithm operates on sequences of equal length. However, when it is implicit that time in \( X \) starts from 0, it is not always explicitly stated what the end of the sequences is. In that case, we can only depend on the last timestamp of a sequence, i.e., the end of the last interval. Thus, the number of rows of the matrix is set to the number of steps resulting from parsing the sequence that contains the highest end timestamp. All other sequences are effectively padded with zeros. Other techniques of ensuring the same size of input matrices are also possible and we describe one such technique in Section VII-B.

B. Drawbacks of the the proposed transformation and classification approach. Rationale behind the new method.

A number of difficulties can arise as a consequence of operating on the data created with the described transformation. Too high a sampling rate unnecessarily increases the volume of data. Too low a frequency, in turn, results in too much information being lost. Selecting the right frequency is difficult and demands domain knowledge from the user. Sometimes even the best rate will give unsatisfactory results because the first variant is not suitable for storing the second type of information efficiently. These not-yet-solved issues motivated us to develop COSTI, a new method for classification of sequences of temporal intervals, operating directly on the interval data. We compare the performance of our solution with the results obtained using transformation between formats and show that not only it allows to omit the problematic sampling stage, it also benefits from operating on the original data in terms of time efficiency. To avoid confusion between different tasks and solutions, Fig. 2 contains information about all relevant methods discussed up to this point.

V. METHOD

In this section, we present our new approach to the task of sequences of temporal intervals classification. The new algorithm can be conceptually summarized as a processing pipeline consisting of the following steps:

1) Transforming interval-based data set to a custom data structure.
2) Fitting model parameters using a training set.
3) Calculating features for training and test sets.
4) Performing features for training and test sets.

The proposed solution can be seen as a generalization of the time series processing steps present in MiniRocketMultivariate algorithm.

A. Introductory notes on the new approach

Calculation of a single feature is performed using a set of randomly chosen parameters: a subset of channels \( Q \), bias \( b \), dilation \( d \) and a convolutional kernel with weights \( \{w_0, \ldots, w_8\} \). Now, let us consider a case when observations \( X \) are defined on a continuous range \( < t_s, t_e > \). This would allow us to introduce the following way of computing features:

\[
a = \frac{1}{t_e - t_s} \int_{t_s}^{t_e} H \left( \sum_{k=0}^{8} \sum_{c \in Q} w_k \cdot X_c[t + dk] - b \right) dt \quad (2)
\]

where \( H(\cdot) \) is the Heaviside step function given as:

\[
H(x) = \begin{cases} 
1, & x > 0 \\
0, & x \leq 0 
\end{cases} \quad (3)
\]

Please note that the value of the integral is effectively the sum of lengths of all intervals where the expression inside \( H(\cdot) \) is larger than zero.

From now on, let us assume that each channel of \( X \) is defined by a number of intervals of constant values, as described in Section III-F. We can utilize this characteristic to create an efficient method of computing the integral in (2). The key approach here will be aligning sequences corresponding to each \( k \) in \( X_c[t + dk] \), and then creating a sorted concatenation using those. This will allow us to calculate the integral by just iterating over a sequence of timestamps between which the value of the expression inside \( H(\cdot) \) does not change. Below the algorithm description, you may find a toy example illustrating this approach.

B. The new algorithm – steps

1) Create a list \( L_0 \) containing all timestamps where the value of all channel changes is sorted according to the timestamp in ascending order. Store information about the difference between the new and the old value and the channel’s id where the change happened.

2) \( L = L_0 \cup L_1 \cup \ldots \cup L_k \), where \( L_k \) is created by adding \( dk \) to each element of \( L_0 \). Save the information about \( k \) which the element originated from. This information will be used to choose the weight of the convolutional kernel.

3) Sort all elements of \( L \) by their timestamps in ascending order. Please note that the individual lists \( L_k \) are already sorted, which can be used to greatly speed up the procedure.

4) Set \( psum = -b \), \( a = 0 \).

5) Traverse \( L \), adding to \( psum \) the changes of values of intervals multiplied by the weight corresponding to the \( k \) from which the change point originated from. Take
into account only changes in the channels belonging to the subset \( Q \). Whenever \( psum \) over the current interval is larger than zero, add the length of the interval to \( a \).

6) Feature = \( a / (t_e - t_s) \).

Please note that the first three points of this algorithm demand only the value of the dilation to be known. As the original MiniRocketMultivariate algorithm uses a maximum of 32 different dilation values, we suggest to analogously compute the lists in points 1-3 for these 32 dilations, and for each feature calculate only points 4-6.

**Algorithm 1** Feature generation in COSTI

Require: \( ts, cs, vs, E, C, ds, ps \)

```plaintext
features = []
L0 = array(length=E)
vprev = array(length=C)
for i in range(0, E) do
    vd = vs[i] - vprev[c]
    L0[i] = (ts[i], vd, cs[i])
    vprev[c][i] = vs[i]
end for
for d in ds do
    L = array(length=9E)
    for k in range(0, 9) do
        for i in range(0, E) do
            t, v, c = L0[i]
            L[k*E+i] = (t+d*k, v, c, k)
        end for
    end for
    sort(L, key=first element, ascending)
    for b, w, Q in ps[d] do
        tprev = zeros(length=C)
psum = -b
        a = 0
        for i in range(0, 9E) do
            t, v, c, k = L[i]
            if c ∈ Q then
                psum = psum + v * w[k]
            if psum > 0 then
                a = a + t - tprev[c]
                tprev[c] = t
            end if
        end if
    end for
features.append(a)
end for
```

**Symbols:**
- \( ts \) = events’ timestamps,
- \( cs \) = events’ channels,
- \( vs \) = events’ values,
- \( E \) = number of events,
- \( C \) = number of channels,
- \( ds \) = dilations,
- \( ps \) = parameters for calculating features, grouped by dilations,
- \( b \) = bias,
- \( w \) = weights,
- \( Q \) = subset of channels

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**Fig. 3.** Example sequence consisting of two events.

**Fig. 4.** Toy case study data structure containing \( L \).

#### C. Illustrative Example

Fig. 4 presents \( L \), a data structure created based on the sequence visualized in Fig. 3. Please note that the consecutive rows in the \( L \) data structure correspond to changes in value of the expression \( psum = \sum_{k=0}^{8} \sum_{c \in Q} w_k \cdot X_c[t + dk] - b \), only in this toy example 8 is replaced by 1. This can be visualized as a convolutional kernel of length equal to 2 moving above the sequence along the time axis. When one of its inputs approaches the next change point, a new value of the expression is computed by adding \( w_k \cdot v \) to \( psum \), where \( v \) is “value” and \( k \) is “weight index”. Before we update \( psum \), we check if the previous value was greater than 0. If it was, the \( H(\cdot) \) function would return 1, so we can add the length of the interval that just ended to the final integral value \( a \).

The data structure created this way can be used to calculate the integral for any given kernel and bias. Moreover, if we want to take a smaller combination, say \( Q = \{ 1 \} \), we can just ignore timestamps that have “channel” field equal to 2 during the structure iteration.

#### D. Selecting final timestamp

We denote the highest timestamp found in the data by \( t_{max} \) and set \( t_{min} \) always equal to 0. These values are then used to determine \( t_s \) and \( t_e \), depending on the usage of padding (see Section V-G). This approach can be seen as padding all sequences with events with intensity equal to zero to meet the duration of the longest sequence.

#### E. Fitting biases

Choice of bias depends on a histogram of \( \sum_{k=0}^{8} \sum_{c \in Q} w_k \cdot X_c[t + dk] \), from which random quantiles are extracted and become biases. Here, \( X \) is an element from the training set, chosen at random for each combination of convolutional kernels and dilation values. Extraction of a random quantile
can be performed using a cumulative sum of duration over intervals of occurring values sorted by values. This technique does not create any serious computational overhead.

**F. Fitting dilations**

The fact that the number of used dilations is limited to 32 directly contributes to achieving high performance using this workflow. The new algorithm can use floating-point numbers for dilation values as observations are no longer defined only in fixed timestamps. Please note that MiniRocketMultivariate dilution distribution starts at 1, as no smaller dilation can be used. No such lower boundary can be established in the presented solution, as timestamps can be of any magnitude. Thus, dilations are taken from evenly divided exponential distribution over interval \[ d_{min}, (t_{max} - t_{min})/8 \], where \( d_{min} \) is the smallest difference between different timestamps found in the training data. Using continuous values means also avoiding duplicates (in MiniRocketMultivariate there can be multiple equal dilations, if \( T \) is small enough). This could, in theory, convey to a higher modelling power. However, experiments on data containing only integer timestamps show that the solution performs better when dilations are rounded to the nearest integers. This may suggest that when there is a strong sampling frequency present in the data, it is beneficial to use only multiples of this frequency as dilations. However, the experimental data was too limited to decide on the significance of this relationship. Also, we did not want to permit the usage of floating-point timestamps. As a compromise, dilation values are rounded to integers only when it is detected that all timestamps in train data are integers.

**G. Padding**

Similarly to MiniRocketMultivariate, we use padding in half of the features. The way features are computed when padding is on corresponds to \( t_s = t_{min} + 4d, t_e = t_{max} + 4d \), so the interval depends on the middle weight remaining in the range of the original data (taking into consideration positive shifts in dilation introduced in (2)). The case with no padding is \( t_s = t_{min} + 8d, t_e = t_{max} \). No padding is used when biases are generated.

**H. Time complexity**

Let us address time complexity of the proposed approach and compare it with MiniRocketMultivariate, which was the starting point for our research.

MiniRocketMultivariate has linear scalability in the number of features \( m \), number of examples \( n \) and time series length \( l_{input} \). The number of channels chosen for computation of a single feature is taken from a logarithmic distribution, which makes the overall complexity equal to \( O(m \cdot n \cdot l_{input} \cdot \log(C)) \), where \( C \) is the number of channels. When used to classify sequences of intervals, MiniRocketMultivariate takes a set of matrices as an input, which normally must be generated from the interval data. The complexity of this operation is proportional to the number of cells in all matrices, which is \( O(n \cdot l_{input} \cdot C) \).

Creating the data structure and calculating features in COSTI comes with time complexity \( O(m \cdot n \cdot E \cdot \log(C)) \), where \( E \) is the number of all events present in the data. Additionally, sorting the input sequences at the beginning is \( O(n \cdot E \cdot \log(E)) \). Please note that step 3 of the algorithm described in Section V-B involves concatenating and sorting 9 sorted lists. This is implemented as iterating over 9 sequences using a priority queue, achieving linear time complexity.

### VI. DATA SETS AND BASIC EXPERIMENT ASSUMPTIONS

The proposed methodology of transitions and generalizations between time series data sets and sequences data sets is a novel contribution of this paper. Furthermore, the literature does not offer any solutions designed to solve the problem of classification of sequences of temporal intervals after generalization we outlined in Section III-F (by adding values corresponding to events).

In Section VI-B and Section VI-C, we present two new data sets that we contribute alongside this paper. They concern real-world phenomena that are best described as sequences of events with meaningful weights (which is an extended temporal data representation format, more versatile than sequences of events with no weights).

**A. Existing data sets**

Nonetheless, to compare the new solution with other algorithms, we also had to perform tests on a constrained version of the classification of event intervals problem, with no values, just events happening or not. There exists a small benchmark data collection for this problem, containing six data sets. Recent algorithms in this field, STIFE [7] and SMILE [6], used this data for accuracy evaluation.

Please note that we did not prepare a specialized version of the algorithm despite using the algorithm for a constrained problem, where timestamps are integers and there are no values associated with intervals. For the use of comparison with other algorithms, all values were set to 1 when an event occurs (and to 0 when it does not, but this is also true for an unconstrained problem). This was also true for the two new data sets.

**B. New dataset for musical key recognition**

The data set contains time series extracted from 240 MIDI files taken from the MASETRO Dataset [33] and from the
public domain. The files contained recordings of piano pieces, either created manually or recorded during live performance. We processed each MIDI file to create a series of events spanning across 12 channels. Each channel corresponds to one of the notes (C, C# = Db, D, D# = Eb, E, F, F# = Gb, G, G# = Ab, A, A# = Bb, B). An event in a channel is present when at least one key corresponding to the channel’s note is being pressed. Timestamps in the time series were taken from the MIDI files, as these also operate on timestamps. However, timestamps in MIDI are purely relative – they cannot be easily converted to time, and it is impossible to compare them across different files. Considering this, we decided to resize all timestamps, so the end of the last interval in each recording would be equal to 24000. This value should be big enough to ensure that all rounding errors are negligible (for a ten minutes-long piece, this is 40 Hz, and in a normal tempo like moderato, one thirty-second note lasts 1/12 of a second). In a version of the data set with values associated with intervals, value of an interval is set to the velocity the key was pressed with. The classification task is to predict which of the 24 key signatures the piece was written in. There are 12 classes with 20 samples each and 12 classes with ten samples each.

C. New dataset concerning weather events

The data set was extracted from a weather events data set, US Weather Events (2016 - 2020), a countrywide data set of 6.3 million weather events. We pulled only the events recorded in 2017 by weather stations in the state of Louisiana. Each channel corresponds to the presence of some specific type of weather conditions. Individual series were made from the recordings for every single weather station and each single month, summing up to 12 series for a single station. Each series contains a month of weather events, with timestamps rounded with 10 minutes step, counting from the month’s start (please note, that even using a one second step would not hinder the performance of the presented algorithm. However, this would be hardly reasonable given the nature of the data). In a version of the data set with values associated with intervals, the value of an interval is equal to the intensity of the weather condition. The classification task is to predict which month the series was recorded in. For a detailed description of the data and levels of intensity, please refer to [34].

VII. RESULTS

A. Comparison with state-of-the-art

There are two state-of-the-art algorithms in the field of classification of sequences of temporal intervals: STIFE and SMILE. No implementation of SMILE was made available to the public up to the time this paper was submitted, so we followed [6] and recreated SMILE using python and Numba. We decided to use our own implementation of STIFE, even though its implementation, written in Java, is available in the Internet. Both hyperparameters of SMILE, that is number of 2-lets and number of e-lets, were set to 75, as suggested in the paper. We used 75 as the number of 2-lets in STIFE. As for the accuracy comparison, we followed the example of [6] and included two extracted parts of SMILE, namely Static features (STAT) and Class-based medoid distance features (MEDOID), described in Section III-H. These were used as individual methods of feature extraction based on which classification was performed. Default version of a Random Forest classifier from scikit-learn package [35] was used as the final step in all four above methods. Additionally, we included IBSM method, which SMILE was also compared to. IBSM method works by 1-nearest-neighbour classification using the IBSM distance. As for the presented algorithm, we used default values of 10 000 features and 32 dilations. All experiments were performed using 10-fold cross-validation. Every experiment was repeated 10 times and mean values of accuracy were calculated. Because we implemented methods other than COSTI with no regard for high performance, we decided not to report their time of execution. For the time of execution of COSTI, see Table III.

Results of experiments are presented in Table II. The new method achieved a significant improvement over the current state-of-the-art in the field of classification of sequences of temporal intervals, reaching the highest scores for all eight data sets. The second best method was STIFE, although the differences between STIFE and SMILE were very small.

B. Comparison with MiniRocketMultivariate

In all the experiments, step during matrices creation \( s = 1 \), so we simulate a situation when no information is discarded. Matrices creation for sequences that have different highest timestamps can be performed by padding shorter sequences with zeros, as explained in Section IV-A. An alternative
approach would be to perform bilinear interpolation on each matrix that has a number of rows smaller than maximal, stretching it to meet the number of rows of the longest matrix. A similar technique is used in IBSM distance [21] when two sequences of different length are compared. Decision on the matrix creation technique could be made with regard to the domain knowledge. We decided to test both approaches on each data set. Please note that this dilemma concerns only MiniRocketMultivariate. COSTI operates directly on the interval data and does not use matrix representation.

In all tests, we used the implementation of MiniRocketMultivariate available through the sktime package. In all algorithms, default numbers of features (10 000) and dilations (32) were used. Time of execution of MiniRocketMultivariate did not include creating a matrix representation of the data. Time of execution of the new algorithm did include sorting the data at the beginning. All experiments were performed using 10-fold cross-validation. Every experiment was repeated 10 times and mean values of accuracy and time were calculated.

Table III presents accuracy and time of execution for all eight data sets. Although the presented solution achieved slightly better results in accuracy than the two variants of MiniRocketMultivariate, the difference is too little to consider it statistically significant. A much bigger difference in favour of the presented solution was observed for the time of execution. Please note that compared algorithms differ significantly in terms of important factors affecting the time complexity. For example, the higher the maximal timestamp is, the longer it takes to execute both versions of MiniRocketMultivariate. On the other hand, dealing with a very high number of events makes COSTI slower. Experimental results confirm this –

| Data set | Accuracy | Scaling | Padding |
|----------|----------|---------|---------|
| Auslan2  | 0.59     | 0.59    | 0.52    |
| Blocks   | 1.00     | 1.00    | 1.00    |
| Context  | 2.18     | 2.25    | 2.28    |
| Hepatitis| 134.44   | 133.39  | 2.80    |
| Pioneer  | 0.76     | 0.80    | 0.67    |
| Skating  | 108.05   | 102.73  | 1.69    |
| Musekey  | 257.54   | 258.27  | 184.47  |
| Weather  | 31.04    | 31.55   | 5.57    |

VIII. CONCLUSION AND CRITICAL DISCUSSION

We believe that the most important conclusion from our work is that the two discussed classification problems are similar enough to share classification techniques that can be successfully applied in both of them. Moreover, we believe that the main contribution presented in COSTI does not come from its speed advantage over MiniRocketMultivariate on a matrix representation, but rather from a new approach to processing information, more fitting to the problem of classification of interval sequences. The possibility of using real values of timestamps and avoiding any type of sampling is important not only when the modelling power is concerned. It can also prove to be advantageous from a practical perspective, making the framework easier to use for users with various levels of experience. We strongly believe there are many ways to improve accuracy of the presented method.

One of possible research directions would be to examine the impact of different distributions of floating-point dilations and other ways of analysing the convolution output sequences. An interesting research direction would be to mimic the data unification process presented in [25], [27] and [28]. Instead of transforming all the data into sequences of temporal intervals, one could use MiniRocketMultivariate and COSTI for feature extraction in corresponding data representations and construct a classifier based on the results.

One weak spot of the proposed approach is the lack of interpretability. The utilized method of feature extraction relies on the proportion of positive values after a given convolutional filter is applied. This value does not correspond to any quality of time series easy to understand for a human. When interpretability is a hard requirement, we recommend using SMILE. Another weakness of the proposed method is its memory complexity. When we consider input data in a typical format (each interval is represented by three real values: start, end, and intensity, and one integer value describing the channel), the size of the data structure used in the algorithm is about 18 times the size of the input. When much bigger data sets emerge in the field, this trait of the method can become a significant downside, making it unsuitable for severely resource-constraint domains, like wearable electronics. However, we believe that mainstream domains of application of the developed approach are not falling into this area.
