Determination of Propagation Times of Finite Ultrasonic Signals in the UFM Measuring Path

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Abstract. This article provides a brief analysis of the error in calculating the discrete cross-correlation function of the transit-time ultrasonic flowmeter signals. Special attention is paid to the study of the influence of the obtained discrete correlation function inaccuracy on the ultrasonic flowmeter’s propagation times determining error. It is known that for real time-limited acoustic signals, the discrete correlation function values are calculated with a significant error. The consequence of this is the appearance of the measurement error of the time delay between signals. The reason for this phenomenon is incorrect truncation of finite length digital sequences of the received acoustic signals. The report presents and describes an improved cross-correlation method for determining the time difference. The new algorithm takes into account the existing discretizing parameters of the received UPS – sampling frequency, sequence size and the truncated signal’s shape. Theoretical analytical expressions for the signals discrete cross-correlation function estimating are obtained as an approximation of a continuous function (the method of trapezoids and Simpson is used as an integral replacement). The numerical simulation by MatLab explains the error formation essence in the signal times difference calculating.

1. Introduction

1.1. The relevance of the flow rate determining

At present, the problem of measuring the volumetric flow rate of liquids and gases [1-3] has a wide area of distribution and a high importance. This is necessary to ensure accounting, monitoring and control of the transported medium, calculating the amount of consumed heat carrier in various aspects of the flow determination application. These include correct operation of heat and hydroelectric power plants, functioning of production and technological processes at factories, pumping installations on main and supply (secondary) pipelines, water treatment plants, housing and communal services, etc.

For these indicated tasks, electronic flowmeters with a non-contact measuring method [2-4] are well suited. Thus, the transit-time (“time-of-flight”, ToF) ultrasonic method for determining the flow rate [2, 5] has become widespread and worldwide recognized.

Modern models of ultrasonic liquid and gas flowmeters (UFM) often have additional applications of digital signal processing (DSP), which improves their metrological (accuracy, resolution, dynamic
range, etc.) and operational characteristics, increase the possible functionality of this devices. For completeness of the presenting research object, we further restrict the area of interest to ultrasonic time-of-flight flowmeters with clamp-on sensors (primary transducers of the piezoelectric type – PZT) [2, 5, 6], designed to work with closed pipelines under pressure of the flow.

1.2. The generalized flow measurement principle

The operation principle of the transit-time (“time-of-flight”) UFM is shown in the corresponding diagram in Fig. 1.1 [2, 5]. Piezoelectric transducers (PZTs), acting as emitters and receivers of an ultrasonic probing signals (UPS), alternately emit and receive UPS, along the carrier medium flow (from PZT1 to PZT2) and against the flow, respectively. The moving stream “enthralls” the ultrasonic wave beam (UWB) and its velocity (for a stationary observer) adds up with the flow velocity, which leads to decrease the UWB propagation time moving upstream – duration \( t_1 \), and to increase in the UWB propagation time moving downstream – duration \( t_2 \) [2, 5]:

\[
t_{1,2} = \frac{L_a}{c \pm v_L \cdot \cos \alpha} + 2t_{\text{PWL}} + 2t_{\text{ATL}}.
\]

where \( t_{1,2} \) is the UWB propagation time in the acoustic path; \( L_a \) — “active base” – part of the path of an ultrasonic signal, at which the flow velocity is different from zero (when UWB is propagating in a medium), where \( L_a = D/\sin \alpha \); \( c \) — ultrasound speed in a motionless medium; \( v_L \) is the average flow velocity along the UWB length; \( \alpha \) is the propagation angle of the UWB in the pipeline relative to the pipe axis; \( l \) is the distance between the sensors along the pipe axis, with \( l = L_a \cdot \cos \alpha \); \( t_{\text{PWL}}, t_{\text{ATL}} \) — the UWB propagation times in the pipe wall and in the material of the sensor’s acoustical transmission line (as a rule, it is made in the form of a wedge or a prism); \( D \) is the pipe inner diameter [2].

![Flow measurement by ultrasonic “transit-time” method for UFM with clamp-on sensors](image)

The flow velocity (proportional to the измеряемому volumetric flow rate) determination is based on a function of the informative parameter \( \Delta t \) of the receiving sensor’s output signal – time shift between UPS, e.g. \( \Delta t = t_2 - t_1 \). With the known \( D \) and \( c \), the velocity \( v_L \) can be defined as [2]:

\[
v_L = \frac{\Delta t \cdot c^2}{2D \cdot \cot \alpha}.
\]

Determination of the flow rate of liquids or gases using UFM significantly depends on the accuracy of the acoustic signals propagation times measuring along the flow \( t_1 \) and against the flow of the controlled medium \( t_2 \) [2, 5]. The final difference in the UPS propagation times \( \Delta t = t_2 - t_1 \) in the general case will be proportional to the flow velocity \( v_L \) of the medium and the measured volumetric flow rate.

The problem complexity of the propagation times high-precision measurement in UFM lies in extremely high requirements for the time intervals meters resolution; as shown in [7-9], the time difference \( \Delta t \) measurement error should not exceed \((20-50) \cdot 10^{-15} \) s, which is usually no more than \( 5 \cdot 10^{-5} \) periods of PZT’s natural oscillations (see below the UPS form on Fig. 2.1).
With the modern development of microelectronics, this high accuracy in measuring the difference in the UPS propagation times can be realized by DSP methods, which, in some cases, turn out to be simpler than traditional analog methods for measuring short time intervals. The most widespread method for these purposes is the correlation functions calculating for signals $Y_1$ and $Y_2$ (see Fig. 2.1).

**1.3. The Cross-Correlation Technique**

To measure the specified time difference in modern UFM, digital signal processing methods are increasingly used, which consist in determining the cross-correlation function (CCF) [10-13] for signals received along the flow – $Y_1(t)$ and against the flow $Y_2(t)$. The maximum’s position of this function corresponds to the measured time difference $\Delta t$. When using DSP, both signals themselves and the calculated correlation function $R(m)$ are obtained as sampled in time [10, 11]:

$$R(m) = \frac{1}{N - N_0} + \sum_{n=N_0}^{N} y_1(n) \cdot y_2(n + m)$$

(1.3)

as $y_i(n) = Y_i(nT_0)$ are numerical sequences of digitizing input signals at the ADC output (see Fig. 2.1).

The values of the discrete CCF (DCCF) with a limited number of samples $(N-N_0+1)$ in the sequences can have significant differences from the corresponding values of the continuous CCF calculated for $\tau = nT_S$ [10, 11]:

$$R(\tau) = \frac{1}{t_{\text{max}} - t_0} \int_{t_0}^{t_{\text{max}}} y_1(t) \cdot y_2(t + \tau)dt$$

(1.4)

where $t_{\text{max}} = nT_S$, $n = N_{\text{max}}$ – sample’s number at which the CCF’s sequence has the largest value; $t_0 = nT_S$ at $n = N_0$; or, in other words, there are the integration limits, beyond which one of the signals is taken to be zero.

When the number of samples in the sequences $Y_1$ and $Y_2$ tends to infinity, the DCCF values will coincide with the corresponding values of the continuous CCF.

To find the exact position of the CCF maximum from the DCCF calculated values, it is interpolated, for example, using a second-order polynomial [14-17]. For example, in the case of the DCCF interpolation by a parabola, the maximum’s position is determined by the polynomial’s coefficients [17]: $\tau_{\text{max}} = -a_1/a_2$, which, in turn, are uniquely determined by the DCCF values in the region of its maximum: $R_0$, $R_1$, $R_2$. As a result, the maximum’s position during interpolation by a parabola is determined by the equation [17]:

$$\tau_{\text{max}} = \frac{3R_0 - 4R_1 + R_2}{2R_0 - 4R_1 + 2R_2}T_S$$

(1.5)

where $\tau_{\text{max}}$ is the bias of the interpolated CCF maximum relative to the point $m_iT_S$; $R_0$, $R_1$, $R_2$ are the DCCF values at samples $m_0$, $m_1$, $m_2$, and these points are selected from the condition $R_0 < R_1 \geq R_2$.

It follows from equation (1.5) that an error in calculating the values of $R_0$, $R_1$, $R_2$ leads to an error in determining the maximum’s position $\tau_{\text{max}}$ and an error in measuring the informative parameter $\Delta t$ (or $d\tau$). Besides, our numerical simulation of the correlation measurement method carried out in Section 3 (see Fig. 3.5) showed that when the values of $R_0$, $R_1$, $R_2$ coincide with the corresponding values of the CCF, i.e. for $R_m = R(mT_S)$, the calculated value of $\tau_{\text{max}}$ exactly coincides with the value of $d\tau$. This coincidence is explained by the fact that both the parabola and the CCF are symmetric functions with respect to their extrema. In this regard, the DCCF error will be determined by the following difference:

$$\Delta R_m = R_2(m) - R(m \cdot T_S)$$

(1.6)

For the purpose of further analysis, it is important to note that the correlation method allows you to effectively suppress the noise contained in the received signals. In this case, the filtering coefficient:
$K_F = a_{DCCF}/a_{UPS}$ [9], where $a_{DCCF}$ is the ratio of the DCCF’s noise component (RMS) to its maximum value, $a_{UPS}$ is the signal-to-noise ratio in the received UPS, increases with an increase in the signal form factor: the ratio of the RMS signal in the range of integration $t_0$, $t_{\text{max}}$ to the signal amplitude. For this reason, the initial and final signal waveforms, with a small amplitude, are uninformative. It is advisable to “truncate” this signals both on the left (at the UPS beginning) and on the right (at the signal’s end). If both signals are truncated in the same way, for example, when the first and third waves cross the zero level, then the CCF maximum’s position for truncated UPS will exactly coincide with the CCF maximum’s position for full UPS, and noise will be suppressed in an optimal way. On the other hand, signal truncation leads to a decrease in the sample size, and, as a consequence, to an increase in the calculation error of the DCCF values.

To reduce the UFM cost, they strive to reduce the signals sampling frequency as much as possible, which, on the other hand, leads to an increased error in calculating the CCF’s maximum and an error in measuring the liquid or gaze flow rate.

2. Influence of the ending sample length and sampling period on the CCF of the finite UPS

2.1. The problem statement

Let’s formulate additional explanations to the algorithm for calculating $dr$:

- a) The CCF (continuous) has a maximum at a point displaced from the reception time moment of one of the signals (the moment of the integration beginning) by $dt$.
- b) The DCCF values are sampled in time and coincide with the CCF corresponding values (at the points $m\cdot T_d$) only at a high sampling rate and at $N \to \infty$.
- c) since the DCCF is sampled in time to find the maximum, it is interpolated (usually using a 2nd order polynomial or a cosine function) [15, 17].

Addition to item (c). It was initially assumed that If the DCCF values were accurate, then one or the other interpolation method would give the same results for $dt$, since the interpolating functions are symmetric, and the CCF is symmetric. However, the CCF in our case (for a truncated sine signal) is a pure cosine. And the cosine is interpolated by a parabola with an error leading to a shift in the maximum. This was previously shown by modeling [17]. i.e., the cosine and parabola pass through three points in different ways, both remain symmetrical, but with different maxima.

So, for real (radio pulse signals, limited in time), the DCCF values are calculated with a significant error, which leads to an error in measuring the propagation time difference – the main informative parameter in the UFM.

The aim of the research was to analyze the influence of the sampling and signal truncation parameters on the accuracy of the DCCF values calculating and on the measurement error $dr$.

In order to carry out this study, namely, how the sampling and signal truncation parameters will affect the accuracy of the DCCF values calculating in the its maximum’s region, we represent the signals $y_1$ and $y_2$ in the form of sinusoidal signals with an arbitrary phase shift relative to the sampling time moments, and the signal $y_2$ is truncated to three full waves.

In Fig. 2.1. two numerical sequences of acoustic signals are presented: UPS samples $Y_1$ and $Y_2$. The size of the second sample is limited to $n < N_{\text{max}} = 38$. The signal $Y_2$ is delayed relative to the first for a time $t_p$, which is not a multiple of the sampling period $T_S$.

A priory, the main maximum coordinate of the CCF in time $\Delta t_{\text{max}}$ will correspond to the designated parameter $dr$ [10, 12, 13]. It is required to determine the influence on the calculation error of the CCF maximum position $\Delta t_{\text{max}} = \tau_{\text{max}} - t_p$, where $\tau_{\text{max}}$ is the calculated position of the CCF maximum from a) the sample length, time $t_d$ and b) the number of sampling points in the 1st period of the UPS: $N_S = T_c / T_S$, where $T_c$, $T_S$ is the period of the sinusoidal signal and the sampling period.

This task is essentially divided into two subtasks:

- the effect determination of sampling parameters on the accuracy of calculating $t_{\text{max}}$ of a quasi-sinusoidal function from three of its values near the maximum [15, 17];
- the error determination in calculating the DCCF values near its CCF global maximum.
Within the framework of this article, we will focus on Problem No. 2, and Problem No. 1 is reduced to interpolating the CCF near the region of its main maximum over three points (samples). So, earlier in [17] the quadratic CCF interpolation (or parabolic) was analyzed, cosine interpolation studied in [15] and will be assumed in future.

2.2. Determination of the error in calculating the DCCF values
The error in the DCCF calculating was determined earlier in equation (1.6) in Section 1. Then we will give a reveal the individual components from which this error is obtained [10, 11]:

\[
R_S(m) = \frac{1}{L} \sum_{k=0}^{L} \sin \left(2\pi \cdot \frac{kT_S}{T_C} \right) \sin \left(2\pi \left( \frac{kT_S - t_d - mT_S}{T_C} \right) \right)
\]  
(2.1)

\[
R(mT_S) = \frac{1}{LT_S} \int_0^{LT_S} \sin \left(2\pi \cdot \frac{t}{T_C} \right) \sin \left(2\pi \left( \frac{t - t_d + mT_S}{T_C} \right) \right) dt
\]  
(2.2)

i.e. this CCF are calculated for finite sinusoidal signals that differ from the real UPS shown in Fig. 2.1.

As follows from the expression (2.1), DCCF: \(R_S(m)\), this is a rough value of the integral in equation (2.2), since the integration is performed by the “rectangles” method (the summed "continuous" function is limited to "0") [18-21]. It will be important to note the negative aspect that the integration error will depend on the time \(t_d\), i.e. will lead to an error in measuring the propagation times difference. The value of this error is best determined by numerical simulation. Integral function graph:

\[
f(t) = \sin \left(2\pi \cdot \frac{t}{T_C} \right) \sin \left(2\pi \left( \frac{t - t_d + mT_S}{T_C} \right) \right) = \frac{\cos \left(2\pi \left( \frac{t_d - mT_S}{T_C} \right) + \cos \left(2\pi \left( \frac{2t - t_d + mT_S}{T_C} \right) \right) \right)}{2}
\]  
(2.3)

shown in Fig. 2.2. Using this formula, it is easy to determine \(R(mT_S)\). Naturally, the integration limits must be multiples of the signal period \(T_C\). So, for a sinusoid, \(LT_S = T_C\) makes sense. In this case, the integral is calculated abstractly “on the fly” and is equal to

\[
R(mT_S) = 0.5 \cdot \cos \left(\frac{2\pi (t_d - mT_S)}{T_C}\right) \Bigg|_{T_C}
\]  

i.e. this is the RMS squared product of sinusoids with a phase shift of
Qualitatively, it can be seen from the graphs above that the value of the integral sum according to equation (2.1) for these functions will give a large error in the DCCF calculation. The best result can be obtained by using the numerical integration methods [18-21], namely, the “trapezoid” formula or the Simpson formula (using quadratic or parabolic function interpolation) [18-21].

Thus, the formula of trapeziums [18-22], applied to the integral functions of the CCF:

$$R_s(m) = \frac{1}{L} \left[ \frac{y(0,m) + y(L,m)}{2} + \sum_{k=1}^{L-1} y(k,m) \right]$$

and Simpson's formula [18-22]:

$$R_s(m) = \frac{1}{3L} \left[ y(0,m) + y(L,m) + 2 \sum_{k=1}^{(L-2)/2} y(2k,m) + 4 \sum_{k=1}^{(L-2)/2} y(2k-1,m) \right]$$

where \( L \) is the total number of points on the curve: \( y(l, m) = \sin(2\pi l/L C) \cdot \sin(2\pi / T_C \cdot (kT_S - t_d + mT_S)) \), while \( L \) must have an even value; \( k \) is an intermediate summation variable, \( k = l/2 \) for even \( l \), and \( k = l/2 + 1 \) for odd \( l \).

For the possibility of further calculations and the adequacy of the obtained results, we introduce a number of reasonable restrictions on the integrable functions:

- a) the UPS signals must be synchronized with respect to the sampling frequency, its period is a multiple of the sampling period, and it ends with a “0” value at \( m+1 \) half periods.
- b) the maximum CCF is observed for values \( \tau \approx t_d \), therefore, we introduce a new variable \( \Delta \tau = \tau - t_d \) and it is natural to assume that \( \Delta \tau \ll \pi \).

A summary brief:

1) if the window for the CCF calculating is limited to an integer number of periods of the support (reference) function, then the calculation error will be minimal, since the function \( y(l, m) \) for arbitrary values of \( m \) and \( t_d \) at the window boundaries turns to “0”.
2) the DCCF calculating accuracy can be increased by using the formulas (2.4) and (2.5), then it will be finite and depend on time \( t_d \) (this is an assumption, and requires confirmation).

2.3. Truncation of the UPS digital sequence when the DCCF calculating

With the pulsed excitation of the transducers, due to the resonant nature of their transfer function, the received signals (UPS) have an extended duration (see Fig. 2.3).
The accuracy of the DCCF calculating is determined by the power of the convolved signals: the higher the signal power, the higher the filtering coefficient of the error random component. Therefore, the “tail” of the UPS digital sequence is uninformative. In this case, it is advisable to truncate the digital sequence. However, simply “removing” part of the digital sequence will result in unequal distortion of the $Y_1$ (downstream) and $Y_2$ (upstream) signals. In this case, the position of the CCF main maximum of the signals $Y_1$ and $Y_2 - \Delta t$, equal to the difference in the UPS propagation times, will be calculated with an error, which will lead to an error in measuring the flow rate.

\begin{equation}
\Delta R(m) = Y_1(n)\cdot Y_2(n-d+m) = Y_1(n)\cdot Y_2(n-d+m) \approx -0.034.
\end{equation}

With a fast increase in the signal, this correction can be significant and will lead to an asymmetry of the CCF relative to its main maximum, which, in turn, will lead to an error in the measurement.
(calculation) of the CCF maximum position, and this error will depend on the displacement of the UPS signals onset relative to the sampling instants.

In addition to this, it is possible to note the reason for the CCF maximum offset for the truncated UPS relative to the maximum of the same signals, but full and not sampled signals. So, when sampling, the \( Y_1 \) and \( Y_2 \) signals become non-identical. It is known a priori that for signals of different shapes, the CCF function will be asymmetric [23].

In order to avoid this error, the UPS signals \( Y_1 \) and \( Y_2 \) after truncation must have the same shape. This truncation (only the right truncation is considered) is possible if both signals are truncated at the time moment when the selected signal’s half-wave crosses the “0”-th level [9, 24].

2.4. The mathematical meaning of the DCCF

The DCCF from a mathematical point of view can be described as follows (the formula is given for the case of a signal’s discrete autocorrelation function – DACF, the case of DCCF was considered earlier in Section 1, equation (1.3)) [10, 11, 13]:

\[
R(m) = \frac{1}{N-N_0+1} \sum_{n=N_0}^{N} y(n) \cdot y(n+m)
\]  

(2.7)

The summation in equation (2.7) is the numerical integration of the function \( Y(n) = y(n) \cdot y(n+m) \) by the trapezoidal method [18-21], if we assume that \( Y(N_0) = Y(0) = 0 \) (see. Fig. 2.5), while the actual DCCF is the average value of this function on the interval from zero to \((N-N_0+1) \Delta t\). Moreover, when the DCCF calculating, the values of \( N \) and \( N_0 \) are selected from the condition that outside boundaries the summed function is identically equal to “0”, therefore, to obtain a real integration interval, “1” is added to the difference \( N-N_0 \) (this follows well from Fig. 2.5). A generalized analytical expression for describing the mathematical meaning of integration by the trapezoidal method [22] is given in (2.8):

\[
I_{0-N \Delta t} = \int_{0}^{N \Delta t} y(x) dx \approx \sum_{n=0}^{N} S_{trap} = \left( \frac{Y_0 + Y_1}{2} + \frac{Y_1 + Y_2}{2} + \ldots + \frac{Y_N}{2} \right) \cdot \Delta t \approx \sum_{n=0}^{N} y(n)
\]

(2.8)

upon condition to \( y(n \leq 0) = 0; y(n \geq N) = 0 \).

If the function \( Y(n) \) reaches “0” not at the sampling points, but at the point \( \delta t_1 \) and at the point \( N+\delta t_2 \), then for the correct (approximate) calculation of the average value \( Y(n) \) it is enough to reduce the averaging interval:

\[
R^*(m) = \frac{1}{N-N_0+1-\delta t_1+\delta t_2} \sum_{n=N_0}^{N} y(n) \cdot y(n+m)
\]

(2.9)

here \( N-N_0 \) is the number of points at which the function \( Y(n) \) is not identically equal to “0”. In Fig. 2.5, these points are numbered as 1 ... 5.

This is the essence of the modernized cross-correlation method for DCCF calculating. Naturally, for this it is necessary to calculate the points \( \delta t_1 \) and \( \delta t_2 \), where the functions \( y(x) \) and \( y(x+m) \) are truncated, and one of them intersects the abscissa’s axis. It is possible to determine these values using linear interpolation (under the assumption that \( y_0 < 0 \) and \( y_5 > 0 \)). So, for \( \delta t_1 \):

\[
\delta t_1 = -\frac{y_0}{y_1-y_0} \Delta t
\]

(2.10)
The intersection point “0” at the end of the sample sequence (when passing from negative values to positive values: \( y_N < 0, y_{N+1} > 0 \); if \( Y_N = 0 \), then this is the zero point and no further search is required) is determined in a similar way (\( y_N \rightarrow y_5, y_{N+1} \rightarrow y_6 \)):

\[
\Delta t_2 = \frac{y_N}{y_{N+1} - y_N} \Delta x = \frac{y_5}{y_5 - y_6} \Delta x
\]

(2.11)

The reason for the high integration accuracy by the trapezoidal method can be illustrated on the following graphs of the CCF generator (see Fig. 2.6 (a) and Fig. 2.6 (b)).

As can be seen from the above graphs on Fig. 2.6, “truncation” of a sinusoid during its linear interpolation does not affect the value of the integral of this function, if the sampling frequency is a multiple of the signal frequency. The “truncation” of the function occurs symmetrically, i.e. the areas of the filled segments (Fig 2.6 (b)) are equal and, when integrated, completely compensate each other (the upper segment increases the integral’s value, and the lower segment decreases).

Thus, for the correct CCF calculation of sinusoidal signals from the functions values at discrete moments (time samples), it is sufficient to fulfill the following conditions:

- 1) Correctly determine the integration limits (taking into account the cut-off of the signals when they are truncated in time).
- 2) The sampling frequency should be at least 4 times higher and a multiple of the UPS signal frequency. For a symmetric cut-off of the generating function, the frequency ratio must be a multiple of four (in our case, \( M = 8 \)): \( M = 4, 8, 12, \) etc.
Figure 2.6. (b). The generating functions of the DCCF for a sinusoidal signal are sinusoids with a frequency $\omega = 2\omega_C$ and are shifted along the Y-axis by $\Delta y = \cos(2\pi m/M)$, at $M = \frac{T_c}{T_S} = 8$ ($m = 2$ corresponds to an offset of signals by $\pi/2$, i.e., $R(m) = 0$), where $\omega_C = \frac{2\pi}{T_c}$.

How can the accuracy of the CCF maximum position calculating be limited? Primarily:

- a) noise in signals (the more $M$, the less the noise compensation effect of the segment areas);
- b) the accuracy of the truncation moments determining of the generating function, i.e. moments of the UPS signals crossing ($y_1$ and $y_2$) through the “0” level, and since these moments are determined in our case by linear interpolation, then this error will also significantly depend on the $a_n$ ratio and on $M$.

The analysis of the indicated errors should begin with the analysis of the error in calculating the maximum position from three samples (values $y_0$, $y_1$, $y_2$) in the CCF maximum region [14-16]. However, within the framework of this article, this issue is not considered in detail. More information about this can be found in [17].

3. Simulation and verification of theoretical provisions

We will produce the simulation indicated in subsection 2.2, taking into account the theses of subsection 2.3, the principle of the CCF calculating by applying the numerical method of taking the integral on the example of sinusoidal signals $y_1(t) = \sin(2\pi T_c(t-\delta t_1))$ and $y_2(t) = \sin(2\pi T_c(t-\delta t_2-2mT_S))$ for the relative phase shift (in fractions of the sampling period) $m = 0, 1, 2, 3$ and $d = 1$. The full period of the UPS signal (in fractions of the sampling period) $M = \frac{T_c}{T_S}$ is chosen equal to 8, the given values of the CCF offset (by argument) in the signal cut-off points $\delta t_1$ and $\delta t_2$ are set to 0.1 and 0.7, respectively. In Fig. 3.1 displays the graphs of the sine signals (a) and the obtained subintegral function (as a product of signals) for the case $m = 0$ (b) and $n = t/T_S$ is the reduced value of the signal argument over time. The values of the calculated CCF (at $m = 0$) are given in Table 1.

| $m$   | 0    | 1    | 2    | 3    |
|-------|------|------|------|------|
| $R^*(m)$ | 0.225057664 | 0.430754966 | 0.523676846 | 0.4121315 |

Another example of the CCF calculating based on the subintegrals is shown in Fig. 3.2. So in Fig. 3.2 (a) shows the graphs of the investigated sine signals and the resulting subintegral function (for $m = 0$), and in Fig. 3.2 (b) plots of the corresponding CCF and DCCF. Detailed values of the calculated correlation functions are given in Table 2.
In Fig. 3.3 shows the curves of the CCF’s subintegral function of sinus signals (for different $m$), where it is seen that the use of linear interpolation leads to symmetric cutting-off of this function’s segments (sinus with doubled frequency). And these “cut-off” segments will compensate each other if the number of positive waves is equal to the number of negative waves, and all these waves must begin and end with zero values.

Table 2. The achieved CCF and DCCF values.

| $m$  |  |  |  |
|------|---|---|---|
| $R(mT_S)$ | 0.245087742 | 0.47801986 | 0.501360055 | 0.31276276 |
| $R(m)^*$ | 0.225057664 | 0.430754966 | 0.523676846 | 0.4121315 |

Interpolation at the “0” point of the DCCF gives relatively small errors, and the main problem is that the DCCF’s values, calculated at a low sampling frequency and in the absence of the UPS synchronization relative to the sampling signal, differ significantly from the analogous values of the “analog” CCF (discrete CCF at high resolution). This error, as shown by the obtained simulation results, depends on the cut-off (truncation) signal’s value: i.e. offset “0” of the time moment of the
UPS crossing from the sampling moment. This principle is well illustrated by the example of the corresponding sinusoid picture (see Fig. 3.4, limiting case). Here the UPS signal is given in the form of an one complete sinusoid’s period; \( \Delta x_i \) – fractional parts of the signals \( y_i(t) \) delay (cut-off) relative to “0” sample (relative phase shift of the \( i \)-th signal), \( \Delta T \) – delay of signal \( y_2 \) relative to signal \( y_1 \) (measured difference in propagation times, expressed as the phase difference of signals \( y_1 \) and \( y_2 \)), \( \tau \) is the time shift estimate of the UPS signals (the CCF’s global maximum for the signals under consideration).

Thus, if the sinusoids samples are “truncated” in different ways at different points, i.e. when the difference in propagation times \( dT \) is not a multiple of the sampling period, then the interpolated DCCF maximum will not coincide with the CCF’s main maximum, namely, this maximum position determines the desired time \( dT \). This explains the cause of the main error for the \( dT \) measuring.

Based on the discussions in Section 2.3, to estimate the DCCF error at \( \delta t_1 = 0.1, \delta t_2 = 0.7, d = 1 \), the following values were obtained: \( R(mT_s) = 0.610127733, R'(m) = 0.954458553 \). In Fig. 3.5 shows the error dependence in determining the CCF’s maximum position with parabolic interpolation in a wide range of variations in the signals time delay. Here \( dr = (t_{max} - n_0T_s) \), \( t_{max} \) is the actual position of the CCF maximum, equal to the difference in the UPS delay times, \( n_0 \) is the number of the zero reference sample: the first one before the global maximum. As you can see, the achieved error rates of the order of 3 ns (at \( F_s = 8 \) MHz) significantly exceed the required values of 20-50 ps. It is possible to reduce the DCCF calculating error using the “modernized” cross-correlation method described in section 2.4.
Figure 3.5. The error in determining the CCF’s maximum with interpolation by a parabola.

It can be noted that Simpson's formula (2.5) for the CCF calculating from the DCCF discrete values does not give a significant advantage over the trapezoidal formula (2.4) for the numerical integration of the subintegrals of signals discrete convolution (the simulation results were omitted here).

4. Conclusions
The article presents generalized analytical expressions for the CCF (1.4) and DCCF (1.3), (2.9) calculating as functions of the cut-off values $\delta t_1$ and $\delta t_2$, but they are very cumbersome and difficult to analyze. Expressions for the CCF are obtained and can be used to calculate the reference values. On the other hand, it is better to use mathematical modeling tools to estimate the DCCF values. Presumably, the differences that can be detected will be systematic, and then they can be unambiguously compensated by the cut-offs value $\delta t_1$ and $\delta t_2$.

The main aspect in the problem of the DCCF correct interpolation is that the obtained interpolating function (in the maximum’s region) coincides as best as possible with the real CCF obtained from signals without time sampling, i.e. without "truncating" signals from the "0" crossing point to the first nonzero value (or from the last nonzero value to "0", if truncated to the right).

Carrying out digital processing of the truncated DCCF in similar way that its interpolated maximum coincides with the signal delay is not fully achievable at the current stage. The main reason for the error occurrence in calculating the interpolated DCCF maximum position can be the difference in the waveforms when they are truncated (these are only preliminary arguments).

However, there is the prospect of modifying the signal truncation algorithm and calculating the “exact” CCF values from a discrete sample. This error can be dealt with by signals digital interpolation, achieving the same shape of truncated UPS, which seems to be non-trivial.

That's why, it is proposed to truncate the UPS, keeping the points of the “0” signals level crossing. For this, it is sufficient to change only the integration limits. Thus, for sinusoidal signals, the calculated DCCF values will be exactly equal to the corresponding CCF values (provided that the zero points are precisely determined), then an error in determining these “zeros” will give an error of the second order DCCF (in fact, this will be an error from errors). But this requires further research.

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