QUANTUM CONTROL IN STRONGLY DRIVEN OPTICAL LATTICES

DONATELLA CIAMPINI  
CNISM and INO-CNR, Dipartimento di Fisica, Università di Pisa, L.go B. Pontecorvo 3  
Pisa, I-56127, Italy  
ciampini@df.unipi.it

OLIVER MORSCH  
INO-CNR, Dipartimento di Fisica, Università di Pisa, L.go B. Pontecorvo 3  
Pisa, I-56127, Italy  
morsch@df.unipi.it

ENNIO ARIMONDO  
CNISM and INO-CNR, Dipartimento di Fisica, Università di Pisa, L.go B. Pontecorvo 3  
Pisa, I-56127, Italy  
arimondo@df.unipi.it

Received Day Month Year  
Revised Day Month Year

Matter waves can be coherently and adiabatically loaded and controlled in strongly driven optical lattices. This coherent control is used in order to modify the modulus and the sign of the tunneling matrix element in the tunneling Hamiltonian. Our findings pave the way for studies of driven quantum systems and new methods for engineering Hamiltonians that are impossible to realize with static techniques.

Keywords: Optical lattices; driven quantum systems; quantum control.

1. Introduction

The experimental realization of quantum degenerate states in ultracold atoms gases in the 1990s opened the possibility to coherently control many-body systems isolated from external perturbations and at temperatures close to absolute zero. These techniques, applied to ultracold atomic gases in optical lattices, allow physicists to operate at the interface between quantum information and many-body systems.

The possibility of easily changing the parameters (depth, lattice constant etc.) of a defect-free periodic potential permits one to observe, for example, Bloch oscillations and the quantum phase transition into a Mott-insulator regime. In strongly driven quantum systems, a time-dependent (in particular periodic) perturbation is introduced into the system in order to probe it or to change its fundamental properties. From a theoretical point of view, under certain circumstances the phe-
nomena resulting from the strong driving can be described by absorbing the effect of the external driving inside one of the parameters of the unperturbed system. This approach bears a strong resemblance to the renormalization of the mass of an electron in the periodic potential of a crystal familiar from condensed matter physics.

Here we present experimental results on Bose-Einstein condensates in driven optical lattices. We show that by periodically forcing the system, the fundamental tunneling properties of the atoms inside the lattice can be adiabatically changed and the system can be carried into novel regimes impossible to reach with static techniques.

In the following Sec. 2 we review the theoretical description of strongly driven optical lattices. In Sec. 3 we describe the experimental setup and the details of how the driving is implemented and report our measurements, which demonstrate the renormalization of the tunneling matrix element and the dynamics of the condensate after an abrupt change of the sign of the tunneling. In the final Sec. 4 we spell out our conclusions.

2. Theory

Our system consisting of a Bose-Einstein condensate inside a sinusoidally shaken one-dimensional optical lattice is approximately described by the Hamiltonian

$$\hat{H}_0 = -J \sum_{\langle i,j \rangle} (\hat{c}_i^\dagger \hat{c}_j + \hat{c}_j^\dagger \hat{c}_i) + \frac{U}{2} \sum_j \hat{n}_j (\hat{n}_j - 1) + K \cos(\omega t) \sum_j j \hat{n}_j, \quad (1)$$

where $\hat{c}_i^{\dagger}$ are the boson creation and annihilation operators on site $i$, $\hat{n}_i = \hat{c}_i^\dagger \hat{c}_i$ are the number operators, and $K$ and $\omega$ are the strength and angular frequency of the shaking, respectively. The first two terms in the Hamiltonian describe the Bose-Hubbard model with the tunneling matrix element $J$ and the on-site interaction term $U$. By using the Houston functions (a different procedure leading to the same result was used by Ref. 9) it can be demonstrated that the presence of a driving force corresponds to a renormalization of the Bose-Hubbard Hamiltonian, leading to an effective tunneling parameter

$$J_{\text{eff}} = J_j\mathcal{J}_0(K_0), \quad (2)$$

where $\mathcal{J}_0$ is the zeroth-order ordinary Bessel function and we have introduced the dimensionless parameter $K_0 = K/\hbar \omega$.

3. Experiment

3.1. Setup

In our experiment we created BECs of about $5 \times 10^4$ 87-rubidium atoms in an optical crossed dipole trap (for details of the experiment see 12,13). After obtaining pure condensates, the powers of the trap beams were adjusted in order to obtain
 elongated condensates with the desired trap frequencies (≈ 20 Hz in the longitudinal direction and 80 Hz radially). Along the axis of one of the dipole trap beams a one-dimensional optical lattice potential was then added by ramping up the power of the lattice beams in 50 ms (the ramping time being chosen such as to avoid excitations of the BEC). The optical lattices used in our experiments were created using two counter-propagating gaussian laser beams (λ = 852 nm) with 120 µm waist and a resulting optical lattice spacing \( d_L = \lambda/2 = 0.426 \mu m \), sketched in Fig. 1.

The depth \( V_0 \) of the resulting periodic potential is measured in units of \( E_{\text{rec}} = \hbar^2 \pi^2/(2m d_L^2) = p_{\text{rec}}^2/2m \), where \( m \) is the mass of the Rb atoms and \( p_{\text{rec}} \) is the recoil momentum. By introducing a frequency difference \( \Delta \nu \) between the two lattice beams (using acousto-optic modulators which also control the power of the beams), the optical lattice could be moved at a velocity \( v = d_L \Delta \nu \) or accelerated with an acceleration \( a = d_L d_L \Delta \nu \). In order to periodically shake the lattice, \( \Delta \nu \) was sinusoidally modulated with angular frequency \( \omega \), leading to an inertial time-varying force in the lattice reference frame

\[
F(t) = m \omega d_L \Delta \nu_{\text{max}} \cos(\omega t) = F_{\text{max}} \cos(\omega t).
\]

The peak shaking force \( F_{\text{max}} \) is related to the shaking strength \( K \) appearing in Eq. 2 by \( K = F_{\text{max}} d_L \), and hence the dimensionless shaking parameter is

\[
K_0 = \frac{K}{\hbar \omega} = \frac{m d_L^2 \Delta \nu_{\text{max}}}{\hbar} = \frac{\pi^2 \Delta \nu_{\text{max}}}{2 \omega_{\text{rec}}},
\]

(4)

Fig. 1. Experimental setup for a driven optical lattice. The two beams originating from the same laser pass trough two separate acousto-optic modulators (AOM). Each AOM is driven by a function generator whose frequency can be modulated both with internal preset functions or by using a triggered external channel.
For a typical shaking frequency $\omega/2\pi = 3\text{kHz}$ the spatial shaking amplitude $\Delta x_{\text{max}} \approx 0.5d_L$ at $K_0 = 2.4$.

### 3.2. Results

After loading the BECs into the optical lattice, the frequency modulation of one of the lattice beams creating the shaking was suddenly switched on. In order to measure the effective tunneling rate $|J_{\text{eff}}|$ between the lattice wells (where the modulus indicates that we are not sensitive to the sign of $J$, in contrast to the time-of-flight experiments described below), we then switched off the dipole trap beam that confined the BEC along the direction of the optical lattice, leaving only the radially confining beam switched on (the trap frequency of that beam along the lattice direction was on the order of a few Hz and hence negligible on the timescales of our expansion experiments, which were typically less than 200 ms). The BEC was now free to expand along the lattice direction through inter-well tunneling and its in-situ width was measured using a resonant flash, the shadow cast by which was imaged onto a CCD chip.

![Graph](image.png)

**Fig. 2.** Dynamical suppression of tunneling in an optical lattice. Shown here are the values of $J_{\text{eff}}/J$ calculated from the expansion velocities as a function of the shaking parameter $K_0$. The sign of $J_{\text{eff}}/J$ has been determined by time-of-flight experiments. The lattice depth and shaking frequency were: $V_0/E_{\text{rec}} = 6$, $\omega/2\pi = 1\text{kHz}$. The dashed line is the theoretical prediction.
The sign of $J_{\text{eff}}$ can be measured by looking at the interference pattern of a BEC released from a modulated lattice, after a sufficiently long time of flight. After shaking the condensate in the lattice for a fixed time between 1 and $\approx 200 \text{ ms}$ we accelerated the lattice for $\approx 1 \text{ ms}$ with an acceleration chosen such that at the end of the process the BEC was in a staggered state at the edge of the Brillouin zone. After switching off the dipole trap and lattice beams and letting the BEC fall under gravity for 20 ms, this resulted in an interference pattern featuring two peaks of roughly equal height for shaking amplitudes $K_0 < 2.4$ for which $J_{\text{eff}} > 0$. The visibility of the interference pattern reflects the phase coherence of the BEC. In the region between the first two zeroes of the Bessel function, where $J_0 < 0$, the interference pattern is shifted by half a Brillouin zone, as predicted from the dependence of the band shape on the sign of $J_{\text{eff}}$. When the shaking is switched on with $K_0$ between 2.4 and 5.9, the atoms, in fact, occupy the new ground state at the edge of the Brillouin zone. Fig. 3 shows how the system evolves from the initial state for which $J_{\text{eff}} > 0$ to the new one with $J_{\text{eff}} < 0$.

4. Conclusions
In summary, we have measured the dynamical suppression of tunneling of a BEC in strongly shaken optical lattices and found excellent agreement with theoretical predictions.

The fact that time-of-flight images with high-contrast interference patterns could be realized shows that the tunneling suppression occurs in a phase-coherent way and can, therefore, be used as a tool to control the tunnelling matrix element and without disturbing the condensate. The condensate then occupies a single Floquet state, just as it occupies a single energy eigenstate when there is no forcing.

---

**Fig. 3.** Interference pattern of a time-of-flight experiment with final acceleration to the zone edge is shown for various times in the modulated lattice at $K_0 = 4$ (where $J_{\text{eff}}$ is negative): (a) $t = 1 \text{ ms}$, (b) $t = 2 \text{ ms}$, (c) $t = 2.5 \text{ ms}$ (on the $x$-axis, the spatial position has been converted into the corresponding momentum in units of the recoil momentum $p_{\text{rec}}$).
The interference patterns also allowed us to verify that strong driving can lead to a negative value of the tunnelling parameter (impossible to create with static potentials). Such systems with a negative value and/or anisotropy of the tunneling parameter may produce new quantum phases. In a triangular lattice, such a negative tunneling energy can, for instance, mimic frustrated spin systems and also simulate other complicated Hamiltonians that are difficult to study in solid state materials [14].

Our results can also be interpreted as a realization of dressed matter-waves, where the shaking becomes a dressing that can take the system into a regime where the time-periodical perturbation behaves like a time-independent parameter [15]. This feature allows us to simulate regimes impossible to reach by changing the lattice depth, the atomic species etc. At the same time the realization of such a system allows us to study the Floquet states for large numbers of atoms and lattice sites, which is extremely complicated from the computational point of view.

Acknowledgments

The authors would like to thank the PhD students and post-docs participating in the experiments described in this article: H. Lignier, C. Sias, A. Zenesini, Y. Singh and J. Radogostowicz. We acknowledge funding by the MIUR (PRIN-2007), CNISM Progetto d’Innesco 2007, OLAQUI (EC FP6-511057), and NAMEQUAM (EC FP7-225187).

References

1. O. Morsch and M. Oberthaler, Rev. Mod. Phys. 78 (2006) 179.
2. D. Jaksch et al., Phys. Rev. Lett. 82 (1999) 1975.
3. R. Augusiak et al., Many body physics from a quantum information perspective, quant-ph/0003.3153v1.
4. O. Morsch et al., Phys. Rev. Lett. 87 (2001) 140402.
5. M.P.A. Fisher et al., Phys. Rev. B 40 (1989) 546.
6. D. Jaksch et al., Phys. Rev. Lett. 81 (1998) 3108.
7. M. Greiner et al., Nature(London) 415 (2002) 39.
8. M. Holthaus, Coherent control of quantum localization, in Coherent Control in Atoms, Molecules, and Semiconductors, eds. W. Pött and W.A. Schroeder (Kluwer, Dordrecht, 1999).
9. D.H. Dunlap and V.M. Kenkre Phys. Rev. B 34 (1986) 3625.
10. A. Eckardt et al., Phys. Rev. Lett. 95 (2005) 260404.
11. C.E. Creffield and T.S. Monteiro, Phys. Rev. Lett. 96 (2006) 210403.
12. H. Lignier et al., Phys. Rev. Lett. 99 (2007) 220403.
13. A. Zenesini et al., Phys. Rev. Lett. 102 (2009) 100403.
14. A. Eckardt et al., Europhys. Lett. 89 (2010) 10010.
15. A. Eckardt and M. Holthaus J. Phys.: Conf. Ser. 99 (2008) 012007.