Nucleosynthesis Constraints on a Scale-Dependent New Intermediate Range Interaction

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ABSTRACT

We derive constraints on the strength of a new intermediate range interaction that couples to baryon number from primordial nucleosynthesis yields. The nucleosynthesis limits here used arise from matching observations and predictions of standard and inhomogeneous primordial scenarios. We show that the standard nucleosynthesis scenario is more restrictive ($\alpha_5 \lesssim 0.2$) when the range of the interaction is greater than about 1 m. We further discuss the implications of considering the scalar particle responsible for the new interaction as the main component of the dark matter in the galactic halo such that its decay can account for the ionization of hydrogen in the interstellar medium and the temperature of Lyman-α clouds.
1. The existence of new fundamental forces beyond the already known four interactions is an exciting possibility that may have profound implications in our understanding of the physics beyond the standard model. The claim, more than a decade ago, of evidence for an intermediate range interaction with sub-gravitational strength \[1\] has led to a great demand of theoretical explanations (see \[2\] for a review and a complete set of references) and, most importantly, has given origin to fresh experiments based on new ideas and to the repetition of well known experiments using new state of the art technology.

In its simplest versions, the putative new interaction or a *fifth force* would arise from the exchange of a light boson coupled to matter with a strength comparable to gravity. There are several schemes through which physics at the Planck scale could give origin to such an interaction and yield a Yukawa type modification in the interaction energy between point masses. This new contribution to the interaction energy can arise, for instance, from extended supergravity theories after dimensional reduction \[2, 3\], from the compactification of 5-dimensional generalized Kaluza-Klein theories that include gauge interactions at higher dimensions \[4\] and also from string theory. On quite general terms, the interaction energy, $V(r)$, between two point masses $m_1$ and $m_2$, can be expressed in terms of the gravitational interaction as

$$V(r) = -\frac{G_\infty m_1 m_2}{r} \left(1 + \alpha_5 \, e^{-r/\lambda_5}\right), \quad (1)$$

where $r = |\vec{r}_2 - \vec{r}_1|$ is the distance between the masses, $G_\infty$ is the gravitational coupling for $r \to \infty$, $\alpha_5$ and $\lambda_5$ are the strength and the range of the new interaction. Of course, $G_\infty$ has to be identified with the Newtonian gravitational constant and the gravitational coupling would be dependent on $r$. Indeed, the force associated with eq. (1) is given by:

$$\vec{F}(r) = -\nabla V(r) = -\frac{G(r) \, m_1 \, m_2}{r^2} \, \hat{r}, \quad (2)$$

where

$$G(r) = G_\infty [1 + \alpha_5 \, (1 + r/\lambda_5) \, e^{-r/\lambda_5}]. \quad (3)$$
The great interest sparked by the suggestion of existence of a new interaction was the recognition that the coupling $\alpha_5$ was not an universal constant, but instead a parameter depending on the chemical composition of the test masses [3]. This dependence comes about if one assumes that the new bosonic field couples to the baryon number $B = Z + N$ which is the sum of protons and neutrons. Hence the new interaction between masses with baryon numbers $B_1$ and $B_2$ can be expressed through a new fundamental constant, $f$, in the following way:

$$V(r) = -f^2 \frac{B_1 B_2}{r} e^{-r/\lambda_5},$$  \hspace{1cm} (4)$$

such that the constant $\alpha_5$ can be written as

$$\alpha_5 = -\sigma \left( \frac{B_1}{\mu_1} \right) \left( \frac{B_2}{\mu_2} \right),$$  \hspace{1cm} (5)$$

with $\sigma = f^2/G_\infty m_H^2$ and $\mu_{1,2} = m_{1,2}/m_H$ ($m_H$ is the hydrogen mass).

Of course, from the above equations it follows that in a Galileo-type experiment an acceleration difference between masses $m_1$ and $m_2$ would be given by:

$$\vec{a}_{12} = \sigma \left( \frac{B}{\mu} \right)_+ \left[ \left( \frac{B_1}{\mu_1} \right) - \left( \frac{B_2}{\mu_2} \right) \right] \vec{F},$$  \hspace{1cm} (6)$$

where $\vec{F}$ is the field strength of the Earth (which is denoted by $\oplus$).

Several experiments (see, for instance, [4] for a list of the most important ones) have been performed in order to establish the parameters of a new interaction based on the idea of a composition-dependence differential acceleration as described in eq. (6) and other composition-independent effects. The current experimental situation is essentially compatible with predictions of Newtonian gravity in either composition-independent or composition-dependent experiments. The bounds on parameters $\alpha_5$ and $\lambda_5$ can be summarised as follows:

- satellite tests probing ranges about $10^5 m < \lambda_5 < 10^7 m$ indicate that $\alpha_5 < 10^{-5}$;
- gravimetric experiments that are sensitive in the range of $10 m < \lambda_5 < 10^3 m$ suggest that $\alpha_5 < 10^{-3}$;
- laboratory experiments deviced to measure deviations from the inverse-square law are sensitive essentially to the range $10^{-2} m < \lambda_5 < 1 m$ and constrain $\alpha_5$ to be smaller than $10^{-4}$. 

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Interestingly, for $\lambda_5 < 10^{-3} \, m$ and $\lambda_5 > 10^{13} \, m$, $\alpha_5$ is essentially unconstrained. The former range is particularly attractive as new forces with sub-millimetric range seems to be favoured from scalar interactions in supersymmetric theories [3] and in the recently proposed theories on $TeV$ scale quantum gravity [4]. This range also arises from assuming that scalar [5] or tensor interactions associated to the breaking of the Lorentz invariance in string theories [9] account for the vacuum energy up to the level $\Omega_V < 0.5$.

In order to close our summary of the experimental situation we should point out that, as discussed in [1], existing experimental data cannot account for certain anomalies such as the one claimed to exist in the original Eötvös experiment [10] and hence these anomalies remain still an open issue. Another quite recent claim concerning the existence of a new interaction, is given by the radio metric data from Pioneer 10/11, Galileo and Ulysses spacecraft, indicating an anomalous constant acceleration of about $8.5 \times 10^{-8} \, cm \, s^{-2}$ acting on the spacecraft directed towards the sun. A new interaction with $\alpha_5 = -1 \times 10^{-3}$ and $\lambda_5 = 200 \, A.U. = 1.49 \times 10^{13} \, m$ seems to account for the anomaly [11] (see however [12]).

In this work we shall establish limits to the coupling $\alpha_5$ and range $\lambda_5$ of the putative new interaction using results from nucleosynthesis in the context of the Big-Bang. As we shall see, these limits appear to be much less stringent than the ones obtained from laboratory experiments. Nevertheless, independently from the theoretical setting from which this interaction may arise, if the coupling of the putative new interaction is a running coupling constant, the limits derived here may turn out to be relevant. Indeed, in this case, the coupling constant at the time of primordial nucleosynthesis $\alpha_5^{prim}$ can be greater than the bounds arising from laboratory experiments, as long as the interaction is a local gauge interaction and its $\beta$-function is positive. This is, for instance, the case of the $U(1)$ coupling constant in QED. Assuming scale-dependence for the new interaction coupling constant is quite a natural assumption, as all gauge coupling constants in the standard model are running. Even the gravitational coupling constants are, at one-loop level, running couplings in higher derivative theories of gravity [13] (including of course the Newton’s constant). This fact has implications for quite a few diversified problems: the problem of the rotation curve of galaxies [14, 15]; the cosmological dark matter problem [17], the large scale structure of the Universe [16] and the cosmic virial
theorem \[18\]. Notice that we are concerned only with zero temperature running effects as those for couplings of the type $g \phi \bar{\psi}$ (cf. eq. (14)). The finite temperature corrections that are proportional to $g^2$ (see eg. \[19\]) become negligible (cf. considerations after eq. (15)) in the limit $T >> \mu$ (since we shall deal with temperatures $T \sim MeV$ and argue that $\mu \sim eV$).

Of course, the limits that we are going to establish can be regarded as independent of any considerations concerning the running of couplings and are on their own of relevance as they are consistent with laboratory experiments and are obtained from an independent line of reasoning than the usual approaches.

Standard primordial nucleosynthesis scenario (see \[20\] for a review) allows obtaining bounds on the variation of the effective gravitational coupling when confronted with the measurements of abundance of light elements in the Universe. Such measurements are in agreement with the standard nucleosynthesis scenario leaving still some room for variations in the effective number of neutrinos, the baryon fraction of the universe and also in the value of the gravitational constant. It is well known that the predicted mass fraction of primordial $^4$He can be parametrised, in theories with a variable gravitational coupling, in the following way \[20, 21\],

$$Y_p = 0.228 + 0.010 \ln \eta_{10} + 0.327 \log \xi,$$

where $\eta_{10}$ denotes the baryon to photon ratio in units of $10^{-10}$ and $\xi$ is the ratio of the Hubble parameter at the time of nucleosynthesis and its present value which is itself proportional to the square root of the gravitational constant. In the fit eq. (7) it is assumed that the effective number of light neutrinos is $N_\nu = 3$ and that the neutron lifetime is $\tau_n = 887.0 \pm 2.0$ seconds \[22\].

A range for the values of the effective gravitational coupling that are compatible with the observations of the primordial D, $^3$He, $^4$He and $^7$Li abundances can be obtained running the nucleosynthesis codes for different values of $G$ \[23\]. It turns out that the permissible range is rather large ($\Delta G/G = 0.2$ at the $1 \sigma$ level), given the large statistical and systematic errors of the observations. We shall use this result to constrain the parameters of a new intermediate range interaction assuming it to be sensitive to baryon number. The maximum range that can
be tested corresponds to the physical horizon distance at the time of primordial nucleosynthesis. At that time, the horizon distance grows from a few light-seconds to a few light-minutes, and therefore is much smaller than a few microparsecs: \( r < 3 \times 10^{10} \) m. Using eq. (3) and considering the 1 \( \sigma \) level result of Ref. [23], we obtain:

\[
\frac{\Delta G(r)}{G_\infty} = \alpha_5 \left( 1 + \frac{r}{\lambda_5} \right) e^{-r/\lambda_5} < 0.2 ,
\]

which we shall use in the next sections to impose constraints on the parameters of the new interaction.

Of course, as a light-second is about the distance to the Moon, constraints on a variation of \( G \) can, at this scale, be set from lunar laser ranging, \( \Delta G/G < 0.6 \) [24]. It is worth mentioning that similar reasoning has been used to impose constraints on the variation of the gravitational coupling due to a possible dependence on scale [14].

2. Let us turn to the discussion of the bounds that can be imposed from the standard primordial nucleosynthesis scenario. First of all we need to estimate a typical distance \( r \) between particles interacting through this new interaction. Only then can we infer on the range and strength of the fifth force.

The period of interest is that immediately after the weak-interaction decoupling, for which temperatures are of the order \( E \sim 1 \) MeV and the total baryon density is \( \rho_B \sim 1.21 \times 10^{-2} \) g cm\(^{-3}\) [23]. We shall assume that, in this epoch, the baryons are in the form of protons and neutrons only: \( \rho_B = \rho_p + \rho_n \).

One can argue that the typical distance between the interacting particles should be smaller or of the same order as their mean free paths \( \lambda_n \) and \( \lambda_p \). In order to estimate \( \lambda_n \) and \( \lambda_p \), we need to calculate the relevant stopping cross sections and the densities of the stopping particles. Within a standard scenario the number of neutrons is slightly smaller than the number of protons due to the mass difference and neutron decay:

\[
\frac{n_n}{n_p} = e^{-\frac{\Delta m c^2}{E}} e^{-\frac{t}{\tau_n}} \approx \frac{1}{e} ,
\]

although, at this temperature \( t << \tau_n \), and hence the neutron decay is not so important.
Using eq. (9) and the charge neutrality condition, we arrive at the following estimates: $\rho_p = 0.9 \times 10^{-2} \text{ g cm}^{-3}$, $\rho_n = 0.3 \times 10^{-2} \text{ g cm}^{-3}$, and $\rho_e = 0.5 \times 10^{-5} \text{ g cm}^{-3}$.

In order to estimate the relevant cross sections we should keep in mind that the proton is essentially scattered by the electrons, whereas for the neutron, both proton and electron scattering processes should in principle be taken into account. For these cases we have reproduced the estimates given in [26].

Let us first consider the proton-electron scattering. Here we use the expression from [26]:

$$
\sigma_{pe} = 2\pi \int_{\theta_0}^{\pi} d\theta \left( 1 - \cos \theta \right) \frac{2\pi \alpha^2 m_e^2}{4k^4 \sin^4 \left( \frac{\theta}{2} \right)} \left( 1 + \frac{k^2}{m_e^2} \cos^2 \left( \frac{\theta}{2} \right) \right),
$$

where $\alpha = \frac{e^2}{\hbar c}$ is the fine-structure constant.

At $E \sim 1 \text{ MeV}$, the Debye shielding of the proton by the electronic cloud inhibits scattering below $\theta_0 \simeq 0.77^\circ$. Evaluating the integral in eq. (10) we obtain $\sigma_{pe} \simeq 1.5 \times 10^{-24} \text{ cm}^2$.

The nucleon-nucleon cross section at these energies is very well known given the large amount of data available for the nucleon-nucleon phase shifts. Typically $\sigma_{np}$ is written as the sum of a singlet and a triplet contribution:

$$
\sigma_{np}(E) = \frac{\pi a_s^2}{(a_s k)^2 + (1 - 0.5r_s a_s k^2)^2} + \frac{3\pi a_t^2}{(a_t k)^2 + (1 - 0.5r_t a_t k^2)^2},
$$

where the parameters for scattering lengths and scattering radii are obtained from fits to the data. We have used $a_s = -23.71 \text{ fm}$; $r_s = 2.73 \text{ fm}$ for the singlet component and $a_t = 5.432 \text{ fm}$; $r_t = 1.749 \text{ fm}$ for the triplet component [27]. This yields the following value for the nucleon-nucleon cross section: $\sigma_{np} \simeq 1.9 \times 10^{-24} \text{ cm}^2$.

The neutrons interact with the electrons through their magnetic moment. At these energies ($E \sim 1 \text{ MeV}$) the cross section is given by [26]:

$$
\sigma_{ne} = 3\pi \left( \frac{\alpha K_{mag}}{m_n} \right)^2 = 8 \times 10^{-31} \text{ cm}^2,
$$

where $K_{mag} = -1.91$ is the anomalous magnetic moment of the neutron in nuclear magnetons.

The mean free path is defined as: $\lambda = \frac{m}{\rho \sigma}$ where $m$ and $\rho$ are the mass and density of the stopping particles and $\sigma$ is the relevant stopping cross section. Taking into account our estimates for the cross sections, we easily conclude that, in this epoch, both neutrons and protons have mean free paths of the same order of magnitude: $\lambda_p \sim \lambda_n \sim \lambda = 1 \text{ m}$. 

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In Figure (1) we present a contour plot of \( \frac{\Delta G(r)}{G_\infty} \) as a function of the interaction parameters \((\alpha_5, \lambda_5)\) by taking the typical distance between nucleons to be \( r \sim \lambda = 1 \text{ m} \). The black, dark grey and light grey regions correspond to \( \frac{\Delta G(r)}{G_\infty} \) within the ranges \([0, 0.1]\), \([0.1, 0.2]\), and \([0.2, 0.3]\) respectively. One can immediately conclude from the contour shapes that, whichever the range for the interaction, the maximum restriction that can be extracted for the coupling constant \( \alpha_5 \) from nucleosynthesis corresponds exactly to the limit given for \( \frac{\Delta G(r)}{G_\infty} \). It is worth underlining that the condition on \( \alpha_5 \) is valid for the primordial epoch (given that primordial nucleosynthesis was used). If \( \alpha_5 \) is assumed to be a running coupling constant, the limits here derived for the strength of this new interaction are quite useful, given that laboratory experiments can never impose constraints on \( \alpha_5^{\text{prim}} \).

Evidently \( \alpha_5^{\text{prim}} \lesssim 0.2 \) is far above the typical values permitted by experiments (\( \alpha_5^{\text{today}} \lesssim 10^{-4} \)). In spite of that, even if the coupling constant is not running, it is interesting to look at the extremes where there are no experimental bounds. If this new interaction is very short range \( \lambda_5 < 10^{-3} \text{ m} \), primordial nucleosynthesis does not constrain the coupling constant at all. However if this is a very long range interaction (\( \lambda_5 > 10^{13} \text{ m} \)), we find that \( \alpha_5^{\text{prim}} \lesssim 0.2 \) in order to satisfy eq. (8).

So far we have approximated the typical distance between the interacting particles by the mean free paths \( r \sim \lambda_n \sim \lambda_p \). Alternatively, a very trivial estimate for the typical distance between nucleons could be extracted directly from the total baryon density normalisation condition:

\[
a = \left( \frac{1}{\rho_B N_A} \right)^{1/3} \tag{13}
\]

where \( N_A \) is the Avogadro’s number. Evaluating eq. (13) yields \( a \sim 10^{-9} \text{ m} \). We find that now the limit \( \alpha_5^{\text{prim}} \lesssim 0.2 \) holds even if the interaction is short range \( a < \lambda_5 < 10^{-3} \text{ m} \). Note again that there are no experimental limits on \( \alpha_5^{\text{today}} \) in this region.

Ultimately we shall consider the reasoning of the previous section to obtain bounds on the parameters of a new intermediate range interaction in the context of inhomogeneous nucleosynthesis scenario. In inhomogeneous Big Bang models one assumes that there are non-linear perturbations of the baryon density producing large proton deficient regions where the production of \(^4\text{He}\) is hindered \[26\]. The main advantage of these models is that they allow for a larger
deuterium abundance, which is one of the directions suggested by observational data [28].

In this picture most neutrons drift into neutron rich regions, and therefore we consider that it is the electron-neutron scattering process (rather than the proton scattering) which determines the neutron mean free path. For the sake of our argument, we shall assume that the electrons remain homogeneously distributed. Using the values of $\sigma_{ne}$, $\sigma_{pe}$ and $\rho_e$ given in the previous section, we estimate a neutron mean free path of $\lambda_n^{inhom} \sim 10^6$ m whereas the proton mean free path remains essentially the same $\lambda_p \sim 1$ m. If we now take the typical distance for interacting particles to be $r = \lambda_n^{inhom}$ the restrictions imposed on the primordial value of the coupling constant are less severe. This aspect is manifest in the contour plot shown in Figure (2).

We can then conclude that for $\lambda_5 \gtrsim 1$ m, the standard nucleosynthesis scenario is more effective in constraining $\alpha_5$ than the inhomogeneous models.

3. An independent estimate of the parameters of the new interaction can be obtained assuming, for instance, that the light boson carrier of the new interaction accounts for the main contribution of the dark matter in the galactic halo, as well as being responsible for the ionization of interstellar hydrogen. Even though this might seem highly speculative, the clumping of this bosonic field around and inside stars derives naturally from the coupling of this field to ordinary matter (cf. eq. (14) below). As discussed in Ref. [29] this implies that masses and couplings of ordinary particles depends on the features of the bosonic field interaction, meaning in turn, that the fundamental coupling constants are altered by the nearby density of matter. Estimates of the effects of this dependence on the cooling of neutron stars, the neutrino burst in the supernova SN1987A and the period of the remnant pulsar, imply bounds for $\alpha_5$ and $\lambda_5$ that are much less stringent than the ones emerging from Eötvös-type experiments and from satellite measurements mentioned in the introduction. However, as we shall see, interesting limits do arise if one assumes that the boson responsible for the new interaction decays into photons and accounts for the dark matter contained in the halo of our galaxy.

Denoting the light scalar field responsible for the new interaction by $\phi$, we assume its coupling to nucleons and photons is of the following form:
\[ L_{\text{int}} = c_n \frac{\phi}{\langle \phi_5 \rangle} m_n N \bar{N} + c_p \frac{\phi}{\langle \phi_5 \rangle} F_{\mu\nu} F^{\mu\nu}, \]  

(14)

where \( \langle \phi_5 \rangle \) is a large scale associated to the new interaction. Of course, it is this scale that establishes the likelihood of creating this field in particle physics accelerators. This interaction yields, at tree level, a modification to the Newtonian potential such as that in eq. (1) where

\[
\alpha_5 = \frac{c_n^2}{4\pi} \left( \frac{M_P}{\langle \phi_5 \rangle} \right)^2,
\]

(15)

and \( M_P \equiv G^{-1/2}_\infty \) is the Planck mass. We see that the existing bounds on \( \alpha_5 \) imply that \( \langle \phi_5 \rangle \sim M_P \). This agrees with what had already been mentioned in the introduction: the new interaction must arise from physics close to the Planck scale. For simplicity we shall assume that \( \langle \phi_5 \rangle = M_P \). Following from our previous conclusion that \( \alpha_5^{\text{prim}} \lesssim 0.2 \) for \( \lambda_5 \gtrsim 1 \, m \), we obtain \( c_n \lesssim 1.58 \).

In order to extract further information on the new interaction, we demand that the scalar field, \( \phi \), decays into photons that are energetic enough to account for the observed ionization of interstellar hydrogen, the temperature of Lyman-\( \alpha \) clouds [30], and the anomaly in the abundance of He I in the three high-redshift Lyman-limit systems of the quasar HS 1700 + 6416 [31]. This implies that \( m_5 \gtrsim 27.2 \, eV \) and hence that \( \lambda_5 \lesssim 7 \times 10^{-9} \, m \). This is consistent with the bounds on \( \alpha_5 \) obtained from supposing that the distance between nucleons during nucleosynthesis is given by eq. (13).

Assuming the scalar particle is stable enough to account for all the dark matter in the galactic halo, then the density of scalar particles is given by \( \rho_5 = m_5 n_5 = \rho_h \), where \( n_5 \) is the scalar particle number density and \( \rho_h = 2 - 13 \times 10^{-25} \, g \, cm^{-3} = 0.1 - 0.7 \, GeV \, cm^{-3} \) [22] is the galactic halo density. This hypothesis yields that:

\[
3.67 \times 10^6 \, cm^{-3} < n_5 < 2.57 \times 10^7 \, cm^{-3}.
\]

(16)

Notice that if we had chosen to account for the observed cosmological energy density, we would have had to replace \( \rho_h \) by \( (0.2 - 0.3) \rho_c \), where \( \rho_c = 1.88 \times 10^{-29} \, h_0^2 \, g \, cm^{-3} = \).
$1.05 \times 10^{-5} h_0^2 \text{GeV cm}^{-3}$ is the critical density and $h_0$ is the Hubble parameter in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (observationally $0.5 < h_0 < 0.8$). Then the number density of scalar particles would be smaller than the estimate eq. (16) by about four orders of magnitude.

To further verify our assumptions we compute the rate of decay of the scalar particle. Since it can only decay into photons, its decay width can be obtained from the last term in eq. (14). A straightforward calculation reveals that:

$$
\Gamma_5 = c_n^2 \frac{m_5^3}{\langle \phi_5 \rangle^2}.
$$

(17)

For $m_5 = 27.2 \text{ eV}$, we have $t_5 \equiv \Gamma_5^{-1} = 3.0 \times 10^{36} \text{ s} \gg t_U \approx H_0^{-1} = h_0^{-1} 9.78 \text{ Gyr}$. Thus the scalar particle responsible for the new interaction is comfortably stable to be a good dark matter candidate.

4. In this paper we have discussed bounds on the parameters of a new interaction that couples with baryon number, arising from primordial nucleosynthesis. We have considered the standard nucleosynthesis scenario and shown that $\alpha_5 \lesssim 0.2$ for $\lambda_5 \gtrsim 1 \text{ m}$ (see Figure 1). The inhomogeneous nucleosynthesis scenario of Ref. [26] allows one to conclude the same for $\lambda_5 \gtrsim 5 \times 10^5 \text{ m}$ (Figure 2). These limits do not require the assumption of a running coupling constant and even though consistent, they are much less stringent than the ones obtained from laboratory experiments and satellite tests. As discussed in the text, they may be relevant for a new interaction whose coupling constant is scale-dependent and is not asymptotically free. We have also derived bounds on the clumpiness of the scalar particle assuming it is the main contributor to the dark matter in the galactic halo, that is $3.67 \times 10^6 \text{ cm}^{-3} < n_5 < 2.57 \times 10^7 \text{ cm}^{-3}$ for $\lambda_5 \lesssim 7 \times 10^{-9} \text{ m}$. This corresponds to $m_5 \gtrsim 27.2 \text{ eV}$ implying that the scalar particle decay can account for the ionization of hydrogen observed in the interstellar medium and the temperature of Lyman-$\alpha$ clouds.

**Acknowledgements**

One of us (F.N.) gratefully acknowledges the support of the JNICT Fellowship BIC 1481.
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Figure 1: Limits imposed on the new force parameters based on the nucleosynthesis constraints for $\frac{\Delta G(r)}{G_\infty}$ within the ranges $[0, 0.1]$, $[0.1, 0.2]$, and $[0.2, 0.3]$ respectively.

Figure 2: Limits imposed on the new force parameters based on the nucleosynthesis constraints and the inhomogeneous primordial nucleosynthesis model for $\frac{\Delta G(r)}{G_\infty}$ within the ranges $[0, 0.1]$, $[0.1, 0.2]$, and $[0.2, 0.3]$ respectively.