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**H∞ Observer for Damper Force in a Semi-Active Suspension**

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**Abstract:** An $H_\infty$ observer for the Semi-Active (SA) force of an Electro-Rheological (ER) damper in a Quarter of Vehicle (QoV) model is proposed. This robust observer is designed in the $H_\infty$ framework to minimize the effect of the unknown road disturbance on the force estimation and includes the damper non-linearities and its dynamic behavior. Simulation and experimental rig tests results using a 1/5 scale car using easily accessible measurements for the observer, such as acceleration sensors, which are relatively cheap and easy to implement in a real environment. The estimated damper force could be used in a state feedback control strategy to improve comfort and road holding performance of a vehicle with a reduced number of sensors.

**Keywords:** $H_\infty$-observer, State Estimation, Semi-active suspension, Linear Matrix Inequalities

1. INTRODUCTION

The vehicle suspension system provides the ride comfort and handling characteristics during different driving situations. A passive suspension design implies a trade-off in the vertical vehicle dynamics behavior and SA suspensions can be used to overcome this compromise. Its main characteristic is the use of a shock absorbers with a variable damping coefficient, modified by an external control input. They bring very important advantages over passive or active systems since they can approximate the performance of an active suspension. They are preferred in the automotive industry since they work without actuators, low energy consumption, are less bulky and at a lower cost.

To achieve the wanted performance, SA suspensions depend upon a control system. Force controllers are frequently designed, these controllers compute the demanded damping force to fulfill the performance specifications. Nonetheless, the damping force computation is not straightforward, mainly due to non-linear characteristics of SA dampers and since the actual damper manipulation is either voltage or current, there is a need to transform from the desired damping force to the needed manipulation.

A FCS of a SA damper is proposed in Besinger et al. (1995), a force feedback control strategy, which adjusts the damping rate according to the measured and desired damping forces. A similar research, where a model of an ER damper under proportional feedback control is derived in Sims et al. (1997), the generated force is measured and fed back via a sensor with a certain gain and then compared with a reference force. Batterbee and Sims (2007) validated the force feedback linearization algorithm for an ER damper in a vehicle suspension under real road disturbance conditions, using a force sensor to measure the damper force and an LVDT sensor to measure the suspension deflection in an experimental facility.

In Vivas-Lopez et al. (2015) a FCS, based on feedback linearization, was proposed to improve a LPV control system. It takes the ER damper non-linear dynamic behavior into account. The FCS adjusts the manipulation to reach the reference force, regardless the uncontrolled variables in the force control loop, however, this scheme requires the force measurement. A methodology to estimate the state variables in a full-car vertical model with the design of an $H_\infty$ observer for suspension control applications was proposed in Dugard et al. (2012), allowing to minimize the unknown ground disturbances effects on the estimated state variables. Experimental results in a real car validates the observer.

Eroglu and Sims (2014) established a control algorithm in a MR damper. The aim is to perform optimal force-feedback linearization of the MR damper using an observation of the feedback force with an accelerometer rather than the measured value. However, this work considers a simplified Single Degree Of Freedom (SDOF) model, and considers the disturbance as a known input, which makes unfeasible for vehicle suspension applications. A robust $H_\infty$ observer to estimate the force in an ER damper is proposed. The observer considers the non linear characteristics of a real ER damper taking its dynamical response into account, it achieves an accurate and reliable force estimation in an ER damper with a reduced number of sensors.

This paper is organized as follows. Section 2 describes the suspension system model. Section 3 presents in detail the observer design approach. Section 4 discusses the simulation and experimental results. Finally, section 5
concludes this research. Table 1 describes the variables used in this paper.

2. SUSPENSION SYSTEM

The Quarter of Vehicle (QoV) model is often used when suspension modeling and control are considered. It allows to study the vertical behavior of a vehicle according to the suspension characteristics, Figure 1. This model offers a suitable representation of the problem to control the wheel load variations and forces in the suspension system.

The model shows the sprung mass \(m_s\), supported above the wheel and suspension assembly, referred to as the unsprung mass \(m_{us}\), which is supported by the tire, with a stiffness coefficient \(k_t\), above the road surface. Between the sprung and unsprung masses are the SA damper with a force \(F_D\) and the suspension spring with a stiffness coefficient \(k_s\).

![Fig. 1. Semi-Active Quarter of Vehicle model.](image)

The dynamical equations that represent the SA QoV masses motion are:

\[
\begin{align*}
\dot{z}_s &= -k_s(z_s - z_{us}) - F_D \\
\dot{z}_{us} &= k_s(z_s - z_{us}) + F_D - k_t(z_{us} - z_r)
\end{align*}
\]

where \(F_D\) is the overall SA damping force from the ER damper, which is inherently nonlinear due to saturation, hysteresis, dynamic effect, etc. When no electric field is applied, the ER damper develops a damping force only produced by the fluid resistance. However, when a certain level of the electric field is applied, the ER damper generates an increased damping force due to the yield stress of the ER fluid. This damping force is able to be constantly adjusted by controlling the strength of the electric field. According to Guo et al. (2006), the damping force is:

\[
F_D = k_0 \dot{z}_{def} + c_0 \dot{z}_{def} + F_{ER}
\]

where \(k_0\) is the effective stiffness due to the gas pressure, \(c_0\) is the effective damping due to the fluid viscosity, and \(F_{ER}\) is the controllable force which is a function of the applied electric field. The field dependent damping force \(F_{ER}\) is modeled as:

\[
F_{ER} = f_e \tanh \left( a_1 \dot{z}_{def} + a_2 \dot{z}_{def} \right) \cdot U
\]

where the coefficients \(f_e, a_1\) and \(a_2\) are damper-dependent parameters, that vary according to each damper model; they can be experimentally identified, and \(U\) is the control input, a PWM signal. To take the dynamic characteristic force, eqn (3) is expressed by:

\[
\tau \frac{d}{dt} F_{ER} + F_{ER} = f_e \tanh \left( a_1 \dot{z}_{def} + a_2 \dot{z}_{def} \right) \cdot U
\]

\[
F_{NL} = f_e \tanh \left( a_1 \dot{z}_{def} + a_2 \dot{z}_{def} \right)
\]

where \(\tau\) stands for the time constant of damping force and \(F_{NL}\) contains the nonlinear behavior of the damper. By rearranging, the controlled damper force is:

\[
F_{ER} = -\frac{1}{\tau} F_{ER} + \frac{1}{\tau} F_{NL} \cdot U
\]

These dynamic equations are shown in Figure 2; the damping force \(F_D\) is presented within the suspension system.

![Fig. 2. ER dynamic model.](image)

Remark. The time constant \(\tau\) depends on the ER fluid and might vary according to some factors (i.e. control input, fluid temperature, etc.); it is considered constant.

3. OBSERVER DESIGN

The model used integrates the controllable damper force \(F_{ER}\) as a system state, the state-space model is given by:

\[
\begin{align*}
\dot{x}(t) &= A \cdot x(t) + dB \cdot w(t) + B \cdot F_{NL} \cdot u(t) \\
y(t) &= C \cdot x(t) + C \cdot w(t)
\end{align*}
\]

where \(x\) is the state vector, \(w\) the unknown road input, \(u\) the control input, \(y\) the measured variables, \(w\) the measurements noise and \(A \in \mathbb{R}^{n \times n}, \ dB \in \mathbb{R}^{n \times p}, \ B \in \mathbb{R}^{n \times p}, \ C \in \mathbb{R}^{q \times n}, \ dC \in \mathbb{R}^{q \times p}\) as follows:

\[
A = \begin{bmatrix}
0 & 1 & 0 & -1 & 0 \\
-(k_s + k_0) & c_0 & 0 & c_0 & -1 \\
0 & m_s & m_s & m_s & -m_s \\
-k_t & c_0 & m_s & -m_s & 0 \\
m_s & m_s & -m_s & m_s & 0 \\
0 & 0 & 0 & 0 & -1/\tau
\end{bmatrix}
\]

\[
dB = \begin{bmatrix}
0 & 0 & -1 & 0 & 0
\end{bmatrix}^T \quad B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0
\end{bmatrix}^T \quad dC = \begin{bmatrix}
0.1 \\
0.005
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
-(k_s + k_0) & c_0 & 0 & m_s & -1 \\
(k_t + k_0) & c_0 & m_s & m_s & -1 \\
m_s & m_s & -m_s & m_s & 0 \\
m_s & m_s & -m_s & m_s & 0 \\
m_s & m_s & -m_s & m_s & 0
\end{bmatrix}
\]

3.1 \(H_\infty\) observer

Instead of estimating disturbances, the \(H_\infty\) observer offers a direct method to reduce the negative effect of disturbances in the states estimation.
The structure of the observer to estimate this model is:

\[
\dot{x}(t) = A \dot{x}(t) + L(y(t) - C \dot{x}(t)) + dB \ w_1(t) + BF_N \ L \cdot u(t) \\
\]

\[
\dot{x}_0 \text{ to be defined} \quad (8)
\]

where \( \dot{x} \in \mathbb{R}^{n \times n} \) is the estimated state of \( x \) and \( L \in \mathbb{R}^{n \times n} \) is the observer matrix to be designed. The system (8) is said to be an \( H_\infty \) observer for the system (7) if:

\[
\lim_{t \to \infty} e(t) \to 0 \quad \text{for} \quad w(t) = 0
\]

\[
\left\| \frac{e(s)}{w(s)} \right\|_\infty = \left\| T_{ew}(s) \right\|_\infty \leq \gamma \quad \text{under} \quad \dot{e}(t) = 0 = 0 \quad (9)
\]

where \( \left\| T_{ew}(s) \right\|_\infty \) is the \( H_\infty \) norm of the transfer function from the disturbances to the estimation error. The \( H_\infty \) estimation error dynamical equation, taking the unknown disturbances into account, can be expressed as:

\[
\dot{e}(t) = \dot{x}(t) - \dot{x}(t) = (A - LC)e + (dB - LdC)w \quad (10)
\]

The estimated state variable \( \hat{x} \), controlled by the error dynamics (10) converges asymptotically to the state \( x \) for any bounded initial conditions \( x(0) \) and \( \dot{x}(0) \) if and only if the following conditions are met, Darouach (2000).

**Stability:**

\[
N = A - LD \quad (11)
\]

where \( N \) is a Hurwitz or stable matrix.

**Disturbance decoupling:**

\[
\begin{align*}
\bar{d}B - LdC &= 0 \\
\end{align*} \quad (12)
\]

The estimation error, described by (10), is driven by the unknown disturbance \( w \). If an exact observer design, it is, where an exact disturbance decoupling is achieved since the estimated variables do not depend on the disturbance, is not possible, the disturbance effect on the estimated state variables can be minimized and is possible to compute an efficient observer. The method resides in applying the Bounded Real Lemma (BRL) to the error equation and apply a change of variables to obtain some LMIs.

Considering (7) and the observer (8). Given a positive scalar \( \gamma \), if there exist \( P = P^T > 0 \) satisfying the inequality:

\[
\begin{bmatrix}
(A - LC)^T P + P(A - LC) & P(dB - LdC) & I_n \\
* & -\gamma I_d & O_{d,n} \\
* & * & -\gamma I_n
\end{bmatrix} < 0
\]

then the observer (8) is an \( H_\infty \) observer according to 9. The BRL applied to the error dynamics (10) gives the solution to (9) and leads to the bilinear matrix inequality (BMI) (13) where \( P = P^T > 0 \) and \( L \) are the unknown matrices to be determined. Thus, the full-order stable observer design problem consists in solving (13). It is possible to transform the BMI into a solvable LMI With a change of variables, let us define \( Y = -PL \), leading to the following LMI:

\[
\begin{bmatrix}
A^T P + PA + YC + C^T Y^T & P dB + Y dC & I_n \\
* & -\gamma I_d & O_{d,n} \\
* & * & -\gamma I_n
\end{bmatrix} < 0
\]

the observer gain will then be:

\[
L = -P^{-1} Y \quad (15)
\]

Finally, the proposed observer is synthesized so that the stability conditions in (11) are fulfilled, and the disturbance decoupling conditions in (12) are approximated by minimizing \( \gamma \) subject to (9).

### 3.2 Pole Placement

Stability is a minimum condition for control and estimation systems. However, in most real situations, a good observer should not only deliver an stability condition, but also to keep sufficiently fast and well-damped time responses. The preceding approach assures the observer stability and the disturbance effect minimization, but the observer poles are achieved through the solution of (13) and may be either very high, with high imaginary parts, or be almost unstable. A traditional approach to ensure suitable transients is to establish closed-loop poles in a convenient region of the complex plane, the idea is to make sure that the observer dynamics will be faster enough to accurately estimate the damper force in a real environment.

It is possible to use the quadratic Lyapunov function \( V(z) = z^T P z \) to settle a lower bound on the decay rate of the system. If:

\[
\frac{dV(x)}{dt} \leq -2\alpha V(x) \quad \text{for all trajectories} \quad (16)
\]

which is equivalent to

\[
A^T P + PA + 2\alpha P \leq 0 \quad (17)
\]

with this, it is possible to situate the observer poles within the region \( D_1 \) in the complex plane, corresponding to a left half plane as represented in Fig. 3. This region is defined by the LMI (18), ensuring that the poles have real parts in \([-\infty, -\alpha]\).

\[
D_1 = z \in C : z + z^* + 2\alpha < 0 \quad (18)
\]

![Fig. 3. LMI \( D_1 \) region in the complex plane.](image)

Then, from (14) and the pole placement approach the LMI to be solved is defined by:

\[
\begin{bmatrix}
\Phi & \Psi & I_n \\
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\[
\begin{bmatrix}
\Phi & \Psi & I_n \\
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* & * & -\gamma I_n
\end{bmatrix} < 0
\]
where $\mathbf{P}$ and $\mathbf{Y}$ are the unknown matrices to be solved and $\alpha$ selected according to desired performance.

4. RESULTS

A QoV model of a 1:5 scale vehicle was used as test bench. An experimental model of an ER damper was considered. Both, simulation and experimental tests results are shown under different road conditions and control inputs. Table 1 shows the used model parameters.

Table 1. Model parameters

| Parameter | Description         | Value | Units |
|-----------|---------------------|-------|-------|
| $m_s$     | Sprung mass         | 2.27  | kg    |
| $m_{us}$  | Unsprung mass       | 0.25  | kg    |
| $k_s$     | Spring stiffness    | 1396  | N/m   |
| $k_t$     | Tire stiffness      | 12270 | N/m   |
| $k_0$     | Damper stiffness coefficient | 186 | N/m |
| $c_0$     | Viscous damping coefficient | 23 | Ns/m |
| $a_1$     | Velocity hysteresis coefficient | 21 | Ns/m |
| $a_2$     | Disp. hysteresis coefficient | 13 | 1/m |
| $f_c$     | Yield force of ER fluid | 42 | N |
| $\tau$    | Damper time constant | 50 | ms |
| $U$       | PWM input           | 10    | %     |

The road input is an unknown disturbance in the observer and both accelerations ($\ddot{z}_s$ and $\ddot{z}_{us}$) are used as the observer inputs. The evaluation of the SA damper force estimation system is composed in two steps:

1. Assessment of the observer system for different tests in the time domain.
2. Evaluation of the force estimation using a performance index.

4.1 Design of Experiments

The observer is tested under six different conditions in simulation and real tests with different road profiles and control inputs as follows:

1. Bumps test with fixed 10% PWM input.
2. Rough road with fixed 35% PWM input.
3. Chirp road input with fixed 20% PWM input.
4. Bumps test with variable PWM (Fig. 4 top).
5. Rough road with variable PWM input (Fig. 4 top).
6. Chirp road input with variable PWM (Fig. 4 bottom).

Two different PWM sequences are used according to the test, these inputs come from a uniformly distributed random signal between 0.05 and 0.40 and is saturated according to the control signal constraints $[0.10, 0.35]$, the first one is used in tests 4 and 5, while the second one is used in the Chirp profile test, see Figure 4.

The implemented road profiles are shown in Fig. 5, the first one is a bumps test, then a rough road profile and finally a Chirp signal as a road input to test the observer under different frequencies.

To quantitatively evaluate the $H_{\infty}$ observer performance, the Error to Signal Ratio (ESR) index was adopted. It is calculated as the ratio of the variance of the damper force estimation error and the variance of the actual force, Savaresi et al. (2005):

$$ESR = \frac{Var(\hat{F}_D - F_D)}{Var(F_D)}$$ (20)

Additionally to the ESR performance index, the Normalized Root-Mean-Square Error (NRMSE) was computed for each test to compare the estimation in terms of percentage. This index is a way of measuring how good our estimated model is over the actual data and is sensitive to outliers. It computes the square root of the Mean Squared Error (MSE) and then normalizes it by dividing by the force estimation range:

$$NRMSE = \frac{\sqrt{MSE(\hat{F}_D, F_D)}}{\max(F_D) - \min(F_D)}$$ (21)
4.2 Simulation

Each of the six conditions listed before are tested in simulation, tests were done with initial values in the observer different than zero, to evaluate the transient state behavior in the initial stage. A qualitative analysis can be made by plotting simulations results. Aiming to show the observer trackability, only results in damper force estimation of test 2 and 4 are shown in Figs. 6-7.

Fig. 6. Damper force simulation in test 2.

Fig. 7. Damper force simulation in test 4.

4.3 Experiments

The performance of the $H_{\infty}$ observer has been investigated with numerical simulations and the proposed observer seems to be effective. Some experimental tests in the real test bench are presented.

The experimental platform consists in a 1:5 scale vehicle that has been developed as part of the INOVE\textsuperscript{1} project. The test bench has been designed to analyze the vehicle vertical behavior with sensors placed to measure some variables, which describe the vehicle dynamics.

The road profile is simulated using 4 linear motors applying vertical displacements to each wheel. Only the front right corner, which consists in a double-wishbone suspension, has been used to test the developed damper force observer. The measured variables are shown in Fig. 8 according to: (1) $ER$ Damper force ($F_D$), (2) Sprung mass acceleration ($z_s$), and (3) Unsprung mass acceleration ($z_{us}$).

Fig. 8. INOVE test-bench and sensors.

Additionally, four $SA$ suspensions of SOBEN\textsuperscript{2} have been mounted thus replacing the original passive suspension. The suspension system comprises four $ER$ dampers, which have a force range of $\pm 30$ N. These dampers are regulated using a manipulation voltage between 0 and 5 $kV$, which is generated by the amplifiers modules. The control input for the modules is a $PWM$ signal at 25 $kHz$. These amplifiers proportionally transform the duty-cycle of the received $PWM$ signal into a voltage.

The decay rate $\alpha$, which is an observer design parameter, has been set to $\alpha = 10$, this value showed the best performance based on experimental tests.

As well as in simulation, a qualitative analysis was done by plotting simulations results in the time domain. For the sake of brevity, and aiming to show the observer trackability, only the accelerations for the first test are shown in Fig. 9, where is clear that the observer follows the acceleration measurements filtering the high noise signals. For the remaining tests only damper force estimation results are shown in Figures 10 and 11.

Fig. 9. Sprung and unsprung mass accelerations in test 1.

Fig. 10. Damper force in test 1 (top), test 2 (middle) and test 3 (bottom).

From tests 1 and 2, it is fair to say that the observer follows the damper force behavior, although, in some parts the estimation seems a little bit underestimated. The test 3 shows the Chirp road profile, the estimation is good in range.

\begin{itemize}
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\end{itemize}

\textsuperscript{1} Integrated approach for observation and control of vehicle dynamics, http://www.gipsa-lab.fr/projet/inove/

\textsuperscript{2} SOBEN is a specialized company in innovative shock absorbers, http://www.soben.fr/
general, despite the fact that estimation does not seem good in low frequency, this is mainly due to the high noise in the sensors and low relative motion in the initial stage.

![Force (N)](image)

**Fig. 11.** Damper force in test 4 (top), test 5 (middle) and test 6 (bottom).

### 4.4 Discussions

A quantitative analysis is shown in Table 2. The ESR and NRMSE values for each test show that the observer is able to track accurately the actual damper force under different road and control input conditions in simulation and real experiments. In the case of simulation tests the table was computed without taking the initial transient phase of the observer into account. There is a deterioration in the damper force estimation when a variable control input is applied in comparison with the fixed PWM.

| Test | ESR  | NRMSE  |
|------|------|--------|
| 1    | 0.0622 | 2.48%  | 11.52% |
| 2    | 0.0104 | 1.86%  | 8.25%  |
| 3    | 0.0386 | 1.64%  | 8.18%  |
| 4    | 0.0622 | 1.99%  | 12.03% |
| 5    | 0.0169 | 1.87%  | 11.32% |
| 6    | 0.0480 | 1.63%  | 10.29% |

Table 2. ESR and NRMSE results for real tests

It is clear that the simulation results are far more accurately, in the case of ESR values real data shows, in average, an error 10 times higher while with the NRMSE index, the values in real tests are around five times higher. When implementing this estimated force in a FCS, the controller should be robust enough to handle this estimation error.

Not much work has been done in SA damper force estimation, Eroglu and Sims (2014) developed an observer based controller, where the damper force estimation is used for control purposes in a SDOF system; nonetheless, the observer requires the disturbance to estimate the force, which would be a costly solution for real applications, and the model does not take the damper dynamics into account, the current work was developed looking to tackle these limitations, considering an unknown road input and the damper dynamics.

### 5. CONCLUSIONS

An $H_{\infty}$ observer for the SA damper force estimation of an ER damper in a vehicle suspension under an unknown road disturbance has been proposed. The ER damper model dynamics has been integrated in the QoV model dynamics and have been structured into a single five states system, handling the non-linearities and dynamic behavior in the damper. Several SA suspension control systems compute the desired damper force rather than the physical control signal for the damper; thus, a control loop is needed to transform the desired force to the damper manipulation signal or an inverse model dynamics of the ER damper is required, which could harm the system controllability due to the ER hysterical behavior.

The observer solves the complexity of the FCS based on feedback linearization theory with acceleration measurements. The design of robust observer within the $H_{\infty}$ framework plays a key role in the performance of the proposed estimation method. The developed methodology includes both the performance specifications and measurement noise filtering. SA suspension control is a very challenging problem in the automotive industry and within the automation research community as well, where the reduced number of sensors and cost implementation are a key point in the design process.

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