SPECTRAL PROPERTIES OF THREE-DIMENSIONAL MAGNETOHYDRODYNAMIC ACCRETION FLOWS

KEN OHSUGA,1,2 YOSHIKI KATO,2 and SHIN MINESHIGE2

Received 2004 September 30; accepted 2005 March 20

ABSTRACT

In spite of a large number of global three-dimensional MHD simulations of accretion flows and jets being made recently, their astrophysical relevance for realistic situations is not well known. In order to examine to what extent the simulated MHD flows can account for the observed SED of Sgr A*, for the first time we calculate the emergent spectra from three-dimensional MHD flows in a wide range of wavelengths (from radio to X-ray) by solving the three-dimensional radiative transfer equations. We use the simulation data by Kato and coworkers and perform Monte Carlo radiative transfer simulations, in which synchrotron emission/absorption, free-free emission/absorption, and Compton/inverse Compton scattering are taken into account. We assume two-temperature plasmas and calculate electron temperatures by solving the electron energy equation. Only thermal electrons are considered. It is found that the three-dimensional MHD flow generally overproduces X-rays by means of bremsstrahlung radiation from the regions at large radii. A flatter density profile, $\rho \propto r^{-a}$ with $a < 1$, than that of the ADAF, $\rho \propto r^{-3/2}$, is the main reason for this. If we restrict the size of the emission region to be as small as $\sim 10r_s$, where $r_s$ is the Schwarzschild radius, the MHD model can reproduce the basic features of the observed SED of Sgr A* during its flaring state. Yet, the spectrum in the quiescent state remains to be understood. We discuss how to resolve this issue in the context of MHD flow models. Possibilities include modifications of the MHD flow structure by the inclusion of radiative cooling and/or significant contributions by nonthermal electrons. It is also possible that the present spectral results may be influenced by particular initial conditions. We also calculate the time-dependent spectral changes, finding that the fluxes fluctuate in a wide range of the frequency and the flux at each wavelength does not always vary coherently.

Subject headings: accretion, accretion disks — black hole physics — Galaxy: center — radiative transfer

1. INTRODUCTION

There is a long research history in the theoretical modeling of black hole accretion flows. The standard-disk picture was first established by Shakura & Sunyaev (1973) after many attempts. In this model the gravitational energy is efficiently converted to radiation energy and is finally radiated away. Then, the disk was predicted to be luminous and relatively cold, exhibiting multi-color blackbody spectra (Mitsuda et al. 1984). The standard-disk model is widely accepted as a model for disks with moderately high accretion rates, $M \lesssim \dot{M}_{\text{Edd}}/c^2$, with $\dot{M}_{\text{Edd}}$ being the Eddington luminosity and $c$ being the light velocity (Esin et al. 1997). In fact, the calculated spectra based on the standard-disk model fit well with the thermal component of the high state of the black hole candidates (BHCs; Ebisawa 1999), including the specific temperature gradient (Mineshige et al. 1994) and probably the big blue bump of the active galactic nuclei (AGNs; Shields 1978; Malkan 1983).

On the other hand, when the mass accretion rate is much less than the critical value of $\sim \dot{M}_{\text{Edd}}/c^2$, the radiation loss is inefficient and thus the accretion flow becomes a radiatively inefficient accretion flow (RIAF). In such a situation, the thermal energy of the gas can be advected inward toward the black hole without much being radiated away. This basic idea, now known as the notion of advection-dominated accretion flow (ADAF), was first presented by Ichimaru (1977) and has been investigated in quite a lot of detail since the 1990s (Narayan & Yi 1994, 1995a, 1995b; Abramowicz et al. 1995; for reviews see Narayan et al. 1998b; Kato et al. 1998). Since the ADAF can reproduce hard, power-law spectra in the X-ray range, as has been observed, this model was thought to be a good representation of disks in low-luminosity AGNs (LLAGNs) and in the BHCs during their low-hard state. Remarkably, the ADAF model can nicely fit the emergent spectrum of Sagittarius A* (Sgr A*) and other black hole objects (Narayan et al. 1995, 1998a; Mannoto et al. 1997; Mannoto 2000; Özel et al. 2000; Oka & Mannoto 2003). (We should keep in mind, however, that most of observational data of Sgr A* only give upper limits except at radio and X-ray wavelengths.)

We should be aware that there are a number of serious problems inherent in the ADAF formulation. For example, the ADAF model cannot properly treat three-dimensional motion because the ADAF model is a one-dimensional model (in the sense that only the radial structure is solved), although both of the simple arguments and hydrodynamical simulations have shown that the occurrence of convections and/or outflow seems to be a natural consequence in the RIAF (Narayan & Yi 1994; Igumenshchev & Abramowicz 2000). The magnetic fields are treated as a single parameter, the plasma $\beta$, to predict synchrotron emissivity, which is too simple and problematic to describe the dynamics of magnetic fields (see a comprehensive discussion in Narayan 2002). Yuan et al. (2003) recently succeeded in reproducing the spectrum of Sgr A* by introducing nonthermal electron components, but they still treat the magnetic fields in a simplified way. Recent MHD simulations have commonly shown the important role of the magnetic fields in the flow dynamics. The MHD flow is intrinsically time varying and exhibits fractal structure (e.g., Kawaguchi et al. 2000; Machida & Matsumoto 2003; hereafter MM03). To summarize, the lack of three-dimensional motion and the oversimplified treatment of magnetic fields lead to self-inconsistencies in the ADAF solution.
The global three-dimensional MHD simulations of RIAFs were first made by Matsumoto (1999) and have been extensively performed recently by several groups (for a compilation of recent works see Mineshige & Makishima 2004). It has been revealed by the three-dimensional MHD simulations that the flow pattern is considerably complicated and differs significantly from that of the ADAF model (Hawley 2000; Machida et al. 2000, 2001, hereafter MMM01; Hawley & Krolik 2001; Hawley & Balbus 2002, hereafter HB02; MM03; Iguменшчев et al. 2003, hereafter INA03). Stone & Pringle (2001, hereafter SP01) established that a complex flow pattern is produced due to a magnetorotational instability (MRI; for a review see also Hawley et al. 2001; Hawley 2001; Balbus 2003). Since the dynamics of magnetic fields is fully solved, the three-dimensional MHD models seem to be more suitable for describing the RIAF than other models without appropriate treatment of magnetic fields. Thus, the MHD models are expected to fit the observations. Surprisingly, however, such a critical test has not been well investigated. Some papers discussing the observational appearance of the simulated MHD flow, HB02 for example, have shown the distribution of the synchrotron emissivity based on their MHD model. Mineshige et al. (2002) reported a preliminary examination, claiming that the radial density profile is significantly flatter than that of ADAF, leading to overproduction of X-rays, but they demonstrated a serious discrepancy between them. A discussion of the method of Monte Carlo radiative transfer in § 3.3, where we examine spectral variations.

2.2. Calculations of Physical Quantities

In the present study we adopt Cartesian coordinates (x, y, z), where the black hole is located at the origin of the coordinate axes, the z-axis is set to be the rotation axis of the accretion flow, and the x-y plane corresponds to the equatorial plane. We employ Cartesian grids with numbers \((N_x, N_y, N_z) = (100, 100, 197)\) of cells. We assume that radiation is generated within the cylindrical region with radius \(R: (x^2 + y^2)^{1/2} \leq R\) and \(z \leq 100R\). We consider the cases of \(R = 10R_S\) and \(30R_S\). The size of the calculating box is \(2X \times 2Y \times 2Z\), where we set \((X, Y, Z) = (R, R, 100R_S)\).

We take three-dimensional data of density, magnetic fields, and proton temperature distributions in the accretion flows from the three-dimensional MHD simulations (KMS04). Since the MHD simulations only give the normalized density, \(\tilde{\rho}\), with the normalization, \(\rho_0\), and the normalized field strength, \(\tilde{B}\), we have one free parameter, \(\rho_0\), to determine absolute values of density and magnetic fields; that is, \(\rho = \rho_0\tilde{\rho}\) and \(B = (\rho_0c^2)^{1/2}\tilde{B}\), where \(\tilde{B}\) is the field strength. The proton temperature does not depend on the density parameter and is given by the MHD simulation as \((\mu m_p c^2/k)\tilde{e}_p\); where \(\mu\) is the mean molecular weight (=0.5), \(m_p\) is the proton mass, \(k\) is the Boltzmann constant, and \(\tilde{e}_p\) is the normalized sound velocity obtained by the simulation.

Although KMS04 assumed a one-temperature plasma, here we adopt two-temperature assumptions. Assuming that the electrons have a Maxwellian distribution, we evaluate the electron temperature, \(T_e\), through the energy balance of the electrons between Coulomb collisions with ions and radiative cooling,

\[
\int_{-Z}^{Z} \int_{-R}^{R} \frac{L_{\nu}}{2\pi} 2\pi dr dz = \int L_{\nu} d\nu, \tag{1}
\]

Here \(L_{\nu}\) is the energy transfer rate from ions to electrons (Stepney & Guilbert 1983) and \(L_{\nu}\) is the luminosity at frequency \(\nu\). In the present study we suppose, for simplicity, the electron temperature to be a function of only the radius, \(r\), and to be independent of the altitude, \(z\). This simplification does not affect the results too much, since the emission from regions around the equatorial plane, where the density is at a maximum, is dominant at each radius. In addition, we solve for the electron temperature by dividing the calculating box into two parts: the inner \((r < 10R_S)\) and outer parts \((r = 10R_S-30R_S)\). The fractional luminosity, \(L_{\nu,\text{f}}\), is obtained by Monte Carlo simulations (see next subsection) for a given \(T_e\), which does not always satisfy equation (1). We thus iteratively calculate the electron temperature and the emergent spectrum so as to meet the condition given by equation (1).

2.3. Monte Carlo Radiative Transfer Calculations

The method of the Monte Carlo simulation is based on Pozdnyakov et al. (1977). In this study, synchrotron emission/absorption, free-free emission/absorption, and Compton/inverse Compton scattering are taken into account. Some of the photons
emitted by synchrotron and free-free emission pass through the calculating box without being scattered or absorbed, while some are scattered and/or absorbed in the accretion flow, depending on the mean free path at the photons emitting position. In order to efficiently calculate the emergent spectra, we introduce a weight, \( w \), as described by Pozdnyakov et al. (1977). At the beginning, the weight, \( w_0 \), is set equal to unity for each emitted photon and then we calculate the escape probability, \( P_0 \). The escape probability of a photon after the \( i \)-th scattering (for \( i \geq 1 \)), \( P_i \), is evaluated as

\[
P_i = \exp \left( -\left( \frac{\rho(x_i, y_i, z_i)}{m_p} \right) \sigma_{\text{KN}}(x_i, y_i, z_i) + \kappa_{\text{abs}}(x_i, y_i, z_i) \right),
\]

where \((x_0, y_0, z_0)\) corresponds to the point where a photon was generated, \((x_i, y_i, z_i)\) is the point where a photon is subject to the \( i \)-th scattering, \( m_p \) is the proton mass, \( \sigma_{\text{KN}} \) is the Klein-Nishina cross section (Rybicki & Lightman 1979), \( \kappa_{\text{abs}} \) is the absorption coefficient, and \( l \) is the distance from the point \((x_i, y_i, z_i)\) to the boundary of the calculating box. Assuming local thermodynamic equilibrium (LTE), we can write the absorption coefficient as

\[
\kappa_{\text{abs}} = \frac{\varepsilon_{\text{syn}} + \varepsilon_{\text{R}}}{4\pi B_j},
\]

where \( \varepsilon_{\text{syn}} \) and \( \varepsilon_{\text{R}} \) are the synchrotron and free-free emissivities, respectively (Pacholczyk 1970; Stepney & Guilbert 1983), and \( B_j \) is the Planck function. The quantity of \( w_0P_0 \) represents the transmitted portion of photons and is recorded to calculate the penetrated spectrum or reprocessed photons according to the escape direction of the photon. A fraction of \( A_0 \) of the remaining portion, \( w_0(1 - P_0)A_0 \), undergoes at least one scattering, while \( w_0(1 - P_0)(1 - A_0) \) describes the absorbed portion, where \( A_0 (t \geq 0) \) is the scattering albedo. The transmitted portion of photons after the \( i \)-th scattering, \( w_i P_i \), is recorded to calculate the transmitted spectrum, and the remaining portion, \( w_i(1 - P_i)A_i \), undergoes the \( (i + 1) \)-th scattering. This calculation is continued until the weight \( w_i \) becomes sufficiently small (\( w_i < 1 \)). The whole process is simulated by the Monte Carlo method. Finally, we suppose the region within the Schwarzschild radius, \((x^2 + y^2 + z^2)^{1/2} < r_s\), to be vacuum. Gravitational lensing and photon redshifts are not considered, since the black hole is much smaller than the calculating box in our study.

3. RESULTS: SPECTRAL FEATURES OF MHD FLOW

3.1. Basic Features

We first display the representative spectra of the MHD flow in Figure 1 (*thick solid lines*) together with the observed data of Sgr A*. The adopted parameters are the radius of the emitting region, \( R = 30 r_s \), the elapsed time at \( t = 5.7 \times 10^7 \) s, and the density normalization, \( \rho_0 = 1.6 \times 10^{-14} \) (upper) and \( 5.6 \times 10^{-16} \) g cm\(^{-3}\) (lower). It might be noted that the emission is predominantly from the equatorial plane and other parts at large altitudes \((|z| > 10 r_s)\) do not contribute very much to the emergent spectra, although we sum up all the contributions from the vertically elongated cylinder between \( Z = \pm 100 r_s \). This is because the density rapidly decreases as \(|z|\) increases. The calculated electron temperatures are listed in Table 1. As expected, electron temperatures are insensitive to radius, but there is a slight tendency that \( T_e \) decreases with an increase of radius. Similarly, relatively flat electron temperature profiles were obtained in two-temperature, hot accretion flows (Narayan & Yi 1995b; Nakamura et al. 1996; Mannoto et al. 1997) and evaporated disk-corona flows (Liu et al. 2002).

It is apparent that the resultant spectra look similar to those of the ADAF model. The lower energy peak in the radio band is a synchrotron peak created because of significant self-absorption of synchrotron emission. The IR emission at around \( \log \nu = 14 \) is due to inverse Compton scattering of synchrotron photons. Since we assume thermal electrons only, the spectral slope at \( \log \nu < 11 \) is \( L_{\nu} \propto \nu^2 \), corresponding to that of Rayleigh-Jeans emission. The electron temperatures do not vary much, even if we change \( \rho_0 \) and, hence, the radio parts (which depend primarily on the electron temperature) are roughly identical among different models, as shown in Figure 1. In contrast, the flux at \( \log \nu > 12 \) and the frequency of the lower energy peak (in the radio band) are both sensitive to \( \rho_0 \).

Here we stress that the magnetic field strength affects the lower energy peak in the radio band and the IR flux, whereas the X-ray

![Figure 1](image-url)

Note.—Our 2D: our two-dimensional MHD simulations.
flux and the slope at \( \log \nu < 11 \) do not depend on the magnetic field strength itself. To check what alters if the magnetic field strengths were under- or overestimated in the simulation, we calculate the spectra with artificially strengthened magnetic fields, keeping the same density profile. The thin solid line in Figure 1 indicates the spectrum; here the magnetic field strength is set to be 10 times stronger than the original values, so as to be compared with the original (lower thick solid line). It is found that the X-ray flux and the slope at \( \log \nu < 11 \) do not change, but the radio peak luminosity and IR flux both increase. This is because the synchrotron emission contributes to the radio flux and the inverse Compton scattering of the synchrotron photons is dominant in the IR band.

A big distinction between the spectra of ADAF and those of simulated MHD flows is found in the X-ray bands; namely, inverse Compton scattering is dominated in the former, while the thermal bremsstrahlung is the dominant mechanism in the latter. This reflects a flatter density profile in the MHD flow than for the ADAF, since a flatter density profile means that more material is present at large radii than for the case with a steep density profile, thereby producing more bremsstrahlung photons (discussed later).

We confirmed that the resultant spectra do not significantly change even if we employ the density and magnetic fields averaged over the azimuth in the simulated data of KMS04. This implies that three-dimensional effects are not essential for the study of the spectra. This is because the flow pattern is almost axisymmetric in the simulations of KMS04, who calculated the evolution of a torus threaded by weak poloidal magnetic fields. In the case that the perturbations in the azimuthal direction increase and nonaxisymmetric structures form, any three-dimensional aspects would clearly appear in the spectra.

### 3.2. Fitting to the Flaring-State Spectrum

A striking fact is that we cannot fit both the radio and X-ray data simultaneously with the current MHD flow model. The MHD flow model can reproduce only a part of the observations. For example, it can fit the observed radio peak, if we assign \( \rho_0 = 1.6 \times 10^{-14} \) g cm\(^{-3} \), but its flux largely exceeds the observed X-ray data and the upper limits in the IR band. This X-ray excess is caused by the strong free-free emission from the outer part of the flow (\( r \geq 10 r_S \)), since the emissivity of free-free emission, which is dominant in X-rays, is \( \varepsilon_{\text{ff}} \propto \rho^2 T_e^{1/2} \propto r^{1.5} \) for a density profile of \( \rho \propto r \) and the \( T_e \) profile of \( T_e \propto r^{-1} \). The entire luminosity is \( \varepsilon_{\text{ff}} d^3 r \propto r^{3.5} \) for a density profile of \( \rho \propto r \).

For comparison, we plotted the spectra for the emission from only the inner region within \( R = 10 r_S \) in Figure 1 (upper dashed line) for the same density parameter, \( \rho_0 = 1.6 \times 10^{-14} \) g cm\(^{-3} \). The electron temperature is \( 5.56 \times 10^7 \) K (see Table 1). As shown in this figure, huge X-ray excesses disappear (cf. the case with \( R = 30 r_S \)) and the model is successful in reproducing radio observations at around \( \log \nu \sim 11-12 \) and is consistent with X-ray observations during the flaring state (upper dashed line).

The discrepancy between the upper thick solid and upper dashed lines represents the contribution from the outer parts (\( r > 10 r_S \)). This clearly demonstrates that a huge X-ray excess is generated at the outer part of the flow and we need to somehow remove these contributions to fit the data.

Alternatively, we can reduce the X-ray flux so as to fit the data by employing a lower density parameter, \( \rho_0 = 5.6 \times 10^{-16} \) g cm\(^{-3} \) (Fig. 1, lower thick solid line), but for such a case, radio peak flux decreases in accordance with the reduced density and, hence, is short of the radio luminosity. We thus see that for this particular density profile, it is totally impossible to reproduce the observations regardless of \( \rho_0 \) (see also Mineshige et al. 2002).

This situation resembles that of the convection-dominated accretion flow (CDAF; see Ball et al. 2001), since it has a flatter density profile like for the MHD flow (MMM01). For \( \rho \propto r^{-1/2} \) and \( T \propto r^{-1} \), we have \( \varepsilon_{\text{ff}} d^3 r \propto r^{0.5} \) dr, indicating significant free-free emission from the outer parts.

Here we stress that the emergent spectra start to resemble the observed data if the magnetic field strengths are systematically underestimated in the three-dimensional MHD simulation; that is, the actual magnetic fields are stronger than in the simulated data. As was already mentioned in the previous subsection, the radio peak luminosity and IR flux both increase with an increase of the magnetic field strength. Therefore, the emergent spectra are consistent with the observations if the actual magnetic fields are 10 times stronger than the simulated data (thin solid line).

Strictly speaking, the calculated spectrum also deviates from the observed data at lower frequencies, \( \log \nu < 10 \). This inconsistency would be resolved if we consider synchrotron emission from nonthermal electrons as was first claimed by Mahadevan (1998). By a careful fitting to the up-to-date spectrum, Yuan et al. (2003) concluded that the low-frequency radio flux can be explained via the emission of nonthermal electrons, which possess a small fraction of the electron thermal energy.

### 3.3. Case of the Quiescent Spectrum

What about the case of the quiescent spectrum? Before attempting any fitting to the quiescent spectrum, we need to remark on the observational constraints of the size of the emission region. By the Chandra X-ray observations, Baganoff et al. (2001, 2003) have claimed that the X-ray emission of Sgr A* is extended on scales of about 0.04 pc in the quiescent state. However, Tan & Draine (2004) suggested that about half of the extended X-ray emission is due to dust scattering from an unresolved source. Therefore, we regard the observed X-ray data in the quiescent state as giving an upper limit for the X-ray luminosity of the accretion flow around the black hole.

To fit the X-ray flux in the quiescent state, we need to adopt a small density parameter, \( \rho_0 = 2.0 \times 10^{-15} \) g cm\(^{-3} \) (lower dashed lines), but then the flux cannot reproduce a radio peak. In addition to this, the X-ray spectral slope also differs. The X-ray excess decreases with a decrease in size of the emission region, since the X-ray emission is dominated by the outer part, as we already mentioned in the previous subsection. We can adjust the X-ray luminosity to the observed value in the quiescent state by setting a much smaller emitting region, \( R < 10 r_S \). However, the X-ray spectral slope is still much steeper than the observed one.

To summarize, we can reproduce the observed spectral energy distribution (SED) during the flaring state if the emitting region of the flow is restricted to be relatively small (\( r < 10 r_S \)), but we cannot account for the quiescent-state spectrum for any choice of parameters. Note that a compact emission region is needed to account for short-term variability, since otherwise the variability timescale will be much longer in accordance with the timescales in the outer zone (Mineshige et al. 2002).

### 3.4. Time Variation

MHD simulations show that MHD flows are highly time variable. It is thus interesting to see how the resultant spectra change with time and check if the calculated trend fits the observational ones. We plot the time variations of the spectra in Figure 2. Here we again set \( R = 10 r_S \) and \( \rho_0 = 1.6 \times 10^{-14} \) g cm\(^{-3} \). The calculated epochs are divided into two phases: the jet phase...
wavelengths and their amplitudes vary a few hundred percent. The jet is produced (nearly constant. The radio and IR luminosities monotonously increase until luminosities, as well as the electron temperature. The electron temperature is automatically with increasing wavelength in the LLAGNs. 

Interestingly, the calculated flux at each wavelength does not reduce the observed spectrum in the flaring state of Sgr A\textsuperscript{*}.

The X-ray time variation predicted by our results is too small to account for the observed large-amplitude variations (X-ray burst) reported by Baganoff et al. (2003); i.e., the X-ray luminosity in the flaring state is about 45 times larger than that in the quiescent state. Such an X-ray flaring event might be produced by the nonthermal electrons, whereas we take only thermal electrons into account in the present study.

4. DISCUSSION: THE IMPLICATIONS OF OUR RESULTS

We have calculated the emergent spectra based on the MHD simulation by KMS04 and demonstrated that the MHD flow model cannot account for both the radio and X-ray observations in the flaring state simultaneously, unless we restrict the emission region to be compact. We have also shown that it is problematic for reproducing the spectrum in the quiescent state. In this section we discuss what these results imply.

4.1. Flaring State

We first focus our discussion on the case of the flaring state. As was already mentioned in § 3.2, the relatively compact emission region (r < 10\textsubscript{r}\textsubscript{s}) is required for the MHD flow model to reproduce the observed spectrum in the flaring state of Sgr A\textsuperscript{*}. There are several possibilities that meet this requirement.

4.1.1. Modifying Density Profiles

A compact emission region would be realized, if the density profile at large radii is much steeper than that obtained by KMS04 or if the broad density peak found in KMS04 is somehow removed. Since KMS04 provides only one specific example, it is necessary to check the density profiles of other MHD simulations. In Table 2 we summarize the density profiles of some representative simulations with brief descriptions of their calculations.

SP01, HB02, and KMS04 adopted similar initial conditions; namely, they started calculations with a torus threaded by weak localized poloidal magnetic fields and assumed no further mass input. They all investigated the evolution of the torus under the pseudo-Newtonian potential. INA03 and Proga & Begelman (2003, hereafter PB03) set a different situation: INA03 continuously injected magnetized matter into the computational domain from near the outer boundary, and PB03 started the simulations with Bondi accretion flow, whose angular momentum is zero except for the outer part of the flow. Note that SP01 and PB03 performed two-dimensional simulations, while others did three-dimensional simulations. Despite some differences in simulations, SP01, INA03, and PB03 all obtained a relatively steep density profile, \( \rho \propto r^{-1} \). No obvious density peak is found in their simulations.

HB02 obtained a similar density profile but with a remarkable peak at \( \sim 5r_s \), which may be a remnant of the initial torus. Caution should be taken here, since this peak may be transient. Similarly, the density peak found at \( \sim 20r_s \) in KMS04, which mainly contributes to X-ray emissions, might be a direct consequence of the initial torus located at 40r\textsubscript{s} at the beginning of the simulations. If this density peak is removed by long-term numerical calculations, we expect that the large X-ray excess may disappear and the resultant spectra can be consistent with the observations, even if we do not artificially restrict the emission region.

To see if this is the case, we made the following test. We performed two-dimensional MHD simulations, starting from an initial condition similar to that of KMS04 but with the initial torus at a much larger radius, around \( \gtrsim 100r_s \). After a long time interval, we have confirmed that the broad density peak, a remnant of the initial torus, disappears. The resultant density...
structure in this two-dimensional model (at $\tilde{t} = 8200$) is plotted by the solid line in the top panel of Figure 3 with that of KMS04. The density profile of our two-dimensional model is roughly proportional to $r^{-1}$ at $r > 7r_\text{s}$ and roughly agrees with the simulations by SP01, INA03, and PB03.

We next calculate the spectrum based on our two-dimensional model and plot it in Figure 3 (bottom panel). The adopted density parameter is $\rho_0 = 3.0 \times 10^{-15}$ g cm$^{-3}$, and the size of the emitting region is $R = 30r_\text{s}$. The calculated electron temperature is shown in Table 1. Clearly, our two-dimensional MHD model can nicely fit the observed data points during the flaring state. Thus, it is very likely that if we continue MHD simulations from the data by KMS04 further, we may be able to fit the observational data. We need caution, however, because behavior of the magnetic fields in two- and three-dimensional simulations may be distinct. We cannot trust the two-dimensional data too much. We need to perform longer three-dimensional simulations to confirm our tentative conclusions.

It is expected that the MHD models by INA03, SP01, and PB03 can also reproduce the observed SED during the flaring state, since these models have similar density profiles to those of our two-dimensional model (see Fig. 3, top panel). We demonstrate it by calculating the spectrum from the regions at $r < 30r_\text{s}$ of the spherical flow model, which is constructed based on the three-dimensional MHD simulations of INA03 (Fig. 3, bottom panel, dotted line). In this model the density and proton temperature profiles are set to be $\rho = 7.0 \times 10^{-15}(r/r_\text{s})^{-1}$ g cm$^{-3}$ and $T_p = 8.6 \times 10^{11}(r/r_\text{s})^{-1}$ K, respectively. The electron temperature and the plasma $\beta$ are assumed to be constant in the calculating box, $T_e = 5 \times 10^9$ K and $\beta = 3$. The INA03 model shows an SED similar to that of our two-dimensional model and is consistent with the observed data in the flaring state. The other MHD models by HB02, MMM01, and MM03, on the other hand, have flatter density profiles, $\rho \propto r^{-\alpha}$ with $\alpha = 0 - \frac{1}{2}$ (see Table 2). Hence, they will produce an X-ray excess as was the case in our simulation.

4.1.2. Moderate Radiative Cooling

A distinct way of possibly producing a steeper density profile is to introduce moderate radiative cooling (Mineshige et al. 2002). By the terminology of RIAF it means that radiative loss is inefficient. However, it may happen that radiative cooling is only partly and transiently important. If we set the cooling timescale to be comparable to the accretion timescale, we expect that only dense parts can efficiently be cooled. Since the density is generally higher in the innermost region, cooling results in

| References | $\Psi$ | I.C./B.C. | Jet | $\rho$ Profile | $\rho$ Peak |
|------------|-------|----------|-----|----------------|-----------|
| SP01       | PN    | $B_s$ torus | Yes | $r^{0.3}(>10r_\text{s})$, $r^{-1}(>10r_\text{s})$ | $10r_\text{s}$ |
| MMM01      | N     | $B_s$ torus | No  | $r^{-1/2}$     | ...       |
| HB02       | PN    | $B_s$ torus | Yes | $r^{0.5}(r < 10r_\text{s})$ | $5r_\text{s}$ |
| INA03      | PN    | Injection | Yes | $r^{-1/2}(>10r_\text{s})$, $r^{-1}(>10r_\text{s})$ | $3r_\text{s}$ |
| MM03       | PN    | $B_s$ torus | No  | $r^{6}$         | $3r_\text{s}$ |
| PB03       | PN    | Bondi flow | Yes | $r^{0.5}(>7r_\text{s})$, $r^{-1}(>7r_\text{s})$ | $7r_\text{s}$ |
| KMS04      | PN    | $B_s$ torus | Yes | $r^{-1}(>20r_\text{s})$, $r^{-1}(>20r_\text{s})$ | $20r_\text{s}$ |
| Our 2D     | PN    | $B_s$ torus | Yes | $r^{-1}(>7r_\text{s})$, $r^{-1}(>7r_\text{s})$ | $7r_\text{s}$ |

Note.—Our 2D: our two-dimensional MHD simulations; $\Psi$: gravitational potential; N: Newtonian; PN: pseudo-Newtonian; I.C./B.C.: initial or boundary condition; $B_s$ or $B_p$: a torus threaded by weak localized toroidal or poloidal magnetic fields is initially assumed; Injection: magnetized matter is continuously injected into the computational domain near the outer boundary.

![Figure 3](image-url)

FIG. 3.—Top: Normalized density profile given by our two-dimensional MHD simulations (thick solid line) and KMS04 (dashed line). The density peak in KMS04 is around 20$r_\text{s}$; in contrast, it is near the black hole ($r \sim 7r_\text{s}$) in the case of our two-dimensional simulations. The thin solid lines indicate the schematic density profile given by INA03 (three-dimensional simulations), SP01, and PB03 (two-dimensional simulations). They showed a similar density profile, $\rho \propto r^{-1}$, with our two-dimensional MHD simulations at the outer part ($r > 10r_\text{s}$). Bottom: The solid line is the spectra of our two-dimensional MHD model. We consider the emission from the extended region of $r < 30r_\text{s}$. The normalization of density is set to be $\rho_0 = 3.0 \times 10^{-15}$ g cm$^{-3}$, and the calculated electron temperature is shown in Table 1. The dotted line represents the spectra of the INA03 model. Here we use the spherical flow model, which is constructed based on the three-dimensional MHD simulations of INA03. In this model, the density and proton temperature profiles are $\rho = 7.0 \times 10^{-15}(r/r_\text{s})^{-1}$ g cm$^{-3}$ and $T_p = 8.6 \times 10^{11}(r/r_\text{s})^{-1}$ K, respectively. We set $T_e = 5 \times 10^9$ K and plasma $\beta = 3.0$. Our two-dimensional MHD model, as well as the INA03 model, can reproduce the observed data during the flaring state.
Further mass concentration toward the center, a preferable condition to fit the observations. However, this is only a possibility and we need three-dimensional MHD simulations to confirm this idea.

Another possibility is that the outer portions of the disk may become radiation efficient flow. Even if the density profile does not change, the emission from the outer parts can be reduced, if the electron temperature there is much lower than that of the inner part. In KMS04 and most of the other MHD simulations, the one-temperature plasma is assumed and the cooling of protons is neglected. However, the proton thermal energy may be converted into the electron energy due to the effective Coulomb collisions and be radiated away in the outer zones. Further study requires three-dimensional MHD simulations coupled with the electron energy equation and radiative cooling.

4.2. Quiescent State

The case of the quiescent spectrum will be much more difficult to reproduce by the MHD model. We have seen that the calculated spectral slope was much steeper than that of the X-ray observations in the quiescent state of Sgr A*. We can reduce the X-ray flux but cannot easily change the spectral slope in the X-ray range. We find two possibilities to make a flatter spectral slope.

4.2.1. Nonthermal Electrons

This difficulty may be removed, if emission from the nonthermal electrons dominates over that from thermal ones. By considering emission from nonthermal electrons, Yuan et al. (2003) nicely reproduced the observed spectrum in the quiescent state of Sgr A*. (However, they prescribe an arbitrary energy distribution of the nonthermal electrons with more free parameters in order to give a good fit to the observations.) This is a very attractive possibility, but it is hard to prescribe an energy distribution of nonthermal electrons because of a lack of good theory of particle acceleration. Thus, we need to introduce parameters describing the electron energy distribution, and the results strongly depend on these parameters.

4.2.2. Compton Scattering in Disk Coronae

The spectral slope at the X-ray band would be flatter if inverse Compton scattering is more enhanced, compared with other processes, and becomes dominant in the X-ray range. Note that the calculated steep spectral slopes shown in Figure 1 are due to bremsstrahlung emissions from the outer parts. The relative importance of the Comptonization inside the corona is proportional to the column density, $\sim \rho H$ (with $H$ being the half-thickness of the corona), while the bremsstrahlung emissivity is proportional to $\rho^2 H$. Thus, the presence of an extended, tenuous corona may resolve the problem. However, the formation mechanism of the corona is still an open issue, and it is not easy to probe this possibility.

5. CONCLUSIONS

In order to test the three-dimensional MHD flow simulation as a model of optically thin, high-temperature accretion flows through comparison with observations, we studied the spectral properties of magnetized accretion flows based on the three-dimensional MHD simulation data. Surprisingly, such comparative studies have not been well investigated so far in spite of a large number of MHD simulations having been performed recently. We summarize our results as follows:

1. We found that the MHD model cannot reproduce the observed SED in the flaring state of Sgr A* without substantial modification. If we use the data by KMS04, we need to require that the emission region is restricted to be compact ($r < 10 r_s$), but this could be due to the particular initial condition. Some other MHD models with a steeper density profile ($\rho \propto r^{-1}$) and without a broad density peak ($r > 10 r_s$) can fit the observations, if the emission from regions of $r \approx 30 r_s$ mainly contributes to the SED.

2. The MHD flow cannot generally reproduce the spectrum (spectral shape, in particular) in the quiescent state. Significant contributions by nonthermal electrons should resolve this issue. It is also possible that observational X-ray data may contain emission from regions other than the vicinity of the black hole.

3. The MHD flow predicts substantial and incoherent time variations in the emergent spectrum. The predicted variation amplitude is, however, too small to account for the large-amplitude X-ray burst.

The authors would like to thank the anonymous referee for important comments and suggestions. The calculations were carried out at Yukawa Institute for Theoretical Physics at Kyoto University and the Department of Physics at Rikkyo University. This work is supported in part by Research Fellowship of the Japan Society for the Promotion of Science for Young Scientists 02796 (K. O.), the Grants-in-Aid of the Ministry of Education, Science, Culture, and Sport (14079205, 16340057), and a Grant-in-Aid for the 21st Century COE “Center for Diversity and Universality in Physics” (S. M.).

REFERENCES

Abramowicz, M. A., Chen, X., Kato, S., Lasota, J.-P., & Regev, O. 1995, ApJ, 438, L37
Baganoff, F. K., et al. 2001, Nature, 413, 45
Balbus, S. A. 2003, ARA&A, 41, 555
Ball, G. H., Narayan, R., & Quataert, E. 2001, ApJ, 552, 221
Ebisawa, K. 1999, in ASP Conf. Ser. 161, High Energy Processes in Accreting Black Holes, ed. J. Poutanen & R. Svensson (San Francisco: ASP), 39
Esin, A., McClintock, J. E., & Narayan, R. 1997, ApJ, 489, 865
Ghez, A. M., et al. 2003, ApJ, 586, L127
Goldston, J. E., Quataert, E., & Igumenshchev, I. V. 2005, ApJ, 621, 785
Hawley, J. F. 2000, ApJ, 528, 462
Hawley, J. F., & Balbus, S. A. 2002, ApJ, 573, 738 (HB02)
Hawley, J. F., Balbus, S. A., & Stone, J. M. 2001, ApJ, 554, L49
Igumenshchev, I. V., & Abramowicz, M. A. 2000, ApJS, 130, 463
Igumenshchev, I. V., Narayan, R., & Abramowicz, M. A. 2003, ApJ, 592, 1042 (INA03)
Kato, S., Fukue, J., & Mineshige, S. 1998, Black-Hole Accretion Disks (Kyoto: Kyoto Univ. Press)
Kato, Y., Mineshige, S., & Shibata, K. 2004, ApJ, 605, 307 (KMS04)
Kawaguchi, T., Mineshige, S., Machida, M., Matsumoto, R., & Shibata, K. 2000, PASJ, 52, L1
Liu, B. F., Mineshige, S., Meyer, F., Meyer-Hofmeister, E., & Kawaguchi, T. 2002, ApJ, 575, 117
Machida, M., Hayashi, M. R., & Matsumoto, R. 2000, ApJ, 532, L67
Machida, M., & Matsumoto, R. 2003, ApJ, 585, 429 (MM03)
Machida, M., Matsumoto, R., & Mineshige, S. 2001, PASJ, 53, L1 (MM01)
Mahadevan, R. 1998, Nature, 394, 651
Malkan, M. A. 1983, ApJ, 268, 582
Manmoto, T., Minehige, S., & Kusunose, M. 1997, ApJ, 489, 791
Matsumoto, R. 1999, in Numerical Astrophysics, ed. S. M. Miyama, K. Tomisaka, & T. Hanawa (Boston: Kluwer), 195
