Label Embedded Dictionary Learning for Image Classification

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Abstract—Recently, label consistent k-svd(LC-KSVD) algorithm has been successfully applied in image classification. The objective function of LC-KSVD is consisted of reconstruction error, classification error and discriminative sparse codes error with $\ell_0$-norm sparse regularization term. The $\ell_0$-norm, however, leads to NP-hard issue. Despite some methods such as orthogonal matching pursuit can help solve this problem to some extent, it is quite difficult to find the optimum sparse solution. To overcome this limitation, we propose a label embedded dictionary learning(LEDL) method to utilise the $\ell_1$-norm as the sparse regularization term so that we can avoid the hard-to-optimize problem by solving the convex optimization problem. Alternating direction method of multipliers and blockwise coordinate descent algorithm are then used to optimize the corresponding objective function. Extensive experimental results on six benchmark datasets illustrate that the proposed algorithm has achieved superior performance compared to some conventional classification algorithms.

Index Terms—Dictionary learning; sparse representation; label embedded dictionary learning; image classification.

I. INTRODUCTION

Image classification has been a classical issue and challenging research topic in computer vision. In the past decades, many image classification algorithms have been proposed [1]–[13]. One major category is the sparse representation based methods which is widely used among different types of image classification methods. Sparse representation is capable of representing the data more adaptively and flexibly by coding the input sample features under over-complete bases.

Wright [2] proposed sparse representation based classifier(SRC) algorithm which is the most representative one in the sparse representation based methods. SRC achieved impressive performance via reconstructing a testing sample feature by a sparse linear combination of the training sample features with $\ell_1$-norm regularization term. Then the residual error from each class can be used to predict label. However, the training sample features are directly used in SRC algorithm which can not contain enough discriminant information, thus the performance of SRC is not good enough. To handle this problem, dictionary learning(DL) is introduced to preprocess the training sample features before classification.

Dictionary learning is a theory which aims to lead to a "simple" description for a wide range of signals to reduce the redundant information. In 1993, Mallat [12] first proposed the concept of over-complete dictionary which is the theoretical principle of DL. Then Olshausen [20] proposed the application of DL on natural images and after that the DL was widely used in many fields such as image denoising [21]–[23], image superresolution [24]–[26], and image classification [9], [12], [17], [27]. In real applications of image classification, it is usually to learn a dictionary by utilising the training sample features before classification. While a well learned dictionary can help to get significant boost in classification accuracy. Therefore, methods for DL in classification are more and more popular in recent years.

Aharon [27] proposed a method named K-SVD to address the problem of efficiently learning a overcompeting dictionary from training sample features. This method achieves a good performance in image classification. However, the label information is ignored while the representational ability is focused on in K-SVD. Thus, the coding procedure cannot get the discriminative information well. In order to solve this problem, two popular strategies for DL are proposed. One is to learn separate dictionaries to promote discrimination for each class such as class specific dictionary learning (CSDL) based representation algorithms [8], [9]. Another one of discriminative DL is to learn a shared dictionary for all classes. For example, Zhang [11] proposed discriminative K-SVD(D-KSVD) algorithm to learn a single dictionary for all classes and formulate a linear classifier as a joint optimization problem. Furthermore, Jiang [12] proposed label consistence K-SVD(LC-KSVD) method which add a label consistence term into the objective function of D-KSVD. The motivation for adding this term is to encourage the training samples from the same class to have similar sparse codes and those from different classes to have dissimilar sparse codes. Thus, the discriminative abilities of the learned dictionary is effectively improved. However, the sparse regularization term in LC-KSVD is $\ell_0$-norm which lead to the NP-hard [28] problem. Although some greedy methods such as orthogonal matching pursuit(OMP) [29] can help solve this problem to some extent, it is usually to find the suboptimum sparse solution instead of the optimal sparse solution. More specifically, greedy method solve the global optimal problems by finding basis vectors in order of reconstruction errors from small to large until $T$(the sparsity constraint factor) times. Thus, the initialized values are crucial. However, it is easy to find a local minimum value instead of global one by using greedy methods so that it is usually to get the suboptimal sparse solution.

In this paper, we propose a novel dictionary learning algorithm named label embedded dictionary learning(LEDL). This method introduces the $\ell_1$-norm regularization term to replace

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the \( \ell_0 \)-norm regularization of LC-KSVD. Thus, we can freely select the basis vectors for linear fitting to get optimal sparse solution. In addition, \( \ell_1 \)-norm sparse representation is widely used in many fields so that our proposed LEDL method can be extended and applied easily. A schematic description of our proposed method is given in Figure 1. We adopt the alternating direction method of multipliers (ADMM) [30] framework and blockwise coordinate descent (BCD) [31] algorithm to optimize LEDL. Our work mainly focuses on threefold.

- We propose a novel dictionary learning algorithm named label embedded dictionary learning that introduces the \( \ell_1 \)-norm regularization term.
- We combine the ADMM framework and BCD method to optimize LEDL.
- We evaluate the proposed LEDL algorithm on six benchmark datasets and demonstrate that LEDL outperforms other three classical approaches.

The rest of the paper is organized as follows. Section II reviews two conventional classifier, specifically sparse representation based classifier and Label Consistent K-SVD algorithms. Section III-A explains the proposed label embedded dictionary learning method. The optimization approach and the convergence are elaborated in Section III-B and then the overall algorithm is shown. The experimental results on six well-known datasets are shown in Section V. Finally, we conclude this paper in Section V.

II. RELATED WORK

In this section, we overview two related algorithms, including sparse representation based classification (SRC) and label consistent K-SVD (LC-KSVD).

A. Sparse representation based classification (SRC)

SRC for robust image recognition was proposed by [2]. Assume that we have \( C \) classes of training samples, denoted by \( \{ \mathbf{x}_c \} \), \( c = 1, 2, \cdots, C \), where \( \mathbf{x}_c \) is the training sample matrix of class \( c \). Each column of the matrix \( \mathbf{X}_c \) is a training sample feature from the \( c_{th} \) class. The whole training sample matrix can be denoted as \( \mathbf{X} = [ \mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_C ] \in \mathbb{R}^{D \times N} \), where \( D \) represents the dimensions of the sample features and \( N \) is the number of training samples. Supposing that \( y \in \mathbb{R}^{D \times 1} \) is a testing sample vector, the sparse representation algorithm aims to solve the following objective function:

\[
\mathbf{s} = \arg \min_{s} \left\{ \| \mathbf{y} - \mathbf{Xs} \|_2^2 + 2\alpha \| \mathbf{s} \|_1 \right\}
\]

(1)

Here, \( \alpha \) is the regularization parameter to control the tradeoff between fitting goodness and sparseness. The sparse representation based classification is to find the minimum value of the residual error for each class.

\[
\text{id}(y) = \arg \min_c \| y - \mathbf{x}_c \mathbf{s}_c \|_2^2
\]

(2)

where \( \text{id}(y) \) represents the predictive label of \( y \), \( \hat{s}_c \) is the sparse codes of \( c_{th} \) class.

The procedure of SRC is shown in Algorithm 1. Obviously, the residual \( e_c \) is associated with only a few images in class \( c \). The impressive results of SRC have been reported in [2].

B. Label Consistent K-SVD (LC-KSVD)

Jiang [12] proposed LC-KSVD to encourage the similarity among representations of samples belonging to the same class in D-KSVD [11]. The authors proposed to combine the discriminative sparse codes error with the reconstruction error and the classification error to form a unified objective function, which gave discriminative sparse codes matrix \( \mathbf{Q} = [q_1, q_2, \cdots, q_C] \in \mathbb{R}^{K \times N} \), label matrix \( \mathbf{H} = [h_1, h_2, \cdots, h_N] \in \mathbb{R}^{C \times N} \) and training sample matrix \( \mathbf{X} \). The objective function is defined as follows:

\[
\begin{align*}
&< \mathbf{B}, \mathbf{W}, \mathbf{A}, \mathbf{S} > = \arg \min_{\mathbf{B}, \mathbf{W}, \mathbf{A}, \mathbf{S}} \| \mathbf{X} - \mathbf{BS} \|_F^2 + \lambda \| \mathbf{H} - \mathbf{WS} \|_F^2 \\
&\quad + \omega \| \mathbf{Q} - \mathbf{AS} \|_F^2 \\
&\quad \text{s.t.} \| \mathbf{s}_i \|_0 < T \quad (i = 1, 2, \cdots, N)
\end{align*}
\]

where \( T \) is the sparsity constraint factor, making sure that \( \mathbf{s}_i \) has no more than \( T \) nonzero entries. The dictionary \( \mathbf{B} = [\mathbf{b}_1, \mathbf{b}_2, \cdots, \mathbf{b}_K] \in \mathbb{R}^{D \times K} \), where \( K > D \) is the number of atoms in the dictionary, and \( \mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_N] \in \mathbb{R}^{K \times D} \) is the sparse codes of training sample matrix \( \mathbf{X} \). \( \mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_K] \in \mathbb{R}^{C \times K} \) is a classifier learned from the given label matrix \( \mathbf{H} \). We hope the \( \mathbf{W} \) can return the most probable class this sample belongs to. \( \mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_K] \in \mathbb{R}^{K \times K} \) is a linear transformation relies on \( \mathbf{Q} \). \( \lambda \) and \( \omega \) are the regularization parameters balancing the discriminative sparse codes error and the classification contribution to the overall objective, respectively. The algorithm is shown in Algorithm 2. Here, we denote \( m (m = 0, 1, 2, \cdots) \) as the iteration number and \( \bullet \) means the value of matrix \( \bullet \) after \( m_{th} \) iteration.

While the LC-KSVD algorithm exploits the \( \ell_0 \)-norm regularization term to control the sparseness, it is difficult to find the optimal sparse solution to a general image recognition. The reason is that LC-KSVD use OMP method to optimise the objective function which usually obtain the suboptimal sparse solution unless finding the perfect initialized values.

III. METHODOLOGY

In this section, we first give our proposed labeled embedded dictionary learning algorithm. Then we elaborate the optimization of the objective function.
Motivated by that the optimal sparse solution can not be found easily with \( \ell_0 \)-norm regularization term, we propose a novel dictionary learning method named label embedded dictionary learning(LEDL) for image classification. This method introduces the \( \ell_1 \)-norm regularization term to replace the \( \ell_0 \)-norm regularization of LC-KSVD. Thus, we can freely select the basis vectors for linear fitting to get optimal sparse solution. The objection function is as follows:

\[
< B, W, A, S > = \arg \min_{B,W,A,S} \| X - BS \|_F^2 + \lambda \| H - WS \|_F^2
\]

\[
+ \omega \| Q - AS \|_F^2 + 2\varepsilon \| S \|_{\ell_1}
\]

\[
s.t. \quad \| B_{\cdot k} \|_2^2 \leq 1, \quad \| W_{\cdot k} \|_2^2 \leq 1, \quad \| A_{\cdot k} \|_2^2 \leq 1 \quad (k = 1, 2, \cdots K)
\]

Here, \((\bullet)_k\) denote the \( k \)th column vector of matrix \((\bullet)\). The \( \ell_1 \)-norm regularization term is utilized to enforce sparsity and \( \varepsilon \) is the regularization parameter which has the same function as \( \alpha \) in Equation (1).

### B. Optimization of The Objective Function

Consider the optimization problem (4) is not jointly convex in both \( S, B, W \) and \( A \), it is separately convex in each \( S \) (with \( B, W, A \) fixed), \( B \) (with \( S, W, A \) fixed), \( W \) (with \( S, B, A \) fixed) or \( A \) (with \( S, B, W \) fixed). To this end, the optimization problem can be recognised as four optimization subproblems which are finding sparse codes \((S)\) and learning bases \((B, W, A)\), respectively. Here, we employ the alternating direction method of multipliers (ADMM) [30] framework to solve the first subproblem and the blockwise coordinate descent (BCD) [31] algorithm for the rest subproblem. The complete process of LEDL is shown in Figure 2.

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**Algorithm 2** Label Consistent K-SVD

**Input:** \( X \in \mathbb{R}^{D \times N}, H \in \mathbb{R}^{C \times N}, Q \in \mathbb{R}^{K \times N}, \omega, \tau, T, K \)

**Output:** \( B \in \mathbb{R}^{D \times K}, W \in \mathbb{R}^{C \times K}, A \in \mathbb{R}^{K \times K}, S \in \mathbb{R}^{N \times K} \)

1: Compute \( B_0 \) by combining class-specific dictionary items for each class using K-SVD [27].
2: Compute \( B_0 \) for \( X \) and \( B_0 \) using sparse coding.
3: Compute \( A_0 \) using \( A = QS^T (SS^T + I)^{-1} \).
4: Compute \( W_0 \) using \( W = HS^T (SS^T + I)^{-1} \).
5: Solve Eq.(3); Use \( B_0, W_0, \sqrt{\sum A_0} \) to initialize the dictionary.
6: Normalize \( B, A, W \):
   \[
   B \leftarrow \begin{bmatrix} b_1 \| b_1 \|_2 \| b_1 \|_2 \cdots \| b_1 \|_2 \| b_K \|_2 \| b_K \|_2 \cdots \| b_K \|_2 \end{bmatrix}
   \]
   \[
   A \leftarrow \begin{bmatrix} a_1 \| a_1 \|_2 \| a_1 \|_2 \cdots \| a_1 \|_2 \| a_K \|_2 \| a_K \|_2 \cdots \| a_K \|_2 \end{bmatrix}
   \]
   \[
   W \leftarrow \begin{bmatrix} w_1 \| w_1 \|_2 \| w_1 \|_2 \cdots \| w_1 \|_2 \| w_K \|_2 \| w_K \|_2 \cdots \| w_K \|_2 \end{bmatrix}
   \]
7: return \( B, W, A, S \)
1) **ADMM for finding sparse codes:** While fixing $B$, $W$, and $A$, we introduce an auxiliary variable $Z$ and reformulate the LEDL algorithm into a linear equality-constrained problem with respect to each iteration has the closed-form solution. The objective function is as follows:

$$
\begin{align*}
\text{min}_{B, W, A, Z} & \quad \|X - BC\|_F^2 + \lambda \|H - WC\|_F^2 + \omega \|Q - AC\|_F^2 + \epsilon \|Z\|_{\ell_1} + \rho \|C - Z\|_F^2 \\
\text{s.t.} & \quad Z = C, \quad \|B_{\cdot k}\|_2^2 \leq 1, \quad \|W_{\cdot k}\|_2^2 \leq 1, \quad \|A_{\cdot k}\|_2^2 \leq 1 \quad (k = 1, 2 \cdots K)
\end{align*}
$$

The closed form solution of $Z$ is

$$
Z_{m+1} = \max \left\{ C_{m+1} + \frac{L_m}{\rho} - \frac{\epsilon}{\rho} I, 0 \right\} + \min \left\{ C_{m+1} + \frac{L_m}{\rho} + \frac{\epsilon}{\rho} I, 0 \right\}
$$

(10)

where $I$ is the identity matrix and $0$ is the zero matrix.

(3) Updating the Lagrangian multiplier $L$:

$$
L_{m+1} = L_m + \rho (C_{m+1} - Z_{m+1})
$$

(11)

where the $\rho$ in Equation (11) is the gradient descent(GD) method, which has no relationship with the $\rho$ in Equation (6). In order to make better use of ADMM framework, the $\rho$ in Equation (11) can be rewritten as $\theta$.

$$
L_{m+1} = L_m + \theta (C_{m+1} - Z_{m+1})
$$

(12)

2) **BCD for learning bases:** Without the sparseness regularization term in Equation (5), the constrained minimization problem of (4) with respect to the single column has the closed-form solution which can be solved by BCD method. The objective function can be rewritten as follows:

$$
\begin{align*}
\text{min}_{B, W, A} & \quad \|X - BC\|_F^2 + \lambda \|H - WC\|_F^2 + \omega \|Q - AC\|_F^2 + 2\epsilon \|Z\|_{\ell_1} + 2L^2 \|C - Z\|_F^2 \\
\text{s.t.} & \quad \|B_{\cdot k}\|_2^2 \leq 1, \quad \|W_{\cdot k}\|_2^2 \leq 1, \quad \|A_{\cdot k}\|_2^2 \leq 1 \quad (k = 1, 2 \cdots K)
\end{align*}
$$

(13)

We initialize $B_0$, $W_0$ and $A_0$ to be random matrices and normalize them, respectively. Then try to find reasonable values for $\lambda$ and $\omega$. After that we use BCD method to update $B$, $W$ and $A$.

(1) Updating $B$ while fixing $C$, $L$, $Z$, $W$ and $A$

$$
B_{m+1} = \langle B_m, W_m, A_m, C_{m+1}, Z_{m+1}, L_m \rangle
$$

(14)
The closed-form solution of single column of $\mathbf{B}$ is
\[
(B_{k*})_{m+1} = \frac{X^T [C_{k*}]_{m+1} - (\mathbf{B})_{m+1}^T C_{k*}_{m+1} [C_{k*}]_{m+1}^T}{\|X^T [C_{k*}]_{m+1} - (\mathbf{B})_{m+1}^T C_{k*}_{m+1} [C_{k*}]_{m+1}^T\|_2^2}
\]
(15)

where $\mathbf{B}^c_k = \{B_{*,p}, p \neq k \cup \{0\}, p = k\}$ denote the $k$th row vectors of matrix $\mathbf{B}$.

(2) Updating $\mathbf{W}$ while fixing $\mathbf{C}$, $\mathbf{L}$, $\mathbf{Z}$, $\mathbf{B}$ and $\mathbf{A}$

\[
W_{m+1} = \langle B_{m+1}, W_m, A_m, C_{m+1}, Z_{m+1}, L_{m+1} \rangle
\]
(16)

The closed-form solution of single column of $\mathbf{W}$ is
\[
(W_{k*})_{m+1} = \frac{H^T [C_{k*}]_{m+1} - (\mathbf{W})_{m+1}^T C_{k*}_{m+1} [C_{k*}]_{m+1}^T}{\|H^T [C_{k*}]_{m+1} - (\mathbf{W})_{m+1}^T C_{k*}_{m+1} [C_{k*}]_{m+1}^T\|_2^2}
\]
(17)

where $\mathbf{W}^c_k = \{W_{*,p}, p \neq k \cup \{0\}, p = k\}$

(3) Updating $\mathbf{A}$ while fixing $\mathbf{C}$, $\mathbf{L}$, $\mathbf{Z}$, $\mathbf{B}$ and $\mathbf{W}$

\[
A_{m+1} = \langle B_{m+1}, W_{m+1}, A_m, C_{m+1}, Z_{m+1}, L_{m+1} \rangle
\]
(18)

The closed-form solution of single column of $\mathbf{A}$ is
\[
(A_{k*})_{m+1} = \frac{Q^T [C_{k*}]_{m+1} - (\mathbf{A})_{m+1}^T C_{k*}_{m+1} [C_{k*}]_{m+1}^T}{\|Q^T [C_{k*}]_{m+1} - (\mathbf{A})_{m+1}^T C_{k*}_{m+1} [C_{k*}]_{m+1}^T\|_2^2}
\]
(19)

where $\mathbf{A}^c_k = \{A_{*,p}, p \neq k \cup \{0\}, p = k\}$

3) Convergence Analysis: Assume that the result of the objective function after $m$th iteration is defined as $f(C_m, Z_m, L_m, B_m, W_m, A_m)$. Since the minimum point is obtained by ADMM and BCD methods, each method will monotonically decrease the corresponding objective function after about 100 iterations. Considering that the objective function is obviously bounded below and satisfies the Equation (20), it converges. Figure 2 shows the convergence curve of the proposed LEDL algorithm by using four well-known datasets. The results demonstrate that our proposed LEDL algorithm has fast convergence and low complexity.

$$f(C_m, Z_m, L_m, B_m, W_m, A_m)$$
$$\geq f(C_{m+1}, Z_{m+1}, L_{m+1}, B_m, W_m, A_m)$$
$$\geq f(C_{m+1}, Z_{m+1}, L_{m+1}, B_{m+1}, W_m, A_m)$$
(20)

4) Overall Algorithm: The overall updating procedures of proposed LEDL algorithm is summarized in Algorithm 3. Here, maxiter is the maximum number of iterations, $1 \in \mathbb{R}^{K \times K}$ is a square matrix with all elements 1 and $\odot$ indicates element dot product. By iterating $\mathbf{C}$, $\mathbf{Z}$, $\mathbf{L}$, $\mathbf{B}$, $\mathbf{W}$ and $\mathbf{A}$ alternately, the sparse codes are obtained, and the corresponding bases are learned.

**Algorithm 3** Label Embedded Dictionary Learning

**Input:** $X \in \mathbb{R}^{D \times N}$, $H \in \mathbb{R}^{C \times K}$, $Q \in \mathbb{R}^{K \times N}$, $\mathbf{A} \in \mathbb{R}^{K \times N}$, $\mathbf{C} \in \mathbb{R}^{K \times N}$

**Output:** $\mathbf{B} \in \mathbb{R}^{D \times K}$, $\mathbf{W} \in \mathbb{R}^{C \times K}$, $\mathbf{A} \in \mathbb{R}^{K \times N}$, $\mathbf{C} \in \mathbb{R}^{K \times N}$

1: $C_0 \leftarrow \text{zeros}(K, N)$, $Z_0 \leftarrow \text{zeros}(K, N)$, $L_0 \leftarrow \text{zeros}(K, N)$

2: $B_0 \leftarrow \text{rand}(D, K)$, $W_0 \leftarrow \text{rand}(C, K)$, $A_0 \leftarrow \text{rand}(K, K)$

3: $B_1 = \mathbf{B}^k_m \mathbf{W}^k_{m+1} = \|\mathbf{W}^k_m\|_2$, $A_1 = \|\mathbf{A}^k_m\|_2$, $Z_1 = \|Z_m\|_2$, $L_1 = \|L_m\|_2$

4: $m = 0$

5: while $m \leq \text{maxiter}$ do

6: $m \leftarrow m + 1$

7: Update $C$:

8: $C_{m+1} = \left( B_m^T B_m + \lambda W_m^T W_m + \omega A_m^T A_m + \rho I \right)^{-1}$

9: $= \left( B_m^T X + \lambda W_m^T H + \omega A_m^T Q + \rho Z_m - L_m \right)$

10: Update $Z$:

11: $Z_{m+1} = \max \left\{ C_{m+1} \odot \frac{L_{m+1}}{Z_{m+1}} \right\}$

12: $= \min \left\{ C_{m+1} \odot \frac{L_{m+1}}{Z_{m+1}} \right\}$

13: Update $L$:

14: $L_{m+1} = \min \left\{ L_{m+1} \odot \left( C_{m+1} - Z_{m+1} \right) \right\}$

15: Update $B$, $W$, $A$

16: Compute $D_{m+1} = (C_{m+1} \odot C_{m+1}^T) \odot (1 - \mathbf{I})$

17: for $k = 1:k \leq K$, $k + 1$ do

18: $(B_{k*})_{m+1} = \frac{X^T (C_{k*})_{m+1}^T - B_m (D_{k*})_{m+1}}{\|X^T (C_{k*})_{m+1}^T - B_m (D_{k*})_{m+1}\|_2}$

19: $(W_{k*})_{m+1} = \frac{H^T (C_{k*})_{m+1}^T - W_m (D_{k*})_{m+1}}{\|H^T (C_{k*})_{m+1}^T - W_m (D_{k*})_{m+1}\|_2}$

20: $(A_{k*})_{m+1} = \frac{Q^T (C_{k*})_{m+1}^T - A_m (D_{k*})_{m+1}}{\|Q^T (C_{k*})_{m+1}^T - A_m (D_{k*})_{m+1}\|_2}$

21: end for

22: Update the objective function:

23: $f = \|X - BC\|_F^2 + \lambda \|Y - WC\|_F^2 + \omega \|Q - AC\|_F^2 + \rho \|Z\|_F$ + $\|Z\|_F$ + $\|Z\|_F$ + $\|Z\|_F$

25: end while

26: return $B$, $W$, $A$

5) LEDL for Label Prediction: The Algorithm 4 shows our proposed LEDL algorithm for predicting the label of a testing sample.

IV. EXPERIMENTAL RESULTS

In this section, we utilize several benchmark datasets (Extended YaleB [32], CMU PIE [33], UC Merced Land Use [34], AID [35], Caltech101 [36] and USPS [37]) to evaluate the
Algorithm 4 Label Prediction of LEDL Algorithm

Input: $B \in \mathbb{R}^{D \times K}, W \in \mathbb{R}^{D \times K}, y \in \mathbb{R}^{D \times 1}, \alpha$
Output: $id(y)$
1: Code $y$ with the dictionary $B$ via $\ell_1$-minimization.
2: $\hat{s} = \arg \min_s \{ ||y - Bs||_2^2 + 2\alpha||s||_1 \}$
3: $id(y) = \max \{ W\hat{s} \}$
4: return $id(y)$

performance of our algorithm and compare it with other state-of-the-art methods such as SRC [2], LC-KSVD [12], CRC [5] and CSDL [9]. In the following subsection, we first give the experimental settings. Then experiments on these six datasets are analyzed. Moreover, some discussions are listed finally.

A. Experimental settings

For all the dataset, in order to eliminate the randomness of the experiment, the data are randomly split into the training set and the testing set 8 times, respectively. The mean of the image recognition rate are reported. And for all the experiments, we randomly select 5 samples per class for training and 10 samples per class for testing. For Extended YaleB and CMU PIE datasets, each image is cropped to $32 \times 32$, pulled into column vector, and $\ell_2$ normalized to form the raw $\ell_2$ normalized features. For UC Merced Land Use dataset, AID dataset, we use resnet model [38] to extract the features. Specifically, the layer $pool5$ is utilized to extract 2048-dimensional vectors for them. For Caltech101 dataset, we use the layer $pool5$ of resnet model and spatial pyramid matching (SPM) with two layers (the second layer include five part, such as left upper, right upper, left lower, right lower, center) to extract 12288-dimensional vectors. And finally, each of the images in USPS dataset is resized into $16 \times 16$ vectors.

For SRC, CSDL and our proposed LEDL algorithms, we use ADMM framework to solve the constrained minimization problem. Here, the penalty parameter $\rho$ in Equation (6) and the gradient $\theta$ in Equation (12) are fixed for all the experiments. We set $\rho = 1$ and initial $\theta = 0.5$, then decrease the $\theta$ in each iteration. Besides the $\rho$ and $\theta$, there are another five parameters required to adjust for our proposed LEDL algorithm. For the dictionary size $K$, we usually set it between once and twice the size of the training samples per class. $\lambda$, $\omega$, $\epsilon$ and $\alpha$ are varying between $2^{-9}$ and $2^{-14}$. The $\alpha$ in LEDL is close to the $\alpha$ in SRC. We show the detail in the following subsection.

B. Extended YaleB dataset

The Extended YaleB dataset contains 2,414 face images from 38 individuals, each having around 64 nearly frontal images under varying illumination conditions. Figure 4 shows some images of the dataset.

In addition, we set $K = 380$, $\lambda = 2^{-3}$, $\omega = 2^{-11}$, $\epsilon = 2^{-8}$ and $\alpha = 2^{-8}$ in our experiment. The experimental results are summarized in Table I. Our proposed LEDL algorithm achieves superior performance to other classical classification methods by an improvement of at least 1%. In order to further illustrate the performance of our method, we choose the first 20 classes samples as a subdataset and show the confusion matrix for LEDL in Figure 5.

C. CMU PIE Dataset

The CMU PIE dataset consists of 41,368 images of 68 individuals with 43 different illumination conditions. Each human is under 13 different poses and with 4 different expressions. In Figure 6, we list several samples from this dataset.

D. UC Merced Land Use Dataset

The UC Merced Land Use Dataset is widely used for aerial image classification. It consists of totally 2,100 land-use images of 21 classes. The dataset is collected from the United
States Geological Survey National Map of 20 U.S. regions and every image in the dataset is cropped into the size of pixels. There are some samples are showed in Figure 7.

In Table I we can see that our proposed LEDL algorithm is only similar with CRC and still outperforms the other methods. Here, we set \( K = 210, \lambda = 2^{10}, \omega = 2^{-9}, \varepsilon = 2^{-6} \) and \( \alpha = 2^{-8} \) to get the optimal result. The confusion matrix of the UC Merced Land Use dataset for all classes is shown in Figure 8.

E. AID Dataset

The AID dataset is a new large-scale aerial image dataset which can be downloaded from Google Earth imagery. It contains 10,000 images from 30 aerial scene types. In Figure 9 we show several images of this dataset.

As can be seen in Table I our proposed LEDL algorithm outperforms all the competing approaches by setting \( K = 1020, \lambda = 2^{-4}, \omega = 2^{-13}, \varepsilon = 2^{-14} \) and \( \alpha = 2^{-13} \) and achieves an improvement of at least 0.7% over other methods. Here, for the Caltech101 dataset, we also choose the first 20 classes to build the confusion matrix. To further validate the difference between our proposed LEDL algorithm and LC-KSVD algorithm, we show the confusion matrices of our proposed LEDL and LC-KSVD algorithms in Figure 11 and Figure 12 respectively.
G. USPS Dataset

The USPS dataset contains 9,298 handwritten digit images from 0 to 9 which come from the U.S. Postal System. We list several samples from this dataset in Figure 13.

![Examples of the USPS dataset](image-url)

Table 1 shows the comparison results of four algorithms and it is easy to find out that our proposed LEDL algorithm outperforms over other well-known methods by an improvement of at least 2.3%. The optimal parameters are $K = 50$, $\lambda = 2^{-4}$, $\omega = 2^{-8}$, $\varepsilon = 2^{-5}$ and $\alpha = 2^{-3}$.

H. Discussions

From the experimental results on six datasets, we can obtain the following conclusions.

(1) All the above experimental results illustrate that, our proposed LEDL algorithm is an effective and general classifier which can achieve superior performance to state-of-the-art methods on various datasets, especially on face datasets and handwritten datasets.

(2) Confusion matrices on three datasets show the classification rate per class of our proposed method. For caltech101 dataset, the results in Figure 11 and Figure 12 show that our proposed LEDL method perform much better than LCKSVD method in some classes such as beaver, binocular, brontosaurus, cannon and ceiling fan.

(3) We take YaleB dataset as the example to illustrate the influence of parameters to the experimental results. Here, we usually set the dictionary size $K$ one to two times to the number of training samples. $\varepsilon$ represents sparsity of sparse codes which is set between $2^{-4}$ and $2^{-6}$. $\lambda$ and $\omega$ all represent the coefficients of discriminative information, however $\lambda$ is much bigger than $\omega$. We usually set $\lambda$ between $2^0$ and $2^{-4}$ while $\omega$ is set $2^{-10}$ to $2^{-14}$. $\alpha$ is close to the regularization parameter of SRC which is usually set $2^{-6}$ to $2^{-10}$.

V. Conclusion

In this paper, we propose a Label Embedded Dictionary Learning(LEDL) algorithm. Specifically, we introduce the $\ell_1$-norm regularization term to replace the $\ell_0$-norm regularization term of LC-KSVD which can help to avoid the NP-hard problem and find optimal solution easily. Furthermore, we propose to adopt ADMM algorithm to solve $\ell_1$-norm optimization problem and BCD algorithm to update the dictionary. Besides, extensive experiments on six well-known benchmark datasets have proved the superiority of our proposed LEDL algorithm.

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