To reduce the risk of voltage violation after gas station networks (GSNs) are attacked, this study investigates an inter-area mobile charging strategy of plug-in hybrid electric vehicles (PHEVs) to decrease the charging load by taking full advantage of charging resources. First, considering the location of the charging station, the waiting time, and the charging fee, an inter-area mobile charging strategy of PHEVs is proposed, and a mobile charging model of PHEVs among regions is established to relieve the charging pressure. Second, the risk index is developed to analyze the risk of voltage violation in terms of the results of probabilistic load flow (PLF). Finally, the proposed strategy is tested on a modified coastal active power distribution network, and simulation results show that the charging load of PHEVs is dispersed among regions and the risk of voltage over-limit can be reduced.

Keywords: gas station network (GSN), attack, plug-hybrid electric vehicle (PHEV), mobile charging strategy, risk of voltage violation

1 INTRODUCTION

The scale of plug-in hybrid electric vehicles (PHEVs) has developed rapidly during the past several years. The high penetration of PHEVs causes the power grids to confront the operation risks (Badawy and Sozer (2017)). Moreover, with the increasing pervasion of communication networks into gas stations, gas station networks (GSNs) have been formed (Chao et al. (2018); Xie et al. (2018); Zhang et al. (2019); Du et al. (2019a)), which are also subject to cyberattacks. Once GSNs are attacked, the charging load will definitely increase with the lack of fuel in PHEVs. Due to the limited number of charging piles, it is an indisputable fact that charging resources will be scarce under GSNs that have been attacked. The increasing of the charging load can raise the voltage violation risks of grids (Li et al. (2020)). It is necessary to investigate the charging strategy of PHEVs to allocate the charging resources reasonably and the grid voltage violation risk to guarantee the safe operation of power grids.

Various charging strategies have been proposed to deal with the problems mentioned above that are caused by the PHEV charging load. Vehicle-to-grid (V2G) strategies are developed (Ko et al. (2018); Liu D. et al. (2018); Zhong et al. (2018); Zeng et al. (2018)). A battery replacement strategy has been proposed (Dong et al. (2016)), which cannot reduce the charging demand under GSNs that have been attacked. A centralized charging strategy of PHEVs has been presented to reduce voltage deviation via a series of fitness functions (Maigha and Mariesa Crow, (2014)). A PHEV smart charging strategy has been proposed to minimize the MG dependency on the main grid (Fouladi et al. (2019)). An orderly charging strategy has been investigated by considering various factors such as the
economic benefits of users (Liu A. et al. (2018)), the working status of charging piles and charging habits of users (Chen and Huang (2019)), the fluctuation of the charging load (Cai et al. (2021)), and so on. However, the surge of the PHEV charging load will lead to a long queue in the charging stations of GSNs’ attacked areas, while the public charging piles in the unattacked area may be in an idle status, as shown in Figure 1. In this regard, the strategies mentioned above are not applicable without investigating the charging optimization among the areas. Moreover, the queuing problem in charging stations is rarely considered.

It is undeniable that the increase in the PHEV charging load in the attacked area will bring another problem, in that the security of the grid operation will be damaged. Various negative effects of electric vehicles connecting to the power grid are investigated, including voltage risks, and power flow risks (Pouladi et al. (2016); Salehi and Abdolahi (2019); Nafisi et al. (2016a); Yang et al. (2020); Wang et al. (2018); Nornagoro et al. (2020)). The impact of electric vehicle access on the voltage drops and power loss of power grids has been analyzed without establishing a relevant index (Pouladi et al. (2016); Salehi and Abdolahi (2019); Nafisi et al. (2016a)). The voltage over-limit and overload risk caused by electric vehicles connecting to the power grid have been presented from the perspective of components and networks (Yang et al. (2020)). The risks caused by the charging and discharging of electric vehicles to microgrids have been evaluated, from the aspects of expected lack of power supply, expected load reduction frequency, and load reduction probability (Wang et al. (2018)). The load loss ratio and overload risks are analyzed (Li et al. (2021); Du et al. (2019b)). The studies mentioned above only focus on the impact on the voltage level of power grids under the normal operation of GSNs. Even so, the risks are seldom quantified (Pouladi et al. (2016); Salehi and Abdolahi (2019); Nafisi et al. (2016a)). Due to the coincidence of the charging peak of the electric vehicle and the daily load peak (Nafisi et al. (2016b)), the surge of the PHEV charging load in the attacked area could greatly reduce the grid voltage, especially during the peak charging period.

To address these problems under GSNs that have been attacked, this study analyzes the risk of voltage violation while using a mobile charging strategy of regional PHEVs. The main contributions of this study are as follows:

1) Considering the location of charging stations, waiting time, and charging fee, an inter-area mobile charging strategy of PHEVs is proposed to choose the optimal charging station. A PHEV mobile charging model is then established to disperse the charging stress in the attacked region.

![FIGURE 1](image.png) | Schematic diagram of the unbalanced charging load.
2) Considering the probability and severity of voltage violation, an index of the voltage violation risk is developed to analyze the impact of PHEV mobile charging on distribution networks under GSNs’ attacked regions.

The rest of this article is organized as follows. Section 2 describes the PHEV mobile charging model based on the mobile charging strategy, including the PHEV charging model and the queue charging model. Section 3 presents the index of the voltage violation risk based on the results of probabilistic load flow (PLF). In section 4, simulation results are shown in detail, followed by discussion in section 5.

2 PROBABILITY MODEL OF PLUG-IN HYBRID ELECTRIC VEHICLE MOBILE CHARGING

The driving mode of PHEVs can be divided into the combustion mode and the electric mode depending on different driven powers. In the traditional charging model of PHEVs, the electric energy required to charge is determined by the daily driving distance and operating status (OS). The OS is the ratio of the distance traveled in the electric mode to the daily driving mileage. Once GSNs are attacked, the PHEV will be more dependent on the electric power, as presented in Figure 2. Moreover, the length of queuing in the charging stations may be longer due to the limitation of charging piles.

To solve the problem mentioned above, a mobile charging strategy is designed by considering the queue length, the charging fee, and the geographical location of each charging station. Then, the PHEV mobile charging model is established based on the mobile charging strategy.

2.1 Mobile Charging Strategy of Plug-In Hybrid Electric Vehicles With Gas Station Networks Attacked

Due to the limited number of charging piles, it is undeniable that there will be a charging congestion in charging stations located in GSNs’ attacked areas, while there are some charging piles available in other charging stations. In this regard, this section proposes a mobile charging strategy for PHEVs between regions by considering charging fee, moving distance, and queuing time, as shown in Figure 3.

First, the expected charging fees for the $i$th PHEV in different charging stations need to be calculated as follows:

$$Q_i' = \frac{E_i'}{\kappa'} \times c_{\text{station}}$$  \hspace{1cm} (1)

where $c_{\text{station}}$ is the charging unit price, $E_i'$ is the power required to fully charge the $i$th PHEV, and $\kappa'$ is the charging efficiency. Charging unit prices of different charging stations vary due to the station scales, electricity prices, and so on. It should be noted that the charging unit prices are varied during peak charging periods and other periods. Moreover, the commercial charging stations will charge additional service fees in addition to the basic charging price. The total charging fee for each PHEV varies according to the electric energy required to charge and the charging unit price.

Next, considering the distance between the charging stations and the queue length of each charging station at the time instant $t$, the comprehensive index of each charging station can be calculated as follows:

$$\text{Index} = \alpha \times Q_i' + \beta \times d_{i-s} + \gamma \times t_{\text{wait}}^i$$  \hspace{1cm} (2)

where $\alpha, \beta, \gamma$ are the weight coefficients, $d_{i-s}$ is the distance between the charging stations, and $t_{\text{wait}}^i$ is the waiting time of the $i$th PHEV. $\alpha, \beta, \gamma$ are decided by PHEV users. For example, some users pay more attention to fees, so the weight of the charging fees is relatively large.

Comparing the indicators of each charging station, the optimal charging station is selected as follows:

$$\text{best} = \min (\text{Index}_A, \text{Index}_B, \text{Index}_C, \ldots),$$  \hspace{1cm} (3)

where $\text{Index}_A, \text{Index}_B, \text{Index}_C$ are the comprehensive index of charging stations A, B, and C, respectively, calculated using Eq. 2.

The optimal charging station for the $i$th PHEV at the time instant $t$ is obtained using Eqs 2, 3. PHEV users are encouraged to choose the optimal charging station to reduce the waiting time and charging cost. If the current charging station is the optimal charging station, the PHEV waits in place; otherwise, the $i$th PHEV moves to the optimal charging station.

2.2 Mobile Charging Model of Plug-In Hybrid Electric Vehicles Under GSNs Attacked

According to the mobile charging strategy proposed above, the mobile charging model of PHEVs is established. In the PHEV
charging model with GSNs that have been attacked in the study by Li et al. (2020), the mismatch between the quantity of charging piles and PHEVs is ignored. There may be no idle charging piles when a PHEV arrives, especially during peak periods, as shown in Figure 4. Therefore, it is profitable for PHEV users to carry out the mobile charging.

To establish a PHEV charging model under GSNs that have been attacked, the daily commuting distance of PHEVs is analyzed. Through the statistical data analysis, the probability characteristic of the daily commuting distance of the vehicle conforms to the lognormal distribution (Li and Zhang (2012); Villanueva et al. (2011)). The probability density function (PDF) of the daily commuting distance for the $i$th PHEV is expressed as follows:

$$f(x_i) = \frac{1}{x_i \sigma_D \sqrt{2\pi}} e^{-\frac{(\ln x_i - \mu_X)^2}{2 \sigma_D^2}},$$

(4)

where $x_i$ is the daily commuting distance of the $i$th PHEV, and $\mu_X$ and $\sigma_D$ represent the mean and the variance of the lognormal distribution, respectively.

According to the driving distance in the combustion mode, the fuel consumption of the day can be obtained as follows:

$$g_{ai} = x_{gn} \times g_{ai} = (1 - OS_i) x_i \times g_{ai},$$

(5)

where $g_{ai}$ is the fuel consumption per kilometer of the $i$th PHEV, $OS_i$ is the OS of the $i$th PHEV, and $x_{gn}$ is the driving distance in the combustion mode.

When GSNs are unavailable, the driving distance in the combustion mode can be calculated using the following:

$$x'_i = \begin{cases} (1 - OS_i) x_i, & (1 - OS_i) x_i \times g_{ai} < g_{ar} \\ g_{ar} / g_{ai}, & (1 - OS_i) x_i \times g_{ai} \geq g_{ar} \end{cases},$$

(6)

where $g_{ar}$ is the remaining fuel in the fuel tank of the $i$th PHEV, which is assumed to obey a uniform distribution. It is worth noting that if the amount of remaining fuel can satisfy the commuting needs of the day, the distance traveled in the combustion mode is determined by the daily travel distance and the OS; otherwise, the distance traveled in the combustion mode depends on the amount of remaining fuel.

The driving distance in the electric mode $x'_e$ can be calculated from the daily travel distance $x_i$ and the driving distance in the combustion mode $x'_i$ as follows:

$$x'_e = x_i - x'_i = \begin{cases} OS_i \times x_i, & (1 - OS_i) x_i \times g_{ai} < g_{ar} \\ x_i - g_{ar}, & (1 - OS_i) x_i \times g_{ai} \geq g_{ar} \end{cases}.$$
where \( c_i \) is the electric energy consumed by the PHEV per kilometer and the unit is kWh/km, \( C \) is the battery capacity and the unit is kWh, and \( AER_i \) represents the maximum distance that can be traveled in the electric mode when the battery is fully charged.

The charging duration of the \( i \)-th PHEV is obtained as follows:

\[
T_i = \frac{E'}{P_i \kappa'} \tag{10}
\]

where \( P_i \) is the charging power, and \( \kappa' \) is the charging efficiency.

For the convenience of modeling, it is assumed that the end of the last trip for PHEVs is the beginning of charging and that PHEVs are only charged once a day (Tian et al. (2010); Clement-Nyns et al. (2010)). As stated by statistics, the time of the start of charging conforms to the normal distribution, and its PDF can be expressed as follows:

\[
f(t'') = \begin{cases} 
\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t'' - \mu_i)^2}{2\sigma^2}}, & (\mu_i - 12) < t'' \leq 24 \\
\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t'' + 24 - \mu_i)^2}{2\sigma^2}}, & 0 < t'' \leq (\mu_i - 12) 
\end{cases} \tag{11}
\]

where \( t'' \) is the time instant of the start of charging for the \( i \)-th PHEV; \( \mu_i \) and \( \sigma_i \) represent the mean value and standard deviation of the starting time of charging, respectively. The time for each PHEV to reach the charging station is independent.

According to the first come first service (FCFS) queuing principle, the charging priority is set depending on the arrival time of PHEVs. To determine whether the PHEV arriving at time instant \( t \) can be charged immediately, it is necessary to know the working status of the charging pile at time instant \( t \). The working status of the charging pile at time instant \( t \) can be expressed as follows:

\[
State = \begin{cases} 
0, 0 \leq N(t) < N_{\text{charging}} \\
1, N(t) \geq N_{\text{charging}} 
\end{cases} \tag{12}
\]

where \( N(t) \) is the number of PHEVs in the charging station at time instant \( t \), and \( N_{\text{charging}} \) denotes the amount of charging piles. The State value of 0 indicates that there are charging piles in the idle state; the State value of one indicates that all charging piles are in the working state.

Then, the remaining working time of all charging piles is sorted in ascending order at each moment. When the State value at time instant \( t \) is 1, the queue duration of the \( i \)-th PHEV, \( t_{\text{wait}} \), is determined by the remaining working time of the charging piles and the required charging duration of PHEVs waiting in the queue. If there are charging piles in an idle status, the PHEV that arrives first will be arranged for charging; otherwise, it will continue to wait until a fully charged PHEV leaves.

According to the mobile charging strategy, if the current charging station is the optimal charging station, the ending time of charging is \( t_i + T_i + t_{\text{wait}} \). When the ending time of charging \( t_i + T_i + t_{\text{wait}} \) is less than 24, the charging load is given by the following:

\[
P^w_i = \begin{cases} 
P_i, 0 \leq t < t_i + T_i + t_{\text{wait}} \\
0, 1 \leq t < t_i + T_i + t_{\text{wait}} < t \leq T 
\end{cases} \tag{13}
\]

otherwise, the charging load is as follows:

\[
P^w_i = \begin{cases} 
P_i, 0 \leq t < t_i + T_i + t_{\text{wait}} \\
0, 1 \leq t \leq t_i + T_i + t_{\text{wait}} \tag{14}
\end{cases}
\]

If the current charging station is not the optimal charging station, the moving time is calculated as follows:

\[
t_{\text{move}} = \frac{d_{\text{gas}}}{v'} \tag{15}
\]

where \( v' \) is the moving speed of the \( i \)-th PHEV. In this study, all PHEVs are assumed to drive at a constant speed.

Then the arrival time of the target charging station is as follows:

\[
t_{\text{arrive}} = t + t_{\text{move}} \tag{16}
\]

However, considering the remaining power of the PHEV including the remaining fuel and remaining power, the maximum travelable distance is obtained as follows:

\[
d_{\max}^i = \max \left( \frac{\text{gas}_i}{V}, \frac{\text{SOC}_i \times C}{e_{\text{Traction}}} \right). \tag{17}
\]
When \( d_{\text{max}}^i > d_{\text{max}}^j \), it is possible for the PHEV to move to the selected target station. Conversely, the next best charging station is selected, that is, the optimal charging station is selected within the allowable range of the maximum driving distance.

The \( i \)-th PHEV may also be required to queue in the arrival station. There will be a new waiting time \( t_{\text{wait},2}^i \) calculated using the remaining working time of the charging piles and the required charging duration of PHEVs waiting in the queue in the arrival station. For the convenience of modeling, the consumption of electric power is neglected, that is, the charging duration \( T_i^0 \) remains the same.

When the ending time of charging \( t_{\text{arrival}}^i + t_{\text{wait},2}^i + T_i^0 \) is less than 24, the charging load in the optimal charging station is given as follows:

\[
P_w^i = \left\{ \begin{array}{ll} P_{\text{station}}^i, & 0.1 \leq t_{\text{arrival}}^i + t_{\text{wait},2}^i + T_i^0 \leq T_i^0 < t_{\text{arrival}}^i + t_{\text{arrive},2}^i + T_i^0 \leq t \leq T \\ 0, & \text{otherwise} \end{array} \right. \tag{18}
\]

otherwise, the charging load is as follows:

\[
P_w^i = \left\{ \begin{array}{ll} P_{\text{station}}^i, & 0.1 \leq t_{\text{arrival}}^i + t_{\text{wait},2}^i \leq t \leq T \end{array} \right. \tag{19}
\]

Then, the total charging load of all PHEVs in a single charging station is as follows:

\[
P_{\text{station},k} = \sum_{i=1}^{N_{\text{total}}} P_w^i \tag{20}
\]

where \( N_{\text{total}} \) is the total number of vehicles charged in a period.

### 3 Risk Index of Voltage Violation Based on Probabilistic Load Flow

Probabilistic load flow (PLF) was proposed by Borkoaka in 1974 to solve the uncertainty in the power system and reflect the influence of random changes of various factors on the operation of the system. Considering the randomness of the distributed power generation and the PHEV load probability model in the grid, the PLF algorithm based on the \( 2m + 1 \) point estimation method (PEM) is employed to solve the voltage, and the risk of voltage violation is analyzed.

#### 3.1 Correlation Transformation

To deal with the correlation of the input variables of PLF (wind speed, light intensity, daily load, etc.), Natraf transformation and elementary transformation (ET) are applied to transform the input variables in the correlated non-normal random vector space (CSNRVS) into the independent standard normal random vector (ISNRSV) for the PEM (Li et al. (2020)). Wind speed, light intensity, and daily load variables can be described by Weibull distribution, Beta distribution, and normal distribution, respectively (Bilir et al. (2015); Cao et al. (2018); Ran and Miao (2016)). The PHEV charging load adopts the mobile charging model established above.

#### 3.1.1 Natraf Transformation

Natraf transformation is used to transform the variables in the CNNRVS into the correlated standard normal random vector space (CSNRVS). It should be noted that Natraf transformation is applicable for variables known by the marginal distribution function and the correlation coefficient matrix. Specific steps are as follows.

As for input vector \( X \), it is assumed that the cumulative distribution function (CDF) of the input variable \( x_i \) \((i = 1, 2, \cdots, n)\) is known. A random vector \( Z \) in the correlated standard normal space is introduced. The CDF is obtained as follows:

\[
\begin{bmatrix} \Phi(z_i) = F(x_i), \\
\end{bmatrix} \tag{21}
\]

where \( \Phi(\cdot) \) and \( \Phi^{-1}(\cdot) \) are the CDF and the inverse CDF of standard normal variables, respectively.

#### 3.1.2 Elementary Transformation

After the Natraf transformation, the random vector \( Z \) in the CSNRVS is obtained, which needs to be transformed into the ISNRVS by ET transformation. ET transformation requires the use of two lemmas:

Lemma 1 (Knapp (2007)) It is assumed that \( R_Z \) is an \( n \)-dimensional non-negative real symmetric matrix. There is always an \( n \)-dimensional invertible matrix \( Q \) so that \( R_Z \) can be converted to a diagonal matrix \( Q^T R_Z Q \) using the following formula:

\[
Q^T R_Z Q = R_Z^* \tag{22}
\]

Lemma 2 (Bauer (1996)) If there is an \( n \)-dimensional invertible matrix that conforms to Eq. 23, then Eq. 24 is also satisfied, as follows:

\[
(Y_1, Y_2, \cdots, Y_n) = (Z_1, Z_2, \cdots, Z_n)\text{diag}(\sigma_{Z_1}^{-1}, \sigma_{Z_2}^{-1}, \cdots, \sigma_{Z_n}^{-1})Q. \tag{23}
\]

\[
\text{COV}(Z) = \left(\text{diag}(\sigma_{Z_1}^{-1}, \sigma_{Z_2}^{-1}, \cdots, \sigma_{Z_n}^{-1})Q\right)^T \text{COV}(Z)\left(\text{diag}(\sigma_{Z_1}^{-1}, \sigma_{Z_2}^{-1}, \cdots, \sigma_{Z_n}^{-1})Q\right). \tag{24}
\]

where \( \text{COV}(Z) \) is the covariance of \( Z \), \( \text{COV}(Y) \) is the covariance of \( Y \), and \( \sigma_{Z_i} \) is the standard deviation of \( z_i \).

The correlation coefficient matrix of the correlated standard normal random vector \( R_Z \) satisfies the following formula:

\[
R_Z = \left(\text{diag}(\sigma_{Z_1}^{-1}, \sigma_{Z_2}^{-1}, \cdots, \sigma_{Z_n}^{-1})\right)^T \text{COV}(Z)\text{diag}(\sigma_{Z_1}^{-1}, \sigma_{Z_2}^{-1}, \cdots, \sigma_{Z_n}^{-1}). \tag{25}
\]

The vector \( Z \) obeys the normal distribution, and the components have correlation.According to the definition of the covariance matrix, it can be known that \( \text{COV}(Z) \) is a non-negative real symmetric matrix and \( \text{diag}(\sigma_{Z_1}^{-1}, \sigma_{Z_2}^{-1}, \cdots, \sigma_{Z_n}^{-1}) \) is an invertible matrix. It can be seen that \( R_Z \) is a non-negative real symmetric matrix (31 (?)).

According to Lemma 1, combining Eq. 25, the following result is obtained:

\[
\left(\text{diag}(\sigma_{Z_1}^{-1}, \sigma_{Z_2}^{-1}, \cdots, \sigma_{Z_n}^{-1})Q\right)^T \text{COV}(Z)\left(\text{diag}(\sigma_{Z_1}^{-1}, \sigma_{Z_2}^{-1}, \cdots, \sigma_{Z_n}^{-1})Q\right) = R_Z^* \tag{26}
\]

According to Lemma 2, the random vector \( Y \) can be obtained. Comparing Eq. 24 with Eq. 26, it can be seen that \( \text{COV}(Y) \) is a
The original space where CSNRVS is transformed into \( X \) in the ISNRVS.

3.2 Probabilistic Load Flow Based on 2m + 1 PEM

The Monte Carlo and PE methods are commonly used to solve PLF. Although the Monte Carlo method is highly accurate, it is computationally intensive. Compared with the Monte Carlo method, the PEM can not only reduce computational burden but also achieve high accuracy by using the generated input variable samples, which becomes a good alternative to the Monte Carlo method. There are several different PEM approaches; the 2m + 1 PEM is widely used in PLF because it involves less calculation and higher accuracy than other PE methods (Che et al. (2020); Xia et al. (2016)). The 2m + 1 PEM is also called the 3m method. For m input variables, 3m estimated points are constructed while m points correspond to the same estimated point vector, so the 3m PEM is transformed to the 2m + 1 PEM. The specific method is as follows.

The estimated value \( y_i \) of the \( i \)-th input variable \( Y_i \) can be obtained as follows:

\[
y_{i,k} = \mu_y + \xi_{i,k} \sigma_y, k = 1, 2, 3,
\]

(27)

where \( \mu_y \) and \( \sigma_y \) are the mean and deviation of \( Y_i \). \( \xi_{i,k} \) is the position measurement coefficient of the estimated value, as shown below:

\[
\begin{align*}
    \xi_{i,k} &= \frac{\gamma_i}{2} + (-1)^{k-1} \sqrt{\lambda_i - \frac{3}{4} \gamma_i^2}, \quad k = 1, 2, 3 \\
    \xi_{i,3} &= 0
\end{align*}
\]

(28)

where \( \gamma_i \) and \( \lambda_i \) are the skewness coefficient and the kurtosis coefficient of \( y_i \). \( \xi_{i,3} = 0 \) indicates that \( y_{i,3} \) is the mean value of \( y_i \).

When other input variables take the mean value, the variable \( y_i \) is replaced by each estimate \( y_{i,k} \) in turn. The original sample matrix \( Y \in \mathbb{R}^{m \times m} \) is reduced to an estimated point matrix \( Y' \in \mathbb{R}^{(2m+1) \times m} \). After the construction of the estimates is completed, the estimated point matrix \( Y' \in \mathbb{R}^{(2m+1) \times m} \) is transformed from the ISNRVS to the original space \( X' \in \mathbb{R}^{(2m+1) \times m} \) by inverse ET and inverse Nataf transformation.

According to the analysis above, the input vector \( X' \in \mathbb{R}^{(2m+1) \times m} \) is obtained, including the charging load of PHEVs, daily load, wind speed, light intensity, and other information. Then the deterministic load flow is operated 2m + 1 times. To the end, the output vector \( W = (W_1, W_2, \cdots, W_{2m+1}) \) composed of branch power flows and nodal voltages is obtained.

The \( p \)-th moment \( W \) can be expressed as follows:

\[
E(W^p) = \sum_{i=1}^{m} \left[ \omega_{i,1}(W_{2i-1})^p + \omega_{i,2}(W_{2i})^p \right] + \omega_{2m+1}(W_{2m+1})^p,
\]

(29)

where \( E \) represents the mean operator, and \( \omega_{ij} \) is the probability concentration. The mean and standard deviation of the output variable can be expressed as follows:

\[
\mu_w = E(W),
\]

(30)

\[
\sigma_w = \sqrt{E(W^2) - [E(W)]^2}.
\]

(31)

The specific procedures of PLF are designated in Algorithm 1:

**Algorithm 1: Procedures of PLF**

**Input:** \( X_w, X_{pg}, X_L, X_{PHEV} \)

**Output:** \( Vol \)

1: \((Z_w, Z_{pg}) = \text{Nataf} (X_w, X_{pg})\)
2: \((Y_w, Y_{pg}, Y_L, Y_{PHEV}) = \text{ET} (Z_w, Z_{pg}, X_L, X_{PHEV})\)
3: \(Y \leftarrow [Y_w, Y_{pg}, Y_L, Y_{PHEV}]\)
4: for \( i = 1:m\) \n5: \( y_{ij} = \text{Skewness} (Y_i)\)
6: \( \lambda_{ij} = \text{Kurtosis} (Y_i)\)
7: \( \xi_{i,k} = \text{Calculate} \) according to (28) \n8: \( \mu_{ij} = \text{Mean} (Y_i)\)
9: \( \sigma_{ij} = \text{Var} (Y_i)\)
10: for \( k = 1:3\) \n11: \( y_{i,k} = \mu_{ij} + \xi_{i,k} \sigma_{ij}\)
12: end
13: \((Y'_w, Y'_{pg}, Y'_L, Y'_{PHEV}) = \text{Ponit} (y_{i,k})\)
14: end
15: \((X'_w, X'_{pg}, X'_L, X'_{PHEV}) = \text{Inverse} (Y'_w, Y'_{pg}, Y'_L, Y'_{PHEV})\)
16: \((P_w, P_{pg}) = \text{Generation} (X'_w, X'_{pg})\)
17: for \( i = 1:2m+1\) \n18: \( v = \text{Flow} (P_w, P_{pg}, X'_L, X'_{PHEV})\)
19: \( Vol = Vol + v\)
20: end

**Input and Output:** \( X_w, X_{pg}, X_L, \) and \( X_{PHEV} \) are wind speeds, light intensity, basic load, and charging load of the PHEV. \( Vol \) is the voltage of the power grid.
3.3 Risk Index of Voltage Violation

The voltage violation risk of the \(i\)th node in the \(k\) period is calculated using the voltage violation probability \(P(V^k_i)\) and the corresponding load loss \(P_{load}(V^k_i)\) as follows:

\[
R(V^k_i) = P(V^k_i)P_{load}(V^k_i),
\]  

where \(V^k_i\) is the voltage of the \(i\)th node in the \(k\) period.

The probability of voltage violation can be obtained from the voltage distribution of PLF results. In general, the voltage distribution obeys normal distribution. The probability of voltage violation can be expressed as follows:

\[
P(V^k_i) = F_i \left( V^k_i \notin \left[ V^k_{imin}, V^k_{imax} \right] \right), I = \left| V^k_i - \mu_{V^k_i} \right| < 3\sigma_{V^k_i},
\]

where \(V^k_{imax}\) and \(V^k_{imin}\) indicate the upper and lower limits of the node voltage and are set as 1.05 and 0.95 of the rated voltage, respectively. \(\mu_{V^k_i}\) and \(\sigma_{V^k_i}\) are the mean and standard deviation of the nodal voltage. \(I\) is the confidence level when the sample data of the voltage follow the “3\(\sigma\) principle.” \(F\) is the CDF of \(V^k_i\). The probability of voltage violation includes the probability of exceeding the upper limit and exceeding the lower limit.

The severity of the voltage violation is presented as follows:

\[
\text{Sev}(V^k_i) = \begin{cases} 
\left| \frac{V^k_i - V^k_{imax}}{V^k_{imax}} \right|, & V^k_i > V^k_{imax} \\
\left| \frac{V^k_i - V^k_{imin}}{V^k_{imin}} \right|, & V^k_i < V^k_{imin} 
\end{cases}
\]

where \(\text{Sev}(V^k_i)\) and \(\text{Sev}(V^k_i)\) indicate the severity index when the node voltage exceeds the upper limit and the lower limit, respectively.

The corresponding load loss can be derived using the severity of the voltage violation. The relationship between severity and load loss is linear within a certain range, as shown in Figure 5. The critical value of the upper severity is 14.29% \((1.2 - 1.05)/1.05)\). The critical value of the lower severity is 15.79% \((0.8 - 0.95)/0.95)\). The action threshold of overvoltage protection is set as 1.2 times the rated voltage value. The threshold of overvoltage protection is set as 1.05 times the rated voltage value. The critical value of the lower severity is the same.

The charging load increases after GSNs are attacked, and it is an indisputable fact that the voltage level decreases and the risk of voltage violation increases. PHEVs in the attacked area are motivated to perform mobile charging using the proposed mobile charging strategy. The charging stress in the...
FIGURE 7 | Load variation of charging station A. (A) Charging load in the first 24 h (1st day). (B) Charging load in the third 24 h (3rd day) (A–M, GSNs are attacked and PHEVs move to charge; A–N, GSNs are attacked and PHEVs do not move to charge; N–N, GSNs work normally and PHEVs do not move to charge).

FIGURE 8 | Load variation of charging station C. (A) Charging load in the first 24 h (1st day). (B) Charging load in the third 24 h (3rd day).
FIGURE 9 | Load variation of charging station B. (A) Charging load in the first 24 h (1st day). (B) Charging load in the third 24 h (3rd day).

FIGURE 10 | Load variation of charging station D. (A) Charging load in the first 24 h (1st day). (B) Charging load in the third 24 h (3rd day).
attacked area is then abated, which reduces the risk of voltage violation.

4 SIMULATIONS

The modified coastal active power distribution network is used as a case study to analyze the impact of the proposed mobile charging strategy on the voltage (Strugar et al. (2013)), as shown in Figure 6. Three wind farms and two photovoltaic systems are installed at nodes 3, 4, 6, 11, and 17, respectively. Four charging stations (A, B, C, and D) are considered in nodes 9, 13, 15, and 20. It is assumed that there are 14,000 residents in the area and the quantities of vehicles per capita is 291 vehicles per 1,000 people (RS.Statistical Office (2016)). Moreover, the penetration rate of PHEVs is set as 35%; it indicates that the number of PHEVs is about 1,426 cases in this simulation. Here, 10 distinct results are calculated, with Table 1 presenting the load variations of stations B and D. It can be seen from Figure 7 that the charging stress of charging stations A and C is dispersed, and the fully loaded duration of charging stations A and C decreases, as shown by the red curves in Figures 7 and 8. It is noteworthy that all charging piles are involved, to work in stations A and C during 17:00–21:00 in case of 72 h of attack. In this case, it does not illustrate that mobile charging has not occurred. As long as there are PHEVs in the queue, the charging stations are still fully loaded. Overall, the congestion in stations A and C is suitably improved.

4.1 The Impact of Mobile Charging Strategy on Charging Load

In general, there is not a lot of surplus of charging piles, especially when GSNs are unavailable. This phenomenon adds congestion to the charging station, as shown in Figures 7 and 8. It is obvious that the charging loads at charging stations A and C increase due to the unavailability of GSNs. Moreover, the charging stations A and C are fully loaded for a long time during peak charging periods. In other words, PHEVs arriving at the peak charging period will have to queue for a long time.

Moreover, the fully loaded duration of the charging station is prolonged by an increase during the attacked time, as depicted in Figures 7B and 8B. As can be observed, the charging load in case of A-N almost remains the highest during 17:00–24:00, which means that the more severe charging congestion occurs in the charging station. The charging jams and greater dependence of PHEVs on electric energy proves that mobile charging is essential.

According to this mobile charging strategy, a portion of PHEV users who originally charged at charging stations A and C are encouraged to move to stations B and D for charging. Hence, the comprehensive economic and time benefits of users are protected. Consequently, the charging stress of charging stations A and C is dispersed, and the fully loaded duration of charging stations A and C decreases, as shown by the red curves in Figures 7 and 8. It is noteworthy that all charging piles are involved, to work in stations A and C during 17:00–21:00 in case of 72 h of attack. In this case, it does not illustrate that mobile charging has not occurred. As long as there are PHEVs in the queue, the charging stations are still fully loaded. Overall, the congestion in stations A and C is suitably improved.

Tables 1, 2, and 3 present the load variations of stations B and D. It is clear that the charging loads of charging stations B and D increase during the peak charging periods via mobile charging of PHEVs. Additionally, the longer the GSNs are attacked, the greater the number of PHEVs going to charging stations B and D. As a result, the charging congestion occurs in stations B and D. The worst case is presented in Figure 8B. In fact, although there is a queue at station D, it is still the optimal choice for some PHEV users. According to Figures 7–10, it is obvious that the mobile charging strategy is effective for relieving the charging stress in attacked areas.

Each PHEV has its own waiting time and charging fee, so there are 1,426 cases in this simulation. Here, 10 distinct results of the waiting time and charging fee before and after mobile charging are presented in Tables 1, 2, and 3. It can be seen from Table 1 that after PHEVs move to the optimal charging station, the waiting time of four PHEVs is zero, while the waiting time of

| TABLE 1 | Comparison of the waiting time before and after mobile charging. |
|------------------|------------------|
| Case             | Waiting time (min) |
| Before mobile charging | 47, 36, 29, 30, 29, 35, 24, 34, 30 |
| After mobile charging | 17, 5, 0, 1, 0, 4, 3, 0, 30 |

| TABLE 2 | Comparison of the charging fee before and after mobile charging. |
|------------------|------------------|
| Case             | Charging fee (RMB) |
| Before mobile charging | 12, 16.25, 26.75, 47.75, 24.25, 24.5, 22.5, 38, 40.75, 35.75 |
| After mobile charging | 14.4, 19.5, 32.1, 54.9, 29.1, 29.4, 27, 18.5, 48.9, 42.9 |

| TABLE 3 | Comparison of the comprehensive index before and after mobile charging. |
|------------------|------------------|
| Case             | Comprehensive index |
| Before mobile charging | 22.4, 20.08, 26.75, 27.75, 24.25, 21.35, 22.5, 28, 20.75, 15.75 |
| After mobile charging | 20.62, 18.35, 22.35, 24.9, 19.1, 18.92, 18.36, 22.5, 18.9, 12.9 |
six PHEVs decreases significantly. Meanwhile, it is found from Table 2 that the charging fee of one PHEV decreases significantly, while the charging fee of nine PHEVs increases slightly after mobile charging. It is seen from Table 3 that although the charging fee increases, the waiting time is shortened, and the comprehensive index is better. Therefore, PHEVs choose mobile charging instead of waiting in place. When the current charging station is optimal, PHEVs will wait in place for charging, so the charging cost and waiting time will not change.

4.2 The Impact of Mobile Charging Strategy on the Voltage Violation Risk

For further analysis, the risk of voltage violation is presented to illustrate the effect of the mobile charging strategy.
Considering the similarity of the load variation shown above, only node 13 (connected by the charging station) and node 15 (connected by charging station C) are taken as the examples to analyze.

Figure 11 depicts the voltage comparison of node 15. A voltage drop can be observed when GSNs are unavailable. In addition, as the attack time is prolonged, the voltage drops more. However, due to the limitation of the charging piles, the voltage does not drop excessively. It is obvious that the duration that the voltage is at a lower level is longer due to the extension of the fully loaded periods. For the mobile charging strategy, the voltage of node 15 at peak periods rises up in the first 24h of attack (first day). Consequently, the risk of voltage violation decreases significantly as shown in Figure 12A. In the case of the first 72h of attack, no matter whether PHEVs move or not, station C is always fully loaded during 17:00–21:00, resulting in a little change in the voltage and the risk of voltage violation. After 21:00, the voltage increases and the risk of voltage violation decreases because of a decrease in the charging load of station C, as shown in Figures 11B, 12B. In conclusion, the charging load of PHEVs is dispersed using the mobile charging strategy, and the voltage level increases and the risk of voltage violation is reduced.

Figure 13 shows the voltage and the violation risk of node 13. After mobile charging of PHEVs, it is obvious that the voltage during the peak charging period decreases because of the increase in the charging demand. However, the risk of voltage violation is always at 0, as shown in Figure 13B, denoting that the risk is not affected by mobile charging of PHEVs. In fact, although the voltage of node 13 drops on using the mobile charging strategy, it does not cause the voltage limit to be exceeded, that is, the increasing load is within its acceptable range.

5 CONCLUSION

For a surge of the PHEV charging load under the regional GSNs being attacked, this study investigates the risk of voltage violation after the mobile charging of PHEVs. In view of the phenomenon of charging congestion under GSNs that have been attacked, the mobile charging strategy is proposed to encourage users to charge movably. The charging fee, queueing time, and location of the charging station are emphasized, and the mobile charging model is developed to relieve the charging stress of charging stations in the attacked areas. Then the PLF based on the 2m + 1 PEM is used to obtain the voltage distribution, and a risk index of voltage violation is set up to analyze the impact of PHEV mobile charging.

The validation of the proposed mobile charging strategy is performed on the modified coastal active power distribution network. The results demonstrate that the charging load in the attacked area of GSNs is significantly reduced, which further indicates that PHEVs are encouraged to move using the mobile charging strategy, and the full-loaded time is sharply decreased. After mobile charging, the charging stress of charging stations located in the attacked area is dispersed and the voltage level rises. Therefore, it indicates that the risk of voltage violation can be reduced using the proposed mobile charging strategy. It is worth mentioning that the security constraint of charging stations located in unattacked areas is neglected. However, the excessive movement of PHEVs may lead to an increase in the risk of voltage violation in normal areas, although the risk in normal areas has always been zero in this work. Therefore, more security constraints can be investigated in future work.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material; further inquiries can be directed to the corresponding authors.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.
**FUNDING**

This research was funded by the National Science Foundation of China (Grant nos. 61773253, 61803252, and 61633016), the Project of Science and Technology Commission of Shanghai Municipality, China (20JC1414000, 19500712300, and 19510750300) and Shanghai Talent development Foundation.

**ACKNOWLEDGMENTS**

This work was supported by the Shanghai Key Laboratory of Power Station Automation Technology, Shanghai University. The authors would like to thank reviewers for their careful work. Their most helpful and constructive comments improve the quality of the article.

**REFERENCES**

Badawy, M. O., and Sozer, Y. (2017). Power Flow Management of a Grid Tied PHEV Battery System for Electric Vehicles Charging. *IEEE Trans. Ind. Appl.* 53, 1347–1357. doi:10.1109/TIA.2016.2633526

Bauer, H. (1996). Probability Theory (Gerke Gmbh). Berlin, Germany.

Bilir, L., Imam, M., Devrim, Y., and Albostan, A. (2015). Seasonal and Yearly Wind Speed Distribution and Wind Power Density Analysis Based on Weibull Distribution Function. *Int. J. Hydrogen Energ.* 40, 15301–15310. doi:10.1016/j.ijhydene.2015.04.140

Cai, Z., Shu, H., Yang, B., and Shan, J. (2021). Research on Regional Pv Capacity Planning for Wind Farm Expansion Based on Nasa Data. *PESGM.2020.9362638*.

Chao, H., Li, J., and Peng, D. (2018). A Charging Strategy for Phevs Based on Maximum Employment of Renewable Energy Resources in Microgrid. In 2019 IEEE International Conference on Environment and Electrical Engineering and 2019 IEEE Industrial and Commercial Power Systems Europe (IEEEIC/I CPS Europe). Genova, Italy. 11–14 June 2019. IEEE. 1–5. doi:10.1109/IEEEIC.2019.8783742

Chao, H., Li, J., and Peng, D. (2018). Model Predictive Control of Batch Processes Based on Two-Dimensional Integration Frame. *Nonlinear Anal. Hybrid Syst.* 28, 75–86. doi:10.1016/j.nahs.2017.11.002

Knapp, A. (2007). Advanced Algebra (Birkhauser). Berlin, Germany: Springer.

Knapp, A., Pack, S., and Leung, V. C. M. (2018). Mobility-aware Vehicle-To-Grid Control Algorithm in Microgrids. *IEEE Trans. Intell. Transport. Syst.* 19, 2165–2174. doi:10.1109/TITS.2018.2816935

Li, G., and Zhang, X.-P. (2012). Modeling of Plug-In Hybrid Electric Vehicle Charging Demand in Probabilistic Power Flow Calculations. *IEEE Trans. Smart Grid* 3, 492–499. doi:10.1109/TSG.2011.2172643

Li, X., Dong, J., Du, D., Wu, L., and Fei, M. (2020). Impact of Phev in Active Distribution Network under Gas Station Network Attack. *ISA Trans.* 104, 192–203. doi:10.1016/j.isatra.2019.02.024

Li, X., Jiang, C., Du, D., Wang, R., Fei, M., Li, X., et al. (2021). Optimization and Control of Cyber-Physical Power Systems under Dual-Network Interactive Cascading Failure. *Control. Eng. Pract.* 111, 104789. doi:10.1016/j.conengprac.2021.104789

Liu, A., Yuan, X., and Li, J. (2018a). Research on Orderly Charging Strategy of Electric Vehicle under Urban Distribution Network. *Electr. Meas. Instrumentation* 55, 55–55+74.

Liu, D., Zhong, Q., Zhong, W., Yang, Y., and Liu, G. (2018b). Modeling and Control of a V2g Charging Station Based on Synchronverter Technology. *Csee Jps* 4, 326–338. doi:10.17775/CSEEJIPES.2016.01430

Maigha and Crow, M. (2014). Economic Scheduling of Residential Plug-In (Hybrid) Electric Vehicle (PHEV) Charging. *Energies* 7, 1876–1898. doi:10.3390/en7041876

Nafisi, H., Askarian Abyaneh, H., and Abedi, M. (2016a). Energy Loss Minimization Using Phevs as Distributed Active and Reactive Power Resources: a Convex Quadratic Local Optimal Solution. *Int. Trans. Electr. Energy Syst. 26* (6), 1287–1302. doi:10.1002/etep.2134

Nafisi, H., Agah, S. M. M., Askarian Abyaneh, H., and Abedi, M. (2016b). Two-stage Optimization Method for Energy Loss Minimization in Microgrid Based on Smart Power Management Scheme of Phevs. *IEEE Trans. Smart Grid* 7, 1268–1276. doi:10.1109/TSG.2015.2480999

Nornagoro, A., Banjar-Nahor, K. M., Rahman, F. S., Rahmani, R., and Hariyanto, N. (2020). Impact of Combined Electric Vehicle and P V Parking Lot to Reliability of Distribution Network Systems and its Mitigations. In 2020 International Conference on Technology and Policy in Energy and Electric Power (ICT-PER), Bandung, Indonesia, 23-24 Sept. 2020. IEEE, 283–288. doi:10.1109/ICT-PER.2020.9249941

Pouladi, J., Banna Sharifian, M. B., and Soleymani, S. (2016). Determining Charging Load of Phevs Considering Hvac System and Analyzing its Probabilistic Impacts on Residential Distribution Network. *Electric Power Syst. Res.* 141, 300–312. doi:10.1016/j.epsr.2016.08.008

Ran, X., and Miao, S. (2016). Three-phase Probabilistic Load Flow for Power System with Correlated Wind, Photovoltaic and Load. *IET Generation, Transmission. Distribution* 10, 3093–3101. doi:10.1049/iet-gtd.2016.0424

RS-Statistical Office (2016). Statistical Yearbook (Statistical Office of the Republic of Serbia).

Salehi, I., and Abdolahian, A. (2019). Optimal Scheduling of Active Distribution Networks with Penetration of Phev Considering Congestion and Air Pollution Using Dr Program. *Sustainable Cities Soc.* 51, 101709. doi:10.1016/j.scs.2019.101709

Strugar, L., Katic, V. A., and Milanovic, J. V. (2013). Generic Model of Coastal Distribution Network for Power System Harmonics Studies. *Przegląd Elektrotechniczny* 89, 149–155.

Tian, L., Shi, S., and Jia, Z. (2010). A Statistical Model for Charging Power Demand of Electric Vehicles. *Power Syst. Technology* 34, 126–130. doi:10.13335/j.1000-1667.2010.11.010

Villanueva, D., Pazos, J. L., and Feijoo, A. (2011). Probabilistic Load Flow Including Vehicle-to-Grid Electric Vehicle under Urban Distribution Network. *IEEE Trans. Sustain. Energ.* 7, 1133–1144. doi:10.1109/TSTE.2016.2520481
Xie, X., Yue, D., and Peng, C. (2018). Relaxed Real-Time Scheduling Stabilization of Discrete-Time Takagi-Sugeno Fuzzy Systems via an Alterable-Weights-Based Ranking Switching Mechanism. *IEEE Trans. Fuzzy Syst.* 26, 3808–3819. doi:10.1109/TFUZZ.2018.2849701

Yang, X., Yi, Y., Zhang, Y., Mo, Y., Huang, T., and Xie, X. (2020). Operation Risk Analysis of Electric Vehicle Integrated to Distribution Network Based on Weighted Distribution Entropy. *Automation Electric Power Syst.* 044, 171–179.

Zeng, M., Leng, S., Zhang, Y., and He, J. (2018). Qoe-aware Power Management in Vehicle-To-Grid Networks: A Matching-Theoretic Approach. *IEEE Trans. Smart Grid* 9, 2468–2477. doi:10.1109/TSG.2016.2613546

Zhang, D., Liu, L., and Feng, G. (2019). Consensus of Heterogeneous Linear Multiagent Systems Subject to Aperiodic Sampled-Data and Dos Attack. *IEEE Trans. Cybern.* 49, 1501–1511. doi:10.1109/TCYB.2018.2806387

Zhong, W., Yu, R., Xie, S., Zhang, Y., and Yau, D. K. Y. (2018). On Stability and Robustness of Demand Response in V2g mobile Energy Networks. *IEEE Trans. Smart Grid* 9, 3203–3212. doi:10.1109/TSG.2016.2629378

**Conflict of Interest:** The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

**Publisher’s Note:** All claims expressed in this article are solely those of the authors and do not necessarily represent those of their affiliated organizations, or those of the publisher, the editors and the reviewers. Any product that may be evaluated in this article, or claim that may be made by its manufacturer, is not guaranteed or endorsed by the publisher.

**Copyright © 2021 Li, Zhang and Du. This is an open-access article distributed under the terms of the Creative Commons Attribution License (CC BY). The use, distribution or reproduction in other forums is permitted, provided the original author(s) and the copyright owner(s) are credited and that the original publication in this journal is cited, in accordance with accepted academic practice. No use, distribution or reproduction is permitted which does not comply with these terms.**