Boundary String Field Theory at One-loop

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(Dated: October 29, 2018)

Abstract

We discuss the open string one-loop partition function in tachyon condensation background of a unstable D-brane system. We evaluate the partition function by using the boundary state formulation and find that it is in complete agreement with the result obtained in the boundary string field theory. It suggests that the open string higher loop diagrams may be produced consistently by a closed string field theory, where the D-brane plays a role of source for the closed string field.
I. INTRODUCTION

The boundary string field theory (BSFT) [1, 2] has been proved to be a useful tool to study the dynamics of the D-branes in string theory. It is this framework where the tachyon condensation on unstable D-brane systems has been discussed extensively in recent works [3, 4, 5]. The BSFT deals with the disk partition function of the open string theory. The relationship between the BSFT action, $S$ and the disk partition function $Z$ is given as

$$S = \left( 1 + \beta^i \frac{\partial}{\partial g^i} \right) Z$$

where $g^i$ are the couplings of the boundary interactions and $\beta^i$ are the corresponding world-sheet $\beta$-functions. If a specific form of tachyon profile is chosen, the BSFT action reduces to the effective action for the tachyon field. The BSFT has been extended to the case of superstring theories [6], where the BSFT action coincides with the disk partition function $Z$. The results obtained in those studies confirm Sen’s conjecture; as the tachyon condensation develops, the unstable open string vacuum spontaneously rolls down to the stable closed string vacuum.

The open string one-loop partition function is also studied recently in refs. [7, 8, 9, 10]. Assuming that the BSFT action may coincide with the partition function beyond the disk diagram in superstring theory, we find that the effective action for tachyon may receive corrections from the open string loop diagrams. The effective action for tachyon at the lowest order is obtained from the disk partition function with a specific form of the tachyon profile on the boundary of the disk. In the string theory we expect that there are some corrections to the disk partition function. While keeping fixed the boundary of the disk and the tachyon profile on it, we may add more boundaries and handles to the string world-sheet diagram. They may give corrections to the disk partition function, thus the effective action for the tachyon field. We may propose that the total partition function may be written schematically as

$$Z = \sum_{b=1, g=0} \int dm Z[b, g; m]$$

where $b$ denotes the number of boundaries and $g$, the number of handles. We collectively denote the moduli parameters by $[m]$. One of the boundaries of the string world sheet, on which the partition function is evaluated, is fixed as in the disk diagram.

The leading correction to the disk partition function may come from the open string one-loop diagram which has two boundaries. One may employ two different schemes to calculate the one-loop partition function. One is to use the open string channel, which
adopts the open string Green function on an annulus \([1, 8, 9]\), and the other is to use the closed string channel \([1, 12, 13]\), which adopts the boundary state formulation \([14]\). In the second scheme the geometry of the string world sheet is given by a cylinder. A of figure \([1]\) depicts the one-loop diagram in the open string channel. If one once calculates the Green function on the annulus, the partition function is readily obtained as

\[
\frac{d}{dy} \ln Z(u, a) = -\frac{1}{8\pi} \int_0^{2\pi} d\sigma \langle X^2 \rangle, \quad Z = \int \frac{da}{a} Z(u, a)
\]  

(3)

where \(u\) is the tachyon profile parameter and \(a\) is the modular on a cylinder. As depicted by B of figure \([1]\), the same diagram can be viewed in the closed string channel as follows; a closed string appears from a D-brane and propagates to another (or the same) D-brane. The initial and the final states of the closed string are described by boundary states \(|B\rangle\) and the one-loop partition function is obtained as

\[
Z(u, \tau) = \langle B | \exp \left[ -\tau (L_0 + \tilde{L}_0) \right] | B \rangle, \quad Z = \int d\tau Z(u, \tau).
\]  

(4)

As is well known, the partition functions obtained in two different schemes agree on-shell thanks to the open-closed string duality. However, it remains to be checked whether two different schemes yield the same result in the presence of the tachyon condensation, i.e., off-shell. Since the boundary state formulation entirely depends on this equivalence, it is important to clarify this issue. At the tree level, it has been shown that the disk partition function in the BSFT appears as the normalization factor of the boundary state. As we shall see that the open string one-loop partition function calculated in the boundary state formulation, Eq.(4) is in complete agreement with that obtained in the BSFT, Eq.(3). It suggests that the open string partition functions may be reproduced in a closed string field theory, where the D-brane plays a role of source for the closed string field. Constructing such a theory, we may have a consistent framework to calculate the total partition function Eq.(2) in a systematic way, thus the corrections to the effective action for tachyon.

In the next section, we discuss the bosonic string theory at open string one-loop in the background of the tachyon condensation and show by an explicit calculation that two different schemes yield the same result. In section 3, we extend it to the supersymmetric theory. Section 4 concludes the paper with some discussions.

II. BOSONIC STRING FIELD THEORY AT ONE-LOOP

We start with the action

\[
S = S_{\text{bulk}} + S_{\text{bndy}} = \frac{1}{8\pi} \int_{\Sigma} d^2x \sqrt{h} h^{ij} \partial_i X^\mu \partial_j X_\mu + \frac{1}{8\pi} \int_{\partial \Sigma} d\sigma \mathcal{V}.
\]  

(5)
\( S_{\text{bndy}} \) describes the interactions between the string and the background, which may be expanded as

\[
\mathcal{V} = T(X) + A_\mu(X) \partial_\sigma X^\mu + C_{\mu\nu} \partial_\sigma X^\mu \partial_\sigma X^\nu + \cdots. \tag{6}
\]

In this paper, we only consider the tachyon field background, which take the following simple form

\[
\mathcal{V} = T(X) \equiv \sum_{i=1}^{p} u_i (X^i)^2. \tag{7}
\]

**Open string channel - Annulus**

At the tree level, the world sheet is a unit disk on which the boundary action is given

\[
S_{\text{bndy}} = \frac{1}{8\pi} \int_{0}^{2\pi} d\sigma \left( uX^2 \right). \tag{8}
\]

Throughout this paper we are only concerned with the string variables in the direction where the tachyon condensation takes place. So we omit the space-time indices hereafter. The geometry of the world sheet for the open string one-loop diagram is an annulus with \( a \leq r \leq b \), which has two boundaries. It leads us to introduce the following boundary action

\[
S_{\text{bndy}} = \frac{1}{8\pi} \left[ \int_{0}^{2\pi} d\sigma (u_a X^2)|_a + \int_{0}^{2\pi} d\sigma (u_b X^2)|_b \right]. \tag{9}
\]

One may consider an alternative form of the boundary action, which makes use of the induced metric on the world sheet

\[
S'_{\text{bndy}} = \frac{1}{8\pi} \left[ \int_{0}^{2\pi} a \, d\sigma (u_a X^2)|_a + \int_{0}^{2\pi} b \, d\sigma (u_b X^2)|_b \right]. \tag{10}
\]
However, if we take $S'_{\text{bndy}}$ as the boundary action, we would encounter difficulties such as the breakdown of the Fischler-Susskind mechanism \[15\] as pointed out in ref. \[10\]. Hence, the theory may become singular. If we turn on the tachyon condensation, the system is no longer conformally invariant. But we expect that the system is invariant under the following transformation, $z \to ab/\bar{z}$, which maps the inner boundary of the annulus onto the outer boundary and vice versa. It is obvious that $S'_{\text{bndy}}$ does not respect this symmetry while $S_{\text{bndy}}$ does. For these reasons we take $S_{\text{bndy}}$ Eq.(9) as the boundary action. As we shall see, the partition function with the boundary action $S_{\text{bndy}}$ in the open string channel agrees with that obtained in the boundary state formulation.

It follows from the action (5) that the world sheet Green’s function

$$\triangle_z G(z, w) = -4\pi \delta^{(2)}(z - w),$$

satisfies the following conditions

$$\left( a \frac{\partial}{\partial r} - u_a \right) G_B(z, w)|_{r=a} = 0,$$

$$\left( b \frac{\partial}{\partial r} + u_b \right) G_B(z, w)|_{r=b} = 0,$$

which read in the complex coordinates $(z, \bar{z}) = (re^{i\sigma}, re^{-i\sigma})$ as

$$\partial_z \bar{\partial}_z G(z, w) = -2\pi \delta^{(2)}(z - w),$$

with the boundary conditions

$$(z \partial + \bar{z} \bar{\partial} - u_a)G_B(z, w)|_{r=a} = 0,$$

$$(z \partial + \bar{z} \bar{\partial} + u_b)G_B(z, w)|_{r=b} = 0.$$ (14)

An explicit form of the Green function is obtained by solving this boundary problem \[7\]. The partition function is then determined by Eq.(3) up to an integration constant $c_B$

$$Z_B(u_a, u_b; a, b) = c_B T_p^2 \left[ u_a + u_b - \frac{u_a u_b}{2} \ln \left( \frac{a^2}{b^2} \right) \right]^{-1/2}$$

$$\prod_{n=1}^{\infty} \left[ (n + u_a)(n + u_b) - (n - u_a)(n - u_b) \left( \frac{a}{b} \right)^{2n} \right]^{-1}$$ (15)

where $T_p$ is the tension of the Dp-brane. Here we set $\alpha' = 2$. We consider the case where both ends of the open string are attached on the same D-brane, $u_a = u_b = u$. Since we are concerned with the corrections to the disk partition function, one of the boundaries should be same as the boundary of the disk. Thus, choosing $b = 1$, we obtain

$$Z_B(u, a) = c_B T_p^2 \sqrt{\frac{1}{2u - u^2 \ln a}} \prod_{n=1}^{\infty} \frac{1}{(n + u)^2 - (n - u)^2 a^{2n}}.$$ (16)
Closed string channel - Cylinder

In the boundary state formulation the open string one-loop diagram is depicted as a cylinder diagram, where a closed string comes out from the D-brane and propagates onto the D-brane. The interaction between the D-brane background and the string is encoded in the initial and final states of the closed string, which are termed the boundary states. Construction of the boundary state begins with defining the boundary states \( \{ \langle X \rangle \} \) which form a basis for the closed string states

\[
\hat{X} \langle X \rangle = X \langle X \rangle, \quad \hat{X}(\sigma) = \hat{x}_0 + \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \left\{ (a_n + \tilde{a}_n^\dagger) e^{2i n \sigma} + (a_n^\dagger + \tilde{a}_n) e^{-2i n \sigma} \right\}
\]

\[
X(\sigma) = x_0 + \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \left( x_n e^{2i n \sigma} + \bar{x}_n e^{-2i n \sigma} \right)
\]

where \( \sigma \in [0, \pi] \) and \( [a_n, a_m^\dagger] = [\tilde{a}_n, \tilde{a}_m^\dagger] = \delta_{nm} \). For a given boundary action \( S_{\text{bndy}} \) the boundary state is constructed as

\[
\langle B \rangle = T_p \int D[x, \bar{x}] e^{i S_{\text{bndy}}[x, \bar{x}]} |x, \bar{x}\rangle \tag{18}
\]

Choosing \( S_{\text{bndy}} \) Eq. (8) as the boundary action in order to describe the tachyon background we obtain the boundary state \([11]\) as

\[
\langle B_B \rangle = T_p \prod_{n=1}^{\infty} \left( 1 + \frac{u}{n} \right)^{-1} \int dx d\bar{x} e^{-\frac{1}{4} \bar{x} u x} \exp \left\{ a_n^\dagger \left( \frac{n-u}{n+u} \right) \tilde{a}_n^\dagger \right\} |x\rangle \tag{19}
\]

where \( |x\rangle \) is defined as

\[
\hat{x}_0 |x\rangle = x |x\rangle, \quad a_n^\dagger |x\rangle = \tilde{a}_n^\dagger |x\rangle = 0, \tag{20}
\]

where \( n = 1, 2, 3, \ldots \) (Note that the tachyon profile parameter \( u \) in the ref. \([11]\) differs from that in the ref. \([7, 8]\) by \( 1/4\pi \).

We can calculate the partition function by making use of Eq.(11)

\[
Z_B(u, \tau) = T_p^2 \langle B_B \rangle \exp \left[ -\tau (L_0 + \tilde{L}_0) \right] \langle B_B \rangle
\]

\[
= T_p^2 \prod_{n=1}^{\infty} \left( 1 + \frac{u}{n} \right)^{-2} \int dx d\bar{x}' \langle x'| e^{-\frac{1}{4} \bar{x}' u x} \exp \left\{ a_n^\dagger \left( \frac{n-u}{n+u} \right) \tilde{a}_n^\dagger \right\} e^{-\frac{1}{4} \bar{x}' u x} |x\rangle
\]

\[
= T_p^2 \sqrt{\frac{2\pi}{u + u^2 \tau / 2}} \prod_{n=1}^{\infty} \left( 1 + \frac{u}{n} \right)^{-2} \frac{1}{1 - e^{-2n\tau} \left( \frac{n-u}{n+u} \right)^2} \tag{21}
\]

where

\[
N = \sum_{n=1}^{\infty} n a_n^\dagger a_n, \quad \tilde{N} = \sum_{n=1}^{\infty} n \tilde{a}_n^\dagger \tilde{a}_n.
\]
Comparing the partition function Eq.(21) in the closed string channel with that Eq.(16) in the open string channel and using the zeta function regularization
\[
\prod_{n=1} \frac{1}{n + \epsilon} = \exp \left\{ \frac{d}{ds} \left( \zeta(s, \epsilon) - \epsilon^{-s} \right) \right\}_{s=0} = \frac{\epsilon \Gamma(\epsilon)}{\sqrt{2\pi}},
\] (22)
we find that they coincide if we choose
\[\tau = -\ln a, \quad c_B = 4\pi^{3/2}.\] (23)

III. SUPERSTRING FIELD THEORY AT ONE-LOOP

In this section, we extend our previous discussion on equivalence of two schemes to the superstring theory. The bulk action in the supersymmetric theory is given as the superstring world sheet action
\[
S_{\text{bulk}} = \frac{1}{8\pi} \int_{\Sigma} d^2z \left( \partial X^\mu \partial X + \psi \partial \psi + \bar{\psi} \partial \bar{\psi} \right),
\] (24)
where \(\psi\) and \(\bar{\psi}\) are the holomorphic and antiholomorphic fermionic fields. The interaction between the tachyon background and the superstring is described by the following boundary action
\[
S_{\text{bndy}} = \frac{1}{8\pi} \int_{\partial\Sigma} d\sigma \left[ (T(X))^2 + (\psi \partial T) \frac{1}{\partial_{\sigma}} (\psi \partial T) + (\bar{\psi} \partial T) \frac{1}{\partial_{\sigma}} (\bar{\psi} \partial T) \right].
\] (25)
In order to produce the same tachyon profile in the bosonic sector as in the bosonic string theory we choose \(T(X) = uX\). The total world sheet action for the superstring is
\[S = S_{\text{bulk}} + S_{\text{bndy}}.\] (26)

Open string channel - Annulus

The bosonic part is exactly the same as in the bosonic string theory discussed in the last section. The superstring has two different fermionic sectors, depending on the boundary conditions for the fermion fields; R-R sector and NS-NS sector. In this paper we only consider the NS-NS sector. The equations for the fermion Green functions, following from the superstring action Eq.(26) are
\[
\bar{\partial} G_F(z, w) = -i \sqrt{z \bar{w}} \delta^{(2)}(z - w),
\]
\[
\partial \bar{G}_F(z, \bar{w}) = +i \sqrt{\bar{z} \bar{w}} \delta^{(2)}(\bar{z} - \bar{w})
\] (27)
\[
\left( 1 - iy_a \frac{1}{\partial_{\sigma}} \right) G_F \bigg|_{r=a} = \left( 1 + iy_a \frac{1}{\partial_{\sigma}} \right) \bar{G}_F \bigg|_{r=a},
\]
\[
\left( 1 + iy_b \frac{1}{\partial_{\sigma}} \right) G_F \bigg|_{r=b} = \left( 1 - iy_b \frac{1}{\partial_{\sigma}} \right) \bar{G}_F \bigg|_{r=b}
\] (28)
where $G(z, w)$ and $\tilde{G}(\tilde{z}, \tilde{w})$ are the holomorphic and antiholomorphic fermion Green function, and $y \equiv u^2$.

Explicit expressions of the fermion Green functions can be found in [8]. Then a simple integration over $y$ yields the partition function in the superstring theory Eq.(3). Up to a constant $c_S$ the partition function is obtained as [8]

$$Z_S(y, a) = c_S(2T_p)^2 \sqrt[2]{\frac{1}{2y - y^2 \ln a}} \prod_{n=1}^{\infty} \frac{(n + 2y)^2 - (n - 2y)^2 a^n}{[(n + y)^2 - (n - y)^2 a^n]^2}.$$ (29)

The normalization of the one-loop partition function taken here is consistent with refs. [6, 16].

Closed string channel - Cylinder

In the superstring theory the boundary state may be given as $|B⟩ = |B_B⟩ \otimes |B_F⟩$. Here $|B_B⟩$, describing the interaction between the bosonic degrees of freedom of the superstring and the tachyon condensation background, is already given by Eq.(19). The interaction between the fermion degrees of freedom and the tachyon condensation background is encoded in $|B_F⟩$. Construction of the fermionic part of the boundary state is similar to that of the bosonic part. It begins with defining the following eigenstate in the NS-NS sector

$$\frac{1}{\sqrt{2}} (\psi + \tilde{\psi}) |\theta, \bar{\theta}\rangle = \sum_{r=1/2} \left( \theta_r e^{2ir\sigma} + \bar{\theta}_r e^{-2ir\sigma} \right) |\theta, \bar{\theta}\rangle.$$ (30)

Here

$$\psi = \sqrt{2} \sum_{r=1/2} \left( b_r e^{2ir\sigma} + b^\dagger_r e^{-2ir\sigma} \right),$$

$$\tilde{\psi} = \sqrt{2} \sum_{r=1/2} \left( \tilde{b}_r e^{2ir\sigma} + \tilde{b}^\dagger_r e^{-2ir\sigma} \right),$$

$$\{b_r, b^\dagger_s\} = \delta_{rs}, \quad \{\tilde{b}_r, \tilde{b}^\dagger_s\} = \delta_{rs}.$$ The fermionic part of the boundary state $|B_F⟩$ is written in terms of $|\theta, \bar{\theta}\rangle$ as

$$|B_F⟩ = \int D[\theta, \bar{\theta}] e^{S_{\text{bndy}}}[\theta, \bar{\theta}] |\theta, \bar{\theta}\rangle.$$ (31)

\footnote{This expression differs by a factor $4^y$ from that of ref. [9] due to a different choice of regularization scheme. Similar to the disk case, this difference does not affect any physical quantities. However, we may easily remove this factor by choosing an alternative regularization scheme as follows: In ref. [9], the divergences of bosonic Green function and fermionic Green function cancel each other and leave a constant $(-8 \ln 2)$, which leads to the $4^y$ factor in the partition function. If we follow the regularization scheme in [9]

$$\langle X(\theta)X(\theta)⟩_{\partial\Sigma} = \lim_{\epsilon \to 0} \left[ X(\theta)X(\theta) - \ln(1 - e^{i\epsilon}) - \ln(1 - e^{-i\epsilon}) \right]$$

for both bosonic and fermionic Green functions separately, we will obtain the expression (29).}
For the background of the tachyon condensation we have

\[ S_{\text{bdy}}^F = \frac{y}{4\pi} \int d\sigma \theta(\partial_\sigma)^{-1}\theta \]

\[ \theta(\sigma) = \sum_{r=1/2} \left( \theta_r e^{2i\sigma} + \bar{\theta}_r e^{-2i\sigma} \right). \]

A simple algebra yields the explicit form of \(|B_F\)

\[ |B_F\rangle = \prod_{r=1/2} \left( 1 + \frac{y}{r} \right) \exp \left\{ \tilde{b}_r^\dagger \left( \frac{r-y}{r+y} \right) b_r \right\} |0_F\rangle \]

where

\[ b_r^\dagger |0_F\rangle = \tilde{b}_r^\dagger |0_F\rangle = 0. \]

As in the bosonic sector the contribution of the fermionic sector to the partition function is written as the expectation value of the superstring propagator

\[ Z_F(y, \tau) = \langle B_F | \exp \left[ -\tau (L_F^F + \tilde{L}_F^F) \right] |B_F\rangle \]

\[ = \prod_{r=1/2} \left( 1 + \frac{y}{r} \right)^2 \langle 0_F | \exp \left\{ \tilde{b}_r^\dagger \left( \frac{r-y}{r+y} \right) b_r \right\} e^{-\tau (N_F + \tilde{N}_F)} \]

\[ \exp \left\{ \tilde{b}_r^\dagger \left( \frac{r-y}{r+y} \right) \tilde{b}_r^\dagger \right\} |0_F\rangle \]

\[ = \prod_{r=1/2} \left( 1 + \frac{y}{r} \right)^2 \left[ 1 - e^{-2\tau \left( \frac{r-y}{r+y} \right)^2} \right] \]

where

\[ N_F = \sum_{r=1/2} r b_r^\dagger b_r, \quad \tilde{N}_F = \sum_{r=1/2} r \tilde{b}_r^\dagger \tilde{b}_r \]

Thus, the total partition function may be written as

\[ Z_S(y, \tau) = 2^2 Z_B(y, \tau) Z_F(y, \tau) \]

\[ = 4T^2 \sqrt{\frac{2\pi}{y+y^2\tau/2}} \prod_{n=1}^{1/2} \left( 1 + \frac{2y}{n} \right)^2 \left( 1 + \frac{y}{n} \right)^{-4} \frac{1 - e^{-2n\tau \left( \frac{n-2y}{n+2y} \right)^2}}{\left[ 1 - e^{-2n\tau \left( \frac{n-y}{n+y} \right)^2} \right]^2} \]

where we use a simple identity

\[ \prod_{r=1/2} f(r) = \frac{\prod_{n=1} f(n/2)}{\prod_{n=1} f(n)}. \]

At a glance, as in the bosonic case, we find that, the partition function evaluated in the boundary state formulation Eq.(36) is the same as the partition function evaluated in the BSFT Eq.(29). Comparison between two results yields

\[ \tau = -\ln a, \quad c_S = 4\pi^{3/2}. \]
IV. DISCUSSION

In this paper we show that the open string one-loop partition function in the BSFT can be calculated in the boundary state formulation as the expectation value of the closed string propagator between the boundary states in the presence of tachyon condensation, off-shell. It is this equivalence that the boundary state formulation is entirely based on. It is also this equivalence that Polchinski [17] utilized to explore the fundamental properties of the D-branes. However, use of this equivalence has been limited to on-shell. Recently the boundary state formulation has been proven to be useful to discuss the tachyon condensation. So, it becomes important to see that this equivalence holds for higher loop partition function if we adopt the boundary state formulation to explore the corrections to the effective tachyon action. By an explicit calculation we show that it holds for the open string one-loop partition function. Extrapolating from the present work, we may propose that all higher loop partition functions may be generated by the following closed string field theory[18], where the D-brane plays a role of source for the closed string field [19]

\[ S = \frac{1}{2} \int \Phi K \Phi + \frac{q^2}{3!} \int \Phi \Phi \Phi + \kappa \int \Phi J, \quad (39) \]

where

\[ K = \hat{L}_0 + \bar{\hat{L}}_0, \]
\[ \Phi = \Phi[x, \bar{x}], \]
\[ J = e^{iS_{\text{bdy}}[x, \bar{x}]} \]

The source term in the closed string field action is written as

\[ \kappa \int \Phi J = \kappa \langle \Phi | B \rangle, \quad (40) \]

and the disk partition function appears as a normalization factor of the boundary state when it is written in a form of coherent state

\[ |B\rangle = \int D[x, \bar{x}]e^{iS_{\text{bdy}}[x, \bar{x}]}|x, \bar{x}\rangle \]
\[ = Z_{\text{Disk}} \exp \left( a^\dagger M a \right) |0\rangle. \quad (41) \]

As we have seen that the open string one-loop partition function corresponds to the closed string propagator. It is interesting to see that all open string higher loop diagrams are generated as tree level diagrams of this closed string field theory. In addition to the open string loops, we find that the closed string field action Eq. (39) generates closed string loops, i.e., the string diagrams with handles. Figure 2. depicts a string diagram with
one handle. It should be appropriate also to take into account the string diagrams with handles when we calculate corrections to the disk partition function. The closed string field theory we propose here Eq.\((39)\) generates the string diagrams with arbitrary number of boundaries and handles, of which boundaries are attached on the D-brane. Hence, it may serve as a consistent framework to calculate corrections Eq.\((2)\) to the disk partition function systematically. One of the good features of the closed string field theory is that we only need to deal with the weak coupling regime in order to discuss the tachyon condensation in contrast to the open string field theory \([20]\).

We conclude this paper with a remark on the infrared fixed point limit. In the infrared fixed limit, \(u \to \infty\), the unstable Dp-brane turns into a lower dimensional D-brane. It has been shown at the tree level by explicit calculations of the partition function on a disk \([11]\). From the explicit calculation given in this paper we also observe this phenomenon at open string one-loop level. It follows from Eq.\((21)\) that the partition function of the bosonic string theory reduces to

\[
\lim_{u \to \infty} Z_B(u, \tau) = T_p^2 \frac{4\pi \sqrt{\tau}}{\sqrt{\tau}} \prod_{n=1}^{\frac{1}{1-e^{-2\pi u}}} \frac{1}{1-e^{-2\pi u}}
\]

in the infrared fixed point limit. Since the propagator of the zero mode with initial and final points fixed is given as

\[
\langle x | e^{-\tau \hat{p}^2} | x \rangle = \frac{1}{2\sqrt{\pi \tau}},
\]

the infrared fixed point limit of \(Z_B\) can be understood as

\[
\lim_{u \to \infty} Z_B(u, \tau) = (8\pi^2 T_p^2) \langle D | e^{-\tau (L_0 + \tilde{L}_0)} | D \rangle
\]

where \(|D\rangle\) corresponds to the boundary state with the Dirichlet boundary condition. Thus, the one-loop partition function of the Dp-brane reduces to that of D(p-1)-brane.
Then it implies that \( 8\pi^2 T_p^2 = T_{p-1}^2 \), i.e., the descent relation between D-brane tensions
\[
T_{p-1} = 2\pi \sqrt{\alpha'} T_p.
\]
(45)

The infrared fixed point limit of the open string one-loop partition function of the superstring Eq.(36) can be discussed in a similar way. The present work can be also extended to the noncommutative string theories [4, 11, 21].

Acknowledgements: This work is supported by a grant from Natural Sciences and Engineering Research Council of Canada. The work of TL was supported by KOSEF (995-0200-005-2) and was done during the PIMS-APCTP Summer Workshop 2001 held at Simon-Fraser University, Canada.

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