Tunable coupling of two mechanical resonators by a graphene membrane

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Coupled nanomechanical resonators are interesting for both fundamental studies and practical applications as they offer rich and tunable oscillation dynamics. At present, the coupling in such systems is mediated by a fixed geometric contact or static clamping between the resonators. Here we show a graphene-based electromechanical system consisting of two physically separated mechanical resonators – a comb-drive actuator and a suspended silicon beam – that are tunably coupled by an integrated graphene membrane. The graphene membrane, moreover, provides a sensitive electrical read-out for the two resonating systems showing 16 different modes in the frequency range from 0.4 to 24 MHz. In addition, by pulling on the graphene membrane with an electrostatic potential applied to the silicon beam, we control the coupling, quantified by the $g$-factor, from 20 kHz to 100 kHz. Our results pave the way for coupled nanoelectromechanical systems requiring controllable mechanically coupled resonators.

Keywords: Graphene, resonators, tunable coupling, NEMS, MEMS

Graphene-based nanoelectromechanical systems are promising candidates for overcoming scaling limitations of silicon-based microelectromechanical systems. Graphene, an atomically thin crystal of carbon atoms, features a high mechanical strength [1, 2], an unprecedented high carrier mobility [3], and unique electromechanical coupling mechanisms. The latter enables highly sensitive electrical read-out schemes and allow for better integration [4, 5]. Moreover, the low mass density and the high Young’s modulus [1, 7] make graphene very interesting for resonator-based sensor applications [8] including for example force sensors [9, 11], (ultra) sound detectors [12, 15] or accelerometers [16, 17], where e.g. graphene is used to hold suspended proof masses and to detect their motion. These prototype demonstrations show that graphene is truly interesting for becoming a serious functional part of next generation nanoelectromechanical resonator systems.

Resonating micro- and nanoelectromechanical systems operate in general over a wide range of frequencies, varying from the kHz to the GHz regime, very much depending on the application, including for example high-quality-factor band pass filters [18–20], signal amplifiers [21, 22], high-precision sensors (incl. biosensors) [23], or even logic gates [24]. Mechanically coupled resonators have recently attracted increasing attention thanks to their interesting dynamics [25–28], which in many applications show better performance and advanced tunability compared to single resonators [29]. The mechanical coupling between different mechanical resonators can be well-designed [30] and can be successfully used as low-noise signal amplifier [31]. Yet, up to now, the coupling has always been mediated by a fixed geometric contact or clamping between the mechanical resonators, which limits their field of application [32]. The implementation of an integrated and independent control of the mechanical coupling is, however, still a major technological challenge.

Here we show that a suspended graphene membrane can be used to strongly couple two physically separated mechanical resonators. Moreover, the membrane simultaneously provides an electrical read-out scheme for the motion of the two resonators and a mechanical coupling between the resonators that can be controlled over a wide range by an electrostatic potential.

The device was designed for applying strain to a graphene membrane in a very controlled way, while retaining the possibility of tuning independently the charge carrier density in the graphene itself (Figures 1a to 1c) [33, 34]. While investigating this device, we discovered a very general way of tuning the mechanically coupling of two different resonators via the integrated graphene membrane also used for reading out the motion of the resonators. The mechanical resonators in this study are the suspended comb-drive (CD) actuator used to strain the graphene membrane and the suspended silicon beam (SB) placed underneath the graphene membrane for tuning its charge carrier density (see Fig. 1d).

The device was fabricated by an electron-beam lithography (EBL) based structuring of a Cr/Au/Cr hard mask on a home-made silicon-on-insulator substrate consisting of 725 $\mu$m silicon, 1 $\mu$m SiO$_2$ and 2 $\mu$m highly p-doped silicon followed by a deep reactive ion etching (DRIE) step [34]. The silicon beam, which also functions as a low-lying electrostatic gate, was fabricated by interrupting the DRIE step after etching 275 nm deep followed by the deposition of an additional Cr mask before etching completely through the highly p-doped silicon layer. After removal of the Cr, a graphene/PMMA stack is
transferred on the patterned comb-drive actuator. Raman spectroscopy confirms the single layer nature of the graphene flake (Supplementary Figure 1). By an additional EBL step we partly cross-link the PMMA to clamp the graphene membrane onto the actuator on one side and to a fixed anchor on the other side (Figures 1a, b and d). Finally, the actuator with the integrated graphene membrane is released from the substrate by removing the SiO$_2$ layer with 10% hydrofluoric (HF) acid solution followed by a critical point drying (CPD) step. In the measurements presented here, the suspended graphene membrane has a length of $L = 2 \, \mu m$ and a width of $W = 3 \, \mu m$ (Supplementary Figure 1). The measurements were performed in a $^3$He/$^4$He dilution refrigerator with a base temperature around 20 mK, unless otherwise stated.

Figure 1c depicts the electrical scheme of the measured device. A potential difference $V_a$ between the asymmetrically placed fingers of the CD actuator gives rise to an electrostatic force, $F_a = \frac{1}{2} \partial_x C_a V_a^2$, that pulls the suspended comb in the $x$-direction (Figure 1h). Here, the capacitance $C_a$ and $\partial_x C_a$ are given by the zeroth and the first order term in the displacement $\delta z$ of the actuator in a series expansion of the parallel plate approximation for the capacitance between its fingers [33].

An AC potential $V_{g,ac}$ on the suspended SB at frequency $\omega/2\pi$ excites a mechanical mode in the graphene membrane. The mechanical displacement of the graphene $\delta z$ perpendicular to the membrane plane ($z$-direction, see Figure 1h) at frequency $\omega/2\pi$ modulates its conductance $G$. As $\omega/2\pi$ is usually in the MHz range, a drain-source bias $V_b$ is applied at a slightly different frequency $(\omega \pm \Delta \omega)/2\pi$ to generate a down-mixed current $I_{\Delta \omega}$ at a low, measurable frequency $\Delta \omega/2\pi$ [35]:

\[
I_{\Delta \omega} = \frac{1}{2} V_b \frac{\partial G}{\partial V_g} \left( V_{g,ac}^2 + V_g \frac{\partial_x C_g}{C_g} \delta z \right),
\]

where $V_{g}$ is the applied DC potential on the SB acting as gate, and $\partial G/\partial V_g$ is the transconductance (Supplementary Figure 2). The capacitance $C_g = 0.19 \, F$ and $\partial_x C_g = -0.70 \, nF/m$ are given by the zeroth and the first order term in $\delta z$ of a series expansion of the parallel plate approximation for the capacitance between the suspended SB and the graphene membrane.

To extract the resonance frequencies of the device, we measure $I_{\Delta \omega}$ as a function of $\omega/2\pi$. The mechanical resonances are observed as dips and peaks in $I_{\Delta \omega}$ (see Figure 1h). We measured in total sixteen resonances in the range from 0.4 to 24 MHz, which is an unusually high number considering reports on graphene resonators in the literature [6, 12, 35]. In this work, we focus on the three main resonances (labelled as I, II, and III) and corresponding close-ups are shown in Figure 1l. We fit the resonances with a nonzero-phase Lorentzian [35] to extract the resonance frequency $\omega_0/2\pi$, the quality factor $Q$, and the effective drive amplitude $A$.

To understand the physical origin of the different resonances, we extract the effective masses and spring con-
FIG. 2: (a) Down-mixing current $I_{\Delta\omega}$ as a function of $\omega/2\pi$ and $V_g$ (raw data) The dashed line highlights the resonance frequency. (b-d bottom) Frequency dependence of the resonances I, II, and III on the applied potentials $V_g$ (blue) and $V_a$ (red). The black lines are quadratic fits in accordance to the applied electrostatic force (see text). (b-d top) Schematic illustrations of the mechanical system. The black arrows indicate the main vibrating component of the corresponding resonance. The red and blue arrow indicate the direction of the force induced by $V_g^z$ and $V_a^z$, respectively.

The effective masses of resonances I and II are comparable to the estimated mass of the CD actuator when taking a density of 2329 kg/m$^3$ for the highly p-doped silicon leading to $m_{\text{CD}} = 1.79 \, \text{ng}$, in good agreement with the effective mass extracted from resonance I. The effective mass extracted from resonance II is roughly one-third of $m_{\text{CD}}$. This reduction in effective mass is comparable to the mass reduction for a doubly clamped beam [41] in its fundamental mode and thus highlights the importance of the mode shape. The effective mass $m_{\text{eff}}$ extracted from resonance III is approximately equal to the estimated mass of the suspended silicon beam, $m_{\text{SB}} = 0.048 \, \text{ng}$. This suggests that the observed resonances can be attributed to resonances of the actuator and of the silicon beam.

To proof this origin and to elucidate the mode shapes, we performed finite element calculations using COMSOL [12]. It is important to included the suspended graphene membrane in the simulations for two reasons: (i) we can only measure modes with an oscillation of the graphene membrane in the $z$-direction (see equation 1) and (ii) the spring constant $k_{\text{eff}} \sim 160 \, \text{GPa}$ for the two-dimensional Young’s modulus of graphene that is expected at cryogenic temperatures [13, 44]. The highly p-doped silicon has a Young’s modulus of $\approx 160 \, \text{GPa}$ [13, 44]. We find excellent agreement between the measured and computed resonance frequencies: the ratio between them is on average $1.04 \pm 0.06$ (see Supplementary Table 2). The top panels in Figures 2b-d schematically illustrate the main vibrating components for resonances I, II, and III. We find that resonances I and II correspond to an out-of-plane and in-plane motion of the CD actuator. Resonance III is an out-of-plane mode of the SB, which is supported by the absence of any tun-
ability with $V_a$. Details of all computed frequencies and mode shapes are provided in Supplementary Table 2 (and Supplementary Figure 4). The computed mode shapes are consistent with the observed capacitive softening. In total we detect fifteen mechanical resonances of the CD actuator and one of the suspended silicon beam.

As the observed frequency-tuning is in agreement with capacitive softening, we can use the extracted effective spring constants to determine the static displacements $\delta x$ and $\delta z$. We compute these displacements by dividing the electrostatic forces $F_a = \frac{1}{2} \partial z C_a V_a^2$ and $F_g = \frac{1}{2} \partial z C_g V_g^2$ by the spring constant of the lowest in-plane and out-of-plane mode, respectively (Supplementary Figure 5). The in-plane displacement $\delta x$ goes up to 3 nm and is in agreement with the ones extracted by Raman spectroscopy on similar devices.

To understand the increase of resonance I with applied $V_a$ in Figure 2b, we now focus on the graphene membrane. As illustrated in the top panel of Fig. 2, resonance I is dominated by a spring along the $z$-direction, i.e. an out-of-plane motion of the CD. Both the actuator and the bending stiffness of the graphene contribute to this spring. The bending stiffness of the graphene membrane, however, is known to be highly sensitive to the induced strain $\Delta \varepsilon$, and thus to a $\delta x$ displacement. As the total spring constant is known from the resonance itself, and the in-plane displacement of the actuator and the dimensions of the graphene membrane are known as well, the only free parameter is the Young’s modulus of the graphene membrane. Requiring the same in-plane actuator displacement for tuning resonance I as for tuning resonance II, we obtain an effective Young’s modulus of $Y_{\text{GD}} = 350 \pm 20 \text{ N/m}$ (see Methods and Supplementary Figure 5), which is in good agreement with values reported in the literature and supports the used value in the finite element calculations.

This type of analysis gives us a complete understanding of the mechanical behavior of the system. Let us next focus on the resonance III attributed to the suspended silicon beam. When measuring this resonance as a function of $V_a$ and $V_g$, we observe the emergence of avoided crossings, which is a clear signature for two strongly coupled modes. Figures 3a and 3b show the raw down-mixing current as function of $V_a$ for various $V_g$ values (separated by dashed lines). Remarkably, we only observe both interacting modes at the avoided crossings as shown in Figure 3. This suggests that the graphene membrane has no measurable motion in the $z$-direction for the mode with which the silicon beam is interacting. However, we can reconstruct the dependence of the mode with which the silicon beam is interacting by tracing the position of the avoided crossing as a function of $V_a$, as indicated by the black dashed parabola in Figure 3c (for more data see Supplementary Figure 6). The absence of a mode at twice or half the frequency with equivalently scaled tuning (see Figure 1b) allows us to exclude parametric effects. Moreover, the computed resonance frequency spectrum of the CD actuator shows a twist mode close to the one of the suspended SB with negligible net graphene motion (displacement) in $z$-direction. We thus attribute the avoided crossing to a strong coupling between the silicon beam and the twist mode of the CD actuator (Fig. 3d).

We extract the coupling strength between the modes, i.e. the so-called $g$-factor, $g = \omega_{\text{sep}} / 2\pi$ and linewidth $\Gamma = \omega_0 / 2\pi Q$ from each individual avoided crossing (Figure 4a). The former shows that the strong coupling between the modes is tunable with applied $V_a$, i.e. by the electrostatic force between the silicon beam and the graphene membrane (Figure 4a). The latter suggests two fully hybridized modes with equal energy transfer between them (Figure 4b). The linewidth comparison was performed on the data at $V_g = -14$ V, which show the twist mode nearly over the full $V_a$-range. The $g$-factor is set by the electrostatic softening between the SB and the graphene membrane (Figure 4b). Both the...
motion of (i) the silicon beam and (ii) the graphene membrane alter their separation, resulting not only in a shift of the resonance frequency, i.e. the well-known electrostatic softening [35], but also in the coupling between the two resonators. Following equation 2 this coupling can be characterized by an effective spring constant \( k_\text{eff} \approx \frac{1}{2} \left( \frac{\partial^2 C_\text{g}}{\partial z^2} \right) V_\text{a}^2 \) (Figure 4(b)). Then the \( g \)-factor can be estimated without making any assumptions on the mode shape by \( g = \sqrt{k_\text{eff}/m_{\text{eff}}} / 2\pi \). The effective mass \( m_{\text{eff}} \) is the one of the hybridized modes. As the mass of the suspended SB is much smaller than that of the twist mode, we set \( m_{\text{eff}} \) equal to the mass \( m_{\text{CD}} \) of the actuator. When using the estimated mass of the actuator \( m_{\text{CD}} = 1.79 \, \text{ng} \) for \( m_{\text{eff}} \) as well as the value for \( \left( \frac{\partial^2 C_\text{g}}{\partial z^2} \right) = 5.1 \, \text{mF/m}^2 \) given above, we find a remarkably good agreement with the experimentally extracted \( g \)-factor and the computed one (see dashed line in Figure 4(c)). The suspended graphene membrane thus strongly couples two physically separated resonators, and even allows for the tuning of this coupling by an applied voltage.

In conclusion, by taking advantage of the high sensitivity of graphene resonators, we implement and quantitatively validate a widely applicable coupling scheme for nanoelectromechanical systems. The coupling strength can be tuned from 20 kHz to 100 kHz and be completely switched off by an electrostatic potential. The resonators themselves are not affected by the light-weighted graphene membrane. This coupling scheme is on-chip, poses no restrictions on the choice of material for the connected masses, it is scalable by means of integrated graphene obtained via chemical vapour deposition, and can be possibly extended using other conducting two-dimensional materials instead. The presented technique enables a platform to study nonlinear dynamics at the smallest scale. For example, the tunable coupling can be used for studying the route to chaos in nonlinear dynamics by systematically measuring the orbit diagram [50]. Additionally, the presented scheme enables one to switch on and off the mechanical coupling, giving rise to read-out schemes for quantum state at a well defined time instant with minimal back-action effects at other times.

**Methods.** Young’s modulus extraction. The effective spring constant \( k_\text{eff} \) is given by the one of the actuator in parallel to the bending stiffness of the graphene membrane. The bending stiffness of the graphene membrane depends on the pre-strain \( \epsilon_0 \) [35].

For a fixed \( V_\text{g} \), we find a direct relation between the strain induced by the actuator \( \Delta \epsilon \) and the change of the out-of-plane spring constant \( \Delta k \) of the graphene membrane:

\[
\Delta k = \frac{16 Y_{2D} W}{3L} \Delta \epsilon. \tag{3}
\]

Here, \( W \) (\( L \)) is the width (length) of the suspended graphene membrane. Note that we neglect the increase in strain by pulling upon the graphene with \( V_\text{g} \) for two reasons: Firstly, \( V_\text{g} \) is constant, and secondly, the in-plane spring constant of the CD actuator (\( \sim 6.7 \, \text{N/m} \)) is much smaller than the expected in-plane spring constant of the graphene membrane (\( \sim 540 \, \text{N/m} \)), thereby minimising the strain induced with \( V_\text{g} \). We extract \( \Delta k \) and \( \Delta \epsilon \) experimentally and determine \( W \) and \( L \) from optical and scanning electron microscope images, which leaves \( Y_{2D} \) as the only free parameter. We extract \( \delta x \) from the in-plane modes of the CD actuator. The induced strain is then simply \( \Delta \epsilon = \delta x / L \). We then determine \( \Delta k \) from the observed increase in resonance frequency of the out-of-plane mode:

\[
\Delta k = k_\text{eff} \left( \frac{\omega_0(V_\text{a})^2}{\omega_0(V_\text{a} = 0 \, \text{V})^2} - 1 \right). \tag{4}
\]

In Supplementary Figure 5, we plot \( 3L \Delta k / 16 W \) as a function of \( \Delta \epsilon \), such that the slope is directly providing \( Y_{2D} \).
Associated Content  The Supporting Information is available free of charge.

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Notes  The authors declare that they have no competing financial interests.

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Supplementary Figure 1: (a) Optical image of the region of the comb-drive actuator where the graphene membrane has been transferred. The graphene membrane is placed inside the red box. (b) Raman spectrum of graphene measured at the position indicated with red cross in panel (a), showing the Raman G- and 2D-peak. The absence of a D-peak shows the good quality of the integrated graphene flake. (c) Width of the Raman 2D-peak measured in the area outlined by the red box in panel (a). The average width of the Raman 2D-peak is 26.4 cm$^{-1}$, which shows single layer nature of the integrated graphene membrane. The 2D-peak broadens or disappears in regions where cross-linked PMMA is present.

Supplementary Figure 2: Measured conductance ($G$) and transconductance ($\partial G/\partial V_g$) of the integrated graphene membrane as a function of the potential $V_g$ applied on the suspended silicon beam. (a) The conductance $G$ measured at 20 mK. The extracted transconductance $\partial G/\partial V_g$ from the measurement in panel (a) is shown in panel (b). The red lines show the smoothing splines of the (trans)conductance.
Supplementary Figure 3: Drive amplitude as a function of drive force. (a)-(c) Measured down-mixing current $I_{\Delta \omega}$ (see main manuscript) as a function of the drive force $F_d = \partial_z C_g V_a V_{ac}^g$ (for more details see main manuscript) and frequency $\omega/2\pi$ close to the resonances I, II, and III introduced in the main manuscript. By fitting each trace to a nonzero phase Lorentzian [35], we find the effective drive amplitude $A$, which we convert into a physical amplitude in nanometer with equation 1 in the main text, using the known transconductance (see Supplementary Figure 2). (d) The drive force $F_d$ as function of the extracted drive amplitude $A$ for resonances I, II, and III. The slopes are equal to the effective spring constant $k_{\text{eff}}$ of the different resonances (see labels).

Supplementary Figure 4: Mode shape of resonances III (left panel) and the twist mode of the comb-drive actuator with which it interacts (right panel). Both are extracted from a finite element analysis.
Supplementary Figure 5: (a) Estimated $\delta x$ of the comb-drive actuator as a function of $V_a$ from resonance II. (b) Estimated $\delta z$ of the comb-drive actuator as a function of $V_g$ from resonances I. (c) The black data points represent the scaled change in spring constant $\frac{3L}{16W} \Delta k$ (see methods) of resonance I as a function of the strain $\Delta \varepsilon$ induced with the comb-drive actuator. The strain $\Delta \varepsilon$ is estimated from the data in panel (a) and $\frac{3L}{16W} \Delta k$ is computed from the change in resonance frequency. The black line connects the data points and serves as a guide to the eye. The red line represents a linear fit to the black data points. The slope of the red line equals the Young’s modulus and has a value of $Y_{2D} = 350 \pm 20$ N/m. The error on the Young’s modulus is given by the fitting error.

Supplementary Figure 6: The left panel shows the raw data of the down-mixing current $I_{\Delta \omega}$ as a function of $\omega/2\pi$ and $V_a$ for $V_g = -17.5$ V (black label). This data set does not reveal any avoided crossings. The colored lines in the right panel indicate the extracted resonance frequencies for different fixed $V_g$ (black labels), including the extracted frequencies at $V_g = -17.5$ V. The dashed black line traces the dependence of the avoided crossings on $V_a$. 
Supplementary Table 1: Overview of the theoretical and experimentally determined effective masses and spring constants. The theoretical values were obtained from finite element analysis of the studied device. $k_x$ denotes the spring constant in the $x$-direction and $k_z$ in the $z$-direction. Experimentally, we determined the effective masses and spring constants (i) from equation 2 in the main text taking into account the capacitive softening (Cap. Soft.) effect and (ii) from the effective drive amplitude $A$ as a function of drive force (Driving). Based on the theoretical masses, we attribute resonances I and II to the comb-drive actuator and resonance III to the suspended silicon beam (see main text).
Supplementary Table 2: Mode shape overview. The first column gives the experimentally measured frequency $f_e$ and the second column gives the corresponding frequency $f_s$ extracted from the finite element simulation. The third column gives the squared ratio between $f_e$ and $f_s$. The fourth and fifth column give a side and top view of the mode shape, respectively. Note that the color scale for each figure in the same column is identical such that the amplitudes can be directly compared to one another. The last column specifies the observed tuning with applied potential $V_a$ to the actuator. All modes tune downwards with applied potential $V_a$ to the silicon beam underneath the graphene. Resonances I and II defined in the main manuscript at 1.461 MHz and 5.09 MHz, respectively, are highlighted in red.