A Measurement of Gas Temperatures in Galaxy Clusters using the Relativistic Sunyaev-Zeldovich Effect

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Abstract. The hot gas in clusters of galaxies creates a distinctive spectral distortion in the cosmic microwave background (CMB) via the Sunyaev-Zeldovich (SZ) effect. To first order, the shape of the spectral distortion is fixed, but relativistic corrections (rSZ) introduce a dependence on the gas temperature. In this paper, we extract fluxes from a sample of 47 clusters in the \textit{Planck} maps and make a \(\sim 5\sigma\) detection of the rSZ effect by measuring the scaling relation between the SZ amplitude (a proxy for cluster mass) and the cluster temperature. Our measurement requires no prior knowledge of the clusters’ gas temperatures and hence is an example of how the rSZ can be used to probe fundamental astrophysics. We find excellent agreement between our measurement and temperatures obtained with X-ray measurements.


1 Introduction

1.1 Theory

The Sunyaev-Zeldovich effect (SZE) is the distortion of the cosmic microwave background (CMB) spectrum due to inverse Compton scattering off of thermal electrons in ionised gas and is most readily observable in the intracluster medium of galaxy clusters [1, 2]. The distortion induces the following temperature change in the CMB:

$$\Delta T_{\text{SZ}}(x) = T_{\text{CMB}} \int dr \left[ \frac{k_B T_e}{m_e c^2} f(x, T_e) - \frac{v_r}{c} \right] n_e \sigma_T$$

where $T_{\text{CMB}}$ is the average temperature of the CMB, $k_B$ is the Boltzmann constant, $m_e$ is the mass of the electron, $c$ is the speed of light and $\sigma_T$ is the Thomson cross-section. The integral is performed along the line-of-sight, along which the cluster temperature, $T_e$, and the gas number density, $n_e$, can both in principle vary. The two terms inside the square brackets
represent the thermal SZE (tSZ) and the kinematic SZE (kSZ), respectively. The kSZ has no frequency dependence and its magnitude, which depends on the cluster’s peculiar velocity, \(v_r\), is also subdominant to the tSZ at most frequencies for typical cluster temperatures. The tSZ has a frequency dependence that is expressed by the function \(f(x, T_e)\), where \(x = h\nu/k_BT_{\text{CMB}}\) is the dimensionless frequency and the dependence on electron temperature, \(T_e\), is due to the presence of relativistic terms that become important for the high gas temperatures typically found in clusters [3]. It can be expanded in a Taylor series about the gas temperature:

\[
f(x, \theta_e) = A_0 + \theta_e A_1 + \theta_e^2 A_2 + \theta_e^3 A_3 + \ldots
\]

(1.2)

where \(\theta_e = k_BT_e/m_ec^2\) is the dimensionless temperature. Various groups have provided analytical expressions for \(A_n\) as a function of \(x\); Ref. [4], whose formulas we use in this paper, provide expansions up to the fourth power.\(^1\)

In the non-relativistic limit, which is equivalent to setting \(\theta_e = 0\),

\[
f_{\text{NR}}(x) = A_0 = x \coth(x/2) - 4.
\]

(1.3)

In this paper, we wish to exploit the dependence of the SZ spectral response on \(\theta_e\) to determine cluster temperatures. This represents a new approach to measuring the gas temperature in clusters, which has typically done using X-ray spectroscopy. Fig. 1 shows how the gas temperature changes the SZ spectrum.

Under the approximation that the gas temperature is constant, Eq. 1.1 simplifies to:

\[
\Delta T_{\text{SZ}}(x) = T_{\text{CMB}} \tau [\theta_e f(x, \theta_e) - \beta],
\]

(1.4)

where \(\tau = \sigma_T \int dr n_e\) is the optical depth of the cluster and \(\beta = v_z/c\) is a dimensionless peculiar velocity.

A quantity of physical interest is the integrated Compton parameter:

\[
Y_\Omega \equiv \theta_e \int_\Omega d\Omega' \tau,
\]

(1.5)

where \(\Omega\) is the solid angle subtended by the cluster. This is often defined with respect to \(R_{500}\), the radius within which its density is 500 times larger than the average density of the Universe at the cluster’s redshift, to which corresponds an angular size \(\theta_{500}\). Because \(Y_\Omega\) is directly proportional to the number of electrons, it is a measure of the mass of the gas in the galaxy cluster; under certain physical assumptions it is also straightforwardly related to the gas temperature, a fact that we exploit in this paper (see Sec. 4.1).

Denoting the SZ temperature integrated over the cluster’s solid angle by \(\Sigma_\Omega \equiv \int_\Omega d\Omega' \delta T\), we can express the SZE flux density in terms of \(Y\):

\[
\frac{\Sigma_\Omega(x)}{T_{\text{CMB}}} = Y_\Omega f(x, \theta_e) - \int_\Omega d\Omega' \beta \tau,
\]

(1.6)

\(^1\)In full generality, there is a relativistic correction due to the cluster’s peculiar velocity for both the tSZ and the kSZ. We ignore it in this paper because in the non-relativistic limit the kSZ is removed by our CMB cleaning (see Sec. 3.1). Any residual will be subdominant to the other terms in the tSZ.
**Figure 1.** The effect of relativistic corrections on the SZ spectrum. Vertical lines indicate the frequencies of the *Planck* data analysed in this paper. The top panel shows the frequency-dependent term $f(x, T_e)$ for various gas temperatures; the bottom panel shows the difference between the rSZ and the non-relativistic SZ. For a temperature of 5 keV, the correction is already sizeable: $\sim 5\%$ in the 353 GHz band. A salient feature of the rSZ is that it alters the location of ‘SZ null’: at 217 GHz there is no SZ power for the non-relativistic approximation, but when the rSZ is properly accounted for this is no longer the case. The frequency of the null is in fact temperature dependent. This makes the 217 GHz channel particularly sensitive to the rSZ.

**1.2 Context**

Over the past five years, CMB surveys have discovered hundreds of new galaxy clusters [5–10] and experiments currently underway, such as SPT3G and Advanced ACTpol, will find many thousands more [11, 12]. Because the amplitude of the SZ effect does not depend on the distance to the cluster, these catalogues have a nearly constant mass threshold as a function of redshift. While such catalogues can in principle provide precise measurements of key cosmological parameters [13, 14], fully exploiting them has proven challenging due to our ignorance of some important details of the astrophysics of the intracluster gas. In particular, the mechanisms by which the intracluster gas is heated as well as the energy transfer rate between the electronic plasma and the photons of the CMB are poorly understood [15].

Furthermore, one physical effect related to the gas temperature is frequently ignored: the relativistic corrections. Nevertheless, it is an effect that is typically on the order of several percent at key frequencies (c.f., Fig. 1) and neglecting it leads to an underestimate of $Y_\Omega$, resulting in a bias in the slope of the $Y$–$M$ relation used for cosmological parameter estimation [16]. While the rSZ is still generally subdominant to the uncertainties in $Y_\Omega$, the error bars are shrinking. In fact, two groups have been able to show evidence for the rSZ signature in ensembles of *Planck* clusters. Ref. [17] (hereafter H2016) stacked over 2000 clusters in 5 bins of $T_e$, as determined by X-ray measurements, and reported a 5.3$\sigma$ detection.
Table 1. Frequency channels used for analysis in this paper. All but the highest frequency are from the Planck experiment (either the LFI or HFI instrument) while the 3000 GHz channel is the IRIS 100 μm map. Also listed are the effective SZ frequencies (see text) and an indication of which channels are used for the rSZ analysis and which are used as dust templates for map cleaning.

| Channel (GHz) | 70 | 100 | 143 | 217 | 353 | 857 | 3000 |
|---------------|----|-----|-----|-----|-----|-----|------|
| LFI           | ✓  | ✓  | ✓   | ✓   | ✓   | X   | X    |
| HFI           | ✓  | ✓  | ✓   | ✓   | ✓   | X   | X    |
| Effective SZ Freq. (GHz) | 70.82 | 103.10 | 144.53 | 222.10 | 355.08 | —   | —    |
| rSZ fit       | ✓  | ✓  | ✓   | ✓   | ✓   | X   | X    |
| Dust template | X  | X  | X   | X   | X   | ✓   | ✓    |

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2 Data

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2.1 Map Products and Frequency Selection

For determining the cluster flux densities, we use the Planck DR2 maps [19] at 70, 100, 143, 217 and 353 GHz. Before the SZ signal can be extracted, though, we need to clean our maps from dust contamination and from the primary CMB anisotropies. We describe the cleaning process below (Sec. 3.1); here we simply introduce the data products used. For dust-removal, we use the IRIS 100 μm map [20] together with the Planck DR2 857 GHz channel. For subtracting the CMB anisotropies, we use the LGMCA (local-generalised morphological component analysis) map, which is obtained by combining Planck DR2 and WMAP 9-year data [21]. L2015, who used the DR1 version of this map [22], demonstrated that the LGMCA product is not measurably affected by tSZ residuals, unlike the Planck DR1 CMB maps. Finally, bright point sources are excluded from the analysis using the SZ union source mask created by the Planck collaboration for their SZ catalogue work [7].

Since the tSZ spectrum differs from that of the CMB, we need to account for its different bandpass response. For the bands from Planck’s High Frequency Instrument (HFI; 100, 143, 217 and 353 GHz) we adopt effective frequencies that correspond to the average $K_{\text{CMB-to-YSZ}}$ of the rSZ. From the detected rSZ signal he infers $T_e$ values that are $1.4\sigma$ higher than the X-ray temperatures. Ref. [16] (hereafter E2018) stacked 821 clusters and did a joint fit for the rSZ spectrum and far infrared (FIR) emission, obtaining a $2\sigma$ measurement of the mean cluster temperature.

Our approach differs from H2016 and E2018 in that we fit directly for the relationship between $Y_\Omega$ and $T_e$ (Sec. 4.1). This means that we do not need to bin or stack our data; nor do we rely on any prior information on the electronic temperature for our fit. In effect, we are making an independent measure of the clusters’ temperatures using the rSZ, even if it is an ensemble fit. With our method, we obtain a $5\sigma$ result using only 47 clusters.

This paper is organised as follows. In Sec. 2 we introduce the Planck and ancillary maps used in our analysis and describe our sample of galaxy clusters. In Sec. 3 we describe how we obtain temperature fluxes from the maps. Our methodology for measuring the $Y - T_e$ scaling relation from the rSZ is presented and verified with simulations in Sec. 4. The results and systematics tests are presented in Sec. 5 and discussed in Sec. 6. We conclude in Sec. 7.
conversion factors listed in Table 6 of Ref. [23]. For the 70 GHz channel, which comes from the Low Frequency Instrument (LFI), we obtain the effective frequency by integrating the SZ spectrum across the bandpass from the 2015 Reduced Instrument Model (c.f., Ref. [24]). Table 1 lists these effective frequencies. For complete accuracy, one would calculate an effective frequency as a function of electron temperature $\theta_e$ since the rSZ corrections alter the spectrum of $f(x, \theta_e)$. However, the fractional difference between $f(x, \theta_e)$ obtained with a fixed effective frequency and by integrating over the bandpass is always less than 1%, except at temperatures above 17 keV in the 217 GHz band. Since in practice our fits do not explore such high temperatures, we consider the effect negligible and use a single effective frequency for each band.

2.2 Galaxy Cluster Sample

Following L2015, we select clusters from the Planck SZ1 cluster catalogue (PSZ1) [7] that appear in the MetaCatalogue of X-ray detected clusters of galaxies (MCXC) [25]. However, we are more selective than L2015. First, we only include clusters for which there is an electron temperature from X-ray spectroscopy available so that we can compare our measurement to these existing measurements. Hence, we include clusters for which $T_e$ is provided in the BAX database for X-ray clusters [26] or, in the case of two clusters, elsewhere in the literature: ACT-CL J0102–4215 (‘El Gordo’) [27] and PSZ1 G266.56–27.31 [28]. This yields 77 clusters. Of these, we cut two (PSZ1 G303.73+33.69, PSZ1 G304.44+32.45) which are close enough to each other so that there is not enough room around the clusters to do the covariance estimate described Sec. 3.2. We also cut PSZ1 G34.37+60.57 and PSZ1 G324.05+48.79 since for some aperture definitions that we used for flux estimation (see Sec. 3.2) we obtained unusually high covariances, perhaps due to artefacts in the dust channels.

The second selection we make is to exclude clusters with $\theta_{500} > 12.0'$. Above this angular size there is a clear trend of increasing correlation between the fluxes we measure in different frequency channels with increasing cluster size. In other words, as the cluster size increases, the effective number of eigenmodes in the noise covariance matrix decreases since it becomes dominated by one or two common modes. We believe this is due to the fact that for larger clusters we need to use larger areas in the map for flux estimation, resulting in less effective map cleaning and higher dust contamination across frequencies (see Sec. 3.1, below). The result is that we are not at all confident in our estimate of the flux covariance for these large clusters and, since our measurement can be sensitive to the interfrequency covariances (see Sec. 5.2.4), we omit them. The remaining 47 clusters with $\theta_{500} < 12.0'$, in which the interfrequency correlation has no dependence on cluster angular size, form the sample used in this paper and still gives us excellent results.

3 Determination of the SZ flux densities

3.1 Map cleaning

It is widely known that the Planck maps include radiation from a variety of astrophysical sources, chiefly the CMB, the SZE, Galactic foregrounds, CO emission, zodiacal light and point sources. In order to separate these components from the tSZ signal we apply to the Planck DR2 data the same recipe used by L2015 on DR1 data, which we summarise in this section.

For the HFI maps (100 to 857 GHz) we use the zodiacal light subtracted maps provided by Planck DR2. Although the Planck Collaboration also produced CO correction maps, we
prefer not to apply these corrections since they are either too noisy or contain significant SZ contamination that could bias our results. We note, however, that the CO contamination is low in the majority of the galaxy clusters in our catalogue. Extragalactic point sources, and regions with strong diffuse Galactic emission are masked out by using the Planck Union mask. To subtract the CMB emission we use the LGMCA map, which has the advantage of containing no significant tSZ residuals. It should be noted that this CMB map was produced by assuming a non-relativistic SZ spectrum, so it may contain tiny rSZ residuals, and using it for CMB removal could also remove some of the rSZ signal we are searching for. However, when L2015 stacked LGMCA maps on their cluster positions they found no detectable SZ signal, demonstrating that this systematic is negligible.

Dust emission is cleaned from our maps by subtracting a dust template multiplied by a constant factor, one per frequency, that minimises the variance of the resulting maps, a method proposed by Ref. [29]. We use two templates: first, we clean with the IRIS 100\(\mu\)m map, and then with the Planck 857\,GHz map. In order to improve the efficiency of this method, instead of performing a full sky dust cleaning, we individually apply the process at the position of each galaxy cluster. Consequently, each cluster at each frequency has its own constant factor for minimising the variance of the resulting maps. During the cleaning process we work at the same angular resolution in all maps. Hence, we work at the original resolutions of the 70, 100 and 143\,GHz maps, and at 5\,arcmin (the angular resolution of the LGMCA map) for the 217\,GHz and higher-frequency maps (for which the original angular resolutions are already very close to 5\,arcmin).

3.2 Aperture Photometry

As in L2015, SZ flux densities are extracted through aperture photometry, i.e., by integrating all pixels enclosed inside a radius \(\theta_1\) and subtracting a constant background level estimated by the median of the pixels inside a larger annulus defined by the radii \(\theta_2\) and \(\theta_3\). To accurately characterise the SZ spectrum it is extremely important to ensure that we capture the same fraction of SZ flux density at all frequencies. One possibility is to work at the lowest angular resolution. However, in order to take the advantage of the finer angular resolution of the high-frequency bands, we instead vary the value of \(\theta_1(\nu)\) in such a way that the fraction of SZ flux inside this radius is the same at every frequency. Whereas in L2015 we used \(\theta'_1(\nu) = \max[\theta_{500}, 0.75\theta_{\text{FWHM}}(\nu)]\) (where \(\theta_{\text{FWHM}}\) is the beam size), in the present analysis we define the aperture radius \(\theta_1(\nu)\) for each frequency such that it contains 66\% of the SZ flux density. This corresponds to the fraction enclosed inside a radius \(\theta_{500}\) in the case of no beam dilution (i.e., assuming a pencil beam). This is an important improvement with respect to L2015, as we avoid a sharp cut in the definition of the aperture radius that particularly affects clusters with \(\theta_{500}\) similar in size to \(\theta_{\text{FWHM}}\).

To define the exact value of \(\theta_1(\nu)\) at each frequency we assumed that the SZ brightness profile is described by a Universal Pressure Profile (UPP) [30]:

\[
P(\zeta) \equiv n_e(\zeta)k_BT_e(\zeta)P_{500} = \frac{P_0}{(c_{500}\zeta)\gamma [1 + (c_{500}\zeta)\alpha]^{(\beta-\gamma)/\alpha}}P_{500},
\]

where \(\zeta = R/R_{500}\) is the dimensionless distance from the centre of the cluster and \(P_{500}\) is the characteristic pressure for the self-similar model (see Appendix A of Ref. [30]). As there is no specific information in the literature to define an individual profile for every cluster of our sample, we are forced to use the same profile for all clusters, for which we adopt the
parameters obtained for the self-similar case and provided in Eq. B.2 of Ref. [30]:

\[
[P_0, c_{500}, \gamma, \alpha, \beta] = [8.130 h_{70}^{-3/2}, 1.156, 0.3292, 1.0620, 5.4807].
\] (3.2)

These values come from a hybrid average profile derived from a combination of simulations and observations.

For each galaxy cluster of our catalogue, we perform the line-of-sight integral of a realisation of its UPP for all pixels enclosed within a radius 5\(\theta_{500}\)—this is considered the cluster boundary because the profile typically flattens beyond this distance [30]. This provides us with a full sky map of the Compton parameter \(y = \theta_\tau\), which we smooth to the angular resolution of each frequency band by assuming perfectly Gaussian and symmetric beams. In order to define \(\theta_1\) (the radius of the aperture within which we determine the flux), let \(Y_{\text{cyl}}\) be the integral of \(y\) over all pixels enclosed in a radius \(\theta\) from the centre of a galaxy cluster, and let

\[
f_{\text{SZ}}(\theta) = \frac{Y_{\text{cyl}}}{Y_{\text{cyl}}^{5\theta_{500}}}.\] (3.3)

We choose the \(\theta_1\) for which \(f_{\text{SZ}}(\theta_1)\) is closest to 0.660. (Due to the effect of the pixelisation, \(f_{\text{SZ}}(\theta_1)\) is never exactly 0.660, nor is it exactly the same for all clusters.) For the region used for estimating the background level to be subtracted from the flux contained in the central aperture, we use an annulus centred on the cluster with inner radius \(\theta_2(\nu) = a_2 \times \theta'_1(\nu)\) and outer radius \(\theta_3(\nu) = a_3 \times \theta'_1(\nu)\), where \(\theta'_1(\nu)\) is the angular size for which \(f_{\text{SZ}}(\theta'_1)\) is closest to 0.956 (corresponding to 3\(\theta_{500}\) for a pencil beam). By default, we choose \(a_2 = 1.33\) and \(a_3 = 2.00\), for which, on average, the fraction of flux in the annulus is \(f_{\text{SZ}}(\theta_2 - \theta_3) \approx 0.005\). This annulus is chosen to be close enough to the cluster centre to have a similar background level while being far enough away to contain little SZ flux. In one of our tests for systematics we explore how varying \(\theta_1\), \(a_2\) and \(a_3\) affects our results (Sec. 5.2.3).

Using the fractions of the SZ flux densities enclosed in the aperture and in the background annulus we estimate a correction factor, \(f_c\), that we apply to our measured flux densities \(S_{\text{ap}}\) to infer an effective value for the SZ flux density within 5\(\theta_{500}\):

\[
S_{5\theta_{500}}(\nu) = S_{\text{ap}}(\nu) \times f_c = S_{\text{ap}}(\nu) \times \frac{1}{f_{\text{SZ1}(\nu)} - f_{\text{SZ23}(\nu)} n_1 / n_{23}},\] (3.4)

where \(n_1\) and \(n_{23}\) are the number of pixels in the aperture and in the background annulus, respectively.

Of course, it would have been more accurate to use a specific density profile for each galaxy cluster rather than an average profile. In order to assess the reliability of our single-profile approximation, we have compared the profiles from our UPP \(y\) map with those derived from the MILCA \(y\) map derived from Planck data [31] (after convolving our Compton \(y\) map to 10 arcmin, which is the angular resolution of the MILCA map). In general we found very good agreement. Further, for the seven clusters in our sample that are also included in the REXCESS catalogue of Ref. [32], and for which Ref. [30] give specific profile parameters, we also produced \(y\) profiles using these parameters. In three of these clusters these new profiles happen to be very similar to the averaged profiles. In the other four clusters there are some differences but, rather surprisingly, the averaged profiles seem to more closely follow the MILCA profiles.
Following L2015 we estimated the uncertainties of our derived flux densities by calculating the standard deviation of the flux densities obtained in 10 non-overlapping apertures located a distance of $3.2 \times \max[\theta_3(\nu)]$ from the cluster centre. In each of these patches we use the same algorithm for flux extraction and adopt the same values of $\theta_1$, $\theta_2$ and $\theta_3$ that are used for SZ flux density estimate at the cluster position. Note that we also calculate the covariances between the fluxes of different frequencies for each cluster, which are not negligible due to common background residuals. Although an analytical estimate of the error bars is also possible, estimating them directly from the maps accounts better for residual background fluctuations and other possible correlated noise components.

4 Methodology for Measuring of Gas Temperature

4.1 Technique

The signal-to-noise of individual clusters is not sufficient to make a detection of the rSZ and so we seek a single parameter describing the temperature dependence of the rSZ in an ensemble of clusters.

To obtain such a parameter, we assume that clusters are spherical and self-similar with the gas in hydrostatic equilibrium. In this case, the relation between the amplitude of the SZ signal and the gas temperature is:

$$Y_\Omega d_A^2 \propto T_e^{5/2} E(z)^{-1}. \quad (4.1)$$

Here, $d_A$ is the angular diameter distance and $E(z) = H(z)/H_0$ is the expansion rate of the Universe. Rearranging and introducing a constant of proportionality, $\gamma$, we have:

$$T_e = \gamma \left[Y_{5R_{500}} E(z) d_A^2\right]^{2/5}. \quad (4.2)$$

Note that we have explicitly defined our $Y_\Omega$ parameter to subtend the flux within a radius $5R_{500}$ to correspond to our flux estimates (Eq. 3.4). Hence, our definition of $\gamma$ is based on a particular choice of $Y_\Omega$. For each cluster $i$ in our sample, we can use the above equation to convert its Compton parameter, $Y_i$, into an estimate its temperature, $\tilde{T}_{e,i}$. Note that due to our method of flux estimation, this will be the average electronic temperature in a region of radius $\sim R_{500}$, i.e., the size of the aperture from which the total flux in $5R_{500}$ is extrapolated. The estimate $\tilde{T}_{e,i}$ allows us to model the cluster’s SZ flux at frequency $x$ as:

$$\widetilde{\tau}_{ix} = T_{\text{CMB}} Y_i \left[f(x, \tilde{T}_{e,i}) + \xi_i\right], \quad (4.3)$$

where we have used Eq. 1.6, but replaced the kSZ term with a ‘pseudo-kSZ’ parameter $\xi_i$ to emphasise the fact that residuals due to the CMB cleaning (see §3.2) might masquerade as kSZ. Hence, when we obtain fits for $\xi_i$ in our analysis, we do not claim to be making a detection of kSZ. Note that $\xi$ can be translated into a pseudo-peculiar velocity:

$$v_{\text{pseudo}} = \xi \theta e c. \quad (4.4)$$

We use the model of Eq. 4.3 to fit for the parameters $\{Y_i, \xi_i, \gamma\}$ given the measured $\widetilde{\tau}_{ix}$. For $N$ clusters there are therefore $2N + 1$ parameters: a Compton $Y$ parameter and a pseudo-kSZ parameter $\xi$ for each cluster plus a single scaling parameter $\gamma$ common to all clusters. The likelihood of the parameters given the data is:
Figure 2. The $Y$–$T_e$ scaling for the cluster sample described in §2.2. Electron temperatures $T_e$ are from X-ray measurements while $Y$ are fit using these X-ray temperatures to calculate $f(x, T_e)$ (see text). The best-fit scaling, shown with the dashed black line, is $\gamma = (0.304 \pm 0.008)\,\text{keV arcmin}^{-4/5}\,\text{Mpc}^{-4/5}$.

\[
\log L = -\frac{1}{2} \sum_i \sum_{x,x'} \left( \xi_{ix} - \bar{\xi}_{ix} \right)^T N_{ixx'}^{-1} \left( \xi_{ix'} - \bar{\xi}_{ix'} \right),
\]

where $N_{ixx'}$ is the noise covariance matrix, which is block-diagonal as there is no covariance between the fluxes of different clusters.

To obtain the best fit parameters, we use an affine invariant Markov chain Monte Carlo (MCMC) sampler [34], as implemented by the emcee python package [35]. We run 1000 chains, using 500 samples after a burn-in time of 1000 samples. We find that the latter is well above 10 autocorrelation times, meaning that the chains will have long converged [35].

We use flat priors for the $Y_i$ parameters, requiring only that $Y_i > 0$. To start the chains, we calculate the following initial estimates:

\[
Y_{i,\text{init}} = \frac{1}{T_{\text{CMB}}} \sum_x \frac{\xi_{ix} f^{-1}_{\text{NR}}(x)/\sigma_i(x)^2}{\sum_x 1/\sigma_i(x)^2},
\]

where $\sigma_i(x) = \sqrt{N_{ixx} f^{-1}_{\text{NR}}(x)}$. Note that this expression simply gives the weighted average of the ratio of the fluxes to the non-relativistic SZ frequency $f_{\text{NR}}$. We give the chains initial values distributed normally around $Y_{i,\text{init}}$ with a width of 5%. For $\xi_i$ we impose a Gaussian prior with a width of 0.5, which corresponds to $v_{\text{pseudo}} \approx 1000\,\text{km s}^{-1}$ for a cluster of $T_e = 3.5\,\text{keV}$ (the lowest X-ray temperature in our sample). This is significantly larger than the expected scatter in $v_{\text{pseudo}}$ (see Sec. 4.2), but our simulations show that using a larger prior does not affect our result and it has the advantage of giving us a very comfortable margin for error. For the scaling parameter $\gamma$ we impose a flat prior in the interval $[0, 23.67]$ keV arcmin$^{-4/5}\,$Mpc$^{-4/5}$.

\footnote{This seemingly arbitrary interval is a relict of our analysis in which we use fluxes that have not been divided by $T_{\text{CMB}}$ to obtain $Y$ values; the range quoted in the text is simply $T^{-2/5}_{\text{CMB}} \times [0, 1]\,\text{keV mK}^{-2/5}\text{arcmin}^{-4/5}\text{Mpc}^{-4/5}$. This range is more than ample for the range of values that the chains explore.}
To get an estimate of the $\gamma$ parameter we may expect to fit, we can use X-ray temperatures to measure the scaling of Eq. 4.2 directly. To do so, we use the procedure described above to fit for $Y_i$, with the difference that we do not include the $\gamma$ parameter in the fit but rather use the X-ray temperatures directly to determine $f(x, T_e)$. Figure 2 shows the resulting scaling of Eq. 4.2, from which we obtain the following result:

$$\gamma_{\text{xray}} = (0.304 \pm 0.008) \text{ keV arcmin}^{-4/5} \text{ Mpc}^{-4/5}. \quad (4.7)$$

Though the data clearly show the scaling relation, the fit itself is poor, having $\chi^2/\text{dof} = 12.1$, reflecting the fact that the self-similar, isothermal model we have assumed (c.f., Eq. 4.1) is not realised in many clusters. For instance, the second-hottest cluster in Fig. 2, A1451, which has a significantly smaller SZ signal than the scaling relation would predict, is known to be dynamically perturbed [36, 37]. Nevertheless, despite the poorness of the scaling approximation in some individual cases, we have enough clusters that the statistics win out and our $\gamma$-scaling approximation is sufficient to make a significant and physically interesting detection of the rSZ.

### 4.2 Verification with Simulations

To test the technique described above, we calculate the $Y_{i,\text{init}}$ parameter using Eq. 4.6 for each of the clusters in our sample. Then, using the cluster’s X-ray temperature, $\theta_{X,i}$, we calculate a fiducial SZ flux, $\tilde{T}_{ix}$, at each frequency using Eq. 4.3; for the pseudo-kSZ term $\xi$ we assign a random value drawn from a Gaussian distribution of pseudo-peculiar velocities $v_{\text{pseudo},i}$ (Eq. 4.4) with a width of 600 km s$^{-1}$. This number is determined by calculating the following residual in the 217 GHz channel for each cluster:

$$v_{i,217} = c\theta_{X,i} \left[ \frac{\tilde{T}_{i}(217 \text{ GHz})}{Y_i} - f(217 \text{ GHz}, \theta_{X,i}) \right], \quad (4.8)$$

where $Y$ are from the fits obtained using the X-ray temperatures $\theta_{X,i}$ directly (see Sec. 4.1). The values $v_{i,217}$ are thus the residuals assuming a pure tSZ signal, converted to a velocity, in the frequency that is most sensitive to kSZ. A Gaussian fit to the histogram of $v_{i,217}$ has a width of $\approx 510$ km s$^{-1}$; we conservatively round up to 600 km s$^{-1}$, though the results of our simulations are not sensitive to the exact width of the pseudo-kSZ distribution.

We then generate simulated SZ temperature flux densities $\tilde{T}_{ix}$ by drawing from the multivariate normal distribution with mean values $\tilde{T}_{ix}$ and the cluster’s measured covariance matrix $N_{\text{xx}}$. The result is a sample of simulated fluxes that include the relativistic effect and have the same uncertainties as the real data. We run our MCMC fits for $\{Y_i, \xi_i, \gamma\}$ on 16 sets of simulations. Fig. 3 compares the input values of $Y_i$ and $v_{\text{pseudo},i}$ to the best-fit values returned by the chains. As can be seen in the figure, the fits of $Y$ agree with the input values to 0.3%; this bias, measured at 3$\sigma$, may be real. The fits of $v_{\text{pseudo},i}$ are on average $(93.5 \pm 1.5)$% the size of the input values, but this reduction is to be expected due to the Gaussian prior preferring values closer to zero. We refrain from further studying these small biases since, as we shall now see, they do not affect our ability to make a good measurement of $\gamma$.

The left panel of Fig. 4 shows the posterior distributions of $\gamma$ for the 16 simulations, with one randomly selected simulation highlighted to better show the width of the distribution.
Figure 3. Input vs. output values for the 16 simulations used to test our method. The input $Y_i$ parameters are the same for each simulation: hence the clumping of points in the left-hand plot. The input pseudo-velocities, $v_{i,p}$, on the other hand, are drawn from a different random distribution in each simulation, as described in the text.

The average maximum-likelihood (ML) value of the simulations is:

$$\gamma_{\text{sim}} = (0.296 \pm 0.006) \text{ keV arcmin}^{-4/5} \text{ Mpc}^{-4/5}. \quad (4.9)$$

The uncertainty quoted here is the error on the mean; the average 1σ width of the posterior distributions is 0.062 keV arcmin$^{-4/5}$ Mpc$^{-4/5}$, indicating that our technique should be able to make a ∼5σ detection of the rSZ. Further, we find excellent agreement with the value obtained from our direct measurement of the $Y$–$T_X$ scaling (Eq. 4.7). We thus conclude that our technique is capable of measuring $\gamma$ well.

One possible concern is that our fits are not able to distinguish sufficiently between the pseudo-kSZ signal and the rSZ, particularly in the 217 GHz channel—the so-called SZ null—which is sensitive to both components. This is also the channel in which our flux uncertainties are generally the lowest and hence is given higher weight in the fits. To assess this possibility, we run four additional suites of simulations, the results of which are shown in the right panel of Fig. 4 (see Table 2 for a summary) and are described here:

1. We perform the fit on simulations that do not have relativistic corrections: i.e., we set $\bar{T}_{e,i} = 0$ in Eq. 4.3. As expected, the MCMC fits prefer $\gamma = 0$ for these simulations.

2. To verify that the pseudo-kSZ included in the simulations is not interpreted as rSZ, we ran our fits on simulations without either rSZ or pseudo-kSZ. Again, the MCMC fits prefer $\gamma = 0$, indicating that there is no such contamination.

3. Next, we omit rSZ again include pseudo-kSZ in the simulations. However, we force $\xi_i = 0$ in the MCMC, leaving only $\{Y_i, \gamma\}$ as free parameters. In some of these simulations, the ML of $\gamma$ has shifted away significantly from zero, since the kSZ is mimicking rSZ and is not being accounted for in the fit. As can be seen, while this test usually returns a $\gamma$ somewhat smaller than the empirical value, it occasionally provides a completely false positive.

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We estimate the ML by calculating the median value of our distributions, which, since they are very symmetric, is a robust method.
Figure 4. Posterior likelihood for $\gamma$, from simulations. Left: The ensemble of 16 simulations with the standard fit, showing that the recovered $\gamma$ is in good agreement with the expected value (Eq. 4.7), indicated by a vertical dashed line. The thick, blue distribution is a randomly selected simulation, plotted simply to show what the shape of the typical distribution is. Right: The results from the tests summarised in Table 2 (see text for more detail). The tests ‘No rSZ’ (black) and ‘No rSZ or kSZ’ (grey) check that the fits do not detect rSZ when it is not present in the simulated data. The tests ‘No rSZ; no $\xi$’ (orange) and ‘No $\xi$’ (green) show that when we neglect the pseudo-kSZ parameter $\xi$ in the fits, we get less reliable results, including false positives.

Table 2. Summary of simulation tests run to ensure that our method reliably detects rSZ without confusing it with pseudo-kSZ. See text for details and the right-hand panel of Fig. 4 for results.

| Test          | Included in Simulation | Included as Fit Parameter |
|---------------|------------------------|---------------------------|
|               | rSZ    | kSZ    | $\xi$ (pseudo-kSZ)     |
| Test 1 (no rSZ) | $\times$ | ✓      | ✓                        |
| Test 2 (no rSZ or kSZ) | $\times$ | $\times$ | ✓                        |
| Test 3 (no rSZ; no $\xi$) | $\times$ | ✓      | $\times$                |
| Test 4 (no $\xi$) | ✓      | ✓      | $\times$                |

4. Finally, we use fiducial simulations in which both rSZ and pseudo-kSZ are included, but, as in Test 3, we force $\xi_i = 0$ in the fits. The results are often consistent with the expected $\gamma$, but the MLs have significant scatter around the expected value and the $\gamma$ distribution is frequently significantly higher than the empirical value. In both this test and Test 3, the large variation in the amount of false rSZ measured is due to sensitivity to the specific values of the random pseudo-kSZ values used from simulation to simulation.

Taken all together these tests show that our inclusion of the pseudo-kSZ parameter $\xi_i$ in the fits is both effective and necessary.

5 Results

5.1 Main Result

Fig. 5 shows the posterior distribution of $\gamma$ for the data from the Planck clusters. The ML value is:
Figure 5. Left: Posterior likelihood for $\gamma$ for our ensemble of Planck clusters; it has a ML of $(0.313 \pm 0.060)$ keV arcmin$^{-4/5}$ Mpc$^{-4/5}$. The thin, black line is the best-fit Gaussian to the distribution. The vertical dashed line is $\gamma_{\text{xray}}$ (see Eq. 4.7), with which our result is perfectly consistent. The non-rSZ hypothesis ($\gamma = 0$) is excluded at 5.2$\sigma$. Right: Posterior distributions for the systematic tests described in the text—each vertical panel is labelled with the section in which the test is described. Each data point is the weighted mean value of the MLs of $\gamma$ obtained in a given suite of tests, with the weights given by the inverse squared 1$\sigma$ width of the distribution. The thick error bars are the weighted standard deviations of the MLs, whereas the thin error bars are the mean 1$\sigma$ widths of the distributions. The solid black vertical line is the ML for the fiducial case (left-hand graph of this figure) and the shaded region is its 1$\sigma$ confidence interval. The vertical dashed line is, as in the left-hand graph, $\gamma_{\text{xray}}$.

$\gamma_{\text{data}} = (0.313 \pm 0.060)$ keV arcmin$^{-4/5}$ Mpc$^{-4/5}$. \hfill (5.1)

The non-relativistic case ($\gamma = 0$) is excluded with 5.2$\sigma$ confidence. Fitting a Gaussian to the distribution (see Fig. 5), we find virtually the same significance: the non-relativistic case is excluded by 5.3$\sigma$. We note that our result is perfectly consistent with the $\gamma$ predicted by the X-ray temperatures (Eq. 4.7).

To further explore the robustness of our results, we perform several tests on our data, as described in the following sections.

5.2 Tests for Systematics

5.2.1 Bootstrap Test

To test whether the posterior distribution of our $\gamma$ parameter is sensitive to the fluxes of one or two individual clusters in the ensemble or whether it is a robust estimate of the true
Figure 6. The amount of residual FIR flux that could be present in a typical cluster in our sample after cleaning for foreground dust (see text for details of the model used). Even if the cluster has the same dust temperature $T_d$ and spectral index $\beta$ (c.f., Eq. 5.2) as dust in the Milky Way, its spectrum is redshifted and thus not all of its FIR emission is removed by our cleaning procedure.

distribution, we employ the bootstrap method \cite{38}, running our fits on 16 random resamplings of our clusters. The mean ML is shown in the bootstrap point of Fig. 5 (right panel). The mean value and its standard deviation are in excellent agreement with our result, indicating that the distribution in the left panel of Fig. 5 is well-estimated.

Of the 16 resamplings, there was one outlier ($3.4\sigma$ higher than the average). This appears to be due to the cluster PSZ1 G021.10+33.24, included four times in this particular bootstrap sample, which happens to have a particularly poor fit to the model due to deficit of flux at 217 and 353 GHz. Since the rSZ decreases the flux at these frequencies, these deficits will work to push $\gamma$ to higher values. When we remove this run from the bootstrap ensemble, the mean value of the ML of $\gamma$ shifts slightly lower to become coincident with the ML of the fiducial result (bootstrap $-1$ in Fig. 5). To ensure that this cluster is not biasing our main result, we removed it and ran our fit on the full sample of clusters again, and found that the result did not significantly change. Evidently, PSZ1 G021.10+33.24 has to be included four times to change the result significantly.

5.2.2 Injecting Far Infrared Signal

In addition to distorting the CMB spectrum via the SZ effect, clusters emit their own electromagnetic radiation. At our frequencies, this is chiefly thermal emission of dust in the member galaxies or in the intracluster media, whose intensity is typically modelled by a modified black body:

$$I_{\text{FIR}}(\nu) = A_d \frac{2\pi h\nu^3}{c^2} \left( \frac{\nu}{\nu_0} \right)^{\beta_d} \frac{1}{e^{h\nu/k_B T_d} - 1},$$

(5.2)

where $A_d$ is a (dimensionless) amplitude factor, $\beta_d$ is the so-called dust emissivity index accounting for opacity effects, $\nu_0$ is a reference frequency, $T_d$ is the temperature of the dust and $h$ is Planck’s constant. Typically, $T_d \sim 20\,\text{K}$, so the emission peaks well above our CMB frequencies; hence, it is our higher frequencies, 353 and 217 GHz which would be most affected by any signal from the Rayleigh-Jeans tail of the FIR emission.
E2018 fit for this term explicitly, including Planck’s 545 and 857 GHz channels in their analysis in order to get leverage where the FIR emission is stronger. H2016, on the other hand, who uses a similar map-cleaning procedure as we do, argues that most of the FIR emission is removed since the emission of dusty galaxies is similar to the Milky Way and the clusters in his sample are at low redshifts. Nevertheless, we wish to ensure that these assumptions truly hold for our sample, and so we simulate the effect that residual FIR has on our fits. Even if the dusty galaxies had exactly the same emission as our own galaxy, their rest-frame emission spectrum (Eq. 5.2) would be altered by their non-zero redshift, and our template subtraction of the IRIS and 857 GHz channels would leave residual emission in our cleaned maps.

The Planck collaboration measured the FIR emission in a stack of 645 clusters [39]. Fixing $\beta_d = 1.5$, they measured $T_d = (19.2 \pm 2.4)\,\text{K}$ [4]. Since the flux is best measured at their highest frequency, we adopt $\nu_0 = 857\,\text{GHz}$, at which they measured an average FIR flux of $\mathcal{T}_{\text{FIR,857}} = (4700 \pm 1700)\,\text{mK\,arcmin}^2$, in a sample with mean redshift of $z = 0.26 \pm 0.17$ and mean $\theta_{500} = (7.4 \pm 5.3)\,\text{arcmin}$. For each cluster in our sample, we scale $\mathcal{T}_{\text{FIR,857}}$ under the assumption that it is proportional to the physical size of the cluster, and use a $\beta$-profile [40] with $\beta = 1$ to estimate the solid angle of the cluster, accounting for the fact that the Planck FIR measurement was made in a 15’ aperture, twice as large as the average $\theta_{500}$ of their sample, while our aperture contains the flux of each cluster within about $\theta_{500}$. We then use the spectral dependence of Eq. 5.2 to calculate how much FIR flux remains in our maps after cleaning, under the assumption that the values $T_d = 19.2\,\text{K (rest-frame)}$ and $\beta_d = 1.5$ hold both for our own galaxy as well as for the cluster. Fig. 6 shows the size of this residual at 217 and 353 GHz for a cluster with the median physical size in our sample, which is 63% of the physical size of the average cluster in the Planck sample. Though the amplitude of contamination grows with redshift, the angular size of the cluster decreases, and so the total flux of FIR leakage is fairly flat as a function of redshift.

We ran simulations in which we added these FIR residuals to our simulated fluxes. The point labelled $1 \times \mathcal{T}_{\text{FIR,857}}$ in the right-hand panel of Fig. 5 shows the mean ML of $\gamma$ for eight such simulations. We also ran simulations in which we increased the amount of FIR by a factor of 2, 4, 10 and 20; the results of eight simulations for each of these scenarios is shown in the same graph. Note that the effect of adding the FIR residuals is to decrease $\gamma$ since at 217 and 353 GHz, the relativistic corrections decrease the amplitude of the SZ signal (see Fig. 1).

Examining the results shown in Fig. 5 we conclude that FIR levels would need to be an order of magnitude greater than that measured by Ref. [39] in order to bias our result by more than 1$\sigma$. Granted, the flux measured by Planck has $\sim 30\%$ uncertainty and we have made several assumptions in our model, such as the galaxy and the clusters having the same rest-frame dust temperature and assuming a $\beta$-profile for the FIR distribution. Nonetheless, it is difficult to imagine that errors in such model choices could conspire to make our estimate of FIR residuals wrong by almost an order of magnitude. We conclude that our analysis is safe against FIR contamination.

5.2.3 Varying Aperture Photometry Parameters

To test how sensitive our results are to the particular definition of annulus used for estimating the background (c.f., Sec. 3.2), we obtained fluxes with five other annuli, listed in the left-
Table 3. Variations of background-subtraction annuli (left) and aperture sizes (right) used for obtaining fluxes, as described in Sec. 5.2.3. See Sec. 3.2 for definitions of $a_2$, $a_3$ and $f_{SZ}(\theta_1)$. The $\theta_1/\theta_{500}$ column is the size of the aperture in the ideal case of a pencil beam.

| Name | $a_2$ | $a_3$ | $f_{SZ}(\theta_1)$ | Ideal $\theta_1/\theta_{500}$ |
|------|------|------|-----------------|-----------------|
| 1.33–2.00 (fiducial) | 1.33 | 2.00 | 0.660 (fiducial) | 1 |
| 1.00–1.33 | 1.00 | 2.00 | 0.481 | 0.65 |
| 1.33–1.67 | 1.33 | 1.67 | 0.883 | 2 |
| 1.67–1.83 | 1.67 | 1.83 | 0.956 | 3 |
| 1.67–2.00 | 1.67 | 2.00 | 0.660 | 1 |
| 2.00–2.33 | 2.00 | 2.33 | 0.660-lowres | 0.660 |

hand series of Table 3. The average ML of $\gamma$ for the six annuli definitions (i.e., including the fiducial annulus) is shown by the point labelled 0.660 in the right-hand panel of Fig. 5. We note that the average is less than the fiducial result, which happens to have the largest ML, but is still consistent with it.

A potential source of bias in our determination of fluxes is that the UPP cluster profile used to determine the aperture size at each frequency is not correct, causing slightly different fractions of the cluster flux to be included at different frequencies. To test for this, we obtained fluxes from maps that had been smoothed to lowest resolution (that of 70 GHz) so that we could use the same size aperture for all frequencies. We performed this procedure using our fiducial aperture of $f_{SZ}(\theta_1) = 0.660$ and the six annuli listed in Table 3. The result is shown by the 0.660-lowres point in the right-hand panel of Fig. 5. The mean ML of $\gamma$ is nearly identical to the case in which UPP-correction with no smoothing is used (0.660). As expected, the uncertainty is greater since the apertures at higher frequencies are larger to account for the lower resolution, and hence more noise—chiefly background fluctuations due to imperfect cleaning—are included in the flux measurements. We conclude that our treatment of the cluster profile creates no significant bias and yields a more precise result.

We also varied the size of the annulus used for flux calculation, using the sizes listed in the right-hand series of Table 3. For each of the additional three aperture sizes, we ran our fit on each of the six annuli sizes; the mean ML of $\gamma$ for each additional aperture size are shown by the points 0.481, 0.883 and 0.956 in the right-hand panel of Fig. 5. The point all-aper shows the mean value for all aperture sizes (including 0.660, but excluding 0.660-lowres). As expected, increasing the aperture size (0.883 and 0.956) increases the error since more noise (again, chiefly due to imperfect cleaning) is included. Decreasing the aperture size from 0.660 to 0.483, on the other hand, does not appear to lower the uncertainty of the fit.

We comment more fully on the results of these tests in our Discussion (Sec. 6).

5.2.4 Omitting 353 GHz from the Fit

Our final test for systematics is to omit the 353 GHz channel in our fit. There are two reasons why this is informative. First, residual foreground dust is strongest at 353 GHz and typically leads to large covariances with the other channels. This can cause a systematic in the result as follows: if the flux at one frequency has a small bias, any interfrequency correlation introduced by the covariance matrix will cause this bias to be spread to the other frequencies. The larger the covariance, the larger this effect is. Now, the most effective way to fit a signal common to all frequencies is with the pseudo-kSZ term $\xi$. Consequently $\xi$ absorbs the bias
that has been shared among all frequencies by the interfrequency correlation, resulting in a
\( \gamma \) that is under-fit. Since most of the interfrequency correlation is with the 353 GHz channel,
running the fits with this channel removed is an effective test for this systematic, as we have
verified with simulations.

The second reason that omitting 353 GHz is informative is that FIR contamination
affects the 217 GHz channel just as much as the 353 GHz channel: even though the residual
FIR flux is significantly smaller at 217 GHz, the SZ signal is also significantly smaller and
the fractional effect is thus commensurate. In fact, in simulations in which we inject FIR
contamination into our data and omit 353 GHz, we find that the effect on the fit for \( \gamma \) is even
stronger. Hence, if for some reason our estimate of the FIR contamination in the previous
section were wrong, and in fact it \textit{was} responsible for the slightly smaller ML for \( \gamma \) found
by our aperture tests, the fits excluding 353 GHz would also exhibit this bias. If, on the
other hand, it were the high level of interfrequency correlation which was reducing \( \gamma \), the fits
excluding 353 GHz would not be affected.

The top section of the right-hand panel of Fig. 5 shows the results of our fits when
353 GHz is excluded. We ran the fits on all our aperture–annuli combinations (see Ta-
ble 3). As expected, the uncertainties of the fits increase somewhat due to having one
fewer channel included in the fit. The best-fit \( \gamma \) is on average closer to the value expected
from the X-ray data than when 353 GHz is included. Of note is that the result from our
fiducial aperture–annulus sizes gives a value that is 1.4\( \sigma \) higher than \( \gamma_{\text{Xray}} \): its ML for \( \gamma \)
is (0.423 \pm 0.087) \text{ keV arcmin}^{-4/5} \text{ Mpc}^{-4/5}, a 4.9\( \sigma \) measurement of rSZ. We discuss these
findings in the following section.

6 Discussion

6.1 Assessment of the Main Result

In our systematics tests we have established that our detection of the rSZ signature is not
contaminated by FIR emission. However, we do see some variations in our result when we
modify the aperture-photometry parameters and when we omit the 353 GHz channel that
bear discussion. Two general features can be noted in the right-hand panel of Fig. 5.

First, when we vary the parameters used for aperture photometry (Sec. 5.2.3), the fits
tend to yield a lower ML for \( \gamma \) than the fiducial case. We stress here that our fiducial
aperture parameters were not chosen \textit{post facto} in order to show a result closer to the value
expected from X-ray temperatures; rather, we selected a fiducial aperture size that gave us
good signal-to-noise in the fluxes and a background-annulus sufficiently far away so as not
to be contaminated by cluster SZ signal and with a significant width so as to sample more
of the background. This happened to be the choice that yielded a ML \( \gamma \) very close to \( \gamma_{\text{Xray}} \).
We note, however, that the differences resulting from modifying the aperture parameters are
within \( \sim 1\sigma \).

Second, when we omit the 353 GHz channel (Sec. 5.2.4), the average results tend to
be \( \sim 1\sigma \) higher, and are in excellent agreement with \( \gamma_{\text{Xray}} \). As reported, the result for the
fiducial aperture parameters when 353 GHz is omitted is 1.4\( \sigma \) higher. There is thus a mild
tension between the results with and without 353 GHz. We speculate that it is due to the
systematic that can be caused by the higher interfrequency covariance introduced by the
353 GHz channel (see Sec. 5.2.4).

Both of these phenomenon are likely due to residuals left over after our map cleaning
(Sec. 3.1), since they can both cause variations in flux when different aperture parameters
are used and introduce interfrequency covariance. The variations are on the \(\sim 1\sigma\) level and so it is difficult to ascertain to what degree they are merely statistical and to what degree they represent an additional, systematic uncertainty. Thus, while we report them to indicate that our map cleaning is not perfect, in the end we do not consider their effect significant enough to alter our main result: i.e., that we have made a \(\sim 5\sigma\) detection of the rSZ and measured a \(Y-T_e\) scaling relationship that is consistent with the gas temperatures determined by X-ray spectroscopy.

### 6.2 Comparison to Previous Work

As mentioned in the introduction, H2016 and E2018 have both reported evidence for the rSZ. Our method of fitting for the \(Y-T_e\) scaling is novel in that it does not require stacking or binning clusters with different gas temperature. One complication this adds is that because we are not stacking, we cannot expect pseudo-kSZ signal to average away. Thus we must also fit for a pseudo-kSZ term, which we have demonstrated to be indispensable in our method (Tests 3 and 4 of Sec. 4.2).

It is worth comparing some of the details of our analysis here with H2016 and E2018. Like H2016, we use a dust template to clean FIR from our maps, but whereas he uses the 217 GHz channel as a template for cleaning the CMB, we use the LGMCA map. This allows us to use the 217 GHz fluxes, which have good sensitivity to the rSZ and increase the significance of our results. Further, we have developed a method for robustly defining apertures that are tailored to each cluster and each frequency, allowing us to retain the full resolution of the higher frequencies. This at least partly addresses the criticism that E2018 raise against aperture photometry, who find that a fixed 15′ aperture provides noisier results than their matched filter method. Finally, we have done careful simulations to ensure that the amount of residual FIR in the maps is negligible (Sec. 5.2.2): the FIR level measured by the *Planck* Collaboration is an order of magnitude lower than that required to significantly bias our result. E2018 wondered whether H2016’s method of using a dust template to clean the maps used for determining SZ fluxes was truly immune to FIR contamination. Given the result of our FIR test, we can respond that, at least for our sample, the template approach is effective.

Turning to our results, we have made a \(5\sigma\) detection using only 47 clusters, similar in significance to H2016 who uses more than 2000 clusters. Our increased sensitivity is likely due to including fluxes from the 217 GHz channel in our analysis. We may also gain sensitivity by not having to group the data into coarse temperature bins. E2018’s measurement of the gas temperature is much less significant (2\(\sigma\)), though this is for a single stack of all 821 clusters, so the intrinsic scatter of the temperatures is necessarily large. Further, rather than relying on a dust template to clean the FIR they explicitly fit for it, so that their rSZ result is marginalised over the dust amplitude and temperature parameters.

H2016 measures temperatures 1.4\(\sigma\) higher than the X-ray temperatures, which he thinks could be due to *Planck* calibration uncertainties. E2018, on the other hand, find an average temperature 1.3\(\sigma\) lower than the average X-ray value. They speculate that this is due to the fact that the X-ray signal depends on the square of the density of gas, \(n_e^2\), whereas the SZ is directly proportional to \(n_e\). The hotter central region of the cluster (see, e.g., Refs. [41, 42]), is thus given more weight in the X-ray measurements than in the rSZ measurements and results in a lower temperature when determined over the whole cluster [30]. Our main result, however, indicates that the rSZ temperature is in good agreement with the X-ray data. This would seem to indicate that within \(R_{500}\)—the approximate size of our flux-extraction
apertures—that any slope or clumpiness in the temperature profiles is not strong enough to manifest this $n_e^2$ effect, at least when averaged over our 47 cluster sample and within our measurement uncertainties.

7 Conclusions

We have investigated the effect of the relativistic SZ corrections in a selection of 47 clusters of galaxies in the Planck data. Our extraction of fluxes from the maps includes careful cleaning of dust contamination and removal of the primary CMB anisotropies, and takes into account beam and pixelisation effects. Using these data, we are able to make a $\sim 5\sigma$ detection of $\gamma$, the scaling relation between the amplitude of the SZ signal, $Y$, and the gas temperature $T_e$ (see Eq. 4.2). Our measurement of $\gamma$ is in excellent agreement with the value obtained using X-ray temperatures.

In continuity with H2016 and E2018, our work provides further evidence that current SZ data are already sensitive to the relativistic effect. As SZ measurements improve, it should become possible to use the rSZ to complement X-ray determinations of cluster gas temperature. And as more precise determinations of the cluster mass are sought from the Compton $Y$ parameter, it will be critical that the rSZ be included in SZ analysis. The time has arrived for SZ science to no longer rely on the non-relativistic approximation as it contributes to advancing precision cosmology.

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