Abstract—Dynamic economic dispatch with valve-point effect (DED-VPE) is a non-convex and non-differentiable optimization problem which is difficult to solve efficiently. In this paper, a hybrid mixed integer linear programming (MILP) and interior point method (IPM), denoted by MILP-IPM, is proposed to solve such a DED-VPE problem, where the complicated transmission loss is also included. Due to the non-differentiable characteristic of DED-VPE, the classical derivative-based optimization methods cannot be used any more. With the help of model reformulation, a differentiable non-linear programming (NLP) formulation which can be directly solved by IPM is derived. However, if the DED-VPE is solved by IPM in a single step, the optimization will easily trap in a poor local optima due to its non-convex and multiple local minima characteristics. To exploit a better solution, an MILP method is required to solve the DED-VPE without transmission loss, yielding a good initial point for IPM to improve the quality of the solution. Simulation results demonstrate the validity and effectiveness of the proposed MILP-IPM in solving DED-VPE.

Index Terms—Dynamic economic dispatch, valve-point effect, non-linear programming, interior point method, mixed integer linear programming

I. INTRODUCTION

Dynamic economic dispatch (DED) is one of the fundamental issues for the optimal economic operation in power system, which aims at allocating the customers’ load demands among the available thermal power generating units in an economic, secure and reliable way at a certain time of interest [1]. Traditionally, the generation cost function of DED is approximated by a convex, quadratic and differentiable polynomials. However, in actual operation, wire drawing effects, occurring as each steam admission valve in a turbine starts to open, produce a rippling effect on the generation cost curve [2], which is known as the valve-point effect (VPE). To model the effect of valve-points, a recurring rectified sinusoid contribution is added to the input-output equation [2], which makes the generation cost function non-convex, non-differentiable and multiple extremal. When VPE is ignored, the rough approximation of the generation cost function will introduce some inaccuracies into the dispatch results. In order to improve the optimality of the solution, a more accurate DED model, dynamic economic dispatch with valve-point effect (DED-VPE) should be considered. However, when VPE is considered, some non-convex, non-differentiable and multiple extremal characteristics are introduced, which makes the solution for DED-VPE more challenging.

In the past decades, a number of optimization methods have been proposed to solve the DED-VPE. Due to the intractability of the problem, most of the currently available approaches for DED-VPE are heuristic optimization techniques [2–26], such as genetic algorithm (GA) [2], evolutionary programming (EP) [3], simulated annealing (SA) [5], particle swarm optimization (PSO) [6], differential evolution algorithm (DE) [7], artificial bee colony algorithm (ABC) [12], artificial immune system (AIS) [13], enhanced cross-entropy (ECE) [14], harmony search (HS) [15], imperialist competitive algorithm (ICA) [20], bee swarm optimization algorithm (BSO) [21], teaching-learning algorithm (TLA) [24], etc. These heuristic optimization techniques are population-based search methods which do not depend on the gradient and Hessian operators of the objective function. So, they can be applied to solve the DED-VPE problem effectively. However, they are quite sensitive to various parameter settings and solution may be different at each trial. Hence, hybrid methods which combine several heuristic techniques or deterministic approaches [3, 27–31] such as hybrid evolutionary programming and sequential quadratic programming (EP-SQP) [3], hybridization of artificial immune systems and sequential quadratic programming (AIS-SQP) [28], hybrid seeker optimization algorithm and sequential quadratic programming (SOA-SQP) [29], hybrid immune-genetic algorithm (HIGA) [30], etc, tend to be more efficient than the individual methods. However, they still have the intrinsic drawbacks of the heuristic method we mentioned above.

Unlike heuristics, deterministic mathematical programming-based optimization techniques can solve to a robust result due to the solid mathematical foundations and the availability of the powerful software tools. Therefore, a strategy recently appeared for DED-VPE is to reformulate the generation cost function, yielding a good optimization model that can be solved by a deterministic method. In [32], the non-convex and non-differentiable cost function caused by VPE is piece-wise linearized, then a mixed integer quadratic programming (MIQP) method can be used to solve DED-VPE. But when the MIQP formulation is directly solved by using a mixed integer programming (MIP) solver, the optimization will suffer convergence stagnancy and run out of memory. As a result,
the multi-step method, the warm start technique and the range restriction scheme are required. However, the range restriction scheme just restricts the solution space to a subspace where the global optimal solution would probably lie in. Consequently, the optimality of the solution for the MIQP cannot be guaranteed. Besides, when the complicated transmission loss is considered, base on the above process, more tedious adjustment techniques are needed. Different from [32], the whole generation cost function is considered for piecewise linearization in [33], then a mixed integer linear programming (MILP) formulation is proposed to solve the DED-VPE, where a global optimal solution within a preset tolerance can be guaranteed. But the transmission loss is not considered in [33]. Therefore, more efforts are worth developing effective deterministic mathematical programming-based optimization methods for DED-VPE to obtain better dispatch results.

In this paper, a hybrid deterministic method that integrates the MILP and interior point method (IPM), denoted by MILP-IPM, is proposed for solving the DED-VPE problem while transmission loss is included. Due to the non-differentiable characteristic of DED-VPE, the classical derivative-based optimization methods can not be used any more. With the help of model reformulation, we derive a non-linear programming (NLP) formulation of DED-VPE, which can be solved by the polynomial time IPM immediately. However, IPM is a local optimization method. If the DED-VPE is solved by IPM in a single step, the optimization will easily trap in a poor local optima due to its non-convex and multiple local minima characteristics. In order to overcome this deficiency, MILP method [33] is combined to generate a good initial point for IPM. And then, solving its NLP formulation via IPM, a good initial point for local optimization method is obtained and then, solving its NLP formulation via IPM, a good initial point for local optimization method is obtained.

The rest of this paper is organized as follows. Section 2 describes the mathematical formulation of DED-VPE. Section 3 derives an NLP formulation for DED-VPE. Section 4 introduces the implementation of MILP-IPM. Section 5 presents the simulation results and analysis. Section 6 draws the conclusions.

II. MATHEMATICAL FORMULATION OF DED-VPE

The DED-VPE problem usually can be formulated as a non-convex and non-differentiable optimization problem. The objective of DED-VPE is to minimize the total generation cost over a scheduled time horizon, which can be written as:

$$\min \sum_{i=1}^{N} \sum_{t=1}^{T} c(P_{i,t})$$

where

$$c(P_{i,t}) = \alpha_i + \beta_i P_{i,t} + \gamma_i P_{i,t}^2 + e_i \sin(f_i(P_{i,t} - P_{i,min}^t))$$

(2)

where $P_{i,t}$ is the power output of unit $i$ in period $t$; $P_{i,min}^t$ is the minimum power output of unit $i$; $\alpha_i$, $\beta_i$, $\gamma_i$, $e_i$ and $f_i$ are positive coefficients of unit $i$; $N$ and $T$ are total number of units and periods, respectively.

The minimized DED-VPE problem should be subjected to the constraints as follows.

- Power balance equations

$$\sum_{i=1}^{N} P_{i,t} = D_t + P_{i,t}^{loss}, \quad \forall t$$

(3)

where $D_t$ is the load demand in period $t$; $P_{i,t}^{loss}$ is the transmission loss in period $t$, which can be calculated based on the B-Matrix loss coefficients and expressed in the quadratic form as given below [34]

$$P_{i,t}^{loss} = \sum_{i=1}^{N} \sum_{j=1}^{N} P_{i,t} B_{i,j} P_{j,t}, \quad \forall t$$

(4)

where $B_{i,j}$ is the $(i,j)$-th element of the transmission loss coefficient matrix.

- Power generation limits

$$P_{i,min} \leq P_{i,t} \leq P_{i,max}, \quad \forall i, t$$

(5)

where $P_{i,max}$ is the maximum power output of unit $i$.

- Ramp rate limits

$$DR_i \leq P_{i,t} - P_{i,t-1} \leq UR_i, \quad \forall i, t$$

(6)

where $DR_i$ and $UR_i$ are the ramp-down and ramp-up rates of unit $i$, respectively.

- Spinning reserve constraints

$$\begin{cases} SR_{i,t} \leq P_{i,t}^{max} - P_{i,t}, \\ SR_{i,t} \leq \tau UR_i, \quad \forall i, t \\ \sum_{i=1}^{N} SR_{i,t} \geq R_t, \quad \forall t \end{cases}$$

(7)

where $SR_{i,t}$ is the spinning reserve provided by unit $i$ in period $t$; $R_t$ is the system spinning reserve requirement in period $t$; $\tau$ is the time duration for units to deliver reserve [22, 32].

III. AN NLP FORMULATION FOR DED-VPE

As we can see from the section II, DED-VPE is a non-convex and non-differentiable optimization problem which is hard to tackle. Due to the non-differentiable characteristic of DED-VPE, the classical mathematical programming-based methods, also known as derivative-based optimization methods, are not suitable any more. To overcome this difficulty, we replace $|\sin(f_i(P_{i,t} - P_{i,min}^t))|$ for (2) with an auxiliary variable $s_{i,t}$, then the objective function (1) can be equivalent to

$$\min \sum_{i=1}^{N} \sum_{t=1}^{T} (\alpha_i + \beta_i P_{i,t} + \gamma_i P_{i,t}^2 + e_i \sin(f_i(P_{i,t} - P_{i,min}^t)))$$

s.t. $s_{i,t} \geq \sin(f_i(P_{i,t} - P_{i,min}^t))$ (10)

$$s_{i,t} \geq -\sin(f_i(P_{i,t} - P_{i,min}^t))$$

(11)

By introducing some slack variables $u_{i,t}^0 \geq 0$, $v_{i,t}^0 \geq 0$, the new inequality constraints (10) and (11) are converted into equality constraints

$$s_{i,t} - \sin(f_i(P_{i,t} - P_{i,min}^t)) - u_{i,t}^0 = 0$$

(12)

$$s_{i,t} + \sin(f_i(P_{i,t} - P_{i,min}^t)) - v_{i,t}^0 = 0$$

(13)
From (12) and (13), we can get the following equations

\[ 2s_{i,t} - u_{i,t}^0 - v_{i,t}^0 = 0 \]
\[ -2\sin(f_i(P_{i,t} - P_{i}^{\text{min}})) - u_{i,t}^0 + v_{i,t}^0 = 0 \]

Then we have

\[ s_{i,t} - \frac{u_{i,t}^0}{2} - \frac{v_{i,t}^0}{2} = 0 \]
\[ \sin(f_i(P_{i,t} - P_{i}^{\text{min}})) + \frac{u_{i,t}^0}{2} - \frac{v_{i,t}^0}{2} = 0 \]

Let \( u_{i,t} = \frac{u_{i,t}^0}{2}, v_{i,t} = \frac{v_{i,t}^0}{2} \), we can obtain that

\[ s_{i,t} - u_{i,t} - v_{i,t} = 0 \]
\[ \sin(f_i(P_{i,t} - P_{i}^{\text{min}})) + u_{i,t} - v_{i,t} = 0 \]
\[ u_{i,t} \geq 0, \quad v_{i,t} \geq 0 \]

Consequently, the DED-VPE can be formulated as the following differentiable NLP formulation.

\[
\begin{aligned}
\min & \quad \sum_{t=1}^{T} \sum_{i=1}^{N} \left( a_{i} + \beta_{i} P_{i,t} + \gamma_{i} P_{i,t}^2 + e_{i}s_{i,t} \right) \\
\text{s.t.} & \quad \text{(5) to (8), (18) to (20)}
\end{aligned}
\]

\[ \geq \text{d} \] is incorporated to find a good initial point for IPM. In step 2, the transmission loss is small compared to the load demand \([32]\). When transmission loss is ignored, the solution gained in Step 1 is “the most” economic. When transmission loss is considered, it means that more outputs are needed and some transmission loss constraints will be added into the original model. Then base on the most economic initial solution, some units outputs will be fine tuned (since the transmission loss is small compared to the load demand) by IPM in step 2, to meet the new constraints and attain a new economic solution. The following simulation results conform well with the above analysis.

\[ \text{Step 1: Solve the MILP formulation (22) by using MILP method to obtain a global optimal solution within a preset tolerance for DED-VPE without transmission loss.} \]

\[ \text{Step 2: Solve the NLP formulation (21) by using IPM, where the initial point is taken at the solution gained in Step 1, to obtain a good local optimal solution for DED-VPE with transmission loss.} \]

In step 1, an MILP formulation for DED-VPE without transmission loss is solved, yielding a solution which is close to a good optimal solution for DED-VPE with transmission loss. This is mainly because, in a DED problem, the transmission loss at each period is small compared to the corresponding load demand \([32]\). When transmission loss is ignored, the solution gained in step 1 is “the most” economic. When transmission loss is considered, it means that more outputs are needed and some transmission loss constraints will be added into the original model. Then base on the most economic initial solution, some units outputs will be fine tuned (since the transmission loss is small compared to the load demand) by IPM in step 2, to meet the new constraints and attain a new economic solution. The following simulation results conform well with the above analysis.

\[ \text{V. SIMULATION RESULTS AND ANALYSIS} \]

To assess the validity and effectiveness of the proposed MILP-IPM, in this section, several test systems which are widely studied for DED-VPE with and without consideration of transmission loss constraints over a scheduled time horizon of 24 h are simulated. For a fairer comparison with most of
the existing methods, the spinning reserve constraints (7) and (8) are not included in our simulations. Actually, it does not reduce the difficulty of the problem since the spinning reserve constraints are some simple linear constraints which is very easy to handle for MILP and IPM.

Since the computation time highly depends on the computer system used, so the CPU execution times for different methods may not be directly comparable due to different computers used. In order to have a fair comparison regarding the computational effort, the CPU chip frequency from the used computer is used to convert the CPU times obtained from different methods into a common base for comparison [36]:

\[
\text{Scaled CPU time} = \frac{\text{Given CPU speed}}{\text{Base CPU speed}} \times \text{Given CPU time}. \quad (24)
\]

In this paper, the base CPU speed is 2.4 GHz and we denote Scaled CPU time by S-time for short.

In this section, we first carry out two experiments on DED-VPE without transmission loss to compare MILP-IPM with MILP and other methods, showing the potential of MILP-IPM for seeking better solutions. Then two experiments on DED-VPE with transmission loss are carried out to demonstrate the validity and effectiveness of MILP-IPM in solving DED-VPE. All cases are performed on an Intel Core 2.5 GHz Dell-notebook with 8 GB of RAM. Our models are coded in YALMIP [37] within the MATLAB R2014a and optimised using CPLEX 12.6.1 [38] for solving the MILP (22) and IPOPT 3.12.6 [39] for solving the NLP (21).

A. DED-VPE without transmission loss

**Case 1:** A 5-unit system without transmission loss

The first test case is incorporated five thermal units. The characteristics of the thermal units and load demands are taken from [5]. Firstly, we directly solve its MILP formulation using CPLEX to 3.2% optimality. In this formulation, segment parameter \( M \) is set to 4. After the optimization, an optimal solution is yielded in 0.82 min with a total generation cost 42563$. Taking such a solution as an initial point in step 2, we solve its NLP formulation using IPOPT with the default options, then a local optimal solution with an optimal value 42524$ is found in 0.04 min.

Table I lists comparison results of the total generation cost obtained by MILP and MILP-IPM. It is clearly seen that the proposed MILP-IPM can make a certain improvement for 5-unit system.

**TABLE I**

| Method    | Total generation cost ($) | S-time(min) |
|-----------|---------------------------|-------------|
| MILP      | 42563                     | 0.82        |
| MILP-IPM  | 42524                     | 0.86        |

The outputs obtained by using MILP-IPM for the 5-unit system are given in Table II for verification.

**Case 2:** A 10-unit system without transmission loss

The second test case contains ten thermal units. The characteristics of the thermal units and load demands are taken from [33]. Firstly, we directly solve its MILP formulation using CPLEX to 0.3% optimality. After the optimization, an optimal solution is yielded in 0.94 min with a total generation cost 1016316$. Using such a solution as an initial point in step 2,
we solve its NLP formulation using IPOPT with the default options, then a local optimal solution with optimal value 101631.1\$ is found in 0.08 min.

The comparison results of the total generation cost obtained by using MILP-IPM and other methods are shown in Table III. It is obvious that the proposed MILP-IPM can solve to the lowest generation cost among all the methods in a reasonable time. But at the same time, we also see that, although MILP-IPM can exploit a better solution in comparison with MILP, but the improvement is small. This is mainly because the solution achieved in step 1 is very well.

The outputs obtained by using MILP-IPM for the 10-unit system are given in Table IV for verification.

### B. DED-VPE with transmission loss

Since transmission loss can not be avoided in a power distribution system and it is critical in real-world DED problem, solution for DED-VPE with transmission loss has more values. Due to the data unavailability, in our simulations, only two cases (5- and 10-unit systems) for DED-VPE with transmission loss are considered.

#### Case 3: A 5-unit system with transmission loss

The third test case is a 5-unit system in which the characteristics of the thermal units and load demands are the same as those in case 1. In this case, the transmission loss is considered. Owing to the limits of space, the loss coefficients are not listed here. One can refer to [8].

Since the transmission loss is included, unlike case 1, it can not be solved by MILP immediately. Fortunately, with the help of model reformulation, we derive a differentiable NLP formulation [21] of DED-VPE, which can be solved by the powerful IPM directly.

In order to demonstrate the validity and effectiveness of the proposed NLP formulation and MILP-IPM for the DED-VPE with transmission loss, on the one hand, we directly solve its NLP formulation using IPOPT with the default options. Then a local optimal solution with an optimal value 43443\$ is obtained in 0.05 min. On the other hand, we solve this system by using MILP-IPM, where the parameters in step 1 are set the same as the case 1. After the optimization, a local optimal solution with an optimal value 43084\$ is found in 0.87 min. The results are compared with other methods in Table V.

As we can see in Table V, although the proposed NLP formulation can be solved to a local optimal solution in a short time, but when the DED-VPE is directly solved by IPM in a single step, the obtaining solution is not the best due to its non-convex and multiple local minima characteristics. While the MILP-IPM which combines MILP with IPM is employed, it can solve to the lowest generation cost among all the methods in a reasonable time.

But, we should note that, not only the optimality but also the feasibility should be considered for assessing the quality of the solution. In Table VI the generation dispatch results obtained
by using MILP-IPM for the 5-unit system are presented, where \( \Delta P_t \) denotes the violation degree of the power balance constraint at interval \( t \), which is calculated by

\[
\Delta P_t = \frac{1}{N} \sum_{i=1}^{N} P_{i,t} - D_t - \frac{1}{N} \sum_{j=1}^{N} \sum_{i=1}^{N} P_{i,t}B_{i,j}P_{j,t}.
\]

Note that, the outputs which have been round-off are used for this calculation (i.e. the outputs in Table VI). Actually, when the original outputs are adopted, all the violations are less than 7e-7. In other words, the solution obtained by using MILP-IPM strictly satisfies the power balance constraints at the same time. This is mainly the result of the rigorous theoretical foundations of interior point algorithm.

### Table VI

**DISPATCH RESULTS (MW) FOR THE 5-UNIT SYSTEM**

| Unit | Unit 2 | Unit 3 | Unit 4 | Unit 5 | Loss | \( \Delta P_t \) |
|------|--------|--------|--------|--------|------|-------------|
| 1    | 526080 | 98.5898| 100.0000| 124.9079| 139.7598| 3.8157 | 0.0000 |
| 2    | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 3    | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 4    | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 5    | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 6    | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 7    | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 8    | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 9    | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 10   | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 11   | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 12   | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 13   | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 14   | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 15   | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 16   | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 17   | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 18   | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 19   | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 20   | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 21   | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 22   | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 23   | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |
| 24   | 92.8835| 66.5972| 124.9079| 139.7598| 139.7598| 4.1239 | 0.0000 |

### Case 4: A 10-unit system with transmission loss

The fourth test case is a 10-unit system in which the characteristics of the thermal units and load demands are the same as those in case 2 and the loss coefficients are taken from [4].

Similar to the case 3, on the one hand, we directly solve its NLP formulation using IPOPT with the default options. Then a local optimal solution with an optimal value 1047294$ is found in 0.22 min. On the other hand, we solve this system by using MILP-IPM, where the parameters in step 1 are set the same as the case 2. After the optimization, a local optimal solution with an optimal value 1040676$ is found in 1.12 min. The results are compared with other methods in Table VII.

As we can see in the Table VII in comparison with IPM, MILP-IPM can solve to a much better solution in a reasonable time. Meanwhile, the total generation cost obtained by MILP-IPM is lower than most of the results reported in the literatures.

In Table VII we provide the generation dispatch results obtained by MILP-IPM for the 10-unit system. In this table, the violations of power balance constraints for CSO[25] and EAPSO[18] are also calculated due to their available solutions. For a fair comparison, we use the outputs shown in the corresponding literatures to compute the corresponding \( \Delta P_t \). From the Table VIII we notice that, although CSO[25]

and EAPSO[18] can obtain lower total generation costs than MILP-IPM, but MILP-IPM outperforms them in terms of the feasibility of the solution for DED-VPE. In fact, when the original outputs are adopted for our case, all the violations are less than 8e-6. It means that, the feasibility of the solution obtained by MILP-IPM can be strictly satisfied.

### C. Results analysis

From the simulation process in this section, we observe that, in the 5-unit system with transmission loss, 66.67% of the solution points obtained by MILP-IPM are the same as those obtained by the individual MILP and differences of the rest range from 0.0960 to 11.72. In the 10-unit system with transmission loss, 70.83% of the solution points obtained by MILP-IPM are the same as those obtained by the individual MILP and differences of the rest range from 0.0001 to 39.77. The details with respect to the solution differences between MILP and MILP-IP for both cases are given in Table IX.

To some extent, this phenomenon indicates that after solving MILP formulation in step 1, an initial point which is close to a good optimal solution for DED-VPE is yielded, and then in step 2, IPM starts its search from this initial point and tunes to a good optimal solution where all the constraints are satisfied, which is consistent with the analysis in the subsection 4.2.

### VI. Conclusion

In this paper, a deterministic MILP-IPM is proposed to solve the non-convex and non-differentiable DED-VPE. To avoid the intractable non-differentiable characteristic of DED-VPE, we derive a differentiable NLP formulation for DED-VPE. Although the NLP formulation can be directly solved by IPM, but the optimization will easily trap in a poor local optima. Therefore, MILP is integrated to generate a good initial point. And then, IPM can be used to exploit a better local optima for DED-VPE. Comparing with the heuristics which are inherently stochastic, MILP-IPM results are much...
when VPE type factors are considered. The proposed MILP-IPM technique is very promising to apply to the practical problems and ensures more stable and the feasibility of the solution can be strictly guaranteed. So, MILP-IPM as a deterministic optimization technique is very promising to apply to the practical problems when VPE type factors are considered.

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