ABSTRACT

We introduce a new information-theoretic measure, fragmentation which can be used to determine how fragmented information is in a system. The concept can be extended to generate fragmentation matrices that can, in turn, illustrate information flows through digital brains, in the form of directed 'information flow' graphs. In addition to introducing Fragmentation we show how it’s application can be used to better understand how digital brains process information and “think”. We show that fragmentation can be used to examine how complex processing arises in neural networks, including differences in lifetime processing and incidents of incidental encryption.

1 Introduction

1.1 Fragmentation

Here we introduce a new information theoretic measure called Fragmentation ($F$) which measures how fragmented the information in a system is. $F$ is measured on a feature (a random variable) and some partitionable system (a partitionable random variable). The systems that $F$ can be applied to include digital brains, gene regulatory networks, neuron firing patterns, and weather predictions. $F$ is measured by determining the smallest partition of the system that can predict the feature to some minimum threshold of accuracy. $F$ is extended with the idea of Fragmentation Matrices, which are generated by calculating the mutual entropy of a set of features and all partitions of the system (see Fig.3).

$F$ is well suited to detecting how and where environment features are stored in digital brains. Prior work has looked at the concepts of Representation ($R$, Marstaller et al. (2013)) and Smearedness ($S$, Hintze et al. (2018)), but where $R$ and $S$ provide general information about a system, $F$ and fragmentation matrices are designed to focus on details. In addition to finding relationships between world states and brain states, fragmentation matrices that use future brain states as features can identify causal and temporal links between a brains current state (inputs and memory) and future states (outputs and updated memory). These relationships can be used to generate information flow diagrams that show how information moves through a digital brain (see Fig.4).

Fragmentation is calculated by comparing the entropy of the feature with the entropy of different partitions of a system. The partitions of a system are all possible subsets of the parts that make up the system, or in other words, the power set of the parts of the system. $F$ is calculated with a threshold and returns the size of the smallest partition that predicts the feature to the accuracy determined by the threshold. Typically calculating $F$ starts by investigating small partitions and stop as soon as a partition with sufficient mutual information is found.

Calculating $F$ and generating Fragmentation Matrices is trivial, but can be costly for even moderately large systems (say, systems with more then 25 partitionable elements). One way to work with larger networks is to only calculate
some subset of partitions (usually the smallest and largest since the intermediate size partitions are generally the least interesting and comprise most of the power set).

1.2 Summery of Work

To provide an intuition for Fragmentation and its uses, we evolve Markov Brains (Hintze et al., 2017) and Recurrent Neural Networks (RNN) in a memory task (NBack) and a behavior task (Block Catch). We use fragmentation to quantify the presence and flow of information within the resulting brains. We also generate information flow diagrams that we use to demonstrate the occurrence of encryption in within simple digital brains and to show how information processing changes in these brains over agents lifetimes.

Finally we show how these methods can be used to detect complexity (especially unnecessary complexity) in evolved structures. In the last section of this document we show that complexity is common in the small networks we evolve and that this complexity is associated with evolutionary costs, as was demonstrated recently in Beslon et al. (2021).

2 Methods

The methods are broken into 2 sections. First we cover in the definition of Fragmentation, how Fragmentation and Fragmentation Matrices are generated, and how Fragmentation Matrices are used to create information flow plots. In the second section we will explain the digital evolution system used to create the digital brains that are used to demonstrate some uses of $F$.

2.1 Entropy

In this document, when we refer to entropy we are speaking about entropy as defined by Claude Shannon (Shannon, 1948), which describes the amount of uncertainty in a system. Given a discrete random variable $X$, with possible outcomes $(x_1, ..., x_n)$, which occur with probability $(p(x_1), ..., P(x_n))$ the entropy of $X$ is formally defined as:

$$H = \sum_{i=1}^{n} p(x_i) \log(p(x_i))$$

We also rely on mutual entropy (sometimes called mutual information, shared entropy, or shared information) which describes how much information one random variable $X$ conveys about another random variable $Y$. The formula for mutual entropy for the random variables $X$ and $Y$ is:

$$H(X; Y) = H(X) + H(Y) - H(X, Y)$$

This says that the entropy that $X$ shares with $Y$ is what’s left when you remove the entropy of the joint variable $(X, Y)$ from the sum of each variables independent entropy. In other words, $H(X) + H(Y)$ counts the mutual entropy between $X$ and $Y$ twice, but $H(X, Y)$ only counts this mutual entropy once.

2.2 Fragmentation

Fragmentation is defined as the minimum number of unique elements in an system that share at least $t$ entropy with a feature. First, we will consider $F$ in the form of an election process. Consider a vote on a measure to raise taxes for a new library in the 8 towns region (some fictional place). Each town in the 8 towns region has a number of votes based on population. Five of the towns have already voted and the count stands at 160 against to 180 for. The three remaining towns, Ford, York, and Jersey, have 5, 30, and 40 votes respectively. The table below shows all possible outcomes once the votes from the last three towns results are counted.

|       | Yea | Nay | Jersey | total for | total against |
|-------|-----|-----|--------|-----------|---------------|
| Yea   | Yea | Yea | 235    | 180       |
| Yea   | Yea | Nay | 195    | 220       |
| Yea   | Nay | Yea | 205    | 210       |
| Yea   | Nay | Nay | 165    | 250       |
| Nay   | Yea | Yea | 230    | 185       |
| Nay   | Yea | Nay | 190    | 225       |
| Nay   | Nay | Yea | 200    | 215       |
| Nay   | Nay | Nay | 160    | 255       |
There are of course only two possible outcomes. Note that the only way for the measure to pass is if both York and Jersey vote yea. Also note that the results in no way depend on Ford.

The Fragmentation of the system with regards to the question “will the measure pass?” is 2.

Now consider if Jersey had 100 votes rather then 40. In this case, only the votes from Jersey would matter to the outcome and the fragmentation would be 1.

2.3 Fragmentation Matrix

A Fragmentation Matrix is a matrix that has features rows and partitions columns. The value in each cell of the Fragmentation Matrix is the mutual entropy of a feature and a partition. Calculating a Fragmentation Matrix is a trivial brute force process. For large systems this process can be time consuming so it may be reasonable to only calculate a subset of all possible partitions - usually this subset includes the smallest and largest partitions as the partitions of intermediate size make up the majority of columns in a Fragmentation Matrix, and are generally of less interest.

2.4 Visualizing a Fragmentation Matrix

When generating a fragmentation matrix it can be useful to also store some additional rows and columns. We tend to save two extra columns (one with the entropy of the feature, and another with the mutual entropy of the feature and the largest partition, i.e. the brains total mutual entropy with the feature) and 1 extra column (with the entropy of each partition). In addition, rather then storing the mutual entropy in each cell it is often more useful to save a value representing the % of the brains total mutual entropy with the feature in the associated cell. Fragmentation Matrices can be visualized directly (see Fig.3 described in detail in the results section).

2.5 Visualizing Data Flow

In cases where the system and features have causal and temporal relationships a Data Flow diagram can be made from the fragmentation matrix where directed lines show which nodes in a system predict which other nodes. For each feature in a Fragmentation Matrix the unique partitions (i.e. current state) that predict each feature (i.e. future states) must first be identified. This is an iterative process. A partition is only included in the set of unique partitions if it shares at least $t$ information with the feature and is not a super set of another unique partition. So, if a partition $[A, B, C]$ was found for a feature, but the partition $[A, C]$ had already been identified then $[A, B, C]$ would be ignored. If there is only one unique partition for a given feature in the plot then this partition can be the only explanation for the feature. Links are rendered in black between the elements of the partition and the feature. If more than one unique partition is found for a feature this means that there is more than one partition that may be responsible for the states of the feature. An addition step is performed were the elements in each partition are determined to either be in all unique partitions or not. Elements in all partitions are necessary and so their links are rendered in black. The remaining elements (those that only appear in two or more partitions) belong to partitions that have the ability to predict the feature, but are not necessarily causal and so are rendered in red. When reading a Data Flow diagram remember that black links are necessary, and if there are red links then at least one of these is necessary as well.

If the structure of the system is known, additional pruning can be applied. For example in the plots shown in Fig.4 and Fig.6 we had access to the connectome of the brains so we could identify and trim “impossible” links - i.e. cases where a partition predicted a feature, but where the necessary connections were not present in the brain.

2.6 Digital Evolution System

In this work we examine two types of artificial cognitive systems (Recurrent Artificial Neural Networks and Markov Brains) in the context of two tasks (NBack and Block Catch). We used the MABE [Bohm et al., 2017] digital evolution research tool to conduct our experiments.

2.6.1 Tasks - NBack

The goal of the NBack task is for the cognitive system to correctly remember and report on a string of bits. Every update a new bit is passed to the brain which must output some "output set" of prior inputs. We defined the output set as $[1, 3, 5, 7, 8]$ [1][a]. Every generation, each agent is tested 25 times on a length 33 bit string. Each test we call a 'lifetime' for this task. The brains outputs corresponding to the first 8 bits are discarded when calculating fitness to allow for a 'seeding' period. See [1][a] for a visualization of NBack task.
Figure 1: Depiction of the two tasks used. [a] illustrates how the NBack task may be solved by an agent. Successive bits provided to the agent at input \( i \) are remembered in and pass through various portions of the memory \( m \) and also output at later times, so that the outputs \( o_1, o_2, o_3, o_4, \) and \( o_5 \) at a given time \( t \) provide the input state from prior time points \( t - 1, t - 3, t - 5, t - 7, \) and \( t - 8 \), respectively. [b] illustrates the Block Catch task, where the blocks, here shown as red, with particular sizes and movement patterns are to be avoided, while other blocks, here shown as green, with different specified sizes and movement patterns are to be caught. The right portion of [b] shows a subsection of the environment at some random moment with a left falling size 4 block (red). The agent is depicted in blue, with the sensors in dark blue and the “blind spot” in light blue. As currently positioned, only the rightmost sensor of the agent would be on. Here the agent should miss the block. Score is determined for a block when the block has dropped to the same level of the agent. The agent "catches" a block if any part of the block intersects any part of the agent.

NBack is a simple task, in that it only requires memory use. No information integration is needed to achieve perfect fitness.

Fitness is a value from 0.0 to 1.0 and is determined by the number of correct answers divided by the total number of answers provided.

### 2.6.2 Tasks - Block Catch

The Active Categorical Perception Task has been used in the connection information theory (Marstaller et al., 2013; Kirkpatrick and Hintze, 2020). In this paper we refer to the task by its more descriptive name, ‘Block Catch’. In the Block Catch task an agent is positioned below a constantly descending object (a ‘block’). Blocks have a size and a left or right drift. Agents must either catch or avoid blocks based on the blocks size and drift direction. An agent catches a block if any part of the block intersects any part of the agent when the block has dropped to the same level of the agent. The agent is given a limited sensor scope with two sensors of width two separated by a central ‘blind spot’ of width 2. Agents can move left or right to identify and track the block as it descends. The environment is a 32 units high, 20 units wide cylinder space (i.e. the left and right sides are “connected”). For each agent, each block type (i.e. size + direction) is dropped from each of the 20 possible starting positions and descends, moving constantly either to the left or to the right until it reaches the level of the agent and is either caught or missed. Each individual combination of size, direction, and position is referred to as a ‘lifetime’. See [1]b for a visualization of the Block Catch task.

With the sensor configuration agents cannot identify all blocks size on a single update. Moreover, in order to infer the blocks drift direction, requires sensor readings over time. So, unlike the NBack Task that only requires memory use, Block Catch requires agents to integrate information over time and over sensor space in addition to memory use.

While previous versions of this task (Marstaller et al., 2013; Kirkpatrick and Hintze, 2020) have focused primarily on a relatively simple decision criteria of catching small blocks and avoiding larger blocks, we increase the difficulty here by manipulating the set of objects that must be caught or avoided to be not only dependent on size but also on direction of...
2.7 Cognitive Systems

In order to illustrate fragmentation we evolve two types of cognitive systems (i.e. digital brains), Recurrent Neural Networks (RNN) and Markov Brains. Both of these brains share the same general structure in that they receive inputs and generate outputs. The number of inputs and outputs is determined by the task. In addition the brains have 8 memory values which can be thought of as additional inputs and outputs. Every time the brain updates in addition to writing outputs, the brain also writes new memory values that will be provided to the brain the next time it updates. The inputs and input memory are labeled at $T_0$ and the outputs and new memory are labeled as $T_1$.

2.7.1 Recurrent Neural Networks

Here we use a simple Recurrent Neural Network (RNN) model. Each value in $T_1$ is determined by a dedicated summation and threshold node that is linked to every value in $T_0$ with a weighted connection. When the RNN updates, each node adds its bias to the summation of the product of each $T_0$ value and its node-specific weight for that value. Tanh is then applied to this sum which results in a value in the range [-1,1]. This result is assigned to the appropriate output. The structure of the brain is fixed, and a genome is used to determine the node weights and biases. In order to improve fragmentation accuracy, memory is discretized before being copied into $T_0$. To perform the discretization, values less than or equal to 0 are set to 0 and values greater than zero are set to 1.

2.7.2 Markov Brains

Markov Brains are made up of wires and gates. Wires connect $T_0$ values to gates and gates to $T_1$ values. Gates contain a lookup table that converts inputs to that gate into outputs. Every update all of the gates in the brain are run (i.e. read from their input wires, and deliver the lookup table determined output to the output wires). Memory works in the same way as in the RNNs. A genome is used to determine the gate definitions using an indirect encoding where a sequence of
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genome values indicates the beginning of a gate description (a start codon) and a set of genome values following the start codon determine the input and output wires and the lookup table.

2.8 Evolutionary Algorithm

The agents’ brains were created from genomes with fixed size of 2000 sites. The mutation operators included point mutations, which randomize the value of a site with a per-site probability of 0.005, point offset mutations, which changes the value of a site within a range of plus or minus 10 with a per-site probability of 0.005, and copy replace mutations, which copies one section of the genome sized between 8 and 500 sites, and replaces another identically sized section with a per-site probability of 0.00005. Population size 100 was used. After population evaluation, the tournament selection method (tournament size 5) was used to select the next population. For those experiments on the NBack Task, the experiments were run for 40000 generations, while for the Block Catch Task, experiments were run for 20000 generations. For each of the 4 experiments, a combination of a brain type (Markov Brain or RNN) and a task (NBack or Block Catch), 250 replicates were run. At the end of each run, the line of descent was reconstructed and used to identify model networks and statistics. The analysis of F and information flow was run using the networks from the lines of descent.

2.9 Testing Mutational Robustness

In the last section of the paper, to serve as a demonstrate application of the methods presented, we compare data flow complexity (the number of links in a data flow plot) to mutational robustness. In order to test mutation robustness we evaluated the final agent from each experiments line of decent, to establish a baseline score. We then produced 100 mutants (using the standard per offspring mutation rates) and determined the average score of these 100 mutants. We calculated a mutation offset as:

\[
\frac{\sum (\text{mutant scores})/100}{\text{baseline score}}
\]

In perfect scoring agents mutation offset is simply the average mutant score (as perfect score is 1.0). Imperfect agents may have scores that are greater then or less then 1.0.

3 Results

Understanding information flow can be complex in agents with rational polices and is often more difficult in irrational agents with imperfect behavior. Since the goal of the paper is to share the methods and potential used of fragmentation, we will primarily rely on results from perfect scoring agents.

Figures 3 and 4 show results from 4 brains evolved on the NBack task that archive perfect fitness. These brains were selected because they represent a range of complexities starting with (a) a brain with the simplest possible solution, and progressing through (b), (c), and (d) which each represent increasingly complex solutions. In figure 3, it can be seen that the simplest solution (a) has fragment size 1 for all features, were as (d) has 2 features with size 1 (o0 and o1), 1 feature with fragmentation 2 (o2) and 2 features with fragmentation 3 (o3 and o4). As the brains increase in complexity, the representations are less concise, resulting in larger fragmentation sizes seen in the fragmentation matrices. In figure 4, increasing complex solutions can be intuitively understood as plots with more node connections, representing the causal connections between input, memory and output. We define “Flow Complexity” simply as the number of connections in a data flow figure.

Figures 5 and 6 show results from 2 brains evolved on the Block Catch task that archive perfect fitness. These brains were selected because they represent a simple solution (a) and (c) and a complex solution (b) and (d). Because the Block Catch task requires agents to determine the block size and drift direction during each lifetime, perfect information in the fragmentation matrix, like we see in the NBack task, is not possible. As a result, in Figure 5, we only see shades of red and orange and no yellow. This figure also shows that the data is fragmented.

It is interesting to note that, apparently, perfect information about the blocks is not needed to achieve perfect performance on the Block Catch task. The two brains shown each have a high level of information in only one feature. In (a) this is the direction the block is moving (the “left” feature) and in (b) this is if the block is moving left and it should be caught (the “catch+left” feature). The reason for the low levels of information may be due to the fact that the brains require much of their lifetime to acquire the information or that it is simply not necessary to have accurate information to preform well, or perhaps both. In figure 6 we also illustrate how data flow changes over the agents lifetime. (a) and (b) show the information flow observed over the entire lifetime of the two agents. (c) and (d) show only the late lifetime (last 25%) information flow. In both cases, the active network simplifies later in life, because once the agent
Figure 3: Fragmentation Matrices for the NBack Task. Matrices from four Markov brains evolved on the NBack Task that evolved perfect performance ([a],[b],[c], and [d]), chosen to represent an variety of complexities. The features (shown on the y axis) are the expected outputs on the next update and the partitions (shown on the x axis) are combinations of the brain’s 8 memory values. The amount of correlation between each feature and each partition is indicated by the value and color. Bright yellow squares indicate high correlation, orange, red and black squares represent successively less correlation. The red arrows identify the smallest partitions that predict each feature. In [a], for example, output o1 is predicted by memory value 1. A portion of each plot, containing partitions of intermediate size, is not shown to save space, indicated by the ellipses. [a] and [b] show brains where each feature is predicted by a single memory value. [c] and [d] show brains that require larger fragments to predict some of the features. Note the darker red colored cells in [d] indicating partial information.

Figure 4: Information Flow Diagrams for the NBack Task. These plots show how data flows though the same four brains shown in Fig. [3] and are labeled identically. Green, White, and Blue nodes indicate inputs (i), memory (h), and outputs (o) respectively. The numbers in the nodes indicate the information in bits that each node receives (when preceded by ‘..’ ) and/or delivers (when followed by ‘..’). The labels accompanying each connecting link and the link’s width both indicate the proportion of the entropy in the down stream node that can be accounted for by that link. The links going into each node represent the connections necessary to account for the total entropy in that node. In this configuration NBack agents were required to report on the outputs correspondent to t-1, t-3, t-5, t-7 and t-8 where t is the current time. Note that xor behavior is indicated in [b] by the fact that the two inputs to h2 account for no information independently, but together account from 1 bit of entropy in h2.
Figure 5: Fragmentation Matrices for the Block Catch Task. Matrices from two Markov brains evolved on the Block Catch Task that evolved perfect performance, selected to represent a simple solution and a complex solution. Each figure shows the fragmentation matrix for a one brain. The features (shown on the y axis) represent various salient features of world state (such as "should I catch this block?", "is this block moving left?", "is this block size 2?" and combinations of some of these states. The partitions (shown on the x axis) are combinations of the brains memory values. Yellow indicates perfect information. Orange, red and black represent progressively lower levels of information. The values here are generally lower then in figure 3 because in this task agents must "learn" the state of the world during their lifetime. In addition, agents can rely on behavioral algorithms that do not require perfect knowledge of world state.

Figure 6: Information Flow Diagrams for the Block Catch Task. These plots show how data flows though the two brains shown in figure 6. Green, White, and Blue nodes indicate inputs (i), memory (h), and outputs (o) respectively. The numbers in the nodes indicate the information in bits that each node receives (when preceded by '..') and/or delivers (when followed by '..'). The labels accompanying each connecting link and the link's width both indicate the proportion of the entropy in the down stream node that can be accounted for by that link. The links going into each node represent the connections necessary to account for the total entropy in that node. Red links indicate connections that can, but which are not necessary to account for downstream information. This occurs when more then one partition can account for the state of a node. (a) and (c) shows the information flow for the brain shown in figure 5a. (a) shows the entirety of all of that brains lifetimes and (c) shows only the last 25% of each lifetime. Likewise, (b) and (d) show entire lifetime and last 25% respectively for the brain shown in figure 5b.
Figure 7: Plots showing the relationship between data flow complexity and the effect of mutation on score in the Block Catch task. [a] and [b] show results from the populations of Markov Brains and RNNs respectively. Red points indicate results from perfect performing agents and blue ‘x’ represent results from imperfect agents. The black lines indicate a line of best fit through the perfect points(solid) and though all points (dotted).

has sufficiently identified the block size and drift direction, the behavior required to execute the correct catch or miss behavior is relatively simple. Note, that these plots do not indicate that the structures of the brains are changing, but rather, just which elements and connections in the brain are being used and how information is moving through these parts.

In figure 7 we look at the relationship between complexity (i.e. the number of connections in the whole lifetime data flow plots) vs mutational robustness on the Block Catch task. We find that there is a strong correlation for both perfect (shown as red points) and imperfect (shown as blue x marks) agents for both RNN and Markov Brains. The simple takeaway is that simpler brains have less things that can go wrong. One explanation for the correlation could be that more complex Markov Brains have more gates, which means that there are more coding sites in the these organisms genomes that can be targeted by mutations. But, this can not be said from RNN which have a fixed number of coding sites. Regardless, in the scope of this paper, we feel that it is sufficient to show that there are measurable correlations exist between previously identified metrics (mutation robustness) and values resulting from our new methods. Further study into this question is definitely warranted!

4 Conclusion

We are very interested in investigations using Fragmentation to look at how digital brains (and perhaps in time biological brains) process information on lifetime scales and how logic develops on evolutionary time scales. We think that one area that these methods may be very well suited for is understanding the evolutionary histories of simple naturally occurring neural circuits (such as the nematode tap withdrawal circuit or the moths’ predator avoidance circuit) as these circuits are comprised of a small number of neurons with interaction networks small enough to be directly evaluated.

We were able to use fragmentation to draw a connection between complexity of data flow and mutational robustness. Here we only looked at small brains on simple tasks. We assume that larger networks will inevitably result in greater encryption and complexities — perhaps suggesting that this should be the expected norm in complex natural systems. We hope that Fragmentation will allow for more thorough explorations into how complexity develops in evolving systems.

From a more practical point of view, the modern world has developed an increasing reliance on artificial cognitive systems (i.e. digital brains) mostly in the form of artificial neural networks. These systems are widely used in emerging technology, such as cloud computing and information security, as well as other fields including agriculture, medicine, and manufacturing ([Abiodun et al.][2018]). As computer power has increased, methods employed to develop digital brains and the resulting digital brains themselves have become more complex. In fact, digital brains are often so complex that they are effectively black boxes — that is, they can perform tasks to a high degree of accuracy, but the
process used to archive the results can not be identified. As vital systems are entrusted to these networks, we must continue to develop better methods for understanding of how they “think” and process information. Fragmentation, and other information theoretic tools like it, may be useful in unlocking the black boxes by providing new views on the operations of these systems. In addition, understanding information flow in deep neural networks could aid in the development of new algorithms that can detect inefficient resource use (i.e. encryption) as well as over reliance on features resulting in over-fitting.

Fragmentation provides new angles from which we can observe complex information networks. With currently available computing power, the application of Fragmentation is limited to small structures. In time, more efficient methods and faster computers will allow us to consider larger systems (such as deep neural networks), and even biological networks (such as gene regulatory networks and brains). In the near term, we are hopeful that Fragmentation will provide insights into the evolution of cognitive processes as well as provide a basis for new theory that can be applied to more complex cognitive structures in the future.

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