Newly observed two-body decays of $B$ mesons in a hybrid perspective

K. Terasaki
Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
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Abstract

In consistency with $\bar{B} \to D^{(*)}\pi$, $J/\psi K$ and $J/\psi\pi$ decays, recently observed $B^0 \to D^+\pi^-$ and $\bar{B}^0 \to D^+K^-$ are studied in a hybrid perspective in which their amplitude is given by a sum of factorizable and non-factorizable ones. (Quasi) two-body decays of $B$ mesons have been studied extensively by using the factorization \cite{1,2}. However, recently measured rates \cite{3} for the color mismatched spectator (CMS) decays, $\bar{B}^0_d \to D^{(*)0}\pi^0$, are much larger than the expectation by the factorization. It suggests that non-factorizable contributions can play an important role in these decays. In addition, very recently, $\bar{B}^0 \to D^+_sK^-$ and $B^0 \to D^+_s\pi^-$ have been observed \cite{4}. The rate for the former is again much larger than the expectation by the factorization, i.e., it has been expected to be strongly suppressed (the helicity suppression) since it is described by an annihilation diagram in the weak boson mass $m_W \to \infty$ limit. It means that the non-factorizable contribution is dominant in this decay. The latter is a pure spectator decay, $\bar{b} \to \bar{u} + (c\bar{s})$, but does not satisfy the kinematical condition of color transparency \cite{5}, so that it is not very clear if the factorization works well in this decay. Therefore, it is meaningful to study a possible role of non-factorizable contributions in the newly observed $\bar{B}^0_d \to D^{(*)0}\pi^0$, $\bar{B}^0 \to D^+_sK^-$ and $B^0 \to D^+_s\pi^-$ in consistency with the $b \to c$ type of decays, $\bar{B} \to D^{(*)}\pi$, $J/\psi K$ and $J/\psi\pi$.

We, first, review briefly our (hybrid) perspective (see Ref. \cite{6} for more details). Our starting point is to assume that the amplitude can be decomposed into a sum of factorizable and non-factorizable ones, ($M_{FA}$ and $M_{NF}$, respectively). $M_{FA}$ is estimated by using the factorization while $M_{NF}$ is assumed to be dominated by dynamical contributions of various hadron states and calculated by using a hard pion (or kaon) approximation in the infinite momentum frame (IMF) \cite{7,8}. In this approximation, $M_{NF}$ is given by a sum of the surface term ($M_S$) which is given by a sum of all possible pole amplitudes and the equal-time commutator term ($M_{ETC}$) which arises from the contribution of non-resonant (multi-hadron) intermediate states \cite{9}. Corresponding to the above decomposition of the amplitude, the effective weak Hamiltonian, $H_w \simeq (G_F/\sqrt{2})\{c_1O_1 + c_2O_2\} + h.c.$, (where $c_1$ and $c_2$ are the Wilson coefficients), is decomposed into a sum of the BSW Hamiltonian \cite{1}, $H_w^{(BSW)}$, and an extra term, $\tilde{H}_w$, i.e., $H_w \to H_w^{(BSW)} + \tilde{H}_w$, by using the Fierz reshuffling, where
The updated values of the decay constants of heavy mesons, $F_B$, factors, $V_f$, of $M_{ETC}$ and $M_S$ with contributions of low lying meson poles are taken into account. In the same way, we can calculate the amplitude for the $b \to u + (c\bar{s})$ decays, $B^0 \to D_s^{(*)}+\pi^-$. These amplitudes, however, include many parameters, i.e., form factors, decay constants of heavy mesons, asymptotic matrix elements of $\tilde{H}_w$ (matrix elements of $H_w$ taken between single hadron states with infinite momentum), phases, $\delta_I$, of $M_{ETC}^I(\bar{B} \to D\pi)$, $(I = \frac{1}{2}$ and $\frac{3}{2})$, relative to $M_S$ and the relative phase ($\Delta$) between $M_{FA}$ and $M_{NF}$ which has not been considered in our previous studies. The other parameters involved are known or can be estimated by using related experimental data and asymptotic flavor symmetries $[11]$. To obtain improved values of the above amplitudes, we update values of parameters involved. Asymptotic matrix elements of axial charges are estimated as follows, i.e., $|\langle \rho | A_\pi^+ | \pi^- \rangle| \approx 1.0$ from $\Gamma(\rho \to \pi\pi)_{\exp} \approx 150$ MeV $[11]$. Here we take $\langle \rho | A_\pi^+ | \pi^- \rangle = 1.0$ and the other parameters can be related to it by using related asymptotic flavor symmetries, for example, $\sqrt{2}|\langle D^{**} | A_\pi^+ | D^0 \rangle| = -\langle \rho | A_\pi^+ | \pi^- \rangle$, etc., as in our previous study $[11]$. As the values of the CKM matrix elements $[12]$ and the decay constants, we take $V_{ts} \approx V_{ud} \approx 0.98$, $V_{cd} \approx -0.22$, $V_{cb} \approx 0.040$, $|V_{ub}/V_{cb}| \approx 0.090$ and $f_\pi \approx 130.7$ MeV, $f_K \approx 160$ MeV from Ref. $[11]$. The decay constant, $f_{J/\psi} \approx 406$ MeV, can be obtained from $\Gamma(J/\psi \to e^+e^-)_{\exp} = 5.26 \pm 0.37$ keV $[11]$. The updated values of the decay constants of heavy mesons, $f_D \approx 0.226$ GeV, $f_{D_s} \approx 0.250$ GeV and $f_B \approx 0.198$ GeV, are taken from the lattice QCD $[13]$, and $f_{D_s^*} \approx f_D$ and $f_{D_{s1}} \approx f_D$, are assumed as expected by the heavy quark effective theory (HQET) $[14]$. The form factors, $F_0^{(DB)}(m_\pi^2)$ and $A_0^{(DB)}(m_\pi^2)$, are estimated by using the HQET and the data on the semi-leptonic decays of $B$ mesons $[11]$ as $F_0^{(DB)}(m_\pi^2) \approx 0.74$ and $A_0^{(DB)}(m_\pi^2) \approx 0.65$. The form factors, $F_0^{(\pi B)}(q^2)$ and $F_1^{(\pi B)}(q^2)$, are estimated by using extrapolation formulas based on the lattice QCD $[13]$. We here take $F_0^{(\pi B)}(m_\psi^2) \approx 0.28$, $F_0^{(\pi B)}(m_{D_s}^2) \approx 0.32$, $F_1^{(\pi B)}(m_{D_s}^2) \approx 0.34$, $F_1^{(\pi B)}(m_{D_s}^2) \approx 0.50$ and $F_1^{(KB)}(m_{\psi}^2) \approx 0.59$. The annihilation amplitudes which contain $F_0^{(D\pi)}(m_{D_s}^2)$ and $A_0^{(D\pi)}(m_{D_s}^2)$ will be small and neglected because of the helicity suppression.

The asymptotic matrix element of $\tilde{H}_w$ is parameterized by

$$\frac{|\langle D^0 | \tilde{H}_w^{(ud;cb)} | B^0_d \rangle|}{V_{ub}V_{cd}f_\pi} = B_H \times 10^{-5} \text{ (GeV)},$$

(1)

where $\tilde{H}_w^{(ud;cb)}$ is a component of $\tilde{H}_w$ which is given by a sum of $\tilde{O}_3^{(ud;cb)} = V_{ud}V_{cb}\{2 \sum_a (\bar{d}t^a u)L(\bar{c}t^a b)L\}$ and $\tilde{O}_2^{(ud;cb)} = V_{ud}V_{cb}\{2 \sum_a (\bar{c}t^a u)L(\bar{d}t^a b)L\}$ with the color $SU_c(3)$ generator $t^a$. To evaluate the $\bar{B} \to D^*\pi$ amplitudes, we assume
as expected by the HQET. All the other asymptotic matrix elements of the non-factorizable amplitudes are combined with the ones in Eq. (2), i.e., by inserting commutation relations, $V_{SU}(\alpha)$ from a quark counting [16], where $\tilde{H}_w$'s denote the low lying mesons.

Table 1. Factorized and non-factorizable amplitudes for the $\bar{B} \to D^{(*)} \pi$, $J/\psi K$, $J/\psi \pi$ and $B^0 \to D_s^{(*)+} \pi^-$ decays. The CKM matrix elements are factored out.

| Decay | $A_{FA}$ ($\times 10^{-5}$ GeV) | $A_{NF}$ ($\times 10^{-5}$ GeV) |
|-------|--------------------------------|---------------------------------|
| $\bar{B}^0 \to D^+ \pi^-$ | $1.94 a_1 e^{i\Delta}$ | $- \left\{ \frac{4}{3} e^{i\delta_1/2} - \frac{1}{3} e^{i\delta_2/2} \right\} B_H$ |
| $\bar{B}^0 \to D^0 \pi^0$ | $-1.14 \left( \frac{f_D}{0.226 \text{ GeV}} \right) a_2 e^{i\Delta}$ | $- \left\{ \frac{2\sqrt{2}}{3} e^{i\delta_1/2} + \frac{\sqrt{2}}{3} e^{i\delta_2/2} \right\} B_H$ |
| $B^- \to D^0 \pi^-$ | $1.94 a_1 \left\{ 1 + 0.48 \left( \frac{f_D^*}{f_D} \right) \left( \frac{a_2}{a_1} \right) \right\} e^{i\Delta}$ | $e^{i\delta_2/2} B_H$ |
| $\bar{B}^0 \to D^{*+} \pi^-$ | $-1.68 a_1 e^{i\Delta}$ | $-0.694 B_H$ |
| $\bar{B}^0 \to D^{*0} \pi^0$ | $1.07 \left( \frac{f_D^*}{0.226 \text{ GeV}} \right) a_2 e^{i\Delta}$ | $0.983 B_H$ |
| $B^- \to D^{*0} \pi^-$ | $-1.68 a_1 \left\{ 1 + 0.52 \left( \frac{f_D^*}{f_D} \right) \left( \frac{a_2}{a_1} \right) \right\} e^{i\Delta}$ | $-0.696 B_H$ |
| $B^- \to J/\psi K^-$ | $-3.60 a_2 e^{i\Delta}$ | $-0.548 B_H$ |
| $\bar{B}^0 \to J/\psi K^0$ | $-3.60 a_2 e^{i\Delta}$ | $-0.548 B_H$ |
| $B^- \to J/\psi \pi^-$ | $-3.08 a_2 e^{i\Delta}$ | $-0.692 B_H$ |
| $\bar{B}^0 \to J/\psi \pi^0$ | $2.18 a_2 e^{i\Delta}$ | $0.489 B_H$ |
| $B^0 \to D_s^+ \pi^-$ | $1.95 a_1 e^{i\Delta}$ | $e^{i\delta_1} B_H$ |
| $B^0 \to D_s^{*+} \pi^-$ | $1.54 a_1 e^{i\Delta}$ | $0.70 B_H$ |

$$\langle D^{*0}\bar{H}_w^{(ud;cb)}|\bar{B}_d^{*0}\rangle = \langle D^{*0}\bar{H}_w^{(ud;cb)}|\bar{B}_d^{*0}\rangle$$

(2)
as expected by the HQET. All the other asymptotic matrix elements of $\tilde{H}_w$ involved in the non-factorizable amplitudes are combined with the ones in Eq. (2), i.e.,

$$\langle J/\psi\bar{H}_w^{(cd;cb)}|\bar{B}_d^{*0}\rangle = \left( \frac{V_{cd}}{V_{cs}} \right) \langle J/\psi\bar{H}_w^{(cs;cb)}|\bar{B}_d^{*0}\rangle$$

$$= \left( \frac{V_{cd}}{V_{uds}} \right) \langle D^{*0}\bar{H}_w^{(ud;cb)}|\bar{B}_d^{*0}\rangle$$

$$= - \left( \frac{V_{cd} V_{cb}}{V_{cs} V_{ub}} \right) \langle D_s^{*+}\bar{H}_w^{(cs;ub)}|\bar{B}_d^{*+}\rangle,$$

(3)

by inserting commutation relations, $[V_{K^0}|\bar{H}_w^{(cs;cb)}] = (V_{cs}/V_{cd})\bar{H}_w^{(cd;cb)}$, $[V_{D^0}, \bar{H}_w^{(cd;cb)}] = (V_{cd}/V_{uds})\bar{H}_w^{(ud;cb)}$, $[V_{D^0}, \bar{H}_w^{(cs;cb)}] = (V_{cs}/V_{ub})\bar{H}_w^{(cs;ub)}$, between related asymptotic states (single hadron states with infinite momentum) and using asymptotic $SU_f(3)$ and $SU_f(4)$ relations, $\langle B_d^{*0}|V_{K^0}|\bar{B}_d^{*0}\rangle = -1$, $\langle D^{*0}|V_{D^0}|J/\psi\rangle = -1$, etc. To obtain the last equality in Eq. (3), we have used the $CP$-invariance which is always assumed in this note and $\langle \{q\bar{q}\}_0|\bar{O}_+|\{q\bar{q}\}_0\rangle = 0$ from a quark counting [16], where $\bar{O}_+ = \bar{O}_1 \pm \bar{O}_2$. The $\{q\bar{q}\}_0$'s denote the low lying mesons.
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reproduce the measured branching ratios for the $\bar{B} \to D^{(*)}\pi$, $J/\psi K$ and $J/\psi\pi$ decays, where the values of the parameters involved are given in the text. $B_{\text{FA}}$ and $B_{\text{tot}}$ are given by $M_{\text{FA}}$ and $M_{\text{tot}}$, respectively. $B_{\exp}$ are taken from Ref. [17].

| Decays              | $B_{\text{FA}}$ | $B_{\text{tot}}$ | $B_{\exp}$ |
|---------------------|-----------------|-----------------|------------|
| $\bar{B}^0 \to D^{+}\pi^-$ | 4.0             | 3.1             | 3.0 ± 0.4  |
| $\bar{B}^0 \to D^0\pi^0$   | 0.10            | 0.24            | 0.27 ± 0.06|
| $B^- \to D^0\pi^-$         | 5.6             | 5.6             | 5.3 ± 0.5  |
| $\bar{B}^0 \to D^{*+}\pi^-$| 3.1             | 2.6             | 2.76 ± 0.21|
| $\bar{B}^0 \to D^{*0}\pi^0$| 0.09            | 0.22            | 0.22 ± 0.10|
| $\bar{B}^0 \to D^{*0}\pi^-$| 4.1             | 4.7             | 4.6 ± 0.4  |
| $B^- \to J/\psi K^-$       | 0.82            | 0.99            | 1.01 ± 0.05|
| $\bar{B}^0 \to J/\psi\bar{K}^0$ | 0.75          | 0.91            | 0.87 ± 0.05|
| $B^- \to J/\psi\pi^-$      | 0.030           | 0.039           | 0.042 ± 0.007|
| $\bar{B}^0 \to J/\psi\pi^0$ | 0.014          | 0.018           | 0.021 ± 0.005|

In this way, we can obtain $M_{\text{FA}}$ and $M_{\text{NF}}$ in the second and third columns, respectively, of Table 1, where we have neglected small contributions of annihilation terms in $M_{\text{FA}}$ and excited meson poles in $M_{\text{NF}}$.

We now look for values of parameters, $a_1$, $a_2$, $\Delta$, $\tilde{\delta}_I$, $(I = \frac{1}{2} \text{ and } \frac{3}{2})$, and $B_H$, which reproduce the measured branching ratios for the $\bar{B} \to D^{(*)}\pi$, $J/\psi K$ and $J/\psi\pi$ decays. $a_1$ and $a_2$ are treated as adjustable parameters with values around $a_1^{(\text{BSW})}$ and $a_2^{(\text{BSW})}$. The phase $\tilde{\delta}_I$ is restricted in the region $|\tilde{\delta}_I| < 90^\circ$ since resonant contributions have already been extracted as pole amplitudes in $M_S$ while $\Delta$ and $B_H$ are treated as free parameters. The result is not very sensitive to $\tilde{\delta}_I$, and the coefficients, $a_1$ and $a_2$, favor values close to the ones taken in Ref. [8] which is based on the factorization. The above implies that the non-factorizable contribution is not very important in the color favored decays. We can reproduce the experimental data ($B_{\exp}$) compiled by the Particle Data Group 2002 [17] taking values of parameters in the range, $1.00 \lesssim a_1 \lesssim 1.13$, $0.28 \lesssim a_2 \lesssim 0.31$, $24^\circ \lesssim |\Delta| \lesssim 32^\circ$, $|\delta_{1/2}| \lesssim 70^\circ$, $10^\circ \lesssim |\delta_{3/2}| \lesssim 90^\circ$ and $0.09 \lesssim B_H \lesssim 0.25$. To see more explicitly a role of the non-factorizable contribution, we list a typical result on the branching ratios (near the best fit to $B_{\exp}$) for $a_1 = 1.08$, $a_2 = 0.29$, $\tilde{\delta}_I = 0.0^\circ$, $\bar{\delta}_3 = \pm 90^\circ$, $\Delta = \pm 28^\circ$ and $B_H = 0.19$ in Table 2, where we have used $\tau(B^-) = 1.67 \times 10^{-12}$ s and $\tau(\bar{B}^0) = 1.54 \times 10^{-12}$ s from Ref. [17]. $B_{\text{FA}}$ and $B_{\text{tot}}$ are given by $M_{\text{FA}}$ and $M_{\text{tot}} = M_{\text{FA}} + M_{\text{NF}}$, respectively. As seen in Table 2, $B_{\text{FA}}$ in which $M_{\text{NF}}$ is discarded is hard to reproduce the data on the CMS decays, $\bar{B} \to D^{(*)0}\pi^0$. If we add $M_{\text{NF}}$, however, we can get a much better fit to the data including the CMS decays. In the color favored $\bar{B} \to D^{(*)}\pi$ decays, $M_{\text{NF}}$ is rather small (but it can interfere efficiently with the main amplitude, $M_{\text{FA}}$). In the $B^- \to D^0\pi^-$ decay, however, it is very small. In the $\bar{B} \to J/\psi K$ and $J/\psi\pi$ decays, the color suppression does not work so
well that $M_{\text{NF}}$ is not dominant in contrast with the $\bar{B} \to D^{(*)0}\pi^0$ although all of them are the CMS decays.

Next, we study the $B^0 \to D_s^{(*)+}\pi^-$ decays comparing with the $B^- \to D^0\pi^-$ which has been studied above. Using the same values of parameters as the above, i.e., $a_1 = 1.08$, $a_2 = 0.29$, $B_H = 0.19$, we obtain

$$|M_{\text{NF}}(B^0 \to D_s^{+}\pi^-)| \simeq 0.09|M_{\text{FA}}(B^0 \to D_s^{+}\pi^-)|,$$

$$|M_{\text{NF}}(B^0 \to D_s^{+}\pi^-)| \simeq 0.08|M_{\text{FA}}(B^0 \to D_s^{+}\pi^-)|,$$

which imply that the factorization works considerably well in these decays although they do not satisfy the condition of the color transparency. Neglecting the rather small $M_{\text{NF}}$ in the $B^0 \to D_s^{+}\pi^-$ and using the same values of parameters as the above, we obtain

$$|M(B^0 \to D_s^{+}\pi^-)| \simeq 0.074|M(B^- \to D^0\pi^-)|,$$

where we have used $|V_{ub}/V_{cb}|_{\text{exp}} \simeq 0.090$ \cite{11}. The measured branching ratio for the $B^- \to D^0\pi^-$ decay \cite{7} leads us to

$$\mathcal{B}(B^0 \to D_s^{+}\pi^-) \simeq 2.7 \times 10^{-5},$$

which reproduces well the recent measurements \cite{4},

$$\begin{cases} \mathcal{B}(B^0 \to D_s^{+}\pi^-)_{\text{BABAR}} = (3.1 \pm 2.0) \times 10^{-5}, \\ \mathcal{B}(B^0 \to D_s^{+}\pi^-)_{\text{BELLE}} = (2.4^{\pm 1.0}_{-0.8} \pm 0.7) \times 10^{-5}. \end{cases}$$

In the same way, we obtain $\mathcal{B}(B^0 \to D_s^{+}\pi^-) \simeq 1.7 \times 10^{-5}$, which is again compatible with the experimental upper limits \cite{6}.

In the $\bar{B}^0 \to D_s^{+}K^-$ decay, $M_{\text{FA}}$ is strongly suppressed because of the helicity suppression, so that $M_{\text{NF}}$ dominates the decay in the present perspective, i.e.,

$$M(B^0 \to D_s^{+}K^-) \simeq M_{\text{NF}}(B^0 \to D_s^{+}K^-) \simeq -iV_{ub}V_{ud}\left(\frac{f_\pi}{f_K}\right)\frac{\langle D^0|\bar{H}_w|B_s^0\rangle}{V_{cb}V_{ud}f_\pi}\epsilon^{\delta_3}.\tag{8}$$

The same value of parameters as the above leads to

$$\mathcal{B}(\bar{B}^0 \to D_s^{+}K^-) \simeq 2.8 \times 10^{-5},$$

which should be compared with the measured values \cite{4},

$$\begin{cases} \mathcal{B}(\bar{B}^0 \to D_s^{+}K^-)_{\text{BABAR}} = (3.2 \pm 2.0) \times 10^{-5}, \\ \mathcal{B}(\bar{B}^0 \to D_s^{+}K^-)_{\text{BELLE}} = (4.6^{+1.2}_{-1.1} \pm 1.3) \times 10^{-5}. \end{cases}$$

In summary, we have studied the recently observed decays, $\bar{B} \to D^{(*)0}\pi^0$, $B^0 \to D_s^{+}\pi^-$ and $\bar{B}^0 \to D_s^{+}K^-$, in consistency with the $b \to c$ type of decays, $\bar{B} \to D^{(*)}\pi$, $J/\psi K$ and $J/\psi\pi$, providing their amplitude by a sum of factorized and non-factorizable ones. To study the non-factorizable amplitudes, we have used the asymptotic $SU_f(3)$ and $SU_f(4)$ symmetries which may be broken. The size of the symmetry breaking can be estimated.
from the value of the form factor, \( f_+(0) \), in the matrix element of related vector current, where \( f_+(0) = 1 \) in the symmetry limit. From the measured values of the form factors, \( f_+^{(\pi D)}(0) = 0.71 \pm 0.06 \) \cite{18} and \( |f_+^{(\pi D)}(0)/f_+^{(K D)}(0)| = 1.00 \pm 0.13 \) \cite{19}, the asymptotic \( SU_f(4) \) symmetry seems to be broken to the extent of 30% while the asymptotic \( SU_f(3) \) still works well. However, such a large symmetry breaking has not caused any serious problem in the present study since \( M_{NF} \) is much smaller than \( M_{FA} \) except for some decays in which \( M_{FA} \) is strongly suppressed and whose experimental errors are still large. For more precise studies, of course, more detailed informations of the symmetry breaking will be needed.

The amplitude with final state interactions has been included in the non-factorizable one. For the color favored \( B \to D^{(*)} \pi \) decays, \( M_{NF} \) has been rather small and, therefore, the final state interactions seem to be not very important (but not necessarily negligible) in these decays. In the \( \bar{B} \to D^{(*)0} \pi^0 \) which are the CMS decays, \( M_{NF} \) has been dominant since \( M_{FA} \) is suppressed because of the color suppression. In the \( \bar{B} \to J/\psi K \) and \( J/\psi \pi \), which also are the CMS decays, however, the color suppression has not worked so well that \( M_{NF} \) has not been dominant in contrast with the \( \bar{B} \to D^{(*)0} \pi^0 \) decays. In the \( B^0 \to D_s^{(*)+} \pi^- \) which are the color favored \( \bar{b} \to \bar{u} + (c\bar{s}) \) type of spectator decays, \( M_{NF} \) has been small. The values of parameters which reproduce the measured branching ratios for the \( \bar{B} \to D^{(*)} \pi \), \( J/\psi K \) and \( J/\psi \pi \) decays have lead to \( B(B^0 \to D_s^{(*)+} \pi^-) \) consistent with the very recent measurements. It means that the factorization works considerably well in these decays although they do not satisfy the condition of color transparency. In the \( B^0 \to D_s^+ K^- \) which is the annihilation decay, \( M_{NF} \) has been dominant and reproduced the very recent measurements within their large errors. All the above suggest that dynamical contributions of hadrons should be carefully treated in hadronic weak decays of \( B \) mesons.

In the CMS decays, \( B^0 \to D_s^{(*)0} \pi^0 \), \( J/\psi K \) and \( J/\psi \pi \), the annihilation decay, \( \bar{B}^0 \to D_s^+ K^- \), and the \( \bar{b} \to \bar{u} + (c\bar{s}) \) type of spectator decay, \( B^0 \to D_s^+ \pi^- \), both of the theoretical and experimental ambiguities are still large although their measured rates have been reproduced considerably well by taking account of the non-factorizable contributions. More theoretical and experimental studies on these decays will be needed.

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