ABSTRACT: The stringent regulations of fuel consumption and exhaust emission require further refinement of the control strategy for diesel engines. In the future, the prediction of the in-cylinder combustion process will become necessary to achieve a more dedicated control performance. Hence, a more precise model able to run in a real-time application is required to predict the nature of multiphase Diesel combustion. This paper presents a modified multi-Wiebe function with a concise parameter structure, which is governed by the center point of the combustion process $\theta_{50}$ and the form factor $m_j$ of each stage. The modified function captures the typical characteristics of the measured heat release rate and avoids the ambiguous determination of several parameters, therefore improving the calibration efficiency. A novel calibration method called “backward-stepwise recursion” is introduced that decomposes the nature of the measured heat release rate and fits the function from the tail stage to the precombustion stage. This method is suitable for large-quantity diesel fuel combustion and dual-fuel combustion cases in which the adjacent combustion stages superimpose one another. The proposed method is applied in the measured heat release rate of a single-cylinder prototype diesel engine from 15% to 100% load conditions. The modified multi-Wiebe function suggests good accordance in heat release prediction at all the load conditions, which demonstrates its ability to be embedded in the control unit for crank-angle-resolved real-time combustion prediction.

1. INTRODUCTION
To achieve the goal of fuel economy and to regulate carbon dioxide emissions, the further development of intelligent control in diesel engines is necessary. Low-temperature combustion (LTC) is an advanced in-cylinder emission control strategy that has been studied extensively. Some studies of fuel properties and alternative fuels have also been published. In recent years, advanced combustion modes have been proposed that correspond to different control factors, and the corresponding combustion models were established as well. The in the near future, the combustion theoretical model will be introduced to the ECU to predict the operating process of the individual cylinder to achieve more refined combustion phasing control. A combustion prediction model with a relatively high accuracy and a low level of computational effort is also required by the hardware-in-the-loop (HiL) test system for ECU validation, where the crank-angle-resolved in-cylinder pressure will be calculated in real-time applications. There are several approaches for control-oriented modeling (COM), including the phenomenological model, the mean-value engine model (MVEM), and the zero-dimensional empirical model. The phenomenological model divides the fuel plumes into a certain number of packets, and fuel injection, atomization, evaporation, ignition, and combustion processes are described by the combination of numerous theoretical submodels. The spatially inhomogeneous distribution can be reproduced well by this method; however, the relatively complicated computational process limits its usage in real-time simulations. The MVEM method has been widely used in the control strategy; however, it generates the mean value over one engine cycle, which makes it unable to provide the crank-angle-resolved information on the combustion process.

The zero-dimensional model shows its advantage by balancing the prediction accuracy and the computational time for the control-oriented application. The representative Wiebe
function has shown comprehensive applicability in many works. Refs 14 and 15 show the fundamental application of a single-Wiebe function on engine combustion. For diesel combustion or multiple injection combustion cases, premixed combustion and diffusive combustion phases present different heat release natures. Thus, double-Wiebe16,17 or multi-Wiebe18 functions have been developed. The main challenges to improving the prediction accuracy of the multi-Wiebe function lie in distinguishing the different combustion phases from the measured results and identifying the numerous Wiebe factors. Yaliana et al.19 discussed that the logarithm function of heat release has a certain functional relationship with the logarithm function of the crankshaft angle, then conducted the estimation of the double-Wiebe parameters by fitting the combustion facts at typical points such as θ10 and θ25. Yang et al.20 found that the third derivative of the mass burned rate could correspond well to the fraction of different combustion phases, as the parameters of each phase are calibrated by least-squares regression. A similar method could also be found in ref 21. Maroteaux et al.22,23 used the nonlinear fitting method to obtain the Wiebe parameters of each stage for a multi-injection strategy, and the calibration step of Wiebe correlations was introduced. However, the method needs to determine the initial values of a group of Wiebe functions in advance, and after some assumptions there are still 10 parameters that need to be calibrated. Nonlinear least-squares fitting (NLSF) is a method commonly used to

Figure 1. Effects of CE, m, and a on the combustion form in a single-Wiebe function.
determine the parameters of Wiebe functions and shows quite good correlations. However, the initial values and the boundary selection may limit the availability of the function in a wide range of operating conditions, as the fitting result is significantly influenced by the presets of the NLSF. It is essential to develop a physical-based approach to determine the optimum results for fractions of multiple combustion phases and Wiebe parameters. Furthermore, because the number of calibrated parameters increases with the complexity of the multi-Wiebe function, the difficulty of determining some parameters such as the combustion duration of each phase also limits the reproduction of the combustion fact.

The objective of this work is to present a modified multi-Wiebe function with a concise parameter structure that aims to capture the typical characteristics of multistage heat release rate and avoids the ambiguous determination of several parameters. Therefore, the calibration efficiency could apparently be improved. Next, a novel calibration method is introduced that decomposes the nature of the measured heat release rate and fits the function from the tail stage to the precombustion stage. Because the method minimizes the number of assumptions for model calibration, the results could better reflect the actual situation of combustion. Finally, the proposed method is validated by the measured in-cylinder pressure profiles at varying engine operations, and a comparison between the proposed model in this paper and some other representative multi-Wiebe approaches is carried out. The detailed information on combustion phases that was derived from the modeling process is also discussed.

2. ANALYSIS AND MODIFICATION OF THE MULTI-WIEBE FUNCTION

The single-stage Wiebe function has been widely used to predict heat release profiles in gasoline and gas engines in which premixed combustion takes a dominant position. As for the typical diesel engine combustion, part of the fuel evaporates before the onset of combustion and burns in a premixed mode, where the flame propagation rate is mainly determined by the chemical reaction kinetics. This is followed by the diffusive combustion mode, where the reacting rate is determined by the mixing process of fuel and ambient gas. Therefore, a multi-Wiebe model is necessary to represent the diesel engine combustion process. The original multistage Wiebe function is usually defined as follows:

\[
X(\theta) = (\text{CE}) \sum_{j=1}^{3} F_j \left\{ 1 - \exp \left( -a \left( \frac{\theta - \text{SOC}}{D_j} \right)^{m+1} \right) \right\}
\]

where \(X(\theta)\) is the mass fraction burned, \text{SOC} is the start of combustion, \(F_j\) is the fraction of each stage, \(D_j\) is the duration of each stage (the real combustion duration), \(m\) is the form factor, \(a\) is the efficiency factor, and CE is the combustion efficiency. The subscript \(j\) is defined as \(p, m, \text{ and } t\), which represent the premix, main, and tail combustion stages, respectively. Basically, \text{SOC} and \(D_j\) can be measured according to the specific operating condition, thus \(\text{CE}, a, \text{ and } m\) are the main calibration parameters with which to adjust the combustion form. By fixing the values of \(\text{SOC}\) and \(D_j\), the effects of the calibration parameters on the combustion form in a single-Wiebe function are shown in Figure 1. Figure 1(a) \((a = 6.908 \text{ and } m = 1.5)\) shows that as \(\text{CE}\) increases, the cumulative burned fraction increases as well as burned fraction rate after the \(\text{SOC}\). However, \(\text{CE}\) has little impact on the position of the peak burned fraction rate. Figure 1(b) \((\text{CE} = 0.99 \text{ and } a = 6.908)\) shows that the rising rate and the peak position of the burned fraction rate are significantly influenced by changing the value of \(m\). Figure 1(c) \((\text{CE} = 0.99, m = 1)\) shows that when \(a\) is increased the combustion does not finish during the predefined \(D_j\); correspondingly, the burned fraction rate also decreases. In many studies, \(\text{CE}\) is considered to be 0.999 at the end of combustion, since \(a = -\ln(1 - 0.999)\) could be derived from eq 1 and \(a\) is defined as a fixed value of 6.9.

When fitting the multiple heat release rate curves of diesel engines, there are three main challenges in parameter calibration: (1) It is difficult to distinguish the real combustion duration \(D_j\) because the slow heat release lasts for a long time at both the beginning and the end of combustion. The measuring noise also impact the accuracy of \(D_j\). (2) The actual fraction of each combustion stage \(F_j\) is unknown, which makes it difficult to fit the multi-Wiebe model for diesel-like multistage combustion. (3) As expressed in Figure 1, \(\text{CE}, a, \text{ and } m\) have cross effects on the combustion profile, so the use of an appropriate calibration method to identify the Wiebe model parameters is critical to the rationality of the predicted results.

As a reference, GT-SUITE proposed a new formula based on the original multi-Wiebe function.

Figure 2. Comparison of the fitting results with the variation of \(m\), where \(\text{Ne} = 3000 \text{ rpm}, \text{ the load was } 10\%, \text{ and the injection timing was } -17^\circ\text{CA ATDC.} \)
\[ X(\theta) = (CE) \sum F_j [1 - \exp(-WC_j)] \]
\[ \theta - SOI - ID = (m+1) \]
\[ WC_j = \left[ \frac{D_j^*}{BEC^{1/m+1} - BSC^{1/m+1}} \right]^{-m+1} \]
\[ BEC = -\ln(1 - BS) \] is the burned start constant; BS is a constant of 0.1, which denotes the first 10% mass fraction burned; \[ \text{BEC} = -\ln(1 - BE) \] is the burned end constant; BE is a constant of 0.9, which denotes the 90% mass fraction burned; SOI is the start of injection; and ID is the ignition delay from the SOI to the start of ignition. Equation 3 is tenable because of the following deduction process. According to eq 1, the crank angle corresponding to the exact mass fraction burned at a random stage could be expressed as follows:
\[ \theta - SOI = D_j \left( -\frac{\ln(1 - X(\theta))}{a} \right)^{1/m+1} \]
By subtracting the crank angle corresponding to 90% mass fraction burned \( \theta_{90} \) from the crank angle corresponding to 10% mass fraction burned \( \theta_{10} \), \( D_j^* \) could be expressed as shown in eq 5.
\[ \theta_{90} - \theta_{10} = D_j^* \]
\[ = D_j \left( -\frac{\ln(1 - BE)}{a} \right)^{1/m+1} - \left( -\frac{\ln(1 - BS)}{a} \right)^{1/m+1} \]
\[ D_j = \left( \frac{\text{BEC}}{a} \right)^{1/m+1} - \left( \frac{\text{BSC}}{a} \right)^{1/m+1} \]
Equation 6 can be further rearranged into eq 7, which supports eq 2.
\[ \frac{a}{D_j^{m+1}} = \left[ \frac{D_j^*}{\text{BEC}^{1/m+1} - \text{BSC}^{1/m+1}} \right]^{-(m+1)} = WC_j \]
\[ \text{X}(\theta) = (CE) \sum F_j [1 - \exp\left(\left(\theta_{90} - \theta_{50} - \text{SOC}\right)^{m_j+1}\right)] \]
There are two advantages to applying the modified multi-Wiebe function in eq 9. First, undefined parameters \( D_j^* \) and \( a \) can be excluded, which considerably improves the calibration efficiency. Second, the combustion characteristics can be easily derived as long as the center of the combustion phase \( \theta_{50} \) is predefined carefully for each stage. As shown in Figure 2 (b), if the position of peak apparent heat release rate is defined as the \( \theta_{50} \) in lower load conditions, the Wiebe function indicates robust predicted results regardless of the selection of the value of \( m \) from 1.5 to 3.
Ref 30 presents an equation similar to eq 9; however, only the single-Wiebe function is discussed, the parameters are determined by the least-squares method, and the calibration strategy is not explained in detail. In the following sections, the proposed modified multi-Wiebe function in eq 9 will be applied to fit the Diesel combustion process under different operating conditions. In particular, the method for determining the parameters in each stage will be introduced (which is named the “backward-stepwise recursion” method). The influence of the selection of \( \theta_{50} \) and \( m_j \) on the combustion prediction results will also be discussed.

3. EXPERIMENTAL SETUP

In this study, a single-cylinder air-cooled naturally aspirated 186FA diesel engine was selected as the experimental prototype

| Table 1. Main Engine Specifications |
|-----------------------------------|
| cylinder diameter (mm)            | 86 |
| stroke (mm)                      | 72 |
| volume (L)                       | 0.418 |
| compression ratio                | 19:1 |
| chamber shape                    | deep \( \omega \) |
| swirl ratio                      | 2.5 |
| number of valves                 | 2 |
| RPM (r/min)                      | 3000 |
| rated power (kW)                 | 5.68 |
| intake valve opening timing (°CA ATDC) | -8.5 |
| intake valve closing timing (°CA ATDC) | -135.5 |
| exhaust valve opening timing (°CA ATDC) | 124.5 |
| exhaust valve closing timing (°CA ATDC) | 8.5 |

to collect diesel combustion data. The main engine parameters are shown in Table 1. The schematic diagram of the engine prototype is shown in Figure 3. The fuel injection pressure and the timing are controlled by a common-rail system and a fuel supplying control unit, respectively. In-cylinder pressure was measured by an AVL GH14DK piezoelectric pressure transducer in conjunction with a Kistler S018A charge amplifier. An optical encoder producing 3600 pulses per revolution was used, which supplied a resolution of 0.1 crank angle degrees (°CA). The apparent heat release rate was calculated according to the measured in-cylinder pressure and the volume history based on the first law of thermodynamics, as shown in eq 10. R is the gas constant, \( C_p \) and \( C_v \) are the specific heat capacities at constant pressure and constant volume, respectively, \( P \) is the cylinder pressure, and \( V \) is the cylinder volume.
The proposed modified multi-Wiebe function was verified in a wide range of loads from 10% to 100% the rated power (Table 2). The engine speed was fixed at the rated speed of 3000 rpm. The larger injection pressure was applied for higher load conditions to maintain a similar injection duration and improve the atomization and evaporation process. To ensure the reliability of the experimental results, more than 100 cycles of pressure history were recorded at each load condition; the averaged value was used for the model analysis.

4. CALIBRATION METHOD

Taking the heat release rate result at 75% load as an example (in Figure 4), the fraction of the diffusive combustion stage becomes apparent as the power output increases because the fuel/air mixing process requires more time than the ignition delay. Therefore, it is necessary to introduce a multi-Wiebe function to describe the precombustion, main combustion, and tail combustion phases with different characteristics. In the previous

\[
\frac{dQ}{d\theta} = C e_{p} \frac{dV}{d\theta} + C e_{T} \frac{dP}{d\theta}
\]

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work, the multi-Wiebe function showed good predictability for multi-injection Diesel combustion and dual-fuel combustion. The fractions of the different phases are usually linked by the weight factor, which is defined by the fuel burned in each stage. For multi-injection operation, the weight factor is relatively easy to calibrate because the separation of the combustion stages is obvious. However, as shown in Figure 4, the different combustion stages superimpose with each other if a large fuel quantity is injected in a short period. As a result, appropriately defining the combustion fraction \( F_j \) becomes difficult, which impacts not only the combustion stage distribution but also the prediction accuracy of the total heat release profile.

To solve the above problem, a novel calibrating method called “backward-stepwise recursion” is introduced. Generally, it is noted that both the premixed combustion process and the main combustion process are impacted by the three stages, while the final phase of combustion is only impacted by the tail.
combustion stage. Therefore, it is favorable to fit the tail combustion stage first according to the descent curve of the total heat release rate. Then, by subtracting the fitted tail combustion stage from the measured total heat release rate, the descent curve of the residual heat release rate represents the characteristics of the main combustion stage. By repeating the same method, the three stages can finally be identified according to heat release characteristics from experimental results. The flowchart for the "backward-stepwise recursion" calibration method is indicated in Figure 5.

The advantages of this method are as follows:

1. The tail combustion stage spans the whole combustion process, and the descent curve of the total heat release rate merely depends on the tail combustion stage. Therefore, it is easy to fit the tail combustion profile as long as the parameters \( m_t \) and the combustion fraction \( F_t \) are calibrated.

2. The main combustion curve and the precombustion curve could be fitted separately using the residual heat release rate. The influence of the superimposition of adjacent combustion stages could be avoided. Only the form factors \( m_m \) and \( m_p \) and the mail combustion fraction \( F_m \) need to be calibrated.

3. This method fully utilizes the characteristics of measured combustion information, avoiding the ambiguous definition of the combustion fraction \( F_f \) for each stage.

Because the proposed method is carried out in reverse, the accuracy of the previous step will affect the next step. Meanwhile, the interpretation of the measured combustion results (such as the identification of the center point of combustion (\( \theta_{50} \)) is critical to the predicted results. In Sections 4.1–4.3, the curve-fitting process for each stage will be presented. In section 4.4, the influence of \( \theta_{50} \) and \( m_t \) on the predicted results will be discussed.

4.1. Tail Combustion Stage. Figure 6 presents the fitting result for the tail combustion stage in the 90% load condition. According to eq 9, the eq 11 is used to fit for the heat release for the tail combustion as follows:

\[
\frac{dQ}{d\theta} = HR \times F_t \times \frac{-\ln(0.5)}{\theta_{50} - SOC} \times (m_t + 1) \times \left( \frac{\theta - SOC}{\theta_{50} - SOC} \right)^{m_t} \\
\times \exp \left( \ln(0.5) \left( \frac{\theta - SOC}{\theta_{50} - SOC} \right)^{m_t + 1} \right)
\]

(11)

where \( HR \) is the heat release rate during the whole combustion process, \( SOC \) is the starting crank angle of combustion, and \( \theta_{50} \) is the center point of the tail combustion stage. According to the definition of the multi-Wiebe function, the tail combustion stage spans the whole combustion period, thus it is easy to determine \( \theta_{50} \) based on the SOC and the end position of combustion.

Iterative calculations were carried out to obtain the optimum value of \( m_t \) in an effort to minimize the difference between the predicted heat release rate from eq 11 and the descent part of the measured total heat release rate. The corresponding \( F_t \) can be determined according to
where $\frac{dQ_{00}}{dt}$ denotes the heat release rate at the center point of the specific combustion stage. The smaller the value of $m$ is, the more the peak value of the predicted tail curve tilts to the left. The fraction of the tail combustion stage is finally adjusted to 0.1, which implies that the tail combustion stage has a minor impact on the whole combustion process.

4.2. Main Combustion Stage. By subtracting the calculated tail combustion curve from the original measured data, the residual part can be used to fit the main combustion curve, as shown in Figure 7. The same as the fitting process in the tail combustion stage, the optimum selection of $m_m$ and $F_m$ is also derived by iterative calculations.

The main combustion stage of the diesel engine is mainly dominated by diffusive combustion, which is governed by the physical mixing process of evaporated fuel and ambient air. The reaction rate of diffusive combustion is much slower than that of the chemical reaction dominated by premixed combustion. As shown in Figure 7, the diffusive combustion phase corresponds to the “stable heat release zone” after the junction point. The region from the junction point to the end of the residual combustion is finally adjusted to 0.1, which implies that the tail combustion stage has a minor impact on the whole combustion process.

Figure 14. Results for the heat release rate predicted using the modified multi-Wiebe function.
combustion curve is used to fit the parameters \( m_m \) and \( F_m \). The key to fitting the main combustion curve lies in the selection of \( \theta_{m50} \). According to the previous statement, the position of \( \theta_{m50} \) should lie in the “stable heat release zone”; the effect of \( \theta_{m50} \) on the predicted results will be discussed in Section 4.4.

4.3. Precombustion stage. Figure 8 presents the fitting result for the precombustion stage. After subtracting the tail combustion curve and the main combustion curve from the original data, it can be noted that the residual combustion curve corresponds to the rapid premixed-controlled heat release process. In this way, the premixed combustion phase can be derived from the experimental data rather than any ambiguous assumption to distinguish between the premixed combustion and the diffusive combustion. The amount of fuel burned in this region is determined by how well the diesel fuel evaporates and mixes before the start of combustion. In this stage, only the parameter \( m_p \) should be calibrated because \( F_p \) can be derived according to the following equation:

\[
F_p + (F_m + F_t) = 1
\]  

(13)

4.4. Analysis of the Fitting Process. From the above analysis, it can be seen that the determination of the main combustion stage has the most influence on the fitting process for the following reasons:

1. Only \( F_t \) and \( m_t \) need to be adjusted when fitting the tail-combustion stage. The value of \( F_t \) is between 0.1 and 0.2, which means the tail combustion curve has little influence on the whole combustion process. Meanwhile, the only parameter needed to be adjusted for the precombustion stage is \( m_p \) because the impact of superposition from other combustion stages has been removed by the proposed “backward-stepwise recursion” method.

2. The fitting of the main combustion curve is affected by three factors, namely the selected \( \theta_{m50} \) and the calibrated values of \( m_m \) and \( F_m \). The calculation of the main combustion stage determines the fraction of the premixed combustion and diffusive combustion phases and eventually impacts the modeling of the whole combustion process. Therefore, the influence of parameter setting for the main combustion stage will be discussed in detail.

As discussed in section 4.2, the main combustion stage mainly occurs during the stable heat release process after the junction point; thus, it is reasonable to assume that \( \theta_{m50} \) should be located around the “stable heat release” region. As shown in Figure 9, six test points from 12.24 to 15.28°C ATDC are selected in the “stable heat release” region to represent \( \theta_{m50} \). The calculated curves indicate the results predicted for the main combustion stage using the optimized values of \( m_m \) and \( F_m \). \( \theta_{m50} \) settings.

Figure 10 presents the tendency of the fraction of the precombustion and main combustion stages by varying the value of \( m_m \) with the different definitions of \( \theta_{m50} \). Figure 10 (a) indicates that the fraction of main combustion \( F_m \) decreases approximately linearly with the increase in \( m_m \) and the advance of \( \theta_{m50} \). Since the total combustion durations of the precombustion and main combustion stages are fixed, the fraction of precombustion \( F_p \) increases correspondingly with the increase of \( m_m \) and the advance of \( \theta_{m50} \) (Figure 10(b)). It is also noted that part of the evaluated \( F_p \) values shows negative value if \( \theta_{m50} \) is retarded too much, which helps limit the available range when defining \( \theta_{m50} \).

Figure 11 presents the calculated combustion duration under the same conditions as those in Figure 10. As long as \( m_m \) is determined, the combustion duration could be derived according to Eqs 7 and 8. Generally, increasing the value of \( m_m \) or \( \theta_{m50} \) tends to tilt the heat release profile to the retarded direction, and the combustion process finishes earlier as a result.

Figure 12 presents the coefficient of determination \( R^2 \) for the heat release rate according to the measured data and the predicted result, which indicates the prediction accuracy of the proposed method. It was found that \( R^2 > 95\% \) could be obtained by adjusting the value of \( m_m \) regardless of if \( \theta_{m50} \) was shifted from 12.75°C ATDC to 14.52°C ATDC, which verified the robustness of the prediction accuracy against the preset value of \( \theta_{m50} \). A too advanced or too retarded definition of \( \theta_{m50} \) will lead to the reduction of \( R^2 \), which provides available evidence for the determination of the \( \theta_{m50} \) range.

After the optimum value of \( m_m \) was determined for each selected value of \( \theta_{m50} \) inside the available range, the fractions of the precombustion and main combustion stages were determined, as shown in Figure 13. As the preset \( \theta_{m50} \) is retarded, the fraction of the main combustion stage decreases, and the fraction of the precombustion stage increases; the difference is less than 10%. The result implies that a relatively robust predicted result could be obtained regardless of the preset parameters. In the following results, a group of \( \theta_{m50} \) and \( m_m \) values with the largest \( R^2 \) will be selected as the final value.

5. RESULTS AND DISCUSSION

The above calibration process was applied to the measured heat release rate of a 186FA single-cylinder diesel engine for 15% to 100% load conditions. The fitted results for the three combustion stages are shown in Figure 14. Generally, the proposed multi-Wiebe function suggests good accordance with the heat release prediction at all the load conditions, and the coefficients of determination \( R^2 \) are approximately 92–98.5%.

The proposed model is compared with several other models. Sun et al.21 used a double-Wiebe functional for a sequential turbocharged diesel engine. They discussed the method by calculating the second derivative to obtain the transition angle, \( \theta_t \), rearranging terms, and twice taking the natural log of the Wiebe function, as shown in equation 14. The transition angle, \( \theta_t \), divides the profile into two parts, which can respectively calculate the heat release and obtain the burnt fraction \( F \).

\[
\ln(\theta - \text{SOC}) = \frac{1}{m + 1} \ln\left(\frac{\ln(1 - x)}{\ln(0.5)}\right) + \ln(\theta_t - \theta)
\]

(14)

The equation clearly shows that \( \frac{1}{m + 1} \) is the slope of the plot of \( \ln(\theta - \text{SOC}) \) versus \( \ln\left(\frac{\ln(1 - x)}{\ln(0.5)}\right) \), where the intercept on the y-axis is \( \ln(\theta_t - \theta) \). Figure 15 shows the calculation of the second derivative and the plot of \( \ln(\theta - \text{SOC}) \) versus \( \ln\left(\frac{\ln(1 - x)}{\ln(0.5)}\right) \).

In Sun’s model, the double-Wiebe function has two different SOCs, SOC1 and SOC2, which are divided by the transition angle. In this way, a sudden change in the heat release curve will inevitably occur, which is shown in Figure 16.

Glewen et al.24 and Yeliana et al.19 both used the least-squares method (LSM) to approach the Wiebe parameters but set different objective functions. Glewen’s model consists of two independent Wiebe functions, \( x_t \) and \( x_p \), but both of them use the same values of \( \theta_0 \) and \( D, F, a_1, a_2, m_1, m_2 \) are the parameters.
to be optimized. The target of optimization in Glewen’s model is the apparent heat release rate. The objective functions are shown as follows:

\[ x = F \times x_1 + (1 - F) \times x_1 \]  
\[ x_1 = x_2 = a_{1,2} \frac{m_{1,2}}{D} + 1 \left( \frac{\theta - \text{SOC}}{D} \right)^{m_{1,2}} e^{-a \left( \frac{\theta - \text{SOC}}{D} \right)^{m_{2,2} + 1}} \]  

In Yeliana’s model, meanwhile, the mass fraction burned is regarded as the target of optimization. The objective functions are shown in eq 18. The parameters that need to be optimized are \( F, D_1, D_2, m_1, \) and \( m_2 \).

\[ \frac{dQ}{d\theta} = Q_{\text{tot}} \times \frac{dx}{d\theta} \]  

The comparison of Glewen’s model and Yeliana’s model is shown in Figure 17. A summary of the comparison of the above three models and the model proposed in this paper is shown in Table 3.

The fitting results determined at the 25% and 90% load conditions using the above models are compared in Figure 18. It is noted that Sun’s model could achieve high precision in both low and high engine loads \((R^2 > 0.95)\), but the sudden change still exists when the combustion stages shift. Both Yelana’s model and Glewen’s model fit well under a low engine load. However, their models do not perform very well under a high engine load, especially when there is an obvious slow-burning period. As for Glewen’s model, because the same combustion duration \( D \) is used for the double-Wiebe function, the two parts are prone to overlap. For Yelana’s model, the initial heat release becomes prominent under high load conditions, which makes the proportion of the first stage of the Wiebe function larger.

Figure 19 presents the calibrated fraction \( F_i \), the combustion duration \( D_i \), and \( \theta_{50i} \) for three stages as a function of the engine load. It is noted that the fraction of the precombustion stage...
decreases from 70% to 20% as the engine load increases, while the fraction of the main combustion stage increases from 20% to 60%. The fraction of the tail combustion stage is also extended at higher load conditions. This is because the increased engine load leads to a shorter ignition delay, as the proportion of diesel fuel mixes with ambient gas and evaporates before the SOC decreases. Since the total combustion duration apparently increases with the increase of the engine load, the duration of precombustion remains almost constant. In comparison, the durations for the main and tail combustion stages increase. The center points of the main combustion and tail combustion stages retard apparently with the increase of the engine load because of the prolonged combustion duration; however, the center point of precombustion $\theta_{50}$ advances in reverse due to the decreased proportion of precombustion and the faster chemical reaction rate.

Xu et al.31 studied the application of a four-stage Wiebe function on the diesel/NG dual-fuel engine, and they also agreed that the premixed and diffusion combustion stages of diesel and the flame propagation of NG happen simultaneously in a shared space. Thus, it is difficult to analyze the combustion process by simply examining the total heat release profile. However, the fraction of each stage is defined by an empirical function governed by the ignition delay and equivalent ratio. According to the calibration results, a reduction in the premixed fraction was observed and the combustion duration was extended as the engine load increased, similar to the results obtained in this work.

The predicted results provide useful information regarding the diesel engine combustion process, such as the timing and proportion of different combustion stages, which are difficult to measure directly in an experiment. The proposed method is expected to be applied in the dual-fuel-like complex combustion system in which the different types of combustion phases gather together.

6. CONCLUSIONS

This paper introduces a modified multi-Wiebe function that is applied to fit the diesel combustion process with distinct premixed combustion and diffusive combustion features. Notably, a “backward-stepwise recursion” calibration method was designed to determine the control parameters of each combustion stage based on the decomposition of the measured heat release process. The proposed method was verified by comparing the predicted results with the experimental results.

### Table 3. Comparison of Several multi-Wiebe Functions

| Model            | Objective Functions | Unknown Parameters |
|------------------|---------------------|--------------------|
| Sun’s model      | double-Wiebe        | $\text{SOC}_{1}$, $\text{SOC}_{2}$, $\theta_{50}$, $m_1$, and $m_2$ |
| Glewen’s model   | double-Wiebe        | $F$, $a_1$, $a_2$, $m_1$, and $m_2$ |
| Yeliana’s model  | double-Wiebe        | $F$, $D_1$, $D_2$, $m_1$, and $m_2$ |
| The authors’ model | triple-Wiebe       | $m_p$, $m_{\text{pre}}$, $m_\theta_{50}$, and $\theta_{50}$ |

### Figure 18. Accuracy comparison with other models.

(a) 25% engine load

(b) 90% engine load

Figure 18. Accuracy comparison with other models.
derived from a prototype diesel engine in a wide range of load conditions. Several findings could be concluded as follows:

(1) Compared to the original multi-Wiebe function, the newly developed function is controlled by the center point of the combustion process $\theta_{50}$ and the form factor $m_i$ of each stage, avoiding the ambiguous determination of the combustion duration $D_j$ and the stage fraction $F_j$. The calibration efficiency could be obviously improved. The predetermination of $\theta_{50}$ helps capture the typical characteristics of the measured heat release rate, which leads to robust predicted results for combustion.

(2) Appropriately distinguishing among each stage becomes difficult because the different combustion stages superimpose with each other if a quantity of large fuel is injected in a short duration. To solve the problem, a novel calibration method called “backward-stepwise recursion” is introduced, which decomposes the nature of the measured heat release rate and fits the function from the tail combustion stage to the precombustion stage. Because the proposed calibration method is carried out by fully considering the measured nature of combustion, the influence of the superimposition of adjacent combustion stages can be avoided.

(3) The calculation of the main combustion stage determines the fraction of the premixed and diffusive combustion phases, which has the most influence on the modeling of the whole combustion process. $\theta_{50}$ is considered to be located in the “stable heat release” region during the diffusive combustion process. The optimum values of $\theta_{50}$ and $m_m$ were obtained through iterative calculations aimed at minimizing the difference between the predicted and measured heat release curve.

(4) The above calibration process is applied to the measured heat release rate of the 186FA single-cylinder diesel engine under 15% to 100% load conditions. The proposed multi-Wiebe function suggests good accordance for the heat release prediction at all the load conditions. The method is expected to be applied in the dual fuel-like complex combustion system in which the different types of combustion phases gather together.

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Figure 19. Predicted combustion characteristics of the three stages as a function of the engine load.
COMBUSTION PROCESS.

Simulation Approach for the Prediction of a Dual-Fuel Pilot Injection Gas Applications: A Review.

NOMENCLATURE

BE burned fuel percentage at duration end
BM burned fuel percentage at the anchor angle
BS burned fuel percentage at duration start
BEC burned end constant
BMC burned midpoint constant
BSC burned start constant
CE combustion efficiency
D combustion duration, °CA ATDC
F fraction of the combustion stage
HR heat release, J
ID ignition delay, °CA
P in-cylinder pressure, Pa
V combustion chamber volume, m³
RMSE root-mean-square error
R² determination coefficient
SOC start of combustion, °CA ATDC
SOI start of injection, °CA ATDC
WC weight coefficient in the GT—power function
m Wiebe form factor
a efficiency factor
j corner marker
p precombustion Stage
m midcombustion Stage
t tail combustion Stage
\( \frac{dQ}{dt} \) heat release rate, J/°CA
θ Crank Angle, °CA
x mass fraction burned

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