Theory of Network Wave

Bo LI*, Mao YANG, and Zhongjiang YAN
School of Electronics and Information, Northwestern Polytechnical University, Xi’an, CHINA, 710072
{libo.npu*, yangmao, zhjyan}@nwpu.edu.cn

Abstract: Aiming at the disorder problem (i.e. uncertainty problem) of the utilization of network resources commonly existing in multi-hop transmission networks, the paper proposes the idea and the corresponding supporting theory, i.e. theory of network wave, by constructing volatility information transmission mechanism between the sending nodes and their corresponding receiving nodes of a pair of paths (composed of two primary paths), so as to improve the orderliness of the utilization of network resources. It is proved that the maximum asymptotic throughput of a primary path depends on its intrinsic period, which in itself is equal to the intrinsic interference intensity of a primary path. Based on the proposed theory of network wave, an algorithm for the transmission of information blocks based on the intrinsic period of a primary path is proposed, which can maximize the asymptotic throughput of a primary path. In the cases of traversals with equal opportunities, an algorithm for the cooperative volatility transmission of information blocks in a pair of paths based on the set of maximum supporting elements is proposed. It is proved that the algorithm can maximize the asymptotic joint throughput of a pair of paths. As for the cases of traversals with unequal opportunities, an algorithm for the cooperative volatility transmission of information blocks in a pair of paths based on the set of maximum supporting elements is also proposed. Moreover, the upper bound of the asymptotic joint throughput of a pair of paths achieved by using this algorithm is obtained. Finally, an algorithm flow that can maximize the end-to-end asymptotic joint throughput of a pair of paths is proposed, which comprehensively covers almost all the key factors that may affect the asymptotic joint throughput of a pair of paths, such as the selection of routes, the determination of the reachable periods of the two primary paths in a
pair of paths, the number of times the two primary paths are traversed within a joint period, and the allocation strategy of time beats and so on. The concept of the Distribution Spectrum of Interference-Spacing of nodes of a primary path is defined, and based on it a simple and general algorithm flow for solving the intrinsic period of a primary path is proposed. The research results of the paper lay an ideological and theoretical foundation for further exploring more general methods that can improve the orderly utilization of network resources.

Keywords: Network Wave, Multi-hop Transmission Networks, Primary Path, Pair of Paths, End-to-end Asymptotic Throughput, Intrinsic Period, Intrinsic Interference Intensity, Subset of Equally Spaced Nodes, Distribution Spectrum of Interference-Spacing of Nodes
I. Introduction

As we all know, in today’s networks with a certain scale (including wired networks and wireless networks), the multi-hop transmission mode based on the idea of “store and forward” is mainly used to transmit information from one end of the network to the other end (in fact, the idea of “store and forward” constitutes the core of packet switching technology [1]). When implementing multi-hop transmissions based on the idea of “store and forward”, the common practice is that an information forwarding node (commonly known as “a router”) schedules its access to network resources according to the distribution of its local resources and the dynamic requirements of network traffic. The advantage of this method for accessing network resources is that it can help to reduce the design and implementation complexity of network protocols as much as possible. However, the problem it brings about is also prominent, that is, because each node only determines the usage of network resources according to its own needs, which can be compared to the way that each node uses shared network resources from the starting point of “myopia” and “selfishness” (Note: as a matter of fact, the multiple access protocol of nodes in a multi-hop network operates based on this basic idea). This will inevitably lead to the disorderly usage of network resources (Note: the “disorder” here actually refers to “uncertainty”, which can be measured by the concept of entropy proposed in information theory [2]). And the disorderly usage of network resources leads to two problems: first, the availability of network resources used to transmit network traffic presents great uncertainty, which makes it difficult to ensure the Quality of Services (QoS) guarantee; second, due to the possible disorderly competition for network resources among different nodes, collisions of resource occupations occur from time to time, which further reduces the utilization of network resources.

Based on the starting point of “myopia” and “selfishness”, with the continuous expansion of the network scale (i.e. the continuous increase of the number of nodes accessing into the network), the uncertainty of the availability of network resources will also increase. In other words, with the continuous expansion of the network scale, the disorderly usage of network resources will be further amplified. Consider the example shown in Fig. I.1, in the subfigure (a), node 1 sends three packets, and to some extent the time intervals between the three packets present some uncertainty (i.e. disorder). It is particularly noteworthy that due to the uncertainty of the intervals of these transmissions, the remaining idle access resources also present some uncertainty. And then, another
node, i.e. node 2, can only, by following the starting point of “myopia” and “selfishness”, access into the remaining resources which have already shown a certain degree of uncertainty (Note: in order to avoid mutual interference, the accesses and transmissions of node 1 and node 2 must be staggered in time), and sends its two packets. As can be seen from the subfigure (b), the access resources occupied by nodes 1 and 2 show greater uncertainty.

![Fig. I.1 The disorder in the usage of network resources](image_url)

With the help of the concepts of entropy and conditional entropy defined in information theory [2], it is easy to explain why the disorderly usage of network resources is gradually amplified. Suppose that the three transmission time instances, which can be regarded as three random variables, corresponding to the three packets sent by node 1 in Fig. I.1 are $T_1, T_2, T_3$, respectively. And the two transmission time instances corresponding to the two packets sent by node 2 are $T_4, T_5$ (which are also regarded as random variables), respectively. The following relationship can be obtained easily according to the information theory:

$$H(T_1, T_2, T_3) = H(T_1, T_2, T_3) + H(T_4, T_5 | T_1, T_2, T_3) \geq H(T_1, T_2)$$

Among them, $H(T_1, T_2, T_3)$ is the joint entropy of the transmission time instances corresponding to the three packets sent by node 1, i.e. the uncertainty presented by node 1 with its transmissions. And, $H(T_4, T_5 | T_1, T_2, T_3)$ denotes the conditional entropy corresponding to the other two packets sent by node 2 on the premise that the access time instances of node 1 are known (that
is, the uncertainty of the accesses of node 2 with the transmissions of node 1 being given. $H(T_1, T_2, T_3, T_4, T_5)$ is the joint entropy of the random variables corresponding to the five transmission time instances of both node 1 and node 2, which measures the uncertainty of the usage of the network resources with the transmissions of the two nodes. It can be seen from Equation (I.1), with the further accesses of node 2, the occupation of network resources shows greater uncertainty (i.e. $H(T_1, T_2, T_3, T_4, T_5) \geq H(T_1, T_2, T_3)$). What is worse, in addition to the gradual “amplification” of the above disorder, this kind of disorder can be rapidly spreaded throughout the network with the interactions between adjacent nodes (just like the spread of viruses!). In short, the access mode based on the starting point of “myopia” and “selfishness” makes the disorderly usage of network resources to be continuously amplified and spreaded out in the whole network.

In the paper, the idea of introducing volatility into the network access mechanism is proposed, which reduces the disorder of the usage of network resources and accordingly improves the orderliness of the usage of network resources. Therefore, network throughput and performance for QoS guarantee can be improved. The contributions of the paper can be summarized as follows:

1) The idea of volatility is introduced into the access mechanism of multi-hop networks;

2) For a single multi-hop transmission path (i.e. a primary path), the corresponding basic theoretical framework of network wave is constructed. It is proved that the maximum asymptotic throughput of a primary path depends on its intrinsic period, and the intrinsic period of a primary path is equal to its intrinsic interference intensity;

3) An algorithm for the transmission of information blocks based on the intrinsic period of a primary path is proposed, which can maximize the asymptotic throughput of a primary path;

4) For a pair of paths composed of two primary paths, the corresponding basic theoretical framework of network wave is constructed, and the reachability of the joint period of a pair of paths is proved;

5) As for the cases of traversals with equal opportunities, an algorithm for the cooperative volatility transmission of information blocks in a pair of paths based on the set of maximum supporting elements is proposed. It is proved that the algorithm can maximize the asymptotic joint throughput of a pair of paths.

6) As for the more general cases of traversals with unequal opportunities, an algorithm for the
cooperative volatility transmission of information blocks in a pair of paths based on the set of maximum supporting elements is also proposed. Moreover, the upper bound of the asymptotic joint throughput of a pair of paths achieved by using the proposed algorithm is obtained.

7) An algorithm flow that can maximize the end-to-end asymptotic joint throughput of a pair of paths is proposed, which comprehensively covers almost all the key factors that may affect the asymptotic joint throughput of a pair of paths, such as the selection of routes, the determination of the reachable periods of the two primary paths in a pair of paths, the number of times the two primary paths are traversed within a joint period, and the allocation strategy of time beats and so on.

8) The concept of the Distribution Spectrum of Interference-Spacing of nodes of a primary path is defined, and based on it a simple and general algorithm flow for solving the intrinsic period of a primary path is proposed.

The paper is organized into nine parts: in Section I, the disorder of the usage of network resources in the current access mechanisms of the multi-hop networks is pointed out. In Section II, the basic idea of the paper is proposed, that is, volatility is introduced into the access mechanism of a multi-hop network. In Section III, the concepts of a primary path and a pair of paths are defined. In Section IV, the relationships of concurrency and interference between nodes are defined. In Section V, the concepts of the intrinsic concurrency intensity and the intrinsic interference intensity are defined, and the related basic properties are analyzed and proved. In Section VI, the concepts of the subset of equally spaced nodes, the reachable period of a primary path and the reachable joint period of a pair of paths are introduced, and the related basic properties are analyzed and proved. In Section VII, the average throughput and the asymptotic throughput of both a primary path and a pair of paths are defined, respectively, and the relevant basic properties are proved. Moreover, algorithms for the volatility transmission of information blocks in a primary path and in a pair of paths are proposed, respectively. In Section VIII, the concept of the Distribution Spectrum of Interference-Spacing of nodes of a primary path is defined, and a simple and general algorithm flow for solving the intrinsic period of a primary path is proposed. Finally, conclusions are given in Section IX.

II. Basic idea
In order to reduce the disorderly usage of network resources, it is necessary to increase the certainty for behaviors of nodes accessing into network resources. In the example shown in Fig. II.1, the access behaviors of both node 1 and node 2 show apparent certainty (i.e. notable periodicity). If most of the nodes access into the network by following such kind of notable periodicity, network resources available to be used will inevitably show good certainty (i.e. orderliness).

![Fig. II.1 Orderliness of the usage of network resources](image)

So, how to achieve the above “attractive” orderliness in networks? There are many possible ways, but the authors’ suggestion is to follow nature! As we all know, wave phenomena occur almost everywhere in nature. We can enumerate many different types of waves, such as water wave, sound wave, light wave, seismic wave, gravitational wave, and even the “material wave” studied in quantum mechanics, and so on. Just as the famous physicist Feynman pointed out in his book “the Feynman structures on Physics (Vol. I)” [3], “…except that wave oscillations appear not only as time-oscillations at one place, but propagate in space as well”. In other words, if one looks along the propagation path of a wave, she will find that the wave is moving forward. Moreover, if a point at a specific position on the propagation path of the wave being observed is fixed, it will be found that the motion of that point shows a strong regularity, that is, the so-called “vibration phenomenon”. As for information network, which is invented by human beings, its core mission is to transmit information from one end of the network to the other. Therefore, is it possible that we imitate the
wave mechanism in nature and build a similar “wave” in the network (let us call it as “network wave”), and, based on that, realize the orderly and efficient transmission of information blocks? Based on the research of the paper, the answer we give is yes. Since the physical world itself operates in this way, why can’t this mechanism give us some important enlightenments?!

In short, the core idea of building network wave can be summarized as follows: make each forwarding node on a multi-hop path regularly switch back and forth between “forwarding state” and “silence state”. And the state switching behaviors between adjacent nodes on the path are well coordinated in time (that is, in the same time beat, we make some nodes with a certain spatial distance on the path to be in the “forwarding state” and the other nodes to be in the “silence state”). In this way, the network wave can be established, and the information can be transmitted from one end to the other along the multi-hop path. Obviously, with the establishment of network wave, the access behaviors of nodes on the path do have good certainty (refer to the subsequent chapters of this paper for the detailed theory of network wave and the corresponding proposed algorithms).

III. The concepts of a primary path and a pair of paths

In this section, the concepts of both a primary path and a pair of paths are given. The set and subsets of sending nodes contained in a primary path and in a pair of paths are defined, respectively. Moreover, the identification method of any sending node in a pair of paths is given.

Definition III.1: A primary path

Consider that in a multi-hop path consisting of \( N+1 \) nodes, information blocks are relayed from the node 1 (i.e. the source node) through the node 2, the node 3, ..., the node \( i, ..., \) and the node \( N \ (\geq 1) \) to the final node \( N+1 \) (i.e. the destination node) (Note: 1, it is assumed that all the former \( N \) nodes have the function of sending/forwarding information blocks, and the last node \( N+1 \) only has the function of receiving information blocks. Therefore, for convenience and consistency of discussion, the node \( N+1 \) is generally not mentioned in the following discussions unless otherwise specified. 2, for the sake of convenience, it is assumed that the source node always has saturated traffic loads waiting to be transmitted). A multi-hop path, which transmits information
blocks from the source node to the destination node, consisting of \( N \) sending nodes (whose sequence numbers are set in turn by using incrementing positive integers) arranged in sequence is called as a primary path, which is denoted as \( P_i \) \( (i \in \mathbb{Z}^+) \) is the identification number of a primary path). In the following part of the paper, for the convenience of description, the node in a primary path with a larger sequence number is said to be the “downstream node” of the node with a smaller sequence number, and, conversely, the node with a smaller sequence number is the “upstream node” of the node with a larger sequence number. Without loss of generality, it is assumed that in a primary path the propagation of an information block from one node to its immediate downstream node takes only one time beat, which is denoted as \( \Delta t \).

**Definition III.2: The set of sending nodes of a primary path and the spacing between nodes**

The set of nodes, consisting of all the sending nodes in a primary path \( P_i \), is defined as the set of sending nodes of \( P_i \), which is denoted as \( S_{P_i} \). The number of elements in the set \( S_{P_i} \) is denoted as \( N_{S_{P_i}} \). As for the primary path \( P_i \), the spacing between a sending node with a sequence number of \( k \) and a sending node with a sequence number of \( l \) on \( P_i \) is defined as:

\[
d_{S_{P_i}} = |k - l|.
\]

**Definition III.3: A subset of sending nodes of a primary path**

A subset of sending nodes of a primary path \( P_i \) is defined as a subset of \( S_{P_i} \), which is denoted as \( S'_{P_i} \) \( (S'_{P_i} \subseteq S_{P_i}) \). The number of elements in the set \( S'_{P_i} \) is denoted as \( N_{S'_{P_i}} \).

**Definition III.4: A pair of paths**

Consider two primary paths, which are denoted as \( P_{[1]} \) and \( P_{[2]} \) respectively, where \( P_{[i]} \) \( (i = 1, 2) \) is a multi-hop transmission path composed of \( N_{S_{P_i}} + 1 \) \( (i = 1, 2) \) nodes. As for the primary path \( P_{[i]} \) \( (i = 1, 2) \), information blocks are relayed from the node 1 of it (i.e. the source node of \( P_{[i]} \)) through the node 2, the node 3, ..., the node \( i \), ..., and the node \( N_{S_{P_i}} \) \( (\geq 1) \) to the final
node \( N_{x_{j+i+1}} \) (i.e. the destination node of \( P_{(i)} \)) (Note: for the sake of convenience, it is assumed that source nodes always have saturated traffic loads waiting to be transmitted). The set composed of two primary paths is defined as a pair of paths, and is denoted as \( P_{[1,2]} \).

**Definition III.5: The set of sending nodes of a pair of paths**

The set of nodes, consisting of all the sending nodes in a pair of paths \( P_{[1,2]} \), is defined as the set of sending nodes of \( P_{[1,2]} \), which is denoted as \( S_{[1,2]} \). The number of elements in the set \( S_{[1,2]} \) is denoted as \( N_{S_{[1,2]}} \). According to the definition of a pair of paths, it is evident that

\[
N_{S_{[1,2]}} = N_{S_{[1]}} + N_{S_{[2]}}.
\]

**Definition III.6: A subset of sending nodes of a pair of paths**

A subset of sending nodes of a pair of paths \( P_{[1,2]} \) is defined as a subset of \( S_{[1,2]} \), which is denoted as \( S'_{[1,2]} \) (\( S'_{[1,2]} \subseteq S_{[1,2]} \)). The number of elements in the set \( S'_{[1,2]} \) is denoted as \( N_{S'_{[1,2]}} \).

**Definition III.7: A specific sending node in a pair of paths**

As for a pair of paths \( P_{[1,2]} \) composed of two primary paths, i.e. \( P_{[1]} \) and \( P_{[2]} \), let

\[
n_{x_{j+i}} \quad (i=1,2; 1 \leq j \leq N_{x_{j+i}})
\]

denote a node belonging to the primary path \( P_{[i]} \) (\( i=1,2 \)) whose sequence number in \( P_{[i]} \) is \( j \).

IV. **Definitions of the relationships of concurrency and interference between nodes**

In this section, an interference model between nodes is defined. The possible relationships of concurrency and interference between nodes in a pair of paths are defined. The mutual exclusion rules between the relationships of concurrency and interference are defined. Moreover, the second-order concurrency subset consisting of some sending nodes in a pair of paths and the related second-order concurrency rules are also defined.
Definition IV.1: Interference model between nodes and effectiveness of information transmission

For a node $n_i$ in the network, define $r_{iTC}^r$, $r_{iIC}^r$, and $r_{iT}^r$ as its transmission coverage radius, interference coverage radius, and interference tolerance radius, respectively. And, generally speaking, the relation $r_{iTC}^r \leq r_{iIC}^r$ holds. Furthermore, we make the following defines:

1) At a given instant, for a node $n_i$ in its receiving state, if there is a node $n_k$ in its transmitting state ($n_k$ is not the corresponding transmitting node $n_j$ of node $n_i$), which brings about the fact that $n_i$ is within the interference coverage range of $n_k$ (assuming it is a circle with its center being $n_k$ and a radius of $r_{iIC}^r$), and at the same time $n_k$ is also within the interference tolerance range of $n_i$ (assuming it is a circle with its center being $n_i$ and a radius of $r_{iT}^r$), it is considered that node $n_k$ interferes with the information transmission from node $n_j$ to node $n_i$;

2) At a given moment, for a node $n_i$ in its receiving state, if there is at least one node $n_k$ in its transmitting state interfering with the information transmission from node $n_j$ to node $n_i$, it is considered that the information transmission from node $n_j$ to node $n_i$ is ineffective;

3) At a given moment, for a node $n_i$ in its receiving state, if $n_i$ is within the transmission coverage range of $n_j$ (assuming it is a circle with its center being $n_j$ and a radius of $r_{iTC}^r$) and at the same time the information transmission from node $n_j$ to node $n_i$ is not interfered by other nodes, it is considered that the information transmission from node $n_j$ to node $n_i$ is effective;

Note: Considering the fact of the unreliability of a wireless transmission channel, effective information transmission does not necessarily mean that the corresponding information transmission must be successful.
Definition IV.2: The concurrency relationship between two nodes

If a node \( n_{i,j} \) \((i = 1, 2; 1 \leq j \leq N_{s_{i,j}})\) and a node \( n_{k,l} \) \((k = 1, 2; 1 \leq l \leq N_{s_{k,l}})\) (possibly they are the same node) in a pair of paths \( P_{[1,2]} \) can be activated within the same time beat (i.e., both of them are in the state of sending information blocks in the same time duration) without interfering with the correct receptions of information blocks of the corresponding receiving nodes, we say that the two nodes can be concurrent within the same time beat, i.e. there is a concurrency relationship between these two nodes, which is denoted as \( n_{i,j} \parallel n_{k,l} \). Specifically, it is defined that any single sending node has a concurrency relationship with itself, i.e. \( n_{i,j} \parallel n_{i,j} \).

Definition IV.3: The interference relationship between two nodes

If two different nodes \( n_{i,j} \) \((i = 1, 2; 1 \leq j \leq N_{s_{i,j}})\) and \( n_{k,l} \) \((k = 1, 2; 1 \leq l \leq N_{s_{k,l}})\) in a pair of paths \( P_{[1,2]} \) (Note: saying that two nodes are different means \( i \neq k \) or \( j \neq l \), which is also denoted as \( n_{i,j} \neq n_{k,l} \)) are activated within a same time beat, which brings about the interference with the correct reception of at least one of the corresponding receiving nodes, we say that the two nodes cannot be concurrent within the same time beat, i.e. there is an interference relationship between the two sending nodes, which is denoted as \( n_{i,j} >\!< n_{k,l} \).

Rule IV.1: The mutual exclusion rules between the relationships of concurrency and interference

1) If two different nodes \( n_{i,j} \) and \( n_{k,l} \) in a pair of paths \( P_{[1,2]} \) have an interference relationship, i.e. \( n_{i,j} >\!< n_{k,l} \) holds, then \( n_{i,j} \parallel n_{k,l} \) must not hold;

2) If two nodes \( n_{i,j} \) and \( n_{k,l} \) (possibly they are the same node) in a pair of paths \( P_{[1,2]} \) have a concurrency relationship, i.e. \( n_{i,j} \parallel n_{k,l} \) holds, then \( n_{i,j} >\!< n_{k,l} \) must not hold.

Definition IV.4: A second-order concurrency subset
If there is a concurrency relationship between any two nodes in a nonempty subset $S'_{[n,2]}$ of sending nodes in a pair of paths $P_{[1,2]}$, i.e. $n_{i,j} \parallel n_{k,l} \ (n_{i,j}, n_{k,l} \in S'_{[n,2]} )$, then $S'_{[n,2]}$ is defined as a second-order concurrency subset of $P_{[1,2]}$. In particular, a subset $S'_{[n,2]}$ containing only one sending node is also regarded as a second-order concurrency subset.

### Rule IV.2: The second-order concurrency rule

It is assumed that for any second-order concurrency subset of nodes concurrent transmissions with all the member nodes in the set participating in it at the same time beat can be achieved (i.e. no interference with each other), which is regarded as the second-order concurrency rule. (Note: this rule is an approximation of higher-order concurrency relationships, and is strictly consistent with the interference model of nodes given in Definition IV.1).

### V. Concepts of the intrinsic concurrency intensity and the intrinsic interference intensity and some related properties

In this section, the concepts of the intrinsic concurrency intensity for a subset of nodes, the intrinsic interference intensity for a subset of nodes, the intrinsic concurrency connection degree of a node and the intrinsic interference connection degree of a node are proposed. The maximum second-order concurrency subset, the second-order interference subset, the maximum second-order interference subset, the subset without interference, the subset with interference, and the subset of nodes with dominant intrinsic interference intensity are defined, respectively. The necessary and sufficient conditions for a nonempty subset of nodes to be a second-order concurrency subset are given (Theorem V.1). The concurrency rules between nodes in a primary path are defined. Both the structural property of a second-order interference subset in a primary path (Theorem V.2) and the continuity of a maximum second-order interference subset (Corollary V.2.1) are proved. The splitting characteristics of a subset of nodes with dominant intrinsic interference intensity are proved (Theorem V.3). Moreover, the relationship between the intrinsic interference intensity and the intrinsic concurrency intensity of a subset of nodes with dominant intrinsic interference intensity is given (Corollary V.3.1).
Definition V.1: The maximum second-order concurrency subset of a nonempty subset of nodes

In all of the second-order concurrency subsets contained in a nonempty subset $S'_{n_{i,j}}$ of nodes in a pair of paths $P_{i,j}$, the second-order concurrency subset with the largest number of elements is defined as the maximum second-order concurrency subset of $S'_{n_{i,j}}$ (Note: there may be more than one such second-order concurrency subset).

Definition V.2: The intrinsic concurrency intensity of a nonempty subset of nodes

The number of elements contained in a maximum second-order concurrency subset of a nonempty subset $S'_{n_{i,j}}$ of nodes in a pair of paths $P_{i,j}$ is defined as the intrinsic concurrency intensity of the set, which is denoted as $C'_{S'_{n_{i,j}}}$. 

Definition V.3: The intrinsic concurrency intensity of a primary path

The intrinsic concurrency intensity of the set $S_{n_{i}}$ composed of all the sending nodes of a primary path $P_{i}$ is defined as the intrinsic concurrency intensity of the primary path $P_{i}$, which is denoted as $C'_{S_{n_{i}}}$. 

Definition V.4: The intrinsic concurrency intensity of a pair of paths

The intrinsic concurrency intensity of the set $S_{n_{i,j}}$ composed of all the sending nodes of a pair of paths $P_{i,j}$ is defined as the intrinsic concurrency intensity of the pair of paths $P_{i,j}$, which is denoted as $C'_{S_{n_{i,j}}}$. 

Definition V.5: The concurrency connection degree of a node

As for a sending node $n_{i,j}$ ($\in S_{n_{i,j}}$), the number of sending nodes (excluding the node $n_{i,j}$ itself) in a nonempty subset $S'_{n_{i,j}}$ of nodes in a pair of paths $P_{i,j}$ having concurrency
relationships with it is defined as the concurrency connection degree of the node \( n_{i,j} \) in \( S'_{i,j} \), which is denoted as \( D_{S'_{i,j}}[n_{i,j}] \).

**Definition V.6: The intrinsic concurrency connection degree of nodes in a nonempty subset of nodes**

The maximum value among the concurrency connection degrees of all the sending nodes \( n_{i,j} \in S'_{i,j} \) in a nonempty subset \( S'_{i,j} \) of nodes in a pair of path \( P_{[1,2]} \) is defined as the intrinsic concurrency connection degree of nodes in \( S'_{i,j} \), which is denoted as \( D'_{S'_{i,j}} \).

**Definition V.7: The intrinsic concurrency connection degree of nodes in a primary path**

The maximum value among the concurrency connection degrees of all the sending nodes \( n_{i,j} \in S_{i,j} \) in the set \( S_{i,j} \) of nodes in a primary path \( P_{[i]} \) is defined as the intrinsic concurrency connection degree of nodes in \( P_{[i]} \), which is denoted as \( D'_{S_{i,j}} \).

**Definition V.8: The intrinsic concurrency connection degree of nodes in a pair of paths**

The maximum value among the concurrency connection degrees of all the sending nodes \( n_{i,j} \in S_{i,j} \) in the set \( S_{i,j} \) of nodes in a pair of paths \( P_{[1,2]} \) is defined as the intrinsic concurrency connection degree of nodes in \( P_{[1,2]} \), which is denoted as \( D'_{S_{i,j}} \).

**Definition V.9: The second-order interference subset**

If any two different nodes in a nonempty subset \( S'_{i,j} \) of nodes in a pair of paths \( P_{[1,2]} \) have interference relationship, i.e. \( n_{i,j} > n_{i,j} \in S'_{i,j}, n_{i,j} \in S'_{i,j}, n_{i,j} \neq n_{i,j} \) holds, then the subset \( S'_{i,j} \) is defined as a second-order interference subset of \( P_{[1,2]} \) (Note: a second-order interference subset contains at least two elements).

**Definition V.10: The subset without interference and the subset with interference**
If a nonempty subset \( S'_{\eta_{[1,2]}} \) of nodes in a pair of paths \( P_{[1,2]} \) does not contain any second-order interference subset, it is defined as a subset without interference, otherwise it is defined as a subset with interference.

**Lemma V.1.1: The subset without interference forms a second-order concurrency subset**

If a nonempty subset \( S'_{\eta_{[1,2]}} \subseteq S_{\eta_{[1,2]}} \) of nodes is a subset without interference, all the sending nodes in it form a second-order concurrency subset.

**Proof:**

**Case 1:** Number of sending nodes in \( S'_{\eta_{[1,2]}} \geq 2 \)

Here, reduction to absurdity is adopted. Assuming that all the sending nodes in \( S'_{\eta_{[1,2]}} \) cannot form a second-order concurrency subset, it is evident that at least two of these sending nodes must have interference relationship with each other. Therefore, these two nodes form a second-order interference subset, which is contrary to the proposition.

**Case 2:** Number of sending nodes in \( S'_{\eta_{[1,2]}} = 1 \)

According to the definition of a second-order concurrency subset (Definition IV.4), a subset containing only one sending node forms a second-order concurrency subset.

**Lemma V.1.2: A second-order concurrency subset is a subset without interference**

If a nonempty subset \( S'_{\eta_{[1,2]}} \) of nodes is a second-order concurrency subset, it is also a subset without interference.

**Proof:**

It can be directly proved based on the mutual exclusion rules between the relationships of concurrency and interference (Rule IV.1), the definition of a second-order concurrency subset (Definition IV.4) and the definition of a subset without interference (Definition V.10).

**Theorem V.1: The necessary and sufficient condition for a nonempty subset to be a second-order concurrency subset**
The necessary and sufficient condition for a nonempty node subset \( S'_{\tilde{\eta},[1]} \) to be a second-order concurrency subset is that the subset is a subset without interference.

**Proof:**

It can be proved based on Lemma V.1.1 and Lemma V.1.2.

**Definition V.11: The maximum second-order interference subset of a subset with interference**

Among all the second-order interference subsets contained in a subset \( S'_{\tilde{\eta},[1]} \) with interference in a pair of paths \( P_{[1,2]} \), the subset with the maximum number of elements is defined as a maximum second-order interference subset of \( S'_{\tilde{\eta},[1]} \) (Note: there may be more than one such second-order interference subset).

**Definition V.12: The intrinsic interference intensity of a nonempty subset of nodes**

If a nonempty subset \( S'_{\tilde{\eta},[1]} \) of nodes in a pair of paths \( P_{[1,2]} \) is a subset with interference, the number of elements contained in its maximum second-order interference subset is defined as its intrinsic interference intensity, which is denoted as \( I'_{S'_{\tilde{\eta},[1]}} \). Moreover, if \( S'_{\tilde{\eta},[1]} \) is a subset without interference, its intrinsic interference intensity is defined as \( I'_{S'_{\tilde{\eta},[1]}} = 1 \).

**Definition V.13: The intrinsic interference intensity of a primary path**

The intrinsic interference intensity of the set \( S_{\tilde{\eta},[1]} \) composed of all the sending nodes of a primary path \( P_{[1]} \) is defined as the intrinsic interference intensity of \( P_{[1]} \), which is denoted as \( I'_{S_{\tilde{\eta},[1]}} \).

**Definition V.14: The intrinsic interference intensity of a pair of paths**

The intrinsic interference intensity of the set \( S_{\tilde{\eta},[2]} \) composed of all the sending nodes of a pair of paths \( P_{[1,2]} \) is defined as the intrinsic interference intensity of \( P_{[1,2]} \), which is denoted as \( I'_{S_{\tilde{\eta},[2]}} \).

**Definition V.15: The interference connection degree of a node**
As for a sending node \( n_{i,j} (\in S'_{P_1}) \), the number of sending nodes in a nonempty subset \( S'_{P_2} \) of nodes in a pair of paths \( P_{[1,2]} \) having interference relationships with it is defined as the interference connection degree of the node \( n_{i,j} \) in \( S'_{P_1} \), which is denoted as \( D_{S_{[P_1,P_2]}^{i,j}} \).

**Definition V.16: The intrinsic interference connection degree of nodes in a nonempty subset of nodes**

The maximum value among the interference connection degrees of all the sending nodes \( n_{i,j} (\in S'_{P_1}) \) in a nonempty subset \( S'_{P_1} \) of nodes in a pair of paths \( P_{[1,2]} \) is defined as the intrinsic interference connection degree of nodes in \( S'_{P_1} \), which is denoted as \( D_{S_{[P_1,P_2]}^{i,j}}^{*} \).

**Definition V.17: The intrinsic interference connection degree of nodes in a primary path**

The maximum value among the interference connection degrees of all the sending nodes \( n_{i,j} (\in S_{P_1}) \) in the set \( S_{P_1} \) of nodes in a primary path \( P_{[1,j]} \) is defined as the intrinsic interference connection degree of nodes in \( P_{[1,j]} \), which is denoted as \( D_{S_{[P_1,P_2]}^{i,j}}^{*} \).

**Definition V.18: The intrinsic interference connection degree of nodes in a pair of paths**

The maximum value among the interference connection degrees of all the sending nodes \( n_{i,j} (\in S_{P_1}) \) in the set \( S_{P_1} \) of nodes in a pair of paths \( P_{[1,2]} \) is defined as the intrinsic interference connection degree of nodes in \( P_{[1,2]} \), which is denoted as \( D_{S_{[P_1,P_2]}^{i,j}}^{*} \).

**Definition V.19: A nonempty subset of nodes with dominant intrinsic interference intensity**

In a nonempty subset \( S'_{P_1} \) of nodes, if its intrinsic interference connection degree \( D_{S_{[P_1,P_2]}^{i,j}}^{*} \) is less than its intrinsic interference intensity \( I_{S_{[P_1,P_2]}^{i,j}} \), i.e. \( D_{S_{[P_1,P_2]}^{i,j}}^{*} < I_{S_{[P_1,P_2]}^{i,j}} \), the subset \( S'_{P_1} \) is defined as a subset of nodes with dominant intrinsic interference intensity.

**Definition V.20: A primary path with dominant intrinsic interference intensity**
As for a primary path $P_{(i)}$, if its intrinsic interference connection degree $D_{s_{(i)}}^{*} < D_{s_{(i)}}^{*}$ is less than its intrinsic interference intensity $I_{s_{(i)}}^{*}$, i.e. $D_{s_{(i)}}^{*} < I_{s_{(i)}}^{*}$, the primary path $P_{(i)}$ is defined as a primary path with dominant intrinsic interference intensity.

**Definition V.21: A pair of paths with dominant intrinsic interference intensity**

As for a pair of paths $P_{(1,2)}$, if its intrinsic interference connection degree $D_{s_{(1,2)}}^{*} < D_{s_{(1,2)}}^{*}$ is less than its intrinsic interference intensity $I_{s_{(1,2)}}^{*}$, i.e. $D_{s_{(1,2)}}^{*} < I_{s_{(1,2)}}^{*}$, the pair of paths $P_{(1,2)}$ is defined as a pair of paths with dominant intrinsic interference intensity.

**Rule V.1: The concurrency rules between nodes in a primary path**

If two different nodes $n_{i,j}$ and $n_{i,k}$ in a primary path $P_{(i)}$ have concurrency relationship (i.e. $n_{i,j} \parallel n_{i,k}$ ($n_{i,j}, n_{i,k} \in S_{t_{i}}, j < k$), and node $n_{i,k+1}$ is another sending node of $P_{(i)}$ (i.e. $n_{i,k+1} \in S_{t_{i}}$), then it can be deduced that node $n_{i,j}$ and node $n_{i,k+1}$ also have concurrency relationship. (Note: this rule can be summarized as: $n_{i,j} \parallel n_{i,k} \Rightarrow n_{i,j} \parallel n_{i,k+1}$).

**Rule V.2: The concurrency rules between nodes in a primary path**

If two different nodes $n_{i,j}$ and $n_{i,k}$ in a primary path $P_{(i)}$ have concurrency relationship (i.e. $n_{i,j} \parallel n_{i,k}$ ($n_{i,j}, n_{i,k} \in S_{t_{i}}, j < k$), and node $n_{i,j-1}$ is another sending node of $P_{(i)}$ (i.e. $n_{i,j-1} \in S_{t_{i}}$), then it can be deduced that node $n_{i,j-1}$ and node $n_{i,k}$ also have concurrency relationship. (Note: this rule can be summarized as: $n_{i,j} \parallel n_{i,k} \Rightarrow n_{i,j-1} \parallel n_{i,k}$).

**Note:**

1) We collectively refer to the above two rules as “the interference reduction rule between nodes”;

2) Rule V.1 can be summarized as: $n_{i,j} \parallel n_{i,k} \Rightarrow n_{i,j} \parallel n_{i,k+1}$;
3) **Rule V.2** can be summarized as: \( n_{i,j} \parallel n_{i,k} \Rightarrow n_{j,i} \parallel n_{k,i} \).

**Definition V.22: The continuous second-order interference subset in a primary path**

Among all the sending nodes in a second-order interference subset \( S'_{i,j} \) of a primary path \( P_{[i]} \), let node \( n_{i,j} (\in S'_{i,j}) \) have the smallest sequence number (the sequence number of node \( n_{i,j} \) is \( j \)), and node \( n_{i,k} (\in S'_{i,j}, k \neq j) \) have the largest sequence number (the sequence number of node \( n_{i,k} \) is \( k \)). If \( S'_{i,j} \) includes all the nodes with their sequence numbers between \( j \) and \( k \) in \( P_{[i]} \), then \( S'_{i,j} \) is regarded as a continuous second-order interference subset in \( P_{[i]} \).

**Theorem V.2: The structural property of a second-order interference subset in a primary path**

Assuming that a primary path \( P_{[i]} \) satisfies both **Rule V.1** and **Rule V.2**, in which two different nodes \( n_{i,j} \) and \( n_{i,k} \) do not have concurrency relationship (i.e. \( n_{i,j} < n_{i,k} (n_{i,j}, n_{i,k} \in S'_{i,j}, j < k) \)), then the node set \( S'_{i,j} = \{ n_{i,j} | n_{i,j} \in S'_{i,j}; j \leq l \leq k \} \) composed of all the nodes (including \( j \) and \( k \)) with sequence numbers between \( j \) and \( k \) in \( P_{[i]} \) is a continuous second-order interference subset.

**Proof:**

Reduction to absurdity is adopted. Assuming that node subset \( S'_{i,j} = \{ n_{i,j} | n_{i,j} \in S'_{i,j}; j \leq l \leq k \} \) is not a second-order interference subset, therefore, there are at least two nodes \( n_{i,j} \) and \( n_{i,m} \) (\( j \leq l < m \leq k \)) that can be concurrent, i.e. \( n_{i,j} \parallel n_{i,m} (j < l < m \leq k) \). According to **Rule V.1**, we know that \( n_{i,j} \parallel n_{i,m} \) holds through several iterations. Since \( n_{i,j} \parallel n_{i,k} \), according to **Rule V.2**, \( n_{i,j} \parallel n_{i,k} \) can be obtained through several iterations, which is contradictory to the premise of \( n_{i,j} > n_{i,k} \) in the proposition. Therefore, \( S'_{i,j} \) must be a second-order interference subset. Furthermore, combined with the definition of the continuous second-order interference subset in a...
primary path (Definition V.22), it can be seen that $S'_{P_{ij}}$ is a continuous second-order interference subset in $P_{ij}$. ■

**Corollary V.2.1: The continuity of a maximum second-order interference subset in a primary path**

On the premise of satisfying both Rule V.1 and Rule V.2, any maximum second-order interference subset in a primary path $P_{ij}$ containing at least one second-order interference subset is a continuous second-order interference subset.

**Proof:**

Reduction to absurdity is adopted. Consider a subset $S'_{P_{ij}}$ is a maximum second-order interference subset in a primary path $P_{ij}$, and the number of elements of $S'_{P_{ij}}$ is $I'_{S'_{P_{ij}}}$. Without losing generality, assuming that node $n_{ij}$ has the smallest sequence number (i.e. $j$) in $S'_{P_{ij}}$ and node $n_{jk}$ has the largest sequence number (i.e. $k$) in $S'_{P_{ij}}$. If $S'_{P_{ij}}$ is not continuous, there must be at least one node in $P_{ij}$ whose sequence number is between $j$ and $k$, and the node does not belong to $S'_{P_{ij}}$. Therefore, it can be deduced from Theorem V.2 that all nodes with sequence numbers between $j$ and $k$ (including $j$ and $k$) can form a continuous second-order interference subset, and the number of elements in this subset must be greater than $I'_{S'_{P_{ij}}}$, which contradicts the premise that $S'_{P_{ij}}$ is a maximum second-order interference subset. ■

**Theorem V.3: The splitting characteristics of a subset of nodes with dominant intrinsic interference intensity**

If the intrinsic interference intensity of a subset $S'_{P_{ij2}}$ of sending nodes with dominant intrinsic interference intensity is $I'_{S'_{P_{ij2}}}$, all the nodes contained in it can be divided into mutually disjoint second-order concurrency subsets of nodes.

**Proof:**
Because the intrinsic interference intensity of $S'_{(1,2)}$ is $I_{S'_{(1,2)}}$, at least $I_{S'_{(1,2)}}$ sending nodes that can jointly form a second-order interference subset can be found in $S'_{(1,2)}$. Starting from these $I_{S'_{(1,2)}}$ sending nodes, we can construct $I_{S'_{(1,2)}}$ subsets of sending nodes which contain only one sending node (from one of these $I_{S'_{(1,2)}}$ sending nodes) each and are mutually disjoint to each other.

On the premise of keeping these subsets of sending nodes as second-order concurrency subsets, by adding the remaining sending nodes in $S'_{(1,2)}$ into these subsets one by one, $I_{S'_{(1,2)}}$ mutually disjoint second-order concurrency subsets of nodes can be finally formed. Assuming that at least one sending node $n$ in $S'_{(1,2)}$ cannot be added into any of these $I_{S'_{(1,2)}}$ second-order concurrency subsets, which indicates that in each of these $I_{S'_{(1,2)}}$ second-order concurrency subsets at least one sending node, which has interference relationship with the node $n$, can be found. Therefore, it can be seen that the interference connection degree of node $n$ in $S'_{(1,2)}$ must not be less than $I_{S'_{(1,2)}}$, which is contrary to the assumption that the set $S'_{(1,2)}$ is dominated by intrinsic interference intensity. Therefore, such a node $n$ does not exist at all. That is to say, all nodes contained in $S'_{(1,2)}$ can be divided into $I_{S'_{(1,2)}}$ mutually disjoint second-order concurrency subsets of nodes. □

Corollary V.3.1: The relationship between the intrinsic interference intensity and the intrinsic concurrency intensity of a set of nodes with dominant intrinsic interference intensity

Let the intrinsic interference intensity and intrinsic concurrency intensity of a set $S'_{(1,2)}$ of nodes with dominant intrinsic interference intensity be $I_{S'_{(1,2)}}$ and $C_{S'_{(1,2)}}$, respectively. If the number of elements in the set $S'_{(1,2)}$ is $N_{S'_{(1,2)}}$, there is $I_{S'_{(1,2)}} \cdot C_{S'_{(1,2)}} \geq N_{S'_{(1,2)}}$.

Proof:

According to Theorem V.3, $S'_{(1,2)}$ can be divided into $I_{S'_{(1,2)}}$ nonempty and mutually disjoint second-order concurrency subsets, and the number of elements in each subset will not
exceed the intrinsic concurrency intensity $C_{S'[\eta, \eta]}'$ (according to the definition of intrinsic concurrency intensity of nonempty subsets of nodes (Definition V.2)), so $I_{S'[\eta, \eta]}' : C_{S'[\eta, \eta]}' \geq N_{S'[\eta, \eta]}'$ is valid.

**Note:** Theorem V.3 and Corollary V.3.1 do not depend on whether Rule V.1 and Rule V.2 are satisfied or not. Therefore, they are more general.

**Corollary V.3.2:** The maximum second-order interference subsets contained in a set of nodes with dominant intrinsic interference intensity do not intersect to each other

If there are multiple different maximum second-order interference subsets in a set $S'[\eta, \eta]$ of nodes with dominant intrinsic interference intensity, these maximum second-order interference subsets do not intersect to each other.

**Proof:**

Reduction to absurdity is adopted. Assuming that the intrinsic interference intensity of $S'[\eta, \eta]$ is $I_{S'[\eta, \eta]}'$. Both node subsets $S'_A (\subseteq S'[\eta, \eta])$ and $S'_B (\subseteq S'[\eta, \eta])$ are maximum second-order interference subsets of $S'[\eta, \eta]$, and $S'_A \neq S'_B$, so there is at least one node $j (\notin S'_A, \in S'_B)$.

Assuming that $S'_A \cap S'_B \neq \emptyset$, there is at least one node $k (\in S'_A \cap S'_B)$. According to the definition of the maximum second-order interference subset (Definition V.11), there is an interference relationship between $k (\in S'_A)$ and $I_{S'[\eta, \eta]}' - 1$ nodes in $S'_A$, and the node $k (\in S'_B)$ and the node $j (\notin S'_A, \in S'_B)$ also interfere with each other. Therefore, it can be deduced that the interference connection degree of node $k$ in $S'[\eta, \eta]$ is not less than $I_{S'[\eta, \eta]}'$, which contradicts with the premise that $S'[\eta, \eta]$ is dominated by its intrinsic interference intensity, so $S'_A \cap S'_B = \emptyset$.

**VI.** The concepts of the subset of equally spaced nodes, reachable period, reachable joint period and related basic properties
In this section, the subset of equally spaced nodes with a given initial phase in a primary path is defined. The concepts of reachable period and intrinsic period of a primary path are given. The concepts of reachable joint period and intrinsic joint period of a pair of paths are proposed. The matrix of joint concurrency relationship between subsets of equally spaced nodes of a pair of paths is defined. The binary matrix and its corresponding set of supporting elements are defined, and the maximum reachable supporting number of a binary matrix is further defined. The reachability of the period of a primary path is proved, and the important conclusion that the intrinsic period of a primary path is equal to its intrinsic interference intensity is obtained (Theorem VI.1). It is further pointed out that subsets of equally spaced nodes corresponding to the intrinsic period of a primary path cannot form a second-order concurrency relationship between each other (Corollary VI.1.1). Moreover, the lower bound (Theorem VI.2) and the upper bound (Theorem VI.3) of the intrinsic interference intensity of a pair of paths are given, respectively. The reachability of the joint period of a pair of paths is proved (Theorem VI.4-Theorem VI.7).

**Definition VI.1:** The subset of equally spaced nodes with a given initial phase in a primary path

As for a primary path $P_{i}$, given the spacing between adjacent nodes as $T_{s_{i}} (T_{s_{i}} \in \mathbb{Z}^{+}, 1 \leq T_{s_{i}} \leq N_{s_{i}}, i = 1, 2)$ and the initial phase as $\theta_{s_{i}} (\theta_{s_{i}} \in \mathbb{Z}^{+}, 1 \leq \theta_{s_{i}} \leq T_{s_{i}}, i = 1, 2)$, respectively. The subset of nodes $\Phi'_{s_{i}} (\theta_{s_{i}}, T_{s_{i}}) \triangleq \{ n_{i;j} | j = \theta_{s_{i}} + k \cdot T_{s_{i}}; 1 \leq j \leq N_{s_{i}}; k \in \mathbb{Z}, k \geq 0 \}$ in $P_{i}$ is defined as a subset of equally spaced nodes with its initial phase being $\theta_{s_{i}}$ and the spacing between adjacent nodes being $T_{s_{i}}$. If the set $\Phi'_{s_{i}} (\theta_{s_{i}}, T_{s_{i}})$ is also a second-order concurrency subset, it is defined as a second-order concurrency subset with equally spaced nodes in a primary path $P_{i}$. (Note: in order to make $\Phi'_{s_{i}} (\theta_{s_{i}}, T_{s_{i}})$ contain at least one valid element of node, only the case of $T_{s_{i}} \leq N_{s_{i}}$ is considered in the paper).

**Definition VI.2:** The reachable period of a primary path

24 / 87
For a certain value \( T_{s_{0}} \) \((T_{s_{0}} \in \mathbb{Z}^{+}, 1 \leq T_{s_{0}} \leq N_{s_{0}}, i = 1, 2)\), if all the subsets \( \Phi_{s_{0}}(\theta_{s_{0}}, T_{s_{0}}) \) of equally spaced nodes in a primary path \( P_{i} \) \( (i = 1, 2) \) with their initial phases between 1 and \( T_{s_{0}} \) (including 1 and \( T_{s_{0}} \)) are second-order concurrency subsets, then \( T_{s_{0}} \) is defined as a reachable period of \( P_{i} \), otherwise \( T_{s_{0}} \) is regarded as an unreachable period of \( P_{i} \).

**Definition VI.3: The reachable joint period of a pair of paths**

Given the corresponding reachable period \( T_{s_{0}} \) \((i = 1, 2)\) of the primary path \( P_{i} \) \((i = 1, 2)\), for a certain value \( T_{s_{0},2} \) \((\in \mathbb{Z}^{+}, \geq 1)\), consider the sequence of continuous \( T_{s_{0},2} \) time beats. If all the following conditions can be satisfied for a pair of paths \( P_{1,2} \), \( T_{s_{0}} \) is defined as a reachable joint period of \( P_{1,2} \):

1) **Non-emptiness:** at least one subset \( \Phi_{s_{0}}(\theta_{s_{0}}, T_{s_{0}}) \) \((1 \leq \theta_{s_{0}} \leq T_{s_{0}}, i = 1, 2)\) of equally spaced nodes is active in each time beat, i.e. all the nodes of the subset are in sending state.

2) **Concurrency:** all the nodes activated simultaneously in each time beat form a second-order concurrency subset.

3) **Uniqueness:** in each time beat, the number of subsets of equally spaced nodes, which belong to the same primary path and are activated at the same time, shall not exceed one.

4) **Uniform ergodicity:** within \( T_{s_{0},2} \) time beats, all subsets of equally spaced nodes \( \Phi_{s_{0}}(\theta_{s_{0}}, T_{s_{0}}) \) \((1 \leq \theta_{s_{0}} \leq T_{s_{0}}, i = 1, 2)\) should be activated at least once, and all subsets of equally spaced nodes belonging to the same primary path should be activated the same number of times (Note: subsets of equally spaced nodes belonging to different primary paths can be activated different number of times).

**Definition VI.4: The intrinsic period of a primary path**

The minimum value of all the reachable periods of a primary path \( P_{i} \) is defined as the
intrinsic period of $P_i$, and it is denoted as $T^*_{S_{i}}$.

**Definition VI.5: The intrinsic period of a pair of paths**

The minimum value of all the reachable joint periods of a pair of paths $P_{[1,2]}$ is defined as the intrinsic joint period of $P_{[1,2]}$, and it is denoted as $T^*_{S_{[1,2]}}$.

**Definition VI.6: The joint concurrency coefficient between subsets of equally spaced nodes of two primary paths**

Given the reachable periods of the primary paths $P_{[1]}$ and $P_{[2]}$ are $T_{S_{[1]}}$ and $T_{S_{[2]}}$, respectively. If a subset $\Phi'_{[1]}(\theta_{S_{1}}, T_{S_{1}})$ in $P_{[1]}$ and a subset $\Phi'_{[2]}(\theta_{S_{2}}, T_{S_{2}})$ in $P_{[2]}$ can jointly form a second-order concurrency subset, the joint concurrency coefficient of these two subsets of nodes is defined as: $c(\theta_{S_{1}}, \theta_{S_{2}}) \equiv 1$, otherwise it is defined as $c(\theta_{S_{1}}, \theta_{S_{2}}) \equiv 0$. (Note: with the variations of the reachable periods $T_{S_{[1]}}$ and $T_{S_{[2]}}$, the corresponding joint concurrency coefficients may vary accordingly.)

**Definition VI.7: The matrix of joint concurrency relationship between subsets of equally spaced nodes of a pair of paths**

Given the reachable periods of the primary paths $P_{[1]}$ and $P_{[2]}$ are $T_{S_{[1]}}$ and $T_{S_{[2]}}$, respectively. Define a matrix $C$ containing $T_{S_{[1]}}$ rows and $T_{S_{[2]}}$ columns of elements as the matrix of joint concurrency relationship between the subsets of equally spaced nodes of $P_{[1,2]}$, and its elements are defined as: $C(\theta_{S_{1}}, \theta_{S_{2}}) \equiv c(\theta_{S_{1}}, \theta_{S_{2}})$ (1 ≤ $\theta_{S_{i}}$ ≤ $T_{S_{i}}$, $i = 1,2$). (Note: with the variations of the reachable periods $T_{S_{[1]}}$ and $T_{S_{[2]}}$, the matrix $C$ may vary accordingly.)
Definition VI.8: The matrix of intrinsic joint concurrency relationship between subsets of equally spaced nodes of a pair of paths

\[
\mathbf{C} \triangleq \begin{pmatrix}
    c(1,1) & c(1,2) & c(1,3) & \ldots & c(1,T_{s_{q_1}}) \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    c(j,1) & c(j,2) & c(j,3) & \ldots & c(j,T_{s_{q_1}}) \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    c(T_{s_{q_1}},1) & c(T_{s_{q_1}},2) & c(T_{s_{q_1}},3) & \ldots & c(T_{s_{q_1}},T_{s_{q_1}})
\end{pmatrix}
\]

(VI.1)

\[
\mathbf{C}^* \triangleq \begin{pmatrix}
    c(1,1) & c(1,2) & c(1,3) & \ldots & c(1,I_{s_{q_1}}^* ) \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    c(j,1) & c(j,2) & c(j,3) & \ldots & c(j,I_{s_{q_1}}^* ) \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    c(I_{s_{q_1}}^*,1) & c(I_{s_{q_1}}^*,2) & c(I_{s_{q_1}}^*,3) & \ldots & c(I_{s_{q_1}}^*,I_{s_{q_1}}^*)
\end{pmatrix}
\]

(VI.2)

Given the intrinsic interference intensities of the primary paths \( P_{[1]} \) and \( P_{[2]} \) are \( I_{s_{q_1}}^* \) and \( I_{s_{q_2}}^* \), respectively. Let us construct \( I_{s_{q_1}}^* \) and \( I_{s_{q_2}}^* \) subsets of equally spaced nodes respectively, i.e. \( \Phi_{s_{q_1}}^* (\theta_{s_{q_1}},I_{s_{q_1}}^*) (1 \leq \theta_{s_{q_1}} \leq I_{s_{q_1}}^* ) \) and \( \Phi_{s_{q_2}}^* (\theta_{s_{q_2}},I_{s_{q_2}}^*) (1 \leq \theta_{s_{q_2}} \leq I_{s_{q_2}}^* ) \). Based on these constructed subsets of equally spaced nodes, we define a matrix \( \mathbf{C}^* \) containing \( I_{s_{q_1}}^* \) rows and \( I_{s_{q_2}}^* \) columns of elements, and its elements are defined as:

\[
\mathbf{C}'[\theta_{s_{q_1}},\theta_{s_{q_2}}] \triangleq c(\theta_{s_{q_1}},\theta_{s_{q_2}}) (1 \leq \theta_{s_{q_1}} \leq I_{s_{q_1}}^*, j = 1,2).
\]

Definition VI.9: A binary matrix and its corresponding set of supporting elements

A matrix whose elements are either 0 or 1 is defined as a binary matrix. Given a binary matrix \( \mathbf{B} \), a set \( S_{B_{\theta}} \) is defined as the set of supporting elements of matrix \( \mathbf{B} \), if it satisfies all the following conditions:

1) Corresponding to non-zero elements: for binary matrix \( \mathbf{B} \) whose elements are all 0, define the set \( S_{B_{\theta}} \triangleq \emptyset \). Moreover, for the binary matrix \( \mathbf{B} \) with non-zero elements, the set \( S_{B_{\theta}} \)
is composed of several 2-tuples, and any element in \( S_{B_{\nu}} \), that is, a 2-tuple \([i_k, j_k]\) \((\in S_{B_{\nu}}, k \in \mathbb{Z}^+)\), corresponds to the non-zero element \( B[i_k, j_k] (=1) \) in the matrix \( B \).

2) **Covering non-zero elements:** for any non-zero element \( B[i, j] (=1) \) in matrix \( B \), there is at least one element \([i_k, j_k]\) in the set \( S_{B_{\nu}} \), which makes \( i_k = i \) or \( j_k = j \) hold.

3) **Not covering with each other:** for any two elements (if any) in \( S_{B_{\nu}} \), i.e. \([i_k, j_k], [i_l, j_l]\) \((\in S_{B_{\nu}}; k, l \in \mathbb{Z}^+, k \neq l)\), both \( i_k \neq i_l \) and \( j_k \neq j_l \) hold.

**Note:** given a binary matrix \( B \), there may be more than one set \( S_{B_{\nu}} \) satisfying the above conditions. That is, for a binary matrix \( B \), it may have multiple sets of supporting elements.

**Definition VI.10: The set of maximum supporting elements of a binary matrix**

Among all possible sets of supporting elements of a binary matrix \( B \), the set with the largest number of elements is defined as the set of maximum supporting elements of \( B \), which is denoted as \( S_{B_{\nu}}^{*} \). (Note: given a binary matrix \( B \), there may be more than one such set.)

**Definition VI.11: The reachable supporting number and the maximum reachable supporting number of a binary matrix**

The number of elements contained in a set \( S_{B_{\nu}} \) of supporting elements of a binary matrix \( B \) is defined as a reachable supporting number of \( B \), which is denoted as \( U_{B_{\nu}} \). The number of elements contained in the set \( S_{B_{\nu}}^{*} \) of maximum supporting elements of \( B \) is defined as the maximum reachable supporting number of \( B \), and it is denoted as \( U_{B_{\nu}}^{*} \).

**Lemma VI.1.1: The unreachability of the period of a primary path**

If the intrinsic interference intensity of a primary path \( P_{i}^{(i)} \) \((i = 1, 2)\) is \( I_{S_{B_{\nu}}}^{*} \), the period of \( T_{S_{B_{\nu}}} \) cannot be reached.

**Proof:**
**Case 1: \( I'_{S_{k(i)}} > 1 \)**

Let the primary path \( P_{(i)} \) be composed of \( N_{S_{k(i)}} \) sending nodes, and the sequence numbers of these nodes are 1, 2, ..., \( N_{S_{k(i)}} \), respectively. Consider the case of \( T_{S_{k(i)}} < I'_{S_{k(i)}} \). First, a matrix \( M \) is constructed:

\[
M \triangleq \begin{pmatrix}
1 & 1 + T_{S_{k(i)}} & 1 + 2T_{S_{k(i)}} & \cdots & 1 + \left[ \frac{N_{S_{k(i)}}}{T_{S_{k(i)}}} \right] T_{S_{k(i)}} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
j & j + T_{S_{k(i)}} & j + 2T_{S_{k(i)}} & \cdots & N_{S_{k(i)}} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
T_{S_{k(i)}} & T_{S_{k(i)}} + T_{S_{k(i)}} & T_{S_{k(i)}} + 2T_{S_{k(i)}} & \cdots & \cdots \\
\end{pmatrix}
\]

(VI.3)

1) The matrix has \( T_{S_{k(i)}} (1 \leq T_{S_{k(i)}} \leq N_{S_{k(i)}}) \) rows and \( \left[ \frac{N_{S_{k(i)}}}{T_{S_{k(i)}}} \right] \) columns;

2) In ascending order, put the sequence number of the node \( j (1 \leq j \leq N_{S_{k(i)}}) \) in \( P_{(i)} \) into the corresponding unit in row \( \left[ j - T_{S_{k(i)}} \right] \) and column \( \left[ j \right] \) of the matrix one by one.

That is, set the value of the unit in row \( \left[ j - T_{S_{k(i)}} \right] \) and column \( \left[ j \right] \) of \( M \) to be \( j \);

3) Ignore the settings of matrix units that may not be filled by sequence numbers of nodes.

According to the definition of the subset of equally spaced nodes, it can be seen that row \( j \) of matrix \( M \) is composed of all the elements of the subset \( \Phi'_{S_{k(i)}}(j, T_{S_{k(i)}}) \) with initial phase being \( j \) and spacing between adjacent nodes being \( T_{S_{k(i)}} \).

Now let us prove that if \( T_{S_{k(i)}} < I'_{S_{k(i)}} \), it can be deduced that not all the \( T_{S_{k(i)}} \) subsets \( \Phi'_{S_{k(i)}}(\theta_{S_{k(i)}}, T_{S_{k(i)}}) (1 \leq \theta_{S_{k(i)}} \leq T_{S_{k(i)}}) \) are second-order concurrency subsets, which suggests that \( T_{S_{k(i)}} < I'_{S_{k(i)}} \) is not reachable. Assuming that \( S_{k(i)} \) is a maximum second-order interference subset.
of $P_i$ with the number of its elements being $I_{s(i)}^*$, and the set is continuous according to Corollary V.2.1. Since the number of rows of matrix $M$ is $T_{s(i)} < I_{s(i)}^*$, the corresponding distributions of the elements of $S_{s(i)}^*$ in $M$ will stretch across (i.e. occupy) at least two adjacent columns of $M$, so that there must be at least two adjacent elements on a row $k$ ($1 \leq k \leq T_{s(i)}$) of matrix $M$ belonging to the set of $S_{s(i)}^*$, which makes it is evident that the two nodes corresponding to these two adjacent elements in $M$ cannot be concurrent. That is to say, the subset $\Phi_{s(i)}^* (k,T_{s(i)})$ of equally spaced nodes corresponding to the row $k$ of matrix $M$ is not a second-order concurrency subset, which indicates that among $T_{s(i)} (< I_{s(i)}^*)$ subsets $\Phi_{s(i)}^* (\theta_{s(i)},T_{s(i)}) (1 \leq \theta_{s(i)} \leq T_{s(i)})$ of equally spaced nodes, at least one subset is not a second-order concurrency subset. Therefore, according to the definition of the reachability of the period of a primary path, it can be concluded that the period $T_{s(i)} (< I_{s(i)}^*)$ is unreachable.

**Case 2:** $I_{s(i)}^* = 1$

According to the definition of a reachable period, the reachable period should not be smaller than one. That is to say, as for the case of $I_{s(i)}^* = 1$, the reachable period $T_{s(i)}$ should not be smaller than $I_{s(i)}^*$. ■

**Note:** Although the proof of the above conclusion is based on the interference reduction rule between nodes (Rule V.1 and Rule V.2), according to the definition of intrinsic interference intensity (Definition V.12), it is easy to know that the above conclusion also applies to more general cases where the interference reduction rule between nodes does not necessarily hold.

**Lemma VI.1.2: The reachability of the period of a primary path**

If the intrinsic interference intensity of a primary path $P_i$ ($i = 1, 2$) is $I_{s(i)}^*$, it is reachable for the period of $I_{s(i)}^* \leq T_{s(i)} \leq N_{s(i)}$.

**Proof:**

---

30 / 87
Case 1: $I_{s_{k|1}}' > 1$

Reduction to absurdity is adopted. Suppose that a period $T_{s_{k|1}} (\geq I_{s_{k|1}}' )$ is not reachable. First, let us construct the matrix $M$ (that is, the matrix has $T_{s_{k|1}}$ rows and $\left\lfloor \frac{N_{s_{k|1}}}{T_{s_{k|1}}} \right\rfloor$ columns) according to the same method adopted in Lemma VI.1.1. Because it is assumed that the period $T_{s_{k|1}} (\geq I_{s_{k|1}}' )$ is unreachable, according to the definition of the unreachable period of a primary path (Definition VI.2), it can be concluded that at least one subset $\Phi'_{s_{k|1}}(k,T_{s_{k|1}})$ corresponding to row $k (1 \leq k \leq T_{s_{k|1}})$ of the matrix $M$ is not a second-order concurrency subset, which suggests that at least two nodes in the subset $\Phi'_{s_{k|1}}(k,T_{s_{k|1}})$ cannot be concurrent. According to Theorem V.2, these two nodes and all the other nodes between them will form a continuous second-order interference subset, and the number of elements contained in the interference subset will not be less than $T_{s_{k|1}} + 1$. That is, the number of elements of the formed interference subset is larger than the intrinsic interference intensity $I_{s_{k|1}}'$. (Note: the case of $T_{s_{k|1}} \geq I_{s_{k|1}}'$ is considered here), which contradicts the premise that the intrinsic interference intensity of $P_{(i)}$ is $I_{s_{k|1}}'$. 

Case 2: $I_{s_{k|1}}' = 1$

In this case, all the $N_{s_{k|1}}$ sending nodes of $P_{(i)}$ constitute a second-order concurrency subset $\Phi'_{s_{k|1}}(1,1)$ (i.e. $T_{s_{k|1}} = 1$ and $\theta_{s_{k|1}} = 1$) with equally spaced nodes.

---

**Theorem VI.1:** The intrinsic period of a primary path is equal to its intrinsic interference intensity

If the intrinsic period of a primary path $P_{(i)} (i = 1,2)$ is $T_{s_{k|1}}'$, and the intrinsic interference intensity is $I_{s_{k|1}}'$, then $T_{s_{k|1}}' = I_{s_{k|1}}'$ holds.

**Proof:**

According to Lemma VI.1.1, the period of $T_{s_{k|1}} < I_{s_{k|1}}'$ is not reachable, and according to
Lemma VI.1.2, the period of \( T_{s_{0_i}} \geq I^*_{s_{0_i}} \) is reachable. Therefore, the minimum reachable period, i.e. the intrinsic period of a primary path \( P_{[i]} \), is \( T_{s_{0_i}} = I^*_{s_{0_i}} \). □

Corollary VI.1.1: The subsets of equally spaced nodes corresponding to the intrinsic period of a primary path cannot form a second-order concurrency relationship between each other.

If the intrinsic period of a primary path \( P_{[i]} \) \((i = 1, 2)\) is \( T_{s_{0_i}} \((>1)\)\), among all of its \( T^*_{s_{0_i}} \) subsets of equally spaced nodes with initial phases being \( \theta_{s_{0_i}} \) \((1 \leq \theta_{s_{0_i}} \leq T^*_{s_{0_i}})\) and spacing between adjacent nodes being \( T^*_{s_{0_i}} \), any two of them cannot jointly form a second-order concurrency subset.

Proof:

Firstly, a matrix \( M \) with the number of rows being \( T^*_{s_{0_i}} \) and the number of columns being \( \left[ N_{s_{0_i}} \atop T^*_{s_{0_i}} \right] \) is constructed by following the method adopted in Lemma VI.1.1. According to Theorem VI.1, the intrinsic interference intensity of the primary path \( P_{[i]} \) is \( I^*_{s_{0_i}} = T^*_{s_{0_i}} \), and according to Corollary V.2.1, all possible maximum second-order interference subsets of \( P_{[i]} \) are continuous. Therefore, if we take any two rows (corresponding to two subsets of equally spaced nodes with different initial phases) from the matrix \( M \), it can be found that these two rows must contain two different elements, each of which belongs to a common maximum second-order interference subset, and the corresponding two nodes must interfere with each other. Therefore, the two subsets of equally spaced nodes corresponding to these two rows selected arbitrarily from the matrix \( M \) contain nodes that interfere with each other. That is to say, the two arbitrarily selected subsets of equally spaced nodes cannot form a second-order concurrency subset. □

Theorem VI.2: The lower bound of the intrinsic interference intensity of a pair of paths

If the intrinsic interference intensity of a pair of paths \( P_{[1,2]} \) is \( I^*_{s_{0,[1,2]}} \), and the intrinsic interference intensities of the primary paths \( P_{[1]} \) and \( P_{[2]} \) are \( I^*_{s_{0,1}} \) and \( I^*_{s_{0,2}} \), respectively,
then there is $I_{s_{[1,2]}}^* \geq \max[I_{s_{[1]}}^*, I_{s_{[2]}}^*]$.

Proof:

Case 1: $\max[I_{s_{[1]}}^*, I_{s_{[2]}}^*] = 1$

According to the definition, the intrinsic interference intensity $I_{s_{[1,2]}}^*$ of a pair of paths $P_{[1,2]}$ must not be less than 1, so $I_{s_{[1,2]}}^* \geq \max[I_{s_{[1]}}^*, I_{s_{[2]}}^*]$ holds in this case.

Case 2: $\max[I_{s_{[1]}}^*, I_{s_{[2]}}^*] > 1$

Without losing generality, assume that $I_{s_{[1]}}^* = \max[I_{s_{[1]}}^*, I_{s_{[2]}}^*]$, in the following reduction to absurdity is adopted. If $I_{s_{[2]}}^* < I_{s_{[1]}}^*$ is true, according to the definition of the intrinsic interference intensity of a pair of paths (Definition V.14), it can be concluded that the maximum second-order interference subset $S_{h_{[2]}}^*$ composed of $I_{s_{[2]}}^*$ nodes in $P_{[1,2]}$ must not be the maximum second-order interference subset in $P_{[1,2]}$, because at least in the primary path $P_{[1]}$ there exists a larger second-order interference subset $S_{h_{[1]}}^*$ composed of $I_{s_{[1]}}^*$ nodes, which contradicts the premise of the proposition. ■

Theorem VI.3: The upper bound of the intrinsic interference intensity of a pair of paths

If the intrinsic interference intensity of a pair of paths $P_{[1,2]}$ is $I_{s_{[1,2]}}^*$, and the intrinsic interference intensities of the primary paths $P_{[1]}$ and $P_{[2]}$ are $I_{s_{[1]}}^*$ and $I_{s_{[2]}}^*$, respectively, then there is $I_{s_{[1,2]}}^* \leq I_{s_{[1]}}^* + I_{s_{[2]}}^*$.

Proof:

Let the nonempty subset $S_{h_{[2]}}^*$ of nodes be a maximum second-order interference subset of $P_{[1,2]}$.

Case 1: All nodes of $S_{h_{[2]}}^*$ are from a primary path (without losing generality, say from $P_{[1]}$), and $I_{s_{[1,2]}}^* = 1$.
According to the definition, both $I_{s_{q(1)}}^*$ and $I_{s_{q(2)}}^*$ are not less than 1, so $I_{s_{q(1)}}^* < I_{s_{q(1)}}^* + I_{s_{q(2)}}^*$ holds in this case.

**Case 2:** All nodes of $S'_{\eta_{(1,2)}}$ are from a primary path (without losing generality, say from $P_{[1]}$), and $I_{s_{q(2)}}^* > 1$:

According to the definition, the nodes constituting $S'_{\eta_{(1,2)}}$ form a second-order interference subset of the primary path $P_{[1]}$. Because the intrinsic interference intensity of $P_{[1]}$ is $I_{s_{q(1)}}^*$, there is $I_{s_{q(2)}}^* \leq I_{s_{q(1)}}^*$. Furthermore, since $I_{s_{q(2)}}^* \geq 1$, there is $I_{s_{q(2)}}^* < I_{s_{q(1)}}^* + I_{s_{q(2)}}^*$.

**Case 3:** One part of the nodes of $S'_{\eta_{(1,2)}}$ comes from the primary path $P_{[1]}$ (assuming the number of these nodes is $m_1$), and the other part (assuming the number of these nodes is $m_2$) comes from the primary path $P_{[2]}$, and $m_1 > 1$, $m_2 > 1$:

According to the definition of the second-order interference subset (Definition V.9), $m_1$ nodes coming from $P_{[1]}$ constitute by themselves a second-order interference subset, and $m_1 \leq I_{s_{q(1)}}^*$; $m_2$ nodes from $P_{[2]}$ also form a second-order interference subset, and $m_2 \leq I_{s_{q(2)}}^*$. Therefore, it can be seen that $I_{s_{q(1,2)}}^* = m_1 + m_2 \leq I_{s_{q(1)}}^* + I_{s_{q(2)}}^*$, i.e. $I_{s_{q(1,2)}}^* \leq I_{s_{q(1)}}^* + I_{s_{q(2)}}^*$ holds.

**Case 4:** One part of the nodes of $S'_{\eta_{(1,2)}}$ comes from the primary path $P_{[1]}$ (assuming the number of these nodes is $m_1$), and the other part (assuming the number of these nodes is $m_2$) comes from the primary path $P_{[2]}$, and $m_1 > 1$, $m_2 = 1$:

According to the definition of the second-order interference subset (Definition V.9), $m_1$ nodes coming from $P_{[1]}$ constitute by themselves a second-order interference subset, and $m_1 \leq I_{s_{q(1)}}^*$; Based on the definition of the intrinsic interference intensity of a nonempty subset of nodes (Definition V.12), we have $m_2 = \leq I_{s_{q(2)}}^* (m_2 = 1)$. Therefore, it can be seen that $I_{s_{q(1,2)}}^* = m_1 + m_2 \leq I_{s_{q(1)}}^* + I_{s_{q(2)}}^*$, i.e. $I_{s_{q(1,2)}}^* \leq I_{s_{q(1)}}^* + I_{s_{q(2)}}^*$ holds.
Case 5: One part of the nodes of $S'_{P_{[1,2]}}$ comes from the primary path $P_{[1]}$ (assuming the number of these nodes is $m_1$), and the other part (assuming the number of these nodes is $m_2$) comes from the primary path $P_{[2]}$, and $m_1 = 1, \ m_2 > 1$:

The proof here is the same as that of case 4 (omitted).

Case 6: One part of the nodes of $S'_{P_{[1,2]}}$ comes from the primary path $P_{[1]}$ (assuming the number of these nodes is $m_1$), and the other part (assuming the number of these nodes is $m_2$) comes from the primary path $P_{[2]}$, and $m_1 = 1, \ m_2 = 1$:

In this case, $I_{P_{[1,2]}} = m_1 + m_2 = 2$, and considering the fact that both $I_{S_{[1]}}$ and $I_{S_{[2]}}$ are not less than 1, so $I_{S_{[1]}} \leq I_{S_{[1]}} + I_{S_{[2]}}$ is established.

---

Theorem VI.4: The unreachability of the joint period of a pair of paths

If the intrinsic interference intensity of a pair of paths $P_{[1,2]}$ is $I_{S_{[1,2]}}$, the joint period $T_{S_{[1,2]}} < I_{S_{[1,2]}}$ cannot be reached.

Proof:

Case 1: $I_{S_{[1,2]}} = 1$

According to the required ergodicity defined by the reachable joint period of a pair of paths (Definition VI.3), it can be seen that in this case, the joint period $T_{S_{[1,2]}}$ should not be less than 1, i.e. $T_{S_{[1,2]}} \geq I_{S_{[1,2]}}$ holds.

Case 2: $I_{S_{[1,2]}} > 1$

Let the nonempty subset $S'_{P_{[1,2]}}$ of nodes be a maximum second-order interference subset of the pair of paths $P_{[1,2]}$, which is composed of $I_{S_{[1,2]}}$ sending nodes. According to the requirement of concurrency in the definition of the reachable joint period of a pair of paths, these $I_{S_{[1,2]}}$ sending nodes must belong to $I_{S_{[1,2]}}$ different subsets of equally spaced nodes respectively (otherwise, the concurrency will not be met). According to the required concurrency and ergodicity by the definition.
of the reachable joint period of a pair of paths, it needs at least \( I^*_{s_{[1,2]}} \) time beats to traverse these \( I^*_{s_{[1,2]}} \) different subsets of equally spaced nodes, that is, the joint period \( T_{s_{[1,2]}} \) should not be less than \( I^*_{s_{[1,2]}} \), that is, \( T_{s_{[1,2]}} \geq I^*_{s_{[1,2]}} \) holds.

\[ \begin{align*}
\text{Theorem VI.5: The reachability of the joint period of a pair of paths (} I^*_{s_{[1,2]}} = 1) \\
\text{If the intrinsic interference intensity of a pair of paths } P_{[1,2]} \text{ is } I^*_{s_{[1,2]}} = 1, \text{ the joint period } T_{s_{[1,2]}} = I^*_{s_{[1,2]}} \text{ is reachable.}
\end{align*} \]

\textbf{Proof:}

In this case, the set \( S_{[1,2]} \) composed of all the sending nodes of the pair of paths \( P_{[1,2]} \) is a second-order concurrency set, so the subsets \( S_{[r]} \) and \( S_{[r]} \) composed of all the sending nodes of the primary paths \( P_{[1]} \) and \( P_{[2]} \) must also be a second-order concurrency subset, respectively. As for \( P_{[1]} \) and \( P_{[2]} \), a subset of equally spaced nodes can be constructed respectively, i.e. \( \Phi^*_{s_{[1]}}, (1,1) \) (including all the nodes of \( S_{[1]} \)) and \( \Phi^*_{s_{[2]}}, (1,1) \) (including all the nodes of \( S_{[2]} \)), and these two subsets of equally spaced nodes can be activated in the same time beat to realize the concurrent activation of all the nodes of \( P_{[1,2]} \). According to the definition of the reachable joint period of a pair of paths, in this case \( T_{s_{[1,2]}} = I^*_{s_{[1,2]}} = 1 \) is reachable.

\[ \begin{align*}
\text{Theorem VI.6: The reachability of the joint period of a pair of paths (} I^*_{s_{[1,2]}} = I^*_{s_{[1]}} + I^*_{s_{[2]}}) \\
\text{If the intrinsic interference intensity of a pair of paths } P_{[1,2]} \text{ is } I^*_{s_{[1,2]}} \text{, and the intrinsic interference intensities of the primary paths } P_{[1]} \text{ and } P_{[2]} \text{ are } I^*_{s_{[1]}} \text{ and } I^*_{s_{[2]}}, \text{ respectively. As for the case of } I^*_{s_{[1,2]}} = I^*_{s_{[1]}} + I^*_{s_{[2]}}, \text{ the joint period } T_{s_{[1,2]}} = I^*_{s_{[1,2]}} \text{ is reachable.}
\end{align*} \]

\textbf{Proof:}

According to \textbf{Theorem VI.1}, in this case, for the primary paths \( P_{[1]} \) and \( P_{[2]} \), we can
construct $I_{S_{[1]}}$ and $I_{S_{[2]}}$ subsets of equally spaced nodes respectively, that is, 

$$\Phi_{S_{[1]}}(\theta_{S_{[1]}}^{[1]}, I_{S_{[1]}}^{[1]}) \quad (1 \leq \theta_{S_{[1]}}^{[1]} \leq I_{S_{[1]}}^{[1]})$$

(including all the nodes of $S_{[1]}$) and 

$$\Phi_{S_{[2]}}(\theta_{S_{[2]}}^{[2]}, I_{S_{[2]}}^{[2]}) \quad (1 \leq \theta_{S_{[2]}}^{[2]} \leq I_{S_{[2]}}^{[2]})$$

(including all the nodes of $S_{[2]}$), and arrange these $I_{S_{[1]}}^{[1]} + I_{S_{[2]}}^{[2]}$ subsets of equally spaced nodes to be activated in consecutive $I_{S_{[1]}}^{[1]} + I_{S_{[2]}}^{[2]}$ time beats without overlapping to each other and without any omission (that is, only one subset of equally spaced nodes is activated in one time beat). By following this way, we can activate all the subsets of equally spaced nodes of the pair of paths $P_{[1,2]}$. According to the definition of the reachable joint period of a pair of paths, in this case $T_{S_{[1,2]}} = I_{S_{[1,2]}}^{[1]} = I_{S_{[1]}}^{[1]} + I_{S_{[2]}}^{[2]}$ is reachable.}

**Theorem VI.7: The reachability of the joint period of a pair of paths** ($1 < I_{S_{[1,2]}}^{[1]} < I_{S_{[1]}}^{[1]} + I_{S_{[2]}}^{[2]}$)

If the intrinsic interference intensity of a pair of paths $P_{[1,2]}$ is $I_{S_{[1,2]}}^{[1]}$, and the intrinsic interference intensities of the primary paths $P_{[1]}$ and $P_{[2]}$ are $I_{S_{[1]}}^{[1]}$ and $I_{S_{[2]}}^{[2]}$, respectively. Let $C$ be the matrix of joint concurrency relationship between subsets of equally spaced nodes of a pair of paths $P_{[1,2]}$ (Definition VI.7). As for the case of $1 < I_{S_{[1,2]}}^{[1]} < I_{S_{[1]}}^{[1]} + I_{S_{[2]}}^{[2]}$, if there are combinations of reachable periods $T_{S_{[1]}}$ and $T_{S_{[2]}}$ of $P_{[1]}$ and $P_{[2]}$ respectively, which makes the maximum reachable supporting number $U_{C_{[1,2]}}^{[1]}$ of the corresponding matrix $C$ equal to $T_{S_{[1]}} + T_{S_{[2]}} - I_{S_{[1,2]}}^{[1]}$, the joint period of $T_{S_{[1,2]}} = I_{S_{[1,2]}}^{[1]}$ is reachable.

**Proof:**

Assuming that the matrix $C$ (which has $T_{S_{[1]}}$ rows and $T_{S_{[2]}}$ columns) of joint concurrency relationship satisfying the relation $U_{C_{[1,2]}}^{[1]} = (T_{S_{[1]}} + T_{S_{[2]}}) - I_{S_{[1,2]}}^{[1]}$ exists (Note: according to the definition of a binary matrix, it is known that $C$ is a binary matrix). Let us determine one set $S_{C_{[1,2]}}^{[1]}$ as the set of maximum supporting elements of $C$ (Definition VI.10), and assign incremental sequential numbers to all of its $U_{C_{[1,2]}}^{[1]}$ elements (i.e. element No. 1, element No. 2,..., and element No. $U_{C_{[1,2]}}^{[1]}$).
First, let us consider the No. 1 element in $S^*_{C_w}$, i.e. $(i_1, j_1) \in S^*_{C_w}$, assume that its corresponding non-zero element in $C$ is $C[i_1, j_1] = 1$, and arrange the subset $\Phi^*_{s_{i_1}}(i_1, T_{s_{i_1}})$ of equally spaced nodes in the primary path $P_{[1]}$ and the subset $\Phi^*_{s_{j_1}}(j_1, T_{s_{j_1}})$ of equally spaced nodes in the primary path $P_{[2]}$ to be activated concurrently within the time beat No. 1 (Since $C[i_1, j_1] = 1$, such concurrency is feasible). Likewise, for the No. 2 element in $S^*_{C_w}$, i.e. $(i_2, j_2) \in S^*_{C_w}$, assume that its corresponding non-zero element in $C$ is $C[i_2, j_2] = 1$, and arrange the subset $\Phi^*_{s_{i_2}}(i_2, T_{s_{i_2}})$ of $P_{[1]}$ and the subset $\Phi^*_{s_{j_2}}(j_2, T_{s_{j_2}})$ of $P_{[2]}$ to be activated concurrently within the time beat No. 2. By analogy, we can allocate $U^*_{C_w}$ continuous time beats to realize the concurrent activation of the subsets of equally spaced nodes respectively from $P_{[1]}$ and $P_{[2]}$ (i.e. $U^*_{C_w}$ subsets are from $P_{[1]}$, and $U^*_{C_w}$ subsets are from $P_{[2]}$).

Through the above processing for arranging time beats, there are still $T_{s_{i_1}} - U^*_{C_w}$ and $T_{s_{j_1}} - U^*_{C_w}$ subsets of equally spaced nodes in $P_{[1]}$ and $P_{[2]}$, respectively, having not been assigned to their corresponding time beats. Therefore, the total number of subsets of equally spaced nodes waiting to be arranged with time beats is $T_{s_{i_1}} + T_{s_{j_1}} - 2U^*_{C_w}$. According to the definition of the set of maximum supporting elements, any of the remaining $T_{s_{i_1}} - U^*_{C_w}$ subsets in $P_{[1]}$ and any of the remaining $T_{s_{j_1}} - U^*_{C_w}$ subsets in $P_{[2]}$ cannot be arranged into a same time beat to form a second-order concurrency relationship. Therefore, in order to avoid mutual interference, the remaining $T_{s_{i_1}} + T_{s_{j_1}} - 2U^*_{C_w}$ subsets of equally spaced nodes are arranged to be activated in the subsequent $T_{s_{i_1}} + T_{s_{j_1}} - 2U^*_{C_w}$ time beats without overlapping to each other and without any omission (that is, only one subset of equally spaced nodes is activated in each time beat).

In summary, based on the above processing for arranging time beats, a total of $T_{s_{i_1}} = U^*_{C_w} + (T_{s_{i_1}} + T_{s_{j_1}} - 2U^*_{C_w}) = I^*_{s_{i_1}}$ time beats are assigned to realize one round activation (i.e. one round traversal) of all the subsets of equally spaced nodes in $P_{[1,2]}$. According to the definition
of the reachable joint period, in this case, $T^{*}_{s_{[1,2]}} = I^{*}_{s_{[1,2]}}$ is reachable.

**Corollary VI.7.1:** The upper bound of the maximum reachable supporting number $U^{*}_{C_p}$ of the matrix $C$ of joint concurrency relationship

If the intrinsic interference intensity of a pair of paths $P_{[1,2]}$ is $I^{*}_{s_{[1,2]}}$, and the reachable periods of the primary paths $P_{[1]}$ and $P_{[2]}$ are $T_{s_{[1]}}$ and $T_{s_{[2]}}$, respectively. Let $C$ be the matrix of joint concurrency relationship between subsets of equally spaced nodes of a pair of paths $P_{[1,2]}$. And, the maximum reachable supporting number of $C$ is $U^{*}_{C_p}$. There is

$$U^{*}_{C_p} \leq T_{s_{[1]}} + T_{s_{[2]}} - I^{*}_{s_{[1,2]}}.$$  

**Proof:**

The reduction to absurdity is adopted. If $U^{*}_{C_p} > T_{s_{[1]}} + T_{s_{[2]}} - I^{*}_{s_{[1,2]}}$ is possible, according to the construction method used in the proof of the above theorem (Theorem VI.7), the case of $T_{s_{[1,2]}} < I^{*}_{s_{[1,2]}}$ can be constructed out, which is contrary to the conclusion about the unreachability of the joint period obtained in **Theorem VI.4**.

**Definition VI.12:** A pair of paths that can form an optimal pairing

As for a pair of paths $P_{[1,2]}$ whose intrinsic interference intensity is $I^{*}_{s_{[1,2]}}$, if its joint period can reach $I^{*}_{s_{[1,2]}}$, it is defined as a pair of paths which can form an optimal pairing.

**Theorem VI.8:** The intrinsic joint period of a pair of paths that can form an optimal pairing

Assume that a pair of paths $P_{[1,2]}$ can form an optimal pairing. If its intrinsic interference intensity is $I^{*}_{s_{[1,2]}}$, its intrinsic joint period $T^{*}_{s_{[1,2]}} = I^{*}_{s_{[1,2]}}$.

**Proof:**

It can be proved directly according to **Theorem VI.4** and **Definition VI.12**.
VII. Definitions of throughput of a pair of paths and some related basic properties

In this section, the average throughput and the asymptotic throughput of a primary path are defined. An algorithm (Algorithm VII.1) for the transmission of information blocks based on the intrinsic period of a primary path is proposed, and an important conclusion is drawn that the proposed transmission algorithm can maximize the asymptotic throughput of a primary path (Theorem VII.1). Moreover, in Corollary VII.1.1, it is found that given the intrinsic interference intensity of a primary path, dividing the set of all its sending node into several second-order concurrency subsets with equally spaced nodes, where the number of subsets is the same as its intrinsic interference intensity, is the only way to achieve the maximum asymptotic throughput in all cases. As for the cases of traversals with equal opportunities, an algorithm (Algorithm VII.2) for the cooperative volatility transmission of information blocks in a pair of paths based on the set of maximum supporting elements is proposed. It is proved that the algorithm can maximize the asymptotic joint throughput of a pair of paths (Theorem VII.3), and the upper bound of the asymptotic joint throughput is given (Theorem VII.4). Moreover, the unreachability of the joint period of a pair of paths under certain conditions is proved (Theorem VII.5). As for the more general cases of traversals with unequal opportunities, an algorithm (Algorithm VII.3) for the cooperative volatility transmission of information blocks in a pair of paths based on the set of maximum supporting elements is also proposed. Moreover, the upper bound of the asymptotic joint throughput of a pair of paths achieved by using this algorithm is obtained (Theorem VII.6 and Corollary VII.6.1). Finally, an algorithm flow (Algorithm VII.4) that can maximize the end-to-end asymptotic joint throughput of a pair of paths is proposed, which comprehensively covers almost all the key factors that may affect the asymptotic joint throughput of a pair of paths, such as the selection of routes, the determination of the reachable periods of the two primary paths in a pair of paths, the number of times the two primary paths are traversed within a joint period, and the allocation strategy of time beats and so on.

Definition VII.1: The specific time interval

Starting from time \( t \), the time duration of lasting for \( b \in \mathbb{Z}, b \geq 0 \) time beats is defined as
the specific time interval, which is denoted as $D_{i,b}^{\text{int}}$.

**Definition VII.2:** The average throughput of a primary path in a specific time interval

If the destination node of a primary path $P_i$ ($i = 1, 2$) receives $m_k$ information blocks in the time interval $D_{i,b}^{\text{int}}$ (Note: for simplicity, assuming that all the information blocks carry the same amount of information, so here only the number of information blocks is used to measure the amount of information received), the value $R_{P_i}^{\text{av}} \triangleq \frac{m_k}{b}$ (unit: number of information blocks/time beat) is defined as the average throughput of a primary path $P_i$ in this time interval.

**Definition VII.3:** The average joint throughput of a pair of paths in a specific time interval

If the destination nodes of a pair of paths $P_{[1,2]}$ receives a total of $m_k$ information blocks in the time interval $D_{[1,2]}^{\text{int}}$, the value $R_{P_{[1,2]}}^{\text{av}} \triangleq \frac{m_k}{b}$ (unit: number of information blocks/time beat) is defined as the average joint throughput of a pair of paths $P_{[1,2]}$ in this time interval. It is evident that $R_{P_{[1,2]}}^{\text{av}} = R_{P_{[1]}}^{\text{av}} + R_{P_{[2]}}^{\text{av}}$.

**Definition VII.4:** The asymptotic throughput of a primary path

Assuming that the destination node of a primary path $P_i$ ($i = 1, 2$) receives $m_k$ information blocks in the time interval $D_{i,b}^{\text{int}}$. The limitation of the average throughput of $P_i$ in the case of the duration $b$ (measured by the number of time beats) of a time interval $D_{i,b}^{\text{int}}$ approaching to infinity is defined as the asymptotic throughput of $P_i$, which is denoted as $R_{P_i}^{\infty} \triangleq \lim_{b \to \infty} \left( \frac{m_k}{b} \right)$.

**Definition VII.5:** The asymptotic joint throughput of a pair of paths

Assuming that the destination nodes of a pair of paths $P_{[1,2]}$ receives a total of $m_k$ information blocks in the time interval $D_{[1,2]}^{\text{int}}$. The limitation of the average joint throughput of $P_{[1,2]}$
in the case of the duration $b$ (measured by the number of time beats) of a time interval $D_{s,b}^m$ approaching to infinity is defined as the asymptotic joint throughput of $P_{[1,2]}$, which is denoted as

$$R^e_{s_{[1,2]}} = \lim_{b \to \infty} \left( \frac{m_i}{b} \right).$$

**Lemma VII.1.1: The upper bound of the asymptotic throughput of a primary path**

Assuming that the intrinsic interference intensity of a primary path $P_{[i]}$ $(i = 1, 2)$ is $I'_{s_{[i]}}$ (according to Theorem VI.1, we have $T'_{s_{[i]}} = I'_{s_{[i]}}$), then the asymptotic throughput of $P_{[i]}$ is

$$R^e_{s_{[i]}} \leq \frac{1}{I'_{s_{[i]}}} = \frac{1}{T'_{s_{[i]}}}.$$  

**Proof:**

**Case 1:** $I'_{s_{[i]}} > 1$

Assuming that the set $S'_{P_{[i]}}$ is a maximum second-order interference subset of $P_{[i]}$. When information blocks are transmitted among the nodes in $P_{[i]}$, they must pass through all the nodes contained in $S'_{P_{[i]}}$ successively. When information blocks are transmitted in $S'_{P_{[i]}}$ one hop after another, in order to avoid interference between adjacent nodes, only one information block can be transmitted simultaneously in $S'_{P_{[i]}}$, i.e. at most one node in $S'_{P_{[i]}}$ is at active state in one time beat. Therefore, as for $S'_{P_{[i]}}$, at most one information block can be output per $I'_{s_{[i]}}$ time beats, so the asymptotic throughput of a primary path $R^e_{s_{[i]}} \leq \frac{1}{I'_{s_{[i]}}}$. Furthermore, from Theorem VI.1, we have $R^e_{s_{[i]}} \leq \frac{1}{I'_{s_{[i]}}} = \frac{1}{T'_{s_{[i]}}}$.  

**Case 2:** $I'_{s_{[i]}} = 1$

In this case, a primary path $P_{[i]}$ can at most output one information block per time beat, so the asymptotic throughput $R^e_{s_{[i]}} \leq \frac{1}{I'_{s_{[i]}}} \leq \frac{1}{T'_{s_{[i]}}}$ is still valid. Furthermore, from Theorem VI.1, it can be
obtained that \( R_{x_{b(i)}}^{\infty} \leq \frac{1}{I_{s_{b(i)}}^{*}} = \frac{1}{T_{s_{x_{b(i)}}}^{*}} \)

Algorithm VII.1: The algorithm for the transmission of information blocks based on the intrinsic period of a primary path

1: BEGIN
2: Set the intrinsic interference intensity of a primary path \( P_{(i)} (i = 1, 2) \) as \( I_{s_{b(i)}}^{*} \)
   \( (T_{s_{b(i)}}^{*} = I_{s_{b(i)}}^{*}) \), the time beat for \( P_{(i)} \) starting the transmission of the first information block is regarded as the “starting beat”, and the corresponding sequence number of the “starting beat” is set to be 1;
3: WHILE (the sequence number of the beat \( \leq \) the upper limit of the sequence number of the beat) DO
4: IF (the sequence number of the beat = 1 + \( k \cdot T_{s_{b(i)}}^{*} \) \( (k = 0, 1, 2, ...) \)) THEN
5: Activate all the nodes in the subset \( \Phi_{s_{b(i)}}^{'} (1, I_{s_{b(i)}}^{*}) \);
6: For node 1 in \( \Phi_{s_{b(i)}}^{'} (1, I_{s_{b(i)}}^{*}) \): gets a new information block from the information source (here, it is assumed that there is always an information block waiting to be transmitted at the information source) and sends it to its adjacent downstream node within the current beat; For all the other nodes in \( \Phi_{s_{b(i)}}^{'} (1, I_{s_{b(i)}}^{*}) \): send the information block (if any) received from their upstream nodes in the previous beat to their adjacent downstream nodes in the current beat;
7: ELSE IF (\( I_{s_{b(i)}}^{*} > 1 \))
   //Note: in this case, the sequence number of the beat is \( j + k \cdot T_{s_{b(i)}}^{*} \) \( (1 < j \leq T_{s_{b(i)}}^{*}, k = 0, 1, 2, ...) \)
8: Activate all the nodes in the subset \( \Phi_{s_{b(i)}}^{'} (j, I_{s_{b(i)}}^{*}) \);
9: For all the nodes in \( \Phi_{s_{b(i)}}^{'} (j, I_{s_{b(i)}}^{*}) \): send the information block (if any) received from their upstream nodes in the previous beat to their adjacent downstream nodes in the
current beat;
10:   END IF
11:   Increase the sequence number of the beat by one;
12:   END WHILE
13:   END

**Lemma VII.1.2: The reachable upper bound of the asymptotic throughput**

Assuming that the intrinsic interference intensity of a primary path $P_i$ $(i = 1, 2)$ is $J^r_{S_{k|i}}$

$(T^r_{S_{k|i}} = T^*_{S_{k|i}})$, then the asymptotic throughput $R^*_{S_{k|i}} = \frac{1}{I^r_{S_{k|i}}} = \frac{1}{T^r_{S_{k|i}}}$ of $P_i$ can be reached.

**Proof:**

At least Algorithm VII.1 can be adopted. That is to say, by using Algorithm VII.1, in steady state, the primary path $P_i$ will achieve the performance of outputting one information block per $T^r_{S_{k|i}}$ beats. In this case, the asymptotic throughput of $P_i$ is $R^*_{S_{k|i}} = \frac{1}{I^r_{S_{k|i}}} = \frac{1}{T^r_{S_{k|i}}}$. Therefore, the upper bound of the asymptotic throughput $R^*_{S_{k|i}} = \frac{1}{I^r_{S_{k|i}}} = \frac{1}{T^r_{S_{k|i}}}$ can be reached. ■

**Theorem VII.1: The asymptotic throughput of a primary path can be maximized by using Algorithm VII.1**

Assuming that the intrinsic interference intensity of a primary path $P_i$ $(i = 1, 2)$ is $J^r_{S_{k|i}}$

$(T^r_{S_{k|i}} = T^*_{S_{k|i}})$, the maximum asymptotic throughput $R^*_{S_{k|i}} = \frac{1}{I^r_{S_{k|i}}} = \frac{1}{T^r_{S_{k|i}}}$ can be achieved by using Algorithm VII.1 for $P_i$.

**Proof:**

It can be obtained by combining Lemma VII.1.1 and Lemma VII.1.2. ■

**Corollary VII.1.1: Given the intrinsic interference intensity of a primary path, dividing the**
set of all its sending node into several second-order concurrency subsets with equally spaced nodes, where the number of subsets is the same as its intrinsic interference intensity, is the only way to achieve the maximum asymptotic throughput in all cases.

Assuming that the intrinsic interference intensity of a primary path $P_{[i]}$ is $I_{s_{[i]}}^*$, dividing the set $S_{s_{[i]}}$ of all its sending node into $I_{s_{[i]}}^*$ second-order concurrency subsets with equally spaced nodes (that is, $\Phi^{*}_{s_{[i]}}(j, I_{s_{[i]}}^*) (1 \leq j \leq I_{s_{[i]}}^*)$), is the only solution which can achieve the maximum asymptotic throughput $R_{s_{[i]}}^* = \frac{1}{I_{s_{[i]}}^*}$ under all the cases where the intrinsic interference intensity of the primary path $P_{[i]}$ is known as $I_{s_{[i]}}^*$. 

Proof:

It can be seen from Theorem VII.1 that by dividing the sending node set $S_{s_{[i]}}$ of $P_{[i]}$ into $I_{s_{[i]}}^*$ second-order concurrency subsets with equally spaced nodes, and using Algorithm VII.1, the maximum asymptotic throughput $R_{s_{[i]}}^* = \frac{1}{I_{s_{[i]}}^*} = \frac{1}{I_{s_{[i]}}^*}$ can be achieved in all the cases (i.e., under the premise that the intrinsic interference intensity of the primary path $P_{[i]}$ is known as $I_{s_{[i]}}^*$).

In the following, it will be proved that the above partition is the only partition that can achieve the maximum asymptotic throughput in all cases. For this purpose, it is only necessary to construct a specific case where the intrinsic interference intensity is $I_{s_{[i]}}^*$, and to prove that in that case there is only one way (i.e. the partition $\Phi^{*}_{s_{[i]}}(j, I_{s_{[i]}}^*) (1 \leq j \leq I_{s_{[i]}}^*)$ which is proposed in the proposition) of dividing the sending node set $S_{s_{[i]}}$ into $I_{s_{[i]}}^*$ second-order concurrency subsets. Suppose that $P_{[i]}$ consists of $N_{s_{[i]}}$ sending nodes, whose sequence numbers are 1, 2,..., $N_{s_{[i]}}$ in turn, and its intrinsic interference intensity is $I_{s_{[i]}}^*$. Furthermore, let us consider the specific case that any two sending nodes $k$ and $l$ ($k \neq l, |k-l|<I_{s_{[i]}}^*$) in $S_{s_{[i]}}$ interfere with each other.

First, in the following equation a matrix $M$ is constructed:
The matrix has \( I'_{s_{01}} \) rows and \( \left\lceil \frac{N_{s_{01}}}{I'_{s_{01}}} \right\rceil \) columns;

2) In ascending order, put the sequence number of the node \( j (1 \leq j \leq N_{s_{01}}) \) in \( R_{i[j]} \) into the corresponding unit in row \( j-I'_{s_{01}} \), \( \left\lceil \frac{j}{I'_{s_{01}}} \right\rceil \) and column \( \left\lceil \frac{j}{I'_{s_{01}}} \right\rceil \) of the matrix one by one.

That is, set the value of the unit in row \( j-I'_{s_{01}} \), \( \left\lceil \frac{j}{I'_{s_{01}}} \right\rceil \) and column \( \left\lceil \frac{j}{I'_{s_{01}}} \right\rceil \) of \( M \) to be \( j \);

3) Ignore the settings of matrix units that may not be filled by sequence numbers of nodes.

Next, consider the grouping of the nodes \( n (1 \leq n \leq I'_{s_{01}}) \). Since these nodes constitute a second-order interference subset in the specific case considered above, in order to ensure that each group of nodes is a second-order concurrency set, these nodes can only be divided into \( I'_{s_{01}} \) groups of nodes (that is, each group contains only one of these \( I'_{s_{01}} \) nodes). Without losing generality and for the convenience of discussion, it is assumed that the node \( n (1 \leq n \leq I'_{s_{01}}) \) is put into the group \( n (1 \leq n \leq I'_{s_{01}}) \) (that is, the group number is the same as the row number corresponding to the node \( n (1 \leq n \leq I'_{s_{01}}) \) in the matrix \( M \)).

In the following, let us consider the grouping of the remaining sending nodes \( n (1+I'_{s_{01}} \leq n \leq N_{s_{01}}) \) in the set \( S_{r_{01}} \). Here, mathematical induction is used to prove that the node...
$n = j + ml^{*}_{s_{(j)}} \quad (1+I^{*}_{s_{(j)}} \leq n \leq N_{s_{(j)}}, 1 \leq j \leq I^{*}_{s_{(j)}}, 1 \leq m \leq \left\lfloor \frac{N_{s_{(j)}}}{I^{*}_{s_{(j)}}} \right\rfloor)$ can and can only be divided into the group $j$ (so as to form a second-order concurrency subset). First, let us consider the grouping of the node $1+I^{*}_{s_{(j)}}$ (this node is located in the first row of the matrix $M$). Considering that we want to find out a way to subdivide the sending node set $S_{(j)}$ into $I^{*}_{s_{(j)}}$ second-order concurrency subsets of nodes, and that $I^{*}_{s_{(j)}}$ groups of nodes have already been generated in the above discussion, hence, in order not to generate more groups, we need to find out a grouping method to put the node $1+I^{*}_{s_{(j)}}$ into one of the existing $I^{*}_{s_{(j)}}$ groups. Since the node $1+I^{*}_{s_{(j)}}$ has interference relationship with nodes $n' \ (1 < n' \leq I^{*}_{s_{(j)}})$ whose sequence numbers are less than $1+I^{*}_{s_{(j)}}$, and only has concurrency relationship with the node 1, the node $1+I^{*}_{s_{(j)}}$ can only be put into the group 1, which is the group of node 1. In this way, we prove that the node $1+I^{*}_{s_{(j)}}$ can only be put into the group 1. Further, assuming that the node $n = j + ml^{*}_{s_{(j)}}$ can and can only be put into the group $j$. With $\lfloor n-n' \rfloor \leq I^{*}_{s_{(j)}} (n-(I^{*}_{s_{(j)}}-1) \leq n' \leq n-1)$, there is an interference relationship between node $n$ and node $n'$. These $I^{*}_{s_{(j)}}-1$ nodes (i.e. nodes $n' \ (n-(I^{*}_{s_{(j)}}-1) \leq n' \leq n-1$) have already been divided into $I^{*}_{s_{(j)}}-1$ groups respectively (the group number is the row number corresponding to these nodes in the matrix $M$). Therefore, in order not to generate more groups of nodes, and to ensure that each group of nodes constituting a second-order concurrency set, the node $n$ can only be put into the group $j$ ($j$ is the row number of the node $n$ in the matrix $M$). It is easy to verify that putting the node $n$ into the group $j$ can indeed form a second-order concurrency subset (the proof is omitted here), which proves that the node $n = j + ml^{*}_{s_{(j)}}$ can and can only be put into the group $j$.

Based on the above proof, it can be seen that the second-order concurrency subsets with equally
spaced nodes grouped according to the rows of the matrix $M$ (that is, the nodes corresponding to each row of $M$ form a second-order concurrency subset with equally spaced nodes) are the only partition to achieve the maximum asymptotic throughput under the above constructed specific case. Therefore, this partition (that is, the partition given in the proposition) is the only one which can achieve the maximum asymptotic throughput in all the cases. 

**Theorem VII.2: The asymptotic joint throughput of a pair of paths given its reachable joint period and activation times**

Given the reachable joint period of a pair of paths $P_{[1,2]}$ is $T_{s_{[1,2]}}$, and in each joint period (i.e. $T_{s_{[1,2]}}$ continuous time beats), the subsets of equally spaced nodes belonging to the primary paths $P_{[1]}$ and $P_{[2]}$ are activated $L_1 (\in \mathbb{Z}^+)$ and $L_2 (\in \mathbb{Z}^+)$ times, respectively. The asymptotic joint throughput of $P_{[1,2]}$ is $R^\infty_{s_{[1,2]}} = \frac{L_1 + L_2}{T_{s_{[1,2]}}}$.

**Proof:**

For a primary path, in steady state, every time all the subsets of equally spaced nodes contained in it are activated once (that is, one traversal of all the subsets of equally spaced nodes contained in the primary path is completed), its corresponding destination node will receive an information block. Therefore, in a reachable joint period (i.e. $T_{s_{[1,2]}}$ continuous time beats), if the subsets of equally spaced nodes belonging to the primary paths $P_{[1]}$ and $P_{[2]}$ are activated $L_1$ and $L_2$ times respectively, it indicates that the corresponding destination nodes in $P_{[1]}$ and $P_{[2]}$ have received $L_1 + L_2$ information blocks in total. Therefore, the average joint throughput of a pair of paths in a reachable joint period is $\frac{L_1 + L_2}{T_{s_{[1,2]}}}$. Considering that the pair of paths $P_{[1,2]}$ transmits information blocks strictly according to the period of $T_{s_{[1,2]}}$, let $n (\in \mathbb{Z}^+)$ represent the number of periods experienced, and according to the definition of the asymptotic joint throughput of a pair of paths, it can be obtained that $R^\infty_{s_{[1,2]}} = \lim_{n \to \infty} \frac{n \cdot (L_1 + L_2)}{n \cdot T_{s_{[1,2]}}} = \frac{L_1 + L_2}{T_{s_{[1,2]}}}$.

48 / 87
Definition VII.6: Equal opportunity traversals of the primary paths contained in a pair of paths

In a reachable joint period, if all the subsets of equally spaced nodes belonging to the primary paths \( P_{[1]} \) and \( P_{[2]} \) contained in a pair of paths \( P_{[1,2]} \) are activated \( L (\in \mathbb{Z}^+) \) times, it is said that the traversals of the primary paths contained in a pair of paths (i.e. the “activation” mentioned above) is of equal opportunity. On the contrary, it is said that the traversals of the primary paths contained in a pair of paths is of unequal opportunity.

Algorithm VII.2: The algorithm for the cooperative volatility transmission of information blocks in a pair of paths based on the set of maximum supporting elements in the case of equal opportunity traversals

Let the intrinsic interference intensity of a pair of paths \( P_{[1,2]} \) be \( I^*_{[1,2]} \), and assume that the reachable periods of the primary paths \( P_{[1]} \) and \( P_{[2]} \) are \( T_{[1]} \) and \( T_{[2]} \), respectively. Let \( C \) be the matrix of joint concurrency relationship between subsets of equally spaced nodes of a pair of paths \( P_{[1,2]} \). And the maximum reachable supporting number of \( C \) is \( U^*_{C_p} \). Determine one set \( S^*_{C_p} \) as the set of maximum supporting elements of \( C \), and assign incremental sequential numbers to all of its \( U^*_{C_p} \) elements (i.e. element No. 1, element No. 2, ..., and element No. \( U^*_{C_p} \)). Allocate \( T_{[1,2]} \) consecutive time beats as follows to equally activate all the subsets of equally spaced nodes from both \( P_{[1]} \) and \( P_{[2]} \) \( L \) times:

1: BEGIN
2: Make the time beat counter \( m \) indicate the sequence number of the current time beat;
3: FOR (the counter of traversal times = 1 to \( L (\in \mathbb{Z}^+) \)) DO
4: As for the No. 1 element in \( S^*_{C_p} \), i.e. \( [i_1, j_1] (\in S^*_{C_p}) \), assume that its corresponding non-zero element in \( C \) is \( C[i_1, j_1] = 1 \), and arrange the subset \( \Phi^*_{[1]} (i_1, T_{[1]}) \) of equally
spaced nodes in the primary path \( P_{(1)} \) and the subset \( \Phi'_{s_{(1)}}(j_i, T_{s_{(1)}}) \) of equally spaced nodes in the primary path \( P_{(2)} \) to be activated concurrently within the time beat No. \( m + 1 \). Likewise, as for the No. 2 element in \( S_{c_p}^* \), i.e. \([i_2, j_2] \in S_{c_p}^*\), assume that its corresponding non-zero element in \( C \) is \( C[i_2, j_2] = 1 \), and arrange the subset \( \Phi'_{s_{(2)}}(j_2, T_{s_{(2)}}) \) of \( P_{(1)} \) and the subset \( \Phi'_{s_{(2)}}(j_2, T_{s_{(2)}}) \) of \( P_{(2)} \) to be activated concurrently within the time beat No. \( m + 2 \). By analogy, we can allocate \( U_{c_p}^* \) continuous time beats to realize the concurrent activation of the subsets of equally spaced nodes respectively from \( P_{(1)} \) and \( P_{(2)} \) (i.e. \( U_{c_p}^* \) subsets are from \( P_{(1)} \) and \( U_{c_p}^* \) subsets are from \( P_{(2)} \)). And mark all these \( U_{c_p}^* \) time beats as time beats of “Category 1”;

5: Through the above processing for arranging time beats, there are still \( T_{s_{(1)}} - U_{c_p}^* \) subsets of equally spaced nodes in \( P_{(1)} \), having not been assigned to their corresponding time beats. In order to avoid mutual interference, these remaining \( T_{s_{(1)}} - U_{c_p}^* \) subsets of equally spaced nodes in \( P_{(1)} \) are arranged to be activated in the subsequent \( T_{s_{(1)}} - U_{c_p}^* \) time beats without overlapping to each other and without any omission (that is, only one subset of equally spaced nodes is activated in each time beat). And mark all these \( T_{s_{(1)}} - U_{c_p}^* \) time beats as time beats of “Category 2”;

6: Through the above processing for arranging time beats, there are still \( T_{s_{(2)}} - U_{c_p}^* \) subsets of equally spaced nodes in \( P_{(2)} \), having not been assigned to their corresponding time beats. In order to avoid mutual interference, these remaining \( T_{s_{(2)}} - U_{c_p}^* \) subsets of equally spaced nodes in \( P_{(2)} \) are arranged to be activated in the subsequent \( T_{s_{(2)}} - U_{c_p}^* \) time beats without overlapping to each other and without any omission. And mark all these \( T_{s_{(2)}} - U_{c_p}^* \) time beats as time beats of “Category 3”;
7: Increase the value of the time beat counter $m$ by $T_{s_{[1]}} + T_{s_{[2]}} - U^{*}_{c_{u}}$;

   //Note: after the above three steps, the total allocated time beats are
   $U^{*}_{c_{u}} + (T_{s_{[1]}} - U^{*}_{c_{u}}) + (T_{s_{[2]}} - U^{*}_{c_{u}})$, i.e. $T_{s_{[1]}} + T_{s_{[2]}} - U^{*}_{c_{u}}$.

8: END FOR;

9: So far, the allocation of $T_{s_{[0,2]}} = L(T_{s_{[1]}} + T_{s_{[2]}} - U^{*}_{c_{u}})$ time beats contained in a reachable joint period has been completed. The next transmissions will be executed in periods of $T_{s_{[0,2]}}$;

10: END

**Theorem VII.3:** In the case of equal opportunity traversals, Algorithm VII.2 can maximize the asymptotic joint throughput of a pair of paths

Let the reachable periods of the primary paths $P_{[1]}$ and $P_{[2]}$, which are contained in a pair of paths $P_{[1,2]}$, be $T_{s_{[1]}}$ and $T_{s_{[2]}}$, respectively. Let $C$ be the matrix of joint concurrency relationship between subsets of equally spaced nodes of a pair of paths $P_{[1,2]}$. And the maximum reachable supporting number of $C$ is $U^{*}_{c_{u}}$. If time beats are allocated so that all the subsets of equally spaced nodes in $P_{[1]}$ and $P_{[2]}$ are activated $L$ times, the asymptotic joint throughput $R_{S_{[0,2]}}^{s}$ of $P_{[1,2]}$ can be maximized by using Algorithm VII.2.

**Proof:**

According to Theorem VII.2, we know that $R_{S_{[0,2]}}^{s} = \frac{2L}{T_{s_{[0,2]}}}$. Therefore, given the reachable periods $T_{s_{[1]}}$ and $T_{s_{[2]}}$, and the activation times of each subset of equally spaced nodes is $L$, if one want to prove that $R_{S_{[0,2]}}^{s}$ is maximized, it is only necessary to prove that the realized joint period $T_{s_{[0,2]}}$ is minimized by using Algorithm VII.2.

According to the definition of reachable joint period of a pair of paths (Definition VI.3), it is known that the time beats contained in a joint period $T_{s_{[0,2]}}$ must belong to one of the following

51 / 87
three categories, that is, in time beats of category 1, one subset of equally spaced nodes from the primary path \( P_{(1)} \) and one subset of equally spaced nodes from the primary path \( P_{(2)} \) are activated at the same time, and the two subsets of nodes together constitute a second-order concurrency subset of nodes; In time beats of category 2, only one subset of equally spaced nodes from \( P_{(1)} \) is activated; In time beats of category 3, only one subset of equally spaced nodes from \( P_{(2)} \) is activated. If Algorithm VII.2 is executed, it can be seen that the number of time beats of category 1 within a joint period is \( LU^{\prime}_{C_p} \); The number of time beats of category 2 is \( L(T_{S(1)} - U^{\prime}_{C_p}) \), and the number of time beats of category 3 is \( L(T_{S(2)} - U^{\prime}_{C_p}) \). Therefore, by executing Algorithm VII.2, the total number of time beats within a joint period is

\[
T_{S(1,2)} = L(T_{S(1)} + T_{S(2)} - U^{\prime}_{C_p}) \tag{VII.2}
\]

Assuming that the allocation of the above time beats is not optimal, that is, after executing Algorithm VII.2, the obtained \( T_{S(1,2)} \) does not reach the minimum. From the above analysis of the categories of \( T_{S(1,2)} \) time beats, it is inevitable that in the current allocation of \( T_{S(1,2)} \) time beats after executing Algorithm VII.2, at least two subsets of equally spaced nodes corresponding to one time beat of category 2 and one time beat of category 3 can be found, which can jointly form a second-order concurrency subset of nodes. Because only in this way can the number of time beats in the current joint period \( T_{S(1,2)} \) be further reduced. However, this means that in the matrix \( C \) of joint concurrency relationship, one can still find out at least one non-zero element in \( C \) which is not covered by the set of maximum supporting elements over which Algorithm VII.2 executes, which is contrary to the definition of the set of maximum supporting elements (see condition 2 of Definition VI.9 for details).

To sum up, one cannot achieve a smaller joint period than \( T_{S(1,2)} = L(T_{S(1)} + T_{S(2)} - U^{\prime}_{C_p}) \), that is, Algorithm VII.2 is optimal in its capable of minimizing the joint period of a pair of paths. ■

**Theorem VII.4:** The upper bound of the asymptotic joint throughput of a pair of paths achieved by Algorithm VII.2
Let the intrinsic interference intensity of a pair of paths \( P_{[1,2]} \) be \( I^*_{S_{[1,2]}} \), and assume that the reachable periods of the primary paths \( P_{[1]} \) and \( P_{[2]} \), which are contained in \( P_{[1,2]} \), are \( T_{S_{[1]}} \) and \( T_{S_{[2]}} \), respectively. Let \( C \) be the matrix of joint concurrency relationship between subsets of equally spaced nodes of \( P_{[1,2]} \). And the maximum reachable supporting number of \( C \) is \( U^*_C \). If time beats are allocated by using Algorithm VII.2, so that the joint period of \( P_{[1,2]} \) is \( T_{S_{[1,2]}} \), and within which all the subsets of equally spaced nodes in \( P_{[1]} \) and \( P_{[2]} \) are activated \( L \) times. It can be concluded that the upper bound of the asymptotic joint throughput \( R^*_C \) of the pair of paths \( P_{[1,2]} \) is independent of the activation times \( L \), and \( R^*_C \leq \frac{2}{T_{S_{[1,2]}}} \) (when \( U^*_C = T_{S_{[1]}} + T_{S_{[2]}} - I^*_{S_{[1,2]}} \), the equality relation holds).

**Proof:**

From Theorem VII.2 and Theorem VII.3, we have

\[
R^*_C = \frac{2L}{T_{S_{[1,2]}}} = \frac{2}{T_{S_{[1]}} + T_{S_{[2]}} - U^*_C} \quad (VII.3)
\]

According to Corollary VI.7.1, it can be found that \( U^*_C \leq T_{S_{[1]}} + T_{S_{[2]}} - I^*_{S_{[1,2]}} \). Therefore, by combing with the above formula, it is obtained that

\[
R^*_C = \frac{2}{T_{S_{[1]}} + T_{S_{[2]}} - U^*_C} \leq \frac{2}{I^*_{S_{[1,2]}}} \quad (VII.4)
\]

Furthermore, the equality relation holds when \( U^*_C = T_{S_{[1]}} + T_{S_{[2]}} - I^*_{S_{[1,2]}} \). ■

**Theorem VII.5:** In the case of equal opportunity traversals, the unreachability of the joint period of a pair of paths

Let the intrinsic interference intensity of a pair of paths \( P_{[1,2]} \) be \( I^*_{S_{[1,2]}} \), and assume that the reachable periods of the primary paths \( P_{[1]} \) and \( P_{[2]} \), which are contained in \( P_{[1,2]} \), are \( T_{S_{[1]}} \) and \( T_{S_{[2]}} \), respectively. Let \( C \) be the matrix of joint concurrency relationship between subsets of equally spaced nodes of \( P_{[1,2]} \). And the maximum reachable supporting number of \( C \) is \( U^*_C \). If time beats are allocated by using Algorithm VII.2, so that the joint period of \( P_{[1,2]} \) is \( T_{S_{[1,2]}} \), and within which all the subsets of equally spaced nodes in \( P_{[1]} \) and \( P_{[2]} \) are activated \( L \) times. It can be concluded that the upper bound of the asymptotic joint throughput \( R^*_C \) of the pair of paths \( P_{[1,2]} \) is independent of the activation times \( L \), and \( R^*_C \leq \frac{2}{T_{S_{[1,2]}}} \) (when \( U^*_C = T_{S_{[1]}} + T_{S_{[2]}} - I^*_{S_{[1,2]}} \), the equality relation holds).
of equally spaced nodes of $P_{[1,2]}$. And the maximum reachable supporting number of $C$ is $U_{c^*}$. If the relation of $U_{c^*} < T_{S_{[1]}} + T_{S_{[2]}} - I_{S_{[1,2]}}^*$ holds for all the possible combinations of the reachable period of $T_{S_{[1]}}$ and $T_{S_{[2]}}$, the joint period $T_{S_{[1,2]}} = I_{S_{[1,2]}}^*$ is not reachable on the premise of equal opportunity traversals.

**Proof:**

Assuming that time beats are allocated, so that the joint period of $P_{[1,2]}$ is $T_{S_{[1,2]}}$, and within which all the subsets of equally spaced nodes in $P_{[1]}$ and $P_{[2]}$ are activated $L$ times. According to Theorem VII.3, for a reachable joint period $T_{S_{[1,2]}}$, the following relationship must be true:

$$T_{S_{[1,2]}} \geq L(T_{S_{[1]}} + T_{S_{[2]}} - U_{c^*}) \quad \text{(VII.5)}$$

From the premise, it is known that $U_{c^*} < T_{S_{[1]}} + T_{S_{[2]}} - I_{S_{[1,2]}}^*$ (i.e. $I_{S_{[1,2]}}^* < T_{S_{[1]}} + T_{S_{[2]}} - U_{c^*}$) and $L \geq 1$, base on the above formula we can further obtain:

$$T_{S_{[1,2]}} \geq T_{S_{[1]}} + T_{S_{[2]}} - U_{c^*} > I_{S_{[1,2]}}^*$$

Therefore, $T_{S_{[1,2]}} > I_{S_{[1,2]}}^*$ is obtained, which suggests that $T_{S_{[1,2]}} = I_{S_{[1,2]}}^*$ is not reachable. ■

**Definition VII.7:** The $L_1 \times L_2$-th continuation matrix $M^{L_1 \times L_2}$ of a matrix $M$

Let $M$ be a matrix of $n$ rows and $o$ columns. Repeat it horizontally for $L_2$ times to form a matrix $M'$ of $n$ rows and $o \cdot L_2$ columns, and then repeat the obtained $M'$ vertically for $L_1$ times. The matrix formed in this way (consisting of $n \cdot L_1$ rows and $o \cdot L_2$ columns) is defined as the $L_1 \times L_2$-th continuation matrix of the matrix $M$, which is denoted as $M^{L_1 \times L_2}$ (see the following formula). If $M$ is a binary matrix, the set of maximum supporting elements of $M^{L_1 \times L_2}$ is denoted as $S_{M^{L_1 \times L_2}}$, and the corresponding maximum reachable supporting number of $M^{L_1 \times L_2}$ is denoted as $U_{M^{L_1 \times L_2}}^*$. 

54 / 87
Algorithm VII.3: The algorithm for the cooperative volatility transmission of information blocks in a pair of paths based on the set of maximum supporting elements in the case of unequal opportunity traversals

Let the reachable periods of the primary paths $P_{[1]}$ and $P_{[2]}$, which are contained in a pair of paths $P_{[1,2]}$, be $T_{S_{[1]}}$ and $T_{S_{[2]}}$, respectively. Let $C$ be the matrix of joint concurrency relationship between subsets of equally spaced nodes of a pair of paths $P_{[1,2]}$. Allocate $T_{S_{[1,2]}}$ consecutive time beats as follows, so as to activate the subsets of equally spaced nodes from $P_{[1]}$ for $L_1$ times, and to activate the subsets of equally spaced nodes from $P_{[2]}$ for $L_2$ times.

1: BEGIN
2: Generate the $L_1 \times L_2$-th continuation matrix $C^{L_1 \times L_2}$ of the matrix $C$;

//Note: According to the definitions of the matrix $C$ of joint concurrency relationship between subsets of equally spaced nodes and its $L_1 \times L_2$-th continuation matrix $C^{L_1 \times L_2}$, any element $C^{L_1 \times L_2}[i,j] (1 \leq i \leq L_1, 1 \leq j \leq L_2)$ in $C^{L_1 \times L_2}$, represents the joint concurrency coefficient (Definition VI.6) between the subset $\Phi^{S_{[1,2]}}((i-1) \mod T_{S_{[1]}} + 1, T_{S_{[1]}})$ of equally spaced nodes in the primary path $P_{[1]}$ and the subset $\Phi^{S_{[1,2]}}((j-1) \mod T_{S_{[2]}} + 1, T_{S_{[2]}})$ of equally spaced nodes in the primary path $P_{[2]}$.

3: Determine one set $S_{C^{L_1 \times L_2}}^*$ as the set of maximum supporting elements of $C^{L_1 \times L_2}$. Let $U_{C^{L_1 \times L_2}}^*$ denote the maximum reachable supporting number of $C^{L_1 \times L_2}$, and assign incremental sequential numbers to all of the $U_{C^{L_1 \times L_2}}^*$ elements of $S_{C^{L_1 \times L_2}}^*$ (i.e. element No.
1, element No. 2,..., and element No. \( U_{c_{x_{ij}}^{*}} \);

4: As for the No. 1 element in \( S_{c_{y_{ij}}^{*}} \), i.e. \([i_1, j_1] \in S_{c_{y_{ij}}^{*}} \), assume that its corresponding non-zero element in \( C_{L_1 \times L_2} \) is \( C_{L_1 \times L_2} [i_1, j_1] = 1 \), and arrange the subset \( \Phi_{y_{ij}}' ( ((i_1 - 1) \mod T_{y_{ij}} + 1, T_{y_{ij}}) ) \) of equally spaced nodes in \( P_{[1]} \) and the subset \( \Phi_{y_{ij}}' ( ((j_1 - 1) \mod T_{y_{ij}} + 1, T_{y_{ij}}) ) \) of equally spaced nodes in \( P_{[2]} \) to be activated concurrently within the time beat No. 1. Likewise, as for the No. 2 element in \( S_{c_{y_{ij}}^{*}} \), i.e. \([i_2, j_2] \in S_{c_{y_{ij}}^{*}} \), assume that its corresponding non-zero element in \( C_{L_1 \times L_2} \) is \( C_{L_1 \times L_2} [i_2, j_2] = 1 \), and arrange the subset \( \Phi_{y_{ij}}' ( ((i_2 - 1) \mod T_{y_{ij}} + 1, T_{y_{ij}}) ) \) of \( P_{[1]} \) and the subset \( \Phi_{y_{ij}}' ( ((j_2 - 1) \mod T_{y_{ij}} + 1, T_{y_{ij}}) ) \) of \( P_{[2]} \) to be activated concurrently within the time beat No. 2. By analogy, we can allocate \( U_{c_{x_{ij}}^{*}}' \) continuous time beats to realize the concurrent activation of the subsets of equally spaced nodes respectively from \( P_{[1]} \) and \( P_{[2]} \), and mark all these \( U_{c_{x_{ij}}^{*}}' \) time beats as time beats of “Category 1”;

5: Through the above processing for arranging time beats, there are still \( L_1 \cdot T_{y_{ij}} - U_{c_{x_{ij}}^{*}}' \) subsets of equally spaced nodes in \( P_{[1]} \), having not been assigned to their corresponding time beats. In order to avoid mutual interference, these remaining \( L_1 \cdot T_{y_{ij}} - U_{c_{x_{ij}}^{*}}' \) subsets of equally spaced nodes in \( P_{[1]} \) are arranged to be activated in the subsequent \( L_1 \cdot T_{y_{ij}} - U_{c_{x_{ij}}^{*}}' \) time beats without overlapping to each other and without any omission (that is, only one subset of equally spaced nodes is activated in each time beat). And mark all these \( L_1 \cdot T_{y_{ij}} - U_{c_{x_{ij}}^{*}}' \) time beats as time beats of “Category 2”;

6: Through the above processing for arranging time beats, there are still \( L_2 \cdot T_{y_{ij}} - U_{c_{x_{ij}}^{*}}' \) subsets of equally spaced nodes in \( P_{[2]} \), having not been assigned to their corresponding time beats. In order to avoid mutual interference, these remaining \( L_2 \cdot T_{y_{ij}} - U_{c_{x_{ij}}^{*}}' \) subsets
of equally spaced nodes in $P_{[2]}$ are arranged to be activated in the subsequent $L_2 \cdot T_{S_{b[2]}} - U^*_{C^o_{S_{b[2]}}} \cdot T_{S_{b[2]}} - U^*_{C^o_{S_{b[2]}}}$ time beats without overlapping to each other and without any omission (that is, only one subset of equally spaced nodes is activated in each time beat). And mark all these $L_2 \cdot T_{S_{b[2]}} - U^*_{C^o_{S_{b[2]}}}$ time beats as time beats of “Category 3”;

7: So far, the allocation of $T_{S_{b[1:j]}} = L_1 \cdot T_{S_{b[1]}} + L_2 \cdot T_{S_{b[2]}} - U^*_{C^o_{S_{b[2]}}}$ time beats contained in a reachable joint period has been completed. The next transmissions will be executed in periods of $T_{S_{b[1:j]}}$.

//Note: From the above step 4 to step 6, the total allocated time beats are $U^*_{C^o_{S_{b[2]}}} + (L_1 \cdot T_{S_{b[1]}} - U^*_{C^o_{S_{b[1]}}}) + (L_2 \cdot T_{S_{b[2]}} - U^*_{C^o_{S_{b[2]}}})$, i.e. $L_1 \cdot T_{S_{b[1]}} + L_2 \cdot T_{S_{b[2]}} - U^*_{C^o_{S_{b[2]}}}$.

8: END

---

**Theorem VII.6: The asymptotic joint throughput of a pair of paths achieved by Algorithm VII.3**

Let the reachable periods of the primary paths $P_{[1]}$ and $P_{[2]}$, which are contained in a pair of paths $P_{[1,2]}$, be $T_{S_{b[1]}}$ and $T_{S_{b[2]}}$, respectively. Let $C$ be the matrix of joint concurrency relationship between subsets of equally spaced nodes of a pair of paths $P_{[1,2]}$. $C^{L_1 \times L_2}$ is the $L_1 \times L_2$-th continuation matrix of $C$, and its maximum reachable supporting number is $U^*_{C^o_{S_{b[2]}}}$. If time beats are allocated by using Algorithm VII.3, so that the joint period of $P_{[1,2]}$ is $T_{S_{b[1:j]}}$, and within which subsets of equally spaced nodes in $P_{[1]}$ and $P_{[2]}$ are activated $L_1$ and $L_2$ times, respectively. It can be concluded that the asymptotic joint throughput of $P_{[1,2]}$ is

$$R_{S_{b[1:j]}}^x = \frac{L_1 + L_2}{L_1 \cdot T_{S_{b[1]}} + L_2 \cdot T_{S_{b[2]}} - U^*_{C^o_{S_{b[2]}}}}.$$ 

**Proof:**

According to the above descriptions of Algorithm VII.3, the reachable joint period achieved by Algorithm VII.3 is $T_{S_{b[1:j]}} = L_1 \cdot T_{S_{b[1]}} + L_2 \cdot T_{S_{b[2]}} - U^*_{C^o_{S_{b[2]}}}$. Furthermore, by combining with
Theorem VII.2, it can be obtained that
\[ R^c_{S_{[1,2]}} = \frac{L_1 + L_2}{L_1 \cdot T_{S_{[1]}} + L_2 \cdot T_{S_{[2]}}} - U^*_{C^*_{[1,2]}}. \]

**Corollary VII.6.1:** The upper bound of the asymptotic joint throughput of a pair of paths achieved by Algorithm VII.3

Let the reachable periods of the primary paths \( P_{[1]} \) and \( P_{[2]} \), which are contained in a pair of paths \( P_{[1,2]} \), be \( T_{S_{[1]}} \) and \( T_{S_{[2]}} \), respectively. Let \( C \) be the matrix of joint concurrency relationship between subsets of equally spaced nodes of a pair of paths \( P_{[1,2]} \). \( C^{L_1 \times L_2} \) is the \( L_1 \times L_2 \)-th continuation matrix of \( C \), and its maximum reachable supporting number is \( U^*_{C^*_{[1,2]}} \).

If time beats are allocated by using Algorithm VII.3, so that the joint period of \( P_{[1,2]} \) is \( T_{S_{[1,2]}} \), and within which subsets of equally spaced nodes in \( P_{[1]} \) and \( P_{[2]} \) are activated \( L_1 \) and \( L_2 \) times, respectively. The asymptotic joint throughput of \( P_{[1,2]} \) is \( R^c_{S_{[1,2]}} \leq \frac{1}{T_{S_{[1]}}} + \frac{1}{T_{S_{[2]}}} \) (when \( U^*_{C^*_{[1,2]}} = L_1 \cdot T_{S_{[1]}} = L_2 \cdot T_{S_{[2]}} \), the equality relation holds).

**Proof:**

**Case 1:** \( L_1 \cdot T_{S_{[1]}} \geq L_2 \cdot T_{S_{[2]}} \)

In this case, \( U^*_{C^*_{[1,2]}} \leq L_2 \cdot T_{S_{[2]}} \) and \( L_2 \leq \frac{L_1 \cdot T_{S_{[1]}}}{T_{S_{[2]}}} \). Therefore, according to Theorem VII.6, we have:

\[
R^c_{S_{[1,2]}} = \frac{L_1 + L_2}{L_1 \cdot T_{S_{[1]}} + L_2 \cdot T_{S_{[2]}}} - U^*_{C^*_{[1,2]}} \leq \frac{L_1 + L_2}{L_1 \cdot T_{S_{[1]}} + L_2 \cdot T_{S_{[2]}}} \leq \frac{1}{T_{S_{[1]}}} + \frac{1}{T_{S_{[2]}}} \quad \text{(VII.8)}
\]

It is evident that when \( U^*_{C^*_{[1,2]}} = L_1 \cdot T_{S_{[1]}} = L_2 \cdot T_{S_{[2]}} \), the equality relation holds.
Case 2: \( L_1 \cdot T_{S_{q[i]}} < L_2 \cdot T_{S_{q[i]}} \)

In this case, \( U^*_{C_{q[i]}} \leq L_1 \cdot T_{S_{q[i]}} \) and \( L_4 < \frac{L_2 \cdot T_{S_{q[i]}}}{T_{S_{q[i]}}} \). Therefore, according to Theorem VII.6, we have:

\[
R^*_{S_{q[i]}} = \frac{L_1 + L_2}{L_1 \cdot T_{S_{q[i]}} + L_2 \cdot T_{S_{q[i]}} - U^*_{C_{q[i]}}} \\
\leq \frac{L_1 + L_2}{L_2 \cdot T_{S_{q[i]}}} < \frac{L_2 \cdot T_{S_{q[i]}}}{T_{S_{q[i]}}} \\
= \frac{1}{T_{S_{q[i]}}} + \frac{1}{T_{S_{q[i]}}}
\]

(VII.9)

The proposition is proved by combining the above two cases. ■

**Corollary VII.6.2: Relationship between \( U^*_{C_q} \) and \( U^*_{C_{q_{LL}}} \)**

Let the reachable periods of the primary paths \( P_{[1]} \) and \( P_{[2]} \), which are contained in a pair of paths \( P_{[1,2]} \), be \( T_{S_{q[i]}} \) and \( T_{S_{q[i]}} \), respectively. Let \( C \) be the matrix of joint concurrency relationship between subsets of equally spaced nodes of a pair of paths \( P_{[1,2]} \). \( C_{LL} \) is the \( L \times L \) -th continuation matrix of \( C \). The maximum reachable supporting numbers of \( C \) and \( C_{LL} \) are \( U^*_{C_q} \) and \( U^*_{C_{q_{LL}}} \), respectively. There is \( U^*_{C_q} = \frac{U^*_{C_{q_{LL}}}}{L} \).

**Proof:**

Consider the case \( L_1 = L_2 = L \), according to Theorem VII.6, we have:

\[
R^*_{S_{q[i]}} = \frac{2L}{L(T_{S_{q[i]}} + T_{S_{q[i]}} - \frac{U^*_{C_{LL}}}{L})} \\
= \frac{2}{T_{S_{q[i]}} + T_{S_{q[i]}} - \frac{U^*_{C_{LL}}}{L}}
\]

(VII.10)

On the other hand, from Theorem VII.2 and Theorem VII.3, we have:

\[
R^*_{S_{q[i]}} = \frac{2}{T_{S_{q[i]}} + T_{S_{q[i]}} - \frac{U^*_{C_q}}{L}}
\]

(VII.11)
Combing equation (VII.10) and (VII.11), it can be concluded that \( U_{c_{sp}} = \frac{U_{c_{sp}}^*}{L} \).

Algorithm VII.4: The algorithm flow of maximizing end-to-end asymptotic joint throughput of a pair of paths

Given the distribution of the geographical locations of network nodes, and given the pairs of source nodes and destination nodes belonging to the two primary paths respectively, the following algorithm flow is executed so as to maximize end-to-end asymptotic joint throughput of a pair of paths:

1: BEGIN
2: WHILE( \( R_{S_{(1,2)}}^e \) is not yet optimal and can be further improved by adjusting routing selections for the pair of paths) DO
3:   For two pairs of source nodes and destination nodes, two multi-hop routes, which are used to support the transmissions of information blocks of two primary paths, are selected to improve the asymptotic joint throughput of the pair of paths;
4:   Based on the generated multi-hop routes, the transmission coverage relationships and the interference relationships between nodes of the pair of paths are formed;
5:   Based on the obtained transmission coverage relationships and interference relationships between nodes, the intrinsic interference intensities \( I_{S_{(1)}}^* \), \( I_{S_{(2)}}^* \) and \( I_{S_{(1,2)}}^* \) corresponding to \( P_{[1]} \), \( P_{[2]} \) and \( P_{[1,2]} \) are calculated, respectively;
6:   WHILE( \( R_{S_{(1,2)}}^e \) is not yet optimal and can be further improved by adjusting the reachable periods of the two primary paths) DO
7:     Based on the obtained intrinsic interference intensities \( I_{S_{(1)}}^* \) and \( I_{S_{(2)}}^* \), the reachable periods \( T_{S_{(1)}} (\geq I_{S_{(1)}}^*) \) and \( T_{S_{(2)}} (\geq I_{S_{(2)}}^*) \) adopted by \( P_{[1]} \) and \( P_{[2]} \) respectively, which can further improve the asymptotic joint throughput of the pair of paths, are obtained;
8:     Based on the obtained reachable periods \( T_{S_{(1)}} \) and \( T_{S_{(2)}} \), the subsets
Based on the obtained subsets of equally spaced nodes and the interference relationships among them, the matrix $C$ of joint concurrency relationship between subsets of equally spaced nodes of $P_{[1]}$ is obtained;

WHILE($R_{S_{[1,2]}}^0$ is not yet optimal and can be further improved by adjusting the activation times of the two primary paths) DO

The activation times $L_1$ and $L_2$ of the subsets of equally spaced nodes in $P_{[1]}$ and $P_{[2]}$, which can improve the asymptotic joint throughput of a pair of paths within a reachable joint period, is obtained respectively;

Based on the obtained matrix $C$ and the selected activation times $L_1$ and $L_2$, generated the corresponding $L_1 \times L_2$-th continuation matrix $C_{L_1 \times L_2}$, and calculated out its maximum reachable supporting numbers $U_{C_{L_1 \times L_2}}$;

Based on the above determined parameters (i.e. end-to-end routings, $T_{S_{[1]}}$, $T_{S_{[2]}}$, $L_1$, $L_2$, and $U_{C_{L_1 \times L_2}}$), calculated the corresponding asymptotic joint throughput of a pair of paths as:

$$R_{S_{[1,2]}}^* = \frac{L_1 + L_2}{L_1 \cdot T_{S_{[1]}} + L_2 \cdot T_{S_{[2]}} - U_{C_{L_1 \times L_2}}}$$

END WHILE

END WHILE

After the relevant parameters of maximizing the asymptotic joint throughput are determined, execute the time beat allocation algorithm (such as the one given in Algorithm VII.3), so as to implement the optimal allocation of $T_{S_{[1,2]} = L_1 \cdot T_{S_{[1]}} + L_2 \cdot T_{S_{[2]}} - U_{C_{L_1 \times L_2}}}$ time beats within a joint period. And based on the optimized allocation of time beats, take $T_{S_{[1,2]}}$ time beats as a cycle to activate the corresponding subsets of equally spaced nodes, so as to realize the
Note: The above algorithm flow can be realized in a centralized way or in a more distributed way (i.e. not a purely distributed implementation). For example, in a possible centralized implementation, a centralized control node can be selected to collect necessary information, complete relevant calculations, and issue time beat allocation instructions and so on. In a more distributed implementation, one possible way is to select one appropriate node from each of the two primary paths to serve as the coordination nodes between the two primary paths. The selected coordination nodes are responsible for: collecting the relevant information of nodes in the corresponding primary path, interacting with the coordination node of another primary path, calculating the time beat allocation of the corresponding primary path according to the same algorithm (such as Algorithm VII.3), and finally issuing the instructions of the time beat allocation in the corresponding primary path.

Definition VII.8: One time end-to-end delay of an information block on a primary path

On a primary path \( P_i \) \((i = 1, 2)\), the total number of time beats needed from the time an information block entering the source node to the time it finally reaching the destination node is defined as the one-time end-to-end delay of an information block on \( P_i \), which is denoted as \( D_{ed_i} \).

Theorem VII.7: The one time end-to-end delay on a primary path can be minimized by using Algorithm VII.1

As for a primary path \( P_i \) \((i = 1, 2)\), if Algorithm VII.1 is adopted, the one time end-to-end delay can be minimized, i.e. \( D_{ed_i} = N_{S_{b_i}} \).

Proof:

For a primary path \( P_i \) with \( N_{S_{b_i}} \) sending nodes, starting from its node 1 (source node), at least \( N_{S_{b_i}} \) beats are needed for an information block reaching the final destination node (i.e. node
through hop-by-hop transmissions. By checking the execution procedure of Algorithm VII.1, it is not difficult to see that for the transmission of each information block, Algorithm VII.1 can achieve the shortest one-time end-to-end delay.

VIII. A general method for solving the intrinsic period of a primary path

In the previous part of the paper, Theorem VI.1 gives an important conclusion that the intrinsic period of a primary path is equal to its intrinsic interference intensity. However, this conclusion is based on the assumption that a primary path should follow the interference reduction rule between nodes (i.e. Rule V.1 and Rule V.2). Considering the fact that in the real world not all the primary paths follow this assumption, in this section we introduce the concept of the Distribution Spectrum of Interference-Spacing of nodes (i.e. DSIS) of a primary path to find out a more general method for solving the intrinsic period of a primary path. In this section, we first define the concept of the DSIS of a primary path and its related basic concepts. Next, we analyze several basic properties related to the DSIS of a primary path (such as Corollary VIII.3.1). Finally, based on the obtained fundamental properties of the DSIS of a primary path, we propose a simple and general algorithm flow for solving the intrinsic period of a primary path (i.e. Algorithm VIII.1).

A. The definition of the DSIS of a primary path

Consider that in a primary path (Definition III.1) consisting of \( N+1 \) nodes, information blocks are relayed from the node 1 (i.e. the source node) through the node 2, the node 3,..., the node \( i \),..., and the node \( N (\geq 1) \) to the final node \( N+1 \) (i.e. the destination node). If two different sending nodes, like \( k (1 \leq k \leq N) \) and \( l (1 \leq l \leq N) \), in \( P \) are activated within a same time beat, which brings about the interference with the correct receptions of information blocks of the corresponding receiving nodes, these two nodes cannot be concurrent within the same time beat, i.e. there is an interference relationship between the two sending nodes (Definition IV.3). According to Definition III.2, the spacing (i.e. “distance”) between these two mutually interfering nodes in the primary path is \( |k-l| \).
Definition VIII.1: The Distribution Spectrum of Interference-Spacing of nodes (i.e. DSIS) of a primary path

For a primary path $P$, let us construct a function mapping relationship $F_{S_P}^I (\eta, d)$, where the independent variable $\eta$ ($1 \leq \eta \leq N - 1, \eta \in \mathbb{Z}$) represents the spacing between mutually interfering sending node pairs on $P$. The value $F_{S_P}^I (\eta, d)$ ($\geq 0$) represents the total number of pairs of mutually interfering sending nodes with a spacing of $\eta$ in the set $S_P$ of all the sending nodes of $P$. (Note: considering that the interference relationship given in Definition IV.3 is mutual, when counting the number of pairs of mutually interfering nodes, the interfering node pair $(k,l)$ and the interfering node pair $(l,k)$ will be treated equally and only counted as one pair of interfering nodes). The function mapping relationship $F_{S_P}^I (\eta, d)$ is referred to as the Distribution Spectrum of Interference-Spacing of nodes (i.e. DSIS) of a primary path $P$. $\eta$ is the sequence number of a spectral line, and the corresponding value $F_{S_P}^I (\eta, d)$ is defined as the intensity of a spectral line with sequence number of $\eta$. For the spectrum $F_{S_P}^I (\eta, d)$, if all the intensities of its spectral lines are greater than 0 (i.e. $F_{S_P}^I (\eta, d) > 0$ ($1 \leq \eta \leq N - 1$)), it is called as a pure non-zero distribution spectrum of interference-spacing of nodes (referred to as “pure non-zero spectrum” for short).

Definition VIII.2: A zero point of the DSIS of a primary path

If the intensity $F_{S_P}^I (\eta, d)$ of a spectral line with sequence number $\eta$ ($1 \leq \eta \leq N - 1$) is 0, the spectral line with sequence number $\eta$ is called as a zero point of the spectrum $F_{S_P}^I (\eta, d)$.

Definition VIII.3: An equally spaced cumulative zero point of the DSIS of a primary path with a spacing of $T$

For a spectral line with sequence number $\eta$ in a spectrum $F_{S_P}^I (\eta, d)$, if

$$
\sum_{i=0}^{\frac{(N-1)-\eta}{T}} F_{S_P}^I (\eta + i \cdot T) = 0 \quad (1 \leq \eta \leq N - 1, i \in \mathbb{Z})
$$

holds (where $T \geq 1, T \in \mathbb{Z}$), then the spectral line...
\( \eta \) is said to be an equally spaced cumulative zero point of the spectrum \( F^{l,d}_{S_p}(\eta) \) with a spacing of \( T \).

**Definition VIII.4:** A periodic equally spaced cumulative zero point of the DSIS of a primary path

If the spectral line \( \eta \) is an equally spaced cumulative zero point of the spectrum \( F^{l,d}_{S_p}(\eta) \) with a spacing of \( T = \eta \), then it is said to be a periodic equally spaced cumulative zero point of the spectrum \( F^{l,d}_{S_p}(\eta) \) with a period of \( \eta \).

**Definition VIII.5:** The minimum periodic equally spaced cumulative zero point of the DSIS of a primary path

Among all the periodic equally spaced cumulative zero points of a spectrum \( F^{l,d}_{S_p}(\eta) \), the spectral line with the minimum sequence number is called as the minimum periodic equally spaced cumulative zero point of the spectrum \( F^{l,d}_{S_p}(\eta) \), and its corresponding sequence number is denoted as \( \eta_{\min} \).

**Definition VIII.6:** A point of returning to zero of the DSIS of a primary path

For a spectrum \( F^{l,d}_{S_p}(\eta) \), if the spectral line \( \eta \) (\( 1 \leq \eta \leq N - 1 \)) satisfies the condition of \( F^{l,d}_{S_p}(m) = 0 \) (\( \eta \leq m \leq N - 1, m \in \mathbb{Z} \)), then the spectral line \( \eta \) is said to be a point of returning to zero of the spectrum \( F^{l,d}_{S_p}(\eta) \).

**Definition VIII.7:** The minimum point of returning to zero of the DSIS of a primary path

For a spectrum \( F^{l,d}_{S_p}(\eta) \) with some points of returning to zero, the spectral line with the minimum sequence number among all its points of returning to zero is said to be the minimum point of returning to zero of the spectrum \( F^{l,d}_{S_p}(\eta) \), and its corresponding sequence number is denoted as \( \eta_{\min} \).
Definition VIII.8: A zeroing primary path and a non-zeroing primary path

If the number of points of returning to zero in a spectrum $F_{S_P}^{l,d}(\eta)$ is greater than 0, then the spectrum $F_{S_P}^{l,d}(\eta)$ is called as a zeroing spectrum, and its corresponding primary path $P$ is called as a zeroing primary path. Otherwise, the spectrum $F_{S_P}^{l,d}(\eta)$ is called as a non-zeroing spectrum, and its corresponding primary path $P$ is called as a non-zeroing primary path.

Definition VIII.9: An interference continuous primary path and an interference non-continuous primary path

As for any two sending nodes, like $k$ and $l$ ($k < l$), in a primary path $P$ having interference relationships between them, if the subset of nodes composed of all the sending nodes (including $k$ and $l$) with their sequence numbers between $k$ and $l$ is a continuous second-order interference subset (Definition V.22), $P$ is defined as an interference continuous primary path, otherwise $P$ is called as an interference non-continuous primary path.

Definition VIII.10: A compact primary path and a non-compact primary path

If the intrinsic period (Definition VI.4) of a primary path $P$ is equal to its intrinsic interference intensity (Definition V.12), $P$ is called as a compact primary path, otherwise $P$ is called as a non-compact primary path.

B. Several basic properties of the DSIS of a primary path

Theorem VIII.1: A basic property of an equally spaced cumulative zero point of a spectrum $F_{S_P}^{l,d}(\eta)$ with a spacing of $T$

Assume that the DSIS of a primary path $P$ (including $N$ sending nodes) is $F_{S_P}^{l,d}(\eta)$. If the spectral line with sequence number $\eta$ is an equally spaced cumulative zero point of the spectrum $F_{S_P}^{l,d}(\eta)$ with a spacing of $T$, and $1 \leq \eta + i \cdot T \leq N - 1 (i \in \mathbb{Z}^+)$, then the spectral line...
with sequence number $\eta + i \cdot T$ is also an equally spaced cumulative zero point of the spectrum $F_{S_p}^{I, d}(\eta)$ with a spacing of $T$.

Proof:

Due to the fact that the spectral line with the sequence number $\eta$ is an equally spaced cumulative zero point of the spectrum $F_{S_p}^{I, d}(\eta)$ with a spacing of $T$, according to Definition VIII.3, the following equation holds:

$$\sum_{j=0}^{\left\lfloor \frac{(N-1)-\eta}{T} \right\rfloor} F_{S_p}^{I, d}(\eta + j \cdot T) = 0 \quad \text{(VIII.1)}$$

According to Definition VIII.1, the intensities of all the spectral lines are greater than or equal to 0, therefore there is

$$\sum_{j=0}^{i-1} F_{S_p}^{I, d}(\eta + j \cdot T) = 0 \quad \text{(VIII.2)}$$

where $i-1 \leq \left\lfloor \frac{(N-1)-\eta}{T} \right\rfloor$. By combining Equation (VIII.1) and Equation (VIII.2), we can further obtain

$$\sum_{j=0}^{\left\lfloor \frac{(N-1)-\eta}{T} \right\rfloor} F_{S_p}^{I, d}(\eta + j \cdot T) = \sum_{j=0}^{i-1} F_{S_p}^{I, d}(\eta + j \cdot T) + \sum_{j=i}^{\left\lfloor \frac{(N-1)-\eta}{T} \right\rfloor} F_{S_p}^{I, d}(\eta + j \cdot T)$$

$$= \sum_{j=i}^{\left\lfloor \frac{(N-1)-\eta}{T} \right\rfloor} F_{S_p}^{I, d}(\eta + j \cdot T)$$

$$= \sum_{j=i}^{\left\lfloor \frac{(N-1)-\eta}{T} \right\rfloor} F_{S_p}^{I, d}((\eta+i \cdot T) + (j-i) \cdot T)$$

$$= 0$$

Let $k = j-i$, and by substituting $k$ into the above equation, it can be obtained that

$$\sum_{j=0}^{\left\lfloor \frac{(N-1)-\eta}{T} \right\rfloor} F_{S_p}^{I, d}(\eta + j \cdot T) = \sum_{k=0}^{\left\lfloor \frac{(N-1)-\eta + T}{T} \right\rfloor} F_{S_p}^{I, d}((\eta+i \cdot T) + k \cdot T)$$

$$= \sum_{k=0}^{\left\lfloor \frac{(N-1)-\eta + T}{T} \right\rfloor} F_{S_p}^{I, d}((\eta+i \cdot T) + k \cdot T)$$

$$= 0$$

According to Definition VIII.3, the spectral line $\eta + i \cdot T$ is also an equally spaced cumulative zero point of the spectrum $F_{S_p}^{I, d}(\eta)$ with a spacing of $T$. ■

Theorem VIII.2: A basic property of a periodic equally spaced cumulative zero point of a
**Theorem VIII.3:** The sufficient and necessary condition for a positive integer $T$ to be a reachable period (Definition VI.2) of a primary path $P$.

Assume that the DSIS of a primary path $P$ (including $N$ sending nodes) is $F_{S_p}^{I,D}(\eta)$. If the spectral line with sequence number $\eta$ is a periodic equally spaced cumulative zero point of the spectrum $F_{S_p}^{I,D}(\eta)$ with a period of $\eta$, and $1 \leq i \cdot \eta \leq N - 1 (i \in \mathbb{Z}^+, i > 1)$, then the spectral line with sequence number $i \cdot \eta$ is also a periodic equally spaced cumulative zero point of the spectrum $F_{S_p}^{I,D}(\eta)$ with a period of $i \cdot \eta$.

**Proof:**

Proof by contradiction is adopted. Assuming that spectral line $i \cdot \eta$ is not a periodic equally spaced cumulative zero point of the spectrum $F_{S_p}^{I,D}(\eta)$ with a period of $i \cdot \eta$, according to Definition VIII.4, it can be inferred that there is at least one spectral line $\eta = i \cdot \eta + j \cdot (i \cdot \eta) (i \cdot \eta \leq \eta \leq N - 1)$ among spectral lines $i \cdot \eta + j \cdot (i \cdot \eta) (0 \leq j \leq \lfloor \frac{(N - 1) - i \cdot \eta}{i \cdot \eta} \rfloor)$, with its corresponding intensity of the spectral line satisfying $F_{S_p}^{I,D}(\eta) > 0$. Therefore, it can be obtained that

$$F_{S_p}^{I,D}(\eta') = F_{S_p}^{I,D}(i \cdot \eta + j \cdot (i \cdot \eta)) = F_{S_p}^{I,D}(\eta + (i - 1) \cdot \eta + j^{*} \cdot i \cdot \eta) = F_{S_p}^{I,D}(\eta + ((j^{*} + 1)i - 1) \cdot \eta) = F_{S_p}^{I,D}(\eta + i^{*} \cdot \eta) > 0$$

where $i^{*} = (j^{*} + 1)i - 1 (i^{*} \geq 1)$. On the other hand, according to Definition VIII.4, since spectral line $\eta$ is a periodic equally spaced cumulative zero point of the spectrum $F_{S_p}^{I,D}(\eta)$ with a period of $\eta$, the intensities of all the spectral lines, corresponding to the sequence numbers which can be represented in the form of $\eta + i \cdot \eta (i \geq 0)$ in the spectrum $F_{S_p}^{I,D}(\eta)$, must be equal to 0. Therefore, it can be found that the above assumption is contradictory.
The sufficient and necessary condition for a positive integer \( T \) \((1 \leq T \leq N-1)\) to be a reachable period of a primary path \( P \) is that the spectral line with sequence number \( \eta = T \) is a periodic equally spaced cumulative zero point of the spectrum \( F_{s_{p}}^{I,d}(\eta) \) with a period of \( T \).

**Proof:**

Firstly, let us prove that if \( T \) is a reachable period of \( P \), then the spectral line with sequence number \( \eta = T \) is a periodic equally spaced cumulative zero point of the spectrum \( F_{s_{p}}^{I,d}(\eta) \) with a period of \( T \). Proof by contradiction is adopted. Assuming that the spectral line \( \eta = T \) is not a periodic equally spaced cumulative zero point of the spectrum \( F_{s_{p}}^{I,d}(\eta) \), according to **Definition VIII.4**, there is at least one spectral line \( \eta' = T + j \cdot T \) \((T \leq \eta' \leq N - 1)\) among the spectral lines \( T + j \cdot T \) \((0 \leq j \leq \lfloor \frac{(N-1) - T}{T} \rfloor, j \in \mathbb{Z})\), with the intensity of its spectral line satisfying \( F_{s_{p}}^{I,d}(\eta') > 0 \).

That is to say, in the set \( S_{p} \) of all the sending nodes of \( P \), there is at least one pair of interfering nodes \((k,l)\) with a spacing between them being \( d_{s_{p}}^{k,l} = |k - l| = \eta' = (1 + j') \cdot T \) (**Definition III.2**). Because the spacing between the two interfering sending nodes is an integer multiple of the reachable period \( T \) of \( P \), these two interfering sending nodes must share a common subset of equally spaced nodes with the spacing between adjacent nodes being \( T \) in the primary path \( P \), which contradicts to the premise that all the subsets of equally spaced nodes with the spacing between adjacent nodes being \( T \) in \( P \) are second-order concurrency subsets.

 Secondly, let us prove that if the spectral line with sequence number \( \eta = T \) is a periodic equally spaced cumulative zero point of the spectrum \( F_{s_{p}}^{I,d}(\eta) \) with a period of \( T \), then \( T \) is a reachable period of \( P \). Again, proof by contradiction is adopted. Assuming that \( T \) is not a reachable period of \( P \), there is at least one subset \( \Phi_{s_{p}}^{'} \) of equally spaced nodes with the spacing between adjacent nodes being \( T \) in the primary path \( P \), which is not a second-order concurrency subset. That is to say, in the set \( \Phi_{s_{p}}^{'} \), there is at least one pair of interfering nodes \((k',l')\) with a spacing \( d_{s_{p}}^{k',l'} = |k' - l'| = T + j' \cdot T \) \((j' \geq 0, j' \in \mathbb{Z})\) between them. According to the definition of a subset of equally spaced nodes with the spacing between adjacent nodes being \( T \) (**Definition VI.1**), it is known that \( T \leq d_{s_{p}}^{k',l'} \leq N-1 \). Therefore, it can be obtained that the intensity
\[ F^{I,d}_{s_P} (d^{I,j}_{s_P}) \] of the spectral line \( \eta = d^{I,j}_{s_P} \) is larger than 0, i.e. \( F^{I,d}_{s_P} (d^{I,j}_{s_P}) = F^{I,d}_{s_P} (T + j \cdot T) > 0 \), which contradicts to the premise that the spectral line with sequence number \( \eta = T \) is a periodic equally spaced cumulative zero point of the spectrum \( F^{I,d}_{s_P} (\eta) \) with a period of \( T \).

**Corollary VIII.3.1: The sufficient and necessary condition for a positive integer \( T \) to be the intrinsic period (Definition VI.4) of a primary path \( P \)**

Assume that the DSIS of a primary path \( P \) (including \( N \) sending nodes) is \( F^{I,d}_{s_P} (\eta) \).

The sufficient and necessary condition for a positive integer \( 1 \leq T \leq N-1 \) to be the intrinsic period of a primary path \( P \) is that the spectral line with sequence number \( \eta = T \) is the minimum periodic equally spaced cumulative zero point of the spectrum \( F^{I,d}_{s_P} (\eta) \).

**Proof:**

Firstly, let us prove that if \( T \) is the intrinsic period of \( P \), then the spectral line with sequence number \( \eta = T \) is the minimum periodic equally spaced cumulative zero point of the spectrum \( F^{I,d}_{s_P} (\eta) \). Proof by contradiction is adopted. Assuming that the spectral line with sequence number \( \eta = T \) is not the minimum periodic equally spaced cumulative zero point of the spectrum \( F^{I,d}_{s_P} (\eta) \), there must exist a spectral line \( \eta' < T \) (\( \eta' \in \mathbb{Z}^+ \)), which is a periodic equally spaced cumulative zero point of the spectrum \( F^{I,d}_{s_P} (\eta) \) with a period of \( \eta' \). According to **Theorem VIII.3**, the positive integer \( \eta' \) must be a reachable period of \( P \), which contradicts to the premise that \( T \) is the intrinsic period of \( P \).

Secondly, let us prove that if the spectral line with sequence number \( \eta = T \) is the minimum periodic equally spaced cumulative zero point of the spectrum \( F^{I,d}_{s_P} (\eta) \), then \( T \) is the intrinsic period of \( P \). Proof by contradiction is adopted. According to **Theorem VIII.3**, since the spectral line with sequence number \( \eta = T \) is the minimum periodic equally spaced cumulative zero point of the spectrum \( F^{I,d}_{s_P} (\eta) \) (i.e. the spectral line \( \eta = T \) is also a periodic equally spaced cumulative zero point of \( F^{I,d}_{s_P} (\eta) \)), \( T \) is at least a reachable period of \( P \). Assuming that \( T \) is not the intrinsic period of \( P \), there must exist a positive integer \( T' < T \) is another reachable period of \( P \).
According to Theorem VIII.3, the spectral line with sequence number \( \eta' = T' \) is a periodic equally spaced cumulative zero point of the spectrum \( F^{I,d}_{S_{\eta'}}(\eta) \) with a period of \( T' \), which contradicts to the premise that the spectral line with sequence number \( \eta = T \) is the minimum periodic equally spaced cumulative zero point of \( F^{I,d}_{S_{\eta'}}(\eta) \).

**Corollary VIII.3.2:** If a positive integer \( T \) is a reachable period of a primary path \( P \), then the positive integer \( i \cdot T \) is also a reachable period of the primary path \( P \).

If a positive integer \( T \) \((1 \leq T \leq N-1)\) is a reachable period of a primary path \( P \) (including \( N \) sending nodes), and \( 1 \leq i \cdot T \leq N-1 \), then the positive integer \( i \cdot T \) is also a reachable period of \( P \).

**Proof:**

According to Theorem VIII.3, if \( T \) is a reachable period of \( P \), then the spectral line with sequence number \( \eta = T \) is a periodic equally spaced cumulative zero point of the spectrum \( F^{I,d}_{S_{\eta}}(\eta) \) with a period of \( T \). Furthermore, according to Theorem VIII.2, the spectral line with sequence number \( \eta = i \cdot T \) \((1 \leq i \cdot T \leq N-1)\) is also a periodic equally spaced cumulative zero point of the spectrum \( F^{I,d}_{S_{\eta}}(\eta) \) with a period of \( i \cdot T \). Finally, again based on Theorem VIII.3, it can be obtained that the positive integer \( i \cdot T \) is also a reachable period of \( P \).

**Corollary VIII.3.3:** The intrinsic period of a primary path with a pure non-zero spectrum is \( N \).

If the DSIS of a primary path \( P \) (including \( N \) sending nodes) is a pure non-zero spectrum (Definition VIII.1), the intrinsic period of \( P \) is \( N \).

**Proof:**

According to the definition of a reachable period of a primary path (Definition VI.2), the positive integer \( N \) is a reachable period of a primary path \( P \) (i.e. it is evident that each sending node in \( P \) can individually serve as a second-order concurrency subset of nodes). Since the spectrum \( F^{I,d}_{S_{\eta}}(\eta) \) is a pure non-zero spectrum, for a positive integer \( T \) \((1 \leq T \leq N-1)\), the intensity of the corresponding spectral line \( \eta = T \) is greater than 0. Therefore, the spectral line
with sequence number \( \eta = T \) is not a periodic equally spaced cumulative zero point of the spectrum \( F_{S_p}^{I,d}(\eta) \). Furthermore, according to Theorem VIII.3, positive integer \( T \ (1 \leq T \leq N - 1) \) is not a reachable period of \( P \). Therefore, the positive integer \( N \) is the minimum reachable period of \( P \), i.e. the intrinsic period of \( P \). 

\[ \] 

**Lemma VIII.4.1:** A point of returning to zero of the DSIS of a primary path is a periodic equally spaced cumulative zero point of it

Assume that the DSIS of a primary path \( P \) (including \( N \) sending nodes) is \( F_{S_p}^{I,d}(\eta) \). If a spectral line with sequence number \( \eta = T \) is a point of returning to zero of the spectrum \( F_{S_p}^{I,d}(\eta) \), it must be a periodic equally spaced cumulative zero point of \( F_{S_p}^{I,d}(\eta) \) with a period of \( T \).

**Proof:**

According to the definition of a point of returning to zero of the DSIS (Definition VIII.6), we have

\[
\sum_{i=0}^{|(N-1)-T|} F_{S_p}^{I,d}(T + i \cdot T) = 0 \tag{VIII.6}
\]

Based on the definition of a periodic equally spaced cumulative zero point of the DSIS (Definition VIII.4), the spectral line with sequence number \( \eta = T \) is a periodic equally spaced cumulative zero point of \( F_{S_p}^{I,d}(\eta) \) with a period of \( T \). 

\[ \]

**Lemma VIII.4.2:** The sequence number of a spectral line of a point of returning to zero of the DSIS of a primary path is a reachable period of the primary path

Assume that the DSIS of a primary path \( P \) (including \( N \) sending nodes) is \( F_{S_p}^{I,d}(\eta) \). If the spectral line with sequence number \( \eta = T \) is a point of returning to zero of the spectrum \( F_{S_p}^{I,d}(\eta) \), then \( T \) is a reachable period of the primary path \( P \).

**Proof:**

According to Lemma VIII.4.1, the spectral line with sequence number \( \eta = T \) is a periodic
equally spaced cumulative zero point of $F_{S_p}^{I,d}(\eta)$ with a period of $T$. Furthermore, according to

**Theorem VIII.3**, $T$ is a reachable period of $P$. ■

**Lemma VIII.4.3**: The intrinsic period of a primary path is not larger than the sequence number of a spectral line of the minimum point of returning to zero of the DSIS of the primary path

Assume that the DSIS of a primary path $P$ (including $N$ sending nodes) is $F_{S_p}^{I,d}(\eta)$. The intrinsic period of $P$ is $T_{S_p}^*$. If the spectrum $F_{S_p}^{I,d}(\eta)$ is a zeroing spectrum (Definition VIII.8), and the corresponding sequence number of its minimum point of returning to zero is $\eta_{Bs}^{min}$, then $T_{S_p}^* \leq \eta_{Bs}^{min}$ holds.

**Proof:**

According to **Lemma VIII.4.2**, $\eta_{Bs}^{min}$ is a reachable period of $P$. Furthermore, based on the definition of the intrinsic period of a primary path (Definition VI.4), it can be concluded that $T_{S_p}^* \leq \eta_{Bs}^{min}$ holds. ■

**Theorem VIII.4**: The intrinsic interference intensity of a primary path is not larger than its intrinsic period, and the intrinsic period of the primary path is not larger than the sequence number of a spectral line of the minimum point of returning to zero of the DSIS

Assume that the DSIS of a primary path $P$ (including $N$ sending nodes) is $F_{S_p}^{I,d}(\eta)$. The intrinsic interference intensity and the intrinsic period of $P$ are $I_{S_p}^*$ and $T_{S_p}^*$, respectively. If the spectrum $F_{S_p}^{I,d}(\eta)$ is a zeroing spectrum, and the corresponding sequence number of its minimum point of returning to zero is $\eta_{Bs}^{min}$, then $I_{S_p}^* \leq T_{S_p}^* \leq \eta_{Bs}^{min}$ holds.

**Proof:**

By combing **Lemma VI.1.1** and **Lemma VIII.4.3**, it can be readily concluded that $I_{S_p}^* \leq T_{S_p}^* \leq \eta_{Bs}^{min}$ holds. ■
**Lemma VIII.5.1:** An interference continuous primary path must satisfy the interference reduction rule between nodes

If a primary path $P$ (including $N$ sending nodes) is an interference continuous primary path, it must satisfy the interference reduction rule between nodes (i.e. Rule V.1 and Rule V.2).

**Proof:**

Firstly, we prove that the primary path $P$ satisfies Rule V.1, i.e. if two different nodes $k$ and $l$ in $P$ have concurrency relationship, and node $l+1$ is another sending node of $P$, then it can be deduced that node $k$ and node $l+1$ also have concurrency relationship. Proof by contradiction is adopted. If node $k$ and node $l+1$ interfere with each other, it can be inferred from the fact of $P$ being an interference continuous primary path that node $k$ and node $l$ must interfere with each other, which contradicts the fact that node $k$ and node $l$ have concurrency relationship.

Secondly, we prove that the primary path $P$ satisfies Rule V.2, i.e. if two different nodes $k$ and $l$ in $P$ have concurrency relationship, and node $k-1$ is another sending node of $P$, then it can be deduced that node $k-1$ and node $l$ also have concurrency relationship. Proof by contradiction is adopted. If node $k-1$ and node $l$ interfere with each other, it can be inferred from the fact of $P$ being an interference continuous primary path that node $k$ and node $l$ must interfere with each other, which contradicts the fact that nodes $k$ and $l$ have concurrency relationship.

**Theorem VIII.5:** The necessary and sufficient condition for a primary path to be an interference continuous primary path is that it satisfies the interference reduction rule between nodes

The necessary and sufficient condition for a primary path $P$ (including $N$ sending nodes) to be an interference continuous primary path is that it satisfies the interference reduction rule between nodes (i.e. Rule V.1 and Rule V.2).

**Proof:**

It can be readily proved by combing Theorem V.2 and Lemma VIII.5.1.
Corollary VIII.5.1: An interference continuous primary path must also be a compact primary path

If a primary path $P$ (including $N$ sending nodes) is an interference continuous primary path, then it must be a compact primary path.

Proof:

Since $P$ is an interference continuous primary path, it must follow the interference reduction rule between nodes. Furthermore, in this case, according to Theorem VI.1, the intrinsic interference intensity of $P$ equals to its intrinsic period, which indicates that $P$ is a compact primary path.

Theorem VIII.6: The non-zero characteristics for intensities of partial spectral lines of the DSIS of an interference continuous primary path

Assume that a primary path $P$ (including $N$ sending nodes) is an interference continuous primary path, and its DSIS is $F_{S_p}^{l,d}(\eta)$. The spectrum $F_{S_p}^{l,d}(\eta)$ is a zeroing spectrum, and the corresponding sequence number of its minimum point of returning to zero is $\eta_{B_i}^{\text{min}} (>1)$, then $F_{S_p}^{l,d}(\eta) > 0 (1 \leq \eta < \eta_{B_i}^{\text{min}})$ holds.

Proof:

According to the definition of the minimum point of returning to zero of the spectrum $F_{S_p}^{l,d}(\eta)$ (Definition VIII.7), the intensity $F_{S_p}^{l,d}(\eta_{B_i}^{\text{min}} - 1)$ of the spectral line with sequence number $\eta = \eta_{B_i}^{\text{min}} - 1$ is larger than 0, which suggests that there is at least one pair of mutually interfering sending nodes $k$ and $l (k < l)$ in $P$ with a spacing between them being equal to $d_{S_p}^{l,k} = |k - l| = \eta_{B_i}^{\text{min}} - 1$. Since $P$ is an interference continuous primary path, the subset of nodes composed of all the sending nodes (including $k$ and $l$) with their sequence numbers between $k$ and $l$ is a continuous second-order interference subset $S'_p \triangleq \{i \mid i \in S_p; k \leq i \leq l\}$ ($S_p$ is the set of nodes composed of all the sending nodes in $P$). Therefore, at least one sending node $m (k < m \leq l)$ that interferes with node $k$ can be found in $S'_p$, so that $d_{S_p}^{l,m} = |k - m| = \eta (1 \leq \eta \leq \eta_{B_i}^{\text{min}} - 1)$, which means that $F_{S_p}^{l,d}(\eta) > 0 (1 \leq \eta < \eta_{B_i}^{\text{min}})$ holds.
Theorem VIII.7: The strict monotonic decreasing property for intensities of partial spectral lines of the DSIS of an interference continuous primary path

Assume that a primary path \( P \) (including \( N \) sending nodes) is an interference continuous primary path, and its DSIS is \( F_{S_p}^{I,P} (\eta) \). The spectrum \( F_{S_p}^{I,P} (\eta) \) is a zeroing spectrum, and the corresponding sequence number of its minimum point of returning to zero is \( \eta_{Bz}^{\min} (>1) \), then \( F_{S_p}^{I,P} (\eta) > F_{S_p}^{I,P} (\eta+1) \) \( (1 \leq \eta < \eta_{Bz}^{\min}) \) holds.

Proof:

Firstly, let us prove the case of \( \eta = \eta_{Bz}^{\min} -1 \). According to the definition of the minimum point of returning to zero of the DSIS of a primary path, in this case,
\[
F_{S_p}^{I,P} (\eta) > F_{S_p}^{I,P} (\eta+1) = F_{S_p}^{I,P} (\eta_{Bz}^{\min}) = 0
\]
must hold.

Secondly, let us prove the case of \( 1 \leq \eta < \eta_{Bz}^{\min} -1 \). Assuming that \( F_{S_p}^{I,P} (\eta+1) = m \) \( (m>0) \), in the set of sending nodes in \( P \), there are \( m \) pairs of mutually interfering node pairs with spacing between them being \( \eta +1 \). Without loss of generality, the \( m \) pairs of mutually interfering node pairs can be respectively expressed as: \((k_1,k_1+\eta+1)\), \((k_2,k_2+\eta+1)\), ..., and \((k_m,k_m+\eta+1)(k_1<k_2<...<k_m)\). Based on these listed node pairs, and combing with the fact that the primary path \( P \) is an interference continuous primary path, therefore, at least \( m+1 \) pairs of mutually interfering node pairs with spacing between them being \( \eta \) can be found. This is because that \( m+1 \) pairs of mutually interfering node pairs can be easily constructed and represented as: \((k_1,k_1+\eta)\), \((k_2,k_2+\eta)\), ..., \((k_m,k_m+\eta)\), and \((k_m+1,k_m+\eta+1)(k_1<k_2<...<k_m)\). Therefore,
\[
F_{S_p}^{I,P} (\eta) > m \quad (m>0)
\]
holds, i.e. \( F_{S_p}^{I,P} (\eta) > F_{S_p}^{I,P} (\eta+1) \).

Corollary VIII.7.1: The intrinsic interference intensity, the intrinsic period, and the sequence number of a spectral line of the minimum point of returning to zero of the DSIS of an interference continuous primary path are equal to each other

Assume that a primary path \( P \) (including \( N \) sending nodes) is an interference continuous primary path, and its intrinsic interference intensity, intrinsic period, and DSIS is \( I_{S_p}^{I} \).
$T_{S_Y}^*$, and $F_{S_Y}^{I,J,d}(\eta)$, respectively. The spectrum $F_{S_Y}^{I,J,d}(\eta)$ is a zeroing spectrum, and the corresponding sequence number of its minimum point of returning to zero is $\eta_{Bc}^{\min}$, then

$$I_{S_Y}^* = T_{S_Y}^* = \eta_{Bc}^{\min} \quad \text{holds.}$$

**Proof:**

According to Theorem VI.1, it can be readily obtained that $I_{S_Y}^* = T_{S_Y}^*$.

Below, we just need to prove that $T_{S_Y}^* = \eta_{Bc}^{\min}$. According to Lemma VIII.4.3, we know that $T_{S_Y}^* \leq \eta_{Bc}^{\min}$. As for the case of $\eta_{Bc}^{\min} = 1$, since an intrinsic period should not less than 1, and $\eta_{Bc}^{\min}$ is a reachable period of $P$ (according to Lemma VIII.4.2), therefore, we conclude that $T_{S_Y}^* = \eta_{Bc}^{\min}$, i.e. $I_{S_Y}^* = T_{S_Y}^* = \eta_{Bc}^{\min}$ holds. As for the case of $\eta_{Bc}^{\min} > 1$, it can be obtained from Theorem VIII.7 that $F_{S_Y}^{I,J,d}(\eta) > 0$ ($1 \leq \eta < \eta_{Bc}^{\min}$). Therefore, for a sequence number $\eta$ ($1 \leq \eta < \eta_{Bc}^{\min}$), its corresponding spectral line should not be a periodic equally spaced cumulative zero point of the spectrum $F_{S_Y}^{I,J,d}(\eta)$ with a period of $\eta$. Furthermore, according to Theorem VIII.3, it can be seen that the positive integer $\eta$ ($1 \leq \eta < \eta_{Bc}^{\min}$) must not be a reachable period of $P$, and on the other hand $\eta_{Bc}^{\min}$ is a reachable period of $P$ (according to Lemma VIII.4.2), therefore, the intrinsic period $T_{S_Y}^*$ of $P$ must be equal to $\eta_{Bc}^{\min}$, i.e. $I_{S_Y}^* = T_{S_Y}^* = \eta_{Bc}^{\min}$ holds.\[\blacksquare\]

### C. A general algorithm flow for solving the intrinsic period of a primary path

In this section, based on Corollary VIII.3.1, a general algorithm for solving the intrinsic period of a given primary path $P$ is given. That is to say, in the proposed algorithm, we no longer restrict a primary path $P$ to follow the interference reduction rule between nodes (i.e. Rule V.1 and Rule V.2). According to Corollary VIII.3.1, the intrinsic period of a given primary path $P$ is equal to the sequence number of the minimum periodic equally spaced cumulative zero point of its DSIS (i.e. the spectrum $F_{S_Y}^{I,J,d}(\eta)$). Therefore, the problem of solving the intrinsic period is transformed into the problem of searching the minimum periodic equally spaced cumulative zero point of the spectrum $F_{S_Y}^{I,J,d}(\eta)$. The detailed algorithm flow is shown in Algorithm VIII.1 as follows.
Algorithm VIII.1: A general algorithm flow for solving the intrinsic period of a primary path

1: BEGIN
2: Initialize the intrinsic period as $T^r_{s_v} = N$;
3: // $N$ denotes the number of sending nodes in $P$
4: // If a periodic equally spaced cumulative zero point cannot be found within the range of
5: // 1 to $N-1$, the intrinsic period must be equal to $N$
6: Initialize the sequence number of the spectral line being considered as $\eta = 1$;
7: WHILE ($\eta \leq N-1$) DO
8: $i = 0$;
9: $FLAG = 1$;
10: // If the indicator $FLAG$ remains to be 1, it shows that the spectral line being
11: // considered is a periodic equally spaced cumulative zero point. Otherwise, it will be
12: // set to be 0
13: WHILE ($i \leq \left\lfloor \frac{(N-1)-\eta}{\eta} \right\rfloor$) DO
14: IF ($F^l_{s_v,\eta}(\eta + i \cdot \eta) > 0$) THEN
15: $FLAG = 0$;
16: BREAK;
17: // It shows that the spectral line $\eta$ being considered is not a periodic equally
18: // spaced cumulative zero point. Therefore, the current WHILE loop should be
19: // early ended
20: END IF
21: $i = i + 1$;
22: END WHILE // The WHILE loop for $i$
23: IF ($FLAG = 1$) THEN
24: $T^r_{s_v} = \eta$;
25: BREAK;
It shows that the minimum periodic equally spaced cumulative zero point is found, and the intrinsic period $T_{s'}$ is set to be the sequence number $\eta$ of the spectral line being considered. Then, the current WHILE loop should be early ended, and the algorithm flow is ended.

```
30: END IF
31: $\eta = \eta + 1$;
32: END WHILE // The WHILE loop for $\eta$
33: END
```

IX. Conclusions

Aiming at the disorder problem (i.e. uncertainty problem) of the utilization of network resources commonly existing in multi-hop transmission networks, the paper proposes the idea and the corresponding supporting theory, i.e. theory of network wave, by constructing volatility information transmission mechanism between the sending nodes and their corresponding receiving nodes of a pair of paths (composed of two primary paths), so as to improve the orderliness of the utilization of network resources. The research results of the paper lay an ideological and theoretical foundation for further exploring more general methods that can improve the orderly utilization of network resources.

Some important conclusions obtained in the paper are summarized as follows:

1) It is proved that the maximum asymptotic throughput of a primary path depends on its intrinsic period, which in itself is equal to the intrinsic interference intensity of a primary path (Theorem VI.1).

2) In Theorem VII.1, it is pointed out that by using Algorithm VII.1, the asymptotic throughput of a primary path can be maximized. Moreover, in Corollary VII.1.1, it is proved that given the intrinsic interference intensity of a primary path, dividing the set of all its sending node into several second-order concurrency subsets with equally spaced nodes, where the number of subsets is the same as its intrinsic interference intensity, is the only way to achieve the maximum asymptotic throughput in all cases.
3) As for the cases of traversals with equal opportunities, an algorithm for the cooperative volatility transmission of information blocks in a pair of paths based on the set of maximum supporting elements is proposed (Algorithm VII.2). It is proved that the algorithm can maximize the asymptotic joint throughput of a pair of paths (Theorem VII.3). Moreover, the upper bound of the asymptotic joint throughput of a pair of paths achieved by Algorithm VII.2 is given in (Theorem VII.4).

4) As for the more general cases of traversals with unequal opportunities, an algorithm for the cooperative volatility transmission of information blocks in a pair of paths based on the set of maximum supporting elements is also proposed (Algorithm VII.3). The upper bound of the asymptotic joint throughput of a pair of paths achieved by using this algorithm is obtained (Theorem VII.6 and Corollary VII.6.1). Furthermore, from the unequal relationship given by Corollary VII.6.1, it can be found that by properly adjusting the number of times that the two primary paths constituting a pair of paths are activated in a joint period and reducing the mutual interference between the two primary paths as much as possible, the equality relationship in Corollary VII.6.1 can be established, and in this case, the asymptotic joint throughput of the pair of paths is maximized, i.e. 

\[ R^P_{(S_1,S_2)} = \frac{1}{T_{S_{21}}} + \frac{1}{T_{S_{12}}} \]

5) An algorithm flow (Algorithm VII.4) that can maximize the end-to-end asymptotic joint throughput of a pair of paths is proposed, which comprehensively covers almost all the key factors that may affect the asymptotic joint throughput of a pair of paths, such as the selection of routes, the determination of the reachable periods of the two primary paths in a pair of paths, the number of times the two primary paths are traversed within a joint period, and the allocation strategy of time beats and so on.

6) We introduce the concept of the Distribution Spectrum of Interference-Spacing of nodes (i.e. DSIS) of a primary path, and several basic properties related to the DSIS of a primary path are analyzed. In Corollary VIII. 3.1, the sufficient and necessary condition for a positive integer \( T \) to be the intrinsic period of a primary path is given. Finally, based on the obtained fundamental properties of the DSIS of a primary path, we propose a simple and general algorithm flow for solving the intrinsic period of a primary path (i.e. Algorithm VIII. 1).

Based on the conclusions obtained in this paper, we can further summarize the advantages of
volatility transmission method as: it can maximize the end-to-end asymptotic throughput of a primary path/a pair of paths while maintaining the orderly utilization of network resources as much as possible. In the future work, based on the work proposed in this paper, we will further study how to extend the theory of network wave to more general cases, such as the cases including any multiple primary paths (i.e. group of paths).

References

[1]  A. S. Tanenbaum, D. J. Wetherall, “Computer networks (5th ed.)”, Prentice Hall, 2011.
[2]  T. M. Cover, J. A. Thomas, “Elements of information theory”, John Wiley & Sons, Inc., 2006.
[3]  P. R. Feynman, R. B. Leighton, and M. Sands, “The Feynman Lectures on Physics”, vol. I, Basic Books, 2010.
### Appendix A: Symbol List

| Symbol | Definition |
|--------|------------|
| $P_{[i]}$ | A primary path with its identification number being $i \in \mathbb{Z}^+$ (refer to the Definition III.1) |
| $S_{P_{[i]}}$ | The set of sending nodes of a primary path $P_{[i]}$ (refer to the Definition III.2) |
| $N_{S_{P_{[i]}}}$ | The number of elements in a set $S_{P_{[i]}}$ (refer to the Definition III.2) |
| $d_{S_{P_{[i]}}}^{k,l}$ | The spacing between a sending node with a sequence number of $k$ and a sending node with a sequence number of $l$ on a primary path $P_{[i]}$ (refer to the Definition III.2) |
| $S'_{P_{[i]}}$ | A subset of sending nodes of a primary path $P_{[i]}$ (refer to the Definition III.3) |
| $N_{S'_{P_{[i]}}}$ | The number of elements in a set $S'_{P_{[i]}}$ (refer to the Definition III.3) |
| $P_{[1,2]}$ | A pair of paths consisting of primary paths $P_{[1]}$ and $P_{[2]}$ (refer to the Definition III.4) |
| $S_{P_{[1,2]}}$ | The set of sending nodes of a pair of paths $P_{[1,2]}$ (refer to the Definition III.5) |
| $N_{S_{P_{[1,2]}}}$ | The number of elements in a set $S_{P_{[1,2]}}$ (refer to the Definition III.5) |
| $S'_{P_{[1,2]}}$ | A subset of sending nodes of a pair of paths $P_{[1,2]}$ (refer to the Definition III.6) |
| $N_{S'_{P_{[1,2]}}}$ | The number of elements in a set $S'_{P_{[1,2]}}$ (refer to the Definition III.6) |
| $n_{i,j}$ | A node belonging to the primary path $P_{[i]}$ ($i = 1, 2$) whose sequence number in $P_{[i]}$ is $j$ (refer to the Definition III.7) |
| $r_i^{TC}$ | Transmission coverage radius of node $i$ (refer to the Definition IV.1) |
| $r_i^{IC}$ | Interference coverage radius of node $i$ (refer to the Definition IV.1) |
| Symbol   | Description                                                                 |
|----------|-----------------------------------------------------------------------------|
| $r_i^{IT}$ | Interference tolerance radius of node $i$ (refer to the Definition IV.1)     |
| $n_{i,j} \parallel n_{k,j}$ | There is a concurrency relationship between two nodes $n_{i,j}$ and $n_{k,j}$ (refer to the Definition IV.2) |
| $n_{i,j} >\!\!< n_{k,j}$ | There is an interference relationship between two nodes $n_{i,j}$ and $n_{k,j}$ (refer to the Definition IV.3) |
| $C_{S_{P_{1,2}}}^*$ | The intrinsic concurrency intensity of a nonempty subset $S_{P_{1,2}}^*$ of nodes (refer to the Definition V.2) |
| $C_{P_{1}}^*$ | The intrinsic concurrency intensity of a primary path $P_{1}$ (refer to the Definition V.3) |
| $C_{P_{1,2}}^*$ | The intrinsic concurrency intensity of a pair of paths $P_{1,2}$ (refer to the Definition V.4) |
| $D_{S_{P_{1,2}}}^*$ | The concurrency connection degree of a node $n_{i,j}$ in a subset $S_{P_{1,2}}^*$ of nodes (refer to the Definition V.5) |
| $D_{S_{P_{1,2}}}^*$ | The intrinsic concurrency connection degree of nodes in a subset $S_{P_{1,2}}^*$ of nodes (refer to the Definition V.6) |
| $D_{P_{1,2}}^*$ | The intrinsic concurrency connection degree of nodes in a primary path $P_{1}$ (refer to the Definition V.7) |
| $D_{P_{1,2}}^*$ | The intrinsic concurrency connection degree of nodes in a pair of paths $P_{1,2}$ (refer to the Definition V.8) |
| $I_{S_{P_{1,2}}}^*$ | The intrinsic interference intensity of a nonempty subset $S_{P_{1,2}}^*$ of nodes (refer to the Definition V.12) |
| $I_{P_{1}}^*$ | The intrinsic interference intensity of a primary path $P_{1}$ (refer to the Definition V.13) |
| $I_{P_{1,2}}^*$ | The intrinsic interference intensity of a pair of paths $P_{1,2}$ (refer to the Definition V.14) |
| Expression | Description |
|------------|-------------|
| $D^*_{S_{[1,2]}^{i,j}}$ | The interference connection degree of a node $n_{i,j}$ in a subset $S'_{n_{[1,2]}}$ of nodes (refer to the Definition V.15) |
| $D^*_{S_{[1,2]}^{i,j}}$ | The intrinsic interference connection degree of nodes in a nonempty subset $S'_{n_{[1,2]}}$ of nodes (refer to the Definition V.16) |
| $D^*_{S_{[1,2]}^{i,j}}$ | The intrinsic interference connection degree of nodes in a primary path $P_{[1]}$ (refer to the Definition V.17) |
| $D^*_{S_{[1,2]}^{i,j}}$ | The intrinsic interference connection degree of nodes in a pair of paths $P_{[1,2]}$ (refer to the Definition V.18) |
| $\Phi'_{S_{[1,2]}^{i,j}}(\theta_{S_{[1,2]}^{i,j}},T_{S_{[1,2]}^{i,j}})$ | The subset of equally spaced nodes with its initial phase being $\theta_{S_{[1,2]}^{i,j}}$ and the spacing between adjacent nodes being $T_{S_{[1,2]}^{i,j}}$ in a primary path $P_{[1]}$ (refer to the Definition VI.1) |
| $T_{S_{[1,2]}^{i,j}}$ | The period of a primary path $P_{[1]}$ (refer to the Definition VI.2) |
| $T_{S_{[1,2]}^{i,j}}$ | The joint period of a pair of paths $P_{[1,2]}$ (refer to the Definition VI.3) |
| $T^*_{S_{[1,2]}^{i,j}}$ | The intrinsic period of a primary path $P_{[1]}$ (refer to the Definition VI.4) |
| $T^*_{S_{[1,2]}^{i,j}}$ | The intrinsic period of a pair of paths $P_{[1,2]}$ (refer to the Definition VI.5) |
| $c(\theta_{S_{[1,2]}^{i,j}},\theta_{S_{[1,2]}^{i'}})$ | The joint concurrency coefficient between subsets of equally spaced nodes of two primary paths $P_{[1]}$ and $P_{[2]}$ (refer to the Definition VI.6) |
| $C$ | The matrix of joint concurrency relationship between subsets of equally spaced nodes of a pair of paths (refer to the Definition VI.7) |
| $C^*$ | The matrix of intrinsic joint concurrency relationship between subsets of equally spaced nodes of a pair of paths (refer to the Definition VI.8) |
| $S_{B_{n}}$ | The set of supporting elements of a binary matrix $B$ (refer to the Definition VI.9) |
| $S^*_{B_{n}}$ | The set of maximum supporting elements of a binary matrix $B$ (refer to the Definition VI.10) |
| Symbol   | Description                                                                                                                                 |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------|
| $U_{B\sigma}$ | The reachable supporting number of a binary matrix $B$ (refer to the Definition VI.11)                                                   |
| $U_{B\sigma}^*$ | The maximum reachable supporting number of a binary matrix $B$ (refer to the Definition VI.11)                                             |
| $D_{t,b}^{int}$ | The time interval starting from time instant $t$ and lasting for $b$ ($b \in \mathbb{Z}, b \geq 0$) time beats (refer to the Definition VII.1) |
| $R_{S[i]}^{av}$ | The average throughput of a primary path $P_{[i]}$ in a specific time interval (refer to the Definition VII.2)                           |
| $R_{S_{[i,j]}}^{av}$ | The average joint throughput of a pair of paths $P_{[i,j]}$ in a specific time interval (refer to the Definition VII.3)                         |
| $R_{S[i]}^{\infty}$ | The asymptotic throughput of a primary path $P_{[i]}$ (refer to the Definition VII.4)                                                   |
| $R_{S_{[i,j]}}^{\infty}$ | The asymptotic joint throughput of a pair of paths $P_{[i,j]}$ (refer to the Definition VII.5)                                          |
| $M_{L_1 \times L_2}$ | The $L_1 \times L_2$-th continuation matrix of a matrix $M$ (refer to the Definition VII.7)                                              |
| $S_{M_{L_1 \times L_2}}^*$ | The set of maximum supporting elements of $M_{L_1 \times L_2}$ (refer to the Definition VII.7)                                         |
| $U_{M_{L_1 \times L_2}}^*$ | The maximum reachable supporting number of $M_{L_1 \times L_2}$ (refer to the Definition VII.7)                                         |
| $D_{S_{[i,j]}}^{ed}$ | One time end-to-end delay of an information block on a primary path $P_{[i]}$ (refer to the Definition VII.8)                           |
| $F_{S_{[i,j]}^{\perp}}^{L_1 \times L_2}(\eta)$ | The Distribution Spectrum of Interference-Spacing of nodes (i.e. DSIS) of a primary path $P$ (refer to the Definition VIII.1)          |
| $\eta$ | The sequence number of a spectral line of the spectrum $F_{S_{[i,j]}^{\perp}}^{L_1 \times L_2}(\eta)$ (refer to the Definition VIII.1) |
| \( \eta_{\text{min}} \) | The sequence number of the minimum periodic equally spaced cumulative zero point of the spectrum \( F_{S_p}^{I,d}(\eta) \) (refer to the Definition VIII.5) |
| \( \eta_{\text{min}} \) | The sequence number of the minimum point of returning to zero of the spectrum \( F_{S_p}^{I,d}(\eta) \) (refer to the Definition VIII.6) |
Appendix B: List of Abbreviations

| DSIS   | the Distribution Spectrum of Interference-Spacing of nodes of a primary path (refer to the Section VIII) |