Analysis of $\tau^{-} \to K_{S}\pi^{-}\nu_{\tau}$ Belle data in a chiral framework

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The recent measurement of the $\tau^{-} \to K_{S}\pi^{-}\nu_{\tau}$ spectrum by the Belle Collaboration provides the frame to set forth a theoretical description of the decay, which is based on the contributing vector $F_{+}^{K\pi}(s)$ and scalar $F_{0}^{K\pi}(s)$ form factors. We show that a good representation of data is obtained through the use of form factors calculated within resonance chiral theory and constrained by dispersion relations and short-distance QCD. Hence we obtain a determination of $K^*(892)$ parameters and the low-energy parameterization of $F_{+}^{K\pi}(s)$.

1. INTRODUCTION

Hadronic decays of the $\tau$ lepton provide excellent setting for the study of hadronization of vector and axial-vector QCD currents [12,13] at $E \lesssim M_{\tau}$. Moreover the experimental separation of the Cabibbo-allowed and Cabibbo-suppressed modes into strange particles [10] are now available.

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We have tackled the description of the $\tau^{-} \to K_{S}\pi^{-}\nu_{\tau}$ data through a thorough construction of the relevant form factors by imposing the model-independent constraints of dispersion relations, chiral symmetry and their asymptotic QCD behaviour. Later on [14] we have applied that study to the analysis of Belle data [10]. Here I collect the summary of our results. Other recent works have also approached the study of this channel [15].

The relevant form factors are defined by the hadronic matrix element :

$$\langle \pi^{-}(p') | \sum_{\mu} \gamma_{\mu} u | 0 \rangle = \left[ \frac{Q_{\mu} Q_{\nu}}{Q^{2}} - g_{\mu \nu} \right] (p - p')^{\mu} F_{+}^{K\pi}(Q^{2})$$

$$- \frac{\Delta_{K\pi}}{Q^{2}} Q_{\mu} F_{0}^{K\pi}(Q^{2}) , \quad (1)$$

where $Q_{\mu} = (p + p')_{\mu}$ and $\Delta_{K\pi} = M_{K_{S}}^{2} - M_{\pi}^{2}$. Here $F_{+}^{K\pi}(Q^{2})$ and $F_{0}^{K\pi}(Q^{2})$ are the vector and scalar $K\pi$ form factors, respectively. The general expression for the $\tau^{-} \to K_{S}\pi^{-}\nu_{\tau}$ differential decay distribution is given by :

$$\frac{d\Gamma_{K_{S}\pi}}{d\sqrt{s}} = \frac{G_{F}^{2} |V_{us}|^{2} M_{\tau}^{3}}{96 \pi^{3} s} S_{EW} \left( 1 - \frac{s}{M_{\tau}^{2}} \right)^{2}$$

$$\times \left[ \left( 1 + 2 \frac{s}{M_{\tau}^{2}} \right) q_{K\pi}^{2} |F_{+}^{K\pi}(s)|^{2}$$

$$+ \frac{3 \Delta_{K\pi}^{2}}{4 s} q_{K\pi} |F_{0}^{K\pi}(s)|^{2} \right] , \quad (2)$$

being $q_{K\pi}$ the kaon momentum in the rest frame.
of the hadronic system, \( s = Q^2 \) and \( S_{\text{EW}} \) is an electro-weak correction factor.

2. THE FORM FACTORS

In Ref. [13] we have studied a theoretical representation of the vector form factor \( F_+^{K\pi}(s) \) in complete analogy to the description of the pion form factor presented in Refs. [10]. This approach includes our present knowledge on phenomenological hadronic Lagrangians, short-distance QCD, the large-\( N_C \) expansion as well as analyticity and unitarity.

The dynamical information of the vector form factor is dominantly carried out by the lightest \( s \)-flavoured vector resonance, namely \( K^* = K^*(892) \). Since the \( \tau \) lepton can also decay hadronically into the second vector resonance \( K^{*'} = K^*(1410) \), we have also included it in our parameterization. Its expression is given by:

\[
F_+^{K\pi}(s) = \left[ \frac{M_{K^*}^2 + \gamma s}{M_{K^*}^2 - s - iM_{K^*}\Gamma_{K^*}(s)} \right. \\
\left. \frac{\gamma s}{M_{K^{*'}}^2 - s - iM_{K^{*'}}\Gamma_{K^{*'}}(s)} \right] \times \\
\exp \left\{ \frac{3}{2} \text{Re} \left[ \bar{H}_{K\pi}(s) + \bar{H}_{K\eta}(s) \right] \right\}.
\]

This incorporates all known constraints from Chiral Perturbation Theory [17] and Resonance Chiral Theory (RχT) [18,19]. The relation of the parameter \( \gamma \) to the RχT couplings is given by \( \gamma = F_V G_V / (F_K F_\pi) - 1 \) when we impose the vanishing of the \( N_C \to \infty \) form factor at \( s \to \infty \).

The expressions of the vector resonances off-shell widths can be seen in Ref. [14]. In Eq. (3) \( \bar{H}(s) \equiv H(s) - 2L_0^2 s / (3F_K F_\pi) \) and \( H(s) \) can be read from Ref. [20]. In the following we will call our proposal in Eq. (3) the Chiral form for \( F_+^{K\pi}(s) \).

For the sake of comparison we have also considered de vector form factor constructed as combinations of Breit-Wigner functions [12]:

\[
\frac{F_+^{K\pi}(s)}{F_+^{K\pi}(0)} = \frac{\Gamma_{K^*}(0) + \beta BW_{K^*}(s)}{1 + \beta},
\]

\[
BW_R(s) = \frac{M_R^2}{M_R^2 - s - iM_R\Gamma_R(s)},
\]

where in the width of the \( K^* \) only the \( K\pi \) contribution is included. Hence \( \beta = 0 \) includes only the \( K^*(892) \) intermediate state.

The scalar form factor \( F_0^{K\pi}(s) \) was calculated in Refs. [21] in the framework of RχT with the implementation of constraints from dispersion theory as well as the short-distance QCD ruled behaviour of the form factor. We will use thoroughly the results of these studies. To proceed we will take the central value of the scalar form factor and will not consider its error. See Ref. [14] for a discussion on this issue.

A still hotly debated issue is the value of the form factors at \( s = 0 \) where \( F_+^{K\pi}(0) = F_0^{K\pi}(0) \) [22]. However from Eq. (2) we notice that the normalization is given by \( |V_{us}| F_+^{K\pi}(0) \). Hence we only need to determine the shape of normalized (at the origin) form factors. From the analyses of semi-leptonic kaon decays we will take [23]:

\[
|V_{us}| F_+^{K\pi}(0) = 0.21664 \pm 0.00048.
\]

Then we will be able to predict the total branching fraction \( B(\tau^- \to K_S\pi^-\nu_\tau) \) from a fit of the shape of the form factors, independent of the normalisation problem.

3. FITS TO THE BELLE SPECTRUM

In order to analyse the Belle data [10] we take the following Ansatz:

\[
0.0115(\text{GeV/bin}) \frac{N_T}{\Gamma_T B_{K\pi}} \frac{d\Gamma_{K\pi}}{ds},
\]

where the bin-width chosen by the experimentalists, \( N_T = 53110 \) is the total number of observed signal events, \( \Gamma_T \) is the total decay width of the \( \tau \) lepton and \( B_{K\pi} \) is a normalisation factor. A perfect agreement between data and the fit function would imply that \( B_{K\pi} = B(\tau^- \to K_S\pi^-\nu_\tau) \). Then we proceed to analyse different fits. A discussion of the numerical input for the vector form factors is accounted for in Ref. [14].

In our first fit we employ the Belle data in the range \( 0.808 - 1.015 \text{ GeV} \) where the vector form factor dominates. The result is shown in the Fig. [1] for both our chiral form factor (3) with \( \gamma = 0 \) and the Breit-Wigner expression (4) with \( \beta = 0 \). In both cases we find \( \chi^2 / \text{n.d.f.} \simeq 1.9 \).
Figure 1. Fit result for the differential decay distribution of the decay $\tau^- \to K_S \pi^- \nu_\tau$, when fitted with a pure $K^*$ vector resonance [dotted (Breit-Wigner) and short-dashed (chiral) curves] or with $K^*$ plus the scalar form factor (long-dashed and solid curves, respectively).

Next we include the scalar form factor $F_{K\pi}^0$ and extend the fitted energy region within $0.636 - 1.015$ GeV. Now the goodness of the fit for both vector form factors stays around $\chi^2/\text{n.d.f.} \simeq 1.6$ and the curves are, again, given in Fig. 1. As can be seen the inclusion of the scalar form factor is all-important in the low-energy region up to $\sqrt{s} \sim 0.9$ GeV where it dominates over the vector one.

Finally we fit the full energy range (but for the three points quoted in footnote 2) by including the $K^*$ in the vector form factors. The results of the fit are collected in Tab. 1 where we also show the results for the normalization $B_{K\pi}$. The corresponding spectra are displayed in Fig. 2. It is relevant to notice that, over the full range, the better agreement with Belle data is achieved by our chiral vector form factor (9) though for both we have $\chi^2/\text{n.d.f.} \simeq 1$. After a thorough study of the error sources (14) we obtain :

$$B[\tau^- \to K_S \pi^- \nu_\tau] = (0.427 \pm 0.024)$$.  

in good agreement with the value obtained by the Belle Collaboration (10) $B[\tau^- \to K_S \pi^- \nu_\tau] = (0.404 \pm 0.013)$%.

The general expansion of the vector form factor around $s = 0$ is given by :

$$\frac{F_{K\pi}^+(s)}{F_{K\pi}^+(0)} = 1 + \lambda'_+ \frac{s}{M^2_{\pi^-}} + \frac{\lambda''_+}{2} \frac{s^2}{M^4_{\pi^-}} + \frac{\lambda'''_+}{6} \frac{s^3}{M^6_{\pi^-}} + \ldots$$

where $\lambda'_+$, $\lambda''_+$ and $\lambda'''_+$ are the slope, curvature and cubic expansion parameters, respectively. From the results in Tab. 1 we find :

$$\lambda'_+ = (25.20 \pm 0.33) \times 10^{-3},$$
$$\lambda''_+ = (12.85 \pm 0.31) \times 10^{-4},$$
$$\lambda'''_+ = (9.56 \pm 0.28) \times 10^{-5},$$

that compare very well with the recent determinations from $K_{e3}$ decays (23) :

$$\lambda'_+ = (25.20 \pm 0.9) \times 10^{-3},$$
$$\lambda''_+ = (16 \pm 4) \times 10^{-4}.$$  

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| BW form for $F_K^+(s)$ | Chiral form for $F_K^+(s)$ |
|-------------------------|--------------------------|
| $B_K^+$ (%)             | $0.421$                  |
| $\bar{B}_K^+$ (%)       | $0.427$                  |
| $M_{K^*}$ (MeV)         | $895.12 \pm 0.19$        |
| $\Gamma_{K^*}$ (MeV)   | $46.79 \pm 0.41$         |
| $M_{K^{*'}}$ (MeV)      | $1598 \pm 25$            |
| $\Gamma_{K^{*'}}$ (MeV) | $224 \pm 47$            |
| $\beta, \gamma$        | $-0.079 \pm 0.010$       |
| $\chi^2/n.d.f.$         | $88.7/81$                |

Table 1
Full fit to the Belle $\tau^- \to K_S^0 \pi^- \nu_\tau$ spectrum with the two $K^*$ and $K'^*$ vector resonances in $F_K^+(s)$ and the central value of the scalar form factor $F_0^+(s)$.

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