Electroweak Form Factors of the $\Delta(1232)$ Resonance

Krzysztof M. Graczyk, Jakub Żmuda, Jan T. Sobczyk

Institute of Theoretical Physics, University of Wrocław, pl. M. Borna 9, 50-204, Wrocław, Poland

(Dated: July 22, 2014)

Nucleon $\rightarrow \Delta(1232)$ transition electroweak form factors are discussed in a single pion production model with nonresonant background terms originating from a chiral perturbation theory. Fits to electron-proton scattering $F_2$ as well as neutrino scattering bubble chamber experimental data are performed. Both $\nu$-proton and $\bar{\nu}$-neutron channel data are discussed in a unified statistical model. A new parametrization of the $N \rightarrow \Delta(1232)$ vector form factors is proposed. Fit to neutrino scattering data gives axial mass $M_{A\Delta} = 0.85^{+0.09}_{-0.08}$ (GeV) and $C_5^A(0) = 1.10^{+0.14}_{-0.16}$ in accordance with Goldberger-Treiman relation as long as deuteron nuclear effects are considered.

I. INTRODUCTION

Weak single pion production (SPP) processes have been studied for many decades, but their importance for the neutrino physics has grown with the development of accelerator neutrino experiments. In the few-GeV energy range characteristic for the experiments such as T2K \cite{1}, MINOS \cite{2}, NOvA \cite{3}, MiniBooNE \cite{4}, and LBNE \cite{5} this interaction channel contributes a large fraction of the total cross section. Rough estimates show, that for an isoscalar target and neutrino energy of around 1 GeV SPP accounts for about 1/3 of the interactions.

The SPP events with pion absorption contribute to the background in measurements of quasi-elastic neutrino scattering on nuclear targets. Neutral current $\pi^0$ production processes add to the background for the $\nu_e$ appearance measurement in water Cherenkov detectors. The detailed estimate of the cross-sections for the SPP is important for a correct extraction of neutrino oscillation parameters in long baseline experiments.

Theoretical modelling of the SPP processes on nuclear targets suffers from extra complications. Any attempt to obtain an information about the Nucleon (N) to $\Delta(1232)$ resonance transition vertex from these data is biased by systematic errors coming from nuclear model uncertainties. On the experimental side, there seems to be a tension between the MiniBooNE and very recent MINERvA SPP data on (mostly) carbon target, see Ref. \cite{6}. For hereby analysis measurements of the neutrino-production on free or almost free targets are desired. At present such data exist only for $\sim$30 years old Argonne National Laboratory (ANL) \cite{7,8} and Brookhaven National Laboratory (BNL) \cite{9,10} bubble chamber experiments, where deuteron and hydrogen targets were utilized. In this case one may hope to reduce the many-body bias in a reasonable manner with a simple theoretical ansatz \cite{11}.

In order to understand the neutrino SPP data it is necessary to have a model of nonresonant background, see Ref. \cite{12}. In more recent studies of weak SPP typically only the neutrino-proton channel $\nu_\mu + p \rightarrow \mu^- + \pi^+ + p$ is discussed in detail \cite{13,14}. This is a big drawback, because simple total cross section ratio analysis shows, that the background contribution is much larger in neutrino-neutron channels. The neutrino-proton SPP channel can be described well within a model that contains the $\Delta(1232)$ resonance contribution only, see e.g. Ref. \cite{15}. In the latter paper it was argued that the $d\sigma/dQ^2$ results in \cite{8} do not include the flux normalization error. Incorporation into the analysis this error and also deuteron effects in both ANL and BNL experiments allowed for a consistent fit for both data sets with $C_5^A(0) = 1.19 \pm 0.08$ and $M_A = 0.94 \pm 0.03$ GeV. The attempt to extract the leading $C_5^A(Q^2) \rightarrow \Delta$ form factor parameters in a model containing nonresonant background has been done in Refs. \cite{14,15}. The results both for a model without \cite{14} and with deuteron effects \cite{15} gave the values of $C_5^A(0)$ far from the Goldberger-Treiman relation estimate of $C_5^A(0) \approx 1.2 \pm 0.20$ \cite{20} ($C_5^A(0) = 0.867 \pm 0.075$ in \cite{14} and $C_5^A(0) = 1.00 \pm 0.11$ in \cite{15}). From the above mentioned models only those in Refs. \cite{14,15} have been directly validated on the electroproduction processes. Some authors use vector form factor parametrization from Ref. \cite{21}, based on the MAID analysis \cite{22}. The authors of Ref. \cite{21} proposed a model containing only $\Delta$ resonance contribution without any background and compared it to the MAID2007 helicity amplitudes. The problem is that the $\Delta$ helicity amplitudes extraction procedure is model-dependent. There are important $\Delta$ – background interference effects and separation procedure depends on the background model details. It is important to have $\Delta$ form factor consistent with the other ingredients of the model.

Keeping in mind the above caveats of previous analyses we propose an improved approach. We adapt and develop the statistical framework of Ref. \cite{19} in order to fit both vector and axial form factors of the $\Delta(1232)$ resonance. We use inclusive electron-proton scattering data for the electromagnetic interaction in the $\Delta(1232)$ region and deuteron bubble chamber data for the weak one. For the latter we expand the previously used statistical approach in order to incorporate the neutron channels, which was never done before. In this manner we include the data sets, that are very sensitive to the nonresonant background.

This paper is organized as follows: section II is devoted to the general formalism of single pion production, section III introduces the statistical model of our analysis, section IV shows our main results and finally section V
contains the conclusions.

II. GENERAL FORMALISM

We discuss the charged current inelastic neutrino scattering off nucleon targets. Three channels for neutrino SPP interactions are:

\begin{align*}
\nu_\mu(l) + p(p) &\rightarrow \mu^-(l') + \pi^+(k) + p(p') \\
\nu_\mu(l) + n(p) &\rightarrow \mu^-(l') + \pi^0(k) + p(p') \\
\nu_\mu(l) + n(p) &\rightarrow \mu^-(l') + \pi^+(k) + n(p')
\end{align*}

with \( l, l', p, p' \) and \( k \) being the neutrino, muon, initial nucleon, final nucleon and pion four momenta respectively. The four momentum transfer is defined as:

\[
q = l - l' = p' + k - p, \quad Q^2 = -q^2, \quad q^\mu = (q^0, \mathbf{q})
\]

and the square of hadronic invariant mass is:

\[
W^2 = (p + q)^2 = (p' + k)^2.
\]

Throughout this paper the metric \( g^{\mu\nu} = \text{diag}(+,-,-,-) \) is used.

For the pion electroproduction we are interested in proton target reactions:

\begin{align*}
\bar{e}^-(l) + p(p) &\rightarrow \bar{e}^-(l') + \pi^+(k) + n(p') \\
\bar{e}^-(l) + p(p) &\rightarrow \bar{e}^-(l') + \pi^0(k) + p(p').
\end{align*}

In the 1 GeV energy region the process (1) is overwhelmingly dominated by the intermediate \( \Delta^{++}(1232) \) state. The dominance of the resonant pion production mechanism makes this channel attractive for the analysis of the \( \Delta(1232) \) properties. The other two channels (Eqs. (2) and (3)) are known to have a large nonresonant pion production contribution and thus present more challenges for theorists.

A. Cross section

The inclusive double differential SPP cross section for neutrino scattering off nucleons at rest has the following form:

\[
\frac{d^2\sigma}{dQ^2dW} = \frac{4\pi^2G_F^2\cos^2\theta_C}{M^3}
\int\frac{d^3k}{(2\pi)^32E_\pi(k)}L_{\mu\nu}H_{\mu\nu}
\]

\[
L_{\mu\nu} = \bar{l}^{\mu}l^{\nu} + \bar{l}'^{\mu}l'^{\nu} - e^{\mu\nu\alpha\beta}l^\alpha l'^\beta
\]

\[
H_{\mu\nu} = \frac{1}{f} \int \frac{d^3p'}{(2\pi)^34E(p')} \sum_{\text{spins}} \langle \pi N' | j^{\mu}_{\pi}(0) | N \rangle
\]

\[
\langle \pi N' | j^{\mu}_{\pi}(0) | N \rangle = \frac{1}{128\pi^3M^2E(p')}A_{\mu\nu}\delta(E(p') + E_\pi(k) - M - q^0),
\]

where \( E \) is incident neutrino energy, \( M \) is the averaged nucleon mass, \( E_{\pi}(k) \) and \( E(p') \) are the final state pion and nucleon energies, \( G_F = 1.1664 \cdot 10^{-11} \text{ MeV}^{-2} \) is the Fermi constant, \( L_{\mu\nu} \) - the leptonic and \( H_{\mu\nu} \) - the hadronic tensors. The Cabibbo angle, \( \cos(\theta_C) = 0.974 \), was factored out of the weak charged current definition.

The information about dynamics of SPP is contained in matrix elements, \( \langle \pi N' | j^{\mu}_{\pi}(0) | N \rangle \), which describe the transition between an initial nucleon state \( |N\rangle \) and a final nucleon-pion state \( |\pi N'\rangle \). One can introduce “reduced current matrix elements" \( s^\mu \) and express the weak transition amplitudes:

\[
\langle N'(p', s')|\pi(k)|j^{\mu}_{\pi}(0)|N(p, s)\rangle = \pi_{\mu'}(p')s^\mu u_\nu(p),
\]

with isospin information hidden inside \( s^\mu \).

After performing the summations over nucleon spins we can rewrite the hadronic tensor as:

\[
A_{\mu\nu} = \text{Tr}\left[(q' + M)s^\mu(\not{q'} + M)\gamma^0 s^\nu\gamma^0\right]
\]

where \( \not{q'} = \gamma_\mu p'^\mu \).
The differential cross section on free nucleons becomes then:

\[
\frac{d^2\sigma}{dWdQ^2} = \frac{G_F^2 \cos^2(\Theta_C)}{512\pi^4 E^2 M} \int d\Omega \int_0^\infty k^2 d|k| \frac{W}{M} L_{\mu\nu} A^{\mu\nu} \delta(E(p') + E_n(k) - M - q^0). (11)
\]

In the model of this paper the dynamics of SPP process is defined by a set of Feynman diagrams (Fig. 1) with vertices determined by the effective chiral field theory. They are discussed in Ref. [14], where one can find exact expressions for the electromagnetic data set allows for an introduction of multiple fit parameters. The size and excellent accuracy of the pion pole diagram, which is purely axial. We call this approach "HNV model" after the names of the authors of Ref. [14].

B. \( N \to \Delta(1232) \) excitation

The \( \Delta(1232) \) resonance excitation is treated within the isobar framework. For positive parity spin-\( \frac{3}{2} \) particles we can write down a general form of the electroweak excitation vertex:

\[
\Gamma^{\alpha\mu}(p, q) = \left[ \gamma^{\mu} - A^{\alpha\mu}_{3/2} \right] \gamma^5
\]

where

\[
V^{\alpha\mu}_{3/2} = \frac{C_5^V(Q^2)}{M} (g^{\nu\alpha} q^\nu - q^\alpha \gamma^\nu) + \frac{C_4^V(Q^2)}{M^2} (g^{\nu\alpha} q \cdot (p + q) - q^\alpha (p + q)^\nu) + \frac{C_3^V(Q^2)}{M^2} (g^{\nu\alpha} q \cdot p - q^\alpha p^\nu) + g^{\nu\alpha} C_6^V(Q^2)
\]

\[
-A^{\alpha\mu}_{3/2} = \left[ \frac{C_3^V(Q^2)}{M} (g^{\nu\alpha} q^\nu - q^\alpha \gamma^\nu) + \frac{C_4^V(Q^2)}{M^2} (g^{\nu\alpha} q \cdot (p + q) - q^\alpha (p + q)^\nu) + C_5^V(Q^2) g^{\nu\alpha} + \frac{C_6^V(Q^2)}{M^2} q^\alpha q^\mu \right] \gamma^5. (13)
\]

C. Conserved vector current and vector form factors

Thanks to conserved vector current (CVC) hypothesis we can express weak vector form factors by electromagnetic ones. There exist several parametrizations of \( C_j^V \) proposed over the course of past five decades, see Refs. [21–24]. In this paper we propose our own model in order to be consistent with the chosen description of the non-resonant background. The size and excellent accuracy of the electromagnetic data set allows for an introduction of multiple fit parameters.

We assume that the \( N \to \Delta \) transition form factors have the same large \( Q^2 \) behaviour as the electromagnetic elastic nucleon form factors. The theoretical arguments [25] suggest that at \( Q^2 \to \infty \) the nucleon form factors fall down as \( 1/Q^4 \) and we adopt appropriate Padé type parametrisation [26]. We allow for a violation from the \( SU(6) \)-symmetry quark model relations \( C_4^V(Q^2) = -\frac{6}{M} C_5^V(Q^2) \) and \( C_5^V = 0 \) between the form factors [27]. Finally, to reduce the number of parameters in \( C_5^V \) we assume the dipole representation. Altogether, our parametrisation has the following form:

\[
C_3^V(Q^2) = \frac{C_3^V(0)}{1 + AQ^2 + BQ^4 + CQ^6} (1 + K_1 Q^2), (14)
\]

\[
C_4^V(Q^2) = -\frac{M_F}{W} C_3^V(Q^2) \cdot \frac{1 + K_2 Q^2}{1 + K_1 Q^2}, (15)
\]

\[
C_5^V(Q^2) = \frac{C_5^V(0)}{\left(1 + D \frac{Q^2}{M_F^2}\right)^2}. (16)
\]

We use the standard value of the vector mass \( M_V = 0.84 \) GeV. This parametrisation reproduces quark model relation between \( C_3^V \) and \( C_4^V \) at \( Q^2 = 0 \) and is consistent with nonzero \( S_{1/2} \) helicity amplitude.

In Sect. [LV] we present the best fit values of parameters \( C_3^V(0), C_5^V(0), A, B, C, D, K_1 \) and \( K_2 \).

D. Partially conserved axial current and axial form factors

In the axial part the leading contribution comes from \( C_5^A(Q^2) \) which is an analogue of the isovector nucleon axial form factor. Partially conserved axial current (PCAC) hypothesis relates the value of \( C_5^A(0) \) with the strong coupling constant \( f^* \) through off-diagonal Goldberger-Treiman relation [20]:

\[
C_5^A(0) = \frac{f^*}{\sqrt{2}} \approx 1.2, (17)
\]
but we will treat \( C_5^A(0) \) as a free parameter. Most often it is assumed, that \( C_5^A \) has a dipole \( Q^2 \) dependence:

\[
C_5^A(Q^2) = \frac{C_5^A(0)}{(1 + Q^2/M_{A\Delta}^2)^2} \tag{18}
\]

The axial mass parameter \( M_{A\Delta} \) is expected to be of the order of 1 GeV. The authors of Refs. [14] and [17] use the parametrization of \( C_5^A(Q^2) \) proposed in Ref. [28]:

\[
C_5^A(Q^2) = \frac{C_5^A(0)}{(1 + Q^2/M_{A\Delta}^2)^2 (1 + Q^2/(3M_{A\Delta}^2))^2} \tag{19}
\]

Other groups, e.g. authors of Ref. [29], occasionally use parametrization from Ref. [30], which contains even more free parameters. In our fits we assume the dipole form of \( C_5^A \).

The \( C_6^A \) form factor is an analogue of the nucleon induced pseudoscalar form factor. It can be related to \( C_5^A \) as:

\[
C_6^A(Q^2) = \frac{M^2}{m_\pi^2 + Q^2} C_5^A(Q^2), \tag{20}
\]

where \( m_\pi \) is average pion mass. The \( C_5^A(Q^2) \) is the axial counterpart of the very small electric quadrupole (E2) transition form factor and we set \( C_3^A = 0 \). For the \( C_4^A \) we use the Adler model relation [31]:

\[
C_4^A(Q^2) = -\frac{1}{4} C_5^A(Q^2). \tag{21}
\]

In this way the axial contribution is fully determined by \( C_5^A(Q^2) \). Altogether there are two free parameters: \( C_5^A(0) \) and \( M_{A\Delta} \). If there were enough experimental data one could drop the Adler relation and treat \( C_5^A(Q^2) \) as an independent form factor. However, the ANL and BNL experimental data do not have sufficient statistics to obtain separate fits of \( C_5^A \) and \( C_4^A \) [32], see also the discussion in Ref. [16].

\[\text{E. Deuteron effects}\]

In this paper we consider a deuteron model based on phenomenological nucleon momentum distribution. The following effects are taken into account:

- Nucleon momentum distribution \( f(p) \) taken from the Paris potential [33] (also used by the authors of Ref. [16]). We verified that other parameterizations (Hulthen [34], Bonn [35]) lead to very similar results.

- Flux correction coming from varying relative neutrino-nucleon velocity:

\[
v_{\text{rel.}} = \sqrt{(l \cdot p)^2} = \left| \frac{(EE(p) - 1 \cdot p)}{EE(p)} \right| = 1 - \frac{1 \cdot p}{EE(p)}. \tag{22}\]

- Realistic energy balance within plane wave impulse approximation (PWIA). It is assumed that the spectator nucleon does not participate in the interaction. In the case of quasielastic neutrino scattering it was shown in [36] that for neutrino energies larger than 500 MeV final state interactions effects violating PWIA are very small. The effective, momentum dependent, binding energy becomes:

\[
B(p) = 2E(p) - M_D, \tag{23}
\]

where \( M_D \) is deuteron mass.

- De Forest treatment of the off-shell matrix elements [37].

The expression for the cross section becomes:

\[
\frac{d\sigma}{dQ^2 dW} = \int d^3p \left| f(p) \frac{G_F^2}{4\pi} \left( \langle \Theta |C| l' \rangle \right) \right|^2 \int d^3k \frac{(2\pi)^3 2E_\pi(k)}{(2\pi)^3 2E_{\pi}(p)} \int \frac{d^3p'}{(2\pi)^3 2E(p')} L_{\mu\nu} A_{\mu\nu}(p, q, k) \delta^4(p + q - k - p'). \tag{24}\]

\[\text{III. STATISTICAL FRAMEWORK}\]

Our main goal is to have a SPP model working for the weak pion production. A natural procedure is to extract the information about vector and axial form factors independently using first respective electron scattering and then neutrino SPP data. In the next paragraphs we describe details of our statistical model.
A. Vector Contribution to Weak SPP

The available electron data set is very prolific and accurate compared to the neutrino data. One can extract the information about the functional form of the vector \( N \rightarrow \Delta \) transition form factors from several observables, including electron/target polarizations. Dedicated electroproduction experiments were performed in JLab and Bonn [28,42]. Our main goal is (due to a poor quality of the neutrino SPP data) to reproduce correctly only the most important characteristics of the neutrino SPP reactions: overall cross sections and distributions in \( Q^2 \). Detailed analysis of the electroproduction data should focus on pion angular distributions but it goes beyond the scope of this paper and is going to be a subject of further studies.

We explore the information contained in electron-proton \( F_2 \) data from [43]. In our fit we include 37 separate series (for different \( Q^2 \) values) of \( F_2 \) data points. Since our final analysis aims at neutrino ANL experiment we have restricted ourselves to data points from the lowest value of \( Q^2 \) (0.225 GeV^2) up to 2.025 GeV^2 only.

The data are for the inclusive structure function, thus we have limited ourselves to values of invariant mass \( W \) up to \( M_p + 2m_\pi \). Beyond that value the experimental data include more inelastic channels, starting from two pion production. Even with this limitation for \( Q^2 \leq 2.025 \) and \( W < M_p + 2m_\pi \) there are still 603 data points.

In order to ensure that the results will reproduce well the data at the \( \Delta(1232) \) peak we decided to expanded our fit to \( W = 1.27 \) GeV. Because there are no exclusive electron SPP data in the region \( W \in (M + 2m_\pi, 1.27 \) GeV) we add to our fit a term in which MAID 2007 model predictions are taken as 228 fake data points. The total errors are identical with those of respective Osipenko [44] points. Additional points help to reproduce better the \( \Delta(1232) \) peak region. From technical reasons we could not apply the MAID model directly to the integrated errors only. Following [19] we explore this fact and make the analysis more complete by considering also a correlated error coming from the overall flux normalization uncertainty. We define the \( \chi^2 \) estimator as:

\[
\chi^2_{A1} = \sum_{i=1}^9 \left( \frac{\sigma^{th}_{i} - \sigma_{i}^{exp}}{p_{ANL} \Delta \sigma_{i}^{exp}} \right)^2 + \left( \frac{p_{ANL} - 1}{\Delta p_{ANL}} \right)^2
\]

with ANL normalization factor \( p_{ANL} \) treated as a free parameter.

The theoretical cross sections are defined as:

\[
\sigma_{i}^{th} = \frac{1}{\Delta Q_i^2} \int_{E_{min}}^{E_{max}} \frac{1}{\Phi(E')dE'} \int_{Q_i^2-\Delta Q_i^2/2}^{Q_i^2+\Delta Q_i^2/2} dQ^2 \int_{E_{min}}^{E_{max}} dE \Phi(E) \cdot \frac{d\sigma_{i}^{th}(E,Q^2)}{dQ^2},
\]

\[
\frac{d\sigma^{th}(E,Q^2)}{dQ^2} = \int_{M+m_\pi}^{1.4 \text{ GeV}} dW \frac{d^2\sigma^{th}(E,Q^2)}{dW}.
\]
For both ANL neutron channels $\nu_\mu + n \rightarrow \mu^- + p + \pi^0$ (denoted as $A_2$) and $\nu_\mu + n \rightarrow \mu^- + n + \pi^+$ (denoted as $A_3$) the data are in a form of event distributions in $Q^2$ denoted as $N_i^{\text{EXP}}$ and also a few overall cross sections.

![Image](image.png)

where

$$N_{A2,3}^{\text{exp}} = \sum_{j=1}^{12} N_{A2,3;j}^{\text{exp}}$$

$$\sigma_{A2,3;\text{tot}}^{\text{exp}} = \sum_{j=1}^{12} \sigma_{A2,3;j}^{\text{exp}}$$

$\sigma_{A2,3;\text{tot}}^{\text{exp}}$ is the total cross section for $A2,3$ channel, $N_{A2,3;i}^{\text{exp}}$ is the number of events in the $i$-th bin of the $A2,3$ channels.

Some of the experimental bins contain too few events for a $\chi^2$-based analysis. We have combined some of the neighbouring bins in order to keep a meaningful event statistics and the number of $Q^2$ bins is 12 in both neutron channels. The upper bound on neutrino energy is now $E_{\nu_{\text{max}}} = 1.5$ GeV and one has to account for that fact by changing the integration limits and normalization factor in Eq. (27).

Eventually, the complete $\chi^2$-function for the ANL data reads,

$$\chi^2_{ANL} = \sum_{k=1}^{3} \chi^2_{Ak}.$$  (33)

IV. RESULTS

A. Electromagnetic fits

The best fit results of our vector form factor parametrization given by Eqs. (14-16) are shown in Table I. For our best fit the value of $C_5^V(0)$ is close to the one from Ref. [21] and we get a clear beyond-dipole $Q^2$ dependence of $C_5^V(Q^2)$ and $C_3^V(Q^2)$. Surprisingly, the $Q^2$ dependence of $C_5^V(Q^2)$ is exactly dipole $\left(1 + Q^2/M_V^2\right)^{-2}$ with $M_V = 0.84$ GeV being the standard vector mass.

Fig. 2 shows that qualitatively in the region below two pion production threshold our fit reproduces the data rather well. In the same figure we show also predictions from the MAID2007 model. In order to compare both results we calculated the $\chi^2$ contribution from data points below the $2\pi$ threshold ($\chi^2_{2\pi}$). The same $\chi^2$ function with the MAID2007 model predictions gives $\chi^2_{2\pi}/NDF \approx 12.1$ and with our best fit results $\chi^2_{2\pi}/NDF \approx 13.6$. Our form factors lead to better agreement with the electron scattering data than the form factors considered in Ref. [21] (with the same HNV background model) giving $\chi^2_{2\pi}/NDF = 16.5$. Inspection of Fig. 2 (and also similar figures not shown in the paper) shows that most of the contribution to $\chi^2$ comes from a region of low $W$. Our fits are going to be used in the analysis of neutrino scattering data and some discrepancy at low $W$ is of no practical importance.

TABLE I. Best fit coefficients for vector form factors given by Eqs. (14-16) to be used in neutrino scattering data analysis. We do not report 1$\sigma$ errors because of hybrid character of our estimator, see explanations in the text.

| $C_5^V(0)$ | $C_5^V(0)$ | $A$ | $B$ | $C$ | $D$ | $K_1$ | $K_2$ |
|-----------|-----------|-----|-----|-----|-----|-------|-------|
| 2.10      | 0.63      | 4.73 | -0.39 | 5.59 | 1.00 | 0.13  | 1.68  |

B. Axial fits

For the axial contribution to $N \rightarrow \Delta$ transition our analysis assumes, that $C_3^A(0)$, $M_{A\Delta}$ and normalization factor $p_{ANL}$ are free fit parameters. We present our results in Tab. I and in Figs. 3 and 5.
In order to illustrate a role of deuteron effects we show the 2π production threshold.

![Graph](image_url)

**FIG. 2.** (Color online) Best fit results for vector form factors given by Eqs. (14-16) plotted against experimental data from Ref. 13 as well as MAID2007 predictions. Vertical lines show the 2π production threshold.

In the Tab. II are the results for fits to all three channels separately, and also the joint fit to three channels together. In each case the number of degree of freedom is calculated as:

\[ NDF = \text{No. } Q^2 \text{ bins} - \text{No. fitted parameters}. \]

In order to illustrate a role of deuteron effects we show also the results for a “model” of deuteron as consisting from free proton and neutron.

In both free target and deuteron target cases we see, that taken separately the \( p\pi^+ \) (A1) and \( p\pi^0 \) (A2) channels are statistically consistent, albeit their predicted scale parameters differ by around 10%. The latter channel seems to carry less information on the \( N \rightarrow \Delta \) transition axial current than the first one, which is reflected in larger uncertainty contours. This could be explained by a bigger background contribution to that channel, which makes it less sensitive to changes in the \( \Delta \) resonance description.

The biggest difficulty is encountered in the \( n\pi^+ \) (A3) channel, where we obtain \( C_2^A(0) \) twice as large as for the other two channels and \( M_{\Delta\Delta} \) significantly smaller. Here the number of events reported by ANL is comparable to \( p\pi^0 \) channel, but theoretical cross section predictions with nonresonant background are smaller, as one can readily see in the Fig. 6. This results in the drastic overestimation of \( C_2^A(0) \). Still, the fits to separate isospin channels give acceptable values of \( \chi^2_{\text{min}} \) for both neutron channels.

Deuteron effects affect mostly the value of \( C_2^A(0) \), by up to 20% depending on interaction channel. The same applies to the joint fit. A significant improvement with respect to previous fits to HNV model done in Refs. 14-16 is that with deuteron target effects we get the best fit value of \( C_2^A(0) \) within 1σ range from the theoretical Goldberger-Treiman relation. The joint fit agrees also on the 1σ level with separate fits on \( p\pi^0 \) and \( p\pi^+ \) channels. Deuteron effects lead to a slight improvement in the values of \( \chi^2_{\text{min}} \).

We have compared total cross section and \( Q^2 \) event distribution from the ANL experiment and our best fit. They are presented in Fig. 4 and in Fig. 7 respectively. They reflect previously described problems with the \( n\pi^+ \) channel. For two other channels we get a good agreement with the data.

Fitted normalizations factors \( p_{\text{ANL}} \) are different for neutron and proton channel as long as one considers separate fits. The proton channel prefers the data to be scaled up and both neutron channels prefer the data to be scaled down. Inclusion of deuteron effects does not change the value of fitted \( p_{\text{ANL}} \). The joint fit uses the same \( p_{\text{ANL}} \) parameter for all channels and seems to prefer the data to be scaled down even more \( (p_{\text{ANL}} \approx 0.90 \text{ both for free and deuteron targets}) \). These values of \( p_{\text{ANL}} \) are all well within the assumed error \( \Delta p_{\text{ANL}} \). This indicates that our fitting procedure is numerically stable. The effect of the fitted overall normalization factor has been shown in Fig. 8 for the total cross sections and in Fig. 7 for the differential cross sections.

Finally, we noticed that the best fit values for \( C_2^A(0) \) and \( M_{\Delta\Delta} \) are different from those obtained in Ref. 19 because in the current analysis the non-resonant background contribution is included.

**TABLE II.** Best fit for the \( \Delta(1232) \) axial form factors. Upper table: free nucleon target, lower table: deuteron target. 1σ contours for physical parameters can be found in Figs. 4 and 6. Errors for \( C_2^A(0) \) and \( M_{\Delta\Delta} \) where obtained after marginalization of \( p_{\text{ANL}} \).

| Fit  | \( C_2^A(0) \) | \( M_{\Delta\Delta}(\text{GeV}) \) | \( p_{\text{ANL}} \) | \( \chi^2/NDF \) | NDF |
|------|----------------|-----------------|-----------------|-----------------|-----|
| Free n+p | | | | | |
| A1   | 0.94±0.30 0.93±0.18 | 0.93±0.13 0.93±0.13 | 1.03 1.03 | 0.15 0.15 | 6 9 |
| A2   | 1.09±0.50 0.94±0.30 | 0.94±0.30 0.94±0.30 | 0.97 0.97 | 1.55 1.55 | 9 9 |
| A3   | 2.48±0.52 0.75±0.14 0.75±0.14 | 0.94 0.94 | 1.56 1.56 | 9 9 |
| Joint| 0.93±0.13 0.81±0.09 0.81±0.09 | 0.89 0.89 | 2.11 2.11 | 30 30 |
| Deuteron | | | | | |
| A1   | 1.11±0.32 0.97±0.17 0.97±0.17 | 1.04 1.04 | 0.20 0.20 | 6 6 |
| A2   | 1.31±0.49 1.00±0.25 1.00±0.25 | 0.93 0.93 | 1.52 1.52 | 9 9 |
| A3   | 2.83±0.62 0.76±0.13 0.76±0.13 | 0.94 0.94 | 1.47 1.47 | 9 9 |
| Joint| 1.10±0.14 0.85±0.09 0.85±0.09 | 0.90 0.90 | 2.06 2.06 | 30 30 |
the same formula for $\chi^2$ function as in Ref. [19], Sec. 5.2 with the normalization factor for the BNL data, $p_{BNL}$, treated as a free fit parameter.

The joint ANL+BNL data fit was done for the $p\pi^+$ channel and the best fit result is: $C_A^A(0) = 1.26^{+0.20}_{-0.21}$ (consistent with Goldberger-Treiman relation) and $M_{AA} = 1.06^{+0.16}_{-0.16}$ GeV ($\chi^2/NDF = 0.74$ with $NDF = 35$). Our results are different from those obtained in Ref. [16] because both studies use distinct estimators $\chi^2$. In Ref. [16] only total cross sections information from the BNL data is utilized. As explained above, we have used an
We first introduced new vector form factors, consistent with the HNV model of the nonresonant background. In the next step we investigated all three neutrino-free nucleon SPP channels, most importantly also neutrino-neutron channels that were never before used in the phenomenological studies.

Our main result is that the obtained value of \( C_{A}^{4}(0) \) agrees, on the 1\( \sigma \) level, with the Goldberger-Treiman relation. Also, our results confirm that there is a strong tension between \( n\pi^{+} \) and remaining two channels in the sense that the same theoretical model does not seem to reproduce all the data in a consistent way. There can be various reasons for that, some of them have been already mentioned:

- ANL data for the neutron SPP channels is of poor statistics.
- The HNV model for the background is well justified only near the pion production threshold and perhaps it is not reliable in the \( \Delta(1232) \) peak region.

V. CONCLUSIONS

In this paper we made a new attempt to get an information about weak \( N \rightarrow \Delta \) transition matrix elements.
FIG. 8. (Color online) Best fit results for the ANL+BNL pπ⁺ data on deuteron target with 1σ error bands.

Still another reason of theoretical difficulties may come from a missing unitarization of the model. The unitarity constraint, following the Watson theorem [47], imposes a relation between phases in weak neutrino-nucleon and constraint, following the Watson theorem [47], imposes from a missing unitarization of the model. The unitarity data on deuteron target with 1

ACKNOWLEDGEMENTS

We thank Luis Alvarez-Ruso for fruitful discussions. JTS and JZ were supported by Grant 4585/PB/IFT/12 (UMO-2011/M/ST2/02578).

Numerical calculations were carried out in Wroclaw Centre for Networking and Supercomputing [http://www.wcss.wroc.pl], grant No. 268.
[26] J. Kelly, Phys. Rev. C 70, 068202 (2004).
[27] J. Liu, N. C. Mukhopadhyay, and L.-s. Zhang, Phys. Rev. C 52, 1630 (1995).
[28] E. A. Paschos, J. Y. Yu, and M. Sakuda, Phys. Rev. D 69, 014013 (2004).
[29] M. Sajjad Athar, S. Chauhan, and S. K. Singh, Eur. Phys. J. A 43, 209 (2010).
[30] P. A. Schreiner and F. Von Hippel, Nucl. Phys. B 58, 333 (1978).
[31] S. L. Adler, Annals Phys. 50, 189 (1968).
[32] K. M. Graczyk, PoS EPS-HEP2009, 286 (2009).
[33] M. Lacombe, B. Loiseau, R. Vinh Mau, J. Cote, P. Pires, and R. de Tourreil, Phys. Lett. B, 139 (1981).
[34] L. Hulthen and M. Sugawara, Handbuch der Physik, Vol. 39 (Springer Verlag, 1957).
[35] R. Machleidt, K. Holinde, and C. Elster, Phys. Rept., 1 (1987).
[36] G. Shen, L. Marcucci, J. Carlson, S. Gandolfi, and R. Schiavilla, Phys. Rev. C 86, 035503 (2012).
[37] T. De Forest, Nucl. Phys. A 392, 232 (1983).
[38] M. Ungaro et al. (CLAS Collaboration), Phys. Rev. Lett. 97, 112003 (2006).
[39] K. Joo et al. (CLAS Collaboration), Phys. Rev. Lett. 88, 122001 (2002).
[40] K. Joo et al. (CLAS Collaboration), Phys. Rev. C 68, 032201 (2003).
[41] K. Joo et al. (CLAS Collaboration), Phys. Rev. C 70, 042201 (2004).
[42] H. Egiyan et al. (CLAS Collaboration), Phys. Rev. C 73, 025204 (2006).
[43] M. Osipenko, G. Ricco, M. Tantula, M. Anghinolfi, M. Battaglieri, et al., (2003), arXiv:hep-ex/0309052 [hep-ex].
[44] R. Davidson, N. Mukhopadhyay, and R. Wittman, Phys. Rev. D 43, 71 (1991).
[45] J. S. O’Connell, W. R. Dodge, J. W. Lightbody, X. K. Maruyama, J. O. Adler, K. Hansen, B. Schroder, and A. M. Bernstein et al., Phys. Rev. Lett. 53, 1627 (1984).
[46] M. Christy and P. E. Bosted, Phys. Rev. D 81, 155213 (2010).
[47] K. M. Watson, Phys. Rev. 88, 1163 (1952).
[48] L. Alvarez-Ruso, E. Hernandez, M. J. Vicente-Vacas, and J. Nieves, “Watson’s theorem, Goldberger-Treiman relation and the π NΔ (0) coupling constant,” (May 19-24 2014), talk at NuInt14, London.