Branching Ratio and CP-asymmetry for $B \to 1^1P_1 \gamma$ decays

M. Jamil Aslam

National Centre for Physics and Department of Physics, Quaid-i-Azam University, Islamabad 1

Riazuddin

National Centre for Physics, Quaid-i-Azam University Campus, Islamabad 2

We calculate the branching ratios for $B_d^0 \to (b_1, h_1) \gamma$ at next-to-leading order (NLO) of $\alpha_s$ where $b_1$ and $h_1$ are the corresponding radially excited axial vector mesons of $\rho$ and $\omega$ respectively. Using the $SU(3)$ symmetry for the form factor, the branching ratio for $B_d^0 \to (b_1, h_1) \gamma$ is expressed in terms of the branching ratio of the $B_d^0 \to K_1 \gamma$ and it is found to be $B(B_d^0 \to b_1 \gamma) = 0.71 \times 10^{-6}$ and $B(B_d^0 \to h_1 \gamma) = 0.74 \times 10^{-6}$. We also calculate direct CP asymmetry for these decays and find, in conformity with the observations made in the literature, that the hard spectator contributions significantly reduces the asymmetry arising from the vertex corrections alone. The value of CP-asymmetry is 10% and is negative like $\rho$ and $\omega$ in the Standard Model.

1 E-mail: jamil@ncp.edu.pk
2 E-mail: riazuddin@ncp.edu.pk
1. Introduction. The Flavor-Changing-Neutral-Current (FCNC) processes which cause $b \to s \gamma$ and $b \to d \gamma$ decays may contain new physics (NP) effects through penguin amplitudes. As the SM effects represent the background when we search for NP effects, we shall compute these effects. In doing so, we can understand the sensitivity of each NP search.

The first experimental evidence of this FCNC transition process in $B$ decay was observed about a decade ago, where the inclusive process $b \to s \gamma$ and exclusive process $B \to K^* \gamma$ were detected, and their branching ratios were measured\[^{11, 12}\]. On the other hand, the expected branching ratio for $b \to d \gamma$ is suppressed by $\mathcal{O}(10^{-2})$ with respect to the $b \to s \gamma$, because of the Cabbibo-Kobayashi-Masukawa quark mixing matrix factor (CKM). The world average for $b \to d$ penguin decays are given as follow\[^{13}\]

$$\mathcal{B} \left( B^0 \to \rho^0 \gamma \right) = (0.38 \pm 0.18) \times 10^{-6}$$

$$\mathcal{B} \left( B^0 \to \omega \gamma \right) = \left( 0.54^{+0.23}_{-0.21} \right) \times 10^{-6}$$

$$\mathcal{B} \left( B^+ \to \rho^+ \gamma \right) = \left( 0.68^{+0.36}_{-0.31} \right) \times 10^{-6}$$

Theoretically, $B \to (\rho, \omega) \gamma$ are widely studied both within and beyond the SM\[^{14, 15}\]. Now after the first measurement of BELLE for the decay $B \to K_1 \gamma$, where $K_1$ are the higher resonances of kaon\[^{16}\], these higher states become a subject of topical interest for the theoreticians. These decays have been studied widely in the literature\[^{17, 18, 19, 20}\]. Recently, the leading twist LCDAs as well as the first few Gegenbauer moments of $1^1P_1$ mesons, $b_1(1235)$ and $h_1(1170)$, which are the axial vector states of the $\rho$ and $\omega$ mesons have been studied\[^{21}\]. It is pointed out that these LCDAs are not only important to explore the tensor-type new-physics in $B$ decays but also for $B \to 1^1P_1 \gamma$ studies.

In this paper the branching ratio for $B_d^0 \to (b_1, h_1) \gamma$ at NLO of $\alpha_s$ are calculated using the LEET approach\[^{12, 13}\]. We follow the same frame work as done by Ali et al.\[^{14}\] for $B \to (\rho, \omega) \gamma$, because $B_d^0 \to (b_1, h_1) \gamma$ shares many things with it. The only difference is the DA for the daughter meson. As $(b_1, h_1)$ is an axial vector and is distinguished by vector by the $\gamma_5$ in the gamma structure of DA and some non perturbative parameters. But the presence of $\gamma_5$ does not alter the calculation, give the same result for the perturbative part. The higher twist terms are also included through the Gegenbauer moments in the Gegenbauer expansion.

At next-to-leading order of $\alpha_s$, $B \to (\rho, \omega) \gamma$ and $B_d^0 \to (b_1, h_1) \gamma$ are characterized by the weak form factor and decay constant, plugged by the common perturbative and kinematical factors. With $\mathcal{B} \left( B \to (\rho, \omega) \gamma \right)$ at hand, we can say that the future experiment will check the structure for $B_d^0 \to (b_1, h_1) \gamma$.

2. Effective Hamiltonian. The effective Hamiltonian for the radiative $b \to d \gamma$ decays (equivalently $B_d^0 \to b_1 \gamma$ and $B_d^0 \to h_1 \gamma$ decays) is obtained from the Standard Model (SM) by integrating out the heavy degrees of freedom (the top quark and $W^\pm$-bosons). The resulting expression at the scale $\mu = \mathcal{O}(m_b)$, where $m_b$ is the $b$-quark mass, is given by

$$\mathcal{H}_{b \to d}^{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{ud}^* \left[ C_1^{(u)}(\mu) \mathcal{O}_1^{(u)}(\mu) + C_2^{(u)}(\mu) \mathcal{O}_2^{(u)}(\mu) \right] + V_{cb}V_{cd}^* \left[ C_1^{(c)}(\mu) \mathcal{O}_1^{(c)}(\mu) + C_2^{(c)}(\mu) \mathcal{O}_2^{(c)}(\mu) \right] - V_{tb}V_{td}^* \left[ C_7^{\text{eff}}(\mu) \mathcal{O}_7(\mu) + C_8^{\text{eff}}(\mu) \mathcal{O}_8(\mu) \right] + \ldots \right\},$$
where $G_F$ is the Fermi coupling constant and only the dominant terms are shown. The operators $O_1^{(q)}$ and $O_2^{(q)}$, $(q = u, c)$, are the standard four-fermion operators and $O_7$ and $O_8$ are the electromagnetic and chromomagnetic penguin operators, respectively. Their explicit expressions are

$$O_1^{(q)} = (\bar{d}_a \gamma_\mu (1 - \gamma_5) q_\beta) (\bar{q}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha), \quad O_2^{(q)} = (\bar{d}_a \gamma_\mu (1 - \gamma_5) q_\alpha) (\bar{q}_\beta \gamma^\mu (1 - \gamma_5) b_\beta),$$

and

$$O_7 = \frac{e m_b}{2 \pi^2} (\bar{d}_a \sigma^{\mu\nu} (1 + \gamma_5) b_\alpha) F_{\mu\nu}, \quad O_8 = \frac{g_s m_b}{2 \pi^2} (\bar{d}_a \sigma^{\mu\nu} (1 + \gamma_5) T^a_\alpha b_\beta) G^a_{\mu\nu}. \quad (4)$$

Here, $e$ and $g_s$ are the electric and colour charges, $F_{\mu\nu}$ and $G^a_{\mu\nu}$ are the electromagnetic and gluonic field strength tensors, respectively, $T^a_\alpha$ are the colour $SU(N_c)$ group generators, and the quark colour indices $\alpha$ and $\beta$ and gluonic colour index $a$ are written explicitly. Note that in the operators $O_7$ and $O_8$ the $d$-quark mass contributions are negligible and therefore omitted. The coefficients $C_1^{(q)}(\mu)$ and $C_2^{(q)}(\mu)$ in Eq. (2) are the usual Wilson coefficients corresponding to the operators $O_1^{(q)}$ and $O_2^{(q)}$, while the coefficients $C_7^{\text{eff}}(\mu)$ and $C_8^{\text{eff}}(\mu)$ include also the effects of the QCD penguin four-fermion operators which are assumed to be present in the effective Hamiltonian [12] and denoted by ellipses there. For details and numerical values of these coefficients, see Ref. [15] and also references therein. We use the standard Bjorken-Drell convention [16] for the metric and the Dirac matrices; in particular $\gamma_5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3$, and the totally antisymmetric Levi-Civita tensor $\varepsilon_{\mu\nu\rho\sigma}$ is defined as $\varepsilon_{0123} = +1$. A point to note is that the three CKM factors shown in $H_{\text{eff}}^{b\rightarrow d}$ are of the same order of magnitude and, hence, the matrix elements in the decays $b \rightarrow d \gamma$ and $B^0_d \rightarrow (b_1, h_1) \gamma$ have non-trivial dependence on the CKM parameters. This is not the case of $b \rightarrow s \gamma$ decay (equivalently the $B \rightarrow K \gamma$ decays), the effective Hamiltonian $H_{\text{eff}}^{b\rightarrow s}$ describing the $b \rightarrow s$ transition can be obtained by the replacement of the quark field $d_\alpha$ by $s_\alpha$ in all the operators in Eqs. (3) and (4) and by replacing the CKM factors $V_{gb} V_{qs}^\ast \rightarrow V_{gb} V_{qs}^\ast (q = u, c, t)$ in $H_{\text{eff}}^{b\rightarrow d}$ [2]. Noting that among the three factors $V_{gb} V_{qs}^\ast$, the combination $V_{ub} V_{us}^\ast$ is CKM suppressed, the corresponding contributions to the decay amplitude can be safely neglected.

3. Theoretical framework for the $B \rightarrow 1^1 P_1 \gamma$ decays. The matrix element for the $B^0 \rightarrow 1^1 P_1 \gamma (1^1 P_1 = b_1, h_1)$ decays, we need to calculate the matrix elements $\langle 1^1 P_1 \gamma | O_i | B \rangle$, where $O_i$ are the operators appearing in $H_{\text{eff}}^{b\rightarrow d}$ and $H_{\text{eff}}^{b\rightarrow s}$. At the leading order in $\alpha_s$, this involves only the operators $O_7$, $O_1^{(u)}$ and $O_2^{(u)}$. The contribution from $O_7$ is termed as the long-distance contribution characterized by the top quark induced amplitude, where $O_1^{(u)}$ and $O_2^{(u)}$ corresponds to the short distance contributions and it includes the penguin amplitude for the $u$ and $c$ quark intermediate states and also the so-called weak annihilation and $W$-exchange contributions. There is also some contribution from annihilation penguin diagrams, which, however, are small. For detailed discussion about these kind of topologies for $B \rightarrow V \gamma$ decays and references to earlier papers, see Ref. [17]. Recently it has been shown that for the higher kaon resonances $K_1$, the branching ratio for $B \rightarrow K_1 \gamma$ has negligible dependence on such kind of annihilation topologies [18].

To calculate $O(\alpha_s)$ corrections, all the operators listed in [3] and [11] have to be included. QCD factorization [19] is most convenient framework to carry out these calculations. This allows to express the hadronic matrix elements in the schematic form:

$$\langle 1^1 P_1 \gamma | O_i | B \rangle = F^{B \rightarrow 1^1 P_1} T_i^I + \frac{d k_+}{2 \pi} \int_0^1 du \phi_{B, +}(k_+) T_i^{II}(k_+, u) \phi_{\perp}^{(1^1 P_1)}(u), \quad (5)$$

3
where \( F^{B \rightarrow 1^+P_1} \) are the transition form factors defined through the matrix elements of the operator \( O_7 \). \( \phi^{(1^+P_1)}_{B,+}(k_+) \) is the leading-twist \( B \)-meson wave-function with \( k_+ \) being a light-cone component of the spectator quark momentum, \( \phi^{(1^+P_1)}(u) \) is the leading-twist light-cone distribution amplitude (LCDA) of the transversely-polarized axial-vector meson, and \( u \) is the fractional momentum of the vector meson carried by one of the two partons. The expressions for these wavefunctions are given in Ref. [8–10], where it was pointed out that vector and axial vector mesons are distinguished by \( \gamma_5 \) in the gamma structure of the decay amplitude and some non perturbative parameters. The quantities \( T^I \) and \( T^{II} \) are the hard-perturbative kernels calculated to order \( \alpha_s \), with the latter containing the so-called hard-spectator contributions. The factorization formula [3] holds in the heavy quark limit, i.e., to order \( \Lambda_{QCD}/M_B \). This factorization framework has been used to calculate the branching fractions and related quantities for the decays \( B \rightarrow K^{*}\gamma \) [20–22] and \( B \rightarrow \rho\gamma \) [20–22] and for \( B \rightarrow K_1\gamma \) [8–10]. The isospin violation in the \( B \rightarrow K^{*}\gamma \) decays in this framework have also been studied [23]. Very recently, the hard-spectator contribution arising from the chromomagnetic operator \( \mathcal{O}_8 \) have also been calculated in next-to-next-to-leading order (NNLO) in \( \alpha_s \) showing that the spectator interactions factorize in the heavy quark limit [21]. However, the numerical effect of the resummed NNLO contributions is marginal and we shall not include this in our update.

It is shown that the the extra \( \gamma_5 \) in the DA of axial vector meson in comparison to the vector meson does not alter the calculation, giving the same result for the perturbative part. As for the non-perturbative parameters, the decay constant is most important. The LCDA for \( b_1 \) and \( h_1 \) meson has recently been calculated in [11]. The transverse decay constant of these mesons as well as the first few Gagenbaur moments of leading twist LCDA are calculated by using QCD sum rule technique. Their numerical values are given in Table 1.

In what follows we shall use the notations and results from Ref. [10], to which we refer for detailed derivations for \( B \rightarrow K_1\gamma \) decay. The final state \( K_1 \) is also the axial vector meson like \( b_1 \) and \( h_1 \) mesons. The only difference is in the quark content and we have to change the \( s \) quark with \( d \) quark every where in the calculation. The branching ratio of the \( B^0 \rightarrow (b_1, h_1) \) decay corrected to \( O(\alpha_s) \) can be written as follows [10]:

\[
B_{th}(B^0 \rightarrow (b_1, h_1) \gamma) = \tau_B \Gamma_{th}(B^0 \rightarrow (b_1, h_1)) = \tau_B \frac{G^2_F \alpha |V_{tb} V_{td}^*|^2}{32\pi^4} m_{b,\text{pole}}^2 M^3 \left[ \xi_{(b_1, h_1)} \right]^2 \left( 1 - \frac{m_{(b_1, h_1)}^2}{M^2} \right)^3 \left| C_{(0)\text{eff}} + A^{(1)}(\mu) \right|^2
\]

(6)

where \( G_F \) is the Fermi coupling constant, \( \alpha = \alpha(0) = 1/137 \) is the fine-structure constant, \( m_{b,\text{pole}} \) is the pole \( b \)-quark mass, \( M \) and \( m_{(b_1, h_1)} \) are the \( B \)- and axial vector-meson masses, and \( \tau_B \) is the lifetime of the \( B^0 \)- or \( B^+ \)-meson. The value of these constants is used from [14–10] and are collected in Table 1, for the numerical analysis. For this study, we consider \( \xi_{(b_1, h_1)} \) as a free parameter and we will extract its value from the current experimental data on \( B \rightarrow K_1\gamma \) decays because \( K_1 \) is also an axial vector meson. This is in analogy with the calculation done for the branching ratio of \( B \rightarrow (\rho, \omega) \gamma \) in terms of the branching ratio of \( B \rightarrow K^{*}\gamma \) by Ali et al. [14].

The function \( A^{(1)} \) in Eq. (6) can be decomposed into the following three components:

\[
A^{(1)}(\mu) = A_{C_7}^{(1)}(\mu) + A_{\text{ver}}^{(1)}(\mu) + A_{\text{sp}}^{(1)}(\mu_{sp})
\]

(7)
Here, $A_{C7}^{(1)}$ and $A_{\text{ver}}^{(1)}$ are the $O(\alpha_s)$ (i.e., NLO) corrections due to the Wilson coefficient $C_7^{\text{eff}}$ and in the $b \to s \gamma$ vertex, respectively, and $A_{sp}^{(1)K_1}$ is the $O(\alpha_s)$ hard-spectator corrections to the $B \to K_1 \gamma$ amplitude computed in this paper. Their explicit expressions are as follows:

$$A_{C7}^{(1)}(\mu) = \frac{\alpha_s(\mu)}{4\pi} C_{eff}^{(1)}(\mu),$$

$$A_{\text{ver}}^{(1)}(\mu) = \frac{\alpha_s(\mu)}{4\pi} \left\{ \frac{32}{81} \left[ 13C_2^{(0)}(\mu) + 27C_7^{(0)eff}(\mu) - 9C_8^{(0)eff}(\mu) \right] \ln \frac{m_b}{\mu} 
- \frac{20}{3} C_7^{(0)eff}(\mu) \right\} + \frac{4}{27} \left( 33 - 2\pi^2 + 6\pi i \right) C_8^{(0)eff}(\mu) + r_2(z) C_2^{(0)}(\mu) \right\},$$

$$A_{sp}^{(1)P_1}(\mu_{sp}) = \frac{\alpha_s(\mu_{sp})}{4\pi} \frac{2\Delta F_{\perp_1}^{(1)P_1}(\mu_{sp})}{9 \xi^{(K_1)}_\perp} \left\{ 3C_7^{(0)eff}(\mu_{sp}) 
+ C_8^{(0)eff}(\mu_{sp}) \left[ 1 - \frac{6a_{\perp_1}^{(1)P_1}(\mu_{sp})}{\langle \bar{u}_{\perp_1}^{(1)P_1} \rangle_{\perp_1}(\mu_{sp})} \right] + C_2^{(0)}(\mu_{sp}) \left[ 1 - \frac{h_{\perp_1}^{(1)P_1}(z,\mu_{sp})}{\langle \bar{u}_{\perp_1}^{(1)P_1} \rangle_{\perp_1}(\mu_{sp})} \right] \right\}. \tag{10}$$

Actually $C_{eff}^{(1)}(\mu)$ and $A_{\text{ver}}^{(1)}(\mu)$ are process independent and encodes the QCD effects only, where as $A_{sp}^{(1)}(\mu_{sp})$ contains the key information about the outgoing mesons. The factor $\frac{6a_{\perp_1}^{(1)P_1}(\mu_{sp})}{\langle \bar{u}_{\perp_1}^{(1)P_1} \rangle_{\perp_1}(\mu_{sp})}$ appear in the Eq. (8) is arising due to the Gegenbauer moments.

| Parameters | Values   | Parameters | Values   |
|------------|----------|------------|----------|
| $M_W$      | 80.423 GeV | $M_Z$      | 91.1876 GeV |
| $M_B$      | 5.279 GeV  | $m_{b_1}$  | 1.229    |
| $G_F$      | $1.166 \times 10^{-5}$ GeV | $m_{h_1}$  | 1.170    |
| $\alpha_s(2\pi)$ | 0.1172 | $\alpha$   | 1/137.036 |
| $m_{t_{\text{pole}}}$ | 178 GeV | $\Lambda_h$ | 0.5 GeV  |
| $|V_{tb}V_{td}^*|$ | $5 \times 10^{-3}$ | $m_{b_{\text{pole}}}$ | 4.27 GeV |
| $f_B$      | 200 MeV   | $\sqrt{z} = m_c/m_{B_d}$ | 0.29    |
| $a_{(b_1)}^{(1)P_1}(1\text{GeV})$ | 0 | $a_{(b_1)}^{(1)P_1}(1\text{GeV})$ | 0.1 |
| $a_{(b_1)}^{(1)P_1}(1\text{GeV})$ | 0 | $a_{(h_1)}^{(1)P_1}(1\text{GeV})$ | 0.35 |
| $f_{(b_1)}$ | 180 MeV | $f_{(h_1)}$ | 200 MeV |
| $\lambda_{B_1}^{-1}$ | $(2.15 \pm 0.50)$ GeV$^{-1}$ | $\sigma_{B_1}^{-1}$ | $(1 \text{ GeV})$ |

| Table: Input quantities and their values used in the theoretical analysis |

### 4.1. Branching ratios

We now proceed to calculate numerically the branching ratios for the $B_d^0 \to b_1 \gamma$ and $B_d^0 \to h_1 \gamma$ decays. The theoretical ratios involving the decay widths on the r.h.s. of these equations can be written in the form:

$$R_{th}(b_1 \gamma/K_1 \gamma) = \frac{\mathcal{B}_{th}(B_d^0 \to b_1 \gamma)}{\mathcal{B}_{th}(B_d^0 \to K_1 \gamma)} = \frac{1}{2} \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{(m_{b_1}^2 - m_{b_1}^2)^3}{(m_{b_1}^2 - m_{K_1}^2)^3} \zeta^2 \left[ 1 + \Delta R(b_1/K_1) \right], \tag{11}$$

$$R_{th}(h_1 \gamma/K_1 \gamma) = \frac{\mathcal{B}_{th}(B_d^0 \to h_1 \gamma)}{\mathcal{B}_{th}(B_d^0 \to K_1 \gamma)} = \frac{1}{2} \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{(m_{h_1}^2 - m_{h_1}^2)^3}{(m_{h_1}^2 - m_{K_1}^2)^3} \zeta^2 \left[ 1 + \Delta R(h_1/K_1) \right], \tag{12}$$

where $m_{b_1}$ and $m_{h_1}$ are the masses of the $b_1$- and $h_1$-mesons, $\zeta$ is the ratio of the transition form factors, which we have assumed to be the same for the $b_1^0$- and $h_1$-mesons. To get the theoretical
branching ratios for the decays $B_d^0 \rightarrow b_1 \gamma$ and $B_d^0 \rightarrow h_1 \gamma$, the ratios $\xi_1$ and $\xi_2$ should be multiplied with the corresponding experimental branching ratio of the $B_d^0 \rightarrow K_1 \gamma$ decay.

It is well known that in vector meson case the theoretical uncertainty in the evaluation of the $R_{\text{th}}(\rho \gamma/K^* \gamma)$ and $R_{\text{th}}(\omega \gamma/K^* \gamma)$ ratios is dominated by the imprecise knowledge of $\zeta = \frac{T_{\rho}^0(0)}{T_{\rho}^{K^*}(0)}$ characterizing the $SU(3)$ breaking effects in the QCD transition form factors. In the $SU(3)$-symmetry limit, $T_{\rho}^0(0) = \frac{1}{2} T_{\rho}^{K^*}(0)$, yielding $\zeta = 1$. We make use of the $SU(3)$ symmetry to relate the form factor of $B \rightarrow b_1 \gamma$ and $B_d^0 \rightarrow h_1 \gamma$ with that of $B_d^0 \rightarrow K_1 \gamma$ decay which is the only unknown parameter involved in the calculation of branching ratio for these decays. We use this symmetry because there is no experimental limit on the branching ratio of these decays. It is reasonable to use $\xi_1 = \xi_2$ because $SU(3)$ symmetry is good for the form factors irrespective of the fact that it is not exact for the masses. Thus in present analyses we use $\xi_1 = 0.32$ together with the values of the other input parameters entering in the calculation of the $B_d^0 \rightarrow (b_1, h_1) \gamma$ decay amplitudes and these are given in Table 1.

The individual branching ratios $B_{\text{th}}(B_d^0 \rightarrow b_1 \gamma)$ and $B_{\text{th}}(B_d^0 \rightarrow h_1 \gamma)$ and their ratios $R_{\text{th}}(b_1 \gamma/K_1 \gamma)$ and $R_{\text{th}}(h_1 \gamma/K_1 \gamma)$ with respect to the corresponding $B \rightarrow K_1 \gamma$ branching ratios are calculated and the corresponding values are:

\[
\begin{align*}
B_{\text{th}}[B_d^0 \rightarrow b_1 \gamma] &= 0.71 \times 10^{-6} \\
B_{\text{th}}[B_d^0 \rightarrow h_1 \gamma] &= 0.74 \times 10^{-6} \\
R_{\text{th}}[b_1 \gamma/K_1 \gamma] &= 0.0166 \\
R_{\text{th}}[h_1 \gamma/K_1 \gamma] &= 0.0167
\end{align*}
\]

To calculated these values we have used the experimental value of the branching ratio of $B \rightarrow K_1 \gamma$. One can easily see that there is very small difference between $B_d^0 \rightarrow b_1 \gamma$ and $B_d^0 \rightarrow h_1 \gamma$ branching fractions, and this is due to the slight change in the hadronic parameters of these decays.

The $SU(3)$-breaking effects in $\rho$ and $K^*$ form factors have been evaluated within several approaches, including the LCSR and Lattice QCD. In the earlier calculations of the ratios for $B \rightarrow \rho \gamma$ and $B \rightarrow K^* \gamma$, the following ranges were used: $\zeta = 0.76 \pm 0.06$ and $\zeta = 0.76 \pm 0.10$, based on the LCSR approach, which indicate substantial $SU(3)$ breaking in the $B \rightarrow K^*$ form factors. There also exists an improved Lattice estimate of this quantity, $\zeta = 0.9 \pm 0.1$. To incorporate the $SU(3)$ symmetry for these axial meson decays we have plotted the branching ratios of $B_d^0 \rightarrow (b_1, h_1) \gamma$ decay with the LEET form factor which is presented in Fig.1. The solid and dashed line show the dependence of the branching ratio of $B_d^0 \rightarrow b_1 \gamma$ and $B_d^0 \rightarrow h_1 \gamma$ on the LEET form factor $\xi_1 = \xi_2$ respectively. The graph shows that in the range $0.76 \leq \zeta \leq 0.9$ the value of the branching ratio (in the units of $10^{-6}$) is $0.4 \leq (B_d^0 \rightarrow 1 \gamma) \leq 0.7$.

4.2. CP-violating asymmetries. The direct CP-violating asymmetries in the decay rates for $B_d^0 \rightarrow (b_1, h_1) \gamma$ decays are defined as follows:

\[
\begin{align*}
A_{CP}^{\text{dir}}(b_1 \gamma) &\equiv \frac{B(B_d^0 \rightarrow b_1 \gamma) - B(B_d^0 \rightarrow b_1 \gamma)}{B(B_d^0 \rightarrow b_1 \gamma) + B(B_d^0 \rightarrow b_1 \gamma)}, \\
A_{CP}^{\text{dir}}(h_1 \gamma) &\equiv \frac{B(B_d^0 \rightarrow h_1 \gamma) - B(B_d^0 \rightarrow h_1 \gamma)}{B(B_d^0 \rightarrow h_1 \gamma) + B(B_d^0 \rightarrow h_1 \gamma)}.
\end{align*}
\]
Before we go for the numerical values of CP-asymmetry, let us discuss the difference in the hadronic parameters involving the $b_1$ and $h_1$ mesons. As these are the axial vector states of $\rho^0$ and $\omega$ mesons so these are also the maximally mixed superpositions of the $\bar{u}u$ and $\bar{d}d$ quark states: $|b_1⟩ = (|dd⟩ - |\bar{u}\bar{u})/\sqrt{2}$ and $|h_1⟩ = (|dd⟩ + |\bar{u}\bar{u})/\sqrt{2}$. Neglecting the $W$-exchange contributions in the decays, the radiative decay widths are determined by the penguin amplitudes which involve only the $|dd⟩$ components of these mesons, leading to identical branching ratios (modulo a tiny phase space difference). The $W$-exchange diagrams from the $O_1^{(u)}$ and $O_2^{(u)}$ operators (in our approach, we are systematically neglecting the contributions from the penguin operators $O_3,...,O_6$) yield contributions equal in magnitude but opposite in signs [for detailed calculation please see [14, 17]]. If we use the notations and expressions given in Ref. [17], the LCSR results are: $\epsilon_A^{(h)} = -\epsilon_A^{(h_1)} = 0.07$. The smallness of these numbers reflects both the colour-suppressed nature of the $W$-exchange amplitudes in $B_d^0 \rightarrow (b_1,h_1)γ$ decays, and the observation that the leading contributions in the weak annihilation and $W$-exchange amplitudes arise from the radiation off the $d$-quark in the $B_d^0$-meson, which is suppressed due to the electric charge.

The explicit expressions of these asymmetries for the charged axial vector meson in terms of the individual contributions in the decay amplitude can be found in Ref. [18], which for $A^{dir}_CP(b_1γ)$ and $A^{dir}_CP(h_1γ)$ may be obtained by obvious replacements. The calculated values of the CP-asymmetry for the above mentioned decays are summarized in Table 2. The CP-asymmetry receives contributions from the hard spectator corrections which tend to decrease its value estimated from the vertex corrections alone. The dependence of the direct CP-asymmetry on the CKM unitarity-triangle angle $α$ is presented in the Fig.2. It should be noted that the predicted direct CP-asymmetries are rather sizable (of order 10%) and is negative like $ρ$ and $ω$ meson case. It is quite unfortunate that the predicted value of CP asymmetry is sensitive to both the choice of the scale as well as the quark mass ratio $z = m_c^2/m_b^2$ used in the calculation.

| $B_d^0 \rightarrow b_1γ$ | $B_d^0 \rightarrow h_1γ$ |
|-------------------------|-------------------------|
| $\mathcal{R}_{th}$     | 0.0166                  |
| $\mathcal{B}_{th}$     | $0.71 \times 10^{-6}$  |
| $A^{dir}_CP$           | $-10.7%$                |

5. Summary We have calculated the branching ratios for $b \rightarrow 1^1P_1γ$ decays at NLO of $α_s$. These $1^1P_1$ are $b_1$ and $h_1$ mesons which are the corresponding radially excited axial vector mesons of $ρ$ and $ω$ respectively. Using the $SU(3)$ symmetry for the form factor, the branching ratio for $B_d^0 \rightarrow (b_1,h_1)γ$ is expressed in terms of the branching ratio of the $B_d^0 \rightarrow K_1γ$ and it is found to be $B(B_d^0 \rightarrow b_1γ) = 0.71 \times 10^{-6}$ and $B(B_d^0 \rightarrow h_1γ) = 0.74 \times 10^{-6}$. Then we have plotted the branching ratio with the LEET form factor which is the only unknown parameter involved in the calculation. It is shown that the corresponding to the range of $SU(3)$ symmetry breaking parameter $ζ$, $0.76 \leq ζ \leq 0.9$ the value of the branching ratio ($10^6$) is $0.4 \leq (B_d^0 \rightarrow 1^1P_1γ) \leq 0.7$. Therefore in future when we have the experimental data on these decays we will be able to extract the value of form factor. Further we have also calculated direct CP asymmetry for these decays and find, in conformity with the observations made in the literature, that the hard spectator contributions significantly reduces the asymmetry arising from the vertex corrections alone. The value of CP-asymmetry is 10% and is negative like $ρ$ and $ω$ in the Standard Model. Thus the measurement of CP-asymmetry will either overconstrain the angle $α$ of the unitarity
triangle, or they may lead to the discovery of physics beyond the SM in the radiative $b \rightarrow d \gamma$ decays.

Acknowledgements. One of the authors (J) would like to thank Prof. Fayyazuddin for valuable discussion. This work was supported by a grant from Higher Education Commission of Pakistan.

References

[1] R. Ammar et al. [CLEO Collaboration], Phys. Rev. Lett. 71, 674 (1993).
[2] M. S. Alam et al. [CLEO Collaboration], Phys. Rev. Lett. 74, 2885 (1995).
[3] The Heavy Flavor Averaging Group (HFAG), [http://www.slac.stanford.edu/xorg/hfag/](http://www.slac.stanford.edu/xorg/hfag/)
[4] C. Greub, H. Simma, D. Wyler, Nucl. Phys. B434, 39 (1995); J. Milana, Phys. Rev. D53, 1403 (1996); A. Ali, V.M. Braun, Phys. Lett. B359, 223 (1995); J.F. Donoghue, E. Golowich, A.A. Petrov, Phys. Rev. D55, 2657 (1997); T. Huang, Z.H. Li, H.D. Zhang, J. Phys. G25, 1179 (1999); D. Pirjol, Phys. Lett. B487, 306 (2000) [arXiv:hep-ph/0006315]; M. Beyer, D. Melikhov, N. Nikitin, B. Stech, Phys. Rev. D64, 094006 (2001); S.W. Bosch, eprint hep-ph/031031; M. Beneke, T. Feldmann and D. Seidel, [arXiv:hep-ph/0412400](http://arxiv.org/abs/hep-ph/0412400).
[5] A. Ali, L.T. Handoko, D. London, Phys. Rev. D63, 014014 (2000); L.T. Handoko, Nucl. Phys. Proc. Suppl. 93, 296 (2001); A. Arhrib, C.K. Chua, W.S. Hou, Eur. Phys. J. C21, 567 (2001); Z. Xiao, C. Zhuang, Eur. Phys. J. C33, 349 (2004); C.S. Kim, Y.G. Kim, K.Y. Lee, eprint hep-ph/0407060.
[6] K. Abe et al. (Belle Collaboration), [hep-ex/0408138](http://arxiv.org/abs/hep-ex/0408138).
[7] A. S. Safir, Eur.Phys.J.direct C3, 15 (2001); [arXiv: hep-ph/0109232](http://arxiv.org/abs/hep-ph/0109232).
[8] Jong-Phil Lee, Phys.Rev. D69, 114007 (2004); [arXiv:hep-ph/0403034](http://arxiv.org/abs/hep-ph/0403034).
[9] Y.J. Kwon, Jong-Phil Lee, Phys.Rev. D71, 014009 (2005); [arXiv:hep-ph/0409133](http://arxiv.org/abs/hep-ph/0409133).
[10] M. Jamil Aslam and Riazuddin, Phys. Rev. D72, 094019 (2005); [arXiv:hep-ph/0509082](http://arxiv.org/abs/hep-ph/0509082).
[11] Kwei-Chou Yang, JHEP 0510 108 (2005); [arXiv: hep-ph/0509337](http://arxiv.org/abs/hep-ph/0509337).
[12] M. J. Dugan and B. Grinstein, Phys. Lett. B 255, 583 (1991).
[13] J. Charles, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D 60, 014001 (1999) [hep-ph/9812358](http://arxiv.org/abs/hep-ph/9812358).
[14] A.Ali, E. Lunghi, A.Y. Parkhomenko, Phys.Lett. B595, 323-338 (2004); [arXiv: hep-ph/0405075](http://arxiv.org/abs/hep-ph/0405075).
[15] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996) [arXiv:hep-ph/9512380](http://arxiv.org/abs/hep-ph/9512380).
[16] J.D. Bjorken and S.D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).
[17] B. Grinstein and D. Pirjol, Phys. Rev. D 62, 093002 (2000) [arXiv:hep-ph/0002216].
[18] M. Jamil Aslam, hep-ph/0604025.
[19] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999) [arXiv:hep-ph/9905312]; Nucl. Phys. B 591, 313 (2000) [arXiv:hep-ph/0006124].
[20] A. Ali and A. Y. Parkhomenko, Eur. Phys. J. C 23, 89 (2002) [arXiv:hep-ph/0105302].
[21] M. Beneke, T. Feldmann and D. Seidel, Nucl. Phys. B 612, 25 (2001) [arXiv:hep-ph/0106067].
[22] S. W. Bosch and G. Buchalla, Nucl. Phys. B 621, 459 (2002) [arXiv:hep-ph/0106081].
[23] A. L. Kagan and M. Neubert, Phys. Lett. B 539, 227 (2002) [arXiv:hep-ph/0110078].
[24] S. Descotes-Genon and C. T. Sachrajda, [arXiv:hep-ph/0403277].
[25] V. M. Braun, D. Y. Ivanov and G. P. Korchemsky, Phys. Rev. D 69, 034014 (2004) [arXiv:hep-ph/0309330].
[26] A. Ali and E. Lunghi, Eur. Phys. J. C 26, 195 (2002) [arXiv:hep-ph/0206242].
[27] A. Ali, V. M. Braun and H. Simma, Z. Phys. C 63, 437 (1994) [arXiv:hep-ph/9401277].
[28] A. Ali and V. M. Braun, Phys. Lett. B 359, 223 (1995) [arXiv:hep-ph/9506248].
[29] P. Ball and V. M. Braun, Phys. Rev. D 58, 094016 (1998) [arXiv:hep-ph/9805422].
[30] S. Narison, Phys. Lett. B 327, 354 (1994) [arXiv:hep-ph/9403370].
[31] D. Melikhov and B. Stech, Phys. Rev. D 62, 014006 (2000) [arXiv:hep-ph/0001113].
[32] D. Becirevic, invited talk at the Flavor Physics and CP Violation (FPCP 2003) Conference, Paris, France, June 3-6, 2003.
**Figure Captions**

Figure 1: Branching ratio for $B \rightarrow 1^1P_1 \gamma$ decay vs LEET form factor; Solid line shows the value for $b_1$ meson and the dashed line is for $h_1$ meson.

Figure 2: CP-asymmetry ($-A_{CP}\%$) vs the Unitarity triangle phase $\alpha$; Solid line is for $h_1$ meson and dashed line is for $b_1$ meson.
