Creating solitons by means of spin-orbit coupling

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Abstract – This mini-review collects results predicting the creation of matter-wave solitons by the spinor system of Gross-Pitaevskii equations (GPEs) with the self-attractive cubic nonlinearity and linear first-order-derivative terms accounting for the spin-orbit coupling (SOC). In 1D, the so-predicted bright solitons are similar to usual ones, supported by the GPE in the absence of SOC. Essentially new results were recently obtained for 2D and 3D systems: SOC suppresses the collapse instability in the multidimensional GPE, creating 2D ground-state solitons and metastable 3D ones of two types: semi-vortices (SVs), with vorticities $m=1$ in one component and $m=0$ in the other, and mixed modes (MMs), with $m=0$ and $m=\pm1$ present in both components.

With the Galilean invariance broken by SOC, moving solitons exist up to a certain critical velocity. The latest result predicts stable 2D “quantum droplets” of the MM type in the presence of the Lee-Huang-Yang corrections to the GPE system, induced by quantum fluctuations, in the case when the inter-component attraction dominates over the self-repulsion in each component.

Introduction. – Atomic gases, cooled to the state of Bose-Einstein condensates (BEC) [1,2], find an important application as a testbed allowing simulation of various effects, which were originally known in complex forms in condensed-matter physics, and may be emulated in a simple and clean form in ultracold quantum gases [3]. BEC also offers a way to reproduce diverse phenomena which were previously discovered in optics [4]. The latter possibility is based on the similarity of the nonlinear Schrödinger equation (NLSE), which is the basic propagation equation in optics [5], and the Gross-Pitaevskii equation (GPE), which is a universal model for ultracold bosonic gases [1]. In the framework of the similarity, the cubic nonlinearity, which represents the Kerr term in optics, accounts for collisional effects in atomic BEC.

In particular, a binary bosonic gas, whose two-component mean-field wave function is considered as a pseudo-spinor, may emulate effects in the fermionic gas of electrons with spin 1/2. In this vein, great attention has been drawn to the experimentally demonstrated [6] BEC emulation of spin-orbit coupling (SOC) in semiconductors, i.e., the interaction between the electron’s momentum and its spin. In terms of the atomic gas, SOC is mapped into a linear interaction of the atoms’ momentum and pseudospin [6–11]. The mapping makes it also possible to include the Zeeman splitting (ZS) between up- and down-states of the electron, which is represented by an energy difference between two components of the BEC wave function [12]. In semiconductors, two fundamental types of SOC are represented by the Dresselhaus [13] and Rashba [14] Hamiltonians, both admitting the emulation in ultracold gases.

While the majority of experimental works on the emulation of SOC were carried out in effectively one-dimensional (1D) settings, the realization of the SOC in the quasi-2D BEC was reported too [15], encouraging the consideration of multidimensional SOC-supported states, such as 2D [16–21] and 3D [22] solitons, vortices [23–26], skyrmions [27], etc. A crucial role in the creation of these nonlinear modes belongs to the sign of the nonlinearity. Thus far, SOC was realized in the $^{87}$Rb gas with repulsive interactions. Nevertheless, in other bosonic species which admit the transition to BEC, the sign can be switched by means of the Feshbach resonance [28,29]. In particular, the application of this method to the condensate of $^7$Li [30,31] and $^{85}$Rb [32] atoms has made it possible the creation of effectively 1D solitons.

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This mini-review is focused on recently elaborated schemes for the creation of solitons in binary BEC with SOC between the components. Such experimental results were not yet reported, but theoretical predictions provide an incentive for the development of the experiment in this direction. While in the 1D setting the addition of SOC does not lead to qualitative changes of the known soliton phenomenology, essentially novel effects have been predicted in 2D and 3D geometries. Arguably, the most noteworthy ones are possibilities to create absolutely stable 2D [16] and metastable 3D [22] solitons in the self-attractive BEC, in spite of the presence of the critical and supercritical collapse, driven in the 2D and 3D space, respectively, by the cubic self-attraction [33,34] (the collapse in SOC systems was considered in [35]). For this reason, the creation of stable multidimensional optical and matter-wave solitons is a challenging problem, which has drawn a great deal of interest, see reviews [4,36] and [37,38]. The above-mentioned recent results make an essential contribution towards the solution of this problem. In particular, the newest addition to the topic [39] is the prediction of stable SOC solitons in the effectively 2D setting stabilized by the Lee-Huang-Yang (LHY) corrections to the GPE [40], which are generated by quantum fluctuations around the mean-field states. Recently, it has been predicted [41,42] and demonstrated [43–45] that the LHY effect readily stabilizes two-component 3D solitons, in the form of “quantum droplets” (QDs), against the collapse (in the system which does not include SOC effects).

The mini-review is structured as follows: basic models of SOC systems are formulated in the following section, the main results for 2D and 3D solitons are summarized in the third section, and the article is completed by the last section.

The models: systems of coupled GPEs. – The system of GPEs in 2D space \((x,y)\) for the pseudo-spinor wave function \((\phi_+,\phi_-)\), which includes the self-attraction with the coefficient scaled to be 1, cross-attraction with relative strength \(\gamma\) with \(\phi_0\), linear SOC terms of the Rashba type is included, with coefficient \(\lambda\), and \(\lambda_D\), and ZS with strength \(\Omega\) \(\geq 0\), is written as [8–11,17]

\[
\begin{align*}
\frac{i}{\hbar} \frac{\partial \phi_+}{\partial t} &= -\frac{1}{2} \nabla^2 \phi_+ - (|\phi_+|^2 + \gamma|\phi_-|^2) \phi_+ \\
&\quad + \left( \lambda D^{[+]} \phi_- - i \lambda_D D^{[+]} \phi_- \right) - \Omega \phi_+,
\end{align*}
\]

\[
\begin{align*}
\frac{i}{\hbar} \frac{\partial \phi_-}{\partial t} &= -\frac{1}{2} \nabla^2 \phi_- - (|\phi_-|^2 + \gamma|\phi_+|^2) \phi_- \\
&\quad - \left( \lambda D^{[+]} \phi_+ + i \lambda_D D^{[+]} \phi_+ \right) + \Omega \phi_-,
\end{align*}
\]

with \(D^{[\pm]} \equiv \partial/\partial x \pm i \partial/\partial y\). The GPE system conserves the Hamiltonian, momentum, and the norm, proportional to the number of atoms in the condensate:

\[
N = \int \int (|\phi_+|^2 + |\phi_-|^2) dx dy \equiv N_+ + N_-.
\]

In the scaled equations (1) the unit length corresponds to the distance \(\sim 1 \mu \text{m}\) and \(N = 1\) is tantamount to \(\sim 3 \times 10^3\) atoms [17]. The spectrum of excitations generated by the linearization of eq. (1), for \(\phi_\pm \sim \exp (i k \cdot r - i \mu t)\), where \(k\) is the wave vector, contains two branches [21]:

\[
\mu = \frac{k^2}{2} \pm \sqrt{(\lambda^2 + \lambda_D^2) k^2 + 4 \lambda \lambda_D k_x k_y + \Omega^2},
\]

where the sign \(\pm\) is unrelated to subscripts \(+/−\) in eq. (1). Solitons may exist at values of \(\mu\) which are not covered by eq. (3) with real \(k_{x,y}\), \(i.e., \at \)

\[
\mu < -\frac{1}{2} \left[ \left( |\lambda| + |\lambda_D| \right)^2 + \Omega^2 \right],
\]

if \((|\lambda| + |\lambda_D|)^2 > |\Omega|\),

\[
\mu < -|\Omega|, \text{ if } (|\lambda| + |\lambda_D|)^2 < |\Omega|.
\]

The 3D model was addressed in [22], with SOC of the Weyl type [7]:

\[
\begin{align*}
\frac{i}{\hbar} \frac{\partial \phi_+}{\partial t} &= -\frac{1}{2} \nabla^2 \phi_+ + \lambda D^{[+]} \phi_+ + g(|\phi_+|^2 - |\phi_-|^2) \phi_+ + \left( \frac{g^2}{4 \pi} \right) \ln \left( |\phi_+|^2 + |\phi_-|^2 \right), \\
\frac{i}{\hbar} \frac{\partial \phi_-}{\partial t} &= -\frac{1}{2} \nabla^2 \phi_- - \lambda D^{[+]} \phi_- + g(|\phi_-|^2 - |\phi_+|^2) \phi_- + \left( \frac{g^2}{4 \pi} \right) \ln \left( |\phi_+|^2 + |\phi_-|^2 \right),
\end{align*}
\]

where \(g > 0\) is the nonlinearity strength, and SOC of the Rashba type is included, with coefficient \(\lambda\).

Stable 2D and 3D solitons in the SOC system: semi-vortices (SVs) and mixed modes (MMs). – Semi-vortices. Basic results for 2D solitons stabilized by SOC are presented here, following works [16,17] and [46], where details can be found. First, eq. (1) admits stationary solutions of the SV type (alias half-vortices [25]). In the absence of the Dresselhaus and ZS terms, \(\lambda_D = \Omega = 0\), SV solitons with chemical potential
μ are bound states of zero-vorticity \((m_+ = 0)\) and vortical \((m_- = 1)\) components, represented by an exact ansatz written in terms of polar coordinates \((r, \theta)\):

\[
\phi_+(r, \theta) = e^{-i\mu t} f_1(r) ,
\]

\[
\phi_-(r, \theta) = e^{-i\mu t + i\lambda_2 \theta} f_2(r^2) ,
\]

Fig. 1: (Color online) (a) Cross-sections of two components of the SV (semi-vortex), \(|\phi_+(x, 0)|\) and \(|\phi_-(x, 0)|\), with \(N = 5\) and \(\lambda = 1\), \(\gamma = \lambda_D = \Omega = 0\) in eq. (1). (b) and (c): chemical potential \(\mu\) and the relative share of the norm in the zero-vorticity component, \(N_\perp/N\) (see eq. (2)), vs. \(N\) for the SV family. The plots are borrowed from [16] and [46].

| \(N\) | \(\mu\) | \(N_\perp/N\) |
|-------|-----|---------|
| 4     | 0.5 | 0.2     |
| 5     | 0.6 | 0.3     |

where \(f_{1,2}(r^2)\) takes finite values at \(r = 0\), and decays at \(r \to \infty\) as \(\exp\left(\sqrt{-2\mu + \lambda^2}r\right)\), hence it exists at \(\mu < -\lambda^2/2\). In the general case, with \(\lambda_D, \Omega \neq 0\), solitons exist in the range of \(\mu\) given by eq. (4). Due to its symmetry, eq. (1) also gives rise to a mode which is a counterpart of SV (4) with vorticities \((m_+, m_-) = (0, 1)\) replaced by \((m_+, m_-) = (-1, 0)\).

A stable SV, obtained as a numerical solution of eq. (1), is displayed in fig. 1(a). Results for the SV family are summarized in figs. 1(b), (c), which display \(\mu\) as a function of \(N\). It is seen that the zero-vorticity component always carries a larger share of the total norm. The negative local slope of the \(\mu(N)\) curve in fig. 1(b) implies that it satisfies the Vakhitov-Kolokolov (VK) criterion [33, 34, 47], which is a necessary condition for the stability of solitons supported by self-attractive nonlinearities. The SV family exists at \(N < N_{\text{max}} \equiv N_T \approx 5.85\), the latter value being the collapse threshold, \(N_T\), i.e., the norm of the 2D *Townes solitons* (TSs) [33, 34] generated by the single GPE in the absence of SOC. Indeed, fig. 1(c) shows that the vortex component \(\phi_-\) vanishes in the limit of \(N \to N_T\) \((\mu \to -\infty)\), when SV degenerates into an unstable single-component TS. At \(N > N_T\), solitons do not exist, as the norm exceeding the threshold gives rise to the collapse. On the other hand, there is no minimum value of \(N\) necessary for the existence of stable SVs.

The fundamental reason for the instability of TSs in the usual 2D NLSE is the scaling invariance of this equation, due to which TSs with all values of \(\mu < 0\) have a single value of the norm, \(N_T\). The SOC terms in eq. (1) break the scaling invariance, pushing the soliton’s norm to \(N < N_T\) and thus preventing the destabilization of the TSs by the collapse, as it may only be initiated by \(N \geq N_T\).

\[
N < N_{\text{max}} \equiv 2N_T/(1 + \gamma),
\]

Fig. 2: (Color online) (a), (b): the same as in fig. 1(a), (b), but for stable MMs (mixed modes) at \(\gamma = 2\). (c) Separation \(\Delta X\) between peak positions of \(|\phi_+(x, 0)|^2\) and \(|\phi_-(x, 0)|^2\) vs. the norm. The plots are borrowed from [16] and [46].

**Mixed modes.** Another type of 2D self-trapped vortical states supported by eq. (1) can be produced by input

\[
(\phi_{\perp})_{\text{MM}} = B_1 e^{-\beta_1 r^2} \mp B_2 r e^{i\beta_2 r^2} ,
\]

with real constants \(B_{1,2} > 0\) (unlike the SV ansatz (4), this one is not compatible with the equations, being used as an initial guess, or a basis for the variational approximation [16]). Modes generated by this input are called MMs as they mix vorticities \((0, -1)\) and \((0, +1)\) in the two components. A stable MM and \(\mu(N)\) dependence for the MM family, which also satisfies the VK criterion, are displayed in figs. 2(a) and (b), respectively (for the system without the Dresselhaus and ZS terms, i.e., \(\lambda_D = \Omega = 0\)). In the absence of the ZS, norms of the two MM’s components are always equal, while their maxima are separated by distance \(\Delta X\), see fig. 2(c). MMs exist at

\[
N < N_{\text{max}} \equiv 2N_T/(1 + \gamma) ,
\]
Dresselhaus terms, with \( \lambda \) in their fusion into a single mode \([16]\). It is possible to simulate collisions between them, which results in very small. Lastly, the availability of mobile MMs makes it possible for GSs of the MM and SV types. The vertical line corresponds to \( \Omega \approx 1.95 \), at which the SV suffers the delocalization. The plots are borrowed from \([17]\) and \([46]\).

breaks the Galilean invariance of eq. (1). Rewriting eq. (1) in the reference frame moving with velocities \((v_x, v_y)\) and looking for the corresponding stationary solutions, it was found \([16]\) that there are no solitons with \( v_x \neq 0 \), while MM can be set in steady motion in the direction of \( y \), with \( |v_y| < v_{\text{max}} \). The amplitude of the MM with a fixed norm decays with the increase of \( v_y \), vanishing at \( v_y = v_{\text{max}} \). For instance, \( v_{\text{max}} \approx 1.8 \) for \( \lambda = 1, \gamma = 2 \) and \( N = 3.1 \). SV may also be set in motion along \( y \), but its limit velocity is very small. Lastly, the availability of mobile MMs makes it possible to simulate collisions between them, which results in their fusion into a single mode \([16]\).

**Combined Rashba and Dresselhaus SOC.** When the Dresselhaus terms, with \( \lambda_D \neq 0 \), are present in eq. (1), an exact ansatz similar to (4) is not available, but a numerical SV solution can be constructed starting from the initial guess \( \phi^{(0)}_+ (x,y) = A_1\exp(-\alpha_1 r^2), \phi^{(0)}_- (x,y) = A_2\exp(-\alpha_2 r^2) \), with constants \( A_{1,2} \) and \( \alpha_{1,2} > 0 \), while MM is generated by the same ansatz (8) as above. As a result, it is found that, again, SV and MM realize GS at \( \gamma < 1 \) and \( \gamma > 1 \), respectively \([17]\).

An essential effect caused by the inclusion of the Dresselhaus terms is the destruction of SV and MM when \( \lambda_D \) exceeds certain critical values, which are growing functions of \( N \) and depend on \( \gamma \). In other words, SV and MM exist in the intervals, respectively, \( N_{\text{SV}}^{(\lambda_D)} < N < N_T \) and \( N_{\text{MM}}^{(\lambda_D)} < N < 2(1 + \gamma)^{-1} N_T \), cf. eq. (9) \([17]\). These results are summarized in fig. 3, which shows stability regions for SVs and MMs in the \((N, \lambda_D)\)-plane at \( \gamma = 0 \) and \( \gamma = 2 \), respectively. The presence of the threshold \( N_{\text{min}} \) necessary for the existence of the solitons is a drastic difference from the system with \( \lambda_D = 0 \), cf. figs. 1(b) and 2(b), while the upper limits, \( N_T \) and \( 2(1 + \gamma)^{-1} N_T \), do not depend on \( \lambda_D \). Note that the critical value of \( \lambda_D \), up to which the solitons persist, is essentially larger for MM than for SV.

**Effects of the Zeeman splitting (ZS).** Through the trend to populate one component and depopulate the other one, ZS essentially affects the SOC soliton phenomenology \([17]\). In particular, the stationary version of eq. (1), with \( \phi_{\pm} = e^{-\mu t} \phi_{\pm} (x, y) \), admits the application of a simple approximation in the limit of large \( \Omega \), setting \( \mu = -\Omega + \delta \mu \), with \( |\delta \mu| \ll \Omega \). In this limit, fixing \( \lambda = 1 \) and \( \lambda_D = 0 \), one can eliminate the depopulated component in favor of the other one: \( u_- \approx (2\Omega^{-1} D)^{1/2} u_+ \), which obeys the stationary NLSE, \( (\delta \mu) u_+ = -(1/2) (1 - \Omega^{-1}) \nabla^2 u_+ - |u_+|^2 u_+ \). Up to rescaling, it gives rise to the TS, while \( u_- \) is a small vortex component of the SV complex. For small \( 1/\Omega \), the respective SV’s norm is \( N = (1 - \Omega^{-1}) N_T + \Omega (\Omega^{-2}) \). Being slightly smaller than the collapse threshold, it keeps SV protected against the collapse. The conclusion is that, at large \( \Omega \), GS is of the SV type, irrespective of the value of \( \gamma \) in eq. (1), which does not appear in this approximation.
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Fig. 6: (Color online) Typical examples of elongated QDs (“quantum droplets”), generated by eq. (6) with \((g,\lambda,N) = (2,1,20)\), are displayed in columns (a)–(c), with the vertical, diagonal, and horizontal orientations, respectively. The first and second rows display density patterns \(|\phi_+(r)|^2\) and \(|\phi_-(r)|^2\), the phase pattern of \(\phi_+\) is presented in the third row, the fourth row shows the total density profile, \(|\phi_+(r)|^2 + |\phi_-(r)|^2\), and the fifth row illustrates the stability of the QDs in direct simulations of eq. (6) (shown by the evolution of the total density). Column (d) is an example of QD with \((g,\lambda,N) = (2,0.2,20)\), which is nearly isotropic, as \(\lambda\) is small. The plots are borrowed from [39].

Because \(N\) takes values close to \(N_T\) at large \(\Omega\), one may expect that SVs with smaller norms disappear with the increase of \(\Omega\) at some critical value \(\Omega_{cr}\). This is indeed revealed by both the variational approximation and numerical results [17], see fig. 4(a) (for instance, \(\Omega_{cr}(\gamma = 0,N = 3) \approx 1.95\)). In line with the above approximation, at \(\Omega < \Omega_{cr}\) stable SV keeps vorticity in the component with a smaller norm. As concerns MM, it is ousted by SV with the increase of \(\Omega\). This is seen in the increase of the value of \(\gamma_{cr}\), above which MM plays the role of GS: while, as mentioned above, \(\gamma_{cr} = 1\) is the universal SV-MM boundary at \(\Omega = 0\), fig. 4(b) demonstrates that \(\gamma_{cr}\) grows with the increase of \(\Omega\). At \(\gamma > \gamma_{cr}\), the symmetry between two MM’s components, which holds at \(\Omega = 0\), is broken, the norm of \(\phi_+\) being larger, with a smaller vortex part in it.

Metastable 3D solitons. As said above, in 2D the SOC system creates the otherwise missing GS at \(N < N_{\text{max}}\).

In the 3D system (5), SOC cannot suppress the supercritical collapse, therefore the 3D system has no GS [22]. Nevertheless, the variational analysis predicts metastable 3D solitons which represent a local energy minimum, \(i.e.,\) such solitons are stable against small perturbations. Numerically, they can be produced by inputs similar to those adopted in the 2D system (see, \(e.g.,\) eq. (8)), with
extra factor $\exp(-\gamma z^2)$ enforcing the localization in $z$ for $\gamma > 0$. As a result, both the variational approximation and numerical analysis generate families of 3D metastable solitons, which are SV and MM at $\gamma < 1$ and $\gamma > 1$, respectively, see fig. 5. Setting $\lambda = 1$ in eq. (5), the conclusion is that, similar to the 2D setting (cf. eq. (9)), the metastable 3D solitons exist at values of the 3D norm $N_{3D} < (N_{3D})_{\text{max}}(\gamma)$, the largest value being $(N_{3D})_{\text{max}}(\gamma = 0) \approx 11.5$ [22].

2D “quantum droplets” (QDs) of the MM type stabilized by the LHY corrections. A general property of absolutely stable 2D solitons and metastable 3D ones is that they exist below a critical value of the norm, see, e.g., eq. (9). On the other hand, as LHY terms may stabilize 3D and 2D solitons against the collapse, in the form of QDs [41,42], the SOC system (6), including the LHY terms in their 2D form, gives rise to a family of stable QMs of the MM type, without any upper boundary $N_{\text{max}}$. As shown in fig. 6, a characteristic feature of the SOC-affected QDs is their elongated shape, which may have arbitrary orientation in the $(x, y)$-plane, due to the azimuthal invariance of eqs. (5): if there is a stationary solution $\{\phi_+, \phi_-(r, \theta)\}$, its rotated version, $\{\phi_+(r, \theta + \theta_0), e^{-i\theta_0}\phi_-(r, \theta + \theta_0)\}$, with arbitrary angle $\theta_0$, is a solution too [39]. All QDs are stable, as illustrated by the bottom row of fig. 6. Lastly, if SOC terms of both the Rashba and Dresselhaus types are included, eq. (6) generates QDs for any ratio $\lambda_D/\gamma$ of their strengths, including $\lambda_D/\gamma = 1$, while, in the absence of the LHY effect, MM exists only at essentially smaller values of $\lambda_D/\gamma$, see fig. 3(b).

**Conclusion.** — This mini-review focuses on recent theoretical results which demonstrate the possibility of the creation of absolutely stable 2D [16,17] and metastable 3D [22] solitons supported by SOC in binary BEC with attractive nonlinearity. An essential prediction is that, on the contrary to the commonly known instability of 2D solitons created by cubic self-atraction, two stable soliton species, SV (semi-vortex) and MM (mixed mode), are supported by SOC. In 2D, SV and MM represent the GS (ground state) when self-attraction in each component is, respectively, stronger or weaker than its cross-interaction counterpart. SV is made more favorable if ZS (Zeeman splitting) is applied. A broad class of stable MMs was recently predicted [39] in the 2D system which includes beyond-mean-field (LHY) corrections. These results suggest novel possibilities for the creation of stable vorticity-bearing solitons in BEC. An analog of SOC can also be implemented in optics, which was used to predict stable spatiotemporal solitons in nonlinear dual-core planar waveguides [21,48].

The work can be extended in other directions. In particular, if SOC acts in a confined 2D or 3D spatial domain (similar to the 1D setting introduced in [49]), it is relevant to identify a minimum size of the domain which is sufficient for the creation of stable solitons [50]. It is relevant too to construct 3D solitons in the SOC system, which includes LHY terms, and to consider effects of ZS on 3D solitons. Proceeding from stationary configurations to nonstationary ones, it may be interesting to apply the concept of the “nonlinearity management” [51] to the SOC system, periodically switching it, by means of the time-dependent Feshbach resonance, between settings in which SV and MM represent GS.

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