Michelson-type all-reflective interferometric autocorrelation in the VUV regime

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Abstract
We demonstrate second-order interferometric autocorrelation of a pulse in the vacuum-ultraviolet (VUV) spectral range using an optical arrangement equivalent to a Michelson interferometer. In an all-reflective design, wavefront splitting is realized with two moveable interdigitated reflective gratings forming a diffraction pattern with well separated orders and an intensity distribution depending on the precisely adjustable path-length difference. An imaging time-of-flight spectrometer is able to spatially select ions created by nonlinear two-photon absorption in the focus of the zeroth diffraction order. This arrangement is used to demonstrate interferometric autocorrelation in krypton with femtosecond VUV pulses at 160 nm wavelength. In addition to the pulse duration, which is already accessible with non-collinear intensity autocorrelation, the full interferometric contrast of the presented approach enables us to extract also information on temporal phases.

Keywords: interferometric autocorrelation, multiphoton ionization, nonlinear processes, high-harmonic generation, ultrafast processes
1. Introduction

Employing high harmonics of ultrafast lasers in pump–probe experiments is continuously proving to be an essential tool for the investigation of ultrafast atomic [1] and molecular dynamics [2–4]. With the advance of laser technology high harmonic generation (HHG) sources have become sufficiently powerful to drive nonlinear processes in gases [5–7], thus enabling the extension of autocorrelation (AC) techniques from the visible into the vacuum-ultraviolet (VUV) and extended-ultraviolet (XUV) regimes. In addition to pulse metrology, the same experimental arrangement can be utilized as an XUV-pump–XUV-probe scheme and has become a widely used technique for the determination of time constants of atomic and molecular processes driven by XUV pulses from HHG sources [8, 9] or free-electron lasers [10–14]. As the intensity AC (IAC) trace does not carry any phase information, it does not allow for reconstruction of a light pulse without making assumptions about its temporal phase [15]. Advanced variants like XUV-FROG [16] are able to fully characterize high harmonic pulses but require a rather complicated setup. The second-order interferometric autocorrelation, on the other hand, has proven to be a valuable technique for light metrology and nonlinear spectroscopy in the visible range [17, 18]. Briefly, two identical replicas of the pulse to be characterized are spatially and temporally superimposed in a collinear geometry, with a variable delay between them, on a two-photon detector possessing an instantaneous, spectrally flat response to the incident electromagnetic field. The resulting nonlinear signal, recorded as a function of the delay between the two pulse replicas, corresponds to the second order fringe resolved autocorrelation function of the pulse. It delivers information about the duration of the pulse as well as about the light field’s temporal phase evolution [15]. The degree to which this information can be extracted depends critically on the experimentally achieved dynamic range and signal-to-noise ratio [19].

The implementation of interferometric AC in the VUV regime implies a couple of experimental challenges:

(i) below 105 nm (the cut-off for LiF) no transparent bulk material is available, i.e., the beam manipulation must be based on reflective optics,

(ii) the autocorrelation itself cannot take place in a solid-state medium, i.e., only gases or surfaces can serve as nonlinear media,

(iii) with the lack of transparent bulk media a conventional Michelson-type beam-splitting is not feasible or

(iv) carried out using transmission gratings that impose high losses and do not deliver the theoretical contrast of 1 : 8 [20, 21]

(v) wavefront-splitting, often utilized instead with a split-mirror arrangement [22–25] leads to a complex volume interference reducing the interferometric contrast.

The objective of this work is to establish interferometric AC in the VUV with a contrast approaching 1 : 8 as realized with Michelson interferometers in the visible range. The solution presented in this work combines an all-reflective beam transport with a dedicated beam-splitting device which produces a widespread interference pattern in a gas target, from which two-photon created ions can be spatially resolved in an imaging time-of-flight (TOF) detector. The concept is tested at 160 nm wavelength but its versatility shall allow an extension into the XUV—or even x-ray range.
We remind ourselves that the second order interferometric autocorrelation or fringe resolved interferometric autocorrelation (FRIAC) trace of a light pulse $E(t)$ can be written as [15]:

$$\text{FRIAC}(\tau) \propto 1 + 2 \text{IAC}(\tau) + 4 \text{Re} \left[ F_1(\tau) \cdot \exp(j\Omega\tau) \right] + \text{Re} \left[ F_2(\tau) \cdot \exp(j2\Omega\tau) \right]$$

(1)

with

$$\text{IAC}(\tau) = \int_{-\infty}^{\infty} dt \ I(t) \cdot I(t - \tau)$$

(2)

$$F_1(\tau) = \frac{1}{2} \int_{-\infty}^{\infty} dt \ (I(t) + I(t - \tau)) E(t) \cdot E^*(t - \tau)$$

(3)

$$F_2(\tau) = \int_{-\infty}^{\infty} dt \ E^2(t) \cdot E^{*2}(t - \tau).$$

(4)

Here $E(t)$ is the electric field of the pulse, $I(t)$ its intensity envelope, $\Omega$ its central frequency and $\tau$ is the optical delay between both branches of the beamsplitting device. The three components $\text{IAC}(\tau)$, $F_1(\tau)$ and the second harmonic field autocorrelation $F_2(\tau)$ are well separated in Fourier space. The IAC trace can be extracted by low-pass filtering the FRIAC trace.

The main experimental difference between intensity and interferometric AC is the mutual angle between the two partial beams. In a traditional interferometric AC geometry utilizing a Michelson-interferometer with a transmissive beam-splitter, the two light pulses travel collinearly, whereas in an intensity AC they have a non-zero angle with respect to each other leading to a spatial phase distribution within the focal plane which does not allow for complete destructive interference.

The split mirror geometry depicted in figure 1(b) represents an intensity autocorrelator as one partial beam emerges from the upper half mirror while the other partial beam comes from the lower half mirror. Both beams have a mutual angle with respect to each other, inducing the closely spaced double spot depicted in figure 2(b) when a $T/2$ optical delay is introduced, with $T$ being the light oscillation period. In a collinear setup one would instead expect complete destructive interference at this delay.

The common tendency of nonlinear processes to scale in efficiency with a high power of the wavelength requires tight focusing with correspondingly small spot sizes in the few $\mu$m or even sub-$\mu$m range [26, 27]. Consequently, the intensity pattern can hardly be resolved and the
The autocorrelation trace is typically obtained by detecting the nonlinear signal integrated over the whole effective volume. For the case of a split-mirror this results not only in a significantly reduced contrast [25] between minima and maxima as depicted in figure 2(d), but also in a loss of additional information. Integral detection over the focal plane lets $\tau F_2^2$ vanish [23] which turns out to be essential in the procedure described by Naganuma [15] to reconstruct the light pulse out of the interferometric AC trace.

In order to exploit all the properties known from Michelson-type interferometric AC, as depicted in figure 2(c), we combine two interdigitated comb-like mirrors forming a reflective beam splitter as shown in figure 1(a). A similar type of beam-splitter has been used for linear Fourier-transform spectroscopy in the x-ray range [28]. Each comb creates a diffraction pattern with clearly separable diffraction orders. Upon longitudinally moving one comb with respect to the other, the superimposed field contributions from both partial waves now form a fully modulated and well-separated zeroth-order spot. To demonstrate that this arrangement acts like a Michelson interferometer we refer the reader to the appendix deriving analytically that in the focus the beams originating from each comb have the same spatial amplitude and phase.

**Figure 2.** Irradiance plots at the focal plane of the two different schemes shown in figure 1 simulated with $\lambda = 160$ nm and $f = 50$ mm, with a beam diameter of $\omega = 3$ mm for (a) the double-comb mirror with a period of $\Theta = 0.57$ mm. In (b) a bisected split mirror is simulated with the same beam size and focusing. The red box in (a) indicates the region of interest that has to be introduced experimentally in the comb mirror setup in order to measure the interferometric trace (c). The split mirror autocorrelation trace (d) is obtained by integration over the same region of interest (red dashed rectangle).
For visualizing the difference in the diffraction patterns generated with either of the schemes 1(a) and 1(b), the corresponding intensity distributions $I(x, y)$ in the focal plane have been simulated using

\[ I(x, y) \propto |U(x, y)|^2 \]  

with $U(x, y)$ being the electric field in the focal plane calculated with equation (A.1). The results are presented in figures 2(a) and (b), respectively. For the case of the bisected split mirror the calculation within the appendix has to be modified accordingly and will not be discussed here; a comprehensive description of this scheme is given in [22, 23]. For the double-comb mirror the zeroth and first diffraction orders are clearly discernible for a delay $\Omega \tau = \pi/2$ between both combs. Using the experimentally realized parameters, the first orders are found $14 \mu m$ out of center. Spatial integration over the displayed region of interest indicated by the red box in figure 2(a) and scanning the delay between both combs will result in the autocorrelation trace displayed in figure 2(c) for a second-order nonlinear process.

For determining the parameters of the optical setup necessary to achieve a certain separation between zeroth and first orders, we can apply simple diffraction theory. For a grating with groove period $d$,

\[ m\lambda = d (\sin \gamma + \sin \delta) \]  

yields the diffraction angle $\delta$ of $m$th order at a given wavelength $\lambda$ and angle of incidence $\gamma$. The smallest manufacturable groove period is still large compared to the considered wavelengths, resulting in $\sin \gamma + \sin \delta \approx \gamma + \delta$. With this approximation and the focal length $f$ of the focusing mirror the displacement $\Delta$ of the $\pm 1^{st}$ orders is

\[ \Delta = \frac{\lambda f}{d}. \]  

Even for moderate groove densities of a few lines/mm any signal emanating from the channel paced by the zeroth order radiation can be selectively analyzed with an imaging detector with moderate—about 10 $\mu m$—resolution.

2. Experiment

In our experimental setup depicted in figure 3 an intense 12 mJ, 40 fs laser pulse centered at 800 nm with a 25 Hz repetition rate is focused into a 9 cm long argon gas cell using a 5 m focal length mirror. A thorough description of the harmonic generation scheme used here is given by Takahashi et al [31].

After leaving the generation region, the high-harmonic pulse is reflected into the experimental chamber using a silicon mirror placed at the Brewster angle, thus suppressing the fundamental beam by a factor of at least $10^{-3}$ while reflecting 60% of the fifth harmonic. Subsequently, a silicon double-comb mirror consisting of two interdigitated combs with 20 mm long and 0.24 mm wide teeth is hit by the VUV pulse with a beam diameter of 3 mm under a 60° angle. The two collinearly propagating partial beams are then further purified with a plane multilayer mirror (Layertec GmbH) which reflects more than 90% at 160 nm and suppresses the fundamental and the other harmonics by a factor of at least $5 \times 10^{-2}$. The radiation is then focused by a 50 mm focal length normal-incidence multilayer mirror with the same properties into a krypton gas medium supplied with a pulsed nozzle. The 5th harmonic pulse energy was
measured with a calibrated XUV photodiode (IRD AXUV100) to be 0.2 $\mu$J on target. We note, that since the zeroth diffraction order does not discriminate wavelengths, an efficient reduction of the fundamental beam is mandatory. Due to an overall IR-suppression of at least $\times -2.5 \times 10^{-6}$, no ionic signal was observed to emerge from the IR pulse only.

Ions created through absorption of two VUV-photons are recorded using an imaging ion-TOF spectrometer, similar to the one described by Schultze et al [32]. Using electrostatic lenses it maps an ion’s origin at a $\times 27$ magnification onto a multichannel-plate (MCP) + Cer:YAG phosphor-screen. While 70% of the screen’s light output is imaged onto a 2D CCD detector, the rest is imaged onto a fast photomultiplier (PMT) read out by a digital oscilloscope. Gating of the MCP discriminates the desired $\text{Kr}^+$ signal from any residual gas contributions with different TOF.

3. Results

Resulting ion images on the camera are presented in figure 4 for different relative displacements of the two combs of the beamsplitter. They represent side-views of the ion-channels created by the propagation of the beam through the focus. The pattern follows exactly the redistribution of intensity between the diffraction orders as expected from the simulation in figure 2. Due to the nonlinearity of the ionization process, orders $> 1$ are not discernible. Zeroth and first orders are clearly separated by approximately 14 $\mu$m enabling the selective evaluation of only the zeroth order by defining an area-of-interest for the 2D image or by introducing a slit in front of the photomultiplier (PMT) that acts as a spatial filter which permits only the zeroth order to be imaged on the PMT and effectively blocks any signal emanating outside of this region of interest. While both detectors can be operated simultaneously, the latter one was found to deliver a better signal-to-noise ratio.
In order to verify that krypton is ionized by a two-photon process we analyzed the intensity-dependence of the $\text{Kr}^+$ ion yield. The fitted slope of $1.89 \pm 0.14$ for the graph in figure 5 is well in accordance with a value of 2 expected for a second-order nonlinearity [33]. The nonlinear autocorrelation is obtained by recording the spatially selected zeroth order contribution on the photomultiplier as a function of the optical delay introduced through the comb beam-splitter.

The shape of the FRIAC trace in figure 6(a) resulting from data points averaged over 25 shots at each delay setting closely resembles a second order interferometric autocorrelation trace obtained with a conventional Michelson interferometer in the visible spectral range (compare figure 2(c)). In particular, the expected contrast of 1 : 8 between the signals at zero and infinite delay, respectively, is almost reached. The limit of the dynamic range is dictated by residual beam pointing instability originating from the driving laser system and wrong phase contributions due to the limited flatness of the comb mirrors. A small intensity loss between

Figure 4. Images of the ion distribution in the focal volume of the fifth harmonic beam taken with the CCD at different positions of the delay stage. As expected from the simulation shown in figure 2(a), the light intensity in the focal volume is completely transferred to the first orders at $T/2$ optical delay. The red box indicates the spatial filter that was introduced to obtain the interferometric trace with the PMT. The images are integrated over 100 shots.

Figure 5. $\text{Kr}^+$ ion yield as a function of fifth harmonic pulse energy on target. For every data point the fifth harmonic pulse energy was measured with a calibrated XUV photodiode (IRD AXUV100). The inset shows a typical measured TOF-spectrum of $\text{Kr}^+$ with the characteristic isotope distribution.
the data acquisition at $-60$ fs and $+60$ fs is compensated by normalizing to an estimated linear drop of 15% over the duration of the experiment.

The favorable noise-level and dynamic range encourages the further extraction of information on the temporal phase $\phi(t)$ from the interferometric AC trace. Assuming a Gaussian pulse electric field for the two overlapping replica pulses

$$E(x, y, t, \tau) = |U(x, y)|\left(f(t) e^{i\Omega t + \phi(t)} + f(t - \tau) e^{i(\Omega t - \tau + \phi(t - \tau))}\right)$$

with the envelope $f(t) = \exp\left(-t^2/(2\sigma^2)\right)$ and restricting the analysis to a second-order temporal phase contribution $\phi(t) = A\tau^2$, i.e., a linear spectral chirp, the formalism introduced in reference [23, 34] can be developed into an analytical expression for the upper and lower envelope of the Michelson-type AC trace,

$$\text{FRIAC}_{\text{env}}^{\text{Gauss}}(\tau) \propto 1 + 2 \text{IAC}_{\text{Gauss}}^{\text{Gauss}}(\tau) \pm S_1 \cdot F_1^{\text{Gauss}}(\tau) + S_2 \cdot F_2^{\text{Gauss}}(\tau)$$

with

$$\text{IAC}_{\text{Gauss}}^{\text{Gauss}}(\tau) = \exp\left(-\frac{\tau^2}{2\sigma^2}\right)$$

$$F_1^{\text{Gauss}}(\tau) = \exp\left(-\left(3 + 4A^2\sigma^4\right)\frac{\tau^2}{8\sigma^2}\right) \cdot \cos\left(\frac{A\tau^2}{2}\right)$$

$$F_2^{\text{Gauss}}(\tau) = \exp\left(-\left(1 + 4A^2\sigma^4\right)\frac{\tau^2}{2\sigma^2}\right)$$

**Figure 6.** Measured krypton ion yield from the zeroth diffraction order as a function of optical delay. The full trace (blue dots) is shown in (a) and a section of 5 fs is plotted in (b). The observed contrast of 1:7 is close to the 1:8 ratio expected for a Michelson-type AC. The black curve represents the corresponding intensity autocorrelation extracted via low-pass filtering. The full red and dashed gray curves in (a) indicate envelopes of interferometric traces of a pulse of 18.4 fs duration, but with and without a chirp of $A = -4.4 \times 10^{-3} \text{ rad fs}^{-2}$, respectively. The full red curve in (b) depicts the calculated trace using the obtained fit values for chirp and pulse duration but including the $\Omega$ and $2\Omega$ oscillations multiplied to the corresponding terms of formula (9).
with $A$ being the chirp parameter. The spatial integration factors $S_1$ and $S_3$ would equal 1 for a perfect interferometer [23] which corresponds to 100% visibility of the respective terms in the experiment.

A least-squares fitting procedure of the experimental data to equation (9) is used to extract $\sigma$ and $A$. To account for the residual imperfections also $S_1$ and $S_3$ are left as fit parameters. The fit (upper and lower red curve in figure 6(a)) yields $|A| = (4.4 \pm 0.6) \times 10^{-3}$ rad fs$^{-2}$, being slightly below a value of $A = -5.5 \times 10^{-3}$ rad fs$^{-2}$ calculated for the 5th harmonic under our experimental conditions [16]. Owing to the symmetry of a second-order AC trace, the sign of $A$ is not accessible.

The same fit also yields the pulse duration of $(18.4 \pm 0.7)$ fs FWHM. In figure 6(a) we show the interferometric AC together with its intensity counterpart, extracted from the former as the IAC-term of equation (2) by low-pass filtering. The sensitivity of the FRIAC trace on the temporal phase is underlined by comparing the fitted envelope with one based on the the same pulse duration, but vanishing chirp (dashed gray curve in figure 6(a)).

A more pronounced temporal chirp was introduced by placing a 1 mm thick CaF$_2$ window into the beam path. The duration of more than 80 fs extracted from the FRIAC trace is compatible with the known linear dispersion of the material.

4. Summary

Using two interdigitated moveable gratings we have realized a beam-splitter for an interferometric autocorrelation in the VUV spectral range. This geometry overcomes the volume averaging effects connected with simple split mirror geometries while keeping the focal size small, thus facilitating the high light intensities necessary to drive nonlinear processes. In the present setup a spatially resolving detector realizes the selection of the zeroth diffraction order. However, in an alternative incarnation an aperture in the focal plane, followed by a second refocusing mirror would allow a volume-integrated detection, and would permit even tighter focusing, which otherwise contradicts an efficient order separation. With a laser-generated wavelength of 160 nm we could demonstrate an interferometric autocorrelation in krypton at a contrast and dynamic range that allows the extraction of phase information. We note that, while temporal phases in the studied case are acknowledged as a property of the VUV pulse, also the ionization process itself may introduce a time-lag, i.e., phase shift [35, 36]. The all-reflective design of the correlator facilitates a further extension to even shorter wavelengths. The reduced diffraction could be counter-balanced by weaker focusing, so as to preserve a few $\mu$m separation between the zeroth and first diffraction orders. More serious is the general tendency of nonlinear processes as well as the high harmonic generation efficiency to decrease with wavelength. Seeded free-electron lasers [37], however, carry the potential to deliver sufficient intensity as well as temporal coherence for driving nonlinear processes at significantly shorter wavelengths. The analytical power of interferometric autocorrelation will therefore aid in improving the expressiveness of nonlinear optics experiments in the soft- and hard x-ray range.
Appendix A

To demonstrate that the two beams originating from each comb have the same spatial amplitude and a flat spatial phase in the focus we analyze the Fresnel diffraction-integral, taken from [29]

\[ U(x, y) = \frac{e^{jkz}e^{j\frac{k}{2z}(\xi^2 + \eta^2)}}{j\lambda z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(\xi, \eta) \exp \left[ \frac{j}{2z} \left( \xi^2 + \eta^2 \right) \right] \exp \left[ -j \frac{k}{z} (x\xi + y\eta) \right] d\xi d\eta \]  \hspace{1cm} (A.1)

for one tooth of the comb and coherently sum over all teeth afterwards. Here \( U(x, y) \) will be the spatial field distribution in the focus and \( U(\xi, \eta) \) denotes the field distribution on the comb mirror. The distance from the comb mirror to the focusing mirror is ignored in terms of diffraction to keep the math readable [30]. The distance \( z \) from the focusing mirror to the focal plane equals \( f \). The wavelength of the light is denoted as \( \lambda \) and \( k = 2\pi/\lambda \).
For the $n$th tooth of the first comb the aperture distribution $U_{1,n} (\xi, \eta)$ consists of three parts

$$
U_{1,n} (\xi, \eta) = \exp \left[-\frac{\xi^2 + \eta^2}{\omega^2}\right] \cdot \exp \left[-j\frac{k}{2f} \left(\xi^2 + \eta^2\right)\right] \cdot [-H (\xi - n\Theta - \Theta/4) + H (\xi - n\Theta + \Theta/4)].
$$

The first factor arises from the Gaussian field distribution with a waist size $\omega$ and an amplitude that was set to 1. The second factor containing a quadratic phase originates from focusing with focal length $f$. The third part finally describes the $n$th tooth of the first comb with the Heaviside function

$$
H (x) = \begin{cases} 
0 & x < 0 \\
1 & x \geq 0
\end{cases}
$$

with $\Theta$ being the period of the comb or twice the width of a tooth.

For one tooth of the second comb the aperture distribution is similar

$$
U_{2,n} (\xi, \eta) = \exp \left[-\frac{\xi^2 + \eta^2}{\omega^2}\right] \cdot \exp \left[-j\frac{k}{2f} \left(\xi^2 + \eta^2\right)\right] \cdot [-H (\xi - n\Theta + \Theta/4) + H (\xi - n\Theta - 3\Theta/4)].
$$

The aperture distributions $U_{1,n} (\xi, \eta)$ and $U_{2,n} (\xi, \eta)$ are illustrated in figure A.1(a). The integrals over $\xi$ and $\eta$ can be evaluated separately, yielding $U (x, y) = U (x) U (y)$. The constants and phase exponentials in front of the integral (A.1) are the same for both combs and do not affect the following discussion thus they are omitted. Integration over $\eta$ yields a Gaussian distribution with $\beta = \frac{k\omega}{2f}$

$$
U (y) = \omega \sqrt{\pi} \exp \left[-\beta^2 y^2\right].
$$

With $\alpha = \Theta/\omega$ and the error function Erf [$x$] the integral over $\xi$ yields

$$
U_{1,n} (x) = \frac{1}{2} \omega \sqrt{\pi} \exp \left[-\beta^2 x^2\right] \cdot \left(\text{Erf} [\alpha/4 - n\alpha - j\beta x] + \text{Erf} [\alpha/4 + n\alpha + j\beta x]\right)
$$

and

$$
U_{2,n} (x) = \frac{1}{2} \omega \sqrt{\pi} \exp \left[-\beta^2 x^2\right] \cdot \left(\text{Erf} [3\alpha/4 - n\alpha - j\beta x] - \text{Erf} [\alpha/4 - n\alpha - j\beta x]\right).
$$

The field distribution of one comb in the focus is described by summation over all teeth

$$
U_i (x, y) = U (y) \cdot \sum_n U_{i,n} (x) \quad i = (1, 2)
$$

Remembering that in a Michelson interferometer two beams are overlapped with equal amplitude and phase we analyze the phase contribution of the Erf terms in $U_{1,n} (x)$ and $U_{2,n} (x)$ by adding the contributions of the $lth$ and $(-ln)th$ teeth for $U_{i,n} (x)$. 
\[ U_{1,1} (x) + U_{1,-1} (x) = \frac{1}{2} \omega \sqrt{\pi} \exp \left[ -\beta^2 x^2 \right] \]

\[
\begin{align*}
&= \text{Erf} \left[ \alpha/4 - n\alpha - j\beta x \right] + \text{Erf} \left[ \alpha/4 + n\alpha + j\beta x \right] + \\
&= \text{Erf} \left[ \alpha/4 + n\alpha - j\beta x \right] + \text{Erf} \left[ \alpha/4 - n\alpha + j\beta x \right].
\end{align*}
\]

(A.9)

With \( \text{Erf} \left[ z \right] = \text{Erf} \left[ \zeta \right] \), it is recognized that \( U_{1,\text{lin}} (x) + U_{1,-\text{lin}} (x) \) is a real quantity and therefore the (complex) phase \( \phi_1 (x, y) \) of

\[ U_1 (x, y) = |U_1 (x, y)| e^{i\phi_1 (x, y)} \]  

(A.10)
is either 0 or \( \pi \) depending on the sign of \( U_1 (x, y) \). The same holds for \( U_{2,\text{lin}+1} (x) + U_{2,-\text{lin}} (x) \).

\( U_1 (x) \) and \( U_2 (x) \) are illustrated in figure A.1(b). Around \( X = 0 \) both fields have almost the same intensity and the same phase (\( \phi_1 = \phi_2 = 0 \)). For the first diffraction orders the amplitude is almost equal as well but the phase is \( \phi_1 = 0 \) and \( \phi_2 = \pi \) leading to a positive amplitude for \( U_1 (x) \) and a negative amplitude for \( U_2 (x) \). Between the zeroth and the first diffraction orders and in regions beyond the first diffraction orders the intensity will be small compared to the contributions of the zeroth and first orders and will not conduce to the nonlinear process. To obtain a Michelson-interferometer we have to spatially select only the zeroth order where amplitude and phase of both beams are equal.

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