Weyl semimetal with a boundary at $z = 0$ - a photoemission study

D. Schmeltzer

Physics Department, City College of the City University of New York, New York 10031, USA

Abstract

We consider a Weyl semimetal Hamiltonian with two nodes and derive the scattering Hamiltonian in the presence of a boundary at $z = 0$. We compute the photoemission spectrum and demonstrate the presence of the Fermi arcs which connect the two nodes. In the presence of an electric field parallel to the scattering surface we observe the one dimensional chiral anomaly.
Weyl fermions represent a pair of particles with opposite chirality described by the massless solution of the Dirac equation \[1\]. Recently it has been proposed that in material with two nondegenerate bands crossing at the Fermi level in three dimensional (3D) momentum space, the low-energy excitations can be described by the Weyl equations, allowing a condensed -matter realization of Weyl fermions quasiparticles \[2, 3\]. The band crossing points are called Weyl points, and material possessing such Weyl points are known as Weyl semimetals (WSMs). The bulk of the WSMs is dominated by Weyl points with linear low-energy excitations. The Weyl points come in pairs with opposite chirality \[4\]. The surface state of the WSMs are characterized by ”Fermi arcs” that link the projection of the bulk Weyl points with opposite chirality in the Brillouine zone. In the presence of a parallel electric and magnetic field the WSMs have a large negative magnetoresistance , due to the Adler - Bell -Jackiw chiral anomaly \[7\]. The WSMs exist in materials where time-reversal symmetry or inversion are broken \[2\]. Recently the noncentrosymmetric and nonmagnetic transition-metal monoarsenide/posphides: TaAs ,TaP, NbAs and NbP have been predicted to be WSMs with 12 pairs of Weyl points \[5\].

The hallmarks of the WSMs is the presence of the Fermi arcs which have been observed by photoemission \[5\] and Scanning Tunneling microscopy \[6\].

It is an important task for theory to compute the photoemission spectrum for the WSMs and to demonstrate the presence of the Fermi arcs. In order to achieve this goal we need to take into account the boundary effect at \(z = 0\) and the photon fermion coupling \(\vec{\sigma} \cdot \vec{A}\). This coupling is different from the coupling \(\vec{A} \cdot \vec{p}\) in non Dirac materials. (For \(\vec{\sigma} \cdot \vec{A}\) there is no matrix elements between the Weyl fermions and the fermions in the vacuum.)

For the Weyl fermions we consider a Hamiltonian which respects time reversal symmetry and has a broken inversion symmetry. We consider a simplified model with a single pair of Weyl nodes :\n
\[
h(\vec{k}) = \tau_3(\sigma_2k_2 + \sigma_3k_3) + \tau_2\sigma_2g(k_2^2 - M^2) = \gamma_0[\gamma_3k_3 + \gamma_2k_2 + i\sigma_2\tau_3g(k_1^2 - M^2)] \quad (1)
\]

The matrices \(\vec{\sigma}\) are used for the spin of the electron and the matrices \(\vec{\tau}\) describe the two orbitals. We introduce the anti commuting \(\gamma\) matrices \(\gamma_0, \gamma_i, i = 0, 1, 2, 3\) \(\vec{\gamma} = \vec{\sigma} \otimes i\vec{\tau}\). The helicity operator \(\gamma_5\) is given by \(\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3\). For simplicity we choose a quadratic function in the momentum \(g(k_1^2 - M^2)\) which introduces two Weyl nodes \(\vec{k} = [k_1 = \pm M, k_2 = 0, k_3 = 0]\). The crystal is restricted to the region \(L \leq z \leq 0\) with the boundary at \(z = 0\). The wave
function $\Psi$ of the Hamiltonian in Eq.(1) must be invariant with respect the rotations around the $z$ axes. The rotation operator around the $z$ axes commute with the matrix $\gamma_3 = \sigma_3 \otimes i\tau_2$. We will choose for the wave function the conditions $\gamma_3 \Psi = \pm \Psi$.

$$\gamma_0 \left[ (-i\partial_z + i\sigma_2 g(k_1^2 - M^2)\tau_3) \right] \Psi = \left[ E - \sigma_2 k_2 \tau_3 \right] \Psi; \quad \Psi = \left[ \Psi_1, \Psi_2 \right]^T$$ \hspace{1cm} (2)

The right hand side of gives us two equations, $\left[ E - \sigma_2 k_2 \right] \Psi_1 = 0$, and $\left[ E + \sigma_2 k_2 \right] \Psi_2 = 0$. We find zero energy solutions which are localized on the boundary $z = 0$ by choosing $\sigma_2 \Psi_{1,s=\frac{1}{2}} = \Psi_{1,s=\frac{1}{2}}$, $\sigma_2 \Psi_{2,s=\frac{1}{2}} = -\Psi_{2,s=\frac{1}{2}}$ with the eigenvalues $E_{1,s=\frac{1}{2}} = k_2$ and $E_{1,s=-\frac{1}{2}} = -k_2$. For the second spinor we have and $\sigma_2 \Psi_{2,s=\frac{1}{2}} = \Psi_{2,s=\frac{1}{2}}$, $\sigma_2 \Psi_{2,s=-\frac{1}{2}} = -\Psi_{2,s=-\frac{1}{2}}$ with the eigenvalues $E_{2,s=\frac{1}{2}} = -k_2$ and $E_{2,s=-\frac{1}{2}} = k_2$. We define new spinors $\hat{C}_1(k_1,k_2,z)$ and $\hat{C}_2(k_1,k_2,z)$ as a linear combination of the original spinor $\Psi_{1,s=\frac{1}{2}}(\sigma;k_1,k_2,z), \Psi_{2,s=\pm\frac{1}{2}}(\sigma;k_1,k_2,z)$. ($\hat{k}_|| = [k_1,k_2]$ is the momentum parallel to the surface.)

$$\frac{1}{\sqrt{2}} \left( \Psi_{1,s=-\frac{1}{2}}(\sigma;k_1,k_2,z) + \Psi_{2,s=-\frac{1}{2}}(\sigma;k_1,k_2,z) \right) = \eta_{s=\frac{1}{2}}(\sigma) \hat{C}_1(k_1,k_2,z)$$

$$\frac{1}{\sqrt{2}} \left( \Psi_{1,s=-\frac{1}{2}}(\sigma;k_1,k_2,z) + \Psi_{2,s=\frac{1}{2}}(\sigma;k_1,k_2,z) \right) = \eta_{s=-\frac{1}{2}}(\sigma) \hat{C}_2(k_1,k_2,z)$$ \hspace{1cm} (3)

Where $\eta_{s=\frac{1}{2}} = \frac{1}{\sqrt{2}} [1, -i]^T$, $\eta_{s=-\frac{1}{2}} = \frac{1}{\sqrt{2}} [1, i]^T$ are two component spinors which obey $\sigma_2 \eta_{s=\pm\frac{1}{2}} = \pm \eta_{s=\pm\frac{1}{2}}$. (Similarly we will introduce $D_1$, $D_2$ for antiparticles.)

The spinors $\hat{C}_1(k_1,k_2,z)$, $\hat{C}_2(k_1,k_2,z)$ obey the equation:

$$\left[ -\partial_z + g(k_1^2 - M^2) \right] \hat{C}_1(k_1,k_2,z) = 0$$

$$\left[ -\partial_z - g(k_1^2 - M^2) \right] \hat{C}_2(k_1,k_2,z) = 0$$ \hspace{1cm} (4)

The solution of equation (4) is given in terms of the normalized spinors $C_1(k_1,k_2)$ and $C_2(k_1,k_2)$:

$$\hat{C}_1(k_1,k_2,z) = \theta[-z] \theta[k_1^2 - M^2] \sqrt{2gM^2((k_1/M)^2 - 1)} e^{gM^2((k_1/M)^2 - 1) z} C_1(k_1,k_2)$$

$$\hat{C}_2(k_1,k_2,z) = \theta[-z] \theta[-k_1^2 + M^2] \sqrt{2gM^2(-(k_1/M)^2 + 1)} e^{gM^2(-(k_1/M)^2 + 1) z} C_1(k_1,k_2)$$ \hspace{1cm} (5)

Where $\theta[-z]$ is the step function which confines the Weyl electrons to $z \leq 0$, $\theta[k_1^2 - M^2]$ and $\theta[-k_1^2 + M^2]$ describes the solutions in the momentum space. We will compute compute
the photoemission for a positive chemical potential $\mu > 0$ (we need to consider only the particle excitations).

Next we derive the Hamiltonian for the photoemission which in the final form is given in Eq.(13). We consider first the projected Weyl Hamiltonian $H^{(W)}$ on the surface $z = 0$:

$$H^{(W)} = \int \frac{d^2k}{(2\pi)^2} \left[ C_1^i(k_1, k_2) \left(k_2\theta[k_2]\theta[k_2^2 - M^2]\right) C_1(k_1, k_2) - C_2^i(k_1, k_2) \left(k_2\theta[-k_2]\theta[M^2 - k_2^2]\right) C_2(k_1, k_2) \right]$$

(6)

$\theta[k_2]$ represents the step function which is one for $k_2 > 0$ and zero otherwise. The coupling between the surface electrons and the free electron $f_\sigma(k_1, k_2, z > 0)$ is given by the tunneling amplitude for the Weyl electrons to propagate as plane waves. The vacuum electrons and the surface electrons. The situation is similar to the topological insulator where the bulk gap gives confined electrons to the surface. For the Weyl fermions the term $\bar{\sigma} \cdot \vec{A}$ through the Dirac form is essential for the photoemission process. The Weyl electrons couples light through the Dirac form $\bar{\sigma} \cdot \vec{A}$ and the electrons in the vacuum region $z > 0$ couples through the term $\vec{A} \cdot \vec{p}$ ($\vec{p}$ is the momentum). There is no direct matrix element between the Weyl electrons and the vacuum electrons. The situation is similar to the topological insulator where the bulk gap gives confined electrons to the surface. For the Weyl fermions the localization on the boundary is induced by the term $\bar{\sigma} \tau_2 g(k_1^2 - M^2)$ for $k_1^2 \neq M^2$ in the Hamiltonian in Eq.(1). The boundary Hamiltonian $Weyl - Vacuum$ is given by $H^{(W,V)}$:

$$H^{(W,V)} = \sum_{\sigma = \uparrow, \downarrow} \int \frac{d^2k}{(2\pi)^2} \int_0^\Lambda \frac{dk_z}{2\pi} \left[ t(k_z) \left( \tau_+(k_1)\theta[k_2]C_1^i(k_1, k_2)\eta_{s=\downarrow}(\sigma) f_\sigma(k_1, k_2, k_z) \right. \right.

+ \left. \tau_-(k_1)\theta[-k_2]C_2^i(k_1, k_2)\eta_{s=\downarrow}(\sigma) f_\sigma(k_1, k_2, k_z) \right] + h.c. \right]$$

$$\tau_+(k_1) = \theta[k_1^2 - M^2], \tau_-(k_1) = \theta[M^2 - k_1^2];$$

(8)

$$t(k_z) = \int_{-d}^0 dz \sqrt{2gM^2(k_1/M)^2 - 1}e^{gM^2(k_1/M)^2 - 1} \frac{1}{\sqrt[4]{L}} e^{ik_z z}$$

$$t(k_z)t^*(k_z) \approx \left( \frac{d}{L} \right) \frac{2g[(k_1/M)^2 - 1]^2}{(k_zd)^2}; gM^2 = \hat{g}d^{-1}$$

(7)

This term is essential for the photoemission process. The Weyl electrons couples with the free electrons in the vacuum region and the electrons in the surface region coupling through the term $\bar{\sigma} \cdot \vec{A}$ (see [10]).
In the photoemission process the momentum parallel to the surface is conserved \( \vec{k}_\parallel = \begin{bmatrix} k_1, k_2 \end{bmatrix} \). The energy of the emitted electrons \( E^{(V)} = \epsilon^{(W)} + W \). (vacuum electrons) is related to the Weyl electron energy \( \epsilon^{(W)} \), and the work function \( W \). When the crystal is excited by a laser beam of frequency \( \Omega \) the energy of the Weyl electrons becomes \( \epsilon^{(W)} + \hbar \Omega \). The vacuum energy obey the relation \( E^{(V)} = \epsilon^{(W)} + W + \hbar \Omega \). The kinetic energy of the emitted electrons determines the momentum \( k_z \) given by [9]:

\[
\begin{align*}
  k_z &= \sqrt{\frac{2m}{\hbar^2}} \left[ (\epsilon^{(W)} + \hbar \Omega) \cos^2(\theta) - W \right], \\
  \epsilon^{(W)} &= k_2 \theta[k_2] + (-k_2) \theta[-k_2]
\end{align*}
\]

(9)

For \( \cos^2(\theta) \approx 1 \) we find \( k_z = \sqrt{\frac{2m}{\hbar^2}} (\epsilon^{(W)} + \hbar \Omega - W) \).

The Hamilonian for the free electrons is a function of the conserved parallel momentum \( H^{(V)} \) and is given by,

\[
H^{(V)} = \int \frac{d^2k}{(2\pi)^2} \sum_\sigma \left[ f^\dagger_\sigma(\vec{k}) \left( k_2 \theta[k_2] + W \right) \theta[k_2^2 - M^2] f_\sigma(\vec{k}) + f^\dagger_\sigma(\vec{k}) \left( (-k_2) \theta[-k_2] + W \right) \theta[-k_2^2 + M^2] f_\sigma(\vec{k}) \right]
\]

(10)

Next we consider the coupling of the photon to the surface electrons. The boundary at \( z = 0 \) determines the structure of the spinor \( \eta_\sigma(\sigma) \). Only the \( y \) component of the photon field couples to the surface. The Weyl electrons confined to the region \( L \leq z \leq 0 \) couple on the boundary to the photon of frequency \( \Omega \) and momentum \( k_z = \frac{\Omega}{c \cos(\theta)} \) through the term \( \vec{A} \cdot \vec{A} \). The photon-matter Hamiltonian is given by \( H^{(ext)} \):

\[
H^{(ext)} = \sqrt{\frac{\hbar}{2 \epsilon \Omega}} \sum_{\alpha = 1, 2} \int \frac{d^2k}{(2\pi)^2} \int_{-L}^{0} dz \left[ 2g(k_1^2 - M^2)e^{2gM^2(k_1^2 - M^2)z} \tau_+(k_1)C_1^\dagger(k_1, k_2)C_1(k_1, k_2)A_\alpha e^{-i\Omega t} \\
e^{ik_z z} + A^\dagger_\alpha e^{i\Omega t} e^{-ik_z z} \right] + 2g(-k_2^2 + M^2)e^{2g(-k_2^2 + M^2)z} \tau_-(k_1)C_2^\dagger(k_1, k_2)C_2(k_1, k_2)A_\alpha e^{-i\Omega t} e^{ik_z z} \\
+ A^\dagger_\alpha e^{i\Omega t} e^{-ik_z z} \right] |k_z = \frac{\Omega}{c \cos(\theta)} \rangle \langle \Omega | e^\Phi(\theta, \varphi)
\]

(11)

The representation of the Weyl fermions given in Eq.(5) determines the functions \( F_+(k_1, \pm \Omega) \), and \( F_-(k_1, \pm \Omega) \):

\[
F_+(k_1, \pm \Omega) = 2g(k_1^2 - M^2) \left( \frac{1 - e^{2g(k_1^2 - M^2) L e^{\pm i(\frac{\Omega}{c \cos(\theta)}) L}}}{2g(k_1^2 - M^2) \pm i(\frac{\Omega}{c \cos(\theta)}) L} \right)
\]
\[
F_-(k_1, \pm \Omega) = 2g(-k_1^2 + M^2) \left( 1 - e^{2g(-k_1^2 + M^2)\epsilon} e^{\pm i \left( \frac{\Omega}{c \cos(\theta)} \right)} \right)
\]

(12)

The \( y \) component of photon field is given by \( e^{y}_{a=1,2}(\theta, \phi) \) see [10] with the two linear polarization \( \alpha = 1, 2 \) are orthogonal to the incident photon propagation direction

\[
\bar{p} = \left[ \sin(\theta) \cos(\phi), \sin(\theta) \sin(\phi), \cos(\theta) \right].
\]

\( \epsilon \) is the dielectric constant and \( c \) is the light velocity. The photon field is in a coherent state \(|\Omega\rangle\) and obeys \( A_\alpha |\Omega\rangle = A_\alpha |\Omega\rangle \) with \( A_\alpha \) being the eigenvalue. Combining the results in Eqs. (6, 8, 10, 11) we obtain the photoemission Hamiltonian \( H \):

\[
H = H^{(W)} + H^{(V)} + H^{(W-V)} + H^{(\text{ext})}
\]

(13)

The spinor structure guaranty that the \( y \) polarization of the photon field couple to the Weyl electrons polarization. The \( y \) polarization of the emitted electrons is given by \( S^y(\vec{k}, k_z) \).

\( S^y(\vec{k}, k_z) \) is expressed in terms of the single particles Green’s function \( G_{\alpha,\beta}(\vec{k}, k_z; \delta t) \) which are computed with respect the exact ground state \(|g\rangle\).

\[
S^y(\vec{k}, k_z) = -i \sum_{\alpha = \uparrow, \downarrow} \sum_{\beta = \uparrow, \downarrow} \left[ \sigma^y \right]_{\alpha,\beta} \int \frac{d\omega}{2\pi} G_{\alpha,\beta}(\vec{k}, k_z = 0; \omega) e^{i\delta t} \Omega_{\alpha,\beta}(\vec{k}, k_z; \omega)
\]

\[
S^y(\vec{k}, k_z; \delta t) = -i \langle g | T(f^{\uparrow}_{\vec{k}, k_z}(t) f^{\dagger}_{\alpha}(\vec{k}, k_z, t + \delta t) | g) = -i \langle O \otimes \Omega | T(f^{\uparrow}_{\vec{k}, k_z, t} f^{\dagger}_{\alpha}(\vec{k}, k_z, t + \delta t) e^{-\frac{i}{\hbar} \int dt' H^{(\text{ext})}(t')} | O \otimes \Omega \rangle
\]

(14)

We compute the Green’s function with respect ground state of the ground state \(| O \otimes \Omega \rangle = |O\rangle |\Omega\rangle\) of the Hamiltonian \( H^{(W)} + H^{(V)} + H^{(W-V)} \).

\[
g_{\downarrow, i}(\vec{k}, k_z; t) = -i \langle O | T(f_{\downarrow}(\vec{k}, k_z; t) C_{i}^{\dagger}(\vec{k}, 0)) | O \rangle, i = 1, 2
\]

\[
g_{\uparrow, i}(\vec{k}, k_z; t) = -i \langle O | T(f_{\uparrow}(\vec{k}, k_z; t) C_{i}^{\dagger}(\vec{k}, 0)) | O \rangle, i = 1, 2
\]

\[
g^{(W_1, W_2)}(\vec{k}, k_z; t) = -i \langle O | T(C_{1}(\vec{k}, t)(\vec{k}, k_z; t) C^{\dagger}_{2}(\vec{k}, 0)) | O \rangle
\]

(15)

The Fourier transform allows to compute the Green’s function by summing up the one loop diagrams: \( g_{\downarrow, i}(\vec{k}, k_z; \omega) = \frac{i}{\sqrt{2}} g^{(V, W_i)}(\vec{k}, k_z; \omega); g_{\uparrow, i}(\vec{k}, k_z; \omega) = \frac{i}{\sqrt{2}} g^{(V, W_i)}(\vec{k}, k_z; \omega); i = 1, 2 \) and \( g^{(W_1, W_2)}(\vec{k}, k_z; \omega) \)
Using the Green’s function for the unperturbed Weyl Hamiltonian in Eq.(6)
\[ g_{11}(\vec{k},\omega)^{-1} = \left[ \omega - (k_2\theta[k_2] - \mu) + i\delta sgn[\omega] \right]^{-1}, \]
\[ g_{22}(\vec{k},\omega)^{-1} = \left[ \omega - (-k_2\theta[-k_2] - \mu) + i\delta sgn[\omega] \right]^{-1} \]
and the unperturbed Green’s function for the vacuum electrons in Eq.(10)
\[ G_{||}^{(0)}(\vec{k}, k'_z; \omega) = \frac{G_{1,1}^{(0)}(\vec{k}, k'_z; \omega) + G_{1,1}^{(0)}(\vec{k}, k'_z; \omega)}{2} \]
we obtain the Green’s function defined in Eq.(15)
\[ g^{(V,W_1)}(\vec{k}, k'_z; \omega) = \frac{i(k_z)\tau_+(k_1)\theta[k_2]G_{||}^{(0)}(\vec{k}, k'_z; \omega)}{[g_{11}(\vec{k}, \omega)]^{-1} - \tau_+^2(k_1)\theta[k_2] \int dk_z \frac{i(k_z)}{2\pi} G_{||}^{(0)}(\vec{k}, k'_z; \omega)} \]
\[ g^{(V,W_2)}(\vec{k}, k'_z; \omega) = \frac{i(k_z)\tau_-(k_1)\theta[-k_2]G_{||}^{(0)}(\vec{k}, k'_z; \omega)}{[g_{22}(\vec{k}, \omega)]^{-1} - \tau_-^2(k_1)\theta[-k_2] \int dk_z \frac{i(k_z)}{2\pi} G_{||}^{(0)}(\vec{k}, k'_z; \omega)} \]
\[ g^{(W_1,W_1)}(\vec{k}, k'_z; \omega) = \frac{1}{[g_{11}(\vec{k}, \omega)]^{-1} - \tau_+^2(k_1)\theta[k_2] \int dk_z \frac{i(k_z)}{2\pi} G_{||}^{(0)}(\vec{k}, k'_z; \omega)} \]
\[ g^{(W_2,W_2)}(\vec{k}, k'_z; \omega) = \frac{1}{[g_{22}(\vec{k}, \omega)]^{-1} - \tau_-^2(k_1)\theta[-k_2] \int dk_z \frac{i(k_z)}{2\pi} G_{||}^{(0)}(\vec{k}, k'_z; \omega)} \]
\[ \hat{t}(k_z) = \hbar\delta^2(0)\hat{t}(k_z) \]

(16)

We find from Eq.(14) the y polarization \( S^y(\vec{k}, k_z) \) for large photon intensities. ( \( |\Omega\rangle, A_\alpha|\Omega\rangle = \sqrt{N_\alpha}|\Omega\rangle \). \( A^\dagger_\alpha A_\alpha = A_\alpha A^\dagger_\alpha + 1 \approx N_\alpha \).

\[ S^y(\vec{k}, k_z; \delta t) = \frac{N_\alpha}{4\epsilon\hbar\Omega} \sum_{\alpha=1,2} \int \frac{d\omega}{2\pi} e^{i\omega \delta t} \left[ F_+(k_1, \Omega)F_+(k_1, -\Omega) \left( g^{(V,W_1)}(\vec{k}, k_z; \omega + \Omega)g^{(V,W_1)}(\vec{k}, k_z; \omega + \Omega)^* + g^{(V,W_1)}(\vec{k}, k_z; \omega - \Omega)g^{(V,W_1)}(\vec{k}, k_z; \omega - \Omega)^* \right) \right] \]

\[ + F_-(k_1, \Omega)F_-(k_1, -\Omega) \left( g^{(V,W_2)}(\vec{k}, k_z; \omega + \Omega)g^{(V,W_2)}(\vec{k}, k_z; \omega + \Omega)^* + g^{(V,W_2)}(\vec{k}, k_z; \omega - \Omega)g^{(V,W_2)}(\vec{k}, k_z; \omega - \Omega)^* \right) \]

\[ \approx \frac{N_\alpha}{4\epsilon\hbar\Omega} \sum_{\alpha=1,2} \left( -i \right) \int \frac{d\omega}{2\pi} e^{i\omega \delta t} \left[ F_+(k_1, \Omega)^2 \left( g^{(V,W_1)}(\vec{k}, k_z; \omega - \Omega)g^{(V,W_1)}(\vec{k}, k_z; \omega - \Omega)^* + g^{(W_1,W_1)}(\vec{k}, k_z; \omega) \right) \right] \]

\[ + |F_-(k_1, \Omega)|^2 \left( g^{(V,W_2)}(\vec{k}, k_z; \omega - \Omega)g^{(V,W_2)}(\vec{k}, k_z; \omega - \Omega)^* + g^{(W_2,W_2)}(\vec{k}, k_z; \omega) \right) \]

\[ \left( e^{\alpha}_\omega(\theta, \varphi) \right)^2 \]

(17)
FIG. 1: The y polarization of the emitted electrons given by $S_y(\vec{k}, k_z)$ as a function of the $k_x$ and $k_y$ momentum. The plot is for the chemical potential $\mu = 0.5$, work function $W$, laser energy $\hbar \Omega$ and no electric field.

Figure 1 shows the y polarization of the emitted electrons $S_y(\vec{k}, k_z; \delta t)$ as a function of the momentum $k_1 = k_x$ and $k_2 = k_y$ for the chemical potential $\mu = 0.5 \, \text{eV}$ and $M = \pm 0.3 \, \text{eV}$ for the location of the nodes. The plot in figure (3) is for the same parameters as in figure (1). The plot in the middle is in the absence of the electric field. We plot is the function $S_y - \mu$. We integrate with respect $k_2$ and observe that the photoemission spectrum shows of the Weyl fermions dispersion as a function of $k_1$ as given in given Eq.(2). The Fermi arc connects the two Weyl nodes at $k_1 = \pm 0.3$. The zero energy path from the node at $k_1 = 0.3$ to the node at $k_1 = -0.3$ goes through $0 = k_2 \rightarrow k_2 = -0.5 \rightarrow k_2 = 0$. Figure 1 shows the contour as a function of the two dimensional momentum for $\mu = 0.5$. This figure can be understood from the Hamiltonian in Eq.(2) for $\mu = 0$, the two nodes are connected through a path $k_2 = 0$ therefore for any finite chemical potential $\mu > 0$ the path which start at $k_1 = M$ and ends at $k_1 = -M$ will be $0 = k_2 \rightarrow k_2 = -\mu \rightarrow k_2 = 0$.

The suggested three dimensional chiral anomaly and the detection in photoemission [11], is realized in our case as a one dimensional chiral anomaly. This is a result of applying an electric field on the boundary at $z = 0$. The occupation number $n_R(k_2), n_L(k_2)$ and the number of electrons $N_R, N_L$ for the right and left chirality is given by, $n_R(k_2) = \int_0^\infty \frac{d\epsilon}{\hbar v(k_2)} n_R(k_2), n_L(k_2) = \int_0^\infty \frac{d\epsilon}{\hbar v(k_2)} n_L(k_2)$, where $\epsilon = \hbar v(k_2)(k_2)$. The one
The Fermi arc - a two dimensional contour $S_y(\vec{k})$ as a function of the two dimensional momentum.

dimensional chiral anomaly is: $\frac{1}{L} \frac{d(N_R-N_L)}{dt} = \left( \frac{1}{e^{-\beta\epsilon F}+1} \right) \left( \frac{-e}{\hbar} E_2 \right)_{T \to 0} = \frac{-e}{\hbar} E_2$.

We assume inter-valley scattering controlled by the scattering time $\tau_v \frac{dN_R}{dt}_{\text{collision}} = -\frac{1}{\tau_v} \left( N_R - N_R^0 \right)$ where $N_R = N_R^0(\epsilon_F + \delta\mu_R)$ and $\frac{-e}{\hbar} E_2 = \frac{d(N_R-N_L)}{dt} = -\frac{1}{\tau_v} \left( N_R^0(\epsilon_F + \delta\mu_R) - N_L^0(\epsilon_F + \delta\mu_L) \right)$.

We obtain: $\delta\mu_R - \delta\mu_L = ev_F\tau_v E_2$.

We have checked the effect of the chiral anomaly on the photoemission spectrum by using the shift of the chemical potential $\delta = \delta\mu_L = -\delta\mu_R = \frac{v_F\tau_v}{2} (\frac{-e}{\hbar}) E_2$. We observe that $\delta$ controls the polarization function $S_y - \mu$. $\delta = 0$ correspond to the plot in the middle of figure (2). When an electric field is applied we obtain the lower plot and the upper plot in figure (2) with $\delta = \pm 0.05ev$. To conclude we have demonstrate theoretically the emergence of the fermi arcs and their manipulation with the help of the one dimensional chiral anomaly. This has been achieved with the help of an Hamiltonian which consider the connection between the two nodes and a wave function which respect the boundary conditions in the presence of a surface at $z = 0$.

[1] H. Weyl Z.Phys.56 330 (1929).

[2] X. Wan, A.M. Turner, A. Vishwanath, and S.Y.Savrasov Phys.Rev.B. 83, 205101 (2011).
FIG. 3: The plot of $S_y(k_x)$ for $\delta = 0.05$, $\delta = 0.0$ and $\delta = -0.05$

[3] G.Xu,H.Weng,Z.Wang,X.Dai and Z.Fang, Phys.Rev.Lett. 107, 186806 (2011).
[4] H.Nielsen and M.Ninomiya, Phys.Lett. B 130, 389 (1983).
[5] Su-Yang Xu, Ilya Belopolski,Nasser Alidoust, Madhab Neupaine, Guang Bian, Chenlong, Raman Sankar, Guoqing Chang, Zhujun Yuan, Chi-Cheng Lee, Shin-Ming Huang, Hao Zheng, Jie Ma, Daniel S.Sanchez, BaoKai Wang, Arun Bansil, Fangcheng Chou, Pavel P.Shibayev, Hsin Lin, Shuang Jia, M.Zahid Hasan Science, vol. 349 ISSUE 6248 (2015)
[6] Rajib Batabyal,Noam Morali, Nurit Avraham, Yan Sun, Marcus Schmidt, Caudia Felser, Ady Stern, Binghai Yan, Haim Beidenkopf Sci. Adv. 2, e1600709 (2016).
[7] Dan Thahn Son and Naoki Yamato Phys.Rev.Lett. 109, 181602 (2012).
[8] D.T.Son and B.Spivak Phys.Rev.B 88,104412 (2013)
[9] T.C. Chiang, J.A.Knapp, M.Aono and D.E. Eastman Phys.Rev.B 21, 3513 (1979).
[10] D.Schmeltzer and A.Saxena Journal of Physics ,Condensed Matter 27 485601 (2015).
[11] Jan Behrends,Adolpho G.Grushin , Teemu Ojanen and Jens H. Badarson Phys. Rev.B 93,075114 (2015)