A general heavy-flavor mass scheme for charge-current DIS at NNLO and beyond

& implications for EW phenomenology

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4th May 2022

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Based on: Phys. Rev. D105 (2022) 1, L011503 [arXiv: 2107.00460]
pQCD at NNLO → next-generation PDF extractions

- theory accuracy now/approaching (N)NNLO in $\alpha_s$ for typical processes
  
  $\rightarrow$ NLO EW corrections, especially for LHC data

- fit a wide assortment of data from various underlying processes; scales
  
  $\rightarrow$ CC DIS (e.g., HERA) important complement in CT; needs NNLO treatment
CC DIS: motivation

- charged-current DIS: central component in several next-generation QCD expt. programs
  - $\nu A$, forward physics: DUNE, FASER$\nu$
  - precision QCD: EIC, LHeC

  **example:** DUNE target sensitivity to $\delta_{CP} \rightarrow$ control over CC DIS for $E_\nu \sim$ few GeV

- $W$-exchange processes probe unique combinations of in-nucleon flavor currents; e.g., nucleon $s(x, Q)$

NuTeV $\mu^+\mu^- \nu$-production, CT18 NNLO

- CC DIS data cover *wide range* in $Q$; higher pQCD accuracy *needed* for perturbative stability, PDF extractions
evolution schemes as general problem in QCD

- higher order(s) in pQCD: improved accuracy in Wilson coeff., control over scale dependence
- at given fixed order, nontrivial relationship with chosen heavy-quark (HQ) scheme

→ fixed flavor-number (FFN): $Q \gtrsim M_Q$; flavor-creation (FC) processes with $n_f = 3$

→ zero-mass (ZM) variable flavor-number: $Q \gg M_Q$; flavor-excitation (FE) processes with $n_f = 4$

- 2 paradigms adapted to different regimes w.r.t. HQ mass scale; ∃ interpolation scheme?
general-mass schemes: S-ACOT-χ

- variable flavor-number scheme to interpolate between ZM and FFN regimes: ACOT
  Aivazis, Collins, Olness, Tung; PRD50 (1994) 3085-3118

→ systematic approach to incorporating HQ mass dependence

- introduce subtraction term(s) to eliminate double counting between FC/FE contributions:

\[
\begin{align*}
Q \gtrsim M_Q & \implies \text{(SUB)} \approx \text{(FE)} \text{ such that } n_f = 3 \text{ FC dominates} \\
Q \gg M_Q & \implies \text{(SUB)} \approx \text{(FC)} \text{ such that } n_f = 4 \text{ FE dominates}
\end{align*}
\]

\[
\chi(x, Q, M_Q) = x \left(1 + \frac{1}{Q^2} \sum_{\text{F.S.}} M_Q^2\right)
\]

- “simplified” ACOT (S-ACOT): neglect full HQ mass dependence in FE graphs
- S-ACOT-χ: smooth HQ thresholds, include approx. HQ mass dependence: \( C_i(x) \rightarrow C_i(\chi) \)

- formulation necessitates careful tracking of diagrams to organize calculation correctly
template calculation: NC DIS at NNLO

- at structure-function level, factorization allows separation of coeff. functions, PDFs:

\[
F(x, Q) = \sum_{i=1}^{N_f^{fs}} e_i^2 \sum_{a=0}^{N_f} \left[ C_{i,a} \otimes \Phi_{a/p} \right](x, Q) \quad (F = F_{2,L})
\]

- compute S-ACOT-\(\chi\) coeff. functions: expand in \(\alpha_s\) each term in auxiliary partonic struct. func.:

\[
F_{i,b}(\hat{x}, Q) = \sum_{a=0}^{N_f} \left[ C_{i,a} \otimes \Phi_{a/b} \right](\hat{x}, Q)
\]

\[
\begin{align*}
C_{i,b}^{(0)}(\hat{x}) &= F_{i,b}^{(0)}(\hat{x}) \\
C_{i,b}^{(1)}(\hat{x}) &= F_{i,b}^{(1)}(\hat{x}) - \left[ C_{i,a}^{(0)} \otimes A_{ab}^{(1)} \right](\hat{x}) \\
C_{i,b}^{(2)}(\hat{x}) &= F_{i,b}^{(2)}(\hat{x}) - \left[ C_{i,a}^{(0)} \otimes A_{ab}^{(2)} \right](\hat{x}) - \left[ C_{i,a}^{(1)} \otimes A_{ab}^{(1)} \right](\hat{x})
\end{align*}
\]

- organize into heavy-, light-quark pieces: \(F = \sum_{l=1}^{N_l} F_l + F_h\)

\[
C_{h,g}^{(2)} = \hat{F}_{h,g}^{(2)} - A_{h_g}^{(2)} - c_{h_h}^{(1)} \otimes A_{h_g}^{(1)}
\]

S-ACOT-\(\chi\): massless FE, \(\chi\)-rescaled

\[
\rightarrow \text{light-quark SFs: additional flavor non-sing. (NS) disconnected graphs:}
\]
conceptually, organization of CC DIS calculation resembles NC

gauge-boson coupling introduces charge/flavor-changing vertices

\[ F_l \iff W\bar{q}_l q_l \]
\[ F_h \iff W\bar{q}_h q_l \text{ or } W\bar{q}_l q_h \]

→ appearance of heavy flavor occurs at different orders w.r.t. NC

- HQ contributions begin only at NNLO for \( F_l \)
- FE and FC diagrams involving HQ start from LO for \( F_h \)

- S-ACOT-\( \chi \) patterns of subtractions, HQ mass dependence in CC NLO Wilson coeff.:

\[
\begin{align*}
C_{l,l}^{(1)} &= c_{l,l}^{(1)}(z), & C_{l,g}^{(1)} &= c_{l,g}^{(1)}(z), & C_{h,h}^{(1)} &= c_{l,l}^{(1)}(\chi) \\
C_{h,l}^{(1)} &= H_l^{(1)}(z) - C_{h,l}^{(0)} \otimes A_{ll}^{(1)} \\
C_{h,g}^{(1)} &= H_g^{(1)}(z) - C_{h,l}^{(0)} \otimes A_{lg}^{(1)} - C_{h,h}^{(0)} \otimes A_{hg}^{(1)}
\end{align*}
\]

- HQ contributions explicitly appear in light-quark SF at NNLO; subtracted NS coeff.:

\[
\begin{align*}
C_{l,g}^{(2)} &= c_{l,g}^{(2)}(z), & C_{l,h}^{(2)} &= c_{l,h}^{(2)}(\chi) \\
C_{l,l}^{(2)} &= c_{l,l}^{(2)}(z) + \tilde{C}_{l,l}^{(NS,2)}(z)
\end{align*}
\]

- careful ordering of diagrams by flavor structure, topology is crucial
CC DIS at NNLO and beyond (ii)

representative CC NNLO subtraction diagrams – identifiable with coeff. expressions

\[ C_{h,h}^{(2)} = c_{h,h}^{(2)}(\chi) \]
\[ C_{h,l}^{(2)} = H_{l}^{(2)}(z) - \Delta C_{h,l}^{(2)} \]
\[ C_{h,g}^{(2)} = H_{g}^{(2)}(z) - \Delta C_{h,g}^{(2)} \]

constructed from 2-loop operator matrix elements

- finally, ZM N^3LO Wilson coeffs. available!
- evaluate approximate N^3LO (i.e., N^3LO´)

- S-ACOT-\(\chi\) patterns of subtractions, HQ mass dependence in CC NLO Wilson coeffs.:

\[
\begin{align*}
C_{l,l}^{(1)} &= c_{l,l}^{(1)}(z), \quad C_{l,g}^{(1)} = c_{l,g}^{(1)}(z), \quad C_{h,h}^{(1)} = c_{h,l}^{(1)}(\chi) \\
C_{h,l}^{(1)} &= H_{l}^{(1)}(z) - C_{h,l}^{(0)} \otimes A_{l,l}^{(1)} \\
C_{h,g}^{(1)} &= H_{g}^{(1)}(z) - C_{h,l}^{(0)} \otimes A_{l,g}^{(1)} - C_{h,h}^{(0)} \otimes A_{h,g}^{(1)}
\end{align*}
\]

- HQ contributions explicitly appear in light-quark SF at NNLO; subtracted NS coeff.:

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\begin{align*}
C_{l,g}^{(2)} &= c_{l,g}^{(2)}(z), \quad C_{l,h}^{(2)} = c_{l,h}^{(2)}(\chi) \\
C_{l,l}^{(2)} &= c_{l,l}^{(2)}(z) + \tilde{C}_{l,l}^{(\text{NS,2})}(z)
\end{align*}
\]

- careful ordering of diagrams by flavor structure, topology is crucial
illustration for arbitrary DIS cross sections

- before expt.-specific predictions, compute generic DIS reduced cross section(s)
  
  → compare calculations of highest available order: FFN vs. ZM vs. GM schemes

\[ e^- p \text{ CC DIS, } \sqrt{s} = 200 \text{ GeV, } x = 0.02 \]

\[ d^2\sigma/dxdQ^2 \]

\( \text{ZM NLO} \)
\( \text{FFN NLO} \)
\( \text{ZM N2LO} \)
\( \text{FFN N2LO} \)
\( \text{ZM N3LO} \)

CT14 NNLO PDF

\( \text{GM N2LO} \)
\( \text{ZM N2LO} \)
\( \text{FFN N2LO} \)

\[ \text{NLO: ZM vs FFN, } \lesssim 6\% \]
\[ \text{NNLO: ZM vs FFN, } \lesssim 3\% \]

GM interpolation between FFN at low \( Q \), ZM at high \( Q \)

\( \text{GM N}^3\text{LO}': \) GM NNLO + ZM N\(^3\)LO

[NNLO PDFs]

scale variations: shift \( \mu_R, \mu_F \) by \( \times 2 \)

→ improved convergence with order!

(illustrate virtuality dependence)
implications for CC DIS at EIC

- EIC will undertake various precision QCD measurements; EIC Yellow Report [arXiv: 2103.05419]

  → Inclusive Reactions Study (YR7.1.1): CC – including positron beam – access to $d, s$ PDFs

![Graph showing $e^{-}p$ CC DIS, $\sqrt{s} = 141$ GeV, $Q^2 = 100$ GeV$^2$]

| $d^2 \sigma/dQ^2$ | $\sigma$ | $\rho$ |
|------------------|---------|-------|
| $GM$ N3LO$'$     | 1.2     | 0.8   |
| $GM$ N2LO        | 1.0     | 0.6   |
| $GM$ NLO         | 0.8     | 0.4   |
| $GM$ LO          | 0.6     | 0.2   |
| CT14 NNLO PDF    | 0.4     | 0.2   |

- consider high-energy EIC collisions
  → reconstruction challenges: CC events restricted to high $Q^2$

- strong perturbative convergence
  → for N$^3$LO$'$, scale variations generally contained to $\lesssim 0.5 - 1\%$

- significantly smaller than PDF-driven uncertainties, which can be as large as $\approx 2\%$

vital ingredient in EIC PDF program
precision QCD will also be necessary for $\nu A$

- forthcoming neutrino-nuclear experiments cover wide range of energies, $E_\nu$

- even at DUNE, events coming from DIS represent $\gtrsim 40\%$

  (important dependence on SF extrapolations, correlations in tunes with low-energy model parameters)

  $\rightarrow$ small DIS cross section variations influence DUNE sensitivity

  $\rightarrow$ at higher energies, significant impact on sensitivity of forward-physics program at FASER$\nu$ ($\sim 100\text{s GeV}$); neutrino telescopes ($>\text{TeV}$)
precise QCD will also be necessary for $\nu A$

- $\nu$ cross sections generally diminished by LO $\rightarrow$ (N)NNLO, by 6% for most $E_\nu$
- as before, NNLO and $N^3\text{LO}^\prime$ corrections greatly reduce scale variations

\[
\begin{align*}
E_\nu > 100 \text{ GeV}: & \text{ negligible} \\
\sim 1 - 3\% \text{ elsewhere}
\end{align*}
\]

- in contrast, PDF uncertainties are $\sim 1 - 2\%$
  \[
  \rightarrow \text{ strong pQCD theory for FASER$\nu$ program}
  \]

- future analyses will witness an interplay between pQCD and nuclear effects
  
  \[
  \rightarrow \text{ assessed nuclear correction using nCTEQ15: } \sim 0.5\% \text{ effect}
  \]
conclusions: next steps, PDF implications

- have extended general-mass HQ scheme to CC DIS at NNLO; approximate N$^3$LO
  - incorporates full HQ threshold dependence; interpolation between FFN, ZM approaches
  - dramatic reductions to dependence on perturbative QCD scale choices
  - consistency across broad ranges of $x, Q^2, E_\nu$

- perturbative uncertainties at EIC reduced to sub-percent level for target kinematics
  - substantially boosts precision of inclusive measurements program; PDF sensitivity

- (N)NNLO accuracy reduces $\nu$DIS scale uncertainties to ~1-3%; less at high energies
  - critical to achieving precision objectives in $\nu$A programs at DUNE

- interfaces with PDF global analyses (and perhaps generators) will be valuable
  - higher pQCD accuracy suggests need for parallel enhancements in, e.g., nuclear modeling, EW corrections, few-GeV nonperturbative theory