MULTI-BOSON EFFECTS IN BOSE-EINSTEIN INTERFEROMETRY

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Multi-boson symmetrization effects on two-particle Bose-Einstein interferometry are studied for ensembles with arbitrary multiplicity distributions. In the general case one finds interesting residual correlations which require a modified framework for extracting information on the source geometry from two-particle correlation measurements. In sources with high phase-space densities, multi-boson effects modify the Hanbury Brown–Twiss (HBT) radius parameters and simultaneously generate strong residual correlations. We clarify their effect on the correlation strength (intercept parameter) and thus explain a variety of previously reported puzzling multi-boson symmetrization phenomena. For event ensembles of (approximately) fixed multiplicity, the residual correlations lead to a minimum in the correlation function at non-zero relative momentum, which can be practically exploited to search, in a model-independent way, for multi-boson symmetrization effects in high-energy heavy-ion experiments.

1 Introduction

Heavy-ion collisions at high energies produce large numbers of secondary particles. Even if these particles are created independently (“chaotic source”) and are free from final state interactions, the measured $n$-body momentum spectra still do not simply factor into a product of single-particle spectra. Instead, symmetrization of the many-body wave function with respect to the exchange of identical particles leads (in the case of bosons) to so-called Bose-Einstein correlations.

Bose-Einstein correlations affect the measured cross sections whenever more than one boson occupies an elementary phase-space cell. This is the basis of Bose-Einstein or intensity interferometry: for a source of size $R$, symmetrization effects lead to an enhancement of, for example, the two-particle cross-section at relative momenta $q < 1/R$. The theory which allows to extract the source size $R$ from two-particle correlation data assumes, in its simplest and usually employed form, that the effect is dominated by symmetrization with respect to an exchange of the two observed particles, and that the additional symmetrization with respect to all the other identical bosons in the multiparticle final state can be neglected.

This is only true if the phase-space density of the source at freeze-out
(i.e. at point of last interaction) is sufficiently small. In regions of high phase-space density, multi-boson symmetrization effects change the shape even of the single-particle spectra and two-particle correlations. They lead to a clustering a low momenta in momentum space and at low separation in coordinate space. For example, in an infinite medium they turn an exponential momentum distribution of distinguishable elementary pion production sources into a Bose-Einstein distribution for the observed pion momenta. And in two-pion interferometry, the effective source extension extracted from the two-pion correlation function is smaller than the r.m.s. width of the spatial distribution of the individual pion emitters.

In addition to these generally accepted phenomena, multi-boson symmetrization effects also cause other, somewhat more confusing effects. For example, it has been noted that in ensembles with fixed total pion multiplicity multi-pion effects reduce the intercept of the two-pion correlation function at vanishing relative momentum. This has been interpreted as a sign of “effective coherence” in the source which becomes significant if its phase-space density is large and which will lead to a “pion laser” once a critical phase-space density is exceeded. This interpretation will be shown to be incorrect: The intercept reduction happens even for completely chaotic sources without any phase coherence, and even the formation of a Bose condensate is not accompanied by “pion lasing”. We found that the reduced intercept reflects residual correlations among the pions which are not directly related to the Hanbury Brown-Twiss correlations which are exploited in Bose-Einstein interferometry. These residual correlations are generic in the sense that they vanish only for very specific cases where the multiplicity distribution of the ensemble takes on a particular form. When present, they render the extraction of the source size from Bose-Einstein interferometry more difficult.

2 Multi-boson formalism

Due to space reasons, I can give here only a very rudimentary account of the mathematics; all technical details can be found elsewhere. Following earlier work by others, the source is parametrized by a density operator constructed out of a superposition of \( n \)-particle states emitted from classical source currents which themselves are Gaussian wave packets. The number of these source current wave packets, their centers in phase-space, and the pion multiplicity \( n \) all have completely arbitrary (although normalizable) distributions. The source currents emit pions with random phases which ensures complete source chaoticity. For technical reasons, the \( n \)-pion states used in the construction are not normalized, but the density operator is. Cal-
Calculating this normalization is the only part of the problem which cannot be done analytically even for the simple case of nonrelativistic, instantaneous Gaussian wave packets, but requires the numerical evaluation of a recursion relation.\footnote{As found by Pratt\cite{Pratt}, all $n$-particle cross sections can be expressed in terms of the so-called “ring integrals”
\begin{equation}
G_i(\vec{p}_1, \vec{p}_2) = \int G_1(\vec{p}_1, \vec{k}_1) d\vec{k}_1 G_1(\vec{k}_1, \vec{k}_2) \cdots d\vec{k}_{i-1} G_1(\vec{k}_{i-1}, \vec{p}_2),
\end{equation}
where $d\vec{k} = d^3k/E_k$. They describe the effect on the two-particle exchange amplitude (with respect to the interchange of momenta $\vec{p}_1$ and $\vec{p}_2$) of the symmetrization with respect to $i-1$ additional particles with momenta $\vec{k}_1, \ldots, \vec{k}_{i-1}$. They can be generated recursively from the elementary two-particle exchange amplitude $G_1(\vec{p}_1, \vec{p}_2)$, the Fourier transform of the source Wigner density $g(x, K)$:
\begin{equation}
G_1(\vec{p}_1, \vec{p}_2) = \int d^4x g(x, \frac{1}{2}(p_1+p_2)) e^{-i(p_1-p_2) \cdot x} = \int d^4x g(x, K) e^{-iq \cdot x}.
\end{equation}
For a non-relativistic spherical Gaussian Wigner density with spatial width $R$ and momentum spread $\Delta$, which emits particles instantaneously $\sim \delta(t)$, the ring integrals can be calculated analytically\footnote{\begin{align*}
G_n(\vec{p}_1, \vec{p}_2) &= \frac{c_n \sqrt{E_1 E_2}}{(2\pi \Delta^2)^\frac{3}{2}} \exp \left( -\frac{\vec{K}^2}{2\Delta_n^2} - \frac{\vec{q}^2}{2} \right), \\
R_n^2 &= a_n R^2, \quad \Delta_n^2 = a_n \Delta^2, \quad v = 2R\Delta \geq 1, \\
a_n &= \frac{1}{v} \frac{(v+1)^n + (v-1)^n}{(v+1)^n - (v-1)^n} \leq 1, \quad c_n = \left( \frac{2^n v}{(v+1)^{2n} - (v-1)^{2n}} \right)^{3/2} \leq 1.
\end{align*}}
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\end{align*}

One sees that the higher order ring integrals are Gaussians with reduced widths $R_n$ and $\Delta_n$ in both coordinate and momentum space, reflecting the bosonic clustering at low momenta and relative distances mentioned in the Introduction. The strength of these clustering effects is controlled by the single parameter $v = 2R\Delta$ which measures the phase-space volume of the source. The above equations remain correct for expanding Gaussian sources (after suitable redefinition\footnote{of the parameters $R$ and $\Delta$), and they are easily generalized for Gaussians with different widths along the three Cartesian directions.}}
3 Bose-Einstein and residual correlations

For this very general class of models, the two-particle correlation function can be written down explicitly as follows:

\[
C_2(\mathbf{p}_1, \mathbf{p}_2) = \frac{\sum_{i,j} h_i h_j G_i(\mathbf{p}_1, \mathbf{p}_1) G_j(\mathbf{p}_2, \mathbf{p}_2)}{\sum_{i,j} h_i h_j G_i(\mathbf{p}_1, \mathbf{p}_2) G_j(\mathbf{p}_2, \mathbf{p}_2)} \times \left( 1 + \frac{\sum_{i,j} h_i h_j G_i(\mathbf{p}_1, \mathbf{p}_2) G_j(\mathbf{p}_2, \mathbf{p}_1)}{\sum_{i,j} h_i h_j G_i(\mathbf{p}_1, \mathbf{p}_1) G_j(\mathbf{p}_2, \mathbf{p}_2)} \right) \equiv C_2^{(\text{res})}(q, K) \left( 1 + R_2(q, K) \right). \tag{6}
\]

The sums over \(i\) and \(j\) take into account symmetrizations with respect to an increasing number of other particles in the final state. The weights \(h_i\) are related to the measured pion multiplicity distribution \(p_n\) by

\[
h_i = \sum_{n=i}^{\infty} p_n \frac{\omega(n-i)}{\omega(n)}, \tag{7}
\]

where \(\omega(n)\) is (up to trivial factors) the factor required to normalize the \(n\)-particle density operator and must be obtained by numerically solving the recursion relation

\[
\omega(n) = \frac{1}{n} \sum_{i=1}^{n} C_i \omega(n-i), \tag{8}
\]

with \(\omega(0) = \omega(1) = 1\) and \(C_i = \int d\mathbf{p} G_i(\mathbf{p}, \mathbf{p}) = c_n a_n^{3/2}\). Obviously, the \(\omega(n)\) and the weights \(h_i\) depend non-trivially on the phase-space volume factor \(v\).

For dilute systems \((v \gg 1)\), the higher order ring integrals can be neglected, and all the sums in (6) are dominated by their first terms. In this case the prefactor \(C_2^{(\text{res})}\) in (3) reduces to a normalization constant \(N = h_2/h_1^2\), and the correlator assumes its standard form

\[
C_2(\mathbf{p}_1, \mathbf{p}_2) = N \left( 1 + \frac{|G_1(\mathbf{p}_1, \mathbf{p}_2)|^2}{G_1(\mathbf{p}_1, \mathbf{p}_1) G_1(\mathbf{p}_2, \mathbf{p}_2)} \right). \tag{9}
\]

In general, however, the prefactor is a non-trivial function of both the pair momentum \(K\) and the relative momentum \(q\). These residual correlations do not involve the exchange of momenta \(\mathbf{p}_1\) and \(\mathbf{p}_2\) and thus are not related to Bose-Einstein interferometry; they cannot be used to extract the source radius \(R\). They do, however, modulate the correlation function and modify its intercept at \(q = 0\). A typical example is shown in Figure 1. One sees the genuine (Gaussian) Bose-Einstein correlation function \(R_2(K, q)\) sitting on
top of the residual correlation; the latter starts out at the value $N = h_2/h_1^2$ at $q = \infty$ but decreases to a much smaller value at $q = 0$.

The source size signal is contained in the function $R_2$ in (6). Note that $R_2$ goes to 1 as $q \to 0$; thus at $q = 0$ the exchange term is as large as the direct term, implying a fully chaotic source (as it should be by construction). The reduced intercept at $q = 0$ thus has nothing to do with coherence.

The residual correlations can be positive or negative, enhancing or decreasing the intercept at $q = 0$ above or below the value $2N$. Negative residual correlations appear, for example, in ensembles with fixed pion multiplicity or with a Poissonian multiplicity distribution; on the other hand, a Bose-Einstein multiplicity distribution can lead to large positive residual correlations. The residual correlations disappear in the dilute limit, $v \to \infty$, but generically become strong for large phase-space densities or small values of $v$. At the moment there are only two special multiplicity distributions known for which residual correlations are completely absent, for all allowed values of $v$.

The limit $v \to 1$ is particularly interesting. In this limit the source is as small as allowed by the uncertainty principle. The result are vanishing
HBT radii and the correlation function becomes completely flat (i.e. \( q \)-independent). [Note that in this limit also the residual correlation function \( C_{2}^{\text{res}} \) become \( q \)-independent.] Superficially this looks like the correlation function from a coherent state ("pion laser") which shows no Bose-Einstein correlations. This impression is, however, quite misleading: The correlation function still has the structure (6), with an exchange term \( R_{2} \equiv 1 \) which is as large as the direct term (i.e. the source is still fully chaotic). Note that this limit can be taken while keeping the pion multiplicity distribution fixed, i.e. the latter does not automatically become a Bose-Einstein distribution when \( v \rightarrow 1 \).

4 Epilogue

Does all this matter for our real lives as heavy-ion physicists? Most likely not. An analysis of average pion phase-space densities at freeze-out from heavy-ion collisions at AGS and SPS energies yields universally low values for all considered collision systems and beam energies. First results from RHIC appear to confirm this even at the much higher collision energy of \( \sqrt{s} = 130 \text{ AGeV} \). Apparently pions stop interacting (freeze out) only after the phase-space density has dropped below a critical value which is low enough for multi-boson symmetrization effects to remain unimportant. This is good news: it means that we can continue to use the standard HBT formalism to extract source sizes from two-pion correlation data. It would be good, though, to confirm this by directly checking for (the absence of) residual correlations in a high-statistics analysis of two-particle correlation data from heavy-ion collisions. This can be done by selecting heavy-ion collisions producing an (approximately) fixed number of, say, \( \pi^{+} \) mesons and searching for a minimum at non-zero \( q \) in the (Coulomb corrected) \( \pi^{+}\pi^{+} \) correlation function. At the same time, it may be interesting to explore other fields of application for the elegant multi-boson symmetrization techniques presented here.

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