Higher spin supermultiplets in three dimensions: 
(2,0) AdS supersymmetry

Jessica Hutomo and Sergei M. Kuzenko

Department of Physics M013, The University of Western Australia 
35 Stirling Highway, Crawley W.A. 6009, Australia 

jessica.hutomo@research.uwa.edu.au, sergei.kuzenko@uwa.edu.au

Abstract

Within the framework of (2,0) anti-de Sitter (AdS) supersymmetry in three dimensions, we propose a multiplet of higher-spin currents. Making use of this supercurrent, we construct two off-shell gauge formulations for a massless multiplet of half-integer superspin \((s + \frac{1}{2})\), for every integer \(s > 0\). In the \(s = 1\) case, one formulation describes the linearised action for (2,0) anti-de Sitter supergravity, while the other gives the type III minimal supergravity action in (2,0) AdS superspace, with both linearised supergravity actions originally derived in \[arXiv:1109.0496\]. We formulate topologically massive higher-spin supermultiplets in (2,0) AdS superspace. Our results admit a natural extension to the case of \(S^3\).
1 Introduction

In four dimensions (4D), there is an interesting correspondence between $\mathcal{N} = 1$ anti-de Sitter (AdS) supergravity\textsuperscript{1} \cite{1} and massless higher-spin supermultiplets in AdS\textsubscript{4} \cite{3}. Specifically, two off-shell formulations are known for pure $\mathcal{N} = 1$ AdS supergravity, the minimal \cite{4,5,6} (see, e.g., \cite{7,8} for pedagogical reviews) and the non-minimal \cite{9} theories. In AdS\textsubscript{4} there exist two series of massless off-shell gauge supermultiplets of half-integer superspin $s + \frac{1}{2}$, with $s = 1, 2, \ldots$ \cite{3,2}. The correspondence consists of the fact that, for the lowest superspin value corresponding to $s = 1$, one series yields the linearised action for minimal AdS supergravity, while the other leads to linearised non-minimal AdS supergravity. It has recently been pointed out \cite{10} that a similar correspondence might

\textsuperscript{1}Townsend’s work on $\mathcal{N} = 1$ AdS supergravity \cite{4} appeared shortly after Freedman and Das constructed $\mathcal{N} = 2$ AdS supergravity \cite{2}. The motivations for \cite{1} and \cite{2} were rather different.

\textsuperscript{2}Such a supermultiplet describes two ordinary massless spin-($s + \frac{1}{2}$) and spin-($s + 1$) fields on-shell.
occur in the case of 3D $\mathcal{N} = 2$ supersymmetry, which is a natural cousin of the 4D $\mathcal{N} = 1$ one.

Unlike four dimensions, where pure $\mathcal{N} = 1$ AdS supergravity is unique on-shell, the feature specific to three dimensions is the existence of two distinct $\mathcal{N} = 2$ AdS supergravity theories \[11\], which are known as the (1,1) and (2,0) AdS supergravity theories, originally constructed as Chern-Simons theories. Two off-shell formulations for (1,1) AdS supergravity have been developed, the minimal \[12, 13, 14, 15, 16, 17, 18\] and the non-minimal \[17, 18\] theories, and one for (2,0) AdS supergravity \[19, 16, 17, 18\]. Since there are three off-shell $\mathcal{N} = 2$ AdS supergravity theories, one might expect the existence of three series of massless higher-spin gauge supermultiplets. In a recent paper \[10\], we have presented two series of massless higher-spin actions which are associated with the minimal and the non-minimal (1,1) AdS supergravity theories, respectively, generalising similar constructions in the super-Poincaré case \[20\]. The present paper is devoted to constructing higher-spin gauge multiplets with (2,0) AdS supersymmetry.

It is worth pointing out that the massless 3D constructions of \[10, 20\], were largely modelled on the 4D results of \[3, 21\]. With respect to 3D (2,0) AdS supersymmetry, unfortunately there is no 4D intuition to guide us, and new ideas are required in order to construct higher-spin gauge supermultiplets. In this paper our approach will be to utilise an observation that has often been used in the past to formulate off-shell supergravity multiplets \[22, 23, 24, 25, 26, 27\]. The idea is to make use of a higher-spin extension of the supercurrent (also known as the multiplet of currents), the concept introduced by Ferrara and Zumino in the case of 4D $\mathcal{N} = 1$ Poincaré supersymmetry \[28\] and extended to 4D $\mathcal{N} = 2$ Poincaré supersymmetry by Sohnius \[29\]. Specifically, for a simple supersymmetric model in (2,0) AdS superspace we identify a multiplet of conserved higher-spin currents. In general, the multiplet of currents is always off-shell. Using the constructed higher-spin supercurrent, we may identify a corresponding supermultiplet of higher-spin fields. The procedure to follow is concisely described by Bergshoeff et al. \[22\]: “One first assigns a field to each component of the current multiplet, and forms a generalized inner product of field and current components.”

Our multiplet of currents is described by the conservation equations

$$D_\beta J_{\alpha_1...\alpha_{2s-1}} = D_{(\alpha_1} T_{\alpha_2...\alpha_{2s-1})} , \quad \bar{D}_\beta J_{\bar{\alpha}_1...\bar{\alpha}_{2s-1}} = \bar{D}_{(\bar{\alpha}_1} \bar{T}_{\bar{\alpha}_2...\bar{\alpha}_{2s-1})}. \quad (1.1a)$$

Here $D_\alpha$ and $\bar{D}_\alpha$ are the covariant spinor derivatives of (2,0) AdS superspace \[17\], $J_{(2s)} := J_{\alpha_1...\alpha_{2s}} = J(\alpha_1...\alpha_{2s}) = \bar{J}_{(2s)}$ denotes the higher-spin supercurrent, and $T_{\alpha(2s-2)}$ the corre-
sponding trace supermultiplet constrained to be covariantly linear:

\[ \mathcal{D}^2 T_{\alpha(2s-2)} = 0, \quad \mathcal{D}^2 \bar{T}_{\alpha(2s-2)} = 0. \]  

(1.1b)

In general, the trace supermultiplet is complex,

\[ T_{\alpha(2s-2)} = \mathcal{Y}_{\alpha(2s-2)} - i\mathcal{Z}_{\alpha(2s-2)}, \quad \text{Im} \, \mathcal{Y}_{\alpha(2s-2)} = 0, \quad \text{Im} \, \mathcal{Z}_{\alpha(2s-2)} = 0. \]  

(1.1c)

In the \( s = 1 \) case, the above conservation equation coincides with that for the (2,0) AdS supercurrent [17].

Our work may have various generalisations and applications. For instance, the massless higher-spin actions constructed in section 4.1 are expected to possess nonlinear completions, say, in the spirit of the bosonic Chern-Simons constructions of [30, 31, 32]. Our results admit a natural extension to the case of \( S^3 \), which may lead to higher-spin applications of the localisation techniques, see, e.g., [33, 34] for reviews. The adequate superspace setting to formulate \( \mathcal{N} = 2 \) supersymmetric theories on \( S^3 \) has been developed [35].

This paper is organised as follows. Section 2 provides a brief review of (2,0) AdS superspace. In section 3 we consider simple models for a chiral scalar supermultiplet and demonstrate how the higher-spin supercurrent (1.1) emerges. In section 4 we develop two off-shell formulations for a massless multiplet of half-integer superspin \((s + \frac{1}{2})\) in (2,0) AdS superspace, with \( s \) a positive integer. Our results and their implications and possible extensions are discussed in section 5. In the appendix we collect important (2,0) AdS identities.

## 2 (2,0) AdS superspace

In this section we give a summary of the most important results concerning (2,0) AdS superspace, see [17] for the details.

The covariant derivatives of (2,0) AdS superspace have the form

\[ \mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \mathcal{D}^\alpha) = E_A + \Omega_A + i\Phi_A J. \]  

(2.1)

Here \( E_A \) and \( \Omega_A \) denote the inverse supervielbein and the Lorentz connection, respectively,

\[ E_A = E_A^M \frac{\partial}{\partial z^M}, \quad \Omega_A = \frac{1}{2} \Omega_A^{bc} M_{bc} = -\Omega_A^b M_b = \frac{1}{2} \Omega_A^{\beta\gamma} M_{\beta\gamma}. \]  

(2.2)

---

3We make use of the blackboard bold letters for covariantly linear superfields, in accordance with the notation adopted in [17].
The Lorentz generators with two vector indices \((M_{ab} = -M_{ba})\), with one vector index \((M_a)\) and with two spinor indices \((M_{\alpha\beta} = M_{\beta\alpha})\) are defined in the appendix. The \(U(1)_R\) generator \(J\) in (2.1) is defined to act on the covariant derivatives as follows:

\[
[J, D_\alpha] = D_\alpha, \quad [J, \bar{D}^\alpha] = -\bar{D}^\alpha, \quad [J, D_a] = 0.
\]  

(2.3)

The covariant derivatives satisfy the following algebra \([17]\):

\[
\{D_\alpha, D_\beta\} = 0, \quad \{\bar{D}_\alpha, \bar{D}_\beta\} = 0, \quad (2.4a)
\]

\[
\{D_\alpha, \bar{D}_\beta\} = -2iD_{\alpha\beta} - 4i\varepsilon_{\alpha\beta}SJ + 4iSM_{\alpha\beta}, \quad (2.4b)
\]

\[
[D_\alpha, D_\beta] = (\gamma_a)_{\beta}^\gamma SD_\gamma, \quad [D_\alpha, \bar{D}_\beta] = (\gamma_a)_{\beta}^\gamma S\bar{D}_\gamma, \quad (2.4c)
\]

\[
[D_\alpha, D_b] = 4\varepsilon_{abc}S^2M^c. \quad (2.4d)
\]

Here the parameter \(S\) is related to the AdS scalar curvature as \(R = -24S^2\).

In accordance with the general formalism of \([8]\), the isometries of \((2,0)\) AdS superspace are generated by those real supervector fields \(\zeta^A E_A\) which obey the superspace Killing equation \([17]\)

\[
[\zeta + i\tau J + \frac{1}{2}l^{bc}M_{bc}, D_A] = 0, \quad (2.5a)
\]

where

\[
\zeta = \zeta^B D_B = \zeta^b D_b + \zeta_\beta D_\beta + \bar{\zeta}_\beta \bar{D}_\beta, \quad \bar{\zeta}^b = \zeta^b, \quad (2.5b)
\]

and \(\tau\) and \(l^{bc}\) are some local \(U(1)_R\) and Lorentz parameters, respectively. Every solution of (2.5) is called a Killing supervector field of \((2,0)\) AdS superspace. As demonstrated in \([17]\), eq. (2.5) implies that the parameters \(\zeta_\alpha\), \(\tau\) and \(l_{\alpha\beta}\) are uniquely expressed in terms of the vector \(\zeta_{\alpha\beta}\),

\[
\zeta_\alpha = \frac{i}{6}\bar{D}^\beta\bar{\zeta}_{\beta\alpha}, \quad \tau = \frac{i}{2}D^\alpha\zeta_\alpha, \quad l_{\alpha\beta} = 2(D_{(\alpha\beta)} - S\zeta_{\alpha\beta}), \quad (2.6)
\]

which obeys the equation

\[
D_{(\alpha\zeta_{\beta\gamma})} = 0. \quad (2.7)
\]

It follows that \(\zeta_\alpha\) is a Killing vector field,

\[
D_a\zeta_b + D_b\zeta_a = 0. \quad (2.8)
\]

One may also prove the following relations

\[
\bar{D}_\alpha\tau = \frac{i}{3}\bar{D}^\beta l_{\alpha\beta} = 4S\zeta_\alpha, \quad \bar{D}_\alpha\zeta_\beta = 0, \quad D_{(\alpha l_{\beta\gamma})} = 0. \quad (2.9)
\]
The Killing supervector fields of (2,0) AdS superspace generate the supergroup OSp(2|2; \mathbb{R}) \times Sp(2, \mathbb{R})$, the isometry group of (2,0) AdS superspace. Rigid supersymmetric field theories on (2,0) AdS superspace are invariant under the isometry transformations. The isometry transformation associated with the Killing supervector field $\zeta^A E_A$ acts on a tensor superfield $U$ (with its indices suppressed) by the rule

$$\delta_\zeta U = (\zeta + i \tau J + \frac{1}{2} t^{bc} M_{bc}) U.$$ (2.10)

Associated with a real scalar superfield $L$ is the following supersymmetric invariant

$$\int d^3 x d^2 \theta d^2 \bar{\theta} E L = -\frac{1}{4} \int d^3 x d^2 \theta \mathcal{E} \bar{D}^2 L, \quad E^{-1} = \text{Ber} \left( E_A^M \right),$$ (2.11)

where $\mathcal{E}$ denotes the chiral integration measure.

## 3 Higher-spin supercurrents for chiral matter

In this section we study higher-spin supercurrents in simple models for a chiral scalar supermultiplet in (2,0) AdS superspace.

### 3.1 Massless models

We first consider a massless model. Its action

$$S = \int d^3 x d^2 \theta d^2 \bar{\theta} E \Phi \bar{\Phi}, \quad \bar{D}_a \Phi = 0$$ (3.1)

is invariant under the isometry transformations of (2,0) AdS superspace for any $U(1)_R$ charge $w$ of the chiral superfield,

$$J \Phi = -w \Phi.$$ (3.2)

The action is superconformal provided $w = \frac{1}{2}$.

As in [10], it is useful to introduce auxiliary real variables $\zeta^\alpha \in \mathbb{R}^2$. Given a tensor superfield $U_{\alpha(m)}$, we associate with it the following field

$$U_{(m)}(\zeta) := \zeta^{\alpha_1} \cdots \zeta^{\alpha_m} U_{\alpha_1 \cdots \alpha_m},$$ (3.3)

which is a homogeneous polynomial of degree $m$ in the variables $\zeta^\alpha$. We introduce operators that increase the degree of homogeneity in the variable $\zeta^\alpha$,

$$\bar{D}_{(1)} := \zeta^\alpha \bar{D}_\alpha, \quad \bar{D}_{(1)}^2 = 0,$$ (3.4a)

$$5$$
\[ \mathcal{D}_{(1)} := \zeta^\alpha \mathcal{D}_\alpha, \quad \mathcal{D}_{(1)}^2 = 0, \quad (3.4b) \]
\[ \mathcal{D}_{(2)} := i\zeta^\alpha \zeta^\beta \mathcal{D}_{\alpha\beta}. \quad (3.4c) \]

We also introduce two nilpotent operators that decrease the degree of homogeneity in the variable \( \zeta^\alpha \), specifically
\[ \mathcal{D}_{(-1)} := \mathcal{D}^\alpha \frac{\partial}{\partial \zeta^\alpha}, \quad \mathcal{D}_{(-1)}^2 = 0, \quad (3.5a) \]
\[ \mathcal{D}_{(-1)} := \overline{\mathcal{D}}^\alpha \frac{\partial}{\partial \zeta^\alpha}, \quad \mathcal{D}_{(-1)}^2 = 0. \quad (3.5b) \]

Let us first consider the superconformal case, \( w = \frac{1}{2} \). The analysis given in [10] implies that the theory possesses a real supercurrent \( J_{(2s)} = \tilde{J}_{(2s)} \), for any positive integer \( s \), which obeys the conservation equation
\[ \mathcal{D}_{(-1)} J_{(2s)} = 0. \quad (3.6) \]
This supercurrent proves to have the same form as in the (1,1) AdS case considered in [10]. Specifically, the higher-spin supercurrent is given by
\[ J_{(2s)} = \sum_{k=0}^{s} (-1)^k \left\{ \frac{1}{2} \left( \frac{2s}{2k+1} \right) \mathcal{D}_{(2)}^k \mathcal{D}_{(1)} \overline{\Phi} \mathcal{D}_{(2)}^{s-k-1} \mathcal{D}_{(1)} \Phi + \left( \frac{2s}{2k} \right) \mathcal{D}_{(2)}^k \overline{\Phi} \mathcal{D}_{(2)}^{s-k} \Phi \right\}. \quad (3.7) \]
Making use of the massless equations of motion, \( \mathcal{D}^2 \Phi = 0 \), one may check that this supermultiplet does obey the conservation equation (3.6).

Now we turn to the non-superconformal case, \( w \neq \frac{1}{2} \). Direct calculations give
\[ \mathcal{D}_{(-1)} J_{(2s)} = \mathcal{D}_{(1)} \mathbb{T}_{(2s-2)}, \quad (3.8a) \]
where we have denoted
\[ \mathbb{T}_{(2s-2)} = 2i(1-2w)S(2s+1)(s+1) \sum_{k=0}^{s-1} \frac{1}{2s-2k+1} (-1)^k \left( \frac{2s}{2k+1} \right) \mathcal{D}_{(2)}^k \overline{\Phi} \mathcal{D}_{(2)}^{s-k-1} \Phi. \quad (3.8b) \]
The trace multiplet \( \mathbb{T}_{(2s-2)} \) is covariantly linear,
\[ \mathcal{D}^2 \mathbb{T}_{(2s-2)} = 0, \quad (3.8c) \]
\[ \mathcal{D}^2 \mathbb{T}_{(2s-2)} = 0, \quad (3.8c) \]

\[ ^4 \text{In the flat superspace limit, the supercurrent (3.7) reduces to the one constructed in [36].} \]
as a consequence of the equations of motion and identities (A.2c). It is seen that $T_{(2s-2)}$ has non-zero real and imaginary parts,

$$T_{(2s-2)} = Y_{(2s-2)} - iZ_{(2s-2)}; \quad \bar{Y}_{(2s-2)} = Y_{(2s-2)}; \quad \bar{Z}_{(2s-2)} = Z_{(2s-2)},$$

except for the $s = 1$ case which is characterised by $\bar{Y} = 0$. For $s = 1$ the above results agree with [17]. The technical details of the derivation of (3.8) are collected in the appendix.

The above results can be used to derive higher-spin supercurrents in a non-minimal scalar supermultiplet model described by the action

$$S = -\int d^3x d^2\theta d^2\bar{\theta} E \bar{\Gamma} \Gamma, \quad \bar{D}^2 \Gamma = 0,$$

with $\Gamma$ being a complex linear superfield. The non-minimal theory (3.9) proves to be dual to (3.1) provided the $U(1)_R$ weight of $\Gamma$ is opposite to that of $\Phi$,

$$J \Gamma = w \Gamma.$$

Replacing $\Phi \rightarrow \bar{\Gamma}$ and $\bar{\Phi} \rightarrow \Gamma$ in (3.8) gives the higher-spin supercurrents in the non-minimal theory (3.9), which is similar to the 4D case [37, 38].

### 3.2 Massive model

Let us add a mass term to the functional (3.1) and consider the following action

$$S = \int d^3x d^2\theta d^2\bar{\theta} E \Phi \Phi + \left\{ \frac{1}{2} \int d^3x d^2\theta E m\Phi^2 + \text{c.c.} \right\},$$

with $m$ a complex mass parameter. In the $m \neq 0$ case, the $U(1)_R$ weight of $\Phi$ is uniquely fixed to be $w = 1$, in order for the action to be $R$-invariant.

Making use of the massive equations of motion

$$-\frac{1}{4} \bar{D}^2 \Phi + \bar{m} \Phi = 0, \quad -\frac{1}{4} D^2 \bar{\Phi} + m \Phi = 0,$$

we obtain

$$D_{(-1)} J_{(2s)} = -2i S(2s + 1)(s + 1) D_{(1)} \sum_{k=0}^{s-1} \frac{1}{2s - 2k + 1} (-1)^k \begin{pmatrix} 2s \\ 2k + 1 \end{pmatrix} \times \bar{D}^k (2) \bar{\Phi} \bar{D}^{s-k-1} \Phi$$

$^5$Unlike eq. (1.1b), the above condition on $\Gamma$ is the only constraint obeyed by $\Gamma$. 


\[ + \bar{m} (-1)^s (2s + 1) \sum_{k=0}^{s-1} \left\{ 1 + (-1)^s \frac{2k + 1}{2s - 2k + 1} \right\} (-1)^k \left( \frac{2s}{2k + 1} \right) \times \mathcal{D}^k \Phi \mathcal{D}^{s-k-1} \bar{\mathcal{D}} \Phi , \]  

(3.13)

where \( J_{(2s)} \) is defined by (3.7). We observe that (3.13) can also be written in the form

\[
\mathcal{D}(-1) J_{(2s)} = \frac{1}{2} (-1)^s \mathcal{D}(-1) \sum_{k=0}^{s-1} (-1)^k \left( \frac{2s}{2k + 1} \right) \mathcal{D}^k \mathcal{D} \Phi \mathcal{D}^{s-k-1} \bar{\mathcal{D}} \Phi \\
- \frac{1}{2} \mathcal{D} \sum_{k=0}^{s-1} (2k + 1)(-1)^k \left( \frac{2s}{2k + 1} \right) \mathcal{D}^k \mathcal{D} \Phi \mathcal{D}^{s-k-1} \mathcal{D} \Phi \\
+ 2i \mathcal{S} \mathcal{D} \sum_{k=0}^{s-1} [(2k + 1) + (-1)^s] \left( \frac{2s}{2k + 1} \right) \times (-1)^k \left( \frac{2s}{2k + 1} \right) \mathcal{D}^k \mathcal{D} \Phi \mathcal{D}^{s-k-1} \mathcal{D} \Phi \\
+ i [1 + (-1)^s] \sum_{k=0}^{s-1} (2k + 1)(-1)^k \left( \frac{2s}{2k + 1} \right) \times \mathcal{D}^k \mathcal{D} \Phi \mathcal{D}^{s-k-1} \mathcal{D} \Phi . \]  

(3.14)

Thus, for all odd values of \( s \),

\[ s = 2n + 1 , \quad n = 0, 1, \ldots , \]  

(3.15a)

we end up with the conservation equation

\[ \mathcal{D}(-1) \hat{J}_{(2s)} = \mathcal{D} \mathcal{D} \hat{T}_{(2s-2)} \]  

(3.15b)

where we have denoted

\[
\hat{J}_{(2s)} = J_{(2s)} - \frac{1}{2} \sum_{k=0}^{s} (-1)^k \left( \frac{2s}{2k + 1} \right) \mathcal{D}^k \mathcal{D} \Phi \mathcal{D}^{s-k-1} \mathcal{D} \Phi , \]  

(3.15c)

\[
\hat{T}_{(2s-2)} = - \frac{1}{2} \sum_{k=0}^{s-1} (2k + 1)(-1)^k \left( \frac{2s}{2k + 1} \right) \mathcal{D}^k \mathcal{D} \Phi \mathcal{D}^{s-k-1} \mathcal{D} \Phi \\
+ 2i \mathcal{S} \sum_{k=0}^{s-1} [(1 - s)(2k + 1) + 2s^2] (-1)^k \left( \frac{2s}{2k + 1} \right) \mathcal{D}^k \mathcal{D} \Phi \mathcal{D}^{s-k-1} \mathcal{D} \Phi . \]  

(3.15d)

The trace multiplet \( \hat{T}_{(2s-2)} \) is covariantly linear,

\[ \mathcal{D}^2 \hat{T}_{(2s-2)} = 0 , \quad \mathcal{D}^2 \hat{T}_{(2s-2)} = 0 . \]  

(3.15e)
The conservation equation defined by eqs. (3.15b) and (3.15e) coincides with that defined by eqs. (3.8a) and (3.8c).

The above consideration demonstrates that in the massive case higher-spin supercurrents \( \hat{J}_{(2s)} \) exist only for the odd values of \( s \), eq. (3.15a). This conclusion is analogous to the earlier results in four dimensions \[39, 40, 38\]. As was demonstrated \[38\] in the 4D case, the even values of \( s \) are also allowed provided there are several massive chiral superfields in the theory. The analysis of \[38\] may be extended to the 3D (2,0) AdS case.

### 4 Massless higher-spin gauge theories

The explicit structure of the higher-spin supercurrent defined by eqs. (3.8a) and (3.8c) allows us to develop two off-shell formulations for a massless multiplet of half-integer superspin \( (\frac{s + 1}{2}) \), for every integer \( s > 0 \). We will call them type II and type III models in order to comply with the terminology introduced in \[17\] for the minimal formulations of \( \mathcal{N} = 2 \) supergravity.

#### 4.1 Type II series

Given a positive integer \( s \geq 2 \), we propose to describe a massless multiplet of half-integer superspin \( (s + \frac{1}{2}) \) in terms of the following dynamical variables:

\[
\mathcal{V}^{(II)}_{(s+\frac{1}{2})} = \left\{ \mathcal{H}_{\alpha_{(2s)}}, \mathcal{L}_{\alpha_{(2s-2)}} \right\} .
\]

(4.1)

Here \( \mathcal{H}_{\alpha_{(2s)}} = \mathcal{H}_{(\alpha_1...\alpha_{2s})} \) and \( \mathcal{L}_{\alpha_{(2s-2)}} = \mathcal{L}_{(\alpha_1...\alpha_{2s-2})} \) are unconstrained real tensor superfields. We postulate gauge transformations for the dynamical superfields:

\[
\delta_{\lambda} \mathcal{H}_{\alpha_{(2s)}} = \mathcal{D}_{(\alpha_1 \lambda \alpha_{2s-1})} - \mathcal{D}_{(\alpha_1 \bar{\lambda} \alpha_{2s-1})} ,
\]

(4.2a)

\[
\delta_{\lambda} \mathcal{L}_{\alpha_{(2s-2)}} = -\frac{i}{2} \left( \bar{\mathcal{D}}^{\beta} \lambda_{\beta\alpha_{(2s-2)}} + \mathcal{D}^{\beta} \bar{\lambda}_{\beta\alpha_{(2s-2)}} \right) ,
\]

(4.2b)

where the gauge parameter \( \lambda_{\alpha_{(2s-1)}} \) is unconstrained complex. In order for \( \delta_{\lambda} \mathcal{H}_{\alpha_{(2s)}} \) and \( \delta_{\lambda} \mathcal{L}_{\alpha_{(2s-2)}} \) to be real, \( \lambda_{\alpha_{(2s-1)}} \) must be charged under the \( R \)-symmetry group \( U(1)_R \):

\[
J \lambda_{\alpha_{(2s-1)}} = \lambda_{\alpha_{(2s-1)}}, \quad J \bar{\lambda}_{\alpha_{(2s-1)}} = -\bar{\lambda}_{\alpha_{(2s-1)}} .
\]

(4.3)

Equation (4.2a) is the gauge transformation law of a conformal superspin-\( (s + \frac{1}{2}) \) gauge multiplet \[10\]. It is natural to interpret \( \mathcal{L}_{\alpha_{(2s-2)}} \) as a compensating multiplet.
We postulate the compensator $\mathcal{L}_{\alpha(2s-2)}$ to have its own gauge freedom of the form
\begin{equation}
\delta_{\xi} \mathcal{L}_{\alpha(2s-2)} = \xi_{\alpha(2s-2)} + \bar{\xi}_{\alpha(2s-2)}, \quad \bar{D}_{\beta} \xi_{\alpha(2s-2)} = 0,
\end{equation}
with the gauge parameter $\xi_{\alpha(2s-2)}$ being covariantly chiral, but otherwise arbitrary. It should be pointed out that in (1,1) AdS superspace covariantly chiral superfields exist only in the scalar case, since the constraint $\bar{D}_{\beta} \Psi_{\alpha(n)} = 0$ is inconsistent for $n > 0$. Therefore, the gauge transformation law (4.4) is specific for the (2,0) AdS supersymmetry.

Associated with $\mathcal{L}_{\alpha(2s-2)}$ is the real field strength
\begin{equation}
\mathbb{L}_{\alpha(2s-2)} = i\bar{D}_{\beta} \bar{D}_{\beta} \mathcal{L}_{\alpha(2s-2)}, \quad \mathbb{L}_{\alpha(2s-2)} = \mathbb{L}_{\alpha(2s-2)},
\end{equation}
which is invariant under the gauge transformations (4.4), $\delta_{\xi} \mathbb{L}_{\alpha(2s-2)} = 0$. It is not difficult to see that $\mathbb{L}_{\alpha(2s-2)}$ is a covariantly linear superfield,
\begin{equation}
\mathcal{D}^2 \mathbb{L}_{\alpha(2s-2)} = 0.
\end{equation}
From (4.2b) we can read off the gauge transformation of the field strength
\begin{equation}
\delta_{\lambda} \mathbb{L}_{\alpha(2s-2)} = \frac{1}{4} \left( \mathcal{D}^\beta \mathcal{D}^2 \lambda_{\alpha(2s-2)} - \bar{\mathcal{D}}^\beta \mathcal{D}^2 \bar{\lambda}_{\alpha(2s-2)} \right).
\end{equation}

Modulo an overall normalisation factor, there is a unique quadratic action which is invariant under the gauge transformations (4.2). It is given by
\begin{equation}
S^{(II)}_{(s+\frac{1}{2})}[\mathcal{F}_{\alpha(2s)}, \mathcal{L}_{\alpha(2s-2)}] = \left( -\frac{1}{2} \right)^s \int d^3x d^2\theta d^2\bar{\theta} E \left\{ \frac{1}{8} \mathcal{F}_{\alpha(2s)} D^\beta \mathcal{D}^\gamma \mathcal{D}_{\beta} \mathcal{F}_{\gamma(2s)} 
\right.

- \frac{s}{8} \left[ (D^\beta , D^\gamma) \mathcal{F}_{\alpha(2s-2)}^{\beta \gamma} \right] [D^\beta , \bar{D}^\rho] \mathcal{F}_{\rho \alpha(2s-2)}

+ \frac{s}{2} \left[ (D_{\beta} , D_{\gamma}) \mathcal{F}_{\beta \gamma} \mathcal{F}_{\alpha(2s-2)} \right] D^\rho \mathcal{F}_{\rho \alpha(2s-2)} + 2is \mathcal{S} \mathcal{F}_{\alpha(2s)} D^\beta \mathcal{D}_{\beta} \mathcal{F}_{\alpha(2s)}

- \frac{2s - 1}{2} \left[ \mathbb{L}_{\alpha(2s-2)} D^\beta , D^\gamma \mathcal{F}_{\beta \gamma \alpha(2s-2)} + 2 \mathbb{L}_{\alpha(2s-2)} \mathbb{L}_{\alpha(2s-2)} \right]

- \frac{(s - 1)(2s - 1)}{4s} \mathcal{D}_{\beta} \mathcal{L}_{\beta \alpha(2s-3)} D^\beta D^\gamma \mathcal{L}_{\gamma \alpha(2s-3)}

\left. - 4(2s - 1) \mathcal{S} \mathcal{L}_{\alpha(2s-2)} \mathbb{L}_{\alpha(2s-2)} \right\}.
\end{equation}

By construction, the action is also invariant under (4.4).

Setting $s = 1$ in (4.8) gives the linearised action for (2,0) AdS supergravity, which was originally derived in section 10.1 of Ref. [17]. It should be remarked that the second last term in (4.8) is not defined in the $s = 1$ case. However, this term contains an overall numerical factor $(s - 1)$ and therefore it does not contribute for $s = 1$.

\textsuperscript{6}Ref. [17] made use of the curvature parameter $\rho$, which is related to our $\mathcal{S}$ as $\rho = 4\mathcal{S}$. 

10
4.2 Type III series

Our second model for the massless superspin-\((s + 1/2)\) multiplet is realised in terms of dynamical variables that are completely similar to (4.1),

\[ V^{(\text{III})}_{(s+1/2)} = \left\{ \mathcal{H}_{\alpha(2s)}, \mathcal{V}_{\alpha(2s-2)} \right\}. \] (4.9)

Here \( \mathcal{H}_{\alpha(2s)} \) and \( \mathcal{V}_{\alpha(2s-2)} \) are unconstrained real tensor superfields. The only difference from the type II case consists in a different gauge transformation law for the compensator \( \mathcal{V}_{\alpha(2s-2)} \). We postulate the following gauge transformation laws:

\[
\begin{align*}
\delta_\lambda \mathcal{H}_{\alpha(2s)} &= \bar{D}(\lambda_{\alpha_1 \ldots \alpha_{2s}} - D(\lambda_{\alpha_1 \ldots \alpha_{2s}}), \quad (4.10a) \\
\delta_\lambda \mathcal{V}_{\alpha(2s-2)} &= \frac{1}{2s} \left( \bar{D}^\beta \lambda_{\beta \alpha(2s-2)} - D^\beta \bar{\lambda}_{\beta \alpha(2s-2)} \right), \quad (4.10b)
\end{align*}
\]

where the gauge parameter \( \lambda_{\alpha(2s-1)} \) is unconstrained complex. The compensator \( \mathcal{V}_{\alpha(2s-2)} \) is required to have its own gauge freedom of the form

\[
\delta_\xi \mathcal{V}_{\alpha(2s-2)} = \xi_{\alpha(2s-2)} + \bar{\xi}_{\alpha(2s-2)}, \quad \bar{D}_\beta \xi_{\alpha(2s-2)} = 0, \quad (4.11)
\]

with the gauge parameter \( \xi_{\alpha(2s-2)} \) being covariantly chiral, but otherwise arbitrary.

A unique gauge-invariant action is given by

\[
S^{(\text{III})}_{(s+1/2)} = \left( -\frac{1}{2} \right)^s \int d^3x d^2\theta d^2\bar{\theta} E \left\{ \frac{1}{8} \mathcal{H}^{\alpha(2s)} D^\beta D^2 D_\beta \mathcal{H}^{\alpha(2s)} \\
- \frac{1}{16} ([D_\beta, \bar{D}_\gamma] \mathcal{H}^{\beta\gamma\alpha(2s-2)}) [D^\delta, \bar{D}^\rho] \mathcal{H}^{\delta\rho\alpha(2s-2)} \\
+ \frac{1}{4} (D_\beta \bar{D}^\gamma \mathcal{H}^{\beta\gamma\alpha(2s-2)}) D^\delta \bar{D}^2 \mathcal{H}^{\delta\gamma\alpha(2s-2)} - iS \mathcal{H}^{\alpha(2s)} D^\delta \bar{D}^2 \mathcal{H}^{\alpha(2s)} \\
- \frac{2s - 1}{2} \left( V^{\alpha(2s-2)} D^\gamma \bar{\mathcal{H}}^{\beta\gamma\alpha(2s-2)} + \frac{1}{2} V^{\alpha(2s-2)} V^{\alpha(2s-2)} \right) \\
+ 2s(2s - 1) S \mathcal{V}^{\alpha(2s-2)} V^{\alpha(2s-2)} \\
+ \frac{1}{8} (s - 1)(2s - 1) D_\beta \mathcal{V}^{\beta\alpha(2s-3)} D^\gamma D^\gamma \mathcal{V}^{\gamma\alpha(2s-3)} \right\}. \quad (4.12)
\]

This action involves the real linear field strength

\[ \mathcal{V}_{\alpha(2s-2)} = iD^\beta D_\beta \mathcal{V}_{\alpha(2s-2)}, \quad (4.13) \]

which is invariant under (4.11). It varies under the transformation (4.10) as

\[
\delta_\lambda \mathcal{V}_{\alpha(2s-2)} = \frac{i}{4s} \left( D^\beta \bar{D}^2 \lambda^{\beta\alpha(2s-2)} + D^\beta \bar{D}^2 \bar{\lambda}^{\beta\alpha(2s-2)} \right). \quad (4.14)
\]
Setting $s = 1$ in (4.12) gives the type III minimal supergravity action in (2,0) AdS superspace, which was originally derived in section 10.2 of [17].

5 Discussion

In this paper we did not carry out a systematic analysis (similar to that given by Dumitrescu and Seiberg [42] for ordinary supercurrents in Minkowski space) of the higher-spin supercurrent (1.1). The explicit form of the multiplet of currents was deduced from the consideration of simple dynamical systems in (2,0) AdS superspace. However, the formal consistency of (1.1) follows from the structure of the massless higher-spin gauge theories constructed in section 4. For instance, within the framework of the type II formulation let us couple the prepotentials $\mathcal{F}_{\alpha(2s)}$ and $\mathcal{L}_{\alpha(2s-2)}$ to external sources

$$S_{\text{source}}^{(s+\frac{1}{2})} = \int d^3x d^2\theta d^2\bar{\theta} E \left\{ \mathcal{F}_{\alpha(2s)} J_{\alpha(2s)} - 2 \mathcal{L}_{\alpha(2s-2)} Z_{\alpha(2s-2)} \right\} .$$

(5.1)

Requiring $S_{\text{source}}^{(s+\frac{1}{2})}$ to be invariant under the gauge transformations (4.4) tells us that the real supermultiplet $Z_{\alpha(2s-2)}$ is covariantly linear,

$$\bar{D}^2 Z_{\alpha(2s-2)} = 0 .$$

(5.2)

If we also require $S_{\text{source}}^{(s+\frac{1}{2})}$ to be invariant under the gauge transformations (4.2), we obtain the conservation equation

$$\bar{D}^\beta J_{\beta\alpha_1...\alpha_{2s-1}} = i\bar{D}_{(\alpha_1} Z_{\alpha_2...\alpha_{2s-1})} .$$

(5.3)

Additionally, taking the type III formulation into account leads to the general conservation equation

$$\bar{D}^\beta J_{\beta\alpha_1...\alpha_{2s-1}} = \mathcal{D}_{(\alpha_1} (Y_{\alpha_2...\alpha_{2s-1}) + iZ_{\alpha_2...\alpha_{2s-1}}) ,$$

(5.4)

where the real trace supermultiplets $Y_{\alpha(2s-2)}$ and $Z_{\alpha(2s-2)}$ are covariantly linear.

An improvement transformation exists for the higher-spin supercurrent multiplet (1.1). Let us introduce

$$J_{\alpha(2s)} := J_{\alpha(2s)} + [\mathcal{D}_{(\alpha_1}, \mathcal{D}_{\alpha_2}] S_{\alpha_3...\alpha_{2s}} + 2 \mathcal{D}_{(\alpha_1} \mathcal{R}_{\alpha_2...\alpha_{2s})} ,$$

(5.5a)

Type III supergravity is known only at the linearised level. In the super-Poincaré case, it is a 3D analogue of the massless superspin-3/2 multiplet proposed in [41].
\[ \tilde{Y}_{\alpha(2s-2)} := \mathcal{Y}_{\alpha(2s-2)} - iD^\gamma \tilde{D}_\gamma \mathbb{R}_{\alpha(2s-2)} + 4(s + 1)S\mathbb{R}_{\alpha(2s-2)} + \frac{2}{s}(s - 1)D^\beta (\alpha_1 \mathbb{R}_{\alpha_2...\alpha_{2s-2}}) \beta, \] (5.5b)

\[ \tilde{Z}_{\alpha(2s-2)} := \mathcal{Z}_{\alpha(2s-2)} - i s + 1 s D^\gamma \tilde{D}_\gamma S_{\alpha(2s-2)} - 4(s + 1)S\mathbb{S}_{\alpha(2s-2)} - \frac{2}{s}(s - 1)D^\beta (\alpha_1 S_{\alpha_2...\alpha_{2s-2}}) \beta. \] (5.5c)

with \( S_{\alpha(2s-2)} \) and \( \mathbb{R}_{\alpha(2s-2)} \) real linear superfields. One may check that \( \tilde{J}_{\alpha(2s)}, \tilde{Y}_{\alpha(2s-2)} \) and \( \tilde{Z}_{\alpha(2s-2)} \) obey the conservation equation and constraints described by (1.1). In the \( s = 1 \) case, we reproduce the result given in section 10.4 of [17].

There is one special feature of the supergravity case, \( s = 1 \), for which the supercurrent conservation equation takes the form [17]

\[ \bar{D}^\beta J_{\beta\alpha} = \bar{D}_\alpha (\mathcal{Y} + i\mathcal{Z}), \] (5.6)

with the real trace supermultiplets \( \mathcal{Y} \) and \( \mathcal{Z} \) being covariantly linear. Building on the thorough analysis of [42], it was pointed out in [17] that there exists a well-defined improvement transformation that results with \( \mathcal{Y} = 0 \). For all the supersymmetric field theories in (2,0) AdS superspace considered in [17], the supercurrent is characterised by the condition \( \mathcal{Y} = 0 \). Actually, this condition is easy to explain. The point is that every 3D \( \mathcal{N} = 2 \) supersymmetric field theory with U(1) \( R \)-symmetry may be coupled to the (2,0) AdS supergravity, which implies \( \mathcal{Y} = 0 \) upon freezing the supergravity multiplet to its maximally supersymmetric (2,0) AdS background. However, in the higher-spin case it no longer seems possible to improve the trace supermultiplet \( \mathcal{Y}_{\alpha(2s-2)} \) to vanish, as our analysis in section 3 indicates.

The massless models (4.8) and (4.12) describe no propagating degrees of freedom. However, in conjunction with the superconformal higher-spin actions in conformally flat backgrounds proposed in [10] they can be used to construct topologically massive higher-spin supersymmetric theories. Specifically, let us consider the following gauge-invariant models:

\[ S^{(II)}_{\text{massive}} = \kappa S_{\text{SCS}}[\mathcal{H}_{\alpha(2s)}] + m^{2s-1}S^{(II)}_{(s+\frac{1}{2})}[\mathcal{H}_{\alpha(2s)}], \] (5.7a)

\[ \text{with } \mathcal{H}_{\alpha(2s)} \text{ being the flat-superspace covariant derivatives. In Minkowski superspace eq. } (5.6) \text{ implies } \partial^\alpha J_{\alpha\beta} = iD^\alpha \tilde{D}_\alpha \mathcal{Y}, \text{ and therefore } \mathcal{Y} = iD^\alpha \tilde{D}_\alpha \mathbb{R} \text{ for some real linear superfield } \mathbb{R}. \text{ If we now apply the flat-superspace version of (5.5) with } \mathbb{S} = 0, \text{ we will end up with } \mathcal{Y} = 0. \]
\[ S_{\text{massive}}^{(III)} = \kappa S_{\text{SCS}}[\mathcal{H}_\alpha(2s)] + m^{2s-1} S_{(s+\frac{1}{2})}^{(III)}[\tilde{\mathcal{H}}_\alpha(2s), \mathcal{W}_\alpha(2s-2)] , \]  

(5.7b)

with \( \kappa \) and \( m \) dimensionless and massive parameters, respectively. Here

\[ S_{\text{SCS}}[\mathcal{H}_\alpha(2s)] = -\frac{(-1)^s}{2s+1} \int d^3x d^2\theta d^2\bar{\theta} E \mathcal{H}_\alpha^{(2s)} \mathcal{W}_\alpha^{(2s)}(\mathcal{S}) \]  

(5.8)

is the superconformal higher-spin action [10], with \( \mathcal{W}_\alpha^{(2s)}(\mathcal{S}) = \bar{\mathcal{W}}_\alpha^{(2s)}(\mathcal{S}) \) being the higher-spin super-Cotton tensor. It is the unique descendant of \( \mathcal{H}_\alpha^{(2s)} \) with the following properties: (i) \( \mathcal{W}_\alpha^{(2s)} \) is invariant under the gauge transformations (4.2a); (ii) \( \mathcal{W}_\alpha^{(2s)} \) obeys the conservation equations

\[ \bar{D}_\beta \bar{W}_{\beta\alpha_1...\alpha_{2s-1}} = 0 , \quad D^\beta \mathcal{W}_{\beta\alpha_1...\alpha_{2s-1}} = 0 . \]  

(5.9)

We believe that the higher-derivative actions (5.7a) and (5.7b) describe the on-shell massive superspin-\((s+\frac{1}{2})\) multiplets formulated in [43].

For a positive integer \( n > 0 \), a massive on-shell multiplet of superspin \((n+1)/2\) is described by a real symmetric rank-\(n\) spinor \( T_\alpha^{(n)} \) subject to the constraints [43]

\[ \bar{D}_\gamma T_{\alpha_1...\alpha_n} = (\bar{D}^\gamma D_a + (n+2)iSD^\gamma \bar{D}_\gamma - n(n+2)S^2)T_{\alpha_1...\alpha_n} , \]  

(5.10a)

\[ \left(\frac{i}{2} \bar{D}^\gamma \bar{D}_\gamma + m\right)T_{\alpha_1...\alpha_n} = 0 . \]  

(5.10b)

It may be shown that

\[ \left(\frac{i}{2} \bar{D}^\gamma \bar{D}_\gamma\right)^2 T_{\alpha_1...\alpha_n} = \left(D^a D_a + (n+2)iSD^\gamma \bar{D}_\gamma - n(n+2)S^2\right)T_{\alpha_1...\alpha_n} , \]  

(5.11)

where the second term on the right can be rewritten as follows:

\[ \frac{i}{2} \bar{D}^\gamma \bar{D}_\gamma T_{\alpha_1...\alpha_n} = \bar{D}_\gamma (\alpha_1 T_{\alpha_2...\alpha_n})\gamma + (n+2)S T_{\alpha_1...\alpha_n} . \]  

(5.12)

At the component level, the equations (5.10) may be shown to describe the on-shell massive fields in AdS3 introduced in [44, 45].

It is possible to construct Lagrangian models that lead directly to the equations (5.10), by generalising the flat-space bosonic constructions of [46, 47]. Such a model is formulated in terms of a real symmetric rank-\(n\) spinor superfield \( \mathcal{H}_\alpha^{(n)} \)

\[ S_{\text{massive}}[\mathcal{H}_\alpha^{(n)}] = -\frac{i^n}{2^{[n/2]+1}} \frac{\kappa}{m} \int d^3x d^2\theta d^2\bar{\theta} E \mathcal{W}_\alpha^{(n)}(\mathcal{S}) \{ m + \frac{i}{2} \bar{D}^\gamma \bar{D}_\gamma \} \mathcal{H}_\alpha^{(n)} , \]  

(5.13)

\footnote{In the case of Minkowski superspace, this may be proved in complete analogy with the analysis given in [20].}
where $\mathfrak{W}_{\alpha(n)}(\mathcal{H})$ is the higher-spin super-Cotton tensor associated with $\mathfrak{H}_{\alpha(n)}$ \cite{10}. The action is invariant under gauge transformations

$$
\delta_\lambda \mathfrak{H}_{\alpha(n)} = \mathcal{D}_{(\alpha_1, \lambda_{\alpha_2...\alpha_n})} - (-1)^n \mathcal{D}_{(\alpha_1, \bar{\lambda}_{\alpha_2...\alpha_n})},
$$

(5.14)

with the gauge parameter $\lambda_{\alpha(n-1)}$ being unconstrained complex. The gauge invariance of (5.13) follows from the properties that $\mathfrak{W}_{\alpha(n)}(\mathcal{H})$ is (i) gauge-invariant; and (ii) transverse linear, $\bar{\mathcal{D}}^\beta \mathfrak{W}_{\beta\alpha_1...\alpha_{n-1}} = \mathcal{D}^\beta \mathfrak{W}_{\beta\alpha_1...\alpha_{n-1}} = 0$. The action (5.13) becomes superconformal in the $m \to \infty$ limit.

It is of interest to carry out $\mathcal{N} = 2 \to \mathcal{N} = 1$ AdS superspace reduction of the massless models (4.8) and (4.12). Following \cite{48}, we can introduce a real basis for the spinor covariant derivatives which is obtained by replacing the complex operators $\mathcal{D}_\alpha$ and $\bar{\mathcal{D}}_\alpha$ with $\nabla^I_\alpha$, where $I = 1, 2$, defined by

$$
\mathcal{D}_\alpha = \frac{1}{\sqrt{2}}(\nabla^1_\alpha - i \nabla^2_\alpha), \quad \bar{\mathcal{D}}_\alpha = -\frac{1}{\sqrt{2}}(\nabla^1_\alpha + i \nabla^2_\alpha).
$$

(5.15)

Defining $\nabla_a = \mathcal{D}_a$, the new $(2,0)$ AdS covariant derivatives satisfy the algebra

$$
\{\nabla^I_\alpha, \nabla^J_\beta\} = 2i \delta^{IJ} \nabla_\alpha \beta - 4i \delta^{IJ} \varepsilon M_{\alpha\beta} + 4 \varepsilon_{\alpha\beta} \varepsilon_{IJ} S J ,
$$

(5.16a)

$$
[\nabla_a, \nabla_I^J] = S(\gamma_a)_{\beta}^\gamma \nabla^J_\beta , \quad [\nabla_a, \nabla_b] = -4 S^2 M_{ab} .
$$

(5.16b)

The graded commutation relations for the operators $\nabla_a$ and $\nabla^I_\alpha$ have the following properties: (i) they do not involve $\nabla_2^\alpha$; and (ii) they are identical to those defining $\mathcal{N} = 1$ AdS superspace, $\text{AdS}^{3|2}$, see \cite{48} for the details. These properties mean that $\text{AdS}^{3|2}$ is naturally embedded in $(2,0)$ AdS superspace as a subspace. The Grassmann variables of $(2,0)$ AdS superspace, $\theta_\mu^I = (\theta_1^\mu, \theta_2^\mu)$, may be chosen in such a way that $\text{AdS}^{3|2}$ corresponds to the surface defined by $\theta_2^\mu = 0$. Every supersymmetric field theory in $(2,0)$ AdS superspace may be reduced to $\text{AdS}^{3|2}$. Carrying out the $\mathcal{N} = 2 \to \mathcal{N} = 1$ AdS superspace reduction of the massless models (4.8) and (4.12) will give a new understanding of the difference between these models. It will also uncover whether one of the massless models (4.8) and (4.12) contain any new $\mathcal{N} = 1$ supersymmetric higher spin actions compared with those derived in \cite{49,50}.

Acknowledgements:

SMK is grateful to Darren Grasso for comments on the manuscript, and to Gabriele Tartaglino-Mazzucchelli for pointing out important references. The work of JH is supported by an Australian Government Research Training Program (RTP) Scholarship. The work of SMK is supported in part by the Australian Research Council, project No. DP160103633.
A (2,0) AdS identities

The Lorentz generators with two vector indices \((M_{ab} = -M_{ba})\), one vector index \((M_a)\) and two spinor indices \((M_{\alpha\beta} = M_{\beta\alpha})\) are related to each other by the rules: \(M_a = \frac{1}{2} \varepsilon_{abc} M^{bc}\) and \(M_{\alpha\beta} = (\gamma^a)_{\alpha\beta} M_a\). These generators act on a vector \(V_c\) and a spinor \(\Psi_\gamma\) as follows:

\[
M_{ab} V_c = 2 \eta_{ca} V_b , \quad M_{\alpha\beta} \Psi_\gamma = \varepsilon_{\gamma(\alpha} \Psi_{\beta)} .
\] (A.1)

The covariant derivatives of (2,0) AdS superspace hold various identities, which can be easily derived from the covariant derivatives algebra (2.4). We have made use of the following identities:

\[
[D^\alpha, D^2] = 4i D^{\alpha\beta} D_\beta + 4i S D^\alpha - 8i S D^\beta M^{\alpha\beta} , \quad (A.2a)
\]

\[
[\bar{D}^\alpha, D^2] = -4i D^{\alpha\beta} D_\beta - 4i S D^\alpha - 8i S D^\beta M^{\alpha\beta} , \quad (A.2b)
\]

\[
[D_a, D^2] = 0 , \quad [\bar{D}_a, D^2] = 0 , \quad (A.2c)
\]

where \(D^2 = D^\alpha D_\alpha\), and \(\bar{D}^2 = \bar{D}_\alpha \bar{D}^\alpha\). These relations imply the identity

\[
D^\alpha \bar{D}^2 D_\alpha = \bar{D}_\alpha D^2 \bar{D}^\alpha ,
\] (A.3)

which guarantees the reality of the actions considered in the main body of the paper.

In deriving eq. (3.8), one may find the following identities useful. We start with the obvious relations

\[
\frac{\partial}{\partial \zeta^\alpha} D_{(2)} = 2i \zeta^\beta D_{\alpha\beta} , \quad (A.4a)
\]

\[
\frac{\partial}{\partial \zeta^\alpha} D_{(2)}^k = \sum_{n=1}^k D_{(2)}^{n-1} 2i \zeta^\beta D_{\alpha\beta} D_{(2)}^{k-n} , \quad k > 1 . \quad (A.4b)
\]

To simplify eq. (A.4b), we may push \(\zeta^\beta D_{\alpha\beta}\), say, to the left provided that we take into account its commutator with \(D_{(2)}\):

\[
[\zeta^\beta D_{\alpha\beta} , D_{(2)}] = -4i S^2 \zeta^\alpha \zeta^\beta \zeta^\gamma M_{\beta\gamma} .
\] (A.5)

Associated with the Lorentz generators are the operators

\[
M_{(2)} := \zeta^\alpha \zeta^\beta M_{\alpha\beta} , \quad (A.6)
\]

where \(M_{(2)}\) appears in the right-hand side of (A.5). This operator annihilates every superfield \(U_{(m)}(\zeta)\) of the form (3.3),

\[
M_{(2)} U_{(m)} = 0 . \quad (A.7)
\]
From the above consideration, it follows that
\[
[\zeta^\beta D_{\alpha\beta}, D^k_{(2)}] U(m) = 0 , \quad (A.8a)
\]
\[
\left( \frac{\partial}{\partial \zeta^\alpha} D^k_{(2)} \right) U(m) = 2i k \zeta^\beta D_{\alpha\beta} D^{k-1} U(m) . \quad (A.8b)
\]

We also state some other properties which we often use throughout our calculations
\[
D^2_{(1)} = 0 , \quad (A.9a)
\]
\[
[D_{(1)}, D_{(2)}] = [\bar{D}_{(1)}, D_{(2)}] = 0 , \quad (A.9b)
\]
\[
[D^\alpha, D_{(2)}] = 2i S \zeta^\alpha D_{(1)} , \quad (A.9c)
\]
\[
[D^\alpha, D^k_{(2)}] = 2i S k \zeta^\alpha D^{k-1} D_{(1)} , \quad (A.9d)
\]
\[
[D^\alpha, \zeta^\beta D_{\alpha\beta}] = 3S D_{(1)} . \quad (A.9e)
\]

References

[1] P. K. Townsend, “Cosmological constant in supergravity,” Phys. Rev. D 15, 2802 (1977);
[2] D. Z. Freedman and A. K. Das, “Gauge internal symmetry in extended supergravity,” Nucl. Phys. B 120, 221 (1977).
[3] S. M. Kuzenko and A. G. Sibiryakov, “Free massless higher-superspin superfields on the anti-de Sitter superspace” Phys. Atom. Nucl. 57, 1257 (1994) [Yad. Fiz. 57, 1326 (1994)] arXiv:1112.4612 [hep-th]].
[4] S. Ferrara, M. T. Grisaru and P. van Nieuwenhuizen, “Poincaré and conformal supergravity models with closed algebras,” Nucl. Phys. B 138, 430 (1978).
[5] S. Ferrara and P. van Nieuwenhuizen, “Tensor calculus for supergravity,” Phys. Lett. 76B, 404 (1978).
[6] W. Siegel, “Solution to constraints in Wess-Zumino supergravity formalism,” Nucl. Phys. B 142, 301 (1978).
[7] S. J. Gates Jr., M. T. Grisaru, M. Roček and W. Siegel, Superspace, or One Thousand and One Lessons in Supersymmetry, Benjamin/Cummings (Reading, MA), 1983, hep-th/0108200.
[8] I. L. Buchbinder and S. M. Kuzenko, Ideas and Methods of Supersymmetry and Supergravity or a Walk Through Superspace, IOP, Bristol, 1995 (Revised Edition: 1998).
[9] D. Butter and S. M. Kuzenko, “A dual formulation of supergravity-matter theories,” Nucl. Phys. B 854, 1 (2012) arXiv:1106.3038 [hep-th].
[10] J. Hutomo, S. M. Kuzenko and D. Ogburn, “N = 2 supersymmetric higher spin gauge theories and current multiplets in three dimensions,” arXiv:1807.09098 [hep-th].
[11] A. Achúcarro and P. K. Townsend, “A Chern-Simons action for three-dimensional anti-de Sitter supergravity theories,” Phys. Lett. B 180, 89 (1986).

[12] M. Roček and P. van Nieuwenhuizen, “$N \geq 2$ supersymmetric Chern-Simons terms as $d = 3$ extended conformal supergravity,” Class. Quant. Grav. 3, 43 (1986).

[13] B. M. Zupnik and D. G. Pak, “Superfield formulation of the simplest three-dimensional gauge theories and conformal supergravities,” Theor. Math. Phys. 77, 1070 (1988) [Teor. Mat. Fiz. 77, 97 (1988)].

[14] H. Nishino and S. J. Gates Jr., “Chern-Simons theories with supersymmetries in three dimensions,” Int. J. Mod. Phys. A 8, 3371 (1993).

[15] E. Bergshoeff, S. Cecotti, H. Samtleben and E. Sezgin, “Superconformal sigma models in three dimensions,” Nucl. Phys. B 838, 266 (2010) [arXiv:1002.4411 [hep-th]].

[16] S. M. Kuzenko, U. Lindström and G. Tartaglino-Mazzucchelli, “Off-shell supergravity-matter couplings in three dimensions,” JHEP 1103, 120 (2011) [arXiv:1101.4013 [hep-th]].

[17] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, “Three-dimensional $N=2$ (AdS) supergravity and associated supercurrents,” JHEP 1112, 052 (2011) [arXiv:1109.0496 [hep-th]].

[18] S. M. Kuzenko, U. Lindström, M. Roček, I. Sachs and G. Tartaglino-Mazzucchelli, “Three-dimensional $N=2$ supergravity theories: From superspace to components,” Phys. Rev. D 89, 085028 (2014) [arXiv:1312.4267 [hep-th]].

[19] P. S. Howe, J. M. Izquierdo, G. Papadopoulos and P. K. Townsend, “New supergravities with central charges and Killing spinors in 2+1 dimensions,” Nucl. Phys. B 467, 183 (1996) [arXiv:hep-th/9505032].

[20] S. M. Kuzenko and D. X. Ogburn, “Off-shell higher spin $N=2$ supermultiplets in three dimensions,” Phys. Rev. D 94, no. 10, 106010 (2016) [arXiv:1603.04668 [hep-th]].

[21] S. M. Kuzenko, V. V. Postnikov and A. G. Sibiryakov, “Massless gauge superfields of higher half-integer superspins,” JETP Lett. 57, 534 (1993) [Pisma Zh. Eksp. Teor. Fiz. 57, 521 (1993)].

[22] E. Bergshoeff, M. de Roo and B. de Wit, “Extended conformal supergravity,” Nucl. Phys. B 182, 173 (1981).

[23] M. F. Sohnius and P. C. West, “An alternative minimal off-shell version of $N=1$ supergravity,” Phys. Lett. B 105, 353 (1981).

[24] M. F. Sohnius and P. C. West, “The new minimal formulation of $N=1$ supergravity and its tensor calculus,” in Quantum Structure of Space and Time, M. J. Duff and C. J. Isham (Eds.), Cambridge University Press, Cambridge, 1982, pp. 187–222.

[25] M. Sohnius and P. C. West, “The tensor calculus and matter coupling of the alternative minimal auxiliary field formulation of $N = 1$ supergravity,” Nucl. Phys. B 198, 493 (1982).

[26] P. S. Howe and U. Lindström, “The supercurrent in five dimensions,” Phys. Lett. B 103, 422 (1981).

[27] P. S. Howe, “Off-shell $N=2$ and $N=4$ supergravity in five-dimensions,” in Quantum Structure of Space and Time, M. J. Duff and C. J. Isham, Cambridge University Press, 1982, pp. 239–253.
[28] S. Ferrara and B. Zumino, “Transformation properties of the supercurrent,” Nucl. Phys. B 87, 207 (1975).

[29] M. F. Sohnius, “The multiplet of currents for N=2 extended supersymmetry,” Phys. Lett. B 81, 8 (1979).

[30] M. P. Blencowe, “A consistent interacting massless higher spin field theory in $D = 2+1$,” Class. Quant. Grav. 6, 443 (1989).

[31] M. Henneaux and S. J. Rey, “Nonlinear $W_{\infty}$ as asymptotic symmetry of three-dimensional higher spin AdS gravity,” JHEP 1012, 007 (2010) [arXiv:1008.4579 [hep-th]].

[32] A. Campoleoni, S. Fredenhagen, S. Pfenninger and S. Theisen, “Asymptotic symmetries of three-dimensional gravity coupled to higher-spin fields,” JHEP 1011, 007 (2010) [arXiv:1008.4744 [hep-th]].

[33] V. Pestun and M. Zabzine, “Introduction to localization in quantum field theory,” J. Phys. A 50, no. 44, 443001 (2017) [arXiv:1608.02953 [hep-th]].

[34] B. Willett, “Localization on three-dimensional manifolds,” J. Phys. A 50, no. 44, 443006 (2017) [arXiv:1608.02958 [hep-th]].

[35] I. B. Samsonov and D. Sorokin, “Superfield theories on $S^3$ and their localization,” JHEP 1404, 102 (2014) [arXiv:1401.7952 [hep-th]].

[36] A. A. Nizami, T. Sharma and V. Umesh, “Superspace formulation and correlation functions of 3d superconformal field theories,” JHEP 1407, 022 (2014) [arXiv:1308.4778 [hep-th]].

[37] K. Koutrolikos, P. Koči and R. von Unge, “Higher spin superfield interactions with complex linear supermultiplet: conserved supercurrents and cubic vertices,” JHEP 1803, 119 (2018) [arXiv:1712.05150 [hep-th]].

[38] E. I. Buchbinder, J. Hutomo and S. M. Kuzenko, “Higher spin supercurrents in anti-de Sitter space,” JHEP 1809, 027 (2018) [arXiv:1805.08055 [hep-th]].

[39] I. L. Buchbinder, S. J. Gates Jr. and K. Koutrolikos, “Higher spin superfield interactions with the chiral supermultiplet: conserved supercurrents and cubic vertices,” Universe 4, no. 1, 6 (2018) [arXiv:1708.06262 [hep-th]].

[40] J. Hutomo and S. M. Kuzenko, “Non-conformal higher spin supercurrents,” Phys. Lett. B 778, 242 (2018) [arXiv:1710.10837 [hep-th]].

[41] I. L. Buchbinder, S. J. Gates Jr., W. D. Linch and J. Phillips, “New 4D, N = 1 superfield theory: Model of free massive superspin-3/2 multiplet,” Phys. Lett. B 535, 280 (2002) [arXiv:hep-th/0201096].

[42] T. T. Dumitrescu and N. Seiberg, “Supercurrents and brane currents in diverse dimensions,” JHEP 1107, 095 (2011) [arXiv:1106.0031 [hep-th]].

[43] S. M. Kuzenko, J. Novak and G. Tartaglino-Mazzucchelli, “Higher derivative couplings and massive supergravity in three dimensions,” JHEP 1509, 081 (2015) [arXiv:1506.09063 [hep-th]].
[44] S. Deger, A. Kaya, E. Sezgin and P. Sundell, “Spectrum of D = 6, N=4b supergravity on AdS in three-dimensions x S**3,” Nucl. Phys. B 536, 110 (1998) [hep-th/9804166].
[45] E. A. Bergshoeff, O. Hohm, J. Rosseel, E. Sezgin and P. K. Townsend, “On critical massive (super)gravity in adS3,” J. Phys. Conf. Ser. 314, 012009 (2011) [arXiv:1011.1153 [hep-th]].
[46] E. A. Bergshoeff, O. Hohm and P. K. Townsend, “On higher derivatives in 3D gravity and higher spin gauge theories,” Annals Phys. 325, 1118 (2010) [arXiv:0911.3061 [hep-th]].
[47] E. A. Bergshoeff, M. Kovacevic, J. Rosseel, P. K. Townsend and Y. Yin, “A spin-4 analog of 3D massive gravity,” Class. Quant. Grav. 28, 245007 (2011) [arXiv:1109.0382 [hep-th]].
[48] S. M. Kuzenko, U. Lindström and G. Tartaglino-Mazzucchelli, “Three-dimensional (p,q) AdS superspaces and matter couplings,” JHEP 1208, 024 (2012) [arXiv:1205.4622 [hep-th]].
[49] S. M. Kuzenko and M. Tsulaia, “Off-shell massive N=1 supermultiplets in three dimensions,” Nucl. Phys. B 914, 160 (2017) [arXiv:1609.06910 [hep-th]].
[50] S. M. Kuzenko and M. Ponds, “Topologically massive higher spin gauge theories,” arXiv:1806.06643 [hep-th].