Bulk and brane gauge propagator on 5d AdS black hole

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ABSTRACT

The bulk gauge fields on 5d AdS black hole are discussed. We construct the bulk (and the corresponding brane) gauge propagator when black hole has large radius. The properties of gauge and ghost propagators are studied in both, minkowski or euclidean signature. In euclidean formulation the propagator structure corresponds to the one of theory at finite temperature (which depends on coordinates). The decoupling of KK modes and localization of gauge fields on flat brane is demonstrated. We show that with such a bulk there is no natural solution of hierarchy problem.

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1. Assuming that observable Universe is multi-dimensional one the fundamental question appears: What is the form (and number) of extra dimensions? Are they planar or orbifolded? Within the recent braneworld scenario [1] the most likely possibility for Universe is 5d AdS space. The braneworld scenario with bulk 5d AdS is very attractive as it may have the superstring origin. Moreover, in such picture the four-dimensional world represents the brane (where graviton and matter fields are trapped) embedded into AdS bulk. The nice way to solve the hierarchy problem on the brane appears.

Nevertheless, one can admit another choice for bulk manifold, for example, AdS black hole. Indeed, like pure AdS space the AdS black hole may be (part of) superstring vacuum. The phase transition which may be interpreted as confinement-deconfinement transition in AdS/CFT set-up may occur between pure AdS and AdS black hole[2]. Moreover, the graviton is localized on the brane embedded in AdS black hole[3]. Hence, it is very interesting to understand the properties of such bulk choice and their possible relation with the phenomenology of the unified theories.

In the present Letter we consider bulk abelian gauge fields in 5d AdS black hole. The bulk (brane) gauge propagator is constructed for large black hole with minkowskii or euclidean signature. The decoupling of KK modes and localization of gauge fields on flat brane is demonstrated. It is shown that unlike to pure AdS bulk there is no natural solution for hierarchy problem. Brief discussion of possible phase transition and its consequences for bulk QFT is given.

2. In order to consider the propagators, we start with the action of the abelian gauge theory in the 5-dimensional curved background:

\[ S_0 = -\frac{1}{4} \int d^5 x \sqrt{-g} g^{\mu\nu} g^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma} , \quad F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} . \]  

The gauge fixing action \( S_f \) and the ghost action \( S_g \) are:

\[ S_f = \frac{1}{2\xi} \int d^5 x \sqrt{-g} (\nabla^\mu A_\mu)^2 , \quad S_g = \int d^5 x \sqrt{-gb} \nabla^\mu \partial_\mu c . \]  

Note that discussion of bulk gauge fields in AdS braneworlds has been given in refs.[4, 5, 6, 7, 8]. As a curved background, we consider the Schwarzschild-
Anti-de Sitter (SAdS) space with the flat horizon:

\[ ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -e^{2\rho}dt^2 + e^{-2\rho}dr^2 + r^2 \sum_{i=1,2,3} (dx^i)^2, \quad e^{2\rho} = \frac{r^2}{l^2} - \frac{\mu}{r^2}. \] (3)

In the following, \( \xi = -1 \). The simplest choice corresponds to the large black hole, that is, the horizon radius \( r_H = \frac{l^2}{l^2} \to \infty \). We now define a new coordinate \( s \) by \( s = r - r_H \), then \( e^{2\rho} \sim \frac{4r_H}{l^2} s \). With new coordinate the action looks as

\[
S \to \int d^5x \left[ A_r \left\{ - \frac{r_H^3}{2} \partial_t^2 A_r + \frac{2r_H^2}{l^2} s \Delta A_r + \frac{8r_H^5}{l^4} s^2 \partial_s^2 (s A_r) \right\} 
+ A_t \left\{ \frac{r_H^4}{32s^2} \partial_t^2 A_t - \frac{r_H^3}{2} \partial_s^2 A_t - \frac{l^2}{8s} \Delta A_t \right\} + \frac{r_H^5}{s} A_r \partial_t A_t + A_t \left\{ - \frac{l^2}{8s} \partial_s^2 A_t \right\} 
+ \frac{2r_H^2}{l^2} \partial_s (s \partial_s A_t) + \frac{1}{2r_H} \Delta A_t \right\} + b \left\{ - \frac{l^2r_H^2}{4s} \partial_s^2 c + r_H \Delta c + \frac{4r_H^4}{l^2} \partial_s (s \partial_s c) \right\} \right].
\]

As we consider the plane waves in the \( t \) and \( x^i \) directions, we replace \( \partial_t \) and \( \Delta \) by \( i\omega \) and \( -k^2 \), respectively. Then the equations of motion corresponding to the action (4) have the following form:

\[
0 = r_H^3 \omega^2 A_r - \frac{4r_H^2 k^2}{l^2} s A_r + \frac{16r_H^5}{l^4} s \partial_s^2 (s A_r) + \frac{r_H^3 \omega}{s} A_t ,
\]

(5)

\[
0 = - r_H^4 \omega^2 A_t - r_H^3 \partial_s^2 A_t + \frac{l^2 k^2}{4s} A_t + \frac{r_H^3 \omega}{s} A_r ,
\]

(6)

\[
0 = \frac{l^2 \omega^2}{8s} A_i + \frac{2r_H^2}{l^2} \partial_s (s \partial_s A_i) - \frac{k^2}{2r_H} A_i ,
\]

(7)

\[
0 = \frac{l^2 \omega^2}{4r_H s} \psi - \frac{k^2}{r_H} \psi + \frac{4r_H}{l^2} s \partial_s (s \partial_s \psi) \quad \psi = b, c .
\]

(8)

We should note the equations (7) and (8) for \( A_i \), \( c \) and \( b \), are identical with each other. If one defines a new complex field \( B \) by \( B \equiv s A_r + \frac{i l^2}{4r_H} A_t \), the equations (5) and (6) can be combined to give

\[
0 = \partial_s^2 B - \frac{l^2 k^2}{4r_H^3 s} B + \frac{1}{s^2} \left( \frac{l^4 \omega^2}{16r_H^2} + \frac{l^2 \omega}{4r_H} \right) B .
\]

(9)

By further defining

\[
B = s^{\frac{3}{2}} \tilde{B} , \quad s = \frac{r_H^3}{l^2 k^2} v^2 ,
\]

(10)
we can rewrite (9) in the following form:

\[
0 = \frac{d^2 \tilde{B}}{dv^2} + \frac{1}{v} \frac{d \tilde{B}}{dv} - \left\{ 1 + \frac{\nu_B^2}{v^2} \right\} \tilde{B}, \quad \nu_B^2 \equiv 1 - \frac{l_4^4 \omega^2}{4r_H^2} - \frac{l_2^2 \omega}{r_H},
\]

which is the modified Bessel’s derivative equation. The solution is given by the linear combination of the modified Bessel functions \( I_{\nu_B}(v) \) and \( K'_{\nu_B}(v) \). Then the general solution for \( B \) is given by

\[
B = s^{\frac{1}{2}} \left[ \alpha_0 I_{\nu_B} \left( \frac{lks_1^{\frac{1}{2}}}{r_H^2} \right) + \beta_0 K_{\nu_B} \left( \frac{lks_1^{\frac{1}{2}}}{r_H^2} \right) \right].
\]

Here \( \alpha_0 \) and \( \beta_0 \) are constants. On the other hand, defining \( v \) by (10), Eq.(7) can be rewritten as

\[
0 = \frac{d^2 \tilde{A}_i}{dv^2} + \frac{1}{v} \frac{d \tilde{A}_i}{dv} - \left\{ 1 + \frac{\nu_A^2}{v^2} \right\} A_i, \quad \nu_A^2 = -\frac{l_4^4 \omega^2}{4r_H^2},
\]

whose solution is again given by the modified Bessel functions. In general, \( \nu_A \) is imaginary. Then the general solution for \( A_i \), and similarly the solutions for (8) are given by (with constant \( \alpha_{c,b} \) and \( \beta_{c,b} \), \( \psi = c, b \))

\[
A_i = \alpha_i I_{\nu_A} \left( \frac{lks_1^{\frac{1}{2}}}{r_H^2} \right) + \beta_i K_{\nu_A} \left( \frac{lks_1^{\frac{1}{2}}}{r_H^2} \right), \quad \psi = \alpha_\psi I_{\nu_A} \left( \frac{lks_1^{\frac{1}{2}}}{r_H^2} \right) + \beta_\psi K_{\nu_A} \left( \frac{lks_1^{\frac{1}{2}}}{r_H^2} \right).
\]

Putting two branes at \( s = s_1 \) and \( s = s_2 \) \((0 < s_1 < s_2)\) one glues two bulk spaces whose boundaries are the branes and imposes \( Z_2 \) symmetry. Then the gauge and ghost fields should satisfy the following boundary conditions

\[
\left. \frac{\partial A_i}{\partial s} \right|_{s=s_{1,2}} = \left. \frac{\partial A_i}{\partial s} \right|_{s=s_{1,2}} = \left. \frac{\partial c}{\partial s} \right|_{s=s_{1,2}} = \left. \frac{\partial b}{\partial s} \right|_{s=s_{1,2}} = A_{r|s=s_{1,2}} = 0.
\]

In general, the boundary conditions at \( s = s_1 \) are not consistent with those at \( s = s_2 \). We denote the fields which satisfy the boundary conditions at \( s = s_a \) \((a = 1, 2)\) by \( A_{(a)\mu}, c_{(a)}, b_{(a)} \), etc. Using the boundary conditions (15), the ratios between the coefficients in (12) and (14) can be determined
as follows:

\[
\frac{\alpha(t)}{\beta(t)} = \left(\frac{1}{2} + \frac{\nu a}{2}\right) K_{\nu B}\left(\frac{1}{r_H^2}\frac{l_k s^2}{r_H^2}\right) + \frac{1}{2r_H^2} K_{\nu B}^{-1}\left(\frac{1}{r_H^2}\frac{l_k s^2}{r_H^2}\right),
\]

\[
\frac{\alpha(r)}{\beta(r)} = -\frac{\frac{1}{2} + \frac{\nu a}{2}}{I_{\nu B}\left(\frac{1}{r_H^2}\frac{l_k s^2}{r_H^2}\right)} I_{\nu B}\left(\frac{1}{r_H^2}\frac{l_k s^2}{r_H^2}\right) + \frac{1}{2r_H^2} I_{\nu B}^{-1}\left(\frac{1}{r_H^2}\frac{l_k s^2}{r_H^2}\right).
\]

\[
\left\{ \begin{array}{l}
\alpha(a) = \frac{\nu A}{2} K_{\nu A}\left(\frac{1}{r_H^2}\frac{l_k s^2}{r_H^2}\right) + \frac{1}{2r_H^2} K_{\nu A}^{-1}\left(\frac{1}{r_H^2}\frac{l_k s^2}{r_H^2}\right) \\
\beta(a) = -\frac{\nu A}{2} I_{\nu A}\left(\frac{1}{r_H^2}\frac{l_k s^2}{r_H^2}\right) + \frac{1}{2r_H^2} I_{\nu A}^{-1}\left(\frac{1}{r_H^2}\frac{l_k s^2}{r_H^2}\right)
\end{array} \right.
\]

(16)

Here \(\alpha_0\) and \(\beta_0\) in (12) are written as \(\alpha(a)_0 = \alpha(a)_t + i \frac{\beta_0^2}{4r_H^2}\alpha(a)_r\), \(\beta(a)_0 = \beta(a)_t + i \frac{\beta_0^2}{4r_H^2}\beta(a)_r\).

We now construct the propagators from the obtained classical solutions by following the procedure given in [9]. One first constructs the propagator \(G_{ij}\) for \(A_i\):

\[
G_{ij}(u, v) = N_1 \delta_{ij} \left\{ A_{(2)}(u) A_{(1)}(v) \theta(u - v) + A_{(2)}(v) A_{(1)}(u) \theta(v - u) \right\},
\]

\[
A_{(a)}(s) \equiv \alpha(a)_u I_{\nu A}\left(\frac{1}{r_H^2}\frac{l_k s^2}{r_H^2}\right) + \beta(a)_u K_{\nu A}\left(\frac{1}{r_H^2}\frac{l_k s^2}{r_H^2}\right).
\]

(17)

Here \(N\) is a normalization constant determined later. We should note that the above propagator satisfies the boundary conditions, on both of the branes, corresponding to (15): \(\frac{\partial G_{ij}(u, v)}{\partial u}\big|_{u=s_{1,2}} = \frac{\partial G_{ij}(u, v)}{\partial v}\big|_{v=s_{1,2}} = 0\). As \(A_{(a)}\) satisfies Eq.(7) one gets

\[
\frac{l^2 \omega^2}{8 u} G_{ij}(u, v) + \frac{2r_H^2}{l^2} \partial_u (u \partial_u G_{ij}(u, v)) - \frac{k^2}{2r_H} G_{ij}(u, v)
\]

\[
= \frac{2r_H^2 N}{l^2} \left( \frac{\partial A_{(2)}(u)}{\partial u} A_{(1)}(u) - A_{(2)}(u) \frac{\partial A_{(1)}(u)}{\partial u} \right) \delta(u - v).
\]

(18)
Then one can find the quantity \( u \left( \frac{\partial A_{(2)}(u)}{\partial u} A_{(1)}(u) - A_{(2)}(u) \frac{\partial A_{(1)}(u)}{\partial u} \right) \) does not depend on \( u \). We now can evaluate the quantity \( u \left( \frac{\partial A_{(2)}(u)}{\partial u} A_{(1)}(u) - A_{(2)}(u) \frac{\partial A_{(1)}(u)}{\partial u} \right) \) at \( u = s_1 \) by using the boundary condition (15) and the expressions in (16):

\[
u \left( \frac{\partial A_{(2)}(u)}{\partial u} A_{(1)}(u) - A_{(2)}(u) \frac{\partial A_{(1)}(u)}{\partial u} \right) = -\frac{1}{2} \left( \beta_{(1)} \alpha_{(2)} - \alpha_{(1)} \beta_{(2)} \right) . \quad (19)
\]

In the last line, we have used a formula for the modified Bessel functions:

\[
I_\nu(z) K_{\nu-1}(z) + I_{\nu-1}(z) K_\nu(z) = \frac{1}{z}.
\]

Since the r.h.s. of Eq. (18) should be \( \frac{1}{2} r_H^2 H \delta(u-v) \) since \( \sqrt{-g} = r_H^3 \), one finds the normalization constant should be

\[
N = -\frac{1}{2 r_H^2} \left( \beta_{(1)} \alpha_{(2)} - \alpha_{(1)} \beta_{(2)} \right).
\]

We now consider the correlation functions for \( B \). From the action (4), we find the correlator \( G_{BB^*}(u,v) \) between \( B \) and its complex conjugate vanishes. One should be careful for the fact that the boundary condition for \( A_r \) is different from \( A_t \) as in (15), we find the correlator between two \( B \) is given by

\[
G_{BB}(u,v) = -\frac{l^4}{16 r_H^2} \left\{ B_{(2)}(u) B_{(1)}(v) \theta(u-v) + B_{(2)}(v) B_{(1)}(u) \theta(v-u) \right\} \left( \beta_{0(1)} \alpha_{0(2)} - \alpha_{0(1)} \beta_{0(2)} \right) . \quad (20)
\]

Denoting the correlators between two \( A_t \)'s, two \( A_r \)'s, \( A_t \) and \( A_r \) by \( G_{tt}(u,v) \), \( G_{rr}(u,v) \), \( G_{tr}(u,v) = G_{rt}(v,u) \), respectively, one gets

\[
G_{BB}(u,v) = -\frac{l^4}{16 r_H^2} G_{tt}(u,v) + uv G_{rr}(u,v)
\]

\[
+ \frac{i l^2}{4 r_H^2} \left( uG_{rt}(u,v) + vG_{tr}(u,v) \right) , \quad 0 = G_{BB^*}(u,v)
\]

\[
= \frac{l^4}{16 r_H^2} G_{tt}(u,v) + uv G_{rr}(u,v) + \frac{i l^2}{4 r_H} ( -uG_{rt}(u,v) + vG_{tr}(u,v) ) . \quad (21)
\]

Then

\[
G_{tt}(u,v) = -\frac{8 r_H^2}{l^4} \Re G_{BB}(u,v) \quad G_{rr}(u,v) = \frac{1}{uv} \Im G_{BB}(u,v) , \quad uG_{rt}(u,v) + vG_{tr}(u,v) = \frac{2 r_H}{l^2} \Im G_{BB}(u,v) . \quad (22)
\]
Here $\Re$ and $\Im$ express the real and imaginary parts, respectively.

Let us investigate the behavior of the obtained Green functions on the branes. In order to investigate the Kaluza-Klein (KK) mode, we consider the case that $\omega = 0$ and investigate where the poles of the Green functions exist with respect to $k^2$. We now consider $G_{ij}(u,v)$ with $u = v = s_1$ and $\omega = 0$. Since $\omega = 0$, $\nu_A$ in (13) vanishes. Since the radius $r_H$ is large, we can use the asymptotic expansion of the modified Bessel functions. Then we find the following expression of $G_{ij}$

$$G_{ij}(s_1, s_1)|_{\omega=0} \sim \delta_{ij} \frac{2}{k^2 r_H^2} \frac{\frac{1}{2} + \frac{2s_2}{s_1 s_2} (s_1 + s_2 \ln \frac{s_2}{s_1})}{s_1 - s_2 + \frac{2s_2}{s_1} \left( s_1^2 - s_2^2 + 2s_1 s_2 \ln \frac{s_2}{s_1} \right)}.$$  \hspace{1cm} (23)

The propagator (23) has a pole at

$$k^2 = 0$$ \hspace{1cm} (24)

and

$$-k^2 = m_1^2 \equiv \frac{8r_H^2 (s_1 - s_2)}{l^2 \left( s_1^2 - s_2^2 + 2s_1 s_2 \ln \frac{s_2}{s_1} \right)}. \hspace{1cm} (25)$$

The pole (24) corresponds to the zero mode and that of $k^2 = -m_1^2$ (25) to the first KK mode. When $k^2 \sim 0$, the propagator behaves as

$$G_{ij} \sim \frac{R_0 \delta_{ij}}{k^2}, \quad R_0 \equiv -\frac{1}{r_H^2 (s_1 - s_2)}, \hspace{1cm} (26)$$

and when $k^2 \sim -m_1^2$

$$G_{ij} \sim \frac{R_1 \delta_{ij}}{k^2 + m_1^2}, \quad R_1 \equiv -\frac{1 - \frac{2(s_1 - s_2) \left( s_1 + s_2 \ln \frac{s_2}{s_1} \right)}{s_1^2 - s_2^2 + 2s_1 s_2 \ln \frac{s_2}{s_1}}}{r_H^2 (s_1 - s_2)}. \hspace{1cm} (27)$$

$R_0$ and $R_1$ correspond to the square of the wave functions of zero mode and the first KK mode, respectively. Then $R_0$ and $R_1$ express the coupling of these modes on the brane. Eq.(26) indicates that the coupling of the KK mode might not be small. Eq.(25) tells, however, the mass of the KK modes is very large since we are considering the large black hole ($r_H \rightarrow \infty$). Then the KK modes decouple.
The above mass of the first KK mode (25) can be compared with the pure AdS case. By choosing the metric of the AdS as
\[ ds^2_{\text{AdS}} = \frac{l^2}{z^2} \left( dz^2 - dt^2 + \sum_{i=1,2,3} (dx^i)^2 \right), \tag{28} \]
we put two branes at \( z = z_1 = l \) and \( z = z_2 \). If \( z_1 < z_2 \) and \( 1/z_2 \) is the order of TeV, the brane at \( z = z_2 \) corresponds to the so-called TeV brane and that at \( z = z_1 \) to the Planck brane. Then the mass of the first KK mode is typically given by [10]
\[ \tilde{m}_1 = \frac{a}{z_2 \sqrt{2 \ln \left( \frac{z_2}{z_1} \right)}} . \tag{29} \]
Here \( a \) is a parameter describing the coupling of the gauge field to the TeV brane. Then if \( a \sim 0.1 \), the mass is weak scale. On the other hand, from (25), the mass corresponding to the first KK mode is given by
\[ \hat{m}_1^2 = \frac{m_1^2}{r_H^2} = \frac{8r_H(s_1 - s_2)}{l^2 \left( s_1^2 - s_2^2 + 2s_1s_2 \ln \frac{s_2}{s_1} \right)} . \tag{30} \]
The denominator \( r_H^2 \) in \( \frac{m_1^2}{r_H^2} \) comes from the factor \( r^2 \sim r_H^2 \) in front of \( \sum_{i=1,2,3} (dx^i)^2 \) in the metric (3). If \( s_1 \sim s_2 \sim l \) and they are of the order of the Planck length, the mass \( \hat{m}_1 \) is \( \hat{m}_1 \sim \frac{1}{\sqrt{lH}} \). Then the mass is much larger than the Planck mass for the large black hole.

It is interesting to understand if the hierarchy problem with AdS BH can be solved in the same way as in [1]. Naively as scale factor is almost constant for the large black hole, it would be difficult to realize the solution of hierarchy problem. Since \( \exp^{2\rho} \sim \frac{4r_H}{l^2} s \), the metric looks as:
\[ ds^2 = -\frac{4r_H}{l^2} dt^2 + \frac{l^2}{4r_H} ds^2 + r_H^2 \sum_{i=1,2,3} (dx^i)^2 . \tag{31} \]
Then on the brane at \( s = s_a \), if we redefine the coordinates by \( \tilde{t} = 2t \sqrt{\frac{r_H}{s}} \), \( \tilde{x}^i = lx^i \), the metric on the brane becomes flat
\[ ds^2_{\text{brane}} = \sum_{m,n=0}^3 \tilde{g}_{mn} dx_m dx_n \equiv \frac{r_H^2}{l^2} \left( -d\tilde{t}^2 + \sum_{i=1,2,3} (d\tilde{x}^i)^2 \right) . \tag{32} \]
We now consider the gauge field $\tilde{A}_m$ and the Higgs fields $\phi$ whose action on the brane is given by

$$S_{\tilde{A}, \phi} = \int d^4x \sqrt{-\tilde{g}} \left\{ \frac{1}{2} \left( \tilde{g}^{mn} \left( \partial_m \phi^* + ie\tilde{A}_m \phi^* \right) \left( \partial_n \phi - ie\tilde{A}_n \phi \right) - g^2 \left( \phi^* \phi - \lambda^2 \right)^2 \right) - \frac{1}{4} \tilde{g}^{mn} \eta^{kl} F_{mk} F_{nl} \right\}.$$

Here $\eta_{mn}$ is the metric of the flat Minkowski space. The couplings $e$ and $g$ are of the order of the unity and $\lambda$ could be of the order of the Planck scale, which gives $\lambda \sim \frac{1}{l}$. If we rescale the Higgs scalar field $\phi$ by $\phi = \frac{l}{r_H} \tilde{\phi}$, the action (33) can be rewritten by

$$S_{\tilde{A}, \phi} = \int d^4x \left\{ \frac{1}{2} \left\{ \eta^{mn} \left( \partial_m \phi^* + ie\tilde{A}_m \phi^* \right) \left( \partial_n \phi - ie\tilde{A}_n \phi \right) - g^2 \left( \phi^* \phi - \lambda^2 \frac{r_H^2}{l^2} \right)^2 \right\} - \frac{1}{4} \eta^{mn} \eta^{kl} F_{mk} F_{nl} \right\}.$$  \hspace{1cm} (34)

As the expectation value of $\langle \tilde{\phi} \rangle \sim \frac{\lambda}{r_H} \sim \frac{r_H}{l}$ gives the mass of the gauge field $\tilde{A}_m$, in order that $\langle \tilde{\phi} \rangle \sim 10^2$ GeV, we have $\frac{1}{r_H} \sim 10^{36}$ (GeV), which may be unrealistic, and unfortunately may contradict with the assumption that the black hole is large. The problem has occurred because we consider the brane outside the horizon where the warp factor becomes the large constant.

It is remarkable that the small $k$ behavior in (26) shows that the propagator behaves as $1/r$ in the position space, where $r$ is the distance between two points on the brane. The behavior is identical with usual propagator in four dimensions but not with that in five dimensions. Then (26) shows that the gauge fields are localized on the brane (for recent discussion of gauge fields localization on brane embedded into pure AdS or dS bulks, see[11]).

So far the brane was considered near the horizon. One may consider the brane near the singularity, say $r \sim l$. Since we can regard the warp factor with $r^2$, the ratio of the scales on the brane and on the brane near the
horizon, where \( r \sim \mu^{\frac{1}{4}} l^{\frac{1}{2}} \), is given by \( \mu^{\frac{1}{4}} l^{\frac{1}{2}} \). Then to reproduce the ratio of the Planck scale and the weak scale, we have \( \frac{\mu^{\frac{1}{4}} l^{\frac{1}{2}}}{\kappa^{r^{2}}} \sim 10^{17} \). Since the mass \( M \) of the black hole is given by \( M \sim \frac{\mu^{\frac{1}{2}}}{\kappa^{r^{2}}} \sim 10^{68} \frac{l}{r} \). Since the Planck mass \( \frac{1}{l} \) is \( \sim 10^{-55} \) gram, if the mass of the bulk black hole is macroscopic and given by \( 10^{65} \) gram = \( 10^{60} \) kg, the hierarchy might be realized. Of course, such huge black hole mass seems to be unrealistic and such effect may occur only if black hole quickly evaporate. Another possibility which we mention in the discussion is that hierarchy problem is solved with pure AdS bulk but then phase transition to AdS black hole occurs.

3. In this section, we Wick-rotate the metric (3) into the Euclidean signature. Then since the black hole has (Hawking) temperature, one arrives at the finite temperature theory. Especially for the large black hole, the euclidean bulk spacetime surely becomes flat.

As a background bulk space, the Schwarzschild-anti de Sitter space with the flat horizon is taken:

\[
\begin{align*}
\text{ds}^2 &= g_{\mu\nu} dx^\mu dx^\nu = -e^{2\rho} dt^2 + e^{-2\rho} dr^2 + r^2 \sum_{i=1,2,3} (dx^i)^2, \quad e^{2\rho} = \frac{r^2}{l^2} - \frac{\mu}{r^2}. \\
\text{(35)}
\end{align*}
\]

For the large black hole the horizon radius \( r_H = l^{\frac{1}{4}} \mu^{\frac{1}{4}} \to \infty \). We now define a new coordinate \( \sigma \) by \( \sigma = r - r_H \), then \( e^{2\rho} \sim \frac{4rH}{l^2} \sigma \) and

\[
\begin{align*}
\text{ds}^2 &\to -\frac{4rH}{l^2} \sigma dt^2 + \frac{l^2}{4rH \sigma} d\sigma^2 + r_H^2 \sum_{i=1,2,3} (dx^i)^2. \\
\text{(36)}
\end{align*}
\]

After Wick-rotating time coordinate and introducing new coordinate \( \theta \) and \( \rho \) by \( \theta = i \frac{2\sigma}{rH^2} \), \( \rho = l \sqrt{\frac{\sigma}{rH}} \), the metric (36) can be rewritten as

\[
\begin{align*}
\text{ds}^2 &= \rho^2 d\theta^2 + d\theta^2 + r_H^2 \sum_{i=1,2,3} (dx^i)^2. \\
\text{(37)}
\end{align*}
\]

In order to avoid the orbifold singularity at \( \rho = 0 \), the coordinate \( \theta \) has a period of \( 2\pi : \theta \sim \theta + 2\pi \). Then the spacetime is flat and locally \( R_2 \times R_3 \sim R^5 \).

We now further rescale the coordinates \( x^i \) by \( r_H x^i \to x^i \) and define a new field \( B : B \equiv A_\theta + i \rho A_\rho \). The part including only \( B \) in the action has a \( U(1) \)
symmetry $B \to e^{i\phi} B$ with constant parameter $\phi$ of the transformation. As the coordinate $\theta$ has a period of $2\pi$, we can replace $\partial_\theta$ by $i n$. Here $n$ is an integer. Similarly, we replace $\partial_i$ by $k_i$ as the Fourier transformation. Then the solutions of the equations of motion are given by the modified Bessel functions with integer index $n$ by a way similar to the previous section. We now assume that there are branes at $\rho = \rho_1, \rho_2$ ($\rho_1 < \rho_2$). One considers two classes of solutions, which satisfy the following boundary conditions at one of the two branes ($a = 1, 2$): $\left. \partial_\rho A_{(a)i} \right|_{\rho = \rho_a} = A_{(a)\theta} \left|_{\rho = \rho_a} = A_{(a)\rho} \right|_{\rho = \rho_a} = \left. \partial_\rho c_{(a)} \right|_{\rho = \rho_a} = \left. \partial_\rho b_{(a)} \right|_{\rho = \rho_a} = 0$. Here we have imposed the same boundary condition for $A_\theta$ and $A_\rho$ in order to preserve the $U(1)$ symmetry of $B$. Then we find

$$
G_{ij}(u, v) = -\frac{\delta_{ij}}{2} \frac{A_{(2)i}(u)A_{(1)j}(v) + A_{(1)i}(u)A_{(2)j}(v)}{\beta_{(1)}(2) - \alpha_{(1)}\beta_{(2)}}
$$

$$
G_{BB^*}(u, v) = -\frac{1}{2} \frac{B_{(2)i}(u)B_{(1)j}(v) + B_{(1)i}(u)B_{(2)j}(v)}{\beta_{(1)^*}(2)0 - \alpha_{(1)0}\beta_{(2)0}}
$$

(38)

Here

$$
A_{(a)} = \alpha_{(a)} I_n(k \rho) + \beta_{(a)} K_n(k \rho),
$$
$$\alpha_{(a)} = -K_n'(k \rho_a) = K_{n-1}(k \rho_a) + \frac{n}{k \rho} K_n(k \rho_a),
$$
$$\beta_{(a)} = I_n'(k \rho_a) = I_{n-1}(k \rho_a) - \frac{n}{k \rho} I_n(k \rho_a).
$$

(39)

The coefficients are given by $\frac{\alpha_{(a)i}}{\beta_{(a)i}} = \frac{\alpha_{(a)\theta}}{\beta_{(a)\theta}} = \frac{\alpha_{(a)\rho}}{\beta_{(a)\rho}} = -\frac{K_n'(k \rho_a)}{I_n(k \rho_a)} = \frac{K_{n-1}(k \rho_a) + \frac{n}{k \rho} K_n(k \rho_a)}{I_{n-1}(k \rho_a) + \frac{n}{k \rho} I_n(k \rho_a)}$, $\frac{\alpha_{(a)\theta}}{\beta_{(a)\rho}} = -\frac{K_{n-1}(k \rho_a)}{I_{n-1}(k \rho_a)}$. Here $\alpha_{(a)0} = \alpha_{(a)\theta} + i \alpha_{(a)\rho}$ and $\beta_{(a)0} = \beta_{(a)\theta} + i \beta_{(a)\rho}$. We also find the propagator between two $B$ or two $B^*$ vanishes.

We now consider the high energy behavior of the propagator when $k, n \to \infty$ but $\frac{k}{n}$ is finite. Using asymptotic expansion of Bessel functions one obtains

$$
G_{ij}(u, v) \sim \frac{\delta_{ij}}{4|n|} \frac{z_1^2}{1 + \sqrt{1 + z_1^2}} + \frac{z_2^2}{1 - \sqrt{1 + z_2^2}} \frac{1}{\left(1 + z_u^2\right)^{1/2} \left(1 + z_v^2\right)^{1/2}}
$$

$$
\times \left\{ e^{i|n|(\eta_u - \eta_v)} \theta(u - v) + e^{i|n|(\eta_u - \eta_v)} \theta(v - u) \right\},
$$

(38)
\[ G_{BB^*}(u, v) \sim -\frac{1}{4|n|} \frac{e^{\frac{n-1}{2}(|\eta_u - \eta_v|)} \theta(u - v) + e^{\frac{n-1}{2}(|\eta_u - \eta_v|)} \theta(v - u)}{(1 + z_u^2)^\frac{n}{4} (1 + z_v^2)^\frac{n}{4}}. \] (40)

Here
\[ z_a \equiv \frac{k \rho_a}{n}, \quad \eta_a \equiv \sqrt{1 + z_a^2 + \ln \frac{z_a}{1 + \sqrt{1 + z_a^2}}}, \]
\[ (a = 1, 2, u, v, \rho_u = u, \rho_v = v). \] (41)

As a result, \( G_{ij}(u, u), G_{BB^*}(u, u) \propto \frac{1}{|n|\sqrt{1 + z^2}} = \frac{1}{u \sqrt{k^2 + \frac{n^2}{\rho^2}}} \). Then the behavior of the Green functions seems to be typical for theory at finite temperature. In fact, if we express the \( D \)-dimensional propagator at the finite temperature by \( D - 2 \) spacial momenta \( k_i \) \((i = 1, 2, \ldots, D - 2)\) and 1 position \( x^{D-1} \) and further consider the case that \( x^{D-1} = 0 \), we obtain
\[ \hat{G} \left( k_1, k_2, \ldots, k_{D-2}, x^{D-1} = 0, n \right) = \frac{\pi}{\sqrt{\sum_{i=1}^{D-1} k_i^2 + \frac{n^2}{\beta^2}}}, \] (42)
which corresponds to above brane propagator. Here \( \beta \) is the inverse of the temperature. We should also note that the brane temperature \( T = \frac{1}{\beta} \) seems to be \( u \)-dependent, \( T \propto u \). Then for two branes, the inner brane is hotter than the outer brane. There will be a thermal radiation from the inner brane to the outer brane.

It is interesting that there may appear the imaginary pole in the propagator. Let us investigate the propagator \( G_{ij}(u, v) \) (38). Taking \( k = 0 \) and extending \( n \) to the complex continuum parameter, the propagator has a pole at \( n = 0 \), which is the usual massless pole. Besides the pole, imaginary poles appear, where the denominator of propagator vanishes: \( n = i \frac{\pi N}{\ln \rho} \), where \( N \) is an integer. It is known that imaginary part of the propagator is related with the damping constant of the corresponding plasma state. The fact that poles depend on the extra coordinates may lead to the following conjecture. The (part of) background radiation (energy) may be due to the extra dimensions (if bulk is some black hole). The effect should be universal and applied to any type of matter. Hence, the temperature of hot matter may be presumably caused by the presence of bulk black hole. This may provide the means for estimations of length of extra dimension(s).
4. We presented the bulk and brane gauge propagator structure in AdS black hole bulk for both, euclidean and minkowski signatures of the metric. As it has been noted earlier for gravitons[3], it is demonstrated that there occurs localization of gauge fields on the brane embedded into 5d AdS black hole. It is shown that KK modes decouple. However, the solution of hierarchy problem when bulk is BH is not realistic.

Clearly, the study of gauge propagator is interesting in relation with possibility to apply it in diagrams for unified theories. Let us make several remarks in this connection. We may introduce interaction considering other bulk matter fields (scalars, spinors) or generalizing theory to non-abelian case. As one can see from (37), the spacetime is locally flat although there are boundaries corresponding to the branes. Since the renormalization is determined by the short distance behavior, the beta-functions are not changed from their flat values. In more than four spacetime dimensions, the gauge coupling constant has dimension. Then the beta-functions show power law behaviour except in four dimensions. If the spacetime dimension is less than four, the non-abelian gauge theory can be still asymptotic free but if the dimension is larger than four, there appears ultra-violet fixed point. (The extra orbifold dimensions may completely change the structure of even trivial brane (scalar) theory and number of fixed points appear or non-AF theory may become AF one[12]). Then we will have a ultra-violet fixed point in the Schwarzschild-AdS background with large black hole. For pure AdS, the gauge theories can be asymptotic free even in five dimensions [9, 13]. Then there might be a phase transition from asymptotic free theory to asymptotic non-free theory at some critical horizon radius. (This may be another effect associated with AdS black hole, similar to Hawking-Page-Witten phase transition [2] interpreted as confinement-deconfinement phase transition via AdS/CFT. This phase transition has also D-brane interpretation [14].) In case of the Schwarzschild black hole in the Minkowski background, the larger black hole has the lower temperature but in case of Schwarzschild-AdS black hole with flat horizon, since the Hawking temperature is given by $T_H = \frac{r_H}{\pi L}$, the larger black hole has the higher temperature. Then the phase transition may be due to the temperature. The confinement would occur for asymptotic free gauge theories but would not occur for asymptotic non-free theories. Then the above phase transition, if it really exists, would correspond to the confinement-deconfinement phase transition.
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