Approach to estimate the ship center of gravity based on accelerations and angular velocities without ship parameters

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Abstract. An important aspect in the stability calculation of sailing ships is the actual center of gravity whereby changing load conditions complicate the calculations. Standard factors for these calculations are ship parameters and the hydrodynamic model of the ship. These calculations depend on the correct input of each individual load. This in turn is reflected in possibly incorrect calculations of the center of gravity and thus of the stability of the ship. This paper presents a new method that is independent of ship parameters describing the form of the ship. The center of gravity during sailing is calculated from the measurement data of motion sensors to avoid sources of error, such as the human input of parameters. In addition, this is a more cost-effective method than conventional methods. A new algorithm is presented which is derived from the heel test and the construction formulas and independent of human inputs and environmental factors such as wind and waves. The purpose of the calculation of the current center of gravity during sailing is given in the further work, which calculates the ship movement and the actual sea state around the ship to predict dangerous situations for man and cargo at sea.

1. Introduction

Already at the beginning of the ship design the calculation of the center of gravity is an important aspect for the stability calculation. During the design phase, the individual weights are calculated and estimated in order to predict the light ship mass and the maximum load case. The weight calculation includes the steel used for hull and superstructure, used light metals, accommodation and equipment, machinery and possible load cases.

Despite 3D designs and calculation tables, the weight and thus also the center of gravity remains an estimate in the design. Therefore, a heeling test of the ship must be carried out after the construction phase. In this heeling experiment, the center of gravity of the built ship is determined experimentally. These results are used to correct the calculated center of gravity. The adjusted center of gravity is used for all stability calculations of the ship. Due to this complexity of several weights and their distribution over the entire hull, the center of gravity is a nearest approximation.

In the life of a ship the weight and thus the center of gravity changes constantly. Every change in weight leads to a change in the center of gravity. As a result, the stability properties also change permanently. Modern algorithms make it possible to calculate the changing center of gravity but are dependent on the manual input of each individual weight or correct ship parameters with indication of the exact position on board the ship. Since the center of gravity represents the best possible approximation to the construction phase and the changing load depends on human influence, the actual calculated center of gravity is ultimately only an estimate and can deviate from reality by up to several meters. This can be devastating for stability calculations.
Nowadays, the center of gravity is determined by special programs or special calculations, all of which require the ship parameters as a prerequisite [1]. Other methods define the turning point instead of the center of gravity by calculating moments around a turning point [2]. In ships, the center of gravity is not at the pivot point because there is a buoyancy force in the water. The buoyancy force acts on the underwater part of the ship and changes with the heel and trim angle of the ship. The pivot point moves in the same way.

This paper proposes a new algorithm based on design formulas and experiments analogous to the heeling experiments for sailing ships to calculate the center of gravity independent of individual weights, their location, ship length, width, light ship mass, draught (ship parameters), waves and wind during sailing. The independence from the ship parameters avoids error inputs and allows an exact calculation based on the actual ship movements. In addition, this method is a cost-effective method that can be used flexibly on different ship sizes and types.

In this paper, the theoretical background is first discussed by explaining the heeling experiment and the design formulas used for the algorithm. In addition, a new algorithm will be presented and experimentally tested. In a concluding discussion, advantages and disadvantages as well as an outlook on further research will be discussed.

2. Nomenclature

| Symbol | Description |
|--------|-------------|
| B      | Center of buoyancy |
| b      | Beam |
| \(c_b\) | Block coefficient |
| D      | Draft |
| e      | Distance of the load to central axis |
| g      | Gravity |
| G      | Center of gravity |
| h      | Heeling arm |
| K      | Keel point |
| \(k'_{xx}\) | Radius of gyration |
| \(L_{oa}\) | Length over all |
| M      | Metacenter, pivot point |
| O      | Body fixed coordinate systems |
| P      | Load |
| \(S_{1,2,3}\) | Sensor 1-3 |
| T      | Roll period |
| \(x,y,z\) | Coordinates |
| \(\omega\) | Frequency |
| \(\varphi\) | Heeling angle |
| \(\psi\) | Pitching angle |
| \(\rho\) | Water density |
| \(\forall\) | Displaced volume |
| \(\Delta\) | Displacement (mass) |
| \(\nabla\) | Waterline |

3. Theoretical background

This section describes the heel tests as a reference and the design formulas from which the calculation of the center of gravity is derived.

3.1. The heeling experiment [3]

In general, the stability of a ship is its ability to return from heeling to the upright floating position. Moments leading to heeling can be asymmetric loads, wind forces, icing, ground contact, turning circles and others. The position of the center of gravity and the center of buoyancy has a decisive influence on stability. The center of gravity of the lift depends on the shape of the ship and the center of gravity on the weight distribution. Loading weights causes both heel and trim. This is used for the heel experiments.

A weight is mounted on the lightship and shifted in height, width and length. The horizontal movement of a weight is shown in figure 1. The heel angle \(\varphi\) is the horizontal movement of the load P at distance e. The center of buoyancy B changes to point B’ and is perpendicular to the transverse metacenter M. The metacenter is the intersection of the buoyancy vector and the center axis of the ship for small heel angles \(\varphi\) (2-4%). In center of gravity G, the weight force counteracts the buoyancy force in B’ vertically.
Whereby: \[ M = P \cdot e \cdot \cos \varphi \] (1)

For small angle: \[ \cos \varphi \cong 1 \]

So that: \[ M \cong P \cdot e \] (2)

\[ \sum M_i = 0: \]
\[ P \cdot e - G\bar{M} \cdot \varphi \cdot \Delta = 0 \] (3)

With:
\[ \Delta = L_{oa} \cdot b \cdot D \cdot c_B \cdot \rho \] (4)

**Figure. 1.** Emigrating of the center of buoyancy B by the weight postponement P – heeling experiment

Here is \( \Delta \) the displacement (masse of displaced water), \( \forall \) the displaced volume, \( L_{oa} \) the length over all, \( b \) the beam of the ship, \( D \) the draft, \( c_B \) the block coefficient and \( \rho \) the water density. \( G\bar{M} \) is than calculated with the equation (3) by measuring the angles:

\[ G\bar{M} = \frac{P \cdot e}{\Delta \cdot \tan \varphi} \] (5)

The weights are shifted, and the corresponding heel angles are measured. These measurements are made depending on the weight and the distance to the axis. In ship design, these results correct the calculated center of gravity.

### 3.2. Design formulas

Design formulas accompany the ship design from the idea to the finished ship. This includes formulas for the most important ship parameters such as length, width, block coefficient as well as the rolling period, the heel angle, the shape of the frames and the relationship between the construction points.

### 3.3. The rolling period in smooth water with damping is [4]:

\[ T_\varphi \approx \frac{2\pi k'_{xx}}{\sqrt{g \cdot G\bar{M}}} \] (6a)

\[ \rightarrow G\bar{M} \approx \frac{1}{g} \cdot \left( \frac{2\pi k'_{xx}}{T_\varphi} \right)^2 \] (6b)

With \( G\bar{M} \) distance of the light metacenter to the center of gravity, \( g \) the gravity, \( T_\varphi \) roll period by an angle \( \varphi \) and \( k'_{xx} \) the radius of gyration taking hydrodynamic mass into account. Hereby the damping hardly influences the rolling period. This is also valid for bigger rolling angles. The radius of gyration is between 0,36b and 0,45b. It is generally recognized at 0,4b with \( b \) as beam. Additionally, the period can be expressed in dependence of the frequency by [4]:

\[ T = \frac{2\pi}{\omega} \] (7)

By inserting the equation (7) into (6b) it results \( G\bar{M} \) as follows:
\[ \frac{GM}{g} \approx 0.16 \omega^2 b^2 \]  

(8)

3.4. The heeling arm for undamped free oscillation with vertical side walls for small angles is [4]:

\[ h(\varphi) = \left( \frac{GM}{g} + \frac{1}{2} BM \cdot \tan^2 \varphi \right) \cdot \sin \varphi \]  

(9)

The figure 2 shows the relations between the distances and center of gravity G, center of buoyancy B, the heeling angle \( \varphi \), the heeling arm h, the metacenter M, the keel point K, the distance to the line h(K) through the center of buoyancy and the metacenter and the coordinates \( y_{B'}, z_{B'} \).

Based on the theoretical background the new simplified method for the calculation of the center of gravity follows.

Figure 2. Geometric relations during the rolling of the ship

Figure 3. Arrangement of IMU’s independent from the center of gravity

4. Proposed new method

A basis idea for this paper is to measure the movement of the ship during sailing using motion sensors. The sensor system consists of simple inertial measurement units (IMU’s), which measure the accelerations in the x, y and z directions as well as the angular velocities around these axes. The big advantage is the flexible application on different ship sizes and types. The arrangement of the sensors \( S_1, S_2 \) and \( S_3 \) is shown in figure 3.

The coordinate system with (x, y, z) forms a body fixed coordinate system independent from the center of gravity. Each sensor lies on one axis and built a fixed coordinate system (x1,2,3; y1,2,3; z1,2,3).

When the ship rolls, an angle of heel is formed. The heeling angle of the ship is analogous to the angle of the sensors as shown in figure 4.

Figure 4. Generation of heel angle and corresponding sensor angle
Consequently, the angles of heeling for each sensor are calculated by the equations:

In horizontal direction: \[ \varphi = \arctan\left(\frac{\text{Acc} \ Z}{\text{Acc} \ Y}\right) \] (10)

In longitudinal direction: \[ \psi = \arctan\left(\frac{\text{Acc} \ Z}{\text{Acc} \ X}\right) \] (11)

In the next step \( \overline{GM} \) is calculated for each sensor individual. Therefor the equation (8) and the received data from the sensors for the angular velocity \( \omega \) in consideration of the measured unit of [rad/s] are used:

\[ \overline{GM} = \frac{0.16 \omega^2 b^2}{g} \] (12)

With \( b = 2 \times S_z S_1 \)

After receiving the values of \( \overline{GM} \) in dependence of \( \varphi \) for each sensor, \( \overline{GM} \) over \( \varphi \) can be approximated. Further \( \overline{GM} \) is calculated in the YZ and XZ plane.

Now the distance of the three sensors to their center of rotation can be geometrically determined. The center of rotation of the sensors is the metacenter \( M \) of the ship. The following example shows the geometrically displacement of the sensors in the YZ plane (figure. 5) for the heeling angle \( \varphi \). The perpendicular at the center of \( S_1 \) intersect at the metacenter \( M \).

From the arrangement of the sensors (figure 3) the coordinates of the fixed body coordinate system \((O: \vec{x}; \vec{y}; \vec{z})\) is known. With \( S_1(0;y;0), S_2(x;0;0) \) and \( S_3(0;0;z) \) and the measured accelerations \( X, Y \) and \( Z \) follows:

The velocity: \[ v(t) = \int_{t_1}^{t_2} a(t) dt + v_0 \] (13)

With the velocity the displacement can be calculated with:

\[ s(t) = \int_{t_1}^{t_2} v(t) dt + v_0 \cdot t + s_0 \] (14)

The perpendicular at the center of \( \overline{S_1 S_1} \) can be drawn for each sensor for the heeling angle \( \varphi \). The intersection designs the metacenter \( M_\varphi(y;z) \). This calculation is repeated for the calculated heeling angle \( \varphi \) and the interpolation of intersection points forms the geometrically calculated metacenter \( M_0 \) for small angle.

With the evaluated \( \overline{GM} \), the coordinates of \( G(y,z) \) in the body fixed coordinate system \((O: \vec{y}; \vec{z})\) are calculable (see figure 6):
In the same way, the calculation is derived in the XZ plane and the coordinates for \( M(\phi; z) \), \( G(x, z) \), \( B(x, z) \) and \( K(x, z) \) in the body fixed coordinate system \((O: \vec{x}; \vec{z})\) are calculable. By combining the results, the complete coordinates for \( M(\phi; x, y, z) \), \( G(x, y, z) \) and \( B(x, y, z) \) are obtained.

This shows the position of the center of gravity \( G(x, y, z) \) as a function of measured movements from IMU’s. The independence from the common ship parameters such as length, beam, draft and displacement support a flexible application of this algorithm in shipping. This mathematical approach must be validated in the next step by experiment and possibly corrected.

5. Experiment

The new method described above for calculating the center of gravity of a sailing vessel is tested in this chapter by means of an experiment. For this 3 IMU’s of the brand XSens [7] were used. Including two of the series MTi-100 and one of the series MTi-10. Its specification includes a gyro bandwidth of 415Hz and an acceleration bandwidth of 375Hz. The meantime between failures is 300.000 hours, the sampling frequency is 10kHz/channel and the latency time is under 2 milliseconds. The sensors were arranged with the following coordinates in the body-fixed coordinate system on board the ferry:

- IMU 1: \( S_1(-2,0; 0; 0,1) \) [m]
- IMU 2: \( S_2(0; 2,0; -0,13) \) [m]
- IMU 3: \( S_3(0; 0; 2,0) \) [m]

In this experiment it was not possible to distribute the sensors over the width of the ferry. The ferry used for the experiment has the following ship parameters [8]:

- Length 178,40 m
- Beam 29,61 m
- Draught 6,22 m
- Speed 21,2 kn
- Tonnage 40,039 GT

The measured data were recorded during a trip from Copenhagen to Oslo. In order to reduce the amount of data was recorded with a frequency of 25Hz over several hours on different days. The measured accelerations are used to calculate the roll angle \( \phi \) from equation (10) in the YZ and XZ plane. Over a period of one hour and twelve minutes, the three sensors produce more than 100,000 calculated heeling angles. The calculations show that angles of -1.5° to 1.5° were created during the experiment. With a ship width of 29.61m this means that the outermost port and starboard sides of the ferry have moved up and down by about 0.4m.
In the next step, the $\overline{GM}$'s are calculated using the measured angular velocities. The data obtained are used to calculate the $\overline{GM}$ by entering the measured angular velocity in equation (12). In analogy to the heeling experiment the results are represented as follows for $\overline{GM}$'s from 0.3 to 1m.

![Figure 7](image_url)

**Figure 7.** Evaluation in analogy to the heeling experiment for $\overline{GM}$ from 0.3m to 1m for IMU 3 in the YZ plane

Figure 7 shows the evaluation based on the known heeling experiment in the YZ plane. In analogy the $\overline{GM}$ is calculated in the XZ plane. In contrast to the heeling experiment [3], $P \ast e / \Delta$ is shown instead of $P \ast e$, since $\Delta$ is an unknown quantity. This results in the lever arm curve estimation in figure 8 for the YZ and the XZ plane by using equation (5). The dashed line is for the XZ plane and the solid line is for the YZ plane. In the XZ plane it is the rotation around the y axis and thus the stamping of the ship (pitch). The flat lever arm is explained by the geometry of the ship (dashed line). The calculated $\overline{GM}$ for the YZ plane is 0.52m and for the ZX plane -0.03m.

![Figure 8](image_url)

**Figure 8.** Lever arm curve estimation in YZ and XZ plane

Now the center of rotation calculation follows by applying the equations (13) and (14). This results in the displacements of the sensors in the YZ and XZ plane. By means of the shift the coordinates of the sensors are calculated after the shift.
\[ S'_{1,2,3} = S_{1,2,3} + s(t) \] 

This is followed by the calculation of the perpendiculars to the center and thus the centers of rotation. For each of the three sensors there is an accumulation of intersections. Their mean values form a triangle (see figure 9). The center of gravity of this triangle is the calculated center of rotation and thus the metacenter.

![Figure 9](image.png)

**Figure 9.** Center of gravity of the triangle, formed by intersections of the perpendicular midpoints of the three sensors for YZ plane

For small angles the middle metacenter is at the point \( M_{yz}(0.57; −0.93) \) in the YZ plane and \( M_{yz}(0.04; 0.19) \). Together with the \( \overline{GM} = 0.52 \) in the YZ plane and \( \overline{GM} = −0.03 \) in the XZ plane and the averaging of the z value, this results in the coordinate \( G(0.04; 0.57; −0.37) \) in the fixed coordinate system for the center of gravity.

6. Discussion and outlook

In this paper, a method for calculating the center of gravity independent of the ship parameters was presented. The advantage of the independence from ship parameters and the environmental is the flexible applicability on different ship types as the functionality as a self-sufficient system. No input from other systems or the ship's command is necessary. Furthermore, by reducing the input parameters by means of low-cost acceleration sensors, a larger conversion or installation of a system is avoided. In this paper the validation of the method was shown. The next step is the verification error estimation of this method. For this purpose, several test runs with the research vessels WEGA and DENEH of the Federal Maritime and Hydrographic Agency [9] are currently being carried out. Here the calculated method is compared with the loading computer. On the other hand, simulation is planned at the OFFIS Institute. Here dynamic models as well as different sea spectra are available.

Further steps lead to the movement of the ship in the sea, whereupon the current sea state around the ship is calculated. This will be used to predict the ship's movement in order to avoid dangerous movements of the ship (strong rolling, parametric rolling, inability to maneuver, cargo losses…).
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