Plate Deflection Analysis with Different Mesh

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Abstract. Ocean structures like a jacket, jack-up, and other floating structures consist of many structural elements to support and strengthen from internal and external loads. The structural element, like a beam, plate, and the stiffened plate, has a significant influence on the ultimate strength of the global structure. So that all these elements components should be evaluated and analyzed for the local and global of the structural strength. The objective of the present study is to analyze the plate deflection due to different mesh dimension. The plate is modeled by shell elements with different mesh dimensions; those are 8 x 8 and 16 x 16. The clamped edges as boundary conditions are applied to all sides of the plated. The applied load is located at the center of the plate. The Finite Element Method is used to analyze the plate deflection with different mesh dimensions under axial load acts at the center of the plate. The result obtained by FE Method is therefore compared to the analytical solution. It is found that the plate deflection obtained by the FE method is in good agreement with the analytical solution, and the behavior of the plate is also presented in this study.

1. Introduction

Generally, the ocean structure is very complex and consists of many subdivision components such as beam, plate, stiffened plate, and so on. All these components must be analyzed and evaluated due to the applied load on the structure. The loads could be internal and external loads, for instance, the structure itself, wind, waves, currents, etc. The ocean environment is severe, complex, and continuously varying, especially waves that always acting periodically in the horizontal direction. On the other hand, the load of the structural component is also applied in the vertical direction. Therefore, due to applied load acting on the structure and the components itself, the behavior in terms of deflection and deformation must be taken into consideration.

The analysis of plate behavior such as deflection or deformation on the structural engineering like a ship and other ocean structures has been conducted by some papers. Vaz [1] presented an experimental campaign and a numerical finite element model to obtain the ultimate strength of tubular structures with circular perforated damage subjected to axial compression. Tekgoz [2] analyzed the effect of structural damage and associated neutral axis translation and rotation of the residual load-carrying capacity of a container ship hull subjected to asymmetrical bending loading. The influence of nonlinear finite element method models on the ultimate bending moment for hull girder was studied by Xu [3]. The residual strength of an Aframax-class double hull oil tanker damaged in the collision had been assessed by Parunov [4]. The ultimate strength of ship hull girder strength was conducted by Muis Alie [5] considering the critical element at the deck part under sagging condition. Muis Alie [6] analyzed the effect of symmetrical and unsymmetrical configuration shapes on buckling and fatigue strength analysis of the fixed offshore platform. Two models of the fixed offshore structure were taken
to be analyzed with the same dimension but different configuration shapes. The numerical calculation was performed to analyze the buckling and fatigue strength of both structures. Muis Alie [7] analyzed the residual strength of asymmetrically damaged ship hull girder using the beam finite element method. The Bulk Carrier ship model was taken to be analyzed.

In addition, a computational model for analysis of local buckling and postbuckling of stiffened panels were derived by Byklum [8]. An approximate analytical method to determine the large-deflection behavior of rectangular simply supported thin plates under transverse loading was described by Bakker [9]. The comparison of the ultimate strength of T and Y stiffeners subjected to lateral load was conducted by Badran [10]. Tanaka [11] performed a series of collapse analyses by applying nonlinear FEM on stiffened panels subjected to longitudinal thrust. Lee [12] applied the nonlinear finite element analysis to examine the ultimate strength characteristics of steel brackets, and to develop a simple design formula to predict the ultimate strength of a steel bracket. The ultimate strength behavior of steel concrete steel sandwich plate under concentrated load was studied by Yan [13]. The development of an assessing formula for ultimate strength of hull plate with pitting corrosion damage under combined loading was done by Zhang [14]. The ultimate strength assessment method of corroded plates based on corroded volume loss had been used proposed by the theoretical derivation and finite element analysis by Zhang [15].

The objective of the present study is to analyze the plate deflection due to different mesh dimension. The plate is modeled by shell elements with different mesh dimensions; those are 8 x 8 and 16 x 16. The clamped edges as boundary conditions are applied to all sides of the plated. The applied load is located at the center of the plate. The Finite Element Method is used to analyze the plate deflection with different mesh dimensions under axial load acts at the center of the plate. The result obtained by FE Method is therefore compared to the analytical solution. It is found that the plate deflection obtained by the FE method is in good agreement with the analytical solution, and the behavior of the plate is also presented in this study.

2. Fundamental Study
The deflection can be analyzed by using an example like a plate with the thickness as a parameter. The thickness is constant to all edges of the plate. Assuming that y-axes are the long axes and z-axes in the positive direction to downward, as shown in Figure 1. If the breadth of the plate is l, so that the small element of the plate may be assumed as a beam with the rectangular cross-section having a length (l) and depth (h), respectively.

![Figure 1. Deflection on plate](image-url)
as \(-\frac{\partial^2 w}{\partial x^2}\) where \(w\) is deflection on beam in the \(z\)-direction, and this is small than beam length (l). The unit strain \(\varepsilon_x\) of the layer in the \(z\)-direction of the center surface, as shown in Figure 1 is \(-\frac{z}{E} \frac{\partial^2 w}{\partial x^2}\). By applying Hooke’s law, the strain \(\varepsilon_x\) and \(\varepsilon_y\) defined in the normal stresses \(\sigma_x\) and \(\sigma_y\) acting on the element as shown in Figure 1 can be determined by the following equation,

\[
\begin{cases}
\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} = 0 \\
\varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E} = 0
\end{cases}
\]

(1)

The lateral strain in \(y\)-direction must be zero to keep continuity on the plate during bending, so that \(\sigma_y = \nu \sigma_x\). Substitute this value into equation (1) yields,

\[
\begin{cases}
\varepsilon_x = \frac{(1-\nu^2)\sigma_x}{E} \\
\sigma_x = \frac{E \varepsilon_x}{1-\nu^2} = \frac{Ez}{1-\nu^2} \frac{d^2 w}{dx^2}
\end{cases}
\]

(2)

After obtaining the bending stress \(\sigma_x\), by integrating the equation the bending moment can be obtained on the element layer,

\[
M = \int_{-h/2}^{h/2} \sigma_x z \, dz = \frac{h}{2} \int_{-h/2}^{h/2} Ez^2 \frac{d^2 w}{dx^2} \, dz = -\frac{Eh^3}{12(1-\nu^2)} \frac{d^2 w}{dx^2}
\]

(3)

where,

\[
\frac{Eh^3}{12(1-\nu^2)} = D
\]

(4)

Therefore, the equation of deflection curve on the element can be determined as follow,
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\[ D \frac{d^2 w}{dx^2} = -M \]  

(5)

Where \( D \) changed to \( EI \) on beam called the flexural rigidity of plate. For the general equation of plate, the notations are expressed in Figure 3 as follow,

\[ \frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial y^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} = \frac{q}{2} \]  

(6)

where,

\[ w = \text{deflection} \]

\[ x, y = \text{distance} \]

\[ q = \text{load} \]

\[ q = q_0 \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \]  

(7)

\[ q_0 = \text{load intensity at the plate center} \]

\[ D = \text{bending stiffness} \]

By giving the load, the equation of the plate becomes,

\[ \frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial y^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} = \frac{q_0}{D} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \]  

(8)

The boundary conditions are given by,

\[ \begin{align*}
  x = 0 \text{ and } x = a, w = 0 & \Rightarrow M_x = 0 \\
  y = 0 \text{ and } y = b, w = 0 & \Rightarrow M_y = 0
\end{align*} \]

Deflection of plate required the boundary conditions is determined by the equation as,
\[ w = c \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \]  

(9)

The constant \( c \) must be calculated by considering the boundary conditions, yields,

\[ c = \frac{q_0}{D \pi^4} \left( \frac{1}{\left( \frac{1}{a^2} + \frac{1}{b^2} \right)^2} \right) \]  

(10)

Finally, the deflection equation becomes,

\[ w = \frac{q_0}{D \pi^4} \left( \frac{1}{\left( \frac{1}{a^2} + \frac{1}{b^2} \right)^2} \right) \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \]  

(11)

3. Results and Discussions

In the present study, plate deflection is analyzed using the Finite Element Method, and the result is compared to the analytical solution. The plate is modeled by two kinds of different mesh dimensions, namely 8 x 8 and 16 x 16. Figures 4 and 5 show the plate deflection obtained by FE Method.

**Figure 4.** Deflection with mesh 8 x 8

**Figure 5.** Deflection with mesh 16 x 16

The comparison of the plate deflections between FEM and analytical solution with different mesh dimensions are summarized in Tables 1 and 2 as follow,
Table 1. Comparison of deflection with meshing 8 x 8

| h     | Analytical Solution | FEM    | Error Percentage |
|-------|---------------------|--------|-----------------|
| 1     | 0.000306            | 0.000322 | 5.31%           |
| 0.8   | 0.000725            | 0.000763 | 5.28%           |
| 0.5   | 0.002446            | 0.002575 | 5.27%           |
| 0.3   | 0.019569            | 0.020600 | 5.27%           |
| 0.1   | 0.305760            | 0.321881 | 5.27%           |
| Mean  |                     |         | 5.28%           |

Table 2. Comparison of deflection with meshing 16 x 16

| h     | Analytical Solution | FEM    | Error Percentage |
|-------|---------------------|--------|-----------------|
| 1     | 0.000306            | 0.000311 | 1.71%           |
| 0.8   | 0.000725            | 0.000738 | 1.83%           |
| 0.5   | 0.002446            | 0.002490 | 1.80%           |
| 0.3   | 0.019569            | 0.019920 | 1.80%           |
| 0.1   | 0.305760            | 0.311245 | 1.79%           |
| Mean  |                     |         | 1.78%           |

4. Conclusions
The plate deflection analysis has been conducted using the Finite Element Method considering different mesh dimensions. The following conclusion can be drawn; the result of plate deflection obtained by the finite element method is almost identical with the analytical solution, the plate model using mesh 16 x 16 is more accurate than mesh 8 x 8 to obtain the deflection value.

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