Active beam steering and afocal zooming by nematic liquid crystal-infiltrated graded index photonic structures

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Abstract
This study presents active beam steering and afocal zooming of light by incorporating liquid crystals (LCs) with graded index photonic crystals (GRIN PCs). The GRIN PC structures are composed of low refractive index polymer annular rods with holes of gradually varying radii. To actively manipulate incident light, the annular rods are infiltrated with nematic LCs. By applying an external voltage to the infiltrated LCs, the effective index profile of the low-index GRIN PC structure is modulated without introducing any mechanical movement. The incident beam deflection and corresponding focal distance modulation are tuned only by controlling the applied bias voltage. In the present work, the hyperbolic secant refractive index profile is chosen to design GRIN PC structures. To design a GRIN PC structure with annular PCs, the Maxwell–Garnett effective medium approximation is employed. We analytically express the relation between infiltrated LCs and the gradient parameter to show the physical background of the tuning ability of the proposed devices. Beam steering and afocal zooming devices are analytically investigated via geometrical optics, and numerically realized with the help of a finite-difference time-domain method. A beam deflection with an angle change of $\Delta \theta_{\text{out}} = 44^\circ$ and a light magnification with maximum $\times 2.15$ are obtained within operating frequencies of $a/\lambda = [0.10–0.15]$ and $a/\lambda = [0.15–0.25]$, respectively, where ‘a’ is the lattice constant and $\lambda$ is the incident wavelength. The corresponding operating frequency bandwidths are calculated as 40% and 50% for the beam steering and afocal zooming applications, respectively. LCs are inexpensive materials and work under low voltage/power conditions. This feature can be used for designing an electro–optic GRIN PC device that has the potential for use in a wide variety of optical applications.

Keywords: photonic crystals, nematic liquid crystals, graded index optics, active or adaptive optics, beam-steering, afocal zooming

(Some figures may appear in colour only in the online journal)
photonic devices. In this regard, developing the ability to dynamically control optical properties with external parameters is crucial, not only to enhance the functionality, efficiency, flexibility and compactness of optoelectronic devices, but also to reduce their cost. In the last decade, there have been several efforts to achieve optical tunability using various means such as microfluidics [7], ferroelectric materials [8], graphene [9], dielectric elastomers [10], liquid crystals (LCs) [11], thermal control [12] and mechanical control [13]. LCs are a good candidate for approaching optical tunability due to their low power consumption/dissipation, fabrication cost efficiency and structural compactness [14]. For instance, LCs have been used to tune band gaps [15–18] and the negative refraction effect [19] in PCs, and to design tunable optical waveguides [20, 21]. Moreover, the use of LCs in the design of diffractive lenses, switchable gratings, and beam deflectors has been proposed in [22–24].

Light focusing and guiding elements occupy an important role in photonic applications. In order to implement the focusing effect, various notable approaches based on negative refraction [25], Fabry–Perot resonances [26], super-lenses [27], aperiodic metallic waveguide arrays [28], metamaterials [29], and photonic nano-jets [30] have been proposed. On the other hand, light guiding can be achieved using photonic-bandgap systems [31], slot-waveguides [32], plasmonic waveguides [33], coupled resonators [34] and dispersion-based optical routing [6, 35]. Apart from these techniques, the manipulation of light propagation behavior is further enhanced by introducing the concept of gradient-index (GRIN) materials. These powerful tools have contributed to numerous nano-photonic and optical applications, including optical mode coupling [36], flat GRIN PC lens [37] and Maxwell fish-eye GRIN PC lens [38] for efficient light focusing, light bending [39], wavelength de-multiplexing [40] and mode order converting [41]. Nevertheless, despite significant progress having been made in these studies, nearly all of the designed light focusing and light guiding structures operate without tunability characteristics. In other words, the optical properties of these structures are fixed and active control is not possible.

In this study, computational results are obtained for beam steering and afocal zooming applications. GRIN PC structures are fabricated by aligning annular polymer rods in an air medium and filling them with nematic LCs. By incorporating LC materials within the GRIN media, active beam steering and afocal zooming of the incident light are achieved. Firstly, the ray transfer analysis of proposed structures is analytically derived via geometrical optics. Then, simulations are performed with the 2D finite-difference time-domain (FDTD) method, using appropriate boundary conditions. Good agreement between the analytical and numerical methods is observed.

2. Problem definition and nematic LCs as a tool for active control

Photonic interconnectors are used for efficient light transmission between different optical elements. Here, the main challenge is misalignment between the components which guide the light input and output. If one considers a fiber-optic communication system, misalignment between the transmitting and receiving components leads to an increase in transmission losses, which decreases the maximum rate of data transmission. In this case, mechanical alignment of the elements can be quite complicated and adversely affect the durability of the system. For this reason, controlling the alignment without any displacement of elements by mechanical intervention is crucial. By using a beam steering mechanism this problem can be solved, and via this approach, photonic interconnectors can be integrated easily to a system [42]. Optical beam steering also plays an important role in light detection [43], near field beam manipulation (light manipulation by multiple split-beam antenna) in microwaves [44], laser micromachining [45] microscopic imaging [46–48] and PC based biosensors for direct detection of molecules in drug screening and label-free high-resolution cell imaging [49] applications.

Another active light control concept studied in this work is afocal zooming, which is an important optical effect for microscopy and imaging systems [50]. An optical lens system can focus, diverge and collimate incoming light; here, for focusing and diverging behaviours of light, imaging properties of the lens system can be modelled via the focal length. On the other hand, if the focal length of collimated light is infinite, then paraxial characteristics of the lens system cannot be modelled. Such systems are referred to as afocal lens systems [51]. Afocal lens systems are mostly used for optical zooming applications; optical zooming can be obtained by using mechanically compensated or optically compensated zoom lenses that are used for beam expansion and compression purposes [52]. Here, optically compensated lens systems depend on the single motion of a lens or lens system, while mechanically compensated zoom lenses operate with multiple motions in multiple directions. Both zooming approaches require complicated simultaneous adjustment of multiple lens pairs. For this reason, employing these systems in micro/nano photonic applications is a challenging task. Nevertheless, in order to solve this issue, a monolithic system with a cascaded construction is proposed [53]. However, for different magnification cases, distinct monolithic systems are required, which makes this approach unfeasible.

In this study, we propose beam steering and afocal zooming concepts using nematic LC infiltrated PC structures. Here, active control of the beam deflection and tuning of spot size is achieved by applying external voltages. Nematic LCs are birefringent materials which provide different refractive indices along different optical axes [54, 55]. When a ray of light is incident upon a birefringent medium, it splits into two mutually perpendicular components, called ordinary rays (o-rays) and extraordinary rays (e-rays). O-rays always lie perpendicular to the optical axis, while e-rays lie in the plane of the optical axis [55]. This phenomenon brings out ordinary ($n_o$) and extraordinary ($n_e$) refractive indices for nematic LCs. Hence, the orientation of the nematic LCs can be tuned by applying an external voltage, and consequently the effective refractive index of the nematic LCs can be altered [56]. For transverse magnetic (TM) polarized light, where the concerned non-zero electric and magnetic field components
are $E_o$, $H_o$, and $H_r$, respectively. The effective permittivity of nematic LCs can be expressed as [57]:

$$\varepsilon_{\text{LC}} = n_{\text{LC}}^2 = \frac{n_0^2 n_{\text{LC}}^2}{n_o^2 \cos^2 (\theta_{\text{LC}}) + n_o^2 \sin^2 (\theta_{\text{LC}})^2} \quad (1)$$

where $\theta_{\text{LC}}$ is the rotation angle of the nematic LCs. Since the rotation angle $\theta_{\text{LC}}$ defines the variation of effective refractive index of nematic LCs, it can be expressed as a function of the applied external voltage as follows:

$$\theta_{\text{LC}} = \begin{cases} 0 & V \leq V_C \\ \frac{\pi}{2} - 2\tan^{-1} \exp \left( -\frac{V - V_C}{V_0} \right) & V > V_C \end{cases} \quad (2)$$

Here, the nematic LC molecules can be oriented by an electric field (or a magnetic field) when the field strength exceeds the Fredericksz transition threshold [58]. In equation (2), $V_C$ is the critical voltage at which the tilting angle starts to change, and $V_0$ is a constant value. Also, the term $(V - V_C/V_0)$ can be defined as the root-mean-square (RMS) voltage, $V_{\text{RMS}}$.

The relationships between the nematic LCs’ rotation angle $\theta_{\text{LC}}$ with respect to refractive index alteration $n_{\text{LC}}$ and $V_{\text{RMS}}$ are represented in figure 1(a). Also, the nematic LC molecules’ orientation in the $xz$-plane is given as an inset in a same plot. In figure 1(b) the schematic representation of a square lattice PC structure infiltrated by nematic LCs is presented. Corresponding important parameters such as height $d_z$, width $d_x$, radii of the dielectric rods $r_{\text{rod}}$ and the holes $r_{\text{hole}}$ are depicted in a same plot. In order to excite nematic LCs by an external voltage, pair of indium tin oxide (ITO) conducting layers are also used to provide the specific initial alignment layers. As can be seen in figure 1(c), in addition to ITO layers, the ‘alignment layers’ are also used to provide the specific initial angle of the nematic LCs’ orientation. This initial angle is crucial to obtain a defect-free alignment and to improve the performance of nematic LCs (response time and viewing angle). Another important parameter that influences the performance of the nematic LCs is the temperature. According to the Clausius-Mossotti and Lorentz-Lorenz equations the temperature dependency of the nematic LCs’ birefringence is expressed as follows [59]:

$$\Delta n = \Delta n_0 \left( 1 - \frac{T}{T_C} \right)^\beta \quad (3)$$

where $T$ is the operating temperature, $T_C$ is the clearing temperature of the nematic LC material, $\beta$ is an exponent and $\Delta n_0$ stands for the nematic LCs temperature dependent birefringence at $T = 0$K. Since a high birefringence is needed to obtain broad range tunability, throughout this manuscript the operating temperature is assumed to be equal to room temperature. It is important to note that in the case of a temperature increase, depending upon the birefringence change, the tuning value will decrease.

3. Active beam steering and afocal zooming design approach

As it is well known, the light within the GRIN medium follows curved trajectories because of a gradual change in refractive index. Hence, for optical phenomena such as focusing, diverging and collimating, the GRIN medium can be used without introducing the curved front and back interfaces for the optical components. In this regard, the main concept of the study lies in merging GRIN optics with nematic LCs to achieve active control of light propagation. Here, the gradient of the refractive index distribution of the GRIN medium can be actively changed to obtain the desired steering and zooming effects.

In order to form the GRIN medium, two-dimensional (2D) annular PCs are employed. The significant design parameters for the annular PCs are the radii of the dielectric rods ($r_{\text{rod}}$) and the holes ($r_{\text{hole}}$) that are drilled into the dielectric rods. By gradually changing air holes’ radii, one can change the dielectric filling ratio of the elementary PC cell, so that a GRIN PC medium with the desired refractive index profile can be obtained.

In this study, the hyperbolic secant (HS) refractive index profile is chosen to design the GRIN PC lens. The HS refractive index distribution is mathematically expressed as follows:

$$n(y) = n_0 \sec h(\alpha y) \quad , \quad (4)$$

where $n_0$ is the refractive index value at the optical axis and $\alpha$ is the gradient parameter that represents the depth of the index distribution. The shape of the HS function closely resembles a parabola and by tuning the gradient parameter $\alpha$ one can control the sign of the parabola [60]. Here, $\alpha$ can take real and
complex values that result in negative (downward facing) and positive (upward facing) parabolas, respectively.

In order to design the GRIN PC structure with annular PCs, the Maxwell-Garnett EM approximation is employed. Here, the EM approximation determines the radii of the air holes according to the defined HS refractive index profile. The formulation of the effective permittivity for the TM polarized wave is expressed as follows:

$$\varepsilon_{\text{eff}} = \varepsilon_{\text{rod}}(f) + \varepsilon_{\text{air}}(1-f),$$

where $f = \pi (r_{\text{rod}}^2 - r_{\text{hole}}^2)/a^2$ is the dielectric filling ratio of annular PC rods. Additionally, $\varepsilon_{\text{rod}}$ and $\varepsilon_{\text{air}}$ correspond to the permittivity of the annular PC rod and air medium. After some algebra, the variation formula of air holes’ radii $r_{\text{hole}}$ for TM polarization can be expressed as follows:

$$r_{\text{hole}}(y) = \sqrt{r_{\text{rod}}^2 + \frac{a^2}{\pi} \left( \frac{\varepsilon_{\text{air}} - r_{\text{eff}}(y)}{\varepsilon_{\text{rod}} - \varepsilon_{\text{air}}} \right)},$$

where ‘a’ is the lattice constant and $r_{\text{eff}}(y)$ stands for the HS refractive index distribution. It should be noted that a gradual variation of the air holes is achieved only along the transverse y-direction and the positions of the annular PC cells are kept constant at unit distance $1a$ along the x- and y- directions. Here, by incorporating the equations (5) and (6), one can design a square lattice GRIN PC structure with a desired refractive index distribution. For the design of the structure, the annular rods are assumed to be made of a polymer with refractive index of $n_p = 1.80$ [61], and the outer radii are fixed to $r_{\text{rod}} = 0.48a$. Also, minimum $r_{\text{hole}} = 0.06a$ and maximum $r_{\text{hole}} = 0.46a$. A detailed schematic of the designed pure annular GRIN PC structure is depicted in figure 2(a). Here, the gradient parameter is calculated as $\alpha = 0.0887a^{-1}$. Also, the structural dimensions such as height $d_x$ and width $d_y$ are presented as insets in a same plot and the real values of them are given in the following text.

In order to design the tunable GRIN PC structure, the air holes of annular PCs are assumed to be infiltrated by nematic LC molecules having ordinary and extraordinary refractive indices of $n_o = 1.59$ and $n_e = 2.22$, respectively. Here, the refractive indices correspond to a phenylacetylene LC material [62]. Infiltration by nematic LCs leads to a change in the effective refractive index distribution of the annular GRIN PC structure according to the formula:

$$n_{\text{eff}}^2 = \varepsilon_{\text{eff}} = \varepsilon_{\text{rod}}(f f_i - f f) + \varepsilon_{\text{LC}}(f f) + \varepsilon_{\text{air}}(1 - f f),$$

where $\varepsilon_{\text{LC}}$ is the permittivity of the nematic LCs, $f f_i = \pi r_{\text{hole}}^2/a^2$ and $f f = \pi r_{\text{rod}}^2/a^2$ are the fill factors of the hole areas drilled into the dielectric rods and annular PC rods, respectively. The schematic representation of the LC infiltrated structure with its corresponding refractive index profile is given in figure 2(b). In this case, the gradient parameter $\alpha$ is appeared to be $0.0477a^{-1}$. As can be seen from the index profiles in figures 2(a) and (b), one can observe that infiltration of LCs into the annular GRIN PC structure affects the gradient parameter. In order to analytically express the relationship between the infiltrated LCs and $\alpha$, the following formula can be obtained by incorporating equations (4) and (7):

$$\alpha = \frac{1}{y} \sec h^{-1} \left( \frac{1}{n_p} \sqrt{\frac{\pi^2 \varepsilon_{\text{LC}}^2}{a^2} (\varepsilon_{\text{LC}} - \varepsilon_{\text{rod}}) + \frac{\pi^2 \varepsilon_{\text{rod}}^2}{a^2} (\varepsilon_{\text{rod}} - \varepsilon_{\text{air}}) + \varepsilon_{\text{air}}} \right).$$

In equation (8), all parameters except $\varepsilon_{\text{LC}}$ are definite constant values, so that $\alpha$ depends solely on $\varepsilon_{\text{LC}}$, which varies with the rotation angle of the nematic LCs, $\theta_{\text{LC}}$. The refractive index profile variation of annular GRIN PC with respect to the rotation angle $\theta_{\text{LC}}$ of the infiltrated nematic LCs is presented in figure 3(a). Here, one can see the transition between positive and negative paraboloid index profiles. This transition is related to the values of $\alpha$, which takes real and imaginary values (this can also be proved by equation (8)). As can be seen in figure 3(b), for $\theta_{\text{LC}}$ in the range between $0^\circ$ ($\varepsilon_{\text{LC}} = 1.59$) and $43^\circ$ ($\varepsilon_{\text{LC}} = 1.80$), the gradient parameter is real-valued and decays to zero with an increase in the rotation angle. On the other hand, $\alpha$ takes imaginary values for rotation angles $\theta_{\text{LC}}$ greater than $43^\circ$, as shown in figure 3(c). Here, the important point is that for $\theta_{\text{LC}} = 43^\circ$ the refractive index profile is close to a straight line (constant along the y-axis), and $\alpha$ is approximately equal to zero.

In light of the discussed gradient parameter tuning concept, we propose beam steering and afocal zooming applications with nematic LCs infiltrated annular GRIN PC structures. In figure 4(a), a perspective view of the proposed beam steering device is presented. In the given schematic, we show the operating principle of the beam steering device schematically where the applied external voltage tunes the directivity of the incident beam. The width and height of the beam steering device are fixed to the values of $d_x = 11.96a$ and $d_y = 9.96a$, respectively. Indeed, the applied voltage changes the gradient parameter $\alpha$ with the help of the nematic LCs, and consequently the paraboloid refractive index profile changes.
its sign as shown in figure 4(b). The proposed afocal-zoom design is presented in figure 4(c) having structural dimensions of $d_x = 32.96a$ and $d_y = 20.96a$, respectively. Here, for afocal zooming, the device designed for beam steering is mirror symmetrized with respect to the propagation $x$-direction as can be seen in figure 4(c). This mirror-symmetry reveals the converging lens characteristics of the design. The afocal-zoom structure consists of three regions. Only the middle section of the PC structure is infiltrated with LCs (the air holes of the PCs are shaded in yellow, see figure 4(c)), and the left and right sections are made of non-infiltrated annular GRIN PC structures. Here, the annular GRIN PCs at the sides operate as converging lenses, and the middle section infiltrated with LCs serves as the lens with a tunable focal point (the applied external voltage manipulates the focusing characteristics). The working principle of beam diameter manipulation can be deduced from the effective refractive index profile shown in figure 4(d). While the LCs’ orientations are in the range of $0^\circ \leq \theta_{LC} < 43^\circ$, the middle section operates as a converging lens and for LCs’ orientations in the range of $43^\circ < \theta_{LC} \leq 90^\circ$, the middle section operates as a diverging lens. Consequently, adaptation of the beam diameter to the particular task can be achieved without introducing any mechanical motion of the lens, such as in [52].

4. Analytical and numerical results

In this section, the operating principle of the proposed beam steering and afocal zooming devices is analytically investigated via ray theory. For this reason, the GRIN media with effective refractive index profiles exactly same as in figures 4(c) and (d) are examined for beam steering and afocal
zooming applications. Afterwards, time-domain analyses of the proposed structures are provided in detail. Conceptual modelling of light propagation through a non-homogeneous medium can be explained by the ray equation also known as the Eikonal equation:

\[ \frac{d}{ds} \left[ n \frac{dr(s)}{ds} \right] = \nabla n, \]

where \( n \) is the refractive index of the medium, \( \nabla n \) is the gradient of \( n \), \( r(s) = [x(s), y(s)] \) is the ray path and \( ds = \sqrt{dx^2 + dy^2} \) is the arc length along the ray path. By solving the ray equation for a given refractive index distribution, ray trajectories can be obtained. In this regard, the ray trajectory function in a continuous HS GRIN medium with the refractive index profile given in equation (4) is expressed as follows (see [60] for detailed derivation of the ray trajectory):

\[ y(x) = \frac{1}{\alpha} \sinh^{-1} \left( \frac{\mu_0 \sin(\alpha x)}{\alpha} + \mu_0 \alpha \cos(\alpha x) \right), \]

where \( y(x) \) is the ray trajectory function with respect to initial position \( \mu_0 \) and incident angle \( \alpha \). Here, for the gradient parameter \( \alpha \), the data calculated in figures 3(b) and (c) are used. It should also be noted that equation (10) is derived in a hyperbolic coordinate system by using the transformation \( u = \sinh(\alpha y) \). By taking the position derivative of equation (10), the slope of the ray can be expressed as follows:

\[ \frac{dy}{dx} = -\frac{\alpha \mu_0 \sin(\alpha x) + \mu_0 \alpha \cos(\alpha x)}{\alpha \cosh \left( \sinh^{-1} \left[ \mu_0 \alpha \cos(\alpha x) + \mu_0 \sin(\alpha x)/\alpha \right] \right)}. \]

By using equations (10) and (11), the ray trajectories through the HS GRIN medium are calculated; the corresponding ray propagations for the beam steering application are plotted in figures 5(a)–(d). In order to visualize the beam steering effect, in figures 5(a)–(c) we show three distinct ray deflection cases, in which the gradient parameter equals \( \alpha = 0.0477 \alpha^{-1} \).

Figure 5. Expected beam bending based on ray theory in the cases of (a) \( \alpha = 0.0477 \alpha^{-1} \), (b) \( \alpha = 0.0098 \alpha^{-1} \) and (c) \( \alpha = 0.0691 \alpha^{-1} \). (d) Beam deflection angle profile in compliance with the rotation angle of the LC molecules. Field intensities for \( \alpha \alpha = 0.15 \) in the cases of (e) \( \theta_{LC} = 0^\circ \), (f) \( \theta_{LC} = 43^\circ \) and (g) \( \theta_{LC} = 90^\circ \). (h) Analytical beam deflection angle profile with respect to the rotation angle of the LC molecules for \( \alpha = 0.100, \alpha = 0.125 \) and \( \alpha = 0.150 \).

| \( \theta_{LC} \) | \( \theta_{out} \) at \( \alpha = 0.100 \) | \( \theta_{out} \) at \( \alpha = 0.125 \) | \( \theta_{out} \) at \( \alpha = 0.150 \) |
|---|---|---|---|
| 0° | 10° | 12° | 14° |
| 43° | 0° | 0° | 0° |
| 90° | 34° | 30° | 21° |

\( \alpha = 0.0098 \alpha^{-1} \), and \( \alpha = 0.0691 \alpha^{-1} \), respectively. Gradient parameters are taken from the data in figures 3(b) and (c). If \( \alpha \) is set in the interval \( 0.0477 \alpha^{-1} \geq \alpha \geq 0.036 \alpha^{-1} \) (0° \( \leq \theta_{LC} < 42° \)), the incident beam is steered downward (see figure 5(a)) and for the interval \( 0.0144 \alpha^{-1} \leq \alpha \leq 0.0691 \alpha^{-1} \) (44° \( < \theta_{LC} \leq 90° \)), the incident beam is steered upwards (see figure 5(c)) by the HS GRIN medium. On the other hand, when \( \alpha \) is set to 0.0098 \alpha^{-1} (\theta_{LC} = 43°), the overall medium becomes uniform, and thus incident rays exit the medium without any deflection, as shown in figure 5(b). The variation in deflection angle \( \theta_{out} \) at the back surface of the GRIN medium with respect to \( \alpha \) (LC rotation angle \( \theta_{LC} \)) is shown in figure 5(d).

After the conceptual explanation of the beam steering application via ray theory, numerical analyses of the proposed LC infiltrated annular GRIN PC structure were conducted by employing the FDTD method. The beam steering structure shown in figure 4(a) is excited by a TM polarized continuous wave source with a Gaussian profile. Figures 5(e)–(g) represent the steady state intensity profiles for cases of \( \theta_{LC} \) = \{0°, 43°, 90°\} under an incident light wave operating at a normalized frequency of \( \alpha \alpha = 0.150 \) where \( \lambda \) is the wavelength of incident light in free space. The beam deflection characteristics of the proposed structure can be clearly observed from the figure plots. Also, the ray theory observations shown in figures 5(a)–(c) are verified by the FDTD results (see figures 5(e) and (g)). In order to analyse the operating frequency bandwidth of the beam steering PC structure, its steering ability

\[ \theta_{out} \alpha = \frac{\sin(\alpha x)}{\alpha} + \mu_0 \alpha \cos(\alpha x) \left( \sinh^{-1} \left[ \mu_0 \alpha \cos(\alpha x) + \mu_0 \sin(\alpha x)/\alpha \right] \right). \]

\[ \frac{dy}{dx} = \frac{\alpha \mu_0 \sin(\alpha x) + \mu_0 \alpha \cos(\alpha x)}{\alpha \cosh \left( \sinh^{-1} \left[ \mu_0 \alpha \cos(\alpha x) + \mu_0 \sin(\alpha x)/\alpha \right] \right)}. \]
is examined under excitation by light waves with normalized frequencies of $a/\lambda = 0.100$, $a/\lambda = 0.125$, and $a/\lambda = 0.150$ (corresponding to the first TM band). With a 40% operating bandwidth, the variation in beam deflection angle with respect to LCs rotation angle is presented. As can be seen in table 1, for the normalized frequencies of $a/\lambda = \{0.100, 0.125, 0.150\}$, the total deflection angle obtained is $\Delta \theta_{\text{out}} = \{44°, 42°, 35°\}$ when varying the rotation angle from 0° to 90°.

The same procedure was also followed for the proposed afocal zooming application. Similarly, in order to explain the behaviour of light rays within the stacked GRIN media, the ray trajectories have been calculated and are schematically represented in figures 6(a)–(c). In figure 6(a), the exiting positions of propagating rays accumulate near the optical axis for a gradient parameter of $\alpha = 0.0477a^{-1}$. This means that if one considers a light beam instead of rays, the output beam diameter is reduced. On the other hand, if $\alpha$ changes from $\alpha = i0.0098a^{-1}$ (see figure 6(b)) to $\alpha = i0.0691a^{-1}$ (see figure 6(c)), the locations of the exiting rays become distant from each other, i.e. beam diameter expansion occurs. Figure 6(d) gives an insight into the afocal zooming effect. In figure 6(d), the distance $d_i$ between the upper and lower exiting ray positions with respect to $\alpha$ is represented.

Under the same simulation conditions given for the beam steering device, the numerical analyses of the afocal annular GRIN PC structure was examined via FDTD. Steady state field intensities of the designed zoom lens were determined for $\theta_{LC} = \{0°, 43°, 90°\}$ and the results are shown in figures 6(e)–(g), respectively. As expected, the results obtained by the ray theory are corroborated by the numerical results. For the normalized frequencies of $a/\lambda = \{0.15, 0.20, 0.25\}$, the beam diameter tuning results presented in table 2. Here, the beam diameter variation of the proposed structure was examined with a 50% operating bandwidth; this is shown in figure 6(h). Here, beam diameter calculations are conducted by extracting cross-sectional profiles of the exiting beam at the output, and measuring the beam width between the two points where the intensity is $1/e^2 = 0.135$ times the peak value. As a result, the beam diameter varies for $a/\lambda = 0.15$ from 1.37 in to 2.79, and $\alpha/\lambda = 0.20$ from 1.64 in to 3.54 in, and for $a/\lambda = 0.25$ from 1.85 in to 4.04 in for $\theta_{LC} = \{0°, 90°\}$, respectively. The beam diameter tuning results presented in table 2. Here, the input and output beam diameters are defined as $D_{\text{in}}$ and $D_{\text{out}}$, respectively.

It is important to note that the performances of beam deflection and light magnification are highly dependent on the total height of the structure (number of annular rods in γ-direction) and the material parameters (the values of ordinary and extraordinary refractive indices) of the LCs. Therefore, the structural height of the devices and the birefringence level of the LCs can be considered as important limiting factors for the beam deflection and light magnification characteristics.

For the practical applications of the proposed devices, annular polymer rods can be fabricated by using electro-spinning [63], direct polymerization [64] or spin-casting methods [65]. Here, the spin-casting method presented by Jin et al [65] is a good candidate for the fabrication of the

![Figure 6](image-url)
presented structures because the thickness and the diameter of the annular rods can be controlled by changing the concentration of the polymer solution. They have introduced annular polymer rods with the diameters ranging from 20 nm to 400 nm and wall thicknesses between 30 nm and 80 nm by gradually increasing the concentration of the polymer solution. Since this technique offers precise control of annular rod’s parameters with short fabrication time and high reproducibility it may provide feasible fabrication approach for the proposed designs. After the fabrication of the structures, LCs can be infiltrated into the annular rods by using a micropipette [66] or capillary action in an evacuated flask [67]. Due to the strong capillary force, complete filling of the annular rods can be provided fairly easily. Also, the surface anchoring results in the self-rotation of the LCs molecules which influences their alignments. Generally, homeotropic and homogenous alignment types are most commonly used to achieve the two different configurations of LCs [68]. In homogenously aligned types, direction of the LCs molecules orients parallel to the alignment layers while the direction of the LCs molecules is perpendicular in homeotropical case, see figure 1(c). In our analysis, we assume homogeneous alignment conditions where the LC-molecules are aligned with the x-axis at the absence of external electric field. To obtain the homogenous alignment, chemical treatment can be applied by using the surfactants such as hexadecyltrimethylammonium bromide (HTAB) or lecithin [68]. As a result, to achieve the rotation of the LCs molecules, the driven voltage can be applied to the ITO plates as illustrated in figure 1(c).

5. Conclusion

In this study, we presented compact structures for tunable beam steering and afocal beam zooming applications by combining LCs and annular PCs. Since there are no moving mechanical parts, the proposed structures are more compact and practical than conventional counterparts. The presented calculations show that for different frequency ranges, the proposed beam steering and afocal zooming mechanisms supply a broad tuning capability. For the beam steering application, a 40% operating frequency bandwidth is achieved, and for the normalized frequencies of $a/\lambda = [0.100, 0.125, 0.150]$, the total deflection angles obtained are $\Delta \theta_{\text{out}} = [44^\circ, 42^\circ, 35^\circ]$. On the other hand, for the beam diameter tuning device the 50% frequency bandwidth is obtained. The beam diameter varied from $1.37\lambda$ to $4\lambda$ within the operating frequencies of $a/\lambda = 0.10$ and $a/\lambda = 0.25$. As a result, beam diameter tuning device can compress incident beam diameter to 27% and expand incident beam to 215%. We investigated the beam steering and afocal zooming concepts both analytically and numerically. The designed LC infiltrated PC structures have the potential to be implemented in various optical applications, such as near-field scanning, vision correction, object tracking, optical communication, medical diagnostics, x-ray optics, space and atmospheric research, holography and imaging systems.

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**References**

[1] Deb Nath K, Bucio T D, Al-Attili A, Khokhar A Z, Saito S and Gardes F Y 2017 Opt. Express 25 3214–21
[2] Beggs D M, White T P, O’Faolain L and Krauss T F 2008 Opt. Lett. 33 147–9
[3] Mirjalili S M and Mirjalili S Z 2017 Neural Comput. Appl. 28 1–7
[4] Zhou W D, Sabarinarathan J, Bhattacharya P, Kochman B, Berg E, Yu P C and Pang S 2001 IEEE J. Quantum Electron. 37 1153–60
[5] Kosaka H and Kawashima T 1999 Appl. Phys. Lett. 74 1212–4
[6] Prather D W, Shi S Y, Pustai D M, Chen C H, Venkataraman S, Shankawy A, Schneider G J and Murakowski J 2004 Opt. Lett. 29 50–2
[7] Domachuk P, Nguyen H C, Eggleton B J, Straub M and Gu M 2004 Appl. Phys. Lett. 84 1838
[8] Tagantsev A K, Sherman V O, Astafiev K F, Venkatesh J and Setter N 2003 J. Electroceramics 11 5–66
[9] Yan H, Li X, Chandara B, Tulevski G, Wu Y, Freitag M, Zhu W, Avouris P and Xia F 2012 Nat. Nanotechnol. 7 330
[10] Wang H W, Chang I L and Chen L W 2012 Opt. Commun. 24 5524–30
[11] Rezaei B, Giden I H and Kurt H 2017 Opt. Commun. 382 28–35
[12] Cunningham J E et al 2010 Opt. Express 18 19055
[13] Park W and Lee J 2004 Opt. Express 11 5
[14] Domachuk P, Nguyen H C, Eggleton B J, Straub M and Gu M 2004 Appl. Phys. Lett. 84 1838
[15] Gourlay J, Love G D, Birch P M, Sharples R M and Purvis A 1997 Opt. Commun. 137 17–21
[16] Busch K and John S 1999 Phys. Rev. Lett. 83 967–70
[17] Liu C Y and Chen L W 2005 Phys. Rev. B 72 045133
[18] Khalkhali T F, Rezaei B and Ramezani A H 2012 Opt. Commun. 285 5254–8
[19] Wang Y Y and Chen L W 2006 Opt. Express 14 10580–7
[20] Zografopoulos D C, Asquini R, Kriezis E E, d’Alessandro A and Beccherelli R 2012 Lab Chip 12 3598–610
[21] Kniehl H and Citrin D S 2008 Opt. Express 16 11995–2001
[22] Li G et al 2006 Proc. Natl Acad. Sci. USA 103 6100–4
[23] Shang X, Trinidad A M, Joshi P, Smet J D, Cuypers D and Smet H D 2016 IEEE Photonics J. 8 1
[24] Liu Y J, Zheng Y B, Shi J, Huang H, Walker T R and Huang T J 2009 Opt. Lett. 34 2351–3
[25] Matsumoto T, Kun-Sun E and Baba T 2006 Opt. Lett. 31 2786–8
[26] Li X, He S and Jin Y 2007 Phys. Rev. 75 045103
[27] Cabukcu E, Aydin K, Ozbay E, Foteinopoulou S and Soukoulis C M 2003 Phys. Rev. Lett. 91 207401
[28] Verslegers L, Catrysse P B, Yu Z and Fan S 2009 Phys. Rev. Lett. 103 033902
[29] Ren G, Lai Z, Wang C, Feng Q, Liu L, Liu K and Luo X 2010 Opt. Express 18 18151–7
[30] Mahariq I, Kuzuoglu M, Tarman I H and Kurt H 2014 IEEE Photonics J. 6 1–14
[31] Joannopoulos J D, Villeneuve P R and Fan S H 1997 Nature 386 143–9
[32] Almeida V R, Xu Q, Barrios C A and Lipson M 2004 Opt. Lett. 29 1209–11
[33] Weeber J C, Lacroute Y and Dereux A 2003 Phys. Rev. B 68 115401
