THE EFFECT OF HYDROSTATIC WEIGHTING ON THE VERTICAL TEMPERATURE STRUCTURE OF THE SOLAR CORONA

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ABSTRACT

We investigate the effect of hydrostatic scale heights \( h \) in coronal loops on the determination of the vertical temperature structure \( T(h) \) of the solar corona. Every method that determines an average temperature at a particular line of sight from optically thin emission (e.g., in EUV or soft X-ray wavelengths) of a multitemperature plasma is subject to the emission measure–weighted contributions \( d\text{EM}(T)/dT \) from different temperatures. Because most of the coronal structures (along open or closed field lines) are close to hydrostatic equilibrium, the hydrostatic temperature scale height introduces a height-dependent weighting function that causes a systematic bias in the determination of the temperature structure \( T(h) \) as function of altitude \( h \). The net effect is that the averaged temperature seems to increase with altitude, \( dT(h)/dh > 0 \), even if every coronal loop (of a multitemperature ensemble) is isothermal in itself. We simulate this effect with differential emission measure distributions observed by SERTS for an instrument with a broadband temperature filter such as Yohkoh/Soft X-Ray Telescope and find that the apparent temperature increase due to hydrostatic weighting is of order \( \Delta T \approx T_e h/R_e \). We suggest that this effect largely explains the systematic temperature increase in the upper corona reported in recent studies (e.g., by Sturrock et al., Wheatland et al., or Priest et al.), rather than being an intrinsic signature of a coronal heating mechanism.

Subject headings: Sun: atmosphere — Sun: corona — Sun: X-rays, gamma rays

1. INTRODUCTION

Attempts to solve the elusive coronal heating problem have been undertaken by determining the heating function \( F_h(h) \) as function of height \( h \), inferred from the vertical temperature structure \( T_r(h) \) of the solar corona. In this context, a systematic temperature increase \( T(h) \) with height \( h \) has been reported from numerous observations of the quiet diffuse corona, coronal arcades, or coronal loops (Mariska & Withbroe 1978; Kohl et al. 1980; Falconer 1994; Foley et al. 1996; Sturrock, Wheatland, & Acton 1996a, 1996b; Wheatland, Sturrock, & Acton 1997; Fludra et al. 1999; Priest et al. 1999, 2000). A common method that is chosen to infer the vertical temperature structure \( T_r(h) \) is the extraction of soft X-ray fluxes in different wavelengths as function of height, say \( F_1(h) \) and \( F_2(h) \) from two different wavelengths 1 and 2, and then to use the filter-ratio method \( Q(h) = F_2(h)/F_1(h) \) to determine the temperature as function of height, \( T(h) \), by inverting the filter-ratio function \( Q(T) \). The filter-ratio method has some obvious limitations, such as the limited range in which the function \( Q(T) \) is unique, and thus permits only an inversion within this range, but the method has also some more subtle drawbacks in the case of a multitemperature plasma, since it exists in the solar corona. In principle, the filter-ratio method is only exact for an isothermal plasma, within the uniqueness range of \( Q(T) \). The solar corona consists of myriads of open and closed field lines filled with plasmas of almost every temperature in the range of \( 10^4 \leq T \leq 10^7 \) K, which is usually quantified with a differential emission measure distribution \( d\text{EM}(T)/dT \). This multithermal nature can cause systematic errors in the determination of an average vertical temperature profile \( T_v(h) \) because of a systematic weighting bias of the temperature-dependent pressure and density scale heights (Fig. 1). The purpose of this Letter is to demonstrate this systematic error in the determination of the vertical temperature profile \( T_v(h) \) for some typical observations of active regions and the quiet corona, using a broadband filter instrument such as the Yohkoh Soft X-Ray Telescope (SXT).

2. MODEL

The soft X-ray flux measured along a given line of sight represents an integral over emission measure contributions from plasmas with different temperatures, which can be expressed by the differential emission measure distribution \( d\text{EM}(T)/dT \), where the emission measure contribution at a given temperature \( [T, T + dT] \) itself represents an integration along the line of sight \( z \),

\[
\frac{d\text{EM}(T)}{dT} dT = \int n_e^2(T, z) dz. \tag{1}
\]

The flux measured by a detector \( i \) is then given by the product of the differential emission measure function \( d\text{EM}(T)/dT \) with the instrumental temperature response function \( R_i(T) \),

\[
F_i = \int \frac{d\text{EM}(T)}{dT} R_i(T) dT. \tag{2}
\]

We characterize now the solar corona by a superposition of many different flux tubes (along open or closed magnetic field lines), each one having its own temperature and density function. For the purpose of this demonstration, we make the simplest assumption that is compatible with observations, namely (1) that each flux tube is near isothermal (as it has been established for many observed EUV loops in the temperature range of \( T_e \approx 1.0-2.0 \) MK; e.g., Neupert et al. 1998; Lenz et al. 1999; Aschwanden et al. 1999, 2000a, 2000b), and (2) that each flux tube is in near-hydrostatic equilibrium (a condition
When the filter-ratio method is applied, one takes the flux ratio of the two fluxes at every pixel (along a chosen altitude path $h$):

$$Q(h) = \frac{F_i(h)}{F_o(h)},$$

which is now height dependent, so that the resulting filter-ratio temperature $T(Q) = T[Q(h)]$ yields the height dependence of the temperature $T(h)$.

3. OBSERVATIONS AND SIMULATION

Some typical differential emission measure distributions $d\text{EM}(T)/dT$ have been determined with the NASA/GSFC Solar EUV Rocket Telescope and Spectrograph (SERTS), using density-sensitive line ratios from eight different ionization states of iron between Fe$^{+8}$ (Fe X) and Fe$^{+13}$ (Fe XVII), during two flights in 1991 and 1993 (Brosius et al. 1996). These line ratios provide density diagnostics between temperatures of $\log(T_e) = 5.0$ and $\log(T_e) = 6.7$ (i.e., $T_e \approx 0.1-5.0$ MK). Brosius et al. (1996, see their Figs. 8 and 9) derived a differential emission measure curve $d\text{EM}(T)/dT$ in the temperature range of $\log(T_e) = 4.8-7.0$, which is reproduced in Figure 2 (top) for two observations of active regions (Active Region [AR] 93, AR 91) and two observations of quiet-Sun regions (Quiet Region [QR] 93, QR 91).

We consider now the instrumental response functions of Yohkoh/SXT. For active regions, the two filters sensitive to the lowest temperatures are the thin aluminum (Al $\lambda 1265$) and the Al/Mg/Mn composite filter (Tsuneta et al. 1991). The corresponding response functions $R_i(T)$ and $R_o(T)$ are shown in Figure 2 (second panel), and their filter ratio $Q(T) = R_i(T)/R_o(T)$ is given in Figure 2 (bottom). In order to understand the temperature contributions to the observed flux, we show the differential soft X-ray flux $dF(T)/dT = [d\text{EM}(T)/dT]R(T)$ (Fig. 1, third panel) for both filters and for all four regions. The differential soft X-ray (SRX) flux exhibits a peak at a temperature of $T_e \approx 10^{6.65} = 4.5$ MK for the active regions and at $T_e \approx 10^{6.3} = 2.0$ MK for the quiet-Sun regions.

We calculate now the fluxes $F_i(h)$ and $F_o(h)$ in the two filters as function of the height $h$ above the limb, using the hydrostatic distribution defined in equations (5)–(6), where each flux tube with (different) temperature $T$ has a (different) density scale height of $\lambda = \lambda(T,T_e)$, while the total ensemble of flux tubes is summed up by an integration over the entire temperature range (i.e., temperature integral in eqs. [5]–[6]). The resulting SXR fluxes as function of height are shown in Figure 3 (top), illustrating that the SXR flux drops exponentially with height. We derive now the filter ratio $Q(h) = F_i(h)/F_o(h)$, shown in Figure 3 (middle) for all four regions. The filter ratio $Q(h)$ clearly varies as function of height $h$, although each flux tube is assumed to be isothermal.

We demonstrate now what effect this filter-ratio variation $Q(h)$ has on the inference of a single-temperature model $T(h)$, as it is assumed in the classical filter-ratio method by definition. To invert the filter ratio $Q(T)$ as function of the temperature $T$, we find the following analytical approximation (accurate within $\pm 0.7\%$) in the temperature range of $T = 1.5-6.0$ MK (see fit in Fig. 2, bottom),

$$Q(T) \approx \frac{R_o(T)}{R_i(T)} = 0.39 + 0.27[\log(T) - 6.18]^{1/2}. \quad (7)$$

This analytical approximation allows us conveniently to invert...
The inverted temperatures $T(Q(h))$ are shown in Figure 3 (bottom) for all four regions. The filter-ratio temperature $T(h)$ shows a height dependence from $T(h=0) \approx 2.1$ to $T(h=0.5 \ r_0) \approx 3.1$ MK for the quiet regions and from $T(h=0) \approx 4.1-4.4$ to $T(h=0.5 \ r_0) \approx 5.4-6.3$ MK for the active regions. Thus, the weighting effect of temperature scale heights over the broadband response function introduces an apparent temperature gradient of $d\bar{T}/dh \approx 0.003 \ K \ m^{-1}$ for the quiet corona regions and about $d\bar{T}/dh \approx 0.005 \ K \ m^{-1}$ for active regions. This corresponds about to a doubling of the apparent temperature over a distance in the range of $Q = 0.4-0.6$, i.e.,

$$\log [T(Q)] = 6.18 + \left( \frac{Q - 0.39}{0.27} \right)^2. \quad (8)$$

The inverted temperatures $T(Q(h))$ are shown in Figure 3 (bottom) for all four regions. The filter-ratio temperature $T(h)$ shows a height dependence from $T(h=0) \approx 2.1$ to $T(h=0.5 \ r_0) \approx 3.1$ MK for the quiet regions and from $T(h=0) \approx 4.1-4.4$ to $T(h=0.5 \ r_0) \approx 5.4-6.3$ MK for the active regions. Thus, the weighting effect of temperature scale heights over the broadband response function introduces an apparent temperature gradient of $d\bar{T}/dh \approx 0.003 \ K \ m^{-1}$ for the quiet corona regions and about $d\bar{T}/dh \approx 0.005 \ K \ m^{-1}$ for active regions. This corresponds about to a doubling of the apparent temperature over a distance.
of a solar radius $r_\odot$.

$$\Delta T^{SXT} \approx T_0 \left( \frac{h}{r_\odot} \right)^\alpha .$$

### 4. DISCUSSION AND CONCLUSIONS

We have investigated the effect of hydrostatic density scale heights in coronal loops on the inference of a filter-ratio temperature from a broadband instrument, in particular for the two thinnest filters of Yohkoh/SXT, which are generally used to derive electron temperatures in active regions and in the quiet corona. The principal effect is that, with increasing altitude $h$ (above the solar surface), the emission measure–weighted temperature $T_e$ becomes systematically more weighted by the larger scale heights $\lambda$ (Fig. 1), which are associated with loops of higher temperature and thus mimic an average temperature increase with height. We used differential emission measure distributions $d EM(T)/dT$ that have been observed in active regions and in quiet-Sun regions and simulated the temperature bias on $T(h)$ for the instrumental response functions of Yohkoh/SXT. The resulting temperature bias can be quantified approximately as $\Delta T^{SXT} \approx T_0 (h/r_\odot)$. We discuss now the consequences of this result.

The radial variation of temperature in the inner corona (out to 0.7 and 0.95 solar radii) has been examined for the diffuse corona from long-exposure Yohkoh/SXT images by Wheatland et al. (1997). These authors find a systematic temperature increase from $T_e \approx 1.6$ MK near the solar surface to $T_e \approx 2.4$ at a height of 0.5 solar radii for the 1992 May 7–9 active region and from $T_e \approx 1.8$ to $T_e \approx 2–3$ MK at 1 $r_\odot$ for the 1992 August 26 region. This systematic temperature increase of the solar corona was interpreted in terms of a downward heat flux, leading to the conclusion of a heat deposition above the observed height. According to our model (eq. [9]), we estimate fully consistent temperature increases $T_e(h=0) = 1.6$ MK $\rightarrow T_e(h=0.5 r_\odot) = 2.4$ MK for the first case, and $T_e(h=0) = 1.8$ MK $\rightarrow T_e(h=r_\odot) = 2.7$ MK for the second case from the emission measure–weighted hydrostatic scale heights alone, even if all flux tubes are isothermal. Therefore, if the hydrostatic weighting effect on the Yohkoh/SXT filter-ratio method would be corrected, no net temperature increase would result and thus no support for a heating function in the upper corona is warranted.

With the same measurement technique, Priest et al. (1999, 2000) analyzed large-scale arcades and loops and found a temperature increase from $T_e(h=0) = 1.6$ to $T_e(h=0.5 r_\odot) = 2.2–2.3$ MK for a first loop observed on 1992 October 3, an increase from $T_e(h=0) = 1.6$ to $T_e(h=500 \text{ Mm}) = 2.4–2.6$ MK in a second loop, and an increase from $T_e(h=0) = 1.6$ to $T_e(h=350 \text{ Mm}) \approx 2.1$ MK in a third loop. The authors fitted three heating models to these temperature increases $T_e(h)$ and found that a uniform heating function provides the best fit for all three cases, while heating functions localized at the loop top were found to be less likely and a heating function localized near the loop footpoints was rejected. From our model (eq. [9]) we can reproduce the same temperature increases for these three cases, so that virtually no net temperature increase remains if the Yohkoh/SXT filter ratios would be corrected for the hydrostatic emission measure weighting. Comparing the corrected temperature profiles $T_e(h) \approx \text{const.}$ with the heating models shown in Figures 8 and 9 in Priest et al. (2000), one would conclude that the data are most consistent with the theoretical model of footpoint heating, a conclusion that would also be more in line with other recent observations from TRACE (Schrijver et al. 1999; Aschwanden et al. 2000b).

In summary, we would like to point out that filter-ratio temperatures from broadband instruments may lead to systematic errors in the determination of vertical temperature profiles $T_e(h)$, which can only be corrected properly by forward-fitting of models that contain both temperature $T_e(h)$ and density profiles $n_e(h)$. The systematic effects are larger for broadband filter ratios (e.g., Yohkoh/SXT) than for narrowband filters (e.g., SOHO/EIT or TRACE). Any detected temperature increase derived from an emission measure–weighted temperature definition is subject to the hydrostatic weighting of a multi-temperature plasma and does not directly describe a variation $(dT/dh)$ of the electron temperature along a magnetic field line.

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