Gamma-Ray Bursts and Topology of the Universe

MAREK BIESIADA
Copernicus Astronomical Center,
Bartycka 18, 00-716 Warsaw, Poland

September 1993

Abstract

In this letter we propose a physical explanation for recently reported correlations between pairs of close and antipodal gamma-ray bursts from publicly available BATSE catalogue. Our model is based on the cosmological scenario in which bursters are located at cosmological distances of order of 0.5–2 Gpc. Observed distribution of gamma-ray bursts strongly supports this assumption. If so gamma-ray bursts may provide a very good probe for investigating the topological structure of the Universe. We notice that correlation between antipodal events may in fact indicate that we live in the so called Ellis’ small universe which has Friedman-Roberston-Walker metric structure and nontrivial topology.
1 Introduction

Gamma ray bursts (GRBs thereafter) are undoubtedly one of the most mysterious phenomena in the sky with enormous diversity of durations, time variability and spectra (for a review see [1]). Although their discovery was announced 20 years ago [2] and despite numerous efforts aimed at detecting these events (PVO, KONUS, SIGNE, SMM, GINGA, BATSE) the nature of GRBs, mechanisms of their \(\gamma\)-emission and the distance scales still remain unknown. The last issue is crucial in a sense that determination of the distance scale may in principle be performed without any detailed understanding of the GRBs and will provide a severe constraint on the set of all possible models of GRBs [3]. Unfortunately all we can currently do is to make inferences about the distance scales to the bursters from their distribution.

Since the work of Schmidt et al. [4] GRBs’ uniform distribution is tested by extracting a quantity \(V/V_{\text{max}}\) which is a quotient of the volume determined by the distance to the source and the maximal volume accessible for the detector. For an isotropic and homogeneous population the distribution of \(V/V_{\text{max}}\) is uniform over a unit interval \([0, 1]\) and has a mean value of \(<V/V_{\text{max}}>=0.5\).

Thirteen years of continuous operating of PVO provided with a very good statistic of strong bursts. They turned out to be distributed isotropically on the celestial sphere (in angle) [5] and uniformly in radial coordinate — \(<V/V_{\text{max}}>_{\text{PVO}}=0.46\pm0.02\) [6]. First results from the Burst and Transient Source Experiment (BATSE) aimed at detection of weak bursters in hope to see them concentrating towards the galactic plane were sensational [7]. They revealed that the distribution of weak GRBs is isotropic and that there is fewer weak bursts than it could be expected from extrapolating the number of strong bursts — \(<V/V_{\text{max}}>_{\text{BATSE}}=0.324\pm0.014\). Hence we are apparently placed at the center of a spherically symmetric distribution of GRBs which is uniform out to some distance and falls off beyond. This finding clearly prefers the hypothesis of cosmological origin of GRBs [8]. Indeed we observe the Universe as isotropic and the deficiency of weak (distant) bursts may be explained in a natural way as a consequence of the Hubble expansion. Although alternative explanations placing GRBs within the extended halo [9] or the Oort cloud [10] cannot be excluded definitely at this moment the cosmological (extragalactic) scenario seems to be the most plausible one.

Recently Quashnock and Lamb [11] analysed the angular distribution of GRBs from the BATSE catalogue using a nearest neighbor analysis. They found that bursts are significantly clustered on an angular scale \(\sim 4^\circ\). Since systematic measurement errors in the BATSE experiment are of order of \(4^\circ\) they conjectured that GRBs typically repeat. If they were correct it would rule out most of current extragalactic models which invoke single violent events such like neutron star – neutron star or neutron star – black hole mergers [12]. Narayan and Piran [13] subsequently reanalysed the data using angular autocorrelation function and repeated the nearest neighbor analysis of Quashnock & Lamb. They have employed the full sample of 260 bursts from BATSE catalogue and defined a subsample (131 bursts) by including only those bursts which have formal positional error smaller than \(4^\circ\). The reason for the latter choice is that in addition to the systematic error of \(4^\circ\) bursts positions have variable error which is estimated in the BATSE catalogue for each event separately as the so called formal positional error (and which may be as large as \(20^\circ\)). In addition a sample denoted in [11] as Type I+II was considered in its
full extent (201 events) and truncated (108 bursts) by demand that the formal positional error be smaller than 4°. What they found was an excess of close pairs with separations smaller than 4° as well as an excess of antipodal pairs with separations larger than 176°. The statistical significance of the excesses was not impressive. For example peaks of the autocorrelation function had amplitudes of 1.75 σ for close pairs and 1.86 σ for antipodal pairs in the full BATSE sample, 0.84 σ and 2.6 σ for truncated BATSE sample, 1.95 σ and 1.75 σ for Quashnock & Lamb sample and finally 0.74 σ and 2.6 σ for truncated Quashnock & Lamb sample. Narayan and Piran concluded that both excesses (of close pairs and antipodal ones) are likely due some unknown selection effect.

In the present letter we propose an explanation of reported excess of antipodal GRBs provided that bursters are of cosmological origin and the Universe possesses nontrivial topology.

2 Topological structure of the Universe

It is commonly accepted that the Universe we live in is extremely well approximated by one of homogeneous and isotropic Friedman-Robertson-Walker (FRW) models [14]. The FRW metric may be written in the form:

\[ ds^2 = c^2 dt^2 - a^2(t) \left( \frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right), \]  

(1)

where \( a(t) \) is the scale factor and \( k = 0, +1, -1 \) is the curvature of constant time hypersurfaces and determines whether the Universe is flat, closed or open respectively. It is obvious that the metric of the space-time is of local nature and gives no information about its topology. In other words a given metric structure such like FRW metric (1) can be realized for different topologies. For sake of illustration let us recall that two-dimensional flat Euclidean metric can equally well be realized on a plane \( \mathbb{R}^2 \), a cylinder or on a torus \( T^2 \). On the other hand all successfull physical predictions of standard big-bang cosmology are based on the local metric structure (1) and the problem of topological structure of the Universe remains open. All we know is that the Universe we observe is locally isotropic and homogeneous i.e. the hypersurfaces of constant time \( \Sigma_t \) are 3-dimensional spaces of constant curvature \( k \) (\( k = 0, +1, -1 \)). Classification of all topologically distinct spaces \( \Sigma_t \) comes from the celebrated theorem of Killing and Hopf that \( \Sigma_t \) are isometric to \( \tilde{\Sigma}_t/\Gamma \) where \( \Gamma \) is a discrete isometry subgroup of \( \tilde{\Sigma}_t \) and \( \tilde{\Sigma}_t \) (the so called covering manifold) is

\[ \tilde{\Sigma}_t = S^3 \] — three-sphere for \( k = +1 \),
\[ \tilde{\Sigma}_t = R^3 \] — Euclidean space for \( k = 0 \),
\[ \tilde{\Sigma}_t = H^3 \] — three-dimensional hyperbolic space for \( k = -1 \).

The full topological classification (equivalent to enumerating all relevant discrete groups \( \Gamma \)) exists for \( k = 0, +1 \) [15] — for \( k = -1 \) only compact spaces can be classified [16]. Although the number of topologically distinct spaces with FRW metric is large (18 for flat and infinite for open and closed models) it is a common procedure to assume for simplicity that \( \Gamma = I \) and hence \( \Sigma_t = \tilde{\Sigma}_t \). Whenever \( \Gamma \) is not equal to identity, the points equivalent under action of \( \Gamma \) are identified. Such a procedure generates a multiply connected space in which geodesics connecting two points are not unique. This fact opens
the possibility of observationally verifying the topology of the Universe provided the scale for multiply connectedness is smaller than present horizon scale $a_0$ (the so called Ellis’ small universe) \[ L \] where $a_0 = c/H_0 \approx 3 \times 10^3 h_{100}^{-1} \text{Mpc}$. An unusual and distinctive feature of a multiply connected small universe is that an observer sees many copies of the same object in different directions at different distances. The crucial point here is the existence of geodesics starting and ending at the same point — the so called main geodesics of this point \[ \Gamma \]. Just to be specific let us restrict our attention to flat FRW universe ($k = 0$) with the topology of a torus $T^3$ generated by a discrete group $\Gamma$ of translations by vectors $e_1, e_2, e_3$. Then geodesics in the directions $d_1 = e_1/|e_1|$, $d_2 = e_2/|e_2|$, $d_3 = e_3/|e_3|$, or some linear combination of them $d_{ij} = n_i d_i + n_j d_j$ are the main geodesics. If an object lies on (or near to) the main geodesic of an observer then two copies of it may be observed in exactly (or nearly) opposite directions. Another important effect is the periodicity of distances determined by the radial coordinate $r$ in the FRW metric (1). Namely if the object is situated close to the main geodesic then the observer sees many copies of it at distances $r + n|e_i|$ where $n$ is an integer in the direction $d_i$ as well as in opposite direction. More detailed discussion of the issue of main geodesics and observations in the small universe can be found in \[ \text{[17]} \] and \[ \text{[18]} \].

3 Discussion and summary

In this section we shall discuss an intriguing possibility that the effect of correlation between antipodal pairs of GRBs reported by Narayan & Piran \[ \text{[13]} \] may be an evidence for nontrivial topology of the Universe. Let us start with known constraints on the scale $L_{\text{top}}$ for multiply connectedness of the Universe. The earliest attempts to confront the idea of multiply connected universe with observations performed by Sokolov & Shvartsman \[ \text{[20]} \] and by Gott \[ \text{[21]} \] constrained this scale from below — $L_{\text{top}}$ should be larger than $200 h_{100}^{-1} \text{Mpc}$ where as usually $h_{100}$ stands for the present Hubble constant in units of 100 km/s/Mpc and is currently believed to lie between 0.4 and 1. This threshold value stems from the fact that we do not observe many copies of familiar objects such like Coma or Virgo clusters. Searching for periodicity in quasar distances Fang et al. \[ \text{[22]} \] found positive effect with periodicity scale (which is of the same order as $L_{\text{top}}$) $\sim 600 \text{ Mpc}$. However they analysed only two small areas in the sky without considering antipodal areas. Demiański and Lapucha \[ \text{[18]} \] investigated the effect of antipodal pairs of galaxies, clusters and quasars and found marginally significant effect but gave no new estimate for $L_{\text{top}}$. Therefore we shall adopt the estimate of Sokolov, Shvartsman & Gott and conclude that the effect of non-trivial topology of the Universe may potentially manifest itself only for objects at distances larger than $\sim 500 h_{100}^{-1} \text{ Mpc}$. This requirement is met for the population of GRBs in the cosmological scenario. The observed isotropy of GRBs combined with their uniform number density out to some distance falling off beyond indicate that GRBs are most likely at distances larger than $\sim 500 h_{100}^{-1} \text{ Mpc}$. Indeed as estimated by Mao & Paczyński \[ \text{[19]} \] cosmological bursters should have redshifts within the range from $z \approx 0.2$ to $z \approx 1.7$ which correspond to the distance scale of $\sim 500 h_{100}^{-1} \text{ Mpc}$ and $\sim 2 h_{100}^{-1} \text{ Gpc}$ respectively (the second estimate comes from the faintest bursts seen in BATSE). Hence the GRBs are very good candidates for probing the topological structure of the Universe.

Although the statistical significance of excess in antipodal pairs of GRBs found in
is not very impressive there is still some slight evidence that this effect may be real. Whereas the significance of close pairs excess depends crucially on the sample chosen the correlation of antipodal pairs displays relative stability within tests performed in [13]. Moreover the significance of excess in oposite pairs increases when the sample is improved by rejecting bursts with large positional error unlike in the case of close pair correlation. Narayan & Piran did not commented on this since they assumed that correlation of antipodal bursts is unphysical.

As already mentioned not every object observed in the small universe will have its antipodal ghost image—this effect manifests only in the direction of a main geodesic. Therefore we may expect that only a certain fraction of GRBs contributes to the net effect. In order to be observed in the BATSE experiment the light coming from an GRB directly and from the antipodes must arrive at approximately the same time. In the other words two arcs of the main geodesic passing through an observer and an GRB must have approximately the same length. This further constrains the fraction of events contributing to antipodal pair correlation. If we knew the topology of the universe in advance we might be able in principle to estimate this fraction. However we may expect that very strong correlation of only a fraction of events should leave an imprint on overall correlation function even though the significance is decreased by contamination of statistical sample by uncorrelated GRBs.

Narayan & Piran noticed that statistical significance would be by far better in the case of combined effect of both close and oposite pairs correlation. Such an effect can easily be explained in terms of periodic distance images close to main geodesics in multiply connected universe. Indeed when the object ($\gamma$-burster) is located near the main geodesic then we should see it accompanied by its ghost images distributed periodically in radial comoving coordinate $r$ and the whole picture should be reproduced at an antipodal locus. In the case of GRBs it would mean in particular that strong bursts should be correlated with faint ones. In fact such a correlation has been reported by Quashnock & Lamb [11]. However their nearest neighbor analysis was insensitive to antipodal correlations. On the other hand Narayan & Piran who discovered antipodal correlations did not investigated correlations between faint and strong bursts. Therefore future analysis of BATSE data in which correlation between close and antipodal as well as between faint and strong bursts is carefully investigated is a natural test for checking the idea proposed in this letter. We can make a crude estimate of how many faint bursts may be expected to accompany a strong event in the BATSE data. Since the strongest GRBs seen by BATSE lie probably at $\sim 500 \ Mpc$ and the detector at Compton GRO is probing the distance out to $\sim 2 \ Gpc$ then assuming the topological periodicity scale $\sim 300 \ Mpc$ (which is a compromise between the lower bound of Sokolov, Shvartsman & Gott and the value reported by Fang) we may conclude that up to about 5 weak events may be correlated with a strong one. This result is however a very rough one depending crucially on the value of unknown scale $L_{\text{top}}$. Note also that topological periodicity scales can be different for different directions.

It is obvious that in this scenario the faint burst must have occured about $(L_{\text{top}}/1 \ Mpc) \times 3 \cdot 10^6 \ yr \sim 10^9 \ yr$ earlier than correlated strong one. Hence they cannot come from the same event. On the other hand according to a merger scenario which is the most popular of extragalactic scenarios GRBs are consequences of neutron star–neutron star or black hole–neutron star mergers. As demonstrated in [12] such a scenario is capable of explaining most of observed features of GRBs. Suppose that a burst occured in certain
galaxy then because the rate of mergers is about $10^{-5} - 10^{-6} \, yr^{-1}$ per galaxy we have a chance of accidental coincidence of a given burst with one which have occured $\sim 10^9 \, yr$ earlier and is now visible in one of ghost images of this galaxy.

In this letter we have proposed an explanation for reported recently correlation between pairs of antipodal GRBs in the BATSE data. Our model invokes a fascinating idea that our Universe may possess nontrivial topological structure known for long as the so called Ellis’ small universe. Unfortunately angular resolution $\sim 4^\circ$ of the BATSE experiment is very poor for verifying the hypothesis of small universe. It is very possible that in the nearest future other space experiments will provide accurate positions, of order of a fraction of an arc minute, for strong bursts \([1]\). This will enable us to test some specific models of GRBs in the cosmological scenario. It will also be of interest to see whether the opposite pairs of GRBs turn to be correlated within this improved accuracy. More extensive statistical analysis of the BATSE data will also tell us a lot. It may well be the case that the original point of view of Narayan & Piran to search an explanation in some selection effect is valid. Even if our hypothesis is disproven it is worth in our opinion to recall from time to time and to reflect upon the fact that even though we know the metric structure of the world we live in yet we can hardly say anything about its topology.

When this letter has been completed a preprint by Maoz \([2]\) appeared in which author proposed an alternative explanation of antipodal pair correlation — the so called ring bias. According to this hypothesis some bursts collapse to $4^\circ$ wide rings around great circles in the celestial sphere because of GRO’s localization procedure. Bursts lying in the intersection of such rings would account for observed correlation in both close and antipodal pairs. It is worth noticing in the context of the present letter that numerical simulations of Maoz clearly demonstrate that correlation between close and antipodal pairs can be reproduced even if biased distribution is mixed with randomly distributed GRBs. This supports our claim that it is sufficient to have only a fraction of events located suitably at main geodesics in the Ellis’ small universe in order to obtain observed features in autocorrelation function.

Acknowledgements

This project was supported by the Foundation for Polish Science and is a contribution to the KBN Grant 2 20447 91 01.

References

[1] Paczyński B., ”Gamma-Ray Bursts: Facts and Fantasies”, 1992 Princeton Observatory Preprint 463; available through an anonymous ftp from astro.princeton.edu as /bp/science.tex

[2] Klebesadel R.W., Strong I.B., Olson R.A., ApJ., 182, L85, 1973.
[3] Paczyński B., "Gamma-Ray Bursts", talk presented at the Texas/PASCOS Symposium, Berkeley Dec.13–18, 1992.

[4] Schmidt M., Higdon J.C. and Huetner G., Ap.J. 329, L85, 1988.

[5] Atteia J.L., Barat C., Hurley K., Niel M., Vedrenne G., Evans W., Fenimore E., Klebesadel R.W., Laros J.G., Cline T., Desai U., Teegarden B., Estulin I., Zhenchenko V., Kuznetsov A. and Kurt V., Ap.J.Suppl. 64, 305, 1987.

[6] Fenimore E.E., Epstein R.I., Ho C., Klebesadel R.W. and Laros J.G., Nature 357, 140, 1992.
Chuang K.W., White R.S., Klebesadel R.W. and Laros J.G., Ap.J. 391, 242, 1992.

[7] Meegan C., Fishman G., Wilson R., Paciesas W., Pendleton G., Horack J., Brock M. and Kouveliotou C., Nature 355, 143, 1992.

[8] Paczyński B., Acta Astronomica 41, 257, 1991.
Piran T., Ap.J. 389, L45, 1992.
Dremer C.D., Phys.Rev.Lett. 68, 1799, 1992.

[9] Paczyński B., Acta Astronomica 41, 157, 1991.
Mao S. and Paczyński B., Ap.J. 389, L13, 1992.
Breinerd J.J., Nature 355, 522, 1992.

[10] Nakamura T., Shibazaki N., Murakami T. and Yoshida A., Prog.Theor.Phys. 87, 879, 1992.
Kuznetsova A.V., Cosmic Res. 20, 72, 1982.

[11] Quashnock J.M. and Lamb D.Q., "Evidence that γ-ray burst sources repeat", submitted to Mon.Not.Roy.Astr.Soc., 1993.

[12] Narayan R., Paczyński B. and Piran T., Ap.J. 395, L83, 1992.
Narayan R., Piran T. and Shemi A., Ap.J 379, L17, 1991.

[13] Narayan R. and Piran T., "Do Gamma-Ray Burst Sources Repeat?", preprint astro-ph/9308007, 1993 (submitted to Mon.Not.Roy.Astr.Soc.).
[14] Kolb E. and Turner M., "The Early Universe", Addison-Wesley Publ. Comp., New York, 1990.

[15] Wolf J., "Spaces of Constant Curvature", McGraw-Hill, New York, 1967.

[16] Thurston W., "The Geometry and Topology of 3-manifolds", Princeton U Press, Princeton NJ, 1978.

[17] Ellis G.F.R., Gen.Rel.Grav. 2, 7, 1971.
Ellis G.F.R., in General Relativity and Gravitation, eds Berotti B.,de Felice F. and Pascolini A., Reidel, Dordrecht, 1983.

[18] Demiański M. and Lapucha M., Mon.Not.Roy.Astr.Soc. 224, 527, 1987.

[19] Mao S. and Paczyński, Ap.J 388, L45, 1992.

[20] Sokolov D.D. and Shvartsman V.F., Sov.Phys.J. ZhETF 39, 196, 1974.

[21] Gott J.R., Mon.Not.Roy.Astr.Soc. 193, 153, 1980.

[22] Fang L.Z., Chu Y.Q. and Liu Y., Astron.Astrophys. 106, 287, 1983.

[23] Maoz E., "A possible explanation for the peculiar correlations in the angular distribution of gamma-ray bursts", Preprint astro-ph/9308040, 1993.