Recursive Prediction for Lossless Image Compression

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Abstract

This paper introduced an algorithm for lossless image compression to compress natural and medical images. It is based on utilizing various casual fixed predictors of one or two dimension to get rid of the correlation or spatial redundancy embedded between image pixel values then a recursive polynomial model of a linear base is used.

The experimental results of the proposed compression method are promising in terms of preserving the details and the quality of the reconstructed images as well improving the compression ratio as compared with the extracted results of a traditional linear predicting coding system.

Keywords: Polynomial Coding , Lossless Image Compression, Fixed Predictor, Image Compression.
reconstructed images by merging several coding techniques. Review on image compressions techniques can be found in [3-5]. The first order Taylor series linear polynomial coding based on modelling the distance between image pixels and the centre was used effectively to compress images by several researchers to remove the redundancy between image pixel neighbours; further information can be found in [6-10]. The primary objective of this paper is to design an efficient algorithm to compress lossless images by combining the fixed predictor with a recursive polynomial model of a linear base.

**Materials and Methods**

All the estimated predictors in this study as illustrated in table (1) were casual; i.e. pixel (i,j) will be predicted from the previous visited pixels as shown in Figure 1 [11,12]; on the other hand; a recursive polynomial coding or top-down levels scheme is adopted to remove the redundancy embedded within the coefficients to improve the compression ratio and preserve image quality, it worked reversely from subsequent levels to construct up layers [2,13]. Figure 2 illustrates the layout of the suggested compression algorithm.

**Table 1-The Models of the tested Predictors**

| Index | Predictor | Description |
|-------|-----------|-------------|
| 1     | x(i-1,j-1) | First order, one dimension Predictor |
| 2     | x(i-1,j)   | First order, one dimension Predictor |
| 3     | (x(i,j-1)+x(i-1,j))/2 | second order, two dimension Predictor |
| 4     | x(i,j-1)+(x(i-1,j)-x(i-1,j-1))/2 | Third order, two dimension Predictor |
| 5     | x(i-1,j)+(x(i,j-1)-x(i-1,j-1))/2 | Third order, two dimension Predictor |
| 6     | x(i,j-1)+(x(i-1,j)-x(i-1,j-1)) | Third order, two dimension Predictor |
| 7     | (x(i,j-1)+x(i-1,j)+x(i-1,j-1)+x(i-1,j+1))/4 | Fourth order, two dimension Predictor |

![Figure 1](image1.png)  
**Figure 1** - The Predicted Pixel and the Casual Region [11]
The proposed algorithm was implemented on each predictor model in table (1) separately as follows:

Step 1- Apply the predictor model on the input image; name the output image Ip.

Step 2- A recursive polynomial linear model is applied on image Ip such that two layers will be generated.

In layer 1, three coefficients are calculated namely $c_0, c_1$ and $c_2$ using equations 1, 2 and 3 [5,6].

$$c_0 = \frac{\sum \sum Ip(i,j)}{n \times n} \quad \ldots \ldots \quad (1)$$

$$c_1 = \frac{\sum \sum Ip(i,j) \cdot (j - xx)}{\sum \sum (j - xx)^2} \quad \ldots \ldots \quad (2)$$

$$c_2 = \frac{\sum \sum Ip(i,j) \cdot (i - yy)}{\sum \sum (i - yy)^2} \quad \ldots \ldots \quad (3)$$

Where

$n$ is the image block size

$$xx = yy = \frac{n-1}{2}$$

$i=j=0, \ldots, n-1$

In layer 2 another three coefficients will be generated $c_{00}, c_{01}$ and $c_{02}$ depending on the $c_0$ coefficient in layer 1 using equations 4, 5 and 6 [13].

$$c_{00} = \frac{1}{n \times n} \sum \sum c_0(i,j) \quad \ldots \ldots \quad (4)$$
For Layer 2:
a- Determine the deterministic part (function formula) \( \tilde{c}_0 \) [13].
\[
\tilde{c}_0 = c00 + c01(j - xx) + c02(i - yy)
\]  
…..(7)

b- Find residual [13].
\[
c00 = c0 - c00\tilde{c}
\]  
……..(8)

c- Build the modeled approximated \( \hat{c}_0 \) [13].
\[
\hat{c}_0 = \tilde{c}_0 - c0\Re sd
\]  
…..(9)

Step 3- Determine the deterministic part \( \mathcal{L} \) [14].
\[
\mathcal{L} = \hat{c}_0 + c1(j - xx) + c2(i - yy)
\]  
…..(10)

Step 4- Find the error (residual) [14].
\[
\Re sd = \mathcal{L} - \mathcal{L} \hat{c}
\]  
……..(11)

Reconstruct the compressed image using the following steps:

a - find the approximation model using [14]:
\[
\mathcal{L} \hat{a} = \mathcal{L} + \mathcal{L} \Re sd
\]  
…..(12)

b- Build the decoded image [10,12]:
\[
\mathcal{L} \hat{a}(i, j) = \mathcal{L} \hat{a}(i, j) + \Re(p(i - 1, j))
\]  
…..(13)

Where \( i=2…N/2 \) \( j=1…N/2 \)

**Results**

In this work, all the tested natural and medical images (as indicated in Figure 3 ) were of size 65536 bytes. The tests were done using block size of 8*8. To measure the efficiency of the lossless compression method; the compression ratio was determined (which is the ratio of the original image size to the compressed size). Table 2 illustrates the results of a traditional coding system and a recursive polynomial linear base without applying the predictors; while Table 3 summarizes the compressed image size and the compression ratio obtained by applying the predictors models listed in table 1 and the recursive linear model.

Any way, it obvious that the suggested recursive linear model (with or without using the predictor) is better than the traditional coding system where the compression ratio was improved.

Figure 4 records the resulting images of the suggested recursive prediction using the predictors in Table 1.
Table 2 - Compression Performance of a Traditional Linear System and Recursive Prediction

| Test image | Size  | Compression Ratio | Size  | Compression Ratio |
|------------|-------|-------------------|-------|-------------------|
| Pepper     | 28674 | 2.2856            | 7391  | 8.8670            |
| Rose       | 26622 | 2.4617            | 6116  | 10.7155           |
| Brain1     | 6967  | 2.2207            | 4150  | 9.4066            |
| Brain2     | 28290 | 2.3166            | 8443  | 7.7622            |

Table 3 - Performance of the Recursive Prediction using the predictor

| Image   | Predictor1 | Predictor2 | Predictor3 | Predictor4 | Predictor5 | Predictor6 | Predictor7 |
|---------|------------|------------|------------|------------|------------|------------|------------|
| Pepper  | 65         | 10.045     | 10.173     | 10.516     | 10.516     | 10.516     | 9.2460     |
| Rose    | 52         | 12.598     | 12.632     | 12.941     | 12.941     | 12.941     | 12.064     |
| Brain1  | 61         | 10.680     | 10.843     | 11.104     | 11.104     | 11.104     | 10.590     |
| Brain2  | 77         | 8.4825     | 8.5467     | 9.0494     | 9.0494     | 9.0494     | 7.9341     |

Figure 3- Input Images: (a) Pepper (b) Rose (c) Brain1 and (d) Brain2
| Predictor 1 | Predictor 2 | Predictor 3 | Predictor 4 | Predictor 5 | Predictor 6 | Predictor 7 |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| ![Pepper Image](image1) | ![Rose Image](image2) | ![Brain Image 1](image3) | ![Brain Image 2](image4) | ![Brain Image 1](image3) | ![Brain Image 2](image4) | ![Brain Image 1](image3) |
| ![Pepper Image](image1) | ![Rose Image](image2) | ![Brain Image 1](image3) | ![Brain Image 2](image4) | ![Brain Image 1](image3) | ![Brain Image 2](image4) | ![Brain Image 1](image3) |
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| ![Pepper Image](image1) | ![Rose Image](image2) | ![Brain Image 1](image3) | ![Brain Image 2](image4) | ![Brain Image 1](image3) | ![Brain Image 2](image4) | ![Brain Image 1](image3) |
| ![Pepper Image](image1) | ![Rose Image](image2) | ![Brain Image 1](image3) | ![Brain Image 2](image4) | ![Brain Image 1](image3) | ![Brain Image 2](image4) | ![Brain Image 1](image3) |
| ![Pepper Image](image1) | ![Rose Image](image2) | ![Brain Image 1](image3) | ![Brain Image 2](image4) | ![Brain Image 1](image3) | ![Brain Image 2](image4) | ![Brain Image 1](image3) |

**Figure 4** - Resulting Images of the Recursive Prediction using the predictors (1-7)
Conclusions

The results obtained from the proposed method showed a promising performance due to the spatial domain utilization to eliminate the redundant image pixels. The best compression results for the tested images was recorded using predictor of order 4; i.e a predictor with four pixels was better than one. Another factor affects the compression ratio in this study is the characteristics of the test image, where high compression rate was recorded for the detailed image (such as rose).

In future, this work can be extended to fit the video compression system

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