Four Fermions Productions at a $\gamma\gamma$ Collider

Mauro Moretti,
(e-mail: moretti@vaxfe.fe.infn.it)

Abstract

Using the recently proposed ALPHA algorithm (and the resulting code) I compute the rate (at tree level) for the process $\gamma\gamma \rightarrow \bar{\nu}_e e^- u\bar{d}$. The bulk of the contribution is due to W pair production and decay. However a non negligible ($\sim 10\%$) contribution comes from other channels, mainly the production and decay of a W and a collinear charged fermion. Requiring that the reconstructed invariant $u\bar{d}$ mass lies in the intervals $M_W \pm 5$ GeV and $M_W \pm 20$ GeV one obtains a rate which is lower, by 25 % and 4 % respectively, than the rate obtained in the narrow width approximation, thus demonstrating the relevance of the finite W width.

1 Introduction

Future high energy $e^+e^-$ colliders are likely to make available the possibility to operate also in the $e\gamma$ and $\gamma\gamma$ mode. This last mode will allow the experimental study of the bosonic sector of the electroweak lagrangian, triple and quartic gauge boson couplings as well as the coupling of gauge bosons with the higgs particle if it is light enough to be produced.

The most abundant final state to be studied will be W pair production. This process is allowed at the tree level and, since the production rate is dominated by $t$ channel virtual W exchange, the cross section becomes nearly constant for a center of mass energy above 400 GeV. With the aimed luminosity in the range of $10^{-12}$ inverse femtobarns per year about one millions of W pairs per year are expected.

Because of the high statistic and of the relatively clean environment provided by a lepton collider, a good accuracy in the theoretical prediction will be necessary.

Since the W boson is shortlived, the experimental signature for W pair production is via its decay products, mostly four fermions in the final state. Therefore one has to compute the rate for the process $\gamma\gamma \rightarrow 4 \text{ fermions}$.

In this paper I compute, at tree level, the rate for the process

$$\gamma\gamma \rightarrow \bar{\nu}_e e^- u\bar{d}$$

(1)

discussing in some detail the comparison with the narrow width approximation and with the calculation which includes only the contribution of the subset of doubly resonant diagrams. I carry on the comparison both at the level of total cross section and of differential distributions.
\[ M_W = 80.23 \text{ GeV} \quad \Gamma_W = 2.03367 \quad \alpha_{QED} = 1./128.07 \quad \sin^2 \theta_W = 0.23103 \]

| Parameter | Value |
|-----------|-------|
| \( m_e \) | 5 MeV |
| \( m_u \) | 10 MeV |
| \( m_d \) | 5 MeV |

Table 1: Input parameters for the electroweak lagrangian. \( m_e, m_u, m_d \) are the electron \( u \) and \( d \) quarks masses respectively, \( M_W \) and \( \Gamma_W \) the W mass and width, \( \theta_W \) is the Weinberg angle and \( \alpha_{QED} \) is the electromagnetic coupling constant. Tree level relationships among the parameters of the standard model electroweak lagrangian are assumed.

## 2 The computation

The amplitude for the process (1) is computed using a new technique which, in collaboration with F. Caravaglios, I have recently developed. Exploiting the relation between the one-particle irreducible Green Functions generator \( \Gamma \) and the connected Green Functions generator \( G \) we have proposed a simple numerical algorithm to compute tree level scattering amplitudes. We have then implemented the algorithm in a FORTRAN code ALPHA which presently uses the standard electroweak lagrangian (QCD is not included yet) and can compute any scattering amplitude in this framework, in a fully automatic way. We have used the ALPHA code to obtain the rates for the processes \( e^+e^- \) into four fermions which are of relevance for the LEP200 experiments and our results are in complete agreement, within the statistical error of the Montecarlo integration (typically less than one per mille), with the results obtained using more conventional methods.

This provides the most stringent test of our algorithm and of our code: it is important to stress here that the calculation of the matrix element of any of these processes is entirely automatic; to study a different process we had to change only an input file specifying the type and number of particles involved.

The input values, which will be used in the present paper, for particles masses, widths and for the electroweak coupling constants are reported in table 1. The running width scheme is used, namely the W propagator \( \pi_W \) is taken as

\[
\pi_W^{\mu \nu} = \frac{-i(g^{\mu \nu} - p^\mu p^\nu/M_W^2)}{p^2 - M_W^2 + i\Gamma_W p^2 \theta(p^2)/M_W} \tag{2}
\]

where \( p \) is the W four momentum, \( M_W \) and \( \Gamma_W \) are the W mass and width respectively, and \( \theta(p^2) \) is the Riemann \( \theta \) function: it is equal to one for positive \( p^2 \) and zero otherwise.

The bulk of the contribution to the rate for (1) is obtained when the process is mediated by two almost on shell W which decay into two pairs of fermions or when one of the charged fermions is collinear to one of the incoming photons and an on shell W is emitted. Therefore, to perform the numerical integration over the phase space variables, one needs to increase the sampling in these two regions. To this purpose I have used the package VEGAS and all the reported results are obtained with at least twenty VEGAS estimates of the integral with a \( \chi^2 \) smaller than two.

To assess the performances of the ALPHA code as an event generator for \( \gamma\gamma \) processes the CPU time required for the evaluation of the scattering matrix element of several final states is reported in table 2.
Table 2: CPU time required by the ALPHA code for the evaluation of some final states from $\gamma\gamma$ fusion. Times are in second per $10^6$ events. These performances are obtained with a DIGITAL machine ALPHA 3000/600 with 64M of memory. Double precision is used. All fermions are massive.

### The Results

One of the most important (and abundant) process to be observed at the proposed $\gamma\gamma$ collider will be $W$ pair production

$$\gamma\gamma \rightarrow W^+W^-.$$ (3)

Because of the finite (and ‘short’) lifetime of the $W$, at the collider one will observe the $W$’s decay products, mostly four final fermions. As a first order approximation (the so called narrow width approximation) one computes the rate for the process (3) and multiplies it for the branching ratios appropriate to the given four fermions final state.

My purpose here is to confront this approximation with the results for the full computation of the process (1) at tree level. There are two main differences: in addition to the diagrams accounting for $W$ pair production and decay, a lot more diagrams contribute to the same final state and, because of the finite $W$ width, the bulk of the cross section occurs for a reconstructed $W$ invariant mass spread over a few $W$ widths, making the definition of a ‘$W$’ via invariant mass cuts more delicate and affecting in a sizable way the cross section.

At a realistic $\gamma\gamma$ collider the photon energy spectrum will not be nearly monochromatic (as the one of the parent $e^+e^-$) and, to account for the experimental features, one should fold the photon spectra with the cross section for (1). However, for the present purposes, a discussion of the main features at a fixed center of mass energy is sufficient.

In Fig. 1 the differential cross section, at a center of mass energy of 500 GeV, for the process (1) is plotted as a function of $\mu = \max(\mu_1, \mu_2)$ where $\mu_1 = |[(p_e + p_\nu)^2]^{1/2} - M_W|$, $\mu_2 = |[(p_u + p_d)^2]^{1/2} - M_W|$ and $p_e$, $p_\nu$, $p_u$, $p_d$ are the electron, neutrino u and d quarks momenta respectively. Some numerical values for the same quantities are reported in table 3. In table 4 the same quantities are given for center of mass energies of 300 and 1000 GeV. I give the results for the full set of diagrams and those obtained using only resonant contributions, namely those accounting for $W$ pair production and decay.

Both the impact of finite $W$ width and of non resonant contribution are clearly seen: i) the cross section for (1) is about 10 % smaller than that for (2) with an invariant mass cut $\mu \leq 15 \div 20$ GeV and for $20$ GeV $> \mu > 10$ GeV the contribution to the cross section...
| Diagrams | Angular Cut | $\sigma(5) \text{ (pb)}$ | $\sigma(10) \text{ (pb)}$ | $\sigma(18) \text{ (pb)}$ | $\sigma(250) \text{ (pb)}$ |
|----------|-------------|----------------|----------------|----------------|----------------|
| all      | $|\cos \theta_f| < 1$ | 2.529(3) | 2.932(3) | 3.153(3) | 3.825(4) |
| resonant | $|\cos \theta_f| < 1$ | 2.508(2) | 2.886(2) | 3.068(2) | 3.374(2) |
| all (fudge) | $|\cos \theta_f| < 1$ | 2.524(3) | 2.926(3) | 3.146(3) | 3.797(4) |
| all      | $|\cos \theta_f| < 0.98$ | 1.748(2) | 2.012(2) | 2.138(2) | 2.290(2) |
| resonant | $|\cos \theta_f| < 0.98$ | 1.753(2) | 2.016(2) | 2.144(2) | 2.378(2) |
| all (fudge) | $|\cos \theta_f| < 0.98$ | 1.752(2) | 2.014(2) | 2.142(2) | 2.294(2) |
| all      | $|\cos \theta_f| < 0.92$ | 0.7575(9) | 0.871(2) | 0.926(1) | 0.993(1) |
| resonant | $|\cos \theta_f| < 0.92$ | 0.759(1) | 0.874(1) | 0.930(1) | 1.053(1) |
| all (fudge) | $|\cos \theta_f| < 0.92$ | 0.760(2) | 0.867(2) | 0.929(2) | 0.996(2) |

Table 3: Rates for $\gamma\gamma \rightarrow \nu_e e^- u\bar{d}$ for a center of mass energy of 500 GeV. $\sigma(a)$ is the cross section with the invariant mass cut $\mu < a$ (GeV). The variable $\mu$ is defined as the maximum value of $\mu_1 = [(p_e + p_\nu)^2]^{1/2} - M_W$ and $\mu_2 = [(p_u + p_d)^2]^{1/2} - M_W$ and $p_e, p_\nu, p_u, p_d$ are the electron, neutrino u and d quarks momenta respectively. The angle $\theta_f$ is the minimal angle of charged fermions with respect to the beam direction. Entries labelled all refer to the full computation (all Feynman Graphs included), those labelled resonant to the resonant one (only doubly resonant Feynman Graphs included) and those labelled fudge to the full computation using the fudge scheme (see eq. (5) ) with the running width. The rate in the narrow width approximation (3) is 3.29 pb.

| Energy | Diagrams | Angular Cut | $\sigma(5) \text{ (pb)}$ | $\sigma(10) \text{ (pb)}$ | $\sigma(18) \text{ (pb)}$ | $\sigma(250) \text{ (pb)}$ |
|--------|----------|-------------|----------------|----------------|----------------|----------------|
| 300    | all      | $|\cos \theta_f| < 1$ | 2.291(3) | 2.648(3) | 2.838(4) | 3.329(9) |
| 300    | resonant | $|\cos \theta_f| < 1.$ | 2.274(2) | 2.616(2) | 2.780(2) | 2.982(2) |
| 300    | all      | $|\cos \theta_f| < 0.98$ | 1.976(2) | 2.273(2) | 2.416(3) | 2.574(3) |
| 300    | resonant | $|\cos \theta_f| < 0.98$ | 1.976(2) | 2.274(2) | 2.417(2) | 2.596(2) |
| 300    | all      | $|\cos \theta_f| < 0.92$ | 1.330(2) | 1.529(2) | 1.623(3) | 1.718(3) |
| 300    | resonant | $|\cos \theta_f| < 0.92$ | 1.332(2) | 1.534(2) | 1.630(2) | 1.759(2) |
| 1000   | all      | $|\cos \theta_f| < 1.$ | 2.619(5) | 3.040(5) | 3.278(5) | 4.132(8) |
| 1000   | resonant | $|\cos \theta_f| < 1.$ | 2.588(3) | 2.978(4) | 3.167(4) | 3.576(5) |
| 1000   | all      | $|\cos \theta_f| < 0.98$ | 0.802(2) | 0.923(2) | 0.981(2) | 1.065(2) |
| 1000   | resonant | $|\cos \theta_f| < 0.98$ | 0.802(2) | 0.922(2) | 0.982(2) | 1.159(2) |
| 1000   | all      | $|\cos \theta_f| < 0.92$ | 0.251(1) | 0.289(1) | 0.307(1) | 0.336(1) |
| 1000   | resonant | $|\cos \theta_f| < 0.92$ | 0.2506(9) | 0.288(1) | 0.306(1) | 0.371(1) |

Table 4: Rates for $\gamma\gamma \rightarrow \nu_e e^- u\bar{d}$ for center of mass energies of 300 and 1000 GeV. Everything is as in table 3. In the narrow width approximation the cross section at 300 and 1000 GeV is 2.99 and 3.40 pb respectively.
section is still at the level of one per cent per GeV;

\( \text{ii) the difference among the computations which do and do not include non-resonant contributions} \)

\( \text{is shown in Fig. 2: already for } \mu \leq 4 \text{ GeV it is about 1 \%} \), it becomes 2 \% at \( \mu \leq 10 \text{ GeV} \) and it is larger than 10 \% for the total cross section.

The effect of angular cuts on the emitted fermions is also manifest. A large fraction of the cross section occurs in correspondence of fermions emitted along the beam direction. This is due to the dominant contribution of \( t \) channel virtual W exchange which increases the amplitude for the emission of \('W'\) collinear to the photons. The two W, in turn, being quite strongly boosted in the laboratory frame, emit their decay products mainly in the forward direction.

When invariant mass cuts are applied, the non resonant contribution is almost unimportant for \( | \cos \theta_f| < 0.98 \), \( \theta_f \) being the smallest of the angles of charged fermions with the beam direction; this demonstrates that the bulk of the contribution of non resonant diagrams occurs for small \( \theta_f \), when an almost on shell virtual fermion is exchanged in the \( t \) channel.

The difference is below 1 \% when angular and invariant mass cuts are applied (\( \mu < 15 \div 20 \) GeV, \( | \cos \theta_f| < 0.98 \)) and becomes a few per cent when no invariant mass cuts are applied (this can be relevant for purely leptonic events since the W invariant mass cannot be reconstructed in this case) perhaps suggesting that for large W virtuality some unitarity violation, induced by the lack of gauge invariance of the subset of resonant diagrams, might play a role.

In Fig. 3 the angular distribution for the \( u \) quark is plotted with various choices of angular and invariant mass cuts. There is a small difference among the ‘full’ (including the contribution of all graphs) and the ‘resonant’ (including only the contribution of doubly resonant graphs) calculation. When angular and invariant mass cuts are applied the difference is likely to be statistically meaningless. In Fig. 4 the relative difference among the two distribution is reported.

To provide a semiquantitative assessment of the difference among the two spectra I have divided the variable \( \cos \theta_u \) (\( \theta_u \) is the angle of the \( u \) quarks with the beam direction) in \( k \) (unequal) bins and defined the following function

\[
f = \frac{1}{N_{\text{res}}} \sum_{j=1}^{k} \frac{(n_{j}^{\text{all}} - n_{j}^{\text{res}})^2}{n_{j}^{\text{res}}} \tag{4}
\]

where \( n_{j}^{\text{all}} \) and \( n_{j}^{\text{res}} \) are the numbers of events in the \( j \)-th bin predicted according to the full and resonant calculation respectively and \( N_{\text{res}} \) is the total number of events according to the resonant calculation. Under the assumption that \( n_{j}^{\text{all}} \) is the prediction of a ‘model’ and \( n_{j}^{\text{res}} \) are ‘experimental’ measurements, the quantity \( N_{\text{res}} f \) would be the \( \chi^2 \) of the experimental measurements versus the ‘model’ and therefore we can interpret \( f \) as a

\[1\text{Although the ALPHA algorithm does not make use of the Feynman graphs technique to compute the scattering amplitudes, it is still possible to isolate the contribution of a subset of graphs: in the present case it can be seen, by direct inspection, that setting to zero the couplings of the fermions with the photon, one indeed isolates the contribution of the doubly resonant diagrams only.}

\[2\text{This is true for large } n_{j}^{\text{res}} \text{ since in this case the error of the experimental measurement can be estimated as } \sqrt{n_{j}^{\text{res}}}. \text{ Moreover the expected spread in the experimental measurements is not accounted for in eq. (4). In view of the modest purpose of the discussion all these details are inessential.} \]
Table 5: The function $f$ defined in (4) is given as a function of various invariant mass and angular cuts. The angle $\tilde{\theta}_f$ is the minimal angle (in absolute value) of $e^-$ or $d$ with the beam direction, the angle $\theta_u$ is the angle of the $u$ quark with respect to the beam direction, $f_{\mu<s}$ is the function $f$ defined in (4) for a data sample to which the cut $\mu \leq s$ has been applied, $N_{\text{bin}}$ is the number of bins in which the data sample has been divided, finally the reported limit value for $N_f$ corresponds to a probability smaller than 0.005 that an experimental distribution is well reproduced by the resonant calculation. See the text for the caveats of this statement. $N$ is the number of events and the definition of $\mu$ is given in the caption of table 3.

| Angular cut $(e, d)$ | Angular cut $(u)$ | $N_{\text{bin}}$ | $f_{\mu<5}$ | $f_{\mu<10}$ | $f_{\mu<20}$ | $\chi^2$ limit |
|----------------------|-------------------|-----------------|-------------|--------------|--------------|---------------|
| $\cos \theta_f < 0.90$ | $\cos \theta_u < 0.98$ | 34 | $1.67 \times 10^{-4}$ | $1.33 \times 10^{-4}$ | $1.26 \times 10^{-4}$ | $N_f < 58$ |
| $\cos \theta_f < 0.98$ | $\cos \theta_u < 0.98$ | 34 | $9.09 \times 10^{-5}$ | $7.62 \times 10^{-5}$ | $7.55 \times 10^{-5}$ | $N_f < 58$ |
| $\cos \theta_f < 0.90$ | $\cos \theta_u < 0.91$ | 24 | $6.96 \times 10^{-5}$ | $5.66 \times 10^{-5}$ | $5.49 \times 10^{-5}$ | $N_f < 44.5$ |
| $\cos \theta_f < 0.98$ | $\cos \theta_u < 0.91$ | 24 | $8.83 \times 10^{-5}$ | $8.02 \times 10^{-5}$ | $8.07 \times 10^{-5}$ | $N_f < 44.5$ |
Table 6: All the definition are the same as in table 5, but the angle $\tilde{\theta}_f$ is now the minimal angle (in absolute value) of the charged fermions with the beam and $\theta_e$ is the angle of the $e^-$ with respect to the beam direction.

$$\begin{array}{|c|c|c|c|c|c|}
\hline
\text{angular cut (}u, d\text{)} & \text{angular cut (}e\text{)} & N_{bin} & f_{\mu<5} & f_{\mu<10} & f_{\mu<20} & \chi^2 \text{ limit} \\
\hline
\cos \theta_f < 0.90 & \cos \theta_e < 0.98 & 34 & 1.52 \times 10^{-4} & 1.40 \times 10^{-4} & 1.46 \times 10^{-4} & N_f < 58 \\
\cos \theta_f < 0.98 & \cos \theta_e < 0.98 & 34 & 8.67 \times 10^{-5} & 5.95 \times 10^{-5} & 5.97 \times 10^{-5} & N_f < 58 \\
\cos \theta_f < 1. & \cos \theta_e < 0.98 & 34 & 8.60 \times 10^{-5} & 2.17 \times 10^{-4} & 5.71 \times 10^{-4} & N_f < 58 \\
\cos \theta_f < 0.90 & \cos \theta_e < 0.91 & 24 & 1.24 \times 10^{-4} & 1.22 \times 10^{-4} & 1.27 \times 10^{-4} & N_f < 44.5 \\
\cos \theta_f < 0.98 & \cos \theta_e < 0.91 & 24 & 7.48 \times 10^{-5} & 5.66 \times 10^{-5} & 5.83 \times 10^{-5} & N_f < 44.5 \\
\cos \theta_f < 0.98 & \cos \theta_e < 0.91 & 24 & 1.02 \times 10^{-4} & 2.40 \times 10^{-4} & 4.95 \times 10^{-4} & N_f < 44.5 \\
\hline
\end{array}$$

Table 7: All the quantities in the table are defined as in table 5 but the angle $\tilde{\theta}_f$ is the minimal angle (in absolute value) of the charged fermions with the beam and $\theta_W$ is the angle of the reconstructed (from $p_u$ and $p_d$) $W^+$ with the beam.

$$\begin{array}{|c|c|c|c|c|c|}
\hline
\text{angular cut (}e, u, d\text{)} & \text{angular cut (}\theta_W\text{)} & N_{bin} & f_{\mu<5} & f_{\mu<10} & f_{\mu<20} & \chi^2 \text{ limit} \\
\hline
\cos \theta_f < 0.90 & \cos \theta_W < 0.98 & 34 & 1.01 \times 10^{-4} & 6.61 \times 10^{-5} & 1.73 \times 10^{-4} & N_f < 58 \\
\cos \theta_f < 0.98 & \cos \theta_W < 0.98 & 34 & 6.31 \times 10^{-5} & 6.05 \times 10^{-5} & 6.03 \times 10^{-5} & N_f < 58 \\
\cos \theta_f < 0.90 & \cos \theta_W < 0.91 & 24 & 6.93 \times 10^{-5} & 5.44 \times 10^{-5} & 5.81 \times 10^{-5} & N_f < 44.5 \\
\cos \theta_f < 0.98 & \cos \theta_W < 0.91 & 24 & 4.59 \times 10^{-5} & 3.95 \times 10^{-5} & 3.26 \times 10^{-5} & N_f < 44.5 \\
\hline
\end{array}$$

is doomed to disappear when purely leptonic final states are considered and the observable final states do not allow to reconstruct the $W$ invariant mass.

### 3.1 Gauge Invariance

As it stands the calculation that I have presented is not gauge invariant. The reason is that the introduction of a finite and running W width as in (2) explicitly breaks gauge invariance. Several strategies have been proposed to restore gauge invariance [7, 8, 9, 10]. The most satisfactory appears to be the one inspired by field theoretical arguments, namely the inclusion in the input lagrangian of the one loop contribution of fermion loops to the imaginary part of the gauge bosons self couplings [9]. However, although straightforward, this strategy is a bit cumbersome to be implemented since it requires the introduction of complicated form factors which considerably slow down the computational speed. Therefore to study the quantitative impact of the breaking of the gauge invariance induced by (2) I have used a trick to restore gauge invariance which is easier to implement and does not affect computational time, the so called $fudge$ [10] scheme, namely one first computes the matrix element with zero W width and therefore in a fully gauge invariant
way, and only at the very end multiplies the result for a common factor $\lambda_{fdg}$,

$$\lambda_{fdg} = \frac{(p_1^2 - M_W^2)(p_2^2 - M_W^2)}{(p_1^2 - M_W^2 + i\Gamma_w p_1^2 \theta(p_1^2)/M_W)(p_2^2 - M_W^2 + i\Gamma_w p_2^2 \theta(p_2^2)/M_W)}$$

(5)

where $p_1 = p_e + p_\nu$ and $p_2 = p_q + p_u, p_e, p_u$ and $p_d$ being the neutrino, electron, $u$ and $d$ quarks four momenta respectively.

The disadvantage of this scheme is that it grossly mistreats the non resonant contribution as well as resonant non-resonant interference when $p_1$ or $p_2$ are close to the W mass. However in view of the good agreement between the resonant and the full calculation when invariant mass cuts are applied one can hope that this does not induce sizable errors.

The results are displayed in table 3 and in Figg. 7 and 8. When angular cuts are applied, no appreciable difference with respect to the full calculation in the running width scheme is observed thus suggesting that the breaking of the gauge invariance from (2) does not affect the numerical results in an important way. If no cut is applied a small difference appears and it is not possible to determine, at this level, whether this is due to the already mentioned caveats of the fudge scheme or to the gauge violation effect. Ultimately one has to perform the calculation in the most appropriate way using the best gauge restoration scheme which is available, however the shown comparison suggests that, to study the experimental data, it is possible to use a computation with the explicit breaking of the gauge invariance (2) and only at the very end of the analysis to check the results with a theoretically more satisfactory calculation.

4 Conclusions and Remarks

I have addressed the calculation of the process $\gamma\gamma \rightarrow e^-\bar{\nu}_e u\bar{d}$ at the tree-level by means of the ALPHA code.

The rate is dominated by W pairs production and decay and a sizable contribution comes also from the exchange of an almost on shell $t$ channel virtual charged fermions (always linked to the emission of a charged fermions collinear to the incoming photons). At the high center of mass energies ($0.5 \div 2$ TeV) typical of future linear colliders the W bosons are boosted in the laboratory frame and, since they are produced mainly in the beam direction because of the dominant contribution from $t$ channel virtual W exchange, also their decay products are emitted mainly along the beam direction.

The result of the full calculations exhibits important differences with respect to the narrow width approximation (see (3) ) which has been used up to now in the discussion in the literature. There is an important dependence on the cuts on the invariant mass of the reconstructed W: requiring this mass to be within a 5, 10 and 18 GeV interval of the measured W mass one obtains differences of about 16 % and 24 % for the total rate. Also the approximation based on the subset of doubly resonant diagrams appears in general to be unsatisfactory leading to discrepancies larger than 10 %. However when both invariant

3This statement is obviously valid only for the unitary gauge which is used here. Since the result is gauge dependent, choosing an appropriate gauge, the running width scheme can lead to an arbitrary value for the cross section.
mass and angular cuts are applied (this is impossible for purely leptonic final states) the complete calculation and the the contribution of doubly resonant diagrams only are much closer to each other.

A final remark is in order here: although the computation has been presented for a fixed center of mass energy it is a straightforward matter to allow for a variable energy for the colliding photons and therefore the whole calculation can easily be adapted to become an event generator for the future $\gamma\gamma$ collider.

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Figure Captions

Fig. 1 Differential cross section in picobarns as a function of the invariant mass cut $\mu$ for various angular cuts. $\mu$ and $\theta_f$ are defined as in table 3. The continuous line is the contribution of doubly resonant diagrams only (referred as resonant in the text) and the dotted line the full calculation (referred as full in the text).

Fig. 2 The relative difference among the full and resonant computation (see text for the definition) as a function of the the invariant mass cut $\mu$ for various angular cuts. $\mu$ and $\theta_f$ are defined as in table 3. $\Delta\sigma = (\sigma_{all} - \sigma_{resonant})/\sigma_{all}$ where the labels all and resonant refers to the full and resonant computation respectively.

Fig. 3 Differential cross section in picobarns as a function of $\cos \theta_u$ where $\theta_u$ is the angle of the $u$ quark with respect to one of the incoming photons. The distribution is given for various invariant mass cuts $\mu$. $\mu$ is defined as in table 3. The dotted line refers to an angular cut $|\cos \hat{\theta}| \leq 1$ the dashed one to $|\cos \hat{\theta}| \leq 0.98$ and the continuous one to $|\cos \hat{\theta}| \leq 0.9$, where $\theta$ is the minimal (in absolute value) among the angles of $e^-$ and $\bar{d}$ quark with the beam direction.

Fig. 4 Relative difference among the full and resonant computation for the differential distribution given in fig. 3. All the quantities are defined as in fig. 3 and $\Delta\sigma$ as in fig. 2.

Fig. 5 Differential cross section in picobarns as a function of $\cos \theta_W$ where $\theta_W$ is the angle of the reconstructed $W$ (from $u$ and $\bar{d}$ four momenta) with respect to one of the incoming photons. The distribution is given for various invariant mass cuts $\mu$. $\mu$ is defined as in table 3. The dotted line refers to an angular cut $|\cos \theta_f| \leq 1$ the dashed one to $|\cos \theta_f| \leq 0.98$ and the continuous one to $|\cos \theta_f| \leq 0.9$, where $\theta_f$ is the minimal (in absolute value) among the angles of charged fermions with the beam direction.

Fig. 6 Relative difference among the full and resonant computation for the differential distribution given in fig. 5. All the quantities are defined as in fig. 5 and $\Delta\sigma$ as in fig. 2.

Fig. 7 Relative difference among the full calculation with the running width for the $W$ propagator as in eq. (2) and the calculation in the fudge scheme for the differential distribution given in fig. 2. $\mu$ and $\theta_f$ are defined as in table 3.

Fig. 8 Relative difference among the full calculation with the running width for the $W$ propagator as in eq. (2) and the calculation in the fudge scheme for the differential distribution given in fig. 3. $\mu$ and $\theta_f$ are defined as in table 3.
Fig. A

$\cos n \theta$

$\pi \leq 5$ GeV

$\pi \leq 10$ GeV

$\pi \leq 20$ GeV

$\Delta \sigma$ (%)
