Cosmic acceleration from second order gauge gravity

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Abstract We construct a phenomenological theory of gravitation based on a second order gauge formulation for the Lorentz group. The model presents a long-range modification for the gravitational field leading to a cosmological model provided with an accelerated expansion at recent times. We estimate the model parameters using observational data and verify that our estimative for the age of the Universe is of the same magnitude than the one predicted by the standard model. The transition from the decelerated expansion regime to the accelerated one occurs recently (at \( \sim 9.3 \) Gyr).

Keywords Cosmic acceleration · Higher order derivative · Gauge theory

1 Introduction

One of the most challenging problems of Physics nowadays is to explain the origin and evolution of the present accelerated expansion of the universe. One way of obtaining a mechanism of acceleration is to modify one of the cornerstones of modern physics, the theory of General Relativity.

Modifications in the scheme of General Relativity are being proposed since its invention, in the beginning of the 20th century, and they are motivated by several reasons, from the quest for agreement with the theory for the inner structure of quantized matter, to the eventual need for extra-dimensions and the desire to obtain unification of the interactions. The first modification of General Relativity was proposed by Einstein through the introduction of the cosmological constant, which is one of the several alternatives to describe the present-day acceleration of the universe. Other proposals associated with renormalizability are the quadratic Lagrangians in the Riemann tensor (Querella 1998; Stelle 1977; Buchbinder and Lyahovich 1987) and the Horava–Lifshitz model (Horava 2009); in the context of Cosmology possible modifications involve the \( f(R) \) Lagrangians (Capozziello and Francaviglia 2008; Li and Barrow 2007; Amendola et al. 2007), the introduction of one (or more) spatial extra-dimension in the braneworld scenario (Durrer 2005; Sahni and Shtanov 2003; Papantonopoulos 2002), or the presence of a self-interacting scalar field, the quintessence models (Caldwell et al. 1998; Steinhardt et al. 1999a, 1999b; França and Rosenfeld 2002; Ratra and Peebles 1988).

Another class of modified gravity theories consists of the inclusion of non-local terms in the gravitational Lagrangians. Non-local terms arise naturally, for instance, if one considers the inverse d’Alembertian operator (Deser and Woodard 2007). This have inspired non-local modifications
in $f(R)$ gravity which apply inverse differential operators to the Ricci scalar (Nojiri and Odintsov 2008) or to the Gauss–Bonnet invariant (Capozziello et al. 2009). Non-local theories can generate late time acceleration in the universe or even flat rotation curves in galaxies (de Melo and Resende 2005, 2006, 2007) or even to investigate the gauge fixing in quantized electromagnetic field (de Melo et al. 2006).

It is worth to emphasize that the choice (2) constitutes a phenomenological model valid within a limited interval of energy (set by the values of the coupling constant $\beta$); it does not hold during all the cosmological history (as we shall see) but only for a certain period. The same phenomenological Lagrangian was used by Gottlöber et al. in another context (Gottlöber et al. 1990), where the authors claimed that the consideration of this higher order term “could be thought of as an attempt to make a further step in understanding the features of (…) non-local interaction”.

The paper is organized as follows. In Sect. 2 the field equations are written for a Friedmann–Lemaître–Robertson–Walker metric. Section 3 is devoted to obtain a perturbative solution of the field equations about the usual dust-matter model of the Einstein–Hilbert theory (as described by the Friedmann equations). The perturbative solution is constructed in such a way that the universe is dominated by a decelerated regime until the time $t^*$ when the additional term $\frac{2}{4}L_p \beta$ begins to be relevant. In Sect. 4 the parameters of the model are related to the observational data through a set of coupled nonlinear equations. Such equations are solved by numerical methods in Sect. 5, and the results are discussed in Sect. 6.

2 Friedmann equations

The invariance of the action (1) with respect to $\delta g_{\lambda\nu}$ yields the field equations:

$$R_{\lambda\nu} - \frac{1}{2} g_{\lambda\nu} R + \beta^2 H_{\lambda\nu} = \chi T_{\lambda\nu},$$

$$H_{\lambda\nu} = \nabla_{\lambda} \nabla_{\nu} [\Box R] + \frac{1}{2} \nabla_{\lambda} R \nabla_{\nu} R$$

$$- R_{\lambda\nu} \Box R - g_{\lambda\nu} \Box[\Box R] - \frac{1}{4} g_{\lambda\nu} \nabla^\rho R \nabla_\rho R,$$  \hspace{1cm} (3)

where $\Box \equiv \nabla_\mu \nabla^\mu$ and $\nabla_\mu$ is the covariant derivative.

Applying the field equations (3) to a homogeneous and isotropic space, described by the Friedmann–Lemaître–Robertson–Walker (FLRW) metric

$$ds^2 = dt^2 - a^2(t) \left( \frac{1}{1 - k r^2} dr^2 + r^2 d\Omega^2 \right)$$
one finds, after some direct but long calculations
\[
-3(\dot{H} + H^2) - \frac{1}{2} R + \beta \left[ -3H\ddot{R} + 3\dot{H}\dot{R} - 6H^2\ddot{R} + 9H^3\dot{R} + \frac{1}{4} \dot{R}^2 \right] = \chi T_{00},
\]
\[
\frac{a^2}{(1 - \kappa r^2)} \left[ \ddot{H} + 3H^2 + \frac{1}{2} R + 2\kappa \frac{\dot{R}}{a} \right] + \beta \left( \dddot{R} + 5H\ddot{R} + 3H^2\dot{R} + 5H\dot{R} + 3H\dot{H}\dot{R} - 9H^3\dot{R} + 3\dot{H}\dot{R} + \frac{1}{4} \dot{R}^2 - 2\kappa \frac{R}{a^2} (\ddot{R} + 3H\dot{R}) \right] = \chi T_{11},
\]
where \( H(t) = \dot{a}/a \) is the Hubble function, \( R(t) = g^{\mu\nu} R_{\mu\nu} \) is the scalar curvature and we are using units such that \( \chi = 8\pi G \). In our notation, dot means derivation with respect to the cosmic time \( t \). These are the higher order Friedmann equations in terms of the Hubble function \( H(t) \) and the scalar curvature \( R(t) \).

Following the standard procedure we use the energy-momentum tensor of a perfect fluid in a comoving coordinate system
\[
T_{\mu\nu} = (\rho + p) \delta_{\mu}^0 \delta_{\nu}^0 - p g_{\mu\nu}.
\]

In order to simplify the treatment, we will be concerned only with the case of a flat spatial section, \( \kappa = 0 \). So, using the relationship between the scalar curvature and the Hubble function
\[
R = -6(\dot{H} + 2H^2),
\]
we get the following modified Friedmann equations:
\[
3H^2 + \beta (18H\dddot{H} + 108H^2\ddot{H} - 18H\dot{H} + 9H^2 + 90H^3\dot{H} + 216H\dot{H}\ddot{H} - 72H^3 + 288(\dot{H}H)^2 - 216H^4\dot{H}) = \chi \rho,
\]
\[
2\dot{H} + 3H^2 + \beta (6H^5 + 54H^2\dot{H} + 138H^2\ddot{H} + 126H\dot{H} + 81\dot{H}^2 + 18H^3\dot{H} + 498H\dot{H}\ddot{H}) + 120H^3 - 216H^4\dot{H} = -\chi p.
\]

Combining the equations, one finds:
\[
2\dot{H} + \beta (6H^5 + 36H^2\dot{H} + 30H^2\ddot{H} + 144H\dot{H} + 72H^2 - 72H^3\dot{H} + 282H\dot{H}\ddot{H} + 192H^3 - 288(\dot{H}H)^2) = -\chi (p + \rho).
\]

Once we want to describe the evolution of the universe, this equation must be complemented with the covariant conservation of energy-momentum
\[
\dot{\rho} + 3H(\rho + p) = 0,
\]
and an equation of state \( f \) relating the energy density \( \rho \), the pressure \( p \) and the Hubble function \( H \)
\[
f(\rho, p, H) = 0.
\]

The dependence on \( H \) is included to account for the general case when one admits interaction among the constituents of the cosmic fluid (Aldrovandi et al. 2008a, 2008b). In this case, there is a possible constraint relating \( p \), \( \rho \) and the scale factor, or equivalently \( H \). On the other hand, the usual equations of state of physical cosmology associate only pressure \( p \) to the energy density \( \rho \), or pressure to the numerical density \( n \). For example, the equation of state for the dust matter is \( p = nkT \ll \rho \) (\( k \) is the Boltzmann constant and \( T \) the temperature), and \( p = \rho/3 \) is the one used for ultra-relativistic particles.

### 3 Solutions of the higher order Friedmann equations

#### 3.1 Dust matter

Our main interest here is to apply the model to the present state of the universe. Therefore, we take as source a perfect fluid composed by dust matter \( p = 0 \) (ordinary or dark). In this case, the continuity equation gives:
\[
\rho(t) = \rho_0 \left( \frac{a_0}{a(t)} \right)^3.
\]

In order to use such result directly, we would have to rewrite (5) in terms of the scale factor, obtaining a nonlinear and much more complicate equation, which we shall avoid. Instead, we consider simultaneously the following pair of coupled equations:
\[
\dot{H} + \beta (3H(5) + 18H\dddot{H} + 15H^2\ddot{H} + 72H\dot{H} + 36H^2 - 36H^3\ddot{H} + 141H\dot{H}\ddot{H} + 96H^2 - 144(HH)^2) = -\frac{\chi}{2} \rho,
\]
\[
\dot{\rho} + 3H\rho = 0.
\]

These equations can be analyzed by several methods, such as the linearization of dynamical systems, spectral analysis or perturbation theory. Here we will consider only this last procedure, leaving the other options for future investigations.
3.2 Perturbation theory

The model is constructed by assuming a standard Friedmann expansion prior to some time \( t^* \) from which the second order effects start to become significant. The strategy is to consider a perturbation series in the coupling parameter \( \beta \), in order to guarantee the accordance of our model with the usual cosmological model (in some region of the space of parameters). Take, for instance, an expansion up to second order terms; it reads:

\[
H(t) = H_F + \beta H_1 + \beta^2 H_2,
\]

\[
\rho(t) = \rho_F + \beta \rho_1 + \beta^2 \rho_2,
\]

where the label \( F \) stands for the standard Friedmann solution of the Einstein equations.

Substituting expansions (7) in the pair (6) and matching the terms order by order, we get:

\[
O(\beta^0) \rightarrow \begin{cases} \dot{H}_F + \frac{\chi}{2} \rho_F = 0, \\ \dot{\rho}_F + 3H_F \rho_F = 0, \end{cases} \tag{8}
\]

and

\[
O(\beta^1) \rightarrow \begin{cases} \dot{H}_1 + \frac{\chi}{2} \rho_1 = S_1(t), \\ \dot{\rho}_1 + 3H_F \rho_1 + 3H_1 \rho_F = 0, \end{cases} \tag{9}
\]

where

\[
S_1(t) \equiv -\left( 3H_F^{(S)} + 18H_F \ddot{H}_F + 15H_F^2 \dot{H}_F \\
+ 72 \dot{H}_F \dddot{H}_F + 36 \dddot{H}_F^2 - 36 \dddot{H}_F^2 \dot{H}_F \\
+ 141H_F \dot{H}_F \dddot{H}_F + 96 \dddot{H}_F^2 - 144H_F^2 \dddot{H}_F^2 \right); \tag{10}
\]

and also

\[
O(\beta^2) \rightarrow \begin{cases} \dot{H}_2 + \frac{\chi}{2} \rho_2 = S_2(t), \\ \dot{\rho}_2 + 3H_F \rho_2 + 3H_2 \rho_F = -3H_1 \rho_1, \end{cases} \tag{11}
\]

with

\[
S_2(t) \equiv -\left[ 3H_1^{(S)} + 18(H_1 \dddot{H}_F + H_F \dddot{H}_F) + 30H_F H_1 \dddot{H}_F \\
+ 72(H_F \dddot{H}_1 + \dot{H}_1 \dddot{H}_F + \dddot{H}_F \dddot{H}_1) \\
- 108H_1 H_F^2 \dddot{H}_F - 36H_F^2 \dddot{H}_F \dddot{H}_1 \right] \\
- \left[ 141H_1 \dot{H}_F \dddot{H}_F + H_F \dot{H}_F \dddot{H}_1 + H_F \dddot{H}_1 \dddot{H}_F \right] \\
+ 288(\dot{H}_1 \dddot{H}_F^2 - H_F^2 \dddot{H}_F \dddot{H}_1 - H_F \dot{H}_1 \dddot{H}_F^2). \tag{12}
\]

This way, we obtained a pair of coupled linear equations in each order. Their previous orders give the source term and the coefficients.

### 3.2.1 Zeroth order solution: the standard cosmological model

The solution for the system of zeroth order in the coupling parameter \( \beta \), (8)

\[
\dot{H}_F + \frac{\chi}{2} \rho_F = 0, \tag{13}
\]

\[
\dot{\rho}_F + 3H_F \rho_F = 0, \tag{14}
\]

can be obtained by direct integration. Solving this coupled system, we have:

\[
H_F = \frac{2}{3} \frac{1}{t}, \tag{15}
\]

\[
\rho_F = \frac{3}{\kappa} H_F^2,
\]

with an appropriate initial condition.

### 3.2.2 First order solution

In the first order approximation, we have the coupled set (9),

\[
\dot{H}_1 + \frac{\chi}{2} \rho_1 = S_1(t), \tag{16}
\]

\[
\dot{\rho}_1 + 3H_F \rho_1 + 3H_1 \rho_F = 0. \tag{17}
\]

These equations can be solved by the Increasing Order Method. Differentiating the first of these equations and using the second one, we obtain

\[
\dddot{H}_1 + 3H_F \dddot{H}_1 - \frac{3\chi}{2} \rho_F H_1 = \dddot{S}_1(t),
\]

\[
\dddot{S}_1(t) = \dddot{S}_1(t) + 3H_F S_1(t). \tag{18}
\]

The general solution of such equation can be obtained in the form of a power law:

\[
H_1(t) = at + bt^{-2} + \frac{4912}{243} t^{-5},
\]

\[
\rho_1(t) = \frac{2}{\kappa} \left( -\frac{15977}{243} t^{-6} - a + 2bt^{-3} \right).
\]

The integration constants \( a \) and \( b \) should be chosen in accordance to the physical situation to be described. Since the zeroth order terms appear as source terms in the first order approximation, one can choose the initial conditions \( H_1(t^*) = \rho_1(t^*) = 0 \). This determines the integration constants leaving the theory with only three free parameters, namely the coupling constant \( \beta \), the age of the universe \( t_0 \) (see below) and the instant of perturbation \( t^* \).
Therefore, in the first order approximation we find the following solution to (6):

\[
H(t) = \frac{2}{3} \frac{1}{t} + \beta \left( \frac{11065}{729} \frac{(t^*)^4}{t} \right) t^{-1} + 4912 \left( \frac{t^*}{t} \right)^4 t^{-1} - \frac{35408}{729} \frac{t}{t^*} (t^*)^{-1}.
\]

\[
8\pi G\rho(t) = \frac{4}{3} \frac{1}{t^2} + 2 \beta \left( \frac{22130}{729} \frac{(t^*)^4}{t} \right) t^{-2} - 15977 \frac{(t^*)^4}{243} t^{-2} + \frac{35408}{729} (t^*)^{-2}.
\]

(16)

4 Observational parameters

Now, let us focus on the problem of estimating the magnitude of the parameters of our theoretical model using the observational data available.

The redshift \( z \) and the luminosity distance \( d_L \) are dependent on the null geodesic equation only, and this is not changed by the second order field equations (3). Therefore, they constitute the ideal data set to be compared with the predictions of our model. The luminosity distance can be directly related to the redshift (Weinberg 1972),

\[ d_L \approx \frac{1}{H_0} \left( z + \frac{1}{2} (1 - q_0) z^2 \right). \]

\( q_0 \) is the deceleration parameter.

The supernovae projects usually measure the curve of \( d_L(z) \) determining the parameter \( q_0 \) with good accuracy. In order to do it, supernovae projects maximize the likelihood function adjusting the model parameters. Instead of following this approach, we shall obtain an initial estimation for \( q_0 \) using the observational data available.

In our model we have three parameters to be found:

1. The age of the universe \( t_0 \)
2. The instant \( t^* \) from which the perturbation coming from the modified gravitational equation becomes important
3. The coupling constant \( \beta \) for the higher derivative terms in the action

We need three independent measurements to find these parameters. We will use \( H_0, q_0 \) and \( \Omega_{m0} \) obtained from the literature.

In the following section, we shall carefully discuss how to use \( H_0, q_0 \) and \( \Omega_{m0} \) to obtain \( t_0, t^* \) and \( \beta \). But, before that, we will add to system (16) the constraint

\[
\dot{H}(t_0) = -H_0^2 (q_0 + 1)
\]

following from the definition of both the Hubble and the deceleration functions in terms of the scale factor: \( H = \dot{a}/a \), \( q = -\ddot{a}/a \). Gathering (16) and (17), we get the new system to be solved:

\[
H(t_0) = \frac{2}{3} \frac{1}{t_0} + \beta \left( \frac{11065}{729} \frac{t^*}{t} \right) t^{-1} - \frac{35408}{729} \frac{t}{t^*} (t^*)^{-1}
\]

\[
+ \frac{11065}{729} \left( \frac{t_0}{t^*} \right)^{-1} t_0^{-1} + 4912 \left( \frac{t_0}{t^*} \right)^{-4} t_0^{-4}.
\]

\[
\dot{H}(t_0) = -\frac{2}{3} \frac{1}{t_0} + \beta \left( \frac{22130}{729} \frac{t^*}{t} \right) t^{-2} - \frac{22130}{729} \left( \frac{t_0}{t^*} \right)^{-1} t_0^{-2} - \frac{24560}{243} \left( \frac{t_0}{t^*} \right)^{-4} t_0^{-4},
\]

\[
8\pi G\rho(t_0) = \frac{4}{3} \frac{1}{t_0^2} + 2 \beta \left( \frac{15977}{243} \frac{t^*}{t} \right) t^{-2} - \frac{15977}{243} \left( \frac{t_0}{t^*} \right)^{-4} t_0^{-2} + \frac{35408}{729} (t^*)^{-2} + \frac{22130}{729} \left( \frac{t_0}{t^*} \right)^{-1} t_0^{-2}.
\]

(17)

The first member of each equation of the system above is given in terms of observational constants, while the right hand side of each equality bears the parameters of the perturbed model. These will be calculated by solving numerically the above transcendental equations. We deal with this task now.

5 Numerical calculations

In order to solve numerically the system of coupled transcendental equations, let us perform some simple manipulations. First, we define new non-dimensional variables,

\[
u \equiv \frac{t^*}{t_0}, \quad b \equiv \frac{\beta}{t^*},
\]

in terms of which the system is rewritten as

\[
H_0 = \frac{2}{3} \frac{1}{t_0} \left( 1 + b \left( -\frac{17704}{243} \frac{1}{u^2} + \frac{11065}{486} u + \frac{2456}{81} u^4 \right) \right);
\]

(19)

\[
H_0^2 (q_0 + 1) = \frac{2}{3} \frac{1}{t_0^2} \left( 1 + b \left( \frac{17704}{243} \frac{1}{u^2} + \frac{11065}{243} u + \frac{12280}{81} u^4 \right) \right);
\]

(20)

\[
3H_0^2 \Omega_{m0} = \frac{4}{3} \frac{1}{t_0^2} \left( 1 + b \left( -\frac{15977}{162} u^4 + \frac{17704}{243} \frac{1}{u^2} + \frac{11065}{243} u \right) \right).
\]

(21)

where

\[
\Omega_{m0} \equiv \frac{8\pi G}{3H_0^2} \rho_0.
\]
Taking the ratio of the two last equations of the system above, one gets:

\[
b(u) = \frac{2(q_0 + 1) - 3\Omega_{m0}}{s(u)},
\]

\[
s(u) \equiv \left(3\Omega_{m0} - 2(q_0 + 1)\right) \times \left(\frac{17704}{243} \frac{1}{u^2} + \frac{11065}{243} \frac{1}{u} + \frac{36840}{243} u^4\right) + \frac{40537}{81} (q_0 + 1) u^4.
\]

We must have \(\beta < 0\) in order to assure the stability of the theory (Cuzinatto et al. 2008; Gottlöber et al. 1990). This establishes a constraint on the sign of \(b\). Substituting \(b(u)\) in the equation for \(H_0\):

\[
\frac{1}{t_0(u)} = \frac{3}{2} \frac{H_0}{(1 + b(u)) \left(\frac{17704}{243} \frac{1}{u^2} + \frac{11065}{243} \frac{1}{u} + \frac{2456}{81} u^4\right)},
\]

Combining (19) and (20) we get

\[
H_0^2\left(2(q_0 + 1) - 3\Omega_{m0}\right) = \frac{4}{3} b(u) \left(\frac{12280}{81} + \frac{15977}{162}\right) u^4,
\]

which is a nonlinear equation for the parameter \(u\). This equation can be solved using the Newton–Raphson method. The result is then used in (22) to obtain the numerical value of \(b\) and both \(b\) and \(u\) are inserted in (23) to get the age of the universe \(t_0\). The time of perturbation \(t^*\) can then be obtained as a simple ratio. Proceeding this way, we are able to find all the parameters of the model given the measurements of \(H_0\), \(q_0\) and \(\Omega_{m0}\) (or \(\rho_0\)).

The Hubble constant is measured with great accuracy by the Hubble Space Telescope Key Project (Freedman et al. 2001). The deceleration parameter is taken from Rapetti et al. (2007) who used a kinematical approach to cosmological expansion. The matter density \(\Omega_{m0}\) was evaluated following the same procedure as Allen et al. (2008) but retaining just the three galaxy clusters with redshift below 0.1 since this minimizes the dependence of the estimation of \(\Omega_{m0}\) on different cosmological models.

With this assumptions and using the experimental values \(H_0 = 0.074\) (Gyr\(^{-1}\)), \(q_0 = -0.81\), \(\Omega_{m0} = 0.276\), we find:

\[u \approx 0.75,\quad b \approx -0.0036,\quad t^* \approx 9.3\) Gyr,\]

which can be used to estimate the parameter \(\beta\) and the age of the universe,

\[\beta \approx -26\) (Gyr\(^4\)),\]

\[t_0 \approx 12.4\) Gyr.\]

By varying \(q_0\) in 20% we obtain numerically a variation less than 2% for \(t_0\) and less than 20% for \(t^*\) what shows that the model is approximately robust.

In spite of its apparently high value, \(\beta\) does not break the meaning of the modified action as proposed in (1). This is so because the first order perturbation is such that the term \(\beta H_t^*\) scales as \(\beta/t^{*4}\) and therefore is small when compared to the usual Friedmann term \(H_F\). Of course, the contribution of the additional term grows for values of \(t\) progressively greater than \(t^*\). But we do not hope that the perturbative approach holds for arbitrary large values of \(t\). The convergence of the perturbative expansion can be qualitatively studied plotting the ratio of the perturbation term by the usual FLRW solution, as given in Fig. 1.

As long as the curves lie within the interval \([-1, 1]\) the perturbative scheme can be assumed as valid. Notice that the Hubble function fits the perturbative scheme for a longer time compared to the behavior of the energy density.

The value of \(t_0\) provided by the WMAP3 data (Yao et al. 2006)—which assumes the \(ΛCDM\) model as the supernovae approach does—is calculated as \(13.7^{+0.1}_{-0.2}\) Gyr (for a flat Universe). The result given by our model is roughly close with the one predicted by the \(ΛCDM\) model. This apparent tension between both results does not impair the model proposed here since we are just looking for a preliminary estimation of our free parameters. Besides, globular clusters data gives 11.2 Gyr, at 95% of confidence level, as inferior limit for the age of the universe (Krauss and Chaboyer 2003).
Using the solution (16), one easily finds the ratio of scale factor at two arbitrary times \( t_i \) and \( t_f \) as

\[
\ln \left( \frac{a_f}{a_i} \right) = \ln \left( \frac{t_f}{t_i} \right)^2 - \frac{\beta}{(t_i^*)^4} \left( \frac{17704}{729} \left( \frac{t_f}{t_i^*} \right)^2 + 11065 \left( \frac{t_f}{t_i^*} \right) + 1228 \left( \frac{t_f}{t_i^*} \right)^4 \right)
\]

which exhibits a very smooth transition from the Friedmann standard regime to the accelerated one.

The same quantity can be used to estimate the red-shift at the transition,

\[1 + z^* = \frac{a_0}{a(t^*)},\]

as

\[1 + z^* = \left( \frac{t_0}{t^*} \right)^2 \times \exp \left( - \frac{\beta}{(t^*)^4} \left( \frac{17704}{729} \left( \frac{t_0}{t^*} \right)^2 + 11065 \left( \frac{t_0}{t^*} \right) + 1228 \left( \frac{t_0}{t^*} \right)^4 - 32453 \right) \right) .\]

With the values of \( \beta, t^* \) and \( t_0 \), one calculates \( z^* \approx 0.27 \).

6 Conclusions

We have constructed a model based on a phenomenological theory of gravitation obtained from the inclusion of a Podolsky-like term scaling with the square of the covariant derivative of the Ricci scalar. This model implies long-range modifications in gravitation, which leads to an accelerated regime for the present-day universe, even in the absence of a dark energy component or cosmological constant. According to our perturbative evaluation, this accelerated expansion started recently as indicated by the values of \( t^* \) or, equivalently, \( z^* \).

The estimations given by the model for the age of the universe and the redshift of transition are close to the supernovae data (Riess et al. 1998; Perlmutter et al. 1999, 2003; de Bernardis et al. 2000; John 2004; Boughn and Chittenden 2004; Cole et al. 2005; Astier et al. 2006; Springel et al. 2006; Wood-Vasey et al. 2007) or the analysis of the cosmic microwave background based on the \( \Lambda CDM \) model (Spergel et al. 2007). Other modified gravity theories—e.g., the \( f(R) \) theories (Capozziello and Francaviglia 2008)—can generate accelerated phases for the expansion of the universe. In this paper, we obtained the same qualitative results by adding a term proportional to \((\nabla R)^2\) to the Einstein–Hilbert Lagrangian. Our future perspectives include to obtain more accurate values for the parameters using the likelihood function fitted by supernovae data. We also intend to study a perturbative solution for a closed (\( \kappa = 1 \)) universe, keeping in mind the exact solution found in the ordinary FRW case (Aldrovandi et al. 2006).

The same phenomenological Lagrangian presented here was applied to describe inflation in Gottlöber et al. (1990), but we emphasize the high order of magnitude of the energies involved there, which would correspond to the early stages of evolution of the Universe. On the other hand, the model presented here engenders sensible dynamical effects at recent cosmic times, where the energy scale is very low.

The fact that the gravitational field is weaker at long distances with a characteristic scale given by the coupling constant \( \beta \) suggests the existence of massive modes in the weak field approximation, but in a way that does not break the coordinate invariance, analogously to what happens in the Podolsky electrodynamics (Cuzinatto et al. 2007). The eventual existence of such massive modes are under investigation, and the results should be compared to other approaches in the same direction (Novello and Neves 2003; Novello 2004; Gazeau and Novello 2006).

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