The ratio of p and n yields in NC $\nu($$\bar{\nu}$$)$ nucleus scattering and strange form factors of the nucleon

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Abstract

We calculate the ratio of proton and neutron yields in NC induced $\nu(\bar{\nu})$–nucleus inelastic scattering, which was proposed a few years ago as a probe of strange form factors of the nucleon. In a previous work we have demonstrated that at neutrino energies of about 1 GeV this ratio depends very weakly on the nuclear model employed. Here we show that in $\nu$ and $\bar{\nu}$ cases the ratios have different sensitivity to the axial and vector strange form factors: a measurement of them could allow a separation of the axial and vector strange form factors; moreover the ratio of $\bar{\nu}$–nucleus cross sections turns out to be very sensitive to the electric strange form factor.

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The determination of the one–nucleon matrix elements of the axial and vector (weak) strange currents has become an important challenge both for theory and experiment: after the measurements of the polarized structure function of the proton, $g_1$, in deep inelastic scattering experiments [1], [2], the value $g_A^s = -0.10 \pm 0.03$ has been set [3], while the strange magnetic form factor of the nucleon has been recently determined at Bates [4] via measurements of the P–odd asymmetry in electron–proton scattering, with the result $G_M^s(0.1 \text{ GeV}^2) = 0.23 \pm 0.37 \pm 0.15 \pm 0.19$. The latter is still affected by large experimental (and theoretical) uncertainties, which are compatible with vanishing magnetic strange form factor; the former seems to indicate a non–zero value of the strange axial constant, but the theoretical analysis of the data leading to the above mentioned result still suffers from some uncertainties and model dependence. Further progress is thus needed in order to assign a reliable quantitative estimate of the strange components of the weak neutral hadronic current.

In previous works [5], [6] we have shown that an investigation of elastic and inelastic neutral current (NC) scattering of neutrinos (and antineutrinos) on nucleons and nuclei is an important tool to disentangle the isoscalar strange components of the nucleonic current. In this letter we focus the attention on one specific observable, namely the ratio between the cross sections of the inelastic processes:

\begin{align}
\nu_\mu(\overline{\nu}_\mu) + (A, Z) &\rightarrow \nu_\mu(\overline{\nu}_\mu) + p + (A - 1, Z - 1), \\
\nu_\mu(\overline{\nu}_\mu) + (A, Z) &\rightarrow \nu_\mu(\overline{\nu}_\mu) + n + (A - 1, Z),
\end{align}

where $(A, Z)$ represents a nucleus with $A$ nucleons and atomic number $Z$; a proton (or a neutron, respectively) is emitted in the final state. This ratio has been first suggested as a probe for strange form factors by Garvey et al. [7], [8], at rather low incident neutrino energies ($E_\nu \simeq 200 \text{ MeV}$), a kinematical condition which is appropriate for LAMPF.

The influence of the nuclear dynamics on this ratio, expressed through various typical nuclear models, has been thoroughly discussed in ref. [9] and [6]. It has been found that at $E_\nu$ of the order of 200 MeV the theoretical uncertainties associated, e.g., with the final states interaction (FSI) of the ejected nucleon with the residual nucleus could introduce ambiguities in the determination of the strange axial and magnetic form factors [6]. The dependence of the ratio of integrated cross sections within LAMPF kinematical conditions upon different assumptions about the strange nucleonic form factors is also considered in ref. [10]: these authors, however, assume a small sensitivity of the ratio to the nuclear model employed for the calculation. In our opinion, incident neutrino energies of the order of 1 GeV appear to be more interesting from the theoretical point of view, since the nuclear model effects are within percentage range and are well under control. Neutrinos with such energies are available at Brookhaven, KEK, Protvino and probably will be available at Fermilab (see BOONE proposal [11]).

The aim of this letter is to discuss a few key points which might help the interpretation of future (in our auspices) data: the main point concerns the interference between the matrix elements of the axial and magnetic strange currents, which plays an important role and has opposite sign in the neutrino and antineutrino processes. Not only the value, but also the sign of the strange magnetic moment are not known at present. We will show that the measurement of both $\nu$ and $\overline{\nu}$ cross sections will help to determine this quantity.
Let us start from a few definitions of the quantities we wish to discuss: for the sake of illustration we shall present here only the expressions for the cross sections obtained, in the plane wave impulse approximation (PWIA), within the relativistic Fermi gas model (RFG). As discussed in ref. [1], for incident neutrino energies \( E_\nu \geq 1 \text{ GeV} \) the nuclear model dependence of the ratio we are interested in is fairly negligible: we shall discuss results evaluated within a relativistic nuclear shell model (RSM), taking into account the effect of FSI through a relativistic optical potential (ROP). Nevertheless the simplicity of the Fermi gas model is better suited to get a feeling of the interplay between the different components which will be examined.

The differential cross sections for the processes (1) and (2) can be written as follows:

\[
\left( \frac{d^2 \sigma}{dT_N d\Omega_N} \right)^{NC}_{\nu(\tau)A} = \frac{G_F^2}{(2\pi)^2} \frac{V}{(2\pi)^3} \frac{|\vec{p}_N|}{4k_0} \int \frac{d^3k'}{k_0'} \frac{d^3p}{p_0} \delta \left( k_0 - k_0' + p_0 - E_N \right) \times \delta^{(3)} \left( \vec{k} - \vec{k}' + \vec{p} - \vec{p}_N \right) \theta(p_F - |\vec{p}|) \theta(|\vec{p}_N| - p_F) \left( L_5^{\mu\nu} + L_6^{\mu\nu} \right) W_{\mu\nu},
\]

where the plus (minus) sign refers to neutrino (antineutrino) scattering, \( V \) is the nuclear volume, \( k \) (\( k' \)) and \( p \) are the four momenta of the incident (outgoing) neutrino and of the initial nucleon, respectively; moreover \( p_N \equiv (\vec{p}_N, E_N) \) is the four momentum of the ejected nucleon, \( T_N \) is its kinetic energy, \( p_F \) is the Fermi momentum (related to the density of a symmetric nucleus by the customary relation \( Z/V = N/V = p_F^3/3\pi^2 \)) and \( L_5^{\mu\nu}, L_6^{\mu\nu} \) are the leptonic tensor and pseudotensor, respectively:

\[
L_5^{\mu\nu} = k^\mu k'^\nu + k'^\mu k^\nu - g^\mu\nu k \cdot k',
\]

\[
L_6^{\mu\nu} = i\epsilon^{\mu\nu\rho\sigma} k_\rho k'_\sigma.
\]

Finally, \( W_{\mu\nu} \) is the single nucleon hadronic tensor:

\[
W_{\mu\nu} = \sum_{s,s_N} < p_N, s_N | \hat{J}_\mu | p, s > < p, s | \hat{J}_\nu | p_N, s_N >,
\]

where \( \hat{J}_\mu \) is the weak nucleonic neutral current, with matrix elements

\[
< p_N, s_N | \hat{J}_\mu | p, s > = \mathcal{U}_{s_N}(p_N) \left[ \gamma_\mu F^z_V(Q^2) + \frac{i}{2M} \sigma_{\mu\alpha} q^\alpha F^z_M(Q^2) + \gamma_\mu \gamma_5 F^z_A(Q^2) \right] U_s(p),
\]

In the above \( Q^2 = -q^2 \) (\( q = p_N - p \)) and \( F^z_V, F^z_M, F^z_A \) are the vector, magnetic and axial nucleonic form factors, respectively: whether they are referred to proton or neutron depends on the nature of the ejected nucleon, represented by the state vector \( |p_N, s_N > \).

After some algebra one can rewrite eq. (3) as:

\[
\left( \frac{d^2 \sigma}{dT_N d\Omega_N} \right)^{NC}_{\nu(\tau)A} = \frac{G_F^2}{(2\pi)^2} \frac{V}{(2\pi)^3} \frac{|\vec{p}_N|}{k_0} \theta(|\vec{p}_N| - p_F) \int \frac{d^3k'}{k_0'} \frac{d^3p}{p_0} \delta \left( k_0 - k_0' + p_0 - E_N \right) \times \delta^{(3)} \left( \vec{k} - \vec{k}' + \vec{p} - \vec{p}_N \right) \theta(p_F - |\vec{p}|) \left\{ 2M^2 (k \cdot k') \left[ \tau (G^z_M)^2 + (1 + \tau) (F^z_A)^2 \right] + \left[ \frac{(G^z_E)^2 + \tau (G^z_M)^2}{1 + \tau} + (F^z_A)^2 \right] \left[ 2(k \cdot p) (k' \cdot p) - M^2 (k \cdot k') \right] \right\} \pm 2F^z_A G^z_M (k' \cdot p + k \cdot p) k \cdot k',
\]
where the upper (lower) sign refers to $\nu(\bar{\nu})$ induced processes. The cross-section (8) is expressed in terms of the weak Sachs’ form factors:

\[ (G_E^Z)_{\nu(n)}(Q^2) = \frac{\varepsilon_W}{2} G_{E}^{\nu(n)}(Q^2) - \frac{1}{2} G_{n}^{n(p)}(Q^2) - \frac{1}{2} G_{s}^{s}(Q^2), \]
\[ (G_M^Z)_{\nu(n)}(Q^2) = \frac{\varepsilon_W}{2} G_{M}^{\nu(n)}(Q^2) - \frac{1}{2} G_{n}^{n(p)}(Q^2) - \frac{1}{2} G_{M}^{s}(Q^2), \]
\[ (F_A^Z)_{\nu(n)}(Q^2) = \pm \frac{1}{2} F_{A}^{n(n)}(Q^2) - \frac{1}{2} F_{A}^{s}(Q^2). \]

In the above $\varepsilon_W = (1 - 4 \sin^2 \theta_W)$ and $\tau = Q^2/4M^2$; the Standard Model relationship between weak and electromagnetic form factors and isospin invariance of the strong interaction have been used. In addition in (9)–(11) three isoscalar strange form factors enter: electric ($G_E^s$), magnetic ($G_M^s$) and axial ($F_A^s$).

Once the expressions (8)–(11) are inserted into (8), one obtains the NC cross sections:

\[ \left( \frac{d^2\sigma}{dT_Nd\Omega_N} \right)^{NC} = \frac{3Z(N)}{8\pi p_F^3} \frac{4|\vec{p}_N| M^4}{k_0} \theta(|\vec{p}_N| - p_F) \]
\[ \times \int \frac{d^3p}{k_0 p_0} \delta(k_0 - k_0' + p_0 - E_N) \mathcal{I}_{\nu(n)}(k, p, Q^2), \]

where (the plus/minus sign always refer to the $\nu$ and $\bar{\nu}$ cases):

\[ \mathcal{I}_{\nu(n)}(k, p, Q^2) = \mathcal{N}_{\nu(n)} + \tau^2 G_{n}^{n(p)} G_{M}^{s} \]
\[ + \frac{1}{2} \left[ (x - \tau)^2 - \tau(\tau + 1) \right] \left[ \frac{G_{E}^{n(p)} G_{E}^{s} + \tau G_{M}^{n(p)} G_{M}^{s}}{1 + \tau} \right] \]
\[ - \delta_{p(n)} \left\{ \frac{1}{2} \left[ (x - \tau)^2 + \tau(\tau + 1) \right] F_{A}^{s} \right\} \]
\[ \pm \tau(x - \tau) \left( G_{M}^{n(p)} F_{A}^{n(p)} + F_{A}^{s} G_{M}^{s} - G_{M}^{n(p)} F_{A}^{s} \right), \]

with $x = k \cdot p/M^2$ and $\delta_p = 1$, $\delta_n = -1$. The terms

\[ \mathcal{N}_{\nu(n)} = \frac{1}{2} \tau^2 \left( G_{M}^{n(p)} F_{A}^{s} \right) \]
\[ + \frac{1}{4} \left[ (x - \tau)^2 - \tau(\tau + 1) \right] \left[ \frac{G_{E}^{n(p)} F_{A}^{s} + \tau G_{M}^{n(p)} F_{A}^{s}}{1 + \tau} \right] + F_{A}^{s} \]

\[1 \text{ In our numerical calculations we took into account terms containing } \varepsilon_W. \text{ Their contribution to the cross sections could be comparable with the contribution of strange form factors.}\]
contain only the electromagnetic and axial form factors and can be assumed to be well under control (though sizeable uncertainties remain in the neutron electric form factor as well as in the high $Q^2$ behaviour of the axial nucleonic form factor). In the remaining expressions of $I_{p(n)}$, eq.(13), all the three unknown isoscalar strange form factors ($G^s_E$, $G^s_M$ and $F^s_A$) enter into play.

Finally we define the ratio of $\nu$ ($\bar{\nu}$) cross sections with the emission of a proton and a neutron, respectively, integrated over the angle $\Omega_N$:

$$R_{\nu p/n} = \frac{\left( \frac{d\sigma}{dT_N} \right)_{\nu p}}{\left( \frac{d\sigma}{dT_N} \right)_{\nu n}}.$$

Even with the simplified formulas (12)–(14) it is difficult to foresee, “a priori”, what is the sensitivity of the ratio $R_{p/n}$ to the various strange form factors of the nucleon, although it has been pointed out in the past (see, for example, refs. [6], [10]) that it is quite affected by $F^s_A$ and, to somewhat minor extent, by $G^s_M$.

Thus in the following we exhibit the results of the numerical evaluation of $R_{p/n}$ together with our conclusions about the usefulness of this quantity for investigating the strange content of the nucleon.

The cross sections entering into (15) have been evaluated within the nuclear models referred to above as RFG (only in PWIA) and RSM (in PWIA and also in DWIA, namely taking into account the effects of final state interactions through the ROP) [6]. Details of the relativistic shell model as well as of the optical model potential utilized for the distorted wave of the ejected nucleon can be found elsewhere (see, for example, refs. [12], [13] and [14]), together with its expected validity in reproducing exclusive $(e,e'N)$ cross sections at fairly large energies.

In Fig. 1a,b we present the ratio $R_{\nu p/n}^\nu$ (a) and $R_{\nu p/n}^{\bar{\nu}}$ (b) for incident neutrino energy $E_\nu = 1$ GeV as a function of the kinetic energy of the final nucleon, $T_N$, at different values of the parameters that characterize the strange form factors. The solid lines correspond to the pure RSM, the dot–dashed lines to the DWIA (RSM+ROP) and the dotted lines to the RFG. The latter almost coincide with the solid ones in Fig. 1a, while small differences are seen in the ratio of $\bar{\nu}$–cross sections (Fig. 1b). Also the effect of FSI appears to be somewhat more relevant in the $\bar{\nu}$ processes, while it is fairly negligible in $R_{p/n}^\nu$. As already noticed in ref. [14], at $E_\nu = 1$ GeV the ratio $R_{\nu p/n}^\nu$ is substantially unaffected by the nuclear model description, even by including the distortion of the knocked out nucleon (in spite of the fact that the FSI sizably reduce the separated cross section with respect to the PWIA); moreover $R_{p/n}^\nu$ is fairly constant as a function of the ejected nucleon energy over the whole interval of kinematically allowed $T_N$ values, thus providing a wide range of energy for testing the effects of the strange form factors.

On the contrary $R_{p/n}^{\bar{\nu}}$ shows a more pronounced dependence upon the energy of the final nucleon, stemming from the fact that the $(\bar{\nu}, n)$ cross sections decrease faster than the $(\bar{\nu}, p)$ ones. As a consequence the range of $T_N$ where the ratio increases appears to be more sensitive to the nuclear model and to FSI (we have partially cut the curves in the large $T_N$ region, the latter being uninteresting for the discussion). If we further restrict to the region
where $R^\nu_{p/n}$ remains fairly constant, the sensitivity of the ratio to the nucleonic strangeness is comparable to the one of $R^\nu_{p/n}$.

In Fig. 1 we display the calculation of $R^\nu_{p/n}$ and $R^{\bar{\nu}}_{p/n}$ for three different choices of the axial and magnetic vector strangeness parameters:

1. $g^s_A = 0$, $\mu_s = 0$ (no strangeness),
2. $g^s_A = -0.15$, $\mu_s = 0$,
3. $g^s_A = -0.15$, $\mu_s = -0.3$,

assuming a dipole form for the $Q^2$ dependence of the corresponding form factors:

\begin{align}
G^s_M(Q^2) &= \frac{\mu_s}{(1 + Q^2/M^2_V)^2}, \\
F^s_A(Q^2) &= \frac{g^s_A}{(1 + Q^2/M^2_A)^2},
\end{align}

(16)

$M_{V(A)}$ being the same cutoff masses of the non–strange vector (axial) form factors.

Fig. 1b shows however an important difference with respect to Fig. 1a: while in $R^\nu_{p/n}$ the effects of axial and magnetic strangeness (assumed to have the same, negative, sign) have a constructive interference, which enhances the global effect of strangeness (if any), the opposite occurs for anti–neutrinos: indeed, for the same choices 1–3 of the strangeness parameters, the ratio $R^{\bar{\nu}}_{p/n}$ corresponding to case 3 falls below case 2, showing the destructive interference between axial and magnetic strangeness if they have the same sign.

Before discussing the case of a positive $\mu_s$, we would like to examine the sensitivity of the ratio to the non–strange nucleonic form factors entering into the considered NC cross sections.

First we remind the reader that the most recent data on the electromagnetic Sach’s form factors have shown significant deviations from the customary dipole form [13], [16] at relatively high $Q^2$ and have been fitted with different $Q^2$ dependence by various authors [17], [18]. We have shown in a previous work [5] how the present uncertainty on the electromagnetic form factors can affect NC neutrino–nucleon scattering. Here we have tested the sensitivity of the ratios $R_{p/n}$ by employing two, rather different, parameterizations of the electromagnetic form factors, namely the simple dipole form (together with the Galster parameterization for $G^n_E$ [19]) and the one proposed by Watanabe and Takahashi [17]. The resulting $R_{p/n}$ do not differ significantly, both for $\nu$ and $\bar{\nu}$ ratios, the biggest deviation being of the order of $1 \div 2\%$.

The second important uncertainty in neutrino processes at high energy concerns the $Q^2$ evolution of the axial form factor $F_A$: for it one commonly utilizes a dipole form with a cutoff $M_A$ of the order of 1 GeV (a frequently used value is $M_A = 1.032 \pm 0.036$ GeV). The analysis of $\nu(\bar{\nu}) - p$ cross sections by Garvey et al. [20] has shown the ambiguities which still concern this form factor, together with its strong correlation (in the separated cross sections) with the axial strange constant $g^s_A$; while $M_A = 1.032$ GeV is consistent with $g^s_A = -0.21 \pm 0.10$, a slightly larger axial cutoff, $M_A = 1.089$ GeV is compatible with $g^s_A = 0$.

We have thus analyzed the influence of $M_A$ on the ratio $R_{p/n}$, by allowing it to vary in the range $1.0 \leq M_A \leq 1.1$ GeV: the effect on the ratio is small (at most of the order of 3%) and does not severely interfere with the much larger effects of the strange components.
Let us now discuss the interference between the axial and magnetic components of the weak strange neutral current: in Fig. 1 we have shown a case where $g_A^s$ and $\mu_s$ have the same (negative) sign. Of course the situation gets inverted if $g_A^s$ and $\mu_s$ have opposite signs: in this case the ratio of $\nu$ cross sections is less sensitive to the presence of both strange components while the ratio of $\bar{\nu}$ cross sections shows larger effects when axial and magnetic strangeness are present. This is illustrated in Fig. 2, which summarizes all possible occurrences, keeping fixed $g_A^s = -0.15$ (according to the data of experiments on deep inelastic scattering of polarized leptons on polarized nucleons the nucleon matrix element of the strange axial current is negative and of the order of 0.1): for $\mu_s$ we show three cases, ($\mu_s = 0, \pm 0.3$), with the opposite behaviour of $R_{\nu}^{p/n}$ (Fig. 2a) and $R_{\bar{\nu}}^{p/n}$ (Fig. 2b) discussed above. We also show, for comparison, the ratios obtained in the absence of strangeness. For simplicity, only the RFG results are shown.

The interest of considering positive $\mu_s$ values stems from the recent measurement of this quantity performed at Bates in parity violating electron scattering on the proton \cite{4}. Though affected by large errors, which give a result still compatible with zero magnetic strangeness, a positive strange magnetic moment of the nucleon is allowed. The value of $G_M^s = +0.23 \pm 0.37 \pm 0.15 \pm 0.19$ at $Q^2 = 0.1$ GeV$^2$ corresponds to a $\mu_s = 0.30 \pm 0.48 \pm 0.20 \pm 0.25$ if extrapolated down to the origin by using form factors of dipole type (the quoted uncertainties are, respectively, the statistical and systematic errors together with the theoretically estimated radiative corrections \cite{21}). This first experimental indication faces a wide range of negative and positive values proposed for $\mu_s$ in a large variety of phenomenological models.

Thus far we have discussed results obtained for the ratio $R_{p/n}$ by setting to zero the electric strange form factor, $G_E^s$: the latter is multiplied by $G_E^n$ in the numerator and by $G_E^p$ in the denominator [see eq.(13)], so that one can expect a residual, unbalanced, effect in the ratio. In order to examine its importance, we have included $G_E^s$ with the following parameterization:

$$G_E^s(Q^2) = \rho_s \tau G_D^V(Q^2)$$

$\rho_s$ being a constant and $G_D^V(Q^2)$ the usual dipole form factor of the vector currents, with cutoff $M_V = 0.84$ GeV.

We have found that, for rather large values of $\rho_s$ (of the order of $\pm 2$)\footnote{The value $\rho_s = 2$ is compatible with the vector strange form factors employed in fit IV of Garvey et al. \cite{21} in the analysis of $\nu(\bar{\nu})$–p cross sections.}, the ratio $R_{p/n}$ is appreciably modified, in particular it is enhanced by a negative $\rho_s$ and reduced by a positive one.

Moreover we have found that the electric strangeness has a quite different impact on $R_{\nu}^{p/n}$ and on $R_{\bar{\nu}}^{p/n}$. In the first case ($R_{\nu}^{p/n}$) the effect of $G_E^s$ does not exceed 25% of the correction associated to the axial strange form factor, which remains the dominant one, while it can be of the order of 50% of the correction associated with a strange magnetic moment $\mu_s = -0.3$.

Instead, for the ratio obtained with antineutrino beams ($R_{\bar{\nu}}^{p/n}$), the interference between the electric and magnetic strange form factors appears to be much more important: it turns
out that \( R_{\nu{p/n}}^\nu \) is even more sensitive to \( G_E^s \) than to \( G_M^s \), although, again, the axial strange form factor plays the major role. This introduces a third unknown in the analysis of \( R_{\nu{p/n}}^\nu \) and \( R_{\bar{\nu}{p/n}}^\bar{\nu} \). However it is worth reminding that it is quite difficult to determine the electric strange form factor in parity violating electron scattering: this component can affect the PV asymmetry by at most 20% at very small scattering angles \([22]\), while it is possible to measure \( G_M^s \), as shown by the SAMPLE experiment and more precise measurements are indeed under way. Thus one can exploit the sensitivity of \( R_{\nu{p/n}}^\nu \) to \( G_E^s \) precisely to extract the relevant information on the electric strangeness.

In order to illustrate this point, we present in Fig. 3a,b the ratio of the integrated NC \( \nu(\bar{\nu}) \)–nucleus cross sections

\[
R_{\nu{p/n}} = \frac{\int_{T_{\text{min}}}^{T_{\text{max}}} dT_N \left( \frac{d\sigma}{dT_N} \right)_{\nu(\bar{\nu}), p}}{\int_{T_{\text{min}}}^{T_{\text{max}}} dT_N \left( \frac{d\sigma}{dT_N} \right)_{\nu(\bar{\nu}), n}},
\]  

where we have chosen, for \( E_{\nu(\bar{\nu})} = 1 \) GeV, \( T_{\text{min}} \approx 27 \) MeV (the minimum kinematically allowed value) and \( T_{\text{max}} = 400 \) MeV (the maximum reliable interval, in which the \( \bar{\nu} \) are fairly stable versus \( T_N \)). The ratio \([18]\) is displayed as a function of \( \mu_s \), fixing \( g_A^s = 0 \) and \( g_A^s = -0.15 \) and showing, around this last value, the “band” associated with a variation of \( \rho_s \) between \(-2\) and \(+2\). This band is rather narrow in Fig. 3a, referring to the ratio measurable with neutrino beams, while it is larger in Fig. 3b, referring to the \( \bar{\nu} \) case: yet, in this last instance, room enough is left to appreciate different values of \( g_A^s \). Concerning the sensitivity of the integrated ratio to the magnetic strange form factor, one can see that the \( \nu \) case shows a perceptible slope with increasing \( \mu_s \), whereas the \( \bar{\nu} \) case appears to be almost independent upon the value of \( \mu_s \): this fact favours the extraction of the electric strange form factor.

In conclusion we have analyzed the ratio of \( \nu \) (and \( \bar{\nu} \))–nucleus NC inelastic cross sections with a proton and a neutron, respectively, in the final state, focussing on \( \nu(\bar{\nu}) \) incident energies in the GeV range.

In agreement with a previous work \([9]\), we have found that, at least for \( E_\nu \geq 1 \) GeV, this quantity is fairly unaffected by the specific nuclear model employed; we have thus thoroughly discussed various uncertainties stemming from the electromagnetic and axial form factors of the nucleon and we have recognized that they are quite small with respect to the expected sensitivity of \( R_{\nu{p/n}}^\nu \) and \( R_{\bar{\nu}{p/n}}^\bar{\nu} \) to the axial and magnetic strange form factors.

Finally we have focussed our analysis on the interplay, in the ratio, between axial, magnetic and electric strange form factors. The largest effect is associated with the axial strange form factor: the interference between \( g_A^s \) and \( \mu_s \) crucially depends on their relative sign and turns out to act in opposite ways on \( R_{\nu{p/n}}^\nu \) and \( R_{\bar{\nu}{p/n}}^\bar{\nu} \). Moreover we have found a strong sensitivity of \( R_{\nu{p/n}}^\nu \) to the electric strange form factor. Thus, by assuming that PV electron scattering experiment will support a more precise (than the present one) determination of the magnetic strange form factor, the combined measurement of \( R_{\nu{p/n}}^\nu \) and \( R_{\bar{\nu}{p/n}}^\bar{\nu} \) could allow an unambiguous and model independent determination of all three strange form factors of the nucleon.
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FIG. 1. The ratio $R_{p/n}^\nu$ (a) and $R_{p/n}^\bar{\nu}$ (b) for NC neutrino processes, versus the kinetic energy of the final nucleon $T_N$, at incident energy $E_{\nu(\bar{\nu})} = 1$ GeV. The solid lines correspond to the RSM calculation, the dot–dashed lines include the effect of FSI accounted for by the ROP model and the dotted lines to the RFG model. Three different choices of the strangeness parameters are shown, as indicated in the figure.
FIG. 2. The ratio $R_{\nu}^{p/n}$ (a) and $R_{\bar{\nu}}^{p/n}$ (b) for NC neutrino processes, as a function of the kinetic energy of the final nucleon $T_N$: all curves are evaluated in the RFG. The incident energy is $E_{\nu(\bar{\nu})} = 1$ GeV. The solid line corresponds to zero strangeness ($g_A^{s} = \mu_s = 0$), in the other three curves [both in (a) and in (b)] we have fixed $g_A^{s} = -0.15$ and chosen $\mu_s$ to be: $\mu_s = 0$ (dashed line), $\mu_s = -0.3$ (dot–dashed line) and $\mu_s = +0.3$ (dotted line).
FIG. 3. The ratio $R_{\nu}^{p/n}$ (a) and $R_{\bar{\nu}}^{p/n}$ (b) of the integrated NC neutrino–nucleus cross sections, as a function of $\mu_s$: all curves are evaluated in the RFG. The incident energy is $E_{\nu(\bar{\nu})} = 1$ GeV. The solid line corresponds to $g_A^s = \rho_s = 0$, in the other three curves [both in (a) and in (b)] we have fixed $g_A^s = -0.15$ and chosen $\rho_s$ to be: $\rho_s = 0$ (dashed line), $\rho_s = -2$ (dot–dashed line) and $\rho_s = +2$ (dotted line).