A Scaled Smart City for Emerging Mobility Systems

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Emerging mobility systems, e.g., connected and automated vehicles (CAVs), shared mobility, mark a paradigm shift in which a myriad of opportunities exist for users to better monitor the transportation network conditions and make optimal operating decisions to improve safety and reduce pollution, energy consumption, and travel delays [1]. Emerging mobility systems are typical cyber-physical systems where the cyber component (e.g., data and shared information through vehicle-to-vehicle and vehicle-to-infrastructure communication) can aim at optimally controlling the physical entities (e.g., CAVs, non-CAVs). The cyber-physical nature of such systems is associated with significant control challenges and gives rise to a new level of complexity in modeling and control [2]. As we move to increasingly complex emerging mobility systems, new control approaches are needed to optimize the impact on system behavior [3] of the interplay between vehicles at different traffic scenarios [4]. It is expected that CAVs will gradually penetrate the market and interact with human-driven vehicles in ways that will improve safety and transportation efficiency over the next several years. However, different levels of vehicle automation in the transportation network can significantly alter transportation efficiency metrics [5] ranging from 45% improvement to 60% deterioration. Moreover, we anticipate that efficient transportation and travel cost reduction might alter human travel behavior causing rebound effects, e.g., by improving efficiency, travel cost is decreased, hence willingness-to-travel is increased. The latter would increase overall vehicle miles traveled, which in turn might negate the benefits in terms of energy and travel time.

Several studies have shown the benefits of CAVs to reduce energy and alleviate traffic congestion in specific transportation scenarios [6]–[8]. There have been two major approaches to utilizing connectivity and automation of vehicles, namely, platooning and traffic smoothing. A platoon is defined as a group of closely-coupled vehicles traveling to reduce their aerodynamic drag, especially at high cruising speeds. The concept of platoon formation is a popular system-level approach to address traffic congestion, which gained momentum in the 1980s and 1990s.
To date, there has been a rich body of research focusing on exploring various methods of forming and/or utilizing platoons to improve transportation efficiency [12]–[23]. Traffic smoothing is another approach that has been explored to mitigate the speed variation of individual vehicles throughout the transportation network, which may be introduced by unnecessary braking and the geometry of the road network. One of the very early efforts in this direction was proposed by Athans [24] for safe and efficient coordination of merging maneuvers with the intention of avoiding congestion. Assuming a given merging sequence, Athans formulated the merging problem as a linear optimal regulator, proposed by Levine and Athans [25], to control a single string of vehicles, with the aim of minimizing the speed errors that will affect the desired headway between each consecutive pair of vehicles. Since then, several studies have been reported in the literature that investigate traffic smoothing to eliminate stop-and-go driving at critical scenarios, such as single intersections [26]–[51], multiple adjacent intersections [52]–[62], merging roadways [63]–[72], roundabouts [73]–[80], speed reduction zones and lane drops [81]–[89], and transportation corridors [90]–[96]. A thorough review of the state-of-the-art methods and challenges in coordination of CAVs is provided in [97], [98].

To thoroughly evaluate the performance of CAVs, they must be tested on a broad spectrum ranging from numerical simulation to real-world public roads. Several commercially simulation environments are currently available that can be used as an essential tool to test the performance of CAVs in a safe and cost-efficient setting. Simulation can help us gather key information about how the system performs in an idealized environment. However, testing the performance of CAVs in a simulation environment is often not as accurate as testing in real world due to the difficulty in modeling the exact physics of the test environment, vehicle dynamics, and driving behavior in simulation. The complexities arising from data loss and transmission latency associated with connectivity and communication networks cannot be captured either. As Grim et al. [99] stated, “the problem with simulations is that they are doomed to succeed.” Although there have been several studies reporting on the impacts of coordination of CAVs at the transportation network, the effectiveness of most of these studies have been only shown in the simulation due to the high costs and safety risks of field tests. Therefore, demonstrating the effectiveness of control techniques for CAVs in a physical experiment is of great importance.

Although several original equipment manufacturers (OEMs) have been demonstrating their self-driving vehicles on public roads to a limited extent, the topic of whether to allow CAVs to operate in public traffic has attracted considerable debates due to the obvious safety concerns [100], [101]. To facilitate a closed and controlled test environment, several smart mobility test centers such as the Mcity [102], the American Center for Mobility [103], and Millbrook Proving Ground [104] have been established over the years. These facilities support CAV testing for
OEMs using vehicle-to-everything (V2X) connectivity across their secure, configurable, and confidential 5G testbeds. Nevertheless, there are several challenges with such facilities related to high cost maintenance. In addition, the significant number of unknown system and environmental factors impose limitations on the repeatability of the testing in some instances. As a result, other platforms have been reported in the literature to complement the capacity of the simulation and real-world testing environments, e.g., hardware-in-the-loop simulation platforms [105], vehicle-in-the-loop simulation platforms [106]. To model vehicle behavior in the field, a virtual copy of the real world is constructed [107], [108] which functions as a parallel traffic system. Such systems can be used to design different test scenarios and evaluate how either conventional or hybrid electric vehicles [109], [110], perform [111]. More recently, a test procedure based on augmented reality has been demonstrated in Mcity [112], [113], where the real-world test vehicle can interact with the virtual vehicles of the simulation environment through V2X communication.

Scaled testbeds for testing CAVs have also attracted considerable attention over the last few years. Such testbeds can be used to safely control scaled CAVs by conducting quick and repeatable experiments. MIT’s Duckietown [114] and Go-CHART [115] focus primarily on local perception and autonomy, whereas the Cambridge Minicars [116] can provide a testbed for emulating cooperative driving in highway conditions. A general-purpose robotics testbed has been developed in Robotarium [117] which features differential drive robots. The Cyber-Physical Mobility Lab [118] is another scaled testbed developed with a focus on decision-making policies and trajectory planning, and includes an intersection and several merging scenarios. For a relatively recent review of similar robotics testbeds, see [119].

In 2017, we developed the Information and Decision Science Lab’s Scaled Smart City (IDS³C) to conduct comprehensive experimentation of CAV technologies at different traffic scenarios. In what follows, we provide a detailed technical exposition on the setup and working principle of IDS³C.

**Information and Decision Science Lab’s Scaled Smart City**

IDS³C (Fig. 1) is a 1:25 scaled testbed spanning over 400 square feet, and it is capable of replicating real-world traffic scenarios in a small and controlled environment using 50 ground and 10 aerial vehicles. IDS³C provides the opportunity to prove concepts beyond simulations and to understand the implications of errors and delays in the vehicle-to-vehicle and vehicle-to-infrastructure communication as well as their impact on energy usage. It can also help visualize how a smart city may look in the future by demonstrating coordination of vehicles in different traffic scenarios, such as merging at roadways and roundabouts, cruising in congested traffic, passing through speed reduction zones, and lane-merging or passing maneuvers. IDS³C can also
be used to understand the implications of emerging mobility systems, consisting of CAVs, electric vehicles, and shared mobility on energy consumption and transportation efficiency. Another facet of research that can be explored using IDS³C is complex missions that include the cooperation of aerial and ground vehicles for logistic problems, such as last-mile delivery. IDS³C has six driver emulation stations [120] interfaced directly with the robotic cars which also allow us to explore human driving behavior.

IDS³C is equipped with a VICON motion capture system and uses eight cameras to track the position of each vehicle with sub-millimeter accuracy. The testbed contains a dozen traffic bottlenecks, including merging roadways, multi-lane roundabouts, adjacent intersections, multi-lane intersections, lane-drops, and speed reduction zones. A central mainframe computer (Processor: Intel Core i7-6950X CPU @ 3.00 GHz x 20, Memory: 125.8 Gb) stores a map of the IDS³C as a database of line and arc segments that make up the road network. The reference trajectory of each CAV is generated on the mainframe using a custom-built multi-threaded C++ application, which allocates a thread to each CAV. The current state and reference trajectory of each CAV is broadcast over WiFi using the user datagram protocol (UDP); this information is used by the CAVs for lane and reference tracking. A schematic of the high, medium, and low-level control loops that run the IDS³C is shown in Fig. 2. We have selected UDP over transmission control protocol (TCP) to increase performance; unlike TCP, a UDP message does not require a response from the recipient. To compensate for data loss and transmission errors, we send messages at up to 50 Hz, and each message contains up to three seconds of reference trajectory data.

The CAVs of IDS³C have been designed using off-the-shelf electrical components and 3D printed parts created at the University of Delaware. The primary microcontroller on the CAV is a Raspberry Pi 3B running Ubuntu Mate and ROS Kinetic. The Raspberry Pi receives current state and reference trajectory information from a high-level control loop on the mainframe computer and passes a desired motor speed and steering angle to a Pololu Zumo 32U4 for low-level control. The Zumo applies PID control to track the desired motor speed while acting as an ad-hoc analog to digital conversion for sensors. Steering is achieved by a custom 3D printed Ackermann-style steering mechanism actuated by a Miuzei micro servo motor controlled directly by the Zumo. The CAVs are equipped with a Pi Camera, accelerometer, gyroscope, and compass to collect experimental data and reduce the overall reliance on VICON. With this hardware configuration, the CAV is able to run and collect experimental data at 20 Hz for approximately 2 hours.

Coordination of the CAVs within the IDS³C is achieved using a multi-level control framework spanning the mainframe computer and the individual CAVs in an experiment. At the start of the experiment, each CAV sets its temporal baseline from which it measures all later
times; this avoids the problem of synchronizing CAV clocks, as all information is calculated relative to the experiment start time. High-level trajectory generation is achieved by a multithreaded C++ program on the mainframe computer, which allocates a thread to each CAV in the experiment; this replicates the decentralized structure of traffic coordination problems. The control strategy for each CAV is defined for each zone in the experiment (for example, optimal control in the control zone and the intelligent driver model everywhere else). The mainframe computer transmits the current state of the CAV as well as reference trajectory data to each CAV through a UDP socket.

Lane and reference trajectory tracking are accomplished onboard each CAV in a purely distributed manner. Using state and trajectory data from the mainframe, each CAV updates its lateral, heading, and distance error at a rate of 50 Hz. Each CAV can operate in one of three modes depending on its control strategy. In trajectory tracking mode, i.e., optimal planning, the CAV uses a modified Stanley controller [121] for lane tracking while a feedforward-feedback PID controller [122] tracks the desired longitudinal speed and position. In lane tracking mode, that is reactive control, the modified Stanley controller is used to control the steering angle, while the desired speed is transmitted directly to the CAV. Finally, in human driver mode [120], the CAV directly receives a steering angle and wheel speed command from a driving base station and transmits back a live feed from the onboard Pi camera.

We designed IDS³C with the capacity to experimentally validate a wide variety of urban traffic scenarios. This includes eco-routing, mixed traffic [7], ride sharing [123], and air-ground coordination [124]. In several recent efforts, we have used IDS³C to implement and validate control algorithms for coordinating CAVs at traffic scenarios, such as merging roadways [68], roundabouts [77], intersections [43], adjacent intersections [56]–[58], and corridors [125]. We have also used IDS³C to generate control actions from neural networks [79], [80] and handle the stochasticity that arises in physical systems [126]. Recently, we introduced a Unity-based virtual simulation environment for emerging mobility systems, called Information and Decision Science Lab’s Scaled Smart Digital City (IDS 3D City), intended to operate alongside its physical peer, IDS³C, and its existing control framework. For a brief summary of IDS 3D City see “The Information and Decision Science Lab’s Scaled Smart Digital City (IDS 3D City)”, while for the detailed discussion see [127].

Our most significant effort in IDS³C has been the optimal coordination of CAVs at transportation bottlenecks. Next, we present a decentralized optimal control framework that can be used to solve the coordination problem of CAVs.
Optimal Coordination

Our main area of research with the IDS^3C has been the development and implementation of decentralized control algorithms aimed at coordinating CAVs at traffic scenarios, such as merging at roadways and roundabouts, cruising in congested traffic, passing through speed reduction zones, and lane-merging or passing maneuvers. By coordinating CAVs in such scenarios implies two immediate benefits. First, safety can be explicitly guaranteed by imposing constraints on each vehicle, including rear-end safety, maximum safe turning speed, and lateral collision avoidance. Second, the trade-off between fuel consumption and travel time can be explicitly included in the analysis. These are, in general, competing objectives. A vehicle that minimizes travel time will generally brake and accelerate at the maximum limits, leading to significant fuel consumption. In contrast, when focusing only on minimizing fuel consumption, a vehicle tends to coast at a constant speed to minimize transient energy operation and conserve fuel. Thus, finding an appropriate trade-off between fuel consumption and travel time is a critical consideration for the coordination of CAVs.

Several research efforts in the literature have considered a two-level optimization framework for coordination of CAVs at traffic bottlenecks consisting of a throughput maximization (upper-level) and an energy minimization problem (lower-level). For each CAV with a given origin and desired destination, the solution of the upper-level problem yields the optimal travel time for the CAV to exit the control zone. The solution of the low-level problem yields, for each CAV, the optimal control input (acceleration/deceleration) to achieve the solution of the upper-level problem while minimizing energy consumption subject to the state, control, and safety constraints. There have been several approaches in the literature to solve the upper-level optimization problem, including the first-in-first-out (FIFO) queuing policy [35], [39], [63], centralized control approaches [33], [34], [37], [50], [51], [53], and scheduling approaches [36], [57]. Given the solution of the upper-level optimization problem, the low-level constrained optimal control problem determines the optimal control input of each CAV. The details of the low-level problem can be found in [35], [48], [128]. Solving a constrained optimal control problem leads to a system of non-linear equations that are often infeasible to solve in real time. For more details on the constrained optimal control and the associated technical challenges, see “The Challenge with Constrained Optimal Control.” We have proposed an alternative approach to avoid the complexities associated with real-time constrained optimal control. Our proposed optimization framework for each CAV consists of a single optimization level aimed at both minimizing energy consumption and improving traffic throughput [48]. Next, we formally describe this framework.
Problem Formulation

Although the framework presented here can be applied to any traffic scenario, e.g., merging at roadways and roundabouts, cruising in congested traffic, passing through speed reduction zones, and lane-merging or passing maneuvers, we use an intersection as a reference to provide the fundamental ideas. This is because an intersection has unique features which makes it technically more challenging compared to other traffic scenarios. However, our analysis can be applied to other traffic scenarios too.

We consider CAVs at a 100% penetration rate crossing a signal-free intersection (Fig. 4). The region at the center of the intersection, called merging zone, is the area of potential lateral collision of CAVs. The intersection has a control zone inside of which the CAVs plan their time trajectories (the time that are in a given position inside the control zone) by communicating with each other and with a coordinator, i.e., a roadside unit that stores the planned time trajectories of each CAV as they pass through the control zone. The distance from the entry of the control zone until the entry of the merging zone is $S_c$, and, although it is not restrictive, we consider to be the same for all entry points of the control zone. We also consider the merging zone to be a square of side $S_m$ (Fig. 4). Note that $S_c$ could be in the order of hundreds of meters depending on the CAVs’ communication range capability, while $S_m$ is the length of a typical intersection. The CAVs crossing the intersection can also make a right turn of radius $R_r$, or a left turn of radius $R_l$ (Fig. 4). The aforementioned values of the intersection’s geometry are not restrictive in our modeling framework, and are used only to determine the total distance traveled by each CAV inside the control zone.

In our problem formulation, we assume that each CAV can communicate with other CAVs and the coordinator without any errors or delays. This assumption is relatively straightforward to relax as long as the noise in the communication, measurements, and/or delays is bounded. We also assume that upon entering the control zone, the initial state of each CAV is feasible, that is, none of the speed or safety constraints are violated. This is a reasonable assumption since CAVs are automated; therefore, there is no compelling reason for them to violate any of the constraints by the time they enter the control zone.

We denote the set of CAVs in the control zone by the set $\mathcal{N}(t) = \{1, \ldots, N(t)\}$, where $N(t) \in \mathbb{N}$ is the total number of CAVs at time $t \in \mathbb{R}_{\geq 0}$. In our framework, for each CAV $i \in \mathcal{N}(t)$, we seek to jointly minimize energy consumption and improve traffic throughput. Upon entering the control zone, CAV $i$ communicates with the coordinator and receives the time trajectory of all CAVs $j \in \mathcal{N}(t) \setminus \{i\}$. Next, CAV $i$ computes the time $t^f_i$ that must exit the control zone which guarantees that its corresponding energy optimal time trajectory does
not activate any of the state, control, and safety constraints. This trajectory is communicated back to the coordinator so as the subsequent CAVs plan their trajectories. If two or more CAVs enter the control zone simultaneously, then the coordinator arbitrarily determines the sequence in which they receive information to plan their trajectories. Ongoing research explores the optimal sequence of decision making for CAVs to plan their trajectories when they enter the control zone simultaneously.

The implications of this framework are that the CAVs do not have to come to a full stop at the intersection, thereby conserving momentum and fuel while also improving travel time. By enforcing the unconstrained energy optimal time trajectory that guarantees that none of the state, control, and safety constraints becomes active, we avoid the challenges associated with the real-time implementation of the constrained optimal control solution.

In our analysis, we consider that each CAV $i \in \mathcal{N}(t)$ is governed by the following dynamics,

\[
\begin{align*}
\dot{p}_i(t) &= v_i(t), \\
\dot{v}_i(t) &= u_i(t), \\
\dot{s}_i(t) &= v_k(t) - v_i(t), \quad t \in [t^0_i, t^f_i],
\end{align*}
\]

where $t^0_i$ and $t^f_i$ correspond to the times that CAV $i$ enters and exits the control zone, respectively; $p_i(t) \in \mathcal{P}_i$ is the position of each CAV $i$ from the entry until the exit of the control zone at $t$; $v_i(t) \in \mathcal{V}_i$ and $u_i(t) \in \mathcal{U}_i$ are the speed and acceleration/deceleration (control input), respectively, of each CAV $i$ inside the control zone at $t$; $s_i(t) \in \mathcal{S}_i$ denotes the distance of CAV $i$ from CAV $k$ which is physically located ahead of $i$, and $v_k(t)$ is the speed of CAV $k$. The sets $\mathcal{P}_i, \mathcal{V}_i, \mathcal{U}_i$, and $\mathcal{S}_i, i \in \mathcal{N}(t)$, are complete and totally bounded subsets of $\mathbb{R}$.

In our framework, we impose the following constraints to ensure the CAV’s control input and state remain within an admissible range,

\[
\begin{align*}
0 \leq u_{i,\text{min}} &\leq u_i(t) \leq u_{i,\text{max}}, \\
0 &< v_{\text{min}} \leq v_i(t) \leq v_{\text{max}},
\end{align*}
\]

for all $t \in [t^0_i, t^f_i]$, where $u_{i,\text{min}}, u_{i,\text{max}}$ are the minimum and maximum control inputs and $v_{\text{min}}, v_{\text{max}}$ are the minimum and maximum speed limit, respectively.

To ensure that no rear-end collisions occur between two CAVs traveling in the same lane, we impose the rear-end safety constraint as

\[
s_i(t) = p_k(t) - \lambda_k - p_i(t) \geq \delta_i(t) = \gamma + \rho_i v_i(t),
\]
where $\lambda_k$ is the length of CAV $k$, $\gamma$ is the standstill distance, and $\rho_i$ is the minimum time headway that CAV $i$ wishes to maintain with preceding CAV $k$.

Finally, let $j \in \mathcal{N}(t) \setminus \{i\}$ correspond to another CAV that has already entered the control zone and may have a lateral collision with CAV $i$, for example, suppose CAV $i$ travels north-south and CAV $j$ that travels east-west (see Fig. 4). We label all such conflict points by the finite set $\mathcal{O} \subset \mathbb{N}$, which is entirely determined by the geometry of the road. By $p^n_i$ and $p^n_j$ we denote the position of the conflict point $n \in \mathcal{O}$ along CAV $i$ and $j$’s path, respectively. Since we do not enforce a FIFO queuing policy, CAV $i$ can either cross this conflict point before, or after CAV $j$. For the first case in which CAV $i$ reaches the conflict point after CAV $j$, we have

$$p^n_i - p_i(t) \geq \delta_i(t), \quad \text{for all } \ t \in [t^n_i, t^n_j],$$

where $t^n_j$ is the known time that CAV $j$ reaches at conflict point $n$, that is, position $p^n_j$. Likewise, for the second case in which CAV $i$ reaches the conflict point $n$ before CAV $j$, we have

$$p^n_j - p_j(t) \geq \delta_k(t), \quad \text{for all } \ t \in [t^n_j, t^n_i],$$

where $t^n_i$ is determined by the trajectory planned by CAV $i$.

Since $0 < v_{\min} \leq v_i(t), the position $p_i(t)$ is a strictly increasing function. Thus, the inverse $t_i(\cdot) = p_i^{-1}(\cdot)$ exists and it is called the time trajectory of CAV $i$, hence we have $t^n_i = p_i^{-1}(p^n_i)$. The closed-form solution of the inverse function is derived in [48]. To guarantee lateral safety between CAV $i$ and CAV $j$ at a conflict point $n$, either (5) or (6) must be satisfied. Therefore, we impose the following lateral safety constraint on CAV $i$,

$$\min \left\{ \max_{t \in [t^n_i, t^n_j]} \{\delta_i(t) + p_i(t) - p^n_i\}, \max_{t \in [t^n_j, t^n_i]} \{\delta_i(t) + p_j(t) - p^n_j\} \right\} \leq 0. \quad (7)$$

When CAV $i \in \mathcal{N}(t)$ enters the control zone it must determine the exit time $t^n_i$ such that the resulting time trajectory does not activate any of (1) - (4) and (7). We start our exposition by considering the unconstrained solution of CAV $i$, namely

$$u_i(t) = 6a_i t + 2b_i,$$

$$v_i(t) = 3a_i t^2 + 2b_i t + c_i,$$

$$p_i(t) = a_i t^3 + b_i t^2 + c_i t + d_i,$$

where $a_i, b_i, c_i, d_i$ are constants of integration. CAV $i$ must also satisfy the boundary conditions

$$\left( p_i(t^n_i), v_i(t^n_i) \right) = \left( 0, v_i^0 \right),$$

$$\left( p_i(t^n_i), u_i(t^n_i) \right) = \left( S_i, 0 \right),$$

$$\left( p_i(t^n_i), v_i(t^n_i) \right) = \left( 0, v_i^0 \right).$$

9
where $S_i$ is the known length of CAV $i$’s path in the control zone. For more details in deriving the unconstrained energy optimal solution and the accompanying boundary conditions, see “Unconstrained Optimal Control.”

There are five unknown variables that determine the optimal time trajectory of CAV $i$, four constants of integration from (8), and the unknown exit time $t_i^f$. Without loss of generality, letting $t_i^0 = 0$ implies that

$$p_i(t_i^0) = d_i = 0, \quad v_i(t_i^0) = c_i = v_i^0, \quad (11)$$

while $u_i(t_i^f) = 0$ yields

$$a_i = \frac{-b_i}{3t_i^f}, \quad (13)$$

and $p_i(t_i^f) = S_i$ gives

$$b_i = \frac{3(S_i - v_i^0t_i^f)}{2(t_i^f)^2}. \quad (14)$$

Furthermore, $t_i^f$ takes a value from a compact set, $[t_i^l, t_i^r]$. See “Bounds for Feasible Exit Time” for more details on the derivation of this compact set based on the speed and control input constraints. This leads to the following optimization problem

$$\min_{t_i^f \in [t_i^l, t_i^r]} t_i^f \quad (15)$$

subject to:

- rear-end safety (4),
- lateral safety (7), and
- boundary conditions (11) – (14).

Substituting the optimal value of $t_i^f$ from (15) into (13) and (14) yields $a_i$ and $b_i$, which, along with $c_i$ and $d_i$, determine the optimal unconstrained trajectory (8). The value of $t_i^f$ guarantees that an unconstrained trajectory satisfies all the state, control, and safety constraints and the boundary conditions. In practice, for each CAV $j \in \mathcal{N}(t)$ the coordinator stores the optimal exit time $t_j^f$ and the corresponding coefficients $a_j, b_j, c_j, d_j$. It has been shown [48] that there is no duality gap in (15), thus the solution can be derived in real time.

Each time a CAV $i$ enters the control zone, then it first requests the time trajectories of all other CAVs already in the control zone to find its optimal exit time $t_i^f$ from (15). Then, CAV
transmits its four coefficients and $t_i^f$ back to the coordinator. Next, we present a brief tutorial on the experimentation procedure of IDS$^3$C by applying our control framework to a multi-lane roundabout scenario.

**Tutorial: Roundabout Case Study**

To illustrate the implementation of our proposed framework in IDS$^3$C, we have performed experiments in the multi-lane roundabout shown in Fig. 5 using three CAVs per path. Figure 5 shows three paths with three conflict points that have a potential for lateral collisions, which we denote as lateral nodes. The length of the control zone for paths are 5.3 m, 5.8 m, and 3.8 m (132.5 m, 145 m, 95 m scaled), respectively. The CAVs initially follow the intelligent driver model controller [129], and switch to our proposed optimal control framework when entering the control zone. Each CAV then determines its optimal trajectory by solving (15) numerically. The CAV follows this optimal trajectory through the control zone. Upon exiting the control zone, it reverts to an intelligent driver model and loops back around toward the control zone entrance.

To facilitate data collection and fast analysis, we define a finite state machine for the experiment consisting of three states: **Starting**, **Running**, and **Waiting**. In the Starting state of the experiment, CAVs are released upstream of the considered traffic scenario (that is, roundabout) and drive towards the control zone. Once the first CAV enters the control zone, we log the current time and the state machine transitions to Running. In the Running state, we track the planned exit time and trajectory coefficients of each CAV that enters the control zone. Simultaneously, we log the state information of each CAV inside the control zone with a corresponding timestamp. Finally, when a CAV exits the control zone, we log the actual exit time, and, if it is the last CAV to exit the experiment, the state machine transitions to Waiting. Once all of the CAVs have exited the experiment and sufficient number of CAVs have circled around to their initial positions, a user-specified amount of time counts down before the CAVs are released to start another experiment.

To automate the testing procedure, we developed a queue sign, which is analogous to a ramp metering signal. We use queue signs to precisely time the release of CAVs on each path during an experiment. The number of CAVs released by each queue sign is determined by user input, and it can be fixed or drawn from a discrete uniform distribution for each experiment. Once the required number of CAVs have formed a queue behind a queue sign, the experiment state machine is notified that it is ready for the experiment to begin. A schematic showing the behavior of the experiment and queue signs, as well as their connections, is shown in Fig. 6. Once all queue signs have a sufficient number of CAVs accumulated, the state machine can transition from Waiting to Starting, wherein it will notify all queue signs to release CAVs. Each
queue sign has a user-specified initial delay, which allows the arrival time of CAV to the control zone to be precisely selected. The initial delay can either be a fixed number or drawn from a user-supplied uniform distribution. Similarly, the time delay between the release of each CAV from a particular queue sign can be set to a fixed number or a uniform distribution. Once the queue sign has released the appropriate number of CAVs it waits for the experiment to complete, and at that point, it selects the number of CAVs for the next experiment. This allows for little to no delay between experiments if the number of CAVs physically present in the testbed is greater than the number of CAVs required for any one experiment—potentially doubling the rate of data collection.

For the roundabout experiments we used the following parameters: $v_{\text{max}} = 0.5 \text{ m/s (28 mph full scale)}$, $v_{\text{min}} = 0.15 \text{ m/s (8.4 mph full scale)}$, $u_{\text{max}} = 0.45 \text{ m/s}^2 (11 \text{ m/s}^2 \text{ full scale})$, and $u_{\text{min}} = -u_{\text{max}}$. To ensure safety, we select a time gap of 1.0 s and a minimum standstill distance of 0.07 m (approx. 1 car length). To quantify the effect of noise and disturbances acting on the system, we repeated the experiment five times. Furthermore, we precisely timed the release of the CAVs into the roundabout such that lateral collisions would occur without intervention. Supplementary videos of the roundabout experiment can be found at https://sites.google.com/view/ud-ids-lab/csm.

Minimum and average speed and travel time results for the five experiments are summarized in Table 1. Note that the minimum speed of all CAVs is 0.12 m/s (7 mph at full scale) across all optimal control experiments, which demonstrates that stop and go driving has been completely eliminated. Additionally, the average speed of CAVs is 0.42 m/s (24 mph at full scale), which implies that most CAVs travel near $v_{\text{max}} = 0.5 \text{ m/s}$. The error between desired and actual exit time varies between $2 - 4\%$, which comes from the tracking error in the CAV’s low-level controller. The exit time data for each CAV is visualized in Fig. 7, where the grey bars represent the feasible space of $t_f^i$, the wide black bars correspond with the planned value of $t_f^i$, and the thin red bars show the actual value of $t_f^i$ achieved by each CAV. This effect of tracking error is visible in Table 1, where the minimum achieved speed is slightly lower than the minimum speed imposed on the reference trajectory.

The position trajectory of an ego-CAV following path 2 is given in Fig. 8. The ego-CAV’s position is denoted by the dashed red line, while the positions of two other CAVs are represented by dotted black lines. The lateral collision constraints are denoted by vertical black bars, and the rear-end safety constraint is the hashed region on the graph. There are two other CAVs shown; one is on path 3 and merges in front of the ego-CAV at node 1 (see Fig. 5) and a second CAV leads the ego-CAV on path 2. Figure 8 demonstrates that, in a physical testbed, noise and disturbances play a significant role in the actual trajectory of the CAVs. The trajectory
generated by the ego-vehicle did not lead to a physical collision, but it does violate the rear-end safety constraint by a car length. However, at that speed, the rear-end safety constraint required a three-car length gap, so a robust control formulation of (15) could likely guarantee collision avoidance. This can also be seen in the lateral collision avoidance constraint in Fig. 8, where a CAV later in the sequence crosses node 3 in a way that violates the time headway constraint (again, without leading to an actual collision).

Finally, the average, maximum, and minimum speed for each CAV across all experiments are given in Fig. 9. Each figure corresponds to a single path (see Fig. 5) and considers 15 CAVs (3 CAVs per path over five experiments). The CAVs’ positions are taken directly from VICON and numerically derived using a first-order method. From Fig. 9, the average speed for CAVs on each path is very close to constant. Path 1 shows the most variance, which is due to the distance between collision nodes 2 and 3 on path 1 (see Fig. 5). In order for a CAV $i \in \mathcal{N}(t)$ that is traveling along path 1 to reduce its arrival time at node 2, it must make a proportionally larger reduction in the value of $t_i^f$. This is a side effect of enforcing the unconstrained trajectory on each CAV over the entire control zone. Additionally, the entrance to the control zone along path 3 follows a sharp right turn. This results in relatively lower average speed in Fig. 9(c), as the dynamics of the CAVs reduce their speed while turning, causing them to enter the control zone at a lower initial speed. Finally, there are instances in Fig. 9(b) where the maximum vehicle speed surpasses the speed limit. This is a result of stochasticity in the vehicle dynamics and sensing equipment, as well as environmental disturbances, on our deterministic controller. This analysis motivates an enhanced trajectory generation framework that accounts for noise, disturbances, communication delay, and low-level tracking errors in the CAVs [58], [126].

Next, we present a high-level overview and analysis of our framework for a full transportation corridor in IDS$^3$C.

**Transportation Corridor**

To demonstrate our optimal coordination framework for a transportation corridor in IDS$^3$C, we use a fleet of 15 CAVs. The corridor is shown in Fig. 10, where 3 ego-CAVs are released along the red path (starting in the northeast of the IDS$^3$C) and travel through a roundabout, intersection, and merging roadway. At each traffic scenario, we release 3 additional CAVs per path (as indicated in Fig. 10) to create congestion. The traffic scenarios were specifically selected so that upon entering the control zone, each CAV would have approximately 3 m (75 m scaled) to adjust their speed before reaching a conflict point. This also allowed us to consider each coordinator and control zone independently, as the control zone length was sufficiently long to neglect the influence of another upstream control zone.
In the baseline case, we replaced the roundabout and merging zone coordinators with yield signs. In both scenarios, the merging vehicles yield to any vehicle within 0.4 m of the merging zone (10 m scaled, approx. 4 car lengths). To manage the intersection, we implemented a four-way stop with a FIFO queue, that is, whenever a vehicle enters a line segment leading up to the intersection, it is added to the queue. When the merging zone contains no vehicles, if the front vehicle has come to a complete stop, it is removed from the queue and allowed to pass through the merging zone. We have taken this approach to the intersection in order to avoid any bias that may get introduced into our results by the timing of a traffic light.

Finally, to ensure a fair comparison, we set the speed limit for the entire city to 0.5 m/s (approximately 30 mph scaled) in both tests. In our framework, we impose a maximum speed of 0.3 m/s (approx. 15 mph scaled) outside of the control zone. This ensures that the vehicles enter the control zone at a speed lower than $v_{\text{max}}$, and gives them the opportunity to accelerate through the control zone. Figure 11 shows that despite the apparent advantage of the baseline case’s higher speed limit, the ego-CAV maintains a higher average speed, and that stop-and-go driving has been completely eliminated in the optimal control case. Furthermore, Fig. 12 shows that the ego-CAVs do not activate any safety constraints throughout the experiment. Additional videos and figures of the experiment can be found at https://sites.google.com/view/ud-ids-lab/csm.

**Conclusion**

In this article, we introduced the IDS$^3$C, a 1:25 scale testbed capable of safely validating control approaches for transportation applications and investigating the implications of emerging mobility systems. We presented the hardware and software architecture of the IDS$^3$C, and provided an overview of our optimal framework that can aim CAVs at coordinating in traffic scenarios such as merging at roadways and roundabouts, cruising in congested traffic, passing through speed reduction zones, and lane-merging or passing maneuvers. We demonstrated the effectiveness of our framework in a multi-lane roundabout using 9 CAVs, and showed that our approach can be implemented in real time. To demonstrate the scalability of our optimal control framework in IDS$^3$C, we used a fleet of 15 CAVs to optimize the flow of traffic along a transportation corridor consisting of a roundabout, an intersection, and a merging roadway. We compared the results of our framework with a baseline scenario that used the intelligent driver model and conventional signage to avoid collisions. We showed that, under our optimal coordination framework, stop-and-go driving is completely eliminated and collisions are avoided. Ongoing research considers uncertainty in the framework originated from the vehicle-level control [58], [126], the effects of errors and delays in vehicle-to-vehicle and vehicle-to-
infrastructure communication, and the impact of mixed traffic coordination [22].

The existence of data and shared information in CAVs is associated with significant technical challenges and gives rise to a new level of complexity in modeling and control. It is expected that CAVs will gradually penetrate the market, interact with human-driven vehicles, and contend with vehicle-to-vehicle and vehicle-to-infrastructure communication limitations, e.g., bandwidth, dropouts, errors and/or delays. However, different penetration rates of CAVs can significantly alter transportation efficiency and safety. As we move to increasingly diverse mobility systems with different penetration rates of CAVs, new approaches are needed to optimize the impact on system behavior of the interplay between CAVs and human-driven vehicles at different traffic scenarios. Scaled testbeds such as IDS$^3$C can help us develop control algorithms and prove new concepts in a safe, controlled, and cost-efficient environment. Using IDS$^3$C we have been able to (a) train graduate students by exposing them to a balanced mix of theory and practice, (b) integrate the research outcomes into existing courses, (c) involve undergraduate students in research, (d) create interactive educational demos, and (e) reach out to high-school students. IDS$^3$C has been an excellent catalyst for motivating interest in undergraduate and high-school students in science, technology, engineering, and mathematics.

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TABLE 1: Minimum and average speed and travel time results for the 5 experiments. Root mean square error (RMSE) is the error between desired and actual exit time from the control zone, calculated over each CAV.

| Experiment | $v_{\text{min}}$ [m/s] | $v_{\text{avg}}$ [m/s] | Travel Time RMSE |
|------------|------------------------|-------------------------|------------------|
| 1          | 0.16                   | 0.41                    | 2.71 %           |
| 2          | 0.27                   | 0.45                    | 1.54 %           |
| 3          | 0.18                   | 0.41                    | 4.03 %           |
| 4          | 0.12                   | 0.43                    | 1.92 %           |
| 5          | 0.21                   | 0.42                    | 1.38 %           |
Figure 1: A view of the IDS Lab’s Scaled Smart City and mainframe control station.
Figure 2: Diagram of the high, medium, and low-level control loops. This schematic shows the high-level (mainframe), medium-level (Raspberry Pi) and low-level (Zumo) control loops in IDS$^3$C and communication structure for each CAV.
Figure 3: Connected and automated vehicle used in the IDS$^3$C with a decorative shell attached.
Figure 4: A signal-free intersection with connected and automated vehicles.
Figure 5: A schematic of the roundabout scenario. The highlighted control zone continues upstream from the roundabout.
Figure 6: State machine diagrams of the experiment and queue sign. The starting states for experiment and queue sign state machines are colored green. Solid lines correspond to state transitions, and hollow lines correspond to communication between state machines.
Figure 7: Planned and achieved exit time for each vehicle over all experiments. The grey bars shows the range of admissible $t_i^f$ from the state and control constraints. Every 9 vehicles corresponds to a single experiment; they are sorted in ascending order by departure time from the control zone.
Figure 8: Position trajectory for the third vehicle entering from path 2 in the 5th experiment. The lateral constraints are shown as vertical lines, and the rear-end safety constraint is the hashed region.
Figure 9: Speed range and average for all CAVs on (a) path 1, (b) path 2, and (c) path 3 across all experiments in the multi-lane roundabout.
Figure 10: Corridor experiment where the ego-CAVs (red path) must navigate a roundabout, intersection, and merging roadway. The paths are only colored where they pass through a control zone, and the segments belonging to the same path have a shared color.
Figure 11: Speed vs position graph for the front ego vehicles in the optimal control and baseline cases. Blue highlighted areas are within each of the control zones in the optimal case, and the vertical dashed lines correspond to the location of stop and yield signs in the baseline case.
Figure 12: Time vs position graph for the ego CAVs in the optimal control case. Solid lines correspond to the CAV trajectories, dashed lines correspond to CAVs that merge onto the ego-path, and orange boxes correspond to time intervals when a lateral conflict point is occupied by another CAV.
Emerging mobility systems, e.g., connected and automated vehicles (CAVs), shared mobility, provide the most intriguing opportunity for enabling users to better monitor transportation network conditions and make better decisions for improving safety and transportation efficiency. However, before connectivity and automation are deployed en masse, a thorough evaluation of CAVs is required—ranging from numerical simulation to real-world public roads. In particular, assessment of CAVs performance in scaled testbeds has recently gained popularity as a robust approach that ensures absolute safety, complete control of the test-environment variables, and quick, repeatable experiments. This article introduces the Information and Decision Science Lab’s Scaled Smart City (IDS$^3$C), a 1:25 scaled testbed that is capable of replicating different real-world urban traffic scenarios. IDS$^3$C was designed with the capabilities to investigate the effect of emerging mobility systems, such as CAVs, electric vehicles, and shared mobility, on energy consumption and transportation efficiency. In our overview of the IDS$^3$C, we provide our framework that allows to optimally coordinate CAVs at traffic scenarios such as merging at roadways and roundabouts, cruising in congested traffic, passing through speed reduction zones, and lane-merging or passing maneuvers. As a tutorial, we present the application of our control framework to a multi-lane roundabout scenario in the IDS$^3$C, and demonstrate the scalability of our testbed in a fleet of 15 CAVs cruising in a transportation corridor.
Sidebar: Information and Decision Science Lab’s Scaled Smart Digital City (IDS 3D City)

The Information and Decision Science Laboratory’s Scaled Smart Digital City (IDS 3D City) is a digital environment corresponding to a full-scale digital recreation of the Information and Decision Science Lab’s Scaled Smart City (IDS$^3$C) in Unity. This digital replica can communicate with the central mainframe computer using the user datagram protocol (UDP), allowing users to debug and evaluate their algorithm’s performance before running a physical experiment in IDS$^3$C. The main benefit of the IDS 3D City is that it enables users to rapidly iterate the design of their control algorithms and experiment parameters before deploying it to the physical city. The IDS 3D City exploits the dynamics of Microsoft AirSim to control the simulated vehicles, which provides further evidence that our proposed control algorithms are applicable to full-scale vehicles.

We have designed the IDS 3D City to integrate seamlessly with the current control code used in the physical testbed. To mimic the behavior of the physical city in our simulation, we receive reference trajectory data over a local UDP socket that contains the desired state of each CAV. After applying the control inputs to each CAV, their position and orientation in the digital environment are broadcast through a ROS to a node designed to mimic our VICON motion capture system. This information is accessed by the mainframe computer, which it uses to execute the control algorithms and sends new commands to the simulation. This setup also enables us to operate some vehicles virtually while others are physically present in the city, and reacting to the virtual vehicles in real time. A schematic of our communication structure is presented in Fig. S1. The end result is that we can seamlessly switch between running any individual car in the physical or virtual environment with minimal changes to our input files, and we have the capability to perform augmented reality experiments with a mixture of physical and virtual vehicles. The IDS 3D City is also capable of replaying experimental data, and we also allow users to directly control a vehicle through joystick input. We are also capable of streaming a live feed of the virtual cameras that are attached to each simulated vehicle.
Sidebar: The Challenge with Constrained Optimal Control

The standard methodology to solve a continuous-time constrained optimal control problem is to employ Hamiltonian analysis with interior point state and/or control constraints [S1]. Namely, we first start with the unconstrained arc and derive the solution of the optimal control problem without considering any of the state or control constraints in (2), (3) and (4). If the unconstrained solution violates any of the state or control constraints, then the unconstrained arc is pieced together with the arc corresponding to the violated constraint. The two arcs yield a set of algebraic equations which are solved simultaneously using the boundary conditions and optimality conditions between the arcs. If the resulting solution, which includes the determination of the optimal switching time from one arc to the next one, violates another constraint, then the last two arcs are pieced together with the arc corresponding to the new violated constraint, and we re-solve the problem with the three arcs pieced together. The three arcs will yield a new set of algebraic equations that need to be solved, and this process is repeated until the solution does not violate any constraints. This iterative process can be computationally intensive for several reasons [S2]. In particular, the Euler-Lagrange equations are numerically unstable for non-conservative systems leading to significant numerical challenges. Second, the number of active constraints is not known a priori, and it may require a significant number of iterations to compute. Third, the boundary conditions and recursive equations may be implicit functions that do not have a closed-form analytical solution.

Excluding cases with terminal speed and relaxing safety constraints, in recent work [S3], [S4], we have introduced a condition-based solution framework for the optimal coordination of CAVs, which leads to a closed-form analytical solution without this iterative procedure. In this framework, we mathematically characterized the activation cases of different state and control constraint combinations, and provided a set of a priori conditions under which different constraint combinations can become active. Although this approach alleviates the computational complexity of the constrained optimal control in the coordination problem to some extent, the aforementioned iterative procedure is still required for cases when safety and terminal speed constraints are included.

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Sidebar: Unconstrained Optimal Control and Boundary Conditions

Let \( t^f_i \) be the specified exit time of CAV \( i \) from the control zone. To minimize the energy consumption of \( i \) inside the control zone, we minimize transient engine operation through the \( L^2 \)-norm of the control input \( u_i(t) \) over the interval \([t^0_i, t^f_i] \); this is known to have direct benefit in fuel consumption and emission in conventional vehicles [S5], [S6]. Namely, CAV \( i \) minimizes the following cost function,

\[
J_i(u_i(t), t^f_i) = \frac{1}{2} \int_{t^0_i}^{t^f_i} u_i(t)^2 \, dt. \tag{S1}
\]

For each CAV \( i \) in the control zone, the unconstrained Hamiltonian is

\[
H_i(t, p_i(t), v_i(t), u_i(t)) = \frac{1}{2} u_i(t)^2 + \lambda^p_i p_i(t) + \lambda^v_i v_i(t), \tag{S2}
\]

where \( \lambda^p_i \) and \( \lambda^v_i \) are costates corresponding to the position and speed of the CAV, respectively. The Euler-Lagrange optimality equations become

\[
\dot{\lambda}^p_i = -\frac{\partial H_i}{\partial p_i} = 0, \tag{S3}
\]

\[
\dot{\lambda}^v_i = -\frac{\partial H_i}{\partial v_i} = -\lambda^p_i, \tag{S4}
\]

\[
\frac{\partial H_i}{\partial u_i} = u_i + \lambda^v_i = 0. \tag{S5}
\]

Since the speed of CAV \( i \) is not specified at the terminal time \( t^f_i \), we have [S7]

\[
\lambda^v_i(t^f_i) = 0. \tag{S6}
\]

Applying the Euler-Lagrange optimality conditions (S3)-(S5) to the Hamiltonian (S2) yields

\[
u_i^*(t) = -\lambda^v_i \cdot \dot{a} = a'_i t + b'_i, \tag{S7}
\]

where \( a'_i \) and \( b'_i \) are constants of integration. By integrating the control input, we can find the optimal position and speed trajectories as

\[
p_i(t) = \frac{1}{6} a'_i t^3 + \frac{1}{2} b'_i t^2 + c'_i t + d'_i, \tag{S7}
\]

\[
v_i(t) = \frac{1}{2} a'_i t^2 + b'_i t + c'_i, \tag{S8}
\]

where \( a'_i, b'_i, c'_i, d'_i \) are constants of integration, which are found by substituting the boundary conditions. The boundary conditions for any CAV \( i \) are,

\[
p_i(t^0_i) = p^0_i, \quad v_i(t^0_i) = v^0_i, \tag{S9}
\]

\[
p_i(t^f_i) = p^f_i, \quad u_i(t^f_i) = 0, \tag{S10}
\]

43
where $p_i$ is known at $t_i^0$ and $t_i^f$ by the geometry of the road, and $v_i^0$ is the speed at which the CAV enters the control zone. The final boundary condition, $u_i(t_i^f) = 0$, arises from substituting (S6) into (S5) at $t_i^f$, that is

$$u_i(t_i^f) + \lambda_i^v(t_i^f) = 0,$$

which implies $u_i(t_i^f) = 0$.

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Sidebar: Derivation of Bounds for Feasible Exit Time

The unconstrained optimal trajectory of CAV $i \in \mathcal{N}(t)$ takes the form
\[
\begin{align*}
    u_i(t) &= 6a_i t + 2b_i, \quad (S12) \\
    v_i(t) &= 3a_i t^2 + 2b_i t + c_i, \quad (S13) \\
    p_i(t) &= a_i t^3 + b_i t^2 + c_i t + d_i, \quad (S14)
\end{align*}
\]
where $a_i, b_i, c_i, d_i$ are constants of integration, which are found by substituting the boundary conditions. We derive the upper and lower bounds on the exit time of the control zone for a CAV $i \in \mathcal{N}(t)$ using the speed and control constraints by exploiting two properties of the optimal trajectory. As the optimal control input is linear and satisfies $u_i(t_f^i) = 0$, it must be zero, strictly decreasing, or strictly increasing. In all three cases $u_i(t)$ achieves its extreme at $t_0^i$, and therefore satisfying $u_{\min} \leq u_i(t_0^i) \leq u_{\max}$ is necessary and sufficient condition to guarantee constraint satisfaction. Likewise, the speed of CAV $i$ starts at $v_i(t_0^i) = v_0^i$ and must be constant, strictly increasing, or strictly decreasing inside the control zone. In all three cases $v_i(t)$ takes its extreme value at $t_f^i$, and thus satisfying $v_{\min} \leq v_i(t_f^i) \leq v_{\max}$ is necessary and sufficient condition to guarantee constraint satisfaction.

Next, without loss of generality, let $t_0^i = 0$ and $p_0^i = 0$. This implies that $p_i(t_0^i) = d_i = 0$ and $v_i(t_0^i) = c_i = v_0^i$, while $u_i(t_f^i) = 0$ implies
\[
a_i = \frac{-b_i}{3t_f^i}, \quad (S15)
\]
and $p_i(t_f^i) = S_i$ yields
\[
b_i = \frac{3(S_i - v_0^i t_f^i)}{2(t_f^i)^2}. \quad (S16)
\]
In order to compute the lower bound on exit time of the control zone for CAV $i$, $t_f^i$, there are two cases to consider:

**Case 1:** CAV $i$ achieves its maximum control input at entry of the control zone, that is, $u_i(t_0^i) = u_{\max}$. In this case, evaluating (S12) at $t_0^i = 0$ yields
\[
u_i(t) = 2b_i = u_{\max}. \quad (S17)
\]
Substituting (S16) into (S17) and solving for $t_f^i$ yields the quadratic equation
\[
u_{\max} t_f^i + 3v_0^i t_f^i - 3S_i = 0, \quad (S18)
\]
which has two real roots with opposite signs, since $t_f^i_{1,2} = -\frac{3S_i}{u_{\max}} < 0$. Thus, $t_{i,u_{\max}}^f > 0$ is
\[
t_{i,u_{\max}}^f = \frac{\sqrt{9v_0^i - 12S_i u_{\max}^3} - 3v_0^i}{2u_{\max}}. \quad (S19)
\]
Case 2: CAV \( i \) achieves its maximum speed at the end of control zone, that is, \( v_i(t_i^f) = v_{\text{max}} \). For this case, by (S13), we have
\[
v_i(t_i^f) = 3a_i t_i^f + 2b_i t_i^f + v_i^0 = v_{\text{max}}.
\] (S20)
Substituting (S15) and (S16) into (S20) yields
\[
v_i(t_i^f) = 3 \left( \frac{-b_i}{3t_i^f} \right) t_i^f + 2b_i^0 t_i^f + v_i^0 = b_i t_i^f + v_i^0 = \frac{3(S_i - v_i^0 t_i^f)}{2t_i^f} + v_i^0 = v_{\text{max}},
\] (S21)
which simplifies to
\[
t_i^{f,max} = \frac{3S_i}{v_i^0 + 2v_{\text{max}}},
\] (S22)
Thus, our lower bound on \( t_i^f \) is given by
\[
t_i^f = \min \left\{ t_i^{f,\text{max}}, t_i^{f,\text{min}} \right\}.
\] (S23)
The upper bound for \( t_i^f \) can be derived following similar steps for the lower bound, and can be broken into two cases.

Case 1: CAV \( i \) achieves its minimum control input at the entry of the control zone, that is, \( u_i(t_i^0) = u_{\text{min}} \). This implies
\[
u_{\text{min}} t_i^f + 3v_i^0 t_i^f - 3S_i = 0,
\] (S24)
which has two positive roots, as \( t_{i,1}^{f} t_{i,2}^{f} = \frac{-3S_i}{u_{\text{min}}} > 0 \). Of these we select the smaller one,
\[
t_i^{f,\text{min}} = \sqrt{\frac{9v_i^0 - 12S_i u_{\text{min}} - 3v_i^0}{2u_{\text{min}}}},
\] (S25)
as the speed of the vehicle should be always greater than zero. Note that when \( 9v_i^0 + 12S_i u_{\text{min}} < 0 \) there is no real value of \( t_i^f \) which satisfies all of the boundary conditions simultaneously, and therefore the constraint \( u(t_i^0) = u_{\text{min}} \) can never become active if (S25) is complex. In that case, the lower bound must be given by Case 2.

Case 2: CAV \( i \) achieves its minimum speed at the entry of the control zone, that is, \( v_i(t_i^0) = v_{\text{min}} \). Evaluating (S13) at \( t_i^f \) yields
\[
v_i(t_i^f) = 3a_i t_i^f + 2b_i t_i^f + v_i^0 = v_{\text{min}}.
\] (S26)
Substituting (S15) and (S16) into (S26) yields
\[
v_i(t_i^f) = 3 \left( \frac{-b_i}{3t_i^f} \right) t_i^f + 2b_i^0 t_i^f + v_i^0 = b_i t_i^f + v_i^0 = \frac{3(S_i - v_i^0 t_i^f)}{2t_i^f} + v_i^0 = v_{\text{min}},
\] (S27)
which simplifies to

\[ t_{i,v_{\min}}^f = \frac{3S_i}{v_i^0 + 2v_{\min}}. \]  

(S28)

Thus, the upper bound on the exit time for CAV \( i \) is

\[
t_{i}^f = \begin{cases} 
  t_{i,v_{\min}}^f, & \text{if } 9v_i^0 + 12S_i u_{i,\min} < 0, \\
  \max\{t_{i,u_{\min}}^f, t_{i,v_{\min}}^f\}, & \text{otherwise.}
\end{cases}
\]  

(S29)

where

\[
t_{i,v_{\min}} = \frac{3S_i}{v_i^0 + 2v_{\min}}, \]  

(S30)

\[
t_{i,u_{\min}} = \frac{\sqrt{9v_i^0 + 12S_i u_{\min} - 3v_i^0}}{2u_{\min}}. \]  

(S31)
Author Biography

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Andreas A. Malikopoulos (S’06–M’09–SM’17) received the Diploma in mechanical
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Figure S1: Comparison of the physical and virtual city environments. The mainframe computer can switch between physical and virtual experiment seamlessly.