Inflation in the warm and cold regimes

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It is now understood that inflation dynamics comes in two forms, isentropic or cold inflation and nonisentropic or warm inflation. In the former, inflation occurs without radiation production, whereas in the latter both radiation production and inflation occur concurrently. Recent, detailed, quantum field theory calculations have shown that many generic inflation models, including hybrid inflation, which were believed only to have cold inflation regimes, in fact have regimes of both warm and cold inflation. These results dispel many foregone assumptions generally made up to now about inflation models and bring to the fore various elementary issues that must be addressed to do reliable calculations from inflation models. Here I review these results and issues. I then show that warm inflation has intrinsic model independent features that makes it natural or equivalently have no “eta problem”. Next density perturbations and observational consequences of warm inflation are discussed. Finally the implications of warm inflation to model building and physics beyond the Standard Model are outlined.

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I. INTRODUCTION

For over two decades, inflation has been a very successful idea. In its earliest days, this success was attributed to the ability for this idea to unite particle physics and cosmology. In the past decade, the success of the inflation idea has been driven by its consistency to observation based on data in particular from precision CMB satellite experiments. The growing success over the years of the inflation idea has also led to an increasing understanding of the underlying dynamics of inflation. By now it is known that there are two dynamical realizations of inflation. One is the original or standard picture, also referred to as isentropic or cold inflation [1, 2, 3]. In this picture inflationary expansion occurs with the universe in a supercooled phase which subsequently ends with a reheating period that introduces radiation into the universe. The fluctuations created during inflation are effectively zero-point ground state fluctuations and the evolution of the inflaton field is governed by a ground state evolution equation. In an alternative class of cold inflation models, inflation has a geometrical origin [4] with adiabatic density perturbations [5]. The other picture of inflation dynamics is nonisentropic or warm inflation [6]. In this picture, inflationary expansion and radiation production occur concurrently. Moreover, the fluctuations created during inflation emerge from some excited statistical state and the evolution of the inflaton has dissipative terms arising from the interaction of the inflaton with other fields.

The dividing point between warm and cold inflation is roughly at \( \rho_{r}^{1/4} \approx H \), where \( \rho_{r} \) is the radiation energy density present during inflation and \( H \) is the Hubble parameter. Thus \( \rho_{r}^{1/4} > H \) is the warm inflation regime and \( \rho_{r}^{1/4} < H \) is the cold inflation regime. This criteria is independent of thermalization, but if such were to occur, one sees this criteria basically amounts to the warm inflation regime corresponding to when \( T > H \). This is easy to understand since the typical inflaton mass during inflation is \( m_{\phi} \approx H \) and so when \( T > H \), thermal fluctuations of the inflaton field will become important. This criteria for entering the warm inflation regime turns out to require the dissipation of a very tiny fraction of the inflaton vacuum energy during inflation. For example, for inflation with vacuum (i.e. potential) energy at the GUT scale \( \sim 10^{15-16} \text{GeV} \), in order to produce radiation at the scale of the Hubble parameter, which is \( \approx 10^{10-11} \text{GeV} \), it just requires dissipating one part in \( 10^{20} \) of this vacuum energy density into radiation. Thus energetically not a very significant amount of radiation production is required to move into the warm inflation regime. In fact the levels are so small, and their eventual effects on density perturbations and inflaton evolution are so significant, that care must be taken to account for these effects in the analysis of any inflation models.

In this paper, we will examine warm inflation dynamics and compare it to cold inflation dynamics. In Sect. II the basic equations of both pictures are reviewed. In Sect. III a particular mechanism for dissipation is presented and the dissipative quantum field theory calculation is done to determine dissipative effects in the inflation effective equation of motion. In Sect. IV density fluctuations in cold and warm inflation are presented. A calculation of inflation for a monomial potential is done, and it is shown that in contrast to cold inflation, in the warm inflation case the mass of the inflaton is bigger than the Hubble parameter and the field amplitude is below the Planck scale. These are two very different outcomes of warm inflation that are not found in the cold inflation case for such potentials. In Sect. V an analysis of the SUSY hybrid model is done. The parameter space

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is shown to divide into regimes of cold and warm inflation. The spectral index is also calculated to examine observational differences in the two cases. Finally some examination is made of particle physics of this model in the warm inflation case. Finally in Sect. VII our conclusions are given.

II. TWO INFLATION DYNAMICS

There are two distinct dynamical realizations of inflation. In the original picture, termed cold, supercooled or isentropic inflation [1, 2, 3], the universe rapidly supercools during inflation and subsequently a reheating phase is invoked to end inflation and put the universe back into a radiation dominated regime. In this picture, inflaton dynamics is governed by the equation

$$\ddot{\phi} + 3H\dot{\phi} + \xi R\phi + \frac{dV_{\text{eff}}(\phi)}{d\phi} = 0,$$

(1)

where $R = 6(\ddot{a}/a + \dot{a}^2/a^2)$ is the curvature scalar. The conditions for slow-roll inflation in this picture are

$$\epsilon_H = \frac{m_P^2}{2} \left( \frac{V'}{V} \right)^2 \ll 1,$$

(2)

and

$$\eta_H = \frac{m_P^2}{2} \frac{V''}{V} \ll 1,$$

(3)

where $m_P \equiv m_{pl}/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. In this slow-roll regime the evolution equation (1) is well approximated by

$$3H\dot{\phi} + \xi R\phi + \frac{dV_{\text{eff}}(\phi)}{d\phi} = 0.$$

(4)

In the other picture, termed warm or nonisentropic inflation [4], dissipative effects are important during the inflation period, so that radiation production occurs concurrently with inflationary expansion. Phenomenologically, the inflaton evolution in simple warm inflation models has the form,

$$\ddot{\phi} + [3H + \Upsilon(\phi)]\dot{\phi} + \xi R\phi + \frac{dV_{\text{eff}}(\phi)}{d\phi} = 0.$$

(5)

For $\Upsilon = 0$, this equation reduces the rich familiar inflaton evolution equation for cold inflation Eq. (4), but for a nonzero $\Upsilon$, it corresponds to the case where the inflaton field is dissipating energy into the universe, thus creating a radiation component. The conditions for slow-roll inflation are modified in the presence of the extra friction term $\Upsilon$, and we have now:

$$\epsilon_\Upsilon = \frac{\epsilon_H}{(1 + r)^2} < 1,$$

(6)

$$\eta_\Upsilon = \frac{\eta_H}{(1 + r)^2} < 1,$$

(7)

where $r \equiv \Upsilon/(3H)$, and $\epsilon_H, \eta_H$ are the slow-roll parameters without dissipation given in Eqs. (2) and (3). In addition, when the friction term $\Upsilon$ depends on the value of the inflaton field, we can define a third slow-roll parameter

$$\epsilon_{\Upsilon H} = \frac{r}{(1 + r)^3} \beta_\Upsilon < 1,$$

(8)

with

$$\beta_\Upsilon = \frac{V'}{3H^2} \frac{\Upsilon'}{\Upsilon}.$$

(9)

In this slow-roll regime of warm inflation the inflaton evolution equation is well approximated by

$$[3H + \Upsilon(\phi)]\dot{\phi} + \xi R\phi + \frac{dV_{\text{eff}}(\phi)}{d\phi} = 0.$$

(10)

The dissipation of the inflaton’s motion is associated with the production of entropy. The entropy density of the radiation $s(\phi, T)$ is defined by a thermodynamic relation in terms of the thermodynamic potential,

$$s = -V_{,T}.$$

(11)

The rate of entropy production can be deduced from the conservation of energy-momentum. The total density $\rho$ and pressure $p$ are given by,

$$\rho = \frac{1}{2}\dot{\phi}^2 + V + Ts,$$

(12)

$$p = \frac{1}{2}\dot{\phi}^2 - V.$$

(13)

Energy-momentum conservation,

$$\dot{\rho} + 3H(p + \rho) = 0,$$

(14)

now implies entropy production. Making use of Eq. (5) (with $\xi = 0$) we get

$$T(\dot{s} + 3Hs) = \Upsilon\dot{\phi}^2.$$

(15)

The zero curvature Friedman equation completes the set of differential equations for $\phi, T$ and the scale factor $a$,

$$3H^2 = 8\pi G(\frac{1}{2}\dot{\phi}^2 + V + Ts).$$

(16)

The entropy production has been described in a slightly different way in the initial warm inflation papers [4, 5]. We can recover an alternative equation in the case when the temperature corrections to the potential are negligible. If we set $\delta m_T = 0$ in the finite temperature effective potential $V_{\text{eff}}(\phi, T)$, then the radiation density $\rho_r = 4sT/3$, and Eq. (15) becomes

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2.$$

(17)

This equation is only valid when $\delta m_T = 0$.

Eq. (10) is the simplest form of warm inflation dynamics. The basic idea of warm inflation is that radiation
production is occurring concurrently with inflationary expansion due to dissipation from the inflaton field system. This dissipation would imply nonconservative terms in the inflaton evolution equation. In general these terms need not be of the simple temporally local form $\dot{\Upsilon}\phi$ as in Eq. (11), but could be nonlocal. One such example would be a form like $\int K(t, t') \varphi^n(t') dt'$. Moreover in general this radiation production could be produced far from equilibrium.

### III. RADIATION PRODUCTION MECHANISM DURING INFLATION

The key question is what types of first principles mechanisms can result in radiation production during inflation. Here we consider the example of $\mathbb{S}$, which develops the mechanism of the scalar inflaton field $\phi$ exciting a heavy bosonic field $\chi$ which then decays to light fermions $\psi_d$,

$$\phi \rightarrow \chi \rightarrow \psi_d.$$  

This mechanism is expressed in its simplest form by an interaction Lagrangian density for the coupling of the inflaton field to the other fields of the form

$$L_I = -\frac{1}{2} g^2 \Phi^2 \chi^2 - g' \Phi \bar{\psi}_d \psi_d,$$  

(19)

where $\psi_d$ are the light fermions to which $\chi$-particles can decay, with $m_\chi > 2m_{\psi_d}$. Aside from the last term in Eq. (19), these are the typical interactions commonly used in studies of reheating after inflation $\mathbb{L}$, $\mathbb{N}$. However a realistic inflation model often can also have additional interactions outside the inflaton sector, with the inclusion of the light fermions $\psi_d$ as depicted above being a viable option; an example of this is the SUSY hybrid model for which a recent study of warm inflation has been done $\mathbb{S}$ and will be discussed in Sect. $\mathbb{V}$. Moreover in minimal supersymmetry (SUSY) extensions of the typical reheating model or multifield inflation models, the interactions of the form as given in Eq. (19) can emerge as an automatic consequence of the supersymmetric structure of the model. Since in the moderate to strong perturbative regime, reheating and multifield inflation models will require SUSY for controlling radiative corrections, Eq. (19) with inclusion of the $\psi_d$ field thus is a toy model representative of many realistic situations.

The effective evolution equation for the inflaton background field arising from mechanism Eq. (19) has been computed in $\mathbb{S}$. The basic idea is we are interested in obtaining the effective equation of motion (EOM) for a scalar field configuration $\varphi = \langle \Phi \rangle$ after integrating out the $\Phi$ fluctuations, the scalars $\chi_j$ and spinors $\psi_j, \bar{\psi}_j$.

By and everything else is the environment, which in particular means the $\Phi$ fluctuation modes, the scalars $\chi_j$ and the spinors $\psi_k, \bar{\psi}_k$ are regarded as the environment bath. In a Minkowski background, at $T = 0$, the EOM for $\varphi$ has been derived in $\mathbb{S}$ using the Schwinger closed time path formalism. Here we follow a completely analogous approach and derive the EOM in a FRW background following $\mathbb{S}$. The field equation for $\Phi$ can be readily obtained from Eq. (19) and it is given by

$$\ddot{\Phi} + \frac{3}{a} \dot{\Phi} - \frac{\nabla^2}{a^2} \Phi + m^2_\Phi \Phi + \frac{\lambda}{6} \Phi^3 + \xi R \Phi$$

$$+ \sum_{j=1}^{N_\chi} g_j^2 \Phi \langle \chi^2_j \rangle = 0.$$  

(20)

In order to obtain the effective EOM for $\varphi$, we use the tadpole method. In this method we split $\Phi$ in $\mathbb{S}$, as usual, into the (homogeneous) classical expectation value $\varphi(t) = \langle \Phi \rangle$ and a quantum fluctuation $\phi(t), \Phi(t) = \varphi(t) + \phi(t)$. In this case, the field equation for $\Phi$, after taking the average (with $\langle \phi(x) \rangle = 0$), becomes

$$\ddot{\varphi}(t) + \frac{3}{a(t)} \dot{\varphi}(t) + m^2_\varphi \varphi(t) + \frac{\lambda}{6} \varphi^3(t)$$

$$+ \xi R(t) \varphi(t) + \frac{\lambda}{2} \varphi(t) \langle \phi^2 \rangle + \frac{\lambda}{6} \langle \phi^3 \rangle$$

$$+ \sum_{j=1}^{N_\chi} g_j^2 [\varphi(t) \langle \chi_j^2 \rangle + \langle \phi \chi_j^2 \rangle] = 0,$$  

(21)

where $\langle \phi^2 \rangle$, $\langle \phi^3 \rangle$, $\langle \chi_j^2 \rangle$ and $\langle \phi \chi_j^2 \rangle$ can be expressed $\mathbb{S}$ in terms of the coincidence limit of the (causal) two-point Green’s functions $G_{\phi}^{++}(x, x')$ and $G_{\chi j}^{++}(x, x')$, for the $\Phi$ and $\chi_j$ fields respectively. In particular

$$G_{\chi j}^{++}(x, x') = i \langle T_+ \chi_j(x) \chi_j(x') \rangle$$

$$G_{\chi j}^{--}(x, x') = i \langle T_- \chi_j(x) \chi_j(x') \rangle$$

$$G_{\chi j}^{+-}(x, x') = i \langle \chi_j(x) \chi_j(x') \rangle$$

$$G_{\chi j}^{-+}(x, x') = i \langle \chi_j(x) \chi_j(x') \rangle,$$  

(22)

and similarly for $G_{\phi}(x, x')$. The momentum space Fourier transfer of $G_{\chi j}(x, x')$ is

$$G_{\chi j}(x, x') = i \int \frac{d^3 q}{(2\pi)^3} e^{i q \cdot (x-x')} \begin{pmatrix} G_{\chi j}^{++}(q, t, t') \\ G_{\chi j}^{+-}(q, t, t') \\ G_{\chi j}^{-+}(q, t, t') \\ G_{\chi j}^{--}(q, t, t') \end{pmatrix},$$  

(23)

where

$$G_{\chi j}^{++}(q, t, t') = G_{\chi j}^{--}(q, t, t') \theta(t - t')$$

$$+ G_{\chi j}^{+-}(q, t, t') \theta(t' - t),$$

$$G_{\chi j}^{-+}(q, t, t') = G_{\chi j}^{--}(q, t, t') \theta(t' - t)$$

and

$$G_{\chi j}^{+-}(q, t, t') = G_{\chi j}^{--}(q, t, t') \theta(t - t').$$
and similarly for $G_a(x,x')$. The Green’s functions $G_{X_j}^{-}(q,t',t)$ are written in terms of the modes of the scalar field as

$$G_{X_j}^{-}(q,t,t') = f_{X_j;1}(q,t)f_{X_j;2}(q,t')\theta(t-t')$$

$$+ f_{X_j;1}^*(q,t)f_{X_j;2}^*(q,t')\theta(t' - t) ,$$

$$G_{X_j}^{<}(q,t,t') = f_{X_j;1}(q,t)f_{X_j;2}(q,t')\theta(t' - t)$$

$$+ f_{X_j;1}^*(q,t)f_{X_j;2}^*(q,t')\theta(t - t') ,$$

where in general there will be two independent solutions $f_{X_j;1,2}(q,t)$ since $X_j$ obeys an equation second order in time. To obtain the evolution equations for the mode functions in Eq. (26), the fermion fields that interact with $X_j$ fields are integrated out following [9], which then leads to

$$\left[ \frac{d^2}{dt^2} + 3\frac{\partial}{\partial t} + \frac{q^2}{a^2} + M^2(q,t) \right] f_{X_j}(q,t)$$

$$+ \int dt'a^3(t')\Pi_{X_j}(q,t',t)f_{X_j}(q,t') = 0 ,$$

where $\Pi_{X_j}(q,t,t')$ is the spatial Fourier transform of the $X_j$ field self-energy term. In particular, in terms of the self-energy matrix on the closed time path $\Pi_{X_j}$ is defined through

$$\Sigma_{X_j}^{\pm}(x,x') = \Sigma_{X_j}^+ (x,x') - \Sigma_{X_j}^- (x,x')$$

$$= 2 \left[ \Sigma_{X_j}^{\pm}(x,x') + \Sigma_{X_j}^{\pm}(x',x) \right]$$

$$= 2\theta(t_1 - t_2) \left[ \Sigma_{X_j}^+ (x,x') - \Sigma_{X_j}^- (x,x') \right]$$

$$= \Pi_{X_j}(x,x') = \Pi_{1,X_j}(x,x') + \Pi_{2,X_j}(x,x') ,$$

where

$$\Pi_{1,X_j}(x,x') = \left[ 2\theta(t_1 - t_2) - 1 \right] \left[ \Sigma_{X_j}^+ (x,x') - \Sigma_{X_j}^- (x,x') \right] ,$$

$$\Pi_{2,X_j}(x,x') = \Sigma_{X_j}^+ (x,x') - \Sigma_{X_j}^- (x,x') ,$$

which have the properties $\Pi_{1,X_j}(x,x') = \Pi_{1,X_j}(x',x)$ and $\Pi_{2,X_j}(x,x') = -\Pi_{2,X_j}(x',x)$.

Typically, equations for the mode functions for an interacting model, of the general form as given by Eq. (26), can be very difficult to solve analytically, in particular for an expanding background. There are a few particular cases, such as for de Sitter expansion $H \sim$ constant, so $a(t) = \exp(Ht)$, and power law expansion $a(t) \sim t^n$, where solutions for the mode equation for free fluctuations are known in exact analytical form (see e.g. Ref. [13]). However, for deriving an approximate solution for the mode functions in the interacting case, we can apply a WKB approximation for equations of the general form Eq. (26) and then check the validity of the approximation for the parameter and dynamical regime of interest to us. As will be seen below, under the dynamical conditions we are interested in studying in this paper, this approximation will suit our purposes. Let us briefly recall the WKB approximation and its general validity regime, when applied to obtaining approximate solutions for field mode equations. An approximated WKB solution for a mode equation like

$$\left[ \frac{d^2}{dt^2} + \omega^2(q,t) \right] f(q,t) = 0 ,$$

is of the form

$$f_{WKB}(q,t) = 1/|\omega(q,t)|^{1/2} \exp \left[ \pm i \int dt'' \omega(q,t'') \right] ,$$

which holds under the general adiabatic condition $\omega(q,t) \ll \omega^2(q,t)$.

Proceeding with our derivation, consider then a differential equation in the form of Eq. (26). Instead of working in cosmic time, it is more convenient to work in conformal time $\tau$, defined by $d\tau = dt/a(t)$, where the metric becomes conformally flat,

$$ds^2 = a(\tau)^2 \left( d\tau^2 - d\mathbf{x}^2 \right) ,$$

By also defining a rescaled mode field in conformal time by

$$\frac{1}{a(\tau)} \tilde{f}(q,\tau) = f(q,t) ,$$

we can then re-express Eq. (26) in the form (generically valid for either $\phi$ or $\chi_j$ scalar fluctuations)

$$\frac{d^2}{d\tau^2} \tilde{f}(q,\tau) + \tilde{\omega}(q,\tau) \tilde{f}(q,\tau)$$

$$+ \int d\tau' \tilde{\Pi}(q,\tau,\tau') \tilde{f}(q,\tau') = 0 ,$$

where we have defined

$$\tilde{\omega}(q,\tau)^2 = q^2 + a(\tau)^2 \left[ M^2 + \left( \xi - \frac{1}{6} \right) R(\tau) \right] .$$

In (34) the conformal symmetry appears in an explicit form, with $\xi = 1/6$ referring to fields conformally coupled to the curvature, while $\xi = 0$ gives the minimally coupled case. Note also that in conformal time the scalar curvature becomes

$$R(\tau) = \frac{6}{a^3} \frac{d^2a}{d\tau^2} .$$
In Eq. (33) we have also defined the self-energy in conformal time as,

\[
\frac{\Pi(q, \tau, \tau')}{a(\tau)^{3/2}a(\tau')^{3/2}} = \Pi(q, t, t') ,
\]

where the self-energy contribution \(\Pi\), coming from the integration over the bath fields, is given by the space Fourier transformed form for Eq. (27). In (27), \(\Pi\) was split into symmetric and antisymmetric pieces with respect to its argument as defined in Eq. (28). Thus based on Eq. (28), the self-energy term in (33) can then be written as \(\Pi(q, \tau, \tau') = \Pi_1(q, \tau, \tau') + \Pi_2(q, \tau, \tau')\). In addition, by writing the self-energy term in a diagonal (local) form

\[
\Pi(q, \tau, \tau') = \Pi(q, \tau) \delta(\tau - \tau') = [\Pi_1(q, \tau) + \Pi_2(q, \tau)] \delta(\tau - \tau') ,
\]

and from the properties satisfied by \(\Pi_1\) and \(\Pi_2\), it results that \(\Pi_1(q, \tau)\) must be real, while \(\Pi_2(q, \tau)\) must be purely imaginary. The real part of the self-energy contributes to both mass and wave function renormalization terms that can be taken into account by a proper redefinition of both the field and mass \(M\). On the other hand, the imaginary term of the self-energy is associated with decaying processes, as discussed previously. So, we can now relate the decay width in terms of the CTP self-energy terms as

\[
\Gamma = -\frac{\text{Im} \Pi}{2}\frac{\Sigma^> - \Sigma^<}{2\omega} ,
\]

and Eq. (33) can be put in the form

\[
\left[ \frac{d^2}{dt^2} + \frac{1}{2}\nu(q, \tau)^2 - 2i\nu(q, \tau)\Gamma(q, \tau) \right] \tilde{f}(q, \tau) = 0 .
\]

We now proceed to obtain a standard WKB solution for Eq. (39). To do this, following the usual WKB procedure, we assume the solution to have the form \(\tilde{f}(q, \tau) = c \exp[i\gamma(q, \tau)]\), where \(c\) is some constant that can be fixed by the initial conditions, given by (44) below. This form of the solution is then substituted into (39) to give

\[
i\gamma'' - \gamma'^2 + \omega^2 - 2i\omega\Gamma = 0 .
\]

Working in the standard WKB approximation, for the zeroth order approximation we neglect the second derivative term in (40). Then, by taking \(\Gamma \ll \omega\), we obtain

\[
\gamma_0 = \mp \int_{\tau_0}^{\tau} d\tau' \left( \omega - i\Gamma \right) ,
\]

which is then used in Eq. (40) for the second derivative term to determine the next order approximation,

\[
\gamma_1 \approx \mp \int_{\tau_0}^{\tau} d\tau' \left[ \omega - i\Gamma + \mathcal{O}(\omega^2/\omega^3) \right] + i \sqrt{\omega} .
\]

The next and following orders in the approximation bring higher powers and derivatives of \(\omega'/\omega^2\), which in the adiabatic regime, \(\omega'/\omega^2 \ll 1\), are negligible and we are then led to the result

\[
\tilde{f}_{1,2}(q, \tau) \approx \frac{c}{\sqrt{\omega}} \exp \left[ \mp i \int_{\tau_0}^{\tau} d\tau' \left( \omega - i\Gamma \right) \right] .
\]

The solutions for the modes of the form Eq. (43) and their complex conjugate are general within the adiabatic, or WKB, approximation regime of dynamics. Finally, we completely and uniquely determine the modes by fixing the initial conditions at some initial reference time \(\tau_0\), which can be chosen such that in the limit of \(k \to \infty\) or \(H \to 0\) we reproduce the Minkowski results. These conditions, which correspond to the ones for the Bunch-Davis vacuum [13], can be written as

\[
\tilde{f}_{1,2}(q, \tau_0) = \frac{1}{\sqrt{2\omega(\tau_0)}} ,
\]

\[
\tilde{f}_{1,2}'(q, \tau_0) = \mp i \sqrt{\omega(\tau_0)/2} ,
\]

which already fixes the constant \(c\) in (43) as \(c = 1/\sqrt{2}\). Using the above results in (25) and after returning to cosmic time \(t\), we obtain the result, valid within the WKB approximation, or adiabatic regime,

\[
G^{(\pm)}(q, t, t') = \frac{1}{|a(t)a(t')|^{3/2}} \tilde{G}^{(\pm)}(q, t, t') ,
\]

where

\[
\tilde{G}^{(\pm)}(q, t, t') = \frac{1}{2|\omega(t)\omega(t')|^{1/2}} \left\{ e^{-i\int_{t'}^{t} dt'' \omega(t'') - i\Gamma(t'')} \theta(t - t') \right. \\
\left. + e^{-i\int_{t'}^{t} dt'' \omega(t'') + i\Gamma(t'')} \theta(t' - t) \right\} ,
\]

\[
\tilde{G}^{(\pm)}(q, t', t') = \tilde{G}^{(\pm)}(q, t', t) ,
\]

where \(\Gamma\) is the field decay width in cosmic time, obtained from (38) and

\[
\omega(t) = \sqrt{\frac{q^2}{a(t)^2} + M^2(t)} ,
\]

with \(M^2(t)\), for \(\Phi\) particles, given by
while for $\chi_j$ particles,

$$M_{\chi_j}^2(t) = m_{\chi_j}^2 + g_j^2 \varphi(t)^2 + \left( \xi - \frac{1}{6} \right) R(t),$$

(49)

The same result Eq. (49) could in principle be inferred in an alternative way by expressing the propagator expressions in terms of a spectral function, defined by a Fourier transform for the difference between the retarded and advanced dressed propagators,

$$G^\text{ret}(x, x') = \theta(t - t') \left[ G^>(x, x') - G^<(x, x') \right]$$

$$= G^{++}(x, x') - G^{-+}(x, x'),$$

(50)

$$G^\text{adv}(x, x') = \theta(t' - t) \left[ G^<(x, x') - G^>(x, x') \right]$$

$$= G^{-+}(x, x') - G^{++}(x, x'),$$

(51)

and approximating the spectral function as a standard Breit-Wigner form with width given by $\Gamma$ and poles determining the arguments of the exponential in (14) and its complex conjugate (16). The validity of this approximation in particular was recently numerically tested and verified in Ref. [17] for a 1 + 1 d scalar field in Minkowski space-time. In the Minkowski space-time case, results analogous to Eq. (46) were explicitly derived in Refs. [8, 10, 11, 16, 20]. Indeed, for the case of no expansion $a(t) = \text{constant}$, Eq. (46) reproduces the same expressions as found in the case of Minkowski space-time.

The result Eq. (46), from the previous approximations used to derive the WKB solution Eq. (44), is valid under the requirements

$$\Gamma_\varphi \ll \omega_\varphi,$$

$$\Gamma_{\chi_j} \ll \omega_{\chi_j},$$

(52)

and the adiabatic conditions,

$$\frac{\dot{\omega}_\varphi}{\omega_\varphi} = \frac{\dot{a}}{a} + \frac{\omega_\varphi}{\omega_\varphi} \ll 1,$$

$$\frac{\dot{\omega}_{\chi_j}}{\omega_{\chi_j}} = \frac{\dot{a}}{a} + \frac{\omega_{\chi_j}}{\omega_{\chi_j}} \ll 1,$$

(53)

where in the second term in the equations (46) we have made the change back to comoving time and used $\tilde{\omega} = a(t) \omega(t) = a \sqrt{q^2 / a^2 + M^2}$.

We now turn our attention to the EOM Eq. (21), where we will work it out in the response theory approximation similar to the treatment in (8). Consider the Lagrangian density in terms of the background (system) field $\varphi(t)$ and the fluctuation (bath) fields,

$$\mathcal{L}[\Phi] = \varphi(t) + \phi(x), \chi_j, \psi_k, \psi_{k\mu}, g_{\mu
u} = \mathcal{L}_\varphi[\varphi(t), g_{\mu
u}] + \mathcal{L}_\text{bath}[\varphi(t), \phi(x), \chi_j, \psi_k, \psi_{k\mu}, g_{\mu
u}],$$

(54)

where

$$\mathcal{L}_\varphi[\varphi(t), g_{\mu
u}] =$$

$$a(t)^3 \left\{ \frac{1}{2} \dot{\varphi}(t)^2 - \frac{m_\varphi^2}{2} \varphi(t)^2 - \frac{\lambda}{4!} \varphi(t)^4 - \frac{\xi}{2} R \varphi(t)^2 \right\},$$

is the sector of the Lagrangian independent of the fluctuation bath fields, while $\mathcal{L}_\text{bath}$ denotes the sector of the Lagrangian that depends on the bath fields and in particular includes the key interaction terms Eq. (19). In the following derivation it will be assumed that the background field $\varphi(t)$ is slowly varying, something that must be checked for self-consistency. Thus, if we consider the decomposition of $\varphi(t)$ around some arbitrary time $t_0$ as $\varphi(t) = \varphi(t_0) + \delta \varphi(t)$, $\delta \varphi(t)$ can be regarded as a perturbation, for which a response theory approximation can be used for the derivation of the field averages in Eq. (21).

In response theory we express the change in the expectation value of some operator $\hat{O}(t)$, $\delta \langle \hat{O}(t) \rangle = \langle \hat{O}(t) \rangle_{\text{pert}} - \langle \hat{O}(t) \rangle$, under the influence of some external perturbation described by $\hat{H}_\text{pert}$ which is turned on at some time $t_0$, as (for an introductory account of response theory, see for instance Ref. [21])

$$\delta \langle \hat{O}(t) \rangle = i \int_{t_0}^t dt' \langle \hat{H}_\text{pert}(t'), \hat{O}(t) \rangle_0,$$

(55)

where the expectation value on the RHS of Eq. (55) is evaluated in the unperturbed ensemble. The response function defined by Eq. (55) can be readily generalized for the derivation of the field averages. Provided that the amplitude $\delta \varphi(t)$ is small relative to the background field $\varphi(t_0)$, perturbation theory through the response function can be used to deduce the expectation values of the fields that enter in the EOM Eq. (21). In this case the perturbing Hamiltonian $\hat{H}_\text{pert}$ is obtained from $\mathcal{L}_\text{int}^{\delta \varphi}$, where $\mathcal{L}_\text{int}^{\delta \varphi}$ is the part of the interaction Lagrangian proportional to $\delta \varphi$. From $\mathcal{L}_\text{int}^{\delta \varphi}$ and Eq. (55) we can then determine the averages of the bath fields, for example $\langle \chi_j^2(t) \rangle$, as an expansion in $\delta \varphi(t)$, starting from the time $t_0$ and in an one-loop approximation, as

$$\langle \chi_j^2 \rangle \simeq \langle \chi_j^2 \rangle_0$$

$$+ \frac{1}{a(t)^3} \int_{t_0}^t dt' 2q_j^2 \left[ \varphi(t')^2 - \varphi(t_0)^2 \right]$$

$$\int \frac{d^3q}{(2\pi)^3} \text{Im} \left[ \tilde{G}^{\chi_j^2+}(q, t, t') |_{\varphi(t_0)} \right]^2,$$

(56)
where
\[
\langle \chi_j^2(x, t), \chi_j^2(x, t') \rangle = 2i \operatorname{Im} \left[ \langle T \chi_j^2(x, t), \chi_j^2(x, t') \rangle \right]
\]
\[
= \frac{4i}{|a(t)a(t')|^2} \int \frac{d^3q}{(2\pi)^3} \operatorname{Im} \left[ \langle G^{++}(q, t, t') \rangle \right] , (57)
\]
Similarly the expression for the other expectation values \( \langle \dot{\phi}^2 \rangle \) and \( \langle \phi \dot{\phi}^3 \rangle \) can be determined.

Substituting these field averages into Eq. (21) we then obtain the \( \varphi \)-effective equation of motion
\[
\ddot{\varphi}(t) + 3H(t) \dot{\varphi}(t) + \frac{dV^\text{eff}_r}{d\varphi}(\varphi(t), R(t)) + \lambda^2 \varphi(t) \int_{t_0}^{t} dt' \varphi(t') \varphi'(t') K_{\varphi}(t, t') + \sum_{j=1}^{N_x} 4g_j^4 \varphi(t) \int_{t_0}^{t} dt' \varphi(t') \varphi'(t') K_{\chi_j}(t, t') = 0 , (58)
\]
where \( V^\text{eff}_r \) stand for the renormalized effective potential,
\[
K_{\chi_j}(t, t') = \int_{t_0}^{t'} dt'' \int \frac{d^3q}{(2\pi)^3} \sin \left[ 2 \int_{t''}^{t'} d\tau \omega_{\chi_j, t}(\tau) \right] \times \exp \left[ -2 \int_{t''}^{t'} d\tau \Gamma_{\chi_j, t}(\tau) \right] , (59)
\]
and similar expression for \( K_{\varphi}(t, t') \).

In the case where the motion of \( \varphi \) is slow, the above equation can be further simplified. In particular, in this case the adiabatic-Markovian approximation can be applied to the nonlocal kernels \( K_{\chi_j}(t, t') \) etc... This converts Eq. (58) into one that is completely local in time, albeit with time derivative terms. The details of this approximation for Minkowski space-time can be found in [3]. Its extension to an expanding FRW background follows analogous lines. The Markovian approximation amounts to substituting \( t' \rightarrow t \) in the arguments of the \( \varphi \)-fields in the kernels in Eq. (58). The adiabatic approximation then requires self-consistently that all macroscopic motion is slow on the scale of microscopic motion, thus \( \dot{\varphi}/\varphi, H < \Gamma_{\chi_j} \). Moreover when \( H < M_{\chi} \), the kernel \( K_{\chi_j}(t, t') \) is well approximated by the nonexpanding limit \( H \rightarrow 0 \). The validity of all these approximations were examined in [3]. The result of these approximations is that, after trivially integrating over the momentum integral in the last term in Eq. (58), the effective EOM Eq. (58) becomes Eq. (4). By setting the couplings \( g_j = g_j' = g, h_{\chi_j} = h \gg \lambda \), the mass \( M_{\chi} \gg \varphi \gg m_{\phi} \), and \( \Gamma_{\chi_j} \approx N_\chi \hbar M_{\chi}^2/[8\pi \omega_{\chi_j}] \), it leads to the friction coefficient \( \Upsilon(\varphi) \) in Eq. (5).

\[
\Upsilon(\varphi) = N_\chi \frac{\sqrt{2g^4\alpha_4\varphi^2}}{64\pi M_{\chi} \sqrt{1 + \alpha_4^2 \sqrt{1 + \alpha_4^2 + 1}}} , \quad \text{for } \alpha_4 \equiv N_\phi \hbar^2/(8\pi) , \quad (60)
\]

FIG. 1: Evolution of \( \varphi(t) \) for \( \lambda = 10^{-13} \), \( g = h = 0.37 \), \( \xi = 0 \), \( \varphi(0) = m_{\phi} \), \( \varphi(0) = 0 \).

Fig. 1 compares the various approximations for a representative case where \( g = h = 0.37 \) and the inflaton potential is that for chaotic inflation \( V^\text{eff}(\varphi) = \lambda \varphi^4/4 \) with \( \lambda = 10^{-13} \). In Fig 1 evolution has been examined at the final stages of chaotic inflation where we start with \( \varphi(t_0 = 0) = m_{\phi} \). The solid line is the exact result based on numerically solving Eq. (58). Plotted alongside this, although almost indiscernible, is the same solution expect using the nonexpanding spacetime kernel (dashed line), obtained by setting \( H \rightarrow 0 \) in Eq. (58), and the solution based on the adiabatic-Markovian approximation of Eq. (5) (dot-dashed line) for the same parameter set. As seen, the expanding and nonexpanding cases differ by very little and the adiabatic-Markovian approximation is in good agreement with the exact solution. This result presented first in [3] confirms simplifying approximations claimed in [3, 13, 21, 22, 23] but up to now had not been numerically verified.

More interestingly, the dotted line in Fig. 1 is the solution that would be found by the conventional approach in which the nonlocal terms in Eq. (58) are ignored. The conventional approach [2, 3, 13, 14, 15] expects the inflaton to start oscillating, which is the precursor to entering various stages of pre/re-heating. However with account for dissipative effects, this never happens for our example in Fig. 1 since the inflaton remains overdamped till the end when it settles at its minima at \( \varphi = 0 \). Moreover, throughout inflation, and not just near the end, the inflaton dissipates energy, which yields a radiation component of magnitude
\[
\rho_r \approx \frac{\Upsilon \varphi^2}{4H} . \quad (61)
\]
IV. DENSITY PERTURBATIONS

The difference between the cold and warm inflationary dynamics implies that the inflaton density perturbations in the two cases also have basic differences. In particular in the warm inflation regime, since a thermalized radiation component is present with $T > m_\phi$, inflaton fluctuations are dominantly thermal rather than quantum. There are two distinct regimes of warm inflation to note. One is the weak dissipative regime \[ \delta \varphi^2 \sim HT \] warm inflation ($\Upsilon < 3H$), $T > m_\phi$, (62) and the other is the strong dissipative regime \[ \delta \varphi^2 \sim \sqrt{H\Upsilon}T \] warm inflation ($\Upsilon > 3H$), $T > m_\phi$. (63)

For comparison, for cold inflation, where inflaton fluctuations are exclusively quantum \[ \delta \varphi^2 \sim H^2, \] cold inflation $T < m_\phi$. (64)

For both cold and warm inflation, density perturbations are obtained by the same expression, $\delta \rho / \rho \sim \dot{H} \varphi / \dot{\varphi}$.

The inherent difference in density perturbations in cold versus warm inflation provides a possible direction for distinguishing between these two inflation dynamics using observation. In [27] an order of magnitude estimate of density perturbations during warm inflation was computed by matching the thermally produced fluctuations to gauge invariant parameters when the fluctuations cross the horizon (for other phenomenological treatments of warm inflation see [28]). This work provided a clear statement of the consistency condition. Cold inflation has three parameters, related to the potential energy magnitude $V_0$, slope $\epsilon_H$ in Eq. 2 and $\eta_H$ in Eq. 3, whereas there are four observable constraints ($\delta_H$, $A_g$, $n_s$, $n_g$). This implies a redundancy in the observations and allows for a consistency relation 29. This is usually expressed as a relationship between the tensor-to-scalar ratio and the slope of the tensor spectrum. Warm inflation has an extra parameter, the dissipation factor, which implies four constraints for four parameters. Hence we do not expect the consistency relation of standard inflation to hold in warm inflation 27. Thus discriminating between warm and standard inflation requires measuring all four observables. The WMAP and upcoming Planck satellite missions should provide strong constraints on the scalar spectrum and having polarization detectors, it is hoped the tensor spectrum also will be measured. At the same level of approximation, nongaussian effects from warm inflation models were computed and found to be of the same order of magnitude as in the cold inflation case, and thus too small to be measured 30.

One interesting feature about warm inflation dynamics is that it offers a solution to the $\eta$-problem 31. In standard inflation models 1, 2, 3, where inflaton evolution is damped by the term $3H \dot{\varphi}$, the slow-roll condition amounts to $\eta_H \sim 1$, which equivalently means the potential can not have mass terms bigger than $\sim H^2 \varphi^2$.

Since Supersymmetry suppresses quantum corrections, thus can preserve the tree level potential, it has been a central idea in realizing such flat inflationary potentials. Of course, since inflation requires a nonzero vacuum energy density, inevitably SUSY must be broken during the inflation period, thus possibly ruining the desired degree of flatness in the potential. In particular, once supergravity effects are included, it becomes very difficult for this symmetry to preserve flatness at the level of $\eta_H < 1$. For F-term inflation, where the nonzero vacuum energy density arises from terms in the superpotential, no symmetry prohibits the appearance of the Planck mass suppressed higher dimensional operators $a_n \varphi^n / m_{pl}^{n+1}$ 32, 33, 34, 35. For the large class of chaotic inflation type models 8, where inflation occurs with the inflaton field amplitude above $m_{pl}$, to control these higher dimensional operators would require the fine-tuning of an infinite number of parameters or choosing only certain types of SUGRA corrections, such as the minimal Kahler potential. Even for models where inflation occurs for field amplitudes below the Planck scale, dimension six operator forms of $V \varphi^2 / m_{pl}^2 \sim H^2 \varphi^2$ can emerge and ruin the desired flatness. Both minimal and nonminimal Kahler potentials can lead to such terms 35.

One possible solution to the $\eta$-problem might be D-term inflation. In such models, the nonzero vacuum energy arises from the supersymmetrization of the gauge kinetic energy. However a closer examination 31, 35 reveals that attaining the required degree of flatness makes such models very restrictive.

Up to now, attempts to solve the eta-problem have sought symmetries that can maintain this desired degree of flatness. One of the few that has proven successful is called the Heisenberg symmetry 32, although it is very restrictive. Another proposal has been a certain shift symmetry 35, which is particularly interesting as it does not require SUSY. In common, all attempts so far have one foregone conclusion, that inflaton dynamics is only viable for $\eta_H < 1$. However, if the inflaton evolution happened to have a damping term larger than $3H \dot{\varphi}$, then clearly slow-roll can be satisfied for $\eta_H > 1$. Such a possibility is precisely what occurs in warm inflationary dynamics.

To see this let us examine the warm inflation solution for the simple potential

$$V = \frac{1}{2} m_\phi^2 \varphi^2.$$ (65)

In the cold inflation case, such a model requires an initial inflaton amplitude $\langle \varphi \rangle = \varphi > m_{pl}$. Moreover, SUSY models that realize a potential like this inevitably lead to an eta-problem based on the reasons discussed above.

Let us now treat this model in the warm inflation case. To focus on the essential points, our calculations here will be purely phenomenological, although they can be readily derived from a first principles quantum field theory calculation as done in Sect. III. We consider the case where the dissipative coefficient in Eq. 45 is independent
of both $\varphi$ and $T$, $\Upsilon = \text{constant}$. The inflaton initially is at a nonzero field amplitude $\varphi \neq 0$, thus supporting a vacuum energy.

The background cosmology for models with constant $\Upsilon$ and monomial potentials has been solved exactly \[6\]. From this we find $N_e \approx \Upsilon / m_{\varphi}^2$. The radiation production is determined from the energy conservation equation \[14\]. During warm inflation $\rho_r \approx 0$ \[22\], so that Eq. \[14\] reduces to Eq. \[61\]. Identifying $\rho_r \sim T^4$ permits determination of the temperature during warm inflation. Finally, once $T$ is determined, Eq. \[63\] allows determination of density perturbations.

Combining these expressions, for model Eq. \[65\] with $\Upsilon = \text{const.}$ in Eq. \[14\], gives

$$N_e \approx 2\sqrt{2} \frac{\Upsilon \varphi_0}{m_{\varphi} m_{pl}} \quad (66)$$

$$T \approx \frac{m_{\varphi}^{3/4} m_{pl}^{1/4} \varphi_0^{1/4}}{\Upsilon^{1/4}} \quad (67)$$

$$\frac{\varphi_0}{m_{pl}} \approx 5.3 \times 10^{-9} \frac{m_{\varphi}}{m_{pl}} \quad (69)$$

$$\Upsilon \approx 4 \times 10^{-8} m_{pl}, \text{ and } T \approx 10^{-4} m_{\varphi}, \text{ with the ratio } m_{\varphi}/m_{pl} \text{ free to set. For } m_{\varphi}/m_{pl} \lesssim 10^{-9}, \text{ it means } \eta_H > 1 \text{ and } \varphi < m_{pl}. \text{ Thus we see for sufficiently small inflaton mass, } m_{\varphi} \lesssim 10^{10} \text{GeV, there is no eta-problem, since } m_{\varphi} \gg H \text{ and } \varphi < m_{pl}. \text{ Since this warm inflation solution works for } \eta_H \gg 1, \text{ SUSY models realizing simple monomial potentials like Eq. \[65\] do not require any special symmetries as is the case for cold inflation models. The "eta" and large \varphi-amplitude problems simply correct themselves once interactions already present in the models are properly treated.}$$

V. HYBRID SUSY MODEL

Following the analysis of \[12\], Let us apply the results of the previous two sections to the SUSY hybrid model matter field $\Delta \bar{\Delta}$ coupled to it

$$W = \kappa S(\Phi_1 \Phi_2 - \mu^2) + g \Phi_2 \Delta \bar{\Delta}. \quad (71)$$

In this model, the inflaton is identified with the bosonic part of $S$, $\phi_S$. The above is a toy model representing an example of how the basic hybrid model, first term on the RHS, is embedded within a more complete particle physics model, in this case through the $\Delta$ fields. We will show that in the above model both cold and warm inflation exist and we will determine the parameter regime for them. This will then explicitly verify the conclusions from the recent papers on dissipation \[8, 9\], that showed both types of inflationary dynamics could exist. In both inflationary regimes, we will calculate the scalar spectral index $n_S - 1$, and its running $dn_S/d\ln k$. With this information, we will then identify the qualitative and quantitative differences arising from the warm versus cold regimes.

In this model the heaviest field with mass $m_+$ can decay into the massless fermionic partners of $\Delta$ and $\bar{\Delta}$, with decay rate:

$$\Gamma_+ = \frac{g^2}{16\pi} m_+. \quad (72)$$

We have $\Gamma_+ \propto \phi_S$, and this can be much larger than the Hubble rate during inflation:

$$\Gamma_+ / H = \sqrt{3} \frac{g^2}{16\pi} \left( \frac{m_p}{\mu} \right) (x_N^2 + 1)^{1/2}, \quad (73)$$

where $x_N \equiv \phi_S / (\sqrt{3}\mu)$. Having $\Gamma_+ / H > 1$, all the way up to the end of inflation, only requires $g > 0.16$ for $\kappa < 0.001 \text{ (} g > 0.01 \text{ for } \kappa = 0.5)$. This allows us to apply the adiabatic-Markovian limit in the effective EOM for the inflaton background field, Eq. \[6\], with the dissipative coefficient given now by:

$$\Upsilon \approx \frac{\pi^2}{2} \left( \frac{\kappa}{4\pi} \right)^3 \left( \frac{g^2}{16\pi} \right) \frac{x_N^2}{(1 + x_N^2)^{1/2} \mu}, \quad (74)$$

and the ratio to the Hubble rate is given by:

$$\frac{\Upsilon}{3H} \approx \frac{k^2}{128\sqrt{3\pi}} \left( \frac{g^2}{16\pi} \right) \frac{x_N^2 m_p}{(1 + x_N^2)^{1/2} \mu}, \quad (75)$$

which behaves like $\Upsilon / (3H) \propto x_N \propto \phi_S$, and so decreases during inflation. That is, the evolution of the inflation field may change from being dominated by the friction term $\Upsilon$ to be dominated by the Hubble rate $H$. Whether the transition between these two regimes happens before or after 60 e-folds will depend on the value of the parameters of the model like $\kappa$ and $g$. The amount of "radiation" obtained through the dissipative term, is given by:

$$\frac{\rho_R}{H^2} \approx \frac{9}{2} \frac{r}{(1 + r)^2} \epsilon_H \frac{m_p^4}{\kappa^2 \mu^4}, \quad (76)$$

which even when $\Upsilon > 3H$, could give rise to a thermal bath with $T > H$. In particular, we can have:

(a) $\Upsilon > 3H$, and $T > H$ ($\phi_S \approx -V_0/\Upsilon$):

$$\frac{\rho_R}{H^2} \approx \frac{36\sqrt{3}}{\pi^2} \frac{1}{g^2} \left( \frac{m_p}{\mu} \right)^5 \frac{1}{x_N^4}. \quad (77)$$
(b) \( \Upsilon < H \) \((\hat{\phi}_S \simeq -V_\phi/(3H)):\)

\[
\frac{\rho_R}{H^4} \simeq \frac{9}{256 \nu^2 \pi^2} \left( \frac{\nu}{4 \pi} \right)^4 \left( \frac{g^2}{16 \pi} \right) \left( \frac{m_p}{\mu} \right)^7 \frac{1}{x_N}. \tag{78}
\]

Note one might expect the presence of this thermal bath may induce thermal corrections to \( \Upsilon \) and \( V_{eff} \) but as shown in \([38]\), these corrections are negligible.

The values of the couplings \( \kappa \) and \( g \) for which we could have cold or warm inflation, and strong or weak dissipative dynamics, are plotted in Fig. (2). In order to get the different regions, we have proceeded as follow: for each pair of values in the plane \( \kappa - g \), the value of the inflaton field at the end of inflation is determined. This is done in the cold and weak dissipative regimes by the condition \( \eta_T = 1 \), Eq. (7). In the strong dissipative regime inflation can end either with \( \eta_T = 1 \) or it may also happen that most of the vacuum energy is already transferred into radiation during inflation, and then inflation will end when \( \rho_R \simeq \kappa^2 \mu^4 \) instead. In this case, whichever occurs first fixes the value of the inflaton field at the end of inflation.

The value of the inflaton field at 60 e-folds of inflation is then obtained from

\[
N_e \simeq -\int_{\phi_{60}}^{\phi} \frac{3H^2}{\Delta V}(1 + r)d\phi. \tag{79}
\]

This in turn fixes the value of the dissipative coefficient \( \Upsilon \), Eq. (73), the temperature of thermal bath, Eq. (20), and therefore the amplitude of the spectrum \( P_R \). The COBE normalization is then used to fix the value of the scale \( \mu \). In order to match the expressions for the spectrum across the different regimes, we have used a simple expression with:

\[
P_R^{1/2} = \frac{3H^2}{\Delta V} \left( 1 + r \right) \left( 1 + \sqrt{\frac{T}{H}} \right) \times \left( 1 + \left( \frac{\pi \Upsilon}{4H} \right)^{1/4} \right) \left( \frac{H}{2\pi} \right). \tag{80}
\]

We can see in Fig. (2) that the strong dissipative regime \( \Upsilon > 3H \) requires large values of the couplings, \( \kappa \sim g \sim O(1) \); for values \( \kappa \simeq g \simeq 0.1 \) we are in the weak dissipative regime; and for values \( \kappa \simeq g \simeq 0.01 \) we recover the cold inflationary scenario. Typically, for a fixed value of the scale \( \mu \) the amplitude of the spectrum in the strong dissipative regime would be larger than the one generated at zero \( T \). The COBE normalization implies then a smaller value of the inflationary scale \( \mu \). For example, for \( \kappa = g = 1 \) we have \( \mu \simeq 10^{13} \) GeV, whereas pushing the coupling toward its perturbative limit, \( \kappa = g = \sqrt{4\pi} \) we would get \( \mu \simeq 2 \times 10^{10} \) GeV. On the other hand, going from the cold to the weak dissipative regime, the value of \( \mu \) only varies by a factor of 2 or 3, and it is still in the range of the GUT scale \( O(10^{15}) \) GeV.

In Fig. (3) we have compared the prediction for the spectral index of the scalar spectrum of perturbations in both the CHI scenario, and warm hybrid inflation (WHI). From the warm inflation scenario we can always recover the CHI prediction by taking \( g \ll 1 \). In standard SUSY GUT hybrid inflation, for small values of the coupling \( \kappa \) the spectrum is practically scale invariant, it reaches a minimum around \( \kappa \simeq 0.01 \), and then rises due to SUGRA corrections up to positive values, which are disfavoured by WMAP results. But in the weak and the strong dissipative regime, due to the different origin of the spectrum, we get that the spectral index is still below 1 even for values of the coupling \( \kappa > 0.01 \). Specially in the strong dissipative regime, where the dynamic is such that the inflaton field is well below the Planck scale and SUGRA corrections are negligible. In that regime the departure from scale invariance is within the observational value, with \( n_s - 1 \simeq -0.022 \).

As a well-motivated example \([12]\), which combines inflation with leptogenesis and light neutrino masses \([37, 38]\), the inflaton can decay into right handed (s)neutrino fields \( \nu_Ri \) (\( i \) = family index). The decay proceed through the non-renormalizable coupling \( \Phi_1 \Phi_1 \nu_Ri \nu_Ri \), with decay rate,

\[
\Gamma_S = \frac{1}{8\pi} \left( \frac{M_i}{\mu} \right)^2 m_S, \tag{81}
\]

---

1 The value of \( \eta_T \) becomes larger than 1 before the other two slow-roll parameters.
where $M_i$ is the RH (s)neutrino mass. In the CHI scenario, with $\mu \simeq O(10^{15})$ GeV, $\kappa \simeq 10^{-2}$, and $m_S \simeq 10^{13}$ GeV, the gravitino constraint $T_{RH} \leq 10^9$ GeV translates roughly into $M_i \simeq 10^{-3} \mu \sim O(10^{12})$ GeV. Those values are also consistent with baryogenesis and light neutrino masses [38]. This kind of scenario is also viable in the warm inflationary regime. Being consistent with the observed baryon asymmetry and the atmospheric neutrino oscillations does not directly constrain the value of $\kappa$ but the value of $m_S \sim 10^{13}$ GeV. In the warm inflationary regime the value of the scale $\mu$ required for successful inflation reduces as we moved into the strong dissipative regime, $m_S$ is of the order of $10^{13}$ GeV for $\kappa \simeq O(1)$, and the gravitino constraint gives now $M_i \simeq 10^{-3} \mu \sim O(10^{10})$ GeV. Therefore, a model of warm inflation and leptogenesis without the need of small couplings would be viable and compatible with observations, in the strong dissipative regime.

Another interesting direction is determining tests that could discriminate warm versus cold inflation from observational data. The nature of the density perturbations in the two dynamics is different, so one should expect to find different observational signatures in the two cases. For one the tensor-to-scalar ratios differ for the two cases. Also, although we have not explored this direction in great detail in this talk, more careful analysis of density perturbations done by evolving the complete set of cosmological perturbation equations was done in [38]. This analysis showed that dissipative effects can produce a rich variety of scalar spectra ranging between red and blue and can also cause oscillations in the scalar power spectrum. Further understanding of all these possibilities is needed before firm criteria can be set of for discriminating between warm and cold inflation behavior in the CMB data.

VI. CONCLUSION

In this talk we have reviewed the basic ideas of warm inflation and compared them to the standard or cold inflation picture. An important point that has been attempted to be conveyed is that interactions in a inflation model Lagrangian can have significant effects during the inflation period. This point has led to the key result that many typical models of inflation, which had been assumed to yield just cold inflation dynamics, in fact have regimes of warm inflation. The realization of this fact leads to many interesting and new questions. From the one direction is model building. In the warm inflation case, there are some new features that do not exist for cold inflation. For one the $\eta$-problem can be eliminated simply by the dissipative dynamics and does not require any further constraints on the particle physics model, in particular the Kahler potential. This means greater flexibility to the high energy properties of the models. Another new feature is that for monomial potentials, observationally consistent warm inflation occurs for field amplitudes below the Planck scale, in contrast to cold inflation, where it is well known chaotic inflation requires $\varphi > m_{pl}$. This difference implies from the perspective of effective theories, monomial potentials are acceptable in the strong dissipative warm inflation regime.

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