Information content of the non-linear matter power spectrum

Christopher D. Rimes$^{1*}$ and Andrew J. S. Hamilton$^{1,2*}$

$^1$JILA, University of Colorado, 440 UCB, Boulder, CO 80309-0440, U.S.A.
$^2$Department of Astrophysical and Planetary Sciences, University of Colorado, 391 UCB, Boulder, CO 80309-0391, U.S.A.

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ABSTRACT

We use an ensemble of N-body simulations of the currently favoured (concordance) cosmological model to measure the amount of information contained in the non-linear matter power spectrum about the amplitude of the initial power spectrum. Two surprising results emerge from this study: (i) that there is very little independent information in the power spectrum in the translinear regime ($k \approx 0.2–0.8 \text{ h Mpc}^{-1}$ at the present day) over and above the information at linear scales and (ii) that the cumulative information begins to rise sharply again with increasing wavenumber in the non-linear regime. In the fully non-linear regime, the simulations are consistent with no loss of information during translinear and non-linear evolution. If this is indeed the case then the results suggest a picture in which translinear collapse is very rapid, and is followed by a bounce prior to virialization, impelling a wholesale revision of the HKLM-PD formalism.

Key words: cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

Measurements of galaxy clustering play a key role in the quest to accurately determine cosmological parameters. As pointed out by Eisenstein, Hu & Tegmark (1998, 1999), constraints from galaxy clustering directly complement those provided by the cosmic microwave background (CMB), enabling the breaking of parameter degeneracies which affect both types of observation when considered in isolation. The power of combining datasets in this way has recently been demonstrated by several groups (Seljak et al. 2004; Tegmark et al. 2004; Efstathiou et al. 2002) and the existence of a so-called ‘concordance’ cosmological model is largely thanks to this type of effort.

On large scales, the power spectrum of galaxy clustering traces the spectrum of primordial density fluctuations on those scales and is therefore a direct test of early-universe models. Many such models (including the simplest forms of the inflationary scenario) predict perturbations to the density field that are Gaussian random distributed, and to date this generic prediction has remained consistent with observation (Komatsu et al. 2003). For Gaussian fluctuations the power spectrum contains all possible statistical information about the perturbations.

At smaller scales, where much of the information in galaxy surveys lies, the extent to which the linear power spectrum can be recovered from the non-linear power spectrum remains unknown. The success of the HKLM-PD formalism (Hamilton et al. 1991; Peacock & Dodds 1994, 1996) in relating the non-linear power spectrum to the linear one suggests that non-linear evolution preserves at least some of the information in the power spectrum, albeit transported from larger to smaller comoving scales by the process of gravitational collapse. On the other hand, the simulations of Meiksin, White & Peacock (1999) indicate that non-linear evolution washes out baryonic wiggles in the power spectrum, suggesting that some information is perhaps irreversibly destroyed.

The purpose of the present paper is to investigate quantitatively, using N-body simulations, whether or not non-linear evolution preserves information in the matter power spectrum.

2 INFORMATION

We measure the Fisher information (Tegmark, Taylor & Heavens 1997) $I$ in the log of the amplitude $A$ of the initial (post-recombination) matter power spectrum:

$$I \equiv -\left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial \ln A^2} \right\rangle. \quad (1)$$

* E-mail: rimes@colorado.edu, Andrew.Hamilton@colorado.edu

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Here \( \mathcal{L} \) denotes the likelihood, which for Gaussian fluctuations takes the form \( \mathcal{L} \propto |C|^{-1/2} \exp \left( -\frac{1}{2} \delta C^{-1} \delta \right) \), where \( \delta \) is the observed data vector of fluctuations, and \( C \) is their expected covariance matrix. If fluctuations are statistically homogeneous and isotropic, then each Fourier mode \( \delta_k \) is independently Gaussianian distributed. The variance of Fourier modes, the diagonal elements of the diagonal covariance matrix, constitute the power spectrum \( \langle |\delta_k|^2 \rangle \propto P(k) \).

Thus, for Gaussian fluctuations, the likelihood depends on parameters only through the power spectrum \( P(k) \), and the information \( I \) defined by equation (1) can be written

\[
I = - \left( \sum_{k,k'} \frac{\partial \ln P(k)}{\partial \ln A} \frac{\partial^2 \ln \mathcal{L}}{\partial \ln P(k) \partial \ln P(k')} \frac{\partial \ln P(k')}{\partial \ln A} \right). \tag{2}
\]

For simplicity, in this paper we use results from simulations only at scales where the shot noise contribution to the power is subdominant, so that \( P(k) \) in equation (2) can be regarded as the cosmic variance.

During linear evolution, the partial derivatives of the log power with respect to log amplitude in equation (2) are just unity, \( \partial \ln P(k) / \partial \ln A = 1 \). A short calculation from the Gaussian likelihood function shows that, for Gaussian fluctuations, the information \( I \) of equation (2) equals half the number \( N \) of Gaussian modes:

\[
I = N/2. \tag{3}
\]

Here \( \delta_k \) and its complex conjugate \( \delta_{-k} \) are counted as contributing two distinct modes, the real and imaginary parts of \( \delta_k \).

At non-linear scales, we continue to define the information \( I \) in the non-linear power spectrum \( P(k) \) by equation (2). Clearly, there is a mapping from the initial linear power spectrum \( P_L(k) \) to the non-linear power spectrum \( P(k) \): to find it, just do an \( N \)-body simulation (the cosmic variance in the non-linear power spectrum should be negligible if the simulation is large enough). On the other hand, it is not clear a priori whether an inverse mapping exists. If it does, then the Fisher information \( I \), equation (2), in the non-linear power spectrum should equal that in the initial linear power spectrum: information is preserved. Conversely, if an inverse mapping does not exist, then the Fisher information in the non-linear power spectrum should be less than that in the initial power spectrum: non-linear evolution destroys information.

The definition (2) of information involves partial derivatives \( \partial \ln P(k) / \partial \ln A \) of the log of the non-linear power with respect to the log of the initial, linear amplitude. In an \( N \)-body simulation, increasing the initial amplitude is equivalent to evolving the simulation further in time. Thus the desired partial derivatives can be measured simply by comparing the amplitudes of non-linear power \( P(k) \) at successive epochs. It is this property that makes the information in the log amplitude especially convenient to measure from simulations: there is no need to perform simulations with different values of cosmological parameters.

The other factor in the definition (2) of information is the second derivative of the log likelihood with respect to the log non-linear powers. This factor is the Hessian of the vector \( \ln P(k) \) of log non-linear powers, the expectation value of which is the Fisher matrix with respect to the log powers. Since each \( P(k) \) involves an expectation over many modes \( \delta_k \), it is reasonable to invoke the central limit theorem to assert that estimates of power will be distributed in a Gaussian fashion about the expectation value, in which case this factor is approximately equal to the inverse of the covariance matrix of power spectrum estimates. This assertion holds even if the density field itself is non-Gaussian. The reliability of the approximation can be checked at linear scales (where there are fewest modes so the central limit theorem is least likely to apply), where the Fisher matrix should be diagonal, with diagonal elements equal to half the number of modes in each wavenumber bin.

3 SIMULATIONS

We generated 400 random realizations of a cubic region of the universe 256 \( h^{-1} \) Mpc on the side. A further 200 realizations with a box size of 128 \( h^{-1} \) Mpc were used to isolate numerical effects close to the mesh scale. The cosmological model used was the ‘vanilla-lite’ model of Tegmark et al. (2004, second-last column of their table 4): \( \Omega_M = 0.29, \Omega_b = 0.71, f_{\text{frac}} = \Omega_b / \Omega_M = 0.16, h = 0.71 \) and \( \sigma_8 = 0.97 \). The matter transfer function was calculated using the fitting formula of Eisenstein & Hu (1998). The boxes were evolved using a particle-mesh (PM) code with 128\(^3\) particles and a 256\(^3\) force mesh. Adaptive time-stepping was used to control the force errors; a typical run required be-
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Figure 2. Correlation matrices of estimates of the non-linear power spectrum for (a) the 256 $h^{-1}$ Mpc boxes and (b) the 128 $h^{-1}$ Mpc boxes. Positive correlations are shown by completely filled bins while diamonds denote anti-correlations. Greyscale represents the magnitude of the correlations, ranging from 0 (no correlation) to 1 (perfect correlation).

tween 800 and 1400 steps, with the 128$h^{-1}$ Mpc boxes requiring more, on average, because of the higher degree of clustering. We also ran 25 realizations of a 128 $h^{-1}$ Mpc box at higher resolution using the adaptive mesh refinement code ART (Kravtsov, Klypin & Khokhlov 1997) with a 128$^3$ root mesh and 3 levels of refinement. The initial conditions for these simulations were set up using the GRAFIC package with the same cosmological parameters as above. Although the small number of realizations is not sufficient to yield a precise estimate of the covariance matrix – we find that at least several hundred simulations are required to achieve convergence – they serve to confirm the results of the much larger set of PM simulations on small scales.

The evolution of the non-linear power spectrum is illustrated by Fig. 1, which shows the mean, shot noise-subtracted, power spectrum of the simulations at three epochs. The power spectrum was measured by calculating the density field on a 256$^3$ cubic mesh, using a cloud-in-cell scheme, and binning the resulting Fourier amplitudes in radial bins in $k$-space. For the ART simulations, ‘chaining’ (Jenkins et al. 1998) was used to reach small scales. Smoothing due to the mass assignment scheme was corrected for by dividing by the square of the Fourier transform of the window function, prior to subtracting the shot noise contribution (Smith et al. 2003). In the following analysis, we use only wavenumbers a factor of at least two away from the Nyquist frequency of the Fourier transform grid to avoid problems resulting from the uncertain mass assignment correction and aliasing of power from smaller scales. The power spectra from the two different box sizes agree well up to approximately 3 times the scale of the force mesh of the larger boxes, but at smaller scales the power in the PM simulations is systematically underestimated relative to the higher resolution ART simulations. The latter agree well with fits to the results of Smith et al. (2003) and Peacock & Dodds (1994). The error bars in Fig. 1 which are statistically independent, are actually attached to the uncorrelated band powers described below.

In Fig. 2 we plot the correlation coefficients between estimates of log-power in each pair of wavenumber bins, measured from the simulations. This is simply the covariance matrix scaled so that the diagonal elements are identically unity and has the advantage that, whereas the elements of the covariance matrix are smaller for larger simulation volumes, the correlation co-efficients are independent of box size. The results for the 200 128 $h^{-1}$ Mpc PM simulations and the 25 ART simulations are consistent, so we combine them into a single covariance matrix for this box size. Our results confirm those of Meiksin & White (1999), who found that (i) correlations between neighbouring band powers grow rapidly in the translinear regime, approaching 100% at non-linear wavenumbers and (ii) even for pairs of bands in which only one of the pair is non-linear there are significant correlations, of the order of 20–40%. The results from the two different box sizes are again broadly consistent.

We assign information to each wavenumber bin by defining a set of uncorrelated band-power estimates:

\[
\hat{B}(k) = \sum_{k'} W(k, k') \frac{\hat{P}(k')}{P(k')}
\]

(4)

(we use hats to denote measurements from individual simulations and hatless symbols for averages over all simulations). $W(k, k')$ is a decorrelation matrix (Hamilton & Tegmark 2000). Of the infinity of possible schemes for decorrelating the power spectrum, we use here the upper Cholesky decomposition of the Fisher matrix of the scaled power spectrum, suitably normalized, as our decorrelation matrix. We experimented with several decorrelation schemes and this was the only one that reproduced the expected amount of information in the linear regime, where it is reasonable to expect that information is con-
Cumulative information in the non-linear power spectrum at three epochs (\(a = 0.5\), 0.67 and 1) as a function of (a) comoving and (b) physical wavenumber. Large symbols are points derived from the 256 \(h^{-1}\) Mpc boxes and small symbols are from the 128 \(h^{-1}\) Mpc boxes (PM+ART). The results for the 128h\(^{-1}\) Mpc boxes have been shifted vertically by a factor 8 to account for the higher density of modes at a given comoving \(k\). The dotted lines show the information in the linear power spectrum. The solid curves are the result of applying the Peacock & Dodds (1996) wavenumber scaling to the linear information.

Mathematically, it is equivalent to taking a matrix composed of all the elements of the covariance matrix up to some wavenumber \(k_{\text{max}}\), inverting this matrix and summing all the elements of the resulting Fisher matrix to arrive at a measure of the accumulated information, \(I(< k_{\text{max}})\), up to that wavenumber. Scaling both sides of equation (1) by \(P(k)\) guarantees that the mean of the uncorrelated band power estimates at each wavenumber is equal to the mean power spectrum. It is the errors on the decorrelated band powers that are plotted in Fig. 1. Notice that some of the band powers have error bars much larger than the actual power that are plotted in Fig. 1. Possible reasons for the flatness of cumulative information: Either information is being lost from the power spectrum rather abruptly as structures enter the translinear regime and begin to collapse, or else information is flowing rapidly from large to small scales. The HKLM-PD formalism predicts that information should indeed flow from large to small scales as structures collapse but, as Fig. 3 shows, if information is preserved then the flow of information is far more rapid than predicted by the PD formula.

The second remarkable feature of the curves in Fig. 3 is that in the highly non-linear regime the cumulative information begins to rise sharply again (at \(k \simeq 0.8 h^{-1}\) Mpc\(^{-1}\) for \(a = 1\)). This upturn occurs consistently at all epochs, in both box sizes and with both codes, making it unlikely to be a numerical effect in the simulations (which ought, at the very least, to scale with the box size). The upturn is difficult to explain if information in the power spectrum is completely destroyed during translinear collapse. The possibility remains that during translinear evolution information is temporarily diverted out of the power spectrum into higher order moments and that it is somehow restored into the power spectrum at non-linear scales, but this explanation seems contrived and we do not explore it further.

The definitive test for whether information is being destroyed is to look at the cumulative information in the highly non-linear regime. At highly non-linear scales, structures virialize and therefore cease to collapse rapidly. The HKLM-PD formalism assumes the stable clustering hypothesis: that following virialization structures remain of fixed size in physical co-ordinates. Probably, the assumption of stable clustering is not precisely true (e.g. Padmanabhan et al. 1996; Smith et al. 2003). Nevertheless, the collapse or expansion of structures in the virialized regime is much slower than the rapid dynamic collapse that takes place in the translinear regime.

It follows that, if translinear and non-linear evolution preserve information in the power spectrum, then the cumulative information at a given fixed physical scale in the highly non-linear regime should remain constant. In other words,
the cumulative information at different epochs, plotted as a function of physical wavenumber $k/a$, should asymptote to a single common line in the highly non-linear regime, as it does in the PD formula. Conversely, if evolution destroys information, then the cumulative information at a fixed non-linear physical scale should decrease systematically with time. Fig. 3(b) shows that the cumulative information in the non-linear power spectrum at the smallest physical scales that can be reliably measured, is consistent, within the uncertainties, with no loss of information. What is more surprising is that on small scales, the cumulative information at the two later epochs exceeds that at the same physical scale at $a = 0.5$, although there are indications that all three curves will eventually asymptote to the same value in the highly non-linear regime. This temporary increase of information suggests that structures bounce back prior to virialization. At $a = 1$, for example, structures at $k \approx 0.2 - 0.8 \, h \, \text{Mpc}^{-1}$ are in rapid collapse, following turnaround. At smaller scales, structures have bounced and are actually expanding, before becoming fully virialized at $k \approx 3 \, h \, \text{Mpc}^{-1}$.

The simulations presented here are consistent with the hypothesis that non-linear evolution largely preserves information in the power spectrum. If this is true, then a wholesale revision of the HKLM-PD formalism is needed. The simulations suggest rapid translinear collapse followed by a bounce and subsequent virialization, in contrast to the rather gentle behaviour predicted by the HKLM-PD formalism.

This rapid collapse can be construed as supporting the alternative picture of non-linear evolution put forward by the halo model (e.g. Ma & Fry 2000; Peacock & Smith 2000; Seljak 2000). In the halo model, the cosmic density field is treated as a set of discrete, collapsed dark matter haloes, whose centres are clustered according to linear theory. The non-linear contribution to the matter power spectrum is determined by the density profiles of the individual haloes. Implicit in this picture is the assumption that the transition between the linear and virialized regimes is an abrupt one. Our direct measurements of the flow of information from large to small scales confirm that this indeed appears to be the case.

The results reported in this paper have several other practical implications beyond those relevant to analytic models of non-linear evolution. Firstly, if non-linear evolution completely preserves information in the power spectrum, then information about baryonic wiggles is preserved. Different and better simulations than carried out for this paper will be necessary to test what actually happens to baryonic wiggles. Secondly, if the translinear collapse to small scales is indeed as rapid as suggested by the simulations in this paper, then baryonic wiggles should be stretched over translinear scales much more than had previously been anticipated. Thirdly, if the translinear power spectrum contains little additional information over and above that in the linear power spectrum, then efforts to measure power at translinear scales may not be as rewarding, in the sense of refining estimates of cosmological parameters, as might have been anticipated. It should be emphasized that one should not interpret our results as implying that the translinear regime contains no information; rather, the information in the translinear regime is degenerate with that in the linear regime.

In future work it will be interesting not only to test the results of the present paper at smaller scales, but also to investigate the extent to which non-linear evolution does or does not preserve information about other cosmological quantities, such as the primordial spectral index, or the baryon fraction.

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