Nonperturbative QCD Effects in Weak Nonleptonic Decays

B. Blok
Institute for Theoretical Physics
University of California, Santa Barbara, CA 93106
and
M. Shifman
Theoretical Physics Institute
University of Minnesota, Minneapolis, MN 55455

Abstract

QCD-based analysis of nonfactorizable parts of weak nonleptonic amplitudes is reported. Nonperturbative effects due to soft gluon exchange play a key role leading to the emergence of a dynamical rule of discarding $1/N_c$ corrections.

1 Introduction

Factorization is used in nonleptonic decays from early sixties. However, as our knowledge of QCD and weak decay phenomenology deepens, the simple idea that one must factorize two V-A currents composing the effective weak hamiltonian evolves towards a rather sophisticated scheme with different ingredients. The purpose of this talk is to review recent progress in calculating deviations from naive factorization. We shall concentrate here on exclusive decays.( Inclusive decays are discussed elsewhere.) We shall show that the rule of discarding $1/N_c$ [1, 2, 3] has a dynamical origin, and is due to nonperturbative QCD effects. The key role is played by soft chromomagnetic gluon exchange. The resulting picture [4, 5, 6, 7, 8] is rather versatile—not all transitions are alike in this respect. QCD effects lead to deviations from the naive factorization, specific for each channels. These deviations can be estimated in a model-independent way. In some channels the situation is close to the predictions of the $1/N_c$ rule , in others — to naive factorization. The degree of cancellation of the naive $1/N_c$-suppressed amplitudes is different for each channel, so we can call our approach a dynamical rule of discarding $1/N_c$.

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2 The method

We shall describe our method using the decay $B^0 \to D^+\pi^-$ as an example. The reader is referred to ref. [4] for details. We are interested in the transitions induced by the color-octet times color-octet part of the weak Lagrangian $L \sim (\bar{c}\Gamma^\mu t^a b)(d\Gamma^\mu t^a u)$. In order to calculate the transition amplitude, we start with the correlator

$$A^\beta = \int d^4x <D|T\{L(x), A^\beta\}|\bar{B}> e^{iqx}$$

where the axial current $A^\beta = \bar{u}\gamma^\beta\gamma^5 d$ annihilates the pion. Two key steps are made in order to calculate the latter correlator: we continue $A^\beta$ into the Euclidean region $-q^2 \sim Q^2 \sim 1 \text{ GeV}^2$, and write in this region (borrowing some ideas from the QCD sum rule method [9]) a sum rule for the amplitude $M_{n.t.}$ governed by $L$:

$$A^\beta(Q^2) = M_{n.t.} \frac{f_\pi q^\beta}{q^2} + ...$$

where $+...$ denotes the contribution of higher resonances produced by the axial current. Second, we calculate $A^\beta$ using the Operator Product Expansion. We immediately obtain

$$A^\beta(Q^2) = -2i \frac{1}{8\pi^2} \frac{q^\alpha q^\beta}{q^2} <D|\bar{c}\Gamma^\mu t^a g\tilde{G}^a_{\alpha\mu} b|B> + ...$$

where $+...$ denotes higher order power corrections and we retained only the kinematical structure proportional to $q^\beta$. Comparing the latter two equations and neglecting the dots we immediately obtain that the ratio of the nonfactorizable and $1/N_c$ naive factorizable parts of the amplitude is

$$r = \frac{m^2_{\sigma H}}{4\pi^2 f_\pi^2}.$$  

Note the key distinction from the standard QCD sum rule method: the matrix elements are taken between hadronic states, not between vacuum states. Using the methods of HQET [10] it is easy to get for $m^2_{\sigma H} = \frac{3}{4}(m_{B^*}^2 - m_B^2)$.

It is instructive to emphasize the assumptions and approximations made in eq. (4). First, we neglected the corrections due to operators with higher dimensions, and contamination with higher resonances. Strictly speaking it is necessary to check that the corresponding window exists. This has not been done yet, although arguments in favor of smallness of the above corrections in a large class of transitions were given in ref. [4]. Second, and this is also important, we started from the theoretical limit where $M_B - M_D \ll \frac{1}{2}(M_B + M_D)$. Only in this limit the expansion in eq. (3) goes in dimensions, not twists. Moreover, in a number of cases the hadronic matrix element in eq. (3) reduces to the known quantity in this limit. Otherwise we would have to introduce an unknown function of recoil. Logarithmic corrections due to anomalous
dimensions are also not included so far (although in the transitions considered in ref. \[4\] they seem to be unimportant). We refer to ref. \[4\] for the detailed discussion of the method and expected uncertainties. The expected accuracy for this particular channel is of order one.

Keeping in mind all these uncertainties—a vast field for future work—one can try to extend the method to other weak hadronic decays in a straightforward way. If the particle that splits away is not a pion, we do not get a simple $1/Q^2$ term in the OPE, but rather a more complicated function. Moreover, the higher power corrections can become more important. For example, for the $B \rightarrow DD$ decays we get the function $F(Q^2) = 1/Q^2 - m_\pi^2/Q^4 \ln(Q^2/m_\pi^2)$ as a coefficient in front of the operator $G_{\mu \nu}$, instead of $1/Q^2$. The relevant sum rule takes the form

$$\frac{m_{\pi H}^2}{4\pi^2} F(Q^2) + ... = f_D \frac{M_{n.f.}}{Q^2 + m_D^2} + ..$$

(5)

(where we once again neglected higher power corrections.) It works well for the Euclidean momenta $Q^2 \geq 1 \text{GeV}^2$.

The amplitudes of decays considered above were proportional to $a_1$ in the BSW language \[1\]. The amplitudes of decays proportional to $a_2$ contain an absolutely unknown formfactor, the matrix element $< B | \bar{b} \gamma^\nu g \tilde{G}_{\alpha \nu} u | \pi >$, which cannot be determined using HQET. The sum rules for the decay $B^0 \rightarrow D^0 \pi^0$ and other decays of the type ”$B \rightarrow D$+light meson” in this group will be similar to the above, (with the function $F(Q^2)$ instead of $1/Q^2$) but will include this new formfactor. For the decay $B \rightarrow J/\psi K$ we have a new function $\tilde{F}(Q^2) = 2m_c^2 \int_{4m_c^2}^{\infty} \frac{ds}{(s+Q^2)^{3/2}(s-4m_c^2)}$ in the sum rule instead of $F$.

## 3 Decay widths

Let us briefly discuss numerical aspects of our results. We shall concentrate on the values of $r$ and the amplitudes $a_1$ and $a_2$ that can be directly compared with the experimental data.

For decays $B^0 \rightarrow D^+ \pi^-$, $B^0 \rightarrow D^+ \rho^-$ we get $r \sim -1.5$ and $-1$ respectively. For the decays $B^0 \rightarrow D^{*+} \pi^-$, $B^0 \rightarrow D^{*+} \rho^-$ we get $r' = r/3$ \[4\]. Taking here and below $c_1 \sim 1.12, c_2 \sim -0.26$, we obtain for these decays $a_1 \sim 1.16, 1.12, 1.08, 1.06$ respectively.

Consider now other decays using the same method. The discussion below is given for orientation only, keeping in mind that the effects unaccounted for in our analysis (see section 2) may be important for these decays. If we neglect these effects, we obtain for them once again the formulae similar to the one in eq. (3). Consider first the decays from the $B \rightarrow DD$ group. Their amplitudes are also proportional to $a_1$ and can be obtained using the sum rule sketched in section 2. We get for $B \rightarrow DD$ decays $r \sim -0.9 \frac{m_{\pi H}^2}{4\pi^2 f_D}$, where $f_D$ is a leptonic decay constant taken to be $\sim 170 \text{ MeV}$. We
immediately see that $r \sim -0.8$. For $B \to D^*D^*$ decays using HQET we obtain $r \sim -0.9 \frac{m^2_{\pi} \mu}{12 \pi^2 f^2_D} \sim -0.16$. For $B \to D^*D$ decays we get $r \sim -0.5$. For the corresponding $a_1$ factors in the amplitudes we find $a_1 \sim 1.1, 1.04, 1.07$ respectively. Note that our results for different channels lie between BSW [1] and naive factorization. The accuracy of these results is lower than for the previous group of decays, since we expect here the perturbative logs and higher corrections can play a bigger role.

Consider now the decays proportional to the factor $a_2$. Here we shall be extremely speculative, since the corresponding analysis is far from being completed. We shall only try to indicate what we expect for these decays at the moment, leaving more solid statements for the future investigation. The main difficulty here is that we do not know the key formfactor $\langle B | \bar{b} g \tilde{G}_{\alpha \mu} \gamma^\mu u | \pi \rangle > P^\alpha$ (and the corresponding formfactor with the $\rho$-meson). We can try to roughly estimate these formfactors from our knowledge of the D meson decays using the symmetry between b and c. (Unfortunately, such estimates are very uncertain, though.) Let us completely ignore the recoil dependence in the formfactor $\langle B | \bar{b} g \tilde{G}_{\alpha \mu} \gamma^\mu u | \pi \rangle >$ (unlike $B^0 \to D^+\pi^-$ there is no justification for that) and parametrize $\langle B | \bar{b} g \tilde{G}_{\alpha \mu} \gamma^\mu u | \pi \rangle >$ by a number $m^2_{\pi H} \sim x m^2_{s H}$, where $x$ is an unknown constant. Then for the decays of the type $B^0 \to D^0\pi^0$ we obtain $r \sim -1.6x$, for $B^0 \to D^{*0}\pi^0$ we obtain $r \sim -0.8x$ (the difference between the values of $r$ for decays to $D$ and $D^*$ is proportional to $f^2_D / f^2_{D^*}$, and we use $f_D = 170$ MeV, $f_{D^*} = 220$ MeV). The value of $x$ is not known, but the experimental data on D seems to indicate that it is below 0.4. If this is indeed the case for B decays, then the value of $a_2$ will be suppressed in comparison with the exact $1/N_c$ rule for this group of B decays, and can even be equal to zero for sufficiently small $x$. Such a suppression is favored by the recent experimental data [1]. Future calculations are needed to establish $x$, and at moment we cannot make any definite theoretical statement about this group of decays.

Finally, we note that the same calculation for $B \to J/\psi K$ leads to small $r$ due to a big leptonic decay constant $F_{J/\psi} \sim 300$ MeV, $a_2 \sim 0.12$. However in this case there exist new difficult problems, due to a large recoil, an enhanced role of higher power corrections and higher twists and big continuum contribution (presumably absent for other modes). Moreover, hard gluons can play a significant role in this decay. (The sum rule from section 2 has no stability “window” in this case). Thus, we cannot exclude the possibility of the rule of discarding $1/N_c$ in this channel yet, neither can we confirm it.

We stress here that the pattern of amplitudes proportional to $a_2$ presented above is speculative and is nothing else than an educated guess. A lot of work, especially on the determination of chromomagnetic nondiagonal formfactors remains to be done.
4 Conclusion

We tried here to draw a general picture for deviations from the naive factorization in B decays which stems from nonperturbative QCD. A few aspects requiring further clarification are as follows. Higher power corrections must be calculated, perturbative logs must be taken into account and the nondiagonal magnetic formfactors must be determined. After all this is done the expected accuracy of our results may be 20-30%.

We also considered $K \rightarrow \pi\pi$ and $K - \bar{K}$ mixing parameter (see ref. [5]), as well as inclusive widths of B and D [6, 7, 8].

Summarizing, the nonperturbative QCD effects play a key role here in weak hadronic decays and can be responsible for the dynamical rule of discarding $1/N_c$.

We now have a general method that allows one to carry out these calculations with reasonable accuracy for all decays of B, D and K.

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