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Abstract. Fiber optical parametric amplifiers (FOPAs) are regarded as one of the candidate optical amplifiers for future ultralong-distance, large-capacity, and high-speed optical fiber communication systems. FOPAs provide high signal gain and low noise figures compared with other optical amplifiers. However, they suffer from gain fluctuation, which restricts commercial application. Optimization of the dispersion parameters of highly nonlinear fibers (HNLFs) employed as gain media is essential because they have an immediate impact on the flatness of the amplifier. An optimization method for multiple HNLF segments using the differential evolution algorithm is proposed to minimize spectral gain variation. We theoretically yield an FOPA with an average signal gain of 20 dB and gain fluctuation <0.5 dB within 400 nm.© The Authors. Published by SPIE under a Creative Commons Attribution 4.0 Unported License. Distribution or reproduction of this work in whole or in part requires full attribution of the original publication, including its DOI. [DOI: 10.1117/1.OE.58.8.086104]

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1 Introduction
With the development of optical fiber communication systems, increasing demand has been placed on the performance of optical fiber amplifiers.1 However, currently available commercial optical fiber amplifiers, such as erbium-doped fiber amplifiers and Raman amplifiers, do not meet the requirements of higher gain, broader bandwidth, and lower noise figures (NFs) simultaneously due to the limitations of their amplification mechanism.2 As a well-known amplifier, fiber optical parametric amplifier (FOPA) has many attractive features, including large signal gain,3 broad bandwidth,4 and a low NF;5 especially noiseless amplification when it is operated in the phase-sensitive regime.6 Thus, it has been applied to many scenarios, such as linear optical signal amplification, transparent wavelength conversion, return-to-zero pulse generation, all-optical sampling, etc.7 It is also considered to be the most promising optical amplifier for next-generation high-capacity and long-distance optical fiber communication systems.8

For an ordinary FOPA, it is difficult to obtain all outstanding gain characteristics at the same time. Recently, it was demonstrated that a signal gain of 20 dB at 2-THz wavelength bandwidth could be achieved using a single-pump FOPA; however, the gain spectrum suffers from serious gain differences as large as 10 dB.9 Thus, a single-pump FOPA faces severe challenges in providing a high and flat gain spectrum over a broad wavelength range.10 To address this issue, many configurations have been designed to ensure low signal gain variation.11–13 Among them, designs for the control of the dispersion characteristic of a nonlinear interacting media, such as highly nonlinear fiber (HNLF), have received the maximum attention. However, obtaining the best parameter sets of HNLFs is complicated and time-consuming, as a larger number of parameters needs to be determined. One may choose these parameters through several repetitive attempts; however, it is uncertain whether the best result can be obtained. It is obvious that this process will become more efficient if a method can dynamically optimize the parameters according to gain fluctuation. Some optimization algorithms based on the principle of biological evolution, such as genetic algorithm and differential evolution (DE) algorithm, are good choices in this regard. Compared with the genetic algorithm, the latter one usually exhibits better convergence properties and reduced complexity.14

In this paper, we propose a single-pump FOPA with multistage dispersion management of HNLFs, which can provide a high gain spectrum with reduced gain variation after optimization. Since several key parameters of the HNLFs of the multistage structure, which immediately affect the spectrum of the gain, need to be set, the DE algorithm is employed to control them effectively. First, we define a mathematical model for the multistage dispersion management of FOPA, which includes all the parameters to be calculated by DE optimization. Then, we analyze the impact of these parameters on the parametric gain spectrum and choose the gain fluctuation as the objective function for the DE algorithm. After applying DE algorithm, we theoretically achieve a single-pump FOPA with a gain spectrum of 20 dB and <0.5-dB fluctuation within 400-nm bandwidth.

The rest of this paper is organized as follows. Section 2 describes the FOPA model and application of the DE algorithm, Sec. 3 presents and discusses the simulation results, and Sec. 4 summarizes the study.

2 Methodology

2.1 Fiber Optical Parametric Amplifiers Model
The single-pump FOPA with multistage structure is shown in Fig. 1. The multistage structure employs m-span HNLFs...
including different parameter sets, which are dynamically controlled by the DE algorithm. First, the amplification of a single-pump FOPA with 1-span HNLF by solving differential equations which involve three waves (that is, pump, signal, and idler) can be written as

$$G_s = \frac{|E_s(L)|^2}{|E_s(0)|^2} = 1 + \left[ \frac{\gamma P_p}{g} \sinh(gL) \right]^2,$$  \hspace{1cm} (1)

where $E_s(0)$ and $E_s(L)$ represent the small signal field at the input and output of the HNLF with length $L$, respectively, $P_p$ is the power of the pump signal, and the nonlinear coefficient $\gamma$ is relative to the HNLF. The parametric gain coefficient $g$ is given by

$$g = \left[ -\Delta \beta\left( \frac{\Delta \beta}{4} + \frac{\gamma P_p}{g} \right) \right]^{1/2},$$  \hspace{1cm} (2)

where $\Delta \beta$ donates the linear phase mismatch between the propagation constant of the signal, idler, and pump, and expressed as

$$\Delta \beta = \beta_s + \beta_i - 2\beta_p.$$  \hspace{1cm} (3)

Expanding $\beta(\omega)$ in Taylor series to the fourth order near the pump, $\Delta \beta$ can be rewritten as

$$\Delta \beta = \beta_2(\omega_s - \omega_p)^2 + \frac{\beta_4}{12}(\omega_s - \omega_p)^4.$$  \hspace{1cm} (4)

Then, for an $m$-stage architecture, the final signal gain at the output can be written as

$$G_m = \prod_{k=1}^{m} G_{s,k}.$$  \hspace{1cm} (5)

In Eqs. (1)–(4), there are five main independent variables that affect the signal gain spectrum of the single-pump FOPA model, namely $P_p$, $L$, $\gamma$, $\beta_2$, and $\beta_4$. Equation (5) is used to calculate the total parametric gain in Sec. 3.

### 2.2 Design of Differential Evolution Optimization Algorithm

As the number of parameters increases, it becomes inefficient for the random trial method to obtain optimal results quickly. Based on the DE optimization algorithm,\textsuperscript{14–16} we designed an algorithm for the FOPA model, which mainly employs the difference between adjacent population vectors to update the subsequent stage, obtaining the final result. The main steps are outlined as follows in detail to clarify the concept.

Step 1: Initialize. Assume there are $N$ segments of HNLF and a one-dimensional search space, $D$, which includes an initial population of $M$ individuals. Then each generation vector is written as

$$X_i(0) = [x_{i1}(0), x_{i2}(0), \ldots x_{iN}(0)],$$  \hspace{1cm} (6)

where $x_i$ are individuals with $L$ and $\beta_i$ values. For the $i$th individual, its $j$th value is set as

$$X_{ij}(0) = L_{j, \text{min}} + \text{rand}(0,1)(L_{j, \text{max}} - L_{j, \text{min}}).$$  \hspace{1cm} (7)

Step 2: Mutate. Within the $D$ search space, every three individuals are mutated. Then the differential vector of the three random individuals is computed, and the $F_c$ factor collected. The differential vector can be written as

$$H_i(t) = X_{s1}(t) + F_c \bullet [X_{s2}(t) - X_{s3}(t)].$$  \hspace{1cm} (8)

where $i$ is the $i$th generation, $X_{s1}$, $X_{s2}$, and $X_{s3}$ represent three different individuals of the $i$th generation, and $F_c$ is a factor that is used to control the DE. The mutation operator is written as

$$V_i(t) = X_{s1}(t) + F_c \bullet [X_{s2}(t) - X_{s3}(t)].$$  \hspace{1cm} (9)

Step 3: Crossover. The definition of crossover is as follows:

$$v_{ij} = \begin{cases} h_{ij}(g), & \text{rand}(0,1) \leq cr \\ x_{ij}(g), & \text{else} \end{cases}$$  \hspace{1cm} (10)

where $cr$ is called the crossover rate and is defined as

$$cr_i = \begin{cases} cr_j + (cr_k - cr_j) \frac{f_i - f_{\text{min}}}{f_{\text{max}} - f_{\text{min}}}, & \text{if } f_i > f_{\text{av}}, \\ cr_j, & \text{else} \end{cases}$$  \hspace{1cm} (11)

and $f_i$ is the fitness of the $X_i$th individual.

**Fig. 2 DE optimization algorithm flowchart.**
Step 4: Select. Here, we reproduce the new next-generation matrix according to the following definition:

\[ X_i(t+1) = \begin{cases} V_i(t), & \text{if } (f(V_i(t)) < f(X_i(t))) \\ X_i(t), & \text{else} \end{cases} \]  

where \( f() \) is the objective function that calculates the gain variation. To demonstrate this optimization algorithm clearly, its flowchart is shown in Fig. 2.

We also show the detailed pseudocode of the DE algorithm in Algorithm 1.

Algorithm 1 DE algorithm in the proposed FOPA architecture.

1. Define Dimensionality: \( D \); Population: \( M \); Generation: \( T \); Segment No.: \( N \); Desired gain variation: \( \epsilon \).
2. Initialize \( t = 0 \);
3. For \( i = 1 \) to \( M \) do
4. For \( j = 1 \) to \( N \) do
5. \[ x_{ij}(t) = x_{ij}^{\text{min}} + \text{rand}(0,1)(x_{ij}^{\text{max}} - x_{ij}^{\text{min}}); \]
6. End
7. End
8. While \( (t \leq T) \) do
9. For \( i = 1 \) to \( M \) do
10. (Mutation and crossover)
11. For \( j = 1 \) to \( N \) do
12. \[ v_{ij}(t) = \text{Mutation}\left(x_{ij}(t)\right); \]
13. \[ u_{ij}(t) = \text{Crossover}\left(x_{ij}(t), v_{ij}(t)\right); \]
14. End
15. (Selection)
16. If \( f(u_{ij}) < f(x_{ij}) \) then
17. \[ x_{ij}(t) = u_{ij}; \]
18. If \( f(x_{ij}) < f(\Delta) \) then
19. \[ \Delta = x_{ij}; \]
20. End
21. Else
22. \[ x_{ij}(t) = x_{ij}; \]
23. End
24. End
25. \( t = t + 1 \);
26. End
27. Return the best result;

3 Simulation Results and Discussion

It is well known that the shape of the parametric gain spectrum is determined by the dispersion properties of the HNLF. It can be found that the \( \beta_2 \) term is the main factor that acts on the linear phase mismatch variable \( \Delta f \) and affects parametric gain fluctuation rather than \( \beta_4 \). In practice, the \( \beta_2 \) term can be precisely adjusted by controlling the dispersion slope of the HNLF. Thus, \( \beta_4 \) can be assumed to be constant within a small wavelength range, and \( \gamma \) can be set to a constant value. Then, we study how the three main parameters \((P_p, \beta_2, \text{ and } L)\) impact the gain characteristics. Table 1 illustrates the basic parameters of the HNLF.

The impact of different pump powers on the parametric gain is shown in Fig. 3. It can be found that, with the increasing pump power from 0.5 to 1.5 W when the length of the HNLF is set to be 100 m, both the peak parametric gain and bandwidth increase. At the same time, the flatness worsens. Figure 4 shows the impact of different pump powers along the HNLF on the peak gain and gain fluctuation. It is observed that with higher pump power and longer HNLF, an FOPA obtains greater peak gain; however, the gain fluctuation increases as well.

Figure 5 shows a set of parametric gain spectrum affected by different values of second-order dispersion \((\beta_2)\). The range of \( \beta_2 \) in these calculations is from \(-0.04 \) to \( 0.04 \) and \( P_p \) and length are set to 1 W and 100 m, respectively. The parametric gain spectrum displays obvious distinct shapes with positive and negative values. A positive \( \beta_2 \) results only

| Parameter                      | Value          |
|--------------------------------|----------------|
| Pump wavelength                | 1557 nm        |
| Zero dispersion wavelength     | 1556.5 nm      |
| Nonlinear coefficient          | 20 W\(^{-1}\) · km\(^{-1}\) |
| Second-order dispersion        | \(-0.02\) ps\(^2\) · km\(^{-1}\) |
| Fourth-order dispersion        | \(-0.02 \times 10^{-4}\) ps\(^4\) · km\(^{-1}\) |

Fig. 3 Parametric gain with different pump power \((P_p)\).
in one peak of the parametric gain while a negative $\beta_2$ shows two peaks. Moreover, the value of $\beta_2$ has a significant effect on the parametric gain bandwidth: the smaller the absolute value of $\beta_2$, the larger the bandwidth. This is because a small value of $\beta_2$ will move the phase-matched wavelength far away from the pump wavelength and as a result, broader bandwidth is achieved. In addition, varying values of $\beta_2$ do not influence the peak gain.

For the single-pump FOPA model with a different number of HNLF segments, we set the power of the pump at 1 W. Then, by assuming $\beta_4$ to be constant ($-0.02 \times 10^{-4} \text{ ps}^4 \cdot \text{km}^{-1}$ in the simulation), our optimization method determines the values of $L$ and $\beta_2$ to achieve optimal results; when the number of HNLF segments is set to $N$, there will be $2(N - 1)$ parameters to optimize. After optimization, the signal gain spectrum and NF calculated after each HNLF segment are shown in Figs. 6(a) and 6(b), respectively. It can be seen that the gain and NF fluctuation decrease obviously after 4-span HNLFs optimization and finally, we obtain a flat parametric
Table 2 Parameters of each HNLF optimized by the DE algorithm.

|      | Segment |
|------|---------|
|      | 1       | 2      | 3      | 4      |
| Length (m) | 148     | 73     | 59     | 29     |
| $\beta_2 \times 10^{-2}$ ps$^2$ km$^{-1}$ | 0.1     | 0.14   | 0.38   | 0.25   |

gain spectrum of 20 dB and fluctuation of <0.5 dB within 400-nm bandwidth. For this four-stage structure, the value of the parameter $\beta_2$ and length of each HNLF optimized by the DE algorithm are listed in Table 2.

4 Conclusions

In this paper, we propose an optimization method for a single-pump FOPA with multiple HNLF segments based on the DE algorithm. The influence of different parameters on the gain spectrum is also studied. When we set the number of HNLF segments to 4, good performance can be achieved by the proposed method. The results show that this method can effectively determine the optimal parameters of several HNLF segments needed for gain fluctuation minimization. The simulation suggests that the proposed method could offer a reference for the design of HNLF as a gain media for FOPAs.

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