Abstract Systematic experimental studies of few-nucleon systems expose various dynamical ingredients which play an important role in correct description of observables, such as three-nucleon force, Coulomb force and relativistic effects. A large set of existing experimental data for $^1H(d, pp)n$ reaction allows for systematic investigations of these dynamical effects, which vary with energy and appear with different strength in certain observables and phase space regions. Moreover, systematic comparisons with exact theoretical calculations, done in variables related to the system dynamics in a possibly direct ways is a very important tool to verify and improve the existing description of the nucleon interaction. Examples of experimental data for a breakup reaction, transformed to the variables based on Lorentz-invariants are compared with modern theoretical calculations.

1 Introduction

The $dp$ breakup is one of the simplest processes for testing the dynamics of three nucleons. The process is characterized by a rich kinematics of the final state which makes it selective with respect to the used model of interaction. In the wide range of energy the properties of few-nucleon systems are very well described with the use of realistic nucleon–nucleon (NN) potentials, coupled-channel (CC) calculations with realistic potential including the excitation of a single nucleon to a $\Delta$ isobar [1] or Chiral Perturbation Theory (ChPT) [2]. With increasing energy of interaction, the dynamical effects of few-nucleons, like Three Nucleon Force (3NF) [3, 4] and the relativistic component [5], start playing an important role and must be included in theoretical calculations. In case of the $dp$ breakup reaction, the Coulomb force has also a very significant influence on the cross section and also should be included in theory. The 3NF models, like Tucson Melbourn (TM99) [3] or Urbana IX (UIX) [4] are combined with realistic NN potentials to calculate the 3N system observables [6]. The 3NF appears naturally in ChPT at a certain order while in case of CC calculations a single $\Delta$-isobar
degree of freedom is explicitly included. Over the last few years a big effort for including all dynamical ingredients in theoretical calculations has been made. Currently calculations combining the 3NF and Coulomb effects are available [7,8] as well as relativistic calculations including 3NF [9,10]. High precision experimental data for the breakup process and exact theoretical calculations give an opportunity to study these very subtle dynamical effects with possible feedback on the force models. The kinematics of the $^1H(d,pp)n$ reaction can be described in many equivalent ways, e.g. using information about the energies and emission angles of registered protons or with the Jacobi momenta, which is a very useful method for the description of reactions with three or more nucleons in the final state [11]. In order to regularly study a large database collected in a wide range of energies, a correct choice of kinematic coordinates is important. That is why an alternative way of the description of breakup reaction kinematics has been proposed [12]. The Mandelstam variables have been rewritten in a convenient way for a breakup reaction, $p + d \rightarrow p^{(1)} + p^{(2)} + n$, where all particles exist in the entrance channel—deuteron is described as a proton–neutron pair flying together with a total 4-momentum of $p_d$, so neglecting a relatively small bounding energy each of these particles has a 4-momentum of $p_d/2$. In the exit channel there are two indistinguishable protons and one cannot say which of them, $p^{(1)}$ or $p^{(2)}$ was free (or bound) before the interaction. Based on 4-momentum of proton $p_p$ and deuteron $p_d$ in the entrance channel and 4-momentum of two protons $p_{p^{(1)}}, p_{p^{(2)}}$ and neutron $p_n$ in the exit channel, we define the following set of coordinates:

\[- s_{pp} = (p_{p^{(1)}} + p_{p^{(2)}})^2 — for protons in the exit channel,\]
\[- s_{pn} = (p_{p^{(1)}} + p_n)^2 — for a proton–neutron pair in the exit channel (a pair with the smallest value of $s_{pn}$),\]
\[- t_n = (p_d/2 − p_n)^2—4-momentum transfer from a bound neutron in entrance channel to a free neutron in the exit channel,\]
\[- t_p = (p_p − p_{p^{(z)}})^2—4-momentum transfer from an unbound proton to one of two protons in the exit channel (we choose the proton which wasn’t used in the calculation of $s_{pn}$).\]

Instead of using rest masses of outgoing particles we propose to use variables corresponding to the kinetic energy of relative motion of two nucleons (proton–proton, proton–neutron) in the final state

\[E_{rel}^{pp} = \sqrt{s_{pp}} - 2m_p, \quad E_{rel}^{pn} = \sqrt{s_{pn}} - m_p - m_n,\]  

and the energy transfer from a bound neutron or free proton in the entrance channel to a neutron or one of two protons, respectively, in the exit channel

\[E_{tr}^n = \frac{-t_n}{2m_n}, \quad E_{tr}^p = \frac{-t_p}{2m_p}.\]  

In this paper the experiment—theory comparison of 3NF effects is performed for cross section presented as a function of coordinates based on Lorentz-invariants.

2 Experiment

In order to study 3NF effects in a systematic way, the breakup of a deuteron in collision with a proton has been used. Experimental studies of the $^1H(d,pp)n$ reactions with deuteron beams of energies 130 and 160 MeV were performed at KVI in Groningen, using SALAD and BINA detectors [12–14]. The SALAD detector consisted of a three-plane MWPC and a scintillator hodoscopes: 24 horizontal detectors (ΔE) and 24 vertical stopping detectors (E), arranged perpendicularly to one another. It covered laboratory polar angles between 10° and 40° and the full range of azimuthal angles. The BINA detector consisted of two parts: Wall which is similar to SALAD and BALL which was built of almost 150 scintillators. BALL covered laboratory polar angles up to 160° and the full range of azimuthal angles.

3 Results

Measurements of the differential cross section of the $dp$ breakup reaction at 130 and 160 MeV have been done in a very wide range of the available phase space. The research shows that the 3NF plays a crucial role in the cross section. Figure 1 presents the comparison of 3NF effects predicted in theory (full circles) and observed in the experiment (open circles) of $dp$ breakup at 130 MeV. The y-axis represents the ratio $\frac{\sigma_{\text{CDB}}}{\sigma_{\text{CD++}}}$, where
\[ \sigma_i \text{ denotes the theoretical (with the 3NF included) or experimental differential cross section. The theoretical calculations are based on the CD-Bonn potential with } (\sigma_i) \text{ and without } (\sigma_{CD}) \text{ Tucson Melbourne 3NF (TM99) included. Generally, experimental data reveal effects beyond pure CD-Bonn NN potential, which are consistent with predicted influence of the 3NF. The disagreement between the data and theory at the largest } E_{rel}^{pn}, \text{ the lowest } E_{rel}^{pp} \text{ and } E_{tr}^p \text{ is due to missing Coulomb force in the calculations. For better localization of 3NF effects two-dimensional spectra have been constructed. Figure 2 presents the net effects in function of two out of four invariants. The shapes of these spectra correspond to the acceptance of the } dp \text{ breakup experiment at 130MeV while the colors code the magnitude of the effect. Figure 3 presents theoretical predictions of the 3NF effects for the } dp \text{ breakup at 160MeV. Calculations have been done by A. Deltuva for the Argonne V18 (AV18) potential combined with Urbana IX (UIX) 3NF [8]. The effect of 3NF is expected to be even twice so large as at energy of 130MeV, particularly for the smallest } E_{rel}^{pn} \text{ and the largest } E_{rel}^{pp}, \text{ which were not accessible in experiment at lower beam energy. For the } E_{tr}^p \geq 90 \text{MeV one could observe an increasing value of the } \frac{\sigma_{AV18+UIX} - \sigma_{AV18}}{\sigma_{AV18}}, \text{ which suggests that the 3NF in this region exists. These predictions will be soon confronted with the experimental data.} \]
4 Outlook

Systematic studies of the breakup reaction in a wide range of the phase space are very important for understanding the interaction between nucleons in few-nucleon systems. In order to verify the existing theoretical approaches, the large and exact experimental database is needed. Results presented in this paper seem to confirm the expectation that the cross section is very useful to perform systematic studies of 3NF effect in the regime of invariant coordinates.

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