Communication through an extra dimension

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Abstract

If our visible universe is considered a trapped shell in a five-dimensional hyper-universe, all matter in it may be connected by superluminal signals traveling through the fifth dimension. Events in the shell are still causal, however, the propagation of signals proceeds at different velocities depending on the fifth coordinate.

PACS numbers: 03.50.-z, 03.50.De
1 The universe as finite size shell

Modern physics advocates for the existence of extra dimensions besides the four we seem to live in. This assumption creates enough room for a geometrical unification of the different interactions between elementary particles and their grouping in symmetry multiplets.

Our apparent blindness to these extra dimensions is explained by resorting to the hypothesis of compactification. It amounts to identifying all the extra dimensions as essentially closed circles with a minuscule (thereby undetectable) radius.[1] Such an explanation originated in the work of Oskar Klein[1], and has arose both support and antagonism over the years.[2]

On the other hand, the possibility of the existence of large extra dimensions has been given considerable attention recently. For compactified large extra dimensions, it appears that, the experimentally safe limit allows for them to be of almost macroscopic size in submillimeter range.[3] This concept was put forth for the first time by Antoniadis in connection to the problem of supersymmetry breaking in string theory.[4]

An even more intriguing possibility of our universe as a shell has been called upon in order to try and solve the hierarchy problem.[5] The latter approach seems to be consistent with Newton’s law of gravitation if the gravitons themselves are confined to the shell.[6] The idea has also prompted cosmological investigations and the scenario can be made consistent with the accepted models of inflation and cosmological evolution.

Alternatively, the induced-matter approach of Wesson[7] resorts to extra dimensions that are not curled-up, in order to generate matter fields. Five-dimensional Einstein gravity of empty space is used to generate our four-dimensional universe both in its geometric (curvature) aspects and its energy-momentum contents. The very existence of mass and charge of elementary particles is claimed to be a consequence of the properties of a universe with extra dimensions, a feature already noticed by Kaluza and Klein in the early part of the century.[1] Mass could be related to the fifth coordinate and charge to the momentum along it. The extra dimension shows up in the physical properties of particles.

It seems that extra dimensions are slowly losing their role as mere ancillary variables.

A few years ago, several authors in the physics literature speculated about the possibility that our universe is a thin shell in a larger dimensional hyperuniverse. In this approach, the extra dimensions are not curled-up, hence their influence might be felt perhaps at an energy scale lower than that needed to explore the Planckian compactified dimensions.[8]

Visser[9] showed that the trapping of our universe in a thin shell can be implemented mathematically by using a large cosmological constant, originating from a five-dimensional electromagnetic field energy.

Squires[10] later explicitly showed the trapping by using a cosmological
constant. Mater is represented by a scalar field that becomes effectively confined to one dimension less than the original space. The solution of Squires in three dimensions is similar to the one obtained by Visser [9].

More formal arguments for multidimensional spaces were given by Beciu [11]. Gogberashvili has shown that trapping also occurs when a homogeneous background of electromagnetic energy fills the shell [12]. Gogberashvili’s solution consists of a metric conformal to flat space with a conformal factor depending on the fifth dimension. In this case, the stability of the shell is satisfied for a four dimensional space-time immersed in a five dimensional manyfold only.

In the recent works aiming to solve the hierarchy problem, our world is a three-brane embedded in a larger dimensional universe. The models resort to bulk a cosmological constant and brane tensions (for more than one brane). Consistency with Newton’s law even when there exist more than four noncompact dimensions is nevertheless maintained [6].

In the present work we will show that, irrespectively of the mechanism of trapping, the very existence of the trapped shell, implies a superluminal connection between all matter in the universe. In other words, the speed of light increases the farther off-center is the propagation of the signal, the center being defined as the bottom of some potential well and identified with our universe. This effect may be regarded as time dilation, time runs slower in the inner regions of the shell. Causality is nevertheless maintained. Light cones will not tip enough in order to generate closed timelike curves, the whole shell is causally connected.

As a consequence of the present results, we will find that all matter and energy in our universe (the center of the shell), is tied up together in a manner that usual normal signal propagation at the center of the shell would prohibit. The speed of propagation of signals is not limited by the speed of light measured on the four dimensional hypersurface. If a signal penetrates inside the shell along the fifth dimension it can leap forward huge distances in extremely short times as compared to the time taken by signals propagating on the hypersurface. Similar findings have recently been described by Chung and Freese [14]. They use more than one brane in a cosmological context, the link between causally disconnected regions may then be achieved by means of signals proceeding through the extra dimension. Hence, there is no need for inflation in order to solve the horizon problem [14]. This is already evidenced in the results of ref. [13].

In the next section we will examine the metric of five dimensional space-time with an extra space-like dimension and find the conditions for solutions to exist. Section 3 will treat the propagation of a signal along geodesics inside the shell and provide some speculative thoughts implied by the present results.
2 Shell metric

Before proceeding to analyze the problem, we show that the effect of hyperfast communication through an extra dimension is already evident in the problem dealt by Visser.[9]

Visser used a cosmological constant modeled in terms of an electric field in the fifth direction yielding a metric

$$ds^2 = \cosh(E\xi) \ dt^2 - d\Omega_3^2 - d\xi^2$$

(1)

where $E$ is a constant, $d\Omega_3^2$ is the volume on the three brane and $\xi$ is the extra dimension.

It is easy to integrate the geodesic equations for metric above. Consider a signal traveling in the $x, \xi$ direction with initial speed at $\xi = 0$, $\dot{\xi}_0 = u, \dot{x}_0 = v$.

We obtain

$$\dot{x} = v \cosh(E\xi)$$
$$\dot{\xi}^2 = \cosh(E\xi) - (1 - u^2) \cosh^2(E\xi)$$

(2)

It is clear from the above equation that, the signal will climb up the potential into the fifth dimension thereby accelerating beyond the normal speed of propagation at $\xi = 0$ in an exponential manner. The signal will stop at a point where $\dot{\xi}$ vanishes. Then will recede, oscillate back and forth between endpoints propagating at great speed in the $x$ direction inside the shell. It will be quite hard to detect such a signal because it spends almost all its time outside the center of the shell.

We will now examine the conditions that a five-dimensional metric has to obey, in order to be appropriate for the trapped shell scenario, and show that even without a cosmological constant, the effect persists.

For simplicity we take a isotropic and homogeneous four-dimensional hypersurface embedded along the fifth dimension.

The line element consistent with this demand reads,

$$ds^2 = a(\xi, t) \ dt^2 - b(\xi, t) \ d\xi^2 - c(\xi, t) \ d\Omega_3^2$$

(3)

where $a, b, c$ depend only on the fifth coordinate $\xi$ and the cosmic time $t$.

The above ansatz corresponds to a hyperspherically symmetric solution of five dimensional Einstein gravity, provided Einstein’s gravity still works in the hyper-universe, as one might hope. We still consider space-time inside the shell as a Riemannian manifold.

Define the center of the hypersurface by $\xi = 0$, such that $a(0, t) = 1, b(0, t) = A(t)$, with $A$ the scale parameter of our expanding universe. This coordinate choice defines what is understood by the cosmic time on the shell, at $\xi = 0$. 

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The shell is allowed to have a finite -albeit minuscule- width. We have chosen a flat four-dimensional metric. Distances in the shell at fixed $\xi$ are not limited. We could have chosen a curved shell, however, locally, the effect of cosmic curvature should be negligible. Moreover, a similar calculation with a closed (or open) universe yields analogous results. We have also chosen the signature of the extra dimension to be space-like. Time-like extra dimensions lead to tachyonic gravitons and other exotic phenomena.\textsuperscript{[15]}

We now simplify the problem by taking all the metric functions to be time independent, namely we choose $t = t_0$, the present epoch. We do so because the scale function of conventional Friedmann-Robertson-Walker expansion is the inverse Hubble constant, that is extremely large as compared to the supposed thickness of the shell. We are not interested in cosmological evolution, but, instead in the effects of communication through the extra dimension in our present time. The cosmological aspect may be found in ref.\textsuperscript{[14]} Time evolution of the shell is not important for the time scales that will be involved in the present investigation.

Inserting the ansatz of eq.(3) into the Einstein tensor we find

$$
G_{00} = \frac{3a}{4c^2}b (2 b'' c - b' c') \\
G_{ii} = -\frac{1}{4} (2 b a'b'c'a - a^2 b'' c + 4 a^2 b'' b c - 2 a^2 b' c' b + 2 b^2 a'' a c - b^2 a^2 c - b^2 a' c' a)/(a^2 b c^2) \\
G_{44} = -\frac{3 b'}{4 a b^2} (a b)'$$

primes denoting derivatives with respect to $\xi$ and $ii = 11, 22, 33$.

Regardless of the trapping mechanism (cosmological constant, negative energy, background fields) the function $a$ related to the gravitational potential, may be expanded around the equilibrium (or quasiequilibrium, due to the time dependence) radius $\xi = 0$. At fixed time, the expansion has to consist of a constant and a quadratic term. The linear term has to vanish due to the equilibrium condition, otherwise, all the mass in the universe would suffer a constant force that will eventually drag it to a new equilibrium position and we can always call this new position $\xi = 0$.

Assuming, that the extension of the shell into the fifth dimension is tiny as compared to any reasonable macroscopic scale length -as it should be if its effect remains undetected in the dynamics of bodies-, our approximation will be valid.

Hence we have

$$a(\xi, t) = a_0 + k \frac{\xi^2}{2}$$

(5)
Space-time is flat at the center of the shell $\xi = 0$. In order to connect to the Minkowski metric of our flat world, we take $a_0 = 1$, without loss of generality. The second assumption is that the solution we are looking for is surrounded by some agent. Hence it is an external approximately vacuum solution. External in the sense, that the brane is an almost empty region of space sandwiched between hypervolumes filled with some unknown substance.

As in the Schwartzschild case we will then have $c(\xi) = a(\xi)^{-1}$. Such an ansatz simplifies the equations radically, and can be viewed as a redefinition of the coordinate $\xi$.

A solution in and around the shell location may be obtained in several manners. (Recall we are not demanding an exact solution to all orders in $\xi$.)

One such solution obtains with $b(\xi) = e^{-\frac{\beta \xi^2}{2}}$. For which, $G_{00} \approx -\frac{3}{2} \beta + O(\xi^2)$, the other components of the Einstein tensor vanishing provided that $\beta = k$.

The energy-momentum tensor implied by eq.(4) has then a vanishing pressure and an energy density that is nonzero, a cloud of dust.

One should bear in mind that the dust we need in order to confine the shell, is not the matter dust of our world. The latter is a hundred of orders of magnitude less dense. This is also the case when one uses a cosmological constant. Particle physics teaches us that the cosmological constant should be of the order of a density of $\Lambda \approx TeV/fm^3$, while, if at all needed for the purposes of cosmology, we should have a cosmological constant 120 orders of magnitude smaller. The fact that both the cosmological constant and the energy density needed in order to confine the shell are enormous, in cosmological terms, is quite disturbing. However, these parameters are merely hiding our ignorance concerning the kind of matter-energy that exists beyond our visible world. Moreover, the huge cosmological constant used in the literature to generate the brane, or, in the present case, the huge energy density, serve only for that specific purpose. We are using the vacuum energy of particle physics to stabilize the brane. In some sense this is an economical approach.

The difference between our solution and those that use a cosmological constant, is that the present solution is analogous to the so called quintessence models, presently adduced in order to explain the acceleration (negative deceleration) parameter of the universe. In those models the pressure is smaller (and perhaps even zero) as compared to the energy density. In any event, we will find the same hyperfast travel effect as with the cosmological constant, supporting the claim that it is quite a general feature of trapped shells.

The basic parameter that determines the curvature of the shell is $k$. It should be large enough, for the thickness of the shell to be smaller than the radii of nuclei, smaller than the deep inelastic scattering scale of the experiments carried until now, otherwise, its imprint should have become
visible by now. Recent works on submillimeter extra dimensions, seem to indicate that for the compactified scenario, the upper limits on the size of these dimensions could be much higher, even macroscopic.\footnote{3} We here opt, however, for a much more conservative approach and assume the shell size parameter is much smaller. The results are in any event independent of the choice, but are much more evident the thinner the shell.

Summing up, we have a metric with

\begin{align}
g_{00} &= 1 + k\xi^2 \\
g_{ii} &= -e^{-k\xi^2/2} \\
g_{44} &= -\frac{1}{1 + k\xi^2}
\end{align}

(6)

Let us now consider signal propagation. We will here use an extremely schematic approach in order to estimate the effect of the shell thickness on the signals. A geodesic method will be exploited in the next section. For this purpose we assume propagation along a path that goes in the $\xi$ direction, then across it and back.

Our measure of time in the shell is the cosmic time $t$, and we refer every process to it. Suppose a massless field signal whose speed in the shell is the velocity of light $c = 1$ in our units, starts traveling from $\xi = 0$, the center of the shell to some fixed $\xi$ inside it, then travels a distance $L$ at fixed $\xi$ and returns from $\xi$ to $\xi = 0$ back to a point at a coordinate distance $L$ from the initial point. Signals emitted in a direction at an angle to the hypersurface, scatter inside it, or may be reflected at some hypothetical edge. The question of the dynamics of radiation inside the shell is here relevant, especially if photons, gravitons, etc., become massive. We here confine ourselves to a kinematical approach. A rough estimate of the superluminal effect is obtained below, while a geodesic approach is taken up in the next section.

Using three dimensional spherical coordinates, $L$ along the radial distance in three-dimensional space, and for fixed angles, we find

\begin{align}
ds^2 = (1 + k\xi^2)dt^2 - \frac{1}{(1 + k\xi^2)}d\xi^2 - e^{-k\xi^2/2}dl^2
\end{align}

(7)

Our choice of $k$ will be of the order of $R_{GUT}^{-2}$, where $R_{GUT} \approx 10^{-31} m$ stands for the grand unified theories scale. We do this because we want to encompass democratically all the known interactions, and, in order to avoid quantum gravity effects that will enter at much smaller scales of the order of $R_{Planck} \approx 10^{-35} m$.

However, any microscopic scale will be a viable choice. We take this value for the sake of exemplification
Hence
\[ t = 2 \int_0^\xi \frac{d\xi}{1 + k \xi^2} + \int_0^L \frac{dl}{\sqrt{1 + k \xi^2}} \approx \frac{L e^{-k \xi^2}}{\sqrt{1 + k \xi^2}} \]  

(8)

Where we neglected the first integral because it is of the order of \( t \approx 10^{-39} \) sec.

If the signal climbs up the harmonic potential and back far enough in \( \xi \), the coordinate time becomes negligible.

With \( \xi = 15 R_{\text{GUT}} \approx 1.5 \times 10^{-30} m \), several times the radius of curvature of the shell, and \( L = 100 \text{Mpc} \), the time taken by radiation to traverse this cosmic distance is \( t = 2.5 \times 10^{-10} \text{sec} \). A ridiculously small time as compared to the 326 Million years needed for the light to traverse this distance along the direction \( \xi = 0 \).

Due to the crudeness of the approximations used in order to derive the above result, we should not attach too much rigor to the actual numbers. The effect is, nevertheless, evident.

The same results are obtained in the eikonal approximation for a scalar field propagating on the shell. This was indeed checked by using a conformal factor in the metric and identifying the departure of the factor from the value of one as the propagating scalar field.

3 Geodesic propagation of signals in the shell

In the previous section we considered a simplified scenario of signal propagating along straight lines inside the shell in order to estimate the effect of superluminal communication between different points in the embedded universe. However, straight lines are not the correct paths of either massive or massless particles. We here proceed one step forward and study the propagation of a signal in the shell using the metric found in the previous section as a background.

Consider the propagation of a signal in the \( \xi, x \) plane, where \( x \) is a coordinate along the shell at \( \xi = 0 \). The geodesic equations for the \( t, \xi, x \) coordinates become

\[ t'' + \frac{2 k \xi}{1 + k \xi^2} t' \xi' = 0 \]
\[ x'' - \beta \xi x' \xi' = 0 \]
\[ \xi'' + t'^2 k \xi (1 + k \xi^2) - \frac{k \xi}{1 + k \xi^2} \xi'^2 + \frac{\beta \xi}{2} (1 + k \xi^2) e^{-\beta \xi^2} x^2 = 0 \]  

(9)

Where the primes represent here derivatives with respect to the parameter of the geodesic equations.
The first equation of the set above may be solved by $t' = (1 + k \xi^2)^{-1}$, fixing the parameter of the geodesics to be the time $t$ at $\xi = 0$ up to an irrelevant constant. The motion along $x$ is then determined by $\frac{dx}{dt} = C(1 + k \xi^2) e^{\frac{\beta \xi^2}{2}}$, with $C$ a constant of integration determined by the projection of the initial velocity on the $x$ axis. The equation for the $\xi$ coordinate then becomes (the equation may be also obtained from variation of the path distance $-\int ds$)

$$\ddot{\xi} = \frac{3k\xi\dot{\xi}^2}{1 + k\xi^2} + k\xi(1 + k\xi^2) + C^2\beta\xi/2 (1 + k\xi)^3 e^{\beta\xi^2/2} = 0 \quad (10)$$

The equation has no analytical solution. If one neglects the nonlinear terms, the solution becomes a harmonic function as expected. However, these terms are extremely important for $\xi$ large enough. The equation for $\xi$ has to be solved numerically. Once $\xi$ is found the displacement of the particle along the shell may be found.

We use $k = \beta = 1$ in units of $\frac{1}{R_{\text{GUT}}}$, and a signal impinging at an angle $\alpha$ on the shell with velocity components $\dot{x}_0, \dot{\xi}_0$. It is found that regardless of the angle of incidence, the particle oscillates back and forth across the shell with a frequency of the order of $\sqrt{k}$. The long-time behavior of the signal requires an extremely fine grid in order to prevent roundoff errors. Long time is still a minute time as it is measured GUT time units. For example, a signal with initial speed $0.9c$ at an angle of 45 degrees with respect to the $\xi = 0$ axis has traversed a distance $L = 4c \, t$ after $t = 7000$ time units $\approx 2 \times 10^{-36}$ sec, It is clear that we recover the results of the simplistic calculation of the previous section. The signal travels superluminally cosmic distances. The gain increases as time goes on.

The possibility of disappearance of signals into the brane and their reappearance in some other part of the universe is quite perplexing. One immediate consequence is that, the flux coming from a source decreases faster than $\frac{1}{r^2}$, flux is lost. The caveat is that we used a geodesic approach. It is not all certain that this is a sound procedure, especially because particles cease to appear pointlike as compared to the size of the brane in the fifth dimension. In order to give some more credibility to the present results, we need a dynamical theory of fields and particles. However, our clues coming from our experience on the brane seem a bit irrelevant in this respect. The use of a geodesic equation despite its limitations is, in some sense, the least biased approach. Any dynamical model, based upon field theory as we know it may fail to describe the physics of the entities in a larger number of dimensions. We need some experimental input in order to proceed. In the absence of such evidence it is preferable to leave the situation as is.

For observers far from the source, the luminosity of a source and therefore its mass, are underestimated. If taken seriously, the present effect has to be included in the calibration of radiating sources both near and far.
Perhaps even for a nearby radiating source we are witnessing only the ‘tip of the iceberg’ of its energy output. It is quite evident that gravitational radiation will also be scattered all over the shell. Wandering radiation and particles inside the shell create a background noise present in the whole universe, perhaps serving as the very source of the energy density used above in order to maintain the shell’s stability.

If a photon does indeed penetrate the thicket of the shell then it will become red shifted and blue shifted enormously, but will arrive at its destination with the same frequency as emitted. However, the very existence of photons inside the shell has to be questioned.

In conventional Kaluza-Klein approaches, the electromagnetic field (potential) is considered a four-vector from the onset, there is no electromagnetic potential in the extra direction. However, if the extra dimension is a physical one and not curled-up, there is a-priori no clear differentiation between, say, the middle of the shell and, nearby positions along the fifth direction. The presence of the electromagnetic field in the fifth direction has to be considered also, at least classically. Moreover, if one relaxes the condition of cylindricity, there arise fields that propagate along the extra dimension, and correspond to the electromagnetic fields in the shell.

The cylinder condition arises in compactified models when one uses the lowest order mode of the metric expanded in terms of Fourier amplitudes. The mechanism of compactification is still unclear. The compactification ansatz was devised by Klein to justify the cylinder condition of Kaluza. Klein himself restricted $g_{44}$ to be a constant. Later [1], it was found that such an assumption is inconsistent. It gives an extra equation for the electromagnetic field that is not satisfied by real fields. The lesson we learn form Klein’s restriction and its failure is that one should not constrain the fields beforehand.

If indeed light might be lost in the depth of the shell, we have an alternative explanation to the missing matter (or missing light) problem. In this case, the velocities of galactic dust clouds are determined by the gravitational masses present in the galaxies, but, the light emitted by the galaxies underestimates the mass, flux is lost. The cosmic distance ladder has to be recalibrated in order to account -at least statistically- for the missing light. Moreover, it could be that our universe-shell is cracked outside the regions where galaxies reside. The velocity of orbiting clouds is then determined not only by the positive mass inside the galaxies, but, by the invading mass (presumably negative) that comes from the regions external to the brane.

Radiation emitted into the shell is detected almost instantaneously by particles far away in the universe in a random-like manner. The fact that we do not violate causality and locality is because both are distorted enormously by the potential of the shell. This in turn might have some bearing to the nonlocality witnessed in quantum mechanics. Instead of having alternative hidden-variable theories we could think about hidden-dimension theories. Another aspect of the superluminal propagation is that there is no need for an
inflationary era of cosmological evolution.\[14\] The homogeneity and isotropy of the universe, evidenced by the microwave background, can be achieved by means of the same mechanism. Far away points in the universe do not have to wait until they enter each other’s horizon, they are communicated at all times through the extra dimension. Differing from ref.\[14\], the brane we use is of finite size. If it were of zero width it would be hard to view a signal (not only photon), as a messenger. (see remark below eq.(2)). However, if matter is extended along the fifth dimension too, as it should if the brane is of finite size, then there is no such problem, matter will detect the signal even if it does not transite through the middle of the shell.

The model addresses what appears to be action at a distance in terms of ultrafast communication. The incessant bombardment by ultrafast radiation that fills the universe’s shell, may be also related to the zeropoint field of quantum theory because of its random character with no definite temperature signature.
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