Muon Anomalous Magnetic Moment and $\mu \rightarrow e\gamma$ in $B - L$ Model with Inverse Seesaw

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We study the anomalous magnetic moment of the muon, $a_\mu$, and lepton flavor violating decay $\mu \rightarrow e\gamma$ in TeV scale $B - L$ extension of the Standard Model (SM) with inverse seesaw mechanism. We show that the $B - L$ contributions to $a_\mu$ are severely constrained, therefore the SM contribution remains intact. We also emphasize that the current experimental limit of $BR(\mu \rightarrow e\gamma)$ can be satisfied for a wide range of parameter space and it can be within the reach of MEG experiment.

I. INTRODUCTION

The anomalous magnetic moment of the muon has been measured at Brookhaven National Laboratory to a precision of 0.54 parts per million. The current average of the experimental results is given by

$$a_{\mu}^{\text{exp}} = 11659208.0(6.3) \times 10^{-10},$$

which is different from the Standard Model (SM) prediction by $3.3 \sigma$ to $3.6 \sigma$.

This discrepancy has been established by the impressive accuracy of recent theoretical and experimental results. Therefore, it is tempting to consider the above result as a strong signature for physics beyond the SM. It is important to note that the SM estimation for $a_\mu$ depends on the low-energy hadronic vacuum polarization, which is the main source of the uncertainty. The above result is obtained when hadronic vacuum polarization is determined directly from the annihilation of $e^+e^-$ to hadrons. However, if hadronic $\tau$ decays are included, substantially larger value for $a_\mu^{\text{had}}$ is derived that reduces the discrepancy to about 2.4 $\sigma$ only.

In addition, non-vanishing neutrino masses confirmed by neutrino oscillation experiments are one of the firm observational evidences for an extension of the SM. The simplest way to account for small neutrino masses is to introduce right-handed neutrinos into the SM, which are Majorana-type particles with very heavy masses. In this case, type I seesaw mechanism can be implemented and an elegant explanation for light neutrinos is obtained. Recently, it has been shown that TeV scale right-handed neutrinos can be naturally implemented in $B - L$ extension of the SM, where three SM singlet fermions arise naturally to cancel the $U(1)_{B-L}$ triangle anomaly. Also, the scale of $B - L$ symmetry breaking can be related to supersymmetry breaking scale, therefore, the right-handed neutrino masses are naturally of order TeV scale.

In order to fulfill the experimental measurements for the light neutrino masses with TeV scale right-handed neutrino, a very small Dirac neutrino Yukawa couplings, $Y_\nu < \mathcal{O}(10^{-7})$ must be assumed. In this case, the mixing between light and heavy neutrinos are negligible, and hence the interactions of right-handed neutrinos with the SM particles are very suppressed.
is proposed to prohibit type I seesaw and allow another scenario for generating the light neutrino masses, namely the inverse seesaw mechanism \cite{3,10}. In this scenario, the neutrino Yukawa coupling is no longer suppressed and can be of order one. Thus, the heavy neutrinos associated to this model are quite accessible and lead to interesting phenomenological implications.

In this paper we analyze the anomalous magnetic moment of the muon in TeV scale $B - L$ extension of the SM with inverse seesaw mechanism. We provide analytical formula for loop contributions due to the exchange of right-handed neutrinos, $B - L$ gauge boson, and extra Higgs. We show that right-handed neutrinos give the dominant $B - L$ contribution to $a_\mu$. However, the unitarity violation limits of the light neutrino mixing matrix restrict this effect significantly. We also consider the impact of the right-handed neutrinos on the Lepton Flavor Violation (LFV) decays $\mu \to e\gamma$. We show that the rate of this decay is enhanced and becomes within the reach of present experiments.

The paper is organized as follows. In section 2 we briefly review the TeV scale gauged $B - L$ extension of the SM with inverse seesaw mechanism. We focus on the neutrino sector and show that the unitarity violation limits of $U_{MNS}$ mixing matrix constrain the mixing between light and heavy neutrinos. In section 3 we study the anomalous magnetic moment of the muon due to the exchange of heavy neutrinos, $B - L$ gauge boson $Z'$ and $B - L$ extra Higgs $H'$. In section 4 we analyze the LFV process $\mu \to e\gamma$ and the constrained imposed by the experimental limit of $BR(\mu \to e\gamma)$ on the heavy neutrino contributions. Section 5 is devoted for the numerical results and possible correlation between $a_\mu$ and $BR(\mu \to e\gamma)$. Finally we give our conclusions in section 6.

II. TEV SCALE $B - L$ WITH INVERSE SEESAW

In this section we briefly review the TeV scale $B - L$ extension of the SM with inverse seesaw mechanism, which has been recently proposed in Ref.\cite{3}. This model is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, where the $U(1)_{B-L}$ is spontaneously broken by a SM singlet scalar $\chi$ with $B - L$ charge $= -1$. Also a gauge boson $Z'$ and three SM singlet fermions $\nu_R$, with $B - L$ charge $= -1$ are introduced for the consistency of the model. Finally, three SM singlet fermions $S_1$ with $B - L$ charge $= -2$ and three singlet fermions $S_2$ with $B - L$ charge $= +2$ are considered to implement the inverse seesaw mechanism. The $B - L$ quantum numbers of fermions and Higgs bosons of this model are given in Table I.

| Particle | $Q$ | $u_R$ | $d_R$ | $L$ | $e_R$ | $\nu_R$ | $\phi$ | $\chi$ | $S_1$ | $S_2$ |
|----------|----|-------|-------|-----|-------|---------|------|-------|-------|------|
| $Y_{B-L}$ | 1/3 | 1/3   | 1/3   | -1  | -1    | -1      | 0    | -1    | -2    | +2   |

TABLE I: $B - L$ quantum numbers of fermions and Higgs particles

The relevant part of the Lagrangian in this model is given by

$$\mathcal{L}_{B-L} = -\frac{1}{4} F'_{\mu \nu} F'^{\mu \nu} + i \bar{L} i \gamma^\mu L + i \bar{e}_R D_\mu e_R + i \bar{\nu}_R D_\mu \nu_R + i \bar{S}_1 D_\mu \gamma^\mu S_1 + i \bar{S}_2 D_\mu \gamma^\mu S_2 + \left( D^\mu \phi \right) (D_\mu \phi) + \left( D^\mu \chi \right) (D_\mu \chi) - V(\phi, \chi) - \left( \lambda e \bar{L} \phi e_R + \lambda_\nu \bar{L} \phi \nu_R + \lambda_S \bar{S}_2 R \chi S_2 + h.c. \right) - \frac{1}{M^2} S_1^4 S_1 - \frac{1}{M^2} S_2^4 S_2, \tag{3}$$

where $F'_{\mu \nu} = \partial_\mu Z'_\nu - \partial_\nu Z'_\mu$ is the field strength of the $U(1)_{B-L}$. The general expression for the covariant derivative $D_\mu$ is defined as

$$D_\mu = \partial_\mu - ig_s T^a G'^a_\mu - ig \tau_i W^i_\mu - ig' Y B_\mu - ig'' Y_{B-L} Z'_\mu, \tag{4}$$
where $g''$ is the $U(1)_{B-L}$ gauge coupling constant. The last two terms in $\mathcal{L}_{B-L}$ are non-renormalizable terms, which are allowed by the symmetries and relevant for generating small mass for $S_1$ and $S_2$ at TeV, are required by inverse seesaw mechanism. Few remarks are in order: i) The $B - L$ symmetry allows a mixing kinetic term $F_{\mu
u}F^{\mu\nu}$. This term leads to a mixing between $Z$ and $Z'$. However due to the stringent constraint from LEP II on $Z - Z'$ mixing, one may neglect this term. In our analysis we assume a minimal model of $B - L$ extension of the SM. ii) In order to avoid other possible non-renormalizable term that may spoil the inverse seesaw mechanism that we adopt, a discrete symmetry like $Z_4$ is imposed. iii) In order to avoid a large mass term $m_\mu S_1 S_2$ in the above Lagrangian, one assumes that the SM particles, $\nu_R$, $\chi$, and $S_2$ are even under a $Z_2$-symmetry, while $S_1$ is an odd particle. Finally, $V(\phi, \chi)$ is the most general Higgs potential invariant under these symmetries and it is given by

$$V(\phi, \chi) = m_1^2 \phi^\dagger \phi + m_2^2 \chi^\dagger \chi + \lambda_1 (\phi^\dagger \phi)^2 + \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 (\phi^\dagger \phi)(\chi^\dagger \chi),$$

where $\lambda_3 > -2\sqrt{\lambda_1 \lambda_2}$ and $\lambda_1, \lambda_2 \geq 0$, so that the potential is bounded from below.

The non-vanishing vacuum expectation value (vev) of $\chi$: $|\langle \chi \rangle| = v'/\sqrt{2}$ is assumed to be of order TeV, which is consistent with the result of radiative $B - L$ symmetry breaking found in gauged $B - L$ model with supersymmetry. The vev of the Higgs field $\phi$: $|\langle \phi^0 \rangle| = v/\sqrt{2}$ breaks the electroweak (EW) symmetry, i.e., $v = 246$ GeV. After the $B - L$ gauge symmetry breaking, the gauge field $Z'$ acquires the following mass:

$$M_{Z'}^2 = g''/v'^2.$$  

The experimental search for $Z'$ LEP II leads to

$$M_{Z'}/g'' > 6 \text{ TeV.}$$

Also, after the $B - L$ and the EW symmetry breaking, the neutrino Yukawa interaction terms lead to the following mass terms:

$$\mathcal{L}_m^\nu = m_D \bar{\nu}_L \nu_R + M_N \bar{\nu}_R^c S_2 + h.c.,$$

where $m_D = \frac{1}{\sqrt{2}} \lambda_\nu v$ and $M_N = \frac{1}{\sqrt{2}} \lambda_s v'$. In addition the second non-renormalizable term in Eq. induces a Majorana mass for $S_2$ fermion. Hence, the Lagrangian of neutrino masses, in the flavor basis, is given by

$$\mathcal{L}_m^\nu = \mu_s S_2 \bar{S}_2 + (m_D \bar{\nu}_L \nu_R + M_N \bar{\nu}_R^c S_2 + h.c.),$$

where $\mu_s = \frac{\nu'}{\sqrt{M}} \sim 10^{-9}$ GeV, hence $M$ is of order intermediate scale $10^7$ GeV and the flavor indices are omitted for simplicity. Therefore, the neutrino mass matrix can be written as $M_\nu \bar{\psi} \psi$ with $\psi = (\nu_L^c, \nu_R, S_2)$ and $M_\nu$ is given by

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_N \\ 0 & M_N^T & \mu_s \end{pmatrix}. $$

The diagonalization of this mass matrix leads to the following light and heavy neutrino masses, respectively:

$$m_{\nu_l} = m_D M_N^{-1} \mu_s (M_N^c)^{-1} m_D^T,$$

$$m_{\nu_h}^2 = m_{\nu_h}^2 = M_N^2 + m_D^2.$$
The light neutrino mass matrix in Eq. (11) must be diagonalized by the physical neutrino mixing matrix $U_{\text{MNS}}$, i.e.,

$$U_{\text{MNS}}^T m_{\nu} U_{\text{MNS}} = m_{\nu}^{\text{diag}} \equiv \text{diag}(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}). \quad (13)$$

Thus, one can easily show that the Dirac neutrino mass matrix can be defined as:

$$m_D = U_{\text{MNS}} \sqrt{m_{\nu}^{\text{diag}}} R \sqrt{M_N}, \quad (14)$$

where $R$ is an arbitrary orthogonal matrix. It is clear that this expression is a generalization to the expression of $m_D$ in type I seesaw, which is given by $m_D = U_{\text{MNS}} \sqrt{m_{\nu}^{\text{diag}}} R \sqrt{M_N}$. Accordingly, the matrix $V$ that diagonalizes the $9 \times 9$ neutrino mass matrix $M_{\nu}$, i.e., $V^T M_{\nu} V = M_{\nu}^{\text{diag}}$, is given by:

$$V = \begin{pmatrix} V_{3 \times 3} & V_{3 \times 6} \\ V_{6 \times 3} & V_{6 \times 6} \end{pmatrix}, \quad (15)$$

where the matrix $V_{3 \times 3}$ is given by

$$V_{3 \times 3} \simeq \left(1 - \frac{1}{2} F F^T\right) U_{\text{MNS}}. \quad (16)$$

It is clear that in general $V_{3 \times 3}$ is not unitary matrix and the unitarity violation, i.e., the deviation from the standard $U_{\text{MNS}}$, is measured by the size of $\frac{1}{2} F F^T$. The matrix $V_{3 \times 6}$ is defined as

$$V_{3 \times 6} = (0_{3 \times 3}, F)V_{6 \times 6}, \quad F = m_D M_N^{-1}. \quad (17)$$

Finally, $V_{6 \times 6}$ is the matrix that diagonalize the $\{\nu_R, S_2\}$ mass matrix. Note that due to the Higgs mixing term in the potential $V(\phi, \chi)$, the physical Higgs scalars $(H, H')$ are given as a linear combination of $\phi$ and $\chi$, with the following masses:

$$m_{H, H'}^2 = \lambda_1 v^2 + \lambda_2 v'^2 \mp \sqrt{(\lambda_1 v^2 - \lambda_2 v'^2)^2 + \lambda_3 v^2 v'^2}. \quad (18)$$

A detailed analysis for the phenomenology of the Higgs bosons of this model at the LHC has been considered in Ref. [16, 17].

### III. $B - L$ CONTRIBUTIONS TO THE MUON ANOMALOUS MAGNETIC MOMENT

In this section we analyze new contributions to the muon anomalous magnetic moment due to the extra particles of the $B - L$ TeV scale model. From the effective Lagrangian of leptonic sector, one finds the following interactions

$$\mathcal{L} = \frac{g}{\sqrt{2}} (V_{\mu i}^\dagger \bar{\nu_i} \gamma^\alpha W^+_\alpha P_L \mu + V_{\mu i} \bar{\mu} \gamma^\alpha W^-_\alpha P_L \nu_i) + g'' \bar{\mu} \gamma^\alpha Z'^\alpha_{\alpha \mu} + \lambda_{\mu} \sin \theta \bar{\mu} H' \mu, \quad (19)$$

where $V$ is $9 \times 9$ extended MNS matrix, as discussed above, $\lambda_{\mu}$ is the Yukawa coupling of the muon, and $\theta$ is the mixing angle between the SM-like Higgs and extra Higgs [12]. Thus, one can easily observe that the new contributions to the anomalous magnetic moment of the muon are generated by one loop diagrams involving the exchange of $W$ gauge boson and heavy neutrino, or $Z'$ boson and $\mu$ exchange, or $H'$ neutral scalar boson and $\mu$, as shown in Fig. [1]. Therefore, one can define $a_{\mu}^{B - L}$ as

$$a_{\mu}^{B - L} = a_{\mu}^{\nu} + a_{\mu}^{Z'} + a_{\mu}^{H'}. \quad (20)$$
FIG. 1: The new contributions of the muon anomalous magnetic moment in $B - L$ extension of the SM.

The calculation of the first diagram in Fig. 1 leads to

$$a_\mu^\nu = \frac{G_F m_\mu^2}{\sqrt{2} 8\pi^2} \sum_{i=1}^9 V_{\mu i}^* V_{\mu i} f(r_{\nu_i}),$$

(21)

where $r_{\nu_i} = (m_{\nu_i}/M_W)^2$ and $f(r)$ is given by

$$f(r) = \frac{10 - 43r + 78r^2 - 49r^3 + 4r^4 + 18r^3 \ln(r)}{3(1-r)^4}.$$  

(22)

In this calculation, we assume that $(m_\mu/M_W)^2 \approx 0$. For $r \approx 0$ one finds that $f(0) = 10/3$, while if $r \gg 1$ then $f(r) \to 4/3$. Thus, in the SM this contribution implies

$$(a_\mu^\nu)^{SM} = \frac{G_F m_\mu^2}{\sqrt{2} 8\pi^2} \times \frac{10}{3} = 3.89 \times 10^{-9},$$

(23)

where the mixing matrix $V$ is given by the unitary $U_{\text{MNS}}$ mixing matrix, i.e., $\sum_{i=1}^3 |V_{\mu i}|^2 = 1$. In our model with TeV scale $B - L$, the $9 \times 9$ mixing matrix $V$ is unitary, however the $3 \times 3$ mixing matrix of light neutrino is no longer unitary. In our analysis, we constrain ourselves with the following non-unitary limits for light neutrino mixing matrix

$$|NN^\dagger| = \begin{pmatrix} 0.994 \pm 0.005 & < 7.0 \times 10^{-5} & < 1.6 \times 10^{-2} \\ < 7.0 \times 10^{-5} & 0.995 \pm 0.005 & < 1.0 \times 10^{-2} \\ < 1.6 \times 10^{-2} & < 1.0 \times 10^{-2} & 0.995 \pm 0.005 \end{pmatrix}.$$  

(24)

In this case, one finds that $0.99 \leq \sum_{i=1}^3 |V_{\mu i}|^2 \leq 1$. Hence the SM-like contribution is slightly reduced to $3.851 \times 10^{-9}$. Since $4/3 \leq f(r) \leq 10/3$, one can easily see that

$$\frac{10}{3} \geq \sum_{i=1}^9 |V_{\mu i}|^2 f(r_{\nu_i}) \geq \frac{10}{3} \sum_{i=1}^3 |V_{\mu i}|^2 + \frac{4}{3} \left(1 - \sum_{i=1}^3 |V_{\mu i}|^2\right) \geq \frac{6}{3} \sum_{i=1}^3 |V_{\mu i}|^2 + \frac{4}{3},$$

Thus, the ratio $R_\mu^\nu = a_\mu^\nu/(a_\mu^\nu)^{SM}$ lies within the tiny range:

$$0.994 \leq R_\mu^\nu \leq 1,$$

(25)

which means that within TeV scale $B - L$ extension of the SM, the discrepancy between the theoretical prediction of the anomalous magnetic moment of the muon and its experimental measurement remains $2.4\sigma$ as in the SM and another source of new physics is required to account for this difference.
Next, we consider the contribution of $Z'$ to $a_\mu$. From the second diagram in Fig. 1, one finds the following result:

$$a^{Z'}_\mu = \frac{g''}{4\pi^2} \frac{m_\mu^2}{M_{Z'}^2} g(r_{m_\mu}),$$

(26)

where we assume that $r_{m_\mu} \equiv (m_\mu / M_{Z'})^2 \simeq 0$, and $g(r_{m_\mu})$ is given by

$$g(r_{m_\mu}) = \int_0^1 dz \frac{z^2(1-z)}{1-z + r_{m_\mu} z^2} \frac{r_{m_\mu} \approx 0}{3},$$

(27)

Hence one finds that

$$a^{Z'}_\mu \simeq \frac{m_\mu^2}{4\pi^2} \left( \frac{g''}{M_{Z'}} \right)^2 \frac{1}{3} < \frac{m_\mu^2}{6000 \text{ GeV}^2} \simeq 2.34 \times 10^{-12}.$$ (28)

This contribution is quite small and one can neglect the effect of the $Z'$ diagram. Finally, we consider the diagram of extra Higgs. We find that the corresponding contribution to $a_\mu$ is given by

$$a^{H'}_\mu \simeq \frac{m_\mu^2}{32\pi^2} \frac{g''}{m_{H'}} h(r_{m_\mu}'),$$

(29)

where $r_{m_\mu}' = (m_\mu / m_{H'})^2 \simeq 0$ is assumed and $\lambda_\mu = m_\mu / v \simeq O(10^{-4})$. The loop function $h(r_{m_\mu}')$ is given by

$$h(r_{m_\mu}') = \frac{2 + 3r_{m_\mu}' - 6r_{m_\mu}'^2 + r_{m_\mu}'^3 + 6r_{m_\mu}' \ln(r_{m_\mu}')} {3(1 - r_{m_\mu}')^4}. (30)$$

Hence one can estimate this contribution, for $m_{H'} \simeq O(100) \text{ GeV}$ and $\sin \theta = 1/\sqrt{2}$, as

$$a^{H'}_\mu \simeq 10^{-16},$$

(31)

which is also quite negligible. Therefore, one concludes that the $B - L$ contributions to $a_\mu$ can not account for the reported discrepancy between the theoretical and experimental expectations. It is worth noting that $B - L$ contribution to $a_\mu$, obtained from the loop diagram mediated by heavy neutrinos can be of order the SM contribution and has a significant effect if the mixing between light and heavy neutrinos is sizable.

However this mixing is strongly constrained by several leptonic processes [18].

**IV. µ → eγ IN TEV SCALE B − L WITH INVERSE SEESAW**

We now consider the LFV decay $\mu \rightarrow e\gamma$ in the TeV scale $B - L$ model with inverse seesaw mechanism. Many experiments have been designed to search for LFV processes, in particular $\mu \rightarrow e\gamma$. The current upper limit is given by the MEG experiment [19]

$$BR(\mu \rightarrow e\gamma) < 2.4 \times 10^{-12}.$$ (32)

New experiments are expected to improve this limit by three order of magnitudes. It is important to note that the SM result for the branching ratio of $\mu \rightarrow e\gamma$, with neutrino masses as in Eqs. (42,43), is given by

$$BR(\mu \rightarrow e\gamma)^{\text{SM}} \simeq 10^{-55}.$$ (33)

Thus, the observation of $\mu \rightarrow e\gamma$ decay will be a clear signal for physics beyond the SM. We perform the
Therefore, the decay rate is given by as shown in Fig. 2. The amplitude of $\mu \to e\gamma$, in the limit of $m_e \to 0$, can be written as

$$A(\mu \to e\gamma) \simeq \frac{m_\mu G_F}{32\sqrt{2}\pi^2} \sum_{i=1}^{9} V_{\mu i}^* V_{ei} f(r_i) \times \bar{u}(p)[2e(p' \cdot e)]u(p').$$  \hspace{1cm} (34)$$

Let us define the coefficient of the amplitude $A$ as

$$a = \frac{em_\mu G_F}{32\sqrt{2}\pi^2} \sum_{i=1}^{9} V_{\mu i}^* V_{ei} f(r_i).$$  \hspace{1cm} (35)$$

Therefore, the decay rate is given by

$$\Gamma(\mu \to e\gamma) = \frac{m_\mu^3}{8\pi} |a|^2.$$  \hspace{1cm} (36)$$

Using the dominant decay mode of $\Gamma(\mu \to e\nu \bar{\nu}) \simeq m_\mu^5 G_F^2/(192\pi^3)$, the branching ratio is given by

$$BR(\mu \to e\gamma) = \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu \bar{\nu})} = \frac{m_\mu^3|a|^2}{192\pi^3} \frac{m_\mu^5 G_F^2}{m_\mu^5 G_F^2} = \frac{3\alpha}{64\pi} \left| \sum_{i=1}^{9} V_{\mu i}^* V_{ei} f(r_i) \right|^2,$$  \hspace{1cm} (37)$$

where $\alpha = e^2/4\pi \simeq 1/137$. From the experiment upper bound in Eq.(38), one finds the following constraint on the light and heavy neutrino mixing:

$$\left| \sum_{i=1}^{9} V_{\mu i}^* V_{ei} f(r_i) \right| < 0.000149.$$  \hspace{1cm} (38)$$

In case of extremely heavy right-handed neutrinos, the lepton mixing matrix $V_{3 \times 3}$ is almost unitary. Therefore, the contribution of light neutrinos, which corresponds to $i = 1, 2, 3$, is almost zero. In addition, the contribution of heavy neutrinos is quite suppressed due to the small mixing between light and heavy neutrinos in this case ($V_{\mu i} \sim V_{ei} \sim m_D/M_N \sim \mathcal{O}(10^{-8})$). Hence the above constraint is satisfied and $BR(\mu \to e\gamma) \ll 10^{-12}$ is obtained.

However, within TeV scale inverse seesaw the lepton mixing matrix is non-unitary. Also the mixing between heavy and light neutrinos are not small, since $m_D/M_N \sim \mathcal{O}(0.1)$. Therefore, the bound in Eq.(38) can be written as

$$\left| \frac{10}{3} \sum_{i=1}^{3} V_{\mu i}^* V_{ei} + \sum_{j=4}^{9} V_{\mu j}^* V_{ej} f(r_j) \right| < 0.000149,$$  \hspace{1cm} (39)$$

where $r_j = (m_{\nu j}/M_W)^2$ and $V_{\mu(i)j}$, as defined in Eq.(17), is given by

$$V_{\mu(i)j} = \left[ (0, m_D M_N^{-1} ) V_{6 \times 6} \right]_{(2),j-3} = \left[ \left( 0, U_{\mu i}^{\text{diag}} \sqrt{m_{\mu i}} R \sqrt{\mu_s^{-1}} \right) V_{6 \times 6} \right]_{(2),j-3}. $$  \hspace{1cm} (40)$$
Thus, for \( r_j \gg 1 \), i.e. \( f(r_j) = 4/3 \), one finds
\[
\sum_{i=1}^{3} V_{\mu i}^* V_{e i} < 0.0000636,
\]
which implies that \((FF^T)_{21,12} \lesssim 10^{-4}\). This bound can be easily satisfied, due to the constraints imposed on the off-diagonal elements of the non-unitary \(U_{MNS}\) mixing matrix.

V. NUMERICAL RESULTS

In our model of \(B - L\) extension of the SM with inverse seesaw mechanism, the relevant input parameters involved in the computation of the anomalous magnetic moment of muon are the following:

1. Three right-handed neutrino masses.
2. Three \(\mu_s\) mass parameters.
3. Three angles of the orthogonal matrix \(R\).

In fact, the form of the Dirac neutrino mass matrix \(m_D\), given in Eq.\((14)\), guarantees that we obtain the correct light neutrino masses and mixing matrix \(U_{MNS}\).

The solar and atmospheric neutrino oscillation experiments provide the following results for the neutrino mass-squared differences with best-fit values within 1\(\sigma\) errors \[20\]:
\[
\Delta m^2_{21} = (7.64 \pm 0.19) \times 10^{-5} \text{eV}^2, \\
|\Delta m^2_{31}| = (2.45 \pm 0.09) \times 10^{-3} \text{eV}^2.
\]

Therefore, one finds
\[
m_{\nu_2} = \sqrt{7.64 \times 10^{-5} + m_{\nu_1}^2},
\]
\[
m_{\nu_3} = \sqrt{2.45 \times 10^{-3} + m_{\nu_1}^2},
\]
with arbitrary \(m_{\nu_1}\). If \(m_{\nu_1}^2 \ll 7.64 \times 10^{-5} \text{eV}^2\), the ansatz of hierarchal the light neutrino masses is obtained.

In this case one gets the following the light neutrino masses:
\[
m_{\nu_1} \lesssim 10^{-5} \text{eV}, \quad m_{\nu_2} = 0.008 \text{eV}, \quad m_{\nu_3} = 0.05 \text{eV}.
\]

In our analysis, we adopt these values for the light neutrino masses and also assume that the neutrino mixing matrix is given by Eq.\((24)\). From Eq.\((40)\), one notices that the mixing element \(V_{\mu i}\), which plays a crucial role in the result of \(a_\mu\), can be enhanced if: (i) \(\mu_{s_3} \lesssim m_{\nu_3}\), (ii) the orthogonal matrix \(R\) is maximally mixing. For example, if \(\mu_{s_3} = 2.7 \times 10^{-9} \text{GeV}, M_{N_1} = 900 \text{GeV}, M_{N_2} = 1500 \text{GeV}, M_{N_3} = 1900 \text{GeV},\) and the other parameters are fixed as in Fig.\(3\) then one finds the following extended MNS mixing matrix:

\[
V \simeq \begin{pmatrix}
0.806 & -0.591 & 0.001 & -0.008 & -0.008 & -0.012 & -0.012 & 0.0001 & -0.0001 \\
-0.418 & -0.569 & 0.701 & 0.020 & 0.020 & -0.012 & -0.012 & 0.068 & -0.068 \\
0.417 & 0.570 & 0.701 & -0.020 & -0.020 & 0.012 & 0.020 & 0.068 & -0.068 \\
0 & 0 & 0 & 0.707 & -0.707 & -0.001 & 0.001 & 0 & 0 \\
0 & 0 & 0 & 0.001 & -0.001 & 0.707 & -0.707 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.707 & -0.707 \\
-0.032 & -0.026 & 0 & -0.707 & -0.707 & 0 & 0 & 0 & 0 \\
0 & 0.029 & 0 & -0.001 & -0.001 & -0.707 & -0.707 & 0 & 0 \\
0 & 0 & 0 & -0.136 & 0 & 0 & 0 & 0 & 0.701 & -0.701
\end{pmatrix}.
\]
FIG. 3: $BR(\mu \to e\gamma)$ versus $\delta m = M_{N_2} - M_{N_1}$, for $M_{N_1} = 100, 500, 900$ GeV from up to down, respectively. The horizontal dashed line refers to the MEG experiment limit of $BR(\mu \to e\gamma)$. The other parameters are fixed as follows: $M_{N_3} = 2000$ GeV, $\mu_{s_1} = 10^{-10}$ GeV, $\mu_{s_2} = 10^{-8}$ GeV, $\mu_{s_3} = 2.62 \times 10^{-9}$ GeV, $m_{\nu_1} = 10^{-13}$ GeV, $m_{\nu_2} = 8.5 \times 10^{-12}$ GeV, $m_{\nu_3} = 5.05 \times 10^{-11}$ GeV.

From this example, one notices that the elements $V_{\mu i}, i = 4, \ldots, 9$ are of order $O(0.01)$ which induce the SM-like contribution $3.846 \times 10^{-9}$ and total contribution of order $3.863 \times 10^{-9}$. Thus, the ratio $R_\mu = a_\mu^e/(a_\mu^{SM})$ is given by $R_\mu = 0.994$.

In the Fig. 3 we present the $BR(\mu \to e\gamma)$ versus $\delta m = M_{N_2} - M_{N_1}$ for $M_{N_1} = 100, 500, 900$ GeV from up to down, respectively. Here we assume that $M_{N_3} = 2000$ GeV, $\mu_{s_1} = 10^{-10}$ GeV, $\mu_{s_2} = 10^{-8}$ GeV, $\mu_{s_3} = 2.62 \times 10^{-9}$ GeV, $m_{\nu_1} = 10^{-13}$ GeV, $m_{\nu_2} = 8.5 \times 10^{-12}$ GeV, $m_{\nu_3} = 5.05 \times 10^{-11}$ GeV. Note that the $BR(\mu \to e\gamma)$ is not sensitive to the value of $M_{N_3}$. For other LFV processes like $\tau \to e\gamma$ and $\tau \to \mu\gamma$, the
present experimental limits of their branching ratios are given by \[21\]

\[
BR(\tau \to e\gamma) < 3.3 \times 10^{-8}, \quad BR(\tau \to \mu\gamma) < 4.4 \times 10^{-8}.
\]

One can easily show that in our models these experimental bounds can be translated into the following constraints:

\[
\left| \sum_{i=1}^{3} V^\ast_{\tau i} V_{ei} \right| < 7.5 \times 10^{-3}, \quad \left| \sum_{i=1}^{3} V^\ast_{\tau i} V_{\mu i} \right| < 8.6 \times 10^{-3}.
\]

This implies that \((FFT)_{31,13} \lesssim 1.5 \times 10^{-2}\) and \((FFT)_{23,32} \lesssim 1.7 \times 10^{-2}\). As mentioned above, \(F = m_D M_N^{-1} \sim \mathcal{O}(0.1)\), therefore these constraints are naturally satisfied in our model, for \(m_D \sim \mathcal{O}(100)\text{GeV}\) and \(M_N \sim \mathcal{O}(\text{TeV})\) and no constraint will be imposed.

In Fig. 4 we plot \(R_{\mu}^\nu\) as a function of \(\mu_s^3\), for \(M_N = (120, 125, 300)\text{GeV}\), \((400, 465, 800)\text{GeV}\) and \((900, 1500, 1900)\text{GeV}\) from up to down, respectively. The other parameters are fixed as in Fig. 3. In addition we vary the angles of the orthogonal matrix \(R\) from 0 to \(\pi\). As can be seen from this figure, there is no significant difference between the SM expectation and the total result of \(g - 2\) in TeV scale \(B - L\) extension of the SM with inverse seesaw.

\section*{VI. CONCLUSIONS}

In this paper we have computed the anomalous magnetic moment of the muon in TeV scale \(B - L\) extension of the SM with inverse seesaw mechanism. The one loop contributions due to the exchange of right-handed neutrinos, \(B - L\) gauge boson, and extra Higgs have been analyzed. We showed that right-handed neutrinos may give a significant contribution to \(a_\mu\), however it turns out that it is quite restricted by sever constraints from leptonic processes. Therefore, the SM contribution in \(B - L\) extension of the SM with inverse seesaw mechanism remains intact. Thus the discrepancy between the theoretical prediction of \(a_\mu\) and its experimental measurement requires a different source of new physics, like supersymmetric models with minimal flavor violation, which usually respect the LFV constraints. We also studied the impact of the right-handed neutrinos on the LFV decay \(\mu \to e\gamma\). We have shown that the rate of this decay is enhanced and is reachable by the MEG experiment.

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