Meson-meson interactions and Regge propagators

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Abstract

By a reformulation of the loop expansion in the Resonance-Spectrum-Expansion amplitude for meson-meson scattering, in terms of $s$-channel exchange of families of propagator modes, we obtain a formalism which allows for a wider range of applications. The connection with the unitarized amplitudes employed in some chiral theories is discussed. We also define an alternative for the Regge spectra and indicate how this may be observed in experiment.

1 Introduction

In Ref. [1] a nonrelativistic Schrödinger model was proposed for the scattering amplitude of non-exotic multi-channel meson-meson scattering, which allows an exact solution in the form of an analytic expression for the scattering amplitude. Bound states and resonances are obtained through the coupling of the two-meson system to a harmonic oscillator (HO), the oscillator frequency being independent of flavor. By fine-tuning the intensity of the coupling, one can transform the oscillator spectrum into the spectrum of mesons for all possible flavor combinations. The model’s results, in particular concerning the resonance and bound-state pole structure in the scalar-meson sector [2,3], are well known. Here, it is our aim to show that this model corresponds to $s$-channel exchange of families of propagator modes, similarly to the exchange of a family of leading and daughter Regge trajectories.

In the naivest quark-model picture, quarks and antiquarks are assumed to be confined to a small region in space by strong forces. The bulk of the interactions of the quarks with the glue is contained in effective, or constituent, quark masses, whereas the remaining dynamics is described by a confining potential of sorts [4,5]. The quantum numbers of the effective $q\bar{q}$ system are given by the total $q\bar{q}$ spin $S$, the relative $q\bar{q}$ orbital angular momentum $L$, and the $q\bar{q}$ radial
excitation $N$. For the lowest radial states of different flavor combinations, i.e., having $N = 0$, the confinement-model parameters can be adjusted to experiment so as to obtain reasonable results [6]. However, for higher radial excitations, the results are poor [7]. Of course, the reason is the absence of meson loops, in combination with a wrong fine-tuning of the model parameters of the $N = 0$ states to the lowest states in the experimental spectrum [1].

More elaborate quark models do include meson loops [1,8–10], and predict physical resonances instead of fictitious real meson masses only. On the other hand, in an effective, non-microscopic picture, Oller and Oset [11] dynamically generated the low-lying scalar mesons $f_0(600)$ (alias $\sigma$ meson), $f_0(980)$, and $a_0(980)$ (the $K^*_0(800)$ or $\kappa$ meson was not studied), by means of coupled-channel Lippmann-Schwinger equations with meson-meson potentials resulting from the lowest-order chiral Lagrangian. Their procedure boils down to using an effective four-meson vertex, summing up the bubbles from the meson loops, and unitarizing the resulting scattering amplitude. Hence, they nicely showed that it only takes a four-point interaction to generate scattering poles associated with light scalar mesons. Earlier, Törnqvist had made a similar suggestion [9], but went one step further by also assuming pre-existing mesons, which correspond to an input spectrum for bare mesons. Unfortunately, his proposal ignored the issue of Adler zeros, and so did not allow to conclude that in his model actually two scalar-meson nonets can be generated. Later, in Ref. [12] he did include Adler zeros, but in the subsequent analysis, together with Roos [13], the $K^*_0(800)$ (alias $\kappa$ meson) was still not found, probably due to the use of an unphysical, negative Adler zero in the $I=1/2$ case [14]. The latter pole of the isodoublet $S$-wave scattering amplitude was then indeed generated by Oller and Oset, together with Peláez [15], in an alternative coupled-channel unitarization scheme. Furthermore, in Ref. [16] Oller and Oset reported the observation of a second nonet, associated with pre-existing meson states, in an $N/D$ approach to unitarization. This issue was thoroughly studied by Boglione & Pennington in Ref. [17], who came to the conclusion that it is indeed possible to generate two states starting from one bare seed (or pre-existing state) only, and that it even may be plausible to dynamically generate many states with the same quantum numbers but different masses. That is exactly what had been proposed almost two decades earlier in Ref. [2].

In Ref. [1] an infinity of seeds was introduced, all related through one parameter, viz. the oscillator frequency $\omega$, being the same for all flavors. When, at low energies, the sum is reduced to an effective constant, one obtains a four-meson vertex which dynamically generates exactly one pole in the case of the scalar mesons. Also, at slightly higher energies, the sum can be approximated by its leading term and an effective constant for the remaining sum [18]. Then one obtains, apart from the dynamically generated resonance, a second pole associated with the one leading seed. However, in general, there is no need to approximate the sum, thus allowing to generate several dynamical resonance poles, [2,3,19] and moreover an infinite number of normal resonances associated with the infinity of seeds. Furthermore, the model of Ref. [1] can be applied to different flavors, including charm and bottom, with just one set of parameters. The low-lying scalar-meson nonet pops up without even being anticipated [2], using the parameters that were fine-tuned for the vector and pseudoscalar spectra in Ref. [20]. However, there is an important difference with the technique of introducing by hand one or more seeds, namely that the coupling constants can be controlled.

The problem of couplings and propagator modes is not new. In Ref. [21], Delbourgo, Rashid, Salam, and Strathdee remarked that it is well-known “that Regge trajectories arise from sums of infinite sequences of Feynman diagrams in conventional field theory”, and, moreover, that this “poses the problem of suitable coupling constants”. The latter problem was solved in Ref. [21], where a strategy was developed for the decomposition of a Regge trajectory in its propagator
modes. Here, we shall follow a different but comparable approach.

In Refs. [1, 20], the couplings of the propagator modes were controlled by Clebsch-Gordans and 9-j symbols, assuming the \(^3P_0\) mechanism [22–25] for quark-pair creation. But this procedure led to inconsistencies in the light-meson sector. However, not only the couplings, but also the choice of the spectrum of propagator modes is of importance. In Refs. [1, 20], such a choice was made involving one free parameter, viz. the universal level splitting, which appeared to be largely flavor independent, as we shall further elaborate in Sect. 3 below. Based on the choice of the spectrum, the couplings issue was solved by the \(^3P_0\) recoupling strategy of Refs. [26–28], leaving only one parameter free, which represents the probability of quark-pair creation. As a consequence, there are only two free parameters, no matter if the whole sum of propagator modes is approximated by just one point vertex, or by the sum of a few propagator modes and one point vertex.

The Schrödinger equation for the meson-meson scattering model was solved with the wave-function approach (WFA) in Refs. [1,20]. Later, in Ref. [29], the same dynamical equations were solved by an iterative method, which we refer to by Resonance-Spectrum Expansion (RSE). The latter method allows more easily a comparison with current theories for meson-meson interactions. However, it should be kept in mind that both approaches, WFA and RSE, lead from the same dynamical equations to the same expression for the meson-meson scattering amplitude.

In Refs. [30,31], besides the usual four-meson interaction, a fictitious bare scalar \(\kappa\) meson was introduced, in a similar fashion as suggested by Törnqvist [9] and Oller & Oset [16], leading to a two-pole description of the \(K\pi\) S-wave interaction for energies up to about 1.6 GeV. The behavior of these poles, as a function of one overall coupling parameter, is comparable to the pole movements of the two lowest lying \(\kappa\) (or \(K^*\)) poles described in Refs. [2,18]. Namely, the lower pole, which is dynamically generated by the \(K\pi-K\pi\) vertex and associated with the \(K^*(800)\), moves towards larger negative imaginary energies, away from the real axis, according as the coupling is decreased, whereas the higher pole, stemming from the seed, approaches the real axis for decreasing coupling, ending up at the mass of the fictitious meson for vanishing coupling. This similarity in pole behavior inspires us to revisit the meson-meson scattering model of Ref. [1], and relate it to the exchange of Regge propagators [21].

\section{The scattering amplitude}

We define the amputated amplitude for non-exotic two-meson scattering by

\[ t = V + V\Omega V + V\Omega V\Omega V + \ldots = V \left[1 - \Omega V\right]^{-1}, \tag{1} \]

where \(V\) stands for the RSE propagator and \(\Omega\) for the two-meson loop function. In Ref. [29], it was shown that the one-channel RSE expression for the \(\ell\)-th partial-wave two-meson scattering amplitude follows from

\[ V_{\ell}(p) = \frac{\lambda^2}{r_0} \sum_{N=0}^{\infty} \frac{|g_{NL}|^2}{E(p) - E_{NL}} \quad \text{and} \quad \Omega_{\ell}(p) = -2i\mu pr_0^2 j_\ell (pr_0) h^{(1)}_\ell (pr_0), \tag{2} \]

where \(p\) is the center-of-mass (CM) linear momentum, \(E(p)\) the total invariant two-meson mass, \(j_\ell\) and \(h^{(1)}_\ell\) the spherical Bessel function and Hankel function of the first kind, respectively, \(\mu\) the reduced two-meson mass, and \(r_0\) a parameter with dimension mass\(^{-1}\), which can be interpreted as the average string-breaking distance. The coupling constants \(\lambda\) and \(g_{NL}\) are discussed in Sect. [4] where in Eq. (16) the relation between \(\ell\) and \(L = L(\ell)\) is expressed.
The kernel $V$, which is graphically represented by Fig. 1, is the result of $s$-channel exchange of a system with internal structure, characterized by resonance modes and their masses $E_{NL}$. In the back of our mind we have, of course, the picture of a resonating quark-antiquark system. But this is not of much importance yet. So we consider the exchange of propagators with structure, rather than of single and pointlike massive objects, in the very same spirit as a Regge propagator.

At the vertices, each mode couples independently to the meson pairs. The constants $g_{NL}$ indicate the intensities of the couplings of the propagator modes to the meson pairs. Now, since we consider an infinity of propagator modes, we would end up with an infinite number of coupling parameters, with which one could describe any two-meson system. However, this freedom is strongly restricted by the $^3P_0$ recoupling scheme [27,28], described in Sect. 4. In fact, within the $^3P_0$ scheme, all but one of the parameters $g_{NL}$ are fully determined. The only remaining free parameter is $\lambda$, i.e., the overall three-meson-vertex coupling constant.

We thus assume that the RSE model can be formulated in terms of $s$-channel exchanges of an infinite spectrum of propagator modes. The expression for the partial-wave loop function $\Omega_\ell(p)$ in Eq. (2) stems from the nonrelativistic two-meson loops of expansion (1). We have depicted $\Omega_\ell(p)$ in Fig. 2. In the present formulation we are free to adopt different schemes for the determination of loop functions. This way, with not too much effort, the formalism can also be extended to non-exotic baryon-meson and baryon-antibaryon scattering and production processes.

Besides the coupling constants, we also assume vertex functions which regularize the loop integrations. In the RSE, one employs a local spherical delta-shell with radius $r_0$. In the CM of the two-meson system, its Fourier transform is given by a spherical Bessel function, i.e.,

$$j_\ell(kr_0),$$

(3)
where $k$ stands for the CM loop momentum. In the CM system, we furthermore adopt for the two-meson loops the techniques developed by Logunov & Tavkhelidze and Blankenbecler & Sugar [32, 33], and also follow the covariant prescription formulated by Cooper and Jennings [34]. This reduces the four-dimensional integration to a three-dimensional one in the CM frame, which, assuming spherical symmetry, takes the form

$$
2\mu r_0^2 \int_0^\infty k^2 dk \frac{J^2_\ell(kr_0)}{p^2 - k^2 + i\epsilon},
$$

(4)

where $k$ and $p$ represent relativistic three-momentum moduli, and $p^2$ is given by the on-shell relative two-meson momentum in the CM frame. This expression straightforwardly leads to the loop function $\Omega_\ell(p)$ in Eq. (2).

One obtains a two-meson scattering amplitude $T$ satisfying unitarity, from the amputated amplitude $t$ defined in Eq. (1), by

$$
T = X^\dagger t X = X^\dagger V_{\ell} \left[ 1 - \Omega_{\ell} V_{\ell} \right]^{-1} X, \quad \text{where} \quad X^\dagger X = XX^\dagger = \Im m(\Omega).
$$

(5)

Consequently, in the loop-expansion approximation of the scattering amplitude, it is sufficient to determine the amputated propagator $V$ and the amputated loop function $\Omega$. The latter, which is depicted in Fig. 2 consists here of two vertices and two free meson propagators.

The partial-wave amplitude $T_\ell(p)$ is related to the amputated partial-wave amplitude $t_\ell(p)$ of Eq. (17) by

$$
T_\ell = X^\dagger_\ell t_\ell X_\ell = X^\dagger_\ell V_{\ell} \left[ 1 - \Omega_{\ell} V_{\ell} \right]^{-1} X_\ell, \quad \text{where} \quad X^\dagger_\ell X_\ell = XX^\dagger_\ell = \Im m(\Omega_\ell),
$$

(6)

ensuring unitarity for each partial wave.

One ends up with an expression for the amplitude of non-exotic two-meson scattering, even in the case of many coupled channels, which contains only three free parameters, viz. $\lambda$, $r_0$, and $\omega$, besides the constituent quark masses. The resulting amplitudes, for different flavors and a variety of orbital quantum numbers, have been confronted with experiment in numerous publications.

### 3 The propagator modes

Soon after T. Regge noticed, in his famous work [35], that bound states and resonances of the scattering amplitude are related to their poles in the complex orbital-angular-momentum plane, Chew and Frautschi made the observation that the squares of the masses of baryonic and mesonic resonances come out on almost linear Regge trajectories [36]. For mesons this has been explored in many models, obtaining relations of the form [37]

$$
E_{NL}^2 = C^2 + 2C\omega \left( 2N + L + \frac{3}{2} \right),
$$

(7)

for meson masses $E_{NL}$. Here, $N$ and $L$ represent the radial and orbital-angular-momentum quantum numbers, respectively, of the $q\bar{q}$ systems. Furthermore, $C$ is a constant which depends on the $q\bar{q}$ constituent flavor masses, and $\omega$ is a universal frequency. However, in Ref. [1] it was assumed that, at least for low energies, the trajectories of propagator modes for $c\bar{c}$ and $b\bar{b}$ systems are linear, not quadratic, in their masses. In Ref. [20], the linear mass dependence was extended to the light quarks $u$, $d$, and $s$, too, while in Ref. [2] the linear propagator modes were shown
to explain the scalar resonances in non-exotic $S$-wave meson-meson scattering as well as the scattering data for energies up to about 2 GeV in the $K\pi + K\eta + K\eta'$ complex [19, 38].

For low excitations, the linear mass relation is obvious from Eq. (7) if

$$ C \gg \omega, \quad (8) $$

in which case one finds

$$ E_{NL} \approx C + \omega \left( 2N + L + \frac{3}{2} \right), \quad (9) $$

but less obvious when $C$ and $\omega$ are of the same order of magnitude. Now, for $\omega$ one finds in Ref. [20] the value 0.19 GeV, whereas for $b\bar{b}$ and $c\bar{c}$ the values 9.45 GeV and 3.12 GeV, respectively, are quoted for $C$, which satisfies well condition (8). For the light quarks, $C \sim 1$ GeV, which still is not in conflict with condition (8) for low energies. Consequently, from the good results of the model in Refs. [1, 2, 19, 20, 38], one cannot exclude that, at high energies, the propagator trajectories will be quadratic in mass. However, since also the results for the $K\pi + K\eta + K\eta'$ complex support linear relations, we will stick here to relation (9) for the model’s Regge trajectories.

In previous work on the RSE, we referred to the modes of the exchange propagator (2) as the confinement bound states. This picture has not been completely abandoned here. On the contrary, it will play an important role in determining the parameters that describe the propagator modes. Mode masses depend in the first place on the quark flavors flowing in the propagator. That information is contained in the constant $C$ of Eq. (9). However, we will argue in the following that the mode level splittings are largely flavor independent.

The mesonic resonances extracted from experiment are organized by flavor content, $J^{PC}I^G$ quantum numbers, mass and width. Based on the $b\bar{b}$ and $c\bar{c}$ spectra, it was concluded in Ref. [1] that, in principle, there must exist an infinity of such states, though most of the excited states are difficult to observe because of the many open two-mesons channels to which they couple, albeit at higher masses obscured from observation because of the many two-meson systems which couple to $q\bar{q}$. Accordingly, we expect an infinite number of scattering poles in meson-meson scattering, here represented by

$$ E = P_0, P_1, P_2, \ldots. \quad (10) $$

Unitarity then requires that in the one-channel restriction, assuming the poles (10) to be simple, the elastic scattering matrix $S$ be given by [9]

$$ S(E) = \frac{(E - P_0^*) (E - P_1^*) (E - P_2^*) \ldots}{(E - P_0) (E - P_1) (E - P_2) \ldots}. \quad (11) $$

If we suppose that the resonances (10) stem from the spectrum of modes of propagator (2), given by the real quantities

$$ E = E_0, E_1, E_2, \ldots, \quad (12) $$

then we may represent the differences $(P_n - E_n)$, for $n = 0, 1, 2, \ldots$, by the complex mass shifts $\Delta E_n$. Thus, we obtain for the unitary $S$-matrix the expression

$$ S(E) = \frac{(E - E_0 - \Delta E_0^*) (E - E_1 - \Delta E_1^*) (E - E_2 - \Delta E_2^*) \ldots}{(E - E_0 - \Delta E_0) (E - E_1 - \Delta E_1) (E - E_2 - \Delta E_2) \ldots}. \quad (13) $$

1 Note that we do not consider here a possible overall phase factor representing a background.
So we assume here that resonances occur in scattering because the two-meson system couples to certain modes of the propagator $\Sigma$, usually of the $q\bar{q}$ type, viz. in non-exotic meson-meson scattering. Let the strength of the coupling be given by $\lambda$. For vanishing $\lambda$, we presume that the widths and real shifts of the resonances also vanish. Consequently, the scattering poles end up at the positions of the mode spectrum $\Delta E_n$, and so

$$\Delta E_n \rightarrow 0 \quad \text{for} \quad n = 0, 1, 2, \ldots$$

As a result, the scattering matrix tends to unity, as expected in case there is no interaction. The scattering amplitude $(1)$ exactly satisfies these requirements for the propagator and loop function $(2)$. As a consequence, it seems that one may only deduce an approximate mode spectrum from experiment. Its precise masses $E_{NL}$ can then be found by comparison to scattering and production data, once the full scattering amplitude has been composed. However, in Sect. 7 we shall see that in production processes the Regge spectrum may become visible.

In order to set out with the task to find a reasonable ansatz for the mode spectrum of our propagator, let us assume that the spectrum of mesonic quark-antiquark systems can be described by flavor-independent HO confinement. Then, for each pair of flavors, an infinite set of mesons exists with all possible spin, angular, and radial excitations. But unfortunately, for most flavor pairs only a few angular and even fewer radial recurrences are known [39]. If we do not distinguish up and down, but just refer to non-strange ($n$) quarks, then we have at our disposal four different flavors: $n$, $s$, $c$, and $b$. These can be combined into ten different flavor pairs, each of which may come in two different spin states: 0 or 1. This gives rise to, in principle, twenty different meson spectra. With some 150 known mesons, this means 7.5 angular plus radial excitations on average, per flavor pair. This is much less than e.g. the known excitations of the positronium spectrum. No wonder that it requires some imagination to guess economic strategies for the description of mesons.

![Figure 3: Nonstrange, charmonium, and bottomonium $J^{PC} = 1^{--}$ states compared to the corresponding states from a harmonic-oscillator spectrum. The level spacing for the oscillator equals 0.38 GeV.](image)

As one may verify from the latest Review of Particle Physics [39], known vector states are more numerous than any other type of mesonic resonances, since they are easier to produce.
Consequently, in order to structure the mode masses, we begin with the vector mesons, carrying quantum numbers $J^{PC} = 1^{--}$. In Fig. (3) we compare the observed $n\bar{n}$, $c\bar{c}$, and $b\bar{b}$ vector states with the HO possible states. Most of the data are taken from Ref. [39]. The $\rho(1250–1290)$ signal was originally reported in Refs. [40–44], and has very recently been confirmed in a coupled-channel data analysis [45]. The $\Upsilon(1D)$ has been observed in Ref. [46].

The charmonium vector states, shown in Fig. (3), bear many similarities with the two-particle HO: a ground state in a $c\bar{c}$ $S$-wave, and higher $c\bar{c}$ radial excitations that are almost degenerate with the $c\bar{c}$ $D$-wave states. Also, except for the ground state, the level spacings are roughly equal. In Refs. [1, 20, 47], the mechanism was discussed which turns the HO spectrum into the charmonium spectrum, also including the ground-state levels.

For the $\rho$ and $\Upsilon$ vector states, also shown in Fig. (3), we see a very similar pattern: the $q\bar{q}$ $S$-$D$ splittings are slightly larger, while the $\rho(770)$ ground state of the $\rho$ spectrum and the $\Upsilon(1S)$ ground state of the $\Upsilon$ spectrum also come out far below the corresponding oscillator ground states. From Fig. (3) one may moreover conclude that, as far as the level splittings are concerned, there is not much reason to separate the light-quark sector from the heavy quarks. The mechanism which turns the oscillator states into the $\rho$ and $\Upsilon$ resonances is discussed in Ref. [20].

What we learn from the above comparison is that the level splittings are largely flavor independent and more or less constant, whereby the ground-state level of the mode spectrum is determined by the effective flavor masses.

| $I = 1$ | $I = \frac{1}{2}$ | $I = 0$ |
|---|---|---|
| $n\bar{n}$ | $n\bar{s}$ | $n\bar{n}$ | $s\bar{s}$ |
| $0^+$ | $a_0(1450)$ | $K_0(1430)$ | $f_0(1370)$ | $f_0(1500)$ |
| | | $K_0(1980)$ | $f_0(1710)$ | $f_0(2020)$ |
| | | | $f_0(2200)$ | |
| $1^+$ | $a_1(1260)$ | $K_1(1270)$ | $f_1(1285)$ | $f_1(1420)$ |
| | $b_1(1235)$ | $K_1(1400)$ | $h_1(1170)$ | $h_1(1380)$ |
| | $a_1(1640)$ | $K_1(1650)$ | $f_1(1510)$ | $h_1(1595)$ |
| $2^+$ | $a_2(1320)$ | $K_2(1340)$ | $f_2(1270)$ | $f_2(1430)$ |
| | $a_2(1700)$ | $K_2(1980)$ | $f_2(1525)$ | $f_2(1640)$ |
| | | | $f_2(1565)$ | $f_2(1640)$ |
| | | | $f_2(1910)$ | $f_2(1810)$ |
| | | | $f_2(2300)$ | $f_2(2150)$ |

Table 1: The experimentally observed light positive-parity mesons.

In Table 1 we show the experimental spectrum of light positive-parity mesons. Only for the $f_2$ states, almost enough resonance data are available, to allow for comparison with HO confinement. We therefore assume that the states in the first two $f_2$ columns contain mostly non-strange $q\bar{q}$ pairs, and in the next two predominantly $s\bar{s}$ pairs. Furthermore, the quark pair may come in a relative $P$-wave (1st and 3rd column) or $F$-wave (2nd and 4th column). From the values for the central mass positions as given in Ref. [39], we collect in Table 2 mass differences
for a selected set [48] of $f_2$ states. For the spectra of Fig. 3 we deduced a level spacing of 0.38

| states                              | mass difference   |
|-------------------------------------|-------------------|
| $m(f_2(1910)) - m(f_2(1525))$      | $0.39 \pm 0.01$ GeV |
| $m(f_2(2300)) - m(f_2(1910))$      | $0.38 \pm 0.03$ GeV |
| $m(f_2(1950)) - m(f_2(1565))$      | $0.40 \pm 0.02$ GeV |
| $m(f_2(2340)) - m(f_2(1950))$      | $0.39 \pm 0.04$ GeV |
| $m(f_2(2010)) - m(f_2(1640))$      | $0.38 \pm 0.05$ GeV |
| $m(f_2(2150)) - m(f_2(1810))$      | $0.34 \pm 0.02$ GeV |

Table 2: The experimentally [39] observed mass differences for isoscalar light positive-parity mesons with $J = 2$.

GeV, which agrees well with the splittings in Table 2. As a first approximation, it thus seems reasonable to adopt for the masses of the propagator modes the expression

$$M(f, \bar{f}; N, L) = m_f + m_{\bar{f}} + \omega \left(2N + L + \frac{3}{2}\right).$$

(15)

Here, $f$ and $\bar{f}$ represent the flavors of the quark and the antiquark, $m_f$ and $m_{\bar{f}}$ their respective masses, and $\omega$ the oscillator frequency.

In the following, let us study some details of formula (15). The vector-meson states have unit total angular momentum, $J = 1$, and unit $q\bar{q}$ total spin, $S = 1$. Hence, since the parity of vector-meson states equals $P = -1$, their orbital angular momentum can be $L = 0$ ($S$-wave) or $L = 2$ ($D$-wave). From formula (15) we then understand that, for HO confinement, the vector-meson states with $(N, L = 2)$ are degenerate with the vector-meson states with $(N + 1, L = 0)$, as shown in Fig. 3. For other flavor and spin excitations similar results emerge. One obtains a very regular, equally spaced spectrum of propagator modes, with an oscillator frequency $\omega$, which comes out at about 0.19 GeV for the data.

Non-strange ($n\bar{n}$) and strange ($s\bar{s}$) configurations double the number of isoscalar states into $SU(3)$-flavor singlets and octets. But one should be be aware that all states are mixed through the meson loops. Hence, like in Nature we will not find pure angular, radial, or flavor excitations for resonances of the scattering amplitude (1). Lattice calculations reveal that it may even be very hard to disentangle the various configurations showing up in $f_0$ systems [49]. Moreover, meson loops influence the precise resonance shapes. Some come out broad, others narrower. Also, the central resonance positions may shift substantially (100–300 MeV [1, 2, 20]) with respect to the propagator mode spectrum.

Now, as discussed in the beginning of this section, we may not exclude the possibility that the trajectories for the propagator modes are quadratic in mass. Hence, we may thus very well assume that relation (15) is only an approximation, valid for low energies, and refer to propagators (11) as Regge Green’s functions [21]. We leave the study of a more precise relation with string theory [50] for future research.
4 Vertices

In this section we study how the propagator modes couple to the meson pair. Thereto, we characterize each meson by its quantum numbers. This is in part guess work, since we only have at our disposal the total spin \(J\), the parity \(P\), and, for flavorless mesons, the \(C\)-parity. But let us suppose here that we also have knowledge of the orbital angular momentum \(L\), the internal spin \(S\), and the radial excitation \(n\) of the quark pair that constitutes the meson. Hence, a two-meson system then consists of the sets of quantum numbers \((J_1, L_1, S_1, n_1)\) and \((J_2, L_2, S_2, n_2)\), characterizing each meson, and also the quantum numbers describing the relative motion of the two mesons, viz. \(J, \ell, s,\) and \(n\). The propagator modes are similarly characterized by a set of meson quantum numbers \((J, L, S, N)\). Hence, the complete coupling is given by the matrix element of the transition operator \(O\)

\[
\langle (J_1, L_1, S_1, n_1); (J_2, L_2, S_2, n_2); J, \ell, s, n | O | J, L, S, N \rangle .
\]  

(16)

Given this form of the coupling constants, it is advantageous to determine the scattering amplitude from the partial-wave expansion. In the two-meson CM system, assuming spherical symmetry, we define

\[
T(\vec{p}) = \sum_{\ell=0}^{\infty} (2\ell + 1) P_{\ell}(\hat{p} \cdot \hat{p}') T_{\ell}(p).
\]  

(17)

An important ingredient for the vertices employed in the WFA and the RSE is the \(^3P_0\) mechanism for quark-pair creation of Micu [22] and Carlitz & Kislinger [23], which has been worked out in more detail by Le Yaouanc, Oliver, Pène & Raynal [24], Chaichian & Kögerler [25], and later by Ribeiro [26]. A complete expression of the latter matrix elements for all possible quantum numbers and different effective quark masses is given in Refs. [27, 28]. The resulting couplings have been employed for perturbative calculus of particle widths and mass shifts in Refs. [51–54]. In the latter works, flavor mass and the universal oscillator frequency were combined to one parameter, different for each flavor.

In Ref. [29] it was shown that the spectral representation of the Green’s function, viz.

\[
\sum_{N=0}^{\infty} \frac{|\mathcal{F}_{NL}(r_0)|^2}{E - E_{NL}} = \frac{2\mu}{\mu_0^2} \frac{F_L(E, r_0) G_L(E, r_0)}{W(F_L(E, r_0), G_L(E, r_0))},
\]  

connects the WFA expression for the full propagator (righthand side [1]) with the RSE iterative result (leighthand side [29]). The full propagator is obtained by constructing an exact solution of a nonrelativistic stationary equation for meson-meson scattering containing a confining part \(H_c\). The expansion in eigensolutions of \(H_c\) is obtained by an iterative method. The set of functions \(\{\mathcal{F}_{NL}; N = 0, 1, 2, \ldots\}\) represents, for orbital angular momentum \(L\), a full set of radial eigenstates, with eigenvalue \(E_{NL}\), of the confining part \(H_c\) of the full Hamiltonian. Furthermore, \(F_L\) and \(G_L\) represent two linearly independent solutions of \(H_c\) for any value of the energy \(E\).

The mode distribution in the CM frame of the two-meson system is contained in \(|\mathcal{F}_{NL}(r_0)|^2\) of Eq. (18). In the RSE, the mode distribution is contained in the matrix elements of Eq. (16). Henceforth, we put aside the WFA description of the propagator modes and its coupling to meson pairs, and concentrate on the iterative RSE description. Hence, when we abbreviate the result of matrix element (16) for two-meson channel \(i\) by \(g_i,\ell(N, L)\), then we propose here to write for the partial-wave Regge propagator connecting the two-meson channels \(i\) and \(j\) the expression

\[
[V_{\ell}(s)]_{ij} = \frac{\lambda^2}{\mu_0^2} \sum_{L=0}^{\infty} \sum_{N=0}^{\infty} \frac{g_i,\ell(N, L) g_j,\ell(N, L)}{E - E_{NL}},
\]  

(19)
where $L = L(\ell)$ and in which form the RSE formalism takes a shape similar to the result of Ref. [21]. The sum in $L$ is usually very much restricted, since most couplings $g_{i,\ell}(N, L)$ vanish, as follows from the details [27,28] of Eq. (10).

The vertices connecting the multi-mode propagator to the two-meson systems, are calculated in the CM system. On a basis of HOs, for each mode one assumes a spatial distribution of two quarks and two antiquarks, one colorless pair for the propagator mode, and an equally colorless $^3P_0$ pair. These spatial distribution functions are decomposed on the basis of two-meson distributions [55]. The coefficients of such a decomposition are called recoupling constants [27,28]. In Table 3 we have collected some of the $^3P_0$ recoupling constants, merely as a demonstration.

| vertex $nJ^{PC} \rightarrow rJ^{PC} + rJ^{PC}$ in $(\ell,s)$ | recoupling coefficients $\{g(n)\}^2 \times 4^n$ |
|----------------------------------------------------------|---------------------------------|
| $n^0++ \rightarrow o^0++ + o^0++$ (0,0)                 | $\frac{1}{144}(2n-3)^2$         |
| $n^0++ \rightarrow o^1-- + o^1--$ (0,0)                 | $\frac{1}{144}(2n-3)^2$         |
| $n^0++ \rightarrow o^1-- + o^1++$ (0,0)                 | $\frac{1}{72}(2n-3)^2$          |
| $n^0++ \rightarrow o^0-- + o^0--$ (1,1)                 | $\frac{1}{24}(2n+3)$            |
| $n^0++ \rightarrow o^1-- + o^1--$ (1,1)                 | $\frac{1}{12}(2n+3)$            |
| $n^0++ \rightarrow o^0++ + o^0++$ (0,0)                 | $\frac{1}{24}(n+1)$             |
| $n^0++ \rightarrow o^0++ + o^0--$ (0,0)                 | $\frac{1}{288}(2n+3)(n-1)^2$    |
| $n^0++ \rightarrow o^1-- + o^1--$ (0,0)                 | $\frac{1}{6x24^2} n(2n+1)(2n+3)(n-3)^2$ |
| $n^0++ \rightarrow o^0++ + o^1++$ (1,1)                 | $\frac{1}{12}$                 |
| $n^0++ \rightarrow o^1-- + o^1--$ (0,0)                 | $\frac{1}{72}(n+1)$             |
| $n^0++ \rightarrow o^1-- + o^1--$ (2,2)                 | $\frac{1}{18}(2n+5)$            |
| $n^0++ \rightarrow o^1-- + o^1--$ (0,0)                 | $\frac{1}{4x67}(2n+3)(n-1)^2$   |
| $n^0++ \rightarrow o^1-- + o^1--$ (0,0)                 | $\frac{1}{5x62}(2n+3)(2n-5)^2$  |
| $n^0++ \rightarrow o^0-- + o^0--$ (1,1)                 | $\frac{1}{12}$                 |
| $n^0++ \rightarrow o^0++ + o^0++$ (0,0)                 | $\frac{1}{132}(2n+3)(n-3)^2$    |
| $n^0++ \rightarrow o^1++ + o^1++$ (0,0)                 | $\frac{1}{144}(2n+3)(n-2)^2$    |
| $n^0++ \rightarrow o^1++ + o^1++$ (0,0)                 | $\frac{1}{144}(2n+3)(n-1)^2$    |

Table 3: Vertex recoupling constants $g(n)$ for the radial excitations $n$ ($n = 0, 1, 2, \ldots$) of pseudoscalar and scalar modes of the propagator (1), for the case of equal effective quark masses [27,28]. We have characterized the two-meson systems by the quantum numbers $rJ^{PC}$ ($r$ for radial excitation), the relative two-meson angular momentum $\ell$, and the total two-meson spin $s$. In one case, we have also indicated the $q\bar{q}$ internal angular momentum of the meson. For the other cases, the lowest possible quantum numbers are assumed.

For $n = 0$, the recoupling constants squared in one column add up to 1, with the proviso that meson pairs with two different mesons count twice, as actually we should have repeated the corresponding lines in the table for the interchanged pair. This result reflects the fact that we consider a properly normalized distribution for each mode, and an orthonormal set of two-meson distributions.
For $n > 0$, the recoupling constants squared in one column do not add up to 1. The reason is that for $n > 0$ more possible two-meson systems couple to the propagator modes. If those were included in the tables, we would obtain unity for all $n$. A full table would have infinite length. Moreover, even when limited to $J < 10$ and only for a few radial excitations, one would easily end up with a table of hundreds of pages. However, with a fast computer code in Fortran, based on the expressions given in Refs. [27,28], and which moreover takes care of the various possible isospin combinations and also allows for unequal effective flavor masses, the absence of such tables is no limitation.

5 Comparison to other models

In their comment [56] on the work of Törnqvist & Roos [13], Isgur & Speth pointed out that, since exotic channels do not couple to the propagator of Eq. (2), the corresponding scattering amplitude vanishes, which is not in agreement with experiment. Furthermore, they argued that in the Jülich model [57] $t$-channel processes lie at the origin of a broad dynamically generated pole in the $I = 0$ $S$-wave pion-pion scattering amplitude, and not $s$-channel propagator modes. Finally, they remarked that also the $a_0(980)$ and $f_0(980)$ are largely due to $t$-channel forces.

Now, it seems indisputable that with zero couplings there is no scattering. Nevertheless, the first observation of Isgur & Speth is incomplete.

Namely, when just arbitrary couplings and seeds are involved, then there is no clear prescription how to handle exotic channels. However, when propagator modes and couplings are related, like in the choice of Ref. [1] and the corresponding recoupling constants of Ref. [28], then quark-interchange processes, and so scattering in exotic channels, make part of the interactions that can be handled in the model.

In Ref. [28], the quark-interchange diagrams were suppressed in order to single out the effect of the propagator modes in $s$-channel exchange via $^3P_0$ pair creation. Consequently, although in principle there is no limitation in the model inhibiting the study of exotic channels, it is just neglected in the RSE approach to non-exotic meson-meson scattering. A study also including the quark-interchange diagrams has been carried out by Bicudo and Ribeiro in Ref. [58].

Moreover, in Ref. [59] Fariborz, Jora & Schechter observed that neglecting the $t$-channel exchange of a $\rho$ meson in $I = 0$ $S$-wave $\pi\pi$ scattering does not remove the $\sigma$ pole from the scattering amplitude of their model, and only shifts it in a modest way. Also note that the dynamical “$\sigma(400)$” pole generated by the Jülich group [57] is considerably lighter and broader than generally accepted.

In Ref. [17], also Boglione & Pennington remarked: “$s$-channel dynamics is not all that controls the scattering”, when referring to Ref. [13]. However, one may not conclude from spicing up with $t$-channel exchange, a model which only accounts for one (or a few) propagator modes, moreover in the ladder approximation, that meson-exchange contributions are really needed. In their replies to the comments on their work, Törnqvist and Roos stressed the concept of duality. Hence, it is not even clear whether $t$-channel exchange should be considered at all, when all propagator modes in $s$-channel exchange are accounted for. Duality [60,61] probably just works the other way around as well. So either take all $t$-exchange contributions into account, ideally in an untruncated fashion and not just in the ladder approximation, or all $s$-exchange contributions, but not both.

The behavior of the $f_0(980)$ and $a_0(980)$ poles, for variations in the RSE couplings, was studied in Refs. [62,63]. No further $t$-channel exchange was needed in order to describe these
scalar resonances by the lowest dynamically generated poles of the model in $K \bar{K}$, for respectively $I = 0$ and $I = 1$. The modelling is just a bit more complicated than for the $\sigma$ and the $\kappa$, because lower lying, but not very strongly coupled, channels are involved as well, namely $\pi\pi$ and $\eta\pi$, respectively. In fact, the correct way to describe the $f_0$ resonances is, of course, by employing a coupled-channel approach to the $I = 0$ S-wave [2,64]. Then one obtains in the RSE both the $f_0(600)$ ($\sigma$) and $f_0(980)$ resonances, in agreement with data, besides other scattering observables like phase shifts, line-shapes, and inelasticities [64].

Recently, the true pole positions of broad states were highlighted [17,65,66]. However, pole positions will always depend on the properties of a specific model. For narrow resonances there will be agreement, to some extent, among different models. But for broad structures it is rather unlikely that different approaches will lead to exactly the same pole positions. Boglione & Pennington wrote: “fitting data along the real axis cannot accurately determine the true pole position of a broad state without an analytic continuation, or a very specific model” [17]. Hence, unless we fully agree on the perfect model to describe the available data, true pole positions for broad structures do not exist. At best, we may agree on whether a specific pole exists.

In Ref. [18], for the description of elastic $P$-wave $\pi K$ scattering and the $K^*(892)$ resonance, the propagator of Eq. (18) was approximated by the first term plus a constant representing the remainder of the sum:

$$\sum_{N=0}^{\infty} \frac{|F_N(r_0)|^2}{E - E_N} \longrightarrow \frac{\alpha}{E - E_0} - \beta .$$

Such a procedure is equivalent to the approaches of Törnqvist [12] and Oset & Oller [16], as discussed in the introduction. In the latter approaches, however, one loses track of the relations among all coupling constants, thus needing to introduce arbitrary parameters, like $\alpha$ and $\beta$ in Eq. (20), while also contact with different flavors and other angular momenta is lost. Nevertheless, in the approximation of Eq. (20), one can study sufficiently well the properties of the dynamically generated and the lowest $q\bar{q}$ resonances.

In the case of elastic $P$-wave $\pi K$ scattering, no dynamically generated resonance is found. So, when the overall coupling is decreased, the $K^*(892)$ pole returns to $E_0$ [18]. But for non-exotic elastic $S$-wave $\pi K$ scattering, the dynamically generated $K_0^*(800)$ resonance appears, besides the $K_0^*(1430)$. We may thus conclude that the contact term indeed absorbs those terms of the Regge propagator which are not accounted for. In the loop sum (1) its contribution is negligible, provided a complete Regge propagator is exchanged. Hence, this result seems to suggest that the contact term is not necessary at all in a microscopic formulation.

The effect of hadron loops on the spectra of mesons and baryons has been studied by various groups, and for a variety of different confinement mechanisms [8,67–69]. For mesons, the procedure usually amounts to the inclusion of meson loops in a $q\bar{q}$ description, or, equivalently, the inclusion of quark loops in a model for meson-meson scattering. This results in resonance widths, central masses that do not coincide with the pure confinement spectrum, mass shifts of bound states, resonance line-shapes that are very different from the usual Breit-Wigner ones, threshold effects and cusps. In particular, it should be mentioned that mass shifts are large and negative for the ground states of the various flavor configurations [20]. Unquenching the lattice is still in its infancy, at least for the light scalars, as we conclude from Ref. [70]. However, its effects should not be underestimated. Hence, ground-state levels of quenched approximations for $q\bar{q}$ configurations in relative $S$-waves must be expected to come out far above the experimental masses.
6 Quark-interchange contributions

As mentioned in the previous section, the model is not limited to the study of non-exotic channels. However, $^3P_0$ pair creation/annihilation, through which process meson pairs couple to the Regge propagator, does not work for exotic channels. Nevertheless, the alternative, which is quark interchange, does give contributions to all possible hadronic final-state interactions, hence also for exotic two-meson channels.

Quark-interchange contributions to meson-meson interactions can be determined [58] with the very same techniques that were developed in Refs. [26–28].

7 Experimental results for the Regge spectrum

As may be concluded from Eq. (1), and from the expressions for $V_\ell$ and $\Omega_\ell$ given in Eq. (2), the dressed partial-wave RSE propagator for strong interactions takes the form (restricted to the one-channel case and leaving out some parts not essential for our discussion in this section)

$$\Pi_\ell(E) = \left\{ 1 - ij_\ell (pr_0) h^{(1)}_\ell (pr_0) \sum_{n=0}^{\infty} \frac{|g_{NL}|^2}{E - E_{NL}} \right\}^{-1}. \quad (21)$$

This propagator has the very intriguing property that it vanishes for $E \rightarrow E_{NL}$. Hence, one may wonder what happens in a physical process when the propagator does not allow any signal to pass. We shall show in the following that this phenomenon can be, and has indeed been, observed in experiment, but not in scattering processes.

The RSE amplitude for strong scattering is given in Eqs. (15). One easily verifies that it does not vanish in the limit $E \rightarrow E_{NL}$. However, for strong production processes, we deduced in Ref. [71], following a similar procedure as Roca, Palomar, Oset, and Chiang in Ref. [72], a relation between the production amplitude $P$ and the scattering amplitude $T$, reading

$$P_\ell = j_\ell (pr_0) + i T_\ell h^{(1)}_\ell (pr_0), \quad (22)$$

which, using Eqs. (21) and (1), can also be written as

$$P_\ell = j_\ell (pr_0) \Pi_\ell(E). \quad (23)$$

For the latter expression we find, by the use of Eq. (21), that the production amplitude of Eq. (23) tends to zero when $E \rightarrow E_{NL}$. This effect must be visible in experimental strong production cross sections.

Actually, the primary question here is not so much if a vanishing $q\bar{q}$ propagator is observable, but rather whether the production amplitude always vanishes when $E \rightarrow E_{NL}$. In order to answer this, we must return to the results of Ref. [71], where we found, for the complete production amplitude in the case of multi-channel processes, that Eq. (23) represents the leading term, and that the remainder is expressed in terms of the inelastic components of the scattering amplitude. The latter terms do not vanish in the limit $E \rightarrow E_{NL}$, as we have discussed above. Hence, the production amplitude only vanishes approximately in this limit, in case inelasticity is suppressed.

However, there are more questions to be responded with respect to the observability of vanishing propagators. Namely, do processes exist where only one partial wave contributes and in which processes other than $s$-channel exchange do not play an important role? Fortunately,
the answer to the latter, pertinent, question can be responded affirmatively, because electron-positron annihilation into multi-hadron final states takes basically place via one photon, hence with $J^{PC} = 1^{--}$ quantum numbers. Consequently, when the photon materializes into a pair of current quarks, which couple via the $q \bar{q}$ propagator to the final multi-hadron state, we may assume that the intermediate propagator carries the quantum numbers of the photon. Moreover, alternative processes are suppressed.

We may thus conclude that, if we want to discover whether the propagator really vanishes at $E \rightarrow E_{NL}$, then the ideal touchstone is $e^+e^-$ annihilation into multi-hadron states. But not only do we have at our disposal a wealth of experimental results on such processes, there also exist predictions for the values of $E_{NL}$, with $L = 0$ or $L = 2$, given by the parameter set of Ref. [20]. As an example, for $c \bar{c}$ one finds in the latter paper $E_{0,0} = 3.409$ GeV and $\omega = 0.19$ GeV. Using Eq. (15), we then get for the higher $c \bar{c}$ confinement states the spectrum $E_{1,0} = E_{0,2} = 3.789$ GeV, $E_{2,0} = E_{1,2} = 4.169$ GeV, $E_{3,0} = E_{2,2} = 4.549$ GeV, . . . .

The latter two levels of the $c \bar{c}$ confinement spectrum can indeed be clearly observed in experiment. For example, the non-resonant signal in $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$ (see Fig. 5 of Ref. [73]) is divided into two substructures [74–76], since the full $c \bar{c}$ propagator (21), dressed with meson loops, vanishes at $E_3 = 4.55$ GeV [20]. In the same set of data, one may observe a lower-lying zero at $E_2 = 4.17$ GeV [20], more distinctly visible in the data on $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ (see Fig. 3 of Ref. [77]). The true $c \bar{c}$ resonances can be found on the slopes of the above-mentioned non-resonant structures [78], unfortunately with little statistical significance, if any [79].

In the light-quark sector, where low statistics does not result in sufficient accuracy, one needs some imagination for the identification of vanishing amplitudes in $e^+e^-$ annihilation processes into multi-hadron states. Thus, from data given in Ref. [80], one may infer that certain four-pion amplitudes vanish at, or near, the predicted [20] 1.097 GeV $n\bar{n}$ ground state, and near the predicted 1.477 GeV $n\bar{n}$ excited state. Some further evidence, also for the predicted higher excited states in $n\bar{n}$ and $s\bar{s}$, is found in data from Refs. [81–85].

So we indeed observe minima in production processes, which confirm vanishing $q\bar{q}$ propagators. Moreover, the $q\bar{q}$ confinement spectrum predicted 25 years ago in Ref. [20] seems to agree well with experimental observations for vector mesons. Accordingly, we expect vector-meson $q\bar{q}$ resonances associated with each of the Regge states: one ground state, dominantly in a $q\bar{q} S$-wave, and two resonances for each of the higher excited Regge states, viz. one dominantly in an $S$-wave, and the other mostly in a $D$-wave (see Fig. 3).

8 Conclusions

We have found that the meson spectrum can be described by $s$-channel exchange of Regge propagators in non-exotic meson-meson interactions, and furthermore that the bound-state and resonance spectrum of mesons is richer than, and different from, the underlying Regge spectrum [2, 19]. For the latter we generally observe very regular and equidistant level spacings, instead of quadratic trajectories. We also have established a link between $s$-channel exchange of Regge propagators and unitarized chiral models constructed for the study of resonances in $S$-wave meson-meson scattering. Moreover, a method is given to relate the coupling constants and seed masses for the latter models. Moreover, we have indicated how the Regge spectrum of $q\bar{q}$ propagator modes may be observed in production processes.

Finally, the use of Regge propagators for meson physics is seen [1,20] not to be restricted to light flavors, but can, without any further effort, be extended to heavy quarkonia with the same
set of parameters, i.e., the effective quark masses, the universal frequency $\omega$, the overall vertex intensity $\lambda$, and the average string-breaking distance $r_0$.

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