The Secret Gauging of Flavor Symmetries in Noncommutative QFT

Ken Intriligator, 1 and Jason Kumar 2

Department of Physics, University of California, San Diego
La Jolla, CA 92093-0354 USA

Abstract

We show that flavor ’t Hooft anomalies automatically vanish in noncommutative field theories which are obtained from string theory in the decoupling limit. We claim that this is because the flavor symmetries are secretly local, because of coupling to closed string bulk modes. An example is the SU(4) R-symmetry of N=4 D=4 NCSYM. The gauge fields, along with all closed string bulk modes, are not on-shell external states but do appear as off-shell intermediate states in non-planar processes; these closed string modes are thereby holographically encoded in the NCFT.

1 email address: keni@ucsd.edu
2 e-mail address: j1kumar@ucsd.edu
1 Introduction

Non-commutative field theories (NCFT) exhibit fascinating similarities with string theory. In the realization of non-commutative field theories via D-branes in a background $B$ field, one sees that the decoupling limit [1] retains some of the stringy behavior, which is usually thrown away in the decoupling limit for commutative QFTs. In particular, diagrams with non-planar momentum exhibit UV suppression, which can lead to UV/IR mixing [2, 3]. This effect was interpreted in [2, 3] as non-decoupling of closed string modes (see also [4]), and can also be viewed as the effect of stretched open strings [5] - [9]. A way to think about this [2] is that the closed string propagator involves $(\alpha')^2 g^{\mu\nu}$, which one might expect to vanish in the decoupling limit $\alpha' \to 0$; however, holding the open string metric fixed to $G^{\mu\nu} = \eta^{\mu\nu}$, we actually have in the decoupling limit

$$(\alpha')^2 g^{\mu\nu} \to -(G^2)^{\mu\nu}. \quad (1)$$

We claim, based on eqn. (1), that all closed string modes remain coupled to the NCFT. In the decoupling limit, the closed string modes do not exist as on-shell external states, but they still remain coupled as intermediate, off-shell contributions to non-planar loop diagrams. As a particular case of this, we argue that worldvolume flavor symmetries are secretly gauged, with the gauge bosons coming from closed strings which remain coupled via eqn. (1). As evidence, we discuss anomalies.

The UV suppression of non-planar diagrams in NCFT, which is related to eqn. (1), ensures that no non-planar diagrams are anomalous [10, 11]. Thus mixed anomalies, which could only arise via non-planar diagrams, always automatically vanish in NCFT. The interpretation of [11] is that these anomalies are cancelled because the NCFT automatically contains the necessary closed string axion fields which cancel the anomaly by the Green-Schwarz mechanism, precisely as in string theory. This can be seen explicitly in string theory constructions of chiral NCFTs. As also pointed out in [11], all ABJ type anomalies, involving both flavor and gauge currents, are also automatically cancelled by a Green-Schwarz type mechanism.

We will here argue that all flavor ’t Hooft anomalies also automatically vanish in NCFTs which are obtained via string theory. This is caused by the same nonplanar UV suppression factors which occur via eqn. (1). Our interpretation of this, as well as the vanishing of the ABJ anomalies, is that the flavor symmetries are secretly gauged, and thus necessarily anomaly free.
String theory constructions, of course, will always lead to a consistent theory. The ’t Hooft anomaly cancellation, and secret gauging, is via closed string modes, which remain coupled essentially due to eqn. (1).

The flavor symmetries of the NCFT living in the worldvolume of a D-brane are the “normal bundle” rotations of the directions transverse to the brane. Before taking the decoupling limit, these are gauge symmetries of the bulk theory, which includes gravity. The worldvolume flavor current couples to bulk gauge fields, which can be written in terms of the bulk metric. For D-branes in a general curved bulk metric background, we’ll have terms in the worldvolume action like

$$\int d^{p+1}x \text{str}[g_{ab}(\Phi^a)D_\alpha \Phi^a D^b + \text{fermion terms}].$$

Here \text{str} is the symmetrized trace over the worldvolume $U(K)$ gauge indices, $D_\alpha$ are the $U(K)$ gauge (and not flavor) covariant derivatives, with $\alpha = 0 \ldots p$ the worldvolume space-time index, and $a, b, c$ run over the bulk transverse directions. Expanding this around a classical solution for $\Phi^a$ gives a coupling

$$\int d^{p+1}x \ g_{ab,c} \text{str}(D_\alpha \Phi^a D^b \Phi^c) + \text{fermion terms},$$

with $g_{ab,c}$ the derivative of the classical bulk metric. These terms lead to a coupling between the worldvolume flavor current $J^{[bc]}_{\alpha} = tr(\Phi^b \partial_\alpha \Phi^c)$+fermion terms (of course, it’s the fermion terms which are relevant for anomalies) and a “bulk gauge field:

$$\int d^{p+1}x J^{[bc]}_{\alpha} A^\alpha_{[bc]} \quad \text{with} \quad A^\alpha_{[bc]} = g_{a[b,c]} \partial^a \phi^a,$$

where $\Phi^a$ is a $U(K)$ adjoint and $\phi^a$ is its eigenvalue, which we take to be all the same for $K$ coincident branes. Simply put, the D-brane couples to angular momentum, which is gauged in the full string theory. The “gauge field” is the Christoffel symbol, i.e. derivatives of the graviton, $g_{a[b,c]}$, pulled back to the worldvolume via $\partial^a \phi^a$.

The above coupling between the flavor current and the bulk closed string modes goes away in the commutative $\alpha' \to 0$ decoupling limit. However, in the NCFT noncommutative decoupling limit, the worldvolume theory retains this coupling to the bulk closed string modes in intermediate channels of non-planar loop amplitudes. In particular, for our present case of interest, the NCFT retains the information that the worldvolume flavor symmetries
correspond to bulk gauge symmetries. Also retained is the cancellation of all
gauge anomalies, including that of the gauged flavor symmetry.

Though our discussion can be generalized, we focus on a particular ex-
ample, non-commutative $\mathcal{N} = 4$ $U(N)$ gauge theory. Both the commuta-
tive and the non-commutative theory respect a $SU(4)_R$ flavor symmetry. In
the commutative case, the $Tr SU(4)^3_R$ 't Hooft anomaly is non-zero: all of
the gauginos are in the $\mathbf{4}$, with none in the $\mathbf{4}'$, so the 't Hooft anomaly is
$Tr SU(4)^3_R = |G|$, the dimension of the gauge group. This is an obstruction
to gauging $SU(4)_R$ in the commutative theory; one would first have to add
some additional massless fields to cancel this obstruction. Though 't Hooft
anomalies are generally expected to remain unchanged by the dynamics of
the theory, we claim that making the theory noncommutative does effectively
change the 't Hooft anomaly, actually making it vanish.

In the full 10d string theory, before taking the decoupling limit, the more
precise statement about the cancellation of the 't Hooft anomaly is that
the $SU(4)_R$ current is indeed conserved, but there is anomaly inflow of this
current between the brane and the bulk spacetime. The natural holographic
analog of this for the NCFT is that current inflow into the bulk should
correspond to current flowing across energy scales.

In section 2 we compare commutative and non-commutative field theory
flavor anomaly calculations to string theory anomaly calculations, and will
demonstrate that the flavor anomalies vanish in non-commutative field the-
ories obtained from string theory in the decoupling limit. In section 3, we
argue that this anomaly cancellation is in fact a consequence of the reintro-
duction of closed string modes, which cause the symmetry to be gauged.
This is outlined in the matrix model description of NCFT.

2 The Cancellation of Flavor 't Hooft Anoma-
lies

Though our discussion applies to any non-commutative gauge theory which
arises as a limit of string theory, we'll focus on the particular case of non-
commutative $\mathcal{N} = 4$ $U(N)$ gauge theory, which arises via D3-branes in the
presence of a background NS-NS B-field. The non-commutative theory pre-
serves the $SU(4)$ flavor symmetry. Before taking the decoupling limit, the
$SU(4)$ symmetry is a gauge symmetry, with the gauge field corresponding
to the graviton, as in eqn. (3). (As seen in 10d IIB sugra on $S^5$ [12], the $SU(4)_R$ gauge field actually comes from a linear combination of the metric and the four-form gauge field.)

Consider the $TrSU(4)_R^3$ 't Hooft anomaly first in the commutative theory before taking the decoupling limit. This anomaly can be computed by a string theory world-sheet annulus diagram which, as we’ll discuss, yields the result that the anomaly vanishes. Of course, this is the expected result, given that $SU(4)_R$ is a gauge symmetry before taking the decoupling limit. Now consider the commutative field theory in the decoupling limit, which has a non-zero $TrSU(4)_R^3$ 't Hooft anomaly. This non-zero result is obtained because, in the commutative decoupling limit one omits the region of the moduli space where the annulus becomes a long, thin, cylinder. This omitted part is the non-zero bulk inflow contribution to the anomaly, coming from closed string modes in the bulk, which would cancel the anomaly if included.

This is similar to the string theory cancellation of reducible anomalies, which arise from non-planar diagrams. These diagrams are regularized by string theory, and thus there is no reducible anomaly, even if the corresponding field theory does have one. Again, the field theory limit omits the region of the annulus moduli space where it’s a long cylinder, which cancels the anomaly. The space-time interpretation is that the long cylinder is the closed string channel, which contains the tree-level diagram involving the Green-Schwarz field responsible for cancelling the anomaly.

Now NCFT, even in the decoupling limit, retains stringy behavior in non-planar diagrams. For example, the stringy automatic Green-Schwarz anomaly cancellation of reducible anomalies occurs in NCFT [12]. We now argue that in NCFT the $TrSU(4)_R^3$ 't Hooft anomaly is also automatically cancelled, even in the decoupling limit, by the “bulk inflow” closed string contributions to the annulus diagram. To do this, we consider the full string theory anomaly diagram, and then take the non-commutative field theory limit. We will see that, unlike the case of the commutative field theory limit, in the NCFT limit the diagram is still finite and hence non-anomalous.

The $TrSU(4)_R^3$ anomaly diagram is an annulus with three insertions of the $SU(4)_R$ gauge field (essentially the bulk graviton, as in eqn. (3)). Because these gauge fields are closed string modes, their vertex operators are to be inserted in the bulk of the annulus diagram, at locations which are to be integrated over. Fortunately, our main conclusion will not require that we actually compute the diagram in detail. Instead, we only need to argue that it is regulated in the UV, as this is sufficient to show that the $TrSU(4)_R^3$
anomaly vanishes. To show that the diagram is regulated in the UV, we
don’t even need to worry about specific form of the gauge boson vertex
operator; all that is needed is the form of the worldsheet propagator between
the closed string vertex operator insertions.

The worldsheet propagator is the Green function on the annulus, with
non-zero B-field and appropriate boundary conditions, which was exhibited
e.g. in [9], [13]. The relevant term is

\[ G^{\mu\nu}(\omega_1, \omega_2) = \frac{\Theta G \Theta}{8\pi^2\alpha'} \left[ \ln \left| \frac{\nu_1}{\nu_2} \left( \frac{\omega_1 - \omega_2}{2\pi t} \right) \right|^2 - \frac{[Re\omega_1 - \pi]^2 + [Re\omega_2 - \pi]^2}{\pi t} + \frac{2\pi}{t} \right] \]

where \( t \) is the modulus of the annulus and \( \omega_{1,2} \) are the worldsheet bulk
insertion points. If there is non-planar momentum \( k_{\mu} \) flowing between these
vertex operator insertions, the annulus diagram will contain a factor of

\[ \exp(-k_{\mu}k_{\nu}G^{\mu\nu}); \]

these closed string factors yield the stringy UV regulator for the diagram,
which imply that the diagram is finite, and thus anomaly free, in the full
string theory before taking the \( \alpha' \to 0 \) decoupling limit.

We now consider the NCFT limit, \( \alpha' \to 0 \) with \( \alpha' t \) fixed. The anomaly will
still vanish because, even in the decoupling limit, the above UV suppression
factors continue to provide a UV regulator for the anomaly diagram. This is
because the above propagator, in the decoupling limit, continues to contain
terms of the form

\[ \propto \frac{(\Theta^2)^{\mu\nu}}{\alpha' t} \]

which lead to the UV regulator in the NCFT limit. There is indeed such
a term in the above propagator whenever the vertex operator insertion loca-
tions satisfy \( Re\omega_1 \neq Re\omega_2 \). A special case of this is all non-planar open
string diagrams, where one vertex operator is on one boundary of the annulus,
\( Re\omega_1 = 0 \), and the other is on the other boundary of the annulus, \( Re\omega_2 = \pi \).
But the above UV suppression factor also occurs for closed string vertex op-
erators, inserted in the bulk of the annulus, as long as \( Re\omega_1 \neq Re\omega_2 \).

For our \( SU(4)_R \) anomaly calculation, we have 3 closed string vertex op-
erators inserted at locations \( \omega_i, i = 1 \ldots 3 \), which should be integrated over
the bulk of the annulus. As long as the insertions are at different \( Re\omega_i \),
the diagram will have the UV damping factors discussed above, and thus no
anomaly. The integration over the $\omega_i$ will include regions where the $Re \omega_i$ are not separated, and thus without the UV damping, but such regions are a set of measure zero. Thus, the integrated diagram remains UV finite, and the SU(4) R-symmetry anomaly is automatically canceled. It is easy to see that this generalizes to all global symmetries in string-derived NCFT, since any global symmetry in such a theory must arise from a gauge symmetry with a closed string gauge boson in the underlying string theory. The associated one-loop annulus anomaly diagram with closed string insertions will be finite, exactly as above.

Note that, although the usual $U(N)$ symmetry is non-commutative, the $SU(4)_R$ gauge symmetry will be commutative. This is because $SU(4)$ gauge boson is a closed string, so the argument in [1] for noncommutivity, based on open string gauge fields, does not apply. Of course the $SU(4)_R$ symmetry could not possibly have been noncommutative in any case, since it’s actually $Spin(6)$, with scalars in the 6 and fermions in the spinor 4.

3 Reappearance of the Bulk Graviton Couplings

The cancellation of the flavor anomalies in the previous section was via the closed string UV suppression factors of the form $e^{\alpha' t_k g^{\mu \nu} k_{\mu} k_{\nu}}$, which do not become trivial in the NCFT decoupling limit thanks to the scaling of eqn. (1). These suppression factors can be considered as a stretched string effect [5] - [9] as they can be written as $e^{-\frac{\Delta x^2}{\alpha'^2}}$.

The anomaly cancellation is clearly very similar to that of the full string theory before taking the decoupling limit. The natural interpretation, as in string theory, is that the annulus diagram differs from the naive field theory diagram by including the closed string propagation region of the moduli space, where the cylinder has non-zero height. Thus the anomaly is naturally regarded as being cancelled by closed string bulk inflow, involving closed string modes which are re-introduced by UV/IR mixing. This is analogous to the claim of [11], that the mixed anomalies are canceled by the coupling of the non-commutative gauge bosons to closed string axion-type fields, which cancel the anomaly by the Green-Schwarz mechanism. (A difference is that there the closed string field is a twisted sector axion, living in the brane worldvolume, whereas here the closed string modes generally live in the bulk.)
So the $TrSU(4)_R^3$ anomaly cancellation mechanism is naturally the same as in the full string theory. Moreover, the reason why this actually had to be the case is also naturally the same as in the full string theory: the flavor symmetries are secretly gauge symmetries. This is because the closed string gauge bosons, which couple to the flavor current as in eqn. (3), actually do not decouple in the NCFT limit. They do not exist as external, on-shell states. But, nevertheless, their presence is seen in non-planar NCFT diagrams, which includes all closed string modes in the intermediate channel via the propagators (or, in other words, UV/IR mixing) discussed in the previous section. Because NCFT retains the couplings, as in eqn. (3), to the closed string gauge fields, the $SU(4)_R$ symmetry is secretly gauged.

This interpretation also explains why the observation of [11], that all ABJ type mixed flavor-gauge anomalies automatically vanish (via Green-Schwarz cancellation) in noncommutative field theories, also actually had to be the case. If the flavor symmetries were really global symmetries, such anomalies need not vanish. But because the flavor symmetries are secretly gauged, consistency of the theory requires these anomalies, along with the 't Hooft anomalies, to all be cancelled. String theory, and also the NCFT decoupling limit, automatically ensure that this is the case.

The secret gauging of the $SU(4)_R$ symmetry, along with the supersymmetry algebra, implies that gravity is also included and one is discussing supergravity. This is consistent with our picture for NCFT, since all of the closed string modes, and in particular the 10d supergravity fields, remain coupled as off-shell, intermediate states in non-planar diagrams. The fact that 10d bulk closed string modes remain coupled via non-planar diagrams also fits with the remark in [13], that the leading non-planar one-loop effective action will lead to a 10d, non-relativistic gravitational force between sources of stress-energy.

The coupling of the worldvolume flavor current to the bulk can also be seen in the matrix model description of NCFT. As an example, consider $Dp$ branes with $p$ even via the M-theory matrix model (one can similarly consider $p$ odd in the IIB matrix model). Follow the discussion in [16], we expand the above action around a solution which captures the noncommutivity: $X^I = X^I_0 + Y^I$, with $X^i_0 = x^i$ and $X_0^{a+p} = 0$, $Y^i = \theta^{ij} \hat{A}_j$, $Y^{a+p} = 2\pi\alpha' \Phi^a$, with $i = 1 \ldots p$, $a = 1 \ldots 9 - p$, and $[x^i, x^j] = i\theta^{ij}$ (with $\theta^{ij}$ of rank $p$). For flat $g_{IJ}$, this leads to the action for the NCFT Yang-Mills theory in the worldvolume of the Dp brane [16].

To see our couplings to the bulk gravitons, we should expand the matrix
model action for a general curved metric $g_{IJ}$ [14]:

$$\int dt \text{STR} \left( \frac{1}{2} g_{IJ}(X) D_t X^I D_t X^J - g_{IK}(X) g_{JL}(X) [X^I, X^J] [X^K, X^L] + \ldots \right),$$

where $\text{STR}$ is the symmetrized trace, $\ldots$ are the fermion terms, and $I, J = 1 \ldots 9$ run over the directions transverse to the D0s. Expanding around the solution corresponding to NCFT, we replace $g_{IJ}(X) = \sum_{n=1}^{\infty} \frac{1}{n!} g_{IJ,R_{1\ldots R_n}}(X_0) Y_{R_1} \ldots Y_{R_n}$ for all occurrences of the metric in the above action. Following [16], this then leads to the NCFT action, along with additional couplings, as in [17], like

$$\int dt \sum_{n=0}^{\infty} \frac{1}{n!} (T^{IJ(i_1\ldots i_n)} \partial_{i_1} \ldots \partial_{i_n} g_{IJ}(0) + \ldots),$$

with $T^{IJ(i_1\ldots i_n)}$ moments of the stress tensor. The $n = 1$ term in the above yields our desired coupling eqn. (3) between the worldvolume flavor currents and the associated bulk gauge field. Even though there is no non-commutivity in the time direction, $J^b_{[c]}$ does couple to an associated bulk gauge field, as seen upon expanding the kinetic term in the action (6).

Finally, we make a general comment about the kinetic terms of the closed string modes. In the NCFT limit, say for D3 brane with the non-commutivity only in the $x^1$ and $x^2$ spatial directions, $[x_1, x_2] = i\theta$, the closed string metric eqn. (1) will have non-zero components only in the non-commutative directions. Thus, the closed string modes with no momentum in the non-commutative directions will decouple (these are a set of measure zero). Since the only closed string modes which couple have momenta along the non-commutative directions, which are taken to be only space directions, those modes will be space-like. The exponential factor $e^{\frac{2\pi}{g_s} k_\mu g_{\mu\nu} k_\nu}$ appearing in the non-planar amplitude will thus have an exponent which is always negative, and thus always damp the UV limit of the integration over non-planar momenta. Though there are no finite energy poles associated with the closed string degrees of freedom, the coupling in the noncommutative directions between the closed string gauge boson modes and the flavor currents are sufficient to ensure finiteness of the one-loop diagram and invariance of the theory under the local flavor symmetry transformations.
Acknowledgments

We would like to thank J. Gomis, A. Rajaraman, L. Susskind, M.M. Sheikh-Jabbari and M. Van Raamsdonk for useful discussions and N. Seiberg for several useful correspondences. J. K. would like to thank the Aspen Center for Physics for its hospitality. This work is supported by DOE-FG03-97ER40546.

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