Dissipative Spin Dynamics in Relativistic Matter

Samapan Bhadury,1,∗ Wojciech Florkowski,2, † Amarendra Jaiswal,1, † Avdhesh Kumar,1, § and Radoslaw Ryblewski3, ‡

1School of Physical Sciences, National Institute of Science Education and Research, HBNI, Jatni-752050, India
2Institute of Theoretical Physics, Jagiellonian University ul. St.Łojasiewicza 11, 30-348 Krakow, Poland
3Institute of Nuclear Physics Polish Academy of Sciences, PL-31342 Krakow, Poland

(Dated: August 26, 2020)

Using classical description of spin degrees of freedom, we extend recent formulation of the perfect-fluid hydrodynamics for spin-polarized fluids to the case including dissipation. Our work is based on the analysis of classical kinetic equations for massive particles with spin 1/2, with the collision terms treated in the relaxation time approximation. The kinetic-theory framework determines the structure of viscous and diffusive terms and allows to explicitly calculate a complete set of new kinetic coefficients that characterize dissipative spin dynamics.

Keywords: perfect and viscous hydrodynamics with spin, energy-momentum and spin tensors, kinetic coefficients, relaxation time approximation

I. INTRODUCTION

In non-central ultra-relativistic heavy-ion collisions, the two colliding nuclei carry large amount of orbital angular momentum \( \mathbf{L} \). Soon after the initial impact, a substantial part of \( \mathbf{L} \) is deposited in the interaction zone and can be further transformed to the spin part \( \mathbf{S} \) (with the total angular momentum \( \mathbf{J} = \mathbf{L} + \mathbf{S} \) being conserved). The latter can be reflected in the spin polarization of the particles emitted at freeze-out. To verify this phenomenon, the spin polarization of various particles (\( \Lambda \), the particles emitted at freeze-out. To verify this phenomenon, the spin polarization of various particles (\( \Lambda \), \( K^\pm \), \( \phi \)) produced in relativistic heavy-ion collisions has been recently measured by the STAR [1, 2], ALICE [3] and HADES [4] experiments.

On the theoretical side, first predictions of a non-zero global spin polarization of the \( \Lambda \) hyperons, based on perturbative-QCD calculations and the spin-orbit interaction, were made in Refs. [5, 6] and [7], respectively (see also Ref. [8]). In these works, a substantial polarization effect of the order of 10% was found. Subsequently, using relativistic hydrodynamics with local thermodynamic equilibrium of the spin degrees of freedom [9–18], a smaller polarization of about 1% was predicted, an effect which was eventually confirmed by STAR [1, 2].

Interestingly, the same hydrodynamic models [18, 19] are not able to describe the experimentally measured longitudinal polarization of \( \Lambda \)’s [20, 21]. For example, the oscillation of the longitudinal polarization of the \( \Lambda \) hyperons measured as a function of the azimuthal angle in the transverse plane [20] has an opposite sign compared to the results obtained with relativistic hydrodynamics with thermalized spin degrees of freedom. This issue is at the moment the subject of very intensive investigations [16, 17, 22–39].

The relativistic hydrodynamic models (perfect or viscous) that have been used so far to describe the global spin polarization of the \( \Lambda \) and \( \bar{\Lambda} \) hyperons [13, 15, 18] make use of the fact that spin polarization effects are governed by the thermal vorticity tensor

\[
\omega_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu).
\]

Here the four-vector \( \beta_\mu \) is defined in the standard way as the ratio of the fluid flow vector \( u_\mu \) and the local temperature \( T \), i.e., \( \beta_\mu = u_\mu / T \). One can notice that the use of (1) does not require any modifications of the existing hydrodynamic codes as spin effects are determined solely by the form of \( u^\mu \) and \( T \).

However, on the general thermodynamic grounds [40], it is expected that the spin polarization effects may be governed by the tensor \( \omega_{\mu\nu} \) (called below the spin polarization tensor) that can be independent of the thermal vorticity (1). This suggests that a completely new hydrodynamic approach including spin dynamics can be constructed, with the spin polarization tensor \( \omega_{\mu\nu} \) treated as an independent hydrodynamical variable. In this context, the concept of local spin equilibrium also changes as one no longer requires that \( \omega_{\mu\nu} = \epsilon_{\mu\nu} \) to have zero entropy production.

First steps to formulate the perfect-fluid version of hydrodynamics of spin polarized fluids that incorporates the spin polarization tensor \( \omega_{\mu\nu} \) have already been made in a series of publications [40–42], for a recent summary see Ref. [43]. However, only in a very recent work [44], the dissipation effects in such systems have been explicitly considered, see also Refs. [45–50].

In this work we continue and significantly extend the results obtained in Ref. [44]. In order to identify the structure of dissipative terms, we use classical kinetic theory for particles with spin 1/2. The collision terms are treated in the relaxation time approximation (RTA) according to the prescription defined in Ref. [44] and, for the sake of simplicity, we restrict our considerations to the Boltzmann statistics. The kinetic-theory framework determines the structure of viscous and diffusive...
terms and allows to explicitly calculate a set of new kinetic coefficients that characterize dissipative spin dynamics. These coefficients describe coupling between a non-equilibrium part of the spin tensor and thermodynamic forces such as the expansion tensor, shear flow tensor, the gradient of chemical potential divided by temperature, and, finally, the gradient of the spin polarization tensor.

The structure of the paper is as follows: In Sec. II we recall the formulation of the perfect-fluid hydrodynamics with spin. Our presentation is based on the classical concept of spin and classical distribution functions in an extended phase space. In Sec. III we introduce kinetic equations with the collision terms treated in the relaxation time approximation and derive the form of the dissipative corrections. This section contains also the explicit form of the new, spin-related kinetic coefficients. We conclude and summarize in Sec. IV. The paper is closed with several appendices where details of our straightforward but quite lengthy calculations are given. We use natural units and the metric tensor with the signature (+ − − −).

II. FORMULATION OF PERFECT FLUID HYDRODYNAMICS FOR SPIN POLARIZED FLUIDS

A. Spin-dependent equilibrium distribution function

We start with the classical treatment of massive particles with spin$^{1/2}$ and introduce their internal angular momentum $s^{αβ}$ [51]. It is connected with the particle four-momentum $p_γ$ and spin four-vector $s_δ$ [52] by the following relation

$$s^{αβ} = \frac{1}{m} e^{αβγδ} p_γ s_δ,$$

where $m$ is the mass of the particle. Equation (2) implies that $s^{αβ} = -s^{βα}$ and $p_α s^{αβ} = 0$. Moreover, from Eq. (2) we find

$$s^α = \frac{1}{2m} c^{αβγδ} p_β s_γ s_δ.$$

This implies that spin four-vector $s^α$ is orthogonal to four-momentum $p^α$, i.e., $s \cdot p = 0$. In the particle rest frame (PRF), where the four-momentum of a particle is $p^μ = (m, 0, 0, 0)$, the spin four-vector $s^α$ has only spatial components, i.e., $s^α = (0, s_s)$, with the length of the spin vector defined by $-s^2 = |s_s|^2 = s^2 = 1/2 (1 + 3 s^2)$. Identification of the so-called collisional invariants of the Boltzmann equation allows us to construct the equilibrium distribution functions $f_{s, eq}^±(x, p, s)$ for particles and antiparticles [43, 44],

$$f_{s, eq}^±(x, p, s) = f_{eq}^±(x, p) \exp \left[ \frac{1}{2} \omega_{μν}(x) s^{μν} \right]. \quad (4)$$

Here $f_{eq}^±(x, p, s) = \exp \left[ -p^μ β_μ(x) ± ξ(x) \right]$ is the Jüttner distribution, with $ξ$ and $β_μ$ traditionally defined as ratios of chemical potential $μ$ to temperature $T$ and four-velocity $u_μ$ to temperature $T$, i.e., $ξ = μ/T$ and $β_μ = u_μ/T$. The spin polarization tensor $ω_{μν}$ has been introduced in Sec. I. It plays a crucial role in our formalism and can be interpreted as the (tensor) potential conjugated to the spin angular momentum.

Before we proceed further we note that in our approach $s^{μν}$ is dimensionless (measured in units of $h$) and so is $ω_{μν}$. Consequently, we can make expansions in $ω_{μν}$ and, in fact, most of our results will be valid in the leading order of $ω_{μν}$.

Ordinary phase-space equilibrium distribution functions can be obtained by integrating out the spin degrees of freedom present in $f_{s, eq}^±(x, p, s)$,

$$\int dS f_{s, eq}^±(x, p, s) = f_{eq}^±(x, p), \quad (5)$$

where [43]

$$dS = \frac{m}{π \delta} d^3s \ δ(s \cdot s + s^2) \ δ(p \cdot s). \quad (6)$$

Different properties of spin integrals done with the integration measure (6) are collected in Appendix A.

B. Perfect fluid hydrodynamics for spin polarized fluids

For a system of particles and anti-particles with spin degrees of freedom included only through degeneracy factors, the relevant conserved quantities are the energy-momentum tensor ($T^{μν}$) and charge current ($N^{μ}$). If spin is explicitly included, one has to consider an additional conserved quantity, namely, the angular-momentum tensor ($J^{λ,μν}$) [43, 53]. This is connected with the fact that the total angular momentum conservation law for particles with spin has a non-trivial form.

The total angular-momentum tensor ($J^{λ,μν}$) can be written as a sum of the orbital ($L^{λ,μν}$) and spin ($S^{λ,μν}$) parts. The latter is known as the spin tensor. It is well known that there are various equivalent forms of the energy-momentum and spin tensors that can be used to define system’s dynamics [46, 54, 55]. The forms used in this work agree with the definitions introduced by de Groot, van Leeuwen, and van Weert in [56]. To emphasize this fact we sometimes use the acronym GLW.

The structures of $T^{μν}$, $N^{μ}$ and $S^{λ,μν}$ can be connected to the behaviour of microscopic constituents of the system through the moments of the phase-space distribution functions $f_{eq}(x, p, s)$. Using the equilibrium distributions

---

1 We note that since we always consider particles being on the mass shell ($p^0 = E_p = \sqrt{p^0 + m^2}$) the distribution $f(x, p, s)$ is in fact a function of $p$ only.
\( f_{eq}(x, p, s) \) defined above, the hydrodynamic quantities such as charge current, energy-momentum tensor, and the spin tensor can be obtained in the similar way as in standard hydrodynamics.

### C. Charge current

The equilibrium charge current is defined by the formula

\[
N^\mu_{eq} = \int dP \ dS \ p^\mu \left[ f^+_{s,eq}(x, p, s) - f^-_{s,eq}(x, p, s) \right],
\]

where the invariant momentum integration measure \( dP \) is

\[
dP = \frac{d^3p}{(2\pi)^3 E_p},
\]

while the measure \( dS \) is defined by Eq. (6). Using the equilibrium functions (4) we obtain

\[
N^\mu_{eq} = 2 \sinh(\xi) \int dP \ p^\mu e^{-p \cdot s} \int dS \exp\left(\frac{1}{2} \tilde{\omega}_{\alpha \beta} s^\alpha s^\beta\right).
\]

Since for large values of the spin polarization tensor the system becomes anisotropic in the momentum space and requires special treatment \([57, 58]\), in most of our calculations we consider only the case of small values of \( \omega \). In this case the last exponential function in (9) can be expanded up to linear order and we find

\[
N^\mu_{eq} = 2 \sinh(\xi) \int dP \ p^\mu e^{-p \cdot s} \int dS \left(1 + \frac{1}{2} \tilde{\omega}_{\alpha \beta} s^\alpha s^\beta\right).
\]

After carrying out integration first over spin and then over momentum we get

\[
N^\mu_{eq} = n u^\alpha,
\]

where

\[
n = 4 \sinh(\xi) \ n_0(T)
\]

is the charge density \([41]\). In Eq. (12) the quantity \( n_0(T) \) is the number density of spinless, neutral massive Boltzmann particles which is defined by the thermal average

\[
n_0(T) = \langle u \cdot p \rangle_0,
\]

where

\[
\langle \cdots \rangle_0 = \int dP(\cdots) e^{-\beta p}.
\]

The explicit calculation gives

\[
n_0(T) = \int dP (u \cdot p) e^{-\beta p} = I^{(0)}_{10} = \frac{1}{2\pi^2} T^3 z^2 K_2(z),
\]

with \( z \equiv m/T \). Thermodynamic integrals \( I^{(r)}_{\mu \nu} \) are defined in Appendix B.

### D. Energy-momentum tensor

The energy-momentum tensor is defined as the second moment in momentum space,

\[
T^{\mu \nu}_{eq} = \int dP \ dS \ p^\mu p^\nu \left[ f^+_{s,eq}(x, p, s) + f^-_{s,eq}(x, p, s) \right].
\]

Using Eq. (4) we can rewrite this formula as

\[
T^{\alpha \beta}_{eq} = 2 \cosh(\xi) \int dP \ p^\alpha p^\beta e^{-p \cdot s} \int dS \exp\left(\frac{1}{2} \tilde{\omega}_{\alpha \beta} s^\alpha s^\beta\right).
\]

Considering the case of small \( \omega \) and carrying out integration over spin and momentum space we get

\[
T^{\alpha \beta}_{eq}(x) = \varepsilon u^\alpha u^\beta - P \Delta^{\alpha \beta},
\]

where

\[
\varepsilon = 4 \cosh(\xi) \varepsilon_0(T)
\]

and

\[
P = 4 \cosh(\xi) P_0(T),
\]

respectively \([41]\). The auxiliary quantities \( \varepsilon_0(T) \) and \( P_0(T) \) are defined as follows

\[
\varepsilon_0(T) = \langle (u \cdot p)^2 \rangle_0
\]

and

\[
P_0(T) = -(1/3) \langle p \cdot p - (u \cdot p)^2 \rangle_0.
\]

Similarly to \( n_0(T) \), they describe the energy density and pressure of spinless, neutral massive Boltzmann particles. In Eq. (18), the tensor \( \Delta^{\alpha \beta} = g^{\alpha \beta} - u^\alpha u^\beta \) is an operator projecting on the space orthogonal to the fluid four-velocity \( u^\alpha \). For the reader’s convenience, the properties of this and other projectors are listed in Appendix C.

With the help of thermodynamic integrals \( I^{(r)}_{\mu \nu} \) defined in Appendix B one obtains

\[
\varepsilon_0(T) = \int dP (u \cdot p)^2 e^{-\beta p} = I^{(0)}_{20} = \frac{1}{2\pi^2} T^4 z^2 \left[ 3K_2(z) + zK_1(z) \right]
\]

and

\[
P_0(T) = -\frac{1}{3} \Delta^{\mu \nu} \int dP p^\mu p^\nu e^{-\beta p} = -I^{(0)}_{21} = \frac{1}{2\pi^2} T^4 z^2 K_2(z) = n_0(T)T.
\]
E. Spin tensor

Now we come to the fundamental object in our formalism, namely, the spin tensor. We adopt the following definition [43]

\[ S_{\text{eq}}^\lambda,\mu = \int dP \, dS \, P^\lambda \, s^{\mu\nu} \left[ f_{s,\text{eq}}^+(x, p, s) + f_{s,\text{eq}}^-(x, p, s) \right] \]

\[ = 2 \cosh(\xi) \int dP \, p^\lambda \exp \left( -p \cdot \beta \right) \times \int dS \, s^{\mu\nu} \exp \left( \frac{1}{2} \omega_{\alpha\beta} s^{\alpha\beta} \right) . \]  

(25)

Expanding the exponential function in the last line, in the leading order in \( \omega \) we obtain

\[ \int dS \, s^{\mu\nu} \exp \left( \frac{1}{2} \omega_{\alpha\beta} s^{\alpha\beta} \right) = \int dS \, s^{\mu\nu} \left( 1 + \frac{1}{2} \omega_{\alpha\beta} s^{\alpha\beta} \right) \]

\[ = \frac{2}{3m^2} \, s^2 \left( m^2 s^{\mu\nu} + 2p^\mu p^\nu \omega^\alpha_{\alpha} \right) . \]  

(26)

Using Eq. (26) in Eq. (25) we find

\[ S_{\text{eq}}^\lambda,\mu = \frac{4s^2}{3m^2} \cosh(\xi) \int dP \, p^\lambda \exp \left( -p \cdot \beta \right) \left( m^2 s^{\mu\nu} + 2p^\mu p^\nu \omega^\alpha_{\alpha} \right) . \]  

(27)

It is interesting to observe that the last result agrees with the formula \( S_{\text{GLW}}^\lambda,\mu \) obtained in the semiclassical expansion of the Wigner functions [53]. This fact supports our use of the definition (25).

After carrying out the momentum integration we get

\[ S_{\text{eq}}^\lambda,\mu = S_{\text{GLW}}^\lambda,\mu \left( \eta_0(T) u^\lambda \omega^{\mu\nu} + S_{\text{GLW}}^\lambda,\mu \right) . \]  

(28)

Here \( \eta_0(T) = (4/3)s^2 \cosh(\xi) \) and the auxiliary tensor \( S_{\text{GLW}}^\lambda,\mu \) is given by the expression

\[ S_{\text{GLW}}^{\alpha,\beta,\gamma} = A_0 \left( u^\alpha u^\beta \Delta^{\gamma}_{\gamma} \right) \]

\[ + B_0 \left( u^\beta \Delta^{\alpha}_{\alpha} \omega^\gamma_{\gamma} + u^\alpha \Delta^{\beta}_{\beta} \omega^\gamma_{\gamma} + u^\delta \Delta^{\alpha}_{\alpha} \omega^\gamma_{\gamma} \right) , \]  

where

\[ B_0 = \frac{2}{3} \frac{\varepsilon_0(T) + P_0(T)}{\varepsilon_0(T)} = \frac{2}{3} s_0(T) . \]  

(30)

and

\[ A_0 = \frac{6}{z} s_0(T) + 2n_0(T) = -3B_0 + 2n_0(T) , \]  

(31)

with \( s_0 \) being the entropy density of spinless, neutral, massive Boltzmann particles satisfying thermodynamic relation \( s_0 = (\varepsilon_0 + P_0) / T \).

We note that since our energy-momentum tensor is symmetric, the spin tensor is separately conserved. The conservation of the spin tensor gives six additional equations which are required to determine the space-time evolution of \( \omega \). We note that this situation may change if non-local effects are included, for a very recent discussion of this point see Refs. [45, 46].

F. Entropy Current

To construct the entropy current we adopt the Boltzmann definition

\[ H^\mu = -\int dP \, dS \, p^\mu \left[ f_{s,\text{eq}}^+(\ln f_{s,\text{eq}}^+ - 1) \right. \]

\[ \left. + f_{s,\text{eq}}^-(\ln f_{s,\text{eq}}^- - 1) \right) . \]  

(32)

Using Eqs. (4), (7), (16), and (25), we find

\[ H^\mu = \beta^\alpha \omega^\mu_{\alpha} + \frac{1}{2} \omega_{\alpha\beta} S_{\text{eq}}^{\mu,\alpha\beta} - \xi N^\mu_{\text{eq}} + n^\mu_{\text{eq}} . \]  

(33)

where

\[ n^\mu_{\text{eq}} = \frac{\cosh(\xi)}{\sinh(\xi)} N^\mu_{\text{eq}} . \]  

(34)

Using Eq. (33) as well as the conservation laws for charge, energy-momentum and spin we obtain the following expression,

\[ \partial^\mu H^\mu = \left( \partial^\mu \beta^\alpha \right) T_{\text{eq}}^{\mu\alpha} - \frac{1}{2} \left( \partial^\mu \omega_{\alpha\beta} \right) S_{\text{eq}}^{\mu,\alpha\beta} \]

\[ - \left( \partial^\mu \xi \right) N_{\text{eq}}^\mu + \partial^\mu N_{\text{eq}}^\mu . \]  

(35)

Now using the conservation laws for charge, energy and momentum one can easily show that the entropy current is conserved, namely

\[ \partial^\mu H^\mu = 0 . \]  

(36)

It should be emphasized that the last result is exact in the sense that it does not depend on the expansion in \( \omega \). Moreover, we see that the contributions to the entropy production coming from the spin polarization tensor are quadratic. This means that there is no effect on the entropy production from the polarization in the linear order. This suggests that we can neglect the effects of polarization on the global evolution of matter, provided we restrict our considerations to the linear terms. For both the conserved charge and the energy-momentum tensor the corrections start with the second order, hence, as long as we restrict ourselves to the linear terms in \( \omega \), we can first solve the system of standard hydrodynamic equations (which are not affected by polarization in the linear order) and subsequently determine the spin evolution (linear in \( \omega \)) on top of such a hydrodynamic background.

III. FORMULATION OF DISSIPATIVE HYDRODYNAMICS FOR SPIN POLARIZED FLUIDS

The formalism presented in the previous section is already well established and may be treated as the definition of the perfect-fluid hydrodynamics with spin. In the next section, we include dissipation effects. This will be done with the help of the relaxation time approximation used for the collision terms in the classical kinetic equations, as originally introduced in Ref. [44].
A. Classical RTA kinetic equation

In the absence of mean fields, the distribution function satisfies the equations

\[ p^\mu \partial_\mu f_s^\pm (x, p, s) = C[f_s^\pm (x, p, s)] , \]  
(37)

where \( C[f_s^\pm (x, p, s)] \) is the collision term. In the relaxation time approximation, the collision term has the form [44]

\[ C[f_s^\pm (x, p, s)] = p \cdot u \frac{f_{s,eq}^\pm(x, p, s) - f_s^\pm(x, p, s)}{\tau_{eq}}. \]  
(38)

We consider now a simple Chapman-Enskog expansion of the single particle distribution function about its equilibrium value in powers of space-time gradients

\[ f_s^\pm(x, p, s) = f_{s,eq}^\pm(x, p, s) + \delta f_s^\pm(x, p, s) , \]  
(39)

Using Eqs. (38) and (39) in Eq. (37) we get

\[ p^\mu \partial_\mu f_{s,eq}^\pm(x, p, s) = -p \cdot u \frac{\delta f_s^\pm(x, p, s)}{\tau_{eq}}. \]  
(40)

After substituting equilibrium distribution function (4) in Eq. (40) we obtain (in linear order in \( \omega \))

\[ \delta f_s^\pm = -\frac{\tau_{eq}}{(u \cdot p)} e^{\pm \xi - p \cdot \beta} \left[ \left( \pm p^\mu \partial_\mu \xi - p^\lambda p^\nu \partial_\nu \beta_\lambda \right) \left( 1 + \frac{1}{2} s^{\alpha \beta} \omega_{\alpha \beta} \right) + \frac{1}{2} p^\mu p^\nu s^{\alpha \beta} (\partial_\nu \omega_{\alpha \beta}) \right] . \]  
(41)

The corrections \( \delta f_s^\pm \) result in dissipative effects in the conserved quantities such as charge current, energy-momentum tensor, and spin tensor. We discuss them now starting from the simplest case of the charge current. The details of rather lengthy calculations are given in Appendix D.

B. Conserved hydrodynamic quantities and dissipative corrections

Taking the appropriate moments of the transport equation (37), the following equations for the charge current \( (N^\mu) \), energy-momentum tensor \( (T^{\mu \nu}) \) and spin tensor \( (S^{\lambda \cdot \mu \nu}) \) can be obtained

\[ \partial_\mu N^\mu(x) = -u_\mu \left( \frac{N^\mu(x) - N_{\mu,eq}^\mu(x)}{\tau_{eq}} \right) , \]  
(42)

\[ \partial_\mu T^{\mu \nu}(x) = -u_\mu \left( \frac{T^{\mu \nu}(x) - T^{\mu \nu}_{eq}(x)}{\tau_{eq}} \right) , \]  
(43)

\[ \partial_\lambda S^{\lambda \cdot \mu \nu}(x) = -u_\lambda \left( \frac{S^{\lambda \cdot \mu \nu}(x) - S^{\lambda \cdot \mu \nu}_{eq}(x)}{\tau_{eq}} \right) , \]  
(44)

respectively.

Conservation of the charge current \( (\partial_\mu N^\mu = 0) \), energy-momentum tensor \( (\partial_\mu T^{\mu \nu} = 0) \), and spin tensor \( (\partial_\lambda S^{\lambda \cdot \mu \nu} = 0) \) implies that the quantities on the right-hand sides of Eqs. (42)–(44) should be zero, i.e., we must have

\[ u_\mu \delta N^\mu = 0 , \]  
(45)

\[ u_\mu \delta T^{\mu \nu} = 0 , \]  
(46)

\[ u_\lambda \delta S^{\lambda \cdot \mu \nu} = 0 , \]  
(47)

Note that Eqs. (45) and (46), satisfied by the corrections \( \delta N^\mu \) and \( \delta T^{\mu \nu} \), are known in the literature as the Landau matching conditions. They are used (and needed) to determine the values of the chemical potential, temperature, and three independent components of the flow four-vector appearing in the equilibrium distributions defined by Eq. (4) — altogether Eqs. (45) and (46) are five independent equations for five unknown functions. A novel feature of our approach is that we introduce an additional matching condition given by Eq. (47). These are in fact six equations that allow us to determine six independent components of the spin polarization tensor \( \omega_{\mu \nu} \). Below we refer to the complete set of Eqs. (45)–(47) as to the Landau matching conditions.

The conserved quantities obtained from the moments of the transport equations (37) can be further decomposed in terms of the hydrodynamic degrees of freedom. The charge current is decomposed into two parts

\[ N^\mu = \int dP \, dS \, p^\mu \left[ f^+_s(x, p, s) - f^-_s(x, p, s) \right] = N_{\mu,eq}^\mu + \delta N^\mu = uu^\mu + \nu^\mu . \]  
(51)

In this decomposition, the quantity \( \nu^\mu \) is known as the charge diffusion current. The presence of the dissipa-
tive corrections implies that the form of the energy-momentum tensor is
\[ T^\mu{}{}^\nu = \int dP \, dS \, p^\mu p^\nu [f^+(x, p, s) + f^-(x, p, s)] \]
\[ = T^\mu{}{}\nu + \delta T^\mu{}{}\nu \]
\[ = \varepsilon u^\mu u^\nu - P \Delta ^\mu{}{}\nu + \pi ^\mu{}{}\nu - \Pi \Delta ^\mu{}{}\nu. \]  
(52)

In this decomposition, \( \varepsilon, P, \pi^\mu{}{}\nu, \) and \( \Pi \) are density, equilibrium pressure, shear stress tensor, and bulk pressure, respectively. We use here the Landau frame, where \( T^\mu{}{}^\nu u_\nu = \varepsilon u^\mu. \) Finally, we define the correction to the spin tensor by the decomposition
\[ S^{\lambda,\mu}{}^\nu = \int dP \, dS \, p^\lambda s^\mu{}{}^\nu \left[ f^+(x, p, s) + f^-(x, p, s) \right] \]
\[ = S_{\text{eq}}^{\lambda,\mu}{}^\nu + \delta S^{\lambda,\mu}{}^\nu. \]  
(53)

The non-equilibrium quantities \( n, \varepsilon, P \) can be obtained by the Landau matching conditions, namely
\[ n = n_{\text{eq}} = u_\mu N_{\text{eq}}^\mu \]  
(54)
\[ = u_\mu \int dP \, dS \, p^\mu \left[ f_{\text{eq}}^+(x, p, s) - f_{\text{eq}}^-(x, p, s) \right], \]
\[ \varepsilon = \varepsilon_{\text{eq}} = u_\mu u_\nu T_{\text{eq}}^\mu{}{}^\nu \]  
(55)
\[ = u_\mu u_\nu \int dP \, dS \, p^\mu p^\nu \left[ f_{\text{eq}}^+(x, p, s) + f_{\text{eq}}^-(x, p, s) \right] \]

and
\[ P = P_{\text{eq}} = -\frac{1}{3} \Delta _\mu{}{}^\nu T_{\text{eq}}^\mu{}{}^\nu \]  
(56)
\[ = -\frac{\Delta _\mu{}{}^\nu}{3} \int dP \, dS \, p^\mu p^\nu \left[ f_{\text{eq}}^+(x, p, s) + f_{\text{eq}}^-(x, p, s) \right]. \]

After carrying out integration over spin and momentum, Eqs. (54), (55), and (56) yield the same results as Eqs. (12), (19), and (20). Here we also note that the choice of Landau frame and matching conditions enforces the following constraints on the dissipative currents
\[ u_\mu \nu^\mu = 0, \]
\[ u_\mu \pi ^\mu{}{}^\nu = 0. \]  
(57)

C. Convective derivatives of hydrodynamic variables

An intermediate step in the calculation of standard kinetic coefficients is the derivation of expressions for the convective derivatives of the hydrodynamic variables \( \xi, \beta, \) and \( u^\mu. \) The convective derivatives are space-time derivatives taken along the streamlines of the fluid. We denote them by a dot or the letter \( D, \) for example,
\[ \dot{\xi} = D\xi = u^\mu \partial_\mu \xi. \]  
(58)

With spin degrees of freedom included, one has to calculate the convective derivative of the spin polarization tensor \( \omega_{\mu\nu} \) as well. In this section we describe the necessary steps needed to determine all those derivatives. The details of the calculations, which are quite lengthy due to complicated tensor structures, are given in the Appendices E-F.

Using the conservation laws for energy and momentum \( (\partial_\mu T^\mu{}{}^\nu = 0) \) as well as charge \( (\partial_\mu N^\mu{}{}^\nu = 0), \) we get the following equations that dictate the evolution of \( T, u^\nu, \) and \( \omega, \)
\[ \dot{\varepsilon} + (\varepsilon + P + \Pi) \frac{\partial}{\partial \theta} \theta - \pi ^\mu{}{}^\nu \sigma ^\mu{}{}^\nu = 0, \]  
(59)
\[ (\varepsilon + P) \ddot{u}^\alpha - \nabla ^\alpha P = 0, \]  
(60)
\[ \dot{n} + n \theta + \partial_\mu n^\mu = 0. \]  
(61)

Here we use the following notation: \( \theta = \partial_\mu u^\mu \) is the expansion scalar, \( \nabla ^\mu = \Delta ^\mu{}{}^\nu \partial_\nu \) denotes the transverse gradient, and \( \sigma ^\mu{}{}^\nu = \frac{1}{2} (\nabla ^\mu u^\nu + \nabla ^\nu u^\mu) - \frac{1}{3} \Delta ^\mu{}{}^\nu (\nabla ^\lambda u^\lambda) \) is the shear flow tensor. In order to determine the space-time evolution of the spin polarization tensor, the above system of equations should be supplemented by the conservation of the spin tensor,
\[ \partial_\lambda S^{\lambda,\mu}{}^\nu = 0. \]  
(62)

Keeping only the terms up to the first order in velocity gradients, the conservation equations (59), (60), (61), and (62) are reduced to
\[ \dot{\varepsilon} + (\varepsilon + P) \frac{\partial}{\partial \theta} \theta = 0, \]  
(63)
\[ (\varepsilon + P) \dot{u}^\alpha - \nabla ^\alpha P = 0, \]  
(64)
\[ \dot{n} + n \theta = 0, \]  
(65)
\[ \partial_\lambda S_{\text{eq}}^{\lambda,\mu}{}^\nu = 0, \]  
(66)

respectively. Furthermore, from Eqs. (12) and (19) we obtain
\[ \dot{n} = 4 \cosh(\xi) \delta I_{10}^{(0)} + 4 \sinh(\xi) \delta I_{10}^{(0)}, \]  
(67)
\[ \dot{\varepsilon} = 4 \sinh(\xi) \delta I_{20}^{(0)} + 4 \cosh(\xi) \delta I_{20}^{(0)}. \]  
(68)

Using Eq. (B16) that connects derivatives of the Bessel functions, the above equations can be written as
\[ \dot{n} = 4 \cosh(\xi) \delta I_{10}^{(0)} - 4 \sinh(\xi) \delta I_{10}^{(0)}, \]  
(69)
\[ \dot{\varepsilon} = 4 \sinh(\xi) \delta I_{20}^{(0)} - 4 \cosh(\xi) \delta I_{20}^{(0)}. \]  
(70)

Substituting \( n, \varepsilon, P, \dot{n} \) and \( \dot{\varepsilon} \) from Eqs. (12), (19), (20), (69) and (70) in Eqs. (63) and (65) we get
\[ \sinh(\xi) \delta I_{20}^{(0)} - \cosh(\xi) \delta I_{10}^{(0)} = - \cosh(\xi) \left( I_{20}^{(0)} - I_{21}^{(0)} \right) \theta, \]  
(71)
\[ \cosh(\xi) \delta I_{10}^{(0)} - \sinh(\xi) \delta I_{20}^{(0)} = - \sinh(\xi) I_{10}^{(0)} \theta. \]  
(72)

2 Equations (59)–(61) do not include the spin polarization tensor, if we consider only linear terms in \( \omega. \)
Using the relations: \( I^{(0)}_{20} = \varepsilon_0, \ I^{(0)}_{21} = -P_0 = -n_0 T, \)
\( I^{(0)}_{30} = n_0, \) and \( I^{(0)}_{31} = \frac{1}{\beta} (P_0 + \varepsilon_0) + z^2 P_0, \) and solving Eq. (71) and (72) for \( \xi \) and \( \beta \) we can get
\[
\dot{\xi} = \xi_0 \theta, \quad (73)
\]
\[
\dot{\beta} = \beta_0 \theta, \quad (74)
\]
where
\[
\xi_0 = \sinh(\xi) \left[ \frac{\varepsilon_0^2 - n_0 T \left( (3 + z^2) P_0 + 2\varepsilon_0 \right)}{n_0 T \cosh^2(\xi) \left( (3 + z^2) P_0 + 3\varepsilon_0 \right) - \varepsilon_0^2 \sinh^2(\xi)} \right],
\]
\[
\beta_0 = \frac{n_0 (\cosh^2(\xi) P_0 + \varepsilon_0)}{n_0 T \cosh^2(\xi) \left( (3 + z^2) P_0 + 3\varepsilon_0 \right) - \varepsilon_0^2 \sinh^2(\xi)}. \quad (75)
\]
Substituting into Eq. (64) the energy density \( \varepsilon \) and pressure \( P \) defined by Eqs. (55) and (56) we get
\[
\cosh(\xi) \left[ I^{(0)}_{20} - I^{(0)}_{21} \right] \dot{u}^\alpha = -\sinh(\xi) \left( \nabla^\alpha \xi \right) I^{(0)}_{21} - \cosh(\xi) \left( \nabla^\alpha I^{(0)}_{21} \right). \quad (77)
\]
Now we can write
\[
\nabla^\alpha I^{(0)}_{21} = \nabla^\alpha \left\{ \frac{1}{3} \Delta_{\mu \nu} \int \! dP \, p^\mu p^\nu e^{-p^\beta \lambda} \right\}
\]
\[
= \frac{1}{3} \Delta_{\mu \nu} \left[ -\nabla^\alpha \xi \right] \int \! dP \, p^\mu p^\nu e^{-p^\beta \lambda}
\]
\[
= -\frac{1}{3} \Delta_{\mu \nu} \left( \frac{1}{T} \nabla^\alpha u_\lambda - \frac{\mu_\lambda}{T^2} \nabla^\alpha T \right) \left[ I^{(0)}_{30} u^\lambda u^\mu u^\nu + I^{(0)}_{31} \Delta^{\lambda \mu} u^\nu + \Delta^{\mu \lambda} u^\nu \right]
\]
\[
= \left( \frac{\mu_\lambda}{T^2} \nabla^\alpha T \right) u^\lambda I^{(0)}_{31}
\]
\[
= -\left( \nabla^\alpha \beta \right) I^{(0)}_{31}, \quad (78)
\]
and using Eq. (78) in Eq. (77) we obtain
\[
\cosh(\xi) \left[ I^{(0)}_{20} - I^{(0)}_{21} \right] \dot{u}^\alpha = -\sinh(\xi) \left( \nabla^\alpha \xi \right) I^{(0)}_{21} + \cosh(\xi) \left( \nabla^\alpha \beta \right) I^{(0)}_{31}. \quad (79)
\]
Now from the recurrence relation (B15) we obtain
\[
I^{(0)}_{31} = -\frac{1}{\beta} \left( I^{(0)}_{20} - I^{(0)}_{21} \right) = -\frac{1}{\beta} (\varepsilon_0 + P_0),
\]
\[
I^{(0)}_{21} = -P_0 = -\frac{n_0}{\beta}. \quad (80)
\]
Using the above expressions for \( I^{(0)}_{31} \) and \( I^{(0)}_{21} \) in Eq. (79), the following equation for \( \dot{u}^\mu \) can be derived
\[
\beta \dot{u}^\alpha = \frac{n_0 \tanh(\xi)}{\varepsilon_0 + P_0} \left( \nabla^\alpha \xi \right) - \left( \nabla^\alpha \beta \right). \quad (81)
\]
Now we turn to the equilibrium spin tensor. With the help of Eq. (28) it can be written as
\[
S_{\text{eq}}^{\lambda \mu \nu} = \frac{4\varepsilon^2}{3} \cosh(\xi) I^{(0)}_{10} \mu^\lambda \nu^\alpha \mu^\nu \quad (82)
\]
\[
+ \frac{4\varepsilon^2}{3m^2} \cosh(\xi) \left[ 2I^{(0)}_{30} \mu^\lambda \nu^\mu u^\alpha u^\nu \right] + 2I^{(0)}_{31} \left( \Delta^{\lambda \alpha} u^\mu \nu^\alpha \right) + u^\lambda \Delta^{\alpha \nu} u^\nu \alpha + u^\nu \Delta^{\mu \nu} \alpha \alpha.
\]
The above equation can further be simplified as
\[
S_{\text{eq}}^{\lambda \mu \nu} = \frac{4\varepsilon^2}{3} \cosh(\xi) I^{(0)}_{10} \mu^\lambda \nu^\alpha \mu^\nu \quad (83)
\]
\[
+ \frac{8\varepsilon^2}{3m^2} \cosh(\xi) \left[ \left( I^{(0)}_{30} - 3I^{(0)}_{31} \right) u^\lambda u^\mu u^\alpha u^\nu \right] + I^{(0)}_{31} \left( u^\mu \nu^\lambda \right) - \mu^\nu u^\lambda + u^\nu \mu^\lambda \alpha \alpha.
\]
Substituting Eq. (83) into Eq. (66), and using Eqs. (73), (74), and (81), the following dynamical equation for the spin polarization tensor \( \omega^{\mu \nu} \) can be obtained
\[
\omega^{\mu \nu} = D^{\mu \alpha \beta \gamma} + (\nabla^\alpha \xi) \left( D^{\mu \nu \alpha} + D^{\nu \mu \alpha} \right) + \frac{2\varepsilon^2}{3m^2} \cosh(\xi) \left[ \left( I^{(0)}_{30} - 3I^{(0)}_{31} \right) u^\lambda u^\mu u^\alpha u^\nu \right] + I^{(0)}_{31} \left( u^\mu \nu^\lambda \right) - \mu^\nu u^\lambda + u^\nu \mu^\lambda \alpha \alpha.
\]
For details see Appendix E, where the explicit expressions for various \( D \)-coefficients are given.

Note that while deriving the dynamical equation (84), we initially encounter the term \( u_\nu \omega^{\mu \nu} \) in the expression for \( \omega^{\mu \nu} \). To eliminate this term we derive another dynamical equation for \( u_\nu \omega^{\mu \nu} \) by taking projection of Eq. (66) along \( u_\nu \). The dynamical equation for \( u_\nu \omega^{\mu \nu} \) is given by the expression
\[
u_\nu \omega^{\mu \nu} = C_{\mu \alpha \beta \gamma} + \frac{C_{\mu \alpha}}{\alpha} (\nabla^\alpha \xi) + C_{\nu \alpha} \sigma^{\alpha \mu} + C_{\nu \mu} \nabla^{\lambda \sigma \nu} \nabla^{\lambda \omega} \sigma^{\alpha \mu}. \quad (85)
\]
The explicit expression for various \( C \)-coefficients appearing above are also given in Appendix E. See also Appendix F, where the Landau matching conditions are presented in more detail.

\section{Transport coefficients}

The dissipative forces arise due to non-zero gradients in the system. In the present case, we will confine ourselves only to first order in gradients and hence the dissipative parts of \( T^{\mu \nu}, N^\mu, \) and \( S^{\lambda \mu \nu}, \) i.e., \( \delta T^{\mu \nu}, \delta N^\mu, \) and \( \delta S^{\lambda \mu \nu}, \) respectively, must be first order in gradients too. The shear stress \( \langle \pi^{\mu \nu} \rangle, \) bulk viscous pressure \( \langle \Pi \rangle \) and particle diffusion current \( (n^\mu) \) can be found from \( \delta T^{\mu \nu} \) and \( \delta N^\mu \) as:
\[
\pi^{\mu \nu} = \delta_{\nu \alpha} \delta T^{\alpha \beta}, \quad \Pi = -\frac{1}{3} \Delta^{\alpha \beta} \delta T^{\alpha \beta}, \quad n^\mu = \Delta^{\alpha \nu} \delta N^\mu. \quad (86)
\]
Hence, using Eqs. (49) and (48), the above dissipative quantities can be written as:

\[
\pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta} \int dP \ dS \ p^\alpha p^\beta (\delta f^+_s + \delta f^-_s),
\]

(87)

\[
\Pi = - \frac{1}{3} \Delta \alpha \beta \int dP \ dS \ p^\alpha p^\beta (\delta f^+_s + \delta f^-_s),
\]

(88)

\[
\nu^\mu = \Delta^{\mu}_{\alpha} \int dP \ dS \ p^\alpha (\delta f^+_s - \delta f^-_s).
\]

(89)

Evaluating the expressions defined by Eqs. (87), (88), and (89), the dissipative quantities are found to be (see Appendix G)

\[
\beta_\pi = 4 \ i_{21}^{(1)} \cosh(\xi),
\]

(91)

\[
\beta_{\Pi} = 4 \left\{ \left[ \frac{n_0 \cosh(\xi)}{\beta} \left[ \sinh^2(\xi) \left( \varepsilon_0 (P_0 + \varepsilon_0) - n_0 T (P_0 (z^2 + 3) + 3\varepsilon_0) \right) \right] \right] - \frac{n_0 \cosh(\xi)}{\beta} \left[ \frac{(P_0 + \varepsilon_0) (P_0 \cosh^2(\xi) + \varepsilon_0)}{n_0 T \cosh^2(\xi) (P_0 (z^2 + 3) + 3\varepsilon_0) - \varepsilon_0^2 \sinh^2(\xi) + \frac{5\beta}{3} i_{42}^{(1)}} \right) \right\},
\]

(92)

\[
\beta_n = 4 \left[ \left[ \frac{n_0 \tanh(\xi)}{\varepsilon_0 + P_0} \right] i_{21}^{(0)} \sinh(\xi) - i_{21}^{(1)} \cosh(\xi) \right].
\]

(93)

Similarly, using Eq. (41) in (50) and then carrying out integration over spin and momentum variables we get,

\[
\delta S^{\lambda,\mu\nu} = \tau_{eq} \left[ B_{\Pi}^{\lambda,\mu\nu} \theta + B_{\sigma}^{\lambda,\mu\nu} \nabla_\pi \xi + B_{\xi}^{(\pi\delta)\lambda,\mu\nu} \delta_{\sigma} + B_{\Sigma}^{\eta\gamma\lambda,\mu\nu} \nabla_\eta \omega_{\beta\gamma} \right].
\]

(94)

Different coefficients appearing on the right-hand side of Eq. (94) are the kinetic coefficients for spin. They have a tensor structures expressed in terms of \( u^\mu \), \( g^{\mu\nu} \), and \( \omega^{\mu\nu} \). Explicit forms of these coefficients are as follows:

\[
B_{\Pi}^{\lambda,\mu\nu} = B_{\Pi}^{(1)} \mu [\mu, \omega^\nu] + B_{\Pi}^{(2)} \mu \omega^\alpha u^[\mu, \omega^\nu]_\alpha + B_{\Pi}^{(3)} \Delta [\mu, \omega^\nu]_\alpha, \tag{95}
\]

\[
B_{\sigma}^{\lambda,\mu\nu} = B_{\sigma}^{(1)}\Delta \[\mu, \omega^\nu\] + B_{\sigma}^{(2)} \Delta [\mu, \omega^\nu]_\alpha + B_{\Pi}^{(3)} \Delta \eta [\mu, \omega^\nu]_\alpha + B_{\Pi}^{(4)} \omega^{\mu\nu} u^\rho \Delta^{\rho\delta}, \tag{96}
\]

\[
B_{\xi}^{\lambda,\mu\nu} = B_{\xi}^{(1)} \Delta \lambda \mu \omega^{\nu} + B_{\xi}^{(2)} \Delta \lambda \omega^{\nu} \mu + B_{\Pi}^{(3)} \Delta \lambda \omega^{\nu} \alpha + B_{\Pi}^{(4)} \lambda \mu \omega^{\nu} \mu + B_{\xi}^{(5)} \Delta \lambda \mu \omega^{\nu} \alpha, \tag{97}
\]

where the scalar coefficients \( B^{(i)}_X \) are explicitly defined in Appendix E.

Equation (94) is our main result. It shows that the dissipative spin effects are connected with the presence of expansion scalar, gradient of the ratio of chemical potential and temperature, the shear-flow tensor, and the gradient of the spin polarization tensor. All these quantities may be interpreted as “thermodynamic forces” that trigger dissipative currents. The first three among them are well known — they lead to appearance of bulk pressure, diffusive current, and shear stress tensor. Interestingly, in the considered case, they also induce the dissipative part of the spin tensor. The fourth term in Eq. (94) describes the induction of the dissipative spin tensor by the gradient of the spin polarization tensor, hence, may be treated as a direct non-equilibrium interaction between spin degrees of freedom.

Finally, we note that all the kinetic coefficients obtained from Eq. (38) are proportional to the same relaxation time \( \tau_{eq} \). This implies that the equilibration times for momenta and spin degrees of freedom are the same. In phenomenological applications it is conceivable to vary the values of the relaxation times that appear in different kinetic coefficients, arguing that they describe independent physical phenomena. However, such modifications require further studies.
IV. SUMMARY AND CONCLUSIONS

In this paper we have significantly extended the results obtained in Ref. [44]. We used classical kinetic theory for particles with spin $1/2$ with Boltzmann statistics to obtain the structure of dissipative terms and the associated transport coefficients. We considered the relaxation time approximation for collision term in order to account for the interactions. This kinetic-theory framework was used to determine the structure of spin-dependent viscous and diffusive terms and explicitly evaluate a set of new kinetic coefficients that characterize dissipative spin dynamics.

Our main result is given by Eq. (94), together with the explicit expressions for the kinetic coefficients $B$ given in the appendices. Equation (94) shows that a non-equilibrium part of the spin tensor is produced by the thermodynamic forces such as expansion scalar, gradient of the ratio of chemical potential and temperature, the shear-flow tensor, and the gradient of the spin polarization tensor. Thus, the spin dissipative phenomena are connected with those leading to formation of bulk pressure, diffusion current, and the shear stress tensor. Probably, the most interesting term in Eq. (94) is the last one, which describes induction of a non-equilibrium spin tensor by a gradient of the spin polarization tensor. In the future investigations, it would be interesting to analyze the role played by various coefficients appearing in Eq. (94) and to find out which kind of corrections they imply for the spin tensor. The complicated tensor structure of the spin kinetic coefficients may lead to various interesting phenomena.

ACKNOWLEDGMENTS

W.F. and R.R. acknowledge the hospitality of National Institute of Science Education and Research where most of this work was done. S.B., A.J. and A.K. would like to acknowledge the kind hospitality of Jagiellonian University and Institute of Nuclear Physics, Krakow, where part of this work was completed. A.J. was supported in part by the DST-INSPIRE faculty award under Grant No. DST/INSPIRE/04/2017/000038. W.F. and R.R. were supported in part by the Polish National Science Center Grants No. 2016/23/B/ST2/00717 and No. 2018/30/E/ST2/00432.

Appendix A: Spin-space integrals

In this appendix several integrals over the spin space are explicitly done. The results obtained here are used throughout the paper in the calculations of the charge current, energy-momentum tensor, and the spin tensor.

1. Normalization of spin integration measure

We start with the calculation of the normalization of the spin integration measure [43]. Since it is a Lorentz invariant quantity depending on the (external) momentum $p$, the calculations can be done in the particle rest frame (PRF) where $p^\mu = (m, 0, 0, 0)$ and $s^\mu = (0, s_\star)$,

$$
\int dS = \frac{m}{\pi^5} \int d^4 s \delta(s \cdot s + s^2) \delta(p \cdot s) = \frac{m}{\pi^5} \int ds_\star \int d|s_\star||s_\star|^2 \int d\Omega \delta(|s_\star|^2 - s^2)\delta(ms_\star).
$$

(A1)

With the normalization \( \int d\Omega = \int \sin \theta d\theta \int d\phi = 4\pi \) we obtain

$$
\int dS = \frac{4\pi}{\pi^5} \int d|s_\star||s_\star|^2 \delta(|s_\star|^2 - s^2) = 2.
$$

(A2)

The factor of 2 reflects here the two possibilities of the spin-$1/2$ projection.

2. Spin average of $s^{\mu\nu}$

While expanding the spin-dependent equilibrium distribution function in powers of $\omega$, we encounter the integrals of the form

$$
\int dS s^{\mu\nu} = \frac{1}{m} \int dS \epsilon^{\mu\nu\alpha\beta} p_\alpha s_\beta = \frac{1}{m} \epsilon^{\mu\nu\alpha\beta} p_\alpha \int dS s_\beta.
$$

(A3)

Since the last integral can be a function of momentum $p_\beta$ only, we can write

$$
\int dS s_\beta = c p_\beta.
$$

(A4)
After contraction with $p$, this equation gives
\[ \int dS (p \cdot s) = cm^2, \] (A5)
which implies that the constant $c$ equals zero, as $p \cdot s = 0$. Hence, throughout the paper we can use the property
\[ \int dS s^{\mu\nu} = 0. \] (A6)

3. Spin average of $s^{\mu\nu}s^{\alpha\beta}$

In the second order of expansions in $\omega$ we deal with the integrals of the form
\[ \int dS s^{\mu\nu}s^{\alpha\beta} = \frac{1}{m^2} \int dS \epsilon^{\mu\nu\rho\sigma} p_{\rho} s_{\sigma} \epsilon^{\alpha\beta\gamma\delta} p_{\gamma} s_{\delta} = \frac{1}{m^2} \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} p_{\rho} p_{\gamma} \int dS s_{\sigma} s_{\delta}. \] (A7)
Since the last integral can be a function of momenta and the metric tensor, we write
\[ \int dS s_{\sigma} s_{\delta} = a g_{\sigma\delta} + b p_{\sigma} p_{\delta}, \] (A8)
where $a$ and $b$ are scalar coefficients. Multiplying Eq. (A8) by $p_{\sigma} p_{\delta}$ in the first case and contracting the indices in Eq. (A8) in the second case, we obtain two equations
\[ \int dS (p \cdot s)^2 = a m^2 + b m^4 \] (A9)
and
\[ \int dS s^2 = 4a + b m^2. \] (A10)
The left-hand sides of Eqs. (A9) and (A10) yield
\[ \int dS (p \cdot s)^2 = 0, \]
\[ \int dS s^2 = \frac{m}{\pi s} \int d^4s (s \cdot s) \delta(s \cdot s + s^2) \delta(p \cdot s) = -\frac{m}{\pi s} \int ds_0 \int d|s_0|^4 \int d\Omega \delta(|s_0|^2 - s^2) \delta(m s_0) = -\frac{m}{\pi s} \frac{4 \pi s^3}{m} = -2s^2. \] (A11)
Thus, from Eqs. (A9) and (A10) we get
\[ a m^2 + b m^4 = 0, \] (A12)
\[ 4a + b m^2 = -2s^2. \] (A13)
Solving these two equations we get $a = -2s^2/3$ and $b = 2s^2/(3m^2)$. Hence we have
\[ \int dS s_{\sigma} s_{\delta} = -\frac{2s^2}{3} \left( g_{\sigma\delta} - \frac{p_{\sigma} p_{\delta}}{m^2} \right) \] (A14)
and
\[ \int dS s^{\mu\nu}s^{\alpha\beta} = \frac{2s^2}{3m^2} \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} p_{\rho} p_{\gamma} \left( g_{\sigma\delta} - \frac{p_{\sigma} p_{\delta}}{m^2} \right). \] (A15)
Appendix B: Thermodynamic integrals

Thermodynamic integrals considered in this work are given by the following expression

\[ I_{nq}^{(r)} = \frac{1}{(2q + 1)!!} \int dP \left( u \cdot p \right)^{n-2q-r} \left( \Delta_0 \beta p^\alpha \rho^\beta \right)^q e^{-\beta p}. \] (B1)

From the above formula, as the special cases, we obtain:

\[ I_{10}^{(0)} = \frac{T^3 z^2}{2\pi^2} K_2(z), \] (B2)
\[ I_{20}^{(0)} = \frac{T^4 z^2}{2\pi^2} \left[ 3K_2(z) + zK_1(z) \right], \] (B3)
\[ I_{21}^{(0)} = -\frac{T^4 z^2}{2\pi^2} K_2(z), \] (B4)
\[ I_{30}^{(0)} = \frac{T^5 z^5}{32\pi^2} \left[ K_5(z) + K_3(z) - 2K_1(z) \right], \] (B5)
\[ I_{31}^{(0)} = -\frac{T^5 z^5}{96\pi^2} \left[ K_5(z) - 3K_3(z) + 2K_1(z) \right], \] (B6)
\[ I_{40}^{(0)} = \frac{T^6 z^6}{64\pi^2} \left[ 4K_5(z) + K_4(z) - K_2(z) - 2K_0(z) \right], \] (B7)
\[ I_{41}^{(0)} = -\frac{T^6 z^6}{192\pi^2} \left[ K_6(z) - 2K_4(z) - K_2(z) + 2K_0(z) \right], \] (B8)
\[ I_{42}^{(0)} = \frac{T^6 z^6}{960\pi^2} \left[ K_6(z) - 6K_4(z) + 15K_2(z) - 10K_0(z) \right]. \] (B9)

Here \( K_\alpha(z) \) denotes the modified Bessel functions of the second kind with the argument \( z = m/T \). They are defined by the integral

\[ K_\alpha(z) = \int_0^\infty dx \cosh(nx) e^{-z \cosh x}. \] (B10)

The other thermodynamic integrals which are needed in our calculation are given by the expressions

\[ I_{21}^{(1)} = -\frac{T^3 z^3}{6\pi^2} \left[ \frac{1}{4} K_3(z) - \frac{5}{4} K_1(z) + K_{1,1}(z) \right], \] (B11)
\[ I_{42}^{(1)} = \frac{T^5 z^5}{480\pi^2} \left[ 22K_1(z) - 7K_3(z) + K_5(z) - 16K_{1,1}(z) \right], \] (B12)

where

\[ K_{1,1}(z) = \int_0^\infty dx \sech x e^{-z \cosh x} = \frac{\pi}{2} \left[ 1 - z K_0(z) L_{-1}(z) - z K_1(z) L_0(z) \right] \] (B13)

is the first-order Bickley-Naylor function with \( L_i \) being the modified Struve function.

Note that here we have not listed the functions \( I_{20}^{(1)}, I_{30}^{(1)}, I_{31}^{(1)}, I_{40}^{(1)}, I_{50}^{(1)}, I_{51}^{(1)} \) and \( I_{52}^{(1)} \) as they all can be written in terms of the integrals listed above, using the following recurrence relations

\[ I_{n,q}^{(r)} = I_{n-1,q}^{(r-1)}, \quad n \geq 2q, \] (B14)
\[ I_{n,q}^{(0)} = \frac{1}{\beta} \left[ (n - 2q)I_{n-1,q}^{(0)} - I_{n-1,q-1}^{(0)} \right], \] (B15)
\[ I_{n,q}^{(0)} = -\beta I_{n+1,q}^{(0)}. \] (B16)
Appendix C: Properties of the projection operators

Herein we list useful relations involving the projection operators and the differential operator $\nabla^\mu \equiv \Delta^{\mu\nu} \partial_\nu$:

$$
\begin{align*}
\Delta_{\mu\nu} \Delta^{\mu\nu} &= 3, \quad u_\mu \Delta^{\mu\nu} = u_\nu \Delta^{\nu\mu} = 0, \quad \Delta^{\mu\nu} \Delta^\lambda_{\nu} = \Delta^\mu_{\lambda\nu}, \quad u_\mu \nabla^\mu = 0, \\
\Delta^\alpha_{\mu\beta} &= \frac{1}{2} \left( \Delta^\alpha_{\mu\nu} \Delta^\beta_{\nu} + \Delta^\alpha_{\nu\beta} \Delta^\beta_{\mu} - \frac{2}{3} \Delta^\alpha_{\mu\nu} \Delta^\mu_{\nu} \right), \\
u^{\mu} \Delta^\alpha_{\mu\beta} &= u_\lambda \Delta^\alpha_{\mu\beta} = u_\alpha \Delta^\alpha_{\mu\beta} = 0, \\
\Delta^{\mu\nu} \Delta^\alpha_{\mu\beta} &= \Delta_{\mu\nu} \Delta^\alpha_{\mu\beta} = 0, \\
\Delta^\mu_{\lambda\nu} \Delta^\alpha_{\mu\beta} &= \Delta^\alpha_{\nu\lambda} \Delta^\mu_{\mu\nu}, \\
\Delta^\alpha_{\beta\gamma\rho} \nabla_{\rho} u_{\lambda} &= \frac{1}{2} \left( \Delta^\alpha_{\beta\gamma} \Delta^\rho_{\lambda} + \Delta^\alpha_{\beta\lambda} \Delta^\rho_{\gamma} - \frac{2}{3} \Delta^\alpha_{\beta\rho} \Delta^\lambda_{\gamma} \right) \nabla_{\rho} u_{\lambda} = \frac{1}{2} \left( \nabla^{\beta} u^{\alpha} + \nabla^{\alpha} u^{\beta} - \frac{2}{3} \Delta^\alpha_{\beta} \nabla^\lambda u_{\lambda} \right) = \sigma^\alpha_{\beta}.
\end{align*}
$$

Appendix D: Calculation of the dissipative corrections $\delta T^{\mu\nu}$, $\delta N^\nu$, $\delta S^{\lambda,\mu\nu}$

1. Dissipative corrections $\delta N^\nu$

The dissipative part of the baryon current can be written as

$$
\delta N^\mu = \int dP \ dS \ p^\mu \left( \delta f_s^+ - \delta f_s^- \right).
$$

From Eq. (41) we obtain

$$
\begin{align*}
\delta f_s^+ - \delta f_s^- &= -\frac{\tau_{eq}}{u \cdot p} e^{\xi - \beta p} \left[ \left( 1 + \frac{1}{2} s_{\alpha\beta} \omega^{\alpha\beta} \right) \left( p^\mu \partial_\mu \xi - p^\lambda \rho \partial_\mu \beta_\lambda + \frac{1}{2} s_{\alpha\beta} p^\mu \partial_\mu \omega^{\alpha\beta} \right) \\
&\quad - \frac{\tau_{eq}}{u \cdot p} e^{-\xi - \beta p} \left[ \left( 1 + \frac{1}{2} s_{\alpha\beta} \omega^{\alpha\beta} \right) \left( p^\mu \partial_\mu \xi + p^\lambda \rho \partial_\mu \beta_\lambda - \frac{1}{2} s_{\alpha\beta} p^\mu \partial_\mu \omega^{\alpha\beta} \right) \\
&\quad = -\frac{2\tau_{eq}}{u \cdot p} e^{-\beta p} \left[ \left( 1 + \frac{1}{2} s_{\alpha\beta} \omega^{\alpha\beta} \right) \left( \cosh \xi \ p^\mu \partial_\mu \xi - \sinh \xi \ p^\lambda \rho \partial_\mu \beta_\lambda \right) + \frac{1}{2} \sinh \xi \ s_{\alpha\beta} p^\mu \partial_\mu \omega^{\alpha\beta} \right].
\end{align*}
$$

Substituting Eq. (D2) into Eq. (D1) we get

$$
\delta N^\mu = -2\tau_{eq} \int dP \ dS \ \frac{p^\mu}{u \cdot p} e^{-\beta p} \left[ \left( 1 + \frac{1}{2} s_{\alpha\beta} \omega^{\alpha\beta} \right) \left( \cosh \xi \ p^\rho \partial_\rho \xi - \sinh \xi \ p^\lambda \rho \partial_\rho \beta_\lambda \right) + \frac{1}{2} \sinh \xi \ s_{\alpha\beta} p^\rho \partial_\rho \omega^{\alpha\beta} \right].
$$

Now using Eqs. (A2) and (A6) we can easily carry out the integration over spin variables, therefore,

$$
\begin{align*}
\delta N^\mu &= -4 \tau_{eq} \int dP \ \frac{p^\mu}{u \cdot p} e^{-\beta p} \left[ \cosh \xi \ p^\rho \partial_\rho \xi - \sinh \xi \ p^\lambda \rho \partial_\rho \beta_\lambda \right] \\
&\quad = -4 \tau_{eq} \cosh \xi \ \partial_\rho \xi \int dP \ \frac{p^\rho p^\rho}{u \cdot p} e^{-\beta p} + 4 \tau_{eq} \sinh \xi \ \partial_\rho \beta_\lambda \int dP \ \frac{p^\rho p^\rho}{u \cdot p} e^{-\beta p}.
\end{align*}
$$

Momentum integration can be carried out using the following useful integral formulas,

$$
\begin{align*}
J_{(r)}^{\mu_1 \mu_2 \ldots \mu_n} &= \int \frac{dP}{(u \cdot p)^r} \left( p^{\mu_1} p^{\mu_2} \ldots p^{\mu_n} e^{-\beta p} \right) \\
&= J_{n_0}^{(r)} u^{\mu_1} u^{\mu_2} \ldots u^{\mu_n} + J_{n_1}^{(r)} \left( \Delta^{\mu_1 \mu_2} u^{\mu_3} u^{\mu_4} \ldots u^{\mu_n} + \text{permutations} \right) + \ldots
\end{align*}
$$

(5)
Thus, in the cases which are of interest for us, the formula (D5) gives:

\[
I_{(r)}^{\mu \rho} = I_{(20)}^{(r)} u^\mu u^\rho + I_{(21)}^{(r)} \Delta^{\mu \rho},
\]

\[
I_{(r)}^{\mu \nu \lambda \rho} = I_{(30)}^{(r)} u^\mu u^\nu u^\rho u^\sigma + I_{(31)}^{(r)} \left( \Delta^{\mu \nu} u^\lambda u^\rho + \Delta^{\nu \rho} u^\mu u^\sigma + \Delta^{\mu \rho} u^\nu u^\lambda + \Delta^{\lambda \rho} u^\mu u^\nu \right) + I_{(32)}^{(r)} \left( \Delta^{\mu \nu} \Delta^{\lambda \rho} + \Delta^{\mu \lambda} \Delta^{\nu \rho} + \Delta^{\nu \rho} \Delta^{\mu \lambda} \right),
\]

\[
I_{(r)}^{\mu \nu \lambda \rho \sigma} = I_{(40)}^{(r)} u^\mu u^\nu u^\lambda u^\rho u^\sigma + I_{(41)}^{(r)} \left( \Delta^{\mu \nu} u^\lambda u^\rho u^\sigma + \Delta^{\nu \lambda} u^\mu u^\rho u^\sigma + \Delta^{\mu \rho} u^\nu u^\lambda u^\sigma + \Delta^{\nu \rho} u^\mu u^\lambda u^\sigma \right) + I_{(42)}^{(r)} \left[ \Delta^{\mu \nu} \Delta^{\lambda \rho} + \Delta^{\mu \lambda} \Delta^{\nu \rho} + \Delta^{\nu \rho} \Delta^{\mu \lambda} \right].
\]

Using the integral formula (D5) in Eq. (D4) we get

\[
\delta N^\mu = -4\tau_{eq} \cosh \xi \left( I_{(1)}^{\mu \rho} \right) \partial_\rho \xi + 4\tau_{eq} \sinh \xi \left( I_{(1)}^{\mu \rho} \right) \partial_\rho \beta_\lambda,
\]

\[
= -4\tau_{eq} \cosh \xi \left( I_{(20)}^{(1)} u^\mu u^\rho + I_{(21)}^{(1)} \Delta^{\mu \rho} \right) \partial_\rho \xi + 4\tau_{eq} \sinh \xi \left[ I_{(30)}^{(1)} u^\mu u^\rho + I_{(31)}^{(1)} \left( \Delta^{\mu \lambda} u^\rho + \Delta^{\nu \rho} u^\lambda + \Delta^{\lambda \rho} u^\nu \right) \right] \partial_\rho \beta_\lambda.
\]

One can express the space-like (transverse) derivative operator as

\[
\nabla_\rho = \Delta^{\alpha \beta} \partial_\alpha = (g^\alpha_\rho - u_\rho u^\alpha) \partial_\alpha = \partial_\rho - u_\rho D.
\]

Using Eq. (D11) we can write

\[
\partial_\rho \xi = (\nabla_\rho + u_\rho D) \xi = \nabla_\rho \xi + u_\rho \xi,
\]

\[
\partial_\rho \omega^{\mu \nu} = (\nabla_\rho + u_\rho D) \omega^{\mu \nu} = \nabla_\rho \omega^{\mu \nu} + u_\rho \omega^{\mu \nu},
\]

\[
\partial_\rho \beta_\lambda = \partial_\rho (\beta u_\lambda) = \beta \partial_\rho u_\lambda + u_\lambda \partial_\rho \beta.
\]

Again using Eq. (D11) in Eq. (D14) we get

\[
\partial_\rho \beta_\lambda = \beta (\nabla_\rho + u_\rho D) u_\lambda + u_\lambda (\nabla_\rho + u_\rho D) \beta = \beta \nabla_\rho u_\lambda + \beta u_\rho \dot{u}_\lambda + u_\lambda \nabla_\rho \beta + u_\lambda u_\rho \dot{\beta}.
\]

Using Eqs. (12) and (15) in Eq. (10) one gets

\[
\delta N^\mu = -4\tau_{eq} \cosh \xi \left( I_{(20)}^{(1)} u^\mu u^\rho + I_{(21)}^{(1)} \Delta^{\mu \rho} \right) \left( \nabla_\rho \xi + u_\rho \xi \right) + 4\tau_{eq} \sinh \xi \left[ I_{(30)}^{(1)} u^\mu u^\rho + I_{(31)}^{(1)} \left( \Delta^{\mu \lambda} u^\rho + \Delta^{\nu \rho} u^\lambda + \Delta^{\lambda \rho} u^\nu \right) \right] \beta \nabla_\rho \beta_\lambda + \beta u_\rho \dot{u}_\lambda + u_\lambda \nabla_\rho \beta + u_\lambda u_\rho \dot{\beta}.
\]

2. Dissipative corrections \( \delta T^{\mu \nu} \)

The dissipative part of the energy-momentum tensor can be written as

\[
\delta T^{\mu \nu} = \int dP \, dS \, p^\mu p^\nu \left( \delta f^+_s + \delta f^-_s \right).
\]

From Eq. (41) we find the sum of the out-of-equilibrium corrections to the distribution functions for particles and antiparticles

\[
\delta f^+_s + \delta f^-_s = -\frac{\tau_{eq}}{u \cdot p} \frac{1}{e^+ \xi - \beta_\rho p} \left[ 1 + \frac{1}{2} s_{\alpha \beta} \omega^{\alpha \beta} \right] \left( p^\mu \partial_\rho \xi - p^\rho \partial_\mu \partial_\rho \beta_\lambda + \frac{1}{2} s_{\alpha \beta} p^\mu \partial_\rho \omega^{\alpha \beta} \right)
\]

\[
+ \frac{\tau_{eq}}{u \cdot p} \frac{1}{e^- \xi - \beta_\rho p} \left[ 1 + \frac{1}{2} s_{\alpha \beta} \omega^{\alpha \beta} \right] \left( p^\mu \partial_\rho \xi + p^\rho \partial_\mu \partial_\rho \beta_\lambda - \frac{1}{2} s_{\alpha \beta} p^\mu \partial_\rho \omega^{\alpha \beta} \right)
\]

\[
= -\frac{2\tau_{eq}}{u \cdot p} \left( 1 + \frac{1}{2} s_{\alpha \beta} \omega^{\alpha \beta} \right) \left( \sinh \xi p^\rho \partial_\rho \xi - \cosh \xi p^\rho \partial_\rho \beta_\lambda + \frac{1}{2} \cosh \xi s_{\alpha \beta} p^\rho \partial_\rho \omega^{\alpha \beta} \right). \]
Substituting Eq. (D18) in Eq. (D17), we obtain
\[ \delta T^{\mu \nu} = -2 \tau_{eq} \int \text{d}P \text{d}S \frac{p^{\mu}p^{\nu}}{u \cdot p} e^{-\beta p} \left[ \frac{1}{2} s_{\alpha \beta} \omega^{\alpha \beta} \right] \left( \sinh \xi \rho \partial_{\rho} \xi - \cosh \xi \rho \partial_{\rho} p^{\sigma} \partial_{\rho} \lambda \right) + \frac{1}{2} \cosh \xi s_{\alpha \beta} p^{\rho} \partial_{\rho} \omega^{\alpha \beta} \right]. \] (D19)

Integration over spin variables and using Eqs. (A2) and (A6) leads to
\[ \delta T^{\mu \nu} = -4 \tau_{eq} \int \text{d}P \frac{p^{\mu}p^{\nu}}{u \cdot p} e^{-\beta p} \left( \sinh \xi \rho \partial_{\rho} \xi - \cosh \xi \rho \partial_{\rho} \lambda \right) \]
\[ = -4 \tau_{eq} \sinh \xi \partial_{\xi} \int \text{d}P \frac{p^{\mu}p^{\nu}}{u \cdot p} e^{-\beta p} + 4 \tau_{eq} \cosh \xi \partial_{\lambda} \int \text{d}P \frac{p^{\mu}p^{\nu} \rho}{u \cdot p} e^{-\beta p}. \] (D20)

Using the integral formula (D5) in the above equation we obtain
\[ \delta T^{\mu \nu} = -4 \tau_{eq} \sinh \xi I_{(1)}^{\mu \nu \rho} \partial_{\rho} \xi + 4 \tau_{eq} \cosh \xi I_{(1)}^{\mu \nu \lambda \rho} \partial_{\rho} \beta_{\lambda}. \] (D21)

Subsequently, using Eqs. (D7) and (D8), we find
\[ \delta T^{\mu \nu} = -4 \tau_{eq} \sinh \xi \left[ I_{(1)}^{(1)} u^\rho u^{\nu} u^{\sigma} + I_{(3)}^{(1)} \left( \Delta^{\mu \nu} u^\rho + \Delta^{\mu \rho} u^{\nu} + \Delta^{\nu \rho} u^{\mu} \right) \right] + 4 \tau_{eq} \cosh \xi \partial_{\rho} \lambda \left[ I_{(1)}^{(1)} u^\rho u^{\nu} u^{\lambda} u^{\rho} + I_{(1)}^{(1)} \left( \Delta^{\mu \nu} u^{\lambda} u^{\rho} + \Delta^{\mu \rho} u^{\lambda} u^{\nu} + \Delta^{\nu \rho} u^{\lambda} u^{\mu} + \Delta^{\lambda \rho} u^{\mu} u^{\nu} \right) \right] \]
\[ + I_{(1)}^{(2)} \left( \Delta^{\mu \nu} \Delta^{\lambda \rho} + \Delta^{\mu \lambda} \Delta^{\nu \rho} + \Delta^{\mu \rho} \Delta^{\nu \lambda} \right). \] (D22)

Furthermore, using Eqs. (D12) and (D15) in the above equation we get
\[ \delta T^{\mu \nu} = -4 \tau_{eq} \sinh \xi \left[ I_{(1)}^{(1)} u^\rho u^{\nu} u^{\sigma} + I_{(3)}^{(1)} \left( \Delta^{\mu \nu} u^\rho + \Delta^{\mu \rho} u^{\nu} + \Delta^{\nu \rho} u^{\mu} \right) \right] \left( \nabla_{\rho} \xi + u_{\rho} \xi \right) + 4 \tau_{eq} \cosh \xi \left[ I_{(1)}^{(1)} u^\rho u^{\nu} u^{\lambda} u^{\rho} + I_{(1)}^{(1)} \left( \Delta^{\mu \nu} u^{\lambda} u^{\rho} + \Delta^{\mu \rho} u^{\lambda} u^{\nu} + \Delta^{\nu \rho} u^{\lambda} u^{\mu} + \Delta^{\lambda \rho} u^{\mu} u^{\nu} \right) \right] \]
\[ + I_{(1)}^{(2)} \left( \Delta^{\mu \nu} \Delta^{\lambda \rho} + \Delta^{\mu \lambda} \Delta^{\nu \rho} + \Delta^{\mu \rho} \Delta^{\nu \lambda} \right) \left( \beta \nabla_{\rho} \lambda + \beta u_{\rho} \beta_{\lambda} + u_{\lambda} \nabla_{\rho} \beta + u_{\lambda} u_{\rho} \beta \right). \] (D23)

### 3. Dissipative corrections \( \delta S^{\lambda, \mu \nu} \)

Dissipative part of the spin-tensor is given by the formula
\[ \delta S^{\lambda, \mu \nu} = \int \text{d}P \text{d}S p^{\lambda} s^{\mu \nu} \left( \delta f_{+}^{s} + \delta f_{-}^{s} \right). \] (D24)

Using Eq. (D18) in Eq. (D24) one gets
\[ \delta S^{\lambda, \mu \nu} = -2 \tau_{eq} \int \text{d}P \text{d}S \frac{p^{\lambda} p^{\mu \nu}}{u \cdot p} e^{-\beta p} \left[ \frac{1}{2} s_{\alpha \beta} \omega^{\alpha \beta} \right] \left( \sinh \xi \rho \partial_{\rho} \xi - \cosh \xi \rho \partial_{\rho} \lambda \right) + \frac{1}{2} \cosh \xi s_{\alpha \beta} p^{\rho} \partial_{\rho} \omega^{\alpha \beta} \right]. \] (D25)

With the help of Eqs. (A6) and (A15) the integration over the spin degrees of freedom in the above equation can be easily performed giving
\[ \delta S^{\lambda, \mu \nu} = - \frac{4 \tau_{eq}^{2}}{3 m^{2}} \int \text{d}P \frac{p^{\lambda}}{u \cdot p} e^{-\beta p} \left( \sinh \xi \rho \partial_{\rho} \xi - \cosh \xi \rho \partial_{\rho} \lambda \right) \left( m^{2} \omega^{\mu \nu} + p^{\alpha} p^{\mu} p^{\nu} \omega^{\alpha} \right). \] (D26)

The above equation can further be written as a sum of six terms
\[ \delta S^{\lambda, \mu \nu} = - \frac{4 \tau_{eq}^{2}}{3 m^{2}} \int \text{d}P \frac{p^{\lambda}}{u \cdot p} \left( \sinh \xi \rho \partial_{\rho} \xi + \cosh \xi \rho \partial_{\rho} \lambda \right) \int \text{d}P \frac{p^{\mu} p^{\nu} \rho^{(\mu}}{u \cdot p} e^{-\beta p} \right]
\[ - \frac{4 \tau_{eq}^{2}}{3 m^{2}} \int \text{d}P \frac{p^{\mu} p^{\nu}}{u \cdot p} e^{-\beta p} - \frac{8 \tau_{eq}^{2}}{3 m^{2}} \int \text{d}P \frac{p^{\mu} p^{\nu} \rho^{(\mu}}{u \cdot p} e^{-\beta p} \right]
\[ + \frac{8 \tau_{eq}^{2}}{3 m^{2}} \left( \sinh \xi \rho \partial_{\rho} \xi + \cosh \xi \rho \partial_{\rho} \lambda \right) \int \text{d}P \frac{p^{\mu} p^{\nu} \rho^{(\mu}}{u \cdot p} e^{-\beta p} \right]. \] (D27)
Now we evaluate one by one each of the terms appearing in this expression.

**Term I:**

\[
I = \frac{4s^2}{3} \tau_{eq} \sinh \xi \partial_\rho \xi \omega^{\mu \nu} \int d\rho \frac{p^\lambda p^\rho}{u \cdot p} e^{-\beta \rho}.
\]  

Using Eqs. (D12), (D5) and (D6), we can get

\[
I = \frac{4s^2}{3} \tau_{eq} \sinh \xi (\nabla_\rho \xi + u_\rho \dot{\xi}) \omega^{\mu \nu} I_{\lambda \rho}^{(1)}
\]

\[
= \frac{4s^2}{3} \tau_{eq} \sinh \xi (\nabla_\rho \xi + u_\rho \dot{\xi}) \omega^{\mu \nu} \left( I_{20}^{(1)} u^\lambda u^\rho + I_{21}^{(1)} \Delta^\lambda \rho \right)
\]

\[
= \frac{4s^2}{3} \tau_{eq} \sinh \xi \omega^{\mu \nu} \left( I_{21}^{(1)} \nabla^\lambda \xi + \dot{\xi} u^\lambda f_{20}^{(1)} \right).
\]  

**Term II:**

\[
II = \frac{4s^2}{3} \tau_{eq} \cosh \xi \partial_\rho \beta_\alpha \int d\rho \frac{p^\lambda p^\rho \omega^{\mu \nu}}{u \cdot p} e^{-\beta \rho}.
\]  

Using Eqs. (D15), (D5), and (D7) this term can be written as

\[
II = \frac{4s^2}{3} \tau_{eq} \cosh \xi \omega^{\mu \nu} \left( \beta \nabla_\rho \beta_\alpha + \beta u_\rho \dot{\beta} + u_\rho \nabla_\beta + u_\rho u_\rho \dot{\beta} \right) \left[ I_{30}^{(1)} \nabla^\lambda \beta + I_{31}^{(1)} \left( \Delta^\lambda \beta + \Delta \beta u^\lambda + \Delta \beta \nabla^\lambda \beta \right) \right].
\]  

This expression simplifies to

\[
II = \frac{4s^2}{3} \tau_{eq} \cosh \xi \omega^{\mu \nu} \left[ I_{30}^{(1)} \beta \nabla^\lambda \beta + I_{31}^{(1)} \left( \beta u^\lambda + \beta \dot{u}^\lambda + \nabla^\lambda \beta \right) \right].
\]  

**Term III:**

\[
III = \frac{4s^2}{3} \tau_{eq} \cosh \xi \partial_\rho \omega^{\mu \nu} \int d\rho \frac{p^\lambda p^\rho}{u \cdot p} e^{-\beta \rho}.
\]  

Using Eqs. (D13), (D5), and (D6) we can write

\[
III = \frac{4s^2}{3} \tau_{eq} \cosh \xi \omega^{\mu \nu} \left( \beta \nabla_\rho \omega^{\mu \nu} + u_\rho \dot{\omega}^{\mu \nu} \right) I_{\lambda \rho}^{(1)}
\]

\[
= \frac{4s^2}{3} \tau_{eq} \cosh \xi \omega^{\mu \nu} \beta \nabla_\rho \omega^{\mu \nu} + u_\rho \dot{\omega}^{\mu \nu} \left( I_{20}^{(1)} u^\lambda u^\rho + I_{21}^{(1)} \Delta^\lambda \rho \right)
\]

\[
= \frac{4s^2}{3} \tau_{eq} \cosh \xi \left( I_{21}^{(1)} \nabla^\lambda \omega^{\mu \nu} + I_{20}^{(1)} u^\lambda \omega^{\mu \nu} \right).
\]  

**Term IV:**

\[
IV = \frac{8s^2}{3 m^2} \tau_{eq} \sinh \xi \partial_\rho \xi \int d\rho \frac{p^\lambda p^\rho p^\rho p^\rho}{u \cdot p} e^{-\beta \rho}.
\]  

Using Eqs. (D12), (D5), and (D8) we find

\[
IV = \frac{8s^2}{3 m^2} \tau_{eq} \sinh \xi (\nabla_\rho \xi + u_\rho \dot{\xi} ) I_{\lambda \rho}^{(1)}
\]

\[
= \frac{8s^2}{3 m^2} \tau_{eq} \sinh \xi (\nabla_\rho \xi + u_\rho \dot{\xi} ) \left[ I_{40}^{(1)} u^\lambda u^\rho u^\mu u^\alpha + I_{41}^{(1)} \Delta^\lambda \rho u^\mu u^\alpha + \Delta^\lambda \rho u^\mu u^\alpha + \Delta^\lambda \rho u^\mu u^\alpha 
\]

\[+ \Delta^\alpha \mu u^\mu u^\lambda + \Delta^\rho \mu u^\mu u^\alpha + \Delta^\alpha \mu u^\mu u^\alpha \right] + I_{42}^{(1)} \left( \Delta^\lambda \rho \Delta^\alpha \mu + \Delta^\alpha \mu \Delta^\rho \mu + \Delta^\lambda \rho \Delta^\alpha \mu \right) \omega^\nu \]

\[
= \frac{8s^2}{3 m^2} \tau_{eq} \left[ I_{41}^{(1)} \left( u^\alpha u^\mu \omega^\nu \right) \nabla^\alpha \xi + u^\lambda u^\mu \omega^\nu \nabla^\alpha \xi + u^\nu \omega^\nu \nabla^\alpha \xi 
\]

\[+ I_{42}^{(1)} \left( \Delta^\alpha \mu \omega^\nu \right) \nabla^\alpha \xi + \Delta^\lambda \alpha \omega^\nu \nabla^\alpha \xi + \Delta^\lambda \mu \omega^\nu \nabla^\alpha \xi \right]

\[
+ \dot{\xi} \left( I_{40}^{(1)} u^\lambda u^\mu u^\alpha u^\nu + I_{41}^{(1)} \left( \Delta^\lambda \mu u^\alpha u^\nu + \Delta^\lambda \mu u^\alpha u^\nu + \Delta^\lambda \mu u^\alpha u^\nu \right) \right). \]  

\]
Term VI:

\[
V = \frac{8s^2}{3m^2} \tau_{eq} \cosh \xi \partial_\rho \beta_\kappa \int dP \frac{p^\lambda p^\rho p^\alpha p^\mu \omega^\alpha}{u \cdot p} e^{-\beta_\rho p}. \tag{D37}
\]

Using Eqs. (D15), (D5) and (D9) this term can be written as

\[
V = \frac{8s^2}{3m^2} \tau_{eq} \cosh \xi \left( \beta \nabla_\rho u_\kappa + \beta u_\mu \partial_\kappa + u_\kappa \nabla_\rho \beta + u_\kappa u_\rho \beta \right) I^{(1)}_{\lambda \rho \alpha \mu \omega} \tag{D38}
\]

Term VII:

\[
VI = \frac{8s^2}{3m^2} \tau_{eq} \cosh \xi \int dP \frac{p^\lambda p^\rho p^\alpha p^\mu \partial_\rho \omega^\alpha}{u \cdot p} e^{-\beta_\rho p} \tag{D39}
\]

Using Eqs. (D13), (D5) and (D8) we obtain

\[
VI = \frac{8s^2}{3m^2} \tau_{eq} \cosh \xi \left( \beta \nabla_\rho u_\kappa + \beta u_\mu \partial_\kappa + u_\kappa \nabla_\rho \beta + u_\kappa u_\rho \beta \right) I^{(1)}_{\lambda \rho \alpha \mu \omega} \tag{D40}
\]

Now substituting Eqs. (D29), (D32), (D34), (D36), (D38), and (D40) into Eq. (D27) one can obtain the following
expression for the dissipative correction to the spin tensor

\[
\delta S^{\lambda,\nu} = \frac{4s^2}{3} \tau_{eq} \left[ -\sinh \xi \left( I_{21}^{(1)} \omega^\mu_\nu \nabla^\lambda \xi + I_{20}^{(1)} \xi u^\lambda \omega^\mu_\nu + \frac{2}{m^2} i I_{41}^{(1)} \left( u^{\alpha u^\beta_\mu \omega^\nu_\alpha} \nabla^\lambda \xi + u^\lambda \omega^{[\mu} \omega^\nu_{\alpha]} \nabla^\alpha \xi + u^\lambda u^\alpha_\omega^{[\nu} \nabla^\mu \xi \right) + \xi i I_{41}^{(1)} \left( u^\lambda \omega^{[\mu} \omega^\nu_{\alpha]} + u^\lambda u^\alpha_\omega^{[\nu} \nabla^\mu \xi \right) + \xi I_{41}^{(1)} \left( u^\lambda u^\alpha_\omega^{[\nu} \nabla^\mu \xi \right) \right) \right] + \cosh \xi \left( I_{30}^{(1)} \beta u^\lambda \omega^\mu_\nu + I_{31}^{(1)} \left( \beta \theta u^\lambda + \beta \dot{u}^\lambda + \nabla^\lambda \beta \right) \omega^\mu_\nu + \frac{2}{m} \beta i I_{50}^{(1)} \right.
\]

\[
+ \frac{2}{m^2} i I_{51}^{(1)} \left( \beta \theta u^\lambda u^{[\mu} \omega^\nu_{\alpha]} + \beta \dot{u}^\lambda + \nabla^\lambda \beta \right) u^{[\mu} u^{\nu]_\alpha} + \left( \beta \dot{u}^\lambda + \nabla^\lambda \beta \right) u^{[\mu} u^{\nu]_\alpha} \right) + \left( \beta \dot{u}^\mu + \nabla^{[\mu} \beta \right) \omega^{\nu]_\alpha u^\lambda u^\alpha + \beta \left( \Delta^\lambda \nu \alpha u^{\omega_\nu} \beta + \Delta^\lambda \nu \alpha u^{\omega_\nu} \beta + \beta \theta u^\lambda \omega^\mu_\nu + \beta u^\lambda \omega^{[\mu} \nabla^\nu_\alpha u^{\omega]_\alpha} + \beta u^\alpha \omega^{[\mu} \nabla^\nu_\alpha u^\lambda + \beta u^\alpha \omega^{[\mu} \nabla^\nu_\alpha u^\lambda + \beta u^\mu_\nu \beta + \beta u^\mu_\nu \beta + \beta u^\mu_\nu \beta \right)
\]

\[
+ \frac{2}{m^2} i I_{51}^{(1)} \left( \beta \theta u^\lambda \omega^\mu_\nu \beta + \beta \theta u^\lambda + \nabla^\lambda \beta \right) \omega^\mu_\nu \beta + \beta u^{[\mu} \omega^{\nu]_\alpha} + \beta \theta u^\lambda \omega^\nu_\alpha + \beta \theta u^\lambda \omega^\nu_\alpha + \beta \theta u^\lambda \omega^\nu_\alpha \right) \left( \beta \dot{u}^\mu + \nabla^{[\mu} \beta \right) \omega^{\nu]_\alpha \beta} - \left( I_{21}^{(1)} \nabla^\lambda \omega^\mu_\nu + I_{20}^{(1)} \nabla^\lambda \omega^\mu_\nu \right) - \frac{2}{m^2} \left( I_{40}^{(1)} u^\mu u^{[\mu} \omega^\nu_\alpha + I_{40}^{(1)} u^\mu \nabla^\nu_\alpha + \Delta^\lambda \nu \alpha u^{\omega_\nu} \beta + \Delta^\lambda \nu \alpha u^{\omega_\nu} \beta + \Delta^\lambda \nu \alpha u^{\omega_\nu} \beta \right)
\]

\[
+ I_{41}^{(1)} \left( \beta \dot{u}^\mu + \nabla^{[\mu} \beta \right) \omega^{\nu]_\alpha + \Delta^\lambda \nu \alpha u^{\omega_\nu} \beta + \Delta^\lambda \nu \alpha u^{\omega_\nu} \beta + \Delta^\lambda \nu \alpha u^{\omega_\nu} \beta + \Delta^\lambda \nu \alpha u^{\omega_\nu} \beta \right) \right) \right].
\]

(D41)

**Appendix E: Eliminating $\dot{\xi}$, $\ddot{\beta}$, $\dot{u}^\mu$ and $\omega^\mu_\nu$ from $\delta S^{\lambda,\mu}$**

Note that the derivation of equations that specify the convective derivatives $\dot{\xi}$, $\ddot{\beta}$, and $\dot{u}^\mu$ has already been done in Sec. III C and our results are reported in Eqs. (73), (74), and (81). Here we present important steps needed for derivation of the dynamical equation for $\dot{\omega}^\mu_\nu$. Substituting Eq. (83) in Eq. (66) we can get

\[
\dot{\omega}^\mu_\nu = -\frac{1}{I_{10}^{(0)} - \frac{2}{m^2} I_{31}^{(0)}} \left( \frac{I_{10}^{(0)} \theta \omega^\mu_\nu + I_{10}^{(0)} \dot{\xi} \omega^\mu_\nu \tanh \xi + \dot{I}_{10}^{(0)} \omega^\mu_\nu} \right)
\]

\[
- \frac{2}{m^2 I_{10}^{(0)} - 2 I_{31}^{(0)}} \left[ \tanh \xi \left( I_{30}^{(1)} - 3 I_{31}^{(1)} \right) \xi u^\alpha u^{[\mu} \omega^\nu_{\alpha]} + I_{31}^{(1)} \partial_\lambda \xi \left( u^{[\mu} \omega^\nu_{\lambda]} - \omega^\mu_\nu u^\lambda + u^\alpha g^{[\mu} \omega^\nu_{\alpha]} \right) \right)
\]

\[
+ \left( I_{30}^{(1)} - 3 I_{31}^{(1)} \right) u^\alpha u^{[\mu} \omega^\nu_{\alpha]} + \left( u^{[\mu} \omega^\nu_{\lambda]} - \omega^\mu_\nu u^\lambda + u^\alpha g^{[\mu} \omega^\nu_{\alpha]} \right) \partial_\lambda \xi
\]

\[
+ \left( I_{30}^{(1)} - 3 I_{31}^{(1)} \right) \theta u^\alpha u^{[\mu} \omega^\nu_{\alpha]} + \left( I_{30}^{(1)} - 3 I_{31}^{(1)} \right) \omega^\nu_{\lambda} \partial_\mu \omega^\lambda + \left( I_{30}^{(1)} - 3 I_{31}^{(1)} \right) u^\alpha \partial_\mu \omega^\lambda + \left( I_{30}^{(1)} - 3 I_{31}^{(1)} \right) u^\alpha \partial_\mu \omega^\lambda \right].
\]

(E1)
Using the relations \( \dot{I}^{(0)}_{10} = -\dot{\beta}I^{(0)}_{20} \), \( \dot{I}^{(0)}_{30} = -\dot{\beta}I^{(0)}_{00} \), \( \dot{I}^{(0)}_{0} = -\dot{\beta}I^{(0)}_{41} \), \( \partial_{\lambda}I^{(0)}_{30} = - (\partial_{\lambda}\beta) I^{(0)}_{41} \), \( \partial_{\lambda}I^{(0)}_{41} = - (\partial_{\lambda}\beta) I^{(0)}_{41} \) and substituting \( \partial_{\lambda} = \nabla_{\lambda} + u_{\lambda}D \) in the above equation we obtain

\[
\dot{\omega}^{\mu\nu} = - \frac{2}{(m^2 I^{(0)}_{10} - 2 I^{(0)}_{31})} \left( \hat{I}^{(0)}_{30} - \hat{I}^{(0)}_{31} \right) u^{\alpha} u^{[\mu} \hat{v}^{\nu]} + \frac{1}{(I^{(0)}_{10} - \frac{2}{m^2} I^{(0)}_{31})} \left( I^{(0)}_{10} \theta \omega^{\mu\nu} + I^{(0)}_{10} \xi \omega^{\mu\nu} \tanh \xi - \tilde{\beta} I^{(0)}_{20} \omega^{\mu\nu} \right)
- \frac{2}{(m^2 I^{(0)}_{10} - 2 I^{(0)}_{31})} \left[ \tanh \xi \left( I^{(0)}_{30} - 3 I^{(0)}_{31} \right) \xi u^{\alpha} u^{[\mu} \hat{v}^{\nu]} + \tanh \xi I^{(0)}_{31} \left( \nabla_{\lambda} \xi + \xi u_{\lambda} \right) \left[ u^{[\mu} \omega^{\nu]} - \omega^{\mu\nu} u^{\lambda} + u^{\alpha} g^{[\lambda}_{\mu}[\omega^{\nu]}] \right] + \hat{\beta} \left( I^{(0)}_{40} - 3 I^{(0)}_{41} \right) u^{\alpha} u^{[\mu} \hat{v}^{\nu]} - \left( \left( \nabla_{\lambda} \beta + \tilde{\beta} u_{\lambda} \right) I^{(0)}_{41} u^{[\mu} \omega^{\nu]} - \tilde{\beta} I^{(0)}_{41} \omega^{\mu\nu} + \left( \nabla_{\lambda} \beta + \tilde{\beta} u_{\lambda} \right) I^{(0)}_{41} u^{\alpha} \omega^{[\lambda}_{\mu}] \right) \right] + \left( I^{(0)}_{30} - 3 I^{(0)}_{31} \right) u^{\alpha} u^{[\mu} \omega^{\nu]} + \left( I^{(0)}_{30} - 2 I^{(0)}_{31} \right) \dot{\omega}^{\mu\nu} + \left( I^{(0)}_{30} - 2 I^{(0)}_{31} \right) u^{\alpha} \dot{u}^{[\mu} \omega^{\nu]} \right] \right].
\]

(E2)

We first eliminate \( u^{\alpha} \left( u^{[\mu} \hat{v}^{\nu]} \right) \) from the above expression. Contracting the resulting equation with \( u_{\nu} \) and using \( I^{(0)}_{30} - 2 I^{(0)}_{31} = -\beta I^{(0)}_{41} \), \( I^{(0)}_{30} - I^{(0)}_{31} = I^{(0)}_{31} - \beta I^{(0)}_{41} \) at appropriate places we obtain

\[
u^{\mu} \omega^{\mu\nu} = - \frac{m^2 I^{(0)}_{10} - \left( I^{(0)}_{30} + I^{(0)}_{31} \right)}{m^2 I^{(0)}_{10} - \left( I^{(0)}_{30} + I^{(0)}_{31} \right)} \left( I^{(0)}_{10} \theta \omega^{\mu\nu} u_{\nu} + I^{(0)}_{10} \xi \omega^{\mu\nu} \tanh \xi u_{\nu} - I^{(0)}_{20} \tilde{\beta} \omega^{\mu\nu} u_{\nu} \right)
- \frac{1}{m^2 I^{(0)}_{10} - \left( I^{(0)}_{30} + I^{(0)}_{31} \right)} \left[ \tanh \xi \left( I^{(0)}_{30} + I^{(0)}_{31} \right) \xi \omega^{\mu\nu} u_{\nu} - \tanh \xi I^{(0)}_{31} \Delta_{\nu}^{\mu} \omega^{\nu\lambda} \nabla_{\lambda} \xi \right.
+ \hat{\beta} \left( I^{(0)}_{40} + I^{(0)}_{41} \right) \omega^{\mu\nu} u_{\nu} + \left( \beta \dot{u}^{\alpha} + \nabla^{\alpha} \beta \right) I^{(0)}_{41} \Delta^{\mu}_{\nu} \omega^{\nu\lambda}
\left. + \left( I^{(0)}_{41} - I^{(0)}_{31} \right) \theta \omega^{\mu\nu} u_{\nu} + I^{(0)}_{31} \left( \omega^{\mu\lambda} u_{\nu} \nabla_{\lambda} u^{\mu} - \Delta_{\nu}^{\mu} \nabla_{\lambda} \omega^{\mu\lambda} + u_{\nu} \omega^{\nu\alpha} \nabla_{\alpha} u^{\mu} + u^{\alpha} u_{\nu} \nabla^{\mu} \omega^{\nu\lambda} \right) \right].
\]

(E3)

Now eliminating \( \dot{\xi} \), \( \dot{\beta} \), and \( \dot{u}^{\mu} \) (with the help of Eqs. (73), (74) and (81)) the above equation can be written as

\[
u^{\mu} \omega^{\mu\nu} = C_{\Pi}^{\mu} \theta + C_{n}^{\mu} \nabla^{\lambda} \xi + C_{\gamma}^{\mu} \theta^{\alpha} + C_{\Sigma}^{\mu} \nabla_{\nu} \omega^{\nu\lambda}.
\]

(E4)

Various \( C \)-coefficients appearing in the above equation are as follows:

\[
C_{\Pi}^{\mu} = C_{\Pi} u_{\nu} \omega^{\mu\nu},
C_{n}^{\mu} = C_{n} \Delta_{\nu}^{\mu} \omega^{\nu\lambda},
C_{\gamma}^{\mu} = C_{\gamma} u_{\nu} \omega^{\nu\lambda},
C_{\Sigma}^{\mu} = C_{\Sigma} \Delta_{\nu}^{\mu}.
\]

(E5)\(\text{E6}\)\(\text{E7}\)\(\text{E8}\)

where

\[
C_{\Pi} = - \frac{1}{m^2 I^{(0)}_{10} - \left( I^{(0)}_{30} + I^{(0)}_{31} \right)} \left[ m^2 \xi_{\theta} \tanh \xi I^{(0)}_{10} - m^2 \beta_{\theta} I^{(0)}_{20} + m^2 I^{(0)}_{10} - \tanh \xi \left( I^{(0)}_{30} + I^{(0)}_{31} \right) \xi_{\theta} \right.
+ \beta_{\theta} \left( I^{(0)}_{40} + I^{(0)}_{41} \right) + \beta I^{(0)}_{41} - \frac{5}{3} I^{(0)}_{31} \right],
\]

(E9)

\[
C_{n} = \frac{\tanh \xi}{m^2 I^{(0)}_{10} - \left( I^{(0)}_{30} + I^{(0)}_{31} \right)} \left( I^{(0)}_{31} - \frac{n_{0} I^{(0)}_{41}}{\varepsilon_{0} + I^{(0)}_{41}} \right),
\]

(E10)

\[
C_{\gamma} = - \frac{2 I^{(0)}_{31}}{m^2 I^{(0)}_{10} - \left( I^{(0)}_{30} + I^{(0)}_{31} \right)},
\]

(E11)

\[
C_{\Sigma} = \frac{I^{(0)}_{31}}{m^2 I^{(0)}_{10} - \left( I^{(0)}_{30} + I^{(0)}_{31} \right)}.
\]

(E12)
Using Eq. (E4) and the recurrence relation $I_{30}^{(0)} - 2I_{31}^{(0)} = -\beta I_{31}^{(0)}$ in (E2) and then eliminating $\dot{\xi}$, $\dot{\beta}$, and $\ddot{u}^\mu$ (using Eqs. (73), (74), and (81)) we obtain

$$\omega^{\mu \nu} = D^{\mu \nu} \theta + D_n^{[\mu \nu]} \sigma^{\lambda},$$

(E13)

The various $D$-coefficients appearing in the above equation are given by the following expressions

$$D_{\Pi}^{\mu \nu} = D_{\Pi 1} \omega^{\mu \nu} + D_{\Pi 2} u^\alpha \omega^{\mu \nu} \sigma^{\lambda},$$

(E14)

$$D_n^{[\mu \nu]} = -D_{n 1} u^\alpha \omega^{\mu \nu} \sigma^{\lambda},$$

(E15)

$$D_{n \lambda}^{[\mu \nu]} = -\omega^{[\mu \nu} \sigma^{\lambda],} - u^\alpha \omega^{\mu \nu} \sigma^{\lambda},$$

(E16)

$$D_{\Sigma 1} = -u^\alpha 2I_{31}^{(0)} \sigma^{\lambda}$$

(E17)

$$D_{\Sigma 2}^{[\mu \nu]} = -u^\alpha 2I_{31}^{(0)} \mu^{\rho},$$

(E18)

where

$$D_{\Pi 1} = \frac{1}{I_{10}^{(0)} - 2I_{31}^{(0)}} \left( \frac{I_{10}^{(0)}}{I_{31}^{(0)}} - 2 \xi_{0} \tan \xi I_{10}^{(0)} - \beta \xi I_{20}^{(0)} + 2 \frac{m^2 \xi_{0}}{I_{10}^{(0)} - 2I_{31}^{(0)}} \right),$$

(E19)

$$D_{\Pi 2} = \frac{2}{m^2 I_{10}^{(0)} - 2I_{31}^{(0)}} \left( \frac{I_{10}^{(0)}}{I_{31}^{(0)}} - 2 \xi_{0} \tan \xi I_{10}^{(0)} - \frac{2m^2 \xi_{0}}{I_{10}^{(0)} - 2I_{31}^{(0)}} \right),$$

(E20)

$$D_{n 1} = \frac{2}{m^2 I_{10}^{(0)} - 2I_{31}^{(0)}} \left( \frac{I_{10}^{(0)}}{I_{31}^{(0)}} - \frac{m^2 \xi_{0}}{I_{10}^{(0)} - 2I_{31}^{(0)}} \right),$$

(E21)

$$D_{n 2} = \frac{2}{m^2 I_{10}^{(0)} - 2I_{31}^{(0)}} \left( \frac{I_{10}^{(0)}}{I_{31}^{(0)}} - \frac{m^2 \xi_{0}}{I_{10}^{(0)} - 2I_{31}^{(0)}} \right),$$

(E22)

Using Eqs. (73), (74), (81), (E4), (E13), and (D41), we finally obtain

$$\delta S^{\lambda, \mu \nu} = \tau_{\mu \nu}^{\lambda} + B_{n 1}^{\lambda, \mu \nu} \sigma^{\lambda},$$

(E23)

where different coefficients appearing on the right-hand side of Eq. (E23) are the kinetic coefficients for spin-related phenomena. These coefficients are listed in Eqs. (95), (96), (97), and (98) where:

$$B_{\Pi 1}^{(1)} = \frac{4}{3} \left( -\frac{2}{m^2 \xi_{0}} \sin \xi I_{141}^{(1)} + \frac{2}{m^2 \xi_{0}} I_{31}^{(1)} \beta \cosh \xi + \frac{10}{3m^2} I_{52}^{(1)} \beta \cosh \xi - \frac{2}{m^2} I_{41}^{(1)} \cosh \xi D_{\Pi 1} \right),$$

(E24)

$$B_{\Pi 1}^{(2)} = \frac{4}{3} \left[ -\frac{2}{m^2 \xi_{0}} \sin \xi I_{40}^{(1)} + \frac{4}{m^2 \xi_{0}} \sin \xi I_{41}^{(1)} + \frac{2}{m^2} I_{50}^{(1)} \beta \cosh \xi + \frac{2}{m^2} I_{51}^{(1)} \beta \cosh \xi - \frac{20}{3m^2} I_{52}^{(1)} \beta \cosh \xi - \left( I_{20}^{(1)} - \frac{3}{m^2} I_{41}^{(1)} \right) \cosh \xi D_{\Pi 2} - \frac{2}{m^2} \left( I_{40}^{(1)} - 2I_{41}^{(1)} \right) \cosh \xi C_{II} \right],$$

(E25)

$$B_{\Pi 1}^{(3)} = \frac{4}{3} \left( -\frac{2}{m^2 \xi_{0}} \sin \xi I_{41}^{(1)} + \frac{2}{m^2} I_{51}^{(1)} \beta \cosh \xi + \frac{10}{3m^2} I_{52}^{(1)} \beta \cosh \xi - \frac{2}{m^2} I_{41}^{(1)} \cosh \xi C_{II} \right),$$

(E26)
In this section we show that \( \delta N^\mu, \delta T^{\mu\nu}, \) and \( \delta S^{\lambda,\mu\nu} \) given by Eqs. (D16), (D23), and (E23) satisfy the relations (45), (46), and (47).

### Appendix F: Landau matching Conditions

Projecting Eq. (D16) along \( u_\mu \) we obtain

\[
u_\mu \delta N^\mu = -4r^{(1)}_{20} \xi \tau_{eq} \cosh \xi + 4 \left( I^{(1)}_{31} \beta \theta + I^{(1)}_{30} \beta \right) \tau_{eq} \sinh \xi.
\]
Using the recurrence relation (B14) we can write down
\[ I_{20}^{(1)} = I_{10}^{(0)} = n_0, \quad I_{31}^{(1)} = I_{21}^{(0)} = -P_0, \quad I_{30}^{(1)} = I_{20}^{(0)} = \varepsilon_0. \] (F2)

Substituting the above values for \( I_{20}^{(1)}, I_{31}^{(1)}, I_{30}^{(1)} \) and the values of \( \dot{\xi} \) and \( \dot{\beta} \) from Eqs. (73) and (74) into Eq. (F1), we can show that the right-hand side of Eq. (F1) vanishes.

2. Proving \( u_\mu \delta T^{\mu\nu} = 0 \)

Projecting Eq. (D23) along \( u_\mu \) we obtain
\[ u_\mu \delta T^{\mu\nu} = -4\tau_{eq} \sinh \xi \left( I_{30}^{(1)} \dot{\xi} u^{\nu} + I_{31}^{(1)} \nabla^{\nu} \xi \right) + 4\tau_{eq} \cosh \xi \left( I_{40}^{(1)} \dot{\beta} u^{\nu} + I_{41}^{(1)} \left( \beta \dot{u}^{\nu} + \nabla^{\nu} \beta + \beta \theta u^{\nu} \right) \right). \] (F3)

Using Eq. (81), the above equation can be written as
\[ u_\mu \delta T^{\mu\nu} = -4\tau_{eq} \left( I_{40}^{(1)} \dot{\beta} \cosh \xi - I_{41}^{(1)} \beta \cosh \xi \right) u^{\nu} - 4\tau_{eq} \left( I_{31}^{(1)} \sinh \xi - I_{41}^{(1)} \cosh \xi n_0 \tanh \xi \right) \nabla^{\nu} \xi. \] (F4)

Using the recurrence relations (B14) and (B15) we can write
\[ I_{30}^{(1)} = I_{20}^{(0)}, \quad I_{31}^{(1)} = I_{31}^{(0)} = n_0, \quad I_{41}^{(1)} = I_{31}^{(0)} = -\frac{1}{\beta} \left( I_{20}^{(0)} - I_{21}^{(0)} \right). \] (F5)

Using the above relations along with the values of \( \dot{\xi} \) and \( \dot{\beta} \) from Eqs. (73) and (74), we see that the first square bracket term on the right-hand side of Eq. (F4) vanishes; see Eq. (71) for details. Using the relations (F2) and (F5), it can also be shown that the second square bracket term in Eq. (F4) is zero.

3. Proving \( u_\mu \delta S^{\lambda,\mu\nu} = 0 \)

Projecting Eq. (D41) along \( u_\lambda \) we obtain
\[
\begin{align*}
    u_\lambda \delta S^{\lambda,\mu\nu} &= \frac{4s^2}{3} \tau_{eq} \left[ -\sinh \xi \left( I_{10}^{(0)} \dot{\xi} \omega^{\mu\nu} + \frac{2}{m^2} \left( I_{41}^{(1)} \left( u^{[\mu} \omega^{\nu]}_\alpha \nabla^\alpha \xi + u^\alpha \omega^{[\mu}_\alpha \nabla^{\nu]}_\xi \right) + \dot{\xi} \left( I_{40}^{(1)} u^{\alpha} u^{[\mu}_\alpha \omega^{\nu]}_\alpha + I_{41}^{(1)} \Delta^{\alpha}_{[\mu} \omega^{\nu]}_\alpha \right) \right) \right] \\
    &+ \cosh \xi \left( I_{31}^{(1)} \beta \theta \omega^{\mu\nu} + I_{30}^{(1)} \beta \omega^{\mu\nu} \right) + \frac{2}{m^2} \dot{\beta} I_{50}^{(1)} u^{\alpha} u^{[\mu}_\alpha \omega^{\nu]}_\alpha + \frac{2}{m^2} I_{52}^{(1)} \left( \beta \dot{\theta} \Delta^{\alpha}_{[\mu} \omega^{\nu]}_\alpha + \beta (\nabla^{[\mu} u^{\alpha} + \nabla^{\alpha} u^{[\mu} \omega^{\nu]}_\alpha) \right) \omega^{\nu]}_\alpha u^{\alpha}_\alpha + \dot{\beta} \Delta^{\alpha}_{[\mu} \omega^{\nu]}_\alpha) \\
    &+ \frac{2}{m^2} I_{51}^{(1)} \left( \beta \theta u^{\alpha} u^{[\mu}_\alpha \omega^{\nu]}_\alpha + (\beta \dot{u}^{\alpha} + \nabla^{\alpha} \beta) u^{[\mu}_\alpha \omega^{\nu]}_\alpha + (\beta \dot{u}^{\alpha} + \nabla^{\mu} \beta) \omega^{\nu]}_\alpha u^{\alpha}_\alpha + \dot{\beta} \Delta^{\alpha}_{[\mu} \omega^{\nu]}_\alpha \right) \\
    &- \left( I_{20}^{(1)} - \frac{2}{m^2} I_{41}^{(1)} \right) \omega^{\mu\nu} + \frac{2}{m^2} \left( I_{52}^{(1)} \left( I_{10}^{(1)} - I_{41}^{(1)} \right) u^{\alpha} u^{[\mu}_\alpha \omega^{\nu]}_\alpha \right) \right] - \frac{2}{m^2} I_{41}^{(1)} \left( u^{[\mu} \nabla^\alpha + u^\alpha \nabla^{[\mu} \right) \omega^{\nu]}_\alpha \right]. \end{align*}
\] (F6)
Using Eq. (E2), the above equation can further be written as

\[
u_\alpha \delta S^{\lambda,\mu\nu} = \frac{4\partial^2}{3} \tau_{eq} \left[ -\sinh \xi I_{10}^{(0)} \xi \omega_{\mu\nu} - \frac{2 \sinh \xi}{m^2} \left( I_{41}^{(1)} \left( u^\nu \omega^\nu \alpha \nabla \xi + u^\alpha \omega^\nu \left( \nabla \mu \right) \xi \right) + \xi \left( I_{40}^{(1)} u^\alpha u^\nu \omega^\nu \alpha + I_{41}^{(1)} \Delta^\alpha \mu \omega^\nu \alpha \right) \right) + \cosh \xi \left( I_{31}^{(1)} \beta \theta \omega_{\mu\nu} + I_{30}^{(1)} \beta \omega_{\alpha\nu} + \frac{2}{m^2} \beta I_{50}^{(1)} u^\alpha \omega^\nu \alpha + \frac{2}{m^2} I_{52}^{(1)} \left( \beta \omega \Delta^\alpha \mu \omega^\nu \alpha + \beta (\nabla \mu \omega^\alpha + \nabla \alpha \omega^\nu \mu) \right) \right) + \frac{2}{m^2} I_{51}^{(1)} \left( \beta \theta u^\alpha u^\nu \omega^\nu \alpha + \left( \beta \theta + \nabla^\alpha \beta \right) u^\mu \omega^\nu \alpha + \left( \beta \theta + \nabla^\nu \beta \right) \omega^\nu \alpha \right) + \left( I_{10}^{(0)} \theta \omega_{\mu\nu} + I_{10}^{(0)} \xi \omega_{\mu\nu} \right) \tan \xi - \frac{\beta I_{20}^{(0)} \omega_{\mu\nu}}{m^2} \right] + \frac{2}{m^2} \left\{ \tan \xi \left( I_{30}^{(0)} - 3I_{31}^{(0)} \right) \xi u^\alpha u^\nu \omega^\nu \alpha + \tan \xi I_{31}^{(0)} \left( \nabla \xi + \xi u^\alpha \right) \left( u^\nu \omega^\nu \lambda - \omega_{\mu\nu} u^\lambda + u^\alpha g^\lambda \mu \omega^\nu \alpha \right) - \left( \beta \xi - I_{40}^{(0)} - 3I_{41}^{(0)} \right) u^\alpha u^\nu \omega^\nu \alpha + \left( \nabla \beta + \beta u^\nu \omega^\nu \alpha \right) I_{41}^{(0)} u^\mu \omega^\nu \lambda - \beta I_{41}^{(0)} \omega_{\alpha\nu} + \left( \nabla \beta + \beta u^\mu \omega^\alpha \right) I_{41}^{(0)} u^\alpha \omega^\nu \lambda \right) + \left( I_{30}^{(0)} - 3I_{31}^{(0)} \right) \theta u^\alpha u^\nu \omega^\nu \alpha + \left( I_{30}^{(0)} - 2I_{31}^{(0)} \right) u^\mu u^\nu \omega^\nu \alpha \right) + I_{31}^{(0)} \left( \omega^\nu \nabla \omega^\lambda \alpha + u^\mu \nabla \omega^\nu \lambda \alpha - \theta \omega_{\mu\nu} + \omega_{\alpha \nu} \nabla \omega^\nu \alpha + u^\alpha \nabla \omega^\nu \alpha \right) \right\} - \frac{2}{m^2} I_{41}^{(1)} \left( u^\nu \omega^\alpha + u^\alpha \nabla \omega^\nu \right) \omega^\nu \alpha \right]. \tag{F7}
\]

Now using Eq. (81) we rewrite the above equation as

\[
u_\alpha \delta S^{\lambda,\mu\nu} = \frac{4\partial^2}{3} \tau_{eq} \left[ \omega_{\mu\nu} \left( -I_{10}^{(0)} \xi \sinh \xi + \frac{2}{m^2} I_{41}^{(1)} \xi \sinh \xi + I_{30}^{(1)} \beta \cosh \xi + I_{31}^{(1)} \beta \theta \cosh \xi \right) - \frac{2}{m^2} I_{51}^{(1)} \cosh \xi \beta \right]
- \frac{2}{m^2} I_{52}^{(0)} \beta \cosh \xi + I_{10}^{(0)} \xi \sinh \xi - I_{20}^{(0)} \beta \cosh \xi + I_{10}^{(1)} \cosh \xi - \frac{2}{m^2} I_{31}^{(0)} \xi \sinh \xi
+ \frac{2}{m^2} I_{41}^{(0)} \beta \cosh \xi \left( -I_{41}^{(0)} \sinh \xi + \frac{n_0 \tanh \xi}{\varepsilon_0 + P_0} I_{41}^{(1)} \cosh \xi + I_{31}^{(0)} \sinh \xi - \frac{n_0 \tanh \xi}{\varepsilon_0 + P_0} I_{41}^{(1)} \cosh \xi \right)
+ \frac{2}{m^2} \left( \nabla \xi \right) u^\nu \omega^\nu \alpha \alpha \left( -I_{41}^{(0)} \sinh \xi + \frac{n_0 \tanh \xi}{\varepsilon_0 + P_0} I_{41}^{(1)} \cosh \xi + I_{31}^{(0)} \sinh \xi - \frac{n_0 \tanh \xi}{\varepsilon_0 + P_0} I_{41}^{(1)} \cosh \xi \right)
+ \frac{2}{m^2} u^\alpha u^\nu \omega^\nu \alpha \alpha \left( -I_{40}^{(0)} \xi \sinh \xi + I_{41}^{(1)} \xi \sinh \xi + I_{50}^{(1)} \beta \cosh \xi + I_{51}^{(1)} \beta \theta \cosh \xi - I_{41}^{(0)} \beta \sinh \xi \right)
- I_{30}^{(0)} - 3I_{31}^{(0)} \xi \sinh \xi + 2I_{31}^{(0)} \xi \sinh \xi - \left( I_{40}^{(0)} - 3I_{41}^{(0)} \beta \cosh \xi - 2I_{41}^{(0)} \beta \cosh \xi + \left( I_{30}^{(0)} - 3I_{31}^{(0)} \right) \theta \cosh \xi \right)
+ \frac{2}{m^2} \left( \nabla \xi \omega^\alpha + \nabla \omega^\alpha \omega^\mu \right) \omega^\nu \alpha \left( I_{52}^{(1)} \alpha \cosh \xi + I_{31}^{(0)} \cosh \xi \right)
+ \frac{2}{m^2} \left( u^\nu \omega^\alpha + u^\alpha \nabla \omega^\nu \right) \omega^\nu \alpha \left( I_{31}^{(0)} \cosh \xi - I_{41}^{(0)} \cosh \xi \right). \tag{F8}
\]

From this equation it can be clearly seen that the coefficient of all the tensor objects on the right-hand side cancels out. Thus, we confirm that

\[
u_\alpha \delta S^{\lambda,\mu\nu} = 0. \tag{F9}
\]

**Appendix G: Calculation of \( \nu^\alpha \), \( \pi^{\alpha\beta} \) and \( \Pi \)**

By contracting Eq. (D16) with \( \Delta^\mu_\alpha \), the following expression for particle diffusion current can be obtained

\[
u^\alpha = \Delta^\alpha_\mu \delta N^\mu
= -4\tau_{eq} \left( \nabla^\alpha \xi \right) \cosh \xi F_{21}^{(1)} + 4\tau_{eq} F_{31}^{(1)} \left( \beta u^\alpha + \nabla^\alpha \beta \right) \sinh \xi. \tag{G1}
\]
Using Eq. (81), the above equation can be cast in the following simpler form

\[ \nu^{\alpha} = -4\tau_{eq} (\nabla^{\alpha} \xi) \left[ \cosh \xi I_{21}^{(1)} - \left( \frac{n_0 \tanh \xi}{\varepsilon_0 + P_0} \right) I_{31}^{(1)} \sinh \xi \right]. \]  

(G2)

Contracting Eq. (D23) with \( \Delta_{\mu\nu}^{\alpha\beta} \) yields

\[ \pi^{\alpha\beta} = \Delta_{\mu\nu}^{\alpha\beta} T^{\mu\nu} = \Delta_{\mu\nu}^{\alpha\beta} \left[ -4\tau_{eq} \sinh \xi \left( \nabla_{\rho} \xi + u_{\rho} \xi \right) \left( I_{30}^{(1)} u^{\mu} u^{\nu} u^\rho + I_{31}^{(1)} (\Delta^{\mu\nu} u^\rho + \Delta^{\rho\mu} u^{\nu} + \Delta^{\rho\nu} u^\mu) \right) \right. \]

\[ + 4\tau_{eq} \cosh \xi \left( \beta \nabla_\rho u_\lambda + \beta u_\rho u_\lambda + \alpha \nabla_{\rho} \beta + u_\lambda u_{\rho} \beta \right) \left( I_{40}^{(1)} u^{\lambda} u^{\mu} u^{\nu} u^\rho \right) \]

\[ + I_{41}^{(1)} (\Delta^{\mu\nu} u^\rho + \Delta^{\nu\mu} u^\rho + \Delta^{\mu\rho} u^\lambda u^\nu + \Delta^{\nu\rho} u^\lambda u^\mu + \Delta^{\rho\mu} u^\lambda u^\nu + \Delta^{\rho\nu} u^\lambda u^\mu) \]

\[ + \left. I_{42}^{(1)} (\Delta^{\mu\nu} \Delta^{\rho\lambda} + \Delta^{\nu\rho} \Delta^{\mu\lambda} + \Delta^{\rho\mu} \Delta^{\nu\lambda}) \right] \].

(G3)

Doing simple algebraic manipulations where Eqs. (C1)–(C6) are used, we find

\[ \pi^{\alpha\beta} = 8\tau_{eq} \cosh \xi \beta I_{42}^{(1)} \sigma^{\alpha\beta}, \]

where \( \sigma^{\alpha\beta} = \frac{1}{2} (\nabla^{\beta} u^{\alpha} + \nabla^{\alpha} u^{\beta} - \frac{2}{3} \Delta^{\alpha\beta} \nabla^\lambda u_\lambda) \) is the shear flow tensor. Thus, the bulk pressure \( \Pi \) can be expressed by the formula

\[ \Pi = -\frac{1}{3} \Delta_{\mu\nu} \delta T^{\mu\nu} = -\frac{1}{3} \Delta_{\mu\nu} \left[ -4\tau_{eq} \sinh \xi \left( \nabla_{\rho} \xi + u_{\rho} \xi \right) \left( I_{30}^{(1)} u^{\mu} u^{\nu} u^\rho + I_{31}^{(1)} (\Delta^{\mu\nu} u^\rho + \Delta^{\rho\mu} u^{\nu} + \Delta^{\rho\nu} u^\mu) \right) \right. \]

\[ + 4\tau_{eq} \cosh \xi \left( \beta \nabla_\rho u_\lambda + \beta u_\rho u_\lambda + \alpha \nabla_{\rho} \beta + u_\lambda u_{\rho} \beta \right) \left( I_{40}^{(1)} u^{\lambda} u^{\mu} u^{\nu} u^\rho \right) \]

\[ + I_{41}^{(1)} (\Delta^{\mu\nu} u^\rho + \Delta^{\nu\mu} u^\rho + \Delta^{\mu\rho} u^\lambda u^\nu + \Delta^{\nu\rho} u^\lambda u^\mu + \Delta^{\rho\mu} u^\lambda u^\nu + \Delta^{\rho\nu} u^\lambda u^\mu) \]

\[ + \left. I_{42}^{(1)} (\Delta^{\mu\nu} \Delta^{\rho\lambda} + \Delta^{\nu\rho} \Delta^{\mu\lambda} + \Delta^{\rho\mu} \Delta^{\nu\lambda}) \right] \].

(G5)

Using the relations defined in Eq. (C1) we obtain

\[ \Pi = 4\tau_{eq} \left[ I_{31}^{(1)} \xi \sinh \xi - \cosh \xi \left( I_{41}^{(1)} \hat{\beta} + \frac{5}{3} I_{42}^{(1)} \beta \nabla^\lambda u_\lambda \right) \right]. \]

(G6)

Now using the recurrence relation (B14) we can write

\[ I_{41}^{(1)} = I_{31}^{(1)} = -\frac{1}{\beta} (I_{20}^{(0)} - I_{21}^{(0)}) = -\frac{1}{\beta} (\varepsilon_0 + P_0), \]

(G7)

\[ I_{31}^{(1)} = I_{21}^{(0)} = -P_0 = -\frac{n_0}{\beta}. \]

(G8)

Substituting \( I_{41}^{(1)} \) and \( I_{31}^{(1)} \) from the above equations and the convective derivatives \( \hat{\xi} \) and \( \hat{\beta} \) from Eqs. (73) and (74) into Eq. (G6) the following result for the bulk pressure can be obtained

\[ \Pi = -4\tau_{eq} \left[ n_0 \left( \cosh \xi \sinh^2 \xi \left( \varepsilon_0 (P_0 + \varepsilon_0) - n_0 T \left( P_0 (z^2 + 3) + 3 \varepsilon_0 \right) \right) \right) \right. \]

\[ - \cosh \xi \left( \frac{n_0 (P_0 + \varepsilon_0) (P_0 \cosh^2 \xi + \varepsilon_0)}{n_0 T \cosh^2 \xi (P_0 (z^2 + 3) + 3 \varepsilon_0) - \varepsilon_0^2 \sinh^2 \xi} \right) + \left. \frac{5\beta}{3} I_{42}^{(1)} \right] \theta. \]

(G9)

[1] **STAR** Collaboration, L. Adamczyk et al., “Global A hyperon polarization in nuclear collisions: evidence for
the most vortical fluid," Nature 548 (2017) 62–65, arXiv:1701.06657 [nucl-ex].

[2] STAR Collaboration, J. Adam et al., “Global polarization of Λ hyperons in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV,” Phys. Rev. C98 (2018) 014910, arXiv:1805.04400 [nucl-ex].

[3] ALICE Collaboration, S. Acharya et al., “Measurement of spin-orbital angular momentum interactions in relativistic heavy-ion collisions,” Phys. Rev. Lett. 125 (2020) no. 1, 012301, arXiv:1910.14408 [nucl-ex].

[4] HADES Collaboration, F. Kornas et al., Lambda Polarization in Au+Au collisions at $\sqrt{s_{NN}} = 2.4$ GeV measured with HADES, talk given at the Strange Quark Matter, Bari, Italy, June 11-15, 2019.

[5] Z.-T. Liang and X.-N. Wang, “Globally polarized quark-gluon plasma in non-central A+A collisions,” Phys. Rev. Lett. 94 (2005) 102301, arXiv:nucl-th/0410079 [nucl-th]. [Erratum: Phys. Rev. Lett.96,039901(2006)].

[6] Z.-T. Liang and X.-N. Wang, “Spin alignment of vector mesons in non-central A-A collisions,” Phys. Lett. B629 (2005) 20–26, arXiv:nucl-th/0411101 [nucl-th].

[7] S. A. Voloshin, “Polarized secondary particles in unpolarized high energy hadron-hadron collisions?,” Phys. Rev. C73 (2006) 015204, arXiv:nucl-th/0505046 [nucl-th].

[8] B. Betz, M. Gyulassy, and G. Torrieri, “Polarization probes of vorticity in heavy ion collisions,” Phys. Rev. C76 (2007) 044901, arXiv:0708.0035 [nucl-th].

[9] F. Becattini, F. Piccinini, and J. Rizzo, “Angular momentum conservation in heavy ion collisions at very high energy,” Phys. Rev. C77 (2008) 024906, arXiv:0711.1253 [nucl-th].

[10] F. Becattini, L. Csernai, and D. J. Wang, “A polarization in peripheral heavy ion collisions,” Phys. Rev. C88 (2013) no. 3, 034905, arXiv:1304.4427 [nucl-th]. [Erratum: Phys. Rev.C93,no.6,069901(2016)].

[11] F. Becattini, V. Chandra, L. Del Zanna, and E. Grossi, “Relativistic distribution function for particles with spin at local thermodynamical equilibrium,” Annals Phys. 338 (2013) 32–49, arXiv:1303.3431 [nucl-th].

[12] F. Becattini and F. Piccinini, “The Ideal relativistic spinning gas: Polarization and spectra,” Annals Phys. 323 (2008) 2452–2473, arXiv:0710.5694 [nucl-th].

[13] F. Becattini, I. Karpenko, M. Lisa, I. Upsal, and S. Voloshin, “Global hyperon polarization at local thermodynamic equilibrium with vorticity, magnetic field and feed-down,” Phys. Rev. C95 (2017) no. 5, 054902, arXiv:1610.02506 [nucl-th].

[14] F. Becattini, G. Inghirami, V. Rolando, A. Beraudo, L. Del Zanna, A. De Pace, M. Nardi, G. Pagliara, and V. Chandra, “A study of vorticity formation in high energy nuclear collisions,” Eur. Phys. J. C75 (2015) no. 9, 406, arXiv:1501.04468 [nucl-th]. [Erratum: Eur. Phys. J.C75,no.5,535(2018)].

[15] I. Karpenko and F. Becattini, “Study of Λ polarization in relativistic nuclear collisions at $\sqrt{s_{NN}} = 7.7$ 200 GeV,” Eur. Phys. J. C77 (2017) no. 4, 213, arXiv:1610.04717 [nucl-th].

[16] Y. Xie, D. Wang, and L. P. Csernai, “Global Lambda polarization in high energy collisions,” Phys. Rev. C95 (2017) no. 3, 031901, arXiv:1703.03770 [nucl-th].

[17] L.-G. Pang, H. Petersen, Q. Wang, and X.-N. Wang, “Vortical Fluid and A Spin Correlations in High-Energy Heavy-Ion Collisions,” Phys. Rev. Lett. 117 (2016) no. 19, 192301, arXiv:1605.04024 [hep-ph].

[18] F. Becattini and I. Karpenko, “Collective Longitudinal Polarization in Relativistic Heavy-Ion Collisions at Very High Energy,” Phys. Rev. Lett. 120 (2018) no. 1, 012302, arXiv:1707.07984 [nucl-th].

[19] F. Becattini and M. A. Lisa, “Polarization and Vorticity in the Quark Gluon Plasma,” arXiv:2003.03640 [nucl-ex].

[20] STAR Collaboration, T. Niida, “Global and local polarization of Λ hyperons in au+au collisions at 200 gev from star,” in Global and local polarization of Λ hyperons in Au+Au collisions at 200 GeV from STAR, vol. 982, pp. 511–514. 2019. arXiv:1808.10482 [nucl-ex].

[21] STAR Collaboration, J. Adam et al., “Polarization of Λ (Λ) hyperons along the beam direction in au+au collisions at $\sqrt{s_{NN}} = 200$ gev,” Phys. Rev.Lett. 123 (2019) no. 13, 132301, arXiv:1905.11917 [nucl-ex].

[22] H. Li, H. Petersen, L.-G. Pang, Q. Wang, X.-L. Xia, and X.-N. Wang, “Local and global Λ polarization in a vortical fluid,” Nucl. Phys. A967 (2017) 772–775, arXiv:1704.03569 [nucl-th].

[23] H. Li, L.-G. Pang, and X.-L. Wang, Qun wand Xia, “Global Λ polarization in heavy-ion collisions from a transport model,” Phys. Rev. C96 (2017) no. 5, 054908, arXiv:1704.01507 [nucl-th].

[24] Y. Sun and C. M. Ko, “A hyperon polarization in relativistic heavy ion collisions from a chiral kinetic approach,” Phys. Rev. C96 (2017) no. 2, 024906, arXiv:1706.09467 [nucl-th].

[25] Y. Sun and C. M. Ko, “Azimuthal angle dependence of the longitudinal spin polarization in relativistic heavy ion collisions,” Phys. Rev. C99 (2019) no. 1, 011903, arXiv:1810.10359 [nucl-th].

[26] R.-h. Fang, L.-g. Pang, Q. Wang, and X.-n. Wang, “Polarization of massive fermions in a vortical fluid,” Phys. Rev. C94 (2016) no. 2, 024904, arXiv:1604.04036 [nucl-th].

[27] W. Florkowski, A. Kumar, R. Ryblewski, and R. Singh, “Spin polarization evolution in a boost invariant hydrodynamical background,” Phys. Rev. C99 (2019) no. 4, 044910, arXiv:1901.09655 [hep-ph].

[28] W. Florkowski, A. Kumar, R. Ryblewski, and A. Mazeliauskas, “Longitudinal spin polarization in a thermal model,” Phys. Rev. C100 (2019) no. 5, 054907, arXiv:1904.00002 [nucl-th].

[29] J.-H. Gao, G.-L. Ma, S. Pu, and Q. Wang, “Recent developments in chiral and spin polarization effects in heavy-ion collisions,” arXiv:2005.10432 [hep-ph].

[30] F. Li and S. Y. Liu, “Anomalous Lorentz transformation and side jump of a massive fermion,” arXiv:2004.08910 [nucl-th].

[31] S. Y. Liu, Y. Sun, and C. M. Ko, “Local spin polarizations in relativistic heavy ion collisions,” in 28th International Conference on Ultrarelativistic Nucleus-Nucleus Collisions. 2, 2020. arXiv:2002.11752 [nucl-th].

[32] S. Y. Liu, Y. Sun, and C. M. Ko, “Spin Polarizations in a Covariant Angular-Momentum-Conserved Chiral Transport Model,” Phys. Rev. Lett. 125 (2020) no. 6, 24
D. Montenegro and G. Torrieri, “Linear response theory for a finite-mass quark/antiquark and the thermal vorticity in relativistic heavy-ion collisions,” arXiv:2003.06545 [nucl-th].

Y. Ivanov, “Global polarization in heavy-ion collisions based on axial vortical effect,” arXiv:2006.14328 [nucl-th].

Y.-C. Liu and X.-G. Huang, “Anomalous chiral transports and spin polarization in heavy-ion collisions,” Nucl. Sci. Tech. 31 (2020) no. 6, 56, arXiv:2003.12482 [nucl-th].

X.-G. Huang, “Vorticity and Spin Polarization — A Theoretical Perspective,” arXiv:2002.07549 [nucl-th].

X.-G. Deng, X.-G. Huang, Y.-G. Ma, and S. Zhang, “Vorticity in low-energy heavy-ion collisions,” Phys. Rev. C 101 (2020) no. 6, 064908, arXiv:2001.01371 [nucl-th].

D. Montenegro and G. Torrieri, “Linear response theory of relativistic hydrodynamics with spin,” arXiv:2004.10195 [hep-th].

Y. B. Ivanov, V. Toneev, and A. Soldatov, “Estimates of hyperon polarization in heavy-ion collisions at collision energies $\sqrt{s_{NN}} = 4–40$ GeV,” Phys. Rev. C 100 (2019) no. 1, 014908, arXiv:1903.05455 [nucl-th].

F. Becattini, W. Florkowski, and E. Speranza, “Spin tensor and its role in non-equilibrium thermodynamics,” Phys. Lett. B789 (2019) 419–425, arXiv:1807.10994 [hep-th].

W. Florkowski, B. Friman, A. Jaiswal, and E. Speranza, “Relativistic fluid dynamics with spin,” Phys. Rev. C97 (2018) no. 4, 041901, arXiv:1705.00587 [nucl-th].

W. Florkowski, B. Friman, A. Jaiswal, R. Ryblewski, and E. Speranza, “Spin-dependent distribution functions for relativistic hydrodynamics of spin-1/2 particles,” Phys. Rev. D97 (2018) no. 11, 116017, arXiv:1712.07676 [nucl-th].

W. Florkowski, R. Ryblewski, and A. Kumar, “Relativistic hydrodynamics for spin-polarized fluids,” Prog. Part. Nucl. Phys. 108 (2019) 103709, arXiv:1811.04409 [nucl-th].

S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar, and R. Ryblewski, “Relativistic dissipative spin dynamics in the relaxation time approximation,” arXiv:2002.03937 [hep-ph].

N. Weickgenannt, E. Speranza, X.-l. Sheng, Q. Wang, and D. H. Rischke, “Generating spin polarization from vorticity through nonlocal collisions,” arXiv:2005.01506 [hep-ph].

E. Speranza and N. Weickgenannt, “Spin tensor and pseudo-gauges: from nuclear collisions to gravitational physics,” arXiv:2007.00138 [nucl-th].

K. Hattori, Y. Hidaka, and D.-L. Yang, “Axial Kinetic Theory and Spin Transport for Fermions with Arbitrary Mass,” Phys. Rev. D 100 (2019) no. 9, 096011, arXiv:1903.01653 [hep-ph].

D.-L. Yang, K. Hattori, and Y. Hidaka, “Quantum kinetic theory for spin transport: general formalism for collisional effects,” arXiv:2002.02612 [hep-ph].

K. Hattori, M. Hongo, X.-G. Huang, M. Matsuo, and H. Taya, “Fate of spin polarization in a relativistic fluid: An entropy-current analysis,” Phys. Lett. B795 (2019) 100–106, arXiv:1901.06615 [hep-th].

S. Shi, C. Gale, and S. Jeon, “Relativistic Viscous Spin Hydrodynamics from Chiral Kinetic Theory,” arXiv:2008.08618 [nucl-th].

M. Mathisson, “Neue mechanik materieller systemes,” Acta Phys. Polon. 6 (1937) 163–2900.

C. Itzykson and J. B. Zuber, Quantum Field Theory. International Series In Pure and Applied Physics. McGraw-Hill, New York, 1980. http://dx.doi.org/10.1063/1.2916419.

W. Florkowski, A. Kumar, and R. Ryblewski, “Thermodynamic versus kinetic approach to polarization-vorticity coupling,” Phys. Rev. C98 (2018) 044906, arXiv:1806.02616 [hep-ph].

F. W. Hehl, “On the Energy Tensor of Spinning Massive Matter in Classical Field Theory and General Relativity,” Rept. Math. Phys. 9 (1976) 55–82.

L. Tinti and W. Florkowski, “Particle polarization, spin tensor and the Wigner distribution in relativistic systems,” arXiv:2007.04029 [nucl-th].

S. R. De Groot, Relativistic Kinetic Theory. Principles and Applications. 1980.

W. Florkowski and R. Ryblewski, “Highly-anisotropic and strongly-dissipative hydrodynamics for early stages of relativistic heavy-ion collisions,” Phys. Rev. C83 (2011) 034907, arXiv:1007.0130 [nucl-th].

M. Martinez and M. Strickland, “Dissipative Dynamics of Highly Anisotropic Systems,” Nucl. Phys. A848 (2010) 183–197, arXiv:1007.0889 [nucl-th].