New prediction of elastic scattering cross sections ratio $R_{e^+e^-}$ based on phenomenological two-photon exchange corrections

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Abstract. The proton’s electric and magnetic form factors (FFs) and their ratio $R_p = \frac{G_{pE}(Q^2)}{G_{pM}(Q^2)}$ are fundamental and essential ingredients needed to parametrize the internal structure of the proton as well as many composite particles. However, the inconsistency in the results reported on these FFs and their ratio as measured by the Rosenbluth method and the polarization transfer method has led the community to believe that such a discrepancy is due to a systematic difference between the two methods. It was proposed that missing higher order radiative correction to the electron-proton elastic scattering cross section $\sigma_R(\epsilon, Q^2)$ and in particular two-photon exchange correction (TPE) should be applied in order to reconcile these measurements. The effect of TPE on electron-proton scattering observables was studied extensively in the last few years. However, the most direct method for measuring TPE contributions is the comparison of positron-proton and electron-proton elastic scattering cross sections ratio $R_{e^+e^-}(\epsilon, Q^2)$. In this proceedings, I present a new prediction of the ratio $R_{e^+e^-}$ as determined using a new parametrization of TPE corrections to $\sigma_R$. I then compare my results to several previous phenomenological extractions, TPE hadronic calculations, and previous and recent direct measurements of $R_{e^+e^-}$.

1. Introduction

The electric, $G_{pE}(Q^2)$, and magnetic, $G_{pM}(Q^2)$, form factors (FFs) of the proton, and their ratio $\mu_p R_p = \mu_p G_{pE}^p / G_{pM}^p$ are essential ingredients needed to parametrize the internal structure of the proton as well as many composite particles [1, 2]. Utilizing electron scattering, these FFs can be extracted primarily using two methods: the Rosenbluth separation method [3, 4], and the polarization transfer method [5]. In the Rosenbluth separation method, the unpolarized electron-proton cross section is measured, and the reduced cross section $\sigma_R(\epsilon, Q^2)$ for electron-proton scattering in the Born or one-photon-exchange (OPE) approximation is given by:

$$\sigma_R(\epsilon, Q^2) = \left[G_{pM}(Q^2)\right]^2 + \frac{\epsilon}{\tau}\left[G_{pE}(Q^2)\right]^2,$$

with $Q^2$ being the four-momentum transferred squared of the virtual photon with longitudinal polarization parameter $\epsilon$, $\tau = Q^2/4M_p^2$ is a kinematics factor, and $M_p$ is the mass of the proton. In the polarization transfer method, measurement of the spin dependent cross section is performed by simultaneous measurements of the transverse $P_t$ and longitudinal $P_l$ polarization.
components of the recoil proton. That allows for a direct extraction of the ratio \( R_p \) defined in the OPE approximation \([5]\) as:

\[
\mu_p R_p = \mu_p \frac{G_E^p}{G_M^p} = -\mu_p \frac{P_t (E + E')}{2M_p} \tan(\frac{\theta_e}{2}),
\]

with \( E \) and \( E' \) being the initial and final energy of the incident electron, respectively, and \( \mu_p \) is magnetic moment of the proton. The two methods yield strikingly different results on the ratio \( \mu_p R_p \) for \( Q^2 > 1.0 \) (GeV/c)\(^2\) with values differing almost by a factor of three at high \( Q^2 \). While the Rosenbluth separation method shows approximate FF scaling, \( \mu_p R_p \approx 1 \), with large uncertainties at high \( Q^2 \), the recoil polarization method, on the other hand, shows roughly a linearly decreasing uncertainties at high \( Q^2 \).

To reconcile these measurements, it was proposed that missing higher order radiative corrections to \( \sigma_R \), and in particular TPE contributions diagrams should be accounted for. This is done by adding the real function \( F(\varepsilon, Q^2) \), which represents the interference of the OPE and TPE amplitudes, to the Born reduced cross section \( \sigma_{\text{Born}} \) or: \( \sigma_R = \sigma_{\text{Born}} + F(\varepsilon, Q^2) \).

SeeRefs. \([6, 7, 8, 9, 10, 11]\) for detailed reviews. However, comparison of positron-proton and electron-proton Bremsstrahlung interference and conventional charge-independent radiative corrections to \( \mu \) and any deviation of \( \mu_p R_p \) with increasing \( Q^2 \), with some hint of flattening out above 5 (GeV/c)\(^2\).

In this section, I discuss the procedure together with the constraints and assumptions used to predict the ratio \( R_p \) to \( \sigma_R \). However, comparison of positron-proton and electron-proton Bremsstrahlung interference and conventional charge-independent radiative corrections to \( \mu \) and any deviation of \( \mu_p R_p \) with increasing \( Q^2 \), with some hint of flattening out above 5 (GeV/c)\(^2\).

2. Two-photon exchange parametrization

In this section, I discuss the procedure together with the constraints and assumptions used to predict the ratio \( R_{e^+e^-}(\varepsilon, Q^2) \) as determined based on a new parametrization of the TPE corrections to electron-proton elastic scattering cross section \( \sigma_R \). Guttmann et al., \([12]\) and based on the theoretical framework of Guichon and Vanderhaeghen \([6]\) expressed \( \sigma_R / G_{3p}^p \) and the ratio \(-\sqrt{\frac{\tau}{(1 + \varepsilon)}}\frac{P_t}{P_i} = -R_p \) in terms of the ratio \( G_E^p / G_M^p \), and the real parts of the TPE amplitudes relative to the magnetic FF or \( Y_M(\varepsilon, Q^2) \), \( Y_E(\varepsilon, Q^2) \), and \( Y_3(\varepsilon, Q^2) \) as:

\[
\frac{\sigma_R}{(G_M^p)^2} = 1 + \frac{\varepsilon}{\tau} \left( \frac{G_E^p}{G_M^p} \right)^2 + 2Y_M + \frac{\varepsilon}{\tau} \frac{G_E^p}{G_M^p} Y_E + 2\varepsilon \left( 1 + \frac{G_E^p}{\tau G_M^p} \right) Y_3 + O(\varepsilon^4),
\]

and

\[
-\sqrt{\frac{\tau}{2\varepsilon}} \frac{P_t}{P_i} = \frac{G_E^p}{G_M^p} + Y_E - \frac{G_E^p}{G_M^p} Y_M + \left( 1 - \frac{2\varepsilon}{1 + \varepsilon} \frac{G_E^p}{G_M^p} \right) Y_3 + O(\varepsilon).
\]

Assuming that the TPE correction is responsible mainly for the discrepancy on the ratio \( R_p \), and knowing that the recoil polarization data were shown to be independent of \( \varepsilon \) \([13]\), I constrain Eq. \((6)\) to its Born value or \( R_p = G_E^p / G_M^p \) by setting the \( \varepsilon \)-dependent term to zero,
allowing for the amplitude $Y_M(\varepsilon, Q^2)$ to be expressed in terms of $Y_E(\varepsilon, Q^2)$ and $Y_3(\varepsilon, Q^2)$ as:

$$Y_M = Y_E + (1 - \frac{2\varepsilon R_p}{1 + \varepsilon}) Y_3 / R_p.$$  

For the ratio $R_p$ and its associated uncertainty, I used the recent parametrization of Ref. [14]. Because $\sigma_R$ shows no (or weak) nonlinearity in $\varepsilon$, and to preserve the linearity of the Rosenbluth plots, I expand the amplitudes $Y_E$ and $Y_3$ as a second-order polynomial of the form $Y_E(\varepsilon, Q^2) = (\alpha_0 + \alpha_1 \varepsilon + \alpha_2 \varepsilon^2)$, and $Y_3(\varepsilon, Q^2) = (\beta_0 + \beta_1 \varepsilon + \beta_2 \varepsilon^2)$ to preserve as possible the linearity of the Rosenbluth plots as well as to account for any possible nonlinearities in the TPE amplitudes. Here $\alpha_i$ and $\beta_i$ ($i = 0, 1, 2$) are functions of $Q^2$ only. By imposing the Regge limit where the TPE correction to $\sigma_R$ vanishes in the limit $\varepsilon \rightarrow 1$ where $Y_E(1, Q^2) = Y_3(1, Q^2) = Y_M(1, Q^2) = 0$, we obtain the following constraints: $\alpha_0 = -(\alpha_1 + \alpha_2)$ and $\beta_0 = -(\beta_1 + \beta_2)$. Using the above discussed assumptions and constraints, $\sigma_R$ as given by Eq. (5) can now be written as:

$$\frac{\sigma_R}{(G_M^p)^2} = 1 + \frac{\varepsilon}{\tau} R_p^2 + \left[ \frac{2}{R_p} + \frac{2\varepsilon R_p}{\tau} \right] \left[ \alpha_1 (\varepsilon - 1) + \alpha_2 (\varepsilon^2 - 1) \right] + \left[ \frac{2}{R_p} \left( 1 - \frac{2\varepsilon R_p}{1 + \varepsilon} \right) + 2\varepsilon \left( 1 + \frac{R_p}{\tau} \right) \right] \left( \beta_1 (\varepsilon - 1) + \beta_2 (\varepsilon^2 - 1) \right),$$  

(7)

with $(G_M^p)^2$, $\alpha_1$, $\alpha_2$, $\beta_1$, and $\beta_2$ being the parameters of the fit, and are functions of $Q^2$ only. We can reduce the number of fitting parameters by fixing the value of $(G_M^p)^2$ in Eq. (7). This is done by realizing that the TPE correction to $\sigma_R$ vanishes in the limit $\varepsilon \rightarrow 1$ or:

$$\sigma_R(\varepsilon = 1, Q^2) = [(G_M^p)^2 + (G_D^p)^2/\tau].$$  

Also, because of the "experimentally" observed linearity of $\sigma_R$, we can fit $\sigma_R$, at a fixed $Q^2$, linearly to the form $\sigma_R = [a(Q^2) + \varepsilon b(Q^2)]$ and extract the constants $a(Q^2)$ and $b(Q^2)$. Equating the two forms of $\sigma_R$ at $\varepsilon = 1$, we get the value of $(G_M^p(Q^2))^2$ as:

$$[G_M^p(Q^2)]^2 = \frac{a(Q^2) + b(Q^2)}{(1 + R_p^2/\tau)}.$$  

(8)

For a fixed $Q^2$ value, and following Eq. (8), I fit the world data on $\sigma_R$ used in the analysis of Ref. [14] to extract $(G_M^p(Q^2))^2$ first. I then constrain the values of $(G_M^p(Q^2))^2$ and the ratio $R_p$ in Eq. (7), and fit the world data on $\sigma_R$ again to extract the TPE amplitudes coefficients $\alpha_i$ and $\beta_i$ ($i = 1, 2$). The coefficients $\alpha_{i(2)}$ and $\beta_{i(2)}$ are then used to determine the coefficients $\alpha_0$ and $\beta_0$ and the three TPE amplitudes. In this work, $\sigma_R$ measured at 93 $Q^2$ points up to $Q^2 = 4.0$ (GeV/c)$^2$ were used in the analysis with $\sigma_R$ measured at a minimum of five $\varepsilon$ points. The TPE amplitudes coefficients $\alpha_k$ and $\beta_k$ ($k = 0, 1, 2$) are found to be at the few-percentage-points level. Several functional forms were used to parametrize the $Q^2$ dependence of these TPE coefficients. The best fit was obtained when each TPE coefficient was parametrized as a second-order polynomial of the form: $a(\beta)(0,1,2)(Q^2) = (\alpha_0 + \alpha_1 Q^2 + \alpha_2 Q^4)$. See Refs. [15, 16, 17] for details.

3. Results and discussion

I use my new parametrization of the TPE corrections to electron-proton elastic scattering $F(\varepsilon, Q^2)$ discussed above to construct the ratio $R_{e^+e^-}(\varepsilon, Q^2)$ as defined in Eq. (4). Figure (1) shows the ratio $R_{e^+e^-}$ as a function of $\varepsilon$ as extracted from this work (solid black line). I also compare my predictions to several previous phenomenological extractions and fits from “Qattan-I” [20] (dashed magenta line), “Qattan-II” [14] (solid magenta line), “Arrington” [19] (dashed blue line), “ABGG” [18] (solid red line), “Bernauer” [21] (dashed-dotted black line), as well as TPE hadronic calculations “AMT” [22] (short-dashed red line), and to previous and recent direct measurements of the ratio $R_{e^+e^-}$ from Refs. [23, 27, 28, 29, 30, 31, 32, 33, 24, 25, 34, 35, 26]. For the world data, the measurements and the $Q^2$ value(s) in (GeV/c)$^2$ are listed in the figure.
Figure 1. The ratio $R_{e^+e^-}$ as a function of $\varepsilon$ as extracted from this work (solid black line) at the $Q^2$ values listed in the figure. Also shown are several previous phenomenological extractions and fits: Qattan-I [20] (dashed magenta line), Qattan-II [14] (solid magenta line), Arrington [19] (dashed blue line), ABGG [18] (solid red line), Bernauer [21] (dashed-dotted black line), and TPE hadronic calculations AMT [22] (short-dashed red line). The data points are direct measurements of $R_{e^+e^-}$ from Refs. [23, 27, 28, 29, 30, 31, 32, 33, 24, 25, 34, 35, 26]. Note that the measurements and the $Q^2$ value(s) are given in (GeV/c)$^2$. 
At low $Q^2$, my results suggest that the ratio $R_{e^+e^-}$ is below unity and behaves linearly with $\varepsilon$. My extractions deviate from the the Arrington, ABGG, and Qattan-I extractions as they predict a ratio above unity with strong nonlinearity at low $Q^2$ and low $\varepsilon$ as can be seen clearly in both the Arrington and the ABGG extractions. On the other hand, my extractions are in a very good agreement with both the Bernauer and the Qattan-II phenomenological extractions as well as with AMT hadronic TPE calculations with the Qattan-II extractions, however, exhibiting strong nonlinearity at low $\varepsilon$. Such a strong nonlinearity in $R_{e^+e^-}$ at low $Q^2$ is expected when the applied TPE correction is linear or roughly linear function times $(G_M^p)^2$. As $Q^2$ increases, the ratio $R_{e^+e^-}$ as extracted from this work starts to increase slowly, change sign (above unity), and behave linearly with $\varepsilon$. While my extractions are in generally good qualitative agreement with previous extractions and TPE hadronic calculations, they deviate from the ABGG fit which shows essentially no $Q^2$ dependence. My extractions are also in generally good agreement with existing world data including the very recent measurements from the CLAS collaboration [24], and VEPP-3 collaboration [25] with my predictions are slightly larger than the later measurement at $Q^2 = 1.0$ (GeV/c)$^2$. These two recent measurements have provided precise measurements of $R_{e^+e^-}$ at $Q^2 \approx 1.0$ and 1.5 (GeV/c)$^2$ with $R_{e^+e^-}$ exhibiting a clear $\varepsilon$ dependence at $Q^2$ values of $1.0-1.6$ (GeV/c)$^2$ consistent with the form factor discrepancy at these $Q^2$ values presenting an evidence for nonzero TPE effect at larger $Q^2$ values as well as a change of sign from the exact calculations at $Q^2 = 0$ (GeV/c)$^2$ [36]. On the other hand, the recent direct measurements by the OLYMPUS collaboration [26] at $Q^2$ values of $0.165-2.038$ (GeV/c)$^2$ are clearly below my predictions at larger $Q^2$ values, as well as TPE calculations and phenomenological extractions and fits shown. Finally, the majority of the world data on the ratio $R_{e^+e^-}$ are taken for $Q^2 < 2.1$ (GeV/c)$^2$ which is really below where the discrepancy on the ratio $\mu_p G_E^p/G_M^p$ is significant. Therefore, the assumption that TPE corrections could account for the discrepancy on the ratio $\mu_p G_E^p/G_M^p$ is still an open question that needs to be addressed, and clearly future measurements of the ratio $R_{e^+e^-}$ at $Q^2 > 2.1$ (GeV/c)$^2$ are highly recommended.

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