Study of Required Thrust Profile Determination of a Three Stages Small Launch Vehicle

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Abstract. The effect of solid rocket motor specifications, i.e. specific impulse and mass flow rate, and coast time on the thrust profile of three stages small launch vehicle is studied. Solid rocket motor specifications are collected from various small launch vehicle that had ever been in operation phase, and also from previous study. Comparison of orbital parameters shows that the radius of apocenter targeted can be approached using one combination of solid rocket motor specifications and appropriate coast time. However, the launch vehicle designed is failed to achieve the targeted orbit nor injecting the satellite to any orbit.

Keywords: thrust profile, small launch vehicle, specific impulse, mass flow rate, coast time

1. Introduction

One of the main factors in space exploration is the capability to transport objects from earth surface to specific positions in space or orbit. The development of launch vehicle plays key roles in providing this capability. Hence the technology to design, develop, and operate launch vehicle is obligatory.

Indonesia, as one the countries that is developing its capability for space exploration, has also an objective to develop launch vehicle for delivering payloads to orbits, in order to support the implementation of some maritime surveillance application in Indonesia. In order to achieve the objective, it is planned to develop a three stages solid small launch vehicle that capable of carrying 50 kg payload to low Earth orbit in 2019 [1]. Some studies related to the planned program has been carried out, one of them is the works by Utama [2] that explores on the design of multi-stage launch rocket and its trajectory control system. However, due to the limitations of the study, iteration of rocket motor specifications has not done yet.

The objective of this paper is to study the possibility to approximate reference trajectory, which is shaped by reference thrust profile. Approximation is done by varying the specifications of solid rocket motor, i.e. specific impulse and mass flow rate, and the duration of coast time. Various solid rocket motor specifications data are collected from various operated small launch vehicle and also from previous research by Utama [2,3]. Two-point boundary value problem developed by Tewari [4] is used to generate optimal trajectory of desired initial and final flight condition. The trajectory generated is used as reference trajectory. The deviation of trajectory to the reference leads a study of how coast time duration affect the flight, as Pontani and Teoffilato [5] use this method to evaluate launch vehicle flight performance. To determine the orbit achieved by the launch vehicle, the orbit parameters, i.e. semi-major axis and eccentricity, are calculated from terminal condition of position, velocity, and flight path angle.
2. Launch Vehicle Configurations and Modelling

2.1. Multistage Rocket

In this paper we distinguish all nomenclature system, except the one used by Cornelisse [6]. The term sub-rocket define a complete rocket vehicle consisting of one or more stages together with the payload and guidance control system. Besides, the term stage defines a complete propulsion unit together with control equipment, which is discarded completely when staging process has occurred. Here is the example of three stages rocket. The total mass of rocket $m_0$ is divided into payload mass $m_u$, useful propellant mass $m_p$, structural mass $m_c$. In case of three-stages rocket with, the formula of mass should be as following.

$$
m_{01} = m_{c1} + m_{p1} + m_{02}
$$

$$
m_{02} = m_{c2} + m_{p2} + m_{03}
$$

$$
m_{03} = m_{c3} + m_{p3} + m_u
$$

(1)

![Figure 1. Nomenclature for a three-stages rocket [6]](image)

2.2. Mass Parameters of Rocket

Given mass parameters of rocket [6] which has range, for single stage rocket in general, presented in Table 1. Here are ratio of masses defined, i.e. Mass Ratio $MR$, Payload Ratio $PR$, Structural Efficiency $SE$, and Propellant Ratio $PrR$. The payload mass, structural mass, and propellant mass of rocket should fullfil the range.

| Parameters | $PR$ | $PrR$ | $SE$ | $MR$ |
|------------|------|-------|------|------|
| Formula    | $PR = m_u/m_0$ | $PrR = m_p/m_0$ | $SE = m_c/(m_c + m_p)$ | $MR = m_0/m_e$ |
| Range      | $0,01 < PR < 0,2$ | $0,5 < PrR < 0,9$ | $0,2 < SE < 0,5$ | $2 < MR < 10$ |

2.3. Solid Rocket Motor

The thrust profile of launch vehicle depends on the rocket motor specifications. A rocket motor, which generates thrust during burning period, delivers a total impulse $I$. The term specific impulse $I_p$ is used to define impulse per unit weight of propellant. Mass flow rate $\dot{m}$ is the rate of propellant mass flowing through the nozzle. Often, steady-state mass flow assumption is used in convergent-divergent nozzle
analysis to deal with ideal motor case [6]. In this case the density, flow speed, and cross-sectional area of exit nozzle remains constant.

\[ I_{sp} = \int_{0}^{t_b} F(t) dt / (m_p g_0) \]  

\[ \dot{m} = \rho V A = constant \]  

Consider one solid rocket motor with its propellant has one value of specific impulse \( I_{sp} \) and constant mass-flow rate, the thrust should be continuous and cannot be throttled over burning period. Thrust \( T \) can be calculated in terms of specific impulse, mass-flow rate, and sea-level gravity acceleration \( g_0 \) as:

\[ T = \dot{m} g_0 I_{sp} \]  

Various solid rocket motor specifications data, referred to previous research and operated small launch vehicle [2,3], are compiled in Table 2. In this study, we consider only the specific impulse and mass-flow rate.

**Table 2. Various Solid Rocket Motor Specifications per Stage [2, 3]**

| Stage | SRM | \( I_{sp} \) (s) | \( \dot{m} \) (kg/s) |
|-------|-----|-----------------|---------------------|
| 1     | 1A  | 192             | 79.719              |
|       | 1B  | 240             | 128.46              |
|       | 1C  | 253             | 202.71              |
|       | 1D  | 263             | 380.228             |
|       | 1E  | 286             | 573.213             |
|       | 2A  | 193             | 41.2                |
|       | 2B  | 203             | 625.961             |
|       | 2C  | 250             | 35.184              |
|       | 2D  | 250             | 85.888              |
|       | 2E  | 255             | 48.052              |
|       | 2F  | 267             | 102.041             |
|       | 2G  | 280             | 178.571             |
|       | 2H  | 285             | 173.613             |
| 3     | 3A  | 211             | 14.218              |
|       | 3B  | 258             | 3.080               |
|       | 3C  | 260             | 120                 |
|       | 3D  | 277             | 33.412              |
|       | 3E  | 278             | 10.021              |
|       | 3F  | 280             | 89.286              |
|       | 3G  | 292             | 41.306              |
|       | 3H  | 301             | 99.668              |

2.4. Coast Time

Coast time \( t_c \) defined as time between burnout of \( n^{th} \) stage to ignition of \( (n+1)^{th} \) stage. In coast time period, there is no thrust generated because the burning does not occurred. Thus, the gravity turn will change the flight path direction, and thus convert the potential energy to kinetic energy [5].
2.5. Equation of Motion

The ascent of launch vehicle from planetary surface through the atmosphere can be modelled as flight in vertical plane. It is usually follows a gravity-turn trajectory, where the angle of attack $\alpha$ is maintained close to zero, leading to negligible lift force. The duration of launch vehicle flight is generally small enough to neglect the effects of planetary rotation. In this study the plane of Earth is assumed to be flat. As a point mass, the equation of motion are written as follows:

\[
\dot{x} = V \cos \gamma \\
\dot{h} = V \sin \gamma \\
\dot{V} = u_1 - g \sin \gamma \\
\dot{\gamma} = (u_2 + (V^2/r - g) \cos \gamma)/V
\]
\[ u_1 = \frac{(T - D)}{m} \]  
\[ u_2 = \frac{T_c}{m} \]  
\[ \dot{m} = -\dot{m}_p \]

Figure 4. Flight in Vertical Plane

with state variables flight path angle \( \gamma \), velocity \( V \), and radius \( r \), input variables thrust \( T \) and input control thrust \( T_c \), gravity acceleration \( g \), and drag \( D \). In this study, the drag coefficient is assumed to be constant due to short duration of flight in atmosphere. If the shape of nose cone is a half-round, the \( C_D \) should be around 0.5 [3]. The drag is formulated as :

\[ D = 0.5 D = 0.5 \rho V^2 S C_D \]

2.6. Orbital Mechanics

In gravitational condition, the two-body problem assumption is used [7]. Gravity force exerted on the body is the function of radius from the inertial frame. On the other hand, Velocity of body on orbit is obtained using Vis-Viva Integral of energy. The formula is as follows :

\[ g = -\frac{\mu}{r^2} \]  
\[ V = \sqrt{\frac{\mu}{2/r - 1/a}} \]

To determine the orbit achieved by the launch vehicle, parameters of orbit, i.e. semi major axis, angular momentum, and eccentricity are calculated in terms of final altitude, velocity, and flight path angle. The formulas are as follows [8].
\[ a = \left( \frac{2}{r} - \frac{V^2}{\mu} \right)^{-1} \] (15)

\[ |h| = |r \times V| = |r||V| \sin \left( \frac{\pi}{2} - \gamma \right) \] (16)

\[ 1 - e^2 = \frac{|h|^2}{\mu} \left( \frac{2}{r} - \frac{V^2}{\mu} \right) \] (17)

2.7. Payload and Mission Specifications

In this study, the three-stages solid launch vehicle is adopted from previous research by Utama [2], where the launch vehicle has total mass 12,800 kg. The allocation of total mass from 1st stage to 3rd stage is 91%, 7.5%, and 1.5%, which are determined to fulfill the mass parameters of rocket [6] and has structural efficiency assumed to be 0.2. The payload mass is 50 kg. The launch vehicle is targeted to inject the payload to low earth orbit with semi major axis \( a = 6528.14 \) km. The flight begins from the ground, with initial velocity at lift-off is 0.001 km/s and elevation angle 70° [2]. The satellite is planned to be injected from apocenter \( r_a = 6578.14 \) km. In order to get into the orbit, the velocity of satellite injected should be 7.75439 km/s and the final flight path angle should be 0°.

Table 3. Mission Specifications

| Parameters | \( r \) (km) | \( V \) (km/s) | \( \gamma \) (°) | \( |h| \) (km²/s) | \( a \) (km) | \( e \) | \( r_p \) (km) |
|------------|--------------|----------------|----------------|----------------|-------------|-----|--------------|
| Initial    | 6378.14      | 0.001          | 70             | -              | -           | -   | -            |
| Target/final | 6578.14      | 7.75439        | 0              | 51009.48       | 6528.14     | 0.0077 | 6478.14      |

3. Thrust Profile Shaping

3.1. Method Description

![Flow Chart of Method](Figure 5)
The method of thrust profile shaping is illustrated in Figure 5. The reference trajectory is generated from flight simulation of launch vehicle using optimal longitudinal acceleration from 2PBVP solution. Approximation to reference trajectory, and thus the reference thrust profile, is done by considering the solid rocket motor and the duration of coast time. To determine the orbit achieved by the launch vehicle, the orbit parameters, i.e. angular momentum, semi-major axis, and eccentricity, are calculated from terminal conditions. The terminal conditions considered are position, velocity, and flight path angle.

3.2. Reference Trajectory

The reference trajectory generated in this paper is obtained from two-point boundary value problem (2PBVP) solution developed by Tewari [4]. It is done by running bvp4c function in MATLAB [9]. The longitudinal acceleration \( u_1 \) is the optimized input. Optimization is based on quadratic cost \( L = u^2 \) and without terminal penalty, which yields the Hamiltonian equation \( H \). The necessary condition of the derivation of Hamiltonian equation by input variables yields the extremal longitudinal acceleration input. Below presented the state and co-state equation, \( f \), and the optimal longitudinal acceleration input, \( u_1^* \).

\[
H = L + \lambda f \tag{18}
\]

\[
dH/du = dL/du + \lambda df/du = 0 \tag{19}
\]

\[
f = \begin{pmatrix}
\dot{r} \\
\dot{\psi} \\
\dot{\gamma} \\
\dot{\lambda}_2 \\
\dot{\lambda}_3
\end{pmatrix} = \begin{pmatrix}
v \sin \gamma \\
u_1 - (\mu/r^2) \sin \gamma \\
(v/r - (\mu/(r^2 v))) \cos \gamma \\
-2(\mu \sin \gamma) \lambda_3/r^3 \\
-(\sin \gamma) \lambda_2
\end{pmatrix} \tag{20}
\]

\[
u_1^* = -\lambda_3/(2R_1) \tag{21}
\]

The function bvp4c required time of flight \( t_f \) as the time interval \( 0 \leq t \leq t_f \) and boundary condition from table 2. The time of flight \( t_f = 555 \) s is chosen as it generates optimal trajectory with initial value of flight path angle 70.01\(^\circ\), which is considered as initial condition of the flight in this study. The optimal trajectory defined by optimal flight state variables and input is depicted in Figure 6. Note that the longitudinal acceleration input will determine the thrust optimal needed as the launch vehicle mass is known during the simulation.
Figure 6. (a) Optimal Longitudinal Acceleration Input, (b) Optimal Flight Path Angle, (c) Optimal Velocity, (d) Optimal Heights solutions by two-point boundary value problem

4. Flight Simulation and Analysis

4.1. Simulation Platform

To perform flight simulations, a flight simulation platform is built in Simulink environment. The Simulink model consists of equations of motion depicted in Figure 7.

Figure 7. Equation of Motion in Simulink
4.2. Simulating Reference Trajectory

By using optimal longitudinal acceleration input from 2PBVP and initial guess of specific impulse and mass flow rate, the reference trajectory is obtained from flight simulation. The reference thrust profile is also generated. The orbital parameters achieved are presented in Table 3.

| Parameters | $r$ (km) | $V$ (km/s) | $\gamma$ (°) | $\dot{h}$ (km$/s$) | $a$ (km) | $e$   | $r_p$ (km) |
|------------|---------|-----------|---------------|-----------------|-------|------|-----------|
| Target     | 6578.14 | 7.75439   | 0             | 51009.48        | 6528.14 | 0.0077| 6478.14   |
| Reference  | 6580.793| 7.75276   | 0.21          | 51018.96        | 6530.656| 0.0085| 6575.208  |

Reference trajectory could be shaped only if the thrust profile is similar to the reference. The reference thrust generated could be achieved if the launch vehicle equipped with liquid rocket motor. Thus, in this section considerations of solid rocket motor and coast time are done to approximate the the reference trajectory, and thus the reference thrust profile.

![Reference Trajectory](image1)

**Figure 8. Reference Trajectory**

![Reference Thrust Profile](image2)

**Figure 9. Reference Thrust Profile**

4.3. Solid Rocket Motor Specifications Consideration

The steps begin with choosing the specifications of 1st stage rocket motor, followed by the 2nd stage and the 3rd stage. The consideration is based on the trajectories formed by three-stages launch vehicle with
various specifications of solid rocket motors from Table 2. The trajectories and thrust plotted in Figure 11 until Figure 16 are analysed in order to get the nearest one to the reference.

**Figure 10.** Flow Chart of Solid Rocket Motor Specifications consideration

**Figure 11.** Trajectories After 1st Stage Burnout

**Figure 12.** Trajectories After 2nd Stage Burnout
For the 1st stage, solid rocket motor with code 1A, 1B, and 1C could not propel the launch vehicle during the burning time. On the other hand, code 1E propels the launch vehicle to the higher altitude than the reference and code 1D trajectory at the same ground distance. Thus, code 1D is chosen as solid rocket motor for 1st stage. For the 2nd stage and 3rd stage, code 2B and 3C are chosen as it increases the altitude of launch vehicle less than the other solid rocket motor.

Figure 13. Trajectories After 3rd Stage Burnout

Figure 14. Thrust Profile Variation of 1st Stage

Figure 15. Thrust Profile Variation of 2nd Stage
Figure 16. Thrust Profile Variation of 3rd Stage

Thrust of each stage plotted in Figure 14 until Figure 16 shows how the thrust affecting the trajectory shaped. 1st stage with specifications code 1A, 1B, and 1C could not propel the launch vehicle along burning time because the thrusts provided are lower than the reference thrust, and thus the impulses generated could not fulfill the minimum to propel the launch vehicle. Thrust generated by code 1D is actually higher than the reference, but it is lower than the code 1E. Thrust generated by code 2B and 3C are the highest among each stage, but the duration is the shortest. This condition leads to the increment of altitude less than the others. Thus, the chosen combination of solid rocket motor specifications is 1D+2B+3C.

4.4. Flight simulation

After choosing the combination of solid rocket motor, flight simulation is conducted in 555 s as the reference trajectory takes 555 s to achieve the orbit. The trajectory and thrust profile of launch vehicle is plotted along the reference. The final conditions and the orbital parameters are presented in Table 4.

Figure 17. Trajectory of Launch Vehicle
Figure 18. Thrust Profile of Launch Vehicle

Table 5. Orbital Parameters achieved by Launch Vehicle

| Parameters | \( r \) (km) | \( V \) (km/s) | \( \gamma \) (°) | \( |h| \) (km²/s) | \( a \) (km) | \( e \) | \( r_p \) (km) | \( r_a \) (km) |
|------------|-------------|---------------|-------------|----------------|--------|-----|------------|------------|
| Target     | 6578.14     | 7.75439       | 0           | 51009.48       | 6528.14 | 0.0077 | 6478.14    | 6578.14    |
| Reference  | 6580.79     | 7.75276       | 0.21        | 51018.96       | 6530.66 | 0.0085 | 6575.21    | 6586.10    |
| Coast 0 s  | 7068.65     | 3.27162       | -8.7        | 22860.59       | 3904.92 | 0.8150 | 722.37     | 7087.48    |

From the flight simulation of launch vehicle without coast time, the launch vehicle fly through a trajectory higher than the reference. From the data presented in Table 4, the radius of pericenter achieved is 722.367 km, which is lower than radius of Earth (6378.14 km). In this case the launch vehicle will fly into the atmosphere and crashing the ground. On the other hand, the radius in final condition is 7068.646 km, higher than the orbit target or reference. Also, the final velocity is slower than the velocity targeted. This shows that it is important to change the direction of flight to convert potential energy to kinetic energy, which can be done by gravity turn. Thus, for the following flight simulation, the duration of coast time after burnout of 1\textsuperscript{st} stage is adjusted.

4.5. Coast Time Consideration

To get the optimal coasting duration after burnout of 1\textsuperscript{st} stage, various coast time are considered. Then the radius of pericenter and radius of apocenter are compared, and the nearest one to the reference is chosen as the appropriate coast time. The orbital parameters of launch vehicle with various coast time are presented in Table 5. The radius of pericenter and apocenter of various coast time plotted in Figure 16.

Table 6. Orbital Parameters with Various Coast Time

| Parameters | \( r \) (km) | \( V \) (km/s) | \( \gamma \) (°) | \( |h| \) (km²/s) | \( a \) (km) | \( e \) | \( r_p \) (km) | \( r_a \) (km) |
|------------|-------------|---------------|-------------|----------------|--------|-----|------------|------------|
| Target     | 6578.14     | 7.75439       | 0           | 51009.48       | 6528.14 | 0.0077 | 6478.14    | 6578.14    |
| Reference  | 6580.79     | 7.75276       | 0.21        | 51018.96       | 6530.66 | 0.0085 | 6575.21    | 6586.10    |
| Coast 0 s  | 7068.65     | 3.27162       | -8.7        | 22860.59       | 3904.92 | 0.8150 | 722.37     | 7087.48    |
| Coast 10 s | 8116.16     | 4.74713       | 26.9        | 34345.3        | 5266.34 | 0.6619 | 1780.746   | 8751.93    |
| Coast 30 s | 8054.67     | 5.03499       | 26.1        | 36425.83       | 5414.11 | 0.6206 | 2053.99    | 8774.22    |
| Coast 50 s | 7885.11     | 5.15608       | 22.9        | 37459.34       | 5349.13 | 0.5847 | 2221.43    | 8476.83    |
| Coast 80 s | 7623.65     | 5.37042       | 17.8        | 38976.27       | 5263.58 | 0.5253 | 2498.68    | 8028.48    |
| Coast 110 s| 7355.62     | 5.62449       | 12.5        | 40386.7        | 5193.84 | 0.4606 | 2801.64    | 7586.04    |
| Coast 150 s| 6995.39     | 6.01810       | 5.1         | 41930.21       | 5127.14 | 0.3738 | 3210.68    | 7043.59    |
| Coast 200 s| 6566.79     | 6.56428       | -4.4        | 42978.21       | 5090.1 | 0.2993 | 3566.48    | 6613.72    |
From Figure 16, the highest radius of pericenter occurred when the duration of coasting is 214 s. The radius of pericenter is 3593.32 km, thus can be concluded as lower than the target and reference. This also shows that the launch vehicle will fail to inject the satellite into the orbit target nor bring the satellite orbiting the Earth. The radius of apocenter of reference and target can be approximated, which is 6585.4 km.

4.6. Analysis

Section 4.3, i.e. choosing the solid rocket motor specifications, shows that the altitude achieved is based on the specifications chosen. Higher mass flow rate will lead to shorter duration of burning, thus the altitude will be lower at burnout. Compared to the reference trajectory, the altitude of launch vehicle rises faster with 1st stage rocket motor specifications chosen (code 1D). Thus, it is reasonable to choose
the 2nd and 3rd stage rocket motor specifications (code 2B and 3C) which increases the altitude less than by the other codes.

By simulating the launch vehicle without coast time, it can be seen from Figure 14 that the launch vehicle will achieve altitude higher that the apocenter of the target (shown by blue marker). This happens because the thrust generated is higher than the reference thrust. It means the impulse is higher than the reference and propel the launch vehicle to a higher trajectory. It is also shown from Table 3 that the radius of pericenter is 722.37 km, which is lower that Earth’s radius. It means that the launch vehicle and the satellite will crash the ground and counted as a failed mission. It is necessary to change the direction of flight, which can be done naturally by gravity turn.

Figure 17 shows that there is change of direction of flight by using coast time. This means that gravity turn effect will be dominant in a situation where there is no thrust generated, and thus convert the potential energy to kinetic energy. From Figure 18, the thrust profile of launch vehicle is adjusted by the coast time, thus in the coast phase the trajectory following gravity turn trajectory. Unfortunately, in this case the highest radius of pericenter is 3593.32 km. It shows that the launch vehicle with the chosen specifications of rocket motor and adjusted coast time could not inject the satellite to any orbit.

5. Conclusions and Future Work

In this paper, a study of thrust profile shaping of small launch vehicle based on solid motor specifications and coast time cosideration was carried out. Varying the solid rocket motor configuration of each stage yields the variety of altitude achieved. The consideration of choosing solid rocket motor code 1D for the 1st stage is done based on the altitude achieved is slightly higher than the reference trajectory, but lower that the code 1E. Solid rocket motor code 2B and 3C are chosen for 2nd and 3rd stage to overcome the
higher altitude achieved. Applying the appropriate coast time yields the higher radius of pericenter, which means that the direction of flight is changing as effect of gravity turn become dominant. This lead to the lower altitude achieved.

For the future work, it is necessary to do some steps. First, the design optimization is using existing multidisiplinary optimization method to solve the design problems. The methods chosen must be applicable to launch vehicle design problems. Second, the propellant grain should be designed in order to generate mass flow rate in which the thrust generated is approximating the reference thrust. Third, configuration of mass of the launch vehicle should be iterated to find the optimal one.

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