Solving the Cosmological Moduli Problem with Weak Scale Inflation

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Abstract

Many models of supersymmetry breaking involve particles with weak scale mass and Planck mass suppressed couplings. Coherent production of such particles in the early universe destroys the successful predictions of nucleosynthesis. We show that this problem may be solved by a brief period of weak scale inflation. Furthermore the inflaton potential for such an inflation naturally arises from the same assumptions which lead to the cosmological problem. Successful baryogenesis and preservation of density fluctuations for large scale structure formation are also possible in this scenario.

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1. Introduction

An often discussed problem with Planck scale physics is the difficulty in deriving definite generic low energy predictions. Nonetheless, high energy scales are indirectly accessible through their effects on cosmological evolution. Although there are many uncertainties in this evolution, at least two constraints seem fairly reliable. First, the remaining mass density in stable particles should not “over-close” the universe given the bounds on its age. Second, the cosmology should allow for a successful theory of nucleosynthesis.

In a recent paper, Banks, Kaplan, and Nelson [1], in a reincarnation of the well known Polonyi problem [2], have shown the relevance of the second of these constraints to models of dynamical supersymmetry breaking, including those motivated by superstring theory. Many string models include directions in field space which are flat in the supersymmetric limit and couple to light fields only through Planck scale suppressed interactions. If the potential for these flat directions is stabilized by the same physics responsible for supersymmetry breaking, a particle with mass of order a TeV and dangerously long lifetime results. Possible examples of such particles are the dilaton of string theory, the massless gauge singlets of string compactifications, a Planck scale coupled singlet responsible for supersymmetry breaking, or a singlet field responsible for communicating supersymmetry breaking to the visible sector. In this paper, we refer to these fields collectively as moduli. If one of the moduli starts with a Planck scale expectation value, coherent oscillations about the minimum of the potential dominate the energy density of the universe at an early epoch. Under naturalness assumptions for the couplings, one finds that after decay of the moduli, the temperature of the universe is too low to allow standard nucleosynthesis. The conclusion is that the mass or couplings of these fields is not consistent with naturalness assumptions or that they are excluded [1,2].

In this paper, we investigate how this constraint may be avoided. We will show that the moduli problem can be solved by a period of inflation with Hubble
constant of order of the weak scale. Furthermore, such a period of inflation is to be expected under similar assumptions to those which give rise to the cosmological problem because one of the moduli or a flat direction in the visible sector can act as the inflaton. The resulting Hubble constant during inflation is dictated by supersymmetry breaking, and is naturally of order the weak scale. Additional mass scales are therefore not introduced into the theory. Furthermore, the same assumptions which yield the dangerous moduli yield potentials of the form required for either a “new” or a “chaotic” inflationary potential. However, unlike the dangerous moduli fields, the field driving inflation is not necessarily weakly coupled. The reheat temperature after inflation can then be much larger than that for the dangerous moduli. In fact, very natural choices for the inflaton parameters reduce the moduli amplitude sufficiently and generate an acceptable reheat temperature. We interpret our result as indicating that cosmological constraints cannot be used to eliminate models containing these Planck scale coupled moduli.

The structure of the paper is as follows. We first review the cosmological problem of Planck scale coupled singlet fields connected with the supersymmetry breaking sector, as discussed in Ref. [1]. We discuss the various potential cosmological problems associated with these TeV scale late decaying particles. We then discuss potential ways to avoid these problems. Our primary focus will be inflation, and we discuss the necessary properties of an inflationary potential which can resolve the cosmological problems. We conclude that a period of relatively late inflation, with the Hubble expansion parameter of order the weak scale, readily solves the problem. In Section 4, we discuss the naturalness of the required inflationary scenario. We show that such a period of inflation might be expected under the same assumptions which imply the existence of dangerous moduli fields. Finally, we show our inflationary scenario could naturally preserve density fluctuations necessary to generate the structure of the universe if the duration of inflation is sufficiently short.
2. The Problem

Supersymmetry provides a natural solution to the large hierarchy between the weak and GUT or Planck scales. In hidden sector models supersymmetry breaking is transmitted to the low energy visible sector through Planck scale suppressed interactions. In nonrenormalizable hidden sector models \[1\], supersymmetry breaking also vanishes when \(M_p \to \infty\). In exact supersymmetric theories “flat” directions in field space for which the potential vanishes are common. If the potential for these “flat” directions is generated through the physics associated with supersymmetry breaking, the potential takes the form

\[
\mathcal{V}(\Phi) = \frac{m^2_3}{2} \frac{G(|\Phi|/M)}{M^2}
\]  

(2.1)

where \(M \equiv M_p/\sqrt{8\pi}\) is the reduced Planck mass, \(m_{3/2}\) is the gravitino mass, \(\Phi\) represents a flat direction and \(G\) is some function. The potential for the dangerous directions vanishes in the flat space limit, \(M \to \infty\), since \(m_{3/2} \to 0\) in this limit. Excitations about the minimum of the potential, \(\Phi_{\text{min}}\), have mass \(m_{\Phi} \sim m_{3/2}\). Notice the essential assumptions are that the potential vanishes in the flat space limit and is proportional to \(m^2_{3/2}\). In reality, it is difficult to construct models for supersymmetry breaking of this sort. However, we will show is that the assumed form of the potential not only leads to cosmological problems for moduli fields, but can also yield an inflaton potential which naturally solves the problem.

Although the potential for flat directions vanishes for \(M \to \infty\), due to the nonrenormalization theorems of supersymmetry, this is not necessarily true of the couplings. This means that the couplings of flat directions to the low energy visible sector can be large, even though all terms in the potential (2.1) are suppressed by powers of \(M\). For example, \(D\)-flat directions in the squark fields have Yukawa couplings to other light fields. However, in hidden sector models it is possible for flat directions which are singlets under the low energy gauge group couple to other fields only through Planck mass suppressed interactions. Examples of such fields include the dilaton and compactification moduli [3] of string theory, and singlet
fields responsible for breaking supersymmetry or communicating supersymmetry breaking \[1\]. These are the potentially dangerous moduli fields.

There are several types of Planck mass suppressed couplings which might be present. The moduli can couple either to gauge fields, chiral fields of a hidden sector, or chiral fields of the visible sector. By assumption, the coupling in each case is suppressed by the Planck mass, but may be further suppressed. For example, the coupling to gauge bosons might arise only at one loop, due to an anomaly. Decay to fermions through a current interaction is suppressed by the fermion mass. Decay through an explicit helicity changing operator will have similar suppression. The decay to scalars can be large but turns out to be suppressed by numerical factors. We conclude that an estimate for the decay rate of the moduli is at most

\[
\Gamma \sim \frac{m^3_{\phi}}{8\pi M^2},
\]

although the rate might be even smaller. The slow decay rate for the moduli is the source of the cosmological problem.

As discussed below, relic moduli produced in the very early universe survive to a dangerously late epoch. These relic moduli could have arisen from two distinct sources. First, coherent oscillations can give rise to a Bose condensate for the scalar component. When the Hubble constant, \( H \), reaches a value \( 3H \sim m_{\Phi} \), the scalar component begins to oscillate coherently about the minimum of the potential. The energy stored in this oscillation subsequently redshifts like matter and can eventually dominate the energy density. If the universe is radiation dominated when the oscillations begin, the ratio of moduli number density in the condensate, \( n_{\Phi} = \frac{1}{2} m_{\Phi} \Phi^2 \), to entropy density, \( s = (2\pi^2/45)g_{*}T^3 \), where \( T \) is the temperature when \( 3H \sim m_{\Phi} \), is

\[
\frac{n_{\Phi}}{s} \sim 10^{-2} \frac{\Phi^2_{0}}{\sqrt{m_{\Phi} M^3}}
\]

(2.3)

where \( \Phi_{0} \) is the initial expectation value of \( \Phi \) relative to \( \Phi_{\text{min}} \) and \( g_{*} \sim 275 \) for the supersymmetric standard model. Without any fine tuning, the natural scale for \( \Phi_{0} \) could reasonably be expected to be of order \( M \) for Planck scale coupled moduli,
which results in $n_\Phi/s \sim 10^6$. If there are no entropy releases after the oscillations begin, the moduli dominate the energy density at a temperature

$$T \sim m_\Phi \frac{n_\Phi}{s}$$  \hspace{1cm} (2.4)

If $\Phi_0 \sim M$, the coherent moduli dominate essentially at the onset of oscillations.

The second source for both scalar and fermionic relic moduli is thermal scattering in the plasma. For $T \sim M$ the moduli are in equilibrium with a thermal number density, $n_\Phi/s \sim 1/g_\ast$. A significant entropy release, such as from a standard inflationary scenario, would dilute this number density. Even so, moduli can be produced after the entropy release by thermal rescatterings. Neglecting any initial number density, the Boltzmann equation for the number density of incoherent moduli is

$$\dot{n}_\Phi + 3Hn_\Phi = \sum_{i \geq j} \langle \sigma v \rangle_{ij} n_i n_j$$  \hspace{1cm} (2.5)

where $\langle \sigma v \rangle_{ij}$ is the thermally averaged cross section for the initial states $i$ and $j$. Typically, the two to two scattering cross section dominates and is of order $\sigma \sim \alpha/M^2$ where $\alpha$ is a gauge coupling. Integrating (2.5) from an initial temperature $T_I$ (an inflaton reheat temperature for example) with this cross section gives an incoherent relic density of

$$\frac{n_\Phi}{s} \sim \frac{\alpha T_I}{M}$$  \hspace{1cm} (2.6)

For $T_I << M$ this is much smaller than a thermal number density, and potentially quite insignificant compared with the coherent number density (2.3). In Section 4, we show that a weak scale inflation can reduce the initial number density of both coherent and incoherent moduli. Thermal rescattering production of incoherent moduli after this inflation would then be the dominant source. However, so long as the reheat temperature after inflation is sufficiently low to evade the similar problem for gravitinos [4], the incoherent production of moduli will also be sufficiently suppressed.
A large relic density of moduli fields, independent of the source, can be dangerous. If they dominate the energy density before decaying, the universe enters a (moduli) matter dominated era. After the moduli decay, the thermalized decay products return the universe to a radiation dominated era. The decay rate sets the scale for the “reheat” temperature of the decay products. In the sudden decay approximation the reheat temperature, $T_R$, is defined by equating the lifetime, $\Gamma^{-1}$, with the expansion time, $t = \frac{2}{3} H^{-1}$, where $H$ is the Hubble constant at the time of decay, $H = \pi \sqrt{g_*/90T_R^2/M}$,

$$T_R \sim \left(\frac{40}{g*_\pi^2}\right)^{1/4} (\Gamma M)^{1/2}$$

(2.7)

Notice that $T_R$ is independent of the initial moduli number density so long as the moduli dominate the energy density before they decay. With the decay rate (2.2)

$$T_R \sim 5 \left(\frac{m_\Phi}{\text{TeV}}\right)^{3/2} \text{ keV}$$

(2.8)

Such a low reheat temperature is inconsistent with successful nucleosynthesis. First, a matter dominated era preceding $^4\text{He}$ synthesis would increase the expansion rate relative to the usual radiation dominated scenario, thereby decreasing the the neutron to proton ratio, $n/p$, and subsequently the $^4\text{He}$ fraction [5,6]. Reducing the initial $n_b/s$, where $n_b$ is the baryon density, can postpone $^4\text{He}$ synthesis, allowing for an acceptable $^4\text{He}$ fraction, but $^3\text{He}$ and $D$ are then overproduced [5,6]. The second important constraint arising for relic decays occurring in the latter stages of nucleosynthesis is from the photodissociation and photoproduction of light elements by decay products [5,6,7]. These problems might be avoided for $T_R > 1\text{MeV}$ since the weak interactions are in equilibrium and the usual $n/p$ ratio can be obtained. Even this is not sufficient though since $n/p$ can be modified for $T_R > 1$ MeV when hadronic decay channels are available (which is inevitable in the thermalization process). In fact, $T_R \gtrsim 6$ MeV is required for $m_\Phi \sim 1$ TeV in order to ensure the $^4\text{He}$ abundance is not too large [7]. Clearly the low reheat temperature (2.8) is not compatible with the requirements for a standard nucleosynthesis
scenario. This is the cosmological problem for Planck scale coupled moduli. It is important to note that this is a problem for every moduli of this type in the theory.

In fact, the problem is even more severe, in that even if the moduli never dominates, the relic abundance is severely restricted. This is because even a particle with small number density which decays during or after nucleosynthesis is completed can destroy existing nuclei, or change their abundance. If the lifetime $\tau \lesssim 10^4$ s hadronic decay modes can increase $n/p$, thereby overproducing $^4$He and $D + ^3$He [6,7]. If the lifetime $\tau \gtrsim 10^4$ s electromagnetic cascades in the decay can photodissociate and photoproduce light elements [6]. These bounds are only relevant if a significant fraction of the moduli decay to the visible sector. It is worth noting that this can be avoided if the moduli never dominate the energy density and decay primarily to hidden sector fields. This might occur for example if the couplings to the visible sector were suppressed by an extra power of $M$. The exact bound in the case the moduli do decay to the visible sector is a function of the mass of the particle and its lifetime, and varies considerably over the parameter space. The most stringent bound arises for $\tau \gtrsim 10^6$ s. In this case for $m_\Phi \sim 1$ TeV, $n_\Phi/s \lesssim 10^{-15}$ is required in order not to overproduce $D + ^3$He [6]. This would require the initial amplitude of each moduli field to be $\Phi_0 \lesssim 10^8$ GeV for nucleosynthesis to proceed as usual.

We have so far considered the constraint on late decaying particles from nucleosynthesis. There are other potential problems, though these are more model dependent. Most notable among these is the necessity for generating baryons. Baryon number might be created by electroweak baryogenesis [8], an Affleck-Dine mechanism [9], or directly in the moduli decay [10]. Generically, the first two possibilities probably require a much higher reheat temperature than that derived from nucleosynthesis. Because sphaleron interactions are suppressed below the electroweak scale, the reheat temperature is required to be at least this order for the first possibility. Although in principle, the Affleck-Dine mechanism can work even with a much lower reheat temperature, it is unlikely to meet all necessary constraints. In particular, in theories with an unbroken R-parity, if the reheat
temperature is lower than the LSP mass by about factor of ten, the LSP will not be able to equilibrate. If sufficiently many baryons are created, under the most plausible assumptions, the mass density of LSP’s will be much too high. Although models can be constructed to avoid this problem, the most likely scenario is that the reheat temperature is high \[11\].

In what follows, we will assume the reheat temperature must be of order 10 MeV, but will also consider the possibility that it should be higher. In fact, a high reheat temperature can result very generally in the context of a natural inflationary scenario discussed in section 4.

The cosmological problems associated with the low moduli reheat temperature can be avoided if the naturalness assumptions are relaxed. The first possibility is that the naive estimate of the decay rate is too low. For example, the bound of \(T_R \gg 6 \text{ MeV}\) can be reached with (2.2) for a moduli mass of somewhat more than 100 TeV \[7\]. This would happen if the moduli potential is stabilized by something other than supersymmetry breaking physics, or if the supersymmetry breaking scale is higher than would be expected on the basis of naive estimates. Alternatively, the rate could be large if the moduli couplings to the visible sector are anomalously large. A coupling a factor of \(\gtrsim 10^3\) larger than that assumed in (2.2) would result in an acceptable reheat temperature. It is important to note that the mass and/or couplings of all moduli must be large in order to solve the problem in this way. This is certainly not what would be expected on the basis of naive estimates.

The second possibility is that the relic moduli density is lower than the naive estimate. If the moduli decay before dominating the energy density of the universe, the requirement that the universe be radiation dominated during nucleosynthesis can be satisfied. For \(m_\Phi \sim 1 \text{ TeV}\) and \(\tau \gtrsim 10^3 \text{ s}\), the effect of the moduli energy density on the expansion rate does not significantly alter the yield of light elements if \(n_\Phi/s \lesssim 10^{-7}\) \[5\]. Including the effects of the dilution of \(n_b/s\), which would result if the moduli decay predominantly to the visible sector, increases this bound to
\( n_{\Phi}/s \lesssim 3 \times 10^{-7} - 1 \times 10^{-8} \) for \( \tau \) in the range \( 10^{3} - 10^{7} \) s [6]. This would require an initial amplitude of the oscillating field to be \( \Phi_{0} \lesssim 3 \times 10^{11} \) GeV. However, in order to avoid the more stringent bound of \( n_{\Phi}/s \lesssim 10^{-15} \) from the D + \(^3\)He abundance discussed above would require an initial amplitude \( \Phi_{0} \lesssim 10^{8} \) GeV. Because the precise bound is a complicated function of the mass and lifetime, in this paper, we will assume that \( n_{\Phi}/s \) must be less than \( 10^{-8} \), but also consider the most stringent possible bound of \( 10^{-15} \).

It is clearly extremely unnatural to assume the initial relic densities of all moduli fields was so small. In the next section, we show that a period of weak scale inflation naturally reduces the relic density of all moduli fields to an acceptable level. Unlike the other possibilities we have mentioned, no unnatural fine tuning of the moduli fields will be necessary.

3. Diluting Moduli with Inflation

The cosmological problems of the moduli will be solved only if 1) any preexisting thermal moduli are diluted, 2) the production of moduli by thermal rescatterings is small, and 3) the amplitudes of all scalar moduli are sufficiently small to eliminate excessive relic coherent moduli. A standard inflationary scenario solves the first two problems; in this section we show that inflation can also solve the last problem, although this is most naturally achieved if the Hubble constant during inflation is low, of order of the electroweak scale. In this section, we demonstrate how inflation solves the cosmological problems. In subsequent sections, we will address the questions of naturalness and density perturbations.

First consider the classical evolution of the scalar moduli. During inflation the Hubble constant is approximately time independent. The classical equation of motion for the zero mode of a moduli is then just that of a damped oscillator

\[
\ddot{\Phi} + \frac{3}{H} \dot{\Phi} + V'(\Phi) = 0
\]

(3.1)

For \( \Phi \lesssim M \) the harmonic part of the potential should be most important. The
solutions to (3.1) in the over-, critically-, and under-damped cases are then

\[
\Phi(N) \simeq \Phi_0 e^{-\left(\frac{m^2}{3H^2}\right)N} \quad (H >> m) \\
\Phi(N) = \Phi_0 \left(1 + \frac{3}{2}N\right) e^{-\frac{3}{2}N} \quad (H = \frac{2}{3}m) \\
\Phi(N) \simeq \Phi_0 e^{-\frac{3}{2}N} \quad (H << m) \tag{3.2}
\]

where \( N \equiv Ht \) is the number of \( e \)-foldings since the beginning of inflation and \( \Phi_0 \) is the moduli expectation value for \( N = 0 \). Depending on the number of \( e \)-foldings, \( \Phi \) can be significantly reduced. In the overdamped case \((H >> m)\), to first approximation \( \Phi \) is unchanged. A standard inflation with \( H \sim 10^{13} \text{ GeV} \) and \( N \sim 60 \) required to solve the horizon and flatness problems therefore does not significantly alter the moduli amplitude. However, if \( N >> (H/m)^2 \) the classical amplitude is reduced. In the highly overdamped case this would require an extremely fine-tuned inflationary potential to obtain such a large number of \( e \)-foldings.* Measured by the number of required \( e \)-foldings, the underdamped \((H << m)\) case is clearly much more efficient in reducing the amplitude. In this case, the coherent moduli begin oscillating before the inflationary phase. The reduction in amplitude then amounts to a dilution of the number density in the coherent condensate by \( e^{-3N} \), just as for any number density.

In addition to the classical evolution, the moduli fields undergo quantum de-Sitter fluctuations in the inflationary phase. For a harmonic potential the classical and quantum evolutions decouple. In the underdamped case the quantum fluctuations, \( \delta \Phi \), are exponentially suppressed and unimportant. In the overdamped case initially the classical evolution is frozen and the fluctuations grow as for a massless field, \( \sqrt{\langle (\delta \Phi)^2 \rangle} \simeq \sqrt{N}H/2\pi \) [13]. For \( N \gtrsim (H/m)^2 \) the growth of fluctuations ceases and the low frequency modes have a thermal occupation with \( \langle (\delta \Phi)^2 \rangle \simeq 3H^4/8\pi^2m_\phi^2 \) [13]. The quantum fluctuations represent a lower bound on the rms value of \( \Phi \) at the end of inflation, thereby constraining the overdamped

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* We note that this solution to the cosmological moduli problem is similar to the solution of the cosmological axion problem presented in Ref. [12]. However, an enormous number of \( e \)-foldings is required when the inflation scale is not extremely low.
scenario. Assuming the moduli begins oscillating in the radiation era following inflaton decay, $H \lesssim 5 \times 10^7$ GeV is required during inflation in order that the rms fluctuations in $\Phi$ satisfy the bound of $n_\Phi/s \lesssim 10^{-8}$. Although an inflation with $m_\Phi \lesssim H \lesssim 5 \times 10^7$ GeV and $N >> (H/m_\Phi)^2$ could in principle solve the moduli problem, there is no natural source for $H$ is this range and a huge number of $e$-foldings would be required.† We therefore disregard the overdamped scenario and focus on the underdamped and critically damped cases in what follows.

The inflationary phase ends when the inflaton starts to undergo coherent oscillations. The universe then enters an inflaton matter dominated era. When the Hubble parameter is of order of the inflaton decay rate, $\Gamma_I$, the inflaton decays, reheating the universe to a temperature $T_r^4 \sim (\Gamma_I M)^{1/2}/g_s^{1/4}$. Clearly this temperature depends on the strength of the inflaton coupling. The reheat temperature should not be too large; otherwise there is a problem with thermal rescatterings producing gravitinos, moduli, and perhaps other relic particles. In the underdamped scenario this bound is readily satisfied; the maximum reheat temperature if the inflaton is strongly coupled and decays immediately following inflation is $T_{r,\text{max}} \sim (HM)^{1/2}/g_s^{1/4} \lesssim 10^{10}$ GeV (since $H \lesssim m_\Phi$). The inflaton is unlikely to be strongly coupled so the actual reheat temperature will be much less. In fact the stronger bound on this scenario will be achieving a sufficiently high reheat temperature to avoid the the nucleosynthesis bounds discussed in the previous section. However we will see in the next section that there exist models with this property.

The relic moduli density after inflation can now be determined in the underdamped scenario. During inflation, the coherent number density is diluted by a factor $e^{-3N}$. After inflation the ratio of moduli to inflaton number density is constant until the inflaton decays. Since the inflaton decay is the source of entropy

† However a chaotic inflationary potential of the form $V = (\mu/M)^2 \phi^4$ has a maximum number of $e$-foldings $M/\mu$ for $V \sim M^4$. The moduli fields will be damped most efficiently when $H$ has fallen to $H \sim \mu$ at which point $N \gtrsim (\mu/m_\Phi)^2$ is required. This works only for $\mu \lesssim 10^8$ Gev.
for the subsequent radiation era, the ratio of moduli to entropy density is

\[
\frac{n_\Phi}{s} \sim \frac{m_\Phi \Phi_0^2 e^{-3N}}{2(2\pi^2/45)g_* T_f^3} \left( \frac{R_{osc}}{R_f} \right)^3 \sim \frac{m_\Phi T_r \Phi_0^2 e^{-3N}}{72m_I^2 M^2}
\]  

(3.3)

where \(m_I\) is the inflaton mass (of order \(H\) during inflation for a natural potential), and \(R_{osc}\) and \(R_f\) are the scale factors at the epoch when the inflaton begins oscillating and decays respectively. The result is essentially the same for the critically damped case. The required number of e-foldings to sufficiently dilute the moduli fields depends on the reheat temperature. For the most natural models described in the next section this lies between 10 MeV and 10^5 GeV. As an example, for the moderately underdamped case of \(m_I \sim H \sim \frac{1}{3} m_\Phi\), and assuming the initial moduli amplitude before inflation was \(\sim M\), the required number of e-foldings is 2-7 for \(T_r \sim 10\) MeV (for the bound on \(n_\Phi/s\) between \(10^{-8}\) and \(10^{-15}\)) and 6-12 for \(T_r \sim 10^5\) GeV. These numbers are low, as a significant dilution occurs during the inflaton reheat stage.

It is important to note that although the inflaton decay contributes to the moduli dilution, it alone is not sufficient. This is because for \(\Phi_0 \sim M\), without the dilution factor \(e^{-3N}\), the moduli and inflaton energy densities are roughly equal after the moduli begin oscillating (irrespective of when the oscillations begin). After the inflaton decays the energy in the thermalized decay products redshifts and the moduli dominate the energy density below \(T \sim T_r\). Although this is a much lower temperature than that from (2.3) and (2.4), \(T_r \gtrsim 6\) MeV is necessary for the inflaton in order to obtain successful nucleosynthesis as discussed in section 2. The requirement that the universe be radiation dominated during nucleosynthesis could then not be satisfied. A brief period of weak scale inflation in which the moduli amplitudes are reduced is therefore necessary to solve the moduli problem.
4. Inevitable Inflation

We next consider the question of the naturalness of the type of inflationary scenario outlined above, namely one with Hubble parameter of order the weak scale and reheat temperature of at least 10 MeV. We will show first of all that the assumed form of the potential for a “flat” direction is one that can readily be associated with “chaotic” inflation [15], although with small fine tuning it can be associated with a “new” inflationary scenario [16]. Furthermore, the expected order of magnitude for the Hubble parameter is $m_{3/2}$. This is exactly what is required to efficiently dilute the dangerous moduli fields. The second requirement of our inflationary scenario was a sufficiently high reheat temperature, which should be at least 10 MeV, and probably higher (to account for the generation of baryons). A sufficiently high reheat temperature is naturally obtained if the inflaton has renormalizable couplings to the visible sector. We will consider examples of such inflatons comprised of either standard model or nonstandard model supersymmetric flat directions.

In order for the Hubble constant during inflation to be of order $m_{3/2}$ the vacuum energy density, $V$, must be of order $m_{3/2}^2 M^2$. Now suppose the inflaton corresponds to a particle with potential (2.1). The initial value of the flat direction is expected to be of order $M$, corresponding to $V \sim m_{3/2}^2 M^2$. If inflation does occur the Hubble parameter, $H \sim V^{1/2}/M$, is then naturally expected to of order $m_{3/2}$, and is precisely of the order of magnitude for which we expect inflation to be most efficient in reducing the number density of potentially dangerous moduli. No new mass scales are introduced into the theory. This is the most compelling feature of weak scale inflation as a solution to the cosmological moduli problem.

Now consider the question of the likelihood of inflation. The existence of compelling inflationary models is not yet satisfactorily resolved. The two favorite scenarios are based on either a “new” inflationary potential [16] which requires the potential be sufficiently flat for slow roll but with sufficient curvature for reasonable reheating, or the “chaotic” inflationary scenario [15] which requires assumptions
about initial conditions, and a small parameter in order to generate sufficiently small density fluctuations. As a measure of inflation consider the number of e-foldings which result with the assumed form of our potential,

$$\mathcal{V} = m_{3/2}^2 M^2 \mathcal{G}(\phi/M)$$  \hspace{1cm} (4.1)

where $\phi$ is now the inflaton. Using the slow roll equation $3H\dot{\phi} \simeq -V'$ the number of e-foldings is

$$N = \int H dt = \int dt \frac{\sqrt{V/3}}{M} = \int d\phi \frac{V}{M^2 V'} \approx \frac{\Delta \phi}{M G'^2}$$  \hspace{1cm} (4.2)

where $\Delta \phi$ is the total change in $\phi$ during inflation. New inflation would obtain sufficiently many e-foldings through a flat potential. Specifically, the conditions for new inflation with the potential (4.1) are $G'' << 3 \mathcal{G}$ and $G' << \sqrt{6 \mathcal{G}}$ [17]. Although the potential must be somewhat fine tuned in order to get inflation, the solution of the cosmological moduli problem only requires a small number of e-foldings, so this might not be very unnatural. Furthermore, we will see in the next section that the most favored scenarios are indeed those with small numbers of e-foldings since density fluctuations generated at an earlier epoch necessary for galaxy formation can then be preserved.

Alternatively, the initial value of the inflaton field might be somewhat greater than $M$. This leads to a “chaotic” inflationary scenario [15]. In general, chaotic inflation requires a small parameter so that density fluctuations are sufficiently small. The assumed form of the potential (4.1) (that which generates both the cosmological problem, and potentially its solution) does contain a small parameter.

For example, an expansion of $\mathcal{G}$ would give a quartic coupling of order $(m_{3/2}/M)^2$. Chaotic inflation makes no assumption about $\mathcal{G}''$, but only requires that $(\mathcal{G}'/\mathcal{G})^2 < 1$ which follows from $\dot{H} < H^2$. For a potential with polynomial behavior at large field this requires $\phi_0 > M$, where $\phi_0$ is the field value during inflation. In fact, if only a few e-foldings are required, the polynomial form is not necessary. We refer to weak scale inflation of this type as “inevitable” inflation since nothing is
assumed other than the form of the potential which follows from the assumptions of a nonrenormalizable hidden sector. Notice that our scenario is distinguished from that of Linde in that we do not assume the potential energy is as large as $M^4$ during inflation [15], but rather only $m_{3/2}^2 M^2$ corresponding to $G \sim 1$. It is hard to estimate the likelihood of one initial condition over another.

If the potential takes a polynomial form, it will allow the possibility of “eternal” inflation [18] so long as the deSitter fluctuations, of amplitude of order $H$, are greater than the change in $\phi$ according to its classical evolution in a Hubble time, $V'/H^2$, which is equivalent to $G'/G^{3/2} < m_{3/2}/M$. Unlike the condition for chaotic inflation, $((G/G')^2 < 1)$, this inequality involves a small number, $m_{3/2}/M$. It can therefore be satisfied by potentials which are large for some field value (e.g. a polynomial potential). So a reasonably flat potential allows the possibility of chaotic inflation but not eternal inflation, whereas a polynomial potential allows both. As discussed in the next section the preservation of density fluctuations requires a small number of $e$-foldings. The chaotic inflation we envision therefore corresponds to initial energy density $V \sim m_{3/2}^2 M^2$ but not much bigger. If eternal inflation is permitted, this initial condition might be unlikely; this question of initial conditions is however beyond the scope of this paper. Nonetheless, we find it compelling that the assumed form of the potential allows for “inevitable” inflation, and automatically contains the small parameter required for consistent chaotic inflation.

Having shown that it is quite natural to expect a flat direction of the supersymmetric potential to play the role of an inflaton with associated Hubble parameter of order $m_{3/2}$, we now turn to the question of the reheat temperature after inflation. Clearly, if the inflaton were identified with one of the dangerous moduli, the reheat temperature would be of order 5 keV which is much too low for successful nucleosynthesis. As with the moduli fields themselves, one can imagine that either the mass or couplings are larger than would be naively estimated on dimensional grounds. This scenario has the advantage that it only requires tuning the parameters of a single field. However, a very massive inflaton would probably mean that
the Hubble parameter during inflation is larger than the mass of the dangerous moduli fields, which would also imply inefficient damping of the moduli amplitude.

A much better scenario would require the inflaton to have renormalizable couplings (not suppressed by $M$) to the visible sector. This is consistent with the fact that the inflaton is a flat direction in the supersymmetric limit because of the supersymmetric nonrenormalization theorems. Gauge couplings would arise for either a nonstandard model particle with color or electroweak gauge couplings, or for a flat direction in the visible sector. Whether the former type of flat direction actually occurs depends on the underlying model. One might worry about the finite temperature corrections to the potential; however, so long as the field value is large, the relevant thermal degrees of freedom are heavy and their thermal abundance is exponentially suppressed. We first show the possibility of flat directions in the standard supersymmetric sector. These can have both gauge and Yukawa couplings.

As discussed in Ref. [9] there are many flat directions which occur in the renormalizable supersymmetric standard model (these are flat with respect to both $F$ and $D$ terms in the potential). However, for our scenario to be viable, the $F$ term must vanish along the flat direction to very high order in $1/M$. In fact, it is sufficient to show that there are no Planck suppressed superpotential terms. This is because all nonPlanck suppressed terms in the potential are explicitly proportional to the $F$ component of the flat direction. It might be that an underlying superstring model protects against the generation of all possible Planck suppressed holomorphic potential terms which are consistent with the symmetries [19]. It is also possible for field theory symmetries to guarantee that a flat direction is flat to arbitrary order in $M$ in the supersymmetric limit. A simple example in terms of the standard supersymmetric model fields is $Q_1^1 = v$, $L_2^1 = v$, and $d^2 = v$, with a conserved global $U(1)_{B-L}$. Lower indices are for $SU(2)_L$, and upper indices are for generation. It is easy to see that any gauge invariant operator with nonzero $F$ component which can be constructed in the superpotential does not respect the global symmetry. The global symmetry may not be gauged in this example since
the direction would then not be $D$ flat. This appears to be generally true; for ordinary (non $R$) symmetries, the conditions for $D$ flatness allow for nonvanishing nonrenormalizable terms in the superpotential, so that these directions will not be $F$ flat.

If the inflaton field has either gauge couplings or a direct Yukawa coupling to visible field particles, a one loop diagram will generate a $D$ type term of the form [9]

$$\frac{g^2}{\langle \phi \rangle} \int d^4 \theta \chi^\dagger \chi \phi$$

which includes component couplings of the form

$$\frac{g^2}{\langle \phi \rangle} \phi \psi \partial \psi^\dagger$$

where the fermion may or may not be the fermionic component of the $\phi$ field, and loop factors are absorbed in the definition of the coupling $g$, which may be a gauge or Yukawa coupling. The inflaton decays when the Hubble parameter, $H \sim m_\phi \langle \phi \rangle / M$ is approximately equal to the width $\Gamma \sim g^4 m_\phi^3 / \langle \phi \rangle^2$. This yields a reheat temperature, $T_r \sim \sqrt{\Gamma M} \sim g^{2/3} m_\phi^{5/6} M^{1/6} \sim g^{2/3} 10^5 \text{GeV}$. This might be as large as an order of magnitude bigger than the weak scale.

When there is a Yukawa coupling $\lambda \phi \phi_1 \phi_2$ in the superpotential which couples the inflaton $\phi$ directly to $\phi_1$ and $\phi_2$, there would also be a direct decay into the associated fields, providing they are lighter than the inflaton. However, so long as $\phi$ is large, $\phi_1$ and $\phi_2$ will be heavy and the decay is kinematically forbidden. Nonetheless, the amplitude of $\phi$ decreases as the number density decreases during the (inflaton) matter dominated era following inflation. When $\langle \phi \rangle$ becomes sufficiently small to kinematically allow the decay, we expect $\phi$ to decay essentially instantaneously, so long as $\Gamma > H$ at this time. Assuming the Yukawa coupling is $\lambda$, the decay rate would be of order $\Gamma \sim \lambda^2 m_\phi / 16\pi^2$. The Hubble constant at this time is of order $m_\phi \langle \phi \rangle / M$. Since $\lambda \langle \phi \rangle < m_\phi$ in order for the decay to be allowed, we find the Hubble constant at the time of decay will be less than $m_\phi^2 / \lambda M$. So long as $\lambda$ is not too small (greater than about $10^{-5}$ so that $\Gamma > H$ when kinematically
allowed), the decay will occur essentially immediately. If this is the dominant \( \phi \) decay mode, the reheat temperature would be expected to be about \( T_r \sim m_\phi / \sqrt{\lambda} \). This can be as large as several orders of magnitude larger than the weak scale.

We conclude that it is very natural to have inflation with inflaton amplitude a few times \( M \), a Hubble constant of order the weak scale, and reheat temperature at or above the weak scale. This does not require fine tuning of parameters. Multiple flat directions may contribute to inflation. The most significant requirement is that the flat direction which inflates the longest is one with renormalizable couplings to the visible sector, so that the reheat temperature is sufficiently high. All other requirements fall naturally from the assumption that inflation is associated with a flat direction of the supersymmetric theory.

The only requirement of inflation which cannot be easily met without fine tuning the potential is the generation of density fluctuations. However, if the number of \( e \)-foldings is not too large, late inflation will preserve density fluctuations which were produced during an earlier cosmological epoch. We discuss this in the following section.

5. Retaining Density Fluctuations During Inevitable Inflation

In the previous section, we showed that it is fairly natural to expect an inflation with \( H \sim m_{3/2} \). Such a late inflation solves the cosmological moduli problem, and could in principle solve the horizon and flatness problems with sufficiently many \( e \)-foldings. However, the observed density fluctuations of \( \delta \rho / \rho \sim 10^{-5} \) could not be generated without an extreme fine tuning of the potential. For such a small Hubble constant the density fluctuations are dominated by the deSitter fluctuations of the inflaton, for which [20]

\[
\frac{\delta \rho}{\rho} \approx \frac{H^2}{\dot{\phi}} \approx \frac{H^3}{V}'
\]  

(5.1)

where the last equality is from the slow roll condition. Without any fine tuning of the potential, \( \delta \rho / \rho \) would be expected to be of order \( m_{3/2} / M \). This means that the
observed density fluctuations require a fine tuning of $G'/G^{3/2}$ at the level of one part in $10^{10}$. This is clearly not natural. A more reasonable assumption is that density fluctuations were created prior to the period of late inflation, presumably by an earlier inflationary epoch with much larger Hubble parameter. In this case, all that is required of late inflation is that it does not destroy the density fluctuations which are necessary to produce the large scale structure of the universe.

All fluctuations which pass outside the horizon during the weak scale inflation have amplitude given by (5.1), which in a natural scenario is quite small. The requirement is therefore that all these fluctuations come back inside the horizon before a time relevant to the growth of large scale cosmological density fluctuations. Any primordial fluctuations on scales larger than $H^{-1}$ at the beginning of weak scale inflation are unaffected.

The smallest objects which could reasonably be related to cosmological density fluctuations are dwarf galaxies and Lyman alpha clouds with mass of order $10^6 M_\odot$ [21]. Of course, the growth of density fluctuations on this scale are not measured directly; the necessity for structure formation from cosmological fluctuations is obtained from assuming a scenario for galaxy formation. It is possible this criterion is more conservative than required.

The largest scale affected by the weak scale inflation is that which goes outside the horizon at the beginning of inflation, with size $H^{-1}$. During inflation it is stretched by a factor $e^N$. After inflation it is further stretched by the expansion of the universe in the inflaton matter dominated era and radiation era following the inflaton decay. The horizon also grows after inflation, eventually encompassing this scale. The requirement is that the size of the largest fluctuation affected by late inflation must be smaller than the Hubble size when $10^6 M_\odot$ of nonrelativistic matter enters the horizon,

$$H^{-1}_{10^6} \gtrsim H^{-1} e^N \left( \frac{V}{g_\ast T_r^4} \right)^{1/3} \left( \frac{T_r}{T_{10^6}} \right)$$

(5.2)

where $V \sim m_{3/2}^2 M^2$ is the vacuum energy at the end of inflation, $H$ is the Hubble
constant during inflation, and $T_{10^6} \sim 5$ keV is the temperature when $10^6 M_\odot$ of nonrelativistic matter passes inside the horizon with associate Hubble constant $H_{10^6} \sim g_s^{1/2} T_{10^6}^2 / M_e$ [22]. The factors in (5.2) are as follows. The first factor is the size of the initial Hubble patch stretched by inflation. The second is the amount by which this patch is stretched during the matter dominated era between the end of inflation and inflaton decay. The third is the stretch during the radiation era between inflaton reheating and $T_{10^6}$.

The criterion (5.2) amounts to a bound on the number of $e$-foldings for a given inflaton reheat temperature. If the reheat temperature is as low as 10 MeV, $N \lesssim 25$ is required. If $T_r \sim 10^5$ GeV, a less stringent bound of $N \lesssim 30$ is obtained. Notice that with this bound on the number of $e$-foldings, the cosmological moduli problem is solved. However, with this small number of $e$-foldings, a prior period of inflation is required to solve the horizon and flatness problems, as well as to imprint density fluctuations on large scales.

6. Conclusions

The cosmological moduli problem presents a potentially serious threat to hidden sector models of supersymmetry breaking with Planck scale suppressed couplings and weak scale masses. The low reheat temperature after moduli decay is not sufficient to obtain a successful period of nucleosynthesis. The bounds from nucleosynthesis might be avoided if naturalness assumptions about all the moduli masses and/or couplings are relaxed. Even then, baryon production could be a significant constraint. In this paper we have show that a period of inflation with small enough Hubble constant can reduce the scalar moduli amplitude to acceptable levels. The most efficient dilution occurs for a Hubble constant of order or smaller than the weak scale. The most important conclusion here is that a “flat” direction can naturally give rise to inflation with just such a Hubble constant. No new mass scales are introduced into the theory. This is the most compelling aspect of weak scale inflation. The flatness of the potential which gives rise to the problem
could also lead to the solution.

The form of the inflation may be associated with a “new” inflationary scenario with a moderate tuning of the potential to achieve slow roll for Planck scale field values. Alternately a “chaotic” scenario can result if the initial field in the flat direction is somewhat larger than the Planck mass. The supersymmetry breaking induced potential for the flat directions therefore leads to an “inevitable” inflationary scenario with weak scale Hubble constant. One of the dangerous moduli can act as the inflaton for such an inflation. Attaining a sufficiently high reheat temperature after inflation then requires relaxing the naturalness assumptions for this field. More appealing is the possibility that a flat direction with renormalizable couplings to the visible sector acts as the inflaton. The reheat temperature is then naturally low enough to avoid reproducing moduli and gravitinos from thermal rescattering, but above the weak scale so that baryogenesis can proceed through weak scale processes or an Affleck-Dine mechanism.

A weak scale inflation can not naturally give rise to cosmological density fluctuations of the magnitude required for large scale structure formation. However if the number of $e$-foldings during inflation is sufficiently small, density fluctuations from an earlier period of inflation can be preserved on large scales. In fact it is perhaps more natural for the weak scale inflation to persist for only a moderate number of $e$-foldings. In conclusion, we have shown that same assumptions which give the cosmological moduli problem also naturally lead to its solution via inevitable weak scale inflation. Within this solution successful baryogenesis and the preservation of large scale density fluctuations are also possible.

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