Estimation of the state of a technical system with substantially non-linear characteristics

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Abstract. The article considers the problem of estimating the state of the technical system described by nonlinear differential equations. An approach is proposed to design reduced-order asymptotic observers for systems where dynamic equations contain products of state variables. The solvability conditions for the synthesis problem in the form of linear matrix equations are determined. The possibility of reducing the original problem to the classical modal control problem is shown. Two modifications of the algorithm for calculating the coefficients of the state observer, which ensures that the estimation error is asymptotically zero, are proposed.

1. Introduction

Asymptotic state observers are often used to estimate the state of various technical systems [1, 2]. At the same time most design methods and algorithms imply the linearity of the model of the object, which is not always followed in practice [3]. Well-known results in designing observers of nonlinear systems state do not cover all known types of nonlinearities. Therefore, the problem of developing new methods of estimating, allowing to restore the state of nonlinear systems can be considered quite actual.

2. Problem specification

Consider the mathematical model of the technical system in the form of a set of equations in the state space. Let us introduce the following notation: $X$ is the state vector of considering system, which includes two components $X_1$ and $X_2$; $x_1$ is the first element of the state vector; $U$ is an input signal; $Y$ is an output signal. We assume that the first part of the state vector forms the output signal of the system and the second part of it cannot be measured directly (respectively, $Y = X_1$; $y_1 = x_1$).

Consider the situation when equations of system dynamics include terms proportional to $x_1$ and other state variables:

$$\begin{align}
\dot{X}_1 &= A_{11}X_1 + A_{12}X_2 + x_1F_{11}X_1 + x_1F_{12}X_2 + B_1U; \\
\dot{X}_2 &= A_{21}X_1 + A_{22}X_2 + x_1F_{21}X_1 + x_1F_{22}X_2 + B_2U; \\
Y &= X_1. 
\end{align}$$

(1)

It is obvious that the model of the system can be represented in the form (1) only in the case when the input signal of the system is not transmitted directly to its output, and the state variable determining nonlinear terms in the dynamic equation arrives at the output signal of the system. It is required to synthesize the observer of the dynamic system state (1), which allows using the known $Y$ and $U$ to
form the vector of assessment Z in the following way:
\[
\lim_{t \to \infty} e(t) = \lim_{t \to \infty} [Z(t) - X_2(t)] = 0. 
\] (2)

3. Synthesis of the observer’s equation

Let us compose the equation of the observer on the basis of the second equation of system (1), adding it to the term proportional to the difference of assessment and the true value of the vector \(X_2\):
\[
\dot{Z} = A_2 Z + a_2 Y + F_{12} Z + B_2 U + L(A_2 Z + a_1 Y + F_{12} Z - A_2 X_2 - a_1 Y_2). 
\] (3)

Substituting true values of terms \(A_1 X_2\) and \(y_1 F_{12} X_2\) from the first equation system (1) in equation (3), we get:
\[
\dot{Z} = (A_2 + LA_{12}) Z + y_1 (F_{22} + LF_{12}) Z + (A_2 + LA_{12}) Y + y_1 (F_{22} + LF_{11}) Y + (B_2 + LB_{12}) U - \dot{Y}. 
\] (4)

Let us form the equation of dynamics of the estimation error of the vector \(X_2\) by subtracting from the equation (4) the second equation of the system (1):
\[
\dot{\epsilon} = (A_2 + LA_{12}) \epsilon + y_1 (F_{22} + LF_{12}) \epsilon. 
\] (5)

In the general case the solution of equation (5) depends both on matrix coefficients on the right-hand side and on the form of the function \(y_1\). It is obvious that for not all values of \(y_1\) the equation (5) will provide the required dynamics of estimation error. To simplify the problem we choose the matrix \(L\):
\[
F_{22} + LF_{12} = 0. 
\] (6)

Whenever the condition (6) is fulfilled, the equation (5) will be the following:
\[
\dot{\epsilon} = (A_2 + LA_{12}) \epsilon. 
\] (7)

The asymptotic stability of the trivial solution of equation (7) can be ensured by the appropriate choice of the matrix \(L\) on the condition of visibility or identity of the pair \((A_{22}, A_{12})\). Obviously, this will satisfy the condition (2).

Let’s write equation (4) with a constraint (6):
\[
\dot{Z} = (A_2 + LA_{12}) Z + (A_2 + LA_{12}) Y + y_1 (F_{22} + LF_{12}) Y + (B_2 + LB_{12}) U - \dot{Y}. 
\] (8)

The equation (8) determines the asymptotic state observer of reduced order for system (1). At the same time in its right part there is a term proportional to the derivative of the output signal. It is known [1] that in practice the introduction of the operation of numerical differentiation can lead to occurrence of significant errors, so it is recommended to avoid it.

To eliminate the operation of differentiation from the observer equation (8), we modify the calculation scheme. We introduce the auxiliary variable \(w(t)\) as follows:
\[
w(t) = Z(t) + LY(t). 
\] (9)

Taking into account the relation (9), the equation (8) can be written as follows:
\[
\dot{w} = (A_2 + LA_{12}) w + (A_2 + LA_{12} - A_2 L - LA_{12} L) Y + y_1 (F_{22} + LF_{12}) Y + (B_2 + LB_{12}) U. 
\] (10)

In this case the equation for assessing the vector \(X_2\) will be the following:
\[
Z(t) = w(t) - LY(t). 
\] (11)

Thus, in the case of visibility or identity of the pair \((A_{12}, A_{12})\) by fulfilling the condition (6), one can form observer equations that ensure that assessment error of unmeasured component of the state vector of a technical system (1) is asymptotically zero. In this case two variants of the design scheme are possible – with differentiation of the output signal (8) and without it (10) - (11).

4. The analysis of solution conditions

Let us define conditions for the existence of an observer described by equations (8) or (10) - (11). Relator (6) is a linear matrix equation. The condition of its solution and expression for full set of solutions are formulated in terms of matrix canonization technology [4].

The solution of equation (6) exists if the condition [4] is satisfied:
\[
F_{12} F_{22} = 0. 
\] (12)

In this case a full set of solutions is determined by the expression:
Here \( \eta \) is an arbitrary numerical matrix of the corresponding dimension. For an arbitrary matrix \( M \) symbols like \( \tilde{M}^R \) and \( \tilde{M}^L \) show nonzero matrices of maximal rank such as \( M \tilde{M}^R = 0 \) and \( \tilde{M}^L M = 0 \), respectively; and \( \tilde{M}^R \) and \( \tilde{M}^L \) matrices of maximum rank \( \tilde{M}^L M \tilde{M}^R = I \); the matrix \( \tilde{M} = \tilde{M}^R \tilde{M}^L \) is also introduced to simplify recording.

Taking into account the expression (13), the matrix of coefficients from the right side of equation (7) will look like:

\[
A_{22} + LA_{12} = A_{22} - F_{22} \tilde{F}_{12} A_{12} + \eta \tilde{F}_{12}^L A_{12}.
\]

By analogy with the well-known result [5], we introduce new notations:

\[
A^* = \left( A_{22} - F_{22} \tilde{F}_{12} A_{12} \right)^T; \quad B^* = \left( \tilde{F}_{12}^L A_{12} \right)^T.
\]

To satisfy the condition (2), it is necessary and sufficient all the eigenvalues of the matrix to be located in the left semi-plane of the complex plane. If the pair \((A^*, B^*)\) is fully Kalman controlled or stabilized, then the solution to the original problem can be obtained using modal control methods [6].

Thus, the solution of the problem is possible if condition (12) is fulfilled, and the pair \((A^*, B^*)\) given by expression (14) is Kalman controlled or stabilized.

5. Algorithm of synthesis

Based on obtained results we formulate an algorithm for calculating coefficients of the state observer of a nonlinear dynamical system, the model of which can be represented in the form (1).

1. Set the matrix of coefficients of the model (1), the required law of distribution of poles of the observer, information on feasibility or non-feasibility of differentiation of the output signal.
2. Check the fulfillment of the condition (12). If it is fulfilled, then you should proceed to the next step, otherwise the proposed method is not applicable.
3. Calculate the matrix of coefficients by formulas (14) and estimate the controllability / stabilizability of the pair \((A^*, B^*)\). If it is fully controllable or uncontrollable eigenvalues are located in the required area of the complex plane, then you should proceed to the next step, otherwise it is recommended to change requirements for the dynamics of the estimation process or use a different calculation algorithm.
4. Solve the problem of modal control by providing the choice of the matrix \( \eta \) to the desired location of eigenvalues of the matrix \( A^* + B^* \eta^T \).
5. Calculate the matrix of observations using the formula (13).
6. Form equations of the observer. If differentiation of the output signal is allowed, then equation (8) is used. Otherwise, the equation of the observer’s dynamics is given by formula (10) and the expression for estimating the vector \( X_2 \) is given by formula (11).

The end of the algorithm.

6. Conclusions

A new method for estimating the state of the technical system is proposed, the model of which includes the product of state variables. The solvability conditions for the problem are formulated, as well as the step-by-step algorithm for calculating the coefficients of the observer, which ensures that the estimation error is asymptotically zero. The obtained results can be used in practice when building diagnostic systems for technical objects with non-linear characteristics.

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