The Apsidal Antialignment of the HD 82943 System

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Abstract. We perform numerical simulations to explore the dynamical evolution of the HD 82943 planetary system. By simulating diverse planetary configurations, we find two mechanisms of stabilizing the system: the 2:1 mean motion resonance between the two planets can act as the first mechanism for all stable orbits. The second mechanism is a dynamical antialignment of the apsidal lines of the orbiting planets, which implies that the difference of the periastron longitudes $\theta$ librates about $180^\circ$ in the simulations. We also use a semi-analytical model to explain the numerical results for the system under study.

Keywords: N-body simulations, mean motion resonance, apsidal antialignment, planetary system (HD 82943, GJ 876, HD 12661, 47 UMa, $\upsilon$ And, 55 Cnc)

1. Introduction

At present, more than one hundred giant extrasolar planets have been discovered in Doppler surveys of solar-type stars (Butler et al., 2001, 2003), among which there are 10 multiple-planet systems, including eight two-planet systems (HD 82943, GJ 876, HD 168443, HD 74156, 47 UMa, HD 37124, HD 38529 and HD 12661) and two three-planet systems (55 Cnc and $\upsilon$ And). The observations indicate the Mean Motion Resonance (MMR) frequently occurs for the planets of the multiple-planet systems: the two planets of the GJ 876 (Laughlin and Chambers, 2001; Kinoshita and Nakai, 2001; Snellgrove, Papaloizou and Nelson, 2001; Lee and Peale, 2002; Ji, Li and Liu, 2002) and HD 82943 (Gozdziewski and Maciejewski, 2001; Beauge, Ferraz-Mello and Michchenko, 2002; Ji and Kinoshita, in preparation) are respectively in the 2:1 MMR, the inner two companions of the 55 Cnc (Marcy et al., 2002; Fischer et al., 2003; Ji et al., 2003) is in the 3:1 MMR and 47 UMa (Laughlin, Chambers and Fischer, 2002) is close to a 7:3 commensurability, which inspire us to study the resonant configurations of the planetary systems.

We utilized N-body codes (Ji et al., 2002) to perform the numerical integrations for the HD 82943 system with RKF7(8) (Fehlberg, 1968). Moreover, we also used symplectic integrators (Wisdom and Holman, 1991) to examine...
the same orbits to assure the results for some cases. In the simulations, the mass of the central star is adopted to be $1.05 \, M_\odot$, and those of the two planets (HD 82943b, HD 82943c) are respectively $1.63$ and $0.88 \, M_{Jup}$ under the assumption of $\sin i = 1$. Next, we introduce the scheme to generate the initial six orbital elements (semi-major axis $a$, eccentricity $e$, inclination $i$, nodal longitude $\Omega$, apsidal argument $\omega$ and mean anomaly $M$) for each planet. Throughout the paper, let us bear in mind the hypothesis that the two planets are always considered to be coplanar. For all the orbits, we assume that the semi-major axes of the two planets always start at $1.16$ AU and $0.73$ AU (see Table 1). We take their initial eccentricities $e_0$ to be centered respectively at $0.41$ and $0.54$, and randomly displaced by the measuring error $\Delta e$. And the initial arguments of periastron are treated in similar way. The remaining two angles of nodal longitudes and mean anomalies are randomly chosen between $0^\circ$ and $360^\circ$. Thus, 100 pairs of coplanar orbits are prepared for the integration. In this paper, we aim to study the dynamical behavior of the HD 82943 system by exploring diverse configurations in the neighborhood of the best-fit solutions, further attempt to discover the possible stabilizing mechanisms of maintaining this system.

Table I. The parameters of the HD 82943 planetary system

| Parameter          | HD 82943b | HD 82943c |
|--------------------|-----------|-----------|
| $M \sin (M_{Jup})$| 1.63      | 0.88      |
| Orbital period $P$| 444.6     | 221.6     |
| $a$ (AU)           | 1.16      | 0.73      |
| Eccentricity $e$   | 0.41      | 0.54      |
| $\Delta e$         | 0.08      | 0.05      |
| $\omega$ (deg)     | 117.8     | 138.0     |
| $\Delta \omega$ (deg)| 3.4  | 10.2      |

2. Apsidal antialignment

In the numerical investigations, our goal is to understand the characteristic of the secular behavior of the HD 82943 planetary system. By using the aforementioned initial orbits, each integration was executed for 10 Myr. As a result, we find that 75% of the orbits are unstable for the timescale of $10^5$ years.

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1 The data are partly adopted from http://cfa-www.harvard.edu/planets/encycl.html, http://obswww.unige.ch/~udry/planet/hd82943syst.html
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yr and only 5% of the experiments survived for $10^7$ yr. It is noteworthy that all the stable cases are involved in the 2:1 MMR and that the stability of a system is obviously sensitive to its initial planetary configuration when we fix the masses of the planets. In this paper, we discuss a stable configuration in association with the 2:1 MMR and apsidal resonance.

At first, we introduce the lowest order critical arguments $\theta_1$ and $\theta_2$ for the 2:1 MMR of the HD 82943 system

$$\theta_1 = \lambda_1 - 2\lambda_2 + \tilde{\omega}_1, \quad (1)$$

$$\theta_2 = \lambda_1 - 2\lambda_2 + \tilde{\omega}_2, \quad (2)$$

where $\lambda_1$, $\lambda_2$ are, respectively, the mean longitudes of the inner and outer planets, and $\tilde{\omega}_1$, $\tilde{\omega}_2$ denote their apsidal longitudes respectively (subscript 1 for the inner planet; 2, the outer planet). Figure 1a shows that one of the resonant arguments $\theta_2$ librates about $0^\circ$ for the timescale of 10 Myr (in fact, $\theta_1$ also librates around $180^\circ$ for the same timescale), indicating the two planets of the system are now locked into the 2:1 MMR. On the basis of the best-fit solutions by Laughlin and Chambers (2001) for GJ 876, Lee and Peale (2002) found that $\theta_1$ and $\theta_2$ both librate about $0^\circ$ with small amplitudes. Moreover, they pointed out that the differential planet migration due to planet-nebular interaction could give rise to capture into the 2:1 MMR. From the Laughlin-Chambers solutions, one can see that the outer planet of the GJ 876 moves on a near circular orbit. By contrast, the HD 82943 system differs in that both of the planets occupy high eccentricities at present-day. Hence, the origin of the 2:1 resonance for the HD 82943 surrounded by two massive eccentric planets should be explained by a new mechanism (S. Peale, private communication), which may be quite different from the resonant origin of the GJ 876.

Most important of all, we find that the periodic coplanar orbits of the HD 82943 apparently cross during the secular orbital evolution because of high eccentricities of two planets. To the best of our knowledge, such kinds of the stable orbits are never reported before. One should make clear that what kinds of the mechanisms make the system stable for tens of millions of years. However, we notice that the planets of the studied system simultaneously undergo apsidal antialignment. The relative apsidal longitude $\theta_3$ is denoted by

$$\theta_3 = \theta_1 - \theta_2 = \tilde{\omega}_1 - \tilde{\omega}_2. \quad (3)$$

From Figure 1b, we see that $\theta_3$ librates about $180^\circ$ with an amplitude of $\pm 30^\circ$, conversely, Lee and Peale (2002) found that $\theta_3$ librates about $0^\circ$ for the GJ 876, which indicates the alignment of the apsidal lines. However, it is the first time to observe the antialignment configuration for the HD 82943 system. Recently, the antialignment for the HD 12661 was independently confirmed by Lee and Peale (2003a) and Gozdziewski (2003). The librations of the relative apsidal longitude were also discovered in $\upsilon$ And (Kinoshita and Nakai,
2000; Chiang, Tabachnik and Tremaine, 2001) and 47 Uma (Laughlin et al., 2002). Besides these symmetric apsidal resonance, Ji et al. (2003) found the asymmetric apsidal librations about 250° or 110° for the 55 Cnc, and the asymmetric configurations were also suggested by Lee and Peale (2003b) and Beauge et al. (2002). As for υ And, Kinoshita and Nakai (2000) reported the mechanism of the apsidal alignment by using the linear secular perturbation theory. Now it is believed that the apsidal resonance significantly plays a part in stabilizing the multiple-planet systems. In the following, we utilize a semi-analytical model to study the dynamics near the apsidal resonance and then compare the analytical results with the numerical outcomes.

The Hamiltonian for the coplanar case (Kinoshita and Nakai, 2002) is

\[ F = F(a_1, a_2, e_1, e_2, \tilde{\omega}_1, \tilde{\omega}_2, \lambda_1, \lambda_2). \] (4)

In order to keep the Hamiltonian form, we should use Jacobi coordinates or canonical heliocentric coordinates to study the system. As the indirect part of the Hamiltonian does not contribute to the secular part, we simply take the direct part. As aforementioned, the orbits of two planets intersect each other, the usual analytical development method of the main part can not be applicable to this case, then we adopt the original form of the main part of the disturbing function and numerically evaluate it. Furthermore, by eliminating short-periodic terms, the new Hamiltonian reads:

\[ F^* = F^*(a_1, a_2, e_1, e_2, \theta_2, \theta_3). \] (5)

The degrees of freedom of the new Hamiltonian (5) are reduced from four to two. However this Hamiltonian is still not integrable. As the amplitude of the critical argument \(\theta_2\) is not so large, we assume \(\theta_2=0\). From this assumption, the semi-major axes \(a_1\) and \(a_2\) are derived as constant and we use \(a_1=0.73\) AU and \(a_2=1.16\) AU in the following discussions. Correspondingly, the degrees of freedom of the new Hamiltonian are reduced from two to one and the new Hamiltonian takes the following form:

\[ F^* = F^*(e_1, e_2, \theta_3). \] (6)

We eliminate \(e_2\) in equation (6) with the conservation of the angular momentum \(H\), then we have

\[ F^* = F^*(e_1, \theta_3, H). \] (7)

Thus we can plot the level curves of the Hamiltonian and understand the global behavior of \(e_1\) and \(\theta_3\). We draw the contour map of the Hamiltonian (7) by taking \(\theta_3\) as the horizontal axis and \(e_1\) or \(e_2\) as the vertical axis with the parameter \(H\), which is determined from the initial conditions. Figure 2 shows the contour map of the Hamiltonian (7) given by thin lines. Because the numerical solution includes the short-periodic terms, the numerically averaged solution represented by broad thick lines is shown in Figure...
2. The figure shows a good agreement between the numerical solution and the semi-analytical secular solution. The eccentricities of both planets are well restricted because of the libration of $\theta_3$ around $180^\circ$. Therefore, we may safely conclude that the stability of the HD 82943 system that is related to the above coplanar crossing orbits can be simultaneously sustained by two mechanisms—the 2:1 MMR and the apsidal antialignment.

3. Conclusions

In final, we summarize some conclusions: we carried out the numerical simulations of the HD 82943 system for $10^7$ yr. In the simulations, we find that all of the stable orbits are associated with the 2:1 MMR. In particular, we underline that a stable case of the coplanar crossing orbits is not only in a 2:1 MMR, but also experiences the apsidal resonance during the secular orbital evolution. The presence of the 2:1 MMR reveals the fact that the resonance can act as one of the mechanisms of stabilizing the HD 82943 system that harbors two eccentric Jupiter-like companions. Additionally, the second mechanism is a dynamical antialignment of the axes of the orbits of the two planets, showing that $\theta_3$ librates about $180^\circ$ for the HD 82943 system. The $\theta_3$-libration makes it possible that the massive planets in the mean motion resonance avoid frequent close encounters. Furthermore, we point out that to date the discovered multiple-planet systems in the MMR (HD 82943, GJ 876, HD 12661, 47 Uma, 55 Cnc) are always followed by apsidal resonance and this can be easily understood that the apsidal libration can result from the librations of the mean motion resonant arguments. This apsidal libration is also responsible for the stability of nonresonant systems, such as $\upsilon$ And and other cases.

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Figure 1. (a) The upper panel shows that one of the resonant arguments of $\theta_2$ librates about $0^\circ$ for 10 Myr, indicating the two planets of the HD 82943 system are now locked into the 2:1 MMR. (b) The lower panel displays that the argument of $\theta_3$ librates about $180^\circ$ for the timescale of 10 Myr, with an amplitude of $\pm 30^\circ$ for the system, which implies the antialignment for this system. Note: the initial conditions for numerical integration: $a_1 = 0.73$, $e_1 = 0.5440$, $\Omega_1 = 8.01^\circ$, $\omega_1 = 132.55^\circ$, $M_1 = 267.66^\circ$, $a_2 = 1.16$, $e_2 = 0.4628$, $\Omega_2 = 223.94^\circ$, $\omega_2 = 117.89^\circ$, $M_2 = 214.50^\circ$. 
Figure 2. The equi-Hamiltonian curves for the eccentricities of two planets versus $\theta_3$. The thin lines are computed from numerically averaged Hamiltonian by assuming that the planetary system is in exact 2:1 MMR. The broad thick line in the libration region around $\theta_3 = 180^\circ$ shows the solution obtained by numerical integration, which suggests a good agreement with the semi-analytical method for the case of the libration.