Neutrino mass hierarchy determination with PINGU

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Abstract. The PINGU (Precision IceCube Next Generation Upgrade) experiment is a proposed project that will probe the neutrino mass hierarchy by studying the energy and zenith spectra of atmospheric neutrinos. PINGU in a few years will collect a huge statistics ($O(10^5)$ events) and its sensitivity will be limited by systematics errors. In this work we discuss the impact of the shape systematics on the hierarchy determination.

1. Introduction

Since the discovery of atmospheric neutrino oscillations in the Super-Kamiokande (SK) experiment in 1998 [1], about twenty years of neutrino oscillation experiments have established the standard framework of $3\nu$ flavor states ($\nu_e, \nu_\mu, \nu_\tau$), mixed with 3 mass eigenstates states ($\nu_1, \nu_2, \nu_3$) having definite masses ($m_1, m_2, m_3$), by means of the three mixing angles ($\theta_{12}, \theta_{13}, \theta_{23}$) and a CP-violating phase $\delta$ [2]. Neutrino oscillations depend on the three angles $\theta_{ij}$ and two independent squared mass differences, which we choose as $\delta m^2 = m_2^2 - m_1^2 > 0$ and [3, 4]

$$\Delta m^2 = \pm \left| m_3^2 - \frac{(m_2^2 + m_1^2)}{2} \right|. \tag{1}$$

The sign of $\Delta m^2$ distinguishes the normal (+) and inverted (−) neutrino mass hierarchy (NH and IH, respectively). For recent oscillation analyses, with updated bounds on the mixing angles and squared mass differences, see [5, 6, 7].

While global data analyses have already shown a slight sensitivity to the hierarchy through various subsets of data [3, 4, 5, 6, 7, 8, 9], the subleading effects that would allow a clear discrimination of the hierarchy (especially through matter effects in atmospheric neutrinos) have not being precisely measured. Indeed, the difference between the two hierarchies was $\Delta \chi^2 \simeq 0.3$ in the analyses before SK [8], and it is $\Delta \chi^2 \simeq 0.9$ in the latest SK Collaboration results [10].

Hierarchy effects will probably emerge in future large-volume atmospheric neutrino experiments [11]. The proposed ice-Cherenkov detector PINGU (Precision IceCube Next Generation Upgrade) [12] is expected to collect about one hundred thousand neutrino events in a few years. Other proposed projects with similar purposes are ORCA (Oscillation Research with Cosmics in the Abyss) [13], Hyper-Kamiokande [14], and the ICAL-INO project (Iron CALorimeter at the India-based Neutrino Observatory) [15] (see [16] for a recent discussion).

In this work we study the PINGU sensitivity to the hierarchy (see also [17]). With respect to previous works [18, 19, 20, 21, 22], we have tried to elucidate several subtle issues in the
calculation of spectral shapes, by estimating all possible systematic uncertainties. In addition, we have studied the interplay between the hierarchy determination and the measurement of $\theta_{23}$ in PINGU.

2. Calculation of the expected neutrino spectra

The expected neutrino spectra in PINGU can be expressed in compact notation as

$$\frac{d^2 N^\alpha}{d \cos \theta' d E'} = \left[ 2 \pi T \frac{\rho V_{e\beta}^\alpha(E')}{m_p} \sigma_{CC}(E') \frac{d^2 \Phi^\alpha(\theta', E')}{d \cos \theta' d E'} \right] P^\alpha(\theta', E') .$$

(2)

In Eq. (2), $E'$ and $\theta'$ are the true neutrino energy and zenith angle for the flavor $\alpha$ ($\alpha = \mu, e$), $T$ is the detector livetime, $\rho V_{e\beta}$ is the effective detector mass, $\sigma_{CC}$ is the neutrino charged-current cross section (\(\bar{\sigma}\) for anti-neutrinos), $\Phi^\alpha$ is the atmospheric neutrino flux and $P^\alpha$ is a factor containing the oscillation probabilities. The last factor is the only one that does depend on the oscillations and can be written as

$$P^\alpha = \left[ P_{\alpha \alpha} + \Phi^\beta \Phi^\alpha P_{\beta \alpha} \right] + \left[ \Phi^\alpha \sigma_{CC} T_{\alpha \alpha} + \bar{\Phi}^\beta \sigma_{CC} T_{\beta \alpha} \right].$$

(3)

In general, the true and the measured neutrino energy and angle will differ because of the detector energy and angle resolutions, so that the neutrino spectra, in terms of the reconstructed variables, $(E, \theta)$, will be

$$\frac{d^2 N^\alpha}{d \theta d E} = \int_0^{2\pi} \sin \theta' d \theta' r^\alpha_{\theta}(\theta, \theta') \int_{E'_{\text{thr}}}^{\infty} d E' r^\alpha_E(E, E') \frac{d^2 N^\alpha}{d \cos \theta' d E'} .$$

(4)

In the last equation, $r^\alpha_E(E, E')$ and $r^\alpha_{\theta}(\theta, \theta')$ are the energy and angular resolution functions. We assume gaussian form for the two resolution functions, with widths $\sigma_E(E')$ and $\sigma_{\theta}(E')$, respectively. By using the PINGU histograms in Fig. 14 of [12], we obtain the following empirical

Figure 1. Widths (at $\pm 1\sigma$) of the resolution functions in energy and angle for $\nu_\mu$-like events in PINGU.
functions:

\[ \sigma^\mu_{\theta E} / E' = 0.266 / (E^{0.171} - 0.604) \, , \]
\[ \sigma^\mu_{\theta E} / E' = 0.369 / (E^{0.247} - 0.508) \, , \]
\[ \sigma^\mu_\theta = 3.65 / (E'^{1.05} + 5.00) \, , \]
\[ \sigma^\mu_\theta = 1.88 / (E'^{0.823} + 1.93) \, , \]

where \( [E'] = \text{GeV} \) and \( [\sigma_\theta] = \text{rad} \). The resulting resolution functions at \( \pm 1\sigma \) are shown in Figure 1, for the three representative energies \( E' = 3, 10, \) and \( 30 \text{ GeV} \) (horizontal bands) and the three angles \( \theta'/\pi = 0.6, 0.75 \) and 0.9 (vertical bands). Upgoing events (\( \theta'/\pi \sim 1 \)) correspond to the left of the zenith scale, and horizontal events (\( \theta'/\pi \sim 0.5 \)) to the right. Figure 1 shows the advantages of using the zenith angle rather than its cosine: the symmetry of the vertical bands (in the right plot the vertical bands are squeezed towards the upgoing directions) and the better coverage of the interesting region of neutrinos traversing the Earth core and the mantle, where matter effects are enhanced [23].

The number of events \( N^\theta_j \) in the \( ij \)-th bin is calculated by integrating the r.h.s. of Eq. (4) over the bin area \( [\theta_i, \theta_{i+1}] \otimes [E_j, E_{j+1}] \) [28]. In view of the energy resolution shown in Figure 1, we adopt a logarithmic energy variable for the true neutrino energy \( E' \) and for the reconstructed energy \( E \). We consider reconstructed energies in the interval \( E \in [1, 40] \text{ GeV} \), divided into 16 bins. We also divide the reconstructed angle range into 10 bins. The bin widths are smaller than the typical resolution widths in Figure 1, and large enough to contain a significant number of events after a few years of exposure.

The main components of the PINGU spectra calculations (first three couples of panels from the left) and final spectra (last couple of panels on the right) are shown in Figure 2. We have
assumed normal hierarchy (NH), $\sin^2 \theta_{23} = 0.5$, and the other oscillation parameters as in the best fit of [5]. The darkest color corresponds to the bins with maximum contents. We estimate a total of $1.9 \times 10^4$ muon and $1.4 \times 10^4$ electron events per year in PINGU. The total number of events will be of order $\sim 10^5$, after few years of data taking. The first three panels on the left in Figure 2 show the product $V_{\alpha \beta}^{\alpha} \Phi^\alpha \sigma^{\alpha}_{\text{CC}}$, the oscillation-dependent factor $P^\alpha$ in Eq.(3) and their binned product, which is proportional to the unsmearred spectrum. The prefactor is suppressed, at high energy, by the neutrino flux ($\Phi^\alpha \sigma^{\alpha}_{\text{CC}} \sim E^{-2}$) and, at low energy, by the effective detector volume, and it is maximum in the few GeV range, relevant for matter effects. Unluckily, the atmospheric neutrino flux is maximum at the horizon ($\theta/\pi \rightarrow 0.5$), where matter effects vanish. The oscillation-dependent factor shows large variations for muons, where a large disappearance “valley” corresponds to the first oscillation minimum, while $P^e$ shows much milder variations, since the $\nu_e$ disappearance and appearance probabilities are suppressed, because of the small value of $\sin^2 \theta_{13}$. The binned product of the prefactor and $P^\alpha$ shows residual structures in the central part of the plot. These structures, however, are largely suppressed around the vertical upgoing direction, where the atmospheric flux is lower. In the smeared spectra, that include resolution effects (right panels), the oscillating structures are largely suppressed, except for the remnants of the large $\nu_\mu$ disappearance valley. Hierarchy effects will only emerge as smooth and subdominant modulations of these spectra.

2.1. Shape uncertainties
The large statistics that PINGU will collect in few years will reduce the statistical uncertainties at few percent level. However, the shape uncertainties of the event spectra may also be affected by comparable systematic errors. These errors depend on the oscillation-independent factors $(V_{\alpha \beta}^{\alpha} \Phi^\alpha \sigma^{\alpha}_{\text{CC}})$, on the oscillation-dependent ones $(P^\alpha)$, on the resolution functions $(r^\alpha_{E,\theta})$, and on the event integration into bins. See also [11, 12, 18, 19, 20, 21, 22] for a discussion of this kind of errors. The effective volume $V_{\text{eff}}^{\alpha}$ is affected by shape uncertainties, since it has been estimated in PINGU via Monte Carlo (MC) simulations [12] with finite statistics. Moreover, the neutrino detection efficiency also depends on $\theta'$ [24] and, to some extent, on the azimuth angle $\phi'$, because of unavoidable anisotropies of the detector and inhomogeneities of the ice. It is thus reasonable that $V_{\text{eff}}^{\alpha}(E', \theta', \phi')$ is affected by systematic errors at the few percent level (possibly in different way for $\mu$ and $e$). The atmospheric neutrino fluxes also depend, in general, on the three variables $(E', \theta', \phi')$ and are affected, at the percent level of accuracy, by a number of irreducible shape uncertainties [25, 26, 27]. The main uncertainties in the calculation of the oscillation probabilities are related to knowledge of the mass-mixing neutrino parameters, particularly of the dominant ones ($|\Delta m^2|$, $\theta_{23}$), whose variation reduces the sensitivity to the hierarchy [11]. Furthermore, the oscillation probabilities are affected by uncertainties in the neutrino production height and in the electron density profile, during the propagation through the Earth. The CC cross sections are poorly known in the (few) GeV range, when deep inelastic scattering is not dominant. The uncertainties on total and differential cross sections affect the spectral normalization and shape, respectively, at a typical few percent level. Concerning the resolution functions, their centroids and shapes are biased by cross section and reconstruction uncertainties. Finally, the multi-dimensional integration into binned spectra may also be a source of numerical errors in itself. Each of these sources gives extra freedom to adjust the event spectra in data fits, thus reducing the sensitivity to spectral differences induced by hierarchy effects. Although each reduction may be small, their sum may become non negligible. In our analysis we study, in a more quantitative way, the impact of the above error sources in PINGU.

3. Statistical analysis
The sensitivity to the hierarchy can be calculated by the marginalized $\Delta \chi^2$ difference between the true hierarchy (TH) and the wrong hierarchy (WH) [29]. The parameter $N_\sigma = \sqrt{\Delta \chi^2}$ will
give the effective number of standard deviations separating the TH and WH hypotheses. The TH and WH spectral event rates are given by

\[ R_{ij}^\alpha(p_k) = \frac{N_{ij}^\alpha(\text{TH}; p_k)}{T}, \]

\[ \tilde{R}_{ij}^\alpha(\tilde{p}_k) = \frac{N_{ij}^\alpha(\text{WH}; \tilde{p}_k)}{T}, \]

where \( T \) is the detector livetime, the \( p_k \) are the (oscillation and systematic) fixed parameters in TH, and \( \tilde{p}_k \) are the corresponding parameters in WH. The TH event rates are affected by statistical errors decreasing as \( \sqrt{T} \),

\[ s_{ij}^\alpha = \sqrt{\frac{R_{ij}^\alpha}{T}} \quad (1\sigma), \]

and by systematic errors on the parameters \( p_k \),

\[ p_k \pm \sigma_k \quad (1\sigma). \]

The \( \Delta \chi^2 \) is defined as

\[ \Delta \chi^2 = \min_{\tilde{p}_k} \sum_{i=1}^{10} \sum_{j=1}^{16} \sum_{\alpha=\mu, e} \left[ \frac{\left( R_{ij}^\alpha(p_k) - \tilde{R}_{ij}^\alpha(\tilde{p}_k) \right)^2}{(s_{ij}^\alpha)^2 + (u_{ij}^\alpha)^2} + \frac{\left( p_k - \tilde{p}_k \right)^2}{\sigma_k} \right]. \] (13)

For the nuisance parameters there are two particular cases: \( \sigma_k = 0 \) (parameter fixed since \( \tilde{p}_k = p_k \)) and \( \sigma_k = \infty \) (\( \tilde{p}_k \) unconstrained). The \( \Delta \chi^2 \) includes possible uncorrelated systematic errors \( u_{ij}^\alpha \) (induced for instance from finite MC statistics in each bin). To minimise the \( \chi^2 \), we have chosen to use a method based on the “pull approach” [30], by assuming a first-order expansion of the \( \tilde{R}_{ij}^\alpha(\tilde{p}_k) \) under small deviations of (some) parameters \( \tilde{p}_k \) around the “true” values \( p_k \):

\[ \tilde{p}_k = p_k + \xi_k \longrightarrow \tilde{R}_{ij}^\alpha(\tilde{p}_k) \simeq \tilde{R}_{ij}^\alpha(p_k) + \xi_k \left( \frac{\partial \tilde{R}_{ij}^\alpha(\tilde{p}_k)}{\partial \tilde{p}_k} \right)_{\tilde{p}_k=p_k}. \] (14)

The linearization of the rates is exact for some parameters, for instance for normalization errors. We have checked that it is a reasonably good approximation for all the other parameters, with the exception of \( \theta_{23} \) and \( \delta \). For any value of the TH parameters \( (\sin^2 \theta_{23}, \delta) \), we vary the WH parameters \( (\sin^2 \tilde{\theta}_{23}, \tilde{\delta}) \) in the full range \([0, 1] \otimes [0, 2\pi]\). For each \( (\sin^2 \tilde{\theta}_{23}, \tilde{\delta}) \) point, we numerically calculate the derivative in Eq. (14) and then minimize the \( \chi^2 \) analytically over the linearized \( \tilde{p}_k \) variations [30]. Finally, we find the absolute \( \chi^2 \) minimum, marginalizing also on \( (\sin^2 \tilde{\theta}_{23}, \tilde{\delta}) \).

The first set of sources of systematic errors that we have considered includes errors related to the calculation of oscillation probabilities and to absolute and relative normalizations. We have assumed \( \sigma(\Delta m^2) = 2.6\% \) and \( \sigma(\sin^2 \theta_{13}) = 8.5\% \). The solar parameters \( \delta m^2 \) and \( \sin^2 \theta_{12} \) are kept fixed. The true parameter \( \delta \) is taken \( \delta = 3\pi/2 \), while it is left free \([0, 2\pi]\) in the tested hierarchy. The true parameter \( \sin^2 \theta_{23} \) is chosen in the range \([0.4, 0.6]\), while it is left free in the WH. We have assumed a 3\% error on the electron density in the Earth’s core, \( \sigma(N_e) = 3\% \) and a 15\% error \( f_N \) to all the event rates, accounting for fiducial volume, flux and cross section uncertainties. The relative normalizations between the \( \mu \) and \( e \) rates, and between the \( \nu \) and \( \bar{\nu} \) components of the rates, are allowed to differ, respectively, up to 8\% and 6\% at 1\( \sigma \) [19, 31].
Figure 3. Case of true normal hierarchy and sin²θ₂₃ = 0.6. Absolute event spectra, statistical deviations with respect to the best-fit spectrum in the wrong (inverted) hierarchy, and deviations in percent values.

Figure 3 shows the absolute spectra $R_{ij}^\mu(p_k)$ and $R_{ij}^e(p_k)$ in the favorable case of true normal hierarchy and sin²θ₂₃ = 0.6. This is a favorable case in PINGU, because for true normal hierarchy matter effects are stronger for neutrinos, which are enhanced by a larger cross section than antineutrinos. Moreover, when sin²θ₂₃ is in the second octant, $P_{\mu e}$ and hence matter effects are larger. The central panels show the statistical differences between NH spectra and the corresponding best-fit spectra in inverted hierarchy, $\tilde{R}_{ij}^\mu(p_k)$ and $\tilde{R}_{ij}^e(p_k)$, after marginalization over oscillation and normalization parameters. The right panels show the differences in terms of percent deviations.

Figure 4 shows what happens for true inverted hierarchy and for sin²θ₂₃ = 0.4. The left panels are basically indistinguishable from Fig. 3, so that the hierarchy cannot be discriminated “by eye”. The deviations in the middle and right panels are, as expected, generally smaller than in Fig. 3 and thus, in this case the hierarchy discrimination will be much more difficult.

Concerning the resolution functions we have considered two kinds of systematics, the first one of the kind
\[
E' \rightarrow E'(1 + f_E) \ , \ \sigma(f_E) = 0.05 
\]
for $\mu$ and $e$ event independently, and the second
\[
r_\alpha \rightarrow r_\alpha^\prime (1 + f_\alpha^\prime) \ , \ \sigma(f_\alpha^\prime) = 0.1 
\]
where $\alpha = \mu, e$ and $z = E, \theta$.

Besides the previous energy scale and resolution width errors, uncertainties in the effective volume and in the neutrino atmospheric fluxes may also lead to an entire set of (few) percent deviations as a function of energy and angle, which are not necessarily well known and under...
good control. Consequently, we have assumed that the observable PINGU spectra have small, additional functional uncertainties with some freedom in their shape. We have considered linear, quadratic, cubic, and quartic polynomial deformations, which introduce from 4 (linear case) to 28 (quartic case) extra degrees of freedom in the fit. The coefficients of the polynomials are allowed to float around a null central value within representative errors, that we choose at the level of 1.5% (default), or of 3% (1.5%) for doubled (halved) errors. Several terms might add up to more than ±1.5% (default), but the results of the analysis show that such a freedom is never really exploited in the fit.

Since our knowledge of the systematics may still be incomplete, we also consider in our study possible residual uncorrelated fractional uncertainties in each bin, at the level of 1.5% as default, or of 3% (1.5%) for doubled (halved) errors.

4. Results
The main results of our work are shown in Figures 5 and 6. Figure 5 shows the PINGU sensitivity to the hierarchy as a function of the time $T$ in years. The bands correspond to the results of the fit obtained by varying $\sin^2 \theta_{23}$ in the range [0.4, 0.6]. On the horizontal axis we have rescaled the time as $\sqrt{T}$, so that the ideal case of no systematic errors would by represented by a straight line. We show the results obtained by including different sets of systematic errors: oscillation and normalization uncertainties, plus energy scale and resolution width errors, plus polynomial shape systematics up to quartic terms, plus uncorrelated systematics at the default level of 1.5%. While with only normalization and systematic errors the sensitivity scales almost linearly with $\sqrt{T}$, the progressive inclusion of correlated shape systematics, both “known” (resolution scale and widths) and “unknown” (polynomial deviations), and eventually of uncorrelated shape systematics, decreases the PINGU sensitivity by up to $\sim 35\%$ ($\sim 40\%$) for 5 (10) years of data taking. Shape uncertainties at the percent level may therefore produce an important overall
Figure 5. PINGU sensitivity to the hierarchy ($N_\sigma$), for either true NH (top panels) or true IH (bottom panels), as a function of the live time $T$ in years.

Table 5 shows the fitted value $\sin^2 \theta_{23}^{\text{fit}}$ (at 1, 2 and 3 $\sigma$) as a function of the true value $\sin^2 \theta_{23}^{\text{true}} \in [0.4, 0.6]$, for the four possible cases with the test hierarchy assumed to be either the true or the wrong one, for 5 years of PINGU data and with polynomial and uncorrelated shape errors at the 1.5% level. Panel (a) refers to the case with true normal hierarchy, assumed to be unambiguously determined by PINGU. By construction, in this case, $\sin^2 \theta_{23}^{\text{fit}} = \sin^2 \theta_{23}^{\text{true}}$. The 1, 2 and 3 $\sigma$ bands give the accuracy of the $\sin^2 \theta_{23}$. Panel (c) is the analogous case for inverted hierarchy: the accuracy for $\sin^2 \theta_{23}^{\text{fit}}$ is worse and the octant degeneracy is not always resolved. Panels (b) and (d) refer to the case of misdiscrimination of the hierarchy. When the true hierarchy is the normal one, $\sin^2 \theta_{23}^{\text{fit}}$ is always higher than the true one: the larger effects induced by matter in the true NH, favor $\sin^2 \theta_{23}^{\text{fit}}$ as large as possible. The opposite happens in panel (d), for true inverted hierarchy. It is therefore mandatory to correctly determine the

In our analysis we have assumed $\sin^2 \theta_{23}^{\text{true}} \in [0.4, 0.6]$, more or less corresponding to the current $\pm 2\sigma$ allowed region. If the $\sin^2 \theta_{23}^{\text{true}}$ allowed range is reduced, for instance, to [0.46, 0.54] (as could happen with the help of the results coming from ongoing and future accelerator experiments), the $N_\sigma$ bands of Figures 5 e 6 would be significantly smaller [17]. PINGU itself would measure $\sin^2 \theta_{23}^{\text{true}}$ with improved precision, but its sensitivity to this mixing angle will be related to the prior knowledge of the true hierarchy.

In the first case, the hierarchy sensitivity can be lower than $\sim 3\sigma$ in the worst cases, even after 10 years of data taking. If shape systematics errors are instead halved, the sensitivity is always above $3\sigma$, after 10 years of data taking. Therefore, spectral shape uncertainties require a careful investigation, since they may significantly lower the PINGU sensitivity, when added to systematics due to oscillation and normalization uncertainties.

The 1, 2 and 3 $\sigma$ bands give the accuracy of the $\sin^2 \theta_{23}$. Panel (c) is the analogous case for inverted hierarchy: the accuracy for $\sin^2 \theta_{23}^{\text{fit}}$ is worse and the octant degeneracy is not always resolved. Panels (b) and (d) refer to the case of incorrect determination of the hierarchy. When the true hierarchy is the normal one, $\sin^2 \theta_{23}^{\text{fit}}$ is always higher than the true one: the larger effects induced by matter in the true NH, favor $\sin^2 \theta_{23}^{\text{fit}}$ as large as possible. The opposite happens in panel (d), for true inverted hierarchy. It is therefore mandatory to correctly determine the

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Figure 8 shows the fitted value $\sin^2 \theta_{23}^{\text{fit}}$ (at 1, 2 and 3 $\sigma$) as a function of the true value $\sin^2 \theta_{23}^{\text{true}} \in [0.4, 0.6]$, for the four possible cases with the test hierarchy assumed to be either the true or the wrong one, for 5 years of PINGU data and with polynomial and uncorrelated shape errors at the 1.5% level. Panel (a) refers to the case with true normal hierarchy, assumed to be unambiguously determined by PINGU. By construction, in this case, $\sin^2 \theta_{23}^{\text{fit}} = \sin^2 \theta_{23}^{\text{true}}$. The 1, 2 and 3 $\sigma$ bands give the accuracy of the $\sin^2 \theta_{23}$. Panel (c) is the analogous case for inverted hierarchy: the accuracy for $\sin^2 \theta_{23}^{\text{fit}}$ is worse and the octant degeneracy is not always resolved. Panels (b) and (d) refer to the case of incorrect determination of the hierarchy. When the true hierarchy is the normal one, $\sin^2 \theta_{23}^{\text{fit}}$ is always higher than the true one: the larger effects induced by matter in the true NH, favor $\sin^2 \theta_{23}^{\text{fit}}$ as large as possible. The opposite happens in panel (d), for true inverted hierarchy. It is therefore mandatory to correctly determine the
hierarchy in order to achieve a precise measurement of $\sin^2 \theta_{23}$ in PINGU. We have verified that the role of the phase $\delta$ is marginal and that its values are never constrained above the 1$\sigma$ level.

5. Conclusions

In this work we have shown that the sensitivity to neutrino mass hierarchy of the proposed PINGU experiment can be affected by shape variations, at the few percent level, of the measured neutrino spectra. Reaching such level of accuracy is not usually required to constrain the dominant oscillation parameters ($\Delta m^2$, $\theta_{23}$), and will constitute an unprecedented challenge for atmospheric neutrino experiments with very high statistics. We have included in our analysis all independent sources of uncertainties coming from models of differential atmospheric fluxes and cross sections, as well as from detector models used for event reconstruction. Furthermore, we have also taken into account residual correlated and uncorrelated systematic uncertainties. We have first studied the PINGU sensitivity to the hierarchy in the presence of the most obvious systematic errors, due to oscillation parameter and normalization uncertainties. Then, we have added in sequence shape systematics related to the resolution functions in energy and angle, generic polynomial shape deviations at the few percent level, and possible uncorrelated systematic errors at the same level. The results of our analysis show that the cumulative effect of all these shape errors can reduce the PINGU hierarchy sensitivity. Finally, we have also discussed the interplay between the PINGU sensitivity to the hierarchy and to $\theta_{23}$.

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