Baryon-poor outflows from rotating black holes

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Cosmological gamma-ray bursts are probably powered by systems harboring a rotating black hole. We show that frame-dragging creates baryon-poor outflows in a differentially rotating gap along an open magnetic flux-tube with dissipation $P = \Omega_T(\Omega_H - 2\Omega_T)A_\phi^2$, where $2\pi A_\phi$ denotes the flux in the open tube, for angular velocities $\Omega_H$ and $\Omega_T$ of the black hole and the torus, respectively. The output in photon-pair outflow will be reprocessed in an intervening hypernova progenitor wind or the ISM.

Subject headings: gamma-rays: bursts, theory

Cosmological gamma-ray bursts may be powered by rotating black holes following the collapse of young massive stars in hypernovae or the coalescence of black hole-neutron star binaries. If all black holes are produced by stellar collapse, they should be nearly maximally rotating, whose mass is relatively high. The torus is expected to form from fallback matter stalled against an angular momentum barrier or as the debris of a neutron star following tidal break-up around a Kerr black hole. The torus will provide a similarly-shaped magnetosphere from the remnant magnetic field. The magnetic field strength derives from conservation of magnetic flux and linear amplification.

GRB/afterglow studies indicate an output in ultrarelativistic baryon poor jets. In this Letter, we show that frame-dragging creates powerful photon-pair outflows in a differentially rotating gap along open magnetic flux-tubes supported by black holes in equilibrium with a surrounding torus magnetosphere. This process is continuous and involves no baryonic load. This analysis represents a non-perturbative extension of earlier calculations on pair-production about the a Wald field.

Pair-creation in magnetospheres around rotating black holes is made possible by the Rayleigh criterion, since radiation from the horizon possesses a specific angular momentum at least twice that of the black hole itself. The black hole may support open flux-tubes by an magnetic moment in equilibrium with a surrounding torus magnetosphere. This is a consequence of a no fourth-hair theorem (see below). Frame-dragging introduces Faraday potential drops across these flux-tubes. This creates a competition between local equilibration into a dissipation-free state and current-continuity as a global constraint. Here, currents are determined by slip-slip boundary conditions on the horizon and infinity. As will be shown, the outcome of this competition is dissipation in a differentially rotating gap. This results in the creation of a photon-pair outflow to infinity.

A torus surrounding a black hole faces the black hole horizon on the inside and infinity on the outside. These “two faces” are each equivalent in poloidal topology to pulsar magnetospheres with commensurate causal interactions, as shown in Fig. 1. The inner face of the torus receives energy and angular momentum, as does a pulsar when infinity wraps around it (see Fig. 2 of [25]); the outer face always loses energy and angular momentum similar to ordinary pulsars. Here, we assign Dirichlet boundary conditions on field-lines.
footed in the torus, and outgoing/ingoing radiative boundary conditions at infinity/on the horizon. The former represent no-slip boundary conditions in the non-perturbative limit of flux-tubes in equilibrium charge-separation, whereas the latter represent slip boundary conditions in the same limit. It will be appreciated that field-lines with either DR— or D\(\bar{R}\)—boundary conditions correspond to open field-lines in pulsar magnetospheres: those which pass through the light cylinder (lc) out to infinity [11]. In the equilibrium charge-separation limit, the inner and outer torus magnetospheres rotate with the approximately Keplerian angular velocity of the torus. The inner torus magnetosphere passes through an inner light surface (ils) [34] close to the horizon, as the limiting surface where particles could stay in closed orbits with the angular velocity \(\Omega_T\) of the torus. The ils and ols serve the same role as the outer light cylinder in a pulsar magnetosphere. This equivalence to pulsars indicates that the torus mediates two Poynting-flux dominated winds, one flowing into the black hole and one flowing out to infinity - each bringing about energy and angular momentum transport. If the black hole spins at the rate of the torus times a geometrical factor, either without [27] or with [30] gravitational radiation, a state of suspended accretion may result in which the net torque vanishes. The suspended accretion state hereby lasts for the life-time of rapid spin of the black hole.

In the lowest energy state, a rotating black hole surrounded by a torus magnetosphere assumes a magnetic moment \(\mu_H\). This is no fourth-hair. Indeed, we have [34,32]

\[
\mu_H = qJ_H/M \tag{1}
\]

with induced horizon flux \(\phi_H = 4\pi qM\Omega_H\) [34,36]. This may be understood to form an azimuthal current \(q\Omega_H/2\pi\) generated by corotation of a horizon charge charge \(q\). Exposed to an external uniform magnetic field aligned with its axis of rotation, the potential energy of the electromagnetic field is

\[
\mathcal{E} \approx q^2/2r_H - \mu_H B. \tag{2}
\]

The ground state assumes \(\partial\mathcal{E}/\partial q = 0\), which defines \(q^e \approx B J_H (r_H/M)\) in view of (1). (It can be shown that \(\delta(J_H/M)/\delta q = O(B^2M^2)\), which is negligible for gravitationally weak magnetic fields.) This equilibrium charge may develop by accretion [32] or by pair-creation [10]. Combined with the horizon flux in the uncharged state, the net flux becomes

\[
\Phi^e_H = 4\pi BM^2 \cos \lambda + \phi^e_H \approx 4\pi BM^2, \tag{3}
\]

where \(\sin \lambda = a/M\). Here, the numerical value on the right hand-side is given for Wald’s value \(q^e = 2BJ_H\), which results from an exact analysis in a source-free Wald-field. The energy arguments used describe black hole coupling to the poloidal magnetic field average \(B = \langle B_z \rangle\) over an \(O(M)\)-neighborhood. The result is therefore expected to be robust within a factor of unity for general, aligned magnetospheres, and is in agreement with an explicit calculation in the force-free limit [17]. In particular, it should not depend too sensitively on the behavior of magnetic winds as they enter the horizon. The magnetic moment in the magnetostatic ground state satisfies \(\mu^e_H \approx 2BMr_H^2(2M\Omega_H)^2\) which, by \(\phi_H\), produces an azimuthal component of the electromagnetic vector potential
The magnetic moment in the ground state hereby serves two purposes, particularly when the black hole rotates rapidly: it ensures strong black hole-torus coupling and it permits the horizon to support an open flux-tube infinity. The open flux-tube is endowed with conjugate radiative-radiative $RR$–boundary conditions, which become slip-slip boundary conditions in the equilibrium charge-separation limit (below). Considerable disorder (e.g.: in spectral energy-density) is expected in the external magnetic field due to turbulent shear within the torus [28]. In contrast, the open field-lines along the axis of rotation are as always well-ordered, since they are supported by $\mu_H^e$ via (4).

Open flux-tubes can be created out of closed loops between the black hole and the torus, as shown in Fig. 2. This change in topology may be compared with coronal mass ejections [31]. Thus, the magnetic flux of the open flux-tube supported by $\mu_H^e$ is equal and opposite to the magnetic flux of a surrounding flux tube supported by the surrounding torus. With reference to the the boundary conditions, we form $D\bar{R} \rightarrow DR + RR$ with fractions of magnetic flux

$$f_{DR} = f_{RR} = f_o,$$ (5)

e.g., relative to the net poloidal flux $2\pi A$ supported by the torus. This transformation is due, in part, to the flow of baryonic matter into the equatorial plane where it settles into a torus and, in part, due to bulging of the inner torus magnetosphere in response to repulsive current-current interactions in the poloidal current loop - a fast magnetosonic wave as a nonlinear feature to the powerful DC Alfvén wave in the torus magnetosphere around the black hole.

Locally, the magnetosphere tends to equilibrate into a non-dissipative state of equilibrium charge-separation. In particular, this will hold true for the inner and outer torus magnetospheres, by their equivalence to pulsar magnetospheres. Recall that a magnetosphere surrounding a pulsar with angular velocity $\Omega_{psr}$ assumes an equilibrium charge-density at the Goldreich-Julian value $\rho = -\Omega_{psr} B/2\pi$ on-axis, where $B$ denotes the poloidal magnetic field-strength [11]. Note that the near-axis field-lines in pulsar magnetospheres are endowed with $DR$–boundary conditions. In general, equilibrium flux surfaces are described by two parameters: an angular velocity and a poloidal current (see [24]). It becomes of interest to consider the competition between these local equilibrium considerations and current continuity as a global constraint.

Locally, equilibrium flux-surfaces $A_\phi$ assumes a uniform electrostatic potential $-A_t = \Omega A_\phi$ with rigid angular velocity $\Omega$ in Boyer-Lindquist coordinates (see [24]). The associated equilibrium charge-density $\rho^e$ can be calculated from Maxwell’s equations. About the axis of rotation, these give

$$\left(\sqrt{-g} g^{ac} g^{td} F_{cd}\right)_{,a} = -4\pi \sqrt{-g} j^t,$$ (6)

where $\sqrt{-g} = \rho^2 \sin \theta$, and, by evaluation in Boyer-Lindquist coordinates,

$$[\sin \theta (\xi^a A_a)_{,\theta}]_{,\theta} \simeq 4\pi \sqrt{-g} \alpha^2 j^t$$ (7)
about the axis of rotation $\theta = 0$. The equilibrium charge-density $\rho^e = \alpha^2 j^i$ (one $\alpha$ for redshift and one for volume density) as seen by zero-angular momentum observers (ZAMOs) assumes the asymptotic values

$$\rho^e = -(\Omega + \beta)B/2\pi = \begin{cases} 
(\Omega_H - \Omega_+)B/2\pi & \text{on } H, \\
-\Omega_-B/2\pi & \text{at } \infty,
\end{cases} \tag{8}$$

where $\Omega_+$ and $\Omega_-$ denote the angular velocities of the equilibrium sections attached to the horizon and infinity, respectively. This on-axis density distribution is distinct from the Goldreich-Julian charge-density in pulsar magnetospheres most notably so in a sign-change, whenever $\Omega_H - \Omega_+ > 0$ and $\Omega_- > 0$. Indeed, when $\Omega_H - \Omega > 0$, the charge-density (8) possesses an interface with $\rho^e = -(\Omega + \beta)B/2\pi = 0$ [13]. This defines an underlying $pn-$structure to the flux-tube. With $B$ parallel to $\Omega_H$, the $n-$section faces the horizon and the $p-$section faces infinity. The $pn-$structure permits the inner flux-cone to conduct a DC current towards the black hole, given a lower electrostatic potential of the $n-$section.

The current in the asymptotically ultrarelativistic flows is convection of the equilibrium charge-density (8). Thus, $j^2 \to 0$ describes the continuum limit of the $R\bar{R}-$boundary conditions, wherein drift currents are suppressed by the high Lorentz factor bulk flow upon going into the horizon [23] as well as going out to infinity [28]. This defines two current-sources in series (Fig. 3). Subject to current continuity, we have:

$$-I = (\Omega_H - \Omega_+)A_\phi = \Omega_-A_\phi. \tag{9}$$

Here, $I < 0$ refers to currents towards the black hole. Thus, $I$ generally arises from differential rotation between the $+$ and $-$ sections. The commensurate Faraday-induced potential difference

$$\Delta V = [\Omega]A_\phi = (\Omega_+ - \Omega_-)A_\phi = \Omega_HA_\phi + 2I, \tag{10}$$

across a dissipative inversion layer in differential rotation. Likewise, the asymptotic state of the outer flux-cone endowed with $DR-$boundary conditions defines a current-source at infinity. Current closure at infinity and (8) give $I = \Omega_T A_\phi$ and hence $\Omega_- = \Omega_T$. It follows that the inversion layer dissipates energy at a rate [28]

$$P = I\Delta V = \Omega_T(\Omega_H - 2\Omega_T)A_\phi^2, \tag{11}$$

provided that $\Omega_H > 2\Omega_T$. This shows that the open field-lines along the axis of rotation contain a region which is out of equilibrium as a consequence of current continuity. This manifests itself in a macroscopic photon-pair outflow. Note that this involves no baryonic load.

The sign change in the charge density in the gap introduces some common ground with outer gaps in pulsar magnetospheres [14], though the boundary conditions are different. The equilibrium charge $\rho^e$ is approximately linear in the inversion layer about the root $r = r^*$ of $\Omega + \beta^* = 0$. Here, $\Omega = (\Omega_+ + \Omega_-)/2$ and so

$$\rho^e(z) \approx -bz, \quad z = r - r^*, \tag{12}$$
where \( b = \beta^*_r \), being positive in the \( n \)-section and negative in the \( p \)-section. In the ultrarelativistic limit, the current density \( j = -n \) in the present sign convention, where \( n = n_+ + n_- \) denotes the sum of the charge densities \( e^+ \) and \( e^- \). In the stationary limit, \( j \) is constant as a function of height upon ignoring curvature drift. Since \( n_\pm \) derive from dissipation in the gap, the associated local charge density satisfies

\[
\rho_j = j \frac{[V]_{-h/2} - [V]_{h/2}}{[V]_{-h/2}^{h/2}},
\]

(13)

where \( E = -V'(z) \) is the electric field in terms of the electrostatic potential \( V \). \( \Delta V \) shall denote \([V]_{-h/2} = [\Omega]A_\phi \leq [\beta]_{h/2}A_\phi \). Thus, \( E \) satisfies Poisson’s equation \( E' = 4\pi(\rho + \rho_j) \), where \( \rho(z) = -\rho_e(z) \). The equations (12) and (13) give

\[
\rho_j \sim 2jz/h \quad (-h/2 < z < h/2).
\]

(14)

No new net charges are created and, in the linear regime (12), the net charge within the inversion layer remains zero. Hence, the electric field is non-zero only within the layer, leaving zero surface-charge density on the two virtual Faraday disks at \( z = \pm h/2 \). For the front at \( z = h/2 \), the outgoing radiation pressure \( P_r \) derives from particles moving upwards, i.e.: \( n_- = -(\rho + j)/2 \) in the present sign convention. Hence, we arrive at an output

\[
L_p = \int_{-h/2}^{h/2} n_- Edz = -\frac{1}{2}j\Delta V = \frac{1}{2}\Omega_T(\Omega_H - 2\Omega_T)A_\phi^2,
\]

(15)

using (11). Thus, one-half the power converted in the inversion layer produces \( e^\pm \gamma \)-outflow. The output power in isotropic equivalent luminosity assumes GRB values for canonical values of \( B = 10^{15} \text{G} \) in the torus magnetosphere. The Lorentz factor of the outflow will be high, whose terminal value depends, in part, on curvature radiation in the open field-lines. At the distance where the GRB takes place, it further depends on the amount of baryonic matter intercepted from the interstellar medium, as described in current theories of GRB-afterglows.

The dissipation (11) results a photon-pair outflow (13) to infinity. Observed emissions result from reprocessing in the intervening medium, notably a hypernova progenitor wind or the interstellar medium (13).

The BATSE GRB catalogue shows a bi-modal distribution of short durations of about 0.3s and long durations of about 30s (14,27). In (27), we associate this bi-modal distribution with hyper-/suspended accretion states onto slowly/rapidly spinning black holes. For black hole angular velocities \( \Omega_H > 2\Omega_T \) while sufficiently slow to assume a state of hyperaccretion, baryon poor outflows will arise similarly as in the suspended accretion state. Thus, we predict that HETE-II could detect afterglows also from short bursts, but with weak or no iron-line emissions (27). A recent analysis of the opening angle from long bursts, inferred from a break in the temporal evolution of the GRB luminosity, suggests a narrow distribution of GRB fluence around \( 5 \times 10^{50} \text{ergs} \) with a considerable spread in the half-opening angles \( \theta_j \). In (29), we attribute this to a standard half-opening angle \( \theta_H \simeq 35^\circ \) of the open flux-tube on the horizon in the presence of a geometrically thick torus. In this scenario, the spread in the
observed half-opening angles $\theta_j$ on the celestial sphere is due variations in the luminosity of
the collimating wind $[18]$. This may be tested in future observations, wherein the estimated
value of $\theta_H$ should provide a cut-off to $\theta_j$.

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Figure captions

Figure 1. Equivalence in poloidal topology of the magnetosphere of torus to that of a pulsar indicates commensurate causal interactions in a black hole-torus system in suspended accretion. The torus magnetosphere is supported by a magnetic moment density $\mu_T$ in the surrounding magnetized matter. In equilibrium, the black hole assumes a magnetic moment $\mu_H$ which supports a tube of open field-lines along its axis of rotation. These open field-lines have conjugate radiative-radiative $\mathcal{RR}$-boundary conditions or, in the equilibrium charge-separated limit, slip-slip boundary conditions on the horizon and infinity. These field-lines have no counterpart in pulsar magnetospheres. Field-lines from the torus extending to the black hole have $\mathcal{DR}$-boundary conditions, and those extending to infinity have $\mathcal{DR}$-boundary conditions. These are equivalent to open field-lines in pulsar magnetospheres. A limited number of field-lines make up closed field lines in an inner and outer bag attached to the two faces of the torus with $\mathcal{DD}$-boundary conditions, delineated by the dashed curves representing the inner light surface and the outer light cylinder. This equivalence shows that the inner face of the torus receives energy and angular momentum as does a pulsar when infinity wraps around it [31], while the outer face always loses energy and angular momentum as do ordinary pulsars. The strength of these interactions is given by the equivalent angular velocities $-\Omega_{psr} = \Omega_H - \Omega_T$ and $\Omega_{psr} = \Omega_T$, where $\Omega_H$ and $\Omega_T$ are the angular velocities of the black hole and the torus. (Reprinted from [28], ©2001, Elsevier B.V.)

Figure 2. Topological diagram of the creation of a co-axial flux-cone in a black hole-torus system. The torus supports a similarly shaped magnetosphere, which comprises upper and lower annular tubes connected to the black hole. Bulging of the these tubes in a poloidal expansion of their upper and outer layers is induced by repulsive current-current interactions – a DC fast magnetosonic wave as a nonlinear feature to strong Alfvén waves associated with the powerful interaction between the black hole and the torus. A pair of co-axial flux-tubes forms following a stretch-fold-cut on the upper annular tube with end points taken to infinity ($\alpha$). This leaves the inner and outer tubes with magnetic flux equal in magnetic and opposite in sign ($\beta$). (Reprinted from [28], ©2001, Elsevier B.V.)

Figure 3. The open flux-tube along the axis of rotation is endowed with slip-slip boundary conditions on the horizon and infinity, described by angular velocities $\Omega_+$ and $\Omega_-$, respectively. The angular velocities as observed by local zero-angular momentum observers define the equilibrium charge-separation densities ($\mathcal{R}$). Ultrarelativistic flows introduce $j^2 \to 0$ on the horizon ($\mathcal{R}$) and to infinity, which introduces current sources $I_+ = (\Omega_H - \Omega_+)A_\phi$ and $I_- = \Omega_-A_\phi$, where $2\pi A_\phi$ denotes the flux in the flux-tube. Current continuity imposes the constraint $I_+ = I_-$. Differential rotation $\Omega_+ \neq \Omega_-$ hereby gives rise to a Faraday-induced potential drop between these lower and upper sections.
\[ -\Omega_{psr} \overset{\wedge}{=} \Omega_H - \Omega_T \quad \Omega_{psr} \overset{\wedge}{=} \Omega_T \]
FIGURE 2

(α)

(β)
\[ j^2 = 0 \]

\[ -(\Omega_H - \Omega_+) \]

\[ \Omega_+ \]

\[ \Omega_- \]

\[ I_- \]

\[ I_+ \]

FIGURE 3