Online Service Caching and Routing at the Edge with Switching Cost

Siqi Fan, I-Hong Hou
Texas A&M University
College Station, USA
{siqifan, ihou}@tamu.edu

Van Sy Mai, Lotfi Benmohamed
National Institute of Standards and Technology
Gaithersburg, USA
{vansy.mai, lotfi.benmohamed}@nist.gov

Abstract—This paper studies a problem of jointly optimizing two important operations in mobile edge computing: service caching, which determines which services to be hosted at the edge, and service routing, which determines which requests to be processed at the edge. We aim to address several practical challenges, including limited storage and computation capacities of edge servers, delay of reconfiguring edge servers, and unknown future request arrival patterns. To this end, we formulate the problem as an online optimization problem, in which the objective function includes both the costs of forwarding requests, processing requests, and reconfiguring edge servers. By leveraging a natural timescale separation between service routing and service caching, namely, the former happens faster than the latter, we propose an online two-stage algorithm and its randomized variant. Both algorithms have low complexity and our fractional solution achieves sublinear regret. Simulation results show that our algorithms significantly outperform other state-of-the-art policies, including one that assumes the knowledge of all future request arrivals.

Index Terms—Edge Network, Online Optimization, Service Caching, Service Routing

I. INTRODUCTION

A growing challenge for mobile computing is the proliferation of data/computation-intensive and delay-sensitive applications, such as cognitive assistance, real-time video/audio processing, and augmented reality (AR). On the one hand, running these applications completely within mobile devices may be infeasible due to the limited computation, storage, and battery capacity of such devices. On the other hand, offloading computation tasks of these applications to remote data centers may result in excessive end-to-end latency and hence poor user experience.

Such a dilemma has given rise to the popularity of mobile edge computing [1], [2]. In mobile edge computing, edge servers are deployed close to wireless base stations. These servers can host some popular services and process the corresponding computation tasks directly without having to forward them to remote data centers. Due to their close proximity to end users, edge servers are able to provide these services with much lower latency.

Despite the obvious advantage of mobile edge computing, there remain multiple important challenges that need to be addressed. First, edge servers can often host only a small number of services and process requests at a moderate rate due to their resource constraints. Second, mobile users generate requests for services in arbitrary, and typically time-varying, patterns. Without knowledge of future requests, an edge server needs to make decisions on which services to host (or cache) and whether to process a request locally at the edge. Third, it is typically time-consuming and expensive for an edge server to change the set of services it hosts, which would involve downloading all necessary data from a remote data center and setting up appropriate virtual machines or containers. The switching cost of changing cached services needs to be explicitly taken into account.

Most existing works only focus on one or two challenges above. Some studies assume that the edge server has infinite computation power, and therefore only address the caching problem. For example, Cao et al. [3] maximize the expected revenue of a service provider by optimizing the content caching strategy. Krolikowski et al. [4] aim to maximize the traffic offloading with minimum caching cost based on the distribution of requests. Bao et al. [5] propose a policy that jointly optimizes file caching probabilities and the request limit. Zhao et al. [6] [7] study an online caching problem by explicitly taking the switching cost into account. Many other studies focus on request processing and offloading, but not service caching. For instance, Wang et al. [8] propose an algorithm that optimize the edge cloud resource allocation. Li et al. [9] propose an optimization framework that minimizes energy consumption and response delay in mobile edge computing. Some other papers consider joint designs of service caching and request processing under the assumption that the request arrival patterns are either predictable or follow a certain stationary random process. For example, Ning et al. [10] and Xu et al. [11] propose online algorithms for optimizing service caching and request routing based on a Lyapunov optimization framework.

In this paper, we aim to address all three challenges by proposing online algorithms for (1) dynamic service caching, which determines the set of services to be hosted at the edge, and (2) service routing, which determines whether to process a request at the edge or route it to a data center. For the design and analysis of online algorithms, we propose an analytical model that jointly considers the unknown future requests, the storage capacity at the edge server, the load-dependent queueing and processing delay of the edge server, and the switching cost of modifying the set of cached services.
We note that there is a natural timescale separation between service caching and service routing, where the former is a much slower operation than the latter. Using this observation, we formulate a two-stage online optimization problem where the decision of service caching is updated periodically, while the decision of service routing can be changed in real time. The two-stage structure without future requests patterns differentiates our online optimization problem from most existing ones.

To solve this two-stage online optimization problem, we employ a fractional relaxation to turn the original problem into a convex one. We then propose a two-stage online policy. Our policy consists of two parts: The first part is a low-complexity algorithm that finds the optimal service routing in real time given the current service caching decision, and the second part is another low-complexity algorithm that updates the service caching decisions periodically by taking into account how service routing will react to future request arrivals. We theoretically prove that, even after taking the switching cost into account, our policy still achieves sublinear regret.

Furthermore, to make the fractional solution from our two-stage online policy implementable, we propose a randomized online algorithm. Under this randomized online algorithm, the probability that the edge server caches a service is the same as the fractional solution by the two-stage online policy. In addition, we prove that the switching cost of this randomized online algorithm is at most twice the switching cost of the two-stage online policy.

Our online algorithms are evaluated through simulations under various of scenarios. We compare them against two other algorithms, including an offline algorithm that knows all the request arrivals in advance. Simulation results show that our randomized algorithm performs much better than the other online algorithm, and performs virtually the same as the offline algorithm.

The rest of the paper is organized as follows: Section II presents our system model and formulates the two-stage online problem. Section III gives an overview of our solutions with sublinear regret and addresses some crucial challenges. Section IV provides our detailed solutions for the online problem. Section V presents a randomized integral solution that ensures sublinear regret and addresses some crucial challenges. Section VI shows our simulation results under a variety of scenarios. Finally, Section VII concludes the paper.

II. SYSTEM MODEL

A. System Overview

We consider an edge system with a backhaul connection. This edge system includes multiple clients, an edge server and remote data centers. Clients generate requests for different services according to some unknown and unpredictable patterns, and then send these requests to the edge server. We use $N$ to denote the total number of different services. The edge server may cache some services and process some requests for these services locally, while forwarding the remaining requests to remote data centers. Requests processed at the edge encounter a processing latency due to the limited computation capacity of the edge server, while requests forwarded to remote data centers encounter a forwarding latency due to network latencies. The goal of the edge server is to determine which services to be cached and which requests to be processed at the edge so as to reduce the total latencies experienced by all requests. Fig. 1 illustrates the topology of the system.

There are two practical challenges for service caching that makes this problem significantly different from traditional data caching. First, due to the limited computation capacity of the edge server, the processing latency increases as more requests are processed at the edge. As a result, the edge server may need to forward some requests for services it caches, especially when it is overloaded. Second, retrieving a new service can be a costly operation, which typically involves downloading codes and databases and setting up virtual machines or containers. These challenges need to be explicitly addressed for the solutions to be practical.

B. Service Caching and Processing

We assume that time is slotted and the system runs for $T$ time slots. Each time slot is denoted by $t = 1, 2, \ldots, T$. The duration of a time slot is chosen so that, in any given time slot, the patterns for the service requests (originating from the different clients) remain roughly the same.

Since the clients’ requests pattern may be different in different time slots, the edge server may change the services it caches in each time slot. As discussed earlier, changing cached services at the edge is a costly and slow process. Thus, we assume that the edge server can only change the services it caches at the beginning of each time slot.

Let $x_{n,t} \in \{1, 0\}$ be a binary decision variable that indicates whether the edge server will cache service $n$ at the beginning of the time slot $t$, and let $X_t := [x_{1,t}, x_{2,t}, \ldots, x_{N,t}]$. To take into account the edge server’s limited storage capacity, we assume that the edge server can cache at most $Z$ services, i.e.,

$$\sum_{n=1}^{N} x_{n,t} \leq Z, \quad \forall t.$$  \hspace{1cm} (1)

We call the problem of determining $X_t$ the service caching problem.
After the edge server determines $X_t$ at the beginning of time slot $t$, it observes the requests from clients and calculates the request arrival rate of each service. We use $\lambda_{n,t}$ to denote the request arrival rate of service $n$ in time slot $t$, and let $\Lambda_t := [\lambda_{1,t}, \lambda_{2,t}, \ldots, \lambda_{N,t}]$. We assume that an upper bound $W$ on the total arrival rate is known, that is,

$$\sum_{n=1}^{N} \lambda_{n,t} \leq W, \quad \forall t. \quad (2)$$

During each time slot $t$, the edge server needs to decide which requests to be processed locally. Due to the limited computation power of the edge server, it may not be desirable to process all requests for services that it caches. For a service $n$, the edge server will process a fraction $y_{n,t}$ of the requests locally, and forward the remaining $(1 - y_{n,t})$ portion of the requests to the data center. Since the edge server can only process requests whose corresponding services have already been cached at the edge, we require that

$$y_{n,t} \leq x_{n,t}, \quad \forall n, \quad \forall t. \quad (3)$$

Let $Y_t := [y_{1,t}, y_{2,t}, \ldots, y_{N,t}]$. We call the problem of determining $Y_t$ the service routing problem. Since the edge server can adjust service routing in real time, we consider that the edge server determines $Y_t$ after it observes $\Lambda_t$.

C. Cost and Problem Formulation

The goal of the edge server is to minimize the total cost of the system, which consists of switching cost and latency cost.

The switching cost refers to the operation cost incurred when the edge server changes the set of services being cached at the beginning of each time slot. Since the total switching cost relates to the number of services the edge server changes, we assume that every cached service change incurs a cost of $\beta$. Hence, the total switching cost over $T$ time slots is

$$\beta \sum_{t=1}^{T} \left\| X_t - X_{t-1} \right\| = \beta \sum_{t=1}^{T} \sum_{n=1}^{N} \left[ x_{n,t} - x_{n,t-1} \right].$$

The latency cost refers to the total latency experienced by all requests. In the system, when a request is forwarded to the remote data center, it experiences a forwarding latency, which is denoted as $d_n$ for service $n$. When a request is processed at the edge, it experiences a computation latency due to the limited computation power of the edge server. It is reasonable to assume that the per-request computation latency at the edge depends on the total computation load, and can be described by a convex, increasing, and differentiable function $C(\cdot)$ with $0 \leq C(0) \leq d_n, \forall n$ and $\lim_{s \to -\infty} C(s) = -\infty$. The assumption $C(0) < d_n$ indicates that the computation latency is smaller than the forwarding latency when the edge server is lightly loaded. Since the total computation load at the edge server is $\sum_{n=1}^{N} \lambda_{n,t} y_{n,t}$, the total latency of all requests can be written as

$$L_t(Y_t) := \sum_{n=1}^{N} \lambda_{n,t} y_{n,t} C\left( \sum_{n=1}^{N} \lambda_{n,t} y_{n,t} \right)$$

$$+ \sum_{n=1}^{N} \lambda_{n,t} (1 - y_{n,t}) d_n, \quad (4)$$

Under the above assumptions on the function $C$, it can be verified that $L_t$ is a convex function on $\mathbb{R}_+^{N}$.

As a result, the total cost over whole $T$ time slots can be written as $\sum_{t=1}^{T} \left( L_t(Y_t) + \beta \left\| X_t - X_{t-1} \right\| \right)$. The edge server aims to find $[X_1, X_2, \ldots, X_T]$ and $[Y_1, Y_2, \ldots, Y_T]$ that minimize the total cost. To make the optimization problem convex, we further relax the integer constraint on $x_{n,t}$ and allow $x_{n,t}$ to be any real number in $[0, 1]$. After the relaxation, the (offline) problem of minimizing the total cost can be written as follows

$$\min_{[X_t][Y_t]} \sum_{t=1}^{T} \left( L_t(Y_t) + \beta \left\| X_t - X_{t-1} \right\| \right), \quad (5)$$

subject to

$$0 \leq x_{n,t} \leq 1, \quad \forall n, \forall t, \quad (6)$$

$$0 \leq y_{n,t} \leq x_{n,t}, \quad \forall n, \forall t, \quad (7)$$

$$\sum_{n=1}^{N} x_{n,t} \leq Z, \quad \forall t. \quad (8)$$

Here, note that a fractional $x_{n,t}$ can be interpreted as the probability that the edge server caches service $n$ in time slot $t$. In Section V, we will propose a randomized algorithm such that the probability of caching $n$ in time slot $t$ is exactly $x_{n,t}$.

While the above problem is a standard convex optimization problem, solving it requires the knowledge of all request arrival rates in every time slot, that is, all $\Lambda_t$, in advance. In practice, however, the edge server needs to make service caching and routing decisions, which is $X_t$ and $Y_t$, without the knowledge of future arrival rates. Moreover, as noted before, the service caching problem and the service routing problem operate on different timescales. The edge server needs to decide $X_t$ at the beginning of each time slot $t$, without any knowledge about arrival rate $\Lambda_t$ in this time slot. In contrast, as changing routing decisions is a fast operation and the request arrival rates remain the same within a time slot, the edge server can decide the value of $Y_t$ after observing the first few requests in a time slot to estimate the arrival rate $\Lambda_t$.

Based on these observations, we formulate an online service caching and routing problem as shown below. An algorithm for this problem is called an online algorithm.

**Online Service Caching and Routing Problem**

for $t = 1, 2, \ldots, T$ do:

choose $X_t$

observe $\Lambda_t$\n
choose $Y_t$\n
receive cost $L_t(Y_t) + \beta \left\| X_t - X_{t-1} \right\|$.

The performance of an online algorithm is evaluated by comparing its cost against the cost of the optimal offline algorithm that knows all $\Lambda_t$ in advance. We assume that the optimal offline algorithm needs to choose a fixed solution for service caching, but can change its solution for service routing dynamically, as the latter is a fast operation. We formally define the optimal offline algorithm as follows:

**Definition 1 (Optimal offline algorithm):** The optimal offline algorithm is the algorithm that, after knowing $\Lambda_t$ for all $t \in$
[1,T], chooses a non-negative vector \( X^o := [x_1^o, x_2^o, \ldots, x_N^o] \) with \( x_n^o \leq 1 \) and \( \sum_{n=1}^{N} x_n^o \leq Z \), and \( T \) non-negative vectors \( Y_1^o, Y_2^o, \ldots, Y_T^o \) with \( y_n^{o,t} \leq x_n^o \), that minimize \( \sum_{t=1}^{T} L_t(Y_t^o) \).

The goal of this paper is to find an online algorithm with provably small regret under any sequence of arrival rates. The regret is defined as the difference of cost under the online algorithm and that under the optimal offline algorithm, given a sequence of arrival rates \( \lambda_1, \lambda_2, \ldots, \lambda_T \). Specifically, let \( \hat{X} := [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_T] \) and \( \hat{Y} := [\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_T] \) be the solutions produced by an online algorithm, then the regret of this online algorithm is denoted by

\[
\text{Reg}(\hat{X}, \hat{Y}) := \sum_{t=1}^{T} (L_t(\hat{Y}_t) - L_t(Y_t^o) + \beta\|\hat{X}_t - \hat{X}_{t-1}\|_1).
\]

III. Solution Overview

In this section, we describe a framework for solving the online service caching and routing problem based on the well-known online gradient descent method. We also highlight challenges that need to be addressed before we can employ this framework.

We first consider the service routing problem, which entails finding \( Y_t \) after \( X_t \) is fixed and \( \Lambda_t \) is observed. Since \( X_t \) and \( \Lambda_t \) are known, finding the optimal \( Y_t \) is equivalent to solving the following standard convex optimization problem:

\[
G_t(X_t) := \min_{Y_t} L_t(Y_t), \quad \text{s.t.} \quad 0 \leq y_{n,t} \leq x_{n,t}, \forall n, \forall t. \tag{9}
\]

We first show that \( G_t \) defined above is a convex function by using the following lemma; see e.g., [12, §3.2.5].

**Lemma 1:** If a function \( l(x,y) \) is convex in \( x \) and \( y \), and \( D \) is a convex set, then the function \( g(x) := \inf_{y \in D} l(x,y) \) is convex in \( x \), provided that \( g(x) > -\infty \) for all \( x \).

**Lemma 2:** \( G_t(X_t) \) is convex in \( X_t \).

**Proof:** Since \( L_t(Y_t) \) is a convex function, the set \( \{X_t, Y_t | 0 \leq y_{n,t} \leq x_{n,t}, \forall n, \forall t \} \) is convex, and \( G_t(X_t) > -\infty \), we conclude that \( G_t(X_t) \) is convex based on Lemma 1.

As a result, the online service caching problem, which entails finding \( X_t \) before \( \Lambda_t \) is revealed, can be expressed as the following online convex optimization problem.

**Algorithm 1** Online Gradient Descent with Routing (OGDR)

- **Initialize:** \( \eta_t, \theta_1 \leftarrow 0 \)
- **for** \( t = 1, 2, \ldots, T \)**
  - \( X_t \leftarrow \) the Euclidean projection of \( \eta\theta_t \) onto the set \( \{X_t | 0 \leq x_{n,t} \leq 1, \sum_{n=1}^{N} x_{n,t} \leq Z\} \)
  - \( Y_t \leftarrow \min_{y_{n,t} \leq x_{n,t}, \forall n} L_t(Y_t) \)
  - \( \theta_{t+1} = \theta_t - \nabla G_t(X_t) \)

We now establish a regret upper bound for OGDR.

**Theorem 1:** If \( \|\nabla G_t(X_t)\|_2^2 \) is upper-bounded by \( H \) for all \( t \), then, by choosing \( \eta = \sqrt{\frac{Z}{2T(H + \beta \sqrt{NH})}} \), the regret of OGDR is bounded by \( \sqrt{2ZT(H + \beta \sqrt{NH})} \).

**Proof:** The proof of upper bound of the regret is similar to the proofs in [13]. The main difference is that we also need to incorporate the switching cost.

Recall that \( X^o \) is the optimal offline solution for the service caching problem. Let \( X^* := [X_1^*, X_2^*, \ldots, X_T^*] \) be the solutions produced by OGDR and \( Y^* := [Y_1^*, Y_2^*, \ldots, Y_T^*] \) be the solutions of (9). The regret is then defined by

\[
\text{Reg}(X^*, Y^*) := \sum_{t=1}^{T} (G_t(X_t^*) - G_t(X^o) + \beta\|X_t^* - X_{t-1}^*\|_1).
\]

We first obtain a bound for \( \sum_{t=1}^{T} (G_t(X_t^*) - G_t(X^o)) \).

By Corollaries 2.13 and 2.17 in [13] and the definition of \( H \), we have

\[
\sum_{t=1}^{T} (G_t(X_t^*) - G_t(X^o)) \leq \frac{\eta}{2\eta} + \eta TH. \tag{11}
\]

Next, we obtain a bound on the switching cost. We have

\[
\sum_{t=1}^{T} \beta\|X_t^* - X_{t-1}^*\|_1 \\
\leq \beta \sum_{t=1}^{T} \sqrt{N}\|X_t^* - X_{t-1}^*\|_2 \quad \text{(Cauchy–Schwarz inequality)} \\
\leq \beta \sqrt{N} \sum_{t=1}^{T} \|Y_t - \theta_t\|_2 \quad \text{(X_t^* is the projection of \( \eta\theta_t \))} \\
\leq \beta \sqrt{N} \sum_{t=1}^{T} \|\nabla G_t(X_t^*)\|_2 \quad \text{(by Alg. 1)} \\
\leq \eta \beta T \sqrt{NH} \quad \text{(by definition of \( H \)).} \tag{12}
\]

Thus, choosing \( \eta = \sqrt{\frac{Z}{2T(H + \beta \sqrt{NH})}} \), we have:

\[
\text{Reg}(X^*, Y^*) \leq \sqrt{2ZT(H + \beta \sqrt{NH})}. \tag{13}
\]

**Theorem 1** shows that OGDR can achieve a regret bound that is sublinear with \( T \). However, there are two major challenges that need to be addressed before one can implement OGDR.

- Running OGDR requires solving the routing problem in (9)–(10) and finding \( \nabla G_t(X_t) \), both of which depend on \( \Lambda_t \). Since these two terms cannot be calculated until the edge server observes the request arrival rates, we need low-complexity algorithms for calculating them.

In particular, we note that \( G_t(X_t) \), being the infimum of \( L_t(Y_t) \), might not have a closed-form expression,

**Online Service Caching at the Edge**

- for \( t = 1, 2, \ldots, T \)
  - choose \( X_t \)
  - determine \( G_t(X_t) \)
  - receive cost \( G_t(X_t) + \beta\|X_t - X_{t-1}\|_1 \)

There exists many online algorithms for solving the above online convex optimization problem. In this paper, we employ the Online Gradient Descent with Lazy Projections for finding \( X_t \). Combining it with the solution of \( Y_t \), we propose the following Online Gradient Descent with Routing (OGDR) algorithm. In the pseudocode, \( \nabla G_t(X_t) \) is the subgradient of the function \( G_t(X_t) \), \( \eta \) is the step size, and \( \theta_t = [\theta_{1,t}, \theta_{2,t}, \ldots, \theta_{N,t}] \) is an internal vector with \( \theta_1 = 0 \).
and therefore its gradient can be difficult to characterize. We also need to characterize the upper-bound of \( \| \nabla G_t(X_t) \|_2^2 \).

- Under OGDR, \( X_t \) can be fractional. Of course, most practical systems cannot cache a partial service. A commonly-used approach of obtaining integer solutions from fractional \( X_t \) is to employ randomized algorithm and cache a service \( n \) with probability \( x_{n,t} \) independently from any prior events. However, such an approach would result in a switching cost of \( \beta \sum_n x_{n,t}(1 - x_{n,t-1}) + (1 - x_{n,t})x_{n,t-1} \), which can be much larger than \( \beta \| X_t - X_{t-1} \|_1 \). We need randomized algorithms whose switching cost is close to \( \beta \| X_t - X_{t-1} \|_1 \).

We address these two challenges in the next two sections.

IV. LOW COMPLEXITY ALGORITHMS FOR OGDR

In this section, we develop a low-complexity algorithm for implementing OGDR.

A. Algorithm for Service Routing

Without loss of generality, we assume that \( d_1 \geq d_2 \geq \cdots \geq d_N \). Recall that the service routing problem is

\[
\begin{align*}
\min_{Y_t} & \quad L_t(Y_t), \\
\text{s.t.} & \quad y_{n,t} \leq x_{n,t}, \quad \forall n, \forall t, \\
& \quad 0 \leq y_{n,t}, \quad \forall n, \forall t,
\end{align*}
\]

where the latency cost is given in (4) with partial derivatives computed as \( \frac{\partial L_t(Y_t)}{\partial y_{n,t}} = \lambda_{n,t}(J_t(Y_t) - d_n) \), where

\[
J_t(Y_t) = C \bigg( \sum_{n=1}^{N} \lambda_{n,t}y_{n,t} \bigg) + \sum_{n=1}^{N} \lambda_{n,t}y_{n,t}C' \bigg( \sum_{m=1}^{N} \lambda_{m,t}y_{m,t} \bigg).
\]

In the following, we show that this routing problem can be solved efficiently by using its KKT conditions. Let \( \nu_n \) and \( \mu_n \) be the Lagrange multipliers associated with (15) and (16), respectively. Thus, the KKT conditions for (14) – (16) are

\[
\begin{align*}
\lambda_{n,t}(J_t(Y_t) - d_n) - \mu_n + \nu_n &= 0, \quad \forall n, \quad (17) \\
\nu_n(y_{n,t} - x_{n,t}) &= 0, \quad \forall n, \quad (18) \\
\mu_n(-y_{n,t}) &= 0, \quad \forall n, \quad (19) \\
\mu_n &\geq 0, \quad \forall n, \quad (20)
\end{align*}
\]

Using the property that \( J_t(Y_t) - d_1 \leq J_t(Y_t) - d_2 \leq \cdots \leq J_t(Y_t) - d_N \) for all \( Y_t \), we propose Alg. 2 below to find \( Y_t, \mu_n, \) and \( \nu_n \) that satisfy the KKT conditions, and thereby solving the service routing problem.

For the algorithm above, we have following results.

**Theorem 2:** Alg. 2 produces the optimal solution for the problem (14) – (16).

**Proof:** It is obvious that Alg. 2 satisfies conditions (17) – (19). Thus, we only need to show that it also satisfies the condition (20). In particular, we need to prove the following two claims: If \( y_{n,t} = x_{n,t} > 0 \), then \( J_t(Y_t) \leq d_n \). If \( y_{n,t} = 0 \) and \( x_{n,t} > 0 \), then \( J_t(Y_t) \geq d_n \).

We first consider the case \( y_{n,t} = x_{n,t} > 0 \). Let \( m^* \) be the last service that chooses \( y_{m^*,t} > 0 \). Since \( y_{n,t} > 0 \), we have \( m^* \geq n \) and hence \( d_{m^*} \leq d_n \). By the design of Steps 1 – 9 in Alg. 2, we have \( J_t(Y_t) \leq d_{m^*} \leq d_n \) when Steps 1 – 9 are completed. This proves the first claim.

Next, we consider the case \( y_{n,t} = 0 \) and \( x_{n,t} > 0 \). By the design of Steps 1 – 9 in Alg. 2, we have \( J_t(Y_t) \geq d_n \) by the end of the \( n \)-th iteration of the for loop. Since \( Y_t \) can only be increased in latter iterations and \( J_t(Y_t) \) is an increasing function, we have \( J_t(Y_t) \geq d_n \) when Steps 1 – 9 are completed. This proves the second claim.

In addition to solving the service routing problem, Alg. 2 also produces \( \nabla G_t(X_t) \).

**Lemma 3:** Let \( Y_t^* \) be the solution given by Alg. 2. A subgradient of \( G_t \) at \( X_t \) is \( \nabla G_t(X_t) = [\frac{\partial G_t(X_t)}{\partial x_{t,1}}, \frac{\partial G_t(X_t)}{\partial x_{t,2}}, \ldots, \frac{\partial G_t(X_t)}{\partial x_{t,N}}] \), where

\[
\frac{\partial G_t(X_t)}{\partial x_{n,t}} = -\nu_n = \begin{cases} 
\lambda_{n,t}(J_t(Y_t^*) - d_n), & \text{if } y_{n,t} = x_{n,t}, \\
0, & \text{otherwise.}
\end{cases}
\]

Moreover, we have the following bound:

\[
\| \nabla G_t(X_t) \|_2^2 \leq W^2 d_t^2
\]  \hspace{1cm} (22)

**Proof:** Eq. (21) is from [12, §5.6]. Then, Eq. (22) holds because \( \| \nabla G_t(X_t) \|_2^2 = \sum_{n=1}^{N} \nu_n^2 \leq \sum_{n=1}^{N} \lambda_{n,t}^2 d_n^2 \leq W^2 d_t^2 \).

In summary, Alg. 2 produces both \( \arg \min_{Y_t} L_t(Y_t) \) and \( \nabla G_t(X_t) \). Thus, it can be used to implement Alg. 1.

**B. Regret and Complexity Analysis**

Since we have \( \| \nabla G_t(X_t) \|_2^2 \leq W^2 d_t^2 \), we can obtain the following regret bound.
Theorem 3: By choosing \( \eta = \sqrt{\frac{2TW d_1(W d_1 + \beta \sqrt{N})}{Z}} \), the regret of applying Alg. 1 and Alg. 2 is bounded by \( \sqrt{2ZW d_1(W d_1 + \beta \sqrt{N})} \).

Proof: This result follows directly from Theorem 1 and Lemma 3 with \( H = W^2 d_1^2 \).

Next, we analyze the complexity of Alg. 1. It is easy to see that the complexity of Alg. 2, which implements steps 3 and 4 of Alg. 1 is \( O(N) \) per time slot.

It remains to determine the complexity of step 2 in Alg. 1, which entails finding the projection of a vector \( \eta \theta_t \) onto the set \( \{X \in \mathbb{R}^N \mid 0 \leq x_n \leq 1, \sum_{n=1}^N x_n \leq Z \} \). To this end, for any vector \( \eta \theta_t \), define \( p(\eta \theta_t) \) as the projection of \( \eta \theta_t \) onto \([0, 1]^N\), i.e., \( p_n(\eta \theta_t) = \min\{\max\{\eta \theta_{n,t}, 0\}, 1\} \). It then follows from [14, p. 150] that we can compute \( X_t \) as

\[
X_t = \begin{cases} 
  \{p(\eta \theta_t), \text{ if } \sum_{n=1}^N p_n(\eta \theta_t) \leq Z, \\
  p(\eta \theta_t) - \gamma^* 1, \text{ otherwise,} 
\end{cases}
\]

where \( \gamma^* \) is any positive root in the interval \([0, \max_n \{\eta \theta_{n,t}\}]\) of the nonincreasing function

\[
\phi(\gamma) = \sum_{n=1}^N p_n(\eta \theta_t - \gamma^* 1) - Z.
\]

This can be done efficiently using a bisection method. Thus, it remains to bound \( p(\eta \theta_t) \). Note from line 4 of Alg. 1 and (21) that \( \|\theta_t\|_\infty \leq \sum_{k=1}^T \|\nabla G_k(X_k)\|_\infty \leq \sum_{k=1}^T \max\{\lambda_n, \eta \theta_{n,k}\}, 1 \leq n \leq N, 1 \leq k \leq T \} \leq TW d_1 \). Thus, given a desired root finding accuracy \( \varepsilon > 0 \), the number of bisection steps is bounded by \( O(\log \frac{\eta TW d_1}{\varepsilon}) \), where each step costs only \( O(N) \).

Let us now show that using this approximate projection in Alg. 1 adds only a negligible error to our regret bound. Following the proof of Theorem 1 and considering approximation errors, the bounds in (11) and (12) are replaced by

\[
\begin{align*}
  \sum_{t=1}^T (G_t(X_t^*) - G_t(X^*) ) &\leq \frac{Z \varepsilon}{2T} + \eta TH \leq \frac{Z \varepsilon}{2T} + \eta TH, \\
  \sum_{t=1}^T \beta \|X_t^* - X_{t-1}^*\| &\leq \beta \sqrt{N} \sum_{t=1}^T (\|\eta \theta_t - \eta \theta_{t-1}\|_2 + 2 \varepsilon \sqrt{\frac{N}{2T} \beta^2}) \\
  &\leq \beta \eta T \sqrt{NH} + 2N T \beta \varepsilon.
\end{align*}
\]

Take \( \varepsilon = \frac{2T \beta}{\sqrt{2} \sqrt{N}} \), where \( \varepsilon' \) is an arbitrary small constant, the regret bound then becomes \( \text{Reg}(X^*, Y^*) \leq \sqrt{2(Z + 1) T (H + \beta \sqrt{N H})} + \varepsilon' \). Note that the number of bisection steps is \( O(\log \frac{\eta T^2 W N d_1^3}{\varepsilon}) \).

Thus, we conclude that complexity of Alg. 1 with approximate projection is \( O(N \log \frac{\eta T^2 W N d_1^3}{\varepsilon}) \) per time slot.

V. RANDOMIZED ALGORITHM FOR SERVICE CACHING

The online algorithms for finding \( X_t \) as proposed in Alg. 1 may produce fractional solutions. When \( X_t \) is fractional, the value \( x_{n,t} \) can be interpreted as the probability that the edge server caches service \( n \) at time \( t \). In this section, we propose a randomized algorithm that satisfies this probability interpretation while guaranteeing a provably small switching cost.

A. Randomized Algorithm

The basic idea of our randomized algorithm is to simultaneously maintain \( K \) sample paths, where each sample path represents a probability mass of \( \frac{1}{K} \). We then quantize each \( x_{n,t} \) into a multiple of \( \frac{1}{K} \). Specifically, let \( X^Q \) be the quantized version of \( X_t \), then we require that \( K x^Q_{n,t} \) to be a non-negative integer and \( \sum_n x^Q_{n,t} \leq Z \).

Let \( r_{k,n,t} \) be the indicator function that service \( n \) is cached at the edge at time \( t \) in sample path \( k \). Let \( R_k,t \) be the vector \( [r_{k,1,t}, r_{k,2,t}, \ldots] \). In every time slot \( t \), our randomized algorithm receives \( X^Q_t \) from Alg. 1. The randomized algorithm then constructs \( R_{k,t} \) based on \( X^Q_t \) and \( R_{k,t-1} \) to ensure three properties: First, the probability of caching service \( n \) is indeed \( x^Q_{n,t} \), that is, \( \sum_k r_{k,n,t} = K x^Q_{n,t} \). Second, the storage capacity constraint is satisfied for all sample paths, that is, \( \sum_n r_{k,n,t} \leq Z, \forall k \). Third, the expected switching cost, which can be expressed as \( \frac{1}{K} \sum_k \|R_k,t - R_{k,t-1}\|_1 \), is bounded. Let \( \Delta_t := \|\delta_{t,1}, \delta_{t,2}, \ldots, \delta_{t,K}\|_1 \) be the difference between \( X^Q_t \) and \( X_{t-1}^Q \). Alg. 3 shows the complete randomized algorithm, including all decisions on service caching and routing.

Algorithm 3 Randomized Service Caching and Routing (RSCR)

**Initialize:** \( K, R_{k,0} \leftarrow 0, \forall k \)

1: Choose \( k^* \) uniformly from \( \{1, 2, \ldots, K\} \).
2: for \( t = 1, 2, \ldots, T \) do
3: //Service Caching
4: Obtain \( X^Q_t \) from Alg. 1.
5: \( R_{k,t} \leftarrow R_{k,t-1}, \forall k. \)
6: \( \Delta_t \leftarrow X^Q_t - X^Q_{t-1}. \)
7: for \( n = 1, 2, \ldots, N \) do
8: if \( \delta_{n,t} > 0 \) then
9: Randomly choose \( K \delta_{n,t} \) sample paths with \( r_{k,n,t} = 0 \), and set \( r_{k,n,t} = 1 \) for them.
else if \( \delta_{n,t} < 0 \) then
10: Randomly choose \( |K \delta_{n,t}| \) sample paths with \( r_{k,n,t} = 1 \), and set \( r_{k,n,t} = 0 \) for them.
end if
end for
end while
14: while \( \exists k \) such that \( \sum_n r_{k,n,t} > Z \) do
15: Find one sample path \( k' \) with \( \sum_n r_{k',n,t} < Z \).
16: Find one service \( n \) with \( r_{k,n,t} = 1 \) and \( r_{k',n,t} = 0 \).
17: Set \( r_{k,n,t} = 0 \) and \( r_{k',n,t} = 1 \).
end while
19: Cache all services with \( r_{k,n,t} = 1 \).
20: //Service routing
21: Observe \( \Lambda_t \).
22: Obtain \( Y_t \) from Alg. 2 by setting \( X_t = R_{k^*,t}. \)
23: Process \( y_{n,t} \) of requests from \( n \) at the edge.
end for

By the design of Alg. 3, we obviously have the first two properties. We show below that Alg. 3 also enjoys a provably small expected switching cost.
B. Performance Analysis

First, we consider the influence of Alg. 3 on the switching cost, which is shown below.

Theorem 4: The expected switching cost at each time slot in Alg. 3 is at most $3\beta\|X^Q_t - X^Q_{t-1}\|1$.

Proof: As the switching cost only happens when we change $r_{k,n,t}$, we aim to bound the number of changes in $r_{k,n,t}$. Under Alg. 3, $r_{k,n,t}$ can be changed either in lines 8 – 12 or in lines 14 – 18. In lines 8 – 12, the total number of changes is $K\sum_{i=1}^{N}|\delta_{i,t}| = K\|X^Q_t - X^Q_{t-1}\|1$. Moreover, every change in lines 8 – 12 can result in at most two changes in lines 14 – 18. Hence, the total number of changes in lines 14 – 18 is at most $2K\|X^Q_t - X^Q_{t-1}\|1$.

Thus, the maximum number of changes in Alg. 3 is $3K\|X^Q_t - X^Q_{t-1}\|1$ over all sample paths. Since each sample path represents a probability mass of $\frac{1}{K}$, the expected switching cost is at most $3\beta\|X^Q_t - X^Q_{t-1}\|1$.

Then, we analyze the complexity of Alg. 3. Since $\sum_n X^Q_t \leq Z$ and $\sum_n X^Q_{t-1} \leq Z$, at most $KZ$ variables will be increased to 1 and at most $KZ$ variables will be decreased to 0 in Steps 7–13. This is a total of $O(KZ)$ changes. To implement the while loop in Steps 14 – 18, we can first divide all sample paths into three groups: those with $\sum_n r_{k,n,t} > Z$, those with $\sum_n r_{k,n,t} = Z$, and those with $\sum_n r_{k,n,t} < Z$. Then, Step 15 is a $O(1)$ operation. Step 16 takes $O(N)$ time. We note that each increase in Steps 7–13 will result in at most one iteration of the while loop in Steps 14 – 18. Hence, steps 14 – 18 will be executed at most $KZ$ times and the overall complexity of this while loop is $O(KZN)$.

Thus, the complexity of Alg. 3 is $O(KZTN)$ per time slot.

VI. Simulation Results

In this section, we present our simulation results. In addition to our OGDR (Alg. 1) and RSCR (Alg. 3), we also evaluate the following two policies:

- Stationary Offline Policy (SOP): This algorithm is based on the Iterative Caching update algorithm (ICE) proposed by Ma et al. [15]. This is an offline policy that has knowledge of all future arrival rates. In the context of this work, ICE is equivalent to one that caches the same $Z$ services with the largest $\frac{1}{T} \sum_{t=1}^{T} \lambda_{n,t} d_n$ in all time slots. Since ICE does not consider the routing problem, we will employ the optimal routing decisions for ICE.

- Online Gradient Ascent (OGA): This is an algorithm proposed by Paschos et al. [16]. It uses online gradient ascent by setting the gradient as the vector $[\lambda_{1,t} d_1, \lambda_{2,t} d_2, \ldots, \lambda_{N,t} d_N]$ in each time slot for the service caching problem. Since OGA does not consider routing procedure, we apply our routing policy in this algorithm to obtain its best performance. OGA produces fractional $X_t$ and its cost is based on the fractional solutions.

We model computation latency at the edge server by assuming that the edge server operates like a $M/M/1$ queueing system with service rate $\phi$. Thus, from [17], we have $C(\sum_{i=1}^{N} \lambda_{i,t} y_{i,t}) = \frac{1}{\phi - \sum_{i=1}^{N} \lambda_{i,t} y_{i,t}}$.

An important parameter of our online algorithms is the step size $\eta$. While we have demonstrated a specific choice of $\eta$ that leads to sublinear regret, we note that this choice may be too conservative because it is based on the upper-bound of $\|\nabla G_1(X_n)\|_2$ and the total number of time slots $T$. In our simulations, we use the time-average empirical value of $\|\nabla G_1(X_n)\|_2$ to determine the step size. Besides, we change the $T$ term to 50. Effectively, this means we aim to achieve a good performance over a time horizon of 50 slots. Specifically, let $E_t := \sum_{n=1}^{t} \|\nabla G_1(X_n)\|_2 / t$, we choose the step size to be $\sqrt{\frac{100\epsilon^2 + \beta \sqrt{N E_t}}{Z}}$ in time slot $t$.

In addition, we select the forwarding latency $d_n$ as a uniform random variable between 0.01 and 0.1 seconds, the accuracy parameter $\epsilon' = 1$, and vary the values of $\phi$, $Z$ and $\beta$.

Our simulations consist of two scenarios. The first scenario is based on a Google Trace data set from [18], containing a sequences of different service requests which we consider as the trace of request arrivals. This data set includes more than three million requests for $N = 9,218$ unique services within a seven-hours timespan. As time is slotted in the data set by 300 seconds, which is a large jump, we divide each interval into 60 different part with equal number of requests following the original sequence. Thus, the duration of a time slot in our experiments is five seconds. In addition, the upper bound for number of requests in each time slot is $W = 893$. The second scenario is based on a synthesis trace consisting of 500 services. Each service generates requests periodically, with different services having different periods.

The simulation results for the two scenarios are shown in Fig. 2 and Fig. 3, respectively. Several important observations can be made. First, our RSCR outperforms OGA significantly in all settings. While both RSCR and OGA are based on online gradient methods, RSCR is able to achieve better performance because it explicitly considers the processing latency at the edge server. This result shows that any online algorithm for edge computing needs to address both memory and computation power constraints of edge servers. Second, our RSCR outperforms SOP in Fig. 3 and has virtually the same performance as SOP in Fig. 2. This result is very surprising when one considers that SOP is an offline policy that knows all future request arrivals.

Finally, we note that RSCR and OGDR have very similar performance in all cases. OGDR produces fractional solutions for the service caching problem, and then RSCR transforms such fractional solutions into rounded solutions with integer solutions on every sample path. As discussed in Section V, by carefully choosing which services to host at the edge on every sample path, RSCR is able to incur a switching cost that is at most three times larger than the switching cost of OGDR. Our simulation results further show that the overall costs of RSCR and OGDR are almost identical in practical scenarios.

VII. Conclusion

This paper studies the problem of service caching and routing without any knowledge about future requests. Motivated by
a practical timescale separation, we formulate this problem as a two-stage online optimization problem that jointly considers the storage and computation constraints of the edge server, as well as the switching cost. We propose a low-complexity online algorithm for this problem that achieves sublinear regret bounds under a fractional relaxation. We further introduce a randomized algorithm that is guaranteed to produce integer solutions with provably small switching cost. Simulation results demonstrate that our RSCR and OGDR algorithms have similar or even better performance compared to other recent proposed policies.

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