Mass spectroscopy of excited light mesons using truncated overlap fermions

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Abstract

We study excited light mesons by quenched lattice quantum chromodynamics (QCD) simulations with a truncated overlap fermion formalism based on domain wall fermions. Truncated overlap fermions satisfy lattice chiral symmetry instead of chiral symmetry in continuum field theory, as for domain wall fermions, but offer lower simulation costs. Our results show good agreement with the experimental values for the excited state of \(a_1\), \(\rho\) and \(\pi\) mesons, and demonstrate that \(a_1\) (1260) and \(a_1\) (1640) are simple two-quark states, whereas \(a_1\) (1420) may have a more complicated structure. The results are similar to those of previous dynamical studies using clover-Wilson fermions or chirally improved fermions, even though our lattice QCD calculations are performed with the quenched approximation. The study shows that lattice QCD simulations using truncated overlap fermions are essential in lattice studies of excited states.

1. Introduction

The spectra of mesons and baryons show a high degree of regularity. The organizational principle that best categorizes this regularity is encoded in the quark model. The quark model assumes that mesons are bounds state of quark \((q)\)-antiquark \((\bar{q})\) pairs. The lightest mesons are built from \(u\) and \(d\) quarks, and \(qq\) spins are coupled together to give a certain total spin \(S\), which is then coupled to the angular momentum \(L\) to obtain the total observed angular momentum \(J\). The suggested spectroscopic notation for meson states is shown in table 1. Recently, a large number of light and heavy mesons have been experimentally observed, and the variety is much richer than that predicted by the quark model. Reconstructing hadron mass spectra from first principles is an important aspect of lattice quantum chromodynamics (QCD). However, analyzing excited mesons using lattice QCD has been challenging because the masses of the excited states have to be extracted from sub-leading exponentials based on spectral decomposition data using the correlation functions for meson states. The early work on excited mesons in lattice QCD has been reviewed by Fodor and Hoelbling [2].

Recently, the restoration of chiral symmetry in excited hadrons has been discussed [3–6] and studies into this phenomenon suggest that it is important to use chiral fermions in lattice studies of excited states. The Ginsparg–Wilson (GW) relation [7],

\[ \gamma_5 D + D \gamma_5 = 2a D \gamma_5 D, \]

where \(D\), \(\gamma_5\), and \(a\) are the fermion matrix, the gamma matrix, and the lattice spacing, respectively, is equivalent to the lattice version of chiral symmetry transformation [8]. Lattice fermions that satisfy the GW relation are called lattice chiral fermions. Presently, there is only one explicit formulation for lattice chiral fermions, for which the overlap fermion operator strictly satisfies the GW relation [9, 10]. Lattice QCD simulations of overlap fermions are much more expensive than for Wilson-like fermions [11]. Therefore, we consider lattice chiral fermions, for which numerical calculations are realistic.
Table 1. Isospin $I = 1$ mesons and their quantum numbers with spectroscopic notation $^{2S+1}L_J$. Isospin $I = 1$ mesons are constructed from $u$ and $d$ quarks to produce $uds$, $sud$, and $\bar{d}d - u\bar{u}$) forms. These three mesons have a positive, negative, and neutral charge, respectively. In the notation $^{2S+1}L_J$, $S$ is the total spin of the quark and anti-quark, $I$ is the angular momentum between the quark and anti-quark, and $J$ is the total angular momentum. [1].

| $^{2S+1}L_J$ | $S$ | $L$ | $J$ | Radial ground states of $I = 1$ mesons | Radial excited states of $I = 1$ mesons |
|---------------|-----|-----|-----|----------------------------|----------------------------------|
| $^1S_0$       | 0   | 0   | 0   | $\pi$                      | $\pi(1300)$                      |
| $^1S_1$       | 1   | 0   | 1   | $\rho$                     | $\rho(1450)$                     |
| $^1P_1$       | 0   | 1   | 1   | $b_1(1235)$                |                                  |
| $^2P_0$       | 1   | 1   | 0   | $a_0(1450)$                |                                  |
| $^2P_1$       | 1   | 1   | 1   | $a_0(1260)$                | $a_0(1640)$                      |

The chirally improved (CI) fermion operator gives an approximate solution to the GW relation for fermions obeying chiral symmetry in a lattice [12, 13]. The general Dirac operator can be expanded on the basis of operators on the lattice, and the expansion coefficients can be obtained by solving the GW equation for the operators. The CI fermionic operators can control deviations from the GW relation. However, this approach is not realistic for determining all eigenmodes in numerical simulations. Lattice QCD simulations using DWF have produced valuable results with CI fermion action [19, 20], while the present study only uses lattice chiral fermions. Some preliminary results for quenched calculations using CI fermion action have been presented in [21]. No other studies have adopted lattice chiral fermions. Our ultimate goal is to perform a dynamical simulation of excited light mesons using a lattice chiral fermion operator. We therefore employ the truncated overlap fermion (TOF) operator based on the DWF formalism [16, 22]. Simulations with the TOF can be faster than those with other lattice chiral fermions.

Previous reports have focused on exciting light mesons based on quenched calculations [17, 18] and dynamical calculations [19, 20]. The Bern-Graz-Regensburg (BGR) collaboration has presented results using CI fermion action [20], while the present study only uses lattice chiral fermions. Some preliminary results for quenched calculations using CI fermion action have been presented in [21]. No other studies have adopted lattice chiral fermions. Our ultimate goal is to perform a dynamical simulation of excited light mesons using a lattice chiral fermion operator. We therefore employ the truncated overlap fermion (TOF) operator based on the DWF formalism [16, 22]. Simulations with the TOF can be faster than those with other lattice chiral fermions.

The aim of this work was to simulate pseudoscalar, vector, and axial-vector ground states along with radially excited states from lattice QCD using a quenched simulation (i.e., omitting quark loop effects) with the TOF. This study represents the first step towards using the TOF to explore these states from first principles and presents the initial mass spectroscopy results for light excited mesons based on lattice QCD in association with lattice chiral symmetry. This work also determined the effects of chiral fermions on the mass spectra of light excited states from lattice QCD using a quenched simulation [10].

2. Truncated overlap fermion

The fermion action of the truncated overlap fermion (TOF) is defined as

$$S_{\text{TOF}} = \bar{\psi} D_{\text{TOF}} \psi,$$

where $\psi$ and $\bar{\psi}$ are four-dimensional fermion fields and $D_{\text{TOF}}$ is the fermion matrix for the TOF [16, 22]. $D_{\text{TOF}}$ is defined as

$$D_{\text{TOF}} = e^{\dag P^1D_{\text{PV}}} D_{\text{DWF}} Pe,$$

where $D_{\text{DWF}}$ is the domain wall fermion operator, $P$ is a five-dimensional projection operator, and $e$ is a five-dimensional unit vector defined by $e_i = \delta_{i5}$. The indices represent the five-dimensional lattice sites defined in $x_5 \in [1, N_5]$ where $N_5$ is the five-dimensional lattice size. The Pauli–Villars matrix $D_{\text{PV}}$ is given by $D_{\text{PV}} = D_{\text{DWF}}(m_f a = 1)$ where $m_f$ and $a$ correspond to the bare quark mass and the lattice spacing, respectively. The five-dimensional projection operator $P$ is constructed from the four-dimensional projection operators $P_{R/L} = (1 \pm \gamma_5)/2$ as

$$P_{R/L} = (1 \pm \gamma_5)/2.$$

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The domain wall fermion operator $D_{DWF}$\cite{14, 15} is defined as

$$D_{DWF}\delta_{x,y} = D_{WF}(x, y)\delta_{x,y} - (P_1\delta_{x+1,y} + P_R\delta_{x,y} + P_L\delta_{x-1,y})\delta_{x,y} + \delta_{x,x}\delta_{x,y} + m_f a(P_R\delta_{x,y}\delta_{y,N} + P_L\delta_{x,y}\delta_{y,-1})\delta_{y,y},$$

where $P_1$ is the Wilson fermion operator. This operator is written as

$$D_{WF}(x, y) = (4 - M_5)\delta_{x,y} - \frac{1}{2}\sum_{m=\pm 1}^4 (1 - \gamma_\mu)U_{\mu}(x)\delta_{x+,\mu,y},$$

where $U_{\mu}(x)$ and $M_5$ are a link variable and the height of the domain wall, respectively. Within the $N_5 \to \infty$ and $m_f a \to 0$ limits, the fermion matrix for TOF satisfies the Ginsparg–Wilson relation \cite{7},

$$\gamma_5D_{TOF} + D_{TOF}\gamma_5 = 2aD_{TOF}\gamma_5D_{TOF} \quad (N_5 = \infty \text{ and } m_f a = 0).$$

Therefore, the TOF action is invariant in the case of lattice chiral symmetry:

$$\begin{cases}
\psi(x) \to e^{i\theta(1 - aD_{TOF})}\psi(x) \\
\bar{\psi}(x) \to \bar{\psi}(x) e^{i\theta(1 - aD_{TOF})}\gamma_5
\end{cases},$$

### 3. Algorithm

There are two approaches to construct the fermion matrix for the TOF and its inverses. One option is to prepare five- and four-dimensional fermion fields and to use the conjugate gradient (CG) method for five-dimensional matrixes. Using this technique, the four-dimensional fermion field, $\psi$, can be projected to the five-dimensional fermion field, $\Psi$, via the projection operator $P$ as

$$\Psi = P\psi,$$

Solving large-scale linear equations with the CG method in five dimensions as

$$D_{PV}\xi = D_{DWF}\Psi$$

gives the TOF action

$$S_{TOF} = \bar{\psi}\epsilon P^\dagger\xi,$$

where $\xi$ is a five-dimensional matrix.

The alternative is to prepare only four-dimensional fermion fields and employ the CG method with four-dimensional matrixes. The fermion matrix associated with the TOF can be rewritten as

$$D_{TOF} = \frac{1 + m_f a}{2} + \frac{1 - m_f a}{2} (H_{W+}^{N_5} - H_{W-}^{N_5})\gamma_5,$$

where $H_{W+}$ and $H_{W-}$ are defined based on the Hermitian matrix $H_W$ as

$$H_{W+} = 1 + H_W, \quad H_{W-} = 1 - H_W, \quad H_W = \gamma_5 D_{WF} D_{WF} + 2.$$

From the formulation with the limits $N_5 \to \infty$ and $m_f a \to 0$, we obtain

$$D_{TOF} = \frac{1}{2}[1 + \gamma_5 \text{ sgn}(H_W)],$$

demonstrating that the TOF action satisfies the Ginsparg–Wilson relationship \cite{7}. Because this approach does not include five-dimensional operators, the CG method can be used in ordinary four-dimensional space as

$$\left[\frac{1 + m_f a}{2} (H_{W+}^{N_5} + H_{W-}^{N_5})\gamma_5 + \frac{1 - m_f a}{2} (H_{W+}^{N_5} - H_{W-}^{N_5})\gamma_5\right] \xi = (H_{W+}^{N_5} + H_{W-}^{N_5})\gamma_5 \psi.$$

By solving the linear equation for $\xi$, we obtain the TOF action

$$S_{TOF} = \bar{\psi}\xi.$$
43 × 8 lattice, $N_f = 4, M_5 = 1.65$, and $m_f a = 0.20$ in conjunction with a random gauge configuration on SX-ACE and SQUID at RCNP and at the Cybermedia Center, Osaka University as shown in table 2. The results of these computations indicated that the first method described above was approximately seven times faster than the second approach, and so the first algorithm was used for the subsequent calculations, as described below.

4. Simulation setup and lattice QCD results

The masses of $\pi$, $\rho$, and $a_1$ mesons were calculated using an $8^3 \times 24$ quenched lattice with plaquette gauge action and a lattice coupling $\beta = 5.7$. Gauge configurations were generated employing the pseudo–heat-bath method and, after 20,000 thermalization iterations, gauge configurations were saved every 1000 sweeps. The fermion parameters for the TOF were set to $N_f = 32$, $M_5 = 1.65$, and $m_f a = 0.04–0.08$. Previous work by our group confirmed that $N_f = 32$ is sufficiently large to be considered infinite when using these parameters [25].

Values for the meson propagator, $G(t)$, were calculated in association with the TOF using gauge configurations of 7864, 3600, 3000, 3000, and 3000 for $m_f a = 0.04, 0.05, 0.06, 0.07, \text{and} 0.08$, respectively. A $\bar{q}q$ point operator was employed as the source and sink for the meson propagator. The results of these calculations are summarized in figure 1. During these calculations, statistical errors were estimated using the jackknife method and the assumption of periodic boundary conditions in the temporal direction improved the statistical accuracy based on the inclusion of temporal symmetry.

The effective masses $m_{\text{eff}}(t)$ for $\pi$, $\rho$, and $a_1$ were derived by fitting with

$$G(t + 1) = \cosh \left\{ m_{\text{eff}}(t) \left( \frac{T}{2} - (t + 1) \right) \right\} \cosh \left\{ m_{\text{eff}}(t) \left( \frac{T}{2} - t \right) \right\},$$

where $T$ is the temporal lattice size. The variations in $m_{\text{eff}}$ over time are plotted in figure 2. Meson masses were obtained using a single-pole fitting form based on the plateaus in these $m_{\text{eff}}$ plots. The resulting meson masses are provided in table 3 and also indicated by the solid lines in figure 2.

Next, this work also examined the extraction of meson masses for the excited states with the double-pole fitting form, such as in the case of

$$G(t) = Z_0 \cosh \left\{ m_0 \left( \frac{T}{2} - t \right) \right\} + Z_1 \cosh \left\{ m_1 \left( \frac{T}{2} - t \right) \right\},$$

where $m_0$ and $m_1$ correspond to the meson masses for the ground and excited states, respectively. We obtained $m_0$ and $m_1$ by fitting the meson propagators with (18). The meson masses for each quark mass are summarized in table 4. Figure 3 summarizes the squared pion mass dependence of the meson masses.

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**Table 2.** Comparison of test computational times for the two methods. We used the first method in the main calculations.

| Machine | Time by first method (s) | Time by second method (s) | Ratio of time for two methods |
|---------|--------------------------|----------------------------|-------------------------------|
| SX-ACE  | 8.3                      | 56.7                       | 6.8                           |
| SQUID   | 3.37                     | 21.74                      | 6.45                          |

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![Figure 1. Propagator values for $\pi$, $\rho$, and $a_1$ mesons for $m_f a = 0.04$, 0.06, and 0.08.](image1.png)
operators. am af

states and sink, meaning that virtual intermediate states such as $\pi^{0}$ states whereas $\rho$ state. It would be of interest to perform calculations of the

four combinations of single-pole and double-pole fitting forms for the $\pi$, $\rho$, and $a_1$ mesons. Specifically, single-pole fitting was used for all mesons in figure 3(a), single-pole fitting was used for the $\pi$ and $\rho$ mesons and double-pole fitting for the $a_1$ meson in figure 3(b), single-pole fitting was used for the $\pi$ meson and double-pole fitting for the $\rho$ and $a_1$ mesons in figure 3(c), and double-pole fitting was used for all mesons in figure 3(d). A linear extrapolation of the meson masses to the chiral limit, $(m_{\pi} a)^2 = 0$, was also performed. By tuning the $\rho$ meson mass in the chiral limit to $m_{\rho} = 775.26$ MeV, lattice spacings $a = 0.1890(13)$, 0.1890(13), 0.1870(8), and 0.1870(8) fm were obtained for each combination, as shown in plots (a), (b), (c), and (d) in figure 3, respectively. In the chiral limit, the quark mass does not become zero but reaches a residual mass $m_{\pi} a \rightarrow m_{\text{res}} a$ because $N_{\pi}$ is finite. The residual masses in the present work were determined to be $m_{\text{res}} a = 13.5(6), 13.5(6), 13.7(5)$, and 13.5(2) MeV for plots (a), (b), (c), and (d), respectively. The meson masses in the chiral limit are summarized in table 5.

In figure 4, the meson masses determined in the present study are compared with the experimental values in the particle data group (PDG) [1] and with previously reported lattice data [17–20]. Our data in figure 4 correspond to the data in table 5. These results demonstrate that the meson masses were unaffected by the fitting combinations of single-pole and double-pole forms.

In the case of the $a_1$ mesons, the present results are seen to be consistent with the experimental values of $a_1(1260)$ and $a_1(1420)$. These simulations were performed in conjunction with the quenched approximation and the $\bar{q}q$ source and sink, meaning that virtual intermediate states such as $\bar{q}q\bar{q}q$ states were highly suppressed, and thus these results suggest that $a_1(1260)$ and $a_1(1420)$ are simple $\bar{q}q$ states whereas $a_1(1420)$ may have a more complicated structure than the $\bar{q}q$ state. It would be of interest to perform calculations of the $a_1(1420)$ mass using other operators such as the $\bar{q}q\bar{q}q$ operators.

Table 3. Masses of the $\pi$, $\rho$, and $a_1$ mesons (as obtained from fitting with the single-pole form), mass ratios, $m_{\pi}/m_{\rho}$, and number of configurations (Confs.) for each quark mass.

| $m_f a$ | $m_{\pi} a$ | $m_{\rho} a$ | $m_{a_1} a$ | $m_{\pi}/m_{\rho}$ | Confs. |
|--------|-------------|-------------|-------------|---------------------|--------|
| 0.08   | 0.6675(10)  | 0.9508(33)  | 1.395(6)    | 0.702(3)            | 3000   |
| 0.07   | 0.6288(12)  | 0.9260(27)  | 1.361(10)   | 0.679(3)            | 3000   |
| 0.06   | 0.5900(7)   | 0.9052(30)  | 1.351(9)    | 0.652(3)            | 3000   |
| 0.05   | 0.5482(8)   | 0.8824(26)  | 1.319(8)    | 0.621(3)            | 3000   |
| 0.04   | 0.5034(7)   | 0.8606(16)  | 1.281(9)    | 0.588(2)            | 7864   |

Table 4. Masses of $\pi$, $\rho$, and $a_1$ mesons for each quark mass as obtained from fitting with double-pole form.

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| $m_f a$ | $m_{\pi} a$ | $m_{\rho} a$ | $m_{a_1} a$ | $m_{\pi}/m_{\rho}$ | Confs. |
|--------|-------------|-------------|-------------|---------------------|--------|
| 0.08   | 0.6649(2)   | 1.673(12)   | 0.9353(12)  | 1.379(13)           | 2.156(69) |
| 0.07   | 0.6265(5)   | 1.642(31)   | 0.9111(12)  | 1.785(12)           | 2.082(79) |
| 0.06   | 0.5882(2)   | 1.654(15)   | 0.8900(14)  | 1.771(15)           | 2.038(60) |
| 0.05   | 0.5467(3)   | 1.613(29)   | 0.8678(13)  | 1.766(14)           | 1.974(48) |
| 0.04   | 0.5019(3)   | 1.560(40)   | 0.8507(15)  | 1.809(17)           | 1.258(26) |
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Figure 2. Effective masses of $\pi$, $\rho$, and $a_1$ mesons for $m_f a = 0.04, 0.06, and 0.08$. Fits based on the single-pole form are indicated by solid lines.
The results for the excited $\rho$ meson exceed the experimental value for $\rho(1700)$, similar to the findings of previous dynamical studies \cite{19,20}. Calculating the $\rho(1450)$ mass may require using the variational method with other operators coupled with $\rho(1450)$, as has been previously performed \cite{20}.

The result obtained in this work for the excited $\pi$ meson lies between the experimental $\pi(1300)$ and $\pi(1800)$ values, which would be expected based on the degenerate masses of the two excited $\pi$ mesons. Our lattice data are also equivalent to or slightly larger than the values obtained from previous dynamical studies. The effect of
the quenched approximation in which sea quarks are infinitely heavy is apparent in the slightly large value for the excited $\pi$ meson.

It should also be noted that the present results for the $a_1$, excited $\rho$, and excited $\pi$ mesons using chiral fermions are in reasonably good agreement with the values from a previous dynamical study using clover-Wilson fermions or chirally improved fermions, even though our calculations were performed with the quenched approximation.

5. Conclusion

This work investigated mass spectroscopy data obtained with quenched lattice QCD using truncated overlap fermion action based on the formalisms of domain wall fermions. The double-pole fitting form was used to obtain masses for the ground and first excited states of $a_1$, $\rho$, and $\pi$ mesons. The results for $a_1$ mesons coincided with the experimental values of $a_1(1260)$ and $a_1(1640)$. Since our simulations were performed with the quenched approximation and the two-quark source and sink, the results suggest that $a_1(1260)$ and $a_1(1640)$ are simple two-quark states, whereas $a_1(1420)$ may have a more complicated structure. The results for the $a_1$, excited $\rho$ and excited $\pi$ mesons using truncated overlap fermions with the quenched approximation (i.e., omitting quark loop effects) were in good agreement with those obtained in prior dynamical studies (i.e., including quark loop effects) using clover-Wilson fermions or chirally improved fermions. Our work supports Glozman’s suggestion that chiral fermions are essential in lattice studies of excited states [4]. In order to reconstruct the experimental values of the $a_1(1420)$ mass, $\rho(1450)$ mass, and the degenerate masses of the two excited $\pi$ mesons, $\pi(1300)$ and $\pi(1800)$, lattice QCD calculations may be required using the variational method with other operators such as $\bar{q}q$ or $\bar{q}\bar{q}qq$ operators. Based on this, we expect that dynamical calculations using the truncated overlap fermion with the variational method will be helpful in future research into the mass spectroscopy of light excited mesons.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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