CAN WE SEE THE SHAPE OF OUR UNIVERSE?∗

G.I. GOMERO†
Centro Brasileiro de Pesquisas Físicas, Rua Dr. Xavier Sigaud 150
22290-180, Rio de Janeiro – RJ, Brazil

Received (Day Month Year)
Revised (Day Month Year)

This is a written version of a talk given at the Fifth Friedmann Seminar on recent work in Observational Cosmic Topology done in partial collaboration with Armando Bernui. We address three relevant questions related to the search for the size and shape of our Universe: (i) How do the actual observation of multiple images of certain cosmic objects, e.g. galaxy clusters, constrain the possible models for the shape of our Universe?, (ii) What kind of predictions can be done once a pair of cosmic objects have been identified to be topological images related by a translation?, and (iii) Is it possible to determine if two regions of space are topologically identified, even when distortions on the distributions of cosmic sources due to observational limitations are not negligible? We give examples answering the first two questions using the suggestion of Roukema and Edge that the clusters RXJ 1347.5-1145 and CL 09104+4109 might be topological images of the Coma cluster. For the third question, we suggest a method based on the analysis of PSH’s noise correlations which seems to give a positive answer.

Keywords: Cosmic topology; observational cosmology; galaxy clusters.

1. Introduction
The talk given at the Fifth Friedmann Seminar was a brief report of recent work in Cosmic Topology done in partial collaboration with Armando Bernui (see Refs. 1–3 for the original papers). This written version briefly summarizes the main results and discusses further work.

What is the shape of our Universe? This is one of the most intriguing questions in observational cosmology, and the last two decades have seen a continuously increasing interest, from the part of cosmologists, in studying problems closely related to it (see Ref. 4 and references therein). Active research is currently being done from the more formal aspects such as the (im)possibility of observing (at least part of) the shape of space up to developing concrete methods to detect (or even determine) a non-trivial topology of our Universe. These methods are all based

∗Talk given at the Fifth Alexander Friedmann International Seminar on Gravitation and Cosmology held in João Pessoa, Paraíba, Brazil from April 24th to April 30th, 2002.
†german@cbpf.br
on the observation of multiple images of discrete cosmic objects\footnote{As in previous papers dealing with Euclidean manifolds, the notation used here is that of Wolf.} or some kind of pattern repetition on maps of CMBR.\footnote{As in previous papers dealing with Euclidean manifolds, the notation used here is that of Wolf.}

Three relevant questions related to the search for the size and shape of our Universe are: (i) How do the actual observation of multiple images of certain cosmic objects, e.g. galaxy clusters, might constrain the possible models for the shape of our Universe?, (ii) What kind of predictions can be done once a pair of cosmic objects have been identified to be topological images related by a translation?, and (iii) Is it possible to determine if two regions of space are topologically identified even when distortions on the distributions of cosmic sources due to observational limitations are not negligible?

In Refs. 1 and 2 I gave examples answering the first two questions using a suggestion of Roukema and Edge that the clusters RXJ 1347.5-1145 and CL 09104+4109 might be topological images of the Coma cluster\footnote{As in previous papers dealing with Euclidean manifolds, the notation used here is that of Wolf.} these results are reviewed in Sections \ref{sec:model-building} and \ref{sec:analysis} respectively. For the third question, in Ref. 3 A. Bernui and I suggested a method based on the analysis of PSH’s noise correlations which seems to give a positive answer. This method is briefly described in Section \ref{sec:noise-correlations}. Finally, Section \ref{sec:conclusions} deals with conclusions and further work.

2. Model Building in Cosmic Topology

Suppose we have identified three clusters of galaxies as being different topological images of the same object. How do these multiple images constrain the possible models for the shape of our Universe? Some time ago Roukema and Edge suggested that the clusters RXJ 1347.5-1145 and CL 09104+4109 might be topological images of the Coma cluster\footnote{As in previous papers dealing with Euclidean manifolds, the notation used here is that of Wolf.} The distances of these clusters to Coma being 970 and 960\,$h^{-1}$\,Mpc respectively (taking $\Omega_0 = 1$ and $\Lambda = 0$), and the angle between them, with the Coma cluster at the vertex, being $\approx 88^\circ$. Roukema and Edge suggested that the clusters RXJ 1347.5-1145 and CL 09104+4109 might be images of Coma by equally spaced pure translations and assumed a right angle between the lines joining each high redshift cluster with Coma. With these considerations they constructed FL cosmological models with compact flat spatial sections of constant time. These spatial sections were (i) 3-torii, (ii) manifolds of class $\mathcal{G}_2$, or (iii) manifolds of class $\mathcal{G}_4$, all with square cross sections, and scale along the third direction larger than the depth of the catalogue of X-ray clusters used in the analysis\footnote{As in previous papers dealing with Euclidean manifolds, the notation used here is that of Wolf.}. In Ref. 1 I analyzed the Roukema-Edge hypothesis in the context of Friedmann-Lemaître cosmological models with flat spatial sections whose matter components are pressureless dust and a cosmological constant, and studied the possibility that at least one of these high redshift clusters is an image of Coma by a skew motion. In this study it was not assumed that the distances from Coma to both of the high redshift clusters are equal, nor that they form a right angle (with Coma at the vertex). It was shown that this configuration of clusters can be accommodated
within any of the six classes of compact orientable 3-dimensional flat space forms, providing a plethora of models for the shape of our Universe. Furthermore, it was also shown that the identification of two more triples of multiple images of clusters of galaxies, in the neighbourhood of the first one is enough to completely determine the topology of space in most of the models proposed.

Although it could also be considered the possibility that one pair of clusters are identified by a glide reflection, thus giving rise to non-orientable manifolds for models for the shape of space, this was not done since these cases do not give qualitatively different results, and the corresponding calculations can be done whenever needed.

3. Possible Topological Images of Nearby Clusters of Galaxies

A common feature of most of the models constructed in Ref. 1 is that one of the topological images is related to Coma by a pure translation. This gives the possibility of making precise predictions for the positions of images of other clusters of galaxies. In Ref. 2 it was reported the angular positions and redshifts of these potential images for 31 nearby clusters of galaxies from the Abell-ACO catalogue. Remarkably, several of the predicted angular positions coincide with angular positions of faint clusters of the same catalogue, within a 1° uncertainty, suggesting that topological images of clusters might have been detected and recorded a long time ago, although they have not been recognized as being so. These faint clusters of galaxies have no measured redshifts, so it was argued to be of considerable importance to measure them in order to verify if they are actually topological images of nearby clusters. Moreover, it was suggested to be also of considerable interest to plan and execute deep redshift surveys looking for clusters of galaxies around the clusters RXJ 1347.5-1145 and CL 09104+4109, and their antipodal points with respect to Coma, in order to identify the (possible) topological images predicted in that work.

4. Local Correlations in Cosmic Topology

Cosmic Crystallography is a statistical method which looks for distance correlations between cosmic sources in pair separation histograms (PSH), i.e. graphs of the number of pairs of sources versus the squared distance between them. These correlations are due to isometries, of the covering group of the manifold that models the spatial sections of the Universe, which give rise to the (observed) multiple images. This method has been extensively studied by diverse groups of researchers, although its present stage of development does not allow its direct application to real current catalogues.

On the other hand, a method proposed by Roukema in Ref. 11, looks for identical quintuplets of quasars in two different regions of space. As it stands, this method is not practical since it looks for identical rigid configurations, and does not consider the possibility that in one of the configurations, one or more of the quasars are not
actually being observed. Nevertheless, an abstraction of the 3-D quasars positions proposal lead us to the idea of looking at correlations between the distributions of sources in different regions of space.

Indeed, in Ref. 3 A. Bernui and I proposed a new method for the search of topology that captures ideas from Cosmic Crystallography and the 3-D quasars positions method of Roukema. The basic idea is to analyse the correlations of the noise of two PSHs corresponding to two small distant regions of space. The Local Noise Correlations method (or LNC for short), as it was called, calculates the coefficient of linear correlation, $r$, of the noise of two PSHs. If the two regions under scrutiny are not related by an isometry, then the correlation coefficient vanishes. On the other hand, if the two regions are identified by an isometry, they should have ideally an identical distribution of images, so the (ideal) coefficient of linear correlation takes the value $r = 1$.

Actually, even if the two regions are identified by an isometry, the distributions of images in both regions are not identical. This is due mainly to the fact that the two regions are at different distances to the observer, and so correspond to different epochs of the Universe. As a consequence, peculiar velocities of the sources, luminosity thresholds and finite lifetimes of the sources contribute to differentiate the two samples of images with which one produces the PSHs. However, and this is very remarkable, the noises of both PSHs remain strongly correlated provided the distortions in the distributions due to observational limitations are not too strong.

The distributions of images in both samples may be non-identical due to another reason. The two regions may not be identified by an isometry, but the image of one of the regions may overlap the other one, thus some portions of both distributions would actually coincide. We showed that the LNC method is also robust under this source of distortion provided the overlapping region is not too small.

Thus, the robustness of the LNC method suggests its potential value for the search of the topology of the Universe in at least two different ways. First, it was argued that the Roukema-Edge hypothesis can be tested in a definitive way by the LNC method provided detailed redshift surveys can be performed in the regions around these two high redshift clusters. In fact, the correlation coefficient of the noise of the PSHs corresponding to these two surveys, together with the same analysis applied to each of these surveys with a survey of local clusters of galaxies would tell if these three regions are actually identified by isometries.

The second suggestion is more speculative. It was shown in Ref. 1 that the identification of three neighboring triples of multiple images of clusters of galaxies is enough to completely determine the topology of space in most of the models proposed there. For the remaining possibilities one can determine just two generators of the covering group and either the axis of rotation of the third generator, or at least its direction. If it turns out that the clusters RXJ 1347.5-1145 and CL 09104-4109 are actually topological images of Coma, then it would be very easy to identify many more triples of images of clusters. If, additionally, it turns out that the shape of space is such that it cannot be determined by these additional clusters,
one can use the LNC method to search for the remaining generator by comparing regions along the axis of rotation, or parallel to it.

5. Discussion and Further Remarks

In this written version of my talk given at the Fifth Friedmann Seminar, I addressed three relevant questions related to the search for the size and shape of our Universe. These questions deal respectively with model building, the prediction power of these models, and the possibility of developing search methods of topology which would test them. I reported examples answering the first two questions using the suggestion of Roukema and Edge that the clusters RXJ 1347.5-1145 and CL 09104+4109 might be topological images of the Coma cluster. As for the third question, I reported a method based on the analysis of PSH’s noise correlations which seems to be of great potential value.

The results reviewed here suggest the importance of developing suitable methods for deciding whether two clusters of galaxies are topological images. Once three clusters had been suggested as being images of the same cluster, the methods discussed here can be used to test the hypothesis. In this respect, it turns out to be of considerable importance to plan and execute deep redshift surveys looking for clusters of galaxies around the clusters RXJ 1347.5-1145 and CL 09104+4109, and their antipodal points with respect to Coma, in order to identify the topological images that may exist if the Roukema-Edge hypothesis turns out to be true.

Despite the physical and philosophical relevance of identifying the shape of our Universe, there are other related topics of practical interest in cosmology and astrophysics. In fact, it may be possible to use the knowledge of the shape of our Universe to improve the determination of cosmological parameters and to measure transverse velocities and radial positions of galaxies in clusters. Although these studies are in their very beginning, one can expect that further developments would open the possibility of considerably improve our knowledge of the time evolution of galaxies and of cluster dynamics. It is then of considerable interest to determine whether our Universe allows the existence of multiple images of these objects.

Acknowledgements

I would like to thank CLAF for the grant under which this work was carried out.

References

1. G.I. Gomero, in preparation.
2. G.I. Gomero, in preparation.
3. A. Bernui and G.I. Gomero, in preparation.
4. M. Lachieze-Rey and J.-P. Luminet, Phys. Rep. 254, 135 (1995); B. Roukema, astro-ph/9801225 (1998); G.D. Starkman, Class. Quantum Grav. 15, 2529 (1998). See also the other articles in
this special issue featuring invited papers from the Topology of the Universe Conference, Cleveland, Ohio, October 1997. Guest editor: Glenn D. Starkman; R. Lehoucq, J.-P. Uzan and J.-P. Luminet, Astron. Astrophys. 363, 1 (2000); V. Blanloeil and B.F. Roukema, Editors of the electronic proceedings of the Cosmological Topology in Paris 1998, astro-ph/0010170; J. Levin, gr-qc/0108043, submitted to Physics Reports (2001); K.T. Inoue, PhD Thesis, Kyoto University, astro-ph/0103158 (2001).

5. G.I. Gomero, M.J. Reboucas and R. Tavakol, Class. Quantum Grav. 18, 4461 (2001);
G.I. Gomero, M.J. Reboucas and R. Tavakol, Class. Quantum Grav. 18, L145 (2001);
E. Gausmann, R. Lehoucq, J.-P. Luminet, J.-P. Uzan and J. Weeks, Class. Quantum Grav. 18, 5155 (2001);
G.I. Gomero and M.J. Reboucas, gr-qc/0202094 (2002);
G.I. Gomero, M.J. Reboucas and R. Tavakol, Int. J. Mod. Phys. A, this issue (2002).

6. R. Lehoucq, M. Lachièze-Rey and J.-P. Luminet, Astron. Astrophys. 313, 339 (1996);
B.F. Roukema, Mon. Not. R. Astron. Soc. 283, 1147 (1996);
G.I. Gomero, A.F.F. Teixeira, M.J. Reboucas and A. Bernui, gr-qc/9811038 (1998);
H.V. Fagundes and E. Gausmann, Phys. Lett. A261, 235 (1999);
R. Lehoucq, J.-P. Luminet and J.-P. Uzan, Astron. Astrophys. 344, 735 (1999);
J.-P. Uzan, R. Lehoucq and J.-P. Luminet, Astron. Astrophys. 351, 776 (1999);
R. Lehoucq, J.-P. Uzan and J.-P. Luminet, astro-ph/0005515 (2000);
G.I. Gomero, M.J. Reboucas and A.F.F. Teixeira, Phys. Lett. A275, 355 (2000);
G.I. Gomero, M.J. Reboucas and A.F.F. Teixeira, Int. J. Mod. Phys. D9, 687 (2000);
G.I. Gomero, M.J. Reboucas and A.F.F. Teixeira, Class. Quantum Grav. 18, 1885 (2001).

7. J.J. Levin, J.D. Barrow, and J. Silk, Phys. Rev. Lett. 79, 974 (1997);
N.J. Cornish, D.N. Spergel and G.D. Starkman, Phys. Rev. D57, 5982 (1998);
J.J. Levin, E. Scannapieco and J. Silk, Phys. Rev. D58, 103516 (1998);
J.J. Levin, E. Scannapieco, E. Gasperis, J. Silk and J.D. Barrow, Phys. Rev. D58, 123006 (1999);
R. Aurich, Astrophys. J. 524, 497 (1999);
J.R. Bond, D. Pogosyan and T. Souradeep, Phys. Rev. D62, 043005 (2000);
J.R. Bond, D. Pogosyan and T. Souradeep, Phys. Rev. D62, 043006 (2000);
R. Aurich and F. Steiner, Mon. Not. Roy. Astron. Soc. 323, 1016 (2001).

8. B.F. Roukema and A. Edge, Mon. Not. R. Astron. Soc. 292, 105 (1997).
9. J.A. Wolf, Spaces of Constant Curvature, (fifth ed., Publish or Perish Inc., Delaware, 1984).
10. G.O. Abell, H.G. Corwin and R.P. Olowin, ApJS 70, 1, (1989).
11. B.F. Roukema, Mon. Not. R. Astron. Soc. 283, 1147 (1996).
12. B.F. Roukema and J.-P. Luminet, Astron. Astrophys. 348, 8 (1999).
13. B.F. Roukema and S. Bajtlik, Mon. Not. R. Astron. Soc. 308, 309 (1999); G.I. Gomero, in preparation.