A Chaotic Ordered Hierarchies Consistency Analysis
Performance Evaluation Model

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Abstract. The Hierarchies Consistency Analysis (HCA) is proposed by Guh in-cooperated along with some case study on a Resort to reinforce the weakness of Analytical Hierarchy Process (AHP). Although the results obtained enabled aid for the Decision Maker to make more reasonable and rational verdicts, the HCA itself is flawed. In this paper, our objective is to indicate the problems of HCA, and then propose a revised method called chaotic ordered HCA (COH in short) which can avoid problems. Since the COH is based upon Guh’s method, the Decision Maker establishes decisions in a way similar to that of the original method.

1. Introduction
A fundamental goal of management is to measure and assess the performance of an organization and/or a system. Such evaluation of performance consists of two parts: the choice of the principle and the establishment of evaluation model [1,2]. The question of how to accurately and correctly estimate the performance has always been the most basic topic of discussion in the field of management. Matters of concern such as system stability, the monitoring of the manufacturing process, and quality of service or the performance evaluation itself in fields such as health work, business and so forth all involve performance evaluation. It is always a major concern for decision-makers (DMs) to establish an effective model for objective assessment in the field of performance evaluation [2]. Many researchers have built performance evaluation models as a hierarchy of goals and means in the shape of a systematic diagram composed of all the criteria elements [3-7]. Hierarchical structure is a good approach for describing a complicated system in which the relationships among all the related criteria are complicated.

However, in most problems, a single framework of an analysis model would provide only one stand point of evaluation [8]. This would have a likely chance in producing an error in evaluation particularly for large-scale complex processes. Hence, a different viewpoint or specific point’s emphasis will result in different hierarchy structures in the analysis of large-scale problems [1,9].

Moreover, with the integration and compromise of analysis results, a more accurate and objective result would surface. Thus, Guh [1] proposed a new weighting methodology called Hierarchies Consistency Analysis (HCA) based on the fact that when a DM transfers a multiple criteria problem into a hierarchy structure, several hierarchy structures may actually be produced from emphasizing different viewpoints [1,9]. Guh et al. improved the HCA furthermore and named it AHP-HCA [10]. AHP-HCA uses AHP to acquire the initial weights and then employs HCA to establish a multiple structures model for the AHP.

HCA and AHP-HCA can provide us with a comprehensive framework for solving hierarchy evaluation problems to aid the DM to make decision more reasonable and rational. Generally, the process of determining the evaluation index for a HCA includes the following steps [10]: The lowest level criteria of the problem are defined and collected, next based on some specified management viewpoints these lowest level criteria are further grouped and clustered.
into multiple level and multiple criteria (hierarchy structure) models, and the DM then assigns directly, or obtains indirectly, the weights for the criteria in all levels of the hierarchy structure. Finally, we calculate the aggregated index by multiplying the weights and the observed values of the criteria for the evaluated unit. If the solutions do not satisfy all DMs, then a new solution is obtained by justifying the weights for the criteria. The above procedure is repeated until all obtained weight are reasonable acceptable, and consistent [1,8-11].

HCA focuses on “reasonable, acceptable, and consistent solution”, the way the DM makes his/her final decision with the upmost accuracy possible. What makes this model more reasonable and acceptable than the previous one is the higher level of acceptability it requires. As DMs who follow the old method (e.g., AHP) employ only one set of results, DMs who apply HCA use two sets of results that can be compared with and sifted out to obtain the more correct results. This need

The HCA is a straightforward method and was proven to be very effective to find a reasonable comprised solution especially for complex and large-scale problem performance evaluation without suffering numerical difficulties and reduce the computational burdens (e.g., eigenvectors) [10]. However, the final result derived from HCA may cause inconsistent with our intuitions about the ranking of the weights for the criteria (see Section 3). In the real world, we can illustrate many examples like the above situations again. Thus, there is still a lots room and need to improve HCA (AHP-HCA) to easy help the DM find acceptability and reasonable solution.

Chaos refers to a state lacking order or predictability. It has been studied from many perspectives [12-16]. To the best of the author’s knowledge, the concept of chaos has never been investigated in the context of performance evaluation models. Thus, in this research, chaos is incorporated into HCA to construct a new method called the chaotic ordered AHP-HCA (COH), where seek consistency for the weights of the criteria of each hierarchy ability of HCA and chaotic searching behavior are reasonably combined. Example results and comparisons demonstrate the effectiveness and efficiency of the proposed COH.

2. Acronym, notations and assumptions

2.1 Acronym

AHP : Analytical Hierarchy Process
HCA : Hierarchies Consistency Analysis
COH : Chaotic Ordered HCA
CI : Consistency Index
CR : Consistency Ratio

2.2 Notations

Pr(•) : the probability of the occurrence of •.
Max(•) : the maximal number of the set •.
Min(•) : the minimal number of the set •.
*i→*j : the path from nodes *i to *j in the corresponding hierarchy.
|•| : the number of the set •.
i : the number of changes made
j : the ith type (hierarchy), i=1,2,…,s.
t : the tth iteration.
ni : the total number of criteria in the ith hierarchy.
I : I=I1I2I3…Is denotes the position in the lth sub-criteria of the (ls)th sub-criteria of the (ks-2)th sub-criteria of the (ks)th sub-criteria.
lj : lj=I1I2I3…Is if I=I1I2I3…Is.
Pj,l : the performance index of node I with respective to ith type (hierarchy).
Pj,lr : the parent node of Pj,l with respective to ith type (hierarchy).
Pj,lr : the bottom node with Pj,l is one of its ancestors with respective to ith type (hierarchy).

wi,l =Pr(Pr(Pj,l→Pj,l)) is the weight corresponding to the sub-criteria in the position I in the tth iteration.

wi,l : the counter-weight corresponding to wi,l. It is defined as wi,l=[the average of
Pr(P_h→P_{i,h}) in the \( t \)th iteration for all \( i\neq h \) and \( I \) is the bottom node), and \( \bar{w}_{t,j}^{i} \) is the average of \( \Pr(P_{j,i}→P_{j,i}^{*}, P_{j,i}→P_{j,i}^{*}) \) in the \( t \)th iteration for all \( i\neq h \).

\( k_t \): The combination parameter in the convex combination

\[
\bar{w}_{t,j}^{i} = k_t w_{i,j}^{t} + (1 - k_t) \bar{w}_{i,j}^{t} 
\]

with \( 0 < k_t < 1 \). Note that \( k_t = k = 0.5 \) in the traditional HCA and AHP-HCA.

\( CR : CR = CI/RI \) [3, 4, 5].

\( (CI)^t : \sum \sum |w_{i,j}^{t} - w_{i,j}^{t'}|/n_t \). Note that when \( CI = 0 \), it indicates that the DM has reached consistency of judgment. Higher \( CI \) suggests more serious inconsistence. Hence, it was suggested that the error can be accepted with \( CI \leq 0.1 \) [3, 4, 5].

2.3 Assumption [3]

1. If attribute A is absolutely more important than attribute B and is rated at 9, then B must be absolutely less important than A and is valued at 1/9.
2. The DM is rational, i.e. if A is preferred to B and B is preferred to C, then A is preferred to C.

3. The proposed COH

To enrich the negotiable, reasonable, and practicable behavior of finding comprised weights, a new HCA-based method called the chaotic ordered HCA (COH in short) is proposed. There are three important assumptions in the proposed COH: 1) each hierarchy is always tried to find possible combination for the comprised weights, 2) each hierarchy is always protected its own profit, 3) the DM must have the ability to discriminate which criterion (sub-criterion) is more important than the others under same parent node. The above concerns are reasonable and practical which are three significant factors need to be included in Guh’s HCA-based method [1].

The above important assumptions are all based on the psychology of the decision making process [2]. The chaotic random number is employed to \( k_t \) in Eq.(1) for the best comprised weights to achieve the maximal profit. A restriction is imposed to \( k_t \) also in Eq.(1) to express that the each hierarchy is always protected its own profit. The more important is a novel method to readjust weights is proposed to keep the order of weights of criteria help the DM. The proposed COH approach is discussed in detail in the rest of this section.

3.1 The Chaotic Random Number

Each hierarchy is always tried to find possible \( k_t \) in Eq.(1) for the best comprised weights to achieve the maximal profit. Theoretically, the optimal \( k_t \) for the best comprised weights can be identified by means of an exhaustive search over the all possible values of \( k_t \). In practice, however, we are not able to check all possible strategies because of the vast size of weights and \( k_t \). Therefore, \( k_t \) is set to a fixed number, say \( k = 0.5 \), in all traditional HCA or AHP-HCA [1, 8, 9, 10]. To correct the shortcoming of using fixed \( k_t \), the chaotic random number is incorporated in the proposed COH to improve the negotiable space of finding comprised weights.

In recently years, growing interests from physics, chemistry, biology and engineering have stimulated the studies of chaos for control, synchronization, and optimization [18, 19]. Due to the easy implementation and special ability to avoid being trapped in local optima, chaos has been a novel optimization technique and chaos-based searching algorithms have aroused intense interests [20]. In general, the chaotic random number is a special ergodicity, pseudo-randomness and irregularity variable [19]. The well-known logistic equation is adapted here to replace \( k_t \) in Eq.(1) such that a slight difference in the initial value of the chaotic random number would result in a significant difference in its long time behavior as follows [18].

\[
k_{t+1} = m k_t (1 - k_t),
\]

where \( m \neq 4 \) is the control parameter, \( 0 < k_t < 1 \) and \( k_t \neq 0.25, 0.5, \) and \( 0.75 \). The logistic equation has the basic characteristic of chaos, i.e., it reveals the sensitive dependence on initial conditions and includes infinite unstable periodic motions.
3.2 A limitation to \( k \)

According to the characteristic of the convex combination shown in Eq.(1), the new generated number \( w_{ij}^{k+1} \) is between \( w_{ij}^t \) and \( w_{ij}^l \). If \( k > 0.5 \), then \( w_{ij}^{k+1} \) is closer to \( w_{ij}^t \) than to \( w_{ij}^l \) and vice versa. Since the priority of each hierarchy is always to protect its own profit in real-life situation. Therefore, to be more reasonable, the \( k \) must not less than \( 0.5 \) in this study, i.e., Eq.(1) is revised as:

\[
 w_{ij}^{k+1} = \begin{cases} 
 k w_{ij}^t + (1-k)w_{ij}^l & \text{if } k_i \geq 0.5 \\
 (1-k)w_{ij}^t + k w_{ij}^l & \text{otherwise} 
\end{cases} 
\]  

(3)

3.3 The Novel Method to Readjustment Weights

Experienced DM may not be able to provide a 100% correct value to each weights of criterion. However, they can tell which criterion is more important. That is why Guh’s AHP-HCA [1] resulted in failed to apply in real-life. To avoid such flaw of AHP-HCA discussed in Section 3.4, a novel equation is proposed to replace Eqs.(1) and (3) to readjustment each weight by a new convex combination after sorting the weights under same parent in decreasing order as follows:

\[
 w_{ij}^t = k_i w_{ij}^t + (1-k_i)w_{ij}^l + \Delta_i . 
\]  

(4)

where

\[
 w_{ij}^t > w_{ij}^l \text{ if and only if } j < l, 
\]  

(5)

\[
 \Delta_i = \begin{cases} 
 \text{Min}[w_{ij}^t, w_{ij}^l] & \text{if } w_{ij}^t < w_{ij}^l, \\
 \text{Max}[w_{ij}^t, w_{ij}^l] & \text{otherwise} 
\end{cases} 
\]  

(6)

\[
 \bar{k}_i = \text{Max}[k_i, (1-k_i)]. 
\]  

(7)

To satisfy the total weight under the same parent is equal to one, a normalization is implemented to Eq.(4):

\[
 w_{ij}^t = \sum_j \frac{o_{ij}^t}{o_{ij}^l} . 
\]  

(8)

The major concept behind Eqs.(4)-(8) is that the order of the importance of criteria can never be change, i.e. the following important relationship must be satisfied for each new generated \( w_{ij}^{k+1} \):

\[
 w_{ij}^{k+1} < w_{ij}^{k+1} < w_{ij}^{k+1}, 
\]  

(9)

if \( w_{ij}^0 < w_{ij}^0 \) and \( w_{ij}^0 < w_{ij}^0 \).

\[
 w_{ij}^{k+1} < w_{ij}^{k+1} < w_{ij}^{k+1}, 
\]  

(10)

3.4 Procedures of the Proposed COH

The procedure of the Proposed COH is presented in detail using the following steps:

**Purpose:** Find the comprised weights for the DM.

**Input:** The performance evaluation reference index.

**Output:** The comprised weights satisfied by the DM.

**STEP 0.** Design types, qualitative and quantitative criteria of evaluation hierarchy structures for a problem by the experienced DMs.

**STEP 1.** Implement the AHP to find the prior values of weights to qualitative and quantitative criteria of each type of hierarchy structure constructed in STEP 0 if necessary.

**STEP 2.** Let \( t = 0 \).

**STEP 3.** Compute counter-weight of each criterion based on Eqs.(3) or (4).

**STEP 4.** Compute \((C^f)/f\) of the various indexes.

**STEP 5.** If \((C^f)/f < 0.001\), then the performance evaluation model would produce a
4. An example

![Diagram of H1 Evaluation Framework and H2 Evaluation Framework]

Figure 1. The hierarchy structure schema for the benchmark problem.

The general procedure is best illustrated with an example. Since the total running time and computational difficulty will grow exponentially with the number of hierarchies, levels, criteria, and sub-criteria in the worst case. Owing to this inherent problem, instead of presenting practically large problem, the famous benchmark problem shown in Figure 1 is selected to demonstrate the proposed COH as in all existing known algorithms [1,8-11].

| Table 1. The detail of 10 runs of the proposed COH for Figure 1. |
|---|---|---|---|---|---|---|---|---|---|---|
|   | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | Avg | Max | Min | Std |
| $w_{1,1}^f$ | 0.499 | 0.499 | 0.495 | 0.482 | 0.493 | 0.498 | 0.495 | 0.500 | 0.499 | 0.495 | 0.500 | 0.482 | 0.006 |
| $w_{1,2}^f$ | 0.501 | 0.501 | 0.505 | 0.518 | 0.507 | 0.502 | 0.505 | 0.511 | 0.500 | 0.501 | 0.505 | 0.518 | 0.006 |
| $w_{1,11}^f$ | 0.702 | 0.711 | 0.692 | 0.702 | 0.695 | 0.708 | 0.729 | 0.680 | 0.707 | 0.708 | 0.703 | 0.729 | 0.013 |
| $w_{1,12}^f$ | 0.298 | 0.290 | 0.308 | 0.298 | 0.306 | 0.292 | 0.271 | 0.320 | 0.294 | 0.292 | 0.297 | 0.320 | 0.013 |
| $w_{1,21}^f$ | 0.537 | 0.526 | 0.543 | 0.525 | 0.531 | 0.529 | 0.502 | 0.541 | 0.551 | 0.529 | 0.531 | 0.551 | 0.013 |
| $w_{1,22}^f$ | 0.463 | 0.474 | 0.457 | 0.475 | 0.469 | 0.471 | 0.498 | 0.459 | 0.449 | 0.471 | 0.469 | 0.449 | 0.013 |
| $w_{2,1}^f$ | 0.619 | 0.618 | 0.617 | 0.610 | 0.612 | 0.618 | 0.614 | 0.610 | 0.629 | 0.619 | 0.616 | 0.629 | 0.610 |
| $w_{2,2}^f$ | 0.381 | 0.382 | 0.383 | 0.390 | 0.388 | 0.382 | 0.386 | 0.391 | 0.371 | 0.382 | 0.384 | 0.391 | 0.371 |
| $w_{2,11}^f$ | 0.567 | 0.575 | 0.557 | 0.556 | 0.560 | 0.572 | 0.589 | 0.548 | 0.563 | 0.572 | 0.566 | 0.589 | 0.548 |
| $w_{2,21}^f$ | 0.434 | 0.426 | 0.443 | 0.444 | 0.440 | 0.428 | 0.411 | 0.453 | 0.437 | 0.428 | 0.434 | 0.453 | 0.411 |
| $w_{2,22}^f$ | 0.389 | 0.378 | 0.397 | 0.368 | 0.387 | 0.380 | 0.347 | 0.400 | 0.395 | 0.382 | 0.382 | 0.400 | 0.347 |
| $t$ | 11   | 12   | 11   | 12   | 11   | 11   | 13   | 12   | 14   | 12   | 11.9 | 14   | 11.994 |
| $10^3(CI)^t$ | 61   | 79   | 94   | 98   | 98   | 80   | 91   | 70   | 86   | 62   | 78   | 98   | 61   |

| $k$ | Table 2. The compromised weights under $k=0$ | Avg | Max | Min | Std |
|-----|---------------------------------|-----|-----|-----|-----|
| $w_{1,1}^f$ | 0.605 | 0.583 | 0.565 | 0.545 | 0.525 | 0.505 | 0.484 | 0.462 | 0.441 | 0.420 | 0.400 | 503 | 600 | 400 | 067 |
includes all the situations of AHP and HCA, but also it improves both AHP and HCA as well.

Furthermore, the proposed COH not only provides a better understanding of the structure of the problem. Furthermore, the proposed COH not only provides a better understanding of the structure of the problem. Thus, utilizing the proposed COH is more reasonable than using HCA-based method for ranking criteria, and the DM gains greater consistency with our intuitions than original one. Thus, utilizing the proposed COH is more reasonable than using HCA-based method for ranking criteria, and the DM gains greater consistency with our intuitions than original one. Therefore, the DM possesses, the better the advantage and success will be.

To the best of the author’s knowledge, the concept of chaos has never been investigated in the context of performance evaluation models. In this paper, we improved Guh’s HCA-based method and presented a revised method named chaotic ordered HCA (COH in short). The proposed COH has the advantage is that the ranking order of the proposed COH is more consistent with our intuitions than original one. Thus, utilizing the proposed COH is more reasonable than using HCA-based method for ranking criteria, and the DM gains greater understanding of the structure of the problem. Furthermore, the proposed COH not only includes all the situations of AHP and HCA, but also it improves both AHP and HCA as well.

5. Conclusions
Performance evaluation method has been applied extensively in many real-world systems. In general, the structure of evaluation models established for large-scale problem resolution seems to be the multiple-criterion and multiple-level type models that are more practical and simulate real problems in management. Hence, the more knowledge the DM possesses, the better the advantage and success will be.

There are 10 runs for the proposed COH to Figure 1, and the result is shown in Table 1. From Table 1, the number of iterations is higher that Guh’s HCA (see Table 2), but it can be ignored since the running times all are less than 0.1 second. Moreover, all the order of weights is still held in the proposed COH. According to the results, we find that there is no significant change when we increase the iteration number of COH.


t & 2 & 3 & 4 & 4 & 4 & 4 & 4 & 3 & 23.46 & 4 & 2.82

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