Short-time behavior of percolation observables at O(3) spin models: a preliminary analysis

Wanzeller W G
Universidade Federal da Fronteira Sul — Campus Laranjeiras do Sul, Paraná - Brazil
E-mail: wanderson@uffs.edu.br

Abstract. We analyzed the short-time behavior of percolation observables at three dimensional O(3) spin models. Following the Metropolis time evolution we can compare the magnetic and percolation order parameters. The magnetic order parameter evolves with a power law $M \sim t^{\theta}$, as expected. However the percolation order parameter follows a different law. We explored this particularity and raised some hypothesis. This paper is a preliminary analysis on this subject.

1. Introduction
In the literature, it is well-known that the magnetic order parameter (magnetization), at the critical point, evolves obeying a power law[1] $M \sim t^{\theta}$, where $\theta$ is a dynamic critical exponent and $t$ is the (Monte Carlo) time. On the other hand, until now, the short-time evolution of percolation order parameter has received a few attention. This is due to, perhaps, these quantities present the same behavior at thermodynamic equilibrium[2] or because the theory of percolation does not have a proper dynamics.

Recently was shown that percolation order parameter and magnetization do not have the same behavior at the heat-bath short-time dynamics in the bidimensional Ising models[3]. This raised some questions about the evolution of the percolation order parameters to more complex spin models. In this work we will compare the short-time evolution of percolation and magnetic order parameters using the classical Heisenberg spin model [O(3)] and present some preliminary results about this question.

2. The classical Heisenberg spin models
The classical Heisenberg spin model is given by the hamiltonian (without external magnetic field)

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j.$$  \hspace{1cm} (1)

The constant $J$ denotes the exchange interaction between nearest-neighbors $\langle ij \rangle$ three-dimensional unit spins $\vec{S}_i$ and $\vec{S}_j$. The magnetic order parameter is defined as the x-component of magnetization

$$M_x = \frac{1}{V} \sum_i S_i^x,$$ \hspace{1cm} (2)
where $V$ is the number of lattice sites (volume). This model presents a magnetic phase transition in a three-dimensional lattice at critical temperature\cite{4},

$$T_c = 1.44292(8),$$  \hspace{1cm} (3)

and many critical exponents\cite{1, 5}.

3. Percolation Observables
The mapping of a percolation problem in continuous spins models is well-known\cite{9, 10} and will not be detailed here. In this work we study the short-time behavior of the percolation order parameter

$$\Omega = \frac{\Delta}{V},$$ \hspace{1cm} (4)

where $\Delta$ is the percolating cluster size (number of connected spins) and $V$ the lattice volume.

4. Short-time dynamics
Using renormalization-group theory, it can be shown\cite{6} that the early time evolution of an order parameter (e.g. the magnetization) already displays universal critical behavior, given by

$$M(t, \epsilon, m_0) = b^{-\beta/\nu} M(t b^{-z}, \epsilon b^{1/\nu}, m_0 b^{x_0}),$$ \hspace{1cm} (5)

where $m_0$ is the initial magnetization, $\epsilon \equiv (T - T_c)/T_c$, $M$ is a universal function and $b$ is a scale factor, which can be taken equal to $t^{1/z}$. We thus expect for $T = T_c$ and small $m_0$ a power-law behavior at early times

$$M(t)_{\epsilon \to 0} \sim m_0 t^\theta,$$ \hspace{1cm} (6)

with \(\theta = (x_0 - \beta/\nu)/z\).

5. Results
We have studied the three-dimensional case of the O(3) model, performing Monte Carlo simulations with 50,000 seeds (samples or histories) and 200 time steps for each seed. To analyse the short-time behavior of magnetic and percolation order parameters we simulated at $T = T_c$ for different values of the $m_0$ initial magnetization (0.002, 0.004 and 0.006). With the lattice initially hot (null magnetization) we randomly flipped some spins to obtain the magnetization $m_0$ (sharp). We can also prepare the initial lattice with $m_0 = 1$. Thus, the magnetization exhibits the power law $M(t) \sim t^{-\beta/z\nu}$. The same was done for the Baxter-Wu model\cite{8}. Here this approach was not done yet.

In the implementation of O(3) algorithms we use the Metropolis one\cite{7}. The x-magnetization [Eq. (2)] can be positive or negative. Thus, the percolating cluster can be positive or negative, so the percolation order parameter [Eq. (4)] is calculated from

$$\Omega = \Omega_+ - \Omega_-, \hspace{1cm} (7)$$

where $\Omega_+$ ($\Omega_-$) is the percolation order parameter for positive (negative) magnetization.

The data from $\Omega$ [Eq. (7)] are fitted by (see tables 1 and 2)

$$f(t)\Omega = A \exp(-B/t),$$ \hspace{1cm} (8)

where $A$ and $B$ are constants. This function is similar to the solution of the diffusion equation [3, 11]. On the other hand the magnetic order parameter follows a power law $M \sim t^\theta$ as expected. In figure 1 we have the time evolution of magnetic and percolation order parameters for $V = 60^3$ and $m_0 = 0.006$.  

2
Table 1. Fit of percolation order parameter from Eq. (8) and $V = 30^3$.

| $m_0$ = 0.002 | $m_0$ = 0.004 | $m_0$ = 0.006 |
|----------------|----------------|----------------|
| $\Delta t$    | A   | B   | $\Delta t$    | A   | B   | $\Delta t$    | A   | B   |
| 10-200         | 0.0020 | 19.9(1.4) | 10-200         | 0.0046 | 22.3(0.7) | 14-200         | 0.0076 | 24.5(6) |
| 28-200         | 0.0022 | 26.0(2.9) | 14-200         | 0.0048 | 24.1(0.8) | 22-200         | 0.0075 | 27.1(7) |
| 38-110         | 0.0624 | 32.9(6.1) | 24-200         | 0.0050 | 26.3(1.0) | 36-200         | 0.0070 | 29.2(1.2) |

Table 2. Fit of percolation order parameter from Eq. (8) and $V = 60^3$.

| $m_0$ = 0.002 | $m_0$ = 0.004 | $m_0$ = 0.006 |
|----------------|----------------|----------------|
| $\Delta t$    | A   | B   | $\Delta t$    | A   | B   | $\Delta t$    | A   | B   |
| 10-200         | 0.0028 | 118.1(3.6) | 32-200         | 0.0057 | 118.4(1.9) | 28-200         | 0.0086 | 119.1(1.7) |
| 32-200         | 0.0030 | 124.4(3.6) | 54-200         | 0.0060 | 124.1(2.8) | 46-200         | 0.0091 | 126.7(1.9) |
| 50-200         | 0.0032 | 134.4(4.8) |                |       |       |                |       |       |

Figure 1. Comparison between time evolution of magnetic and percolation order parameters for $V = 60^3$ and $m_0 = 0.006$. 
6. Conclusions
In this work we present a preliminary study about the short-time behavior of magnetic and percolation order parameters in the three dimensional Heisenberg model adopting Metropolis algorithm. From the data it is evident that the two observables do not exhibit the same behavior. Indeed, the magnetization follows a power law while the percolation order parameter follows a function solution similar to the diffusion equation. This fact has already been observed for the Ising model[3]. We must conclude our simulations soon using largest lattices and heat-bath algorithm[7, 12].

7. ACKNOWLEDGMENTS
The author thanks R. S. Marques de Carvalho for helpful comments and suggestions and Departamento de Informática em Saúde of Escola Paulista de Medicina - Universidade Federal de São Paulo, where part of the codes was executed.

References
[1] Zheng B 1998 Int. J. Mod. Phys. B 1419
[2] Wanzeller W G, Cucchieri A, Mendes T and Krein G 2004 Braz. J. Phys. 34 247
[3] Wanzeller W G, Mendes T and Krein G 2006 Phys. Rev. E 74 051123
[4] Chen K, Ferrenberg A M and Landau D P 1993 Phys. Rev. E 48 3249
[5] Holm C and Janke W 1993 Phys. Rev. B 48 936
[6] Janssen H K, Schaub B and Schmittmann B 1989 Z. Phys. B 73 539
[7] Loison D, Qin C L, Schotte K D and Jin X F 2004 Eur. Phys. J. B 41 395
[8] Santos M and Figueiredo W 2001 Phys. Rev. E 63 042101
[9] Wolff U 1989 Phys. Rev. Lett. 62 361
[10] Dimitrović I, Hasenfratz P, Nager J and Niedermayer F 1991 Nucl. Phys. B 350 893
[11] Doremus R H 1995 Rates of Phase Transformations (New York: Academic Press).
[12] Miyatake Y, Yamamoto M, Kim J J, Toyonaga M and Nagai O 1996 J.Phys. C 19 2539