T. Zhang, Y. H. Zhang, Y. Yan, Z. H. Zhang, Z. Y. Zhang, Z. Y. Zhang, G. Zhao, J. Zhao, J. Y. Zhao, J. Z. Zhao, L. Zhao, M. G. Zhao, Q. Zhao, S. J. Zhao, Y. B. Zhao, Y. X. Zhao, Z. G. Zhao, A. Zhemchugov, B. Zheng, J. P. Zheng, Y. H. Zheng, B. Zhong, C. Zhong, X. Zhong, H. Zhou, L. P. Zhou, X. Zhou, X. K. Zhou, X. R. Zhou, X. Y. Zhou, Y. Z. Zhou, J. Zhu, K. Zhu, K. J. Zhu, L. X. Zhu, S. H. Zhu, S. Q. Zhu, T. J. Zhu, W. J. Zhu, Y. C. Zhu, Z. A. Zhu, B. S. Zou, J. H. Zou.

(BESIII Collaboration)

1 Institute of High Energy Physics, Beijing 100049, People’s Republic of China
2 Beijing University, Beijing 100191, People’s Republic of China
3 Beijing Institute of Petrochemical Technology, Beijing 102617, People’s Republic of China
4 Bochum Ruhr-University, D-44780 Bochum, Germany
5 Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA
6 Central China Normal University, Wuhan 430079, People’s Republic of China
7 China Center of Advanced Science and Technology, Beijing 100190, People’s Republic of China
8 COMSATS University Islamabad, Lahore Campus, Defence Road, Off Raiwind Road, 54000 Lahore, Pakistan
9 Fudan University, Shanghai 200433, People’s Republic of China
10 G.I. Budker Institute of Nuclear Physics SB RAS (BINP), Novosibirsk 630090, Russia
11 GSI Helmholtzcentre for Heavy Ion Research GmbH, D-64291 Darmstadt, Germany
12 Guangxi Normal University, Guilin 541004, People’s Republic of China
13 Hangzhou Normal University, Hangzhou 310036, People’s Republic of China
14 Helmholtz Institute Mainz, Staudinger Weg 18, D-55099 Mainz, Germany
15 Henan Normal University, Xinxiang 453007, People’s Republic of China
16 Henan University of Science and Technology, Luoyang 471003, People’s Republic of China
17 Henan University of Technology, Zhengzhou 450001, People’s Republic of China
18 Huangshan College, Huangshan 245000, People’s Republic of China
19 Hunan Normal University, Changsha 410081, People’s Republic of China
20 Hunan University, Changsha 410082, People’s Republic of China
21 Indian Institute of Technology Madras, Chennai 600036, India
22 Indiana University, Bloomington, Indiana 47405, USA
23 INFN Laboratori Nazionali di Frascati, (A)INFN Laboratori Nazionali di Frascati, I-00044, Frascati, Italy; (B)INFN Sezione di Perugia, I-06100, Perugia, Italy; (C)University of Perugia, I-06100, Perugia, Italy
24 INFN Sezione di Ferrara, (A)INFN Sezione di Ferrara, I-44122, Ferrara, Italy; (B)University of Ferrara, I-44122, Ferrara, Italy
25 Institute of Modern Physics, Lanzhou 730000, People’s Republic of China
26 Institute of Physics and Technology, Peace Ave. 54B, Ulaanbaatar 13330, Mongolia
27 Jilin University, Changchun 130019, People’s Republic of China
28 Johannes Gutenberg University of Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany
29 Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia
30 Justus-Liebig-Universitaet Giessen, I. Physikaisches Institut, Heinrich-Buff-Ring 16, D-35392 Giessen, Germany
31 Lanzhou University, Lanzhou 730000, People’s Republic of China
32 Liaoning Normal University, Dalian 116029, People’s Republic of China
33 Liaoning University, Shenyang 110036, People’s Republic of China
34 Nanjing Normal University, Nanjing 210023, People’s Republic of China
35 Nanjing University, Nanjing 210093, People’s Republic of China
36 Nankai University, Tianjin 300071, People’s Republic of China
37 National Centre for Nuclear Research, Warsaw 02-093, Poland
38 North China Electric Power University, Beijing 102206, People’s Republic of China
39 Peking University, Beijing 100871, People’s Republic of China
40 Qufu Normal University, Qufu 273165, People’s Republic of China
41 Shandong Normal University, Jinan 250014, People’s Republic of China
42 Shandong University, Jinan 250100, People’s Republic of China
43 Shanghai Jiao Tong University, Shanghai 200240, People’s Republic of China
44 Shanxi Normal University, Linfen 041004, People’s Republic of China
45 Shandong University, Taiyuan 030006, People’s Republic of China
46 Sichuan University, Chengdu 610064, People’s Republic of China
47 Soochow University, Suzhou 215006, People’s Republic of China
48 South China Normal University, Guangzhou 510006, People’s Republic of China
49 Southeast University, Nanjing 211100, People’s Republic of China
50 State Key Laboratory of Particle Detection and Electronics, Beijing 100049, Hefei 230026, People’s Republic of China
51 Sun Yat-Sen University, Guangzhou 510275, People’s Republic of China
52 Suranaree University of Technology, University Avenue 111, Nakhon Ratchasima 30000, Thailand
53 Tsinghua University, Beijing 100084, People’s Republic of China
We present the first search for the semileptonic decay $D_s^+ \to \pi^0 e^+ \nu_e$ using a data sample of electron-positron collisions recorded with the BESIII detector at center-of-mass energies between 4.178 and 4.226 GeV, corresponding to an integrated luminosity of 6.32 fb$^{-1}$. This decay is expected to be sensitive to $\pi^0$-$\eta$ mixing. No significant signal is observed. We set an upper limit of $6 \times 10^{-5}$ on the branching fraction at the 90% confidence level.

I. INTRODUCTION

Neutral mesons that have hidden flavors and the same quantum numbers can mix via the strong and electromagnetic interactions. Meson mixing is an interesting phenomenon that can be used to explain some specific decay processes of heavy mesons. Many mixing effects are being widely studied, such as in the systems $\pi^0$-$\eta$ [1], $\rho$-$\omega$ [2], $\omega$-$\phi$ [3], and $\eta$-$\eta'$ [4]. This analysis searches for $\pi^0$-$\eta$ mixing in semileptonic $D_s^+$ decays. The semileptonic decay $D_s^+ \to \pi^0 e^+ \nu_e$ can only occur via $\pi^0$-$\eta$ mixing, as shown in Fig. 1, and nonperturbative weak annihilation effects, as shown in Fig. 2, where the two gluons can be emitted from the $c$ quark or $s$ quark, or one gluon from each quark [1]. However, the radiation of a $\pi^0$ from the weak annihilation effect is suppressed not only by the Okubo-Zweig-Iizuka (OZI) rule but also by isospin conservation. Consequently, the weak annihilation contribution to the $D_s^+ \to \pi^0 e^+ \nu_e$ decay is relatively small compared to that from $\pi^0$-$\eta$ mixing. The contribution to the branching fraction (BF) of $D_s^+ \to \pi^0 e^+ \nu_e$ from the weak annihilation effect is expected to be only of the order of $10^{-7} - 10^{-8}$, while the contribution from $\pi^0$-$\eta$ mixing is expected to be (2.65 ± 0.38) × 10$^{-5}$ [1]. Therefore, this decay provides an excellent opportunity to study the $\pi^0$-$\eta$ mixing effect.

In this paper, we present the first search for the semileptonic decay $D_s^+ \to \pi^0 e^+ \nu_e$ in a data sample corresponding to an integrated luminosity of 6.32 fb$^{-1}$, which was recorded by the BESIII detector at center-of-mass
FIG. 1. Feynman diagram of the semileptonic decay $D_s^+ \rightarrow \pi^0 e^+ \nu_e$ through $\pi^0$-$\eta$ mixing.

FIG. 2. Feynman diagrams of the semileptonic decay $D_s^+ \rightarrow \pi^0 e^+ \nu_e$ through the weak annihilation effect with the radiation of a $\pi^+$ meson.

full solid angle and consists of a helium-based multilayer drift chamber (MDC), a plastic scintillator time-of-flight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T magnetic field. The solenoid is supported by an octagonal flux-return yoke with resistive plate counter muon identification modules interleaved with steel. The charged-particle momentum resolution at 1 GeV/$c$ is 0.5%, and the specific energy loss ($dE/dx$) resolution is 6% for electrons from Bhabha scattering. The EMC measures photon energies with a resolution of 2.5% (5%) at 1 GeV in the barrel (end cap) region. The time resolution in the TOF barrel region is 68 ps, while that in the end cap region is 110 ps. The end cap TOF system was upgraded in 2015 using multi-gap resistive plate chamber technology, providing a time resolution of 60 ps [8].

The data samples used in this analysis correspond to an integrated luminosity ($\mathcal{L}_{\text{int}}$) of 6.32 fb$^{-1}$ taken in the range of $\sqrt{s} = 4.178$ to 4.226 GeV, as listed in Table I. All data samples except the 4.226 GeV one benefit from the improved time resolution in the endcaps. In these energies, $D_s^+ D_s^- \bar{D}_s^+$ events provide a large sample of $D_s^+$ mesons. The cross section of $D_s^+ D_s^- \bar{D}_s^+$ production in $e^+ e^-$ annihilation is about a factor of twenty larger than that of $D_s^+ D_s^- [9]$, and $D_s^+ \bar{D}_s^+$ decays to $\gamma D_s^+$ with a dominant BF of $(93.5 \pm 0.7)\%$ [10]. Therefore, we use $D_s^+ D_s^- \bar{D}_s^+$ events in this analysis.

**TABLE I.** Integrated luminosity $\mathcal{L}_{\text{int}}$ and the recoil mass $M_{\text{rec}}$ requirements for various energies, where $M_{\text{rec}}$ is defined in Eq. (5). The first and second uncertainties are statistical and systematic, respectively. The data collected at $\sqrt{s} = 4.178-4.219$ GeV (which corresponds to about 83.3% of total data sample) use the updated TOF [11, 12].

| $\sqrt{s}$ (GeV) | $\mathcal{L}_{\text{int}}$ (pb$^{-1}$) | $M_{\text{rec}}$ (GeV/$c^2$) |
|-----------------|-----------------|-----------------|
| 4.178           | 3189.0 $\pm$ 0.2 $\pm$ 31.9 | [2.050, 2.180]  |
| 4.189           | 5267.6 $\pm$ 0.1 $\pm$ 2.2 | [2.048, 2.190]  |
| 4.199           | 526.0 $\pm$ 0.1 $\pm$ 2.1 | [2.046, 2.200]  |
| 4.209           | 517.1 $\pm$ 0.1 $\pm$ 1.8 | [2.044, 2.210]  |
| 4.219           | 514.6 $\pm$ 0.1 $\pm$ 1.8 | [2.042, 2.220]  |
| 4.226           | 1056.4 $\pm$ 0.1 $\pm$ 7.0 | [2.040, 2.220]  |

Large samples of Monte Carlo (MC) simulated events produced with GEANT4-based [13] software, which includes the geometric description of the BESIII detector and the detector response, are used to determine the detection efficiency and to estimate the background contributions. The simulation includes the beam-energy spread and initial-state radiation (ISR) in the $e^+ e^-$ annihilation modeled with the generator KKMC [14]. Inclusive MC samples with 40 times the size of data are used to simulate the background contributions. The inclusive MC samples, which contain no signal $D_s^+ \rightarrow \pi^0 e^+ \nu_e$ decays, include the production of open-charm processes, the ISR production of vector charmonium(-like) states, and the
continuum processes incorporated in KKMC. The known decay modes are modeled with EVTGEN [15] using world averaged BF values [10], and the remaining unknown decays from the charmonium states with LUNDCHARM [16]. Final-state radiation from charged final-state particles is incorporated with PHOTOS [17]. The signal detection efficiencies and signal shapes are obtained from signal MC samples, in which the signal $D_s^+ \rightarrow \pi^+ e^+ \nu_e$ decay is simulated using the ISGW2 model [18, 19].

III. DATA ANALYSIS

The process $e^+ e^- \rightarrow D_s^{*+} D_s^- + c.c. \rightarrow \gamma D_s^{*+} D_s^-$ allows the study of semileptonic $D_s^*$ decays with a tag technique [20] since only one neutrino escapes undetected. There are two types of samples used in the tag technique: single tag (ST) and double tag (DT) events. In the ST sample, a $D_s^-$ meson is reconstructed through a specific hadronic decay without any requirement on the remaining measured tracks and EMC showers. In the DT sample, a $D_s^-$, designated as the “tag”, is reconstructed through a hadronic decay mode first, and then the decay $D_s^- \rightarrow \pi^0 e^+ \nu_e$, designated as the “signal”, is reconstructed with the remaining tracks and EMC showers. For a specific tag mode, the ST yield is given by

$$N_{\text{tag}}^{\text{ST}} = 2 N_{D^* D^-} B_{\text{tag}}^{\text{ST}},$$

and the DT yield is given by

$$N_{\text{tag,sig}}^{\text{DT}} = 2 N_{D^* D^-} B_{\text{tag}}^{\text{DT}} B_{\text{sig}}^{\text{ST}},$$

where $N_{D^* D^-}$ is the total number of $D_s^{*+} D_s^- + c.c.$ pairs produced, $B_{\text{tag}}^{\text{ST}}$ is the BF of the signal decay (the tag mode), $B_{\text{tag}}^{\text{DT}}$ is the BF of $D_s^- \rightarrow \gamma D_s^-$ ($\pi^0 \rightarrow \gamma \gamma$), and $\epsilon_{\text{tag}}^{\text{ST}}$ ($\epsilon_{\text{tag,sig}}^{\text{DT}}$) is the corresponding ST (DT) efficiency. By isolating $B_{\text{tag}}^{\text{ST}}$, one obtains

$$B_{\text{tag}}^{\text{ST}} = \frac{N_{\text{tag,sig}}^{\text{DT}}}{N_{\text{tag}}^{\text{ST}} B_{\text{tag}}^{\text{DT}}},$$

where the yields $N_{\text{tag}}^{\text{ST}}$ and $N_{\text{tag,sig}}^{\text{DT}}$ are obtained from data samples, while $\epsilon_{\text{tag}}^{\text{ST}}$ and $\epsilon_{\text{tag,sig}}^{\text{DT}}$ are obtained from inclusive and signal MC samples, respectively. For multiple tag modes and energy points, the above equation is generalized as

$$B_{\text{tag}}^{\text{ST}} = \frac{N_{\text{tag,sig}}^{\text{DT}}}{B_{\gamma} B_{\pi^0} \sum_{\alpha,i} \epsilon_{\alpha,i}^{\text{ST}} \epsilon_{\alpha,i}^{\text{DT}}},$$

where $\alpha$ represents tag modes, $i$ represents different energy points, and $N_{\text{total,sig}}^{\text{DT}}$ is the total signal yield.

The tag candidates are reconstructed with $K^\pm$, $\pi^\pm$, $\pi^0$, $\rho^0$, $\eta$, $\eta'$, and $K_S^0$ mesons that satisfy the particle selection criteria detailed below. Twelve tag modes are used, and the requirements on the invariant masses of tagged $D_s^-$ candidates ($M_{\text{tag}}$) are summarized in Table II.

Photon candidates are reconstructed from isolated clusters found in the EMC. The EMC shower time is required to be within [0, 700] ns from the event start time in order to suppress fake photons due to electronic noise or $e^+ e^-$ beam background. Photon candidates within $|\cos \theta| < 0.80$ (barrel) are required to deposit more than 25 MeV of energy, and those with $0.86 < |\cos \theta| < 0.92$ (end cap) must deposit more than 50 MeV, where $\theta$ is the polar angle with respect to the $z$ direction (the positive direction of the MDC axis). To exclude showers that originate from charged tracks, the angle between the position of each shower in the EMC and the closest extrapolated charged track must be greater than 10 degrees. The $\pi^0$ ($\eta$) candidates are reconstructed through $\pi^0 \rightarrow \gamma \gamma$ ($\eta \rightarrow \gamma \gamma$) decays, with at least one barrel photon. The diphoton invariant masses for the identification of $\pi^0$ and $\eta$ decays are required to be in the ranges of $[0.115, 0.150]$ GeV/$c^2$ and $[0.500, 0.570]$ GeV/$c^2$, respectively. The $\chi^2$ of the kinematic fit constraining $M_{\gamma\gamma}$ to the $\pi^0$ or $\eta$ known mass [10] is required to be less than 30.

Charged particle candidates reconstructed using the information of the MDC must satisfy $|\cos \theta| < 0.93$ with the distance of closest approach to the interaction point (IP) less than 10 cm in the $z$ direction and less than 1 cm in the plane perpendicular to $z$. Particle identification (PID) of charged kaons and pions is implemented by combining the information of $dE/dx$ from the MDC and the time-of-flight from the TOF system. For charged kaon (pion) candidates, the likelihood for the kaon (pion) hypothesis is required to be larger than that for a pion (kaon). Electron PID uses EMC information along with $dE/dx$ and time-of-flight to construct likelihoods for electron, pion, and kaon hypotheses ($L_e$, $L_\pi$, and $L_K$). Electron candidates must satisfy $L_e/(L_e + L_\pi + L_K) > 0.8$. Additionally, the energy deposited in the EMC by the electron candidate must be more than 80% of the track momentum measured by the MDC.

Candidate $K_S^0$ mesons are reconstructed with pairs of two oppositely charged particles, whose distances of closest approach to the IP along $z$ are less than 20 cm. These two particles are assumed to be pions without PID applied. Primary and secondary vertices are reconstructed, and the decay length between the two vertices is required to be greater than twice its uncertainty. This requirement is not applied for the $D_s^- \rightarrow K_S^0 K^-$ decay due to the low combinatorial background. Candidate $K_S^0$ mesons are required to have the $\chi^2$ of the vertex fit less than 100 and be inside an invariant-mass window $[0.487, 0.511]$ GeV/$c^2$, which is about three times the resolution. The invariant mass of the $\pi^+ \pi^-$ pair of the $D_s^- \rightarrow K^- \pi^+ \pi^-$ decay is required to be outside of the $K_S^0$ invariant mass window to prevent an event being doubly counted in selecting the $D_s^- \rightarrow K_S^0 K^-$ and $D_s^- \rightarrow K^- \pi^+ \pi^- \pi^+ \pi^-$ tag modes. The $\rho^0$
candidates are selected via the process $\rho^0 \rightarrow \pi^+\pi^-$ with an invariant mass window $[0.620, 0.920]$ GeV/$c^2$, which is about two times the $\rho^0$ width. The $\eta'$ candidates are formed from $\pi^+\pi^-\eta$ and $\gamma\rho^0$ combinations with invariant masses falling within the range of $[0.946, 0.970]$ GeV/$c^2$, about three times the resolution.

In order to identify the process $\pi^+e^- \rightarrow D_s^+D_s^-\gamma$, the signal windows, listed in Table I, are applied to the recoiling mass ($M_{\text{rec}}$) of the tag candidate. The definition of $M_{\text{rec}}^2$ is

$$\sqrt{(E_{\text{cm}} - \sqrt{c^2|\vec{p}_{\text{tag}}|^2 + c^4m_{D_s}^2})^2 - c^2|\vec{p}_{\text{tag}}|^2},$$

where $E_{\text{cm}}$ is the energy of the $e^+e^-$ CM system, $(\sqrt{|\vec{p}_{\text{tag}}|^2 + c^2m_{D_s}^2}, \vec{p}_{\text{tag}})$ is the measured four-momentum of the tag candidate, and $m_{D_s}$ is the known $D_s^-$ mass [10]. If there are multiple candidates for a tag mode, the one with $M_{\text{rec}}$ closest to the known $D_s^\pm$ mass [10] is chosen.

The ST yields for various tag modes $N_{\text{tag}}^{\text{ST}}$ are obtained by fitting the $M_{\text{tag}}$ distributions of the accepted ST $D_s^-$ candidates. Example fits to the data sample at 4.178 GeV are shown in Fig. 3. The description of the signal shape is based on the MC-simulated shape convolved with a Gaussian function accounting for the resolution difference between data and MC. The background is described by a second-order Chebyshev polynomial. The only two significant peaking backgrounds in all the signal windows, listed in Table II, is described by a second-order Chebyshev polynomial. The background is the energy of the $e^+e^-$ CM system, $(\sqrt{|\vec{p}_{\text{tag}}|^2 + c^2m_{D_s}^2}, \vec{p}_{\text{tag}})$ is the measured four-momentum of the tag candidate, and at least one more photon to reconstruct the transition photon of $D_s^\pm$ meson mass-squared before the kinematic fit for signal $D_s^\pm$ decays. To separate the two peaking backgrounds are fixed based on their $M_{\text{rec}}$ to satisfy $3.83 < M_{\text{rec}} < 3.96$ GeV/$c^2$, as shown in Fig. 4(a). Studies of the inclusive MC sample show that there is a large background coming from $D_s^0 \rightarrow K^-e^+\nu$ decays versus a hadronic $D_s^0$ decay with a $\pi^0$ meson in the final state, where the $K^-$ and the $\pi^0$ mesons are interchanged between the two decays. In order to remove this background, for $D_s^-$ tag modes with a $K^-$, the invariant mass of the final-state particles of the reconstructed tag $D_s^-$ except the $K^-$ and the $\pi^0$ in the reconstructed signal $D_s^+$ is calculated, called $M_{K\pi\nu}$. A veto $1.835 < M_{K\pi\nu} < 1.890$ GeV/$c^2$ is applied as shown in Fig. 4(b). This veto removes more than 90% of this background (about 20% of the total background) and sacrifices only about 4% efficiency. The DT efficiencies are obtained using the signal MC samples and listed in Table III.

The missing-mass squared of the neutrino is defined as

$$MM^2 = \frac{1}{c^2}(p_{\text{cm}} - p_{\text{tag}} - p_{\pi^0} - p_e - p_{\gamma})^2,$$

where $p_{\text{cm}}$ is the four-momentum of the $e^+e^-$ CM system, and $p_i$ ($i = \pi^0, e, \gamma$) is the four-momentum of the final-state particle $i$ on the signal side. The $MM^2$ distribution of accepted candidate events is shown in Fig. 4(c). Unbinned maximum-likelihood fits to the $MM^2$ distribution are performed, where the signal and background shapes are modeled by MC-simulated shapes obtained from the signal and inclusive MC samples, respectively. The fit result is shown in Fig. 5, and the fitted signal yield is $-6.9 \pm 7.2$. Since no significant signal is observed, an upper limit is determined with the likelihood distribution, shown in Fig. 6, as a function of assumed BFs. The upper limit on the BF at the 90% confidence level, obtained by integrating from zero to 90% of the resulting curve, is $0.57 < M_{D_s^0} < 6.4 \times 10^{-5}$. The method to incorporate systematic uncertainty is discussed in the next section.

### IV. SYSTEMATIC UNCERTAINTY

The likelihood distribution used in the upper limit measurement covers a range of BFs, as shown in Fig. 6 (or signal events yields). The sources of systematic uncertainties on the BF measurement are classified into two types: additive (or independent of the measured BF central value) and multiplicative (proportional to the BF).
FIG. 3. Fits to the $M_{\text{tag}}$ distributions of the ST $D_s^-$ candidates at $\sqrt{s} = 4.178$ GeV. The points with error bars are data, red solid lines are total fits, and blue dashed lines are the fitted backgrounds. The pairs of pink arrows denote signal regions. The peaking background MC-simulated shapes of $D^- \to K^0_S \pi^-$ and $D^- \to \eta\pi^+\pi^-\pi^-$ decays are added to the background polynomials in the fits of $D_s^- \to K^0_S K^-$ and $D_s^- \to \pi^-\eta'$ decays to account for the peaking background, respectively.

TABLE II. Requirements on $M_{\text{tag}}$, the ST yields ($N_{\text{tag}}^{\text{ST}}$) and ST efficiencies ($\epsilon_{\text{tag}}^{\text{ST}}$) at $\sqrt{s} = (I) 4.178$ GeV, (II) 4.189-4.219 GeV, and (III) 4.226 GeV, where the subscripts of $\eta$ and $\eta'$ denote the decay modes used to reconstruct $\eta$ and $\eta'$ candidates. The efficiencies for the energy points 4.189-4.219 GeV are averaged based on the luminosities. The BF of the sub-particle ($K^0_S$, $\pi^0$, $\eta$ and $\eta'$) decays are not included. Uncertainties are statistical only.

| Tag mode        | $M_{\text{tag}}$ (GeV/$c^2$) | (I) $N_{\text{tag}}^{\text{ST}}$ | (I) $\epsilon_{\text{tag}}^{\text{ST}}$ (%) | (II) $N_{\text{tag}}^{\text{ST}}$ | (II) $\epsilon_{\text{tag}}^{\text{ST}}$ (%) | (III) $N_{\text{tag}}^{\text{ST}}$ | (III) $\epsilon_{\text{tag}}^{\text{ST}}$ (%) |
|-----------------|-------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| $D_s^- \to K^0_S K^-$ | [1.948, 1.991]                | 31941 ± 312                      | 47.36 ± 0.07                     | 18559 ± 261                      | 47.26 ± 0.09                     | 6582 ± 160                       | 46.37 ± 0.16                     |
| $D_s^- \to K^+ K^-\pi^-$ | [1.950, 1.986]                | 137240 ± 614                     | 39.47 ± 0.03                     | 81286 ± 505                      | 39.32 ± 0.04                     | 28439 ± 327                      | 38.38 ± 0.07                     |
| $D_s^- \to K^0_S K^-\pi^0$ | [1.946, 1.987]                | 11385 ± 529                      | 16.12 ± 0.11                     | 6832 ± 457                       | 15.71 ± 0.16                     | 2227 ± 220                       | 15.93 ± 0.29                     |
| $D_s^- \to K^+ K^-\pi^-\pi^0$ | [1.947, 1.982]                | 39306 ± 799                      | 10.50 ± 0.03                     | 23311 ± 659                      | 10.58 ± 0.05                     | 7785 ± 453                       | 10.39 ± 0.08                     |
| $D_s^- \to K^0_S K^-\pi^-\pi^+$ | [1.958, 1.980]                | 8093 ± 326                       | 20.40 ± 0.12                     | 5269 ± 282                       | 20.19 ± 0.17                     | 1662 ± 217                       | 19.50 ± 0.31                     |
| $D_s^- \to K^0_S K^+\pi^-\pi^-$ | [1.953, 1.983]                | 15719 ± 289                      | 21.83 ± 0.06                     | 8948 ± 231                       | 21.63 ± 0.09                     | 3263 ± 172                       | 21.29 ± 0.15                     |
| $D_s^- \to \pi^-\pi^-\pi^+$ | [1.952, 1.982]                | 37077 ± 859                      | 51.43 ± 0.15                     | 21999 ± 776                      | 50.35 ± 0.22                     | 7511 ± 393                       | 49.32 ± 0.41                     |
| $D_s^- \to \pi^-\eta_{\gamma\gamma}$ | [1.930, 2.000]                | 17049 ± 403                      | 43.58 ± 0.15                     | 10025 ± 339                      | 43.00 ± 0.22                     | 3725 ± 252                       | 41.83 ± 0.41                     |
| $D_s^- \to \pi^-\pi^0\eta$ | [1.920, 2.000]                | 42618 ± 1397                     | 18.09 ± 0.11                     | 26067 ± 1196                     | 18.40 ± 0.16                     | 10513 ± 1920                     | 17.69 ± 0.30                     |
| $D_s^- \to \pi^-\eta_{\eta^+\eta^-\eta_{\gamma\gamma}}$ | [1.940, 1.996]                | 7759 ± 141                       | 19.12 ± 0.06                     | 4428 ± 111                       | 19.00 ± 0.08                     | 1648 ± 74                        | 18.56 ± 0.13                     |
| $D_s^- \to \pi^-\eta_{\eta'}$ | [1.939, 1.992]                | 20610 ± 538                      | 26.28 ± 0.10                     | 11937 ± 480                      | 26.09 ± 0.14                     | 3813 ± 335                       | 25.94 ± 0.27                     |
| $D_s^- \to K^-\pi^+\pi^-$ | [1.953, 1.986]                | 17423 ± 666                      | 47.46 ± 0.22                     | 10175 ± 448                      | 47.19 ± 0.32                     | 4984 ± 458                       | 45.66 ± 0.59                     |
are statistical only. The signal MC sample is normalized arbitrarily for visualization purposes. An additional requirement of $|MM^2| < 0.20\text{ GeV}^2/c^4$ has been applied.

TABLE III. DT efficiencies ($\epsilon_{\text{tag,sig}}^{\text{DT}}$) at $\sqrt{s} = (\text{I}) 4.178\text{ GeV}, (\text{II}) 4.189-4.219\text{ GeV},$ and (III) 4.226 GeV. The efficiencies for the energy points 4.189-4.219 GeV are averaged based on the luminosities. The BF of the $\pi^0$ decay is not included. Uncertainties are statistical only.

| Tag mode          | (I) $\epsilon_{\text{tag,sig}}^{\text{DT}}$(%) | (II) $\epsilon_{\text{tag,sig}}^{\text{DT}}$(%) | (III) $\epsilon_{\text{tag,sig}}^{\text{DT}}$(%) |
|-------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| $D_s^+ \rightarrow K^+K^−\pi^−$ | 13.94 ± 0.11 | 13.18 ± 0.06 | 12.20 ± 0.11 |
| $D_{s}^− \rightarrow K_S^0K^−$ | 10.31 ± 0.04 | 9.77 ± 0.02 | 9.02 ± 0.04 |
| $D_s^− \rightarrow K_S^0K^−\pi^0$ | 4.78 ± 0.07 | 4.56 ± 0.03 | 4.34 ± 0.06 |
| $D_s^− \rightarrow K^+K^−\pi^−\pi^0$ | 2.89 ± 0.02 | 2.79 ± 0.01 | 2.66 ± 0.02 |
| $D_s^− \rightarrow K_S^0K^−\pi^−\pi^+$ | 5.38 ± 0.09 | 5.03 ± 0.04 | 4.71 ± 0.08 |
| $D_s^+ \rightarrow K_S^0K^+\pi^−\pi^−$ | 5.40 ± 0.07 | 5.15 ± 0.03 | 4.84 ± 0.06 |
| $D_s^− \rightarrow \pi^0\pi^−\pi^−$ | 16.92 ± 0.12 | 15.79 ± 0.06 | 14.51 ± 0.12 |
| $D_s^− \rightarrow \pi^−\eta$ | 13.98 ± 0.14 | 13.02 ± 0.07 | 12.02 ± 0.13 |
| $D_s^+ \rightarrow \pi^−\pi^0\eta$ | 6.52 ± 0.04 | 6.07 ± 0.02 | 5.52 ± 0.04 |
| $D_s^− \rightarrow \pi^−\eta_{\pi^0}^{\prime}\pi^−\eta$ | 5.60 ± 0.09 | 5.31 ± 0.04 | 4.87 ± 0.09 |
| $D_s^+ \rightarrow \pi^−\eta_{\pi^0}^{\prime}\pi^0$ | 7.89 ± 0.08 | 7.59 ± 0.04 | 7.05 ± 0.08 |
| $D_s^− \rightarrow K^−\pi^+\pi^−$ | 13.33 ± 0.14 | 12.51 ± 0.07 | 11.51 ± 0.13 |

The multiplicative ones are summarized in Table IV. Note that most systematic uncertainties on the tag side cancel due to the DT technique.

Additive uncertainties affect the signal yield determination, which is dominated by the imperfect background shape description. This systematic uncertainty is studied by altering the nominal MC background shape with two methods. First, alternative MC samples are used to determine the background shape, where the relative fractions of backgrounds from $q\bar{q}$ and non-$D_s^+D_s^−$ open-charm are varied within their uncertainties, and the BFs of the major $D_s^+D_s$ background sources, i.e. $D_s^+ \rightarrow \eta\pi^0\nu_{\pi}$, $D_s^+ \rightarrow f_0\pi^+\nu_e$, $D_s^+ \rightarrow K_0^0\pi^+\nu_e$, and $D_s^+ \rightarrow \pi^+\nu_e$, are varied by their listed uncertainties [10]. Second, the background shape is obtained from the inclusive MC samples using a Kernel estimation method [22] implemented in RooFit [23]. The smoothing parameter of RooKeysPdf is varied between 0 and 2 to obtain alternative background shapes. An alternative signal shape based on the simple pole model [24] is tested, but the associated uncertainty is negligible.

Multiplicative uncertainties are from the efficiency determination and the quoted BFs. The uncertainties in the total number of the ST $D_s^+$ mesons is assigned to be 0.5% by examining the changes of the fit yields when varying the signal shape, background shape, and taking into account the background fluctuation in the fit. The uncertainty from the BFs of $D_s^+ \rightarrow \gamma\gamma$ and $\pi^0 \rightarrow \gamma\gamma$ decays are 0.7% and 0.03%, respectively, according to the known values [10]. The systematic uncertainty related to $e^+\nu_e$ tracking or PID efficiency is assigned as 1.0% from studies of a control sample of radiative Bhabha events. The systematic uncertainties associated with reconstruction efficiencies of the transition photon and $\pi^0$ are studied by using control samples of the decay $J/\psi \rightarrow \pi^+\pi^-\pi^0$ and the process $e^+e^- \rightarrow K^+K^−\pi^+\pi^−\pi^0$, respectively.
The efficiency difference between data and MC samples is then determined to be 1.0% for the transition photon and 2.0% for the final state $π^0$. The uncertainties due to the maximum energy of photons not used in the DT event selection criteria, and requiring one charged track are assigned as 0.5% and 0.9%, respectively. We determine these uncertainties by analyzing DT hadronic events in which one $D^-_s$ decays into one of the tag modes and the other $D^-_s$ decays into $K^+K^-π^−$ or $K^0_S K^−$. The uncertainty due to the limited MC sample size is obtained by

$$\sum_\alpha (f_\alpha σ_{\epsilon,\alpha})^2 \approx 0.5\%,$$

where $f_\alpha$ is the tag yield fraction, and $\epsilon_\alpha$ and $σ_{\epsilon,\alpha}$ are the signal efficiency and the corresponding uncertainty of tag mode $\alpha$, respectively. The acceptance efficiencies of the kinematic fit requirement are studied with the control sample $D_s^+ → π^+π^0η$ from the DT hadronic $D^-_s D^+_s + c.c.$ events due to its similar topology and large BF. We take into account the difference of the acceptance between data and MC simulation and the statistical uncertainty of this control sample, and assign 0.8% as the corresponding uncertainty. We test an alternative simple pole model in place of the ISGW2 model in generating the signal MC sample for the determination of detection efficiency. The form factor of the generic and the signal MC samples differ slightly in quadrature, the total multiplicative systematic uncertainty $σ_ε$ is estimated to be 3.9%.

![FIG. 5. Fit to $MM^2$ distribution of data samples. The data are represented by points with error bars, the total fit result by the violet dashed line, the background by the blue solid line, and signal by the red filled histogram.](image)

To take into account the additive systematic uncertainty, the maximum-likelihood fits are repeated using different alternative background shapes as mentioned in the previous Section and the one resulting in the most conservative upper limit is chosen. Finally, the multiplicative systematic uncertainty $σ_ε$, is incorporated in the calculation of the upper limit via [25, 26]

$$L(B) \propto \int_0^1 L(B, ε) \exp \left[ -\frac{(ε/ε_0 - 1)^2}{2(σ_ε^2)} \right] dε,$$

where $L(B)$ is the likelihood distribution as a function of assumed BFs; $ε$ is the expected efficiency and $ε_0$ is the averaged MC-estimated efficiency. The likelihood distributions with and without incorporating the systematic uncertainties are shown in Fig. 6.

![FIG. 6. Likelihood distributions versus BF of the data samples. The likelihood of each bin is denoted as $L_i$ and the maximum of the likelihood is $L_{i,\text{max}}$. The results obtained with and without incorporating the systematic uncertainties are shown with red solid and blue dashed curves, respectively. The pink arrow shows the result corresponding to the 90% confidence level.](image)
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