Kinetic Model for Solute Diffusion in Liquid Membrane Systems

Juan Abdul-Wahid O. Al-Naisani* Eman T. Kareem**
Yousif I. AbuZaid*** Taki A. Himdan***

* Dept. of Chemistry, College of Education, University of Samarra, Samarra, Iraq
** Dept. of Chemistry, College of Veterinary Medicine, University of Karbala, Karbala, Iraq
*** Dept. of Chemistry, College of Education for Pure Science, Ibn Al-Haytham, University of Baghdad, Baghdad, Iraq

E-mail: yousif.abuzaid@gmail.com

Received 20/9/2015
Accepted 20/12/2015

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License

Abstract:
In this study, a mathematical model for the kinetics of solute transport in liquid membrane systems (LMSs) has been formulated. This model merged the mechanisms of consecutive and reversible processes with a “semi-derived” diffusion expression, resulting in equations that describe solute concentrations in the three sections (donor, acceptor and membrane). These equations have been refined into linear forms, which are satisfying in the special conditions for simplification obtaining the important kinetic constants of the process experimentally.

Key words: kinetic model, diffusion, liquid membranes, mathematical model, solute diffusion in a LMS.

Introduction:
A liquid membrane system LMSs can be defined as a system containing a definite liquid split by another immiscible liquid; in general, it is a phase split by another phase. Despite of the simplicity of this definition, it does not reduce the wide spread cases that are represented by it and their importance. The cell membranes of living organisms can be considered as one of these systems [1-3]. Additionally, these systems are widely used in research and industry for selective separation and purification of many important materials [4-9], and the capability of using membrane selective electrodes in qualitative and quantitative identification carries a latent possibility for their use for sensing and controlling many processes [10]. In environmental fields of application, LMSs are currently attracting great interest, and this interest will increase in the future [11-14]. For the reasons mentioned above, the knowledge of the material transport kinetics in LMSs is a critical point in
understanding and developing applications of these systems. In LMSs, the kinetic theoretical treatment is not as simple as most of its practical procedures because it includes many complex stages based on reversible, consecutive and diffusion phenomena. Many researchers describe the process as an ordinary consecutive mechanism for the material transport from the donor phase to the receptor phase through the membrane [15-25]. Others have merged Fick’s second law of diffusion with the kinetics of reversible reactions [26-28], and some researchers have also described the process kinetics as first order mechanisms [29-34]. These schemes have sometimes merged the diffusion process in flux terms [35-40]. These schemes are still deficient because they address certain aspects of the state and neglect others. The driving aim of this study is to drive a mathematical expression that includes consecutive, reversible and diffusion mechanisms describing material transport in LMSs.

Model formulation

Note: The model below was conjugated with practical work about atropine transport using LMSs, but for the generality of the model, that work is published separately [41].

The kinetic model for the transition of atropine (solute) dissolved in benzene (donor light phase) in the first arm of a U-shaped glass tube across a section of water (heavy phase) containing the copper ion Cu⁴⁺ as a carrier to the second arm containing benzene (acceptor light phase) has been derived according to the following equation:

$$k_1 \quad k_2$$

$$(A)_{B1} \rightleftharpoons (A)_{H} \rightleftharpoons (A)_{B2}$$

$$k_2 \quad k_1$$

Where $(A)_{B1}, (A)_{B2}$ and $(A)_{H}$ are the atropine concentrations dissolved in benzene in the first arm, second arm and aqueous phase, respectively, $k_1$ is the rate constant of atropine transmission from benzene to the water and $k_2$ is the rate constant of the reverse process. Panaggio in reference [42] described a similar equation with four rate constants, which we reduced to two rate constants due to the state of symmetry between the first and second arm. In addition, the final solution in the above reference is limited and includes many problems in the determination of the constants of the process. The general equations that describe the changes in the atropine concentration in the three segments with time are as following [43]:

$$\frac{d(A)_{B1}}{dt} = -k_1(A)_{B1} + k_2(A)_{H} \quad \ldots \quad (1)$$

$$\frac{d(A)_{H}}{dt} = k_1(A)_{B1} - k_2(A)_{H} - k_2(A)_{H} + k_1(A)_{B2} \quad \ldots \quad (2)$$

$$\frac{d(A)_{B2}}{dt} = k_2(A)_{H} - k_1(A)_{B2} \quad \ldots \quad (3)$$

The atropine concentration in the aqueous phase will increase until it reaches a maximum value. At that point, a steady-state will be established in the aqueous segment, and it will proceed to equilibrium. This phenomenon can be written by setting equation (2) equal to zero as following [44]:

$$\frac{d(A)_{H}}{dt} = k_1(A)_{B1} - k_2(A)_{H}^{\text{max}} - k_2(A)_{H}^{\text{max}} + k_1(A)_{B2} = 0 \quad \ldots \quad (4)$$

$$k_1(A)_{B1} + k_1(A)_{B2} = 2k_2(A)_{H}^{\text{max}} \quad \ldots \quad (5)$$

Where $(A)_{H}^{\text{max}}$ is the concentration of atropine in the aqueous phase when the steady-state is established and also when the full equilibrium of the system is achieved. The substitution of equation (5) into equation (2) produces the following equation:

$$\frac{d(A)_{H}}{dt} = 2k_2\{(A)_{H}^{\text{max}} - (A)_{H}\} \quad \ldots \quad (6)$$

This equation is similar to elementary first order equations and can be solved at the boundary conditions, which state that the concentration of atropine in the aqueous phase is zero at
time zero, leading to the following equation:
\[
\ln \left(\frac{(A)_{H}^{\text{max}}}{(A)_{H}^{\text{max}}(t)}\right) = -2k_{2}t \quad \ldots \quad (7)
\]
Equation (7) can be written in the following form:
\[
(A)_{H}^{\text{max}} = (A)_{H}^{\text{max}}(1 - e^{-2k_{2}t}) \quad \ldots \quad \ldots \quad (8)
\]
At \( t = 0 \), the limit exponential will be one, and therefore, \((A)_{H}^{\text{max}}\) will be zero at the beginning of the process. When the system reaches equilibrium, a steady-state or \( t = \infty \), then the limit exponential will be zero, and thus, the concentration of atropine in the aqueous phase will adhere to the following:\( (A)_{H} = (A)_{H}^{\text{max}} \). This is a logical description of what should be going on in the system.

Returning to equations (1) and (3), the substitution of equation (8) in these equations will produce two linear first order differential equations with just two variables. These equations can be solved using the method of integration coefficient (integrating factor) [45]. For example we can rewrite equation (1) after substitution in the following form:
\[
\frac{d(A)_{B1}}{dt} + k_{1}(A)_{B1} = k_{2}(A)_{H}^{\text{max}} \{1 - e^{-2k_{2}t}\} \quad \ldots \quad (9)
\]
Where the integration factor (IF) of this equation is \( e^{k_{1}t} dt \). By multiplying this parameter with the above equation, we obtain the following equation:
\[
d(A)_{B1} e^{k_{1}t} + e^{k_{1}t} k_{1} dt(t)_{B1} = k_{2}(A)_{H}^{\text{max}} \{e^{k_{1}t} dt - e^{k_{1}(1-2k_{2})t} dt\} \quad \ldots \quad (10)
\]
Carrying out the integration process of this equation produces the following equation:
\[
(A)_{B1} e^{k_{1}t} = k_{2}(A)_{H}^{\text{max}} \left\{ \frac{e^{k_{1}t} - e^{k_{1}(1-2k_{2})t}}{k_{1}} \right\} + I \quad \ldots \quad (11)
\]
Where \( I \) is the constant of integration, which can be obtained by the substitution of the boundary condition \((A)_{B1} = (A)_{B1}^{0}\) when \( t = 0 \), and the concentration of atropine is equal to the initial concentration at time zero leading to equation (12).
\[
I = (A)_{B1}^{0} - k_{2}(A)_{H}^{\text{max}} \left\{ \frac{1}{k_{1}} - \frac{1}{(k_{1} - 2k_{2})} \right\} \quad \ldots \quad (12)
\]
By substituting the constant of integration into equation (11) we obtain the following:
\[
(A)_{B1} e^{k_{1}t} = (A)_{B1}^{0} + k_{2}(A)_{H}^{\text{max}} \left\{ \frac{e^{k_{1}t} - 1}{k_{1}} - \frac{e^{k_{1}(1-2k_{2})t} - e^{-k_{1}t}}{k_{1}} \right\} \quad \ldots \quad (13)
\]
By rearranging the above equation, equation (14) is obtained.
\[
(A)_{B1} = (A)_{B1}^{0} e^{-k_{1}t} + k_{2}(A)_{H}^{\text{max}} \left\{ \frac{1 - e^{-k_{1}t}}{k_{1}} - \frac{e^{-2k_{2}t} - e^{-k_{1}t}}{(k_{1} - 2k_{2})} \right\} \quad \ldots \quad (14)
\]
When you reach a state of equilibrium \((t = \infty)\), the exponential terms in the previous equation will become zero, and thus, the concentration of atropine remaining in the first arm in the equilibrium state will be as following:
\[
(A)_{B1}^{\infty} = \frac{k_{2}(A)_{H}^{\text{max}}}{k_{1}} \leftrightarrow (A)_{H}^{\text{max}} = \frac{k_{1}}{k_{2}} (A)_{B1}^{\infty} \quad \ldots \quad \ldots \quad (15)
\]
\[
(A)_{H}^{\text{max}} = K_{e} (A)_{B1}^{\infty} \quad \ldots \quad \ldots \quad (16)
\]
Where \( K_{e} \) is the thermodynamic equilibrium constant or distribution coefficient of atropine between the aqueous phase and benzene. Similarly we can obtain an equation that expresses the concentration of atropine in the second arm \((A)_{B2}\) by the integration and application of appropriate boundary conditions, resulting in equation (17).
\[
(A)_{B2}^{\infty} = k_{2}(A)_{H}^{\text{max}} \left\{ \frac{1 - e^{-k_{1}t}}{k_{1}} - \frac{e^{-2k_{2}t} - e^{-k_{1}t}}{(k_{1} - 2k_{2})} \right\} \quad \ldots \quad \ldots \quad (17)
\]
At time zero, it is clear that the concentration of atropine dissolved in benzene in the second arm is zero. At equilibrium \((t = \infty)\), the exponential terms in equation (17) become zero, resulting in equation (18).
\[
(A)_{B2}^{\infty} = \frac{k_{2}}{k_{1}} (A)_{H}^{\text{max}} \quad \ldots \quad \ldots \quad (18)
\]
By the comparison of the previous equation with equation (15), it can stated that total equilibrium is established when the equilibrium concentration of atropine dissolved in benzene in the first arm is equal to the concentration of atropine dissolved in benzene in the second arm. In other words, the operation will continue until the concentration atropine dissolved in benzene is equal in both arms of the U-

The above discussion describes the kinetic model through equations (8), (14) and (17) or the changes in the concentrations of atropine in the three sections with time. when variation in the concentration of atropine with location in one section (this situation exists when there is no efficient stirring of the solutions in these sections) is imposed, the transmission of atropine occurs from the site of a high concentration to the low concentration through the diffusion process.

**Kinetic model based on the diffusion process**

The solution of the second order differential equation for the diffusion process depends on the specific boundary conditions, which include the time and the size and shape of the system being studied in addition to the initial and final concentrations of the solute [46]. Diffusion in one dimension within a limited volume fits the following general empirical formula [47]:

\[
C(x, t) = a + b \cdot e^{-at} \{c \cdot \sin(\beta x) + d \cdot \cos(\beta x)\} \ldots \ldots \ldots (19)
\]

where (a, b) are constants with the units of concentration, and (c, d) are constants that are determined through the normalization of a concentration distribution spread function over the space of diffusion. Constants (a, β) are identified by making the general above formula obey the second order Fick’s law. In this equation, it is clear that at a certain point the solute concentration decreases or increases exponentially with time depending on the sign of constant (β). For the distribution of the solute concentration along the variable (x), it is clear that it is determined by the section containing the trigonometric functions in the diffusion equation. It is not necessary for these trigonometric functions to give periodic behavior with a change of location, but rather they describe the behavior of decline or increase in the function depending on the period of change specified with constant (β).

The constants (a, b) can be determined by determining the initial and final concentrations of the solution at the times (0) and (∞) in the source point (x = 0). When a equilibrium (t = ∞), the second term of equation (19) becomes zero so(C_e = a). On the other hand, when (t = 0) and at the source point (x = 0), the exponential term in equation (19) is one as well as the distribution of the trigonometric function because all of the solute will be at this point in time (t = 0). Thus,(C(x, t) = C_0), and so we obtain (b = C_0 - C_e).

Constant (c) takes a zero value because the behavior of the diffusion function did not agree with the sin(βx) function. This comes from the fact that 100% of the solute will be at the point (x = 0) at time zero, while the sin(βx) function makes the amount of solute at this point equal to zero, and this contrasts with reality. Thus, equation (19) can be written as following:

\[
C(x, t) = C_e + (C_0 - C_e)e^{-at} d \cos(\beta x) \ldots \ldots (20)
\]

By applying the second law of one-dimensional diffusion on this relationship, we obtain the following:

\[
\frac{\partial^2 C(x, t)}{\partial t^2} = D \frac{\partial^2 C(x, t)}{\partial x^2} \ldots \ldots (21)
\]

\[
\alpha = D \cdot \beta^2 \ldots \ldots (22)
\]
Where \((D)\) is the diffusion coefficient in the units \((m^2/sec)\). The constant \((\beta)\) can be determined by defining the behavior of \(\cos(\beta x)\), which fits with the experiment throughout the following states: the concentration of the substance is at its maximum at the starting point and then decreases to a value close to zero with increasing distance far from the source point. The concentration resembles the probabilistic distribution function along the distance, and the probability distribution functions do not take a negative value. Thus, this behavior is similar to the behavior of the \(\cos\) function in the first quarter of its period \((0 \rightarrow \pi / 2)\). This behavior is happening along the field of diffusion \((l)\); therefore, the function \(\cos(\beta x)\) will take the form \(\cos(\frac{\pi}{2l} x)\), and we obtain the following:

\[
\beta = \frac{\pi}{2l} \quad \text{and} \quad \alpha = D \frac{\pi^2}{4l^2} \ldots \ldots (23)
\]

In other words, the form of equation (20) becomes as follows:

\[
C(x, t) = C_e + (C_0 - C_e) e^{\frac{-\pi ^2}{4l^2} d t} \cos \left( \frac{\pi}{2l} x \right) \ldots (24)
\]

This format is compatible with the second order diffusion equation and consistent with the general formula for diffusion within a limited volume [48]. Returning to the practical way in which the concentration of dissolved atropine change over time was recorded by taking samples from both sides of the tube during the definite time periods, and then they were homogenized to measure the concentration of atropine in the sample with UV-vis. Apparatus. This means that the measurement process was beyond the problem of a concentration gradient along the distance through shaking the sample, which gives a unified concentration along length of the sample during the measurement process. It is clear here that the concentration that was being measured is the mean concentration along the length of the sample. Thus, from the mean value theory [49-51], the measured mean concentration in the specified section is as following:

\[
\bar{C}(t) = \frac{\int_{0}^{l} C(x, t) dx}{\int_{0}^{l} dx} \ldots \ldots (25)
\]

Carrying out the integration process, we obtain equation (26):

\[
\bar{C}(t) = C_e + (C_0 - C_e) e^{\frac{-\pi ^2}{4l^2} d t} \cdot \frac{2l}{\pi} \ldots (26)
\]

Because the integration above is equivalent to the normalization of the function (in the sense that the amount of material scattered along the space must be 100% or 1 in all sections) and because the constant \(d\) contributes to this condition, the remainder of the extent of trigonometric functions after achieving this condition must be equal to 1, which means that \(\frac{d.2l}{\pi} = 1\). Thus the value of the constant is \(d = \frac{\pi}{2l}\).

The use of the average concentration method that we performed practically allows simplifying the \((x)\) dependent part of the concentration function to just sin and cos trigonometric functions that appear in equation (19), which is required to satisfy the Fick’s second law and results in no need to insert a Fourier series as in reference [47].

It remains to be noted that the primary concentration here, \(C_0\) replaced with concentration \(C_i\) of acquired from the equations (8), (14) and (17). This is similar to the case in [27, 28] that qualifies \(\frac{dC}{dt}\) term in the diffusion equation with \(\frac{dC}{dt}\) in the kinetic equation, but the qualifying step in this work was performed after integration. This will give the concentration at the three sections with the contribution of the diffusion process so resulting in the following:

\[
\bar{C}(t) = C_e + (C_i - C_e) e^{\frac{-\pi ^2}{4l^2} d t} \ldots (27)
\]
\[
\frac{(A)_{B1}(t)}{k_2(A)_H} = \frac{(A)_{B1}^0 e^{-k_1 t} + \frac{[1-e^{-k_1 t}]}{k_1} - \frac{[e^{-2k_2 t} - e^{-k_1 t}]}{(k_1-2k_2)}}{e^{4.1b_1} D_B t} \quad \ldots (28)
\]

\[
\frac{(A)_{B1}}{k_2(A)_H} = \frac{(A)_{B1}^0 e^{-k_1 t} + \frac{[1-e^{-k_1 t}]}{k_1} - \frac{[e^{-2k_2 t} - e^{-k_1 t}]}{(k_1-2k_2)}}{e^{4.1b_1} D_B t} \quad \ldots (29)
\]

The linear form of equation (30) can be obtained by rearranging and taking the logarithm and is as following:

\[
\ln \left[ \frac{(A)_{B1}^0 - (A)_{B1}(t)}{k_2(A)_H} \right] = \left( 2k_2 + \frac{\pi^2}{4.1b_1^2 D_H} \right) t \quad \ldots \quad (32)
\]

By drawing \( \ln \left[ \frac{(A)_{B1}^0 - (A)_{B1}(t)}{k_2(A)_H} \right] \) versus time, a straight line going through the origin point will result in its slope being equal to \( 2k_2 + \frac{\pi^2}{4.1b_1^2 D_H} \). In contrast, the mean concentration of dissolved atropine in benzene in both arms of the tube does not give a simple expression that can be used to calculate the constants in an experimental way to compare the proposed kinetic model with experimental results, but it can simplify equations (28) and (31) through the study of these equations during the initial time periods (primary sections of the curves), where in the initial time periods, the exponential terms can be replaced by \( e^{-x} \approx 1 - x \) according to the Taylor series [52]. Therefore, equation (28) becomes the following:

\[
\frac{(A)_{B1}(t)}{k_2(A)_H} = \frac{(A)_{B1}^0 e^{-k_1 t} + \left( (A)_{B1}^0 (1-k_1 t) - \frac{\pi^2}{4.1b_1^2 D_B t} \right)}{e^{4.1b_1^2 D_H} t} \quad \ldots (33)
\]

This can be arranged into the form

\[
\frac{(A)_{B1}(t)}{k_2(A)_H} = \frac{(A)_{B1}^0 e^{-k_1 t} + \left( (A)_{B1}^0 (1-k_1 t) - \frac{\pi^2}{4.1b_1^2 D_B t} \right)}{e^{4.1b_1^2 D_H} t} \quad \ldots (34)
\]

The linear form becomes as follows:

\[
\frac{(A)_{B1}^0 - (A)_{B1}(t)}{k_2(A)_H} = \left( (A)_{B1}^0 e^{-k_1 t} + \frac{\pi^2}{4.1b_1^2 D_B t} (A)_{B1}^0 - (A)_{B1}^0 \right) \quad \ldots (35)
\]

By plotting \( \frac{(A)_{B1}^0 - (A)_{B1}(t)}{k_2(A)_H} \) versus time, a straight line is obtained with the intercept

\[
\frac{(A)_{B1}^0 - (A)_{B1}(t)}{k_2(A)_H} = \left( (A)_{B1}^0 e^{-k_1 t} + \frac{\pi^2}{4.1b_1^2 D_B t} (A)_{B1}^0 - (A)_{B1}^0 \right) \quad \ldots (36)
\]

The above equation states that when the primary section of the concentration \( \frac{(A)_{B1}^0 - (A)_{B1}(t)}{k_2(A)_H} \) is plotted versus time, we will obtain a straight line through the origin point that has the slope \( (A)_{B1}^0 - (A)_{B1}^0 \) with the intercept \( (A)_{B1}^0 e^{-k_1 t} \). In addition to the equilibrium constant relationship which is:

\[
K_e = \frac{k_1}{k_2} = \frac{(A)_{B1}^0}{(A)_{B1}^0} \quad \ldots \quad (37)
\]

All of the kinetic constants for this system can be determined, thus allowing the supposed kinetic model to be compared with the experimental results for the purpose of determining the efficiency of this model to describe the overall process.

References
[1] Araki, T. and Tsukube, H. 1990. Liquid membranes: Chemical Applications. CRC press. USA. PP4.
[2] Westmark, P.; Gardiner, S. and Smith, B. 1996. Selective Monosaccharide Transport through
Lipid Bi-layers Using Boronic Acid Carriers. J. Am. Chem. Soc. 118: 11093-11100.

[3]Mishrarrahul, P. and Pancholi, S. 2013. Liquid Membrane Phenomenon in the Biological Action of Venlafaxine. International Journal of Pharmacy and Pharmaceutical Sciences, 5(1): 383-387.

[4]Ferraz, H.; Duarte, L.; Di Luccio, M.; Alves, T.; Habert, A. and Borges, C. 2007. Recent Achievements in Facilitated Transport Membranes for Separation Processes. Brazilian Journal of Chemical Engineering, 24(1): 101-118.

[5]Schlosser, S. and Martak, J. 2009. Separation of Mixtures by Pertraction or Membrane–Based Solvent Extraction and New Extractants. 123-152. in Wyklady Monograficzne I Specjalistyczne. Membrany Teorja I Praktyka Zeszyt iii. Torun.

[6]Parhi, P. 2013. Supported Liquid Membrane Principle and Its Practices: A Short Review. Journal of Chemistry, 2013: 1-11.

[7]Noble, R.; Koval, C. and Pellegrino, J. 1989. Facilitated Transport Membrane Systems. Chemical Engineering Progress, 43: 58-70.

[8]Sahoo, G. and Dutta, N. 2002. Perspectives in Liquid Membrane Extraction of Cephalosporin Antibiotics. Advances in Biochemical Engineering-Biotechnology. 75: 209-242.

[9]Riedl, W. and Thomas, T. 2008. Membrane-supported extraction of bio-molecules with aqueous two-phase systems. Desalination. 224: 160-167.

[10]Lajos, H. 2009. Investigation and modeling the mass transport properties of ionophore based liquid membrane electrodes and nano-pore sensors. PhD thesis, Budapest University of Technology and Economics Faculty of Chemical and Bioengineering. Department of Inorganic and Analytical Chemistry, Budapest. 1-16(in English).

[11]Kaminski, W. and Kwapisinski, W. 2000. Applicability of Liquid Membranes in Environmental Protection. Polish Journal of Environmental Studies, 9(1): 37-43.

[12]Maşu, S.; Botau, D. and Manea, F. 2005. Application of Emulsion Liquid Membrane Technique for MB R 12 Red Reactive Dye-Containing Simulated Wastewater Treatment. Chem. Bull. Polithnica Univ. Timisoara, 50(64): 9-13.

[13]Alpoguz, H.; Kaya A. and Deligöz, H. 2006. Liquid Membrane Transport of Hg(II) by an Azocalix[4]arene Derivative. Separation Science and Technology, 41: 1155-1167.

[14]Teng, T. and Talebi, A. 2012. Green Liquid Membrane: Development and Challenges. J Memb Sci Technol, 2(3): 1-2.

[15]Demircioglu, N.; Levent, M.; Koby, M. and Topcu, N. The Effects of Stirring Speed on Coupled Transport of Nitrite Ions through Liquid Membranes. Chem. Biochem. Eng. Q, 14(4): 109-116.

[16]Alpoguz, H.; Kaya, A. and Karakus, M. 2005. Mechanism and Kinetics of Copper (II) Transport through a Liquid Membrane Containing a Dithiophosphonate Derivative as Carrier. Turk J. Chem, 29: 345-353.

[17]Religa, P.; Gawroński, R. and Gierycz, P. 2009. Kinetics of Chromium (III) Transport through a Liquid Membrane Containing DNNSA as a Carrier. Int. J. Mol. Sci, 10: 964-975.

[18]Gubbuk, H.; Gungor, O.; Alpoguz, H.; Ersoz, M. and Yilmaz, M. 2010. Kinetic study of mercury (II) transport through a liquid membrane containing calix[4]arene nitrile derivatives as a carrier in chloroform. Desalination, 261: 157-161.

[19]Karakus, M.; Alpoguz, H; Kaya, A.; Acar, N.; Görgülü, A. and Mustafa,
Arslan, M. 2011. A kinetic study of mercury (II) transport through a membrane assisted by new transport reagent. Chemistry Central Journal, 5: 43-49.

[20]Diaconu, I.; Zaharia, I.; Ruse, E. and Radu, D. 2012. Kinetic Aspects of Transport para-aminophenol through Agitated Bulk Liquid Membrane. Rev. Chim. Bucharest, 63(2): 153-158.

Zaharia, I.; Diaconu, I.; Ruse, E. and Nechifor, G. 2012. The Transport of 3-Aminophenol through Bulk Liquid Membrane in the Presence of Aliquat 336. Digest Journal of Nanomaterials and Bio-structures, 7(3): 1303-1314.

[22]Zaharia, I.; Aboul-Enein, H.; Diaconu, I.; Ruse, E.; Bunaciu, A. and Nechifor, G. 2013. Facilitated transport of 5-aminosalicylic acid through bulk liquid membrane. J. Iran Chem. Soc, 10: 1129-1136.

[23]Rahman, A.; Hameed, K.; Salman, M. and Al-Hassani, M. 2013. Kinetic of Atropine pertraction from the seeds of Datura metel linn plant using liquid-liquid membrane technique. Diyala Journal of Engineering Sciences, 6(1): 1-16.

[24]Muthuraman, G. and Ibrahim, M. 2013. Removal of anionic dye from aqueous solution using a cationic carrier. International Journal of Industrial Chemistry, 4: 15-22.

[25]Jalhoom, M.; Dhababe, J. and Mussa, Y. 2013. Transport of Metal Ions across Bulk Liquid Membrane with Macro Cycle Compounds. Iraqi National Journal of Chemistry, 51:247-263.

[26]Benzal, G.; Kumar, A.; Delshams, A. and Sastre, A. 2004. Mathematical modeling and simulation of co-transport phenomena through flat sheet-supported liquid membranes. Hydrometallurgy, 74: 117-130.

[27]Ganesan, S.; Anitha, S.; Subbiah, A. and Rajendran, L. 2013. Mathematical Modeling of a Carrier-Mediated Transport Process in a Liquid Membrane. J. Membrane Biol, 246: 435-442.

[28]Pei, L.; Wang, L. and Guo, W. 2013. Modeling of Ce (IV) transport through a dispersion supported liquid membrane including P204 as the carrier. Desalination and Water Treatment, 51: 2193-2201.

[29]Fu J.; Nakamura S. and Akiba K. 1994. Liquid-Membrane Transport of Gold by a Trioctylamine Mobile carrier. Analytical Science, 10: 935-938.

[30]Narayanan, J. and Palanivelu, K. 2008. Recovery of acetic acid by supported liquid membrane using vegetable oils as liquid membrane. Indian Jour. of chem. Tech, 15: 266-270.

[31]Nowik-Zajac, A.; Kozłowski, C. and Walkowiak, W. 2010. Transport of Perrhenate anions across Plasticizer Membranes with Basic Ion Carriers. Physicochem. Probl. Miner. Process, 44: 179-186.

[32]Pei, L.; Yao, B.; Wang, L. and Ma, Z. 2011. Tb (III) Transport in Dispersion Supported Liquid Membrane System with D2EHPA as Carrier in Kerosene. Chem. Res. Chinese Universities, 27(1): 132-139.

[33]Kiswando, A.; Siswanta, D.; Aprilita, N. and Santosa, S. 2012. Transport of Phenol Through Inclusion Polymer Membrane (PIM) Using Copoly(Eugenol-DVB) as Membrane Carriers. Indo. J. Chem, 12 (2): 105-112.

[34]Nosrati, S.; Jayakumar, N.; Hashim, M. and Mukhopadhyay, S. 2013. Performance evaluation of vanadium (IV) transport through supported ionic liquid membrane. Journal of the Taiwan Institute of Chemical Engineers, 44: 337-342.

[35]Venkateswaran, P.; Navaneetha, Gopalakrishnan, A. and Palanivelu, K. 2007. Di(2-ethylhexyl)phosphoric
acid-coconut oil supported liquid membrane for the separation of copper ions from copper plating wastewater. Journal of Environmental Sciences, 19. 1446-1453.

[36] Touaj, K.; Tbeur, N.; Hor, M.; Verchère, J. and Hlaibi, M. 2009. A supported liquid membrane (SLM) with resorcinarene for facilitated transport of methyl glycopyranosides: Parameters and mechanism relating to the transport. Journal of Membrane Science, 337: 28-38.

[37] Raizada, P.; Vyas V. and Sharma, U. 2010. Liquid membrane extraction and transport of amino acids using calyx[6]arene. Indian Jour. Of Chem. Tech, 17: 267-273.

[38] Hor, M.; Riad, A., Benjjar, A.; Lebrun, L. and Hlaibi, M. 2010. Technique of supported liquid membranes (SLMs) for the facilitated transport of vanadium ions (VO$^{2+}$) Parameters and mechanism on the transport process. Desalination, 255: 188-195.

[39] Benjjar, A.; Hor, M.; Riri, M.; Eljaddi, T.; Kamal, O.; Lebrun, L. and Hlaibi, M. 2012. A new supported liquid membrane (SLM) with methyl cholate for facilitated transport of dichromate ions from mineral acids: parameters and mechanism relating to the transport. J. Mater. Environ. Sci, 3(5): 826-839.

[40] Benjjar, A.; Eljaddi, T.; Kamal, O.; Lebrun, L. and Hlaibi, M. 2013. Methyl Cholate and Resorciniarene New Carriers for the Recovery of Cr (III) Ions by Supported Liquid Membranes (SLM)s. Open Journal of Physical Chemistry, 3: 103-114.

[41] Al-Naisani, J. A. O. 2015. Kinetic and Thermodynamic study of Atropine transfer across the Liquid membrane. M.Sc. thesis, University of Samarra, Iraq, PP66- 83 (in Arabic).

[42] Panaggio, A. 1982. An investigation of some factors controlling solute transport across liquid membranes. PhD thesis, University of Rhodes Island, Rhodes, PP123-166 (in English).

[43] Hammet, L. 1970. Physical organic chemistry. Mc Grow Hill Kogakusha, 2ed Edition, Japan. PP 46

[44] Dybkov, V. 2013. Chemical Kinetics. IPMS Publications. Kyiv. PP 76.

[45] Bowman, F. 1968. Elementary Calculus. Longmans. UK. PP 463.

[46] Crank, J. 1975. The Mathematics of Diffusion. 2ed edition. Clarendon Press Oxford. UK. PP 421.

[47] Deserno, M. 2004. One-dimensional diffusion on a finite region. International Journal of Industrial Chemistry, 9: 33-45.

[48] Benzal, G.; Kumar, A.; Delshams, A. and Sastre, A. 2004. Mathematical modeling and simulation of co-transport phenomena through flat sheet-supported liquid membranes. Hydrometallurgy, 74: 117-130.

[49] Manzanares, J.; Lahtinen, R.; Quinn, B.; Kontturi, K. and Schiffrin, D. 1998. Determination of rate constants of ion transfer kinetics across immiscible electrolyte solutions. Electrochimica Acta, 44: 59-71.

[50] George, S. and Thomas, S. 2001. Transport phenomena through polymeric systems. prog. Polym. Sci, 26: 985-1017.

[51] Elliot, P. 2013. Probabilistic Number Theory I: Mean-Value Theorems. Springer, New York. PP 203.

[52] Lefcourt, T. 2013. Calculus AB & BC. Kaplan publishing, New York. PP 267.
نموذج حركي لانتشار المذاب في أنظمة الاغشية السائلة

جوان عبد الواحد النيساني
إيمن طالب كريم
يوسف ابراهيم ابوزيد
تقي الدين عبد الهادي حمدان

قسم الكيمياء، كلية التربية، جامعة سامراء، سامراء، العراق
قسم الكيمياء، كلية الطب البيطري، جامعة كربلاء، كربلاء، العراق
قسم الكيمياء، كلية التربية للعلوم الصرفة/ ابن الهيثم، جامعة بغداد، بغداد، العراق

الخلاصة:
في هذه الدراسة تم صياغة نموذج حركي لانتشار المذاب في أنظمة الاغشية السائلة. دمج هذا النموذج بين حركتي العمليات المتتابعة و الانعكاسية مع تعبير شبه مشتق للانتشار، منتجا معادلات تصف تركيز المذاب في المقاطع الثلاثة (الواهب و المستقبل و الغشاء السائل). تم تعدل المعادلات الناتجة إلى الشكل الخطي و الذي يتحقق تحت ظروف خاصة لغرض تبسيط الحصول على الثوابت الحركية المهمة للعملية بصورة تجريبية.

الكلمات المفتاحية: نموذج حركي، الانتشار، الاغشية السائلة، نموذج رياضي، انتشار المذاب في LMS