Cepheid Kinematics and the Galactic Warp

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Abstract—The space velocities of 200 long-period ($P > 5$ days) classical Cepheids with known proper motions and line-of-sight velocities whose distances were estimated from the period–luminosity relation have been analyzed. The linear Ogorodnikov-Milne model has been applied, with the Galactic rotation having been excluded from the observed velocities in advance. Two significant gradients have been found in the Cepheid velocities, $\partial W/\partial Y = -2.1 \pm 0.7 \text{ km s}^{-1} \text{ kpc}^{-1}$ and $\partial V/\partial Z = 27 \pm 10 \text{ km s}^{-1} \text{ kpc}^{-1}$. In such a case, the angular velocity of solid-body rotation around the Galactic $X$ axis directed to the Galactic center is $-15 \pm 5 \text{ km s}^{-1} \text{ kpc}^{-1}$.

INTRODUCTION

As analysis of the large-scale structure of neutral hydrogen showed, a warp of the gas disk is observed in the Galaxy (Westerhout 1957). The results of studying this structure using the currently available data on the HI and HII distributions are presented in Kalberla and Dedes (2008) and Cersosimo et al. (2009), respectively. The warp is seen in the distribution of stars and dust (Drimmel and Spergel 2001), pulsars (Yusifov 2004), OB stars from the Hipparcos catalogue (Miyamoto and Zhu 1998), and in the distribution of 2MASS red-giant-clump stars (Momany et al. 2006). The system of Cepheids also exhibits a similar feature (Fernie 1968; Berdnikov 1987; Bobylev 2013).

Of great interest are the attempts to find a relationship between the kinematics of stars and the disk warp (Miyamoto et al. 1993; Miyamoto and Zhu 1998; Drimmel et al. 2000; Bobylev 2010). In particular, based on the proper motions of O–B5 stars, Miyamoto and Zhu (1998) found a positive rotation of this system of stars around the Galactic $x$ axis with an angular velocity of about $+4 \text{ km s}^{-1} \text{ kpc}^{-1}$. In contrast, based on the proper motions of about 80 000 red-giant-clump stars, Bobylev (2010) found an opposite rotation of this system of stars around the $x$ axis with an angular velocity of about $-4 \text{ km s}^{-1} \text{ kpc}^{-1}$.

The stellar proper motions alone do not allow complete information to be obtained. In this respect, although the Cepheids are not all that many, they are a unique tool for studying the three-dimensional kinematics of the Galaxy: the distances, proper motions, and line-of-sight velocities are known for them.

A number of models were proposed to explain the Galactic warp: (1) the interaction between the disk and a nonspherical dark matter halo (Sparke and Casertano 1988);
(2) the gravitational influence from the Galaxy's nearest satellites (Bailin 2003); (3) the interaction of the disk with the flow near the Galaxy formed by high-velocity hydrogen clouds that resulted from mass exchange between the Galaxy and the Magellanic Clouds (Olano 2004); (4) the intergalactic flow (López-Corredoira et al. 2002); and (5) the interaction with the intergalactic magnetic field (Battaner et al. 1990).

Note that the term “warp” implies some nonlinear dependence. However, we attempt to find a relationship between the kinematics of stars and the warp of the hydrogen layer in the form of a simple linear approach. For this purpose, we search, for example, for the rotation of the symmetry plane of the system of stars around some axis. Since the symmetry plane of the Cepheid system is inclined to the Galactic plane at an angle of \( \approx -2^\circ \) in a direction of \( \approx 270^\circ \) (Bobylev 2013), the most suitable manifestation of the relationship is the rotation of the system around the Galactic \( x \) axis.

The goal of this study is to reveal the relationship between the Cepheid velocities and the warp of the stellar-gaseous Galactic disk. For this purpose, we use a sample of long-period classical Cepheids with measured proper motions and line-of-sight velocities and estimate their distances from the period–luminosity relation. We apply the linear Ogorodnikov-Milne model for our analysis and exclude the Galactic rotation from the observed velocities in advance, focusing our attention on the motion in the \( XZ \) and \( YZ \) planes.

**DATA**

We use Cepheids of the Galaxy’s flat component classified as DCEP, DCEPS, CEP(B), CEP in the GCVS (Kazarovets et al. 2009) as well as CEPS used by other authors. To determine the distance based on from the period–luminosity relation, we used the calibration from Fouqué et al. (2007): \( \langle M_V \rangle = -1.275 - 2.678 \log P \), where the period \( P \) is in days. Given \( \langle M_V \rangle \), taking the period-averaged apparent magnitudes \( \langle V \rangle \) and extinction \( A_V = 3.23E(\langle B \rangle - \langle V \rangle) \) mainly from Acharova et al. (2012) and, for several stars, from Feast and Whitelock (1997), we determine the distance \( r \) from the relation

\[
10^{10-0.2(\langle M_V \rangle - \langle V \rangle - 5 + A_V)}.
\]

For a number of Cepheids (without extinction data), we used the distances from the catalog by Berdnikov et al. (2000) determined from infrared photometry.

Data from Mishurov et al. (1997) and Gontcharov (2006) as well as from the SIMBAD and DDO databases served as the main sources of line-of-sight velocities for Cepheids. As a rule, the proper motions were taken from the UCAC4 catalog (Zacharias et al. 2013) and, in several cases, from TRC (Hog et al. 2000).

Proceeding from the goals of our study, we concluded that it would be better not to use several stars located above the Galactic plane by more than 2 kpc and deep in the inner Galaxy. Thus, we used the constraints

\[
\begin{align*}
|Z| & < 2 \text{ kpc}, \\
P & > 5^d, \\
|V_{pec}| & < 100 \text{ km s}^{-1}, \\
\sigma_V & < 80 \text{ km s}^{-1},
\end{align*}
\]
satisfied by 205 Cepheids. When calculating the velocity errors, we assumed the distance error to be 10%. In particular, the constraint for $\sigma_V$ in (2) is the random error in the total space velocity of a star. The constraint on the pulsation period $P$ was chosen from the following considerations. Our analysis of the distribution of classical Cepheids (Bobylev 2013) shows that the oldest Cepheids with periods $P < 5^d$ have a significantly different orientation than younger Cepheids.

Bobylev et al. (2008) found the parameters of the Galactic rotation curve containing six terms of the Taylor expansion of the angular velocity of Galactic rotation $\Omega_0$ for the Galactocentric distance of the Sun $R_0 = 7.5$ kpc. Data on hydrogen clouds at tangential points, on massive star-forming regions, and on the velocities of young open star clusters were used for this purpose. The more up-to-date value of $R_0$ is 8 kpc (Foster and Cooper 2010). Therefore, the parameters of the Galactic rotation curve were redetermined using the same sample but for $R_0 = 8$ kpc:

$$
\begin{align*}
\Omega_0 &= -27.4 \pm 0.6 \text{ km s}^{-1} \text{ kpc}^{-1}, \\
\Omega_1 &= 3.80 \pm 0.07 \text{ km s}^{-1} \text{ kpc}^{-2}, \\
\Omega_2 &= -0.650 \pm 0.065 \text{ km s}^{-1} \text{ kpc}^{-3}, \\
\Omega_3 &= 0.142 \pm 0.036 \text{ km s}^{-1} \text{ kpc}^{-4}, \\
\Omega_4 &= -0.246 \pm 0.034 \text{ km s}^{-1} \text{ kpc}^{-5}, \\
\Omega_5 &= 0.109 \pm 0.020 \text{ km s}^{-1} \text{ kpc}^{-6}.
\end{align*}
$$

Based on a sample of Cepheids, Bobylev and Bajkova (2012) found $\Omega_0 = -27.5 \pm 0.5$ km s$^{-1}$ kpc$^{-1}$, $\Omega'_0 = 4.12 \pm 0.10$ km s$^{-1}$ kpc$^{-2}$ and $\Omega''_0 = -0.85 \pm 0.07$ km s$^{-1}$ kpc$^{-3}$, which are in good agreement with the corresponding values (3). At the same time, the parameters (3) allow the Galactic rotation curve to be constructed in a wider range of Galactocentric distances $R$. This rotation curve is shown in Fig. 1. The parameters (3) were used to analyze the peculiar velocity $V_{pec}$ in (2). The constraint on the magnitude of $V_{pec}$ is an indirect constraint on the radius of the sample, which is $r \approx 6$ kpc is our case.
THE MODEL

We use a rectangular Galactic coordinate system with its axes directed from the observer toward the Galactic center (the $x$ axis or axis 1), in the direction of Galactic rotation (the $y$ axis or axis 2), and toward the North Galactic Pole (the $z$ axis or axis 3).

We apply the linear Ogorodnikov–Milne model (Ogorodnikov 1965), where the observed velocity $V(r)$ of a star with a heliocentric radius vector $r$ is described, to terms of the first order of smallness $r/R_0 \ll 1$, by the vector equation

$$V(r) = V_\odot + Mr + V',$$  

(4)

Here, $V_\odot(X_\odot, Y_\odot, Z_\odot)$ is the Sun’s peculiar velocity relative to the stars under consideration, $V'$ is the star’s residual velocity, $M$ is the displacement matrix (tensor) whose components are the partial derivatives of the velocity $u(u_1, u_2, u_3)$ with respect to the distance $r(r_1, r_2, r_3)$, where $u = V(R) - V(R_0)$, $R$ and $R_0$ are the Galactocentric distances of the star and the Sun, respectively. Then,

$$M_{pq} = \left(\frac{\partial u_p}{\partial r_q}\right)_\odot, \quad p, q = 1, 2, 3,$$  

(5)

taken at $R = R_0$. All nine elements of the matrix $M$ can be determined using three components of the observed velocities — the line-of-sight velocities $V_r$ and stellar proper motions $\mu_l \cos b, \mu_b$:

$$V_r = -X_\odot \cos b \cos l - Y_\odot \cos b \sin l - Z_\odot \sin b +$$

$$+ r [\cos^2 b \cos^2 l M_{11} + \cos^2 b \cos l \sin l M_{12} + \cos b \sin b \cos l M_{13} +$$

$$+ \cos b \sin l \cos l M_{21} + \cos^2 b \sin^2 l M_{22} + \cos b \sin l M_{23} +$$

$$+ \sin b \cos b \cos l M_{31} + \cos b \sin b \sin l M_{32} + \sin^2 b M_{33}],$$

$$\mu_l \cos b = X_\odot \sin l - Y_\odot \cos l +$$

$$+ r [- \cos b \cos l \sin l M_{11} - \cos b \sin^2 l M_{12} - \sin b \sin l M_{13} +$$

$$+ \cos b \cos^2 l M_{21} + \cos b \sin l \cos l M_{22} + \sin b \cos l M_{23},$$

$$\mu_b = X_\odot \cos l \sin b + Y_\odot \sin l \sin b - Z_\odot \cos b +$$

$$+ r [- \sin b \cos b \cos^2 l M_{11} - \sin b \cos l \sin l M_{12} - \sin^2 b \cos^2 l M_{13} -$$

$$- \sin b \cos b \sin l \cos l M_{21} - \sin b \cos b \sin^2 l M_{22} - \sin^2 b \sin l M_{23} +$$

$$+ \cos^2 b \cos l M_{31} + \cos^2 b \sin l M_{32} + \sin b \cos b M_{33}].$$

(6)

It is useful to divide the matrix $M$ into its symmetric, $M^+$ (local deformation tensor), and antisymmetric, $M^-$ (rotation tensor), parts:

$$M^+_{pq} = \frac{1}{2} \left(\frac{\partial u_p}{\partial r_q} + \frac{\partial u_q}{\partial r_p}\right)_\odot, \quad M^-_{pq} = \frac{1}{2} \left(\frac{\partial u_p}{\partial r_q} - \frac{\partial u_q}{\partial r_p}\right)_\odot, \quad p, q = 1, 2, 3,$$  

(7)

where the subscript 0 means that the derivatives are taken at $R = R_0$. The quantities $M^+_{12}, M^+_{13}$ and $M^-_{13}$ are the components of the solid-body rotation vector of a small solar neighborhood around the $x, y, z$ axes, respectively. In accordance with our chosen rectangular coordinate system, the positive rotations are those from axis 1 to axis 2 ($\Omega_z$), from
Table 1: Kinematic parameters of the Ogorodnikov–Milne model

| Parameter | Without correction | Wit correction |
|-----------|--------------------|----------------|
| $X_\odot$ | $6.1 \pm 1.4$      | $6.3 \pm 1.5$  |
| $Y_\odot$ | $11.0 \pm 1.4$     | $10.0 \pm 1.5$ |
| $Z_\odot$ | $5.3 \pm 1.4$      | $6.0 \pm 1.5$  |
| $M_{11}$  | $-0.5 \pm 1.0$     | $0.1 \pm 1.0$  |
| $M_{12}$  | $-2.2 \pm 0.7$     | $-2.9 \pm 0.7$ |
| $M_{13}$  | $-8.0 \pm 11.7$    | $1.9 \pm 11.9$ |
| $M_{21}$  | $1.6 \pm 1.0$      | $0.8 \pm 1.0$  |
| $M_{22}$  | $-0.9 \pm 0.7$     | $-1.1 \pm 0.7$ |
| $M_{23}$  | $34.4 \pm 11.7$    | $32.4 \pm 11.9$|
| $M_{31}$  | $-2.4 \pm 1.0$     | $-2.8 \pm 1.0$ |
| $M_{32}$  | $-1.1 \pm 0.7$     | $-2.1 \pm 0.7$ |
| $M_{33}$  | $11.7 \pm 11.7$    | $13.2 \pm 11.9$|

Note. The velocities $X_\odot$, $Y_\odot$, and $Z_\odot$ are in km s$^{-1}$; the remaining parameters are in km s$^{-1}$ kpc$^{-1}$.

axis 2 to axis 3 ($\Omega_x$), and from axis 3 to axis 1 ($\Omega_y$):

$$M^- = \begin{pmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{pmatrix}. \quad (8)$$

The quantity $M_{21}^-$ is equivalent to the Oort constant $B$. Each of the quantities $M_{12}^+$, $M_{13}^+$ and $M_{23}^+$ describes the deformation in the corresponding plane; in particular, $M_{12}^+$ is equivalent to the Oort constant $A$. The diagonal elements of the local deformation tensor $M_{11}^+$, $M_{22}^+$ and $M_{33}^+$ describe the general local compression or expansion of the entire stellar system (divergence). The set of conditional equations (6) includes twelve sought-for unknowns to be determined by the least-squares method.

RESULTS AND DISCUSSION

The table gives the parameters of the Ogorodnikov–Milne model found by simultaneously solving the set of equations (6) using a sample of 200 Cepheids. Two solutions are presented. The point is that the ICRS/Hipparcos (1997) system, whose extension is the UCAC4 catalog we use, has a small residual rotation relative to the initial frame of reference. The equatorial components of this vector are $(\omega_x, \omega_y, \omega_z) = (-0.11, 0.24, -0.52) \pm (0.14, 0.10, 0.16)$ mas yr$^{-1}$ (Bobylev 2010). Therefore, the table gives the parameters calculated for two cases: when the Cepheid proper motions were not corrected and when they were derived from the stellar proper motions after applying the correction $\omega_z = -0.52$ mas yr$^{-1}$.

There are no significant differences between the two solutions. However, it can be noted that with the corrected proper motions, the parameter $M_{13}$ decreased to zero and
the parameter $M_{32}$ slightly increased, which is important to us. Therefore, below we will use the results from the last column of the table.

**The XY plane.** Since $M_{12} = -\Omega_0$, the value of $M_{12} = -2.9 \pm 0.7 \text{ km s}^{-1} \text{ kpc}^{-1}$ found shows that the Cepheid velocities were slightly overcorrected (we should have used $\Omega_0 \approx -26 \text{ km s}^{-1} \text{ kpc}^{-1}$ precisely for this sample). This is of no serious importance for the goals of our study, because this is just a linear shift, while the nonlinear character of the Galactic rotation curve was taken into account well. The remaining parameters describing the kinematics in the XY plane, $M_{11}$, $M_{21}$ and $M_{22}$, are close to zero.

**The XZ plane.** As can be seen from the table, none of the coefficients $M_{11}$, $M_{13}$, $M_{31}$ and $M_{33}$, describing the kinematics in this plane differs significantly from zero. Figure 2 displays the corresponding distributions of stars.

**The YZ plane.** Figure 3 shows the distributions of stars; the solid lines indicate two dependences plotted according to the data from the table: $M_{23} = \partial V/\partial Z = 32.4 \pm 11.9 \text{ km s}^{-1} \text{ kpc}^{-1}$ and $M_{32} = \partial W/\partial Y = -2.1 \pm 0.7 \text{ km s}^{-1} \text{ kpc}^{-1}$. We refined the coefficient $M_{23} = 26.8 \pm 10.2 \text{ km s}^{-1} \text{ kpc}^{-1}$ using a graphical method. For this purpose, we calculated the dependence $V = f(Z)$ from the data of the corresponding graph in Fig. 3 with the constraint $|Z| > 0.040 \text{ kpc}$ (136 stars were used).
Let us now consider the displacement tensor $M_W$ that we associate with the influence of the disk warp on the motion of the Cepheid system:

$$M_W = \begin{pmatrix} \frac{\partial V}{\partial Y} & \frac{\partial V}{\partial Z} \\ \frac{\partial W}{\partial Y} & \frac{\partial W}{\partial Z} \end{pmatrix}. \tag{9}$$

According to the data from the table, both of its diagonal elements can be set equal to zero. This means that there are no motions like expansion–compression in this plane. Then,

$$M_W = \begin{pmatrix} 0 & 26.8_{(10,2)} \\ -2.1_{(0,7)} & 0 \end{pmatrix}, \tag{10}$$

the deformation tensor (7) takes the form

$$M^+_{W} = \begin{pmatrix} 0 & 12.4_{(5,1)} \\ 12.4_{(5,1)} & 0 \end{pmatrix}, \tag{11}$$

and the rotation tensor (7) is

$$M^-_{W} = \begin{pmatrix} 0 & 14.5_{(5,1)} \\ -14.5_{(5,1)} & 0 \end{pmatrix}. \tag{12}$$
Based on (12), we may conclude that the angular velocity of solid-body rotation of the Cepheid system around the \( X \) axis is \( \Omega_W = M_{32} = -15 \pm 5 \text{ km s}^{-1} \text{ kpc}^{-1} \). This is the minimum (but more reliable) estimate. If the deformations \( (M_{23}^{+}) \) are assumed to be also related to the effect under consideration, then the maximum angular velocity of rotation can be estimated as \( \Omega_W = M_{32} - M_{23} = -27 \pm 10 \text{ km s}^{-1} \text{ kpc}^{-1} \).

It is important that the direction of the rotation found (minus sign) is in agreement with the result of our analysis of the proper motions for red-giant clump stars (Bobylev 2010), where we used photometric distance estimates with errors \( e\pi/\pi \approx 30\% \). The sign of the angular velocity \( \Omega_W \) depends on the sign of \( M_{32} \) (Eqs. (7)–(8)), which was determined from Cepheids rather reliably owing to the wide range of coordinates \( \Delta Y \approx 10 \text{ kpc} \). Since the range of coordinates \( \Delta Z \approx 1.2 \text{ kpc} \) is small when determining \( M_{23} \), the influence of random fluctuations in Cepheid velocities can be significant.

The value of \( \Omega_W = -15 \pm 5 \text{ km s}^{-1} \text{ kpc}^{-1} \) derived from Cepheids exceeds \( \Omega_W \approx -4 \pm 0.5 \text{ km s}^{-1} \text{ kpc}^{-1} \) obtained from red-giant-clump stars by Bobylev (2010) by a factor of 4. Such a difference may be related to the sample ages: the mean age of our sample of Cepheids is 77 Myr, while the mean age of the red-giant-clump stars is approximately 1 Gyr. However, this question requires a further study based on larger volumes of more accurate data.

CONCLUSIONS

We considered the space velocities of about 200 long-period (with periods of more than 5 days) classical Cepheids with known proper motions and line-of-sight velocities whose distances were estimated from the period–luminosity relation.

We applied the linear Ogorodnikov–Milne model to analyze their kinematics. The Galactic rotation that we found based on a more complex model was excluded from the observed velocities in advance. Two significant gradients were detected in the Cepheid velocities: \( \partial W/\partial Y = -2.1 \pm 0.7 \text{ km s}^{-1} \text{ kpc}^{-1} \) and \( \partial V/\partial Z = 27 \pm 10 \text{ km s}^{-1} \text{ kpc}^{-1} \). This leads us to conclude that the angular velocity of solid-body rotation around the Galactic x axis is \( \Omega_W = -15 \pm 5 \text{ km s}^{-1} \text{ kpc}^{-1} \), which we associate with a manifestation of the warp of the stellar–gaseous Galactic disk. Indeed, the relationship between the spatial distribution of Cepheids and the warp of the stellar–gaseous Galactic disk may be considered to have been firmly established (Fernie 1968; Berdnikov 1987; Bobylev 2013). The results of our study show that the kinematic relationship of Cepheids to this phenomenon is also highly likely.

The method considered here can be useful for a future analysis of large volumes of data, for example, from the GAIA space experiment or on masers with their trigonometric parallaxes measured by VLBI.

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