Spectral Representation Theory for Dielectric Behavior of Nonspherical Cell Suspensions

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Abstract

Recent experiments revealed that the dielectric dispersion spectrum of fission yeast cells in a suspension was mainly composed of two sub-dispersions. The low-frequency sub-dispersion depended on the cell length, while the high-frequency one was independent of it. The cell shape effect was simulated by an ellipsoidal cell model but the comparison between theory and experiment was far from being satisfactory. Prompted by the discrepancy, we proposed the use of spectral representation to analyze more realistic cell models. We adopted a shell-spheroidal model to analyze the effects of the cell membrane. It is found that the dielectric property of the cell membrane has only a minor effect on the dispersion magnitude ratio and the characteristic frequency ratio. We further included the effect of rotation of dipole induced by an external electric field, and solved the dipole-rotation spheroidal model in the spectral representation. Good agreement between theory and experiment has been obtained.

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I. INTRODUCTION

The polarization of biological cells has a wide scope of practical applications like manipulation, trapping or separation of biological cells [1,2], and thus an accurate characterization of the polarization is needed. While the polarization of biological cells can be investigated by the method of dielectric spectroscopy [3] as well as by the measurement of field-induced cell movements [4,5], the former method has a much higher resolution [1]. For biological cells, the main contribution to the dielectric dispersion is the structural (Maxwell-Wagner) polarization effects [1]. Because of the practical applications, there is a strong need for intuitive models as well as simplified equations which describe the parameter dependence of the polarization. Thus, various cell models have been proposed for the analysis of the polarization mechanisms. However, due to the complexity of existing theories, these methods have not yet found broader acceptance.

In this work, we propose the use of the spectral representation [6] for analyzing the cell models. The spectral representation is a rigorous mathematical formalism of the effective dielectric constant of a two-phase composite material [6]. It offers the advantage of the separation of material parameters (namely the dielectric constant and conductivity) from the cell structure information, thus simplifying the study. From the spectral representation, one can readily derive the dielectric dispersion spectrum, with the dispersion strength as well as the characteristic frequency being explicitly expressed in terms of the structure parameters and the materials parameters of the cell suspension (see section II.B below). The actual shape of the real and imaginary parts of the permittivity over the relaxation region can be uniquely determined by the Debye relaxation spectrum, parametrized by the characteristic frequencies and the dispersion strengths. So, we can study the impact of these parameters on the dispersion spectrum directly.

The plan of the paper is organized as follows. In the next section, we will review the spectral representation theory [6] and show that the dielectric dispersion spectrum of a cell suspension can be expressed in terms of the spectral representation. In section III, we
will apply the spectral representation to the various cell models and present an alternative approach. We show that a better agreement with the experimental data can be achieved. In section IV, we discuss the effects of dipole rotation. We will show that the dipole rotation effect has a strong impact on the dispersion spectrum when the cells are sufficiently long. Discussion on further applications of our theory will be given.

II. FORMALISM

We regard a cell suspension as a composite system consisting of biological cells of complex dielectric constant $\tilde{\varepsilon}_1$ dispersed in a host medium of $\tilde{\varepsilon}_2$. A uniform electric field $E_0 = E_0 \hat{z}$ is applied along the $z$-axis. We briefly review the spectral representation theory of the effective dielectric constant to establish notations.

A. Spectral representation

The spectral representation is initiated by solving the differential equation

$$\nabla \cdot \left[ \left( 1 - \frac{1}{s} \eta(\mathbf{r}) \right) \nabla \phi(\mathbf{r}) \right] = 0,$$

where $s = \tilde{\varepsilon}_2/(\tilde{\varepsilon}_2 - \tilde{\varepsilon}_1)$ denotes the relevant material parameter and $\eta(\mathbf{r})$ is the characteristic function of the cell structure. The electric potential $\phi(\mathbf{r})$ can be solved formally

$$\phi(\mathbf{r}) = -E_0 z + \frac{1}{s} \int d\mathbf{r}' \eta(\mathbf{r}') \nabla' G_0(\mathbf{r} - \mathbf{r}') \cdot \nabla' \phi(\mathbf{r}'),$$

where $G_0(\mathbf{r} - \mathbf{r}') = 1/4\pi|\mathbf{r} - \mathbf{r}'|$ is the free space Green’s function. By denoting an operator

$$\Gamma = \int d\mathbf{r}' \eta(\mathbf{r}') \nabla' G_0(\mathbf{r} - \mathbf{r}') \cdot \nabla',$$

and the corresponding inner product

$$\langle \phi | \psi \rangle = \int d\mathbf{r} \eta(\mathbf{r}) \nabla \phi^* \cdot \nabla \psi,$$
it is easy to show that $\Gamma$ is a Hermitean operator. Let $s_n$ and $|n\rangle$ be the eigenvalue and eigenfunction of $\Gamma$ such that $\Gamma |n\rangle = s_n |n\rangle$, where $0 \leq s_n < 1$ is a real eigenvalue. The integral equation can be solved symbolically:

$$|\phi\rangle = -\frac{s}{s - \Gamma} |z\rangle E_0.$$  \hspace{1cm} (5)

From the solution, we obtain the electric field and hence compute the effective dielectric constant $\tilde{\varepsilon}_e$ in the spectral representation. We further define the reduced effective dielectric function $F(\tilde{s})$:

$$F(\tilde{s}) = 1 - \frac{\tilde{\varepsilon}_e}{\varepsilon_2} = -\frac{1}{s V E_0} \langle z | \phi \rangle.$$  \hspace{1cm} (6)

By inserting the complete set $1 = \sum_n |n\rangle \langle n|$, we find

$$F(\tilde{s}) = \frac{1}{V} \sum_n \langle z | n \rangle \langle n | z \rangle = \sum_n \frac{F_n}{\tilde{s} - s_n}.$$  \hspace{1cm} (7)

$F_n$ is defined as the spectral function:

$$F_n = \frac{1}{V} \langle z | n \rangle \langle n | z \rangle,$$  \hspace{1cm} (8)

which satisfies a sum rule $F$:

$$\sum_n F_n = \frac{1}{V} \sum_n \langle z | n \rangle \langle n | z \rangle = \frac{1}{V} \langle z | z \rangle = V_1 / V = p,$$  \hspace{1cm} (9)

where $V_1$ is the total volume of the suspending cells and $p$ the volume fraction of the cells.

**B. Dielectric dispersion spectrum**

For cells of arbitrary shape, the eigenvalue problem of the $\Gamma$ operator can only be solved numerically. However, analytic solutions can be obtained for isolated spherical and ellipsoidal cells. For dilute suspensions of prolate spheroidal cells, the cells can be regarded as noninteracting. The problem is simplified to the calculation of $s_n$ and $|n\rangle$ with a single cell, which can be solved exactly. Only two of the $F_n$ are nonzero, due to the orthogonality of $|n\rangle$ with $|z\rangle$. 
Thus, in subsequent studies, we restrict ourselves to two poles \((n = 1, 2)\). From Eq.\((\text{7})\), the effective dielectric constant is written in the spectral representation:

\[
\tilde{\varepsilon}_e = \tilde{\varepsilon}_2 \left( 1 - \frac{2}{n=1} \frac{F_n}{s - s_n} \right).
\]  

(10)

After substituting \(\tilde{\varepsilon}_1 = \epsilon_1 + \sigma_1 / j 2 \pi f\) and \(\tilde{\varepsilon}_2 = \epsilon_2 + \sigma_2 / j 2 \pi f\) into Eq.\((\text{10})\), where \(\epsilon\) and \(\sigma\) are the real and imaginary parts of the complex dielectric constant, we rewrite the effective dielectric constant after simple manipulations:

\[
\tilde{\varepsilon}_e = \epsilon_H + \sum_{n=1}^{2} \frac{\Delta \epsilon_n}{1 + j f / f_c^n} + \frac{\sigma_L}{j 2 \pi f},
\]

(11)

where \(\epsilon_H\) and \(\sigma_L\) are the high-frequency dielectric constant and the low-frequency conductivity respectively, while \(\Delta \epsilon_n\) are the dispersion magnitudes, \(f_c^n\) are the characteristic frequencies of the nth sub-dispersion. We obtain the dispersion magnitudes \(\Delta \epsilon_n\) and the characteristic frequencies \(f_c^n\), respectively \([7]\):

\[
\Delta \epsilon_1 = F_1 \epsilon_2 \frac{s_1(s - t)^2}{s(s - s_1)(t - s_1)^2};
\]

\[
\Delta \epsilon_2 = F_2 \epsilon_2 \frac{s_2(s - t)^2}{s(s - s_2)(t - s_2)^2};
\]

\[
f_1^c = \frac{\sigma_2 s(t - s_1)}{2 \pi \epsilon_2 l(s - s_1)},
\]

\[
f_2^c = \frac{\sigma_2 s(t - s_2)}{2 \pi \epsilon_2 l(s - s_2)}.
\]

where \(s = \epsilon_2 / (\epsilon_2 - \epsilon_1)\) and \(t = \sigma_2 / (\sigma_2 - \sigma_1)\). To compare with experiment data \([8]\), we express the dispersion magnitude ratio and characteristic frequency ratio as

\[
\frac{\Delta \epsilon_1}{\Delta \epsilon_2} = \frac{F_1}{F_2} \cdot \frac{s_1(s - s_2)(t - s_2)^2}{s_2(s - s_1)(t - s_1)^2},
\]

(12)

\[
\frac{f_2^c}{f_1^c} = \frac{(t - s_2)(s - s_1)}{(t - s_1)(s - s_2)}.
\]

(13)

### III. APPLICATIONS TO VARIOUS CELL MODELS

In a recent work \([7]\), we adopted the spheroidal model (SM) to analyze the cell suspensions. Here we briefly review the analytic results of the model:

\[
s_1 = L_z, \quad s_2 = L_{xy}, \quad F_1 = \frac{1}{3} p, \quad F_2 = \frac{2}{3} p.
\]

5
where

\[ L_z = -\frac{1}{q^2 - 1} + \frac{q}{(q^2 - 1)^{3/2}} \ln(q + \sqrt{q^2 - 1}), \]

\[ L_{xy} = (1 - L_z)/2 \]

are the depolarization factors along the z-axis and x-(or y-) axis of the prolate spheroid and \( q \) is the ratio of length \( L \) to diameter \( D \).

In the spheroidal model, we neglected the presence of a cell membrane. To study this effect, we put forward the shell-spheroidal model (SSM) here. In this case, the biological cells are modelled as shell-spheroidal ones with a spheroidal core of complex dielectric constant \( \varepsilon_1 \), covered with a confocal spheroidal shell of \( \varepsilon_s \). For a small volume fraction \( p \) of shelled spheroidal cells embedded in a host medium of complex dielectric constant \( \varepsilon_2 \), the effective dielectric constant \( \tilde{\varepsilon}_e \) is given by the dilute-limit expression:

\[ \tilde{\varepsilon}_e = \tilde{\varepsilon}_2 + p\tilde{\varepsilon}_2 (b_z + 2b_{xy}). \]

where \( b_z \) is the dipole factor for a single-shelled spheroidal cell along the z-axis [9]:

\[ b_z = \frac{1}{3} \frac{(\tilde{\varepsilon}_s - \tilde{\varepsilon}_2)[\tilde{\varepsilon}_s + L_z(\tilde{\varepsilon}_1 - \tilde{\varepsilon}_s)] + (\tilde{\varepsilon}_1 - \tilde{\varepsilon}_s)y[\tilde{\varepsilon}_s + L_z(\tilde{\varepsilon}_2 - \tilde{\varepsilon}_s)]}{(\tilde{\varepsilon}_s - \tilde{\varepsilon}_1)(\tilde{\varepsilon}_2 - \tilde{\varepsilon}_s)yL_z(1 - L_z) + [\tilde{\varepsilon}_s + L_z(\tilde{\varepsilon}_1 - \tilde{\varepsilon}_s)][\tilde{\varepsilon}_2 + L_z(\tilde{\varepsilon}_2 - \tilde{\varepsilon}_s)]}, \]

where \( y \) is the volume ratio of core to the whole shelled spheroid, while \( b_{xy} \) indicates the dipole factor along the x- (or y-) axis, which can be obtained by replacing the subscript \( z \) with \( xy \) in the expression of \( b_z \). As a matter of fact, the cell suspension consisting of shell-spheroidal cells dispersed in a host medium is a three-phase system. Although the spectral representation was generally valid for two-phase composites, we have recently shown that it applies to composites of coated spheres as well as to coated spherical particles randomly embedded in a host medium [10]. Note the sum rule \( \sum F_n = p \) is no longer valid. Similarly, one can show that the spectral representation also applies to the present system consisting of spheroidal cells with shells of complex dielectric constant \( \tilde{\varepsilon}_s \) dispersed in a host. The effective dielectric constant is then given by

\[ \tilde{\varepsilon}_e = \tilde{\varepsilon}_2 \left[ 1 - \left( \sum_{n=1}^{2} \frac{F_n}{s_n - s_n} + N.P. \right) \right] \]

(14)
with \( N.P. \) being the nonresonant part which vanishes in the limit of unshelled spheroidal inclusions, where

\[
\begin{align*}
s_1 &= \frac{L_z[1 + (x - 1)y + L_z(-1 + x + y - xy)]}{x - L_z(x - 1)^2(y - 1) + L_z^2(x - 1)^2(y - 1)}, \\
s_2 &= \frac{L_{xy}[1 + (x - 1)y + L_{xy}(-1 + x + y - xy)]}{x - L_{xy}(x - 1)^2(y - 1) + L_{xy}^2(x - 1)^2(y - 1)}, \\
F_1 &= \frac{px^2y}{3[x - L_z(x - 1)^2(y - 1) + L_z^2(x - 1)^2(y - 1)]^2}, \\
F_2 &= \frac{2px^2y}{3[x - L_{xy}(x - 1)^2(y - 1) + L_{xy}^2(x - 1)^2(y - 1)]^2},
\end{align*}
\]

where \( x = \tilde{\epsilon}_s/\tilde{\epsilon}_2 \). We omit the complicated expression for the nonresonant part here. In what follows, for the sake of convenience, we assume: (1) \( y \) is a constant for all coated spheroid; (2) \( x \) is a real number.

In Fig.1, we plot the structure parameters \( F_n \) and \( s_n \) versus \( x \) for various \( y \) and for (a) \( q = 3.46 \), (b) 7.17 and (c) 10.24, respectively. In all case, \( p = 0.01 \). We find \( F_n \) is strongly dependent on \( y \) for \( x > 0.5 \), whereas it is not the case for \( s_n \). It may be concluded that the dielectric property of the cell membrane has a minor effect on the dispersion magnitude ratio, but plays no role in the characteristic frequency ratio.

To investigate the validity of these models, we compare to experimental data, which was extracted by using a temperature sensitive cell division cycle mutant of fission yeast, \textit{cdc25-22} [8]. Asami’s theory [8] results are also plotted for comparison. From Fig.2, it is evident that our model gives a better comparison with experimental data than Asami’s theory. The reason for the improvement lies in the introduction of the conductivity contrast \( t \) by using of the spectral representation. As stated in Ref. [8], the large difference between our model and Asami’s theory is due to a large \( \sigma_1 \gg \sigma_2 \) used in contrast to Asami’s claim \( \sigma_1 \approx \sigma_2 \) [8]. We further find that SSM provides a better fit than SM for the dispersion magnitude ratio \( \Delta \epsilon_1/\Delta \epsilon_2 \), while SSM yields the same results as those of SM for the frequency ratio \( f_2^c/f_1^c \) (both curves overlap in the right panel of Fig.2), indicating that the dielectric property of a cell membrane is indeed unimportant.
IV. EFFECTS OF DIPOLE ROTATION

According to the numerical results, we find that the SSM provides a better fitting with previous experimental data than SM, but this improvement is actually too small. In other words, the dielectric properties of a cell membrane does not play an important role in dielectric dispersion spectrum. But, those numerical result will also show that both SM and SSM cannot obtain a good agreement with experimental data. In the presence of an electric field, cells of large length may rotate in favor of the applied field, thus we propose another model, namely the dipole-rotation spheroidal model (DRSM) to obtain a better fitting.

When the cells are long enough, the rotation of dipole becomes very important with the external electric field under consideration, and the system is in general anisotropic. We have to take into account the effect of dipole rotation on $F_1$ and $F_2$, even for a weak electric field. Let us compute them from a thermodynamic consideration. We will show that they in general depends on $q$.

Consider a spheroidal cell in an electric field $E_0$. Its long axis makes an angle $\theta$ with the field. The dipole energy of the cell is

$$E_d(q, \theta) = -\text{Re} \left[ \frac{\tilde{\epsilon}_2 D^3 E_0^2}{16} q (b_z \cos^2 \theta + b_{xy} \sin^2 \theta) \right],$$  \hspace{1cm} (15)

where $b_z$ and $b_{xy}$ are dipole factors along and perpendicular to the long axis:

$$b_z = \frac{1}{3(L_z - \bar{s})}, \quad b_{xy} = \frac{1}{3(L_{xy} - \bar{s})}.$$

Eq.\,(15) can be understood by the energy approach. For simplicity, suppose the major axes of the cells all lie along the electric field, i.e., $\theta = 0$, then the induced dipole moments of the cells give a contribution to the effective dielectric constant. In the dilute limit,

$$\tilde{\epsilon}_e = \tilde{\epsilon}_2 + 3p\tilde{\epsilon}_2 b_z,$$

where $p = V_1/V$ is the volume fraction of the cells. For a fixed external field condition, the total electrostatic energy density of the suspension is given by $E_t = -\text{Re}(\tilde{\epsilon}_e E_0^2/8\pi)$, which is equal to $-\text{Re}(\tilde{\epsilon}_2 E_0^2/8\pi) + E_d/V$, and hence the desired results.
We showed that the conductivity contrast $t$ attains a small negative value \[7\] and thus the complex material parameter $\tilde{s}$ can be approximated by $t$. Consequently, both $b_z$ and $b_{xy}$ have positive values. The probability is given by the Boltzmann factor

$$\rho(q, \theta) = A e^{-E_d(q, \theta)/k_B T}$$

where $A$ is a normalization factor such that $\int \rho(q, \theta) d\Omega = 1$, where $\Omega$ is the solid angle, $d\Omega = \sin \theta d\theta d\varphi$. We can calculate $F_1$ and $F_2$ by the following integrals

$$F_1(q) = p \int \rho(q, \theta) \cos^2 \theta d\Omega, \quad F_2(q) = p \int \rho(q, \theta) \sin^2 \theta d\Omega.$$  \hspace{1cm} (17)

The $F_1(q)/F_2(q)$ ratio may be obtained by integrating with respect to $\theta$ from 0 to $\pi/2$ by symmetry. In the absence of an electric field, $E_d(q, \theta) = 0$ and $\rho(q, \theta)$ equals to a uniform distribution. In which case, we obtain $F_1 = p/3$ and $F_2 = 2p/3$, and hence $F_1/F_2 = 0.5$. When the electric field is weak enough, the ratio is still constant and $F_1/F_2 = 0.5$. Otherwise, the ratio will increase rapidly with $q$. For $q = 1$, $b_z = b_{xy}$ and $F_1/F_2 = 0.5$ always. The above result implies that both $F_1$ and $F_2$ depend strongly on $q$ when there is an electric field. For large $q$, $b_z \gg b_{xy}$, the spheroids tend to align with the applied field and hence $F_1/F_2$ becomes very large.

It is found that the mean cell length depends on the cultivation time, whereas the diameter is almost unchanged in an experiment \[8\], which will be applied to compare the different models. In the following numerical calculation, without loss of generality, we neglect the small imaginary part of $\tilde{\varepsilon}_2$, and define a new parameter $\xi$:

$$\xi = \frac{\varepsilon_2 D^3 E_0^2}{16 k_B T}$$ \hspace{1cm} (18)

which characterizes the electric field strength.

We can readily obtain the dispersion magnitude ratio $\Delta \varepsilon_1/\Delta \varepsilon_2$ as well as the dispersion frequency ratio $f_2^c/f_1^c$ by substituting the results of $F_1(q)/F_2(q)$ into Eqs.(12) and (13), and setting $s_1 = L_z$ and $s_2 = L_{xy}$. In Fig.3, $F_1/F_2$ is plotted versus $q$. It is shown that $F_1/F_2$ depends strongly on the axial ratio $q$, especially for large $q$ or strong magnitude of external electric field.
To compare with experimental data in Fig.2, we obtain good agreement in the DRSM with $\xi = 0.017$, which corresponds to a weak field $E_0 \approx 0.1$ V/m. The results show that dipole rotation indeed plays an important role in the dielectric dispersion – we cannot neglect the effect of the rotation of dipole induced by the applied electric field, especially when the average length of cell is large. In addition, good agreement exists only for large cytoplasmic conductivity, as attributed to a higher ion concentration in their cytoplasm to avoid the shrinkage of cells due to a loss of water across the cell membrane.

V. DISCUSSION AND CONCLUSION

Here we would like to make a few comments. At low frequencies, the cell membrane effectively insulates the interior of cell. In other words, a potential builds up entirely over the cell membrane, leaving the interior of cell rather inactive to the field [1]. Thus $\epsilon_1 \ll \epsilon_2$ and $s = 1^+$ and it is reasonable to use $s = 1.001$ as fitting parameter. On the other hand, we assume that the host medium has low loss and $\sigma_2 \approx 0$ and at the same time a large cytoplasmic conductivity $\sigma_1 \gg \sigma_2$. Thus $t = 0^-$ and it is reasonable to use $t = -0.0001$ to fit the data.

The resulting equations [Eqs.(12) and (13)] are indeed simple equations arising from the spectral representation. These equations serve as a basis which describe the parameter dependence of the polarization and thereby enhances the applicability of various cell models for the analysis of the polarization mechanisms. In this connection, the shell-spheroid model may readily be extended to multi-shell cell model. However, we believe that the multi-shell nature of the cell may have a minor effect on the dispersion magnitude ratio as well as on the characteristic frequency ratio.

In the presence of external electric fields, field-induced motions such as rotation of cells, dielectrophoretic motion or vibrational motion may have a significant impact on the dielectric dispersion spectrum. With the recent advent of experimental techniques such as automated video analysis [11] as well as light scattering methods [2], the cell movements can
be accurately monitored. For purely rotational motions, the distribution of surface charge on the cell surfaces may deviate significantly from the equilibrium distribution for cells at rest, leading to a change in the polarization relaxation and in the dielectric dispersion spectrum. In this regard, our recent work on dynamic electrorheological effects [12], in which the suspended particles can have rotational motions, may be applied to cell rotational motions. In Ref. [12], we found that the particles’ rotational motions do change the polarization relaxation substantially.

In this work, we considered a monodisperse cell suspension, in which the cells are of the same shape (i.e., same length and diameter). While the diameter of the cells may remain constant during the cultivation process, the cells may possess a wide distribution of cell lengths [8]. A modified theory, which takes the distribution of length into account, is urgently needed and our spectral representation theory will certainly help. In this connection, we may apply a strong dc electric field (in addition to the ac probe field) to help separating the long cells from the short ones. Our results indicate that even in a moderate field, the long cells can easily be aligned with the applied field, while the short ones remain essentially randomly oriented. In this way, an emphasis of dispersion spectrum of the long cells can be made possible.

In summary, prompted by the discrepancy between recent theory and experiment on fission yeast cells, we have proposed the use of spectral representation to analyze more realistic cell models. We adopted a shell-spheroidal model to analyze the effects of the cell membrane. It is found that the presence of a membrane has only a minor effect on the dispersion ratio, but plays no role in the frequency ratio. We further included the effect of rotation of dipole induced by an external electric field. It has been found that the dipole-rotation effect plays an important role in the dispersion magnitude, but it does not change the characteristic frequency ratio. We obtained good agreement between theory and experiment when dipole-rotation effect is included.
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REFERENCES

[1] For a review, see J. Gimsa and D. Wachner, Biophys. J. 77, 1316 (1999).

[2] J. Gimsa, Ann. NY Acad. Sci. 873, 287 (1999).

[3] K. Asami, T. Hanai and N. Koizumi, Jpn. J. Appl. Phys. 19, 359 (1980).

[4] G. Fuhr, J. Gimsa and R. Glaser, Stud. Biophys. 108, 149 (1985).

[5] J. Gimsa, P. Marszalek, U. Lowe and T. Y. Tsong, Biophys. J. 73, 3309 (1991).

[6] D. J. Bergman, Phys. Rep. 43, 379 (1978).

[7] Jun Lei, Jones T. K. Wan, K. W. Yu and Hong Sun, J. Phys.: Condens. Matter 13, 3583 (2001); Phys. Rev. E, July 2001, to be published.

[8] K. Asami, Biochim. Biophys. Acta 1472, 137 (1999).

[9] L. Gao, Jones T. K. Wan, K. W. Yu and Z. Y. Li, J. Phys.: Condens. Matter 12, 6825 (2000).

[10] K. P. Yuen and K. W. Yu, J. Phys.: Condens. Matter 9, 4669 (1997).

[11] G. De Gasperis, X.-B. Wang, J. Yang, F. F. Becker and P. R. C. Gascoyne, Meas. Sci. Technol. 9, 518 (1998).

[12] Jones T. K. Wan, K. W. Yu and G. Q. Gu, Phys. Rev. E 62, 6846 (2000).
FIGURES

FIG. 1. For SSM, $F_1$, $F_2$, $s_1$ and $s_2$ are plotted versus the dielectric constant ratio $x$ for different thickness parameter $y$: (a) $q = 3.46$; (b) $q = 7.17$; (c) $q = 10.24$.

FIG. 2. Ratios of the dispersion magnitudes and the characteristic frequencies are plotted versus $q$. Asami’s theory: $\sigma_1 \approx \sigma_2$; SM: $t = -0.0014$, $s = 5.0$; SSM: $t = -0.0014$, $s = 5.0$, $x = 2$, $y = 0.8$; DRSM: $t = -0.0001$, $s = 1.001$, $\xi = 0.017$ (i.e., $E_0$ is about 0.1V/m). Note that the curves of SM and SSM overlap in the right panel, while they are quite close in the left panel.

FIG. 3. For DRSM, the ratio $F_1/F_2$ is plotted versus $q$ for different electric field strength parameter $\xi$. 
Fig. 1(a) / Huang, Yu, Lei, Sun
FIG. 1(b) / Huang, Yu, Lei, Sun

Fig.1(b) / Huang, Yu, Lei, Sun
FIG. 1(c)
Asami's theory
SM
SSM
DRSM

Fig. 2/Huang, Yu, Lei, Sun
Fig. 3/Huang, Yu, Lei, Sun