Testing Bell’s inequality in constantly coupled Josephson circuits by effective single-qubit operations

L.F. Wei,1,2 Yu-xi Liu,1 and Franco Nori1,3

1 Frontier Research System, The Institute of Physical and Chemical Research (RIKEN), Wako-shi, Saitama, 351-0198, Japan
2 Institute of Quantum Optics and Quantum Information, Department of Physics, Shanghai Jiaotong University, Shanghai 200030, P.R. China
3 Center for Theoretical Physics, Physics Department, CISC, The University of Michigan, Ann Arbor, Michigan 48109-1120, USA

(Dated: April 1, 2022)

PACS number(s): 03.65.Ud, 03.67.Lx, 85.25.Dq.

I. INTRODUCTION

Non-locality (i.e., entanglement) is one of the most profound features of quantum theory and plays an important role in quantum information processing, including quantum computation and quantum communication [1]. Mathematically, an entangled pure state of a composite system is defined as a state that cannot be factorized into a direct tensor product of the states associated with individual subsystems. The presence of multi-partite entanglement is necessary for implementing quantum algorithms that are exponentially faster than classical ones [2]. Also, entanglement may offer some new information transfer modes, e.g., teleportation and quantum cryptography [1]. Therefore, generating and verifying entanglement between qubits are of great practical importance [2].

The existence of entanglement can be verified by using various experimental methods, e.g., quantum tomographic techniques, Bell-state analysis, quantum jump measurements, etc. (see, e.g., [3, 4]). Also, spectral analysis has been used to probe the existence of two-qubit entanglement in coupled Josephson qubits [5]. However, the degree of entanglement between the two always-present interacting qubits changes rapidly [7] and, at certain times, two-qubit states can be almost separable. Thus, verifying the instantaneously generated entangled state in coupled systems [8], and using it to realize quantum information processing, e.g., teleportation and quantum memory, are very important challenges.

Historically, Bell’s inequality always served as one of the most important witnesses of entanglement: its violation implies that entanglement must be shared by the separate parts. Numerous experimental tests of Bell’s inequality have been made with entangled photons separated far apart (e.g., up to 500 m) [9] and entangled closely-spaced trapped ions (e.g., separated a few micrometers apart) [10]. A Bell-like inequality has also been tested via single-neutron interferometry by measuring the correlations between two entangled degrees of freedom (comprising spatial and spin components) of single neutrons [11]. The results from these experiments strongly violate the tested Bell’s inequalities, and thus agree with quantum mechanical predictions. Very recently, preliminary proposals have been explored for testing Bell’s inequalities with switchable Josephson qubits [12].

Almost all proposals for manipulating quantum information rely on the execution of both single-qubit and two-qubit gates. In some cases (e.g., trapped ions, QED cavities, and quantum dots [13]), single-qubit operations are easy to realize by applying certain controllable local fields. However, the interbit couplings are fixed and uncontrollable in current experimental Josephson circuits [4, 5] and NMR systems [14]. Nevertheless, qubits in NMR are individually addressable, because the coupling constants $J_{ij}$ between the ith and jth qubits are sufficiently weak (i.e., much smaller than the differences $\Delta \omega_i \equiv |\omega_i - \omega_j|$ between the eigenfrequencies of the qubits, e.g., $J_{ij}/\Delta \omega_{ij} \ll 10^{-4}$ [14]). However, the usual capacitive coupling in Josephson circuits [4, 5] is relatively strong, thus making it difficult to perform local single-qubit operations on individual qubits. The method [15] of physically arranging qubits (i.e., using two or three physical qubits to encode a logical qubit) cannot be directly used in the present two-qubit system.

Here we develop a new approach to overcome the serious “fixed-interaction” difficulty and effectively implement desired single-qubit operations on selected qubits. This dynamical decoupling method can be used to generate long-lived entangled states in coupled Josephson qubits. The long-lived entanglement obtained here, assisted by the proposed local single-qubit operations, should allow to test Bell’s inequality with a pair of capacitively-coupled Josephson qubits. This provides a powerful approach, besides spectral-analysis [Nature 421, 823 (2003); Science 300, 1548 (2003)], to verify the existence of macroscopic quantum entanglement between two fixed-coupling Josephson qubits.
The Hamiltonian of this circuit is
\[ 2 \epsilon^2 C_m / \epsilon \text{m.} \] The effective charge energy \( E_C \) is \( E_C = E_{C,j} (1/2 + n_{g_j}) + E_m (1/4 + n_{g_j}/2) \) (j, k = 1, 2) with
\[ n_{g_j} = (C_g V_{g_j} + C_p V_p)/(2e) \] \( \text{and} \ E_{C,j} = 4e^2 C_{S,j} / \epsilon \). The effective Josephson energy of the SQUID is \( E_j = 2e^2 / 2 \cos (\pi \Phi_j / \Phi_0) \) with Josephson energy \( E_j \) of the single junction. Above, \( C_{S,j} = C_{S,1} - C_{S,2}, \) and \( C_{S,j} \) is the sum of all capacitances connected to the jth box. The pseudospin operators are defined as \( \sigma_z = |0_j \rangle \langle 0_j | - |1_j \rangle \langle 1_j | \) and \( \sigma_x = |0_j \rangle \langle 1_j | + |1_j \rangle \langle 0_j | \).

First, let us consider the circuit working at the co-resonance point (i.e., \( n_{g_1} = n_{g_2} = 0.5 \), yielding \( E_C^{(1)} = E_C^{(2)} = 0 \)), and the applied fluxes are set as \( \Phi_0 = \Phi_0 / 2, k \neq j \) (yielding \( E_j^{(2)} = 2e / 2 \)). In this case, the circuit has the Hamiltonian
\[ \hat{H}_1 = -\varepsilon_j \sigma_z + E_{12} \sigma_z^{(1)} \sigma_z^{(2)}. \] The corresponding time-evolution operator reads
\[ \hat{U}(t) = \exp \left( -i \frac{t}{\hbar} \hat{H}_1 \right) = \exp \left( i \frac{t}{\hbar} \varepsilon_j \sigma_z \right) \hat{U}_\text{int}(t), \] where the operator \( \hat{U}_\text{int}(t) \) is determined by
\[ i \hbar \frac{\partial \hat{U}_\text{int}(t)}{\partial t} = \hat{H}_\text{int}(t) \hat{U}_\text{int}(t), \] with
\[ \hat{H}_\text{int}(t) = E_{12} \exp \left( -i \frac{t}{\hbar} \varepsilon_j \sigma_z \right) \sigma_z^{(1)} \sigma_z^{(2)} \exp \left( i \frac{t}{\hbar} \varepsilon_j \sigma_z \right). \]

Neglecting the higher-order terms of \( \zeta_j \), since it is small, the Hamiltonian of the system can be effectively rewritten as
\[ \hat{H}_\text{eff}^{(j)} = -\varepsilon_j \sigma_z + \frac{E_{12}^2}{2 \varepsilon_j} \sigma_z \otimes I^{(k)}. \] Above, the first-order expansion term \( \hat{U}_\text{int}^{(1)}(t) = (-i / \hbar) \int^t dt' \hat{H}_\text{int}(t') \) practically does not contribute to the time evolution, due to its small probability (proportional to \( \zeta_j^2 \)). Under this approximation, the fixed interaction between the qubits has been effectively eliminated, except resulting in a shift of the relatively strong Josephson energy. Thus, the system effectively undergoes an evolution:
\[ \hat{R}_j^{(j)}(\varphi_j) = \exp \left[ i \varphi_j \sigma_z^{(j)} \right], \] \( \varphi_j = \frac{\varepsilon_j t}{\hbar} (1 + 2 \zeta_j^2), \)

FIG. 1: Two capacitively coupled charge qubits. The quantum states of two Cooper-pair boxes (i.e., qubits) are manipulated by controlling the applied gate voltages \( V_{g_1}, V_{g_2} \) and external magnetic fluxes \( \Phi_1, \Phi_2 \) (penetrating the SQUID loops). \( P_1 \) and \( P_2 \) (dashed line parts) read out the final qubit states.

Energy. The two Cooper-pair boxes are coupled via the capacitance \( C_m \). The qubits work in the charge regime, with
\[ k_B T \ll E_j \ll E_C, \] where both quasi-particle tunnelling and excitations are effectively suppressed and the number \( n_j \) (with \( n_j = 0, 1, 2, \ldots \)) of Cooper-pairs in the boxes is a good quantum number. Here, \( k_B, T, E_j, E_C \), and \( E_j \) are the Boltzmann constant, temperature, superconducting gap, the charging and Josephson energies of the \( j \)th qubit, respectively. Following Refs. 4, 5, the system is assumed to work near the co-resonant point and its quantum dynamics can be restricted to the subspace spanned by the four lowest charge states: \( |00 \rangle, |10 \rangle, |01 \rangle \) and \( |11 \rangle \). Thus, the Hamiltonian of this circuit is
\[ \hat{H} = \frac{1}{2} \sum_{j=1,2} \left[ E_C \sigma_z - E_j \sigma_x \right] + E_{12} \sigma_z^{(1)} \sigma_z^{(2)}. \]
which reduces to the single-qubit $\sigma^{(j)}_z$-rotation (i.e., qubit-flip) on the $j$th qubit, if the duration is set by $\cos \varphi_j = 0$.

The robustness of this dynamical decoupling can be verified by testing the difference of the corresponding physical effects, e.g., the transition probabilities $P$ between two selected states, due to the present approximate time-evolution

$$\hat{U}_{\text{appr}}(t) = R_x^{(j)}(\varphi_j) \otimes I^{(k)}$$

and the exact one

$$\hat{U}_{\text{ex}}(t) = \exp(-it\hat{H}_j/\hbar) = A(t)\sigma^{(j)}_x \otimes I^{(k)} + B(t)(|0_j0_k\rangle\langle 0_j0_k| + |1_j1_k\rangle\langle 1_j1_k|) + B^*(t)(|1_j0_k\rangle\langle 1_j0_k| + |0_j1_k\rangle\langle 0_j1_k|),$$

respectively. Here,

$$A(t) = i\rho_j(t), \quad B(t) = [1 - \rho^{(j)}_z(t)]^{1/2}\exp[-i\xi_j(t)],$$

with

$$\rho_j(t) = \nu_j^{-1}\sin(\nu_j t/h), \quad \nu_j = [1 + (E_{12}/\varepsilon^{(j)}_j)^2]^{1/2}, \quad \xi_j(t) = \arctan[2\nu_j^{-1}\tan(\varepsilon^{(j)}_j \nu_j t/h)].$$

Figure 2 shows the probabilities for the transition $|1_j0_k\rangle \leftrightarrow |0_j0_k\rangle$ due to the evolutions $\hat{U}_{\text{appr}}(t)$ (solid lines) and $\hat{U}_{\text{ex}}(t)$ (dotted lines) possess the same oscillating period. Also, the difference between the probabilities decreases when deceasing the coupling strength $\xi_j$: the largest differences are less than 0.06 and 0.04 for coupling strengths $\xi_j = 1/8$ and $\xi_j = 1/10$, respectively.

Similarly, if the system works far from the co-resonance point (e.g., $n_{g_j} < 0.25$, \(\xi_j \approx 1/4\)), then both $E^{(j)}_j$ and the fixed coupling $E_{12}$ are the small perturbative quantities, compared to the charging energy $E^{(j)}_C$. Thus, the Hamiltonian (1) can be effectively approximated by

$$\hat{H}_2 = \sum_{j=1,2} E_j \sigma^{(j)}_z + E_{12} \sigma^{(1)}_z \otimes \sigma^{(2)}_z,$$

with $E_j = E^{(j)}_C[1 + \xi_j^2/(1 - \xi_j^2)]$, and $\xi_j = E^{(j)}_j/(2E^{(j)}_C)$, $\xi_{12} = E_{12}/E^{(j)}_C$. The evolution corresponding to this effective Hamiltonian results in a two-qubit operation

$$R^{(12)}_z(\chi) = \exp[-i\chi_{12}\sigma^{(1)}_z\sigma^{(2)}_z] \otimes \prod_{j=1,2}\exp[-i\chi_j\sigma^{(j)}_z],$$

where $\chi = \{\chi_j, \chi_{12}\}$. Now, we have $E^{(j)}_j = E^{(j)}_{12}/$. By using a refocusing technique, like in NMR [13], we can effectively realize another important single-qubit operation:

$$R^{(j)}_z(\phi_j) = [R^{(12)}_z(\chi)\sigma^{(j)}_z]^2 = \exp[-i\phi_j\sigma^{(j)}_z],$$

with $\phi_j = 2\chi_j$. The inverse of this operation, i.e., the gate $R^{(j)}_z(-\phi_j) = \exp[i\phi_j\sigma^{(j)}_z]$ can be obtained by changing the signs of $E_j$ via controlling the applied gate voltage.

The single-qubit gates $R^{(j)}_z(\varphi_j)$ and $R^{(j)}_z(\phi_j)$ do not commute, and thus constitute a universal single-qubit gate set, which can assist the realization of two-qubit gates to implement any unitary operation on this circuit. For example, a Hadamard-like operation

$$R_j(\theta_j) = R^{(j)}_z(\theta_j/2)R^{(j)}_z(\pi/4)R^{(j)}_z(\theta_j/2) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \exp(i\theta_j) \\ -i \exp(-i\theta_j) & 1 \end{pmatrix},$$

can be implemented, which will take an important role for testing Bell’s inequality.

**III. TESTING BELL’S INEQUALITY BY USING EFFECTIVE SINGLE-QUBIT LOCAL OPERATIONS**

By using the above dynamical decoupling procedure, we now show that Bell’s inequality may be tested with fixed-coupling Josephson-qubits.

First, the desired entanglement between these SQUID-based qubits can be created in a repeatable way. Initially, the system works sufficiently far from the co-resonance point and remains at the state $|\psi_0\rangle = |00\rangle$, $\Phi_j = 0$. Now, a pair of gate voltage pulses brings the system to the co-resonance point and lets the system undergo the evolution $\hat{U}_3(t) = \exp(-it\hat{H}_3)$, with

$$\hat{H}_3 = -\varepsilon_j \sum_{j=1,2} \sigma^{(j)}_z + E_{12} \sigma^{(1)}_z \otimes \sigma^{(2)}_z.$$

For simplicity, here we assume that $\varepsilon^{(1)}_j = \varepsilon^{(2)}_j = \varepsilon_j$. We analytically derive the time-dependent degree of entanglement
or concurrence \( C_E(t) \) of this circuit

\[
C_E(t) = \frac{1}{2} \sqrt{P^2(t) + Q^2(t)},
\]

with

\[
P(t) = \cos^2 \vartheta(t) - \cos \varrho(t) + \sin^2 \vartheta(t) \left( \frac{1}{1 + \zeta^2} - \frac{1}{1 + \zeta^{-2}} \right),
\]

\[
Q(t) = \frac{\sin^2[2\vartheta(t)]}{\sqrt{1 + \zeta^{-2}}} - \sin \varrho(t),
\]

and

\[
\vartheta(t) = \gamma(t)(1 + \zeta^2)^{1/2}, \quad \varrho(t) = 2\zeta \gamma(t),
\]

\[
\gamma(t) = 2\epsilon_i t / \hbar, \quad \zeta = E_{12}/(2\epsilon). \]

Figure 3 shows this evolution, showing some plateaus near the times \( t_e \) when \( \sin \vartheta(t_e) = 0 \). At these times, the system is in the following compact entangled state

\[
|\psi_e \rangle = \alpha|00\rangle + \beta|11\rangle,
\]

with

\[
C_E(t_e) = 2|\alpha + \alpha^-| = |\sin(E_{12}t_e / \hbar)|.
\]

and

\[
\alpha = [1 \pm \exp(\pm i t E_{12} / \hbar)]/2.
\]

These states are very adjacent to the eigenstates of \( \hat{H}_3 \), and almost do not evolve for several very short time intervals. Thus, periodically, their degrees of entanglement are almost unchanged, shown by the short plateaus in Fig. 3. The maximally entangled states (corresponding to the top plateaus in Fig. 3) occur when the pulse durations \( t_e \) are set properly such that the condition \( \cos(E_{12}t_e / \hbar) = 0 \) is further satisfied.

Next, using dynamically-generated single-qubit operations, the controllable variables \( \{\theta_j\} \) can be encoded into the generated entangled states, keeping the degree of entanglement unchanged. The change of concurrence of the two-qubit entangled state can be effectively suppressed by continuously applying controllable single-qubit operations. This is similar to the approaches for suppressing decoherence in open quantum systems by using the quantum Zeno effect and the “bang-bang” decoupling method [16]. Thus, the new entangled state

\[
|\psi_e \rangle = \prod_{j=1,2} \hat{R}_j(\theta_j)|\psi_e \rangle = \sum_{m,n=0,1} a_{mn}|mn\rangle,
\]

with

\[
a_{00} = [\alpha - \beta \exp(\i \theta_1 + \i \theta_2)]/2,
\]

\[
a_{10} = [-\i \alpha \exp(-\i \theta_1) - \i \beta \exp(\i \theta_2)]/2,
\]

\[
a_{01} = [-\i \alpha \exp(-\i \theta_2) - \i \beta \exp(\i \theta_1)]/2,
\]

\[
a_{11} = [\alpha \exp(-\i \theta_1 - \i \theta_2) - \beta]/2,
\]

has the same degree of entanglement as that of the \( |\psi_e \rangle \) generated above.

Finally, the correlations between the classical variables \( \{\theta_j\} \) can be measured by simultaneously detecting the populations of qubits in the excited \( |1\rangle \) or ground states \( |0\rangle \) [17]. Experimentally, the above steps can be repeated many times for the same classical variables and then the correlation function \( E_c(\theta_1, \theta_2) \) can be measured as

\[
E_c(\theta_1, \theta_2) = \frac{N_{\text{same}}(\theta_1, \theta_2) - N_{\text{diff}}(\theta_1, \theta_2)}{N_{\text{same}}(\theta_1, \theta_2) + N_{\text{diff}}(\theta_1, \theta_2)},
\]

with \( N_{\text{same}}(\theta_1, \theta_2) \) \((N_{\text{diff}}(\theta_1, \theta_2))\) being the number of events with two qubits found in the same (different) logic states. Theoretically, the above projected measurements can be expressed via

\[
\hat{P}_\theta = |11\rangle\langle11| + |00\rangle\langle00| - |10\rangle\langle10| - |01\rangle\langle01|,
\]

\[
= \sigma_x^{(1)} \otimes \sigma_x^{(2)},
\]

and the correlation in the outcomes can be calculated as

\[
E(\theta_1, \theta_2) = \langle \psi_e' | \hat{P}_\theta | \psi_e' \rangle = \pm \sin(t_{\text{e}}E_{12}/\hbar) \sin(\theta_1 + \theta_2).
\]

For the sets of angles: \( \{\theta_j, \theta_j'\} = \{-\pi/8, 3\pi/8\} \), the Clauser, Horne, Shimony and Holt (CHSH) [5] function

\[
f(\psi_e') = |E(\theta_1, \theta_2) + E(\theta_1', \theta_2) + E(\theta_1, \theta_2') - E(\theta_1', \theta_2')| = 2\sqrt{2} |\sin(t_{\text{e}}E_{12}/\hbar)|
\]

is larger than 2 for

\[
|\sin(t_{\text{e}}E_{12}/\hbar)| > \frac{1}{\sqrt{2}}.
\]

Therefore, properly setting the pulse duration \( t_e \) to prepare the desired entangled state (whose plateau-like concurrence is larger than 0.707), the CHSH-type Bell’s inequality [3]

\[
f_c(\{\psi_e'\}) < 2
\]

can be effectively tested by experimentally measuring the CHSH function: \( f_c(\{\psi_e'\}) = |E(\theta_1, \theta_2) + E_c(\theta_1, \theta_2) + E_c(\theta_1, \theta_2') - E_c(\theta_1', \theta_2')|\)

**IV. DISCUSSIONS AND CONCLUSIONS**

The simplest dynamical-decoupling process proposed above consists of two \( \sigma_x^{(j)} \)-pulses and two delays

\[
\hat{U}_d(\tau) = \exp[-iE_{12}\tau \sigma_z^{(1)} \sigma_z^{(2)}].
\]

The duration of the \( \sigma_x^{(j)} \)-pulse is calculated [5] as \( t_x = (2l + 1)t_0, \) \( l = 0, 1, 2, \ldots \) and \( t_0 = \pi/\sqrt{2}E_{12}(1 + 2\zeta^2) \approx 31 \text{ ps} \). Consequently, the longest time delay \( \tau \) between the two \( \sigma_x\)-pulses could be estimated [5] as \( \tau \sim 270 \text{ ps} \) for \( l = 0 \), \( \sim 200 \text{ ps} \) for \( l = 1 \), \( \sim 145 \text{ ps} \) for \( l = 2 \), and \( \sim 98 \text{ ps} \) for \( l = 3 \), etc. Thus,
FIG. 3: Dynamical evolution of the degree of entanglement $C_E(t)$ for the circuit (Fig. 1) with couplings: $E_m / \varepsilon J = 1/2$ (dashed line), $1/4$ (solid line), respectively. Here, $\varepsilon J = 30 \mu eV$. The $C_E = 0.707$ (dotted line) gives the threshold for the violation of Bell’s inequality for the entangled state (8), whose degree of entanglement slowly changes during several short time intervals (see the plateaus in the figure).

it is easy to experimentally check this simplest proposal for eliminating the fixed interbit coupling by using the pulse sequence: $\hat{U}_d(\pi) \sigma_z^{(j)} \hat{U}_d^{-1}(\pi) \sigma_z^{(j)}$, or $\sigma_z^{(j)} \hat{U}_d(\pi) \sigma_z^{(j)} \hat{U}_d^{-1}(\pi)$. After these operations the two qubits should return to their initial states. Furthermore, an universal two-qubit controlled-$\sigma_z^{(j)}$ gate could be implemented by using the operational sequence: $\hat{U}_d(-\pi/4 E_{12}) R_z^{(k)}(\pi/4) R_z^{(j)}(\pi/4)$.

Similar to other theoretical schemes (see, e.g., [18]), the realizability of the present proposal also faces the technological challenge of rapidly on/off switching the Josephson energy of the qubit by using fast magnetic pulses [19]. This experimental difficulty could be relaxed by increasing the durations of the applied pulses. Especially, the decoherence time of the two-qubit capacitively-coupled Josephson circuit [20] could be increased by decreasing the coupling capacitance $C_m$. In principle, the lifetime of the generated entangled state (19) adequately allows to perform the required operations for testing Bell’s inequality (27), since such a state is very adjacent to the eigenstates of the circuit’s Hamiltonian $\hat{H}_3$. In fact, the decay time of a two-qubit excited state is long (up to ~0.6 ns), even for very strong interbit coupling (e.g., $E_m \approx 2e \varepsilon J$ in the recent experiment [3]). In addition, the influence of the environmental noise and operational imperfections is not fatal, as the nonlocal correlation $E(\theta_i, \theta_j)$ in Bell’s equality is statistical—i.e., its fluctuations could be effectively suppressed by averaging over several repeatable experiments.

In summary, we propose an effective dynamical-decoupling approach to overcome the fixed-interaction difficulty in superconducting nanocircuits. The dynamically-generated single-qubit operations may be used to test Bell’s inequality, providing another way to verify the existence of entanglement between two capacitively coupled Josephson qubits. The proposed approach can be easily modified to manipulate quantum entanglement in other “fixed-interaction” solid-state systems, e.g., the capacitively (inductively) coupled Josephson phase (flux) circuits, and the Ising (Heisenberg)-spin chain.

Acknowledgments

We thank Drs. J. S. Tsai, Y. Pashkin, and X. Hu for useful discussions. This work was supported in part by the National Security Agency (NSA) and Advanced Research and Development Activity (ARDA) under Air Force Office of Research (AFOSR) contract number F49620-02-1-0334, and by the National Science Foundation grant No. EIA-0130383.

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