Theoretical aspects of the CEBAF 89-009 experiment on inclusive scattering of 4.05 GeV electrons from nuclei

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Abstract

We compare recent CEBAF data on inclusive electron scattering of 4.05 GeV electrons on nuclei with predictions, based on a relation between structure functions (SF) of a nucleus, a nucleon and a nucleus of point-nucleons. The latter contains nuclear dynamics, e.g. binary collision contributions in addition to the asymptotic limit. The agreement with data is good, except in low-intensity regions. Computed ternary collision contributions appear too small for an explanation. We perform scaling analyses in Gurvitz’s scaling variable and found that for $y_G \geq 0$, ratios of scaling functions for pairs of nuclei differ by less than 15-20% from 1. Scaling functions for $\langle y_G \rangle < 0$ are, for increasing $Q^2$, shown to approach a plateau from above. We observe only weak $Q^2$-dependence in FSI, which in the relevant kinematic region is ascribed to the diffractive nature of $NN$ amplitudes appearing in FSI. This renders it difficult to separate asymptotic from FSI parts and seriously hampers the extraction of $n(p)$ from scaling analyses in a model-independent fashion.
In the following we discuss some aspects of inclusive scattering of high-energy electrons from nuclear targets with the following cross section per nucleon

$$\frac{d^2\sigma_{eA}(E;\theta,\nu)/A}{d\Omega d\nu} = \frac{2}{A}\sigma_M(E;\theta,\nu)\left[\frac{xM^2}{Q^2}F_2^A(x,Q^2) + \tan^2(\theta/2)F_1^A(x,Q^2)\right]$$

(1)

The inclusive, as well as the Mott cross for point-nucleons $\sigma_M$, are measured as functions of the beam energy $E$, the scattering angle $\theta$ and the energy loss $\nu$. The above $F_{1,2}^A(x,Q^2)$ are two nuclear structure functions (SF) which account for the inclusive scattering of unpolarized electrons from randomly oriented targets. These depend on the squared 4-momentum transfer $Q^2 = q^2 - \nu^2$ and the Bjorken variable $x$, corresponding to the nucleon mass $M$ with range $0 \leq Q^2/2M\nu \leq A$.

We concentrate on the recent CEBAF $E=4.05$ GeV inclusive scattering data on nuclear targets, notably Fe. These cover a range of squared 4-momenta transfer $Q^2 = q^2 - \nu^2$, $1 \lesssim Q^2(\text{GeV}^2) \lesssim 7$ and Bjorken variable $x = Q^2/2M\nu$, $0.20 \lesssim x \lesssim 4.2$, and vastly extend the angular and energy-loss ranges of the older NA3 SLAC$^3$ and related experiments$^3$.

Predictions for the above CEBAF experiment have been made before$^5$. These were based on a model which connects the structure functions (SF) of a nucleus $F_k^A$ and $F_k^N$ of a nucleon

$$F_k^A(x,Q^2) = \int_0^1 \frac{dz}{z^{2-k}} f^{PN}(z,Q^2) F_k^N\left(\frac{x}{z},Q^2\right)$$

(2)

A calculation of the above nuclear SF requires two input elements. The first one is $F_k^N \equiv F_k^{p,n}$, properly weighted with proton and neutron numbers in the nuclear target. The $F^N$ contain nucleon elastic (NE) part in terms of the standard static electric and magnetic form factors $G_E^N, G_M^N$, and nucleon inelastic (NI) parts which are available in parametrized and tabulated forms$^7$. The second element is the structure function $f^{PN}$ for a nucleus of point-nucleons, which accounts for the nuclear dynamics in $F_k^A$. Calculations employ a relativistic extension of the non-relativistic (NR) Gersch-Rodriguez-Smith (GRS) series in $1/q^8$. The latter contains an asymptotic limit (AL), related to the single-nucleon momentum distribution $n(p)$, and Final State Interactions (FSI) which are dominated by binary collisions between the knocked-on nucleon and a nucleon from the core.
Expressions, similar to (2), have previously been suggested with light-cone momentum fractions instead of Bjorken variables. The two coincide for \( Q^2 \to \infty \) and Eq. (2) conjectures the approximate equality to hold for \( Q^2 \), in excess of some \( Q^2_c \).

The model incorporated in (2) locates weak \( A \)-dependence of \( F^A_k(x, Q^2) \) only in the neutron excess \( \delta N/2A \), and in \( f^{PN} \), thus

\[
F^A_k(x, Q^2) \approx F_k(x, Q^2) + \mathcal{O}(1/A); \quad A \gtrsim 12, \quad (3)
\]

with the above restriction on \( A \), due to the neglect of nucleon recoil in calculations of \( f^{PN} \) in (2). In Fig. 1 we compare Fe data and the above predictions and conclude:

i) There is good agreement in the deep-inelastic region \( \nu \geq Q^2/2M \), and satisfactory correspondence on the immediate NE side of the quasi-elastic peak (QEP), \( x \gtrsim 1 \).

ii) Data are for given \( \theta \), and not for fixed \( Q^2 \), but from the fact that \( Q^2 \) increases with \( \theta \), and for given \( \theta \) with decreasing \( \nu \), one observes that discrepancies grow with decreasing \( \theta \), i.e. for decreasing \( Q^2 \). For \( Q^2 \lesssim 1.6 \text{ GeV}^2 \) this may in part be due to variation in some ill-determined \( NN \) scattering parameters, which enter the FSI calculations. However, it is more likely, that \( Q^2 \lesssim Q^2_c(x, \theta) \), below which the representation (2) becomes progressively flawed.

iii) For all \( \theta \), computed cross sections overestimate the data by a factor up to 2-3 for the lowest energy losses \( \nu \), where cross sections have dropped orders of magnitude from their maximum.

In an attempt to understand the above discrepancy we worked out more complex FSI contributions, generated by ternary collisions (TC). These are on general grounds expected to effect low-intensity regions far from the QEP, where they compete with both the AL and FSI, due to BC. Their small, but noticeable effect has recently been established in the response of liquid \(^4\text{He} \). We have used a relativistic extension of the above TC contribution and found that these indeed contribute in the above kinematic regions, but only with insignificant weight.

At this point we recall that it is not at all clear that the above is a real discrepancy. Using
different acceptable $n(p)$ produce results which range over the area of the above mentioned local discrepancies (see Figs. 5,6 in Ref. 3).

The NA3 experiment\textsuperscript{3} has also been analysed by means of versions of the Plane Wave Impulse Approximation (PWIA) in terms of a spectral function, occasionally supplemented by additional FSI\textsuperscript{12}–\textsuperscript{15}. We recall attempts by Benhar et al to ascribe to color transparency a desired lowering of cross sections\textsuperscript{12}. Conventional $2p - 1h$ FSI on the PWIA apparently produce the same effect\textsuperscript{16}.

A comparison with the above mentioned generalized GRS results shows excellent agreement in inelastic regions. For decreasing energy losses Ciofi and Simula somewhat underestimate intensities\textsuperscript{16}, whereas our approach overestimates them. In fact, one wonders how two entirely different series, GRS and IA tend to produce comparable answers. The explanation is a recent demonstration for NR dynamics of a surprising, order-by-order correspondence in $1/q\textsuperscript{17}$.

Next we turn to a few selected scaling analyses, previously applied to the NA3 data\textsuperscript{18},\textsuperscript{4}. We had used there a relativistic West-GRS scaling variable suggested by Gurvitz\textsuperscript{19} $y_G$, related to $x$ by $y_G \approx (M\nu/q)(1 - x)$. We investigated ratios of inclusive cross sections

$$\xi^{A_1,A_2} = \left( \frac{d^2\sigma^{eA_1}}{A_1} \right) / \left( \frac{d^2\sigma^{eA_2}}{A_2} \right)$$

and in particular $\xi^{A,<N>}(y_G < 0, q)$ in the NE region. For the latter, data for selected nuclei, nuclear matter and in fact for all investigated targets treated simultaneously, showed universal coarse scaling in $y_G$. The above then also holds for the ratios $\xi^{A_1,A_2}(y_G < 0, Q^2)$. The same results\textsuperscript{20} when a relativistic Fermi gas scaling variable is used\textsuperscript{21}.

We report below on such an analysis of the new CEBAF data for the pairs C,Fe and Fe,Au. A novel element is the extension of the $y_G$-range into the entire NI range $y_G \geq 0$. Although for all $\theta$ or $Q^2$, cross sections per nucleon for given $y_G$ span 4-5 decades (or about 3 decades for $\xi^{A,<N>}$), $\xi^{A_1,A_2} \approx 1$ within 15-20% and frequently better (see Table I). This agrees with the approximate $A$-independence (3) of nuclear SF. Occasional larger deviations from 1 are ascribed to experimental uncertainties, in particular for data of lowest intensity.
Next we consider the EMC ratio $\xi_{NE}^{A,N}$, which is not a special case of $\xi^{A_1,A_2}$ with $A_2 \to \langle N \rangle = D/2$. Density, momentum distribution and pair-distribution function for $D$ differ and $A \gtrsim 12$. Although the NE and the NI regions both contain information on the single-nucleon momentum distribution $n(p)$, implicit in $f^{PN}$, Eq. (2), simple expressions for $\xi_{A,N}$ can only be given for $y_G < 0$. However, that portion is not directly observable. In particular for high $Q^2$, NI contributions compete with the NE ones, even on the elastic side $y_G \approx 0$ ($x \geq 1$) of the QEP (cf. Ref. 4, Fig. 4), as has already been realized in the analysis of early high-$E$ experiments on the lightest nuclei $D$, $^{3}\text{He}$, $^{4}\text{He}$.

In order to isolate the NE part of $\xi_{A,N}(y_G < 0, Q^2)$, one has to remove NI parts from the data, which can be done in several ways. For instance (2) in conjunction with (1), provides a model for the NI parts, which gives perfect agreement with the data for $y_G \geq 0$. Although this need not be the same for $y_G \leq 0$, we assume this to be the case. The procedure ultimately becomes impractical, because both total and NI parts rapidly decrease with $y_G$ beyond $y_G \approx -0.25$. One has then to rely on directly calculated NE parts, using again (2), now with NE parts for $F_{k}^{N}$. The result is

$$F_{1}^{N(NE)}(x, Q^2) = \frac{x}{2}[G_{M}^{N}(Q^2)]^2 \delta(x - 1)$$

$$F_{2}^{N(NE)}(x, Q^2) = \frac{[G_{E}^{N}(Q^2)]^2 + \eta[G_{M}^{N}(Q^2)]^2}{1 + \eta} \delta(x - 1)$$

(5a)

$$F_{1}^{A(NE)}(x, Q^2) = \frac{1}{2}f^{PN}(x, Q^2)[G_{M}^{N}(Q^2)]^2$$

$$F_{2}^{A(NE)}(x, Q^2) = xf^{PN}(x, Q^2)\frac{[G_{E}^{N}(Q^2)]^2 + \eta[G_{M}^{N}(Q^2)]^2}{1 + \eta}$$

(5b)

Substitution in (1) yields for $\xi_{A,N}$, Eq. (1) ($\eta = Q^2/4M^2$)

$$\xi_{NE}^{A,N}(x, Q^2) = f^{PN}(x, Q^2) \left[ \frac{(x^2m^2/Q^2)([G_{E}^{N}]^2 + \eta[G_{M}^{N}]^2)/(1 + \eta) + \tan^2(\theta/2)([G_{M}^{N}]^2)}{(m^2/Q^2)[G_{E}^{N}]^2 + \eta[G_{M}^{N}]^2)/(1 + \eta) + \tan^2(\theta/2)[G_{M}^{N}]^2} \right]$$

(6a)

$$\xi_{NE}^{A,N}(x \approx 1, Q^2) = f^{PN}(x \approx 1, Q^2)$$

(6b)

where for better readability, we have dropped the arguments on $G^{N}$.

In Fig. 2 we plot $\xi_{NE}^{Fe,N}(y_G < 0, Q^2)$ against $Q^2$ for a number of narrowly binned $y_G$. Whenever possible, we give the two results, which with the exception of $y_G = 0$, approximately agree. For all $y_G \leq 0$ in the kinematic range of the experiment, $\xi_{NE}$ approaches
the AL $Q^2 \to \infty$ from above, or can confidently be extrapolated to $Q^2$ beyond the observed ones. For the largest $|y_G|$, there is hardly any $Q^2$ dependence.

We mention here an analysis of a PWIA scaling function which differs from the above $\xi^{A,N}$ primarily by a kinematic factor and by the use of a different scaling variable $y_0 = y^{PWIA}$. Arrington et al found that for $y_0 \leq -0.3$ GeV, $\xi^{PWIA}$ approaches a plateau from above, with persistent structures for large $Q^2$. However, for $y_0 = 0.0, -0.1$, $\xi^{PWIA}$ increases with growing $Q^2$. Assuming that NI contributions have been properly removed, the spelled-out differences may be due to the use of different scaling variables. Those amount to the implicit retention of different FSI parts which are not easily compared. Since the authors of Ref. plan studies in still different variables, we do not pursue the issue here.

A plateau is conventionally related to the AL, from which one extracts the single-nucleon momentum distribution $n(p)$. However, the standard argument becomes invalid, if parts of the FSI happen to be weakly $Q^2$-dependent, causing the plateau to also contain FSI parts. There are strong indications that this is approximately the case for the kinematic range of the CEBAF experiment, which seems to contradict the behavior of $\xi^{A,N}$ for low $Q^2$, seen in Fig. 1.

We give here only an outline of the resolution of the apparent contradiction: details may be found in Ref. The argument follows the construction of $f^{PN}(x, Q^2)$ from the NR SF $\phi(q, y_G)$. For smooth underlying $NN$ interactions the $q$-dependence of FSI may as follows be written and parametrized

$$\phi(q, y) = F_0(y) + \phi^{odd}(q, y) + \Delta^{even}\phi(q, y)$$  

$$F_0(y) = \lim_{q \to \infty} \phi(q, y) = \frac{1}{4\pi^2} \int_{|y|}^{\infty} dppn(p, \gamma_k),$$  

$$\phi^{odd}(q, y) = U^{(o)}(q) y \sum_n a_n^{(o)} y^{2n} \exp(-[A^{(o)} y]^2)$$  

$$\phi^{even}(q, y) = U^{(e)}(q) \sum_n a_n^{(e)} y^{2n} \exp(-[A^{(e)} y]^2)$$

$F_0$ and $\Delta\phi$ are parts even in $y$, $\phi^{odd}$ is odd.

The notion of a local, energy-independent $V_{NN}$ breaks down for increasing lab momentum $q$, and $V$ has to be replaced $V \to V_{eff} = t_q$, the latter being the off-shell elastic $T$ matrix
for small momentum transfer. It is the dominant \( \text{Im}[t(q, Q^2 = 0)] \propto \sigma_{qN,tot}^N \) which is hardly \( q \)-dependent in the relevant range. The link of \( \phi \) with \( f^{PN} \) in (2) reads

\[
f^{PN}(x, Q^2) = MD(x, Q^2) \phi \left( q(x, Q^2), y_G(x, Q^2) \right),
\]

\[
\lim_{Q^2 \gg 4M^2 x^2} D(x, Q^2) = 1,
\]

with \( D \) a purely kinematic factor. From (6b) one concludes that for \( y \approx 0 \) \( (x \gtrsim 1) \)

\[
\xi_{A,N}^{A,N}(y_G \lesssim 0, Q^2) \approx f^{PN}(y_G \lesssim 0, Q^2) = MD\phi(q, y_G \lesssim 0)
\]

Consequently a weakly \( q \)-dependent \( \phi(q, y_G) \) acquires \( Q^2 \) dependence through \( D \) for relatively low \( Q^2 \), which gets weaker for increasing \( Q \), as indeed observed.

It is of interest to recall here a suggested parametrization of \( \xi^{IA}(y_0, Q^2) \), function of \( y_0 \)

\[
\xi^{IA}(y_0) = \frac{C_1 \exp(-a^2 y_0^2)}{\alpha^2 + y_0^2} + C_2 \exp(-b|y_0|)
\]

The above appears to provide a universal fit for \( q \)-dependent data, with practically \( A \)-independent parameters for \( A \gtrsim 12 \), which follows from Eq. (3). Moreover we note that an expansion of (10) in \( y_0 \) leads to a form similar to (7d). Without entering an interpretation of the parameters, this implies that either all the data are in the AL region, or that FSI in addition to the AL are hardly \( q \)-dependent. If the latter is the case, the extraction of \( n(p) \) from the AL, without some knowledge of dynamics, becomes close to impossible.

Summarizing we put into focus the relation (3) which enables the determination of nuclear SF \( F_k^A \) for computed SF \( f^{PN} \) of a nucleus, composed of point nucleons. The latter contains the AL, expressed by the single-nucleon momentum distribution \( n(p) \), and FSI terms for finite \( Q^2 \). Those FSI, induced by binary collisions between the struck and a core nucleon, depend on effective \( NN \) interactions, matter properties, none of which are known with great precision.

A first application is for inclusive cross and the agreement with data is excellent for NI parts and also for the immediate NE side \( x \gtrsim 1 \) of the QEP. It deteriorates for the most elastic regions \( x \geq 1 \) for all angles where cross sections are smallest. We found that ternary
collision contributions do not account for the above discrepancies. However, different single-nucleon momentum distributions easily cover the range of the problematic parts of the cross sections.

We performed a scaling analysis for ratios $\xi^{A_1:A_2}(x, Q^2)$ of cross sections for different nuclei and observed the predicted $A$-independence over the entire NE and NI regions. A second ratio $\xi^{A,N}(y_G, Q^2)$ was found to approach a plateau for all $y_G < 0$ with negative slope, decreasing for increasing $Q^2$. We also studied FSI and concluded that those are only weakly $q$-dependent. Consequently the plateau contains the asymptotic limit as well as FSI. The same seems also to be compatible with results of a direct fit of scaling functions. Should the above be confirmed, there may be less incentive to study nuclear scaling in the high-$Q^2$ regime as a tool to extract nucleon momentum distributions. From the agreement between predictions for $d^2\sigma$ and data (Fig. 1) we conclude that a weakly $q$-dependent FSI, which does not contradict any principle, is empirically correct.

In our conclusion we assemble results obtained for a variety of inclusive scattering observables, which cover a wide range of kinematic ranges, and which all are related to computed nuclear form factors. In addition to the above mentioned inclusive cross sections, we mention $R$ ratios and ratios of moments of $F_k^A(x, Q^2)$. Generally there is agreement with data, which frequently span broad ranges of variation. All prediction use the underlying relation (4) between structure functions of a nucleus and of nucleons. Of course, the agreement does not prove the relation but neither does it disclose definite flaws. In fact, one wonders whether (2) is the result of some effective theory which, at least for the chosen observables, successfully replaces the fundamental QCD.

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Figure captions

Fig. 1 Data\textsuperscript{18} and predictions\textsuperscript{19} for the CEBAF 89-001 experiment.

Fig. 2. The NE part of the GRS-type scaling function $\xi_{Fe,N}^{\text{FE,N,E}}(y_G \leq 0, Q^2)$\textsuperscript{23} as function of $Q^2$. Dots connect extracted values, drawn lines data, with calculated NI parts removed.
Table I

| $\langle y_G \rangle$ (GeV) | $\theta$ | $x$ | $Q^2$ (GeV$^2$) | $\xi_{C,Fe}$ | $\xi_{Fe,Au}$ |
|--------------------------|---------|-----|----------------|-------------|-------------|
| 23                       | 2.30    | 2.26| 0.81           | 1.03        |
| -0.4                     | 30      | 1.95| 3.38           | 0.70        | 0.84        |
| 45                       | 1.67    | 5.46| 0.97           | -           |
| 15                       | 2.49    | 1.05| 0.82           | 1.00        |
| -0.2                     | 30      | 1.37| 3.09           | 0.98        | 1.19        |
| 55                       | 1.30    | 5.78| 0.87           | 1.24        |
| 15                       | 1.02    | 0.97| 1.18           | 1.05        |
| 0.0                      | 30      | 1.01| 2.79           | 1.04        | 1.16        |
| 74                       | 1.01    | 5.77| 1.28           | 0.84        |
| 15                       | 0.65    | 0.91| 0.97           | 1.02        |
| 0.2                      | 30      | 0.72| 2.43           | 1.00        | 1.10        |
| 74                       | 0.74    | 4.54| 1.08           | -           |
| 0.4                      | 15      | 0.43| 0.83           | 1.00        | 1.03        |

Selection of cross section ratios $\xi^{A_1,A_2}$, Eq. (1). For each selected, narrowly-binned $\langle y_G \rangle$, available data of the ratios are given for the smallest, some medium and largest ($x, Q^2$) in the data sets.
Fe
$\xi_{Fe,N(NE)}(y_G, Q^2)$ [fm]

- $y_G = -0.0$
- $y_G = -0.1$
- $y_G = -0.2$
- $y_G = -0.3$
- $y_G = -0.4$
- $y_G = -0.5$

$Q^2 (\text{GeV}^2)$

- NI Removed
- Extracted

Graph showing the variation of $\xi_{Fe,N(NE)}(y_G, Q^2)$ with $Q^2$ for different $y_G$ values.