Holes in the walls: primordial black holes as a solution to the cosmological domain wall problem

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We propose a scenario in which the cosmological domain wall and monopole problems are solved without any fine tuning of the initial conditions or parameters in the Lagrangian of an underlying field theory. In this scenario domain walls sweep out (unwind) the monopoles from the early universe, then the fast primordial black holes perforate the domain walls, change their topology and destroy them. We find further that the (old vacuum) energy density released from the domain walls could alleviate but not solve the cosmological flatness problem.

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Domain walls arise in a wide class of cosmological models. Any early-universe phase transition, in which a discrete symmetry of classical field theory is spontaneously broken, results in domain walls \textsuperscript{1}. According to standard cosmology, domain walls, once formed, quickly come to dominate the energy density of the universe and severely violate many observational astrophysical constraints including measurements of the cosmic background radiation. The domain walls must disappear before the epoch of nucleosynthesis at the latest. Various solutions to this cosmological problem have been put forward, notably a period of inflation \textsuperscript{2}, but it remains useful to examine new solutions.

Recently, two of us proposed a solution to the cosmological monopole problem: primordial black holes, produced in the early universe, can accrete magnetic monopoles within the horizon before the relics dominate the energy density of the universe \textsuperscript{3}. Here we propose that primordial black holes can be used in yet another way. We will show that primordial black holes can perforate domain walls, and that the resulting holes in the walls can grow to destroy the domain walls altogether. We also note a variant of this picture in which monopoles and domain walls can both be destroyed: following the scenario presented in \textsuperscript{1,3}, it is possible for the domain walls to sweep up and eliminate monopoles before the domain walls themselves are destroyed (if they are sufficiently long-lived).

We imagine that domain walls are formed in the early universe, during some phase transition at an energy scale \( \sigma \). The energy per unit area of these domain walls is \( \sigma \approx \eta^3 \). By the Kibble mechanism \textsuperscript{4}, we expect to form one domain wall per cosmological horizon at the formation time. Subsequently, the cosmological horizon grows to encompass many previously-disconnected horizon volumes. The domain wall network evolves. For walls arising from a remnant \( Z_2 \) symmetry, the network is usually dominated by one infinite wall of very complicated topology. In addition there are some finite closed walls. The structure can be more complicated for \( Z_N \) and non-Abelian walls. \( Z_N \) (with large \( N \)) and non-Abelian walls show a tendency for frustration (freezing into a static structure). While \( Z_2 \) walls do not tend to frustration, their motion can still be severely damped if their interaction with surrounding matter is strong. Thus, in most cases the network of domain walls is slowly evolving and thus very slowly dissipates its energy.

As the scale factor \( a(t) \) of the universe increases, the number density of domain walls decreases as the volume, \( n \propto a^{-3} \), but the mass of a slowly evolving domain wall increases approximately as its area, \( m \propto a^2 \). The net effect is that the energy density of domain walls redshifts only as fast as \( a^{-1} \). Meanwhile, matter and radiation redshift as \( a^{-3} \) and \( a^{-4} \) respectively. Eventually (at least in the absence of an even more slowly evolving component), the domain walls come to dominate the energy density of the universe.

The characteristic time for wall domination is given by a generic value \( t_{\text{WD}} = (G\sigma)^{-1} \). Since a domain-wall dominated expansion has \( a(t) \propto t^2 \), rather than the \( a(t) \propto t^{1/2} \) of radiation domination, it would drastically change the abundance of light elements produced during nucleosynthesis. Therefore, if the universe produces domain walls, it must get rid of them before nucleosynthesis.

The early universe may also produce large numbers of primordial black holes. Such black holes can be formed by many processes \textsuperscript{5,6,7,8,9,10,11,12,13,14,15}. The earliest mechanism for black hole production can be fluctuations in the space-time metric at the Planck epoch. Large number of primordial black holes can also be produced by nonlinear density fluctuations due to oscillations of some (scalar) field. If within some region of space density fluctuations are large, so that the gravitational force overcomes the pressure, we can expect the whole region to collapse and form a black hole. In
the early universe, generically, black holes of the horizon size are formed, although it is possible to form much smaller black holes. Black holes can also be produced in first and second order phase transitions in the early universe. Gravitational collapse of cosmic string loops and closed domain walls can also yield black holes. The mass range of primordial black holes formed in the above mentioned processes ranges roughly from $M_{Pl}$ (black holes formed at the Planck epoch) to $M_{sun}$ (black holes formed at the QCD phase transition).

As the black holes move through the universe, they encounter domain walls. The outcome of the encounter depends on the relative velocity of the domain wall and the hole. If the relative black hole-domain wall velocity is small, the black hole gets stuck on the domain wall and its kinetic energy goes into oscillatory modes of the wall (configuration (1) in Fig. 1). If the black hole kinetic energy is large enough, the black hole can pierce a hole in the wall as it passes through (configuration (3) in Fig. 1). Depending on the underlying field theory, the boundary of a hole made in this way can be either a cosmic string (if the field theory admits the existence of such strings) or a black string (a one-dimensional generalization of a black hole). Even at intermediate velocities, where one might have expected the wall to smoothly reconnect behind the black hole after it passes through (configuration (2) in Fig. 1), the hole in the domain wall formed by its intersections with the black hole event horizon might instead begin to expand away from the black hole, bounded by a cosmic or black string. This will happen if the intermediate states of the system are suitably unstable to such expansion.

Under certain generic conditions (shown below), the hole in the wall expands. If there is only one hole in a large domain wall, it may not be sufficient to entirely destroy the wall (which is expanding itself). However, if at least four holes are made in the wall and expand outwards, Chamblin and Eardley have shown very generally that the domain wall will be utterly destroyed. It is this mechanism that we will employ to rid the universe of excess domain walls.

The probability of cosmic/black string formation here can be large due to thermal activation. By contrast, in situations where the process of cosmic/black string is spontaneous (quantum mechanical tunneling or instanton), it can be highly suppressed. The process we want to use here is induced by a black hole and it is "over the barrier" process rather than tunneling. The kinetic energy of the black holes gets converted into a thermal bath; if this energy is large enough, it can cause the process of string formation to be classically allowed. On dimensional grounds we expect the probability for this process to be proportional to $e^{-m_s/T}$, where $m_s$ is the mass of the created string, while $T$ is the temperature of the thermal bath. When $T \sim m_s$, the process becomes unsuppressed. For an effective thermal bath one needs finite energy distributed over some finite volume relevant for the process — a condition that is rarely satisfied in collisions of point like particles. Since both the black hole and the domain wall are extended objects we expect that most of the change in kinetic energy of the black hole in a black hole-wall collision is converted into a thermal bath. For the exact fraction of the kinetic energy that goes into thermal bath, one would have to perform a detailed analytical (or numerical) analysis.

We have mentioned that is possible for black strings to be formed at the black hole/domain wall encounter. Domain walls tend to decay by nucleating string loops on their world-sheet. Even topologically stable domain walls can be unstable to nucleating black strings when we include gravity. The situation here has an analog in one dimension less. It is well known that a topological string can break by nucleating a pair of black holes. Unfortunately, there is no appropriate calculation for the case of a hole bounded by a black string within a domain wall sheet. Here, we assume that such a configuration is allowed. It would be very interesting to find an appropriate metric that describes this configuration.

The probability for black string creation should also be greatly enhanced (in comparison with spontaneous black string nucleation), not only due to thermal bath...
enhancement but also due to the physical setup. A black string solution is just a direct product of a black hole and one extra dimension. There are known solutions of \((3 + 1)\)-dimensional black strings in string theory where there are some additional fields beside gravity. Alternatively, we can construct a black string solution as a direct product of a BZT black hole (the only known black hole solution in \((2 + 1)\) dimensions) and an extra spatial dimension. There are no asymptotically flat (real) black hole solutions in \((2 + 1)\) dimensions. The BZT black hole is a solution in the presence of the cosmological constant. The momentum energy tensor of the domain wall is such that the interior of the wall is a region with negative pressure due to the (old vacuum) energy density that acts similarly to a cosmological constant. One can easily imagine that in the black hole-domain wall encounter, the black hole horizon gets deformed and stretched by the domain wall tension into a black string configuration on the boundary of a hole made in the wall. We leave detailed description of this process for the future.

A comprehensive study of the interaction of the domain walls and black holes in the weak field regime and/or some other approximations has been done in [15]. A proper treatment of the present problem in the strong field regime, near the black hole horizon, is merited; however, in the absence of such a treatment, we will rely on simple back-of-the-envelope estimates.

We first consider the kinematics of a black hole perforating a domain wall. This is a classically allowed process if the black hole has sufficient kinetic energy to cause the production of a (black or cosmic) string bounding the perforation. In the non-relativistic limit this is crudely:

\[
M_{BH}(1 + v^2/2) + \pi R_p^2 \sigma \geq M'_{BH}(1 + v'^2/2) + 2\pi R_p \mu
\]

where \(M_{BH}\) and \(M'_{BH}\) are the mass of the black hole before and after the interaction and \(v\) and \(v'\) its velocity. \(R_p\) is the initial radius of the perforation in the wall, \(\sigma\) is the energy per unit area of the domain wall, and \(\mu\) is the energy per unit length of the string bounding the perforation. Generically, \(\sigma \sim \eta_w^2\) where \(\eta_w\) is the energy scale at which the walls are formed. Similarly, \(\mu \sim \eta_w^2\) where \(\eta_w\) is the energy scale characteristic of the strings.

(For definiteness we consider the situation where domain walls can be bounded by a cosmic string. The calculation with black strings is analogous with \(\mu_s\) replaced by \(\mu_{bs}\) — mass per unit length of the black string.)

One can take for \(R_p\) the Schwarzschild radius of the black hole after the interaction. In the interaction, a black hole swallows a piece of the domain wall. Accounting for the negative pressure inside the wall is difficult in our heuristic calculation of energy conservation. Consequently we confine ourselves to the case where the energy of the eaten disc of domain wall is small compared to the black hole mass,

\[
M_{BH} \gg \pi R_p^2 \sigma.
\]  

In this case we can still take the radius of the perforation in the domain wall to be the Schwarzschild radius of the black hole after the interaction, \(R_p = 2M'_{BH}/M_{Pl}^2\). The mass of the black hole is enhanced by an amount equal to the mass of the eaten disk,

\[
M'_{BH} = M_{BH} + \pi R_p^2 \sigma \sim M_{BH} \left(1 + 4\pi \frac{\eta_w^2 M_{BH}}{M_{Pl}^4}\right)
\]

(3)

to first non-trivial order in small quantities.

Then the condition in Eq. (2) is satisfied as long as

\[
M_{BH} < \frac{1}{16\pi} \frac{M_{Pl}^4}{\eta_w^2} \left(\frac{M_{BH}}{M_{Pl}}\right)^3.
\]

(4)

For \(\eta_w \sim 10^{17}\) GeV (motivated by GUTs), the mass must be

\[
M_{BH} \leq 2 \times 10^4 M_{Pl} \left(\frac{\eta_w}{10^{17}\text{GeV}}\right)^{-3}.
\]

(5)

We note that, at these very early stages of the universe, such small black hole masses are extremely plausible (the horizon size of the universe at the time was extremely small).

We can now estimate \(v'\), the post-collision velocity of the black hole (in the frame of the domain wall) by assuming that all the momentum of the incoming black hole is carried off by the outgoing black hole, and none is transferred to the domain wall. (We expect this too to be a good approximation if [14] holds.) From momentum conservation \(M_{BH}'v' = M_{BH}v\) and using Eq. (3), we have

\[
v' \sim v \left(1 + 4\pi \frac{\eta_w^2 M_{BH}}{M_{Pl}^4}\right)^{-1}.
\]

(6)

Under these assumptions, the condition (1) can be rewritten more usefully:

\[
2\pi M_{BH} \frac{\eta_w^3}{M_{Pl}^4} \geq \frac{\eta_w^2 M_{Pl}^2}{\eta_w^2 M_{BH}} \frac{v^2}{2}
\]

(7)

This is algebraically satisfied if either

\[
\frac{M_{BH}}{M_{Pl}} \geq \frac{1}{\sqrt{8\pi}} \frac{\eta_w^2 M_{Pl}^2}{\eta_w^3}
\]

(8)

or

\[
\frac{1}{2} M_{BH} v^2 \geq \frac{\eta_w^2 M_{Pl}^2}{\eta_w^3}.
\]

(9)

We will concentrate our analysis on these two conditions.

It may be worth noting that perforation of the wall may also occur by Hawking radiation of a closed string from the black hole as it traverses the domain wall. This is possible only if the mass of the string loop does not greatly
If formed, grows. Therefore, if these conditions are met, then string loop formation and expansion is energetically favored. It is therefore likely to occur in at least a reasonable fraction of hole-wall collisions.

It is useful to write the condition (10) (for validity of our approximation) together with the conditions for formation of a hole in the wall (9) and (12) as

\[
\min \left( \frac{1}{\sqrt{8\pi}} \frac{\eta_h M_{pl}^3}{\eta_w}, \frac{2 \eta_h^2 M_{pl}^3}{\eta_w^3} \right) \leq \frac{M_{BH}}{M_{pl}} \leq \frac{M_{pl}^3}{16\pi \eta_w^2}. \tag{13}
\]

For \( \eta_h \sim \eta_w \sim 10^{17} \text{GeV} \), this is

\[
\min \left( 2 \times 10^3, \frac{2 \times 10^2}{v^2} \right) \leq \frac{M_{BH}}{M_{pl}} \leq 2 \times 10^4. \tag{14}
\]

This condition (14) is independent of \( \eta_w \) and can be satisfied for the right black hole mass. This is a strong indication that there are no serious problems for the scenario to work. Now, we put the scenario into a cosmological framework.

The observational constraints on the abundance of primordial black holes have been studied earlier (14). These constraints are sensitive to the assumptions of the model but for the range of parameters of interest for GUT domain walls, observations do not practically imply any constraints (unless the endpoint of hole black hole evaporation is a Planck mass relic).

The maximal mass of a primordial black hole is limited by the total mass within the cosmological horizon, i.e., \( M_{BH} = M_{pl}^3/\Lambda^2 \) at any given energy scale \( \Lambda \) at which the black hole forms. This is also the expected mass scale of a black hole in most early universe scenarios for the production of black holes. (Stellar black holes are a clear counterexample from the present universe.) Taking

\[
M_{BH} = f M_{pl}^3/\Lambda^2, \tag{15}
\]

where \( f \) is the fraction of the horizon mass that equates to the mass of a black hole, equation (11) becomes

\[
\frac{\Lambda}{M_{pl}} > \sqrt{16\pi f} \left( \frac{\eta_w}{M_{pl}} \right)^{3/2}. \tag{16}
\]

We remind the reader that perforation of the domain wall is kinematically allowed if either Eq. (8) or (9) is satisfied. Let us examine each of these in turn. Satisfying the condition (8) would require that

\[
\frac{\Lambda}{M_{pl}} \leq \sqrt{\frac{8\pi}{f}} \left( \frac{M_{pl}}{\eta_w} \right)^{1/2} \left( \frac{\eta_w}{M_{pl}} \right)^{3/2}. \tag{17}
\]

The combination of Eqs. (16) and (17) can be satisfied if \( \eta_w \leq \sqrt{\frac{v^2}{8\pi}} M_{pl} \).

The other possibility for satisfying the kinematics of the collision is given by Eq. (9). We now examine the consequences of this condition. The kinetic energy \( M_{BH} v^2/2 \) of a black hole of mass \( M_{BH} \) formed at energy scale \( \Lambda \) would be expected to be of order \( \Lambda \),

\[
\frac{1}{2} M_{BH} v^2 \sim \Lambda. \tag{18}
\]

For \( M_{BH} = f M_{pl}^3/\Lambda^2 \), this means that we expect

\[
U_{\text{thermal}} \sim \sqrt{\frac{2}{f}} \left( \frac{\Lambda}{M_{pl}} \right)^{3/2}. \tag{19}
\]

This is quite small for \( f \sim 1 \) if \( \Lambda \ll M_{pl} \); however, if the mass of the black hole is much smaller than the horizon mass \( (f \ll 1) \) its velocity can be relativistic. (Black holes formed by the gravitational collapse of cosmic string loops would be expected to be relativistic.) To obtain estimates we will take the black hole's velocity at its formation \( v_i \) to be as given by (19) with \( f = 1 \). Imposing the constraint (9) would then require
\[ \frac{\Lambda}{M_{Pl}} \geq \left( \frac{\eta_s}{M_{Pl}} \right)^2 \left( \frac{M_{Pl}}{\eta_w} \right)^3 \]  \tag{20} 

Requiring \( \Lambda \leq M_{Pl} \), we then have

\[ \frac{\eta_s}{M_{Pl}} \leq \left( \frac{\eta_w}{M_{Pl}} \right)^{3/2} \]  \tag{21} 

a rather mild hierarchy if \( \eta_w \sim 10^{15} \text{GeV} \).

The black holes may form either before the domain walls (\( \Lambda \geq \eta_w \)), or after them (\( \Lambda \leq \eta_w \)). If they form after the walls, there remain two possibilities – the black holes can form either before or after the walls come to dominate the energy density of the universe. Black holes form before wall domination if \( M_{Pl}/\Lambda^2 \sim t_{\Lambda} \leq t_{WD} \sim (G\sigma)^{-1} \), i.e., if

\[ \frac{\Lambda}{M_{Pl}} \geq \left( \frac{\eta_w}{M_{Pl}} \right)^{3/2} \]  \tag{22} 

If black holes form before wall domination, then, as long as they don’t decay, they characteristically travel a distance \( d_\leq = \int_{t_{\Lambda}}^{t_{WD}} v dt \). Taking \( v_t \) to be the initial thermal velocity of the black hole given by \( 19 \), and using the fact that velocity scales as \( a^{-1} \) during the expansion of the universe and that the universe is radiation dominated before the wall domination, we have \( v = v_t \frac{a(t)}{a(t_{\Lambda})} = v_t (\frac{t}{t_{\Lambda}})^{1/2} \). Then

\[ d_\leq = d(t \leq t_{WD}) \sim 2v_t \sqrt{t_{WD}t_{\Lambda}}. \]  \tag{23} 

The number of black holes formed at time \( t_{\Lambda} \) that are inside a horizon volume at a later time \( t_{WD} \) is

\[ N_{BH} = F \left( \frac{t_{WD}}{t_{\Lambda}} \right)^3, \]  \tag{24} 

where \( t_{WD} = \sqrt{M_{Pl}/\Lambda^2} = (\eta_w/M_{Pl})^{3/2} M_{Pl} \) is the temperature at time \( t_{WD} \), and \( F \) is the number of black holes per horizon volume at time \( t_{\Lambda} \). If black holes are formed by density perturbations, we can expect to have one black hole per horizon at the time of formation. In general, \( F \) may be greater or less than 1, although \( Ff \leq 1 \). \( N_{BH} \) is easily evaluated:

\[ N_{BH} = F \left( \frac{\Lambda}{M_{Pl}} \right)^3 \left( \frac{M_{Pl}}{\eta_w} \right)^{9/2}. \]  \tag{25} 

The number of perforations which one expects in a domain wall by the time \( t_{WD} \) is therefore

\[ N_{\text{perforations}} \sim \frac{d_\leq}{t_{WD}N_{BH}} \approx \frac{F}{\sqrt{f}} \left( \frac{\Lambda}{M_{Pl}} \right)^{7/2} \left( \frac{M_{Pl}}{\eta_w} \right)^3. \]  \tag{26} 

Since most of the energy density is concentrated in one infinite wall of complicated geometry/topology, it is sufficient to destroy only this one. The other finite closed pieces of walls will collapse within some short time due to their own tension. We remind the reader that as long as there are at least four black hole perforations into the domain wall, the wall can generically be destroyed \( 10 \).

With generic values \( F \sim 1 \) and \( f \sim 1 \), the condition \( N_{\text{perforations}} \geq 4 \sim O(1) \) becomes

\[ \frac{\Lambda}{M_{Pl}} \geq \left( \frac{\eta_w}{M_{Pl}} \right)^{6/7}. \]  \tag{27} 

This condition is stronger than the one in \( 22 \), which is required for black holes to form before domain walls come to dominate (we have assumed this latter condition in deriving Eq. \( 27 \)).

Again, we now split our discussion according to the two possible conditions in Eqs.\( 8 \) and \( 9 \) for satisfying the kinematics required for a black hole to perforate a domain wall. If we are to satisfy condition \( 17 \) (required by Eq. \( 8 \)) together with \( 24 \) we must have

\[ \left( \frac{M_{Pl}}{\eta_s} \right)^{1/2} \left( \frac{\eta_w}{M_{Pl}} \right)^{3/2} \geq \frac{\Lambda}{M_{Pl}} \geq \left( \frac{\eta_w}{M_{Pl}} \right)^{6/7}, \]  \tag{28} 

where we have dropped factors of order unity. The consistency condition (the upper limit is greater than the lower) is

\[ \frac{\eta_s}{M_{Pl}} \leq \left( \frac{\eta_w}{M_{Pl}} \right)^{9/7}. \]  \tag{29} 

Condition \( 29 \) is the necessary condition for the solution of the domain wall problem, under the assumption of Eq. \( 8 \) for satisfying the kinematic requirements.

We now turn to the second possible kinematic requirement of Eq. \( 9 \). Then we need to satisfy \( 29 \) together with \( 24 \), so that

\[ \left( \frac{\Lambda}{M_{Pl}} \right) \geq \max \left[ \left( \frac{\eta_w}{M_{Pl}} \right)^{6/7}, \left( \frac{\eta_w}{M_{Pl}} \right)^2 \left( \frac{M_{Pl}}{\eta_w} \right)^{3} \right]. \]  \tag{30} 

This equation, together with the constraint on \( \eta_s \) given in Eq. \( 21 \) provides the set of conditions for the solution of the domain wall problem given the second possibility in Eq. \( 9 \) for satisfying the kinematic requirements for perforation.

The black hole lifetime \( (t_{BH} \approx M_{BH}^3/M_{Pl}^4) \) is very short. For example, a black hole of \( M_{BH} = 10^5 M_{Pl} \) evaporates within about \( 10^{-13} \text{s} \). Therefore, just making the required black holes is not enough; we must also ensure that they do not evaporate before they perforate.
the domain walls. The above calculation assumed that the black holes did not decay before the wall domination era, i.e.,

$$t_{BH} = M_{BH}^3/M_{Pl}^4 \geq \frac{M_{Pl}^2}{\eta_w^6} = t_{WD}. \quad (31)$$

Using Eq. \ref{eq:30}, this equation becomes

$$\frac{\Lambda}{M_{Pl}} \leq \left( \frac{f \eta_w}{M_{Pl}} \right)^{1/2}. \quad (32)$$

Again we split our discussion into the two ways the kinematics of wall perforation can be satisfied. The above equation can be consistent with \ref{eq:22} and \ref{eq:27} if \( f \geq (\eta_w/M_{Pl})^{5/7} \), which is satisfied if \( f \sim 1 \) as expected. Thus, from \ref{eq:22} and \ref{eq:27} we have (for \( f \sim 1 \))

$$\min \left[ \left( \frac{M_{Pl}}{\eta_w} \right)^{1/2}, \left( \frac{\eta_w}{M_{Pl}} \right)^{1/2}, \left( \frac{\eta_w}{M_{Pl}} \right)^{1/2}, \left( \frac{\eta_w}{M_{Pl}} \right)^{1/2} \right] \geq \frac{\Lambda}{M_{Pl}} \geq \left( \frac{\eta_w}{M_{Pl}} \right)^{6/7}. \quad (33)$$

This condition takes care that black holes are capable of perforating the domain wall and do not decay before the wall domination. The constraint on \( \eta_w \) that goes together with this one is given in \ref{eq:24}.

If, alternatively, we assume the second condition in Eq. \ref{eq:29} for the kinematics, the alternative set of necessary conditions can be obtained from \ref{eq:30} and \ref{eq:32}

$$\left( \frac{\eta_w}{M_{Pl}} \right)^{1/2} \geq \left( \frac{\Lambda}{M_{Pl}} \right) \geq \max \left[ \left( \frac{\eta_w}{M_{Pl}} \right)^{6/7}, \left( \frac{\eta_w}{M_{Pl}} \right)^{2}, \left( \frac{M_{Pl}}{\eta_w} \right) \right]. \quad (34)$$

The constraint on \( \eta_w \) that goes together with the above equation is given in \ref{eq:27}.

The black holes that evaporate before wall domination are still capable of destroying walls. In this case, the condition \ref{eq:33} could be somewhat relaxed. Alternatively, it is also possible that black holes form only after wall domination. However, in this case it becomes more and more difficult for the black holes to solve the domain wall problem. Since in the wall dominated universe \( a(t) \sim t^2 \), black hole velocities will be redshifted very quickly: in addition the walls rapidly move away from one another. These two effects make the solution of the domain wall problem unlikely if the black holes form only after wall domination.

In summary, there are two alternative sets of conditions for the solution of the domain wall problem, depending on which condition we use to satisfy the required kinematics for a black hole to perforate a domain wall. If either set of conditions is satisfied, domain walls disappear. The first set is determined by the condition \ref{eq:29}. For this set, the necessary condition for the solution of the domain wall problem is given by Eq. \ref{eq:29} if the black holes are formed at energy scales given in Eq. \ref{eq:33}. The second set is determined by the condition \ref{eq:29}. For this set, the necessary condition for the solution of the domain wall problem is given by Eq. \ref{eq:29} for the black holes that are formed at energy scales given in Eq. \ref{eq:34}. It is enough to satisfy either one of these sets of conditions in order for our scenario to work. Thus, combining the righthand side of Eq. \ref{eq:33} (which is less restrictive than that of Eq. \ref{eq:41}) with the left hand side of Eq. \ref{eq:41} (which is less restrictive than the left hand side of Eq. \ref{eq:43}), we conclude that the black holes that are formed at energy scales

$$\left( \frac{\eta_w}{M_{Pl}} \right)^{1/2} \geq \frac{\Lambda}{M_{Pl}} \geq \left( \frac{\eta_w}{M_{Pl}} \right)^{6/7}. \quad (35)$$

are generically capable of destroying walls, as long there is the following hierarchy between the black/cosmic string scale and domain wall scale:

$$\frac{\eta_w}{M_{Pl}} \leq \left( \frac{\eta_w}{M_{Pl}} \right)^{9/7}. \quad (36)$$

Here, the right hand side of the equation is obtained from Eq. \ref{eq:29} (which is less restrictive than the right hand side of Eq. \ref{eq:21}) and there is no lower bound (since there is none in Eq. \ref{eq:21}). It is not difficult to envision satisfying these bounds for GUT domain walls with \( \eta_w \sim (10^{16} - 10^{17})\text{GeV} \).

Outside of the range of parameters given by eq. \ref{eq:35} and \ref{eq:36}, it is still possible that the walls get destroyed under some special circumstances. For example, it is quite likely for a primordial black hole to form near the domain wall; in this case, the black hole does not have to travel an entire horizon distance to encounter the wall, so that the probability of wall destruction is significantly enhanced. Alternatively, if a black hole is formed with (or gets accelerated to) a velocity much larger than the average thermal velocity, then again the probability of wall destruction is enhanced. Either of these two scenarios is in fact quite plausible.

We have seen that, within the range of parameters given by eq. \ref{eq:35} and \ref{eq:36}, domain walls have little chance to survive. While there are several constraints on this mechanism for black hole destruction of domain walls, it seems to work quite generally for GUT-scale domain walls. The specific constraints can easily be re-calculated for domain walls formed by a phase transition at any other energy scale.

**Solving the monopole problem:** In order to complete the scenario we mention an interesting result presented in \ref{eq:4} and \ref{eq:5}. Namely, it was shown that the interaction between the monopoles and domain walls can be such that monopoles unwind while passing through the domain wall and their topological charge gets spread all over
the wall. Thus, the domain walls can sweep up monopoles from the universe. The basic concept of the idea as well as details about the interaction can be found in \[1\]. For the reasonable range of the parameters in the model, the domain walls clean the universe of monopoles before the critical time $t_{WD}$ when the walls come to dominate the universe. This naturally fits into our scenario.

**Solving other cosmological problems?:** The equation of state of domain walls depends on their velocity. If they are relativistic, then the equation of state is $p = 1/3\rho_w$, as for ordinary radiation. But, if their velocity is small, the equation of state is $p = -2/3\rho_w$ \[1\]. This result is not surprising since the pressure inside the domain wall is negative. This is consistent with the scaling $\rho_w \sim 1/a$. Since this is a slower redshifting that even the curvature term in the Friedmann equation, it suggests that a period of wall domination in the early universe could probably solve the flatness problem. If the energy density of the universe is dominated by the walls energy density the scale factor grows quadratically with time, i.e. $a \sim t^2$, so what we have here is nothing more than domain-wall-driven power law inflation. During the expansion of the hole in a domain wall, the vacuum energy contained in the wall gets released. This process recalls post-inflationary re-heating.

However, a rough analysis indicates that reheating after domain wall inflation would not work. After the period of inflation needed for solving the cosmological flatness and horizon problems, the domain walls end up very far apart. The energy released when the domain walls disappear is at first located only near the domain walls. Even if this energy travels away from the walls at the speed of light, thermalization of a big enough patch to encompass our observable universe takes too long. If we require that the thermalization is finished at some high enough temperature (say $T > $ MeV), then the amount of inflation would not be sufficient to completely solve the flatness and horizon problems.

For completeness we note here that the mechanism of destroying domain walls by primordial black holes can also fit naturally in models where cosmologically dangerous domain walls appear after inflation (see for example brane inflation models \[18\]).

We would like to conclude with the summary of the cosmological scenario we proposed here:

1. Domain walls sweep out (unwind) the monopoles
2. Black holes perforate the domain walls and destroy them
3. Energy density released from the domain walls could alleviate but not solve the cosmological flatness and homogeneity problems.

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\[1\] A. Vilenkin and E.P.S. Shellard, *Cosmic Strings and other Topological Defects*, Cambridge University Press, Cambridge (2000).
\[2\] A. Guth, Phys. Rev. D 23, 347 (1981); A.D. Linde, Phys. Lett. 108B, 389 (1982); A. Albrecht and P.J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982).
\[3\] D. Stojkovic and K. Freese, Phys. Lett. B606, 251 (2005); hep-ph/0403248
\[4\] G. R. Dvali, H. Liu and T. Vachaspati, Phys. Rev. Lett. 80 2281 (1998); L. Pogosian and T. Vachaspati, Phys. Rev. D62 105005 (2000); Phys. Rev. D62 123506 (2000)
\[5\] S. Alexander, R. H. Brandenberger, R. Easther, hep-ph/9903254; hep-ph/0008014
\[6\] T.W.B. Kibble, J. Phys. A9, 1387, (1976)
\[7\] B.J. Carr and S.W. Hawking, Mon. Not. Roy. Astron. Soc. 168 399 (1974)
\[8\] B.J. Carr, Lect. Notes Phys 631 301 (2003)
\[9\] K. Jedamzik, Phys. Rev. D55 5871 (1997)
\[10\] J.C. Niemeyer and K. Jedamzik, Phys. Rev. Lett. 80 5481 (1998)
\[11\] M.Yu. Khlopov, R.V. Konoplich, S.G. Rubin and A.S. Sakharov, Grav. Cosmol. 2:81 (1999)
\[12\] S.G. Rubin, M.Yu. Khlopov and A.S. Sakharov, Grav. Cosmol. 56 51 (2000)
\[13\] A. Polnarev and R. Zembowicz, Phys. Rev. D43 1106 (1991)
\[14\] B.J. Carr, J.H. Gilbert and James E. Lidsey, Phys. Rev. D50 4853 (1994); A. M. Green and A. R. Liddle, Phys. Rev. D56 6166 (1997)
\[15\] T. Matsuda, hep-ph/0509062 hep-ph/0509061
\[16\] A. Chamblin and D. Eardley, Phys.Lett. B475, 46 (2000)
\[17\] V. Frolov, M. Snajdr and D. Stojkovic, Phys. Rev. D68 044002 (2003); V. P. Frolov, D. V. Fursaev, D. Stojkovic, Class. Quant. Grav. 21 3483 (2004); JHEP 0406 057 (2004); D. Stojkovic, JHEP 0409 061 (2004);Phys. Rev. Lett. 94 011603 (2005)
\[18\] T. Matsuda, JHEP 0410 042 (2004); D. A. Easson, M. Trodden, hep-th/0505098, R. Brandenberger, D. A. Easson and A. Mazumdar, Phys. Rev. D 69, 083502 (2004); N. Barnaby, A. Berndsen, J. M. Cline, H. Stoca hep-th/0412095;