Research on Decision-making of Demand-side Energy Consumption Plan Based on Game Theory

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Abstract. A Based on the features of information sharing and energy interaction within smart grid, a decision-making model of energy consumption based on Nash equilibrium solution is developed to analysis the problem of competition and benefit conflict among different kinds of power users on demand-side. The payoff functions of three kinds of player are built, which are flexible, non-flexible and storable, relatively. The Nash equilibrium is solved to calculate the respective energy consumption and benefit in a non-cooperative game by Nikaido–Isoda function optimization. Taking a certain area in Western China as a case, simulation analysis of the game process of three kinds of users with the actual load is carried out. The results show that the game between power users can in-crease the system load rate, reduce the peak and valley difference of the system load, and reduce the cost of power production.

1. INTRODUCTION
In the face of traditional primary energy shortage and slow growth in conversion efficiency, demand-side management has already caused wide attention. Demand Side Source can be regarded as a negative power generation resource, whose utilization is now relatively low but with great potential in the future. As a new type of intelligent service network, the smart grid created an open information sharing model. So that the smart grid can efficiently integrate data in the power system, optimize the operation and management of the power infrastructure, and promote interaction among users. Smart grid integrates advanced information, control and communication technologies to facilitate dynamic management of grid supply and demand. Smart grid provides not only technical support for system interaction, but also the best application platform for information and power interaction.

Game theory provides effective concepts, models and methods for the analysis of the interaction between different participants in a competitive environment and the conflict analysis when there is interaction between participants. With the establishment of electricity market, game theory has been widely applied in the electricity market. Document [5] uses Shapley value to analyse the allocation method of transmission network loss; document [6] uses relaxation algorithm to calculate Nash-Cournot equilibrium in power market, and analyses the non-cooperative and cooperative game of generators; document [7] based on game theory, analyses the market power of generators in power market; literature [8] analyses bidding strategies of power suppliers and establishes game model under stochastic load. Literature [9] establishes demand side management autonomy model under smart grid to control electrical appliances. At present, with the large-scale new energy generation grid-connected,
how to mobilize the enthusiasm of different types of users to participate in grid-connected peak shaving, the game decision-making between stakeholders has become a hot topic of current research. Reference [10] proposes a non-cooperative game pricing method under incomplete information in deregulated electricity market. Literature [11] applies the equilibrium theory of game theory to study the game model of power plant-grid-user demand response based on smart grid platform, discusses the equilibrium game of demand response when supply and demand are relaxed under complete information, and gives the market rules of demand response game. Literature [12] introduced the penalty cost of pollution emission into the traditional economic dispatch model of power generation, constructed the optimal model of contract generation displacement under the minimization of pollution emission cost and generation cost, and used the Shapley model of cooperative game benefit distribution to optimize the profit distribution among all units after cooperation. Literature [13] discussed two game problems in DSM when users equipped with energy storage devices, one of which is non-cooperative game between residents and the other is Stackelberg game between power suppliers and power users. “Through simulation results, the relationship between system performance measured by energy storage capacity, power demand, number of users, total cost and PAR index were elaborated.

As electricity power market participant, the electric users play an important role in the market competition. The rise of smart grid has injected new vitality for the reform of electricity market. The smart grid has brought a more comprehensive system information, promoted information interaction. In mastering each other on the basis of electricity usage, power users can adjust their way in which they use the electricity and also can make the best decision according to the information. Based on smart grid information sharing and power supply and demand interaction, this paper establishes a demand side energy consumption plan decision model based on Nash equilibrium, and analysis the game behavior of power consumption among three types of users.

2. Game Theory

After Game can be described with these elements: N represents the participants, Xi is on behalf of the strategy of participant i, which will be paid for.

Suppose you have n participants to participate in the game, other participants are given a strategy, they choose their own optimal strategy (individual optimal strategy may depend on or may not depend on the strategy of others), so as to maximize their own interests, all participants’ strategy constitute a (Strategy Profile) Nash equilibrium refers to such a Strategy Profile, the Strategy Profile is composed of all participants in the optimal strategy. No one have enough reasons to break this equilibrium, when other participants are also given strategies, so it can be substantially defined as a state of non-cooperative game.

Assuming a game with N participants, each player's strategy is represented with vector Xi. All players’ strategy combination is represented with vector, represents the set of participants i, represents the payoff function, represents the set of collective strategies:

\[ x = (x_1, x_2, \ldots, x_n) \quad \text{and} \quad y = (y_1, y_2, \ldots, y_n) \]

are all the elements from collective strategies is one element of the whole set of collective strategies, which means only participant I changes his strategy without other participant’s strategy changed. Point is called Nash Equilibrium Point, if this following function can be satisfied for participant i:

\[ \phi_i(x^*) = \max_{x_i \in x^*} \phi_i(x_i | x^*) \]  

3. POWER CONSUMPTION PLAN DECISION GAME MODEL

After Market-oriented pricing implementation, the cost of power production is a function of the total power load and power load is decided by individual users (actor). Therefore, users of electricity energy consumption behavior have important influence to the market price. Regardless of the situations of power generation side competition, price will be decided jointly by all electricity users.

Reference [9], the relationship between the electricity consumption of participants in each period and the system electricity price can be expressed by formula (2)-(5).
The cost function of power production is expressed as the quadratic cost function of thermal power unit:

$$C_h(L_h) = a_h L_h^2 + b_h L_h + c_h$$  \hspace{1cm} (2)$$

The total electric quantity $L_h$ of all users in the h period is the sum of the electric quantity $l_h$ of each user in the h period.

$$L_h = \sum_{n \in N} l_h^n$$ \hspace{1cm} (3)$$

Power companies charge power users for their electricity in proportion to the cost of power supply, this proportion is represented with $k$. When $k$ is equal to 1, the power company will break even, and when $k>1$, the power company will have a surplus.

$$k = \frac{\sum_{n \in N} B_n}{\sum_{h=1}^{H} \sum_{l=1}^{L} C_l} \geq 1$$ \hspace{1cm} (4)$$

$B_n$ represents the electricity charges of power users n. Based on the definition of the revenue and expenditure ratio of power companies, $p_h$ value of electricity price in period h can be expressed as the ratio of the total electricity charges of each user in that period to the total electricity consumption.

$$p_h = \frac{kC_h \left( \sum_{n \in N} l_h^n \right)}{\sum_{n \in N} l_h^n}$$ \hspace{1cm} (5)$$

3.1 Flexible adjustable load (participant 1) model

This kind of load changes over time and fluctuate obviously, residential load and commercial load have the same characteristics, its specific performance is the load can increase or decrease and be transferable in transferable. In this way, the load adjustment should increase the energy storage equipment during the trough period, and there will be some energy loss. The payment function (income) of participant 1 can be obtained.

$$R_1 = \sum_{h=1}^{H} p_h l_h^b - \sum_{h=1}^{H} \sum_{l=1}^{L} K^{h,l} \left| s_{p}^{l,h} \right|$$ \hspace{1cm} (6)$$

$$P_h = a_h l_h^2 + b_h l_h^b$$ \hspace{1cm} (7)$$

$$a_1 < 0, b_1 > 0$$

$$S_{p}^{l,h} = -S_{p}^{l,h}$$ \hspace{1cm} (8)$$

$$l_h = l_h^b + \sum_{i=1}^{H} S_{p}^{i,h}$$ \hspace{1cm} (9)$$

3.2 Flexible non-adjustable load (participant 1) model

Taking industrial load as the representative, industrial production is generally continuous production, and the load curve is relatively flat with less transferable load. The electricity production function:

$$R_2 = KF - \sum_{h=1}^{H} p_h l_h^2 - M \sum_{h=1}^{H} l_h^b$$ \hspace{1cm} (10)$$

$$Q_b = \sum_{h=1}^{H} l_h^b$$ \hspace{1cm} (11)$$

$$F = a_2 Q_b^2 + b_2 Q_b + c_2$$ \hspace{1cm} (12)$$

$l_h^b$ on behalf of the participants 2 in the h period of electricity consumption, it’s the decision variable of participants2. K is price of production, $Q_2$ is the electricity of participant 2, F is the output, presented by quadratic function of electricity consumption, $a_2$, $b_2$, $c_2$ are the yield function
coefficient. Assuming that other production factors and electric quantity are input in proportion, \( M \) is the ratio of cost and electric quantity of other production factors.

3.3 Energy storage load model (participant 3)

With the popularization of electric vehicles and so on, battery loads account for an increasing proportion in the system. Charging stations will play an important role in the game of demand side, and such load transfer will not affect the production and operation of enterprises, so adjustment costs need not be considered. This kind of payoff matrix (profit) which demand side can autonomously adjust the load of participant 3 is:

\[
R_3 = \sum_{h=1}^{H} \sum_{l=1}^{L} P_h l_h
\]

(13)

Literature [14] proved the existence of Nash equilibrium solution of the quasi concave n-person game. In the respective strategy space, the payment function of participant 1 and participant 2 is strictly concave, and the payment function of participant 3 is quasi-concave. According to the Nash equilibrium existence theorem, the game must have Nash equilibrium solution we build a user game decision model to maximize the payment function of each participant.

\[
\text{max} \sum_{l=1}^{L} \sum_{h=1}^{H} P_h l_h
\]

\[
\text{s.t.}
\]

\[
L^h \leq L_{\max} (h=1,2,\ldots,H) \quad (C1)
\]

\[
-s_{\max} \leq S_p \leq S_{\max} \quad (C2)
\]

\[
l_{2\min} \leq l_{2\max} \leq l_{2\max} \quad (C3)
\]

\[
l_3^h \leq l_{3\max} (h=1,2,\ldots,H) \quad (C4)
\]

\[
E_{\min} \leq \sum_{h=1}^{H} E_{\max} \quad (C5)
\]

(14)

All participants need to satisfy their own and common constraints when making decisions, Constraint \( C_1 \) is the system capacity constraint and is the common constraint of all participants. \( L_{\max} \) is the system capacity; constraint \( C_2 \) is the power transfer constraint of participant 1, \( S_{\max} \) is the maximum of transferable electricity of participants. The constraint \( C_3 \) is the power consumption constraint of participant 2, \( l_{2\min}, l_{2\max} \) are the minimum and maximum electricity consumption of participant 2. \( C_4 \) is the power constraint of participant 3, and \( l_{3\max} \) is the maximum charging power of participant 3, constraint \( C_5 \) is the power consumption constraint of participant 3, \( E_{\min}, E_{\max} \) are the participant 3 energy storage equipment to meet the minimum remaining power and capacity. Because the benefits of each user are mutually restricted, it is impossible to make the benefits of each user reach the absolute maximum at the same time.

4. ALGORITHM

The Nikaido Isoda function is introduced to calculate the Nash equilibrium and transform the equilibrium problem into an optimization problem. The Nikaido-Isoda function definition is below:

\[
\psi(x,y) = \sum_{i=1}^{n} \left[ \phi_i(y_i) - \phi_i(x) \right]
\]

(15)

Each item that fulfils the sum \((x,y) = 0\) represents an improvement in the payment set by one participant in the other participant's strategy, and represents an improvement in the overall payment. Notice that the maximum value of the function is non-negative, for a given \( x \), and at the Nash equilibrium point, the maximum value of the function is non-positive, because no participant has the ability to improve their payments so at the equilibrium point \( x^* \), the maximum value of each of the terms of the sum is zero. Therefore, the optimal response function is introduced when Max intervals \( \max \psi(x^*,y) = 0 \) are satisfied, which is the result when all participants strive to improve their payment to maximize the Nikaido Isoda function. At point \( x \), the optimal response function is defined as follows.

\[
Z(x) = \text{arg} \max \psi(x,y), \quad x,Z(x) \in X
\]

(16)

relaxation algorithm adopted
\[ x^{k+1} = (1 - \alpha^k) x^k + \alpha^k Z(x^k) \]  \hfill (17)

In that formula, step \( 0 < \alpha^k \leq 1 \). The step length can be the Uber length (the step length that minimizes the maximum value of Nikaido-Isoda function), or the fixed step length can be adopted, usually taking \( \alpha^k = 0.5 \). Literature [15] proved the convergence of the algorithm under the condition that Nikaido Isoda function meets the weak concavo-convex conditions. Literature [16] compares the Uber length with the fixed step length. It is found that although the Uber length is improved in the convergence speed, the calculation time is costly. A good calculation effect is obtained by taking a fixed step length of 0.5, which is also adopted in this paper. The process of this algorithm is shown in figure 2.

![Figure 1. The flowchart of algorithm](image)

5. CALCULATION CASE AND ANALYSIS

The original load of participant 1 is listed in table 1, and the system capacity is 15000MW. The information system in smart grid is open and adopts the mode of information sharing, therefore, it is assumed that the game information below is complete information and common knowledge.

| Time slot | MW  | Time slot | MW  |
|-----------|-----|-----------|-----|
| 1         | 1090| 13        | 1136|
| 2         | 1079| 14        | 1129|
| 3         | 1074| 15        | 1160|
| 4         | 1075| 16        | 1174|
| 5         | 1075| 17        | 1189|
| 6         | 1117| 18        | 1237|
| 7         | 1157| 19        | 1250|
| 8         | 1170| 20        | 1224|
| 9         | 1186| 21        | 1200|
| 10        | 1200| 22        | 1165|
| 11        | 1205| 23        | 1131|
| 12        | 1208| 24        | 1099|

The load trough time of participant 1 was from 23:00-6:00 and 13:00-14:00, and the load peak time was from 9:00-12:00 and 16:00-21:00. Appendix table 1 lists the load transfer of participant 1, after the game, in addition to the power transfer, since the utility generated by the increase of power consumption is higher than the corresponding power consumption cost, participant 1 actively increased the power consumption in each period to represent the transferred load in figure 3, which
can be represented by the load transfer in the above-mentioned period. As shown in figure 4, after each of the participants in the game, player 1 load of happened in the process of game, as a rational actor, can maximize their own interests to make energy consumption plans, according to the electricity price and load, the relationship between the participants will reduce peak power, or by transferring power to lower prices, the peak load and improve their utilization efficiency.

Figure 2. Participant 1 load transfer
Figure 3. The load of Participant1

Figure 5 shows the interest balance process in the game of each participant. As shown in the figure, the interests of each participant can converge rapidly during the game.

As figure 6 shows, after the game, the system overall load curve is more smooth, system load rate is higher, not only raise the load rate, also reduces the production cost, has played a very good peak shaving effect Players 1 and player 3 have strong ability of load adjustment, in combination of the two, the system load rate has reached a higher level Figure 7 shows the system electricity price changes before and after the game, because all participants to adjust the electric energy consumption plan, after the game system of electricity price also along with the load more flatten out.

Figure 4. Participant game equalization process
6. CONCLUSION
Based on the game theory and analysis method, this paper studies the cooperative competition among related stakeholders and establishes a demand side user energy consumption plan decision model based on Nash equilibrium solution. The results show that the game among power users makes the load curve of each users and the system load curve become smooth, so that the system load rate is improved and the production cost is reduced. The game of demand side causes the competition of power users. The competition of power users not only improves their own benefits, but also improves the production efficiency of the system.

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