Quantum Tunnelling of non-zero mass particle and evolution of the horizon for a static black hole

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Abstract

In this work, we study Hawking radiation from a general static black hole due to tunnelling of particle having nonzero mass. Hawking temperature has been calculated using both the tunnelling method and the Hamilton-Jacobi method and the results do not depend on the mass of the particle. Due to complicated form of the equations involved quantum corrections are evaluated only up to first order and it is possible to interpret the correction term as loop corrections of the back reaction effects in the space time. Finally, modified expressions for entropy and surface gravity has been evaluated and it is found that the leading order correction term to the entropy is not necessarily logarithmic in nature.

Keywords : Hawking Temperature, Quantum correction, Tunnelling.

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1 Introduction

Classically, nothing can escape from a black hole across its event horizon. But in 1974, there was a drastic change in view when Hawking \([1, 2]\) showed that black holes are not totally black, there is a continuous emission of radiation from a black hole identical to black body radiation \([2, 3]\) having temperature \(T = \frac{\kappa}{2\pi}\), \(\kappa\) being the surface gravity of the black hole. Subsequently, there has been lot of attraction to this issue and various approaches has been developed to derive Hawking radiation and corresponding temperature \([3, 4, 5, 6, 7]\). In recent past a semiclassical approach \([8, 9]\) has been formulated considering Hawking radiation as tunnelling phenomena across the horizon. The basic idea behind the semiclassical approaches is to evaluate the imaginary part of the action (\(S\)) for the (classically forbidden) process of \(s\)-wave emission across the horizon and is then related to the Boltzmann’s factor to obtain Hawking radiation. In fact, The studies of black hole radiation can be classified into two groups. The first approach developed by Parikh and Wilczek \([8]\) is based on the heuristic pictures of visualization of the source of radiation as tunnelling and is known as radial null geodesic method. The essence of this method is to calculate the imaginary part of the action for the \(s\)-wave emission using the radial null geodesic equation. This method is limited to massless particles only and is applicable to such coordinate system only in which there is
no singularity across the horizon. The alternative way of looking into this aspect is known as complex paths method. Srinivasan et. al. [10, 11] did the pioneering work for these Hamilton Jacobi(HJ) approach considering emitted particle without self-gravitation and assuming the action satisfies the HJ equation. Tunnelling of both massless and non-zero mass particles are possible in this approach and it is applicable to any coordinate system to describe the black hole.

Most of the studies [12, 13] dealing with Hawking radiation are concentrated to semiclassical analysis and are confined to massless particles. Recently, Banerjee et. al. [14] initiated the calculation of Hawking temperature with quantum correction and have evaluated the modified Hawking temperature, surface gravity and entropy. Also they have interpreted the parameters involved in quantum corrections by loop corrections to backreaction effects in the space time.

In the present work, we consider tunnelling of a nonzero mass particle using both the approaches in the semiclassical approach of Parikh et. al. [8](which if so far applicable for massless particles only) we have evaluated the radial velocity of the wave front of the de Broglie’s wave as spherical wave and evaluated the imaginary part of the action. The semiclassical Hawking temperature is then evaluated in section 2 by comparing with Boltzmann factor. Quantum correction to the Hawking temperature has been evaluated using HJ approach in section 3 and the semiclassical part of the temperature agrees with that in section 2. The evolution of the horizon is studied in section 4. We have shown that the static black hole in the tunnelling phase behaves like Vaidya type black hole and tunnelling of massless and massive particles have identical radial velocity. Finally, in section 5 we have given modified expressions for surface gravity and entropy with quantum corrections. Also the parameters are analysed from the point of view of loop corrections to back reaction effect of the space time. The paper ends with a summary in section 6.

## 2 Semiclassical Tunnelling approach

The section deals with tunnelling of nonzero mass particle considering the picture of Hawking radiation. In fact the method correlates the imaginary part of the action for the classically forbidden process of de Broglie wave emission across the horizon with the Boltzmann factor for the black body radiation at the Hawking temperature.

A general static black hole metric has the form

$$ ds^2 = -A(r)dt^2 + \frac{1}{B(r)}dr^2 + r^2d\Omega_2^2 $$

(1)

where the location of the horizon $r = r_h$ is given by $A(r_h) = 0 = B(r_h)$. To remove the coordinate singularity at the horizon we consider the above metric in Painleve coordinates [15] as

$$ ds^2 = -A(r)dt^2 + 2\sqrt{\frac{A(r)}{B(r)}}(1-B(r))dtdr + dr^2 + r^2d\Omega_2^2 $$

(2)

which is obtained from equation (1) by the transformation

$$ dt \rightarrow dt - \sqrt{\frac{1-B(r)}{A(r)B(r)}}dr $$

(3)

The Painleve coordinates has a number of interesting features, namely (i) the spatial geometry of any $t$=constant hypersurface is flat,
(ii) the boundary geometry for any fixed radius is identical to that of the metric in equation (11) and 
(iii) the metric has time reversal asymmetry.

Unlike the massless particle moving along the null geodesic a non-zero mass particle is treated as a de Broglie wave (s-wave). Further, to preserve the spherical symmetry of the space-time during the tunnelling process, we treat it as a spherical wave with velocity of the wave front as

$$\dot{r} = -\frac{1}{2} \frac{g_{00}}{g_{01}} = \frac{1}{2} \sqrt{\frac{A(r)B(r)}{1 - B(r)}}$$

(4)

Then according to Parikh and Wilczek [8] the imaginary part of the action is obtained as

$$\text{Im}S = \text{Im} \int_{r_{in}}^{r_{out}} p_r dr = \text{Im} \int_{r_{in}}^{r_{out}} \int_{0}^{p_r} dp'_r dr = \text{Im} \int_{r_{in}}^{r_{out}} \left\{ \int_{0}^{H} \frac{dH'}{dr} \right\} dr$$

(5)

where \((r, p_r)\) are canonical pair and Hamilton’s equation \(\dot{r} = \frac{dH}{dp_r} |_{r}\) has been used in the last step.

In ordinary quantum mechanics, the action of a tunnelled particle in a potential barrier having energy larger than the energy of the particle will be imaginary as \(p_r = \sqrt{2m(E-V)}\). For the present static black hole, the tunnelling of a nonzero particle of energy ‘\(\omega\)’ implies \(dH'\) integration over all possible values of energy of outgoing particle from zero to \(\omega\). Further, as we are dealing with tunnelling across the black hole horizon so using Taylor series expansion about the horizon \(r_h\) we write

$$A(r) \simeq \frac{\partial A}{\partial r} |_{r=r_h} (r - r_h) + O (r-r_h)^2 \right\}$$

$$B(r) \simeq \frac{\partial B}{\partial r} |_{r=r_h} (r - r_h) + O (r-r_h)^2 \right\}$$

(6)

So from (4) the velocity of the wave front in the neighbourhood of the horizon can be approximated as

$$\frac{dr}{dt} \simeq \frac{2 \sqrt{A'(r_h)B'(r_h)} (r - r_h)}{2 \sqrt{1 - B'(r_h) (r - r_h) + O (r-r_h)^2}}$$

(7)

where dash denotes differentiation w.r.t. ‘\(r'\).

Using (7) in (5) and evaluating the contour integration over ‘\(r'\) we obtain

$$\text{Im}S = \frac{2\pi\omega}{\sqrt{A'(r_h)B'(r_h)}}$$

(8)

where the contour for \(r\)-integration is chosen on the upper half complex plane to avoid the coordinate singularity at \(r_h\). Thus the tunnelling probability is given by

$$\Gamma \sim \exp \left\{ \frac{-2\pi}{\hbar} \text{Im}S \right\} = \exp \left\{ -\frac{4\pi\omega}{\hbar \sqrt{A'(r_h)B'(r_h)}} \right\}$$

(9)

which in turn matching with the Boltzmann factor \(\exp \left\{ -\frac{\omega}{T} \right\}\), we obtain the Hawking temperature as

$$T_H = \frac{\hbar \sqrt{A'(r_h)B'(r_h)}}{4\pi}$$

(10)

This is the wellknown semiclassical Hawking temperature for tunnelling of a particle across the static black given by equation (11). Note that \(T_H\) is independent of the mass of the particle.
3 Hamilton-Jacobi method: WKB approximation

The HJ method will be employed to consider tunnelling of particle having nonzero mass beyond semiclassical approximation. We shall restrict ourselves to only loading order quantum correction due to complicated form of the equations involved.

The Klein Gordon (KG) equation for a scalar field $\psi$ describing a scalar particle of mass $m_0$ has the form

$$\left(\Box + \frac{m_0^2}{\hbar^2}\right) \psi = 0$$

where the $\Box$ (box) operator is evaluated in the background of the black hole metric. Due to spherical symmetry, without any loss of generality we may restrict ourselves only to $(t, r)$-sector on the space-time, i.e., we fix our attention to two dimensional black hole problem. So the explicit form of the above KG equation is

$$\frac{1}{A} \frac{\partial^2 \psi}{\partial t^2} - B \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{2A} \frac{d}{dr}(AB) \frac{\partial \psi}{\partial r} + \frac{m_0^2}{\hbar^2} \psi(t, r) = 0$$

In W.K.B. approximation, we substitute the standard ansatz for the semiclassical wave-function namely,

$$\psi(t, r) = \exp\left\{-\frac{i}{\hbar} S(r, t)\right\}$$

then the action $S$ will satisfy the following differential equation

$$\left[\frac{1}{A} \left(\frac{\partial S}{\partial t}\right)^2 - B \left(\frac{\partial S}{\partial r}\right)^2 - m_0^2\right] - \frac{\hbar}{i} \left[\frac{1}{A} \frac{\partial^2 S}{\partial t^2} - B \frac{\partial^2 S}{\partial r^2} - \frac{1}{2A} \frac{d}{dr}(AB) \frac{\partial S}{\partial r}\right] = 0$$

Now, quantum corrections are introduced by expanding $S$ in powers of Planck constant $\hbar$ as

$$S(r, t) = S_0(r, t) + \sum_k \hbar^k S_k(r, t)$$

where $S_0$ is the semiclassical action and $k$ is a positive integer. Using this ansatz for $S$ into the differential equation and equating different powers of $\hbar$ we obtain the following set of partial differential equations (p.d.e.) :

$$\hbar^0 : \quad \frac{1}{A} \left(\frac{\partial S_0}{\partial t}\right)^2 - B \left(\frac{\partial S_0}{\partial r}\right)^2 - m_0^2 = 0$$

$$\hbar^1 : \quad \frac{2}{A} \frac{\partial S_0}{\partial t} \frac{\partial S_1}{\partial t} - 2B \frac{\partial S_0}{\partial r} \frac{\partial S_1}{\partial r} + i \left[\frac{1}{A} \frac{\partial^2 S_0}{\partial t^2} - B \frac{\partial^2 S_0}{\partial r^2} - \frac{1}{2A} \frac{d}{dr}(AB) \frac{\partial S_0}{\partial r}\right] = 0$$

$$\hbar^2 : \quad \frac{1}{A} \left(\frac{\partial S_1}{\partial t}\right)^2 + \frac{2}{A} \frac{\partial S_0}{\partial t} \frac{\partial S_2}{\partial t} - B \left(\frac{\partial S_1}{\partial r}\right)^2 - 2B \frac{\partial S_0}{\partial r} \frac{\partial S_2}{\partial r} + i \left[\frac{1}{A} \frac{\partial^2 S_1}{\partial t^2} - B \frac{\partial^2 S_1}{\partial r^2} - \frac{1}{2A} \frac{d}{dr}(AB) \frac{\partial S_1}{\partial r}\right] = 0$$

and so on.

The differential equation for the semiclassical action $S_0$ can be solved by choosing the separable ansatz

$$S_0(r, t) = \omega_0 t + D_0(r)$$
where

\[ D_0(r) = \pm \int_0^r \sqrt{\frac{\omega_0^2 - A m_0^2}{A B}} \, dr = \pm I_0 \text{ (say)} \]  

(20)

Here +/− sign corresponds to ingoing/outgoing scalar particle substituting this choice for \( S_0 \) into equation (17). We have the differential equation for the first order correction \( S_1 \) as

\[ \omega_0 \frac{\partial S_1}{\partial t} \mp \sqrt{A B (\omega_0^2 - m_0^2 A)} \frac{\partial S_0}{\partial r} \pm \frac{1}{i} \frac{m_0^2}{4} \sqrt{\frac{A B}{\omega_0^2 - m_0^2 A}} \, dr \]  

(21)

using similar ansatz for \( S_1 \) namely,

\[ S_1(r, t) = \omega_1 t + D_1(r) \]  

(22)

we have from equation (21)

\[ D_1(r) = \pm \omega_0 \omega_1 \int \frac{dr}{\sqrt{A B (\omega_0^2 - m_0^2 A)}} - \frac{i}{4} \ln |\omega^2 - m_0^2 A| \]  

(23)

Due to complicated form, retaining terms up to first order quantum corrections, i.e.,

\[ S = S_0 + \hbar S_1 = (\omega_0 + \hbar \omega_1) t + \{ D_0(r) + \hbar D_1(r) \} \]  

(24)

Thus the wave functions denoting ingoing and outgoing solutions of the KG equation (12) using (13) and (24) are of the form

\[ \psi_{in} = \exp \left[ -\frac{i}{\hbar} \left\{ (\omega_0 + \hbar \omega_1) t + I_0 + \hbar I_1 - \frac{i}{4} \ln |\omega^2 - m_0^2 A| \right\} \right] \]  

(25)

\[ \psi_{out} = \exp \left[ -\frac{i}{\hbar} \left\{ (\omega_0 + \hbar \omega_1) t - I_0 - \hbar I_1 - \frac{i}{4} \ln |\omega^2 - m_0^2 A| \right\} \right] \]  

(26)

As during tunnelling across the horizon \((r, t)\) changes their coordinate nature so to overcome this difficulty we take into consideration that the time coordinate has an imaginary part in crossing the horizon and accordingly the temporal part has contribution to the probabilities \[14, 17\]. Thus the incoming and outgoing probabilities are given by

\[ P_{in} = |\psi_{in}|^2 = \exp \left[ \frac{2}{\hbar} \left\{ \text{Im} [(\omega_0 + \hbar \omega_1) t] + \text{Im} [I_0 + \hbar I_1] - \text{Im} \left[ \frac{i}{4} \ln |\omega^2 - m_0^2 A| \right] \right\} \right] \]  

(27)

and

\[ P_{out} = |\psi_{out}|^2 = \exp \left[ \frac{2}{\hbar} \left\{ \text{Im} [(\omega_0 + \hbar \omega_1) t] - \text{Im} [I_0 + \hbar I_1] - \text{Im} \left[ \frac{i}{4} \ln |\omega^2 - m_0^2 A| \right] \right\} \right] \]  

(28)

As in the classical limit the horizon behaves as a one way membrane, no barrier for incoming particles so in the limit, i.e.,

\[ \lim_{\hbar \to 0} P_{in} = 1 \]  

(29)

Hence from (27) we have

\[ \text{Im} (\omega_0 t) = \text{Im} I_0 \]  

and

\[ \text{Im} (\omega_1 t + I_1) = \text{Im} \left[ \frac{i}{4} \ln |\omega^2 - m_0^2 A| \right] \]  

(30)
As a result $P_{out}$ turns out to be in simple form as

$$
P_{out} = \exp \left\{ \frac{-\frac{1}{2} I m (J_0 + h I_1)}{\hbar} \right\}
$$

$$
= \exp \left\{ \frac{-4\omega_0 I m}{\hbar} \int_0^r \sqrt{\frac{1 - \frac{\Lambda m^2}{r^2}}{AB}} \, dr + \frac{\hbar \omega_1}{\omega_0} \int_0^r \frac{dr}{\sqrt{AB \left(1 - \frac{\Lambda m^2}{r^2}\right)}} \right\}
$$

$$
= \exp \left\{ \frac{-4\omega_0}{\hbar} \left(1 + \frac{4\omega_1}{\omega_0} \right) \sqrt{\frac{\pi}{A'(r_h)B'(r_h)}} \right\}
$$

So from the “detailed balance” principle \[10, 11, 18\] namely

$$
P_{out} = \exp \left\{ \frac{-E}{T} \right\} P_{in} = \exp \left\{ -\frac{E}{T} \right\}
$$

the temperature of the black hole is given by

$$
T_B = \frac{\hbar}{4\pi} \frac{E}{\omega_0} \left[ \left(1 + \frac{\hbar \omega_1}{\omega_0} \right) \frac{1}{\sqrt{A'(r_h)B'(r_h)}} \right]^{-1}
$$

$$
= \left(1 + \frac{\hbar \omega_1}{\omega_0} \right)^{-1} T_H
$$

where energy of the radiated particle is taken to be $\omega_0$ (i.e., $E = \omega_0$). Thus the temperature of radiation does not depend on the radiated particle at least upto the first order quantum correction and it agrees with the claim in the literature.

4 Evolution of the horizon during tunnelling

At the beginning we try to address the question: “Do static black holes evolve to Vaidya black hole during Hawking radiation?”

To have a possible answer to this question we start with a general static spherically symmetric black hole metric

$$
ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2d\Omega_2^2
$$

where $f(r)$ without any loss of generality can be chosen as

$$
f(r) = 1 - \frac{2M(r)}{r}
$$

so that at the horizon $r = r_h$, $f(r_h) = 0$ and we have

$$
2M(r_h) = r_h
$$

As before in Painleve coordinates (i.e., $dt \rightarrow dt - \sqrt{1 - f(r)} \, dr$) the metric \([33]\) can be written in singularity free form as

$$
ds^2 = -f(r)dt^2 + 2\sqrt{1 - f(r)}dt \, dr + dr^2 + r^2d\Omega_2^2
$$
Now to match with the Vaidya black hole we make another transformation similar to the ingoing Eddington-Finkelstein type coordinates \((v, r, \theta, \phi)\) with

\[
v = t + \int \frac{dr}{1 + \sqrt{1 - f(r)}}.
\]

(39)

Accordingly, the metric \(ds^2\) becomes

\[
ds^2 = -f(r)dv^2 + 2dvdr + dr^2 d\Omega^2_2
\]

(40)

If \(M\) in equation (36) is now a function of \(v\) then equation (40), which is the line element of the static black hole, can be identified as the Vaidya type black hole metric.

Basically, the tunnelling process described by Parikh and Wilczek [8] is the image of that originally described by Hawking. According to Hawking [1], a virtual particle pair is created just outside the horizon due to vacuum fluctuations. The negative energy virtual particle, which is forbidden in the exterior to the black hole, tunnels to the interior while the positive energy particle materializes as a real particle and escape to infinity. On the other hand, Parikh and Wilczek considered pair creation inside the event horizon and the positive energy virtual particle tunnels out to the exterior. However in both the processes the positive energy virtual particle becomes real one outside the black hole and seems to be radiated (Hawking radiation) from the black hole while the negative energy particle is absorbed by the black hole. As a result, there is a decrease in the mass of the black hole and consequently, the black hole will shrink due to the effect of self-gravitation. The barrier in the tunnelling process is set due to contraction of the event horizon or the self gravity effect of the tunnelling particle and the two turning points can be identified as

\[
t_i = r_h_i \quad \text{and} \quad t_f = r_h_f
\]

(41)

where \(r_h_i\) and \(r_h_f\) are the positions of the event horizon before and after the tunnelling.

For example in case of Schwarzschild black hole

\[
t_i = r_h_i = 2M \quad \text{and} \quad t_f = r_h_f = 2(M - \omega)
\]

(42)

where \(\omega\) is the mass (or energy) of the tunnel particle.

Thus in course of tunnelling, the static black hole becomes a dynamic one and then they can be identified as Vaidya type black hole given in equation (40).

We shall now show that the radial velocity of the escaping particle does not depend on its mass- it is same for both massless or non-zero mass particle. A massless particle moves along a radial null geodesic which is characterized by \(ds^2 = d\theta = d\phi = 0\), the velocity of the wave front is given by

\[
\frac{dr}{dv} = \frac{1}{2} f(r)
\]

(43)

As a nonzero mass particle moves along a time-like geodesic so we can treat it as a de Broglie wave. Further, to maintain the spherical symmetry of the space time in course of the tunnelling process, we assume it as a spherical wave and consequently, the velocity of the wave front of the de Broglie s-wave is

\[
\frac{dr}{dv} = -\frac{1}{2} \frac{g_{00}}{g_{01}} = \frac{1}{2} f(r)
\]

(44)

Hence the velocities of the wave fronts for the massless or massive particles are identical.
Further, due to self gravitation, the mass \( M \) in \( f(r) \) given by equation (36) corresponding to the Vadya metric (40) will become \( M - \omega \) where a particle with a mass of \( \omega \) tunnels out the horizon. Hence the rate of decrease of the horizon is given by

\[
\dot{r}_h = -\dot{r}_{\mid r=2m} = -\frac{1}{2} f(v, M - \omega) = -\frac{1}{2} \left\{ 1 - \frac{2(M(v) - \omega)}{r} \right\} \tag{45}
\]

Thus in the original position of the event horizon (i.e., \( r = r_h \)) the shrinking velocity of the event horizon is

\[
\dot{r}_h = -\dot{r}_{\mid r=2m} = -\frac{\omega}{2M(v)} = -\frac{\omega}{r_h} \tag{46}
\]

But for Vaidya black hole given by equation (10), the local event horizon (a null hypersurface, preserving the symmetry of the space time) is given by [19]

\[
r_h = \frac{2M(v)}{1 - 2\dot{r}_h} \tag{47}
\]

So using (46) the position of the event horizon (after tunnelling) is given by (neglecting square and higher powers of \( \omega \) in comparison to \( M \))

\[
r_h = \frac{2M(v)}{1 + \frac{\omega}{M}} \simeq 2(M - \omega) = r(1 - f(r)) - 2\omega \tag{48}
\]

Thus we have \( t_i = r_h \) and \( t_f = r_h - 2\omega \) are the two turning points of the tunnelling process and the contraction velocity of the event horizon is given by (46).

5 Modified Surface gravity and entropy of the black hole

In this section we derive expressions for surface gravity and entropy of the black hole considering to leading order quantum correction for the Hawking temperature. Using the standard relation between the surface gravity (\( \kappa_c \)) and the Hawking temperature \( T_H \), i.e.,

\[
T_H = \frac{\kappa_c}{2\pi}
\]

the quantum corrected surface gravity is given by

\[
\kappa_Q = 2\pi T_B = \kappa_c \left( 1 + \frac{\hbar \omega_1}{\omega_0} \right)^{-1} \tag{49}
\]

Assuming

\[
\frac{\omega_1}{\omega_0} = \frac{\epsilon}{M^2} \tag{50}
\]

where \( \epsilon \) is a small dimensionless parameter then if we retain terms upto first order in \( \epsilon \) then from (49)

\[
\kappa_Q = \kappa_c \left( 1 - \frac{\epsilon \hbar}{M^2} \right) \tag{51}
\]

According to ref. [14, 20] the above expression can be considered as the one loop back reaction effects in the space time. Also the dimensionless parameter \( \epsilon \) can be related to the trace anomaly [14].
Further, if in the expansion in equation (49) we retain terms up to second order in $\epsilon$ then we get
\[
\kappa_Q = \kappa_c \left( 1 - \frac{\epsilon h}{M^2} + \frac{\epsilon^2 h^2}{M^4} \right)
\]
(52)

This is nothing but the correlated form of the surface gravity of a black hole due to two loop back reaction effects in the space time and $\epsilon^2$ is related to the contribution from the second loop. Therefore, expanding in higher order terms one can reproduce other higher order loop back reaction effects in the space time.

The usual Bekenstein-Hawking area law
\[
S_c = \frac{A}{4\hbar}
\]
(53)

where $A$, the area of the horizon, is valid at the semiclassical level. To find the modified form of the entropy when quantum corrections are taken into consideration, we take care of the first law of thermodynamics. Essentially, it is an energy conservation relation in which a change in the black hole mass $M$ is related to the change in its entropy by the relation (assuming uncharged and nonrotating black hole)
\[
dM = T_B dS_B
\]
(54)
i.e.,
\[
S_B = \int \frac{dM}{T_B} = \int \frac{dM \left( 1 + \frac{\epsilon h}{\omega_0} \right)}{T_H} \left( 1 + \frac{\epsilon h}{\omega_0} \right)
\]
(55)

where $T_H$ is given by equation (10). Writing explicitly we have
\[
S_B = \int \frac{dM}{T_H} + \hbar \int \left( \frac{\omega_1}{\omega_0} \right) \frac{dM}{T_H} = S_c + \epsilon \hbar \int \frac{dM}{T_H M^2}
\]
(56)

Note that the second term on the r.h.s. of equation (56) which is the leading order correction term cannot always give rise to the logarithmic correction -- it depends on the nature of the derivative of the metric coefficients at the horizon. Therefore, in general for a static black hole the leading order quantum correction may not give rise to a logarithmic form.

### 6 Summary

In this work, we consider tunnelling of a non-zero mass particle across the horizon of a general static black hole. First of all, we have used the semiclassical tunnelling approach of Parikh and Wilczek to determine the Hawking temperature. Normally, this approach is used for zero-mass particle travelling along radial null geodesic and hence this method is also known as radial null geodesic approach. In section 2 we have used the tunnelling method for no zero mass particle where we have treated the particle as a de Broglie wave and the radial velocity of the wave front is evaluated considering it as a spherical wave. Then HJ method is used to incorporate quantum corrections. Due to complicated form of the equations involved we restrict ourselves to the leading order quantum corrections only. The results do not depend on the mass of the tunnelling particle and the semiclassical Hawking temperature agrees with the result of the previous section. Subsequently, we have shown that all static black holes with $g_{00} = g_{rr}^{-1}$, otherwise general become Vaidya-type black hole during tunnelling process and we have shown that the radial velocity of the wave front corresponding to particle of zero mass or non-zero mass are identical. Finally, we have evaluated modified surface gravity and entropy of the black hole with quantum correction. We have seen
that for general static black hole first order quantum correction may not be logarithmic in nature. Also with the proper choice of the arbitrary parameters involved it is possible to show different order loop corrections of the back reaction effects in the space-time and the dimensionless parameter may be related to the trace anomaly.

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