Research Article

Nonlinear Effect of Urbanization on the Gap between Urban and Rural Elementary Education in China

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The urban-rural gap of elementary education seriously affects social fairness, so the study on the urban-rural gap of elementary education can help promote social fairness in China. China’s urbanization is an important variable affecting the social process, and its impact on the urban-rural gap of elementary education is worthy of further study. Based on China’s provincial panel data from 2006 to 2017, this paper uses the Theil index to measure the urban-rural gap in different aspects of elementary education and uses principal component analysis (PCA) to construct a comprehensive index to objectively measure the urban-rural gap of overall elementary education in China. Our results show that the gap between urban and rural areas in China’s elementary education showed an upward trend from 2006 to 2010, and the gap decreased year by year from 2010 to 2017. Then we used the panel smooth transition regression (PSTR) model to study the impact of urbanization on the urban-rural gap of elementary education in China and find evidence that urbanization has a nonlinear effect on the urban-rural gap of elementary education. That is, in economically underdeveloped areas, urbanization exacerbates the gap between urban and rural areas in elementary education, while in economically developed areas, urbanization narrows the gap between urban and rural areas in elementary education. Therefore, developing economy and continuing to promote urbanization are effective measures to narrow the gap between urban and rural areas in elementary education.

1. Introduction

Education is not only the main driving force of economic development but also the main way for individuals to become talents [1, 2]. Fairness and justice are important parts of the fundamental values of modern society. Fairness in elementary education is an important aspect of fairness and justice. It is the cornerstone of social fairness and justice [3]. It is also a fundamental measure to eliminate intergenerational transmission of poverty and cultivate human capital for future. Although different countries have different national conditions, children’s priority and social equity are commonly accepted [4].

However, due to historical reasons, China’s urban-rural dual structure has existed for a long time. The wealth gap between urban and rural areas in China is severe. China’s financial investment in education is obviously biased towards cities and towns [4, 5]. A large number of institutional inequalities have resulted in an obvious gap between urban and rural areas in terms of teachers and school-running conditions of elementary education in China. This issue has aroused heated discussion among people from all walks of life in the society and has also aroused great attention from the Chinese government. As an irresistible trend, urbanization can fundamentally disintegrate the urban-rural dual structure and promote China’s educational equity and social progress. However, there are great differences in economy and culture among Chinese provinces. Then, what is the current situation of the urban-rural gap of elementary education across China? Does urbanization in different regions help to narrow the gap between urban and rural areas in China’s elementary education? Is there any difference in the degree of the impact? This paper will study these questions. The answers to the above questions will help solve the problem of fairness in elementary education in China.
2. Literature Review and Institutional Background

Due to the restriction of the household registration system, before the reform and opening up, China’s urban and rural areas were two relatively separate systems. Like many developing countries, China adopted the “city bias” strategy to accumulate capital and gave priority to the development of heavy industry, which led to a widening gap between urban and rural areas [6–8]. At the same time, given the huge economic gap between urban and rural areas, China implemented the policy of “urban education paid by the government and rural education paid by the countryside,” resulting in a huge gap between urban and rural elementary education [4, 5]. Although China achieved free nine-year compulsory education in 2006, the hierarchical school-running system was retained. Emi [9] believed that China’s hierarchical school-running system handed over the main responsibility of compulsory education to grass-roots governments and families, and the huge resource gap between urban and rural areas in China led to a huge gap in elementary education. The gap resulted in the inflow of rural talents into cities and towns, and the urban-rural gap in China’s elementary education got widened [10–12].

Zhang and Shi [11] compared the data of Chinese families’ investment in elementary education and found that urban families not only had higher economic investment than rural families but also had more frequent educational participation behaviors than rural families. Previous studies also found similar findings [13, 14]. Zhao [15] found that the gap between urban and rural financial expenditure on elementary education in China showed a narrowing trend from 1997 to 2005, while the inequality of expenditure on primary education decreased much faster than that of junior secondary education. Zhai and Sun [16] constructed an educational equilibrium index of elementary education. The results showed that although there was a gap between urban and rural areas in China’s elementary education, the gap tended to narrow. Jiang and Luo [17] used the Theil index to investigate the urban-rural gap of compulsory education in financial funds, high-quality teachers, and hardware equipment. It was found that the urban-rural gap of financial funds first increased and then decreased, the urban-rural gap of teachers decreased, and the urban-rural gap of hardware equipment increased. Wen and Jiang [18] used the bivariate Theil index to study and found that the overall gap in compulsory education in China did not narrow but showed an expanding trend.

Urbanization could eliminate the urban-rural dual structure and reduce the gap between urban and rural areas in elementary education. Tian and Wang [2] analyzed the impact of urbanization on the investment gap between urban and rural compulsory education from the three dimensions such as population, economy, and land by using GMM of dynamic panel data. They pointed out that urbanization can significantly reduce the investment gap between urban and rural elementary education. Chen and Shi [19] showed that urbanization had a significant impact on narrowing the gap between urban and rural elementary education. Zhao et al. [20] also confirmed this conclusion. However, Zhang [21] showed that due to the restriction of compulsory education system caused by the household registration system, the children of migrant workers who cannot obtain urban household registration are prone to fall into the new education-poverty trap even if they live in cities. Rao and Ye [4] sorted out the history of China’s rural education system from ancient times to the present, and their conclusion was that the Chinese government should strive to develop a rural-oriented and rights-based rural elementary education system.

The previous literature provided a good basis for us to explore the gap between urban and rural elementary education, but there were still some areas where improvements can be made. Firstly, in the past, the gap between urban and rural education was usually measured by single indicators, which contained less information. At the same time, they usually do not take into account changes in the proportion of the population in urban and rural areas. Therefore, we used the Theil index to measure the urban–rural gap of eight types of elementary education resources in three aspects in China. Using the principal component analysis (PCA) method to reduce dimension, we constructed a comprehensive score of elementary education gap between urban and rural areas. Secondly, previous studies mostly adopted linear models, without considering the heterogeneous impact of urbanization and other influencing factors on the urban-rural gap of elementary education. Therefore, we used the panel smooth transition regression model (PSTR) [22, 23] to study the nonlinear effects of urbanization and other factors on the urban-rural gap of elementary education in regions with different economic development levels.

3. Empirical Methods

3.1. Theil Index. Taking the urban and rural population of each region as the weight, the Theil index measuring the urban-rural gap of elementary education can be defined as

\[
\text{Theil}_t = \frac{Y_{ut}}{Y_t} \ln \left( \frac{Y_{ut}/Y_t}{P_{ut}/P_t} \right) + \frac{Y_{rt}}{Y_t} \ln \left( \frac{Y_{rt}/Y_t}{P_{rt}/P_t} \right),
\]

where Theil, is the Theil index of year t, \( Y_{ut} \) is the amount of elementary education resources enjoyed by urban students in year t, \( Y_{rt} \) is the amount of elementary education resources enjoyed by rural students in year t, \( Y_t \) is the total amount of elementary education resources in urban and rural areas in year t, \( P_{ut} \) is the number of urban students receiving elementary education in year t, and \( P_{rt} \) is the total number of rural students receiving elementary education in year t. The Theil index takes into account both the changes in the absolute amount of urban and rural elementary education resources and the changes in the demographic structure of urban and rural areas. The larger the Theil index, the greater the gap between urban and rural elementary education.
3.2. Panel Smooth Transition Regression Model (PSTR). In order to obtain the nonlinear effect of the explanatory variable on the explained variable, we need to use a non-linear econometric model instead of the traditional linear econometric model. Nonlinear econometric models include nonparametric/semiparametric models, threshold regression models, and smooth transition regression models. In the threshold regression model, the effect of the explanatory variable on the explained variable changes in a jumping manner before and after the threshold value, which does not conform to reality. The nonparametric/semiparametric models require large sample sizes. Therefore, we use a panel smooth transition regression model to study the nonlinear effect of urbanization on the urban-rural gap of elementary education.

The panel smooth transition regression model can be expressed as

\[ y_{it} = \mu_i + \beta_0 x_{it} + \sum_{j=1}^{r} \beta'_j x_{it} g_j(q_{jt}; \gamma, \epsilon_j) + \epsilon_{it}, \]  

where \( i = 1, \ldots, N \) and \( t = 1, \ldots, T \) represent the cross-section and time of panel data, respectively, and where \( y_{it} \) is the explained variable, \( x_{it} \) denotes a \( k \)-dimensional vector of explanatory variable, i.e., \( x_{it} = (x_{i1t}, \ldots, x_{ikt})' \). Then, \( \beta_0 = (\beta_{01}, \ldots, \beta_{0k}) \) and \( \beta_j = (\beta_{j1}, \ldots, \beta_{jk}) \) are the parameter vectors of the linear and nonlinear parts of the explanatory variable, respectively. \( \mu_i \) is the individual effect, and \( \epsilon_{it} \) represents the random error. The given error term \( (\epsilon) \) is assumed to be normally distributed at zero mean value and constant variance [24]. The transition function takes the form of the following logic function:

\[ g(q_{jt}; \gamma, \epsilon) = \left( 1 + \exp(-\gamma(q_{jt} - \epsilon)) \right)^{-1}. \]  

Therefore, equation (3) is a continuous function with a value range of \((0,1)\). \( \gamma > 0 \) determines the slope of the transition function, and \( \epsilon_1 \leq \cdots \leq \epsilon_m \) are the location parameters at which the model switches, generally \( m \) takes 1 or 2. \( q_{jt} \) is called the transition variable, which can be the lag term of the explained variable or the function of one or several elements of the explanatory variable vector.

A nonlinearity test is required before modeling with a PSTR model. When the nonlinearity is confirmed, the grid search method can be used to select the initial values of the slope parameter and the location parameters in the transition function. Then the nonlinear least squares method is used to estimate the coefficient vector of the explanatory variables.

3.3. Data Sources and Variable Description

3.3.1. Data Sources. In view of the availability of data, this paper adopts the data of 30 provinces/municipalities/autonomous regions in Chinese mainland except Tibet from 2006 to 2017. The data used in this paper are from Educational Statistics Yearbook of China (2007–2018), China Educational Finance Statistical Yearbook (2007–2018), and China Statistical Yearbook (2007–2018). Specifically, the relevant data of the explained variable are from Educational Statistics Yearbook of China and China Educational Finance Statistical Yearbook. The data of other variables are from China Statistical Yearbook. These statistical yearbooks can be accessed from the website of CNKI (https://data-cnki-net.s.vpn3.fjnu.edu.cn/Yearbook/Navi?type=type&code=A). All nominal variables are adjusted to real variables based on 2006.

3.3.2. Urban-Rural Gap of Elementary Education (EG). Referring to the existing literature, the gap between urban and rural elementary education is mainly reflected in the quality of teachers, school-running conditions, and education investment. Specifically, this paper analyzes the gap between urban and rural elementary education in China by using eight subindicators of three aspects: teachers (teacher-student ratio and well-educated teacher-student ratio), school-running conditions (number of books, number of computers, fixed assets, and playground area) and financial investment (education expenditure per student and public budget education expenditure per student). We first calculate the Theil index of eight subindicators and then synthesize them into a comprehensive index by using the principal component score method.

Using the principal component analysis method, the cumulative variance contribution rate of the first three principal components reaches 95.526%, and their variance contribution rates are 64.816%, 21.174%, and 9.535%, respectively. Among them, the first principal component is mainly the synthesis of four subindicators, namely, the number of books per student, the number of computers per student, the total value of fixed assets per student, and the area of playground per student. It represents the gap between urban and rural school-running conditions. The second principal component is mainly the combination of education expenditure per student and public budget education expenditure per student. It represents the gap between urban and rural compulsory education investment. The third principal component is mainly the ratio of teacher-student ratio and well-educated teacher-student ratio, which represents the gap between urban and rural teachers. Therefore, we extract the first three principal components, and then weigh each principal component according to the variance contribution rate so as to calculate the comprehensive score of the urban-rural gap of elementary education in each province.

Because the principal component comprehensive score of the urban-rural gap of elementary education has positive and negative values, which is not easy to observe, the Min-Max standardized method is adopted to standardize the comprehensive score of each province from 2006 to 2017. Then, we can get the provincial average gap.
between urban and rural elementary education in China from 2006 to 2017, as shown in Figure 1.

As can be seen from Figure 1, the gap between urban and rural elementary education in China showed an upward trend from 2006 to 2010, and the gap gradually narrowed from 2010 to 2017. In the decade before 2009, China was in the stage of unbalanced development of elementary education. In 2010, China clearly proposed to build an integrated development mechanism of urban and rural compulsory education and to increase compensation for rural areas. Subsequently, a series of relevant policies were issued to promote the integration of urban-rural elementary education. Therefore, the urban-rural gap of compulsory education was gradually narrowed.

3.3.3. Transition Variable. In regions at different levels of economic development, governments and households tend to behave differently. The impact of urbanization on the gap between urban and rural elementary education may be quite different. Therefore, per capita GDP is selected as the transition variable.

3.3.4. Explanatory Variable. We focus on the impact of urbanization (Urb) on the gap between urban and rural elementary education. Referring to the existing literature, the control variables selected in this paper include income gap between urban and rural residents (IG), government spending on education per student (GSE), intensity of family education expenditure (EEI), and local average education level (Edu).

Urbanization (Urb): It is measured by the urbanization rate. In underdeveloped areas, urbanization is accompanied by the net outflow of a large number of rural labor forces. These rural workers contribute to urban development but leave their minor children in the countryside. This results in a net loss of rural wealth and the phenomenon of “taking it from the countryside and giving it to the city,” thus widening the gap between urban and rural areas in elementary education. In areas with high level of economic development, urbanization is accompanied by the net inflow of rural labor force in the local province and other provinces, thus increasing the level of local economic development and financial resources. In these areas, cities can feed the countryside so as to narrow the gap between urban and rural elementary education [25].

Income gap between urban and rural residents (IG): It is measured by the Theil index of urban-rural income. The advantage of this index is that it takes the change of population proportion into account and can dynamically reflect the urban-rural gap. Generally speaking, the income gap between urban and rural residents will increase the urban-rural gap of elementary education. The higher the level of economic development, the more obvious this trend may be [2, 26].

Government spending on education per student (GSE): The data of this variable can be obtained from Educational Statistics Yearbook of China. When there are differences in provincial government expenditure on education, there will also be great differences in the distribution of elementary education. Therefore, GSE should be controlled.

Family education expenditure intensity (EEI): It is measured by the ratio of family’s expenditure on education and entertainment to the total expenditure every year. Although compulsory education in China is free, families need to fund their children’s education in other aspects as well. Family education expenditure is a supplement to school education expenditure. Generally speaking, families will adjust their expenditure according to needs to prevent their children from falling behind. Therefore, family education expenditure may narrow the gap between urban and rural areas in elementary education.

Local average education level (Edu): Based on the measurement standard of China’s National Bureau of Statistics, it is measured by the ratio of educated population to the total population. The level of education affects people’s perception of the importance of education. Generally, the higher the education level, the more the people can realize the importance of education, so they pay more attention to the urban-rural gap of elementary education. Under the supervision of the people, the government will take more effective measures to narrow the urban-rural gap of elementary education. In the more economically developed areas, the implementation of the policy to narrow the urban-rural gap of elementary education may be stronger. Thus, its impact may be greater.

Based on the above analysis, we can construct the following PSTR model:

\[ E_{G, it} = \mu_i + \beta_0'X_{it} + \sum_{j=1}^{5} \beta_j'X_{it} G_{jt} + \gamma_j c_m + \epsilon_{it}, \]

where \( X_{it} = (\text{Urb}_{it}, \text{IG}_{it}, \text{GSE}_{it}, \text{EEI}_{it}, \text{Edu}_{it}) \) denotes the explanatory variable of PSTR model, and \( \text{GSE}_{it} \) is the transition variable. The estimated parameters are \( \beta_0 = (\beta_{01}, \ldots, \beta_{05}) \) and \( \beta_j = (\beta_{j1}, \ldots, \beta_{j5}) \). \( \gamma_j \) is the transition parameter, where \( j = 1, \ldots, r \) is the number of transition parameters. \( c_m \) is the location parameter or threshold parameter, and \( m \) is the number of location parameters.
4. Results and Discussion

4.1. Stationarity Test. To avoid the "spurious regression," we need to test the stationarity of the data before estimation. The Levin Lin Chu test (LLC) and Im Pesaran Shin test (IPS) are used, respectively. In the test, we eliminate the time trend and cross-sectional mean of the data. The results are shown in Table 1.

As can be seen from Table 1, at the confidence level of 5%, all variables significantly reject the null hypothesis of the existence of unit root under different test methods. Therefore, we believe that the variables used in this study are stationary.

4.2. Nonlinearity Test of the Model. Before adopting the nonlinear PSTR model, it is necessary to test whether the simple linear model is enough to reveal the relationship between variables. González et al. [27] tested it by constructing auxiliary regression. The transition function will not work when \( \gamma = 0 \), and the model (2) will degenerate to a linear model, i.e., \( r = 0 \). Under the weakest assumption \( r = 1 \), we carry out the first-order Taylor expansion of the transition function at \( \gamma = 0 \), and substitute it into (4), then we can get

\[
EG_u = \mu_i + \beta_0^* X_{it} + \beta_2^* X_{it} \text{GDP}_{it} + \ldots + \beta_m^* X_{it} \text{GDP}_{it}^m + \epsilon_{it}.
\]  

where the vectors \( \beta_0^*, \ldots, \beta_m^* \) are the product of the original coefficient and \( \gamma \), and \( \epsilon_{it} = \epsilon_i + O(y)^m \). For equation (2), the test of \( H_0: \gamma = 0 \) is equivalent to the test of \( \beta_0^* = \ldots = \beta_m^* = 0 \). Under the null hypothesis, \( \{\epsilon_i\} \sim \{\epsilon_i\} \), so the Taylor expansion does not affect the asymptotic distribution of statistics. Therefore, three statistics and their asymptotic distribution can be obtained:

\[
LM = TN \frac{(SSR_0 - SSR_m)}{SSR_0} \xrightarrow{d} \chi^2(mK), \quad LMF = \frac{|(SSR_0 - SSR_m)/(mK)|}{(TN - N - mK)} \xrightarrow{d} \chi^2(mK),
\]

where \( SSR_0 \) represents the sum of squares of residuals of fixed effect model under null hypothesis \( H_0 \); \( SSR_m \) represents the sum of squares of residuals of auxiliary regression under alternative hypothesis \( H_1 \). \( K \) is the number of explanatory variables (here, \( K = 5 \)).

In the nonlinearity test, the number of location parameters \( m \) should be assumed in advance. The number of location parameters in the transition function can be determined by a sequential test [28, 29] and then testing the null hypothesis, \( H_{01}: \beta_3^* = 0, H_{02}: \beta_2^* = 0 | \beta_3^* = 0, \) and \( H_{03}: \beta_1^* = 0 | \beta_2^* = \beta_3^* = 0. \) If \( H_{02} \) is rejected most strongly, choose \( m = 2 \); otherwise, \( m = 1 \). Here, we take all the cases of \( m = 1 \) and \( m = 2 \) into account, and report the results of nonlinearity test in Table 2.

According to the test results in Table 2, it can be found that whether the number of location parameters is \( m = 1 \) or \( m = 2 \), the PSTR model significantly rejects the null hypothesis. That is, it rejects the hypothesis of linear model and accepts the panel smooth transition regression model. This shows that the impact of urbanization on the urban-rural gap of elementary education does have a nonlinear effect because of the different levels of economic development in different provinces. Therefore, it is appropriate to use the panel smooth transition regression model in this paper.

4.3. The Number of Transition Functions and Location Parameters. After confirming that the model is nonlinear, in order to determine the specific form of PSTR model, we need to determine the number of location parameters \( m \) and the number of transition functions \( r \). Generally, \( m = 1 \) or \( m = 2 \).

We first confirm the number of location parameters of the model in this paper. We only need to estimate the panel smooth transformation regression model when \( m = 1 \) and \( m = 2 \) and get the values of AIC and BIC. Comparing the values of AIC and BIC when \( m = 1 \) and \( m = 2 \), the number of optimal location parameters of PSTR model can be determined. The results are shown in Table 3.

It can be seen from Table 3 that the values of AIC and BIC are smaller when \( m = 1 \) than those when \( m = 2 \). Therefore, the optimal number of location parameters of PSTR model in this paper should be \( m = 1 \).

After confirming the number of location parameters \( m \), it is necessary to further determine the number of transition functions \( r \). The test method is similar to the nonlinearity test. First, test the null hypothesis \( H_2: r = 1 \) and the alternative hypothesis \( H_1: r = 2 \). If the null hypothesis \( H_2 \) is rejected, we will accept the alternative hypothesis \( H_1 \). Then we continue to test the null hypothesis \( H_0: r = 2 \) and the alternative hypothesis \( H_1: r = 3 \). If the null hypothesis is rejected, we will accept the alternative hypothesis and continue to test the next hypothesis and so on. Until you cannot reject the null hypothesis for the first time, the obtained \( r \) is the optimal number of transition functions. In that case, the PSTR model covers all nonlinearity. According to the test results reported in Table 3, \( H_0: r = 1 \) is rejected at 1% significance level when \( m = 1 \), but \( H_0: r = 2 \) cannot be rejected. Therefore, the number of transition functions of PSTR model should be \( r = 2 \). Therefore, we choose \( r = 2 \) and \( m \) in the PSTR model of this paper.

4.4. Estimated Results. After confirming the model form, we select the transition parameter \( \gamma \) and location parameter \( c \) through the two-dimensional grid search method. Then, we use the nonlinear least squares method to estimate the parameters of PSTR model. The results are shown in Table 4:
Substituting the estimated coefficients into the PSTR model (4) and combining the similar items, the specific form of the PSTR model in this paper is as follows:

\[
\begin{align*}
\text{EG}_{it} &= \mu_i + (2.2950 - 7.2688G_1 + 0.5099G_2)Urb + (0.2789 + 0.827G_1 - 0.4301G_2) \\
\text{IG}_{it} &= (0.0263 + 0.0017G_1 - 0.3051G_2)GSE + (-0.0023 + 0.3044G_1 - 0.855G_2) \\
\text{EEI}_{it} &= (-3.7604 - 10.1324G_1)Edu + \epsilon_{it},
\end{align*}
\]

Notes: \(p^{*} < 0.10, p^{**} < 0.05, \) and \(p^{***} < 0.01, \) and the corresponding \( p \) value of test statistics is given in brackets.
where the transition functions are

\[
G_1 = \left(1 + \exp\left(-46.4199\left(\text{GDP}_t - 4.6811\right)\right)\right)^{-1},
\]

\[
G_2 = \left(1 + \exp\left(-7.4439\left(\text{GDP}_t - 4.8740\right)\right)\right)^{-1}.
\]

In order to observe the nonlinear impact of each explanatory variable on the urban-rural gap of elementary education, we calculate the partial derivative of the dependent variable with respect to the independent variable. Then we calculate the coefficients of each explanatory variable at different values of per capita GDP, that is, their effect on the urban-rural gap of elementary education. We visually depict the impact of Urb, IG, GSE, EEI, and Edu on the urban-rural gap of elementary education in Figure 2–6.

Figure 2 shows the nonlinear effect of urbanization on urban-rural gap of elementary education. It can be seen that in underdeveloped provinces, urbanization exacerbated the urban-rural gap of elementary education. When the GDP per capita reached about 4.6 (¥ 46x104), the impact of urbanization on the urban-rural gap of elementary education began to decline rapidly. As the GDP per capita increased to about 4.75, the impact of urbanization on the urban-rural gap of elementary education turned into a negative impact. In other words, in developed provinces, urbanization narrowed the urban-rural gap of elementary education. This may be because the main form of urbanization in underdeveloped areas was the net outflow of rural labor force, but the children of migrant workers stayed in rural areas. Such urbanization widened the financial gap between urban and rural areas, thus increasing the gap between urban and rural areas in elementary education. In areas with high levels of economic development, urbanization was often accompanied by the net inflow of rural labor force, which increased local fiscal revenue. Developed provinces could provide financial subsidies to rural areas so as to narrow the gap between urban and rural elementary education.

Figure 3 shows the nonlinear effect of IG on the urban-rural gap of elementary education. It can be seen that IG was positively correlated with the urban-rural gap of elementary education, and the nonlinear characteristics of its impact were very obvious with the change of GDP per capita. With the increase of GDP per capita, the positive effect of IG on the gap between urban and rural elementary education increased. When the GDP per capita reached about 4.6, the positive impact of IG increased rapidly. In other words, IG intensified the urban-rural gap of elementary education. In high-income areas, the absolute income gap between urban and rural residents was large. The gap between government investment in education was large, as was the gap between families’ investment in education. In low-income areas, the amount of financial investment and family investment in education was small, so the effect of urban-rural income gap was relatively weak. Therefore, with the increase in income level, the gap between urban and rural areas in elementary education became larger.

Figure 4 shows the nonlinear effect of GSE on the urban-rural gap of elementary education. In underdeveloped areas, China’s urban expenditure bias led to more financial investment in urban elementary education, exacerbating the gap between urban and rural elementary education. On the contrary, in high-income areas, cities could feed the countryside. The government gave more financial subsidies to rural areas, which narrowed the gap between urban and rural elementary education.

Figure 5 shows the nonlinear relationship between EEI and the urban-rural gap of elementary education with the change of GDP per capita. It can be seen that there was a negative correlation between EEI and the gap between urban and rural elementary education, and the nonlinear characteristics were obvious. With the growth of per capita GDP, the inhibitory effect of EEI on the gap between urban and rural elementary education first decreased and then increased. Because family expenditure was the supplement of school education expenditure, families spontaneously adjusted the intensity of education expenditure to narrow the
education gap and prevent their children from falling behind in school. In backward areas, household income was limited, so its role in suppressing the gap between urban and rural areas in elementary education was limited. In developed regions, the effect was relatively large.

Figure 6 shows the nonlinear effect of education level on the gap between urban and rural elementary education. As can be seen from the figure, the level of education was negatively correlated with the gap between urban and rural areas in elementary education. With the increase of GDP per capita, the inhibitory effect of education level on the gap between urban and rural elementary education became stronger and stronger. This might be because highly educated people pay more attention to education. With the increase of income, people had more time and money to promote educational equity.

5. Conclusions and Policy Suggestions

This paper used the Theil index to measure the urban-rural gap in different aspects of elementary education in China, and used principal component analysis (PCA) to construct a comprehensive index to objectively measure the overall urban-rural gap of elementary education. Then, the panel smooth transition regression model (PSTR) was used to study the nonlinear effects of urbanization and other factors on the urban-rural gap of elementary education in areas with different economic development levels. The results can be concluded as follows: (1) In economically underdeveloped areas, urbanization exacerbated the gap between urban and rural elementary education. In economically developed areas, urbanization narrowed the gap between urban and rural elementary education; (2) the urban-rural income gap increased the urban-rural gap in elementary education. This trend was more obvious in economically developed areas; (3) in economically underdeveloped areas, government expenditure on education exacerbated the urban-rural gap in elementary education, while in areas with high economic development level, government expenditure on education narrowed the urban-rural gap in elementary education; (4) with the increase of income level, the intensity of family education expenditure played a stronger role in narrowing the gap between urban and rural areas in elementary education; and (5) the improvement of education level helped narrow the gap between urban and rural areas in elementary education. With the improvement of economic level, this narrowing effect became stronger.

Based on the above conclusions, we put forward the following policy suggestions: (1) Continue to improve China’s urbanization rate, especially in the developed eastern regions; (2) gradually abolish the household registration system, reduce the restriction of population migration, and encourage population migration; (3) China needs to take measures to reduce the income gap between urban and rural areas, and financial subsidies should be provided for rural elementary education; (4) families should be encouraged to invest in education in order to avoid rural children falling behind in competition; and (5) continue to improve the educational level of the Chinese people and
promote the consensus on educational equity, so as to reduce the gap between urban and rural areas in elementary education.

This study also has some limitations. Firstly, we have not constructed a mathematical economic model to reveal the relationship between urbanization and the urban-rural gap of elementary education, which is not rigorous enough. Secondly, we use macro data to study the urban-rural gap of elementary education, but some valuable micro data have not been used. Therefore, in future research, mathematical models can be established to rigorously deduce the relationship between economic variables. In addition, with the advent of the era of big data, we can collect data through various methods and technologies and use big data to study issues related to the urban-rural gap of elementary education.

Data Availability
The data used to support the findings of this study can be obtained from the corresponding author upon request.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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