Tachyonic vs quintessence dark energy: linear perturbations and CMB data

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Abstract

We use linear perturbation theory to study perturbations in dynamical dark energy (DE) models. We compare quintessence and tachyonic DE models with identical background evolution. We rewrite the perturbation equations for quintessence and tachyonic models in a form that makes it easier to see that these models are very hard to distinguish in the linear regime, especially when the equation of state $w$ is close to that of $\Lambda$ (i.e. in the limit $(1 + w) \ll 1$). We use cosmic microwave background data and parametric representations for the two models to illustrate that current observations cannot distinguish between models with the same background evolution. Further, we constrain tachyonic models with the Planck data. We do this analysis for exponential and inverse square potentials and find that the intrinsic parameters of the potentials remain very weakly constrained. In particular, this is true in the regime allowed by low redshift observations.

Keywords: cosmology, dark energy, cosmological perturbations, quintessence, tachyonic field

(Some figures may appear in colour only in the online journal)

1. Introduction

Observational evidence [1–3] of the accelerated expansion of the Universe spurred the search for theoretical models to explain the phenomenon. In these models, the candidate that drives this acceleration is an exotic, non-luminous, negative pressure medium, and it contributes approximately two-third of the energy budget of the Universe at present. This component called
the dark energy (DE) [4–8] can be one of several theoretical possibilities including modifications to the theory of gravity [8, 9]. The standard model of cosmology (ΛCDM) represents DE by the cosmological constant: this is equivalent to an effective density and pressure that are constant in space-time. All other models are dynamical in nature. They can have a time-varying equation of state \( w(z) \). Observations sensitive to background expansion have been used by various authors to study these models [10–12]. The existence of perturbations is a generic feature of dynamical DE models and hence offers a probing ground for theories. One of the most well-studied models of DE is quintessence [13–16]: the minimally coupled scalar field. Another example is the tachyonic scalar field model [17, 18]. These and other DE models have a scale-dependent response to perturbations in the matter. Linear theory of cosmological perturbations [19–21] is used for studying the evolution of fluctuations in DE. These can be used to compute transfer functions, cosmic microwave radiations anisotropies [22], and other observables.

DE perturbations have been studied in detail for quintessence [23–26]. Perturbation theory employs a split between background and perturbations over that background. Comparisons of dynamical DE models with standard ΛCDM show deviations in expansion history. Presence of DE perturbations affects clustering of matter, propagation of CMB photons, etc. These can be related to observables like galaxy power spectrum, CMB angular spectrum, redshift space distortions, etc. For models allowing perturbations, the evolution is parameterized by a combination of background evolution (or expansion history) and other characteristics of the model. This point needs careful consideration when comparing DE models. Their potentials often have tunable parameters that can be adjusted to get the same expansion history so that any observation based on background cosmology (e.g. supernova data) cannot distinguish between these. In that case, a question that can be asked is if perturbations-based observations show any difference between such models? We address this question from the perspective of linear perturbation theory. A related work with similar results is Sergijenko and Novosyadlyj [27]. They consider two parametrizations of equation of state \( (w(a)) \) and adiabatic sound speed \( (c_s^2) \), which make the calculations more tractable. They find that the two types of scalar field models (tachyonic and quintessence) are indistinguishable for a constant \( w = -0.972 \) background, while there are differences for \( c_s^2 = 0 \) case. This \( c_s^2 = 0 \) case corresponds to a form of \( w(a) \) which is initially close to dust-like \( w \sim 0 \). As we will see in later sections, our results are consistent with these results. We generalise this approach and we show that if \( (1 + w) \) is very small, then the tachyonic and quintessence models are indistinguishable at the linear order irrespective of specific details of \( w \). We also study the possibility of distinguishing them using a parametrized form of effective speed of sound \( c_s^2 \) for the scalar field. Tachyonic models have been constrained with low redshift observations by Singh et al [11, 28]. We constrain tachyonic models with CMB. Here we use data from Planck 2018 in addition to other observational datasets to constrain tachyonic models.

We present the results for quintessence and tachyonic models where we compare the growth of perturbations in linear theory. In the next two sections, we describe the formalism for writing perturbation equations that demonstrate explicitly that the difference between these two models diminish as we go towards \( w = -1 \). Specifically, it depends on the factor \( (1 + w) \). We demonstrate that differences between the two classes of models are suppressed by the factor \( (1 + w) \) and hence diminish rapidly as we approach the cosmological constant model. We also relate our formalism with earlier work as well as with the fluid description. In section 4, we present the numerical results. In section 5, we use an approximate parametric representation for quintessence and tachyonic models to use CMB data to check if these can be distinguished by current data. We find that if the respective potentials for tachyonic and quintessence models are chosen such that the background evolution is identical then it is not possible to distinguish
these with present observations. Finally, in section 6, we constrain parameters of two different tachyonic models with CMB data. We summarize and comment on prospects in the last section.

2. Perturbation theory

We consider scalar metric perturbations in the Newtonian gauge with the following form of the metric:

$$d s^2 = (1 + 2\psi) d t^2 - (1 - 2\xi) a^2 (d x^2 + d y^2 + d z^2).$$  \(1\)

The problem of DE perturbations and its relevance for observables has been studied by many authors [23–33]. Often, a fluid form for DE is assumed. This fluid, at the background level, is characterized by the equation of state parameter ($\bar{w}(z)$). To properly treat perturbations in a fluid, one needs to know how energy density perturbations as well as pressure perturbations evolve. One can obtain dynamical equations for density perturbations and velocity perturbations either directly from Lagrangian density or from continuity equations (see Kodama and Sasaki [19], Mukhanov et al [20], Ma and Bertschinger [21] or Bean and Dore [31]). The set of equations derived in this manner is complete if additional information is provided for term $\delta p/\delta \rho$. In Newtonian gauge, ignoring anisotropic stress, we have [21]:

$$\dot{\delta} = -3H \left( \frac{\delta p}{\delta \rho} - \bar{w} \right) \delta + 3(1 + \bar{w})\dot{\psi} - \frac{1 + \bar{w}}{a} \theta$$  \(2\)

$$\dot{\theta} = -(1 - 3\bar{w})H \theta - \frac{\dot{\bar{w}}}{1 + \bar{w}} + \frac{k^2 \psi}{a} + \frac{k^2 \delta}{(1 + \bar{w})a} \left( \frac{\delta p}{\delta \rho} \right)$$  \(3\)

where $\delta$ is the fluid energy density contrast, $\theta$ is defined as:

$$\theta \rho(1 + \bar{w}) = -i k(\delta T)_{ij}.\$$  \(4\)

For adiabatic perturbations in fluids, there exists a relation between pressure perturbations and density perturbations. Using this relation we can eliminate pressure perturbations and solve for density perturbations. In general, one has to solve for both the perturbations. In cases where density and pressure are effective quantities, e.g., scalar fields, the underlying system of equations has to be solved. A common approach is to quantify the variation of pressure perturbations using a gauge-invariant quantity called the effective speed of sound $c_s^2$. In an arbitrary gauge, pressure perturbations are written as [31, 34]:

$$(\delta p) = c_s^2 (\delta \rho) + 3a(1 + \bar{w})(c_s^2 - c_\perp^2)\frac{\theta}{k^2}.$$  \(5\)

The second term on the right in equation (5) represents non-adiabatic perturbations [35–39] and, in general, is a non-vanishing quantity. There are a few subtle points that need to be considered while using this definition:

- In order to ensure gauge invariance, $c_s^2$ is defined in terms of $\delta p/\delta \rho$ in a frame comoving with fluid, i.e., frame in which $\theta$ is zero. Then from equation (5), $c_s^2$ is just $\delta p/\delta \rho$ in the frame comoving with fluid.
- In general, there are entropy perturbations as well. The gauge invariant amplitude of entropy perturbation is [19, 31]:

$$\bar{w} \Gamma = (c_s^2 - c_\perp^2) \delta,$$  \(6\)
where \( c_a^2 \) is a quantity determined by quantities related to evolution of the background:

\[
c_a^2 = \bar{w} - \frac{\dot{\bar{w}}}{3H(1 + \bar{w})}.
\]  \hfill (7)

For adiabatic perturbations, \( \Gamma \) vanishes, and \( c_a = c_s \).

- Equation (5) can be derived \([31]\) starting from equation (6). \( \delta \) in general is not gauge invariant, but \( \delta \) in the frame comoving with the fluid is a gauge invariant quantity. \( \delta \) and \( \theta \) can be combined to form a gauge invariant quantity:

\[
\delta_t = \delta + 3\dot{a}(1 + \bar{w})\frac{\theta}{k^2}.
\]  \hfill (8)

In equation (6), the left-hand side is gauge invariant and hence the combination on the right-hand side is gauge invariant too.

Then in any frame we can substitute for rest frame \( \delta \) using equation (8) in equation (6) and then obtain equation (5). For multicomponent systems, there can be an additional entropy perturbation besides intrinsic entropy perturbations \([19]\). This can be due to difference in dynamics (different \( c_a^2 \)) or due to non-minimal coupling. In such cases, working in terms of field variables is simpler and less prone to ambiguities.

- For scalar fields \([40]\), let \( L(X, \phi) \) be the Lagrangian density, where \( X = \frac{1}{2} \partial_\mu \phi \partial^{\mu} \phi \) is the kinetic term and \( \phi \) is the scalar field. Rest frame for the field is defined as the one in which \( (\delta \phi) \) vanishes. In an arbitrary frame:

\[
(\delta p) = \frac{\partial p}{\partial X} (\delta X) + \frac{\partial p}{\partial \phi} (\delta \phi)
\]  \hfill (9)

with similar equation for \( (\delta \rho) \). In rest frame:

\[
(\delta p) = \frac{\partial p}{\partial X} (\delta X).
\]  \hfill (10)

Combining equations for \( (\delta p) \) and \( (\delta \rho) \) in the rest frame, we get

\[
c_r^2 = \frac{(\delta p)}{(\delta \rho)} = \frac{p_X}{\rho_X} \quad  \hfill (11)
\]

where \( p_X \) is partial derivative w.r.t. \( X \).

Earlier work \([30–33, 40, 41]\) along these lines has assumed a functional form for \( c_r^2 \) and then constrained \( c_r^2 \) and other parameters. These forms are assumed independent of \( \bar{w}(z) \), thus the model is described by two functions. One general result from these studies is that the effects of different \( c_r^2 \) but same \( \bar{w}(z) \) on observables are significant only in cases where DE has some significant contribution (at least a few percent) at time of recombination \([32, 40]\). But this itself means that \( \bar{w}(z) \) should be of such a form that DE has a significant contribution at early times. For scalar fields, given the form of Lagrangian density, there is no need of using any ad hoc approximate \( c_r^2 \). Equations for systems with scalar field perturbations can be written entirely in terms of field perturbations (gauge-invariant) and perturbations in other constituents. However, studies with effective parametrization of \( c_r^2 \) are useful because they provide a general framework to compare different types of Lagrangian densities. Different Lagrangian densities may have different effective speeds of sound. For example, canonical scalar field Lagrangian of the form:

\[
L = X - V(\phi)
\]  \hfill (12)
always have \( c_s^2 = 1 \), while \( k \)-essence ones with the form:

\[
L = V(\phi)F(X)
\]  

(13)

can have a time-dependent \( c_s^2 \).

In this work, we consider the question whether two different scalar field Lagrangians (tachyonic and quintessence) with the same background evolution can be distinguished at the level of linear perturbations. Instead of working with an assumed form of \( c_s^2 \) and using fluid equations, we directly work with scalar fields and their perturbations. Our model space is limited as we choose two specific Lagrangians, but our calculations are concrete with very few assumptions. We consider models with the same expansion history with this formalism. We explore if these models can be distinguished by observations sensitive to linear perturbations?

3. Basic equations for scalar fields with effective fluid approach

In this section, we derive equations for quintessence and tachyonic fields. For establishing correspondence between field description and the effective fluid description we define a new perturbation quantity: \( u \) which is the deviation in the equation of state from the background homogeneous fluid. The fluid description we employ is slightly different from the standard approach but is useful in highlighting differences in quintessence and tachyonic models. We also give relations between standard fluid variables and the variables used here.
Figure 2. The plots show the potential $\psi$ and its time derivatives for $\bar{w} = -0.975$ case. Clearly, the relative differences are much smaller than in the case of $w = -0.5$.

Let $\Phi$ be the field for a scalar field representing DE. Then its stress–energy tensor can be written as:

$$T_{\mu\nu} = (\rho + P)v_{\mu}v_{\nu} - P g_{\mu\nu}, \quad (14)$$

where

$$v_{\nu} = \frac{\partial_{\nu} \Phi}{\sqrt{\partial^{\alpha} \Phi \partial_{\alpha} \Phi}}. \quad (15)$$

We define first order quantities, density contrast and the corresponding variation in the equation of state parameter

$$\rho = \bar{\rho}(1 + \delta) \quad W = \bar{w}(1 + u), \quad (16)$$

where variables with a bar are background quantities dependent on time only, while the first order variations ($\delta$ & $u$) can vary in space-time. We also define

$$\omega = 1 + \bar{w}. \quad (17)$$

Effective pressure ($P$) for a scalar field theory (with identification of $P$ as per equation equation (14)) is the Lagrangian ($L_\Phi$) of field while the effective density $\rho$ is:

$$\rho = 2g^{\mu\nu} \frac{\partial L_\Phi}{\partial g_{\mu\nu}} - L_\Phi. \quad (18)$$
Writing the field as the sum of background and perturbation:

$$\Phi = \phi + (\delta \phi)$$

and substituting it in equation equation (14), retaining only the first order terms, we get the first order stress energy tensor using metric equation (1):

$$T^0_0 = \bar{\rho} \delta$$

$$T^i_j = \bar{\rho} (u + \delta)(1 - \omega) \text{ for } i = j$$

$$T^j_i = \frac{\bar{\rho} \omega}{\dot{\phi}} \frac{\partial (\delta \phi)}{\partial x^i}.$$  \hspace{1cm} (20)

Off-diagonal spatial components of stress–energy tensor of both dark matter and field vanish at this order, hence the two metric potentials can be taken to be equal. We choose to work with $\psi$. The first order Einstein equation

$$G^1_1 = 8 \pi G T^1_1$$

can be used to obtain

$$\ddot{\psi} + \frac{4}{a} \dot{\psi} + \psi \left[ \frac{2 \ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right] = -4 \pi G \bar{\rho} (u + \delta)(-\bar{w}).$$  \hspace{1cm} (22)
We obtain the dynamical equations for $\delta$ and $u$ by requiring that the four divergence of stress energy tensor vanishes, i.e.

$$T^\mu_{\nu,\mu} = 0$$  \hspace{1cm} (23)

$$\dot{\delta} = 3u(1 - \omega) \frac{\dot{a}}{a} + \omega \left[ 3\dot{\psi} + \frac{\nabla^2(\delta\phi)}{a^2\phi} \right].$$  \hspace{1cm} (24)

Making use of the following off-diagonal Einstein equation:

$$\frac{\dot{a}}{a} \frac{\partial\psi}{\partial x^l} + \frac{\partial\dot{\psi}}{\partial x^l} = \frac{4\pi G \bar{\rho}\omega}{\phi} \frac{\partial(\delta\phi)}{\partial x^l} + 4\pi G T^0_{j(dm)}$$  \hspace{1cm} (25)

with dark matter stress energy contribution as

$$T^0_{j(dm)} = -a^2 \rho_{dm} \frac{\partial U}{\partial x^l},$$  \hspace{1cm} (26)

where $U$ is dark matter velocity potential, we rewrite equation (24) as:

$$\dot{\delta} = 3u(1 - \omega) \frac{\dot{a}}{a} + 3\dot{\psi}\omega + \frac{1}{4\pi G \bar{\rho}\omega^2} \nabla^2 \left[ \frac{\dot{a}}{a} \phi + \psi \right] + \frac{\rho_{dm}}{\bar{\rho}} \nabla^2 U.$$  \hspace{1cm} (27)

**Figure 4.** DE density contrast for $\bar{w} = -0.5$ case. It shows growth of DE normalized by present value.
Figure 5. DE density contrast for $\bar{w} = -0.975$ case. Relative differences between quintessence and tachyonic models are small in comparison with $w = -0.5$ case.

We also get a constraint equation for $u$

\[
(-1 + \omega) \frac{\partial u}{\partial x^j} = (1 - \omega) \frac{\partial \delta}{\partial x^j} + \frac{\omega}{\phi} \frac{\partial (\delta \phi)}{\partial x^j} \left[ \frac{\dot{a}}{a} + \frac{\dot{\rho}}{\rho} + \frac{\dot{\omega}}{\omega} \right] + \frac{\omega}{\phi} \left[ -\frac{\ddot{\phi}}{\phi} \frac{\partial (\delta \phi)}{\partial x^j} + \frac{\partial (\delta \phi)}{\partial x^j} \right] - \omega \frac{\partial \psi}{\partial x^j}.
\]

(28)

We observe that equations (22) and (27) do not have explicit dependence on details of scalar field (whether it is quintessence or tachyonic), but equation (28) does have such a dependence. Therefore any differences between models will arise from this equation. We rewrite these equations in a less ‘field-specific’ form and find that the equations in one of the theories have more terms. For tachyonic field, equation (28) can be written as:

\[
\frac{(-1 + \omega)}{2} \frac{\partial u}{\partial x^j} = (1 - \omega) \frac{\partial \delta}{\partial x^j} + \left[ 3(1 - \omega) \frac{\dot{a}}{a} + \frac{\dot{\omega}}{2\omega} \right] \times \left[ \frac{1}{4\pi G\rho} \left( \frac{\dot{a}}{a} \frac{\partial \psi}{\partial x^j} + \frac{\partial \psi}{\partial x^j} \right) + a^2 \frac{\rho_{dm}}{\rho} \frac{\partial U}{\partial x^j} \right].
\]

(29)
Figure 6. DE density contrast for the CPL case. Curves are neatly clustered here by field type. This suggests that for this particular background evolution, tachyonic field and quintessence field evolve similarly for a particular lengthscale.

While quintessence has extra terms in addition to those present in equation (29):

\[
\frac{(-1 + \omega)}{2} \frac{\partial u}{\partial x^j} = (1 - \omega) \frac{\partial \delta}{\partial x^j} + \left[ 3(1 - \omega) \frac{a}{\dot{a}} + \frac{\dot{\omega}}{2}\right] \\
\times \left[ \frac{1}{4\pi G \bar{\rho}} \left( \frac{\dot{a}}{a} \frac{\partial \psi}{\partial x^j} + \frac{\partial \dot{\psi}}{\partial x^j} \right) + a^2 \frac{\rho_m \bar{\rho} U}{\bar{\rho} \partial \phi} \right] \\
+ \omega \left[ \frac{3\dot{a}}{8\pi G \rho a} \frac{\partial}{\partial x^j} \left( \frac{\dot{a}}{a} \psi + \dot{\psi} \right) + \frac{3\rho_m \bar{\rho} a \dot{a} \bar{\rho} U}{2\bar{\rho} \dot{a} \phi} \frac{\partial}{\partial x^j} + \frac{1}{2} \frac{\partial \delta}{\partial x^j} \right].
\]

(30)

Observing the third line in the above equation and comparing it with the equation for tachyonic counterpart (29), we find that the difference between two models is encoded in the terms multiplied by \( \omega = (1 + \bar{w}) \). For the models constrained by observations, this number is small, much smaller than unity. Effectively this makes the differences between two models a second order term.

We relate \( u \) to familiar quantities:

\[
\frac{(u + \delta)\dot{\bar{w}}}{\delta} = \frac{(\delta \rho)}{(\bar{\rho} \rho)}.
\]

(31)
The effective ‘velocity’ perturbation (coming from 4) for scalar field is:

$$\theta = \frac{k^2(\delta \phi)}{a\phi}$$  \hspace{1cm} (32)

and the effective speed of sound is:

$$c_s^2 = \frac{(\alpha + \delta)\bar{w} + 3\frac{\bar{w}}{a}(1 + \bar{w})c_s^2 \frac{\delta \phi}{\phi}}{\delta + 3\frac{\bar{w}}{a}(1 + \bar{w})\frac{\delta \phi}{\phi}}$$  \hspace{1cm} (33)

As stated earlier, we do not need to incorporate an effective $c_s^2$ while working with fields because we have an analytical expression that can be evaluated. For comparison of models, we derive approximate effective $c_s^2$ for tachyonic field. The value of $c_s^2$ for the quintessence field is unity.
For tachyonic field the Lagrangian density is:

$$L(X, \Phi) = -V(\Phi)\sqrt{1 - 2X}. \quad (34)$$

In the comoving frame of a scalar field:

$$c_s^2 = \frac{p_X}{\rho_X} = \frac{L_X}{L_X + 2L_{XX}X}. \quad (35)$$

In the case of tachyonic field:

$$c_s^2 = (1 - 2X) = -\bar{w} - (1 + \bar{w})(\delta g^{00})_{ef} \approx -\bar{w}. \quad (36)$$

In linear theory approximation $c_s^2$ is just $-\bar{w}$ as $(1 + \bar{w}) \ll 1$ for models allowed by observations and the second term in equation (36) is effectively of second order. Most of the comparisons of $c_s^2$ in literature are between very different values of $c_s^2$ within the allowed range. For models allowed by background observations, tachyonic $c_s^2$ is not very different from quintessence value of 1.

In the following sections, we study the differences in the two models in linear theory using field perturbations. Note that one can either directly use equations derived from field perturbations or the fluid perturbations ($\mu$ and $\delta$) equations derived in this section as these are equivalent approaches.
Figure 9. 2-d plots for present day matter density parameter $\Omega_{\text{cdm}}$ and Hubble parameter ($h$). Left panel is for exponential potential while right one is for inverse square. Red lines show the best-fit values for $\Lambda$CDM model from Planck 2018 [54]. CMB data, as already known, shrinks the constrained region. $H_0$ tension is not resolved by tachyonic models considered here.

4. Results for field-based comparisons

We divide our discussion here into two subsections. In first we show comparisons for quantities, that influence observables, like metric potential ($\psi$) and its derivative ($\dot{\psi}$). In the second subsection, we show differences in DE perturbations.

4.1. Influence on observables

All observables are affected by metric coefficients. The influence of these coefficients on dark matter linear growth rate is used in calculating observational effects like matter clustering, $\sigma_8$, growth index, etc. Rate of change of potential ($\dot{\psi}$) affects CMB photons and causes observable effects like ISW [26, 42].

We present $\psi$ and $\dot{\psi}$ for the following background evolutions (characterized by $\bar{w}(a)$):

- Constant $\bar{w}(z)$ for values: $-0.5$ and $-0.975$.
- Chevallier–Polarski–Linder (CPL) parameterization [43, 44]

$$\bar{w} = w_0 + w_a \left(1 - \frac{a}{a_0}\right).$$  (37)

$\bar{w}(z)$ with parameters: $w_0 = -0.9$ and $w_a = -0.099$

Since differences in growth rate of perturbations with scale has been seen mainly at very large scales [24, 45, 46] we present results for length-scales: 2000 Mpc and 10 000 Mpc. The differences between two models peak approximately around 10 000 Mpc. At small scales, the growth of perturbations is suppressed, and at very large scales the growth rate is independent of the speed of sound. It is only in the intermediate region that we can expect to capture some differences between models with the same expansion history but with a different $c^2_s$.

We show $\psi$ and $\dot{\psi}$ in figures 1–3. In the notation used to annotate the curves, we use ‘quin’ for quintessence models and ‘tach’ for the tachyon models. We find that tachyonic and quintessence models for $\bar{w} = -0.975$ and CPL are almost indistinguishable with differences of about 0.01% in most cases. Corresponding differences for $\bar{w} = -0.5$ are more significant.
Figure 10. Triangle plot using four combinations of data, for exponential potential. Potential parameter $\phi_a$ is slightly constrained to be greater with certain minima. While $\phi_a$ appears to have nonlinear correlations with $w_0$, $\bar{w}_0$ is constrained be close to $-1$. These differences grow in a monotonic and continuous manner as we move away from $w = -1$. We plot for $\bar{w}_0 = -0.5$ that is observationally ruled out but gives an indication of the order of differences between the two classes of models. CPL parameterization and $\bar{w}_0 = -0.975$ are observationally allowed [10, 47] but differences between the models are extremely small at all scales. For $w = -0.5$, differences in potentials and its time derivatives can be of the order 10%, $w = -0.975$ case shows negligible differences (of the order 0.01%) while CPL case has differences of approximately 0.1%.

4.2. Scalar fields

While DE perturbations show more differences (figures 4–6) than their respective potentials, their effects on observables are not very significant as shown in the previous subsection. Fluctuations are stronger for cases that are significantly removed from $w = -1$. Since DE
Figure 11. Triangle plot using four combinations of data, for inverse square potential. Results are somewhat similar to that for exponential case as potential parameter $n$ is slightly constrained and is correlated to $w_0$. A particular value of $w_0$ is favoured, which is not $-1$, but is close to it.

perturbations are not directly observable, the significance of fluctuations can only be evaluated through observables. We have shown comparisons for DE perturbations in figures 4–6. Although there are visible differences (between tachyon and quintessence cases) in the evolution of DE perturbations, these remain insignificant as the amplitude of perturbations is very small.

5. Constraining models with CMB anisotropy data

There are two popular public codes available for CMB anisotropy calculations: CAMB [48] and CLASS [49–51]. Both have support for implementing fluid models with effective $c_s^2$. Here we use CLASS to calculate CMB anisotropy power spectra for effective $c_s^2$ corresponding to tachyon models and quintessence models. This requires some minor modifications in the
default CLASS code as the standard version does not include time-dependent $c_s^2$. We modified the code to allow for a time-varying form of $c_s^2$ for tachyon models. There are various ways tachyonic models can be included in CLASS. We can write effective potentials for the tachyon field in terms of a chosen background DE (particular $w(a)$), or we can have an effective fluid description with $c_s^2$ as derived in equation (36). While the former is a more apt and clean approach, the latter is easier to implement and is expected to give same results for $(1 + \bar{w}) \ll 1$, which is the region already constrained by background cosmology probes. In cases where one has well-motivated forms for potentials, these tachyonic models can be implemented in CLASS. We do this in the next section where we constrain tachyon models for two well-studied potentials.

We adopt the following parametric form for $c_s^2$:

$$c_s^2 = c1 * w + c0.$$  

(38)
This is the simplest form that can capture both quintessence and tachyonic models. For quintessence, we have $c_1 = 0$ and $c_0 = 1$ and for tachyonic models $c_1 = -1$ and $c_0 = 0$. We then do an MCMC sampling using CLASS with MontePython [52, 53]. We use CMB (Planck 2018 high-l TT, TE, EE, low-l EE, low-l TT, lensing) [54] and BAO data (Boss data release 12 [55, 56], small-$z$ BAO data from 6dF Galaxy Survey [57] and SDSS DR7 main Galaxy sample [58]). We find that the two parameters $c_1$ and $c_0$ remain unconstrained. In figure 7, we show triangle plots for 2d marginalized credible intervals. Parameters relating to DE speed of sound are unconstrained. This result is similar to analyses with a constant $c_s^2$ have obtained earlier [30–33, 59]. While the previous work deals with either a constant $c_s^2$ or some particularly chosen form, here we have chosen an explicit parameterized form for it, which encapsulates both quintessence and tachyonic field. In figure 8, we plot the marginalised posteriors.

6. CMB data and tachyonic models

We modify the CLASS code to implement tachyonic models as a scalar field at linear level, where equations are obtained from Lagrangian corresponding to tachyonic DE (the equations and modifications related information is provided in appendix A). Two potentials which we code in CLASS are:

- Exponential potential
  \[ V(\phi) = V_0 \exp \left( -\frac{\phi}{\phi_a} \right). \]  

- Inverse square potential
  \[ V(\phi) = \frac{n^4}{4\pi G} \left( 1 - \frac{2}{3n} \right)^{\frac{1}{2}} \frac{1}{\phi^2}. \]  

These two potentials have some interesting features and have been studied in detail [17, 60]. These potentials have been constrained using low red-shift data in [11, 28]. In [11], tachyonic models were constrained using low redshift data from supernova, Hubble parameter measurements, and BAO data. Evolution of perturbations was considered in [28] and redshift space distortion data was used for model comparisons. These models have not yet been constrained using CMB data. We use CLASS with MontePython to constrain the tachyonic models (with the above-mentioned potentials) using Planck 2018 data [61]. We use the following combinations of data:

- CMB (Planck 2018 high-l TT, TE, EE, low-l EE, low-l TT, lensing) [54].
- BAO (Boss data release 12 [55, 56], small-$z$ BAO data from 6dF Galaxy Survey [57] and SDSS DR7 main Galaxy sample [58]).
- Combination of the above-mentioned CMB and BAO data.
- JLA data [62].

6.1. Results

We first consider constraints on parameters that only concern background evolution and are needed irrespective of potentials: density parameter for dark matter and the Hubble constant. In figure 9, we plot contours for present-day matter density contrast ($\Omega_{cdm}$) and dimensionless Hubble constant ($h$). CMB data provides tight constraints. The best fit values of these parameters, from the ΛCDM model based CMB constraint in Planck 2018 cosmological parameters
Figure 13. Triangle plot with $\sigma_8$ for the inverse square potential. The constraints are comparable with those for exponential potential as well as $\Lambda$CDM.

paper, is represented by red lines in this figure. We find that the best-fit value lies in the 1-sigma region of the JLA data, but it is out of 2-sigma regions for CMB and BAO data constraints. While $h$ is consistent (within 2-sigma regions), it is $\Omega_{cdm}$ which is lower for these tachyonic field based cosmological models. Therefore, inference of dark matter content of the Universe shows dark energy model dependence, when considering extensions beyond $\Lambda$. In figure 10, we present the triangle plot for exponential (exp) potential with potential parameter $\phi_a$ and present-day equation of state $w_0$, included with density and Hubble parameter. $w_0$ is constrained to be close to $-1$. The potential parameter $\phi_a$ is not constrained by any of the data used here. Triangle plot for inverse-square (insq) potential is presented in figure 11. Again, a large range is allowed in the potential parameter $n$ and $w_0$ is very close to $-1$. Plots with $\sigma_8$ are shown in figures 12 and 13. The constraints for $\sigma_8$ for two potentials agree with each other as well as with those for $\Lambda$CDM. This is again a manifestation of the fact that the models which have same background evolution and are close to $\Lambda$ are extremely difficult to distinguish.
7. Summary & prospects

We have studied the prospects of using linear perturbation theory to distinguish two different models of dark energy, namely the quintessence and the tachyonic field. Specifically, we investigate the differences in dynamics of perturbations for the same background expansion in both models. This helps us separate the effects coming from different background expansions and differences due to perturbations.

We recast linear theory equations in a form that provides insight into how the systems of perturbations differ in two theories. We show that when the equations for both are written in fluid terms, substituting for corresponding field terms, one of the equations has extra terms for quintessence. These first-order terms have a factor $\omega \equiv (1 + w)$. This implies that if the background expansion is close to $w = -1$, differences between the two models diminish.

We calculated and showed the evolution of quantities like $\psi$ and its derivative, which affects the observables. These numerical calculations demonstrate the theoretical dependence on the factor of $(1 + w)$.

We find that the differences between models are small at all scales and are the largest around the scale of 10,000 Mpc. This is due to the difference between the effective speed of sound in two models and that this difference is seen in the transition scales from suppression of perturbations at small scales to growth at large scales.

We used the definition of effective $c_s^2$ for two models to write a parametric form for $c_s^2(\equiv c_1 w + c_0)$ which incorporates both fields as special cases. We then used CMB data to constrain this parametric form to understand if we can distinguish two models and find that two parameters $c_0$ and $c_1$ remain unconstrained.

We modified the CMB anisotropy code CLASS to incorporate tachyon models. We then used it to constrain common tachyonic potentials: $V \propto \exp(-\frac{\phi}{\phi_0})$ and $V \propto \phi^{-2}$ using CMB and other data. We find that the parameters are very weakly constrained.

We have shown that it is very difficult to distinguish between these two classes of models at large scales where linear perturbation theory is applicable. We have also shown that this is primarily because only models with $(1 + w) \ll 1$ are allowed and in this regime, the differences between the two classes of models are effectively of second order. Combined with our earlier work where we have explored these models at small scales using spherical collapse, it appears that there are no obvious observables available at present that may be used to distinguish between these two classes of models if the expansion history is the same. We can conclude that at least for these two classes of dark energy, as also for a fluid model of dark energy, the choice of class of models is irrelevant and calculation of observables may be done in any model. On the one hand, this is a potential simplification of calculations, on the other hand, it means that we cannot know which of the models is the true model for dark energy.

We are exploring the following to broaden the scope of our conclusions.

- Working out forecasts to find out the sensitivity of observations required to differentiate between the class of models. This will allow us to see whether future observations can potentially distinguish between the two classes of models. A comparison with the capabilities of upcoming surveys is required to ascertain whether the problem can be solved in the coming years.
- Generalize the analysis to other classes of models to check whether the factor of $(1 + w)$ in model-dependent terms is generic or specific to the classes studied here. We would like to be able to check at least some classes of models with minimal coupling.
• Going beyond linear theory to see if a higher-order calculation or a general numerical relativity calculation at small scales can bring out some features that are not accessible in the two limiting cases we have used so far.

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Data availability statement

No new data were created or analysed in this study.

Appendix A. Equations in synchronous gauge

Here we present the equations required for modification of CLASS for tachyonic field. Since synchronous gauge is the default gauge in CLASS, we write the equations in this gauge. Quintessence with some potentials are already implemented in CLASS, one can simply follow the same structure for incorporating the tachyonic models. Here, we present equations for both quintessence and tachyonic field because this helps on modifications comparing with quintessence implementation.

Note: in this section, we use conformal time and prime represents derivative w.r.t. to conformal time.

Field dynamic equation: for tachyonic models

\[
(\delta \phi)'' = \left[1 - \frac{\phi'^2}{a^2}\right] \left[\alpha(\delta \phi) \left\{ \frac{(V_{\phi})^2}{V^2} - \frac{(V_{\phi\phi})^2}{V^2} \right\} + \nabla^2 (\delta \phi) - \frac{\phi' h'}{2} \right]
+ (\delta \phi)' \left[ \frac{2a'}{a} + 9\frac{a'}{a^2} \phi'^2 + 2\frac{V_{\phi}}{V^2} \phi'^2 \right].
\] (A.1)

For quintessence

\[
(\delta \phi)'' = -a^2(\delta \phi)(V_{\phi\phi}) - \frac{\phi' h'}{2} - \frac{2a'}{a} (\delta \phi)' + \nabla^2 (\delta \phi).
\] (A.2)

Density \((\delta \rho)\) and pressure perturbations \((\delta p)\): for tachyonic field

\[
(\delta \rho) = \frac{(\delta \phi)(V_{\phi})}{\sqrt{1 - \frac{\phi'^2}{a^2}}} + \frac{V_{\phi} (\delta \phi)'}{a^2 \left[1 - \frac{\phi'^2}{a^2}\right]^{3/2}}
\] (A.3)

\[
(\delta p) = -\frac{\delta p}{(\delta \phi)(V_{\phi})} \sqrt{1 - \frac{\phi'^2}{a^2}} + \frac{V_{\phi} (\delta \phi)'}{a^2 \sqrt{1 - \frac{\phi'^2}{a^2}}}
\] (A.4)

For quintessence
\[(\delta \rho) = (\delta \phi)(V_{\phi}) + \frac{\phi'(\delta \phi)'}{a^2} \quad \text{(A.5)}\]

\[(\delta p) = - (\delta \phi)(V_{\phi}) + \frac{\phi'(\delta \phi)'}{a^2}. \quad \text{(A.6)}\]

Effective velocity perturbations:

For tachyonic field

\[ (\bar{\rho} + \bar{\rho}) \theta = ik^j (\delta T) = \frac{\phi'}{a^2} k^2 (\delta \phi) \frac{V}{\sqrt{1 - \frac{\dot{\phi}^2}{a^2}}}. \quad \text{(A.7)}\]

For quintessence

\[ (\bar{\rho} + \bar{\rho}) \theta = \frac{\phi'}{a^2} k^2 (\delta \phi). \quad \text{(A.8)}\]

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