Evidence for $CP$ violation in $B^+ \rightarrow K^+(892)^+\pi^0$ from a Dalitz plot analysis of $B^+ \rightarrow K_S^0\pi^+\pi^0$ decays

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I. INTRODUCTION

The Cabibbo-Kobayashi-Maskawa (CKM) mechanism [1, 2] for quark mixing describes all weak charged current transitions between quarks in terms of a unitarity matrix with four parameters: three rotation angles and an irreducible phase. The unitarity of the CKM matrix is usually expressed as triangle relationships among its elements. The interference between tree-level and loop ("penguin") amplitudes can give rise to direct CP violation, which is sensitive to the angles of the Unitarity Triangle, denoted \( \alpha, \beta, \) and \( \gamma \). Measurements of the parameters of the CKM matrix provide an important test of the Standard Model (SM) since any deviation from unitarity or discrepancies between measurements of the same parameter in different decay processes would imply a possible signature of new physics. Tree amplitudes in \( B \to K^\pi \) decays are sensitive to \( \gamma \), which can be extracted from interferences between the intermediate states that populate the \( K^\pi \) Dalitz plane. However, these amplitudes are Cabibbo-suppressed relative to contributions carrying a different phase and involving radiation of either a gluon (QCD penguin) or photon (electroweak penguin or EWP) from a loop.

QCD penguin contributions can be eliminated by constructing a linear combination of the weak decay amplitudes for \( B^+ \to K^\ast \pi \) to form a pure isospin \( I = \frac{3}{2} \) state [3]:

\[
A_{\frac{3}{2}} = A(K^{*0} \pi^+) + \sqrt{2}A(K^{*+} \pi^0). \tag{1}
\]

Since all transitions from \( I = \frac{3}{2} \) to \( I = \frac{5}{2} \) states occur via only \( \Delta I = 1 \) operators, \( A_{\frac{5}{2}} \) is free from QCD contributions. The weak phase of \( A_{\frac{5}{2}} \) is often denoted as

\[
\Phi_{\frac{5}{2}} = -\frac{1}{2} \text{Arg} \left( \frac{A_{\frac{5}{2}}}{A_{\frac{3}{2}}} \right), \tag{2}
\]

where \( A_{\frac{5}{2}} \) is the CP conjugate of the amplitude in Eq. (1). The phase \( \Phi_{\frac{5}{2}} \) in Eq. (2) is the CKM angle \( \gamma \) in the absence of EWP contributions [4].

Measurements of the rates and CP asymmetries in \( B \to K \pi \) have generated considerable interest because of possible hints of new-physics contributions [4, 5]. Of particular interest is the difference, \( \Delta A_{CP} \), between the CP asymmetry in \( B^+ \to K^+ \pi^0 \) and the CP asymmetry in \( B^0 \to K^+ \pi^- \), which in the SM is expected to be consistent with zero within the theoretical uncertainties assuming U-spin symmetry and in the absence of color-suppressed tree and electroweak amplitudes [7, 8]. Using the average values of \( A_{CP} \) of \( K^+ \pi^0 \) and \( K^+ \pi^- \) decays [3, 6], \( \Delta A_{CP} (K\pi) \) is

\[
\Delta A_{CP} (K\pi) = A_{CP} (K^+ \pi^0) - A_{CP} (K^+ \pi^-) = 0.122 \pm 0.022, \tag{3}
\]

which differs from zero by 5.5 standard deviations. Unfortunately, hadronic uncertainties prevent a clear interpretation of these results in terms of the new-physics implications [3, 10]. Additional information can be obtained through studies of the related vector-pseudoscalar

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decays $B \to K^*\pi$ and $B \to K \rho$, for which the ratios of tree-to-penguin amplitudes are expected to be two to three times larger than for $B \to K \pi$ decays. Hence, $B \to K^*\pi$ and $B \to K \rho$ decays could have considerably larger CP asymmetries.

In this article, we present the results from an amplitude analysis of $B^+ \to K^{*0}(892)^0\pi^0$ decays. The inclusion of charge conjugate processes is implied throughout this article, except when referring to CP asymmetries. This is the first Dalitz plot analysis of this decay by BaBar: the only previous BaBar analysis of this decay was restricted to measuring the branching fraction and CP asymmetry of $B^+ \to K^{0}\rho^+$. An upper limit on the branching fraction for $B^+ \to K^{0}\pi^+\pi^0$ was set by the CLEO Collaboration: $B(B^+ \to K^{0}\pi^+\pi^0) < 6.6 \times 10^{-6}$.

Two contributions to the $K^{0}\pi^+\pi^0$ final state arise from the resonant decays $B^+ \to K^{*}(892)^0\pi^+$ and $B^+ \to K^{+}(892)^0\pi^0$. Although both the rate and CP asymmetries for $B^+ \to K^{*}(892)^0\pi^+$ have been measured, with $K^{0}\to K^{*}\pi^-$, by both the BaBar and Belle Collaborations, the measurements of the rate and CP asymmetry for $B^+ \to K^{*}(892)^0\pi^0$ have significant statistical uncertainties and could benefit from the additional information provided by a full amplitude analysis. In Table I, we review the existing measurements of the rates and CP asymmetries in the $B \to K^{*}(892)\pi$ system.

| Mode               | $B(10^{-6})$ | $A_{CP}$ | References |
|--------------------|--------------|----------|------------|
| $K^{*+}\pi^-$      | 8.5 $\pm$ 0.7 | 0.23 $\pm$ 0.06 | 19, 22     |
| $K^{*+}\pi^0$      | 8.2 $\pm$ 1.8 | 0.06 $\pm$ 0.24 | 18         |
| $K^{*0}\pi^+$      | 9.9 $^{+0.9}_{-0.9}$ | 0.038 $\pm$ 0.042 | 16, 17     |
| $K^{*0}\pi^0$      | 2.5 $\pm$ 0.6 | 0.15 $\pm$ 0.13 | 19, 23     |

This article is organised as follows. The isobar model used to parameterize the complex amplitudes describing the intermediate resonances contributing to the $K^{0}\pi^+\pi^0$ final state is presented in Section II. A brief description of the BaBar detector and the dataset is given in Section III. The event reconstruction and selection are discussed in detail in Section IV. The background study in Section V and a description of the extended maximum likelihood fit in Section VI. The results are given in Section VII and a study of the systematic uncertainties is presented in Section VIII. In Section IX, we provide a summary and conclusion, discussing the results and combining the branching fractions and CP asymmetries for the decays $B^+ \to K^{*}(892)^0\pi^+$, $B^+ \to K_0^{*0}(1430)^0\pi^+$, and $B^+ \to K^{*}(892)^0\pi^0$ with previous BaBar results obtained from the final states $B^+ \to K^{+}\pi^{-}\pi^+$ and $B^+ \to K^{+}\pi^0\pi^0$.

### II. AMPLITUDE ANALYSIS FORMALISM

A number of intermediate states contribute to the decay $B^+ \to K^{*0}_\pm\pi^0_\mp$. Their individual contributions are measured by performing a maximum likelihood fit to the distribution of events in the Dalitz plot formed from the two variables, $m_{K^{0}\pi^+}^2$ and $m_{K^0\pi^0}^2$. We use the Laura++ software to perform this fit.

The total signal amplitudes for the $B^+$ and the $B^-$ decays are given in the isobar formalism by

$$A \left( m_{K^{0}_\pm\pi^0}^2, m_{K^0\pi^0}^2 \right) = \sum_j c_j F_j \left( m_{K^{0}_\pm\pi^0}^2, m_{K^0\pi^0}^2 \right),$$

$$\tilde{A} \left( m_{K^{0}_\mp\pi^0}^2, m_{K^0\pi^0}^2 \right) = \sum_j \tilde{c}_j \tilde{F}_j \left( m_{K^{0}_\mp\pi^0}^2, m_{K^0\pi^0}^2 \right),$$

where $c_j$ is the complex coefficient for a given resonant decay mode $j$ contributing to the Dalitz plot. This complex coefficient contains the weak-interaction phase dependence that is measured relative to one of the contributing resonant channels. In this article we report results for the relative phases between each pair of amplitudes.

The function $F_j$ describes the dynamics of the decay amplitudes and is the product of a resonant lineshape ($R_j$), two Blatt-Weisskopf barrier factors ($X_L$), and an angular-dependent term ($T_{j,L}$):$$F_j = R_j \times \frac{Y_L}{Y_L} \times X_L(|\vec{q}|, |\vec{p}_0|) \times X_L(|\vec{q}|, |\vec{q}_0|) \times T_{j,L}(\vec{p}, \vec{q}),$$

where $L$ is the orbital angular momentum between the intermediate resonance and the bachelor particle (the bachelor particle is the daughter of the $B$ decay that does not arise from the resonance), $\vec{q}$ is the momentum of one of the daughters of the resonance in the rest frame of the resonance, $\vec{p}$ is the momentum of the bachelor particle in the rest frame of the resonance, and $\vec{p}_0$ and $\vec{q}_0$ are the values of $\vec{p}$ and $\vec{q}$, respectively, at the nominal mass of the resonance. The Blatt-Weisskopf barrier factors are given by

$$X_{L=0}(|\vec{u}|, |\vec{u}_0|) = 1,$$

$$X_{L=1}(|\vec{u}|, |\vec{u}_0|) = \sqrt{\frac{1 + z_0}{1 + z}},$$

$$X_{L=2}(|\vec{u}|, |\vec{u}_0|) = \sqrt{\frac{(z_0 - 3)2 + 9z_0}{(z - 3)2 + 9z}},$$

where $z = (|\vec{u}| r_{BW})^2$, $z_0 = (|\vec{u}_0| r_{BW})^2$, $\vec{u}$ is either $\vec{q}$ or $\vec{p}$, and $r_{BW}$ = 4.0 (GeV/c)$^{-1}$ is the meson radius parameter. The uncertainty in $r_{BW}$, used for systematic variations, is $\pm 2$ (GeV/c)$^{-1}$ for the $K^*$ resonances, and ranges from $-1.0$ to $+2.0$ (GeV/c)$^{-1}$ for the $\rho(770)^+$.

The angular term depends on the spin of the resonance and is given by

$$T_{j,L=0} = 1,$$

$$T_{j,L=1} = -2\vec{p} \cdot \vec{q},$$

$$T_{j,L=2} = \frac{4}{3} \left[ 3 (\vec{p} \cdot \vec{q})^2 - (|\vec{p}| \cdot |\vec{q}|)^2 \right].$$
TABLE II: Parameters of the Dalitz plot model for $B^+ \to K_S^0 \pi^+ \pi^0$ used in the nominal fit. The mass and width of the $\rho(770)^+$ and their uncertainties are taken from the analyses by the ALEPH [31] and CMD2 [32] Collaborations. All other parameters are taken from Ref. [28]. The resonance shapes are a Gounaris-Sakurai (GS) function, a relativistic Breit-Wigner (RBW) function, or based on measurements by the LASS Collaboration [33], with $a$ the scattering length and $r$ the effective range of the LASS parametrization.

| Resonance Lineshape | Parameters                  | Resonance mass ($\text{MeV}/c^2$) | Width ($\text{MeV}$) |
|---------------------|----------------------------|-------------------------------------|----------------------|
| $\rho(770)^+$        | GS                         | 775.5 ± 0.6                        | 148.2 ± 0.8          |
| $K^*(892)^+$         | RBW                       | 891.7 ± 0.3                        | 50.8 ± 0.9           |
| $K^*(892)^0$         | RBW                       | 896.1 ± 0.2                        | 50.7 ± 0.6           |
| $(K\pi)_0^0/^+$      | LASS                      | 1412 ± 50                          | 294 ± 80             |

The choice of which resonance daughter is defined to carry the momentum $q$ is a matter of convention. However, its definition is important when comparing measurements from different experiments. In Fig. 1 we illustrate the momentum definitions used for the $K_S^0\pi^+$, $K_S^0\pi^0$, and $\pi^+\pi^0$ resonance combinations.

Table II lists the resonances used to model the signal. We determine a nominal model from data by studying changes in the log likelihood values for the best fit when omitting or adding a resonance to the fit model, as described in Section VII.

For the $K^*(892)^0$ and $K^*(892)^+$ resonances, we use a relativistic Breit-Wigner (RBW) lineshape [28]:

$$R_j^{\text{RBW}}(m) = \frac{1}{m_0^2 - m^2 - im_0\Gamma(m)},$$

where $m$ is the two-body invariant mass and $\Gamma(m)$ is the mass-dependent width. In general, for a resonance decaying to spin-0 particles, $\Gamma(m)$ can be expressed as

$$\Gamma(m) = \Gamma_0 \left( \frac{|q|}{|q_0|} \right)^{2L+1} \left( \frac{m_0}{m} \right) X_L(|q|, |q_0|)^2,$$

where $m_0$ and $\Gamma_0$ are the nominal mass and width of the resonance.

The Gounaris-Sakurai (GS) parametrization [34] is used to describe the lineshape of the $\rho$ resonance decaying into two pions. The parametrization takes the form

$$\rho_j^{\text{GS}} = \frac{1 + \Gamma_0 \cdot d/m_0}{m_0^2 - m^2 + f(m) - im_0\Gamma(m)},$$

where $\Gamma(m)$ is given by Eq. (14). Expressions for $f(m)$, in terms of $\Gamma_0$ and $m$, and the constant $d$ can be found in Ref. [34]. The parameters specifying the $\rho$ lineshape are taken from Refs. [31, 32], which provides lineshape information derived from fits to $e^+e^-$ annihilation and $\tau$ lepton decay data.

For the $J^P = 0^+$ component of the $K\pi$ spectrum, denoted $(K\pi)_0^{*0/+}$, we make use of the LASS parametrization [33], which consists of a $K_0^*$ resonant term together with an effective-range, nonresonant component to describe the slowly increasing phase as a function of the $K\pi$ mass:

$$R_j^{\text{LASS}} = \frac{m}{|q| \cot \delta_B - i |q|} + e^{2i\delta_B} \frac{m_0\Gamma_0}{m_0^2 - m^2 - im_0\Gamma_0} \frac{m_0}{m}|\omega_m|,$$

where $\cot \delta_B = \frac{1 - Q}{\sigma_0} + \frac{i}{2} Q |q|$. The values used for the scattering length $a$ and the effective range $r$ are given in Table II. The effective-range component has a cutoff imposed at $1800 \text{MeV}/c^2$ [16]. Integrating separately the resonant term, the effective-range term, and the coherent sum, we find that the $K_0^*(1430)^0$ and the $K_0^*(1430)^+$ resonances account for 88% of the sum, and the effective range component 49%; the 37% excess is due to destructive interference between the two terms. The LASS parametrization is the least-well-determined component of the signal model; we discuss the impact of these uncertainties in Section VIII.

The complex coefficients $c_j$ and $\bar{c}_j$ in Eqs. (15) can be parametrized in different ways; we follow the parametrization used in Ref. [16] as it avoids a bias in the measurement of amplitudes and phases when the resonant components have small magnitudes:

$$c_j = (x_j + \Delta x_j) + i (y_j + \Delta y_j),$$

$$\bar{c}_j = (x_j - \Delta x_j) + i (y_j - \Delta y_j),$$

where $x_j \pm \Delta x_j$ and $y_j \pm \Delta y_j$ are the real and imaginary parts of the amplitudes. The quantities $\Delta x_j$ and $\Delta y_j$ parametrize the $CP$ violation in the decay. The $CP$
asymmetry for a given intermediate state is given by

\[
A_{CP,j} = \frac{|\bar{c}_j|^2 - |c_j|^2}{|\bar{c}_j|^2 + |c_j|^2} \quad (18)
\]

\[
= -\frac{2(x_j \Delta x_j + y_j \Delta y_j)}{x_j^2 + \Delta x_j^2 + y_j^2 + \Delta y_j^2}. \quad (19)
\]

The results quoted for the resonances in the following analysis use fit fractions (FF) as phase-convention-independent quantities representing the fractional rate of each contribution in the Dalitz plot. The FF for mode \(j\) is defined as

\[
FF_j = \frac{\int \int \left|\mathcal{F}_j\right|^2 \, dm_{K^0} \, dm_{\pi^\pm \pi^\mp}}{\int \int \left|A\right|^2 \, dm_{K^0} \, dm_{\pi^\pm \pi^\mp}}. \quad (20)
\]

The sum of all the fit fractions does not necessarily yield unity due to constructive and destructive interference, as quantified by the interference fit fractions given by [30]

\[
FF_{ij} = \frac{\int \int 2Re\left[c_i c_j^* \mathcal{F}_i \mathcal{F}_j^* \right] \, dm_{K^0} \, dm_{\pi^\pm \pi^\mp}}{\int \int |A|^2 \, dm_{K^0} \, dm_{\pi^\pm \pi^\mp}}. \quad (21)
\]

The parameters \(x_j, \Delta x_j, y_j, \) and \(\Delta y_j\) are determined in the fit, except for the reference amplitude. Fit fractions, relative phases, and asymmetries are derived from the fit parameters and their statistical uncertainties determined from pseudo experiments generated from the fit results.

III. THE Babar DETECTOR AND MC SIMULATION

The data used in the analysis were collected with the Babar detector at the PEP-II asymmetric-energy e^+e^− collider at SLAC National Accelerator Laboratory. The sample consists of 429 fb\(^{-1}\) of integrated luminosity recorded at the \(\Upsilon(4S)\) resonance mass (“on-peak”) and 45 fb\(^{-1}\) collected 40 MeV below the resonance mass (“off-peak”) [32]. The on-peak sample corresponds to the full Babar \(\Upsilon(4S)\) dataset and contains 470.9 ± 2.8 million \(B\bar{B}\) events [30]. A detailed description of the Babar detector is given in Refs. [33, 34]. Charged-particle tracks are measured by means of a five-layer double-sided silicon vertex tracker (SVT) and a 40-layer drift chamber (DCH), both positioned within a solenoid that provides a 1.5 T magnetic field. Charged-particle identification is achieved by combining the information from a ring-imaging Cherenkov detector (DIRC) and specific ionization energy loss (dE/dx) measurements from the DCH and SVT. Photons are detected and their energies measured in a CsI(Tl) electromagnetic calorimeter (EMC). Muon candidates are identified in the instrumented flux return of the solenoid.

We use GEANT4-based software to simulate the detector response and account for the varying beam and experimental conditions [38, 39]. The EvtGen [40] and Jetset7.4 [41] software packages are used to generate signal and background Monte-Carlo (MC) event samples in order to determine efficiencies and evaluate background contributions for different selection criteria.

IV. EVENT SELECTION

We reconstruct \(B^+ \to K_S^0 \pi^+ \pi^0\) candidates from one \(\pi^0\) candidate, one \(K_S^0\) candidate reconstructed from a pair of oppositely charged pions, and a charged pion candidate. The \(\pi^0\) candidate is formed from a pair of neutral energy clusters in the EMC with laboratory energies above 0.05 GeV and lateral moments [42] between 0.01 and 0.6. We require the invariant mass of the reconstructed \(\pi^0\) to lie in the range 0.11 < \(m_{\gamma\gamma}\) < 0.16 GeV/c\(^2\). The \(K_S^0\) candidate is required to have a \(\pi^+\pi^-\) invariant mass within 15 MeV/c\(^2\) of the \(K_S^0\) mass [23], and a proper decay time greater than 0.5 × 10\(^{-11}\)s. To reduce combinatorial background, we also require that the \(K_S^0\) candidates have a vertex probability greater than 10\(^{-6}\) and that the cosine of the angle between the \(K_S^0\) momentum direction and the \(K_S^0\) flight direction (as determined by the interaction point and the \(K_S^0\) vertex) be greater than 0.995. For the \(\pi^+\) candidate, we use information from the tracking systems, the EMC, and the DIRC to select a charged track consistent with the pion hypothesis. We constrain the \(\pi^+\) track and \(K_S^0\) candidate to originate from a common vertex.

Signal events that are misreconstructed with the decay products of one or more daughters completely or partially exchanged with other particles in the rest of the event have degraded kinematic resolution. We refer to these as “self-cross-feed” (SCF) events. This misreconstruction has a strong dependence on the energy of the particles concerned and is more frequent for low-energy particles, i.e., for decays in the corners of the Dalitz plot. Because of the presence of a \(\pi^0\) in the final state, there is a significant probability for signal events to be misreconstructed due to low-energy photons from the \(\pi^0\) decay. Using a classification based on MC information, we find that in simulated events the SCF fraction depends strongly on the resonant substructure of the signal and ranges from 34% for \(B^+ \to K^+(892)^+ \pi^0\) to 50% for \(B^+ \to \rho(770)^+ K_S^0\). In events simulated uniformly in phase space, hereafter referred to as nonresonant MC, the SCF fraction varies from less than 10% in the center of the Dalitz plot to almost 70% in the two corners of the Dalitz plot, where either the \(\pi^0\) or the \(\pi^+\) has low energy. We describe how the SCF events are handled in Section Vf.

In order to suppress the dominant background, due to continuum \(e^+e^- \to q\bar{q} (q = u, d, s, c)\) events, we employ a boosted decision tree (BDT) algorithm that combines four variables commonly used to discriminate jet-like \(q\bar{q}\) events from the more spherical \(B\bar{B}\) events in the \(e^+e^-\) center-of-mass (CM) frame. The first of these is the ratio
of the second-to-zeroth order momentum-weighted Legendre polynomial moments,

\[
\frac{L_2}{L_0} = \frac{\sum_{i \in \text{ROE}} \frac{1}{2} (3 \cos^2 \theta_i - 1) p_i}{\sum_{i \in \text{ROE}} p_i},
\]

where the summations are over all tracks and neutral clusters in the event, excluding those that form the \( B \) candidate (the “rest of the event” or ROE); \( p_i \) is the particle momentum, and \( \theta_i \) is the angle between the particle and the thrust axis of the \( B \) candidate, hereafter also referred to as the \( B \). The three other variables entering the BDT are the absolute value of the cosine of the angle between the \( B \) direction and the collision axis, the zeroth-order momentum-weighted Legendre polynomial moment, and the absolute value of the output of another BDT used for “flavor tagging”, i.e., for distinguishing \( B \) from \( \bar{B} \) decays using inclusive properties of the decay of the other \( B \) meson in the \( \Upsilon(4S) \rightarrow B\bar{B} \) event [33]. The momentum-weighted Legendre polynomial moments and the cosine of the angle between the \( B \) direction and the beam axis are calculated in the \( e^+e^- \) CM frame. The BDT is trained on a sample of signal MC events and off-peak data. We apply a loose criterion on the BDT output where a \( B \) candidate, hereafter also referred to as the \( B \), is defined by \( \cos \theta \rightarrow 0.6 \), which retains approximately 70% of the signal while rejecting 92% of the \( q\bar{q} \) background.

In addition to \( B \) and \( \bar{B} \) decay products, we use two kinematic variables to distinguish the signal from the background:

\[
m_{ES} = \sqrt{E_X^2 - p_B^2},
\]

\[
\Delta E = E_B^* - \sqrt{s}/2,
\]

where

\[
E_X = (s/2 + p_{e^+e^-} \cdot p_B) / E_{e^+e^-},
\]

and where \( s \) is the total \( e^+e^- \) CM energy, with \( (E_{e^+e^-}, p_{e^+e^-}) \) and \( (E_B, p_B) \) the four-momenta of the initial \( e^+e^- \) system and the \( B \) candidate, respectively, both measured in the lab frame, while the star indicates the \( e^+e^- \) CM frame. The signal \( m_{ES} \) distribution for correctly reconstructed events is approximately independent of their position in the \( K_0^0\pi^+\pi^0 \) Dalitz plot and peaks near the \( B \) mass with a resolution of about 3.4 MeV/c^2.

We retain all candidates satisfying the following selection criteria: \( 5.23 < m_{ES} < 5.29 \text{ GeV}/c^2 \) and \( -0.3 < \Delta E < 0.3 \text{ GeV} \). The signal region, where the final fit to data is performed, is defined by the tighter criteria \( 5.260 < m_{ES} < 5.287 \text{ GeV}/c^2 \) and \( -0.20 < \Delta E < 0.15 \text{ GeV} \). We also use candidates in the sideband region of \( m_{ES} \) defined by \( 5.23 < m_{ES} < 5.26 \text{ GeV}/c^2 \) and \( -0.20 < \Delta E < 0.15 \text{ GeV} \) and subtract from distributions for these events the \( B\bar{B} \) background contributions predicted by MC simulations. We then add these distributions to the off-peak data distributions to increase the statistical precision of our model of the Dalitz plot distribution for continuum background.

Each of the \( B \) candidates is refit to determine the Dalitz plot variables. In these fits the \( K_0^0\pi^+\pi^0 \) invariant mass is constrained to the world average value of the \( B \) mass [28] to improve position resolution within the Dalitz plot.

We find that 20% of the remaining events in nonresonant MC have two or more candidates. We choose the best candidate in multiple-candidate events based on the highest \( B \)-vertex probability. This procedure is found to select a correctly reconstructed candidate more than 60% of the time and does not bias the fit variables.

The reconstruction efficiency over the Dalitz plot is modeled using a two-dimensional (2D) binned distribution based on a generated sample of approximately \( 2 \times 10^6 \) simulated \( B^+ \rightarrow K_0^0\pi^+\pi^0 \) MC events, where the events uniformly populate phase space. All selection criteria are applied except for those corresponding to a \( K\pi \) invariant-mass veto described below, which is taken into account separately. The 2D histogram of reconstructed MC events is then divided by the 2D histogram of the generated MC events. In order to expand regions of phase space with large efficiency variations, the Dalitz plot variables are transformed into “square Dalitz plot” coordinates. We obtain an average efficiency, for nonresonant MC events, of approximately 15%. In the likelihood fit we use an event-by-event efficiency that depends on the Dalitz plot position.

V. \( B\bar{B} \) BACKGROUNDs

In addition to continuum events, background arises from non-signal \( B\bar{B} \) events. A major source of \( B\bar{B} \) background arises from \( B^+ \rightarrow \Upsilon(1S) \rightarrow K_0^0\pi^+\pi^0 \) decays. To suppress this background, we veto events with \( 1.804 < m_{K_0^0\pi^0} < 1.924 \text{ GeV}/c^2 \).

The remaining \( B\bar{B} \) backgrounds are studied using MC simulations and classified based on the shape of the \( m_{ES}, \Delta E, \) and Dalitz plot distributions. We identify nine categories of \( B\bar{B} \) backgrounds: categories 1, 2 and 3 include different types of three- and four-body \( B \) decays involving an intermediate \( D \) meson; categories 4 and 5 include charmless four-body \( B \) decays to intermediate resonances where a \( \pi^0 \) in the final state is not reconstructed; categories 6 and 7 include two-body \( B \) decays with a radiated photon misreconstructed as a \( \pi^0 \) decay product or where the \( \pi^0 \) arises from the other \( B \) decay; category 8 includes charmless three-body \( B \) decays where a charged pion is interchanged with a \( \pi^0 \) meson from the other \( B \); and finally category 9 includes all other simulated \( B\bar{B} \) background contributions. Within each category, each of the \( m_{ES}, \Delta E, \text{BDT}_{out}, \) and Dalitz plot distributions are formed by combining the contributions of all decay modes in the category. The combinations are done by normalizing the distributions for each decay mode to the expected number of events in the recorded data sample, which is estimated using reconstruction efficiencies determined from MC, the number of \( B\bar{B} \) pairs in the
recorded data sample, and the branching fractions listed in Refs. [3, 28]. For each category, the histograms of \( m_{ES}, \Delta E, \text{BDT}_{out} \), and the Dalitz plot variables are used as the probability density functions (PDF) in the likelihood fit to data to model the \( B\bar{B} \) background.

VI. THE MAXIMUM LIKELIHOOD FIT

The extended likelihood function is given by

\[
L = \exp \left( -\sum_k N_k \right) \times \prod_{i=1}^{N_c} \left[ \sum_k N_k P_k \left( m_{K_S^0\pi^+}, m_{\pi^0\pi^0}, m_{ES}, \Delta E, \text{BDT}_{out}, q_B \right) \right],
\]

where \( N_k \) is the number of candidates in each signal or background category \( k \), \( N_c \) is the total number of events in the data sample, and \( P_k \) (the PDF for category \( k \) and event \( i \)) is the product of the PDFs describing the Dalitz plot, \( m_{ES}, \Delta E, \) and \( \text{BDT}_{out} \) distributions, with \( q_B \) the charge of the \( B \) candidate.

To avoid possible biases in the determination of the fit parameters \([45]\), we use MC samples to study correlations between the fit variables and the Dalitz plot parameters, \( m_{K_S^0\pi^+} \) and \( m_{\pi^0\pi^0} \). We find that for correctly reconstructed signal candidates, the \( \Delta E \) distribution is strongly dependent on \( m_{K_S^0\pi^+} \). This is mostly due to a dependence of the energy resolution of the \( B \) candidate on the \( \pi^0 \) momentum. For SCF signal candidates, both the \( m_{ES} \) and \( \Delta E \) distributions depend on all three two-body invariant masses: \( m_{K_S^0\pi^+}, m_{K_S^0\pi^0}, \) and \( m_{\pi^0\pi^0} \).

The \( m_{ES}, \Delta E, \) and \( \text{BDT}_{out} \) distributions for continuum and \( B\bar{B} \) backgrounds have negligible correlations with the Dalitz plot parameters.

For correctly reconstructed signal candidates, the \( m_{ES} \) and \( \Delta E \) PDFs are parameterized by a Cricuiff function, which is given by (omitting normalization factor)

\[
f_{\text{Cricuiff}}(x) = \exp \left[ \frac{-(x-m)^2}{2\sigma^2_{L,R} + \alpha L,R (x-m)^2} \right],
\]

where \( m \) gives the peak of the distribution and the asymmetric width of the distribution is given by \( \sigma_L \) for \( x < m \) and \( \sigma_R \) for \( x > m \). The asymmetric modulation is similarly given by \( \alpha_L \) for \( x < m \) and \( \alpha_R \) for \( x > m \). The \( \Delta E \) PDF parameters are calculated on an event-by-event basis in terms of the \( K_S^0\pi^+ \) invariant mass, as a linear function for \( m_{K_S^0\pi^+} \leq 20 \text{GeV}/c^2 \) and as a quadratic function for \( m_{K_S^0\pi^+} > 20 \text{GeV}/c^2 \). These functions are determined by fitting the \( \Delta E \) distribution in large non-resonant MC samples. For the SCF signal, in order to follow the rapid shape variations across the Dalitz plot of the \( m_{ES} \) and \( \Delta E \) distributions, we divide the Dalitz plot into several regions as illustrated in Fig. 2. Each letter indicates whether the dependence is on \( m_{\pi^0\pi^0}, m_{K_S^0\pi^+}, \) or \( m_{K_S^0\pi^0} \). The regions are chosen based on the distribution in the Dalitz plot of the SCF fraction and the mean difference between the true and reconstructed position in the Dalitz plot; we include more regions in areas of the Dalitz plot where these quantities are largest. We use \( m_{ES} \) and \( \Delta E \) PDFs specific to each region, as listed in Table III.

Some of the PDFs used in the parametrization of the SCF include Cricuiff functions, Chebychev polynomials, Gaussian functions, and two-piece Gaussian (BGauss) functions. A two-piece Gaussian function is an asymmetric Gaussian described by the following functional form (omitting normalization factor)

\[
f_{\text{BGauss}}(x) = \exp \left[ \frac{-(x-m)^2}{2\sigma^2_{L,R}} \right].
\]

For the continuum background, we use an ARGUS function \([46]\) to parameterize the \( m_{ES} \) shape. The \( \Delta E \) distribution is described by a linear function, and the \( \text{BDT}_{out} \) distribution by an exponential function. The \( m_{ES}, \Delta E, \) and \( \text{BDT}_{out} \) PDFs for \( B\bar{B} \) backgrounds are defined by the sums of the histograms from the MC simulations for decay modes in each background category, as described in Section VII.

The continuum and \( B\bar{B} \) background Dalitz plot distributions are included in the likelihood as two-dimensional histograms. For \( B\bar{B} \) backgrounds, we use MC samples. For continuum background, we combine events from the off-peak data and the \( m_{ES} \) sideband in on-peak data, after subtracting contributions from \( B \) decays, as described in Section VII. For the 2D histograms, we use the square Dalitz plot coordinates. A linear interpolation between bin centers is applied.

The free parameters in the fit are the yields for signal,
TABLE III: List of PDFs used to describe the $m_{ES}$ and $\Delta E$ self-crossed signal distributions in each of the regions of the $B^+ \to K_S^0\pi^+\pi^0$ Dalitz plot shown in Fig. 2. The abbreviations correspond to the following functional forms: Crujiff function described in Eq. (27) (Crujiff), Chebychev polynomial (Cheb), Gaussian (Gauss), two-piece Gaussian described in Eq. (28) (BGauss), and exponential (Exp).

| Dalitz plot region | $m_{ES}$ PDF | $\Delta E$ PDF |
|--------------------|--------------|---------------|
| $m_{K_S^0\pi^+}$ (A1) | Crujiff | Crujiff |
| $m_{K_S^0\pi^+}$ (B1) | Cheb+Gauss | Exp+SigSigmoid |
| $m_{K_S^0\pi^+}$ (B2) | Cheb+Gauss | linear+BGauss |
| $m_{K_S^0\pi^+}$ (B3) | Crujiff | Exp+SigSigmoid |
| $m_{K_S^0\pi^+}$ (C1) | Cheb+Gauss | Cheb |
| $m_{K_S^0\pi^+}$ (C2) | Cheb+Gauss | Cheb |
| $m_{K_S^0\pi^+}$ (C3) | Crujiff | Cheb |
| $m_{K_S^0\pi^+}$ (C4) | Crujiff | Crujiff |
| Central region (D1) | Crujiff | Crujiff |

continuum background, and $B\bar{B}$ background categories 1 and 9. The yields for the remaining $B\bar{B}$ background categories are fixed to the estimated values. All the PDF parameters for the correctly reconstructed $m_{ES}$ and $\Delta E$ PDFs, except for the tail parameters, are determined in the fit. All SCF signal PDF parameters are fixed to values obtained from fits to nonresonant MC events. The endpoint of the ARGUS function is fixed to 5.289 GeV/c$^2$ while the shape parameter is determined in the fit. The slope for the linear function of the $\Delta E$ PDF and the exponent for the exponential function of the BDT$_{out}$ PDF for continuum background are similarly determined in the fit. The isobar coefficients, $x$ and $y$ in Eq. (18), for all but one of the isobar components are fitted parameters in the fit and are measured relative to the fixed isobar component. The coefficients for the reference isobar are fixed to $x = 1$ and $y = 0$. In total, the fit is performed with 21 free parameters.

TABLE IV: Fit fractions obtained from the fit to data when each additional isobar is added to the fit model one at a time.

| Additional isobar | Fit fraction |
|-------------------|--------------|
| $\rho(1450)^+$     | 0.042 ± 0.044 |
| $K_2^0(1430)^0$    | 0.038 ± 0.017 |
| $K_2^0(1430)^+$    | 0.012 ± 0.020 |
| $K^+(1680)^0$      | 0.032 ± 0.034 |
| $K^+(1680)^+$      | 0.005 ± 0.030 |

We determine a nominal signal Dalitz plot model based on information from previous studies [16,18,22], and on the changes in the log likelihood in the fit to data when resonances are added to, or removed from, the list shown in Table III. In these fits to the combined $B^+$ and $B^-$ data samples, the CP coefficients $\Delta x$ and $\Delta y$ are fixed to zero. We do not find significant contributions in the fit when adding the resonances $\rho(1450)^+$, $K_2^0(1430)^0$, $K_2^0(1430)^+$, $K^+(1680)^0$, or $K^+(1680)^+$, one at a time to the default model. We observe that the fit fractions for these additional resonances, reported in Table V, are consistent with zero. The most statistically significant of these fit fractions is FF $(K_2^0(1430)^0\pi^+)$ = 0.038 ± 0.017; since the statistical significance is less than 2 standard deviations, we do not include any of the additional resonances in the nominal fit.

We do not observe an excess of events for invariant masses greater than 2 GeV/c$^2$, suggesting that a nonresonant component, in addition to that included in the LASS parametrization, is not necessary. We observe that if we add a nonresonant component to the fit, the change in log likelihood for the binned data and the fit projections for the $K_2^0\pi^+$, $K_2^0\pi^0$, and $\pi^+\pi^0$ invariant masses are consistent with the expected change due to the additional free parameters in the fit, and do not indicate any statistically significant nonresonant component. We therefore conclude that, with the current level of statistical sensitivity, the base model, which includes the $\rho(770)^+$, $K^+(892)^0$, $K^+(892)^0$, $(K\pi)^0_0$, and $(K\pi)^0_0$ resonances, provides an adequate description of the data.

VII. RESULTS

We apply the fit described in Section VI to the 31,876 selected $B^+ \to K^0_0\pi^+\pi^0$ candidates. A first fit is performed on the combined $B^+$ sample. We obtain yields of 1014 ± 60 signal events, 24,381 ± 200 continuum events, 2745 ± 70 $B\bar{B}$ events in category 1, and 1768 ± 140 $B\bar{B}$ events in category 9. The results of the fit are shown in Fig. 3. For the purpose of this figure, the contributions of signal events are enhanced by applying the more restrictive selection criteria listed in Table V.

TABLE V: Selection criteria imposed to enhance the contributions of signal events for the results presented in Figs. 3 and 4.

| Projection plot | Selections |
|-----------------|------------|
| $m_{ES}$        | $-0.05 < \Delta E < 0.05$ GeV, BDT$_{out} > 0.1$ |
| $\Delta E$      | $m_{ES} > 5.27$ GeV/c$^2$, BDT$_{out} > 0.1$ |
| BDT$_{out}$     | $m_{ES} > 5.27$ GeV/c$^2$, $-0.05 < \Delta E < 0.05$ GeV |
| $m_{K_2^0\pi^+}$, $m_{K_2^0\pi^0}$, $m_{K^+(1680)^0}$, $m_{K^+(1680)^+}$ | $m_{ES} > 5.27$ GeV/c$^2$, $-0.05 < \Delta E < 0.05$ GeV, BDT$_{out} > 0.1$ |

The branching fraction for $B^+ \to K^0_0\pi^+\pi^0$ is determined from the number of signal events, the efficiency estimated from MC events, and the total number of $B\bar{B}$ events in data. We take into account differences between the $\pi^0$ reconstruction efficiency in data and MC events, determined from control samples with either $\tau$ leptons or initial-state radiation, as a function of $\pi^0$ momentum ($\epsilon_{MC} = 97.2\%$, averaged over $\pi^0$ momentum). We correct for small biases in the branching fraction, as determined from MC pseudo experiments generated with the
FIG. 3: Combined $B^\pm$ fit: Measured distributions and fit projections for $B^\pm \rightarrow K_\pm^0 \pi^\pm \pi^0$ candidates; (a) $m_{ES}$, (b) $\Delta E$, (c) $BDT_{out}$, (d) $m_{K_1^0 \pi^\pm}$, (e) $m_{K_{L}^0 \pi^0}$, and (f) $m_{\pi^+ \pi^-}$. The points with error bars correspond to data, the solid (blue) curves to the total fit result, the dashed (green) curves to the total background contribution, and the dotted (red) curves to the continuum background component. The dash-dotted curves represent the signal contribution. The projected distributions are obtained from statistically precise pseudo experiments generated using the fit results. For all distributions in each panel, the signal-to-background ratio is increased by applying tighter selection requirement on $m_{ES}$, $\Delta E$, and/or $BDT_{out}$, listed in Table V.

same number of signal events and resonance composition as found in the fit to data. We divide the partial branching fraction of $B^+ \rightarrow K^0_{\pm} \rightarrow \pi^0 \rightarrow \pi^+ \pi^- \pi^0 \pi^0$ by the branching fraction for $K^0_{\pm} \rightarrow \pi^+ \pi^-$, and multiply the result by a factor of 2 to account for $K^0_L$ decay, to obtain the branching fraction result $B (B^+ \rightarrow K^0_{\pm} \pi^0 \pi^0) = (45.9 \pm 2.6 \pm 3.0^{+8.6}_{-8.0}) \times 10^{-6}$, where the first uncertainty is statistical, the second is systematic, and the third is due to assumptions made concerning the signal model. The latter two uncertainties are described in Section VIII and the breakdown of the systematic uncertainties is detailed in Table XI.

We measure amplitudes and phases relative to each of the five two-body decays in the signal model to take advantage of the smaller uncertainty observed when measuring the relative phases of the two pairs of decays with same-charge $K^*$ resonances. Table VI lists the relative phase, $\phi$, between each pair of two-body decays in the
TABLE VI: Combined $B^\pm$ fit: Relative phases, $\phi$, for the isobar amplitudes as measured from five fits to data, where each of the five isobar amplitudes is in turn taken as the reference. All phases are quoted in degrees. The uncertainties are statistical only.

| Reference amplitude | Resonant contribution | Relative phase (degrees) |
|---------------------|-----------------------|--------------------------|
| $B^+ \rightarrow K^+(892)^0\pi^+$ | $K^+(892)^0\pi^+$ | 0 $-95 \pm 43$ |
| $B^+ \rightarrow K^+(892)^+\pi^0$ | $K^+(892)^+\pi^0$ | $-90 \pm 42$ |
| $B^+ \rightarrow (K\pi)_0^0\pi^+$ | $(K\pi)_0^0\pi^+$ | $96 \pm 42$ |
| $B^+ \rightarrow (K\pi)_0^+\pi^0$ | $(K\pi)_0^+\pi^0$ | $0 \pm 32 \pm 25$ |
| $B^+ \rightarrow \rho(770)^+K^0_S$ | $\rho(770)^+K^0_S$ | $0 \pm 32 \pm 25$ |

TABLE VII: Combined $B^\pm$ fit: Results for the fit fractions FF$_j$ (diagonal terms) and interference terms FF$_{ij}$ in data for each resonant contribution. The uncertainties are statistical only.

| Resonant contribution | FF$_j$ and FF$_{ij}$ |
|-----------------------|----------------------|
| $B^+ \rightarrow K^+(892)^0\pi^+$ | $K^+(892)^0\pi^+$ $0.10 \pm 0.03$ $0.0004 \pm 0.0028$ $0.007 \pm 0.0005$ $-0.008 \pm 0.0007$ |
| $B^+ \rightarrow K^+(892)^+\pi^0$ | $K^+(892)^+\pi^0$ $-0.10 \pm 0.07$ $0.14 \pm 0.02$ $-0.010 \pm 0.007$ $(-3 \pm 1) \times 10^{-6}$ $0.012 \pm 0.008$ |
| $B^+ \rightarrow (K\pi)_0^0\pi^+$ | $(K\pi)_0^0\pi^+$ $0.36 \pm 0.05$ $1.56 \pm 1.01 \times 10^{-5}$ $0.04 \pm 0.02$ |
| $B^+ \rightarrow (K\pi)_0^+\pi^0$ | $(K\pi)_0^+\pi^0$ $0.27 \pm 0.03$ $-0.02 \pm 0.02$ |
| $B^+ \rightarrow \rho(770)^+K^0_S$ | $\rho(770)^+K^0_S$ $0.19 \pm 0.04$ |

signal model and its uncertainty. The statistical uncertainty in the relative phase is smallest ($\approx 10^\circ$) for the resonances that decay to the same-charge $K\pi$ state. This is due to a larger overlap in the Dalitz plot between the same-charge $K^+$ resonances than occurs for other pairs of resonances that only overlap in the corners of the Dalitz plot.

Since the statistical uncertainties of the fit fractions do not depend on the reference mode, we quote in Table VII only the fit fractions from a fit relative to the $K^+(892)^0\pi^+$ amplitude. The fit fractions for the $K^0_S(1430)^0\pi^+$ and $K^0_S(1430)^+\pi^0$ modes are the product of the $(K\pi)_0^0$ S-wave fit fraction, shown in Table VII and the fraction due to the resonant contribution in the LASS parametrisation (88%). The off-diagonal fit fractions are small compared to the diagonal elements. We calculate the branching fractions for the resonant contributions shown in Table VIII as the product of the total branching fraction and the fit fractions returned by the fit to data, including appropriate Clebsch-Gordan coefficients.

To determine the overall CP asymmetry as well as the CP asymmetries for the contributing isobar components, we simultaneously fit the separate $B^+$ and $B^-$ data samples. The overall $A_{CP}$ value is calculated from the integrals of the positive and negative signal Dalitz plot distributions. The $\Delta x$ and $\Delta y$ parameters from Eq. (13) are allowed to vary in the fit for all components except the reference isobar, for which the $\Delta y$ parameter is fixed to zero (the relative phase of the $B^+$ and $B^-$ Dalitz plots cannot be determined since they do not interfere). To account for possible differences in the reconstruction and particle identification efficiencies for $B^+$ and $B^-$, the efficiency map as a function of the Dalitz plot position is determined separately for $B^+$ and $B^-$. The asymmetry for the continuum background is allowed to vary in the fit. The CP asymmetries of the $B\bar{B}$ backgrounds are expected to be small and so are fixed to zero in the nominal fit. They are varied within reasonable ranges based on world average experimental results [28] in order to determine the associated systematic uncertainty.

We find an overall CP asymmetry of $A_{CP} (B^+ \rightarrow K^0\pi^+\pi^0) = 0.07 \pm 0.05 \pm 0.03$, where the first uncertainty is statistical, the second is systematic, and the third is due to the signal model. This is consistent with zero CP asymmetry. Invariant mass projections for the fit to data allowing for direct CP violation are shown in Fig. 11.

Table VIII shows the results for the branching fractions and CP asymmetries obtained from the fit to data. The first uncertainty is statistical, the second is system-
FIG. 4: The CP fit: Measured distributions and fit projections for $B^+ \rightarrow K^0\pi^+\pi^0$ (left column) and $B^- \rightarrow K^0\pi^-\pi^0$ (right column) candidates; (a) $m_{K^0\pi^+}$, (b) $m_{K^0\pi^-}$, (c) $m_{K^0\pi^0}$ (from $B^+ \rightarrow K_S^0\pi^+\pi^0$), (d) $m_{K^0\pi^0}$ (from $B^- \rightarrow K_S^0\pi^-\pi^0$), (e) $m_{\pi^+\pi^0}$, and (f) $m_{\pi^-\pi^0}$. The points with error bars correspond to data, the solid (blue) curves to the total fit result, the dashed (green) curves to the total background contribution, and the dotted (red) curves to the continuum background component. The dash-dotted curves represent the signal contribution. The projected distributions are obtained from statistically precise pseudo experiments generated using the fit results. For all distributions in each panel, the signal-to-background ratio is increased by applying the tighter selection requirements on $m_{ES}$, $\Delta E$, and/or BDT$_{out}$, listed in Table V.

We observe a significant asymmetry between the $m_{K^0\pi^+}$ and $m_{K^0\pi^-}$ distributions in the region of the $K^+(892)^+$ resonance; see Figs. (a) and (b). We determine the statistical significance, $S$, of a non-zero CP asymmetry in $B^+ \rightarrow K^+(892)^+\pi^0$ from the difference between the best-fit value of the likelihood, $L_{ACP}$, and the value when the CP asymmetry is fixed to zero, $L_0$:

$$S = \sqrt{-2 \ln \left( \frac{L_0}{L_{ACP}} \right)}.$$  \hspace{1cm} (29)

Using this method, we measure a statistical significance of 3.6 standard deviations for a non-zero $A_{CP}$ in $B^+ \rightarrow K^+(892)^+\pi^0$. We obtain a consistent result of 3.7 standard deviations for the statistical significance.
Table IX presents the results, measured with respect to amplitudes, other resonances the CP deviations of zero. defined in Eq. (5), for \( B \) between 70 and 90 amplitudes, the statistical uncertainty ranges between 0.1 and 2 standard deviations. The contours are estimated by calculating the uncertainty and correlation between the two \( CP \) parameters. The stars indicate the central values of the \( CP \) parameters and the cross sign the origin of the plot.

by dividing the central value of the \( CP \) asymmetry by the statistical uncertainty, indicating that the log-likelihood function is close to parabolic. Figure 5 displays the contours in the complex plane of the coefficients \( c = (x + \Delta x, y + \Delta y) \), defined in Eq. (4), for \( B^+ \rightarrow K^+(892)^+\pi^0 \) decays, and of \( \bar{c} = (x - \Delta x, y - \Delta y) \), defined in Eq. (5), for \( B^- \rightarrow K^+(892)^-\pi^0 \) decays. For other resonances the \( CP \) asymmetry is within 2 standard deviations of zero.

We also express the complex isobar coefficients \( c \) and \( \bar{c} \) of Eq. (18) in terms of amplitudes and phases,

\[
c = A_+ e^{i\phi_+},
\]

\[
\bar{c} = A_- e^{-i\phi_-}.
\]

Table 10 presents the results, measured with respect to the \( B^\pm \rightarrow K^*(892)^0\pi^\pm \) reference amplitude. The statistical uncertainties of the separate \( B^+ \) and \( B^- \) decay amplitudes, \( A_+ \) and \( A_- \), vary between 0.1 and 0.3. We thus obtain significant statistical precision for these terms. With respect to the phases, \( \phi_+ \) and \( \phi_- \), only the \( (K\pi)_0^0 \) amplitude yields a statistically precise result. For the other amplitudes, the statistical uncertainty ranges between 70° and 170°, and only the statistical uncertainty is quoted. For the more precisely determined variables, systematic uncertainties are evaluated as well. For the phases of the \( B^\pm \rightarrow (K\pi)_0^0 \) decays relative to the \( B^\pm \rightarrow K^*(892)^0\pi^0 \) amplitude, we obtain

\[
\phi_+ ((K\pi)_0^0 + \pi^0) - \phi_+ (K^*(892)^+\pi^0) = (14 \pm 18 \pm 9_{-6}^{+4})^o,
\]

\[
\phi_- ((K\pi)_0^0 - \pi^0) - \phi_- (K^*(892)^-\pi^0) = (11 \pm 19 \pm 10_{-7}^{+17})^o.
\]

VIII. SYSTEMATIC UNCERTAINTIES

We evaluate systematic uncertainties to account for effects that could affect the branching fractions, phases, and asymmetries, by varying the fixed parameters. The systematic uncertainties described in this section are summarized in Tables XI through XVI of Appendix A. The uncertainties associated with the branching fractions are listed in Table XI. To estimate the uncertainty related to the modeling of the SCF PDFs, we implement a simpler model consisting of only four regions in the Dalitz plot. The PDFs are redefined using MC events to match the distributions found in the newly defined regions. We then fit the data using the new SCF model and take the uncertainties to be the change in the fit parameters compared to those obtained from the nominal fit to data. All relative systematic uncertainties due to the SCF uncertainties \( m_{ES} \) and \( \Delta E \) PDFs range from approximately 1% to 4%, except for the relative systematic uncertainty for the \( B^+ \rightarrow \rho(770)^+K^0 \) decay, which is 7.5%. This is consistent with expectations from simulation that more than half the \( B^+ \rightarrow \rho(770)^+K^0 \) events are due to SCF.

The uncertainties associated with the number of \( B \bar{B} \) background events are evaluated by varying the estimates within their uncertainties, which are primarily due to uncertainties in the branching fractions. The uncertainties related to the \( B \bar{B} \) background \( m_{ES} \), \( \Delta E \), and \( BDT_{\text{out}} \) PDFs are accounted for by varying the histogram bin contents according to their statistical uncertainties. The uncertainty is then taken as the RMS of the distribution of the difference in the fit parameters. The uncertainties related to the limited statistical precision of the MC and data-sideband samples are similarly accounted for by varying the results in the corresponding histogram bins by their uncertainties.

The uncertainty in the BDT_{\text{out}} histogram PDFs for correctly reconstructed and SCF signal events is determined by varying the bin contents in accordance with the observed data/MC difference. For correctly reconstructed signal events, the tails of the asymmetric Gaussian PDFs for \( m_{ES} \) and \( \Delta E \) are fixed. To account for an associated uncertainty, we allow the relevant parameters to vary in a fit to data and use the variation in the fit parameters to define the uncertainty.

To validate the fitting procedure, 500 MC pseudo experiments are generated, using the PDFs with parameter values found from the fit to data. Small fit biases are found for some of the fit parameters and are included in the systematic uncertainties.

We also account for uncertainties in the following parameters describing the signal model: the mass and width of each resonance and the value of the Blatt-Weisskopf barrier radius. The associated uncertainties are determined by varying the parameters within their uncertainties (some of which are given in Table I) and refitting.

The uncertainties in the branching fractions related to particle identification, tracking efficiency, and the total number of \( B \bar{B} \) events are 1.0%, 1.0%, and 0.6%, re-
TABLE IX: Results for the relative phases \( \phi \) obtained from the combined \( B^{\pm} \) fit, the \( \text{CP} \) amplitudes \( A_+ \) and \( A_- \), and the \( \text{CP} \) phases \( \phi_+ \) and \( \phi_- \) obtained from the \( \text{CP} \) fit. All parameters are measured relative to the \( B^{\pm} \rightarrow K^*(892)^0 \pi^{\pm} \) reference amplitude. The first uncertainty is statistical, the second is systematic, and the third is due to the signal model. Note that for the \( \text{CP} \) phases of all contributions except for \( B^{\pm} \rightarrow (K\pi)_0^{\pm} \pi^{\pm} \), only statistical uncertainties are quoted.

| Isobar                                    | \( \phi (^\circ) \) | \( A_+ \)     | \( A_- \)     | \( \phi_+ (^\circ) \) | \( \phi_- (^\circ) \) |
|-------------------------------------------|----------------------|--------------|--------------|----------------------|----------------------|
| \( K^*(892)^0 \pi^0 \)                   | \(-95 \pm 43^{+38}_{-36}^{+38} \) | 1.46 \pm 0.22 \pm 0.05 \pm 0.05 | 0.82 \pm 0.18 \pm 0.05 \pm 0.06 | \(-10 \pm 112 \) | \(-98 \pm 97 \) |
| \( (K\pi)_0^{\pm} \pi^{\pm} \)          | 174 \pm 11 \pm 11^{-6} \pm 10^7 \pm 6 | 1.74 \pm 0.21 \pm 0.11 \pm 0.03 | 2.00 \pm 0.27 \pm 0.13 \pm 0.02 | 165 \pm 19 \pm 9^{-1} \pm 1 | 190 \pm 21 \pm 11^{-1} \pm 1 |
| \( (K\pi)_0^{\pm} \pi^{0} \)            | \(-89 \pm 43^{-5}_{-40}^{+5} \pm 17 \) | 1.44 \pm 0.22 \pm 0.13 \pm 0.06 | 1.88 \pm 0.25 \pm 0.14 \pm 0.06 | 4 \pm 111 | \(-109 \pm 92 \) |
| \( \rho(770)^+ K^0 \)                    | \(-122 \pm 43^{-3}_{-47}^{+5} \pm 66 \) | 1.24 \pm 0.01 \pm 0.09 \pm 0.09 | 1.54 \pm 0.01 \pm 0.09 \pm 0.09 | \(-50 \pm 168 \) | \(-120 \pm 71 \) |

respectively. We estimate systematic uncertainties in the branching fractions associated with the \( \pi^0 \) and \( K^0 \) reconstruction efficiencies to be 1.0% and 1.1%, respectively.

Uncertainties from all the above sources are added in quadrature to yield the total systematic uncertainties, which are listed in Table XI.

We determine changes in the branching fractions, \( \Delta B \), when the signal model is varied. The systematic uncertainties in the branching fractions due to the \( (K\pi)_0^{\pm} \pi^{\pm} \) parametrization are estimated by replacing the LASS model with another phenomenologically inspired parametrization [17]. We take the differences in branching fractions with respect to the nominal fit as the systematic uncertainty. This is the largest contribution to the uncertainty due to the model. Another uncertainty reflects any changes in the fit parameters for the nominal model when including components that are omitted in the nominal fit, such as the \( \rho(1450)^0 \), \( K_2^0(1430)^0 \), and \( K_2^+(1430)^+ \). Positive and negative variations are added separately in quadrature to obtain the systematic uncertainties due to the model, listed in Table XII.

We determine systematic uncertainties in the phases averaged over \( B^+ \) and \( B^- \) decays from the same sources as considered for the branching fractions. The variations in the phases are measured relative to the \( K^*(892)^0 \pi^+ \) amplitude. Since the differences between positive and negative shifts in the phases, shown in Table XIII, are large in some cases, we quote for those phase shifts asymmetric systematic uncertainties.

Reconstruction and particle identification efficiencies cancel to first order in the fit to \( \text{CP} \) asymmetries; therefore the only uncertainties that are included for \( A_{\text{CP}} \) are those coming from the fit and signal model. In addition to this, we do not evaluate any of the uncertainties that are found to be negligible for the branching fractions.

An additional uncertainty for \( A_{\text{CP}} \) arises from having fixed the \( \text{CP} \) asymmetries for individual \( B\bar{B} \) background components to the mean asymmetry averaged over all such components. We take the largest variation of each background asymmetry as the corresponding uncertainty.

The uncertainty related to the efficiency model is determined by exchanging the efficiency maps for the positive and negative Dalitz plots and refitting the data. We then take the difference in \( \text{CP} \) asymmetry with respect to the nominal fit as the uncertainty.

We list in Table XIV the systematic uncertainties associated with the signal \( \text{CP} \) asymmetries and the variations in the asymmetry due to changes in the signal composition.

We evaluate systematic uncertainties for the \( \text{CP} \) amplitudes and \( \text{CP} \) phases from the same sources as for the \( \text{CP} \) asymmetries. We list the variations to the amplitudes \( A_+ \) in Table XV and to the amplitudes \( A_- \) in Table XVI including the uncertainties due to changes to the signal model. Table XVII lists the systematic variations and model uncertainties for \( \phi_+(-) \) \( (K\pi)_0^{\pm} \pi^{\pm} \) and \( \phi_+(-) \) \( (K\pi)_0^{\pm} \pi^{\pm} \).

**IX. SUMMARY AND CONCLUSION**

The measured branching fractions and \( \text{CP} \) asymmetries are summarized in Table VIII and the amplitude and phase values in Table IX including statistical, systematic, and model uncertainties. We have measured for the first time the branching fraction and \( \text{CP} \) asymmetry for the decay \( B^+ \rightarrow K^0 \pi^+ \pi^0 \). We obtain first evidence for direct \( \text{CP} \) violation in the intermediate decay \( B^+ \rightarrow K^*(892)^0 \pi^0 \), with a total significance of 3.4 standard deviations determined by adding statistical, systematic, and signal-model uncertainties in quadrature and dividing the measured \( A_{\text{CP}} \) by the total uncertainty.

In addition, we have measured the branching fractions, \( \text{CP} \) asymmetries, and relative \( \text{CP} \)-averaged phase values of the decays \( B^+ \rightarrow K^*(892)^0 \pi^+, B^+ \rightarrow K^*(892)^0 \pi^0, B^+ \rightarrow K_2^0(1430)^0 \pi^+, B^+ \rightarrow K_2^+(1430)^+ \pi^0, \) and \( B^+ \rightarrow \rho(770)^+ K^0 \). The results for the branching fractions and \( \text{CP} \) asymmetries for \( B^+ \rightarrow K^*(892)^0 \pi^+ \) are consistent with the previous measurement from \( B^+ \rightarrow K^+ \pi^- \pi^+ \) decays by the Belle and \( \text{BaBar} \) Collaborations and the results for \( B^+ \rightarrow K_2^0(1430)^0 \pi^+ \) are within two standard deviations from the previous \( \text{BaBar} \) measurement [16, 17]. The branching fraction for \( B^+ \rightarrow K^*(892)^0 \pi^0 \) is consistent with the previous measurements from the \( \text{BaBar} \) Collaboration for the decay mode \( B^+ \rightarrow K^+ \pi^0 \pi^0 \) and the result for \( A_{\text{CP}} \) is within 2 standard deviations of the previous measurement [18]. The branching fraction and \( A_{\text{CP}} \) results for \( B^+ \rightarrow \rho(770)^+ K^0 \) supersede the previous \( \text{BaBar} \) measurements [14]. The \( \text{CP} \) asymmetries of \( B^+ \rightarrow K^*(892)^0 \pi^+, B^+ \rightarrow K_2^0(1430)^0 \pi^+, \) and \( B^+ \rightarrow \rho(770)^+ K^0 \) are all consistent with zero.
TABLE X: Combined measurements of branching fractions and CP asymmetries from $B^+ \to K^0\pi^+\pi^0$ (this analysis) and from $\bar{B}\bar{A}\bar{B}$ analyses of $B^+ \to K^+(892)^0\pi^+$ and $B^+ \to K^0_{\rm S}(1430)^0\pi^+$ from $B^+ \to K^+\pi^+\pi^-$ [16], and of $B^+ \to K^+(892)^0\pi^0$ from $B^+ \to K^+\pi^0\pi^0$ [15]. The first uncertainty is statistical and the second is systematic.

| Decay channel | $\mathcal{B}$ ($10^{-6}$) | $A_{CP}$ |
|---------------|----------------|----------|
| $K^+(892)^0\pi^+$ | $11.6 \pm 0.5 \pm 1.1 \pm 0.025 \pm 0.050 \pm 0.016$ |          |
| $K^+(892)^0\pi^0$ | $8.8 \pm 1.0 \pm 0.6 \pm 0.39 \pm 0.12 \pm 0.03$ |          |
| $K^0_{\rm S}(1430)^0\pi^+$ | $43.4 \pm 1.2 \pm 5.8 \pm 0.040 \pm 0.033 \pm 0.033$ |          |

as expected. We obtain the first measurements of the branching fraction and CP asymmetry for $B^+ \to K^0_{\rm S}(1430)^0\pi^0$, with a significance of 5.4 standard deviations for the branching fraction.

We combine our results for the branching fractions and CP asymmetries of $B^+ \to K^+(892)^0\pi^+$, $B^+ \to K^0_{\rm S}(1430)^0\pi^+$, and $B^+ \to K^+(892)^0\pi^0$ with the previous $\bar{B}\bar{A}\bar{B}$ measurements. The statistical uncertainties and all systematic uncertainties for the CP asymmetries are uncorrelated between the measurements. For the branching fractions, we account for possible correlations when combining the systematic uncertainties. If the systematic uncertainties are asymmetric, the average systematic uncertainty is calculated from the largest limit. The combined results from $\bar{B}\bar{A}\bar{B}$ for these decay modes are presented in Table X.

Using the world average value for direct CP violation in $B^0 \to K^*(892)^+\pi^-$ [16] and the final $\bar{B}\bar{A}\bar{B}$ result for direct CP violation in $B^+ \to K^+(892)^0\pi^0$, we calculate $\Delta A_{CP}$ for the $K^*\pi$ system to be

$$\Delta A_{CP}(K^*\pi) = A_{CP}(K^{*+}\pi^0) - A_{CP}(K^{*+}\pi^-) = -0.16 \pm 0.13. \quad (33)$$

Thus the value of $\Delta A_{CP}$ in $K^*\pi$ is found to be consistent with zero. The uncertainty in the $\Delta A_{CP}(K^*\pi)$ result remains large, rendering the comparison to $\Delta A_{CP}(K\pi)$, given in Eq. (3), inconclusive at present and motivating improved determinations in future experiments.

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Appendix A: Tables of Systematic and Model Uncertainties

Table [XI] lists the uncertainties in the branching fractions due to systematic effects, efficiency corrections, and changes to the signal model. Table [XII] lists uncertainties in the relative phase values (for $B^+$ and $B^-$ decays combined) due to systematic effects and changes to the signal model. Tables [XIII] [XIV] [XV] and [XVI] list the systematic and signal model uncertainties for the $CP$ asymmetries, the amplitudes for the $B^+$ and $B^-$ Dalitz plots, $A_+$ and $A_-$, respectively, and the corresponding phases $\phi_+$ and $\phi_-$ for the $B^+ \rightarrow K^*_0(1430) \pi^+$ amplitude relative to that for $B^+ \rightarrow K^*(892)^0 \pi^+$, and the phase values for the $B^+ \rightarrow (K\pi)^0 \pi^+$ amplitude relative to that for $B^+ \rightarrow K^*(892)^0 \pi^+$.
TABLE XI: Combined $B^\pm$ fit: Systematic uncertainties for the branching fraction measurements, including uncertainties due to the signal model.

| Source                                                                 | Resonant contribution | Relative Variations of branching fraction (%) |
|-----------------------------------------------------------------------|-----------------------|---------------------------------------------|
| Correctly reconstructed $m_{ES}$ and $\Delta E$ PDF (fixed parameters) | 0.8 1.1 0.6           | Inclusive $K^*(892)^0$ $K^{*+}(892)$ $K_0^*(1430)^+$ $K_0^*(1430)^+$ $\rho(770)^+$ |
| Correctly reconstructed and self crossfeed signal BDT_{out} PDFs      | 3.3 3.3 3.4           |                                             |
| Self crossfeed signal $m_{ES}$ and $\Delta E$ PDF models              | 3.0 4.3 3.1           |                                             |
| Fit bias                                                              | 0.3 0.9 0.6           |                                             |
| $B\bar{B}$ background $m_{ES}$, $\Delta E$ and BDT_{out} PDFs        | 0.3 0.4 0.2           |                                             |
| $B\bar{B}$ background yields                                         | 0.7 1.2 0.6           |                                             |
| Background model in Dalitz plot                                      | 1.5 3.7 2.8           |                                             |
| Signal efficiency model                                               | 0.3 1.8 1.0           |                                             |
| $K^+(892)$ mass and width                                            | 0.1 0.7 0.3           |                                             |
| $K_0^*(1430)$ mass and width                                         | 3.2 3.8 2.1           |                                             |
| $\rho(770)^+$ mass and width                                         | <0.1 0.2 0.1          |                                             |
| Blatt-Weisskopf radius                                               | 2.3 4.4 2.9           |                                             |
| Subtotal                                                             | 6.3 9.1 6.6           | 12.0 8.5 11.0                               |
| Neutral pion efficiency                                              | 1.0 1.0 1.0           | 1.0 1.0 1.0                                 |
| $K_0^*$ efficiency                                                   | 1.1 1.1 1.1           | 1.1 1.1 1.1                                 |
| Charged particle identification efficiency                             | 1.0 1.0 1.0           | 1.0 1.0 1.0                                 |
| Tracking efficiency                                                  | 1.0 1.0 1.0           | 1.0 1.0 1.0                                 |
| $N_{B\bar{B}}$                                                      | 0.6 0.6 0.6           | 0.6 0.6 0.6                                 |
| Total                                                                | 6.6 9.4 7.0           | 12.2 8.7 11.2                               |
| Changes due to signal model                                          |                        | $\Delta B (10^{-6})$                        |
| ($K\pi)^0_0/(K\pi)^+_0$ parametrization                              | +8.0 −0.3 −0.3        |                                              |
| $\rho(1450)^+$                                                       | +2.3 +0.3 −0.4        | +2.7 −0.8 −2.0                              |
| $K_0^*(1430)^0$ and $K_0^*(1430)^+$                                  | +1.4 −0.3 +0.3        | −2.6 −0.8 −0.3                              |
| $K^*(1680)^0$ and $K^{*+}(1680)^+$                                   | +1.8 −0.1 −0.1        | +0.6 −1.4 −0.2                              |
| Total (+)                                                            | +8.6 +0.3 +0.3        | +2.7 +0.0 +0.0                               |
| Total (−)                                                            | −0.0 −0.4 −0.5        | −2.6 −1.8 −2.4                              |
TABLE XII: Combined $B^{\pm}$ fit: Systematic uncertainties due to the fit model, fixed shapes in the parametrization, and changes to the signal model for the relative phases (in degrees) measured relative to the $K^*(892)^0$ amplitude.

| Systematic                                                                 | Resonant contribution | Systematic Variations (°) |
|----------------------------------------------------------------------------|-----------------------|---------------------------|
|                                                                            | $K^*(892)^0 \pi^0$ | $K^*_0(1430)^0 \pi^+$ | $K^*_0(1430)^+ \pi^0$ | $\rho(770)^+ K_S^0$ |
| Self crossfeed PDFs and mapping                                           | 7.4                   | 1.4                       | 9.6                  | 6.7                |
| Correctly reconstructed and self crossfeed BDT$_{out}$ PDFs              | 1.4                   | 0.9                       | 1.3                  | 1.3                |
| $B\overline{B}$ background yields                                        | 1.7                   | 0.5                       | 1.8                  | 2.1                |
| Correctly reconstructed $m_{ES}$ and $\Delta E$ PDF                      | 0.4                   | 0.2                       | 0.9                  | 1.2                |
| Background DP PDF                                                        | 5.7                   | 2.2                       | 4.5                  | 5.4                |
| $B\overline{B}$ background $m_{ES}$, $\Delta E$, BDT$_{out}$ PDF         | 0.3                   | 0.2                       | 0.3                  | 0.3                |
| Signal efficiency model                                                  | 0.5                   | 0.4                       | 0.7                  | 0.7                |
| Fit bias                                                                 | 9.3                   | 7.8                       | 7.3                  | 23.5               |
| $K^*(892)$ mass                                                          | 1.4                   | 0.2                       | 1.2                  | 1.4                |
| $K^*(892)$ width                                                         | 0.3                   | 0.1                       | 0.1                  | < 0.1              |
| $K^*_0(1430)$ mass                                                        | $^{+43}_{-33}$         | $^{+5.2}_{-4.7}$           | $^{+48}_{-37}$        | $^{+46}_{-36}$      |
| $K^*_0(1430)$ width                                                       | 5.2                   | 4.0                       | 5.7                  | 14.6               |
| $\rho(770)^+$ mass                                                       | 0.3                   | 0.1                       | 0.4                  | 1.0                |
| $\rho(770)^+$ width                                                      | 0.6                   | 0.1                       | 0.6                  | 0.2                |
| Blatt-Weisskopf radius                                                   | $^{+15}_{-2}$          | $^{+1.0}_{-1.4}$           | $^{+17}_{-3}$         | $^{+0.3}_{-0.5}$   |
| Total                                                                    | $^{+11}_{-36}$         | $^{+11}_{-11}$             | $^{+53}_{-40}$        | $^{+55}_{-47}$      |

Changes due to signal model

| $(K\pi)_0^0 / (K\pi)_0^{*+}$ parametrization                            | $-$67.0                | $-$                       | $-$                   | $-$60.3             |
| $\rho(1450)^+$                                                          | $-$18.4                | $-$2.8                    | $-$11.8               | $-$27.4             |
| $K^*_2(1430)$                                                           | $+$7.8                 | $-$3.1                    | $+$5.5                | $+$11.3             |
| $K^*(1680)$                                                             | $-$7.8                 | $-$4.9                    | $-$12.9               | $+$11.8             |
| Total (+)                                                               | $+$7.8                 | $+$0.0                    | $+$5.5                | $+$16.4             |
| Total (-)                                                               | $-$69.9                | $-$6.5                    | $-$17.5               | $-$66.2             |
TABLE XIII: Contributions to the uncertainties in the $CP$ asymmetries for the overall and resonant isobar contributions, including uncertainties due to changes to the signal model.

| Systematic | Resonant contribution | Systematic Variations of $A_{CP}$ (%) |
|------------|-----------------------|--------------------------------------|
|            | Inclusive $K^*(892)^0\pi^+$ | $K^*(892)^+\pi^0$ | $K_0^*(1430)^0\pi^+$ | $K_0^*(1430)^+\pi^0$ | $\rho(770)^+K_0^0$ |
| Self crossfeed PDFs and mapping | 2.0 | 6.0 | 1.0 | 1.2 | 0.9 | 5.0 |
| Correctly reconstructed and self crossfeed $BDT_{out}$ PDFs | 0.7 | 1.7 | 1.8 | 1.6 | 1.1 | 2.8 |
| $B\bar{B}$ background asymmetries | 2.5 | 1.4 | 1.7 | 1.8 | 7.5 | 2.3 |
| Background DP PDF | 0.7 | 2.7 | 2.1 | 2.8 | 2.0 | 2.5 |
| $B\bar{B}$ background $m_{ES}$, $\Delta E$, $BDT_{out}$ PDFs | 0.2 | 0.1 | 0.2 | 0.3 | 0.5 | 0.4 |
| Signal efficiency model | 0.2 | 3.9 | 2.1 | 0.1 | 0.9 | 1.3 |
| Fit bias | 0.3 | 1.4 | 0.8 | 1.9 | 1.0 | 1.2 |
| $K^*(892)$ mass and width | 0.1 | 0.2 | 0.4 | 0.2 | 0.2 | 0.2 |
| $K_0^*(1430)$ mass and width | 1.1 | 4.4 | 0.5 | 3.0 | 2.8 | 2.2 |
| $\rho(770)^+$ mass and width | $<0.1$ | 0.1 | 0.1 | 0.2 | 0.1 | 0.1 |
| Blatt-Weisskopf radius | $<0.1$ | 0.9 | 0.3 | 0.8 | 1.1 | 0.5 |
| **Total** | 3.4 | 8.1 | 4.1 | 4.3 | 8.0 | 6.9 |

Changes due to signal model

| Systematic | Resonant contribution | Variation of $A_+$ |
|------------|-----------------------|-------------------|
|            | $K^*(892)^+\pi^0$ | $(K\pi)_0^+\pi^0$ | $(K\pi)_0^+\pi^+\pi^0$ | $\rho(770)^+K_0^0$ |
| $(K\pi)_0^+/(K\pi)_0^+$ parametrization | $-0.7$ | $+6.2$ | $+2.0$ | $-$ | $-$ | $-8.1$ |
| $\rho(1450)^+$ | $+3.3$ | $+1.5$ | $-3.4$ | $-10.5$ | $-11.6$ | $-21.3$ |
| $K_0^*(1430)$ | $-0.2$ | $+5.7$ | $-1.5$ | $-7.5$ | $-2.7$ | $+14.4$ |
| $K^*(1680)$ | $-2.2$ | $+6.3$ | $+0.5$ | $+4.8$ | $-1.9$ | $+12.3$ |
| **Total (+)** | $+2.4$ | $+0.0$ | $+3.7$ | $+13.0$ | $+12.0$ | $+22.8$ |
| **Total (−)** | $-3.3$ | $-10.6$ | $-2.0$ | $-4.8$ | $-0.0$ | $-19.0$ |

TABLE XIV: Variations in the $CP$ amplitude, $A_+$, including uncertainties due to changes to the signal model. In the fits, the amplitudes are measured relative to the $K^*(892)^0$ amplitude.

| Systematic | Resonant contribution | Variation of $A_+$ |
|------------|-----------------------|-------------------|
|            | $K^*(892)^+\pi^0$ | $(K\pi)_0^+\pi^0$ | $(K\pi)_0^+\pi^+\pi^0$ | $\rho(770)^+K_0^0$ |
| Self crossfeed PDFs and mapping | 0.02 | 0.02 | 0.04 | 0.02 |
| Correctly reconstructed and self crossfeed $BDT_{out}$ PDFs | 0.01 | 0.03 | 0.02 | 0.02 |
| $B\bar{B}$ background asymmetries | 0.01 | 0.03 | 0.07 | 0.02 |
| Background DP PDF | 0.02 | 0.06 | 0.04 | 0.04 |
| $B\bar{B}$ background $m_{ES}$, $\Delta E$, $BDT_{out}$ PDFs | $<0.01$ | 0.01 | 0.01 | $<0.01$ |
| Signal efficiency model | 0.01 | 0.01 | $<0.01$ | 0.01 |
| Fit bias | 0.01 | 0.03 | 0.02 | 0.03 |
| $K^*(892)$ mass | 0.01 | 0.01 | 0.01 | $<0.01$ |
| $K^*(892)$ width | $<0.01$ | 0.01 | 0.01 | $<0.01$ |
| $(K\pi)_0^+$ mass | 0.02 | 0.02 | 0.09 | 0.06 |
| $(K\pi)_0^+$ width | 0.02 | 0.06 | 0.01 | 0.02 |
| $\rho(770)^+$ mass | $<0.01$ | $<0.01$ | $<0.01$ | $<0.01$ |
| $\rho(770)^+$ width | $<0.01$ | $<0.01$ | $<0.01$ | $<0.01$ |
| Blatt-Weisskopf radius | 0.02 | 0.03 | 0.02 | 0.02 |
| **Total** | 0.05 | 0.11 | 0.13 | 0.09 |

Changes due to signal model

| Systematic | Resonant contribution | Variation of $A_+$ |
|------------|-----------------------|-------------------|
| $(K\pi)_0^+/(K\pi)_0^+$ parametrization | $-0.03$ | $-$ | $-$ | $-$ | $-0.17$ |
| $\rho(1450)^+$ | $<0.01$ | $-0.01$ | $-0.02$ | $-0.12$ |
| $K_0^*(1430)$ | 0.05 | $-0.08$ | $-0.08$ | $-0.04$ |
| $K^*(1680)$ | $-0.02$ | 0.07 | $-0.05$ | $-0.05$ |
| **Total (+)** | $+0.05$ | $+0.07$ | $+0.00$ | $+0.00$ |
| **Total (−)** | $-0.03$ | $-0.12$ | $-0.10$ | $-0.21$ |
TABLE XV: Variations in the CP amplitude, $A_\pm$, including uncertainties due to changes to the signal model. In the fits, the amplitudes are measured relative to the $K^{*}(892)^0$ amplitude.

| Systematic                                                                 | Resonant contribution | $K^{*}(892)^-\pi^0$ | $(K\pi)_0^{\pm}\pi^- - (K\pi)_0^{\pm}\pi^0$ | $\rho(770)^-\pi^0 - K_S^0$ |
|---------------------------------------------------------------------------|-----------------------|----------------------|-----------------------------------------------|----------------------------|
| Self crossfeed PDFs and mapping                                          | $<0.01$               | 0.05                 | 0.04                                          | 0.05                       |
| Correctly reconstructed and self crossfeed BDT$_{out}$ PDFs             | 0.02                  | 0.02                 | 0.02                                          | 0.03                       |
| $B\bar{B}$ background asymmetries                                       | 0.01                  | 0.01                 | 0.06                                          | 0.02                       |
| Background DP PDF                                                        | 0.02                  | 0.05                 | 0.04                                          | 0.03                       |
| $B\bar{B}$ background $m_{ES}, \Delta E$, BDT$_{out}$ PDFs              | $<0.01$               | $<0.01$              | $<0.01$                                       | $<0.01$                    |
| Signal efficiency model                                                  | 0.02                  | $<0.01$              | 0.02                                          | 0.02                       |
| Fit bias                                                                 | 0.02                  | 0.01                 | 0.02                                          | 0.02                       |
| $K^*(892)$ mass                                                          | $<0.01$               | 0.01                 | $<0.01$                                       | $<0.01$                    |
| $K^*(892)$ width                                                         | $<0.01$               | 0.01                 | 0.01                                          | 0.01                       |
| $(K\pi)_0^0$ mass                                                        | 0.01                  | 0.08                 | 0.08                                          | 0.04                       |
| $(K\pi)_0^0$ width                                                       | 0.01                  | 0.07                 | 0.05                                          | 0.03                       |
| $\rho(770)^+$ mass                                                       | $<0.01$               | $<0.01$              | $<0.01$                                       | $<0.01$                    |
| $\rho(770)^+$ width                                                      | $<0.01$               | $<0.01$              | $<0.01$                                       | $<0.01$                    |
| Blatt-Weisskopf radius                                                   | 0.01                  | 0.01                 | 0.05                                          | 0.01                       |
| Total                                                                    | 0.05                  | 0.13                 | 0.14                                          | 0.09                       |

| Changes due to signal model                                              |                       |                      |                                               |                            |
|--------------------------------------------------------------------------|-----------------------|----------------------|-----------------------------------------------|----------------------------|
| $(K\pi)_0^0/(K\pi)_0^+$ parametrization                                  | $-0.04$               | $-0.02$              | $-0.02$                                       | $-0.04$                    |
| $\rho(1450)^+$                                                           | 0.04                  | 0.12                 | 0.22                                          | 0.22                       |
| $K^*_2(1430)$                                                           | 0.05                  | 0.07                 | 0.05                                          | 0.03                       |
| $K^*(1680)$                                                             | $-0.01$               | $-0.02$              | $-0.02$                                       | $-0.04$                    |
| Total $(+)$                                                              | $+0.06$               | $+0.14$              | $+0.22$                                       | $+0.23$                    |
| Total $(-)$                                                              | $-0.04$               | $-0.02$              | $-0.06$                                       | $-0.09$                    |

TABLE XVI: Variations in the CP phase values $\phi_\pm$ (in degrees) measured for the $(K\pi)_0^0\pi^\pm$ amplitude relative to the $K^{*}(892)^0\pi^\pm$ amplitude, and for the $(K\pi)_0^0\pi^\pm$ amplitude relative to the $K^{*}(892)^0\pi^0$ amplitude.

| Systematic                                                                 | Absolute variations of CP phase values |
|---------------------------------------------------------------------------|---------------------------------------|
| $(K\pi)_0^0\pi^\pm - K^{*}(892)^0\pi^\pm$ | $(K\pi)_0^0\pi^- - K^{*}(892)^0\pi^0$ |
| $\phi_+$                                                                  | $\phi_-$                              | $\phi_+$                              | $\phi_-$                  |
| Self crossfeed PDFs and mapping                                          | 0.6                                   | 4.5                                   | 1.6                       | 3.1                       |
| Correctly reconstructed and self crossfeed BDT$_{out}$ PDFs             | 0.9                                   | 1.6                                   | 1.4                       | 2.6                       |
| $B\bar{B}$ background asymmetries                                       | 1.4                                   | 1.3                                   | 0.7                       | 1.0                       |
| Background DP PDF                                                        | 2.5                                   | 3.0                                   | 2.0                       | 2.7                       |
| $B\bar{B}$ background $m_{ES}, \Delta E$, BDT$_{out}$ PDFs              | 0.2                                   | 0.2                                   | 0.2                       | 0.4                       |
| Signal efficiency model                                                  | 0.3                                   | 1.4                                   | 0.8                       | 1.2                       |
| Fit bias                                                                 | 2.4                                   | 5.2                                   | 1.7                       | 0.9                       |
| $K^*(892)$ mass                                                          | 0.2                                   | 0.2                                   | 0.4                       | 0.6                       |
| $K^*(892)$ width                                                         | 0.3                                   | 0.3                                   | 0.2                       | 0.2                       |
| $(K\pi)_0^0$ mass                                                        | 6.1                                   | 5.3                                   | 5.9                       | 5.5                       |
| $(K\pi)_0^0$ width                                                       | 4.2                                   | 5.0                                   | 5.2                       | 4.5                       |
| $\rho(770)^+$ mass                                                       | 0.2                                   | 0.2                                   | $<0.1$                    | 0.2                       |
| $\rho(770)^+$ width                                                      | 0.2                                   | 0.2                                   | 0.1                       | 0.2                       |
| Blatt-Weisskopf radius                                                   | 1.0                                   | 3.3                                   | 1.3                       | 3.8                       |
| Total                                                                    | 8.5                                   | 11.3                                  | 8.7                       | 9.6                       |

| Changes due to signal model                                              |                                       |                                       |                             |                            |
|--------------------------------------------------------------------------|-----------------------|----------------------|-----------------------------------------------|----------------------------|
| $\rho(1450)^+$                                                           | $+3.7$                 | $-0.9$                | $-3.2$                                       | $+9.4$                     |
| $K^*_2(1430)$                                                           | $-2.6$                 | $+0.5$                | $+3.6$                                       | $-8.8$                     |
| $K^*(1680)$                                                             | $+1.9$                 | $+3.3$                | $+2.2$                                       | $+14.6$                    |
| Total $(+)$                                                              | $+4.2$                 | $+0.5$                | $+4.2$                                       | $+17.4$                    |
| Total $(-)$                                                              | $-2.6$                 | $-3.4$                | $-3.2$                                       | $-8.8$                     |