The Angular Bispectrum of
The Cosmic Microwave Background

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ABSTRACT

COBE has provided us with a whole-sky map of the CBR anisotropies. However, even if the noise level is negligible when the four year COBE data are available, the cosmic variance will prevent us from obtaining information about the Gaussian nature of the primordial fluctuations. This important issue is addressed here by studying the angular bispectrum of the cosmic microwave background anisotropies. A general form of the angular bispectrum is given and the cosmic variance of the angular bispectrum for Gaussian fluctuations is calculated. The advantage of using the angular bispectrum is that one can choose to use the multipole moments which minimize the cosmic variance term. The non-Gaussian signals in most physically motivated non-Gaussian models are small compared with cosmic variance. Unless the amplitudes are large, the non-Gaussian signals are only detectable in the COBE data in those models where the angular bispectrum is flat or increases with increasing multipole moment.

Subject headings: cosmic microwave background — cosmology: theory
The cosmic microwave background radiation (CBR) provides one of the most useful tools for studying the primordial fluctuations that seed large scale structures today. Valuable information about the physical processes that generate primordial fluctuations in the early universe can be obtained by studying the statistics of temperature anisotropies in the CBR: are they Gaussian or non-Gaussian? Cosmic inflation (Guth 1981; Linde 1982; Albrecht & Steinhardt 1982; Liddle & Lyth 1993) predicts a Gaussian pattern of anisotropies on all angular scales. Spontaneous symmetry breaking, on the other hand, will lead to the formation of topological defects, and the pattern of temperature anisotropies produced by defect-networks tends to be non-Gaussian (Bouchet et al. 1988; Turok & Spergel 1991; Turner et al. 1991). After COBE’s detection of CBR anisotropies on large angular scales (Smoot et al. 1992), both skewness and the three point temperature correlation function (which contain the lowest order deviations from a Gaussian) were proposed to test the Gaussian nature of the CBR (Luo & Schramm 1992, 1993; Falk et al. 1993). However, there is a problem in applying these statistics to test for Gaussianity in the COBE map even with all four years of data available (Hinshaw et al. 1993): namely the cosmic variance (Abbot & Wise 1984; Scaramella & Vittorio 1990, 1991; Cayon et al. 1991; White et al. 1993), i.e. the fact that one can only measure CBR temperature anisotropies in one single universe, which introduces theoretical uncertainty in the skewness and three point function. Theoretical studies (Srednicki 1993) and Monte-Carlo simulations (Scaramella & Vittorio 1990, 1991) show that the cosmic variance is large. These results cast doubt on whether one could ever get any information about the non-Gaussian nature of CBR from large scale experiments such as COBE, simply because of the limitations from cosmic variance. The goal of this paper is to address this important issue in the context of the angular bispectrum. We will introduce and discuss several properties of the angular bispectrum, including the relationship to the bispectrum of the perturbations of the gravitational potential, our modeling of the
angular bispectrum, and the cosmic variance of the angular bispectrum when the fluctuations are Gaussian. Limits on model parameters and the observability of the angular bispectrum are also discussed.

1 Angular Power Spectrum

Before we move on to discuss the angular bispectrum, let us briefly review two point statistics first. In the case when the temperature anisotropies are Gaussian, the two-point statistic is all we need to quantify the temperature anisotropy patterns observed in the experiments (such as COBE).

The CBR temperature anisotropy is a 2D random field defined on the two-sphere. One can perform a spherical harmonic expansion of the temperature anisotropy on the sky:

$$\frac{\Delta T}{T_0}(\theta, \phi) = \sum_{lm} a_l^m Y_l^m(\theta, \phi).$$  \hspace{1cm} (1)

Then the spherical harmonic expansion coefficients $a_l^m$, $l \neq 0$, are random variables, and their statistics (usually called angular statistics) will completely specify the statistics of the temperature anisotropy itself. The angular power spectrum $C_l$, is the $l$th component of the Legendre polynomial expansion of the two-point temperature correlation function (Bond and Efstathiou, 1987):

$$C(\theta) = \left\langle \frac{\Delta T(\vec{m})}{T_0} \cdot \frac{\Delta T(\vec{n})}{T_0} \right\rangle = \frac{1}{4\pi} \sum_l (2l + 1)C_l P_l(\cos \theta).$$  \hspace{1cm} (2)

Here the angled brackets denote an ensemble average and $C_l$ is related to the spherical harmonic expansion coefficients through:

$$\langle a_l^m a_l^{m'} \rangle = C_l \delta_{ll'} \delta_{mm'}.$$  \hspace{1cm} (3)

In realistic experimental settings there are two major factors which modify the theoretical angular power spectrum defined in Eq. (3). One is the finite beam effect, which filters all
the high multiple moments. For large scale experiments such as COBE, which have a finite beam and full sky coverage, the angular power spectrum is modified as (Silk & Wilson 1980; Bond & Efstathiou 1984):

\[
\tilde{C}_l = C_l W_l = C_l \exp(-l(l+1)\sigma^2),
\]

where \(\sigma\) is the Gaussian beam width. The other important factor is that one can only measure CBR fluctuations in a single universe (our own universe), thus the observed \(C_l^{\text{obs}}\) is still a stochastic quantity, which has intrinsic fluctuations. Here \(C_l^{\text{obs}}\) is the rotationally invariant sum

\[
C_l^{\text{obs}} = \frac{1}{2l+1} \sum_{m=-l}^{l} a_{lm} a_{lm}^*.
\]

The cosmic variance of \(C_l^{\text{obs}}\) is then given by:

\[
\sigma_l^2 = \langle (C_l^{\text{obs}})^2 \rangle - \langle (C_l^{\text{obs}}) \rangle^2 = \frac{2}{2l+1} C_l^2.
\]

For a Harrison-Zeldovich (H-Z) primordial spectrum, \(C_l\) is given by (Peebles 1982)

\[
\frac{C_l}{4\pi} = \frac{6Q^2}{5l(l+1)},
\]

where \(Q\) is the rms ensemble-averaged quadruple. For large \(l\), \(\sigma_l^2 \propto l^{-5}\) is a rapid decreasing function of \(l\). The cosmic variance dominates the lower multipole moments, and for high \(l\)-multipoles it is negligible. We expect this is also true for the cosmic variance of the skewness and three point function, and we would expect that the theoretical uncertainties due to cosmic variance will be reduced by subtracting out lower multipole moments. We will return to this point in section 3.

2 Angular Bispectrum

If the CBR is Gaussian, the odd moments of the spherical harmonic coefficients \(a_l^{\text{obs}}\) will vanish. Thus, the non-vanishing odd moments will be a clear signature of deviation from Gaus-
sianity. The lowest order of such moments, the angular bispectrum, $B_3(l_1m_1, l_2m_2, l_3m_3)$, is defined as:

$$B_3(l_1m_1, l_2m_2, l_3m_3) = \langle a_{l_1m_1} a_{l_2m_2} a_{l_3m_3} \rangle. \quad (8)$$

The three point temperature correlation is related to the angular bispectrum through:

$$\xi(\hat{l}, \hat{m}, \hat{n}) = \langle \frac{\Delta T}{T_0}(\hat{l}) \frac{\Delta T}{T_0}(\hat{m}) \frac{\Delta T}{T_0}(\hat{n}) \rangle = \sum_{l_i, m_i} B_3(l_1m_1, l_2m_2, l_3m_3) Y_{l_1}^{m_1}(\hat{l}) Y_{l_2}^{m_2}(\hat{m}) Y_{l_3}^{m_3}(\hat{n}). \quad (9)$$

Because the three point function is rotationally invariant, the angular bispectrum $B_3(l_1m_1, l_2m_2, l_3m_3)$ is non-zero only if $l_i m_i, i = 1, 2, 3$ satisfy the following conditions [1]:

1. $l_1, l_2, l_3$ satisfy the triangle rule, i.e. $l_i \leq |l_j - l_k|$,  
2. $l_1 + l_2 + l_3 = \text{even}$, and  
3. $m_1 + m_2 + m_3 = 0$.

The angular bispectrum can be calculated for any given non-Gaussian model. In this paper we consider only the case where the perturbations are adiabatic, so that the temperature anisotropy is related to the gravitational potential $\phi$ at the last scattering surface through the Sachs-Wolfe formula (Sachs & Wolfe 1967),

$$\frac{\delta T}{T} = \frac{\phi}{3}. \quad (10)$$

The three point temperature correlation function is related to the bispectrum of the gravitational potential $\phi$, $P_\phi(k_1, k_2) \equiv \langle \phi_{k_1} \phi_{k_2} \phi_{k_3} \rangle |(k_1 + k_2 + k_3 = 0)$, through

$$\xi_T(\hat{l}, \hat{m}, \hat{n}) = \frac{1}{27} \cdot \int P_\phi(k_1, k_2) e^{i \theta(k_1 \hat{m} + k_2 \hat{n} + k_3 \hat{l})} \delta^3(k_1 + k_2 + k_3) \frac{d^3k_1 d^3k_2 d^3k_3}{(2\pi)^9}, \quad (11)$$

1. These conditions can be derived by choosing special beam configurations. Condition (1) is a result of the rotational invariance of $\xi$ when two beams coincide. Condition (2) results from the invariance of $\xi$ under spatial inversion: $\hat{l} \rightarrow -\hat{l}, \ \hat{m} \rightarrow -\hat{m}, \ \hat{n} \rightarrow -\hat{n}$. Condition (3) results from choosing beam configurations so that the rotational axis is perpendicular to the plane defined by the three beam directions.
where $\eta_0 = 2c/H_0$ is the distance to the last scattering surface and $\hat{l}, \hat{m}, \hat{n}$ are the beam directions. The general relation between the angular bispectrum $B_3$ and the bispectrum $P_\phi$ is rather complicated and we will not give it here. The expression simplifies if $P_\phi(k_1, k_2)$ depends only upon the amplitudes of $\vec{k}_1$ and $\vec{k}_2$. This is what we expect for the angular bispectrum from non-standard inflationary scenario after proper symmetrization. In this case, by using the following expansions,

$$e^{ik \cdot \eta_0} = 4\pi \sum_l j_l(k\eta_0) \sum_m Y_{lm}(\hat{k})Y_{lm}^*(\hat{m}),$$

(12)

and

$$Y_{l_1 m_1}^{m_2} Y_{l_2 m_2}^{m_2} = \sum_{l_3 m_3} A(l_1 l_2 l_3, m_1 m_2 m_3) Y_{l_3 m_3}^{m_3},$$

(13)

we find that

$$B_3 = \frac{4}{27\pi^2} \int dk_1 dk_2 k_1^2 k_2^2 P_\phi(k_1, k_2) j_{l_1}^2(k_1) j_{l_2}^2(k_2) A(l_1 l_2 l_3, m_1 m_2 - m_3),$$

(14)

where

$$A(l_1 l_2 l_3, m_1 m_2 - m_3) = \left[\frac{(2l_1 + 1)(2l_2 + 1)}{4\pi(2l_3 + 1)}\right]^{1/2} C(l_1 l_2 l_3, 000) C(l_1 l_2 l_3, m_1 m_2 - m_3)$$

(15)

and $C(l_1 l_2 l_3, m_1 m_2 m_3)$ are Clebsh-Gordan coefficients. The expression $A(l_1 l_2 l_3, m_1 m_2 - m_3)$ is nonzero only if $m_1 + m_2 + m_3 = 0$, $l_1 + l_2 + l_3 = \text{even}$ and $l_1, l_2, l_3$ satisfy the triangle rule. Thus, the rotational invariance of the three point function is guaranteed.

Let us recall that the angular power spectrum $C_l$ is related to the power spectrum $P_\phi(k)$ of fluctuations in $\phi$ through (Kolb & Turner 1990)

$$C_l = \frac{2}{9\pi} \int dk k^2 P_\phi(k) j_l^2(k\eta_0).$$

(16)

The functional form of $B_3$ given by Eq.(14) and Eq.(16) motived us to model the angular bispectrum as the following:

$$B_3(l_1 m_1, l_2 m_2, l_3 m_3) = C^{3/2}(0) A(l_1 l_2 l_3, m_1 m_2 - m_3) \times$$

$$\{\alpha[\bar{C}(l_1)\bar{C}(l_2) + \bar{C}(l_2)\bar{C}(l_3) + \bar{C}(l_3)\bar{C}(l_1)] + \beta[\bar{C}(l_1)\bar{C}(l_2)\bar{C}(l_3)]\gamma\},$$

(17)
where $\alpha, \beta, \gamma$ are three dimensionless parameters, $\tilde{C}(l) = C_l/C(0)$ is the normalized angular power spectrum and $C(0) = \frac{1}{4\pi} \sum \tilde{C}_l(2l + 1)$. For COBE, where the FWHM beam width is $7^\circ$, $C(0) = 4.63Q^2$ and $\tilde{C}(l) = \frac{3.26}{l(l+1)}$.

Apart from guaranteeing the rotational invariance of the three point function, the bispectrum modeled by Eq.(17) has another advantage: several physically motivated non-Gaussian scenarios give distinctive predictions for the constants $\alpha, \beta, \gamma$. These constants can be calculated directly from Eq.(14) and Eq.(16) once the perturbation bispectrum $P_\phi(k_1, k_2)$ is given for a specific non-Gaussian model. In practice, it was found that in several non-Gaussian models of cosmological interest, it is easier to derive an expression for the three point correlation function $\xi(\hat{r}_1, \hat{r}_2, \hat{r}_3)$ in terms of the two point function $C(\theta)$ than to find an expression for $P_\phi(k_1, k_2)$ (Luo 1993). It is convenient to use the normalized two point functions $\psi$ and three point function $\eta$, where

$$\psi(|\hat{r}_1 - \hat{r}_2|) = C_2(\hat{r}_1, \hat{r}_2)/C_2(0), \quad \psi(0) = 1,$$  \hspace{1cm} (18)

$$\eta(\hat{r}_1, \hat{r}_2, \hat{r}_3) = \frac{\psi(|\hat{r}_1 - \hat{r}_2|)\psi(|\hat{r}_1 - \hat{r}_3|) + \psi(|\hat{r}_1 - \hat{r}_2|)\psi(|\hat{r}_2 - \hat{r}_3|) + \psi(|\hat{r}_1 - \hat{r}_3|)\psi(|\hat{r}_2 - \hat{r}_3|)}{C_2(0)^{1.5}}, \quad \eta(0) = \mu_3,$$  \hspace{1cm} (19)

and $\mu_3$ is the skewness. The theoretical predictions for three point function in specific scenarios are the following:

**1) Inflation**

Various non-standard inflation models will generate a non-zero three point correlation function (Falk et al. 1993; Luo 1993; Gangui et al. 1993). The generic form of the three point function in most inflationary models is

$$\eta(\hat{r}_1, \hat{r}_2, \hat{r}_3) = \frac{\mu_3^2}{3}(\psi(|\hat{r}_1 - \hat{r}_2|)\psi(|\hat{r}_1 - \hat{r}_3|) + \psi(|\hat{r}_1 - \hat{r}_2|)\psi(|\hat{r}_2 - \hat{r}_3|) + \psi(|\hat{r}_1 - \hat{r}_3|)\psi(|\hat{r}_2 - \hat{r}_3|)), \quad (20)$$

where $\mu_3$ is a dimensionless constant. For slow-roll inflation models with one field and a cubic self-coupling, $\mu_3 \sim 10^{-6}$ (Falk et al. 1993). Non-linear gravitational evolution will produce a three point function of similar form with $\mu_3 \sim 0.01$ (Luo & Schramm 1993).
Consider the $\chi_n^2$ field $Y = \sum_{i=1}^{n} X_i^2$, which describes the $O(N)\sigma$ model of global topological defects (Turok & Spergel 1991) in the large $N$ limit, with $n = 4N$. The three point function is found to be

$$\eta(\hat{r}_1, \hat{r}_2, \hat{r}_3) = \sqrt{2/N} (\psi(|\hat{r}_1 - \hat{r}_2|)\psi(|\hat{r}_1 - \hat{r}_3|)\psi(|\hat{r}_2 - \hat{r}_3|))^{3/2}. \quad (21)$$

(3) Late-time Phase Transitions

In this model (Hill et al. 1989), due to the conformal invariance of the system at the critical point (Polyakov 1970), the three point function has the following form (Luo & Schramm 1993):

$$\eta(\hat{r}_1, \hat{r}_2, \hat{r}_3) = \alpha (\psi(|\hat{r}_1 - \hat{r}_2|)\psi(|\hat{r}_1 - \hat{r}_3|)\psi(|\hat{r}_2 - \hat{r}_3|))^{3}, \quad (22)$$

where $\alpha$ is a dimensionless constant of order unity.

Once the three point function is found, the angular bispectrum $B_3$ and constants $\alpha, \beta, \gamma$ can be found from Eq. (9) by expanding the three point function and two point function in spherical harmonics. Non-standard inflation and non-linear gravitational evolution predict $\beta = 0, \alpha = \mu_3/3$, where $\mu_3$ is the skewness; Late-time phase transitions (LTPT) will predict $\alpha \sim 0, \gamma \sim 1$ and $\beta$ of order unity; $O(N)\sigma$ models predict $\alpha \sim 0, \gamma \sim 1/2$ and $\beta \sim \sqrt{2/N}$ (Luo 1993). Another possibility is that the underlying CBR fluctuations are non-Gaussian only on smaller angular scales. In this case, the angular bispectrum will increase slowly with the increasing of $l$, and will peak around $l_c$ corresponding to the characteristic scale $\theta_c$ where the CBR is highly non-Gaussian. If $\theta_c \sim 1^\circ$, then $l_c \sim 100$. Since the observed anisotropies in large angle experiments (such as COBE) are the beam-smoothed fluctuations, we would expect the angular bispectrum to be flat, i.e. $\alpha = 0, \gamma = 0$, at low $l$. Thus, if future analysis of COBE map reveals a non-vanishing angular bispectrum, fitting the experimental
results to the functional form given by Eq. (17) will help to discriminate between different non-Gaussian models.

3 Cosmic Variance of the Angular Bispectrum

The CBR anisotropies are stochastic in nature, which will give rise to a theoretical uncertainty in the particular realization of our sky. This effect is severe in large scale CBR experiments, and limits our ability to extract information about non-Gaussian nature of CBR from current experimental data. In this section we will address this important issue and calculate the cosmic variance of the bispectrum when the fluctuations are Gaussian. We will show how to minimize the cosmic variance by choosing the appropriate pairs of multipole moments. In order for the non-Gaussian signals to rise above the cosmic variance, bounds on the model parameters $\alpha, \beta, \gamma$ of the bispectrum defined in the previous section are also discussed.

The cosmic variance of the angular bispectrum is given by

$$\sigma_3^2 = \langle a_{m_1} a_{m_2} a_{m_3}, a_{m_1\ast}, a_{m_2\ast}, a_{m_3\ast} \rangle$$

(23)

When $a_l^{m}$ is Gaussian, by using Wick’s theorem, the expression above reduces to

$$\sigma_3^2 = C_{l_1} C_{l_2} C_{l_3} + 8\delta_{l_1 l_2 l_3} \delta_{m_1 m_2 m_3} C_{l_1}^3 + \delta_{l_1 l_2} [\delta_{m_1, -m_2} + \delta_{m_3 m_2}] C_{l_1}^2 C_{l_3} +$$

$$+ \delta_{l_2 l_3} [\delta_{m_2, -m_3} + \delta_{m_2 m_3}] C_{l_2} C_{l_1} + \delta_{l_3 l_1} [\delta_{m_1, -m_3} + \delta_{m_1 m_3}] C_{l_3}^2 C_{l_2}$$

(24)

where $C_l = \langle a_l^m a_l^{m\ast} \rangle$ is the angular power spectrum. From the above, one also finds that the variance of $B_3$ depends heavily upon the choice of multipole moment pairs $(l_i m_i)$. The variance of the terms where $l_1 \neq l_2 \neq l_3$ is given by

$$\sigma_3^2 = C_{l_1} C_{l_2} C_{l_3},$$

(25)
while for \( l_1 = l_2 = l_3 = l, \ m_1 = m_2 = m_3 = 0 \), it is given by

\[
\sigma^2_3 = 15C_{l_1}C_{l_2}C_{l_3} = 15C_l^3.
\]  

(26)

The variance of the bispectrum can differ by more than one order of magnitude by choosing different multipole pairs. It is easy to see why the cosmic variance of skewness is large: by writing it in terms of the angular bispectrum, the variance of the skewness is given by

\[
\eta^2 \approx \sum_{l, m_i} \sigma^2_3(l_1m_1, l_2m_2, l_3m_3)A^2(l_1l_2l_3, m_1m_2m_3).
\]  

(27)

This is a sum over all different multipole moments, and is dominated by terms such as \( m_i = 0, l_1 = l_2 = l_3 = l \). The advantage of using the angular bispectrum is that one can choose multipole moment pairs to minimize the cosmic variance, and at the same time, to maximize model predictions for the angular bispectrum.

One way to reduce cosmic variance is to subtract out the lower multipole moments. Part of the CBR signals will then also be removed so that the signal-noise ratio will be reduced. However, this will not be a serious problem for COBE map with four years of data. The noise levels in COBE maps diminish rapidly \( \propto \text{time}^{-3/2} \) with additional data (Hinshaw et al. 1993). In the four year maps, the noise level will be roughly eight times smaller than in the first year skymap. For a scale invariant H-Z initial fluctuation spectrum, the CBR signal is reduced by merely a factor of 3 by removing the quadrupole. Thus, the signal-noise ratio in the quadrupole-removed four-year map will still be a factor of three better than the first year map (with quadrupole).

A more serious problem with subtracting out lower moments is that part of the non-Gaussian signal will also be removed. The amount of non-Gaussian signal removed is model-dependent. We list in Table 1 the angular bispectrum \( B_3 \), modeled by Eq. (17), in various non-Gaussian models, compared with the cosmic variance \( \sigma_3 \). In non-standard inflation or late-time phase transition models, most of the non-Gaussian signal will be removed by
subtracting out the lower multipole moments. In these types of model, one should focus on the lowest multipole moment ($l = 2$). The low limits on the model constants are: $\alpha > 0.6$ in non-standard inflation, where $\beta = \gamma = 0$; $\beta > 3.2$ for late-time phase transitions, where $\alpha = 0, \gamma = 1$. In models where $\alpha = 0, \gamma = 1/2$, the ratio of non-Gaussian signal to the cosmic variance is approximately the same for the low $l$ moments, but since the instrumental noise is relatively high for large moments, one will also lose information by subtracting out lower moments. The low limit on $\beta$ is $\beta > 1.5$. For most physically motivated models (non-standard inflation, non-linear gravitational evolution, the late-time phase transition or the $O(N)$ $\sigma-$models) where the model constants $\alpha, \beta$ are less than unity, all terms of the angular bispectrum are smaller than the cosmic variance term. Since the cosmic variance term $\sigma_3 \propto l^{-3}$ for the H-Z initial fluctuation spectrum, if the angular bispectrum falls off faster than $l^{-3}$, it will be impossible for the experiments to detect deviations from Gaussian behavior, unless the amplitude of the non-Gaussian signature is unrealistically large. This suggests that the decisive test of the Gaussian nature of fluctuations has to come from degree scale experiments, as we have stressed before (Luo 1993). Nevertheless, in models where the angular bispectrum is flat ($\gamma = 0$), even a very small deviation from Gaussianity ($\beta \sim 0.02$) can stand out above the cosmic variance, by analyzing the angular bispectrum at higher multipole moments ($l \sim 8$).

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TABLE CAPTION

Table 1: Comparison of the angular bispectrum $B_3$ in various non-Gaussian models with the cosmic variance of the angular bispectrum for Gaussian fluctuations, weighted by $A(l_i, m_i)$. The multipole pairs $(l_i, m_i)$ are chosen to obtain the maximum value for $B_3$ in non-Gaussian models, and at the same time, to keep the cosmic variance as small as possible.
| Multipole pairs: $(l_1l_2l_3, m_1m_2m_3)$ | Gaussian: $B_3 = 0$ | Non-Gaussian Models: $l^3 B_3(l_i m_i)/C^3(0)$ |
|---------------------------------------------|----------------------|------------------------------------------------|
| Cosmic variance: $\frac{l^3 \sigma_{\ell A(l_i m_i)}}{C^3(0)}$ | $\alpha \neq 0$ | $\alpha = 0$  |
| $\beta = 0$ | $\gamma = 1$ | $\gamma = 1/2$ |
| $\gamma = 0$ | |
| (222, 11-2) | 1.00 | 1.56 $\alpha$  |
| | | $0.28 \beta$  |
| | | $0.70 \beta$  |
| | | 1.77 $\beta$  |
| (444, 22-4) | 1.14 | 0.97 $\alpha$  |
| | | $0.05 \beta$  |
| | | $0.80 \beta$  |
| | | 12.2 $\beta$  |
| (666, 33-6) | 1.15 | 0.70 $\alpha$  |
| | | $0.02 \beta$  |
| | | $0.81 \beta$  |
| | | 37.6 $\beta$  |
| (888, 44-8) | 1.13 | 0.51 $\alpha$  |
| | | $0.007 \beta$  |
| | | $0.80 \beta$  |
| | | 83.4 $\beta$  |
| (101010, 55-10) | 1.11 | 0.40 $\alpha$  |
| | | $0.004 \beta$  |
| | | $0.76 \beta$  |
| | | 154.1 $\beta$  |