Hadron structure at small momentum transfer.

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Abstract

Giving three examples, the form factors of the nucleon, the polarisability of the charged pion and the interference of the $S_{11}(1535)$ with the $D_{13}(1520)$ excitation of the nucleon in the $\eta p$-decay channel, it is argued that the hadron structure at low momentum transfer is highly significant for studying QCD.

1 Introduction: Significance of low $Q^2$

There is little agreement about the most significant questions and the final aim of the field of hadron physics. One formulation due to an old NSAC definition is: “Understand how hadrons are made up of quarks and gluons”. However, from this definition it is not clear whether one has a study, or even a check, of QCD proper in mind or one wants just to understand bound systems of quarks and gluons with “QCD inspired” models. Since QCD is not yet solvable for low $Q^2$, the region where the hadrons in nature reside, the skeptics turn away and join more promising fields.

Some who stay, put their hope into lattice QCD which, however, is so far good for large quark masses only. The other limit, valid for vanishing quark masses, is chiral dynamics. It reflects directly QCD symmetries and vacua. However, this approximation fails frequently to describe the data, in particular at finite momentum transfers, even if they are small.

An alternative view of hadron physics is to seek the aim in the understanding of the effective degrees of freedom, emerging from the many body system of quarks and gluons. This means one takes QCD for the given right theory of strong interactions and is interested in understanding the peculiar facets of hadrons as confinement, spontaneous symmetry breaking and the strong coupling constant at small momentum transfer.

The characteristic feature of many body systems is that they produce emerging structure which are equivalent to effective degrees of freedom. These degrees of freedom are e.g. constituent quarks, di-quarks, pions or vector mesons allowing an approximate description of many observables. If one establishes the salient features of such degrees of freedom by significant experiments one may have finally the chance to extrapolate between the mentioned low and large quark mass approximations of QCD.

The most burning question in hadron physics is therefore: What are the most significant experiments for studying the effective degree of freedom. Many believe that old and new structure functions like spin-structure function, transversity or generalised parton distributions are the right experiments. It is the conviction of this author that the experiments at low $Q^2$, i.e. $Q^2 \lesssim 5 \text{ Gev}^2$, are at least as significant for such studies. These experiments study gross properties of hadrons as form factors, polarisabilities,
resonances, and sum rules which can be directly confronted with models based on effective degrees of freedom.

In this article three examples of significant observables are given: the nucleon form factors, the polarisability of the charged pion, and the $S_{11}(1535)$ and the $D_{13}(1520)$ excitations of the nucleon in the $\eta p$-decay channel. All three examples hint to deviations from our standard understanding of hadron structure. The significance is just in the deviations between theory and experiment since they offer the chance to learn something new.

For a more detailed account of the arguments in this introduction see ref. [1].

2 Form factor of the nucleons, revisited

Our knowledge of form factors has greatly improved over the last decade due to new experimental opportunities, mainly at Jlab, Bates-MIT and MAMI. The most remarkable features are probably the dramatic deviation of the electric form factor of the proton from the dipole form and the much improved accuracy for the electric form factor of the neutron. Particularly at low momentum transfers the possible appearance of bumps and dips with large wave length in r-space has stirred some controversy about their theoretical interpretation. Before we comment this controversy, we want to show the world data of the space like form factors again. Figure 1 shows these data together with a fit of a somewhat simple minded model. The model assumes a core of constituent quarks ($m_{cq} \approx 300$ MeV) distributed in Q-space as the sum of two dipole forms and surrounded by a pion cloud represented by a Gaussian bump. An alternative model uses a nonrelativistic p-state wave function for the pion cloud. The fits of these two models in ref. [2] imply clearly that the splitting of the charge distribution into a “bare nucleon” and a “pion cloud” is highly model dependent. Therefore, these models must not be taken to seriously as a realistic physical picture of the relativistic system nucleon. Rather they are a convenient way to describe the data by a smooth curve allowing to Fourier-transform it for a visualisation of the charge distribution in the Breit frame.

The particularly interesting charge distribution of the neutron is shown in Fig. 2 which is based on a new fit to the world date for the electric form factor $G_{En}$ depicted in Fig. 2(a). A good fit of the existing data indicates charge at large radii. It is natural to identify this charge with an effective pion, the lightest colour neutral hadron which may reach out beyond the confinement radius of approximately 1 fm.

The idea behind this phenomenological analysis has been harshly criticised recently [3] since the analysis based on the fundamental dispersion-relations is in clear disagreement (see Fig. 2(b)). However, the explanation of this disagreement as a purely statistical problem in ref. [4] is not valid. The dispersion relation fits of all nucleon form factors in the space like and time like region gives a $\chi^2$/dof $\approx 1.8$ for about 200 degrees of freedom (dof). The statistical probability for such a large $\chi^2 P(\chi^2$/dof $\approx 1.8$, dof $\approx 200)$ is smaller than $10^{-10}$. Therefore, the 1-σ bands in [4] determined by adding 1 to the absolute $\chi^2$ are meaningless. There exist more discrepancies between the dispersion relations and the data as discussed in some detail in ref. [1]. These disagreements between a fundamental theory and several different experiments are intriguing and, therefore, their further study is very significant.

3 Pion polarisability, revisited

Another significant observable for which stringent theoretical predictions from Chiral Perturbation Theory (ChPTh) exist [5] is the polarisability of the charged pion. On the experimental side the situation was until recently completely confused. The values determined ranged from 2 to $20 \times 10^{-4}$ fm$^3$ [6]. Therefore, the new determination at MAMI [6] promised hope to clarify the issue. However, the new value of $(\alpha - \beta)_{\pi^+} = (11.6 \pm 1.5_{\text{stat}} \pm 3.0_{\text{syst}} \pm 0.5_{\text{model}}) \times 10^{-4}$ fm$^3$ is in contradiction to the theoretical
value of ref. [5] \((\alpha + \beta)_{\pi^+} = (0.16 \pm 0.1) \times 10^{-4} \text{fm}^3\) and \((\alpha - \beta)_{\pi^+} = (5.7 \pm 1.0) \times 10^{-4} \text{fm}^3\). Such a deviation meant a dramatic problem for ChPTh. But, since the deviation has a 2-\(\sigma\) significance only and a not well controlled model dependence, this determination was taken with scepticism. This scepticism seemed to be confirmed by a new determination of the COMPASS collaboration at CERN using the Primakoff scattering of a pion from nuclei. This collaboration got a preliminary value of \((\alpha - \beta)_{\pi^+} = (5.0 \pm 3.4_{\text{stat}} \pm 1.2_{\text{syst}} \pm ?_{\text{model}}) \times 10^{-4} \text{fm}^3\) [7] much more in line with ChPTh. However, this value is at clear variance with the older determination at Serpukov based on the same Primakoff scattering [8] yielding \((\alpha - \beta)_{\pi^+} = (13.6 \pm 2.8_{\text{tot}} \pm ?_{\text{model}}) \times 10^{-4} \text{fm}^3\). Also the recent reanalysis of \(\gamma\gamma \rightarrow \pi^+\pi^-\) by Fil’kov and Kasheharov results in the large value \((\alpha - \beta)_{\pi^+} = (13.0^{+2.6}_{-1.9}) \times 10^{-4} \text{fm}^3\) [9] consistent with the one at Serpukov and MAMI.

It is, therefore, interesting to trace possible differences between the Serpukov and the COMPASS experiment. It turned out in several talks given by members of the COMPASS collaboration ([10] and this author) that one decisive difference is in the cut on the maximal \(Q_{\text{max}}^2\) of the differential cross section of the pion scattering. This cut has to be chosen small enough to guarantee that the contribution due to the strong interaction below the Coulomb peak is negligible. Whereas Serpukov used the very small limit of \(Q_{\text{max}}^2 \approx 6 \times 10^{-4} \text{GeV}^2\) the COMPASS collaboration allowed \(Q_{\text{max}}^2 \approx 5 \times 10^{-3} \text{GeV}^2\). If one cuts the preliminary COMPASS data at the same low value as Serpukov the COMPASS value increases by about a factor of 2 and agreement between the two Primakoff experiments is established.

It is not difficult to understand this behaviour. The differential cross section of the Primakoff scattering can be described by two amplitudes: the Coulomb amplitude and a diffractive strong interaction
amplitude. In order to get at the Compton scattering effect one selects the $\gamma\pi^-$ channel by means of the detectors. Both amplitudes contain contributions to this final state and the selected statistical ensembles will interfere. Here enters a frequent misunderstanding. The diffraction at the high energies given here will produce many unobserved particles and can be expanded in terms of Feynman diagrams with different final states. Some believe that such “diagrams” would not interfere because they can be distinguished “in principle”. However, such an idea is in conflict with quantum scattering theory which requests that one has to sum over all unobserved channels coherently (see e.g. [11]). In many practical cases of the application of quantum mechanics the interference terms cancel or vanish because of orthogonality, but here they do not.

A useful simple model showing the interference between the Coulomb amplitude and the diffraction amplitude is due to Locher [12]. It has been successfully used to describe the Coulomb-strong interference at high [13] and at low energies [14]. Figure 3 shows the results of the Locher model applied to the kinematical situation of the COMPASS experiment.

The free parameters of the diffractive part are the absolute height and the so called slope parameter $b$. They scale with the mass number $A$ as $d\sigma/d\Omega|_{\text{diffractive}} \propto A$ and $b \propto A^{1/3}$. This as well as the known electric form factor of the proton has been included in the calculation producing Fig. 4. The only parameter adjusted “by hand” is the relative phase between the Coulomb and the diffractive amplitude. Once adjusted to reproduce the marked interference minimum at $Q^2 \approx 3 \times 10^{-3}$ GeV$^2$ for light nuclei, clearly visible in the Serpukov data [8] as well as in the COMPASS data for carbon, it was kept fixed. If one now goes from light nuclei to heavy nuclei the Coulomb phase changes and the destructive interference becomes constructive. This has the consequence that for Pb the total differential cross section contains a considerable contribution from strong interaction down to $Q^2 \approx 0.001$ GeV$^2$. Therefore, the cut at the maximal $Q^2_{\text{max}} < 0.001$ GeV$^2$ was well considered in the Serpukov experiment and the final analysis of COMPASS should respect this point.

In summary, the question of the polarisability of the charged pion is unsettled. However, the majority of the experimental results point to a value deviating by a factor of two from ChPT. Since this prediction is one of the most stringent of ChPT, the final result of COMPASS will be very significant indeed. The recent criticism of Fäldt [15] performing a calculation of the Primakoff scattering using Glauber theory is fortunately not valid for the situation at Serpukov and COMPASS since the cuts in

Figure 2: The dotted curve represents the old Galster parametrisation, the dashed curve the dispersion relation calculation of Belushkin, Hammer and Meißner [4], and the solid curve is a new fit of the phenomenological model of ref. [2].

(a) The world data for the electric form factor of the neutron $G_E^n$.

(b) Visualization of the charge distribution $4\pi r^2 \rho(r)$ in the Breit frame.
both experiments exclude the vector meson contributions this calculation includes.

4 Double polarised $\eta$ production with MAMI C

As the last example of a significant observable we want to present a new determination of the relative phase between the $S_{11}(1535)$ and the $D_{13}(1520)$ excitations of the nucleon. This observable offers a sensitive check whether the hypothesis that the nucleon excitation spectrum is composed of many broad overlapping Breit-Wigner resonances sitting on a non resonant “background” is correct.

In the constituent quark model these resonances are explained by three quarks bound in a QCD inspired, ad hoc confining potential. The excited state are by construction narrow and a non-resonant background can not emerge from such a model. The next best modelling is in the introduction of a coupling to the meson decay channels. If one performs a full coupled channel calculation one can model the width of the resonances and, it is believed, also of the background.

However, generally a Hamiltonian in quantum mechanics has a spectrum consisting of discrete states and a continuum both reflecting the dynamics of the considered system. The decay of the discrete states is mostly described by some imaginary potential, somewhat outside the laws of quantum mechanics, or as just mentioned, by coupled channel calculations. This means that it is rather unclear whether the continuum spectrum of the Hamiltonian is modelled or not by these approaches. It may very well be that some dynamics, actually present in nature, is missed because it is not included in the model Hamiltonian.

This problem is well known from nuclear physics and has limited the ability to explain the excitation spectrum of nuclei at high energies in the region of the “giant resonances”. Here it is quite clear that
much strength is not due to giant resonances and the notion of missing resonances has never been considered. Rather it came as a surprise that the strength of some states was so much localised in energy in the many body system nucleus [16]. In the nucleus one distinguishes between the “spreading width” representing the spectrum of the continuum and the “decay width” representing the coupling to some energetically open decay channels. The situation in nucleons is even more problematic since its constituents, the constituent quarks, are only very approximate effective degrees of freedom. The excitation energy is of the same size as the constituent mass and one can not expect that their mass and dynamics will not change with the excitation energy. For a more detailed account of these aspects see ref. [1].

The only way to make progress here is to go beyond the fitting of the excitation spectrum with overlapping resonances. One has to perform experiments sensitive to all amplitudes including their phases in a given excitation energy region. This means the need to study interference terms as they are accessible with spin degrees of freedom. Through the coming into operation of the 1.5 GeV stage of the Mainz Microtron MAMI (MAMI C) beginning of the year 2007, it became possible to study the double polarisation cross section for the $\eta$ electroproduction on the proton with high precision [17].

MAMI C is a double sided harmonic microtron having all the attractive features of a modern electron accelerator: dc operation with high intensity of more than 100 $\mu$A current, a small phase space providing a clean beam with very small diameter, and polarised electrons and photons. A more detailed description can be found in ref. [18].

It is known that the $S_{11}(1535)$ has a branching ratio of 50% into the $\eta p$ decay channel, whereas the $D_{13}(1520)$ with a branching ratio of 0.06% couples only very weakly to this channel. A group at the Bonn electron storage ring ELSA had found a very peculiar angular behaviour of the target polarisation asymmetry at the energy of these overlapping resonances [19] which could not be explained by the phenomenological model of ref. [20] without changing the relative phase of these two resonances. This means that one gives up the assumption of two interfering resonances described by Breit-Wigner shapes. A particularly intriguing explanation supporting the concerns of the introduction to this section, may be the dynamical model of ref. [21]. In this model the $S_{11}$ partial wave is described without the assumption of a resonance in the frame work of chiral dynamics with coupled channels.

The experiment at MAMI C measured the recoil polarisation of the $p(e, e' p)\eta$ reaction in the mass region of the $S_{11}(1535)$ and $D_{13}(1520)$ partial waves. The double polarisations $P_{h}^{x}$ and $P_{h}^{z}$ agree well with the eta-MAID [22] assuming normal resonance behaviour. However, for the $P_{y}'$ single polarisation a three sigma deviation from the eta-MAID prediction was found [17]. The single polarisation in the electro production $P_{y}'$ contains the same multipole combination $Im\{E_{0+}^2(E_{2+} + M_{2-})\}$ as the target polarisation asymmetry in the photo production experiment at ELSA [19]. Consequently, as for the ELSA experiment, eta-MAID can only be made to agree by adjusting the relative phase between the $S_{11}(1535)$ and $D_{13}(1520)$ states. More such examples should be studied in order to learn better how to distinguish between resonances and background.

5 Conclusions

With three example we have shown that by selecting significant observables one can trace the best effective degrees of freedom. The deviations between theory and experiment indicated may force to go beyond the much used constituent quarks and Goldstone pions. Other effective degrees of freedom may be the observed real ($K$, $\rho$, $\omega$ or virtual ($\sigma$, di-quarks) states. The aspect making observables significant is a deep theoretical understanding and the existence of reliable calculations based on this understanding on the one side. On the other side, the experiments have to provide a matching accuracy to distinguish between different models. Thriving for observables fulfilling these requirements is more significant than the investigation of more and more reactions with limited precision.
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