Optimal Mechanism Design for Single-Minded Agents

KIRA GOLDNER, COLUMBIA UNIVERSITY

NIKHIL DEVANUR
AMAZON

RAGHUVANSH SAXENA
PRINCETON

ARIEL SCHVARTZMAN
PRINCETON -> RUTGERS

MATT WEINBERG
PRINCETON

CONTACT: kgoldner@cs.columbia.edu | ARXIV: 2002.06329 | PAPER: www.kiragoldner.com
bundle $G$ = service or set of goods desired

(value $v$ = how much getting their bundle is worth)

FedEx options

1 day  2 days  3 days

Service options

(v, $G$) ~ $F$

Results

General case:
Characterization via dual properties.
Menu complexity unbounded. (But finite!)

For any $M$, $\exists F$ over $(v, G)$ s.t. the optimal mechanism has $\geq M$ different options.

DMR: $vf(v) - [1 - F(v)] = f(v)\phi(v)$ increasing

$F$ is DMR:
Algorithmic characterization, deterministic.

Out-degree $\leq 1$:
FedEx solution [FGKK '16] applies.
Complexity Spectrum: Characterization of the Optimal Mechanism
Number of Distinct Options to the Buyer

| Menu Complexity | Characterization | Dual Properties | Dual Properties | Dual Properties |
|-----------------|-----------------|----------------|----------------|----------------|
| 1 item          | closed form     | explicit       | dual properties| (open, seems harder) |
| [Mye ‘81]       | [MV ‘07, DDT ‘15] | [Mye ‘81]      | [MV ‘07, DDT ‘15] | |
| FedEx: $2^n - 1$ | 1, 2, 3-day shipping | [FGKK ‘16, SSW ‘18] | [FGKK ‘16, SSW ‘18] | |
| 0(2^n)          | unbounded (but finite) | ≥ unbounded | countably infinite | uncountably infinite |
| [DGSSW ‘20]     | Single-Minded | [DGSSW ‘20] | [DGSSW ‘20] | [DGSSW ‘20] |
| [DHP ‘17, DGSSW ‘20] | Multi-Unit Pricing | 1,2,3-cap for documents | 1,2,3-cap for documents | |
| [DW ‘17]       | Coordinated Valuations | Wifi, +TV, +Cable [w/ g(v)] | Wifi, +TV, +Cable [w/ g(v)] | |
| [DGSSW ‘20]     | [DGSSW ‘20] | [DGSSW ‘20] | [DGSSW ‘20] | [DGSSW ‘20] |
| 2 items         | none            | none           | none           | none          |

DMR => deterministic
Key Idea for Menu Complexity Bound

For any M:

\[ a_A(x_1) > 0 \quad A \]

\[ > a_B(x_2) \]

\[ a_A(x_3) > 0 \]

\[ > a_B(x_4) \]

\[ \vdots \]

\[ a_A(r_A) > 0 \quad A \]

\[ > a_B(r_B) \]

\[ \text{Complementary Slackness} \]

\[ + \implies a = 1 \quad \text{and} \quad - \implies a = 0 \]

\[ 0 \lambda_G(v) > 0 \implies \text{allocation constant} \]

\[ \leftarrow a_{c,A}(v) > 0 \implies A \text{ is preferable to } B \text{ at } v \]

(at least as much area under A than B)

\[ > M \text{ distinct options.} \]

\[ a_B(x_1) = 1 \]

\[ a_B(x_2) > 0 \]

\[ > a_A(x_3) \]

\[ a_B(x_4) > 0 \]

\[ > a_A(x_5) \]

\[ \vdots \]

\[ \vdots \]

\[ x_M \]

\[ x_{M-1} \]

\[ x_1 \]

\[ x_2 \]

\[ x_3 \]

\[ x_4 \]

\[ x_M \]

\[ r_A \]

\[ r_B \]

\[ a_B(r_B) > 0 \]

Master Theorem:
For any dual given only by signs (+/−) and nonnegative variables (0 and \( \alpha \) ←), there exists a distribution that causes this dual.

Corollary: The “bad dual” exists.

Upper Bound:
Our algorithm gives menu complexity length of the sequence \((x_1, A), (x_2, B), (x_3, A), \ldots\)

An infinite such sequence:
Bounded and monotone sequence Converges to \( x^* \), can set this price.