Measurement of Muon Neutrino Quasi-Elastic Scattering on a Hydrocarbon Target at $E_\nu \sim 3.5$ GeV

G.A. Fiorentini, D.W. Schmitz, P.A. Rodrigues, L. Aliaga, O. Altinok, B. Baldin, A. Baumbaugh, A. Bodek, D. Boehnlein, S. Boyd, R. Bradford, W.K. Brooks, H. Budd, A. Butkevich, D.A. Martinez Caicedo, C.M. Castromonte, M.E. Christy, H. Chung, J. Chvojka, M. Clark, H. da Motta, D.S. Damiani, I. Danko, M. Datta, M. Day, R. DeMaat, J. Devan, E. Draeger, S.A. Dytman, G.A. Díaz, B. Eberly, D.A. Edmondson, J. Felix, L. Fields, T. Fitzpatrick, A.M. Gago, H. Gallagher, C.A. George, J.A. Gielata, C. Gingu, B. Bobbi, R. Gran, N. Grossman, J. Hanson, D.A. Harris, J. Heaton, A. Higuera, I.J. Howley, K. Hurtado, M. Jerkins, T. Kafka, J. Kaisen, M.O. Kanter, C.E. Keppel, J. Kilmer, M. Kordosky, A.H. Krajewski, S.A. Kulagin, T. Le, H. Lee, A.G. Leister, G. Locke, E. Maher, W.A. Mann, C.M. Marshall, K.S. McFarland, C.L. McGivern, A.M. McGowan, A. Mislivec, J.G. Morfin, J. Mousseau, D. Naples, J.K. Nelson, G. Niculescu, I. Niculescu, N. Ochoa, C.D. O’Connor, J. Olsen, B. Osmanov, J. Osta, J.L. Palomino, V. Paolone, J. Park, C.E. Patrick, G.N. Perdue, C. Peña, L. Rakotondravohitra, R.D. Ransome, H. Ray, L. Ren, C. Rude, K.E. Sassin, H. Schellman, R.M. Schneider, E.C. Schulte, C. Simon, F.D. Snider, M.C. Snyder, J.T. Sobczyk, C.J. Solano Salinas, N. Tagg, W. Tan, B.G. Tice, G. Tzanakos, J.P. Velásquez, J. Walding, T. Walton, J. Wolcott, B.A. Woltthus, N. Woodward, G. Zavala, H.B. Zeng, D. Zhang, L.Y. Zhu, and B.P. Ziemer (The MINERvA Collaboration)

Departamento de Física, Universidad Técnica Federico Santa María, Avda. España 1680 Casilla 110-V, Valparaíso, Chile

We report a study of $\nu_\mu$ charged-current quasi-elastic events in the segmented scintillator inner tracker of the MINERvA experiment running in the NuMI neutrino beam at Fermilab. The events were selected by requiring a $\mu^-$ and low calorimetric recoil energy separated from the interaction vertex. We measure the flux-averaged differential cross-section, $d\sigma/dQ^2$, and study the low energy particle content of the final state. Deviations are found between the measured $d\sigma/dQ^2$ and the expectations of a model of independent nucleons in a relativistic Fermi gas. We also observe an excess of energy near the vertex consistent with multiple protons in the final state.

PACS numbers: 13.15.+g, 25.30.Pt, 21.10.-k
Charged-current neutrino quasi-elastic scattering, $\nu_p n \rightarrow \mu^- p$, distinguishes neutrino flavor and is valuable for neutrino oscillation experiments at energies near 1 GeV where it is responsible for a large fraction of the total reaction cross-section \[1\,2\]. For free nucleons the scattering process may be described by the standard theory of weak interactions with the inclusion of nucleon form factors \[3\]. Electron scattering \[4\] and neutrino scattering on deuterium \[5\,6\] determine the most important form factors with good precision \[7\]. However, neutrino oscillation experiments typically use detectors made of heavier nuclei such as carbon \[8\,9\], oxygen \[10\], iron \[11\], or argon \[12\,13\,14\] where interactions with nucleons are modified by the nuclear environment. These effects are commonly modeled using a relativistic Fermi gas \[15\,16\] (RFG) description of the nucleus as quasi-free, independent nucleons with Fermi motion in a uniform binding potential. Neutrino interaction generators \[17\,18\,19\,20\,21\] additionally simulate interactions of final state hadrons inside the target nucleus. The MiniBooNE experiment recently observed that this prescription, utilizing the free deuterium value for the axial form factor, does not accurately describe their measurements of quasi-elastic scattering of neutrinos and antineutrinos on a hydrocarbon target \[22\,23\].

The RFG approach may be supplemented by accounting for correlations between nucleons within the nucleus. Evidence for these correlations has been observed in electron-nucleus scattering \[24\]. Processes that produce multiple final state nucleons are thought to lead to enhancements in the cross-section \[25\,26\,27\]. These contributions are modeled using different approaches \[28\,29\] which produce qualitatively similar though not quantitatively identical results. The RFG model may also be replaced by an alternate spectral function (SF) model that calculates the joint probability distribution of scattering off a nucleon of given momentum and binding energy inside a nucleus \[30\]. These nuclear effects may be significant for oscillation experiments seeking to measure the neutrino mass hierarchy and CP violation \[31\,32\,33\].

In this Letter we report the first study of muon neutrino quasi-elastic interactions at energies between 1.5 and 10 GeV from the MINERvA experiment, which uses a finely segmented scintillator detector at Fermilab to measure muon neutrino interactions on nuclear targets. The analysis technique is similar to the one employed to study the antineutrino reaction \[34\]. The signal has a $\mu^-$ in the final state along with one or more nucleons (typically with a leading proton), and no mesons. We reject events in which mesons are produced by requiring that the hadronic system recoiling against the muon has a low energy. That energy is measured in two spatial regions. The vertex energy region corresponds to a sphere around the vertex with a radius sufficient to contain a proton (pion) with 225 (100) MeV kinetic energy. This region is sensitive to low energy protons which could arise from correlations among nucleons in the initial state or final state interactions of the outgoing hadron inside the target nucleus \[35\]. We do not use the vertex energy in the event selection but study it for evidence of multi-nucleon processes. The recoil energy region includes energy depositions outside of the vertex region and is sensitive to pions and higher energy nucleons. We use the recoil energy to estimate and remove inelastic backgrounds.

The MINERvA detector was exposed to the NuMI neutrino beam at Fermilab, configured for this analysis to produce a beam consisting of $>95\% \nu_\mu$ at the peak energy of 3 GeV. The neutrino flux is predicted using a Geant4-based simulation tuned to hadron production data \[37\] as described in Ref. \[35\]. This analysis uses data taken between March and July 2010 with $9.42 \times 10^{19}$ protons on target.

The MINERvA detector consists of a fine-grained scintillator tracker surrounded by electromagnetic and hadronic calorimeters \[38\]. The detector enables reconstruction of the neutrino interaction point, the tracks of outgoing charged particles, and the calorimetric reconstruction of other particles produced in the interaction. MINERvA is located 2 m upstream of the MINOS near detector \[12\], which is used to reconstruct the momentum and charge of muons. The hadronic energy scale is set using data from through-going muons and a scaled down MINERvA detector exposed to a hadron test beam \[38\]. The detector’s performance is simulated by a Geant4-based hit-level simulation and a readout model tuned to match the data \[38\]. Event pile-up causes a decrease in the muon track reconstruction efficiency which we studied in both MINERvA and MINOS by projecting tracks found in one of the detectors to the other and measuring the misreconstruction rate. This resulted in a -9.1% (-4.8%) correction to the simulated efficiency for muons with momenta below (above) 3 GeV/c in MINOS. Neutrino interactions in the detector are simulated using the GENIE neutrino event generator \[17\]. Details of the cross section models and associated parameters are described in Ref. \[35\].

Event reconstruction and selection for this analysis is nearly identical to that used in the MINERvA antineutrino quasi-elastic measurement \[35\] with small modifications to account for the likelihood of a leading proton in the final state instead of a neutron. We require events to have a $\mu^-$ originating in the 5.57 metric ton fiducial volume and assign remaining clusters with energies $>1$ MeV to the recoiling hadronic system. The aforementioned vertex region corresponds to a sphere with $30 \mu$g/cm$^2$ of material centered on the vertex. The recoil system outside the vertex region is required to have $\leq 2$ isolated

---

1 The MINERvA scintillator tracking region is 95% CH and 5% other materials by weight.
Events / 20 MeV

Q

tum transferred to the nucleus, tions [39], and the proton and neutron masses are inter-
of +30 MeV used in Ref. [35] due to Coulomb correc-
ing energy correction is taken to be +34 MeV instead
the muon momentum and angle using a quasi-elastic hy-
groups of spatially contiguous energy depositions.

The neutrino energy and the square of the four momen-
tum transferred to the nucleus, $Q_{QE}^2$, are estimated from
the muon momentum and angle using a quasi-elastic hypo-
thesis, as in the antineutrino analysis [35]. The bind-
ing energy correction is taken to be +34 MeV instead
of +30 MeV used in Ref. [35] due to Coulomb corrections
[39], and the proton and neutron masses are inter-
changed.

Figure 1 shows that the quasi-elastic signal preferen-
tially populates lower recoil energies. However, since
the proton’s kinetic energy is $\approx Q^2 / 2M_{\text{neutron}}$ for quasi-
elastic scattering, the recoil energy is expected to scale
with the momentum transfer as the final state proton be-
comes increasingly energetic and escapes the vertex re-
gion. We account for this by varying a cut on the maxi-
num allowed recoil energy as a function of $Q_{QE}^2$ bin.

The background in each $Q_{QE}^2$ bin is estimated from
the data by fitting the relative normalizations of signal
and background recoil energy distributions whose shapes
are taken from the simulation. This procedure reduces
the relative background prediction by 15% below $Q_{QE}^2$
of 0.8 GeV$^2$ and 5% between 0.8 and 2.0 GeV$^2$. The purity
of the resulting sample ranges from 65% at low $Q_{QE}^2$
to 40% at higher $Q_{QE}^2$. Figure 2 compares the $Q_{QE}^2$
distribution of the 29,620 events which satisfy the selection
criteria to the simulation after rescaling the background
according to the fit. The cross-section as a function of
$Q_{QE}^2$ is extracted by subtracting the backgrounds, cor-
recting for detector resolution and acceptance, and di-
viding by the number of neutrons in the fiducial vol-
(1.65 ± 0.02 × 10$^{30}$) and by the flux, as described
in Ref. [35]. The total neutrino flux integrated between
1.5 and 10 GeV is estimated by the simulation to be
2.91 × 10$^{-8}$/cm$^2$ per proton on target.

The same systematic uncertainties which affect the an-
tineutrino analysis [35] are evaluated in this analysis.

2 Isolated energy depositions are created directly by the leading
proton or by secondary hadronic interactions in the detector.

3 See Supplemental Material SuppLocation for the flux as a func-
tion of energy and for correlations of uncertainties among bins
for the cross-section and shape measurement.
TABLE I: Fractional systematic uncertainties on \(d\sigma/dQ_{QE}^2\) associated with (I) muon reconstruction, (II) recoil reconstruction, (III) neutrino interaction models, (IV) final state interactions, (V) flux and (VI) other sources. The rightmost column shows the total fractional systematic uncertainty due to all sources.

| \(Q_{QE}^2\) (GeV\(^2\)) | Cross-section \((10^{-38}\text{cm}^2/\text{GeV}^2/\text{neutron})\) | Fraction of Cross-section (%) |
|-------------------------|---------------------------------|-----------------------------|
| 0.0 – 0.025             | 0.761 ± 0.035 ± 0.097          | 2.15 ± 0.10 ± 0.17         |
| 0.025 – 0.05            | 1.146 ± 0.047 ± 0.137          | 3.24 ± 0.13 ± 0.22         |
| 0.05 – 0.1              | 1.343 ± 0.034 ± 0.156          | 7.60 ± 0.19 ± 0.50         |
| 0.1 – 0.2               | 1.490 ± 0.028 ± 0.170          | 16.85 ± 0.32 ± 1.04        |
| 0.2 – 0.4               | 1.063 ± 0.019 ± 0.120          | 24.06 ± 0.43 ± 1.06        |
| 0.4 – 0.8               | 0.582 ± 0.013 ± 0.074          | 26.33 ± 0.58 ± 0.85        |
| 0.8 – 1.2               | 0.242 ± 0.014 ± 0.053          | 10.95 ± 0.64 ± 1.45        |
| 1.2 – 2.0               | 0.097 ± 0.008 ± 0.024          | 8.81 ± 0.71 ± 1.43         |

TABLE II: Flux-averaged differential cross-sections and the fraction of the cross-section in bins of \(Q_{QE}^2\). In each measurement, the first error is statistical and the second is systematic.

The measured differential cross-section \(d\sigma/dQ_{QE}^2\) is shown in Table I and Fig. 3. Integrating over the flux from 1.5 to 10 GeV, we find \(\sigma = 0.93 ± 0.01\text{(stat)} ± 0.11\text{(syst)} \times 10^{-38}\text{cm}^2/\text{neutron}\). Figures 3 and 4 and Table III compare the data to the RFG model in the GENIE event generator and a set of calculations made with the NuWro generator.

Different models of nuclear effects in quasi-elastic scattering can differ significantly in the shape of \(d\sigma/dQ^2\) from the expectation of the RFG model. In particular, correlations between nucleons not considered in the mean field RFG approach are predicted to contribute to the cross-section at neutrino energies below 2 GeV [28, 30]. Figure 3 compares the shape of the measured cross section to five different models of the quasi-elastic process on carbon. The GENIE prediction, based on a RFG nuclear model and dipole axial form factor with \(M_A = 0.99\text{GeV}\), is taken as a reference; the data and other models are normalized to have the same total cross section across the range shown before forming the ratio. The NuWro calculations utilize an axial-vector form factor parameterized with a dipole form that has one free parameter, the axial mass \(M_A\), and also incorporate different corrections for the nuclear medium. There is little sensitivity to replacement of the Fermi gas with a spectral function (SF) model of the target nucleon energy-momentum relationship [31]. The neutrino data are marginally more compatible, at least in \(Q_{QE}^2\) shape, with a higher axial mass extracted from fits of the MiniBooNE neutrino quasi-elastic data in the RFG model \((M_A = 1.35\text{GeV}/c^2)\) [27] than with that extracted from deuterium data \((M_A = 0.99\text{GeV}/c^2)\). As with the corresponding antineutrino results [35], our data are in best agreement with a transverse enhancement model (TEM) with \(M_A = 0.99\text{GeV}/c^2\). This model implements an enhancement of the magnetic form factors of bound nucleons that has been extracted from electron-carbon scattering data [27], and is the only one of this type that is applicable at neutrino energies above 2 GeV. Table III shows a comparison using \(\chi^2\) values between the mea-
TABLE III: Comparisons between the measured \(d\sigma/dQ^2_{QE}\) (or its shape in \(Q^2_{QE}\)) and different models implemented using the NuWro neutrino event generator, expressed as \(\chi^2\) per degree of freedom (d.o.f.) for eight (seven) degrees of freedom. The \(\chi^2\) computation in the table accounts for significant correlations between the data points caused by systematic uncertainties.

| NuWro Model | RFG | RFG TEM | RFG SF |
|-------------|-----|---------|--------|
| \(M_A (\text{GeV}/c^2)\) | 0.99 | 0.99 1.35 | 0.99 |
| Rate \(\chi^2/d.o.f.\) | 3.5 | 2.4 3.7 | 2.8 |
| Shape \(\chi^2/d.o.f.\) | 4.1 | 1.7 2.1 | 3.8 |

FIG. 5: Reconstructed vertex energy of events passing the selection criteria in the data (points with statistical errors) compared to the GENIE RFG model (shown with systematic errors) for \(Q^2_{QE} < 0.2\) GeV\(^2/c^2\) (top) and for \(Q^2_{QE} > 0.2\) GeV\(^2/c^2\) (bottom).

This work was supported by the Fermi National Accelerator Laboratory under US Department of Energy contract No. DE-AC02-07CH11359 which included the MINERvA construction project. Construction support also was granted by the United States National Science Foundation under Award PHY-0619727 and by the University of Rochester. Support for participating scientists was provided by NSF and DOE (USA) by CAPES and CNPq (Brazil), by ColAcyT (Mexico), by CONICYT (Chile), by CONCYTEC, DGI-PUCP and IDI/IGI-UNI (Peru), by Latin American Center for Physics (CLAF) and by RAS and the Russian Ministry of Education and Science (Russia). We thank the MINOS Collaboration for use of its near detector data. Finally, we thank the staff of Fermilab for support of the beamline and detector.

\* Deceased
\† now at the Thomas Jefferson National Accelerator Facility, Newport News, VA 23606 USA
\‡ now at Vrije Universiteit Brussel, Pleinlaan 2, B-1050 Brussels, Belgium
\§ also at Department of Physics, University of Antananarivo, Madagascar
\¶ now at Temple University, Philadelphia, Pennsylvania 19122, USA
\** now at Dept. Physics, Royal Holloway, University of London, UK
[1] A. A. Aguilar-Arevalo et al. (MiniBooNE Collaboration), Phys.Rev.Lett. 105, 181801 (2010) arXiv:1007.1150 [hep-ex].
[2] A. A. Aguilar-Arevalo et al. (MiniBooNE Collaboration), Phys.Rev. 102, 101802 (2009) arXiv:0812.2243 [hep-ex].
[3] K. Abe et al. (T2K Collaboration), Phys.Rev.Lett. 107, 041801 (2011) arXiv:1106.2822 [hep-ex].
[4] D. S. Ayres et al. (NOvA Collaboration), (2004), arXiv:hep-ex/0503053 [hep-ex].
[5] C. H. Llewellyn Smith, Phys.Rept. 3, 261 (1972).
[6] R. Bradford, A. Bodek, H. S. Budd, and J. Arrington, Nucl.Phys.Proc.Suppl. 159, 127 (2006) arXiv:hep-ex/0602017 [hep-ex].
[7] A. Bodek, S. Avvakumov, R. Bradford, and H. S. Budd, J.Phys.Conf.Ser. 110, 082004 (2008) arXiv:0709.3538 [hep-ex].
[8] K. S. Kuzmin, V. V. Lyubushkin, and V. A. Naumov, Eur.Phys.J. C54, 517 (2008) arXiv:0712.4384 [hep-ph].
[9] M. Day and K. S. McFarland, Phys.Rev. D86, 053003 (2012) arXiv:1206.6745 [hep-ph].
[10] A. A. Aguilar-Arevalo et al. (MiniBooNE Collaboration), Nucl.Instrum.Meth. A599, 28 (2009) arXiv:0806.4201 [hep-ex].
[11] K. Abe et al. (T2K Collaboration), Nucl.Instrum.Meth. A599, 106 (2011) arXiv:1106.1238 physics.ins-det.
[12] D. G. Michael et al. (MINOS Collaboration), Nucl.Instrum.Meth. A596, 190 (2008) arXiv:0805.3170 [physics.ins-det].
[13] H. Chen et al. (MicroBooNE Collaboration), (2007).
[14] T. Akiri et al. (LBNE Collaboration), (2011), arXiv:1110.6249 [hep-ex].
[15] R. A. Smith and E. J. Moniz, Nucl.Phys. B43, 605 (1972).
[16] A. Bodek and J. L. Ritchie, Phys.Rev. D23, 1070 (1981).
[17] C. Andreopoulos, A. Bell, D. Bhattacharya, F. Cavanna, J. Dobson, S. Dyman, H. Gallagher, P. Guzowski, R. Hatcher, P. Kehayias, A. Meregaglia, D. Naples, G. Pearce, A. Rubbia, M. Whalley, and T. Yang, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 614, 87 (2010) Program version 2.6.2 used here.
[18] Y. Hayato, Acta Phys.Polon. B40, 2477 (2009).
[19] T. Golan, C. Justczak, and J. T. Sobczyk, Phys.Rev. C86, 015505 (2012) arXiv:1202.4197 [nucl-th].
[20] O. Buss, T. Gaitanos, K. Gallmeister, H. van Hees, M. Kaskulov, et al., Phys.Rept. 512, 1 (2012) arXiv:1106.1344 [hep-ph].
[21] O. Lalakulich, K. Gallmeister, and U. Mosel, Phys.Rev. C86, 014607 (2012) arXiv:1205.1061 [nucl-th].
[22] A. A. Aguilar-Arevalo et al. (MiniBooNE Collaboration), Phys.Rev.Lett. 100, 032301 (2008) arXiv:0706.0926 [hep-ex].
[23] A. A. Aguilar-Arevalo et al. (MiniBooNE Collaboration), (2013), arXiv:1301.7067 [hep-ex].
[24] R. Subedi, R. Shineor, P. Monaghan, B. D. Anderson, K. Aniol, et al., Science 320, 1476 (2008) arXiv:0908.1514 [nucl-ex].
[25] J. Carlson, J. Jourdan, R. Schiavilla, and I. Sick, Phys.Rev. C65, 024002 (2002) arXiv:nucl-th/0106047 [nucl-th].
[26] G. Shen, L. E. Marcucci, J. Carlson, S. Gandolfi, and R. Schiavilla, Phys.Rev. C86, 035503 (2012) arXiv:1205.4337 [nucl-th].
[27] A. Bodek, H. S. Budd, and M. E. Christy, Eur.Phys.J. C71, 1726 (2011) arXiv:1106.0340 [hep-ph].
[28] M. Martini, M. Ericson, G. Chanfray, and J. Marteau, Phys.Rev. C81, 045502 (2010) arXiv:1002.4538 [hep-ph].
[29] M. Martini and M. Ericson, (2013), arXiv:1307.1157 [nucl-th].
[30] J. Nieves, M. Valverde, and M. J. Vicente Vacas, Phys.Rev. C73, 025504 (2006) arXiv:hep-ph/0511204 [hep-ph].
[31] O. Benhar, A. Fabrocini, S. Fantoni, and I. Sick, Nucl.Phys. A579, 493 (1994).
[32] J. Nieves, F. Sanchez, I. Ruiz Simo, and M. J. Vicente Vacas, Phys.Rev. D85, 113008 (2012) arXiv:1204.5404 [hep-ph].
[33] O. Lalakulich, U. Mosel, and K. Gallmeister, Phys.Rev. C86, 054606 (2012) arXiv:1208.3678 [nucl-th].
[34] M. Martini, M. Ericson, and G. Chanfray, Phys.Rev. D87, 013009 (2013) arXiv:1211.1523 [hep-ph].
[35] L. Fields, J. Chiovarza, et al. (MINERvA Collaboration), (2013) arXiv:1305.2234 [hep-ex].
[36] J. T. Sobczyk, Phys.Rev. C86, 015504 (2012) arXiv:1201.3673 [hep-ph].
[37] C. Ait et al. (NA49 Collaboration), Eur.Phys.J. C49, 897 (2007) arXiv:hep-ex/0606028 [hep-ex].
[38] L. Allaga et al. (MINERvA Collaboration), (2013) arXiv:1305.5199 [physics.ins-det].
[39] T. Katori, A Measurement of the muon neutrino charged current quasielastic interaction and a test of Lorentz violation with the MiniBooNE experiment, Ph.D. thesis, Indiana University (2008).
[40] T. W. Donnelly and I. Sick, Phys.Rev. C60, 065502 (1999) arXiv:nucl-th/9905060 [nucl-th].
[41] M. Martini, M. Ericson, G. Chanfray, and J. Marteau, Phys.Rev. C80, 065501 (2009) arXiv:0910.2622 [nucl-th].
Appendix: Supplementary Material

| $Q_{QE}^2$ (GeV$^2$) Bins | 0.0 - 0.025 | 0.025 - 0.05 | 0.05 - 0.1 | 0.1 - 0.2 | 0.2 - 0.4 | 0.4 - 0.8 | 0.8 - 1.2 | 1.2 - 2.0 |
|-----------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Cross-section in bin (10^{-38}cm^2/GeV^2/neutron) | ± 0.104 | ± 0.144 | ± 0.160 | ± 0.172 | ± 0.122 | ± 0.075 | ± 0.055 | ± 0.025 |
| $Q_{QE}^2$ (GeV$^2$) | 1.000 | 0.869 | 0.882 | 0.873 | 0.832 | 0.690 | 0.415 | 0.327 |
| 0.0 - 0.025 | 0.025 - 0.05 | 1.000 | 0.905 | 0.917 | 0.882 | 0.727 | 0.457 | 0.357 |
| 0.05 - 0.1 | 0.1 - 0.2 | 1.000 | 0.945 | 0.928 | 0.751 | 0.455 | 0.356 |
| 0.2 - 0.4 | 0.4 - 0.8 | 1.000 | 0.946 | 0.788 | 0.481 | 0.385 |
| 0.8 - 1.2 | 1.000 | 0.865 | 0.600 | 0.514 |
| 1.2 - 2.0 | 1.000 | 0.756 | 0.741 |

TABLE IV: The measurement of the neutrino differential cross-sections in $Q_{QE}^2$, their total (statistical and systematic) uncertainties, and the correlation matrix for these uncertainties

| $Q_{QE}^2$ (GeV$^2$) Bins | 0.0 - 0.025 | 0.025 - 0.05 | 0.05 - 0.1 | 0.1 - 0.2 | 0.2 - 0.4 | 0.4 - 0.8 | 0.8 - 1.2 | 1.2 - 2.0 |
|-----------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| % of Cross-section in bin | +0.20 | ±0.26 | ±0.53 | ±1.09 | ±1.14 | ±1.03 | ±1.58 | ±1.60 |
| $Q_{QE}^2$ (GeV$^2$) | 1.000 | 0.689 | 0.712 | 0.684 | 0.557 | -0.175 | -0.585 | -0.623 |
| 0.0 - 0.025 | 0.025 - 0.05 | 1.000 | 0.745 | 0.770 | 0.653 | -0.211 | -0.631 | -0.694 |
| 0.05 - 0.1 | 0.1 - 0.2 | 1.000 | 0.817 | -0.173 | -0.780 | -0.825 |
| 0.2 - 0.4 | 0.4 - 0.8 | 1.000 | -0.129 | -0.752 | -0.795 |
| 0.8 - 1.2 | 1.000 | 0.760 |
| 1.2 - 2.0 | 1.000 |

TABLE V: The measurement of the shape of the neutrino differential cross-sections for $Q_{QE}^2 < 2.0$ GeV$^2$, their total (statistical and systematic) uncertainties, and the correlation matrix for these uncertainties

| $E_e$ in Bin | 1.5 - 2 | 2 - 2.5 | 2.5 - 3 | 3 - 3.5 | 3.5 - 4 | 4 - 4.5 | 4.5 - 5 | 5 - 5.5 |
|-------------|---------|---------|---------|---------|--------|--------|--------|--------|
| $\nu_e$ Flux (neutrinos/cm$^2$/POT ($\times 10^{-8}$) | 0.310 | 0.409 | 0.504 | 0.526 | 0.423 | 0.253 | 0.137 | 0.081 |

| $E_e$ in Bin | 5.5 - 6 | 6 - 6.5 | 6.5 - 7 | 7 - 7.5 | 7.5 - 8 | 8 - 8.5 | 8.5 - 9 | 9 - 9.5 | 9.5 - 10 |
|-------------|---------|---------|---------|---------|--------|--------|--------|--------|--------|
| $\nu_e$ Flux (neutrinos/cm$^2$/POT ($\times 10^{-8}$) | 0.055 | 0.043 | 0.036 | 0.031 | 0.027 | 0.024 | 0.021 | 0.019 | 0.017 |

TABLE VI: The calculated muon neutrino flux per proton on target (POT) for the data included in this analysis