Big Bounce Baryogenesis

Neil D. Barrie

Kavli IPMU (WPI), UTIAS, University of Tokyo, Kashiwa, Chiba 277-8583, Japan
Email: neil.barrie@ipmu.jp

Abstract

We explore the possibility of an Ekpyrotic contraction phase, induced by a fast-rolling pseudoscalar field, harbouring a mechanism for Baryogenesis. Chern-Simons couplings between the pseudoscalar and the Standard Model Hypercharge and Weak gauge fields enable the generation of a non-zero Chern-Simons number density during the contracting phase. The resulting baryon number produced is consistent with observation for a range of couplings and high bounce scales. The gauge field production provide the seeds for galactic magnetic fields and may be a source of Gravitational Waves.
1 Introduction

The origin of the baryon asymmetry of the universe is one of the most important mysteries of particle physics and cosmology. The size of the observed baryon asymmetry is parametrized by the asymmetry parameter $\eta_B$ [1],

$$\eta_B = \frac{n_B}{s} \simeq 8.5 \times 10^{-11},$$  

where $n_B$ and $s$ are the baryon number and entropy densities of the universe, respectively. To generate this asymmetry, in a $\mathcal{CP}$-$\mathcal{T}$ conserving theory, the Sakharov conditions must be satisfied [2]. The Standard Model has all the ingredients for producing a baryon asymmetry in the early universe but it is orders of magnitude smaller than that required to explain observations, necessitating the existence of new physics [3].

The Inflationary scenario is a well-established paradigm in standard cosmology due to its success at solving various observational problems such as flatness, horizon, and monopole problems, as well as providing measurable predictions in the form of primordial perturbations [4–7]. Many models have been proposed and significant effort expended in the pursuit of experimental verification, but the exact mechanism for inflation is unclear [8]. An Inflationary setting provides a unique venue in which Baryogenesis could occur, and has been an area of interest in recent years [9–14]. One possibility is through coupling a pseudoscalar inflaton, $\phi$, to Standard Model gauge bosons through a Chern-Simons term [15–17],

$$\frac{\phi}{4\Lambda} Y_{\mu\nu} \tilde{Y}^{\mu\nu}, \quad \frac{\phi}{4\Lambda} W^a_{\mu\nu} \tilde{W}^{a\mu\nu},$$  

where $Y_{\mu\nu}$ and $W^a_{\mu\nu}$ correspond to the Standard Model Hypercharge and Weak gauge field strength tensors, and the dual of the field strength tensor is defined as $\tilde{F}^{\mu\nu} = \frac{1}{2\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$. This form of interaction can be present in low energy effective field theories associated with a Stueckelberg field [18], or the Green-Schwartz mechanism [19], with corresponding UV cut-off $\Lambda$. The coupling of a pseudoscalar inflaton to the Hypercharge term has been found to generate the observed baryon asymmetry [15–17]. This mechanism provides unique connections between the cosmological background evolution and particle physics through not only Baryogenesis but possible Gravitational Wave signatures through gauge field production, and the seeding of large scale magnetic fields. Thus, this mechanism provides multiple avenues for observational and experimental verification.

Despite the successes of inflationary cosmology, it suffers from many unsolved issues. These include the questions of initial conditions, fine-tuning, the singularity problem, degeneracy of model predictions, trans-Planckian field values and violation of perturbativity [20]. These problems have provided motivation for considering alternative cosmologies such as Bounce Cosmologies [20–25]. The Bounce scenario postulates a period of space-time contraction prior to the onset of standard Big Bang cosmology, with these two epochs separated by a bounce through which the Universe transitions from a period of contraction to the usual expansion phase. One well-studied example of a Bounce Cosmology is the Ekpyrotic universe which involves a period
of ultra-slow contraction \( (\omega \gg 1) \) prior to a bounce \[26, 36\]. A period of Ekpyrotic contraction can be induced by a fast-rolling of a scalar field with a negative exponential potential, which quickly dominates the universe diluting other energy densities, including anisotropies. This model solves the flatness, horizon and monopole problems, and is capable of generating the perturbations observed in the Cosmic Microwave Background (CMB). There has been significant research and advancement in the details of this alternative cosmology model, with current observational results being in tension with the simplest models \[24\]. Ekpyrotic models that consist of two scalars can help to alleviate these issues, and this may be further resolved with improved theoretical understanding \[33, 34\]. This cosmological model provides the interesting background dynamics of a phase of cosmological contraction and has motivated investigations into applications to other open cosmological issues - the origin of dark matter, Magnetogenesis and Gravitational Waves \[37, 42\].

In this work, we propose a Baryogenesis mechanism that takes place during an Ekpyrotic contraction phase in a Bounce Cosmology, inspired by pseudoscalar Inflation Baryogenesis scenarios \[13–17\]. As proof of concept, a single field Ekpyrotic model will be considered, which consists of a pseudoscalar field coupled to the Standard Model Weak and Hypercharge Chern-Simons terms. The fast-rolling of the pseudoscalar will lead to the generation of a net Chern-Simons number carried by the gauge fields, providing the possibility for successful Baryogenesis. The paper will be structured as follows, in Section 2, we describe the properties of the Ekpyrotic phase. The model framework and gauge field evolution will be discussed in Section 3. In Section 4, the Chern-Simons number density accumulated in the \( SU(2) \) sector, prior to the bounce, and the subsequent baryon number generated will be calculated, with the parameter space surveyed for consistency with observation. Section 5 will discuss Baryogenesis via the helicity generated in the Hypermagnetic field during the contracting phase. Finally, in Section 6, we will conclude with a discussion of the results and future directions for investigation.

## 2 Ekpyrotic Contraction and Model Description

The known issues with inflation have led to the exploration of possible alternatives to the usual inflationary paradigm, such as string gas cosmology, bounce, and cyclic models. As with inflation, these models attempt to solve the flatness, horizon, and monopole problems, and must be able to source the nearly scale invariant spectrum of temperature fluctuations observed in the CMB. In what follows, we will focus on a well-known type of bounce cosmology, the Ekpyrotic bounce, which solves the known cosmological problems, and can potentially resolve various issues with other bounce models, while providing the benefits over inflation of geodesic completion, sub-Planckian field values, and removal of the singularity problem. This type of contraction phase is a feature of recent studies into cyclic universe models \[36\].

The period of contraction in Ekpyrotic cosmology is characterised by a large equation of state \( \omega \gg 1 \) prior to a bounce. This can be induced when the universe is dominated by a stiff form of matter such as a fast-rolling scalar field. During such a contracting phase, the stiff
matter will come to dominate the total energy density of the universe,

\[ \rho_{\text{Total}} = \frac{\rho_k}{a^2} + \frac{\rho_{\text{mat}}}{a^3} + \frac{\rho_{\text{rad}}}{a^4} + \frac{\rho_a}{a^6} + \ldots + \frac{\rho_{\phi}}{a^{3(1+\omega_{\phi})}} + \ldots \]  

(3)

where \( \rho_a \) is the energy density associated with anisotropies, and \( \rho_{\phi} \) is the energy density of the fields responsible for the Ekpyrotic contraction. From Eq. (3) it is clear that in a contracting space-time background the \( \rho_{\phi} \) term will quickly increase and come to dominate the energy density of the universe if \( \omega_{\phi} \gg 1 \). Consequently, a sufficiently long period of \( \omega_{\phi} \gg 1 \) contraction naturally leads to the suppression of any initial or generated curvature and anisotropy perturbations, while also diluting the initial radiation and matter densities. This is how the Ekpyrotic phase is able to solve the known cosmological problems, and remove the problem of the rapid growth of initial anisotropies and any anisotropic instabilities, which can occur in other bounce scenarios.

To see how an Ekpyrotic contracting epoch can be induced by a rolling scalar field, consider the following relation for the equation of state parameter for a scalar \( \phi \),

\[ \omega = \frac{\frac{1}{2} \dot{\varphi}^2 - V(\varphi)}{\frac{1}{2} \dot{\varphi}^2 + V(\varphi)}. \]  

(4)

The equation of state can be \( \omega \gg 1 \) if,

\[ \frac{1}{2} \dot{\varphi}^2 + V(\varphi) \approx 0 \quad \text{and} \quad \frac{1}{2} \dot{\varphi}^2 - V(\varphi) \gtrsim 0. \]  

(5)

A simple way to achieve this, is to have the scalar \( \varphi \) fast-roll down a negative exponential potential, leading to an approximate cancellation in the denominator. That is, a scalar potential given by,

\[ V(\varphi) \approx -V_0 e^{-\sqrt{2} \epsilon |\varphi|/M_p}, \]  

(6)

where the \( \epsilon \) parameter shall be referred to as the fast-roll parameter, and where \( M_p = 2.4 \cdot 10^{18} \) GeV is the reduced Planck mass. The relation between the \( \epsilon \) and \( \omega \) is found to be,

\[ \epsilon = \frac{3}{2}(1 + \omega). \]  

(7)

The fast-roll parameter can be considered analogous to the inflationary slow-roll parameter where \( \epsilon_{\text{inf}} \ll 1 \), while instead here \( \epsilon \gg 1 \) is required with corresponding fast-roll conditions [43]. Interestingly, there is a seeming duality between the Ekpyrotic and Inflationary regimes through the respective fast and slow-roll parameters, which is the motivation for the Baryogenesis mechanism considered here.

The Ekpyrotic action is of the following form,

\[ S_{\text{EKP}} = \int d^4x \sqrt{-g} \left( -\frac{1}{2} M_p^2 R + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + V_0 e^{-\sqrt{2} |\varphi|/M_p} \right), \]  

(8)
from which can be derived the scale factor during the Ekpyrotic Contraction,

$$a = (\epsilon H_b t)^{\frac{1}{\epsilon}} = (\epsilon H_b \tau)^{\frac{1}{\epsilon-1}}$$, (9)

and Hubble rate,

$$H = \frac{H_b}{(\epsilon H_b |\tau|)^{\frac{1}{\epsilon-1}}} \simeq \frac{1}{\epsilon \tau}$$, (10)

where we fix the bounce point to be at $$t_b = \tau_b = \frac{1}{\epsilon H_b}$$ such that $$a(t_b) = a(\tau_b) = 1$$, and $$t, \tau \in (-\infty, \frac{1}{\epsilon H_b})$$ during the Ekpyrotic contraction. For large $$\epsilon$$, the contraction rate is very slow such that $$t \sim \tau$$. Through inspection of Eq. (9) and Eq. (10), it is clear that for $$\epsilon \gg 1$$ the Hubble rate can increase exponentially, while the scale factor shrinks by only an $$\mathcal{O}(1)$$ factor. Thus, for $$\epsilon \sim \mathcal{O}(100)$$ only a single e-fold of contraction is needed to generate 60 e-folds worth of perturbations.

In the case of $$\epsilon \gg 1$$, the equations of motion for the $$\varphi$$ scalar are solved by the scaling solution [33],

$$\varphi \simeq M_p \sqrt{\frac{2}{\epsilon}} \ln(-\sqrt{\epsilon V_0} \tau/M_p)$$, (11)

and subsequently,

$$\varphi' \simeq \sqrt{\frac{2}{\epsilon}} \frac{M_p}{\tau}$$, (12)

which is expressed in conformal time.

Despite the advantages and simplicity of the scenario described above, the single field Ekpyrotic scenario leads to a strongly blue-tilted spectral index which is in significant tension with current CMB observations [1, 24]. This is one of the main issues of the original formulation of the Ekpyrotic model. It is solved through the introduction of an additional Ekpyrotic scalar field. If the Ekpyrotic contraction is followed by a period of kinetic dominated contraction ($$\omega = 1$$) prior to the bounce, the observed CMB spectral index can be produced for $$\epsilon \sim \mathcal{O}(100)$$ through the conversion of isocurvature perturbations into adiabatic perturbations by the additional scalar [44], and produce a nearly scale invariant scalar power spectrum in the CMB [45]. This is known as the New Ekpyrotic model, in which the background evolution is induced by two Ekpyrotic scalars and consists of a non-singular bounce sourced by a ghost condensate [32, 36, 46]. In this work we will mainly focus on the simplest single field form of the Ekpyrotic scenario, but the possibility of a two-field New Ekpyrotic scenario is parametrised through allowing an early cut-off to Chern-Simons number production prior to the bounce point.

Ekpyrotic scenarios tend to predict relatively large non-gaussianities in the CMB. This can provide constraints on the background evolution in combination with the scalar power spectrum, for which current observational bounds are $$2 - 3\sigma$$ from the central observed values for typical parameter choices. The current best constraints on the non-gaussianity, from the Planck observations [47], are,

$$f_{NL}^{\text{local}} = 0.9 \pm 5.1$$, (13)
while the $f_{NL}$ predicted by the two scalar Ekpyrotic scenario, with a period of kination prior to the bounce, is \[48\]–\[50\],

$$f_{NL} \propto \sqrt{\epsilon},$$ (14)

where $\epsilon \sim \mathcal{O}(100)$ can successfully produce a nearly scale-invariant scalar power spectrum. This discrepancy may be resolved by future theoretical improvements in the understanding of the period around the bounce point \[51\].

The Ekpyrotic model predicts a blue-tilted tensor power spectrum with a small tensor-to-scalar ratio $r$ on CMB scales, below current sensitivities and difficult to measure within the near future. It is possible that the dynamics of other fields such as the gauge field production during the Ekpyrotic phase, through Chern-Simons couplings considered in this work, could lead to Gravitational Wave signatures \[40\]–\[41\].

3 Gauge Field Dynamics during Contracting Phase

In our model the field $\varphi$ will be taken to be a pseudoscalar field with Chern-Simons couplings to the Standard Model Weak and Hypercharge fields given in Eq. (2). The fast-rolling of $\varphi$ induces $\mathcal{CP}$ violating dynamics in the gauge field sector generating a Chern-Simons number density.

Now that $\varphi$ is taken to be a pseudoscalar, it is necessary to reconsider the form of the potential such that it preserves $\mathcal{P}$ transformations. One possibility is the following,

$$V(\varphi) = \frac{V_0}{2 \cosh \left(\sqrt{2\epsilon} \frac{\varphi}{M_p} \right)},$$ (15)

which for large $\varphi$ converges to,

$$V(\varphi) \approx -V_0 e^{-\sqrt{2\epsilon} \frac{\varphi}{M_p}},$$ (16)

satisfying the scaling solution provided in Eq. (11).

In what follows, the assumption will be made that the motion of $\varphi$ is only negligibly affected by the gauge field sector. This approximation will be justified in the subsequent Sections 4 and 5 through the requirement of a negligible gauge field energy density. We utilise the action in Eq. (8), and the subsequent scaling solution given in Eq. (12) to describe the evolution of $\varphi$.

We require that $\dot{\varphi} > 0$, such that the positive frequency gauge field modes are enhanced and a positive $B$ number is generated.

We investigate the dynamics of the pseudoscalar $\varphi$ coupled to Chern-Simons terms of the Standard Model $U(1)_Y$ and $SU(2)_W$ fields in an Ekpyrotic contracting background. The gauge field Lagrangian is given by,

$$\frac{1}{\sqrt{-g}} \mathcal{L}_g = -\frac{1}{4} g^\mu\nu g^\alpha\beta Y_{\mu\nu} Y_{\alpha\beta} - \frac{1}{4} g^\mu\nu g^\alpha\beta W^a_{\mu\nu} W^a_{\alpha\beta} - \frac{\varphi}{4\Lambda} Y_{\mu\nu} \tilde{Y}^{\mu\nu} - \frac{\varphi}{4\Lambda} W^a_{\mu\nu} \tilde{W}^{a\mu\nu},$$ (17)

where $Y_{\mu\nu}$ and $W^a_{\mu\nu}$ denote the Hypercharge and Weak field strength tensors respectively, with corresponding coupling constants $g_1$ and $g_2$, and we assume that the two Chern-Simons terms
share the same UV cut-off $\Lambda$. Summation over the weak isospin indices $a = 1, 2, 3$ shall be assumed throughout. The background dynamics are due to the rolling of $\varphi$ with scale factor and Hubble rate given in Eq. (9) and Eq. (10), respectively.

In conformal coordinates, the metric can be expressed as: $g_{\mu\nu} = a^2(\tau)\eta_{\mu\nu}$, so that the Lagrangian in Eq. (17) becomes,

$$\mathcal{L} = -\frac{1}{4} \eta^{\mu\rho} \eta^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - \frac{\varphi}{8\Lambda} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}, \quad (18)$$

where we use the notation $F_{\mu} = Y_{\mu}$ and $W_{\mu}^a$. To allow analytical treatment, we will make the simplified assumptions of linearised equations of motion for the gauge fields, and that the back-reaction on the motion of $\varphi$ due to the production of the gauge field $F_i$ is negligible. The non-abelian nature of the $W^a_{\mu}$ gauge field may lead to additional effects, but here we only consider a first order approximation for calculability.

The Lagrangian above leads to the following equation of motion for the $F$ gauge field,

$$\left(\partial_\tau^2 - \vec{\nabla}^2\right) F^i + \frac{\varphi'(\tau)}{\Lambda} \epsilon^{ijk} \partial_j F_k = 0 \ , \quad (19)$$

where the gauge $F_0 = \partial_0 F_i = 0$ has been chosen.

The Ekpyrotic scalar motion $\varphi'$ in Eq. (19) is defined by the scaling solution,

$$\varphi' = \sqrt{\frac{2}{\epsilon}} \frac{M_p}{\tau}, \quad (20)$$

which upon substituting into the linearised equation of motion for the $F_\mu$ gauge field gives,

$$\left(\partial_\tau^2 - \vec{\nabla}^2\right) F^i + \frac{2\kappa}{-\tau} \epsilon^{ijk} \partial_j F_k = 0 \ , \quad (21)$$

where

$$\kappa = \frac{M_p}{\sqrt{2\epsilon \Lambda}}. \quad (22)$$

Note, the similarity to the inflationary scenario, in which the instability parameter is defined as $\xi = \sqrt{\frac{2}{\epsilon \Lambda}} \frac{M_p}{\Lambda} [15]$.

To quantize this model, we promote the $F$ gauge boson fields to operators and assume that the boson has two possible circular polarisation states,

$$F_i = \int \frac{d^3k}{(2\pi)^{3/2}} \sum_\alpha \left[ G_\alpha(\tau, k) \epsilon_{ia} \hat{a}_a e^{i k \cdot x} + G^*_\alpha(\tau, k) \epsilon^*_{ia} \hat{a}_a^* e^{-i k \cdot x} \right] , \quad (23)$$

where $\epsilon_{\pm}$ denotes the two possible helicity states of the $F$ gauge boson ($\epsilon_+ = \epsilon_-$) and the creation, $\hat{a}_\alpha(k)$, and annihilation, $\hat{a}_\alpha(k)$, operators satisfy the canonical commutation relations,

$$\left[ \hat{a}_\alpha(k), \hat{a}_\beta^*(k') \right] = \delta_{\alpha\beta} \delta^3(k - k'), \quad (24)$$
and
\[ \hat{a}_n^\dagger (\vec{k}) |0\rangle_\tau = 0 , \] (25)
where \(|0\rangle_\tau\) is an instantaneous vacuum state at time \(\tau\).

The mode functions in Eq. (23) are described by the following equation, from Eq. (19),
\[ G''_\pm + \left( k^2 \mp \frac{2\kappa k}{-\tau} \right) G_\pm = 0 . \] (26)

Interestingly, this wave mode function equation is equivalent to the case when \(\epsilon = 3\) and \(V(\varphi) \approx 0\) case, which is a contracting kinetic domination epoch with \(\omega = 1\). In this case, whether the background evolution is dictated by \(\varphi\) or not, the above wave mode equation is valid, with \(\kappa = \frac{\varphi'}{6\Lambda H_b}\), where \(\varphi_b\) is the velocity of the scalar at the bounce point. If we were to consider the case of sub-dominant pseudoscalar with \(V(\varphi) = 0\), and \(\varphi'(\tau) = \varphi_b'/a(\tau)^2\). An upper limit on the value of \(\varphi_b\), and hence \(\kappa\), is provided by the requirement that the energy density does not dominate. Therefore, the results in Section 4 and 5 can be easily reinterpreted to this case.

Solving for the mode functions \(G_\pm\) in Eq. (26) gives,
\[ G_\pm = e^{-ik\tau} \sqrt{2k} e^{\pm\pi\kappa/2} U(\pm i\kappa, 0, 2ik\tau) , \] (27)
where \(U\) is a Confluent Hypergeometric functions. This solution has been derived using the Wronskian normalisation and matched to \(\mathcal{CP}\)-invariant planewave modes at \(\tau \to -\infty\), described by,
\[ A_\pm(\tau, k) = \frac{1}{\sqrt{2k}} e^{-ik\tau} . \] (28)

Now that we have derived the dynamics of the gauge fields during the Ekpyrotic contracting phase, it is possible to evaluate the net Chern-Simons number generated during the epoch via a Bogoliubov transformation with the adiabatic vacuum state. Firstly, in Section 4, we will investigate the possibility for successful Baryogenesis within this framework through the generation of a net Chern-Simons number density through the \(\varphi W \tilde{W}\) coupling. In Section 5, we explore the production of the observed baryon asymmetry though helical Hypermagnetic fields induced by the \(\varphi Y \tilde{Y}\) coupling, which can lead to generation of baryon number at the Electroweak Phase Transition (EWPT) through conversion of the Hypermagnetic fields.

4 Generated Baryon Asymmetry from \(\varphi W \tilde{W}\)

From the derived wave mode functions, we evaluate the Chern-Simons number generated by the rolling of the pseudoscalar field at a given \(\tau\). The relation between the baryon number density and Chern-Simons number density is,
\[ \partial_\mu \left( \sqrt{-g} f_B^\mu \right) = \frac{3g_2^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \bar{W}_\nu^\alpha W_\rho^\alpha = \frac{3g_2^2}{16\pi^2} \partial_\mu \left( \sqrt{-g} K^\mu \right) , \] (29)
where $K^\mu$ is the topological current.

We perform a Bogoliubov transformation between the wave mode functions of Eq. (27) and Eq. (28), and only consider wave numbers that satisfy $-k\tau < 2\kappa$, the modes which contribute significantly to the asymmetry between circular polarisations. The generated baryon number density at a given $\tau$ is,

$$\frac{n_B(\tau)}{a(\tau)^3} = \frac{3g_2^2}{32\pi^2} \int_0^\tau d\tau \langle 0 | W_\mu \bar{W}^{\mu\nu} | 0 \rangle$$

$$\simeq \frac{9g_2^2}{8\pi^4} \int_0^{2\kappa |\mathcal{H}|} k^3 (|G_+(\tau)|^2 - |G_-(\tau)|^2) dk$$

$$\simeq \frac{9g_2^2}{16\pi^4} (-\epsilon |\mathcal{H}|)^3 C(\kappa) , \quad (30)$$

where $z = -k\tau$, and summation over $a = 1, 2, 3$ is included. The integral term $C(\kappa)$ takes the following form,

$$C(\kappa) = \int_0^{2\kappa} z^2 (e^{2\pi\kappa}|U(i\kappa, 0, -2iz)|^2 - e^{-2\pi\kappa}|U(-i\kappa, 0, -2iz)|^2) dz , \quad (31)$$

where we will consider values of $\kappa \geq \frac{1}{\sqrt{2\epsilon}}$, which correspond to $\Lambda < M_p$. An approximate form of Eq. (31) can be determined,

$$C(\kappa) \sim 0.007 \frac{e^{2\pi\kappa}}{\kappa^4} , \text{ for } \kappa > 1 . \quad (32)$$

Before proceeding to the calculation of the asymmetry parameter, we shall derive constraints on possible parameters such that the gauge field energy density generated by the rolling of $\varphi$ does not come to dominate the background dynamics, that is $3M_p^2 H^2 \gg \rho_{CS}(\tau)$. The energy density produced by the Chern-Simons term is,

$$\rho_{CS}(\tau) = \langle 0 | W_\mu \bar{W}^{\mu\nu} | 0 \rangle \simeq \frac{12}{\pi} D(\kappa) (\epsilon |\mathcal{H}|)^4 , \quad (33)$$

where

$$D(\kappa) = \kappa \int_0^{2\kappa} z^3 (e^{2\pi\kappa} [U(i\kappa, 0, -2iz)U(1 - i\kappa, 1, 2iz) + U(-i\kappa, 0, 2iz)U(1 + i\kappa, 1, -2iz)]$$

$$+ e^{-2\pi\kappa} [U(-i\kappa, 0, -2iz)U(1 + i\kappa, 1, 2iz) + U(i\kappa, 0, 2iz)U(1 - i\kappa, 1, -2iz)]) dz , \quad (34)$$

which has the following approximate form, for $\kappa > 1$,

$$D(\kappa) \sim 0.01 \frac{e^{2\pi\kappa}}{\kappa^4} \sim 1.5 C(\kappa) . \quad (35)$$

To ensure that the dynamics of the gauge fields do not effect the background evolution induced by $\varphi$ we require that,

$$M_p \gg \sqrt{\frac{6C(\kappa)}{\pi}} \epsilon^2 |\mathcal{H}_c| , \quad (37)$$
where we have fixed the Hubble rate at the maximum value $|H_c|$ for validity over the entire Ekpyrotic epoch. The parameter $H_c$ is the Hubble rate when the acceleration of $\phi$ ends, and subsequently the asymmetry generation ends. This is either at the bounce point ($H_c = H_b$), or at the onset of a period of kination prior to the bounce, where the background dynamics become dominated by an additional Ekpyrotic scalar ($|H_c| < |H_b|$). For $\kappa \leq 1$ this relation is easy to satisfy as $C(\kappa)$ becomes small, and the Hubble parameter is already required to be much less than the Planck scale. This requirement is compared with the model parameters necessary to successfully produce the observed baryon asymmetry.

The Chern-Simons number generated in the $SU(2)_W$ gauge fields by the rolling of the pseudoscalar field $\phi$ corresponds directly to a non-zero density of $B+L$ charge, as calculated in Eq. (30). To ensure this is not washed out by sphalerons in the thermal plasma after reheating, we require the presence of heavy right-handed neutrinos with mass greater than the equilibrium temperature of the sphaleron processes, $T_{\text{spal}} \sim 10^{12}$ GeV. Hence, we must consider reheating temperatures greater than $10^{12}$ GeV. After reheating, the excess $L$ is removed by the lepton violating Majorana mass term of the right-handed neutrinos, which are in thermal equilibrium before the sphaleron processes are in equilibrium. The remaining net $B$ is redistributed by equilibrium sphaleron processes in the known way, conserving the non-zero $B-L$ charge density.

The sphaleron redistribution gives the following relation for the remaining baryon number, $B = \frac{28}{79} (B - L)$ \[52\].

At the bounce point, we match the Chern-Simons number calculated in Eq. (30) with that at the onset of reheating, assuming instantaneous reheating. The exact dynamics of the bounce are model-dependent, and are expected to have only a minor effect on the generated asymmetry, as the bounce is assumed to be smooth and entropy conserving. Any such effects are parametrised in the case $|H_c| < |H_b|$. Assuming no significant entropy production after reheating, the entropy density is given by $s \simeq \frac{\pi^2}{45} g_\ast(T_{\text{rh}}) T_{\text{rh}}^3$, where $T_{\text{rh}}$ is the reheating temperature and $g_\ast(T_{\text{rh}}) \simeq 106.75$. Thus, from Eq. (30) the baryon asymmetry parameter generated by this mechanism is,

$$\eta_B = \frac{28}{79} \frac{n_B(\tau_c)}{s} \simeq \frac{5}{8 \pi^2 g_\ast} C(\kappa) \left( \frac{\epsilon |H_c|}{T_{\text{rh}}} \right)^3,$$

and hence,

$$\frac{\eta_B}{\eta_B^{\text{obs}}} \simeq 2 \cdot 10^5 C(\kappa) \left( \frac{\epsilon |H_c|}{T_{\text{rh}}} \right)^3.$$

Firstly, consider the case of $H_c = H_b$ and assume instantaneous reheating. We then determine constraints on both the parameters $H_b$ and $T_{\text{rh}}$ such that successful Baryogenesis occurs, namely,

$$|H_b| \simeq \frac{2 \cdot 10^{14} \text{ GeV}}{\epsilon^2 C(\kappa)^{2/3}} \quad \text{and} \quad T_{\text{rh}} \simeq \frac{10^{16} \text{ GeV}}{\epsilon^2 C(\kappa)^{1/3}}.$$

The allowed parameter space is depicted in Figure \[\text{I}\]. From the lower bounds on the reheating temperature $T_{\text{rh}} > 10^{12}$ GeV and fast-roll parameter $\epsilon > 10$ we obtain an upper bound on the
parameter $\kappa \sim 5.5$. Interestingly, these can be easily substituted into the energy density constraint derived in Eq. (37) to provide a limit solely on $\kappa$,

$$C(\kappa) \gg 2.5 \cdot 10^{-28},$$

where if we now consider $\kappa \gtrsim 1$ we obtain the approximate form,

$$e^{\pi \kappa/3} \gg 10^{-4} \kappa^{2/3} \text{ or } 1 \gg \epsilon \left(\frac{T_{rh}}{10^{25} \text{ GeV}}\right)^{3},$$

which is satisfied for all $\kappa \gtrsim 1$ and observationally consistent $\epsilon$, hence the limit from the minimum reheating temperature and fast-roll parameter remains. Large $\kappa$ may lead to inconsistency with the linearised approximation, and possible suppression of the baryon asymmetry generated.

Figure 1: The shaded region denotes allowed parameter space of $H_b$, $\epsilon$ and $\kappa$ that leads to successful Baryogenesis and prevents washout by sphaleron processes. The dashed grey line corresponds to the minimum Bounce scale $H_b$ and reheating temperature $T_{rh}$, assuming instantaneous reheating. The black dot-dashed line is the limit $\Lambda < M_p$ or $\kappa > \frac{1}{\sqrt{2}}$, while the solid, dotted and dashed black lines depict the $\kappa = 1$, 3, and 5 cases respectively.

Consider the scenario where the generation of Chern-Simons number density ends prior to the bounce point, $|H_c| < |H_b|$. A bend in the trajectory of the scalar field space, as two field Ekpyrotic contraction comes to an end, can lead to a kinetic dominated contraction phase prior to the bounce, during which one of the fields stops accelerating. If this is the pseudoscalar, the time dependence of the $\mathcal{CP}$ violating term disappears ending the gauge field production. In this case, the asymmetry can be written as,

$$\frac{\eta_B}{\eta_{B}\text{ps}} \simeq 2 \cdot 10^{5}C(\kappa) \left(\frac{\epsilon |H_b|}{T_{rh}}\right)^{3} \left(\frac{H_c}{H_b}\right)^{3},$$

where we have assumed that the number of e-folds of contraction between $\tau_c$ and $\tau_b$ is small.
Figure 2: The shaded region depicts the allowed parameter space of $H_c$, $\epsilon$ and $\kappa$ that leads to successful Baryogenesis and does not over produce energy density in the $W$ field, where $T_{rh} = 10^{15}$ GeV has been selected. The dashed grey line corresponds to the maximum energy density constraint given in Eq. (45). The solid, dotted and dashed black lines depict the $\kappa = 1$, 3 and 5 cases respectively, while the upper limit on $H_c$ is where $H_c = H_b$. The shaded region above the $\kappa = 1$ line corresponds to scenarios with $\kappa < 1$.

From requiring successful Baryogenesis and instantaneous reheating at the bounce point, we obtain the following relation for $H_c$,

$$|H_c| = 2 \cdot 10^{-2} \frac{T_{rh}}{\epsilon C(\kappa)^{1/3}}, \quad (44)$$

which can be substituted into the energy density constraint found in Eq. (37) to obtain a $\kappa$ independent bound on $H_c$,

$$\frac{H_c}{H_b} > \epsilon \frac{T_{rh}}{4 \cdot 10^{22}GeV}, \quad (45)$$

where we have taken $\rho_{CS}(\tau_c) < 0.03 M_p^2 H_c^2$.

In Figure 2 we have plotted the allowed parameter space for successful Baryogenesis when $|H_c| < |H_b|$, with lines of constant $\kappa$, for the reheating temperature $T_{rh} = 10^{15}$ GeV and Bounce scale $|H_b| \simeq 10^{12}$ GeV. Note, that for lines of fixed $\kappa$, increasing $\epsilon$ requires decreasing cut-off scale $\Lambda$; see Eq. (22). The upper limit on $\kappa$ in this scenario is $\kappa \sim 9$, for which the required $\epsilon$ values are too small to be consistent with an Ekpyrotic phase.

In this scenario, there is ample parameter space for which successful Baryogenesis can occur through the coupling of the pseudoscalar Ekpyrotic field to the Standard Model weak gauge field Chern-Simons term. The calculation performed here has been simplified by using the linearised approximation for the weak gauge fields. This approximation will likely break down for large $\kappa$ and require a more detailed analysis which is beyond the scope of this work. The results presented here provide evidence that Baryogenesis is possible through this mechanism.
5 Hypermagnetic Field Generation and Baryogenesis

The possibility of Magnetogenesis and Baryogenesis having their origin in the Inflationary epoch has been studied for many years [15–17, 53–67]. For the baryon asymmetry to be generated through the hypercharge Chern-Simons term, the helicity produced during the Ekpyrotic phase must survive until the EWPT. At the EWPT the large scale helical hypermagnetic fields are converted into magnetic fields and a $B + L$ asymmetry. As the evolution of the hypermagnetic fields after reheating will be analogous to that studied in the Inflationary Baryogenesis scenario, we will follow the analysis therein [16].

Before deriving the generated baryon asymmetry parameter and the associated magnetic fields, we find the constraint on the model parameters such that the energy density of the gauge fields does not become important to the background evolution during the Ekpyrotic phase. The energy density induced by the Chern-Simons coupling is approximately given by,

\[ \rho_{YCS}(\tau) = \langle 0 | Y_{\mu\nu} \tilde{Y}^{\mu\nu} | 0 \rangle \simeq \frac{6}{\pi} C(\kappa)(\epsilon H_c)^4, \]

and hence, to ensure that the dynamics of the hypermagnetic fields do not effect the background evolution induced by $\varphi$ we require that,

\[ M_p \gg \sqrt{\frac{2C(\kappa)}{\pi}} \epsilon^2 |H_c|, \]

which is the same as in Eq. [37] without the factor from summation over the isospin indices of $W_i^a$. This constraint will be compared with the requirements on $H_c$, $\epsilon$, and $\kappa$ for successful Baryogenesis.

The Hypermagnetic helicity generated during the Ekpyrotic phase is assumed to be unchanged across the bounce point, and is matched to the end of the reheating epoch, which is taken as instantaneous and characterised by temperature $T_{rh}$. The magnetic field at the end of reheating is defined as,

\[ B_{rh}(\tau_b)^2 = \frac{1}{2\pi^2} \int_{\mu}^{2\epsilon \kappa |H_c|} k^4(|G_+(\tau)|^2 - |G_-(\tau)|^2) dk \]

\[ \simeq \frac{1}{4\pi^2} (\epsilon H_c)^4 B(\kappa), \]

with the possibility that the generation of Hypermagnetic field helicity is cut-off prior to the bounce point parametrised by $H_c$, and,

\[ B(\kappa) = \int_0^{2\kappa} z^3(e^{\pi \kappa}|U(i\kappa, 0, -2iz)|^2 - e^{-\pi \kappa}|U(-i\kappa, 0, -2iz)|^2) dk, \]

which for $\kappa > 1$ is approximately given by,

\[ B(\kappa) \sim 0.015 \frac{e^{2\pi \kappa}}{\kappa^3} \sim \frac{2}{\kappa} C(\kappa), \]
so the magnetic field at the end of reheating is,

$$B_{rh}(\tau_b) \simeq \frac{1}{2\pi} (\epsilon H_c)^2 \sqrt{\frac{2C(\kappa)}{\kappa}} ,$$  \hspace{1cm} (51)

while the correlation length of these magnetic fields can be approximated as an average of the wavelengths, according to the size of their contributions to the magnetic field energy density,

$$\lambda_{rh}(\tau_b) = 2\pi \int_{\mu_{2\kappa[H_c]}}^{\mu_{2\kappa[H_c]}} N^2 k^4 (|G_+(\tau)|^2 - |G_(\tau)|^2) dk$$

$$\simeq \frac{4\pi\kappa}{\epsilon H_c} .$$  \hspace{1cm} (52)

The analysis of the evolution of the magnetic field in the thermal plasma from reheating to the EWPT is beyond the scope of this work, and as such we utilise the results from the pseudoscalar Inflationary Baryogenesis scenario \[16,66\]. The baryon asymmetry parameter produced at the EWPT from the magnetic field and correlation length given in Eq. (51) and (52) respectively, is,

$$\eta_B \simeq 5 \cdot 10^{-12} f(\theta_W, T_{BAU}) C(\kappa) \left( \frac{H_c}{H_b} \right)^3 \left( \frac{\epsilon^2 |H_b|}{10^{14} \text{ GeV}} \right)^{3/2} ,$$  \hspace{1cm} (53)

where $f(\theta_W, T_{BAU})$ parametrises the time dependence of the hypermagnetic helicity as baryon number is generated during the EWPT. There is significant uncertainty in the value this function takes during the EWPT, with expected values lying within the range \[16\],

$$5.6 \cdot 10^{-4} \lesssim f(\theta_W, T_{BAU}) \lesssim 0.32 ,$$  \hspace{1cm} (54)

for the Baryogenesis temperature $T_{BAU} \sim 135$ GeV. Hence, the generated baryon asymmetry in our model lies in the range,

$$C(\kappa) \left( \frac{H_c}{H_b} \right)^3 \left( \frac{\epsilon^2 |H_b|}{10^{17} \text{ GeV}} \right)^{3/2} < \eta_B < C(\kappa) \left( \frac{H_c}{H_b} \right)^3 \left( \frac{\epsilon^2 |H_b|}{1.5 \cdot 10^{15} \text{ GeV}} \right)^{3/2} .$$  \hspace{1cm} (55)

The parameters required for successful Baryogenesis can be tested against the energy density constraint given in Eq. (47). First consider the constraints on $\epsilon^2 H_b$ from Eq. (55) for the $H_c = H_b$ case,

$$\frac{1.5 \cdot 10^{15} \text{ GeV}}{C(\kappa)^{2/3}} < \epsilon^2 |H_b| < \frac{10^{17} \text{ GeV}}{C(\kappa)^{2/3}} ,$$  \hspace{1cm} (56)

hence, considering the maximum value, Eq. (47) becomes,

$$C(\kappa) \gg 10^{-9} ,$$  \hspace{1cm} (57)

which is always true for $\kappa > 1$. Therefore, there is no constraint on the parameter space due to the energy density constraint when considering hypermagnetic field helicity generation up
Figure 3: Examples of the parameter regions of $H_b$, $\epsilon$ and $\kappa$ that lead to successful Baryogenesis. The shaded regions subtended by the solid, dotted and dashed black lines depict the $\kappa = 1, 3, \text{and} 5$ results respectively. The upper bounds correspond to the lower expectation of hypermagnetic field helicity to baryon number conversion, and vice versa.

The parameter space for successful Baryogenesis and consistency with Eq. (59) is plotted, where the reheating temperature has been fixed to $T_{rh} \sim 10^{15}$ GeV. In this case, the energy density constraints used are,

$$\frac{H_c}{H_b} \gtrsim 4 \cdot 10^{-4} \epsilon, \quad \text{and} \quad \frac{H_c}{H_b} \gtrsim 10^{-6} \epsilon,$$

which leaves significant parameter space that can produce the observed baryon asymmetry.

These two scenarios generate large scale galactic magnetic fields. The evolution of the magnetic field and correlation length after the EWPT, to the present day, is derived as,

$$B_0^2 \approx 2 \cdot 10^{-18} \ G \ C(\kappa)^{1/3} \frac{H_c}{H_b} \left( \frac{\epsilon^2 H_b}{10^{13} \text{ GeV}} \right)^{1/2}.$$
Figure 4: The allowed parameter space of $H_c$, $\epsilon$ and $\kappa$ that leads to successful Baryogenesis and satisfies the energy density constraint, where $T_{\text{rh}} = 10^{15}$ GeV and $H_b \simeq 10^{12}$ GeV has been selected. The left-hand and right-hand plots correspond to the lower and upper limits of Eq. (55), respectively. The dot-dashed grey line corresponds to the lower and upper limits of Eq. (45), while the upper limit on $H_c$ is where $H_c = H_b$. The solid, dotted and dashed black lines depict the $\kappa = 1$, 3, and 5 cases respectively.

and

$$\lambda_p^0 \simeq 6 \cdot 10^{-5} \text{ pc } C(\kappa) \left( \frac{H_c}{H_b} \right)^{1/3} \left( \frac{\epsilon^2 H_b}{10^{13} \text{ GeV}} \right)^{1/2}.$$ (62)

Applying the parameters required for successful Baryogenesis given in Eq. (55), the present day magnetic fields have magnitude and correlation length within the following ranges,

$$2.4 \cdot 10^{-17} \text{ G } < B_p^0 < 2 \cdot 10^{-16} \text{ G},$$ (63)

and

$$7 \cdot 10^{-4} \text{ pc } < \lambda_p^0 < 6 \cdot 10^{-3} \text{ pc},$$ (64)

with the upper and lower bound corresponding to the lower and upper limits of Eq. (54), respectively. These magnetic fields can lead to interesting observational signatures due to their helical nature [68]. The magnitude of these magnetic fields are below the current upper limits, but unfortunately are too small to explain the current Blazar results. It should be noted that, the present day magnetic fields that result from successful Baryogenesis in this Ekpyrotic mechanism are consistent with those for the Inflationary Baryogenesis scenario.
6 Conclusion

The Ekpyrotic cosmological model is able to provide solutions to the cosmological problems as well as those associated with inflationary cosmology. We have proposed a mechanism for Baryogenesis that takes place during the contracting phase prior to the onset of the standard radiation dominated epoch. If the evolution of the universe becomes dominated by a fast-rolling pseudoscalar with a negative exponential potential, a period of Ekpyrotic contraction can begin with equation of state $\omega \gg 1$. Coupling this pseudoscalar to the Standard Model Weak and Hypercharge gauge group, through Chern-Simons terms, leads to the generation of a non-zero Chern-Simons number density prior to the bounce point. Through both the Weak and Hypercharge couplings the size of the produced Chern-Simons number density is found to explain the observed baryon asymmetry for a wide range of reasonable parameter choices.

A more detailed analysis is required to understand the full gauge field dynamics during the contracting phase, and possible back-reaction or suppression effects, particularly in the case of the non-abelian weak gauge fields. Despite this, the large range of allowed parameters in the linearised approximation bodes well for successful Baryogenesis. The non-gaussianities generated in Ekpyrotic models can be too large to be consistent with observation, unless a period of kination is induced by a secondary scalar field prior to the bounce. The presence of a second Ekpyrotic scalar motivated the consideration of the case $|H_c| < |H_b|$ in our analysis, as the Chern-Simons number generation will end when the background trajectory becomes dominated by this scalar. This scenario gives a wide validity region in the parameter space for Baryogenesis, with constraints derived from the requirement of sub-dominance of the gauge field energy density.

The Hypercharge Chern-Simons coupling is able to generate large scale magnetic fields in this mechanism, although the resultant magnitude and correlation length present today is unable to explain the Blazar results and Baryogenesis simultaneously. To do this would require an extra source of suppression of the baryon asymmetry generation at the EWPT. The final magnetic field predictions in our model, for the Hypercharge case, are the same as the pseudoscalar inflation scenario, pointing to the duality of the slow and fast-roll parameters. This may make it difficult to distinguish between the two scenarios through this observable. Similarly to the inflationary mechanism, the gauge field production in this Ekpyrotic scenario leads to the generation of Gravitational Waves. This may provide differentiating observational features between the Ekpyrotic and Inflationary scenarios, warranting further investigation.

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References

[1] N. Aghanim et al. [Planck Collaboration], arXiv:1807.06209 [astro-ph.CO].
[2] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. 5, 32 (1967).
[3] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Ann. Rev. Nucl. Part. Sci. 43 (1993) 27
   doi:10.1146/annurev.ns.43.120193.000331 [hep-ph/9302210].
[4] A. H. Guth, Phys. Rev. D 23 (1981) 347.
[5] A. D. Linde, Phys. Lett. B 108, 389 (1982);
[6] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982).
[7] V. F. Mukhanov and G. V. Chibisov, JETP Lett. 33, 532 (1981) [Pisma Zh. Eksp. Teor.
   Fiz. 33, 549 (1981)].
[8] J. Martin, C. Ringeval and V. Vennin, Phys. Dark Univ. 5-6 (2014) 75
   doi:10.1016/j.dark.2014.01.003 [arXiv:1303.3787 [astro-ph.CO]].
[9] S. H. -S. Alexander, M. E. Peskin and M. M. Sheikh-Jabbari, Phys. Rev. Lett. 96, 081301
   (2006) [hep-th/0403069];
[10] S. Alexander, A. Marciano and D. Spergel, JCAP 1304, 046 (2013) arXiv:1107.0318
    [hep-th]];
[11] A. Maleknejad, M. Noorbala and M. M. Sheikh-Jabbari, arXiv:1208.2807 [hep-th];
[12] A. Maleknejad, Phys. Rev. D 90 (2014) 2, 023542 arXiv:1401.7628 [hep-th]].
[13] N. D. Barrie and A. Kobakhidze, JHEP 1409 (2014) 163 arXiv:1401.1256 [hep-ph]];
[14] N. D. Barrie and A. Kobakhidze, Mod. Phys. Lett. A 32 (2017) no.14, 1750087
    arXiv:1503.02366 [hep-ph]].
[15] M. M. Anber and E. Sabancilar, Phys. Rev. D 92 (2015) no.10, 101501
    doi:10.1103/PhysRevD.92.101501 arXiv:1507.00744 [hep-th]].
[16] D. Jiménez, K. Kamada, K. Schmitz and X. J. Xu, JCAP 1712 (2017) 011
    doi:10.1088/1475-7516/2017/12/011 arXiv:1707.07943 [hep-ph]].
[17] V. Domcke, B. von Harling, E. Morgante and K. Mukaida, JCAP 1910 (2019) no.10, 032
    doi:10.1088/1475-7516/2019/10/032 arXiv:1905.13318 [hep-ph]].
[18] E. C. G. Stueckelberg, Helv. Phys. Acta 11, 225 (1938).
[19] M. B. Green and J. H. Schwarz, Phys. Lett. **149B** (1984) 117. doi:10.1016/0370-2693(84)91565-X

[20] R. Brandenberger and P. Peter, Found. Phys. **47** (2017) no.6, 797 doi:10.1007/s10701-016-0057-0 [arXiv:1603.05834 [hep-th]].

[21] M. Novello and S. E. P. Bergliaffa, Phys. Rept. **463** (2008) 127 doi:10.1016/j.physrep.2008.04.006 [arXiv:0802.1634 [astro-ph]].

[22] J. L. Lehners, Phys. Rept. **465** (2008) 223 doi:10.1016/j.physrep.2008.06.001 [arXiv:0806.1245 [astro-ph]].

[23] R. H. Brandenberger, Int. J. Mod. Phys. Conf. Ser. **01** (2011) 67 doi:10.1142/S2010194511000109 [arXiv:0902.4731 [hep-th]].

[24] D. Battefeld and P. Peter, Phys. Rept. **571** (2015) 1 doi:10.1016/j.physrep.2014.12.004 [arXiv:1406.2790 [astro-ph.CO]].

[25] A. Ijjas and P. J. Steinhardt, Class. Quant. Grav. **35** (2018) no.13, 135004 doi:10.1088/1361-6382/aac482 [arXiv:1803.01961 [astro-ph.CO]].

[26] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, Phys. Rev. D **64** (2001) 123522 doi:10.1103/PhysRevD.64.123522 [hep-th/0103239].

[27] D. H. Lyth, Phys. Lett. B **524** (2002) 1 doi:10.1016/S0370-2693(01)01374-0 [hep-ph/0106153].

[28] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, Phys. Rev. D **66** (2002) 046005 doi:10.1103/PhysRevD.66.046005 [hep-th/0109050].

[29] R. Brandenberger and F. Finelli, JHEP **0111** (2001) 056 doi:10.1088/1126-6708/2001/11/056 [hep-th/0109004].

[30] D. H. Lyth, Phys. Lett. B **526** (2002) 173 doi:10.1016/S0370-2693(01)01438-1 [hep-ph/0110007].

[31] P. J. Steinhardt and N. Turok, Phys. Rev. D **65** (2002) 126003 doi:10.1103/PhysRevD.65.126003 [hep-th/0111098].

[32] J. L. Lehners, P. McFadden, N. Turok and P. J. Steinhardt, Phys. Rev. D **76** (2007) 103501 doi:10.1103/PhysRevD.76.103501 [hep-th/0702153 [HEP-TH]].

[33] E. I. Buchbinder, J. Khoury and B. A. Ovrut, Phys. Rev. D **76** (2007) 123503 doi:10.1103/PhysRevD.76.123503 [hep-th/0702154].

[34] E. I. Buchbinder, J. Khoury and B. A. Ovrut, JHEP **0711** (2007) 076 doi:10.1088/1126-6708/2007/11/076 [arXiv:0706.3903 [hep-th]].
[35] Y. F. Cai, D. A. Easson and R. Brandenberger, JCAP 1208 (2012) 020 doi:10.1088/1475-7516/2012/08/020 [arXiv:1206.2382 [hep-th]].

[36] A. Ijjas and P. J. Steinhardt, [arXiv:1904.08022 [gr-qc]].

[37] J. M. Salim, N. Souza, S. E. Perez Bergliaffa and T. Prokopec, JCAP 0704 (2007) 011 doi:10.1088/1475-7516/2007/04/011 [astro-ph/0612281].

[38] C. Li, R. H. Brandenberger and Y. K. E. Cheung, Phys. Rev. D 90 (2014) no.12, 123535 doi:10.1103/PhysRevD.90.123535 [arXiv:1403.5625 [gr-qc]].

[39] Y. K. E. Cheung, J. U. Kang and C. Li, JCAP 1411 (2014) 001 doi:10.1088/1475-7516/2014/11/001 [arXiv:1408.4387 [astro-ph.CO]].

[40] I. Ben-Dayan, JCAP 1609 (2016) 017 doi:10.1088/1475-7516/2016/09/017 [arXiv:1604.07899 [astro-ph.CO]].

[41] A. Ito and J. Soda, Phys. Lett. B 771 (2017) 415 doi:10.1016/j.physletb.2017.05.017 [arXiv:1607.07062 [hep-th]].

[42] R. Koley and S. Samtani, JCAP 1704 (2017) 030 doi:10.1088/1475-7516/2017/04/030 [arXiv:1612.08556 [gr-qc]].

[43] J. Khoury, P. J. Steinhardt and N. Turok, Phys. Rev. Lett. 92 (2004) 031302 doi:10.1103/PhysRevLett.92.031302 [hep-th/0307132].

[44] A. Ijjas and P. J. Steinhardt, Class. Quant. Grav. 33 (2016) no.4, 044001 doi:10.1088/0264-9381/33/4/044001 [arXiv:1512.09010 [astro-ph.CO]].

[45] A. M. Levy, A. Ijjas and P. J. Steinhardt, Phys. Rev. D 92 (2015) no.6, 063524 doi:10.1103/PhysRevD.92.063524 [arXiv:1506.01011 [astro-ph.CO]].

[46] R. Kallosh, J. U. Kang, A. D. Linde and V. Mukhanov, JCAP 0804 (2008) 018 doi:10.1088/1475-7516/2008/04/018 [arXiv:0712.2040 [hep-th]].

[47] Y. Akrami et al. [Planck Collaboration], [arXiv:1905.05697 [astro-ph.CO]].

[48] E. I. Buchbinder, J. Khoury and B. A. Ovrut, Phys. Rev. Lett. 100 (2008) 171302 doi:10.1103/PhysRevLett.100.171302 [arXiv:0710.5172 [hep-th]].

[49] J. L. Lehners and P. J. Steinhardt, Phys. Rev. D 78 (2008) 023506 Erratum: [Phys. Rev. D 79 (2009) 129902] doi:10.1103/PhysRevD.78.023506, 10.1103/PhysRevD.79.129902 [arXiv:0804.1293 [hep-th]].

[50] J. L. Lehners, Adv. Astron. 2010 (2010) 903907 doi:10.1155/2010/903907 [arXiv:1001.3125 [hep-th]].
[51] A. Ijjas, J. L. Lehners and P. J. Steinhardt, Phys. Rev. D 89 (2014) no.12, 123520 doi:10.1103/PhysRevD.89.123520 [arXiv:1404.1265 [astro-ph.CO]].

[52] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. 155B (1985) 36. doi:10.1016/0370-2693(85)91028-7

[53] M. S. Turner and L. M. Widrow, Phys. Rev. D 37 (1988) 2743. doi:10.1103/PhysRevD.37.2743

[54] W. D. Garretson, G. B. Field and S. M. Carroll, Phys. Rev. D 46 (1992) 5346 doi:10.1103/PhysRevD.46.5346 [hep-ph/9209238].

[55] M. Joyce and M. E. Shaposhnikov, Phys. Rev. Lett. 79 (1997) 1193 doi:10.1103/PhysRevLett.79.1193 [astro-ph/9703005].

[56] M. Giovannini and M. E. Shaposhnikov, Phys. Rev. D 57 (1998) 2186 doi:10.1103/PhysRevD.57.2186 [hep-ph/9710234].

[57] M. Giovannini, Phys. Rev. D 61 (2000) 063502 doi:10.1103/PhysRevD.61.063502 [hep-ph/9906241].

[58] M. M. Anber and L. Sorbo, JCAP 0610 (2006) 018 doi:10.1088/1475-7516/2006/10/018 [astro-ph/0606534].

[59] K. Bamba, Phys. Rev. D 74 (2006) 123504 doi:10.1103/PhysRevD.74.123504 [hep-ph/0611152].

[60] J. Martin and J. Yokoyama, JCAP 0801 (2008) 025 doi:10.1088/1475-7516/2008/01/025 [arXiv:0711.4307 [astro-ph]].

[61] V. Demozzi, V. Mukhanov and H. Rubinstein, JCAP 0908 (2009) 025 doi:10.1088/1475-7516/2009/08/025 [arXiv:0907.1030 [astro-ph.CO]].

[62] R. Durrer, L. Hollenstein and R. K. Jain, JCAP 1103 (2011) 037 doi:10.1088/1475-7516/2011/03/037 [arXiv:1005.5322 [astro-ph.CO]].

[63] C. Caprini and L. Sorbo, JCAP 1410 (2014) 056 doi:10.1088/1475-7516/2014/10/056 [arXiv:1407.2809 [astro-ph.CO]].

[64] P. Adshead, J. T. Giblin, T. R. Scully and E. I. Sfakianakis, JCAP 1610 (2016) 039 doi:10.1088/1475-7516/2016/10/039 [arXiv:1606.08474 [astro-ph.CO]].

[65] K. Kamada and A. J. Long, Phys. Rev. D 94 (2016) no.6, 063501 doi:10.1103/PhysRevD.94.063501 [arXiv:1606.08891 [astro-ph.CO]].

[66] K. Kamada and A. J. Long, Phys. Rev. D 94 (2016) no.12, 123509 doi:10.1103/PhysRevD.94.123509 [arXiv:1610.03074 [hep-ph]].
[67] C. Caprini, M. C. Guzzetti and L. Sorbo, Class. Quant. Grav. 35 (2018) no.12, 124003
doi:10.1088/1361-6382/aac143 [arXiv:1707.09750 [astro-ph.CO]].

[68] H. Tashiro, W. Chen, F. Ferrer and T. Vachaspati, Mon. Not. Roy. Astron. Soc. 445 (2014) no.1, L41 doi:10.1093/mnrasl/slu134 [arXiv:1310.4826 [astro-ph.CO]].