Light-cone Distribution Amplitudes of Ξ and their Applications

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We present the light-cone distribution amplitudes of the Ξ baryons up to twist six on the basis of QCD conformal partial wave expansion to the leading order conformal spin accuracy. The nonperturbative parameters relevant to the DAs are determined in the framework of the QCD sum rule. The light-cone QCD sum rule approach is used to investigate both the electromagnetic form factors of Ξ and the exclusive semileptonic decay of Ξ as applications. Our estimations on the magnetic moments are \(\mu_{\Xi^0} = -(1.92 \pm 0.34)\mu_N\) and \(\mu_{\Xi^-} = -(1.19 \pm 0.03)\mu_N\). The decay width of the process \(\Xi_c \rightarrow \Xi^+ \nu_e\) is evaluated to be \(\Gamma = 8.73 \times 10^{-14}\) GeV, which is in accordance with the experimental measurements and other theoretical approaches.

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I. INTRODUCTION

Light-cone distribution amplitudes (DAs), which denote the momentum fraction distributions of partons in a hadron, play important roles in hard exclusive processes [1, 2] and the light-cone QCD sum rule applications. Furthermore, DAs are of eminent importance themselves because of their ability to reveal the internal structure of the composite particles. Studies on the nucleon octet DAs were first carried out early in late 1980s with QCD sum rules on the moments up to the leading twist [3]. More than ten years later, Braun et al. gave a systematic investigation on the higher order DAs of the nucleon based on the conformal expansion [4]. Recently, an overview was given in Ref. [5], in which the authors present a comparison of the nucleon DAs with various approaches and models. In our previous work [6], we have given the DAs of the Σ and Λ baryons on the conformal partial wave expansion to the leading order conformal spin accuracy. As composite particles with two s-quarks, Ξ have the same Lorentz structures as other \(J^P = \frac{1}{2}^+\) octet baryons in the \(SU(3)\)-flavor symmetry limit, but the \(SU(3)\)-flavor breaking effects may play more important roles than in the cases of the other octet baryons, provided that the masses of the s-quarks are considered. Thus investigations on the Ξ baryon DAs are of interest and somewhat complicated.

The higher order twist contributions to DAs have several origins, among which the main contribution comes from “bad” components in the wave function and in particular of components with “wrong” spin projection for the case of baryons [1, 6]. We focus on higher order twist contributions from “bad” components in the decomposition of the Lorentz
structure in this paper. The general description of DAs is based on the conformal symmetry of the massless QCD Lagrangian dominated on the light cone. The conformal partial wave expansion of the DAs can be carried out safely in the limit of the $SU(3)$-flavor symmetry approach. However, when terms connected with the $s$-quark mass are considered, the $SU(3)$-flavor breaking effects need to be included. We assume that the conformal expansion of the nucleon DAs can be extrapolated to our cases, which is similar to the arguments for mesons and baryons with a single $s$-quark $[4,7]$. In the present work, effects from the $SU(3)$-flavor symmetry breaking are considered as the corrections, which originate from two sources: isospin symmetry breaking and corrections to the nonperturbative parameters.

The related processes containing the baryons can be investigated in the framework of the light-cone QCD sum rule (LCSR)$[8,9,10]$ with the DAs. LCSR is a developed nonperturbative method of the traditional QCD sum rule $[11]$. Its main idea is to expand the correlation function between the vacuum and the hadron state on the light cone $x^2 = 0$, while the nonperturbative effects, which correspond to the condensates in traditional QCD sum rules, are described by the DAs connected with the final hadron state. LCSR has been widely used to investigate the electromagnetic (EM) form factors and various decay processes related to the baryons $[6,12,13,14,15,16,17,18]$. In the applications, we take advantage of the LCSR approach to investigating the EM form factors of $\Xi$ and form factors of the weak transition $\Xi_c \to \Xi$. Both the magnetic moments of $\Xi$ baryons and decay width of the semileptonic transition $\Xi_c \to \Xi e^+\nu_e$ will be estimated.

The paper is organized as follows. Section II is devoted to give the definitions of the DAs and necessary parameters. In Sec. III the conformal partial wave expansion of the DAs is carried out by use of the conformal symmetry. The nonperturbative parameters connected with the DAs are determined in Sec. IV. Section V is the application part, in which the EM form factors of $\Xi$ and the semileptonic decay of $\Xi_c$ baryons are investigated in the framework of LCSR. The summary and conclusion are given in Sec. VI. Finally, we give the explicit expressions of the $\Xi$ baryon DAs in the Appendix.

II. THE LIGHT-CONE DISTRIBUTION AMPLITUDES OF $\Xi$

Our discussion of the $\Xi$ baryon DAs in this section is an extension and a complement of $[4,6]$, so we just list the results following their procedures, and it is recommended that the original papers be consulted for details. Generally, DAs of the $J^P = \frac{1}{2}^+$ octet baryons can be defined by the matrix element of the three-quark operator between the vacuum and the baryon state $|B(P)\rangle$ in the limit of $SU(3)$-flavor symmetry:

$$\langle 0 | \epsilon^{ijk} q^i_\alpha | a_1 z \rangle q^j_\beta | a_2 z \rangle q^k_\gamma | a_3 z \rangle | B(P) \rangle,$$

where $\alpha$, $\beta$, $\gamma$ refer to the Lorentz indices and $i$, $j$, $k$ the color indices, $q_i$ represents the light quark, $z$ is a lightlike vector which satisfies $z^2 = 0$, and $a_i$ are real numbers denoting coordinates of valence quarks.
For the case of $\Xi$, after taking into account the Lorentz covariance, spin and parity of the baryons, the matrix element (1) is decomposed as

$$
4\langle 0|\epsilon^{ijk}s^i_s(a_1z)s^j_s(a_2z)q^k_s(a_3z)|\Xi(P)\rangle = \sum_i F_i \Gamma_{11}^{\alpha\beta}(\Gamma_{21}\Xi)^{\gamma}, \quad (2)
$$

where $\Xi_\gamma$ is the spinor of the baryon with the quantum number $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ ($I$ is the isospin, $J$ is the total angular momentum and $P$ is the parity), $\Gamma_{1(2)i}$ are certain Dirac structures over which the sum is carried out, and $F_i = S_i, P_i, A_i, V_i, T_i$ are the distribution amplitudes which depend on the scalar product $P \cdot z$.

As these “calligraphic” invariant functions do not have a definite twist in the above definition (2), the twist classification is carried out in the infinite momentum frame. With the aid of the definition of light-cone DAs with a definite twist,

$$
4\langle 0|\epsilon^{ijk}s^i_s(a_1z)s^j_s(a_2z)q^k_s(a_3z)|\Xi(P)\rangle = \sum_i F_i \Gamma_{11}^{\alpha\beta}(\Gamma_{21}\Xi)^{\gamma}, \quad (3)
$$

the invariant functions $S_i, P_i, V_i, A_i, T_i$ can be expressed in terms of the DAs $F_i = S_i, P_i, V_i, A_i, T_i$. A simple derivation leads to the following relations between these two sets of definitions for scalar and pseudo-scalar distributions,

$$
S_1 = S_1, \quad 2p \cdot z S_2 = S_1 - S_2, \quad (4)
$$

$$
P_1 = P_1, \quad 2p \cdot z P_2 = P_2 - P_1,
$$

for vector distributions,

$$
V_1 = V_1, \quad 2p \cdot z V_2 = V_1 - V_2 - V_3, \quad (5)
$$

$$
2V_3 = V_3, \quad 4p \cdot z V_4 = -2V_1 + V_3 + V_4 + 2V_5,
$$

$$
4p \cdot z V_5 = V_4 - V_3, \quad (2p \cdot z)^2 V_6 = -V_1 + V_2 + V_3 + V_4 + V_5 - V_6,
$$

for axial-vector distributions,

$$
A_1 = A_1, \quad 2p \cdot z A_2 = -A_1 + A_2 - A_3, \quad (6)
$$

$$
2A_3 = A_3, \quad 4p \cdot z A_4 = -2A_1 - A_3 - A_4 + 2A_5,
$$

$$
4p \cdot z A_5 = A_3 - A_4, \quad (2p \cdot z)^2 A_6 = A_1 - A_2 + A_3 + A_4 - A_5 + A_6,
$$
and, finally, for tensor distributions,

\[ T_1 = T_1, \quad 2p \cdot zT_2 = T_1 + T_2 - 2T_3, \]
\[ 2T_3 = T_7, \quad 2p \cdot zT_4 = T_1 - T_2 - 2T_7, \]
\[ 2p \cdot zT_5 = -T_1 + T_5 + 2T_8, \quad (2p \cdot z)^2 T_6 = 2T_2 - 2T_3 - 2T_4 + 2T_5 + 2T_7 - 2T_8, \]
\[ 4p \cdot zT_7 = T_7 - T_8, \quad (2p \cdot z)^2 T_8 = -T_1 + T_2 + T_5 - T_6 + 2T_7 - 2T_8. \]

The classification of the DAs \( F_i \) with a definite twist are listed in Table I. The explicit expressions of the definition can be found in Refs. [4, 6]. Each distribution amplitude \( F_i \) can be represented as

\[ F(a, p \cdot z) = \int \mathcal{D}x e^{-ipz} \sum_i x_i a_i F(x_i), \]

where the dimensionless variables \( x_i \), which satisfy the relations \( 0 < x_i < 1 \) and \( \sum_i x_i = 1 \), correspond to the longitudinal momentum fractions carried by the quarks inside the baryon. The integration measure is defined as

\[ \int \mathcal{D}x = \int_0^1 dx_1 dx_2 dx_3 \delta(x_1 + x_2 + x_3 - 1). \]

There are some symmetry properties of the DAs from the identity of the two \( s \)-quarks in the \( \Xi \) baryons, which are useful to reduce the number of the independent DAs. Taking into account the Lorentz decomposition of the \( \gamma \)-matrix structure, it is easy to see that the vector and tensor DAs are symmetric, whereas the scalar, pseudoscalar and axial-vector structures are antisymmetric under the interchange of the two \( s \)-quarks:

\[ V_i(1, 2, 3) = V_i(2, 1, 3), \quad T_i(1, 2, 3) = T_i(2, 1, 3), \]
\[ S_i(1, 2, 3) = -S_i(2, 1, 3), \quad P_i(1, 2, 3) = -P_i(2, 1, 3), \]
\[ A_i(1, 2, 3) = -A_i(2, 1, 3). \]

The similar relationships hold for the calligraphic structures in Eq. (2).

In order to expand the DAs by the conformal partial waves, we rewrite the DAs in terms of quark fields with definite chirality \( q'^{(1)} = \frac{1}{2}(1 \pm \gamma_5)q \). The classification of the DAs in this presentation can be interpreted transparently: projection on the state with the two \( s \)-quarks antiparallel, i.e. \( s^\uparrow s^\downarrow \), singles out vector and axial-vector structures, while parallel ones, i.e. \( s^\uparrow s^\uparrow \) and \( s^\downarrow s^\downarrow \), correspond to scalar, pseudoscalar and tensor structures. The explicit expressions of the DAs by chiral-field representations are presented in Table II as an example for \( \Xi^0 \). The counterparts of \( \Xi^- \) can be easily obtained under the exchange \( u \leftrightarrow d \).

Note that in the case of the nucleon, the isospin symmetry can be used to reduce the number of the independent DAs to eight. However, there are no similar isospin symmetric
TABLE I: Independent baryon distribution amplitudes in the chiral expansion.

| Lorentz structure | Light-cone projection | Nomenclature |
|-------------------|-----------------------|--------------|
| Twist 3 \((C \not{p} \otimes \not{q})\) | \(s_1^+ s_1^- u_1^+\) | \(\Phi_3(x_1) = [V_1 - A_1](x_1)\) |
| \((C \sigma_{\perp R} \otimes \gamma^+ \not{p})\) | \(s_1^+ s_1^- u_1^+\) | \(T_1(x_1)\) |
| Twist 4 \((C \not{p} \otimes \not{p})\) | \(s_1^+ s_1^- u_1^-\) | \(\Phi_4(x_1) = [V_2 - A_2](x_1)\) |
| \((C \not{p} \gamma_{\perp} \not{p} \otimes \gamma^+ \not{p})\) | \(s_1^+ s_1^- u_1^+\) | \(\Psi_4(x_1) = [V_3 - A_3](x_1)\) |
| \((C \not{p} \not{p} \otimes \not{p})\) | \(s_1^- s_1^+ u_1^+\) | \(\Xi_4(x_1) = [T_3 - T_7 + S_1 + P_1](x_1)\) |
| \((C \sigma_{\perp L} \otimes \gamma^+ \not{p})\) | \(s_1^- s_1^+ u_1^+\) | \(\Xi'_4(x_1) = [T_3 + T_7 + S_1 - P_1](x_1)\) |
| Twist 5 \((C \not{p} \otimes \not{q})\) | \(s_1^- s_1^+ u_1^+\) | \(\Phi_5(x_1) = [V_5 - A_5](x_1)\) |
| \((C \not{p} \gamma_{\perp} \not{p} \otimes \gamma^+ \not{p})\) | \(s_1^- s_1^+ u_1^+\) | \(\Psi_5(x_1) = [V_4 - A_4](x_1)\) |
| \((C \not{p} \not{p} \otimes \not{p})\) | \(s_1^- s_1^+ u_1^-\) | \(\Xi_5(x_1) = [-T_4 - T_8 + S_2 + P_2](x_1)\) |
| \((C \not{p} \not{p} \otimes \not{p})\) | \(s_1^- s_1^+ u_1^-\) | \(\Xi'_5(x_1) = [S_2 - P_2 - T_4 + T_8](x_1)\) |
| \((C \sigma_{\perp R} \otimes \gamma^+ \not{p})\) | \(s_1^- s_1^+ u_1^-\) | \(T_5(x_1)\) |
| Twist 6 \((C \not{p} \otimes \not{p})\) | \(s_1^- s_1^+ u_1^-\) | \(\Phi_6(x_1) = [V_6 - A_6](x_1)\) |
| \((C \sigma_{\perp L} \otimes \gamma^+ \not{p})\) | \(s_1^- s_1^+ u_1^-\) | \(T_6(x_1)\) |

relationships that exist when the Ξ baryon is considered, which is the same as the cases of the Λ and Σ baryons. Therefore, we need altogether 14 chiral-field representations to express all the DAs.

III. CONFORMAL EXPANSION

In this section we will give the explicit expressions of the DAs with the aid of the conformal expansion approach. The conformal expansion of the DAs is based on the conformal symmetry of the massless QCD Lagrangian, which makes it possible to separate longitudinal degrees of freedom from transverse ones. The properties of transverse coordinates are described by the renormalization scale that is determined by the renormalization group, while the longitudinal momentum fractions that are living on the light cone are governed by a set of orthogonal polynomials, which form an irreducible representation of the collinear subgroup \(SL(2, R)\) of the conformal group.

The algebra of the collinear subgroup \(SL(2, R)\) is determined by the following four generators:

\[
L_+ = -iP_+, \quad L_- = \frac{i}{2}K_-, \quad L_0 = -\frac{i}{2}(D - M_-), \quad E = i(D + M_-),
\]

(11)
where $P_\mu$, $K_\mu$, $D$, and $M_{\mu \nu}$ correspond to the translation, special conformal transformation, dilation and Lorentz generators, respectively. The notations are used for a vector $A$: $A_+ = A_\mu z^\mu$ and $A_- = A_\mu p^\mu/p \cdot z$. Let $L^2 = L_0^2 - L_0 + L_+ L_-; \text{ then a given distribution amplitude with a definite twist can be expanded by the conformal partial wave functions that are the eigenstates of } L^2 \text{ and } L_0$.

For the three-quark state, the distribution amplitude with the lowest conformal spin $j_{\min} = j_1 + j_2 + j_3$ is \cite{13,14}

$$\Phi_{\alpha}(x_1, x_2, x_3) = \frac{\Gamma[2j_1 + 2j_2 + 2j_3]}{\Gamma[2j_1] \Gamma[2j_2] \Gamma[2j_3]} x_1^{2j_1} x_2^{2j_2} x_3^{2j_3},$$

where $j_i$ represents the conformal spin of the quark field. Contributions with higher conformal spin $j = j_{\min} + n \ (n = 1, 2, \ldots)$ are given by $\Phi_{\alpha}$ multiplied by polynomials that are orthogonal over the weight function \cite{12}. In our approach the calculation just contains the leading order conformal spin expansion. For DAs in Table \text{Table I}, we give their conformal expansions:

$$\Phi_3(x_i) = 120 x_1 x_2 x_3 \phi_3^0(\mu), \quad T_1(x_i) = 120 x_1 x_2 x_3 \phi_3^0(\mu),$$

for twist three and

$$\Phi_4(x_i) = 24 x_1 x_2 \phi_4^0(\mu), \quad \Psi_4(x_i) = 24 x_1 x_3 \psi_4^0(\mu),$$

$$\Xi_4(x_i) = 24 x_2 x_3 \xi_4^0(\mu), \quad \Xi'_4(x_i) = 24 x_2 x_3 \xi_4^0(\mu),$$

$$T_2(x_i) = 24 x_1 x_2 \phi_2^0(\mu),$$

for twist four and

$$\Phi_5(x_i) = 6 x_3 \phi_5^0(\mu), \quad \Psi_5(x_i) = 6 x_2 \psi_5^0(\mu),$$

$$\Xi_5(x_i) = 6 x_1 \xi_5^0(\mu), \quad \Xi'_5(x_i) = 6 x_1 \xi_5^0(\mu),$$

$$T_5(x_i) = 6 x_3 \phi_5^0(\mu),$$

for twist five and

$$\Phi_6(x_i) = 2 \phi_6^0(\mu), \quad T_6(x_i) = 2 \phi_6^0(\mu),$$

for twist six. There are altogether 14 parameters which can be determined by the equations of motion.

To the leading order, the normalization of the $\Xi$ baryon DAs is determined by the matrix element of the local three-quark operator without derivatives. The Lorentz decomposition of the matrix element can be expressed explicitly as follows:

$$4(0)e^{ijk}s^i_\alpha(0)s^j_\beta(0)q^k(0)|\Xi(P)\rangle = \mathcal{V}^0_1(PC)_{\alpha\beta}(\gamma_5 \Xi)_\gamma + \mathcal{V}^0_3(\gamma_\mu C)_{\alpha\beta}(\gamma_\mu \gamma_5 \Xi)_\gamma$$

$$+ T^0_1(P^\mu i \sigma_\mu \nu C)_{\alpha\beta}(\gamma^\mu \gamma_5 \Xi)_\gamma + T^0_3 M(\sigma_{\mu \nu} C)_{\alpha\beta}(\sigma^{\mu \nu} \gamma_5 \Xi)_\gamma.$$  \hspace{1cm} (17)
There are altogether four parameters to be determined. To this end, we introduce the four decay constants defined by the following matrix elements:

\[
\langle 0| \epsilon^{ijk} [s^i(0) C \not{s}^j(0)] \gamma_5 \not{q}^k(0)|P\rangle = f_\Xi P \cdot z \Xi(P),
\]

\[
\langle 0| \epsilon^{ijk} [s^i(0) C \gamma_\mu s^j(0)] \gamma_5 \gamma^\mu q^k(0)|P\rangle = \lambda_1 M \Xi(P),
\]

\[
\langle 0| \epsilon^{ijk} [s^i(0) C \sigma_{\mu\nu} s^j(0)] \gamma_5 \sigma^{\mu\nu} q^k(0)|P\rangle = \lambda_2 M \Xi(P),
\]

\[
\langle 0| \epsilon^{ijk} [s^i(0) C \not{q}^\nu \sigma_{\mu\nu} s^j(0)] \gamma_5 \gamma_\mu q^k(0)|P\rangle = \lambda_3 M q \Xi(P).
\]

A simple calculation gives the expressions of the local nonperturbative parameters \( \mathcal{V}_1^0, \mathcal{V}_3^0, T_1^0, \) and \( T_3^0 \) in terms of the four decay constants defined in Eq. (18):

\[
\mathcal{V}_1^0 = f_\Xi, \quad \mathcal{V}_3^0 = \frac{1}{4} (f_\Xi - \lambda_1),
\]

\[
T_1^0 = \frac{1}{6} (4\lambda_3 - \lambda_2), \quad T_3^0 = \frac{1}{12} (2\lambda_3 - \lambda_2).
\]

With the above relations, the coefficients of the operators in Eqs. (13)-(16) can be expressed to the leading order conformal spin accuracy as

\[
\phi_3^0 = \phi_6^0 = f_\Xi, \quad \psi_4^0 = \psi_5^0 = \frac{1}{2} (f_\Xi - \lambda_1),
\]

\[
\phi_4^0 = \phi_5^0 = \frac{1}{2} (f_\Xi + \lambda_1), \quad \phi_3^0 = \phi_6^0 = -\xi_5^0 = \frac{1}{6} (4\lambda_3 - \lambda_2),
\]

\[
\phi_4^0 = \xi_4^0 = \frac{1}{6} (8\lambda_3 - 3\lambda_2), \quad \phi_5^0 = -\xi_5^0 = \frac{1}{6} \lambda_2,
\]

\[
\xi_4^0 = \frac{1}{6} (12\lambda_3 - 5\lambda_2).
\]

IV. QCD SUM RULES FOR THE NONPERTURBATIVE PARAMETERS

The nonperturbative parameters \( f_\Xi, \lambda_1, \lambda_2 \) and \( \lambda_3 \) of the \( \Xi \) baryons can be determined in the QCD sum rule approach. The derivations are carried out from the following two-point correlation functions,

\[
\Pi_i(q^2) = i \int d^4 x e^{i q \cdot x} \langle 0 | T j_i(x) j_i(0) | 0 \rangle,
\]

with the definitions of the currents:

\[
j_1(x) = \epsilon^{ijk} [s^i(x) C \not{s}^j(x)] \gamma_5 \not{q}^k(x),
\]

\[
j_2(x) = \epsilon^{ijk} [s^i(x) C \gamma_\mu s^j(x)] \gamma_5 \gamma^\mu q^k(x),
\]

\[
j_3(x) = \epsilon^{ijk} [s^i(x) C \sigma_{\mu\nu} s^j(x)] \gamma_5 \sigma^{\mu\nu} q^k(x),
\]

\[
j_4(x) = \epsilon^{ijk} [s^i(x) C \not{q}^\nu \sigma_{\mu\nu} s^j(x)] \gamma_5 \gamma_\mu q^k(x).
\]

In compliance with the standard technique of the QCD sum rule, the correlation functions (21) need to be expressed both phenomenologically and theoretically. By inserting
a complete set of states with the same quantum numbers as those of \( \Xi \), the hadronic representations of the correlation functions are given as follows:

\[
\Pi_1(q^2) = 2f_\Xi^2(q \cdot z)^3 \frac{\bar{q}}{M^2 - q^2} + \int_{s_0}^{\infty} \frac{\rho_1^h(s)}{s - q^2} ds,
\]

\[
\Pi_2(q^2) = M^2 \lambda_1^2 \frac{q + M}{M^2 - q^2} + \int_{s_0}^{\infty} \frac{\rho_2^h(s)}{s - q^2} ds,
\]

\[
\Pi_3(q^2) = M^2 \lambda_2^2 \frac{q + M}{M^2 - q^2} + \int_{s_0}^{\infty} \frac{\rho_3^h(s)}{s - q^2} ds,
\]

\[
\Pi_4(q^2) = q^2 M^2 \lambda_3^2 \frac{q + M}{M^2 - q^2} + \int_{s_0}^{\infty} \frac{\rho_4^h(s)}{s - q^2} ds.
\]  

(26)

On the theoretical side, we carry out the operator product expansion taking into account condensates up to dimension 6. Then, as the usual procedure of the QCD sum rule, we utilize the dispersion relationship and the quark-hadron duality assumption. After taking Borel transformation on both sides of the hadronic representation and QCD expansion and matching the two sides, we arrive at the following sum rules:

\[
4(2\pi)^4 f_\Xi^2 \frac{M^2}{M_B^2} = \int_{4m_s^2}^{8m_s^2} \left\{ \frac{s}{3} (1 - x)^5 - 8m_s a_s \frac{1}{s} x^2 (1 - x) + \frac{1}{12} m_s m_0^2 a_s \frac{1}{s^2} x (-2 + 3x) - \frac{1}{4} b x (1 - x)^2 \right\} e^{-\frac{s}{M_B^2}} ds,
\]

(27)

and

\[
4(2\pi)^4 \lambda_1^2 M^2 e^{-\frac{M^2}{M_B^2}} = \int_{4m_s^2}^{8m_s^2} \left\{ s^2 [(1 - x) (1 - 13x - x^2 + x^3) - 12x (1 + x) \ln x] - 2m_s a_s (1 - x) (1 + 5x) - \frac{1}{3} m_s a_s m_0^2 \frac{11 + 8x - 7x^2}{1 - x} + \frac{1}{6} b (3 + 2x - 7x^2) \right\} e^{-\frac{s}{M_B^2}} ds,
\]

(28)

and

\[
(2\pi)^4 \lambda_2^2 M^2 e^{-\frac{M^2}{M_B^2}} = \int_{4m_s^2}^{8m_s^2} \left\{ \frac{3}{2} s^2 [(1 - x^2) (1 - 8x + x^2) - 12x \ln x] - 12m_s a_s (1 - x^2) - \frac{2}{3} m_s a_s m_0^2 \frac{x}{s} + 3b (1 - x)^2 \right\} e^{-\frac{s}{M_B^2}} ds,
\]

(29)

and

\[
(4\pi)^4 \lambda_3^2 M^2 e^{-\frac{M^2}{M_B^2}} = \int_{4m_s^2}^{8m_s^2} \left\{ s^2 [(1 - x) (1 + 17x + 53x^2 - 11x^3) + 60x^2 \ln x] + \frac{4}{5} s^2 (1 - x)^5 - 8m_s a_s (1 - x) (1 + x + 4x^2) \right.
\]

\[
+ 2m_s a_s m_0^2 \frac{1}{s(1 - x)} (8 - 19x + 12x^2 - 3x^3) - \frac{2}{3} b (1 - x) (2 - 13x - x^2) \right\} e^{-\frac{s}{M_B^2}} ds,
\]

(30)
where the notation $x = m_s^2/s$ is adopted for convenience. In the numerical analysis, the parameters employed are the standard values: $a = -(2\pi)^2\langle \bar{u}u \rangle = 0.55 \text{ GeV}^3$, $b = (2\pi)^2\langle \alpha_s G^2/\pi \rangle = 0.47 \text{ GeV}^4$, $a_s = -(2\pi)^2\langle s\bar{s} \rangle = 0.8a$, $\langle \bar{u}g_s \sigma \cdot G u \rangle = m_0^2\langle \bar{u}u \rangle$, and $m_0^2 = 0.8 \text{ GeV}^2$. The mass of the strange quark is used as the central value provided by the particle data group (PDG) [21] $m_s = 0.10 \text{ GeV}$.

Another important parameter in the QCD sum rule is the auxiliary Borel parameter $M_B^2$, which is introduced to suppress both higher resonance and higher order dimension contributions simultaneously. At the same time, there should be a proper region in which the results of the sum rules vary mildly with it. The numerical analysis shows that the working window of the Borel parameter is $0.8 \text{ GeV}^2 \leq M_B^2 \leq 1.2 \text{ GeV}^2$, in which our results are acceptable.

It can be seen that sum rules from Eq. (27) to Eq. (30) can only give the absolute values of the parameters. To determine the relative sign of $f_\Xi$ and $\lambda_1$, we give the sum rule of $f_\Xi\lambda_1^*$:

\[
(2\pi)^4 f_\Xi\lambda_1^* M^2 e^{-\frac{M^2}{M_B^2}} = \int_{4m_s^2}^{s_0} \left\{ -\frac{1}{2} s^2 x[(1 - x)(2 + 5x - x^2) + 6x \ln x] - \frac{1}{2} m_s a_s (1 - x)(3 - 5x) \right. \\
\left. - \frac{1}{6} m_s a_s m_0^2 \frac{1 + 8x - 7x^2}{s(1 - x)} - \frac{b}{16} x(1 - x)(5 - 9x) \right\} e^{-\frac{s}{M_B^2}} ds. \tag{31}
\]

Similarly, the relative signs of $\lambda_2$ and $\lambda_3$ to $\lambda_1$ are determined by the following two sum rules:

\[
(2\pi)^4 (\lambda_1\lambda_2^* + \lambda_1^*\lambda_2) M^2 e^{-\frac{M^2}{M_B^2}} = \int_{4m_s^2}^{s_0} \left\{ -3ms^2[(1 - x)(2 + 5x - x^2) + 6x \ln x] + 3a_s m_0^2 \frac{x}{s} - 12a_s s(1 - x)(1 - 2x) + \frac{3}{2} a_s m_0^2 (4 + x) \right. \\
\left. - \frac{b}{4} m_s \frac{(1 - x)(2 + 5x)}{x} \right\} e^{-\frac{s}{M_B^2}} ds, \tag{32}
\]

and

\[
(2\pi)^4 (\lambda_1\lambda_3^* + \lambda_1^*\lambda_3) M^2 e^{-\frac{M^2}{M_B^2}} = \int_{4m_s^2}^{s_0} e^{-\frac{s}{M_B^2}} ds \left\{ -\frac{ms}{2} s^2 [(1 - x)(3 + 15x + 4x^2)] + 12x(1 + x) \ln x - \frac{1}{2} a_s s(1 - x)(8 - 5x - 5x^2) \right. \\
\left. + \frac{1}{24} a_s m_0^2 \frac{1}{1-x} [2x(2 + x + x^2) + 3(2 - x^2 + 3x^3)] + \frac{1}{64} m_s b[-9(2 - x)(1 - x) + 7 \ln x] \\
\left. - \frac{1}{24} m_s b \frac{1}{x} [(1 - x)(8 + 11x + 11x^2) - 18x \ln x] \right\}. \tag{33}
\]

The numerical analysis shows that if $f_\Xi$ is taken to be positive, the final numerical values of the coupling constants of $\Xi$ are given as follows:

\[
f_\Xi = (9.9 \pm 0.4) \times 10^{-3} \text{ GeV}^2, \quad \lambda_1 = -(2.8 \pm 0.1) \times 10^{-2} \text{ GeV}^2, \\
\lambda_2 = (5.2 \pm 0.2) \times 10^{-2} \text{ GeV}^2, \quad \lambda_3 = (1.7 \pm 0.1) \times 10^{-2} \text{ GeV}^2. \tag{34}
\]
V. APPLICATION: FORM FACTORS OF THE BARYONS WITH LIGHT-CONE QCD SUM RULES

A. Electromagnetic form factors

1. LCSRs for the EM form factors

Electromagnetic form factors, which characterize the internal structure of the hadron, have received great attention in the past decades. There have been a lot of investigations both experimentally and theoretically for mesons. However, as three-body composite particles, baryons have more complex structure in comparison with mesons. Thus studies on the baryon EM form factors are much fewer than those of the meson’s. The existing investigations are mainly focused on the nucleon \[13, 14\] (recent status can be found in Ref. 5 and references therein). We have given an investigation on the EM form factors of the Λ and Σ baryons in previous work 6,12,22. In this subsection, the EM form factors of the Ξ baryon are studied by use of the light-cone QCD sum rule method; furthermore, the magnetic moments of the same baryons are estimated by fitting our results with the dipole formula.

The definition of the EM form factors is connected with the matrix element of the EM current of a baryon between the baryon states:

$$\langle \Xi(P,s) | j_{\mu}^{em}(0) | \Xi(P',s') \rangle = \bar{\Xi}(P,s) [\gamma_\mu F_1(Q^2) - i \frac{\sigma_\mu q'\cdot s}{2M} F_2(Q^2)] \Xi(P',s'),$$  (35)

where $Q^2 = -q^2 = -(P - P')^2$ is the squared momentum transfer, $F_1(Q^2)$ and $F_2(Q^2)$ are the Dirac and Pauli form factors, respectively, and $j_{\mu}^{em} = e_u \bar{u}\gamma_\mu u + e_s \bar{s}\gamma_\mu s$ is the electromagnetic current relevant to the hadron. $P, s$ and $P', s'$ are the four-momenta and the spins of the initial and the final Ξ baryon states, respectively. Experimentally, the EM form factors are usually expressed by the electric $G_E(Q^2)$ and magnetic $G_M(Q^2)$ Sachs form factors:

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2),$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2).$$  (36)

The normalization of the magnetic form factor $G_M(Q^2)$ at the point $Q^2 = 0$ gives the magnetic moment of the baryon:

$$G_M(0) = \mu_\Xi.$$  (37)

In the following we mainly focus on the Ξ$^0$ baryon, and the calculation of Ξ$^-$ is similar. The derivation begins with the following correlation function:

$$T_\mu(P,q) = i \int d^4xe^{iq\cdot x} \langle 0 | T \{ j_{\Xi^0}(0) j_{\mu}^{em}(x) \} | \Xi^0(P,s) \rangle,$$  (38)
where the interpolating current is used as the form (22). The hadronic representation of the correlation function is acquired by inserting a complete set of states with the same quantum numbers as those of $\Xi^0$:

$$z^\mu T_\mu(P, q) = \frac{1}{M_{\Xi^0}^2 - P^2} \int \Xi^0(P' \cdot z)[2(P' \cdot z F_1(Q^2) - \frac{q \cdot z}{2} F_2(Q^2)) \delta^\mu] + (P' \cdot z F_2(Q^2) + \frac{q \cdot z}{2} F_2(Q^2)) \frac{\delta^\mu}{M_{\Xi^0}^2} \Xi^0(P, s) + \ldots,$$

in which $P' = P - q$, and the dots stand for the higher resonances and continuum contributions. Herein the correlation function is contracted with the light-cone vector $z^\mu$ to get rid of contributions proportional to $z^\mu$ which are subdominant on the light cone. On the theoretical side, the correlation function (38) can be calculated with the aid of the DAs obtained above to the leading order of $\alpha_s$:

$$z_\mu T^\mu = 2(P \cdot z)^2 (\xi \Xi) \gamma \left\{ e_u \int_0^1 d\alpha_3 \frac{1}{s - p^2} \left\{ B_0(\alpha_3) + \frac{M^2}{(s - p^2)^2} B_1(\alpha_3) \right\} - 2 \frac{M^4}{(s - p^2)^2} B_2(\alpha_3) \right\} + 2 e_s \int_0^1 d\alpha_2 \frac{1}{s_2 - p^2} \left\{ C_0(\alpha_2) + \frac{M^4}{(s_2 - p^2)^2} C_2(\alpha_2) \right\} + 2(\xi \cdot z)^2 M (\xi \cdot \Xi) \gamma \left\{ e_u \int_0^1 d\alpha_3 \frac{1}{\alpha_3 (s - p^2)^2} (-D_1(\alpha_3)) + 2 \frac{M^2}{(s - p^2)^2} B_2(\alpha_3) \right\} + 2 e_s \int_0^1 d\alpha_2 \frac{1}{\alpha_2 (s_2 - p^2)^2} (-E_1(\alpha_2)) + 2 \frac{M^2}{(s - p^2)^2} C_2(\alpha_2) \right\},$$

where $s = (1 - \alpha_3)M^2 + \frac{(1 - \alpha_3)Q^2}{\alpha_3}$, $s_2 = (1 - \alpha_2)M^2 + \frac{(1 - \alpha_2)Q^2 + m_2^2}{\alpha_2}$, and

$$B_0(\alpha_3) = \int_0^{1-\alpha_3} d\alpha_1 V_1(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3),$$
$$B_1(\alpha_3) = (\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_5)(\alpha_3),$$
$$B_2(\alpha_3) = (-\tilde{V}_1 + \tilde{V}_2 + \tilde{V}_3 + \tilde{V}_4 + \tilde{V}_5 - \tilde{V}_6)(\alpha_3),$$
$$C_0(\alpha_2) = \int_0^{1-\alpha_2} d\alpha_1 V_1(\alpha_1, \alpha_2, 1 - \alpha_1 - \alpha_2),$$
$$C_1(\alpha_2) = (\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3 - \tilde{V}_4 - \tilde{V}_5)(\alpha_2),$$
$$C_2(\alpha_2) = (-\tilde{V}_1 + \tilde{V}_2 + \tilde{V}_3 + \tilde{V}_4 + \tilde{V}_5 - \tilde{V}_6)(\alpha_2),$$
$$D_1(\alpha_3) = (\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3)(\alpha_3),$$
$$E_1(\alpha_2) = (\tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3)(\alpha_2),$$
in which

$$
\tilde{V}_i(\alpha_2) = \int_0^{\alpha_2} d\alpha_2' \int_0^{1-\alpha_2'} d\alpha_1 V_i(\alpha_1, \alpha_2', 1 - \alpha_1 - \alpha_2'),
$$

$$
\tilde{V}_i(\alpha_2) = \int_0^{\alpha_2} d\alpha_2' \int_0^{\alpha_2'} \int_0^{\alpha_2''} d\alpha_1 V_i(\alpha_1, \alpha_2'', 1 - \alpha_1 - \alpha_2''),
$$

$$
\tilde{V}_i(\alpha_3) = \int_0^{\alpha_3} d\alpha_3' \int_0^{1-\alpha_3'} d\alpha_1 V_i(\alpha_1, 1 - \alpha_1 - \alpha_3', \alpha_3'),
$$

$$
\tilde{V}_i(\alpha_3) = \int_0^{\alpha_3} d\alpha_3' \int_0^{\alpha_3'} \int_0^{1-\alpha_3''} d\alpha_1 V_i(\alpha_1, 1 - \alpha_1 - \alpha_3', \alpha_3'').
$$

(42)

By matching both sides of the Borel transformed version of the hadronic and theoretical representations with the assumption of quark-hadron duality, the final sum rules are given as follows,

$$
f_{\Xi^0} F_1(Q^2) e^{-\frac{M^2}{M_B^2}} = e_u \int_{\alpha_{30}}^{1} d\alpha_3 e^{-\frac{M^2}{M_B^2}} \left\{ B_0(\alpha_3) + \frac{M^2}{M_B^2} B_1(\alpha_3) - \frac{M^4}{M_B^4} B_2(\alpha_3) \right\}
$$

$$
+ e_u e^{-\frac{M^2}{M_B^2}} \frac{\alpha_{30}^2 M^2}{\alpha_{20}^2 M^2 + Q^2} \left\{ B_1(\alpha_{30}) - \frac{M^2}{M_B^2} B_2(\alpha_{30}) \right\}
$$

$$
+ e_u e^{-\frac{M^2}{M_B^2}} \frac{\alpha_{30}^2 M^4}{\alpha_{30}^2 M^2 + Q^2} \frac{d}{d\alpha_{30}} B_2(\alpha_{30}) - \frac{\alpha_{30}^2}{\alpha_{20}^2 M^2 + Q^2} \frac{d}{d\alpha_{20}} B_2(\alpha_{20})
$$

$$
+ 2e_s \int_{\alpha_{20}}^{1} d\alpha_2 e^{-\frac{M^2}{M_B^2}} \left\{ C_0(\alpha_2) + \frac{M^2}{M_B^2} C_1(\alpha_2) - \frac{M^4}{M_B^4} C_2(\alpha_2) \right\}
$$

$$
+ 2e_s e^{-\frac{M^2}{M_B^2}} \frac{\alpha_{20}^2 M^2}{\alpha_{20}^2 M^2 + Q^2 + m_s^2} \frac{d}{d\alpha_{20}} C_2(\alpha_{20}) - \frac{\alpha_{20}^2}{\alpha_{20}^2 M^2 + Q^2 + m_s^2} \frac{d}{d\alpha_{20}} C_2(\alpha_{20})
$$

(43)

$$
f_{\Xi^0} F_2(Q^2) e^{-\frac{M^2}{M_B^2}} = M^2 \left\{ 2e_u \int_{\alpha_{30}}^{1} d\alpha_3 e^{-\frac{M^2}{M_B^2}} \frac{1}{\alpha_{30} M_B^2} \left\{ - D_1(\alpha_3) + \frac{M^2}{M_B^2} B_2(\alpha_3) \right\}
$$

$$
- 2e_u e^{-\frac{M^2}{M_B^2}} \frac{\alpha_{30}}{\alpha_{30}^2 M^2 + Q^2} \left\{ D_1(\alpha_{30}) - \frac{M^2}{M_B^2} B_2(\alpha_{30}) \right\}
$$

$$
- 2e_u e^{-\frac{M^2}{M_B^2}} \frac{\alpha_{30}^2 M^2}{\alpha_{30}^2 M^2 + Q^2} \frac{d}{d\alpha_{30}} B_2(\alpha_{30}) - \frac{\alpha_{30}}{\alpha_{30}^2 M^2 + Q^2} \frac{d}{d\alpha_{20}} B_2(\alpha_{20})
$$

$$
+ 4e_s \int_{\alpha_{20}}^{1} d\alpha_2 e^{-\frac{M^2}{M_B^2}} \frac{1}{\alpha_{20} M_B^2} \left\{ - E_1(\alpha_2) + \frac{M^2}{M_B^2} C_2(\alpha_2) \right\}
$$

$$
- 4e_s e^{-\frac{M^2}{M_B^2}} \frac{\alpha_{20}}{\alpha_{20}^2 M^2 + Q^2 + m_s^2} \left\{ E_1(\alpha_{20}) - \frac{M^2}{M_B^2} C_2(\alpha_{20}) \right\}
$$

$$
- 4e_s e^{-\frac{M^2}{M_B^2}} \frac{\alpha_{20}^2 M^2}{\alpha_{20}^2 M^2 + Q^2 + m_s^2} \frac{d}{d\alpha_{20}} C_2(\alpha_{20}) - \frac{\alpha_{20}}{\alpha_{20}^2 M^2 + Q^2 + m_s^2} \frac{d}{d\alpha_{20}} C_2(\alpha_{20})
$$

(44)
FIG. 1: Dependence of the magnetic form factor $G_M(Q^2)$ of $\Xi$ baryons on the Borel parameter at different momentum transfer. The lines correspond to the points $Q^2 = 1, 2, 3, 5$ GeV$^2$ from the bottom up for $\Xi^0$ (left) and $\Xi^-$ (right), respectively.

where $\alpha_{i0}$ corresponds to the continuum threshold $s_0$ by the following expressions:

$$\alpha_{20} = \frac{-(Q^2 + s_0 - M^2) + \sqrt{(Q^2 + s_0 - M^2)^2 + 4(Q^2 + m^2_s)M^2}}{2M^2},$$

$$\alpha_{30} = \frac{-(Q^2 + s_0 - M^2) + \sqrt{(Q^2 + s_0 - M^2)^2 + 4Q^2M^2}}{2M^2}. \quad (45)$$

2. Numerical analysis

In the numerical analysis, the continuum threshold is chosen to be $s_0 = (2.8 - 3.0)$ GeV$^2$; the masses of the $\Xi$ baryons are from Ref. [21]: $M_{\Xi^0} = 1.315$ GeV and $M_{\Xi^-} = 1.322$ GeV. To choose the working window of the Borel parameter, we first give the dependence of the EM form factors on the Borel parameter at different points of $Q^2$ in Fig. 1. It can be seen from the panels that there is an acceptable "stability window" in the range $2 \text{ GeV}^2 \leq M_B^2 \leq 4 \text{ GeV}^2$. Hereafter, the Borel parameter is set to be $M_B^2 = 3 \text{ GeV}^2$ in the following analysis.

The $Q^2$-dependent magnetic and electric form factors are shown in Fig. 2 for $\Xi^0$ and Fig. 3 for $\Xi^-$. It is known that LCSR needs the momentum transfer $Q^2$ to be sufficiently large which is due to the convergence of the light-cone expansion. The calculation shows that higher twist contributions are suppressed efficiently above the point $Q^2 = 1 \text{ GeV}^2$, and the higher resonance contributions are subdominant after Borel transformation below $Q^2 = 7 \text{ GeV}^2$. Therefore we carry on the numerical analysis in the range $1 \text{ GeV}^2 \leq Q^2 \leq 7 \text{ GeV}^2$. In comparison with results from the constituent quark model in Ref. [23], our calculations are in accordance with the authors’ but for the electronic form factor of $\Xi^0$. In our calculation, the electric form factor turns from positive to negative when $Q^2$ increases while their conclusion is the opposite. Taking into account calculations from the chiral quark/soliton model in Ref. [24], in which behaviors of the EM form factors below $Q^2 = 1 \text{ GeV}^2$ are given, our result is reasonable.
FIG. 2: $Q^2$ dependence of the magnetic form factor (a) and the electronic form factor (b) of $\Xi^0$. The lines correspond to the threshold $s_0 = 2.8, 2.9, 3.0 \text{ GeV}^2$ from the bottom up (a) and from the top down (b).

FIG. 3: $Q^2$ dependence of the magnetic form factor (a) and the electronic form factor (b) of $\Xi^-$. The lines correspond to the threshold $s_0 = 2.8, 2.9, 3.0 \text{ GeV}^2$ from the top down.

Similar to the cases of the nucleon and other octet baryons \[6, 12, 13\], the magnetic form factors are assumed to be expressed by the dipole formula:

$$
\frac{1}{\mu_{\Xi}} G_M(Q^2) = \frac{1}{(1 + Q^2/m_{B_0}^2)^2} = G_D(Q^2).
$$

(46)

As there is no information about the parameter $m_B^2$ from experimental data so far, the two parameters $m_B^2$ and $\mu_{\Xi}$ are estimated simultaneously by the dipole formula (46) fitting of the magnetic form factor. The simulation is shown by the dashed line in Fig. 4 for $\Xi^0$ and in Fig. 5 for $\Xi^-$. Our numerical values are $\mu_{\Xi^0} = -(1.92 \pm 0.34) \mu_N$ and $\mu_{\Xi^-} = -(1.19 \pm 0.03) \mu_N$. The uncertainties come from the different choice of the threshold $s_0$ with the Borel parameter variation in the range $2.5 \text{ GeV}^2 \leq M_B^2 \leq 3.5 \text{ GeV}^2$. The numerical analysis shows that the calculation result for $\Xi^-$ is not sensitive to the threshold and the Borel parameter. Note that, as the estimations are from the dipole formula fits with the sum rules as the input data, the uncertainty due to the variation of the input parameters, such as $f_\Xi$ and $\lambda_\Xi$, is not included; it may reach 5% or more. In comparison with the values given by PDG \[21\], our estimations are larger in absolute values, which may partly lie in
FIG. 4: Fits of the magnetic form factor $G_M(Q^2)$ of $\Xi^0$ by the dipole formula $\mu_{\Xi^0}/(1+Q^2/m_0^2)^2$. The dashed lines are the fits, and (a), (b) correspond to the threshold $s_0 = 2.8, 3.0$ GeV$^2$, respectively.

FIG. 5: Fits of the magnetic form factor $G_M(Q^2)$ of $\Xi^-$ by the dipole formula $\mu_{\Xi^-}/(1+Q^2/m_0^2)^2$. The dashed lines are the fits, and (a), (b) correspond to the threshold $s_0 = 2.8, 3.0$ GeV$^2$, respectively.

the accuracy of the DAs that are calculated only to leading order conformal spin expansion. Another possibility is that in the dipole formula the uncertainty of the parameter $m_0^2$ gives a deviation from the formula.

Finally, we plot the physical value $G_M/(\mu_{\Xi}G_D)$ versus $Q^2$ in Fig. 6. The parameters in the dipole formula are chosen as follows: the magnetic moments are used as the central value provided by PDG, $\mu_{\Xi^0} = -1.25\mu_N$ and $\mu_{\Xi^-} = -0.65\mu_N$; the other parameter $m_0^2$ is the central value from the fits, $m_0^2 = 0.94$ for $\Xi^0$ and $m_0^2 = 0.96$ for $\Xi^-$. Future experiments are expected to provide more information about the EM form factors.
B. Semileptonic decay $\Xi_c \rightarrow \Xi e\nu_e$

1. LCSRs for weak transition form factors

The semileptonic decays of charmed hadrons are important decay modes in heavy flavor physics since they can give useful information on the Cabibbo-Kobiyasch-Maskawa matrix elements. However, because of the strong interaction in the hadronic bound state, the calculation of the processes needs some nonperturbative theoretical approaches. Hereafter, we make use of LCSR technique to investigate the exclusive semileptonic decays of $\Xi_c$ involving $\Xi$ baryons.

One of the main decay channels for the baryon $\Xi_c$ is the weak decay $\Xi_c \rightarrow \Xi e^+\nu_e$, which was observed experimentally early in the 1990s \cite{25}. Theoretically, the process has been studied with various models \cite{26,27,28,29,30}. In the theoretical calculations, the hadronic part of the process is generally parametrized by the nonperturbative form factors which are defined by the matrix element of the weak current between the baryon states:

$$\langle \Xi_c(P')|j_\mu|\Xi(P)\rangle = \bar{\Xi}_c(P')f_1\gamma_\mu - i\frac{f_2}{M}\sigma_{\mu\nu}q^\nu - (g_1\gamma_\mu + ig_2\sigma_{\mu\nu}q^\nu)\gamma_5\Xi(P),$$

in which the mass of the positron is neglected. The differential decay rate is given as

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2|V_{cs}|^2}{192\pi^3 M_{\Xi_c}^2} q^2 \sqrt{q^2 + \frac{m_+^2}{q_+^2}} \left\{ -6f_1f_2 M_{\Xi_c} m_+ q_-^2 + 6g_1g_2 M_{\Xi_c} m_- q_+^2 
+ f_1^2 M_{\Xi_c}^2 \left( \frac{m_+^2 m_-^2}{q_+^2} + m_-^2 - 2(q^2 + 2M_{\Xi_c} M_{\Xi}) \right) 
+ g_1^2 M_{\Xi_c}^2 \left( \frac{m_+^2 m_-^2}{q_+^2} + m_+^2 - 2(q^2 - 2M_{\Xi_c} M_{\Xi}) \right) 
- f_2^2 [-2m_+^2 m_-^2 + m_+^2 q^2 + q^2 (q^2 + 4M_{\Xi_c} M_{\Xi})] 
- g_2^2 [-2m_+^2 m_-^2 + m_-^2 q^2 + q^2 (q^2 - 4M_{\Xi_c} M_{\Xi})] \right\},$$

where $m_\pm = M_{\Xi_c} \pm M_\Xi$ and $q_\pm^2 = q^2 - m_\pm^2$ are used for convenience.
The form factors in Eq. (47) can be estimated in the framework of LCSR. The calculation begins with the following correlation function:

\[ T_\mu(P, q) = i \int d^4 x e^{iq \cdot x} \langle 0| T\{ j_{\Xi_c}(0) j^w_\mu(x)\} | \Sigma^+(P, s) \rangle, \]  

where the interpolating current for the \( \Xi_c \) baryon is chosen as

\[ j_{\Xi_c}(x) = \epsilon^{ijk}[s^i(x)C \not\! q^j(x)] \gamma^5 \not\! q^k(x), \]  

and the weak current is

\[ j^w_\mu(x) = \bar{c}(x) \gamma_\mu(1 - \gamma_5)s(x). \]  

Following the standard philosophy of the QCD sum rule, the hadronic representation of the correlation function is written as

\[ z_\mu T^\mu = \frac{2 f_{\Xi_c}(P' \cdot z)^2}{M^2_{\Xi_c} - P'^2} [f_1(q^2) \not\! q + \frac{1}{M} f_2(q^2) \not\! q \not\! \not\! q - (g_1(q^2) \not\! \gamma_5 - \frac{1}{M} g_2(q^2) \not\! q \not\! \gamma_5)] \Xi(P, s) + \ldots, \]  

On the theoretical side, the correlation function is expanded on the light cone and can be expressed by the DAs. After the standard process of LCSR, we get the following results:

\[ f_{\Xi_c} f_1(q^2)e^{-\frac{M^2}{M_B^2}} = \int_{\alpha_2}^1 d\alpha_2 e^{-\frac{\alpha_2}{M_B^2}} \left\{ C_0(\alpha_2) + \frac{M^2}{M_B^2} C_1(\alpha_2) - \frac{M^4}{M_B^4} C_2(\alpha_2) \right\} \]  

\[ + e^{-\frac{\alpha_2}{M_B^2}} \frac{\alpha_2^2 M^2}{M^2 \alpha_2^2 - q^2 + m_s^2} \left\{ C_1(\alpha_2) - \frac{M^2}{M_B^2} C_2(\alpha_2) \right\} \]  

\[ + e^{-\frac{\alpha_2}{M_B^2}} \frac{\alpha_2^2 M^4}{M^2 \alpha_2^2 - q^2 + m_s^2} d \frac{\alpha_2}{M^2 \alpha_2^2 - q^2 + m_s^2} C_2(\alpha_2), \]  

\[ f_{\Xi_c} f_2(q^2)e^{-\frac{M^2}{M_B^2}} = -\int_{\alpha_2}^1 d\alpha_2 e^{-\frac{\alpha_2}{M_B^2}} \frac{M M_{\Xi_c} 1}{M_B^2 \alpha_2} \left\{ E_1(\alpha_2) - \frac{M^2}{M_B^2} C_2(\alpha_2) \right\} \]  

\[ - e^{-\frac{\alpha_2}{M_B^2}} \frac{\alpha_2^2 M^2}{M^2 \alpha_2^2 - q^2 + m_s^2} \left\{ E_1(\alpha_2) - \frac{M^2}{M_B^2} C_2(\alpha_2) \right\} \]  

\[ - e^{-\frac{\alpha_2}{M_B^2}} \frac{\alpha_2^2 M^4}{M^2 \alpha_2^2 - q^2 + m_s^2} d \frac{\alpha_2}{M^2 \alpha_2^2 - q^2 + m_s^2} C_2(\alpha_2), \]  

\[ f_{\Xi_c} g_1(q^2)e^{-\frac{M^2}{M_B^2}} = \int_{\alpha_2}^1 d\alpha_2 e^{-\frac{\alpha_2}{M_B^2}} \left\{ H_1(\alpha_2) + \frac{M^2}{M_B^2} H_2(\alpha_2) + \frac{M^4}{M_B^4} H_3(\alpha_2) \right\} \]  

\[ + e^{-\frac{\alpha_2}{M_B^2}} \frac{\alpha_2^2 M^2}{M^2 \alpha_2^2 - q^2 + m_s^2} \left\{ H_2(\alpha_2) + \frac{M^2}{M_B^2} H_3(\alpha_2) \right\} \]  

\[ - e^{-\frac{\alpha_2}{M_B^2}} \frac{\alpha_2^2 M^4}{M^2 \alpha_2^2 - q^2 + m_s^2} d \frac{\alpha_2}{M^2 \alpha_2^2 - q^2 + m_s^2} H_3(\alpha_2), \]
The quark mass is taken as the central value from PDG: \( m_q = 27 \text{ GeV} \)

where \( x \) in which the following additional notations are used for convenience, the baryons can be found in Ref. [21]. As in the sum rules, the two \( \Xi_c \) are used the same ones given in Sec. IV. Our estimation for the coupling constant is subsection. The continuum threshold is chosen as \( M_{B} \), Thus we take the mass of \( \Xi_c \) the same isomultiplet, effects from the isospin symmetry breaking can be neglected safely.

Before the numerical analysis, we first specify the choice of the parameters used in this subsection. The continuum threshold is chosen as \( s_0 = (7 - 9) \text{ GeV}^2 \), and the masses of the baryons can be found in Ref. [21]. As in the sum rules, the two \( \Xi_c \) baryons belong to the same isomultiplet, effects from the isospin symmetry breaking can be neglected safely. Thus we take the mass of \( \Xi_c \) as \( M_{\Xi_c} = 2.471 \text{ GeV} \).

As to the coupling constant \( f_{\Xi_c} \), we use the QCD sum rule method to estimate it. Complying with the standard procedure of the QCD sum rule, we get the following expression:

\[
 f_{\Xi_c} = \int_{s_0}^{s_B} d\alpha_2 e^{-\frac{\alpha_2}{M_{B}}} \frac{M_{\Xi_c}}{M_{B}} \frac{H_4(\alpha_2)}{\alpha_2} \left\{ H_4(\alpha_2) + \frac{M^2}{M_{B}^2} H_3(\alpha_2) \right\}
\]

in which the following additional notations are used for convenience,

\[
 H_1(\alpha_2) = \int_0^{\alpha_2} d\alpha_1 A_1(\alpha_1, \alpha_2, 1 - \alpha_1 - \alpha_2),
\]

\[
 H_2(\alpha_2) = (\tilde{A}_2 - \tilde{A}_3 + \tilde{A}_4 - \tilde{A}_5)(\alpha_2),
\]

\[
 H_3(\alpha_2) = (\tilde{A}_1 - \tilde{A}_2 + \tilde{A}_3 + \tilde{A}_4 - \tilde{A}_5 + \tilde{A}_6)(\alpha_2),
\]

\[
 H_4(\alpha_2) = (-\tilde{A}_1 + \tilde{A}_2 - \tilde{A}_3)(\alpha_2).
\]

The threshold-related parameter \( \alpha_{20} \) is the same as that in Eqs. [45] with the interchange \( m_s \leftrightarrow m_c \).

2. Numerical analysis

Before the numerical analysis, we first specify the choice of the parameters used in this subsection. The continuum threshold is chosen as \( s_0 = (7 - 9) \text{ GeV}^2 \), and the masses of the baryons can be found in Ref. [21]. As in the sum rules, the two \( \Xi_c \) baryons belong to the same isomultiplet, effects from the isospin symmetry breaking can be neglected safely. Thus we take the mass of \( \Xi_c \) as \( M_{\Xi_c} = 2.471 \text{ GeV} \).

As to the coupling constant \( f_{\Xi_c} \), we use the QCD sum rule method to estimate it. Complying with the standard procedure of the QCD sum rule, we get the following expression:

\[
 (4\pi) \frac{4}{4} f_{\Xi_c}^2 = \int_{(m_c + m_b)^2}^{s_0} ds e^{-\frac{s - M_{\Xi_c}^2}{M_{B}^2}} \left\{ \frac{4}{5} s(1 - x)^5 \right\} - 8 m_s a_s \frac{1}{s} x^2 (1 - x)
\]

\[
 + \frac{1}{12} m_s a_s m_0^2 \frac{1}{s^2} x^2 (2 - 3x) - \frac{1}{6} \frac{1}{s} x(1 - x)^2 \right\}, \tag{58}
\]

where \( x = m_c/s \) is used for convenience. In the numerical analysis, the threshold is \( 7 \text{ GeV}^2 \leq s_0 \leq 9 \text{ GeV}^2 \) and the Borel window is \( 1.5 \text{ GeV}^2 \leq M_{B}^2 \leq 2 \text{ GeV}^2 \). The c-quark mass is taken as the central value from PDG: \( m_c = 1.27 \text{ GeV} \). Other parameters are used the same ones given in Sec. IV. Our estimation for the coupling constant is \( f_{\Xi_c} = (8.6 \pm 0.9) \times 10^{-3} \text{ GeV}^2 \).

The Borel parameter is chosen to suppress both higher resonance and higher twist contributions. The calculations show that the results are acceptable in the range \( 7 \text{ GeV}^2 \leq M_{B}^2 \leq 9 \text{ GeV}^2 \). In the following analysis, the Borel parameter is set to be \( M_{B}^2 = 8 \text{ GeV}^2 \).
FIG. 7: $q^2$ dependence of the weak form factors. The lines correspond to the threshold $s_0 = 7, 8, 9$ GeV$^2$ from the top down for $f_i(q^2)$ and from the bottom up for $g_i(q^2)$.

We plot the weak form factors depending on the momentum transfer $q^2$ in Fig. 7. Note that unlike the case for EM form factors in which the momentum transfer is spacelike, in the decay the physical process can only occur in the timelike region. At the same time, LCSR needs the momentum transfer to satisfy the relation $q^2 - m_c^2 < 0$ so that the main contribution is dominated on the light cone. Hence we choose the range of $q^2$ as $0 \leq q^2 \leq 1.0$ GeV$^2$ in the sum rules.

The thorough investigation of the decay process needs to determine behaviors of the form factors in the whole physical region, $0 \leq q^2 \leq (M_{\Xi_c} - M_{\Xi})^2$. To this end, we recast the form factors as explicit functions of timelike momentum transfer by the dipole formula fits in the sum rule allowed range $0 \leq q^2 \leq 1.0$ GeV$^2$, and then extrapolate them to the whole kinematical region. The three-parameter dipole formula is written as

$$f(q^2) = \frac{f(0)}{1 + a(q^2/M_{\Xi_c}^2) + b(q^2/M_{\Xi_c}^2)^2},$$

(59)

where $f(q^2)$ represents the form factors $f_i(q^2)$ or $g_i(q^2)$ ($i = 1, 2$). The coefficients are listed in Table II.

With Eq. (58) and the dipole formula (59), we can give the $q^2$-dependent differential decay rate, which is shown in Fig. 8. In the analysis the following parameters are central values from PDG: $|V_{cs}| = 1.04$ and $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$. The total decay width can be obtained by integrating out $q^2$ over the whole kinematical region $0 \leq q^2 \leq (M_{\Xi_c} - M_{\Xi})^2$. The final estimation of the total decay width is $\Gamma = 8.73 \times 10^{-14}$ GeV. For a comparison in detail, we first turn to results from the experiments. As there are no absolute branching
TABLE II: Fits of the weak form factors by the dipole formula (59).

| $f_i(0)$ | $a_1$ | $a_2$ |
|----------|-------|-------|
| $f_1$    | 0.87  | -2.45| 4.06 |
| $f_2$    | 0.21  | -3.14| 3.95 |
| $g_1$    | 0.14  | -0.83| 1.55 |
| $g_2$    | 0.33  | -2.45| 2.64 |

FIG. 8: $q^2$ dependence of the differential decay rate of $\Xi_c \to \Xi^+ \nu_e$.

fractions available in experiments so far, we only consider the relative branching ratios herein. In Ref. [25], the relative branching ratios of $B(\Xi_c^+ \to \Xi^- \pi^+ \pi^+)/B(\Xi_c^+ \to \Xi^0 \nu_e)$ and $B(\Xi_c^0 \to \Xi^- \pi^+)/B(\Xi_c^0 \to \Xi^- \nu_e)$ have been measured. Making use of the up bounds of the channels $B(\Xi_c^+ \to \Xi^- \pi^+ \pi^+) \leq 2.1 \times 10^{-2}$ and $B(\Xi_c^0 \to \Xi^- \pi^+) \leq 4.3 \times 10^{-3}$, we give our estimations: $B(\Xi_c^+ \to \Xi^0 \nu_e)/B(\Xi_c^+ \to \Xi^- \pi^+ \pi^+) = 2.7$ and $B(\Xi_c^0 \to \Xi^- \nu_e)/B(\Xi_c^0 \to \Xi^- \pi^+) = 3.4$, which agree with the values provided by PDG [21] $B(\Xi_c^+ \to \Xi^0 \nu_e)/B(\Xi_c^+ \to \Xi^- \pi^+ \pi^+) = 2.3$ and $B(\Xi_c^0 \to \Xi^- \nu_e)/B(\Xi_c^0 \to \Xi^- \pi^+) = 3.1$. In addition, we present theoretical results from various models in Table III. It can be seen from the table that our estimation is acceptable.

VI. SUMMARY

In this paper, the $\Xi$ baryon distribution amplitudes are investigated up to twist six based on the conformal symmetry. It is found that 14 independent DAs are needed to describe

TABLE III: Decay widths from various models in units of $10^{10} s^{-1}$.

| $\Xi_c \to \Xi^+ \nu_e$ | [26] | [27] | [28] | [29] | [30] | this work |
|------------------------|------|------|------|------|------|-----------|
|                        | 28.8 | 18.1 | 8.5  | 8.55 | 7.4  | 8.16      | 13.26     |
the structure of the baryon. In the calculations, the DAs are expanded by the conformal partial waves, and the nonperturbative parameters are determined in the QCD sum rule approach. The calculation on the conformal expansion of the DAs is to the leading order conformal spin accuracy.

As applications, the electromagnetic form factors of Ξ are investigated in the range $1 \text{ GeV}^2 \leq Q^2 \leq 7 \text{ GeV}^2$ with the aid of the DAs obtained. Our calculations show that the magnetic form factor can be well described by the dipole formula. By fitting the result with the dipole law, the magnetic moments of the baryons are estimated to be $\mu_{\Xi^0} = -(1.92 \pm 0.34)\mu_N$ and $\mu_{\Xi^-} = -(1.19 \pm 0.03)\mu_N$. In comparison with the values given by PDG [21], our results are larger in absolute values. This shows that our calculations need more detailed information on the DAs, which may come from higher order conformal spin contributions, and at the same time the choice of the interpolating currents may also affect the estimations to some extent [13, 14, 22].

We also study the semileptonic weak decay $\Xi_c \to \Xi e^+ \bar{\nu}_e$. The weak transition form factors are calculated within LCSR method. Our estimation of the decay width is $\Gamma = 8.73 \times 10^{-14} \text{ GeV}$. We give premature estimations of the relative branching ratios, which are in accordance with the present experimental data. More experiments are expected to test the calculations and give us more information.

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APPENDIX

In the appendix we give the explicit expressions of the Ξ baryon DAs. As to the definition in (3), our results are listed in this section.

Twist-3 distribution amplitudes of Ξ are:

\[
\begin{align*}
V_1(x_i) &= 120 x_1 x_2 x_3 \phi_3^0, \\
T_1(x_i) &= 120 x_1 x_2 x_3 \phi_3^{0}. \\
A_1(x_i) &= 0,
\end{align*}
\]

Twist-4 distribution amplitudes are:

\[
\begin{align*}
S_1(x_i) &= 6(x_2 - x_1)x_3(\xi_4^0 + \xi_4^{0}), \\
V_2(x_i) &= 24 x_1 x_2 \phi_4^{0}, \\
V_3(x_i) &= 12 x_3(1 - x_3)\psi_4^{0}, \\
T_2(x_i) &= 24 x_1 x_2 \phi_4^{0}, \\
T_7(x_i) &= 6 x_3(1 - x_3)(\xi_4^{0} - \xi_4^{0}).
\end{align*}
\]
Twist-5 distribution amplitudes are:

\[
S_2(x_i) = \frac{3}{2}(x_1 - x_2)(\xi_5^0 + \xi'_5^0), \quad P_2(x_i) = \frac{3}{2}(x_1 - x_2)(\xi_5^0 - \xi'_5^0),
\]
\[
V_4(x_i) = 3(1 - x_3)\psi_5^0, \quad A_4(x_i) = 3(x_1 - x_2)\psi_5^0,
\]
\[
V_5(x_i) = 6x_3\phi_5^0, \quad A_5(x_i) = 0,
\]
\[
T_4(x_i) = -\frac{3}{2}(x_1 + x_2)(\xi'_5^0 + \xi_5^0), \quad T_5(x_i) = 6x_3\phi'_5^0,
\]
\[
T_8(x_i) = \frac{3}{2}(x_1 + x_2)(\xi'_5^0 - \xi_5^0).
\] (62)

And finally twist-6 distribution amplitudes are:

\[
V_6(x_i) = 2\phi_6^0, \quad A_6(x_i) = 0,
\]
\[
T_6(x_i) = 2\phi'_6^0.
\] (63)

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