Abstract—Continual learning requires the model to maintain the learned knowledge while learning from a non-i.i.d data stream continually. Due to the single-pass training setting, online continual learning is very challenging, but it is closer to the real-world scenarios where quick adaptation to new data is appealing. In this paper, we focus on online class-incremental learning setting in which new classes emerge over time. Almost all existing methods are replay-based with a softmax classifier. However, the inherent logits bias problem in the softmax classifier is a main cause of catastrophic forgetting while existing solutions are not applicable for online settings. To bypass this problem, we abandon the softmax classifier and propose a novel generative framework based on the feature space. In our framework, a generative classifier which utilizes replay memory is used for inference, and the training objective is a pair-based metric learning loss which is proven theoretically to optimize the feature space in a generative way. In order to improve the ability to learn new data, we further propose a hybrid of generative and discriminative loss to train the model. Extensive experiments on several benchmarks, including newly introduced task-free datasets, show that our method beats a series of state-of-the-art replay-based methods with discriminative classifiers, and reduces catastrophic forgetting consistently with a remarkable margin.

I. INTRODUCTION

Humans excel at continually learning new skills and accumulating knowledge throughout their lifespan. However, when learning a sequential of tasks emerging over time, neural networks notoriously suffer from catastrophic forgetting [1] on old knowledge. This problem results from non-i.i.d distribution of data streams in such a scenario. To this end, continual learning (CL) [2], [3] has been proposed to bridge the above gap between intelligent agents and humans.

In common CL settings, there are clear boundaries between distinct tasks which are known during training. Within each task, a batch of data are accumulated and the model can be trained offline with the i.i.d data. Recently, online CL [4], [5] setting has received growing attention in which the model needs to learn from a non-i.i.d data stream in online settings. At each iteration, new data are fed into the model only once and then discarded. In this manner, task boundary is not informed, and thus online CL is compatible with task-free [6], [7] scenario. In real-world scenarios, the distribution of data stream changes over time gradually instead of switching between tasks suddenly. Moreover, the model is expected to quickly adapt to large amount of new data, e.g. user-generated content. Online CL meets these requirements, so it is more meaningful for practical applications. Many existing CL works deal with task-incremental learning (TIL) setting [8], [9], in which task identity is informed during test and the model only needs to classify within a particular task. However, for online CL problem, TIL is not realistic because of the dependence on task boundary as discussed above and reduces the difficulty of online CL. In contrast, class-incremental learning (CIL) setting [10] requires the model to learn new classes continually over time and classify samples over all seen classes during test. Thus, online CIL setting is more suitable for online data streams in real-world CL scenarios [11].

Most existing online CIL methods are based on experience replay (ER) [12], [13] strategy which stores a subset of learned data in a replay memory and uses the data in memory to retrain model thus alleviating forgetting. Recently, in CIL setting logits bias problem in the last fully connected (FC) layer, i.e. softmax classifier, is revealed [14], which is a main cause of catastrophic forgetting. In Figure 1a we show in online CIL, even if ER is used, logits bias towards newly learned classes in the softmax classifier is still serious and the forgetting on old tasks is dramatic (See Figure 1d). Although some works [14]–[16] propose different methods to reduce logits bias, they all depend on task boundaries and extra offline phases during training so that not applicable for online CIL setting.

In this paper, we propose to tackle the online CIL problem without the softmax classifier to avoid logits bias problem. Instead, we propose a new framework where training and inference are both in a generative way. We are motivated by the insight that generative classifier is more effective in low data regime than discriminative classifier which is demonstrated by [17]. Although the conclusion is drawn on simple linear models [17], similar results are also observed on deep neural

Fig. 1: Logits bias phenomenon of softmax classifier (left) and accuracy & forgetting on different tasks using softmax vs. generative NCM classifier (right). The results are obtained with ER and 1k replay memory on 5-task Split CIFAR10.
networks (DNNs) \cite{18, 19} recently. It should be noticed that in online CIL setting the data is seen only once, not fully trained, so it is analogous to the low data regime in which the generative classifier is preferable. In contrast, the commonly used softmax classifier is a discriminative model.

Concretely, we abandon the softmax FC layer and introduce nearest-class-mean (NCM) classifier \cite{20} for inference, which can be interpreted as classifying in a generative way. The NCM classifier is built on the feature space on the top of previous network layers. Thanks to ER strategy, NCM classifier can utilize the replay memory for inference. As for training, inspired by a recent work \cite{21}, which shows pair-based deep metric learning (DML) losses can be interpreted as optimizing the feature space from a generative perspective, we introduce Multi-Similarity (MS) loss \cite{22} to obtain a good feature space for NCM classifier. Meanwhile, we prove theoretically that MS loss is an alternative to a training objective of the generative classifier. In this way, we can bypass logits bias.

To strengthen the model’s capable of learning from new data in complex data streams, we further introduce an auxiliary proxy-based DML loss \cite{23}. Therefore, our whole training objective is a hybrid of generative and discriminative losses. During inference, we ignore the discriminative objective and classify with the generative NCM classifier. By tuning weight of the auxiliary loss, our method can work well in different data streams.

In summary, our contributions are as follows:

1) We make the first attempt to avoid logits bias problem in online CIL setting. In our generative framework, a generative classifier is introduced to replace softmax classifier for inference and for training, we introduce MS loss which is proven theoretically to optimize the model in a generative way.

2) In order to improve the ability of MS loss to learn from new data, we further introduce an auxiliary loss to achieve a good balance between retaining old knowledge and learning new knowledge.

3) We conduct extensive experiments on four benchmarks in multiple online CIL settings, including a new task-free setting we design for simulating more realistic scenarios. Empirical results demonstrate our method outperforms a variety of state-of-the-art replay-based methods substantially, especially alleviating catastrophic forgetting significantly.

II. RELATED WORK

Current CL methods can be roughly divided into three categories: which are regularization, parameter isolation and replay-based respectively \cite{3}. **Regularization** methods retain the learned knowledge by imposing penalty constraints on model’s parameters \cite{8} or outputs \cite{9} when learning new data. They work well in TIL setting but poor in CIL setting \cite{24}. **Parameter isolation** methods assign a specific subset of model parameters, such as network weights \cite{25} and sub-networks \cite{26} to each task to avoid knowledge interference and thus the network may keep growing. This type of method is mainly designed for TIL as task identity is usually necessary during test. The mainstream of **Replay-based** methods is ER-like \cite{10}, which stores a subset of old data and retrains it when learning new data to prevent forgetting of old knowledge. In addition, generative replay method trains a generator to replay old data approximately \cite{27}.

In the field of online CL, most of methods are on the basis of ER. \cite{28} first explored ER in online CL settings with different memory update strategies. Authors suggested ER method should be regarded as an important baseline as in this setting it is more effective than several existing CL methods, such as A-GEM \cite{29}. GSS \cite{4} designs a new memory update strategy by encouraging the divergence of gradients of samples in memory. MIR \cite{5} is proposed to select the maximally interfered samples from memory for replay. GMED \cite{30} edits the replay samples with gradient information to obtain samples likely to be forgotten, which can benefit the replay in the future. \cite{11} focus on online CIL setting and adopt the notion of Shapley Value to improve the replay memory update and sampling. All of the above methods are replay-based with softmax classifier. A contemporary work \cite{31} proposes CoPE, which is somewhat similar to our method. CoPE replaces softmax classifier with a prototype-based classifier which is non-parametric and updated using features of data samples. However, the loss function of CoPE is still discriminative and the way to classify is analogous to softmax classifier.

Apart from the above ER based methods, \cite{32} propose an online regularization method, however it performs very badly in online CL settings \cite{30}. \cite{7} propose a parameter isolation method in which the network is dynamically expanded and a memory for storing data is still required. Therefore, the memory usage is not fixed and potentially unbounded.

A recent work \cite{33} proposes SDC, a CIL method based on DML and NCM classifier. However, SDC requires an extra phase to correct semantic drift after training each task. This phase depends on task boundaries and the accumulated data of a task, which is not applicable for online CIL. In contrast, our method is based on ER and classifies with replay memory and thus need not correct the drift.

III. ONLINE CLASS-INCREMENTAL LEARNING WITH A GENERATIVE FRAMEWORK

A. Preliminaries and Motivations

1) **Online Class-Incremental Learning**: CIL setting has been widely used in online CL literature, e.g. \cite{5}. \cite{11}, and a softmax classifier is commonly used. A neural network \( f(\cdot; \theta) : \mathcal{X} \rightarrow \mathbb{R}^d \) parameterized by \( \theta \) encodes data samples \( x \in \mathcal{X} \) into a \( d \)-dimension feature \( f(x) \) on which an FC layer \( g \) outputs logits for classification: \( o = Wf(x) + b \). At each iteration, a minibatch of data \( \mathcal{B}_n \) from a data stream \( \mathcal{S} \) arrives and the whole model \( (f,g) \) is trained on \( \mathcal{B}_n \) only once. The training objective is cross-entropy (CE) loss:

\[
\mathcal{L}_{CE} = - \sum_{c=1}^{C} t_c \log \hat{y}_c, \quad \hat{y}_c = \frac{e^{y_c}}{\sum_{c=1}^{C} e^{y_c}} \quad (1)
\]


where $\tilde{C}$ is the number of classes seen so far. $t$ is one-hot label of $x$ and the subscript $:c$ denotes the $c$-th component. The new classes from $S$ emerge over time. The output space of $g$ is the number of seen classes and thus keeps growing. At test time, the model should classify over all $C$ classes seen.

2) Experience Replay for Online Continual Learning: ER makes two modifications during online training: (1) It maintains a replay memory $M$ with limited size which stores a subset of previously learned samples. (2) When a minibatch of new data $B_n$ is coming, it samples a minibatch $B_r$ from $M$ and uses $B_n \cup B_r$ to optimize the model with one SGD-like step. Then it updates $M$ with $B_n$. Recent works, e.g. [5], [11] regard ER-reservoir as a strong baseline, which combines ER with reservoir sampling [34] for memory update and random sampling for $B_r$. See [28] for more details about it.

3) Logits Bias in Softmax Classifier: Some recent works [14]–[16] show in CIL scenarios, even with replay-based mechanism the logits outputted by model always have a strong bias towards the newly learned classes, which leads to catastrophic forgetting actually. In preliminary experiments, we also observe this phenomenon in online CIL setting. We run ER-reservoir baseline on 5-task Split CIFAR10 (each task has two disjoint classes) online CIL benchmark. In Figure 1a, we display the average logits of each already learned tasks over samples in test data after learning each task. The model outputs much higher logits on the new classes (of the task just learned) than old classes.

Following [35], we examine the CE loss in Eq (1), the gradient of $L_{CE}$ w.r.t. logit $\alpha_{c,z}$ of class $c$ is $y_{c,z} - \|t_c = 1\|$. Thus, if $c$ is the real label $y$, i.e. $t_c = 1$, the gradient is non-positive and model is trained to increase $\alpha_{c,z}$, otherwise the gradient is non-negative and model is trained to decrease $\alpha_{c,z}$. Therefore, logits bias problem is caused by the imbalance between the number of samples of the new classes and that of the old classes with a limited size of $M$. As mentioned in Section I, existing solutions [14]–[16] designed for conventional CIL need task boundaries to conduct extra offline training phases and even depend on the accumulated data of one task. They are not applicable for online CIL setting where task boundaries are not informed or even do not exist in task-free scenario.

B. Inference with a Generative Classifier

Proposed generative framework is based on ER strategy, and aims to avoid the intrinsic logits bias problem by removing the softmax FC layer $g$ and build a generative classifier on the feature space $Z : z = f(x)$. If the feature $z$ is well discriminative, we can conduct inference with samples in $M$ instead of a parametric classifier which is prone to catastrophic forgetting [10]. We use NCM classifier firstly suggested by [10] for CL and show it is a generative model. We use $M_c$ to denote the subset of class $c$ of $M$. The class mean $\mu_c$ is computed by $\mu_c^M = \frac{1}{|M_c|} \sum_{x \in M_c} f(x)$. During inference, the prediction for $x^*$ is made by:

$$y^* = \arg \min_c \| f(x^*) - \mu_c^M \|_2$$

(2)

In fact, the principle of prediction in Eq (2) is to find a Gaussian distribution $\mathcal{N}(f(x^*)|\mu_c^M, I)$ with the maximal probability for $x^*$. Therefore, assuming the conditional distribution $p(z|y = c) = \mathcal{N}(z|\mu_c^M, I)$ and the prior distribution $p(y)$ is uniform, NCM classifier virtually deals with $p(y|x)$ by modeling $p(z|y)$ in a generative way. The inference way is according to Bayes rule: $\arg \max p(y|x^*) = \arg \max p(f(x^*)|y)p(y)$. The assumption about $p(y)$ simplifies the analysis and works well in practice. In contrast, softmax classifier models $p(y|x^*)$ in a typical discriminative way.

As discussed above, online CIL is in a low data setting where generative classifiers are preferable compared to discriminative classifiers [17]. Moreover, generative classifiers are more robust to continual learning [18] and imbalanced data settings [19]. At each iteration, $B_n \cup B_r$ is also highly imbalanced. Considering these results, we hypothesis generative classifiers are promising for online CIL problem. It should be noted our method only models a simple generative classifier $p(z|y)$ on the feature space, instead of modeling $p(x|y)$ on the input space using DNNs [18], which is time-consuming and thus is not suitable for online training.

C. Training with a Pair-based Metric Learning Loss from a Generative Perpective

To train the feature extractor $f(\theta)$ we resort to DML losses which aim to learn a feature space where the distances represent semantic dissimilarities between data samples. From the perspective of mutual information (MI), [21] theoretically show the equivalence between CE loss and several pair-based DML losses, such as contrast loss [36] and Multi-Similarity (MS) loss [22]. The DML losses maximize MI between feature $z$ and label $y$ in a generative way while CE loss in a discriminative way, which motivates us to train $f(\theta)$ with a pair-based DML loss to obtain a good feature space $Z$ for the generative classifier.

Especially, we choose the MS loss as a training objective. MS loss is one of the state-of-the-art methods in the field of DML. [22] point out pair-based DML losses can be seen as weighting each feature pair in the general pair weighting framework. As MS loss requires the feature $f(x)$ to be $\ell_2$-normalized first, from now on, we use $z_i$ to denote the $\ell_2$-normalized feature of $x_i$ and a feature pair is represented in the form of inner product $S_{ij} := z_i^T z_j$. To weight feature pairs better, MS loss is proposed to consider multiple types of similarity. MS loss on a dataset $D$ is formulated as follows:

$$L_{MS}(D) = \frac{1}{|D|} \sum_{i=1}^{|D|} \left[ \frac{1}{\alpha} \log [1 + \sum_{j \in \mathcal{P}_i} e^{-\alpha(S_{ij} - \lambda)}] + \frac{1}{\beta} \log [1 + \sum_{j \in \mathcal{N}_i} e^{\beta(S_{ij} - \lambda)}] \right]$$

(3)

where $\alpha$, $\beta$ and $\lambda$ are hyperparameters and $\mathcal{P}_i$ and $\mathcal{N}_i$ represent the index set of positive and negative samples of $x_i$.\footnote{The positive samples have the same labels as $x_i$, while the negative samples have different labels from $x_i$.}
During inference, we use approximate class means settings. MS loss can be efficiently computed as it does not change with the update of 

\[ \mathcal{L}_{MS} = \{ j | S_{ij} < \min_{y_a=y_i} S_{ik} - \epsilon \} \]

where \( \epsilon \) is another hyperparameter in MS loss. At each iteration we use MS loss on the union of new samples and replay samples \( \mathcal{L}_{MS}(B_n \cup B_r) \) to train the model. The sampling of \( B_r \) and update of \( \mathcal{M} \) are the same as ER-reservoir.

To show the connection between \( \mathcal{L}_{MS} \) and the generative classifier in Eq 4, we conduct some theoretical analyses.

**Proposition 1.** Assume dataset \( \mathcal{D} = \{ (x_i, y_i) \}_{i=1}^{n_0} \) is class-balanced and has \( C \) classes each of which has \( n_0 \) samples. For a generative model \( p(z|y) \), assume \( p(y) \) actually obeys the uniform distribution and \( p(z|y=c) = N(z|\mu_c^D, I) \) where \( \mu_c^D = \frac{1}{n_0} \sum_{i=1}^{n_0} z_i[y_i = c] \). For MS loss assume hard mining in Eq 4 is not employed. Then we have:

\[ \mathcal{L}_{MS} \geq \mathcal{L}_{Gen-Bin} \]  

where \( \geq \) stands for upper than, up to an additive constant \( c \) and \( \mathcal{L}_{Gen-Bin} \) is defined in the following:

\[ \mathcal{L}_{Gen-Bin} = - \frac{1}{n} \sum_{i=1}^{n} \log p(z_i|y = y_i) \]

\[ + \frac{1}{nC} \sum_{i=1}^{n} \sum_{c=1}^{C} \log p(z_i|y = c) \]  

The proof of Proposition 1 is in Appendix. Proposition 1 shows \( \mathcal{L}_{MS} \) is an upper bound of \( \mathcal{L}_{Gen-Bin} \) and thus is an alternative to minimizing \( \mathcal{L}_{Gen-Bin} \). The first term of \( \mathcal{L}_{Gen-Bin} \) aims to minimize the negative log-likelihood of the class-conditional generative classifier, while the second term maximizes the conditional entropy \( H(Z|Y) \) of labels \( Y \) and features \( Z \). It should be noticed \( \mathcal{L}_{Gen-Bin} \) depends on modeling \( p(z|y) \). With uniform \( p(y) \), classifying using \( p(z|y) \) equals to classifying using \( p(z, y) \), and \( H(Z|Y) \) is equivalent to \( H(Z, Y) \), which can be regarded as a regularizer against features collapsing. Thus, \( \mathcal{L}_{Gen-Bin} \) actually optimizes the model in a generative way. The assumptions in Proposition 1 are similar with those in Section III-B about NCM classifier. The difference lies in that NCM classifier uses \( \{ \mu_c^M \} \) computed on replay memory \( \mathcal{M} \) to approximate \( \{ \mu_c^D \} \). Therefore, Proposition 1 reveals that MS loss optimizes the feature space in a generative way and it models \( p(z|y) \) for classification which is consistent with the NCM classifier.

The real class means \( \{ \mu_c^D \} \) depend on all training data and change with the update of \( f(\theta) \) so that are intractable in online settings. MS loss can be efficiently computed as it does not depend on \( \{ \mu_c^D \} \) thus the model can be trained efficiently. During inference, we use approximate class means \( \mu_c^{M} \) to classify. In Figure 1B on 5-task Split CIFAR10 benchmark, we empirically show compared to softmax classifier, on old tasks, our method achieves much higher accuracy and much lower forgetting, which implies MS loss is an effective objective to train the model \( f \) and class mean \( \mu_c^{M} \) of replay memory is a good approximation of \( \mu_c^D \).

With discriminative loss like CE loss, the classifier models a discriminative model \( p(y|x) \). Therefore, if training with discriminative loss and inference with NCM classifier based on the generative model \( p(z|y) \), we can not expect to obtain good results. In the next section, experiments will verify this conjecture. In contrast, the way to train and inference are coincided in proposed generative framework.

**D. A Hybrid Generative/Discriminative Loss**

However, when addressing classification tasks, generative classifier has natural weakness, since modeling joint distribution \( p(x, y) \) is much tougher than modeling conditional distribution \( p(y|x) \) for NNs. Moreover, in preliminary experiments, we found if only trained with MS loss, the NCM classifier’s performance degenerates as the expected number of classes in \( B_n \) at each iteration increases. This phenomenon is attributed to the inadequate ability to learn from new data, instead of catastrophic forgetting. We speculate because the size of \( B_n \) is always fixed to a small value (e.g. 10) in online CIL settings, the number of positive pairs in \( B_n \) decreases as the expected number of classes increases.

To remedy this problem, we take advantage of discriminative losses for fast adaptation in online setting. To this end, we introduce Proxy-NCA (PNCA) [23], a proxy-based DML loss, as an auxiliary loss. For each class, PNCA loss maintains “proxies” as the real feature to utilize the limited data in a minibatch better, which leads to convergence speed-up compared to pair-based DML losses. Concretely, when a new class \( c \) emerges, we assign one trainable proxy \( p_c \in \mathbb{R}^d \) to it. PNCA loss is computed as:

\[ \mathcal{L}_{PNCA}(\mathcal{D}) = -\frac{1}{|\mathcal{D}|} \sum_{(x,y) \in \mathcal{D}} \log \frac{e^{-\|f(x)-p_c\|^2}}{\sum_{c=1}^C e^{-\|f(x)-p_c\|^2}} \]  

[23] suggest all proxies have the same norm \( N_P \) and all features have the norm \( N_F \). The latter satisfies as in MS loss the feature is \( \ell_2 \)-normalized, i.e. \( N_F = 1 \). We also set \( N_P = 1 \) by normalizing all proxies after each SGD-like update. In this way, \( \mathcal{L}_{PNCA} \) is equivalent to a CE loss with \( \ell_2 \)-normalized row vectors of \( \mathbf{W} \) and without bias \( \mathbf{b} \), and thus we use PNCA instead of CE loss to keep utilizing the normalized features of MS loss. Our full training objective is a hybrid of generative and discriminative losses:

\[ \mathcal{L}_{Hybrid} = \mathcal{L}_{MS} + \gamma \mathcal{L}_{PNCA} \]  

where \( \gamma \) is a hyperparameter to control the weight of \( \mathcal{L}_{PNCA} \). In general, generative classifiers have a smaller variance but higher bias than discriminative classifiers, and using such a hybrid loss can achieve a better bias-variance tradeoff [37]. Thus we think introducing the discriminative loss \( \mathcal{L}_{PNCA} \) can reduce the bias of model so that boost the ability to learn from new data.

It should be noticed we train the model with \( \mathcal{L}_{Hybrid}(B_n \cup B_r) \), while we only use NCM classifier in Eq 2 for inference.
In all experiments, we set $\alpha = 2$, $\beta = 50$, $\epsilon = 0.1$ following [22] and $\lambda = 0.5$ which always works well in online CIL settings. We only need to tune hyperparameters $\gamma$ in Eq (8) for different experiment settings.

### IV. Experiments

#### A. Experiment Setup

**Datasets** First, we conduct experiments on Split datasets which are commonly used in CIL and online CIL literature. On Split MNIST and CIFAR10, the datasets are split into 5 tasks each of which comprises 2 classes. On CIFAR100 and miniImageNet with 100 classes, we split them into 10 or 20 tasks. The number of classes in each task is 10 or 5 respectively. For MNIST we select 5k samples for training following [5] and we use full training data for other datasets. To simulate a task-free scenario, task boundaries are not informed during training [30].

To conduct a thorough evaluation in task-free scenarios, we design a new type of data streams. For a data stream with $C$ classes, we assume the length of stream is $n$ and $n_0 = n/C$. We denote $p_c(t)$ as the occurrence probability of class $c$ at time step $t$ and assume $p_c(t) \sim \mathcal{N}(t(2c - 1)n_0/2, n_0/2)$. At each time step $t$, we calculate $p(t) = (p_1(t), \ldots, p_C(t))$ and normalize $p(t)$ as the parameters of a Categorical distribution from which a class index $c_t$ is sampled. Then we sample one data of class $c_t$ without replacement. In this setting, data distribution changes smoothly and there is no notion of task. We call such data streams as *Smooth* datasets. To build Smooth datasets, we set $n = 5k$ on CIFAR10 and $n = 40k$ on CIFAR100 and miniImageNet, using all classes in each dataset. For all datasets, the size of minibatch $B_n$ is 10. In Figure 2 we plot the probability distribution on class at each time step in the data stream generation process for Split CIFAR10 and Smooth CIFAR10. For better comparison, we set the length of data stream of two datasets both 5k. Split CIFAR10 has clear task boundaries and within one task the distribution on class is unchanged and uniform. However, On Smooth CIFAR10, the distribution on class keeps changing and there is no notion of task.

**Baselines** We compare our method against a series of state-of-the-art online CIL methods, including: ER-reservoir, A-GEM, GSS, MIR, GMED, CoPE and ASER. We have briefly introduced them in Section II. Specially, we use GSS-greedy and ASER$_\mu$ which are the best variants in the corresponding paper. For GMED, we evaluate both GMED-ER and GMED-MIR. We also evaluate fine-tune baseline without any CL strategy. For all baselines and our method, the model is trained with 1 epoch, i.e. online CL setting. In addition, the performances of i.i.d online and i.i.d offline are also provided, by training the model 1 and 5 epochs respectively on i.i.d data streams. We reimplement all baselines except ASER$_\mu$, whose results are from the original paper.

**Model** Following [5], the model $f$ is a 2-layer MLP with 400 hidden units for MNIST and a reduced ResNet18 for other datasets. For baselines with ER strategy, the size of replay minibatch $B_n$ is always 10. The budget of memory $M$ is 500 on MNIST and 1000 on others. We use a relatively small budget $M$ to mimic a practical setting. All models are optimized by SGD. The *single-head evaluation* is always used for CIL. More details about datasets, hyperparameter selection, and training settings are in the experiments section.
except on MNIST, where GMED-MIR is competitive with our method. An interesting phenomenon is all existing methods do not have a substantial improvement over ER-reservoir on CIFAR100 and miniImageNet, except ASER. We argue for online CIL problem, we should pay more attention to complex settings. Nevertheless, our method is superior to ASER obviously, especially on CIFAR10 and miniImageNet. Table III shows the forgetting of our method is far lower than other methods based on the softmax classifier, except on MNIST. Figure 4 shows our method is almost consistently better than all baselines during the whole learning processes. More results with various memory sizes can be found in Appendix.

**Ablation Study** We also conduct ablation study about training objective and classifier in Table III. The ER-reservoir corresponds to the first row and our method corresponds to the last row. Firstly, we find for ER-reservoir, replacing softmax classifier with NCM classifier makes a substantial improvement on CIFAR10. However, it has no effect on more complex CIFAR100. Secondly, only using MS loss works very well on CIFAR10 while on CIFAR100 poor ability to learn from new data limits its performance (row 3&6). Lastly, when hybrid loss is used, the NCM classifier is much better than proxy-based classifier (row 5&6), and MS loss is critical for NCM classifier (row 4&6). Note that hybrid loss does not outperform MS loss much on Split-CIFAR10. This is because in Split-CIFAR10, a minibatch of new data contains a maximum of two classes, and thus the positive pairs are enough for MS loss to learn new knowledge well. These results verify our statement in Section 3.2.

**Time Comparison** In Figure 4b, we report the training time of different methods. The training time of our method is only a bit higher than ER-reservoir.

Most baselines, such as GSS, MIR, and GMED, improve ER-reservoir by designing new memory update and sampling strategies which depend on extra gradient computations and thus are time-consuming. The inference costs of softmax classifier and our NCM classifier are displayed in Figure 4b.

**Training and Evaluation Metrics** and evaluation metrics are in Appendix.

### Table II: Average Forgetting of 15 runs on Split datasets. Lower is better. * indicates the results are from the original paper.

| Methods                  | MNIST (5-task) | CIFAR10 (5-task) | CIFAR100 (5-task) | CIFAR100 (10-task) | miniImageNet (10-task) | miniImageNet (20-task) |
|--------------------------|----------------|------------------|-------------------|-------------------|------------------------|------------------------|
| fine-tune                | 99.24±0.09     | 85.45±0.63       | 51.60±0.77        | 65.51±0.78        | 41.12±0.82             | 52.99±0.89             |
| ER-reservoir             | 18.33±1.77     | 52.72±1.90       | 45.94±0.55        | 57.31±0.71        | 36.05±0.78             | 47.70±0.90             |
| A-GEM                    | 89.90±2.02     | 82.80±0.73       | 54.15±0.42        | 67.61±0.53        | 43.31±0.52             | 54.47±0.78             |
| GSS-Greedy               | 15.13±0.99     | 49.96±2.82       | 44.30±0.57        | 53.87±0.54        | 36.17±0.58             | 45.91±0.79             |
| MIR                      | 9.71±1.39      | 44.34±2.65       | 46.52±0.52        | 56.58±0.62        | 36.98±0.78             | 45.84±1.11             |
| GMED-ER                  | 16.21±2.70     | 44.93±1.68       | 46.35±0.50        | 57.76±0.94        | 35.22±1.16             | 45.08±1.28             |
| GMED-MIR                 | 12.52±1.05     | 39.88±2.23       | 46.56±0.65        | 58.14±0.55        | 34.79±1.01             | 45.50±1.49             |
| CoPE                     | 9.51±1.15      | 40.01±1.80       | 36.51±0.86        | 43.82±0.62        | 29.43±0.98             | 40.99±1.02             |
| ASER, *                  | -              | 47.90±1.60       | 45.00±0.70        | -                 | 28.00±1.30             | -                      |
| Ours                     | 9.36±0.37      | 35.37±1.35       | 21.79±0.69        | 27.10±1.10        | 21.26±0.59             | 24.98±0.87             |

### Table III: Ablation study on Split CIFAR10 and 20-task Split CIFAR100. We show the performances of different combinations of losses and inference ways. Dis: Discriminative, Gen: Generative.

| Method                  | CIFAR10          | CIFAR100         | miniImageNet     |
|-------------------------|------------------|------------------|------------------|
| Training                | Inference        |                   |                  |
| CE loss                 | Softmax (Dis)    | 39.88±1.52       | 8.95±0.26        |
| CE loss                 | NCM (Gen)        | 44.46±0.95       | 8.96±0.38        |
| MS loss                 | NCM (Gen)        | 51.72±1.02       | 9.99±0.32        |
| PNCA loss               | NCM (Gen)        | 41.91±1.78       | 9.31±0.58        |
| Hybrid loss             | Proxy (Dis)      | 48.16±1.21       | 7.02±0.75        |
| Hybrid loss             | NCM (Gen)        | 51.84±0.91       | 13.65±0.35       |

### Table IV: Final accuracy of 15 runs on Smooth datasets.

| Method                  | CIFAR10          | CIFAR100         | miniImageNet     |
|-------------------------|------------------|------------------|------------------|
| fine-tune               | 10.02±0.03       | 1.02±0.03        | 1.02±0.04        |
| ER-reservoir            | 20.89±2.07       | 3.84±0.42        | 6.85±0.70        |
| MIR                     | 18.75±2.53       | 4.35±0.53        | 6.09±1.04        |
| GMED-MIR                | 18.78±2.31       | 3.68±0.48        | 7.22±0.81        |
| Ours                    | 34.18±0.81       | 10.54±0.38       | 12.24±0.19       |
| i.i.d online            | 31.23±2.11       | 18.08±0.62       | 17.23±0.42       |
| i.i.d offline           | 31.23±2.11       | 18.08±0.62       | 17.23±0.42       |

**B. Main Results on Split Datasets**

On Split datasets, we use Average Accuracy and Average Forgetting after training all tasks [28] for evaluation, which are reported in Table II and Table III respectively. For each metric, we report the mean of 15 runs and the 95% confidence interval.

In Table I we can find our method outperforms all baselines on all 6 settings. The improvement of our method is significant
C. Results on Task-free Smooth Datasets

In newly designed task-free Smooth datasets, the new classes emerge irregularly and the distribution on class changes at each time step. In Table IV we compare our method with several baselines on three smooth datasets. The metric is final accuracy after learning the whole data stream. We can find these datasets are indeed more complex as fine-tune can only classify correctly on the last class, which is due to the higher imbalance of data streams. For this reason, baselines such as ER-reservoir and MIR degrade obviously compared with split datasets. However, our method performs best consistently.

V. Conclusion

In this work, we tackle with online CIL problem from a generative perspective to bypass logits bias problem in commonly used softmax classifier. We first propose to replace softmax classifier with a generative classifier. Then we introduce MS loss for training and prove theoretically that it optimizes the feature space in a generative way. We further propose a hybrid loss to boost the model’s ability to learn from new data. Experimental results show the significant and consistent superiority of our method compared to existing state-of-the-art methods.

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APPENDIX

A. Proof of Proposition 1

**Proof.** For simplicity, we ignore the coefficient $\frac{1}{2}$ both in $\mathcal{L}_{MS}$ and $\mathcal{L}_{Gen-Bin}$ in the proof. We denote the two parts in the
summation of $\mathcal{L}_{MS}$ as $\mathcal{L}_{MS}^+$ and $\mathcal{L}_{MS}^-$ respectively, i.e.:  
\begin{align}
\mathcal{L}_{MS}^+ &= \sum_{i=1}^{n} \frac{1}{\alpha} \log[1 + \sum_{j \in P_i} e^{-\alpha(S_{ij} - \lambda)}] \\
\mathcal{L}_{MS}^- &= \sum_{i=1}^{n} \frac{1}{\beta} \log[1 + \sum_{j \in N_i} e^{\beta(S_{ij} - \lambda)}]
\end{align}

Without hard mining, for $\mathcal{L}_{MS}^+$ we have:
\begin{align}
\mathcal{L}_{MS}^+ &= \sum_{i=1}^{n} \frac{1}{\alpha} \log[1 + \sum_{j : y_j = y_i} e^{-\alpha(S_{ij} - \lambda)}] \\
&\geq \sum_{i=1}^{n} \frac{1}{\alpha} \log[\sum_{j : y_j = y_i} e^{-\alpha(S_{ij} - \lambda)}] \\
&\geq \sum_{i=1}^{n} \frac{1}{\alpha} \left[ \frac{1}{n_0} \sum_{j : y_j = y_i} \log e^{-\alpha(S_{ij} - \lambda)} \right] \\
&\quad \uparrow \text{Jensen’s inequality} \\
&= \sum_{i=1}^{n} \frac{1}{n_0} \left[ \sum_{j : y_j = y_i} -(S_{ij} - \lambda) \right] \\
&= -\sum_{i=1}^{n} \frac{1}{n_0} \sum_{y_j = y_i} S_{ij}
\end{align}

where $\sum$ stands for equal to, up to an additive constant. For $\mathcal{L}_{MS}^-$ we can write:
\begin{align}
\mathcal{L}_{MS}^- &= \sum_{i=1}^{n} \frac{1}{\beta} \log[1 + \sum_{j \neq y_i} e^{\beta(S_{ij} - \lambda)}] \\
&\geq \sum_{i=1}^{n} \frac{1}{\beta} \log[\sum_{j \neq y_i} e^{\beta(S_{ij} - \lambda)}] \\
&\geq \sum_{i=1}^{n} \frac{1}{\beta} \left[ \frac{1}{n_0(C - 1)} \sum_{j \neq y_i} \log e^{\beta(S_{ij} - \lambda)} \right] \\
&\quad \uparrow \text{Jensen’s inequality} \\
&= \sum_{i=1}^{n} \frac{1}{n_0(C - 1)} \sum_{j \neq y_i} S_{ij}
\end{align}

According to Eq (10) and Eq (11), we have:
\begin{align}
\mathcal{L}_{MS} \geq \sum_{i=1}^{n} \left\{ -\frac{1}{n_0} \sum_{y_j = y_i} S_{ij} + \frac{1}{(C - 1)n_0} \sum_{j \neq y_i} S_{ij} \right\}
\end{align}

Now we consider $\mathcal{L}_{Gen-Bin}$. Firstly, it can be written:
\begin{align}
\mathcal{L}_{Gen-Bin} &= \sum_{i=1}^{n} \left\{ -\log p(z_i | y = y_i) \right\} \\
&\quad + \frac{1}{C-1} \sum_{c=1, c \neq y_i}^{C} \log p(z_i | y = c) \\
&= \sum_{i=1}^{n} \left\{ -\log p(z_i | y = y_i) \right\} + \frac{1}{C-1} \sum_{c=1, c \neq y_i}^{C} \log p(z_i | y = c)
\end{align}

For convenience, we denote the features of data samples whose labels are $c$ as $\{\mathbf{z}_c\}_{i=1}^{n_0}$. For the first part in the right hand of Eq (13), we have:
\begin{align}
\sum_{i=1}^{n} -\log p(z_i | y = y_i) \\
&= \frac{1}{2} \sum_{i=1}^{n} \|z_i - \mu_c^D\|_2^2 > p(\mathbf{z} | y = c) = \mathcal{N}(\mathbf{z} | \mu_c^D, \mathbf{I}) \\
&= \frac{1}{2} \sum_{c=1}^{C} \sum_{i=1}^{n_0} \|z_{e_c} - \mu_c^D\|_2^2 \\
&= \frac{1}{2} \sum_{c=1}^{C} \sum_{i=1}^{n_0} \left\{ \|z_{e_c}\|_2^2 - 2z_{e_c}^T \mu_c^D + \|\mu_c^D\|_2^2 \right\} \\
&= \frac{1}{2} \sum_{c=1}^{C} \sum_{i=1}^{n_0} \left\{ \|z_{e_c}\|_2^2 - 2\|\mu_c^D\|_2^2 \right\} > \|\mathbf{z}\|_2^2 = 1 \\
&= \frac{1}{2} \sum_{c=1}^{C} \sum_{i=1}^{n_0} \|\mu_c^D\|_2^2 \\
&= \frac{1}{2} \sum_{c=1}^{C} \sum_{i=1}^{n_0} \sum_{j=1}^{n_0} -\frac{1}{n_0} z_{e_c}^T z_{e_c} \\
&= -\sum_{i=1}^{n} \sum_{j : y_j \neq y_i} \frac{1}{n_0} S_{ij}
\end{align}

For the second part in the right hand of Eq (13), we have:
\begin{align}
\sum_{i=1}^{n} \sum_{c=1, c \neq y_i}^{C} \log p(z_i | y = c) \\
&= \frac{1}{2} \sum_{i=1}^{n} \sum_{c=1, c \neq y_i}^{C} -\|z_i - \mu_c^D\|_2^2 \\
&= \frac{1}{2} \sum_{c=1}^{C} \sum_{i=1, c \neq y_i}^{n_0} \left\{ -\|z_i\|_2^2 + 2z_i^T \mu_c^D - \|\mu_c^D\|_2^2 \right\} \\
&= \frac{1}{2} \sum_{c=1}^{C} \sum_{i=1, c \neq y_i}^{n_0} \left\{ 2z_i^T \mu_c^D - (C - 1)n_0 \sum_{c=1}^{C} \|\mu_c^D\|_2^2 \right\} \\
&= \frac{1}{2} \sum_{c=1}^{C} \sum_{i=1, c \neq y_i}^{n_0} \left\{ 2z_i^T \mu_c^D - (C - 1)n_0 \sum_{c=1}^{C} \|\mu_c^D\|_2^2 \right\} > \|\mathbf{z}\|_2^2 = 1 \\
&= \frac{1}{2} \sum_{c=1}^{C} \sum_{i=1, c \neq y_i}^{n_0} \left\{ 2z_i^T \mu_c^D - (C - 1)n_0 \sum_{c=1}^{C} \|\mu_c^D\|_2^2 \right\} \\
&= \sum_{i=1}^{n} \sum_{j : y_j \neq y_i} \frac{1}{n_0} S_{ij} - \sum_{j : y_j = y_i} \frac{C - 1}{2n_0} S_{ij}
\end{align}

According to Eq (14) and Eq (15), we have:
\begin{align}
\mathcal{L}_{Gen-Bin} &= \sum_{i=1}^{n} \left\{ -\log p(z_i | y = y_i) \right\} + \frac{1}{C-1} \sum_{c=1, c \neq y_i}^{C} \log p(z_i | y = c) \\
&= \sum_{i=1}^{n} \left\{ -\log p(z_i | y = y_i) \right\} + \frac{1}{C-1} \sum_{c=1, c \neq y_i}^{C} \log p(z_i | y = c)
\end{align}
According to Eq (12) and Eq (16), we can obtain:
\[
L_{MS} \geq L_{Gen-Bin}
\] (17)

B. Details about Split Datasets

In Table V we show the detailed statistics about Split datasets. On MNIST [38], we randomly select 500 samples of the original training data for each class as training data stream, following previous works [4, 5, 30]. On CIFAR10 and CIFAR100 [39], we use the full training data from which 5% samples are regarded as validation set. The original miniImageNet dataset is used for meta learning [40] and 100 classes are divided into 64 classes, 16 classes, 20 classes respectively for meta-training, meta-validation and meta-test respectively. We merge all 100 classes to conduct class-incremental learning. There are 600 samples per class in miniImageNet. We divide 600 samples into 456 samples, 24 samples and 120 samples for training, validation and test respectively. We do not adopt any data augmentation strategy.

### Table V: Details about Split datasets.

| Dataset | MNIST | CIFAR10 | CIFAR100 | miniImageNet |
|---------|-------|---------|----------|--------------|
| Image Size | (1,32,32) | (3,32,32) | (3,32,32) | (3,84,84) |
| #Classes | 10 | 10 | 10 | 100 |
| #Train Samples | 5000 | 47500 | 47500 | 45600 |
| #Valid Samples | 10000 | 2500 | 2500 | 2400 |
| #Test Samples | 10000 | 10000 | 10000 | 12000 |
| #Task | 5 | 5 | 10 or 20 | 10 or 20 |
| #Classes per Task | 2 | 2 | 10 or 5 | 10 or 5 |

C. Details about Smooth Datasets

For a data stream with $C$ classes, we assume the length of stream is $N$ and $n_0 = \frac{N}{C}$. We denote $p_c(t)$ as the occurrence probability of class $c$ at time step $t$ and assume $p_c(t) \sim N\left(t\left(\frac{2-1}{n_0}, \frac{n_0}{\tau}\right) \right)$. At each time step $t$, we calculate $p(t) = (p_1(t), ..., p_C(t))$ and normalize $p(t)$ as the parameters of a Categorical distribution from which a class index $ct$ is sampled. Then we sample one data of class $ct$ without replacement. In this setting, data distribution changes smoothly and there is no notion of task. We call such data streams Smooth datasets.

Here we show the characteristics of Smooth datasets in detail. In Table VI we list the detailed statistics about Smooth datasets. Due to the randomness in the data stream generation process, the number of samples of each class is slightly imbalanced. On CIFAR100 and miniImageNet we set the mean number of samples of each class $n_0 = 400$ to avoid invalid sampling when exceeding the maximum number of sample of one class in the original training set. The last rows of Table VI show the range of number of samples of each classes in our experiments. For example, in 15 repeated runs, 15 different Smooth CIFAR10 data streams are established. The number of samples of each class is in the interval [464, 536]. It should be emphasized that in all experiments we use the same random seed to obtain the identical 15 data streams for fair comparison.

### Table VI: Details about Smooth datasets.

| Dataset | CIFAR10 | CIFAR100 | miniImageNet |
|---------|---------|----------|--------------|
| #Classes (C) | 10 | 100 | 100 |
| #Train Samples (length of data stream) ($\alpha$) | 5000 | 46000 | 40000 |
| #Valid Samples | 2500 | 2500 | 2400 |
| #Test Samples | 10000 | 10000 | 12000 |
| Mean number of samples of one class ($n_0$) | 500 | 400 | 400 |
| Minimum number of samples of one class | 464 | 357 | 354 |
| Maximum number of samples of one class | 536 | 450 | 446 |

D. Hyperparameter Selection

In our method, although there are several hyperparameters in $L_{MS}$ and $L_{PNCA}$, we only need to tune $\gamma$ which is the weight of $L_{PNCA}$ in $L_{Hybrid}$. The value of other hyperparameters in $L_{Hybrid}$ is fixed as stated in the main text. We select $\gamma$ from $[0, 0.1, 0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 2.0]$. Especially, $\gamma = 0$ makes $L_{Hybrid}$ become $L_{MS}$.

We follow previous works to use SGD optimizer in all experiments. However, we find compared to CE loss, a wider range of learning rate $\eta$ should be searched in for DML losses. Previous works are based on CE loss and always set $\eta = 0.05$ or $\eta = 0.1$ [4, 5, 11, 30]. We find the optimal $\eta$ for $L_{Hybrid}$ is often larger than 0.1. However, when a larger $\eta$ (e.g. 0.2 on MNIST and CIFAR10) is used, baselines based on CE loss will degrade obviously because of the unstable results over multiple runs. Thus, for baselines we select $\eta$ from $[0.05, 0.1, 0.15, 0.2]$. For our method, we select $\eta$ from $[0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5]$. The hyperparameters of our method are displayed in Table VII.

### Table VII: Hyperparameters of PNCA loss weight $\gamma$ and learning rate $\eta$ for our method on different datasets.

| Dataset | Split Datasets | Smooth Datasets |
|---------|---------------|----------------|
| MNIST | CIFAR10 | CIFAR100 | miniImageNet | MNIST | CIFAR10 | CIFAR100 | miniImageNet |
| $\gamma$ | 0.1 | 0.1 | 0.5 | 1 | 0.1 | 1.0 | 1.0 |
| $\eta$ | 0.05 | 0.2 | 0.35 | 0.2 | 0.25 | 0.5 | 0.2 |

E. Evaluation Metrics

Following previous online CL works [5, 11], we use Average Accuracy and Average Forgetting [28] on Split datasets after training all $T$ tasks. Average Accuracy evaluates the overall performance and Average Forgetting measures how much learned knowledge has been forgotten. Let $a_{k,j}$ denote the accuracy on the held-out set of the $j$-th task after training...
the model on the first $k$ tasks. For a dataset comprised of $T$ tasks, the Average Accuracy is defined as follows:

$$ A_T = \frac{1}{T} \sum_{j=1}^{T} a_{T,j} $$  \hspace{1cm} (18)$$

and Average Forgetting can be written as:

$$ f_j^T = \max_{t \in \{1, \ldots, T-1\}} a_{t,j} - a_{T,j}, \forall j < T $$  \hspace{1cm} (19)$$

$$ F_T = \frac{1}{T-1} \sum_{j=1}^{T-1} f_j^T $$  \hspace{1cm} (20)$$

We report Average Accuracy and Average Forgetting in the form of percentage. It should be noted that a higher Average Accuracy is better while a lower Average Forgetting is better.

On Smooth datasets, as there is no notion of task, we evaluate the performance with the accuracy on the whole held-out set after finishing training the whole training set.

F. Code Dependencies and Hardware

The Python version is 3.7.6. We use the PyTorch deep learning library to implement our method and baselines. The version of PyTorch is 1.7.0. Other dependent libraries include Numpy (1.17.3), torchvision (0.4.1), matplotlib (3.1.2) and scikit-learn (0.22). The CUDA version is 10.2. We run all experiments on 1 NVIDIA RTX 2080ti GPU. We will publish our codes once the paper is accepted.

G. Results with Different Memory Sizes

In Figure 5, we report the performance of our method and ER-reservoir with different memory sizes. Our method outperforms ER-reservoir consistently which shows broad applicability of our method. The improvements are relatively small when memory size is 500 on CIFAR100 and miniImageNet, which is due to the fact that when size of replay memory $M$ is too small (5 samples per class on average), class mean $\mu_c^M$ cannot approximate the real class mean $\mu_c^D$ well and thus proposed NCM classifier degrades.

![Fig. 5: Average Accuracy on valid set of ER-reservoir and our method with different size of $M$.](image-url)