Dispersion relations and the $\Delta$ contributions into the amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}$ from new VPI partial-wave analysis of pion photoproduction

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Within fixed-t dispersion relations the results of new VPI partial-wave analysis for the multipole amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}$ are successfully described, and the resonance and nonresonance contributions into these amplitudes are separated in correspondence with the interpretation on the language of diagram approach, dynamical models and effective Lagrangian approach. The amplitudes $A_{\Delta}^{3/2}$ and $A_{\Delta}^{1/2}$ corresponding to the $\Delta$ contributions into $M_{1+}^{3/2}, E_{1+}^{3/2}$ are obtained. They are in better agreement with quark model predictions than the amplitudes extracted without subtraction of the nonresonance contributions in $M_{1+}^{3/2}, E_{1+}^{3/2}$. The obtained value of the ratio $E2/M1$ for the $\gamma N \rightarrow P_{33}(1232)$ transition is: $E2/M1 = -0.022 \pm 0.004$.

I. INTRODUCTION

It is known that the investigation of the transition $\gamma N \rightarrow P_{33}(1232)$, using the experimental data on the pion photoproduction on the nucleons, is connected with the problem of separation of the resonance and nonresonance contributions in the multipole amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}$, which carry information on this transition. These amplitudes may contain significant nonresonance contributions, the fact which was clear with obtaining the first accurate data on the amplitude $E_{1+}^{3/2}$. The energetic behaviour of this amplitude, in fact, is incompatible with the resonance behaviour. The first investigations of this problem have shown that it is closely related to the problem of fulfilment of the unitarity condition, which for photoproduction multipole amplitudes (let us denote them as $M(W)$) in the $P_{33}(1232)$ resonance region means the fulfilment of the Watson theorem:

$$M(W) = \exp[i\delta(W)] |M(W)|.$$  \hspace{1cm} (1.1)

Here $\delta$ is the phase of the corresponding $\pi N$ scattering amplitude:

$$h(W) = \sin[\delta(W)] \exp[i\delta(W)].$$  \hspace{1cm} (1.2)

There are different approaches for the extraction of an information on the $\gamma N \rightarrow P_{33}(1232)$ transition from the pion photoproduction data with different forms of the unitarization of the multipole amplitudes. These approaches can be subdivided into the following groups: the phenomenological approaches including the approaches based on the K-matrix formalism, the effective Lagrangian (EL) approaches with different phenomenological form of the unitarization of the amplitudes, the dynamical models (DM), and the approaches based on the fixed-t dispersion relations.

In Refs. it was shown that fixed-t dispersion relations used within the approach of Refs. can be useful for the separation of the resonance and nonresonance contributions in the amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}$. In Sec.2 we specify this separation, making correspondence between the contributions in EL approaches and DM and the solutions of the integral equations for $M_{1+}^{3/2}, E_{1+}^{3/2}$, which follow from dispersion relations for these amplitudes within the approach of Refs. These solutions are used in Sec.3 as the input for the description of the results of VPI partial-wave analysis for $M_{1+}^{3/2}, E_{1+}^{3/2}$, and for separation of the resonance and nonresonance contributions in these amplitudes.
II. CORRESPONDENCE BETWEEN CONTRIBUTIONS IN DISPERSION RELATION, EFFECTIVE LAGRANGIAN AND DYNAMICAL APPROACHES

The results of EL approaches and DM can be interpreted on the diagram language, which is most suitable for comparison with the predictions of existing models, because current hadron models and approaches (quark model, bag model, QCD sum rules ...) operate only with vertices and can not predict the whole amplitudes of the processes. In EL approaches and DM the amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}$ are described in terms of the diagrams corresponding to the $N$ exchange in the $u$-channel, $\Delta$ exchange in the $s$- and $u$-channels, and $\pi$ and $\omega$ exchanges in the $t$-channel. The proper phase of the amplitudes is obtained via taking into account final state interaction. This procedure is carried out in EL models phenomenologically using the Olsson [3], Noelle [29] and K-matrix approaches. In DM the unitarization of the amplitudes is made within some method for calculation of the diagrams corresponding to the final state interaction. These calculations are made using different approaches for fulfillment of relativistic and gauge invariance with different methods of cutoff and incorporation of the off-shell effects in the integrals.

Let us denote their contribution into $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$ as $M_{NR}(W)$. In the quantum mechanics, rescattering effects in these amplitudes lead to the following replacement (see Ref. [30], Chapter 9):

$$M_{NR} \rightarrow M_{NR,\text{rescat}} = M_{NR} + \frac{1}{\pi} \frac{1}{D(W)} \int_{W_{thr}}^{\infty} \frac{D(W') h(W') M_{NR}(W')}{W' - W - i\varepsilon} dW' =$$

$$= \exp[i\delta(W)] \left[ M_{NR}(W) \cos\delta(W) + e^{a(W)} r(W) \right], \quad (2.1)$$

where

$$r(W) = \frac{P}{\pi} \int_{W_{thr}}^{\infty} \frac{e^{-a(W')}}{W' - W} \frac{W \delta(W')}{W(W' - W)} dW', \quad (2.2)$$

$$a(W) = \frac{P}{\pi} \int_{W_{thr}}^{\infty} \frac{W \delta(W')}{W(W' - W)} dW'. \quad (2.3)$$

In Eq.(2.1) it is supposed that the unitarity condition (1.1) can be used in the whole range of integration, $D(W)$ is the Jost function:

$$1/D(W) = \exp \left[ \frac{W}{\pi} \int_{W_{thr}}^{\infty} \frac{\delta(W')}{W'(W' - W - i\varepsilon)} dW' \right] = \exp[i\delta(W)] e^{a(W)}. \quad (2.4)$$

The contributions analogous to both terms in Eqs. (2.1),(2.2) exist in all DM. These models reproduce exactly the first term in Eq. (2.3), second one being model dependent and different in different models. In EL approaches the unitarization made via the Noelle and K-matrix ansatzes corresponds to taking into account only the first term in Eq. (2.2) (see Ref. [13]). The unitarization via the Olsson ansatz in these approaches has no analogy with the above formulas. It is interesting that just the first term in Eq. (2.2) determines the nonresonance behaviour of the multipole amplitude $E_{1+}^{3/2}$ (see below the curve 5 in Fig. 2).

In the absence of background contribution into $E_{1+}^{3/2}$, incorporation of the $\pi N$ rescattering in the resonance parts ($M^R$) of the amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}$ leads in the vicinity of $\Delta$ to the following replacements:

$$M^R = \frac{f_{\pi N, \Delta} f_{\Delta, \gamma N}}{s - m_0^2 + i\Gamma_0} \rightarrow \frac{f_{\pi N, \Delta} f_{\Delta, \gamma N}}{s - m_\Delta^2 - im\Delta} = \frac{f_{\pi N, \Delta} f_{\Delta, \gamma N}}{m_\Delta\Gamma_\Delta} \sin\delta_Re^{i\delta_R}. \quad (2.6)$$
Here \( f_{xN, \Delta}, f_{\Delta, \gamma N} \), \( \Gamma_\Delta \) and \( m_\Delta \) are dressed vertices and \( \Delta \) width and mass; the corresponding values containing "0" are bare ones.

The modification of \( M^R \), due to the presence of the background contribution in \( \delta^{3/2}_{1+} \), can be taken into account only phenomenologically. One can estimate the magnitude of this modification using the results of Ref. \[13\] obtained within the Noelle and K-matrix forms of the unitarization of the amplitudes. At the resonance position, where \( \delta^{3/2}_{1+} = 90^\circ \) (for the phase shift analysis of Refs. \[31,32\], it is \( W_R = 1.229 \text{ GeV} \)), the unitarization of \( M^R \) within these methods leads to the same results; namely, in Eq. \( (2.6) \) the replacement \( e^{i\delta_R} \rightarrow e^{i\delta^{3/2}_{1+}} \) should be made, \( \sin \delta_R \) being equal to 1 in the K-matrix approach and to 0.97 in the Noelle approach. This difference in 3\% we will consider as the uncertainty of \( M^R \) coming from the incorporation of the background contribution into \( \delta^{3/2}_{1+} \) at \( W = W_R \).

Let us turn now to the dispersion relations. Dispersion relations for multipole amplitudes follow from dispersion relations for invariant amplitudes, defined in accordance with the hadron current, which obeys the requirements of the relativistic and gauge invariance and the crossing invariance under the replacement \( s \leftrightarrow u \). Let us write these dispersion relations for the multipole amplitudes \( M^{3/2}_{1+}, E^{3/2}_{1+} \) in the form:

\[
M(W) = M^B(W) + M^{\text{high}}(W) + \frac{1}{\pi} \int_{W_{\text{thr}}}^{W_{\text{max}}} \frac{h^*(W') M(W')}{W' - W - i\varepsilon} dW' + \frac{1}{\pi} \int_{W_{\text{thr}}}^{W_{\text{max}}} K(W, W') h^*(W') M(W') dW',
\]

(2.7)

where we have divided dispersion integrals into two parts: from threshold up to \( W_{\text{max}} = 1.55 \text{ GeV} \) (the region which is dominated by the \( \Delta \) contribution), and from \( W_{\text{max}} \) up to \( \infty \). Such division of the dispersion integrals, with the consideration of \( M^{\text{high}}(W) \) as a nonsingular function, is possible only in the case, if \( h(W) \rightarrow 0 \) when \( W \rightarrow W_{\text{max}} \). This condition was not taken into account in Ref. \[23\]. By this reason the solutions of integral equations, obtained in \[23\], are divergent at \( W \rightarrow W_{\text{max}} \).

For the multipole amplitudes \( M^{3/2}_{1+}, E^{3/2}_{1+} \) one can introduce the condition: \( h(W) \rightarrow 0 \) when \( W \rightarrow W_{\text{max}} \), at \( W_{\text{max}} \approx 1.55 \text{ GeV} \), because at \( W = 1.5 \text{ GeV} \) we have \( \delta^{3/2}_{1+} = 164^\circ \) \[31,32\]. From the \( \pi N \) phase shift analyses (see, for example, \[31,32\]) it is known that the amplitude \( h^{3/2}_{1+} \) is elastic in the first integration region; by this reason in the integrals over the region \( (W_{\text{thr}}, W_{\text{max}}) \) the imaginary parts of the multipoles amplitudes are written in the form: \( \text{Im} M(W) = h^*(W) M(W) \), which follow from Eqs. \( (1.1,1.2) \). Therefore, the dispersion relations \( (2.7) \) for the amplitudes \( M^{3/2}_{1+}, E^{3/2}_{1+} \) can be considered as integral equations for these amplitudes in the region \( (W_{\text{thr}}, W_{\text{max}}) \).

In Eq. \( (2.7) \), \( M^B(W) \) is the Born term which corresponds to the \( N \) and \( \pi \) exchanges, with pseudoscalar coupling for the \( NN\pi \) vertex. The corresponding term in EL approaches and DM is obtained using pseudovector coupling for this vertex; it differs from the Born contribution by the nonsingular term which contributes only into the \( B_{1^{+},0^+} \) Ball amplitude:

\[
B_{1^{+},0^+}(s, t) = \frac{g_{e^2/4}}{4m_N^2} g^{(v, s)},
\]

(2.8)

where \( m_N \) is the nucleon mass, and

\[
e^2/4\pi = 1/137, \quad g^2/4\pi = 14.5, \quad g^{(e)} = 3.7, \quad g^{(s)} = -0.12.
\]

(2.9)

The contribution of this term into our final results is negligibly small. \( K(W, W') \) is a nonsingular kernel arising from the \( u\)-channel contribution into the dispersion integral and the nonsingular part of the \( s\)-channel contribution. In the integrand of the relation \( (2.7) \), we did not write the couplings of \( M(W) \) to other multipoles; by our estimations their contributions into our final results are negligibly small.

The values of the high energy integrals in Eq. \( (2.7) \) can be evaluated using the results of analyses of pion photoproduction on nucleons at high energies. In our estimations we have used the results obtained in Ref. \[23\], where different variants of the description of these data are considered within the approach based on the Regge poles and cuts. Our estimations have shown that the high energy integrals in Eq. \( (2.7) \) can be roughly approximated by the \( \omega \) exchange, which contributes to the following Ball amplitudes:

\[
B_{6}^{(+)} = \frac{2g_{\omega \gamma N} g_{\omega NN}}{t - m_\omega^2}, \quad B_{1}^{(+)} = m_N B_{6}^{(+)},
\]

(2.10)
where $m_\omega$ is the $\omega$ mass, and $g_{\gamma\omega\pi}$ is related to the $\omega \rightarrow \pi\gamma$ decay width by:

$$\Gamma(\omega \rightarrow \pi\gamma) = \frac{g_{\gamma\omega\pi}^2 k^3}{12\pi}, \quad (2.11)$$

$k$ is the pion 3-momentum in the $\omega$ rest frame. From the data on $\Gamma(\omega \rightarrow \pi\gamma)$ [33] we get $g_{\gamma\omega\pi} = 0.73$ GeV$^{-1}$. In Eq. (2.10) we have presented only the contribution corresponding to the vector coupling in the vertex $\omega NN$, because the role of the tensor $\omega NN$ coupling in our final results is negligibly small. For the vector coupling constant we have: $g_{\omega NN} = 8 - 14$ [33]. The results presented below in Figs. 1,2 correspond to the mean value of $g_{\omega NN}$ in this interval.

At $K(W,W') = 0$, the integral equation (2.7) has a solution in an analytical form (see Refs. [26,27] and the references therein):

$$M_{K=0}(W) = M_{part,K=0}^B(W) + c_M M_{hom}^K(W). \quad (2.12)$$

Here $M_{part,K=0}^B(W)$ is the particular solution of Eq. (2.7) generated by $M^B$ and $M^\omega$. It is described by Eqs. (2.1) with the replacement $M^{NR} \rightarrow M^B + M^\omega$. With this, in all integrals of Eqs. (2.1)-(2.5) at $W' > W_{max}$, one should take $\delta(W') = \pi$. So, $M_{part,K=0}^B(W)$ reproduces the nonresonance contributions into the amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}$ generated by the $N, \pi$ and $\omega$ exchanges, when the final state interaction, caused by the $\pi N$ rescattering in the $\Delta$ region, is taken into account in accordance with Eq. (2.1).

$M_{hom}^{K=0}(W) = 1/D(W)$ is the solution of the homogeneous equation, which follow from (2.7) at $M^B = M^\omega = 0$. It enters Eq. (2.12) with an arbitrary weight, i.e. multiplied by an arbitrary constant $c_M$. If, following EL approach and DM, we describe the amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}$ in terms of the contributions corresponding to the $N, \Delta, \pi$ and $\omega$ exchanges, then $c_M M_{hom}$ should be considered as the $\Delta$ contribution. In order to obtain the contribution, corresponding to the $\Delta$ exchange in the $s$-channel, one should subtract from $c_M M_{hom}$ the contribution of the $\Delta$ exchange in the $u$-channel. Using final results for the contributions of $c_M M_{hom}$ into $M_{1+}^{3/2}, E_{1+}^{3/2}$, one can estimate this contribution. It appeared that the $\Delta$ contribution, corresponding to the $u$-channel, is negligibly small in comparison with $c_M M_{hom}$ and $M_{part,K=0}^B(W)$. By this reason, the $\Delta$ contribution in the $s$-channel we identify with $c_M M_{hom}$.

Let us note, that our final results correspond to the solutions of the integral equations (2.7) with $K(W,W') \neq 0$, i.e. they satisfy the requirement of the crossing invariance. These solutions were obtained numerically, using the formulas for the amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}$ presented in details in Ref. [24]. At $W_{thr} < W < 1.5$ GeV, the phase $\delta_{1+}^{3/2}$ was taken in the analytical form

$$\sin^2 \delta_{1+}^{3/2} = \frac{(4.27 q^3)^2}{(4.27 q^3)^2 + (q^2 - q_0^2)^2[1 + 40(q^2 - q_0^2)^2 + 21.4q^2]^2}, \quad (2.13)$$

which describe well the experimental data from [31,32] with $q_0 = 0.225$ GeV; $q$ is the 3-momentum of the pion in the $\pi N$ c.m.s.

### III. RESULTS AND DISCUSSION

In this Section we present our results on the description of the data for the multipole amplitudes $M_{1+}^{3/2}, E_{1+}^{3/2}$ which are extracted with high accuracy from existing experimental data in the partial- wave analysis of Ref. [28]. In the dispersion relation approach, presented in the previous Section, these data should be described as sums of the particular and homogeneous solutions of the integral equations (2.7) for the amplitudes $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$. The particular solutions have definite magnitudes fixed by $M^B$ and $M^\omega$, i.e. by the $N, \pi$ and $\omega$ contributions into $M_{1+}^{3/2}, E_{1+}^{3/2}$. The solutions of the homogeneous parts of the integral equations (2.7) with $M^B = M^\omega$, have definite shapes, fixed by the integral equations, and arbitrary weights. These weights are the only unknown parameters which should be found from the requirement of best description of the data on $M_{1+}^{3/2}, E_{1+}^{3/2}$. For this aim we have used fitting procedure.
The obtained results together with the data from Ref. [28] are presented in Figs.1,2. In order to demonstrate the role of different contributions, they are presented in these figures separately.

The curves 4 and 6 are the particular solutions of Eq. (2.7) generated by \( M^B \) and \( M^c \), respectively. They represent the nonresonance contributions into \( M_1^{3/2}, E_1^{3/2} \), caused by the \( N, \pi \) and \( \omega \) exchanges. The curves 5 represent the first term in Eq. (2.2) with \( M^{NR} = M^B \). They are given in order to demonstrate the difference between the nonresonance contributions, generated by the Born term in the EL aproach of Ref. [13] and our approach. This difference is caused by the second term in (2.2); with this term, the nonresonance contributions, generated by the Born term, satisfy dispersion relations.

The curves 3 represent the contributions of the homogeneous solutions, obtained by fitting the weights of these solutions, when the nonresonance contributions are generated by the Born term and \( \omega \) exchange. These curves represent the \( \Delta \) contributions into \( M_1^{3/2}, E_1^{3/2} \). As it was mentioned in Sec.2, our estimations have shown that the \( \omega \)-channel \( \Delta \) contributions are negligibly small in comparison with \( s \)-channel ones. By this reason we identify the contributions of the homogeneous solutions (curves 3) with the \( \Delta \) exchange in the \( s \)-channel.

The summary results are presented by the curves 1, which correspond to the case, when the nonresonance contributions are caused by the \( N, \pi \) and \( \omega \) exchanges. It is seen that the agreement with the VPI data is good for both amplitudes \( M_1^{3/2}, E_1^{3/2} \). In order to demonstrate the role of high energy contributions into dispersion integrals which are approximated in our approach by the \( \omega \) exchange, we present also the curves 2. They are obtained by fitting the homogeneous solutions, when the nonresonance contributions are generated by the Born terms only. It is seen that the \( \omega \) contribution is small; however, in the case of \( M_1^{3/2} \) its role in obtaining the good agreement with experiment is important.

In Table 1 we present the helicity amplitudes \( A_{3/2}^p \) and \( A_{1/2}^p \) and the ratio \( E2/M1 \) for the transition \( \gamma N \rightarrow P_{33}(1232) \), which are obtained from the resonance contributions into \( M_1^{3/2}, E_1^{3/2} \) (the curves 3 in Figs.1,2) at the resonance position. First errors are obtained assuming that the data in Figs.1,2, corresponding to the energy-dependent analysis of Ref. [28] have 2% errors. Second errors come from the uncertainties of the model. They are connected with the cuttof in the dispersion integrals (2.7); with the uncertainties in the \( \omega \) contribution; with neglecting the couplings of the multipole amplitudes with each other in (2.7); and with the uncertainties in the extraction of the resonance amplitudes from the curves 3, discussed in the previous Section.

| Resonance contributions, \( \omega \)-exchange |
|---------------------------------------------|
| \( A_{3/2}^p(10^{-3} \text{Gev}^{-1/2}) \) \( \times 10^3 \) | \( A_{1/2}^p(10^{-3} \text{Gev}^{-1/2}) \) \( \times 10^3 \) | \( E2/M1(\%) \) |
|---------------------------------------------|
| Resonance contributions, our results | \(-110 \pm 2 \pm 6\) | \(-209 \pm 4 \pm 12\) | \(-2.2 \pm 0.1 \pm 0.3\) |
| Total amplitudes, Ref. [28] | \(-135 \pm 5\) | \(-250 \pm 8\) | \(-1.5 \pm 0.5\) |
| Nonrelativistic \( \omega \)-exchange | \(-101\) | \(-175\) | \(0\) |
| Relativistic quark model [36,37] | \(-111\) | \(-207\) | \(-2.1\) |

In Table 1 we present also the results obtained from the total amplitudes \( M_1^{3/2}, E_1^{3/2} \) at the resonance position in Ref. [28]. The amplitudes, extracted in such way, are larger than quark model predictions. As is seen from our results, this disagreement is removed due to taking into account the nonresonance background contributions generated by the \( N, \pi \) and \( \omega \) exchanges.

### Acknowledgments

I am grateful to I.I.Strakovsky for communications and providing the results of the VPI partial-wave analysis in the numerical form. I also acknowledge communications with B.L.Ioffe and O.Hanstein.
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Figure Captions

**Fig. 1** Multipole amplitude \( M_{3+}^{3/2} \). Our results for the imaginary parts of the amplitude (curve 1) in comparison with VPI data [28]: the solid circles represent the results of the energy-dependent analysis, the open circles correspond to the energy-independent analysis. Curve 3 correspond to the resonance contribution. Other contributions are discussed in the text.

**Fig. 2** Multipole amplitude \( E_{3+}^{3/2} \). The legend is as for Fig.1.
