Asymmetric Sneutrino Dark Matter in the NMSSM with Minimal Inverse Seesaw

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Abstract

We dynamically realize the supersymmetric inverse seesaw mechanism in the next to the minimal supersymmetric standard model (NMSSM) in a minimal form. The crucial observation is that the inclusion of a dimension-five operator $\bar{N}_2 S^2 / M^*$, which violates lepton number and is suppressed by the fundamental scale $M^*$, can readily produce a light neutrino mass scale $m_\nu \sim 0.01$ eV. This model is protected by $Z_4^R \times Z_2^M$ discrete symmetry and a very predictive parameter space is restricted by the requirement of naturalness. Then we study some interesting phenomenological consequences: (i) A light sneutrino ($\sim 8$ GeV), which is the lightest supersymmetric particle (LSP) and then the dark matter candidate, can elegantly explain the DAMA/CoGeNT results and even account for the FERMI-LAT gamma ray signals; (ii) It is possible that the sneutrino is an asymmetric dark matter and thus the $\Omega_{DM} : \Omega_b \approx 5 : 1$ naturally predicts a light dark matter; (iii) Higgs physics may dramatically changed due to the significant coupling $y_N LH_u N$ and the light sneutrinos out of the inverse seesaw sector, consequently the lightest Higgs boson is likely dominantly decay to a pair of light sneutrinos. In the DAMA/CoGeNT region, the Higgs typically decays to a pair of LSPs with branching ratio $> 90\%$.

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I. INTRODUCTION AND MOTIVATION

Theoretically, imposing supersymmetry on the standard model (SM) leads to the minimal supersymmetric standard model (MSSM), which is the most promising way to stabilize the weak scale $M_Z \sim 100$ GeV $\ll M_{Pl}$. But phenomenologically the MSSM suffers the $\mu-$problem and the little hierarchy problem. As a popular solution of these problems, the next to the minimal supersymmetric standard model (NMSSM) [1] has recently attracted much attention. Its superpotential and soft terms for the Higgs sector are given by

$$W_{\text{NMSSM}} = \lambda S H_u \cdot H_d + \frac{\kappa}{3} S^3,$$

$$-\mathcal{L}_{\text{soft}} = m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 + \left( \lambda A\lambda S H_u \cdot H_d + \frac{\kappa}{3} A\kappa S^3 + \text{h.c.} \right),$$

where $H_u \cdot H_d \equiv H_u^1 H_d^2 - H_u^2 H_d^1$ (for simplicity the dot will be omitted hereafter). Once the singlet Higgs field develops a non-zero vacuum expectation value (VEV) $v_s \equiv \langle S \rangle$ and breaks the $Z_3$ symmetry around the TeV scale, the $\mu$ term is generated as $\mu = \lambda v_s$, which gives an elegantly dynamical solution to the $\mu-$problem (and also the $\mu/B\mu$ problem).

However, the NMSSM cannot possibly be the whole story because a tiny non-zero neutrino mass $\lesssim O(0.1)$ eV is needed to be generated. Classically, such a tiny mass parameter can be addressed by the dimension-five operators $(H_u L_i)(H_u L_j)/\Lambda$, with $\Lambda \sim 10^{14}$ GeV. To understand the UV origin of these high-dimensional operators, the elegant seesaw mechanism is proposed [2]:

$$W_{\text{seesaw}} = y_{ij}^N H_u L_i N_j + M_{ij}^N N_i N_j^\dagger.$$ 

In this mechanism, three right-handed neutrinos (RHNs) with large Majorana masses $M_{ij}^N \sim \Lambda$ (the seesaw scale) are introduced. Although this mechanism is attractive for its simplicity and well motivated in the $SO(10)$ grand unification theory (GUT), generically speaking it is not testable at the future colliders. If the seesaw scale is lowered to TeV, the RHNs will have very weak Yukawa couplings $y_{ij}^N \sim 10^{-6}$ and thus it is still irrelevant for collider experiments (note, however, in the NMSSM with $SN^2$ coupling to account for the TeV-scale seesaw, the RHNs may be accessible [3]). Moreover, the origin of such an intermediate seesaw scale remains a puzzle.

The inverse seesaw [4] can lower the seesaw scale while keep the large Yukawa couplings for the left-handed neutrinos. The superpotential is given by

$$W_{\text{IVS}} = y_N H_u L N + M_N N \tilde{N} + \frac{M_N^\kappa}{2} \tilde{N}^2.$$ 

Here, for simplicity, we neglect the family indices. The light neutrinos are the three lightest
eigensates of the 9×9 mass matrix, which, in the basis \((\nu_L, N, \bar{N})\), is given by

\[
M_{\text{inverse}} = \begin{pmatrix}
0 & m_T^D & 0 \\
m_D & 0 & M_N^L \\
0 & M_N & M_{\bar{N}}
\end{pmatrix},
\]  

(5)

where each block denotes a 3 × 3 matrix with Dirac neutrino mass matrix \(m_D = Y_N \langle H^0_u \rangle\).

In this paper, to illustrate our main idea, we consider one family of neutrinos for simplicity. Then the effective light neutrino, which is dominated by \(\nu_L\), obtains a mass

\[
m_{\nu}^{\text{eff}} = -\frac{m_D^2}{M_N^2 + m_D^2} M_{\bar{N}}.
\]  

(6)

We are interested in the case with \(M_N\) at the weak scale and \(y_N \sim O(1)\). So, the smallness of neutrino masses are generated by small Majorana masses \(M_{\bar{N}}\). Since this mass term violates lepton number, we see that the small neutrino masses may be ascribed to the degree of lepton number violation.

However, such an inverse seesaw mechanism suffers from a fine-tuning problem, namely the Majorana mass term must be very small. Some efforts have been tried to construct dynamical models to solve this naturalness problem. Commonly, they start from a lepton number \((L)\) conserving model and then the smallness of \(M_{\bar{N}}\) is a result of the spontaneous breaking of \(U(1)_L\) [5]. Usually, some new complicated structures and degrees of freedom are introduced to realize the dynamical goal. In this work we start from a \(L\)–violating theory and present a very simple model to realize the inverse seesaw without introducing any new extra fields or any new ad hoc terms. Thus, our framework is a kind of minimal dynamical realization for the inverse seesaw mechanism.

Our model has rich and interesting phenomenology. First of all, a left-right mixing sneutrino as the lightest supersymmetric particle (LSP) may serve as a dark matter (DM) candidate. In fact, the recent DM direct detection results from DAMA [6] and CoGeNT [7] can be explained by a light DM with a mass around \(\sim 8\) GeV and a rather large DM-nucleon scattering cross section \(\sigma_{SI} \sim 10^{-40} \text{cm}^2\) [8] (a recent interesting work on light DM with isospin violation can recoil DAMA/CoGeNT with other experiments is explored in the [9]). Such a light DM candidate is absent in the MSSM [10], while in the NMSSM its presence requires some fine-tuning in the parameter space [11] (some other exotic extensions allowing for a light DM can be found in [12]). However, in our model, if the light sneutrino LSP has a proper doublet sneutrino component, it can possibly explain the DAMA/CoGeNT signals by the \(Z\)–boson mediated recoil [13]. In some cases, it may have leptonic annihilation into \(\tau^+\tau^-\) with a rate \(\sim 10^{-27} \text{cm}^3/\text{s}\), which, according to the analysis in [14], may be favored by the FGST (FERMI Gamma-Ray Space Telescope) observation of gamma ray in the
region $0.3-100$ GeV (please see some following study in [15]). On the other hand, since the sneutrino LSP carries lepton number, it can be the asymmetric dark matter (ADM) candidate [16–18]. Finally, provided that the sneutrino is so light, the Higgs boson may mainly decay invisibly into a pair of light sneutrinos. Such an interesting phenomenology not only affects the collider physics about Higgs but also may provide a way to alleviate the little hierarchy problem. And in the DAMA/CoGeNT region, the Higgs typically decays to a pair of LSPs with branching ratio $>90\%$. All these phenomenological consequences will be discussed in our analysis.

This work is organized as follows. In Section II a dynamical model for the inverse seesaw is constructed and some discussions about the naturalness of the model are presented. In Section III, we explore some interesting phenomenology for this model. The conclusion is given in Section IV. Some necessary interactions of the sneutrinos are presented in the appendix.

II. MINIMAL DYNAMICAL MODEL FOR THE INVERSE SEESAW

A. The Model and Its Symmetry

As stated in the preceding section, the inverse seesaw mechanism suffers from the fine-tuning problem. To avoid this problem, we now construct a simple dynamical model based on the NMSSM. First, we see from Eq. (6) that with the assumption $m_D \gg M_N$, the light neutrino mass is then an absolute result of the smallness of $M_N \sim \mathcal{O}(0.05) \text{ eV}$. In particle physics, it is very illustrative to get such a small mass scale from some 'fundamental' mass scale, namely

$$\frac{(5 \text{ TeV})^2}{M_*} \simeq 0.01 \text{ eV}, \quad (7)$$

where $M_*$ is the fundamental scale such as the Planck scale or the string scale. In the usual GUTs, the effective fundamental scale can be decrease by a factor about 3 due to the large degrees of freedom at the GUT scale [19]. So $M_*$ can be around $10^{17}$ GeV in principle. Therefore, it naturally guides us to conjecture that $M_N$ may simply open a window to the fundamental scale (UV) physics. In this spirit, we construct the following simple model to dynamically realize the inverse seesaw mechanism

$$W_{DIS} = (y_N L H_u N + \lambda_1 S N \tilde{N}) + \frac{\lambda_2}{4M_*} S^2 \tilde{N}^2, \quad (8)$$

$$-\mathcal{L}_{DIS}^{soft} = m_L^2 |\tilde{L}|^2 + m_N^2 |\tilde{N}|^2 + m_{\tilde{N}}^2 |\tilde{\tilde{N}}|^2 + \left(y_N A_N H_u \tilde{L} \tilde{\tilde{N}} + \lambda_1 A_1 S \tilde{N} \tilde{\tilde{N}} + \text{h.c.} \right). \quad (9)$$
Such a very simple model can provide all the elements we need and it must be the minimal framework to dynamically realize the inverse seesaw mechanism because we do not introduce any new fields (only the NMSSM plus right-handed neutrinos). With the singlet $S$ developing a TeV-scale VEV $v_s$, both $M_N$ and $M_N^\ast$ can be generated

$$M_N = \lambda_1 v_s \sim 100 \text{ GeV}, \quad (10)$$
$$M_N^\ast = \frac{\lambda_2 v_s^2}{M_\ast} \sim 0.01 \text{ eV}, \quad (11)$$

where the operator coefficient $\lambda_2$ is set to be 1. So, $M_N$ can be naturally at weak scale and $M_N^\ast$ can be close to the light neutrino mass scale.

Now, the fermion spectrum contains a light Majorana neutrino with mass given by Eq. (6), whose corresponding eigenstate is

$$\nu_1 \approx \sin \theta_1 \nu_L - \cos \theta_1 \bar{N}, \quad (12)$$

with $\sin \theta_1 \approx M_N/\sqrt{m_D^2 + M_N^2}$. The other two orthogonal Weyl fermions form a Dirac fermion:

$$\nu_{2,3} \approx \frac{1}{\sqrt{2}} \left( \cos \theta_1 \nu_L \pm N + \sin \theta_1 \bar{N} \right), \quad (13)$$
$$M_2 = \sqrt{M_{N}^2 + m_D^2 + O(M_N)} \approx M_N. \quad (14)$$

We make a comment on the dimension-five operator introduced in our model. In fact, this operator closely resembles the classical dimension-five operator in the SM or MSSM:

$$\frac{(LH_u)(LH_u)}{\Lambda} \sim \frac{\tilde{N} \tilde{N} S S}{M_\ast}. \quad (15)$$

The canonical seesaw scale is a somewhat peculiar scale $\Lambda \sim 10^{14}$ GeV because of the VEV $v_u \lesssim 174$ GeV. But by virtue of the inverse seesaw mechanism, the smallness of neutrino mass is transmitted to the small Majorana mass of the extra singlet $\tilde{N}$. Combined with the NMSSM singlet Higgs field $S$ that has a TeV-scale VEV, then we can obtain the neutrino mass scale naturally via the dimension-five operator introduced in our model without invoking new unknown scales.

We make a comment here. The assumption $m_D \gg M_N$ is not realistic since it will lead to the light neutrino dominated by the $\tilde{N}$ component while the major doublet component as heavy as $M_N$. As a result, electroweak phenomenology such as $W, Z$ boson decay will be inconsistent with the current experimental data. On the other hand, if $M_N/m_D$ is too large, $M_N^\ast$ will not be sufficiently generated in our scenario. So in the actual calculation, $M_N \sim 5m_D$ will be assumed.
Note that our dynamical scheme is highly constrained. First, we need the NMSSM with $v_s \gg v$ where $v \equiv \sqrt{\langle H_u^0 \rangle^2 + \langle H_d^0 \rangle^2}$. It is clear that the larger $M_N$, the smaller $m_\nu$. In the fully inverse seesaw limit, $v_s$ takes a value around 5 TeV to produce a neutrino mass scale $m_\nu \sim 0.05$ eV which is the heaviest neutrino mass observed by the neutrino oscillation [20]. As a result, if the little hierarchy problem does not appear, we have $\mu = \lambda v_s \simeq 100$ GeV with $\lambda \sim 10^{-2}$. Further, we are interested in $M_N$ is several hundred GeVs, so typically $\lambda_1 \lesssim 10^{-1}$ and in most cases we shall ignore the role of this parameter.

Since the small neutrino mass scale is simply a relic of fundamental scale physics, this inspires us to investigate the possible model at the fundamental scale. We find that a new $U(1)_R'$ symmetry can guarantee the general form of our model. At the renormalizable level, the supersymmetric model described by Eq. (1) plus Eq. (8) possess an accidental $U(1)_B \times U(1)_L \times U(1)_R$ symmetry with the field charges assigned as

\[
\begin{align*}
L & : \ H_u[0], \ H_d[0], \ S[0], \ L[1], \ E^c[-1], \ N[-1], \ \bar{N}[1], \\
B & : \ H_u[0], \ H_d[0], \ S[0], \ Q[1], \ U^c[-1], \ D^c[-1], \ \bar{N}[0], \\
R & : \ H_u[2/3], \ H_d[2/3], \ S[2/3], \ L[2/3], \ E^c[2/3], \cdots
\end{align*}
\]

(16)

where the dots denote that all other fields carry the same charge $2/3$. In fact, the $U(1)_R$ charge assignment is not fixed according to this superpotential and in the above we simply choose one as an example, which is consistent with $SU(5)$ GUT. Note that the $Z_3$ symmetry simply is an accidental result of $U(1)_R$ symmetry, which forbids the bare mass terms. At the dimension-five level, the operator $S^2 \bar{N}^2$ violates the global symmetry $U(1)_L$ and $U(1)_R$ simultaneously, but still leaves a discrete $Z_2^L \subset U(1)_L$ and a new $U(1)_R'$ invariance:

\[
R' \equiv R - \frac{1}{3} L.
\]

(17)

If $U(1)_R'$ is generated to all orders, then it is not difficult to prove that one of its consequence is that $U(1)_B$ and the matter parity $Z_2^M \equiv (-1)^{3(B-L)}$ conserve to all orders [21].

Some comments are in order. First, we only utilize the universal Abelian $U(1)_L$ charge invariance at the renormalizable model. But $R-$charge does not re-scale, since we fix it to be 2 for the superpotential, not zero, which is quite different from the ordinary $U(1)$. Second, we may directly use the $U(1)_R$ alone to accomplish this task, namely to pick out some required operators and forbid the others, rather than using such a redefined $U(1)_R'$ ( This means the basic symmetry of the whole model is only $U(1)_R \times G_{gauge}$. This is an interesting conjecture, even if taking supersymmetry (SUSY) breaking into account, where the $U(1)_R$ is required [22]. Of course, according to the well known conclusion that no continuous global symmetry is a fundament symmetry, we may finally break it by higher dimension operators). Finally, our model does not conserve the $U(1)_L$ and thus the $U(1)_{B-L}$, which is an essential
difference from the dynamical inverse seesaw model proposed in the literature [5]. Although the $U(1)_{B-L}$ is not conserved, the matter parity is still well defined, which is from the fact $Z_2^L$ (equally $Z_2^{B-L}$) is left. We shall see below that this symmetry is important when it is regarded as an exact symmetry even if taking non-renormalizable operators into account.

B. Tadpole Term for $S$

A light gauge singlet at low energy may destabilize the weak scale. Without any fundamental symmetry like gauge symmetry to protect the singlet, some dangerous tadpole terms with quadratic divergence may be generated by the Planck-scale-suppressed operators [23]. Although in this section we consider the Planck scale suppressed operators as in the literatures, the results hold if we change the Planck scale to the effective fundamental scale.

We first briefly review how the NMSSM can avoid the dangerous tadpole and meanwhile generate a safe tadpole of $S$ to solve the domain-wall problem associated with the spontaneous breaking of the low-energy accidental $Z_3$ discrete symmetry. In particular, the odd-dimensional operators in the superpotential and even-dimensional operators in the Kähler potential will not destabilize the electroweak scale [24].

The study in [25] utilizes a $Z_2^M \times Z_4^R$ symmetry as the fundamental symmetry of the NMSSM superpotential and Kähler potential even at the Planck scale. The $Z_4^R$ symmetry is a discrete subgroup of $U(1)_R$, under which all NMSSM superfields transform as $\Phi \rightarrow i\Phi$ and the superpotential transforms as $W \rightarrow -iW$. This symmetry forbids any terms with odd powers of superfields in the Kähler potential and even powers of superfields in the superpotential. When coupling the theory to the supergravity, the $Z_4^R$ symmetry will be broken by SUSY-breaking effect and consequently $Z_2^M$ singlet $S$ will acquire a tadpole $\mu_S^2S$ induced by the non-renormalizable operators. However, according to the conclusion in [24], such a tadpole is safe and able to solve the domain-wall problem since $\mu_S^2S$ breaks $Z_3$ explicitly at a proper level.

In our work, the situation is different in two aspects. On the one hand, the new singlets $N$ and $\bar{N}$ are charged under the matter parity $Z_2^M$, which is regarded as an exact symmetry in our model (no $R$–parity breaking operators are introduced) and thus the corresponding tadpoles are forbidden. In other words, $Z_2^M$ protects $N$ and $\bar{N}$ from tadpole. On the other hand, one essential point of our model is the operator $S^2N^2/M_{Pl}$ with even powers of superfields in the superpotential. So, we have to check whether it will induce a harmful tadpole
of $S$. In fact, the $Z_4^R \subset U(1)_R$ symmetry is still conserved, with the charge assignment as

\[ S, L, Q \rightarrow -(S, L, Q), \]
\[ H_d, N, U \rightarrow -i(H_d, N, U), \]
\[ H_u, \bar{N}, E^C, D \rightarrow i(H_u, \bar{N}, E^C, D). \]  

Then $\mathcal{W} \rightarrow -\mathcal{W}$ under this transformation. In the following we shall argue that the theory is safe from tadpole up to dimension-eight level.

First of all, it is not difficult to find out that under the symmetry $Z_2^M \times Z_4^R$ the allowed high-dimensional terms with $S$ and Higgs bilinear term $H_uH_d$ take forms of $S^{2n+1}(H_uH_d)^m$ (with $n$ and $m$ being non-negative integers) which are odd powers of fields. So, according to the above discussions, they are safe operators. Next, let us consider the coupling of $S$ with $(N, \bar{N})$. There are two kinds of operators: one type are in the forms of $S^{2n+1}(N\bar{N})^m$, which are safe; the other type are $S^{2n}(N^2)^{2m+1}(\bar{N}^2)^{2k}$ (exchanging $N$ with $\bar{N}$ will not give anything new), which are potentially dangerous. But following the method of calculation given in [24], we can explicitly check that up to dimension-eight level (the next order is dimension-ten), the dangerous tadpole is absent. In fact, the inclusive potentially harmful terms are

\[ S^2\bar{N}^2, S^2N^2, S^2N^4, S^6N^2, S^2N^6, S^2N^2\bar{N}^4. \]  

Then, due to the fact that in the super-Feynman diagram calculation one cannot connect two lines coming from the same vertex to form a loop (the superfield propagator which is proportion to $\delta(\theta_1 - \theta_2)$ vanish), we conclude that they do not generate any tadpoles. For example, Fig. 1 will be the potential contribution to tadpole coming from the high-dimensional operator $S^2N^2$, but vanishes. Although we cannot ensure the safety of the theory up to arbitrary high orders, we believe that the sufficiently suppressed higher dimension operators will not destabilize the electroweak scale.

![FIG. 1: The vanishing tadpole from operator $S^2N^2$.](image)
III. PHENOMENOLOGICAL CONSEQUENCES

In our model some new light degrees of freedom come into play around the weak scale and thus may cause some interesting phenomenology. In this section, we give a preliminary study for such phenomenology. We will mainly discuss those phenomenological aspects which are relevant to the recent experimental results. Some similar analysis has been carried out, albeit based on different models [26].

A. Light Sneutrino Dark Matter

It is well known that in the MSSM the sneutrino has sizable coupling with $Z$–boson, which renders it not a good candidate for the cold DM [27]. To make it viable, several ways have been tried. One is to include lepton flavor violation and consequently the sneutrino is real and does not couple to $Z$ diagonally [28]. Introducing right-handed neutrinos can also make the mixed $\tilde{\nu}$ a good thermal DM candidate [29] (note that the analysis in [3] found that the almost pure right-handed sneutrinos can be thermal WIMP in the low scale seesaw, where the right-handed neutrinos have extra coupling to the NMSSM singlet $S$ in a renormalizable way $SN^2$ and this can not only explain the TeV Majorana mass scale but also open effective annihilation channels for the right-handed sneutrinos). Another way is to abandon the canonical seesaw, and instead take the inverse seesaw to make $\tilde{\nu}$ a viable DM candidate. A previous study (albeit not inspired by the recent detection experiments) has been carried out in [26]. In our following study we will show that in our model the $\tilde{\nu}$ LSP can easily explain the DAMA/CoGeNT signals. Further, since the $\tilde{\nu}$ LSP has distinct leptonic property, it may serve as a very special DM candidate favored by the recent indirect detection of DM.

First, we check the slepton mass and pay special attention to a light DM candidate motivated by DAMA/CoGeNT. In the basis $\Phi^\dagger = (\tilde{\nu}_L^\ast, \tilde{N}, \tilde{\bar{N}}^\ast)$, from Eq. (A1) we obtain the mass square matrix

$$M_{\tilde{\nu}}^2 \approx \begin{pmatrix} m_L^2 + \frac{1}{2}M_Z^2 \cos 2\beta + m_D^2 & (-m_D A_N + \mu m_D \cot \beta) & -m_D M_N \\ -m_D A_N + \mu m_D \cot \beta & m_N^2 + M_{\tilde{N}}^2 + m_D^2 & B_N \\ -m_D M_N & B_N & m_N^2 + M_N^2 \end{pmatrix},$$

(20)

where $B_N = A_1 M_N + \kappa M_N^2 / \lambda_1 - \lambda_1 \lambda v^2 \sin 2\beta / 2$. For simplicity, we have assumed all parameters to be real. The contribution from the small Majorana mass term for $\tilde{N}$ and the corresponding soft bilinear term $B_N \tilde{N}^2$ can be safely ignored for almost all of the study on DM. However, it will induce a small but non-zero mass splitting for the CP-eigenstates, typically at the order $B_N/m_{\tilde{\nu}} \sim 10^{-11}$ GeV, provided that a small bilinear parameter from
ordinary SUSY-breaking $B_N \equiv \lambda_1 A_0 v_s^2/M_{Pl}$. This has important implication on the DM detection and will be discussed later.

\[ \tilde{\nu}_1 \]

\[ \text{FIG. 2: The first two diagrams are the } \tilde{\nu}_1 - N \text{ scattering. The third diagram is the } t-\text{channel annihilation of } \tilde{\nu}_1 \text{ into a pair of } \tau \text{ leptons. The last diagram is the } t-\text{channel annihilation of } \tilde{\nu}_1 \text{ into a pair of neutrinos via exchanging a neutralino (suppressed by neutralino mass).} \]

Now we focus on a light LSP $\sim 8$ GeV. We denote its fraction in various interaction eigenstates as

\[ \tilde{\nu}_L = c_{L1} \tilde{\nu}_1 + \cdots, \quad \tilde{N}^* = c_{N1} \tilde{\nu}_1 + \cdots, \quad \tilde{N} = c_{N1} \tilde{\nu}_1 + \cdots, \quad \text{(21)} \]

where dots denote other irrelevant heavy components. Typically, our desired LSP should have the doublet component $c_{L1} \lesssim 0.2$ so as to explain the DAMA/CoGeNT signals by the Z–boson mediated large scattering rate $\sigma_{\tilde{\nu}_1,p}^{SI} \sim 10^{-40}$ cm$^2$ [13] (note that the latest CDMS results seem to disfavor the DAMA/CoGeNT region [30]) and meanwhile give an effective coupling between the LSP and Z–boson $g_{Z11} Z \tilde{\nu}_1 \tilde{\nu}_1^* \approx g_2 c_{L1}^2 < 0.023$ to satisfy the constraint from the Z–boson invisible decay width [31].

Two comments are in order. First, it is important that the LSP is complex so the diagonal coupling to the Z–boson is preserved. Consequently, the Z–boson just mediates a large elastic cross section $\sigma_{\tilde{\nu}_1,p}$. This makes it distinguishable from other real or inelastic sneutrino LSP candidates. Second, although a large DM-nucleon scattering rate can be achieved via
Z–boson mediation, in principle the Higgs bosons such as $h$ can also act as the mediator:

$$\sigma_{p,n}^{SI} \approx \frac{m_{p,n}^4}{4\pi m_h^4} \left( \frac{\mu_{h11}}{m_{p,n} + m_{\tilde{\nu}_1}} \right)^2 \left[ \sum_{q=u,d,s} f^{(p,n)}_{T_q} g_{qqh} + \frac{2}{27} \sum_{q=c,b,t} f^{(p,n)}_{T_G} g_{qqh} \right]$$

$$\approx 0.67 \times 10^{-40} \times \left( \frac{h_{d1}}{1.0} \right)^2 \left( \frac{\mu_{h11}}{5\text{GeV}} \right)^2 \left( \frac{100\text{GeV}}{m_h} \right)^4 \text{cm}^2,$$  

(22)

where $g_{qqh} = h_{d1}/\cos \beta$ for down-type quarks and $g_{qqh} = h_{u1}/\sin \beta$ for up-type quarks. In the estimation, we assumed a large $\tan \beta = 30$ and $f_{T_q} = 0.14$. Note that if the DM is light, compared to a fermionic DM (as heavy as $\tilde{\nu}_1$) scattering with the nucleon, a scalar DM-nucleon scattering cross section is potentially enhanced by the coupling in the case of $\mu_{h11} > \lambda_{fh} m_{\tilde{\nu}_1}$, where $\lambda_{fh}$ is the Yukawa coupling between the Higgs and fermionic DM. However, in this case the DM annihilates to the SM fermions like $b\bar{b}$ quite effectively (without $p$–wave suppressing, see Eq. (25)). As a result, it induces antiproton excess in the low energy spectra and may be disfavored by the PAMELA data [32].

We now discuss the way to give such a light sneutrino LSP with a doublet fraction $c_{L1} \sim 0.2$. If the SUSY-breaking is mediated by gravity, then all of the matrix elements are expected to be around the weak scale. Moreover, for the favored parameter pattern we have $M_N \sim \mathcal{O}(500) \text{GeV}$, so the scalar singlets $\tilde{N}$ and $\tilde{N}$ both have rather large mass term. As a result, the generation of a light sparticle needs some fine-tuning (reducing the mass eigenvalue by some proper non-diagonal elements). There are quite a few free parameters to tune so as to produce a light LSP required by DAMA/CoGeNT. Based on the structure of the mass matrix in Eq. (20), the following observations are useful to understand the origin of such a light LSP:

- The complex LSP generically has significant $\tilde{\nu}_L$ component because various states are well mixed with each other. In practice, since $M_N$ is larger than $m_L$ and $m_D$, in general $\tilde{\nu}_L$ is the dominant component of $\tilde{\nu}_1$. Increasing $m_L^2$ helps to increase the eigenvalue of the LSP and decrease the $\tilde{\nu}_L$ component.

- To reduce the $\tilde{\nu}_L$ component, the element $B_N$ should be properly large to reduce the eigenvalues of the $(2,3)$-block so that $\tilde{\nu}_1$ can mainly come from $\tilde{N}$ and $\tilde{N}$. A large $B_N$ implies a large $A_1$ or and $\kappa/\lambda_1$. But the element $-m_D A_N + \mu m_D \cot \beta \simeq -m_D A_N$ is also important to determine the mixing. We assume $A_N = A_1$ and $\kappa \sim \lambda \ll 1$. By the way, a small $\kappa$ may be required to generate a multi-TeV $\nu_s$.

- Although the soft masses $m_N^2, m_{\tilde{N}}^2 \ll M_N^2$, it is important to tune them to obtain a small mass of $\tilde{\nu}_1$ without significantly affecting $c_{L1}$.
Now we give a sample point: \( \tan \beta = 3.0, \lambda = 0.03, \kappa = 0.08, y_N = 0.848, \lambda_1 = 0.14, A_N = A_1 = 341 \text{ GeV}, m_N = m_{\tilde{N}} = 171.2 \text{ GeV}, m_L = 400 \text{ GeV}, \mu = 150 \text{ GeV}. \) Other soft parameters such as squark masses are important to Higgs system but not important to the sneutrino sector, and thus they are fixed as 400 GeV. Then we get a sneutrino LSP:

\[
m_{\tilde{\nu}_1} \approx 8.0 \text{ GeV}, \quad c_L \approx 0.24.
\]  

(23)

Such a LSP does not lead to antiproton excess because its annihilation is mediated by Z–boson and is \( p–\)wave suppressed (see Eq. (24)).

Now we check the annihilation rate of the sneutrino LSP pair, which determines the freeze-out relic density of LSP and its indirect signals. As a convention, we label \( \tilde{\nu}_1 \) as 1 and its antiparticle as 2. Then we have to distinguish the annihilation modes: (anti)particle-(anti)particle self-annihilation rate \( \sigma_{11} = \sigma_{22} \), and the antiparticle-particle annihilation rate \( \sigma_{12} = \sigma_{21} \). In our interested parameter space, the contribution to \( \sigma_{12} \) from \( s–\)channel Z–boson mediation is suppressed by \( (c_L)^4 \lesssim 10^{-3} \) and also is \( p–\)wave suppressed, which is negligible. Explicitly we have

\[
\langle \sigma Z v_{\text{rel}} \rangle \approx \frac{7}{24\pi} (c_L g_2)^4 m_{\tilde{\nu}_1}^2 |v|^2 (1 - 4x_f)^{1/2}.
\]  

(24)

The contribution from the \( s–\)channel Higgs mediation is chirality suppressed. For example, the annihilation to \( \bar{b}b \) is given by

\[
\langle \sigma_s v_{\text{rel}} \rangle_{\bar{b}b} \approx \frac{h_{d1}^2}{4\pi} \frac{\mu_{h11}^2}{v^2 \cos^2 \beta} \frac{m_b^2}{m_h^2} (1 - m_b^2/m_{\tilde{\nu}_1}^2)^{3/2} \approx 7.7 \times 10^{-26} \times \left( \frac{h_{d1}}{1.0} \right)^2 \left( \frac{\mu_{h11}}{5 \text{ GeV}} \right)^2 \left( \frac{2 \text{ GeV}}{m_h} \right)^4 \text{ cm}^3/s,
\]  

(25)

where we fixed \( \tan \beta = 30 \). If we take \( \tan \beta = 3 \), then \( \langle \sigma_s v_{\text{rel}} \rangle_{\bar{b}b} \) will be reduced by about two orders. Thus, the antiproton production from DM annihilation is suppressed.

The sneutrino LSP may have the leptonic annihilation property due to the significant \( \tilde{N} \) component and the survived vertex \( y_N LH_{\nu}N \) at low energy. This vertex tends to play a major role in the annihilation of LSP for \( y_N \sim 1 \). Explicitly, the average rate of annihilation into lepton pairs via \( t–\)channel chargino mediation is given by (the amplitude square is given in Eq. (A5))

\[
\langle \sigma_t v_{\text{rel}} \rangle_{ff} \approx \frac{1}{16\pi} c_{N1}^4 g_N^4 \frac{s}{M_C^4} \left( \frac{1 + 2x_f}{3} v^2 + 4x_f (x_{\tilde{\nu}_1} - x_f) \right) (1 - 4x_f)^{1/2},
\]  

(26)

where in the CM frame \( v_{\text{rel}} = 2\sqrt{1 - 4m_{\tilde{\nu}_1}^2/s} \) and \( 1 - v_f^2 = 4x_f \approx m_f^2/m_{\tilde{\nu}_1} \sim 10^{-1} \) for the GeV scale lepton. It is suppressed by an explicit chiral factor or \( p–\)wave suppression, but
still it dominates over the Higgs exchanging annihilation rate given in Eq. (25) in case of a light chargino $M_C \sim m_h$ and/or a rather small $\mu_{h_{11}}$ as well as a small $\tan \beta$.

The annihilation rate presented in Eq. (27) has some implications. On the one hand, when we consider the DM annihilation today, DM is at the non-relativistic limit and thus it is safe to drop the terms $\propto v^2 \sim 10^{-6}$. Then the chiral suppressed mode is barely able to account for the FGST:

$$\langle \sigma v_{\text{rel}} \rangle_{\chi} \simeq 0.72 \times 10^{-26} \times \left( \frac{c_{N_1}}{0.5} \right)^4 \left( \frac{y_{N_2}}{2.0} \right)^4 \left( \frac{100 \text{GeV}}{M_C} \right)^4 \text{cm}^3/\text{s}. \quad (27)$$

On the other hand, at the DM decoupling era, the DM is semi-relativistic with $v \sim 1/4$ (for a typical WIMP $m_{\chi}/T_f \sim 20 - 25$). Consequently, the $p-$wave annihilation mode is active and it is dominant for light leptons $e$ and $\mu$. So even the annihilation channel mediated by Higgs (to $\bar{b}b$) is suppressed for the sake of suppressing antiprotons, we can still allow DM to have significant annihilation rate to the states such as $\bar{e}e$ (required by ADM discussed later), in additional to the $\bar{\tau}\tau$ channel with annihilation rate fixed by FGST. In other words, our model can realize the ambitious scenario that, the sneutrino LSP is an ADM thus predicted to be light, which supports both DAMA/CoGeNT and FGST.

By the way, the rate $\sigma_{11}$ is from $t-$channel neutralino exchanging. It is suppressed by small mixing factor from $(c_{N_1} \sin \theta_1)^4$ and thus the annihilation to light neutrinos is negligible in our model.

### B. Sneutrino as Asymmetric Dark Matter

The sneutrino LSP can serve as the asymmetric dark matter (ADM). ADM is the most attractive framework to understand the coincidence between the density of the baryonic matter and dark matter, $\Omega_{DM} : \Omega_b \simeq 5 : 1$ [16]. In some recent studies [17], usually one has to introduce extra DM candidate which interacts with leptons or quarks at high order so as to extend the lepton number or baryon number to the dark sector and moreover maintain the chemical equilibrium between the two sectors. DM and baryon number densities usually are roughly equal by virtue of the chemical equilibrium. Turn to our model, the sneutrino LSP carries lepton number and thus in principle it can serve as an ADM candidate (a previous study on such a LSP can be found in [18], which is based on the canonical seesaw equipped with special SUSY-breaking). This is a unique property of the sneutrino LSP because the other LSP candidates, the neutralino or gravitino DM does not carry lepton or baryon number.

In our scenario we can assume the matter-antimatter asymmetry has been generated through some mechanism such as GUT-baryogenesis, Affleck-Dine mechanism or even EW-baryogenesis at the early universe. Especially, the lepton number asymmetry has been
generated and some is transferred to the (s)RHN system \((N, \bar{N})\). The (s)RHNs decouple around \(T_f \sim m_{\tilde{\nu}}/20 \ll T_{sph}\), the out-of-equilibrium temperature of electroweak sphaleron process. From then on the baryon and lepton symmetry conserves respectively. In such a way, the asymmetry stored in the sneutrino sector is calculable according to the well established chemical equilibrium condition.

Obviously, it is crucial to determine the chemical equilibrium condition at \(T_{sph}\). To do this, we follow the way developed in [33]. First, assign a chemical potential \(\mu_\phi\) to various particle \(\phi\) in our model. Then at temperature \(T\), the asymmetry of particle \(\phi\) in thermal equilibrium can be expressed with \(\mu_\phi\) (lower index is ignored) [34]

\[
n_+ - n_- = \frac{g T^3 \mu}{\pi^2 T} \int_0^\infty dx \frac{x^2 \exp[-\sqrt{x^2} + (m/T)^2]}{\theta + \exp[-\sqrt{x^2} + (m/T)^2]^2} \equiv \left\{
\begin{array}{ll}
    f_b(m/T) \times \frac{g T^3}{3} \left(\frac{\mu}{T}\right), & \text{for bosons} \\
    f_f(m/T) \times \frac{g T^3}{6} \left(\frac{\mu}{T}\right), & \text{for fermions}
\end{array}\right.
\]  

(28)

where the Boltzmann suppressed factor \(f_{b,f}(m/T)\) denotes the threshold effect for heavy particle in the plasma. If the particle are in the ultra-relativistic limit \(m \ll T\), it tends to \(1\); in the opposite limit, a decoupled particle has vanishing asymmetry density as expected.

In this work, for simplicity, we take all sparticles, except for three lightest complex sleptons, to be highly Boltzmann suppressed and thus do not contribute to total charge asymmetry. In other words, as a quite crude approximation, at \(T_{sph}\) only SM and RHNs fermions as well as three light \(\tilde{\nu}\) particles store the charge asymmetry.

Then the chemical equilibrium establishes relations between \(\mu_\phi\). Around \(T_{sph}\), at first all Yukawa couplings (including CKM mixing and dimensionless coupling in this model) have entered chemical equilibrium. As a result, the Higgs mediated flavor changing processes force all families to carry the same chemical potential. Next, the \(W\)-boson and the charged Higgs mediated processes force the up and down components of the \(SU(2)_L\) doublets to share the same potential (due to from those constraints, the quark/squark sector carries a potential \(\mu_{u_L}\) while the lepton/slepton sector including all (s)RHNs carry \(\mu_{\nu_L}\)). Third, the gauginos mediate equilibrium for SM particle and its superpartner, while the soft trilinear terms make all sfermion families carry the same potential. Finally, the EW-sphaleron process is effective and thus the left-handed quarks and leptons satisfy

\[
3\mu_{u_L} + \mu_{\nu_L} = 0.
\]

(29)

Totally, there are two independent chemical potentials, one is \(\mu_{\nu_L}\) and the other is the neutral Higgs \(\mu_0\). Especially, we have \(\mu_{\tilde{\nu}_L} = \mu_{\tilde{N}} = \mu_{\nu_L}\). Then the total baryon number is calculated to be

\[
B \simeq 4T^2 \mu_{u_L}.
\]

(30)
while the sneutrino asymmetry is given by

$$S = \frac{2T^2}{3} \sum_{i=1}^{k} \mu_{\nu L_i} = \frac{2kT^2}{3} \mu_{\nu L},$$

(31)

with $i$ summing over all sneutrino states lighter than $T_{sp}$ (since all of their asymmetry will be finally transferred to LSP). Using Eq. (29) we get their net charge ratio

$$\frac{n_{\tilde{\nu}_1}}{n_B} = -\frac{S}{B} = \frac{k}{2},$$

(32)

which determines the relic number density ratio. Then from the observed energy ratio, we obtain

$$m_{\tilde{\nu}_1} = \frac{2 \Omega_{DM}}{k \cdot \Omega_B} m_p \simeq 3.3 \text{ GeV} - 10 \text{ GeV}.$$  

(33)

Taking the Boltzmann factor into account, one may get the right mass value of DM ($\sim 8$ GeV) required by the DAMA/CoGeNT and FGST.

Of course, if the $\tilde{\nu}$ is the viable ADM, its symmetric part should almost annihilate away during its freeze-out through $\tilde{\nu}_1 + \tilde{\nu}_1^* \to f \bar{f}$ whose cross section is denoted as $\sigma_{12}$, while the particle-particle annihilation rate $\sigma_{11} = \sigma_{22}$ should be small (moreover, the particle-antiparticle oscillation by virtue of the dark lepton number violating term is negligible at this era, which will be addressed later). The only mode proceeds via exchanging a neutralino as stated before, which is highly suppressed. This is also consistent with a large Yukawa coupling $y_N$. Note that this does not violate the FGST requirement since it only requires $\tau^{\pm}$-pair mode dominates over the $q \bar{q}$ modes while allows even a larger annihilation rate to the neutrinos as well as electrons (they do not produce hard gamma spectrum).

One problem with $\tilde{\nu}_1$ as the ADM may be that presently it may have no pair annihilation modes and thus has nothing to do with the gamma-ray signals. Amazingly, recall that $\tilde{\nu}_1$ and $\tilde{\nu}_1^*$ are just approximate degenerate states and they are split by the small sneutrino mass term coming from the Planck-scale-suppressed soft mass term. In the CP-basis, $\tilde{\nu}_1$ and $\tilde{\nu}_1^*$ oscillate with a rate $\sim \delta m_{\pm}$. It just regenerates the symmetric DM after the LSP decouples. Note that the oscillating time scale $t_{os} \sim \delta m_{\pm}^{-1}$ should be much larger than the Hubble time scale $t_H \sim H^{-1}$ during the LSP freeze-out by symmetric annihilation $\sigma_{12}$. Otherwise, the asymmetric part will be erased by significant oscillation effect, rendering the symmetric relic density determined by ordinary freeze-out dynamics again. In conclusion, the ADM sets a nontrivial constraint

$$H(T = T_f) = \frac{1.66g^*_{s1/2}T^2_f}{M_{Pl}} < \delta m_{\pm}$$

(34)

where $\delta m_{\pm} = B_{N}/m_{\tilde{\nu}}$ and $z_1 = m_{\tilde{\nu}_1}/T_{os} \gg 1$ are used. Roughly, it requires the soft term of $S^2 \tilde{N}^2/M_{Pl}$ suppressed by two orders compared to ordinary soft parameters.
C. Impact on Higgs physics

In our model the Higgs phenomenology may be quite peculiar. First of all, $v_s$ is required to be several TeV in order to produce a proper neutrino mass scale (the NMSSM is near the decoupling limit). Given that $\mu = \lambda v_s \sim 100$ GeV, we immediately have $\lambda \ll 1$ and thus the lightest CP-even Higgs boson mass will not obtain enough enhancement from the tree-level quartic term via the coupling $\lambda S H_u H_d$. Actually, in the NMSSM the lightest CP-even Higgs boson mass square is approximately given by

$$m^2_h \simeq M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta + \frac{3m_t^4}{4\pi^2 v^2} \log \left( \frac{m_{\tilde{t}}^2}{m_D^2} \right),$$

(35)

where the last term is the dominant radiative correction (in the MSSM this term lifts up $m_h^2$ to meet the LEP lower bound and thus needs a heavy stop quark $m_{\tilde{t}} \sim 1$ TeV which causes the little fine-tuning problem). In principle, in our model a new contribution $\Delta m_{H_u}^2 \propto y_N^2 m_D^2 \log(m_{\tilde{N}}^2/m_D^2)/16\pi^2$ from the coupling $y_N LH_u N$ may be significant in some part of parameter space (e.g., $y_N \sim 1$) and thus help to alleviate the little fine-tuning problem (however, in the MSSM with the canonical seesaw, such a large Yukawa coupling of neutrinos just produces a negative effect and lowers the $m_h$ value [35]).

But in this paper we focus on another more interesting effect on the Higgs, that may alleviate the little hierarchy problem. In the previous section, we have demonstrated the possibility of accommodating a rather light sneutrino LSP in the model, although at the price of some kind of fine tuning. In this section, starting from light sneutrino states (but not confined in the DM phenomenology), we pay special attention on the Higgs physics. Then, in the presence of SM singlets interacting with the Higgs sector, the decay modes of Higgs will be affected substantially if some states are sufficiently light. So the invisible Higgs decay scenario can relax the LEP2 lower bound on the Higgs mass [36, 37]. Notice that the sneutrinos are $R$-parity odd, their dominant decay mode is $\tilde{\nu}_i \to$ LSP + $X$, where $X$ is a neutrino for neutralino/gravitino LSP and a light Higgs (if kinetically allowed) or a pair of fermions for sneutrino LSP (this is natural when family structure is introduced).

We denote the lightest Higgs component contained in the three CP-even Higgs as $R_{H_u} = h_{u1} h + \cdots$ (similar for $H_d$ and $S$). Then the decay mode to $b\bar{b}$ has a width given by

$$\Gamma_h \to b\bar{b} \approx N_c h_{d1}^2 \frac{m_b^2}{v^2 \cos^2 \beta} \frac{m_h}{16\pi},$$

(36)

$N_c = 3$ is the color factor. This tree level value will be significantly reduced by the (supersymmetric) QCD correction at the NLO order. This decay mode is the dominant one in the MSSM. But in our model, new invisible decay modes are open: $h \to \nu_i \nu_j$ and $h \to \tilde{\nu}_1 \tilde{\nu}_1^*$. The former is suppressed by small $y_N^2$ and mixing factor, but the latter generically can be
rather significant. The decay to a pair of kinetically allowed sneutrinos has a width given by
\[
\Gamma_{h\rightarrow\tilde{\nu}_1\tilde{\nu}_1} = \frac{1}{32\pi} \frac{\mu_{h11}^2}{m_h} \left( 1 - \frac{4m_{\tilde{\nu}_1}^2}{m_h^2} \right)^{1/2}
\]
where the coupling \(\mu_{h11}\) is defined in Eq. (A2). From Eq. (A2) we see that this mode is potentially important, especially when the decays to neutrinos are suppressed by \(h_{u1}^2 \ll 1\). Compared to the tree level \(b\bar{b}\) decay mode in Eq. (36), this mode is enhanced by a factor \((\mu_{h11}/h_{d1}y_b m_h)^2\). Even without taking QCD correction into account, when \(\tan \beta\) is in the small region \(\tan \beta \sim 3\) the Higgs invisible decay can be dominant decay mode.

In light of the above general analysis, to show the possibility for the Higgs boson \(h\) to have a sizable invisible decay we consider the parameter space with the following characteristics:

- To reduce the number of free parameters, all the new soft terms are assumed to be zero unless specified. This is well justified if the soft terms origin from gauge mediation (GMSB) or anomaly mediation (AMSB), where the soft terms involving singlets are proportional to Yukawa couplings of the singlets (in GMSB they are generated only through RGE effect and thus suppressed further by loop factor). Anyway, in the interested region \(100\text{ GeV} \sim m_D \ll M_N\), even if soft terms are generated at the weak scale, they do not play major role in our discussion.

- At least one sneutrino \(\tilde{\nu}_i\) is lighter than \(m_h/2\) and has significant coupling \(\mu_{hii}h\tilde{\nu}_i\tilde{\nu}_i\). For simplicity, we use all the calculation in the previous section, and denote \(\tilde{\nu}_i\) as \(\tilde{\nu}_1\). \(\tilde{\nu}_1\) is not necessarily the DM candidate and consequently many constraints are relaxed, such as the fraction of the \(\tilde{\nu}_L\) component in \(\tilde{\nu}_1\).

- To keep \(\mu_{h11}\) significantly large, saying \(\sim 5\) GeV, then from Eq. (A2) it can be clearly seen that \(m_D\) and \(M_N\) can not be too small. Generically, the lightest Higgs is \(h_{dR}\) dominated, and the NMSSM decoupling limit shows a hierarchy \(h_{d1} \gg h_{u1} \gg h_{s1}\). Then there are two obvious cases with physical simplicity. One is characterized by the \(\tilde{\nu}_L\)-dominated \(\tilde{\nu}_1\), but usually the singlet components in \(\tilde{\nu}_1\) are still appreciable and then the massive coupling is approximated to be
\[
\mu_{h11} \sim c_{L1} m_D (h_{u1} m_D/v + c_{N1} h_{d1} \mu/v),
\]
which is rather large and sensitive to \(\mu\). This case can be realized clearly with a somewhat small \(m_L \sim 150\) GeV, properly reduced by the small mixing with heavy singlets. But \(m_{\tilde{\nu}_1} > M_Z/2\) is required to satisfy \(Z\)-decay width. Oppositely, \(m_L \sim 300\) GeV is larger compared to singlet mass term, leading to singlets dominated \(\tilde{\nu}_1\) and
\( \mu_{h11} \sim c_L c_N h d_1 m_D \mu/v. \) By keeping \( c_L \lesssim 0.25 \) and \( \mu \simeq 250 \text{ GeV}, \) it takes value around 10 GeV. In this case the \( m_{\tilde{\nu}_1} \) does not have to be tuned to set in the narrow window \((M_Z/2, m_h/2)\). But since \( m_D \) is smaller than the previous case, it is expected that the Higgs invisible decay width will be suppressed more or less compared to small \( m_L \) scenario.

Define \( R \equiv \Gamma_{h \rightarrow \tilde{\nu}_1 \tilde{\nu}^*_1}/\Gamma_{h \rightarrow \bar{b}b} \) for a typical Higgs with mass \( m_h \simeq 100 \text{ GeV}, \) where the decay rate is using the numerical result calculated in the NMSSMtools [38]. For the two cases, we consider the \( R \) varies with \( \tan \beta \) with input parameters respectively given by: on the left panel of Fig. 3 we set \( m_L = 160 \text{ GeV}, \lambda = 0.024, \kappa = 0.0593, \) and \( y_N = 0.5, \lambda_1 = 0.06; \) on the right panel, \( m_L = 320 \text{ GeV}, \lambda = 0.024, \kappa = 0.0729, \) and \( y_N = 0.3, \lambda_1 = 0.0866. \) From the figure it is seen \( R \) decreases as (in larger \( \tan \beta \) region) \( \tan \beta \) increases, and the Higgs invisible decay width benefits from a larger \( \mu. \)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{The lightest CP-even Higgs boson \( h \) decays to a pair of light sneutrino with a ratio \( R \) defined as \( R = \Gamma_{h \rightarrow \tilde{\nu}_1 \tilde{\nu}^*_1}/\Gamma_{h \rightarrow \bar{b}b}. \) From top to bottom, the curves corresponds to fixed \( \mu = 220, 190, 160, 130, 100 \text{ GeV}, \) respectively.}
\end{figure}

- There is a third interesting case related with sneutrino DM. Typically, \( m_L < M_N, \) but we can still get a singlet-dominated \( \tilde{\nu}_1 \) by some proper tuning ( please see the detailed discussion in Section III A). In this dark matter inspired parameter scenario, both \( m_D \) and \( M_N \) take large values, leading to a rather large \( \mu_{h11}. \) It implies that the Higgs inevitably decays to such LSPs with overwhelming branching ratio, typically \( > 90\%. \)

However, it should be stressed that to suppress the sneutrino LSP annihilation into \( \bar{b}b, \) a small \( \tan \beta \) is favored.

In conclusion, once there is light sneutrino states from this sector, we have to investigate their probably significant effect on the Higgs decay and their detection. By the way, we point out that a recent paper [39] studied an interesting phenomenology of a light CP-odd
Higgs and found it can decay to light neutrinos so as to relax the collider constraints on this particle.

IV. CONCLUSION

In this work, we dynamically realize the supersymmetric inverse seesaw mechanism in the NMSSM with a minimal field content. The crucial observation is that a $Z_4^R \times Z_2^M$ symmetry allows for a Planck-scale-suppressed lepton number violating dimension-five operator $\bar{N}^2 S^2 / M_{Pl}$, which can just produce a light neutrino mass scale $m_\nu \sim 0.01$ eV. The theory accommodates the NMSSM $Z_3$-breaking domain-wall problem solution and is safe up to dimension-eight level. Naturalness and neutrino mass constrained the NMSSM parameters as $v_s \sim 5 \times 10^3$ GeV and $\lambda \sim 0.03$. We then preliminarily explored some phenomenology of this model. In addition to the neutrino physics, the most salient thing is that the left-right mixed (with doublet slepton component $c_{L1} \simeq 0.2$) light sneutrinos with mass $\sim 8$ GeV can be the LSP candidate. Such a LSP dark matter can readily explain the DAMA/CoGeNT events and even explain the FGST gamma-ray signals.

We also discussed the possibility that the sneutrino LSP is an asymmetric dark matter (ADM) and thus the ratio $\Omega_{DM} : \Omega_b \approx 5 : 1$ naturally predicts a light DM. Further, the Higgs physics is also dramatically changed in our model, owing to the significant low energy coupling $y_N L H_u N$ and the appearance of light sneutrinos. We found that the lightest CP-even Higgs boson probably dominantly decay to a pair of light sneutrinos and thus substantially relax the LEP bound. It provides a simple way to solve little hierarchy problem. Especially, in the region with light sneutrino LSP required by DAMA/CoGeNT, the Higgs boson invisibly decay with a dominant branching ratio (while the branching ratio to $bb$ typically is less than 10%), which will affect significantly the hunting for the Higgs boson at LHC.

Note that in the inverse seesaw mechanism the lepton flavor violating process like $\mu \rightarrow e \gamma$ may be enhanced [40]. But in our model it depends on the flavor structure of $(y_N)_{ij} H_u L_i N_j$ when three families are incorporated. In principle, we can work in a limit $(y_N)_{ij} \sim \delta_{ij}$ to avoid such large lepton flavor violating processes.

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Appendix A: Interactions and Annihilation of Sneutrinos

In this appendix, we will present some necessary interactions involving the sneutrinos, including the mass terms, the coupling to Higgs as well as some amplitude square of the sneutrino annihilation.

Collect all the relevant $F$–term and soft terms for the sneutrinos, and write the doublets as $H_u^T = (H_u^+, H_u^0)$, $H_d^T = (H_d^0, H_d^-)$, $\tilde{L}^T = (\tilde{\nu}_L, \tilde{e}_L)$, then the relevant scalar potential for the sneutrino system reads

$$V = |\lambda_1 \tilde{N} \tilde{N} + \kappa S^2 - \lambda H_d^0 H_d^0|^2 + |\lambda S H_d^0 + y_N \tilde{\nu}_L \tilde{N}|^2$$

$$+ | - y_N H_u^0 \tilde{\nu}_L + \lambda_1 S \tilde{N}|^2 + |\lambda_1 S \tilde{N}|^2 + |y_N H_u^0 \tilde{N}|^2$$

$$+ m_L^2 |\tilde{\nu}_L|^2 + m_N^2 |\tilde{N}|^2 + m_N^2 |\tilde{N}|^2 + \left( - y_N A_N H_u^0 \tilde{\nu}_L \tilde{N} + \lambda_1 A_1 S \tilde{N} \tilde{N} + h.c. \right), \quad (A1)$$

where the $D$–term for $\tilde{L}$ is not written. From the above we can read the mass matrix for the sneutrinos. We pay attention to the coupling between the lightest Higgs $h$ and $\tilde{\nu}_1 \tilde{\nu}_1$, namely the vertex $\frac{1}{\sqrt{2}} \mu_{h11} h \tilde{\nu}_1 \tilde{\nu}_1$ (1/$\sqrt{2}$ factor stems from the kinetic normalization for the real Higgs $h$) with the coupling given by

$$\mu_{h11} = c_{N1} \mu \cos \beta \left[ c_{L1} h_{s1} \frac{M_D}{v_s} - c_{N1} \frac{v_M}{v_s} (h_{d1} \tan \beta - h_{u1}) \right]$$

$$+ h_{u1} \frac{m_D^2}{v} \left[ c_{L1}^2 + c_{N1}^2 - c_{L1} c_{N1} \left( \frac{A_N}{m_D} - \frac{h_{d1}}{m_D h_{u1}} + \frac{M_N c_{N1}}{m_D c_{N1}} \right) \right]$$

$$+ h_{s1} M_N \left[ \frac{M_N}{v_s} \left( c_{N1}^2 + c_{N1}^2 \right) + c_{N1} c_{N1} \left( \frac{A_1}{v_s} - \frac{c_{L1} m_D}{c_{N1} v_s} + 2 \kappa \right) \right]. \quad (A2)$$

Here the coefficients such as $h_{u1}$ are the $h$–component contained in the real part of neutral Higgs $R_u$. This coupling controls the strength of the LSP interacting with SM fermions via the Higgs $h$ and plays a significant role in direct and indirect detection of DM.

Next we calculate some amplitude square for $\tilde{\nu}_1$ annihilation. First of all, we give the contribution from the $t$–channel chargino exchange, via a vertex in Eq. (8),

$$\mathcal{L}_{\tilde{\nu}_1} \supset y_N c_{N1} \tilde{\nu}_1 \tilde{\nu}_1 C P_L \tau^- + h.c., \quad (A3)$$

with $(\tilde{\tau}^-)^T = (\tilde{\tau}_d^-, \tilde{\tau}_u^+)$ and $(\tau^-)^T = (\tau_d^-, \tau_R^+)$ denoting the chargino and tau lepton, respectively. Here we neglected the contribution from the vertex $g_2 \tilde{L}^T \tilde{W} L + c.c.$ because the light chargino is dominated by Higgsino and it also suppressed by smaller $\tilde{\nu}_1$ from $\tilde{\nu}_L$. In
the center mass frame, we denote the kinetics of initial DM pair and final fermion pairs as

\[ p_1 = E(1, \vec{\beta}), \quad p_2 = E(1, -\vec{\beta}), \]
\[ k_1 = E'(1, \vec{\beta}'), \quad k_2 = E'(1, -\vec{\beta}'). \]  

(A4)

For the particle \( i \) with mass \( m_i \), the velocity is defined as \( \beta_i = \sqrt{1 - 4m_i^2/s} \) with \( s = (k_1 + k_2)^2 \). Sum over the final spin states, the amplitude square is given by

\[ |M|^2 \approx \frac{s^2}{4M_C^4} \left[ 8x_f(x_{\nu_1} - x_f) + v^2(1 - v_f^2 \cos^2 \theta) - 4vv_fx_f \cos \theta \right], \]  

(A5)

where the \( t- \) channel propagator factor is approximated to a contact form, and the terms linear in \( v \) will disappear after integrating over the angle to get the cross section.

For the \( Z- \) mediated process, from the vertex \( c_{L1}^2 g_2 (\bar{\nu}_1 \partial_\mu \bar{\nu}_1^* - \bar{\nu}_1^* \partial_\mu \bar{\nu}_1) Z^\mu \), we have the following \( p- \) wave suppressed (understood from the momentum dependent vertex proportional to \( (p_1 - p_2) \sim v \)) amplitude square

\[ |M|^2 \approx \frac{4}{M_Z^2} \left[ 2(p_1 - p_2) \cdot k_1 (p_1 - p_2) \cdot k_2 - \frac{s}{2} (p_1 - p_2)^2 \right] \]

\[ \approx \frac{8}{M_Z^2} \left[ (p_1^* \cdot k_1^*)^2 + s|p_1^*|^2 \right] \approx \frac{s^2 v^2}{M_Z^2} (1 + 2 \cos^2 \theta). \]  

(A6)

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