Sensitivity to $\theta_{13}$ and $\delta$ in the Decaying Astrophysical Neutrino Scenario

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We have previously shown that the decay of high-energy neutrinos from distant astrophysical sources would be revealed by flavor ratios that deviate strongly from the $\phi_\nu : \phi_\mu : \phi_\tau = 1 : 1 : 1$ expected from oscillations alone. Here we show that the deviations are significantly larger when the mixing angle $\theta_{13}$ and the CP phase $\delta$ are allowed to be nonzero. If neutrinos decay, this could allow measurement of $\theta_{13}$ and $\delta$ in IceCube and other near-term neutrino telescopes.

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FIG. 1: Variation of the ratio $\phi_\tau / \phi_\mu$ with $|U_{e3}|^2 = s_{13}^2$ in the allowed range. From bottom to top, the solid curves correspond to $\delta = (0, \pi/4, \pi/2, 3\pi/4, \pi)$, with the hatched region showing the full allowed range. The atmospheric angle is $\theta_{23} = 45^\circ$ and the solar angle is $\theta_{12} = 32.5^\circ$. The strongest lifetime limits, from solar neutrinos, are too weak to restrict the possibility of astrophysical neutrino decay by a factor of about $10^7$. Other scenarios, small-$\delta m^2$ active-sterile mixing in pseudo-Dirac or mirror models, or CPT violation, produce more subtle deviations from the expected 1 : 1 : 1. While neutrino decay can be tested with IceCube and other near-term detectors, the latter scenarios may require future detectors.

The most interesting decay scenario is that in which $\nu_3$ and $\nu_2$ decay (either into active $\nu_1$ or sterile states), but the lightest neutrino $\nu_1$ is stable. In this case, the beam contains just the single mass eigenstate $\nu_1$, and the flavor ratios at Earth are simply

$$\phi_e : \phi_\mu : \phi_\tau = |U_{e1}|^2 : |U_{\mu 1}|^2 : |U_{\tau 1}|^2 .$$  (2)

We present here further analysis of this scenario, taking into account the broken $\nu_\mu - \nu_\tau$ symmetry that arises when $U_{e3} = \sin \theta_{13} e^{-i \phi}$, which was set to zero for simplicity in our earlier work. We assume a normal neutrino mass hierarchy. For the case of an inverted hierarchy (or no decay), varying $\theta_{13}$ has little effect. Note that in this decay scenario, there is no dependence on the initial astrophysical flux ratios, and hence on the production mechanism of the ultra high energy neutrinos, since the flux reaching Earth consists only of the lightest neutrino $\nu_1$.

To express the flavor ratios in Eq. 2 in terms of the mixing parameters (using the conventions of Ref. 12 and $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$),

$$\phi_\tau = \frac{s_{12}^2(1 - \cos 2\theta_{23}) + s_{13}^2 c_{12}^2(1 + \cos 2\theta_{23}) - 4 \tilde{J}}{s_{12}^2(1 + \cos 2\theta_{23}) + s_{13}^2 c_{12}^2(1 - \cos 2\theta_{23}) + 4 \tilde{J}} .$$  (3)

We have defined

$$\tilde{J} \equiv \frac{1}{4} \sin 2\theta_{12} \sin 2\theta_{23} (s_{13} \cos \delta) ,$$  (4)

which is related to the Jarlskog invariant $J$ according to

Performing a fixed $\phi_\mu = \phi_\tau$ for varying $\phi_\nu = \phi_e$, we find

where $w_\alpha$ are the weights of the flavors produced in the astrophysical source. As is well-known, the weights from a pion-nucleon decay chain, $w_e : w_\mu : w_\tau = 1 : 2 : 0$, lead to flavor ratios at Earth of $1 : 1 : 1$. Variation of the mixing angles from the assumed $\nu_\mu - \nu_\tau$ symmetry limit ($\theta_{23} = 45^\circ$ and $\theta_{13} = 0$) leads to only small ($\lesssim 20\%$) deviations.

We have previously shown that the expected flavor ratios would be dramatically altered if neutrinos decay. The most interesting decay scenario is one in which $\nu_3$ and $\nu_2$ decay (either into active $\nu_1$ or sterile states), but the lightest neutrino $\nu_1$ is stable. In this case, the beam contains just the single mass eigenstate $\nu_1$, and the flavor ratios at Earth are simply

$$P_\beta = \sum_{\alpha} w_\alpha \sum_j |U_{\alpha j}|^2 |U_{\beta j}|^2 ,$$  (1)

where $U_{\alpha j}$ are elements of the neutrino mixing matrix, and $w_\alpha$ are the weights of the flavors produced in the astrophysical source. The probability to measure a neutrino flavor $\beta$ at Earth is therefore

$$P_\beta = \sum_{\alpha} w_\alpha \sum_j |U_{\alpha j}|^2 |U_{\beta j}|^2 ,$$  (1)

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We present here further analysis of this scenario, taking into account the broken $\nu_\mu - \nu_\tau$ symmetry that arises when $U_{e3} = \sin \theta_{13} e^{-i \phi}$, which was set to zero for simplicity in our earlier work. We assume a normal neutrino mass hierarchy. For the case of an inverted hierarchy (or no decay), varying $\theta_{13}$ has little effect. Note that in this decay scenario, there is no dependence on the initial astrophysical flux ratios, and hence on the production mechanism of the ultra high energy neutrinos, since the flux reaching Earth consists only of the lightest neutrino $\nu_1$.

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We have defined

$$\tilde{J} \equiv \frac{1}{4} \sin 2\theta_{12} \sin 2\theta_{23} (s_{13} \cos \delta) ,$$  (4)

which is related to the Jarlskog invariant $J$ according to
$J = J_{e3}^2 \tan \delta$. It is known, and is evident in Eq. 4, that either a nonzero \( \cos 2\theta_{23} \) or \( s_{13} \) breaks \( \nu_e - \nu_\tau \) symmetry. In Fig. 1, we show the ratio \( \phi_e / \phi_\mu \) as a function of \( |U_{e3}|^2 \) (within the range allowed by reactor experiments [9]), with \( \theta_{23} = \pi/4 \) fixed. Even a small \( \theta_{13} \) has a relatively large effect, particularly when the CP phase \( \delta \) is allowed to be nonzero. However, direct measurement of the ratio \( \phi_e / \phi_\mu \) is very difficult, since events which are unique to \( \nu_\tau \) (double-bang and lollipop events) have much lower detection probabilities [8]. In contrast, the \( \phi_e / \phi_\mu \) ratio can be directly probed in a detector like IceCube by comparing the rate of muon events to muon events [8]. This flavor ratio is

$$
\frac{\phi_e}{\phi_\mu} = \left( \frac{|U_{e3}|^2}{1 - |U_{e3}|^2} \right) \left( 1 + \frac{\phi_e}{\phi_\mu} \right). \tag{5}
$$

The \( \nu_e \) fraction in the \( \nu_1 \) mass eigenstate is insensitive to values of \( \theta_{13} \) in the allowed range, shown by the first factor in Eq. 5, where \( |U_{e1}| \equiv (c_{12}c_{13})^2 \). However, the broken \( \nu_e - \nu_\tau \) symmetry affects the \( \phi_e / \phi_\mu \) ratio through the second factor in Eq. 5, and this is a large effect. When \( \cos \delta \) is negative (positive) it decreases (increases) the \( \nu_\mu \) fraction of \( \nu_1 \) with respect to the \( \nu_\tau \) fraction, resulting in an enhanced (suppressed) \( \nu_e / \nu_\mu \) ratio. This is shown in Fig. 2. The curve with \( \delta = 0 \) is as in Ref. 2, though we have updated the solar angle \( \theta_{12} \). For nonzero \( \delta \), new to this work, the flavor ratio is significantly farther from the no-decay value of 1.

Note that the dependence of Eqs. 3 and 5 on the CP phase \( \delta \) occurs only through \( \cos \delta \), since this is a CP-conserving observable. Therefore, it is not necessary to separate astrophysical neutrinos and antineutrinos, which would be very difficult. The phase \( \delta \) is of crucial importance in terrestrial long-baseline oscillation experiments since CP-violating observables, based on the comparison of neutrino and antineutrino oscillation probabilities, are proportional to \( \sin \delta \) [13]. Farzan and Smirnov have shown that a nonzero \( \sin \delta \) may in principle also be inferred by direct construction of the leptonic unitarity triangle [14]. A key distinction is that the terrestrial experiments use a beam of flavor eigenstates, whereas neutrino decay can produce a pure mass eigenstate, allowing for very large variation with \( \delta \). Since measurement of \( \delta \) in terrestrial experiments will be an extremely challenging task, it is intriguing to find an example where the effect of varying \( \delta \) is huge. Finally, the flavor ratios are sensitive only to the “Dirac” phase \( \delta \), and not the “Majorana” phases; Majorana phases are relative phases between mass eigenstates, and the beam consists of the single mass eigenstate \( \nu_1 \).

Variation of the atmospheric mixing angle \( \theta_{23} \) away from \( 45^\circ \) also breaks the \( \nu_\mu - \nu_\tau \) symmetry, as shown in Eq. 4, and has a similar effect on the flavor ratios. Variation of the solar mixing angle changes the \( \phi_e / \phi_\mu \) ratio as it alters the \( \nu_e \) fraction of \( \nu_1 \). In Fig. 2 we show how the variations of these angles within their one-
sigma allowed ranges affects $\phi_e/\phi_\mu$. Note that the size
of the variation due to uncertainties in the solar and at-
tmospheric angles are quite similar. These uncertainties
will be reduced by existing or planned solar and long-
baseline experiments. The angle $\theta_{13}$ may be measured
by future long baseline [13] or reactor [13] experiments,
and measuring $\delta$ may require a neutrino factory.

To conclude, IceCube and other detectors have an ex-
cellent chance of detecting astrophysical neutrinos and
measuring their flavor ratios in the next several years.
If neutrinos decay, the flavor ratio $\phi_e/\phi_\mu$ will be much
larger than its no-decay value of 1, and this effect is sig-
nificantly enhanced by nonzero $\theta_{13}$ and $\delta$. Thus there
may be a new opportunity to measure the last unknown
values in the neutrino mixing matrix.

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