Abstract: Since the inherent intermittency and uncertainty of renewable energy resources complicates efficient Microgrid operations, a Demand Response (DR) scheme is implemented for customers in the grid to alter their power-usage patterns. However, for a real-time pricing model at the current DR, the automated decision on the energy price is not trustworthy because of artificial interferences to the power generation. As opposed to energy price, an operational cost-based prosumer scheduling approach would be able to protect the integrity of the power grid operations from deceptive market transactions and assist in robust energy management. To investigate the operational challenges associated with the costs and prosumers in the Microgrid, we focus on formulating the problem mathematically and designing approximation algorithms to solve the problem of how to optimally identify suppliers to minimize the total operational costs associated with providing electricity. We prove the hardness of the scheduling as one of the NP-Hard problems and propose polynomial time algorithms for approximating optimal solutions. With a proper resilience level for reliable power services, the scheduling algorithms include ways to construct not only robust supplier networks, but also group energy communities in terms of black start while minimizing the operational costs. A series of theoretical performances and experimental evaluations also demonstrates the practical effectiveness of this scheduling model for the operations.

Keywords: Microgrid; scheduling prosumers; minimum operational cost

1. Introduction

The Microgrid represents a way to propel utilities and their electricity-delivery systems into the 21st century by enabling them to autonomously and dynamically coordinate local generation and demands. A typical Microgrid configuration consists of distributed renewable energy sources (e.g., wind turbine and Photo-Voltaic (PV) panels) and co-generation technologies. Reliable solutions to integrate distributed generation units can save money, reduce air pollutant emissions, and increase the resilience of electric-power services. Due to these benefits, Microgrids help establish local clean-energy delivery systems that meet the specific demands of the constituents being served.

Using augmentation strategies for local energy systems, autonomous cooperation with renewable power sources, and electricity storage with smart technologies improves both the efficient usage of power and environmental sustainability. However, the intermittent effects that arise from the uncontrollable renewable resources raise issues related to power quality, reliability, and efficiency [1,2]. In the Microgrid concept, a group of interconnected power loads and distributed energy sources should operate as a unified controllable energy community for power services. The DR reflects the interaction between the electric grids and customers in energy markets. Interestingly, the bidirectional association enriches the benefits of cooperation by reducing the gap between peak load and peak valley. Shifting energy consumption from hours with higher prices to hours with lower prices would alleviate the pressure of extreme load demands on the grid. In contrast to customers with conventional
power systems, Microgrid customers play a crucial role by participating in controlling their loads. Consequently, the flexibility from the DR of the demand side would help stabilize the power services and reliability on the power grid systems as well [3–5].

Although the DR has been gradually integrated into the Microgrids, methods to persuade customers to alter their power consumption patterns might allow for energy prices to be manipulated. Ideally, the local power-grid systems should adjust the prices of their offerings based on the variability and uncertainty of renewable energy resources. They would then remain legitimately self-balanced in terms of prices and self-confined from the main external grids. However, as mentioned by Navid et al. [6], the malicious curtailment of power generation by the aggregators of Distributed Energy Resources would potentially create significant opportunities for the manipulation of energy prices. As a result of the lack of monitoring or regulatory means to verify the availability of various power sources, cyber-attackers can manipulate the spot prices in energy markets by compromising the measurements of the energy grids via corrupted data-injection attacks [7,8]. In such scenarios, the Demand Response (DR), as a real-time pricing scheme, may account for a critical portion of the manipulation of energy prices to discourage prosumers from participating in the energy markets. As a result, the potential disruption of energy pricing mechanisms could end up threatening the success or even existence of the Microgrid, so regulatory technologies should take appropriate steps to mitigate this as necessary.

Previous studies have investigated the problem of finding the optimum power-operation plan in a Microgrid. Different sets of experimental configurations to emulate the small-scale power networks have been used to validate the performances of the proposed approaches. However, their solutions only show optimality within certain physical conditions, since the constraints in the simulations could not consider all possible scenarios to derive generalizable performance evaluations. Realistically, energy management is regulated separately inside each local power system by the cooperative characteristics in Distributed Energy Resources (DERs), making it difficult to construct baseline scenarios. If an optimal scheduling approach not only depends on the specific operational setups but also constructs solutions within the experimental environments, the solutions would affect not the performances but the physical system conditions. Our study’s proposed scheduling strategy is notably independent of the physical constraints in the Microgrids. Only prosumers in the power networks and the operational costs of the associations are parameterized in our algorithms, and even with the dynamic changes in costs with the different physical conditions, the scheduling plans guarantee theoretically bounded performances. Further, the use of a pricing strategy in Microgrid scheduling methods would nullify the efficiency of the solutions, because of the problem of fraudulent trading to manipulate the price in local energy markets. Thus, in this study, we devise scheduling algorithms to allow the operation plans of local power grids to adopt the generation cost, not the vulnerable price, as the value of power.

The proposed scheduling scheme builds on the idea of classifying all prosumers into suppliers and consumers by imposing a role of either providing electric power to the grid or using the energy from the power networks to determine the optimal operational cost. On the other side of the operator, a prosumer is an autonomous cooperative energy-trading platform, and a group of interconnected prosumers to manage their energy sources is considered a Microgrid. Accordingly, we propose approximation algorithms that can produce optimized solutions for the prosumer scheduling problem to provide electric power services with minimum costs. Then, the use of an optimal scheduling scheme based on the operational cost, instead of financial profit, would protect the grid from deliberate attempts to artificially disrupt or distort the energy markets.

To support the feasibility of our scheme, the hardness assessment of the authoritative scheduling problem and the development of approximation algorithms for the problem are as follows:

- Proving the hardness of the scheduling problem by reduction from the Weighted Dominating Set (WDS) problem, a well-known NP-Hard problem.
• Designing an approximation algorithm to construct a robust supplier network with minimized operational costs by employing a solution for the Directed Steiner Tree (DST) problem.

• Designing an approximation algorithm to minimize the operational cost with a constraint on the number of suppliers by employing a solution for the $k$-center problem.

Further, our scheduling approach considers unique constrained situations, such as the islanding mode or black start, in the efficient operation of local energy systems. The black start is a process of restoring an electric-power station or a part of an electric grid to operate without relying on the external electric-power transmission network to recover from a total or partial shutdown. The Microgrid in the islanded mode is defined as when a distributed generation remains energized in a local energy community while the remainder of the electrical-energy system loses power [9–11].

The rest of this paper is organized as follows. Section 2 introduces a Microgrid prosumer scheduling scheme and an explanation for the scheduling problem, along with motivations in Section 2.1. In Section 2.2, we discuss the general idea with which we design on our algorithms to address the scheduling problem. We define our scheduling problem in Section 3.1 and prove its hardness through the reduction of one of the well-known NP-Hard problems in Section 3.2. The hardness of the problem leads us to a study of the approximated solutions in Section 3.3. Section 4 analyzes the performances of the proposed approaches, such as the minimum operational costs, runtime overhead, and the number of energy suppliers in various Microgrid power networks. Finally, the conclusion is discussed in Section 5.

2. An Optimal Prosumer Scheduling Model for Microgrids

Most studies attempting to establish the optimal Microgrid operation plans on a wide range of DERs and Energy Storage Systems (ESSs) have been devoted to highlighting the efficiency of the DR mechanisms. That is, much of the literature on small-scale energy networks has merely targeted the baseline functions of power charging and discharging optimality. In this section, we briefly review the previous studies examining the optimum use of local electrical energy systems to minimize the dependence on primary energy grids.

The optimal operation of the Microgrid would extend beyond the state-of-the-art methods by introducing a selective operational power-exchanging model that is based on determining the minimum operational costs. After Section 2.1 presents a brief investigation of the neglected problems from early studies coping with the optimality of power operation plans, Section 2.2 introduces a dynamically associative power exchanging model for prosumers (e.g., residential buildings with solar PV) in Microgrids.

2.1. Motivation of Prosumer Scheduling for Microgrids

Due to the recent proliferation of local power markets with community energy models, the large volume of distributed small-scale DERs in smart residential communities requires more proactive arbitration between operational cost efficiencies. Advances in communication technologies with considerable computing power would provide efficient methods for accurately predicting volatile energy demands. In other words, the advent of those services ancillary to conventional power systems can help analyze the energy consumption profiles in the loads and track the various kinds of relevant information from the grids. These days, prosumers in the local energy grid play a crucial role in the reformation of power markets and the evolution of Microgrid technologies [12–14].

Unlike customers in conventional power-grid systems, a prosumer has been defined as a customer who can participate in the local energy markets by providing electricity to the Microgrid. To elucidate, the power grid is a coalition of prosumers who are not only physically connected but also logically associated with each other to reduce their operational costs and achieve high economic benefits [15,16]. The direct reciprocity of peer-to-peer (P2P) energy trading by the association of prosumers contributes to a dramatic decrease in transmission losses and the minimization of the operational costs in smart residential communities. However, most studies regarding the productive management of Microgrids
have been dedicated merely to economic incentives for financial sustainability and energy interchanges with the Smart Grids [17,18].

Although the DR may smooth the fluctuation of the load curve because of its flexibility, it does not improve the self-consumption and self-sufficiency rates for customers in the Microgrid. Under deregulated power markets, the interaction between the power supplied to power grids and the demands from customers can primarily act as a sponge to absorb energy surplus and provide it to others who need it. Accordingly, to engage consumption in the supply operation of the power grid, the DR shifts the usages of end-use consumers during peak periods in response to real-time-based billing options for retail electricity [19,20]. In other words, this interaction, called an incentive program through which the residential customers can gain more financial profits, would alleviate the burdens of the power-supply systems through peak-shaving by reducing electricity consumption. The evolution of various DR schemes exploring financial incentives and economic benefits from Microgrids would not help customers, but would increase sustainability by reducing the effects of the fluctuations in renewable energy generation [21]. In addition, the allocation of higher prices during on-peak hours to flatten the load profile does not follow the technological principles of customer satisfaction, system availability, or transparency, given the uncertain power generation.

Although the use of DR in Microgrids is expected to reform existing power systems by means of having an innovative technical design, the electricity pricing framework could be the primary target of cyber-attacks by financially motivated criminals. Local energy markets must set the prices of electricity from DERs without any malicious interference. Nevertheless, having rightful ownership of the energy would not be beneficial with a volatile pricing mechanism, even though it is one of the most vital bases for satisfying the fundamental human needs in modern society, because the distributed and localized power grids do not require a centralized authority to settle every transaction in the markets. In addition, the histories of various economies also lead us to conclude that investor attractiveness might have significant effects on the energy prices [22]. As a result of the relatively lawless nature of the nascent Microgrid energy markets, fraudulent trading to artificially make the price match the higher pricing pattern is a significant concern. All types of price manipulation can potentially harm legitimate customers, but the manipulation of energy consumption is more severe because electricity can directly and indirectly influence human welfare.

The relationship between customers’ satisfaction and the manipulation of prices has not attracted much attention from researchers, although there have been sufficient theoretical or empirical studies related to the efficient operation of power systems. For all these reasons, it is now essential to investigate a new optimization scheduling framework for energy consumption that is satisfactory despite the uncertainties of both generation and demand.

The contributions of this work are as follows:

- Revealing the issues that arise from the suppression of power demand and the manipulation of energy prices in existing Microgrid models.
- Introducing an operational cost-based optimization scheduling problem to decrease the volatility of energy prices in a lower pattern.
- Designing approximation algorithms to sort prosumers into suppliers and consumers based on authoritative production or consumption of energy, respectively.
- Evaluating the proposed algorithms in terms of both mathematical optimality and experimental performance.

2.2. Description of the General Idea

Given the nature of distributed power systems, Microgrids are needed to achieve economic optimization of the operational costs, such as those associated with generation, spinning reserve, and purchasing [23–26]. In a general sense regarding Microgrids, a wide range of electrical power-generation units are not only physically but also logically aggregated to provide cost-effective
power services. The efficient cost management for the services is rooted in intercommunication and collaboration between the units, also known as prosumers in Microgrids. The market prices valued without any interference can be accepted as fair, but one of the most critical distortions of prices is manipulation. Since the operational costs reflect the actual value of the energy, trading energy by considering only the costs would help decide fair prices for customers in energy markets. Consequently, we intend for our scheduling scheme to ensure reliability over the volatility and uncertainty in Microgrids in order to minimize the operational cost by the coalitions and to synchronize the prosumers. The operational cost associated with a prosumer comprises the functions derived from power generation, energy exchange, electricity service participation, maintenance, and start-up/shut-down procedures.

The purpose of our study is to devise inline scheduling algorithms for prosumers in Microgrids to exchange electricity by employing the dependence relationships of operational costs. This approach’s primary intuition leaves us with three challenges: what to optimize for the efficient electric-power trading between the prosumers, how to define the hardness of the scheduling problem for Microgrids, and how to evaluate the effectiveness of the solution from a mathematical perspective. The prosumer scheduling model in this paper is based on a cost optimization problem, which is proved by preserving the inapproximability from one of the well-known NP-Hard problems. Although the scheduling problem does not have a polynomial-time solvable solution, we design algorithms upon polynomial-time approximation approaches within specific mathematical performance ratios.

3. Hardness and Approximation Algorithms

In this section, we will show that the scheduling prosumers problem falls into the NP-Hard domain and would not be approximated within \((1 - \epsilon)\ln |\mathcal{V}|.\) To cope with the inapproximability of the problem, we further propose approximation algorithms within theoretically bounded performance ratios.

3.1. Problem Statement

Given an operational cost graph \(G = (\mathcal{V}, \mathcal{E}, \omega)\) where \(v \in \mathcal{V}\) and \(e \in \mathcal{E}, v\) refer to a vertex of a prosumer, and a notation of \(e(v, u)\) represents an edge between two prosumers \(v\) and \(u\) with an operational cost \(\omega\). Our objective is to efficiently classify each of the prosumers into either a consumer that uses electric power or a supplier that provides energy into the grid with the minimum total operational costs. The identification algorithm to schedule the prosumers’ role would be able to coordinate the power generation and transmission of the suppliers with the optimally minimized costs. Consequently, the prosumers can trade energy without considering the price, which improves the self-consumption and self-sufficiency rates in local energy communities.

3.2. Inapproximability of Prosumer Scheduling Problem

Energy sources with relatively higher operational costs should not be encouraged to integrate their power into the local power supply chain, but should be encouraged to charge it until it is beneficial or transferable. The approximation algorithms for the classification procedures above are to be explained after building the proof of the hardness to our problem.

In order to account for the practicality of the scheduling model, there should be a common understanding regarding the cost variations involved when the suppliers generate and deliver their energy to others. As described in Figure 1, the implementation of an efficient scheduling algorithm should consider the different operational costs for the purpose of minimization in local energy communities. After all, the curtailment of costs over the uncertainty in renewable energy sources and a demand load would be achievable by the synchronous coordination of prosumers.

Theorem 1. For any \(\epsilon \geq 0\), the prosumer scheduling problem to minimize the operational cost would not be approximable within \((1 - \epsilon)\ln |\mathcal{V}|\), unless \(NP \subseteq DTIME(\sqrt[\ln |\mathcal{V}|}{V})\) in the cost graphs.
Proof. The reduction from the WDS problem to the prosumer scheduling problem preserves the inapproximability within \((1 - \epsilon)\ln|V|\) for any \(\epsilon \geq 0\), unless \(NP \subseteq DTIME(|V|^{\log \log |V|})\) in the cost graphs.

Considering a given instance of a general graph \(G = (V, E, w)\) where \(w : V \rightarrow \mathbb{Z}^+\), the WDS problem aims to minimize the total weight of vertices \(v\) in the subset that dominates all other nodes in the graph. The total weight of a dominating set \(D\) is defined as:

\[
\text{Weight}(D) = \sum_{v \in D} \text{Weight}(v) \tag{1}
\]

As depicted in Figure 1, the reformation of a vertex-weighted graph from a WDS problem to an edge-weighted graph would leverage the reduction from an instance of the WDS problem to our operational cost optimization scheduling problem. As proven by Feige [27], the WDS problem is inapproximable within \((1 - \epsilon)\ln|V|\) for any \(\epsilon \geq 0\), unless \(NP \subseteq DTIME(|V|^{\log \log |V|})\). Consider a dominating set \(D\) with a minimum weight \(m_{vw}\) from a given vertex-weighted graph \(G_{vw}\); here, it is clear that a set \(C\) comprised of operational costs to cover all prosumers, in the reduced edge-weighted graph \(G_{ew}\), also maintains the same value of cost \(m_{ew}\). In accordance with the construction of \(G_{ew}\), there is a set of operational costs \(C\) in the graph \(G_{ew}\) iff vertices have to be selected into the group of minimum weight dominating set \(D\) from the graph \(G_{vw}\), and vice versa. As a result, prosumers with the minimum operational cost problem would not be approximated within \((1 - \epsilon)\ln|V|\) for any \(\epsilon \geq 0\), unless \(NP \subseteq DTIME(|V|^{\log \log |V|})\) in a Microgrid cost graph.

The prosumer scheduling problem reduced from the WDS problem has been discovered as one of the NP-Hard problems, and a common approach for practically coping with such problems is to use an approximation algorithm. An approximation algorithm requires polynomial time to produce an optimal solution for its NP-Hard problem. An approximation framework will then be evaluated by performance ratio for an input size \(n\) if the cost of the solution from the approximation algorithm is within a factor of \(\rho(n)\) compared to the cost \(OPT\) of an optimal solution.

3.3. Approximation Algorithms for Prosumer Scheduling Problem

As a result of the hardness of the prosumer scheduling problem, it is difficult to find an optimal solution in polynomial time. In this section, we propose approximation algorithms to return near-optimal solutions for the prosumer scheduling problem within polynomial time.

3.3.1. Approximating Prosumer Scheduling Using Directed Steiner Tree Algorithm

To propose approximated scheduling algorithms, the basic definition of the partial solution for the prosumer problem needs to be explained. Then, we will show that the algorithm for the DST
problem can be explored for the scheduling problem by preserving the identical performance ratio, $O(\log^2 k / \log \log k)$, with a runtime complexity of $n^{\text{polylog}(k)}$, where $k$ is the number of leaf nodes, vertex with a single edge.

**Definition 1** (Directed Steiner Tree problem). In a directed graph $G = (V, E)$ with a weight associated with each edge, given a root $r \in V$ and a set $D \subseteq V$, the Directed Steiner Tree problem asks us to construct a tree with the minimum weight rooted at $r$, ensuring that there is at least a path from the root to each node in the set $D$.

Figure 2 shows how an algorithm for the DST works to impose the role of either a supplier or a consumer, on each prosumer to maintain the minimal operating cost for the Microgrids’ energy services. The prosumers are represented as vertices that are directly related to others by the edges with operational costs in the directed graph $G = (V, E)$. That is, a prosumer $v_i$, as a supplier, generates and transmits a certain amount of energy to a prosumer $v_j$, a customer, iff there exists a directed edge from $v_i$ to $v_j$ with a certain operational cost $w$. The procedure in Algorithm 1 begins its construction of a tree as a Directed Steiner Tree rooted by a node $v$, where $v \in V$, to reach every other node from the cost graph. Hence, at most $n$ trees with roots from $v_1$ to $v_n$ are generated, and the algorithm terminates with a tree in the minimum weight. Then, the internal nodes on a path downward in the tree are grouped into suppliers, and the leaves are turned into consumers, which helps in the selection of the minimum operational cost to provide electricity to prosumers. Since an identified supplier has at least one edge associated with another supplier, in the occurrence of power-generation failures, one edge could still deliver electricity from its parent suppliers. That is, the structure of this solution to provide electricity also has confirmed fault tolerance to support the increased availability of the Microgrid operations.

An approximation algorithm generates the optimal solution of the minimal operational cost to find a DST problem. Since every tree is rooted by $v$ in the cost graph, the solution of the scheduling algorithm preserves the same performance ratio on the optimal cost from the approximation algorithm for the DST problem. As a result, our algorithm has $w.o.l.g$, the identical approximation ratio on the minimum operational cost in Microgrids, because it uses an approximation algorithm for the DST problem. Fabrizio et al. [28] introduced the $O(\log^2 k / \log \log k)$ approximation algorithm for the DST problem which runs in quasi-polynomial-time, i.e., in time $n^{\text{polylog}(k)}$, where $k$ is the number of leaves.

Figure 2. Identifying suppliers from prosumers in a Microgrid network using an algorithm for the Directed Steiner Tree problem in order to provide energy with minimum operational costs.
3.3.2. Approximating Prosumer Scheduling Using a $k$-Center Algorithm

The approximation Algorithm 1 constructs trees rooted by every single $v$ in the graph to achieve the minimum cost with fault tolerance. However, the Microgrid optimal scheduling has the subproblem of the islanding mode which is used to provide energy services with limited power sources. Further, the process of electric-power restoration from a part of the Microgrid requires a contingency plan to reboot the local energy service in the event of a significant system collapse or a system-wide blackout. In both cases, the local power grids do not choose but rely on the limited number of power suppliers, which have to be started by flipping the right switches to bring the entire grid back online. From this point of view, a prosumer-scheduling algorithm with a bounded number of power suppliers is highly needed to minimize the operational cost.

Algorithm 1: Approximation Algorithm to Identify Prosumers with Minimum Operational Cost using an Algorithm for the Directed Steiner Tree Problem.

Identify Prosumers with Minimum Operational Cost Procedure

Input: Microgrid network $M$

Output: A Set of Prosumers with Minimum Operational Cost

Create an operational cost graph $G = (V, E)$ from $M$ using bidirectional edges

$P ← ∅$ /* Initialize a list to save sets of prosumers with each of the operational costs in the set */

$P ← ∅$ /* Initialize a set to save prosumers on their total operational costs */

forall $v ∈ V$ do

$P ← \text{Operational cost tree from } G\text{ using an algorithm for the Directed Steiner Tree problem}$

$P ← P ∪ P$

end

$P ← \text{Min}_\text{Weight}(P)$ /* A set of prosumers with the minimum operational cost */

forall $v ∈ P$ do

if $|\text{Neighbor}(v)| ≥ 2$ then

/* When $v$ is an internal node */

$\text{Status}(v) ← \text{Producer}$

end

else /* When $v$ is a leaf node */

$\text{Status}(v) ← \text{Consumer}$

end

end

return $P$

The vertex $k$-center problem is one of the well-known classical NP-Hard optimization problems in computer science which involves the selection of the locations of facilities and clustering, among others. In this work, we employ an approximation algorithm for the problem of selecting a limited number of suppliers with a minimum operational cost.

Definition 2 ($k$-center problem). In an input graph $G = (V, E)$, the $k$-center problem is to find a subset $C ⊆ V$, such that the distance from the farthest vertex in $V$ to its nearest center in $C$ is minimized, where $|C| ≤ k$, with $k ∈ \mathbb{Z}^+$ as part of the input.

As shown in Figure 3, our scheduling scheme based on the $k$-center problem consists of two techniques: constructing an undirected cost graph and discovering $k$ suppliers in the minimal operating cost, within linear runtime complexity. In order to apply an approximation algorithm for the $k$-center problem to our prosumer scheduling approach, we construct a polynomial-time reduction between the problems by using Algorithm 2.
Since the approximated algorithms for the $k$-center problem consider general undirected graphs in [29–31], a slight modification of the problem would help in the design of an efficient algorithm for identifying $k$-vertices, called suppliers, with a minimal cost from the operational graph. For the scheduling problem in the directed Microgrid cost graph $G = (V, E, w)$, we build a new undirected operational cost graph $G'$ by introducing a dummy connector $d$ with edges $e_{vd}(v, d, w_{vd})$ and $e_{ud}(u, d, w_{ud})$, where $v$ and $u$ are already associated by directional edges in $G$. Throughout Algorithm 2, the construction of graph $G'$ leads to the transformation of a bidirectional weighted graph into a selective cost graph without direction. Then, our scheduling problem would be solvable by the approximation algorithms for the $k$-center problem in which the identified $k$ suppliers would be able to dominate all other dummy vertices. Further, each of the dummy nodes is associated with exactly one single vertex at the other side of the edge in the graph $G'$.

**Algorithm 2: Constructing an Operational Cost Graph from a Microgrid Network.**

Construct Operational Cost Graph Procedure ($M$)

**Input:** Microgrid network $M$

**Output:** Operational Cost Graph $G = (V, E)$

$G \leftarrow \emptyset$

/* $P$ is a set of prosumers $p$ in the Microgrid network */

forall $p_v \in M$ do

forall $p_u \in M$ where $p_v \neq p_u$ and $p_u \notin \text{Neighbor}(p_v)$ do

if $p_v$ and $p_u$ can directly trade electricity then

$v \leftarrow p_v$

$u \leftarrow p_u$

Create dummy node $d$

$V \leftarrow V \cup \{v \cup d \cup u\}$

$e_{vd} = (v, d)$ /* Create an edge between $v$ and $d */$

$e_{ud} = (u, d)$ /* Create an edge between $d$ and $u */$

$\text{Weight}(e_{vd}) \leftarrow \text{Operational cost to supply power from node } v \text{ to } u$

$\text{Weight}(e_{ud}) \leftarrow \text{Operational cost to supply power from node } u \text{ to } v$

$E \leftarrow E \cup \{e_{vd} \cup e_{ud}\}$

$\text{Neighbor}(p_v) \leftarrow \text{Neighbor}(p_v) \cup p_u$

end

end

$P \leftarrow P \setminus p_v$

end

return $G$

Algorithm 3 is motivated by the idea of transforming the prosumer scheduling problem into the $k$-center problem, then using the algorithms in [32,33] to obtain the minimum operational cost, where $k$ is the number of suppliers in the optimal solution. The best possible polynomial-time algorithm for the $k$-center problem currently is to provide 2-approximated solutions; finding a solution in the well-known NP-Hard problems, such as the minimum dominating-set problem. Note that after the transformation, we need to find the right $k$ prosumers to be converted into suppliers, since the performance ratio for the algorithm on the $k$-center problem is also preserved if the $k$-center of vertices is located in the graph $G'$. 
Algorithm 3: Approximation Algorithm to Identify Prosumers with Minimum Operational Cost using an Algorithm for the $k$-center Problem.

Identify Prosumers with Minimum Operational Cost Procedure

Input : Operational Cost Graph $G = (V, E, w)$

Output: A Set of Prosumers with Minimum Operational Cost

$P \leftarrow \emptyset$ /* Initialize a list to save sets of prosumers with each of the operational costs in the set */

$P \leftarrow \emptyset$ /* Initialize a set to save prosumers on their total operational costs*/

forall $k = 0$ to $|E|$ do

$P \leftarrow k$ producers with minimum operational cost from $G$ using an algorithm for $k$-center problem to cover all dummy vertices $d$

/* Replace a possible dummy in $P$ with a neighbor vertex in the minimum cost edge */

forall $v \in P$ do

if $\text{IsDummy}(v) = \text{TRUE}$ then

$e(v, u) \leftarrow$ An edge with minimum cost between the two edges which are associated with $v$

$P \leftarrow P \cup u$

$P \leftarrow P \setminus v$

end

end

$P \leftarrow P \cup P$

end

$P \leftarrow \text{MIN}_\text{WEIGHT}(P)$ /* A set of prosumers with the minimum operational cost */

forall $v \in V$ do

if $v \in P$ then

$\text{Status}(v) \leftarrow \text{Producer}$

forall $w \in \text{Neighbor}(v)$ do

if $\text{IsDummy}(w) = \text{TRUE}$ then

Reconnect $e(v, w)$

Remove $e(v, w)$ and $e(w, u)$

end

end

else

$\text{Status}(v) \leftarrow \text{Consumer}$

end

end

return $P$

Lemma 1. If $v$ is in the optimal set of $P$, then the set introduced by the solution for the $k$-center problem preserves the same optimal weight. The approximation algorithm for the $k$-center problem will obtain the same performance ratio for the $k$-center problem.

Proof. Suppose there is a set of optimal $k$-center $C^*$ in the graph $G'$, then all nodes in the set $C^*$ should appear in the group of suppliers $P^*$ in the graph $G$. For each node $d$, two weighted edges are connected to it without direction, and at least one of its neighbors must be located in $C^*$. In accordance with the construction from the Algorithm 2, a dummy node $d$ is generated from the directed edges $e(v, u, w_{vu})$ and $e(v, u, w_{wu})$, where $e \in E$ in the graph $G$, and breaks the association between $v$ and $u$ into $e(v, d, w_{vd})$ and $e(u, d, w_{ud})$ in the graph $G'$. In a sporadic case, the vertex $d$ might be selected into the $C^*$ by means of an approximation algorithm for the $k$-center problem in $G'$, which implies that the dummy node provides electric power to its neighbors. However, the approximated algorithms that
are operating according to greedy strategies generally make the optimal choice at each step as they attempt to identify a vertex dominating the maximum number of neighbors. In the case of a dummy node \( d \) found in the set of \( C^* \), Algorithm 2 removes it from the set and inserts one of its neighbors onto the edge with minimum weight. This follow-up adjustment corrects the dummy energy supplier problem without affecting the total cost, because either of the neighbors with the minimum weight can be a supplier for the other one. Then, the total weight to supply power from the group of \( \mathcal{P} \) in \( G \) is greater than the cost from the optimal \( k \)-centers in the graph \( G' \). In another case, it is trivial to prove that a supplier group with the minimum operational cost in \( \mathcal{P}^* \) of the graph \( G \) is also located in graph \( G' \) with a weight greater than the minimum cost. Consequently, the operational cost of \( \mathcal{P}^* \) and the weight of \( C^* \) are equivalent, indicating that the scheduling problem’s performance ratio is the same as the ratio for the \( k \)-center problem.

Figure 3. Identifying suppliers from prosumers in a Microgrid network using an algorithm for the \( k \)-center problem in order to provide energy with minimum operational costs.

4. Performance Evaluation

We now present our performance evaluations to verify the theoretical performances of the proposed algorithms and validate the feasibility of the scheduling approach. The proposed algorithms indeed return solutions within the prespecified error range in a reasonable polynomial time. We start by describing our simulation setups, which reflect the practicality of the Microgrid implementation.

According to the literature on the DR, prosumers’ economic profit has been considered the most general optimization objective. However, because of the potential vulnerability of the pricing scheme in the Microgrid, we designed the approximated scheduling algorithms to optimize the operational cost to provide beneficial energy services for the customers rather than financial profits to the energy suppliers. The operational cost described by Sharmistha et al. [34] includes the installation/maintenance of power resources, various inputs to generate electricity, and even start-up/shut-down costs for the distributed generation units in the Microgrid. Sudipta and M. Godoy [35] also mentioned the direct costs, consisting of the equipment installation, the fuel cost to run the equipment, the expenses for non-fuel operation and maintenance, and bills that are imposed by power utility companies. Based on the earlier description of the operational costs, optimizing the power delivery from the distributed generation to the customers in the local energy grids lies in minimizing the volatile costs against uncertainty in future fuel prices, given the intermittent and seasonal nature of renewable power generation. Consequently, for the performance evaluation of the dynamic scheduling for the
Microgrids, our simulations focus on the uncontrollable operating costs, which differ from the random behavior of nature, such as solar radiation and wind speed.

The simulations conducted in this paper do not build specifically composite cases or specific scenarios to validate the theoretical performances of the proposed algorithms. However, the assessment explores a series of simulations based on parameters from officially measured weather data [36,37]. That is, the proportional relationship of, for instance, solar power, between the power generation and solar irradiance is evident for us to measure the level of solar energy. Through observing this relationship, the operational costs should consider reliable weather data to show feasibility and randomized factors to express various conditions of Microgrids. Accordingly, our costs to generate electric power are basically obtained from the officially measured weather data, which are multiplied by random factors between 1 and 50. A simplification of the generation of operational costs for the simulation would emphasize the performance of the scheduling model by eliminating the other irrelevant variables. Figure 4 depicts an example of the collected weather dataset from Korea Institute of Energy Research (KIER) in [38].

4.1. Operational Performance Assessment

Figure 5 shows a performance evaluation comparing the approximation algorithms in terms of the optimal costs. More than 100 minimum operational costs by the scheduling algorithms are averaged at each point, where as the input Microgrid network configurations differ from the specific number of prosumers $|V| \in [50, 150]$ in Figure 5a. As depicted in the figure, the total operational costs for the power generation tend to increase with the number of vertices, prosumers, in increments. Notably, the minimum cost from the approximation algorithm based on the $k$-center problem is at most 30% less than the cost produced by other algorithms, which implies a structural difference in their solutions. The suppliers from the DST-based algorithm are all connected in a tree without any isolated groups of vertices, but the outcomes of suppliers by means of other algorithms allow for isolation. The number of members connected within an isolated group from the $k$-center-based algorithm would be the optimal size of energy units that guarantees the minimum operation cost [17]. Aside from the higher operational costs, the DST-based approach supports the reliability better than the other approach does, since each supplier from the solution has more than one connection to another supplier with only a small increment in the total operational cost.

Figure 4. Weather data examples from the Korea Institute of Energy Research (KIER) to generate operational costs for the performance evaluation.
It is also evident that there is a proportional relationship between the size of the power-supplier group and the number of prosumers in the Microgrid. In addition, an increment of suppliers results in higher operational costs to provide energy service into the grid. Accordingly, the approximation algorithm using the DST approach introduces the operational cost variation close to the supplier size increment, which implies that the cost is affected by the number of suppliers. However, Figure 5b reveals an interesting fact: the growing trend of the number of suppliers from the $k$-center-based algorithm does not follow the inclination as steeply as the increment of its operational costs, because the algorithm for the $k$-center problem operates under a greedy strategy to cover all nodes in a graph while minimizing the total edge weights from the $k$-centers to other vertices.

4.2. Runtime Assessment

The running time curves for the approximation algorithms to complete the identification of power suppliers represent the computational performances. Figure 6 illustrates the comparison of the overheads with varying numbers of prosumers, whereas the algorithms return the suppliers with minimum costs. As mentioned earlier, the runtime complexity of obtaining an optimal answer from an NP-Hard problem is exceptionally high in polynomial-time solvable approaches. Throughout this paper, we understand that the scheduling problem is another NP-Hard problem, and that it requires an approximated approach to obtain optimal polynomial-time solutions. Figure 5a validates the theoretically proven computational overhead by presenting the running time of the two approximated scheduling algorithms. As the number of prosumers increases, the runtime of the $k$-center problem-based algorithm grows faster than the other approximated scheduling algorithm does, even though it works better for the operational cost and the size of the suppliers. In accordance with Algorithm 3, this is because of the exhaustive search strategy used to find a specific number of energy suppliers from the solution for the $k$-center problem, where $k$ ranges from 0 to $|E|$.

![Figure 5](image_url)

**Figure 5.** Comparing operational performances from the approximated scheduling algorithms. (a) Total operational costs obtained by the approximation algorithms when there are varying number of prosumers. (b) Number of suppliers identified by the approximation algorithms when there are varying number of prosumers.

However, the differences between the run times of the individual scheduling algorithms can be turned over by first defining the number of suppliers in the Microgrid. As shown in Figure 6b, the computational overheads of run time drop by at most 60% when the number of suppliers is given as 70 out of 150 prosumers in the power network. The scheduling algorithm with the predefined number of power generation units by the minimum cost aids in the efficient recovery from a black start until the Microgrid is fully functional.
5. Conclusions

The substantial reformations being made in electric power systems in response to the drastic increase in energy demand brings an unprecedented challenge to residential DR scheduling in local energy communities. We designed approximation algorithms for identifying a set of suppliers from the prosumers in a Microgrid network while minimizing the total operational costs to provide energy into the grid. The designs of our algorithms to find the optimal configuration of prosumers began by proving the hardness of the scheduling problem as a NP-Hard problem. Since it is extremely difficult to obtain an optimal answer from an NP-Hard problem within polynomial time, we devise approximation algorithms to solve the scheduling problem with theoretically bounded performance ratios. Significantly, one of the approximation algorithms we employed from an DST problem approach increases fault tolerance while pursuing the minimum operational cost. The other approximated scheduling algorithms using a solution for the $k$-center problem-based approximation algorithm when there are varying number of suppliers out of 200 prosumers.

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