Diagnostics of electromagnetic waves in the vacuum

L. V. Vshivkova¹, V. A. Vshivkov¹ and G. I. Dudnikova²

¹ Institute of Computational Mathematics and Mathematical Geophysics SB RAS, Lavrentiev ave., 6, Novosibirsk, 630090, Russia
² Institute of Computational Technologies SB RAS, Lavrentiev ave., 6, Novosibirsk, 630090, Russia

E-mail: lyudmila.vshivkova@parbz.sscc.ru

Abstract. Under interaction of plasma flows there are electromagnetic waves generated in the plasma. They can be distributed in different directions, interact with the plasma and can be changed by amplitude and motion direction. The study of the generating electromagnetic waves in the plasma is impossible due to the nonlinear character of such interaction when modeling. In many problems to solve this problem one places a vacuum domain next to the plasma region, where the generated by the plasma electromagnetic waves are studied. Waves in vacuum follow to the linear Maxwell equations and, therefore, it is easy to determine their frequencies and amplitudes using the Fourier analysis. To study this matter, it is sufficient to consider the problem in the 2D case. The aim of the current work is the development of the technique to determine directions and amplitudes of all electromagnetic waves in the vacuum which are present in a computational domain at definite time.

1. Introduction
Under interaction of plasma flows there are electromagnetic waves generated in the plasma. These waves can be distributed in different directions, interact with the plasma and can be changed by amplitude and motion direction. When modelling, the study of the generating electromagnetic waves in the plasma is impossible due to the nonlinear character of such interaction. In different problems to solve this matter one places a vacuum domain next to the plasma region, where there are studied the electromagnetic waves generated by the plasma. For instance, the electromagnetic radiation appearing due to the interaction of an electron beam with the plasma is studied in [1, 2]. As an illustration of the generating electromagnetic field, a figure of one component of electric field intensity had been presented. From these articles one can see that the radiation moves by a normal to the boundary of a computational domain, however, the wave characteristics are not defined. In [3] there is considered the generating radiation when modeling by the particle-in-cell (PIC) method, and, as a characteristic of the radiation, the density of the electromagnetic energy is considered only. In [4] the PML method for the absorbing boundary conditions is considered. Here the efficiency of the method had been illustrated by the comparison with an accurate solution only. Nevertheless, the appearing distortion of the electromagnetic waves had not been studied well. In the work of G. Mur [5] the nonreflecting boundary conditions had been proposed, however, the quality of the boundary waves had been characterized by the comparison with an exact solution, also, and the characteristics of reflected waves had not been...
studied. It all leads to the necessity to study more careful the characteristics of the electromagnetic waves in the vacuum.

The aim of the current paper is to develop the technique to determine directions and amplitudes of all electromagnetic waves taking place in a computational domain at a definite moment of time. Waves in vacuum follow to the linear Maxwell equations and break apart in noninteracting with each other electromagnetic waves of different frequencies and amplitudes. In numerical computations it is easy to determine the frequencies and amplitudes of all six components of the electromagnetic field intensities using the Fourier analysis. Unfortunately, all waves are mixed up and it is impossible to determine the characteristics of separate waves. Moreover, it is impossible to separate by the Fourier analysis two waves having the same wave numbers but moving in opposite directions.

2. Algorithm description

To study this matter, let us consider the problem in a uniform rectangular grid with the nodes $(x_i, y_k, t^m)$, where $x_i = i h_x$, $y_k = k h_y$, $t^m = m \tau$, is introduced. Here $h_x, h_y$ are grid steps in the directions $x$ and $y$, correspondingly, $\tau$ is a time step, $t$ and $k$ are node numbers. Approximating the system of equations (1), we obtain the scheme

\[
\begin{align*}
\frac{(B_x)_{i+1/2,k}^{m+1/2} - (B_x)_{i-1/2,k}^{m-1/2}}{\tau} &= \frac{(E_z)_{i+1/2,k+1/2}^{m} - (E_z)_{i-1/2,k-1/2}^{m}}{h_y} \\
\frac{(B_y)_{i,k+1/2}^{m+1/2} - (B_y)_{i,k-1/2}^{m-1/2}}{\tau} &= \frac{(E_x)_{i+1/2,k-1/2}^{m} - (E_x)_{i-1/2,k+1/2}^{m}}{h_x} \\
\frac{(E_z)_{i-1/2,k+1/2}^{m} - (E_z)_{i-1/2,k-1/2}^{m}}{\tau} &= \frac{(B_x)_{i+1/2,k}^{m} - (B_x)_{i-1/2,k}^{m}}{h_x} \\
\frac{(B_y)_{i,k}^{m+1/2} - (B_y)_{i,k}^{m-1/2}}{\tau} &= \frac{(B_y)_{i+1/2,k}^{m} - (B_y)_{i-1/2,k}^{m}}{h_x} \\
\end{align*}
\]
The similar scheme can be written for the system (2). It is known as the FDTD scheme [6] and has the second order of approximation in all directions.

Solving a problem at any time, we have the grid functions $E_x, E_y, E_z, B_x, B_y, B_z$ obeying the grid equations (3) in the vacuum. Our aim is to find the specification of waves and their directions while they move by the known grid fields.

Let us consider the waves dealing with the first subsystem. A wave satisfying the system (1) can be written as

$$
\begin{align*}
E_x &= \omega A \sin(-\omega t + k_x x + k_y y + \varphi_0), \\
B_x &= k_y A \sin(-\omega t + k_x x + k_y y + \varphi_0), \\
B_y &= -k_x A \sin(-\omega t + k_x x + k_y y + \varphi_0),
\end{align*}
$$

where $A$ is a wave amplitude, $\omega$ is a wave frequency, $k_x$ and $k_y$ are components of a wave vector, $\varphi_0$ is a wave shift by space. The components of the wave vector are bound with the frequency by the following formula

$$
\omega^2 = k_x^2 + k_y^2.
$$

It permits us to introduce a slope $\alpha$ of the wave vector to the axis $x$

$$
k_x = \omega \cos \alpha, \quad k_y = \omega \sin \alpha.
$$

In the case of the limited domain

$$
0 \leq x < L_x, \quad 0 \leq y < L_y,
$$

the components of the wave vector can take a counting number of the values

$$
k_x = n_x \frac{2\pi}{L_x}, \quad k_y = n_y \frac{2\pi}{L_y},
$$

where $n_x, n_y$ are natural numbers.

Any function in the 2D case can be represented in the form of the doubled Fourier series

$$
\sum_{n_x=0}^{\infty} \sum_{n_y=0}^{\infty} \left[ R_{1,n_x,n_y} \cdot \cos(k_x x) \cos(k_y y) + R_{2,n_x,n_y} \cdot \cos(k_x x) \sin(k_y y) + \\
+ R_{3,n_x,n_y} \cdot \sin(k_x x) \cos(k_y y) + R_{4,n_x,n_y} \cdot \sin(k_x x) \sin(k_y y) \right],
$$

where the coefficients $R$ are computed by the Euler-Fourier formulas [7].

The problem is in that at any time $t$, knowing the values of the functions $E_x, E_y, E_z, B_x, B_y, B_z$, to find what kind of waves are in the computational domain, i.e. for each $k_x, k_y$ it is necessary to find a wave amplitude corresponding to this wave vector. As all waves are linearly independent, let us consider below only waves which have the same $|k_x|$ and $|k_y|$.

### 2.1. Expansion into the Fourier series of $E_x$

The 2D Fourier analysis for the functions $E_x, B_y, B_y$ gives the amplitudes of all harmonics. However, this expansion mixes up four waves which have the equal $|k_x|$ and $|k_y|$, but have a different motion direction. For example, for the function $E_x$ there will be mixed up the following four waves having different amplitudes and phase shifts

$$
\begin{align*}
E_x^{(1)} &= \omega A_1 \sin(k_x x + k_y y + \varphi_1), \\
E_x^{(2)} &= \omega A_2 \sin(k_x x - k_y y + \varphi_2), \\
E_x^{(3)} &= \omega A_3 \sin(-k_x x + k_y y + \varphi_3), \\
E_x^{(4)} &= \omega A_4 \sin(-k_x x - k_y y + \varphi_4),
\end{align*}
$$

As the function analysis is carried out at a fixed moment of time, here we add $-\omega t$ to the phase shift $\varphi$ (see the formula (4)).

Thus, the mixture of four waves, mentioned above, has the following form

$$
E_x = E_x^{(1)} + E_x^{(2)} + E_x^{(3)} + E_x^{(4)}.
$$

The Fourier transform of the function $E_x$ gives four coefficients for the harmonics
International Conference on Mathematical Modelling in Physical Sciences

where

the result we get the system of four equations

Substituting these expansions into equality (7), we obtain

Writing down the expansions of all functions $E_z^{(i)}$ we get

where $R_1, R_2, R_3, R_4$ are the coefficients of the expansion into the Fourier series of the function $E_z$. As the result we get the system of four equations

The solution of this system gives the following equalities

However, we need more of them to determine the coefficients $p_i$ and $q_i$. For a unique definition of the unknown $p_i$ and $q_i$ it is necessary to use the expansion coefficients for the functions $B_x$ and $B_y$.

2.2. Expansion into the Fourier series of $B_x$. Let us consider the function $B_x$. As for the function $E_z$, let us present it as a sum of four functions having the equal $|k_x|$ and $|k_y|$

$$B_x = B_x^{(1)} + B_x^{(2)} + B_x^{(3)} + B_x^{(4)},$$

where

$$p_1 = \omega A_1 \sin(\varphi_1), \quad q_1 = \omega A_1 \cos(\varphi_1), \quad i = 1, 2, 3, 4.$$
\[ B_x^{(1)} = k_y A_1 \sin(k_x x + k_y y + \varphi_1), \]
\[ B_x^{(2)} = -k_y A_2 \sin(k_x x - k_y y + \varphi_2), \]
\[ B_x^{(3)} = k_y A_3 \sin(-k_x x + k_y y + \varphi_3), \]
\[ B_x^{(4)} = -k_y A_4 \sin(-k_x x - k_y y + \varphi_4). \]

The expansion of the functions \( B_x^{(i)} \) gives the following equalities
\[ B_x^{(1)} = \sin \alpha \left[ p_1 \cos(k_x x) \cos(k_y y) + q_1 \cos(k_x x) \sin(k_y y) + q_1 \sin(k_x x) \sin(k_y y) \right], \]
\[ B_x^{(2)} = \sin \alpha \left[ -p_2 \cos(k_x x) \cos(k_y y) + q_2 \cos(k_x x) \sin(k_y y) - q_2 \sin(k_x x) \sin(k_y y) \right], \]
\[ B_x^{(3)} = \sin \alpha \left[ p_3 \cos(k_x x) \cos(k_y y) + q_3 \cos(k_x x) \sin(k_y y) - q_3 \sin(k_x x) \sin(k_y y) \right], \]
\[ B_x^{(4)} = \sin \alpha \left[ -p_4 \cos(k_x x) \cos(k_y y) + q_4 \cos(k_x x) \sin(k_y y) + q_4 \sin(k_x x) \sin(k_y y) \right], \]

where \( \sin \alpha \) is the sine of the slope of the wave vector to the axis \( x \). In a similar manner, summing up these functions, we get
\[
\sum B_x^{(i)} = \sin \alpha \cdot (p_1 - p_2 + p_3 - p_4) \cdot \cos(k_x x) \cos(k_y y) + \\
+ \sin \alpha \cdot (q_1 + q_2 + q_3 + q_4) \cdot \sin(k_x x) \sin(k_y y) + \\
+ \sin \alpha \cdot (q_1 - q_2 - q_3 + q_4) \cdot \sin(k_x x) \cos(k_y y) + \\
+ \sin \alpha \cdot (-p_1 - p_2 + p_3 + p_4) \cdot \sin(k_x x) \sin(k_y y) = \\
= B_x = U_1 \cos(k_x x) \cos(k_y y) + U_2 \cos(k_x x) \sin(k_y y) + \\
+ U_3 \sin(k_x x) \cos(k_y y) + U_4 \sin(k_x x) \sin(k_y y),
\]

where \( U_1, U_2, U_3, U_4 \) are the coefficients of the expansion into the Fourier series of the function \( B_x \). Here we obtain again the system consisting of four equations
\[
\begin{align*}
  p_1 - p_2 + p_3 - p_4 &= U_1 / \sin \alpha, \\
  -p_1 - p_2 + p_3 + p_4 &= U_4 / \sin \alpha, \\
  q_1 + q_2 + q_3 + q_4 &= U_2 / \sin \alpha, \\
  q_1 - q_2 - q_3 + q_4 &= U_3 / \sin \alpha.
\end{align*}
\]

It gives the solution
\[
\begin{cases}
  p_2 - p_2 &= (U_1 + U_4) / (2 \sin \alpha), \\
  p_2 - p_4 &= (U_1 - U_4) / (2 \sin \alpha), \\
  q_1 + q_4 &= (U_2 + U_3) / (2 \sin \alpha), \\
  q_2 + q_3 &= (U_2 - U_3) / (2 \sin \alpha).
\end{cases}
\] (10)

Solving these equations along with the system (8), we obtain the values of all coefficients \( p_i \) and \( q_i \)
\[
\begin{align*}
  p_1 &= \frac{1}{4} \left( R_1 - R_4 + \frac{U_1 - U_4}{\sin \alpha} \right), \\
  p_2 &= \frac{1}{4} \left( R_1 + R_4 - \frac{U_1 + U_4}{\sin \alpha} \right), \\
  p_3 &= \frac{1}{4} \left( R_1 + R_4 + \frac{U_1 + U_4}{\sin \alpha} \right), \\
  p_4 &= \frac{1}{4} \left( R_1 - R_4 - \frac{U_1 - U_4}{\sin \alpha} \right), \\
  q_1 &= \frac{1}{4} \left( R_2 + R_3 + \frac{U_2 + U_3}{\sin \alpha} \right), \\
  q_2 &= \frac{1}{4} \left( R_3 - R_2 + \frac{U_2 - U_3}{\sin \alpha} \right).
\end{align*}
\]
The solution of this system is as follows

\begin{align*}
q_3 &= \frac{1}{4} \left(R_2 - R_3 + \frac{U_2 - U_3}{\sin \alpha}\right), \\
q_4 &= \frac{1}{4} \left(-R_2 - R_3 + \frac{U_2 + U_3}{\sin \alpha}\right).
\end{align*}

Knowing all \( p_i \) and \( q_i \), it is possible to find the amplitudes of all waves having the equal \(|k_x|\) and \(|k_y|\)

\[ A_i = \frac{1}{\omega} \sqrt{p_i^2 + q_i^2}, \quad i = 1, \ldots, A. \]

The phase shift is defined by the formulas

\[ \tan \varphi_i = \frac{p_i}{q_i}, \quad i = 1, \ldots, A. \]

2.3. Expansion into the Fourier series of \( B_y \). In some cases, it is necessary to use the expansion into the Fourier series of \( B_y \) instead of \( B_x \). As the amplitudes of waves should be defined uniquely, the use of the function \( B_y \) instead of \( B_x \) should lead to the same result. In the similar way, let us present the function \( B_y \) as the sum of four functions having equal \(|k_x|\) and \(|k_y|\)

\[ B_y = B_y^{(1)} + B_y^{(2)} + B_y^{(3)} + B_y^{(4)}. \]

Here

\begin{align*}
B_y^{(1)} &= -k_x A_1 \sin(k_x x + k_y y + \varphi_1), \\
B_y^{(2)} &= -k_x A_2 \sin(k_x x - k_y y + \varphi_2), \\
B_y^{(3)} &= k_x A_3 \sin(-k_x x + k_y y + \varphi_3), \\
B_y^{(4)} &= k_x A_4 \sin(-k_x x - k_y y + \varphi_4).
\end{align*}

As in the previous case, let us write down the expansions

\begin{align*}
B_y^{(1)} &= \cos \alpha \left[-p_1 \cdot \cos(k_x x) \cos(k_y y) - q_1 \cdot \cos(k_x x) \sin(k_y y) - \\
&\quad -q_1 \cdot \sin(k_x x) \cos(k_y y) + p_1 \cdot \sin(k_x x) \sin(k_y y)\right], \\
B_y^{(2)} &= \cos \alpha \left[-p_2 \cdot \cos(k_x x) \cos(k_y y) + q_2 \cdot \cos(k_x x) \sin(k_y y) - \\
&\quad -q_2 \cdot \sin(k_x x) \cos(k_y y) - p_2 \cdot \sin(k_x x) \sin(k_y y)\right], \\
B_y^{(3)} &= \cos \alpha \left[p_3 \cdot \cos(k_x x) \cos(k_y y) + q_3 \cdot \cos(k_x x) \sin(k_y y) - \\
&\quad -q_3 \cdot \sin(k_x x) \cos(k_y y) + p_3 \cdot \sin(k_x x) \sin(k_y y)\right], \\
B_y^{(4)} &= \cos \alpha \left[p_4 \cdot \cos(k_x x) \cos(k_y y) - q_4 \cdot \cos(k_x x) \sin(k_y y) - \\
&\quad -q_4 \cdot \sin(k_x x) \cos(k_y y) - p_4 \cdot \sin(k_x x) \sin(k_y y)\right].
\end{align*}

Substituting these expansions into (12) we get

\[ B_y = V_1 \cdot \cos(k_x x) \cos(k_y y) + V_2 \cdot \cos(k_x x) \sin(k_y y) + \\
+V_3 \cdot \sin(k_x x) \cos(k_y y) + V_4 \cdot \sin(k_x x) \sin(k_y y) = \\
= \sum_{i} B_y^{(i)} = \cos \alpha \cdot (-p_1 - p_2 + p_3 - p_4) \cdot \cos(k_x x) \cos(k_y y) + \\
+\cos \alpha \cdot (-q_1 + q_2 + q_3 - q_4) \cdot \cos(k_x x) \sin(k_y y) + \\
+\cos \alpha \cdot (-q_1 - q_2 - q_3 + q_4) \cdot \sin(k_x x) \cos(k_y y) + \\
+\cos \alpha \cdot (p_1 - p_2 + p_3 - p_4) \cdot \sin(k_x x) \sin(k_y y). \]

Over again, there can be written the system of four equations to determine eight unknowns

\begin{align*}
-p_1 - p_2 + p_3 - p_4 &= V_1 / \cos \alpha, \\
p_1 - p_2 + p_3 - p_4 &= V_2 / \cos \alpha, \\
-q_1 + q_2 + q_3 - q_4 &= V_3 / \cos \alpha, \\
q_1 + q_2 + q_3 + q_4 &= -V_4 / \cos \alpha.
\end{align*}

The solution of this system is as follows
\[
\begin{align*}
    p_3 - p_2 &= (V_1 + V_4)/(2\cos\alpha), \\
p_1 - p_4 &= (V_4 - V_1)/(2\cos\alpha), \\
q_1 + q_4 &= -(V_2 + V_3)/(2\cos\alpha), \\
q_2 + q_3 &= (V_2 - V_3)/(2\cos\alpha).
\end{align*}
\]

Summing up the equalities of this system along with the equations (8) one obtains
\[
\begin{align*}
p_1 &= \frac{1}{4}(R_1 - R_4 + \frac{V_4 - V_1}{\cos\alpha}), \\
p_2 &= \frac{1}{4}(R_1 + R_4 - \frac{V_1 + V_4}{\cos\alpha}), \\
p_3 &= \frac{1}{4}(R_1 + R_4 + \frac{V_4 - V_1}{\cos\alpha}), \\
p_4 &= \frac{1}{4}(R_1 - R_4 + \frac{V_1 + V_4}{\cos\alpha}), \\
q_1 &= \frac{1}{4}(R_2 + R_3 - \frac{V_2 + V_3}{\cos\alpha}), \\
q_2 &= \frac{1}{4}(R_3 - R_2 + \frac{V_2 - V_3}{\cos\alpha}), \\
q_3 &= \frac{1}{4}(R_2 - R_3 + \frac{V_2 - V_3}{\cos\alpha}), \\
q_4 &= \frac{1}{4}(-R_2 - R_3 - \frac{V_2 + V_3}{\cos\alpha}).
\end{align*}
\]

Now, as in the previous case, one can get the wave amplitudes \(A_i\) and the angles \(\varphi_i\) by the formulas (11) and (12).

3. Algorithm implementation

Let us show the algorithm implementation on some examples.

3.1. Example 1. Using the formulas (7), (9) and (13), let us assign the sum of four waves having numbers \(|n_x| = 7, |n_y| = 3\) and the following amplitudes and phase shifts:
\[
A_1 = 2, \quad A_2 = 2.5, \quad A_3 = 3.5, \quad A_4 = 3; \\
\varphi_1 = 0.1, \quad \varphi_2 = 0.2, \quad \varphi_3 = 0.3, \quad \varphi_4 = 0.4.
\]

The results of the implementation of the described algorithm for the current example are shown on figure 1. Here the axis are the harmonic numbers.

![Figure 1. Amplitudes of electromagnetic waves.](image)

Let us note in this example two couples of waves are set and they move in different directions:

1) \((n_x, n_y) = (7,3)\) and \((n_x, n_y) = (-7, -3)\);
2) \((n_x, n_y) = (7, -3)\) and \((n_x, n_y) = (-7, 3)\).

From the figure we can see that all waves have been broken apart as it should be and the wave amplitudes coincide with the given one.

3.2. Example 2. Let us consider the algorithm implementation for the problem on the motion of an electromagnetic pulse in the vacuum. The problem statement is as follows. At initial time an electromagnetic pulse with the circular polarization enters the computational domain through the left boundary \(x = 0\) and moves in the direction of the right boundary, and, then, leaves the domain. There is introduced the uniform rectangular grid in the computational domain. The equations (1) and (2) are solved by the FDTD scheme [6] with the steps \(h_x = h_y = 0.05\). The time step is \(\tau = 0.005\). There are the periodic boundary conditions on the boundary \(y = 0\) and \(y = 15\). On the boundary \(x = 0\) the functions \(E_x, E_y, E_z, B_x, B_y, B_z\) are set, and on the boundary \(x = 8\) there are the nonreflecting Mur boundary conditions of the first order [5]. The density of the electromagnetic wave energy at time \(t = 12.5\) is presented on figure 2.

![Figure 2. Density of the electromagnetic wave energy.](image)

Figure 3 shows the dependence of the full energy of the electromagnetic field in the computational domain depending on time.

![Figure 3. Full energy of the electromagnetic field.](image)

Here one can see that up to time \(t = 6\) there are no electric and magnetic fields. To the moment of time \(t = 9\) the pulse has entered the domain completely, and up to time \(t = 16\) it has been in the domain. Then, to the moment of time \(t = 19\) it has left the domain in full. However, some electromagnetic energy has been left there (approximately 0.6% of the full pulse energy).
Let us consider what wave structure was at different times. Figure 4 represents the amplitudes of the electromagnetic waves for the system of equations (1) at time \( t = 12.5 \) (for the system (2) the representation is similar).

![Figure 4. Wave amplitudes and contours at time \( t = 12.5 \).](image)

One can see that, despite of the main energy is concentrated in the waves moving in the positive direction of the axis \( x \), there are waves moving in the opposite direction. Their energy is small, and from figure 2 the presence of such waves is imperceptible.

After the electromagnetic pulse leaves the computational domain, there is a small quantity of the electromagnetic energy left, the distribution of which by the wave numbers is shown on figure 5.

![Figure 5. Wave amplitudes at time \( t = 25 \).](image)

3.3. Example 3. Let us consider the implementation of the Mur boundary conditions on the example given in [5]. In the 2D vacuum domain \((0 \leq x \leq 3.5, 0 \leq y \leq 3.5)\) at the point \((x_0 = 0.5, y_0 = 0.5)\) there is the current source \( I_z = C \sin(2\pi t/\lambda) \) exciting the electromagnetic field in the whole computational domain. At initial time there is no electromagnetic fields. The Maxwell equations are solved by the same way as in the previous example. There are nonreflecting Mur conditions of the first order on all boundaries. The space steps are the same and equal to \( h_x = h_y = 0.05 \), the time step is \( \tau = 0.002 \). The computation had been carried out up to time \( t = 1.6 \). The component of the electric field intensity \( E_z \) at this time has the structure shown on figure 6.
Figure 6. Electric field intensity $E_z$ at time $t = 1.6$.

From this figure we can see that the wave has not reached all boundaries. The analysis of the electromagnetic waves, excited in the computational domain, gives the following picture (see figure 7).

Figure 7. Surface and contours of the electromagnetic wave amplitudes at time $t=1.6$ when using the Mur boundary conditions.

Figure 8. Surface and contours of the electromagnetic wave amplitudes when using the exact boundary conditions.
It can be seen that there is the predominance of the waves moving in the positive direction of the axis $x$ and $y$. It means that the Mur boundary conditions of the first order do not let pass the electromagnetic waves well, i.e. reflect them partially. When there are no influence of the boundary conditions, the structure is more symmetric (see figure 8). The absence of the circular symmetry is due to the scheme noninvariance by directions.

**Conclusion**

The method of computation of electromagnetic wave amplitudes for 2D grid functions has been considered in the paper. This method permits, using the values of electric and magnetic fields at some moment of time, to obtain amplitudes and phase shifts for all waves. Also, it allows to separate the waves having equal wave vectors but moving in different directions. The implementation of the algorithm has been demonstrated on a few examples.

**Acknowledgements**

The work has been supported by the Russian Science Foundation (RSF) under the grant 16-11-10028.

**References**

[1] Timofeev I.V., Annenkov V.V., Volchok E.P. Generation of highfield narrowband terahertz radiation by counterpropagating plasma wakefields // Physics of Plasmas, 2017, v. 24, p. 103106.

[2] Nuter R., Grech M., Martinez P., Bonnau G., Humieres E. Maxwell solvers for the simulations of the laser-matter interaction // Eur. Phys. J. D (2014) 68: 177.

[3] Vranic M., Martins J.L., Fonseca R.A., Silva L.O. Classical radiation reaction in particle-in-cell simulations // Computer Physics Communications, v. 204, 2016, pp. 141-151.

[4] Abarbanel S., Gottlieb D., J.S. Hesthaven J.S. Long Time Behavior of the Perfectly Matched Layer Equations in Computational Electromagnetics // Journal of Scientific Computing, December 2002, Volume 17, Issue 1–4, pp. 405–422.

[5] Mur G. Absorbing Boundary Conditions for the Finite-Difference Approximation of the Time-Domain Electromagnetic-Field Equations // IEEE Transactions on Electromagnetic Compatibility, Volume: EMC-23, Issue: 4, Nov. 1981.

[6] Taflove A., Hagness S. C. Computational Electrodynamics: The Finite-difference Time-domain Method. — Artech House, 2005. — ISBN 978-1-58053-832-9.

[7] Korn G., Korn T. Mathematical Handbook for Scientists and Engineers // Published by McGraw-Hill (1961).