Magnetic flux detection with an Andreev Quantum Dot

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The charge of the subgap states in an Andreev quantum dot (AQD; this is a quantum dot inserted into a superconducting loop) is very sensitive to the magnetic flux threading the loop. We study the sensitivity of this device as a function of its parameters for the limit of a large superconducting gap ∆. In our analysis, we account for the effects of a weak Coulomb interaction within the dot. We discuss the suitability of this setup as a device detecting weak magnetic fields.

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Introduction. The Josephson effect [1] has been intensively studied during the past 45 years; its main characteristic is the presence of a tunable non-dissipative current when two bulk superconductors are joined via a normal or insulating layer and subjected to a superconducting phase difference \( \varphi \). Recently, it has been realized that in a metallic junction the charge of the normal island in between the superconducting leads depends on \( \varphi \) as well [2, 3]. This dependence is sufficiently strong [3] to use this effect in a magnetic flux detector, although our estimates below give a sensitivity somewhat below the sensitivity of the best SQUIDs.

Usually, small magnetic fields are measured by superconducting quantum interference devices (SQUIDs) [4, 5]. While SQUIDs are based on the dependence of the Josephson current on the superconducting phase difference \( \varphi \) (and hence on the magnetic flux \( \Phi \) threading the loop), here we propose to use the charge-dependence in an Andreev quantum dot for the flux measurement. As shown in Ref. [3], the charge \( Q \) of a single-channel Andreev quantum dot can be fractional \( -|e| < Q < |e| \) and depends on \( \varphi \) (here \( e = -|e| \) is the charge of one electron).

The charge of an Andreev quantum dot can be measured by a sensitive charge detector, e.g., by a single-electron transistor (SET). Today, the best single electron transistors have a sensitivity of the order of \( 10^{-5} |e|/\sqrt{Hz} \) (e.g., see [6]). Using results of Ref. [3], simple estimates tell that an AQD can convert a change in flux \( \delta \Phi \) to a change in charge \( \delta Q \) with a ratio \( \delta Q/\delta \Phi \sim 2|e|/\Phi_0 \), where \( \Phi_0 = 2\pi \hbar/2|e| \) is the superconducting flux. Assuming a superconducting loop area \( \sim 1 \text{ mm}^2 \), we obtain the sensitivity \( 10^{-14} \text{T}/\sqrt{Hz} \), which is comparable with the sensitivity \( 10^{-14} \div 10^{-15} \text{T}/\sqrt{Hz} \) of today’s best SQUIDs [4, 5]. Below, we study in detail the sensitivity ratio \( \delta Q/\delta \Phi \).

Setup. Our Andreev quantum dot is realized by a small metallic dot connecting two superconducting banks joined in a loop, see Fig. 1. Our AQD is assumed to be a quasi one-dimensional normal metal (N) island separated from the superconductors (S) by thin insulator layers (I), generating normal scattering on top of the Andreev scattering characteristic of the normal-metal superconductor junction. The position of the normal resonance in this SINIS system can be tuned by the gate voltage \( V_g \) applied to the normal region of the AQD. The magnetic flux threading the loop \( \Phi \) induces a superconducting phase drop \( \varphi \) across the AQD. Since

![Fig. 1. Andreev quantum dot inserted into the superconducting loop. The Andreev quantum dot is connected to a single electron transistor (SET) and a gate electrode through capacitive couplings. The flux \( \Phi \) produces a phase difference \( \varphi = 2\pi \Phi/\Phi_0 \) across the Andreev quantum dot. The charge of the AQD can be tuned by the gate voltage \( V_g \) and the flux \( \Phi \) threading the loop.](image-url)
the phase drop in the bulk superconductor is negligible as compared to the phase drop \( \varphi \) across the AQD one may relate the latter to the flux \( \Phi \) threading the loop, 
\( \varphi = 2\pi \Phi / \Phi_0 \). In order to measure the charge trapped on the AQD, a single electron transistor is capacitively coupled to the normal metal island. Experimentally, such AQDs have recently been fabricated by coupling carbon nanotubes to superconducting banks [7–10]. In the following, we concentrate on the properties of the key element in the setup — the Andreev quantum dot.

**Energy and charge of the AQD without Coulomb interaction** The Andreev states give rise to new opportunities for tunable Josephson devices, e.g., the Josephson transistor [11–13]; here, we are interested in their charge properties. We will consider the case of one transverse channel such that the problem effectively becomes one dimensional. We consider the case of a large separation \( \delta_n \) between the resonances in the associated NININ problem (where the superconductors S have been replaced by normal metal leads N), \( \delta_n \gg \Delta \), such that a single Andreev level \( \varepsilon_{\alpha} \) is trapped within the gap region. We are interested in sufficiently well isolated dots with a small width \( \Gamma_N \) of the associated NININ resonance, \( \Gamma_N \ll \Delta \). In this section, we neglect charging effects \( E_C = 0 \). In summary, our device operates with energy scales \( \Gamma_N \ll \Delta \ll \delta_n \).

The resonances in the NININ setup derive from the eigenvalue problem \( \mathcal{H}_0 \Psi = E \Psi \) with \( \mathcal{H}_0 = -\hbar^2 \partial_x^2 / 2m + U(x) - \varepsilon_{\varphi} \) with the potential \( U(x) = U_{ps,1}(x + L/2) + U_{ps,2}(x - L/2) + eV_g \theta(L/2 - |x|) \) describing two point-scat terners\(^2\) (with transmission and reflection amplitudes \( T_{1/2}^{\varepsilon_{\alpha}} \), \( R_{1/2}^{\varepsilon_{\alpha}} \); \( R_t = 1 - T_t \), \( t = 1, 2 \)) and the effect of the gate potential \( V_g \), which we assume to be small as compared to the particle’s energy \( E \) (measured from the band bottom in the leads), \( eV_g \ll E \).

Resonances then appear at energies \( E_n = \varepsilon_L (n\pi - \chi_1^2/2 - \chi_2^2/2)^2 \); they are separated by \( \delta_n^n = (E_{n+1} - E_n - 1/2) \approx 2E_n/n \) and are characterized by the width \( \Gamma_n^n = T\delta_n^n / \pi \sqrt{R} \), where \( \varepsilon_L = \hbar^2 / 2mL^2 \). The bias \( V_g \) shifts the resonances by \( eV_g \); we denote the position of the \( n \)-th resonance relative to \( \varepsilon_{\varphi} \) by \( \varepsilon_{\alpha}^n = E_n + eV_g - \varepsilon_{\varphi} \). In the following, we choose a specific resonance in the gap by selecting an appropriate \( n \) and drop the index \( n \), \( \varepsilon_{\alpha}^n \rightarrow \varepsilon_{\alpha}, \delta_{\alpha}^n \rightarrow \delta_{\alpha}, \Gamma_{\alpha}^n \rightarrow \Gamma_{\alpha} \).

We go from a normal- to an Andreev dot by replacing the normal leads with superconducting ones. In order to include Andreev scattering in the SINIS setup, we have to solve the Bogoliubov-de Gennes equations (we choose states with \( \varepsilon_{\alpha} \geq 0 \))

\[
\begin{bmatrix}
\hat{\mathcal{H}}_0(x) & \hat{\Delta}(x) \\
\hat{\Delta}^*(x) & -\hat{\mathcal{H}}_0(x)
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
= \varepsilon_{\alpha}
\begin{bmatrix}
u \\
v
\end{bmatrix},
\]

(1)

with the pairing potential \( \hat{\Delta}(x) = \Delta \theta(-x - L/2)e^{-ix/2} + \theta(x - L/2)e^{ix/2} \); \( u(x) \) and \( v(x) \) are the electron- and hole-like components of the wave function. The discrete states trapped below the gap derive from the quantization condition (in Andreev approximation)

\[
(R_1 + R_2) \cos(2\pi \varepsilon_{\alpha} / \delta_{\alpha}) - 4\sqrt{R_1 R_2} \sin^2 \alpha \cos(2\pi \varepsilon_{N} / \delta_{N}) + 2T_1 T_2 \cos \varphi = \cos(2\alpha - 2\pi \varepsilon_{\alpha} / \delta_{\alpha}) + R_1 R_2 \cos(2\alpha + 2\pi \varepsilon_{\alpha} / \delta_{\alpha}).
\]

The phase \( \alpha = \arccos(\varepsilon_{\alpha}/\Delta) \) is acquired at an ideal NS boundary due to Andreev reflection with \( \varphi = 0 \); the above formula can be directly obtained using results from Refs. [13,14].

We concentrate on the regime \( \Gamma_N, |\varepsilon_{\varphi}| \ll \Delta \), the so-called \( \Delta \rightarrow \infty \) limit. In this limit, the quantization condition can be expanded and we obtain the expression (\( A \) is the asymmetry parameter)

\[
\varepsilon_{\alpha} = \sqrt{\varepsilon_{\varphi}^2 + A^2}, \quad A = \frac{|T_1 - T_2|}{2\sqrt{T_1 T_2}}.
\]

The energy \( \varepsilon_{\alpha} \) of the Andreev state is phase sensitive when \( \varepsilon_{\varphi} \) is close to the chemical potential, \( |\varepsilon_{\varphi}| \lesssim \Gamma_N \), which can be achieved by tuning the gate potential \( V_g \).

In the limit \( \Delta \rightarrow \infty \), both the \( u(x) \) and \( v(x) \) components of the wave function are nonzero only in the normal region,

\[
\begin{array}{c}
\begin{bmatrix}
u(x) \\
v(x)
\end{bmatrix} = \begin{cases}
0, & |x| > L/2, \\
\begin{cases}
C_{e}^{-} e^{ik_{e} x} + C_{e}^{+} e^{-ik_{e} x}, & |x| < L/2,
C_{h}^{+} e^{ik_{h} x} + C_{h}^{-} e^{-ik_{h} x},
\end{cases}
\end{cases}
\end{array}
\]

where \( k_{e,h} = [2m(\varepsilon_{\varphi} \pm \varepsilon_{\alpha})]^{1/2} / \hbar \) are the wave vectors of electrons and holes, respectively. The coefficients are defined by \( C_{e}^{\pm} = C_{e,h}^{\pm} = [(1 \pm \varepsilon_{\alpha} / \varepsilon_{\varphi})/4L]^{1/2} \).

The ground state of the system is the state |0\rangle with energy

\[
\varepsilon_0 = \varepsilon_{N} - \varepsilon_{\alpha}
\]

(5)

(counted from the Fermi energy \( \varepsilon_{\varphi} \)), where we have subtracted the energy of filled resonances below the Fermi surface; the latter are not modified by the superconductivity in the leads and hence do not depend on the phase \( \varphi \). The first excited state with one Bogoliubov

\[\text{')}\]
quasiparticle is doubly degenerate in spin \( |1\rangle = \hat{a}^\dagger_{\uparrow}|0\rangle \) and \( |1\rangle = \hat{a}^\dagger_{\downarrow}|0\rangle \) and has energy \( \epsilon_1 = \epsilon_0 + \epsilon_\Lambda = \epsilon_N \). The doubly excited state with two quasiparticles \( |2\rangle = \hat{a}^\dagger_{\uparrow}\hat{a}^\dagger_{\downarrow}|0\rangle \) has an energy \( \epsilon_2 = \epsilon_0 + 2\epsilon_\Lambda = \epsilon_N + \epsilon_\Lambda \).

The charge of the state \( |\nu\rangle \) (\( \nu = 0, 1, 1, 2 \)) can be obtained by differentiation of the corresponding energy \( \epsilon_\nu \) with respect to the gate voltage, \( q_\nu = \partial \epsilon_\nu / \partial V \), or by averaging the charge operator \( \bar{Q} = e \int_{-L/2}^{L/2} \tilde{\Psi}_\sigma(x) \tilde{\Psi}_\sigma(x) \, dx \) over the state \( |\nu\rangle \), \( q_\nu = \langle \nu | \bar{Q} | \nu \rangle \). Both methods give the identical results

\[
q_0 = e \left( 1 - \frac{\xi_N}{\xi_\Lambda} \right), \quad q_1 = e, \quad q_2 = e \left( 1 + \frac{\xi_N}{\xi_\Lambda} \right). \quad (6)
\]

Below, we will also need the off-diagonal matrix elements of the charge operator \( \bar{Q} \); the only non-vanishing term is \( q_{02} = \langle 0 | \bar{Q} | 2 \rangle = e(1 - \xi_N^2 / \xi_\Lambda^2)^{1/2} \).

**AQD with Coulomb interaction** In order to find the effect of weak Coulomb interaction \( E_C \ll \Delta \) in the limit \( \Gamma_N, |\xi_N| \ll \Delta \), we can disregard the continuous states with energies above the superconducting gap \( \Delta \) and assume that the four levels of the discrete spectrum form the entire basis of the system’s Hilbert space. The interaction is given by the operator

\[
\hat{V} = E_C \frac{\hat{Q}^2}{e^2}. \quad (7)
\]

Given the basis with these four states, we can diagonalize the Hamiltonian exactly. The non-zero matrix elements of the operator \( \hat{V} \) are

\[
V_{00} = E_C (q_0^2 + q_\alpha^2) / e^2, \quad V_{11} = E_C,
\]

\[
V_{22} = E_C (q_2^2 + q_\alpha^2) / e^2, \quad V_{02} = 2E_C q_\alpha / e. \quad (8)
\]

The energy levels are defined by the eigenvalue problem

\[
\begin{pmatrix}
\hat{\epsilon}_0 - E \\
\hat{\epsilon}_{1\uparrow} - E \\
\hat{\epsilon}_{1\downarrow} - E \\
\hat{\epsilon}_2 - E
\end{pmatrix}
= \begin{pmatrix}
V_{00} & V_{02} & 0 & 0 \\
V_{10} & V_{11} & 0 & 0 \\
V_{20} & V_{22} & 0 & 0 \\
0 & 0 & 0 & V_{02}
\end{pmatrix}
\begin{pmatrix}
D_0 \\
D_1 \\
D_2 \\
D_1^\ast
\end{pmatrix}
= 0,
\]

where \( \hat{\epsilon}_\nu = \epsilon_\nu + V_{\nu \nu}, \) \( \nu = 0, 1, 1, 2 \). The energy of the level with one Bogoliubov quasiparticle \( |1\rangle \) is given by the (shifted) constant

\[
E_1 = \epsilon_N + E_C, \quad (9)
\]

and does not mix with the other states; furthermore, the spin degeneracy of this Kramers doublet remains. The ground state \( |0\rangle \) and the doubly excited state \( |2\rangle \) mix due to Coulomb interaction and produce two new states, the singlet states \( \langle - \rangle \) and \( \langle + \rangle \); \( |\pm\rangle = D_0^\dagger |0\rangle + D_2^\dagger |2\rangle \), \( D_0^\dagger / D_2^\dagger = -V_{02}/(\hat{\epsilon}_0 - E_\pm) \), \( |D_0|^2 + |D_2|^2 = 1 \). The energies of these new states are

\[
E_\pm = \epsilon_N + 2E_C \pm \sqrt{(\epsilon_N + 2E_C)^2 + \epsilon_\varphi^2}. \quad (10)
\]

The energies of the doublet and singlet states depend on \( \epsilon_N \) and \( \varphi \) in a different way and may cross; thus the ground state can be formed by either the singlet \( \langle - \rangle \) or by the doublet \( \langle + \rangle \). The state \( \langle + \rangle \) always remains the second excited state, see Fig. 2. When \( E_C > \epsilon_\varphi \geq \Gamma_N A / 2 \equiv E_C^\ast \) (with \( A \) the asymmetry parameter) the ground state is the doublet \( |1\rangle \) in the region

\[
-2E_C - \sqrt{E_C^2 - \epsilon_N^2} < \epsilon_N < -2E_C + \sqrt{E_C^2 - \epsilon_\varphi^2} \quad (11)
\]

and remains \( \langle - \rangle \) at all other values of \( \epsilon_N \) [15].

The origin of this level crossing can be traced to the different shifts in energies with \( E_C \). While \( E_1 \) is shifted up by \( E_C \), \( E_- \) quickly approaches 0 with increasing \( E_C \).

\footnote{In realistic nanodevices the Coulomb energy can be larger then \( \Gamma_N \) and smaller or of the order of \( \delta_N \), but in principle can be made much smaller than both \( \delta_N \) and \( \Delta \) (see the discussion in [3, 7]).}
Note that the terms \( \propto q_0^2 \) and \( \propto q_0 \) in the matrix elements \( V_{\mu \nu} \) lead to the crossing of the energies \( E_- \) and \( E_1 \), while preventing the crossing of the level \( E_+ \) with the others.

At the edge of the region \([11]\) a sharp singlet to doublet crossover takes place, with a jump appearing as a function of \( \varepsilon_N(V_g) \) or \( \varepsilon_\Gamma(\varphi) \) in the charge of the Andreev dot and in the current across (see below). The charges of the new states \(|\mu\rangle\) (\( \mu = 1, \pm \)) can be calculated as in the previous section, \( Q_\mu = \partial E_\mu / \partial V_g \), and are given by

\[
Q_\pm = e \left( 1 \pm \frac{\varepsilon_N + 2E_C}{\sqrt{(\varepsilon_N + 2E_C)^2 + \varepsilon_\Gamma^2}} \right), \quad Q_1 = e. \tag{12}
\]

The charge \( Q_1 \) is integer and does not fluctuate; the charges \( Q_\pm \) are fractional in the region \( |\varepsilon_N + 2E_C| \sim \varepsilon_\Gamma \) and fluctuate strongly (see also the discussion of fluctuations in Ref. [3] where Coulomb effects have been ignored)

\[
\delta Q_\pm \equiv |(\pm \hat{Q}^2)| - (\pm \hat{Q})^2|^{1/2} = e \frac{\varepsilon_\Gamma}{\sqrt{(\varepsilon_N + 2E_C)^2 + \varepsilon_\Gamma^2}}. \tag{13}
\]

Note that the Coulomb interaction merely shifts the regime of \( \varepsilon_N \) where the charges \( Q_\pm \) are fractional. Everywhere outside the doublet region the ground state charge is given by \( Q_- \), while within the Kramers doublet region the charge is pinned to the value \( Q_1 = e \). As illustrated in Figs. 3a and 3b, for \( E_C > E_C^* \) a sharp crossover occurs and the charge jumps by the value \( \delta Q_{\text{cr}} = Q_- - Q_1 \). This jump is smeared at finite temperatures, see Figs. 3c and 3d.

The groundstate charge at finite temperature \( \Theta \) is

\[
Q_{\text{gs}} = e - \frac{e}{E_A} \Theta [E_C < E_A],
\]

where \( E_A = [(\varepsilon_N + 2E_C)^2 + \varepsilon_\Gamma^2]^{1/2} \) and \( E_{\text{gs}} = \varepsilon_N + 2E_C \) denotes the energy of the shifted normal state resonance. The equilibrium charge at finite temperature \( \Theta \) is

\[
Q_{\text{eq}} = \frac{Q_- e^{-E_-/\Theta} + 2Q_1 e^{-E_1/\Theta} + Q_+ e^{-E_+/\Theta}}{e^{-E_-/\Theta} + e^{-E_1/\Theta} + e^{-E_+/\Theta}}; \tag{15}
\]

here and below we set Boltzmann’s constant \( k_B = 1 \).

The currents in the states \(|\mu\rangle\) are defined by relationship \( J_\mu = \partial E_\mu / \partial \Psi \) which provides the results

\[
J_\pm = \frac{2\pi \Gamma_\Psi^2 \sin \varphi}{\Phi_0} \frac{\Theta}{16E_A}, \quad J_1 = 0. \tag{16}
\]

The groundstate current is

\[
J_{\text{gs}} = \frac{2\pi \Gamma_\Psi^2 \sin \varphi}{\Phi_0} \frac{\Theta [E_C < E_A]}{16E_A};
\]

note that the current vanishes throughout the doublet region.

The thermal equilibrium current is

\[
J_{\text{eq}} = \frac{J_- e^{-E_-/\Theta} + J_+ e^{-E_+/\Theta} + J_1}{e^{-E_-/\Theta} + e^{-E_1/\Theta} + e^{-E_+/\Theta}}. \tag{18}
\]

**Differential sensitivity** The differential sensitivity of the equilibrium charge to the magnetic flux threading the superconducting loop is defined by the absolute value of the derivative \( \partial Q_{\text{eq}} / \partial \Phi \) taken at the given value of flux, \( J \equiv |\partial Q_{\text{eq}} / \partial \Phi| \). By using (15) we obtain

\[
S = \left| \frac{F_\Theta \frac{\partial F_\Theta}{\partial \Phi} + Q \frac{\partial F_\Theta}{\partial \Phi}}{\Phi_0} \right|. \tag{19}
\]

Note that the sensitivity of the charge-to-flux converter \( S \equiv S_{\phi \to \Psi} \) coincides with the voltage-to-current sensitivity of the Josephson transistor described in Ref. [13] \( S_{V \to I} = |\partial J_{\text{eq}} / \partial V_g| \).
where \( Q \equiv (Q_+ - Q_-)/2 \), the derivative

\[
\frac{\partial Q}{\partial \Phi} = \frac{2\pi}{\Phi_0} \frac{E_0 \Gamma_N^2 \sin \phi}{16E_A^3},
\]

(20)

the function

\[
F_\Phi = \frac{e^{-E_+ / \Theta} - e^{-E_- / \Theta}}{e^{-E_- / \Theta} + 2e^{-E_+ / \Theta} - e^{-E_- / \Theta} + e^{-E_+ / \Theta}} = -\frac{\sinh(E_\Lambda/\Theta)}{\cosh(E_\Lambda/\Theta) + e^{E_\Lambda/\Theta}},
\]

(21)

and its derivative

\[
\frac{\partial F_\Phi}{\partial \Phi} = \frac{e^{E_\Lambda/\Theta} \sinh(E_\Lambda/\Theta) + 1}{[\cosh(E_\Lambda/\Theta) + e^{E_\Lambda/\Theta}]^2} J_-. \tag{22}
\]

As illustrated in Fig. 3 there are two intervals where the \( Q_{\text{eq}}(\phi) \) dependence is steep. As \( \phi \) increases from \( \phi = 0 \), the charge increases (decreases) and reaches a maximum (minimum). For \( E_C < E^*_c \) the maximum (minimum) of the charge is always at \( \phi = \pi \), while for \( E_C > E^*_c \) the extremum splits and a second interval with a steep dependence \( Q_{\text{eq}}(\phi) \) emerges in between the two new extrema. The first interval (interval I in what follows) corresponds to the singlet state of the AQD, the second (interval II in what follows) corresponds to the doublet state. We start with a description of the first interval. We fix the parameters \( \Gamma_N, A, \) and \( E_C \) and search for the maximum sensitivity \( S \) as a function of \( \varepsilon_N \) and \( \phi \). The non-trivial symmetries \( Q_{\text{eq}}(\phi, \varepsilon_N) = Q_{\text{eq}}(2\pi - \phi, \varepsilon_N), Q_{\text{eq}}(\phi, \varepsilon_N) - Q_{\text{eq}}(\phi, 0) = -Q_{\text{eq}}(\phi, \varepsilon_N - 4E_C) + Q_{\text{eq}}(\phi, 0) \) allow us to restrict the search to the region \( 0 \leq \phi \leq \pi, \varepsilon_N + 2E_C \geq 0 \). Subsequently, we analyze the maximum as a function of \( E_C \) keeping \( A \) and \( \Gamma_N \) constant.

**Interval I:** For \( E_C < [3(1 + A^2)/(1 + 2A^2)]^{1/2}E^*_c \) and zero temperature \( \Theta = 0 \) the sensitivity is determined by the derivative \( \partial Q / \partial \Phi \) (20). The function \( \partial Q / \partial \Phi \) has a maximum at \( \varepsilon_N + 2E_C = [(1 + A^2)/(1 + 2A^2)]^{1/2}E^*_c \) and \( \phi = \pi - 2\arcsin[A/(1 + 2A^2)^{1/2}] \), where the differential sensitivity is given by

\[
S_{\text{max}}^I = |e| \frac{2\pi}{\Phi_0} \frac{1}{6\sqrt{3A\sqrt{1 + A^2}}}. \tag{23}
\]

One observes that the smaller \( A \) is, the larger is the sensitivity. In other words, a symmetric SINIS structure provides a better sensitivity \( S_{\text{max}}^I(A \to 0) \to \infty \), but at the same time the region in \( \phi \) with this large sensitivity vanishes as \( A \to 0 \).

In the opposite case \( E_C \geq [3(1 + A^2)/(1 + 2A^2)]^{1/2}E^*_c \) the doublet region covers all of the interval I and the maximum at zero temperature is always realized at the edge of the doublet region \( \{11\} \), with a sensitivity given by

\[
S_{\text{max}}^{II} \approx |e| \frac{2\pi}{\Phi_0} \frac{\Gamma_N^2}{16E_C^3} \times \sqrt{2(\lambda^2 - \lambda + 1)^{3/2} - (\lambda + 1)(\lambda - 2)(2\lambda - 1)} \tag{24}
\]

realized at \( \varepsilon_N + 2E_C = (\Gamma_N/2)[(2\lambda - 1 + (\lambda^2 - \lambda + 1)^{1/2})/3]^{1/2} \) and \( \phi = 2\arccos[\lambda + 1 - (\lambda^2 - \lambda + 1)^{1/2}/2] \), where \( \lambda = (E^2 - E^*_c^2)/\Gamma_N^{1/2} \). This result reduces to

\[
S_{\text{max}}^{II} \approx |e| \frac{2\pi}{\Phi_0} \frac{\Gamma_N^2}{64E_C^3}. \tag{25}
\]

In the limit \( E_C \gg \Gamma_N \), and remains approximately correct for \( E_C \approx \Gamma_N/2 \). For \( E_C \gg \Gamma_N \), the maximum sensitivity is reached at \( \varepsilon_N + 2E_C \approx E_C - \Gamma_N^2/16E_C^3 \), and \( \phi \approx \pi/2 + \Gamma_N^2/16E_C^3 \).

**Interval II:** At zero temperature there is a jump in the charge at the edges of interval II and thus the sensitivity diverges in these points. A finite temperature smears the jump and the sensitivity becomes finite. If \( E_C \gg \Theta, \Gamma_N, E_C \), the sensitivity \( S \) reaches the maximum near the point \( \varepsilon_N + 2E_C = E_C, \phi = \pi/2 \) where it equals to

\[
S_{\text{max}}^{II} \approx |e| \frac{2\pi}{\Phi_0} \frac{\Gamma_N^2}{64E_C^3}. \tag{26}
\]

The expression for \( S_{\text{max}}^{II} \) is too cumbersome for an arbitrary Coulomb energy \( E_C \) and we plot the numerical result \( S_{\text{max}}^{II}(E_C) \) in Fig. 3. In the same plot, we also present the maxima of the sensitivity from the interval I. One easily notes that for a large Coulomb interaction the charge jump smeared by temperature provides the sharper \( Q_{\text{eq}}(\phi) \) dependence.
Conclusion. In this article, we have pointed out that the $\varphi$-dependence of the charge trapped within an Andreev quantum dot may be used for the implementation of a new type of magnetometer which operates along the pathway ‘magnetic flux–AQD charge–SET–current’ instead of the usual direct SQUID scheme ‘magnetic flux–current’. We have analyzed the charge sensitivity as a function of magnetic flux, gate voltage, Coulomb interaction, dot asymmetry, and temperature. The sensitivity of our setup can be further increased by adding an electromechanical element [16]: Applying a large electric field to the charged nanowire, the change in charge will lead to a mechanical shift of the wire. This shift can then be detected due to the change in the capacitance of the compound setup as in Ref [16]. In the present work, we have concentrated on a single-channel wire in order to demonstrate the effect; the case of an $n$-channel wire ($n = 2$ or $n > 2$) can be analyzed using the same technique and we plan to study this case in the near future.

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