Generation of optical solitons produced by QDs propagated in an optical fiber. A theoretical study

L. Espejo-Bayona¹, S D Horta², D A Avila², J. Sierra-Ortega³ and C. O. Torres²
²Student of Mathematics and Physics Undergraduate Academic Program, Popular of Cesar University, Colombia
²Professor of Physics Departament and researcher of Grupo óptica e Informática, Popular of Cesar University, Colombia.
³Grupo de Investigación en Teoría de la Materia Condensada, Universidad del Magdalena, Santa Marta, Colombia

E-mail: shorta@unicesar.edu.co

Abstract. This research, study the generation of optical solitons from an assembly of conical morphology quantum dots of GaAs. The interaction between a linearly polarized plane wave coming from a light source on a quantum dots assembly was considered for the development of the theoretical analysis. The non-linear Schrödinger equation is solved numerically in Matlab and the propagation behavior of the optical solitons in a non-linear optical fiber was described.

1. Introduction
Quantum dots (QDs) are low dimensional semiconductor nanostructures, in which the motion of conduction band electrons, valence band holes, or excitons are restrict in all three spatial directions. The Russian scientist A. I. Ekimov discovered these nanostructures for the first time in 1981, where he reported CuCl quantum dots in a transparent dielectric matrix [1]. Semiconductor materials of groups III, IV and V of the periodic table compose the quantum dots. Its novel properties, which are different from that of bulk materials, have attracted the attention of researchers in the area of low dimensionality systems. At present, the growth of quantum dots in different matrices, such as glasses, solutions, polymers or even cavities of zeolites, has been reported through different manufacturing processes with materials of groups II-VI or I-VII.

On the other hand, optical fibers become one of the most important and effective mechanisms for light waves propagation with low losses, which are an indispensable tool for the propagation of electromagnetic waves in the infrared and ultraviolet range. However, there are other notable applications for optical fibers in the area of sensors and in the study of nonlinear optical phenomena. Nowadays, it has been possible to combine nanostructures with other polymeric materials, such as optical fibers, giving rise to nanocomposites, which generally consist of several phases, such as SiO₂, where one or several of its dimensions are founds at the nanoscale [2-4]. The semiconductor quantum dots embedded in polymeric matrices are of great interest at present in the realization of photonic devices [5], their strong applications are in the manufacture of lasers [6] of low consumption, high optical coherence and quasi light sources monochromatic.

The optical solitons were first observed in mediums that had a high absorption, known as resonant media, but under certain field strengths, the medium became transparent. Currently this field is known
as electromagnetically self-induced transparency SIT. The solitons generated by a third-order susceptibility, that is, when the refractive index is modified with the intensity of light, are known as the Kerr type, and are described by the non-linear Schrödinger equation. In addition, these solitons are subdivided into spatial and temporal, depending on what type of compensation in the beam of light is being given temporary widening or spatial widening. In the study of nonlinear optical phenomena, some investigations have been developed for the propagation of solitonic optical pulses, because they can propagate along nonlinear optical media with low losses. For the understanding of the nonlinear optical phenomena that a wave can experience in an optical fiber, it is necessary to consider a theory of propagation of waves in dispersive media, in this sense, the non-linear equation of Schrödinger NLSE provides a complete description of a variety of localized non-linear effects that have been widely studied in various contexts of science and whose theory can be directly applied to the propagation of intense optical pulses in non-linear optical fibers that give rise to optical solitons. At the present, different solitonic solutions of the NLSE equation have been theoretically studied, among which are solutions of type Akhmediev, Peregrine, Kuznetsov-Ma, among others, and at the same time it has been reported the observation of certain types of finite base solitons. Solitons in finite background SFB in optical fibers, as solitons of the Kuznetsov-Ma type. Although there may be some experimental advances in the propagation of solitons in optical fibers, it has not been possible to propagate soliton pulses over long distances at a commercial level due to different technical limitations [7, 8].

2. Theoretical Analysis.

Different experimental researchers who implement crystal growth techniques such as MBE, CVD, PLD, electrodeposition have synthesized quantum dots with unusual characteristics where the small size, morphology and choice of the materials that make them, have opened the possibility of implementing this nanostructure in the optical, photonic and communications devices of the future.

When an assembly of quantum dots perturbed by an external laser light source, electronic transitions appear due to the dipole moments present by the interaction with an electromagnetic field, which causes the interaction between the assembly of quantum dots and the optical pulse to increase significantly modifying some properties with interest for applications in the area of nonlinear optics. Semiconductor quantum dots usually have extremely large dipole moments, which makes the non-linear interaction between the quantum dot assembly and optical excitation greater compared to atomic systems. An important consequence in the interaction of intense light with quantum dot systems is the formation of non-linear optical waves such as optical solitons, which were first observed in media with high absorption at specific wavelengths known as media resonant optical, but that at a certain minimum power the medium presented a transparent behavior, this behavior is well known as self-induced transparency (SIT). The SIT effect in the quantum dot systems is different in comparison to the atomic systems since the solitons formation in principle will be linked with the characteristics of the incident light in the assembly of quantum dots, their size, morphology and therefore of the materials that compose.

Investigations reveal that assemblies of quantum dots, when disturbed under certain conditions by an external light source, can generate optical solitons [9 - 11]. To explain this effect, identical GaAs quantum dots, with conical morphology manufactured using the MBE crystal growth technique, will be considered. In the analytical development the quantum dots is considered as a three-level energy system with unique dipolar transitions in a single direction, the Figure 1 shows a scheme of the system.
The electric field of the incident optical pulse in the assembly of quantum dots considered as:
\[ E = \sum_{l=1}^{l} \tilde{E}_l \exp\left[i\left(kz - \partial t\right)\right] \] (1)

The wave equation for the electric field of the plane wave, which affects the assembly of quantum dots and produces a secondary dipolar field that propagates in the same direction as the incident field \( \tilde{E} \) is:
\[-c^2 \frac{\partial^2 E}{\partial z^2} + n_{\text{ref}}^2 \frac{\partial^2 E}{\partial t^2} - \frac{\partial^2 E}{\partial x^2} = -4\pi \frac{\partial^2 P}{\partial t^2} \] (2)

where
\[ \bar{P}(x,z,t) = N \int d\Delta g(\Delta) \langle \tilde{\mu} \rangle(\Delta, x, z, t) + c.c \] (3)

and \( \mu \) the dipole moments of the transitions between the levels. Various alternative methods such as the Bloch Vector, the Bloch optical equations that relate the optical phenomena that occur in this type of transitional problems with the density matrix [5], however, other alternative methods such as the approximation of effective mass, the adiabatic approach help in solving the problem of the quantum dots. The analytical and numerical methods such as the separation of variables and the Split-Step Fourier SSF method respectively allow us to reach the double Sine-Gordon equation, whose solution is possible by changing the variable over time obtaining the solution known as the 2\( \pi \) pulse or solitonic solution
\[ \chi(z,t) = \frac{2\hbar}{\mu_x T} \text{sech} \left( \frac{\xi}{T} \right) \] (4)

with \( T \) the width of the pulse.

3. results.

The theoretical results of equation solitonic that were obtained from the double-Sine-Gordon equation show that the excitation of SQDs from an intense nonlinear wave can produce optical solitons as consequence of the interaction of light with the SQDs. During the non-linear interaction process, the SQDs considered as a three-level energy quantum system, in which the optical transitions are given from the ground state to the excitonic and biexcitonic states. The allowed transitions between the ground state and the excitonic and biexcitonic states have a much lower dipole moment than the transition between the ground state and the background of the exciton band. On the other hand, the characteristics of the light reemitted by the assembly of quantum dots in the form of optical solitons will depend on the intensity of the incident light, whose minimum value to form the optical solitons could be determined specifically depending on the nature of the SQDs. In this way, the soliton remitted will depend on the refractive index of the semiconductor \( n_{\text{ref}} \), the dipole moment
corresponding to the transitions between the base state and the excitonic state or between the excitonic state and the biexcitonic state \( \mu_{ex} \), taking into account that we have considered an energy system of three equidistant levels. On the other hand, there is also a dependence on the external excitation frequency \( f \) and the density of quantum dots \( N \). On the other hand, for the study of the propagation of short optical pulses through non-linear optical fibers, the non-linear Schrödinger NLSE equation is used, which takes into account the effects of the length of the fiber, the dispersion effect of group speed and non-linear optical effects as consequence of the high intensity of light. With respect to the simulation that describes the behavior of the light pulse that emerges from the assembly of quantum dots and that is subsequently coupled to the interior of a non-linear single-mode optical fiber, this phenomenon can be studied through the influence of non-linear, dispersive and attenuation effects, which notoriously influence in the shape of the pulse and its spectrum. This requires solving the equation that governs the propagation of a pulse through a non-linear optical fiber, known as nonlinear Schrödinger's equation NLSE and which is expressed as:

\[
\frac{\partial A}{\partial T} + \alpha + i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - \frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} = i \gamma \left[ |A|^2 A + \frac{i}{\omega_o} \frac{\partial}{\partial T} \left( |A|^2 A \right) - T_\nu A \frac{\partial |A|^2}{\partial T} \right] \tag{5}
\]

where \( A \) represents the envelope of the pulse, \( z \) is the distance propagated along the fiber, \( \beta_2 \) is the second order dispersion parameter, \( \beta_3 \) is the third order dispersion parameter, \( \alpha \) is the absorption coefficient, \( \gamma \) is the non-linear parameter, \( \omega_o \) is the frequency of the carrier and \( T_\nu \) is the non-linear response function. The NLSE requires the application of a numerical technique to determine its solution, in this case, we have chosen the method of Split-Step Fourier for being one of the fastest methods compared to its counterpart: finite difference methods. The SSF method allows obtaining an approximate solution of the equation \( 5 \), assuming the propagation of light over a small distance \( l \), where is assumed that dispersive effects and non-linear effects act independently. In this case, equation \( 5 \) can be expressed in terms of operators in the form \( \partial A / \partial z = (\hat{D} + \hat{N}) A \), where \( \hat{D} \) is the differential operator that contains the dispersive effects and \( \hat{N} \), is the differential operator that contains the non-linear effects. These operators expressed as

\[
\hat{D} = -i \frac{\beta_2}{2} \frac{\partial^2}{\partial T^2} + \frac{\beta_3}{6} \frac{\partial^3}{\partial T^3} - \frac{\alpha}{2} \tag{6}
\]

\[
\hat{N} = i \gamma \left[ |A|^2 A + \frac{i}{\omega_o} \frac{\partial}{\partial T} \left( |A|^2 A \right) - T_\nu \frac{\partial |A|^2}{\partial T} \right] \tag{7}
\]

In the process, we consider that the propagation of light from a distance \( z \) to \( z + l \) is done in two steps: In the first step, non-linearity acts alone and \( \hat{D} = 0 \). In the second step, the dispersion acts and \( \hat{N} = 0 \). In these particular situations, the resulting differential equation has an analytical solution and therefore, the general solution can be approximated by

\[
A(z + l, T) \approx \exp \left( h\hat{D} \right) \exp \left( h\hat{N} \right) A(z, T) \tag{8}
\]

In this case, the exponential operator \( \exp \left( h\hat{D} \right) \) can be evaluated in the Fourier domain using the expression \( \exp \left( h\hat{D} \right) B(z, T) = F^{-1} \exp \left[ hF(D) \right] F_B(z, T) \), where \( F \) represents the transformed Fourier operator. This last step allows the use of the algorithm of the Fast Fourier Transform FFT, allowing the numerical evaluation of the NLS to be fast. The accuracy of the SSF method was improved by replacing the equation \( 8 \) with an approximate expression of the form \([12]\)
The SSF method, has been applied by different authors to solve the NLS, obtaining good results. The numerical technique was applied to the pulse of light emerging from the assembly of quantum dots with specific parameters of the quantum dots, the incident beam and the parameters of the nonlinear optical fiber using a code developed in Matlab and that allowed to obtain the profile of the emerging pulse of fiber. This technique was used to simulate the evolution of solitaires that are re-emitted by the SQD. In the proposal that we propose in this research, we assume that the assembly of quantum dots is coupled to the optical fiber, which allows the light to re-emit by the QDs as a consequence of non-linear interaction between the light and the SQDs is directly coupled to the sea the core of the fiber and in this way the light will be guided by total internal reflection on the interior of the fiber, which can be clearly seen in figure 1. For the simulation, we have considered. In the figure 2, the profile of the soliton re-emitted by the SQDs system for different concentrations of quantum dots is observed. In the results, it is observed that the peak intensity decays at higher density values. An explanation to this effect can realize that it can increase the QDs per unit of volume, it can increase the effects of the absorption of the SQD, re-emitting with a lower intensity. On the other hand, we must bear in mind that the mathematical modeling proposed requires that the density $N_0$ of quantum dots is lower, so that the interactions of QDs in the Hamiltonian are omitted. For the simulation we have proposed SQDs with pyramidal morphology of InAs/GaAs manufactured with a cylindrical symmetry with the parameters: $\eta = 3.3, \mu_z = 1.9 \times 10^{-28} \text{Cm}, \lambda = 850 \text{nm}, \nu = 1.7 \times 10^{8} \text{m}$.

For the simulation of the soliton evolution in the optical fiber we have used the split-step Fourier method on a non-linear standard optical fiber with the parameters: fiber attenuation $\alpha = 0.1 \text{dB/km}$, nonlinear fiber parameter $\gamma = 0.3 \text{W/m}$ and second order dispersion $\beta_2 = -20 \times 10^{-27} \text{m}^2$. From the simulation results, figure 3a shows the input pulse in the fiber while in figure 3b the pulse is observed at a distance traveled of 1000.0 m.
4. Conclusions

Through the present investigation, it can be concluded that it is possible to produce solitonic pulses in quantum dots embedded in nonlinear optical fibers with the purpose of propagating fields without losses over long distances. The analytical treatment formulated in this work, would guarantee the generation of solitonic optical pulses, which would be modulated according to the amplitude of the pulse and its morphology. Likewise, it is proposed the manufacture of this type of nanostructures with different morphologies inserted in optical fibers that allow the propagation of light without losses over long distances.

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