Quantum Theory and the Nature of Gravitation*

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This is an essay sketching the line of thinking which has led the present author to propose the constituent or atomic model of gravitation more than a decade ago. It turns out that viewing the problem of gravitation as a quantum many body problem could be quite useful when addressing some old unsolved problems such as the cosmological constant problem. I have applied this idea in 1996 to the problem of the largest cold gravitating system, the finite Universe itself. The result was the prediction of a small, positive vacuum energy density, now known, after its experimental discovery in 1998, as ‘dark energy’. The smallness of this quantity was understood as the finite size effect in the cold quantum many body system, and I quote here from [5], “The smallness of the cosmological constant in natural Planck units is a

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result of an almost perfect thermodynamical limit. This is to say that the smallness of the cosmological constant is an effect due to an enormous number \( N \) of hypothetical \textit{gravitational atoms}. The present upper bound on the cosmological constant \( \Lambda \) allows us to draw the conclusion about the lower bound on a number of \textit{gravitational atoms} in the observed Universe, \( N \sim 10^{122} \). The large numbers’ coincidences noticed long time ago have something in it after all. We have only one reservation to add: elementary particles are not the same as the hypothetical \textit{gravitational atoms}, and, therefore, have little to do with the cosmological constant. The existence of the latter must be inferred indirectly from phenomena as it was previously done for atoms and elementary particles. We find an unusual coordination between the gravitational atomistic aspect of physical reality in the regime usually called infrared, or large distance scale, and the regime usually called ultraviolet, or short distance scale.” The old cosmological constant problem is a man-made problem because the vacuum energy density has nothing to do with the quartic divergences (zero point energies) in the interacting relativistic quantum field theories (i.e. in the Standard Model of elementary particles). The actual value of the vacuum energy density of the vacuum finite size de Sitter Universe is \( \frac{E_0}{V} \), where \( E_0 \) is the ground state energy of \( N \) \textit{gravitational atoms}, which are spin-zero bosons of mass \( M \sim M_{\text{Pl}} \), contained in the finite volume \( V \).
I wish to describe to you the line of thought which led me to the conclusion that gravitation may, in fact, be the most spectacular demonstration of quantum phenomena on the macroscopic scale.

What is the central idea of this direction of research?

The central idea is that gravitational phenomena are best understood as quantum many body problem. The microscopic theory of gravitation should be based on the idea of *gravitational constituents*. Gravitational constituents are heavy bosons. The bosonic nature of these constituents follows from the nature of gravitation perceived as a macroscopic quantum phenomenon. The appellation ‘heavy’ refers to the extreme feebleness of gravitational forces. A large number of heavy bosons in three spatial dimensions can behave macroscopically in a classical way so that the concept of gravitational field in the large can be defined. The situation here is not unlike the one we have encountered previously in the development of theoretical physics.

Why is the concept of macroscopic electromagnetic fields applicable to quantum states of a large number of photons?

This is because photons are bosons.

Why is this way of looking at gravitation useful and what reasons do we have to think that the constituent picture may be the correct one?

The rationale here is the prediction and explanation of new phenomena some of which have been discovered relatively recently.

Reading Einstein’s *The Meaning of Relativity* one may come to the conclusion that gravitation and hydrodynamics have much in common. The question then was:

What kind of hydrodynamics?

Regarding gravitation as a kind of hydrodynamics led me to look for vortices in the gravitational setting of general relativity \[1, 2\]. My thinking about gravitation in these terms was also strongly influenced by some work on two-dimensional gravitation \[3\] which my PhD Thesis advisor Andrzej Staruszkiewicz has done in the early 60s. It turns out that there are no perturbative excitations (gravitons) in the lower dimensional gravitation. However, one finds that there are non-perturbative excitations such as vortices. This essential difference between Einsteinian gravitation in three spatial dimensions and the Staruszkiewicz gravitation \[3\] on the two-dimensional plane, regarded as an idealization of the thin surface layer...
in three-dimensional space, has alerted me very early on to the possibility that gravitational phenomena may have something to do with properties of some peculiar medium. I started to look for examples of ‘atomic’ media where one sees such dramatic dependence on the dimensionality of space. It is known that a system of heavy bosons interacting via short range forces may undergo Einstein condensation in three spatial dimensions but it cannot condense at even very small temperatures in two spatial dimensions. But how can one show that gravitation has something to do with the presence of heavy bosonic constituents? This is where the study of thermal properties of quantum black holes helps. I have found out that thermodynamical properties of black holes and cosmological spaces with event horizons indeed do suggest the existence of heavy constituents responsible for the feebleness of gravitational forces. It turned out that the low temperature thermal properties of quantum black holes are dominated by the low energy long wavelength collective excitations that behave like gapless phonons. However, this in itself does not imply that the heavy constituents must be fundamental bosons. One can easily envisage the situation where one has composite bosons such as the Cooper pairs in the p- or the d-wave superconductors or in the superfluid He3. The isotropy of the velocity of light puts severe constraints on such scenario. The medium must be isotropic as far as the propagation of light and gravitons is concerned. This follows from the unusually precise tests of special relativity. Hence, one may conclude that there is no emergent gravitation in superfluid phases of He3, contrary to some statements one encounters in the literature.

The origins of the attempt at the microscopic theory of gravitation go back to my studies of gravitational vortices [1, 2], perceived as analogs of Abrikosov vortices in superconductors rather than analogs of the Onsager-Feynman vortices in superfluid He4 [12], and the seemingly separate from it problems of gravitational collapse of matter and the non-vanishing positive vacuum energy density [5]. The latter problem is also known as the cosmological constant problem. The distinction between the two kinds of vortices that may serve as analogs of the gravitational vortex becomes clear-cut once one takes into account the basic fact of experience that there exist inertial frame dragging forces. Already in his first papers on the relativistic theory of gravitation Einstein has asked the following question: Is there an analog of magnetic fields in gravitation? These days we have the name for it: gravimagnetic fields. In fact, the Gravity Probe B mission, launched belatedly in April 2004, is dedicated to the detection of the gravimagnetic field of the Earth. We now know about their presence
in the vicinity of pulsars (neutron stars).

If gravitation has something to do with Einstein condensation of bosons then we should single out the best theoretical laboratory where this hypothesis can be tested. It turns out to be the gravitational vortex which I have discovered long time ago. The breakdown of quantum fluid rigidity (the breakdown of the Einstein condensate of bosons) should then lead to the easily seen pathologies in the general relativity description of physical situations. One does not need to look too far. It is known that general relativity fails in the regions of extremely high tidal forces of the type of a Big Bang or the interior of analytically continued black hole solutions of general relativity. It also fails on the event horizons of black holes and on the cosmological event horizons. A third kind of the failure of general relativity (also on the macroscopic length scales) is associated with the occurrence of closed time-like curves (CTC) or ‘time machines’. CTCs occur frequently in analytically extended space-times described by general relativity once there is rotation present in a physical system under consideration, which is quite common in nature.

These three, seemingly separate until relatively recently, instances of the failure of general relativity are now seen as intrinsically connected. One should always look for other instances of breakdown of a given theory after one finds out that this theory breaks down in one instance. From this point of view it is clear that black hole event horizons and cosmological horizons were to be seen as the natural candidates of instances of such a breakdown of the theory. But this realization has been long in coming. I have reached this conclusion quite early but the actual demonstration of it in technical terms came only in the 90s [4, 5, 6, 7, 8, 9]. The later work on quasi black hole objects (QBHOs or gravastars) has brought the point in to the broader scientific community [10, 11].

It is remarkable that some of my early results on the thermal properties of remnants of gravitational collapse of (quantum) matter and the early prediction of the small and positive vacuum energy density permeating our finite Universe [5, 8], discovered by observational astronomers and sometimes called the Dark Energy, have been noticed by others only relatively recently. This is to say, after the actual discovery of the Dark Energy in 1998. The smallness of this quantity was understood as the finite size effect in the quantum many body system [5, 8], and I quote here from [5], “The smallness of the cosmological constant in natural Planck units is a result of an almost perfect thermodynamical limit. This is to
say that the smallness of the cosmological constant is an effect due to an enormous number $N$ of hypothetical gravitational atoms. The present upper bound on the cosmological constant $\Lambda$ allows us to draw the conclusion about the lower bound on a number of gravitational atoms in the observed Universe, $N \sim 10^{122}$. The large numbers' coincidences noticed long time ago have something in it after all. We have only one reservation to add: elementary particles are not the same as the hypothetical gravitational atoms, and, therefore, have little to do with the cosmological constant. The existence of the latter must be inferred indirectly from phenomena as it was previously done for atoms and elementary particles. We find an unusual coordination between the gravitational atomistic aspect of physical reality in the regime usually called infrared, or large distance scale, and the regime usually called ultraviolet, or short distance scale.”

The old cosmological constant problem is a man-made problem because the vacuum energy density has nothing to do with the quartic divergences (zero point energies) in the interacting relativistic quantum field theories (i.e. in the Standard Model of elementary particles). The actual value of the vacuum energy density of the vacuum finite size de Sitter Universe is $\frac{E_0}{V}$, where $E_0$ is the ground state energy of $N$ gravitational atoms, which are spin-zero bosons of mass $M \sim M_{Pl}$, contained in the finite volume $V$. The tower of excited states of such a system of $N$ gravitational atoms, which merges into the continuum in the thermodynamic limit of $N \to \infty$, $V \to \infty$, and $\frac{N}{V} \to n = \text{const}$, in this limit is most conveniently described by the effective field theory of ‘phonons’ with an almost relativistic dispersion relation. The usual, and incorrect, computation of the contribution of these degrees of freedom to the vacuum energy would involve summing over the zero point energy of ‘phonons’ which is at least quartically divergent. I am sure everybody would agree that such a procedure is equivalent to the double-counting of degrees of freedom and as such has nothing to do with the actual value of the ground state energy of the system. I have put it succinctly in the following phrase [8], I quote, “...DO NOT QUANTIZE WHAT IS ALREADY QUANTIZED...” extracted from a footnote there [14].

This approach to the resolution of the old cosmological constant problem has been later popularized and extended by G. E. Volovik [13].

A connection between the breakdown of quantum fluid rigidity and the appearance of closed time-like curves is most easily seen in the case of the ”spinning cosmic string” or the gravitational vortex solution of the Einstein field equations [1, 2]. It is the solution of
these equations only outside the vortex core; inside the core one finds CTCs. This instance of breakdown of general relativity is also an example of our ideal theoretical laboratory we were looking for. It turns out that the gravimagnetic field associated with the gravitational vortex is essentially the velocity field in the quantum fluid surrounding the vortex core [12]. This velocity satisfies the Onsager-Feynman quantization condition, which actually amounts to the Dirac-like quantization condition for the mass-energy. The quantum fluid velocity becomes comparable to the velocity of sound (the speed of gravity or velocity of light) when the distance from the axis of the vortex is close to the quantum coherence length in the quantum fluid. Therefore quantum fluid rigidity and hydrodynamics (general relativity) breaks down as one enters the core of the vortex. This is closely related to the Landau criterion for the breakdown of superfluidity. Remarkably, this breakdown of a classical description of the quantum fluid seems to be closely related to the breakdown of causality in classical General Relativity associated with the formation of closed time-like curves [12].

The spectacular topological effect of scattering of matter on the gravitational vortex has been found [2, 4]. It was found that for very slow particles the scattering amplitude vanishes precisely for quantized values of the mass-energy of those particles. I have concluded that if such gravitational vortices do exist, and if they have fixed vorticity, then there must exist a quantum of mass. Remarkably one finds here the famous Onsager-Feynman relation which expresses the quantum of vorticity in superfluid He4 in terms of the Planck constant and the mass of a He4 atom. The single parameter which appears in the gravitational vortex metric [1, 2] has a very simple meaning: it is the vorticity of a vortex in some kind of isotropic quantum fluid. The Onsager-Feynman type of quantization condition can be read in the reverse order. From the quantization of vorticity one infers the quantization of mass in the quantum fluid. This is the way one can see the presence of He4 atoms in the superfluid phase. It follows then that there must also exist atoms in the normal phases of He4. The remarkable feature of mass-energy quantization implied by the possible existence of my gravitational vortices (‘spinning cosmic strings’) may also be regarded as the harbinger of the microscopic theory of gravitation.

The hypothesis that all gravitational phenomena, and the recently discovered Dark Energy corresponding to the cosmological vacuum energy density, Cosmic Microwave Background Radiation (CMBR), primordial density fluctuations, and the most remarkable pre-
diction of the closely related phenomenon of global rotation of our finite Universe, should
follow from the existence of the underlying constituent heavy particles is a bold one. It must
be supported by the line of investigation that is independent from the arguments based on
the peculiar properties of gravitational vortices. The phenomenon of gravitational collapse
is an example of such an opening we were looking for.

It is the inconsistency of the fundamental physical principles with the behavior of the
time variable on an analytically extended classical space-times of black holes and cosmolog-
cal models that leads to all the paradoxes of quantum theory applied to black holes and
cosmological models of our Universe, including the so-called Hawking effect of black hole
radiance and the Bekenstein black hole thermodynamics. While this point of view does
not seem to be universally accepted yet it is clear that accepting the possibility of the dis-
appearance of quantum matter past the event horizon during the process of gravitational
collapse and the failure of the existence of the quantum ground state of gravitating matter
is not an option. Indeed, the Bekenstein-Hawking proposal for the black hole entropy, which
essentially corresponds to a classical high-temperature thermodynamical regime, does not
even seem to be compatible with the basic statistical physics one learns from the introd-uc-
tory university textbooks (Landau and Lifshitz, Kittel) because it leads to the occurrence
of negative heat capacities. A quantum many body system is by necessity characterized by
the positive heat capacity, the property intimately related to the unitary evolution of states
of matter in quantum mechanics. The most natural resolution of the paradoxes related to
black hole thermodynamics is the observation that the Bekenstein-Hawking proposal cannot
be correct. I have made this elementary observation a long time ago [5]. This came about as
the result of critical analysis of the above proposal along the lines which will become easily
understood by following the line of thought described below.

One cannot underestimate the importance of the crucial role scientific analogies play in
the process of creative thinking. The situation here should be familiar to all physicists. In
the case of black hole thermodynamics the analogy that invites itself somewhat insistently
is the analogy between the Rayleigh-Jeans description of the cavity black body radiation
and the Bekenstein-Hawking proposal for the thermodynamics of classical black holes. The
reason being simply that each describes the high temperature behavior of a corresponding
system. The role the Maxwell equations play in the derivation of the Rayleigh-Jeans law in
the former case is analogous to that played by the Einstein equations in the latter. Exploring this obvious analogy I followed it to the final conclusion but here I was helped by my earlier work on gravitational vortices mentioned above.

Once one accepts the possibility that the pathological thermodynamical properties of classical black holes cannot be described by a quantum mechanical system there is no other option left but to find a quantum system whose thermal properties at high temperatures would correspond to the Bekenstein-Hawking thermodynamics of the classical black hole once a certain improper order of limits is taken. The paradoxes of BH thermodynamics should be then seen in the completely new light. Indeed, we encounter here an instance of non-commuting limits being taken.

One is therefore led to look for the correct thermodynamics of cold ultra-compact gravitating systems which is compatible with quantum mechanical principles. It is also prudent to be careful how one takes two possible limits here because they do not commute. The simplest way to proceed is to observe that the heat capacity of a system is intrinsically connected with the statistical fluctuations in the observable mass-energy of this system. One computes the statistical fluctuations in the mass-energy of a black hole which follow from the heat capacity suggested by the Bekenstein-Hawking proposal and amends it in the proper way to make them non-negative [5, 8]. The physical picture of the final state of gravitational collapse of matter, a quasi black hole (QBH), which has emerged from these investigations is that of a cold and compact object with a surface layer separating two regions characterized by different values of vacuum energy density whose thermodynamics is dominated by the presence of gapless low energy collective excitations that are not unlike phonons in quantum fluids [5, 8, 9, 10, 11]. These are collective excitations in a medium consisting of heavy gravitational constituents [5, 8].

The corollary of this line of investigation was the conclusion that there should exist other phenomena where the atomic nature of gravitation should demonstrate itself directly. I have immediately applied this idea (in the same 1996 paper) to the problem of the largest cold gravitating system, the finite Universe itself [5, 8]. The result was the prediction of a small, positive vacuum energy density, now known, after its experimental discovery in 1998, as the Dark Energy. The smallness of this quantity was understood as the finite size effect in the quantum many body system [5, 8].
If there indeed exist gravitational constituents then one should explore the implications of this idea to the full extent. New phenomena can be predicted on the basis of a simple hypothesis. I have made a prediction to the effect that if the finite Universe is rotating, which is a natural thing to contemplate because rotation is common in nature, then this effect should be seen as an asymmetry in the CMB spectrum for some broad range of multipoles. This asymmetry should be most prominent for the quadrupole and the octupole, which is to say at the largest distance scales.

After all if our finite Universe rotates then we should be able to detect the presence of an axis of rotation and the resulting asymmetries. The importance of the presence of quantized vortices in a rotating finite Universe cannot be underestimated. They make the whole proposal work once one gets to the technical details.

What motivated me to think about the possibility of a rotating Universe?

The reason for this were three facts of experience. One was that galaxies have angular momentum that seems to be related to their mass in the specific way. The second was the problem of the origin of primordial density fluctuations and the third was that there seems to exist a residual (Birch) rotation of polarization of electromagnetic waves coming from distant radio sources, although the latter continues to be considered controversial.

There was only a small step to make which I have already taken in the 90s. The natural idea that gravitation is the atomic phenomenon which should be best understood as the quantum many body problem requires consideration of the second quantized Schrodinger equation for a system of heavy bosons with its characteristic property which is the presence of the universal time and the quantum entanglement (the EPR property of quantum many body wave functions—the non-factorability of the many-body wave function into the product of single particle ‘orbitals’). One could then start to compute things on the basis of a rather simple idea I have described above.

It appears that gravitation has something to do with the Einstein condensation of massive bosons after all.
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[14] The analogy which invites itself quite naturally is this: in the limit \( \hbar \to 0 \) in quantum statistical mechanics of ideal gases when we neglect the fact of identical nature of quantum particles we obtain the classical Boltzmann gas plagued with its Gibbs paradox and etc.. We should perhaps recall here the story of the now well understood phenomenon of superfluidity in the liquid Helium II. The moral of this very well known story, as told by R. P. Feynman,
is this: **DO NOT QUANTIZE WHAT IS ALREADY QUANTIZED.** This basic observation which was found valid for superfluidity in the past is equally valid today for quantum black holes. Landau’s *quantum hydrodynamics* was only formally ‘quantum’ in view of quantum commutators appearing in the nonrelativistic current algebra. Feynman has discovered that the real problem with *quantum hydrodynamics* was that hydrodynamical description of the superfluid Helium II failed to take into account the Bose statistics of Helium 4 atoms. Now, it is our opinion that we seem to be facing the same dilemma posed by the currently fashionable description of black holes. Incidentally, the cases for quantization of general relativity (GRT) and its later fermionic deformation known as ‘supergravity’ seem to follow the same pattern which was so well understood by Feynman in the context of experimentally rich phenomenon of superfluidity. It seems that general relativity, ‘supergravity’, ‘superstrings’ and recently ‘supermembranes’ were also quantized in the same way the superfluid hydrodynamics was quantized with the well known results. Again, the moral of the story as told by R. P. Feynman when adopted to the present situation of quantum black holes seems to be that we should “give to Caesar what is Caesar’s”. The Atomic Hypothesis and Quantum Statistics rule. Gravitation is the many-body phenomenon. It is our responsibility now to find the physically correct Hamiltonians for systems of gravitational atoms in the framework of the new gravitational noncommutative mechanics.