New channels of prompt-photon production by magnetic fields in heavy-ion collisions

Alejandro Ayala\textsuperscript{1,2}, Jorge David Castaño-Yepes\textsuperscript{1}, Isabel Dominguez Jimenez\textsuperscript{3}, Jordi Salinas San Martín\textsuperscript{1}, María Elena Tejeda-Yeomans\textsuperscript{4*}

\textsuperscript{1}Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Apartado Postal 70-543, México Distrito Federal 04510, Mexico. \textsuperscript{2}Centre for Theoretical and Mathematical Physics, and Department of Physics, University of Cape Town, Rondebosch 7700, South Africa. \textsuperscript{3}Facultad de Ciencias Físico-Matemáticas, Universidad Autónoma de Sinaloa, Avenida de las Américas y Boulevard Universitarios, Ciudad Universitaria, C.P. 80000, Culiacán, Sinaloa, México. \textsuperscript{4}Facultad de Ciencias - CUICBAS, Universidad de Colima, Bernal Díaz del Castillo No. 340, Col. Villas San Sebastián, 28045 Colima, Mexico.

E-mail: *matejeda@ucol.mx

Abstract. In this work I present recent results on the photon yield and elliptic flow coefficient in relativistic heavy-ion collisions for different centralities. In our calculation, we take into account gluon fusion and splitting channels which become available due to an intense and short-lived magnetic field. We show that with reasonable values for the saturation scale, our calculation helps to better describe the experimental results obtained at RHIC energies for the lowest part of the transverse photon momentum at different centralities.

1. Introduction

Photons have proven to be a versatile tool to study relativistic heavy-ion collisions. They are created in various mechanisms occurring from the initial moments of the fireball and through the space-time evolution of this strongly interacting system. Under these conditions, photons are a special probe of the system since they mostly escape without much interaction, but this means that the information they carry is hard to disentangle. One of the current open questions in the field is precisely the puzzling situation in which direct photon yields are large and show strong azimuthal asymmetries, which were both theoretically unexpected [1]. The anomalous excess of direct photons produced in these reactions [2, 3, 4], referred to as the direct photon puzzle, can be studied in terms of the new open channels of photoproduction that become available with the intense magnetic fields generated in peripheral heavy-ion collisions [5]. These magnetic fields become both a source for an excess in photon yield, and they enhance the second harmonic coefficient ($v_2$) of the Fourier expansion in the azimuthal photon distribution [6]. In the literature, there are some hydrodynamic [7, 8] and transport [9] calculations that show partial agreement with ALICE and PHENIX measurements of low and intermediate transverse momentum photons, but further studies are still needed to have a more robust result for different observables [10]. Also, in Au+Au and Cu+Cu collisions at different centralities and beam energies, PHENIX [11] has found that the yield of low $p_T$ photons ($\lesssim 2$ GeV) scales with a given power of $N_{\text{coll}}$, which may mean that the source of these photons is very similar across beam energies and colliding species.
In this work I present a summary of our findings as reported in Refs. [12, 13]. We use UrQMD [14] to obtain the time evolution of the magnetic field and the volume produced by the collision participants and spectators in Au+Au and Cu+Cu collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \) for different centrality ranges. We use these results for the magnetic field and interaction volume to compute both the photon yield and \( v_2 \) through the gluon fusion and gluon splitting channels, and we perform a centrality dependence study for the excess photon yield and \( v_2 \) comparing to the recent PHENIX measurements. We find a relatively good agreement for the low \( p_T \) region which improves for peripheral collisions and in the case where the magnetic field strength includes contributions from both spectators and participants.

2. Gluon fusion and splitting channels in the presence of a strong magnetic field

In this work we include both the gluon fusion and gluon splitting channels, depicted in Table 1, where each row of diagrams includes the different charge flow inside the fermion loop. These diagrams contribute to the total photon production amplitude at lowest order in the strong, \( \alpha_s = g^2/4\pi \), and electromagnetic, \( \alpha_{em} = e^2/4\pi \), couplings. Due to the magnetic field, translational invariance is broken and we calculate the total amplitude for photon production in coordinate space and we later integrate over space-time. I will summarize the results of this calculation but further details can be found in Ref. [12, 13]. We consider a constant magnetic field in the \( z \)-direction, obtained from a vector potential \( A^\mu \) in the symmetric gauge \( A^\mu = \frac{B}{2}(0, -y, x, 0) \). Also, for a four-momentum \( p^\mu \), we have defined \( p^\mu = (0, p_1, p_2, 0) \), \( p_\parallel = (p_0, 0, 0, p_3) \), \( p_\perp^2 = p_1^2 + p_2^2 \), \( p_\parallel^2 = p_0^2 - p_3^2 \), and therefore \( p^2 = p_\parallel^2 - p_\perp^2 \).

Following the Feynman diagrams of Table 1 we can write the expression for the amplitude
\( \widetilde{M}_{gg \rightarrow \gamma} \), and this is explicitly given by

\[
\widetilde{M}_{gg \rightarrow \gamma} = -i(2\pi)^4 \delta^{(4)}(q - k - p) \frac{e_{qf} g_s^2 \delta_{cd} e_f(p_{\perp}, k_{\perp})}{32\pi (2\pi)^6} \left\{ \left( g^{\mu\alpha} - \frac{p_\mu p_\alpha}{p^2} \right) h^\nu(a) \right. \\
- \left( g^{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) h^\alpha(a) + \left( g^{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) h^\alpha(b) - \left( g^{\alpha\nu} - \frac{k_\alpha k_\nu}{k^2} \right) h^\mu(b) \\
+ \left( g^{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) h^{\alpha(c)} - \left( g^{\mu\alpha} - \frac{q_\mu q_\alpha}{q^2} \right) h^\nu(c) \right\} \epsilon_\mu(\lambda_p) \epsilon_\nu(\lambda_k) \epsilon_\alpha(\lambda_q),
\]

where the trace over the Gell-Mann matrices has also been performed and \( h^\mu(a) = -(i/\pi) \epsilon_{ij} a^i g_{j\mu} \), \( a_i = p_i + 2k_i + i\epsilon_{im}p_m, b_i = 2p_i + k_i - i\epsilon_{im}k_m, c_i = k_i - p_i + i\epsilon_{im}(p_m + k_m) \), with \( f(p_{\perp}, k_{\perp}) = \frac{1}{8|e_{qf}|} (p_m - k_m + i\epsilon_{mj}(p_j + k_j))^2 - \frac{1}{2|e_{qf}|^2} (p_m^2 + k_m^2 + 2i\epsilon_{jm}p_m k_j) \), where \( g_\perp = \text{diag}(1, 1) \) and \( g_\parallel = \text{diag}(1, -1) \) are the metric tensors in the transverse and longitudinal spaces.

The matrix element for gluon splitting \( \widetilde{M}_{g \rightarrow g\gamma} \), can be calculated with the diagrams on the right columns of Table 1. This process is related to the gluon fusion channel, \( \widetilde{M}_{gg \rightarrow \gamma} \), by the crossing symmetry \( \widetilde{M}_{g \rightarrow g\gamma}(p, k, q) = \widetilde{M}_{gg \rightarrow \gamma}(p, -k, q) \). Since the processes do not interfere, we sum coherently the two matrix elements squared, obtaining

\[
\sum_{c, p, f} |\widetilde{M}|^2 = V \Delta \tau (2\pi)^4 \sum_{c, p, f} \left[ \delta^{(4)}(q - k - p) |\widetilde{M}_{gg \rightarrow \gamma}|^2 + \delta^{(4)}(q + k - p) |\widetilde{M}_{g \rightarrow g\gamma}|^2 \right],
\]

1 The more general case is currently being explored and will be reported elsewhere.
where
\[
\sum_{c.p,f} |M_{gg\to\gamma}|^2 = \sum_{c.p,f} |M_{g\to\gamma}|^2 = \frac{2\alpha_{em}\alpha_s^2 q_f^2}{\pi\omega_q^2} \sum_f q_f^2 \left( 2\omega_p^2 + \omega_k^2 + \omega_p\omega_k \right)
\times \exp \left\{ -\frac{q_f^2}{(eq_fB)\omega_q^2} \left( \omega_p^2 + \omega_k^2 + \omega_p\omega_k \right) \right\}.
\]

Notice that the factor \(\mathcal{V}\Delta\tau\) in Eq. 3 comes from squaring the delta function for energy-momentum conservation in Eq. (2). This is the space-time volume where the reaction takes place. We will discuss more details about this factor in Sec. 4.

### 3. Photon yield and \(v_2\)

The invariant photon momentum distribution is given by
\[
\frac{dN_{\text{mag}}}{d^3q} = \mathcal{V}\Delta\tau \frac{2}{(2\pi)^3} \int \frac{d^3p}{(2\pi)^32\omega_p} \int \frac{d^3k}{(2\pi)^32\omega_k} \left\{ (2\pi)^4 \delta^{(4)} (q - k - p) n(\omega_p)n(\omega_k) \sum_{c.p,f} |M_{gg\to\gamma}|^2 \right\}
\times \left\{ (2\pi)^4 \delta^{(4)} (q + k - p) n(\omega_p) [1 + n(\omega_k)] \sum_{c.p,f} |M_{g\to\gamma}|^2 \right\},
\]
where \(n(\omega)\) represents the distribution of gluons coming from the shattered glasma. We use for this distribution a simple model that accounts for the high occupation gluon number given by \([15]\) \(n(\omega) = \eta/(e^{\omega/\Lambda_s} - 1)\), where \(\eta\) represents the high gluon occupation factor. So the initial state gluon with energy \(\omega_k\) comes weighed with an occupation factor \(n(\omega_k)\) and the final state one comes weighed with an enhanced occupation factor \(1 + n(\omega_k)\). With this in mind, we finally arrive to the expression we will use to compare with data
\[
\frac{1}{2\pi\omega_q} \frac{dN_{\text{mag}}}{d\omega_q} = \mathcal{V}\Delta\tau \frac{\alpha_{em}\alpha_s^2 \pi}{2(2\pi)^6\omega_q} \sum_f q_f^2 \int_0^{\omega_q} d\omega_p \left( 2\omega_p^2 + \omega_q^2 + \omega_p\omega_q \right) e^{-g_f(\omega_p,\omega_q)}
\times \left\{ I_0 [g_f(\omega_p)] - I_1 [g_f(\omega_p)] \right\} \{ n(\omega_p)n(\omega_q - \omega_p) \} + n(\omega_p) [1 + n(\omega_q - \omega_p)] \right\},
\]
where \(g_f(\omega_p,\omega_q) = (\omega_p^2 + \omega_q^2 - \omega_p\omega_q)(2|eq_fB|)\), and \(I_0, I_1\) are the modified Bessel function of the first kind. Notice that \(\Delta\tau\) in Eq. (6) corresponds to one of the small time intervals that we used to divide the whole time interval \(\Delta T\) over which the magnetic pulse is appreciable. The whole yield thus corresponds to the sum of the yields in each of the small time intervals represented by \(\Delta\tau\). Furthermore, given that the azimuthal distribution with respect to the reaction plane can be written in terms of a Fourier decomposition as
\[
\frac{dN_{\text{mag}}}{d\phi} = \frac{N_{\text{mag}}}{2\pi} \left[ 1 + \sum_{n=1}^{\infty} 2v_n(\omega_q) \cos(n\phi) \right] \quad \text{where} \quad N_{\text{mag}} = \int d^3q \frac{dN_{\text{mag}}}{d^3q},
\]
where
\[
\mathcal{V}\Delta\tau = \left\{ I_0 [g_f(\omega_p)] - I_1 [g_f(\omega_p)] \right\} \{ n(\omega_p)n(\omega_q - \omega_p) \} + n(\omega_p) [1 + n(\omega_q - \omega_p)] \}
\]
Figure 1. On the left, the mean magnetic field strength produced by spectators at the middle of the interaction region as a function of time and on the right, the volume $V$ as a function of time. Both results are shown for three centrality classes 0-20%, 20-40% and 40-60% in Au+Au collisions and one centrality class 0-40% in Cu+Cu collisions at $\sqrt{s_{NN}} = 200$ GeV.

we find that the second flow coefficient is given by

$$v_2^\text{mag}(\omega_q) = \frac{\alpha_{em}^2 \pi V \Delta \tau}{2(2\pi)^3 N_{mag}} \sum_f \int_{\omega_q}^{\omega_q'} d\omega_q' \int_0^{\Delta \tau} d\omega_p$$

$$\times \left( 2\omega_p^2 + \omega_q^2 - \omega_p\omega_{q}' \right) e^{-g_f(\omega_{p},\omega_{q}')} \left\{ I_0[g_f(\omega_p)] - \left[ 1 + \frac{1}{g_f(\omega_p)} \right] I_1[g_f(\omega_p)] \right\}.$$

$$\times \left\{ n(\omega_p)n(\omega_{q}' - \omega_p) + n(\omega_p) \left[ 1 + n(\omega_{q}' - \omega_p) \right] \right\} . \quad (8)$$

We use the results of Eqs. (6) and (8) to compare with data in Sec. 5, after we discuss the procedure we use to obtain estimates for $V$ and $\Delta T$ and the magnetic field strength, from simulations.

4. Magnetic field pulse and space-time volume from simulations

In Eq. 3 the factor $V \Delta \tau$ appeared and consists of the product of the spatial volume of the nuclear overlap region $V(t)$ at time $t$ and the time interval $\Delta \tau$. In this time interval, the magnetic field can be taken as having a constant intensity $B(t)$. Furthermore, for a given centrality class, the spatial volume can be estimated from the fraction of the number of participants to the total number of nucleons. Since the reaction stops as soon as the magnetic field becomes negligible, the overall lifetime $\Delta T$ can be estimated calculating the duration of the magnetic pulse. We perform Monte Carlo simulations using UrQMD [14] to estimate these factors, and to have a blueprint of the magnetic field pulse. First we simulate relativistic Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV for three centrality classes, 0-20%, 20-40% and 40-60%. Next, for Cu+Cu collisions at the same collision energy, we simulate events in the centrality class 0-40%. Then for each event, the magnetic field at position $x$ and at time $t$ can be calculated using the Liénard-Wiechert
Figure 2. On the left, the invariant momentum distribution, and on the right $v_{2}^{\text{mag}}$ for Au+Au collisions in the 0-20%, 20-40% and 40-60% centrality classes and Cu+Cu collisions in the 0-40% centrality class at $\sqrt{s_{\text{NN}}} = 200$ GeV considering only the magnetic field generated by spectators.

The magnetic field generated by non-accelerated charges moving along the beam direction [16] as

$$e\mathbf{B}(\mathbf{x}, t) = \alpha_{\text{em}} \sum_{j} \frac{(1 - v_{j}^2)}{R_{j}^3} \frac{\mathbf{v}_{j} \times \mathbf{R}_{j}}{1 - \frac{(\mathbf{v}_{j} \times \mathbf{R}_{j})^2}{R_{j}^2}}^{3/2},$$

where $\mathbf{R}_{j} = \mathbf{x} - \mathbf{x}_{j}(t)$, $\mathbf{x}_{j}(t)$ is the position of the j-th charge moving with velocity $\mathbf{v}_{j}$, $R_{j}$ is the magnitude of $\mathbf{R}_{j}$ and the sum runs over charged particles per event. The magnetic field computed in the middle of the interaction region $\mathbf{x} = 0$ is shown on the left of Figure 1 produced by spectators, for different centrality ranges in Au+Au and Cu+Cu collisions at $\sqrt{s_{\text{NN}}} = 200$ GeV. On the right hand side of the same figure we show the volume $V(t)$ calculated for each centrality class as $V(t) = 2\pi r_{A}^{2}(N_{\text{part}}/2N)^{2/3}$, where $\pi r_{A}^{2}$ corresponds to the transverse area of a cylinder having a radius $r_{A}$ ($r_{\text{Au}} = 6.38$ fm, $r_{\text{Cu}} = 3.9$ fm [18]).

5. Results

We now summarize some of our findings where we use throughout $\alpha_{s} = 0.3$, $\Lambda_{s} = 2$ GeV, $\eta = 3$. We use the magnetic field strength generated only by spectators$^{2}$ and we calculate the net yield by adding the yields for all time intervals within $\Delta T$ given by $dN_{\text{mag}}^{\text{net}}/d\omega_q = \sum_{i=1}^{\Delta T} \left[ dN_{\text{mag}}^{\text{net}}/d\omega_q \right]_{i}$, where $\left[ dN_{\text{mag}}^{\text{net}}/d\omega_q \right]_{i}$ is the yield corresponding to the ith-time interval $\Delta \tau_{i}$, given by Eq. (6). Also, we calculate the corresponding net $v_{2}$ as a weighted average which accounts for the time-varying yield, as

$$v_{2}^{\text{mag}}(\omega_q) = \frac{\sum_{i=1}^{\Delta T} \left[ dN_{\text{mag}}^{\text{net}}/d\omega_q (\omega_q) \right]_{i} \left[ v_{2}^{\text{mag}}(\omega_q) \right]_{i}}{\sum_{i=1}^{\Delta T} \left[ dN_{\text{mag}}^{\text{net}}/d\omega_q \right]_{i}},$$

$^{2}$ The results for an analysis taking into account both participants and spectators, can be found in Ref. [13].
Figure 3. On the left, difference between PHENIX data [2] and the hydrodynamical calculation of Ref. [7] (open symbols) for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV for the 20-40% centrality classes, compared to the calculation of the invariant yield (filled symbols) considering only the contribution to the magnetic field strength produced by the spectators. On the right, $v_2$ as a function of the photon energy $\omega_q$ where we show the contributions from the hydrodynamical calculation of Ref. [7] (open squares) and the magnetic field-dependent calculation summed as a weighed average including the field strength coming from the spectators. The experimental data (open symbols) correspond to Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV in the 20-40% centrality class from Ref. [2].

where $[v_2^{mag}(\omega_q)]_i$ is the harmonic coefficient corresponding to the $ith$-time interval $\Delta \tau_i$, given by Eq. (8).

Figure 2 shows the invariant momentum distribution and corresponding $v_2^{mag}$, for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV in the 0-20%, 20-40% and 40-60% centrality ranges and Cu+Cu collisions in the 0-40% centrality range at the same energy.

In order to compare these results to experimental data, we note that since the magnetic contribution represents an excess over calculations not including these effects, we can compare the magnetic yield we calculated, to the difference between data and a hydrodynamical simulation of the direct photon yield. For $v_2$, we use a weighed average between $v_2$ from photons produced by the magnetic field and $v_2$ contribution from the direct photons as

$$v_2(\omega_q) = \frac{\sum_{i=1}^{m} \left[ dN/d\omega_q \right]_i \left[ v_2^{mag}(\omega_q) \right]_i + dN^{direct}/d\omega_q(\omega_q) \ v_2^{direct}(\omega_q)}{\sum_{i=1}^{m} \left[ dN/d\omega_q \right]_i + dN^{direct}/d\omega_q(\omega_q)},$$

(11)

where $dN^{direct}/d\omega_q$ and $v_2^{direct}$ are the ($\omega_q$-dependent) spectrum and second harmonic coefficient of direct photons from Ref. [7], respectively.

Figure 3 shows on the left, the difference between PHENIX data [2] and the hydrodynamical calculation of Ref. [7] –open symbols– compared to our calculation of the invariant yield – filled symbols– considering only the contribution to the magnetic field strength produced by the spectators, in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV for the 20-40% centrality class. On the right, this figure shows $v_2$ as the weighted average accounting for the magnetic and direct...
photons, compared to the experimental data for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV in the 20-40% centrality class from Ref. [2] with magnetic field effects coming only from the spectators. Notice that the magnetic field contribution improves the agreement with experimental data for the low part of the spectrum helping to describe the rise of $v_2$ as the photon energy decreases.

6. Summary and conclusions

We have computed the contribution to the photon yield and $v_2$ from gluon fusion and splitting induced by a magnetic field during the early stages of a relativistic heavy-ion collision, where there is a large gluon occupation number below the saturation scale $\Lambda$. The magnetic field strength and volume are computed using UrQMD simulations and the results compared with recent data from PHENIX. For the yield, the excess coming from the magnetic field induced processes is compared with the difference between PHENIX data and the hydrodynamical calculation of Ref. [7]. $v_2$ is computed as a weighed average accounting for magnetic and direct photons. The results show a relatively good agreement for the lower part of the spectra which improves for peripheral collisions. Our results point into the direction of enhancing the yield and $v_2$ for $p_T \lesssim 1$ GeV/c, a trend not incompatible with data [1]. Several avenues for improvement of our work and extensions of these results to other experimental regimes are currently being pursued and will be reported elsewhere.

Acknowledgements

The authors thank L.A. Hernández and S. Hernández-Ortiz for useful discussions. Support for this work has been received in part by UNAM-DGAPA-PAPIIT grant number IG100219 and by Consejo Nacional de Ciencia y Tecnología grant numbers A1-S-7655 and A1-S-16215.

References

[1] Gabor David, Rept.Prog.Phys. 83 (2020) 4, 046301.
[2] A. Adare et al. [PHENIX Collaboration], Phys. Rev. C 91, 064904 (2015); A. Adare et al. [PHENIX Collaboration], Phys. Rev. C 98, 054902 (2018).
[3] A. Adare et al. [PHENIX Collaboration], Phys. Rev. C 94, 064901 (2016).
[4] J. Adam et al. [ALICE Collaboration], Phys. Lett. B 754, 235 (2016).
[5] D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A 803, 227 (2008); V. Skokov, A. Y. Illarionov, V. Toneev, Int. J. Mod. Phys. A 24, 3925 (2009); V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya, V. P. Konchakovski, S. A. Voloshin, Phys. Rev. C 83, 054911 (2011); L. McLerran, V. Skokov, Nucl. Phys. A 929, 184-190 (2014); A. Bzdak, V. Skokov, Phys. Lett. B 710, 171-174 (2012). G. Basar, D. E. Kharzeev, V. Skokov, Phys. Rev. Lett. 109, 202303 (2012); G. Basar, D. E. Kharzeev, E. V. Shuryak, Phys. Rev. C 90, 014905 (2014). B. G. Zakharov, Eur. Phys. J. C 76, 609 (2016). K. Tuchin, Phys. Rev. C 91, 014902 (2015).
[6] A. Adare et al. [PHENIX Collaboration], e-Print: arXiv:1509.07758 [nucl-ex]; D. Lohner, et al. (ALICE Collaboration), e-Print: arXiv:1212.3995 [nucl-ex].
[7] J.-F. Paquet, C. Shen, G. S. Denicol, M. Luzum, B. Schenke, S. Jeon, C. Gale, Phys. Rev. C 93, 044906 (2016).
[8] H. van Hees, M. He, R. Rapp, Nucl. Phys. A 933, 256-271 (2015).
[9] O. Linnyk, V. Konchakovski, T. Steinert, W. Cassing, E. L. Bratkovskaya, Phys. Rev. C 92, 054914 (2015).
[10] For a recent review see C. Shen, Nucl. Phys. A 956, 184-191 (2016).
[11] A. Adare et al. [PHENIX Collaboration], e-Print: arXiv:1805.04084 [hep-ex] (2018); A. Adare et al. [PHENIX Collaboration], e-Print: arXiv:1805.04084 [hep-ex] (2018); T. Sakaguchi, J. Phys. Conf. Ser. 1070 (2018) no.1, 012012 (2018).
[12] A. Ayala, J. D. Castaño-Yepes, C. A. Domínguez, L. A. Hernández, S. Hernández-Ortiz, M. E. Tejeda-Yeomans, Phys. Rev. D 96, 014023 (2017) [Erratum: Phys.Rev. D 96, 119901 (2017)].
[13] A. Ayala, J. D. Castaño-Yepes, I. Domínguez Jimenez, J. Salinas San Martín, M. E. Tejeda-Yeomans. Eur.Phys.J. A 56 (2020) 2, 53.
[14] S. A. Bass, M. Bellacem, M. Bleicher, M. Brandstetter, L. Bravina, C. Ernst, L. Gerland, M. Hofmann, S. Hofmann, J. Konopka, G. Mao, L. Neise, S. Sof, C. Spiesle, H. Weber, L. A. Winkelmann, H. Stocker, W. Greiner, C. Hartnack, J. Aichelin and N. Amelin Prog. Part. Nucl. Phys. 41, 225-370 (1998); M. Bleicher,
E. Zabrodin, C. Spieles, S.A. Bass, C. Ernst, S. Soff, H. Weber, H. Stöcker, W. Greiner. J. Phys. G25, 1859-1896 (1999).

[15] L. McLerran and B. Schenke, Nucl. Phys. A 929, 71-82 (2014).

[16] L. D. Landau, E.M. Lifshitz, The Classical Theory of Fields, Pergamon Press, Oxford, England, (1984).

[17] C. Shen, U. W. Heinz, J.-F. Paquet, C. Gale, Phys. Rev. C 89, 044910 (2014).

[18] H. De Vries, C.W. De Jager, and C. De Vries, Atom. Data Nucl. Data Tabl. 36, 495 (1987).