Rotating-Moving D-Branes with Background Fields in the Superstring Theory

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Abstract

Using the boundary state formalism we study rotating and moving Dp-branes in the presence of the following background fields: Kalb-Ramond, $U(1)$ gauge potential and the tachyon field. The rotation and motion are in the brane volumes. The interaction amplitude of two Dp-branes will be studied, and specially contribution of the superstring massless modes will be segregated. Because of the tachyon fields, rotations and velocities of the branes, the behavior of the interaction amplitude reveals obvious differences from what is conventional.

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1 Introduction

D-branes as essential ingredients of the superstring theory [1] have important applications in different aspects of theoretical physics. These objects are classical solutions of the low-energy string effective action and hence can be described in terms of closed strings. Besides, D-branes with nonzero background internal fields have shown several interesting properties [2]-[7]. For example, these fields affect the emitted closed strings of the branes and therefore modify the branes interactions.

On the other hand, we have the boundary state formalism for describing the D-branes [8]-[15] which is a useful tool in many complicated situations. This is due to the fact that the boundary state encodes all relevant properties of the D-branes. Therefore, in the past years it has been widely used for studying properties of D-branes in the string theory. A boundary state can describe creation of closed string from vacuum, or equivalently it can be interpreted as a source for a closed string, emitted by a D-brane. Among achievements in this formalism it is its extension to the superstring theory and considering the contribution of the conformal and super-conformal ghosts. The overlap of two boundary states corresponding to two D-branes, via the closed string propagator, gives the amplitude of interaction of the branes. So far this adequate method has been applied to the various configurations in the presence of different background fields. For instance, some of these configurations are: stationary branes, moving branes with constant velocities, angled branes [16]-[20], various configurations in the compact spacetime [16], in the presence of the tachyon field [20]-[21], bound state of two D-branes [14], and so on.

Previously we studied a general configuration of rotating and moving Dp-branes of the bosonic string theory in the presence of the the following background fields: the Kalb-Ramond field, $U(1)$ gauge potentials which live in the D-branes worldvolumes and tachyon fields [21]. In this paper the same setup will be considered in the superstring theory. We shall see that the novelty of the results is considerable. Our procedure is as follows. For this setup we obtain the boundary state, associated with the brane, then we compute the interaction between two such Dp-branes as a closed superstring tree-level diagram in the covariant formalism. The generality of the setup strongly recasts the feature of the boundary states and interaction of the branes. We shall observe that the interaction amplitude and its long-range part, which occurs between the distant branes, exhibit some appealing behaviors.
Note that we shall consider rotation of each brane in its volume and its motion along the brane directions. Due to the various fields inside the brane there are preferred directions which indicate the breaking of the Lorentz symmetry and hence such rotation and motion are meaningful.

This paper is organized as follows. In Sec. 2, the boundary state of a closed super-string, corresponding to a rotating-moving Dp-brane with various background fields will be constructed. In Sec. 3, interaction of two Dp-branes in the NS-NS and R-R sectors of the superstring will be calculated. In Sec. 4, the long-range force of the interaction will be extracted. Section 5 is devoted to the conclusions.

2 Boundary state associated with a rotating-moving Dp-brane with background fields

We use the following sigma-model action for closed string to describe a rotating and moving Dp-brane, in the presence of the Kalb-Ramond, photonic and tachyonic fields

$$S = -\frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\sigma (\sqrt{-\eta_{\mu\nu}} G_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + \varepsilon^{ab} B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu)$$

$$+ \frac{1}{2\pi\alpha'} \int_{\partial\Sigma} d\sigma (A_\alpha \partial_\sigma X^\alpha + \omega_{\alpha\beta} J^{\alpha\beta}_\tau + T(X^\alpha)),$$  \hspace{1cm} (1)

where $\Sigma$ is the worldsheet of the closed string, emitted (absorbed) by the brane, and $\partial\Sigma$ shows the boundary of the worldsheet. Besides, “$\alpha$” and “$\beta$” are indices along the brane worldvolume while “$\tau$” will be used for the directions perpendicular to it. In addition, the background fields $G_{\mu\nu}$, $B_{\mu\nu}$, $A_\alpha$ and $T$, and also the antisymmetric variables $\omega_{\alpha\beta}$ and $J^{\alpha\beta}_\tau$ are the spacetime metric, Kalb-Ramond (an antisymmetric tensor), gauge field, tachyon field, angular velocity and angular momentum density of the brane, respectively. Here we consider $G_{\mu\nu}$ as the flat spacetime metric with the signature $\eta_{\mu\nu} = diag(-1, 1, \cdots, 1)$ and the Kalb-Ramond field $B_{\mu\nu}$ to be a constant field.

In the presence of a Dp-brane the 10-dimensional $U(1)$ gauge field $A_\mu$ is decomposed into a longitudinal $U(1)$ gauge field $A_\alpha$, which lives in the worldvolume of the Dp-brane, and a transverse part $A_i$ associated with the $9-p$ scalar fields, from the worldvolume point of view. These scalars represent coordinates of the brane. We shall keep them to be fixed, that is, the branes do not have transverse motion. For the gauge field we choose the gauge $A_\alpha = -\frac{1}{2} F_{\alpha\beta} X^\beta$ with the constant field strength. Now look at the tachyon. Usually in the
literature the tachyon field is nonzero just in one dimension and its effects are studied on a space-filling brane, while in the present article we consider a Dp-brane with an arbitrary value for p. Besides, the square form of tachyon profile is used to produce a Gaussian integral, i.e. \( T(X) = \frac{1}{2} \eta_{\alpha\beta} X^\alpha X^\beta \) in which the symmetric matrix \( U_{\alpha\beta} \) is constant. Thus, the tachyon field possesses components along all directions of the brane worldvolume. The gauge and tachyon fields are in the open string spectrum, which are attached to the Dp-brane. The brane’s rotation-motion term, that contains antisymmetric angular velocity \( \omega_{\alpha\beta} \) and angular momentum density \( J_{\alpha\beta} \), is given by \( \omega_{\alpha\beta} J_{\alpha\beta} = 2 \omega_{\alpha\beta} X^\alpha \partial_\tau X^\beta \).

In fact, the components \( \{ \omega_{0\bar{\alpha}} | \bar{\alpha} = 1, 2, \ldots, p \} \) denote the velocity of the brane, while the elements \( \{ \omega_{\bar{\alpha}\bar{\beta}} | \bar{\alpha}, \bar{\beta} = 1, 2, \ldots, p \} \) represent its rotation. Note that in the presence of the antisymmetric field and the local gauge field there are preferred alignments in the brane, and hence the rotation and motion of the brane in its volume is sensible.

2.1 Bosonic part of the boundary state

In the closed string operator formalism the D-branes of the Type IIA and Type IIB theories can be described by the boundary states. These are closed string states which insert a boundary on the closed string worldsheet and enforce on it appropriate boundary conditions. Now we extract the corresponding boundary state for our setup. By vanishing of the variation of the action with respect to the closed string coordinates \( X^\mu(\sigma, \tau) \) the following boundary state equations are acquired

\[
[ (\eta_{\alpha\beta} + 4 \omega_{\alpha\beta}) \partial_\tau X^\beta + \mathcal{F}_{\alpha\beta} \partial_\tau X^\beta + U_{\alpha\beta} X^\beta ]_{\tau=0} |B_{\text{bos}}\rangle = 0,
\]

\[
(\delta X^i)_{\tau=0} |B_{\text{bos}}\rangle = 0,
\]

(2)

where \( \mathcal{F}_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha - B_{\alpha\beta} \) is the total field strength. Note that we have assumed the following mixed elements vanish, i.e. \( B_{\alpha i} = U_{\alpha i} = 0 \).

It is worthwhile to show that along the worldvolume of the brane the Lorentz symmetry is broken. The Eqs. (2) leads to

\[
J_{\alpha\beta}^{\text{bos}} |B_{\text{bos}}\rangle = \int_0^\pi d\sigma \left[ (A^{-1} F)^\alpha_\gamma X^\beta \partial_\sigma X^\gamma - (A^{-1} F)^\beta_\gamma X^\alpha \partial_\sigma X^\gamma 
+ (A^{-1} U)^\alpha_\gamma X^\beta X^\gamma - (A^{-1} U)^\beta_\gamma X^\alpha X^\gamma \right] |B_{\text{bos}}\rangle,
\]

(3)

where \( A_{\alpha\beta} = \eta_{\alpha\beta} + 4 \omega_{\alpha\beta} \). We observe that for restoring the Lorentz invariance all elements of the tachyon matrix \( U_{\alpha\beta} \) and the total field strength \( \mathcal{F}_{\alpha\beta} \) must vanish. We demonstrated
this for the bosonic part of the boundary state. This procedure can also be applied for
the total boundary state, which includes the bosonic and fermionic parts, to prove the
breakdown of the Lorentz invariance along the worldvolume of the brane.

Introducing the closed string mode expansion into Eq. (2) gives

\[
\left[ \left( \eta_{\alpha\beta} + 4\omega_{\alpha\beta} - F_{\alpha\beta} + \frac{i}{2m} U_{\alpha\beta} \right) \partial_m^\beta + \left( \eta_{\alpha\beta} + 4\omega_{\alpha\beta} + F_{\alpha\beta} - \frac{i}{2m} U_{\alpha\beta} \right) \bar{\partial}_{-m}^\beta \right] |B_{\text{bos}}\rangle^{\text{(osc)}} = 0,
\]

\[
\left[ 2\alpha'(\eta_{\alpha\beta} + 4\omega_{\alpha\beta} - U_{\alpha\beta}) p^\beta + U_{\alpha\beta} x^\beta \right] |B_{\text{bos}}\rangle^{(0)} = 0,
\]

\[
(\alpha_m^i - \bar{\alpha}_{-m}^i) |B_{\text{bos}}\rangle^{\text{(osc)}} = 0,
\]

\[
(x^i - y^i) |B_{\text{bos}}\rangle^{(0)} = 0,
\]

(4)

where the set \( \{ y^i | i = p + 1, \cdots, 9 \} \) indicates the position of the brane. Besides, for the
boundary state \( |B_{\text{bos}}\rangle = |B_{\text{bos}}\rangle^{(0)} \otimes |B_{\text{bos}}\rangle^{\text{(osc)}} \) the components \( |B_{\text{bos}}\rangle^{(0)} \) and \( |B_{\text{bos}}\rangle^{\text{(osc)}} \) represent boundary states for the zero modes and oscillating modes, respectively.

The solution of the oscillating part, which can be found by the coherent state method, is given by

\[
|B_{\text{bos}}\rangle^{\text{(osc)}} = \prod_{n=1}^{\infty} [\det Q_{(n)}]^{-1} \exp \left[ - \sum_{m=1}^{\infty} \frac{1}{m} \alpha_{-m}^\mu S_{(m)\mu\nu} \bar{\alpha}_{-m}^\nu \right] |0\rangle_\alpha \otimes |0\rangle_{\bar{\alpha}} ,
\]

(5)

where the matrices are defined as in the following

\[
Q_{(m)\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta} - F_{\alpha\beta} + \frac{i}{2m} U_{\alpha\beta},
\]

\[
S_{(m)\mu\nu} = \left( \frac{1}{2} \left[ \Delta_{(m)} + \left( \Delta_{(-m)}^T \right)^{-1} \right] \right)_{\alpha\beta} \delta_{ij},
\]

\[
\Delta_{(m)\alpha\beta} = (Q_{(m)}^{-1} N_{(m)})_{\alpha\beta},
\]

\[
N_{(m)\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta} + F_{\alpha\beta} - \frac{i}{2m} U_{\alpha\beta}.
\]

(6)

Since the mode-dependent matrix \( \Delta_{(m)} \) generally is not orthogonal the matrix \( \left( \Delta_{(-m)}^T \right)^{-1} \)
also appears in the definition of \( S_{(m)\mu\nu} \). In the Eq. (5) the normalization factor \( \prod_{n=1}^{\infty} [\det Q_{(n)}]^{-1} \)
can be deduced from the disk partition function.

The boundary state for the zero modes finds the feature

\[
|B_{\text{bos}}\rangle^{(0)} = \int_{-\infty}^{\infty} \exp \left\{ i \alpha' \left[ \sum_{\alpha=0}^{p} (U^{-1} A)_{\alpha\alpha} (p^{\alpha})^2 + \sum_{\alpha,\beta=0,\alpha\neq\beta}^{p} (U^{-1} A + ATU^{-1})_{\alpha\beta} p^{\alpha} p^{\beta} \right] \right\} \times \left( \prod_{\alpha} |p^{\alpha}\rangle dp^{\alpha} \right) \otimes \prod_{i} \delta(x^i - y^i) |p^i = 0\rangle.
\]

(7)
The integration on the momenta indicates that the effects of all values of the momentum components have been taken into account. As we see, unlike the oscillating part, the total field strength did not entered in the Eq. (7).

It should be noted that for calculating the interaction amplitude the contribution of the conformal ghosts $b, c, \tilde{b}$ and $\tilde{c}$ in the bosonic boundary state also will be taken into account.

### 2.2 Fermionic part of the boundary state

Since the supersymmetric version of the action (1) is invariant under the global world-sheet supersymmetry, we can perform the supersymmetry transformations on the bosonic boundary Eqs. (2) and transform them into their fermionic partners. Therefore, one can use the following replacements

$$
\partial_\pm X^\mu (\sigma, \tau) \rightarrow -i\eta \psi^\mu_\pm (\sigma, \tau),
\partial_- X^\mu (\sigma, \tau) \rightarrow \psi^\mu_\pm (\sigma, \tau),
$$
where $\eta = \pm 1$ has been introduced for the GSO projection of the boundary state. As it was seen in the bosonic boundary state equations, due to the presence of the tachyon field, a replacement for $X^\mu$ in terms of the fermionic components is also needed. To obtain that, by using the replacements (8) and $\partial_\pm = \frac{1}{2}(\partial_\sigma \pm \partial_\tau)$ and integration, we receive

$$
X^\mu (\sigma, \tau) \rightarrow \sum_k \frac{1}{2k} \left( i\psi^\mu_k e^{-2i k (\tau - \sigma)} + \eta \tilde{\psi}^\mu_k e^{-2i k (\tau + \sigma)} \right).
$$

Now by introducing the replacements (8) and (9) into the Eqs. (2), for the closed string boundary at $\tau = 0$, we obtain

$$
\left[ (\eta_{\alpha\beta} + 4\omega_{\alpha\beta} - \mathcal{F}_{\alpha\beta} + \frac{i}{2k} U_{\alpha\beta}) \psi^\beta_k - i\eta (\eta_{\alpha\beta} + 4\omega_{\alpha\beta} + \mathcal{F}_{\alpha\beta} - \frac{i}{2k} U_{\alpha\beta}) \tilde{\psi}^\beta_{-k} \right] |B^{(osc)}_{\text{ferm}}, \eta\rangle = 0,
$$

$$
(\psi^i_k + i\eta \tilde{\psi}^i_k) |B^{(osc)}_{\text{ferm}}, \eta\rangle = 0,
$$
for the oscillating parts of the R-R and NS-NS sectors, and

$$
[(\eta_{\alpha\beta} + 4\omega_{\alpha\beta} - \mathcal{F}_{\alpha\beta}) \psi^0_0 - i\eta (\eta_{\alpha\beta} + 4\omega_{\alpha\beta} + \mathcal{F}_{\alpha\beta}) \tilde{\psi}^0_0] |B, \eta\rangle^{(0)}_R = 0,
$$

$$
(\psi^i_0 + i\eta \tilde{\psi}^i_0) |B, \eta\rangle^{(0)}_R = 0,
$$
for the zero-mode part of the R-R sector. As we see in this sector the tachyon has been omitted from the zero-mode boundary state. The importance of this portion will
be revealed in the R-R sector of the boundary state. The Eqs. (10) and (11) can be rewritten in the following features

$$\left(\psi_k^\mu - i\eta \ S^\mu_{(k)} \, \bar{\psi}^\nu \right) \left| B_{\text{ferm}}^{(\text{osc})}, \eta \right\rangle = 0,$$

(12)

for oscillating parts of both sectors, and

$$\left( d_0^\mu - i\eta \ \bar{S}_\mu^\nu \ d_0^\nu \right) \left| B, \eta \right\rangle^{(0)}_R = 0,$$

(13)

for the zero-mode part of the R-R sector. The matrix $\bar{S}_\mu^\nu$ is defined by

$$\bar{S}_\mu^\nu = (\bar{\Delta}_{\alpha\beta}, -\delta_{ij}),$$

$$\bar{\Delta}_{\alpha\beta} = (\bar{Q}^{-1}\bar{N})_{\alpha\beta},$$

$$\bar{Q}_{\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta} - \mathcal{F}_{\alpha\beta},$$

$$\bar{N}_{\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta} + \mathcal{F}_{\alpha\beta}.$$

(14)

Note that in the fermionic parts we should also consider the boundary states associated with the super-conformal ghosts which will be needed for calculating the interaction amplitude.

2.2.1 The Neveu-Schwarz sector

Similar to the bosonic section, with the help of the coherent state method, the oscillating part of the fermionic boundary state including both sectors can be calculated. Thus, the Eq. (12) implies that the NS-NS sector boundary state has the form

$$\left| B_{\text{ferm}}^{\text{NS}}, \eta \right\rangle_{\text{NS}} = \prod_{r=1/2}^\infty \left[ \det Q_r \right] \exp \left[ i\eta \sum_{r=1/2}^\infty \left( \theta_{r}^\mu \ S^\nu_{(r)} \bar{\theta}_{r}^\nu \right) \right] \left| 0 \right\rangle_{\text{NS}}.$$  

(15)

When the path integral is computed the determinant is reversed in comparing to the bosonic Eq. (5). This is due to the Grassmannian property of the fermionic variables [8].

2.2.2 The Ramond-Ramond sector

Solving the Eqs. (12) and (13) in the R-R sector yields the following boundary state

$$\left| B_{\text{ferm}}^{\text{R}}, \eta \right\rangle_{\text{R}} = \prod_{n=1}^\infty \left[ \det Q_{(n)} \right] \exp \left[ i\eta \sum_{m=1}^\infty \left( d_{-m}^\mu \ S_{(m)}^\mu \bar{d}_{-m}^\nu \right) \right] \left| B, \eta \right\rangle^{(0)}_R.$$

(16)
The explicit form of the zero-mode state both in the Type IIA and Type IIB theories is

\[ |B, \eta\rangle^{(0)}_R = \left[ C \Gamma^0 \Gamma^1 \cdots \Gamma^p \left( \frac{1 + i \eta \Gamma_{11}}{1 + i \eta} \right) \right]_{AB} |A\rangle \otimes |\tilde{B}\rangle, \] (17)

where \( A \) and \( B \) denote the 32-dimensional indices for the spinors and \( \Gamma \)-matrices in the 10-dimensional spacetime, \( |A\rangle \otimes |\tilde{B}\rangle \) is the vacuum of the zero modes \( \tilde{d}_0^\mu \) and \( \tilde{d}_0^\mu \), \( C \) is the charge conjugate matrix, and

\[ \Omega = \ast \exp \left( \frac{1}{2} \Phi_{\alpha \beta} \Gamma^\alpha \Gamma^\beta \right) \ast, \]

\[ \Phi_{\alpha \beta} = \left( (\bar{\Delta} - 1)(\bar{\Delta} + 1)^{-1} \right)_{\alpha \beta}. \] (18)

The notation \( \ast \ast \) implies that one should expand the exponential and then antisymmetrize the indices of the \( \Gamma \)-matrices. Therefore, since all terms in the expansion with repeated Lorentz indices are dropped, there are a finite number of terms for each value of \( p \). As an example, for the D3-brane the matrix \( \Omega \) takes the form

\[ \Omega = 1 + \frac{1}{2} \sum_{\alpha,\beta=0}^3 \Phi_{\alpha \beta} \Gamma^\alpha \Gamma^\beta + (\Phi_{01} \Phi_{23} - \Phi_{02} \Phi_{13} + \Phi_{03} \Phi_{12}) \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3. \]

In fact, this convention implies that the matrix \( \bar{\Delta} \) should be orthogonal which gives a restriction that the matrices \( \omega \) and \( F \) should anticommute with each other. For the D1-brane there is an electric field along the brane. Thus, according to this restriction, the only element of the matrix \( \omega \), i.e. the speed of the brane along itself, vanishes. This is an expected result, because of the direction of the electric field, motion of the D-string along itself is not sensible. The other branes can have both rotation and motion.

### 3 Interaction of the branes

Unbroken supersymmetry ensures that the Casimir energy of open superstrings is zero. Therefore, D-branes in supersymmetric configurations exert no net force on each other. A rotating/moving brane can break generically all the supersymmetries, and leads to orientation/velocity-dependent forces.

In this section we calculate the interaction of two rotating and moving parallel Dp-branes, equipped by background fields, via the closed string exchange. For both the NS-NS and R-R sectors the complete boundary state can be written as the following
product

\[ |B, \eta\rangle_{NS,R} = \frac{T_p}{2} |B_{\text{bos}}\rangle \otimes |B_{\text{gh}}\rangle \otimes |B_{\text{ferm}}, \eta\rangle_{NS,R} \otimes |B_{\text{sgh}}, \eta\rangle_{NS,R}, \]

where the overall normalization factor \( T_p \) is the Dp-brane tension. Note that the ghost and superghost boundary states are not affected by the rotation, motion and the background fields. The explicit expressions of \( |B_{\text{gh}}\rangle \) and \( |B_{\text{sgh}}\rangle_{NS,R} \) can be found in the literature, and hence we do not write them here.

For eliminating unwanted states, e.g., the closed string tachyon, and in the same time making the number of spacetime bosonic and fermionic physical excitations equal at each mass level, as it is needed for supersymmetry, one should use the GSO projection. Therefore, the total boundary states which will be used for calculation of the interaction find the forms

\[
|B\rangle_{NS} = \frac{1}{2} (|B, +\rangle_{NS} - |B, -\rangle_{NS}),
\]

\[
|B\rangle_R = \frac{1}{2} (|B, +\rangle_R + |B, -\rangle_R). \tag{19}
\]

One can obtain the interaction amplitude of two D-branes either by the open string one-loop or the closed string tree-level diagram. Thus, the former is a quantum process while the latter is a classical process. In the closed string picture the interaction between two D-branes is viewed as the exchange of a closed string between two boundary states, geometrically describing a cylinder. From this standpoint, the interaction is computed with a tree-level diagram. In this process a closed string is created by one D-brane, it propagates in the transverse space between the two D-branes, and then the other D-brane absorbs it. Therefore, the interaction amplitude between two D-branes in each sector is given by the following overlap of the boundary states \( A_{NS-R-R} =_{NS,R}\langle B_1|D|B_2\rangle_{NS,R} \), where \( D \) is the closed string propagator. In other words, we have

\[
A_{NS-R-R} = 2\alpha' \int_0^{\infty} dt_{NS,R}\langle B_1|e^{-\tau H_{NS,R}}|B_2\rangle_{NS,R}.
\]

The total closed superstring Hamiltonian \( H_{NS,R} \) is sum of the Hamiltonians of the world-sheet bosons, fermions, conformal ghosts and super-conformal ghosts in each sector. The complete interaction amplitude is given by the following combination

\[ A_{total} = A_{NS-NS} + A_{R-R}. \]
According to this formula the boundary states are convenient tools for summing over all forces between two D-branes, which are mediated by the NS-NS and R-R states of closed superstring.

### 3.1 The NS-NS sector interaction

For maintaining the generality let’s consider the $d$-dimensional spacetime instead of $d = 10$. Using the GSO projected boundary states (19), we obtain the interaction amplitude, between two parallel D$p$-branes in the NS-NS sector, as follows

$$
\mathcal{A}_{\text{NS-NS}} = \frac{T_p^2 V_{p+1}\alpha'}{8(2\pi)^{d-p-1}} \prod_{m=1}^{\infty} \frac{\det(Q_{(m-1/2)}^\dagger Q_{(m-1/2)})}{\det(Q_{(m)}^\dagger Q_{(m)})} \int_0^\infty \frac{dt}{\sqrt{\det(R_1^\dagger R_2)}} \left( \frac{\pi}{\alpha' t} \right)^{d-p-1} \exp \left( -\frac{1}{4\alpha'} \sum_i (y_i^2 - y_i^1)^2 \right) 
$$

$$
\times \frac{1}{q} \prod_{n=1}^{\infty} \left[ \left( \frac{1 - q^{2n}}{1 + q^{2n-1}} \right)^{3+p-d} \frac{\det(1 + H_{(n)}^\dagger H_{(n)} q^{2n-1})}{\det(1 - H_{(n)}^\dagger H_{(n)} q^{2n})} \right] 
$$

$$
- \prod_{n=1}^{\infty} \left[ \left( \frac{1 - q^{2n}}{1 - q^{2n-1}} \right)^{3+p-d} \frac{\det(1 - H_{(n)}^\dagger H_{(n)} q^{2n-1})}{\det(1 - H_{(n)}^\dagger H_{(n)} q^{2n})} \right] \right), \quad (20)
$$

where the indices “1” and “2” refer to the first brane or $|B_1\rangle$ and the second brane or $|B_2\rangle$, $V_{p+1}$ is the common worldvolume of the two D$p$-branes, $q = e^{-2t}$, $H_{(n)} = (\Delta_{(n)a} + [\Delta_{(-n)a}^\dagger])^2/2$ with $a = 1, 2$, and the symmetric matrices $R_1$ and $R_2$ contain nonzero elements only along the branes worldvolumes

$$
(R_a)_{\alpha\beta} = 2\alpha' (-i M_a - i U_{a}^{-1} A_a - i A_a^T U_{a}^{-1} + t 1)_{\alpha\beta}, \quad a = 1, 2,
$$

$$
M_a = \begin{pmatrix}
(U_{a}^{-1} A_a)_{00} & \cdots & 0 \\
0 & \ddots & \\
\vdots & \ddots & 0 \\
0 & \cdots & (U_{a}^{-1} A_a)_{pp}
\end{pmatrix},
$$

$$
(A_a)_{\alpha\beta} = \eta_{\alpha\beta} + 4(\omega_a)_{\alpha\beta}. \quad (21)
$$

In addition, we applied the relations $\langle p^\alpha | p^\beta \rangle = 2\pi \delta(p^\alpha - p^\beta)$ and $(2\pi)^{p+1} \delta^{(p+1)}(0) = V_{p+1}$. In this amplitude the exponential is a damping factor with respect to the distance of the branes. In the last two products: the determinant in the denominators reflects the portion of the bosons oscillators along the branes worldvolumes, the determinants in the numerators are due to the fermions oscillators again along the branes worldvolumes.
The other factors in the products are contributions of the bosons and fermions oscillators, perpendicular to the brane worldvolume, and also of the conformal ghosts and superconformal ghosts. Explicitly, the power $3 + p - d = 2 - (d - p - 1)$ is decomposed as: 2 in the numerators for the ghosts, 2 in the denominators for the superghosts, $-(d - p - 1)$ in the numerators for transverse oscillators of the bosons and $-(d - p - 1)$ in the denominators for transverse oscillators of the fermions. The remaining part of the integrand of the amplitude is overlap of the boundary states of the bosonic zero modes, i.e. the Eq. (7). This part completely is influenced by the internal tachyon fields, the motion and rotation of the branes.

Contributions of all closed superstring states in the NS-NS sector that the two branes can emit, are gathered in the amplitude (20). A part of the strength of the interaction is given by the constant overall factor of this amplitude, i.e. the first line of Eq. (20), which possesses contributions from the field parameters, linear and angular velocities and the branes tensions.

### 3.2 The R-R sector interaction

Applying the total GSO projected boundary states (19) we acquire the following interaction amplitude in the R-R sector

$$ A_{R-R} = \frac{T_p^2 V_p \alpha'}{8(2\pi)^{d-p-1}} \int_0^\infty dt \left\{ \left( \kappa \prod_{n=1}^\infty \left[ \begin{pmatrix} 1 - q^{2n} \\ 1 + q^{2n} \end{pmatrix} \begin{pmatrix} \det(1 + H_{(n)}^\dagger H_{(n)}q^{2n}) \\ \det(1 - H_{(n)}^\dagger H_{(n)}q^{2n}) \end{pmatrix} \right] + \kappa' \right) \right\} \times \frac{1}{\sqrt{\det(R_1^T R_2)}} \frac{\pi^{d-p-1}}{\alpha'} \exp \left( -\frac{1}{4\alpha'} \sum_i (y^i_2 - y^i_1)^2 \right), \quad (22) $$

where

$$ \kappa \equiv \frac{1}{2} (-1)^{p+1} \Tr[\Omega_1 C^{-1} \Omega_2^T C], $$

$$ \kappa' \equiv i (-1)^p \Tr[\Omega_1 C^{-1} \Omega_2^T C \Gamma_{11}]. \quad (23) $$

In above relations the matrices $\Omega_{1,2}$ have been defined by the Eq. (18) via the matrices $\omega_{1,2}$ and $F_{1,2}$ for the first and second branes. As we can see in the R-R sector boundary state, and hence in the corresponding amplitude, the normalizing determinant factors of the bosons and fermions cancel each other.

Now we are interested in the total amplitude, i.e. the combination of the amplitudes in the NS-NS and R-R sectors. In the total amplitude of the described system, the
attraction due to the exchange of the NS-NS states of closed string is not compensated by the repulsion of the R-R states. Thus, we can conclude that our setup does not satisfy the BPS no-force condition. This is due to the fact that this configuration of the two D-branes does not preserve enough value of the spacetime supersymmetries of the Type IIA and Type IIB theories. In fact, in the absence of the background fields, motions and rotations, the total amplitude vanishes, because this setup of the branes preserves half of the supersymmetry.

A special feature of the non-BPS branes is presence of the tachyon field in their worldvolumes. In fact, it is not evident how the spacetime supersymmetry is realized with the tachyons, and existence of the broken supersymmetry in the presence of the tachyons has never been explicitly proven [22]. However, setting the branes in relative motion (or rotating them) breaks generically all the supersymmetries, and leads to velocity- or orientation-dependent forces [23].

We observe that in the amplitudes of both sectors, for a system of two D(d−3)-branes, the effect of the ghosts (superghosts) eliminates the contribution of the transverse oscillators of the bosons (fermions).

3.3 An example

To clarify our described system, let study a special case, i.e. parallel D2-branes. Consider the a-th brane (a = 1, 2) with the linear velocity \((v_a)_{\bar{a}} = 1, 2\)\), the angular velocity \((\omega_{12})_a = \bar{\omega}_a\) and the fields \((F_{0\bar{a}})_a = (E_a)_{\bar{a}}, (F_{12})_a = B_a\) and \((U_{\alpha\beta})_a\). Therefore, the interaction amplitude for the NS-NS sector is given by

\[
\mathcal{A}_{\text{NS-NS}} = \frac{T_3^2 V_3 \alpha'}{8(2\pi)^{d-3}} \prod_{m=1}^{\infty} \frac{\det[Q^\dagger_{(m-1/2)1} Q_{(m-1/2)2}]}{\det[Q^\dagger_{(m)1} Q_{(m)2}]} \times \int_0^\infty dt \left\{ \frac{1}{\sqrt{\det(R_1 R_2)}} \left( \frac{\alpha'}{\sqrt{\pi t}} \right)^{d-3} \exp \left( -\frac{1}{4\alpha'} t \sum_{i=3}^{d-1} (y^i_2 - y^i_1)^2 \right) \right\} = \frac{1}{q} \left( \prod_{n=1}^{\infty} \left[ \left( \frac{1 - q^{2n}}{1 + q^{2n+1}} \right)^{5-d} \frac{\det[(1 + H_{(n)1} H_{(n)2} q^{2n-1})]}{\det[(1 - H_{(n)1} H_{(n)2} q^{2n})]} \right] \right) \times \prod_{n=1}^{\infty} \left[ \left( \frac{1 - q^{2n}}{1 - q^{2n-1}} \right)^{5-d} \frac{\det[(1 - H_{(n)1} H_{(n)2} q^{2n-1})]}{\det[(1 - H_{(n)1} H_{(n)2} q^{2n})]} \right] \right\},
\]

(24)
where the matrix $H_{(n)a}$ is defined in terms of $Q_{(\mp n)a}$ and $N_{(\mp n)a}$, as before, in which

$$Q_{(n)a} = \begin{pmatrix} -1 + \frac{i\nu_{10}}{2n} & 4v_1 - E_1 + \frac{i\nu_{10}}{2n} & 4v_2 - E_2 + \frac{i\nu_{10}}{2n} \\ -4v_1 + E_1 + \frac{i\nu_{10}}{2n} & 1 + \frac{i\nu_{11}}{2n} & 4\mathcal{M} - B + \frac{i\nu_{12}}{2n} \\ -4v_2 + E_2 + \frac{i\nu_{10}}{2n} & 4\bar{\mathcal{M}} + B + \frac{i\nu_{11}}{2n} & 1 + \frac{i\nu_{12}}{2n} \end{pmatrix}, \quad a = 1, 2,$$

$$N_{(n)a} = \begin{pmatrix} -1 - \frac{i\nu_{10}}{2n} & 4v_1 + E_1 - \frac{i\nu_{10}}{2n} & 4v_2 + E_2 - \frac{i\nu_{10}}{2n} \\ -4v_1 - E_1 - \frac{i\nu_{10}}{2n} & 1 - \frac{i\nu_{11}}{2n} & 4\mathcal{M} + B - \frac{i\nu_{12}}{2n} \\ -4v_2 - E_2 - \frac{i\nu_{10}}{2n} & 4\bar{\mathcal{M}} - B - \frac{i\nu_{11}}{2n} & 1 - \frac{i\nu_{12}}{2n} \end{pmatrix}, \quad a = 1, 2. \quad (25)$$

The matrix elements of the symmetric matrix $R_a$ are as in the following

$$\begin{align*}
(R_a)_{00} &= -2i\alpha' \left[ (U^{-1})_{00} - 4v_1 (U^{-1})_{01} - 4v_2 (U^{-1})_{02} + it \right]_a , \\
(R_a)_{01} &= -2i\alpha' \left[ 2(U^{-1})_{01} - 4v_1 ((U^{-1})_{00} + (U^{-1})_{11}) - 4\mathcal{M}(U^{-1})_{02} - 4v_2 (U^{-1})_{21} + it \right]_a , \\
(R_a)_{02} &= -2i\alpha' \left[ 2(U^{-1})_{02} - 4v_2 ((U^{-1})_{01} + (U^{-1})_{22}) + 4\mathcal{M}(U^{-1})_{01} - 4v_1 (U^{-1})_{12} + it \right]_a , \\
(R_a)_{11} &= -2i\alpha' \left[ (U^{-1})_{11} - 4v_1 (U^{-1})_{10} - 4\mathcal{M}(U^{-1})_{12} + it \right]_a , \\
(R_a)_{12} &= -2i\alpha' \left[ 2(U^{-1})_{12} + 4\mathcal{M}((U^{-1})_{11} + (U^{-1})_{22}) - 4v_2 (U^{-1})_{10} - 4v_1 (U^{-1})_{02} + it \right]_a , \\
(R_a)_{22} &= -2i\alpha' \left[ (U^{-1})_{22} - 4v_2 (U^{-1})_{20} + 4\bar{\mathcal{M}}(U^{-1})_{21} + it \right]_a , \quad (26)
\end{align*}$$

with $a = 1, 2$. Also, for the R-R sector the amplitude finds the feature

$$A_{\text{R-R}} = \frac{T_2^2 V_3}{8(2\pi)^{d-3}} \int_0^\infty dt \left\{ \frac{1}{\sqrt{\det(R_1 R_2)}} \left( \sqrt{\frac{\pi}{\alpha'}} \right)^{d-3} \exp \left( -\frac{1}{4\alpha'} \sum_{i=3}^{d-1} (y_i^2 - y_i')^2 \right) \right\} \left( \kappa \prod_{n=1}^\infty \left[ \left( 1 - q^{2n} \right)^{5-d} \frac{\det[(1 + H_{(n)1} H_{(n)2} q^{2n})]}{\det[(1 - H_{(n)1} H_{(n)2} q^{2n})]} + \kappa' \right] \right), \quad (27)$$

where

$$\begin{align*}
\kappa &= 16 \left( -1 + \Phi_{(1)01} \Phi_{(2)01} + \Phi_{(1)02} \Phi_{(2)02} - \Phi_{(1)12} \Phi_{(2)12} \right), \\
\kappa' &= -\frac{1}{4} \sum_{\alpha,\beta=0}^2 \sum_{\alpha',\beta'=0}^2 \Phi_{(1)\alpha\beta} \Phi_{(2)\alpha'\beta'} \text{Tr}(\Gamma^\alpha \Gamma^\beta \Gamma^\alpha' \Gamma^\beta' \Gamma_{11}). \quad (28)
\end{align*}$$

Note that we have used of $(\Gamma^\mu)^T = -C \Gamma^\mu C^{-1}$. In fact, the D2-brane is the simplest brane which its rotation and motion along its directions is sensible. We see that for this simple case the interaction amplitudes also are very complicated.
4 Interaction between distant D-branes

For distant D-branes only the closed superstring massless states have a considerable contribution on the interaction. In other words, after long enough time, which is equivalent to the large distance of the branes, the massless states become dominant. Technically, the contribution of these states on the interaction amplitude is obtained by taking the limit of the oscillators portions of Eqs. (20) and (22).

Let $P_n \in \{-1, 1, H^1_{(n)1}H_{(n)2}, H^1_{(n)1}H_{(n)2} \}$ and $q_n \in \{q^{2n}, -q^{2n}, q^{2n-1}, -q^{2n-1} \}$. By applying the following relation

$$\prod_{n=1}^{\infty} (\det(1 + q_n P_n)) = \exp \left\{ \sum_{k=0}^{\infty} \left( \frac{(-1)^k}{k+1} \sum_{n=1}^{\infty} \text{Tr}(g_n P_n)^{k+1} \right) \right\},$$

into the amplitudes (20) and (22) and sending $q$ to zero, the contribution of the massless states can be acquired. Therefore, in the 10-dimensional spacetime, we receive the following amplitudes

$$A_{\text{NS-NS}}^{\text{(massless)}} = \frac{T_p^2 V_{p+1} \alpha'}{4(2\pi)^{9-p}} \prod_{m=1}^{\infty} \frac{\det[Q_{(m-1/2)1}^1 Q_{(m-1/2)2}]}{\det[Q_{(m)1}^1 Q_{(m)2}]} \int_0^\infty dt \left\{ \left( \frac{\pi}{\alpha'} t \right)^{9-p} \right\} \times \left[ 7 - p + \text{Tr}(H^1_{(1)1}H_{(1)2}) \right],$$

for the NS-NS sector, and

$$A_{\text{R-R}}^{\text{(massless)}} = \frac{T_p^2 V_{p+1} \alpha'}{8(2\pi)^{9-p}} \left( \kappa + \kappa' \right) \times \int_0^\infty dt \left\{ \left( \frac{\pi}{\alpha'} t \right)^{9-p} \right\} \frac{1}{\sqrt{\det(R_1^1 R_2)}} \exp \left( -\frac{1}{4\alpha'} \sum_i \left( y_i^2 - y_i^1 \right)^2 \right),$$

for the R-R sector. We did not put the limit on the exponential factors and the two other time dependent parts $\left( \frac{\pi}{\alpha' t} \right)^{9-p}$ and $\frac{1}{\sqrt{\det(R_1^1 R_2)}}$ in the Eqs. (30) and (31). The exponential parts indicate the locations of the branes, while closed string emission (absorption) does not depend on the positions of the branes. The other two factors possess origin in the zero modes, but not in the oscillators which define the closed string states. The provenance of the factor $1/\sqrt{\det(R_1^1 R_2)}$ is the tachyon fields which for large time weakens the interaction amplitudes. Precisely, since the presence of the open string tachyon makes the system unstable, after a long enough time the tachyon will roll down.
towards its minimum potential which causes a decreasing amplitude. In the absence of
the tachyonic fields this slowing down factor disappears.

We observe that for large distance branes the amplitude of the NS-NS sector depends
on the total field strengths $F_1$ and $F_2$ while these fields are absent in the R-R sector. In
other words, the internal electric and magnetic fields of the branes impress the exchange
of the graviton, dilaton and Kalb-Ramond states but do not modify the R-R repulsion
force between the distant branes.

The total amplitude

$$A^{\text{(massless)}} = A^{\text{NS-NS}} + A^{\text{R-R}}$$

$$= \frac{1}{(\alpha')^{3(p+1)/2}} \frac{T_p^2 V_{p+1}}{4(2\pi)^{9-p}} \left[ \frac{1}{2} (\kappa + \kappa') + \left( 7 - p + \text{Tr}(H_{(1)}^1 H_{(1)}^2) \right) \right] \times \prod_{m=1}^{\infty} \frac{\text{det}(Q_{(m-1/2)}^1 Q_{(m-1/2)}^2)}{\text{det}(Q_{(m)}^1 Q_{(m)})} \times \int_0^\infty dt \left\{ \left( \frac{\pi}{t} \right)^{9-p} \frac{1}{\sqrt{\text{det}(\tilde{R}_1^1 \tilde{R}_2^2)}} \exp \left( - \frac{L^2}{4\alpha' t} \right) \right\},$$

(32)

exhibits the long-range force between the D$p$-branes interaction, where $T_p = T_p|_{\alpha' = 1}$,
$\tilde{R}_{1,2} = R_{1,2}|_{\alpha' = 1}$ and $L^2 = \sum_i (y_i^2 - y_i^1)^2$ is the the square distance between the branes.
The NS-NS part indicates the exchange of the graviton, dilaton and Kalb-Ramond fields,
in which the dilaton and the graviton give attraction force while the Kalb-Ramond gives
repulsion one. In the same way, the R-R part indicates the repulsive contribution of the
$(p + 1)$-form potentials in the R-R sector. The net result force for the static branes with
zero background fields vanishes, since the branes are BPS states. But when the branes
possess velocity, rotation and background fields the total force is nonzero, i.e. the various
contributions are not balanced.

5 Conclusions

In this article we constructed a closed superstring boundary state corresponding to a
rotating and moving D$p$-brane which incorporates configurations of electric, magnetic
and tachyonic background fields. The bosonic boundary state includes an exponential
factor which is absent in the conventional boundary states, i.e. that one without tachyon.
This factor originates from the bosonic zero modes, rotation-motion and tachyon terms
in the boundary action.

It should be mentioned that in this article we considered the rotation axis perpendicular to the branes. In addition, the branes move along their volumes. According to the background fields we have preferred directions in the branes which break the Lorentz invariance. Therefore, these rotations and motions are meaningful.

According to the eigenvalues in the boundary state equations we deduce the following constraint equation

$$p^\alpha = -\frac{1}{2\alpha'}[(\eta + 4\omega)^{-1}U]^{\alpha}_{\beta}x^\beta.$$ 

This implies that along the worldvolume of the brane, momentum of an emitted (absorbed) closed string depends on its center of mass position. Fountain of this relation completely is the tachyon field. Thus, in the presence of the tachyon a closed string feels an exotic potential which affects its evolution.

The boundary states enabled us to calculate the interaction amplitude of two moving-rotating Dp-branes with background fields. This amplitude exponentially decreases with the square distance of the branes, but it is a very complicated function of the setup parameters. The variety of the adjustable parameters controls the treatment of the interaction. For example, for two D(d − 3)-branes, which can have different background fields and different motions, the contribution of the (super-)ghosts removes the effects of all transverse oscillators. It was shown that even for co-dimension parallel branes with similar fields, the total amplitude is nonzero. That is, our system does not satisfy the BPS no-force condition. This is due to the presence of the rotations, velocities and tachyonic and photonic fields on the branes.

The long-range part of the interaction was extracted. In this domain the instability of the branes, due to the background tachyon fields, weakens the interaction. This decreasing behavior can be understood by dissipation of the branes to the bulk modes because of the rolling of the tachyon to its minimum potential in long time regime. Finally, we observed that the internal electric and magnetic fields of the branes do not impress the R-R repulsion force between the large separated branes.

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