Thermodynamic Magnon Recoil for Domain Wall Motion

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We predict a thermodynamic magnon recoil effect for domain wall motions in the presence of temperature gradients. All current thermodynamic theories assert that a magnetic domain wall (DW) can move under a temperature gradient. This question has attracted much attention owing to its applicability in magnetic insulators for potential applications in logic devices and data storage technology. The conventional approach using static magnetic fields is well established with high DW velocities, but does not allow for the synchronous motion of multiple domain walls. Synchronous current-induced domain wall motion due to spin-transfer torques and/or spin-orbit torques formed an alternative way to efficiently manipulate the magnetization configuration, but the required high-current densities cause problems, such as Joule heating owing to Ohmic losses. Heat itself has been proposed as an efficient control parameter to overcome the problems during the emergence of spin caloritronics. Fully understanding and predicting new controlled ways to move the domain walls by magnonic heat currents is paramount to exploiting fully all their future device possibilities.

There is at present an theoretical incomplete understanding of temperature-gradient-driven DW motion. There are two types of theories, i.e., a macroscopically thermodynamic theory and a microscopically magnonic one. The theories contradict each other in certain regimes. In previous thermodynamic theories, a magnetic domain wall at finite temperature $T$ is treated as a thermodynamic object with free energy, $F = U - TS$, where $U$ is its internal energy and $S$ is its entropy. The free energy of the DW can also be expressed as the difference between a system with a DW minus that of the same system without the wall $\Delta F = \Delta U - T \Delta S$. Thermodynamic calculations show that, far below the Curie temperature, the entropy $\Delta S (T)$ increases and the free energy $\Delta F (T)$ decreases with the temperature. This leads to a conclusion that the DW must move toward regions with higher temperatures due to the entropic force, with the propagation velocity proportional to the temperature gradient $\nabla T$. This tendency has been observed in experiments.

On the other hand, there are microscopic angular momentum transfer and linear momentum transfer theories for the magnon-driven DW motion. The DW moves along the opposite direction of current flows if the former mechanism dominates, while along the same direction when the latter one is more important, i.e., there are strong spin-wave reflections by the wall. The proposed mechanism of magnonic linear momentum transfer has been confirmed in various systems including ferromagnets, antiferromagnets, and spin textures with Dzyaloshinskii-Moriya interaction.

In this Letter, we show that the inconsistency between these two types of theories can be resolved by augmenting the free energy by a term from the heat current which always flows in a non-equilibrium steady state in the presence of a temperature gradient. The heat current gets modulations by the DW with momentum-conserving back-scatterings. It then leads to a force that pushes a DW toward the colder region. We predict a new thermodynamic magnon recoil effect for the domain wall motion in temperature gradients. Under conditions of a strong backscattering, a high magnon thermal conductivity, and a slow magnon group velocity, this magnon recoil effect surpasses the previously identified entropic force. Such a regime can be achieved in yttrium iron garnet (YIG) and other ferromagnetic insulators, as we show below.

![FIG. 1: (Color online) Heat currents in a 1D magnetic wire in the presence of a temperature gradient $\nabla T$ without (upper panel) and with (lower panel) a domain wall with width $\delta_D$, denoted by $j^I$ and $j^II$, respectively. The presence of the domain wall causes the reflection of heat currents with a probability $R$.](https://example.com/fig1.png)
We consider a one dimensional (1D) magnetic wire connecting two thermal reservoirs with temperatures $T_b$ and $T_i$ ($T_b > T_i$) as shown in Fig. 1. The temperature gradient $\nabla T$ then drives magnon heat currents $j^1$ and $j^\Pi$ in a uniformly magnetized wire (as shown in the upper panel in Fig. 1) and in a wire with a domain wall (shown in the lower panel in Fig. 1), respectively, at non-equilibrium steady states. The presence of the domain wall causes finite magnon reflections [26], thereby reducing the heat current, i.e., $j^\Pi < j^1$. They are equal only when the potential generated by the domain wall is reflectionless [22,23]. A heat current $j^\Pi$ can modify the rate of the change of the entropy [30,31]

$$\frac{dS^\Pi}{dt} = - \int d\lambda j^\Pi \cdot \nabla T \frac{T}{T^2},$$

(1)

and this entropy change modifies the free energy. We therefore write the free energy of the wall as

$$\Delta F = \Delta F_e + \Delta F_{ne}.$$ 

(2)

Here $\Delta F_e$ is the equilibrium part as treated in previous thermodynamic theories [2,4], which may originate from the modified magnon density of states due to the wall [3,4,25,22], while $\Delta F_{ne}$ is the non-equilibrium part proportional to the heat current. Its change rate is then

$$\frac{d(\Delta F_{ne})}{dt} = - \int d\lambda \left( S^\Pi - S^1 \right) \frac{\nabla T}{T} dx,$$

(3)

where we introduced a parameter $R(T)$ denoting the temperature-dependent reflection probability of magnon heat currents by the wall. The magnon heat current in the presence of the domain wall is thus modified, i.e., $j^\Pi = (1 - R) j^1$. The heat current is connected with the temperature gradient by $j^1 = -\kappa \nabla T$, with a positive-definite magnon thermal conductivity $\kappa$. We then obtain

$$\frac{d(\Delta F_{ne})}{dt} = -R(T) \frac{\nabla T}{T} \frac{(\nabla T)^2}{T} dx > 0,$$

(4)

for finite reflections of heat currents. The temperature dependence of the free energy $\Delta F(T)$ is crucial to drive the DW propagation. It has been shown that the equilibrium part $\Delta F_e$ decreases with an increasing temperature $T$ [2,4], which leads to a conclusion that the DWW must move to the hotter region to reduce the free energy due to the entropic force. However, the present system under the temperature gradient $\nabla T$, strictly speaking, is not in an equilibrium state. Moreover, the non-equilibrium part $\Delta F_{ne}$ monotonically increases with time [for $R > 0$ in Eq. (4)], thus the arguments based on either maximization of entropy or minimization of free energy are not always valid [2,4] since transport processes matter a lot in non-equilibrium thermodynamics. In the following, we study the magnon transport, in particular its backscattering by a domain wall, and predict a thermodynamic magnon recoil effect in the presence of a temperature gradient, competing with the entropic force.

The thermal properties of magnons crucially depend on their dispersion relations and lifetime. To calculate the magnon thermal conductivity $\kappa$, we consider the heat current carried by the magnon flow due to the temperature gradient $\nabla T$ in the absence of domain walls (as the upper panel in Fig. 1), $j^1_k = L^{-1} \sum k \delta \omega_k \hbar v(k)$, where $L$ is the wire length, $k$ is the magnon wave-vector, $\delta \omega_k = n_k - \bar{n}_k$ is the magnon number in excess of equilibrium value $\bar{n}_k = 1/\left[ e^{\hbar \omega_k/(k_B T)} - 1 \right]$ being the Bose-Einstein distribution with Boltzmann constant $k_B$, $\hbar \omega(k)$ is the magnon energy, and $v(k) = \partial \omega / \partial k$ is the magnon group velocity. Using the Boltzmann approach we can write a first-order expression for the excess magnon number in the steady state and in the relaxation time approximation, $\delta \omega_k = -\tau_k (\partial \omega_k / \partial T) v(k) \nabla T$, where $\tau_k$ is the magnon relaxation time. One thus obtains the magnon thermal conductivity

$$\kappa = \frac{1}{2\pi} \sum_{n=1}^N \int_{\omega_{\min}}^{\omega_{\max}} \tau_k \hbar \omega (\partial \omega_k / \partial T) v(k) d\omega,$$

(5)

by using one dimensional magnon density of states (DOS) $\rho(k) = L/2\pi$. Here $N$ is the number of energy bands and $\omega_{\min/\max}$ is the lowest (highest) frequency of each band $n$.

The presence of domain wall may lead to a strong spin-wave reflection, and thus a reduction of magnon heat currents $j^\Pi = (1 - R) j^1$. The reported mean free path of thermal magnons in insulating ferromagnets, e.g., YIG, usually is $\sim 1 - 100 \mu m$ [25,34], which is much larger than the domain wall width $\delta_D \sim 10 - 100 \text{ nm}$, the scattering of spin waves by the wall can thus be treated as a ballistic process, thereby conserving the total momentum. We then can derive the reflection probability $R$ of magnon heat currents by the wall via the Landauer-Büttiker formula [26,35]

$$R(T) = \frac{\sum_{n=1}^N \int_{\omega_{\min}}^{\omega_{\max}} F(\omega, T) |r(k)|^2 d\omega}{\sum_{n=1}^N \int_{\omega_{\min}}^{\omega_{\max}} F(\omega, T) d\omega},$$

(6)

where $r(k)$ is the $k$-dependent reflection coefficient of magnons by the wall and $F(\omega, T) = \hbar \omega (\partial \omega / \partial T)$. Here we do not consider the modification of the magnon DOS due to the wall [26], which is relevant to reflectionless magnons treated in equilibrium thermodynamic theories [4,32] but causes only negligible effects to our results here. In the momentum-conserving scattering process between spin waves and the domain wall, the change rate of the linear momentum of a DW is $dp_{DW}/dt = 2\hbar M_s / \gamma$ [23] which must be compensated by that from magnons (with wave vector $k$) $dp_{magnons}/dt = (\partial \omega_k / \partial T) v(k)^2 \hbar k$. Here $\phi$ is the tilted angle of the DW plane, $M_s$ is the saturation magnetization, and $\gamma$ is the gyromagnetic ratio. Spin-wave reflections thus lead to a precession of the domain wall plane with the angular velocity

$$\phi_k = \frac{\gamma}{2M_s} (\partial \omega_k) v(k)^2 \hbar k.$$

(7)

The equivalent magnetic field responsible for the above precession velocity is then $H_k = \phi_k / \gamma$, giving rise to an effective
field along the wire axis after a summation of all states

\[ \mathbf{H}_{\text{ne}} = L^{-1} \sum_{k} \mathbf{h}_k \]

\[ = -\frac{\nabla T}{4\pi M_s} \sum_{k=1}^{N} \frac{c_k^{\text{max}}}{\omega_k} \tau_k (\partial \tilde{n}_k/\partial T) v_k |r(k)|^2 \hbar \omega. \] (8)

which is the effective field or force acting on the wall due to the thermodynamic magnon recoil in a temperature gradient.

Equation (8) is quite a general formula that can be used to calculate the effective field under any magnon dispersion relations and relaxation mechanisms. Experiment data in YIG, for instance, show an acoustic branch with frequency that rises from nearly zero at the Brillouin zone center to a value at the zone boundary that varies from 6 to 9.5 THz. These values correspond to temperatures of approximately 300 and 500 K. Since the lowest optical branch lies above the zone-boundary value, the calculation of the thermal properties up to room temperature can be done considering only the acoustic branch, i.e., \( N = 1 \).

At low wave numbers the dispersion relation can be approximately by a quadratic form \( \omega = \omega_1^{\text{min}} + Jk^2 \), where \( \omega_1^{\text{min}} \) is the magnetic band gap depending on materials parameters, such as the magnetic anisotropy, dipole-dipole coupling, Dzyaloshikii-Moriya interaction, etc., and \( J \) is the exchange constant. The cut-off frequency \( \omega_1^{\text{max}} \) thus is \( \omega_1^{\text{max}} = \omega_1^{\text{min}} + Jk^2 \) with \( k_0 \) the maximum wave vector depending on the magnon propagation direction. The magnon group velocity is then \( v_g = 2\sqrt{J(\omega - \omega_1^{\text{min}})} \). It has been shown that the quadratic dispersion agrees very well with the actual dispersion up to a wave vector \( k = 0.6k_0 \) in YIG [56]. Under the above conditions, we obtain

\[ \mathbf{H}_{\text{ne}} = -\frac{\kappa \nabla T}{2\pi M_s} \frac{|\mathbf{r}|^2}{v_g}, \] (9)

with the average reflection probability

\[ |\mathbf{r}|^2 = \int_{\omega_1^{\text{min}}}^{\omega_1^{\text{max}}} \tau_k F(\omega, T) |r(k)|^2 d\omega / \int_{\omega_1^{\text{min}}}^{\omega_1^{\text{max}}} \tau_k F(\omega, T) d\omega, \] (10)

and the average group velocity

\[ \bar{v}_g = \int_{\omega_1^{\text{min}}}^{\omega_1^{\text{max}}} \tau_k v_g F(\omega, T) d\omega / \int_{\omega_1^{\text{min}}}^{\omega_1^{\text{max}}} \tau_k F(\omega, T) d\omega. \] (11)

We then obtain the DW velocity along the direction of heat currents due to their recoil effect, below Walker breakdown [9],

\[ v_{\text{ne}}(T) = \frac{\gamma_D}{\alpha} \frac{\mathbf{H}_{\text{ne}}}{v_g} = \frac{|\mathbf{r}|^2}{v_g} \frac{\gamma_D \kappa \nabla T}{\alpha} \frac{1}{2\pi M_s}. \] (12)

Schlickeiser et al. [3] derived an effective magnetic field based on equilibrium thermodynamics, \( \mathbf{H}_e = \nabla T (J_0/T_c) / (\delta \delta_{\text{m}} M_s k_B) \) with nearest-neighbor exchange energy \( J_0 \), Curie temperature \( T_c \), equilibrium local magnetization \( m_{\text{eq}} \), and lattice constant \( a \), which is believed to exceed the magnonic spin transfer torque proportional to \( 1 - |\mathbf{r}|^2 \) [3, 22]. So, the final DW propagation direction depends on the competition between \( \mathbf{H}_{\text{ne}} \) and \( \mathbf{H}_e \). The condition to observe a DW propagation toward the colder regime is therefore

\[ \frac{|\mathbf{r}|^2}{v_g} > \frac{2\pi}{\delta \delta_{\text{m}} M_s k_B T_c} (J_0/T_c), \] (13)

which requires a good heat conduction in magnetic domains (a large \( \kappa \) without the domain wall), a strong magnon backscattering (a large \( |\mathbf{r}|^2 \)), a slow magnon group velocity (a small \( v_g \)), and a broad domain wall (a large \( \delta \)). The forces induced by non-equilibrium thermal fluctuations under temperature gradients cause a Brownian motion [37, 38] of the domain wall and could be another reason to push its propagation toward the colder regime [39]. However it is still an open question how valid the classical fluctuation-dissipation theorem for equilibrium states is [40], particularly when it is applied to nonequilibrium steady states in the presence of temperature gradients [41–44]. The Brownian motion effect is however negligible in the presence of strong magnon backscatterings.

In order to evaluate the parameters in criterion (13), we now can make either of two plausible assumptions about the behavior of \( \tau_k \). Model I: If one considers that the relaxation time \( \tau_k \) is independent of both the wave number and the temperature (the simple average-lifetime model) [45] and takes \( \tau_k = \tau_k \), we obtain \( |\mathbf{r}|^2 = R \) the same as the reflection probability of magnon heat currents [Eq. (6) when \( N = 1 \)], \( \bar{v}_g = \int_{\omega_1^{\text{min}}}^{\omega_1^{\text{max}}} v_g F(\omega, T) d\omega / \int_{\omega_1^{\text{min}}}^{\omega_1^{\text{max}}} F(\omega, T) d\omega \), and \( \kappa \propto \tau_k \). Model II: If we consider the Gilbert damping but neglect higher order processes such as magnon-magnon interactions, the relaxation time is then \( \tau_k = 1/(2\alpha \omega) \) with Gilbert damping constant \( \alpha \) [46]. We thus have

\[ |\mathbf{r}|^2 = \int_{\omega_1^{\text{min}}}^{\omega_1^{\text{max}}} (\partial \tilde{n}_k/\partial T) v_g |r(k)|^2 d\omega / \int_{\omega_1^{\text{min}}}^{\omega_1^{\text{max}}} (\partial \tilde{n}_k/\partial T) d\omega, \]

\[ \bar{v}_g = \int_{\omega_1^{\text{min}}}^{\omega_1^{\text{max}}} (\partial \tilde{n}_k/\partial T) v_g d\omega / \int_{\omega_1^{\text{min}}}^{\omega_1^{\text{max}}} (\partial \tilde{n}_k/\partial T) d\omega, \text{ and } \kappa \propto 1/\alpha. \]

It has been shown that both dipole-dipole [23] and Dzyaloshikii-Moriya [29] interactions can result in strong magnon reflections in the presence of a domain wall in ferromagnets. A precessing domain wall in antiferromagnet can also lead to significant magnon reflections [28, 47]. A common feature of the reflection probability function \( |r(k \omega)|^2 \) is the sharp transition from 1 at lower frequencies to 0 at higher frequencies [23, 24, 47], satisfying ansatz \( |r(k \omega)|^2 = |\omega - (\omega - \omega_i)/\Delta \omega|^2 \), with the transition frequency \( \omega_i \) and the spectrum width \( \Delta \omega \). Function \(|\mathbf{r}|^2\) reduces to 1 for \( \omega \ll \omega_i \), and 0 for \( \omega \gg \omega_i \). The form of the function \( \omega \) depends on material parameters such as the domain wall width [26], the Dzyaloshikii-Moriya interaction strength [29], etc., and scattering details such as the incident angle of magnons [23]. However, for a very narrow spectrum (\( \Delta \omega \ll 1 \)) which is often the case [23, 47], it can be approximately described by the Heaviside step function, i.e., \(|w|^2 \approx s(\omega - \omega_i)) with

\[ s(x) = \begin{cases} 0, & x < 0 \\ 1, & x = 0 \\ 1, & x > 0 \end{cases} \] (14)
It indicates that magnons are completely reflected by the wall when their frequencies are lower than \( \omega_c \), while they essentially pass through the domain wall without any reflection above \( \omega_c \). By denoting \( A = \hbar \omega_c^{\text{min}}/k_B \), \( B = \hbar \omega_c^{\text{max}}/k_B \), \( x_A = A/T \), and \( x_C = C/T \) with \( C = \hbar \omega_c/k_B \), Eqs. (5), (10), and (11) can be calculated analytically and yield

\[
\kappa = \frac{\bar{v}_e}{\pi} \sqrt{\frac{Jk_B^2T^3}{\hbar}} \int \frac{\sqrt{\chi - x_0^x e^x}}{(e^x - 1)^2} dx, \tag{15}
\]

\[
\bar{v}_e = 2 \sqrt{\frac{Jk_B T}{h}} \int \frac{f(A/T) - f(B/T)}{f(A/T)} dx, \tag{16}
\]

\[
|\bar{r}|^2 = \frac{R}{f(A/T) - f(B/T)} \int \frac{\left(- (x - x_c) \right) e^{x} }{(e^{-1})^2} dx, \tag{17}
\]

with \( \pi \)

\[
f(x) = -2x \ln(1 - e^{-x}) + \frac{x^2}{e^x - 1} + 2 \sum_{p=1}^{\infty} \frac{e^{-px}}{p^2}, \tag{18}
\]

in Model I (\( \tau_k = \bar{\tau}_k \)), and

\[
\kappa = \frac{1}{2\pi \alpha} \sqrt{\frac{Jk_B^2T^3}{\hbar}} \int \frac{\sqrt{\chi - x_0^x e^x}}{(e^x - 1)^2} dx, \tag{19}
\]

\[
\bar{v}_e = 2 \sqrt{\frac{Jk_B T}{h}} \int \frac{f(A/T) - f(B/T)}{g(A/T) - g(B/T)} dx, \tag{20}
\]

\[
|\bar{r}|^2 = \frac{R}{g(A/T) - g(B/T)} \int \frac{\left(- (x - x_c) \right) e^{x} }{(e^{-1})^2} dx, \tag{21}
\]

with \( \pi \)

\[
g(x) = -\ln(1 - e^{-x}) + \frac{x}{e^x - 1}. \tag{22}
\]

in Model II (\( \tau_k = 1/(2\alpha \omega) \)).

The following three cases are of potential interest. (i) For a low transition frequency (\( \omega_c < \omega_c^{\text{min}} \)), there is no reflection, and Eqs. (17) and (21) yield \( |\bar{r}|^2 = 0 \). (ii) For an intermediate transition frequency (\( \omega_c^{\text{min}} < \omega_c < \omega_c^{\text{max}} \)), Eqs. (17) and (21) reduce to \( |\bar{r}|^2 = \left[ f(A/T) - f(C/T) \right] / \left[ f(A/T) - f(B/T) \right] \) and \( \left[ g(A/T) - g(C/T) \right] / \left[ g(A/T) - g(B/T) \right] \), respectively. (iii) For a high transition frequency (\( \omega_c > \omega_c^{\text{max}} \)), all magnons are reflected by the wall. Thus, Eqs. (17) and (21) reduce to \( |\bar{r}|^2 = 1 \).

Figure 2(a) shows the temperature dependence of the magnon heat conductivity \( \kappa \) in both Models I and II. It increases as the elevated temperature in two cases. In Fig. 2(b) we calculate the average group velocity \( \bar{v}_e \). It monotonically increases with the temperature and saturates at high temperatures. The temperature dependence of \( |\bar{r}|^2 \) for different transition temperatures \( C \) are shown in Fig. 2(c), with a monotonically decreasing manner. It is because higher temperatures make more magnons populate higher energy levels, which leads to a smaller magnon reflection subsequently. We also observe that a higher transition temperature leads to a stronger magnon reflection. Figure 2(d) demonstrates a monotonically increasing dependence on the temperature of parameter \( \kappa |\bar{r}|^2 / \bar{v}_e \) in both models. As shown in Eq. (13) with quadratic magnon dispersion relations, parameter \( \kappa |\bar{r}|^2 / \bar{v}_e \), which is independent of exchange constant \( J \), is crucial to determine if the domain wall can move toward the colder region. According to our calculations, this condition should be easily satisfied at elevated temperatures (\( T \gtrsim 400 \) K) with a small magnon damping (\( \bar{\tau}_k \approx 1 \) ns or \( a \lesssim 10^{-4} \)) for a broad domain wall (\( \partial \theta \approx 100 \) nm) in a weak ferromagnet (\( J_0/a \approx 3 \times 10^{-12} \) J/m) under any temperature gradient that can overcome the pinning force produced by defects or impurities. Other relaxation models considering three- and four-magnon scattering processes are expected not to modify our conclusions significantly.

To summarize, we predict a thermodynamic magnon recoil effect for domain wall motion under temperature gradients. We correct the previous thermodynamic theories by including a heat current term for entropy and/or free-energy generations, which is always presents in non-equilibrium steady states in the presence of a temperature gradient. The heat current gets modulations by the DW with momentum-conserving backscatterings. It then leads to a recoil force on the wall, which competes with the previously identified entropic force. Our theory thereby closes the inconsistency between macro-
scopic and microscopic theories for the domain wall motion, and we propose experiments to test it. We also expect the similar thermodynamic magnon recoil effect to play an important role in other magnetic structures, e.g., magnetic vortices, bubbles, or Skyrmions, and other materials like antiferromagnets or multiferroics.

After the completion of this work, we became aware of one recent report \cite{49} on a DW thermophoresis in antiferromagnets using the classical fluctuation-dissipation relation without considering any magnon backscattering. Our results should also be applicable to their work.

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