Abstract

In this paper we discuss the formal relationship between the classes of languages generated by Tree Adjoining Grammars and Head Grammars. In particular, we show that Head Languages are included in Tree Adjoining Languages and that Tree Adjoining Grammars are equivalent to a modification of Head Grammars called Modified Head Grammars. The inclusion of MHL in HL, and thus the equivalence of HG's and TAG's, in the most general case remains to be established.

1. Introduction

This paper discusses the relationship between Tree Adjoining Grammars (TAG's) and Head Grammars (HG's). TAG's and HG's, introduced to capture certain structural properties of natural languages, were developed independently. TAG's deal with a set of elementary trees which are composed by means of an operation called adjoining. HG's are like Context-free Grammars, except for the fact that besides concatenation of strings, string wrapping operations are permitted. TAG's were first introduced in 1975 by Joshi, Levy and Takahashi [3]. Joshi [2] investigated some formal and linguistic properties of TAG's with local constraints. The formulation of local constraints was then modified and formal properties were investigated by Vijay-Shanker and Joshi [9]. The linguistic properties were studied in detail by Kroch and Joshi [5]. HG's were first introduced by Pollard [6] in 1983 and their formal properties were investigated by Roach [7]. It was observed that the two systems seemed to possess similar generative power and since they also appear to have the same closure properties [7,9] as well as similar parsing algorithms [6,9] a significant amount of indirect evidence existed to suggest that they were formally equivalent. In the present paper, we will attempt to provide a characterization of the formal relationship between HG's and TAG's. In [10] we consider various linguistic aspects of the relationship: in particular what might be referred to as the strong equivalence of the two formalisms.

Vijay-Shanker and Joshi [9] provided a brief description of the intuition behind the inclusion of Tree Adjoining Languages (TAL) in the class of languages generated by a variant of HG's called Modified Head Grammars (MHG's). In the present paper, we give a proof of this result as well as a proof for the inclusion of Modified Head Languages (MHL) in TAL: hence we show that MHG's and TAG's are equivalent. This result is presented in section 3. In section 2, we discuss the relationship between MHG's and HG's.

1.1. Tree Adjoining Grammars

Definition 1. A TAG is a 5-tuple \( G = (V_N, V_T, S, I, A) \) where \( V_N \) and \( V_T \) are finite sets of nonterminals and terminals respectively, \( S \) is a distinguished nonterminal, \( I \) is a finite set of initial trees and \( A \) is a finite set of auxiliary trees.

Initial trees and auxiliary trees have the following form:

\[
\begin{align*}
\text{Initial:} & \quad w \\
\text{Auxiliary:} & \quad w_1, w_2 \in V_T^n, x \in V_N
\end{align*}
\]

Except for one node, which is called the foot node, all the nodes in the frontier of an auxiliary tree are labelled by terminal symbols. The foot node is labelled by the same nonterminal symbol as the root. All initial and auxiliary trees, referred to as elementary trees, have a height of at least one.

We now define the adjoining operation. Let \( \eta \) be some node labelled \( X \) in a tree \( \gamma \). Let \( \beta \) be an auxiliary tree with root and foot labelled by \( X \). The tree obtained by adjoining \( \beta \) at \( \eta \) is given in figure 1.2.

The subtree under \( \eta \) is excised from \( \gamma \) the auxiliary tree \( \beta \) is inserted in its place and the excised subtree is inserted at the foot of \( \beta \).

As defined above it is possible to adjoin any auxiliary tree at a node as long as the label of the node was the same as that of the root and the foot of the auxiliary tree. However, in general, adjoining will be constrained as follows. Associated with each node is a selective adjoining (SA) constraint specifying that subset of the auxiliary tree which can be adjoined at this node. Trees can only be included in the SA constraint associated with a particular node if their root and foot are labelled with the same nonterminal that labels the node. There are two special cases: (a) the subset specified by SA is the entire set of auxiliary trees, in this case the adjoining operation is defined as

\[
\begin{align*}
\text{Subtree:} & \quad w \\
\text{Auxiliary:} & \quad w_1, w_2 \in V_T^n
\end{align*}
\]

The subtree under \( \eta \) is excised from \( \gamma \), the auxiliary tree \( \beta \) is inserted in its place and the excised subtree is inserted at the foot of \( \beta \).

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case the entire set need not be explicitly listed; (b) the sub-
set specified is empty i.e., no adjoining is possible. We call
this the null adjoining (NA) constraint. A node may be
associated with a so-called obligatory adjoining (OA)
constraint which can be used to ensure that an adjunction
is obligatorily performed at a node.

Example 1.1 We now present an example TAG G, which
illustrates the notation used to specify the constraints as-
sociated with a node. There is one initial tree α and two
auxiliary trees β₁ and β₂:

\[
\begin{align*}
α & : S \circ  \\
β₁ & : S \circ  b  \\
β₂ & : S \circ  \gamma \circ  \\
\end{align*}
\]

\[\text{figure 1.3}\]

Having introduced SA and OA constraints, we must ex-
tend the definition of the adjunction operation. Suppose we
adjoin an auxiliary tree β at a node η of a tree γ producing
the tree γ'. For those nodes in γ' that do not correspond
to nodes of β, the constraints remain the same as those in
γ. The remaining nodes in γ' have the same constraints as
those for the corresponding nodes of β. For example, con-
sider a sample derivations in the grammar G as given below
in figure 1.4. We use an * to indicate the node at which
adjunction is performed.

\[
\begin{align*}
\gamma_0 & : S \circ  \\
\gamma' & : b \circ  \gamma \circ  \circ  \\
\gamma'' & : b \circ  b \circ  \gamma \circ  \circ  \\
\end{align*}
\]

\[\text{figure 1.4}\]

We will now present an alternative (yet equivalent) def-
ition of the adjoining operation. Suppose we
adjoin an auxiliary tree β at a node η of a tree γ producing
the tree γ'. For those nodes in γ' that do not correspond
to nodes of β, the constraints remain the same as those in
γ. The remaining nodes in γ' have the same constraints as
those for the corresponding nodes of β. For example, con-
sider a sample derivations in the grammar G as given below
in figure 1.4. We use an * to indicate the node at which
adjunction is performed.

\[
\begin{align*}
\gamma_0 & : S \circ  \\
\gamma' & : b \circ  \gamma \circ  \circ  \\
\gamma'' & : b \circ  b \circ  \gamma \circ  \circ  \\
\end{align*}
\]

\[\text{figure 1.4}\]

Now we can see that the language \(L_1\), generated by the
example grammar G, is \(L_1 = \{ a^n b^n f c^n h \mid n \geq 0 \}\).

It is useful to further generalize the notion of a derived tree
to include trees derived from subtrees of elementary trees. If
η is a node in some elementary tree, then \(D(\eta)\) represents
the set of trees derived from the subtree rooted at η. Nodes
are represented using an extension of the tree addressing
scheme of Gorn [1]. Each node in an elementary tree is
given a unique name in the following manner: the pair \((γ, i)\)
denotes the root of \(γ\); if \((γ_i, j)\) is a node in \(γ\), then \((γ_i, j)\)
represents the \(j^{th}\) daughter of this node.

1.2. Head Grammars

Before giving the formal definition of Head Grammars,
the notion of a headed string will be described. A headed
string is a string of symbols containing one distinguished
symbol referred to as the head of the string. Formally, this
can be represented as a pair consisting of a string \(w\) and an
integer that indicates the position of the head in the string.
In this paper, we use one of two notations to denote this
string: when we wish to explicitly mention the head we use
the representation \(w_1 w_2\) where \(w_1 w_2 = w\); alternatively,
we can simply denote the headed string by \(\overline{w}\). This allows
us to denote the headed empty string as \(\overline{\lambda}\).

Definition 2. A Head Grammar, G, is given by a 4-tupk;
\((\mathcal{V}_N, \mathcal{V}_T, S, P)\). Productions in P are of the form: \(A \rightarrow f(\alpha_1, \ldots, \alpha_n)\) or \(A \rightarrow \alpha_1\) where \(A \in \mathcal{V}_N, \alpha_i\) either belongs
in \(\mathcal{V}_N\) or is a headed string.

\(f \in \bigcup_{i \geq 1} \{ LC_i, LL_i, LR_i, RC_i, RL_i, RR_i \}\)

We now define the operations \(LC_i, LL_i,\) and \(LR_i\) for \(i \in \{1, 2\}\). Definitions of the other operations can be found in [6]
and are not given here, since Roach [7] has shown that there
is a normal form for Head Grammars which only uses these
operations.

\[
\begin{align*}
LC_1(u_1 \overline{u}_1 u_2, v) &= u_1 \overline{u}_1 u_2, \\
LC_2(u, u_1 \overline{u}_2 v) &= u_1 \overline{u}_2 v, \\
LL_1(u_1 \overline{u}_1 u_2, v) &= u_1 \overline{u}_1 u_2, \\
LL_2(u, u_1 \overline{u}_2 v) &= u_1 \overline{u}_2 v, \\
LR_1(u_1 \overline{u}_1 u_2, v) &= u_1 \overline{u}_1 u_2, \\
LR_2(u, u_1 \overline{u}_2 v) &= u_1 \overline{u}_1 u_2
\end{align*}
\]
Both Pollard [6] and Roach [7] define these operations as partial functions. Pollard’s definition of headed strings includes the headed empty string (\(\lambda\)). However, mathematically, \(\lambda\) does not have the same status as other headed strings: for example, \(LC1(\lambda, w)\) is undefined. In general, the term \(f(j[w_1, \ldots, w_n])\) is undefined when \(w_1 = \lambda\). This nonuniformity has led to difficulties in proving certain formal results about Head Grammars [7], and has caused problems in showing the equivalence of MHG’s and HG’s (see section 2).

The language generated by a HG \(G\) is defined as follows: 
\[L(G) = \{ w \mid S \Rightarrow^* w \}\]

**Example 1.2** We now present a sample Head Grammar for the language \(L_1 = \{ a^n b^n a^m c^m | n \geq 0 \}\)

\[
S \rightarrow LL2(S_1), \\
S_1 \rightarrow LC2(a, S_2), \\
S_2 \rightarrow LL2(S_1, bc), \\
S_3 \rightarrow LL2(S_2, bc)
\]

**1.3. Modified Head Grammars**

We find it convenient to consider a formalism that closely resembles HG’s: referred to as Modified Head Grammars (MHG’s). Instead of headed strings, MHG’s have split strings. A split string has a distinguished position between two strings in \(V_T\), about which it may be split. We will denote a split string as \(w_1w_2\) where \(w_1, w_2 \in V_T\). Notice that we can represent the split empty string as \(\lambda_1\lambda_2\), though this will be denoted \(\lambda\) whenever the context makes it obvious that we are referring to a split string. In MHG’s, there are 3 operations on split strings: \(W, C1,\) and \(C2\), defined as follows:

\[
W(w_1w_2, u_1u_2) = w_1u_1u_2w_2 \\
C1(w_1w_2, u_1u_2) = w_1u_1w_2u_2 \\
C2(w_1w_2, u_1u_2) = w_1u_1w_2u_2
\]

The operations \(C1\) and \(C2\) correspond to the operations \(LC1\) and \(LC2\) in HG’s. The operation \(W\) has been defined such that the split point of its second argument becomes the split point of the string resulting from application of the operation (like the HG operations \(LL2\) and \(LR2\)).

Since the split point is not a symbol but a position between strings, separate operations corresponding to \(LL2\) and \(LR2\) are not needed. In addition, unlike HG’s, which distinguish the two wrapping operations \(LL1\) and \(LL2\), \(W\) suffices as a substitute for both of these operations. Suppose \(Y \Rightarrow w_1w_2\) and \(Z \Rightarrow w_1w_2\) and we want \(X\) to derive \(w_1w_2w_2\). This can be achieved with the following two productions: \(Z' \Rightarrow C1(\lambda, Z)\) and \(X \Rightarrow W(Y, Z')\).

**Example 1.3** We now give a MHG generating \(L_1\).

\[
S \rightarrow W(S_1, f_1), \\
S_1 \rightarrow C2(a_1, S_2), \\
S_2 \rightarrow W(S_1, b_1c), \\
S_3 \rightarrow W(S_2, b_1c)
\]

We will defer the discussion of both the formal and linguistic relationship between HG’s and MHG’s until section 2. It is worth noting at this point that the definition of MHG’s given here coincides with the definition of HG’s given in Rounds [8]. As we shall see in section 2, these formalisms are very closely related.

**1.4. Tree Adjunction and Wrapping**

Before showing the formal equivalence of MHG’s and TAG’s, it is instructive to consider the relationship between the wrapping operation \(W\) of MHG’s and the adjoining operation of TAG’s. Suppose that we have the production \(p = X \rightarrow W(Y, Z)\) in a MHG \(G\), and that we have two derivations from the nonterminals \(Y\) and \(Z\) deriving the headed strings \(w_1w_2\) and \(w_1w_2\) respectively. Given the production \(p\), we can derive the split string \(w_1w_1w_2w_2\) from \(X\).

Suppose there is a derived auxiliary tree \(\gamma\) corresponding to the above derivation of \(w_1w_2\), from \(Y\) where the foot node appears at the split point, as shown in figure 1.6 below. Also assume that there is a node \(\eta\) dominating a subtree that corresponds to a derivation of \(w_1w_2\) from \(Z\) where, as before, we assume that the foot node appears at the split point. Consider the tree resulting from the adjunction of \(\gamma\) at the node \(\eta\), also shown in figure 1.6. The resulting tree can be thought of as corresponding to the derivation of the split string \(w_1w_1w_2w_2\) from \(X\).

\[\gamma:\]

\[
\eta:
\]

\[
v_1 \uparrow w_1
\]

\[
v_2 \uparrow w_2
\]

\[
v_3 \uparrow w_3
\]

\[
x \uparrow n
\]

\[
f \uparrow x
\]

\[
v_4 \uparrow v
\]

**figure 1.6**

This example illustrates the basic intuition behind the constructions involved in the following proofs showing the equivalence of MHG’s and TAG’s.

**2. Head Grammars and Modified Head Grammars**

In this section, we shall discuss the relationship between MHG’s and HG’s. First we present the outline of a construction showing that for every HG \(G\) there is an equivalent MHG \(G'\). We then briefly discuss the linguistic relationship between MHG’s and HG’s.

Suppose \(X \Rightarrow w_1h\). This headed string can be split in two ways: into the substrings \(w_1\) and \(h\), or \(w_1\) and \(w_2\). This depends on whether \(X\) is used in a left or right wrapping operation. Since in MHG’s we can only split a string in one place, we use two nonterminals, \(X^l\) and \(X^r\) deriving \(w_1h\) and \(w_2\) respectively. Thus, for example, the production \(Z \rightarrow W(X, Y)\) can be used in place of \(Z \Rightarrow LL2(X, Y)\). A further complication arises when a headed string is split first to the right of its head and then the resulting string is split to the left of the same head. The problem is resolved by introducing nonterminals \(X'^l\), that derive split strings of the form \(w_1w_2\) whenever \(X\) derives \(w_1h\). We can reintroduce the missing head with the following productions:

\[
X' \rightarrow W(X'^l, h) \\
X^r \rightarrow W(X'^r, h)
\]

Complete details of this proof are given in [4].
We are unable to prove the inclusion of MH1's in HL's. The problems faced when attempting to find such a proof are a result of the operations in HG's not being total functions. For example, \( C_1(\lambda, w) \) is defined in MHG's, whereas \( LC_1(\lambda, w) \) is undefined in the HG's framework. We have not found any way of getting around this technical problem in a systematic manner. All TAG's considered by the authors so far have an equivalent HG. We feel that the problem of the empty headed string in the HG formalism does not result from an important difference between the formalisms.

In the following discussion, we propose that MHG's can be given a linguistic interpretation if we retain the notion of a head terminal in a split string. The split point should be viewed as determining the position of the head. As far as the authors are aware, Head Grammars for natural languages use only one kind of wrapping operation: either only the left wrapping operations \( LL_i \), or only the right wrapping operations \( LR_i \). Thus, any headed string can be split on only one side of the head. For example, if \( w_1 h w_2 \) is a headed string, and only the left wrapping operations were used, then the headed string can only be split as \( w_1 h \) and \( w_2 \). For any HG using only left wrapping operations there exists an equivalent HG such that split strings will have their split points immediately to the right of the actual head. However, obviously not every MHG can be given a linguistic interpretation in this way.

3. Equivalence of MHG’s and TAG’s

We will now show that the class MH is equal to the class TAL. The complete proofs for the results presented here are given in [4].

3.1. Inclusion of TAL in MH

Based on the earlier observation concerning the similarity between the wrapping and adjoining operations, we shall now present a scheme for transforming a given TAG \( G = (V_T, V_R, S, I, A) \) to an equivalent MHG \( G' = (V_M, V_R, S, P) \). In this section, we have generalized the concatenation operations of MHG’s to be of the form \( C_j \) for \( j \geq 1 \). It is obvious that these operations can always be simulated using just \( C_1 \) and \( C_2 \).

We shall first describe the algorithm \( \text{convert} \) informally. If \( \eta \) is a node of some elementary tree \( \gamma \), applying \( \text{convert} \) to \( \eta \) returns a sequence of productions in the MHG formalism capturing the structure of the subtree of \( \gamma \) rooted at \( \eta \). The wrapping operation is used to simulate the effect of adjoining; the concatenation operations \( C_j \) concatenate the strings derivable from the daughters of a node. The choice of \( i \) depends on which child is the ancestor of the foot node. The exact structure of a tree can be captured by using nonterminals that are named by the addresses of nodes of elementary trees rather than the nonterminals labelling the nodes.

The main idea of our scheme is as follows. Let \( (\beta, i) \) be the address of a node in an auxiliary tree \( \beta \) and \( \gamma \) belongs to \( D((\beta, i)) \) with a frontier \( w_1 X w_2 \). We have a nonterminal corresponding to this node (denoted by \( [\beta, i] \)) which derives the split string \( w_1 i w_2 \). In particular, when \( (\beta, i) \) is the root of \( \beta \) (i.e., \( i = e \)), then the nonterminal \( [\beta, e] \) should derive the split strings \( w_1 e w_2 \) wherever there is a tree in \( D(\beta) \) with frontier \( w_1 X w_2 \). That is, the split point appears in a position corresponding to the foot node.

Thus, the wrapping operation \( W \) can be used to simulate the effect of adjoining in the following manner. If \( (\gamma, i) \) is a node at which \( \beta \) is adjoinable, we have a production corresponding to adjoining of \( \beta \) at \( (\gamma, i) \).

\[ [\gamma, i] \rightarrow W([\beta, e], [\gamma, i]) \]

where \([\gamma, i]\) derives strings derivable from the children of \((\gamma, i)\). We also have the rule \([\gamma, i] \rightarrow [\gamma, i] \) for the case when no adjunction takes place at \((\gamma, i)\). Since \([\gamma, i]\) is supposed to derive strings derivable by the concatenation of the frontiers of subtrees dominated by the children of \((\gamma, i)\), we have the production,

\[ [\gamma, i] \rightarrow C_j([\gamma, i \cdot 1], \ldots, [\gamma, i \cdot j], \ldots, [\gamma, i \cdot k]) \]

where \((\gamma, i \cdot 1), \ldots, (\gamma, i \cdot j), \ldots, (\gamma, i \cdot k)\) correspond to the \( k \) children of \((\gamma, i)\) and where the \( j^{th} \) child is the ancestor of the foot node. The operation \( C_j \) is used so that the split point appears in the same position as the foot node. By convention, we let \( j = 1 \) when \((\gamma, i)\) is not the ancestor of the foot node.

We are now in a position to define the conversion process. The algorithm is as follows:

- For each initial tree \( a \), let \( S \rightarrow [a, e] \in P \).
- For each elementary tree \( \gamma \), call \( \text{convert}((\gamma, i)) \)

where the procedure \( \text{convert} \) is as defined below.

\[ \text{define } \text{convert}((\gamma, i)); \]
\[ \text{case 1: } (\gamma, i) \text{ is a leaf node} \]
\[ \text{if } (\gamma, i) \text{ has label } a \in V_T \cup \{ \lambda \} \text{ then} \]
\[ \text{step 1: add } [\gamma, i] \rightarrow a \text{ to } P \]
\[ \text{else } [\gamma, i] \text{ is the foot node} \]
\[ \text{step 2: add } [\gamma, i] \rightarrow W([\beta, e], [\gamma, i]) \]
\[ \quad \text{for each } \beta \text{ in SA constraint of } (\gamma, i) \]
\[ \text{step 3: add } [\gamma, i] \rightarrow X \]
\[ \quad \text{if } (\gamma, i) \text{ does not have an OA constraint} \]
\[ \text{case 2: } (\gamma, i) \text{ is an internal node and has } k \text{ children} \]
\[ \text{step 4: add } [\gamma, i] \rightarrow W([\beta, e], [\gamma, i]) \]
\[ \quad \text{for each } \beta \text{ in SA constraint of } (\gamma, i) \]
\[ \text{if } (\gamma, i) \text{ does not have OA constraint then} \]
\[ \text{step 5: add } [\gamma, i] \rightarrow [\gamma, i] \]
\[ \text{if } (\gamma, i) \text{ is ancestor of foot node then} \]
\[ \text{step 6: add } [\gamma, i] \rightarrow C_j([\gamma, i \cdot 1], \ldots, [\gamma, i \cdot k]) \]
\[ \quad \text{for each } j \text{th child dominates foot node;} \]
\[ \text{step 7: add } [\gamma, i] \rightarrow C_1([\gamma, i \cdot 1], \ldots, [\gamma, i \cdot k]) \]
\[ \quad \text{for } 1 \leq j \leq k \text{ do } \text{convert}((\gamma, i \cdot j)) \]

We prove the inclusion of \( L(G) \) in \( L(G') \), by induction on the height of the trees derived from all subtrees of elementary trees, where the inductive hypothesis states:

For all elementary trees \( \gamma \), and addresses \( i \) in \( \gamma \), if there is a tree \( \gamma' \) in \( D((\gamma, i)) \) of height less than \( k \), and the frontier of \( \gamma' = w_1 X w_2 \) or \( w_1 w_2 \), then \([\gamma, i] \rightarrow w_1 w_2 \).

It will be easy to show the inclusion of \( L(G) \) in \( L(G') \) by induction, considering steps 4, 5, 6 and 7. The base cases correspond to steps 1, 2 and 3.
We show the inclusion in the other direction by induction on the length of derivation of split strings in $G'$. The induction hypothesis is given by:

\[
\text{if } (\gamma, i) \rightarrow w_1 w_2 \text{ in } k \text{ steps, then there is a } \gamma' \in D((\gamma, i)) \text{ such that the frontier of } \gamma' \text{ is } w_1 X w_2 \text{ or } w_1 w_2, \text{ depending on whether the foot node of } \gamma, \text{ labelled by } X \text{ if it exists, is a descendant of } (\gamma, i) \text{ or not.}
\]

3.2. Inclusion of MHL in TAL

When we convert a TAG into a MHG, each elementary tree generates a set of productions. The sets generated by any two distinct elementary trees are disjoint and, furthermore, have a constrained form encoding the hierarchical structure of the tree. The task of converting a MHG to a TAG cannot simply involve the inversion of this construction, since it is not in general possible to find groupings of productions in a MHG that have the required structure.

The approach used to convert MHG's to TAG's is based on satisfying the following requirement: for each derivation in the MHG there must be a derived tree in the TAG for the same string, in which the foot is positioned at the split point: i.e., $X \rightarrow w_1 w_2$ in MHG if and only if there is a derived auxiliary tree having no OA constraints, with root labelled $X$ and frontier $w_1 X w_2$.

Suppose we had derived trees corresponding to derivations for $B$ and $C$ (as shown in the center of figure 3.1) that satisfied the above requirement.

We can capture the effect of each MHG production directly by associating exactly one elementary tree with each production. For example, figure 3.1 illustrates trees associated with the productions $A \rightarrow C1(B, C)$ (on the left) and $A \rightarrow C2(B, C)$ (on the right). We position the foot node in the elementary trees to ensure that the split point and foot node appear at the same position. When the tree corresponding to wrapping is used the string derived from $B$ is wrapped around the string derived from $C$. The foot of the resulting tree will appear immediately under the foot of the derived tree for $C$.

We prove that $L(G) \subseteq L(G')$ by induction on the length of the MHG derivation. We show that if $X \Rightarrow w_1 w_2$ then there is a derived auxiliary tree having no OA constraints, with root labelled $X$ and frontier $w_1 X w_2$.

We prove that $L(G') \subseteq L(G)$ by induction on the height of the TAG derivation tree. We show that if $\gamma \in D(\beta)$, has no OA constraints, and has frontier $w_1 X w_2$ then $X \Rightarrow w_1 w_2$.

While straightforward, the proof given above does not capture the linguistic motivation underlying TAG's. The auxiliary trees directly reflect the use of the concatenation and the wrapping operations. It is also interesting to note that a consequence of the equivalence of MHG's and TAG's and the construction used in proving the inclusion of MHL in TAL is that we have the following normal form for TAG's. For any TAG there is an equivalent TAG with exactly one initial tree and auxiliary trees which are of five possible forms shown above.

4. Conclusion

In this paper we have shown the equivalence of TAL and MHL. Since we have also established the inclusion of IL in MHL we have shown that ILs are included in TALs. The inclusion of MHL in IL, and thus the equivalence of HG's and TAG's, in the most general case remains to be established. We briefly discuss the relationship between MHG's and HG's and argue that it is close, both linguistically and formally. Figure 4.1 provides a summary of these results.
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