Demand Response with Cooperating Rational Consumers

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Abstract—The performance of an energy system under a real-time pricing mechanism depends on the consumption behavior of its customers, which involves uncertainties. In this paper, we consider a system operator that charges its customers with a real-time price that depends on the total realized consumption. Customers have unknown and heterogeneous consumption preferences. We propose behavior models in which customers act selfishly, altruistically or as welfare-maximizers. In addition, we consider information models where customers keep their consumption levels private, communicate with a neighboring set of customers, or receive broadcasted demand from the operator. Our analysis focuses on the dispersion of the system performance under different consumption models. To this end, for each pair of behavior and information model we define and characterize optimal rational behavior, and provide a local algorithm that can be implemented by the consumption scheduler devices. Numerical comparisons show that communication model is beneficial for the expected aggregate customer utility while it does not affect the expected net revenue of the system operator. Additional information to customers reduces the variance of demand. While communication is beneficial overall, behavioral change from selfish to altruistic has a stronger positive impact on the system welfare.

I. INTRODUCTION

Demand response management (DRM) emerges as a prominent method to alleviate the complications in power balancing caused by uncertainties both on the consumer and the supply side. Changes in user consumption preferences create the uncertainty on the consumer side while the uncertainty on the supply side is due to renewable resources. DRM refers to the system operator’s effort to improve system performance by shaping consumption through pricing policies. Smart meters that can control the power consumption of customers, and enable information exchange between meters and the system operator (SO) provide the infrastructure to implement these policies.

Real-time pricing (RTP) is a pricing policy where the price depends on instantaneous consumption of the population [1]–[3]. In RTP, the SO shares part of the risk and reward with its customers by setting price based on the total consumption. In these models, it is natural to propose game-theoretic models of consumption behavior, where users strategically reason about the behavior of others to anticipate price and determine their individual consumption [1]–[8]. The specifics of the behavior model and the information available impact the system welfare and is critical in assessing the benefits or disadvantages of a pricing scheme [8]. Given an RTP mechanism, our goal in this paper is to characterize rational price-anticipatory behavior models under different information exchange schemes, and comparatively assess their impact on system performance measures.

We consider an RTP scheme in which customers agree to a price function that increases linearly with total consumption and that depends on an unknown renewable energy parameter (Section II-A). The individual customer utility at each time depends on the individual’s consumption preference and price both of which are in general unknown to others (Section II-B). Initially, the SO sends public information on its estimate of population’s consumption preferences and renewable source generation. Customers use the public information and their self-preferences to anticipate total consumption and renewable source’s effect on price, and respond rationally by consuming according to a Bayesian Nash equilibrium (BNE) strategy. In [9], based on this energy market model, we propose and show the effectiveness of a peak-to-average ratio (PAR)-minimizing pricing strategy.

In this paper we explore the effects of different consumer behavior models, where consumers respond rationally regarding their individual utility, the population’s aggregate utility or the welfare (Section II-C). As time progresses, past consumption decisions contain information about the preferences of others which individuals can use to make more informed decisions in the current time. Based on this observation, we propose three information exchange models, namely, private, action-sharing and broadcast (Section II-D). In the private model, users do not receive any information besides the initial public signal by the SO. In action-sharing there exists a communication network on which users exchange their latest consumption decisions with their immediate neighbors. In broadcasting, the SO broadcasts the total consumption after each time step. We assume that the customer’s power control scheduler can adjust the load consumption between time steps according to its preferences and information. That is, we are interested in modeling consumption behavior for shiftable appliances, e.g., electric vehicles, electronic devices, air conditioners, etc. [10]. We formulate each consumer behavior model and information exchange model pair as a repeated game of incomplete information and characterize BNE behavior (Section IV). We use the explicit characterization to rigorously analyze the effects of each pair of behavior and information exchange model on total consumption, aggregate user utility and the SO’s net revenue (Section V). In [11], we partially present some of the results presented here. In this paper, in addition to an extended discussion of the effects of behavior and information exchange models in Section V, we present a local algorithm for the computation of the BNE for a given behavior and information exchange model in Section IV, and discuss the algorithm’s computational demand in Section IV-A.

Our findings can be summarized as follows. Providing more information to the consumers through action-sharing or broadcasting models does not lower the expected net revenue of the SO and increases the expected aggregate consumption utility. Furthermore, this information reduces the uncertainty in total demand. Action-sharing and broadcasting information exchange models eventually achieve the expected utility under full in-
formation when the communication network is connected. The positive effects of additional information are reduced with growing correlation among preferences. Increasing correlation among consumption preferences has a decreasing effect on the expected aggregate utility for all behavior models. Finally, welfare-maximizing behavior with broadcasted information achieves the highest expected welfare, and the inefficiency due to selfish behavior diminishes with growing number of customers.

II. DEMAND RESPONSE MODEL

There are $N$ customers, each equipped with a power consumption scheduler. Individual power consumption of $i \in N := \{1, \ldots, N\}$ at time $h \in H := \{1, \ldots, H\}$ is denoted by $l_{i,h}$. The total power consumed by $N$ customers at time $h$ is $L_h := \sum_{i \in N} l_{i,h}$.

A. Real-time pricing

The SO implements an adaptive pricing strategy whereby customers are charged a slot-dependent price $p_h$ that varies linearly with the total power consumption $L_h$. The SO has a set of renewable source plants at its dispatch and incorporates renewable generation into the pricing strategy by a random renewable power term $\omega_h \in \mathbb{R}$ that depends on the amount of renewable power produced at time slot $h$. The per-unit price in time slot $h$ is set as

$$ p_h(L_h; \omega_h) = \gamma_h(L_h + \omega_h) \tag{1} $$

where $\gamma_h > 0$ is a policy parameter to be determined by the SO based on its objectives. The random variable $\omega_h$ is such that $\omega_h = 0$ when renewable sources operate at their nominal benchmark capacity $\bar{W}_h$. If the realized production exceeds this benchmark, $W_h > \bar{W}_h$, the SO agrees to set $-L_h < \omega_h < 0$ to discount the energy price and to share its revenue from the windfall. If the realized production is below benchmark, i.e., $W_h < \bar{W}_h$, the SO sets $\omega_h > 0$ to reflect the additional charge on the customers. The specific dependence of $\omega_h$ on the realized energy production and the policy parameter, $\gamma_h$, are part of the supply contract between the SO and its customers. We assume that the SO uses a model on the renewable power generation to estimate the value of $\omega_h$ at the beginning of time slot $h$. The mean estimate $\bar{\omega}_h := E_{\omega_h}[\omega_h]$ of the corresponding probability density function $P_{\omega_h}$ is made available to all customers prior to the time slot.

The operator’s price function maps the amount of energy demanded to the market price. Observe that the price $p_h(L_h; \omega_h)$ at time $h$ becomes known after the end of the time slot. This is because price value depends on the total demand $L_h$ and the value of $\omega_h$ which are unknown a priori. The SO can employ the pricing policy in (1) to achieve certain system performances, e.g., minimizing PAR, maximizing welfare, etc., by picking its policy parameter $\gamma_h > 0$.

B. Power consumer

User $i$’s consumption at time slot $h$, $l_{i,h}$, depends on his consumption preference $g_{i,h} > 0$, modeled as a random variable that may vary across time slots. When user $i$ consumes $l_{i,h}$, its consumption utility increases linearly with its preference $g_{i,h}$ and decreases quadratically with a constant term $\alpha_h$, described as $g_{i,h}l_{i,h} - \alpha_h l_{i,h}^2$. The utility of $i$ at time slot $h \in H$ is then captured by the difference between the consumption utility of $i$ and the monetary cost of consumption $l_{i,h}p_h(L_h; \omega_h)$:

$$ u_{i,h}(l_{i,h}, L_h; g_{i,h}, \omega_h) = -l_{i,h}p_h(L_h; \omega_h) + g_{i,h}l_{i,h} - \alpha_h l_{i,h}^2. \tag{2} $$

Note that even if the SO’s policy parameter is set to $\gamma_h = 0$, the utility of user $i$ is maximized by $l_{i,h} = \bar{g}_{i,h}/2\alpha_h$ — see [2], [12] for similar formulations. Note that we choose $\alpha_h$ to be homogeneous among the consumers. Our results extend to the case where the constant $\alpha_h$ is heterogeneous but known.

The utility of user $i$ depends on the total power, $L_h$, consumed at $h$, which implies that it depends on the powers that are consumed by other users in the current slot, denoted by $L_{-i,h} := \{l_{j,h}: j \in N \setminus i\}$. Power consumption of others, $L_{-i,h}$, depends partly on their respective self-preferences, i.e., preferences $g_{-i,h} := \{g_{j,h}\}_{j \neq i}$, which are, in general, unknown to user $i$. We assume, however, that there is a probability density function $P_{g_i}(g_h)$ on the vector of self-preferences $g_h := [g_{1,h}, \ldots, g_{N,h}]^T$ from which these preferences are drawn. We further assume that $P_{g_i}$ is normal with mean $\bar{g}_h 1$ where $\bar{g}_h > 0$ and $1$ is an $N \times 1$ vector with one in every element, and covariance matrix $\Sigma_h$:

$$ P_{g_i}(g_h) = \mathcal{N}(g_h; \bar{g}_h 1, \Sigma_h). \tag{3} $$

We use the operator $E_{\omega_h}[\cdot]$ to signify expectation with respect to $P_{\omega_h}$, and $\sigma_{ij}^h$ to denote the $(i, j)$th entry of the covariance matrix $\Sigma_h$. Having mean $\bar{g}_h 1$ implies that all customers have equal average preferences in that $E_{g_i}[g_{i,h}] = \bar{g}_h$ for all $i$. If $\sigma_{ij}^h = 0$ for some pair $i \neq j$, it means that the self-preferences of these customers are uncorrelated. In general, $\sigma_{ij}^h \neq 0$ to account for correlated preferences due to, e.g., common weather. We assume that if there is a change in the consumption preferences from one time slot to the other, then the self-preferences $g_{i,h}$ and $g_{k,h}$ for different time slots $h \neq k$ are independent.

At the beginning of time slot $h$, we assume that $P_{g_i}$ in (3) is correctly predicted by the SO based on past data and is announced to the customers. The SO also announces its policy parameter $\gamma_h$ and its expectation of the renewable term $\bar{\omega}_h$. In addition, each customer knows its own consumption preference $g_{i,h}$.

C. Consumer behavior models

Consumption behavior $\{l_{i,h}\}_{i=1,\ldots,N}$ determines the population’s aggregate utility at time $h$,

$$ U_h(l_{i,h}, l_{-i,h}) := \sum_i u_{i,h}(l_{i,h}, L_h; g_{i,h}, \omega_h). \tag{4} $$

The net revenue of the SO is its revenue minus the cost

$$ NR_h(L_h; \omega_h) := p_h(L_h; \omega_h) L_h - C_h(L_h), \tag{5} $$

where $C_h(L_h)$ is the cost of supplying $L_h$ Watts of power. When the generation cost per unit is constant, $C_h(L_h)$ is a linear function of $L_h$. More often, increasing the load $L_h$ results in increasing unit costs as the SO needs to dispatch power from more expensive sources. This results in superlinear cost functions with an approximate model being the quadratic form:

$$ C_h(L_h) = \frac{1}{2} \kappa_h L_h^2 \tag{6} $$

for given constants $\kappa_h > 0$ that depend on the time slot $h$. The cost in (6) has been experimentally validated for thermal generation.

It is possible to add linear and constant cost terms to $C_h(L_h)$ and have all the results in this paper still hold. We exclude these terms to simplify notation.
The welfare of the overall system at time $h$ is the sum of the aggregate utility with the net revenue,

$$W_h(l_{i,h}, l_{-i,h}) := U_h(l_{i,h}, l_{-i,h}) + NR_h(l_{i,h}, l_{-i,h}). \quad (7)$$

Consumer behavior can be selfish, altruistic or welfare-maximizing. User $i$ is selfish when it wants to maximize its individual utility in (2). It is altruistic when it considers the well-being of other users, that is, aims to maximize $U_i$ in (4). Finally, user $i$ might also consider the well-being of the whole system and aim to choose his consumption behavior to maximize the welfare $W_h$ in (7) given its information. We use the superscript $\Gamma \in \{S, U, W\}$ in $u^\Gamma_{i,h}(l_{i,h}, l_{-i,h})$ to indicate that the consumer $i$ maximizes its selfish payoff $S$, aggregate utility $U$ or the welfare $W$. All of these behavior models require strategic reasoning about the behavior of others which constitutes a Bayesian game. Bayesian games model interactions where users have incomplete information about the utility of others. Below we formalize a range of information exchange models.

D. Information models

Consumption preference profile $g_h$ is partially known by the individuals and consumption decisions of individuals at time $h$ can provide valuable information about the consumption preferences $g_h$. This information is of use to the consumer $i$ in estimating consumption for the next time slot $h+1$ if the preferences of the users do not change in that time slot, that is, $g_h = g_{h+1}$. Otherwise, the information at time $h$ is not helpful in estimating the behaviors of others for time slot $h+1$ because we assume the change in the preference distribution to be independent. We let an uninterrupted sequence of time slots in which agents have the same consumption preference profile define a time zone. Formally, a time zone is defined as $T = \{h \in H : g_h = g \wedge (g_{h-1} = g) \lor (g_{h+1} = g)g\}$ for a preference profile $g := [g_1, \ldots, g_N]^T$ with prior probability density function $P_g$ where $\wedge$ is ‘and’ operator and $\lor$ is ‘or’ operator. Next, we present a set of possible information exchange models within a time zone $T$. We use $I^\Omega_{i,h}$ to denote the set of information available to consumer $i$ at time slot $h \in T$ for the information exchange model $\Omega$.

Private. The information specific to consumers is the most private when it consists of the private preference $g_i$, $I^P_{i,h} = \{g_i\}$ for $h \in T$.

Action-Sharing. Power control schedulers are interconnected via a communication network represented by a graph $G(N, E)$ with its nodes representing the customers $N = \{1, \ldots, N\}$ and edges belonging to the set $E$ indicating the possibility of communication. User $i$ observes consumption levels of his neighbors in the network $N_i := \{j \in N : (j, i) \in E\}$ after each time slot. The vector of $i$’s $d(i) := \#N_i$ neighbors is denoted by $[i_1, \ldots, i_{d(i)}]$. Given the communication set-up, the information of user $i$ at time slot $h \in T$ contains its self-preference $g_i$ and the consumption of his neighbors up to time $h-1$, that is, $I^{AS}_{i,h} = \{g_i, \{N_i, t\}_{t=1,\ldots,h-1}\}$ where we define the actions of $i$’s neighbors at time $t$ by $l^i_{N_i, t} := [l_{i_1,t}, \ldots, l_{i_{d(i)},t}]$ and denote the starting time slot of $T$ with $t = 1$. We assume that the power consumption schedulers keep the information received from neighbors private and know the network structure $G$.

SO Broadcast. The SO collects all the individual consumption behavior at each time $h$ and broadcasts the total consumption to all the customers, that is, $I^{SO}_{i,h} = \{g_i, L_{i,h} - 1\}$.

When the time zone $T$ ends, we restart the information exchange process. The prediction of renewable source term $P_{\omega,h}$ is allowed to vary for $h \in T$. Behavior model, $\Gamma \in \{S, U, W\}$, and the information exchange model, $\Omega \in \{P, AS, B\}$, determine the consumption decisions of user $i$. In the following, we define the rational consumer behavior in Bayesian games within a time zone $T$ and then characterize the rational behavior for each behavior and information exchange model pair $(\Gamma, \Omega)$.

III. Bayesian Nash equilibria

User $i$’s load consumption at time $h \in T$ is determined by his strategy $s_{i,h}$, which maps his information to a consumption level. This map depends on the belief of $i$’s $q_{i,h}$ which is a conditional probability on $\omega$ and given $I^\Gamma_{i,h}$, $q_{i,h}(\omega) := P_{\omega,g_i}(|I^\Gamma_{i,h})$. We use $E^\Omega_{i,h}[\cdot] := E_{\omega,g_i}[|I^\Gamma_{i,h}]$ to indicate conditional expectation with respect to $q_{i,h}$. While the model can account for the correlation between the random variables $\omega_k$ and $g_i$, we assume that they are independent. In order to second-guess the consumption of other customers, user $i$ forms beliefs on preferences given the common prior $P_g$ and its information $I^\Omega_{i,h}$. User $i$’s load consumption at time $h \in T$ is determined by its strategy $s_{i,h}$, which is a conditional probability on $\omega$ and given $I^\Gamma_{i,h}$, $q_{i,h}(\omega) := P_{\omega,g_i}(|I^\Gamma_{i,h})$. We use $E^\Omega_{i,h}[\cdot] := E_{\omega,g_i}[|I^\Gamma_{i,h}]$ to indicate conditional expectation with respect to $q_{i,h}$. While the model can account for the correlation between the random variables $\omega_k$ and $g_i$, we assume that they are independent. In order to second-guess the consumption of other customers, user $i$ forms beliefs on preferences given the common prior $P_g$ and its information $I^\Omega_{i,h}$. User $i$’s load consumption at time $h \in T$ is determined by its strategy $s_{i,h}$, which is a conditional probability on $\omega$ and given $I^\Gamma_{i,h}$, $q_{i,h}(\omega) := P_{\omega,g_i}(|I^\Gamma_{i,h})$. We use $E^\Omega_{i,h}[\cdot] := E_{\omega,g_i}[|I^\Gamma_{i,h}]$ to indicate conditional expectation with respect to $q_{i,h}$. While the model can account for the correlation between the random variables $\omega_k$ and $g_i$, we assume that they are independent. In order to second-guess the consumption of other customers, user $i$ forms beliefs on preferences given the common prior $P_g$ and its information $I^\Omega_{i,h}$. User $i$’s load consumption at time $h \in T$ is determined by its strategy $s_{i,h}$, which is a conditional probability on $\omega$ and given $I^\Gamma_{i,h}$, $q_{i,h}(\omega) := P_{\omega,g_i}(|I^\Gamma_{i,h})$. We use $E^\Omega_{i,h}[\cdot] := E_{\omega,g_i}[|I^\Gamma_{i,h}]$ to indicate conditional expectation with respect to $q_{i,h}$. While the model can account for the correlation between the random variables $\omega_k$ and $g_i$, we assume that they are independent. In order to second-guess the consumption of other customers, user $i$ forms beliefs on preferences given the common prior $P_g$ and its information $I^\Omega_{i,h}$.
A BNE strategy (10) is computed using beliefs formed according to Bayes’ rule. Note that the BNE strategy profile is defined for all time slots. No user at any given time slot within $T$ has a profitable deviation to another strategy.

In (10), consumers estimate consumption decisions of others to respond optimally. Equivalently, a BNE strategy is one in which users play best response strategy given their individual beliefs as per (9) to best response strategies of other users – see [15], [16] for similar equilibrium concepts. As a result, the BNE strategy is defined by the following fixed point equations:

$$s_{i,h}^\Gamma(I_{i,h}) = BR(I_{i,h}; s_{i,h}^\Gamma)$$

for all $i \in \mathcal{N}$, $h \in \mathcal{T}$, and $I_{i,h}^\Omega$. We denote $i$’s realized load consumption from the equilibrium strategy $s_{i,h}$, and information $I_{i,h}^\Omega$ with $i_{i,h} = s_{i,h}(I_{i,h}^\Omega)$. Using the definition in (11), we characterize the unique linear BNE strategy in the next section for any information exchange and consumer behavior model.

IV. Consumers’ Bayesian Game

It suffices for customer $i$ to estimate the self-preference profile $g$ in order to estimate consumption of other users [15]. We define the self-preference profile augmented with mean $\tilde{g}$ as $\tilde{g} = [g^T, \tilde{g}]^T$. The mean and error covariance matrix of $i$’s belief $q_i$ at time $h$ are $E_i^{\Omega}(\tilde{g})$ and $M_{\tilde{g}^\Omega}(h) := E_i^{\Omega}([\tilde{g} - E_i^{\Omega}(\tilde{g})])^2[I_{i,h}^\Omega]^T$, respectively. The next result shows that there exists a unique BNE strategy that is a linear weighting of the mean estimate of $\tilde{g}$ for any information exchange model $\Omega$. Furthermore, the weights of the linear strategy are obtained by solving a set of linear equations specific to the behavior model $\Gamma$.

Proposition 1 Consider the Bayesian game defined by the payoff $u_i^{\Omega}$ for $\Gamma \in \{S, U, W\}$. Let the information of customer $i$ at time $h \in T$ $I_{i,h}^\Omega$ be defined by the information exchange model $\Omega \in \{P, AS, B\}$. Given the normal prior on the self-preference profile $g$, user $i$’s mean estimate of the preference profile at time $h \in T$ can be written as a linear combination of $\tilde{g}$. That is, $E_i^{\Omega}(\tilde{g}) = [\tilde{g}^T, \tilde{g}]$ where $I_{i,h}^\Omega \in \mathbb{R}^{N+1 \times N+1}$ for all $h \in T$, and the unique equilibrium strategy for $i$ is linear in its estimate of the augmented self-preference profile,

$$s_{i,h}^\Gamma(I_{i,h}) = v_{i,h}^T E_i^{\Omega}(\tilde{g}) + r_{i,h}$$

where $v_{i,h} \in \mathbb{R}^{N+1 \times 1}$ and $r_{i,h} \in \mathbb{R}$ are the strategy coefficients. The strategy coefficients are calculated by solving the following set of equations for the consumer behavior models $\Gamma \in \{S, U, W\}$

$$v_{i,h}^T \Gamma_{i,h}^\Omega + \rho_i^\Gamma \sum_{j \in \mathcal{N} \setminus i} v_{j,h} \Gamma_{i,j,h}^\Omega = \rho_i^\Gamma e_i, \quad \forall i \in \mathcal{N},$$

and

$$r_{i,h} + \rho_i^\Gamma \sum_{j \in \mathcal{N} \setminus i} r_{j,h} = -\rho_i^\Gamma h \Gamma_{i,h}^\Omega \tilde{w}_h, \quad \forall i \in \mathcal{N},$$

where $\lambda_i^\Gamma, \mu_i^\Gamma, \rho_i^\Gamma$ are as defined in Lemma [2] for $\Gamma \in \{S, U, W\}$, $\rho_i^\Gamma = (2(\tau_i^h + \alpha_i))^{-1}$ and $e_i \in \mathbb{R}^{N+1 \times 1}$ is the unit vector.

Proof: Our plan is to propose a linear strategy and use the general form of the best response function (9) in the fixed point equations (11) to obtain the set of linear equations. We prove by induction. Assume that users have linear estimates at time $h$,

$$E_i^{\Omega}(\tilde{g}) = T_i^{\Omega} \tilde{g}$$

for all $i \in \mathcal{N}$. We propose that users follow a strategy that is linear in their mean estimate as in (12). Using the fixed point definition of BNE strategy in (11), we have

$$v_{i,h}^T E_i^{\Omega}(\tilde{g}) + r_{i,h} = \frac{g_i - \rho_i^\Gamma h \tilde{w}_h - \sum_{j \neq i} E_j^{\Omega}(\tilde{g}) v_{j,h}^T E_j^{\Omega}(\tilde{g}) + r_{j,h}}{2(\tau_i^h + \alpha_i)}$$

for all $i \in \mathcal{N}$ from Lemma [1]. The summation above includes user $i$’s expectation of user $j$’s expectation of the augmented preferences. By the induction hypothesis, we write this term as

$$E_i^{\Omega}(\tilde{g}) T_i^{\Omega} \tilde{g}.$$
where the mean estimate is \( E \left[ \tilde{g} | T_{i,h+1}^g \right] = T_{i,h+1}^g \), completing the induction argument. Similarly, the updates for error covariance matrices follow Kalman updates [17, Ch. 12]

\[
M_{g g}^i(h + 1) = M_{g g}^i(h) - K_{g g}^i(h)H_{g g}^{i,T}M_{g g}^i(h). \tag{24}
\]

At the starting time slot \( h = 1 \), we have \( E[ g_j | g_i ] = (1 - \sigma_{ij}/\sigma_{ii}) \bar{g} + (\sigma_{ij}/\sigma_{ii}) g_i \). Hence the induction assumption is true initially and \( E[ g_j | g_i ] = T_{i:j} \bar{g} \) for all \( \Omega \in \{ P, A S, B \} \).

Since the stage game has the same pay-off structure and the information is normal, it suffices to show uniqueness for the stage game. The uniqueness of the stage game is proven in Proposition 1 in [9]. See also the proof of Proposition 2.1 in [18].

Proposition 1 presents how BNE consumption strategies are computed at each time slot. Accordingly, the scheduler repeatedly determines its consumption strategy given consumption behavior model \( \Gamma \) and available information, receives information based on the information exchange model \( \Omega \) at the end of the time slot, and propagates its beliefs on self-preference profile to be used in the next time slot. For each consumption behavior \( \Gamma \in \{ S, U, W \} \) the user solves a different set of equations in (13)-(14) derived from the fixed point equations of the BNE (11). For Private information exchange model, users do not receive any new information within the horizon hence their mean estimate of \( \bar{g} \) do not change, that is, \( T_{i,h}^g = T_{i,h}^g \) for \( h \in T \), which implies the set of equations (13)-(14) need to be solved only once at the beginning to determine the strategy for the whole time horizon.

For Action-Sharing information exchange model, upon observing actions of its neighbors, user \( i \) has new relevant information about the preference profile which it can use to better predict the total consumption in future steps. Similarly in SO Broadcast model, each user receives the total consumption at each time which is useful in estimating total consumption in the following time slot.

The Bayesian belief propagation for Gaussian prior beliefs corresponds to Kalman filter updates at each step for any information exchange model. In particular, beliefs remain Gaussian and the mean estimates are linear combinations of private signals at all times for any information exchange model. In order to compute the BNE strategy, it does not suffice for scheduler \( i \) to form beliefs on the preference \( \bar{g} \). It also needs to keep track of beliefs of others. Knowing the estimate of all the other schedulers is not possible for \( i \). However, this is not required to compute an estimate of other schedulers’ estimates. It is only required that user \( i \) knows how other schedulers compute their mean estimates which implies knowing the estimation weights \( T_{i,j}^g \). Even though scheduler \( i \) does not know \( P_{g}(\tilde{g} | T_{j,h}^g) \), it can keep track of \( T_{i,j}^g \) via the weight recursion equation in (23), which can be computed using public information. Note that \( i \) cannot compute self-mean estimate of preferences, \( E[ \tilde{g} | T_{i,h}^g ] \), via multiplying \( T_{i,j}^g \) by \( \bar{g} \) since this computation would require knowledge of \( \bar{g} \). Instead, user \( i \) computes its mean estimate by a Kalman filter. We detail the local computations of a scheduler in Algorithm 1

In Algorithm 1 we provide a local algorithm for user \( i \) to compute its consumption level and propagate its belief given a behavior model \( \Gamma \in \{ S, U, W \} \) and the information exchange model \( \Omega = AS \). We point to modifications specific to the other information exchange models here in our explanation. User \( i \) initializes its belief on \( \bar{g} \) at the beginning of the time zone \( T \) according to the preference distribution in (3). It also determines the estimation weights \( T_{i,j}^g \) and error covariance matrix \( M_{g g}^i(1) \) at the beginning for \( j \in N \). Note that user \( i \) does not need any local information from other users in this initialization. Using the estimation weights \( T_{i,j}^g \), it can locally construct the equations in (13) and (14), and solve for the strategy coefficients \( \{ v_{j,h}, r_{j,h} \}_{j \in N} \). At time slot \( h \), it propagates its belief on the self-preference profile given the new information. The propagation of beliefs starts by computing observation matrices of all the users in Step 3 based on the information exchange model \( \Omega \). When the model is action-sharing, \( \Omega = AS \), each observed action \( \{ v_{j,h}^{AS} \}_{j \in N} \) is a linear combination of \( \bar{g} \) with the observation matrix \( H_{j \bar{g}}^{i,T} \) computed by (18). If the model is broadcast, \( \Omega = B \), the observation matrix is a vector computed by (19). If the model is private, \( \Omega = P \), there is no new information available hence scheduler \( i \) goes back to Step 2 with the same strategy coefficients. Next, \( i \) uses these observation matrices in computing the gain matrices in Step 4 of all the users.

In Step 5, \( i \) propagates the estimation weights \( T_{i,h+1}^B \) and error covariance matrix \( M_{g g}^i(h+1) \). Note that in Steps 3-5 user \( i \) does a full network simulation in which it emulates the Kalman filter estimates of everyone using public information, that is, estimation weights \( \{ T_{i,j}^g \}_{j \in N} \), strategy coefficients \( \{ v_{j,h} \}_{j \in N} \) and network topology \( G \). Finally in Step 6, \( i \) propagates its own mean estimate \( E[ \tilde{g} | T_{i,h}^g ] \) by using its own local observation, which is \( \tilde{g}_{N,i} \) for \( \Omega = AS \) or \( L^i_{N} \) for \( \Omega = B \).

### A. Private and complete information games

In Step 2 of Algorithm 1 the user solves a set of \( N^2 \) linear equations. This computation can be avoided in situations where the information of each consumer remains the same. The information is static in two obvious cases. The first one is when the information exchange model is private. Second is when all the users have complete information. For the private information case, there exists a closed-form solution to the set of equations

\[
\text{Algorithm 1 Sequential Game Filter for } \Omega = AS \text{ at User } i
\]

**Require:** Consumer behavior model \( \Gamma \in \{ S, U, W \} \).

**Require:** Posterior distribution on \( g \) at time slot \( h = 1 \) and \( \{ T_{i,j}^g, M_{g g}^i(1) \}_{j \in N} \) according to (3).

while \( g_h = g \) do

1. Equilibrium \( \Gamma \): Solve \( \{ v_{j,h}, r_{j,h} \}_{j \in N} \) using (13)-(14).
2. Play: Compute \( s_{i,h}^\Gamma(T_{i,h}^g) = v_{i,h}^\Gamma E[ \tilde{g} | T_{i,h}^g ] + r_{i,h} \).
3. Construct observation matrix \( \{ H_{j \bar{g}} \}_{j \in N} \): Use (18).
4. Gain matrices: Compute \( K_{g g}^i(h) \)
   \[
   K_{g g}^i(h) := M_{g g}^i(h)H_{g g}^{i,T}H_{g g}^{-1}.
   \]
5. Estimation weights: Update \( \{ T_{j,h+1}^g, M_{g g}^j(h+1) \}_{j \in N} \)
   \[
   T_{j,h+1} = T_{j,h}^g + K_{g g}^i(h) \left( H_{g g}^{i,T} - H_{g g}^{j,T}T_{j,h}^g \right).
   \]
6. Bayesian estimates: Calculate \( E[ g | T_{i,h+1} ] = E[ g | T_{i,h} ] + K_{g g}^i(h) \cdot ( T_{i,h} - E[ g | T_{i,h} ] ) \).

end while
in (13)-(14) that is symmetric when the preference correlation is homogeneous; i.e., the off-diagonal elements of \( \Sigma \) are the same \( \sigma_{ij} = \sigma \) for all \( i, j = 1, \ldots, N \) and \( \sigma \in \{ 0, 1, 2, 3 \} \) – see Proposition 2 in [9]. The complete information is achieved when the SO broadcasts total consumption \( L_{h}^{T} \) and the preference correlation is homogeneous. That is, for each customer, his private preference and the cumulative realized preference \( \{ g_{i}, \sum_{j} g_{j} \} \) is a sufficient statistic of the realized preferences \( g_{i} \) for the homogeneously correlated preference games \( \Gamma \in \{ S, U, W \} \) – see [19]. Furthermore, the total consumption \( L_{h}^{T} \) conveys the cumulative realized preference \( \sum_{j} g_{j} \). This means that in the broadcast information exchange model, \( \Omega = B \), in the first time slot consumers play a private information game and from the second time slot onwards they have complete information.

### B. Price-taking consumers

In all of the behavior models above, users anticipate price which depends on the total consumption. When the price \( p_{h} \) is given or when they do not anticipate their effect on price, they are price takers. Then the utility in (2) depends only on self consumption \( l_{i,h} \) and price \( p_{h} \),

\[
u_{i,h}(l_{i,h}) = -l_{i,h}p_{h} + g_{i,h}l_{i,h} - \alpha_{h}l_{i,h}^{2}.
\]

Given the price at time \( h \), consumers maximize their pay-off by \( l_{i,h}^{K} = (-p_{h} + g_{i,h})/2\alpha_{h} \) where we indicate the price-taking behavior model with \( \Gamma = K \). Consumers are charged with hourly prices \( p_{h} \) that are determined by maximizing hourly expected net revenue, that is, \( p_{h} = \max_{\{ P, AS, B \}} E[pL_{h}^{K} - C_{h}(L_{h}^{K})] \) where \( L_{h}^{K} = \sum_{j=1}^{N} l_{j,h}^{K} \). Maximization of expected net revenue results in \( p_{h} = (2\alpha_{h} + \kappa_{h})g_{h}/(4\alpha_{h} + 2N\kappa_{h}) \). The price taker model provides a benchmark to compare with the price anticipating models presented in the previous section. Note that information exchange models do not affect behavior in the price-taking model. In the following section we numerically compare the effects of the behavior and the information exchange models.

### V. COMPARISON OF BEHAVIOR AND INFORMATION EXCHANGE MODELS

We explore the performance of the smart grid model in two orthogonal axes. In the first we consider consumer behavior models \( \Gamma \in \{ S, U, W, K \} \). In the second we vary the information exchange models \( \Omega \in \{ P, AS, B \} \). For each pair of price anticipating behavior and information exchange model, users follow Algorithm 1. Price takers follow the model in Section IV-B. We consider average consumption \( \bar{L} := \sum_{h} L_{h}/H \) (kWh), aggregate utility \( \bar{U} := \sum_{h} U_{h}/H \) ($), net revenue \( \bar{NR} := \sum_{h} NR_{h}/H \) ($) and welfare \( \bar{W} := \sum_{h} W_{h}/H \) ($) as the performance measures of the system.

In the set-up, there is a single time zone \( T \) which lasts for \( H = 5 \) hours. The cost function of the SO is as given in (6) with the parameter values \( \kappa_{h} = 1 \) for \( h \in T \). The price policy parameter is chosen as \( \gamma_{h} = 1.28/kWh^{2} \) for all time slots. Unless otherwise stated, we consider \( N = 10 \) users. For the AS information model the communication network is determined by randomly placing \( N \) individuals on a 3-mile \times 5-mile area and connecting them if they are closer than the threshold connectivity of 2 miles. The decay parameter of the utility function in (2) is equal to \( \alpha_{h} = 1 \) for \( h \in T \). The mean of the preferences \( g_{i} \) is equal to 30 for \( i \in N \). We let the standard deviation of the preference to be identical for all consumers as \( \sigma_{ii} = 4 \) and the correlation among preferences \( \sigma_{ij} \) is homogeneous among the population. We consider the effect of the correlation coefficient on the mean and variance of the performance measures by varying \( \sigma_{ij} \in \{ 0, 1, 2, 3 \} \). We let the renewable power term \( \omega \) be normal-distributed with mean \( \bar{\omega} = 0 \) and variance \( \sigma_{\omega} = 2 \). We consider 20 instantiations of the random variables \( g \) and \( \omega \) for each \( \sigma_{ij} \in \{ 0, 1, 2, 3 \} \). We compute the expected values of average consumption, aggregate utility and net revenue (\( \bar{EL}, \bar{EU}, \bar{ENR} \)) by taking an average of all runs for a given correlation coefficient \( \sigma_{ij} \). We discuss the effects of \( \Gamma \) and \( \Omega \) summarized in Table I in the following section.

#### A. Effect of consumer behavior

Expected average consumption \( \bar{EL} \) is the largest when consumers are selfish (\( \Gamma = S \)) and lowest when they maximize aggregate utility (\( \Gamma = U \)). The price-taker (\( \Gamma = K \)) and welfare-maximizer (\( \Gamma = W \)) consumption levels lie in between these two behaviors where price-taker behavior attains an expected average consumption close to selfish behavior.

While \( S \) behavior attains a higher aggregate utility than \( K \) behavior, the consumers expect a higher utility when they follow \( U \) or \( W \) behavior. As their names imply, \( U \) behavior achieves the highest \( EU \) and \( W \) behavior achieves the highest \( EW \) for all correlation coefficients \( \sigma_{ij} \in \{ 0, 1, 2, 3 \} \) for a given \( \Omega \).

The net revenue of the SO is the largest when \( \sigma_{ij} = 0 \) and consumers follow \( K \) behavior. However, increasing correlation significantly drops the SO’s expected net revenue for \( K \) behavior from \( \bar{ENR} = \$122 \) when \( \sigma_{ij} = 0 \) to \( \bar{ENR} = \$6.3 \) when \( \sigma_{ij} = 3 \). Moreover, we observe that the variance of \( \bar{ENR} \) increases from 55 to 274 when the correlation coefficient changes from \( \sigma_{ij} = 0 \) to \( \sigma_{ij} = 3 \). On the other hand, among price anticipatory behavior models, \( S \) attains the highest \( \bar{ENR} \) under \( S \) behavior. Furthermore, when the behavior is price anticipatory, the effect of the correlation coefficient on SO’s \( \bar{ENR} \) is small. Under altruistic behavior, the \( \bar{ENR} \) drops significantly, e.g., the \( \bar{ENR} \) drops to \$20 on average when \( \Gamma = U \). For price anticipatory models, the effect of correlation on the variance of \( \bar{NR} \) is insignificant.

Among the price anticipatory behavior models, the lowest expected welfare values are registered for \( S \) behavior. Keeping the information exchange model the same, the difference in expected welfare between \( W \) and \( S \) behaviors shrink with increasing preference correlation. This implies that at high preference correlation, the loss due to selfishness, which does not disappear at any positive value of \( \sigma_{ij} \in [0, 4] \), is less.

#### B. Effect of information exchange

For each consumer behavior model, \( AS \) and \( B \) information exchange models influence the expected consumer utility \( EU \) positively with no significant effect on the expected average consumption and net revenue when compared to the \( P \) information exchange model. Consequently, the \( AS \) and \( B \) models improve expected welfare. We observe that the expected improvement in \( AS \) model is always less than or equal to \( B \) model. This is because in \( AS \) consumers learn about others’ consumption preferences through their neighbors while in the \( B \) model each consumer learns about the sufficient statistic of the price in the next time step. It takes longer in \( AS \) for all the consumers to reach full information for a connected network, which yields a higher expected utility. As can be guessed, the impact of \( AS \) and \( B \) information exchange models vanishes as the preference...
correlation approaches $\sigma_{ij} = 4$, i.e., at full correlation, P, AS, and B all attain the same performance.

The positive effect of communication on expected welfare is intuitively expected because information exchange helps rational users estimate behavior of others better over time. However, the AS model does not improve the utility of all the consumers [19], [20]. Hence, a viable question beyond the scope of this paper is to consider how to incentivize consumers to share their consumption behaviors with others for the well-being of the population.

We further consider the variance of average consumption $\bar{L}$ as a measure of deviation from expectations. We observe that the variance of average consumption among runs grows for AS and B models as preference correlation $\sigma_{ij}$ increases. On the other hand, the variance decreases for the P model. Note that at full correlation ($\sigma_{ij} = 4$), the information exchange models are identical. This implies that for the P model, the variance of average consumption is always higher. That is, in AS and B models total demand predictions have higher certainty.

### C. Effect of population size

Figs. 1(a)-(d) exhibit the total consumption with respect to hours for the population size $N = \{3, 5, 10, 15\}$, respectively. Given a population size plot, each line corresponds to a different information exchange model for the selfish consumer behavior model – see the legend in Fig. 1(d). The diameter of the network is displayed in the horizontal axis with the population size for each plot. We observe that when the network is connected (Figs. 1(b)-(d)), the total consumption in AS model converges to the total consumption in the B model. Furthermore, convergence time is equal to the diameter of the network. When the network is not connected (Fig. 1(a)), convergence does not necessarily happen.

We further examine the effect of population size on the expected welfare loss per capita in Fig. 2. Expected welfare loss, $EWL$, is the difference between the expected welfare for welfare-maximizing consumers with full information, i.e., $(\Gamma, \Omega) = (W, B)$ and the expected welfare for selfish consumers with private information, i.e., $(\Gamma, \Omega) = (S, P)$, $EWL := EW(\{s^W_{i,h}(I^B_{i,h})\}_{i=1,...,N}) - EW(\{s^S_{i,h}(I^P_{i,h})\}_{i=1,...,N})$. Expected welfare loss per capita normalizes $EWL$ by the number of consumers, that is, $EWL/N$. The expected welfare loss incorporates inefficiencies due to selfish behavior and lack of information.

From Fig. 2 we observe that the inefficiency disappears as the number of consumers $N$ increases. Furthermore, the correlation coefficient $\sigma_{ij}/\sigma_{ii}$ can increase welfare loss for small values ($\leq 0.2$); otherwise, its increase has a decreasing effect on expected welfare loss. From Table I we know that an increase

| $\sigma_{ij}$ | $\Omega$ | $\bar{L}$ (kWh) | $EU$ ($) | $ENR$ ($) | $\bar{L}$ (kWh) | $EU$ ($) | $ENR$ ($) | $\bar{L}$ (kWh) | $EU$ ($) | $ENR$ ($) |
|--------------|--------|----------------|--------|--------|----------------|--------|--------|----------------|--------|--------|
| 0            | P      | 19.93          | 100.8  | 67.0   | 11.74          | 186.8  | 19.9   | 13.85          | 181.3  | 29.5   |
|              | AS     | 19.80          | 106.5  | 66.3   | 11.57          | 190.9  | 19.7   | 13.70          | 186.2  | 29.1   |
|              | B      | 19.79          | 106.8  | 66.3   | 11.57          | 191.1  | 19.7   | 13.68          | 186.6  | 29.1   |
| 1            | P      | 19.83          | 99.4   | 66.3   | 11.60          | 183.8  | 19.5   | 13.72          | 178.9  | 28.9   |
|              | AS     | 19.78          | 104.6  | 66.1   | 11.56          | 188.4  | 19.6   | 13.68          | 184.0  | 28.9   |
|              | B      | 19.78          | 104.9  | 66.0   | 11.56          | 188.7  | 19.6   | 13.67          | 184.3  | 28.9   |
| 2            | P      | 19.79          | 99.2   | 66.0   | 11.57          | 182.9  | 19.4   | 13.67          | 178.3  | 28.7   |
|              | AS     | 19.77          | 102.8  | 65.9   | 11.56          | 186.3  | 19.5   | 13.66          | 181.7  | 28.7   |
|              | B      | 19.77          | 103.0  | 65.9   | 11.56          | 186.5  | 19.5   | 13.66          | 182.3  | 28.7   |
| 3            | P      | 19.77          | 99.2   | 66.0   | 11.56          | 182.5  | 19.4   | 13.66          | 178.2  | 28.7   |
|              | AS     | 19.76          | 101.1  | 65.9   | 11.55          | 184.1  | 19.4   | 13.65          | 179.5  | 28.8   |
|              | B      | 19.76          | 101.1  | 65.9   | 11.55          | 184.4  | 19.5   | 13.65          | 179.9  | 28.8   |

**Table I**

Performance of $\bar{L}$ (kWh), $EU$ ($\$\$), $ENR$ ($\$\$) for behavior $\Gamma$ and information exchange models $\Omega$.
We considered rational consumption behavior and information exchange models for an energy system with a set customers and a SO. Each customer has a time-dependent consumption preference which is unknown to the other entities in the system. The SO exercised a RTP policy which set up a non-cooperative game of incomplete information among its users. In these settings user optimal behavior is a BNE strategy. We characterized the BNE strategy for each pair of behavior and information exchange model. Given this characterization, we comparatively analyzed the combined effects of these models on the system performance. In our comparisons we showed that information dissemination among consumers is beneficial to the system overall. Furthermore, we observed that a change in user behavior to altruistic has a stronger positive effect than the case when all user have complete information.

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