RESEARCH-BASED LEARNING TECHNIQUES
IN MATHEMATICS — THE EMPERORS NEW CLOTHES?

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\textbf{Abstract.} Research-based learning implements the idea that learning takes place in the format of research into actual university education, see (Wildt, 2009, Forschendes Lernen: Lernen im “Format” der Forschung, Journal Hochschuldidaktik, 20, no. 2, 4–7, 2009). Currently, this concept is widely discussed among experts in education science and among university teachers who seek for using corresponding methods in their classrooms. In (Schlicht, Slepcevic-Zach, 2016, Research-based learning and service learning: Two versions of problem based learning, Zeitschrift für Hochschulentwicklung, 11, no. 3, 85–105) research-based learning was discussed in the context of business education and the question was raised to what extent their more application-oriented approach can be transferred to areas dealing with foundational research. In this paper we survey a mathematical context. We present concrete research-based learning arrangements that we used in our teaching practise and analyse the students’ responses. As a matter of fact, already in the Bourbaki textbooks of the 1960s tasks can be found that challenge the reader—without further instructions—to find a counterexample or to generalise a previous theorem. This leads us to discuss the question how new the idea of research-based learning in mathematics really is.

1. \textbf{Introduction}

Teaching and learning formats that put the activity of the learners in the center of attention are a hot topic in the practice of and research on education, see, e.g., (Dürnberger, Reim and Hofhues, 2011), (Healey and Jenkins, 2009), (Huber, Hellmer and Schneider, 2009), (Hutchings, 2007), (Kay and Kletskin, 2012), (Reitlinger, 2013) and (Wilson, 1996). There exists a variety of names for various teaching concepts that follow the above idea. Problem-based learning, enquiry- and inquiry based learning, research-based learning, and undergraduate research are only the most prevalent ones and already for these few terms there is no unified definition available in the literature, see, e.g., (Huber, 2014) and (Reimann and Jenert, 2011). The general assessment of learning arrangements that focus on the learners’ activity is very positive, see e.g., (Hmelo-Silver, 1993), (Hmelo-Silver, Duncan and Chi, 2007) and (Stepien and Gallagher, 2008). There are however also critical surveys pointing out that the concepts have disadvantages and may fail if they are applied without the appropriate preparations, see, e.g., (Kirschner, Sweller and Clark, 2006), (Hmelo-Silver et al., 2007), and (Reimann and Jenert, 2011).

In this paper we focus on the method of research-based learning whose leitmotif according to (Wildt, 2009, p. 5) is given by the idea that “learning takes place in the format of research”. According to (Schlicht and Slepcevic-Zach, 2016, p. 89) the goal of research-based learning is to “enable the learners [...] to understand research procedures, to evaluate them according to given criteria, to outline them independently and to implement them with guidance as well as to document the generated knowledge”, see (Schlicht and Slepcevic-Zach, 2016, p. 89). In order to achieve this goal, (Schlicht and Slepcevic-Zach, 2016, p. 90) suggest a problem-based approach during which the learners “together with experts from research [...] define and precise problems, acquire theoretical knowledge by independently accessing the relevant literature in order to be able to thoroughly analyse, understand and treat problems, present and discuss their results, reflect on their solutions and solution processes, and develop generalisations [...]”. (Healey and Jenkins, 2009, p. 23) state that learners learn “the epistemologies and forms of discipline-based inquiry, [...] the particular disciplinary research methodologies, [...] and produce] work that mimics the forms of knowledge creation and dissemination in their disciplines and professional areas” in order to define their concept of undergraduate research. (Wildt, 2009) specifies his concept for the area of empirical social research and (Schlicht...
and Slepcevic-Zach, 2016) for the area of business education. Therefore they add to the above the assumption that the initial point of the learning process is a real-world problem, for instance from an existing company that cooperates with the university. In turn the latter authors add the aim that the learners relate their results to applications in the real world. (Schlicht and Slepcevic-Zach, 2016, p. 100) ask however, to what extent their concept of research-based learning can be transferred to areas dealing with foundational research.

To our knowledge, in mathematics education up to now only very little experience with research-based learning has been documented in the literature. (Narli and Baser, 2008) propose a problem-based approach in teacher education. (Erickson, 1999) restricts on school education and processes classical geometric interpretations of the logarithm to serve in problem-based learning. (Clarke, Breed and Fraser, 2004) also focus on high school education. (Halverscheid, 2005) gives an overview of problem-based learning in German high school education. (Flórez and Mukherjee, 2017) worked with college students on tasks mentioned in the problem section of the Fibonacci Quaterly intended for interested readers. (Lithner, 2017) discusses principles for the design of tasks that foster imitative and creative reasoning in mathematics.

In this paper we will give a possible answer to the question of (Schlicht and Slepcevic-Zach, 2016, p. 100) in the case of tertiary education in mathematics. Our idea is to use custom-made homework assignments that do not just ask for computational applications of results from the lectures but demand to extend the results by requiring students to prove or disprove generalisations. It is crucial that in these tasks it is not clearly stated which result has to be proved but that it is part of task to find out what could be true and what could not. In Section 2 we explain our interpretation of research-based learning and we give two examples of assignments that we used in the classroom. By also showing possible variants we outline how our idea can be used to regulate research-based learning according to the students abilities and needs in the classroom. We discuss grading outcomes and exemplary student solutions from two courses on Ordinary Differential Equations and Real Analysis and we explain how working on our tasks imitates typical research activities in mathematics. In Section 3 we present feedback that we collected from our students and in Section 4 we discuss our findings. Moreover, we illustrate that exercises following our interpretation of research-based learning can be found already in the Bourbaki textbooks of the 1960s.

2. Research-based learning in mathematics

Mathematical research is difficult as long as the researcher is still searching for the correct result and the corresponding proof. The only thing that seems to be even more difficult, is to explain to others how the final insight that gave rise to a new theorem was in detail obtained. Every teacher knows that the students’ suggestion “Show us the solution, then we will learn how to find it” does not lead to an effective teaching strategy when it comes to higher mathematics. In this article we would like to report on practical experiences with certain types of exercises that, when used systematically, can support the students to acquire precisely those competencies, which are difficult to put into words, but which determine a successful researcher. In view of the literature cited in Section 1 we see our approach as an instance of research-based learning. As an orientation for non-mathematicians, we point out that it is neither feasible to involve undergraduate students into current mathematical research, nor is it possible to let the students find the mathematical concepts, that have been invented and improved by the world’s most capable scientists during the last 2000 years, within a 3-years BSc education from scratch and on their own. This work originates in the attempt of the authors to implement nevertheless elements in their courses that allow the students already at the beginning of their studies to acquire those competencies that in the end make the difference between the ability to apply known techniques to standard problems and the ability to find new techniques for new problems. The starting point was a course given in the winter term 2016/2017 at the University of Wuppertal (Germany) about Ordinary Differential Equations. We gave several non-standard tasks like to find a certain example or to discover the relation between given conditions without that explicit instructions on how to tackle the task were provided. The following exercise is a prototype example.

**Exercise 1.** Let $Q := [a, b] \times [c, d]$ and let $f: Q \rightarrow \mathbb{R}, (x, y) \mapsto f(x, y)$ be a map. Compare the following three conditions.

(i) $f$ is Lipschitz continuous.

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(ii) $f$ satisfies a Lipschitz condition with respect to $y$.

(iii) $f(x, \cdot) : [c, d] \to \mathbb{R}$ is Lipschitz continuous for every $x \in [a, b]$.

Below, a corresponding solution by one of our students can be seen. Due to space limitations we are not able to provide a complete translation. But also non-german speakers can see that the student first proved the implications (i)$\Rightarrow$(ii) and (ii)$\Rightarrow$(iii). Then he gave two counterexamples to show that the implications (ii)$\Rightarrow$(iii) and (iii)$\Rightarrow$(ii) are in general not true.

Exercise 1'. Let $Q := [a, b] \times [c, d]$ and let $f : Q \to \mathbb{R}$, $(x, y) \mapsto f(x, y)$ be a map. Prove that (i)$\Rightarrow$(ii)$\Rightarrow$(iii) holds where

(i) $f$ is Lipschitz continuous.

A classical version of this exercise would have been the following.

Exercise 1. Let $Q := [a, b] \times [c, d]$ and let $f : Q \to \mathbb{R}$, $(x, y) \mapsto f(x, y)$ be a map. Prove that (i)$\Rightarrow$(ii)$\Rightarrow$(iii) holds where

(i) $f$ is Lipschitz continuous.
(ii) \( f \) satisfies a Lipschitz condition with respect to \( y \).

(iii) \( f(x, \cdot) : [c, d] \to \mathbb{R} \) is Lipschitz continuous for every \( x \in [a, b] \).

Find a function \( f : [0, 1] \times [0, 1] \to \mathbb{R} \) that satisfies (ii) but not (i). Show that \( g : [0, 1] \times [0, 1] \to \mathbb{R}, g(x) = \sqrt{x+y\sqrt{x}}, \) satisfies (iii) but not (ii).

For our students the formulation “Compare the conditions…” was very unusual, and it was hard for them to understand what Exercise 1 was about. Indeed they were used to formulations like “Show…”, “Prove…”, “Find…” etc. as in Exercise 1’. In addition to this, it is of course considerably harder to prove the correct implications and find counterexamples for the wrong implications if one does not know a priori which ones are correct and which ones are wrong. As an orientation for non-mathematicians, we point out that to show (i) \( \Rightarrow \) (ii) \( \Rightarrow \) (iii) and (i) \( \not\Rightarrow \) (ii) \( \not\Rightarrow \) (iii), is not a teacher-defined model solution. It it a mathematical fact that precisely this is true. There is however a big choice of counterexamples that one can construct and there might also be other arguments that show that the three statements are not equivalent.

We emphasise, that comparing existing conditions, by either trying to prove implications between them or by constructing examples to disprove implications, is a typical task in mathematical research. If one for instance wants to generalize an existing result by relaxing the assumption one has to be sure that the new assumption is really weaker than the old one. Of course one can think of even more research-like versions of Exercise 1, for instance the following variant.

**Exercise 1”.** Discuss the notion of a Lipschitz condition with respect to \( y \).

This exercise would include then the task to firstly find reasonable other conditions that one can compare with the condition to be discussed. Exercises that include such a high amount research-like activities we did not yet use in practice.

The second part of this section is devoted to another example of an exercise that we gave in a course on Real Analysis taught at the University of Wuppertal (Germany) in the summer term 2017. The course Real Analysis II is the middle part of the sequence of courses Real Analysis I-III which are all mandatory for mathematics majors and which cover continuity, differentiation and integration of functions first in one and then in several real variables. The exercise is again to find out the relation between central concepts introduced in the lecture. In this case we performed a detailed analysis of the 30 solutions that were handed in.

**Exercise 2.** Let \( U \subseteq \mathbb{R}^n \) be open and \( x_0 \in U \). For \( f: U \to \mathbb{R}^m \) we consider the statements (i)–(vi) below. What is the relation of these statements? Justify your answer.

(i) \( f \) is continuously partially differentiable at \( x_0 \).

(ii) \( f \) is totally differentiable at \( x_0 \).

(iii) \( f \) is continuous and partially differentiable at \( x_0 \).

(iv) \( f \) is partially differentiable at \( x_0 \).

(v) \( f \) is continuous at \( x_0 \).

(vi) There is a unique matrix \( A \in \mathbb{R}^{m \times n} \) such that \( \lim_{x \to x_0, x \neq x_0} f(x) - f(x_0) - A(x-x_0) \frac{|x-x_0|}{|x-x_0|} = 0 \) holds.

For the solution the students firstly needed to find out that (i) \( \Rightarrow \) (ii) \( \Rightarrow \) (iii) \( \Rightarrow \) (iv) and (iii) \( \Rightarrow \) (v) had been shown already in the lecture or are trivial. Secondly, they had to provide counterexamples in order to show that none of these implications is an equivalence. Then the students had to prove that (ii) and (vi) are equivalent and finally they had to show that neither (iv) \( \Rightarrow \) (v) nor (v) \( \Rightarrow \) (iv) is true, again by giving suitable counterexamples. The following diagram

\[
\begin{align*}
&\text{(i) } \equiv \text{(ii)} \equiv \text{(iii)} \iff \\
&\text{(iv)} \iff \text{(vi)} \iff \text{(v)}
\end{align*}
\]
contains the full picture of all valid and non valid implications.

Thirty students worked on this exercise. The first part, citing corresponding results from the lecture, was indeed done correctly by 26. Only three students gave the counterexamples that we mentioned above as the second step, while the rest of the students did not discuss the converse directions at all. Only six students properly showed that (ii) and (vi) are equivalent. The other students only mentioned that (iii) $\Rightarrow$ (vi) follows directly from the definition but did not give further arguments. In particular, they did not show that (ii) determines the matrix $A$ uniquely. The last question about the relation of (v) and (iv) was not discussed at all. We mention that all students ignored that most of the statements from the lecture were only stated for functions $f: U \rightarrow \mathbb{R}$ but that here vector-valued functions appear which requires to use the arguments of the lecture for each coordinate function.

The rather poor grading results of the exercise are given in the following table.

|                              | N  | AV | SD  |
|------------------------------|----|----|-----|
| Grading results for Exercise 2 | 30 | 2.5| 2.0 |

Table 1. Grading results for Exercise 2

Below a student’s solution for Exercise 2 is depicted.

$$
(i) \quad \Leftrightarrow \quad (iii) \quad \Rightarrow \quad (vi) \quad \Rightarrow \quad (v) \quad \Rightarrow \quad (iv) \quad \Rightarrow \quad \text{trivial}
$$

Figure 2. Student’s solution for Exercise 2.

We emphasise that the fact that this student mentioned the trivial statement (iii) $\Leftrightarrow$ (iv)$\land$ (v) but did not discuss the majority of the non-trivial statements illustrates how much freedom this type of exercises leaves to the students. Although the solution in Figure 2 is by far not satisfactory, it shows that the student analysed the properties and recognised at least this trivial equivalence and the results from the lecture.

3. Student’s feedback

During the course on Ordinary Differential Equations we gave in addition to Exercise 1 several tasks involving the idea of research based learning. Other tasks were the request to construct a certain example (five times), and the question if an effect that was discovered earlier still can take place if the assumptions are changed (one time).

At the end of the course we asked the participants to rate these “non-classical” assignments using scales from 1 to 6, by indicating the reason for their evaluation, and by adding free comments. The questions and the outcome read as follows.

|                              | N  | AV | SD  |
|------------------------------|----|----|-----|
| After passing the oral exam, please rate the non-classical exercises (1=not helpful, 6=very helpful). | 11 | 4.82 | 1.17 |
| Please rate how the non-classical exercises supported your learning during the semester (1=not at all, 6=very much). | 11 | 4.36 | 1.36 |
| Please compare the non-classical exercises to classical ones (1=like classical ones more, 6=prefer non-classical ones). | 11 | 4.27 | 0.90 |
| Would you like to work on non-classical exercises again in the future (1=don’t want, 6=want very much)? | 11 | 4.91 | 0.94 |
Table 2. Students’ rating of the research-based learning exercises.

In the individual feedback the students mentioned explicitly, that the RBL tasks encourage to study intensively (seven times mentioned), that they require to work independently (four times mentioned), and that they foster creativity (three times mentioned), e.g.:

These exercises [the RBL tasks] encourage to think more intensively about the theorems, lemmas etc. and foster the ‘creativity’.

The students mentioned that the tasks are difficult (four times mentioned), e.g.:

During the semester it was difficult to let oneself in for such exercises [the RBL tasks], in particular when the topic was still very fresh and oneself had not been able to think it through completely.

The answers however indicate that a suitable preparation by standard tasks helps to solve the RBL tasks successfully. Indeed, it was mentioned explicitly that RBL tasks are not helpful without suitable preparation (three times mentioned), but helpful if prepared properly (four times mentioned), e.g.:

After several standard exercises these exercises [the RBL tasks] were well-manageable and helped also to comprehend the topic.

In view of the formal assessment, the students mentioned that the exercises are helpful to prepare for the oral exam (three times mentioned) but it was remarked that with these tasks it is difficult to obtain extra credit, e.g.:

If the exercises are only for the preparation of the [oral] exam, then these types of tasks [the RBL tasks] are very helpful. However, if one wants to collect credit, then they are rather annoying.

It was pointed out that both types of exercises are important as they address different levels of understanding. In particular, the students mentioned that the RBL tasks help to obtain a deeper understanding (seven times mentioned), e.g.:

Conventional exercises [are] good for the first understanding, but [. . .] the RBL tasks are good] for a deeper understanding.

Finally, several students suggested to give more hints in order to help them to get started with the RBL tasks (five times mentioned), e.g.:

A tip/hint about what to start with, would be very helpful
The last comment shows that the students grasp exactly what is the point about research-based learning. In turn it is crucial for the teacher not to give too many hints as this would upset the whole concept.

4. The emperors new clothes?

In this article we documented experiences with research-based learning methods in tertiary mathematics education. In a 3rd year university course on Ordinary Differential Equations and a 1st year course on Real Analysis we used exercises of a certain type in order to emulate research-like activities in regular classes. In these exercises a given mathematical problem has to be solved but the approach how to solve the problem, or even the precise statement that should be proved, has to be discovered by the students themselves. The aforementioned type of exercise is neither new nor is it a panacea for all difficulties that appear in mathematics education. Indeed, already in the mid-century Bourbaki textbooks exercises of this type appear. For instance, in (Bourbaki, 1958, §2, no. 3, Exercise 1)

“Donner un exemple de module noethérien et non artinien M tel que tout endomorphisme ≠ 0 de M soit injectif et qu’il existe des endomorphismes de M non bijectifs et non nuls.”

the students are asked to construct a certain example, in (Bourbaki, 1961, §I.4, Exercise 10(b) on p. TG I.96)

“Soit $\left( X_0, f_{\alpha}^{\prime} \right)$ un second système projectif d’espaces topologiques ayant même ensemble d’indices supposé filtrant, et soit, pour tout $\alpha, u_\alpha : X_\alpha \rightarrow X_{0 \alpha}$ une application continue telle que les $u_\alpha$ forment un système projetif d’applications. Les $u_\alpha(X_\alpha)$ forment un système projectif des sous-espaces des $X_{0 \alpha}$; montrer que si $u = \lim_{\alpha} u_\alpha$ et si on suppose les $f_{\alpha}$ surjectives, $u(X)$ est dense dans l’espace $\lim_{\alpha} u_\alpha(X_\alpha)$. La proposition est-elle encore exacte lorsque les $f_{\alpha}$ ne sont plus supposées surjectives (cf. E, III, p. 94, exerc. 4)?”

they are asked if a previous theorem remains true if one of the assumptions is dropped, and in (Bourbaki, 1970, §I.2, Exercise 16(d) on p. A I.121)

“Pour tout $a \in E$, montrer que la translation à gauche $\gamma_a$ appartient à $\Phi$; soit $\varphi_a$ sa classe (mod. R). Montrer que l’application $x \mapsto \varphi_x$ est un isomorphisme de $E$ sur un sous-monoïde de $\Psi$, et que si $x \in E^*$, $\varphi_x$ est inverse dans $\Psi$ (considérez l’application réciproque de $\gamma_x$, montrer qu’elle appartient à $\Phi$, et que sa classe (mod. R) est inverse de $\varphi_x$). En déduire une généralisation du th. 1 de I, p. 18.”

a previous theorem has to be generalised. In view of these examples one might consider research-based learning as the new clothes of a classical educational technique. Indeed, in modern tertiary mathematics education, exercises of purely computational nature appear rather seldom. The majority of the tasks is problem-based and usually each exercise requires an idea. However, we feel that formulating exercises as we illustrated in Section 2 makes a substantial difference and can enhance the learning process tremendously. We believe that students gain more insight when they first have to formulate conjectures and then try to prove them instead of following a list of things they should prove that was worked out by the teacher. Our experiences explained in Section 2 and the feedback summarised in Section 3 underline however that finding challenging problems is not the end of the story of research-based learning but only its starting point. The potential of our concept can only be attained by a systematic and reflected use of corresponding methods and it has to be further developed. Our investigation shows that its success depends strongly on a suitable preparation and more generally on the whole setup of the learning arrangement in which it is employed.

We conclude our discussion with the remark that research-based learning can never be the same as actual research. We also emphasise that we consider a philosophical discussion about the precise definition of research-based learning, or the difference between the latter and, e.g., problem-based learning as not helpful for improving concrete aspects of university education in mathematics. Every modern course on Real Analysis illustrates drastically that today students within one year have to get used to concepts that trace back to names like Weierstraß, Bolzano, or Cauchy who developed them at the climax of their careers and during years of work and even fights about their actual eligibility. What we propose in this article is a concept that accepts the constraints of a modern, highly structured, and complex education system but makes nevertheless already in an early stage of mathematics education the experience of research-like discoveries possible for the students.
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