The effect of the stressed state of the ring gear due to interference with the base part in calculating the flexural strength of the tooth

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Abstract. Strain intensity state (SIS) of a non-rigid toothing in which the shape of the hole is distorted during the clean-working stage due to elastic deformations from clamping forces of the self-centering chuck in research by photomechanics method. For this reason, when the toothing is placed on the base part with interference, there is uneven pressure on the contour of its hole, and also a preliminary uneven SIS is formed in the transitional fillets of the interdental space, which reduces the service life of the toothing by flexural strains.

1. Introduction
In the design of engineering products for the solution of emerging problems in simulation conditions by mathematical methods, physical modeling, in particular, the method of photomechanics is used [1]. In this work we investigate the non-rigid toothing on the optical SIS models in connection with the uneven interference along the contour of the base hole. Using of terms non-rigid and rigid toothing in this study is rather arbitrary. Models differ only in the size of the hole, all other geometric parameters are identical. We introduce the notation for the nonrigid toothing - d_{32} (d = 32 mm) and for the rigid toothing - d_{20} (d = 20 mm). For a visual assessment of the SIS of nonrigid and rigid toothings in Fig. 1 shows the photograms in the same scale of the diagrams of the contour strains of the hole. Compare the diagrams of contour strains for the hole for the most unfavorable case of processing a toothing in a 3-jaw chuck while basing on its outer contour. We find out that the total strain amplitude of compressed and stretched fibers for a non-rigid toothing d_{32} is n = 7 stripes, and for a rigid toothing d_{20} - n = 2.5 stripes, i.e. in 2.8 times less.

As a result of machining in the chuck in the clamped state, the correct shape of the aperture of the toothing is ensured. After releasing the toothing from the clamping forces, the elastic deformations are restored. The correct shape of the hole obtained during machining is distorted in a mirror manner, in comparison with the version at the time the treatment was started. It was established in [2] that the largest distortions of the shape of the hole of a nonrigid toothing occur in the case of clamping in a 3-jaw chuck when basing along its outer contour (Fig. 1, b, d). For clarity we expand the ring-shaped nonrigid toothings with the constructed diagrams (Fig. 1, c, d) to the horizontal plane of the coordinate system (Fig. 2), and add-up the diagram of the rigid toothing (Fig. 1, e). In the variant of basing on the involute of the tooth of the nonrigid toothing d_{32} on the hole contour, there are 12 zero points.
Figure 1. Photograms in the circularly polarized light of models of toothings clamped in a 3-jaw chuck: a, d - basing on the involute tooth; a, b, d, f - nonrigid toothing - d32; b, d, c, e - basing on the outer contour of the toothing tooth - d32 and d20, respectively; c, e - rigid toothing d20; 1 - 12 - zero points on the contour of the toothing hole when basing on the involute tooth; 1- 6 - zero points on the contour of the aperture of the toothing d32 when basing along its outer contour; s - the stretching zone; c - zone of compression; a, b, c - white light; d, e, f - monochromatic light of a mercury lamp.

Stretched fibers are observed between points 1 - 2, 3 - 4, 5 - 6, 7 - 8, 9 - 10, 11 - 12, and between points 12 - 1, 2 - 3, 4 - 5, 6 - 7, 8 - 9, 10 - 11 - compressed fibers of the contour of the toothing hole. For the variant of basing on both the external contour of both a nonrigid and rigid toothing, only 6 zero points on the contour of the hole take place and, in a comparable version of the differing ridge stiffnesses, these points are not combined. In Fig. 2 for a rigid toothing d20, the zero points (1, 2, 3, 4, 5, 6) are visualized by the italic and are located above the contour of the toothing hole, while for the non-rigid toothing d32 the zero points (2, 4, 6, 8, 10, 12) are located below the hole contour.

On the basis of the resulted diagrams obtained as a result of processing the experimental data of physical experiments, it can be asserted that with a larger number of zero points on the hole contour in the version based on the involute of the tooth (12 points), the total radial motion of the contour strains is less than for option of basing on the outer contour of the toothing (6 points). For this reason, the results of a physical experiment with the clamping of a nonrigid toothing in a 3-jaw chuck and basing on its outer contour are given in the following discussion. On black-and-white photograms (Fig. 1), strip counting and a quantitative evaluation of the strain state of the toothing model as a whole were performed. The main strains ± σ1 were plotted on the contour of the toothing hole from the clamping forces in the 3-jaw chuck before processing. The stretching and compression zones on the diagrams are indicated by the symbols (+) and (-), respectively.
Figure 2. Diagrams of the contour strains of the hole $\sigma$ in MPa and in the strips of $n$ toothings clamped in a 3-jaw chuck and unfolded onto a plane:
- basing on an external contour of a non-rigid toothing - $d32$;
- basing on the involute tooth profile of a non-rigid toothing - $d32$;
- basing on an external contour of a rigid toothing - $d20$; 1 - 12 - zero points on the hole contour of the non-rigid toothing $d32$, basing on the involute; 1 - 6 - zero points on the hole contour of the rigid toothing $d20$.

To plot the main strains on the corresponding section of the hole contour, the orders of the stripes were determined and, at the chosen scale of the strip $n$, contours strains were plotted. The obtained diagrams in the $n$ stripes can be converted into the strain dimension by the relation:

$$\sigma_{\text{cont}} = \sigma_0^c \cdot n,$$

where $\sigma_{\text{cont}}^\pm$ is the tensile or compressive normal strain on the contour of the toothing hole; $\sigma_0^c$ is the scale interval of the model from the calibration of its material in MPa or in kgf / cm$^2$. Note that in the method of photomechanics it is most convenient to select arbitrarily the scales of geometric and force similarity starting from the physico-mechanical properties of the material of the models and the conditions of the problem being solved. The geometric scale of the examined toothings models was chosen taking into account the size of the light field of the PPU-7 (polarization projection unit), so that it was possible to simultaneously photograph the whole surface of the strain toothing model. Also, the applied force was observed in the place of application, direction and magnitude of forces, taking into account these two coefficients of similarity. The work simulates the SIS of a non-rigid toothing having a correct (No.1) and non-regular (No.2) hole shape, acquired due to elastic deformations from the clamping forces at the stage of i Material of the model and technique of physical experiments final boring.

2. Material of the model and technique of physical experiments
An optically sensitive material based on ED-6 epoxy resin was used in the production of toothing models. Taring of the optical properties of the material from which the toothing models were made was carried out by loading a flat disc of the same material as the toothing models. Based on the experimental data obtained during calibration, the optical constant is calculated from the expression:

$$\sigma_0^{1,0} = \frac{(8P_t)}{(\pi \cdot d \cdot n_c)}$$

(1)

$\sigma_0^{1,0}$ is the optical constant, in MPa·cm /strip or in kgf /cm/strip; $P_t$ - the load applied to the disk during calibration, in N or in kgf; $n_c$ is the number of interference fringes in the center of the disk of diameter $d$, cm, counted from its free contour in a horizontal diametrical cross-section, on which the order of the strip is always $n = 0$. The optical constant of the toothing material is $\sigma_0^{1,0} = 19.35$ kgf /cm/strip. Taking into account the thickness of the material from which toothings are made $t = 0.48$ cm, the scale interval of the model is $\sigma_0^c = \sigma_0^{1,0} / t = 40.3$ kgf /cm$^2$ /strip = 4.1 MPa /strip. It was revealed by the physical experiment on optical models that the effect of the distortion of the shape of the toothing hole on the unevenness of its tension with the base part after their assembly into a separate unit. Due to uneven interference, an uneven tension is created in the body of the toothing, which extends to the fillets of the interdental space. In this regard, the part of the fillet quill (depending on the loading
scheme), located in sectors with a high level of interference, is previously prone to more high tangential tensile strains compared to other sectors of the strained model, in which the tension is less. As a result of this phenomenon, there will be an unaccounted for in calculations the decrease in the life of the bending strains at the base of the toothing teeth. In the process of modeling the prestressed state in the fillets of the inter-dental space, taking into account the distortion of the shape of the aperture of the toothing from the clamping forces, a loading device was developed and manufactured at the finishing stage (Fig. 3) - mandrel. The use of the mandrel provided the parameters of the force closure with the contour of the hole in the model of the toothing, which made it possible to create an interference with the characteristics of the distorted shape of its hole. To do this, the stress diagram of the clamping forces must be transformed into a diagram of the displacement of the outline points of the hole in the radial direction.

The most important element of the loading mandrel is the elastic ring 2, by means of which it is possible to set the interference between the toothing hole and its distance by convergence of the cones of the flanges 1, 3 due to the tightening of the bolt 5 into the body of the guide part of the mandrel 4. A slight deviation from the alignment of the hole in the ring with a two-sided symmetrical cone on the inner contour relative to its outer cylindrical contour leads to an uneven cross-section in the radial direction in its various sections. This creates variable rigidity of the ring, the asymmetry of the tapered facets of the hole and affects the nature of the tightness between the aperture of the toothing and the elastic ring.

Two toothing models were investigated. In the first model of the toothing No.1, distortion of the shape of the holes obtained during the removal of the circular log was minimal and had practically no effect on the uniformity of the interference (Fig. 4, a, b). For this reason, we arbitrarily consider model No.1 without distortion of the hole contour, and we take model No.2 with a distorted contour of the hole (Fig. 4, c, d). Parameters of the distortion of the shape of the toothing hole in the radial direction of the model No.2 are taken from the nature of the hole contour strains from the clamping forces in the 3-jaw chuck (Fig. 1, b, d) for the case of basing along the outer contour of the nonrigid toothing. At the selected scale of linear quantities, these distortions are introduced into the working drawing of the toothing of the optical model No.2 and realized when processing its hole. Consequently, with similar distortions of the shape of the hole model leading to its SIS (Fig. 4, c, d), the crenellated rim subsequently arrives at the assembly with the base part.
Figure 4. Photograms of the SIS of the optical model of the toothing due to the interference along
the hole contour: a - the enlightened toothing contour of the model No.1 because of the uniform
minimum contact between the elastic bushing and the toothing hole of the correct shape; b - under
conditions of uniform interference and correct form of the toothing of the model No.1; c, d – the
model No.2 with a non-uniform interference, taken in white and monochromatic light.

During the physical experiment, the model of the toothing was mounted on the table of the PPU-7
and was illuminated by the polarized monochromatic light of the mercury lamp. When the expected
result $P_1$ was achieved on loading the model, a digital camera was installed in the polarized light
stream after the analyzer, and photograms were taken in the direct rays of the light flux. Photograms
taken in white light (colour) were used to study zero and isotropic zones, and photograms taken in
monochromatic light of a mercury lamp (green filter) were preliminarily transferred to a black and
white format and used to decode the pattern of strips and assess the strain state the toothing model as a
whole. Loading of the toothing model No.2 with a distorted shape of the hole revealed an uneven
strain state over the entire area of the photogram of the toothing model (Fig. 4, c, d).

However, this uneven distribution of strains obeys the character of the distribution of the elastic
strains caused by clamping forces in the 3-jaw chuck. On the photogram it is possible to distinguish 3
sectors located at an angle of 120° relative to each other, in each of which there are 2 teeth with a
prestrained transitive fillet in the interdental space. And between these prestrained teeth can be
identified 3 more sectors with 4 teeth in each sector, which are also located at an angle of 120° relative
to each other. In the transitional fillets of these teeth, the strains are either close to zero, or the tensile
strain is replaced by compression due to the appearance of an isotropic zone in front of the fillets in
these local sections. Distortion of the shape of the hole in the toothing model is modeled by the law of
pressure of the flanges of the loading mandrel structure on the elastic sleeve. Obviously, with the
correct shape of the cones of the flanges and the correct shape of the hole in the model No.1, a uniform
pressure is created along the contour of the elastic bush in the case of the flanges approaching. This
uniform pressure from the elastic bushing is transferred to the contour of the toothing hole, which
thickness is $t = 0.48$ cm. The thickness of the elastic bush is $0.7$ cm with a margin to ensure a full
contact of the surface of the hole of the model of the toothing with the loading mandrel. The contours
of the maximum tangential strains $2\tau_{\text{max}} = \sigma_1 - \sigma_2$ are formed along the entire surface of the toothing
equidistantly to the hole contour (Fig. 4, b).

The isolines, which are located closer to the toothing hole, acquire a concentric hole shape. The
isolines, remote from the toothing hole, acquire a wavy form with a pitch equal to the pitch of the
teeth. The amplitude of the isochromatic wave in the radial direction depends on its distance from the
outline of the hole, and the greater its magnitude than it is further from the hole contour, i.e. closer to
the toothing. The change in the shape of the stripes (isochromes) is explained by the change in the
rigidity of the ring-shaped part of the toothing when the contour lines approach the base of the
toothing teeth. At high strains, isolines move to the body of the teeth, and zones of increased strain
concentration in the fillets of the interdental space are formed, creating a preliminary SIS, which
should be taken into account when assessing the service life of the toothing for flexural strength.
3. Results and discussions
A series of experiments on the loading of the model of the toothing with the distorted shape of hole No.2 was performed with different values of the interference. Tensions were assigned taking into account the nature of the contour strains of the hole from the clamping force of the toothing in the 3-jaw chuck when basing on its outer diameter during the hole processing phase. At a selected scale of linear magnitudes, the distortions of the shape of the holes were made on one of the investigated models of the toothing. An NC program was developed for processing a distorted hole shape and then realized on an EMCO Mill milling machine. Before and after machining of the hole, circular logs were taken - the parameters of the hole shape of the toothing on the round gauge MMQ 400.

We consider two variants of loading a distorted hole in the model No.2 of the toothing by the pressure $P$, which is extended by the cones of the mandrel of the elastic sleeve and creates an interference on the hole contour of the toothing. The first variant of model loading under tension, which is minimal in a series of performed experiments (Fig. 5, a). The toothing model No.2 was mounted on the loading mandrel and the hole contour was associated with the pressure $P$, due to the expansion of the elastic ring as the cones of the flanges of the loading mandrel approached. A convincing confirmation of the absence of zero points on the hole contour is a color photogram (Fig. 5, b), taken in white light with circular polarization. The effect of these strains, taking into account their unevenness, which distorted the hole contour in model No.2 after removing the clamping forces, is manifested at the level of the transitional fillets of the interstitial space. Let us pay attention to the change in the sign of the strains in the cross section $a_1-a_2$. In the radial direction, compressive strains are exerted on the hole contour due to the tightness to the isotropic zone, which cuts off the part of the transition fillet in which tension strains are formed both in the radial and in the tangential directions.

Due to the distorted hole shape, an uneven SIS appears on its contour, depicted in diagram 2 (Fig. 5, a) of the tangential contour strains with minimal interference from the processing of the first experiment's photogram. The diagram of contour strains 2 indicates that a gap has been chosen over the entire contact surface of the elastic ring and the model hole. There are no zero points on its contour and the character of the diagram 2 is mirror-like the strain diagram on the free contour (Fig. 1, d). In the second series of experiments, the interference was increased on the hole contour in model No.2 of the toothing, which, when compared with the data of the first series of experiments on the maximum stripes of contour strains of the hole, increased about 1.7 times (Fig. 6, a).

![Figure 5. Photograms of the model of the toothing No.2 with the distorted shape of the hole with minimum interference](image-url)
Figure 6. Photograms of the tothing model No.2 with the distorted shape of the hole with increased tension (≈ 1.7 times); a - diagrams of contour strains of the tothing in the fillets of the interdental space - 5 and on the hole contour - 4; b - color photogram of the model in white light; c, d - are the diagrams of $\tau_{\text{max}}$ at the scale of the stripes along the section $a_3$-$a_4$ and the contour of fillets $b_3$-$b_4$; ○○○○○○○○○○ - stretching (+); ×××××××××× - compression (-).

The density of the stripes (isolines) on the photogram of the model increased. Maximum order of stripes in the transitive fillets increased. If for the variant of the first series of experiments the maximum order of the strip in the transitive fillet was $n = 3.4$ stripes, then in the second series for model No.2 is $n = 5$ stripes. Comparison of diagrams 2 and 4 of the contour strains of the hole at two levels of interference testifies to the influence of the amount of interference on the magnitude of the strains in the transitional fillets of the interdental space. With increasing tension in the pair, the aperture of the tothing - the expansion mandrel, the strain level in the calculation points for bending in the fillet halves of the teeth increases. The magnitude of the change in the interference is controlled by the photogram obtained during the experiment. In the case under consideration, for 2 values of interference, the range of its variation was 70% with respect to the minimum value in the variant.

The initial choice of the minimum value of the interference was regulated by the conditions that were reduced to providing a clear visualization of the preliminary uneven tensed state at the design point for the bending of the transitional fillet of the tothing of the tooth. This requirement was fulfilled step by step. First, a gap was chosen in the pair of the hole of the tothing - the elastic ring of the expansion mandrel, which was inspected on the laboratory polariscope by a slight enlightenment of the tothing model when it reported minor deformations. In this case, the material of the model acquires the property of artificial anisotropy and becomes birefringent. Further loading of the tothing model was carried out on a PPU-7 installation in monochromatic light of a mercury lamp, where the change in the strained state was controlled well as the interference was increased. For model No.2, this condition consisted in preserving the original zero zone on fillets with a minimum level of deformation, and tensile strains in the most deformed fillets were visualized in the tangential direction. Such a variant of the photogram (Fig. 5, a) clearly visualizes the uneven SIS in the transverse halves of the roots of the tothing teeth.

Attention should be paid to the consequences for the resource of the toothed contour of the tothing in the calculation of flexural strength. On the photogram the maximum variation (change) in the strains in the transitive tiles is $n \approx 3.5$ stripes. On separate fillets there are insignificant sections with deformation of compression in the tenth part of the strip. The total travel of the tensile strains alone is $n = 3.4$ stripes. Let us give the photograms of experiments and their processing in the variants of the maximum interference of models No.2. In this case, as in the variant of minimum interference, there is an insignificant (share of the band) local strained state in the transition fillet with compression deformation ($a_3$-$a_4$, Fig. 6, c). The maximum tensile strains in the transition fillets reach $n = 5$ stripes.
A tension that varies in the range $n_{\text{min}} = 3$ strips and $n_{\text{max}} = 5$ stripes created this prestrained state in fillets. For clarity and a comparative evaluation of the obtained experimental information, we will present all the obtained diagrams on the unfolding of the photograms of the model of the toothing No.2 with the distorted shape of the hole with 2 variants of interference (Fig. 7).

![Diagram showing the development of the diagrams in the stripes ($n$) of the photograms of the model of the toothing No.2 with the distorted shape of the hole with 2 variants of interference: 1 - diagram of strains on a free hole contour of a nonrigid toothing clamped in a 3-jaw chuck along the outer contour (Fig. 1, d); 2 - diagram of the tangential contour strains of the distorted hole of the toothing from the pressure in connection with the minimum interference when assembling with the base part; 3 - strain diagram in the fillets of the interdental space from the pressure on the contour of the toothing hole in connection with the minimum interference; 4 - the diagram of the tangential contour strains of the distorted toothing hole from the pressure in connection with the maximum interference received in the experiment during assembly with the base part; 5 - the strain diagram in the fillets of the interdental space from the pressure on the contour of the toothing hole in connection with the maximum interference.]

Figure 7. Development of the diagrams in the stripes ($n$) of the photograms of the model of the toothing No.2 with the distorted shape of the hole with 2 variants of interference: 1 - diagram of strains on a free hole contour of a nonrigid toothing clamped in a 3-jaw chuck along the outer contour (Fig. 1, d); 2 - diagram of the tangential contour strains of the distorted hole of the toothing from the pressure in connection with the minimum interference when assembling with the base part; 3 - strain diagram in the fillets of the interdental space from the pressure on the contour of the toothing hole in connection with the minimum interference; 4 - the diagram of the tangential contour strains of the distorted toothing hole from the pressure in connection with the maximum interference received in the experiment during assembly with the base part; 5 - the strain diagram in the fillets of the interdental space from the pressure on the contour of the toothing hole in connection with the maximum interference.

It is possible to estimate these experimentally obtained values by comparing them with the calculated operating strains on the flexural strength in the transitional groove of the base of the toothing, which are performed by the product designer. Similar calculation parameters were not planned in the present work due to the fact that the work of the examined toothed crown does not belong to any structure or mechanism. The aim was to draw the attention of designers (developers of mechanisms with nonrigid toothings) to the presence of a problem of uneven strain in the transition fillets during the operation of the product in connection with the design and technological support of the quality of such non-rigid toothings that could have consequences for the resource of the product as a whole.

References
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