Top Quark Physics at the ILC: Methods and Meanings

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The physics case for studying top-quark physics at the International Linear Collider is well established. This summary places in context the top-quark physics goals, examines the current state-of-the art in understanding of the top-quark mass, and identifies some areas in which the study of the top-quark mass enhances our understanding of new techniques.

1 Introduction

The measurement of top-quark production and decay at the International Linear Collider (ILC) will provide precision access to properties of the top quark, as well as to multiple signals of physics beyond the Standard Model. Recently, a strong case has been made for study of top-quark production at the ILC [1]. Scans at both threshold for \( t \overline{t} \) production, and in the continuum, provide an opportunity to search for both direct and virtual effects of physics Beyond the Standard Model (BSM).

Arbitrary couplings of the form \( \gamma/Z - t - t \) can be measured to the 1% level [2, 3], with improvements beyond coming from additional positron polarization, or collider energy. The left-handed coupling of between \( W - t - b \) might be measured to 3% [4], though beamstrahlung and ISR effects may reduce this reach [5]. These couplings are relevant in models with new generations, top-flavor, Little Higgs with \( T \)-parity, and more. With as little as 500 fb\(^{-1}\) of data at \( \sqrt{S} = 500 \) GeV, vector-like top quarks \( T \) can be indirectly probed through their modification of the \( Z - t - t \) up to 1 TeV [6]. Many scenarios have been considered in detail, and are summarized in the Reference Design Report [1].

Methods for obtaining the top-quark Yukawa coupling, width, and mass have been are also described in detail in the Design Report. However, the last year has seen significant improvement in the predictions of event distributions used to extract the top-quark mass. As this is where the interest has been, I focus here (and in my talk [7]) on reexamining issues regarding the top-quark mass, from where we have come, to where we are going.

2 Top-quark mass: What is it?

The primary focus of top-quark studies at the ILC has been on developing methods to extract the most precise top-quark mass possible [1, 8]. Before addressing the question of why we need a precise top-quark mass, we should be certain we understand what it is we are trying to find, and how we plan to find it.

As the first evidence of the top-quark came to light, it was clear that the top-quark width was small, and, hence, the top quark decays before it has time to hadronize. Given the prospect of probing a quark directly, the question arose: what is the top-quark mass? Several reasonable responses are: a parameter of the Lagrange density \( \mathcal{L} \sim m_t \), an effective Yukawa coupling between a Higgs boson and two top quarks \( m_t = Y_t/(2\sqrt{2}G_F)^{1/2} \approx 1 \) in the Standard Model, or even the kinematic mass seen by experiments. The “pole mass” of the top quark was quickly shown to be ambiguous at the order of \( \Lambda_{QCD} \sim 150 \text{ MeV} \) from
a calculation of renormalon corrections [9]. Unfortunately, the experimental expectation for extraction of the top-quark mass at the ILC is 100 MeV. Hence, there was a theoretical declaration that the $\overline{\text{MS}}$ mass, which suffers much smaller ambiguities, should be used as the standard.

The $\overline{\text{MS}}$ mass is useful for reporting results. However, masses are not actually measured directly in experiments. What is measured are distributions of events from which a mass is inferred. The two distributions at the ILC currently under review are the 1S (“threshold”) mass, and the top-quark jet mass (“continuum”). Distributions are fit using these proxy-masses; and perturbative expansions are used to work back to the $\overline{\text{MS}}$ mass. Recent progress in predicting the distributions to be fit in the data was presented at this conference [5, 10, 11].

The primary method of extracting the top-quark mass at the ILC is to fit a theoretical shape prediction to a scan across the $t\bar{t}$ threshold. An early study pointed out the theoretical prediction of the shape was unstable when using the pole mass (or $\overline{\text{MS}}$ mass) due to non-relativistic corrections near threshold [12]. Use of the pseudo bound-state 1S mass allowed a systematic improvement of the threshold region as a non-relativistic expansion of the cross section of the form

$$
\sigma_{t\bar{t}} \propto v \sum \left( \frac{\alpha_s}{v} \right) \times \left\{ \frac{1}{\sum (\alpha_s \ln v)} \right\} \times \left\{ \text{LO}(1) + \text{NLO}(\alpha_s, v) + \text{NNLO}(\alpha_s^2, \alpha_s v, v^2) \right\}.
$$

(1)

Normalizations change at each order, but use of the 1S mass provides a fairly stable threshold peak distribution shape. This systematic approach leads to an estimated uncertainty $\delta\sigma_{t\bar{t}} = \pm 6\%$ before initial-state radiation (ISR) or beamstrahlung are included — we’ll return to this. Based on this alone, it appears extraction of $\delta m_t \sim 100$ MeV is attainable [1].

Recent theoretical work has extended these calculations to include even smaller effects. Corrections due to the unstable nature of the top quark [13, 14] have been found to be 3–10% across the threshold region. Figure 1 shows that most of the NNNLO term of the expansion in Eq. 1 has now been calculated [15]. At this conference we heard about the electroweak-strong correction of order $\alpha\alpha_s$ [10, 16] which is at the level of $\delta\sigma_{t\bar{t}} \sim 0.1\%$. The overall corrections examined here are well below the expected experimental precision. Does this mean that everything is ready for the ILC to turn on?

There remain several effects on the order of 50 MeV each that remain to be understood. The most pressing issue will be the inclusion of beamstrahlung and ISR. It is apparent from Fig. 2 shown at this conference [5], that beam and ISR effects will dramatically smooth the threshold region. It has been suggested [1] that a forward tracker that can observe Bhabha scattered events could help model some of these effects and maintain a $\delta m_t \sim 100–200$ MeV. What has not been studied, is how to correctly match these ISR effects to the resummed calculations such that there is no double-counting of ISR. I suggest that the likely the matching uncertainty is at least 50 MeV, and could ultimately dominate the uncertainty in the top-quark mass extraction. Significantly

**Figure 1:** Ratio of NNNLO (and NNLO) to LO corrections across the $t\bar{t}$ threshold [15]
more work is needed to understand how to combine these theoretical cross sections and ISR at the ILC.

A new approach to extracting a competitive value for the top-quark mass away from threshold has grown out of the recent successes of effective field theories (EFTs) of QCD. The “continuum” top-quark mass is extracted by a fit to the mass of the jet formed by decay products of the top quark. Demonstration of a formal factorization of the production and decay process into strongly ordered scales \( (Q \gg m_t \gg \Gamma_t \gg \Lambda_{QCD}) \) \[17, 18\], allows for a thrust-like object called the “hemisphere” mass to be constructed. For details, see the work of Ref. \[11\] at this conference. This represents the beginning of a new class of calculations that describe a complete event — from production through decay and hadronization.

3 Top-quark mass: Why do we care?

Now that we have established what it is that we measure, it is useful to examine in a bit more depth why we care. Explicitly, why does it matter that we measure the top-quark mass to 100 MeV, as opposed to 1 GeV? The typical reason given for studying the top-quark mass is that it provides a strong constraint on electroweak physics. Specifically, since the top-quark and Higgs boson contribute at one loop to the \( W \) and \( Z \) propagators, the \( W \) mass is quadratically sensitive to the top-quark mass, and logarithmically dependent on the Higgs mass. Inverting this dependence leads to constraints on the Higgs mass based on fits to the electroweak precision data, and the direct measurement of the \( W \) and top-quark masses at the Fermilab Tevatron. By summer 2008, the top quark was measured to be \( 172.4 \pm 1.2 \) GeV \[19\].

A better way to look at the Higgs constraint was pointed out in Ref. \[20\]. Assuming that the Higgs mass is known, and \( m_W \) is measured to 20 MeV (the LHC target), our current understanding of \( \sin^2 \theta_W \) implies we only need to know \( m_t \) to \( \sim 3 \) GeV at the LHC. The ILC can measure \( m_W \) to \( \sim 6 \) MeV \[1\]. Giga-Z proposes to measure \( \sin^2 \theta_W \) to \( \sim 10^{-5} \) (see the talk of Ref. \[21\] for a method to use \( Z \)-calibration data to attain \( 3 \times 10^{-5} \)). This optimal scenario cannot make use of a top-quark mass uncertainty better than 1 GeV in the Standard Model, a value already saturated by the current Tevatron measurement!

The drive to extract an extremely accurate top-quark mass at the ILC, then, is from physics Beyond the Standard Model. Many models of new physics are very sensitive to the exact top-quark mass. For example, in supersymmetry, the shift in the Higgs mass is

\[
\Delta m_H^2 \approx \frac{3G_F m_t^4}{\sqrt{2} \pi^2 \sin^2 \beta} \ln \left( \frac{\overline{m}_t^2}{m_t^2} \right),
\]

where \( \tan \beta \) is the ratio of vacuum expectation values of the two Higgs doublets, and \( \overline{m}_t \)

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is the average top-squark mass. In this case, the theoretical error in the Higgs mass will be directly proportional to the error in the top-quark mass. Hence, the desire to reach $\delta m_t \sim 100$ MeV. In addition to the Higgs mass, several other parameters of supersymmetry may be strongly constrained, such as $M_A, A_t, m_{1/2}$, etc.

Given that we expect some new physics must exist that explains electroweak symmetry breaking, the motivation to measure a small top-quark mass is strong. It also provides significant theoretical motivation to do higher-order loop corrections. Simple power counting suggests that four-loop corrections will contribute at the same order numerically as the measurement. Hence, the industry of loop calculations in the Standard Model and supersymmetry has a significant effort ahead of it before we can make use of the phenomenal precision that will be achieved at the ILC.

4 Status and remaining questions

The physics case for an extended study of top-quark production and decay at threshold and in the continuum is already strong [1]. The primary focus of the last several years has been to improve techniques for extracting the top-quark mass. Questions have been raised as to whether the best-measured mass will be the 1S mass from threshold, or a top-quark jet mass from the continuum. In the end, neither mass will be measured. Rather, we will measure line shapes or particle flow, or invariant masses with explicit cuts — all under the influence of ISR/FSR radiation effects. Our ability to extract the top-quark mass under these conditions will, therefore, be constrained by our ability to understand and control the systematic uncertainties introduced by our modeling of these phenomena.

This leads to the following questions:

1. Do we need to work out initial-state subtraction terms to merge formal calculations correctly with ISR estimates for incoming elections? Box diagrams and $\alpha^2$ diagrams may be significant on the level of the 0.01% theory error we want for $m_t$ and other observables.

2. The hemisphere-like top-jet definitions are exciting theoretical constructs, but does the factorization demonstrated hold in the presence of hard cuts? Once regions are eliminated due to finite-size detector effects, the current definitions look similar to very large fixed-cone definitions, which are not infrared-safe. More work needs to be done to understand whether experimental implementation of these theoretical constructs is viable.

3. One jet algorithm expected to be used is particle flow. This procedure estimates the energy in the neutral particles in the event from the charged ones. Does this invalidate any assumptions in the theoretical calculations? If we answer in the negative, we must be certain to better than 0.01%.

The greatest opportunity and challenge going forward will be to ensure that the experimental and theoretical definitions agree. Here, a significant opportunity presents itself in the application of the new effective field theories. The merging of perturbation theory (PT), soft-collinear effective theory (SCET), heavy-quark effective theory (HQET), and other derivative theories that better describe jet structure, will produce the tools we need to achieve a theoretical precision in QCD comparable to the amazing precision that will be obtained experimentally at the International Linear Collider.
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