Neutrino masses and LFV from minimal breaking of \(U(3)^5\) and \(U(2)^5\) flavor symmetries

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Abstract. We analyze neutrino masses and Lepton Flavor Violation (LFV) in charged leptons with a minimal ansatz about the breaking of the \(U(3)^5\) flavor symmetry, consistent with the \(U(2)^5\) breaking pattern of quark Yukawa couplings, in the context of supersymmetry. Neutrino masses are expected to be almost degenerate, close to present bounds from cosmology and \(0\nu\beta\beta\) experiments. We also predict \(s_{13} \approx s_{23}|V_{ud}/|V_{ts}| \approx 0.16\), in perfect agreement with the recent DayaBay result. For slepton masses below 1 TeV, barring accidental cancellations, we expect \(\mathcal{B}(\mu \to e\gamma) > 10^{-13}\) and \(\mathcal{B}(\tau \to \mu\gamma) > 10^{-9}\), within the reach of future experimental searches.

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1 Introduction

The Yukawa sector of the Standard Model (SM), minimally extended with a Majorana mass terms for the left-handed neutrinos, provides an excellent description of all the observed phenomena of flavor mixing, both in the quark and in the lepton sector. Still, the peculiar pattern of quark and lepton masses seems to point toward some new dynamics responsible for this highly non-generic flavor pattern. Moreover, the instability of the Higgs sector under quantum corrections suggests the presence of physics beyond the SM close to the electroweak scale.

The absence of any deviation from the SM predictions in flavor-violating processes is one of the most severe problems in building realistic extensions of the model at the TeV scale. This is why minimal flavor-breaking hypotheses for SM extensions, such as the ansatz of Minimal Flavor Violation [1], have been proposed. However, the MFV ansatz does not provide any clue about the origin of the hierarchical nature of the Yukawas, and does not help in understanding the recent severe challenge to SM extensions posed by the absence of direct signals at the LHC.

A TeV extension of the SM aimed to address, at least in part, both the stability of the electroweak sector and the flavor problem is supersymmetry with heavy squark masses for the first two families of squarks, in short split-family SUSY. While the stabilization of the Higgs sector requires mostly the third generation squarks to be light, the tight constraints from CP- and flavor-violating processes are loosened in presence of a squark mass hierarchy [2]. In addition, while the bounds on first generation squark masses are already exceeding 1 TeV, the third generation squarks can still be significantly lighter [3]. As pointed out in a series of recent works [4], such a split spectrum can be achieved with realistic ultraviolet completions of the model.

A hierarchical squark spectrum is not enough to suppress flavor violation to a level consistent with experiments. This is why split-family SUSY with a minimally broken \(U(2)^3 = U(2)_q \times U(2)_q \times U(2)_u\) flavor symmetry, acting on the first two generations of quarks (and squarks), has been considered in Ref. [5]. This set-up has the following advantages: i) it provides some insights about the structures of the Yukawa couplings (along the lines of \(U(2)\) models proposed long ago [6]); ii) it ensures a sufficient protection of flavor-changing neutral currents; iii) it leads to an improved CKM fit with tiny and correlated non-standard contributions to \(\Delta F = 1\) observables. Possible signatures of this framework in the \(\Delta F = 2\) sector have been discussed in Ref. [7] (see also Ref. [8], where the same symmetry with additional dynamical assumption has been considered). More general discussions about the \(U(2)^3\) flavor symmetry beyond supersymmetry has recently been presented in Ref. [9,10].

The purpose of this article is to extend the idea of a minimally broken flavor symmetry acting on the first two generations to the lepton sector. The extension is straightforward in the case of charged leptons, enlarging the flavor symmetry from \(U(2)^3\) to \(U(2)^5 = U(2)^3 \times U(2)_l \times U(2)_e\). However, the situation is more involved in the neutrino sector, whose mass matrix has a rather different flavor
structure: no large hierarchies in the eigenvalues, and large mixing angles \[11\]. A simple ansatz to circumvent this problem is to assume a two-step breaking in the neutrino sector: first, a leading breaking of the maximal flavor mixing angles \[11\], such that the mass eigenstates are ordered following a normal hierarchy (\(\nu_3 < \nu_2 < \nu_1\)) or an inverted one (\(\nu_3 < \nu_1 < \nu_2\)). To distinguish between them, one defines \(\Delta m^2_{ij} = m^2_{\nu_i} - m^2_{\nu_j}\), such that \(\Delta m^2_{31} = \pm \Delta m^2_{\text{atm}}\) and \(\Delta m^2_{21} = \Delta m^2_{\text{sol}}\), where \(\Delta m^2_{\text{atm}}\) denotes the (positive) squared mass differences deduced from atmospheric and solar neutrino data. It is straightforward to deduce that the plus (minus) sign of \(\Delta m^2_{31}\) corresponds to the normal (inverted) hierarchy.

Experimental data on neutrino oscillations indicate the presence of (at least) two small parameters in \(M^2_\nu\),

\[
\zeta = \left| \frac{\Delta m^2_{\text{atm}}}{\Delta m^2_{\text{sol}}} \right|^{1/2}, \quad \text{or } \exp = 0.174 \pm 0.007, \quad (4)
\]

\[
s_{13} = |(U_{\text{PMNS}})_{13}|, \quad s_{\exp} = 0.15 \pm 0.02, \quad (5)
\]

where the value of \(s_{13}\) has been determined from the recent result of the DayaBay experiment \[14\]. Expanding to lowest order in these two parameters (or in the limit \(s_{13} \rightarrow 0\)) we are left with the following structure

\[
(M^2_\nu)^{(0)} = m^2_{\text{light}} \cdot I + \Delta m^2_{\text{atm}} \cdot \Delta \quad (6)
\]

where \(I\) is the identity matrix, \(m^2_{\text{light}}\) is the lightest neutrino mass, and

\[
\Delta_{[\text{n.h.}]} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & s^2_{23} & s_{23}c_{23} \\ 0 & s_{23}c_{23} & c^2_{23} \end{pmatrix}, \quad \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \quad (7)
\]

\[
\Delta_{[\text{l.h.}]} = I - \Delta_{[\text{n.h.}]}, \quad (8)
\]

In order to define a starting point for the neutrino mass matrix in the limit of unbroken flavor symmetry we need to specify the hierarchy between \(m^2_{\text{light}}\) and \(\Delta m^2_{\text{atm}}\), or among the two terms in Eq. \((6)\). We thus have three natural possibilities:

I. \((M^2_\nu)^{(0)} \propto I, \text{ if } m^2_{\text{light}} \gg \Delta m^2_{\text{atm}}\),

II. \((M^2_\nu)^{(0)} \propto \Delta_{[\text{n.h.}]}, \text{ if } m^2_{\text{light}} \ll \Delta m^2_{\text{atm}}\) and \(\Delta m^2_{31} > 1\),

III. \((M^2_\nu)^{(0)} \propto \Delta_{[\text{l.h.}]}, \text{ if } m^2_{\text{light}} \ll \Delta m^2_{\text{atm}}\) and \(\Delta m^2_{31} < 1\).

### 3 Flavor symmetries and symmetry breaking

#### 3.1 \(U(2)_L \times U(2)_e\)

The \(U(2)^2 = U(2)_L \times U(2)_e\) flavor symmetry, under which the lepton superfields of the first two families transform as

\[
L_L \equiv (L_{L1}, L_{L2}) \sim (2, 1), \quad (9)
\]

\[
e^c \equiv (\epsilon_1^c, \epsilon_2^c)^T \sim (1, 2), \quad (10)
\]

offers a natural framework to justify the hierarchal structure of the charged-lepton Yukawa coupling, in close analogy to the \(U(2)^2\) symmetry introduced in Ref. \[3\] for the quark sector. In the limit of unbroken symmetry we recover the result in Eq. \((9)\). Assuming a symmetry-breaking pattern for \(Y_e\) similar to the one adopted for the quark Yukawa couplings, we get

\[
Y_e = y_\tau \begin{pmatrix} \Delta Y_{13} \nu_{13} \\ 0 \end{pmatrix}, \quad (11)
\]
where we have absorbed $O(1)$ couplings in the definition of the breaking terms $V \sim (2, 1)$ and $\Delta Y_e \sim (2, 2)$. Introducing the unitary matrices $U_{eL}$ and $U_{eR}$, such that

$$U_{eL}Y_eU_{eR}^T = \text{diag}(y_e, y_\mu, y_\tau) \, ,$$

and proceeding as in [3], we find that $U_{eR}$ becomes the identity matrix in the limit $m_{\nu,1}/m_{\nu} \to 0$, while $U_{eL}$ assumes the following parametric form

$$U_{eL} \approx \begin{pmatrix} c_\epsilon & s_\epsilon e^{i\alpha_\epsilon} & -s_\epsilon e^{-i\alpha_\epsilon} \\ -s_\epsilon e^{-i\alpha_\epsilon} & c_\epsilon & s_\epsilon e^{i\alpha_\epsilon} \\ 0 & s_\epsilon e^{i\phi_\epsilon} & c_\epsilon \end{pmatrix} \, ,$$

in the $(U(2)_l)$ basis where $V^T \sim (0, 1)$. Here $\alpha_\epsilon$ and $\phi_\epsilon$ are generic $O(1)$ phases, while $s_{\epsilon, \tau}$ are small mixing angles ($c_\epsilon^2 + s_\epsilon^2 = 1$). If the analogy with the quark sector holds, we expect $s_{\epsilon, \tau}$ to be of the order of $s_{d_t} = |V_{t3L}|/|V_{ts}| \approx 0.22$ and $s_{\epsilon, \tau}$ of the order of $\epsilon = |V_{e3L}| \approx 0.04$.

From the point of view of the $(U(2)^2)$ symmetry, the neutrino mass matrix can be decomposed as

$$m_\nu = \begin{pmatrix} m_2 + m_1 & m_2 \nu_1 \\ m_2 \nu_1 & m_1 \end{pmatrix} \, ,$$

where $m_3 \sim (3, 1)$, $m_2 \sim (2, 1)$, and $m_1 \sim (1, 1)$. This decomposition does not match well with any of the potential starting points identified in Eqs. [4]-[5]: they can be obtained only assuming specific relations among terms with different $(U(2)_l)$ transformation properties. This suggests that we need to consider a larger flavor symmetry, whose breaking to $(U(2)_l)$ (or some of its subgroups) could explain such relations. From this point of view the degenerate case is the one that offers the most interesting prospects: on the one hand it requires a special relation only among two of the terms appearing in Eq. [14]: $m_3 = \text{diag}(m_{11}, m_{12})$. On the other hand, it requires $m_2 \ll 1$, as expected given that $m_2$ transforms as the breaking spurion $V$ of $O(e)$ appearing in the charged-lepton Yukawa coupling. As we discuss in the following, the degenerate case can easily be obtained embedding $(U(2)_l)$ in $(U(3)_l)$.

3.2 $U(3)_l \times U(3)_e$

The group $U(3)_l \times U(3)_e$ is the largest flavor symmetry of the lepton sector allowed by the SM gauge Lagrangian. The degenerate configuration for $m_\nu$ is achieved assuming that $U(3)_l$ and the total lepton number,

$$U(1)_{\text{LN}} = U(1)_{l+e} \, ,$$

are broken by a spurion $m_{\nu}^{(0)}$ transforming as a 6 of $U(3)_l$ and leaving invariant a subgroup of $U(3)_l$ that we denote $O(3)_l$. By a proper basis choice in the $(U(3)_l)$ flavor space we can set

$$m_{\nu}^{(0)} \propto \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \, .$$

We shall also require that $U(3)_l \times U(3)_e$ is broken by $U(1)_{\text{LN}}$ invariant spurions to the subgroup $U(2)_l \times U(2)_e$ relevant to the charged-lepton sector. However, it is essential for our construction that this (sizable) breaking does not spoil the Majorana Yukawa coupling. This can be achieved in a supersymmetric context introducing a new spurion $Y_e^{(0)} \sim (3, 3)$ that breaks $U(3)_l \times U(3)_e$ to $U(2)_l \times U(2)_e$ leaving unbroken the $(O(3)_l \times U(2)_e)$ subgroup of both of $O(3)_l$ and $U(2)_e$. By means of $Y_e^{(0)}$ we can have a non-vanishing Yukawa coupling for the third generation in the superpotential

$$L_e Y_e^{(0)} e^c \to y_{\nu}^{(0)} L_3 e_{L3}^c \, ,$$

and, in first approximation, the Majorana mass matrix is unchanged. Note that supersymmetry is a key ingredient for the latter statement to hold. Indeed, if the mass operator was not holomorphic, a Majorana term of the type $L_Y Y^T m_{\nu}^{(0)} L_3^c$ could also be included and this would spoil the degenerate configuration.

Summarizing, introducing the two spurions $m_{\nu}^{(0)}$ and $Y_e^{(0)}$ we recover phenomenologically viable first approximations to both the neutrino and the charged-lepton mass matrices and we are left with an exact $O(2)_l \times U(2)_e$ symmetry that leaves invariant both $m_\nu$ and $Y_e$. Moreover, thanks to supersymmetry, the two sector considered separately are invariant under larger symmetries: $O(3)_l$ for the neutrinos and $U(2)_l \times U(2)_e$ for the charged leptons.

We can then proceed introducing the small $O(2)_l \times U(2)_e$ breaking terms responsible for the subleading terms in $Y_e$ in Eq. [11]. In order not to spoil the leading structure of the neutrino mass matrix, the spurion $V$ in Eq. [12] should be regarded as a doublet of $(O(2)_l)$, rather than a doublet of $(U(2)_l)$. We can also regard it as the $(O(2)_l)$ component of an appropriate 8 of $(U(3)_l)$ with the following structure

$$X = \begin{pmatrix} \Delta_L & V^T \\ \sqrt{1 - \Delta_L^2} & x \end{pmatrix} \, .$$

This allows to write the additional Yukawa interaction $L_e X Y_e^{(0)} e^c$ that, combined with the leading term in [17] and with a proper redefinition of $y_{\nu}$ and $V$ implies

$$Y_e^{(1)} = y_{\nu} \begin{pmatrix} 0 \sqrt{1 - \Delta_L^2} \\ 0 \end{pmatrix} \, .$$

All the components of $X$ do appear in the Majorana sector, via the terms $L_e X m_{\nu}^{(0)} L_3$ and $L_e m_{\nu}^{(0)} X^T L_3^c$. These imply the following structure

$$m_{\nu} = m_{\nu}^{(0)} \left[ I + a \begin{pmatrix} \Delta_L & V^T \\ \sqrt{1 - \Delta_L^2} & x \end{pmatrix} \right] \, ,$$

where $a$ is a $(O(1))$ complex coupling. Assuming that all the entries of $X$ are at most of $O(e)$ does not spoil the degenerate configuration of $m_{\nu}$ in first approximation. In addition, since $\Delta_L$ could enter linearly in the sfermion

1 We denote in bold $(U(3))$ vectors and representations.
mass matrices and induce sizable FCNCs, we expect a small mis-alignment between $\Delta_{\ell}$ and $V$ in the $O(2)_l$ space. Pursuing the analogy with the squark sector, we are forced to assume $(\Delta_{\ell})_{12}$ at most of $O(\epsilon^2)$ in the basis where $V_1 = 0$. In other words, we are lead to the following assignment for the various components of $X$ in the $O(2)_l$ basis where $V^T \propto (0, 1)$:

$$V = \begin{pmatrix} 0 \\ O(\epsilon) \end{pmatrix}, \quad \Delta_{\ell} = \begin{pmatrix} 0 & O(\epsilon^2) \\ O(\epsilon^2) & O(\epsilon) \end{pmatrix}, \quad x = O(\epsilon).$$

(21) 

In the same basis, redefining the unknown parameters, we arrive to the following parametric expression

$$m_{\nu} = \bar{m}_{\nu_{V}} \begin{pmatrix} I + e^{i \phi} \begin{pmatrix} -\sigma \epsilon \gamma & 0 \\ \gamma & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ 0 \end{pmatrix},$$

(22)

where $\phi$, $\sigma$, $\delta$, $\gamma$, and $r$ are real parameters expected to be of $O(1)$. The final step for the construction of a realistic charged-lepton Yukawa coupling is the introduction of the $U(2)_l \times U(2)_e$ bi-doublet $\Delta Y_{e}$. The most economical way to achieve this goal in the context of $U(3)_l \times U(3)_e$ is to introduce a bi-triplet with the following form,

$$\Delta Y_{e} = \begin{pmatrix} \Delta Y_{e}(0) \\ 0 \\ 0 \end{pmatrix},$$

(23)

which provides the desired correction to $Y_{e}$ and has no relevant impact on $m_{\nu}$. Notice that the requirement of having $Y^{(0)}$ and $X$ acting on $O(3)$ and $O(2)$ subspaces can be naturally accomplished by demanding an exact CP symmetry acting on both spurions. The CP symmetry would be broken only by the $\Delta Y_{T}$ spurions, which would provide all CP violation phases.

Our parametrical decomposition of the neutrino mass matrix is therefore the expression in Eq. (22). As can be seen, the latter contains only a CP-violating phase, $\phi_e$, which does not contribute to the PMNS matrix. However, this does not imply that there are no CP-violating phases in $U_{PMNS}$: a non-vanishing phase arises by the diagonalization of the charged-lepton mass matrix. Indeed, to leading order in $\epsilon$, the parametric decomposition of $M_{\nu} = m_{\nu}^T m_{\nu}$ in the basis where $Y_{e}$ is diagonal is

$$M_{\nu}^2 = \bar{m}_{\nu_{V}} U_{eL}^T \begin{pmatrix} 1 - 2\sigma \epsilon & -2\gamma \epsilon \gamma & 2 - 2\epsilon \\ -2\gamma \epsilon \gamma & 2 - 2\epsilon & 1 \\ 2 - 2\epsilon & 1 \end{pmatrix} U_{eL}^T,$$

(24)

where we have redefined $\sigma$, $\delta$, $\gamma$, and $r$, absorbing a cos($\phi_e$) term and $U_{eL}$ is given in Eq. (13). Despite the presence of four $O(1)$ free parameters, the expression of $M_{\nu}^2$ in Eq. (24) is quite constrained by the smallness of $\epsilon$. As we discuss in the next section, it provides a good fit to all the available neutrino data for a natural range of the free parameters and leads to a few unambiguous predictions. We finally stress that we arrived to this decomposition using essentially two main assumptions:

I. An approximate degenerate neutrino spectrum, that fits well with present data if $m_{\nu_{light}}^2 \gg \Delta m_{\text{atm}}^2$.

II. A symmetry-breaking pattern with respect to a purely degenerate spectrum closely related to the minimal $U(2)^5$ symmetry breaking pattern of quark and lepton Yukawa couplings.

4 Predictions for neutrino masses and mixings

We are now ready to analyze the predictions of the $M_{\nu}^2$ parameterization in Eq. (24). We first discuss a few simple analytic results, valid to leading order in $\epsilon$. Given the neutrino spectrum is almost degenerate, the ordering of the eigenvalues has no physical implications. However, for the sake of simplicity, we present analytic results only in the case of normal ordering ($m_{\nu_{1}} > m_{\nu_{2}} > m_{\nu_{3}}$). We then proceed with a systematic numerical scan of the four $O(1)$ free parameters to investigate the stability of the analytic conclusions.

4.1 Mass eigenvalues

From the decomposition of $M_{\nu}^2$ in Eq. (24) we derive the following expressions for the eigenvalues,

$$m_{\nu_{1}}^2 = \bar{m}_{\nu_{1}}^2 (1 - 2\sigma \epsilon),$$

(25)

$$m_{\nu_{2}}^2 = \bar{m}_{\nu_{2}}^2 \left[ 1 - \delta \epsilon - (\delta^2 + 4\epsilon^2)^{1/2} \right],$$

(26)

$$m_{\nu_{3}}^2 = \bar{m}_{\nu_{3}}^2 \left[ 1 - \delta \epsilon + (\delta^2 + 4\epsilon^2)^{1/2} \right],$$

(27)

up to $O(\epsilon^2)$ corrections. The normal ordering of the spectrum is obtained for $\sigma > (\delta^2 + 4\epsilon^2)^{1/2}$ and in this case we find

$$\frac{\Delta m_{\text{atm}}^2}{m_{\nu_{1}}^2} = \left[ \frac{2\sigma - \delta + (\delta^2 + 4\epsilon^2)^{1/2}}{2\sigma - \delta + (\delta^2 + 4\epsilon^2)^{1/2}} \right] \epsilon,$$

(28)

$$\zeta = \left[ \frac{2\sigma - \delta - (\delta^2 + 4\epsilon^2)^{1/2}}{2\sigma - \delta + (\delta^2 + 4\epsilon^2)^{1/2}} \right] \epsilon^{-1/2}.$$

(29)

As can be seen, $\epsilon$ controls the overall scale of neutrino masses, whose natural scale is

$$O[(\Delta m_{\text{atm}}^2)^{1/2} / \epsilon^{-1/2}] = O(0.3 \text{ eV}).$$

(30)

just below the existing bounds (see discussion at the end of this Section). Our parametric decomposition of $M_{\nu}^2$ does not necessarily imply $\zeta \ll 1$. However, the experimental value in Eq. (11) is easily obtained with a modest tuning of the free parameters, especially if $\delta$ is small. As we discuss next, the latter is a condition necessary to reproduce the maximal 2-3 mixing in the PMNS matrix.
4.2 \( \theta_{23} \)

In order to determine the \( \theta_{23} \) and \( \theta_{13} \) mixing angles of the PMNS matrix it is sufficient to expand \( M_{
u}^2 \) up to \( O(e^2, s_{23}^2 e^2) \):

\[
M_{
u}^2 = m_{\nu_1} \left( 1 - 2e\sigma - 2s_{\nu_1}\sigma + 2 \sigma + 2e\delta - 2s_{\nu_1}\delta \right) + O(e^2, s_{23}^2 e^2). \tag{31}
\]

As can be seen, at this level \( \gamma \) does not appear. Note also that for \( s_{\nu_1} \to 0 \) the 2-3 sector decouples. In this limit we obtain the following simple expression for the 2:3 mixing:

\[
t_{23} = \frac{s_{23}}{c_{23}} - \frac{\delta \pm \sqrt{d^2 + 4r^2}}{2r}, \tag{32}
\]

where the sign in the numerator is chosen depending on the sign of \( r \), such that \( t_{23} \) remains positive. As we have explicitly checked by means of the numerical scan, this result is stable with respect to the inclusion of the subleading terms of \( O(s_{\nu_1} e) \) and \( O(e^2) \).

From Eq. (32) it is clear that \( t_{23} \) is naturally expected to be \( O(1) \), and the experimental evidence of maximal mixing, \( t_{23} \approx 1 \), is obtained for \( \delta \to 0 \).

4.3 \( \theta_{13} \) and the PMNS phase

The value of \( \theta_{13} \) can be obtained via the following general relation

\[
\frac{(M_{
u}^2)_{31}}{(M_{
u}^2)_{32}} \approx \frac{s_{13}}{s_{23}} e^{i\delta_P}, \tag{33}
\]

that can be derived expanding \( M_{\nu}^2 \) in the basis where \( Y_e \) is diagonal—up to the first order in \( \zeta \) and \( s_{13} \) (here \( \delta_P \) denotes the PMNS phase in the standard parameterization). Applying this result to the approximate from in Eq. (33) leads to

\[
s_{13} e^{i\delta_P} = s_{\nu_1} s_{23} e^{i\alpha_2 + \pi}. \tag{34}
\]

Assuming \( s_{\nu_1} = s_d = |V_{td}|/|V_{ts}| \), and the experimental value of \( s_{23} (s_{23} \approx 0.52 \pm 0.06 \) [13]), we predict

\[
s_{13} = 0.16 \pm 0.02, \tag{35}
\]

in remarkable agreement with the recent DayaBay result in Eq. (5). This prediction is affected by a theoretical error (not explicitly shown) due to possible deviations from the relation \( s_{\nu_1} = s_d \). On the other hand, as we have checked by means of the numerical scan, it is quite stable with respect to subleading corrections in the expansion of \( M_{\nu}^2 \).

As anticipated, the PMNS phase is completely determined in terms of the CP-violating phase from the rotation of the charged-lepton Yukawa coupling: \( \delta_P = \alpha_c + \pi \). However, we are not able to determine this phase even assuming \( \alpha_c = \alpha_a \), where \( \alpha_a \) is the corresponding phase appearing in the diagonalization of \( Y_e \). On general grounds, we expect \( \delta_P \) to be a generic \( O(1) \) phase.

4.4 \( \theta_{12} \)

Contrary to the case of \( \theta_{23} \) and \( \theta_{13} \), the determination of \( \theta_{12} \) involve subleading terms in \( M_{\nu}^2 \) and thus is more unstable.

As an illustration, consider \( M_{\nu}^2 \) in the limit \( s_{\nu_1} \to 0 \). In this simplified case we obtain the relation

\[
\tan 2\theta_{12} = \frac{2\gamma c_{23}}{\sigma - \delta c_{23} - 2r s_{23} c_{23}}, \tag{36}
\]

that seems to imply \( \theta_{12} \approx 0, \pi/2 \). However, once we impose the constraints from the squared mass differences, we find a cancellation in the denominator leading to generic \( O(1) \) values for \( \theta_{12} \). More explicitly, expressing \( s_{23} \) and \( c_{23} \) in terms of \( \delta \) and \( r \) by means of Eq. (32) we find

\[
\tan 2\theta_{12} = \frac{4\gamma e}{2\sigma - \delta - (\delta^2 + 4\sigma^2)^{1/2}} c_{23} = O(1) \times \frac{e}{\zeta^2}, \tag{37}
\]

which is manifestly a generic \( O(1) \) number. This general conclusion remains valid when subleasing terms of \( O(s_{\nu_1} e) \) and \( O(e^2) \) are taken into account, although the explicit analytical expression for \( \theta_{12} \) becomes more involved.

4.5 Parameter Scan

In order to check the stability of the above conclusions we have performed a numerical scan allowing the four free real parameters in Eq. (24) to vary in the range \([-2, 2]\). The results are summarized in Fig. 1. In all plots the blue points are the allowed points after imposing the squared mass constraints only, while the points in brown are those where both squared mass and mixing constraints, as resulting from the global fit in Ref. [13], are satisfied.

The two plots in Fig. 1 illustrate the role of \( \sigma, \delta, \) and \( r \), in reproducing the mass spectrum. For illustrative purposes, only the points giving rise to normal hierarchy are shown: the inverted case give rise to identical distributions provided \( \sigma \to -\sigma \) and \( \delta \to -\delta \). As can be seen from both plots, there is a wide range of values giving rise to the correct mass spectrum and no serious tuning of the parameters is needed to explain the (modest) hierarchy between |\(\Delta m_{\text{atm}}\)| and \(\Delta m_{\text{sol}}\). The latter emerge naturally provided \(\sigma\) is not too small.

The results for \(\theta_{13}\) and \(\theta_{23}\) as a function of the corresponding most relevant free parameters are illustrated in Figure 2 (again only the normal hierarchy case is explicitly shown). On the top panel we show \(t_{23}\) as a function of \(\delta/r\); the two bands are those expected by the analytical expression in Eq. (32). As can be seen, we cannot claim to predict \(t_{23}\) to be 1, but this is a value perfectly allowed by our parameterization without particular fine tuning. On the contrary, very small or very large values of \(t_{23}\) are

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\footnote{Although the global fit of Ref. [13] does not include the recent Daya-Bay result [14], the resulting value for \(\theta_{13}\) turns out to be in good agreement with the direct determination in Eq. (5). Similar results for all the neutrino parameters but for \(\theta_{13}\) can be found also in Ref. [15].}
disfavored after imposing the mass constraints. On the bottom panel we show $s_{12}$ as a function of $\gamma$. As anticipated in the analytical discussion, $s_{12}$ is very difficult to be predicted (any value is essentially allowed). The only clear pattern emerging is the need of a non-vanishing $\gamma$ in order to reproduce the experimental value of $s_{12}$.

In Figure 3 we show the correlation of $s_{13}$ and $s_{23}$, which is the only clear prediction of our decomposition, as far as the mixing angles are concerned. Also in this case we find a clean confirmation of what expected by means of the approximate analytical result in Eq. (32). The width of the band correspond to the uncertainty in the value of $s_{e}$, that we have varied in the range $[0.19, 0.25]$.

Finally, in Fig. 4 we illustrate our predictions for the absolute values of neutrino masses. We find that typical values for the sum of neutrino masses lie around $0.2 - 1$ eV, in agreement with current bounds. Interestingly, this means that neutrinoless double beta decay should be observed in the upcoming experiments, with the parameter $m_{\beta\beta}$ around $0.02 - 0.4$ eV. Current bounds for the matrix element, as well as the sensitivity of future experiments, are shown in Table 1.

6 The WMAP bound of [17] for the sum of neutrino masses varies between $1.3$ eV (WMAP-only) and $0.58$ (WMAP + Baryon Acoustic Oscillations + Hubble constant measurements)

5 The slepton sector and LFV

5.1 Slepton Structure

Having identified the minimal set of spurions necessary to build the lepton Yukawa coupling and the neutrino mass matrix, we can now turn to study the consequences of this symmetry-breaking pattern in the slepton sector.
transforms as $O_8$ with all constants being real and relevant to $0$.

**Table 1.** Current bounds and prospects on $m_{33}$. Intervals in bounds are due to uncertainty in the nuclear matrix elements. The “reach” assume reference values for $\beta\beta$ isotope masses and a 10-year data taking period.

| Experiment         | Bound (eV), C.L. | Experiment         | Reach (eV) |
|--------------------|-----------------|--------------------|------------|
| KamLAND-Zen ($^{136}$Xe) | $< 0.3 - 0.6, 90\%$ | KamLAND-Zen ($^{136}$Xe) | 0.062      |
| CUORICINO ($^{130}$Te)   | $< 0.19 - 0.68, 90\%$ | CUORE ($^{130}$Te)    | 0.062      |
| NEMO3 ($^{100}$Mo)     | $< 0.7 - 2.8, 90\%$ | NEXT ($^{136}$Xe)    | 0.071      |
| Heidelberg-Moscow ($^{76}$Ge) | $0.32 \pm 0.03, 68\%$ | EXO ($^{136}$Xe)     | 0.072      |

In the fermion sector, the main difference between the $U(3)^5$ set-up we are considering, and that based on a $U(2)^5$ symmetry, lies on the fact that in the latter case one can naturally have sfermions of the first two families considerably heavier than those of the third family [5]. As discussed in the introduction, this possibility is attractive due to the lack of experimental signals for supersymmetry at LHC and, at the same time, the need of relatively light squarks of the third generation in order to stabilize the Higgs sector (hierarchy problem). In the $U(3)^5$ set-up we can also have third-family sfermions substantially lighter than those of the first two generations. However, this can happen only at the price of some fine-tuning of the symmetry-breaking terms. In the case of \( \bar{m}_{33}^2 \), this happens if \( 1 + c_2 |y_\tau|^2 \ll 1 \). However, it is worth to stress that in the sfermion sector the requirement of a sizable mass splitting among the families is less motivated: the sfermions play a minor role in the hierarchy problem and there are no stringent direct experimental bounds on any of the sfermion families.

Let’s start from the $LL$ soft sfermion mass matrix, which transforms as $8 \oplus 1$ under $U(3)_L$ and is invariant under $U(3)_e$. The LN conserving spurions at our disposals are $Y^{(0)}$ and $\Delta Y_e$, both transforming as bi-triplets of $U(3)_L \times U(3)_e$, and $X$, transforming as an $8$ of $U(3)_e$. Given the smallness of neutrino masses, we can safely neglect LN-conserving terms obtained by the contraction of two $m^{(0)}_l$ terms. Expanding to the first non-trivial order in these spurions, the $LL$ soft mass matrix assume the following form

\[
\bar{m}_{LL}^2 = \begin{pmatrix}
(m^{2}_{33})_{hh} & \tilde{c}_3 V^*_{e} & \tilde{c}_3 V^*_{\tau} \\
\tilde{c}_3 V^{*}_{e} & c_3 & 0 \\
\tilde{c}_3 V^{*}_{\tau} & 0 & c_3
\end{pmatrix} \tilde{m}_L^2,
\]

\[
(m^2_{33})_{hh} = I + c_3 \Delta L + c_4 \Delta Y_e \Delta Y_e^T,
\]

\[
(m^2_{33})_{33} = 1 + c_2 |y_\tau|^2 + c_3 x,
\]

with all constants being real and $O(1)$. Since $X$ is at most of $O(\epsilon)$ and $\Delta Y_e$ is at most of order $(y_\mu/y_\tau)$, we can approximate the above expression to

\[
\bar{m}_{LL}^2 = \begin{pmatrix}
1 & c_2' \epsilon^2 & 0 \\
c_3' \epsilon^2 & 1 + c_3 \epsilon & c_5 \epsilon \\
0 & c_5 \epsilon & 1 + c_3 |y_\tau|^2
\end{pmatrix} \bar{m}_L^2,
\]

where we have distinguished $c_3$, $c_2'$ and $c_5'$ due to the possibility of additional $O(1)$ factors from the spurions themselves. With this definition, $c_3'$ and $c_5'$ are complex. In principle, these parameters are related to the parameters appearing in the neutrino mass matrix by

\[
\Re(c_3') = \frac{1}{2 \sigma - \delta} c_3, \quad \Re(c_5') = \frac{1}{2 \sigma - \delta} c_3.
\]

However, we have explicitly checked that these relations do not provide very stringent constraints. For this reason, in the numerical analysis we have treated $c_3$, $c_3'$, and $c_5'$ as independent free parameters.

Here all off-diagonal terms are heavily suppressed by the first and second generation Yukawa couplings and, to a good approximation, can be neglected.

Finally, let’s consider the trilinear soft-breaking term $A_e$, responsible for the $LR$ entries in the sfermion mass matrices. The symmetry breaking structure of $A_e$ is identical to that of the Yukawas, albeit with different $O(1)$ factors:

\[
A_e = \left( a_1 \Delta Y_e', a_2 \Delta Y_e', a_3 \right) y_\tau A_0.
\]
The relevant mixing terms are then diagonalized by \( \Lambda \):

\[
\begin{pmatrix}
\begin{array}{c}
0 \\
0 \\
a_3
\end{array}
\end{pmatrix}
\begin{pmatrix}
\begin{array}{c}
a_1 \\
2
\end{array}
\end{pmatrix}
\begin{pmatrix}
\begin{array}{c}
a_1 \ell_1 \\
0 \\
a_2 - a_3
\end{array}
\end{pmatrix}
\begin{pmatrix}
\begin{array}{c}
a_1 \ell_2 \\
0 \\
2 - a_3 c e
\end{array}
\end{pmatrix}
\begin{pmatrix}
\begin{array}{c}
y_{\tau} A_0 \\
0 \\
0
\end{array}
\end{pmatrix},
\]

where \( \ell_1 = (y_{\tau} / y_{\tau}) \) and \( \ell_2 = (y_{\mu} / y_{\tau}) \). This implies a negligible \( LR \) contribution to the 1–2 sector, and suppressed contributions to the 1–2, 3 sectors.

### 5.2 Lepton Flavor Violation

Given the structure of the soft-breaking terms illustrated above, the leading contributions to LFV processes are induced by LL terms.

Inspired by the symmetry-breaking pattern of the squark sector analyzed in Ref. [5], and in order to simplify the discussion, we start analysing the case where the third generation of sleptons is substantially lighter than the first two. In other words, we assume the existence of an approximate cancellation in the \((3,3)\) element of the \( LL \) slepton mass matrix. Under this assumption, the leading contributions to LFV processes are dominated by the exchange of third-family sleptons.

Before analyzing the predictions of LFV rates by means of a numerical scan of the parameter space, we draw a few analytical considerations. In the limit where we assume the dominance of chargino contributions (as expected because of the larger coupling compared to neutralinos), we only need to analyze the \( LL \) mass matrix of Eq. (38). This is diagonalized by [5]:

\[
W_L^c = \begin{pmatrix}
\begin{array}{ccc}
0 & s_c e^{i \alpha_c} & -c_c s_L e^{i \gamma} e^{-i \beta_c} \\
-s_c e^{-i \alpha_c} & c_c & s_c s_L e^{i \gamma} \\
0 & s_C e^{-i \gamma} & 1
\end{array}
\end{pmatrix},
\]

where

\[
s_L e^{i \gamma} = s_{\tau} e^{-i \phi_{\tau}} + c_{\beta} = O(\epsilon).
\]

The relevant mixing terms are then

\[
\begin{align*}
\mathcal{R}_{13}^c &= -s_c s_L e^{i (\gamma - \alpha_c)}, \\
\mathcal{R}_{23}^c &= -c_c s_L e^{i \gamma}, \\
\mathcal{R}_{33}^c &= 1.
\end{align*}
\]

### Table 2. Bounds and prospects for LFV searches.

| Channel          | Bound (90\% C.L.) | Prospects |
|------------------|-------------------|-----------|
| \( B(\mu \to e\gamma) \) | \(< 2.4 \times 10^{-12} \) [24] | \( 10^{-13} \) |
| \( B(\tau \to e\gamma) \) | \(< 3.3 \times 10^{-8} \) [26] | \( 10^{-9} \) |
| \( B(\tau \to \mu\gamma) \) | \(< 4.4 \times 10^{-8} \) [26] | \( 10^{-9} \) |

Fig. 5. Correlation between \( B(\tau \to \mu\gamma) \) and \( B(\mu \to e\gamma) \) in the case of hierarchical slepton mass spectrum (\( m_{\tilde{\tau}_3} \ll m_{\tilde{\tau}_{1,2}} \)). See text for more details.

This allows us to make the approximate predictions:

\[
\left( \frac{B(\mu \to e\gamma)}{B(\tau \to \mu\gamma)} \right)^{\pm} \approx \left( \frac{m_{\mu}}{m_{\tau}} \right)^5 \frac{\Gamma_{\tau}^{\mu}}{\Gamma_{\mu}^{\tau}} \left| \frac{\mathcal{R}_{13}^c \mathcal{R}_{23}^c}{\mathcal{R}_{13}^c \mathcal{R}_{23}^c} \right|^2 \approx 5.1 s_c^2 s_L^2,
\]

\[
\left( \frac{B(\tau \to e\gamma)}{B(\tau \to \mu\gamma)} \right)^{\pm} \approx \frac{\mathcal{R}_{13}^c \mathcal{R}_{23}^c}{\mathcal{R}_{13}^c \mathcal{R}_{23}^c}^2 \approx s_c^2,
\]

which turn out to be good approximations to the full results in the limit where third generation of sleptons is light.

In our numerical simulation, we include both chargino- and neutralino-mediated contributions. We perform a complete diagonalization of the full \( 6 \times 6 \) slepton mass matrix and the \( 3 \times 3 \) sneutrino mass matrix [4]. We take the \((3,3)\) and \((6,6)\) elements in the range \( (200 \text{ GeV})^2 \)–\((1000 \text{ GeV})^2\), while we assume values between \( 5^2 \) and \( 100^2 \) times heavier for the other mass eigenvalues. The \( A_0 \) parameter is assumed to be proportional to the heavy sfermion mass with a proportionality constant in the range \([-3,3]\). The chargino soft mass is fixed to \( M_2 = 500 \text{ GeV} \), and we use gaugino unification arguments to set \( M_1 = 0.5 M_2 \). We also fix \( \tan \beta = 10 \), and \( \mu = 600 \text{ GeV} \).

The results of this numerical analysis are shown in Figure 5. In Figure 5 we show the correlation between \( B(\tau \to \mu\gamma) \) and \( B(\mu \to e\gamma) \), while on the bottom panel we show the correlation of the former with \( B(\tau \to e\gamma) \). We show the current bounds for each branching ratio with the approximate LEP bounds on chargino, stau and sneutrino masses [25].
density value one order of magnitude smaller than the respective inner contour.

Figure 6 shows that, although a small part of the parameter space is ruled out already, there exist a significant number of points that can be probed by $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ and possibly $\mu \rightarrow e$ conversion experiments in the near future. It is interesting to note that current and future sensitivities of $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ are quite comparable in constraining the model, even if the experimental sensitivity on $\tau \rightarrow \mu\gamma$ is much weaker. The fact that $\tau \rightarrow \mu\gamma$ has such an important role can easily be understood from Eq. 48, where it is clear that $\mu \rightarrow e\gamma$ receives an additional suppression due to $s_{\mu}^2$. Similar conclusions have recently been reached also in Ref. 10. On the other hand, the correlation between $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ in Figure 6 is quite different with respect what expected in various models of Minimal LFV 27, where there is no connections between quark and lepton flavor structures.

Figure 6 shows that $\tau \rightarrow e\gamma$ does not provide additional bounds on the model, as most points that can be probed by this decay mode are already ruled out by $\tau \rightarrow \mu\gamma$. Still, the Figure shows a very strong correlation, as expected from Eq. 49. This correlation could provide a very significant test of the model if it could be verified experimentally.

Finally, in order to test how these conclusions are modified if the slepton spectrum is not hierarchical, we have performed a independent scan without assuming a cancelation in the (3,3) and (6,6) entries of the slepton mass matrix. In particular, we vary all the diagonal entries in the range $(1000 \text{ GeV})^2 - (2000 \text{ GeV})^2$, while keeping all the other parameters (gaugino and chargino masses) fixed as in the previous scan. The result of this second numerical analysis are shown in Figure 7. As can be seen, in this case the correlation between $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ is quite different: the contribution of the sleptons of the first two families is not negligible in $\mu \rightarrow e\gamma$ and, as a consequence, the approximate relation Eq. 48 is no longer valid. In this framework, the recent MEG bound 24 on $\mu \rightarrow e\gamma$ provide a very severe constraint. In particular, it rules out the possibility of visible effects in the $\tau \rightarrow \mu\gamma$ case. The correlation between $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ is not modified with respect to Figure 6 but both modes are beyond the experimental reach after imposing the $\mu \rightarrow e\gamma$ bound.

6 Conclusions

We have proposed an ansatz for the neutrino mass matrix and the charged lepton Yukawa coupling based on a minimal breaking of the $U(3)^5$ flavor symmetry, consistent with the $U(2)^5$ breaking pattern of the quark Yukawa couplings discussed in Ref 8. The key hypothesis that allows us to relate the non-hierarchical neutrino sector to the Yukawa sector is the assumption of a two-step breaking structure in the neutrino case: a leading breaking of the maximal flavor symmetry, $U(3)_l \times U(3)_e$, giving rise to a fully degenerate neutrino spectrum, followed by a sub-leading hierarchical breaking similar to the one occurring in the Yukawa sector. According to this hypothesis, the large 2-3 mixing in the neutrino sector arises as a small perturbation of an approximately degenerate spectrum.

On the other hand, the ratio between $\theta_{13}$ and $\theta_{32}$ is predicted to be of the order of the Cabibbo angle, similarly to the quark sector, in good agreement with the recent DayaBay result.

As we have shown, our framework is able to reproduce all the neutrino oscillation parameters without particular tuning of the free parameters. The neutrino masses are predicted to be almost degenerate: the sum of all the eigenvalues is expected to be around $0.2 - 1 \text{ eV}$, close to the present cosmological bounds, and the $\delta\theta_{13}$ parameter $m_{12}$ is expected in the range $0.02 - 0.4 \text{ eV}$, observable in next generation of experiments.

Our framework can naturally be implemented in supersymmetric extensions of the SM and, more explicitly, within the well-motivated set-up with heavy masses for the first two generations of squarks. We have analyzed the consequences of this flavor-symmetry breaking ansatz in the supersymmetric case, assuming a split family spectrum also in the slepton sector. The model can satisfy the existing constraints on LFV in charged leptons, with the
most significant bounds coming from $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$. For third-generation sleptons masses below 1 TeV both decay modes are expected to be within the reach of future experimental searches.

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