Model-Based Deep Learning of Joint Probabilistic and Geometric Shaping for Optical Communication

Vladislav Neskorinuk1,3, Andrea Carnio2, Domenico Marsella2, Sergei K. Turitsyn3, Jaroslav E. Prilepsky3, Vahid Aref1
1Nokia, 70469 Stuttgart, Germany 2Nokia, Vimercate 20871, Italy 3Aston Institute of Photonic Technologies, Aston University, B4 7ET Birmingham, UK v.neskorinuk@aston.ac.uk

Abstract: Autoencoder-based deep learning is applied to jointly optimize geometric and probabilistic constellation shaping for optical coherent communication. The optimized constellation shaping outperforms the 256 QAM Maxwell-Boltzmann probabilistic distribution with extra 0.05 bits/4D-symbol mutual information for 64 GBd transmission over 170 km SMF link.

© 2022 The Author(s)

1. Introduction

Geometric (GS) and probabilistic (PS) constellation shaping, i.e. the optimization of, respectively, locations and occurrence probabilities of the constellation points, can improve considerably the transmission rate of coherent fiber-optic communication systems. While for linear channels, the family of Maxwell-Boltzmann (MB) distributions leads to the optimal PS [1], finding the optimal shaping for nonlinear-dispersive optical fiber channels is a subtle problem [2]. Usually, a time-consuming numerical statistical modeling is used for the optimization of either GS [3], PS [2], or constrained joint geometric and probabilistic shaping (JS) [4].

End-to-end (E2E) learning [5] offers a different solution to this problem with a gradient-based optimization of constellation shaping which does not require any constraints on the spatial or probabilistic distributions of constellation points. Nonetheless, the previous works on E2E learning of coherent systems were focused on the optimization of GS [6–8]. Here, we apply the E2E learning technique proposed in [9] to jointly optimize geometric and probabilistic shaping in simulation for high symbol-rate coherent transmission over single-span fiber link. Using training through the nonlinear interference noise (NLIN) model [10] in the numerical simulation, we demonstrate that the optimized JS constellation leads to a mutual information (MI) gain over the MB PS of a quadrature-amplitude modulation (QAM) constellation.

2. Joint Learning of Geometric and Probabilistic Shaping

In an autoencoder-based E2E learning, the trainable transmitter (TX), receiver (RX) and the channel are implemented by cascades of neural networks (NN) which are jointly trained to best reproduce the TX inputs from the RX outputs (Fig. 1) [5]. Following [9], the considered E2E architecture starts with the trainable TX sampling a symbol sequence X. We implemented the TX utilizing a trainable sampler based on the Gumbel-Softmax trick [9]. The TX has two sets of trainable parameters: the set of the constellation symbols location, denoted by $S = \{ s_1, s_2, \ldots, s_N \}$, and their occurrence probabilities, denoted by $P_S = \{ p_1, p_2, \ldots, p_N \}$. Each symbol location $s_k$ and probability $p_k$ is trained separately with the only constraint on the average power: $\sum_k p_k |s_k|^2 = 1$.

Next, a pre-defined stochastic channel model, $p(y_i|x_i)$, maps the transmitted symbols $X = \{ x_i \}$, $x_i \in S$ to the received ones, $Y = \{ y_i \}$. We modeled the channel by a nonlinear interference noise (NLIN) model [10]. In details, we approximated the symbol distortion as a Gaussian noise with the variance $\sigma^2 = \sigma_{ASE}^2 + P^4 (\chi_0 + \chi_1 (\mu_4 - 2) + \chi_2 (\mu_6 - 9 \mu_4 + 12) + \chi_3 (\mu_4 - 2)^2)$, where $\sigma_{ASE}^2$ is the variance of amplified spontaneous emission (ASE) noise injected by optical amplifiers (OAs), $P$ is the average launch power, $\mu_4$, $\mu_6$ are the 4th-order

![Fig. 1: The principal scheme of an end-to-end learning algorithm.](Image)
Fig. 2: Numerical estimations of (a) mutual information (MI) over 4D symbols, (d) effective signal-to-noise ratios (SNRs), 
and 6th-order standardized moments of the input sequence $X$, and $X_0, X_1, X_2, X_3$, are the modulation-format-
independent NLIN model coefficients. Then, RX maps each received symbol $y_i$ to the vector of posterior probabilities $p(s_k|y_i)$ for all $s_k \in S$, estimating how likely $s_k$ was sent when $y_i$ is received. Our RX estimates the posteriors in closed form via Bayesian rule from the conditional density defining the channel $p(y_i|s_k)$ and the symbol probabilities $P_S$. As shown in [9], the E2E MI, $I(X;Y)$, can be characterized in terms of $p(s_k|y_i)$ and $P_S$. To numerically maximize MI, the batch gradient descent algorithm with Adam optimizer is used to optimize the training parameters, i.e. $S$ and $P_S$. The gradients are computed using back-propagation through the autoencoder.

To assess the learned constellations, we generated two independent random sequences of constellation symbols sampled according to the optimized $P_S$. These sequences are the transmission data over two polarizations. The transmission is simulated using a precise Manakov equations split-step (MSS) solver. Finally, the E2E mutual information of received symbols are calculated using Gaussian kernel density estimator which numerically estimates the MI with high accuracy.

3. Results and Conclusions

In order to obtain a strong dependence of the nonlinear distortion on the modulation format a single-span link was studied [2]. Particularly, we considered a dual-polarized single-channel 64 G Bd transmission over a 170 km standard single-mode fiber (SSMF) span, followed by an ideal OA.

The effectiveness of the E2E-learned 256-symbol JS constellation compared with MB-shaped 256QAM and unshaped 256QAM was evaluated in terms of MI and reported in Figure 2. The optimization process of E2E-learned and MB constellations was done separately for each power level considering the NLIN channel model, and the MSS solver, respectively. The E2E-learned constellation provided a peak-to-peak MI gain of 0.05 bits/4D over MB-shaped 256QAM and 0.45 bits/4D over unshaped 256QAM. The learned JS offers an effective trade-off between linear regime performance and tolerance to nonlinear distortion: in linear regime, similarly to the MB PS, it leads to a better MI performance than the unshaped constellation, while in the nonlinear regime it produces higher effective SNR (Fig. 2d) than the MB PS, due to lower moments $\mu_2, \mu_4$ (Figs. 2e, 2f). Note that a small mismatch between the optimal launch powers in terms of SNR and MI curves is because the MI is computed by density estimator in complex domain but SNR computation neglects correlations in complex domain.

Acknowledgements: This project has received funding from EU Horizon 2020 program under the Marie Skłodowska-Curie grant agreement No. 766115 (FONTE). SKT acknowledges the support of EPSRC project TRANSNET.

References

1. F. R. Kschischang et al. “Optimal nonuniform signaling for Gaussian channels”. IEEE Trans. Inf. Theory 39.3 (1993), pp. 913-929.
2. T. Fehenberger et al. “On probabilistic shaping of quadrature amplitude modulation for the nonlinear fiber channel”. IEEE JLT 2020.
3. B. Chen et al. “Increasing achievable information rates via geometric shaping”. EOCOC 2020, pp. W4F4.
4. J. Cai et al. “Performance comparison of probabilistically shaped QAM formats and hybrid shaped APSK formats with coded modulation”. IEEE JLT 38.12 (2020), pp. 3280–3288.
5. T. O’Shea et al. “An introduction to deep learning for the physical layer”. IEEE Trans. Cogn. Commun. Netw. 3.4 (2017), pp. 563–575.
6. R. T. Jones et al. “End-to-end learning for GMI optimized geometric constellation shape”. ECOC 2019, pap. W1.B.3.
7. T. Uhrmann et al. “Deep-learning Autoencoder for Coherent and Nonlinear Optical Communication”. arXiv:2006.15027v2
8. J. Song et al. “End-to-end Autoencoder for Superchannel Transceivers with Hardware Impairment”.
9. M. Stark et al. “Joint learning of geometric and probabilistic constellation shaping”. arXiv:1906.07748v3 (2019).
10. R. Dar et al. “Accumulation of nonlinear interference noise in fiber-optic systems”. In: Opt. Express 22.12 (2014), pp. 14199–14211.
11. D. P. Kingma et al. “Adam: A method for stochastic optimization”. arXiv:1412.6980 (2014).