Exploring physical properties of compact stars in $f(R,T)$–gravity: An embedding approach

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Abstract: Solving field equations exactly in $f(R,T)$ gravity is one of the difficult task. To do so, many authors have adopted different methods such as assuming both the metric functions, an equation of state (EoS) and a metric function etc. However, such methods may not always lead to well-behaved solutions and thereby rejection of the solutions may happen after complete calculations. Indeed, very recent works on embedding class one methods suggested that the chances of arriving at the well-behaved-solution is very high thereby inspired us to used it. In class one approach, we have to ansatz one of the metric potentials and the other can be obtain from the Karmarkar condition. In this paper, we are proposing new class one solution which is well-behaved in all physical points of view. We have analyzed the nature of the solution by tuning the $f(R,T)$–coupling parameter $\chi$ and found that the solution results into stiffer EoS for $\chi = -1$ than $\chi = 1$. This is because for lesser values of $\chi$, velocity of sound is more, higher $M_{\text{max}}$ in $M-R$ curve and the EoS parameter $\omega$ is larger. The solution satisfy the causality condition, energy conditions, stable and static under radial perturbations (static stability criterion) and in equilibrium (modified TOV-equation). The resulting $M-R$ diagram from this solution is well fitted with observed values of few compact stars such as PSR J1614-2230, Vela X-1, Cen X-3 and SAX J1808.4-3658. Therefore, for different values of $\chi$, we have predicted the corresponding radii and their respective moment of inertia from the $M-I$ curve.

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1 Introduction

One of the greatest challenges in modern cosmology is the late-time comportment of our Universe. We use several sets of high-precision observational data gathered from various cosmic sources such as Cosmic Microwave Background (CMB) [1–3], SuperNova type Ia (SNe Ia) [4–8], Large Scale Structure (LSS) [9–11], Weak Lensing (WL) [12] and Baryon Acoustic Oscillations (BAO) [13], as standard candles which discovered that our Universe is undergoing an accelerated expansion. The fascinating part is that the Universe expansion is believed to be made from an obscure energy named dark energy, which is around two-thirds of the complete energy budget of the Universe. On account of the puzzling nature of the dark sector (Dark Energy (DE) and Dark Matter (DM); DE yields a late-time speeding up of the cosmological foundation while DM carries on as an undetectable residue matter supporting the procedure of gravitational clustering.), and the way that their existence is construed only through their gravitational impacts, it is entirely to verify whether there is a need to contemplate these elements; Specifically, regardless of whether there is any deflection from ordinary General Relativity (GR) on enormous scales. By utilizing the Einsteinâ€™s Field
Equations (EFEs), there exists an accelerated expansion portrayed by a positive constant, which is extremely little, in the edge work of GR, named the $\Lambda$–CDM model \cite{14}. In the present situation, this little positive constant is related with dark energy in void space, which is utilized to clarify the ongoing case of coeval accelerating expansion of the Universe against the alluring impacts of gravity. In widespread, there are two main methodologies that could supply a surroundings for a theoretical clarification of the accelerated expansion of the Universe. The first methodology be made up of altering the matter substance of the Universe by introducing a DE area, beginning either with a phantom field, a standard scalar field, or with the mixture of the two fields in a unified model and then progressing towards more complicated scenarios; see \cite{15,16} and references therein for more subtleties and audits. The second methodology consists in modifying the gravitational area itself (see e.g. \cite{17–20}), which can likewise be well-respected as one of the great candidate for clarifying the accelerated expansion of the Universe. Prompted by this basic hypothesis that at great astrophysical and cosmological scales, usual GR may not depict effectively the dynamical evolution of the Universe. To manage this problem, several endeavors have been made, among which gravity theories broadening GR have aroused much enthusiasm over the previous decades. With regards to modified gravity theories, a geometric depiction for DM can likewise be well-respected as one of the great candidate for clarifying the accelerated expansion of the Universe.

Different techniques have been suggested up until now, so as to modify the gravitational action (see \cite{32} for more details), offering rise to various classes of alternative gravitational theories. In this regard, some leading models incorporate $f(R)$ gravity \cite{18,20,33–52}, $f(T)$ gravity \cite{53–57}, $f(T)$ gravity \cite{58,59}, $f(G)$ gravity \cite{63–65}, $f(R,T)$ gravity \cite{66–81}, $f(T,T)$ gravity \cite{82–87}, $f(R,G)$ gravity \cite{88}, where $R$, $T$ and $G$ are Ricci’s scalar, trace of stress-energy tensor and Gauss-Bonnet scalar respectively. In $f(R)$ gravity theory, to a greater extent general expression of the Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}$ is utilized instead of $R$ though $f(T)$ gravity is a general type of teleparallel gravity. Spurred by the achievement of cosmological constant as a straightforward and great candidate of DE and DM, a few matter field is additionally combined with the expression of the Ricci scalar $R$ in the action geometry sector in a few alternatives theories of gravitation ($f(R,\mathcal{L}_m)$ theory, where $\mathcal{L}_m$ is the matter Lagrangian density).

In the context of interest of including certain components of matter into the geometry of action, the $f(R,T)$ theory was suggested by \cite{89} which, generally, has been a fascinating framework for studying acceleration models. The $f(R,T)$ gravity theory generalizes $f(R)$ gravity theories by introducing the trace of stress-energy tensor added to the Ricci scalar. The validation for the reliance on $T$ originates from enlistments emerging from a few exotic fluid or quantum impacts. Actually, this enlistment perspective encompasses or connects to the recommendations mentioned, for instance, geometrical curvature prompting matter, a geometrical portrayal of physical powers and a geometrical source for the matter substance of the Universe. In Ref. \cite{89}, the field equations of a few specific models are
introduced, and especially, scalar field models $f(R, T^\Phi)$ are examined in detail with a concise account of their cosmological ramifications. Likewise, the motion equation of the test particle and the Newtonian boundary of this equation are additionally studied in Ref. [89]. Until nowadays, the problems which have been explored alongside this alternative theory are the thermodynamics [90–92], the energy conditions [93], anisotropic cosmology [94, 95], the cosmology where the portrayal uses a helper scalar field [96], reconstruction of some cosmological models [97], the wormhole solution [98, 99], the scalar perturbations [100] and some other relevant aspects [81, 101–103]. Additionally, a more generalization of this theory has been suggested lately in Refs. [104, 105].

Consequently, it is not reasonable to affirm or to refute such theories dependent on the outcomes of cosmology and contrast them with the observational datum, for instance, the challenge of the viability of $f(R, T)$ as an alternative modification of gravity that discussed in [106]. In any case, to set up an agreeable theory of gravitation, it is essential to consider at the astrophysical level, for instance by utilizing the relativistic stellar structures. A few contentions for these modified theories originate from the presumption that relativistic stellar structures in the powerful gravitational sector could distinguish usual gravity from its generalizations. In the scenario of $f(R, T)$ gravity, an enormous number of contributions on the evolution of compact stellar structures are accessible in various literature. In this context, hydrostatic equilibrium structure of strange stars and neutron stars have been investigated [75]. The configuration of compact stellar structures in $f(R, T)$ gravity was explored latterly in Refs. [69, 70, 73, 107–109], though gravastars (GRAVitational VAcuum STARS) resolution has been gotten in [110].

To understand the inside geometry and evolutionary phases of relativistic stellar structures, the distribution of anisotropic fluids acts an important role. As the compact stellar systems have ultra-dense cores and their density surpass the nuclear density, consequently, pressure ought to be anisotropic in the inside of compact stellar systems [111]. In anisotropic relativistic astrophysical systems, it is seen that pressure is apportioned into radial and tangential components. In this specific circumstance, numerous researchers explored attributes of compact and dense stellar systems involving anisotropic fluid structure. [112] first offer the anisotropy concept for static spherically symmetric structures and subsequently, numerous astrophysicists have included this parameter for the compact stellar structures modeling. As a compact stellar structure is shaped with very dense matter, a very-high magnetic domain is related with it due to the magnetic flux preservation. The enormous magnetic domain may produce pressure anisotropy interior the stellar structure [113]. The stage progress at higher densities may likewise prompt anisotropy [114, 115]. The anisotropy aspect furnishes us with one more factor to be incorporated in EFEs and prompting progressively realistic models of the spherically symmetric compact stellar structures. If the anisotropy parameter is positive an outward repulsive force will be applied on the stellar structure in this way, making it increasingly compact and steady. The primary explanation behind emerging anisotropy in a $f(R, T)$ gravity theory model could be the anisotropic type of the two fluid system without interaction. Also, it ought to be emphasized that from a quantum point of view, the $T$–dependent Lagrangian might be identified with the formation of particles which normally portray the presence of bulk viscosity and other flaws in the alluded fluid. Conse-
quently, we suggest the models where transverse pressure surpasses the radial pressure. To provide the accurate solutions of EFEs, two dissimilar methodologies are frequently follow: it is possible that we learn the space-time metric elements first, afterward establish the matter profile, or we first portray the material features in terms of certain state equations i.e. the relationship in the form $p = p(\rho)$ and subsequently investigate the metric potentials. Whereas searching a well-defined solution the constants arising in the solution perform an important role. The slight difference in the values of the constants can distance the stellar structure from its position of equilibrium. Subsequently, several authors have been studied compact stellar structure models employing the Karmarkar condition. At the moment when the Karmarkar condition [116] is exploited on the gravitational components $g_{rr}$ and $g_{tt}$, the issue of establishing the spatio-temporal elements obtain make simple to a great area and the 4-dimensional Riemannian spatio-temporal variety can be described graphically in the 5-dimensional pseudo-Euclidean spatio-temporal variety without any change in its intrinsic characteristics. The solutions fulfilling Karmarkar conditions alongside the condition suggested by [62] are well-known as embedding class one solutions. It is intriguing to observe that the internal solution of [117] is the only structure of bounded neutral matter with a disappearing anisotropy parameter fulfilling the Karmarkar condition. For a more in-depth survey, one may seek advice from alluded literature [61, 118, 119] where authors have clearly involved and examined the impacts of the procedure of embedding of 4-dimensional Riemannian spatio-temporal variety into the 5-dimensional pseudo-Euclidean spatio-temporal variety in the scenario of GR and alternative gravity.

In this paper, we study anisotropic spherically symmetric solutions in the domain of alternative gravity theories, especially, $f(R, T)$ theory of gravity. In this respect, we have considered that the matter Lagrangian density $L_m$ (defined as $L_m = -\mathcal{P} = (p_r + 2p_t)/3$ i.e the isotropic pressure) can be asserted as the linear function of the Ricci scalar $R$ and the trace of the energy-momentum tensor $T$, i.e. $f(R, T) = R + 2\chi T$, where $\chi$ is a dimensionless coupling constant, in order to depict the global set of modified EFEs for the anisotropic matter distribution. We also consider the embedding class I procedure, by means for embedding within 4-dimensional space-time into a 5-dimensional flat Euclidean space to obtain a complete space-time representation interior the relativistic stellar system. Moreover, for investigating physical availability of the acquired solutions, we have analyzed four different compact stars namely PSR J1614-2230, Vela X-1, Cen X-3 and SAX J1808.4-3658 linked physical parameters analytically and graphically. Accordingly, the familiar Darmois-Israel [120, 121] coordinating conditions can be used to calculate all the physical and constant ingredients of the stellar system.

The paper is organized as follows: Beginning with a brief introduction in Sec. 1, we make a review of the concept $f(R, T)$– gravity theory in Sec. 2, next in Sec. 3, the basic EFEs for anisotropic matter distributions in $f(R, T)$–gravity is described. In Sec. 4 we will show the Karmarkar condition well-known as embedding class one solution, then in Sec. 5 the complete stellar system under embedding class one technique in the arena of $f(R, T)$–gravity is gained and its thermodynamic description is given. In Sec. 6 we will analyze the new solutions through various physical tests such as hydrostatic equilibrium, causality condition, stability factor, adiabatic index and stability, static stability criterion

and energy conditions. In Sec. 7 we coordinate the acquired stellar system with the outside spacetime given by Schwarzschild metric, so as to get the constant parameters. Furthermore, the stiffness of EoS, \( M - R \) and \( I - M \) diagram are discussed in Sec. 8. Finally, we conclude our investigation with a short discussion of the result in Sec. 9.

2 Concepts of \( f(R,T) \)-gravity

In the Einstein-Hilbert action, if one replace the Ricci scalar \( R \) by a function of \( R \) and the trace of stress-energy tensor \( T \) we arrived at the modified action in \( f(R,T) \)-gravity as

\[
S = \frac{1}{16\pi} \int f(R,T) \sqrt{-g} \, d^4x + \int L_m \sqrt{-g} \, d^4x
\]

(2.1)

where \( \det(g_{\mu\nu}) = g \). The source term of the matter Lagrangian density \( L_m \) defines a stress tensor as

\[
T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_m)}{\delta g^{\mu\nu}}.
\]

(2.2)

Following [89] approach, the Eqs. (2.2) reduce to

\[
T_{\mu\nu} = g_{\mu\nu} L_m - 2 \frac{\partial L_m}{\partial g^{\mu\nu}}.
\]

(2.3)

Variation of the action w.r.t \( g_{\mu\nu} \) implies the field equations

\[
(R_{\mu\nu} - \nabla_{\mu} \nabla_{\nu}) f R(R,T) + g_{\mu\nu} \Box f R(R,T) - \frac{1}{2} f(R,T) g_{\mu\nu} = 8\pi T_{\mu\nu} - f T(R,T) \left( T_{\mu\nu} + \Theta_{\mu\nu} \right),
\]

(2.4)

provided \( f R(R,T) = \partial f(R,T)/\partial R \) and \( f T(R,T) = \partial f(R,T)/\partial T \). The \( \nabla_{\mu} \) denotes covariant derivative while the box operator \( \Box \) is defined by

\[
\Box \equiv \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left( \sqrt{-g} \, g^{\mu\nu} \frac{\partial}{\partial x^\nu} \right) \quad \text{with} \quad \Theta_{\mu\nu} = g^{\alpha\beta} \frac{\partial T_{\alpha\beta}}{\partial g^{\mu\nu}}.
\]

(2.6)

The conservation equation [122] yields

\[
\nabla^\mu T_{\mu\nu} = \frac{f T(R,T)}{8\pi - f T(R,T)} \left[ (T_{\mu\nu} + \Theta_{\mu\nu}) \nabla^\mu \ln f T(R,T) + \nabla^\mu \Theta_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \nabla^\mu T \right].
\]

(2.5)

Therefore, \( \nabla^\mu T_{\mu\nu} \neq 0 \) implies the conservation equation no longer holds in \( f(R,T) \) theory. By using Eq. (2.3), the tensor \( \Theta_{\mu\nu} \) is found to be

\[
\Theta_{\mu\nu} = -2 T_{\mu\nu} + g_{\mu\nu} L_m - 2 g^{\alpha\beta} \frac{\partial^2 L_m}{\partial g^{\mu\nu} \partial g^{\alpha\beta}}.
\]

(2.6)

To complete the field equations, we assumed an anisotropic fluid source

\[
T_{\mu\nu} = (\rho + p_r) u_{\mu} u_{\nu} - p_t g_{\mu\nu} + (p_r - p_t) g_{\mu\nu},
\]

(2.7)
with \( u_\nu \) is the four velocity, satisfying \( u_\mu u^\mu = -1 \) and \( u_\nu \nabla^\mu u_\mu = 0 \), \( \rho \) is the matter density, \( p_r \) and \( p_t \) are the radial and transverse pressures. If we defined the isotropic pressure as \( -\mathcal{P} = \mathcal{L}_m = (p_r + 2p_t)/3 \) \cite{89}, then (2.6) reduces to

\[
\Theta_{\mu\nu} = -2T_{\mu\nu} - \mathcal{P} g_{\mu\nu}.
\] (2.8)

Further, the functional \( f(R, T) \) is chosen to be \( f(R, T) = R + 2\chi T \) \cite{89}, where \( \chi \) the coupling constant. Now the field equations (2.5) takes the form

\[
G_{\mu\nu} = 8\pi T_{\mu\nu} + 2\chi (T_{\mu\nu} + \mathcal{P} g_{\mu\nu}).
\] (2.9)

For \( \chi = 0 \), one can recover the general relativistic field equations. The linear expression in \( f(R, T) \) solved many cosmological and astrophysical related problems. By substituting \( f(R, T) = R + 2\chi T \) and (2.8) in Eq. (2.5), we obtain

\[
\nabla^\mu T_{\mu\nu} = -\frac{\chi}{2(4\pi + \chi)} \left[ g_{\mu\nu} \nabla^\mu T + 2 \nabla^\mu (\mathcal{P} g_{\mu\nu}) \right].
\] (2.10)

Therefore, the conservation equation in Einstein’s gravity can be recovered for \( \chi = 0 \).

### 3 Field equations in \( f(R, T) \)–gravity

To determine the field equations we assume an interior spacetime of the form

\[
ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2).
\] (3.1)

For the spacetime given in (3.1), the field equation (2.9) becomes

\[
8\pi \rho_{\text{eff}} = e^{-\lambda} \left( \frac{\nu'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2}
\] (3.2)

\[
8\pi p_{\text{reff}} = e^{-\lambda} \left( \frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2}
\] (3.3)

\[
8\pi p_{t\text{eff}} = \frac{e^{-\lambda}}{4} \left( 2\nu'' + \nu'^2 + \frac{2(\nu' - \lambda')}{r} - \nu' \lambda' \right)
\] (3.4)

where,

\[
\rho_{\text{eff}} = \rho + \frac{\chi}{24\pi} \left( 9\rho - p_r - 2p_t \right)
\]

\[
p_{\text{reff}} = p_r - \frac{\chi}{24\pi} \left( 3\rho - 7p_r - 2p_t \right)
\]

\[
p_{t\text{eff}} = p_t - \frac{\chi}{24\pi} \left( 3\rho - p_r - 8p_t \right).
\]
Now the decoupled field equations (3.2)-(3.4) are as follows:

\[
\begin{align*}
\rho &= \frac{e^{-\lambda}}{48\pi^2(\chi + 2\pi)(\chi + 4\pi)} \left[ r\lambda' \left\{16(\chi + 3\pi) - r\chi'\right\} + 16(\chi + 3\pi)(e^\lambda - 1) \\
&\quad + r\chi \left\{2r\chi'' + \nu' \left(r\nu' + 4\right)\right\} \right] \\
p_r &= \frac{e^{-\lambda}}{48\pi^2(\chi + 2\pi)(\chi + 4\pi)} \left[ r\chi' \left(r\nu' + 8\right) - 2r\chi'\nu'' + \nu' \left(20\chi - r\nu' + 48\pi\right) \right] \\
&\quad - 16(\chi + 3\pi)(e^\lambda - 1) \\
p_t &= \frac{e^{-\lambda}}{48\pi^2(\chi + 2\pi)(\chi + 4\pi)} \left[ r\chi' \left(5\chi + 12\pi\right)\nu' + 4(\chi + 6\pi) \right] + 2r(5\chi + 12\pi)\nu'' \\
&\quad + r(\chi + 12\pi)\nu' \left(8\chi + 3\pi\nu'\right) + 8\chi (e^\lambda - 1) \right].
\end{align*}
\] (3.5) (3.6) (3.7)

To solve the field equations, we need to assume some of the physical quantities which satisfy a strict physical constraints.

### 4 Concepts of embedding class one

Kasner coordinate transformation [123] shows that the exterior Schwarzschild vacuum is class two spacetime. Using the transformations given below

\[ X = \frac{R \sin t}{\sqrt{R^2 + 16m^2}}, \quad Y = \frac{R \cos t}{\sqrt{R^2 + 16m^2}}, \quad Z = \int \sqrt{1 + \frac{256m^4}{(R^2 + 16m^2)^3}} \, dR, \]

where \( R = \sqrt{8m(r - 2m)} \) and \( r^2 = x^2 + y^2 + z^2 \), the well-known Schwarzschild vacuum reduces to

\[ ds^2 = -dx^2 - dy^2 - dz^2 + dX^2 + dY^2 - dZ^2. \] (4.1)

This means that the Schwarzschild exterior spacetime can be embedded in six dimensional pseudo-Euclidean manifold. This method was extended for general four dimensional spacetime of the form (3.1) by [124]. The chosen coordinate transformations were

\[ z_1 = ke^{\nu/2} \cosh \left(\frac{t}{k}\right), \quad z_2 = ke^{\nu/2} \sinh \left(\frac{t}{k}\right), \quad z_3 = f(r), \]

\[ z^4 = r \sin \theta \cos \phi, \quad z_5 = r \sin \theta \sin \phi, \quad z_6 = r \cos \theta, \]

which transforms (3.1) into

\[ ds^2 = (dz_1)^2 - (dz_2)^2 - (dz_3)^2 - (dz_4)^2 - (dz_5)^2 - (dz_6)^2, \] (4.2)

with \( |f'(r)|^2 = \mp \left[-(e^\lambda - 1) + k^2e^\nu\nu'^2/4\right] \).

Equation (4.2) also implies that the interior line element (3.1) can be embedded in six dimensional pseudo-Euclidean space, however, if \( (dz_3)^2 = |f'(r)|^2 = 0 \), it can be embedded in 5-D Euclidean space i.e.

\[ |f'(r)|^2 = \mp \left[-(e^\lambda - 1) + k^2e^\nu\nu'^2/4\right] = 0, \] (4.3)
which implies

\[ e^\lambda = 1 + \frac{k^2}{4} \nu'^2 e^\nu \]  

(4.4)
i.e. (4.2) will reduce to

\[ ds^2 = (dz_1)^2 - (dz_2)^2 - (dz_4)^2 - (dz_5)^2 - (dz_6)^2, \]

(4.5)
a class one spacetime.

The same condition (4.4) was originally derived by [116] in the form of components of Riemann tensor as

\[ R_{r\theta r\theta} = R_{r\theta r\phi}R_{\theta\phi\theta} + R_{r\theta \theta}R_{r\phi \phi}. \]

(4.6)

[62] pointed out that Karmarkar condition is only the necessary condition to become a class one, they discovered the sufficient condition as \( R_{\theta\phi\theta\phi} \neq 0 \). Hence, the necessary and sufficient condition to be a class one is to satisfy both Karmarkar and Pandey-Sharma conditions. In terms of the metric components, (4.6) can be written as

\[ \frac{2\nu''}{\nu'} + \nu' = \frac{\lambda' e^\lambda}{e^\lambda - 1} \]

(4.7)

which on integration one gets the

\[ e^\nu = \left( A + B \int \sqrt{e^\lambda - 1} \, dr \right)^2. \]

(4.8)

where \( A \) and \( B \) are constants of integration. In general relativity there is no class one exterior as the existing Schwarzschild’s exterior itself is a class two spacetime. It has been shown that the two class one isotropic-neutral solutions in general relativity i.e. Schwarzschild uniform density model and Kohler-Chao infinite boundary model are the only two possible solutions [125]. However, [126] have also shown that these two isotropic-neutral solutions still exist in \( f(R,T) \)–theory as well. Although, the constant density model in GR can have decreasing density in \( f(R,T) \)–gravity and the infinite boundary Kohler-Chao solution can have finite boundary where the pressure vanishes, due to the \( f(R,T) \)–term.

Figure 1. Variation of metric functions and density with radial coordinate for Vela X-1 (\( M = 1.77 \pm 0.08 \, M_\odot \), \( R = 9.56 \pm 0.08 \, km \)) with \( b = 0.0005/\text{km}^2 \) and \( c = 0.000015/\text{km}^4 \).
χ = -1 Red
χ = -0.5 Blue
χ = 0 Black
χ = 0.5 Orange
χ = 1 Green

Figure 2. Variation of pressures and anisotropy with radial coordinate for Vela X-1 (M = 1.77 ± 0.08 M⊙, R = 9.56 ± 0.08 km) with b = 0.0005/km² and c = 0.000015/km⁴.

5 Embedding class one background in \( f(R, T) \)-gravity

Solving the field equations in \( f(R, T) \)-gravity exactly is a challenging task because of the highly coupled non-linear differential equations. To simplify the problem, we have adopted the embedding class one approach, which is application to all four dimensional spacetime. Here, we propose a new metric function

\[ e^\lambda = 1 + ar^2e^{br^2+cr^4}. \] (5.1)

When ansatz \( e^\lambda \), one must keep in mind that it must be increasing function of radial coordinate and unity at the center. Using (5.1) in (4.8), we get

\[ e^\nu = \left[ A + \frac{B}{\sqrt{2c}} \sqrt{ae^{br^2+cr^4}} F\left(\frac{2cr^2 + b}{2\sqrt{2c}}\right)\right]^2 \] (5.2)

where \( F(x) \) is the Dawson’s integral defined by

\[ F(x) = e^{-x^2} \int_0^x e^{\tau^2} d\tau = \frac{\sqrt{\pi}}{2} e^{-x^2} \text{erfi}(x). \]

Here \( \text{erfi}(x) \) is the usual imaginary error function. The variations of the metric functions are shown in Fig. 1 (left).

Plugging the metric functions into the field equations (3.5)-(3.7), one can write

\[ \rho = \frac{\sqrt{ae^{br^2+cr^4}}}{6(\chi + 2\pi)(\chi + 4\pi)f_3(r)} \left\{ (\chi + 3\pi)f_2(r) F\left(\frac{2cr^2 + b}{2\sqrt{2c}}\right) + \sqrt{c}f_1(r) \right\} \] (5.3)

\[ p_r = \frac{\sqrt{ae^{br^2+cr^4}}}{6(\chi + 2\pi)(\chi + 4\pi)f_3(r)} \left\{ \sqrt{c}f_4(r) + 2\sqrt{2c}bf_5(r) \right\} e^{br^2+cr^4} F\left(\frac{2cr^2 + b}{2\sqrt{2c}}\right) \] (5.4)
\[ \Delta = \frac{r^2 \sqrt{ae^{br^2 + cr^4}}}{2(\chi + 4\pi)} f_3(r) \left( \frac{ar^2 e^{br^2 + cr^4} + 1}{2\sqrt{2c}} \right)^2 \left[ F \left( \frac{2cr^2 + b}{2\sqrt{2c}} \right) \sqrt{2aBe^{br^2 + cr^4}} \right. \\
+ 2\sqrt{c} \left( A \sqrt{ae^{br^2 + cr^4}} - B \right), \] (5.5)

\[ p_t = p_r + \Delta. \] (5.6)

where,

\[ f_1(r) = 4A(\chi + 3\pi) \left( ar^2 e^{br^2 + cr^4} + 2br^2 + 4cr^4 + 3 \right) \sqrt{ae^{br^2 + cr^4}} \]
\[ + B\chi \left( 2ar^2 e^{br^2 + cr^4} + br^2 + 2cr^4 + 3 \right) \]
\[ f_2(r) = 2\sqrt{2aBe^{br^2 + cr^4}} \left( ar^2 e^{br^2 + cr^4} + 2br^2 + 4cr^4 + 3 \right) \]
\[ f_3(r) = \sqrt{2B} \sqrt{ae^{br^2 + cr^4}} F \left( \frac{2cr^2 + b}{2\sqrt{2c}} \right) + 2A\sqrt{c} \]
\[ f_4(r) = B \left[ \chi \left( 10ar^2 e^{br^2 + cr^4} - br^2 - 2cr^4 + 9 \right) + 24\pi \left( ar^2 e^{br^2 + cr^4} + 1 \right) \right] \]
\[ - 4A\sqrt{ae^{br^2 + cr^4}} \left[ 3\pi \left( 1 + ar^2 e^{br^2 + cr^4} \right) - \chi \left( 9ar^2 e^{br^2 + cr^4} + 2cr^4 \right) \right] \]
\[ f_5(r) = r^2 \chi \left( b - ae^{br^2 + cr^4} + 2cr^4 \right) - 3\pi \left( ar^2 e^{br^2 + cr^4} + 1 \right). \]

The variations of the density, pressures, anisotropy and EoS parameter are shown in Figs. 1 (right), 2, 3 (left).

6 Physical analysis on the new solution

Any new solutions must be analyze through various physical tests. After satisfying all the physical constraints one can proceed further for modeling physical systems.

6.1 Hydrostatic equilibrium

All the physical compact stars are believed to be in equilibrium state. Such equilibrium state can be tested by using equation of hydrostatic equilibrium or the modified TOV-equation which is given by

\[ -\nu' \frac{(\rho + p_r)}{2} - \frac{dp_r}{dr} + \frac{2\Delta}{r} + \frac{\chi}{3(8\pi + 2\chi)} \frac{d}{dr} \left( 3\rho - p_r - 2p_t \right) = 0. \] (6.1)

Here, the first term is gravity \( (F_g) \), second term is pressure gradient \( (F_h) \), third term is the anisotropic force \( (F_a) \) and the last term is the additional force \( (F_m) \) in \( f(R, T) \) - gravity. The fulfillment of the modified TOV-equation is shown in Fig. 3 (right). It shows that the forces due to gravity, pressure gradient and \( F_m \) are highest in \( \chi = -1 \), however, anisotropic force is lowest. This will enable to hold more mass than other for lesser values of \( \chi \). As \( \chi \) increases the \( F_g, F_m \) and \( F_h \) decreases although the \( F_a \) slightly increase thereby the maximum mass that the can be hold by the system will also reduces.
Figure 3. Variation of equation of state parameters and forces in TOV-equation with radial coordinate for Vela X-1 ($M = 1.77 \pm 0.08 \, M_\odot$, $R = 9.56 \pm 0.08 \, km$) with $b = 0.0005/\text{km}^2$ and $c = 0.000015/\text{km}^4$.

6.2 Causality condition and stability factor

We all aware of $f(R,T)$–gravity as an extension of general relativity, which provides a constraint on maximum speed limit. All the particle with non-zero rest mass much travel at subluminal speeds i.e. less than the speed of light (causality condition). The velocity of sound in a medium must also satisfy the causality condition and it determines the stiffness of the related EoS. Therefore, one can determine the sound speed in stellar medium to relate its stiffness. The most stiff EoS is the Zeldovich’s fluid ($p_z = \rho_z$) where the sound travels exactly at light speed. The sound speed can be determine as

$$v_r^2 = \frac{dp_r}{d\rho}, \quad v_t^2 = \frac{dp_t}{d\rho}.$$ \hspace{1cm} (6.2)

In Fig. 4 (left), we plot the speed of sound with the radial coordinate. It can be seen that the speed of sound is maximum for $\chi = -1$ and decreases with increase in $\chi$. This imply that the solution leads to a stiffer EoS with lesser values of $\chi$.

The speed of sound can also related to the stability of the configuration. As per [60], the stability factor can be defined as $v_t^2 - v_r^2$. So long as $v_r > v_t$, the system is generally considered stable, or in other form $-1 \leq v_t^2 - v_r^2 \leq 0$, otherwise unstable. The variation of stability factor is also shown in Fig. 4 (right) which clearly indicates the solution is stable.

6.3 Adiabatic index and stability

Another parameter that determines the stability and stiffness of EoS is the adiabatic index which is defined as the ratio of specific heat at constant pressure to the specific heat at constant volume. For any fluid distribution the adiabatic index can be determine as [127]

$$\gamma = \frac{p_r + \rho v_r^2}{p_r}.$$ \hspace{1cm} (6.3)

As per Bondi’s perceptions, the stellar fluid distribution is stable if $\gamma > 4/3$ in Newtonian limit. If $\gamma \leq 1$ contraction is possible and catastrophic if $\gamma < 1$. This is no longer valid for anisotropic fluids. This was extended by [128] to anisotropic fluid. For anisotropic fluids,
the stable limit of $\gamma$ depends on the nature of anisotropy and its initial configuration. If anisotropy $\Delta > 0$, the stable limit will be still $\gamma > 4/3$, however, if $\Delta < 0$ stability is still possible even if $\gamma < 4/3$. The variation of adiabatic index is shown in Fig. 5 (left). For different values of $\chi$ the central adiabatic index is accumulated around 2.

6.4 Static stability criterion

This criterion analyze the stability of stellar configurations under radial perturbations originally established by [129]. Further, [130] and [131] simplifies this method. The static stability criterion imposed the condition that if $\partial M/\partial \rho_c$ is greater than zero, the system is stable otherwise unstable. To see it, we have calculate the mass as a function of $\rho_c$ given as

$$M(\rho_c) = \frac{R}{2} \left( 1 - \frac{1}{1 + aR^2bR^2 + cR^4} \right). \quad (6.4)$$

Here $a$ is a very complicated function of $\rho_c$ and therefore we avoid to mention. The variation of mass with respect to the central density is shown in Fig. 5 (right). From this, one can conclude that the stability is enhance with increase in $\chi$. This is because the range central density is more for saturating the mass when $\chi = 1$ than $\chi = -1$. This implies that the stable range of density during radial oscillation is more for higher values of $\chi$. This can conclude that that solution is stable under radial perturbations.

6.5 Energy conditions

After confirming all the stability tests, the nature of matter content i.e. either normal (baryonic, hadronic etc.) or exotic (dark matter, dark energy etc.) can be identified by using energy conditions. Satisfaction or violation of certain energy conditions will imply the nature of the matter. These energy conditions are given as

Null : $\rho + p_r \geq 0$, $\rho + p_t \geq 0$,
Weak : $\rho + p_r \geq 0$, $\rho + p_t \geq 0$, $\rho \geq 0$,
Strong : $\rho + p_r \geq 0$, $\rho + p_t \geq 0$, $\rho + p_r + 2p_t \geq 0$,
Dominant : $\rho \geq |p_r|$, $\rho \geq |p_t|$.
Figure 5. Variation of adiabatic index with radial coordinate and variation of mass with central density for Vela X-1 ($M = 1.77 \pm 0.08 \, M_\odot$, $R = 9.56 \pm 0.08 \, km$) with $b = 0.0005/km^2$ and $c = 0.000015/km^4$.

Figure 6. Variation of energy conditions and redshift with radial coordinate for Vela X-1 ($M = 1.77 \pm 0.08 \, M_\odot$, $R = 9.56 \pm 0.08 \, km$) with $b = 0.0005/km^2$ and $c = 0.000015/km^4$.

From Fig. 6 (left), it is found that all the energy conditions are satisfied by the solution and therefore, the matter content is normal.

7 Boundary conditions

Boundary conditions ensure that the interior and exterior spacetime are connected. As usual, we will assume the exterior as Schwarzschild vacuum given as

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2). \quad (7.1)$$

However, we must keep in mind that to avoid singularity, one must satisfy $r > 2m$.

At the surface $r = R$, we get $ds^2|_{r=R} = ds^2_\text{int}|_{r=R}$ which imply

$$e^{-\lambda(R)} = 1 - \frac{2M}{R} = e^{\nu(R)}. \quad (7.2)$$
On using (7.2), we get

\[ a = \frac{2Me^{-R^2(b+cR^2)}}{R^2(R - 2M)} \]  

(7.3)

\[ A = \sqrt{1 - \frac{2M}{R}} - \frac{B}{\sqrt{2c}} e^{bR^2+cR^4} F \left( \frac{2cR^2 + b}{2\sqrt{2c}} \right). \]  

(7.4)

Generally, in modeling compact stars the pressure at the surface needs to vanish i.e. \( p_r(R) = 0 \). This condition allow us to determine one more constant as

\[
B = 4\sqrt{c} \sqrt{1 - \frac{2M}{R}}\sqrt{aR^2e^{bR^2+cR^4}} \left[ \right. \left. R^2 \chi \left( 2cR^2 - ae^{bR^2+cR^4} + b \right) - 3\pi \left( aR^2e^{bR^2+cR^4} + 1 \right) \right] \\
- \left. \sqrt{c}R \left\{ \chi \left( bR^2 + 2cR^4 - 9 - 10aR^2e^{bR^2+cR^4} \right) - 24\pi \left( aR^2e^{bR^2+cR^4} + 1 \right) \right\} + 2\sqrt{2a}R \right. \\
\left. e^{bR^2+cR^4} F \left( \frac{2cR^2 + b}{2\sqrt{2c}} \right) \left\{ 3\pi \left( aR^2e^{bR^2+cR^4} + 1 \right) - R^2 \chi \left( b - ae^{bR^2+cR^4} + 2cR^2 \right) \right\} \right]^{-1}. \]  

(7.5)

The parameter \( b \) and \( c \) will be treated as free whereas \( M \) and \( R \) will be taken from the observational evidences.

8 Stiffness of EoS, \( M - R \) and \( I - M \) curve

There are several ways of determining the stiffness of an EoS e.g. by determining adiabatic index, sound speed etc. However, the sensitivity to stiffness is found to be very sharp in \( M - R \) and \( M - I \) graphs. In fact, \( M - I \) graph is the most effective and sensitive to the stiffness of an EoS. In Fig. 7 (left) we shown the variation of mass with respect to the radius. Since, from the above sections we have already noted that the EoS is stiffest for \( \chi = -1 \) and as \( \chi \) increases the stiffness reduces. Due to this, the mass that can hold by the corresponding EoS will also be reduces as \( \chi \) increases. The same nature can be seen from the \( M - R \) curve in Fig. 7 (left). To compare with the \( M - I \) curve, one must establish how to determine the moment of inertia \( (I) \). Adopting the [13] formula one can determine the \( I \) corresponding to a static solution. It is given by

\[ I = \frac{2}{5} \left( 1 + \frac{(M/R) \cdot km}{M_\odot} \right) MR^2. \]  

(8.1)

The change in \( I \) with respective to mass is shown in Fig. 7 (right). Again, we can verify that the EoS is most stiff for lesser values of \( \chi \). The transition at the peak in \( M - I \) curve is sharper than in \( M - R \) curve indicates the sensitivity to the stiffness of the EoS.

Further, our generated \( M - R \) curve is also fit with observational results for few well-known compact stars. As examples, we have matched for PSR J1614-2230, Vela X-1,
Figure 7. Variation of total mass with radius and total with moment of inertia.

Cen X-3 and SAX J1808.4-3658. Since the $M - R$ curve fit with these compact stars, one possibility arises from $M - I$ curve to predict the possible range of $I$ for the above mentioned objects.

9 Discussion and conclusion

In this article, we have successfully devised the method of embedding class one into $f(R, T)$–gravity. This procedure not only simplify in exploring new exact solutions within $f(R, T)$–theory but also to investigate the theory of compact stars in the same realm. The solution was analyzed through various physically stringent conditions like causality condition, energy conditions, satisfaction of TOV-equation, stability criteria via Bondi’s condition, Abreu et al. condition, static stability criterion etc. As we increase the coupling constant $\chi$ from $-1$ to $1$, the density, pressures, $\omega$, speed of sound and interior redshift increases. This increase in energy density may leads to generation of exotic particles such as quarks, hyperons [133, 134], kaon condensation [135] that soften the equation of state. Therefore, in $M - R$ and $M - I$ curves one can see that $M_{\text{max}}$ and $I_{\text{max}}$ increases with decrease in coupling constant, which is the direct consequence of stiffening the equation of state at low $\chi$. This outcomes can also be cross check with the velocity of sound at the interior. From Fig. 4 (left), we can see that $v_{r/t}(\chi = -1)$ is always greater than $v_{r/t}(\chi = 1)$, i.e. the equation of state is stiffer for the former than the later.

Although the stiffness of the EoS is enhanced by the low coupling constant, however, the stability is compromised. In Fig. 5 (left) the central values $\Gamma_c(\chi = 1) = 1.977$ is more than at $\Gamma_c(\chi = -1) = 1.848$. For $\chi = -1$, the value $\Gamma_c(\chi = -1) = 1.848$ is comparatively closer to the Bondi limit i.e. $\Gamma = 1.333$. Therefore, $\chi = -1$ is more sensitive towards radial oscillations and hence the stability. On the other hand, the range of central density is less for $\chi = -1$ than $\chi = 1$. This means that, during radial perturbations the range of stable density perturbation is more for $\chi = 1$ than $\chi = -1$ thereby enhancing its stability. Further, the solution fulfilled all the energy conditions. The anisotropy decreases with decrease in coupling parameter. This leads to the conclusion that anisotropy in pressure reduces with increase in stiffness of the EoS. The surface redshifts predicted form the solution in far within the Ivanov’s limit i.e. $z_s = 3.842$ [136].
In Table 1, we have presented 4 compact stars and few their corresponding physical parameters. Here we have also provide how the radius, central and surface densities, central pressure and moment of inertia varies with $f(R, T)$—coupling constant. Since the stiffness increases with decrease in $\chi$, the central and surface densities, central pressure and radius increases while the stability is compromised. For $-1 \leq \chi \leq 1$, we have predicted the radii of the compact stars. All the above good results and in agreement with the observational values of masses and radii, one can undoubtedly conclude that the solution might have astrophysical significance.

For the graphical test and investigation of the physical reasonable grounds of the accomplished solutions, we have well-respected the physical profile of four well-known compact stellar systems viz., PSR J1614-2230, Vela X-1, Cen X-3 and SAX J1808.4-3658. In this regard, we have supposed the radius-radius element of the metric function ($e^\lambda$) in the new form $e^\lambda = 1 + ar^2e^{br^2+cr^4}$ and provided the time-time element of the metric function ($e^\nu$) as expressed explicitly in Eq. (5.2), as well as exhibiting the anisotropic impacts on the physical systems imposed by the $\chi$—coupling constant of the $f(R, T)$ gravity theory. As establish the most important salient features that describe the stellar system fulfills all the general necessities to guarantee a respectful framework.

The comportment of physical amounts of time-time and radius-radius components, namely, $e^\nu$ and $e^\lambda$, respectively, regarding the radial coordinate $r$, for Vela X-1 ($M = 1.77 \pm 0.08 \ M_\odot$, $R = 9.56 \pm 0.08 \ km$) are represented in Fig. 1 (left). This figure shows that both metric functions are limited at the origin and monotonically expanding towards the point of confinement at the surface. Moreover, it very well may be seen from the Eqs.(5.1) and (5.2) that $e^\lambda(r = 0) = 1$ and $e^\nu(r = 0) \neq 0$ which shows that this stellar system is realistic and agreeable. Hence, Figs. 1 (right) and 3 clearly show that all the thermodynamic observable, i.e energy density $\rho$, radial pressure $p_r$ and transverse pressure $p_t$ are well-defined within the stellar structure. In this respect, is worth mentioning that all quantities mentioned have their maximum values at the core of the stellar structure and monotonically decreasing comportment with increasing radius towards the surface. At this stage it merits referencing that the present stellar model shows a positive anisotropy factor $\Delta$, it can be observed in Fig. 2 (right) where $p_t > p_r$ then $\Delta > 0$. In this way the stellar structure is uncovered to a repulsive force that neutralizes the gravitational slant, this reality permits the building of a progressively compact stellar configuration. We affirm again from Figs. 1 and 2, that the stellar system is absolutely without physical or geometrical singularities for all chosen values of $\chi$—coupling constant of the $f(R, T)$—gravity theory. Fig. 2 (right) exhibit the variation of anisotropy parameter $\Delta$ with radial coordinate for Vela X-1. It vanishes at the center, and consequently, positive defined and increasing function towards the stellar structure surface. Additionally Fig. 3 (left) manifest that the EoS parameters viz., $\omega_r = p_r/\rho$ and $\omega_t = p_t/\rho$ are under 1, demonstrating Zeldovichâ€™s condition is fulfilled wherever within the stellar structure in the context of $f(R, T)$—gravity theory.

Besides, equilibrium study of the model for stellar system is established using generalized TOV-equation originates from the modified type of the energy conservation equation for the energy-momentum tensor in the arena of the $f(R, T)$—gravity theory given in Eq.
In this regard, it is easy to see from Fig. 3 (right) that the modified TOV-equation permits the exploration under various forces that perform on the stellar structure, in this event, the stellar structure is under four various forces, namely, gravity \(F_g\), pressure gradient \(F_h\), the anisotropic force \(F_a\) and the additional force \(F_m\) in \(f(R,T)\)-gravity. The forces due to gravity, pressure gradient and the additional term are highest in \(\chi = -1\), however, anisotropic force is lowest. This will enable to hold more mass than other for lesser values of \(\chi\). As \(\chi\) increases the \(F_g\), \(F_m\) and \(F_h\) decreases although the \(F_a\) slightly increase thereby the maximum mass that the can be hold by the system will also reduces. On the other hand, we investigate the stability of realistic and compact stellar structure solutions via causality condition and stability factor, stability criteria via Bondiâ€™s condition and Harrison-Zeldovich-Novikov static stability criterion corresponding to \(\chi\)–coupling constant of the \(f(R,T)\)-gravity. According to these criteria, In Fig. 4 (left) the performances of the radial \((v_r)\) and transverse \((v_t)\) speed of sound with respect to the radial coordinate \(r\), for the compact stellar configuration have been described and it is remarked clearly that they remain within their predetermined range \([-1, 0]\) throughout the stellar framework, which affirms the causality condition and furthermore valid the acceptability of the subsequent anisotropic solution of our stellar system. Moreover, It can be seen that the speed of sound is maximum for \(\chi = -1\) and decreases with increase in \(\chi\), which implies that our solution leads to a stiffer EoS with lesser values of \(\chi\)–coupling constant. The solution can also appear for static and stable astrophysical structures as the stability factor can be defined as \(v_t^2 - v_r^2\) lies between the bounds \([-1, 0]\) for different values of \(\chi\)–coupling parameter, which are shown in Fig. 4 (right). Furthermore, for a non-collapsing stellar fluid distribution, the adiabatic index should also be greater than \(4/3\) for \(\Delta > 0\) according to stability criteria via Bondiâ€™s perceptions, which can be observed clearly from Fig. 5 (left), so our stellar system is generally stable. In Fig. 5 (right), we plot the variation of mass with respect to the central density which satisfies the Harrison-Zeldovich-Novikov static stability criterion. From this figure, one can conclude that the stability is enhance with increase in \(\chi\). This is on the grounds that the range central density is more for saturating the mass when \(\chi = 1\) than \(\chi = -1\). This infers that the stable range of density during radial oscillation is more for higher values of \(\chi\). This can conclude that our solution is completely stable under radial perturbations. Furthermore, we have investigated the profile of the thermodynamic quantities that prompts a well-behaved and positive defined energy-momentum tensor throughout interior the compact stellar structure which satisfies at the same time by the inequalities that governs them named ECs. Hence, In Fig. 6 (left), we have plotted the L.H.S of these inequalities which checks that all the ECs are achieved at the astrophysical inside and consequently corroborates that the physical accessibility of the compact stellar inside solution. The fulfillment of the redshift with radial coordinate \(r\), for Vela X-1 is illustrated in Fig. 6 (right). From this figure, it can see that the surface redshift within typical value resulting by Ivanovâ€™s which strongly proves the agreement of our compact stellar system.

On the other hand, we have generated the \(M-R\) curves from our solutions in the arena of \(f(R,T)\) gravity theory and we found a perfect fit for a certain compact stellar spherical systems such as PSR J1614-2230, Vela X-1, Cen X-3 and SAX J1808.4-3658. Therefore, we have predicted the corresponding radii and their respective moment of inertia from the
Table 1. Prediction of radius for few well-known compact stars and their corresponding central densities and pressures for different values of $\chi$.

| Objects   | $\chi$ | $M$ ($M_\odot$) | Predicted Radius (km) | $\rho_0$ (MeV/fm$^3$) | $\rho_s$ (MeV/fm$^3$) | $\rho_0$ (MeV/fm$^3$) | $I \times 10^{44}$ (g cm$^{-2}$) |
|-----------|--------|-----------------|------------------------|-------------------------|------------------------|-------------------------|-------------------------------|
| PSR J1614-2330 | -1.0   | 10.13           | 621.11                 | 376.32                  | 72.57                  | 18.51                   |
|           | -0.5   | 9.89            | 581.58                 | 353.52                  | 63.20                  | 17.52                   |
|           | 0      | 1.97            | 549.65                 | 330.71                  | 56.10                  | 16.77                   |
|           | 0.5    | $\pm$ 0.04      | 9.47                   | 519.24                  | 313.98                 | 50.28                   | 16.04                         |
|           | 1.0    | 9.28            | 491.87                 | 295.74                  | 45.44                  | 15.59                   |
| VELA X-1  | -1.0   | 9.76            | 551.66                 | 362.96                  | 56.48                  | 15.29                   |
|           | -0.5   | 9.55            | 518.29                 | 339.95                  | 49.12                  | 14.62                   |
|           | 0      | 1.77            | 488.38                 | 319.24                  | 43.36                  | 13.94                   |
|           | 0.5    | $\pm$ 0.08      | 9.15                   | 461.91                  | 303.13                 | 38.76                   | 13.56                         |
|           | 1.0    | 8.99            | 437.75                 | 285.87                  | 34.85                  | 13.01                   |
| SAX J1808.4-3658 | -1.0   | 7.91            | 431.10                 | 339.05                  | 20.76                  | 4.74                    |
|           | -0.5   | 7.69            | 403.98                 | 317.68                  | 18.14                  | 4.51                    |
|           | 0      | 0.9             | 380.96                 | 298.78                  | 16.12                  | 4.34                    |
|           | 0.5    | $\pm$ 0.3       | 7.42                   | 359.60                  | 282.34                 | 14.50                   | 4.17                          |
|           | 1.0    | 7.26            | 340.69                 | 266.73                  | 13.19                  | 4.00                    |
| CEN X-3   | -1.0   | 9.29            | 502.10                 | 352.97                  | 41.72                  | 11.24                   |
|           | -0.5   | 9.03            | 471.03                 | 330.02                  | 36.47                  | 10.85                   |
|           | 0      | 1.49            | 443.42                 | 311.65                  | 31.83                  | 10.42                   |
|           | 0.5    | $\pm$ 0.08      | 8.66                   | 418.10                  | 294.44                 | 28.60                   | 10.11                         |
|           | 1.0    | 8.53            | 397.39                 | 278.37                  | 25.98                  | 9.72                    |

$M - I$ curve by varying the coupling constant $\chi$ as a free variable. Furthermore, the $M - R$ and $I - M$ curves are represented in Figs. 7. From these curves, one can approve that our solution predicted the radii in good agreement with the observational data.

Finally, we wish to remark that all anisotropic spherically symmetric solutions establish in this work satisfying obtained well-behaved stellar interiors in the area of $f(R,T)$ gravity theory by using the embedding class I procedure are fulfilling and sharing all the physical and mathematical features necessary in the study of compact stellar spherical systems, which provide to understand the evolution of realistic compact stellar spherical systems. In this regard, the $f(R,T)$ gravity theory is a promising scenario to envisage the existence of compact stellar spherical systems performed by an anisotropic matter distribution, whose effects can be contrasted with the well-described GR, and meets the notable and tried general necessities.

References

[1] C. L. Bennett et al., First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Preliminary Maps and Basic Results, Astrophys. J. Suppl. 148 (2003) 1.

[2] D. N. Spergel et al., First-year Wilkinson Microwave Anisotropy Probe (WMAP)* observations: determination of cosmological parameters, Astrophys. J. Suppl. 148 (2003) 175.
[3] D. N. Spergel et al., *Three-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: implications for cosmology*, Astrophys. J. Suppl. 170 (2007) 377.

[4] S. Perlmutter et al., *Measurements* of the Cosmological Parameters $\Omega$ and $\Lambda$ from the First Seven Supernovae at $z \geq 0.35$, Astrophys. J. 483 (1997) 565.

[5] S. Perlmutter et al., *Discovery of a supernova explosion at half the age of the Universe*, Nature 391 (1998) 51.

[6] S. Perlmutter et al., *Measure of $\Omega$ and $\Lambda$ from 42 High-Redshift Supernovae*, Astrophys. J. 517 (1999) 565.

[7] A.G. Riess et al., *Type Ia Supernova Discoveries at $z > 1$ from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution*, Astrophys. J. 607 (2004) 665.

[8] A.G. Riess et al., *New Hubble Space Telescope Discoveries of Type Ia Supernovae at $z \geq 1$: Narrowing Constraints on the Early Behavior of Dark Energy*, Astrophys. J. 659 (2007) 98.

[9] S. Cole et al., *The 2dF Galaxy Redshift Survey: power-spectrum analysis of the final data set and cosmological implications*, Mon. Not. Roy. Astron. Soc. 362 (2005) 505.

[10] E. Hawkins et al., *The 2dF Galaxy Redshift Survey: correlation functions, peculiar velocities and the matter density of the Universe*, Mon. Not. Roy. Astron. Soc. 346 (2003) 78.

[11] M. Tegmark et al., *Cosmological parameters from SDSS and WMAP*, Phys. Rev. D 69 (2004) 103501.

[12] B. Jain and A. Taylor, *Cross-correlation tomography: measuring dark energy evolution with weak lensing*, Phys. Rev. Lett. 91 (2003) 141302.

[13] D.J. Eisenstein et al., *Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies*, Astrophys. J. 633 (2005) 560.

[14] N. A. Bahcall et al., *The cosmic triangle: Revealing the state of the universe*, Science 284 (1999) 1481.

[15] Y.F. Cai, E.N. Saridakis, M.R. Setare and J.Q. Xia, *Quintom cosmology: theoretical implications and observations*, Phys. Rept. 493 (2010) 1.

[16] E.J. Copeland, M. Sami and S. Tsujikawa, *Dynamics of dark energy*, Int. J. Mod. Phys. D 15 (2006) 1753.

[17] S. Capozziello et al., *Hydrostatic equilibrium and stellar structure in f(R) gravity*, Phys. Rev. D 83 (2011) 064004.

[18] A. De Felice and S. Tsujikawa, *f(R) theories*, Living Rev. Relat. 13 (2010) 3.

[19] F.S.N. Lobo, *The dark side of gravity: modified theories of gravity*, arXiv:0807.1640 [gr-qc].

[20] S. Nojiri and S. D. Odinstov, *Unified cosmic history in modified gravity: from f(R) theory to Lorentz non-invariant models*, Phys. Rept. 505 (2011) 59.

[21] C.R. Fadragas, G. Leon and E.N. Saridakis, *Dynamical analysis of anisotropic scalar-field cosmologies for a wide range of potentials*, Class. Quant. Gravit. 31 (2014) 075018.

[22] G. Kofinas, G. Leon and E.N. Saridakis, *Dynamical behavior in f(T, T_G) cosmology*, Class. Quant. Gravit., 31 (2014) 175011.

[23] G. Leon and E.N. Saridakis, *Phase-space analysis of HoÅŹava-Lifshitz cosmology*, J. Cosmol. Astropart. Phys. 0911 (2009) 006.
[24] G. Leon and E.N. Saridakis, Dynamics of the anisotropic Kantowsky–Sachs geometries in R_n gravity, *Class. Quant. Gravit.* 28 (2011) 065008.

[25] G. Leon, J. Saavedra and E.N. Saridakis, Cosmological behavior in extended nonlinear massive gravity, *Class. Quant. Gravit.* 30 (2011) 135001.

[26] M. Skugoreva, E.N. Saridakis and A. Toporensky, Dynamical features of scalar-torsion theories, *Phys. Rev. D* 91 (2015) 044023.

[27] C. Xu, E.N. Saridakis and G. Leon, Phase-space analysis of teleparallel dark energy, *J. Cosmol. Astropart. Phys.* 07 (2012) 005.

[28] C. Xu, E.N. Saridakis and G. Leon, Dynamical analysis of generalized Galileon cosmology, *J. Cosmol. Astropart. Phys.* 1303 (2013) 025.

[29] C. Xu, E.N. Saridakis and G. Leon, Dynamical behavior in mimetic F(R) gravity, *J. Cosmol. Astropart. Phys.* 1504 (2015) 031.

[30] C. Xu, E.N. Saridakis and G. Leon, Cosmology in time asymmetric extensions of general relativity, *J. Cosmol. Astropart. Phys.* 1511 (2015) 11009.

[31] S. Carloni, F.S.N. Lobo, G. Otalora and E.N. Saridakis, Dynamical system analysis for a nonminimal torsion-matter coupled gravity, *Phys. Rev. D* 93 (2016) 024034.

[32] T. Clifton, P.G. Ferreira, A. Padilla and C. Skordis, Modified gravity and cosmology, *Phys. Rep.* 513 (2012) 1.

[33] A. V. Astashenok et al., Further stable neutron star models from f(R) gravity, *J. Cosmol. Astropart. Phys.* 2013 (2013) 040.

[34] A. V. Astashenok et al., Magnetic neutron stars in f(R) gravity, *Astrophys. Space Sci.* 355 (2015) 333.

[35] A. V. Astashenok et al., The realistic models of relativistic stars in f(R) = R + αR^2 gravity, *Class. Quant. Grav.* 34 (2017) 205008.

[36] S. Capozziello and M.D. Laurentis, Extended theories of gravity, *Phys. Rept.* 509 (2011) 167.

[37] S. Capozziello et al., Spherical symmetry in f(R)–gravity, *Class. Quant. Grav.* 25 (2008) 085004.

[38] S. Capozziello et al., Axially symmetric solutions in f(R)–gravity, *Class. Quant. Grav.* 27 (2010) 165008.

[39] S. Capozziello et al., New spherically symmetric solutions in f(R)–gravity by Noether symmetries, *Gen. Relativ. Gravit.* 44 (2012) 1881.

[40] S. Capozziello et al., Modelling clusters of galaxies by f(R) gravity, *Mon. Not. Roy. Astron. Soc.* 394 (2009) 947.

[41] S. Capozziello et al., F(R) theories of gravitation, *Scholarpedia* 10 (2015) 31422.

[42] S. Capozziello et al., Mass-radius relation for neutron stars in f(R) gravity, *Phys. Rev. D* 93 (2016) 023501.

[43] S. Capozziello et al., The role of energy conditions in f(R) cosmology, *Phys. Lett. B* 781 (2018) 99.

[44] S. Capozziello, C. A. Mantica, and L. G. Molinari, Cosmological perfect-fluids in f(R) gravity, *Phys. Lett. B* 781 (2018) 99.
[45] S. Capozziello and R. DâĂŹAgostino, *Kinematic model-independent reconstruction of Palatini f(R) cosmology*, Gen. Relat. Gravit. **51** (2018) 2.

[46] S. V. Chervon et al., *Kinetic scalar curvature extended f(R) gravity*, Nucl. Phys. B **936** (2018) 597.

[47] V.B. Jovanovic, S. Capozziello, P. Jovanovic and D. Borka, *Recovering the fundamental plane of galaxies by f(R) gravity*, Physics of the Dark Universe **14** (2016) 73.

[48] R. Maartens and R. Durrer, *Dark energy and modified gravity*. Cambridge University Press (2010).

[49] S. Nojiri, S.D. Odintsov and D. Saez-Gomez, *Cosmological reconstruction of realistic modified f(R) gravities*, Phys. Lett. B **681** (2009) 74.

[50] S. D. Odintsov and V. K. Oikonomou, *Effects of spatial curvature on the f(R) gravity phase space: no inflationary attractor?*, Class. Quant. Grav. **36** (2019) 065008.

[51] S. D. Odintsov and V. K. Oikonomou, *f(R) gravity inflation with string-corrected axion dark matter*, Phys. Rev. D **99** (2019) 064049.

[52] C. S. Santos, J. Santos, S. Capozziello and J. S. Alcaniz, *Strong energy condition and the repulsive character of f(R) gravity*, Gen. Relativ. Gravit. **49** (2017) 50.

[53] S. Capozziello and R. DâĂŹAgostino, *Exact charged black-hole solutions in D-dimensional f(T) gravity: torsion vs curvature analysis*, J. High Energy Phys. **2013** (2013) 039.

[54] C. G. Böhmer A. Mussa and N. Tamanini, *Existence of relativistic stars in f(T) gravity*, Class. Quant. Grav. **28** (2011) 245020.

[55] M. H. Daouda et al., *New static solutions in f(T) theory*, Eur. Phys. J. C **71** (2011) 1817.

[56] M. Sharif and S. Rani, *Wormhole solutions in f(T) gravity with noncommutative geometry*, Phys. Rev. D **88** (2013) 123501.

[57] T. Wang, *Static solutions with spherical symmetry in f(T) theories*, Phys. Rev. D **84** (2011) 024042.

[58] G.R. Bengochea and R. Ferraro, *Dark torsion as the cosmic speed-up*, Phys. Rev. D **79** (2009) 124019.

[59] E. V. Linder, *EinsteinâĂŹs Other Gravity and the Acceleration of the Universe*, Phys. Rev. D **81** (2010) 127301.

[60] H. Abreu et al., *Sound speeds, cracking and the stability of self-gravitating anisotropic compact objects*, Class. Quantum Grav. **24** (2007) 4631.

[61] M. M. Akbar, *Embedding FLRW geometries in pseudo-Euclidean and antiâĂŞde Sitter spaces*, Phys. Rev. D **95** (2017) 064058.

[62] S. N. Pandey, S. P. Sharma , *Insufficiency of Karmarkar’s condition*, Gen. Relativ. Gravit. **14** (1981) 113.

[63] K. Bamba et al., *Equivalence of the modified gravity equation to the Clausius relation*, Euro-phys. Lett. **89** (2010) 50003.

[64] K. Bamba et al., *Finite-time future singularities in modified GaussâĂŚBonnet and F(R,G) gravity and singularity avoidance*, Eur. Phys. J. C **67** (2010) 295.

[65] M.E. Rodrigues, M.J.S. Houndjo, D. Momeni and R. Myrzakulov, *A type of LeviâĂŚCivita solution in modified GaussâĂŚBonnet gravity*, Can. J. Phys. **92** (2014) 173.
[66] E. H. Baou et al., $f(R, T)$ models applied to baryogenesis, arXiv: 1808.01917.

[67] E. Barrientos et al., Metric-affine $f(R, T)$ theories of gravity and their applications, Phys. Rev. D 97 (2018) 104041.

[68] R. A. C. Correa and P. H. R. S. Moreas, Configurational entropy in $f(R, T)$ brane models, Eur. Phys. J. C 76 (2016) 100.

[69] D. Das et al., Compact stars in $f(R, T)$ gravity, Eur. Phys. J. C 76 (2016) 654.

[70] D. Deb, F. Rahaman, S. Ray and B. K. Guha, Strange stars in $f(R, T)$ gravity, J. Cosmol. Astropart. Phys. 1803 (2018) 044.

[71] S. Hansraj and A. Banerjee, Dynamical behavior of the Tolman metrics in gravity, Phys. Rev. D 97 (2018) 104020.

[72] S. Hansraj, Spherically symmetric isothermal fluids in $f(R, T)$ gravity, Eur. Phys. J. C 78 (2018) 700.

[73] S. K. Maurya, A. Errehymy, D. Deb et al., Study of anisotropic strange stars in $f(R, T)$ gravity: An embedding approach under the simplest linear functional of the matter-geometry coupling, Phys. Rev. D 100 (2019) 044014.

[74] P. H. R. S. Moraes et al., The Starobinsky model within the $f(R,T)$ formalism as a cosmological model, arXiv: 1701.01027v1.

[75] P. H. R. S. Moraes, J. D. V. Arbanil and M. Malheiro, Stellar equilibrium configurations of compact stars in $f(R,T)$ theory of gravity, J. Cosmol. Astropart. Phys. 1606 (2016) 005.

[76] P. K. Sahoo, P. H. R. S. Moraes, Parbati Sahoo and G. Ribeiro, Phantom fluid supporting traversable wormholes in alternative gravity with extra material terms, Int. J. Mod. Phys. D 28 (2019) 1950004.

[77] H. Shabani and A. Hadi Ziaie, Bouncing cosmological solutions from $f(R,T)$ gravity, Eur. Phys. J. C 78 (2018) 397.

[78] J. K. Singh et al., Bouncing cosmology in $f(R,T)$ gravity, Phys. Rev. D 97 (2018) 123536.

[79] Z. Yousa et al., Influence of modification of gravity on the dynamics of radiating spherical fluids, Phys. Rev. D 93 (2016) 064059.

[80] Z. Yousa et al., Existence of compact structures in $f(R,T)$ gravity, Eur. Phys. J. C 78 (2018) 307.

[81] Z. Yousa, K. Bamba and M. Z. u. H. Bhatti, Causes of irregular energy density in $f(R,T)$ gravity, Phys. Rev. D 93 (2016) 124048.

[82] E.L.B. Junior et al., Reconstruction, Thermodynamics and Stability of $Λ$CDM Model in $f(T,T)$ Gravity, Class. Quant. Gravit. 33 (2015) 125006.

[83] D. Momeni and R. Myrzakulov, Cosmological reconstruction of $f(T,T)$ gravity, Int. J. Geom. Meth. Mod. Phys. 11 (2014) 1450077.

[84] S. B. NassurM. J. S. HoundjoM. E. Rodrigues, A. V. Kpadonou and J. Tossa., From the early to the late time universe within $f(T,T)$ gravity, Astrophys. Space Sci. 360 (2015) 60.

[85] M. Pace and J. L. Said, Quark stars in $f(T,T)$ gravity, Eur. Phys. J. C 77 (2017) 62.

[86] D. Saez-Gomez, C.S. Carvalho and F.S.N. Lobo, Constraining $f(T,T)$ gravity models using type Ia supernovae, Phys. Rev. D 94 (2016) 024034.
[87] I.G. Salako, A. Jawad and S. Chattopadhyay, Holographic dark energy reconstruction in $f(R,T)$ gravity, Astrophys. Space Sci. 358 (2015) 13.

[88] S. Nojiri and S. D. Odintsov, Modified Gauss-Bonnet theory as gravitational alternative for dark energy, Phys. Lett. B 631 (2005) 1.

[89] T. Harko, F.S.N. Lobo, S. Nojiri and S.D. Odintsov, $f(R,T)$ gravity, Phys. Rev. D, 84 (2011) 024020.

[90] M.J.S. Houndjo, Thermodynamics in Little Rip cosmology in the framework of a type of $f(R,T)$ gravity, arXiv:1207.1646v2 [gr-qc].

[91] M. Jamil, D. Momeni, and M. Ratbay, Violation of the first law of thermodynamics in $f(R,T)$ gravity, Chin. Phys. Lett. 29 (2012) 109801.

[92] M. Sharif and M. Zubair, Thermodynamics in $f(R,T)$ theory of gravity, J. Cosmol. Astropart. Phys. 03 (2012) 028.

[93] F.G. Alvarenga et al., Testing some $f(R,T)$ gravity models from energy conditions, J. Mod. Phys. 04 (2013) 130.

[94] V. Fayaz, H. Hossienkhani, M. Amirabadi and N. Azimi, Anisotropic cosmological models in $f(R,T)$ gravity according to holographic and new agegraphic dark energy, Astrophys. Space Sci. 353 (2014) 301.

[95] P. K. Sahoo, P. Sahoo and B. K. Bishi, Anisotropic cosmological models in $f(R,T)$ gravity with variable deceleration parameter, Int. J. Geom. Meth. Mod. Phys. 14 (2017) 1750097.

[96] M.J.S. Houndjo, Reconstruction of $f(R,T)$ gravity describing matter dominated and accelerated phases, Int. J. Mod. Phys. D 21 (2012) 1250003.

[97] M. Jamil, D. Momeni, R. Muhammad and M. Ratbay, Reconstruction of some cosmological models in $f(R,T)$ cosmology, Eur. Phys. J. C 72 (2012) 1999.

[98] P. H. R. S. Moraes and P. K. Sahoo, Modeling wormholes in $f(R,T)$ gravity, Phys. Rev. D 96 (2017) 044038.

[99] P. K. Sahoo, P. H. R. S. Moraes and P. Sahoo, Wormholes in $R^2$--gravity within the $f(R,T)$ formalism, Eur. Phys. J. C 78 (2018) 46.

[100] F.G. Alvarenga et al., Dynamics of scalar perturbations in $f(R,T)$ gravity, Phys. Rev. D 87 (2013) 103526.

[101] D. Momeni, R. Myrzakulov and E. Gudekli, Cosmological viable mimetic $f(R)$ and $f(R,T)$ theories via Noether symmetry, Int. J. Geom. Meth. Mod. Phys. 12 (2015) 1550101.

[102] I. Noureen and M. Zubair, Dynamical instability and expansion-free condition in $f(R,T)$ gravity, Eur. Phys. J. C 75 (2015) 62.

[103] B. Saha, Interacting Scalar and Electromagnetic Fields in $f(R,T)$ Theory of Gravity, Int. J. Theor. Phys. 54 (2015) 3776.

[104] Z. Haghani, T. Harko, F.S.N. Lobo, H.R. Sepangi and S. Shahid, Further matters in space-time geometry: $f(R,T,R_{\mu\nu}T^{\mu\nu})$ gravity, Phys. Rev. D 88 (2013) 044023.

[105] S.D. Odintsov and D. Saez-Gomez, $f(R,T,R_{\mu\nu}T^{\mu\nu})$ gravity phenomenology and ΛCDM universe, arXiv:1304.5411v3 [gr-qc].

[106] H.Velten and T. R. CaramÃls, Cosmological inviability of $f(R,T)$ gravity, Phys. Rev. D 95 (2017) 123536.
D. Deb et al., Exploring physical features of anisotropic strange stars beyond standard maximum mass limit in $f(R, T)$ gravity, Mon. Not. Roy. Astron. Soc. 485 (2019) 5652.

S. K. Maurya and F. Tello-Ortiz, Anisotropic fluid spheres in the framework of $f(R, T)$ gravity theory, arXiv:1906.11756 [gr-qc].

M. Zubair, G. Abbas and I. Noureen, Possible formation of compact stars in $f(R, T)$ gravity, Astrophys. Space Sci. 361 (2016) 8.

D. Das et al., Gravastars in $f(R, T)$ gravity, Phys. Rev. D 95 (2017) 124011.

L. Herrera, Cracking of self-gravitating compact objects, Phys. Lett. A 165 (1992) 206.

R. Ruderman, Pulsars: structure and dynamics, Rev. Astron. Astrophys. 10 (1972) 427.

F. Weber, Pulsars as Astrophysical Observatories for Nuclear and Particle Physics. Institute of Physics (1999).

R.F. Sawyer, Condensed $\pi^-$ phase in neutron-star matter, Phys. Rev. Lett. 29 (1972) 382.

A.I. Sokolov, Phase transformations in a superfluid neutron liquid, J. Exp. Theor. Phys. 79 (1980) 1137.

K.R. Karmarkar, Gravitational metrics of spherical symmetry and class one, Proc. Ind. Acad. Sci. 27 (1948) 56.

K. Schwarzschild, Über das Gravitationsfeld einer Kugel aus inkompressibler Flüssigkeit nach der Einsteinschen Theorie, Sitz. Deut. Akad. Wiss. Math. Phys. Berlin 24 (1916) 424.

A. Barnes, Space times of embedding class one in general relativity, Gen. Relativ. Grav. 5 (1974) 147.

P. K. F. Kuhfittig, Two diverse models of embedding class one, Ann. Phys. 392 (2018) 63.

G. Darmois, Les Équations de la gravitation einsteinienne, Mémorial des Sciences Mathematiques (Gauthier-Villars, Paris), Fasc. 25 (1927) 58.

W. Israel, Singular hypersurfaces and thin shells in general relativity, Nuovo Cim. B 44 (1966) 1.

O. J. Barrientos and G. F. Rubilar, Comment on $f(R, T)$ gravity, Phys. Rev. D 90 (2014) 028501.

E. Kasner, Geometrical theorems on Einstein’s cosmological equations, Am. J. Math. 43 (1921) 130.

Y. K. Gupta and M. P. Goel, Class two analogue of TY Thomas’s theorem and different types of embeddings of static spherically symmetric space-times, Gen. Relativ. Gravit. 6 (1975) 499.

J. K. Singh et al., Physical viability of fluid spheres satisfying the Karmarkar condition, Eur. Phys. J. C 77 (2017) 100.

G. Mustafa, M. Zubair, S. Waheed and X. Tiecheng, Realistic stellar anisotropic model satisfying Karmarker condition in $f(R, T)$ gravity, Eur. Phys. J. C 80 (2020) 26.

H. Bondi, The contraction of gravitating spheres, Proc. R. Soc. Lond. A 281 (1964) 39.

R. Chan, L. Herrera and N. O. Santos, Dynamical instability for radiating anisotropic collapse, Mon. Not. R. Astron. Soc. 265 (1993) 533.
[129] S. Chandrasekhar, *A General Variational Principle Governing the Radial and the Non-Radial Oscillations of Gaseous Masses*, Astrophys. J. 139 (1964) 664.

[130] B. K. Harrison, K. S. Thorne, M. Wakano, J. A. Wheeler, *Gravitational theory and gravitational collapse* (University of Chicago Press) (1965) 194.

[131] Ya B. Zeldovich and I. D. Novikov, , *Relativistic astrophysics stars and relativity* Vol. 1. University of Chicago Press (1971).

[132] M. Bejger and P. Haensel, *Moments of inertia for neutron and strange stars: Limits derived for the Crab pulsar*, A & A 396 (2002) 917.

[133] J. L. Zdunik, P. Haensel, E. Gourgoulhon and M. Bejger, *Hyperon softening of the EOS of dense matter and the spin evolution of isolated neutron stars*, A & A 416 (2004) 1013.

[134] S. Balberg et al., *Roles of hyperons in neutron stars*, Astrophys. J. Supp. 121 (1999) 515.

[135] Y. Lim et al., *Kaon condensation in neutron stars with Skyrme-Hartree-Fock models*, Phys. Rev. C 89 (2014) 055804.

[136] B. V. Ivanov, *Maximum bounds on the surface redshift of anisotropic stars*, Phys. Red. D 65 (2002) 104011.

[137] A. V. Astashenok et al., *Nonperturbative models of quark stars in $f(R)$ gravity*, Phys. Lett. B 742 (2015) 160.

[138] N. J. Poplawski, *A Lagrangian description of interacting dark energy*, arXiv:gr-qc/0608031.