Magnetic hysteresis of individual Janus particles with hemispherical exchange biased caps

S. Philipp,1 B. Gross,1 M. Reginka,2 M. Merkel,2 M. Claus,1 M. Sulliger,1 A. Ehresmann,2 and M. Poggio1

1) Department of Physics, University of Basel, 4056 Basel, Switzerland
2) Institute of Physics, University of Kassel, 34132 Kassel, Germany

We use sensitive dynamic cantilever magnetometry to measure the magnetic hysteresis of individual magnetic Janus particles. These particles consist of hemispherical caps of magnetic material deposited on micrometer-scale silica spheres. The measurements, combined with corresponding micromagnetic simulations, reveal the magnetic configurations present in these individual curved magnets. In remanence, ferromagnetic Janus particles are found to host a global vortex state with vanishing magnetic moment. In contrast, a remanent onion state with significant moment is recovered by imposing an exchange bias to the system via an additional antiferromagnetic layer in the cap. A robust remanent magnetic moment is crucial for most applications of magnetic Janus particles, in which an external magnetic field actuates their motion.

Janus particles (JPs) are nano- or micronsized particles that possess two sides, each having different physical or chemical properties. There is a multitude of types of JPs1, differing in shape, material, and functionalization. As a subgroup of micron and sub-micron sized magnetic particles, which are discussed as a multifunctional component in lab-on-chip or micro-total analysis systems2,3, magnetic JPs, consisting of a hemispherical cap of magnetic material on a non-magnetic spherical template, allow not only a controlled transversal motion, but also a controlled rotation by rotating external magnetic fields4,5. Such JPs can be mass-produced via the deposition of magnetic layers on an ensemble of silica spheres. The transversal and rotary motion of these particles can be controlled via external magnetic fields, which exert magnetic forces and torques6. This ability to externally actuate magnetic JPs has led to applications in microfluidics, e.g. as stirring devices7, as microprobes for viscosity changes8, or as cargo transporters in lab-on-chip devices9–11. Magnetic JPs have also been proposed as an in vivo drug delivery system12.

Although, in general, a transversal controlled motion can be achieved by both superparamagnetic particles or particles with a permanent magnetic moment, a control over the rotational degrees of freedom can only be achieved, if the particles possess a sufficiently large permanent magnetic moment. Streubel et al.13 analyzed the remanent magnetic state of magnetic JPs with ferromagnetic (fm) magnetic caps. Their simulations show that permalloy JPs with diameters larger than 140 nm host a global vortex state at remanence. Because this flux-closed state has a vanishing net magnetic moment, magnetic JPs hosting such a remanent configuration are unsuited for applications involving magnetic actuation. Thus, for JPs larger than this critical diameter, strategies to overcome this limitation need to be developed.

Here, we make use of exchange bias14–16, which, in a simplified picture, imposes a preferred direction on the magnetic moments of the fm layer and is thereby able to prevent the formation of a global vortex at remanence. We apply an exchange bias to the fm layer by adding an antiferromagnetic (afm) layer beneath the fn layer. In order to verify that this addition leads to a remanent configuration with large magnetic moment, we measure the magnetic hysteresis of individual JPs with and without this layer. The measurement of individual JPs is necessary in order to eliminate the effects of interactions between neighboring JPs. For this task, we employ dynamic cantilever magnetometry (DCM) and analyze the results by comparison to corresponding micromagnetic simulations. This technique overcomes the limitation of earlier measurements, that were restricted to ensembles of interacting JPs on a substrate17. These measurements relied on the longitudinal magneto-optical Kerr effect and magnetic force microscopy. They found an onion state with a large remanent magnetization in JPs with an afm layer. Nevertheless, given that the measurements were done on close-packed ensembles of JPs, they do not exclude effects due to the interaction between the particles and, therefore, cannot be used to infer the behavior of isolated JPs.

We fabricate the magnetic JPs by coating a self-assembled template of 1.5 μm-sized silica spheres with thin layers of different materials via sputter-deposition. The non-magnetic silica spheres are arranged on a silica substrate using entropy minimization18, which allows the formation of hexagonal close-packed monolayers. JPs with two different layer stacks, shown in Fig. 1 (b), are produced. Ferromagnetic JPs (fmJPs) are fabricated by depositing a 10 nm-thick Cu buffer layer directly on the silica spheres, followed by a 10 nm-thick layer of ferromagnetic CoFe. The film is sealed by a final 10 nm-thick layer of Si. A second type of JP, which we denote exchange-bias JPs (ebJPs), includes an additional 30 nm-thick afm layer of Ir83Mn17 between the Cu buffer and the fm layer. Layer deposition is performed by sputtering in an external magnetic field of 28 kA/m applied in the substrate plane, i.e. in the equatorial plane of the JPs, in order to initialize the exchange bias by field growth. This fabrication process is described in detail in Tomita et al.17. Individual JPs are then attached to the apex of a cantilever for magnetic characterization in a last fabrication step.
A magnetic specimen is attached to the tip of a cantilever, which is driven in a feedback loop at its resonance frequency $f$ with a fixed amplitude, actuated by a piezoelectric transducer. A uniform external magnetic field $H$ is applied to set the magnetic state of the specimen under investigation. $H$ can be rotated within the plane perpendicular to the cantilever’s rotation axis ($xz$-plane) with a span of $117.5^\circ$ and a maximum field amplitude of $H = \pm 3.5 \, \text{T}$. Its orientation is set by the angle $\theta_b$ as defined and indicated in Fig. 1 (a). The magnetic torque acting on the sample results in a deflection of the cantilever as well as a shift in its resonance frequency, which is given by

$$ \Delta f = f - f_0 = \frac{f_0}{2 k_0 l_c^2} \left( \frac{\partial^2 E_m}{\partial \theta_c^2} \right)_{\theta_c=0}. \tag{1} $$

$f_0$ and $k_0$ are the resonance frequency and spring constant of the cantilever at zero applied field, respectively, $l_c$ is the effective length of the cantilever, $\theta_c$ the oscillation angle, and $E_m$ the magnetic energy of the specimen. Properties of the cantilevers used in the experiments can be found in section I of the appendix.

In the limit of large applied magnetic field, such that the Zeeman energy dominates over the anisotropy energy, all magnetic moments align along $H$. In this limit $\Delta f$ asymptotically approaches a value determined by the involved anisotropies, their respective directions, the total magnetic moment, $\theta_b$, and the mechanical properties of the cantilever$^{19,20}$. In particular, the determination of these asymptotes allows us to extract the direction of magnetic easy and hard axes by measuring the angular dependence of the frequency shift at high field $\Delta f_{\text{hf}}$. The maximum positive (minimum negative) $\Delta f_{\text{hf}}(\theta_b)$ indicates the easy (hard) axis. For these measurements, shown in Fig. 2 (c), we apply $\mu_0 H = 3.5 \, \text{T}$, where we expect to be in the high field limit. See section VI and VII of the appendix for more details. We also measure the magnetic hysteresis of $\Delta f(H)$ by sweeping the applied field $H$ up and down for $\theta_b$ fixed to the values determined for the magnetic easy and hard axes, respectively. This procedure reveals signatures of the JPs’ magnetic reversal, as shown in Figs. 2 (d), (e) and 3.

In order to analyze both these types of measurements, we perform micromagnetic simulations using the finite-element software Nmag$^{21}$. We calculate $\Delta f(H)$ from the micromagnetic state for parameters set by the experiment$^{19,22}$. Matching these simulations to the measurements gives us a detailed understanding of the progression of the magnetic configurations present in the JPs throughout the reversal process. Events, such as the nucleation of a magnetic vortex, can be identified and associated with features in $\Delta f(H)$ measured via DCM.

Fig. 2 (a) and (b) shows false color SEMs of the measured fmJP and ebJP, respectively, each attached to the tip of a cantilever. The orientation of the particles

---

**Figure 1.** (a) Sketch of a cantilever with a JP attached to its tip and definition of the coordinate system. (b) Cross-sectional SEM of a JP showing the gradient of the layer thickness. The two investigated layer stacks of the hemispherical cap are shown in the insets. (c), (d) Definition of the angles defining the orientation of a JP on the cantilever ($\theta_{JP}$ and $\varphi_{JP}$).

Note that the values given for thicknesses are nominal and that the film thickness gradually reduces towards the equator of the sphere with respect to the top, as shown in Fig. 1 (b), because of the deposition process$^{17}$. Furthermore, the touching points of the next neighbors in the hexagonal closed packed arrangement of the silica spheres on the substrate template impose a lateral irregularity on the equatorial line of the capping layers. This is best seen in Fig. 2 (a) and (b).

We measure the magnetic hysteresis of each an individual fmJP and an individual ebJP via DCM. DCM is a technique to investigate individual, nano- to micrometer-sized magnetic specimens, similar to a standard vibrating superconducting quantum interference device (SQUID) magnetometer. The key differences are that DCM is sensitive enough to measure much smaller magnetic volumes than a vibrating SQUID magnetometer and that it measures magnetic properties with respect to rotations of the external magnetic field, rather than modulations of its amplitude as in measurements of magnetic susceptibility. Details on the technique and measurement setup can be found in Refs. 19 and 20.
Hence, we can expect a magnetic JP to relax to its ground state configuration over time. The simulation of the fmJP shows a symmetric asymptotic behavior with $\Delta f_{\text{hf}}$ values differing by about 0.9 Hz for $\mu_0 H = \pm 3.5$ T, as seen in the brown curve of Fig. 2 (d). Furthermore, after a full hysteresis cycle, we observe a reduction in the difference of $\Delta f_{\text{hf}}$ at $\pm 3.5$ T by about 0.4 Hz, which is evidence for magnetic training\textsuperscript{23,24}. Measurements of the ebJP also show a highly asymmetric magnetic reversal, which occurs at $\mu_0 H = -44$ mT when sweeping the field down and at $\mu_0 H = 12$ mT when sweeping the field up. All of these findings are characteristic of an exchange bias imposed on the fm layer by the afm layer.

In order to draw conclusions about the magnetic state of the JPs, we establish a micromagnetic model for each of the two types of JPs over many iterations of comparison to measured $\Delta f(H)$ and variations of the parameters for $\mathbf{H}$ applied along the magnetic easy and hard axis, respectively. This iterative process, information from literature, and observations from SEMs lead to a final set of parameters (see appendix, section I) and a model that reproduces the measured $\Delta f(H)$. The model assumes that the magnetic JPs are made from a hemispherical shell with a thickness gradient from the pole towards the equator, which accounts for the gradual reduction of the shell thickness away from the pole, as shown in Fig. 1 (b). The hemisphere is also truncated\textsuperscript{25} by a latitudinal belt around the equator, reflecting observations from the SEM images in Figs. 2 (a) and (b). For simplicity, in the simulations, we do not account for the magnetic film’s irregular edge at the equator and a possible change in the crystallographic texturing with respect to the particle surface as a function of position within the cap. The orientation of a JP with respect to the cantilever rotation axis and $\mathbf{H}$ is set by inferring the orientation from the SEMs and followed by an iterative tuning of the angles ($\theta_{\text{JP}}, \varphi_{\text{JP}}$), as defined in Fig. 1 (d), to match the measured $\Delta f(H)$.

Figs. 3 (a) and (b) shows agreement between the measured and simulated $\Delta f(H)$ curves for the fmJP for $\mathbf{H}$ applied along the magnetic easy and hard axis, respectively. Similar curves are shown for the ebJP in Figs. 3 (c) and (d). Details on the progression of the magnetization configurations as a function of $H$, as indicated by the simulations, are found in section II of the appendix. Here, we focus on the remanent magnetization configurations in both types of particles, as shown on the right of Fig. 3.

For the fmJP, an onion state is realized after sweeping $\mathbf{H}$ along the easy axis while a global vortex state is found after sweeping along the hard axis. In applications, magnetic JPs are subject to considerable disturbances from the outside, including thermal activation, interactions with other nearby magnetic particles, and alternating external magnetic fields for actuation. Hence, we can expect a magnetic JP to relax to its ground state configuration over time. The simulation of the fmJP

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{False color SEMs of the (a) fmJP and (b) ebJP attached to the tip of a cantilever, respectively. The coordinate system is shown on the right. (c) $\Delta f_{\text{hf}}(\theta_h)$ measured at $\mu_0 H = 3.5$ T for the fmJP (blue) and ebJP (brown). Black circles indicate $\theta_h$ of the hysteresis measurements, which are shown in (d), (e), and Fig. 3.}
\end{figure}

in the images can be correlated with the angle $\theta_h$ of the maxima and minima found in the high-field frequency shift $\Delta f_{\text{hf}}(\theta_h)$ in dependence of the magnetic field angle $\theta_h$, shown in Fig. 2 (c). Doing so, we find a magnetic easy direction in the equatorial plane of the particles and a hard direction along the axis of the pole. The 90° angle between easy and hard direction is a clear indication that uniaxial anisotropy is the dominant anisotropy in the system. We ascribe the latter to the shape of the JPs, because no other strong anisotropies are expected. The field-dependent frequency shift $\Delta f(H)$ for $\mathbf{H}$ aligned along the easy axis, see Fig. 2 (d), shows a typical hysteretic, V-shaped curve, that approaches a horizontal asymptote for high field magnitudes\textsuperscript{19}. The fmJP shows a symmetric asymptotic behavior for $\mu_0 H = 3.5$ T and $-3.5$ T (blue curve). Magnetic reversal at low fields, $\mu_0 H$ around $\pm 20$ mT, is symmetric upon reversal of the field sweep direction, as shown in Fig. 2 (e). This behavior is expected for a ferromagnetic particle with a magnetic field applied along its easy axis.
show that when the magnetization is in the global vortex state, its magnetic energy is 101 aJ lower than when it is in the onion state. The global vortex state is therefore energetically more favorable than the onion state, which is also true if compared to any other remanent state that we have found in simulations for fmJPs, as discussed in appendix, section IV. This analysis suggests that a remanent global vortex state, which has a vanishing total magnetic moment, is realized in fmJPs over time, independent of magnetic history. If we normalize the magnetic moment of this state by the magnetic moment of the fmJP for both the easy and hard-axis alignments, as shown in Fig. 3 (c) and (d). This value of $K_{eb}$ is also consistent with results from Ref. 28, once the influence of a reduced sample size is considered.

At remanence, the simulations show that the ebJP hosts an onion state irrespective of its magnetic history, as shown in Figs. 3 (c) and (d). To exclude the presence of an equilibrium global vortex state, we test this state’s stability by initializing the ebJP in a global vortex state at remanence and then relaxing the system to a local energetic minimum. Following this procedure, the system relaxes to the onion state. As a result, we can exclude the global vortex state as a possible equilibrium remanent state in this ebJP.

For the simulations of the ebJP we find a total magnetic moment at remanence, normalized by its maximum value of $M_s$, of 0.89 and 0.71 depending on whether $H$ is applied along the hard or easy direction of the external field, respectively. This remanent moment represents an increase of more than one order of magnitude compared to the remanent moment of the fmJP.
Hence, introducing exchange bias to magnetic JPs, if strong enough, succeeds in stabilizing a high-moment onion state in remanence.

To conclude, micrometer-sized JPs capped with an antiferromagnetic/ferromagnetic or purely ferromagnetic thin film system have been mass produced through a sputter-deposition process. We have investigated the magnetic reversal and remanent magnetic configurations of individual specimens of these JPs using DCM and corresponding micromagnetic simulations. Although the fmJPs host a global vortex state in remanence with a vanishing magnetic moment, the addition of an antiferromagnetic layer in ebJPs successfully changes the remant configuration to a stable high-moment onion state. Unlike previous measurements on close packed particle arrays, our measurements on individual JPs show that the stability of this high magnetic moment texture in remanence is a property of the individual particles and present in absence of interparticle interactions.

We thank Sascha Martin and his team in the machine shop of the Physics Department at the University of Basel for help building the measurement system. We acknowledge the support of the Canton Aargau and the Swiss National Science Foundation under Grant No. 200020-159893, via the Sinergia Grant Nanoskyrmionics (Grant No. CRSII5-171003), and via the National Centre for Competence in Research Quantum Science and Technology. Calculations were performed at sciCORE (http://scicore.unibas.ch/) scientific computing core facility at University of Basel.

The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

1. Andreas Walther and Axel H. E. Müller. Janus Particles: Synthesis, Self-Assembly, Physical Properties, and Applications. Chem. Rev., 113(7):5194–5201, July 2013.
2. D. Issadore, Y. I. Park, H. Shao, C. Min, K. Lee, M. Liong, R. Weisssleder, and H. Lee. Magnetic sensing technology for molecular analyses. Lab Chip, 14:2385–2397, 2014.
3. Arno Ehresmann, Iris Koch, and Dennis Holzinger. Manipulation of superparamagnetic beads on patterned exchange-bias layer systems for biosensing applications. Sensors, 15(11):28854–28888, 2015.
4. Brandon H. McNaughton, Karen A. Kelheim, Jeffrey N. Anker, and Raoul Kopelman. Sudden breakdown in linear response of a rotationally driven magnetic microparticle and application to physical and chemical microsensing. The Journal of Physical Chemistry B, 110(38):18958–18964, 2006. PMID: 16986890.
5. Brandon H. McNaughton, Paivo Kinnunen, M. Shlomi, Co-drin Cionca, Shao Ning Pei, Roy Clarke, Panos Argyrakis, and Raoul Kopelman. Experimental system for one-dimensional rotational brownian motion. The Journal of Physical Chemistry B, 115(18):5212–5218, 2011. PMID: 21500841.
6. Randall M. Erb, Joshua J. Martin, Rasam Soheiliyan, Chunzhou Pan, and Jabulani R. Barber. Actuating Soft Matter with Magnetic Torque. Advanced Functional Materials, 26(22):3859–3880, 2016. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/adfm.201504699.
7. Gabi Steinbach, Michael Schreiber, Dennis Nissen, Manfred Albrecht, Sibyllie Gemming, and Artur Erbe. Anisotropy of colloidal components propels field-activated stirrers and movers. Physical Review Research, 2(2):023092, April 2020. Publisher: American Physical Society.
8. K. V. T. Nguyen and Jeffrey N. Anker. Detecting de-gelation through tissue using magnetically modulated optical nanoprobes (MagMOONs). Sensors and Actuators B: Chemical, 205:313–321, December 2014.
9. L. Baraban, M. Tasinkevych, M. N. Popescu, S. Sanchez, S. Dieterich, and O. G. Schmidt. Transport of cargo by catalytic Janus micro-motors. Soft Matter, 8(1):48–52, 2012.
10. Leonardo F. Valadares, Yu-Guo Tao, Nicole S. Zacharia, Vladimir Kitaev, Fernando Galenbeck, Raymond Kapral, and Geoffrey A. Ozin. Catalytic Nanomotors: Self-Propelled Sphere Dimers. Small, 6(4):565–572, 2010.
11. Ramn Golestanian, Tanniemola B. Liverpool, and Armand Ajdari. Propulsion of a Molecular Machine by Asymmetric Distribution of Reaction Products. Phys. Rev. Lett., 94(22):220801, June 2005.
12. Ugur Bozuyuk, Yunus Alapan, Amirreza Aghakhani, Muhammad Yunusa, and Metin Sitti. Shape anisotropy-governed locomotion of surface microrollers on vessel-like microtopographies against physiological flows. Proceedings of the National Academy of Sciences, 118(13), March 2021. Publisher: National Academy of Sciences Section: Physical Sciences.
13. Robert Streubel, Volodymyr P. Kravchuk, Denis D. Sheka, Denys Makarov, Florian Kronast, Oliver G. Schmidt, and Yuri Gaididei. Equilibrium magnetic states in individual hemispherical perellloy caps. Applied Physics Letters, 101(13):132419, September 2012. 00007 tex.ids: streubel_equilibrium_2012-1.
14. W. H. Meiklejohn and C. P. Bean. New Magnetic Anisotropy. Physical Review, 105(3):904–913, February 1957. Publisher: American Physical Society.
15. J. Nogués and Ivan K Schuller. Exchange bias. Journal of Magnetism and Magnetic Materials, 192(2):203–232, February 1999.
16. L Stamps. Mechanisms for exchange bias. Journal of Physics D: Applied Physics, 34(3):444–444, Jan 2001.
17. Andreaa Tomita, Meike Reginka, Rico Huhnstock, Maximilian Merkel, Dennis Holzinger, and Arno Ehresmann. Magnetic textures in hemispherical thin film caps with in-plane exchange bias. Journal of Applied Physics, 129(1):015305, January 2021.
18. R. Micheleotto, H. Fukuda, and M. Ohtsu. A simple method for the production of a two-dimensional, ordered array of small latex particles. Langmuir, 11(9):3335–3336, 1995.
19. B. Gross, D. P. Weber, D. Rüffer, A. Buchter, F. Heimbach, A. Fontcuberta i Morral, and M. Poggio. Dynamic cantilever magnetometry of individual CoFeB nanotubes. Phys. Rev. B, 93(6):064409, February 2016.
20. B. Gross, S. Philipp, E. Josten, J. Leliaert, E. Weterskog, L. Bergström, and M. Poggio. Magnetic anisotropy of individual maghemite mesocrystals. Phys. Rev. B, 103:014402, Jan 2021.
21. T. Fischbacher, M. Franchin, C. Bordignon, and H. Fauquhar. A systematic approach to multiphysics extensions of finite-element-based micromagnetic simulations: Nmag. IEEE Transactions on Magnetics, 43(6):2896–2898, 2007.
22. A. Mehin, B. Gross, M. Wyss, T. Schefer, G. Tüüüncicüoglu, F. Heimbach, A. Fontcuberta i Morral, D. Grundler, and M. Poggio. Observation of end-vortex nucleation in individual ferromagnetic nanotubes. Phys. Rev. B, 97:134422, Apr 2018.
23. A. Buchter, R. Wilbing, M. Wyss, O. F. Kieler, T. Weimann, J. Kohlmann, A. B. Zorin, D. Rüffer, F. Matteini, G. Tüüüncicüoglu, F. Heimbach, A. Kleibert, A. Fontcuberta i Morral, D. Grundler, R. Kleeer, D. Koelle, and M. Poggio. Magnetization reversal of an individual exchange-biased nanotub
nanotube. *Phys. Rev. B*, 92(21):214432, December 2015.

24. Amitesh Paul, C. Schmidt, N. Paul, A. Ehresmann, Stefan Mattauch, and Peter Böni. Symmetric magnetization reversal in polycrystalline exchange coupled systems via simultaneous processes of coherent rotation and domain nucleation. *Phys. Rev. B*, 86:094420, Sep 2012.

25. A. Mourkas, A. Zarlaha, N. Kourkoumelis, and I. Panagiotopoulos. Curvature induced stabilization of vortices on magnetic spherical sector shells. *Journal of Magnetism and Magnetic Materials*, 524:167676, April 2021.

26. Nicolas David Müglich, Alexander Gaul, Markus Meyl, Arno Ehresmann, Gerhard Götz, Günter Reiss, and Timo Kuschel. Time-dependent rotatable magnetic anisotropy in polycrystalline exchange-bias systems: Dependence on grain-size distribution. *Phys. Rev. B*, 94:184407, Nov 2016.

27. J. Geshev, L. G. Pereira, and J. E. Schmidt. Rotatable anisotropy and coercivity in exchange-bias bilayers. *Phys. Rev. B*, 66:134432, Oct 2002.

28. Maximilian Merkel, Rico Huhnstock, Meike Reginka, Dennis Holzinger, Michael Vogel, Arno Ehresmann, Jonas Zehner, and Karin Leistner. Interrelation between polycrystalline structure and time-dependent magnetic anisotropies in exchange-biased bilayers. *Physical Review B*, 102(14):144421, October 2020. Publisher: American Physical Society.

29. J. Nogués, J. Sort, V. Langlais, V. Skumryev, S. Suriñach, J. S. Muñoz, and M. D. Baró. Exchange bias in nanostructures. *Physics Reports*, 422(3):65–117, December 2005.
I. CANTILEVER PROPERTIES AND SIMULATION DETAILS

Cantilevers are fabricated from undoped Si. They are 75 µm-long, 3.5 µm-wide, 0.1 µm-thick with a mass-loaded end and have a 11 µm-wide paddle for optical position detection. The resonance frequency $f_0$ of the fundamental mechanical mode used for magnetometry is on the scale of a few kilohertz. Spring constant $k_0$ and effective length $l_e$ are determined using a finite element approximation. For the cantilever used for the fmJP we find $f_0 = 5285.8$ Hz, $k_0 = 249$ µN/m and $l_e = 75.9$ µm. For the cantilever of the ebJP $f_0 = 5739.3$ Hz, $k_0 = 240$ µN/m and $l_e = 75.9$ µm.

Micromagnetic simulations are performed with the finite-element software package Nmag. As an approximated geometry for the JPs a semi-sphere shell with a thickness gradient from the pole towards the equator is used, that is truncated at the equator, as discussed in the main text. The exchange constant is set to $A_{ex} = 30$ pJ/m$^3$.

In case of the fmJP, opposing to the SEM image in Fig. 2 (a) in the main text, which suggests a truncation of the fm layer by about 250 nm, it needs to be set to 350 nm or even more to match the high field progression of $\Delta f(H)$. For the same reason the nominal thickness of 10 nm of the fm layer needs to be increased to at least 12 nm at the pole, which is then gradually reduced to 0 at the equator. These two geometric constraints are necessary to keep $M_s$ at a reasonable value below the bulk value of 1.95 MA/m. This suggests, that significantly more fm material than anticipated is deposited on the region around the pole of the JPs, which is the most directly exposed area of the sphere during deposition.

For the simulation of the fmJP we set the following parameters: saturation magnetization $M_s = 1.8$ MA/m, silica sphere diameter of 1.5 µm, particle orientation ($\theta_{JP}, \varphi_{JP}$) = (91°, 2°) and maximum allowed mesh cell size 7.5 nm.

For the ebJP, the geometric parameters have to be adjusted less from their nominal values than for the fmJP, in order to match between the micromagnetic model to the experiment. This result suggests that the afm layer, which is deposited before the fm, acts as an adhesive for the fm, and the ebJP is coated more homogeneously than the fmJP.

For the simulation of the ebJP we set the following parameters: $M_s = 1.44$ MA/m, fm layer thickness of 10 nm at the pole, gradually reduced to 0 at the equator, silica sphere diameter of 1.5 µm, ($\theta_{JP}, \varphi_{JP}$) = (85°, 10°), maximum allowed mesh cell size 7.0 nm, ($\theta_{eb}, \varphi_{eb}$) = (−90°, 0°), unidirectional anisotropy constant $K_{eb} = 22.5$ kJ/m$^3$.

For the generic simulations in sections V and IV we have used $M_s = 1.8$ MA/m, fm layer thickness of 10 nm at the pole, gradually reduced to 1 nm at the equator, a silica sphere diameter of 500 nm, ($\theta_{JP}, \varphi_{JP}$) = (0°, 0°), and ($\theta_{eb}, \varphi_{eb}$) = (−90°, 0°). Parameters that are not mentioned here are given in the main text.

II. PROGRESSION OF THE MAGNETIC STATE WITH EXTERNAL FIELD

A. Ferromagnetic Janus particles

$\Delta f(H)$, measured for $H$ parallel to the magnetic easy (blue data) and hard axis (orange data), respectively, as shown in Fig. 1 (a), gives direct information on high field behavior and magnetic reversal of the JPs. For $H$ parallel to the magnetic easy axis, an overall V-shape suggest Stoner-Wohlfarth like behavior for most of the field range in Fig. 1 (a). As seen in the close-up in Fig. 1 (b), magnetic reversal appears to take place through a few sequential switching events at small negative reverse fields. The simulated $\Delta f(H)$, also shown in Fig. 1 (green points) together with a few exemplary configurations of the simulated magnetic state of the JP, can give more insight into what happens during the field sweep. Starting from full saturation, most magnetic moments stay aligned with the easy direction down to very low reverse fields, nicely seen in Fig. 1, configuration 1 at 3.5 T and configuration 2 at remanence. The latter is an onion state. This progression of configurations is consistent with the Stoner-WoLFarth-like behavior of the experimental $\Delta f(H)$. Magnetic reversal takes place through the occurrence of a so-called S-state, for which the magnetization follows the curvature of an S. The reversal is shown in configurations 3 and 4 in Fig. 1. Then, until full saturation is reached in reverse field, only magnetic moments in proximity to the equator of the JP are slightly canted away from the direction of the external field (and the easy plane). This progression is robust in simulation, even though sometimes, depending on slight variations of simulation parameters, a vortex appears in reverse field instead of the S-state. The observation of several, individual switching events during magnetic reversal in experiment may originate...
in vortex hopping, or switching of different regions in the JP due to variations in material and geometric parameters. Magnetic reversal through a vortex rather than an S-state may also explain the big difference of the coercive fields between experiment ($H_c \approx 32 \text{ mT}$) and simulation ($H_c = 6 \text{ mT}$).

For $H$ parallel to the magnetic hard axis, the experimental data has an inverted V-shape, and there is no easily identifiable sign of magnetic reversal. Yet, for relatively large fields, around 1.5 T, switching events are observed, and exist up to negative fields of similar magnitude. Typically, W-shaped curves are observed for measurements with the field aligned with the hard direction, and can be understood in a simple Stoner-Wohlfarth model, which is discussed in section III. The inverted V-shape instead of the W-shape is a peculiarity of the fact that the fm layer of the JP is curved everywhere. The angle between the local surface normal and the direction of the external magnetic field is different for every polar coordinate of the JP, which leads to a dependence of the local demagnetizing field on the polar coordinate. In consequence, the magnitude of the external magnetic field, for which the local magnetic moments start to rotate towards their local easy direction depends strongly on the position in the magnetic cap. This leads to the observed curve shape of $\Delta f(H)$, a more detailed discussion can be found in section IV. The magnetic progression in simulation for external field alignment with the hard direction can be summarized as follows: Starting from full saturation, the magnetic moments start rotating towards the easy plane with decreasing field magnitude due to the competition between shape anisotropy and Zeeman energy. This takes place for different magnitudes of $H$ depending on where a magnetic moment is located in the JP, as discussed earlier. Configuration 5 in Fig. 1 shows a state for which magnetic moments at the pole have already started to rotate, while magnetic moments in proximity to the equator remain aligned with the external field. Superimposed to this rotation, a minimization of the system’s energy by formation of a magnetic vortex localized at the pole for around 1.8 T takes place, which grows in size with decreasing field, see configuration 6. In simulation, this is a gradual evolution, and only for fields below about 300 mT jumps in $\Delta f$ due to vortex movement are observed. This process is in contrast to the discontinuities that occur at around 1.5 T in the experiment, but can be explained by vortex hopping from pinning site to pinning site. The latter may be present due to fabrication inhomogeneities in the JPs. For zero field the vortex dominates the magnetic configuration of the JP and has evolved into a global vortex state, as shown in configuration 7. Configuration 8 shows the vortex in reverse field, which has changed polarity, and has

---

Figure 1. Data for the ferromagnetic Janus particle. (a) Measured $\Delta f(H)$ for easy (blue) and hard (orange) alignment of the external field, as well as the simulated magnetic configurations in green (easy) and red (hard). (b) Close-up of (a) for low fields. Data with easy orientation is offset by 2 Hz for better visibility. Colored arrows indicate field sweep directions. Numbers in (a) and (b) denote the field values for the configurations of the magnetic state.
jumped to a slightly off-centered position. The latter is too small to be visible in the figure. Further decreasing the field, the vortex sits centrally in the JP and shrinks in size, and vanishes around $-1.82 \, \text{T}$. At the same time, the magnetic moments rotate towards the field direction depending on their position in the JP, as described earlier. Note, that by slightly changing simulation parameters, we find that features due to vortex entrance and hopping may manifest themselves in $\Delta f (H)$ with strongly differing magnitude and for different field values. Introducing artificial pinning cites in simulation can be used to adjust the vortex hopping to match the observed signals more precisely\(^5\), but consumes vast amounts of computational time and should still be understood only as an exemplary progression of the magnetic state.

B. Exchange-biased Janus particles

The progression of the magnetic state for the ebJP is very similar to the fmJP, yet, there are crucial differences. See Fig. 2 for the DCM data, simulation results, and configurations of some magnetic states. For the field oriented in the magnetic easy direction the nearly polarized state, shown in Fig. 2 configuration 1, is similar to that shown in Fig. 1, configuration 1. Reducing the field down to remanence, as shown in Fig. 2 configuration 2, we find an onion state just as for the fmJP. Magnetic reversal occurs again through an S-state, rather than via vortex formation, as shown in configuration 3. However, the reversal is shifted towards negative fields, and occurs for $-15 \, \text{mT}$ for the down sweep, and for $-17.5 \, \text{mT}$ for the up sweep of the magnetic field. This does not match the experimentally observed values, especially for the latter case, for which the switching occurs for positive field. This is no surprise, since the employed model does not account for the contribution of the exchange bias to the coercivity. Yet, both simulation and experiment show a shift of the hysteresis loop towards negative fields as compared to the fmJP.

We only observe a single switching event in experiment for the magnetic reversal, which is consistent with the behavior of the S-state in simulation. For the alternative magnetic reversal process through vortex formation, we would expect several switching events due to vortex hopping. We find such a situation e. g. for a few reversal processes without unidirectional anisotropy, where geometrical parameters of the JPs have been varied, see section V. Yet, it is also possible, that a strong pinning site favors the formation of a vortex, and keeps it in place for all field magnitudes up to the reversal point.

If the magnetic field is swept from negative saturation up to remanence (not shown here), an onion state is present, that has its total magnetic moment pointing opposite to the exchange bias direction. For applications, this is an undesirable state. It is energetically less favorable than the state of parallel alignment, and if the energy barrier between the two states is overcome by an external influence, the JP will switch.

If the external field is aligned with the hard direction, and starts at full saturation, the magnetic moments rotate towards the easy plane depending on their position in the JP for decreasing field just as for the fmJP. Further, the same, superimposed vortex formation takes place, starting for $1.34 \, \text{T}$. Again the vortex occupies more and more volume of the JP with further decreasing field.

Superimposed, a local vortex forms at the pole of the ebJP for an applied field of $1.34 \, \text{T}$. As for the fmJP, the vortex occupies more and more volume of the JP with further decreasing field. However, upon further reducing the field, the vortex, rather than inhabiting the whole JP as a global vortex centered at the pole of the fmJP, it prefers to move to the side of the ebJP, as shown in configuration 4 of Fig. 2. Moving down from the pole towards the equator, the vortex exits from the JP through the equator for $5 \, \text{mT}$, and an onion state is formed at remanence, as shown in configuration 5. The orientation of the onion state is governed by $\hat{u}_{\phi}$. For a small reverse field a domain wall state forms, as shown in configuration 6. With further decreasing field, the domain wall is rotated with respect to the polar axis of the JP. This state seems to be a precursor of the vortex state, and the wall is subsequently replaced by the vortex, sitting again in the center of the JP, as shown in configuration 7. The vortex vanishes for $-1.36 \, \text{T}$. Whether such a domain wall state is indeed realized in the ebJPs for reverse fields, or if a vortex enters from the equator and moves back to the center of the JP, as seen for simulations of smaller JPs (see section V), remains an open question. The DCM signal shows in both experiment and simulation many irregularities for the lower field range, which does not allow us to draw clear conclusions on the magnetic state present in the JPs. Nevertheless, the simulations clearly suggest that an onion state should be realized at remanence, irrespective of the states present during the hysteresis. This situation is markedly different than that of the fmJP and is a direct consequence of the presence of exchange bias.

III. STONER-WOHLARTH MODEL FOR EASY PLANE TYPE ANISOTROPY

The shape of the magnetic material of the fmJPs is, at least in a first approximation, rotationally symmetric around the pole axis. Further, the thickness gradient of the CoFe layer, as shown in the cross-sectional SEM in Fig. 1 (b) in the main manuscript, suggest that the magnetic material is concentrated in proximity to the pole, and that there is less material towards the equator. This material distribution suggests that a uniaxial anisotropy of easy plane type is imposed on the sample by its shape.

The most basic approach to describe such a system is a Stoner-Wohlfarth model, in which a single macro magnetic moment replaces the ensemble of distributed mag-
netic moments. Following Ref. 7, it is straightforward to calculate the magnetic hysteresis and connected DCM response for easy plane anisotropy, where the latter is manifested in a positive, effective demagnetizing factor $D_u$, opposing to a negative $D_u$ for easy axis anisotropy. The model is a good approximation for high fields, where all magnetic moments are aligned with the external field, and essentially behave like a single macro spin. At lower fields deviations from the curve of the SW-model indicate inhomogeneous spin orientation. We show the magnetic hysteresis of the components $m_x$, $m_y$ and $m_z$ of the normalized magnetization $m$ and the DCM response $\Delta f$ in Fig. 3 for several different orientations ($\theta_u$, $\phi_u$) of the uniaxial anisotropy axis $\hat{u}$, which is perpendicular to the easy plane. The external field $H$ has to be fixed in the $z$-direction in the model for technical reasons, which is why $\hat{u}$ is varied rather than $H$, contrary to the situation in experiment. Hence, the angles $\theta_{JP} - \theta_h$ and $\phi_{JP}$, describing the equivalent situation in experiment, have to be compared to $\theta_u$ and $\phi_u$, respectively. However, this does not limit the validity of the model.

For $\theta_u = 0^\circ$, for which $H$ is perpendicular to the easy plane, we find the typical W-shape for $\Delta f$ for a magnetic hard orientation of $H^7$. The columnar arrangement of the components of the magnetization $m_x$, $m_y$ and $m_z$ together with $\Delta f$ allow to correlate changes in magnetic behavior with features in $\Delta f$, such as e.g. the transition from field alignment to the beginning of a rotation of $m$ towards the easy axis.

For $\theta_u = 90^\circ$, for which $H$ lies in the easy plane, the typical V-shaped curve for an easy orientation of $H$ is found, just as expected. This V-shape is connected to a perfect square hysteresis of $m$. Intermediate values of $\theta_u$ lead to intermediate curve progression, which are a combination of the two extrema described above.

Compared to experiment (see Figs. 2 and 3 in the main manuscript) we find, that while in the case of the external field $H$ in the easy plane (last panel in the last row in Fig. 3) the model generates relatively similar curve shapes for $\Delta f$, this is not true for $H$ perpendicular to the easy plane (first panel in the last row). The vertical asymptotes for $h \approx \pm 0.5$ are missing in experiment. Further, the horizontal asymptote at high field is approached from negative rather than positive values, as it is seen in experiment. A more detailed analysis in the following section will show, that the latter originates in the curvature of the magnetic shell, which cannot be captured by a single spin model. Even though typically a good starting point, the SW model seems to be of rather limited validity for the description of the relatively specific geometry of a spherical cap.
Figure 3. Components of $\mathbf{M}$ normalized by $M_s$ (first 3 rows) and $\Delta f$ normalized by $\frac{f_{0\mu_0V\mu_0V^2}{2k_0l_0^2}}{h}$ (last row) vs. normalized magnetic field $h = \frac{H}{H_M}$ for different orientations of the anisotropy axis. Valid for uniaxial anisotropy with $D_u > 0$ (here $D_u = 0.5$). $\theta_u$ is increased from $0^\circ$ in the first column by $30^\circ$ per column up to $90^\circ$. $\phi_u$ is changed in the same steps, given by the different, color-coded graphs within each column (blue for $\phi_u = 0^\circ$, orange for $30^\circ$, green for $60^\circ$ and red for $90^\circ$). Arrows indicate switching of the magnetization. $\Delta f$ has been scaled by a factor 0.5 and offset by 0.5 for $\theta_u = 0^\circ$, and scaled by 2 for the other values of $\theta_u$.

IV. SHAPE ANISOTROPY OF A TRUNCATED SPHERICAL HALFSHELL

As a measure for the strength of the shape anisotropy present in the JPs, and as a parameter used for SW-modeling, the knowledge of the effective demagnetization factor $D_u$ of the geometry is of value. If possible, $D_u$ is determined using an analytical expression\(^8\), which, however, is not known for the given geometry. Using micromagnetic simulations as discussed in the main text, we can extract a good approximation to the demagnetization factor of a given geometry, without necessity for an analytical formula. Here, we analyze a generic, truncated spherical halfshell as defined in section I for different degrees of truncation $d$.

We quickly recall the definition of the effective demagnetization factor $D_u = D_z - D_x$, where $D_z$ and $D_y = D_x$ are the demagnetization factors of a magnetic object that is rotationally symmetric in the $xy$ plane. $-0.5 < D_u < 1$ is true, and $D_u < 0$ is valid for prolate and $D_u > 0$ for oblate bodies. $D_u = 0$ is the case for a perfectly spherical body. We find a minimum $D_u$ of approximately 0.25 for the smallest truncation, and $D_u$ increases with truncation as shown in Fig. 4 (a). Hence, shape anisotropy gets stronger as $d$ is increased, which can be roughly understood as the transformation from a spherical halfshell to a disc. Note, that a spherical halfshell without thickness gradient and without truncation leads to a $D_u$ just slightly large than zero. This implies, that pointing in the easy plane is not very much more favorable than any other direction for the magnetization averaged over the whole sample, see the orange dots in Fig. 4.

The high field frequency shift, $\Delta f_{hf}$, which is of relevance for extracting anisotropy constants from experiment\(^7,9\), first increases with truncation, but later decreases again, see Fig. 4 (c). This is owed to the loss
of magnetic material for increasing truncation as seen in Fig. 4 (b).

In contrast to the SW model the micromagnetic simulations are able to correctly reproduce the experimental curve shape of $\Delta f$ for the hard axis orientation, see Fig. 6 (d). To understand the reason for this, the $x$ component of the demagnetizing field $H_{\text{demag},x}$ within the magnetic layer is visualized for a cut through the geometry in Fig. 5 at high applied field in $x$ direction. It shows a gradual change of the demagnetizing field magnitude with $z$ position, which is a good measure of the preferred orientation of a magnetic moment (top part with $H_{\text{demag},x} \approx 0$ prefers $x$ orientation, opposing to bottom part with maximum $H_{\text{demag},x}$, which needs maximum external field to be aligned in $x$ direction). This shows, that magnetic moments in proximity to the pole need the smallest field magnitude to be aligned with a field in $x$ direction. The required field magnitude gradually increases the closer a magnetic moment is situated to the equator. This explains the gradual change of $\Delta f$ with increasing field in this orientation, opposing to what is evident in the SW model, where all magnetic moments rotate in unison.

V. GENERIC SIMULATIONS OF TRUNCATED SPHERICAL HALFSHELLS WITH AND WITHOUT EXCHANGE BIAS

The speedup of simulations due to a reduced size of the JPs, as defined in section I, allows us to analyze the influence of simulation parameters such as the truncation on the magnetic hysteresis. In Fig. 6 a set of data for JPs with different truncations is shown. Besides the differences in high field asymptotes as already discussed in section IV, the truncation also significantly changes the curvature of $\Delta f(H)$, see Figs. 6 (a) and (d). By adjusting the truncation, this allows to fit the curvature of a given experimental $\Delta f(H)$ in the simulation.

The magnetic state evolves very much as discussed in section II for field alignment in both easy ($H \parallel \hat{x}$) and hard ($H \parallel \hat{z}$) orientation. Yet, especially for the former we observe some distinct differences for a truncation of $d > 100$ nm: Rather than through the formation of an S-state close to remanence, magnetic reversal occurs through a vortex state, that only appears for small reverse fields. This manifests itself e.g. in the magnitude of the total magnetic moment $|\mu|$, which is significantly less for the vortex state as compared to the S-state, as shown in Fig. 6 (b).

For the external magnetic field perpendicular to the easy plane, we find that magnetic reversal takes place via vortex formation for all values of the truncation. Consequently, the total magnetic moment at remanence is always small, as seen in Figs. 6 (b) and (c).

There is a big difference between the magnetic moments at remanence depending on through which progression remanence has been reached. Hence, in order to find the most stable state, it is instructive to compare the energies of the different progressions as shown in Figs. 6 (c) and (f). A state with significant $|\mu|$ at remanence always seems to possess higher energy than the vortex state. Compare, for example, the energy at remanence for the JP with 200 nm truncation for easy (no vortex) and hard (vortex state) field alignment. The former has about 40 aJ, while the latter is more favorable with only 7 aJ.

As discussed in the main text, exchange bias, which forces magnetic moments into a certain direction, can avoid a global vortex state at remanence. We simulate the magnetic hysteresis of reduced-size JPs with global unidirectional anisotropy and vary its strength $K_{eb}$. As an example, we pick a JP with a cut of 100 nm, and match the orientation of the unidirectional anisotropy to the parallel field direction, which gives the maximum effect in the DCM signal.

In Fig. 7, we plot the same set of data for the JP with exchange bias as for the purely ferromagnetic JPs.

Figure 4. (a) Effective demagnetization factor $D_u$ of a truncated spherical halfshell with a gradient shell thickness in dependence of the truncation $d$ (blue) and of a full halfshell (orange). (b) Volume and (c) high field frequency shift $\Delta f_{hf}$ of the same geometries as in (a).

Figure 5. Cut through the geometry of the JP at the position of the $xz$ plane showing the $x$ component of the demagnetizing field within the magnetic layer for $H \parallel \hat{x}$ and $\mu_0 H = 20$ T.
in Fig. 6. The frequency shift for high fields for \( \mathbf{H} \parallel \mathbf{\hat{x}} \) develops an increasing asymmetry for increasing strength \( K_u \) of the unidirectional anisotropy, as shown in Fig. 7 (a). In turn, for perpendicular alignment this asymmetry is not evident, as shown in Fig. 7 (d). Hence, the experimentally observed asymmetry in the asymptotes can serve as good indicator for the strength of the unidirectional anisotropy, given that its orientation is known. The main influence of the unidirectional anisotropy on the progression of the magnetic state with external field for easy field alignment is to shift magnetic reversal in magnetic field magnitude. This can e.g. be seen in \( |\mu| \), see Fig. 7 (b). Magnetic reversal takes place around zero field for \( K_{eb} = 1 \text{kJ/m}^3 \), and is shifted to happen around 50 mT for \( K_{eb} = 100 \text{kJ/m}^3 \). The dominant state during reversal remains an onion state for all values of \( K_{eb} \). For hard alignment, a vortex appears in the JP for all values of \( K_{eb} \), just as described in section II B. The vortex is centered in the JP for larger field magnitudes, but moves to the side of the JP, if the field is reduced. Depending on \( K_{eb} \), the vortex moves very little (small values for \( K_{eb} \)), moves significantly to the side of the JP (intermediate \( K_{eb} \)), or even escapes the JP through the equatorial line (large values of \( K_{eb} \)). If the latter happens, the vortex reenters the JP in reverse field from the other side of the JP and then moves back to the center for increasing reverse field. The larger the \( K_{eb} \), the larger is the total magnetic moment \( \mu \) at remanence, for \( K_{eb} = 100 \text{kJ/m}^3 \) we find almost full saturation in direction of the anisotropy vector, see Fig. 7 (e).

A clear trend is observable, if we consider the energy that sets which remanent state is more likely over long time scales, as shown in Figs. 7 (c) and (f): The difference in energy between the remanent states gets smaller for increasing \( K_{eb} \), and hence diminishes the relevance of the chosen orientation of an applied field to magnetize the JP. The trend for the total magnetic moment is, irrespective of the which orientation for the external field is chosen, to be larger in magnitude if \( K_{eb} \) is increased.

VI. DCM IN THE HIGH-FIELD LIMIT WITH UNIDIRECTIONAL ANISOTROPY

The high-field limit in DCM is reached for \( |H| \gg |\sum_i K_i/(\mu_0 M_s)| \), where \( K_i \) are the different anisotropy contributions for a given direction. The frequency shift of the cantilever resonance is determined in this limit by the competition of the different anisotropy contributions, and can be calculated analytically \(^{7,9} \). For the presence of unidirectional anisotropy of strength \( K_{eb} \) this is given by:

\[
\Delta f_{\text{unidir}} = \frac{f_0 V K_{eb}}{2k_0 l_z^2} \left( \cos \theta_u (\sin \theta_h \sin \theta_{eb} \cos \phi_u \cos \phi_{eb} + \cos \theta_h \cos \theta_{eb}) - \sin \theta_u (\sin \theta_u \cos \theta_h \cos \phi_{eb} + \sin \theta_h \sin \phi_u \sin \phi_{eb}) + \sin \theta_u \sin \theta_h \cos \phi_{eb} \cos \phi_u \right) \tag{1}
\]
Figure 7. Frequency shift $\Delta f$, total magnetic moment $\mu$ and total energy $E_{\text{total}}$ for JPs with 100 nm truncation and different strengths of the unidirectional anisotropy as indicated in the legend in $J/\text{m}^3$. Top row: External field applied in the easy plane ($H \parallel \hat{x}$). Bottom row: External field applied in the hard direction ($H \parallel \hat{z}$).

Here, $(\theta_h, \phi_h = 0)$ define the orientation of the external field, $(\theta_u, \phi_u)$ of the axis of the uniaxial shape anisotropy as defined in Ref. 9, and $(\theta_{eb}, \phi_{eb})$ of the unidirectional anisotropy vector. The latter is oriented first, and rotated by $(\theta_u, \phi_u)$ in a second step to be consistent with the situation in experiment. Cantilever and magnetic parameters are as defined before.

We evaluate the high-field limit for the sum of shape and unidirectional anisotropy for an exemplary situation as it may be present for the exchange biased JPs, using similar parameters as in section IV. However, we increase $K_{eb}$ significantly to magnify its effects. The angles are set to be $(\theta_u, \phi_u) = (-3^\circ, 0^\circ)$ and $(\theta_{eb}, \phi_{eb}) = (-90^\circ, 0^\circ)$, while $\theta_h$ is varied as in experiment. The result is shown in Fig. 8 (a), together with the individual contributions from shape and unidirectional anisotropy. This shows, that the sum of the two contributions may lead to a periodicity that deviates slightly from $180^\circ$, which would be given for pure uniaxial shape anisotropy. Furthermore, the magnitude of maxima and minima may differ significantly. Here we find 1.9 Hz for the maxima and 2.4 Hz for the minima, respectively. We have indeed observed such large asymmetries in experiment for low temperatures (not shown here), however, the origin is different, as will be discussed in the following. Nevertheless, for strong unidirectional anisotropies these findings should be observable in experiment.

The SW model, as described in section III, can be used to calculate $\Delta f(\theta_h)$ for a fixed field magnitude, as done in experiment for 3.5 T. This allows to compare the result of the SW model with the high field limit as discussed above, see Fig. 8 (b). The curve of the high field limit follows a (negative) cosine with $2\theta_h$ in the argument. In turn, for the SW model at 3.5 T, minima are deeper and maxima are shallower in $\Delta f$, respectively. However, there is no deviation from the $180^\circ$ periodicity. Further, maxima are wider than minima, which is a consequence of the fact that positive and negative asymptotes are approached with a different curvature when ramping up the external field in the SW model, compare curves in the last

Figure 8. (a) $\Delta f(\theta_h)$ in the high field limit with shape and unidirectional anisotropy. $\Delta f(\theta_h)$ in the SW model for 3.5 T applied field magnitude, and in the high field limit from (a).
panel in the first column with those in the last panel in the last column in Fig. 3. In experiment this behavior of the maxima and minima is inverted, which is caused by the extreme curvature of the magnetic layer JPs, as discussed in section IV.

REFERENCES

1. COMSOL AB. Comsol multiphysics®, 2021.
2. T. Fischbacher, M. Franchin, G. Bordignon, and H. Fangohr. A systematic approach to multiphysics extensions of finite-element-based micromagnetic simulations: Nmag. IEEE Transactions on Magnetics, 43(6):2896–2898, 2007.
3. D. V. Berkov, C. T. Boone, and I. N. Krivorotov. Micromagnetic simulations of magnetization dynamics in a nanowire induced by a spin-polarized current injected via a point contact. Physical Review B, 83(5):054420, February 2011. Publisher: American Physical Society.
4. J. M. D. Coey. Magnetism and Magnetic Materials. Cambridge University Press, 2010.
5. N. Rossi, B. Gross, F. Dirnberger, D. Bougeard, and M. Poggio. Magnetic force sensing using a self-assembled nanowire. Nano Letters, 19(2):930–936, 2019.
6. Allan H. Morrish. The Physical Principles of Magnetism. Wiley, January 2001.
7. B. Gross, D. P. Weber, D. Rüffer, A. Buchter, F. Heimbach, A. Fontcuberta i Morral, D. Grundler, and M. Poggio. Dynamic cantilever magnetometry of individual CoFeB nanotubes. Phys. Rev. B, 93(6):064409, February 2016.
8. A. Hubert and R. Schäfer. Magnetic Domains: The Analysis of Magnetic Microstructures. 1998.
9. B. Gross, S. Philipp, E. Josten, J. Leliaert, E. Wetterskog, L. Bergström, and M. Poggio. Magnetic anisotropy of individual maghemite mesocrystals. Phys. Rev. B, 103:014402, Jan 2021.