Polarization bispectrum for measuring primordial magnetic fields

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Abstract. We examine the potential of polarization bispectra of the cosmic microwave background (CMB) to constrain primordial magnetic fields (PMFs). We compute all possible bispectra between temperature and polarization anisotropies sourced by PMFs and show that they are weakly correlated with well-known local-type and secondary ISW-lensing bispectra. From a Fisher analysis it is found that, owing to E-mode bispectra, in a cosmic-variance-limited experiment the expected uncertainty in the amplitude of magnetized bispectra is 80\% improved in comparison with an analysis in terms of temperature auto-bispectrum alone. In the Planck or the proposed PRISM experiment cases, we will be able to measure PMFs with strength 2.6 or 2.2 nG. PMFs also generate bispectra involving B-mode polarization, due to tensor-mode dependence. We also find that the B-mode bispectrum can reduce the uncertainty more drastically and hence PMFs comparable to or less than 1 nG may be measured in a PRISM-like experiment.
1 Introduction

Several cosmological and astrophysical observations support existence of finite magnetic fields in galaxies, cluster of galaxies or large voids (e.g., [1–6]). There are a variety of studies where these origin is linked with primordial vector field in the very early Universe (e.g., refs. [7–10]). Despite a fact that such models are strongly constrained by conditions not to contradict inflation or the high energy physics [16–20], these provide phenomenologically interesting outputs (e.g., refs. [21–32]).

In this paper, we focus on magnetized non-Gaussian signals in the cosmic microwave background (CMB). Primordial magnetic fields (PMFs) create not only scalar-mode but also vector-mode and tensor-mode CMB anisotropies via energy density and anisotropic stress fluctuations [33–41]. Recent analyses using CMB power spectra suggest nearly scale-invariant PMFs with strength less than about 3 nG [42–45]. On the other hand, under an assumption of Gaussianity of PMFs, CMB polyspectra also be generated due to quadratic dependence of the stress fluctuations on Gaussian PMFs [46–55]. They have diverse shapes unlike CMB bispectra from standard scalar non-magnetized non-Gaussianities since in PMF case the vector-mode and tensor-mode non-Gaussianities can be enhanced [50, 52–54]. The magnetized bispectrum has provided a new observational constraint on PMFs consistent with bounds from the power spectrum [56].

These previous studies have analyzed effects of temperature auto-bispectrum alone. On the other hand, it is known that polarization bispectra can also help to determine non-Gaussianity parameters [57–59] and they will be utilized in data analysis of the Planck or the proposed PRISM experiment [60, 61]. In this sense, studying impacts of PMFs on the polarization bispectra will be useful and timely.

1 At the same time, several papers have also discussed possibilities of magnetic field production in the late-time Universe (e.g., refs. [11–15]).
On the basis of these motivations, this paper investigates the potential of the polarization bispectra sourced by PMFs. We compute magnetized auto- and cross-bispectra between temperature, E-mode and B-mode anisotropies, and forecast the uncertainty of the amplitude of these bispectra, which depends on PMF strength, via the Fisher analysis. As observations, we assume the *Planck* and the proposed PRISM experiments. In computation of the CMB bispectra, we consider the dependence on the scalar and tensor modes and ignore the vector mode because of its smallness on scales where we focus on. Then, we confirm that owing to tensor-mode contribution, the polarization bispectra reduce the uncertainty of the magnetized bispectra more drastically in comparison with a forecast from the temperature auto-bispectrum alone. We also find that the existence of local-type non-Gaussianity and secondary ISW-lensing signal does not bias an error estimation of the amplitude of the magnetized bispectra. We follow the formulae and computational procedure in ref. [54].

This paper is organized as follows. In the next section, we analyze signatures of all possible magnetized bispectra composed of the temperature, E-mode and B-mode anisotropies. In section 3, through the Fisher analysis, we discuss the detectability of the magnetized bispectra. The final section is devoted to summary and discussion of this paper. In appendices A and B, we summarize instrumental noise information utilized in section 3 and the uncertainty of the local-type non-Gaussianity.

## 2 Temperature and polarization bispectra originating from primordial magnetic fields

In this section, we examine the dependence of all possible temperature and polarization bispectra generated from PMFs for $\ell < 2000$. The notations and conventions are consistent with refs. [50, 52–54].

### 2.1 Magnetized CMB fluctuation

Let us start from a cosmological model with large-scale magnetic fields which are created at very early stages of the Universe and stretched beyond horizon by the inflationary expansion. With assumptions of Gaussianity of PMFs and their evolution like radiations: $B_i \propto a^{-2}$, the PMF power spectrum normalized at the present epoch is given as

$$\langle B_i(k)B_j(k') \rangle = (2\pi)^3 \frac{P_B(k)}{2} P_{ij}(\hat{k}) \delta(k + k'), \quad (2.1)$$

where $P_{ij}(\hat{k}) = \delta_{ij} - \hat{k}_i \hat{k}_j$ is a projection tensor which reflects the divergenceless nature of PMFs. The shape of $P_B(k)$ depends strongly on models of primordial magnetogenesis. In order to find observational clues, it is often parametrized as the power-law type:

$$P_B(k) = A_B k^{n_B}, \quad (2.2)$$

where the amplitude depends quadratically on the PMF strength smoothed on $r$ as

$$A_B = \frac{(2\pi)^{n_B+5} B_r^2}{\Gamma\left(\frac{n_B+2}{2}\right) r^{n_B+3}}. \quad (2.3)$$
PMFs create energy-momentum tensor as

\[ T^i_j(k, \tau) \equiv \rho_\gamma(\tau) \left[ \delta^i_j \Delta_B(k) + \Pi^i_{Bj}(k) \right], \]

\[ \Delta_B(k) = \frac{1}{8\pi \rho_{\gamma,0}} \int \frac{d^3k'}{(2\pi)^3} B^i(k') B_j(k - k'), \]

\[ \Pi^i_{Bj}(k) = -\frac{1}{4\pi \rho_{\gamma,0}} \int \frac{d^3k'}{(2\pi)^3} B^i(k') B_j(k - k'), \]

where \( \rho_\gamma = \rho_{\gamma,0} \) is energy density of photons. These stress fluctuations can behave as a source of the CMB anisotropies as follows. The first contribution (called passive mode) comes from gravitational interaction via the Einstein equations. In deep radiation dominated era, anisotropic stress fluctuation \( \Pi^i_{Bj} \) can enhance superhorizon metric perturbations. After neutrinos decouple, \( \Pi^i_{Bj} \) is compensated by anisotropic stress fluctuation of neutrinos and such growth ends. Resulting superhorizon curvature perturbations and gravitational waves are estimated as \[ \zeta(k) = R_\gamma \ln \left( \frac{\tau_\nu}{\tau_B} \right) \frac{3}{2} O^{(0)}_{ij}(k) \Pi_{Bij}(k), \]

\[ h^{(\pm 2)}(k) = 6R_\gamma \ln \left( \frac{\tau_\nu}{\tau_B} \right) \frac{1}{2} O^{(\pm 2)}_{ij}(k) \Pi_{Bij}(k), \]

where \( R_\gamma = 0.6, \tau_\nu, \tau_B, O^{(0)}_{ij} \) and \( O^{(\pm 2)}_{ij} \) are ratio of \( \rho_\gamma \) divided by total radiation energy density, conformal times of neutrino decoupling and PMF generation, and scalar-mode and tensor-mode projection tensors, respectively [54]. These re-enter horizon just before recombination and generate CMB scalar and tensor fluctuations. Note that vector-mode metric perturbation decays after neutrino decoupling. These passive-mode anisotropies have similar shapes as the CMB fluctuations in non-magnetized standard cosmology [62] since the changes of metric perturbations mentioned above do not affect radiation transfer functions. The second contribution (called compensated mode) is due to Lorentz force at around recombination. The Lorentz force induces baryon velocity via the Euler equations and enhances the CMB scalar and vector fluctuations [33, 35, 36, 38, 40, 41]. Unlike the passive mode, the compensated-mode fluctuations are amplified on small scales and hence they differ from standard CMB patterns. From the analyses of these effects by the CMB power spectra, the PMF strength smoothed on 1 Mpc and the spectral index of the PMF power spectrum have been estimated as \( B_1 < 3.4 \) nG and \( n_B < 0 \) preferred at 95% CL [45].

The tensor and scalar passive modes dominate over the temperature and E-mode fluctuations for \( \ell \lesssim 2000 \) [40, 54]. Even in the B-mode fluctuation, the vector compensated mode is hidden by the presence of the tensor passive mode up to \( \ell \sim 500 \). In the following discussions, we are interested in scales which are not so small; therefore we shall take into account the effects of the scalar and tensor passive modes.

### 2.2 CMB bispectra

PMF-induced metric perturbations (2.5) obey chi-square statistics because of Gaussianity of PMFs. These induce large squeezed-type curvature and tensor bispectra and will be observed as the temperature and polarization bispectra at the present time. In general, these bispectra have very complicated spin and angle dependence due to contraction of \( O^{(0)}_{ij}, O^{(\pm 2)}_{ij} \) and the bispectrum of \( \Pi^i_{Bj} \) [31, 53, 54]. In addition, owing to the dependence of the CMB bispectra on...
$B_i^6$, we are enforced to deal with loop computation. Using a suitable approximation picking up poles, ref. [54] has derived complete formulae applicable to the bispectra composed of not only temperature ($I$) but also E-mode ($E$) and B-mode ($B$) anisotropies.

Computing on the basis of their formalism, we depict the CMB bispectra in figures 1 and 2. Here we distinguish between six parity-even bispectra ((III), (IIE), (IEE), (EEE), (IBB) and (EBB)) and four parity-odd ones ((IIB), (IEB), (EEB) and (BBB)) because they are located at completely different multipole configurations, namely $\ell_1 + \ell_2 + \ell_3 = \text{even}$ and odd, respectively. The vertical axes express absolute values of reduced bispectra: $\langle IB \rangle, \langle EB \rangle, \langle BB \rangle, \langle EE \rangle$. Furthermore, in $\langle III \rangle$ panel, we also describe a CMB bispectrum $b_{\ell_1 \ell_2 \ell_3} = G_{\ell_1 \ell_2 \ell_3}^{-1} B_{\ell_1 \ell_2 \ell_3}$, where

$$G_{\ell_1 \ell_2 \ell_3} = \frac{1}{6} \prod_{n=1}^{3} \left[ \frac{2 \sqrt{\ell_3 (\ell_3 + 1) \ell_2 (\ell_2 + 1)}}{\ell_1 (\ell_1 + 1) - \ell_2 (\ell_2 + 1) - \ell_3 (\ell_3 + 1)} \right] \times \sqrt{\frac{(2\ell_n + 1) (2n + 1)}{4\pi}} \left[ \begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ 0 & -1 & 1 \end{array} \right] + 5 \text{ perms.}.$$  

Note that a relation: $G_{\ell_1 \ell_2 \ell_3} = \sqrt{\frac{(2\ell_1 + 1)(2\ell_2 + 1)(2\ell_3 + 1)}{4\pi}} \left[ \begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{array} \right]$ holds when $\ell_1 + \ell_2 + \ell_3 = \text{even}$ [63–65]. The magnetized bispectrum consists of auto- and cross-correlations between the scalar and tensor anisotropies (i.e., TTT, STT, TST, TTS, SST, STS, TSS and SSS).

In these figures, we express the magnetized bispectrum composed of every conceivable combination in these eight modes as total mode, which means

$$\text{total} = \left\{ \begin{array}{ll} TTT + TTS + TST + STT + SST + STS + TSS + SSS & : (III), (IIE), (IEE), (EEE) \\
TTT + TST + STT + SST & : (IIB), (IEB), (EEB) \\
TTT + STT & : (IBB), (EBB) \\
TTT & : (BBB) \end{array} \right.$$  

Here the difference in the number of terms by each line is due to a fact that the scalar mode cannot generate the B-mode polarization. We also plot each mode to clarify its contribution to the total spectrum.

From these figures, we can confirm that the tensor mode dominates on large scales and the scalar mode catches up with the tensor mode on small scales. These are consistent behaviors with the magnetized power spectra and temperature auto-bispectrum [40, 54]. Especially, we can observe that the TTT modes are $O(10^2)$ times larger than the SSS modes on sufficient large scales. This amplification directly reflects a magnitude relationship between magnetized gravitational waves and curvature perturbations of eq. (2.5), namely, $(h^{(\pm 2)} / \zeta)^3 \sim 6^3$. Overall behaviors of the magnetized bispectra are consistent with the CMB power spectra predicted by the standard cosmology because their transfer functions are same. In four panels for the temperature and E-mode bispectra, the standard local-type bispectra are also plotted. Furthermore, in $\langle III \rangle$ panel, we also describe a CMB bispectrum
Figure 1. Parity-even magnetized bispectra: $\langle III \rangle$, $\langle IIE \rangle$, $\langle IEE \rangle$, $\langle EEE \rangle$, $\langle IBB \rangle$ and $\langle EBB \rangle$ for $\ell_1 = \ell_2 = \ell_3$. For comparison, the local-type bispectra with $f_{\text{NL}} = 2.7$ and the ISW-lensing bispectrum are also plotted. The total spectrum means the bispectrum involving all possible scalar and tensor combinations, and TTT, SSS or STT corresponds to a part of the total bispectrum. The PMF parameters are taken as $B_1 = 3 \text{ nG}$, $\tau_\nu/\tau_B = 10^{17}$ and $n_B = -2.9$. Other cosmological parameters are fixed as values consistent with the Planck results [45]. These spectra obey $\ell_1 + \ell_2 + \ell_3 = \text{even}$.

from a correlation between the late-time ISW effect and weak lensing, i.e., the ISW-lensing bispectrum [66–74]. It is well known that the ISW-lensing bispectrum highly correlates with the local-type bispectrum. We can see that these two types of bispectra resemble the magnetized SSS bispectra, while they are quite different from the total spectra because of the tensor-mode contributions. Therefore, the magnetized bispectrum signals will not be biased in a multi-parameter fitting (for details see the next section). In the next section, we
evaluate the detectability of these signals.

3 Fisher forecast

In this section, through the Fisher analysis, we evaluate the expected error bar of the magnitude of the magnetized bispectra for $n_B = -2.9$, which depends on the PMF strength (smoothed on 1 Mpc) and PMF generation epoch:

$$A_{\text{bis}} = \left( \frac{B_1}{3 \text{ nG}} \right)^6 \left[ \frac{\ln(\tau_B/\tau_{\nu})}{\ln(10^{17})} \right]^3. \quad (3.1)$$

We assume noise information of temperature and polarizations in the Planck and PRISM experiments [60, 61] (for details see appendix A).

3.1 Temperature and E-mode bispectra

Here we focus on the parity-even signals arising from $\langle III \rangle$, $\langle IIE \rangle$, $\langle IEE \rangle$ and $\langle EEE \rangle$. The Fisher matrix element of the normalized bispectra including E-mode polarizations is defined

\[ \text{Figure 2. Parity-odd magnetized bispectra: } \langle IIB \rangle, \langle IEB \rangle, \langle EEB \rangle \text{ and } \langle BBB \rangle \text{ for } \ell_1 + 4 = \ell_2 + 2 = \ell_3. \text{ The } \text{total} \text{ bispectrum consists of the auto- and cross-bispectra of the scalar and tensor anisotropies such as } TTT \text{, } STT + TST \text{ and } SSS. \text{ The settings for the PMF parameters and other cosmological parameters are same as figure 1. These spectra obey } \ell_1 + \ell_2 + \ell_3 = \text{ odd}. \text{ Rapidly-oscillating behavior seen in } \langle BBB \rangle \text{ is due to antisymmetric property of the parity-odd bispectrum.} \]
as \[57, 58\]

\[ F_{ij} = \sum_{X_1X_2X_3} \sum_{X'_1X'_2X'_3} \frac{1}{\Delta_{\ell_1\ell_2\ell_3}} \tilde{B}^{(i)}_{X_1X'_2X'_3,\ell_1\ell_2\ell_3} \left[ \prod_{n=1}^{3} (C^{-1})_{\ell_nX_n} \right] \tilde{B}^{(j)}_{X_1X_2X_3,\ell_1\ell_2\ell_3}, \tag{3.2} \]

where

\[ \Delta_{\ell_1\ell_2\ell_3} = (-1)^{\ell_1+\ell_2+\ell_3} (1 + 2\delta_{\ell_1,\ell_2}\delta_{\ell_2,\ell_3}) + \delta_{\ell_1,\ell_2} + \delta_{\ell_2,\ell_3} + \delta_{\ell_3,\ell_1}, \tag{3.3} \]

and \(X_1X_2X_3\) and \(X'_1X'_2X'_3\) run over eight modes \(III, IIE, IEI, EII, IEE, EIE, EEE\). The inverse matrix of the power spectrum is explicitly written as

\[ (C^{-1})_{XX'} = \begin{pmatrix} C^{II}_{\ell} & C^{IE}_{\ell} \\ C^{EI}_{\ell} & C^{EE}_{\ell} \end{pmatrix}^{-1}, \tag{3.4} \]

where \(C^{XX'}_{\ell} = \tilde{C}^{XX'}_{\ell} + N^{XX'}_{\ell}\) is the CMB power spectrum involving information of cosmic variance \(\tilde{C}_{\ell}\) and instrumental noise \(N_{\ell}\). We want to estimate the signals of the magnetized bispectrum \((B^{(M)})\) under the contamination of the local-type bispectrum \((B^{(L)})\) or the ISW-lensing bispectrum \((B^{(\phi)})\) and accordingly \(\tilde{B}^{(i,j)} = B^{(M)}/A_{\text{bis}}, B^{(L)}/f_{\text{NL}}, B^{(\phi)}\).

Firstly, let us clarify the dependence of the magnetized temperature and polarization bispectra on the scalar and tensor modes under the cosmic-variance-limited ideal experiment. In figure 3 we plot the signal-to-noise ratio, which is given as

\[ \frac{S}{N} = \sqrt{F_{MM}}. \tag{3.5} \]

From this figure, we can see that contribution of the tensor mode is quite larger than that of the scalar mode and therefore the \(TTT\) mode dominates over the total spectrum. However, due to rapidly decaying nature, the tensor mode is saturated for \(\ell \gtrsim 100\) and the scalar mode also contributes to a bit of amplification of the total spectrum. Note that the total spectrum falls below the \(TTT\) mode due to sign difference of each mode. These features have also been observed in the analysis of \((III)\) \[54\] and are quite different from the local-type bispectrum signatures that behave as simple increasing functions of \(\ell_{\max} [57]\).

Next, to estimate the uncertainty of \(A_{\text{bis}}\) under the presence of the contamination of the local-type bispectrum, we introduce the Fisher submatrix as

\[ (2) F = \begin{pmatrix} F_{MM} & F_{ML} \\ F_{LM} & F_{LL} \end{pmatrix}. \tag{3.6} \]

Then, the 1σ errors are given by

\[ (\delta A_{\text{bis}}, \delta f_{\text{NL}}) = \left( \sqrt{(2)F^{-1}_{11}}, \sqrt{(2)F^{-1}_{22}} \right). \tag{3.7} \]

Numerical results of \(\delta A_{\text{bis}}\) are described in figure 4. We will also present \(\delta f_{\text{NL}}\) in appendix A. From this figure, it is found that if we use all information of the temperature and E-mode bispectra, \(\delta A_{\text{bis}}\) is 80% improved in comparison with the analysis in terms of \((III)\) alone under the ideal case. This is an interesting result since in estimation for the local-type bispectrum \(\delta f_{\text{NL}}\) is only 50% reduced (See refs. [57, 58] or figure 7). This indicates that the tensor-mode polarization bispectra are quite informative. As described in this figure,
measuring $A_{\text{bis}}$ with this accuracy is hard in the Planck experiment due to lack of sensitivity of polarizations (see appendix A), while it can be done in the PRISM experiment. In the Planck, PRISM and ideal experiments for $\ell_{\text{max}} = 2000$, we obtain $\delta A_{\text{bis}} = 0.46, 0.17$ and 0.17, respectively (table 1).

To quantify resemblance between $B^{(M)}$ and $B^{(L)}$, we may compute a shape correlator given by

$$ r_{ML} \equiv \frac{F_{ML}}{\sqrt{F_{MM} F_{LL}}} . $$

A numerical result for $\ell_{\text{max}} = 2000$, i.e., $r_{ML} = -0.17$, guarantees that the magnetized bispectrum is weakly correlated with the local-type bispectrum and its contamination is very small. As this result, $(S/N)^{-1}$ of the total spectrum in figure 3 coincides with $\delta A_{\text{bis}} = 0.17$.

Finally, let us evaluate the bias by the ISW-lensing bispectrum. This contaminates only $\langle III \rangle$. In the same manner as the above discussion, we compute $\delta A_{\text{bis}}$ by following

$$ \delta A_{\text{bis}} = \sqrt{(2) F_{11}^{\prime -1}} , $$

and find that the values for $\ell_{\text{max}} = 2000$ become 0.92 (Planck), 0.92 (PRISM) and 0.83 (ideal), respectively. These are almost identical to the values of $\delta A_{\text{bis}}$ from $\langle III \rangle$ in figure 4 (or table 1) and hence we can conclude that the ISW-lensing bispectrum is also a tiny bias comparable to the local-type bispectrum in the $\langle III \rangle$ analysis.

\textbf{Figure 3.} Signal-to-noise ratios of the magnetized bispectra (3.5): $\langle III \rangle + \langle IIE \rangle + \langle IEE \rangle + \langle EEE \rangle$ when $A_{\text{bis}} = 1$. We neglect any instrumental noises; hence the signal-to-noise ratios are determined by the cosmic variance alone.
3.2 B-mode bispectra

In this subsection, we shall consider a possibility of the bispectra including B-mode polarization. Such bispectra are divided into both the parity-even (⟨BBB⟩) and the parity-odd (⟨IBB⟩, ⟨IEB⟩, ⟨EEB⟩ and ⟨BBB⟩) combinations. Although a complete analysis with both these all contributions and the temperature and E-mode bispectra may reduce δA_{bis} more drastically, it will be quite complicated. Accordingly, here let us concentrate on the Fisher analysis with ⟨BBB⟩ alone.

For \ell \gtrsim 500, the compensated vector mode will exceeds the passive tensor mode. Furthermore, on such scales, lensed CMB fluctuations also generate secondary B-mode fluctuations and may contaminate the magnetized bispectrum \[40, 69, 75\]. While the consideration of these sources is important, in this paper we work on large scales up to \ell_{\text{max}} = 500 where these are negligible.

Despite the parity-odd case, we can define the Fisher matrix like the parity-even case:

\[
F \equiv \sum_{\ell_1 \leq \ell_2 \leq \ell_3 \leq \ell_{\text{max}}} \frac{\tilde{B}_{BBB,\ell_1 \ell_2 \ell_3}^2}{\Delta_{\ell_1 \ell_2 \ell_3}} \prod_{n=1}^{3} C_{\ell_n}^{BB},
\]

where \(\tilde{B} = B^{(M)}/A_{\text{bis}}\). Then the 1σ error becomes

\[
\delta A_{\text{bis}} = \sqrt{F^{-1}}.
\]

Figure 5 describes the numerical results of \(\delta A_{\text{bis}}\). As the cosmic-variance spectrum \(\bar{C}_{\ell}^{BB}\), we adopt non-magnetized tensor-mode power spectrum in the standard cosmology, whose amplitude is determined by the tensor-to-scalar ratio \(r\). Especially for the ideal case (\(N_{\ell}^{BB} = \).
Figure 5. Expected 1σ errors of $A_{\text{bis}}$ (3.12) estimated from $\langle BBB \rangle$ if we assume the Planck, PRISM or ideal noise spectrum with $r = 0.05$ or $5 \times 10^{-4}$.

| experiment | $III$ | $EEE$ | all $I + E$ | $BBB$ ($r = 0.05$) | $BBB$ ($r = 5 \times 10^{-4}$) |
|------------|-------|-------|-------------|-------------------|-------------------|
| Planck     | 0.89  | 1.5   | 0.46        | $2.0 \times 10^{-2}$ | $0.71 \times 10^{-3}$ |
| PRISM      | 0.89  | 0.46  | 0.17        | $1.4 \times 10^{-2}$ | $1.6 \times 10^{-3}$ |
| ideal      | 0.79  | 0.44  | 0.17        | $3.4 \times 10^{-4}$ | $3.4 \times 10^{-7}$ |

Table 1. Expected 1σ errors of $A_{\text{bis}}$ at $\ell_{\text{max}} = 2000$ ($III$, $EEE$ and all $I + E$) and 500 ($BBB$) for each experiment.

0, $\delta A_{\text{bis}}$ is then simply proportional to $r^{3/2}$ and therefore we can write $\delta A_{\text{bis}} \approx 0.03 r^{3/2}$ for $\ell_{\text{max}} = 500$. Interestingly, unlike the estimation with the temperature and E-mode bispectra, $\delta A_{\text{bis}}$ in the ideal experiment does not saturate even for high $\ell_{\text{max}}$. This is due to damping behavior of $\bar{C}^B B$ for $\ell \gtrsim 100$, which cannot be seen in $\bar{C}^I I$, $\bar{C}^I E$ and $\bar{C}^E E$ (see figure 6). For $r = 0.05$, owing to this effect, $\delta A_{\text{bis}}$ reaches 0.014 under the PRISM noise level. On the other hand, the Planck experiment is too noisy to reduce the error so much like the $\langle EEE \rangle$ case. If $r = 5 \times 10^{-4}$, the noise dominates completely and $\delta A_{\text{bis}}$ saturates for all $\ell_{\text{max}}$ in both the experiments.

Finally, we summarize the value of $\delta A_{\text{bis}}$ for each case in table 1.

4 Summary and discussion

In this paper we examined how the polarization bispectra of the CMB anisotropies affect constraining PMFs. Firstly, we confirmed that the tensor-mode signals dominate over the bispectrum for $\ell \gtrsim 2000$ and the scalar mode contributes on very small scales in the auto- and cross-bispectra with the polarizations. Owing to this dependence, the magnetized bispectra
are weakly correlated with the standard local-type bispectra and the ISW-lensing bispectrum, and hence from observations the information of PMFs will be able to be extracted efficiently without any contamination.

From the error analyses via the Fisher forecast, we found that potentially, if we utilize all the temperature and E-mode bispectra, the uncertainty of the magnitude of magnetized bispectra can be 80% improved in comparison with the analysis with respect to $\langle III \rangle$ alone. This is interesting since in the analysis of the local-type non-Gaussianity, the improvement is only 50%. The proposed PRISM experiment will be able to reach this precision, while the Planck experiment cannot. If we assume the GUT-scale generation of PMFs, namely $\tau_N/\tau_B = 10^{17}$, the expected 1σ errors on the PMF strength from all the temperature and E-mode bispectra are given as $\delta B_1/nG = 2.6$ and 2.2 in the Planck and PRISM (or ideal) experiments, respectively.

We also considered the possibility of the analysis involving the B-mode bispectrum. In this case, we focused on the Fisher forecast using $\langle BBB \rangle$ and found that the uncertainty keeps on reducing as $\ell_{\text{max}}$ increases due to the damping behavior of the B-mode cosmic-variance spectrum for $\ell \gtrsim 100$. In the ideal experiment, we have a relationship with the tensor-to-scalar ratio: $\delta B_1/nG \approx 1.7r^{1/4}$ for $\ell_{\text{max}} = 500$; therefore we will be able to estimate with $O(0.1)$ nG accuracy if $r \lesssim 0.1$. In practice, the Planck and PRISM instrumental noises relax the value as $\delta B_1/nG = 3.4$ (2.8) and 1.5 (1.0) for $r = 0.05$ ($5 \times 10^{-4}$), respectively.

One may be concerned about comparison with bounds from the power spectrum analysis. According to recent literature [42–45], upper bounds on $B_1$ from the temperature and E-mode power spectra are around 3 nG. As shown above, the bispectrum analysis will provide comparable or tighter constraints on $B_1$. Concerning the B-mode power spectrum, the magnetized passive-mode signals are indistinguishable from the non-magnetized ones from primordial gravitational waves and hence $B_1$ may be not determined accurately in a multi-parameter fitting. In this sense, the information of the B-mode bispectrum will be more useful.

For $\ell > 500$, where this paper has not focused on, the vector compensated mode will dominate over the magnetized B-mode bispectrum. To reduce the uncertainty, we must evaluate such vector-mode contribution. Then, more comprehensive analysis including cross-bispectra between temperature, E-mode and B-mode fluctuations will be required. These informative but complex works remain as future issues.

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A Noise spectra

Here, we summarize the temperature and polarization noise spectra expected in the Planck and PRISM experiments.
Assuming Gaussian random detector noise, each noise spectrum is estimated as \[ N_{XX}^{\ell} = \left( \sum_c \frac{1}{\theta_c^2 \sigma_{X,c}^2} e^{-\ell(\ell+1)\theta_c^2/(8 \ln 2)} \right)^{-1} \] (A.1)

where \( \theta_c \) is the Full Width Half Maximum (FWHM) per the frequency channel \( c \) in radians and \( \sigma_{X,c} \) is the dimensionless sensitivity per \( c \). One can find these values (in arcminutes and \( \mu K \)) in table 2.

Figure 6 shows numerical results of \( N_{EE}^{\ell} \), \( N_{BB}^{\ell} \), and \( N_{EE}^{\ell} \). Here we assume \( N_{IE}^{\ell} = 0 \). We can see that in the \( EE \) mode, the cosmic-variance spectrum is comparable to the Planck noise spectrum, i.e., \( \bar{C}_{EE}^{\ell} \sim N_{EE}^{\ell} \), for \( \ell \gtrsim 10 \). This is a reason why the Planck experiment does not improve \( \delta A_{\text{bis}} \) so much in the analysis including the \( E \)-mode polarization as described in figure 4. Likewise, in the PRISM experiment \( N_{BB}^{\ell} \) exceeds \( \bar{C}_{BB}^{\ell} (r=0.05) \) for \( \ell \gtrsim 100 \) and hence \( \delta A_{\text{bis}} \) never be reduced beyond \( \ell \sim 100 \) when \( r=0.05 \) (figure 5).

### B \ Errors of the local-type non-Gaussianity

In figure 7, we describe the 1\( \sigma \) errors of the local-type nonlinearity parameter \( f_{NL} \) estimated from the two-dimensional Fisher analysis involving \( \delta A_{\text{bis}} \) discussed in subsection 3.1. Thanks to the weak correlation with the magnetized bispectrum, \( \delta f_{NL} \) is in good agreement with the results from the one-dimensional Fisher analysis fitting \( f_{NL} \) alone, i.e., \( 1/\sqrt{F_{LL}} \) [58]. We confirm that in the ideal experiment, the analysis containing both the temperature and E-mode bispectra reduces the value of \( \delta f_{NL} \) to half in comparison with the analysis by \( (III) \) alone. In the PRISM experiment, \( \delta f_{NL} \) can reach 2.3 for \( \ell_{\text{max}} = 2000 \).

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Figure 6. Noise spectra $N^{II}_l$ (red) and $N^{EE/BB}_l$ (green) (A.1) assuming the Planck (top) and PRISM (bottom) experiments. For comparison, we also depict $\bar{C}^{II}_l$ (blue), $|C_l^{IE}|$ (magenta), $C_l^{EE}$ (cyan) and $\bar{C}_l^{BB}$ for $r = 0.05$ (yellow).

Figure 7. Expected $1\sigma$ errors of $f_{NL}$ (3.7) estimated from all possible temperature and E-mode bispectra (red), only $\langle EEE \rangle$ (green) and only $\langle III \rangle$ (blue) if we assume the Planck, PRISM and ideal noise spectra.
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