The effect of long range gravitational perturbations on the first acoustic peak of the cosmic microwave background

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ABSTRACT

In the standard cosmological model, the temperature anisotropy of the cosmic microwave background is interpreted as variation in the gravitational potential at the point of emission, due to the emitter being embedded in a region $C$ of over- or under-density spanning the length (or size) scale $\lambda$ on which the anisotropy is measured. If the Universe is inhomogeneous, however, similar density contrasts of size $\lambda$ are also located everywhere surrounding $C$. Since they are superposition states of many independent Fourier modes with no preferred direction, such primordial clumps and voids should not be configured according to some prescribed spatial pattern. Rather, they can randomly trade spaces with each other while preserving the Harrison-Zeldovich character of the matter spectrum. The outcome is an extra perturbation of the potential when averaged over length $\lambda$ at the emitter, and consequently an additional anisotropy on the same scale, which has apparently been overlooked. Unlike the conventional application of the Sachs-Wolfe effect to the WMAP observations, this extra effect is not scale independent over the $P(k) \sim k$ part of the matter spectrum, but increases towards smaller lengths, as $\sqrt{k}$. The consequence is a substantial revision of the currently advertised values of the key cosmological parameters, unless one postulates a more rapid decrease in the gravitational force with distance than that given by the inverse-square law.

1. Introduction and preliminaries

In their seminal paper, Sachs and Wolfe (1967) discussed the effect of an inhomogeneous Universe at the decoupling epoch on the temperature anisotropy of the cosmic microwave background (CMB). They considered random density perturbations $\delta \rho/\rho$ over some characteristic length scale $\lambda$ - a phenomenon which was subsequently conjectured to have originated from scale invariant fluctuations in the primordial plasma (Harrison 1970, Zeldovich 1972, Peebles & Yu 1970). This behavior, and its manifestation as CMB temperature anisotropy,
has been investigated in detail both theoretically (Peebles 1982, Bond & Efstathiou 1984 and 1987) and observationally (Bennett et al 2003, Page et al 2003, and Spergel et al 2003 provided results from the latest all-sky measurements by WMAP). The purpose of the present work is to point out that the conversion from matter inhomogeneity to CMB anisotropy, adopted by conventional interpretation of the WMAP TT cross correlation data, is a restricted version of the Sachs-Wolfe effect which does not take into account all contributions (to the anisotropy) of comparable magnitude.

Let us first set up the necessary preliminary framework. At a spatial coordinate $\mathbf{r}$ and referring all physical quantities to their values at the present epoch where the expansion parameter is $a_0 = 1$ let the matter density be $\rho(\mathbf{r}) = \rho_0 + \delta \rho(\mathbf{r})$ with the perturbation term having zero spatial average over some large volume, i.e. $\langle \delta \rho(\mathbf{r}) \rangle = 0$. The Fourier transform of $\rho$, and its inverse transform, are

$$\tilde{\rho}(\mathbf{k}) = \int d^3r \, e^{-i\mathbf{k} \cdot \mathbf{r}} \rho(\mathbf{r}), \quad \rho(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{r}} \tilde{\rho}(\mathbf{k}), \quad (1)$$

respectively. The deviation in the gravitational potential at $\mathbf{r}$, due to the inhomogeneous Universe, is given by

$$\delta \Phi(\mathbf{r}) = -G \int d^3r' \frac{\delta \rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}. \quad (2)$$

We shall return to Eq. (2) soon. For the moment, let us examine $\delta \tilde{\rho}(\mathbf{k})$, because the power spectrum $P(k)$ is characterized by an equation which involves $\delta \tilde{\rho}$:

$$\langle \delta \tilde{\rho}(\mathbf{k}) \delta \tilde{\rho}(\mathbf{k}') \rangle = \rho_0^2 P(k)(2\pi)^3 \delta(\mathbf{k} + \mathbf{k'}). \quad (3)$$

For the Harrison-Zeldovich (HZ) spectrum, $P(k) \sim k$.

We now define a smoothing function (or filter) $W_\lambda(\mathbf{r})$ corresponding to any chosen scale $\lambda$, satisfying $W_\lambda(\mathbf{r}) \approx 1$ for $r \ll \lambda$ and $W_\lambda(\mathbf{r}) \approx 0$ for $r \gg \lambda$. The effective smoothing volume $V_\lambda$ is:

$$V_\lambda = \int d^3r \, W_\lambda(\mathbf{r}). \quad (4)$$

For example, a convenient choice is $W_\lambda(r) = e^{-r^2/2\lambda^2}$ for which $V_\lambda = (2\pi)^3/2 \lambda^3$ and $\tilde{W}_\lambda(\mathbf{k}) = V_\lambda e^{-\lambda^2k^2/2}$. Then the smoothed mass over length scale $\lambda$ will be

$$\delta M_\lambda(\mathbf{r}) = \int d^3r' \, W_\lambda(\mathbf{r} - \mathbf{r}') \delta \rho(\mathbf{r}'), \quad (5)$$

or equivalently

$$\delta \tilde{M}_\lambda(\mathbf{k}) = \tilde{W}_\lambda(\mathbf{k}) \delta \tilde{\rho}(\mathbf{k}). \quad (6)$$
The averages of $M_\lambda$ are $\langle \delta M_\lambda(r) \rangle = 0$ and

$$\langle \delta M_\lambda(r) \delta M_\lambda(r') \rangle = \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot(r-r')} |\tilde{W}_\lambda(k)|^2 \rho_0^2 P(k).$$

Likewise for the smoothed density $\delta \rho_\lambda$ we have $\langle \delta \rho_\lambda(r) \rangle = 0$ and

$$\langle \delta \tilde{\rho}_\lambda(k) \delta \tilde{\rho}_\lambda(k') \rangle = \rho_0^2 e^{-\lambda^2 k^2} P(k)(2\pi)^3 \delta(k + k').$$

In particular when $P(k) = Ak^n$, the mean square value of the smoothed mass can be evaluated by the saddle-point approximation. The $k$ integral is dominated by the region near $k = k_\lambda = \sqrt{1+n^2}/\lambda$, to give

$$\langle (\delta M_\lambda)^2 \rangle = c_n M_\lambda^2 k_\lambda^3 P(k_\lambda),$$

where $M_\lambda = \rho_0 V_\lambda$ and $c_n$ is a numerical constant. Here-and-after we shall define the standard deviation in $\delta M_\lambda(r)$ as

$$\delta M_\lambda^{\text{rms}} = \left( \langle (\delta M_\lambda)^2 \rangle \right)^{1/2}.$$  

A note of caution is already in order here. Although the r.m.s. mass and density over some length scale $\lambda$ at position $r$ concern the distribution of matter local to $r$, the same cannot be said about r.m.s. values of the potential fluctuation $\delta \Phi(r)$, Eq. (2). Owing to the long range nature of the gravitational force, density contrasts of size $\lambda$ but spreading over distances far greater than $\lambda$ can also contribute towards $\delta \Phi_\lambda^{\text{rms}}$.

2. CMB temperature anisotropy from primordial density fluctuations: any missing component?

If at some point $r$ on the last scattering surface there is a mass excess (say) of $\delta M_\lambda^{\text{rms}}$ over length scale $\lambda$, the most obvious contribution to the CMB anisotropy in this scale will be a perturbation in the gravitational potential of the form

$$\delta \Phi_\lambda(r) \approx \frac{G \delta M_\lambda}{\lambda} \approx \frac{G \delta M_\lambda}{\lambda M_\lambda} \rho_0 V_\lambda.$$  

Since $V_\lambda \sim \lambda^3$ and from Eq. (8) we have $\delta M_\lambda/M_\lambda \sim k_\lambda^2 \sim \lambda^{-2}$ for $P(k) \sim k$, it is then apparent that

$$\left( \frac{\delta T}{T} \right)_\lambda \approx \delta \Phi_\lambda = \text{constant},$$

i.e. for primordial HZ fluctuations the value of smoothed temperature anisotropy is independent of the length scale of the smoothing filter. This is the well known Sachs-Wolfe effect as applied to a HZ matter spectrum.

Let us however query whether the standard result described above, which concerns anisotropies caused by fluctuation in the mass surrounding the site of photon emission and
extending to the length scale $\lambda$ under consideration, represent all the possibilities. Of course, on scales $\gg \lambda$ any total mass variation will, in the restricted context of Eqs. (10) and (11), lead to anisotropies over correspondingly larger angular separations. Yet Eq. (10) is not the only way to perturb $\Phi_\lambda(\vec{r})$. A simple analogy is that the potential at some point on the earth surface depends solely on the total mass of the earth if there is perfect spherical symmetry in the density function. Deviations in $\Phi$ from point to point on the surface can occur even when the total mass is fixed, if non-uniformities in the matter distribution exists in any part of the earth’s interior, far or near the location in question. Moreover, the distance over which potential excursions occur equals the mean spacing between mass concentrations. In the present problem, obviously beyond the distance $\lambda$ from a CMB emitter at position $\vec{r}$, space remains just as inhomogeneous on the scale of $\lambda$. Then, different configurational realizations (or random placement) of these size $\lambda$ density contrasts (here-and-after referred to as lumps) within the, horizon at decoupling, i.e. the causal sphere $R$ of radius $R \gg \lambda$ centered at the emission point $\vec{r}$, can also result in a finite $\delta \Phi_\lambda(\vec{r})$ which is unrelated to Eq. (10).

Thus our contention is that when the distribution of primordial matter is smoothed at resolution $\lambda$, the resulting lumps of over- and underdensity, being superposition states of many independent Fourier modes with no preferred direction, can randomly trade spaces with each other and the HZ nature of the matter spectrum (on scales upwards of $\lambda$, of course) will still be preserved. This leads to an additional component of $\delta \Phi_\lambda(\vec{r})$ from the configurational arbitrariness of an entire ensemble of lumps, all in causal contact with position $\vec{r}$.

It is in fact quite easy to estimate the magnitude of the additional contribution. If primordial lumps of various masses and size $\lambda$ randomly pack the Universe with 100% filling factor, the resulting total mass fluctuation in any multiple-lump sub-region will remain in compliance with $\delta M/M \sim k^2$, yet there will be a perturbation on the potential $\Phi_\lambda(\vec{r})$, given by $\delta \Phi^{\text{rms}}_\lambda/\Phi_\lambda \sim 1/\sqrt{N}$ where $N \approx R^3/\lambda^3$ is the total number of lumps in our causal sphere, although we shall derive the precise value of $\delta \Phi^{\text{rms}}_\lambda$ in the next section. Thus we deduce that there is more CMB temperature anisotropy to be reckoned with, of order

$$
\left( \frac{\delta T}{T} \right)_\lambda \approx \delta \Phi^{\text{rms}}_\lambda \approx \frac{G\sqrt{N}\delta M_\lambda}{R},
$$

which has not been taken into account by conventional (single lump) application of the Sachs-Wolfe effect, Eq. (11). For a HZ matter spectrum, use of Eq. (8) enables us to write

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1If the matter is of primordial origin the fluctuation in the total mass of any multiple lump region will remain consistent with the HZ spectrum, irrespective of how the lumps are re-arranged, provided that the volume filling factor (by the lumps) is 100%, and the procedure of re-configuration ensures randomness of the lump positions within some very large volume.
Eq. (12) as
\[
\left( \frac{\delta T}{T} \right)_\lambda \approx \text{constant} \sqrt{\frac{R}{\lambda}},
\]
where the constant factor is the same as that in Eq. (11). Bearing in mind that \( R > \lambda \), the new effect presented here appears at least as important as the standard result.

3. Anisotropy from analysis in configurational space

A slightly different way of looking at the physics described in section 2 is afforded by working in real space. After spatial filtering the excursions in the potential, Eq. (2), can be written as a discrete sum over the resolution elements, viz.
\[
\delta \Phi_\lambda(r) = \sum_i -G \frac{\delta M^i_\lambda}{|r - r_i|}.
\]
(14)

Here the mass fluctuation at position \( i \) is given by \( \delta M^i_\lambda \), a quantity with zero mean and standard deviation \( \delta M^{\text{rms}}_\lambda \) where
\[
\delta M^{\text{rms}}_\lambda = \frac{4}{3} \pi \lambda^3 \delta \rho^{\text{rms}}_\lambda.
\]
(15)

Now the contribution to \( \delta \Phi_\lambda(r) \) from the density contrast in the vicinity of the point \( r \), i.e. the ‘local lump’ at \( r_1 = r_0 \) where \( |r - r_0| \sim \lambda \), is
\[
\delta \Phi_\lambda(r) = \frac{G \delta M^{\text{rms}}_\lambda}{\lambda} = \frac{G \delta M^{\text{rms}}_\lambda}{M_\lambda} \frac{4}{3} \pi \rho \lambda^3 = \frac{1}{2} \Omega_m H^2_0 \lambda^2 \frac{\delta \rho^{\text{rms}}_\lambda}{\rho_\lambda},
\]
(16)

where \( \Omega_m \) is the mean matter density of the present Universe in units of the critical density \( \rho_c = 3H_0^2/(8\pi G) \). This yields a temperature change (as the emitted CMB radiation leaves its own region of potential excursion) of \( \delta T/T \approx \delta \Phi_\lambda \), in agreement with Eq. (48) of Sachs & Wolfe (1967). Note that because in the case of a HZ spectrum the quantity \( \delta \rho^{\text{rms}}_\lambda /\rho_\lambda \sim \delta M^{\text{rms}}_\lambda /M_\lambda \sim 1/\lambda^2 \), we have from Eq. (8) \( \delta \Phi_\lambda(r) = \text{constant} \), in agreement with the conclusion of the previous section that single lump Sachs-Wolfe effect leads to the standard result of scale invariant CMB anisotropy for primordial matter distributions.

The important point, however, is that Eq. (14) also depicts effects beyond that of the local lump. As stated earlier, the contribution to \( \delta \Phi_\lambda(r) \) from the random placement all other lumps within the horizon should be included with the summation procedure. It is emphasized again that this effect is not because of the Poisson statistics in the total mass of many independently varying density contrasts belonging to the causal sphere \( R \) (which is suppressed by the HZ spectrum of \( P(k) \sim k \)), but because of the Poisson process in the configurational arrangement of the same primordial lumps inside \( R \).
Our task initially is to compute the deviation $\delta \Phi^\text{rms}_\lambda$ in the gravitational potential of a sphere of radius $R$ when it contains a constant number $N$ of smaller spherical lump of radius $\lambda$, each randomly placed and filled with matter accounting for a mass of $\delta M^\text{rms}_\lambda$ per lump, such that the remaining matter-free regions occupy half the volume of the causal sphere. Next, it is realized that this actually means neglecting the contribution to $\delta \Phi^\text{rms}_\lambda$ from the underdense regions (each of mass $-\delta M^\text{rms}_\lambda$), when it is included $\delta \Phi^\text{rms}_\lambda$ will increase by a factor of two because the gravitational effects of these two types of regions are anti-correlated. The potential deviation at the position $\mathbf{r}$ ($r \leq R$) due to the overdense lumps is

$$
\delta \Phi_\lambda (\mathbf{r}) = - \sum_j \frac{G \delta M^\text{rms}_\lambda}{|\mathbf{r} - \mathbf{r}_j|}.
$$

(17)

Now each of the $N$ overdense lumps has the same probability distribution, in which the probability $p(\mathbf{r}) d^3 \mathbf{r}$ of finding the lump in a small volume $d^3 \mathbf{r}$ is given by

$$
p(\mathbf{r}) d^3 \mathbf{r} = \frac{n(\mathbf{r})}{N} d^3 \mathbf{r}.
$$

(18)

where in Eq. (18) we adopted the continuum approximation appropriate to the limit of many lumps. Hence the mean potential $\langle \Phi(\mathbf{r}) \rangle$ is

$$
\langle \delta \Phi_\lambda (\mathbf{r}) \rangle = -NG\delta M^\text{rms}_\lambda \left( \frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) = -G\delta M^\text{rms}_\lambda \int \frac{n(\mathbf{r}') d^3 \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}.
$$

(19)

Relevant to the present problem is a fixed set of overdense lumps within $\mathcal{R}$, i.e. $n(\mathbf{r}) = 3N/(4\pi R^3)$ for $r \leq R$, and $n(\mathbf{r}) = 0$ for $r > R$ since the matter lying beyond $\mathcal{R}$ plays no part. The integral of Eq. (19) can readily be evaluated under this scenario to yield

$$
\langle \delta \Phi_\lambda (\mathbf{r}) \rangle = - \frac{GN\delta M^\text{rms}_\lambda}{2R} \left( 3 - \frac{r^2}{R^2} \right),
$$

(20)

where Eq. (20) is valid for the range $0 < r \leq R$. Thus at $r = R$ one recovers the usual Newtonian potential $\langle \delta \Phi_\lambda \rangle = -GN\delta M^\text{rms}_\lambda /R$, which on average cancels exactly the contribution from the underdense lumps.

To obtain the variance $\langle (\delta \Phi_\lambda)^2 \rangle$, the quantity by which the potential for points in space separated by lengths $> 2\lambda$ differ from each other, we need the mean square of $\delta \Phi_\lambda$ defined as

$$
\langle [\delta \Phi_\lambda (\mathbf{r})]^2 \rangle = \sum_{j,k} \frac{G^2 (\delta M^\text{rms}_\lambda)^2}{|\mathbf{r} - \mathbf{r}_j||\mathbf{r} - \mathbf{r}_k|}.
$$

(21)

When we subtract from this the ‘square of the mean’, viz. the quantity

$$
\langle \delta \Phi_\lambda (\mathbf{r}) \rangle^2 = \left( \sum_j \frac{G\delta M^\text{rms}_\lambda}{|\mathbf{r} - \mathbf{r}_j|} \right)^2,
$$

(22)
it is clear that all the terms with $j \neq k$ will cancel, and we are left with

$$\langle [\delta \Phi_\lambda(r)]^2 \rangle - \langle \delta \Phi_\lambda(r) \rangle^2 = G^2(\delta M^{\text{rms}}_\lambda)^2 N \left( \frac{1}{|r-r'|^2} - \frac{1}{|r-r'|^2} \right)^2.$$  \hspace{1cm} (23)

Now the average related to the first term on the right side of Eq. (23) may also be computed analytically for the form of $n(r)$ already discussed. By this we mean

$$\langle \frac{1}{|r-r'|^2} \rangle = \frac{3}{2R^2} \int_0^r r' dr' \ln \left( \frac{r + r'}{|r-r'|} \right)$$ \hspace{1cm} (24)

At the surface itself $r = R$, and the integral reduces to the simple form

$$\langle \frac{1}{|r-r'|^2} \rangle = \frac{3}{2R^2} \text{ at } r = R.$$ \hspace{1cm} (25)

The 2nd term on the right side of Eq. (23) is, by Eq. (19), equal to $\langle \delta \Phi_\lambda \rangle^2/N$.

Thus, altogether and after including the aforementioned contribution to $\delta \Phi^{\text{rms}}_\lambda$ from the underdense lumps, we arrive at a standard deviation of

$$\delta \Phi^{\text{rms}}_\lambda(r) = 2 \left[ \frac{3G^2(\delta M^{\text{rms}}_\lambda)^2 N}{2R^2} - \frac{G^2(\delta M^{\text{rms}}_\lambda)^2 N}{R^2} \right]^{\frac{1}{2}} = \frac{\sqrt{2NG\delta M^{\text{rms}}_\lambda}}{R}.$$ \hspace{1cm} (26)

Eq. (26) depicts the spatial variation in the potential between two points located at a distance $\geq 2\lambda$ apart.

The CMB anisotropy is finally obtained from Eq. (26) by observing that within the causal sphere there are altogether $R^3/\lambda^3$ lumps of radius $\lambda$, half of which are overdense. Thus $N = R^3/(2\lambda^3)$, and we have

$$\left( \frac{\delta T}{T} \right)_\lambda \approx \delta \Phi^{\text{rms}}_\lambda = G\delta M^{\text{rms}}_\lambda \lambda^{-\frac{3}{2}} R^\frac{1}{2} = \frac{1}{2} \Omega_m H_0^2 \lambda^2 R^\frac{3}{2} \delta \rho^{\text{rms}}_\lambda \rho_\lambda \text{ for } \lambda < R.$$ \hspace{1cm} (27)

This gives the new contribution which should explicitly be invoked\(^2\) as a separate anisotropy term in the ‘Boltzmann solver’ codes like CMBFAST (the routine used to fit the standard cosmological model to WMAP data, see Zaldarriaga & Seljak 2000), but is not. More elaborately, for self-consistency of the standard model this contribution must be included, even though it has hitherto been ignored. When compared with Eq. (16), we see that although conventional Sachs-Wolfe effect yields a constant $\delta T/T$ at all scales, the extra

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\(^2\)That is to say, the term will not arise ‘naturally’ as solution of a large number of coupled differential equations unless the physics behind it is in place.
anisotropy \((\delta T/T)_\lambda\) equals this same constant multiplied by the factor \(\sqrt{R/\lambda}\) (in agreement with the earlier analysis of section 3), i.e. it is not scale invariant. The reason why CMB temperatures must exhibit the variation of Eq. (27) is that two points on the last scattering surface separated by a distance \(\geq 2\lambda\) experience statistically independent potential deviation, due to the different realization (or pattern) of lumps in the causal spheres centered at these points.

It should also be mentioned that no matter how many more contrains unassumed by us were to be applied to restrict the number of permitted lump configurations within \(\mathcal{R}\), so long as space is not totally homogeneous beyond the emitter’s own ‘local lump’ the incorporation of effects of remote density contrasts via the long range force of gravity must remove the constancy of \((\delta T/T)_\lambda\) by introducing a factor that scales with \(R/\lambda\) in some way. The fact that the standard model predicts a constant \((\delta T/T)_\lambda\) at large (but sub-horizon) \(\lambda\) offers the clearest indication of it’s failure in recognizing the phenomenon presented here.

4. Have the CMB anisotropy observations been interpreted correctly?

At sufficiently low spherical harmonics \(\ell < 90\) where the smoothing length \(\lambda > R\), lack of causal linkage between lumps renders Eq. (27) inapplicable, and the only anisotropy would be the conventional single lump Sach-Wolfe effect of Eq. (16). From the WMAP TT cross correlation plot (Bennett et al 2003) one sees that \((\delta T/T)_\lambda \approx 10^{-5}\) at \(\ell \ll 90\). Thus we conclude that

\[
\frac{1}{2} \Omega_m H_0^2 \lambda^2 \frac{\delta \rho_{\text{rms}}}{\rho_\lambda} \approx 10^{-5}
\]

on all scales \(\lambda\) so long as the matter distribution follows the primordial HZ spectrum of \(P(k) \sim k\).

For sub-horizon features at decoupling which were super-horizon in size during matter-radiation equipartition\(^3\), like the first acoustic peak, the extra anisotropy of Eq. (27) must now be included. Since \(R \approx 490 \text{ Mpc}\) (present value for the causal radius at decoupling) and, in the case of the first peak the full size of these lumps is \(2\lambda \approx 147 \text{ Mpc}\), we see from Eq. (27) and (28) that the total expected anisotropy with the conventional and new component added in quadrature is

\[
\frac{\delta T}{T} \approx 2.77 \times 10^{-5}
\]

\(^3\)Structures that fit inside the comoving equipartition horizon have according to the standard model a density contrast commensurate with \(P(k) \sim k^{-3}\) at the primordial level. Thus our present conclusion regarding the first peak, which is however based upon a \(P(k) \sim k\) spectrum, does not apply to the higher peaks.
for the first peak. Obviously, Eq. (29) in conjunction with the WMAP observations pose a
dilemma for the standard cosmological model, because the detected first acoustic anisotropy
is only $\approx 2.5 \times 10^{-5}$, it already equals the expected value as given by Eq. (29), which is based
upon a purely primordial spectrum before enhancement of density contrasts by sound waves.
This leaves no room for interpreting the acoustic features as compression and rarefaction of
the plasma by sound propagation.

Is there a way of restoring the standard cosmological model, which matches the WMAP
data very well? We could supposedly alter the cosmological parameters, e.g. the primordial
spectral index could substantially increase from $n = 1$ to reduce the impact of Eq. (27),
though the epistemological justification of such changes will almost certainly be artificial.
One less damaging possibility remains, however. The gravitational force beyond some $> 100$
Mpc distance scale could wane more quickly than the inverse-square law. This would appear
to the author as a remedy of ‘least resistance’ for several reasons. Firstly, the standard
model can immediately be reinstated, as the Friedmann equations are constructed in the
context of General Relativity without appealing to the existence of a gravitational influence
on Hubble scales. Secondly, structure formation theory is unaffected, because numerical
codes that simulate the building galaxies, groups, and even clusters do not depend on the
inverse-square law persisting to beyond 100 Mpc. Thirdly, there is currently no reliable
experimental constraint on the behavior of gravity at these very large distances.

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