A perfect teleportation protocol requires pure maximally shared entangled states. While in reality the shared entanglement is drastically degraded due to the inevitable interaction with the noisy environment. Here, we propose a teleportation protocol with W states to teleport an unknown qubit through the amplitude (phase) damping channels with fidelity up to one in a probabilistic manner. In our protocol, we utilize environment-assisted measurement during the entanglement distribution and further modify the standard teleportation protocol to apply weak measurement in the last step of teleportation. To show the significant improvement of the fidelity of our proposed teleportation protocol, the comparison results with the standard teleportation with no protection and the probabilistic teleportation protocol based on weak measurement are presented. Furthermore, we study a controlled teleportation protocol, where all the three qubits of the W state pass through the amplitude (phase) damping channel. Our results show that in this case, by applying environment-assisted measurement the W state cannot be decohered and no weak measurement is required to attain the teleportation fidelity equal to one with a certain success probability, which is favorable for quantum communication and computation.

Quantum teleportation is a quantum communication task which sends an unknown qubit from a sender to a receiver by using shared entanglement and classical communications [1, 2]. The original protocol proposed by Bennett et al. [3] uses Einstein-Podolsky-Rosen (EPR) as the shared entangled state. Other teleportation protocols are also proposed by using two types of inequivalent three qubit entangled state: GHZ states and W states [4, 5, 6], among which W states are preferable since they are robust against loss of particles. If we trace out any one particle from the W state, then there is some genuine entanglement between the remaining two. While in GHZ state by tracing out one particle, no entanglement remains between other two particles. In general, the shared entangled state is prepared by the sender (receiver) and is sent to the receiver (sender), which has been extensively studied [7, 8, 9, 10, 11, 12]. Additionally, controlled teleportation is an indispensable branch of quantum teleportation, where the shared entangled state is prepared by a third party and is sent to the sender and receiver, respectively [13, 14, 15, 16, 17]. As the controller of the whole teleportation protocol, the third party can terminate the teleportation process in time when he notices something aberrant or insecure.

In any realistic implementation of the teleportation protocol, noise is unavoidably present and affects the entangled state during its transmission to teleportation parties [18]. Therefore, the entanglement degree of the quantum channel degraded which seriously deteriorates the performance of the teleportation as a result [9]. A large number of quantum state protection schemes have been developed to overcome the effects of the environmental noise on quantum states such as quantum error correction [19, 20, 21, 22, 23], dynamical decoupling [24, 25, 26, 27, 28], quantum control in the decoherence-free subspace [29, 30, 31] and quantum control via weak measurements (WMs) [18, 32, 33, 34, 35, 36]. Fortunately, some of these schemes have bright application prospects in protected quantum teleportation as well. For instance, Grassl et al. [37] applied quantum error correction to protect the entangled state through the noisy channel; however, it needs redundant qubits in entanglement. Recently, special attention has been paid to protect
quantum teleportation via WMs [12, 38, 39, 40]. Yang et al. [38] utilized the weak measurement and quantum measurement reversal (WMQMR) to protect the quantum entanglement from amplitude damping noise in the bidirectional quantum teleportation, revealing a considerable enhancement of the teleportation fidelity. Li et al. [39] enhanced the teleportation fidelity through correlated amplitude damping channels via WMQMR, which illustrated that this type of decoherence can be suppressed completely by applying well-designed WM and quantum measurement reversal. Also, Harraz et al. [12] applied the quantum feed-forward control and its reversal scheme via the WM and environment-assisted measurement (EAM) to protect the quantum channel from the amplitude damping noise. In all the above schemes, the focus is on applying proper control operations during the entanglement distribution process and protecting the quantum channel to eliminate the effects of the environmental noises. In this respect, applying unitary operations of the standard teleportation protocol by receiver may not always be optimal which limits the fault-tolerance of quantum teleportation [41, 42]. While modifying the teleportation protocol itself is also a feasible choice to enhance the performance of the quantum teleportation [11, 40].

In this paper, we propose a teleportation protocol by utilizing EAM and WM (TP-EW) to transmit an unknown qubit through noisy channels. Different from our previous work [12], here we modify the teleportation protocol itself to cancel the effects of the noisy channels. First, we assume that Alice (sender) prepares a class of W states which are suitable for perfect teleportation [43], and sends one qubit to Bob (receiver) through the noisy channel. We apply EAM and modify the standard teleportation protocol to reverse the effects of the noise with designed WM operators applied in the last step of teleportation. It should be noted that the TP-EW is applicable for teleportation through arbitrary decoherence channels with at least one invertible Kraus operator. However, in this paper we only consider two typical types of noisy channels: amplitude damping channel (ADC) and phase damping channel (PDC). The comparison results with the standard teleportation protocol with no protection demonstrate that the proposed TP-EW improves the average teleportation fidelity significantly for both ADC and PDC. For further comparison we study a pioneer probabilistic teleportation protocol based on weak measurement reversal (WMRTP) and demonstrate the improvement of the performances of TP-EW for all amounts of decaying rate of the noisy channel. We show that by considering designed WM operators the proposed TP-EW is also able to achieve teleportation fidelity equal to one independent of decaying rate. Afterwards, the final expression of average success probability of teleportation in presence of noise is derived. Subsequently, we assume that a third party (controller) prepares the shared W state, and sends the first two qubits to Alice and the third qubit to Bob through noisy channels. In this case, we show that one just need to apply EAM during the entanglement distribution process and no WM is required to achieve the teleportation fidelity equal to one.

This paper is organized as follows. In Section II, we present the proposed TP-EW through noisy channels and analyze its performance in details. In Section III, we apply our protection scheme in the controlled teleportation protocol. Finally, our conclusion is given in Section IV.

1 Teleportation protocol with W states through noisy channels by utilizing environment-assisted measurement and weak measurement

In this section, we present the details of our proposed teleportation protocol with W states through noisy channels by utilizing EAM and WM. First, Alice prepares a class of W state as the entangled shared state. She keeps two qubits for herself and sends one qubit to Bob (receiver) through the noisy channel. We apply EAM and modify the standard teleportation protocol to reverse the effects of the noise with designed WM operators applied in the last step of teleportation. It should be noted that the TP-EW is applicable for teleportation through arbitrary decoherence channels with at least one invertible Kraus operator. However, in this paper we only consider two typical types of noisy channels: amplitude damping channel (ADC) and phase damping channel (PDC). The comparison results with the standard teleportation protocol with no protection demonstrate that the proposed TP-EW improves the average teleportation fidelity significantly for both ADC and PDC. For further comparison we study a pioneer probabilistic teleportation protocol based on weak measurement reversal (WMRTP) and demonstrate the improvement of the performances of TP-EW for all amounts of decaying rate of the noisy channel. We show that by considering designed WM operators the proposed TP-EW is also able to achieve teleportation fidelity equal to one independent of decaying rate. Afterwards, the final expression of average success probability of teleportation in presence of noise is derived. Subsequently, we assume that a third party (controller) prepares the shared W state, and sends the first two qubits to Alice and the third qubit to Bob through noisy channels. In this case, we show that one just need to apply EAM during the entanglement distribution process and no WM is required to achieve the teleportation fidelity equal to one.

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where

\begin{align*}
\rho_n &= |\psi_n\rangle\langle\psi_n| = \begin{bmatrix}
|\alpha|^2 & \alpha\beta^* \\
\alpha^*\beta & |\beta|^2
\end{bmatrix} \quad (1)
\end{align*}

where \(|\alpha|^2 + |\beta|^2 = 1\) and "*" denotes complex conjugation.

Alice prepares the following class of W state as the shared entangled state \([13]\)

\begin{align*}
|W_n\rangle_{123} &= \frac{1}{\sqrt{2 + 2n}} \\
|100\rangle_{123} + \sqrt{n}e^{i\gamma}|010\rangle_{123} + \sqrt{n + 1}e^{i\delta}|001\rangle_{123}
\end{align*} \quad (2)

where \(n\) is a real number, and \(\gamma\) and \(\delta\) are phases.

She keeps the qubits 1 and 2, and sends the qubit 3 of the entangled shared state to Bob through an ADC (PDC). The ADC is defined by the well-known Kraus operators as

\begin{align*}
e_{0}^{ADC} &= \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-r} \end{bmatrix},
e_{1}^{ADC} &= \begin{bmatrix} 0 & \sqrt{r} \\ 0 & 0 \end{bmatrix} \quad (3)
\end{align*}

And the Kraus operators of the PDC is presented as

\begin{align*}
e_{0}^{PDC} &= \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-r} \end{bmatrix},
e_{1}^{PDC} &= \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{r} \end{bmatrix} \quad (4)
\end{align*}

where \(0 \leq r \leq 1\) is the decaying rate.

Since only the third qubit of the shared entangled state passes through the noisy channel, the applied Kraus operators for entangled state is

\begin{align*}
E_0 = I \otimes I \otimes e_{0}^{ADC(PDC)},
E_1 = I \otimes I \otimes e_{1}^{ADC(PDC)}
\end{align*} \quad (5)

where \(I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\) is the identity operator.

Then Bob performs the EAM and tells the result to Alice. If the channel is in unexcited states \((e_0)\), the entanglement distribution is successfully done, and they can start the teleportation process. Otherwise, he discards the entanglement distribution at this time and restarts the process. By considering three-qubit Kraus operators in Eq. (5), we only keep the results corresponding to \(E_0\). According to Eqs. (3) and (4), the invertible Kraus operator of ADC and PDC \((e_0)\) is the same. Hence, by applying EAM and discarding the results corresponding to \(e_{1}^{ADC}\) and \(e_{1}^{PDC}\), the ADC and PDC have the same effects on the entangled state. That is why our TP-EW is applicable for both PDC and ADC.

In this way, the quantum channel between two partners has been successfully constructed and the shared entangled state can be described as

\begin{align*}
|W_{n}\rangle_{123}^{E_0} &= E_0|W_{n}\rangle_{123} = \frac{1}{\sqrt{2 + 2n}} \\
&\left([100]\right)_{123} + \sqrt{n}e^{i\gamma}|010\rangle_{123} + \sqrt{n + 1}e^{i\delta}\sqrt{1-r}|001\rangle_{123}
\end{align*} \quad (6)

To start the teleportation, Alice interacts the input qubit with her qubit of the entangled shared state. The state of the whole system becomes

\begin{align*}
|\psi_{n}^{foo}\rangle &= (\alpha|0\rangle + \beta|1\rangle)_{a} \otimes \left(\frac{1}{\sqrt{2 + 2n}} [100]_{123} \right.
\end{align*} \quad (7)

And it can be rewrote as

\begin{align*}
|\psi_{n}^{foo}\rangle &= \frac{1}{\sqrt{2 + 2n}} [\alpha|010\rangle_{a12}0\rangle_{3}
\end{align*} \quad (8)

Figure 1: The schematic diagram of proposed TP-EW. The double lines indicate the classical communications.
where $|\eta_n^\pm\rangle$ and $|\xi_n^\pm\rangle$ are a set of orthogonal states in the W state category defined as

\[
|\eta_n^\pm\rangle = \frac{1}{\sqrt{2+2n}}(|010\rangle + \sqrt{n}e^{i\gamma}|001\rangle) \\
|\xi_n^\pm\rangle = \frac{1}{\sqrt{2+2n}}(|110\rangle + \sqrt{n}e^{i\gamma}|101\rangle) 
\]

(9)

From now on, for simplicity we set $\gamma = \delta = 0$. Now Alice performs a von Neumann measurement in the basis $\{|\eta_n^\pm\rangle, |\xi_n^\pm\rangle\}$ on her two qubits of the entangled shared state and the input state, and sends the results of her measurement to Bob through classical channel. The probability of occurrence of each measurement operator $\phi_i^{\pm}(i = \eta, \xi) = |\phi_i^\pm\rangle\langle \phi_i^\pm|$ outcome is

\[
P^{\pm} = \langle \psi^a_{\eta,\xi}|\phi_i^{\pm}\rangle\langle \phi_i^{\pm}|\psi^a_{\eta,\xi}\rangle = \frac{1}{4} \tag{10}
\]

By following the standard teleportation protocol and applying unitary operations by Bob, the amount of teleportation fidelity will decrease dramatically. Therefore, we design the WM operators to be applied in the last step of the teleportation to convert the state of Bob’s particle to that of particle $a$. The WM operators $M$ are from the complete measurement set $\{M, M^\dagger\}$ as [44]

\[
M = \begin{bmatrix}
\sqrt{1-q} & 0 \\
0 & 1
\end{bmatrix}, \quad M^\dagger = \begin{bmatrix}
\sqrt{q} & 0 \\
0 & 0
\end{bmatrix}
\]

(11)

where $0 \leq q \leq 1$ is the strength of the WM and $M^\dagger M + M^\dagger M = I$. In our teleportation protocol, we only preserve the result of $M$, discard the result of $M^\dagger$ and normalize the final state at the end of the teleportation process. Hence, the WM operators according to different Alice’s measurement results are given in Table 1.

Generally, the output state of Bob corresponding to different measurement outcomes of Alice can be described as follows

\[
\rho_3^{\pm}(i = \eta, \xi) = \frac{M_i^\dagger|\psi_3^{i\pm}\rangle\langle \psi_3^{i\pm}|M_i^\dagger}{g_i^{\pm}} = \frac{1}{g_i^{\pm}} \begin{bmatrix}
\frac{1}{4}||\alpha|^2(1-q) & \frac{1}{4}\alpha\beta\sqrt{1-q}\sqrt{1-r} \\
\frac{1}{4}\alpha^*\beta\sqrt{1-q}\sqrt{1-r} & \frac{1}{4}||\beta|^2(1-r)
\end{bmatrix} 
\]

(12)

where $|\psi_3^{i\pm}\rangle$ is Bob’s state corresponding to different measurement results of Alice, $M_i^\pm$ is the corresponding WM operators given in Table 1, and $g_i^{\pm}$ is the success probability of gaining the state $\rho_3^{i\pm}$ as

\[
g_i^{\pm} = \langle \psi_3^{i\pm}|M_i^{\dagger\pm}M_i^{\pm}|\psi_3^{i\pm}\rangle = \frac{1}{4}(||\alpha|^2(1-q) + ||\beta|^2(1-r)) 
\]

(13)

Therefore, the total teleportation success probability of TP-EW over all possible input states can be defined as

\[
g_{tot}^{TP-EW} = \int dp \sum_{i=\eta,\xi} g_i^{+} + g_i^{-} = \int dp \sum_{i=\eta,\xi} \langle \psi_3^{i\pm}|M_i^{\dagger\pm}M_i^{\pm}|\psi_3^{i\pm}\rangle = \int dp(|\alpha|^2(1-q) + ||\beta|^2(1-r)) = 1 - \frac{1}{3}(2r + q) 
\]

(14)

To evaluate the performance of the proposed TP-EW, we also consider the fidelity between input state Eq. (1) and the output state of TP-EW in Eq. (12) as

\[
fid^{\pm} = Tr(\rho_3^{i\pm}|\psi_3^{i\pm}\rangle\langle \psi_3^{i\pm}|) = \frac{||\beta||^4(1-r) + ||\alpha||^4(1-q) + 2||\alpha||^2||\beta||^2\sqrt{1-q}\sqrt{1-r}}{||\alpha||^2(1-q) + ||\beta||^2(1-r)} 
\]

(15)

Hence, the average teleportation fidelity of proposed TP-EW over all possible input states is

\[
Fid_{av}^{TP-EW} = \int dp \sum_{i=\eta,\xi} P^{\pm} fid^{\pm} = \int dp\frac{1}{1 - \frac{1}{3}(2r + q)} \frac{||\beta||^4(1-r) + ||\alpha||^4(1-q) + 2||\alpha||^2||\beta||^2\sqrt{1-q}\sqrt{1-r}}{||\alpha||^2(1-q) + ||\beta||^2(1-r)} 
\]

(16)

1.1 Comparison with standard teleportation protocol with no protection

In this subsection we compare the proposed TP-EW with standard teleportation protocol through the ADC and PDC with no protection. As we mentioned before, since PDC and ADC has the same invertible Kraus operator, the performances of TP-EW are the same for these two types of noisy channels.

First, we study the average teleportation fidelity of standard teleportation protocol through the ADC with no protection. In this case, the shared entangled W state in Eq. (2) after passing through the ADC is defined as

\[
|W_{n}\rangle_{123}^{ST} = E_0|W_{n}\rangle_{123} + E_1|W_{n}\rangle_{123} 
\]

(17)
Table 1: Alice’s measurement results and corresponding Bob’s WM operators to reconstruct the input state in TP-EW.

| Alice’s result | Bob’s state | Bob’s WM operator |
|----------------|-------------|------------------|
| $|\eta_n^+\rangle$ | $|\psi_3^+\rangle = \frac{1}{2}(|0\rangle_3 + \beta \sqrt{1-r}|1\rangle_3)$ | $M^+_\eta = \sqrt{1-q}|0\rangle\langle 0| + |1\rangle\langle 1|$ |
| $|\eta_n^-\rangle$ | $|\psi_3^-\rangle = \frac{1}{2}(|0\rangle_3 - \beta \sqrt{1-r}|1\rangle_3)$ | $M^-_\eta = \sigma_z|\sqrt{1-q}|0\rangle\langle 0| + |1\rangle\langle 1|$ |
| $|\xi_n^+\rangle$ | $|\psi_5^+\rangle = \frac{1}{2}(|\beta| |0\rangle_3 + \alpha \sqrt{1-r}|1\rangle_3)$ | $M^+_\xi = \sigma_x|\sqrt{1-q}|0\rangle\langle 0| + |1\rangle\langle 1|$ |
| $|\xi_n^-\rangle$ | $|\psi_5^-\rangle = \frac{1}{2}(|\beta| |0\rangle_3 - \alpha \sqrt{1-r}|1\rangle_3)$ | $M^-_\xi = \sigma_x \sigma_z|\sqrt{1-q}|0\rangle\langle 0| + |1\rangle\langle 1|$ |

where $E_0$ and $E_1$ are the applied Kraus operators given in Eq. (5).

Furthermore, by considering the standard teleportation protocol with W state and applying unitary operations in the last step, Bob’s qubit corresponding to different Alice’s measurement becomes

$$
\rho^{\text{ST-ADC}}_{out_{\eta,\xi}} = \begin{bmatrix}
|\alpha|^2 + |\beta|^2 r & \alpha \ast \beta \ast \sqrt{1-r} \\
\alpha \ast \beta \ast \sqrt{1-r} & |\beta|^2 (1-r)
\end{bmatrix}
$$

(18)

with corresponding probabilities $g^{\text{ST-ADC}}_{\eta,\xi}(i = \eta, \xi) = \frac{1}{4}$.

The teleportation fidelity corresponding to different Alice’s measurement results is presented as

$$
fid_{\eta,\xi}^{\text{ST-ADC}} = \text{Tr}(\rho_{in} \rho^{\text{ST-ADC}}_{out_{\eta,\xi}}) = |\alpha|^4 + |\beta|^4 (1-r) + |\alpha|^2 |\beta|^2 (r + 2 \sqrt{1-r})
$$

(19)

Hence, the average teleportation fidelity of standard teleportation with no protection through the ADC is calculated as

$$
Fid_{av}^{\text{ST-ADC}} = \frac{1}{30} \sum_{i=1}^{4} g_i f:id_{i}^{\text{ST-ADC}}
$$

(20)

Furthermore, by considering PDC the final state of Bob after applying unitary operations corresponding to different Alice’s measurement results is the same and is presented as

$$
\rho^{\text{ST-PDC}}_{out} = \begin{bmatrix}
|\alpha|^2 & \alpha \ast \beta \ast \sqrt{1-r} \\
\alpha \ast \beta \ast \sqrt{1-r} & |\beta|^2
\end{bmatrix}
$$

(21)

Hence, the average teleportation fidelity of standard teleportation with no protection through PDC is calculated as

$$
Fid_{av}^{\text{ST-PDC}} = \int d\rho \text{Tr}(\rho_{in} \rho^{\text{ST-PDC}}_{out}) = \int d\rho (|\beta|^4 + |\alpha|^4 + 2 |\alpha|^2 |\beta|^2 \sqrt{1-r}) = \frac{1}{15} (4 \sqrt{1-r} + 11)
$$

(22)

To show the improvement of teleportation fidelity of TP-EW through PDC and ADC, we define the fidelity of difference as

$$
Fid_{diff}^{\text{PDC}} = Fid_{av}^{\text{TP-EW}} - Fid_{av}^{\text{ST-PDC}}
$$

(23)

$$
Fid_{diff}^{\text{ADC}} = Fid_{av}^{\text{TP-EW}} - Fid_{av}^{\text{ST-ADC}}
$$

where $Fid_{av}^{\text{TP-EW}}$ is the average teleportation fidelity of TP-EW in Eq. (16), $Fid_{av}^{\text{ST-ADC}}$ is the average teleportation fidelity of standard teleportation with no protection through ADC in Eq. (20) and $Fid_{av}^{\text{ST-PDC}}$ is the average teleportation fidelity of standard teleportation with no protection through PDC in Eq. (22). The contour plot of $Fid_{diff}^{\text{ADC}}$ and $Fid_{diff}^{\text{PDC}}$ by varying the measurement strength $q$ and decaying rate $r$ is given in Fig. 2.

As Fig. 2 demonstrates, the proposed TP-EW improves the teleportation fidelity in both ADC and PDC compared to no protection standard teleportation protocol for all decaying rates and a wide range of WM strength $q$. For smaller $r$, by applying smaller $q$ the proposed TP-EW has better performance compared to standard teleportation with no protection; but for larger $r$, a larger $q$ is needed to attain a higher fidelity of difference. Also, the TP-EW has better performance in case of ADC, where the maximum value of $Fid_{diff}^{\text{ADC}}$ is 0.47 when $r = 1$ and $q = 1$ while the maximum value of $Fid_{diff}^{\text{PDC}}$ for $r = 1$ and $q = 1$ is 0.21.

To better study the performance of TP-EW compared to standard teleportation with no protection, we give the maximum amount of fidelity...
A GHZ state is considered as a GHZ state (WMRTP) protocol based on weak measurement reversal. We discuss another probabilistic teleportation of the performances of our proposed TP-EW. In this subsection, to show the improvement of difference and the corresponding WM strength $q$ for different decaying rates in Table 2.

According to Table 2, the TP-EW has better performance for intense decaying rates compared to standard teleportation with no protection. Also, the optimum value of $q$ to gain the maximum fidelity of difference is the same for ADC and PDC; hence, once the optimum $q$ is found, it is applicable for both ADC and PDC. Moreover, in consistent with Fig. 2, the TP-EW has better performance in case of ADC for all decaying rates.

1.2 Comparison with weak measurement reversal teleportation protocol

In this subsection, to show the improvement of the performances of our proposed TP-EW, we discuss another probabilistic teleportation protocol based on weak measurement reversal (WMRTP) [11]. In WMRTP the shared entangled state is considered as a GHZ state $|\psi\rangle_{ab} = \cos \frac{\theta}{2} |0\rangle_a |0\rangle_b + \sin \frac{\theta}{2} |1\rangle_a |1\rangle_b$ which is a maximally entangled state (product state) when $\theta = \frac{\pi}{2}$ $(\theta = 0)$, respectively. Alice prepares the shared entangled state and send one qubit to Bob through ADC. In WMRTP, Bob applies weak measurement reversal (WMR) instead of unitary operations to suppress the effects of the noisy channel.

The average teleportation fidelity of WMRTP is presented as

$$Fid_{av}^{WMRTP} = \int d\psi \left( \frac{1 + r|\alpha|^2|\beta|^2\tan^2 \frac{\theta}{2}}{1 + r|\beta|^2\tan^2 \frac{\theta}{2}} \right)$$

$$Fid_{av}^{ADC} = \int d\psi \left( \frac{1 + r|\alpha|^2|\beta|^2\tan^2 \frac{\theta}{2}}{1 + r|\alpha|^2|\beta|^2\tan^2 \frac{\theta}{2}} \right)$$

FIG. 2: The fidelity of difference between TP-EW and standard teleportation with no protection in case of (a) teleportation through ADC ($Fid_{av}^{ADC}$) (b) teleportation through PDC ($Fid_{av}^{PDC}$).

For fair comparison, we consider the WMRTP with maximally entangled shared state ($\theta = \frac{\pi}{2}$). In Fig. 3(a), we plot the average teleportation fidelity of TP-EW $Fid_{av}^{TP-EW}$ in Eq. (16), the average teleportation fidelity of WMRTP $Fid_{av}^{WMRTP}$ in Eq. (24) (the lavender plate) and the average teleportation fidelity of standard teleportation with no protection through ADC in Eq. (14) (the gray plate) as a function of decaying rate $r$ and the WM strength $q$. Moreover, the total teleportation success probability of TP-EW $g_{av}^{TP-EW}$ in Eq. (14) and the total teleportation success probability of WMRTP $g_{av}^{WMRTP}$ in Eq. (25) (the lavender plate) as a function of the decaying rate $r$ and the WM strength $q$ is given in Fig. 3(b).

As Fig. 3 depicted, by selecting the appropriate amount of WM strength $q$, the proposed TP-EW is capable of gaining the average teleportation fidelity more than 0.85 even for intense decaying rates with acceptable total success probability more than 0.4. The maximum average teleportation fidelity of TP-EW is illustrated by the black line which is equal to one. It is shown in Fig. 3(a) that the proposed TP-EW significantly improves the average teleportation fidelity compare to WMRTP (the lavender plate) and the standard teleportation with no protection (the gray plate). We show the difference between the
fidelity of our proposed protocol and the standard teleportation with no protection (the gray plate) in Fig. 2(a). The average teleportation fidelity of TP-EW can be higher than WMRTP for all amounts of decaying rates by choosing the appropriate WM strength \( q \). However, as Fig. 3(b) illustrates, the total teleportation success probability of our proposed TP-EW is lower than WMRTP total teleportation success probability for smaller decaying rates. By contrasting Fig. 3(a) with Fig. 3(b) on the same \( r \), it can be inferred that a smaller \( q \) leads to improvement in average teleportation fidelity with less decreasing in the amount of total teleportation success probability. Particularly, for \( q = 0 \) (the ridge lines of the TP-EW curves in Figs. 3(a) and (b)) the average teleportation fidelity is improved without loss of the total teleportation success probability compared to WMRTP.

Thus, we introduce two schemes according to specific amounts of WM strength \( q \) in TP-EW: 1) The EAM scheme which only applies EAM during the entanglement distribution process with no WM applied in the last step of teleportation (\( q = 0 \)). The average teleportation fidelity of EAM scheme \( F_{\text{Id}}^\text{EAM} \) can be calculated by setting \( q = 0 \) in Eq. (16), and the total teleportation success probability of EAM scheme \( P_{\text{tot}}^{\text{EAM}} = 1 - \frac{2}{3} r \) is calculated by setting \( q = 0 \) in Eq. (14). 2) The maximal TP-EW (MTP-EW) which is gaining the maximal average teleportation fidelity of TP-EW by setting the optimum \( q \). To gain the maximum teleportation fidelity, the input state in Eq. (1) and the output state of TP-EW in Eq. (12) should be equal. Hence, according to Eq. (12), the optimum WM strength \( q_{\text{opt}} \) is obtained as

\[
1 - q = 1 - r = \sqrt{1 - q} \sqrt{1 - r} \Rightarrow q_{\text{opt}} = r \tag{26}
\]

The MTP-EW average teleportation fidelity is illustrated by the black line in Fig. 3(a).

By considering the optimum WM strength given in Eq. (26), the output state of Bob in Eq. (12) after applying designed WM becomes

\[
\rho_{3}^{\pm}(i = \eta, \xi) = \frac{M_{i}^{\pm} |\psi_{3}^{\pm} i \langle \psi_{3}^{\pm} | M_{i}^{\pm}}{g_{i}^{\pm}} = \frac{1}{4 \alpha^{2} + |\beta|^{2}} \begin{bmatrix} \frac{1}{4} |\alpha|^{2} (1 - r) & \frac{1}{4} \alpha^{*} \beta^{*} (1 - r) \\ \frac{1}{4} \alpha \beta (1 - r) & \frac{1}{4} |\beta|^{2} (1 - r) \end{bmatrix}
\]

which is exactly equal to the input state and the teleportation fidelity becomes \( F_{\text{Id}}^{\pm} = \text{Tr}(\rho_{in} \rho_{3}^{\pm}) = 1 \). Hence, by considering the optimum amount of WM strength \( q \) in Eq. (26), the output state becomes equal to the input state as shown in Eq. (27), which gain the maximum fidelity equal to one. We call the TP-EW scheme with optimum \( q \) as maximized TP-EW (MTP-EW). By considering the probability of occurrence of each measurement outcome of Alice’s measurement operators \( \hat{P}_{i}^{\pm} \) in Eq. (10), the maximized average teleportation fidelity of TP-EW becomes

\[
F_{\text{Id}}^{MTP-EW} = \int dp \sum_{i=\eta,\xi} (p_{i}^{\pm} F_{\text{Id}}^{\pm}) = 4 \times \frac{1}{4} = 1 \tag{28}
\]

Furthermore, the corresponding total teleportation success probability of MTP-EW by considering the optimum \( q \) is

\[
S_{\text{tot}}^{MTP-EW} = \sum_{i=\eta,\xi} g_{i}^{+} + g_{i}^{-} = \sum_{i=\eta,\xi} \langle \psi_{3}^{\pm} | M_{i}^{\pm} M_{i}^{\pm} | \psi_{3}^{\pm} \rangle
\]

\[
= 4 \times \frac{(1 - r)(|\alpha|^{2} + |\beta|^{2})}{4} = 1 - r \tag{29}
\]

According to Eq. (28) and Eq. (29), it can be concluded that the fidelity of proposed MTP-EW is always equal to one, while by decreasing the amount of decaying rate of the noisy channel the teleportation success probability decreases.

| Decaying rate | Max(\( F_{\text{Id}}^{\text{ADC}} \)) | Corresponding \( q \) | Max(\( F_{\text{Id}}^{\text{PDC}} \)) | Corresponding \( q \) |
|---------------|-----------------|----------------|-----------------|-----------------|
| \( r = 0.3 \) | 0.11            | 0.33           | 0.042           | 0.3             |
| \( r = 0.6 \) | 0.23            | 0.61           | 0.096           | 0.62            |
| \( r = 0.9 \) | 0.38            | 0.9            | 0.17            | 0.9             |

Table 2: Maximum fidelity of difference and corresponding \( q \) for different decaying rates.
The differences between the control process of our proposed TP-EW, the EAM scheme and previous teleportation protocol WMRTP are illustrated in Table ??.

Comparison results of three schemes (MTP-EW, EAM and WMRTP) is shown in Fig. 4. In Fig. 4(a) we plot: 1) the average teleportation fidelity of MTP-EW $Fid^{\text{MTP-EW}}_{av}$ in Eq. (28) which is the maximized fidelity of TP-EW by setting the optimum WM strength $q$; 2) the average teleportation fidelity by only considering EAM $Fid^{\text{EAM}}_{av}$ in Eq. (16); and 3) the average teleportation fidelity of WMRTP $Fid^{\text{WMRTP}}_{av}$ in Eq. (24). Moreover, Fig. 4(b) is the comparison result of: 1) the corresponding total teleportation success probability of MTP-EW $g^{\text{MTP-EW}}_{tot}$ in Eq. (29); 2) the total teleportation success probability by only considering EAM $g^{\text{EAM}}_{tot}$ by setting $q = 0$ in Eq. (14); and 3) the total teleportation success probability of WMRTP in Eq. (25).
teleportation fidelity increases and the total success probability decreases.

2 Controlled teleportation with W states through noisy channels by utilizing environment-assisted measurement

In this section, we study a controlled teleportation protocol through noisy channels by utilizing EAM (CTP-EAM). Different from Section 2, here we assume that the shared W state is prepared by a third party (Charlie) who delivers the first two qubits to Alice and the third qubit to Bob through noisy channels. For simplicity, we assume that the decoherence process occurs for three qubits locally and independently but with the same damping rate. Hence, the decoherence process can be described by eight Kraus operators as

\[
E_0 = e_0^{\text{ADC(PDC)}} \otimes e_0^{\text{ADC(PDC)}} \otimes e_0^{\text{ADC(PDC)}}, \\
E_1 = e_0^{\text{ADC(PDC)}} \otimes e_0^{\text{ADC(PDC)}} \otimes e_1^{\text{ADC(PDC)}}, \\
E_2 = e_0^{\text{ADC(PDC)}} \otimes e_1^{\text{ADC(PDC)}} \otimes e_0^{\text{ADC(PDC)}}, \\
E_3 = e_0^{\text{ADC(PDC)}} \otimes e_1^{\text{ADC(PDC)}} \otimes e_1^{\text{ADC(PDC)}}, \\
E_4 = e_1^{\text{ADC(PDC)}} \otimes e_0^{\text{ADC(PDC)}} \otimes e_0^{\text{ADC(PDC)}}, \\
E_5 = e_1^{\text{ADC(PDC)}} \otimes e_0^{\text{ADC(PDC)}} \otimes e_1^{\text{ADC(PDC)}}, \\
E_6 = e_1^{\text{ADC(PDC)}} \otimes e_1^{\text{ADC(PDC)}} \otimes e_0^{\text{ADC(PDC)}}, \\
E_7 = e_1^{\text{ADC(PDC)}} \otimes e_1^{\text{ADC(PDC)}} \otimes e_1^{\text{ADC(PDC)}}
\]

(30)

After Alice interacts the input qubit with her qubit of the entangled shared state, the state of the whole system becomes

\[
|\psi_s^{\text{EAM}}\rangle = (\alpha|0\rangle + \beta|1\rangle)_a \otimes \left(\frac{1}{\sqrt{2+2n}}(|1 - r|00\rangle_{123} + \sqrt{1 - r} \sqrt{n e^{i\gamma}}|01\rangle_{123} \right) + \sqrt{n + 1}\sqrt{1 - re^{i\delta}}|00\rangle_{123})
\]

(31)

Next, Alice performs a von Neumann measurement in the basis \{\{\eta_n^+, \xi_n^+\}\} on her two qubits of the entangled shared state and the input state, and sends the results of her measurement to Bob through a classical channel. As we find out, when all the three qubits of W state pass through the ADC (PDC) with a successful EAM, we do not need to perform the WM in the last step. In other words, the EAM scheme is the optimal teleportation protocol which obtain the average teleportation fidelity equal to one. The schematic diagram of CTP-EAM is given in Fig. 5.

\[
|\psi_s^{\text{EAM}}\rangle = (\alpha|0\rangle + \beta|1\rangle)_a \otimes \left(\frac{1}{\sqrt{2+2n}}(|1 - r|00\rangle_{123} + \sqrt{1 - r} \sqrt{n e^{i\gamma}}|01\rangle_{123} \right) + \sqrt{n + 1}\sqrt{1 - re^{i\delta}}|00\rangle_{123})
\]

(32)

where the definition of \{\eta_n^+, \xi_n^+\} are the same as Eq. (9).

Alice and Bob perform the EAM and tell the results to Charlie. If the channels are in unexcited state \(E_0\), they can continue the teleportation process; otherwise, they need to discard the results and restart the entanglement distribution process. It is clear only \(E_0\) in Eq. (30) is invertible. As a result, we only keep the results corresponding to \(E_0\) and discard measurement results of \(E_1 \sim E_7\).

In this way, the quantum channel between two partners has been successfully constructed and the shared entangled state can be described as

\[
|W_{n_{123}}^{E_0}\rangle = E_0|W_{n_{123}}\rangle_{123}
\]

\[
= \frac{1}{\sqrt{2+2n}}(|1 - r|00\rangle_{123} + \sqrt{1 - r} \sqrt{n e^{i\gamma}}|01\rangle_{123} \right) + \sqrt{n + 1}\sqrt{1 - re^{i\delta}}|00\rangle_{123})
\]

(31)

Figure 5: The schematic diagram of CTP-EAM. The double lines indicate the classical communications.
outcomes of Alice as follows

$$\rho_3^{\pm}(i = \eta, \xi) = \frac{M_i^{\pm} \langle \psi_3^{\pm} | M_i^{\pm} | \psi_3^{\pm} \rangle}{g_i^{\pm}}$$

$$= \frac{1}{g_i^{\pm}} \left[ \frac{1}{2} \alpha \beta^*(1 - r) + \frac{1}{2} |\beta|^2(1 - r) \right]$$

$$= \left[ \frac{1}{2} \alpha^2 + \frac{1}{2} \beta^2 \right] = \rho_{in}$$

which is exactly equal to the input state, i.e., the teleportation fidelity is $$f_{id}^{\pm} = \text{Tr}(\rho_{in} \rho_3^{\pm}) = 1$$. Hence, the average teleportation fidelity $$\bar{F}_{ctp}$$ is also equal to one. And $$g_i^{\pm}$$ is the success probability of gaining the state $$\rho_3^{\pm}$$ as

$$g_i^{\pm} = \langle \psi_3^{\pm} | M_i^{\pm} | \psi_3^{\pm} \rangle = \frac{(1 - r)(|\alpha|^2 + |\beta|^2)}{4} = \frac{1}{4}(1 - r)$$

Therefore, the total teleportation success probability of the controlled teleportation protocol via EAM scheme can be calculated as

$$g_t^{\eta, \xi} = \sum_{i=\eta, \xi} g_i^{+} + g_i^{-} = 1 - r$$

According to Eq. (33) and Eq. (35), the CTP-EAM can achieve the teleportation fidelity equal to one with total success probability of $$g_t^{\eta, \xi} = 1 - r$$. This is because the three qubits of W state and the ADC (PDC) after a successful EAM, are completely symmetric which is favorable for quantum communication and computation.

2.1 Comparison with standard controlled teleportation protocol with no protection

In this subsection, we study the standard controlled teleportation (SCT) protocol with no protection in order to highlight the improvement of the average teleportation fidelity by applying the proposed CTP-EAM.

First, we study the average teleportation of the standard teleportation protocol through ADCs with no protection. In this case, the shared entangled W state in Eq. (2) after passing through ADCs becomes

$$|W_n\rangle_{123}^{\text{SCT}} = \sum_{i=0}^7 E_i |W_n\rangle_{123}$$

where $$E_i$$ are applied Kraus operators given in Eq. (30).

Furthermore, by considering the standard teleportation protocol with W states and applying unitary operations in the last step, Bob’s qubit corresponding to different Alice’s measurement results becomes

$$\rho_{out_{\eta, \xi}}^{\text{SCT-ADC}} = \frac{1}{|\alpha|^2(1 - r) + |\beta|^2(1 + r)}$$

$$\left[ |\alpha|^2(1 - r) + 2r|\beta|^2 \alpha^*(1 - r) \right]$$

$$\alpha^* \beta(1 - r) \left[ |\beta|^2(1 - r) \right]$$

$$\frac{1}{4}(1 - r) + |\alpha|^2(1 + r))$$

$$\frac{1}{4}|\beta|^2(1 - r) + |\alpha|^2(1 + r))$$

with corresponding probabilities

$$g_{\eta, \xi}^{\text{SCT-ADC}} = \frac{1}{4}(|\alpha|^2(1 - r) + |\beta|^2(1 + r))$$

$$g_{\xi, \eta}^{\text{SCT-ADC}} = \frac{1}{4}|\beta|^2(1 - r) + |\alpha|^2(1 + r))$$

Thus, the teleportation fidelity corresponding to different Alice’s measurement results can be calculated as

$$f_{id}^{\eta, \xi} = \text{Tr}(\rho_{in} \rho_{out_{\eta, \xi}}^{\text{SCT-ADC}})$$

$$= \frac{2|\alpha|^2|\beta|^2 + |\alpha|^4(1 - r) + |\beta|^4(1 - r)}{|\alpha|^2(1 - r) + |\beta|^2(1 + r)}$$

$$f_{id}^{\xi, \eta} = \text{Tr}(\rho_{in} \rho_{out_{\xi, \eta}}^{\text{SCT-ADC}})$$

$$= \frac{2|\alpha|^2|\beta|^2 + |\alpha|^4(1 - r) + |\beta|^4(1 - r)}{|\alpha|^2(1 - r) + |\beta|^2(1 + r)}$$

Hence, the average teleportation fidelity of SCT with no protection through ADCs is calculated as

$$F_{id}^{\text{SCT-ADC}} = \int d\rho \sum_{i=\eta, \xi} \left( \rho_i^{\text{SCT-ADC}} f_{id}^{\text{SCT-ADC}} \right)$$

$$= 2|\alpha|^2|\beta|^2 + |\alpha|^4(1 - r) + |\beta|^4(1 - r)$$

where $$\rho_i^{\text{SCT-ADC}} = \frac{1}{4}$$ is the probability of occurrence of each measurement outcome of Alice and $$f_{id}^{\text{SCT-ADC}}$$ are given in Eq. (39).

However, the SCT through PDCs with given W states introduces some errors which may lead to the failure of the teleportation protocol. To be specific, as we present in the subsequent derivation, the total success probability of the SCT through PDCs is not equal to one. After Alice applies the von Neumann measurements in the basis $$\{|\eta^\pm\rangle, |\xi^\pm\rangle\}$$ in Eq. (9), the probabilities of
gaining different measurement results by setting \( n = 1 \) are presented as

\[
P_{\eta \pm}^{\text{SCT-PDC}} = \frac{1}{4}(|\beta|^2(1 - \frac{r}{2}) + |\alpha|^2) \\
P_{\xi \pm}^{\text{SCT-PDC}} = \frac{1}{4}(|\beta|^2(1 - \frac{r}{2}) + |\alpha|^2)
\]

(41)

Also, the output state of Bob corresponding to different Alice’s measurement results are

\[
\rho_{\text{out} \eta \pm}^{\text{SCT-PDC}} = \frac{1}{2|\beta|^2 + |\alpha|^2(2 - r)} \begin{bmatrix}
|\alpha|^2(2 - r) & 2\alpha\beta^*(1 - r) \\
2\alpha^*\beta(1 - r) & 2|\beta|^2
\end{bmatrix}
\]

\[
\rho_{\text{out} \xi \pm}^{\text{SCT-PDC}} = \frac{1}{2|\alpha|^2 - |\beta|^2(2 - r)} \begin{bmatrix}
2|\alpha|^2 & 2\alpha^*\beta(1 - r) \\
2\alpha^*\beta(1 - r) & |\beta|^2(2 - r)
\end{bmatrix}
\]

(42)

with corresponding probabilities

\[
g_{\eta \pm}^{\text{SCT-PDC}} = P_{\eta \pm}^{\text{SCT-PDC}} \\
g_{\xi \pm}^{\text{SCT-PDC}} = P_{\xi \pm}^{\text{SCT-PDC}}
\]

(43)

where \( P_{\eta \pm}^{\text{SCT-PDC}} \) are given in Eq. (41).

Thus, the total success probability of CTP through PDCs becomes

\[
g_{\text{tot}}^{\text{SCT-PDC}} = \sum_{i = \eta, \xi} g_{i \pm}^{\text{SCT-PDC}} + g_{i -}^{\text{SCT-PDC}} = 1 - \frac{r}{4}
\]

(44)

Furthermore, the teleportation fidelity corresponding to different Alice’s measurement results are presented as

\[
fid_{\eta \pm}^{\text{SCT-PDC}} = \text{Tr}(\rho_{\text{in}}\rho_{\text{out} \pm}^{\text{SCT-PDC}}) = \frac{2|\beta|^4 + |\alpha|^4(2 - r) + 4|\alpha|^2|\beta|^2(1 - r)}{2|\beta|^2 + |\alpha|^2(2 - r)}
\]

\[
fid_{\xi \pm}^{\text{SCT-PDC}} = \text{Tr}(\rho_{\text{in}}\rho_{\text{out} \pm}^{\text{SCT-PDC}}) = \frac{2|\alpha|^4 + |\beta|^4(2 - r) + 4|\alpha|^2|\beta|^2(1 - r)}{2|\alpha|^2 - |\beta|^2(r - 2)}
\]

(45)

Hence, the average teleportation fidelity of SCT with no protection through PDCs is calculated as

\[
Fid_{av}^{\text{SCT-PDC}} = \int dp \sum_{i = \eta, \xi} (P_{i \pm}^{\text{SCT-PDC}} f_{i \pm}^{\text{SCT-PDC}})
\]

\[
= \int dp (|\alpha|^4(1 - \frac{r}{4}) + |\beta|^4(1 - \frac{r}{4}) + 2|\alpha|^2|\beta|^2(1 - r))
\]

(46)

Fig. 6 shows the average teleportation fidelity as a function of the decaying rate \( r \) for our proposed CTP-EAM, SCT through ADCs in Eq. (40) and SCT through PDCs in Eq. (46).

It is easy to see from Fig. 6 that the average teleportation fidelity of SCT through ADCs and PDCs decreases dramatically by increasing the decaying rate of the noisy channels, while our proposed CTP-EAM gains the fidelity equal to one independent of the amounts of the decaying rate. Therefore, in the case of controlled teleportation, only by applying EAM in entanglement distribution process of \( W \) states, we could gain the teleportation fidelity equal to one with a certain success probability. In other words, by applying a successful EAM, the \( W \) states and the noise become symmetric and no WM is required in the last step of the teleportation protocol.

3 Conclusion

We proposed a teleportation protocol with \( W \) states via the EAM and WM through noisy channels. The EAM is applied in entanglement distribution step to collect system states corresponding to invertible Kraus operators of the noisy channel. Afterwards, we design WM operators to be applied in the last step of teleportation to reverse the effects of the noise and obtain the teleportation fidelity equal to one. We derived the final expression of the average teleportation fidelity and total teleportation success probability. For comparison, we studied the standard teleportation with no protection through the ADC (PDC) and derived the final expression of the average teleportation fidelity. Numerical simulation re-
sults demonstrated the significant improvement of the teleportation fidelity of our proposed TP-EW in comparison with the standard teleportation with no protection. For further comparison we also study another probabilistic teleportation protocol via weak measurement reversal and show the improvement of the performances of the teleportation of our proposed TP-EW. In controlled teleportation where all qubits of the W state experience the uniform ADC (PDC), respectively, by applying the EAM the optimal teleportation fidelity equal to one is attained. These results will contribute to the distribution of multi-qubit entanglement in noisy channels and the protection of quantum communication.

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