Numerical studies of the two- and three-dimensional gauge glass at low temperature

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(Dated: September 21, 2018)

We report results from Monte Carlo simulations of the two- and three-dimensional gauge glass at low temperature using parallel tempering Monte Carlo. In two dimensions, we find strong evidence for a zero-temperature transition. By means of finite-size scaling, we determine the stiffness exponent \( \theta = -0.39 \pm 0.03 \). In three dimensions, where a finite-temperature transition is well established, we find \( \theta = 0.27 \pm 0.01 \), compatible with recent results from domain-wall renormalization group studies.

PACS numbers: 75.50.Lk, 75.40.Mg, 05.50.+q

I. INTRODUCTION

The gauge glass is a model often used to describe the vortex glass transition in high-temperature superconductors. Still, there are areas which need to be understood. In two dimensions, there is an ongoing controversy as to whether a spin-glass transition occurs at finite temperature or not. In three dimensions, a finite-temperature transition is well established, although there is no consensus on the value of the stiffness exponent. One group of the stiffness exponent found low but finite temperatures to provide a good estimate. In addition, Monte Carlo simulations by Fisher et al., Choi and Park ining resistively shunted junction (RSJ) dynamics, and by Granato et al., who study the scaling of the spin-glass susceptibility via Monte Carlo simulations, have been made. In contrast, Granato and Hyman et al., who also use RSJ dynamics find evidence of a zero-\( T \) transition. In addition, Monte Carlo simulations by Fisher et al. and Reger and Young show evidence of a \( T = 0 \) transition, although the simulations were not performed at low enough temperatures.

Here we perform Monte Carlo simulations of the two- and three-dimensional gauge glass, using parallel tempering, to go to significantly lower temperatures than was possible in earlier work. In particular, we are now able to cover the temperature range in two dimensions where the claimed spin-glass transition takes place. We find strong evidence that \( T_c = 0 \).

In three dimensions, we study the gauge glass at very low but finite temperatures to provide a good estimate of the stiffness exponent \( \theta \). Earlier estimates, which were obtained from ground-state methods, are inconsistent. One group finds values consistent with \( \theta \approx 0 \), whereas another finds \( \theta \) in the range 0.26 – 0.31. We find that \( \theta = 0.27 \pm 0.01 \), which agrees with the results of Refs. 11 and 14-17.

II. MODEL, OBSERVABLES, AND METHOD

The Hamiltonian of the gauge glass is given by

\[
\mathcal{H} = -\sum_{(i,j)} \cos(\phi_i - \phi_j - A_{ij}),
\]

where the sum ranges over nearest neighbors on a hyper-cubic lattice in \( D \) dimensions of size \( N = L^D \), and \( \phi_i \) represent the angles of the XY spins. Periodic boundary conditions are applied. The \( A_{ij} \) are quenched random variables uniformly distributed between \([0,2\pi]\). Because \( A_{ij} \) represent the line integral of the vector potential between sites \( i \) and \( j \), we have the constraint that \( A_{ij} = -A_{ji} \).

Traditionally, one uses the Binder ratio to estimate the critical temperature \( T_c \). As for the gauge glass the Binder ratio cannot exceed unity, the splaying of the curves is small, and \( T_c \) is difficult to establish. In order to avoid this problem, we use a method introduced by Reger and Young in which one calculates the current \( I \) defined as the derivative of the free energy with respect to an infinitesimal twist to the boundaries, i.e.,

\[
I(L) = \frac{1}{L} \sum_i \sin(\phi_i - \phi_{i+\hat{x}} - A_{i+\hat{x}}).
\]

In this case, the twist is applied along the \( \hat{x} \)-direction. As \( \langle I(L) \rangle_{av} = 0 \), we actually calculate the root-mean-square current \( I_{rms} \). Here \( \langle \cdots \rangle \) represents a thermal average, whereas \( [\cdots]_{av} \) represents a disorder average. Note that the current scales as

\[
I_{rms} = \bar{I}(L^{1/\nu}(T - T_c)),
\]

for \( T_c > 0 \), whereas for \( T_c = 0 \),

\[
I_{rms} = L^{-1/\nu} \bar{I}(L^{1/\nu}T).
\]

Equation 4 indicates that the \( T = 0 \) stiffness exponent \( \theta \) is negative and equal to \(-1/\nu \) (as \( I_{rms} \sim L^\theta \)). Equation 3 shows that if \( T_c > 0 \), the curves for \( I_{rms} \) for different sizes intersect at the critical point, whereas Eq. 4 shows that if \( T_c = 0 \) the data decrease with increasing size at \( T = 0 \). For a finite-temperature transition we expect \( \theta > 0 \), since then the ordered state at \( T = 0 \) is “stiff” on large scales, and so will presumably resist small thermal fluctuations. On the other hand, for a zero-temperature transition, the system will then easily break up under the influence of thermal fluctuations and therefore \( \theta < 0 \).

We also have calculated the spin-glass susceptibility, \( \chi_{SG} \), defined by

\[
\chi_{SG} = N[\langle q^2 \rangle]_{av},
\]
where $q$ is the spin-glass order parameter:

$$ q = \frac{1}{N} \sum_i^N \exp[i(\phi_i^a - \phi_i^b)] . \quad (6) $$

Here, $\alpha$, $\beta$ are two replicas of the system with the same disorder. According to standard finite-size scaling, the spin-glass susceptibility, defined in Eq. (5), behaves as

$$ \chi_{SG} = L^{2-\eta} \chi_{SG}(L^{1/\nu}(T - T_c)) , \quad (7) $$

meaning that at criticality ($T = T_c$) it diverges with a power law, i.e., $\chi_{SG} \sim L^{2-\eta}$. Note that the power-law prefactor in Eq. (4) with an unknown exponent complicates the analysis of $\chi_{SG}$ compared with that for $I_{rms}$.

For the simulations, we use parallel tempering Monte Carlo, as it allows us to study systems at lower temperatures than with conventional methods. For details about the implementation, equilibration tests, and detailed parameters of the simulations we refer the reader to Ref. [21]. In two dimensions, the highest temperature is 1.058, whereas the lowest temperature is 0.13 for $L = 4$, 6, 8, 12, and 16 (for $L = 24$ the lowest temperature is 0.20). The number of temperatures $N_T = 30$ is chosen to give satisfactory acceptance ratios for the Monte Carlo moves between the temperatures ($N_T = 24$ for $L = 24$). In 3D the lowest temperature studied is 0.05 (note that $T_c \approx 0.45$) whereas the highest temperature is 0.947, and $N_T = 53$ for $L = 3, 4, 5, 6, 7$, and 8.

### III. RESULTS FOR $D = 2$

Figure 1 shows a finite-size scaling plot of the data for $I_{rms}$ in two dimensions according to Eq. (4). The plot shows that the data collapse if $L^{1/\nu}T$ is small for all values of $L$, and over the whole range of $L^{1/\nu}T$ for the largest sizes. The inset shows results of the unscalled data. At all temperatures, the data decrease with increasing $L$ indicating, from Eq. (3), that $T_c$ must be less than the range of temperatures studied. The data in Fig. 1 are therefore consistent with a zero-temperature transition, but with significant corrections to scaling at intermediate temperatures. We estimate the stiffness exponent to be $\theta = -0.39 \pm 0.03$. The above error bar is estimated by varying $\theta$ slightly until the data do not collapse well. This result is consistent with recent work of Akino and Kosterlitz[11] who find $\theta = -0.36 \pm 0.01$. Choi and Park[12] parameters ($T_c = 0.22, 1/\nu = 0.88$) yield very poor scaling, especially near the proposed $T_c$. In fact, we are unable to get a reasonable fit to the data for $I_{rms}$ according to Eq. (3) for any finite $T_c$.

Figure 2 shows a scaling plot of the data for $\chi_{SG}$ according to Eq. (5) for $T_c = 0$ and $1/\nu = 0.39$, the same parameters found in the scaling of $I_{rms}$, together with $\eta = 0.0 \pm 0.1$, which is expected at a zero-temperature transition. The data at low $T$ and for the largest sizes scale well, but the data away from this range show deviations. Allowing $1/\nu$ to vary, we find $1/\nu = 0.50 \pm 0.03$.

The inset shows data for this optimal value, where only the $L = 4$ and 6 data are not part of the scaling function for all $L^{1/\nu}T$. The fact that the best values of $1/\nu$ are not precisely the same when obtained from $\chi_{SG}$ and $I_{rms}$ presumably indicates that scaling is only valid for fairly low temperatures and large sizes, and that, despite our working at quite low temperatures, we have only a limited range of data which are fully in the scaling regime.

We also scale the data for $\chi_{SG}$ with Choi and Park’s
parameters. The data collapse is poor near $T_c$. The
scaling of $I_{\text{rms}}$ is also much worse, indicating that $I_{\text{rms}}$
distinguishes between a finite $T_c$ and $T_c = 0$ much better
than $\chi_{SG}$ because of its simpler finite-size scaling form.
An attempt to scale the data for $\chi_{SG}$ with $T_c = 0.13$, the
lowest temperature simulated, shows that the best fit is
obtained with $1/\nu = 0.68$ and $\eta = 0.19$. While the data
scale acceptably well, the data for $I_{\text{rms}}$ scale poorly.

According to Eq. (7) for $T = T_c$, the data for $\chi_{SG}$
should lie on a straight line at $T_c$. However, the data
in the vicinity of $T = 0.22$, the transition temperature
claimed by Kim and Choi and Park, show a strong down
ward curvature in our analysis, indicating that this is actually above $T_c$. Only around the lowest tem-
perature where we have data, $T = 0.13$, is the curvature
small, although it still greatly exceeds the error bars.

This indicates that $T_c < 0.13$, which is compatible with
our data for $I_{\text{rms}}$.

IV. RESULTS FOR $D = 3$

Olson and Young have investigated the critical region
of the three-dimensional gauge glass obtaining a lower
bound for the stiffness exponent of $\theta \geq 0.18$. Some previ-
ous results find $\theta$ in the range $0 \leq \theta \leq 0.077$
whereas others find a much larger value, $0.26 \leq \theta \leq 0.31$.

We can estimate $\theta$ from our data for $I_{\text{rms}}$ since $I_{\text{rms}} \sim L^\theta$
when $L^{1/\nu}(T - T_c)$, the argument of the scaling function
in Eq. (8), tends toward infinity. To obtain an esti-
mate of $\theta$, we perform a linear least-squares fit of $\ln(I_{\text{rms}})$
against $\ln(L)$ for each temperature in order to obtain an
effective stiffness exponent $\theta_{\text{eff}}$ which depends on the
temperature. Figure 3 shows that $\theta_{\text{eff}}$ can be fitted
well to a linear form at low temperatures. Extrapolating
to $T = 0$, we obtain $\theta = 0.27 \pm 0.01$, which is clearly posi-
tive and consistent with the results of Refs. 10–13.

V. CONCLUSIONS

Our results from Monte Carlo simulations of the two-
dimensional gauge glass are consistent with a $T = 0$
transition with a stiffness exponent $\theta = -0.39 \pm 0.03$.
These results are incompatible with the prediction made
by Kim and Choi and Park that $T_c = 0.22$. In three
dimensions we report the first reliable estimate of the
stiffness exponent from finite-temperature Monte Carlo
simulations. We find $\theta = 0.27 \pm 0.01$, which agrees with
the results of Refs. 10–13.

HGK acknowledges support from the National Science
Foundation under Grant No. DMR 9985978 and would
like to thank A. P. Young for a fruitful collaboration.

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