Application of the extended P+QQ force model to $N \approx Z$ fp shell nuclei

M. Hasegawa $^a$, K. Kaneko $^b$ and S. Tazaki $^c$

$^a$Laboratory of Physics, Fukuoka Dental College, Fukuoka 814-0193, Japan
$^b$Department of Physics, Kyushu Sangyo University, Fukuoka 813-8503, Japan
$^c$Department of Applied Physics, Fukuoka University, Fukuoka 814-0180, Japan

Abstract

To study collective motion, the extended pairing plus QQ force model proposed recently is applied to $A=46, 48$ and 50 nuclei in the fp shell region. Exact shell model calculations in the truncated model space ($f_{7/2}, p_{3/2}, p_{1/2}$) prove the usefulness of the interaction. The simple model with the pairing plus quadrupole pairing plus QQ force and J-independent isoscalar proton-neutron force reproduces unexpectedly well observed binding energies, energy levels of collective (yrast) states and reduced $E2$ transition probabilities in $^{46}$Ti, $^{46}$V, $^{48}$V, $^{48}$Cr, $^{50}$Cr and $^{50}$Mn. The correspondence between theory and experiment is almost comparable to that attained by the full fp shell model calculations with realistic effective interactions. Some predictions are made for energy levels and variations of $B(E2)$ in the yrast bands, in these nuclei. Characteristics of the interaction are investigated by comparing with the realistic effective interactions.

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Key words: extended pairing plus quadrupole force; p-n interactions; shell model calculation; A=46, 48, 50 nuclei; ground-state energies; energy levels; B(E2).

1 Introduction

Two approaches starting from different pictures are crossing in fpq shell region: one is in progress along the pairing plus quadrupole ($P_0 + QQ$) force model [1–6] and the other is the shell model approach using various effective interactions [7–15]. With the advancement of computer, the shell model approach has been extending the scale of calculation and the sphere of study [16–23]. The full fp shell model calculations with the KB3 interaction [16,21–23] have successfully explained the microscopic structure of the $f_{7/2}$ shell nuclei,
and are pushing the front of study to $A \approx 52$. On the other hand, the $P_0 + QQ$ force written in the isospin invariant form seems to qualitatively explain some properties of nuclei in the $fp$ shell region [24–26]. One could expect a relationship between the two approaches and between the two types of interactions. In fact, it was shown [27] that an important part of the realistic interactions is approximated by the $P_0 + QQ$ force. The $P_0 + QQ$ force accompanied by the quadrupole pairing ($P_2$) force has succeeded in explaining the backbending mechanism of $^{48}\text{Cr}$ [28]. We note here that including both of $T = 0$ and $T = 1$ proton-neutron ($p$-$n$) interactions in the $P_0 + QQ$ force is recommended [25].

In a recent paper [29], we have shown that the $P_0 + QQ$ force can be extended so as to get near to realistic effective interactions. The extended interaction is composed of the four typical forms of interactions, i.e., the isospin invariant $P_0 + P_2 + QQ$ force and $J$-independent isoscalar $p$-$n$ force ($V_{\pi \nu}^{\tau=0}$). This form very harmonizes with the design of Ref. [27]. The calculations [29,30] showed that the isoscalar $p$-$n$ force $V_{\pi \nu}^{\tau=0}$ plays a decisive role in reproducing the gross nuclear properties such as the binding energy, symmetry energy, etc., and that the $P_0 + P_2 + QQ + V_{\pi \nu}^{\tau=0}$ interaction describes considerably well the $f_{7/2}$ and $g_{9/2}$ shell nuclei in which a subshell dominates. The usefulness of the model, however, was studied within the single $j$ shell model calculations there. The $P_0 + QQ$ force model originally aims to describe nuclear collective motion. The next crucial step is, therefore, to examine whether the extended $P_0 + QQ$ force model is quantitatively applicable to real situations having many $j$ shells and is capable of describing the collective motion or not. This examination will show to what extent the picture of the $P_0 + QQ$ force model persists by a modification in various regions. We have confirmed that our interaction can describe the global features of $p$-$n$ interactions and symmetry energy in another paper [31]. The purpose of this paper is to investigate in detail the applicability of the $P_0 + P_2 + QQ + V_{\pi \nu}^{\tau=0}$ interaction model by a realistic treatment of the $f_{7/2}$ shell nuclei.

We carry out exact shell model calculations in even-$A$ nuclei with $A = 44−50$ in order to see products of the interaction without any disturbance due to approximate treatment. This makes it easy to compare our interaction with the realistic effective interactions. We must determine the four force strengths when dealing with the $P_0 + P_2 + QQ + V_{\pi \nu}^{\tau=0}$ interaction. Unfortunately, the authors do not have a fast computer code enough for searching appropriate force strengths in the full $fp$ shell. We therefore adopt the configuration space ($f_{7/2}$, $p_{3/2}$, $p_{1/2}$) as a model space. Calculations prove the usefulness of this model space especially for our interaction including the $V_{\pi \nu}^{\tau=0}$ force (the reason will be explained in Sec. IV). Our simple model with only four force strengths in the small configuration space gives unexpectedly good results in $A=46$, 48 and 50 nuclei with $N \approx Z$, which almost match the full $fp$ shell model calculations with the KB3 interaction [16,21–23], so that we can discuss properties of these nuclei in our model and give information unreported previously.
This paper is organized as follows. In Sec. II, we briefly review the model. The results of calculations on the binding energies, energy levels and electric transition probabilities $B(E2)$ are shown for $^{46}$Ti, $^{46}$V, $^{48}$Cr, $^{50}$Cr and $^{50}$Mn in Sec. III. The dependence of the results on the model, the relation of our interaction to the realistic effective interactions and properties of the collective yrast bands are discussed in Sec. IV. Concluding remarks are given in Sec. V.

2 The model

We have proposed the following isospin-invariant Hamiltonian composed of the four forces, $P_0$, $P_2$, $QQ$ and $V_{\tau=50}^{\pi\nu}$, and have discussed the basic properties of it in Ref. [29]:

$$H = H_{sp} + V(P_0) + V(P_2) + V(QQ) + V_{\tau=0}^{\pi\nu},$$

$$H_{sp} = \sum_a \epsilon_a (\hat{n}_{a\pi} + \hat{n}_{a\nu}),$$

$$V(P_J) = -\frac{1}{2} g_J \sum M_a \sum_{a\leq b \leq c \leq d} p_J(ab)p_J(cd) A^J_{JM1\kappa}(ab) A_{JM1\kappa}(cd),$$

$$V(QQ) = -\frac{1}{2} \chi' \sum M \sum_{ac \rho} \sum_{bd \rho'} q'_{ac} q'_{bd} : B^\dagger_{2M_{\rho}(ac)} B_{2M_{\rho'}(bd)} :$$

$$V_{\pi\nu}^{\tau=0} = -k^0 \sum_{a \leq b} A^J_{JM00}(ab) A_{JM00}(ab),$$

with

$$A^J_{JM\tau\kappa}(ab) = [c^\dagger_a c^\dagger_b]_{JM\tau\kappa} / \sqrt{1 + \delta_{ab}}, \quad B^J_{JM\rho}(ac) = [c^\dagger_{ac} c_{c\rho}]_{JM},$$

$$p_0(ab) = \sqrt{2 j_a + 1} \delta_{ab}, \quad p_2(ac) = q'_{ac} = (a || r^2 Y_2 / b_0^2 || c) / \sqrt{5}.$$  

Here $\hat{n}_{a\pi}$ and $\hat{n}_{a\nu}$ are the proton- and neutron-number operators for a single-particle orbit $a$, the symbol $:$ in $V(QQ)$ means the normal order product of four nucleon operators, $\rho$ denotes the $z$-components of isospin $\pm \frac{1}{2}$ and $b_0$ is the harmonic-oscillator range parameter. This Hamiltonian has only the four force parameters, $g_0$, $g_2$, $\chi'$, and $k^0$, in addition to the single-particle energies $\epsilon_a$. We call the $P_0 + P_2 + QQ + V_{\pi\nu}^{\tau=0}$ interaction “functional effective interaction (FEI)” in this paper.

As shown in Ref. [29], the $J$-independent isoscalar $p-n$ force $V_{\pi\nu}^{\tau=0}$ is reduced to

$$V_{\pi\nu}^{\tau=0} = -\frac{1}{2} k^0 \{ \hat{n} \cdot \hat{n} \} \left( \frac{\hat{n}}{2} + 1 \right) - \hat{T}^2,$$
where the operator $\hat{n}$ stands for the total number of the valence nucleons and $\hat{T}$ for the total isospin, i.e., $\hat{n} = \hat{n}_p + \hat{n}_n = \sum_a (\hat{n}_{a\pi} + \hat{n}_{a\nu})$ and $\hat{T} = \sum_a \hat{T}_a$. For the states with the isospin $T = (n_n - n_p)/2$, this force can be rewritten as

$$\langle V_{\tau=0}^{\pi\nu} \rangle = -\frac{1}{2} k^0 n_p (n_n + 1). \quad (9)$$

In our isospin invariant Hamiltonian, two conjugate nuclei with $T = (n_n - n_p)/2 = (n_p - n_n)/2$ are equivalent to each other, and the $T = 1$ states in odd-odd nuclei with $N = Z = A/2$ are equivalent to those in even-even nuclei with $Z = A/2 - 1$ and $N = A/2 + 1$. The $p$-$n$ force $V_{\tau=0}^{\pi\nu}$ is indispensable for the binding energy but does not change the wave functions determined by the $P_0 + P_2 + QQ$ force. The $V_{\tau=0}^{\pi\nu}$ force is the main origin of the symmetry energy [30] and brings a shift to the energy space between the states with different isospins in a nucleus. The shell model Hamiltonian is originated in the Hartree-Fock (HF) picture and the single-particle energies $\epsilon_a$ stand for the single-particle mean field determined by the HF theory. The $p$-$n$ force $V_{\tau=0}^{\pi\nu}$ in the simple expression (9) suggests that $V_{\tau=0}^{\pi\nu}$ is probably related to the HF variation and may be an average term by which the mean field is accompanied. If we extract $V_{\tau=0}^{\pi\nu}$, residual interaction in the mean field comes from the $P_0$, $P_2$ and $QQ$ forces in our model. This interpretation is similar to that of Dufour and Zuker [27]. The residual $P_0 + P_2 + QQ$ interaction can be understood based on the Bohr-Mottelson picture [4]. This problem how to explain the phenomenologically introduced $p$-$n$ force $V_{\tau=0}^{\pi\nu}$ should be discussed further.

The Hamiltonian (1-5) describes the energy of valence nucleons outside the doubly-closed-shell core ($^{40}$Ca) excluding the Coulomb energy. The corresponding experimental energy is evaluated as

$$W_0(Z, N) = B(Z, N) - B(^{40}\text{Ca}) - \lambda(A - 40) - \Delta E_C(n_p, n_n), \quad (10)$$

where $B(Z, N)$ is the nuclear binding energy and $\lambda$ is the base level of single-particle energies. We fix $\lambda = -8.364$ MeV so that $W_0(^{41}\text{Ca}) = W_0(^{40}\text{Ca}) = 0.0$ and the lowest single-particle energy $\epsilon_{7/2}$ becomes zero, and evaluate the Coulomb energy correction $\Delta E_C(n_p, n_n)$ by the function [32]

$$\Delta E_C(n_p, n_n) = 7.279 n_p + 0.15 n_p(n_p - 1) - 0.065 n_p n_n. \quad (11)$$

We shall compare calculated ground-state energies with the experimental energies $W_0(Z, N)$.

We adopt the model space $(f_{7/2}, p_{3/2}, p_{1/2})$ as mentioned in Introduction. Results of calculations depend on the single-particle energies $\epsilon_a$ and the four force
parameters \( g_0, g_2, \chi' \) and \( k^0 \). We take \( \epsilon_a \) from the experimental spectrum of \(^{41}\)Ca (we fix \( \epsilon_{7/2} = 0 \) mentioned above) as follows:

\[
\epsilon_{7/2} = 0.0, \quad \epsilon_{3/2} = 1.94, \quad \epsilon_{1/2} = 3.61 \quad \text{in MeV.} \tag{12}
\]

We employed fixed numbers for \( g_0, g_2 \) and \( \chi' \) in the single \( j \) model in Refs. [29,30]. Calculations, however, recommend using \( A \)-dependent parameters for the \( P_0 \), \( P_2 \) and \( QQ \) forces in the present many \( j \) shell case as the ordinary treatment of these forces. We put the same \( 1/A \) dependence on \( k^0 \) as in Refs. [29,30]. A rough parameter search in \( A = 46 - 50 \) nuclei leads us to the values

\[
\begin{align*}
g_0 &= 0.48(42/A), \quad g_2 = 0.36(42/A)^{5/3}, \\
\chi' &= 0.30(42/A)^{5/3}, \quad k^0 = 2.23(42/A), \quad \text{in MeV.} \tag{13}
\end{align*}
\]

We do not readjust the seven parameters in each nucleus.

The present calculations are considerably realistic and are expected to provide good information about wave functions. To test the wave functions, we calculate the reduced quadrupole transition probabilities \( B(E2) \) and compare them with observed values. Following Caurier et al. [16], we also calculate the intrinsic quadrupole moment \( Q_0 \) by the equations

\[
\begin{align*}
Q_0 &= \frac{(J + 1)(2J + 3)}{3K^2 - J(J + 1)} Q_{\text{spec}}(J) \quad \text{for } K \neq 1, \tag{14}
\end{align*}
\]

\[
B(E2 : J_i \rightarrow J_f) = \frac{5}{16\pi} e^2 \langle J_i K 20 | J_f K \rangle^2 Q_0^2, \tag{15}
\]

where \( Q_{\text{spec}}(J) \) is the spectroscopic quadrupole moment \( \sqrt{16\pi/5 \langle JJ|Q_{20}^I|JJ\rangle} \) with \( Q_{2M\rho}^I = e^2 b_0^2 \sum_{ac} q'(ac)|c_{0p}^I c_{2\rho}^I| \) and \( K \) is the projection of the total spin on the intrinsic axis. We use the harmonic-oscillator range parameter \( b_0 = 1.01A^{1/6} \) fm, and effective charges of \( 1.5e \) for protons and \( 0.5e \) for neutrons in the calculations of \( B(E2) \) and \( Q_0 \). These values are the same as those used in Refs. [16,19,21–23]. We denote the intrinsic quadrupole moment calculated from Eq. (14) by \( Q_0^{(s)} \) and that calculated from Eq. (15) by \( Q_0^{(t)} \) following Martínez-Pinedo et al. [22].

### 3 The results of calculations

We have carried out exact shell model calculations with the model shown in Sec. II in \( A=46, 48 \) and \( 50 \) systems. The ground-state energies obtained are listed in Table 1, where the experimental energies \( W_0 \) are calculated
from mass excesses [33] using Eqs. (10) and (11). Our simple interaction
\[ P_0 + P_2 + QQ + V_{\pi\nu}^{\tau=0} \]
reproduces the experimental energies well. The agreement is excellent for the \( N = Z \) nuclei, \(^{46}\text{V},^{48}\text{Cr} \) and \(^{50}\text{Mn} \). Note that our model systematically reproduces the energies of different-number nuclei within a single set of parameters. The results confirm the essential role of the \( J \)-independent isoscalar \( p-n \) force \( V_{\pi\nu}^{\tau=0} \) in the binding energies of \( N \approx Z \) nuclei which is stressed in our previous papers [29,30]. It is certain that an important part of nucleon-nucleon interactions can be written as \( V_{\pi\nu}^{\tau=0} \).

### Table 1

| Ground-state energies of \( A = 46, 48, 50 \) nuclei. |
|-----------------------------------------------|
| \( ^{46}\text{Ti}(^{46}\text{V},0^+) \) | \( ^{48}\text{Ti} \) | \( ^{48}\text{V}(4^+) \) | \( ^{48}\text{Cr} \) | \( ^{50}\text{V}(6^+) \) | \( ^{50}\text{Cr}(^{50}\text{Mn},0^+) \) |
| \( W_0 \) | -20.28 | -23.82 | -26.71 | -32.38 | -30.48 | -38.71 |
| Calc. | -20.17 | -23.44 | -26.79 | -32.38 | -30.00 | -38.72 |

The \( P_0 + P_2 + QQ + V_{\pi\nu}^{\tau=0} \) interaction, however, cannot give enough binding energies as \( N \) separates from \( Z \). Especially, the disagreement becomes large when the number of valence protons or neutrons is 0 and 1. This is in contrast to that the KB3 interaction overbinds the \( A = 48 \) nuclei about 0.78 MeV but provides very good relative binding energies of all the \( A = 48 \) nuclei [16]. We shall discuss the difference of our interaction from the realistic interactions in the next section.

In the following subsections, we show calculated excitation energies and electric quadrupole properties in \(^{46}\text{Ti},^{46}\text{V},^{48}\text{V},^{48}\text{Cr},^{50}\text{Cr} \) and \(^{50}\text{Mn} \) with \( Z, N \geq 42 \) where the \( P_0 + P_2 + QQ + V_{\pi\nu}^{\tau=0} \) interaction works well. Only positive-parity states are considered in our model space. It should be remembered here that the energy spectra and wave functions are determined by the \( P_0 + P_2 + QQ \) force in our model, and the \( V_{\pi\nu}^{\tau=0} \) force causes only an energy shift depending on the valence nucleon number and isospin.

### 3.1 \(^{46}\text{Ti}\)

Calculated excitation energies and \( E2 \) transition probabilities in \(^{46}\text{Ti} \) are compared with observed ones [33–35] in Fig. 1 and Table 2. The yrast levels (the lowest-energy state of each spin) are satisfactorily reproduced in spite of the simple model. We can regard the \( 0^+_1, 2^+_1, 4^+_1, ... 14^+_1 \) states as the ground-state band. They are connected by the large \( E2 \) transition probabilities, especially up to the \( 10^+_1 \) state. The observed \( B(E2: J \to J - 2) \) values in the ground-state band are considerably well reproduced by our model though the model space does not include the \( f_{5/2} \) orbit. The spectroscopic quadrupole moments \( (Q_{\text{spec}}) \) of the \( 0^+_1, 2^+_1, ... 14^+_1 \) states have a minus sign and their values are in the range 19-26 \( e \text{ fm}^2 \). The intrinsic quadrupole moments \( Q_0^{(s)} \) and \( Q_0^{(t)} \)
are large and roughly constant up to the $10_1^+$ state, which tells us that these states are collective and have the same nature. The observed energy levels of the ground-state band show the backbending at the $10_1^+$ state. The calculated energy levels also show a similar tendency, though the position of the $10_1^+$ state is not very good. The calculated intrinsic quadrupole moment $Q_0^{(t)}$ suggests different structure at least above the $10_1^+$ state.

![Diagram of energy levels](image)

**Fig. 1.** Calculated and observed energy levels of $^{46}$Ti.

From the good reproduction of the $1_1^+$ and $11_1^+$ levels as well as the ground-state band, the calculated energy levels of the $J=$odd yrast states probably provide a good prediction for their positions. This will be tested in future experiments.

In Fig. 1, we also compare calculated low energy levels below 4 MeV and a couple of levels above it with observed ones, where we omit 4 levels assigned as $J^\pi = 2^+$ and 7 other levels with indefinite spin and parity [33] from the experimental column in order to avoid overcrowding. We can suppose that some of the observed states including the $0_2^+$ state at 2.661 MeV are core-excited states (probably 8-particle-2-hole ones) on the analogy of the core-excited states in $^{44}$Ti. The calculated energy levels of non-collective states lie at a good position roughly speaking and the level density looks good if we exclude the core-excited states.

The state at 10.040 MeV, which was assigned as $J=12$ before [36], was recently identified as $J=14$ [35], because the transition to $11_1^+$ was not detected. The calculated $B(E2)$ values, $B(E2 : 14_1 \rightarrow 12_1)=26$, $B(E2 : 12_2 \rightarrow 12_1)=3.3$ and $B(E2 : 13_1 \rightarrow 12_1)=0.1$ in $e^2$ fm$^4$, suggest that the two levels at 10.040 MeV and 10.380 MeV are probably $14_1^+$ and $12_2^+$, though their order is inverse in
our prediction. The present calculation also provides candidates for the two observed levels at 12.974 MeV and 13.169 MeV. They are possibly the $14^+_2$ and $15^+_1$ states from the calculated $B(E2)$ values.

Table 2

Electric quadrupole properties of the yrast and other states in $^{46}$Ti. The observed $B(E2)$ values are taken from Refs. [30] (Expt.1) and [31] (Expt.2).

| $J_n \rightarrow J'_m$ | $B(E2: J_n - J'_m)$ in $e^2 \text{fm}^4$ | calculated $Q_{spec}$, $Q_0$ in $e \text{fm}^2$ |
|-----------------------|---------------------------------|---------------------------------|
|                       | Expt.1                          | Expt.2                          | Calc.  | $Q_{spec}(J_n)$ | $Q_0^{(s)}$ | $Q_0^{(f)}$ |
| $2_1 \rightarrow 0_1$ | 197±32                          | 119                             | -18.9  | 66              | 77           |
| $4_1 \rightarrow 2_1$ | 192±32                          | 159                             | -24.5  | 67              | 75           |
| $6_1 \rightarrow 4_1$ | 158±28                          | 141                             | -21.3  | 53              | 67           |
| $8_1 \rightarrow 6_1$ | 119±38, 62±30                   | 126                             | -23.7  | 56              | 62           |
| $8_2 \rightarrow 6_1$ | 0.03                            | -11.1                           | 27     |                 |              |
|                       |                                  |                                 | 8.8    |                 |              |
| $10_1 \rightarrow 8_1$ | 152±15, 77±36                   | 104                             | -26.3  | 61              | 56           |
| $10_2 \rightarrow 10_1$ |                                  |                                 | 3.8    | -12.1           | 28           |
|                       |                                  |                                 | 3.7    |                 |              |
|                       |                                  |                                 | 46     |                 |              |
| $12_1 \rightarrow 10_1$ | 25±10, 22±7                     | 53                              | -22.3  | 50              | 39           |
| $12_2 \rightarrow 10_1$ |                                  | 17                              | -18.9  | 43              |              |
|                       |                                  |                                 | 7.8    |                 |              |
|                       |                                  |                                 | 3.3    |                 |              |
|                       |                                  |                                 | 0.3    |                 |              |
| $13_1 \rightarrow 12_1$ |                                  | 0.1                             |        |                 |              |
| $14_1 \rightarrow 12_1$ | 46±30, >44                      | 26                              | -22.8  | 51              | 27           |
|                       |                                  |                                 | 9.5    |                 |              |
| $15_1 \rightarrow 14_1$ |                                  | 2.6                             |        |                 |              |
|                       |                                  |                                 | 3.0    |                 |              |
| $14_2 \rightarrow 14_1$ |                                  | 6.2                             | -23.1  | 51              |              |
|                       |                                  |                                 | 12     |                 |              |
3.2 $^{46}$V

Recently, the $N = Z$ odd-odd nucleus $^{46}_{23}$V$_{23}$ has been studied by elaborate experiments [37,38]. Let us compare calculated energy levels of the $T = 0$ yrast states with observed ones in Fig. 2. Our interaction somewhat overbinds the $T = 0$ low-spin states as the realistic effective interactions do [37,38].

It is still notable that such a simple force $V_{\tau=0}$ improves so well both the binding energy and relative energy between the $T = 0$ and $T = 1$ states. If we regard the $T = 0$ yrast states $3^+_1$, $4^+_1$, $5^+_1$ ... $15^+_1$ as a collective band based on the $3^+_1$ state, the theoretical pattern of excitation is very similar to the observed pattern. The order of spin in the band is correctly reproduced. The correspondence of the excitation energies measured from the $3^+_1$ level between theory and experiment is good.

![Fig. 2. Calculated and observed energy levels of $^{46}$V. Only the yrast states with $T = 0$ are shown.](image)

The calculated $B(E2)$ values listed in Table 3 support that the yrast states $3^+_1$, $4^+_1$, $5^+_1$, $6^+_1$ and $7^+_1$ in order of spin $J$ are members of a collective band, because they are connected by large $B(E2 : J \rightarrow J - 1)$. The spectroscopic quadrupole moment $Q_{\text{spec}}$ shows a rapid change as $J$ increases till $8^+_1$, and then becomes nearly constant (-20 to -25 e fm$^2$). Above $7^+_1$, the pair states with even spin $2J$ and odd spin $2J + 1$ reverse their order in energy and the two series of states with odd $J$ and even $J$ connected by large $B(E2 : J \rightarrow J - 2)$ stand out. We can guess a structure change in the band between $7^+_1$ and $9^+_1$. Is it possible to regard the two series, $9^+_1$, $11^+_1$ ... $15^+_1$ and $8^+_1$, $10^+_1$ ... $14^+_1$, as two quasi-bands? The $\gamma$ transitions observed in Ref. [38] seem to display this feature.
Table 3
Calculated $B(E2)$ (in $e^2\text{ fm}^4$) and $Q_{\text{spec}}$ (in $e\text{ fm}^2$) with respect to the $T=0$ yrast states in $^{46}\text{V}$.

| $J \rightarrow J'$ | $B(E2)$ | $Q_{\text{spec}}$ | $J \rightarrow J'$ | $B(E2)$ | $Q_{\text{spec}}$ |
|-------------------|----------|-----------------|-------------------|----------|-----------------|
| 3                 | 33.2     |                 | 10 $\rightarrow$ 8 | 91       | -25.2           |
| 4 $\rightarrow$ 3 | 249      | 12.0            | $\rightarrow$ 9   | 24       |                 |
| 5 $\rightarrow$ 3 | 60       | -4.1            | $\rightarrow$ 11  | 31       |                 |
|                  | 207      |                 | 11 $\rightarrow$ 9 | 129      | -21.9           |
| 6 $\rightarrow$ 4 | 85       | -10.6           | 12 $\rightarrow$ 10| 71       | -21.2           |
|                  | 165      |                 | $\rightarrow$ 11  | 7.2      |                 |
| 7 $\rightarrow$ 5 | 46       | -14.8           | $\rightarrow$ 13  | 22       |                 |
|                  | 122      |                 | 13 $\rightarrow$ 11| 99       | -23.3           |
| 8 $\rightarrow$ 6 | 111      | -21.5           | 14 $\rightarrow$ 12| 33       | -24.0           |
|                  | 68       |                 | $\rightarrow$ 13  | 7.6      |                 |
| 9 $\rightarrow$ 7 | 142      | -20.3           | $\rightarrow$ 15  | 44       |                 |
| 8 $\rightarrow$ 9 | 75       |                 | 15 $\rightarrow$ 13| 65       | -22.0           |

In Ref. [37], the low-lying states of $^{46}\text{V}$ were examined by the full $fp$ shell model calculations with the realistic effective interactions, KB3 [16] and FPD6 [14]. We compare the result obtained using our interaction FEI with the results of Ref. [37], in Fig. 3. In this figure, all the energy levels below 3.25 MeV including unknown-parity levels but excluding the negative-parity ones are shown ($6_1^+$ and $4_2^+$ with $T=1$ are added). The calculated $T=0$ levels are shifted so that $3_1^+$ is situated at the observed position 0.801 MeV as done in Ref. [37]. The shift 0.56 MeV for the FEI is a little larger than 0.436 MeV for the KB3 and 0.498 MeV for the FPD6. For the $T=0$ states, the FEI result seems to be better than the FPD6 one and considerably match the KB3 one. The FEI result for the $T=1$ levels is not very bad as compared with the KB3 and FPD6 results. The non-collective states except the yrast states obtained by the FEI have a tendency to go down (this tendency is observed in other systems). The number of the energy levels obtained by the FEI may be too many below 3.25 MeV but is smaller than that of the observed levels. The levels shown in the column Expt possibly include core-excited states [37]. The $8p-2h$ states with positive parity in addition to the negative-parity states could be at low energy, because the ($fp)^8$ configurations corresponding to $^{48}\text{Cr}$ gain very large binding energy due to $\alpha$-like four nucleon correlations discussed in the next section. The observed energy levels with $T=0$ seem to be in a middle situation between the KB3 and FEI results.
3.3 $^{48}$Cr

The $^{48}$Cr nucleus has been attracting notice to the rotational band and backbending. In Fig. 4, we illustrate calculated and observed energy levels [33,39–41]. The observed yrast levels, all levels with $J \leq 6$ below 5 MeV and some others above 5 MeV are shown in the column Expt and corresponding levels obtained by the present model are shown in the column Calc. In Table 4, we compare electric quadrupole properties between theory and experiment [39] (the observed value of $B(E2 : 2^+_1 \rightarrow 0^+_1)$ is from Ref. [42]) and also show the full $fp$ shell model result of Caurier et al. [16] for comparison.

Our model reproduces satisfactorily well the observed levels of the ground-state band and the backbending at the $12^+_1$ level, though there are slight deviations for the $6^+_1$, $8^+_1$ and $10^+_1$ states. The agreement is nearly comparable to that of the full $fp$ shell model calculation with the KB3 interaction [16]. The calculated $E2$ transition probabilities $B(E2 : J \rightarrow J - 2)$ in the ground-state band, which are also comparable to those of Caurier et al., correspond well with the observed very large $B(E2)$ values in spite of the lack of the $f_{5/2}$ orbit. This suggests that our interaction including the $QQ$ force is in harmony with the
collective motion of the ground-state band and yields the large $E2$ transition probabilities. If we add the $f_{5/2}$ orbit to the present model, a smaller effective charge can probably reproduce the observed $B(E2)$ values. The spectroscopic quadrupole moment $Q_{spec}$ indicates the structure change of the ground-state band at $12I^+$, corresponding to the backbending. The relatively large $B(E2)$ values between the states with $J=$odd and $J=$even, $B(E2 : 11_1 \rightarrow 12_1)$ and $B(E2 : 13_1 \rightarrow 12_1)$ at the $12I^+$ state are interesting. Our model predicts that the odd-spin state $(2J - 1)^+_{1}$ lies near above the even-spin state $(2J)^+_{1}$ when $2J=10, 12$ and $14$ but the $(2J - 1)^+_{1}$ state is lower than the $(2J)^+_{1}$ state when $2J > 14$. The quality of our model will be examined by the experiment on these points.

The present model can give information on high-spin states except the ground-state band. Brandolini et al. [39] recently identified an energy level $16^+$ at 15.733 MeV. Near there, our model provides two candidates with $16^+$. Cameron et al. [40] detected two states $13^+$ and $14^+$ at 10.430 MeV and 10.610 MeV before. One of them possibly corresponds to the $13^+_1$ level in the column Calc. The position of the calculated low-lying levels except the yrast states seems to be not so bad. The calculated level density is, however, thicker than that observed until now. The present model has a tendency to give too many non-yrast levels at low energy, as seen in $^{46}$V. This could be attributed not only to the insufficiency of the $P_0 + P_2 + QQ + V_{\pi\nu}^{*=0}$ interaction but also to the small model space. Since the extension of the model space lowers selectively the
Table 4
Electric quadrupole properties of the yrast states in $^{48}$Cr: $B(E2)$ in $e^2$ fm$^4$; $Q_{spec}$ and $Q^{(t)}$ in $e$ fm$^2$.

| $J \rightarrow J'$ | Expt. | present work | Caurier et al. |
|---------------------|-------|--------------|----------------|
|                     | $B(E2)$ | $B(E2)$ | $Q_{spec}$ | $Q^{(t)}_0$ | $B(E2)$ | $Q_{spec}$ | $Q^{(t)}_0$ |
| 2 $\rightarrow$ 0   | 321(41)* | 217 | -29.7 | 104 | 228 | -29.5 | 107 |
| 4 $\rightarrow$ 2   | 329(110) | 305 | -38.8 | 104 | 312 | -39.2 | 105 |
| 6 $\rightarrow$ 4   | 301(78) | 300 | -38.9 | 98  | 311 | -39.7 | 100 |
| 8 $\rightarrow$ 6   | 230(69) | 285 | -40.7 | 93  | 285 | -38.9 | 93  |
| 7 $\rightarrow$ 5   |        | 92    |        |      |      |        |      |
| 10 $\rightarrow$ 8  | 195(54) | 231 | -35.6 | 83  | 201 | -22.5 | 77  |
| 9 $\rightarrow$ 10  |        | 38    |        |      |      |        |      |
| $\rightarrow$ 7     |        | 124   |        |      |      |        |      |
| 12 $\rightarrow$ 10 | 167(25) | 130 | -11.9 | 62  | 146 | -5.3  |      |
| 11 $\rightarrow$ 12 |        | 78    |        |      |      |        |      |
| $\rightarrow$ 9     |        | 37    |        |      |      |        |      |
| $\rightarrow$ 10    |        | 9     |        |      |      |        |      |
| 14 $\rightarrow$ 12 | 105(18) | 105 | -10.8 | 55  | 116 |       |      |
| 13 $\rightarrow$ 11 |        | 43    |        |      |      |        |      |
| $\rightarrow$ 12    |        | 43    |        |      |      |        |      |
| $\rightarrow$ 14    |        | 26    |        |      |      |        |      |
| 15 $\rightarrow$ 13 |        | 49    |        |      |      |        |      |
| $\rightarrow$ 14    |        | 22    |        |      |      |        |      |
| 16 $\rightarrow$ 14 | 37(8)  | 61   | -12.6 | 42  | 56  |       |      |
| 17 $\rightarrow$ 15 |        | 69    |        |      |      |        |      |
| $\rightarrow$ 16    |        | 15    |        |      |      |        |      |
| 18 $\rightarrow$ 16 |        | 0.0  | -29.2 | 0.2 |      |        |      |
| $\rightarrow$ 17    |        | 16    |        |      |      |        |      |

collective states rather than non-collective states, the full $fp$ shell calculation deserves to be tested.

Figure 4 illustrates the isobaric analogue states, $4^+$ and $2^+$ with $T = 1$ (dotted lines) and $0^+$ with $T = 2$ (dashed line). Our simple interaction including the isoscalar $p-n$ force $V^{\tau=0}_{\pi\nu}$ lays them at roughly good positions, if we neglect the
inverse order of the 4$^+$ and 2$^+$ levels in the calculated result.

3.4 $^{48}V$

In Fig. 5, we show calculated energy levels of the odd-odd nucleus $^{48}_{23}V_{25}$, compared with observed ones [33,40]. Only the yrast levels and 7 other levels with definitely assigned spin at low energy are shown in the column Expt, and corresponding energy levels obtained by the present model are shown in the column Calc. The present model gives lower energies to the 2$^+$ and 1$^+$ states than the 4$^+$ ground state. The states except the yrast states come to low energy region as compared with the observed ones. We have already stated the overbinding of non-collective states. The $P_0 + P_2 + QQ + V_{\pi\nu}^{\tau=0}$ interaction is still able to describe the yrast band 4$^+_1$, 5$^+_1$, 6$^+_1$ ... 15$^+_1$ based on the 4$^+_1$ state as seen in Fig. 5. The correspondence between calculated and observed yrast levels is quite well. The quality is the same as that of the full $fp$ shell model calculation with the KB3 interaction [16]. There is, however, a discrepancy between theory and experiment about the order of the levels 14$^+_1$ and 15$^+_1$.

Table 5 shows theoretical and experimental values of $B(E2)$ in $^{48}V$. The present results on the $B(E2)$ values agree well with those of the full $fp$ shell model calculation with the KB3 interaction [16], except the 2$^+$ and 4$^+_2$ states. Although there are little experimental data, our model seems to be able to explain the observed large values of $B(E2)$ between the yrast states 4$^+_1$, 5$^+_1$, 6$^+_1$ etc. to the same extent as the full $fp$ shell model calculation of Caurier et al. The calculated results suggest a structure change in the yrast band.

![Fig. 5. Calculated and observed energy levels of $^{48}V$.](image-url)
The spectroscopic quadrupole moment $Q_{\text{spec}}$ changes rapidly from the positive value at the $4_1^+$ state to the negative value at the $8_1^+$ state. Coincidentally, $B(E2 : J \rightarrow J - 1)$ is larger than $B(E2 : J \rightarrow J - 2)$ up to $7_1^+$, while $B(E2 : J \rightarrow J - 2)$ is larger than $B(E2 : J \rightarrow J - 1)$ from $8_1^+$ till $15_1^+$. There seem to be two series of states with even $J$ and odd $J$ connected by the large

Table 5
Quadrupole transition probabilities $B(E2)$ (in $e^2$ fm$^4$) and $Q_{\text{spec}}$ (in $e$ fm$^2$) in $^{48}$V. The upper part shows calculated $B(E2)$ values of the present work and Caurier et al., compared with observed ones. The lower part shows the results of present work for high-spin yrast states.

| $J_n \rightarrow J_m'$ | $B(E2)$ | $B(E2)$ | $Q_{\text{spec}}$ | $B(E2)$ |
|------------------------|--------|--------|----------------|--------|
| 4_1                    | 29.5   |        |                |        |
| 2_1 \rightarrow 4_1    | 28.59(17) | 105 | 48.1          |
| 5_1 \rightarrow 4_1    | 104(42)  | 183 | 20.7          | 209.0  |
| 4_2 \rightarrow 4_1    | 63(25)   | 182 | 28.9          |
| \rightarrow 5_1        | <41     | 76   | 32.0          |
| 6_1 \rightarrow 5_1    | 186(73)  | 164 | -0.3          | 191.0  |
| \rightarrow 4_1        | 46(6)   | 78   | 52.0          |
| 5_2 \rightarrow 4_2    | <176(124) | 34  | 41.0          |
| 2_2 \rightarrow 2_1    | >1.3(19) | 4.7 | 10.7          |

| $J_1$ | $Q_{\text{spec}}$ | $\rightarrow J_m'$ | $B(E2)$ | $\rightarrow J_m'$ | $B(E2)$ |
|-------|-------------------|---------------------|--------|---------------------|--------|
| 7_1   | -6.5              | $\rightarrow 5_1$  | 70     | $\rightarrow 6_1$  | 114    |
| 8_1   | -14.0             | $\rightarrow 6_1$  | 138    | $\rightarrow 7_1$  | 84     |
| 9_1   | -15.8             | $\rightarrow 7_1$  | 137    | $\rightarrow 8_1$  | 66     |
| 10_1  | -21.3             | $\rightarrow 8_1$  | 137    | $\rightarrow 9_1$  | 51     |
| 11_1  | -15.6             | $\rightarrow 9_1$  | 128    | $\rightarrow 10_1$ | 31     |
| 12_1  | -23.7             | $\rightarrow 10_1$ | 95     | $\rightarrow 11_1$ | 22     |
| 13_1  | -17.7             | $\rightarrow 11_1$ | 95     | $\rightarrow 12_1$ | 18     |
| 14_1  | -7.9              | $\rightarrow 12_1$ | 24     | $\rightarrow 13_1$ | 2.2    |
| 15_1  | -13.7             | $\rightarrow 13_1$ | 48     | $\rightarrow 14_1$ | 11     |
| 16_1  | -18.2             | $\rightarrow 14_1$ | 2.3    | $\rightarrow 15_1$ | 9.1    |
| 17_1  | -22.8             | $\rightarrow 15_1$ | 1.5    | $\rightarrow 16_1$ | 5.7    |
$B(E2)$ above the $7_1^+$ level, which is consistent with the observed strong $E2$ transitions $15_1^+ \rightarrow 13_1^+ \rightarrow 11_1^+ \rightarrow 9_1^+$ (the calculated $Q_{spec}$ values is nearly constant for these states). This is similar to the case of $^{46}$V. There is, however, a difference between $^{46}$V and $^{48}$V in the calculated results. The order of the two levels $(2J)_1^+$ and $(2J + 1)_1^+$ is reversed for $2J \geq 8$ in $^{46}$V, while it is in order of spin in $^{48}$V except $14_1^+$ and $15_1^+$. Future experiments will judge the quality of our model. The discrepancy between theory and experiment [40] with respect to the order of the two levels $14_1^+$ and $15_1^+$ should be examined further.

3.5 $^{50}$Cr

Let us compare calculated energy levels of $^{50}$Cr with observed ones [33,35,39,43] in detail. Figure 6 shows low-lying states below 4 MeV and Fig. 7 illustrates a gross level scheme, mainly the yrast states and high-spin states. In Table 6, we compare electric quadrupole properties between theory and experiment. The observed values of $B(E2)$ are from Refs. [39] (Expt.1) and [35] (Expt.2) and $B(E2 : 2_1^+ \rightarrow 0_1^+)$ is from Ref. [44]. The theoretical results of Martínez-Pinedo et al. [21] and Zamick et al. [19] are listed for comparison.

Figures 6, 7 and Table 6 demonstrate that the present model excellently describes the energies and electric quadrupole properties of the low-spin collective states $0_1^+$, $2_1^+$, $4_1^+$, $6_1^+$ and $8_1^+$ before the backbending in the ground-state
Fig. 7. Calculated and observed yrast levels, high-spin states and $T = 2$ isobaric analogue states of $^{50}$Cr.

band. (Remember that the ground-state energy is reproduced very well.) Figure 6, at the same time, reveals the insufficiency of the present model for non-collective states. The calculated low-lying states except the collective states lie lower than the observed levels, though there seems to be a certain correspondence between theory and experiment.

As confirmed in Fig. 7, the correspondence between theory and experiment is very well for the ground-state band, except for the $16^+$ level. The calculated $E2$ transition probabilities $B(E2 : J \rightarrow J - 2)$ in the ground-state band, which are comparable to those of the full $fp$ shell model calculation with the KB3 interaction, well explain the observed $B(E2)$ values except that our model suggests a structure change at $10^+_1$ by the small value of $B(E2 : 10^+_1 \rightarrow 8^+_1)$. The spectroscopic quadrupole moment $Q_{spcc}$, which is also comparable to that of Martínez-Pinedo et al., abruptly changes to the positive value at $10^+_1$ from the negative values up to $8^+_1$, indicating the structure change. Our model, on the other hand, reproduces the large value of $B(E2 : 10^+_2 \rightarrow 8^+_1)$ as well as the energy of the $10^+_2$ state. The nice reproduction of the energy levels $8^+_1$, $10^+_1$, $10^+_2$, $11^+_1$, $12^+_1$, $12^+_2$, $13^+_1$ and $14^+_1$ manifests that our model describes the backbending phenomenon of the ground-state band well. It is interesting that the $E2$ transition probability $B(E2 : J \rightarrow J - 1)$ is larger than $B(E2 : J \rightarrow J - 2)$ for the $11^+_1$, $12^+_1$ and $13^+_1$ states near the backbending.
Table 6
Electric quadrupole properties of the yrast and other states in $^{50}$Cr: $B(E2)$ in $e^2$ fm$^4$; $Q_{spec}$, $Q^{(s)}$ and $Q^{(t)}$ in e fm$^2$.

| $J_n \rightarrow J_m'$ | $B(E2)$ | $Q_{spec}$ | $Q^{(s)}$ | $Q^{(t)}$ | $B(E2)$ | $Q_{spec}$ | $B(E2)$ | $Q_{spec}$ |
|------------------------|---------|------------|-----------|-----------|---------|------------|---------|------------|
|                        | Expt.1  | Expt.2     | present work | Martínez et al. | Zamic et al. |
| $2_1 \rightarrow 0_1$  | 217±25  | 215        | -29       | 101       | 104     | -27        | 173     | -24.8      |
| $4_1 \rightarrow 2_1$  | 204±57  | 302        | -38       | 103       | 103     | -33        | 246     | -30.0      |
| $6_1 \rightarrow 4_1$  | 235±47  | 211        | -18       | 44        | 82      | -18        | 215     | -15.6      |
| $8_1 \rightarrow 6_1$  | 205±51  | 200        | -25       | 59        | 78      | -17        | 192     | -14.7      |
| $8_2 \rightarrow 6_1$  | 0.3     | 4.3        |           |           |         |            |         |            |
| $10_1 \rightarrow 8_1$ | 72±14   | 66$^{\pm102}_{-31}$ | 14 | 47 | -108 | 20 | 54 | 30 | 81 | 26.5 |
| $10_2 \rightarrow 8_1$ | 131±26  | 145        | -14       |           |         |            |         |            |
| $11_1 \rightarrow 9_1$ | 41      | 29         |           |           |         |            |         |            |
| $12_1 \rightarrow 10_1$ | 52±7   | 0±23       | 29        | 17        | -37     | 29        | 30      | 13         | 69 | 13.0 |
| $12_2 \rightarrow 10_1$ | 1.0    | -17        |           |           | 0.0     | -6.6      |         |            |
| $13_1 \rightarrow 11_1$ | >5      | >30        | 42        | 8.6       |         |           |         |            |
| $14_1 \rightarrow 12_1$ | 43±7   | 31±7       | 52        | 10        | -22     | 39        | 60      | 8          | 67 | 8.2 |
| $15_1 \rightarrow 13_1$ | >10     | >21        | 11        | -6.4      |         |           |         |            |
| $16_1 \rightarrow 14_1$ | >10    |            |           |           |         |           |         |            |
| $17_1 \rightarrow 15_1$ | >66     |            |           |           |         |           |         |            |
| $18_1 \rightarrow 16_1$ | 50±20  | 0.5        | -25       | 53        | 3.9     | 76        | 9       | 83         | -8.0 |
| $19_1 \rightarrow 17_1$ |        |            |           |           |         |           |         |            | 17   |
There is a disagreement between theory and experiment on the energies of the $16^+$ and $17^+$ states and $B(E2: 18^+_2 \rightarrow 16^+_1)$. A little possibility that the two levels $16^+$ and $17^+$ observed at 15.032 MeV and 16.048 MeV are not the yrast states remains. Our model predicts a bundle of levels with the same $J$ near each high-spin yrast level. Actually, there are calculated energy levels corresponding well to the observed levels with $10^+, 12^+, 13^+, 14^+$ and $15^+$ in addition to the yrast levels. The observed $16^+$ and $17^+$ states at 15.032 MeV and 16.048 MeV have such candidates at the same energy region in the calculated energy spectra. The calculated values $B(E2: 18^+_2 \rightarrow 16^+_1) = 0.5$ and $B(E2: 18^+_2 \rightarrow 16^+_1) = 1.7$ in $e^2$ fm$^4$ are much smaller than the observed one. The absence of the $f_{5/2}$ orbit may have the influence on the energies and $B(E2)$ of the high-spin states $16^+, 17^+$ and $18^+$, in the present model.

Our model reproduces the isobaric analogue states $6^+, 4^+$ and $2^+$ with $T=2$ (broken lines) at good energy, though the order of the calculated levels ($6^+, 4^+$) and $2^+$ is reverse to that of the observed ones. This is a nice work of the isoscalar $p$-$n$ force $V_{\pi\nu}$. 

### 3.6 $^{50}$Mn

A recent experiment by Svensson et al. [45] identified high-spin states in the $N = Z$ odd-odd nucleus $^{50}_{25}$Mn$_{25}$. We compare the energy scheme obtained by the present model with the observed one in Fig. 8. Like $^{48}$V, our model lays another $T=0$ state ($1^+_1$) at lower energy than the $5^+_1$ state which is the lowest $T=0$ state in experiment. Moreover, these two calculated states $1^+_1$ and $5^+_1$ lie
lower than the ground state $0^+_1$ with $T=1$. The discrepancy between theory and experiment for the $T=0, 5^+_1$ level, however, is only 0.23 MeV. For such a simple model, the obtained energies of the $T=1, 0^+$ and $T=0, 5^+_1$ states are rather satisfactory. We see again that the calculated non-collective states except the yrast states lie lower than the observed ones.

Table 7
Electric quadrupole properties of the yrast states in $^{50}$Mn: $B(E2)$ in e$^2$ fm$^4$ and various $Q$ values in e$^2$ fm$^2$.

| $J \rightarrow J'$ | $B(E2)$ | $Q_{spec}$ | $Q_0^{(s)}$ | $Q_0^{(t)}$ | $B(E2)$ | $Q_{spec}$ | $Q_0^{(s)}$ | $Q_0^{(t)}$ |
|-------------------|---------|------------|-------------|-------------|---------|------------|-------------|-------------|
| $3 \rightarrow 1$ | 259     | -33        | -19         | 240         |         |            |             |             |
| $5$               | 59      | 102        |             |             |         |            |             |             |
| $6 \rightarrow 5$ | 294     | 32         | 103         | 99          | 258     | 32         | 102         | 93          |
| $7 \rightarrow 6$ | 301     | 13         | 94          | 92          | 251     | 14         | 100         | 84          |
| $\rightarrow 5$  | 39      |             | 90          | 42          | 92      |             |             |             |
| $8 \rightarrow 7$ | 276     | 3.4        | 191         | 90          | 140     | 5          | 285         | 64          |
| $\rightarrow 6$  | 89      |             | 95          | 74          | 87      |             |             |             |
| $9 \rightarrow 8$ | 217     | -5.6       | 78          | 84          | 142     | -0.6       | 9           | 68          |
| $\rightarrow 7$  | 127     |             | 95          | 133         | 97      |             |             |             |
| $10 \rightarrow 9$| 177     | -8.1       | 59          | 81          |         |            |             |             |
| $\rightarrow 8$  | 128     |             | 85          |             |         |            |             |             |
| $11 \rightarrow 9$| 97      | 5.4        | -28         | 69          | 130     | 0.5        | -2.6        | 80          |
| $\rightarrow 10$ | 62      |             | 51          |             |         |            |             |             |
| $12 \rightarrow 11$| 119    | -0.4       | 1.5         | 76          |         |            |             |             |
| $\rightarrow 10$ | 73      |             | 57          |             |         |            |             |             |
| $13 \rightarrow 11$| 83      | -1.5       | 5.6         | 58          | 99      | 6          | -23         | 64          |
| $\rightarrow 12$ | 61      |             | 59          |             |         |            |             |             |
| $14 \rightarrow 13$| 69      | -1.8       | 6.2         | 65          |         |            |             |             |
| $\rightarrow 12$ | 43      |             | 41          |             |         |            |             |             |
| $15 \rightarrow 13$| 27      | 33         | -105        | 32          | 47      | 25         | -80         | 41          |
| $\rightarrow 14$ | 47      |             | 41          |             |         |            |             |             |
It is impressive that the observed $T=0$ yrast band on the $5^+_1$ state is reproduced so well by our simple model. The correspondence between theory and experiment is better than that of the full $fp$ shell model calculation using the KB3 interaction and single-particle energies taken from $^{41}$Ca [45]. Calculated $E2$ transition probabilities between the yrast states are compared with those of Ref.[45] in Table 7. The $B(E2)$ values in the present work are larger up to $9^+_1$ and smaller above $10^+_1$ than those of Ref. [45].

The theoretical $B(E2)$ values between the yrast states in $^{50}$Mn show a different feature from those in $^{46}$V and $^{48}$V. Namely, every $B(E2 : J \rightarrow J - 2)$ except the $15^+_1$ state in $^{50}$Mn, while the relative magnitudes of $B(E2 : J \rightarrow J - 1)$ and $B(E2 : J \rightarrow J - 2)$ are reversed above $7^+_1$ corresponding to the structure change in $^{46}$V and $^{48}$V. In $^{50}$Mn, the spectroscopic quadrupole moment $Q_{\text{spec}}$ changes one after another but the intrinsic quadrupole moment $Q_{0}^{(t)}$ keeps relatively large values up to high spin. There is probably a delicate difference between $^{46}$V and $^{50}$Mn which are cross-conjugate systems ($f_{7/2}^6p$ and $f_{7/2}^6h$) within the single $j$ model space. The present result suggests a considerably large contribution of the upper orbits $p_{3/2}$ and $p_{1/2}$ in $^{50}$Mn. Our model yields different level schemes in $^{46}$V and $^{50}$Mn. The inverse order of the levels $(2J)^+_1$ and $(2J+1)^+_1$ happens above $2J = 8$ in $^{46}$V but happens only at $2J = 14$ in $^{50}$Mn. This interesting difference will be examined by detecting the even-J high-spin states.

4 Discussions

4.1 Dependence of the calculated results on the model

We have seen the success of the present model in the small model space $(f_{7/2}, p_{3/2}, p_{1/2})$. To see the dependence on the model space, we made calculations with our interaction in the full $fp$ shell for $^{44}$Ti where the calculations are easy. We took $\epsilon(f_{5/2})=4.88$ MeV from $^{41}$Ca and changed the force strengths as little as possible so as to reproduce the observed energy levels of $^{44}$Ti, i.e., $g_0 = 0.42(42/A)$, $\chi' = 0.29(42/A)^{5/3}$, and $g_2$ and $k^0$ being unchanged. Obtained energy levels (column B) are compared with those in the $(f_{7/2}, p_{3/2}, p_{1/2})$ space (column A) and also with the results obtained using the FPD6 [14] and KB3 interactions in the full $fp$ space, in Fig. 9. This figure teaches that the extension of the model space to the full $fp$ space modifies little the energy levels of the collective states in our model. This is probably true for the collective states of $A=46, 48$ and 50 nuclei, as guessed from the successful results in Sec. III. We shall see the reason in the next subsection.

According to Fig. 9, our interaction FEI is roughly comparable to the FPD6
Fig. 9. Energy levels obtained using various effective interactions in $^{44}\text{Ti}$, compared with observed levels. The ground-state energy is written below every $0^+_1$ level in MeV. The dotted lines denote the $T = 1$ states and dashed lines denote the $T = 2$, $0^+$ state. The space A is $(f_7/2,p_3/2,p_1/2)$ and B is the full $fp$ space.

and KB3 interactions also in $^{44}\text{Ti}$. The KB3 interaction yields rather compressed energy levels for the ground-state band and a shallow binding energy. This could be attributed to the lack of the $g_{9/2}$ orbit which is included in the model space of the original Kuo-Brown (KB) interaction [7]. If the $g_{9/2}$ orbit is added, the additional interactions such as the pairing force will lower the low-lying collective states $0^+_1$, $2^+_1$, ..., and recover the binding energy. This situation presents a contrast to the success of the KB3 interaction in the $A=46$, 48 and 50 nuclei.

The extension of the model space, of course, changes the wave functions. Table 8 lists the $B(E2 : J^+_1 \rightarrow (J-2)^+_1)$ values calculated in the $(f_7/2,p_3/2,p_1/2)$ space and full $fp$ space, which are compared with the observed $B(E2)$ values and those obtained by the $\alpha$-cluster model [48]. The extension of the model space enlarges the $B(E2)$ values between the most collective states $2^+_1$ and $4^+_1$, when our interaction is used. The same does not happen for the realistic effective interactions FPD6 and KB3. The two interactions rather reduce the $B(E2)$ values between the $2^+_1$, $4^+_1$, $6^+_1$ and $8^+_1$ states. An advantage of the $\alpha$-cluster model [48] was the reproduction of the large $B(E2)$ values without the effective charge as compared with the reduced $B(E2)$ values of the shell models using the realistic effective interactions. The shell model using the FEI yields the $B(E2)$ values comparable with the $\alpha$-cluster model, though the former employs the effective charge $e_{eff} = 0.5e$. The $^{44}\text{Ti}$ nucleus is more
Table 8
Dependence of $B(E2 : J^+_I \rightarrow (J-2)^+_I)$ on model spaces and effective interactions in $^{44}$Ti. The space A is $(f_{7/2}, p_{3/2}, p_{1/2})$ and B is the full $fp$ space. The $B(E2)$ values obtained by the $\alpha$-cluster model is also shown.

| J  | A       | B       | A  | B  | A  | B  | FEI | Expt. | FPD6 | KB3 | $\alpha$- |
|----|---------|---------|----|----|----|----|-----|-------|------|-----|-----------|
| 2  | 102     | 103     | 120±37 | 80  | 60  | 73  | 64  | 107   |
| 4  | 125     | 134     | 277±55 | 101 | 72  | 94  | 75  | 146   |
| 6  | 82      | 74      | 157±28 | 99  | 63  | 91  | 57  | 140   |
| 8  | 77      | 73      | >18    | 91  | 58  | 84  | 50  | 118   |
| 10 | 76      | 67      | 138±28 | 79  | 67  | 70  | 62  | 75    |
| 12 | 52      | 52      | 40±8   | 50  | 52  | 44  | 47  | 34    |

or less in an $\alpha$-like four-nucleon correlated state outside the doubly-closed-shell $^{40}$Ca core [49]. The large $B(E2)$ values are related to the $\alpha$-like four-nucleon correlations. The $P_0 + P_2 + QQ + V_{\pi\nu}^\tau=0$ interaction, which is suitable for the description of the nuclear collective motions (the quadrupole vibration and deformation), is considered to have an affinity also for the $\alpha$-like four-nucleon correlations [50] as compared with the FPD6 and KB3 interactions as seen in Table 8. Anyhow, our interaction including the $QQ$ force can give larger $B(E2)$ values than the FPD6 and KB3 interactions, when the same model space is used. In the configuration $(f_{7/2}, p_{3/2}, p_{1/2})^8$ for $^{48}$Cr, for instance, the KB3 interaction gives $B(E2 : 4^+_I \rightarrow 2^+_I) = 196 e^2 fm^4$, while the FEI gives $B(E2 : 4^+_I \rightarrow 2^+_I) = 305 e^2 fm^4$. Thus our interaction in the smaller model space $(f_{7/2}, p_{3/2}, p_{1/2})$ reproduces large $B(E2)$ values comparable to the FPD6 and KB3 interactions in the full $fp$ space, in the $A=46$, 48 and 50 nuclei. If we extend the model space using our interaction, a smaller effective charge will serve in the $A=46$-50 nuclei.

There is another sign that the three interactions FEI, FPD6 and KB3 give different wave functions. For $^{44}$Ti, the expectation values of nucleon number in respective orbits of the $fp$ space are as follows: $n_{7/2}=3.082$, $n_{3/2}=0.701$, $n_{1/2}=0.103$, $n_{3/2}=0.114$ for FEI; $n_{7/2}=3.061$, $n_{3/2}=0.464$, $n_{1/2}=0.124$, $n_{5/2}=0.351$ for FPD6; $n_{7/2}=3.367$, $n_{3/2}=0.262$, $n_{1/2}=0.085$, $n_{5/2}=0.286$ for KB3. The contribution of the $p_{3/2}$ orbit is large in FEI. Nucleons are distributed over all the orbits most in FPD6. It should be noticed that interactions related to the $f_{7/2}$ orbit are strengthened in KB3 by the modification from the KB interaction [16], resulting in the large value of $n_{7/2}$.

The results of calculations depend also on the single-particle energies. We examined the dependence by shell model calculations using Kuo-Brown’s single-particle energies, $\epsilon_{7/2}=0.0$, $\epsilon_{3/2}=2.1$ and $\epsilon_{1/2}=3.9$ in MeV. In Table 9, calcu-
lated energies and $B(E2)$ for the ground-state bands of $^{46}$Ti and $^{48}$Cr are compared with those of Sec. III (where $\epsilon_{7/2}=0.0$, $\epsilon_{5/2}=1.94$ and $\epsilon_{1/2}=3.61$ in MeV). Table 9 says that the change of the single-particle energies tried here does not significantly affect the energies and $B(E2)$. We, therefore, need not change the basic understanding in Sec. III, as long as we use ordinary single-particle energies.

Table 9
Effects of varying the single-particle energies on the excitation energies $E_x$ (or ground-state energies $E_{gs}$) and $B(E2)$ values. The single-particle energies ($\epsilon_{7/2}, \epsilon_{5/2}, \epsilon_{1/2}$) are taken from $^{41}$Ca in I and are Kuo-Brown’s in II, respectively.

| $A \ J^\pi$ | $E_x$ or $(E_{gs})$ | $B(E2: J \rightarrow J-2)$ |
|------------|-----------------|---------------------|
| $^{46}$Ti 0$^+_1$ | (-20.169) | (-19.987) |
| 2$^+_1$ | 1.110 | 1.133 | 119 | 116 |
| 4$^+_1$ | 2.212 | 2.220 | 159 | 152 |
| 6$^+_1$ | 3.237 | 3.198 | 141 | 133 |
| 8$^+_1$ | 4.984 | 4.934 | 126 | 119 |
| 10$^+_1$ | 6.570 | 6.479 | 104 | 100 |
| 11$^+_1$ | 7.548 | 7.420 | 51 | 48 |
| 12$^+_1$ | 8.177 | 8.049 | 53 | 51 |
| 14$^+_1$ | 10.491 | 10.330 | 26 | 25 |
| $^{48}$Cr 0$^+_1$ | (-32.380) | (-32.131) |
| 2$^+_1$ | 0.785 | 0.802 | 217 | 211 |
| 4$^+_1$ | 1.837 | 1.847 | 306 | 297 |
| 6$^+_1$ | 3.222 | 3.217 | 301 | 287 |
| 7$^+_1$ | 5.554 | 5.524 | 92 | 89 |
| 8$^+_1$ | 4.887 | 4.866 | 285 | 274 |
| 10$^+_1$ | 6.670 | 6.606 | 232 | 219 |
| 12$^+_1$ | 8.238 | 8.095 | 130 | 127 |
| 13$^+_1$ | 10.878 | 10.770 | 43 | 32 |
| 14$^+_1$ | 10.429 | 10.248 | 105 | 103 |
4.2 Characteristics of the interaction matrix elements

The effective interactions can be directly checked in $^{42}\text{Sc}$. In Fig. 10, we compare energy levels obtained using the three interactions (FEI, FPD6 and KB3) in $^{42}\text{Sc}$ with observed energy levels, where the model space $(f_{7/2}, p_{3/2}, p_{1/2})$ is employed for FEI. This figure shows that the isoscalar ($T = 0$) $p$-$n$ interactions of FEI are best but the isovector ($T = 1$) interactions of FEI are worst among the three interactions. We now understand the reason why our interaction is not good for the Ca isotopes where only the isovector interactions take action. Probably, the isovector interactions play dominant roles in low-lying states of nuclei with \(n_p=1\) or \(n_n=1\). The success of our interaction FEI for the collective states in the \(A=46, 48\) and \(50\) nuclei with \(n_p \geq 2\) and \(n_n \geq 2\) suggests leading roles of the isoscalar \(p\)-\(n\) interactions there.

The compression of the \(T = 1\) energy levels \((0^+, 2^+, 4^+, 6^+)\) in the KB3 result of Fig. 10 is due to the lack of the \(g_{9/2}\) orbit (compare with the figure of Ref. [7]), as mentioned in the previous subsection. The KB3 interaction lays the \(T = 0\) states \((7^+, 1^+, 5^+, 3^+)\) lower than the FPD6. This is caused by the modification of interaction matrix elements from the original KB interaction [7] to the KB3 [16], where the most important modification is to strengthen the isoscalar \(p\)-\(n\) interactions.

In Table 10, we list typical interaction matrix elements of the four interactions FPD6, KB, KB3 and FEI. For FEI, the force parameters fixed in $^{44}\text{Ti}$

![Image of Fig. 10](image-url)

Fig. 10. Energy levels obtained using various effective interactions in $^{42}\text{Sc}$, compared with observed levels. The ground-state energy is written below every $0_1^+$ level in MeV. The solid lines denote the \(T = 1\) states and dotted lines denote the \(T = 0\) states.
are applied to the \( A = 42 \) system, i.e., \( g_0 = 0.42, g_2 = 0.36, \chi' = 0.30 \) and \( k^0 = 2.23 \) in MeV. Let us denote diagonal matrix elements as \( V(\alpha\beta\gamma\delta) = \langle \alpha\beta\gamma\delta | V | \alpha\beta\gamma\delta \rangle \). The KB3 interaction enlarges \( V(f_{7/2}f_{7/2}f_{7/2}J = odd, T = 0) \) more than \( V(f_{7/2}f_{7/2}f_{7/2}J = even, T = 1) \), especially \( V(f_{7/2}f_{7/2}f_{7/2}J = 1, T = 0) \) and \( V(f_{7/2}f_{7/2}f_{7/2}J = 3, T = 0) \) as well as \( V(f_{7/2}f_{7/2}f_{7/2}J = 0, T = 0) \) with \( r = (p_{3/2}, p_{1/2}, f_{5/2}) \).

These modifications bring the KB3 interaction a little near to the FEI. In our interaction FEI, the \( V_{\pi\nu}^{\tau=0} \) force strengthens just the isoscalar \( (T = 0) \) \( p-n \) interactions, and the matrix elements \( V(f_{7/2}f_{7/2}f_{7/2}J = 1, T = 0) \) and \( V(f_{7/2}f_{7/2}f_{7/2}J = 3, T = 0) \) are comparable with \( V(f_{7/2}f_{7/2}f_{7/2}J = 7, T = 0) \), which makes the \( 1^+ \) state lowest among the \( T = 0 \) states of \(^{42}\text{Sc}\).

The modification of the centroids of interaction matrix elements in the KB3 interaction is discussed in terms of the monopole Hamiltonian in Refs. \([16, 51]\). That discussion is intimately related to the roles of the \( V_{\pi\nu}^{\tau=0} \) force. We have shown in the previous papers \([30, 31]\) that the \( V_{\pi\nu}^{\tau=0} \) force plays an essential role for reproducing the binding energy and symmetry energy in a very wide range of nuclei. The \( V_{\pi\nu}^{\tau=0} \) force which has a simple and definite form (see Eq. (8) or (9)) can play the part for Zuker’s monopole Hamiltonian. According to Dufour and Zuker \([27]\), the residual interactions after extracting the monopole Hamiltonian or the \( V_{\pi\nu}^{\tau=0} \) force must resemble the \( P_0 + P_2 + QQ \) force.

The \( J \)-independent isoscalar \( p-n \) force \( V_{\pi\nu}^{\tau=0} \) gives the average value -2.23 MeV to the diagonal matrix elements \( V(\alpha\beta\gamma\delta | T = 0) \) (see Table 10). The absolute values of \( V(f_{7/2}p_{3/2}p_{3/2}p_{1/2}, T = 0) \) of our interaction FEI are larger than those of the realistic effective interactions. The FPD6 and KB3 interactions have very large matrix elements \( V(f_{7/2}f_{5/2}f_{5/2}p_{3/2}p_{1/2}, T = 0) \), to which the matrix elements of FEI are comparable. The centroid of \( V(f_{7/2}f_{5/2}f_{5/2}p_{3/2}p_{1/2}, T = 0) \) is -1.846 for FPD6, -1.934 for KB3 and -2.241 for FEI. Seeing the large diagonal matrix elements, one naturally hesitates to omit the \( f_{5/2} \) orbit from the model space and hence deals with the full \( fp \) shell space. Why the truncated space \( (f_{7/2}, p_{3/2}, p_{1/2}) \) works well for the \( P_0 + P_2 + QQ + V_{\pi\nu}^{\tau=0} \) interaction? The secret is in a special work of \( V_{\pi\nu}^{\tau=0} \). The \( J \)-independent \( p-n \) force \( V_{\pi\nu}^{\tau=0} \), which is a function of only the number and isospin of valence nucleons in Eq. (8), is independent of the model space in fact. The average contribution of \( V(f_{7/2}f_{5/2}f_{5/2}p_{3/2}p_{1/2}, T = 0) \) is equivalently taken into account in our model. The residual interactions excluding \( V_{\pi\nu}^{\tau=0} \) are, of course, desired to be taken up. The inclusion of the \( f_{5/2} \) orbit will improve wave functions and \( B(E2) \) as seen in the previous subsection. There are large off-diagonal matrix elements \( \langle f_{7/2}f_{7/2}f_{7/2}J, T = 0 | V | f_{7/2}f_{5/2}f_{5/2}J, T = 0 \rangle \) and \( \langle f_{7/2}f_{7/2}f_{7/2}J, T = 0 | V | f_{5/2}f_{5/2}f_{5/2}J, T = 0 \rangle \) in the realistic effective interactions. Their effects on the energies of low-lying states, however, may be secondary as guessed from our results.

We have seen the defects of the present model on non-collective states and also the insufficiency of the isovector interactions. The present interaction remains room for improvement, which could be made by comparing with the
Table 10
Comparison of interaction matrix elements $\langle abJT|V|cdJT \rangle$ between various effective interactions. The orbits are labeled by numbers, 1: $f_{7/2}$, 2: $p_{3/2}$, 3: $p_{1/2}$, 4: $f_{5/2}$.

| a   | b   | c   | d   | J, T | FPD6 | KB   | KB3 | FEI |
|-----|-----|-----|-----|------|------|------|-----|-----|
| 1 1 1 1 | 1 | 0 | -0.177 | -0.525 | -1.175 | -2.680 |
| | 3 | 0 | -0.499 | -0.208 | -0.858 | -2.257 |
| | 5 | 0 | -1.046 | -0.502 | -0.852 | -1.939 |
| | 7 | 0 | -2.474 | -2.199 | -2.549 | -2.490 |
| 1 1 1 1 | 0 | 1 | -2.268 | -1.807 | -1.917 | -2.236 |
| | 2 | 1 | -0.888 | -0.785 | -1.095 | -0.812 |
| | 4 | 1 | -0.144 | -0.087 | -0.197 | 0.185 |
| | 6 | 1 | 0.168 | 0.226 | 0.116 | 0.185 |
| 1 2 1 2 | 2 | 0 | 0.003 | -0.298 | -0.593 | -2.552 |
| | 3 | 0 | -0.823 | -0.604 | -0.904 | -2.178 |
| | 4 | 0 | -0.547 | -0.164 | -0.464 | -1.763 |
| | 5 | 0 | -2.522 | -2.165 | -2.465 | -2.801 |
| 1 3 1 3 | 3 | 0 | -1.824 | -1.484 | -1.784 | -2.230 |
| | 4 | 0 | -0.787 | -0.746 | -1.046 | -2.230 |
| 1 4 1 4 | 1 | 0 | -4.665 | -3.621 | -3.921 | -2.716 |
| | 2 | 0 | -2.950 | -2.731 | -3.031 | -2.427 |
| | 3 | 0 | -1.263 | -0.985 | -1.285 | -2.207 |
| | 4 | 0 | -2.188 | -1.886 | -2.186 | -1.919 |
| | 5 | 0 | -0.008 | -0.112 | -0.412 | -2.019 |
| | 6 | 0 | -2.402 | -2.217 | -2.517 | -2.490 |
| 1 1 1 4 | 1 | 0 | 1.977 | 1.894 | 1.894 | -0.130 |
| | 3 | 0 | 1.322 | 1.005 | 1.005 | -0.172 |
| | 5 | 0 | 1.307 | 0.901 | 0.901 | 0.042 |
| 1 1 4 4 | 1 | 0 | 2.080 | 1.071 | 1.071 | -0.061 |
| | 3 | 0 | 1.260 | 0.517 | 0.517 | 0.004 |
| | 5 | 0 | 0.596 | 0.170 | 0.170 | 0.047 |
realistic effective interactions. Another way may be to add hexadecapole and hexadecapole-hexadecapole forces (octupole and octupole-octupole forces for negative-parity states) as an extension of the \( P_0 + P_2 + QQ \) force.

4.3 Properties of the yrast bands in \( A=46, 48 \) and \( 50 \) nuclei

In order to look at the structure of the yrast bands in \( A=46, 48 \) and \( 50 \) nuclei from a different angle, we illustrate the spin-energy relation in Figs. 11, 12 and 13. We clearly see the good applicability of the \( P_0 + P_2 + QQ + V_{\pi\nu}^{rr=0} \) interaction to the collective yrast states in these nuclei. The rotational behavior and backbending in the ground-state bands in even-even nuclei are well reproduced. It is interesting that in \( ^{50}\text{Cr} \) (Fig. 13) the \( J=\text{odd} \) line touches the ground-state band with \( J=\text{even} \) near \( 11^+_1 \) where the backbending occurs and the values of \( B(E2 : J \to 11^+_1) \) become large (Table 6). The staggering gait depending on odd spin and even spin observed in the \( T = 0 \) yrast bands of the odd-odd nuclei is almost traced by our model. We have already indicated that the magnitudes of staggering are not the same in the cross-conjugate nuclei (within the \( f_{7/2} \) space) \( ^{46}\text{V} \) and \( ^{50}\text{Mn} \). Figure 14 shows that the \( T = 0 \) yrast bands of \( ^{46}\text{V} \) and \( ^{50}\text{Mn} \) do not resemble at low spin but resemble at \( J=\text{odd} \) high spin. This suggests the dominant contribution of the \( f_{7/2} \) orbit in the high-spin states of \( ^{46}\text{V} \) and \( ^{50}\text{Mn} \).

Figures 11, 12 and 13 suggest that there are similarities between the ground-states bands of the even-even nuclei \( ^{46}\text{Ti} \), \( ^{48}\text{Cr} \) and \( ^{50}\text{Cr} \), and between the yrast bands of the odd-odd nuclei \( ^{46}\text{V} \), \( ^{48}\text{V} \) and \( ^{50}\text{Mn} \). In Fig. 14, the observed yrast bands in these nuclei are compared in a sheet of drawing. The slopes at the beginning of the bands resemble in the even-even nuclei and also in the

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![Fig. 11. Spin-energy relation in the yrast bands of \( ^{46}\text{Ti} \) and \( ^{46}\text{V} \).](image-url)
odd-odd nuclei $^{48}$V and $^{50}$Mn. This implies a similar mechanism of excitation in low-lying collective states of the even-even or odd-odd systems, which is considered to be the nuclear rotation $[54–57,28]$. The $N = Z$ odd-odd nuclei $^{46}$V and $^{50}$Mn have $T = 1$ and $T = 0$ bands as the lowest and second lowest bands. (Our Hamiltonian gives the same energies to the $T = 1$ states of $^{46}$Ti and $^{46}$V ($^{56}$Cr and $^{50}$Mn). The $T = 1$ band observed in $^{46}$V or $^{50}$Mn, actually, corresponds well to that observed in $^{46}$Ti or $^{50}$Cr.) Recently, a band crossing which is different from the ordinary one caused by the spin alignment of a high $j$ neutron or proton pair was observed in a heavy $N = Z$ nucleus $^{74}$Rb $[46]$. The crossing frequency which corresponds to $J = 5$
or $J = 7$ is smaller than that in the ordinary band crossing. This phenomenon can be interpreted to be the crossing of the $T = 1$ and $T = 0$ bands [46,47,25]. Figures 11 and 13 show a similar behavior in $^{46}\text{V}$ and $^{50}\text{Mn}$, where the $T = 0$ band appears to cross the $T = 1$ band near $J = 2$. We can suppose a similar situation for the $N = Z$ odd-odd nuclei both in the beginning and middle regions of $fpg$ shell. It is interesting that the crossing frequency in the former region is smaller than that in the latter where nuclei are considered to be certainly deformed.

The backbending phenomena in $^{48}\text{Cr}$ and $^{50}\text{Cr}$ have been studied by the cranked Hartree-Fock-Bogoliubov method [54–57]. A typical interaction for studying the rotation of deformed nucleus by the cranking approach is just the $P_0 + QQ$ force which is the main part of our residual interactions excluding the $V_{\tau\nu=0}$ force. It was shown by the projected quasi-particle shell model in the Nilsson base [6] that the $P_0 + P_2 + QQ$ force successfully describes not only the rotational band in medium-heavy nuclei [5] but also the backbending in $^{48}\text{Cr}$ [28]. The study of Ref. [28] indicates that the $P_0 + P_2 + QQ$ force is comparable to the realistic effective interaction KB3 in reproducing the rotational band of $^{48}\text{Cr}$. It should be noticed here that the $P_0 + P_2 + QQ$ force without the $p-n$ force $V_{\pi\nu=0}$ cannot well reproduce the binding energy, and the $p-n$ interactions as well as like-nucleon interactions are taken into account in our $P_0 + P_2$ force. The $P_0 + P_2 + QQ + V_{\pi\nu=0}$ interaction is, therefore, a very suitable interaction to investigate the rotational band and its backbending in the $N \approx Z$ $fp$ shell region, in parallel with the shell model approach.

One of the key words to understand the excitation mechanism at the beginning of the $fp$ shell nuclei seems to be $\alpha$-like four-nucleon correlations [49]. The
$^{44}$Ti nucleus is an $\alpha$-like correlated state outside the $^{40}$Ca core. The $^{48}$Cr nucleus, in zero order approximation, is described as a correlated state of two $\alpha$-like clusters (quartets) $\sum I \psi_I (\alpha_{I,T=0}^+)^2 |^{40}$Ca$\rangle$ where $\alpha_{I,M,T=0}^+$ being the lowest-energy Tamm-Dancoff modes of four nucleons with spin $I M$ and $T = 0$ [52]. The approximate description gives the ground-state energy -32.05 MeV against the exact energy -32.38 MeV in the present model. Within the single $j = f_7/2$ shell model, the states of $A = 4n + 2$ nuclei are roughly approximated as follows [50,53]:

$$|^{46}$Ti : $J_1^+ \rangle \approx \frac{1}{\sqrt{C_1}} A_{0011}(f_7/2f_7/2)|^{44}$Ti : $J_1^+ \rangle \quad (J = 0, 2),$$

$$|^{50}$Cr : $J_1^+ \rangle \approx \frac{1}{\sqrt{C_2}} A_{0011}(f_7/2f_7/2)|^{48}$Cr : $J_1^+ \rangle \quad (J = 0, 2),$$

(16)

where $C_1$ and $C_2$ are normalization constants. The overlaps of the approximate states (16) with the exact states are more than 0.98 for $J = 0$ and are more than 0.90 for $J = 2$. In other words, the excitation $0_1^+ \rightarrow 2_1^+$ in $^{46}$Ti and $^{50}$Cr resembles the excitation $0_1^+ \rightarrow 2_1^+$ in $^{44}$Ti and $^{48}$Cr, respectively. The excitation $0_1^+ \rightarrow 2_1^+$ in $^{44}$Ti is the change of four nucleon structure $\alpha_{I=0,T=0}^+ \rightarrow \alpha_{I=2,T=0}^+$. A similar change of structure is possibly dominant in the excitation $0_1^+ \rightarrow 2_1^+$ in $^{48}$Cr. We can suppose a common excitation mechanism induced by the $\alpha$-like four-nucleon correlations in $^{46}$Ti, $^{48}$Cr and $^{50}$Cr. We have already discussed the intimate relation of our interaction $P_0 + P_2 + QQ + V_{\tau=0}^{\pi\nu}$ to the $\alpha$-like four-nucleon correlations in Ref. [50]. The $P_0 + P_2 + QQ + V_{\tau=0}^{\pi\nu}$ interaction having strong $p$-$n$ interactions may underlie commonly in the nuclear rotation and $\alpha$-like four-nucleon correlations in the $fp$ shell nuclei.

5 Concluding remarks

We have applied a functional effective interaction extended from the pairing plus $QQ$ force by adding the $J$-independent isoscalar $p$-$n$ force $V_{\tau=0}^{\pi\nu}$ and quadrupole pairing force to $^{46}$Ti, $^{46}$V, $^{48}$V, $^{48}$Cr, $^{50}$Cr and $^{50}$Mn. The exact shell model calculations in the truncated model space ($f_7/2, p_3/2, p_1/2$) demonstrate the usefulness of the interaction for the yrast states in these nuclei with $n_p \geq 2$ and $n_n \geq 2$. The model reproduces well the experimental binding energies, energy levels of the yrast states and $B(E2)$ between them. We have also analyzed the relationship between our interaction and the realistic effective interactions KB3 and FPD6. The analysis clarified the reason why the truncated model space works well especially for our interaction. This work as well as the previous ones [29–31] supports that an important part of nucleon-nucleon interactions can be written as $V_{\tau=0}^{\pi\nu}$. The foundation of the $p$-$n$ force $V_{\tau=0}^{\pi\nu}$ could be discussed in the framework of the HF theory.
The good reproduction of the yrast states made it possible to discuss their structure in the $A=46$, 48 and 50 nuclei. We have given some predictions about the energy levels and characteristic variations of $B(E2)$ in the yrast bands, in these nuclei. The extended $P_0 + QQ$ interaction is excel in describing the collective nature of nuclei, and is expected to be most suitable for studying the rotational properties of the yrast bands. The backbending phenomena in $^{48}\text{Cr}$ and $^{50}\text{Cr}$, which are well described in terms of the shell model with our interaction, can be investigated by a cranking model with the same interaction. The $P_0 + P_2 + QQ + V_{\pi\nu}^{\tau=0}$ interaction composed of typical forms of forces is also useful for the study of competition between the $T=0$ and $T=1$ $p$-$n$ pairing and like-nucleon pairing which is one of current topics. There is a sign that the $P_0 + P_2 + QQ + V_{\pi\nu}^{\tau=0}$ interaction has an affinity for the $\alpha$-like four-nucleon correlations important in $N \approx Z$ nuclei.

The success of the extended $P_0 + QQ$ interaction by means of the exact shell model gives strong evidence that the extended picture of the pairing plus $QQ$ force model holds in lighter nuclei. This supports the discussion by the projected quasi-particle shell model [5,6,28] that the $P_0 + P_2 + QQ$ interaction describes well the rotational states not only in medium-heavy nuclei but also in the $fp$ shell nucleus $^{48}\text{Ca}$. We can probably say that the success of the pairing plus $QQ$ force model in heavier nuclei with $N > Z$ did not depend on approximate treatments made there. Our calculations in this paper and others [29–31] clarified the essential role of the $p$-$n$ force $V_{\pi\nu}^{\tau=0}$ in the binding energy. If $V_{\pi\nu}^{\tau=0}$ is added to the treatment of the pairing plus $QQ$ plus quadrupole force model in $N > Z$ nuclei, binding energies will be well reproduced too.

The present model, however, is not sufficiently good for non-collective states except the yrast states and for nuclei with $n_p \leq 1$ or $n_n \leq 1$. One cause may be attributed to the absence of the $f_{5/2}$ orbit in the present calculations and another to the insufficiency of the isovector interactions which take action in like-nucleon systems. The present interaction remains room for improvement. One way to improve it is to add hexadecapole and hexadecapole-hexadecapole forces (octupole and octupole-octupole forces for negative-parity states) to the $P_0 + P_2 + QQ$ force.

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