Polarization-based Speckle Nulling Using a Spatial Light Modulator to Generate a Wide-field Dark Hole

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Abstract

Direct detection of exoplanets requires a high-contrast instrument called a coronagraph to reject bright light from the central star. However, a coronagraph cannot perfectly reject the starlight if the incoming stellar wave front is distorted by aberrations due to the Earth’s atmospheric turbulence and/or the telescope instrumental optics. Wave-front aberrations cause residual stellar speckles that prevent detection of faint planetary light. In this paper, we report a laboratory demonstration of a speckle-nulling wave-front control using a spatial light modulator (SLM) to suppress the residual speckles of a common-path visible nulling coronagraph. Because of its large format, the SLM potentially has the ability to generate a dark hole over a large region or at a large angular distance from a star of up to hundreds of \( \lambda/D \). We carry out a laboratory demonstration for three cases of dark hole generation: (1) in an inner region (3–8 \( \lambda/D \) in horizontal and 5–15 \( \lambda/D \) in vertical directions), (2) in an outer region (70–75 \( \lambda/D \) in horizontal and 65–75 \( \lambda/D \) in vertical directions), and (3) in a large region (5–75 \( \lambda/D \) in both directions). As a result, the residual speckles are rejected to contrast levels on the order of 10\(^{-8}\) in cases 1 and 2. In cases 2 and 3, we can generate dark holes at a large distance (up to >100 \( \lambda/D \)) and with a large size (70 \( \lambda/D \) square), both of which are out of the Nyquist limit of currently available deformable mirrors.

Unified Astronomy Thesaurus concepts: Direct imaging (387); Coronagraphic imaging (313); Astronomical techniques (1684)

1. Introduction

Since the first discovery of a giant planet around a Sun-like star more than 25 yr ago (Mayor & Queloz 1995), thousands of exoplanets have been discovered by means of various detection methods. Among them, direct detection methods using high-contrast techniques such as coronagraphs, nulling interferometry, and differential imaging (e.g., Traub & Oppenheimer 2010, and references therein) are powerful approaches for probing planetary atmospheres using their spectra. In order to directly detect an exoplanet, a high-contrast instrument is required to reject light from the bright central star. Coronagraphs are a promising approach for rejecting stellar diffraction light. Many kinds of coronagraphs have been proposed (see, e.g., the references in Guyon et al. 2006, in which theoretical detection limits are comprehensively studied for various advanced coronagraphs).

Ground-based large telescopes equipped with several state-of-the-art high-contrast instruments have started operations, including the Gemini Planet Imager, Spectro-Polarimetric High-Contrast Exoplanet Research, and Subaru Coronagraphic Extreme Adaptive Optics (Beuzit et al. 2008; Macintosh et al. 2014; Lozi et al. 2018). Future extremely large telescopes are planned with the goal of discovering Earth-like exoplanets in habitable zones around late-type stars (Kasper et al. 2010; Crossfield 2013; Guyon et al. 2018) and detecting potential biosignatures, such as the oxygen absorption feature (Kawahara et al. 2012).

In space, an advanced coronagraphic instrument will be mounted on the Nancy Grace Roman Space Telescope, formerly called WFIRST, for the purpose of technology demonstration (Spergel et al. 2015). The Roman Space Telescope is expected to be a promising precursor for subsequent planet-finding missions with the aim of searching for habitable Earth-like exoplanets around Sun-like stars and detecting biosignatures in their atmospheres (Bolcar et al. 2016; Mennesson et al. 2016).

For these ultimate scientific goals, extremely high contrasts of 10\(^{-8}\) and 10\(^{-10}\) are required for detecting Earth-like exoplanets in habitable zones around late-type and Sun-like stars, respectively. However, a major problem of coronagraphs is residual stellar speckled light, which prevents the detection of faint planetary light. On the ground, residual speckles are caused by phase and amplitude aberrations due to residual atmospheric turbulence after wave-front correction by adaptive optics systems. In particular, noncommon path aberrations between the imaging and wave-front-sensing paths generate slowly evolving residual speckles that cannot be corrected by adaptive optics. Even for observations in space, telescope instrumental optics cause wave-front aberrations resulting in quasi-static residual speckles. Residual speckles generate false positives in exoplanet detection because their typical size is comparable to a planetary point-spread function (PSF).
A technique called speckle nulling has been proposed and developed to suppress stellar speckles (e.g., Malbet et al. 1995; Bordé & Traub 2006). The technique is based on wave-front control to eliminate the speckles over a specified region, called a dark hole. The technique can be applied to any coronagraph. An “Earth-twin” detection has successfully been demonstrated in the laboratory by using a deformable mirror (DM) to generate a dark hole against the residual speckles of the Lyot-type coronagraph (Trauger & Traub 2007).

When developing high-contrast instruments, it is important to pursue not only deep dark holes but also wide controllable fields. A deep dark hole at wide separation has the potential to shed light on new areas of exoplanets and planet formation that have not been previously explored. For instance, a Jupiter-sized planet candidate, Epsilon Eridani b, was found by radial velocity at 3.4 au, corresponding to a separation of 1”1 (Hatzes et al. 2000; Mawet et al. 2019). A dark hole at ~1” with a contrast of ≤10^-9 is expected to enable direct characterization of this gas giant around the snow line through direct spectroscopy of its reflected light. Indeed, many gaseous planets at several astronomical units have been discovered by radial velocity surveys (e.g., Cumming et al. 2008; Fernandes et al. 2019) and even by transit planet surveys using the data set of Kepler (Kawahara & Masuda 2019). The latter showed that there is an abundant population of Neptune-sized planets at several astronomical units that are not observed in our solar system. A deeper contrast at a wide separation has the potential to explore such smaller exoplanets, whose composition is likely to be different from Jupiter-sized planets. In addition, the ability to generate a large dark hole is helpful for investigating debris disks in detail, such as the inner belt of the debris disk in Epsilon Eridani at 14 au (Greaves et al. 2014), which is expected to be connected with the asteroid belt in our solar system.

The largest possible size of the dark hole is determined by the number of DM actuators (the Nyquist limit). Therefore, it is necessary to use DMs with a large number of actuators in order to generate dark holes over a wide field. In this paper, we report a laboratory demonstration of polarization-based speckle nulling using a liquid-crystal-on-silicon spatial light modulator (SLM) installed in a prism-based common-path visible nulling coronagraph (VNC; Murakami & Baba 2010). The SLM is an attractive device for high-contrast techniques owing to its large format. Speckle-nulling wave-front control using an SLM has the ability to generate dark holes over a large region or at a large distance of up to hundreds of λ/D (where D is the diameter of the telescope primary mirror and λ is the wavelength of light).

In Section 2, we show the principles of the common-path VNC and describe how wave-front aberrations cause residual speckles. Next, we describe the principles of polarization-based speckle-nulling wave-front control using an SLM. In Section 3, we report the results of laboratory demonstration of polarization-based speckle-nulling wave-front control. In Section 4, we discuss several issues, such as limitations on the achievable contrast and chromaticity of speckle nulling using an SLM and the perspective of on-sky observations. Finally, we summarize our conclusions in Section 5.

2. Principle

2.1. Common-path VNC

The common-path VNC, which is also called the Savart-plate lateral-shearing interferometric nuller for exoplanets, is derived from the original VNC (Mennesson et al. 2003). The system is based on a Savart plate and theoretically realizes achromatic suppression of starlight by using a simple and stable optical configuration (Murakami & Baba 2010). The Savart plate is a polarizing beam displacer made of two crossed birefringent prisms. As shown in Figure 1, the common-path VNC consists of the Savart plate, crossed polarizers P1 and P2, and a diaphragm called a Lyot stop.

For simplicity, we assume that complex amplitudes of light are all expressed in the same coordinate system (u, v) from the entrance pupil to the Lyot plane. Now consider the case where a tilted-plane wave front without aberrations is incoming from direction (θi, ϕi) into the coronagraphic system. Assuming unpolarized light, half of the incoming photons are lost at the first polarizer, P1. We note that the loss of the photons can be avoided by replacing the polarizers with polarizing beam splitters to construct a dual-beam configuration (Kitou et al. 2014). After passing through polarizer P1, the complex amplitude E0 (u, v) of the polarized wave front of amplitude A0 can be written as

\[ E_0(u, v) = A_0 P_0(u, v) e^{i \frac{2\pi}{\lambda} (\theta_s u + \phi_s v)}, \]  

where P0(u, v) is the pupil function of the telescope, which takes values of 1 and zero inside and outside the pupil, respectively.

The polarized wave front E0(u, v) is then split into ±45° linearly polarized components by the Savart plate (E1 and E2 in Figure 1). The Savart plate also laterally displaces one wave front by s in the u direction with respect to the other wave front. The Savart plate is regarded as a ±45° polarizer for each light beam. According to the Jones calculus, the two wave fronts, after passing through the second polarizer, P2, can be calculated as

\[ E_{N1}(u, v) = \frac{A_0}{2} P_0(u + \frac{s}{2}, v - \frac{s}{2}) e^{i \frac{2\pi}{\lambda} \left( \frac{\pi}{2} - \theta_s u + \phi_s v \right)}, \]  

and

\[ E_{N2}(u, v) = -\frac{A_0}{2} P_0(u - \frac{s}{2}, v - \frac{s}{2}) e^{i \frac{2\pi}{\lambda} \left( -\frac{\pi}{2} + \theta_s u + \phi_s v \right)}. \]

These two laterally displaced wave fronts interfere at the Lyot plane as EN1 + EN2. The complex amplitude of the interfered wave front after the Lyot stop can then be written as

\[ E_L(u, v) = i e^{-i \frac{\pi \theta_s}{\lambda}} A_0 \sin \left( \frac{\pi \phi_s}{\lambda} \right) P_2(u, v) e^{i \frac{2\pi}{\lambda} (\theta_s u + \phi_s v)}, \]

where PL(u, v) denotes the pupil function of the Lyot stop, which is used for extracting only the interfered region, and the coefficient \( i e^{-i \frac{\pi \theta_s}{\lambda}} \) expresses the initial phase of light, which is not physically significant here. The complex amplitude EL(u, v) has another phase term, \( e^{i \frac{2\pi}{\lambda} (\theta_s u + \phi_s v)} / \lambda \), which suggests that the output interfered light forms a tilted-plane wave front just like the incoming one E0(u, v).

The amplitude of the output wave front is modulated by the term \( \sin(\pi \phi_s / \lambda) \). The interferometric intensity is then proportional to \( \sin^2(\pi \theta_s / \lambda) \) as a function of the incident angle θs. Thus, on-axis starlight (θs = ϕs = 0) is rejected.
be detected at the final focal plane. The interferometric intensity of off-axis planets becomes a maximum at the incident angle $\theta_e = \lambda/(2s)$. We define the inner working angle (IWA) as the incident angle $\theta_i = \lambda/(4s)$ at which the interferometric intensity becomes half of the maximum.

We assume that the final focal plane $(x, y)$ is associated with the Lyot plane $(u, v)$ by a two-dimensional Fourier transform. The complex amplitude of the final image on the detector, $E_f(x, y)$, is written by the Fourier transform of $E_L(u, v)$ as

$$E_f(x, y) = ie^{-i\pi x^2/\lambda}A_0 \sin\left(\frac{\pi s \theta_i}{\lambda}\right) F[P_L(u, v)e^{i\frac{\pi}{2}\sin(\theta_i u + \theta_i v)}],$$

(5)

where $F$ is a Fourier transform operator.

In the above discussion, we assume that the polarization components used in the system are perfectly achromatic. In practice, however, polarizers and Savart plates have finite extinction ratios (e.g., $10^5$ over a designed wavelength range and lower outside this range). The finite extinction ratios cause contamination of incoherent light in their polarized outputs, resulting in degradation of the achievable contrast and achromaticity.

### 2.2. Residual Speckles in Common-path VNC

In the previous section, we showed that on-axis starlight is perfectly rejected by an aberration-free coronagraph with ideal polarizing optics. In practice, however, the stellar wave front is distorted by phase and amplitude aberrations mainly due to telescope instrumental optics in space observations and time-varying residual atmospheric turbulence after correction by adaptive optics in ground-based observations. In both cases, the starlight cannot be perfectly rejected, and the residual speckles prevent detection of faint planetary light.

The aberrations are expressed by a two-dimensional complex function in the pupil plane $\phi_p(u, v)$ whose real and imaginary parts correspond to phase and amplitude errors, respectively. The distorted stellar wave front from an on-axis source can then be written as

$$E_p(u, v) = A_0 P_o(u, v) e^{i\phi_p(u, v)}.\quad(6)$$

The complex amplitudes of the two wave fronts after passing through polarizer P2 are calculated as

$$E_{E_2}'(u, v) = \frac{A_0}{2} P_0\left(u - \frac{s}{2}, v - \frac{s}{2}\right) e^{i\phi_o(u, v)},$$

(7)

$$E_{E_{22}}'(u, v) = \frac{A_0}{2} P_0\left(u + \frac{s}{2}, v + \frac{s}{2}\right) e^{i\phi_o(u + s, v + s)},$$

$$E_{E_{22}}'(u, v) = \frac{A_0}{2} P_0\left(u - \frac{s}{2}, v - \frac{s}{2}\right) e^{i\phi_o(u - s, v - s)}.$$
These two wave fronts interfere at the Lyot plane.

Assuming that the phase terms $\phi_i$ and $\phi_S$ are both very small, we can use a linear approximation and neglect higher-order terms as $e^{i\phi} \approx 1 + i\phi$. The complex amplitude in the final focal plane then simply becomes the sum of the residual speckles and a correction term generated by the SLM as

$$E_f(x, y) = E_0'(x, y) + E_{\text{cor}}(x, y),$$

where the correction term $E_{\text{cor}}(x, y)$ is written as

$$E_{\text{cor}}(x, y) = \frac{iA_0}{2} \mathcal{F}[P_L(u, v) \phi_S(u, v)].$$

We note that the phase modulation of the SLM, $\phi_S(u, v)$, and the correction term $E_{\text{cor}}(x, y)$ are simply connected by the Fourier transform. The correction term can be used to suppress the residual speckles over a target dark hole region in the final focal plane.

Let us consider the case where an optimal modulation is provided by the SLM for removing one speckle at a target position. For this purpose, a cosine-wave modulation with optimal amplitude, phase, spatial frequency, and orientation angle needs to be generated by the SLM. This modulation is written as

$$\phi_S(u, v) = \alpha \cos\{2\pi\beta(u \cos \omega + v \sin \omega) + \gamma\},$$

where the parameters $\beta$ and $\omega$ define the target position $(X, Y)$ in the final focal plane, while $\alpha$ and $\gamma$ set the amplitude and phase of the correction term $E_{\text{cor}}(X, Y)$.

Applying the SLM modulation given by Equation (15), twin spots are obtained at $(X, Y)$ and $(-X, -Y)$. These spots can be regarded as a PSF given by

$$E_{\text{PSF}}(x, y) = \frac{A_0}{2} \mathcal{F}[P_L(u, v)]$$

where $f$ is the focal length of the imaging lens. The coefficient of the Fourier transform is not taken into account because it is not physically significant in the discussion here. By substituting Equation (15) into Equation (14) and using Equation (16), the twin spots generated by the SLM are calculated as

$$E_{\text{cor}}(x, y) = \frac{i\alpha}{4} \left\{ e^{-i\gamma}E_{\text{PSF}}(x - X, y - Y) + e^{-i\beta}E_{\text{PSF}}(x + X, y + Y) \right\},$$

where the target position $(X, Y)$ is given by $(\lambda/\beta \cos \omega, \lambda/\beta \sin \omega)$. At this target position, the correction term can be written approximately as

$$E_{\text{cor}}(X, Y) = \frac{i\alpha}{4} e^{-\gamma}E_{\text{PSF}}(0, 0),$$

assuming that the contribution from the counterpart PSF at $(-X, -Y)$, the second term in Equation (17), is negligible. The correction parameters to be applied to the SLM for rejecting the speckle $E_f'(X, Y)$ are estimated as

$$\alpha = \frac{4\sqrt{\mathcal{R}[E_f'(X, Y)]^2 + \mathcal{I}[E_f'(X, Y)]^2}}{E_{\text{PSF}}(0, 0)},$$

$$\beta = \frac{\sqrt{X^2 + Y^2}}{\lambda},$$

$$\gamma = \tan^{-1}\left( \frac{\mathcal{R}[E_f'(X, Y)]}{-\mathcal{I}[E_f'(X, Y)]} \right),$$

$$\omega = \tan^{-1}\left( \frac{Y}{X} \right),$$

where $\mathcal{R}$ and $\mathcal{I}$ describe the real and imaginary parts of the complex value.

In our laboratory demonstration described in the following section, the phase of the wave front is modulated by the SLM such that several speckles can be rejected simultaneously with each control iteration. In order to reject $M$ speckles over the target region simultaneously, the phase modulation by the SLM needs to be

$$\phi_S(u, v) = \sum_{m=1}^{M} \alpha_m \cos\{2\pi\beta_m(u \cos \omega_m + v \sin \omega_m) + \gamma_m\}.$$
hole generation, we assume that the complex amplitude of the residual stellar speckles $E'_f(x, y)$ can be perfectly measured by an ideal wave-front sensor.

Figure 3 shows images acquired (a) before and (b) after speckle nulling using the SLM whose phase control resolution is arbitrarily set to $\lambda/10,000$. In the numerical simulation, the 500 brightest speckles are selected in each iteration, i.e., $M = 500$ in Equation (23). As can be seen, the D-shaped dark hole, with an inner limit of $3 \lambda/D$ in the horizontal direction and an outer radius of $125 \lambda/D$ (close to the Nyquist limit of the assumed SLM), is successfully generated. The mean contrast over the dark hole region is improved from $1.9 \times 10^{-7}$ to $7.4 \times 10^{-10}$ after 100 iterations.

The four red circles in Figure 3(a) show the distances 4, 20, 37, and 116 $\lambda/D$ from the central star. These distances roughly correspond to the orbits of Earth, Jupiter, Saturn, and Neptune as seen from 10 pc away assuming $D = 4$ m and $\lambda = 500$ nm. The green square represents the Nyquist limit of a 32 $\times$ 32 DM.

3. Laboratory Demonstration

3.1. Optical Setup

In this section, we show several results from the laboratory demonstration of polarization-based speckle-nulling wave-front control. Figure 4 shows the optical setup of the laboratory demonstration. We employ a He-Ne laser ($\lambda = 633$ nm) or a laser diode ($\lambda = 635$ nm) as a model star. The light source is coupled with a single-mode optical fiber (SM fiber) and collimated by a lens, L1 (focal length of 300 mm). A circular diaphragm (diameter of $D = 5$ mm) is placed as a telescope entrance pupil. Behind the entrance pupil, a polarizer, P1, and Savart plate, SP1, are placed as part of the common-path VNC. Savart plate SP1 causes a lateral shift of $s_1 = 0.65$ mm, which corresponds to 0.13$D$. The IWA defined in Section 2 becomes about $2 \lambda/D$.

The entrance pupil plane is reimaged by lenses L2 and L3 (focal lengths of 200 mm), and an SLM (Meadowlark Optics, Inc.) is placed in this reimaged pupil plane. The SLM is operated by a 16-bit driver that implements 65536-level voltage control. Behind the SLM, the entrance pupil is reduced to 1 mm.
Figure 5. Target dark hole regions registered for the laboratory demonstration. The target regions for cases 1, 2, and 3 are indicated by the black rectangles and square. The asterisks and dotted squares indicate the positions of a model star and a Nyquist limit of a 32 × 32 DM, respectively.

Table 1
Summary of Target Dark Hole Regions and Experimental Results for Three Cases

| Case   | Target Dark Hole Regions       | Initial Contrast | Final Contrast |
|--------|--------------------------------|------------------|---------------|
| Case 1 | 3–8 λ/D Horizontal (x) 5–15 λ/D Vertical (y) 5.8λ/D 17λ/D | 3.0×10⁻⁶       | 7.7×10⁻⁸     |
| Case 2 | 70–75 λ/D Horizontal (x) 65–75 λ/D Vertical (y) 96λ/D 106λ/D | 5.9×10⁻⁷       | 6.3×10⁻⁸     |
| Case 3 | 5–75 λ/D Horizontal (x) 5–75 λ/D Vertical (y) 7.1λ/D 106λ/D | 5.2×10⁻⁷       | 1.7×10⁻⁷     |

Note. The θ_m and θ_out mean angular separations from the model star to the innermost and outermost positions of the target dark hole regions.

3.2. Estimation of the Correction Parameters

To reject a residual speckle at a target position (X, Y), we need to estimate the optimal correction parameters α and γ in Equation (23). Equations (19) and (21) suggest that the residual speckle field E_r(X, Y) needs to be measured in order to estimate the parameters α and γ. The parameters β and ω are determined unambiguously for each target position (X, Y) by using Equations (20) and (22).

First, the SLM generates a cosine-wave phase modulation φ_α(μ, ν), as shown by Equation (15), and then twin spots are generated at positions (±X, ±Y) according to Equation (17). Second, the amplitude of the phase modulation α is set so that the intensity of the generated spot becomes comparable to that of the original speckle E_r(X, Y) to be rejected. Third, the phase of the generated spot at the target position (X, Y) is modulated by changing the parameter γ. The original speckle E_r(X, Y) then interferes with the correction term E_corr(X, Y), and the interferometric intensity is modulated as a function of γ. We change the parameter γ eight times in steps of π/4, and the measured interferometric intensities are fit with a model function to search for the optimal value of γ that minimizes the interferometric intensity. At each control iteration, M speckles to be rejected are selected over the target region where the dark hole is generated. The above procedure is conducted for the M speckles simultaneously. Finally, the estimated optimal parameters are applied to the SLM according to Equation (23). The estimation and correction processes are conducted iteratively.

3.3. Results

Figure 5 shows the target dark hole regions for the laboratory demonstration. We carried out the laboratory demonstration for three cases as summarized in Table 1. First, we demonstrate speckle nulling over a region within the Nyquist limit of currently available DMs, such as a 32 × 32 DM. In Figure 5(a), the target dark hole is indicated by a black rectangle, while the asterisk and dotted square indicate the positions of the model star and a Nyquist limit of a 32 × 32 DM, respectively. The target dark hole region is registered over 3–8 λ/D in the horizontal direction and 5–15 λ/D in the vertical direction from the model star (case 1).

Second, we demonstrated the speckle nulling at a large distance of roughly 100 λ/D from the model star by taking advantage of the large format of the SLM (case 2). As indicated
in Figure 5(b), the target dark hole region is registered over 70–75 \( \lambda/D \) in the horizontal direction and 65–75 \( \lambda/D \) in the vertical direction. We note that the super-Nyquist wave-front control technique was proposed for generating dark holes beyond the Nyquist limits of DMs in the context of exoplanet detection in multiple-star systems (Thomas et al. 2015; Sirbu et al. 2017). In our demonstration, we conducted dark hole generation in the above region without special techniques like this.

Third, we demonstrated the speckle nulling over a square region as large as 70 \( \lambda/D \) (case 3). As indicated in Figure 5(c), the dark hole region is registered over 5–7 \( \lambda/D \) in both directions. The target regions for cases 2 and 3 are not accessible by 32 x 32 (or even 64 x 64) DMs.

Figure 6 shows the experimental results of speckle-nulling wave-front control of case 1. As the model star, we used a He-Ne laser (\( \lambda = 633 \) nm). Figures 6(a) and (b) show the observed coronagraphic images before and after speckle nulling. In each iteration, the five brightest speckles (\( M = 5 \)) are selected for speckle nulling. After 40 iterations, the mean contrast improves from \( C = 3.0 \times 10^{-6} \) to \( 7.7 \times 10^{-8} \). The mean contrasts were calculated over a smaller region of size 4–7 \( \lambda/D \) in the horizontal direction and 6–14 \( \lambda/D \) in the vertical direction in order to exclude bright speckles close to the dark hole boundaries. The contrast gain, which is defined as the ratio of achieved contrasts before and after speckle nulling, was about 40 in case 1.

Figure 6(c) shows a numerically simulated image of case 1 after 100 iterations. The phase and amplitude errors and the phase resolution of the SLM are all assumed to be identical to the numerical simulation in Figure 3. The complex amplitude of the residual stellar speckle \( E'(x, y) \) is assumed to be perfectly measured by an ideal wave-front sensor. In the numerical simulation, the mean contrast improves from \( C = 4.8 \times 10^{-8} \) to \( 1.3 \times 10^{-8} \) (a contrast gain of 370).

Figure 7 shows the experimental results of speckle-nulling wave-front control of case 2, in which a laser diode (\( \lambda = 635 \) nm) is used as the model star. Figures 7(a) and (b) show the coronagraphic images observed before and after speckle nulling. Magnified images around the dark hole region are also shown in the bottom row. In each iteration, the five brightest speckles (\( M = 5 \)) are selected. After 30 iterations, the mean contrast improves from \( C = 5.9 \times 10^{-7} \) to \( 6.3 \times 10^{-8} \) (a contrast gain of roughly 10). The mean contrast was calculated over a smaller region of size 71–74 \( \lambda/D \) in the horizontal direction and 66–74 \( \lambda/D \) in the vertical direction.

Figure 7(c) shows a numerically simulated image of case 2 after 100 iterations. The assumed phase and amplitude errors and the phase resolution of the SLM are all identical to the numerical simulation in Figure 3. In this simulation, the mean contrast improves from \( C = 3.9 \times 10^{-8} \) to \( 1.6 \times 10^{-10} \) (a contrast gain of 240).

Figure 8 shows the experimental results of speckle-nulling wave-front control of case 3, in which a laser diode (\( \lambda = 635 \) nm) is used as the model star. Figures 8(a) and (b) show the coronagraphic images observed before and after speckle nulling. In each iteration, the 300 brightest speckles (\( M = 300 \)) are selected. After 15 iterations, the mean contrast over a smaller region (of size 6–74 \( \lambda/D \) in both directions) improves from \( C = 5.2 \times 10^{-7} \) to \( 1.7 \times 10^{-7} \), corresponding to a contrast gain of roughly 3.

Figure 8(c) shows a numerically simulated image of case 3 after 100 iterations, where the phase and amplitude errors and the phase resolution of the SLM are all assumed to be identical to the numerical simulation in Figure 3. In this simulation, the mean contrast improves from \( C = 2.6 \times 10^{-7} \) to \( 2.2 \times 10^{-10} \) (a contrast gain of 1200).

4. Discussion

4.1. Limitations on Achievable Contrasts

Figure 9 shows the obtained contrasts as a function of the iteration number for three cases, and Table 1 summarizes their initial and final contrasts. Unlike cases 1 and 2, a contrast level below \( 10^{-7} \) was not obtained in case 3.

Atmospheric turbulence in the laboratory causes decorrelation of speckles, which degrades the achievable contrast. Figure 10 (left) shows the speckle patterns acquired after the first and last (30th) iterations in case 2. The bottom two panels show close-up images of the regions indicated by the red solid squares. As can be seen, the speckle patterns seem to be stable in spite of the large time difference of about 80 minutes between the images (i.e., about 2.6 minutes per iteration) due to the manual control in which we used several application
Before SN ($C = 5.9 \times 10^{-7}$) 

After SN (30 iterations, $C = 6.3 \times 10^{-8}$) 

After SN (100 iterations, $C_{\text{sim}} = 1.6 \times 10^{-10}$) 

Figure 7. Laboratory demonstration of speckle-nulling wave-front control of case 2. Coronagraphic images acquired (a) before and (b) after speckle nulling (“SN” in the figure) are shown together with (c) a numerically simulated image. Magnified images around the dark hole region are also shown in the bottom row. The plus sign indicates the position of the model star. The color scales differ between the experimentally acquired and simulated images.

programs separately, not integrated into one system, for acquiring the images, estimating the correction parameters, and controlling the SLM. We quantitatively evaluated decorrelation of the speckle pattern by calculating the degrees of cross-correlation between the acquired speckle patterns of adjacent iterations. The cross-correlations are evaluated over a local square region of size $5 \lambda/D \times 5 \lambda/D$ centered on $(x, y)$ (denoted by $A_{x,y}$). The cross-correlation $CC_i(x, y)$ between the $i$th and $(i+1)$th iterations is then defined as

$$CC_i(x, y) = \frac{\sum_{(x', y') \in A_{x,y}} [I_i(x', y') - \bar{I}_i] [I_{i+1}(x', y') - \bar{I}_{i+1}]}{\sqrt{\sum_{(x', y') \in A_{x,y}} [I_i(x', y') - \bar{I}_i]^2} \sum_{(x', y') \in A_{x,y}} [I_{i+1}(x', y') - \bar{I}_{i+1}]^2},$$

(24)

where $I_i$ is the speckle pattern at the $i$th iteration, and $\bar{I}_i$ is its mean intensity over the local region $A_{x,y}$. All three summations in Equation (24) are taken over the region $A_{x,y}$. If two speckle patterns at the $i$th and $(i+1)$th iterations are exactly the same, the cross-correlation becomes unity, i.e., $CC_i(x, y) = 1$. The cross-correlation maps $CC_i(x, y)$ are calculated over one quadrant region (bottom right region of the model star) at a distance of up to $120 \lambda/D$, indicated by the dashed squares in the images in Figure 10 (left). Figure 10 (right) shows the radially averaged profiles of the calculated $CC_i(x, y)$ maps. Although the profile of $CC_{12}$ exhibits a remarkably lower value, the speckle patterns seem to be highly stable, with cross-correlations close to unity (higher than 0.995) from the inner (several $\lambda/D$) to the outer (>100 $\lambda/D$) region.

Cross talk in the SLM might also limit the achievable contrast. Ideally, the SLM can be regarded as an array of variable retarders whose phase response is uniform over a pixel area and does not affect the responses of adjacent pixels. This simple idealized model is assumed in the numerical simulations of speckle nulling reported above. In practice, however, wave-front control suffers from cross talk between responses of the adjacent pixels, particularly in a region close to the Nyquist limit. In the laboratory demonstration of case 3, the distance of roughly 100 $\lambda/D$ from the model star is about 1.6 times smaller than the Nyquist limit. In such a situation, the residual speckles might not be able to be controlled and rejected as designed if an effect of cross talk spreads over $\approx 1.6$ pixels. Cross talk in the SLM, as well as speckle decorrelation due to the laboratory environment, is expected to cause errors in wave-front control and measurement of the speckle fields. The effects of these phenomena on the achievable contrast, particularly in case 3, need to be further investigated in order to pursue higher contrast. We note that cross talk in SLMs and compensation for it have been studied with the aim of achieving the desired wave-front control (e.g., Moser et al. 2019).

Even in cases 1 and 2, the achieved contrasts were still limited to the order of $10^{-8}$ at best. Figure 11 shows the estimated correction parameters $\alpha_m$ for case 1 as a function of the iteration number. The parameter $\alpha_m$ is the applied phase of the SLM, expressed in units of voltage level, for rejecting the $m$th-brightest speckles. The five lines show the estimated...
Figure 9. Measured mean contrasts over dark hole regions for the three cases as a function of iteration number.

values of $\alpha_m$ for rejecting the five brightest speckles in each iteration (i.e., $m$ from 1 to 5). The values of $\alpha_m$ decrease as a function of the iteration number because the intensity levels of the speckles decrease. The estimated applied voltage was only $\alpha_m = 2.1$ in the last (40th) iteration, even for the brightest speckle ($m = 1$), and less than the phase resolution limit $\alpha_m = 0.9$ for the faintest one ($m = 5$). It seems that the SLM can no longer reject such faint residual speckles. However, the numerical simulation of case 3 exhibits a high contrast on the order of $10^{-10}$, as shown in Figure 8(c). In addition, another numerical simulation of case 1 based on the electric field conjugation algorithm (Give'on et al. 2007) assuming an SLM phase resolution of $\lambda/10,000$ shows much better contrast of $4.5 \times 10^{-11}$ after only 10 iterations. The SLM used in the experiments is operated by a 16-bit driver, and laboratory testing of the SLM in amplitude-only modulation mode suggests an estimated phase resolution of roughly $\lambda/16,000$. Therefore, we suppose that the phase resolution of the SLM is not the main contribution to the contrast levels reported above.

4.2. Chromaticity of the Speckle Nulling Using the SLM

Unlike DMs, SLMs modulate the phase of the wave front by making use of the birefringence of a liquid crystal. An SLM has two refractive indices, $n_e$ and $n_o$, for two orthogonal polarized lights (i.e., extraordinary and ordinary rays). An SLM modulates the phase of only the extraordinary ray with an effective refractive index $n_{\text{eff}}$ that takes a value of between $n_e$ and $n_o$, depending on the applied voltage. An extended Cauchy model has been derived for describing the wavelength-dependent refractive indices of the liquid crystal (Li & Wu 2004a, 2004b; Li et al. 2005). According to the model, the refractive indices for the extraordinary and ordinary rays are estimated as $n_{e,o} = A_{e,o} + B_{e,o}/\lambda^2 + C_{e,o}/\lambda^3$, with three coefficients $A_{e,o}$, $B_{e,o}$, and $C_{e,o}$ specific to each material. The phase modulation term $\phi_S$ in Equation (11) of a reflective SLM is approximately written as $\phi_S(V, \lambda) = 4\pi n_{\text{eff}}(V, \lambda) d_S / \lambda$, where $d_S$ is the thickness of the SLM and $V$ is the applied voltage that is spatially variant in the pupil plane as $V = V(u, v)$.

Assuming a simple model in which phase error is only caused by an irregular surface of the reflecting optics $d(u, v)$ placed in the entrance pupil plane, the real part of the aberration in Equation (6) is written as $\phi_f(u, v, \lambda) = 4\pi d(u, v) / \lambda$. If a deep dark hole is generated by the SLM at an optimized wavelength $\lambda_{\text{opt}}$, the corresponding phase error and correction phase term are written as $\phi_f(u, v, \lambda_{\text{opt}})$ and $\phi_S(V, \lambda_{\text{opt}})$, respectively. These phase terms at an arbitrary wavelength $\lambda$ can then be written as $\epsilon_{\text{eff}} \phi_f(u, v, \lambda_{\text{opt}})$ and $\epsilon_S \phi_S(V, \lambda_{\text{opt}})$ with appropriate coefficients $\epsilon_{\text{eff}} = \lambda_{\text{opt}} / \lambda$ and $\epsilon_S = \lambda_{\text{opt}} \phi_{\text{eff}}(V, \lambda) / (\lambda n_{\text{eff}}(V, \lambda_{\text{opt}}))$.

In order to demonstrate the chromatic effect of the liquid crystal on the achieved contrast, we carried out numerical simulations of speckle-nulling wave-front control for case 3 by adopting this simple model. We arbitrarily choose the average values of the nine materials listed in Li et al. (2005) for the Cauchy coefficients. The result from Figure 8(c) was used for the acquired dark hole at the optimized wavelength ($\lambda_{\text{opt}} = 633$ nm is assumed). The phase error $\phi_f$ and correction phase $\phi_S$ were then scaled by the coefficients $\epsilon_{\text{eff}}$ and $\epsilon_S$ to simulate dark holes at the other wavelengths. Figure 12 (left) shows the resultant images at wavelengths $\lambda = 500$ and 700 nm. Evolution of the physical scale of the speckle patterns depending on wavelength is not considered here. That is, the images are displayed with the identical $\lambda / D$ scale. In these images, speckles can be observed over the dark hole regions, particularly close to the central star. Figure 12 (right) shows the estimated contrast before and after speckle nulling. Before
speckle nulling, the contrast slightly depends on the wavelength. Better contrast is obtained at longer wavelengths because the input phase error depends on \( \lambda^{-1} \). After speckle nulling, the best contrast of \( 2.2 \times 10^{-10} \) was obtained at \( \lambda_{\text{opt}} = 633 \text{ nm} \), which corresponds to the result shown in Figure 8(c). The contrast degrades as a function of wavelength due to the chromatic refractive indices of the liquid crystal.

A multiwavelength algorithm has been proposed for achromatizing wave-front control with DMs (e.g., Give’on et al. 2007). It might be possible to also apply the idea to SLMs by taking into account the chromatic effect of the refractive indices of the liquid crystal.

It is also important to mention that the phase term of the ordinary ray \( \phi^o_2(\lambda) \approx 4\pi n_s(\lambda) d_s / \lambda \) is added in Equation (12). Although this phase term ideally does not depend on the applied voltage \( V(u, v) \), it causes an unwanted additional phase difference between the two wave fronts split by the Savart plate. This additional phase difference needs to be compensated for to provide stellar suppression over a broad wavelength range.

4.3. Perspective of On-sky Observations

We think that this laboratory demonstration of high-contrast techniques using an SLM demonstrates an attractive approach for developing wave-front control algorithms for future large-format DMs. Wave-front correction with a large number of degrees of freedom has been explored using an SLM toward future ground-based extreme adaptive optics systems (Pourcelot et al. 2021). It will also be interesting to investigate the possibility of using an SLM for ground- and space-based observations. When SLMs are applied to ground-based observations, high-speed modulation is required to correct fast wave-front errors due to atmospheric fluctuations. It has been pointed out, in the context of digital adaptive coronagraphy, that recent high-speed SLMs can operate at a temporal bandwidth of up to 500–700 Hz, which is an interesting level for ground-based applications (Kühn et al. 2018). Further development of higher-speed SLMs and faster control algorithms will be important for future ground-based speckle-nulling wave-front control. On the other hand, these might be less critical for space-based applications, where speckles are expected to be quasi-static. When using an SLM in space, however, tolerance of the space environment, such as radiation, is an important issue to be investigated.

Speckle-nulling wave-front control by an SLM is applicable to any type of coronagraph. The SLM can correct the wave front of a single light polarization, that is, extraordinary rays. The uncontrollable phase term of the ordinary rays mentioned above is not a problem for other coronagraphs, such as phase-mask and shaped-pupil coronagraphs. Unlike a common-path VNC, the SLM is placed in front of these coronagraphs, and a linear polarizer is used in front of the SLM to make only the controllable extraordinary ray hit the SLM. Thus, we do not need to consider the uncontrollable ordinary rays. The necessity of having a polarizer is compatible with achromatization of the coronagraphic phase masks based on the Pancharatnam–Berry phase, such as photonic crystal coronagraphic masks (Murakami et al. 2010, 2013). Laboratory demonstration of speckle-nulling wave-front control using an SLM combined with an eight-octant phase-
mask coronagraph has been conducted at a test bed recently constructed in Japan (Murakami et al. 2020).

5. Conclusion

In this paper, we report on a laboratory demonstration of polarization-based speckle nulling using an SLM installed in a common-path VNC. The laboratory demonstration of speckle nulling was carried out for three cases of dark hole generation: (1) in a region covering 3–8 \( \lambda / D \) in the horizontal direction and 5–15 \( \lambda / D \) in the vertical direction, (2) in a region covering 70–75 \( \lambda / D \) in the horizontal direction and 65–75 \( \lambda / D \) in the vertical direction, and (3) in a region covering 5–75 \( \lambda / D \) in both directions. As a result, we achieved high contrasts on the order of \( 10^{-8} \) over moderate-sized rectangular regions of 5 \( \lambda / D \) and 10 \( \lambda / D \) in the horizontal and vertical directions (cases 1 and 2). More importantly, we generated dark holes at distances of up to >100 \( \lambda / D \) from the model star (cases 2 and 3) and as large as 70 \( \lambda / D \) square (case 3), both of which exceed the Nyquist limit of typical DMs. We investigated speckle decorrelation due to atmospheric turbulence in the laboratory. As a result, speckle patterns seem to be highly stable through wave-front control over a timescale of about 80 minutes (about 2.6 minutes per iteration). We also discussed other possible limitations, such as cross talk and the control-phase resolution and chromaticity of the SLM. In addition, the perspective of on-sky observations is discussed in terms of both ground- and space-based applications. We expect the laboratory demonstration presented in this paper to be an important step for developing a high-contrast wave-front control scheme for future large-pixel DMs and considering the use of SLMs for on-sky applications in both ground- and space-based telescopes.

The SLM used in the experiment has a 512 \( \times \) 512 pixel format and thus potentially the ability to generate dark holes as large as 256 \( \lambda / D \) or at a large distance of up to 256 \( \lambda / D \) from the model star. This capability is expected to be useful for observing not only Earth-like exoplanets in habitable zones but also planets at various distances. The ability to generate large dark holes will also be interesting for observing debris disks in depth and exoplanets in multiple-star systems. These observations will provide us with valuable information for revealing mechanisms of planetary formation, diversity of planetary architecture, and, ultimately, the habitability of inner Earth-like planets in various planetary systems.

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