Structure of the Nuclear Force in a Chiral Quark-Diquark Model

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We study the structure of the nuclear force using a Lagrangian derived through the hadronization of a chiral quark and diquark model. A generalized trace-log formula is expanded in powers of meson and nucleon fields. It is shown that the nuclear force is composed of long and medium range parts of chiral meson exchanges and short range parts of a quark-diquark loop of nonlocal nature. In a local approximation, the quark-diquark loop contains, among many terms, two types of interactions; one is of an attractive scalar iso-scalar type, and the other is of a repulsive vector iso-scalar type. The ranges of the scalar and vector interactions are similar to those of sigma (σ) and omega (ω) meson exchanges if the size of the nucleon core of the quark-diquark bound state is adjusted appropriately.

§1. Introduction

A microscopic description of the meson-nucleon interaction is one of central issues in hadron-nuclear physics. Although lattice calculations seem to have reached levels of the quantitative description of hadron properties, interpretation based on effective models is still important in order to understand the underlying physical mechanisms. Symmetry is primarily a powerful tool when one attempts to understand the meaning of physical processes. In hadron physics, flavor and chiral symmetries are the two important symmetries that should be incorporated. In particular, inclusion of the pion degrees of freedom associated with the spontaneous breaking of chiral symmetry is a crucially important ingredient. In addition, for baryon dynamics, their internal structure is also important, as various form factors of finite size imply. These aspects can be intuitively understood by the fact that the pion Compton wavelength is larger than the nucleon size and that nucleon size is larger than the nucleon Compton wavelength: $1/m_\pi >> \langle r_N^2 \rangle^{1/2} >> 1/M_N$.

In a recent publication, a method was proposed, which incorporated the above two aspects of chiral symmetry and the internal structure of hadrons. This method employs an extended model of the Nambu-Jona-Lasinio model with the interactions not only of the quark-antiquark pair but also of the quark-diquark pair. It was shown that the method works well for both meson and baryon dynamics. The quark and diquarks were path-integrated out to generate an effective Lagrangian for mesons and baryons while maintaining important symmetries, such as the gauge and chiral symmetries. The hadron structure was then described in terms of their constituents: a quark and an antiquark for mesons and a quark and a diquark for baryons. It was also pointed out that the resulting effective Lagrangian contains various in-

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interactions among hadrons, such as meson-meson, meson-baryon and baryon-baryon interactions.

In this paper, we investigate, among many interactions, the nucleon-nucleon (NN) interaction\(^*\) derived from the previous framework.\(^2\) The NN interaction below the meson production threshold is phenomenologically understood well, being derived from the phase shift analysis, but its microscopic understanding is still needed. While the long range part is described well by the meson exchange picture, there are several different approaches for the description of the short range part, including meson exchanges and quark exchanges.\(^3\)–\(^8\) In our model, all components of the NN force are contained in an effective Lagrangian that is written in a concise of trace-log form. Then, the expansion of the trace-log terms produces an NN force that is described as meson exchanges for long and medium ranges and quark-diquark exchanges for short ranges. The latter is then shown to contain various types of interactions, including a scalar iso-scalar type and a vector iso-scalar type with non-locality.

As we explain in the next section, we need two types of diquarks: a scalar, isoscalar diquark and the an axial-vector, isovector diquark. In the present paper, we perform analytic calculations including both diquarks and study the general structure of the NN interaction. However, for numerical computations, we consider only the scalar diquark, because the structure becomes quite complicated when the axial-vector diquark is included. We then study interaction ranges in more detail than the interaction strengths, because we expect that the former would be less sensitive to the type of the diquark included.

We organize this paper as follows. In §2, we briefly give the derivation of the trace-log formula in the path-integral hadronization of the NJL model with quark-diquark correlations. In §3, terms containing the NN interaction are investigated in detail, where the general structure of the NN amplitude is presented. We present a sample numerical calculation only for the case in which there is a scalar diquark. The present study of the NN interaction is not quantitatively complete, but it will be useful in demonstrating some important aspects of the nuclear force, in particular that the range of the short range interaction is related to the intrinsic size of the nucleon. The final section is devoted to a summary.

§2. Effective Lagrangian for mesons and nucleons

We now briefly review the method to derive an effective Lagrangian for mesons and nucleons from a quark and diquark model of chiral symmetry following the previous work of Abu-Raddad et al.\(^2\) Let us start from the NJL Lagrangian,

\[
\mathcal{L}_{\text{NJL}} = \bar{q}(i\slashed{\partial} - m_0)q + \frac{G}{2} \left[ (\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2 \right].
\]  

(2.1)

Here, \(q\) is the current quark field, \(\vec{\tau}\) represents the isospin (flavor) Pauli matrices, \(G\) is a dimensional coupling constant, and \(m_0\) is the current quark mass. In this paper,

\(^*\) In what follows, we use “nucleon” rather than “baryon,” as we consider flavor \(SU(2)\) for the nucleon sector.
we set $m_0 = 0$ in the chiral limit. As usual, the NJL Lagrangian is bosonized by introducing collective meson fields as auxiliary fields in the path-integral method.\(^9\)\(^-\)\(^11\)

At an intermediate step, we find the following Lagrangian:

$$L'_{q\sigma\pi} = \bar{q} (i\not\!D - (\sigma + i\gamma_5 \vec{\pi} \cdot \vec{\pi})) q - \frac{1}{2G} (\sigma^2 + \vec{\pi}^2). \quad (2.2)$$

Here $\sigma$ and $\vec{\pi}$ are scalar-isoscalar sigma and pseudoscalar-isovector pion fields, as generated from $\sigma \sim \bar{q}q$ and $\vec{\pi} \sim i\bar{q}\vec{\tau}\gamma_5 q$, respectively. For our purpose, it is convenient to work in a non-linear basis rather than a linear one.\(^12\),\(^13\)

This is realized through the chiral rotation from the current ($q$) to constituent ($\chi$) quark fields:

$$\chi = \xi_5 q, \quad \xi_5 = \left(\frac{\sigma + i\gamma_5 \vec{\pi} \cdot \vec{\pi}}{f}\right)^{1/2}. \quad (2.3)$$

where $f^2 = \sigma^2 + \vec{\pi}^2$. Thus, we find

$$L'_{\chi\sigma\pi} = \bar{\chi} (i\not\!D - f - \vec{\phi} - 2\gamma_5) \chi - \frac{1}{2G} f^2, \quad (2.4)$$

where

$$v_\mu = -\frac{i}{2} \left(\partial_\mu \xi^\dagger \xi + \partial_\mu \xi \xi^\dagger\right), \quad a_\mu = -\frac{i}{2} \left(\partial_\mu \xi^\dagger \xi - \partial_\mu \xi \xi^\dagger\right) \quad (2.5)$$

are the vector and axial-vector currents written in terms of the chiral field,

$$\xi = \left(\frac{\sigma + i\vec{\pi}}{f}\right)^{1/2}. \quad (2.6)$$

The Lagrangian (2.4) describes not only the kinetic term of the quark, but also quark-meson interactions such as Yukawa type, Weinberg-Tomozawa type and etc.

In the model we consider here, we introduce diquarks and their interaction terms with quarks. We assume local interactions between quark-diquark pairs to generate the nucleon fields. As inspired by a method to construct local nucleon fields, it is sufficient to consider two types of diquarks, a scalar, isoscalar diquark, $D$, and an axial-vector, isovector diquark, $\vec{D}_\mu$.\(^14\) Hence, our microscopic Lagrangian for quarks, diquarks and mesons is given by

$$\mathcal{L} = \bar{\chi} (i\not\!D - f - \vec{\phi} - 2\gamma_5) \chi - \frac{1}{2G} f^2 + D^\dagger (\partial^2 + M_D^2) D$$

$$+ \vec{D}^\dagger \mu \left[(\partial^2 + M_A^2)g_{\mu\nu} - \partial_\mu \partial_\nu\right] \vec{D}^\nu$$

$$+ \tilde{G} \left(\sin \theta \bar{\chi} \gamma^\mu \gamma^5 \vec{D}_\mu^\dagger + \cos \theta \bar{\chi} D^\dagger \right) \left(\sin \theta \vec{D}_\nu \cdot \vec{\tau} \gamma^\nu \gamma^5 \chi + \cos \theta D \chi \right). \quad (2.7)$$

In the last, term $\tilde{G}$ is a coupling constant for the quark-diquark interaction, and the angle $\theta$ controls the mixing ratio of the scalar and axial-vector diquarks in the nucleon wave function.

Now, the hadronization procedure can be carried out straightforwardly by introducing the baryon fields as auxiliary fields, $B \sim \sin \theta \vec{D}_\nu \cdot \vec{\tau} \gamma^\nu \gamma^5 \chi + \cos \theta D \chi$, and
by eliminating the quark and diquark fields in (2.7). The final result is written in a compact form as

$$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{2\mathcal{G}} f^2 - i \text{tr} \ln(i\partial - f - \phi \gamma_5) - \frac{1}{\mathcal{G}} \overline{B}B + i \text{tr} \ln(1 - \Box). \quad (2.8)$$

Here, the trace is taken over space-time, color, flavor and Lorentz indices, and the operator \( \Box \) is defined by

$$\Box = \begin{pmatrix} A & \mathcal{F}_2 \\ \mathcal{F}_1 & S \end{pmatrix}, \quad (2.9)$$

where

$$A^{\mu i, \nu j} = \sin^2 \theta \overline{B} \gamma_\rho \gamma^5 \tau_k (\tilde{\Delta}^T)^{\rho k, \mu i} S \tau^j \gamma^\nu \gamma^5 B, \quad (2.10a)$$

$$S = \cos^2 \theta \overline{B} \Delta^T S B, \quad (2.10b)$$

$$(\mathcal{F}_1)^{\nu j} = \sin \theta \cos \theta \overline{B} \Delta^T S \tau^j \gamma^\nu \gamma^5 B, \quad (2.10c)$$

$$(\mathcal{F}_2)^{\mu i} = \sin \theta \cos \theta \overline{B} (\tilde{\Delta}^T)^{\rho k, \mu i} \gamma^\rho \gamma^5 \tau_k S B. \quad (2.10d)$$

In these equations, \( S = (i\partial - f - \phi \gamma_5)^{-1}, \Delta = (\partial^2 - M_S^2)^{-1}, \tilde{\Delta}^{\rho k, \mu i} = \delta^{ki}((\partial^2 + M_A^2)g_{\rho \mu} - \partial_{\rho} \partial_{\mu})^{-1} \) are the propagators of the quark, scalar diquark and axial-vector diquark, respectively. Transposed diquark propagators, as denoted by the superscript \( T \), are employed. Though the effective meson-nucleon Lagrangian (2.8) looks simple, it contains many important physical ingredients when the trace-log terms are expanded:

- It generates a meson Lagrangian in a chirally symmetric manner. Up to fourth order in the meson fields, it produces precisely the Lagrangian of the linear sigma model with the realization of the spontaneous breaking of chiral symmetry. Hence, the vacuum expectation value of \( f \) turns out to be the pion decay constant \( f_\pi \).
- From the second trace-log term, a nucleon effective Lagrangian is derived. In a previous paper, the kinetic term of the nucleon was investigated, and the mass of the nucleon was computed at the one-loop level.\(^2\)
- In the nucleon effective Lagrangian, meson-nucleon couplings appear through the diagrams, as shown in Fig. 1. Their strengths and form factors can be computed with use of the underlying quark-diquark dynamics. Using these vertices, meson-exchange interactions are constructed.
- There are diagrams that contain many nucleon fields. For instance, \( NN \) interactions are expressed as one-loop diagrams, as shown in Fig. 2. This term describes the short range part of the \( NN \) interaction. In this paper, we focus our attention mostly on the \( NN \) interaction derived from the one-loop diagrams.

One could construct a nucleon field in the linear basis. However, this makes the chiral transformation properties complicated. In fact, if we replace \( \chi \) by \( q \), then more than two terms are necessary to maintain chiral symmetry for the interaction term in (2.7). By contrast, in the non-linear basis, the transformation property becomes simple; both quark and nucleon fields are in an isospin multiplet which are subject
§3. The structure of the $NN$ interaction

As we have explained, the $NN$ interaction in the quark-diquark model consists of two parts, the meson exchange term for long and medium ranges, and that described by the quark-diquark one-loop diagram, as shown in Fig. 2, for short ranges. The latter diagram is equivalent, when the flow of nucleon lines are rearranged appropriately, to a diquark exchange diagram, as shown in Fig. 2. In this picture, the nucleon goes into a quark-diquark pair through an interaction (at each blob in the diagram). Because of this, the quark-diquark loop differs from the quark exchanges appearing in the norm kernel of the quark cluster model.\textsuperscript{5,6}

In order to investigate in detail the results of our present work, we first study briefly the size of the nucleon in §3.1. It is then related to the form factor of the Yukawa vertex and the ranges of the short-range interaction as discussed in §§3.2 and 3.3.

3.1. Nucleon size

To start, we explain our parameters and the regularization scheme. We follow the scheme presented in Ref. 2), except for the treatment of $\tilde{G}$. The $\Lambda_{PV}$ is the cutoff to the non-linear transformation

$$\chi(x) \rightarrow h(x)\chi(x), \quad B(x) \rightarrow h(x)B(x).$$

(2.11)

Here $h$ is a non-linear function of the chiral transformations and the chiral field at a point $x$.\textsuperscript{15,16}

Fig. 1. A quark-diquark one-loop diagram for the meson-nucleon Yukawa vertex. The solid, double, dotted and triple lines represent the quark, diquark, meson and nucleon. The blobs represent the three point quark-diquark-baryon interaction derived from Eq. (2.7).

Fig. 2. A loop diagram for the $NN$ interaction (left) and the equivalent diquark exchange diagram.

$\Lambda_{PV}$ is the cutoff
mass of the Pauli-Villars method used in this work to regularize divergent integrals. We note that our model is unrenormalizable. The NJL coupling constant $G$ and the cutoff mass $\Lambda_{PV}$ are fixed to generate the constituent quark mass $m_q$ and the pion decay constant through the NJL gap equation in the meson sector.\(^{11,20,21}\) The mass of the scalar diquark $M_S$ is determined in the NJL model by solving the Bethe-Salpeter equation in the diquark channel.\(^{22,23}\) Then, we have one free parameter, the quark-diquark coupling constant $\tilde{G}$. The strength of $\tilde{G}$ controls the binding or size of the nucleon and generates the mass of the nucleon $M_N$. In Table I, we list typical parameter values as used in Ref. 2).

Let us define the nucleon size through the probability distribution of the quark and diquark. These are related to the charge distributions by

$$\langle r^2_q \rangle = -\frac{6}{G_E^q(q^2)/Q_q + G_E^D(q^2)/Q_S} \frac{\partial}{\partial q^2} G_E^q(q^2),$$

$$\langle r^2_D \rangle = -\frac{6}{G_E^q(q^2)/Q_q + G_E^D(q^2)/Q_S} \frac{\partial}{\partial q^2} G_E^D(q^2),$$

where $G_E^q(q^2)$ and $G_E^D(q^2)$ are the quark and diquark contributions to the nucleon electric form factor as functions of the four momentum transfer $q^2$, and $Q_q$ and $Q_S$ are the charges of the quark and scalar diquark. Then, the nucleon size is defined as the sum of the quark and diquark distributions:

$$\langle r^2 \rangle = \langle r^2_q \rangle + \langle r^2_D \rangle.$$\(^{3.3}\)

In Table II we list the binding energy (B.E.), the mass of the nucleon $M_N$, and the nucleon radius $\langle r^2 \rangle^{1/2}$. As $\tilde{G}$ and the binding energy become larger, the nucleon size becomes smaller. This shows that for larger $\tilde{G}$, the quark-diquark system is more tightly bound, while for smaller $\tilde{G}$, it is more loosely bound. For a small coupling constant $\tilde{G}$, where the system becomes very loosely bound, the size of the nucleon becomes unphysically large. At this point and beyond, the present model should not be taken seriously, because it does not describe the dynamics of confinement.

### 3.2. One pion exchange interaction

Let us first examine the long range part of the $NN$ force, which is dominated by the one pion exchange. In this subsection, we calculate the $\pi NN$ Yukawa vertex as shown in Fig. 1. We use the pion-quark interaction in the non-linear representation, as given in (2.4):

$$\mathcal{L}_{\pi qq} = \frac{1}{2f_\pi} \overline{\chi} \gamma_5 \pi \chi.$$\(^{3.4}\)

| $M_N$     | $m_q$      | $M_S$     | $\Lambda_{PV}$ | $\tilde{G}$   |
|-----------|------------|-----------|----------------|---------------|
| 0.94 GeV  | 0.39 GeV   | 0.60 GeV  | 0.63 GeV       | 271.0 GeV\(^{-1}\) |
Table II. Basic nucleon properties when $\tilde{G}$ is varied. B. E. is defined as B. E. = $m_q + M_S - M_N$. $M_N$ is determined from $\tilde{G}$. The other model parameters, $m_q$, $M_S$ and $\Lambda_{PV}$, are fixed.

| $\tilde{G}$ (GeV$^{-1}$) | B. E. (MeV) | $M_N$ (MeV) | $\langle r^2 \rangle^{1/2}$ (fm) |
|--------------------------|-------------|-------------|----------------------------------|
| 156.4                    | 10          | 980         | 1.40                             |
| 271.0                    | 50          | 940         | 0.77                             |
| 445.9                    | 140         | 850         | 0.54                             |
| 529.4                    | 190         | 800         | 0.49                             |
| 609.2                    | 240         | 750         | 0.46                             |
| 687.4                    | 290         | 700         | 0.44                             |

Here, we have used the expanded form

$$a_{\mu} = -\frac{1}{2f_{\pi}} \partial_{\mu}\pi + O(\pi^3).$$  \hspace{1cm} (3.5)

The diagram of Fig. 1 is then regarded as a nucleon matrix element of (3.4) and is given by

$$L_{\pi NN} = -iN_c ZB(p') \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - M_S^2} \frac{\not{\mathbf{p}}' - \not{k} + m_q}{\not{\mathbf{p}}' - \not{k} - m_q} \gamma_{\mu} \gamma_{\nu} a_{\mu} \frac{\not{\mathbf{p}} - \not{k} + m_q}{(p - k)^2 - m_q} B(p),$$

$$= \tilde{B}(p')(g_A(q^2)\gamma^\mu \gamma_5 + \tilde{h}(q^2)q^\mu \gamma_5)a_{\mu}B(p),$$ \hspace{1cm} (3.6)

where $N_c$ is the number of colors and $Z$ is the wave function renormalization constant of the nucleon field. In the second line of this equation, the $\gamma^\mu$ and $q^\mu$ dependent terms are separated. The second term of $\tilde{h}(q^2)$, however, does not contain the pion pole term. From Eq. (3.6) with Eq. (3.5) inserted, we can extract the pion source term in the momentum space,

$$J^a_\pi = \tilde{B}(g_A q^2 + q^2 \tilde{h}_A) \gamma_5 \frac{\tau^a}{2} B q^\mu,$$ \hspace{1cm} (3.7)

which is followed by

$$\frac{g_{\pi NN}(q^2)}{M_N} = \frac{g_A}{f_{\pi}} + \frac{q^2}{2M_N f_{\pi}} \tilde{h}_A.$$ \hspace{1cm} (3.8)

This equation relates the $\pi NN$ coupling constant with the axial coupling $g_A$ and the higher-order term $\tilde{h}_A$ for arbitrary $q^2$ in the present model. At $q^2 = 0$, this equation reduces to the Goldberger-Treiman relation, as expected from the underlying chiral symmetry of the model.

Now, the pion exchange process in the present model is supplemented by the form factor due to the quark-diquark structure of the nucleon, as shown in Fig. 1. In momentum space, the one pion exchange interaction is written, in the non-relativistic approximation, as

$$V_{\text{OPEP}}(\vec{q}) = \frac{g_{\pi NN}(q^2)}{2M_N} \frac{\vec{\sigma}_1 \cdot q \vec{\sigma}_2 \cdot \vec{q}}{q^2 + m_\pi^2} g_{\pi NN}(q^2),$$ \hspace{1cm} (3.9)
Fig. 3. Pion-nucleon form factor $g_{\pi NN}$ normalized at $q^2 = 0$ as a function of $Q^2 = -q^2$. The three curves correspond to $M_N = 0.98$ (loosely bound), 0.85 and 0.70 (tightly bound) GeV from bottom to top.

Table III. The cutoff parameter and the corresponding range of the $\pi NN$ form factor.

| $M_N$ (MeV) | $\Lambda$ (GeV) | $\langle r^2 \rangle_A^{1/2}$ (fm) |
|------------|----------------|-------------------------------|
| 980        | 0.29           | 1.66                          |
| 940        | 0.54           | 0.90                          |
| 850        | 0.78           | 0.62                          |
| 800        | 0.85           | 0.57                          |
| 750        | 0.90           | 0.53                          |
| 700        | 0.93           | 0.52                          |

where $\vec{\sigma}_{1,2}$ are the spin matrices for the nucleon 1 and 2. In this paper, $m_\pi = 0$, since we have set $m_0 = 0$. If we take into account the fact that $m_0 \neq 0$, we can generate the mass of the pion with the correct value.

In the present paper, we include only the scalar diquark, due to the rather complicated structure of the quark-diquark loop integral when the axial-vector diquark is included. A full calculation with both the scalar and axial-vector diquarks included requires careful treatment of the regularization procedure. We postpone such a complete study for future works. It is, however, not very difficult to estimate the effect of the axial-vector diquark for one-body matrix elements; it mostly contributes to the absolute values of the spin-isospin ($\vec{\sigma} \cdot \vec{\tau}$) matrix elements, which appear in the axial-vector coupling constant $g_A$ (and hence in the $\pi NN$ coupling constant $g_{\pi NN}$) and isovector magnetic moments.

In Fig. 3, we plot the $\pi NN$ form factor as a function of the four momentum transfer $Q^2 = -q^2$ for different binding energies, where the form factor is normalized to 1 at $Q^2 = 0$. If we use the parameter set given in Table I, $g_{\pi NN}(0) = 8.8$, and hence the axial coupling constant is $g_A(0) = 0.87$. Although these values are small compared to the experimental values, $g_{\pi NN}(0) \sim 13$ and $g_A(0) = 1.26$, we do not attempt to reproduce them, because our interest here is in determining how the $Q^2$ dependence of the form factors is related to the nucleon radius.
As $\tilde{G}$ is increased so that the size of the nucleon is reduced, the $q$-dependence of the form factors becomes weak. We extract the cutoff parameters of the form factors when fitted by the monopole function $\Lambda^2/(\tilde{G}^2 + \Lambda^2)$ and the ranges of the form factors, $(r^2)_A^{1/2}$. The results are listed in the second and third columns of Table III. When the parameter set in Table I is used, so that the mean square radius of about 0.77 fm is reproduced, the form factor becomes rather soft, specifically, $\Lambda \sim 540$ MeV. However, if we choose a smaller nucleon size, as expected from the fact that there are contributions from meson clouds, then the present nucleon size of the quark-diquark extension may be smaller than the observed one. If we choose, for instance, about 0.5 fm, then the cutoff value $\Lambda$ becomes about 800 MeV. In both cases, however, the cutoff values are smaller than those used in the $NN$ force. However, the cutoff parameters in meson-nucleon vertices depend on the modeling of the meson exchanges. In fact, smaller values are consistent with those extracted from electroproduction of the pion.

3.3. Short range interaction

Let us turn to the short range interaction described by the quark-diquark loop, as shown in Fig. 2. Using the interaction vertices given in Eq. (2.9), it is straightforward to compute the amplitude for the quark-diquark loop:

$$\mathcal{M}_{NN} = -iN_c Z^2 \times \int \frac{d^4k}{(2\pi)^4} B(p_1')(k + m_q)B(p_1)\bar{B}(p_2')(p_2 - p_1' + k + m_q)B(p_2) \left[ (p_1 - k)^2 - M_S^2 \right][k^2 - m_q^2][(p_1' - k)^2 - M_S^2][(p_2 - p_1' + k)^2 - m_q^2].$$

(3.10)

In this equation, the momentum variables are assigned such that $p_1$ and $p_1'$ ($p_2$ and $p_2'$) are for the pair of contracted baryon fields, $\bar{B}(p_1')\cdots B(p_1) (\bar{B}(p_2')\cdots B(p_2))$, and the momentum transfer is defined by $q = p_1' - p_1$. Note that this momentum $q$ is carried by the diquark pair. The amplitude defined in this way can be interpreted as a direct term in the local potential approximation. When computing physical quantities such as phase shifts, we need to include the exchange term that is obtained by interchanging the momentum variables $p_1' \leftrightarrow p_2'$. Although the one-loop integral (3.10) converges when the scalar diquark is included, we keep the counter terms of the Pauli-Villars regularization. Because our model is a cutoff theory with a relatively small cutoff mass, $\Lambda_{PV} = 0.63$ GeV, the counter-terms play a significant role. However, because we include only the scalar diquark, we do not attempt to make a comparison at the quantitative level. Rather, in the following, we study some basic properties of the amplitude itself, mostly the interaction ranges extracted from Eq. (3.10).

Let us evaluate the integral in the center-of-mass system for elastic scattering:

$$p_1 = (E_{\bar{p}}, \bar{p}), \quad p_2 = (E_{\bar{p}}, -\bar{p}),$$

$$p_1' = (E_{\bar{p}'}, \bar{p}'), \quad p_2' = (E_{\bar{p}'}, -\bar{p}'), \quad |\bar{p}| = |\bar{p}'|.$$

To proceed, we write the amplitude (3.10) as

$$\mathcal{M}_{NN} = F_S(\vec{P}, \vec{q})(\bar{B}B)^2 + F_V(\vec{P}, \vec{q})(\bar{B}\gamma_\mu B)^2 + \cdots,$$

(3.11)
Crude, the $q$-dependence expresses the interaction range in the $t$-channel, whose Fourier transform is interpreted as the $r$-dependence of the local potential, while the $P$-dependence expresses the non-locality of the interaction.

In Eq. (3.11), we have defined the coefficients $F_S$ and $F_V$ as scalar and vector interactions, respectively. The dots in Eq. (3.11) then include terms involving external momenta $p_i$, such as $\bar{B} \Gamma(p_i) B (\bar{B} B)$ and $(\bar{B} B) (\bar{B} \Gamma(p_i) B)$, where $\Gamma(p_i)$ is a $4 \times 4$ matrix involving $p_i$. These momentum dependent terms are, however, expected to play a less important role than the dominant components of the scalar and vector terms, as in boson exchange models. Therefore, we consider here the corresponding terms of scalar and vector types. It turns out that the interaction coefficients defined in this way are attractive for $F_S$ and repulsive for $F_V$.

As anticipated, the amplitude is highly non-local, as the quark-diquark loop diagram implies. As a matter of fact, the interaction range for the diquark exchange in the $t$-channel is shorter than that for the quark exchange in the $s$-channel. This is shown explicitly in Fig. 4. Nevertheless, we proceed further and carry out an expansion in $\vec{P}$. We obtain

$$F_i(\vec{P}, \vec{q}) = F_i(\vec{P}, \vec{q})|_{\vec{P}=0} + \vec{P} \cdot \frac{\partial}{\partial \vec{P}} F_i(\vec{P}, \vec{q})|_{\vec{P}=0} + \cdots ,$$

(3.13)

where $i$ stands for $S$ or $V$. The resulting $\vec{q}$ dependent function, in particular the first term, can be interpreted as the Fourier transform of a local potential as a function of the relative coordinates $\vec{r} = \vec{x}_1 - \vec{x}_2$. Then, the general structure of the $NN$ potential can be studied by performing the non-relativistic reduction of the amplitude, Eqs. (3.11) and (3.13). It contains central, spin-orbit and tensor components that accompany functions of non-locality $\vec{P}$.

We now discuss the functions $F_S$ and $F_V$ in Eq. (3.11) for the leading-order terms of Eq. (3.13). Because we cannot write the resulting $q$ dependent functions in an
analytic form, we have carried out numerical calculations employing several different $\tilde{G}$ parameters for different binding energies and the size of the quark-diquark bound state.

First, we determine the strengths of the interactions by extracting coupling constant squares $g_i^2$ as

$$|F_i(0,0)| = \frac{g_i^2}{m_i^2}, \quad g_i > 0,$$

where the effective meson masses $m_i$ are evaluated using the inverse of (3.15) and are plotted in Fig. 7. The results are displayed in Fig. 5 as functions of the size of the nucleon, $\langle r^2 \rangle^{1/2}$. The coupling strengths can be compared with the empirical values $g_S \sim 10$ and $g_V \sim 13$. The present results are strongly dependent on $\langle r^2 \rangle$. When only a scalar diquark is included, the scalar interaction becomes much stronger than the vector interaction. Phenomenologically, the vector (omega meson) coupling is stronger than the scalar (sigma meson) coupling.

We should comment on the effect of the axial-vector diquark. The loop integral containing the axial diquarks diverges and has a large numerical value even when they are regularized. Therefore, it significantly affects the absolute values of the loop integrals as well as that of the baryon self energy, which is necessary to extract the normalization factor $Z$. Therefore, it is expected that the strengths are affected significantly, but the ranges and the corresponding masses, which are computed from the momentum dependence of the normalized loop integrals, are not affected as greatly.

Let us discuss the interaction ranges. In Fig. 6, we show the $q$-dependence of the zeroth order coefficients of (3.13) for three different sizes of the nucleon. It is obvious from Fig. 6 that as the size of the nucleon becomes smaller the interaction ranges become shorter. More quantitatively, we define the interaction ranges $R_i$ by

$$R_i^2 \equiv -6 \frac{1}{F_i(q^2)} \frac{\partial F_i}{\partial q^2} \bigg|_{q^2 \to 0},$$

which are related to the mass parameters of the interaction ranges as $m_i \equiv \sqrt{6/R_i}$. It is interesting that the range (and hence mass) parameters of the interactions are approximately proportional to the size of the nucleon $\langle r^2 \rangle^{1/2}$, as shown in Fig. 7. When the parameter set listed in Table I is used, the mass parameters are about 650 MeV and 800 MeV for the scalar and vector interactions, respectively, which are very close to the masses of the sigma and omega mesons. If, however, we use a parameter set for a nucleon size of about 0.5 fm, then the two masses become $m_S \sim 800$ MeV and $m_V \sim 1000$ MeV.
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Fig. 6. The scalar (left panel) and vector (right panel) form factors $F_{S,V}(q^2, P^2 = 0)/F_{S,V}(q^2 = 0, P^2 = 0)$, where the form factors are normalized to 1. The curves correspond to $M_N = 0.98, 0.85, 0.70$ GeV from bottom to top.

Fig. 7. The interaction range $R_i$ (left panel) and the corresponding mass parameter $m_i$ (right panel) as functions of the nucleon size (distribution of a quark and a diquark).

In the present analysis, apart from the absolute values of the interaction strengths, the interaction ranges for the scalar and vector interactions have been produced appropriately with the quark-diquark loop diagram. One question concerns the small (but non-negligible) difference between the ranges in the scalar and vector channels, which is consistent with empirical results. This can be crudely understood from the dimensionality of the loop integral. As seen from Eq. (3.10), the integrand for the vector interaction is of higher order with respect to the loop momentum than the scalar interaction. Because of this, the vector part reflects shorter distant dynamics and produces a shorter interaction range.

§4. Summary

In this paper we have studied the $NN$ interaction using a microscopic theory of quarks and diquarks. Nucleons were described as quark-diquark bound states. The quark and diquark degrees of freedom were integrated out in the path-integral method, and an effective Lagrangian was derived for mesons and baryons. The resulting trace-log formula contains various meson and nucleon interaction terms, including the $NN$ interaction in the short range region expressed by a quark-diquark loop. Hence, the $NN$ interaction can be naturally expressed as meson exchanges at
long ranges and quark or diquark exchanges at short ranges.

For the long range interaction, we have considered the $\pi NN$ interaction and computed the form factor for the Yukawa vertex. The resulting cutoff mass for the monopole type form factor turned out to be about $\Lambda \sim 0.54$ GeV when the size of the nucleon was chosen around 0.77 fm. For short range interactions, we have computed a quark-diquark loop corresponding to the direct term of the two-nucleon interaction. The resulting interaction is highly nonlocal. We have extracted scalar and vector type interactions in the local potential approximation. It turns out that the scalar term is attractive, while the vector is term repulsive.

In the present paper our numerical calculations contained only scalar diquarks, as a first step toward full calculations. Therefore, we have concentrated our study mostly on the interaction ranges or, equivalently, the masses, because the magnitude of strengths is affected by the axial-vector diquark. The interaction ranges were better studied in the present study including only scalar diquarks, reflecting the size of the nucleon. Consequently, the mass parameters of the interaction ranges for the scalar and vector interactions were found out to be about 650 MeV for the scalar type and about 800 MeV for the vector type, once again when the nucleon size is set at around 0.77 fm. Hence in our model, the scalar-isoscalar interaction emerges from the quark-diquark loop at an energy scale similar to that of the sigma and omega meson exchanges. For actual applications, we need to add exchange terms, due to the antisymmetrization of the two nucleon system. The existence of the two components in the nuclear force, boson exchanges and quark-diquark exchanges, is a general feature when we consider a model of nucleons that is composed of a quark (and diquark) core surrounded by meson clouds.

The present result encourages us to further study baryon properties by extending the model to include the axial-vector diquark. We have already started such a study. When the axial vector is included, due to the massive vector nature of the propagator, loop integrals diverge more strongly than in the case of the scalar diquark. Although this causes the numerical study to be more complicated than in the present case, we hope to make progress along this line in the future.

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References

1) Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961), 345; ibid. 124 (1961), 246.
2) L. J. Abu-Raddad, A. Hosaka, D. Ebert and H. Toki, Phys. Rev. C 66 (2002), 025206.
3) M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Cote, P. Pires and R. De Tourreil, Phys. Rev. C 21 (1980), 861.
4) R. Machleidt, K. Holinde and C. Elster, Phys. Rep. 149 (1987), 1.
5) H. Toki, Z. Phys. A 294 (1980), 173.
6) M. Oka and K. Yazaki, Prog. Theor. Phys. 66 (1981), 556; ibid. 66 (1981), 572.
7) S. Takeuchi, K. Shimizu and K. Yazaki, Nucl. Phys. A 504 (1989), 777.
8) Y. Fujiwara, T. Fujita, M. Kohno, C. Nakamoto and Y. Suzuki, Phys. Rev. C 65 (2001), 014002.
9) T. Eguchi, Phys. Rev. D 14 (1976), 2755.
10) A. Dhar and S. R. Wadia, Phys. Rev. Lett. 52 (1984), 959.
11) D. Ebert and H. Reinhardt, Nucl. Phys. B 271 (1986), 188.
12) D. Ebert and T. Jurke, Phys. Rev. D 58 (1998), 034001.
13) N. Ishii, Nucl. Phys. A 689 (2001), 793.
14) D. Espriu, P. Pascual and R. Tarrach, Nucl. Phys. B 214 (1983), 285.
15) S. Weinberg, The Quantum Theory of the Fields (Cambridge University Press, 1995).
16) A. Hosaka and H. Toki, Quarks, Baryons and Chiral Symmetry (World Scientific, 2001).
17) M. L. Goldberger and S. B. Treiman, Phys. Rev. 110 (1958), 1178.
18) C. Izykuson and J. B. Zuber, Quantum Field Theory (McGraw-Hill, 1980).
19) A. Liesenfeld et al. [A1 Collaboration], Phys. Lett. B 468 (1999), 20.
20) T. Hatsuda and T. Kunihiro Phys. Rep. 247 (1994), 221.
21) D. Ebert, H. Reinhardt and M. K. Volkov, Prog. Part. Nucl. Phys. 33 (1994), 1.
22) U. Vogl and W. Weise, Prog. Part. Nucl. Phys. 27 (1991), 195.
23) R. T. Cahill, C. D. Roberts and J. Praschifka, Phys. Rev. D 36 (1987), 2804.
24) A. Hosaka and H. Toki, Phys. Rep. 277 (1996), 65.